An Adaptive Maintenance Policy With Nonlinear Degradation Modeling Based on Prognostic Information

JIAFEI ZHENG, HANXIAO MU, XUANJUN WANG, TIAN-MEI LI, QI ZHANG, AND XI WANG

Department of Automation, Xi’an Institute of High-Tech, Xi’an 710025, China
Corresponding author: Jianfei Zheng (zjf302@126.com)

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ABSTRACT  Prognostics and health management (PHM) technology is an extremely important research focus in the field of reliability engineering. The ultimate goal of applying PHM technology is health management. Aiming at nonlinear degradation systems, an adaptive maintenance policy based on prognostic information is proposed herein. First, a nonlinear degradation model with an adaptive updating mechanism is used to predict the remaining useful life (RUL) of the degrading system. Then, based on the predicted RUL distribution, a multi-objective optimization model is established to address the trade-off between operating cost and availability through a constructed decision boundary, instead of the approach used in previous studies, which considers cost as a single indicator. Using this multi-objective optimization model, an adaptive decision criterion is proposed to evaluate the advantages and disadvantages of different replacement policies, in order to determine the optimal replacement time and dynamic condition monitoring (CM) interval of the degrading system. Finally, an example of gyroscope in an inertial navigation system (INS) is used to verify the effectiveness of the proposed method.

INDEX TERMS  Health management, prognostic information, multi-objective optimization, replacement.

I. INTRODUCTION
Owing to the continuous improvements in modern production technology and industrial manufacturing, engineering systems have undergone rapid developments in terms of integration, automation, precision, and intelligence [1]. As production efficiency continues to improve, novel engineering systems such as aero engines and gyroscopes are being introduced in core fields, including aerospace and weaponry. During the operation of engineering systems, due to interactions with environmental factors such as temperature, pressure, and vibration shocks as well as internal factors such as wear pitting and fatigue cracks, the performance, reliability, and safety of these systems are degraded [2]–[4]. When this degradation reaches a certain magnitude, the system fails. To reduce the risk of sudden failures and ensure the reliability and safety of engineering systems, prognostics and health management (PHM) technology has received widespread attention in recent years.

PHM technology aims to evaluate system reliability, and formulate a reasonable maintenance policy based on the condition information monitored in real time to further improve the reliability and safety of the systems [5]–[9]. In general, PHM is composed of two aspects: remaining useful life (RUL) prediction and health management. Health management is the ultimate goal of applying PHM technology, which is based on prognostic information, for scientifically and reasonably arranging maintenance activities such as inspections, repairs, replacement of key components, and ordering spare parts [10], [11].

As the core content of maintenance policy, replacement is usually categorized as preventive replacement or failure replacement. Failure replacement refers to the replacement activity conducted after system failure occurs. However, in practice, a sudden failure of the system may result in unpredictable consequences or major losses. Therefore, to reduce
the risk of sudden failure and the losses incurred, preventive replacement is typically employed in engineering practice. Preventive maintenance can be further categorized as scheduled maintenance and condition-based maintenance (CBM). Scheduled maintenance is a traditional maintenance policy whereby preventive maintenance activities are performed at predetermined times. The CBM policy, based on prognostic information, makes full use of real-time degradation data. Through an established decision model, dynamic maintenance is implemented as the system health changes; this approach makes it possible to overcome the shortcomings of traditional maintenance policies \cite{12}. Therefore, CBM policies based on prognostic information have undergone rapid developments in recent years.

Scarf \cite{13} adopted a multi-dimensional Pareto analysis to classify the components of the target system and then suggested appropriate maintenance measures through classification, which served as a guide for technicians to reasonably arrange maintenance activities. Ahmadi \cite{14} assessed the impact of imperfect maintenance on system degradation and determined an optimal replacement policy, considering the lowest cost as the objective. The decision model was based on a periodic inspection policy and determined if replacement activities are required at the current condition monitoring (CM) point, based on the system state and the predefined replacement threshold. Under the CBM framework, Chen et al. \cite{15} determined the optimal CM interval and the optimal replacement policy to minimize total cost during the operation of the system, including the inspection, replacement, and downtime costs. Castro et al. \cite{16} studied maintenance decisions under the combined effects of internal degradation and external shocks. In view of the competing failure model, a condition-based periodic inspection maintenance model was proposed. By optimizing the CM cycle and preventive replacement time, the expression for the expected cost ratio was obtained. On this basis, Rafiee et al. \cite{17} considered the maintenance decision under a generalized mixed shock mode, with the goal of minimizing the expected cost ratio, and optimizing the CM interval of the system.

