Fracture Functions and Jet Calculus

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Abstract

By using Jet Calculus as a consistent framework to describe multiparton dynamics we explain the peculiar evolution equation of fracture functions by means of the recently introduced extended fracture functions.

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1 Introduction

Fracture functions [1] have been introduced to interpret within the framework of perturbative QCD semi-inclusive deep inelastic processes in the target fragmentation region. The formulation of these processes by means of fracture functions does allow to extend the interpretation in terms of QCD-improved parton model to the complementary region of target fragmentation. This implies a description of deep inelastic processes in terms of the novel fracture functions in addition to the usual factorizable structure and fragmentation functions convoluted with hard point-like cross sections.

The introduction of these new objects requires however the formulation of an additional factorization hypothesis whose validity has been recently argued on the basis of the cut vertex formalism together with the use of infrared power counting. These results have been obtained within the theoretical framework of $(\phi^3)_6$ field theory [2, 3] and have been later confirmed in QCD in Ref. [4].

In order to show that the new factorization hypothesis is correct, in Ref. [2] new objects have been defined called extended fracture functions, which depend also on the momentum transfer $t$ between the incoming and outgoing hadron. By an explicit one-loop calculation it has been verified that these objects do factorize and it has been also observed that they show a new logarithmic dependence on the ratio of the scales $Q^2$ and $t$ [5]. As a result extended fracture functions obey an evolution equation different from the one followed by ordinary fracture functions.

The aim of this note is to show in a simple and direct way that extended and ordinary fracture functions are closely connected. As a consequence the corresponding evolution equation do follow one from the other. We work within the framework of Jet Calculus [6] by applying the corresponding rules to the evolution equations. The set of Jet Calculus rules allows the interpretation in terms of QCD-improved parton model of the results obtained by using a different approach [2]. By using this method we can give in fact a definition of the extended fracture function in the region where $t$ is a perturbative scale $\left(\frac{\alpha_s(t)}{2\pi} \approx 1\right)$. As
a consequence a DGLAP evolution equation for extended fracture functions follows and the inhomogeneous evolution equation for the ordinary fracture function is recovered.

2 Evolution pattern

In a deep inelastic semi-inclusive reaction, a hadron $A$ with momentum $p$ is struck by a far off-shell spacelike photon with momentum $q$ and a hadron $A'$ with momentum $p'$ is inclusively observed in the final state. Let us define as usual

$$Q^2 = -q^2 \quad x = \frac{Q^2}{2pq}$$

and choose a frame in which $p = (p_+, p_-, 0)$ with $p_+ \gg p_-$ and $pq \simeq p_+ q_-$. 

As far as we keep away from the target fragmentation region the cross section factorizes as follows [7]

$$\sigma_J = \int \frac{dx'}{x'} \frac{dz'}{z'} F_A^i(x', Q^2) \hat{\sigma}_{ij}(x/x', \bar{z}/\bar{z}', Q^2) D_{A'}^j(\bar{z}', Q^2)$$

where $F_A^i(x, Q^2)$ and $D_{A'}^j(\bar{z}, Q^2)$ are the structure and fragmentation function respectively, $\hat{\sigma}_{ij}(x, \bar{z}, Q^2)$ is the hard semi-inclusive cross section and we have defined $\bar{z} = pp'/pq \simeq p_+/q_-$. 

Eq. (2) expresses the fact that, as long as the produced hadron has a large transverse momentum $p_T' \sim Q^2$ (i.e. $\bar{z}$ is finite) it can be thought as a product of the fragmentation of the active parton. However in the last few years, especially after the appearance of diffractive deep inelastic events at HERA, particular attention has been payed to hadron production in the target fragmentation region (i.e. the region $\bar{z} \to 0$), where eq. (2) fails. In Ref.[1] it has been proposed for target fragmentation the additional factorized term

$$\sigma_T = \int \frac{dx'}{x'} M_{AA'}^i(x', z, Q^2) \hat{\sigma}_i(x/x', Q^2)$$

where $z = p' q/pq \simeq p'_+/p_+$ represent the momentum fraction of the hadron $A'$ with respect to $A$. Here $M_{AA'}^i(x, z, Q^2)$ is the fracture function, giving the probability of finding a parton
with momentum fraction \(x\) in the hadron \(A\) while another hadron \(A'\) with momentum fraction \(z\) is detected.

The fracture function is expected to satisfy an inhomogeneous evolution equation \([1]\), and this fact has been verified at one-loop level in Ref.\([8]\).

