Quantum correlation of light mediated by gravity

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We consider using the quantum correlation of light in two optomechanical cavities, which are coupled to each other through the gravitational interaction of their end mirrors, to probe the quantum nature of gravity. The optomechanical interaction coherently amplifies the correlation signal, and a unity signal-to-noise ratio can be achieved within one-year integration time by using high-quality-factor, low-frequency mechanical oscillators.

Introduction.—Constructing a consistent and verifiable quantum theory of gravity is a challenging task of modern physics [1–3], which is partially due to the difficulty in observing quantum effects of gravity. This, to certain extents, motivates some theoretical models that treat gravity as a fundamental classical entity [4–11] or being emerged from some yet-to-known underlying fundamental microphysics [12–15]. Probing the quantum nature of gravity experimentally is therefore essential for providing hints towards constructing the correct model [16, 17]. Recently, there are two experimental proposals about demonstrating gravity-induced quantum entanglement between two mesoscopic test masses in matter-wave interferometers [18, 19], motivated by an early suggestion of Feynman [20]. The setup involves two interferometers located close to each other and their test masses are entangled through the gravitational interaction. The key requirement is to prepare the test mass in a quantum superposition state for a sufficient amount of time so that the accumulated quantum phase from the gravitational interaction becomes significant. There are some discussions regarding whether the gravity-mediated entanglement in the Newtonian limit proves the quantuness of gravity or not [21–24], because the radiative degrees of freedom—graviton, are not directly probed in these experiments. Given the lack of experimental evidence, such experiments are important first steps towards understanding gravity in the quantum regime. Interestingly, they are also sensitive to gravity-induced decoherence models for explaining the quantum-to-classical transition [25–30].

In this paper, we propose to use linear optomechanical devices [31, 32] to realise gravity-mediated quantum correlation of light—an intermediate step towards demonstrating the entanglement. The setup is shown schematically in Fig. 1. Two optomechanical cavities are placed close to each other with their test-mass mirrors (mechanical oscillators) interacting through gravity. Different from the single-photon nonlinear regime studied by Balushi et al. [33], we are considering the linear optomechanical interaction with the cavity driven by a coherent laser, and having the light (optical field) and mechanical oscillators in Gaussian states. The quantum correlation of light is measured by cross-correlating the homodyne readouts of two photodetectors. With the system being in a steady state, the signal-to-noise ratio (SNR) for the correlation measurement grows in time. As shown later in Eq. (16), the integration time for achieving a unity SNR is

$$\tau \approx 1.0 \text{ year} \left( \frac{n_{th}/C}{0.4} \right) \left( \frac{\omega_m/2\pi}{1 \text{ Hz}} \right)^3 \left( \frac{10^6}{Q_m} \right) \left( \frac{19 \text{ g/cm}^3 \pi^2}{\rho} \right)^{3/2},$$

(1)

where $n_{th}$ is the thermal occupation number, $C$ is the optomechanical cooperativity, $\omega_m$ is the mechanical resonant frequency, $Q_m$ is the quality factor, and $\rho$ is the matter density of the test mass comparable to that of Gold or Tungsten. To constrain the integration time within one-year scale given $\omega_m/2\pi = 1 \text{ Hz}$ and $Q_m = 10^6$, we require the cooperativity $C$ being higher than $n_{th}$—the quantum radiation pressure limited regime. Several optomechanical experiments have achieved such a regime but with high-frequency mechanical oscillators above 1 Hz [34–37]. Advancing the experimental techniques towards low frequencies, also an effort in the gravitational-wave community [38–41], is the key to measure the gravity-mediated quantum correlation.

The same correlation is also responsible for the entanglement of the optical fields in these two cavities. However, a direct observation of such an optical entanglement demands more experimental efforts than verifying the quantum correlation. It is as challenging as observing the steady-state Gaussian entanglement between two test masses mediated by gravity, which was recently considered by Qvarfort et al. [1] in the case of two levitating nanobeads [42]. It requires the thermal decoherence rate $\dot{n}_{th}\omega_m/Q_m$ to be smaller than the coherent gravitational coupling rate $G\rho/\omega_m$ ($G$ is the gravitational constant), as shown explicitly by Eq. (24). In the text that follows,

![FIG. 1. Schematics showing the setup of two optomechanical cavities with their end mirrors coupled to each other through gravity. The quantum correlation of light is inferred by cross-correlating the readouts of two photodetectors.](image-url)
we provide the details for deriving our main result.

