Application of Improved Grey Model Based on Cumulative Method to Deformation Prediction of Tunnel Surrounding Rock

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Abstract. The traditional GM(1,1) model was improved by using the cumulative to increase its prediction accuracy. Based on the measured data of the Laomu tunnel in section 10 of shiqian to yoping expressway, the traditional grey prediction model and the improved grey prediction model were used to predict the surrounding rock deformation of the tunnel. The research shows that the improved model prediction can reduce the probability of outliers and the data series fluctuation of traditional gray prediction model, and makes the predicted value closer to the measured value. Although the improved gray prediction model has a slightly single modeling step, it can effectively increase the prediction accuracy of the model in a short period of time, and has certain reference value for similar tunnel surrounding rock deformation monitoring projects.

1. Introduction

During the tunnel construction process, scorpion-shaped surrounding rock often appears in the tunnel vault, and stell-shaped surrounding rock is prone to collapse and other phenomena, causing serious consequences. Therefore, it is necessary to monitor and predict the surrounding rock of the tunnel grate. Through safe and stable monitoring and high-precision prediction, the construction party can take effective measures as soon as possible to prevent the collapse of surrounding rocks and cause safety accidents [1]. However, due to the shortcomings of its general prediction effect and insufficient prediction accuracy, related experts and scholars have improved it. Wu Wenze, Zhang Tao [2], etc. based on the traditional gray Bernoulli model, combined with optimized initial values, modified the gray prediction model through three aspects of rolling modeling mechanism, Guass-Newton algorithm and optimal model parameters. Wang Lu, Sha Xiuyan [3] proposed the two optimization methods of combination optimization and segmentation optimization by optimizing the background value and the solution method. The comparison of the examples proved that the improved gray prediction model was more feasible and reliable. Li Lu, Hong Youtang [4] During the observation and simulation of the ground settlement of a residential building, based on the gray prediction model, the original data was processed in equal time series. The resulting line chart of the simulated height values more closely matches the real value.

Combining the cumulative method with the gray prediction model can not only effectively avoid
the problem of outliers in the gray prediction model, but also increase the stability of the model. After the modeling process system is optimized, the traditional whitening formula is replaced by the connotative prediction formula to improve the traditional gray prediction model. The grey prediction model curve, the improved grey prediction model curve and the curve obtained from the measured data are used to analyze and compare the residual average value and the predicted value, which proves that the improved grey prediction model has higher prediction accuracy and higher prediction effect.

2. Improvement of Grey System Model Based on Cumulative Method

In 1982, since Deng Julong and others first proposed the grey system theory, experts and scholars in various fields have conducted in-depth use and research of it. Based on the analysis and processing of the known data, some changes in the data are derived to accurately predict the development of the next data. When faced with less research data and uncertain information, researchers often use gray theory models to solve problems.

The gray prediction model is an important part of the gray theory system. Building a gray system model is an effective way to effectively solve uncertain problems, and it can predict events that contain both known and location information. The gray prediction method finds a representative data sequence by processing the original data and looking for the change law over time within a certain range. And build a differential equation model to predict the future development of the event.

The main idea of the GM (1,1) model is: first of all, you need to analyze and judge the original data before modeling. Only the data that meets the requirements of non-negative and equal intervals are allowed to be modeled. Then accumulate or subtract the data that meets the requirements to generate a data sequence [5]. Then, a gray prediction model is established, and the development coefficient and the amount of gray interaction are solved. Finally, the development of the data is predicted according to the model. Its predicted value is:

$$\hat{x}(0)(k+1) = \hat{x}(0)(k) - \hat{x}(0)(k), k = 1,2,...,n-1,$$

(1)

After modeling, an accuracy test is required to ensure the feasibility of the prediction model. The main inspection methods are: residual error inspection method, posterior error inspection method, etc. This time, the posterior difference inspection method is used for inspection. In the inspection process, the posterior difference ratio C and small error probability P are two important indicators for evaluating the accuracy of the prediction model. The comparison table of the prediction accuracy levels is shown in Table 1.

| Forecast accuracy level | P    | C    |
|------------------------|------|------|
| fine                   | >0.95| <0.35|
| qualified              | >0.80| <0.45|
| unconvincing           | >0.70| <0.50|
| Disqualified           | ≦0.70| ≧0.65|

The smaller the C value, the smaller the degree of dispersion, and the better the degree of fitting. The inspection process is as follows:

Let the residual be E (k), then:

$$E(k) = x(0)(k) - \hat{x}(0)(k), k = 2,3,...,N,$$

(2)

Its relative residual is:

$$e(k) = \frac{(x(0)(k) - \hat{x}(0)(k))}{x(0)(k)}, k = 2,3,...,N;$$

(3)

$$C = \frac{S_1}{S};$$

The posterior error ratio is:
Among them, $S_1$ is the variance of the residual and $S$ is the variance of $x(0)$.

