Dynamical stresses in the high class wind turbine blades caused in nonhomogeneous nonstationary wind field.
Part 1 Dynamical model

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Abstract. The wind turbines of a high class are with considerable sizes. For this reason, the consideration of the fluid field as a homogeneous when performing the aerodynamic analysis is not quite correct. In addition, due to the gusts and horizontal turbulence, the wind field is also nonstationary. All this causes dynamic, time-varying loads and fatigue. The determination of these loads is a major aim of this publication. The turbine blades are considered as Euler-Bernoulli beams under the impact of the aerodynamic thrust and torque forces as well as from this of the centrifugal force. The finite element method is applied to solve the partial differential equations describing the model under consideration. Two-node elements with four degrees of freedom for each node and a cubic, polynomial approximation of modal shapes were used. The deformations are relatively small in the separated finite elements and this makes it possible to apply the superposition principle.

1. Introduction
The current study performs dynamic stress analysis directed to the loads caused by the thrust, torque, and centrifugal forces acting on the blades of the wind turbine. In [1-4] the authors proposed modification, that takes into account the vertical wind speed gradient, in the Blade Element Momentum (BEM) theory [5-9] used for modeling the aerodynamic interaction. Based on this modification it is possible to calculate the distribution of the forces along the blade's length, as function of time. This approach renders an account both the azimuth angle of the blade position (i.e. the vertical wind speed gradient) and the speed turbulence in a horizontal direction. Euler-Bernoulli beam model is used for the determination of the dynamical stresses and deformations in the turbine blades i.e. the section's plane normal to the neutral beam axis remains normal after deformation and the shear deflection is neglectable. The mathematical model takes into account the complex implementation of the turbine blades, consisting sections with different airfoils, variable chords' length, mass, stiffness and damping characteristics distributed along the blade length. Approaches studying different aspects of loads in the turbine blades are discussed in many references [10-13], but the main aim in this study is the investigation on the influence of the vertical distribution of wind speed combined with longitudinal turbulence, on the dynamic stresses in the blades of the turbines of high class. This is a premise for additional fatigue loads of the turbine’s blades and in this manner are important factors for the design and maintenance of the turbines.
2. Dynamical model
The model and the acting forces on the turbine blade and on its element with length $dy$ are shown in figure 1. The axes of the used coordinate system, rotating with the turbine blade, are defined as: ‘x’ is parallel to the hub axis, ‘oz’ is coincident with the axis of the turbine blade, and ‘y’ is obtained by cross product of the basis vectors of x and y.

![Figure 1. Loads acting on the single blade element considered as Euler-Bernoulli’s beam](image)

The intensity functions of the aerodynamic thrust force $F_{Thrust}(z,t)$ and torque force $F_{Torque}(z,t)$ in dependence of the position - z along the turbine blade axis (or radius r of the turbine’s disk) and of the time - t, with considered the vertical and longitudinal turbulence are:

$$f_{Thrust}(z,t) = \frac{\partial F_{Thrust}(z,t)}{\partial z}, \quad f_{Torque}(z,t) = \frac{\partial F_{Torque}(z,t)}{\partial z}$$

(1)

The centrifugal force and its intensity function are:

$$F_{Centr}(y,t) = \int_{R_{in}}^{R_{out}} \rho A(z) \omega_{\text{rot}}^2(t) dz, \quad f_{Centr}(z,t) = \frac{\partial F_{Centr}(z,t)}{\partial z} = \rho A(z) z \omega_{\text{rot}}^2(t),$$

(2)

where $\rho(y)$ is the density of the blade’s compose material; $A(y)$ is the cross section’s area and $\omega_{\text{rot}}(t)$ the angular velocity of the turbine.

The inertial force is:

$$\tilde{F}_{\text{inert}}(z,t) = \tilde{F}_{\text{inert}}^z(z,t) + \tilde{F}_{\text{inert}}^y(z,t) = -\int_{R_{in}}^{R_{out}} \rho A(z) \ddot{w}(z,t) dz - \int_{R_{in}}^{R_{out}} \rho A(z) \ddot{v}(z,t) dz,$$

where $w(z,t)$ and $v(z,t)$ are the transverse displacements of the elastic line.