Dynamics.—The derivation follows the linear-dynamics analysis in quantum optomechanics [31, 32]: solving the linear Heisenberg equations of motions for dynamical variables, which are the oscillator position and quadratures of the outgoing optical fields, and representing them in terms of external fields, which are the ingoing optical fields and the thermal bath field.

Specifically, the total Hamiltonian of the system is \( \hat{H}_{\text{tot}} = \hat{H}_A + \hat{H}_B + \hat{H}_{AB} \). The individual cavity is quantified by the standard linearised optomechanical Hamiltonian \( \hat{H}_{AB} \), which describes the radiation pressure coupling between the optical field and the mechanical oscillator in the presence of a coherent laser. More explicitly, the interaction part of \( \hat{H}_A \) for cavity A is (similarly for B):

\[
\hat{H}^\text{int}_A = \hbar \omega_q \hat{X}_A \hat{Q}_A. \tag{2}
\]

We denote \( \hat{X}_A \) as the amplitude quadrature of the cavity mode, which is conjugate to the phase quadrature \( \hat{Y}_A \): \( [\hat{X}_A, \hat{Y}_A] = i \), and \( \hat{Q}_A \) as the oscillator position with the zero-point motion \( \sqrt{\hbar/(2m\omega_0)} \). The parameter \( \omega_q \) describes the optomechanical coupling strength:

\[
\omega_q \equiv \sqrt{\frac{2P_{\text{cav}}\omega_0}{mcL\omega_m}}, \tag{3}
\]

which depends on the intra-cavity optical power \( P_{\text{cav}} \), the laser frequency \( \omega_0 \), the oscillator mass \( m \), and the cavity length \( L \).

The interaction Hamiltonian \( \hat{H}_{AB} \) between two oscillators mediated by gravity goes as follows:

\[
\hat{H}_{AB} = \hbar \omega_q \hat{Q}_A \hat{Q}_B. \tag{4}
\]

We have assumed two oscillators having the same mass and frequency, and ignored high-order terms of \( \hat{Q}_{A,B} \). Here \( \omega_q \) is equal to \( \sqrt{GM/d^3} \) when the two oscillators have a mean separation \( d \) much larger than their size, which is the case for macroscopic levitating test masses considered in Refs. [18, 19, 42]. For macroscopic test-mass oscillators of gram or kilogram scale, their separation can be made comparable to their size and yet not affected by e.g. the Casimir effect. In this case, we have

\[
\omega_q = \sqrt{\Lambda G \rho}, \tag{5}
\]

which does not depend explicitly on the size of the oscillators. The form factor \( \Lambda \) is determined by the geometry of two oscillators. It is \( \pi/3 \) for two spheres with the mean separation equal to twice of the radius, and we assume it is 2.0, which is a good approximation for two closely-located disks with the radius being 1.5 times its thickness (see Appendix A for more details).

Solving the Heisenberg equations of motions results in the following frequency domain input-output relation for cavity A (similarly for cavity B):

\[
\dot{\hat{X}}^\text{out}_A(\omega) = \hat{X}^\text{in}_A(\omega), \tag{6}
\]

\[
\dot{\hat{Y}}^\text{out}_A(\omega) = \hat{Y}^\text{in}_A(\omega) + \sqrt{2/\gamma} \omega_q \hat{Q}_A(\omega), \tag{7}
\]

where we have assumed that the cavity bandwidth \( \gamma \) is much larger than the frequency of interest so that the cavity mode can be adiabatically eliminated, cf. Eq. (2.68) in Ref. [31]. The position of oscillator A satisfies

\[
\dot{\hat{X}}_A = \chi_{qq} \sqrt{\gamma/2} \omega_q \hat{X}^\text{in}_A - (\omega_q^2/\omega_m) \hat{Q}_B + 2 \sqrt{\gamma} \hat{Q}_B^\text{th}. \tag{8}
\]

