GLEASON'S PROBLEM IN WEIGHTED BERGMAN SPACE ON EGG DOMAINS

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Abstract. In the paper, we discuss on the egg domains:

\[ \Omega_a = \left\{ \xi = (z, w) \in \mathbb{C}^{n+m} : z \in \mathbb{C}^n, w \in \mathbb{C}^m, |z|^2 + |w|^{2/a} < 1 \right\}, \quad 0 < a \leq 2. \]

We show that Gleason’s problem can be solved in the weight Bergman space on the egg domains. The proof will need the help of the recent work of the second named author on the weighted Bergman projections on this kind of domain. As an application, we obtain a multiplier theorem on the egg domains.

§1. Introduction

In the paper, we consider the egg domains:

\[ \Omega_a = \left\{ \xi = (z, w) \in \mathbb{C}^{n+m} : z \in \mathbb{C}^n, w \in \mathbb{C}^m, |z|^2 + |w|^{2/a} < 1 \right\}, \quad 0 < a \leq 2, \]

where \( z = (z_1, \ldots, z_n), w = (w_1, \ldots, w_m) \), \( |z|^2 = \sum_{j=1}^n |z_j|^2, |w|^2 = \sum_{j=1}^m |w_j|^2. \)

We also write \( \xi = (\xi_1, \ldots, \xi_{n+m}) \). If \( 0 < a \leq 2 \), then \( \Omega_a \) is a pseudoconvex domain with \( C^1 \) boundary and \( m \) pseudoconvex directions.

For \( \xi \in \Omega_a \), we put

\[ h(\xi) = h(z, w) = (1 - |z|^2)^a - |w|^2, \]

and for \( \sigma > -1, \quad 1 \leq p < \infty \), let \( L^p(\Omega_a, dv_\sigma) \) denote the space of measurable functions on \( \Omega_a \) for which

\[ \int_{\Omega_a} h^\sigma(\xi)|f(\xi)|^p dv(\xi) < \infty, \]

where \( dv \) is the volume measure on \( \Omega_a, dv_\lambda = h^\lambda dv. \)

As usual, \( H(\Omega_a) \) is the space of all holomorphic functions on \( \Omega_a \),

\[ A^p_\sigma(\Omega_a) = L^p(\Omega_a, dv_\sigma) \cap H(\Omega_a) \]

1991 Mathematics Subject Classification : 32A10, 32A30

Key words and phrases. Gleason problem, Bergman space.

Supported by the National Natural Sciences foundation of China and the National Education Committee Doctoral foundation of China.
denotes the weighted Bergman space.

Let \( X \) be some class of holomorphic functions in a domain \( \Omega \subset \mathbb{C}^N \). Gleason’s problem, denoted as \((X, a, \Omega)\), is the following:

For any given \( a \in \Omega \), \( f \in X \) and \( f(a) = 0 \), do there exist functions \( f_1, \ldots, f_N \in X \), such that
\[
 f(z) = \sum_{k=1}^{N} (z_k - a_k) f_k(z). \]

The difficulty of the Gleason’s problem depends on \( \Omega \) and function space \( X \). Gleason originally asked the problem for \((\mathcal{C}^0, \Omega)\), where \( \mathcal{C}^0 \) is the unit ball of \( C^0 \). This problem was solved by Leibenson. Subsequently, in the unit ball, Rudin\[5\], Zhu\[7\], Ren and Shi\[4\] respectively discussed the following Gleason’s problem in Lipschitz space and \( \mathcal{C}^k \) space.

In this paper, we will prove that Gleason’s problem \((\Omega_a, 0, A^p_\lambda(\Omega_a))\) \((1 < p < \infty, \lambda \geq 0 \ or \ p = 1, \lambda > -1)\) can be solved. Its proof based on the recent work of the second named author \[6\] on the weighted Bergman projections on the egg domains. Our main result is the following:

**Theorem A.** Gleason’s problem can be solved on \( A^p_\lambda(\Omega_a) \) \((1 < p < \infty, \lambda \geq 0 \ or \ p = 1, \lambda > -1)\). Furthermore, for any \( k \geq 1 \), there exist bounded linear operators \( A_\alpha \) \(|\alpha| = k\) on \( A^p_\lambda(\Omega_a) \), such that if \( f \in A^p_\lambda(\Omega_a) \), \( D^\alpha f(0) = 0 \) \(|\alpha| \leq m - 1\), then
\[
 f(z) = \sum_{|\alpha| = m} z^\alpha A_\alpha f(z) \]
on \( \Omega_a \), where \( \alpha = (\alpha_1, \ldots, \alpha_n) \) be multiindex, \(|\alpha| = \alpha_1 + \cdots + \alpha_n\).

