Domain Adaptation with $L_2$ constraints for classifying images from different endoscope systems

Toru Tamaki$^a$,*, Shoji Sonoyama$^a$, Takio Kurita$^a$, Tsubasa Hirakawa$^a$, Bisser Raytchev$^a$, Kazufumi Kaneda$^a$, Tetsushi Koide$^b$, Shigeto Yoshida$^c$, Hiroshi Mieno$^c$, Shinji Tanaka$^d$, Kazuaki Chayama$^e$

$^a$Department of Information Engineering, Graduate School of Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8527, Japan
$^b$Research Institute for Nanodevice and Bio Systems (RNBS), Hiroshima University, 1-4-2 Kagamiyama, Higashi-Hiroshima 739-8527, Japan
$^c$Department of Gastroenterology, JR Hiroshima Hospital, 3-1-3 Fatabanosato, Higashiku, Hiroshima 732-0057, Japan
$^d$Department of Endoscopy, Hiroshima University Hospital, 1-2-3 Kasumi, Minami-ku, Hiroshima 734-8551, Japan
$^e$Department of Gastroenterology and Metabolism, Hiroshima University Hospital, 1-2-3 Kasumi, Minami-ku, Hiroshima 734-8551, Japan

Abstract

This paper proposes a method for domain adaptation that extends the maximum margin domain transfer (MMDT) proposed by Hoffman et al., by introducing $L_2$ distance constraints between samples of different domains; thus, our method is denoted as MMDTL2. Motivated by the differences between the images taken by narrow band imaging (NBI) endoscopic devices, we utilize different NBI devices as different domains and estimate the transformations between samples of different domains, i.e., image samples taken by different NBI endoscope systems. We first formulate the problem in the primal form, and then derive the dual form with much lesser computational costs as compared to the naive approach. From our experimental results using NBI image datasets from two different NBI endoscopic devices, we find that MMDTL2 is more stable than MMDT and better than support vector machines without adaptation.

Keywords: Domain adaptation; Dual formulation; Kernels; NBI endoscopy

1. Introduction

In many hospitals, endoscopic examinations (i.e., colonoscopies) using narrow band imaging (NBI) systems are widely performed to diagnose colorectal cancer [1], which is a major cause of cancer deaths worldwide [2]. During examinations, endoscopists observe and examine a polyp based on its visual appearance, including via NBI magnification findings [3, 4], as shown in Figure 1.
To support proper diagnosis during examinations, a computer-aided diagnostic system based on the textural appearance of polyps would be helpful; thus, numerous patch-based classification methods for endoscopic images have been proposed [5, 6, 7, 8, 9, 10, 11].

This paper focuses on the inconsistencies between training and test images. As with other frequently used machine learning approaches, training classifiers assumes that the distribution of features extracted from both training and test image datasets are the same; however, different endoscope systems may be used to collect training and test datasets, causing such an assumption to be violated. Further, given the rapid development of medical devices (i.e., endoscopies in this case), hospitals can introduce new endoscopes after training images have already been taken. In addition, classifiers may be trained with a training dataset collected by a certain type of endoscope in one hospital, while another hospital might use the same classifiers for images taken by a different endoscope. In general, such inconsistencies lead to the deterioration of classification performance; hence, collecting new images for a new training dataset may be necessary or is at least preferred. However, this is not the case with medical images. It is impractical to collect enough sets of images for all types and manufacturers of endoscopes.

Figure 2 shows an example of differences between textures captured by different endoscope systems. More specifically, the images shown in Figures 2(a) and 2(b) are the same scene from a printed sheet of a colorectal polyp image taken by different endoscope systems at approximately the same distance to the sheet from the endoscopes. Even for the same manufacture (e.g., Olympus) and the same modality (e.g., NBI), images may differ in terms of factors such as resolution, image quality, sharpness, brightness, and viewing angle. These differences may impact classification performance.

To address this problem, Sonoyama et al. [12] proposed a method based on transfer learning [13, 14, 15, 16] to estimate a transformation matrix between feature vectors of training and test datasets captured by different (i.e., old and new) devices. In this prior study, we formulated the problem as a constraint optimization problem and developed an algorithm to estimate a linear transformation; however, a key limitation is that corresponding datasets are required, i.e., each test image (i.e., taken by a new device) must have a corresponding training image (i.e., taken by an old device). Further, these images must capture the same polyp to properly estimate the linear transformation. These restrictions are rather strong, causing our system to be somewhat impractical.

Therefore, this paper proposes an improved method for a task that does not require image-by-image correspondences between training and test datasets. More specifically, we extend the transfer learning method proposed by Hoffman et al. [20, 21], called maximum margin domain transfer (MMDT). Their approach was a domain adaptation technique to handle the domain shift problem, in which distributions of classes (or categories) in one domain, called the source, change in another domain, called the target. This situation occurs in various applications, and hence, domain adaptation and transfer learning have already been widely studied.
Figure 1: NBI magnification findings [3].

Compared to previous studies, MMDT had the following advantages: (1) applicability to multiclass problems with one-vs-rest support vector machines (SVMs); (2) the ability to use different feature dimensions in the source and target domains; (3) the ability to handle unlabeled target samples by estimating global class-independent affine transformation matrix $W$; and (4) scalability to the number of constraints, i.e., MMDT solves $KL_s$ constraints as compared to $L_1L_s$ in the previous studies, where $K$ is the number of classes and $L_s$ and $L_t$ are the feature dimensions of the source and target domains.

In this paper, we therefore propose a non-trivial extension to MMDT for handling inconsistencies between NBI endoscopic devices.\footnote{A conference version of this paper was presented [22]. This paper extends that version with more rigorous derivations, the compact form of the dual formulation, and the kernelization of the method, as well as experiments from different aspects.} First, we add $L_2$ distance constraints to MMDT, thus calling our method MMDTL2. The original formulation of MMDT uses the Frobenius norm of transformation $W$ as a regularizer, but pulling transformation $W$ into a zero matrix is not intuitive and might not have a good effect on transferring samples. Other regularizers were discussed by [21], e.g., an identity matrix when $L_s = L_t$, but no examples were given for cases of $L_s \neq L_t$. Instead of exploring another regularizer, we add more intuitive constraints. Further, target samples in one category should be transformed into the same category of the source domain. To this end, we use the $L_2$ distances between the source and transformed target samples as constraints. Second, we explicitly formulate MMDTL2 as a quadratic programming (QP) problem. In [20, 21], the MMDT problem was described but not in a QP form. In this paper, we explicitly formulate MMDTL2 in the standard QP form, which includes MMDT as a special case (i.e., where no $L_2$ constraints are used). Third, we derive the dual problem of MMDTL2. The QP form men-
Figure 2: Example showing differences in appearance of images captured by different endoscope systems: (a) an image taken by an older system (i.e., video system center: OLYMPUS EVIS LUCERA CV-260 [17]; endoscope: OLYMPUS EVIS LUCERA COLONOSCOPE CF-H260AZI [18]); (b) an image of the same scene taken by a newer system (i.e., video system center: OLYMPUS EVIS LUCERA ELITE CV-290 [19]; endoscope: OLYMPUS CF-H260AZI [18]).

Note that our dual form is different from the dual form derived by Rodner et al. [23]. Their motivation was to make MMDT scalable in terms of the number of target samples, because in their study, they attempted to adapt large datasets such as ImageNet. Therefore, their dual form still suffers from the large feature dimensions of the source and target samples. In contrast, our formulation is scalable in terms of feature dimensions.

The rest of the paper is organized as follows. We formulate problems of MMDT and MMDTL2 in Section 2, and then derive the primal form in Section 3. In Section 4, we show the dual form, and in Section 5, we obtain the primal solution from the dual solution. In Section 6, we present our experimental results using numerical examples and datasets of actual NBI endoscopic images.
Finally, in Section 7, we conclude this paper and provide avenues for future work.

2. Problem formulation

In this section, we introduce the problems of MMDT [21] and our proposed MMDTL2.

**Problem 1 (MMDT).** Suppose we are given a training set $\chi^s = \{x^s_n, y^s_n\}_{n=1}^N \subset \mathbb{R}^{L_s} \times \{1, 2, \ldots, K\}$ in the source domain and another set $\chi^t = \{x^t_m, y^t_m\}_{m=1}^M \subset \mathbb{R}^{L_t} \times \{1, 2, \ldots, K\}$ in the target domain for a $K$-class classification problem.