The internal loads in the blade’s cross section are: $Q_s(z,t), Q_y(z,t)$ - shear forces, $N(y,t)$ - normal force, $M_x(y,t), M_y(y,t)$ - bending moments.

Considering the Euler-Bernoulli’s assumptions, the angular displacements $\Theta_s(z,t), \Theta_y(z,t)$ are relatively small, therefore we can assume:
\[ \Theta_s(z,t) = \tan\left(\Theta_s(z,t)\right) = \frac{\partial w(z,t)}{\partial z} = w'(z,t), \quad \sin\left(\Theta_s\right) \approx \frac{\partial w(z,t)}{\partial z}, \quad \cos\left(\Theta_s\right) \approx 1 \]

\[ \Theta_s(z,t) = \tan\left(\Theta_s(z,t)\right) = \frac{\partial v(z,t)}{\partial z} = v'(z,t), \quad \sin\left(\Theta_s\right) \approx \frac{\partial v(z,t)}{\partial z}, \quad \cos\left(\Theta_s\right) \approx 1 \]  \tag{4}

Considering the balance of the forces and moments (figure 1), taking into account (4) and eliminating the infinitely small by a second order, it is obtained:

\[ \frac{\partial^2}{\partial z^2} \left( E I(z) \frac{\partial^2 w(z,t)}{\partial z^2} \right) + \rho A(z) \frac{\partial^2 w(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial w(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z,t) \frac{\partial w(z,t)}{\partial z} \right) = f_{\text{thrust}}(z,t) \]

\[ \frac{\partial^2}{\partial z^2} \left( E I(z) \frac{\partial^2 v(z,t)}{\partial z^2} \right) + \rho A(z) \frac{\partial^2 v(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial v(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z,t) \frac{\partial v(z,t)}{\partial z} \right) = f_{\text{torque}}(z,t) \]  \tag{5}

where:

\[ E \] – Young’s modulus;

\[ I_y(z), I_x(z) \] – area moments of inertia for the cross section centroidal axes,

\[ A(z) \] – cross sections area,

\[ l_{\text{bl}} \] is the length of the blade and \( n \) is coefficient of an attenuation.

The boundary and initial conditions are:

\[ w(0,t) = 0, \quad w'(0,t) = 0 \]

\[ w''(l_{\text{bl}},t) = 0, \quad w'''(l_{\text{bl}},t) = 0 \]

\[ w(x,0) = w_0(x), \quad v(x,0) = v_0(x), \quad v'(0,t) = 0 \]

\[ v''(l_{\text{bl}},t) = 0, \quad v'''(l_{\text{bl}},t) = 0 \]

\[ \dot{w}(x,0) = \dot{w}_0(x), \quad \dot{v}(x,0) = \dot{v}_0(x) \]  \tag{6}

The application of the Hamilton’s variational approach to (5) derives:

\[ \int_0^l \mu_u(x) \left[ \frac{\partial^2}{\partial z^2} \left( E I(z) \frac{\partial^2 w(z,t)}{\partial z^2} \right) + \rho A(z) \frac{\partial^2 w(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial w(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z,t) \frac{\partial w(z,t)}{\partial z} \right) \right] dz = 0 \]

\[ \int_0^l \mu_v(x) \left[ \frac{\partial^2}{\partial z^2} \left( E I(z) \frac{\partial^2 v(z,t)}{\partial z^2} \right) + \rho A(z) \frac{\partial^2 v(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial v(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z,t) \frac{\partial v(z,t)}{\partial z} \right) \right] dz = 0 \]

where \( \mu_u(x), \mu_v(x) \) are weight functions.  \tag{7}

Executing some transformations and integrating by parts leads to:

\[ \int_0^l \left[ E I(z) \frac{\partial^2}{\partial z^2} \left( w(x,t) \frac{\partial^2 w(z,t)}{\partial z^2} \right) + \rho A(z) \frac{\partial^2 w(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial w(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z,t) \frac{\partial w(z,t)}{\partial z} \right) \right] dz - \]

\[ - \left[ \frac{\partial}{\partial z} \left( E I(z) \frac{\partial w(z,t)}{\partial z} \right) - N(z,t) \frac{\partial w(z,t)}{\partial z} \right] \right]_{0}^{l_{\text{bl}}} = 0 \]

