1. INTRODUCTION

The dynamical state of a system of galaxies may be characterized by three intimately related physical quantities: the bolometric X-ray luminosity $L_X$, the average emission-weighted plasma temperature $T$, and the average dark matter velocity dispersion $\sigma$. There is an observed correlation among these parameters, crudely given by $L_X \propto T \propto \sigma^3$, and roughly in agreement with the predictions of physical models. My goal in this Letter is to show that the value of the slope $\alpha_3$ at $z = 0$ can constrain the spectrum of the primordial density fluctuations, $P(k) \propto k^n$.

One powerful tool for linking the X-ray properties of galaxy systems with the cosmological parameters already exists. The cluster temperature function, which is directly related to the cluster mass distribution, can be used at zero redshift to measure the characteristic mass $M_{200}$, or with the substitution of equation (1),

$$Z(z, \Omega) = (1 + z)^3 \frac{\Omega_0}{\Omega(z)},$$

where $\Omega_0$ is the Hubble constant, $\Omega$ is the density parameter, and $z$ is the redshift. The halo’s characteristic circular velocity is $V_{200} = (GM_{200}/r_{200})^{1/2}$, or, with the substitution of equation (1),

$$V_{200} = 10H_0 r_{200} Z^{1/2}.$$
halo mass. This implies that $\delta_2$ should scale with $M$ and $n$ the same way as the average background density scales with $M_*$ and $n$:

$$\delta_2 \propto M_{200}^{-3/2}. \quad (8)$$

The slope of this scaling relation is better than 5% accurate for all $\Omega \leq 1$ cosmologies with either $\Lambda = 0$ or $\Omega + \Lambda = 1$, where $\Lambda$ is the cosmological constant in units of $3 H^2$. With the substitution $\gamma = -(n + 3)/2$ and the use of equations (1), (4), and (6),

$$\frac{c^2}{C(c)} \propto V_{200}^{7/2} Z^{-\gamma/2}. \quad (9)$$

Next, I compute the circular velocity profile of the halo,

$$V_{\text{circ}}(y) = V_{200} \frac{c C(y)}{y C(c)}. \quad (10)$$

Note that the dark matter velocity dispersion profile, $\sigma_{\text{dm}}(y)$, will not in general have the same shape as $V_{\text{circ}}(y)$, contrary to the assumption in similar derivations (e.g., Eke et al. 1998). Rather, the velocity dispersion of the halo is given by the Jeans equation for a spherical, nonrotating system of collisionless particles (Binney & Tremaine 1987):

$$\frac{d (\sigma_r^2 \rho)}{dr} + 2 \beta_1 \rho \sigma_t^2 = - \frac{G M \rho}{r^2}. \quad (11)$$

Here $\sigma_r$ is the radial velocity dispersion, $\beta_1$ is the velocity anisotropy parameter, and $M$ is the mass inside the radius $r$. For systems with isotropic velocity dispersion tensors, $\beta_1 = 0$, and locally $\sigma_{\text{dm}}^2 = 5 \sigma_r^2$. In terms of the previously defined quantities, equation (11) has the solution

$$\sigma_r^2(y) = V_{200}^2 \frac{c}{C(c)} \tilde{\sigma}_r^2(y), \quad (12)$$

$$\tilde{\sigma}_r^2(y) = \frac{1}{\bar{\rho}(y)} \int_y^\infty \frac{\tilde{\rho}(t) C(t)}{t^2} dt. \quad (13)$$

Finally, consider the average dark matter velocity dispersion, $\langle \sigma_{\text{dm}}^2 \rangle$, within the virial radius $r_{200}$:

$$\langle \sigma_{\text{dm}}^2 \rangle = \int_0^{\infty} 3 \sigma_r^2(y) \bar{\rho}(y) y^2 dy \int_0^{\infty} \bar{\rho}(y) y^2 dy \quad (14)$$

$$= V_{200}^2 \frac{c D(c)}{C(c)^2}, \quad (15)$$

$$D(y) = \int_0^y 3 \tilde{\sigma}_r^2(t) \tilde{\rho}(t) t^2 dt. \quad (16)$$

2.2. X-Ray Luminosity

The X-ray luminosity of a ball of plasma is

$$L_X = \int \lambda(T) n_e n_i dV. \quad (17)$$

Here $\lambda(T)$ is the bolometric emissivity as a function of the local temperature $T$, $n_e$ is the electron number density, $n_i$ is the ion number density, and the integral is over the entire volume of the system. Now if (1) $n_e = n_i$, (2) the system is spherically symmetric, and (3) the gas density is related to the dark matter distribution by $\rho_{\text{gas}}(r) = f \rho_{\text{dm}}(r)$, where $f$ is the gas mass fraction, then

$$L_X = 4\pi \int_0^{r_{200}} \lambda(T) \left( \frac{\rho_{\text{dm}}}{\mu m_p} \right)^2 r^2 dr, \quad (18)$$

where $\mu \equiv \rho_{\text{gas}}/(n_e m_p)$ is the mean molecular weight. Substitution of equation (5) yields

$$L_X = 4\pi \left( \frac{f c}{\mu m_p} \right)^2 \frac{c^3}{G} V_{200}^3 \int_0^{r_{200}} \lambda(T) \bar{\rho}(y) y^2 dy. \quad (19)$$

