Yaw Moment Compensation by Actuator-Based Control of Brake System in Automobiles

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Abstract. The growing use of brake systems in enhancing vehicle performance along with safety has promised a greater class of innovation. The presented research first discusses a vehicle dynamics model that accounts for the longitudinal and lateral forces acting on the wheels to compute the yaw moment acting on the vehicle based on the steering angle. Dynamic load transfer due to combined action of gravitational load, gyroscopic couple and centripetal force for real-time behavior is considered in the vehicle dynamics model. With a difference in theory and practice, work then focuses on the various disturbances that affect the dynamics of any vehicle under yawing action. The effect of wind disturbance is considered in-depth for the research and other factors like undesirable tire behavior have been discussed as well. For yaw moment compensation, a unique brake system layout with two circuits (primary & secondary), making the use of shuttle valves has been devised and a closed-loop control system is implemented. Each wheel is autonomously provided a brake force by an electromechanical actuator, computed by its respective PID controller to compensate for the variation in yaw moment caused due to external disturbances.

1. Introduction
Growing decades of socio-economic development have turned the automotive industry into a knowledge and capital-intensive sector [1] that constitutes 5.7% of the global GDP (Gross Domestic Product) [2]. These landmark changes have now prompted a boon in automotive technology, primarily backed by capable analysis and simulation technologies of today, playing an instrumental role in enhancing the performance of multiple of our conventional mechanical systems. This trend of market growth has created an economy of scale, making vehicular technology available to more and more individuals [3].

This trend of growth has also granted a fair share of promise to the advancement of vehicle braking systems, ones which we once considered to only be a vehicle safety apparatus. Sophisticated frameworks of Advanced Driver Assistance Systems (ADAS) and a growing focus on autonomous vehicle technology have widened multi-fold with an ever-greater use in enhancing vehicle performance as well. A well-established testament to this fact lies in the successful implementation of technologies like Anti-lock Braking System (ABS), Electronic Brakeforce Distribution (EBD) systems, and Traction Control (TC), found increasingly in vehicles on the road.

Another critical technology added to this scheme has been vehicle yaw moment control, a concept of effective control of the longitudinal, lateral and vertical dynamics of a vehicle to ensure...
ease of handling [4] and stability, varying from moving on straight paths to drifting on the highly treacherous road [5]. It is concerning this promise that such technology rightfully finds use in several applications, from normal city roads to the most critical racing tracks.

2. Literature Review
The majority of cars obtain the effect of yaw moment control using the method of torque vectoring, where the output torque on each wheel can be varied as per requirement [5]. Many vehicles have this technology in the name of ‘Vehicle Stability Control’ where the brake force on different wheels is handled by the solenoid valves used for ABS. Similarly, the technology of ‘Vehicle Chassis Control’ has also been used in internal combustion engine vehicles. This concept functions on control of yaw moment by a synchronous manipulation of the engine output torque and brake force acting on each wheel [6-8].

With electric vehicles on the rise today [9-10], this technology of yaw moment control has been made for effective control of the electric drivetrains using different methods. Torque vectoring in an electric vehicle by the control of a dedicated Torque Slip Limiter (TSL), a device that controls the torque output of the installed motor has been studied in depth [11]. Similarly, a similar implementation where it tries to manipulate the motor torque output of a Hybrid Vehicle to control lateral dynamics for torque vectoring using a feed-forward controller has also been learned about [12].

Yaw moment control is also obtained in ways other than torque vectoring, one of which includes the use of 4-Wheel Steering (4WS) where the wheels turn based on a proportional relationship between steering angle and tire lateral force. Though this relationship turns uncertain at greater values of steering angle [4].

Yaw moment control in electric vehicles is a very complex process due to complicated motor controller design procedures or the multiple numbers of motors (to be controlled) themselves in electric drivetrains [13]. Similarly, in technologies like ‘Vehicle Stability Control’ and ‘Vehicle Chassis Control’, the used solenoid valves are purpose-specific and have long-haul design procedures with the study of flow fields, magnetic behavior of materials, etc. [14].