Here \( \chi_{qq} \equiv -\omega_m/(\omega_q^2 - \omega_m^2 + i\gamma_m\omega) \) is the mechanical susceptibility and \( \gamma_m \) is the mechanical damping rate; \( \hat{Q}_B^\text{th} \) is the normalised thermal Langevin force, of which the double-sided spectral density is equal to \( \hat{Q}_B + (1/2) \) with the thermal occupation number \( \hat{n}_m \equiv k_B T/(\hbar\omega_m), \) where \( T \) is the environmental temperature.

The final input-output relation involving both cavities is given by

\[
\begin{bmatrix}
\dot{\hat{X}}^\text{out}_A \\
\dot{\hat{Y}}^\text{out}_A \\
\dot{\hat{X}}^\text{out}_B \\
\dot{\hat{Y}}^\text{out}_B
\end{bmatrix} =
\begin{bmatrix}
\mathcal{K}_A & 1 & 0 & 0 \\
0 & \mathcal{G} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \beta_B
\end{bmatrix}
\begin{bmatrix}
\hat{X}^\text{in}_A \\
\hat{Y}^\text{in}_A \\
\hat{X}^\text{in}_B \\
\hat{Y}^\text{in}_B
\end{bmatrix} +
\begin{bmatrix}
\alpha_A & 0 & 0 & 0 \\
0 & \alpha_B & 0 & 0 \\
0 & 0 & \alpha_B & 0 \\
0 & 0 & 0 & \alpha_B
\end{bmatrix}
\begin{bmatrix}
\hat{Q}_A^\text{th} \\
\hat{Q}_B^\text{th}
\end{bmatrix}. \tag{9}
\]

Here \( \mathcal{K} \equiv -4\omega_q^2\chi_{qq}/\gamma \) quantifies the correlation between the amplitude quadrature and the phase quadrature in the individual cavity and is responsible for the optomechanical squeezing [43–47]. The two parameters \( \alpha \equiv 2 \sqrt{2\gamma_m/\gamma} \omega_q \chi_{qq} \) and \( \beta \equiv \alpha \chi_{qq}(\omega_q^2/\omega_m) \) quantify the output response to the thermal noise. The dimensionless parameter \( \mathcal{G} \) quantifies the mutual correlation between two cavities and is defined as \( \mathcal{G} \equiv 4\omega_q^2\chi_{qq}^2/\gamma (\omega_m) \). Its magnitude reaches the maximum at the mechanical resonant frequency and can be rewritten as

\[
|\mathcal{G}(\omega_m)| = 2 \sqrt{\mathcal{C} \mathcal{G} B \omega_m \omega_q^2/\gamma \omega_m^2}. \tag{10}
\]

The mechanical quality factor \( Q_m \) is related to \( \omega_m \) and \( \gamma_m \) by \( Q_m = \omega_m/\gamma_m \). The optomechanical cooperativity \( \mathcal{C} \) is defined as [32]:

\[
\mathcal{C} \equiv \frac{2\omega_q^2}{\gamma \omega_m}, \tag{11}
\]

which is proportional to the number of intra-cavity photons. The fact that \( |\mathcal{G}| \) is proportional to \( \sqrt{\mathcal{C} \mathcal{G} B} \) shows that the optomechanical interaction coherently enhances the gravity-induced correlation by amplifying the quantum fluctuation of light.

