Absolute neutrino mass from helicity measurements

C. C. Nishi∗

Institute of Physics “Gleb Wataghin”
University of Campinas, UNICAMP
13083-970, Campinas, SP, Brazil and
Instituto de Física Teórica, UNESP – São Paulo State University
Rua Pamplona, 145, 01405-900 – São Paulo, Brasil

Abstract

The possibility to access the absolute neutrino mass scale through the measurement of the wrong helicity contribution of charged leptons is investigated in pion decay. Through this method, one may have access to the same effective mass $m_\nu^2$ extractable from the tritium beta decay experiments for electron neutrinos as well as the analogous effective mass $(m_{\nu_\mu}^2)_{\text{eff}}$ for muon neutrinos. In the channel $\pi^- \rightarrow e^- \bar{\nu}$, the relative probability of producing an antineutrino with left helicity is enhanced if compared with the naive expectation $(m_\nu/2E_\nu)^2$. The possibility to constrain new interactions in the context of Two-Higgs-Doublet models is also investigated.

PACS numbers: 14.60.Pq, 13.15.+g, 13.20.Cz

∗Electronic address: ccnishi@ifi.unicamp.br
I. INTRODUCTION

After the confirmation that neutrinos have non-null masses and non trivial mixing among the various types, the knowledge of the absolute scale of neutrino masses is one of the most urgent questions in neutrino physics. In recent times, the greatest advances in the understanding of neutrino properties were boosted by neutrino oscillation experiments which are only capable of accessing the two mass squared differences and, in principle, three mixing angles and one Dirac CP violating phase of the Maki-Nakagawa-Sakata (MNS) leptonic mixing matrix.

Too large masses for the light active neutrinos may alter significantly the recent cosmological history of the Universe. Indeed, the most stringent bounds for the value of the sum of neutrino masses come from Cosmology:

$$\sum \nu m_{\nu} < 0.17 \text{eV}$$

at 95% confidence level [1].

Another issue of great interest refers to the existence of heavy right-handed neutrinos which could explain at the same time the tiny active neutrinos masses through the seesaw mechanism as well as the matter-antimatter asymmetry of the Universe through the implementation of the mechanism of bariogenesis through leptogenesis [2].

Despite of the stringent bound (1) coming from cosmological analyses, terrestrial direct search experiments establish much more looser bounds [3]:

$$m_{\nu_e} \leq 2 \text{eV}$$,  
$$m_{\nu_\mu} \leq 190 \text{keV}$$,  
$$m_{\nu_\tau} \leq 18.2 \text{MeV}$$.

These bounds are based on ingeniously planned experiments [4], but their intrinsic difficulties rely on the fact that they should probe, essentially, the kinematical effects of neutrino masses which are negligible compared to other typical quantities involved in processes with neutrino emission.

Although the bounds in Eqs. (2)–(4) are not as stringent as the cosmological bound in Eq. (1), it is always desirable to have a direct measurement of neutrino masses. Two more reasons can be listed in favor of direct terrestrial searches: (a) cosmological bounds may
be quite model dependent and (b) we may have access to mixing parameters through the effective neutrino masses. For the electron neutrino, there are ongoing experiments planning to reduce the respective bound to 0.2 eV [5].

The main goal of this article is to investigate the possibility of accessing the absolute neutrino mass scale through one of the most natural consequences of massive fermions, i.e., the dissociation of chirality and helicity. Consider the pion decay $\pi^- \to \mu^- \bar{\nu}_\mu$. Since the pion is a spin zero particle, in its rest frame, the decaying states should have the following form from angular momentum conservation,

$$|\pi\rangle \to |\mu:\leftarrow\rangle|\bar{\nu}:\rightarrow\rangle + \delta |\mu:\leftarrow\rangle|\bar{\nu}:\rightarrow\rangle,$$