Small error probability:

$$P = P \left\{ |E(k) - \bar{E}| < 0.6745S \right\},$$

Where, $\bar{E}$ is the mean of the residual.

Cumulative method is a method of accumulating physical quantities that are difficult to measure directly with conventional instruments, and amplifies some small values. This method is called cumulative method [6]. Combining the cumulative method with the gray prediction model can effectively avoid problems such as error outliers in the data analysis process of the gray prediction model, and increase the stability of the model. The cumulative method can directly estimate parameters based on the sample secretary, without the assumption of fitting errors, which is a linear unbiased minimum variance estimation.

Establishing the initial sequence and performing cumulative accumulation of order $k$ on the original time series, we can get:

$$\sum_{i=1}^{n} (k)x^{(0)}(i) = \sum_{i=1}^{n} \sum_{j=1}^{(k-1)} x^{(0)}(j) = \sum_{i=1}^{n} C_{n-1}^{i-1}x^{(0)}(i) \quad (4)$$

After the two cumulative sums on both sides are obtained:

$$\sum_{i=2}^{n} (2)x^{(0)}(i) + a \sum_{i=2}^{n} z^{(1)}(i) = b \sum_{i=2}^{n} (2) \quad (5)$$

Where:

$$\sum_{i=2}^{n} (2)x^{(0)}(i) = \sum_{i=2}^{n} (n-i+1)x^{(0)}(i)$$

$$\sum_{i=2}^{n} z^{(1)}(i) = \sum_{i=2}^{n} (n-i+1)z^{(1)}(i)$$

$$\sum_{i=2}^{n} = \frac{n^2}{2} - n$$

After parameter accumulation, its basic equation can be changed to:

$$x^{(0)}(i) + a[\lambda x^{(1)}(i-1) + (1-\lambda)x^{(0)}(i)] = b \quad (6)$$

Among them, $\lambda \in (0,1)$ is a regulating factor. Its constructed background value is:

$$z^{(1)}(i) = \lambda x^{(1)}(i-1) + (1-\lambda)x^{(1)}(i) \quad (7)$$

Will bring $x^{(1)}(i) = x^{(1)}(i-1) + x^{(0)}(i)$ into the basic equation:

$$x^{(0)}(i) + a[(1-\lambda)x^{(0)}(i) + x^{(1)}(i-1)] = b \quad (8)$$

Then, the prediction formula of the GM (1,1) model improved by the cumulative method can be:

$$\hat{x}^{(0)}(i) = \frac{(1-a\lambda)^{(i-2)}(b-ax^{(0)}(1))}{[1+a(1-\lambda)]^{i-3}} \quad (9)$$

3. Engineering case analysis

The old wooden tunnel on the 10th section of the Shiqian to Yuping highway is a long left-right split highway tunnel. The left and right holes at the entrance end of the tunnel are cut bamboo doors, and the left and right holes at the exit end are end-wall holes. The maximum buried depth of the tunnel is approximately 139.8m, the elevation of the tunnel entrance is approximately 689.6m, and the exit elevation is approximately 673.7m. This tunnel is divided into three surrounding rock levels of grades V, IV, and III. The proportion of the length of the left and right lines of the surrounding rock grades as a proportion of the tunnel length is as follows: Grade V surrounding rocks account for approximately 9.8% (215m), and grade IV surrounding rocks account for approximately 58.3% (1280m) and Grade
III surrounding rocks account for about 31.9% (700m).

When excavated to a certain section, the rock formation was produced in a monoclinic state with the occurrence of 70°-131° ∠5°-8°. The main joint fracture occurrences were L1: 250° ∠80° and L2: 325° ∠82°. The axis direction of the tunnel is 82°-108°, which is approximately perpendicular to the strike of the rock formation, but the inclination angle of the rock formation is small, and the stability of the surrounding rock of the old tunnel has little influence. Surrounding rocks may drop off or collapse at any time during the construction process, which seriously threatens the life safety of construction workers on site. Therefore, monitoring and prediction of surrounding rocks during tunnel excavation plays an important role.