Cross correlation.—The gravitational interaction correlates different quadratures of the outgoing fields of the two cavities, with the magnitude quantified by \( \mathcal{G} \). To infer such a quantum correlation, we apply the data analysis technique used for searching the stochastic gravitational-wave background [48–50], which also has been recently applied to analyse the quantum correlation in the large-scale gravitational-wave detector [51] and the Holometer [52]. We correlate the measurement results of the amplitude quadrature \( \hat{X}^\text{out}_A \) of cavity A and of the phase quadrature \( \hat{Y}^\text{out}_B \) of cavity B, and construct the following estimator:

\[
\dot{\mathcal{C}}_{XY} \equiv \int_{-\tau/2}^{\tau/2} dt dt' \dot{\hat{X}}^\text{out}_A(t) \mathcal{F}(t-t') \hat{Y}^\text{out}_B(t'). \tag{12}
\]
where $\tau$ is the integration time, and $\mathcal{F}$ is some filter function. The correlation signal is given by the expectation value of $\hat{C}_{XY}$ averaged over the quantum state $|\phi\rangle$ of the optical field. The estimation error is determined by the noise terms in $\hat{Y}_{B}^{\text{out}}$ that are uncorrelated with $\hat{X}_{B}^{\text{out}}$, which we shall sum together as $\hat{N}_{B}^{\text{out}}$. The SNR in general depends on the choice of the filter function $\mathcal{F}$ [48], and, as shown in the Appendix B, maximising the SNR over $\mathcal{F}$ leads to

$$\text{SNR} = \sqrt{\tau} \left[ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left| S_{XX}(\omega)S_{NN}(\omega) \right| \right]^{1/2},$$

where the cross spectral density $S_{XX}$ is the Fourier transform of $\langle \phi | \hat{X}_{A}^{\text{out}}(t) \hat{Y}_{B}^{\text{out}}(0) | \phi \rangle$, and similarly $S_{XX}$ and $S_{NN}$ are the spectral densities for $\hat{X}_{A}^{\text{out}}$ and $\hat{N}_{B}^{\text{out}}$, respectively. In obtaining the above result, we have assumed that the integration time is much longer than the damping time of the mechanical oscillator, namely $\tau \gg 2\pi/\gamma_m$. The SNR grows in time, which is a generic feature of such a correlation measurement.

Since the linearised optomechanical interaction Eq. (2) already accounts for the coherent amplitude of the field, the coherent state $|\phi\rangle$ of the optical field is transformed unitarily into the vacuum state $|0\rangle$ [31], and we have

$$S_{XX} = \mathcal{G}^2/2, \quad S_{NN} = 1/2,$$

$$S_{NN} = [1 + |\mathcal{K}_B|^2 + (2\bar{n}_B + 1)(|\alpha_B|^2 + |\beta_B|^2)]/2.$$

Since both $\mathcal{G}$ and $\mathcal{K}_B$ depends on the optical power of cavity $B$, increasing $P_{\text{cav}}^B$ does not necessarily increase the SNR. Maximising the SNR over $P_{\text{cav}}^B$ leads to $\omega_{q,\text{opt}}^B = \sqrt{\gamma/(4|\mathcal{K}q|)}$ and

$$\text{SNR} = \sqrt{\tau} \left[ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \gamma \omega^2 \left| S_{XX}(\omega)S_{NN}(\omega) \right| \right]^{1/2},$$

where we have ignored $|\beta_B|^2$ as it is much smaller than $|\alpha_B|^2$. Since the integrand reaches its maximum at $\pm \omega_m$, we can replace $S_{XX}(\omega) = \gamma \omega^2 \left| S_{XX}(\omega)S_{NN}(\omega) \right|$ in the denominator as $\gamma \omega_m^2 = \gamma_m$. Finally, by completing the integration we obtain

$$\text{SNR} = \sqrt{\tau} \left[ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \gamma \omega^2 \left| S_{XX}(\omega)S_{NN}(\omega) \right| \right]^{1/2}. $$

To reach a unity SNR, the integration time therefore needs to satisfy Eq. (1) for $\bar{n}_B \gg 1$ and given the form factor $\Lambda = 2.0$. We have also assumed $\bar{n}_B$ is equal to $\bar{n}_h^B$ in Eq. (1). In principle, these two cavities can operate at different temperatures, and the requirement of having cavity A quantum-radiation-pressure limited is no longer needed, as long as $\bar{n}_B$ is much smaller than $\bar{n}_h^A$ and $\mathcal{C}_A/\bar{n}_h^B < 1$.

