Combined search for light dark matter with electron
and muon beams at NA64

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Abstract

We discuss prospects for searching for dark photon mediator ($A'$) of light dark matter (LDM) production by using the combined results from the NA64 experiment at the CERN SPS running in high-energy electron (NA64e) and muon (NA64$\mu$) mode. We discuss the most natural values and upper bounds on the $A'$ coupling constant to LDM and show they are lying in the range accessible at NA64. While with $5 \times 10^{12}$ electrons on target (EOT) NA64e is able to probe the scalar and Majorana LDM scenarios, the combined NA64e and NA64$\mu$ results with $\simeq 10^{13}$ EOT and a few $10^{13}$ MOT, respectively, will allow to cover almost fully the parameter space of the most interesting LDM models. This makes NA64e and NA64$\mu$ extremely complementary to each other and greatly increases the discovery potential of sub-GeV DM.
1 Introduction

At present the most striking evidence in favour of new physics beyond the Standard model is the observation of Dark Matter (DM). Among the numerous DM models, for a review, see e.g. [1] - [5], those that motivate the existence of light DM messengers with a mass $m_d \leq O(1)$ GeV are rather popular now [3, 4]. The main idea is that in addition to gravity a new interaction between visible and dark sector exists which is mediated by a new sub-GeV vector or scalar boson, as a review of the current and future efforts towards light Dark Matter and other New Physics, see e.g. Refs. [7]-[11].

Among several renormalizable light DM extensions of the SM, the model with dark photon, where dark sector includes an abelian gauge field $A'_\mu$ (dark photon) is the most popular now. In these dark photon models, dark sector interacts with the SM particles only through nonzero kinetic mixing of the ordinary photon and dark photon, $-\frac{\epsilon}{2} F_{\mu\nu}' F^{\mu\nu}$. In renormalizable models the DM particles interacting with the $A'$ have spin 0 or 1/2. Spin 1/2 DM particles can be Majorana or pseudo-Dirac particles [7, 12]. The annihilation cross section for scalar or Majorana DM has $p$-wave suppression that allows to escape the GMB bound [13, 14] while for Dirac fermions the annihilation cross section is $s$-wave that contradicts to the GMB bound [13, 14, 15]. For the model with pseudo Dirac fermions [16] it is also possible to avoid the GMB bound.

Let us consider, as an example, charged scalar dark matter interacting with dark photons. The charged dark matter field $\phi_d$ interaction with the $A'$ dark photon field is

$$L_{\phi Z'} = (\partial^\mu \phi - i e_d Z'^\mu \phi)^* (\partial_\mu \phi - i e_d Z'_\mu \phi) - m^2_{DM} \phi^* \phi - \lambda \phi^* \phi)^2$$  

The nonrelativistic DM annihilation cross section $\phi_d \bar{\phi}_d \rightarrow e^- e^+$ has the form$^1$

$$\sigma_{an} v_{rel} = \frac{8 \pi \epsilon^2 \alpha_{DM}^2 m_{DM}^2 v_{rel}^2}{3 (m_{A'}^2 - 4 m_{DM}^2)^2}. $$  

Here $\alpha_D = \frac{e^2}{4 \pi}$ is an analog of the fine-structure constant $\alpha = 1/137$ for the DM particles interacting with DM photon. We shall use standard assumption that in the hot

\footnote{Here we consider the case $m_{A'} > 2m_{DM}$, $m_{A'} \gg m_e$.}
early Universe DM is in equilibrium with ordinary matter \[5\]. During the Universe expansion the temperature decreases and at some \(T_d\) the thermal decoupling of the DM starts to work. Namely, at some freeze-out temperature \(T_d\) the cross-section of annihilation \(DM\) particles \(\rightarrow\) \(SM\) particles becomes too small to obey the equilibrium of DM particles with the SM particles and DM decouples. The experimental data are in favour of scenario with cold relic for which the freeze-out temperature \(T_d\) is much lower than the mass of the DM particle. In other words DM particles decouple in the non-relativistic regime. The value of the DM annihilation cross section at the decoupling temperature determines the value of the current DM density in the Universe. Too big annihilation cross section leads to small DM density and vice versa too small annihilation cross section leads to DM overproduction. The observed value of the DM density \(\frac{\rho_{DM}}{\rho_c}\approx 0.23\) (here \(\rho_{DM}, \rho_c\) is the dark matter density, and the total energy density of the Universe, respectively) allows to estimate the DM annihilation cross-section into the SM particles and hence to estimate the discovery potential of light dark matter both in direct underground and accelerator experiments. Very crude estimate for the DM annihilation cross section is \[2\]

\[<\sigma_{an}v_{rel}> = O(1) \text{ pb} \cdot c.\]