It is considering this literature that the proposed study intends to develop a cost and design efficient yaw moment control system based on the use of brake system, with a new and simpler brake circuit layout for the intended purpose.

3. Vehicle Modeling
Whenever a vehicle makes a turn at a given angle, there is a pair of lateral and longitudinal forces acting on each of the wheels that create a moment about the vertical axis (yaw moment) necessary for accomplishing that specific turn. Therefore a mathematical model of the vehicle, emphasising on the lateral, longitudinal and yawing motion of the vehicle is constructed.

The longitudinal force acting on each of the wheels (marked by index i) is defined [15] and given by (1)

\[ F_{\text{Longitudinal},i} = \mu(s)_i F_{Vi} \]  

(1)

Here, \( F_{Vi} \) depicts that normal load acting on each wheel whereas \( \mu(s)_i \) is the coefficient of friction between each wheel and the surface. It is a function of the longitudinal wheel slip (s). \( \mu(s)_i \) is defined [16] and given by (2)

\[ \mu(s)_i = (C_1*(1 - e^{-C_2*s}) - C_3*s) \]  

(2)

\( C_1, C_2 \) and \( C_3 \) vary depending on the type of surface and are shown in Table 1.
Table 1. Constants depending on type of surface

| Road Condition     | $C_1$  | $C_2$  | $C_3$  |
|--------------------|--------|--------|--------|
| Dry Asphalt        | 1.2801 | 23.990 | 0.5200 |
| Wet Asphalt        | 0.8570 | 33.822 | 0.3470 |
| Dry Cement         | 1.1973 | 25.168 | 0.5373 |
| Wet Cobblestone    | 0.4004 | 33.708 | 0.1204 |
| Snowy              | 0.1946 | 94.129 | 0.0646 |
| Icy                | 0.0500 | 306.39 | 0.0010 |

Similarly, the lateral force, also called cornering force acting on each of the wheels (marked by index $i$) is defined [17] and given by (3),

$$F_{\text{Lateral}, i} = C_S * F_{V_i} * \alpha_i$$  \hspace{1cm} (3)

Here, $\alpha_i$ depicts the slip angle of each wheel whereas $C_S$ is the cornering stiffness coefficient of the tire. The value of $C_S$ is a characteristic property of the tire and is $0.12^\circ$ for bias-ply tires whereas $0.16^\circ$ for radial-ply tires [18].

As can be seen for lateral as well as longitudinal force, the normal load on each of the wheels is a necessary input for the computational procedure. Considering a vehicle in motion, the normal load depends on three factors, namely the gravitational load, gyroscopic couple, and centrifugal force.

The gravitational load depicts the load acting due to the weight of the vehicle along with the passengers and luggage, collectively called the gross weight of the vehicle. This study considers a uniform gravitational load on each wheel at all times. Marked by index $i$, the gravitational load acting on each wheel is given by (4),

$$F_{V_i,G} = \frac{W}{4}$$  \hspace{1cm} (4)

Here, $W$ is the gross weight of the vehicle.

The gyroscopic couple and centrifugal force come into play only when the vehicle is taking a turn, creating a variable load distribution on the inner and outer wheels. This study considers only a lateral load transfer due to these factors, i.e., mass transfer about the longitudinal axis.

Every vehicle has multiple rotating components that include the wheels, axles, gears and, engine crankshaft. Whenever the vehicle accomplishes a turn, these components undergo a change in angular momentum that forms an active gyroscopic couple acting on the surrounding. In return, a reactive gyroscopic couple then acts on the vehicle, producing an overturning moment.

It is essential to know that all components in direction of rotation of the wheels create the overturning moment in the outward direction and vice-versa.

This study only considers alignments and components that have their axes parallel to that of the wheels, therefore creating only a lateral mass transfer. Hence, the vehicle is taken to have a lateral engine and gearbox orientation such that the gears, flywheels and, crankshaft have axes parallel to that of the wheels.