However, these previous methods are based on the periodic inspection policy, wherein replacement activities are performed during each inspection and the CM interval remains fixed. Thus, the CM interval cannot be dynamically determined according to the degradation data of the system. In practice, to accurately grasp the health status of the degrading system, it is necessary to shorten the CM interval in order to ensure the safety and reliability of the system. However, previous literature \cite{18} indicates that frequent detection accelerates the degradation of the system, reduces the RUL of the system, increases the cost of testing, and ultimately leads to unnecessary losses. Therefore, a more reasonable CM interval needs to be determined dynamically and non-periodically based on degradation data of the system.

Jing et al. \cite{19} proposed a joint inventory management policy under the framework of CBM and implemented a joint decision optimization of the CM interval and spare parts ordering, using cost minimization as the only indicator. Golmakani and Fattahipour \cite{20} considered age-based inspection scheme and proposed that the CM interval should be adjusted according to the operation time of the system. Lam and Banjevic \cite{21} dynamically optimized the CM interval based on the CBM policy, considering the lowest cost per unit time of the system as the objective. Li et al. \cite{22} established an imperfect maintenance model based on the concept of delay-time in multiple failure modes, thereby deducing reliability and cost models. Subsequently, the minimum average cost per unit time was used as an indicator to determine the optimal CM interval and maximize resource allocation. Letot et al. \cite{23} proposed an adaptive maintenance policy with the goal of minimizing long-run costs per unit time and formulated a decision criterion to determine if preventive replacement activities need to be performed at the current state of the system. Qiang et al. \cite{24} established a model of the first CM time and CM period based on CBM, aimed at cost and availability, and presented a method for calculating the optimal CM period. On this basis, Jiang \cite{25} reported that testing during the early stages of system degradation increase the testing costs and downtime. Therefore, the first CM time and CM period of the system are also considered in the optimization model.

The abovementioned studies considered the change in system state with the operation time and optimized the CM interval according to degradation data. However, a majority of these methods employ the lowest cost as the sole indicator, neglecting the requirements for the long-run availability of the system. In engineering systems, although low cost is an important objective, it is more essential to ensure the reliability of the system.

In view of the abovementioned problems, this article proposes an adaptive maintenance policy based on prognostic information for nonlinear degradation system, in order to obtain the optimal dynamic CM interval and preventive replacement time. First, a nonlinear degradation model with an adaptive updating mechanism is used to predict the RUL of the degrading system. Thereafter, based on the RUL prognostic information of the system, a multi-objective optimization model involving joint cost and availability is established, and the trade-off between cost and availability is determined through the construction of the decision boundary. To obtain the optimal dynamic CM interval and preventive replacement time, an adaptive decision criterion is proposed to evaluate the advantages and disadvantages of different replacement policy for the degrading system. Finally, the proposed method is verified based on the actual degradation data of gyroscope.

The primary contributions of this study are as follows. First, unlike previous studies that only considered cost, a multi-objective optimization model considering the long-run average cost and the long-run average availability is established. Moreover, actual engineering requirements are considered from the perspective of ensuring the security and economy of the system. Second, unlike previous maintenance policies based on the periodic inspection policy, an
II. PROBLEM DESCRIPTION AND DEGRADATION MODELING

The proposed method is an adaptive maintenance policy that considers both cost and availability to determine the optimal replacement time and dynamic CM interval. Under the PHM technology framework, the primary task of implementing maintenance decision is to obtain prognostic information related to the probability density function (PDF), cumulative distribution function (CDF), and reliability function (RF) of the RUL.

A. PROBLEM DESCRIPTION

The degradation of the system operating in a complex environment is highly nonlinear. Therefore, based on the adaptive prognostic approach using nonlinear degradation modeling [26], this study determines the PDF of the RUL. Specifically, a state-space model is constructed using the nonlinear degradation model, and the key parameter in the drift function is updated via Kalman filtering. Using the expectation maximization (EM) algorithm and Kalman smoother, the hidden states and other parameters in the state-space model are recursively updated. Thus, this method can adaptively estimate model parameters and realize online updating of the RUL. Based on prognostic information of the RUL, a multi-objective optimization model and adaptive decision criterion are established to ensure that the maintenance cost is minimized, on the premise that the system availability is met. On this basis, the optimal replacement time and the dynamic CM interval for degrading system are determined to arrange scientific and reasonable maintenance activities and to ensure the safety and reliability of the system.

