RCELF: A Residual-based Approach for Influence Maximization Problem

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Abstract—Influence Maximization Problem (IMP) is selecting a seed set of nodes in the social network to spread the influence as widely as possible. It has many applications in multiple domains, e.g., viral marketing is frequently used for new products or activities advertisement. While it is a classic and well-studied problem in computer science, unfortunately, all those proposed techniques are compromising among time efficiency, memory consumption, and result quality. In this paper, we conduct comprehensive experimental studies on the state-of-the-art IMP approximate approaches to reveal the underlying trade-off strategies. Interestingly, we find that even the state-of-the-art approaches are impractical when the propagation probability of the network have been taken into consideration. With the findings of existing approaches, we propose a novel residual-based approach (i.e., RCELF) for IMP, which i) overcomes the deficiencies of existing approximate approaches, and ii) provides theoretical guaranteed results with high efficiency in both time- and space- perspectives. We demonstrate the superiority of our proposal by extensive experimental evaluation on real datasets.

1 INTRODUCTION

Social networks (e.g., Facebook, Twitter, Weibo) are becoming an essential media for the public recently. Viral marketing is widely used in social networks to promote new products or activities. For example, new products are advertised by some influential users in social networks to other users by “word-of-mouth” effect. Therefore, the problem of selecting small but effective influential user set, a.k.a., Influence Maximization Problem (IMP), is the key for successfully viral marketing, and it has been widely studied in literature [1]–[25]. Mathematically, given a social network G, a positive integer k and a diffusion model M, the Influence Maximization Problem (IMP) returns a size-k nodes subset S (in G) which has the maximum expected influence in G. The diffusion model M defines the exact “word-of-mouth” effect manner, e.g., each influential/activated user can activate its inactive neighbours with a probability in [0, 1].

It is NP-hard to find the optimal size-k set for IMP with Independent Cascade (IC) and Linear Threshold (LT) diffusion models [1]. Due to the hardness of the IMP problem, many approximation approaches [2], [4], [7], [8], [10]–[14] and heuristic solutions [9], [15]–[22], [24], [26] have been proposed and extensive studied. However, all existing approaches (cf. Figure 1) are trading-off among time efficiency, memory consumption, and result quality [27]. Since the empirical performance of the state-of-the-art approximation approaches (e.g., IMM [11], DSSA [13]) are comparable to, even outperform the state-of-the-art heuristic solutions (e.g., IRIE [20], IMRank [25]), we focus on the approximated solutions for IMP in this work.

We classify existing approximation approaches into three categories, Monte-Carlo Simulation, Snapshots and Reverse Influence Sampling, respectively. We summarize the representative approaches of each category in Figure 2.

Monte-Carlo Simulation-based approaches: [1] is the first to apply Monte-Carlo Simulation techniques to solve IMP. It achieves \((1 - 1/e -\epsilon)\)-approximation ratio with probability \(1 - 1/n\) if the number of Monte-Carlo simulation times is \(\Theta(e^{-2k^2n\log(n^2/k)})\). CELF [2] improves the performance of [1] by exploiting the submodularity property of IMP with the same result approximation guarantee. However, CELF and its variant (i.e., CELF++) are not practical for the large social networks as the cost of Monte-Carlo simulations is rather expensive. In summary, Monte-Carlo simulation approaches achieve approximation guaranteed results for IMP by incurring extremely expensive time cost.

Snapshots-based approaches: They are proposed to improve the time efficiency of the Monte-Carlo Simulation-based approaches. SG [4] is the first to employ snapshots to address IMP problem. It samples subgraphs \(G_i\) (a.k.a., snapshots) of social network G in advance by retaining each edge with a probability of its weight. The influence of a node is estimated by averaging its influence on all snapshots. PMC [7] shrinks snapshots in SG into vertex-weighted directed acyclic graphs (DAGs) by using strongly connected
components (SCCs) of snapshots as DAGs' nodes. Through this, it reduces the memory consumption of SG. Both SG and PMC guarantee \((1 - 1/e - \epsilon)\)-approximation ratio by sampling enough snapshots. However, the memory overheads of SG and PMC are infeasible when the size of social network \(G\) is large. In conclusion, snapshots-based approaches provide approximation guaranteed results for IMP by incurring dramatically high memory consumption (to store snapshots).

**Reverse Influence Sampling-based approaches:** The first to propose Reverse Influence Sampling-based approaches for IMP. The core idea of reverse influence sampling is: (i) construct reverse reachable sets for the nodes, (ii) employ a greedy max-coverage algorithm to select the seed node iteratively. The representative approaches are TIM[10], IMM[11] and DSSA[3]. All of them guarantee \((1 - 1/e - \epsilon)\)-approximation ratio with a certain number of generated reverse reachable sets. In general, the memory consumptions of reverse influence sampling-based approaches are much smaller than snapshots-based approaches. However, it also may be unaffordable as (i) reverse reachable set size is sensitive to the propagation probability of each edge in the social network, e.g., IMM incurs 12.3GB memory footprints for 45.6MB DBLP dataset when the weighted cascade setting is slightly revised, we will elaborate it in Section[2] (ii) it needs to generate a large number of reverse reachable sets to guarantee the theoretical bound when \(\epsilon\) is small[28]. To make matters worse, all reverse reachable sets are stored in the memory for the max-cover-approximation.

**Probability assignment in diffusion models:** In general, diffusion models define how the node can switch its status from inactive to active on a weighted graph, where the weight of each edge is the influence probability. For example, an active node \(u\) has single chance to influence its inactive neighbor \(v\) with probability \(w(u, v)\) in IC model. In literature, a commonly-used influence probability assignment method in both IC (i.e., WC) and LT models is setting \(w(u, v) = 1/|\ln(v)|\)[28], where \(|\ln(v)|\) is the in-degree of \(v\). This assignment method assumes a user probably can be activated if all her incoming neighbors are active in both WC and LT models. However, it may not be practical in some real-world applications. For example, many users in Twitter (e.g., the users who use Twitter less than once per day) probably are not be influenced even all the neighbors are active. Interestingly, there is also another kind of social network where the users can be influenced even only one or few of its neighbors are active, e.g., users in Pinduoduo[1](an online shopping website in China) can be easily influenced as they can form a shopping team to get a lower price for their purchase.