For determining the reactive gyroscopic couple on the vehicle, we are first required to find its precessional angular velocity ($\omega_p$), the angular velocity of the vehicle when moving about the precessional axis at turning radius, R and with linear velocity $V$. It is given by (5)
The turning radius, R is given by (6)

\[ R = \frac{W}{\sqrt{2-2\cos(2\theta)}} \]  

Here, W is the wheelbase whereas \( \theta \) is the steering angle of the vehicle, i.e., the angle by which the wheels turn under the given condition.

The study considers gyroscopic couples due to rotation of wheels \( (C_W) \) and rotation of the parts of the powertrain, namely the gears, axles, and crankshaft \( (C_E) \). \( C_W \) is defined and given by (7),

\[ C_W = 4I_w\omega_w\omega_p \]  

Here, \( I_w \) is the moment of inertia of each wheel and \( \omega_w \) is the angular velocity of each wheel. \( I_w \) and \( \omega_w \) are taken to be the same for all the wheels for this research. \( C_E \) is defined and given by (8)

\[ C_E = I_EG\omega_E\omega_p \]  

Here, \( I_E \) is the moment of inertia of the parts of the powertrain, \( G \) is the gear ratio and \( \omega_E \) is the angular velocity of the crankshaft. Any part that has a rotation opposite to that of the wheels (as would be seen in the gearbox) will contribute a negative amount to the value of \( C_E \).

The net gyroscopic couple is therefore given by (9)

\[ C = C_W + C_E \]  

The net lateral load transfer due to this couple is given by (10)

\[ M_G = \pm \frac{C_W + C_E}{T} \]  

Here, T is the trackwidth whereas the sign convention \( (\pm) \) is to depict the normal load mathematically being added to the outer wheels while simultaneously being reduced from the inner wheels.

Therefore, the load increased or decreased on each of the wheels is given by (11)

\[ M_G^* = \pm \frac{C_W + C_E}{2T} \]  

Another factor is the centrifugal force that primarily acts on a vehicle due to inertia, tending to push it outwards while accomplishing a turn. Since the wheels are still in contact with the ground due to friction, it is the chassis that encounters the overturning moment about the center of gravity, prompting a mass transfer about the longitudinal axis as given in the aforementioned discussion.

The centrifugal force acting on the vehicle is given by (12)

\[ F_C = \frac{Mv^2}{R} \]  

Here, M is the mass of the vehicle.

Due to the centrifugal force, net load transfer is given by (13)

\[ M_C = \pm \frac{h F_C}{T} \]
The sign convention \((\pm)\) is to depict the load mathematically being added to the outer wheels while simultaneously being reduced from the inner wheels. Therefore, the load increased or decreased on each of the wheels due to centrifugal force is given by (14)

\[
M_{C^*} = \pm \frac{h_{\text{cm}}}{2\pi T} M_{C^*} \tag{14}
\]

Therefore, the normal load acting on each of the outer wheels (index depending on direction of turn) when taking a turn is given by (15)

\[
F_{V_{1,\text{Outer}}} = \frac{W}{4} + M_{G^*} + M_{C^*} \tag{15}
\]

Whereas, the normal load acting on each of the inner wheels (index depending on direction of turn) when taking a turn is given by (16)

\[
F_{V_{1,\text{Inner}}} = \frac{W}{4} - M_{G^*} - M_{C^*} \tag{16}
\]

Since the vehicle is considered for lateral mass transfer, the sections of trackwidth to the left \((T_L)\) and right \((T_R)\) of the center of gravity (CG) of the vehicle will not be constant. These are determined by the given model using (17) and (18).

\[
T_L = \frac{\text{Normal Load (Right) \cdot Trackwidth}}{\text{Normal Load (Right) + Normal Load (Left)}} \tag{17}
\]

\[
T_R = \frac{\text{Normal Load (Left) \cdot Trackwidth}}{\text{Normal Load (Right) + Normal Load (Left)}} \tag{18}
\]