MMDT solves the following optimization problem:

$$
\min_{W, \hat{\Theta}} \frac{1}{2} \|W\|_F^2 + \frac{1}{2} \|\hat{\Theta}\|_F^2 + c_t L(W, \hat{\Theta}, \chi^t) + c_s L(\hat{\Theta}, \chi^s),
$$

(1)

where $c_t$ and $c_s$ are weights and

$$
L(W, \hat{\Theta}, \chi^t) = \sum_{k,m} \max \left( 0, 1 - y^t_{km} \hat{\theta}^T_k \left( W \hat{x}^t_m \right) \right)
$$

(2)

and

$$
L(\hat{\Theta}, \chi^s) = \sum_{k,m} \max \left( 0, 1 - y^s_{kn} \hat{\theta}^T_k \hat{x}^s_n \right)
$$

(3)

are hinge loss functions. Here, $\hat{\theta}_k \in \mathbb{R}^{L_s+1}$ is an SVM hyperplane parameter (including weights $\theta_k \in \mathbb{R}^{L_s}$ and bias $b_k \in \mathbb{R}$, i.e., $\hat{\theta}_k = (\theta_k^T, b_k)^T$) for the $k$th class stacked into a matrix $\hat{\Theta} = (\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_K)$, $y^t_{km} = 2 \delta(y^t_{m,k}) - 1 \in \{-1, 1\}$ is a label in terms of the $k$th hyperplane, and $c_s$ and $c_t$ are weights.

Note that $\hat{x}$ denotes an augmented vector with 1 as its last element, i.e., $\hat{x} = \begin{pmatrix} x \\ 1 \end{pmatrix}$.

In this problem, we simultaneously obtain $K$ SVM classifiers and transformation $W \in \mathbb{R}^{L_s \times L_t}$. One-vs-all SVMs are used for multiclass classification; thus, $K$ hyperplane parameters $\hat{\theta}_k$ are obtained in the source domain. Target samples $x^t_m$ are transformed by $W$ from the target domain to the source domain, and then the loss function causes them to be classified by the SVMs.

Because this problem is non-convex, an alternating optimization approach was used in [21].

**Problem 2 (MMDT with iteration).** MMDT solves problem 1 by iteratively solving subproblems

$$
\min_{W} \frac{1}{2} \|W\|_F^2 + c_t L(W, \hat{\Theta}, \chi^t),
$$

(4)

and

$$
\min_{\hat{\Theta}} \frac{1}{2} \|\hat{\Theta}\|_F^2 + c_t L(W, \hat{\Theta}, \chi^t) + c_s L(\hat{\Theta}, \chi^s),
$$

(5)
initializing $\hat{\Theta}$ with
\[
\arg\min_{\hat{\Theta}} \frac{1}{2} \|\hat{\Theta}\|_F^2 + c_s \mathcal{L}(\hat{\Theta}, \chi^s). \tag{6}
\]

As noted in the Introduction, the use of Frobenius norm $\|W\|_F^2$ is not intuitive for the transformation matrix. Further, it might not be a good choice for small values of $c_t$ because the obtained solution is pulled toward a zero matrix; however, the use of large values of $c_t$ impacts the SVM subproblem [5] because C-SVM solvers are known to be unstable for large values of parameter $C$.

In this paper, we therefore propose the following problem, which we call MMDTL2. MMDTL2 incorporates additional constraints of $L_2$ distances to pull target samples to source samples of the same category.

**Problem 3 (MMDTL2).** MMDT with $L_2$ constraints solves problem [2] by iteratively solving subproblems
\[
\min_{W} \frac{1}{2} c_f \|W\|_F^2 + c_t \mathcal{L}(W, \hat{\Theta}, \chi^t) + c_d \mathcal{L}(W, \hat{\Theta}, \chi^s), \tag{7}
\]
and
\[
\min_{\hat{\Theta}} \frac{1}{2} \|\hat{\Theta}\|_F^2 + c_t \mathcal{L}(W, \hat{\Theta}, \chi^t) + c_s \mathcal{L}(\hat{\Theta}, \chi^s) \tag{8}
\]
with the same initialization as problem [2] where
\[
c_d \mathcal{L}(W, \chi^s, \chi^t) = \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} \|W \hat{x}_m^t - x_n^s\|_2^2, \tag{9}
\]
where $s_{nm} = c_d \delta(y_n^s, y^t_m)$ is a weight between samples $x^t_m \in \mathbb{R}^{L_t}$ and $x^s_n \in \mathbb{R}^{L_s}$, and $c_d$ is the balance weight.

MMDTL2 reduces to MMDT when $c_f = 1$ and $c_d = 0$.

The SVM subproblem [5] can be solved by common SVM solvers, as is done for [5] by MMDT. Therefore, in the following sections, we focus on deriving the primal and dual forms of subproblem [7] as standard QP problems.

### 3. Primal problem

In this section, we rephrase subproblem [7] with inequality constraints instead of loss functions.

**Problem 4 (Estimation of $W$).** We want to find $W \in \mathbb{R}^{L_s \times (L_t+1)}$ that minimizes the following objective function:
\[
\min_{W, \{\xi^l_m\}} \frac{1}{2} c_f \|W\|_F^2 + c_T \sum_{k=1}^{K} \sum_{m=1}^{M} \xi^l_m + \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} \|W \hat{x}_m^t - x_n^s\|_2^2 \tag{10}
\]
\[ s.t. \]
\[ \xi_{km}^t \geq 0, \quad (11) \]
\[ y_{km}^t \theta_k^T \left( \begin{array}{c} \hat{x}_m^t \\ 1 \end{array} \right) - 1 + \xi_{km}^t \geq 0. \quad (12) \]

First, we rewrite the objective function in a matrix form. To this end, we introduce the vec operator and some formulas below.

3.1. Operator vec
Here, we define a vectorized operator for rearranging matrix-vector products.

**Definition 1.** For a given matrix \( W \in \mathbb{R}^{L_s \times (L_t+1)} \), denoted by a set of row vectors \( w_i \in \mathbb{R}^{L_t} \) as

\[
W = \left( \begin{array}{c}
   w_1^T \\
   w_2^T \\
   ... \\
   w_{L_s}^T
\end{array} \right), \quad (13)
\]

we define operator vec, which vectorizes \( W \) in the row-major order as

\[
vec(W) = \left( \begin{array}{c}
   w_1 \\
   w_2 \\
   ... \\
   w_{L_s}
\end{array} \right) \in \mathbb{R}^{L_s(L_t+1)}. \quad (14)
\]

This definition is different from the one used in the literature, which is defined in the column-major order, for example, in [24].

Next, we can rewrite matrix-vector multiplications using the vec operator, as summarized in the following lemma.

**Lemma 1.** For given matrix \( W \in \mathbb{R}^{L_s \times (L_t+1)} \) and vectors \( \hat{x} \in \mathbb{R}^{L_t+1} \) and \( z \in \mathbb{R}^{L_s} \), the following equations hold:

\[
W \hat{x} = (I_{L_s} \otimes \hat{x}^T)w \quad (15)
\]
\[
\hat{x}^T W^T W \hat{x} = w^T (I_{L_s} \otimes \hat{x} \hat{x}^T)w \quad (16)
\]
\[
z^T W \hat{x} = vec(z \hat{x}^T)^T w \quad (17)
\]

Here, \( w = vec(W) \), \( I_{L_s} \in \mathbb{R}^{L_s \times L_s} \), is an identity matrix and \( \otimes \) is the tensor product.

**Proof.** First, we have

\[
W \hat{x} = \left( \begin{array}{c}
   w_1^T \hat{x} \\
   w_2^T \hat{x} \\
   ... \\
   w_{L_s}^T \hat{x}
\end{array} \right) = \left( \begin{array}{c}
   \hat{x}^T w_1 \\
   \hat{x}^T w_2 \\
   ... \\
   \hat{x}^T w_{L_s}
\end{array} \right) = \left( \begin{array}{c}
   \hat{x}^T \\
   \hat{x}^T \\
   ... \\
   \hat{x}^T
\end{array} \right) \left( \begin{array}{c}
   w_1 \\
   w_2 \\
   ... \\
   w_{L_s}
\end{array} \right). \quad (18)
\]
\[(I_L, \otimes \hat{x}^T)w. \]  

Using this equation, we have

\[
\hat{x}^T W^T W \hat{x} = w^T \begin{pmatrix}
\hat{x} & \hat{x} & \cdots \\
\hat{x} & \hat{x} & \cdots \\
\hat{x} & \hat{x} & \cdots \\
\end{pmatrix} w
\]

\[
= w^T \begin{pmatrix}
\hat{x} \hat{x}^T & \cdots \\
\hat{x} \hat{x}^T & \cdots \\
\hat{x} \hat{x}^T & \cdots \\
\end{pmatrix} w = w^T (I_L, \otimes \hat{x} \hat{x}^T) w. \tag{20}
\]

Also, we have

\[
z^T W \hat{x} = z^T \begin{pmatrix}
\hat{x}^T & \cdots \\
\hat{x}^T & \cdots \\
\end{pmatrix} w = (z_1 \hat{x}^T, z_2 \hat{x}^T, \ldots) w \tag{22}
\]

\[
= \text{vec} \begin{pmatrix}
z_1 \hat{x}^T \\
z_2 \hat{x}^T \\
\end{pmatrix} w = \text{vec}(z \hat{x}^T)^T w = w^T \text{vec}(z \hat{x}^T). \tag{23}
\]

For later use, we also define

\[
U(\hat{x}) = (I_L, \otimes \hat{x} \hat{x}^T). \tag{24}
\]

### 3.2. Rewriting terms with \text{vec} operator

In this subsection, we rewrite the \(L^2\) term in the objective function using the lemma below.