As a direct corollary, we obtain a multiplier theorem.

**Theorem B.** For \( k = 1, \ldots, n + m \), the transformation
\[
 \sum_{\alpha} c_\alpha \xi^\alpha \rightarrow \sum_{|\alpha| \neq 0} \frac{\alpha_k}{|\alpha|} c_\alpha \xi^\alpha \]
maps \( A^p_\lambda(\Omega_a) \) into \( A^p_\lambda(\Omega_a) \). In other words, the complex sequence \( \{\frac{\alpha_k}{|\alpha|}\} \) is a multiplier of \( A^p_\lambda(\Omega_a) \) into \( A^p_\lambda(\Omega_a) \).

§2. SOME LEMMAS

For \( \sigma > -1 \), let \( K_\sigma \) be the Bergman Kernel function on \( A^2_\sigma(\Omega_a) \), then from \[6\],
\[
 K_\sigma(\xi, \xi') = \sum_{k=0}^{n+1} c_k \frac{(1 - < z, z' >)^{ak-n-1}}{(1 - < z, z' >)^a - < w, w' >)^{\sigma+m+k}}, \tag{1} \]
where \( \xi = (z, w) \), \( \xi' = (z', w') \) are points in \( \Omega_a \), and, \( c_k \) are constants only depending on \( m, n, \sigma, a \).

If \( 0 < a \leq 1 \), denote
\[
 G_\sigma(\xi, \xi') = \frac{(1 - < z, z' >)^{(a-1)(n+1)/2}}{(1 - < z, z' >)^a - < w, w' >)^{(\sigma+m+n+3)/2}}; \]
Lemma 1.

If \( 1 < a \leq 2 \), denote

\[
G_{\sigma}(\xi, \xi') = \frac{(1 - <z, z'>)^{(a-1)(n+1)/2}}{((1 - <z, z'>)^a - <w, w'>)^{(\sigma+m+n+2)/2}}.
\]

We will only discuss the case \( 0 < a \leq 1 \), since the case \( 1 < a \leq 2 \) is similar.

As usual, the symbol \( A \lesssim B \) means that there exists a constant \( C \) such that \( A \leq CB \).

**Lemma 1.**

\[
\left| \frac{\partial K_{\sigma}}{\partial \xi_k}(\xi, \xi') \right| \lesssim |G_{\sigma}(\xi, \xi')|^2, \quad (1 \leq k \leq n + m).
\]

**Proof.** If \( \xi = (z, w), \xi' = (z', w') \in \Omega_a \), then

\[
|z|^2 + |w|^{2/a} < 1, \quad |z'|^2 + |w'|^{2/a} < 1.
\]

Namely

\[
1 - <z, z'> |^{2a} \geq (1 - |<z, z'>|)^{2a} \geq (1 - |z||z'|)^{2a} \geq (1 - |z|^2)(1 - |z'|^2) > |w|^2|w'|^2,
\]

thus

\[
< w, w' > < |1 - < z, z' > |^a.
\]

Differentiating the both sides of the formula in (1), we obtain by (3)

\[
\left| \frac{\partial K_{\sigma}}{\partial z_k} \right| \lesssim \sum_{j=0}^{n+1} \frac{(1 - <z, z'>)^{aj-n+1+a-1}}{((1 - <z, z'>)^a - <w, w'>)^{\sigma+m+j+1}}.
\]

Again by (3), the \( j \)-th (\( 0 \leq j \leq n + 1 \)) summand can be controlled by the \( j + 1 \)-th summand, therefore, can be controlled by the \( n + 1 \)-th summand. This means (2) holds for any \( 1 \leq k \leq n \).

