Measurement of Decay Parameters for $\Xi^- \to \Lambda \pi^-$ Decay

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Abstract

Based on 1.35 million polarized $\Xi^-$ events, we measure the parameter $\phi_{\Xi}$ to be $-1.61^\circ \pm 2.66^\circ \pm 0.37^\circ$ for the $\Xi^- \to \Lambda \pi^-$ decay. New results for the parameters $\beta_{\Xi}$ and $\gamma_{\Xi}$ are also presented. Assuming that the CP-violating phase-shift difference is negligible, we deduce the strong phase-shift difference between the P-wave and S-wave amplitudes of the $\Lambda \pi$ final state to be $3.17^\circ \pm 5.28^\circ \pm 0.73^\circ$. This strong phase-shift difference reduces the theoretical uncertainty in estimating the level of CP violation in $\Xi$-hyperon decay.
Breakdown of \( CP \) invariance is well known in the weak decays of the neutral \( K \) meson [1,2] and has been recently established for the \( B \) meson as well [3]. A deeper understanding of this effect, which is evidence of a subtle difference between the dynamics of particles and antiparticles, is one of the central issues of current-day particle physics. Complementary to \( CP \) violation in the mesons is a similar effect in the nonleptonic hyperon decays. This has been the subject of theoretical discussion [4,5] and ongoing experimental searches [6–8]. Current models generally predict the breakdown of \( CP \) symmetry in strange-baryon decays due to the different dynamics in the decay of a hyperon and its antiparticle. In general, observing such a \( CP \)-odd effect requires a strong phase-shift difference, which is the subject of this paper.

For the \( \Xi^- \to \Lambda \pi^- \) decay, the orbital angular momentum of the \( \Lambda \pi^- \) must be either \( L = 0 \) (S-wave) or \( L = 1 \) (P-wave) [9]. We write the S- and P-wave amplitudes as [4,10]:

\[
S = |S| \exp[i(\delta_S + \eta_S)], \quad P = |P| \exp[i(\delta_P + \eta_P)],
\]

(1)

where \( \delta_S, \delta_P \) are the strong rescattering phases and \( \eta_S, \eta_P \) are weak \( CP \)-violating phases.

The \( CP \) asymmetry of the \( \Xi \) decay [4,5],

\[
A_{\Xi} = \frac{\alpha_{\Xi} + \alpha_{\Xi}^-}{\alpha_{\Xi} - \alpha_{\Xi}^-},
\]

(2)

with \( \alpha_{\Xi} (\alpha_{\Xi}^-) \) being a Lee-Yang parameter of the hyperon (antihyperon) decay [9], is given by:

\[
A_{\Xi} \simeq -\tan(D) \sin(d_{CP}),
\]

(3)

where \( D = \delta_P - \delta_S \) and \( d_{CP} = \eta_P - \eta_S \) are phase-shift differences. Theoretical estimates of \( D \) vary between \(-3^\circ\) and \(16^\circ\) [10,11]. The model-dependent \( CP \)-violating phase shifts are generally estimated to be a couple of orders of magnitude smaller than the strong phase shifts [5]. \( D \) can thus be deduced from the decay parameter \( \beta_{\Xi} \) expressed as

\[
\beta_{\Xi} = \alpha_{\Xi} \tan(D + d_{CP}) \simeq \alpha_{\Xi} \tan(D).
\]

(4)

The parameter \( \beta_{\Xi} \) is conveniently determined in terms of a phase \( \phi_{\Xi} = \tan^{-1}(\beta_{\Xi}/\gamma_{\Xi}) \), where \( \gamma_{\Xi} \) is another decay parameter of the decay such that \( \alpha_{\Xi}^2 + \beta_{\Xi}^2 + \gamma_{\Xi}^2 = 1 \). The world average is \( \phi_{\Xi} = 4^\circ \pm 4^\circ \), taken over several measurements with the largest sample using 20,865 events [12]. In this Letter we report a new measurement of \( \phi_{\Xi} \) from a data set of 1.35 million polarized \( \Xi^- \to \Lambda \pi^- \) decays.