**Lemma 2.** The \(L^2\) constraint term of MMDTL2 can be written as

\[
\frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} s_{mn} \|W \hat{x}_m - x_n^s\|_2^2 = \frac{1}{2} (w^T U w - 2q^T w + s), \tag{25}
\]

where \(U\), \(q\), and \(s\) are given in the proof below.

**Proof.** A single \(L^2\) term can be rewritten with lemma 1 as

\[
\|W \hat{x}_m - x_n^s\|_2^2 = (W \hat{x}_m - x_n^s)^T (W \hat{x}_m - x_n^s)
\]

\[
= (\hat{x}_m^T) W^T W \hat{x}_m - 2(x_n^s)^T W \hat{x}_m + \|x_n^s\|_2^2
\]

\[
= w^T U(\hat{x}_m) w - 2 \text{vec}(x_n^s(\hat{x}_m^T))^T w + \|x_n^s\|_2^2 \tag{26}
\]

\[
\frac{1}{2} (w^T U w - 2q^T w + s), \tag{25}
\]

where \(U\), \(q\), and \(s\) are given in the proof below.
\[ w^T U_m w - 2q_{nm}^T w + \|x_n^s\|^2_2, \]  

where

\[ U_m = U(\hat{x}_m^t) = (I_{L_s} \otimes \hat{x}_m^t (\hat{x}_m^t)^T) \]  

and

\[ q_{nm} = \text{vec}(x_n^s(\hat{x}_m^t)^T). \]  

By summing the terms with weights, we have

\[ \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm}(w^T U_m w - 2q_{nm}^T w + \|x_n^s\|^2_2) \]  

\[ = \frac{1}{2} w \left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} U_m \right) w - \left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} q_{nm}^T \right) w + \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} \|x_n^s\|^2_2 \]  

\[ = \frac{1}{2}(w^T U w - 2q^T w + s). \]  

Here, \( U, q, \) and \( s \) are the corresponding factors; we further rewrite them into the simpler forms shown below.

\[ U = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} U_m = \sum_{m=1}^{M} s_m U_m \]  

\[ = \sum_{m=1}^{M} s_m (I_{L_s} \otimes \hat{x}_m^t (\hat{x}_m^t)^T) = (I_{L_s} \otimes \sum_{m=1}^{M} s_m \hat{x}_m^t (\hat{x}_m^t)^T) \]  

\[ = (I_{L_s} \otimes X^t S M (X^t)^T) \]  

\[ q = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} q_{nm} = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} \text{vec}(x_n^s(\hat{x}_m^t)^T) \]  

\[ = \text{vec}(\sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} x_n^s (\hat{x}_m^t)^T) \]  

\[ = \text{vec}(X^s S (X^t)^T) \]  

\[ s = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} \|x_n^s\|^2_2 = \sum_{n=1}^{N} s_n \|x_n^s\|^2_2 \]  

Here, we use data matrices

\[ X^s = (x_1^s, x_2^s, \ldots, x_N^s) \in \mathbb{R}^{L_s \times N} \]  

and

\[ X^t = (\hat{x}_1^t, \hat{x}_2^t, \ldots, \hat{x}_M^t) \in \mathbb{R}^{(L_t+1) \times M} \]  

9
and weights
\[ S = \begin{pmatrix} s_{11} & \cdots & s_{1M} \\ \vdots & & \vdots \\ s_{N1} & \cdots & s_{NM} \end{pmatrix}, \quad (44) \]
\[ s_m = \sum_{n=1}^{N} s_{nm}, \quad s_n = \sum_{m=1}^{M} s_{nm}, \quad S_M = \text{diag}(s_1, \ldots, s_m, \ldots, s_M). \quad (45) \]

Next, we rewrite the conditions in the problem as shown below.

**Lemma 3.** The condition in problem 4 can be written as
\[ y_{km} (\phi_{km}^T w + b_k) - 1 + \xi_{km}^t \geq 0, \quad (46) \]
where \( \phi_{km} = \text{vec}(\theta_k (\tilde{x}_m^t)^T) \).

**Proof.**
\[ y_{km}^T \hat{\theta}_k^T \begin{pmatrix} W \tilde{x}_m^t \\ 1 \end{pmatrix} - 1 + \xi_{km}^t \geq 0 \quad (47) \]
\[ y_{km}^T \theta_k^T W \tilde{x}_m + b_k - 1 + \xi_{km}^t \geq 0 \quad (48) \]
\[ y_{km}^T (\text{vec}(\theta_k (\tilde{x}_m^t)^T) w + b_k) - 1 + \xi_{km}^t \geq 0 \quad (49) \]
\[ y_{km}^T (\phi_{km}^T w + b_k) - 1 + \xi_{km}^s \geq 0 \quad (50) \]

\[ 3.3. \text{Primal QP problem} \]

In this subsection, we write the problem in the form of a canonical QP problem.

**Lemma 4.** Problem 4 can be written as
\[ \min_{w, \xi} \frac{1}{2} (w^T, \xi^T) \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} w \\ \xi \end{pmatrix} + (-q^T, c_T 1_{KM}^T) \begin{pmatrix} w \\ \xi \end{pmatrix} + \frac{1}{2} s \quad (51) \]
s.t.
\[ \begin{pmatrix} 0 & I_{KM} \\ Y \Phi^T & I_{KM} \end{pmatrix} \begin{pmatrix} w \\ \xi \end{pmatrix} \geq \begin{pmatrix} 0 \\ 1_{KM} - Y \tilde{b} \end{pmatrix}, \quad (52) \]

where variables are defined in the proof below.

**Proof.** First, we define two matrices \( V \in \mathbb{R}^{L_t(L_t+1) \times L_t(L_t+1)} \) and \( A \in \mathbb{R}^{(L_t+1) \times (L_t+1)} \) as follows:
\[ V = c_f I_{L_t(L_t+1)} + U \quad (53) \]
\[ c_f I_{L_t+1} + (I_{L_t} \otimes X^t S_M (X^t)^T) = I_{L_t} \otimes (c_f I_{L_t+1} + X^t S_M (X^t)^T) = I_{L_t} \otimes A \]
\[ A = c_f I_{L_t+1} + X^t S_M (X^t)^T \]

Then, we rewrite the objective function (10) as
\[ \frac{1}{2} c_f \|w\|^2 + c_T \sum_{k=1}^K \sum_{m=1}^M \xi_{km}^t + \frac{1}{2} (w^T U w - 2q^T w + s) = \frac{1}{2} w^T V w - q^T w + \frac{1}{2} s + c_T 1_{K_M}^T \xi \]
\[ = \frac{1}{2} (w^T, \xi^T) \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} w \\ \xi \end{pmatrix} + (-q^T, c_T 1_{K_M}^T) \begin{pmatrix} w \\ \xi \end{pmatrix} + \frac{1}{2} s, \]
where
\[ \xi = (\xi_{11}^t, \xi_{12}^t, \ldots, \xi_{1M}^t, \xi_{21}^t, \ldots, \xi_{KM}^t)^T. \]

To rewrite conditions (11) and (12), we turn these constraints into vector form with a generalized inequality. The first constraint, i.e., (11), can be written as follows:
\[ \xi \geq 0 \]
\[ \begin{pmatrix} 0 \\ I_{K_M} \end{pmatrix} \begin{pmatrix} w \\ \xi \end{pmatrix} \geq 0. \]

The second constraint, i.e., (12), is
\[ Y (\Phi^T w + \tilde{b}) - 1_{K_M} + \xi \geq 0 \]
\[ \begin{pmatrix} Y \Phi^T, I_{K_M} \end{pmatrix} \begin{pmatrix} w \\ \xi \end{pmatrix} \geq 1_{K_M} - Y \tilde{b}, \]
where \( 1_{K_M} \) is a vector of \( K \) ones and
\[ \Phi = (\phi_{11}, \phi_{12}, \ldots, \phi_{1M}, \phi_{21}, \ldots, \phi_{KM}) \]
\[ Y = \text{diag}(y^1_{11}, y^1_{12}, \ldots, y^1_{1M}, y^1_{21}, \ldots, y^1_{KM}). \]

By combining these inequalities (63) and (65), we have the conditions in a single form, as claimed.