Similarly

\[
\left| \frac{\partial K_{\sigma}}{\partial w_k} \right| \lesssim \frac{(1 - <z, z'>)^{(a-1)(n+1)}}{((1 - <z, z'>)^a - <w, w'>)^{\sigma+m+n+2}} \left| \frac{(1 - <z, z'>)^{(a-1)(n+2)}}{((1 - <z, z'>)^a - <w, w'>)^{\sigma+m+n+2}} \right| = |G_{\sigma}(\xi, \xi')|^2.
\]

Here we use the condition of \( 0 < a \leq 1 \). This proves Lemma 1.

Put

\[
\psi_{k, r}(\xi, \xi') = \frac{<w, w'>^k}{(1 - <z, z'>)^r} \quad (k \in \mathbb{N} \cup \{0\}, \quad r > 0).
\]

Shi [6] proved that for \( s > -1 \),

\[
\int_{\Omega_a} h^s(\xi')|\psi_{k, r}(\xi, \xi')|^2dv(\xi') \leq \frac{\pi^{n+m}k!\Gamma(s+1)\Gamma(a(s+k+m)+1)}{\Gamma^2(r)\Gamma(s+k+m+1)|w|^{2k}} \sum_{j=0}^{\infty} \frac{\Gamma^2(j+r)|z|^{2j}}{\Gamma(a(s+k+m)+j+n+1)j!}.
\]
Lemma 2. If $0 < d < \sigma + 1$, then
\[
\int_0^1 \int_{\Omega_a} h'^{-d}(\xi')|G_\sigma(t\xi,\xi')|^2 dv(\xi')dt \lesssim h^{-d}(\xi). \tag{4}
\]

Proof. By the formula
\[
\frac{1}{(1-t)^s} = \sum_{k=0}^{\infty} \frac{\Gamma(k+s)}{k!\Gamma(s)} t^k \quad (|t| < 1, \ s > 0),
\]
we obtain
\[
G_\sigma(\xi,\xi') = \frac{1}{(1-<\xi,\xi'>)^{\frac{\sigma+m+n+2}{2}}} \frac{1}{(1-<w,w'>)^{\frac{\sigma+m+n+2}{2}}}
\]
\[
= \sum_{k=0}^{\infty} \frac{\Gamma(k+c)}{k!\Gamma(c)} <w,w'>^k (1-<\xi,\xi'>)^{ak+b}
\]
\[
= \sum_{k=0}^{\infty} \frac{\Gamma(k+c)}{k!\Gamma(c)} \psi_{k,ak+b}(\xi,\xi')
\]
where $b = \frac{a(\sigma+m)+n+2}{2}$, $c = \frac{\sigma+m+n+2}{2}$.

Denote $\mu = \sigma - d$. Since $\{\psi_{k,ak+b}\}_{k=0}^{\infty}$ is a orthogonal basis on $\Omega_a$ [6], Parseval equality tells us that

Left side of (4) = \[
\int_0^1 \int_{\Omega_a} h'^{\mu}(\xi')|G_\sigma(t\xi,\xi')|^2 dv(\xi')dt
\]
\[
= \sum_{l=0}^{\infty} \frac{\Gamma^2(l+c)}{(l!)^2\Gamma^2(c)} \int_0^1 \int_{\Omega_a} h'^{\mu}(\xi')|\psi_{l,al+b}(t\xi,\xi')|^2 dv(\xi')dt
\]
\[
= \sum_{l=0}^{\infty} \frac{\Gamma^2(l+c)}{(l!)^2\Gamma^2(c)} \frac{\pi^{n+m}l!\Gamma(\mu+1)\Gamma(a(\mu+l+m)+1)}{\Gamma^2(a+l+b)\Gamma(\mu+l+m+1)} |w|^{2l}
\]
\[
\sum_{j=0}^{\infty} \frac{\Gamma^2(j+la+b)}{\Gamma(a(l+m)+j+n+1)j!2(l+j)+1}
\]
Right side of (4) = \[
\frac{1}{((1-|z|^2)^a - |w|^2)^d}
\]
\[
= \frac{1}{(1-|z|^2)^{ad}} \frac{1}{(1-\frac{|w|^2}{1-|z|^2})^d}
\]
\[
= \sum_{l=0}^{\infty} \frac{\Gamma(l+d)}{\Gamma(l+d)} \sum_{j=0}^{\infty} \frac{\Gamma(j+a(l+d))}{\Gamma(a(l+d))j!} |z|^{2j}|w|^{2l}
\]
Comparing the coefficient of $|z|^{2j}|w|^{2l}$, we only need to prove the following inequality
\[
\frac{1}{2(l+j)+1} \frac{\Gamma^2(l+c)\Gamma(a(\mu+l+m)+1)\Gamma^2(j+al+b)}{\Gamma^2(la+b)\Gamma(\mu+l+m+1)\Gamma(a(\mu+l+m)+j+n+1)}
\]
\[
\lesssim \Gamma(l+d)\Gamma(j+a(l+d))
\]
\[
\lesssim \Gamma(l+d)\Gamma(l+a(l+d))
\]
while this can be changed to prove