**Entanglement.—**The quantum correlation discussed so far is also the one responsible for the entanglement between the optical fields in the two cavities, which is encoded in the the entire covariance matrix $\mathbf{V}$ of the outgoing fields with the correlation being the off-diagonal term of $\mathbf{V}$. Since the correlation reaches the maximum at the mechanical resonance frequency and has a narrow bandwidth in frequency for high-quality-factor oscillators, we can focus on the optical modes specifically around $\omega_m + \omega_m$ and $\omega_m - \omega_m$. The relevant quadrature operators for these two modes in the two-photon formalism [53] are defined as

$$\hat{X} \equiv \sqrt{\Delta \omega/\pi} \hat{X}_{B}^{\text{out}}(\omega_m), \quad \hat{Y} \equiv \sqrt{\Delta \omega/\pi} \hat{Y}_{B}^{\text{out}}(\omega_m),$$

where $\hat{X}_{B}^{\text{out}}(\omega_m)$ is the Fourier transform of $\hat{X}_{B}^{\text{out}}(t)$ at $\omega_m$. They satisfy $[X, Y] = 2i$. We have used the approximation of Dirac delta function $\delta(0) \approx 1/\Delta \omega$. For this approximation to be valid, the bandwidth $\Delta \omega$ needs to be at least of the same order as $\gamma_m$—the smallest frequency scale in our system, which also implies a measurement time longer than the mechanical damping time when one tries to verify such an entanglement. The covariance matrix is defined as

$$\mathbf{V} = \langle \phi | \hat{X}_A \hat{Y}_A \hat{X}_B \hat{Y}_B | \phi \rangle [\hat{X}_A \hat{Y}_A \hat{X}_B \hat{Y}_B | |\phi\rangle_{\text{sym}}$$

where superscript “$T$” means transpose and subscript “sym” means symmetrisation: $\langle \phi | \hat{X}_{B}^{\text{out}}(\omega_m) | \phi \rangle_{\text{sym}} = \langle \phi | \hat{X}_{B}^{\text{out}}(\omega_m) \hat{Y}_{B}^{\text{out}}(\omega_m) | \phi \rangle/2$. The total covariance matrix is

$$\mathbf{V} = \begin{bmatrix} V_A & V_{AB} \\ V_{AB}^T & V_B \end{bmatrix}.$$ (18)

The diagonal components $V_A = V_B$ are the autocorrelation:

$$V_A = \begin{bmatrix} 1 & \mathcal{K}^* \\ |\mathcal{K}|^2 + |\mathcal{G}|^2 + (2\bar{n}_B + 1)(|\alpha|^2 + |\beta|^2) \end{bmatrix}.$$ (19)

where, for simplicity, we have assumed that two cavities have the same optical power: $\omega_{q} = \omega_{qB}$ so that $K_A = K_B = K$. The off-diagonal one, describing the cross correlation, is

$$V_{AB} = \begin{bmatrix} 0 & \mathcal{G}^* \\ \mathcal{G} & 0 \end{bmatrix}.$$ (20)

All the above quantities $\mathcal{K}, \mathcal{G}, \alpha$ and $\beta$ are referring to their values at $\omega_m$.

The figure of merit for quantifying such a bipartite Gaussian entanglement is the so-called logarithmic negativity $\mathcal{E}_N$ [54, 55], which can be derived from $\mathbf{V}$:

$$\mathcal{E}_N = \text{max} \left[ -\frac{1}{2} \ln \left( |\Sigma - \sqrt{\Sigma^2 - 4\text{det } \mathbf{V}}| / 2 \right), 0 \right],$$ (21)

where $\Sigma \equiv \det V_A + \det V_B - 2 \det V_{AB}$. A nonzero $\mathcal{E}_N$ implies the existence of entanglement. In our case, the first term is

$$-\ln \left( \sqrt{1 + |\mathcal{G}|^2 + (2\bar{n}_B + 1)(|\alpha|^2 + |\beta|^2) - |\mathcal{G}|^2 } \right).$$ (22)