Let us consider the case in which the momentum transfer \(t = -(p - p')^2 \ll Q^2\) is also measured. The current and target fragmentation contributions are in this case

\[
\sigma_J = \int \frac{dx' dx'}{x' z'} F_A^i(x', Q^2) \hat{\sigma}_{ij}(x/x', z/x' z', tx'/z', Q^2) D_{A'}^j(z', Q^2) \tag{4}
\]

and

\[
\sigma_T = \int \frac{dx' dx'}{x'} \mathcal{M}_{A,A'}^i(x', z, t, Q^2) \hat{\sigma}_i(x/x', Q^2) \tag{5}
\]

The function \(\mathcal{M}_{A,A'}^i(x, z, t, Q^2)\) is the extended fracture function. In Ref. \([2]\) it has been shown that this object can be defined just in terms of a new cut vertex. Whereas \(M_{A,A'}^i(x, z, Q^2)\) is expected to satisfy an inhomogeneous evolution equation \([1]\), the extended fracture function obeys a simple DGLAP evolution equation \([2]\).

This facts can be understood by making the following observations. In the region \(\Lambda_{QCD}^2 \ll t \ll Q^2\) the hard semi-inclusive cross section \(\hat{\sigma}_{ij}\) in eq. \((4)\) develops large \(\log Q^2/t\) corrections which have to be resummed \([3]\). Instead of absorbing these logs in \(\hat{\sigma}_{ij}\) we choose to move them in the extended fracture function which, by using Jet Calculus, can be defined in the perturbative region of \(t\) as

\[
\mathcal{M}_{A,A'}^i(x, z, t, Q^2) = \frac{\alpha_S(t)}{2\pi t} \int \frac{dw}{w} F_A^i(w, t) \hat{P}_{i,k}^l(u) \left( \frac{z}{w - x/r}, t \right) E_k^i(r, t, Q^2) \tag{6}
\]

\[
\times \int \frac{dr}{r} \int_{1-z/r}^{1} \frac{dw}{w} \left( \frac{z}{w - x/r} \right) D_{l,A'} \left( \frac{z}{w - r}, t \right) E_k^j(x/r, t, Q^2)
\]

3
where the integration limits have been obtained implementing momentum conservation. We work within the leading logarithmic approximation. Here $P_i^j(u)$ and $\hat{P}^j_{ik}(u)$ are regularized and real Altarelli-Parisi vertices respectively, and the function $E^j_k(x, Q_0^2, Q^2)$ is the evolution kernel from $Q_0^2$ to $Q^2$, which obeys the DGLAP evolution equation:

$$Q^2 \frac{\partial}{\partial Q^2} E^j_i(x, Q_0^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u} P^j_{ik}(u) E^k_i(x/u, Q_0^2, Q^2).$$  (7)

Eq. (6) is represented in Fig. 1. The evolution kernel (the black blob in Fig. 1) resums the dependence on $Q^2$ and $t$ in terms of logarithms of the form $\log \frac{Q^2}{t}$ and at first order in $\alpha_s$ has the following expression

$$E^j_i(x, t, Q^2) \simeq \delta^j_i \delta(1-x) + \frac{\alpha_s}{2\pi} P^j_i (x) \log \frac{Q^2}{t}.$$  (8)

In this way we have absorbed these large logarithms in the definition of $\mathcal{M}^i_{A,A'}(x, z, t, Q^2)$ which in the perturbative region of $t$ is resolved in the convolution of the objects contained in the dotted box in Fig 1.
From eq. (3) it appears that the $Q^2$ dependence of the extended fracture function is completely given by the evolution kernel $E^k_i(x, Q^2_0, Q^2)$ and therefore the evolution equation is

$$Q^2 \frac{\partial}{\partial Q^2} M^j_{A,A'}(x, z, t, Q^2) = \frac{\alpha_S(Q^2)}{2\pi} \int_{\frac{1}{1-x}}^1 \frac{du}{u} P^j_i(u) M^i_{A,A'}(x/u, z, t, Q^2)$$

at least in the perturbative region of $t$, i.e. $t \gtrsim \Lambda^2_{QCD}$. Therefore the anomalous dimension associated to the extended fracture function is the same that controls the evolution of ordinary structure functions.

