Research Article

Linearly Time-Varying Channel Estimation and Symbol Detection for OFDMA Uplink Using Superimposed Training

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We address the problem of superimposed training- (STs-) based linearly time-varying (LTV) channel estimation and symbol detection for orthogonal frequency-division multiplexing access (OFDMA) systems at the uplink receiver. The LTV channel coefficients are modeled by truncated discrete Fourier bases (DFBs). By judiciously designing the superimposed pilot symbols, we estimate the LTV channel transfer functions over the whole frequency band by using a weighted average procedure, thereby providing validity for adaptive resource allocation. We also present a performance analysis of the channel estimation approach to derive a closed-form expression for the channel estimation variances. In addition, an iterative symbol detector is presented to mitigate the superimposed training effects on information sequence recovery. By the iterative mitigation procedure, the demodulator achieves a considerable gain in signal-interference ratio and exhibits a nearly indistinguishable symbol error rate (SER) performance from that of frequency-division multiplexed trainings. Compared to existing frequency-division multiplexed training schemes, the proposed algorithm does not entail any additional bandwidth while with the advantage for system adaptive resource allocation.

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1. Introduction

Orthogonal Frequency-Division Multiplexing Access (OFDMA) is a promising technique for future high-speed broadband wireless communication systems, and it has recently been proposed or adopted in many industry standards (e.g., IEEE 802.16e [1], 3 GPP Long Term Evolution (LTE) [2]). In OFDMA, subcarriers are grouped into sets, each of which is assigned to a different user. Interleaved, random, or clustered assignment schemes can be used for this purpose. Such a system, however, relies on the knowledge of propagating channel state information (CSI). Explicitly, in many mobile wireless communication systems, transmission is impaired by both delay and Doppler spreads [3–10], resulting in inside- and out-of-band interferences.

Channel estimation in OFDMA uplinks is challenging, however, since different channel responses for the individual user need to be tracked simultaneously at the base station (BS). OFDMA systems with adaptive resource allocation are even more critical since the uplink channels have to be estimated over the whole frequency band. In conventional pilot-aided approaches wherein the pilot symbols are frequency-division multiplexed (FDM) with the data symbols [3–8, 10–15]; however, channel estimation can only be performed within each subband of individual user separately since each user is only assigned a subset of the whole frequency band. This may be a great disadvantage for OFDMA systems with adaptive resource allocation. In addition, extra bandwidth is required for transmitting known pilot symbols. In recent years, an alternative and promising approach, referred to as superimposed training (ST), has been widely studied in [9, 16–24]. In the idea of ST, additional periodic training sequences are arithmetically added to information sequence in time or frequency domain, and the channel transfer function can thus be estimated by using the first-order statistics. The advantage of the scheme is that there is no loss in information rate and thus enables higher bandwidth efficiency. In this scheme, however, the information sequences are viewed as interference to channel estimation since pilot symbols are superimposed at a low power to the information sequences at the transmitter. To
circumvent the problem, it was recommended in [16–22, 24] that a periodic impulse train of the period larger than the channel order is superimposed in time-domain, and the channel is thus estimated by averaging the estimations of multiple training periods to reduce the information sequence interference. For a multicarrier systems, that is, SISO/OFDM system, [19] suggested a similar scheme that superimposes the periodic impulse training sequences on time-domain modulated signals, while for single-carrier systems, a novel block transmission method is proposed in frequency domain in [23], where an information sequence dependent component is added to the superimposed training so as to remove the effect of the information sequence on the channel estimation at receiver. In [24], an iterative approach is provided where the information sequence is exploited to enhance the channel estimation performance. These above-mentioned schemes, however, are restricted to the case that the channel is linearly time-invariant (LTI), and cannot be extended to the linearly time-varying (LTV) channel since the variation of channel coefficients may degrade the simple average-based solution extensively. A combined approach is developed in [9, 11] to solve the problem of channel estimation of LTV channels. However, it is only suitable for single-carrier transmission. In addition, some useful power is wasted in ST which could have otherwise been allocated to the information sequence. This lowers the effective signal-to-noise ratio (SNR) for information sequence and affects the symbol error rate (SER) at receiver. This may be a great disadvantage to wireless communication systems with a limited transmission power. On the other hand, the interference to information sequence recovery due to the embedded training sequences may degrade the SER performance severely at receiver. Previous papers merely focus on the information sequence interference suppression; whereas few researches are contributed to the superimposed training effect cancellation for information sequence recovery.

In this paper, we propose a new ST-based channel estimator that can overcome the aforementioned shortcomings in estimating LTV channel for OFDMA uplink systems. In contrast to the previous works, the main contributions of this paper are twofold. First, we extend conventional LTI-based ST schemes [16–24] to the case where the channel coefficient is linearly time-varying. By resorting to the truncated Fourier bases (DFBs) to model the LTV channel, we adopt a two-step approach to estimate the time-varying channel coefficients over multiple OFDMA symbols. Unlike conventional FDM training strategy [12–15] where channel estimation can only be performed within each subband of individual user separately, the LTV uplink channel transfer functions over the whole frequency band can be estimated directly by using specifically designed superimposed training. Furthermore, we present a performance analysis of the channel estimator. We demonstrate by simulation that the estimation variance, unlike that of conventional ST-based schemes of LTI channel [16–22, 24], approaches to a fixed lower bound as the training length increases. Second, an iterative symbol detection algorithm is adopted to mitigate the superimposed training effects on information sequences recovery. In simulations presented in this paper, we compare the results of our approaches with that of the FDM training approaches [12–15] as latter serves as a “benchmark” in related works. It is shown that the proposed algorithm outperforms FDM trainings, and the demodulator exhibits a nearly indistinguishable SER performance from that of [14].