B. DEGRADATION MODELING

Reference [26] uses a nonlinear Wiener process to describe the stochastic degradation process; its degradation model is expressed as

$$X(t) = x_0 + \lambda \cdot \int_0^t \mu(t; \vartheta) \, dt + \sigma_B B(t)$$  \(1\)

where the potential degradation process $X(t)$ is driven by a standard Brownian motion (BM) $B(t)$ with a nonlinear drift $\lambda \cdot \mu(t; \vartheta)$; $\lambda \cdot \mu(t; \vartheta)$ and $\sigma_B$ are the drift and diffusion coefficients, respectively; $\mu(t; \vartheta)$ is a nonlinear function over $t$ with unknown parameter vector $\vartheta$. $\lambda$ is a proportional parameter that controls the rate of nonlinear degradation; and $\vartheta$ is used to determine the shape of the degradation process.

Without a loss of generality, the initial potential degradation state is assumed to be $X(0) = x_0 = 0$, and the model parameters are represented as $\theta = [\lambda, \vartheta, \sigma_B]$.

Based on the concept of the first hitting time (FHT) [27]–[29], when the potential degradation state that characterizes the health level reaches the failure threshold $\omega$ for the first time, the life of the degrading system comes to an end. Therefore, the life $T$ of the degrading system is defined as

$$T = \inf \{t : X(t) \geq \omega | X(0) < \omega \}$$  \(2\)

The RUL $L_k$ of the degrading system at moment $t_k$ is defined as the effective duration from the current moment to the end of its life [30]–[32]. Therefore, the RUL of the degrading system at $t_k$ can be expressed as

$$L_k = \inf \{l_k > 0 : X(l_k + t_k) \geq \omega \}$$  \(3\)

Based on the established degradation model (1), the CM data are used for estimating the state and parameters. The degradation equation can be reconstructed as a state-space model, as follows:

$$\begin{align*}
\dot{\lambda}_k &= \lambda_{k-1} + \eta \\
X_k &= x_{k-1} + \lambda_{k-1} \Omega_k (\vartheta) + \sigma_B \epsilon_k
\end{align*}$$  \(4\)

where $\Omega_k (\vartheta) = h(t_k; \vartheta) - h(t_{k-1}; \vartheta); h(t_k; \vartheta) = \int_0^{t_k} \mu(t; \vartheta) \, dt$; the error term in the state equation is distributed as $\eta \sim N(0, Q)$, and $\epsilon_k = [B(t_k) - B(t_{k-1})] \sim N(0, \sigma^2_B)$. Assume that the initial drift coefficient $\lambda_0$ follows a normal distribution with mean $\mu_0$ and variance $P_0$. It is well known that $\lambda_k$ follows a Gaussian distribution, which can be estimated using a recursive filter based on historical information $X_{1:k} = \{x_1, x_2, \ldots, x_k\}$. Its mean is denoted as $\hat{\lambda}_k = E(\lambda_k | X_{1:k})$ and variance as $P_{k|k} = \text{var}(\lambda_k | X_{1:k})$. Under the framework of Bayesian filtering, $\hat{\lambda}_k$ and $P_{k|k}$ can be obtained via recursive Kalman filtering; the detailed derivation can be found in [26].

Based on the established degradation model (1), the PDF of the RUL can be obtained as follows:

$$f_{L_k|X_{1:k}}(l_k | X_{1:k}) \approx \frac{\alpha_k \Lambda(l_k) - \alpha_k (l_k; \vartheta) \Delta(l_k)}{\sqrt{2\pi l_k^2 \Lambda^3(l_k)}} \exp \left[ -\frac{(\alpha_k - \hat{\lambda}_k \nu(l_k))^2}{2\Delta(l_k)} \right]$$  \(5\)

where $\alpha_k (l_k; \vartheta) = \nu(l_k) - l_k \mu(l_k + t_k; \vartheta)$, $\alpha_k = \omega - x_k$, $\nu(l_k) = \int_0^{l_k + t_k} \mu(t; \vartheta) \, dt$, $\Lambda(l_k) = P_{k|k} \nu(l_k)^2 + \sigma^2_B l_k$, and $\Delta(l_k) = P_{k|k} \nu(l_k) \alpha_k + \hat{\lambda}_k \sigma^2_B l_k$.