To overcome the above limitations of common-used probability assignment method, i.e., \(1/|\ln(v)|\). We propose a generalized probability assignment method in this work. Specifically, the probability that node \(u\) can activate node \(v\) at edge \((u, v)\) is \(w(u, v) = \rho/|\ln(v)|\), where \(\rho\) reflects the activeness of the users. It is worth to note that the generalized IC and LT models are the conventional IC and LT models when \(\rho = 1\). Our approach: In this paper, we propose a novel residual-based algorithm RCELF to overcome the deficiencies of existing approximation approaches. The core of RCELF is the novel marginal gain computation method based on probability theory. Specifically, we define the residual capacity of a node as the maximal contribution of that node can make to influence spread value of the seed set. Initially, the residual capacity of each node is 1. During each seed node selection process, the residual capacity of each node will diminish by either being selected as a seed node or being influenced by other selected seed node. RCELF approach achieves excellent time efficiency as (i) it enjoys the benefits of the submodularity of the residual-based influence function and cost-effective lazy forward node selection manner, however, it requires much fewer Monte-Carlo simulations; (ii) the number of nodes under consideration (i.e., their residual capacities are large than 0) falls quickly during seed set selection process; and (iii) two optimizations are devised to speedup RCELF. Meanwhile, RCELF guarantees \((1 - 1/e)\)-approximation of the result, as elaborated in Section[3].

From memory consumption perspective, RCELF only stores the raw social network data. It does not have any extra memory consumption when comparing with snapshot-based approaches and reverse influence sampling-based approaches. Thus, the space complexity of RCELF is optimal, i.e., \(O(n + m)\). Moreover, RCELF is robust to a generalized probability assignment method, i.e., \(\rho/|\ln(v)|\), in both WC and LT models. \(\rho\) is a tunable parameter and reflects the influence degree of each user in the social network. In summary, RCELF achieves excellent time efficiency, low memory consumption and approximation guaranteed result quality for IMP in widely used diffusion models (i.e., as shown in the center of Figure[1]).

We summarize the comparison among our proposal RCELF and existing approximate approaches for IMP in Table[1]. Specifically, the contributions of this paper are summarized as follows.

1. [https://www.pinduoduo.com/](https://www.pinduoduo.com/)

| Category                      | Time Efficiency | Memory Overhead | Result Quality |
|-------------------------------|-----------------|-----------------|----------------|
| Monte-Carlo Simulation        | Low             | Small           | High           |
| Snapshots                    | Median          | Large           | High           |
| Reverse Influence Sampling    | High            | Median          | High           |
| **Our Approach (RCELF)**      | **High**        | **Small**       | **High**       |
and approximation guaranteed results concurrently (Section [5]).

- We evaluate the effectiveness and efficiency of our proposal by extensive experiments on real-world benchmark datasets (Section [4]).

The remainder of this paper is organized as follows. Section [2] describes the preliminaries and related works of IMP and conducts comprehensive experiments on the state-of-the-art approximate approaches to reveal their underlying issues. Section [3] presents our residual-based approach RCELF for IMP. Section [4] verifies the superiority of our proposal by extensive experiments, followed by the conclusion in Section [5].

## 2 Influence Maximization Problem

In this section, we first define the influence maximization problem (IMP) formally. Then, we conduct extensive preliminary experiments on the representative approximate approaches, and present the findings of existing approaches.

### 2.1 Problem Definition

We introduce several fundamental concepts for influence maximization problem (IMP) first.

**Definition 1.** (Social Network) A social network is a graph $G(V, E, W)$, where $V (|V| = n)$ is the set of nodes, and $E$ is the set of directed edges, $E \subseteq V \times V, (|E| = m)$, and $W$ is the set of weights of each edge in $E$.

The weight of edge $(u, v)$ is $w(u, v)$, and $u$ is the incoming neighbor of $v$, vice versa, $v$ is the outgoing neighbor of $u$. $\text{In}(v)$ and $\text{Out}(v)$ are the incoming and outgoing neighbor sets of node $v$, respectively. Given a social network $G$, the IMP is selecting a small but effective influential user set which could spread the influence in social network $G$ as widely as possible. We formally define seed node (i.e., influential user) in Definition [2].

**Definition 2.** (Seed Node) Node $v \in V$ is a seed node if it acts as the source of information diffusion in the social network $G(V, E, W)$. The set of seed nodes is called seed set, denoted by $S$.

Given a social network $G(V, E, W)$ and seed set $S$, the influence of seed set $S$ in $G$ is the total number of activated nodes with a specified diffusion model $M$, denoted by $\text{I}(S)$. $\text{I}(S)$ includes both newly activated node during information diffusion process and the initial seed set $S$. The information diffusion process (M) is a stochastic process, the goal of IMP is to maximize the expected influence value, as stated in Problem [1].

**Problem 1.** (Influence Maximization Problem, IMP) Given a social network $G(V, E, W)$, an integer $k$, and diffusion model $M$, the influence maximization problem IMP is selecting a size-$k$ seed set $S \subseteq V$, such that the expected influence value $\sigma(S) = E(\text{I}(S))$ is maximized.

The information diffusion model $M$ defines the exact information spread manner of seed set $S$. For example, each active user $u$ in step $t$ will activate each of its inactive outgoing neighbor $v$ in step $t-1$ with an influence probability $p_{u,v}$ in Independent Cascade (IC) and Weighted Cascade (WC) model. In Linear Threshold (LT) model, each edge $(u, v)$ has a weight $p_{u,v}$, and each node $v$ has a threshold $\theta_v$. The node $v$ can be activated if a "sufficient" number of its incoming neighbors are active, i.e., $\sum\text{active neighbors } u, p_{u,v} \geq \theta_v$.