The rest of the paper is organized as follows. Section 2 presents the channel and system models. In Section 3, we estimate the LTV channel coefficients by using the proposed channel estimator. In Section 4, we present the closed-form

Figure 1: System model.
expression of the channel estimation variances of Section 3. An iterative symbol detector is provided in Section 5. Section 6 reports on some simulation experiments carried out in order to test the validity of theoretic results, and we conclude the paper with Section 7.

Notation 1. The letter \( t \) represents the time-domain variable, and \( k \) is the frequency-domain variable. Bold letters denote the matrices and column-vectors, and the superscripts \([\ast]^T\) and \([\ast]^H\) represent the transpose and conjugate transpose operations, respectively. \( I_K \) denotes the identity matrix of size \( K \), and \([\ast]_{k,t} \) denotes the \((k,t)\) element of the specified matrix.

2. Channel and System Model

Consider an OFDMA uplink system with \( N \) active users sharing a bandwidth of \( Z \) as shown in Figure 1. Although there are many subcarrier assignment protocols, in this paper, we assume that a consecutive set of subcarriers is assigned to a user. This assumption is especially feasible when adaptive modulation and coding (AMC) protocol is employed rather than partial usage of subchannels (PUSCs) protocol [12–15]. The \( i \)th symbol of \( n \)th user is denoted by

\[
S_n(i) = [0, \ldots, s_n(i,0), \ldots, s_n(i,k), \ldots, s_n(i,K-1), 0, \ldots, 0]^T,
\]

(1)

where \( s_n(i,k), k = 0, \ldots, K - 1 \) is the transmitted data symbol, \( K \) is the subcarrier number allocated to the \( n \)th user, \( B = NK \) is the OFDM symbol-size.

At transmit terminals, an inverse fast Fourier transform (IFFT) is used as a modulator. The modulated outputs are given by

\[
X_n(i) = [x_n(i,0), \ldots, x_n(i,t), \ldots, x_n(i,B-1)]^T = F^{-1}S_n(i),
\]

(2)

where \( F^{-1} \) is the IFFT matrix with \([F^{-1}]_{k,t} = e^{j2\pi kt/B} \) and \( j^2 = -1 \). Then, \( X_n(i) \) is concatenated by a cyclic-prefix (CP) of length \( L \), propagated through respective channel. At receiver, the received signals, discarding CP, can be written as

\[
y(i,t) = \sum_{n=1}^{N} X_n(i) \otimes h(t) + v(t)
\]

(3)

\[
= \sum_{n=1}^{N} \sum_{t=0}^{L-1} h(t)x_n(i,t - l) + v(i,t), \quad t = 1, \ldots, B,
\]

where \( h(t) = [h_0(t), \ldots, h_{L-1}(t), 0, \ldots, 0]^T \) is the \( B \times 1 \) impulse response vector of the propagating channel with the channel coefficients \( h_l(t), l = 0, \ldots, L-1 \) being the functions of time variable \( t \). The notation \( \otimes \) represents the cyclic convolution, and \( v(i,t) \) is the additive noise with variance \( E_v \).

As mentioned in [3], the coefficients of the time- and frequency-selective channel can be modeled as Fourier basis expansions. Thereafter, this model was intensively investigated and applied in block transmission, channel estimation, and equalization (e.g., [4–8]). In this paper, we extend the block-by-block process [4–8] to the case where multiple OFDMA symbols are utilized. Consider a time interval or segment \( \{t : (l - 1)\Omega \leq t \leq l\Omega\} \), the channel coefficients in (3) can be approximated by truncated discrete Fourier bases (DFBs) within the segment as

\[
h_l(t) \approx \sum_{q=0}^{Q} h_{l,q}e^{j2\pi((q-Q/2)i)/\Omega)},
\]

(4)

\[
(l - 1)\Omega \leq t \leq l\Omega, \quad l = 1, 2, \ldots,
\]

where \( h_{l,q} \) is a constant coefficient, \( l = 0, \ldots, L - 1 \) is the multipath delay, \( Q \) represents the basis expansion order that is generally defined as \( Q \geq 2fd\Omega/f_s \), \( \Omega > B \) is the segment length, and \( l \) is the segment index. Unlike [4–8], the approximation frame \( \Omega \) covers multiple OFDM symbols, denoted by \( i = 1, \ldots, L \), where \( l = \Omega/B' \) and \( B' = B + L \).

Stacking the received signals in (3) to form a vector and then performing FFT operation, we obtain the demodulated signals as

\[
U(i) = [u(i,0), \ldots, u(i,k), \ldots, u(i,B-1)]^T = F[y(i,0), \ldots, y(i,t), \ldots, y(i,B-1)]^T.
\]

(5)

From (3)-(4) and the duality of time and frequency, the FFT demodulated outputs in (5) can be written as

\[
u(i,k) = \text{FFT}\left\{\sum_{n=1}^{N} \sum_{t=0}^{L-1} h_l(t)x_n(i,t-l) + v(i,t)\right\}
\]

(6)

\[
= \sum_{n=1}^{N} \sum_{t=0}^{L-1} \text{FFT}\{h_l(t)\} \otimes \text{FFT}\{x_n(i,t)\} + v(i,k)
\]

\[
= \sum_{n=1}^{N} \sum_{t=0}^{L-1} \sum_{q=0}^{Q} h_{l,q}e^{j2\pi((q-Q/2)i)/\Omega)} \otimes S_n(i) + v(i,k),
\]

(6)

where \( \text{FFT}\{\cdot\} \) represents the FFT vector of the specified function with a length \( B \), and \( v(i,k) \) is the frequency-domain noise. Note that the vectors \( \text{FFT}\{h_l(t)\} \) in (6) should be computed corresponding to the variations of the propagating channel during an OFDM symbol time interval. Specifically, the variation of LTV channel is associated with the OFDM symbol-size as well as the Doppler frequency or mobile velocity.