This primal QP problem involves very large matrices that are impractical to compute. More precisely, \( V \) is a matrix of size \( L_s (L_t + 1) \times L_s (L_t + 1) \), which can be very large when the dimensions of features \( (L_s \) and \( L_t) \) are large. In the next section, we therefore derive the dual form of the problem, which we expect to be less expensive to compute.
4. Dual problem

In this section, we derive the dual of the problem.

4.1. Lagrangian

**Lemma 5** (Lagrangian). *The Lagrangian of problem* (51) *is given by*

\[ L = -\frac{1}{2} a^T Y \Phi^T V^{-1} \Phi Y a + (1^T - \tilde{b}^T Y - q^T V^{-1} \Phi Y) a - \frac{1}{2} q^T V^{-1} q + \frac{1}{2} s. \]  

(70)

**Proof.** The Lagrangian of problem (51) is given by

\[ L = \frac{1}{2} w^T V w - q^T w + \frac{1}{2} s + c^T 1^T \sum_{k=1}^K \sum_{m=1}^M \xi_{km}^l - \sum_{k=1}^K \sum_{m=1}^M \mu_{km} \xi_{km}^l - \sum_{k=1}^K \sum_{m=1}^M a_{km}(y_{km}(\Phi_{km}^T w + b_k) - 1 + \xi_{km}^l), \]  

(71)

where \( a_{km} \geq 0 \) and \( \mu_{km} \geq 0 \) are Lagrange multipliers.

To simplify the derivation, we convert it into vector form

\[ L = \frac{1}{2} w^T V w - q^T w + \frac{1}{2} s + c^T 1^T K_M \xi - \mu^T \xi - a^T (Y(\Phi^T w + \tilde{b}) - 1_K M + \xi) \]  

(72)

\[ = \frac{1}{2} w^T V w - (q + \Phi Y a)^T w + (c^T 1_K M - \mu - a)^T \xi + a^T (1_K M - Y \tilde{b}) + \frac{1}{2} s, \]  

(73)

where

\[ a = (a_{11}, a_{12}, a_{1M}, a_{21}, \ldots, a_{KM})^T \]  

(74)

and

\[ \mu = (\mu_{11}, \mu_{12}, \mu_{1M}, \mu_{21}, \ldots, \mu_{KM})^T, \]  

(75)

with \( a \geq 0 \) and \( \mu \geq 0 \).

Next, we take the derivatives of the Lagrangian as follows. For \( w \), we have

\[ \frac{\partial L}{\partial w} = V w - (q + \Phi Y a) = 0 \]  

(76)

\[ V w = q + \Phi Y a \]  

(77)

\[ w = V^{-1}(q + \Phi Y a). \]  

(78)

For \( \xi \), we have

\[ \frac{\partial L}{\partial \xi} = c^T 1 - \mu - a = 0 \]  

(79)
\[ c_T 1 - a = \mu \geq 0 \]  
\[ c_T 1 - a \geq 0 \]  
\[ c_T 1 \geq a \geq 0 \]

for \( \mu \geq 0 \) and \( a \geq 0 \).

By incorporating \( w \) and \( c_T \) into the Lagrangian and using \( V^T = V \), we have

\[
L = \frac{1}{2} w^T V w - (q + \Phi Y a)^T w + (c_T 1 - \mu - a)^T \xi + a^T (1 - Y \tilde{b}) + \frac{1}{2} s
\]

\[
= \frac{1}{2} (V^{-1}(q + \Phi Y a))^T V (V^{-1}(q + \Phi Y a))
\]

\[
- (q + \Phi Y a)^T (V^{-1}(q + \Phi Y a)) + a^T (1 - Y \tilde{b}) + \frac{1}{2} s
\]

\[
= -\frac{1}{2} (q + \Phi Y a)^T V^{-1}(q + \Phi Y a) + a^T (1 - Y \tilde{b}) + \frac{1}{2} s
\]

\[
= -\frac{1}{2} (q^T V^{-1} q + 2q^T V^{-1} \Phi Ya + a^T Y \Phi^T V^{-1} \Phi Ya) + a^T (1 - Y \tilde{b}) + \frac{1}{2} s
\]

\[
= -\frac{1}{2} a^T Y \Phi^T V^{-1} \Phi Ya + (1^T - \tilde{b}^T Y - q^T V^{-1} \Phi Y) a - \frac{1}{2} q^T V^{-1} q + \frac{1}{2} s.
\]

\[ (83) \]

This is indeed the dual form, but it still involves a large matrix \( V \). In the next subsection, by utilizing the structure of \( V \), we will write \( L \) in such a way that it involves only smaller matrices.

### 4.2. Lagrangian with a compact form

To remove the large matrix \( V \), we use the structure of \( V \) and rewrite terms involving \( V \) \((q^T V^{-1} \Phi \) and \( q^T V^{-1} \Phi \).

First, the inverses of \( V \) and \( A \) are as follows:

\[
V^{-1} = (I_{L_s} \otimes A)^{-1} = I_{L_s} \otimes A^{-1}
\]

\[ (88) \]

\[
A^{-1} = (cf I_{L_t+1} + X^T S_M (X^T)^{-1})^{-1}
\]

\[ (89) \]

\[
= \frac{1}{cf} I_{L_t+1} - \frac{1}{cf} X^T (S_M^{-1} + \frac{1}{cf} (X^T X^t)^{-1} (X^t)^T)
\]

\[ (90) \]

Note that the second form of \( A^{-1} \) is obtained using Woodbury’s formula only if \( S_M \) is non-singular and \( cf \neq 0 \); this is usually the case, because diagonal elements of \( S_M \) are sums of (non-negative) weights.

**Lemma 6.** Given \( V \) of the structure above and vectors \( a, c \in \mathbb{R}^{L_s} \) and \( b, d \in \mathbb{R}^{L_t} \), we have

\[
\text{vec}(ab^T)^T V^{-1} \text{vec}(cd^T) = (a^T c) b^T A^{-1} d.
\]

\[ (91) \]
Proof.

\[
\text{vec}(ab^T)V^{-1}\text{vec}(cd^T) = (a_1b^T,a_2b^T,\ldots) \begin{pmatrix} A^{-1} & & \\ & \ddots & \\ & & A^{-1} \end{pmatrix} \begin{pmatrix} c_1d \\ \vdots \\ c_2d \end{pmatrix}
\]

(92)

(93)

\[
= (a_1b^T,a_2b^T,\ldots) \begin{pmatrix} c_1A^{-1}d \\ c_2A^{-1}d \\ \vdots \end{pmatrix}
\]

(94)

\[
= \sum_i a_ic_i b^TA^{-1}d = (a^Tc)b^TA^{-1}d
\]

(95)

Next, we explore \(\phi^TV^{-1}\phi\) via the lemma below.

Lemma 7. Given matrix \(\Phi \in \mathbb{R}^{L(L+1)\times KM}\), we have

\[
\Phi^TV^{-1}\Phi = (\Theta^T\Theta) \otimes G,
\]

(96)

where \(\Theta = (\theta_1,\theta_2,\ldots,\theta_K) \in \mathbb{R}^{Ls\times K}\) and \(G \in \mathbb{R}^{M\times M}\), the latter given in the proof below.

Proof. Using the above lemma, we have

\[
\phi_{k,m}^TV^{-1}\phi_{k,m'} = \text{vec}(\theta_k(\hat{x}_m^t)^TV^{-1}\text{vec}(\theta_{k'}(\hat{x}_{m'}^t)^T)
\]

\[
= (\theta_k^T\theta_{k'})(\hat{x}_m^t)^TA^{-1}\hat{x}_{m'}^t.
\]

(97)

(98)

By stacking the above equation for \(m = 1,\ldots,M\), we have

\[
\begin{pmatrix} \phi_{k,1}^T \\ \vdots \\ \phi_{k,M}^T \end{pmatrix} V^{-1} (\phi_{k,1} \cdots \phi_{k,M}) = (\theta_k^T\theta_{k'})(\hat{x}_1^t)^TA^{-1}\hat{x}_1^t \cdots (\hat{x}_M^t)^TA^{-1}\hat{x}_M^t
\]

(99)

(100)

\[
= (\theta_k^T\theta_{k'})(X^t)^TA^{-1}X^t
\]

(101)

\[
= (\theta_k^T\theta_{k'})G,
\]

(102)

where \(G = (X^t)^TA^{-1}X^t\).
Finally, by stacking the above equation for \( k = 1, \ldots, K \), we obtain compact form

\[
\Phi^T V^{-1} \Phi = \begin{pmatrix}
\phi^T_{11} \\
\vdots \\
\phi^T_{1M} \\
\vdots \\
\phi^T_{KM}
\end{pmatrix} V^{-1} \left( \phi_{11}, \ldots, \phi_{1M}, \phi_{21}, \ldots, \phi_{KM} \right)
\]

(103)

\[
= \begin{pmatrix}
(\theta^T_1 \theta_1) G & (\theta^T_1 \theta_2) G & \cdots & (\theta^T_1 \theta_K) G \\
(\theta^T_2 \theta_1) G & (\theta^T_2 \theta_2) G & \cdots & (\theta^T_2 \theta_K) G \\
\vdots & \vdots & \ddots & \vdots \\
(\theta^T_K \theta_1) G & (\theta^T_K \theta_2) G & \cdots & (\theta^T_K \theta_K) G
\end{pmatrix}
\]

(104)

\[
= (\Theta^T \Theta) \otimes G,
\]

(105)

where \( \Theta = (\theta_1, \theta_2, \ldots, \theta_K) \).