Now eliminate $r_{200}, \delta_2$, and $\rho_{\text{gas}}$ using equations (2), (4), and (6):

$$L_X = \frac{5 H_0 Z^{1/2}}{2 G^{1/2}} \left( \frac{f c}{\mu m_p} \right)^2 \frac{c^3}{G} V_{200}^3 \int_0^{r_{200}} \lambda(T) \bar{\rho}(y) y^2 dy. \quad (20)$$

Next, note that thermal bremsstrahlung is the dominant cooling process for rich clusters of galaxies; hence $\lambda(T) \propto T^{1/2}$. Then, if the gas is in local hydrostatic equilibrium with the dark matter, $\bar{T}(r) \propto \sigma_{\text{dm}}(r)^2$, and hence $\lambda(T, r) \propto \bar{T}(r)$. Leaving out the constants and substituting equation (12) yields

$$L_X \propto f^2 Z^{1/2} \frac{c^{3/2}}{G^{1/2}} V_{200}^4 \int_0^{r_{200}} 3 \tilde{\sigma}_r^2(y) \bar{\rho}(y) y^2 dy. \quad (21)$$

The integral on the right-hand side is just a number and may be dropped.

Now one must replace $V_{200}$, which is not observable, with the projected galaxy velocity dispersion $\sigma_p$, which can be determined from optical surveys. In the cores of relaxed clusters $\sigma_r^2$ is proportional to $\langle \sigma_{\text{dm}}^2 \rangle$, the average dark matter velocity dispersion (eq. [14]). Then

$$L_X \propto f^2 Z^{1/2} c^{3/2} \frac{C(c)^{3/2}}{D(c)} \sigma_r^2. \quad (22)$$

If $f$ and $c$ remain constant, the above expression reduces to the traditional $L_X \propto \sigma^4$ scaling law from simpler, dimensional arguments (e.g., Quintana & Melnick 1982). However, equation (9) tells us that $c$ is a function of the velocity dispersion and the slope of the primordial spectrum. In fact, if $\bar{\rho}(y)$ is specified, equations (7), (9), (15), and (16) may be used to eliminate $V_{200}, c, C(c),$ and $D(c)$, and one may thus obtain the dependence of $L_X$ on $\sigma_p$ and $n$. Specifically, suppose that within the range of interest for $c, C(c)$, and $D(c)$ exhibit a power-law
behavior of the form $C(c) \propto c^{p}$ and $D(c) \propto c^{q}$. Then

$$L_X \propto f^2 Z^{12+x/6} \sigma_p^{4-x},$$

(23)

where

$$\xi = \frac{3(3 + n)(3 + 3p - 4q)}{3 + 14p - 9q + n(6p - 3 - 3q)}.$$ 

(24)

### 2.3. Model Dependency

I consider dark matter density profiles of the form $\rho(y) = y^{\gamma-1}(1 + y)^{-d}$. Thus each profile’s shape may be specified by a set of three numbers, $(a, b, d)$. Some common profiles and their properties are listed in Table 1.

Once the set $(a, b, d)$ is specified, $C(c)$ and $D(c)$ are readily computable. The relevant range of $c$ for systems of galaxies comes from measurements of surface density profiles and of $r_{200}$ in clusters and groups of galaxies (Carlberg et al. 1997; Mahdavi et al. 1999). In these works, halos with masses in the range $10^{14}$ to $10^{16} M_\odot$ have concentrations $c \approx 2.5$ to $9.5$, in good agreement with N-body simulations (e.g., NFW). For all sets $(a, b, d)$ in Table 1, the power-law approximations $C(c) \propto c^{p}$ and $D(c) \propto c^{q}$ are better than 8% accurate everywhere within $c = 2.5$ to $9.5$.

Figure 1 shows $\xi(n)$ from equation (24) for various profiles. In all cases, $\xi(n)$ is positive and approaches zero as $n \rightarrow -3$. As $n \rightarrow 0$, the models all predict a significant flattening of the $L_X-\sigma$ relation. This is understandable through equation (8): the characteristic density $\delta_c$ is highly anticorrelated with halo mass in the $n \approx 0$ universes, and hence the emission measure increases quite slowly or not at all with mass. As $n \rightarrow -3$, $\delta_c$ becomes nearly independent of the halo mass, and therefore $L_X$ increases rapidly with the velocity dispersion.