In this paper, we focus on the slowly time-varying channel estimation. Following the slowly time-varying assumption where the time-varying channel coefficients can be approximated as LTI during one OFDM symbol period but vary significantly across multiple symbols [25]. Accordingly,
the channel transfer function during an OFDMA symbol can be approximated as

$$h_k(t) = \sum_{q=0}^{Q} h_{l,q} e^{j2\pi (q-Q/2)c/\Omega},$$

where $t_i = (i-1)\Omega + (i-1)B' + B/2$ is the mid-sample of the $i$th OFDMA symbol. In (7), the LTV channel coefficients are in fact approximated by the mid-values of the LTV channel model (4) at the $i$th symbol. Since the proposed channel estimation will be performed within one single frame $\Omega$, we omit the frame index $l$ and thus have $t_i = (i-1)B' + B/2$ for simplification.

Accordingly, the vectors $\text{FFT}[h_k(t)]$ in (6) are thus computed as $\delta$-sequences, and the FFT demodulated signals at the subcarrier $k$ of the $i$th OFDMA symbol can be rewritten as

$$u(i,k) = \sum_{n=1}^{N} \sum_{l=0}^{L-1} \left[ \sum_{q=0}^{Q} h_{l,q} e^{j2\pi (q-Q/2)c/\Omega} \right] e^{-j2\pi k l/K} s_n(i,k) + \nu(i,k)$$

where $h_{l,q} = \sum_{q=0}^{Q} h_{l,q} e^{j2\pi (q-Q/2)c/\Omega}$.

In conventional FDM training schemes [12–14] where each user is only assigned a subset of the whole subcarriers, the channel estimation, however, cannot be performed over the whole frequency band. This may be a great disadvantage for OFDMA systems with adaptive resource allocation.

### 3. Superimposed Training-Based Solution

In this section, we propose an ST-based two-step approach to estimate the channel transfer functions over the whole frequency band and, meanwhile, overcome the above-mentioned shortcoming of conventional ST-based schemes in estimating LTV channels.

#### 3.1. Channel Estimation over One OFDMA Symbol

In this paper, the new ST strategy in estimating LTV channel of OFDMA uplink system is illustrated in Figure 2. Accordingly, the transmitted symbol in (2) can be rewritten by

$$S_n(i) = [p_n(i,0), \ldots, p_n(i,(n-1)K-1), s_n(i,0) + p_n(i,(n-1)K), \ldots, s_n(i,K-1) + p_n(i,nK-1), p_n(i,nK), \ldots, p_n(i,B-1)]^T$$

where $p_n(i,k), k = 0, \ldots, B-1$ is the superimposed pilots of $n$th user. By (8), we notice that the signal at receiver end is overlapped across different users. To circumvent this problem, we adopt the training scheme as

$$p_n(i,k) = \sqrt{E_p} e^{-j2\pi k(n-1)L/B}, \quad k = 0, \ldots, B-1,$$

where $E_p$ is the fixed power of the pilot symbols.

Note that the pilot symbols in (10) are complex exponential functions superimposed over the whole subcarriers, the corresponding time-domain signals of various users are in fact a $\delta$-sequence as $p_n(i,t) = \sqrt{E_p} B\delta(t-(n-1)L), n = 1, \ldots, N$, that follows a disjoint set with an interval $L$. Therefore, using the specifically designed training sequence (10), the training signals of various users are decoupled. The sequence (10), however, possibly leads to high signal peaks at the instant samples $t = (n-1)L, n = 1, \ldots, N$. One of the simple ways to suppress the above undesired signal peaks may refer to the scrambling procedure [25] (details will not be addressed here since it is beyond the scope of this paper).

Substituting the specifically designed pilot sequence (10) into (8), we have

$$u(i,k) = \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_{l}(i)e^{-j2\pi kl/B} p_n(i,k)$$

$$+ \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_{l}(i)e^{-j2\pi kl/B} s_n(i,k) + \nu(i,k)$$

$$= \sqrt{E_p} \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_{l}(i)e^{-j2\pi kl/B} e^{-j2\pi k(n-1)L/B} + w(i,k)$$

$$= \sqrt{E_p} \sum_{n=1}^{N} \sum_{k=0}^{L-1} \lambda_k(i)e^{-j2\pi kL/B} + w^{\text{int}}(i,k),$$

where $w(i,k) = \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_{l}(i)e^{-j2\pi kl/B} s_n(i,k) + \nu(i,k)$. In (11), the channel transfer functions are in fact incorporated into a single vector following the relationship

![Figure 2: Superimposed training sequences with training power $\sqrt{E_p}$](image-url)
where $x_n(i, t)$ is the IFFT modulated signals of the information sequence $s_n(i, k)$ . The received signals (3) in time-domain can be thus obtained as

$$y(i, t) = \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_n(i) \sqrt{E_p} B \delta(t - (n - 1)L - l)$$

$$+ \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_n(i) x_n'(i, t - l) + \nu(i, t)$$