Our data come from Experiment 756 at Fermi National Accelerator Laboratory [13–16]. Polarized \( \Xi^- \) hyperons with transverse momenta \( p_T \) spanning 0.5 to 1.5 GeV/c and momentum fraction \( x_F \) from 0.3 to 0.7 were produced by the collision of unpolarized 800-GeV/c protons incident upon a beryllium target at an angle with respect to the vertical axis. The secondary beam was momentum- and sign-selected by a curved collimator inside a dipole magnet. The current in the magnet was set to yield a field integral \( \int Bdl \) of \( 15.30 \pm 0.15 \) T-m, as measured with a Hall probe, during collection of the data used in the present analysis.

The momenta of the proton and the pions from the decay sequence \( \Xi^- \to \Lambda \pi^-, \Lambda \to p\pi^- \) were measured with eight planes of silicon strip detectors, nine multiwire proportional chambers, and two dipole analyzing magnets that deflected charged particles in the horizontal
plane. The polarity of these magnets could be reversed by switching the direction of the applied current. The $\Xi^-$ triggers and veto scintillation counters have been described in [6]. Data were taken for two sets of production angles averaging $+2.4$ and $-2.4$ mrad, respectively, and the analyzing magnet currents were set to $\pm 2500$A. The $\Xi^-$ polarization and magnetic moment from these data have been reported [15,16].

The events were analyzed off-line using a reconstruction program that determined tracks and kinematic variables from the chamber hits. Events were required to satisfy the three-track, two-vertex topology corresponding to a $\Xi^-$ decay-sequence hypothesis. The geometric $\chi^2$ for the topological fit was required to be less than 100 for a mean of 30 degrees of freedom. The proton and pion from $\Lambda$ decay were required to have a $p\pi$ invariant mass within 3.5 standard deviations (8 MeV/$c^2$) of the $\Lambda$-decay peak at 1.116 GeV/$c^2$, and the $\Lambda\pi$ invariant mass was required to be within 5 standard deviations (14 MeV/$c^2$) of the $\Xi^-$-decay peak at 1.321 GeV/$c^2$. The momenta of the reconstructed $\Xi^-$ candidates were required to be between 240 and 450 GeV/$c$. The $\Xi^-$ momentum was required to trace back to within 0.63 cm of the center of the beryllium target. The $\Lambda$-decay vertex was required to be downstream of the $\Xi^-$-decay vertex, and both vertices were required to be in a fiducial region 0.25 m downstream of the collimator exit and 0.31 m upstream of the multiplicity counter located at 23.31 m from the collimator exit.

The decay parameters are related to the experimental observables through the polarization and angular distribution of the daughter baryon. For $\Xi^- \rightarrow \Lambda\pi^-$, we have [9]:

$$\vec{P}_\Lambda = \frac{(\alpha_\Xi + \vec{P}_\Xi \cdot \hat{\Lambda})\hat{\Lambda} + \beta_\Xi(\vec{P}_\Xi \times \hat{\Lambda}) + \gamma_\Xi \hat{\Lambda} \times (\vec{P}_\Xi \times \hat{\Lambda})}{1 + \alpha_\Xi \vec{P}_\Xi \cdot \hat{\Lambda}},$$

(5)

where $\vec{P}_\Lambda$ is the polarization of the $\Lambda$ hyperon in its rest frame, $\hat{\Lambda}$ is the momentum direction of the $\Lambda$ in the $\Xi^-$ rest frame, and $\vec{P}_\Xi$ is the $\Xi^-$ polarization in the $\Xi^-$ frame. The helicity-frame axes, specified event-by-event in the $\Xi^-$ rest frame, are defined as

$$\hat{X} = \frac{\vec{P}_\Xi \times \hat{\Lambda}}{|\vec{P}_\Xi \times \hat{\Lambda}|}, \quad \hat{Z} = \hat{\Lambda}, \quad \hat{Y} = \hat{Z} \times \hat{X}.$$  

(6)

Then, for the $\Lambda \rightarrow p\pi^-$ decay, the angular distribution of the proton in the $\Lambda$ rest frame can be projected onto the helicity-frame axes to give [17]

$$\frac{dn}{d \cos \theta_{pZ}} = \frac{1}{2}(1 + \alpha_\Lambda \alpha_\Xi \cos \theta_{pZ}),$$  

(7)

$$\frac{dn}{d \cos \theta_{pX}} = \frac{1}{2}(1 + \alpha_\Lambda \beta_\Xi \cos \theta_{pX}),$$  

(8)

$$\frac{dn}{d \cos \theta_{pY}} = \frac{1}{2}(1 + \alpha_\Lambda \gamma_\Xi \cos \theta_{pY}),$$  

(9)

where $\theta_{pi}$ is the angle between the proton momentum in the $\Lambda$ rest frame and the $i$th helicity-frame axis. The ratio of the slopes of the $\cos \theta_{pX}$ and $\cos \theta_{pY}$ distributions provides a measurement of $\tan \phi_\Xi = \beta_\Xi / \gamma_\Xi$. 