where the arrows represent the momentum direction (longer arrow) and the spin direction (shorter arrow); the combination $\rightarrow$, for example, means a rightgoing fermion with a right helicity. On later calculations the right (left) helicity will be denoted simply as $h=+1$ ($h=-1$). The normalization of the state is arbitrary and the coefficient $\delta$ is of the order of $m_\nu/E_\nu$, which will be calculated in Sec. II. For massless neutrinos there is no second term in Eq. (5) since the antineutrino is strictly right-handed in helicity and chirality. Thus by measuring the wrong helicity contribution of the charged lepton it is possible to have access to the neutrino mass. Such possibility was already suggested in Refs. [6, 7] but we intend here a focused reanalysis of the possibility considering the present experimental bounds. The precision in the polarization measurement necessary to extract the wrong helicity is also calculated while the possibility to constrain new interactions in the context of the two-Higgs-doublet model (2HDM) is investigated as well.

It is interesting to notice that there has been 50 years since the first measurement of the helicity of the electron neutrino [8]. At that time the primary concern was to confirm the V-A theory of weak interactions. Nowadays, we can try to invert their roles to obtain new information about the neutrinos from the well established weak interaction part of the Standard Model (SM). Measurements of the muon neutrino and the tau-neutrino helicities can be also found in the literature [9, 10, 11].
II. PION DECAY

Pion decay $\pi^- \rightarrow l_i^- \bar{\nu}_j$ can be effectively described by the four-point Fermi interaction Lagrangian

$$\mathcal{L}_{CC} = 2\sqrt{2}G_F \bar{l}_i \gamma^\mu LU_{ij} v_j J_\mu + \text{h.c.},$$

where $L = \frac{1}{2} (1 - \gamma_5)$, $\{U_{ij}\}$ denotes the MNS matrix while $J_\mu$ is the hadronic current that in the case of pion decay reads

$$J_\mu = V_{ud} \bar{u}_L \gamma^\mu d_L.$$

From Eq. (6) it is straightforward to calculate the amplitude for $\pi(p) \rightarrow l_i(q) \nu_j(k)$, $i, j = 1, 2$,

$$-i\mathcal{M}(\pi \rightarrow l_i \bar{\nu}_j) = 2G_F F_\pi V_{ud} \bar{u}_L(q) \gamma^\mu LU_{ij} v_j(k)$$

by using [12]

$$\langle 0|\bar{u}\gamma_5 \gamma^\mu d|\pi^- \rangle = ip\gamma^\mu \sqrt{2}F_\pi,$$

where $F_\pi \approx 92\text{MeV}$ is the pion decay constant. Let us denote the spinor dependent amplitude as

$$\tilde{\mathcal{M}}_{ij} = \bar{u}_{l_i}(q) \gamma^\mu LU_{ij} v_{\nu_j}(k).$$

$$\sum_{\text{spins}} |\tilde{\mathcal{M}}_{ij}|^2 = 4(p \cdot q_i)(p \cdot k_j) - 2p^2(q_i \cdot k_j)$$

$$= M_i^2(M_\pi^2 - M_j^2)$$

$$+ m_i^2(M_\pi^2 + 2M_i^2 - m_j^2).$$

The last expression (12) is exact and follows when $p = q_i + k_j$ (four-vector), $p^2 = M_\pi^2$, $q_i^2 = M_i^2$ and $k_j^2 = m_j^2$. The first expression (11) does not assume energy-momentum conservation.

We can calculate the amplitude squared, summed over the neutrino spin, but dependent on the polarization $\hat{n}$ of the charged lepton in its rest frame:

$$P_{ij}(n_i) \equiv \sum_{\nu_j \text{spin}} |\tilde{\mathcal{M}}_{ij}|^2$$

$$= M_i^2[q_i \cdot k_j + M_i(k_j \cdot n_i)] + 2M_i^2 m_j^2$$

$$+ m_j^2[q_i \cdot k_j - M_i(k_j \cdot n_i)].$$
where

\[ n_i'' = \left( \frac{q \cdot \hat{n}}{M_i}, \hat{n} + \left( \frac{E_i}{M_i} - 1 \right)(\hat{n} \cdot \hat{q})\hat{q} \right) \]  \hspace{1cm} (15)

For the particular directions \( \hat{n} = h_i \hat{q} \) we single out the positive (\( h_i = 1 \)) and negative (\( h_i = -1 \)) helicity for the charged lepton [13].