As a consequence of the formulae (2,3) for fixed values \(m_{A'}\) and \(m_{DM}\) we can estimate the product \(\epsilon^2 \alpha_D\). Note that fixed target NA64 experiment \[17, 18\] uses the reaction of the dark photon electroproduction on nuclei allowing to obtain only upper bounds on \(\epsilon^2\) vs \(m_{A'}\). Therefore, to test the prediction for the \(\epsilon^2 \alpha_D\) we have to know either the \(\alpha_D\) value or at least an upper bound \(\alpha_D \leq \alpha_o\) on the \(\alpha_D\). The arguments based on the use of the renormalization group and the assumption of the absence of the Landau pole singularity up to some scale \(\Lambda\) allow to obtain upper limit on the coupling constant \(\alpha_D\) \[19\] in the formula (2) for the annihilation cross section. The bound on \(\alpha_D\) depends on the scale \(\Lambda\) logarithmically. Moreover, the scale \(\Lambda\) has to be larger than 1 TeV \[19\]. So for fixed values of \(m_{A'}\) and \(m_{DM}\) the knowledge of the upper bound on \(\alpha_D\) together with the requirement that the dark photon model correctly reproduces the observed Dark matter density allows to obtain lower bound on \(\epsilon^2\) as a function of \(m_{A'}\) or \(m_{DM}\).
In this paper we discuss prospects for searching for $A'$ dark photon mediator of light dark matter (LDM) production at the NA64 experiment at CERN SPS by using $\simeq 100$ GeV electron (NA64e) and muon (NA64$\mu$) beams. The rest of the paper is organized as follows. In Sec. 2 we discuss upper bounds on $\alpha_D$ obtained from the requirement of the absence of Landau pole singularity for the effective coupling constant $\tilde{\alpha}_D(\mu)$ up to some scale $\Lambda$. In Sec. 3 we estimate the NA64e discovery potential of LDM and show that with $\simeq 5 \times 10^{12}$ electrons on target (EOT) the experiment is able to probe the scalar and Majorana scenarios of the LDM models. In Sec. 4 we estimate the NA64$\mu$ discovery potential of LDM. We show that NA64$\mu$ has better sensitivity to the $\gamma - A'$ kinetic mixing for the $A'$ masses $m_{A'} \gtrsim 100$ MeV in comparison with NA64e, and that the combined NA64e and NA64$\mu$ results obtained with $\simeq 10^{13}$ EOT and a few $10^{13}$ MOT, respectively, will allow to cover almost fully the most interesting and natural parameter space of the LDM models. This makes the two approaches extremely complementary to each other and greatly increases the discovery potential of NA64. Sec. 5 contains concluding remarks. In Appendix we collect the main formulae used for approximate DM density calculations.

2 Upper bound and range of $\alpha_D$

One can obtain upper bound on $\alpha_D$ by the requirement of the absence of Landau pole singularity for the effective coupling constant $\tilde{\alpha}_D(\mu)$ up to some scale $\Lambda$ [19]. One loop $\beta$ function for $\tilde{\alpha}_D(\mu)$ is

$$\beta(\tilde{\alpha}_D) = \frac{\tilde{\alpha}_D^2}{2\pi} \left[ \frac{4}{3} (Q_F^2 n_F + Q_S^2 n_S) \right]. \tag{4}$$

Here $\beta(\tilde{\alpha}_D) \equiv \mu \frac{d \tilde{\alpha}_D}{d \mu}$ and $n_F (n_s)$ is the number of fermions (scalars) with the $U'(1)$ charge $Q_F (Q_S)$. For the model with pseudo-Dirac fermion we must have additional scalar with $Q_d = 2$ to realize the splitting between fermion masses, so one loop $\beta$ function is $\beta(\tilde{\alpha}_D) = \frac{4\tilde{\alpha}_D^2}{3\pi}$. For the model with Majorana fermions we also must have an additional scalar field with the charge $Q_S = 2$ and additional Majorana field to cancel $\gamma_5$ anomalies, so the $\beta$ function coincides with the $\beta$ function for the model with pseudo-Dirac fermions. For the model with charged scalar matter in order to create nonzero dark photon mass we have
to introduce additional scalar field with $Q_S = 1$ so one loop $\beta$ function is $\beta = 2 \ast \alpha^2 / 6\pi$.