Note that we can rewrite \( G \) further as

\[
G = (X^T) A^{-1} X^t 
\]

(106)

\[
= (X^T) \left( \frac{1}{c_f} I_{L_t+1} - \frac{1}{c_f} X^t (S_M^{-1} + \frac{1}{c_f} (X^t) T X^t)^{-1} (X^t)^T X^t \right)
\]

(107)

\[
= \frac{1}{c_f} (X^t)^T X^t - \frac{1}{c_f} (X^t)^T X^t (S_M^{-1} + \frac{1}{c_f} (X^t)^T X^t)^{-1} (X^t)^T X^t
\]

(108)

\[
= \frac{1}{c_f} K^t - \frac{1}{c_f} K^t (S_M^{-1} + \frac{1}{c_f} K^t)^{-1} K^t 
\]

(109)

where \( K^t = (X^T) X^t \) is a kernel matrix. If \((K^t)^{-1}\) exists, we obtain

\[
G = (c_f (K^t)^{-1} + S_M)^{-1}
\]

(110)

by applying the Woodbury formula.

In summary, \( G \in \mathbb{R}^{M \times M} \) is

\[
G = \begin{cases}
(\frac{1}{c_f} K^t)^{-1} + S_M, & \text{if } (K^t)^{-1} \text{ exists,} \\
\frac{1}{c_f} K^t - \frac{1}{c_f} K^t (S_M^{-1} + \frac{1}{c_f} K^t)^{-1} K^t, & \text{if } (S_M^{-1} + \frac{1}{c_f} K^t)^{-1} \text{ exists,} \\
(X^T) A^{-1} X^t, & \text{otherwise,}
\end{cases}
\]

(111)

depending on the existence of the inverse of kernel matrix \( K^t \in \mathbb{R}^{M \times M} \). It exists when the dimension of the column space of \( X^t \) is \( M \), i.e., when the target samples are linearly independent. If not, the second option can be used, i.e., the inverse of \( S_M^{-1} + \frac{1}{c_f} K^t \), which can be interpreted as the regularization of kernel \( K^t \) with diagonal weight matrix \( S_M^{-1} \).

In the next lemma, we rewrite \( \Phi^T V^{-1} \Phi \).
Lemma 8. Given matrix $\Phi \in \mathbb{R}^{L_s(L_t+1) \times KM}$ and vector $q \in \mathbb{R}^{L_s(L_t+1)}$, we have

$$q^T V^{-1} \Phi = \text{vec} \left( \Theta^T X^s SG \right)^T. \quad (112)$$

Proof. Using the above lemma in a similar way as with the previous lemma, we have

$$q_{nm}^T V^{-1} \phi_{k'm'} = \text{vec}(x_n^s(\hat{x}_{m'}^t)^T V^{-1} \text{vec}(\theta_{k'}(\hat{x}_{m'}^t))^T) = ((x_n^s)^T \theta_{k'}(\hat{x}_{m'}^t))^T A^{-1} \hat{x}_{m'}^t, \quad (113)$$

$$= (\theta_{k'}^T x_n^s)(\hat{x}_{m'}^t)^T A^{-1} \hat{x}_{m'}^t. \quad (114)$$

By adding and stacking the equation, we have

$$\left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} q_{nm}^T \right) V^{-1} (\phi_{k'1}, \cdots, \phi_{k'M}) = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} (\theta_{k'}^T x_n^s)(\hat{x}_{m'}^t)^T A^{-1} \hat{x}_{m'}^t = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} (\theta_{k'}^T x_n^s)(\hat{x}_{m'}^t)^T A^{-1} \hat{x}_{m'}^t = \theta_{k'}^T \left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} x_n^s \hat{x}_{m'}^t \right)^T A^{-1} X^t = \theta_{k'}^T X^s S (X^t)^T A^{-1} X^t = \theta_{k'}^T X^s SG. \quad (123)$$

Finally, by stacking the equation, we have

$$q^T V^{-1} \Phi = (\theta_1^T X^s A^{-1} X^t, \cdots, \theta_K^T X^s A^{-1} X^t) = (\theta_1^T X^s SG, \cdots, \theta_K^T X^s SG) = \text{vec} \left( \begin{pmatrix} \theta_1^T \\ \vdots \\ \theta_K^T \end{pmatrix} X^s SG \right)^T = \text{vec} \left( \Theta^T X^s SG \right)^T. \quad (127)$$

$\square$
We now have the final form of the dual and present it in the corollary below.

**Corollary 1.** The dual form of the original primal problem is given by

$$\max_a -\frac{1}{2} a^T Y^T ((\Theta^T \Theta) \otimes G) Y a + (1^T - (Y \tilde{b})^T - \text{vec } (\Theta^T X^s SG)^T Y) a$$  \hspace{1em} (128)

s.t. \( c^T 1 \geq a \geq 0 \).  \hspace{1em} (129)

**Proof.** According to the lemmas derived above, we can write the Lagrangian as

$$L = -\frac{1}{2} a^T Y^T \Phi^T V^{-1} \Phi Y a + (1^T - q^T V^{-1} \Phi Y) a + \frac{1}{2} q^T V^{-1} q + \frac{1}{2} s$$  \hspace{1em} (130)

$$= -\frac{1}{2} a^T Y^T ((\Theta^T \Theta) \otimes G) Y a + (1^T - (Y \tilde{b})^T - \text{vec } (\Theta^T X^s SG)^T Y) a$$

$$+ \frac{1}{2} q^T V^{-1} q + \frac{1}{2} s. \hspace{1em} (131)$$

By omitting the last two terms, which do not involve \( a \), we have the dual problem as claimed.

Note that this dual form involves matrices of size at most \( KM \times KM \), which is reasonable when the number of categories \( K \) and the number of samples in the target domain \( M \) are both small.

If all \( s_{nm} = 0 \), then the problem reduces to MMDT, i.e.,

$$L = -\frac{1}{2} a^T Y^T ((\Theta^T \Theta) \otimes G) Y a + (1^T - (Y \tilde{b})^T) a, \hspace{1em} (132)$$

where \( G = (X^t)^T X^t \), since \( A = I \).

5. Retrieving the primal solution

After solving the dual problem with a QP solver, we need to convert the dual solution \( a \) to \( w \) and \( b \) by

$$w = V^{-1} (q + \Phi Y a), \hspace{1em} (133)$$

then finally to \( W \).

Here, we again face the problem of large matrix \( V \). We, therefore, derive the core parts \( V^{-1} q \) and \( V^{-1} \Phi \) as shown in the lemmas below.

**Lemma 9.** Given \( V \) of the structure above and vectors \( c \in \mathbb{R}^{L_2} \) and \( d \in \mathbb{R}^{L_2} \), we have

$$V^{-1} \text{vec}(cd^T) = c \otimes (A^{-1} d). \hspace{1em} (134)$$

**Proof.**

$$V^{-1} \text{vec}(cd^T) = \begin{pmatrix} A^{-1} & A^{-1} & \cdots & \cdots \\
A^{-1} & A^{-1} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} c_1 d \\ c_2 d \\ \vdots \\ \vdots \end{pmatrix} = \begin{pmatrix} c_1 A^{-1} d \\ c_2 A^{-1} d \\ \vdots \\ \vdots \end{pmatrix} = c \otimes (A^{-1} d) \hspace{1em} (135)$$
Lemma 10. Given $V$, $q$, and $\Phi$, we have

$$V^{-1}q = \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm}x_n^s \otimes (A^{-1}\hat{x}_m^t)$$ \hspace{1cm} (136)

and

$$V^{-1}\Phi = \Theta \otimes (A^{-1}X^t).$$ \hspace{1cm} (137)

Proof. For (136), we have

$$V^{-1}q = V^{-1} \left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm}q_{nm} \right)$$ \hspace{1cm} (138)

$$= V^{-1} \left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm}\text{vec}(x_n^s(\hat{x}_m^t)^T) \right)$$ \hspace{1cm} (139)

$$= \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm}V^{-1}\text{vec}(x_n^s(\hat{x}_m^t)^T)$$ \hspace{1cm} (140)

$$= \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm}x_n^s \otimes (A^{-1}\hat{x}_m^t).$$ \hspace{1cm} (141)

