THE BLOOM-GILMAN DUALITY AND LEADING LOGARITHMS

Carl E. Carlson$^a$ and Nimai C. Mukhopadhyay$^b$

$^a$ Physics Department, College of William and Mary
Williamsburg, Virginia 23187

$^b$ Physics Department, Rensselaer Polytechnic Institute
Troy, NY 12180-3590

(July, 1994)

Abstract

The existing inclusive electroproduction data base allows us a look at the issue of the relative behaviors of background and resonance excitations, a part of the Bloom-Gilman duality. These data lack accuracy at high $Q^2$, but establish PQCD scaling in the resonance region and even allow us a glimpse at the leading logarithmic corrections due to the gluon radiation and its possible quenching at large $W$ and $x$. These should inspire better quality experimental tests at facilities like CEBAF II.

PACS numbers: 12.38, 14.20.D

Typeset using REVTeX
I. INTRODUCTION

Now that CEBAF is here, it is quite appropriate to ponder over the question, what if CEBAF had an electron beam of 8 GeV energy and higher. Before such a prospect becomes a realistic proposal, this workshop seeks to identify interesting physics issues that need critically such a higher energy facility. Purpose of this paper is to point out one such high-$Q^2$ physics problem that could be helped at such a facility: investigation of the leading log effects from QCD in the resonance electroweak form factors [1]. One of most fundamental issues in hadron physics is the possibility of reaching the perturbative domain of quantum chromodynamics (PQCD): in such a domain, the PQCD rules the behavior of the electroweak structure function [2]. There is still considerable debate on the domain of validity of the PQCD.

II. THE PQCD RULES FOR HELICITY AMPLITUDES FOR ELECTROEXCITING A RESONANCE

For the helicity amplitudes $G_{\pm,0}$, defined in the Breit frame, for the electroproduction of a resonance, the PQCD rules tell us [2]

$$G_i = \frac{g_i}{Q^n}$$

(1)

where for $i = +, 0$ and $-$ respectively, $n = 3, 4$ and 5, where $Q^2$ is the invariant mass squared and $g_i$’s are constants modulo log$Q^2$, dependent on the distribution amplitude of the relevant hadrons. In the amplitudes by Chernyak and Zhitnitsky, there is an accidental cancellation, making $g_+$ really small [3]. Checking such a prediction is a fundamental enterprise in the resonance physics. Stoler’s analysis [4] leads support to this particular prospect for the Delta(1232), but this is not necessarily the case, according to Davidson and Mukhopadhyay [5] in a different approach, indicating intrinsic model dependence in this sort of analysis.

Another interesting point in the PQCD rule has been pointed out by the present authors [6]. The above rules have basically assumed three quarks as the leading Fock components
for the baryons. In the case of hybrids, which contain valence gluons, the transverse electroproduction is small, compared to “normal” baryons. but the longitudinal one is not, and it scales like that for the normal baryons. Similarly, characteristic high momentum transfer signatures of PQCD can tell whether hadrons such as the $\Lambda(1405)$, $f_0(575)$ and $a_0(980)$ have normal three-quark (baryon), quark-antiquark(mesons) leading Fock configurations.

III. THE BLOOM-GILMAN DUALITY

First noted by Bloom and Gilman [7], more than two decades ago, this duality contains two parts:

(a) Resonances excited in the e-p scattering fall at roughly same rate as the background underneath them with increasing $q^2 = -Q^2$.

(b) The smooth scaling limit seen at high $Q^2$ and $W$ for the structure function $\nu W_2(\omega')$, where $\omega' = 1 + W^2/Q^2$, is an accurate average for bumps seen at lower $Q^2$ and $W$, but at same $\omega'$.

It is the second observation that connects the above behavior with the classical definition of duality, originally proposed by Dolan, Horn and Schmid [8].

The BG duality can be explained by the PQCD rules, as was shown by DeRújula, Georgi and Politzer [9] and the present authors [2]. DeRújula et al. showed that the corrections to the lower moments of the structure function due to final state interactions (or higher twist effects) are small, while the corrections to higher moments are large. Thus, the average value of the structure function cannot be very different from its values at high $Q^2$. The present authors extended this concept to the longitudinal structure function as well.

The BG duality has a strong significance for testing of the PQCD rules in the resonance region: Due to the first part of the duality, there is no need to separate the background and resonance, as has been done by Stoler [4] and other authors [10] to test the PQCD rule. The structure function as a whole can be checked for scaling.

Carlson and Mukhopadhyay [1] have examined the dual relationship between the reso-
nance and the scaling curves for the inelastic structure function in the $Q^2$ range of 1 to 17 $(GeV/c)^2$ and found no exception to this rule. Thus, $W_2$, averaged over a suitable range of the Bjohen $x$ (or the equivalent Nachtmann $\xi$), yields the smooth curve seen at higher values of $Q^2$ at the same value of $x$, for all resonance regions. Looked this way, even the Delta(1232), earlier suspected of “misbehaving”, appears quite “normal” or “unexceptional”.

We have done the above analysis by dividing the resonance regions into three zones, $1.12 \leq W \leq 1.38$ the domain of the Delta(1232); $1.38 \leq W \leq 1.62$, the region containing $N^*(1520)$ and $N^*(1535)$; $1.61 \leq W \leq 1.80 GeV$, with resonance bumps around 1.7 GeV. The analysis computes the integrals

$$I_i = \int_{\Delta x_i} dx F_2(x, Q^2), \quad (2)$$

and

$$S_i = \int_{\Delta x_i} dx F_2^{scaling}(x), \quad (3)$$

where $\Delta x_i$ is the region of $x$ corresponding to the above $W$ intervals. We have then computed the ratios $R_i = I_i/S_i$. We have used the constancy of each $R_i$ as the test for the BG duality. We have also used the Nachtmann variable $\xi = 2x/(1+\sqrt{1+Q^2/\nu^2})$, better for the inclusion of the target mass effect and we have tested various scaling functions [1]. Our conclusion: The BG duality works nicely above $Q^2$ of the order of four GeV$^2$, in all three resonance regions!

