Betz, Volker; Taggi, Lorenzo
Scaling limit of ballistic self-avoiding walk interacting with spatial random permutations.
(English) Zbl 1434.82014
Electron. J. Probab. 24, Paper No. 74, 37 p. (2019).

Summary: We consider nearest neighbour spatial random permutations on $\mathbb{Z}^d$. In this case, the energy of the system is proportional to the sum of all cycle lengths, and the system can be interpreted as an ensemble of edge-weighted, mutually self-avoiding loops. The constant of proportionality, $\alpha$, is the order parameter of the model. Our first result is that in a parameter regime of edge weights where it is known that a single self-avoiding loop is weakly space filling, long cycles of spatial random permutations are still exponentially unlikely. For our second result, we embed a self-avoiding walk into a background of spatial random permutations, and condition it to cover a macroscopic distance. For large values of $\alpha$ (where long cycles are very unlikely) we show that this walk collapses to a straight line in the scaling limit, and give bounds on the fluctuations that are almost sufficient for diffusive scaling. For proving our results, we develop the concepts of spatial strong Markov property and iterative sampling for spatial random permutations, which may be of independent interest. Among other things, we use them to show exponential decay of correlations for large values of $\alpha$ in great generality.

MSC:
82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
82B41 Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics
82C41 Dynamics of random walks, random surfaces, lattice animals, etc. in time-dependent statistical mechanics
82B26 Phase transitions (general) in equilibrium statistical mechanics

Keywords:
self-avoiding walk; random spatial permutations

Full Text: DOI arXiv Euclid

References:
[1] N. Berestycki: Emergence of giant cycles and slowdown transition in random transpositions and k-cycles. Electron. J. Probab. 16, 152-173 (2011). · Zbl 1228.60079
[2] V. Betz: Random permutations of a regular lattice. J. Stat. Phys., Volume 155, Issue 6, pp 1222-1248, (2014). · Zbl 1302.82038 · doi:10.1007/s10955-014-0945-7
[3] V. Betz, D. Ueltschi, Y. Velenik: Random permutations with cycle weights. Ann. Appl. Probab. Volume 21, pp 312-331, (2011). · Zbl 1226.82003 · doi:10.1214/10-AAP697
[4] V. Betz, D. Ueltschi: Spatial random permutations and Poisson-Dirichlet law of cycle lengths. Electron. J. Probab. Volume 16, Issue 41, (2011). · Zbl 1231.60108 · doi:10.1214/EJP.v16-901
[5] V. Betz, D. Ueltschi: Spatial random permutations and infinite cycles. Comm. Math. Phys. 285, 469-501 (2009) · Zbl 1155.82022 · doi:10.1007/s00220-008-0584-4
[6] V. Betz, D. Ueltschi: Spatial random permutations with small cycle weights. Probab. Th. Rel. Fields 149, 191-222 (2011). · Zbl 1226.82003 · doi:10.1007/s00440-009-0248-0
[7] V. Betz, D. Ueltschi: Critical temperature of dilute Bose gases. Phys. Rev. A 81, 023611 (2010).
[8] L. V. Bogachev, D. Zeindler: Asymptotic statistics of cycles in surrogate-spatial permutations. Comm. Math. Phys. 334, 1, p. 39-116. · Zbl 1318.82016 · doi:10.1007/s00220-014-2110-4
[9] M. Campanino, D. Ioffe, and Y. Velenik: Fluctuation theory of connectivities for subcritical random cluster models Ann. Probab., Vol. 36, No. 4 (2008), 1287-1321 · Zbl 1160.60026 · doi:10.1214/07-AOP359
[10] M. Campanino, D. Ioffe, and Y. Velenik: Ornstein-Zernike theory for finite range Ising models above $T_c$. Probability Theory and Related Fields 125(3):305-349 (2003) · Zbl 1032.60093 · doi:10.1007/s00440-002-0229-4
[11] M. Campanino and D. Ioffe: Ornstein-Zernike theory for the Bernoulli bond percolation on $\mathbb{Z}$. Ann. Probab.,
A. Glazman, I. Manolescu: Uniform Lipschitz functions on the triangular lattice have logarithmic variations. ArXiv: 1810.05592.