In the helicity basis

\[ q \cdot k_j \pm M_i (k_j \cdot n_i) = [E_{\mu}(k) \pm h_i |q||E_{\nu_j} \mp h_i \hat{q} \cdot k] \]  \hspace{1cm} (16)

Therefore, for the pion at rest we obtain

\[ P_{ij}(h_i = 1) = M_i^2 (M_{\pi}^2 - M_i^2) + O(m_j^2) \]  \hspace{1cm} (17)

\[ P_{ij}(h_i = -1) = m_j^2 \frac{M_i^4}{M_{\pi}^2 - M_i^2} + O(m_j^4) \]  \hspace{1cm} (18)

Without approximations we obtain

\[ P_{ij}(h_i) - P_{ij}(-h_i) = h_i M_{\pi} (M_i^2 - m_j^2) |q|, \]  \hspace{1cm} (19)

while the sum is given by Eq. (12). One can see this results are in accordance with Eq. (2.38) of Ref. [7] where the polarization \( [P_{ij}(+) - P_{ij}(-)]/[P_{ij}(+) + P_{ij}(-)] \) is calculated.

The ratio between the squared amplitudes for left-handed and right-handed helicities is

\[ R_{ij} = \frac{P_{ij}(h_i = -1)}{P_{ij}(h_i = 1)} = \frac{m_j^2 \frac{M_i^4}{M_{\pi}^2 (M_{\pi}^2 - M_i^2)^2}}{M_i^2 (M_{\pi}^2 - M_i^2)^2}. \]  \hspace{1cm} (20)

Considering numerical values we obtain for \( M_i = M_{\mu} \),

\[ R_{\mu j} = \frac{m_j^2}{(100 \text{keV})^2} \times 4.92 \times 10^{-6}, \]  \hspace{1cm} (21)

while for \( M_i = M_e \),

\[ R_{e j} = \frac{m_j^2}{(1 \text{eV})^2} \times 3.83 \times 10^{-6}. \]  \hspace{1cm} (22)

Considering the actual direct bounds for the neutrino masses [3]: \( m_{\nu_\mu} < 190 \text{keV} \) and \( m_{\nu_e} < 2 \text{eV} \), we need a precision of \( 10^{-6} \) in the helicity measurement to reach those bounds either in the case of muons or electrons. Although the branching ratio to produce muons from pion decay is much bigger than to produce electrons, the required dominant versus wrong helicity probability ratios are similar.

Therefore, the coefficient \( \delta \) in Eq. (5) has exactly the modulus

\[ |\delta_{\mu j}|^2 = R_{\mu j} = \frac{m_j^2}{M_{\mu}^2} \frac{M_{\pi}^4}{(M_{\pi}^2 - M_{\mu}^2)^2}. \]  \hspace{1cm} (23)
If we rewrite
\[ |\delta_{\mu j}| = \frac{m_j}{2E_{\nu_j}} \frac{M_{\pi}}{M_{\mu}}, \]
where \( E_{\nu_j} = \frac{M_{\pi}^2 - M_{\mu}^2}{2M_{\pi}} + O(m_j^2) \), we see that \( |\delta_{\mu j}| \) is modified by the factor \( \frac{M_{\pi}}{M_{\mu}} \) when compared to the naive estimate \( m_j/2E_{\nu} \) \cite{14}. We can also conclude that for the channel \( \pi \to e\bar{\nu} \) the real factor is enhanced considerably (\( \sim 274\times \)).