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Conservation of parity in the strong interactions dictates that any $\Xi^-$ polarization be normal to the production plane at the target. The precession angle $\Phi$ of this polarization relative to the $\Xi^-$ momentum at the collimator exit, after the $\Xi^-$ hyperons pass through the vertically directed magnetic field in the collimator, is given by

$$\Phi = \frac{q}{\beta m_\Xi c^2} \left\{ -\frac{\mu_\Xi m_\Xi}{\mu_N m_p} - 1 \right\} \int B dl ,$$

(10)

where $q$, $\mu_\Xi$ and $m_\Xi$ are the electric charge, magnetic moment and mass of the $\Xi^-$ respectively, $\beta \approx 1$, $\mu_N$ is the nuclear magneton, and $m_p$ is the proton mass. As reported in [15], the polarization of $\Xi^-$ for this data set was confined in the horizontal plane and approximately 10% in magnitude. We used $\mu_\Xi = -0.6505 \pm 0.0025 \mu_N$ [18] to determine $\Phi = -16.68^\circ \pm 0.86^\circ$ and hence the orientation of the $\Xi^-$ polarization in the spectrometer. The helicity-frame axes defined in Eq. 6 were calculated from the reconstructed $\Lambda$ momentum.

The slopes of the $\cos \theta_{pX}$ and $\cos \theta_{pY}$ distributions in Eqs. 8 and 9 were measured using the Hybrid Monte Carlo (HMC) method [19] which factored out the acceptances. Up to 200 HMC events were generated per real event for each distribution. These HMC events were uniformly distributed in $\cos \theta_{pX}$ ($\cos \theta_{pY}$), with the rest of the kinematic quantities such as decay vertices and momentum of the $\Lambda$ taken from the associated real event. The event was included in the measurement when ten of its associated HMC events satisfied all of the requirements used to simulate the triggers, the geometry and inefficiencies of the spectrometer. A $\chi^2$ minimization based on the comparison of weighted HMC data with real events was used to determine the slope of each $\cos \theta$ distribution.

The data were analyzed separately as four streams: data taken at positive and negative production angles for each polarity of the analyzing magnets. The measured slope for a given distribution is comprised of the true slope plus a bias term [13]. The bias, resulting from imperfections in the experimental apparatus and reconstruction effects not fully simulated in the analysis, is not sensitive to the reversal of the production angle. The polarization, however, changes sign when the production angle is reversed. The helicity-frame axes in Eq. 6 transform as $\hat{X} \to -\hat{X}$, $\hat{Y} \to -\hat{Y}$, and $\hat{Z} \to \hat{Z}$ when the polarization changes sign. A bias term in the $X$-direction ($B_X$) will thus reverse sign in the transformed helicity frame. If the slope measured for the positive (negative) production angle is $S_+$ ($S_-$), the true slope and bias are extracted using

$$\frac{\pi}{4} \alpha_\Lambda \beta_\Xi P_\Xi = \frac{S_+ + S_-}{2}, \quad B_X = \frac{S_+ - S_-}{2}.$$  

(11)

Similar expressions can be written for $\alpha_\Lambda \gamma_\Xi P_\Xi$ and the $Y$ bias ($B_Y$). Fig. 1 shows the $\cos \theta_{pX}$ and $\cos \theta_{pY}$ distributions for each stream and the corresponding weighted HMC fit. The HMC analysis was repeated with 26 different random number seeds and the results were averaged. Table I shows the extracted slopes and bias terms of the $\cos \theta_{pX}$ and $\cos \theta_{pY}$ distributions for each polarity of the analyzing magnet.

To show that we were able to determine the correct axes, we measured the slope $\alpha_\Lambda \alpha_\Xi$ of the distribution in Eq. 7. Our result, which does not include systematic studies, was $\alpha_\Lambda \alpha_\Xi = -0.305 \pm 0.002$, consistent with the world average [18]. As a further check, we measured the magnitude of the $\Xi^-$ polarization by projecting the angular distribution of the proton onto the laboratory axes and summing the components (a technique discussed
in [13,15]). We then calculated $\frac{2}{4} \alpha_\Lambda \gamma_\Xi P_\Xi = 0.043 \pm 0.003$, in agreement with the results in Table I and in [15].