\[ \int_0^l \left[ E I(z) \frac{\partial^2}{\partial z^2} \left( v(x,t) \frac{\partial^2 v(z,t)}{\partial z^2} \right) + \rho A(z) \frac{\partial^2 v(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial v(z,t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z,t) \frac{\partial v(z,t)}{\partial z} \right) \right] dz - \]

\[ - \left[ \frac{\partial}{\partial z} \left( E I(z) \frac{\partial v(z,t)}{\partial z} \right) - N(z,t) \frac{\partial v(z,t)}{\partial z} \right] \right]_{0}^{l_{\text{bl}}} = 0 \]  \tag{8}

In order to solve the system (5)-(6) different methods can be used:

- The Galerkin method is based on the minimization of the error between the deflection function and the boundary conditions. The approach is simple, but it is difficult to form a function that satisfies all of the boundary conditions. The method has slowly convergence for the higher frequencies and modes;
Rayleigh–Ritz method is based on the energy principles. The deflected shape is express as a linear combination of known functions with unknown coefficients, which are obtained by minimization of the total energy of the system;

Finite element (FE) method. This method is more convenient in the cases for non-uniform distributed mass-inertial, elasticity and damping characteristics as well in cases of nonstationary acting forces and respectively of a variable angular velocity of the turbine.

Considering all the complexity of the discussed problem the application of Finite Element Method (FEM) is most appropriate approach.

3. Finite element method approach

The blade is divided into $n$ numbers of finite elements. One finite element – figure 2 has two nodes, each node with four degrees of freedom:

- transverse displacements $w(z_i, t)$, $v(z_i, t)$ and
- angular displacements: $\Theta_{i+1}(z_i, t) \approx \frac{\partial^2 w(z_i, t)}{\partial z^2} = w'(z_i, t)$; $\Theta_{i+1}(z_i, t) \approx \frac{\partial^2 v(z_i, t)}{\partial z^2} = v'(z_i, t)$, $i = 1, 2,..(n+1)$

$$
\begin{align*}
\text{(i)} & \quad \Theta(z_i, t) \\
\text{(i+1)} & \quad \Theta(z_{i+1}, t)
\end{align*}
$$

The elastic line can be expressed in terms of them as:

$$
w(z, t) = \Phi_i(z) q_i(t), \quad z \in [z_i, z_{i+1}],
$$

$$
v(z, t) = \Phi_i(z) p_i(t), \quad i=1,2,..n+1,
$$

where the nodal functions vectors are: $\Phi_i(z) = [\phi_{i,1}(z), \phi_{i,2}(z), \phi_{i,3}(z), \phi_{i,4}(z)]$.

At the same time its presentation by separation of variables is:

$$
w(z, t) = X(z) T_x(t), \quad z \in [z_i, z_{i+1}], \quad i=1,2,..n+1,
$$

$$
v(z, t) = Y(z) T_y(t), \quad z \in [z_i, z_{i+1}], \quad i=1,2,..n+1,
$$

where $T_x(t)$, $T_y(t)$ are time dependence functions, and $X(z)$, $Y(z)$ are the modal shapes approximated by cubic polynomials:

$$
X(z) \approx a_1 + a_2 z + a_3 z^2 + a_4 z^3 = [1, z, z^2, z^3] \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T, \quad z \in [z_i, z_{i+1}]
$$

$$
Y(z) \approx b_1 + b_2 z + b_3 z^2 + b_4 z^3 = [1, z, z^2, z^3] \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}^T, \quad z \in [z_i, z_{i+1}]
$$

$$
\begin{align*}
\Phi_i(z) & \approx [1, z, z^2, z^3] a T_x(t) \\
\Phi_i(z) & \approx [1, z, z^2, z^3] b T_y(t)
\end{align*}
$$
Considering equations (13), the general coordinates (9) are obtained in the form:

\[
q_i(t) = \begin{bmatrix} w(z_i, t) \\ w'(z_i, t) \\ w(z_i + \Delta z, t) \\ w'(z_i + \Delta z, t) \end{bmatrix} = \begin{bmatrix} X(z_i) \\ X'(z_i) \\ X(z_{i+1}) \\ X'(z_{i+1}) \end{bmatrix} aT_i(t) = B_i aT_i(t)
\]