Having it larger than zero requires

$$(2\bar{n}_B + 1)(|\alpha|^2 + |\beta|^2) < 2|\mathcal{G}|^2.$$ (23)

When using the fact that $|\alpha| \gg |\beta|$ and $\bar{n}_B \gg 1$, we arrive at the following condition with $\Lambda = 2.0$:

$$\frac{T}{\Omega_m} \leq \frac{hG\rho}{k_B\omega_m} \approx 1.5 \times 10^{-18} K \left( \frac{1 \text{ Hz}}{\omega_m/2\pi} \right) \left( \frac{\rho}{19 \text{ g/cm}^3} \right).$$ (24)

This requirement for creating entanglement is more stringent than the one shown in Eq. (1) for measuring the quantum correlation. It is beyond the state-of-the-art of quantum optomechanics, and requires further experimental efforts.
Conclusions and Discussions.—To summarise, our approach for probing the quantum nature of gravity takes advantage of new advancements in quantum optomechanical experiments, and is complimentary to other approaches based upon matter-wave interferometers. It requires quantum radiation pressure limited systems with high-quality-factor, low-frequency mechanical test masses. Even though the correlation signal does not explicitly depend on the mass of the test mass, having a low resonant frequency usually implies macroscopic test masses. For illustration, we provide a possible set of sample parameters to reach $\bar{n}_m/C$ of the order of 0.4 mentioned in Eq. (1) with $\omega_m/(2\pi) = 1$ Hz and $Q_m = 10^5$:

$$\frac{\bar{n}_m}{C} \approx 0.4 \left( \frac{m}{1 \text{ g}} \right) \left( \frac{2 \text{ kW}}{P_{\text{cav}}} \right) \left( \frac{6000}{\text{Finesse}} \right) \left( \frac{T}{300 \text{ K}} \right), \quad (25)$$

which corresponds to a suspended high-finesse cavity with a gram-scale test-mass mirror at room temperature, close to what has been achieved by the MIT group [56]. The gravity experiments with milligram test masses [57, 58] are also promising if they can be pushed to the low-frequency regime.

Finally, we would like to make a comment on the consequence of different outcomes of such experiments. If we do not detect a predicted level of quantum correlation signal after a sufficiently long integration time, it will imply that the assumption on the gravity sector is problematic, cf. Eq. (4), because the quantum optomechanics experiments have already tested the quantum model of the optomechanical interaction. One compelling possibility is then gravity is classical, so that it does not appear in the quantum interaction Hamiltonian. If we do observe a non-zero correlation and calibrate properly the contributions from other classical sources of correlation, we will be able to rule out Schrödinger-Newton type of classical gravity models—the gravity is sourced by the expectation value of quantum matters [4–11], which does not lead to the quantum correlation. This is because the corresponding Schrödinger-Newton two-body interaction in our setup would be $\hbar(\omega_{fl}/\omega_m)(\bar{Q}_A)(\bar{Q}_B)$, and according to Eq. (8), the quantum part of $(\bar{Q}_A)$ is zero, as the expectation value (the first moment) of the quantum fluctuation $\hat{X}_A^{\text{in}}$ is zero. However, we cannot make a conclusive statement about emergent classical gravity theories. Since we only access the quantum correlation of the light, there could be hidden degrees of freedom that lead to decoherence of the entire system, and still allow the existence of the quantum correlation of the subsystem. Testing these theories would require the verification of quantum entanglement, which, as shown by Eq. (24), is much more challenging given the setup considered here.

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Depending on the geometry of the two test masses, the form factor in defining $\omega_{fl}$ in Eq. (5) is different. The simplest case is having two identical spheres with a uniform density, and $\Lambda = \pi/3$ when their mean separation is equal to twice of their radius. Here we consider two test masses that have the shape of a disk which is usually the geometry for mirrors of optical cavities. Since there is no analytical expression for the Newtonian force between two disks, we perform numerical integration of the force for disks with different ratios between the radius $R$ and the thickness $h$. We then take the derivative numerically with respect to their mean separation $d$ along the optical axis to obtain $\Lambda$ for different mean separations and the maximum $\Lambda$ is achieved when their surfaces are close to each other with $d$ approximately equal to $h$. Fig. 2 shows the result, and we can see the maximum value of $\Lambda$ for $R/h = 1.5$ is around 2.0, which is the one we assumed in the main text.