**Influence Probabilities of Edges:** One of the core components in diffusion model $M$ is determining the influence probabilities/weights of each edge in social network. The commonly-used influence probability assignment method is weighted cascade (WC) [1], [4], [7], [8], [10]–[12]. In particular, all incoming neighbors of $v$ influence $v$ with equal probability $1/|\text{In}(v)|$. However, this assignment method ignores the activeness of the users in practical social networks. e.g., there exists 73% of Twitter users who use Twitter less than once per day. Such kind of users probably cannot be influenced even all her incoming neighbors are activated. Interestingly, users can be easily influenced by one or few of her neighbors in other applications. For example, users in Pinduoduo, an online shopping website, can invite their friends to form a shopping team to get a lower price for their purchase. Thus, an inactive user can be easily influenced by one of her activated friend.

In order to overcome the above limitations of common-used probability assignment method, i.e., $1/|\text{In}(v)|$. We propose a generalized probability assignment method in this work. Specifically, the probability that node $u$ can activate node $v$ at edge $(u, v)$ is $p_{u,v} = \rho/|\text{In}(v)|$, where $\rho$ reflects the activeness of the users. The advantage of the generalized probability assignment method is two-fold: (i) $\rho$ is tunable. It is the conventional setting when $\rho = 1$, and it is more general as $\rho$ can be set by users or learnt from training data, and (ii) it still enjoys the properties of conventional information diffusion models (e.g., IC, WC, and LT).

**Properties of IMP:** In order to facilitate the subsequent discussion, we briefly summarize the properties of IMP in this section.

**Theorem 1** (Hardness of IMP). The problem of influence maximization, as defined in Problem [1], is NP-hard under IC and LT model.

In addition to the above hardness of IMP, we present two nice properties of IMP, monotonicity and submodularity in Theorem [2] and [3] respectively.

**Theorem 2** (Monotonicity). The resulting influence function $\sigma(\cdot)$ is monotone as for any $S' \subseteq S$, we have $\sigma(S') \leq \sigma(S)$.

**Theorem 3** (Submodularity). For an arbitrary instance of the IC or LT model, the resulting influence function $\sigma(\cdot)$ is submodular. In other words, for any $S' \subseteq S$ and $v \notin S$, we have $\sigma(S \cup v) - \sigma(S) \leq \sigma(S' \cup v) - \sigma(S')$.

The marginal gain of node $v$ w.r.t. seed set $S$ is $mg(v|S) = \sigma(S \cup v) - \sigma(S)$. We omit the proofs of Theorem [1]-[3] and refer interested reader to [1].

### 2.2 Approximate Approaches for IMP

Due to the hardness to find the optimal solution for IMP (cf. Theorem [1]), a plethora of techniques [2]–[12] and refer interested reader to [1].
have been proposed to IMP with theoretical approximate bound. In this section, we briefly introduce the key ideas of each category of approximate approaches.

For each approach, we conduct extensive preliminary experiments and present the experimental findings to reveal the underlying issues.

Monte-Carlo Simulation-based GREEDY and CELF: GREEDY [1] is the first approach which employs Monte-Carlo simulation method to address IMP. The sketch of GREEDY is shown in Algorithm 1. GREEDY selects the node which has the largest marginal gain by Monte-Carlo simulation (Line 4) during each node selection iteration. In order to reduce the pain from unguaranteed submodularity [4] during Monte-Carlo simulations, GREEDY runs Monte-Carlo simulation $r$ times for each node $v$, typically, $r$ is 10,000 or 20,000 [1,2,4]. Thus, the time cost of GREEDY is extremely expensive.

Algorithm 1 Greedy($G(V, E, W)$, $k, M$)

1. Initialize seed set $S \leftarrow \emptyset$
2. $i \leftarrow 1$
3. for $i$ from 1 to $k$ do
4. $\sigma(S \cup v) - \sigma(S)$
5. Return $S$

CELF [2] is devised to improve the time efficiency of GREEDY. It exploits the submodularity of IMP (cf. Theorem 3) to reduce a lot of unnecessary marginal gain computations. Consider social network $G$ in Figure 3(a), CELF finds the node with the largest marginal gain at the beginning (i.e., $S = \emptyset$). CELF maintains a max-heap for the marginal gains of each node w.r.t. seed set $S$. Seed set $S$ is $\{c\}$ after the first seed selection iteration as $mg(c|\emptyset)$ is the largest. CELF maintains the rest max-heap by removing node $c$, as illustrated in Figure 3(b-I). At the second iteration, CELF gets the root of the max-heap $mg(b|\emptyset) = 4$. CELF computes node $b$’s marginal gain with seed set $S = \{c\}$, i.e., $mg(b|\{c\}) = 1$, and updates the max-heap accordingly (cf. Figure 3(b-II)). The max-heap root turns to $mg(d|\emptyset) = 3$, CELF then updates it to $mg(d|\{c\}) = 3$. For any descendant of the root in max-heap, their marginal gains must be smaller than 3 due to the submodularity of IMP. Thus, $d$ has the largest marginal gain with seed set $\{c\}$, and it is selected at the second seed selection iteration, i.e., $S = \{c, d\}$. In summary, CELF works in a lazy manner. It only computes the marginal gain of node $v$ with the latest seed set $S$ when it is necessary, e.g., CELF only computes the marginal gains of $b$ and $d$ at the second iteration in the above example, it reduces lots of unnecessary marginal gain computations. Hence, CELF is faster than GREEDY many orders of magnitude (i.e., 700 times [2]). However, CELF is still not feasible to large networks, e.g., a social network with 1 million nodes.

Experimental evaluations and findings: We test CELF on NetHEPT with 15K nodes and 62K edges, and it does not return the size-50 seed set within 30 hours. Monte-Carlo simulation based approaches (i.e., GREEDY and CELF) incur expensive computation time and low memory consumption, and provide theoretical guaranteed approximate solutions for IMP.