**Figure 1.** Vehicle dynamics model
Using the longitudinal ($F_{\text{Longitudinal}}$, $i$) and lateral ($F_{\text{Lateral}}$, $i$) forces that account for the variable normal loading condition, the yaw moment ($M_Z$) acting on a vehicle while taking a turn at steer angle $\theta$ (to the left in this case) is defined [19-20] and given by (19),

$$M_Z = I_Z \hat{r} = W_F^* (F_{y1}^* \cos \theta + F_{y2}^* \cos \theta + F_{x1}^* \sin \theta + F_{x2}^* \sin \theta) - W_R^*(F_{y3} + F_{y4}) - T_L^*(F_{x3} + F_{x1}^* \cos \theta) + T_R^*(F_{x4} + F_{x2}^* \cos \theta)$$

(19)

Here, $I_Z$ is the moment of inertia of the vehicle about the perpendicular axis and $\hat{r}$ is the rate of yawing of the vehicle. $F_{xi}$ represents the longitudinal force whereas $F_{yi}$ stands for the lateral force. $W_F$ and $W_R$ are the sections of the wheelbase to the front and rear of the center of gravity respectively. The vehicle dynamics model is shown in Figure 1.

![Vehicle dynamics model diagram](image)

**Figure 2. Vehicle dynamics model on Simulink®**

The mathematical model of the entire vehicle dynamics system on Simulink® for computing the yaw moment is depicted in Figure 2. Using the steering angle as input, the four subsystems (one for each respective wheel) account for the moment generated by that specific wheel by solving the respective blocks for longitudinal and lateral forces. These individual moments are then added to attain the final value of the yaw moment as output. Depending on the situation, right and left wheels will variably be outer or inner wheels and the outputs of ‘Normal Reaction (Right)’ and ‘Normal
Reaction (Left)’ in the shown model will account for this change using the constant for sign convention.

Following the sign convention, any yaw moment for a left turn (anticlockwise direction) is positive whereas the moment for a right turn (clockwise direction) is negative. The constant for sign convention in the respective subsystems accounts for this condition.

4. Disturbances
Moving in real-time conditions, a vehicle faces multiple disturbances that could affect its yaw moment acting under ideal conditions. One major disturbance observed is wind, creating an undesirable addition/reduction to the yaw moment due to existent longitudinal and lateral forces.

The yaw moment created due to wind ($M_Y$) is defined [21] and given by (20)

$$M_Y = 0.5 \times C_{YM} \times \rho \times V_{\text{Resultant}}^2 \times A \times W$$  \hspace{1cm} (20)

Here, $\rho$ is the density of incident air, $A$ is the frontal projection area and $W$ is the wheelbase of the vehicle, $V_{\text{Resultant}}$ is the resultant of the wind velocity ($V_{\text{Wind}}$) and vehicle velocity ($V_{\text{Vehicle}}$) whereas $C_{YM}$ is the respective coefficient for yawing moment and is given by (21)

$$C_{YM} = -0.00002360 \times \beta^2 - 0.0246 \times \beta + 0.08322$$  \hspace{1cm} (21)

Here, $\beta$ is the angle between the direction of incident wind and direction of motion of vehicle.

![Wind Disturbance Model on Simulink®](image)

**Figure 3.** Wind Disturbance Model on Simulink®

Frontal projection area, $A$ is defined [22] and given by (22)

$$A = -1.23609 + 0.00011 \times M + 1.304851 \times W \times T - 0.05398 \times (W \times T)^2$$  \hspace{1cm} (22)

Here, $M$ is the mass, $T$ is the trackwidth and $W$ is the wheelbase of the vehicle.

Following a similar convention as in modeling the vehicle dynamics, any moment in anticlockwise direction is positive whereas in the clockwise direction is negative.
A Simulink® model of the disturbance is formulated for the purpose of the research and shown in Figure 3.

Other disturbances/factors can be unaccounted for tire behavior that may come into existence due to reasons like aging, wear or use of tires of different kinds in different wheels.