From the requirement that $\Lambda \geq 1 \text{ TeV}$ \[19\], we find that $\alpha_D \leq 0.2$ for pseudo-Dirac and Majorana fermions and $\alpha_D \leq 0.8$ for charged scalars \[2\]. Here $\alpha_D$ is an effective low energy coupling at scale $\mu \sim m_{A'}$, i.e. $\alpha_D = \bar{\alpha}_D(m_{A'})$. In our calculations we used the value $m_{A'} = 10 \text{ MeV}$. In the assumption that dark photon model is valid up to Planck scale, i.e. $\Lambda = M_{PL} = 1.2 \times 10^{19} \text{ GeV}$, we find that for pseudo-Dirac and Majorana fermions $\alpha_D \leq 0.05$ while for scalars $\alpha_D \leq 0.2$. In the SM the $SU_c(3), SU_L(2)$ and $U(1)$ gauge coupling constants are equal to $\sim (1/30 - 1/50)$ at the Planck scale. It is natural to assume that the gauge coupling $\bar{\alpha}_D(\mu = M_{PL})$ also lies within the interval $\sim (1/30 - 1/50)$. As a consequence we find that the values in the range $\sim (0.014 - 0.02)$ are possibly the most natural for the low energy coupling constant $\alpha_D$. In the next section, we shall use the values $\alpha_D = 0.1, 0.05$ and $0.02$ for numerical estimates.

The expression (2) for the annihilation cross section is proportional to factor $K = (m_{A'}^2 / m_{DM}^2 - 4)^{-2}$. As a consequence in the resonance region $m_{A'} \approx 2m_{DM}$ the annihilation cross section has resonance peak and the dark matter density bound on $\epsilon^2$ is proportional to $K^{-1}$ and for $m_{A'} \approx 2m_{DM}$ the bound on $\epsilon^2$ becomes very weak \[20\] escaping current and future accelerator bounds. It should be mentioned that in general the values of $m_{A'}$ and $m_{DM}$ are arbitrary, so the case $m_{A'} = 2m_{DM}$ could be considered as some fine tuning. So it is natural to require the absence of significant fine tuning. Namely, we shall require that $|m_{A'}^2 / 2m_{DM}^2 - 1| \geq 0.25$, i.e. $m_{A'} \geq 2.5m_{DM}$, and we will use two values $m_{A'} / m_{DM} = 2.5$ and $m_{A'} / m_{DM} = 3$ in our further calculations for comparison. Besides we considered the models with charged scalar, Majorana fermion dark matter and the model with pseudo-Dirac fermion dark matter \[12\]. We have considered the model with pseudo-Dirac fermion DM for the most difficult case of small mass splitting $|\delta| \ll 1^3$. Our calculations are based on the approximate formulae presented in the appendix. The limit $\delta \to 0$ corresponds to the case of Dirac fermion.

\[2\] For smaller values of $\Lambda$ we shall have some charged particles with masses $\leq 1 \text{ TeV}$ that contradicts to the LHC bounds.

\[3\] For $|\delta| \ll 1$ the calculations coincide with the corresponding calculations for Dirac fermion DM.
3 Projected LDM sensitivity of NA64e

The NA64e experiment is designed for a sensitive search for the $A'$ mediator of sub-GeV dark matter particle ($\chi$) production in the missing energy events from the reaction of 100 GeV electron scattering on heavy nuclei:

$$e^- + Z \rightarrow e^- + Z + A'; A' \rightarrow \chi\chi$$

at the CERN SPS \[21, 22\]. After the long shutdown (LS2) stop at CERN the experiment plan to accumulate $\gtrsim 5 \times 10^{12}$ EOT. The NA64e limits on mixing strength $\epsilon$ obtained from the 2016 run with $4.3 \times 10^{10}$ EOT \[18\] and expected after the LS2 period assuming the zero-background case \[10\] are shown in the upper l.h.s. panel in Fig.1. The rest of the plots shows the required number of EOT for the 90% C.L. exclusion of the $A'$ with a given mass $m_{A'}$ in the $(m_{A'}, n_{EOT} \times 10^{-12})$ plane for pseudo-Dirac with $\delta \ll 1$ (red), Majorana (blue), and Scalar (green) dark matter models for $\frac{m_{A'}}{m_{DM}} = 2.5$ (solid), and $= 3$ (dashed), and $\alpha_{DM} = 0.1$ (the upper r.h.s panel), 0.05 (the lower l.h.s. panel) , and 0.02 (the lower r.h.s. panel) As one can see, NA64e is able to exclude the interesting scalar and Majorana LDM scenarios in the $A'$ mass range $1 \text{ MeV} \leq m_{A'} \leq 150 \text{ MeV}$ except the most difficult case of pseudo-Dirac DM with $\alpha_D = 0.1$ and $\alpha_D = 0.05$, $\frac{m_{A'}}{m_{DM}} = 2.5$.