According to the modeling method combining the cumulative method and the gray prediction model, a total of 9 settlement observation points were buried on the surface of a section of the Laomu Tunnel to monitor the tunnel settlement in real time. Because the data is too large, this article only takes 2 points of data to analyze the 11 periods of settlement detection data, and the observation time for each period is 20 days. Modeling is performed using the first 9 periods of settlement monitoring data, and the second 2 periods are used to compare the prediction values of the traditional gray prediction model with the improved prediction methods. The monitoring data is shown in Table 2.

| Monitoring period | date       | Elevation of monitoring point 1/cm | Elevation of monitoring point 2/cm |
|-------------------|------------|------------------------------------|------------------------------------|
| 1                 | 2018.05.07 | 758.138                            | 766.997                            |
| 2                 | 2018.05.20 | 758.135                            | 767.037                            |
| 3                 | 2018.05.31 | 758.096                            | 766.975                            |
| 4                 | 2018.06.12 | 758.053                            | 766.942                            |
| 5                 | 2018.06.25 | 758.022                            | 766.959                            |
| 6                 | 2018.07.08 | 757.911                            | 766.822                            |
| 7                 | 2018.07.19 | 757.801                            | 766.686                            |
| 8                 | 2018.07.31 | 757.725                            | 766.554                            |
| 9                 | 2018.08.22 | 757.663                            | 766.443                            |
| 10                | 2018.09.10 | 757.370                            | 766.151                            |
| 11                | 2018.10.01 | 757.293                            | 766.042                            |

The elevations of points 1 and 2 are drawn into a line chart, as shown in Figures 1 and 2. It is not difficult to see from the figure that the settlement of point 1 is relatively stable, but there is a clear settlement on the 160th day. However, the settlement trend of No. 2 monitoring point only stabilized in the later period. Therefore, No. 1 monitoring point was selected as the research object.
After the tunnel construction is completed, the ratio of the previous actual elevations and the elevation calculated by the finite element method is used as the original data of the sample, and the data is predicted by the ordinary gray model and the improved gray model. Since the residuals can intuitively and effectively reflect whether the prediction effect of the model is accurate, the residuals and predicted values can be selected as comparison parameters, as shown in Tables 3 and 4.

### Table 3. Comparison of residuals

| model         | Residual mean/cm | Sum of squared residuals/cm² |
|---------------|------------------|------------------------------|
| GM (1,1) model | -0.0226          | 0.0039                       |
| Improved model | -0.0201          | 0.0037                       |

From Tables 3 and 4, it can be seen that the GM (1,1) gray prediction model has a good effect on the prediction of surrounding rock in the tunnel vault. The gray prediction model based on the cumulative method has a higher prediction effect on the tunnel vault surrounding rock prediction, and its relative error is lower: the average residual error is -0.0201cm, and the accuracy is improved by 11.06%; the sum of squared residual errors is 0.0037/cm², The accuracy increased by 5.13%.