Let us suppose now that, as argued in Ref. [2], the evolution equation (9) applies also within the small $t$ region and define the integrated fracture function as an integral over $t$ up to a $Q^2$-dependent cut-off of order $Q^2$, say $\epsilon Q^2$, with $\epsilon < 1$

$$M^j_{A,A'}(x, z, Q^2) = \int_0^{\epsilon Q^2} dt \ M^j_{A,A'}(x, z, t, Q^2).$$

By taking the logarithmic derivative of eq. (10) we have that

$$Q^2 \frac{\partial}{\partial Q^2} M^j_{A,A'}(x, z, Q^2) = \int_0^{\epsilon Q^2} dt \ Q^2 \frac{\partial}{\partial Q^2} M^j_{A,A'}(x, z, t, Q^2) + \epsilon Q^2 M^j_{A,A'}(x, z, \epsilon Q^2, Q^2)$$

$$= \frac{\alpha_S(Q^2)}{2\pi} \int_0^{\epsilon Q^2} dt \int_{\frac{1}{1-x}}^1 \frac{du}{u} P^j_i(u) M^i_{A,A'}(x/u, z, t, Q^2) + \epsilon Q^2 M^j_{A,A'}(x, z, \epsilon Q^2, Q^2)$$

$$= \frac{\alpha_S(Q^2)}{2\pi} \int_{\frac{1}{1-x}}^1 \frac{du}{u} P^j_i(u) M^i_{A,A'}(x/u, z, Q^2) + \epsilon Q^2 M^j_{A,A'}(x, z, \epsilon Q^2, Q^2).$$

We see appearing an inhomogeneous term in the evolution equation of $M^j_{A,A'}(x, z, Q^2)$ which arises from the $Q^2$ dependence of the upper integration limit. This inhomogeneous term depends on the value of $M^j_{A,A'}(x, z, t, Q^2)$ in the perturbative region of $t$ where eq. (3) holds. By using the boundary condition

$$E^j_k(x, Q^2, Q^2) = \delta^j_k \delta(1-x)$$

(12)
on eq. (6) we get for the inhomogeneous term, up to log $\epsilon$ corrections,

$$
\epsilon Q^2 M^j_{A,A'}(x, z, \epsilon Q^2, Q^2) = \\
= \frac{\alpha_s(Q^2)}{2\pi} \int_x^{1+z} \frac{dr}{r} \int_{x+r}^{1} \frac{dw}{w(w-r)} F^i_A(w, Q^2) \hat{P}^j_i \left( \frac{r}{w} \right) D_{l,A'} \left( \frac{z}{w-r}, Q^2 \right) \delta(x/r - 1)
$$

$$
= \frac{\alpha_s(Q^2)}{2\pi} \int_{z+x}^{1} \frac{dw}{w(w-x)} F^i_A(w, Q^2) \hat{P}^j_i \left( \frac{x}{w} \right) D_{l,A'} \left( \frac{z}{w-x}, Q^2 \right)
$$

$$
= \frac{\alpha_s(Q^2)}{2\pi} \int_x^{x+z} \frac{du}{u} \frac{u}{x(1-u)} F^i_A \left( x/u, Q^2 \right) \hat{P}^j_i(u) D_{l,A'} \left( \frac{zu}{x(1-u)}, Q^2 \right).
$$

By substituting eq. (13) in eq. (11) the evolution equation for $M^j_{A,A'}(x, z, Q^2)$ appears to be

$$
Q^2 \frac{\partial}{\partial Q^2} M^j_{A,A'}(x, z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^{1} \frac{du}{u} P^j_i(u) M^i_{A,A'}(x/u, z, Q^2)
$$

$$
+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^{x+z} \frac{du}{u} \frac{u}{x(1-u)} F^i_A \left( x/u, Q^2 \right) \hat{P}^j_i(u) D_{l,A'} \left( \frac{zu}{x(1-u)}, Q^2 \right),
$$

that is exactly the one proposed in Ref.[1].

Furthermore the perturbative definition (4) can be used to derive the evolution with $t$ of the extended fracture function. In order to make the notation less cumbersome, it is convenient to define

$$
\int_0^1 dzz' \int_0^{1-z} dx x^n M^j_{A,A'}(x, z, Q^2) = M^j_{mn}(Q^2).
$$

(15)

One can verify that eq. (4) becomes

$$
\mathcal{M}^j_{mn}(t, Q^2) = \frac{\alpha_s(t)}{2\pi t} P^{kli}_{mn} F^i_{m+n}(t) D^k_m(t) E^j_l(t, Q^2)
$$