5. Yaw Moment Compensation

Considering the different external disturbances and associated factors, the acting yaw moment \( M_Z^* \) is bound to be different from the desired yaw moment \( M_Z \) at any given steering angle of the vehicle. To compensate for this difference in yaw moment, as shown in Figure 4, the use of the brake system is brought into effect in the performed research.

\[
M_Z = M_Z^* - (F'_{B1} \sin \theta \cdot W_F + F'_{B1} \cos \theta \cdot T_R + F'_{B2} \cdot T_R)
\]

(23) represents the mathematical computation for reducing \( M_Z^* \) to \( M_Z \). Here, \( F'_{B1} \) and \( F'_{B2} \) are the respective brake forces acting on the two outer wheels to achieve the effect of yaw moment compensation.

Similarly, in the case of \( M_Z^* < M_Z \), brakes on both inner wheels are actuated to add to the acting moment \( M_Z^* \) and bring it to the desired value \( M_Z \).
The model is made such that due to certain physical and mathematical constraints, brakes on wheel-pairs on the opposite side may also be actuated in order to satisfy the condition for yaw moment compensation.

6. Brake System
The proposed brake system is based on a hydraulic disc brake setup and consists of two circuits, one primary and one secondary, as shown in Figure 6.

![Figure 6. Brake system layout](image)

The primary circuit functions upon the press of the brake pedal by the driver and generates brake force on all four wheels. A hydraulic disc brake system is adopted for this purpose.

Whereas the secondary circuit functions for autonomous braking for yaw moment compensation. Mimicking the chain of components in the primary circuit, the secondary circuit also runs using the same principle where an electromechanical actuator provides a force that acts on a hydraulic piston (same as brake master cylinder) for leverage and transmission of pressure to the rest of the circuit. Each of these actuators is controlled by a controller which is a part of the Brake Control Unit (BCU) and generates a signal as per the requirement of force for yaw moment compensation.

Since each of the wheels is operating with two circuits, a shuttle valve is installed to divide outflow of hydraulic pressure between the two.

![Figure 7. Shuttle Valve Layout](image)
The ball is located at the center, blocking the output in case of no transmission from either of the inputs. In case of pressure coming from one end (inlet), for instance, the primary circuit, as shown in Figure 7, the ball moves to the other end, exposing the outlet for pressure to be transmitted to the caliper.

7. Controller Design
Adopting the conventional control technique, PID control has been chosen for this research. Considering the multiple associated varying variables, simpler and trivial types of controllers, for instance, the on-off controller have not been considered. Also, with intelligent systems, particularly the fuzzy logic under consideration as well, it was learned that such systems are likely to lose accuracy and this can create undesirable circumstances especially when dealing with a critical arrangement like that of brake systems.

Depending on the purpose of this research, PID controller promises a comparatively simpler design and ease of tuning [23-25]. This ease of tuning is attributed to having only three parameters to tune in a PID, namely proportional term ($K_p$), integral term ($K_i$) and derivative term ($K_d$) [26]. The output of PID ($y(t)$) based on the error ($e(t)$) in terms of the tuning parameters is given by (24)

$$y(t) = [k_p*e(t)] + [k_i*\int e(t).dt] + [k_d*de(t)/dt]$$

(24)

A fairly linear behaviour observed in the model over the course of this research along with the operation of plant linearisation with the adopted transfer function based tuning method on Simulink® justifies the use of PID control.

Using four PID controllers (one for each wheel) that form the Brake Control Unit (BCU), a closed-loop control scheme is followed for the benefit of system stability and accuracy due to the constantly running feedback mechanism.

![Diagram](image)

**Figure 8.** Adopted closed-loop control system layout

As seen in Figure 8, different yaw moments are desired and acting (due to disturbance) on the system depending on the steering angle, $\theta$. Considering this observation, the setpoint for the loop is referred to using a lookup table where different values of desired yaw moment ($M_Z^*$) corresponding to the different steer angles are logged. For this research, the lock-to-lock angle is taken to be 70°, i.e. the
wheel can be steered from $-35^\circ$ (left) to $35^\circ$ (right). In the block for the wind disturbance, the angle of incidence of the wind is used to determine the orientation of its yawing action and compute the yaw moment (as disturbance) it creates.