$$\frac{\Gamma^2(j + la + b)}{(2(l + j) + 1)\Gamma(a(\mu + l + m) + j + n + 1)\Gamma(j + a(l + d))} \lesssim 1, \quad (5)$$

$$\frac{\Gamma^2(l + c)}{\Gamma(l + d)\Gamma(\mu + l + m + 1)} \lesssim 1,$$

where $b = \frac{a(\sigma + m) + n + 2}{2}$, $c = \frac{\sigma + m + n + 2}{2}$, $\mu = \sigma - d$.

Thus it remains to prove (5) and (6) for bigger enough $j, l$. This is not too hard to prove by the well known properties of $\Gamma$ function,

(i). $\Gamma(x + 1) = x\Gamma(x) \quad (x > 0)$,

(ii). $\frac{\Gamma^2(x + t)}{\Gamma(t)\Gamma(2x + t)} \leq 1 \quad (x \geq 0, \ t > 0)$,

(iii). $\lim_{t \to \infty} \frac{\Gamma^2(x + t)}{\Gamma(t)\Gamma(2x + t)} = 1 \quad (x \geq 0)$.

In fact,

Left side of (5) $\lesssim \frac{\Gamma^2(j + la + b)}{\Gamma(a(\mu + l + m) + j + n + 1)\Gamma(j + a(l + d) + 1)}$

$$\quad = \frac{\Gamma^2(j + la + \frac{a(\sigma + m) + n + 2}{2})}{\Gamma(a(\sigma - d + l + m) + j + n + 1)\Gamma(j + a(l + d) + 1)} \leq 1,$$

where the last step follows from (ii) in the properties of $\Gamma$ function, since in the fraction of the last inequality the sum of the two numbers in the brackets of the denominator doubles the number in the bracket of the numerator; similarly we obtain

Left side of (6)

$$\lesssim \frac{\Gamma^2(l + c)}{\Gamma(l + d + n + 1)\Gamma(\mu + l + m + 1)} \lesssim 1.$$ 

This completes the proof of Lemma 2.

§3. Proofs of the theorems

To prove the main theorem, we also need the following known results.

Let $K_\sigma$ be weighted Bergman Kernel function on the space $A^2_\sigma(\Omega_a)$, define the operator

$$(T_\sigma f)(\xi) = C_\sigma \int h^\sigma(\xi')K_\sigma(\xi, \xi')f(\xi')dv(\xi').$$
where \( C_\sigma = \left\{ \int_{\Omega_a} h^\sigma(\xi') K_\sigma(\xi, \xi') dv(\xi') \right\}^{-1} \), this is a constant independent of \( \xi \) [6].

Lemma 3 ([6]). For \( 1 \leq p < \infty \), \( T_\sigma \) is a linear bounded operator on \( L^p(\Omega_a, dv_\lambda) \) iff
\[
0 < \lambda + 1 < p(\sigma + 1) \tag{7}
\]
and when (7) holds, \( T_\sigma \) is a bounded projection operator from \( L^p(\Omega_a, dv_\lambda) \) to \( A^p_\lambda(\Omega_a) \).