### 3. Application

For low-redshift clusters of galaxies, $Z(z, \Omega) \approx 1$ with high accuracy. The dependence of the gas mass fraction $f$ on $(T)$ and hence $\sigma_g$ is poorly determined. Some, using various analysis, claim it should increase slightly with $(T)$ (e.g., David, Jones, & Forman 1995); others say that accounting for cooling flows should cause it to decrease slightly with $(T)$ (e.g., Allen & Fabian 1998). Mohr, Mathiesen, & Evrard (1999), in their detailed study of clusters with ROSAT pointings, find that within 1 Mpc $f$ is nearly independent of $(T)$. Because of the range of contrasting findings, I adopt $f \propto (T)^{0.0 \pm 0.2} \propto \sigma_p^{0.0 \pm 0.4}$ at the 68% confidence level.

There is more agreement among observers regarding the empirical value of $\alpha_s$: Quintana & Melnick (1982) have $\alpha_s = 4.0 \pm 0.7$ for data from the Einstein satellite; theirs are 2–10 keV luminosities, which scale similarly to bolometric luminosities for rich clusters. More recently, Mulchaey & Zabludoff (1998, hereafter MZ98) combined ROSAT observations of groups and clusters with deep optical spectroscopy to obtain $\alpha_s = 4.29 \pm 0.37$ for bolometric luminosities. I adopt the average value, $\alpha_s = 4.15 \pm 0.4$.

Assuming normally distributed errors, the adopted values of $f$ and $\alpha_s$ constrain $\xi$ to be less than 1.0 at the one-sided 90% confidence level, or less than 1.93 at the one-sided 99% confidence level. Figure 1 shows the $\xi = 1.0, 1.93$ boundaries. To accommodate all the models I consider in this scenario, $n$ must be less than $-2.0$ at the 90% confidence level and less than $-1.1$ at the 99% confidence level.

At least two systematic effects could bias these results. Preheating of the plasma in $k(T) \approx 4$ keV clusters (Ponman, Cannon, & Navarro 1999) might suppress their luminosities, while leaving those of keV clusters unchanged. One can avoid this bias by ignoring all clusters with $k(T) < 4$ keV. I find that removing these clusters, which make up $\approx 25\%$ of the MZ98 sample, does not significantly affect the slope of the observed $L_X-\sigma$ relation.

Also, cooling flows might affect the luminosities. Whereas Markevitch (1998) finds that the slope of the $L_X-\langle T \rangle$ relation does not change as a result of removing cooling flows, Allen & Fabian (1998) find that $\alpha_s = 3.1 \pm 0.6$ changes to $\alpha_s = 2.3 \pm 0.4$ after including cooling flows. The Markevitch (1998) method, which removes the cooling flows altogether, is more appropriate for probing the gravitational potential than the Allen & Fabian (1998) method, which includes the luminosity of the cooling component.

To constrain the effect of cooling flows on $\alpha_s$, I conduct the following test. Of the MZ98 clusters, 34% are also contained in Markevitch (1998). I fit $\alpha_s$ for just these clusters, using the $L_X$ which excludes the cooling component. I find that the slope changes to $\alpha_s = 3.85 \pm 0.3$; with this, the upper limits on $n$ become $n < -1.7$ and $n < -0.9$ at the one-sided 90% and 99%

### Table 1: Density Profiles Considered

| Model          | $a$, $b$, $d$ | $p$  | $q$  |
|----------------|---------------|------|------|
| Hernquist      | 1, 1, 3       | 0.35 | 0.16 |
| NFW            | 1, 1, 2       | 0.74 | 0.58 |
| Plummer sphere | 0, 2, 5/2     | 0.14 | 0.05 |
| King           | 0, 2, 3/2     | 0.74 | 0.59 |
| Jaffe          | 2, 1, 2       | 0.17 | 0.03 |

Note.—Hernquist: Hernquist (1990); NFW: Navarro et al. 1997; King: King (1962); Jaffe: Jaffe (1983).

### Figure 1

The correction to the slope of the $L_X-\sigma$ relation, $\xi(n)$, for various density profiles. The dashed lines represent the one-sided 90% and 99% upper bounds on $\xi$ from available observations. The arrows indicate the same confidence intervals on $n$. 
confidence levels. The chief result of this Letter—that if $n$ were much greater than $-1$, $\alpha$, should be $\approx 2$ instead of the observed value, $\approx 4$—is therefore not affected by the 10% correction due to cooling flows.

4. CONCLUSION

If the characteristic densities of clusters of galaxies trace the background density of the universe at the time of each cluster’s formation, there should be a relatively simple relationship between the X-ray luminosity $L_X$, the observed velocity dispersion $\sigma_v$, and the slope of the primordial power spectrum $n$. This relationship, given by equations (23) and (24), depends slightly on the density profile of the clustered dark matter, but should not significantly depend on $\Omega$ or $\Lambda$ when applied to low-redshift clusters of galaxies. For a wide range of assumed density profiles, the observations imply $n < -2.0$ and $n < -1.1$ at the one-sided 90% and 99% confidence levels, respectively. This is consistent with the bounds from the cluster temperature function, which give $n$ between $-2.3$ and $-1.15$. Improving this constraint depends largely on a better understanding of preheating, cooling flows, and the variation of the gas mass fraction with $\sigma_v$.

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