It should be noted that, in (5), all unknown parameters, including $\mu_0$, $P_0$, $\sigma^2_B$, $Q$, and $\vartheta$, need to be estimated. The selection of these parameters will affect the performance of RUL estimation. Specific parameter estimation method and
the derivations are given in [26]. This study uses the prognostic information in [26] to model the maintenance decision; thus, the detailed parameter estimation is not reiterated here.

Based on the PDF of the RUL, the CDF $F_{l_k|X_{1:k}} (l_k | X_{1:k})$ and the RF $R_{l_k|X_{1:k}} (l_k | X_{1:k})$ can be further determined as follows:

$$F_{l_k|X_{1:k}} (l_k | X_{1:k}) = \int_0^{l_k} f_{l_k|X_{1:k}} (\tau | X_{1:k}) \ d\tau$$ (6)
$$R_{l_k|X_{1:k}} (l_k | X_{1:k}) = 1 - F_{l_k|X_{1:k}} (l_k | X_{1:k})$$ (7)

The abovementioned equations utilize the CM data of the degrading system and are adaptively updated with new CM data. The adaptive maintenance policy proposed herein is based on the analysis of the abovementioned RUL prognostic information of the degrading system. The modeling of the maintenance decision is introduced in the following section.

### III. MULTI-OBJECTIVE OPTIMIZATION MODEL

To reduce the risk of sudden failures and the associated losses, preventive replacement activities are often performed. A majority of the previous studies only use the lowest long-run average cost as the sole indicator when optimizing the preventive replacement time; however, for systems with higher requirements for reliability and security, it is more important to ensure long-run average availability. Therefore, this study establishes a multi-objective optimization model that considers both long-run average cost and long-run average availability to determine the optimal preventive replacement time.

#### A. LONG-RUN AVERAGE COST MODEL

According to the preventive maintenance policy [33], a degrading system requires preventive replacement with a cost $C_p$ and failure replacement with a cost $C_f$. Generally, the cost of preventive replacement is less than that of failure replacement, i.e., $C_p < C_f$. In general, it is assumed that, after a preventive or failure replacement, the degrading system is restored to its original state. According to the renewal-reward theorem [34], [35], the long-run average cost model can be constructed as follows:

$$C(T) = \frac{C_p R(T) + C_f F(T)}{\int_0^T \tau f(\tau) \ d\tau + TR(T)}$$ (8)

where the numerator represents the expected cost incurred during a life cycle, the denominator represents the expected length of a life cycle, and $T$ is the preventive replacement time that needs to be optimized.

For nonlinear degradation system, based on CM data with $X_{1:k}$ monitored at the current CM point $t_k$, (8) can be further expressed as (9), as shown at the bottom of the page, and the denominator can be simplified as

$$C(T) = \frac{C_p R(T - t_k | X_{1:k}) + C_f F(T - t_k | X_{1:k})}{t_k + \int_0^{T-t_k} \tau f(\tau | X_{1:k}) \ d\tau + (T - t_k) \left[ 1 - \int_0^{T-t_k} f(\tau | X_{1:k}) \ d\tau \right]}$$ (9)

In this model, RUL distribution is used to estimate the expected length of the renewal cycle and the expected cost of the degrading system. Moreover, the RUL distribution can be updated at each CM point, and the long-run average cost $C(T)$ can be adaptively updated.