In general, experiments cannot achieve a perfect accuracy in helicity measurements because they usually involve polarization distributions \cite{8, 9, 10}. Thus we have to consider the accuracy necessary to be able to measure the wrong, non-dominant helicity amplitude. Parametrizing Eq. (13) using \( \hat{n} \cdot \hat{q} = -\cos \theta \) yields
\[ P_{ij}(\theta) = M_i^2(M_{\pi}^2 - M_i^2)^{1/2}(1 - \cos \theta) \]
\[ + \frac{m_j^2}{2} \left( M_{\pi}^2 + 2M_i^2 \right) \]
\[ + \cos \theta \left[ M_{\pi}^4 + 2M_i^2 M_{\pi}^2 - M_i^2 \right] \].

(26)

For \( \cos \theta = 1 \) (\( \cos \theta = -1 \)) we recover the negative (positive) helicity for the charged lepton. Expanding around \( \theta = 0 \), we obtain
\[ P_{ij}(\delta \theta) \approx M_i^2(M_{\pi}^2 - M_i^2) \frac{\delta \theta^2}{4} + m_j^2 \frac{M_{\pi}^4}{M_{\pi}^2 - M_i^2}. \]

(27)

Therefore we need an angular resolution of
\[ \delta \theta = 2\sqrt{R} \sim 10^{-3} \]

(28)

to be able to probe the ratio \( R = R_{ij} \) in Eq. (20).

Considering the leptonic mixing the measurement of the wrong helicity for muons probes
\[ |\mathcal{M}(\pi \to \mu\bar{\nu} : h_{\mu} = -1)|^2 = |C|^2 \frac{M_{\pi}^4}{M_{\pi}^2 - M_{\mu}^2}(m_{\nu_{\mu}}^2)_{\text{eff}}, \]

(29)

where \( C \equiv 2G_F F_{\pi}V_{ud} \) and
\[ (m_{\nu_{\mu}}^2)_{\text{eff}} \equiv \sum_j |U_{\mu j}|^2 m_j^2, \]

(30)
is an effective mass for the muon neutrino, analogous to \( m_{\beta}^2 \) \cite{4} inferred from the tritium beta decay experiments for the electron neutrino. The effective electron neutrino mass \( m_{\beta}^2 \) is defined as the expression in Eq. (30) using \( |U_{ej}|^2 \) instead of \( |U_{\mu j}|^2 \). In fact, \( m_{\beta}^2 \) can be extracted from \( \pi^- \to e^-\bar{\nu} \) by measuring the electron with negative helicity.
To obtain the decay rate, we must multiply the amplitude squared by the factor

$$\frac{1}{4\pi} \frac{1}{2M_\pi} \left[ \frac{v_\nu v_l}{v_{\nu_j} + v_l} \right] \approx \frac{1}{4\pi} \frac{1}{4M_\pi} \left( 1 - \frac{M_\nu^2}{M_\pi^2} \right) + O(m_\nu^2),$$

arising from the phase space. We then obtain

$$\Gamma_\alpha(h_\alpha = 1) = \frac{G_F^2}{4\pi} F_\pi^2 |V_{ud}|^2 M_\alpha^2 \left( 1 - \frac{M_\alpha^2}{M_\pi^2} \right)^2 + O(m_\nu^2),$$

$$\Gamma_\alpha(h_\alpha = -1) = \frac{G_F^2}{4\pi} F_\pi^2 |V_{ud}|^2 (m_{\nu_{\alpha}}^2)_{\text{eff}} + O(m_\nu^4),$$

where $\alpha = e, \mu$ and $(m_{\nu_{\alpha}}^2)_{\text{eff}} = m_\beta^2$. The expression in Eq. (32) is the ordinary tree level decay rate for the pion decaying into $l_\alpha \nu$. \[12\]