Snapshots-based SG and PMC: Snapshots-based approaches (SG [4] and PMC [7]) are proposed to improve the time efficiency of Monte-Carlo simulation based approaches. SG is the first approach that applies snapshots idea to address IMP. Instead of running lots of Monte-Carlo simulations in GREEDY and CELF, SG samples $r$ snapshots of input social network $G$ by coin flip technique. SG flips all coins with bias $p_{u,v}$ to produce several snapshots in advance, e.g., Figure 4(a) and (b) are the snapshots of the original social graph in Figure 3(a). SG selects seed nodes iteratively by (1) computing the marginal influence of each node $v$ by averaging the total reachable nodes of $v$ in all snapshots, (2) selecting the node with the largest average marginal influence, and (3) remove the node and all its reachable nodes in all snapshots. For example, In Figure 4 node $c$ has largest average reachable nodes (i.e., 5) as its reachable nodes in snapshots $G'_1$ and $G'_2$ are $\{c, f, e, g, a, h\}$ and $\{c, f, e, h\}$, respectively. Then SG selects node $c$ and removes its reachable nodes in Figure 4(a) and (b) before the second iteration.

PMC improves SG by reducing the memory consumption overhead of the snapshots in SG. Particularly, PMC generates the Directed Acyclic Graph (DAG) of each snapshot by identifying the strongly connected components (SCC) in it. However, the space consumption improvement extent of PMC depends on the connectivity of original graph and its snapshots. SG and PMC guarantee $(1 - 1/e - \epsilon)$ approxima-
Reverse Influence Sampling-based IMM and DSSA: Borgs et al. [6] is the first to propose reverse influence sampling (RIS) method for IMM under the IC and WC model. The core concept in RIS is reverse reachable set (RR set). Formally, the RR set of node $v$ is the set of nodes in $G$ that can reach $v$, i.e., $\forall u \in$ RR($v$), there is a path from $u$ to $v$ in $G$. RIS method includes two phases: (i) RR sets generation phase, and (ii) node selection phase. Consider that we run RIS on the social network $G$ in Figure 5(a). For the RR set generation phase, we first transpose $G$ in Figure 5(a) to $G^T$ in Figure 6(a). RIS randomly picks a node in $G^T$ and run Monte-Carlo simulation from it to generate its reachable set, i.e., RR($e$) = {e, f, c}. RIS repeats the above procedure several times to generate the RR sets, as shown in Figure 6(b). For node selection phase, RIS solves the max-coverage problem [29] to select $k$ nodes to cover the maximum number of generated RR sets in above phase. For example, node $c$ is selected as it covers the maximum number of RR sets (i.e., 3) in Figure 6(b). The reverse reachable set which covers node $c$ is marked to be ignored in subsequent seed selections. Theoretically, RIS returns $(1-1/e-\epsilon)$-approximation result [8] with at least a constant probability if the total examined number of nodes and edges reaches a pre-defined threshold $\tau$, which reflects the number of generated RR sets indirectly.

Since there is a large hidden constant factor in the asymptotic time complexity of RIS, which bound the practical efficiency of RIS. In order to address that, [10] proposed Two Phase Influence Maximization (a.k.a., TIM), which returns $(1-1/e-\epsilon)$-approximation solution with at least $(1-n^\epsilon)$ probability, and it runs in $O((k+l)(m+n)\log n/\epsilon^2)$ times. TIM samples pre-decided $\theta$ RR sets, instead of using threshold $\tau$ on computation cost to indirectly control the number in RIS. Later, IMM exploits a classical statistical tool (martingales [30]) to improve the parameters estimation phase in TIM. Since the number of generated RR sets can be arbitrarily larger than theoretical thresholds $\theta$ in TIM and IMM, Nguyen et al. [13] (i) unify the necessary sampled RR sets size in [8], [10], [11] to guarantee $(1-1/e-\epsilon)$-approximation ratio, and (ii) propose DSSA to achieve the minimum number of RR set samples. Technically, IMM and DSSA adopt a bootstrap strategy to probe the sampling size of RR sets. The procedure is: (1) initialize $r$ RR sets based on a given formula; (2) select a size-$k$ seed set $S$ by max-coverage algorithm; (3) evaluate the coverage ratio of $S$. If the coverage ratio is under the stopping condition, increase RR sets size and repeat (2) and (3). Otherwise, terminate and return $S$. The time efficiency and memory consumptions of IMM and DSSA heavily depend on the number of generated RR sets. Theoretically, the number of generated RR sets is decided by two parameters: (i) $\epsilon$, a large number of RR sets will be generated to guarantee the theoretical bound when $\epsilon$ is small [28]; and (ii) $\rho$ (i.e., the influence probability of each edge $\rho/\ln(v)$). Specifically, IMM and DSSA perform pretty good in conventional WC model (i.e., $\rho = 1$) as the expected number of node $v$’s direct reverse reachable neighbors is $1$ as the influence probability from node $u$ to $v$ is $1/\ln(v)$. However, their running times increase dramatically when $\rho$ scale up to 1.5 [23].

Experimental evaluations and findings: We evaluate the performance of IMM and DSSA methods in WC model by varying the influence probability $\rho/\ln(v)$ on Twitter and DBLP. The memory consumption of both IMM and DSSA are unaffordable when $\rho$ is scaling up (or down) as
illustrated in Figure (a) and (b). For example, the memory consumption of IMM with \(\rho = 0.1\) is almost 12.3X and 16.8X over the cost of IMM with \(\rho = 1.0\) in Twitter and dblp, respectively. The time costs of IMM and DSSA by varying \(\rho\) are shown in Figure (c) and (d). Obviously, both approaches are degenerating seriously when scaling up or down the influence probability in each edge \((u, v)\), i.e., \(w(u, v) = \rho/[\ln(v)]\). Both IMM and DSSA perform pretty good in terms of time efficiency and memory consumption in common-used probability assignment method (i.e., \(1/[\ln(v)]\)) in diffusion models. The reason is reverse influence sampling (RIS) technique exploits the expected number of node \(v\)'s direct reverse reachable neighbors is 1 in conventional diffusion models implicitly. The running time and memory consumption of IMM and DSSA are sensitive to the propagation probabilities (i.e., \(\rho/[\ln(v)]\)). Both IMM and DSSA are impractical when \(\rho\) scales up (or down), as the results shown in Figure (e).