\[
p_i(t) = \begin{bmatrix} v(z_i, t) \\ v'(z_i, t) \\ v(z_i + \Delta z, t) \\ v'(z_i + \Delta z, t) \end{bmatrix} = \begin{bmatrix} Y(z_i) \\ Y'(z_i) \\ Y(z_{i+1}) \\ Y'(z_{i+1}) \end{bmatrix} bT_i(t) = B_i bT_i(t)
\]

Substituting (14) in (10) gives:

\[
w_i(z, t) = (\Phi_i(z) B_i aT_i(t) = X_i(z) X_i(t) = \left[\begin{array}{lll} 1, z_i, z_i^2, z_i^3 \end{array}\right] aT_i(t) \Rightarrow \Phi_i(z) = \left[\begin{array}{lll} 1, z_i, z_i^2, z_i^3 \end{array}\right] B_i^{-1}
\]

\[
v_i(z, t) = (\Phi_i(z) B_i bT_i(t) = Y_i(z) Y_i(t) = \left[\begin{array}{lll} 1, z_i, z_i^2, z_i^3 \end{array}\right] bT_i(t) \Rightarrow \Phi_i(z) = \left[\begin{array}{lll} 1, z_i, z_i^2, z_i^3 \end{array}\right] B_i^{-1}
\]

Respectively the nodal function vector and its derivations in developed forms are:

\[
\Phi_i(z) = \left[1 - 3u_i^2 + 2u_i^3, (u_i - 2u_i^2 + u_i^3)z, 3u_i^2 - 2u_i^3, (u_i^3 - u_i^2)z \right]
\]

\[
\Phi_i'(z) = \left[-6u_i + 6u_i^2, (1 - 4u_i + 3u_i^2)z, 6u_i - 6u_i^2, (3u_i^2 - 2u_i)z \right]/\Delta z
\]

\[
\Phi_i''(z) = \left[-6 + 12u_i, (-4 + 6u_i)z, 6 - 12u_i, (6u_i - 2u_i)z \right]/\Delta z^2
\]

\[
\Phi_i'''(z) = \left[12, 6z, -12, 6z \right]/\Delta z^3
\]

The values at the elements boundaries are:

\[
\Phi_i(z) = [1, 0, 0, 0] \quad \Phi_i(z_{i+1}) = [0, 0, 1, 0]
\]

\[
\Phi_i'(z) = [0, 1, 0, 0] \quad \Phi_i'(z_{i+1}) = [0, 0, 0, 1]
\]

\[
\Phi_i''(z) = [-6, -4z, 6, -2z] \quad \Phi_i''(z_{i+1}) = [6, 2z, -6, 4z]
\]

\[
\Phi_i'''(z) = [12, 6z, -12, 6z] \quad \Phi_i'''(z_{i+1}) = [12, 6z, -12, 6z]
\]

If the discretization step \(\Delta z\) is small enough, so the mass, the elastic and the damping characteristics, as well as the external forces for each finite element can be assumed for constant:

\[
\rho A(z) \approx \text{const} = \rho A(z_i), \quad \rho A(z) \approx \text{const} = \rho A(z_{i+1}), \quad \rho A(z) \approx \text{const} = \rho A(z_{i+1})
\]

\[
n(z) \approx \text{const} = n(z_i), \quad f_{\text{Thrust}}(z, t) \approx f_{\text{Thrust}}(z_i, t), \quad f_{\text{Torque}}(z, t) \approx f_{\text{Torque}}(z_i, t)
\]

\[
z \approx [z_i, z_{i+1}], \quad i = 1, 2, n \quad \text{(18)}
\]