(16)

where we have defined

$$
P^{kli}_{mn} = \int_0^1 du u^m(1-u)^n \hat{P}^j_i(u)
$$

(17)

and

$$
E^j_l(t, Q^2) = \int_0^1 du u^n E^j_l(u, t, Q^2).
$$

(18)
The \( t \) evolution equation contains several terms: the first comes from the canonical scale dependence of the extended fracture function, the second from the scale dependence of \( \alpha_s \), and the third from the scale dependence of the evolution kernel. There are then two inhomogeneous terms which follow from the \( t \) dependence of structure and fragmentation functions respectively. By explicitly deriving eq. (16) with respect to \( t \) we get

\[
\frac{d}{dt} M_{mn}^{ij}(t, Q^2) = - \left( \delta^{ip} (1 + \beta_0 \alpha_s(t)) + \frac{\alpha_s(t)}{2\pi} A_n^{ip} \right) M_{mm}^{ip}(t, Q^2)
+ \frac{\alpha_s^2(t)}{(2\pi)^2 t} P_{n}^{kli} A_{m+n}^{ip} F_{m+n}(t) D_{m}^{k}(t) E_n^{ji}(t, Q^2)
+ \frac{\alpha_s^2(t)}{(2\pi)^2 t} P_{n}^{kli} F_{m+n}(t) A_{m}^{kp} D_{m}^{p}(t) E_n^{jl}(t, Q^2)
\]

where

\[
A_n^{ij} = \int_0^1 du u^n P_i^j(u).
\]

We conclude this section by stressing that one can define the fracture function in a more general way as an integral up to an arbitrary \( Q^2 \)-dependent integration limit

\[
M_{A,A'}^{ij}(x, z, Q^2) = \int_0^{t_2(Q^2)} dt \ M_{A,A'}^{ij}(x, z, t, Q^2)
\]

In this case the evolution equation reads

\[
Q^2 \frac{d}{dQ^2} M_{A,A'}^{ij}(x, z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 \frac{du}{u} P_i^j(u) M_{A,A'}^{ij}(x/u, z, Q^2)
+ \frac{\alpha_s(t_2(Q^2))}{2\pi} \frac{Q^2 t_2(Q^2)}{t_2(Q^2)} \int_0^{1-z} \frac{dr}{r} \int_0^{r+1-z} \frac{du}{u} F_A^i(r/u, t_2(Q^2)) \hat{P}_i^{kl}(u)
\times D_{t,A'} \left( \frac{zu}{r(1-u)} , t_2(Q^2) \right) E_k^j (x/r, t_2(Q^2), Q^2)
\]

or, by taking double moments,

\[
Q^2 \frac{d}{dQ^2} M_{mn}^{ij}(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} A_n^{ip} M_{mn}^{ip}(Q^2)
+ \frac{\alpha_s(t_2(Q^2))}{2\pi} \frac{Q^2 t_2(Q^2)}{t_2(Q^2)} P_{n}^{kli} F_{m+n}^{i}(t_2(Q^2)) D_{m}^{k}(t_2(Q^2)) E_n^{jl}(t_2(Q^2), Q^2)
\]
In the case in which \( t_2(Q^2) = \epsilon Q^2 \) the evolution equation (14) is of course recovered.

One could even define a more general fracture function introducing also a lower integration limit \( t_1(Q^2) \)

\[
M_{A,A'}^j(x, z, Q^2) = \int_{t_1(Q^2)}^{t_2(Q^2)} dt \, M_{A,A'}^j(x, z, t, Q^2).
\]

and the equation would have an additional inhomogeneous term taking into account the \( Q^2 \) dependence of the lower integration limit

\[
Q^2 \frac{\partial}{\partial Q^2} M_{mn}^j(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} A_{m}^{jp} M_{mn}^p(Q^2)
\]

\[
+ \frac{\alpha_s(t_2(Q^2))}{2\pi} \frac{Q^2 t_2(Q^2)}{t_2(Q^2)} P_{nm}^{kli} F_{m+n}^{i} \left( t_2(Q^2) \right) D_m^k \left( t_2(Q^2), Q^2 \right) E_n^j \left( t_2(Q^2), Q^2 \right)
\]

\[
- \frac{\alpha_s(t_1(Q^2))}{2\pi} \frac{Q^2 t_1(Q^2)}{t_1(Q^2)} P_{nm}^{kli} F_{m+n}^{i} \left( t_1(Q^2) \right) D_m^k \left( t_1(Q^2), Q^2 \right) E_n^j \left( t_1(Q^2), Q^2 \right).
\]

Phenomenologically these two cases would correspond to the production of hadrons within the target fragmentation region with transverse momentum below \( t_2(Q^2) \) and between \( t_1(Q^2) \) and \( t_2(Q^2) \) respectively.