By taking the ratio of the measured values of $\frac{2}{4} \alpha_\Lambda \beta_\Xi P_\Xi$ and $\frac{2}{4} \alpha_\Lambda \gamma_\Xi P_\Xi$, the phase $\phi_\Xi$ was determined to be $-1.28^\circ \pm 3.86^\circ$ for the $+2500A$ data and $-1.93^\circ \pm 3.68^\circ$ for the $-2500A$ data. The good agreement for two settings of the analyzing magnets provides a systematic check of the analysis method.

The measured $\phi_\Xi$ displays no significant dependence on either the $\Xi^-$ momentum, $p_\Xi$, or the transverse momentum, $p_T$ (Fig. 2). A study of the biases $B_X$ and $B_Y$ as a function of the $\Xi^-$ momentum showed no significant dependence [20]. Variation of some of the data-selection criteria allowed us to study systematic effects due to background, poorly measured events, goodness of the track-fitting, and resolution at the target. The systematic effect of the random number seed in the HMC program was investigated. In addition, we studied the effect of changing the precession angle on the measured $\phi_\Xi$ by varying the value of $\Phi$ by one standard deviation. Table II summarizes our estimates of the systematic uncertainties.

By averaging the measurements for the two settings of the analyzing magnet, and adding the systematic errors in quadrature, we obtain our final result:

$$\phi_\Xi = -1.61^\circ \pm 2.66^\circ \pm 0.37^\circ,$$

where the first error is statistical and the second systematic. Our measurement is consistent with the current world average of $4^\circ \pm 4^\circ$ [18] and with zero. Using

$$\beta_\Xi = (1 - \alpha_\Xi^2)\frac{1}{2} \sin \phi_\Xi, \quad (12)$$
$$\gamma_\Xi = (1 - \alpha_\Xi^2)\frac{1}{2} \cos \phi_\Xi, \quad (13)$$

where $\alpha_\Xi = -0.458 \pm 0.012$ [18], we calculate

$$\beta_\Xi = -0.025 \pm 0.042 \pm 0.006,$$
$$\gamma_\Xi = +0.889 \pm 0.001 \pm 0.007.$$

Both statistical and systematic errors on $\beta_\Xi$ are dominated by the uncertainties in our measurement of $\phi_\Xi$, whereas the systematic error on $\gamma_\Xi$ is due to the uncertainty in $\alpha_\Xi$.

Using Eq. 4, the phase-shift difference in the $\Xi^-$ decay process is also deduced:

$$D + d_{CP} = 3.17^\circ \pm 5.28^\circ \pm 0.73^\circ.$$  

This measurement is consistent with zero and indicates that the strong phase-shift difference of the final state in $\Xi^- \rightarrow \Lambda \pi^-$ decay is small, in agreement with recent calculations [10] but disagreeing with that of [11].

In conclusion, we have measured the parameter $\phi_\Xi$ using 1.35 million $\Xi^- \rightarrow \Lambda \pi^-$ decays. Our result has a precision that is $\approx 1.5$ times better than the world average and $\approx 3.3$ times better than the previous best single measurement [21]. With this result, we obtain new values for $\beta_\Xi$ and $\gamma_\Xi$. The strong phase-shift difference deduced from this measurement is consistent with zero and thus imposes a limit on the level of $CP$ violation in $\Xi^- \rightarrow \Lambda \pi^-$ hyperon decay.

We would like to acknowledge support by the U.S. Department of Energy. K.B.L. was also partially supported by the Miller Institute. The excellent assistance of the Fermilab staff was essential for the completion of the experiment.
### TABLE I. Extracted slopes and biases

| Anal. Magnet Current (A) | No. of Magnet Events | Mean Ξ Momentum (GeV/c) | $\frac{\pi}{4} \alpha \beta_\Xi P_\Xi$ | $B_X$ |
|------------------------|----------------------|-------------------------|---------------------------------|-------|
| +2500                  | 646774               | 309                     | $-0.0009 \pm 0.0028$            | $+0.0042 \pm 0.0028$ |
| -2500                  | 696337               | 326                     | $-0.0014 \pm 0.0026$            | $-0.0015 \pm 0.0026$ |

| Anal. Magnet Current (A) | No. of Magnet Events | Mean Ξ Momentum (GeV/c) | $\frac{\pi}{4} \alpha \gamma_\Xi P_\Xi$ | $B_Y$ |
|------------------------|----------------------|-------------------------|---------------------------------|-------|
| +2500                  | 647029               | 309                     | $+0.0410 \pm 0.0024$            | $+0.0037 \pm 0.0024$ |
| -2500                  | 696633               | 326                     | $+0.0400 \pm 0.0022$            | $+0.0012 \pm 0.0022$ |