Appendix A: Dependence of $\Lambda$ on the test mass geometry

Here we show the details for deriving Eq. (13) by optimising the filter function $\mathcal{F}$. The basic logic follows Ref. [48]; a difference is that, in our case, the readout of the amplitude quadrature $\hat{X}_A^{\text{out}}$ of cavity A is effectively noiseless while both readouts considered in Ref. [48] have the noise.

The amplitude $\mu$ of the correlation signal is obtained by taking the expectation value of the estimator $\hat{C}_{XY}$ over the quantum state $\psi$ of the optical field:

$$\mu \equiv \langle \hat{C}_{XY} \rangle = \left\langle \psi \int_{-\tau/2}^{\tau/2} dt \int_{-\tau/2}^{\tau/2} dt' \hat{X}_A^{\text{out}}(t)\mathcal{F}(t-t')\hat{Y}_B^{\text{out}}(t') \psi \right\rangle = \int_{-\tau/2}^{\tau/2} dt \int_{-\tau/2}^{\tau/2} dt' \mathcal{S}_{XY}(t-t')\mathcal{F}(t-t'). \quad (B1)$$

FIG. 2. The form factor $\Lambda$ as a function of distance for different ratios between the radius $R$ and the thickness $h$ of the disk. As a reference, we also show the case of two spheres in dashed line. The lower bound of the distance for different curves are defined by the one when the two disks touch each other.

Appendix B: Optimal filter function
where $S_{XY}$ is the inverse Fourier transform of the cross spectral density $S_{XY}$. The estimation error $\sigma$ is given by

$$\sigma \equiv \left( \psi \left[ \int_{-\tau/2}^{\tau/2} dt \int_{-\tau/2}^{\tau/2} dt' \hat{S}_{\Delta X}^\text{out}(t) \hat{F}(t-t') \hat{N}_B^\text{out}(t') \right]^2 \right)^{1/2} = \left[ \int_{-\tau/2}^{\tau/2} \int_{-\tau/2}^{\tau/2} dt \int_{-\tau/2}^{\tau/2} dt'' \int_{-\tau/2}^{\tau/2} dt''' \hat{F}(t-t') \hat{S}_{XX}(t-t'') \hat{S}_{NN}(t'-t''') \hat{F}(t''''-t') \right]^{1/2},$$

(B2)

The SNR for the correlation measurement is defined as the ratio of the signal amplitude $\mu$ and the estimation error $\sigma$, namely,

$$\text{SNR} \equiv \frac{\mu}{\sigma},$$

(B3)

which is a functional of the filter function $\mathcal{F}$. If we view $\mathcal{F}$, $C_{NN}$, and $\mathcal{G}$ as matrices in the $L^2$ function space, we can symbolically rewrite SNR as

$$\text{SNR} = \frac{\text{Tr}[S_{XY}\mathcal{F}]}{\text{Tr}[\mathcal{F}S_{XX}S_{NN}\mathcal{F}]}^{1/2},$$

(B4)

where we have used the fact that $\mathcal{F}$ is symmetric. Similar to the derivation of the optimal matched filter in signal processing, the maximum SNR can be derived as

$$\text{SNR}_{\text{max}} = \sqrt{\text{Tr}[S_{XY}(S_{XX}S_{NN})^{-1}S_{XY}]}.$$  

(B5)

If the integration time $\tau$ is longer than the mechanical damping time $2\pi/\gamma_m$, we can approximate $\sin(\omega\tau-\omega\prime\tau)/(\omega-\omega')$ as the Dirac delta function $\delta(\omega-\omega')$, as $\gamma_m$ defines the smallest frequency scale of interest, and the above expression of SNR can then be written as Eq. (13) in the frequency domain.

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