(13)

$$= \lambda_{(n-1)L+i}(i) \sqrt{E_p} B \delta(t - (n - 1)L - l)$$

$$+ \epsilon_{n,i}(i, t) + \nu(i, t), \quad n = 1, \ldots, N,$$

where $\epsilon_{n,i}(i, t) = \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_n(i) x_n'(i, t - l)$ is the interference to channel estimation due to the information sequence. Consequently, the channel estimation can be performed in time-domain as

$$\hat{\lambda}_{(n-1)L+i}(i) = \hat{h}_n(i)$$

$$= h_n(i) + \sum_{n=1}^{N} \sum_{l=0}^{L-1} h_n(i) x_n(i, (n - 1)L - \kappa)$$

$$\sqrt{E_p} B$$

$$+ \nu(i, (n - 1)L - l) \sqrt{E_p} B, \quad i = 1, \ldots, I.$$

(14)

3.2 Channel Estimation over Multiple OFDMA Symbols.

From (14), we note that the information sequence interference vector (the second entry of (14)) can hardly be neglected unless using a large pilot power $E_p$. The conventional ST trainings stated in [16–22, 24] employ averaging the channel estimates over multiple OFDM symbols (or training periods) to suppress the information sequence interference in the case that the channel is linearly time-invariant during the record length. This arithmetical average operation in [16–22, 24], however, is no longer feasible to the channel assumed in this paper wherein the channel coefficients are time-varying over multiple OFDMA symbols.

In this section, we develop a weighted average approach to suppress the abovementioned information sequence interference over multiple OFDMA symbols, and thus overcoming the shortcoming of conventional ST-based schemes for linearly time-varying channel estimation.

We take the LTV channel coefficient estimation of each OFDMA symbol $\hat{h}_n(i)$, $i = 1, \ldots, I$ (14) as a temporal result, and then form a vector $\hat{h}_n = [\hat{h}_n(1), \ldots, \hat{h}_n(I)]^T$. Following the channel model in (7), we have

$$\hat{h}_n = \eta h_{L,q} = \begin{bmatrix} e^{j2\pi(0-Q/2)\frac{t}{\Omega}} & \ldots & e^{j2\pi(Q-Q/2)\frac{t}{\Omega}} \
0 & \ddots & \vdots 
\vdots & \ddots & \vdots 
e^{j2\pi(Q-Q/2)\frac{t}{\Omega}} \
\end{bmatrix} \begin{bmatrix} h_{0} \\
\vdots 
\vdots 
h_{L,q} \end{bmatrix}$$

(15)

where $h_{L,q} = [h_{0}, \ldots, h_{L,q}]^T$ is the complex exponential coefficients modeling the LTV channel, and $\eta$ is a $\Omega \times (Q + 1)$ matrix with $[\eta]_{q,l} = e^{j2\pi q(Q-Q/2)/\Omega}$. Thus, when $I \geq Q + 1$, the matrix $\eta$ is of full column rank, and the basis exponential model coefficients can be estimated by

$$h_{L,q} = \eta^+ \hat{h}_n, \quad l = 0, \ldots, L - 1.$$

(16)

Substituting $t = (i - 1)B' + B/2$ into the matrix $\eta$, we have the pseudoinverse matrix

$$[\eta^+]_{L,q} = e^{-j2\pi(q-Q/2)((i-1)B'+B/2)/\Omega}.$$

(17)

By (16)-(17), the modeling coefficients are estimated over the whole frame OFDMA symbols and can be rewritten by

$$\hat{h}_{L,q} = \frac{1}{I} \sum_{i=1}^{I} e^{-j2\pi(q-Q/2)\frac{t}{\Omega}} \hat{h}_n(i)/I.$$

(18)

In fact, (18) is estimated over multiple OFDMA symbols with a weighted average function of $e^{-j2\pi(q-Q/2)\frac{t}{\Omega}}/I$. Similar to the average procedure of LTI case [16–22, 24], it is thus anticipated that the weighted average estimation may also exhibit a considerable performance improvement for the time-varying channels over a long frame $\Omega$.

Compared with the conventional STs that are generally limited to the case of LTI channels [16–22, 24], the proposed weighted average approach can be performed to estimate the LTV channels of OFDMA uplink systems. In fact, the proposed channel estimation is composed of two steps: first, with specially designed training signals in (10), we estimate the channel coefficients during each OFDMA symbol as temporal results. Second, the temporal channel estimates are further enhanced over multiple OFDMA symbols by using a weighted average procedure. That is, not only the target symbol, but also the OFDMA symbols over the whole frame are invoked for channel estimation.

On the other hand, the proposed ST-based approach can be utilized to estimate the uplink channel over the whole frequency band, thus overcome the shortcoming of FDM training methods [12–14] where channel estimation can only be performed within each subband of individual user, separately.

4. Channel Estimation Analysis

In this section, we analyze the performance of the proposed channel estimator in Section 3 and derive a closed-form
expression of the channel estimation variance which can be, in turn, used for superimposed training power allocation. Before going further, we make the following assumptions.

(H1) The information sequence \( s_n(i) \) is equi-powered, finite-alphabet, i.i.d., with zero-mean and variance \( E_s \), and uncorrelated with additive noise \( \{v_n(i, t)\} \).

(H2) The LTV channel coefficients \( h_l \) are i.i.d. complex Gaussian variables.