#### B. LONG-RUN AVERAGE AVAILABILITY MODEL

When availability is an important indicator for evaluating the system [34], it is necessary to establish a long-run average availability model to ensure that the system has sufficient uptime and to avoid failure replacement activities. According to the renewal-reward theorem, the long-run average availability can be constructed as follows:

$$A(T) = \frac{\int_0^T \tau f(\tau) \ d\tau + T \left[ 1 - \int_0^T f(\tau) \ d\tau \right]}{\int_0^T (\tau + T) f(\tau) \ d\tau + (T + T_f) \left[ 1 - \int_0^T f(\tau) \ d\tau \right]}$$ (12)

Here, the numerator represents the expected total uptime, and the denominator represents the sum of the expected total uptime and the expected total downtime. $T$ is the preventive replacement time that needs to be optimized, $T_p$ is the average length of time required to complete the preventive replacement activity, and $T_f$ is the average time required to complete the failure replacement activity. In practice, $T_p < T_f$. Similar to the long-run average cost model, considering CM data and assuming that the current monitoring point is $t_k$, (12) can be further expressed as (13), as shown at the bottom of the next page, simplifying this equation through mathematical derivations yields (14), as shown at the bottom of the next page. Similar to the long-run average cost model $C(T)$, the
RUL distribution is used to estimate the expected total uptime and expected total downtime of the degrading system. Moreover, the RUL distribution can be updated at each CM point, and the long-run average availability $A(T)$ can be adaptively updated.

C. MULTI-OBJECTIVE OPTIMIZATION MODEL

For the maintenance policy to be suitably applied in practical engineering, it is necessary to determine the optimal preventive replacement time $T^*$, while minimizing the long-run average cost and maximizing the long-run average availability. Thus, this problem becomes a multi-objective optimization problem. According to the long-run average cost model and the long-run average availability model established in the previous two subsections, at the CM point $t_k$, a multi-objective optimization model is constructed through the following objective function and constraint (15), as shown at the bottom of the page.

The multi-objective optimization model established herein makes full use of prognostic information. Based on the CM data $X_{1:k}$, the adaptive update of $C(T)$ and $A(T)$ can be achieved at each CM point. To find a solution to this multi-objective optimization problem, a decision boundary is constructed to address the trade-off between cost and availability to help managers make more scientific and reasonable replacement decisions. Specifically, under the constraint that the long-run average availability $A(T)$ meets at least the minimum level $\zeta$, the long-run average cost $C(T)$ is minimized. Therefore, the multi-objective optimization model can be expressed as follows:

$$\begin{align*}
\text{Minimize } & C(T) \\
\text{Subject to } & A(T) \geq \zeta \\
& T \geq t_k
\end{align*}$$

where $\zeta$ is the availability threshold, which is generally set according to actual requirements. To satisfy the constraint in the abovementioned equation, a decision range for the preventive replacement time $T$ is required, that is, $T \in [T_{\min}, T_{\max}]$. Therefore, the objective function can be optimized within this decision range, and the optimal preventive replacement time $T^*$ can be calculated using the following equation:

$$T^* = \arg \min C(T), \quad T \in [T_{\min}, T_{\max}]$$

Based on (17), the optimal preventive replacement time is obtained through the multi-objective optimization model established herein; this model ensures the availability of the degrading system and minimizes costs. In the subsequent section, based on the optimal preventive replacement time, an adaptive decision criterion is proposed to determine the optimal dynamic CM interval for the degrading system.

IV. ADAPTIVE DECISION CRITERION

In practical applications, to obtain the RUL or to implement CBM, the most basic premise is to obtain CM data that reflect the health status of the system. Related literature indicates that the CM interval of degradation data is generally periodic. However, a more reasonable CM interval should be
dynamically determined with the degradation process. This is because the system has a relatively high level of reliability during the early stages of degradation; therefore, it may not be necessary to implement frequent detection during this period, which may accelerate degradation and increase detection costs [18]. In the later stages of degradation, due to the low reliability, it is necessary to shorten the CM interval to grasp the health status of the degrading system and reduce the risk of sudden failure. Therefore, it is necessary to determine the optimal dynamic CM interval to arrange maintenance activities based on the degradation data, thereby increasing the service life of the degrading system.

To optimize the CM interval of the degrading system, the first CM time is generally determined based on RUL data and expertise. The optimal preventive replacement time determined in the previous section is only for the current CM point \( t_k \), and not for the entire life cycle. Therefore, each CM point would have a corresponding optimal preventive replacement time. For subsequent analyses, let the optimal preventive replacement time at CM point \( t_k \) be \( T_{k}^* \).