Accounting (5) and (18) the elastic line can be described by next system of differential equations:

\[
\begin{align*}
\frac{\partial^3 w(z, t)}{\partial z^3} + \rho A(z) \frac{\partial^3 w(z, t)}{\partial t^3} + 2n \rho A(z) \frac{\partial w(z, t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z, t) \frac{\partial w(z, t)}{\partial z} \right) &= f_{\text{Thrust}}(z, t) \\
\frac{\partial^3 v(z, t)}{\partial z^3} + \rho A(z) \frac{\partial^3 v(z, t)}{\partial t^3} + 2n \rho A(z) \frac{\partial v(z, t)}{\partial t} - \frac{\partial}{\partial z} \left( N(z, t) \frac{\partial v(z, t)}{\partial z} \right) &= f_{\text{Torque}}(z, t)
\end{align*}
\]

\[
\begin{align*}
\left. \int z_i \left[ \frac{\partial^2 \mu(z) \partial w(z, t)}{\partial z^2} + \mu(z) \left( \rho A(z) \frac{\partial^2 w(z, t)}{\partial t^2} + 2n \rho A(z) \frac{\partial w(z, t)}{\partial t} - f_{\text{Thrust}}(z, t) \right) \right] \right|_{z_i}^{z_{i+1}} &+ \left. \int z_i \frac{\partial \mu(z) \partial w(z, t)}{\partial z} \right|_{z_i}^{z_{i+1}} - \left. \int z_i \frac{N(z, t) \partial w(z, t)}{\partial z} \right|_{z_i}^{z_{i+1}} - \left. \int z_i \frac{\partial \mu(z) \partial w(z, t)}{\partial z} \right|_{z_i}^{z_{i+1}} = 0
\end{align*}
\]
\[
\int_{z_i}^{z_f} \left\{ EI(z) \frac{\partial^2 \mu(z) \partial^2 v(z,t)}{\partial z^2} + \mu(z) \left[ \rho A(z) \frac{\partial^2 v(z,t)}{\partial t^2} + 2n \rho A(z) \frac{\partial v(z,t)}{\partial t} - f_{\text{load}}(z,t) \right] + N(z,t) \frac{\partial \mu(z) \partial v(z,t)}{\partial z} \right\} dz - E I(z_i) \left\{ \mu(z) \left[ \frac{\partial^3 v(z,t)}{\partial z^3} - N(z,t) \frac{\partial^2 v(z,t)}{\partial z} - \frac{\partial \mu(z) \partial^3 v(z,t)}{\partial z^2} \right] \right\}_{z_i}^{z_f} = 0.
\]

Establishing the internal loads vector in the nodes

\[
\Psi_i(t) = \begin{bmatrix} Q_i(z_i) \\
M_i(z_i) \\
Q_{y_i}(z_{i+1}) \\
M_{y_i}(z_{i+1}) \end{bmatrix} = EI(z_i) \begin{bmatrix} \frac{1}{\Delta z^2} \left[ \begin{array}{cccc} 6 & 3\Delta z & -6 & 3\Delta z \\ -3\Delta z & 2\Delta z^2 & 3\Delta z & -\Delta z^2 \\ 2\Delta z^2 & -3\Delta z & 2\Delta z & -2\Delta z^2 \\
-3\Delta z & -2\Delta z^2 & 3\Delta z & -\Delta z^2 \\
3\Delta z & \Delta z^2 & -3\Delta z & 2\Delta z^2 \end{array} \right] \end{bmatrix} \begin{bmatrix} q_i(t) \\
p_i(t) \end{bmatrix} \tag{21}
\]

and respectively:

\[
\Xi_i(t) = \begin{bmatrix} M_i(z_i) \\
Q_{y_i}(z_{i+1}) \\
M_{y_i}(z_{i+1}) \end{bmatrix} = EI(z_i) \begin{bmatrix} \frac{1}{\Delta z^2} \left[ \begin{array}{cccc} 6 & 3\Delta z & -6 & 3\Delta z \\ -3\Delta z & 2\Delta z^2 & 3\Delta z & -\Delta z^2 \\ 2\Delta z^2 & -3\Delta z & 2\Delta z & -2\Delta z^2 \\
-3\Delta z & -2\Delta z^2 & 3\Delta z & -\Delta z^2 \\
3\Delta z & \Delta z^2 & -3\Delta z & 2\Delta z^2 \end{array} \right] \end{bmatrix} \begin{bmatrix} p_i(t) \\
n_i(t) \end{bmatrix} \tag{22}
\]

substituting the weight functions:

\[
\mu_i(z) = \mu_i(z) = \phi_k(z), \quad k=1,\ldots,4, \tag{23}
\]

and taking into account (18) the equations (20) are transformed into:

\[
\begin{align*}
&\left| \rho A(z) \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \phi_j \, du \right| \dot{q}_j(t) + 2n \rho A(z) \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \phi_j \, du \dot{q}_j(t) + \\
&+ \left| E I(z) \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \phi_j \, du \right| \Delta \sum_{j=0}^{4} \int_{0}^{1} N(z) \phi_i \phi_j \, du \dot{q}_j(t) = \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \, du f_{\text{load}}(z,t)+Wx_i, \\
&\left| \rho A(z) \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \phi_j \, du \right| \dot{p}_j(t) + 2n \rho A(z) \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \phi_j \, du \dot{p}_j(t) + \\
&+ \left| E I(z) \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \phi_j \, du \right| \Delta \sum_{j=0}^{4} \int_{0}^{1} N(z) \phi_i \phi_j \, du \dot{p}_j(t) = \Delta \sum_{j=0}^{4} \int_{0}^{1} \phi_i \, du f_{\text{torque}}(z,t)+Wy_i, \\
\end{align*}
\]

where the internal loads are:

\[
Wx_i(z_i) = \left[ \phi_i(z_i), -\phi_i(z_i), -\phi_i(z_{i+1}), \phi_i(z_{i+1}) \right] \Psi_i(t) - \left( \phi_i(z_i) N(z,t) \frac{\partial v(z,t)}{\partial z} \right)_{z_i}^{z_f},
\]

\[
Wy_i(z_i) = \left[ \phi_i(z_i), -\phi_i(z_i), -\phi_i(z_{i+1}), \phi_i(z_{i+1}) \right] \Xi_i(t) - \left( \phi_i(z_i) N(z,t) \frac{\partial v(z,t)}{\partial z} \right)_{z_i}^{z_f}, \tag{25}
\]

and the normal force is:

\[
N(z,t) = \omega_m(t)^2 \int_{z}^{z_f} A \rho(z) r \, dr = \omega_m(t)^2 \left[ \sum_{j=0}^{4} \int_{z_j}^{z_{j+1}} \rho A(z) \, r \, dr - \rho A(z) \right]_{z_j}^{z_{j+1}} = \tag{26}
\]

6
\[ \omega_m(t)^n \left[ \frac{\Delta z}{s_1(z_i)} \sum_{j=1}^{n} \rho A(z_j) (z_j + 0.5 \Delta z) - \frac{\Delta z \rho A(z_i)}{m(z_i)} \left( u_{z_i} + 0.5 u^2 \Delta z \right) \right], z \in [z_i, z_i + \Delta z]. \]

Processing (24) gives the next system of differential equations:

\[
\begin{align*}
\Phi_m(z_i) \dot{q}_i(t) + \Phi_m(z_i) \dot{q}_i(t) + \Phi_m(z_i) \dot{q}_i(t) &= \Phi_d(T) + Wx_i(t) \\
\Phi_m(z_i) \ddot{p}_i(t) + \Phi_m(z_i) \ddot{p}_i(t) + \Phi_m(z_i) \ddot{p}_i(t) &= \Phi_d(T) + Wy_i(t) \\
\end{align*}
\]

where:

\[
\Phi_m(z_i) = \frac{\Delta z \rho A(z_i)}{m(z_i)} - \frac{1}{420} \begin{bmatrix} 156 & 22 \Delta z & 54 & -13 \Delta z \\ 22 \Delta z & 4 \Delta z^2 & 13 \Delta z & -3 \Delta z ^2 \\ -13 \Delta z & -3 \Delta z ^2 & -22 \Delta z & 4 \Delta z ^2 \end{bmatrix} = m(z_i) \Phi_m(z_i);
\]

\[
\phi(z_i) = \frac{EI(z_i)}{2} \begin{bmatrix} 6 & 3 \Delta z & -6 & 3 \Delta z \\ 3 \Delta z & 2 \Delta z^2 & -3 \Delta z & \Delta z ^2 \\ -6 & -3 \Delta z & 6 & -3 \Delta z ^2 \\ 3 \Delta z & -3 \Delta z ^2 & 2 \Delta z & 2 \Delta z ^2 \end{bmatrix} + \omega_m(z_i) ^2 S(z_i)
\]