The interference vector caused by the information sequence in (13)-(14) can be rewritten as

\[
\epsilon(i) = \left[ \epsilon_{1,0}(i), \ldots, \epsilon_{1,L-1}(i), \ldots, \epsilon_{N,0}(i), \ldots, \epsilon_{N,L-1}(i) \right]^T
\]

\[= \frac{1}{\sqrt{E_p}} B \left[ \sum_{n=1}^{N} \sum_{k=0}^{L-1} h_k(i) x_n^T(i, B - k), \ldots, \sum_{n=1}^{N} \sum_{k=0}^{L-1} h_k(i) x_n^T(i, (N - 1)L + L - k) \right]^T. \tag{19}\]

The additive noise vector is also given by

\[\nu(i) = [\nu(i, 0), \ldots, \nu(i, NL - 1)]^T\]

\[= \frac{1}{\sqrt{E_p}} \left[ \nu(i, 0), \ldots, \nu(i, (n - 1)L + l), \ldots, \nu(i, NL - 1) \right]. \tag{20}\]

By (H1), \( \nu(i, t) \) is also independent of \( \epsilon_n(l) \). We first calculate the variance of \( \nu(i, t) \) in (20) by

\[\text{var}(\nu(i, t)) = \frac{1}{BE_p} E \left[ |\nu(i, t)|^2 \right] = \sigma^2_{\nu} \frac{BE_p}{BE_p}. \tag{21}\]

We also note that the estimation error \( \epsilon_n(l) = \sum_{m=1}^{N} \sum_{q=1}^{L-1} h_q x_n(i, (n - 1)L + \kappa) \) is approximately Gaussian distributed for large symbol-size \( B \). The estimation variance due to the information sequence interference, therefore, can be obtained as

\[\text{var}(\epsilon_n(l)) = E \left[ |\epsilon_n(l)|^2 \right] = \frac{1}{BE_p} \sum_{l=0}^{L-1} |h_l(i)|^2 E_p. \tag{22}\]

Since (22) depends upon the channel transfer functions (equivalently, the channel impulse response), we define the normalized variance as

\[\text{nvar}(\epsilon_n(l)) = \frac{1}{|h_l(i)|^2} \text{var}(\epsilon_n(l)), \tag{23}\]

where \(|h_l(i)|^2 = \sum_{i=0}^{L-1} |h_l(i)|^2 / L\). Following the definition of (23), we obtain the normalized variance as

\[\text{nvar}(\epsilon_n(l)) = \frac{\text{var}(\epsilon_n(l))}{|h_l(i)|^2} = \frac{E_p \sum_{i=0}^{L-1} |h_l(i)|^2}{BE_p |h_l(i)|^2} = \frac{L E_p}{B E_p} \tag{24}\]

From (24), we can find that the estimation variance due to the information interference is directly proportional to the information-to-pilot power ratio \( E_s/E_p \), thereby resulting in an inaccurate solution for the general case that \( E_p \ll E_s \).

We then analyze the estimation performance (16)–(18) over multiple OFDMA symbols. Neglecting the modeling error, we use \( h_{iq} \) to evaluate the channel estimation variance. Define

\[\epsilon_{nl}(l) = [\epsilon_{nl}(1), \ldots, \epsilon_{nl}(I)]^T\]

\[\nu = [\nu(1), \ldots, \nu(I)]^T. \tag{25}\]

By (H1)-(H2), the MSE of the weighted average estimator is given by

\[\text{MSE}(\text{ave}) \quad \overset{\text{def}}{=} \quad E \left[ \left| h_{iq} - \hat{h}_{iq} \right|^2 \right] = E \left[ \left| \eta^T (\epsilon_{nl} + \nu) \right|^2 \right] \]

\[= \text{tr} \left\{ \eta^T E \left[ \epsilon_{nl} (\nu) \right] \right\} + \text{tr} \left\{ \eta^T E \left[ \nu (\nu) \right] \right\} \eta^T \]

\[= \frac{L}{I} \sum \left\{ \text{var}(\nu(i)) + \text{var}(\epsilon_n(i)) \right\} \eta^T \eta^{-1}. \tag{26}\]

Note that the column vectors of the matrix \( \eta \) in (15) are in fact the FFT vectors of a \( I \times I \) matrix, we thus have \( \eta^T \eta = I (Q + 1) \) and \( \text{tr}[\eta^T \eta^{-1}]^{-1} = (Q + 1)/I \). Substituting (21)-(22) into (26), we then obtain the variance of the weighted average estimation \( \hat{h}_{iq} \) associated with \( \epsilon_{nl}(i), i = 1, \ldots, I \) as

\[\rho_{iq} = \frac{(Q + 1)E_s}{BI^2 E_p} \sum_{i=1}^{I} \sum_{l=0}^{L-1} |h_l(i)|^2 = \frac{(Q + 1)E_s}{\Omega E_p} \sum_{i=1}^{I} \sum_{l=0}^{L-1} |h_l(i)|^2. \tag{27}\]

By analogy, the variance of the additive noise \( \nu(i), i = 1, \ldots, I \) can be also derived as

\[E \left[ |\nu(i)|^2 \right] = \frac{(Q + 1)E_s}{BE_p} E_s = \frac{(Q + 1)E_s}{\Omega E_p}. \tag{28}\]

Combining the variances in (27) and (28), we have the weighted average estimation variances

\[\text{MSE}(\text{ave}) = \frac{(Q + 1)E_s}{\Omega E_p} \sum_{i=1}^{I} \sum_{l=0}^{L-1} |h_l(i)|^2 + \frac{(Q + 1)E_s}{\Omega E_p}. \tag{29}\]

In (29), the last term is due to the additive noise. In general, since the LTV channel model satisfies \((Q + 1)/\Omega \ll 1\), the additive noise is greatly suppressed by the weighted average procedure. On the other hand, estimation variance due to the information sequence interference (the first term in (29)) may be the dominant component of the channel estimation error, especially for high SNR. Similar to (23), we derive the normalized variance of information sequence interference by removing the channel gain by

\[\text{nvar}(\rho_{iq}) = \frac{1}{\Omega |h_l(i)|^2} \text{var}(\rho_{iq}), \tag{30}\]
\[