To obtain the optimal dynamic CM interval and the optimal preventive replacement time of the system throughout its life cycle, an adaptive decision criterion is proposed to determine whether preventive replacement activities are performed at the current CM point or postponed to the next CM point. Specifically, based on the long-run average cost model in (11), by incorporating the average cost \( C_m \) of each CM, the general expression of the constructed decision criterion is formulated as follows:

\[
K_C(t_k) = \frac{C_{p} + (k-1)C_{m}}{t_k + \int_{t_k}^{t_{k+1}} R(k(x_{t+1})) \, dt} + C_f(t_{k+1}(x_{t+1}) + kC_{m}/t_{k+1}}
\]

(18)

where \( t_{k+1} \) is the next CM point predicted by the multi-objective optimization model (15) at the CM point \( t_k \); in other words, \( t_{k+1} = T_{k}^* \). This decision criterion indicates the cost ratio between the replacement policy at the CM point \( t_k \) and the replacement policy deferred to the next CM point \( t_{k+1} \). If \( K_C \geq 1 \), the replacement policy is more expensive at the current CM point; thus, a better policy would be to postpone the preventive replacement activity to the next CM point. If \( K_C < 1 \), the preventive replacement activity is performed at the current CM point.

Although the decision criterion in (18) only considers the ratio between costs, the next CM point \( t_{k+1} \) is predicted using the multi-objective optimization model (15), thereby ensuring that availability and cost are considered simultaneously. In addition, in this study, the online update of the RUL distribution can be realized, and the established multi-objective optimization model can be adaptively updated. Hence, the decision criterion based on the multi-objective optimization model can be adaptively updated with new CM data.

The process flow of the proposed adaptive maintenance policy is presented in Fig. 1. First, the parameters in the cost and availability model are initialized according to the actual condition of the system. Subsequently, based on the CM data \( X_{1:k} \), the distribution of the RUL is adaptively estimated and updated according to the model proposed in [26]. Furthermore, the optimal preventive replacement time \( T_{k}^* \) at the current CM point \( t_k \) is calculated according to the established multi-objective optimization model of joint cost and availability. Finally, the proposed decision criterion \( K_C(t_k) \) is evaluated to determine whether replacement activities should be performed at the current CM point. By repeating the abovementioned process, the adaptive maintenance policy can be realized. Moreover, as \( k \) increases, a series of CM points \( \{t_1, t_2, t_3, \ldots \} \) can be obtained, based on this, the optimal dynamic CM interval and the optimal CM times of the degrading system are obtained.

V. EMPIRICAL STUDY

The gyroscope fixed on the inertial platform is a key component in aerospace and weapon missile systems. Its health is directly related to the precision of guidance and the performance of control systems. When the inertial platform operates, the rotor of the gyroscope rotates at a high speed. With the increase in the operation time, the mechanical wear of the rotating shaft inevitably occurs; this increases the gyroscope drift coefficient, which, on reaching a certain degree, causes gyroscopic fault. For gyroscopes installed on inertial platforms, the number is small and the monitoring cost is high. Hence, if detection is excessively frequent, the degradation process will be accelerated and maintenance costs will also be significantly high. However, if the CM number is excessively low, the replacement activity may not be conducted.
in a timely manner after gyroscope failure; this can lead to severe consequences. Therefore, it is necessary to determine the optimal preventive replacement time and the optimal dynamic CM interval of the gyroscope through scientific and reasonable methods.

The validity of the proposed method is verified based on the degradation data monitored by a type of gyroscope in an inertial navigation system (INS) in [36]. The drift coefficients of the inertial platform in the INS mainly include $K_{0X}$, $K_{0Y}$, $K_{0Z}$, $K_{SX}$, $K_{SY}$, and $K_{IZ}$. The term $K_{SX}$ along the sense axis in the measured degradation drift data plays a dominant role in assessing the health status of the inertial platform. Based on $K_{SX}$, the RUL of the INS is predicted to enable maintenance decisions.

In the experiment, the dataset collects 73 CM points of degradation data with a sampling interval of 2.5 hours, and its life is 180.5 hours of electrification. Based on the technical indicators for this type of gyroscope, the failure threshold of the drift coefficient is set at $\omega = 0.37 (^{\circ}/h)$. Using the nonlinear degradation modeling adaptive prognostic approach proposed in [26], the actual observed degradation path and the predicted degradation path for this type of gyroscope are obtained, as shown in Fig. 2(a). The parameter updating process and the PDF of the RUL at different CM points are shown in Fig. 2(b) and Fig. 2(c), respectively.