Lemma 4 (Schur Lemma [5]). Let \((X, \mu)\) be a measurable space. Suppose \( Q \) is a non-negative measurable function on \( X \times X \), \( 1 < p < \infty \), \( \frac{1}{p} + \frac{1}{q} = 1 \). For the integral operator \( T \) induced by \( Q \), that is, \( Tf(x) = \int_X Q(x, y) f(y) d\mu(y) \), if there exists a non-negative measurable function \( g \) on \( X \) and constant \( C \) such that
\[
\int_X Q(x, y) g^q(y) d\mu(y) \leq C g^q(x), \quad a.e. \quad x \in X, \tag{8}
\]
\[
\int_X Q(x, y) g^p(x) d\mu(x) \leq C g^p(y), \quad a.e. \quad y \in X, \tag{9}
\]
then \( T \) is the bounded operator on \( L^p(X, d\mu) \), and \( \|T\| \leq C \).

Now we set to give the proof of the Main Theorem.

Proof of Theorem A. We only need to prove in the case \( |\alpha| = 1 \), the general case can be proved by induction as in Zhu [7].

For \( 0 < a \leq 2 \), \( \Omega_a \) is a convex Reinhardt domain. By the Leibenzon decomposition [5] on convex domain, for \( f \in H(\Omega_a) \), \( f(0) = 0 \),
\[
f(\xi) = \sum_{k=1}^{n+m} \xi_k \int_0^1 \frac{\partial f}{\partial \xi_k}(t\xi) dt.
\]
Denote
\[
T_k f(\xi) = \int_0^1 \frac{\partial f}{\partial \xi_k}(t\xi) dt.
\]
Clearly we only need to demonstrated that \( T_k \) is a bounded operator in \( A^p_\lambda(\Omega_a) \) for each \( k \).

Due to Lemma 3, for \( f \in A^p_\lambda(\Omega_a) \), there exists the reproducing formula:
\[
f(\xi) = C_\sigma \int_{\Omega_a} h^\sigma(\xi') K_\sigma(\xi, \xi') f(\xi') dv(\xi') \tag{10}
\]
where
\[
0 < \lambda + 1 < p(\sigma + 1). \tag{11}
\]
Hence
\[
T_k f(\xi) = \int_0^1 \frac{\partial f}{\partial \xi_k}(t\xi) dt
\]
\[
= C_\sigma \int_0^1 \int_{\Omega_a} h^\sigma(\xi') \frac{\partial K_\sigma(t\xi, \xi')}{\partial \xi_k} f(\xi') dv(\xi') dt
\]
\[
= C_\sigma \int_{\Omega_a} f(\xi') \left( \int_0^1 h^\sigma(\xi') \frac{\partial K_\sigma(t\xi, \xi')}{\partial \xi_k} dt \right) dv(\xi') \tag{12}
\]
\[
= \int_{\Omega_a} f(\xi') Q(\xi, \xi') dv(\xi').
\]
Here
\[ Q(\xi, \xi') = C_\sigma \int_0^1 h^\sigma(\xi') \frac{\partial K_\sigma(t\xi, \xi')}{\partial \xi_k} dt. \]

So \( T_k \) is just the integral operator induced by \( Q \).

To prove the boundedness of \( T_k \) in \( A^p_\lambda(\Omega_a) \), we treat the two cases \( 1 \leq p < \infty \) and \( p = 1 \) separately.

Case 1. \( 1 < p < \infty, \lambda \geq 0 \).

Choose \( \sigma \), such that \( \sigma > \frac{\lambda + 1}{p} - 1 \), i.e. (11) holds. Since \( \lambda \geq 0 \), then \( \sigma > \frac{1}{p} - 1 \), so the intersection of the two intervals \( (0, \sigma + 1) \cap (-\frac{\sigma}{p-1}, \frac{1}{p-1}) \) is nonempty. Pick

\[ d \in (0, \sigma + 1) \cap (-\frac{\sigma}{p-1}, \frac{1}{p-1}). \]