**IV. LEADING LOGS: ARE GLUON RADIATIONS DAMPED AT HIGH X?**

We have recently studied [1] leading log corrections to the inelastic scattering structure function at high Bjorken $x$. We first investigate these corrections on the parton distribution function. Starting with the Altarelli-Parisi equation having unsuppressed gluon radiation, we find

$$q(x, t) = N_0(1 - x)^{4+\frac{16}{3}(\ln\ln Q^2/\beta_1)}, \quad (4)$$
where \( q(x, t) \) is the quark distribution function of a given flavor, starting with the form

\[
q(x, t_0) = N_0(1 - x)^b,
\]

where \( b \) is a constant, \( t_0 \) corresponds to some benchmark \( Q^2_0 \); symbols here have the usual meaning:

\[
t = \ell n(Q^2/\Lambda^2),
\]

\[
\beta_1 = 11 - \left( \frac{2}{3}n_f \right)
\]

\( n_f \), the number of fermion flavors,

\[
\ell n\ell nQ^2 = \ell n\left( \frac{\ell n(Q^2/\Lambda^2)}{\ell n(Q^2_0/\Lambda^2)} \right) \equiv T(Q).
\]

We find, for \( Q^2_0 = 4 \text{ GeV}^2 \) and \( Q^2 \approx 20\text{ GeV}^2 \), \( F_2 \) changes by about 0.57 due to the logarithmic corrections in the Delta region. Similar conclusions are reached for other resonances.

Moral: *Logs are important!*

V. THE BLOOM-GILMAN DUALITY, LEADING LOGS AND HIGHER TWISTS

Our observations can be summarized as follows:

1. The inclusion of logarithmic effects helps to make the BG duality idea work *better*.

2. For distribution amplitudes due to Chernyak and Zhitnitsky and King and Sachrajda, just to mention two we have examined, the BG duality is logarithmically violated.

3. At \( W > 2\text{ GeV} \) and high \( x \) (\( x > 0.70 \)), the *uncorrected* \( (1 - x)^3 \) form fits the data better, in agreement with an argument due to Brodsky *et al.* [12] that the logarithms are *healed* in the region where \((1 - x)Q^2 \) is small.

Points (1) and (3) will not contradict each other if we introduce a \( W \)-dependent higher twist correction such that we have

\[
F_2 \propto (1 - x)^{3 + \frac{16}{3}T(Q)/\beta_1}(1 + C_2 \frac{m_N^2}{W^2}).
\]
From the available data, we get

\[ C_2 = 1.7. \] (10)

VI. CONCLUDING REMARKS

The existing inclusive electroproduction data in the resonance region, poor though they are, still give valuable insights into the Bloom-Gilman duality and the effects of the gluonic radiation via logarithms. Log corrections seem to be important in the resonance region, but at high \( x \) and \( W > 2GeV \), a plain \( (1 - x)^3 \) fits the data. This requires a hypothesis of evolution healing or the presence of higher twist effects. Measuring structure function over a range of \( x \) at fixed values of \( W \) and \( Q^2 \) respectively would deepen our insight into these mechanisms.

That brings us to CEBAF II. Amen to that!

VII. ACKNOWLEDGEMENT

We are grateful for our research support from the NSF(CEC) and the U. S. Department of Energy (NCM). We thank S. Brodsky and P. Stoler for many helpful discussions.

* Invited talk at the 1994 CEBAF workshop, presented by N. C. Mukhopadhyay.
REFERENCES

[1] C. E. Carlson and N. C. Mukhopadhyay, *Preprint RPI-94-N90* (WM-94-106), (May 1994), to be published.

[2] C. E. Carlson and N. C. Mukhopadhyay, *Phys. Rev.* **D41** (1990) 2343 and refs. therein.

[3] V. L. Chernyak and I. R. Zhitnitsky, *Nucl. Phys.* **B246** (1984) 52; *Phys. Rep.* **112** (1984) 173.

[4] P. Stoler, *Phys. Rep.* **226** (1993) 103 and refs. therein.

[5] R. M. Davidson and N. C. Mukhopadhyay, *Preprint RPI-94-N93*, (June 1994), to be published.

[6] C. E. Carlson and N. C. Mukhopadhyay, *Phys. Rev. Lett.* **67** (1991) 3745.

[7] E. D. Bloom and F. J. Gilman, *Phys. Rev. Lett.* **25** (1970) 1140.

[8] R. Dolan, D. Horn and C. Schmid, *Phys. Rev.* **166** (1968) 1768.

[9] A. De Rújula, H. Georgi and H. D. Politzer, *Ann. Phys.* (N. Y.) **103** (1977) 315.

[10] C. Koeppel, *Ph. D Thesis* (American Univ., 1994), unpublished.

[11] C. E. Carlson and N. C. Mukhopadhyay, *Phys. Rev.* **D47** (1993) R1737.

[12] S. J. Brodsky, private communication, (1994). S. J. Brodsky, T. Huang and G. P. Lepage, *Particle and Fields* **2**, ed. A. Z. Capri and A. N. Kamal (Plenmm, New York, 1982).