4 NA64$\mu$ projections for the $\gamma - A'$ mixing strength

The NA64$\mu$ experiment \[23, 24\] is proposed to search for dark sector particles weakly coupled to the muon, which could explain the muon $(g-2)_\mu$ anomaly \[25, 26\]. One of the good examples of such a particle, is a new light vector $L_{\mu} - L_{\tau}$ $Z'$ boson \[27 - 34\], which interacts predominantly with the $L_{\mu} - L_{\tau}$ current \[^4\] Interestingly, the $Z'$ could also serve as a new leptophilic mediator of dark force between the ordinary and dark matter, which is charged with respect to $U(1)_{L_{\mu}-L_{\tau}}$, and explain the relic DM abundance \[35, 36\], see also \[37\]. Another interesting possibility involves muon-specific scalar mediator which could connect the visible and dark sectors and also account for the $(g-2)_\mu$ anomaly \[38, 39, 40\].

[^4]: One loop corrections lead to nonzero interactions with electron, and other quarks and leptons \[35\].
Figure 1: The upper l.h.s. panel shows NA64 90% C.L. current (solid) [18] and expected bounds for $5 \times 10^{11}$ and $5 \times 10^{12}$ EOT in the $(m_{A'}, \epsilon^2)$ plane (dashed curves). The rest of the plots shows the required number of EOT for the 90% C.L. exclusion of the $A'$ with a given mass $m_{A'}$ in the $(m_{A'}, n_{EOT} \times 10^{-12})$ plane for pseudo-Dirac with $\delta \ll 1$ (red), Majorana (blue), and Scalar (green) dark matter models for $\frac{m_{A'}}{m_{DM}} = 2.5$ (solid), and $= 3$ (dashed), and $\alpha_{DM} = 0.1$ (the upper r.h.s panel), 0.05 (the lower l.h.s. panel), and 0.02 (the lower r.h.s. panel).

The NA64$\mu$ plans to perform a sensitive search for $L_\mu - L_\tau$ $Z'$ as a mediator of sub-GeV dark matter particle ($\chi$) production in missing energy events from the reaction of 100-160 GeV muon scattering on heavy nuclei:

$$\mu^- + Z \rightarrow \mu^- + Z + Z'; Z' \rightarrow \nu\nu, \chi\chi$$  \hspace{1cm} (6)
at the CERN SPS [23, 24].

In the $A'$ models the interaction of dark photon with the leptons and quarks is given by $L_{A'} = e \epsilon A'_\mu J_{SM}^\mu$. Here, $J_{SM}^\mu$ is the electromagnetic current. So, we see that muons and electrons interact with the dark photon universaly, with the same coupling constant.

Hence, similar to the reaction of Eq. (5), the dark photons will be also produced in the reaction of Eq. (6) with the same experimental signature of the missing energy. For the $A'$

$$\frac{\epsilon m_{A'}}{[\text{GeV}]}$$

BaBar E787, E949

(g-2)
e

NA64e, $4.3 \times 10^{10}$ EOT

NA64e, $5 \times 10^{12}$ EOT

NA64

$\mu$, $5 \times 10^{13}$ MOT

Figure 2: The NA64e 90% C.L. current [18] and expected exclusion bounds obtained with $4.3 \times 10^{10}$ EOT and $5 \times 10^{12}$ EOT, respectively, in the $(m_{A'}, \epsilon)$ plane. The NA64$\mu$ projected bounds calculated for $n_{MOT} = 5 \times 10^{12}$ and $5 \times 10^{13}$ are also shown.