Next, we derive (137) by first stacking

$$V^{-1}\phi_{k'm'} = V^{-1}\text{vec}(\theta_{k'}(\hat{x}_{m'}^t)^T)$$ \hspace{1cm} (142)

$$= \theta_{k'} \otimes (A^{-1}\hat{x}_{m'}^t)$$ \hspace{1cm} (143)

for $m = 1, \ldots, M$ to obtain

$$V^{-1} (\phi_{k'1} \cdots \phi_{k'M})$$ \hspace{1cm} (144)

$$= (\theta_{k'} \otimes (A^{-1}\hat{x}_{1}^t), \ldots, \theta_{k'} \otimes (A^{-1}\hat{x}_{M}^t))$$ \hspace{1cm} (145)

$$= \theta_{k'} \otimes (A^{-1}\hat{x}_{1}^t, \ldots, A^{-1}\hat{x}_{M}^t)$$ \hspace{1cm} (146)

$$= \theta_{k'} \otimes (A^{-1}(\hat{x}_{1}^t, \ldots, \hat{x}_{M}^t))$$ \hspace{1cm} (147)

$$= \theta_{k'} \otimes (A^{-1}X^t).$$ \hspace{1cm} (148)

Then, we further stack the above equation for $k = 1, \ldots, K$ to obtain

$$V^{-1}\Phi = V^{-1} (\phi_{11}, \cdots, \phi_{1M}, \phi_{21}, \cdots, \phi_{KM})$$ \hspace{1cm} (149)

$$= (\theta_1 \otimes (A^{-1}X^t), \ldots, \theta_K \otimes (A^{-1}X^t))$$ \hspace{1cm} (150)

$$= (\theta_1, \ldots, \theta_K) \otimes (A^{-1}X^t)$$ \hspace{1cm} (151)

$$= \Theta \otimes (A^{-1}X^t).$$ \hspace{1cm} (152)

\[\square\]
Finally, we show primal solution $W$ directly, i.e., avoiding conversions from $a$ to $w$, and then to $W$. Instead, in the corollary below, we construct matrix $W$ from $a$ with much less computational costs.

**Corollary 2.** The solution to the primal problem is given by

$$W = (X^s S + \Theta(Y \odot \Lambda)^T) (X^t)^T A^{-1},$$

(153)

where $\odot$ is element-wise multiplication and variables are given in the proof below.

**Proof.** We first use the lemmas above to obtain

$$w = V^{-1} (q + \Phi Y a) = (\sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} x^s_n \otimes (A^{-1} x^t_m)) + (\Theta \otimes (A^{-1} X^t)) Y a.$$  

(154)

For the $i$th part of $w$, we have

$$w_i = (\sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} x^s_{n,i} (A^{-1} x^t_m)) + ((\theta_1, i, \ldots, \theta_K, i) \otimes (A^{-1} X^t)) Y a.$$  

(155)

The first term here can be written as

$$A^{-1} \left( \sum_{m=1}^{M} \sum_{n=1}^{N} s_{nm} x^s_{n,i} x^t_m \right) = A^{-1} X^t S T \begin{pmatrix} x^s_{1,i} \\ x^s_{2,i} \\ \vdots \\ x^s_{N,i} \end{pmatrix}$$

(156)

and the second term as

$$A^{-1} X^t (Y \odot \Lambda) \begin{pmatrix} \theta_1, i \\ \vdots \\ \theta_K, i \end{pmatrix},$$

(157)

where

$$a_k = \begin{pmatrix} a_{k1} \\ a_{k2} \\ \vdots \\ a_{kM} \end{pmatrix}.$$  

(158)
\[ \Lambda = (a_1, \ldots, a_K) = \begin{pmatrix} a_{11} & \cdots & a_{K1} \\ \vdots & & \vdots \\ a_{1M} & \cdots & a_{KM} \end{pmatrix} \]  

(164)

\[ Y_k = \text{diag}(y_{k1}, \ldots, y_{kM}) \]  

(165)

\[ \Upsilon = \begin{pmatrix} y_{11} & \cdots & y_{K1} \\ \vdots & & \vdots \\ y_{1M} & \cdots & y_{KM} \end{pmatrix} \]  

(166)

Combining these two terms, we obtain

\[ w_i = A^{-1}X^TS^T \begin{pmatrix} x_{1,i}^s \\ x_{2,i}^s \\ \vdots \\ x_{N,i}^s \end{pmatrix} + A^{-1}X^T(Y_1a_1, \ldots, Y_Ka_K) \begin{pmatrix} \theta_{1,i} \\ \vdots \\ \theta_{K,i} \end{pmatrix} \]  

(167)

By stacking the \( i \)th part for \( i = 1, \ldots, L_s \), we yield the matrix directly as

\[ W^T = (w_1, \ldots, w_{L_s}) = A^{-1}X^TS^T(X^s)^T + A^{-1}X^T(Y_1a_1, \ldots, Y_Ka_K)\Theta^T \]  

(168)

\[ = A^{-1}X^T(S^T(X^s)^T + (Y_1a_1, \ldots, Y_Ka_K)\Theta^T) \]  

(169)

\[ = A^{-1}X^T(S^T(X^s)^T + (\Upsilon \odot \Lambda)\Theta^T) \]  

(170)

\[ \blacksquare \]

6. Kernelization

In this section, we derive the kernel version of the dual formulation. The obtained transformation is further rewritten as

\[ W = (X^s + \Theta(\Upsilon \odot \Lambda)^T)(X^t)^T A^{-1} \]  

(171)

\[ = (X^s + \Theta(\Upsilon \odot \Lambda)^T)(X^t)^T \left( \frac{1}{c_f} I_{L_s+1} - \frac{1}{c_f} X^t(S^{-1}_M + \frac{1}{c_f}(X^t)^TX^t)^{-1} (X^t)^T \right) \]  

(172)

\[ = (X^s + \Theta(\Upsilon \odot \Lambda)^T) \left( \frac{1}{c_f} (X^t)^T - \frac{1}{c_f}(X^t)^TX^t(S^{-1}_M + \frac{1}{c_f}(X^t)^TX^t)^{-1}(X^t)^T \right) \]  

(173)

\[ = (X^s + \Theta(\Upsilon \odot \Lambda)^T) \left( \frac{1}{c_f} I_M - \frac{1}{c_f} K^t(S^{-1}_M + \frac{1}{c_f} K^t)^{-1} \right) (X^t)^T. \]  

(174)

We apply \( W \) to target sample \( x^t \) by multiplying it from the left, i.e., \( W \hat{x}^t \); Therefore, all computations with target samples are inner products, which means we can use kernels to replace the inner products.
In the dual form, we write matrix $G$ with the kernel version using kernel matrix $K^t$ as

$$K^t = \begin{pmatrix} k(x^t_1, x^t_1) & \cdots & k(x^t_1, x^t_M) \\ \vdots & \ddots & \vdots \\ k(x^t_M, x^t_1) & \cdots & k(x^t_M, x^t_M) \end{pmatrix}, \quad (175)$$

where $k()$ is a kernel function. To transform target sample $x^t$ with $W$, we have

$$W \hat{x}^t = (X^t S + \Theta(Y \odot \Lambda)^T) \left( \frac{1}{c_f} I_M - \frac{1}{c_f^2} K^t (S^{-1}_M + \frac{1}{c_f} K^t)^{-1} \right) \begin{pmatrix} k(x^t_1, x^t) \\ \vdots \\ k(x^t_M, x^t) \end{pmatrix}, \quad (176)$$

Note that the nonlinearity introduced by this kernelization appears only in the transformation part; target samples are transformed nonlinearly to the source domain. Only linear SVMs in the source domain can be used here because the primal solutions (i.e., hyperplane parameters) of the source-domain SVMs are explicitly used in the estimation of $W$. Target samples are therefore linearly classified in the source domain after being nonlinearly transformed from the target domain.

7. Results and discussions

In this section, we show two sets of experimental results. Our first experiment uses a standard dataset of supervised domain adaptation to compare our proposed MMDTL2 with MMDT. Our second experiment uses datasets consisting of two NBI endoscopic devices to understand how our proposed method behaves given differing numbers of target domain training samples.