### TABLE II. Systematic errors for the measurement of $\phi_\Xi$

| Study                              | Estimated Maximum Systematic Error |
|------------------------------------|-----------------------------------|
| Momentum Dependence                | $< 0.1^\circ$                      |
| $p_T$ Dependence                   | $< 0.1^\circ$                      |
| Selection Criteria                 | $0.25^\circ$                       |
| Random number seed                 | $0.25^\circ$                       |
| Precession Angle                   | $0.11^\circ$                       |
| Total                              | $0.37^\circ$                       |
FIG. 1. $\cos \theta_{px}$ and $\cos \theta_{py}$ data distributions (points) with corresponding weighted HMC events (histograms) for a) the +2500A and b) the -2500A current setting of the analyzing magnets.
FIG. 2. $\phi_\Xi$ as a function of $p_\Xi$ (left) and $p_T$ (right) for +2500A (top) and -2500A (bottom) settings of the analyzing magnets. Solid and dotted lines indicate the central measurement and its statistical error.
REFERENCES

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[1] J.H. Christenson et al., Phys. Rev. Lett. 13, 138 (1964).
[2] A. Alavi-Harati et al., Phys. Rev. Lett. 83, 2128 (1999); V. Fanti et al., Phys. Lett. B 465, 335 (1999).
[3] A. Abashian et al., Phys. Rev. Lett. 86, 2509 (2001); B. Aubert et al., Phys. Rev. Lett. 86, 2515 (2001).
[4] X.-G. He et al., Phys. Rev. D 61, 071701 (2000); J.F. Donoghue, X.-G. He, S. Pakvasa, Phys. Rev. D 34, 833 (1986).
[5] S. Pakvasa, preprint hep-ph/9910232 (1999); J. Tandean and G. Valencia, preprint hep-ph/0211165 (2002).
[6] K.B. Luk et al., Phys. Rev. Lett. 85, 4860 (2000).
[7] P. Chauvat et al., Phys. Lett. B 163, 273 (1985); M.H. Tixier et al., Phys. Lett. 212, 523 (1988); P.D. Barnes et al., Phys. Rev. C 54, 1877 (1996).
[8] C. Dukes et al., Nucl. Phys. Proc. Suppl. 75B, 281 (1999).
[9] T.D. Lee and C.N. Yang, Phys. Rev. 108, 1645 (1957).
[10] M. Lu, M.B. Wise, M.J. Savage, Phys. Lett. B 337, 134 (1994); A. Datta and S. Pakvasa, Phys. Lett. B 344, 430 (1995); A.N. Kamal, Phys. Rev. D 58, 077501 (1998); J. Tandean et al., Phys. Rev. D 64, 014005 (2001); U.G. Meissner and J.A. Oller, Phys. Rev. D 64, 014006 (2001).
[11] R. Nath and A. Kumar, Nuovo Cimento 36, 669 (1965).
[12] J.R. Bensinger et al., Nucl. Phys. B 252, 561 (1985).
[13] P.M. Ho et al., Phys. Rev. D 44, 3402 (1991); P.M. Ho, Ph.D. Thesis, University of Michigan (1991).
[14] H.T. Diehl et al., Phys. Rev. Lett. 67, 804 (1991); H.T. Diehl, Ph.D. Thesis, Rutgers University (1990).
[15] J. Duryea et al., Phys. Rev. Lett. 67, 1193 (1991); J. Duryea, Ph.D. Thesis, University of Minnesota (1990).
[16] J. Duryea et al., Phys. Rev. Lett. 68, 768 (1992).
[17] R.L. Cool et al., Phys. Rev. D 10, 792 (1974).
[18] Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).
[19] G. Bunce, Nuc. Instr. and Meth. 172, 553 (1980).
[20] A. Chakravorty, Ph.D. Thesis, Illinois Institute of Technology (2000).
[21] C. Baltay et al., Phys. Rev. D 9, 49 (1974).