III. DISCUSSIONS

The wrong helicity contribution come from the right-handed (helicity) contribution in the left-handed (chirality) component of the neutrino which, sometimes, can be explicitly decomposed as in the second term of

$$\bar{u}_i(p) \not{p} L v_{\nu_j}(k) = M_i \bar{u}_i(p) L v_{\nu_j}(k) - m_j \bar{u}_i(p) R v_{\nu_j}(k),$$

where $p = q_i + k_j$ and Dirac equations are used. Thus the wrong helicity contribution may receive new contributions from operators containing the right-handed neutrino $\nu_R$. Therefore, we can try to infer the presence of operators containing terms such as

$$\bar{l}_i R \nu_j$$

coming from new interactions. In the seesaw scenario, however, $\nu_{jR} \rightarrow N_{jR}$ would be heavy Majorana fermions and their production is not possible. The production of active neutrinos through active-sterile mixing is also suppressed since the MNS matrix is unitary to a good extent [15]. The only scenario we can hopefully test here is the Dirac neutrino case with neutrino masses and mixing coming from the Yukawa interactions in complete analogy of the quark sector. The mystery of leptonic mass patterns and mixing have to be explained by the same mechanism acting on the quark sector.

In the SM with three $\nu_{jR}$, without Majorana mass terms, there is no physical interaction containing the term (35). The simplest model containing such term would be the Two-Higgs-doublet model (2HDM) which contains the interaction with the physical charged scalar $h^\pm$:

$$- \mathcal{L}(l, \nu, h^\pm) = \left[ (\Gamma_1^{e*})_{ij} \bar{e}_{jR} \nu_{iL} - (\Gamma_1^\nu)_{ij} \bar{e}_{iL} \nu_{jR} \right] h^- + h.c.$$
The analogous interaction for quarks yields

\(- L(Q, h^\pm) = \left[ (\Gamma_1^d)_{ij} \bar{u}_{iL} d_{jR} - (\Gamma_1^u)_{ij} \bar{u}_{iL} \nu_{iL} \right] h^+ + h.c. \) (37)

These interactions can be read off from the Yukawa interactions before electroweak symmetry breaking (EWSB). For quarks, for example, we have

\(- L_Y(Q) = \left[ (\Gamma' d\ )_{ij} \bar{Q}_{iL} d_{jR} \Phi - \left( \Gamma'_u\right)^{\ast}_{ij} \bar{Q}_{iL} u_{jR} \tilde{\Phi} \right], \ a = 1, 2 \) (38)

where \( \tilde{\Phi}_a = i\sigma_2 \Phi_a^* \) and

\[ \Phi_1 = \begin{pmatrix} h^+ \\ \frac{t_1 - i t_2}{\sqrt{2}} \end{pmatrix}, \]

\[ \Phi_2 = \begin{pmatrix} G^+ \\ \frac{v - t_3 + i c^0}{\sqrt{2}} \end{pmatrix}, \]

in the Higgs basis [16]; \( v = 246 \text{GeV} \) is the electroweak vacuum expectation value. After EWSB, we choose the basis for the fermionic fields such that \( \Gamma_{d\ 2}^{\ast} = \sqrt{2} v \text{diag}(m_d, m_s, m_b) \), \( \Gamma_{u\ 2}^{\ast} = \sqrt{2} v \text{diag}(m_u, m_c, m_t) \), \( \Gamma_{\nu\ 2}^{\ast} = \sqrt{2} v \text{diag}(M_e, M_\mu, M_\tau) \) and \( \Gamma_{\nu\ 2}^{\ast} = \sqrt{2} v \text{diag}(m_1, m_2, m_3) \).

Using the relation [12]

\[ \langle 0 | \bar{u} \gamma_5 d | \pi^-(p) \rangle = -i \sqrt{2} F_\pi \frac{M_\pi^2}{m_u + m_d} , \]

we obtain the amplitude for \( \pi^- \rightarrow l_i^- \nu_j \) for the contribution coming from the exchange of \( h^\pm \)

\[- i M(\pi^- \rightarrow l_i^- \nu_j) = -C_{ij}' \bar{u}_i(q) Rv_{\nu_j}(k) , \] (42)

where

\[ C_{ij}' \equiv \frac{(\Gamma_1^d + \Gamma_1^u)^{\ast}_{11} \sqrt{2} F_\pi M_\pi^2 (\Gamma_1^\nu)_{ij}}{m_{h^\pm}^3 - m_u + m_d} . \] (43)