mass region $m_{A'} \gg m_e$, the total cross section of the dark photon electroproduction $eZ \rightarrow eZ A'$ scales as $\sigma^e_{A'} \sim \epsilon_e^2 / m_{A'}^2$. On the other hand, for the dark photon masses, $m_{A'} \lesssim m_\mu$, the similar $\mu Z \rightarrow \mu Z A'$ cross section can be approximated in the bremsstrahlung-like limit as $\sigma^\mu_{A'} \sim \epsilon_\mu^2 / m_{\mu}^2$. Let us now compare expected sensitivities of the $A'$ searches with NA64$e$ and NA64$\mu$ experiments for the same number $\simeq 5 \times 10^{12}$ particles on target. Assuming the same signal efficiency the number of $A'$ produced by the 100 GeV electron and muon beam can approximated, respectively, as follows

$$N^e_{A'} \approx n_{EOT} L^e \sigma^e_{A'}, \quad N^\mu_{A'} \approx n_{MOT} L^\mu \sigma^\mu_{A'},$$

(7)
where $L^e \simeq X_0$ and $L^\mu \simeq 40X_0$ are the typical distances that are passed by an electron and muon, respectively, before producing the $A'$ with the energy $E_{A'} \gtrsim 50$ GeV in the NA64 active Pb target of the total thickness of $\simeq 40$ radiation length ($X_0$) [18, 23]. The detail comparison of the calculated $A'$ sensitivities of NA64e and NA64$\mu$ is shown in Fig. 2 where the 90% C.L. limits on the mixing $\epsilon$ are shown for different number of particles on target for both the NA64e and NA64$\mu$ experiments. The limits were obtained for the background free case by using exact-tree-level (ETL) cross-sections rather than the Weizsacker-Williams (WW) ones calculated for NA64e in Ref. [11], and for the NA64$\mu$ case in this work. The later are shown in Fig. 3 as a function of $E_{A'}/E_\mu$ for the Pb target and mixing value $\epsilon = 1$. One can see that in a wide range of masses, $20$ MeV $\lesssim m_{A'} \lesssim 1$ GeV, the total WW cross-sections are larger by a factor $\simeq 2$ compared to the ETL ones. As the result, the typical limits on $\epsilon$ for the ETL case are worse by about a factor $\simeq 1.4$ compared to the WW case. One can see that, e.g. for $n_{EOT} = n_{MOT} = 5 \cdot 10^{12}$ the sensitivity ob

![Graph](image_url)

Figure 3: Cross-section of Dark Photon production by muons as a function of $x = E_{A'}/E_\mu$ for various masses $m_{A'}$ and $\epsilon = 1$. Solid lines represent ETL cross-sections and dashed lines show the cross-sections calculated in WW approach.

NA64e is enhanced for the mass range $m_e \ll m_{A'} \simeq 100$ MeV. While for the $A'$ masses $m_{A'} \gtrsim 100$ MeV NA64$\mu$ allows to obtain more stringent limits on $\epsilon^2$ compared to NA64e.
5 Combined LTM sensitivity of NA64e and NA64\(\mu\)

The reported previously limits on the \(\gamma - A'\) mixing strength, allow us to set the combined NA64e and NA64\(\mu\) constraints on the LDM models, which are shown in the \((y; m_\chi)\) plane in Fig.4. As discussed in Sec. I, as a result of the \(\gamma - A'\) mixing the cross-section of

\[ \alpha_D = 0.1 \]

\[ \alpha_D = 0.005 \]

\[ m_{A'} = 3 m_\chi \]

The results are also shown in comparison with bounds obtained from the results of the LSND [42, 43, 44], E137 [45], BaBar [46] and MiniBooNE [47] experiments.
DM particle annihilation into SM particles, which determines the relic DM density, is proportional to $\epsilon^2$. Hence using constraints on the cross section of the DM annihilation freeze out (resulting in Eq.(2), and obtained limits on mixing strength of Figs. 1, 2 one can derive constraints in the $(y;m_\chi)$ plane, which can also be used to restrict models predicting existence of LTDM for the masses $m_\chi \lesssim 1$ GeV. Here, the variable $y = \epsilon^2 \alpha_D (m_\chi / m_A')^4$.