In the control loop, for a given steering angle, the net yaw moment under the effect of disturbances (acting yaw moment/ $M_z$) is sent as feedback signal to the comparator for comparison with the setpoint. The error signal is computed by using the difference of the two and is sent to the respective controllers which accordingly account for the force to be generated by the electromechanical actuators (manipulated variable(s)) to alter the plant yaw moment (controlled variable) and bring the acting yaw moment to the desired value ($M_z$).

8. Results & Discussion
The devised system is tested for a vehicle moving on dry asphalt with the constant system operating parameters shown in Table 2.

| Parameter             | Value   |
|-----------------------|---------|
| Vehicle Speed, v      | 5.56 m/s|
| Longitudinal Slip Ratio, s | 0.2   |
| Mass of Vehicle, M    | 1825 kg |
| Wheelbase, W          | 2.68 m  |
| Trackwidth, T         | 1.88 m  |

The model is tested for the vehicle turning left at $10^\circ$, i.e., $\theta = -10^\circ$. The desired yaw moment ($M_z$) in this case is $+2386$ Nm. The behavior of the vehicle is noted for 10s using appropriate tools given in Simulink®. The wind is incident at an angle of $5^\circ$ from the east (positive direction on the $X$-axis in the coordinate plane) in all the respective cases.

In Case-1, wind at a velocity of 10 m/s is taken from $t=0$ to $t=10$ s, as can be seen in Figure 9 (a).

As per the system response in Figure 9 (b), it can be seen that the Brake Control Unit (BCU) promptly brings the otherwise increasing acting yaw moment to steady state in $t<1$s and maintains that value for the rest of the time period.
In Case-2, the wind is instantaneously incident at a velocity of 30 m/s at t=5s as shown in Figure 10 (a).

![Figure 10. Case-2](image)

In such a case, even when the wind is at 0 m/s for t<5s, there exists a certain aerodynamic drag due to the yet existent air present in the atmosphere.

From system response in Figure 10 (b), it can be learned that the system maintains the acting yaw moment at the desired value \( M_z^* = M_z = +2386\text{Nm} \) in the first 5 seconds \((t<5s)\), easily overcoming the minimal yet existent aerodynamic drag. Upon sudden exposure to the incident disturbance at \( t=5s \), the Brake Control Unit (BCU) quickly brings the increasing yaw moment down to the desired value and maintains it at that for the rest of the period.

Case-3 represents the scenario when the wind is incident at the same angle and the velocity is rising from 0 m/s to 30 m/s in the last 5 seconds \((t \geq 5s)\) of the time period as shown in Figure 11 (a).

![Figure 11. Case-3](image)
With the minimal disturbance due to no flow of wind in the first 5 seconds (t < 5s), the system can be seen to be in a constant physical state of having the acting yaw moment at the desired value in Figure 11 (b). Upon the rising disturbance from t=5s, the BCU alters the system response to maintain the acting yaw moment at a stable value even under a proportionally rising disturbance. The acting yaw moment is observed to have an offset error of 0.08 Nm.

9. Conclusion
The proposed research concludes by devising a model for compensating for yaw moment in automobiles while in motion. An in-depth study of the vehicle’s behavior and associated dynamics on turning helped reach the primary target of modeling for yaw moment compensation. Playing a key role in enhancing vehicle performance and safety in this study, a unique and independent brake circuitry for each wheel has been successfully devised and installed. The use of this layout can also be extended for use in emergency braking (in case of primary line failure) and other autonomous braking applications with growing understanding and use of technology. Demanding a system of closed-loop control for efficient, real-time operation, PID controllers have been used to attain the desired effect. Considering the number of real-time vehicle process variables that such models use, this study can efficiently be implemented on a practical basis for further tuning and eventual full-scale practical implementation.

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