\left| H \right|^2 = \sum_{i=1}^{L} \sum_{n=0}^{L-1} |h(i)|^2/L. \quad (29)
\]

From (29) and (30), it follows that
\[

\text{var}(\rho_{ii}) = \frac{(Q + 1)E_s \sum_{i=1}^{L} \sum_{n=0}^{L-1} |h(i)|^2}{B E_p |H|^2} \approx \frac{L(Q + 1)E_s B^2}{\Omega E_p} \frac{L(Q + 1)}{\Omega E_p}. \quad (31)
\]

From (31), the normalized variance is directly proportional to the information-pilot power ratio \(E_s/E_p\) and the ratio of the unknown parameter number \(L(Q + 1)\) over the frame length \(\Omega\). In particular, for the specifically designed training sequence (10), the closed-form estimation variance (31) may provide a guideline for signal power allocation at transmitter, for example, for a given threshold of the estimation variance \(\phi\) (channel gain has been normalized), the minimum training power \(E_p\) should at least satisfy the approximated constraint as \(E_p \geq \phi \Omega E_s /NL(Q + 1)\).

Compared with the variances of channel estimation over one OFDMA symbol as in (22)–(24), the estimation variances (29)–(31) of the weighted average estimator (15)–(18) are significantly reduced owing to the fact that \(\Omega/B(Q + 1) \gg 1\). Theoretically, the weighted average operation can be considered as an effective approach in estimating LTV channel, where the information sequence interference can be effectively suppressed over multiple OFDMA symbols. As stated in the conventional ST-based schemes [16–22, 24], channel estimation performance can be improved along with the increment of the recorded frame length \(\Omega\), that is, the estimation variance approaches to zero as \(\Omega \rightarrow \infty\). This can be easily comprehended that larger frame length \(\Omega\) means more observation samples, and hence lowers the MSE level. From the LTV channel model (4), however, we note that as the frame length \(\Omega\) is increased, the corresponding truncated DFB requires a larger order \(Q\) to model the LTV channel (maintain a tight channel model), and the least order should be satisfied \(Q/2 \geq f_d \Omega/f_s\), where \(f_d\) and \(f_s\) are the Doppler frequency and sampling rate, respectively [1–8]. Consequently, as the frame length \(\Omega\) increases, the LTV channel estimation variance (31) approaches to only a fixed lower-bound associate with the system Doppler frequency as well as the information-pilot power ratio. This is quite different from the ST trainings in [16–22, 24].

5. Iterative Symbol Detector

Unlike the FDM trainings [10, 12–15, 25], the pilot sequences in (10) are superimposed on the information sequences and thus produce interferences on the information sequences recovery. The existing ST approaches [9, 11, 16–22, 24] merely focus on the information sequence interference suppression; whereas few researches are contributed to the ST effect cancellation for information sequence recovery. In this section, we provide a new iterative symbol detector to cancel the residual training effects on symbol recovery.

As in the symbol detection of conventional ST-based approach, the contribution of the training sequences is firstly removed at OFDMA uplink receiver before recovering the data symbols
\[

\mathbf{U}(i) = \mathbf{U}(i) - \sum_{n=1}^{N} \hat{H}(n) \mathbf{P}_n(i) = \mathbf{H}(i) \mathbf{S}(i) + \mathbf{E}(i) + \mathbf{v}(i), \quad (32)
\]

where \(\hat{H}(i)\) is an \(M \times M\) matrix with the diagonal elements being the estimated channel frequency-domain transfer function, that is, diag(\(H(i)\)) = \([\hat{H}(i, 0), \ldots, \hat{H}(i, k), \ldots, \hat{H}(i, B - 1)\]^T\) (with \(\hat{H}(i, k) = \sum_{n=1}^{L} \hat{h}(i)e^{-j2\pi nk/B}\)) and the remaining entries being zeros. \(\mathbf{E}(i) = [\mathbf{H}(i) - \hat{H}(i)]\mathbf{P}(i)\) is the residual error of the superimposed pilots.

Note that \(\mathbf{E}(i)\) is distributed over the whole frequency tone; whereas owing to the specifically designed training signals in (10), the time-domain received signals affected by the residual error are concentrated only during a sequence of sample periods \(y(i, (n - 1)L + \kappa), \kappa = 0, \ldots, L - 1, n = 1, \ldots, N\). In order to mitigate the residual error, a natural idea is to reconstruct the above time-domain signals of \(t = (n - 1)L + \kappa, \kappa = 0, \ldots, L - 1, n = 1, \ldots, N\). In our proposed iterative method, we carry out the following steps.

Step 1. By (32), we perform zero-forcing equalization by
\[

\hat{\mathbf{S}}(i) = \left[\hat{\mathbf{S}}_1(i), \ldots, \hat{\mathbf{S}}_N(i)\right]^T = (\hat{\mathbf{H}}(i))^\dagger \mathbf{U}(i). \quad (33)
\]

The information symbols, owing to the finite alphabet set property, can be recovered by a hard detector as
\[

s_{n}(i, k) = \arg \min_{s_{n}(i, k) \in \Phi} \{||\hat{s}_n(i, k) - \hat{s}_n(i, k)||^2\}, \quad (34)
\]

where \(\Phi\) is the finite alphabet set from which the transmitted data takes, for example, 4-PSK and 8-PSK signals, and so forth.