Specifically, when \(\rho < 1\), IMM and DSSA require more bootstrap iterations, however, each iteration generates double RR sets. For \(\rho > 1\), the number of node in each generated RR set by IMM and DSSA will increase dramatically as the strongly connected properties of the social network.

2.3 Other Related Works

Influence maximization problem IMP is first solved in algorithmic perspective by probability [31]. Beyond above discussed approximate approaches, there are many heuristic-based approaches [9], [15]–[22], [24]. We omit the details discussed approximate approaches, there are many heuristic-rithmic perspective by probability [31]. Beyond above discussed approximate approaches, there are many heuristic-based approaches [9], [15]–[22], [24]. We omit the details here and refer the interested readers to a recent survey [28]. Very recently, several works [5], [6] are proposed for IMP variants (e.g., online and adaptive IMP), we skip the discussion as the scope of this work is conventional IMP.

3 RESIDUAL-BASED APPROACH

Existing approximate approaches for IMP are compromising either time efficiency or memory overhead for result quality. In this section, we propose a novel residual-based approach (i.e., RCELF) for IMP to overcome this dilemma. We present the fundamental concepts of RCELF approach in Section 3.1. In Section 3.2, we describe the backbone of RCELF and devise two performance optimization techniques for it. We conduct correctness, complexity, and approximate analysis of RCELF in Section 3.3.

3.1 RCELF Approach

Generally, each node \(v \in V\) in social network contributes to the influence spread value of seed set \(S\), i.e., \(\sigma(S)\), by either being selected as a seed node or being influenced by other seed nodes. In this work, we propose a novel concept, node residual capacity, to capture the contribution of each node to the influence spread value. RCELF selects the node \(v \in V - S\) with the largest marginal gain (based on node residual capacity) as a seed node at each iteration. The residual capacity of each node diminishes during the seed node selection process. In order to capture the contribution of each node, we define residual-based social network formally in Definition 3.

Definition 3 (Residual based Social Network). Residual based social network \(rG(V, E, W, C)\) is a social graph \(G(V, E, W)\) (cf. Definition 1) with residual capacity set \(C\). Initially, the residual capacity of each node \(v \in V\) in \(C\) is \(RC(v) = 1\).

Given a diffusion model, the core subroutine of IMP is marginal gain computation. Given seed set \(S\), the marginal gain of node \(u\) is computed by \(mg(u|S) = \delta(S \cup \{u\}) - \delta(S)\) in literature. In this work, RCELF computes the marginal gain \(mg(u|S)\) by exploiting the residual capacity of every nodes during each node selection iteration, i.e., the contribution of each node \(v \in V - S\) to \(mg(u|S)\).

Given residual based social graph \(rG(V, E, W, C)\) and seed set \(S\). The marginal gain \(mg(u|S)\) is contributed by two parts: (i) node \(u\), and (ii) the nodes which can be influenced by node \(u\). Intuitively, the contribution of node \(u\) to \(mg(u|S)\) is its residual capacity \(RC(u)\). The contribution of other nodes \((\text{i.e.,} \forall v \in V - (S \cup u))\) to \(mg(u|S)\) is a bit more intricate. In subsequent sections, we present the marginal gain computations of RCELF with LT and IC model, respectively.

3.1.1 RCELF Marginal Gain Computation in LT Model

In LT model, each node \(v\) uniformly chooses a threshold \(\theta_v\) from the range \([0, 1]\). It can be activated if \(\sum_{u \in \text{activated neighbors}} w(u, v) \geq \theta_v\). Take Figure 3(a) as an example, the probability of node \(b\) can be activated by node \(a\) is \(Pr_{a,b} = Pr_{a,b} = Pr_{b} = 0.7\), as \(\theta_b\) uniformly chooses from \([0, 1]\). In LT model, \(\forall v \in V, \theta_v\) is independent and identically distributed. Hence, given a path \(P = \{v_1, v_2, \ldots, v_m\}\), the probability of node \(v_m\) can be activated by node \(v_1\) is \(Pr_{v_1}(P) = \prod_{i=1}^{m-1} w(v_i, v_{i+1})\). Definition 4 defines the active / influence probability in LT model formally.

Definition 4 (Influence Probability in LT). Given residual network \(rG(V, E, W, C)\), the probability of \(v_i\) activates \(v_m\) through path \(P = \{v_1, v_2, \ldots, v_m\}\) is \(Pr_{v_1}(P) = \prod_{i=1}^{m-1} w(v_i, v_{i+1})\). Set \(P\) includes all paths from node \(v_1\) to \(v_m\) in \(rG\), the overall probability that node \(v_i\) influences node \(v_m\) is \(Pr(v_1, v_m) = \sum_{P \in P} Pr_{v_1}(P)\).

Example: In Figure 3(a), node \(a\) can reach node \(d\) in three paths, i.e., \(P_1 = \{a, d\}, P_2 = \{a, c, d\}\) and \(P_3 = \{a, b, c, d\}\). The probability of node \(a\) can influence node \(d\) is \(Pr_{a,d} = Pr_a(P_1) + Pr_a(P_2) + Pr_a(P_3) = 0.4 + 0.3 + 0.2 + 0.7 + 0.5 + 0.2 = 0.53\).

Definition 5 shows the contribution of node \(v\) to marginal gain \(mg(u|S)\) in LT model.
**Definition 5** (Node Contribution in LT). The contribution of node \( v \) to marginal gain \( mg(u|S) \) is \( \Phi(u, v) = RC(u) \times Pr(u, v) \). \( \Phi(u, v) = RC(v) \) if \( RC(u) \times Pr(u, v) \geq RC(v) \).

**Example:** In Figure 8(b), node \( c \)'s contribution to marginal gain \( mg(b\{a\}) \) is \( \Phi(b, c) = RC(b) \times Pr(b, c) = 0.3 \times 0.5 = 0.15 \).

Formally, given a residual social network \( rG(V, E, W, C) \) and seed set \( S \), the marginal gain of node \( u \) consists of i) the residual capacity of node \( u \), and ii) node contributions from other influenced nodes. Specifically,

\[
mg(u|S) = RC(u) + \sum_{v \in V - (S \cup \{u\})} \Phi(u, v).
\]

**Example:** In Figure 8(b), the marginal gain \( mg(b\{a\}) \) is \( RC(b) + \Phi(b, c) + \Phi(b, d) = 0.3 + 0.3 \times 0.5 + 0.3 \times 0.5 + 0.2 = 0.48 \).