From Fig. 2(a), it is evident that the predicted degradation path is quite close to the actual observed degradation path. Fig. 2(b) indicates that the model parameters quickly converge with the accumulation of degradation data. In addition, if the degradation data vary significantly, the estimated parameters reflect the expected changes in the data. For instance, the parameter $\vartheta$ in Fig. 2(b) changes significantly at approximately the 130th and 170th hour. From Fig. 2(a), it is evident that the actual degradation data change considerably at this time. These observations reflect the adaptive ability of the parameter estimation algorithm. In Fig. 2(c), the red line represents the actual RUL, whereas the black line represents the estimated RUL. It can be seen that the estimated RUL is in good agreement with the actual RUL. Once the parameters in the model are updated at each CM point, the PDF of the corresponding estimated RUL can be calculated by (5). At around the 170th hour, the estimated RUL has a certain deviation, because the parameters change significantly at this time. With the accumulation of degradation data, the PDF curve of the RUL becomes sharper, which suggests that the uncertainty of its prediction constantly declines. This proves that the nonlinear adaptive method can effectively estimate the RUL of degrading system and subsequently obtain the RUL distribution, including the PDF, CDF, and RF.

Here, considering the actual maintenance and the price of the same type of high precision gyroscope, parameters in the multi-objective optimization model are set as $C_p = 5000\$, $C_f = 15000\$, $C_m = 100\$, $T_p = 5\ hours$, $T_f = 200\ hours$, and $\zeta = 0.96$. 

**FIGURE 2.** Results of RUL estimation. (a) The actual observed degradation path and the predicted degradation path. (b) Adaptive updating of model parameters. (c) PDFs of the RUL at different CM time.
Using the nonlinear degradation model in [26] and applying the proposed method, the illustrative example of the proposed maintenance decision is presented in Fig. 3.

As shown in Fig. 3, due to the low wear and high reliability of the gyroscope during the early stages of degradation, the first CM point is determined using the minimum long-run average cost as the only indicator, and \( t_1 = 50 \) hour is obtained as the first CM point in the adaptive maintenance stage. Next, based on the degradation data monitored from the initial time to \( t_1 \) and the multi-objective optimization model, the optimal preventive replacement time \( T^*_1 \) corresponding to \( t_1 \) is obtained. Finally, whether to replace at the current CM point is determined according to the proposed decision criterion \( K_C \) in (18). If \( K_C (t_1) < 1 \), the system is replaced at point \( t_1 \), otherwise, the next CM point is scheduled to \( t_2 = T^*_1 \). This process is repeated for each CM point \( t_k \), until \( K_C (t_k) < 1 \).

As shown in Fig. 3, the CM interval obtained using the proposed method is dynamically determined based on the degradation state of the gyroscope. Due to the high reliability of the gyroscope during the early stages, frequent inspection tends to accelerate its degradation and increase the cost, thus, the CM intervals obtained during the early stages of degradation are larger. Conversely, the CM intervals obtained during the final stages of degradation tend to decrease gradually, this is due to the low reliability of the gyroscope at the end of electrification. It is necessary to shorten the CM interval to grasp the health status of the system, thereby reducing the risk of sudden failures and avoiding unnecessary losses. As shown in Fig. 3, five inspections are performed, and a preventive replacement action is performed at the fifth monitoring point. Accordingly, the optimal CM interval is obtained, and each scheduled CM point is jointly determined based on both cost and availability.

Considering the current CM point of 130th hour as an example, the long-run average cost \( C(T) \) and long-run average availability \( A(T) \) are plotted in Fig. 4.

From Fig. 4, it can be seen that, at the 130th hour, if the long-run average cost is used as the only indicator, the preventive replacement time to achieve the lowest long-run average cost is the 155th hour; conversely, if the long-run average availability is used as the only indicator, the preventive replacement time to achieve the highest long-run average availability is the 141th hour. Thus, the results for cost and availability do not coincide, offering different optimal preventive replacement time. Therefore, to precisely determine the optimal preventive replacement time, a multi-objective optimization model considering both joint cost and availability is established. However, for a multi-objective optimization problem, if the objectives compete with each other, only one of the objective functions is realized to the maximum possible extent; this leads to losses in the remaining objective functions. In this study, the two competing objective functions are the minimizing long-run average cost \( C(T) \) and the maximizing long-run average availability \( A(T) \). To address this trade-off between the two conflicting objectives, a decision boundary is constructed to assist managers in making reasonable maintenance decisions.