Step 2. Reconstruct the time-domain received signal vectors with the estimated channel coefficients in (16) and data sequences in (34), respectively, we obtain
\[

\mathbf{Y}(i) = \left[\hat{y}(i, 0), \ldots, \hat{y}(i, t), \ldots, \hat{y}(i, B - 1)\right]^T = \mathbf{F}^{-1}\mathbf{U}(i). \quad (35)
\]

Step 3. Replace the contaminated signals \(y(i, (n - 1)L + \kappa)\) by the reconstructed signals \(\hat{y}(i, (n - 1)L + \kappa)\) in (35), the received signal vector is then updated by
\[

\hat{\mathbf{Y}}(i) = \left[\hat{\mathbf{Y}}(i, 0), \hat{\mathbf{Y}}(i, 1), \ldots, \hat{\mathbf{Y}}(i, (n - 1)L + \kappa), \ldots, \hat{\mathbf{Y}}(i, (N - 1)L + L - 1), y(i, NL), \ldots, y(i, B - 1)\right]^T. \quad (36)
\]

Step 4. Using the updated signals in (36), we detect the information symbols by (32)–(36) in the forthcoming iteration.

Step 5. Repeat the Steps 1–4 until the increment changes of the improved SER performance over successive iterations are below a given threshold.
When the SER of the initial hard detector in (34) is lower than a certain threshold, the reconstructed signals in the current iteration should approach to the original signals $\hat{y}(i, (n - 1)L + k)$ more than that of the previous iteration, that is, $|\hat{y}_{\text{cur}}(i, (n - 1)L + k) - \hat{y}(i, (n - 1)L + k)| < |\hat{y}_{\text{pre}}(i, (n - 1)L + k) - \hat{y}(i, (n - 1)L + k)|$, where $\hat{y}(i, t)$ is the pure IFFT modulated information signals of $U(i) = \sum_{n=0}^{N-1} H(i)S_n(i)$, $\hat{y}_{\text{cur}}(i, (n - 1)L + k)$ and $\hat{y}_{\text{pre}}(i, (n - 1)L + k)$ are the reconstructed signals by (36) in the current and previous iterations, respectively. Additionally, the iteration index depends crucially on the size of the reconstructed signals over one OFDMA symbol period, that is, $\tau = NL/B$. Base on experiment studies, the proposed iterative method should satisfy the constraint of $\tau \leq 0.2$. Commonly, such constraint for practical implementation can be satisfied freely by simply adjusting the total frequency bandwidth and the number of active users.

6. Simulation Results and Discussion

In this section, we present the numerical examples to validate our analytical results. We assume the OFDMA uplink system with $B = 512$ and all subcarriers are equally divided into $N = 4$ subband that assigned to four users. The transmitted data symbol $s_n(i, k)$ is QPSK signals with symbol rate $f_s = 10^9$/second. The channel is assumed with $L = 10$, and the coefficients $h_n(t)$ are generated as low-pass, Gaussian, and zero-mean random processes and correlated in time with the correlation functions according to Jakes’ mode $r_n(\tau) = \mu^2_n h_0(2\pi f_d \tau)$, $n = 1, \ldots, 4$, where $f_d$ is the Doppler frequency associated with the $n$th user. CP length is chosen to be 15 to avoid intersymbol interferences. The additive noise is a Gaussian and white random process with a zero mean.

We run simulations with the Doppler frequency $f_d = 300$ Hz that corresponds to the maximum mobility speed of 162 km/h as the users operate at carrier frequency of 2 GHz. In order to model the LTV channel, the frame is designed as $\Omega = B' \times 256 = (B + CP - \text{length}) \times 256 = 136192$, that is, each frame consists of 256 OFDMA symbols. During the frame, the channel variation is $f_d \Omega / f_s = 4.1$. Notice that the channel variation during an OFDM symbol is $f_d B' / f_s = 0.0154$, and thus can be neglected. Over the total frame $\Omega$, we utilize the truncated DFB of order $Q = 10$ to model the LTV channel coefficients. The LTV channel modeled by the truncated DFB, however, exhibits modeling errors at the outmost samples. A possible explanation is that as the Fourier basis expansions are truncated in (4), an effect similar to the Gibbs phenomenon, together with spectral leakages, may lead to modeling inaccuracy at the beginning and the end of the frame [3, 5, 7–9]. To circumvent the problem, the frames are designed to be partially overlap, for example, $(t - 1) \Omega - yB' \leq t \leq f\Omega$, $t = 2, 3, \ldots$, where $y$ is a positive integer. By the frame-overlap, the LTV channel at the beginning and the end of the frame can be modeled and estimated accurately from the neighboring frames.

To evaluate the proposed channel estimator, we resort to the MSE of channel estimation to measure the estimation performance, which is defined as

$$\text{MSE} = \frac{\sum_{i=1}^{\Omega/B} \text{MSE}(i)}{\Omega / B}$$

$$= \frac{B}{\Omega} \sum_{i=1}^{\Omega/B} \left\{ \sum_{t=0}^{B-1} \sum_{l=0}^{L-1} \left( \hat{h}(i, t) - \sum_{q=0}^{Q-1} \hat{h}(q \Omega + l) e^{j 2 \pi (q - Q/2) \Omega t / \Omega} \right)^2 \right\}^{1/2}$$

where $\text{MSE}(i)$ denotes the MSE of the $i$th OFDMA symbol.