\[
\begin{bmatrix} 1, 2 & 0,1 \Delta z & -1,2 & 0,1 \Delta z \\ 0,1 \Delta z & 2 \Delta z / 15 & -0,1 \Delta z & -\Delta z ^2 / 30 \\ -1,2 & -0,1 \Delta z & 1,2 & -0,1 \Delta z \\ 0,1 \Delta z & -\Delta z ^2 / 30 & -0,1 \Delta z & 2 \Delta z ^2 / 15 \end{bmatrix} - \begin{bmatrix} \omega_m(z_i) ^2 S(z_i) \\ \omega_m(z_i) \end{bmatrix}
\]

The corresponded matrices, for the \(q\) and \(p\) coordinates, are calculated with the corresponding elasticity coefficients: \(E_I(y)\) and \(E_I(x)\).

When we consider the whole blade, we should take into account that the coordinates of the 2\(^{nd}\) node of the 1\(^{st}\) element coincides with those of the 1\(^{st}\) node of the 2\(^{nd}\) of the second, the coordinates of the 2\(^{nd}\) node of the 2\(^{nd}\) element coincides with the those of the 1\(^{st}\) node of the 3\(^{rd}\) element and so on. The generalized coordinates of the whole blade are 2(n+1):

\[
\begin{bmatrix} q_{bl} \end{bmatrix} = \begin{bmatrix} w(z_i, t), \Theta_z(z_i, t), z_i = (i-1) \Delta z, i=1,..,n+1 \end{bmatrix}^T
\]

\[
\begin{bmatrix} p_{bl} \end{bmatrix} = \begin{bmatrix} v(z_i, t), \Theta_y(z_i, t), z_i = (i-1) \Delta z, i=1,..,n+1 \end{bmatrix}^T
\]

Considering this relation between the elements coordinates, the system of differential equations can be expressed as:
\[
\begin{align*}
\Phi^{gl}(q) + \Phi^{(q)}_c(q) + \Phi^{(q)}_k(q) &= \Phi^{gl}_d f_{gl} \text{ Thrust} (t) = U^{gl(q)}(t) \\
\Phi^{gl}_a + \Phi^{(p)}_c(p) + \Phi^{(p)}_k(p) &= \Phi^{gl}_d f_{gl} \text{ Torque} (t) = U^{gl(p)}(t) ,
\end{align*}
\]

(29)

where the global matrixes are formed by the locals using the next algorithm:

\[
\Phi^{gl} = \begin{bmatrix}
\Phi(1:2,:)\alpha(1), & 0_{2 \times 2(n-1)} \\
0_{2 \times 2(1:2)}, & \Phi(3:4,1:2)\alpha(i-1), & \Phi(3:4,3)\alpha(i-1) + \Phi(1:2,1)\alpha(i), & \Phi(3:4,4)\alpha(i-1) + \\
& + \Phi(1:2,2)\alpha(i), & \Phi(1:2,3:4)\alpha(i), & 0_{2 \times 2(n-1)} \\
0_{2 \times 2(n-1)}, & \Phi(3:4,:)\alpha(n)
\end{bmatrix}
\]

(30)

with substitutions for: \( \Phi^{gl}_m \vdots \alpha = m, \Phi^{gl}_c \vdots \alpha = c, \Phi^{gl}_k \vdots \alpha = k \).

The generalized force matrix is:

\[
U^{gl} = \begin{bmatrix}
\Phi_d(3:4,1)f(i-1) + \Phi_d(1:2,1)f(i), & i = 2, .., n \\
\Phi_d(3:4,1)f(n)
\end{bmatrix}
\]

(31)

The clamped end boundaries are:

\[
\begin{align*}
w(0,t) &= q_{bl}(1) \equiv 0 & \Theta_j(x_i,t) &= q_{bl}(i) \equiv 0 \\
\Theta_j(x_i,t) &= q_{bl}(2) \equiv 0 & \Theta_j(x_i,t) &= p_{bl}(2) \equiv 0
\end{align*}
\]

(32)

This leads to removal of the 1st and 2nd rows and columns of \( \Phi^{gl}_m, \Phi^{gl}_c, \Phi^{gl}_k \) and the 1st and 2nd elements of \( U^{gl} \).