\section{Sum rules}

In Ref.\[1\] it was shown that fracture functions obey the following momentum sum rule

\[
\sum_{A'} \int dz \, z \, M_{A,A'}^j(x, z, Q^2) = (1 - x) F_A^j(x, Q^2)
\]

which accounts for momentum conservation in the \( s \)-channel. In this section we want investigate if a momentum sum rule holds also in the \( t \)-channel. Taking moments of eq. (14) with \( m = 0, n = 1 \) and summing over \( j \) we get [3]

\[
Q^2 \frac{\partial}{\partial Q^2} \sum_j M_{01}^{j,AA'}(Q^2) = \frac{\alpha_s(Q^2)}{2\pi} F_A^i(x, Q^2) \int_0^1 du \, u \, P^k_i (1 - u) D_k^{0,AA'}(Q^2)
\]
where we made use of the well-known property of the splitting function \[3\]

\[
\sum_j A_1^{ij} = 0
\]  

(28)

and of the relation

\[
\int_0^1 du (1-u) \sum_k \hat{P}_i^{jk}(u) = \int_0^1 du (1-u) P_i^j(u).
\]  

(29)

From eq. (27) it appears that the derivative of \(M_{01}^{j,AA'}(Q^2)\) receives contribution from the inhomogeneous term in the evolution equation. This fact suggests that the sum rule could be violated because at values of \(t\) of order \(Q^2\) the current contribution becomes important and the two production mechanisms cannot be disentangled.

As far as the extended fracture function is concerned, since \(\mathcal{M}_{j,AA'}^{j}(x, z, t, Q^2)\) obeys the DGLAP evolution equation, it follows that

\[
Q^2 \frac{\partial}{\partial Q^2} \sum_j \int_0^{1-z} x \, dx \, \mathcal{M}_{j,AA'}^{j}(x, z, t, Q^2) = 0
\]  

(30)

and momentum conservation in the \(t\) channel is recovered. In the perturbative region of \(t\), by using eq. (3), we find

\[
\sum_j \mathcal{M}_{01}^{j,AA'}(t, Q^2) = \frac{\alpha_s(t)}{2\pi t} F_1^{i,A}(t) \int_0^1 du P_i^k(1-u) B_0^{k,A'}(t).
\]  

(31)

4 Summary

In this note we showed that a formulation of perturbative processes in the target fragmentation region can be given by using the formalism of Jet Calculus [3].

A perturbative evaluation in terms of evolution equation and anomalous dimensions of extended fracture function is consistent with the evolution equations proposed in Ref.[1] and [2] for ordinary and extended fracture functions respectively. We showed that the inhomogeneous term in eq. (14) precisely stems from the integration over momentum transfer \(t\).
Moreover in this formalism the extension to next-to-leading order seems quite natural and suggests that eqs. (9) and (14) keep the same structure in the two-loops approximation, analogously to what happens for ordinary structure and fragmentation functions [11].

In the region $\Lambda_{QCD}^2 \ll t \ll Q^2$ the hard semi-inclusive cross section develops large $\log Q^2/t$ corrections which need to be resummed. Our approach is based on the idea of absorbing such corrections in the definition of $\mathcal{M}^i_{AA'}(x, z, t, Q^2)$, namely, $\log Q^2/t$ are fully contained in the evolution kernel $E^i_t(x, t, Q^2)$. These logs, which can be treated with standard renormalization group techniques, are new and potentially useful to understand the dynamics of hadro-production in the target fragmentation region. These logs could affect significantly observables like multiplicities and particle distributions and we believe that a phenomenological study of the role of these corrections would be useful.

As originally proposed in Ref. [1] and in Ref. [11], there now seems to be a widespread consensus that the $Q^2$ evolution in the diffractive regime can be described in terms of the perturbative QCD formalism [4, 12, 13]. The data taken by H1 collaboration [14], showing an evidence of a consistent logarithmic scaling violation in diffractive channels have further stressed that such an interpretation is not far from the experimental observation. All this is consistent with a description in terms of fracture functions. Our results agree with such an expectation, although in the limit $z \to 1$ our formulae will show the appearance of large $\log(1-z)$ factors which should be resummed as well. For the treatment of such a singular region we believe an approach can be used close to the one followed in the inclusive case.

As a concluding remark we want to stress that the results presented here completely agree with those of Ref. [2], obtained within a quite different framework, namely, that of a generalized cut vertex expansion which, as a generalization of Operator Product Expansion, appears to be founded on a firmer theoretical standpoint. Once more the equivalence of parton-like and OPE-like approaches is verified.
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