In the Schur Lemma, take \( X = \Omega_a, \ d\mu = dv_\lambda, \ g = h^{-\frac{d}{q}} \). Then (8) and (9) turn into

\[ \int_{\Omega_a} |Q(\xi, \xi')| h^{-d+\lambda}(\xi') dv(\xi') \lesssim h^{-d}(\xi), \tag{13} \]
\[ \int_{\Omega_a} |Q(\xi, \xi')| h^{-d(p-1)+\lambda}(\xi) dv(\xi) \lesssim h^{-d(p-1)}(\xi'). \tag{14} \]

Note that \( \lambda \geq 0, \ h^\lambda \lesssim 1 \), thus if (13) and (14) hold for \( \lambda = 0 \), they must be hold for any \( \lambda \geq 0 \). On the other hand

\[ |Q(\xi, \xi')| = \left| C_\sigma \int_0^1 h^\sigma(\xi') \frac{\partial K_\sigma(t\xi, \xi')}{\partial \xi_k} dt \right| \]
\[ \lesssim \int_0^1 h^\sigma(\xi') |G_\sigma(t\xi, \xi')|^2 dt. \tag{15} \]

Then it remains to prove:

\[ \int_0^1 \int_{\Omega_a} h^{-d}(\xi') |G_\sigma(t\xi, \xi')|^2 dv(\xi') dt \lesssim h^{-d}(\xi), \]
\[ \int_0^1 \int_{\Omega_a} h^{-d(p-1)}(\xi) |G_\sigma(t\xi, \xi')|^2 dv(\xi) dt \lesssim h^{-d(p-1)-\sigma}(\xi'). \tag{16} \]

By Lemma 2, it only needs to show that (16) holds.

By the definition of \( G_\sigma \),

\[ |G_\sigma(t\xi, \xi')| = |G_\sigma(t\xi', \xi)|. \]

Again use Lemma 2,

\[ \int_0^1 \int_{\Omega_a} h^{-d(p-1)}(\xi) |G_\sigma(t\xi, \xi')|^2 dv(\xi) dt \]
\[ = \int_0^1 \int_{\Omega_a} h^{-d(p-1)}(\xi) |G_\sigma(t\xi', \xi)|^2 dv(\xi) dt \]
\[ \lesssim h^{-d(p-1)-\sigma}(\xi'). \]
Case 2: \( p = 1, \lambda > -1 \).
Choose \( \sigma \), such that \( \sigma > \lambda > -1 \), then (11) holds. As a consequence of Lemma 2, (12) and (15), we have

\[
\int_{\Omega_a} |Q(\xi, \xi')| dv_\lambda(\xi) \lesssim \int_{\Omega_a} \int_0^1 h^\sigma(\xi') |G_\sigma(t\xi, \xi')|^2 dt dv_\lambda(\xi)
\]
\[
= h^\sigma(\xi') \int_0^1 \int_{\Omega_a} |G_\sigma(t\xi, \xi')|^2 h^\lambda(\xi) dv(\xi) dt
\]
\[
\lesssim h^\sigma(\xi') h^{\lambda-\sigma}(\xi') = h^\lambda(\xi').
\]

Therefore

\[
\int_{\Omega_a} |T_k f(\xi)| dv_\lambda(\xi) \leq \int_{\Omega_a} \int_{\Omega_a} |f(\xi')||Q(\xi, \xi')| dv(\xi') dv_\lambda(\xi)
\]
\[
= \int_{\Omega_a} |f(\xi')| \left( \int_{\Omega_a} |Q(\xi, \xi')| dv_\lambda(\xi) \right) dv(\xi')
\]
\[
\lesssim \int_{\Omega_a} |f(\xi')| dv_\lambda(\xi').
\]

This completes the proof of Theorem A.

**Proof of theorem B.** Since \( \Omega_a \) is a Reinhardt domain, every holomorphic function on it has Taylor expansion. By the proof of Theorem A, \( f(\xi) = \sum_\alpha c_\alpha z^\alpha \in A^p(\Omega_a) \) implies

\[
\xi_k T_k f(\xi) = \xi_k \int_0^1 \frac{\partial f}{\partial \xi_k}(t\xi) dt = \sum_\{|\alpha| \neq 0\} \frac{\alpha_k}{|\alpha|} c_\alpha \xi^\alpha \in A^p(\Omega_a)
\]

This completes the proof of Theorem B.

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