These limits obtained from the data sample of the 2016 run [18] and expected from the run after the LS2 are shown in the top panels of Fig. 4 together with combined limits from NA64 and NA64\(\mu\) for $10^{13}$ EOT and $10^{13}$ MOT (dashed green curve) and $10^{13}$ EOT and $2 \times 10^{13}$ MOT (dashed dark blue curve), respectively. The favoured parameter curves for scalar, pseudo-Dirac (with a small splitting) and Majorana scenario of LDM taking into account the observed relic DM density, see e.g. [9, 10]. The limits are calculated by using Eq.(5) from Ref.[18] under the assumption $\alpha_D = 0.1, 0.005$, and $m_A' = 3m_\chi$, here $m_\chi$ stands for the LDM particle’s masses, either scalars or fermions. The plot shows also the comparison of our results with limits from other experiments. The choice of $\alpha_D = 0.1$ is based on arguments for the running of the dark gauge coupling, presented in Sec. I. It should be noted that the $\chi$-yield in the NA64 case scales as $\epsilon^2$, not as $\epsilon^4 \alpha_D$. Therefore, for sufficiently small values of $\alpha_D$ the NA64 limits will be much stronger. This is illustrated in the upper right panel of Fig. 4 where the NA64 limits are shown for $\alpha_D = 0.005$. One can see, that for this, or smaller, values of $\alpha_D$, the direct search for LDM in NA64 excludes models of LDM production via vector mediator for the full mass region $m_\chi \lesssim 0.05$ GeV. While being combined with the NA64\(\mu\) limit, the NA64 results for the coupling $\alpha_D = 0.1$ exclude the models for the entire mass region $m_\chi \lesssim 1$ GeV.

The upper bounds on $\epsilon$ also allow to obtain lower bounds on coupling constant $\alpha_D$ by using a relation among the parameters derived from the requirement of the thermal freeze-out of DM annihilation into visible matter through $\gamma - A'$ kinetic mixing [19]:

$$\alpha_D \simeq 0.02 f \left( \frac{10^{-3}}{\epsilon} \right)^2 \left( \frac{m_A'}{100 \text{ MeV}} \right)^4 \left( \frac{10 \text{ MeV}}{m_\chi} \right)^2$$

which are shown in lower panels of Fig. 4 in the $(\alpha_D;m_\chi)$ plane. For the case of pseudo-Dirac fermions and small splitting, the limits in the lower left panel of Fig. 4 were calcu-
lated by taking the value $f = 0.25$. One can see that for the full mass range $m_\chi \lesssim 0.05$ GeV the obtained combined NA64e and NA64$\mu$ bounds are more stringent than the limits obtained from the results of NA64e allowing to probe almost the full parameter space. The limits for the Majorana case shown in the lower right panel of Fig. 4 were calculated by setting $f = 3$, see [18]. Similar to pseudo-Dirac case the the combined NA64e and NA64$\mu$ limits exclude the full remained parameter area. Note, that new constraints for the large pseudo-Dirac fermion splitting can also be derived. They will be more stringent than for the case of the small splitting and similar to the one obtained for the Majorana case.

6 Conclusions

In this paper we considered the NA64 perspectives for discovery of sub-GeV dark matter by running the experiment in electron and muon modes at the CERN SPS. While with $5 \times 10^{12}$ EOT NA64e is able to test the scalar and Majorana LDM scenarios, the combined NA64e and NA64$\mu$ results with $\simeq 10^{13}$ EOT and a few $10^{13}$ MOT, respectively, will allow to cover almost fully the parameter space of the all most interesting LDM models. This makes NA64e and NA64$\mu$ extremely complementary to each other, as well as to the planned LDMX experiment [48], and greatly increases the NA64 discovery potential of sub-GeV DM.

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Appendix. Basic formulae for DM density

The relic density of DM in the standard scenario is obtained by solving the Boltzmann equation

$$\frac{dn_d}{dt} + 3H(T)n_d = - <\sigma v_{\text{rel}}>(n_d^2 - n_{d,\text{eq}}^2).$$

(9)

Here

$$n_d(T) = \int \frac{d^3p}{2\pi^3} f_d(p, T)$$

(10)
and \( f_d(p, T) \) is the dark matter distribution function.

The dark matter relic density can be numerically estimated as \[\Omega_{DM} h^2 = 0.1 \left( \frac{(n + 1)x_f^{n+1}}{(g_{*s}/g_*^{1/2})} \right) \frac{0.856 \times 10^{-9} \text{GeV}^{-2}}{\sigma_0}, \quad (11)\]

where \( <\sigma v_{\text{rel}} > = \sigma_0 x_f^{-n} \) and

\[ x_f = c - (n + \frac{1}{2}) \ln(c), \quad (12) \]

\[ c = \ln(0.038(n + 1)\frac{g}{\sqrt{g_*}} M_{Pl} m_{DM} \sigma_0). \quad (13) \]

Here \( x_f = \frac{m_{DM}}{f_d} \) and \( n = 0 \) for s-wave and \( n = 1 \) for p-wave annihilation. If DM particles differ from DM antiparticles \( \sigma_o = \sigma_a^2 \).

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