To quantify the contribution of Eq. (42) we have to compare it to the similar contribution coming from the second term of Eq. (34), namely,

\[ \frac{|C_{ij}'|}{m_j C'} = \left| \frac{(\Gamma_1^d + \Gamma_1^u)^{\ast}_{11} (\Gamma_1^\nu)_{ij}}{(m_u + m_d)V_{ud}} \right| \frac{M_\pi^2}{\sqrt{2} m_{h^\pm}^3 G_F m_j} . \] (44)

To estimate Eq. (44) we can assume the Yukawa couplings \( \Gamma_1^u \) and \( \Gamma_1^d \) are of the same order as the couplings responsible for giving the quark masses:

\[ (\Gamma_1^u)_{11} \sim (\Gamma_1^d)_{11} = \frac{\sqrt{2}}{v} m_u , \ (\Gamma_1^u)_{11} \sim (\Gamma_1^d)_{11} = \frac{\sqrt{2}}{v} m_d . \] (45)
Assuming the same for $\Gamma_1^\nu$, i.e.,

$$ (\Gamma_1^\nu)_{ij} \sim \frac{\sqrt{2}}{v} m_j \delta_{ij} , $$

we obtain

$$ \frac{|C'_{jj}|}{|m_j C|} \sim 10^{-3} \times \left( \frac{100\text{GeV}}{m_{h^\pm}} \right)^2. $$

Since $m_{h^\pm}$ is unlikely to be smaller than $100\text{GeV}$\cite{17}, the contribution from the physical charged Higgs exchange is generally suppressed compared to the SM contribution unless the Yukawa couplings $\Gamma_1^d, \Gamma_1^u$ or $\Gamma_1^\nu$ are much stronger than the ones responsible for the respective masses. Another natural possibility is

$$ (\Gamma_1^\nu)_{ij} \sim \frac{\sqrt{2}}{v} m_i U'_{ij} , $$

and $m_i/m_j \sim 10^3$ or larger. Possibly, $U'_{ij}$ can be constrained by flavor changing processes involving leptons\cite{18}.

To summarize, we can have access to the absolute neutrino mass scale beyond the present direct search bounds if one can achieve a precision of $10^{-6}$ in the measurement of the electron or muon helicity. In terms of polarization, an angular resolution of $10^{-3}$ is necessary in the rest frame of the charged particle. Although a large precision is required to perform such measurements, it must be emphasized that the measurement should be performed only on the charged lepton, without the need to detect the neutrinos directly. For instance, another possible alternative method of accessing the absolute neutrino mass scale would be the detection of flavor violating processes such as $\pi \rightarrow \mu \bar{\nu}_e$ which is also proportional to the square of neutrino masses but depends on the detection of both charged lepton and neutrino\cite{19}.

Moreover, contributions from the charged physical Higgs in 2HDMs are suppressed compared to the SM contribution unless unnaturally large Yukawa couplings are present. Nevertheless, the wrong helicity contribution for the channel $\pi \rightarrow e \bar{\nu}_e$ is considerably enhanced if compared to naive estimates. In this channel one can measure the same effective mass $m^2_\beta$ obtainable from the tritium beta decay experiments. On the other hand, the analogous effective mass $(m_{\nu_\mu})_{\text{eff}}$ may be accessible in the dominant channel $\pi \rightarrow \mu \bar{\nu}_\mu$. The alternative method of constraining new physics from the branching ratio fraction $\text{Br}(\pi \rightarrow e \nu) / \text{Br}(\pi \rightarrow \mu \nu)$ is investigate in Ref.\cite{20}.
Acknowledgments

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (Fapesp). The author is thankful to Prof. R. Shrock for pointing out Refs. 6, 7.