3.1.2 **RCELF Marginal Gain Computation in IC Model**

Comparing to the contribution of node \( v \) to \( mg(u|S) \) in LT model, it is more complex in IC model. The reason is that the active node \( u \) will definitely influence its inactive neighbor \( v \) in LT model, i.e., \( \theta_v = 1 - w(u, v) \). However, the active node \( u \) may not influence its inactive neighbor \( v \) as the influence process is a random coin-flip process with bias \( w(u, v) \) in IC model.

To illustrate, consider the probability that node \( c \) can be activated by node \( a \) in Figure 8(a) in both LT and IC model. There are two paths from \( a \) to \( c \), i.e., \( P_1 = \{a, c\} \) and \( P_2 = \{a, b, c\} \) respectively. In LT model, the influence probability \( \Phi(a, c) = Pr_a(P_1) + Pr_a(P_2) = 0.3 + 0.35 = 0.65 \) by Definition 4. I.e., the node \( c \) will be activated if \( \theta_c \leq 0.65 \). In IC model, however, node \( a \) influence node \( c \) with probability \( Pr_1 = w(a, c) = 0.3 \) via path \( P_1 \) and with probability \( Pr_2 = w(a, b) \times w(b, c) = 0.35 \) via path \( P_2 \). Thus, the total activated probability of node \( c \) by node \( a \) is \( 1 - (1 - Pr_1)(1 - Pr_2) = 1 - 0.7 \times 0.65 = 0.545 \) according to conditional probability theory.

To facilitate the discussion of node marginal gain contribution in IC model, we divide the reachable nodes of \( u \) into two groups: (i) \textit{shared-nothing} set SN and (ii) \textit{shared-edge} set SE. For example, Figure 8(a), node \( a \)'s reachable node \( c \) is \textit{shared-nothing} node as the paths from \( a \) to \( c \) does not share any edge, i.e., \( \{a, c\} \) and \( \{a, b, c\} \), \( b \) also is \( a \)'s \textit{shared-nothing} nodes. However, \( d \) is a \textit{shared-edge} node as paths \( \{a, b, c, d\} \) and \( \{a, c, d\} \) shared a common edge \( e(c, d) \). The node contribution of each node \( v \) in \textit{shared-nothing} set SN to the marginal gain \( mg(u|S) \) as follows.

**Definition 6** (Influence Probability in Single Path). Given a path \( P = \{v_1, v_2, \ldots, v_m\} \) in \( rG(V, E, W, C) \), seed set \( S \). The probability of node \( v_1 \) influences \( v_m \) is \( Pr_{v_1}(P) = \prod_{i=1}^{m-1}(RC(v_i)w(v_i, v_{i+1})) \) in IC model.

Suppose there is a set of paths \( P \) in which node \( u \) can influence node \( v \) in SN (i.e., \( v \) in \textit{shared-nothing} node set). According to conditional probability theory, the probability node \( u \) influence node \( v \) is

\[
Pr(u, v) = 1 - \prod_{p \in P}(1 - Pr_u(p)).
\]

The total contribution of node \( v \) to \( mg(u|S) \) in IC model is \( \Phi(u, v) = RC(v) \times Pr(u, v) \).

**Example:** In Figure 8(a), there are two paths from node \( a \) to node \( c \), \( P_1 = \{a, b, c\} \) and \( P_2 = \{a, c\} \). The contribution of \( c \) to \( mg(a|\emptyset) \) is equivalent to \( RC(a) \times Pr(a, c) = 1.0 \times (1 - 0.3 \times 0.7) = 0.545 \).

For the nodes in \textit{shared-edge} set SE, it is quite difficult to analyze the active probability from an active node \( u \) at step \( t \). Fortunately, inspired by the SG approach, the active probability of these nodes in \textit{shared-edge} set could be obtained by running \( r \) times Monte-Carlo simulations. We then define the marginal gain contribution of every node in \textit{shared-edge} set in Definition 7.

**Definition 7** (\textit{Shared-edge} Node Contribution). Given residual-based graph \( rG(V, E, W, C) \) and seed set \( S \). For each node \( v \in SE \), the influence probability \( \Phi(u, v) \) is obtained by Monte-Carlo simulation. The node contribution of node \( v \) to \( mg(u|S) \) in IC model is \( \Phi(u, v) = RC(v) \times (RC(u)Pr(u, v)) \).

Finally, the marginal gain \( mg(u|S) \) in IC model can be computed by \( mg(u|S) = RC(u) + \sum_{v \in V - (S \cup \{u\})} \Phi(u, v) \).

3.1.3 **Updating Node Residual Capacity**

During RCELF seed node selection procedure, suppose node \( u \) is selected as seed node at step \( t \) (i.e., \( u = \operatorname{argmax}_{u \in V - S} mg(u|S) \)), the residual capacity of every \( u \)'s reachable node \( v \) will be updated by \( RC(v) = RC(v) - \Phi(u, v) \) accordingly, it will be ignored if \( RC(v) \leq 0 \) in subsequent seed selections.

3.2 **Implementation and Optimizations**

In this section, we present the sketch of RCELF approach with two performance optimization techniques.

**RCELF Approach:** The sketch of our residual-based approaches RCELF for IMP with LT and IC model as follows:

1. Init influence max-heap \( H \) it builds a max-heap by using the marginal gain as key value.
2. Identify seed node \( u \) (i.e., \( u = \operatorname{argmax}_{u \in V - S} mg(u|S) \)) by \( H \) efficiently, insert it into seed set \( S \leftarrow S \cup \{u\} \).
3. Update residual capacity of each node \( v \) in \( rG \), i.e., \( \forall v \in V - S, RC(v) \leftarrow RC(v) - \Phi(u, v) \). Node \( v \) will be discarded in \( rG \) if \( RC(v) \leq 0 \).
4. Repeat Step (2) and (3), until \( k \) seed nodes are selected.