M. Draief and L. Massouli: Epidemics and Rumours in Complex Networks. London Mathematical Society Lecture Note Series: doi:10.1007/s10955-014-1153-1

D. Ioffe: Multidimensional random polymers: a renewal approach, in Random Walks, random fields, and disordered systems, Zbl 1130.60302

G. Slade: The self-avoiding walk: a brief survey. in Surveys in Stochastic Processes, J. Blath, P. Imkeller, S. Roelly (eds.), Zbl 1359.82007

D. Gandolfo, J. Ruiz, D. Ueltschi: On a model of random cycles. J. Statist. Phys. 129, 663-676 (2007).

Zbl 1338.60215

Zbl 0071.44701

Zbl 1115.60039

D. Ioffe, S. Schlosman, F. Toninelli: Interacting versus entropic repulsion for low temperature Ising polymers, J. Stat. Phys.

Y. Kovchegov: Brownian bridge in percolation, self-avoiding walks and related processes. Ph.D thesis, Department of Mathematics, Stanford University, 2002.

B. Nienhuis: Exact Critical Point and Critical Exponents of $O(n)$ Models in Two Dimensions. Phys. Rev. Lett., Volume 49, Number 15, (1982).

N. Madras, G. Slade: The Self-Avoiding Walk. Birkhäuser (2013), reprint of the 1996 Edition. DOI 10.1007/978-1-4614-6025-1.

O. Penrose, L. Onsager: Bose-Einstein condensation and liquid Helium. Phys. Rev. 104, 576 (1956) - Zbl 0071.44701

O. Schramm: Compositions of random transpositions. Isr. J. Math. 147, 221-243 (2005). - Zbl 1130.60302

G. Slade: The self-avoiding walk: a brief survey. in Surveys in Stochastic Processes, J. Blath, P. Imkeller, S. Roelly (eds.), European Mathematical Society (2011). - Zbl 1238.60215

D. Ioffe: Ornstein-Zernike Behaviour and Analyticity of Shapes for Self-Avoiding Walks On Zd. Markov Processes and Related Fields 4 (1998), 324-350. - Zbl 0924.60086

R. Kotecky, P. Milos, D. Ueltschi: The random interchange process on the hypercube. Electron. Comm. Probab. 21, no. 4 (2016). - Zbl 1338.60019

Y. Kovchegov: Brownian bridge in percolation, self-avoiding walks and related processes. Ph.D thesis, Department of Mathematics, Stanford University, 2002.

B. Nienhuis: Exact Critical Point and Critical Exponents of $O(n)$ Models in Two Dimensions. Phys. Rev. Lett., Volume 49, Number 15, (1982).

N. Madras, G. Slade: The Self-Avoiding Walk. Birkhäuser (2013), reprint of the 1996 Edition. DOI 10.1007/978-1-4614-6025-1.

O. Penrose, L. Onsager: Bose-Einstein condensation and liquid Helium. Phys. Rev. 104, 576 (1956) - Zbl 0071.44701

O. Schramm: Compositions of random transpositions. Isr. J. Math. 147, 221-243 (2005). - Zbl 1130.60302

G. Slade: The self-avoiding walk: a brief survey. in Surveys in Stochastic Processes, J. Blath, P. Imkeller, S. Roelly (eds.), European Mathematical Society (2011). - Zbl 1238.60215

L. Taggi: Shifted critical threshold for the loop $O(n)$ model at arbitrarily small $\delta n$, Electronic Comm. Probab. 2018, Vol. 23, paper no. 96, 1-9. - Zbl 1419.82018

B. Toth: Improved lower bound on the thermodynamic pressure of the spin $1/2$ Heisenberg ferromagnet. Lett. Math. Phys. 28, 75-84 (1993) - Zbl 0772.60103

D. Ueltschi: Relation between Feynman Cycles and Off-Diagonal Long-Range Order. Phys. Rev. Lett. 97, 170601 (2006). - Zbl 1228.82016

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.