[1] U. Seljak, A. Slosar and P. McDonald, “Cosmological parameters from combining the Lyman-alpha forest with CMB, galaxy clustering and SN constraints,” JCAP 0610 (2006) 014 [arXiv:astro-ph/0604335].

[2] M. Fukugita and T. Yanagida, “Baryogenesis Without Grand Unification,” Phys. Lett. B 174 (1986) 45.

[3] S. Eidelman, et al. (Particle Data Group), Phys. Lett. B592, 1 (2004).

[4] C. Giunti, “Absolute neutrino masses,” Acta Phys. Polon. B 36 (2005) 3215 [arXiv:hep-ph/0511131].

[5] L. Bornschein [KATRIN Collaboration], “KATRIN: Direct measurement of neutrino masses in the sub-eV region,” In the Proceedings of 23rd International Conference on Physics in Collision (PIC 2003), Zeuthen, Germany, 26-28 Jun 2003, pp FRAP14 [arXiv:hep-ex/0309007].

[6] R. E. Shrock, “New Tests For, And Bounds On, Neutrino Masses And Lepton Mixing,” Phys. Lett. B 96 (1980) 159;

[7] R. E. Shrock, “General Theory Of Weak Leptonic And Semileptonic Decays. 1. Leptonic Pseudoscalar Meson Decays, With Associated Tests For, And Bounds On, Neutrino Masses And Lepton Mixing,” Phys. Rev. D 24 (1981) 1232.

[8] M. Goldhaber, L. Grodzins and A. W. Sunyar, “Helicity of neutrinos,” Phys. Rev. 109 (1958) 1015.

[9] L. P. Roesch, V. L. Telegdi, P. Truttmann, A. Zehnder, L. Grenacs and L. Palfy, “Direct Measurement Of The Helicity Of The Muonic Neutrino,” Am. J. Phys. 50 (1982) 931;

[10] W. Fetscher, “Helicity Of The Muon-Neutrino In π⁺ Decay: A Comment On The Measurement Of Pµ Ξ ∆/ρ In Muon Decay,” Phys. Lett. B 140 (1984) 117.

[11] A. Heister et al. [ALEPH Collaboration], “Measurement of the Michel parameters and the ντ helicity in tau lepton decays,” Eur. Phys. J. C 22 (2001) 217.

[12] J. F. Donoghue, E. Golowich, and B. R. Holstein, Dynamics of the Standard Model (Cambridge
[13] C. Itzykson and J. B. Zuber, Quantum Field Theory, (McGraw-Hill, 1980), p. 60.

[14] P. Langacker and J. Wang, “Neutrino anti-neutrino transitions,” Phys. Rev. D 58 (1998) 093004 [arXiv:hep-ph/9802383].

[15] S. Antusch, C. Biggio, E. Fernandez-Martinez, M. B. Gavela and J. Lopez-Pavon, “Unitarity of the Leptonic Mixing Matrix,” JHEP 0610 (2006) 084 [arXiv:hep-ph/0607020].

[16] C. C. Nishi, “Physical parameters and basis transformations in the Two-Higgs-Doublet model,” Phys. Rev. D 77 (2008) 055009 [arXiv:0712.4260 [hep-ph]].

[17] M. Krawczyk and D. Sokolowska, “The charged Higgs boson mass in the 2HDM: decoupling and CP violation,” arXiv:0711.4900 [hep-ph].

[18] J. A. Casas and A. Ibarra, “Oscillating neutrinos and $\mu \rightarrow e\gamma$,” Nucl. Phys. B 618 (2001) 171 [arXiv:hep-ph/0103065].

[19] C. C. Nishi and M. M. Guzzo, “Flavor mixing in a Lee-type model,” arXiv:0803.1422 [hep-ph].

[20] B. A. Campbell and D. W. Maybury, “Constraints on scalar couplings from $\pi^\pm \rightarrow l^\pm \nu_l$,” Nucl. Phys. B 709 (2005) 419 [arXiv:hep-ph/0303046].