### Table 4. Model No.1 monitoring point prediction result

| Types         | Date | Predicted value of traditional gray model | Improved grey model predictions |
|---------------|------|------------------------------------------|---------------------------------|
|               |      | Predictive value/cm | Residual/cm | Predictive value/cm | Residual/cm |
|               |      |                         |             |                     |             |
| Analog value  | 0    | 758.138                  | 0.000        | 758.138              | -0.034      |
|               | 20   | 758.167                  | -0.038       | 758.154              | -0.001      |
|               | 40   | 758.081                  | -0.028       | 758.078              | 0.036       |
|               | 60   | 757.998                  | 0.034        | 757.997              | 0.065       |
|               | 80   | 757.901                  | 0.063        | 757.901              | -0.028      |
|               | 100  | 757.820                  | -0.012       | 757.851              | -0.052      |
|               | 120  | 757.721                  | -0.054       | 757.751              | 0.001       |
|               | 140  | 757.637                  | -0.002       | 757.651              | 0.013       |
|               | 160  | 757.547                  | 0.011        | 757.559              | -0.157      |
| Predictive value | 180 | 757.472                  | -0.159       | 757.461              | -0.098      |
|               | 200  | 757.385                  | -0.090       | 757.382              | -0.035      |
In the 10th and 11th actual measurements, the vault elevations were 757.293cm and 766.042cm, and the predicted elevations in the GM (1,1) model were 757.472cm and 757.385cm. In the model, the predicted dome elevations are 757.461cm and 757.382cm. It is not difficult to find that the predicted value of the ordinary GM (1,1) gray model differs from the actual value by 0.179cm and 0.343cm, and the predicted value of the improved gray model differs from the actual value by 0.167cm and 0.341cm. The residuals calculated in the GM (1,1) model are -0.159 and -0.90, respectively. In the improved gray prediction model, the residuals are -0.098 and -0.035. By comparison, it can be found that the predicted value of the improved gray prediction model is closer to the actual measurement value, and its residual value is smaller, which further proves that the improved gray prediction model has better accuracy and higher prediction ability. A comparison of the measured values with the GM (1,1) model and the improved grey prediction model is shown in Figure 3.

![Figure 3 Deformation curve of arch surrounding rock](image)

It is not difficult to see from the deformation curve that the measured values of the arched surrounding rock are unevenly distributed, with no regularity and certain errors. Therefore, there will be some errors from the predicted values. However, the 10th and 11th predictions have less error values from the original measurement values, less degree of fit, and their errors are within the allowable range. Through comparison, it is found that the predicted value curve of the improved gray model and the GM (1,1) model predicted value curve are basically consistent with the measured value curve trend, but the predicted value curve of the improved gray model coincides with the measured value curve, which is closer to measurement Value polyline. The maximum error between the predicted value of the improved gray model and the actual measured elevation is 0.091 cm, which meets the engineering requirements.

4. Conclusion
Taking the old wooden tunnel of the 10th section of the Shiqian to Yuping highway in Guizhou as the research object, based on the research related to the GM (1,1) model, the cumulative method is combined with the GM (1,1) model An improved gray prediction model was obtained, and the model was used to predict the surrounding rock deformation during the construction of the old wooden tunnel. The following conclusions were mainly obtained:

(1) Through the optimization of the background value, the adjustment factor \( \lambda \) is introduced, and the cumulative method is combined with the ordinary gray model to eliminate the structural error of the GM (1,1) model.

(2) The GM (1,1) model's predicted values will have outliers at some points. In the improved gray model, the probability of outliers is reduced, and it is closer to the measured values, which has better accuracy Degree, and solves the problem of morbidity in the traditional gray prediction model.
(3) The improved grey prediction model uses the old wooden tunnel to predict the surrounding rock of the vault, and compared with the traditional grey prediction model, the predicted value curve of the improved grey model fits the measured value better. The average accuracy of the residual error of monitoring point 1 on site increased by 11.06%, and the maximum error between the predicted value of the improved gray model and the actual measured elevation was only 0.091 cm, and the prediction accuracy was high.

References
[1] Guo Yunkai, Xie Teng, Cheng Gang, et al. Application of Non-equidistant Grey Forecasting Model in Monitoring of Multi-arch Tunnels [J]. Modern Tunnel Technology, 2013, 50 (01): 73-79.
[2] Wu Wenze, Zhang Tao, Jiang Jianming. Improvement and Application of Nonlinear Grey Bernoulli Model [J]. Mathematics in Practice and Theory, 2019, 49 (10): 73-79.
[3] Wang Lu, Sha Xiuyan, Xue Ying. Improved GM (1,1) Grey Forecasting Model and Its Application [J]. Statistics and Decision, 2016 (10): 74-77.
[4] Li Lu, Hong Youtang, Ma Xiaofeng, et al. Application of improved grey system model to settlement prediction [J]. Beijing Surveying and Mapping, 2019, 33 (07): 754-758.
[5] Wen Jianhua, Lu Jing, Chen Zijun, et al. An Improved GM (1,1) Prediction Model for Convergence and Deformation of Tunnel Surrounding Rock [J]. China Science and Technology Paper, 2014, 9 (05): 587-589 + 593.
[6] Yuan Quan, Zeng Xiangyan. GM (0, N) Model Based on Innovation First Accumulation and Its Application [J]. Statistics and Decision, 2018, 34 (12): 79-81.