7.1. Office-Caltech dataset

In this experiment, we compare our proposed MMDTL2 method with MMDT using the Office-Caltech dataset, a standard dataset of domain adaptation [20, 21, 25, 26]. In this dataset, there are four domains, i.e., Amazon, webcam, digital SLR (dslr), and Caltech, each of which has 10 categories. For each pair of domains, each sample is represented by an 800-dimension descriptor. One training set has eight source training samples per category (80 samples in total except the domain Amazon which has 200 source training samples (20 samples per category)), and three target training samples per category (30 samples in total). Further, 20 training/test sets are given (and provided by [25, 27]) by randomly selecting samples from each domain and category, which are used to report average performance measures. Note that different target domains use differing numbers of test samples; more specifically, dslr has 127, webcam has 265, Amazon has 928, and Caltech has 1093.

Table 1 shows results of MMDT and MMDTL2. The second column shows a reproduction of results from [20, 21] using publicly available code [27] with the
Table 1: Experimental results for the Office-Caltech dataset. Each row shows the average accuracy for a pair of source and target domains.

| source → target | MMDT (20dim) | MMDT (800dim) | MMDTL2 (800dim) |
|-----------------|--------------|---------------|-----------------|
| amazon → webcam:| 65.04 ± 1.3  | 51.11 ± 0.9   | 62.08 ± 0.9     |
| amazon → dslr:  | 54.57 ± 1.0  | 48.66 ± 0.9   | 56.34 ± 0.9     |
| amazon → caltech:| 39.96 ± 0.5  | 41.56 ± 0.3   | 31.54 ± 0.7     |
| webcam → amazon:| 50.55 ± 0.8  | 45.36 ± 0.5   | 45.89 ± 0.7     |
| webcam → dslr:  | 62.52 ± 1.1  | 69.25 ± 0.8   | 54.72 ± 0.9     |
| webcam → caltech:| 34.67 ± 0.8  | 34.66 ± 0.5   | 29.42 ± 0.8     |
| dslr → amazon:  | 50.20 ± 0.8  | 44.25 ± 0.5   | 44.69 ± 0.8     |
| dslr → webcam:  | 73.75 ± 0.8  | 80.34 ± 0.3   | 62.34 ± 0.8     |
| dslr → caltech: | 35.44 ± 0.7  | 36.36 ± 0.3   | 31.47 ± 0.6     |
| caltech → amazon:| 50.89 ± 0.8  | 46.42 ± 0.5   | 45.75 ± 0.8     |
| caltech → webcam:| 62.55 ± 1.1  | 53.79 ± 0.8   | 59.51 ± 0.9     |
| caltech → dslr: | 53.31 ± 1.0  | 51.54 ± 1.2   | 55.94 ± 0.9     |
| averages        | 52.79 ± 0.9  | 50.27 ± 0.6   | 48.31 ± 0.7     |

provided parameters and necessary preprocessing (i.e., dimensions were reduced from 800 to 20 using principal component analysis (PCA)). In the third column, results are shown for MMDT without the dimensionality reduction via PCA. Finally, results of MMDTL2 are shown in the rightmost column.

Clearly, in comparison with MMDT, our proposed method is not suitable for this dataset. More investigation is necessary here, though a possible reason for this is that the number of training samples (i.e., eight in the source domain and three in the target domain per category) might be too small for the $L_2$ constraint to behave properly. In a more realistic scenario, there are a sufficient number of source training samples, whereas fewer target samples are available. We therefore focus on this revised situation in the next experiment.

7.2. NBI endoscopic image dataset

The NBI dataset used in this experiment consisted of two domains. For the first (i.e., source) domain, we used the NBI image dataset consisting of 908 NBI patches collected from endoscopic examinations at Hiroshima University by using OLYMPUS EVIS LUCERA endoscope system [17]; patches were labeled based on NBI magnification findings [3, 4], which categorizes appearances of tumors into types A, B, and C, with type C further sub-classified into C1, C2, and C3 based on microvessel structures (see Figure 1). In this study, we used only types A, B, and C3 in accordance with our previous work [11, 28, 12, 29]. In general, a patch is trimmed from a larger frame of the entire endoscopic image such that the trimmed rectangular region represents the typical texture pattern of the colorectal polyp appearing in the frame. To align the size of patches between source and target domains, we further trimmed the center of each patch to $180 \times 180$ and created 734 patches with 289 in type A, 365 in type B, and 80 in type C3. Note that this study was conducted with the approval from the Hiroshima University Hospital ethics committee, and informed consents...
Figure 3: Experimental results for the NBI datasets with the fc6 feature.

were obtained from the patients and/or family members for the endoscopic examinations.

For the second (i.e., target) domain, we used another NBI image dataset consisting of 279 patches with 92 in type A and 187 in type B. These images were taken by OLYMPUS EVIS LUCERA ELITE endoscope system [19], which is a newer model than that of the system of the source domain. Due to the limited number of endoscopic examinations using this newer endoscope system, we trimmed the center square to $180 \times 180$ from video frames of 41 NBI examination videos; hence, there are two factors of domain shift here: (1) the NBI endoscopic devices and (2) the differences between still images (i.e., source) and video frames (i.e., target).

From these two domains, we computed convolutional neural network (CNN) features extracted using CaffeNet [30]; more specifically, this is the fc6 feature of 4096 dimensions, which is known to work well for many tasks [31]. These features were used without dimensionality reduction.

To see the effect of the number of target samples, we prepared training/test sets as follows. First, we randomly split the source and target domain samples into half; we therefore had a source training set of 367 source samples and a target training set of 139 target samples. Next, we kept a specified number of target samples per category (up to 40) in each of the target training sets, discarding the rest. For the test set, we used 140 target samples. We created 10 training/test sets and reported average performance.

Figure 3 shows performance results of different methods over the different
numbers of target training samples. As a baseline, we also show two SVM results, i.e., source SVM and target SVM. Source SVM estimates an SVM classifier with source and target training samples (and only source samples when no target samples are used). Target SVM does the same, but only with target training samples. MMDT results were obtained using the code provided by [27], just as in our first experiment. There were three results for MMDTL2, i.e., a linear version and two kernel versions with RBF and polynomial kernels.

Even with no target samples, source SVM performed relatively better and increased its performance as the number of target training samples increased. Target SVM started below 70%, but caught up to source SVM when 20 training target samples were given. The results indicate that our proposed MMDTL2 behaves between source and target SVMs. When one or two target samples are given, MMDTL2 behaves similarly to target SVM and far below the source SVM, but it quickly gains from the benefits of adaptation. With the RBF kernel, MMDTL2 is the best when 10 and 15 training samples are given, but the linear and polynomial kernels become better when more samples are given. MMDTL2 with the linear kernel and target SVM approach one another, which is expected because a sufficient number of target training samples are considered to be the best for classifying target domain samples. Overall, MMDTL2 with a polynomial kernel works the best.

Note that the results of MMDT seem to be unstable for this dataset given differing numbers of training samples. In the extreme case, e.g., the left half of the plot, where only a couple of training samples are available, MMDT might be a good choice, and this is one possible reason why MMDT works better than MMDTL2 in the first experiment.

7.3. NBI endoscopic image dataset with high-dimension features

Figure 4 shows performance results of the given methods as in Figure 3 with the same protocols for training and evaluation. The difference here is the set of features used; more specifically, we use the conv3 features of 64,896 dimensions rather than fc6. We have shown that conv3 features are expected to work better than fc6 features for NBI patch classification problems [32]: however, transformation matrix $W$ for the conv3 features could be very large without our efficient dual formulation. The ability to handle such large dimensions of features is the key advantage of our proposed method. Figure 4 shows that the source SVM is the best when a few target samples are available, just as in Figure 3. Further, our proposed MMDTL2 with a polynomial kernel becomes better at the right half of the plot, and the differences between the source and target SVMs are much more significant than those in Figure 3.

8. Conclusions

In this paper, we proposed MMDT with $L_2$ constraints, i.e., MMDTL2, deriving the dual formulation with much lesser computational costs as compared to the naive QP problem. Further, we showed the kernelization of our method.
Experimental results with NBI datasets from two different endoscopic devices showed that our proposed MMDTL2 with linear and polynomial kernels performed better than the given baselines (i.e., source and target SVMs). Our future work includes using other loss functions for problem formulation. We observed that the one-vs-rest multiclass classification by SVMs was a performance bottleneck of MMDTL2 in our experiments using the Office-Caltech dataset. Given 10 categories, we had 10 SVMs, each classifying test target samples with 80-90% accuracy measures as 10 separated and independent binary classifiers; however, the total performance for multiclass classification was sometimes lower than 40%. Therefore, instead of relying on maximum margin loss functions, multiclass logistic loss might be better here. In the future, we plan to explore this idea and report performance results for the NBI dataset as well.

Acknowledgment

Part of this work was supported by Grant-in-Aid for Scientific Research (B) JSPS KAKENHI, Grant Numbers 26280015 and 14J00223, and was with the help of a grant by Chugoku Industrial Innovation Center, respectively.