For more intuitive decision results, it is assumed that the long-run average availability should always exceed 0.96, i.e., \( \zeta = 0.96 \), which is the availability threshold.

As shown in Fig. 4, when the availability threshold is met, the decision range of the preventive replacement time is \( T \in [130, 148] \). Evidently, using (17), it can be determined that the optimal preventive replacement time at the 130th hour is the 148th hour. Therefore, this is an appropriate solution for the trade-off between cost and availability, because it minimizes the cost while meeting availability requirements. However, it should be noted that the 148th hour is not the optimal preventive replacement time of the degrading system throughout its life cycle, because every CM point will have a different optimal preventive replacement time. The optimal preventive replacement time of the entire life cycle is the 155th hour.
In summary, the proposed method can be used to obtain the optimal preventive replacement time and the optimal CM interval of the system for the entire life cycle. Each CM time obtained is determined using the multi-objective optimization model, which reduces system maintenance costs and also ensures the security of the system.

**VI. CONCLUSION**

This article proposed an adaptive maintenance policy for the system featuring nonlinear degradation, to optimize system health management to a certain extent. To overcome the limitations of existing maintenance policies, a multi-objective optimization model combining both cost and availability was established, and the trade-off between these two parameters was addressed using a decision boundary. On this basis, to determine the optimal preventive replacement time and the optimal CM interval of a degrading system throughout its life cycle, an adaptive decision criterion was proposed to evaluate whether maintenance activities need to be performed at the current CM point or postponed to the next CM point. Finally, the effectiveness of the proposed method was verified using actual degradation data of the gyroscope.

The experimental results indicated that the adaptive maintenance policy proposed herein effectively reduces the total cost, while also ensuring the availability of the system; this policy also determines the optimal dynamic CM interval and the optimal preventive replacement time. Therefore, the proposed method is expected to guide and assist managers in arranging scientific and reasonable performance testing and maintenance activities.

In future works, we plan to extend the proposed method to repairable systems and multi-unit systems, assess the impact of measurement uncertainties on maintenance decisions, determine the optimal decision method, extend the system operating cycle, and improve the accuracy of maintenance decisions.

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JIANFEI ZHENG received the B.Eng., M.Eng., and Ph.D. degrees from the Department of Automation, Xi’an Institute of High-Tech, China, in 2004, 2008, and 2016, respectively. He is currently an Associate Professor with the Xi’an Institute of High-Tech. He has published over 20 articles in several journals. His research interests include prognostics and health management, predictive maintenance, and inertial navigation systems.

HANXIAO MU received the B.Eng. degree from North China Electric Power University, in 2019. She is currently pursuing the master’s degree in control science and engineering with the Xi’an Institute of High-Tech, China. Her research interests include prognostics and health management, predictive maintenance, and deep neural networks.

XUANJUN WANG received the B.Eng. and M.Eng. degrees from the Xi’an Institute of High-Tech, China, in 1987 and 1993, respectively, and the Ph.D. degree in energetic materials from the Nanjing University of Science and Technology (NJUSt), in 1988. He became a Professor with the Xi’an Institute of High-Tech, in 2001. His research interests include special energy science and technology.

TIAN-MEI LI received the B.Eng. degree from Xi’an Jiaotong University, in 2002, and the M.Eng. and Ph.D. degrees from the National University of Defense Technology, in 2004 and 2010, respectively, all in mechanical engineering. She is currently a Lecturer in control science and engineering with the Xi’an Institute of High-Tech. Her current research interests include testability design, demonstration, and evaluation.

QI ZHANG received the B.S. degree from the China University of Petroleum, Dongying, China, in 2002, and the M.S. degree and the Ph.D. degree in control science and engineering from the Xi’an Institute of High-Tech, China, in 2005 and 2009, respectively. She is currently an Associate Professor of control science and engineering with the Xi’an Institute of High-Tech. Her research interests include prognostics and health management, and nonlinear filtering.

XI WANG received the B.Eng. degree in control science and management from the Harbin University of Science and Technology, Harbin, China, in 2017, and the M.Eng. degree from the Xi’an Institute of High-Tech, China, in 2020. His research interests include prognostics and health management, reliability estimation, predictive maintenance, and lifetime estimation.

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