6.1. Channel Estimation. We firstly examine the ST-based weighted channel estimation scheme under different IPR to verify the channel estimation variance analysis in Figure 3. From Figure 3, the curve of the MSE are almost independent of the additive white Gaussian noises, especially as SNR $> 5$ dB since the additive noise has been greatly suppressed by the weighted average procedure. In addition, the results shown in Figure 3 are consistent with the closed-form estimation variance as formulated in (29)–(31), wherein the estimation variances are directly proportional to the unknown parameter $L(Q + 1)$ and inversely proportional to information-to-pilot power ratio $E_0/E_p$, respectively.

Then, we compare the developed channel estimators with the conventional ST-based method under the different

![Figure 3: MSE versus SNR.](image)
Doppler frequencies. It shows clearly in Figure 4 that our estimation approach achieves indistinguishable performance with the conventional ST-based scheme in estimating the LTI channel of $f_n = 0$ Hz, and the MSE level is significantly reduced as the average length increases. However, the shortcoming of conventional ST appears when the channel being estimated is linearly time-varying. Comparatively, by using the weighted average procedure, our proposed approach performs well for the LTV channel estimation of different Doppler frequencies, that is, $f_n = 100$ Hz/300 Hz. On the other hand, we also observe that as the frame-length $\Omega$ increases, the MSE approaches to a constant (lower-bound) that associated with the Doppler frequency. The theoretical analysis has been proved by Section 4.

Figure 5 displays the comparison between the proposed algorithm and the channel estimator [14]; wherein the uplink channel over the whole frequency band is reconstructed with the aid of estimated subband channel transfer functions. Owing to the time-variation of channel coefficients between OFDMA symbols, channel estimation performed in [14] is required in each separate OFDMA symbol. Since the total number of known pilots should be larger than or at least equal to the total channel unknowns $NL = 40, 64$ pilot tones (with 16 pilot symbols in each subband of individual user) are utilized within one OFDMA symbol. Correspondingly, 12.5% of total bandwidth is wasted in transmitting the pilot symbols. Comparatively, the proposed ST-based channel estimation approach, without entailing any additional bandwidth or constraint, outperforms the FDM training-based estimator [14] by using a small pilot power of $E_p = 0.02E_s$. Furthermore, the iterative method developed in [24] can be directly employed herein to further improve the estimation performance of our algorithm.

6.2. Symbol Detection. As aforementioned, symbol detection in demodulator of ST-based schemes [9, 11, 16–22, 24] is affected by the residual contribution of embedded pilot symbols. Herein, we carry out simulation experiments to assess the effectiveness of the proposed iterative symbol detector.

Figure 6 illustrates the SER performance versus SNR with IPR as $E_p = 0.01E_s$. As shown in Figure 6, although the
The channel estimator achieves well estimation performance in estimating the LTV channel coefficients, the conventional demodulator still shows a poor SER performance owing to the effects of the residual error of embedded training sequences. In contrast, by the proposed iterative mitigation procedure, the demodulator achieves a considerable gain than that of conventional ST-based method. It thus confirms that the above-mentioned residual interference can be effectively mitigated with the developed iterative approach. As a comparison, we also list the SER performance based on the FDM training scheme [14] where information sequences and pilot symbols are of frequency-division multiplexed and the symbol detection can be thus performed without additional pilot interference. We observe that the performance of two demodulators is in general indistinguishable (15 dB ~ 25 dB), which confirms that the effects of the above-mentioned residual training on information sequence recovery have been effectively cancelled by the proposed iterative approach.

Figure 7 depicts the SER performance under different reconstructed signal-size over one OFDMA symbol period, that is, \( \tau = NL/B \). As stated in Section 5, the minimum iterations utilized to achieve a steady SER performance depend crucially on the above constraint \( \tau \). It observed that when \( \tau = NL/B \leq 10\% \), a significant SER performance improvement is achieved in the very first iterations (the first 2~3 iterations). Meanwhile, the iterations required to achieve the steady-state solution of SER performance increase along with the increment of \( \tau \). For the situation that \( NL/B > 20\% \), the iterative cancellation may not convergent and the SER still keeps at a high level. Therefore, \( \tau \leq 0.2 \) can be approximately considered as the upper-bound for the implementation of the proposed iterative detection approach.

6.3. Complexity Analysis. The description of the proposed channel estimation method in Section 3 shows that the overall complexity comes from the complex matrix pseudo-inverse operation in (16). Note that (16) can be deduced into a weighted average process in (18). Thus, compared to the ST-based estimator within one OFDMA symbol (13), only \((Q+I+1)\) additional complex multiplication and \((Q+I)\) complex additions are required to obtain the accurate time-domain CSI \( h_l(t) \) of uplink OFDMA systems.

7. Conclusion

In this paper, we have developed a new method for estimating the LTV channels of uplink OFDMA systems by using superimposed training. We extend conventional LTI-based ST schemes to the case where the channel coefficient is linearly time-varying. By resorting to the truncated Fourier bases (DFBs) to model the LTV channel, we adopt a two-step approach to estimate the time-varying channel coefficients over multiple OFDMA symbols. We also present a performance analysis of the channel estimation approach and derive a closed-form expression for the channel estimation variances. It is shown that the estimation variances, unlike conventional superimposed training, approach to a fixed lower-bound that can only be reduced by increasing the pilot power. In addition, an iterative symbol detector was presented to mitigate the superimposed training effects on information sequence recovery, thereby offering an alternative to enhance the channel estimation performance by using a large training power while without sacrificing SER performance degradation. Compared with the existing FDM training schemes, the new estimator can estimate the channel transfer function over the whole frequency band without a loss of rate, and thus enables a higher efficiency with the advantage for system adaptive resource allocation.

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