References

[1] S. Tanaka, T. Kaltenbach, K. Chayama, R. Soetikno, High-magnification colonoscopy (with videos), Gastrointest Endosc 64 (4) (2006) 604–13.
[2] Cancer Research UK. [Bowel cancer statistics] [online] (2016) [cited July, 2016].

[3] H. Kanao, S. Tanaka, S. Oka, M. Hirata, S. Yoshida, K. Chayama, Narrow-band imaging magnification predicts the histology and invasion depth of colorectal tumors., Gastrointest Endosc 69 (3 Pt 2) (2009) 631–636.

[4] S. Oba, S. Tanaka, S. Oka, H. Kanao, S. Yoshida, F. Shimamoto, K. Chayama, Characterization of colorectal tumors using narrow-band imaging magnification: combined diagnosis with both pit pattern and microvessel features, Scand J Gastroenterol 45 (9) (2010) 1084–92.

[5] M. Häfner, A. Gangl, M. Liedlgruber, A. Uhl, A. Vécsei, F. Wrba, Classification of endoscopic images using delaunay triangulation-based edge features, in: A. Campilho, M. Kamel (Eds.), Image Analysis and Recognition, Vol. 6112 of Lecture Notes in Computer Science, Springer Berlin Heidelberg, 2010, pp. 131–140.

[6] M. Häfner, A. Gangl, M. Liedlgruber, A. Uhl, A. Vecsei, F. Wrba, Endoscopic image classification using edge-based features, in: Pattern Recognition (ICPR), 2010 20th International Conference on, 2010, pp. 2724–2727.

[7] R. Kwitt, A. Uhl, M. Häfner, A. Gangl, F. Wrba, A. Vécsei, Predicting the histology of colorectal lesions in a probabilistic framework, in: Computer Vision and Pattern Recognition Workshops (CVPRW), 2010 IEEE Computer Society Conference on, 2010, pp. 103–110.

[8] S. Gross, T. Stehle, A. Behrens, R. Auer, T. Aach, R. Winograd, C. Trautwein, J. Tischendorf, A comparison of blood vessel features and local binary patterns for colorectal polyp classification, in: Proc. SPIE, Vol. 7260, 2009, pp. 72602Q–72602Q–8.

[9] T. Stehle, R. Auer, S. Gross, A. Behrens, J. Wulff, T. Aach, R. Winograd, C. Trautwein, J. Tischendorf, Classification of colon polyps in nbi endoscopy using vascularization features, in: Proc. SPIE, Vol. 7260, 2009, pp. 72602S–72602S–12.

[10] J. J. W. Tischendorf, S. Gross, R. Winograd, H. Hecker, R. Auer, A. Behrens, C. Trautwein, T. Aach, T. Stehle, Computer-aided classification of colorectal polyps based on vascular patterns: a pilot study, Endoscopy 42 (3) (2010) 203–7.

[11] T. Tamaki, J. Yoshimuta, M. Kawakami, B. Raytchev, K. Kaneda, S. Yoshida, Y. Takemura, K. Onji, R. Miyaki, S. Tanaka, Computer-aided colorectal tumor classification in NBI endoscopy using local features, Medical Image Analysis 17 (1) (2013) 78–100.

[12] S. Sonoyama, T. Hirakawa, T. Tamaki, T. Kurita, B. Raytchev, K. Kaneda, T. Koide, S. Yoshida, Y. Kominami, S. Tanaka, [Transfer learning for]
Bag-of-Visual words approach to NBI endoscopic image classification, in: 2015 37th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), IEEE, 2015, pp. 785–788. doi:10.1109/EMBC.2015.7318479.

URL http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=7318479

[13] S. J. Pan, Q. Yang, A survey on transfer learning, Knowledge and Data Engineering, IEEE Transactions on 22 (10) (2010) 1345–1359.

[14] R. Raina, A. Battle, H. Lee, B. Packer, A. Y. Ng, Self-taught learning: transfer learning from unlabeled data, in: Proceedings of the 24th international conference on Machine learning, ACM, 2007, pp. 759–766.

[15] W. Dai, Q. Yang, G.-R. Xue, Y. Yu, Boosting for transfer learning, in: Proceedings of the 24th international conference on Machine learning, ACM, 2007, pp. 193–200.

[16] D. Silver, G. Bakir, K. Bennett, R. Caruana, M. Pontil, S. Russell, P. Tadepalli, Nips workshop on “inductive transfer: 10 years later”, Whistler, Canada.

[17] OLYMPUS. EVIS LUCERA SPECTRUM Endoscopic Video Imaging System [online] (5 2006) [cited 2016-07-05].

[18] OLYMPUS. EVIS LUCERA colonovideoscope instructions [online] (12 2006).

[19] OLYMPUS. CV-290 EVIS LUCERA ELITE Processor [online] (5 2012) [cited 2016-07-05].

[20] J. Hoffman, E. Rodner, T. Darrell, J. Donahue, K. Saenko, Efficient learning of domain-invariant image representations, In Proceedings of the 1st International Conference on Learning Representations (2013) 1–9 arXiv:1301.3224v5.

[21] J. Hoffman, E. Rodner, J. Donahue, B. Kulik, K. Saenko, Asymmetric and category invariant feature transformations for domain adaptation, International Journal of Computer Vision 109 (1-2) (2014) 28–41. doi:10.1007/s11263-014-0719-3.

[22] S. Sonoyama, T. Tamaki, T. Hirakawa, B. Raytchev, K. Kaneda, T. Koide, S. Yoshida, H. Mieno, S. Tanaka, Transfer Learning for Endoscopic Image Classification, in: The Korea-Japan joint workshop on Frontiers of Computer Vision (FCV2016), 2016, pp. 258–262.

[23] E. Rodner, J. Hoffman, J. Donahue, T. Darrell, K. Saenko, Towards Adapting ImageNet to Reality: Scalable Domain Adaptation with Implicit Low-rank Transformations (2013). arXiv:1308.4200 URL http://arxiv.org/abs/1308.4200
[24] D. A. Harville, Matrix Algebra From a Statistician’s Perspective, Springer-Verlag, New York, 1997. doi:10.1007/b98818
URL http://www.springer.com/us/book/9780387949789

[25] B. Gong, Y. Shi, F. Sha, K. Grauman, Geodesic flow kernel for unsupervised domain adaptation, in: Computer Vision and Pattern Recognition (CVPR), 2012 IEEE Conference on, IEEE, 2012, pp. 2066–2073.

[26] K. Saenko, B. Kulis, M. Fritz, T. Darrell, Adapting Visual Category Models to New Domains, Springer Berlin Heidelberg, Berlin, Heidelberg, 2010, pp. 213–226. doi:10.1007/978-3-642-15561-1_16
URL http://dx.doi.org/10.1007/978-3-642-15561-1_16

[27] J. Hoffman, E. Rodner, J. Donahue, K. Saenko, T. Darrell, B. Kulis, Domain Adaptation Project (2013).
URL https://www.eecs.berkeley.edu/~jhoffman/domainadapt/

[28] T. Hirakawa, T. Tamaki, B. Raytchev, K. Kaneda, T. Koide, Y. Kominnami, S. Yoshida, S. Tanaka, SVM-MRF segmentation of colorectal NBI endoscopic images, in: 2014 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, Vol. 2014, IEEE, 2014, pp. 4739–4742. doi:10.1109/EMBC.2014.6944683
URL http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=6944683

[29] S. Sonoyama, T. Tamaki, T. Hirakawa, B. Raytchev, K. Kaneda, T. Koide, Y. Kominnami, S. Yoshida, S. Tanaka, Trade-off between speed and performance for colorectal endoscopic NBI image classification, in: S. Ourselin, M. A. Stynery (Eds.), Proc. SPIE, Vol. 9413, 2015, p. 94132D. doi:10.1117/12.2081928
URL http://dx.doi.org/10.1117/12.2081928http://proceedings.spiedigitallibrary.org/proceeding.aspx?doi=10.1117/12.2081928

[30] Y. Jia, E. Shelhamer, J. Donahue, S. Karayev, J. Long, R. Girshick, S. Guadarrama, T. Darrell, Caffe: Convolutional architecture for fast feature embedding, arXiv preprint arXiv:1408.5093.

[31] A. Sharif Razavian, H. Azizpour, J. Sullivan, S. Carlsson, CNN Features Off-the-Shelf: An Astounding Baseline for Recognition, in: The IEEE Conference on Computer Vision and Pattern Recognition (CVPR) Workshops, 2014.

[32] T. Tamaki, S. Sonoyama, T. Hirakawa, B. Raytchev, K. Kaneda, T. Koide, S. Yoshida, H. Mieno, S. Tanaka, Computer-Aided Colorectal Tumor Classification in NBI Endoscopy Using CNN Features, in: The Korea-Japan joint workshop on Frontiers of Computer Vision (FCV2016), 2016, pp. 61–65.