Thus the generalized coordinate’s matrix size becomes 2n:

\[
\overline{\Phi}^{gl} = q_{bl}(3:2(n+1))
\]

(33)

and respectively the equation’s matrix are:

\[
\overline{\Phi}^{gl}_m = \Phi^{gl}_m(3:2(n+1), 3:2(n+1)) \\
\overline{\Phi}^{gl}_c = \Phi^{gl}_c(3:2(n+1), 3:2(n+1)) \\
\overline{\Phi}^{gl}_k = \Phi^{gl}_k(3:2(n+1), 3:2(n+1))
\]

(34)

\[
\overline{\Phi}^{gl} = U^{gl}(3:2(n+1),1)
\]

Defying the state space and the output variables vectors:

\[
\begin{bmatrix}
\overline{q}^{gl}_n \\
\overline{p}^{gl}_n
\end{bmatrix} = \begin{bmatrix}
X \\
\dot{X}
\end{bmatrix}, \quad Y = \begin{bmatrix}
\overline{q}^{gl}_n \\
\overline{p}^{gl}_n
\end{bmatrix}
\]

(35)

the state space presentation of the system of differential equations is:

\[
\begin{bmatrix}
\dot{X} \\
Y
\end{bmatrix} = \begin{bmatrix}
AX + BU \\
CX + DU
\end{bmatrix},
\]

(36)

where the state matrices are:

\[
A = \begin{bmatrix}
0_{nxn}, & E_n, & 0_{nxn}, & 0_{nxn} \\
0_{nxn}, & 0_{nxn}, & 0_{nxn}, & E_n \\
-(\overline{\Phi}^{gl}_m)^{-1} \overline{\Phi}^{gl(q)}_m, & -(\overline{\Phi}^{gl}_c)^{-1} \overline{\Phi}^{gl(q)}_c, & 0_{nxn}, & 0_{nxn} \\
0_{nxn}, & 0_{nxn}, & -(\overline{\Phi}^{gl}_m)^{-1} \overline{\Phi}^{gl(p)}_m, & -(\overline{\Phi}^{gl}_c)^{-1} \overline{\Phi}^{gl(p)}_c
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0_{nxn} \\
\overline{\Phi}^{gl}_m^{-1} \\
0_{nxn} \\
\overline{\Phi}^{gl}_m^{-1}
\end{bmatrix}, \quad C = \begin{bmatrix}
E_n \\
A(n+1:2n,:) \\
E_n \\
A(3n+1:4n,)
\end{bmatrix}, \quad D = \begin{bmatrix}
0_{nxn} \\
B(n+1:n,:) \\
0_{nxn} \\
B(3n+1:4n,)
\end{bmatrix}
\]

If the nodes displacements are denoted with: \( q_n = q \) (from 1, with step 2, to 2n-1), the shear force and torque at the base of the blade are:
The sheared force and torque dependencies for each element can be calculated iteratively:

\[ Q_x(i) = Q_x(i-1) - f_{\text{thrust}}(i)\Delta z + \ddot{q}_w(i)m(i) \]
\[ M_y(i) = M_y(i-1) - z(i)f_{\text{thrust}}(i)\Delta z + z(i)\ddot{q}_w(i)m(i) \], \quad i=2,..n . \tag{38} 

The stresses are:

\[ \sigma_y(i) = M_y(i)h(i) / I_y(i), \quad i=1,..n , \tag{39} \]

where \( h(i) \) is the thickness of the element.

The calculation of the stress distribution by the other direction is identically.

4. Conclusions

It has been obtained a dynamical model of the loads, stresses, and deformations in wind turbine blades under the action of thrust, torque and centrifugal forces, using the Euler-Bernoulli beam model. Basing on the finite element method the model is discretized and reduced to the system of ordinary differential equations.

Acknowledgements. This work was supported by the Bulgarian Ministry of Education and Science under the National Research Program „Low carbon energy for transport and live”, DCM №577/2018.

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