In the subsequent section, we improve the performance of RCELF by (1) proposing an efficient marginal gain computation algorithm and (2) reducing max-heap update cost.

**Efficient Marginal Gain Computation:** Intuitively, we compute the marginal gain of each node, i.e., for node \( u \) and seed set \( S \), \( mg(u|S) \) is initialized to \( RC(u) \). We enumerate all the paths from \( u \) to each node \( v \in V - S \) and calculate \( \Phi(u, v) \), then accumulate \( \Phi(u, v) \) to \( mg(u|S) \) in LT model. However, the computation cost is exponential to the number of edges in \( rG \). Hence, it is impractical in median or large social networks. In addition, the contribution of \textit{shared-edge} nodes cannot compute exactly as \textit{shared-nothing} nodes in IC model. To address the above issues, we devise a Monte-Carlo simulation based marginal gain computation algorithm (cf. Algorithm 2). The main idea of Algorithm 2 is that it incorporates all \( \Phi(u, v) \) computations in one batch.
Algorithm 2 MCSMG(rG(V, E, W, C), u, S, r)

1. Initialize map Φ ← ∅  
2. for i from 1 to r do  
3. Queue q.enqueue(u)  
4. while !IsEmpty(q) do  
5. node tmp ← q.dequeue()  
6. for each v in Out(tmp) and it is inactive do  
7. if RAND() < RC(tmp) · w(tmp, v) then  
8. q.enqueue(v)  
9. Φ(u, v) ← Φ(u, v) + 1  
10. mg(u|S) ← RC(u)  
11. for each Φ(u, v) in Φ do  
12. Φ(u, v) ← Φ(u, v)/r + RC(u)  
13. mg(u|S) ← mg(u|S) + Φ(u, v)  
14. Return mg(u|S) and Φ

Monte-Carlo simulation process. Algorithm 2 shows the exact steps about mg(u|S) computation in IC model. In each Monte-Carlo simulation, it takes the residual capacity of each inactive node v into consideration by flipping coins with probability RC(c) · w(u, v) (cf. Line 7) instead of only w(u, v). Φ(u, v) counts the number of activated times of node v among r times simulation (cf. Line 2). Finally, for each node v, its Φ(u, v) compute as Definition 5 from Line 11 to Line 13. It is worth noting that Algorithm 2 is applicable to LT models. For example, we only need revise the node activation manner (cf. Line 7) for LT model.

Heap Updates Optimization: RCELF identifies the seed node u (i.e., argmax_u∈V−S mg(u|S)) with max-heap H. The marginal gain of nodes in max-heap H need recompute as it is out-of-date after each seed node selection iteration. i.e., the current marginal gain of node u is computed with an out-of-date seed set, denoted by S_o. However, it should be mg(u|S), where S is latest seed set. Thus, the performance of RCELF approach is very sensitive to the number of node marginal gain computations in Step (2) to identify the next seed node argmax_u∈V−S mg(u|S). Here, we propose an upper bound for the marginal gain of node v with latest seed set S, denote by mg(u|S). It reduces the number of marginal gain computations significantly.

Lemma 1 (Upper Bound of mg(u|S)). For each node v ∈ V, its residual capacity and marginal gain are RC_o(v) and mg(v|S_o) when the seed set is S_o at step t − 1. The seed set is S at step t (i.e., S_o ⊂ S), the upper bound mg(u|S) is RC(u) + RC(u)·∑_v∈Out(u) w(u, v) · RC_o(v) · mg(v|S_o).

Proof. For each node v ∈ V, RC_o(v) ≥ RC(v) as the residual capacity is diminishing during seed selection process. With mg(v|S_o) ≥ mg(v|S) where S_o ⊂ S (by submodularity), we have:

mg(u|S) = RC(u) + RC(u)·∑_v∈Out(u) w(u, v) · RC_o(v) · mg(v|S_o)
≥ RC(u) + RC(u)·∑_v∈Out(u) w(u, v) · RC(v) · mg(v|S)
≥ RC(u) + Φ(u, v)

Thus, we have mg(u|S) ≥ mg(u|S).

Consider mg(u|S_o) with seed set S_o, the marginal gain upper bound mg(u|S) will be computed with constant cost at first, then the max-heap H is updated with mg(u|S). Algorithm 2 will be incurred to compute the exact marginal gain mg(u|S) if and only if the root is mg(u|S). Inherently, Lemma 1 works as a filter which reduces lots of expensive exact marginal gain computations.

3.3 RCELF Analysis

We analyze the property of influence spread function δ(·) in RCELF at Theorem 4, then prove the result accuracy guarantee of RCELF in Lemma 2.

Theorem 4. The influence spread function δ(·) in RCELF with IC model is (i) non-negative, (ii) monotone, and (iii) submodular.

Proof. Suppose the selected seed nodes from 1st iteration to k-th iteration are v_1, v_2, · · · , v_k. The corresponding seed sets are S_1, S_2, · · · , S_k, and S_0 = ∅. Thus, δ(S_k) = mg(v_k|S_{k−1}) + δ(S_{k−1}) = ∑_{i=1}^{k} mg(v_i|S_{i−1}).

∀i ∈ [1, k], mg(v_i|S_{i−1}) ≥ 0 in RCELF approach. Thus, δ(S) ≥ 0 and ∀S ⊂ S, δ(S) ≤ δ(S‘). Hence, δ(·) in RCELF is (i) non-negative and (ii) monotone.

Since the residual capacity of each node v will be diminished, cf. Step (3), during seed node selection process, ∀S ⊂ S we have

mg(u|S) ≥ mg(u|S‘)
⇒ δ(S) + mg(u|S) − δ(S) ≥ δ(S‘) + mg(u|S‘) − δ(S‘)
⇒ δ(S ∪ {u}) − δ(S) ≥ δ(S‘ ∪ {u}) − δ(S‘)

Thus, δ(·) in RCELF is submodular, the proof complete.

We then show the result accuracy guarantee of RCELF in Lemma 2 as follows.

Lemma 2. Let size-k set S* be optimal set of IMP, i.e., δ(S*) has maximal value of all k-element sets. RCELF returns size-k set S, which guarantees

δ(S) ≥ (1 − 1/e) · δ(S*)

Proof. Since the influence spread function δ(·) in RCELF is non-negative, monotone and submodular (cf. Theorem 4), it guarantees that S, returned from RCELF, provides a (1 − 1/e)-approximation ratio, as proved in [1, 32].

Inherently, RCELF works similar with SG and PMC. For example, RCELF is built upon the residual-based graph. The residual capacity diminishing process is similar to removing nodes and edges in generated snapshots in SG and PMC. Inspired from [7], we analyze the number of necessary Monte-Carlo simulations in RCELF to guarantee a (1 − 1/e − e)-approximation result as follows.

Lemma 3. By setting Monte-Carlo simulation times r = O(n² log n log (n/r)), RCELF achieves a (1 − 1/e − e)-approximation ratio with probability 1 − 1/n.

Proof. Let S be the set of every possible result set S, |S| = C_n^k. For any result set instance S ∈ S, δ_i(S) is S’s influence spread value of the i-th Monte-Carlo simulation at G, δ_i(S) ∈ [1, n]. Let δ(S) = ∑_{i=1}^{n} δ_i(S). By
TABLE 2: Dataset characteristics

| Datasets      | n   | m   | Direct | Source          |
|---------------|-----|-----|--------|-----------------|
| NetHEPT       | 15K | 62K | No     | arxiv.org       |
| twitter       | 81K | 1.7M| Yes    | snap.stanford.edu|
| DBLP          | 317K| 2M  | No     | snap.stanford.edu|
| LiveJournal   | 4.8M| 69M | Yes    | snap.stanford.edu|
| twitterLarge  | 417M| 1.3G| Yes    | an.kast.ac.kr   |

applying Hoeffding’s inequality (Theorem 3 in [7]), for any \( S \subseteq V, \frac{1}{n}|\delta(S) - \delta(S)| \leq \epsilon \) holds with at least probability \( 1 - 2e^{-2\epsilon^2(n)} \). By choosing \( \epsilon = \frac{n \log n \log(\frac{1}{\delta})}{2} \), the above conclusion holds with at least probability \( (1 - 1/n) \). By applying Lemma 2 we have \( \delta(S) \geq (1 - 1/e - \epsilon) \cdot \delta(S') \) with \( r \) Monte-Carlo simulation times.

4 EXPERIMENTAL EVALUATION

In this section we evaluate RCELF and present our empirical findings. In Section 4.1 we describe the experimental setting. In Section 4.2 we compare our proposal with existing approximate approaches in real datasets, and investigate the effectiveness of the optimization technique.

4.1 Experimental Setting

Datasets: We use five widely-used benchmarks for IMP. Table 2 provides details on the number of nodes, edges, average degree, data type and sources of each dataset.

Algorithms: We compare our proposal RCELF with four methods, namely, SG [4], PMC [7], IMM [11] and DSSA [13]. We omit CELF and CELF++ as they are infeasible for datasets in Table2. The source code of PMC, IMM and DSSA are from the homepage of the authors. We implement SG and RCELF by C++. All experiments run on Centos 7.4 with Intel Xeon E5-2620(2.1GHz) and 80GB memory.

Diffusion Models: We consider the widely used Weighted Cascade (WC), Independent Cascade (IC) and Linear Threshold (LT) models. In conventional WC and LT models, the weight of each edge \( w(u, v) \) is set as \( 1/\ln(v) \). We use \( \rho/|\ln(v)\) in this work, where \( \rho \) is a tunable parameter. For IC model, we follow the settings in [4], e.g., the weight of each edge is 0.001.

Parameter Setting: In order to guarantee the same result quality, we set \( \epsilon = 0.1 \) in IMM [14], DSSA [13] and our proposal RCELF. For DSSA \( \delta = 1/|V|, r = 200 \) for SG and PMC. The performance metrics of IMP are execution time, memory consumption. Each plotted value corresponds to the average of measurements observed over 20 times.

4.2 Performance Evaluation

Effect of \( \rho \) in WC: Figure 9, 10 and 11 show the execution time, memory consumption and result quality of each approach by varying \( \rho \) from 0.1 to 1.3, respectively. The execution time of RCELF outperforms SG, PMC, DSSA and IMM for all \( \rho \) values in four datasets, as illustrated in Figure 9. However, IMM and DSSA perform as better as RCELF when \( \rho = 1 \) in Twitter and LiveJournal (see Figure 9c and (d)). It confirms our analysis in Section 2.2 i.e., IMM and DSSA are pretty good when \( \rho = 1 \).

We measure the memory consumption of each approach by varying \( \rho \) in four datasets in Figure 10. Since the space complexity of RCELF is \( O(n + m) \), the memory consumption of RCELF performs better than all other competitors by varying \( \rho \) from 0.1 to 1.3 in all datasets. Interestingly, the memory consumptions of IMM and DSSA are rising when \( \rho \) varies from 1 to 0.1, and from 1 to 1.3 in two large datasets (see Figure 10c and (d)). The reason is that the memory consumptions of IMM and DSSA are heavily relying on the prorogation probability of each edge. Specifically, IMM and DSSA requires more bootstrap iterations when \( \rho \) is small. Thus the memory consumptions are rising from 1 to 0.1. When \( \rho \) is rising from 1.0 to 1.3, the number of nodes in each generated RR set by IMM and DSSA will increase dramatically as the strongly connected properties of the underlying social network. Thus the size of RR set grows up.

The result qualities of all approaches by varying \( \rho \) in four datasets are illustrated in Figure 11. The practical spread results of different approaches are similar as all approaches guarantees the same approximation ratio of result.

Effect of \( k \) in WC: We test the effect of \( k \) by fixing \( \rho = 0.1 \) in all four datasets. Figure 12 shows the execution cost of each method, varying \( k \) from 1 to 100. Observe that our proposal RCELF consistently outperforms all competitors. Specifically, our proposal RCELF is up to 86.1X, 149.7X, 72.1X and 47.8X faster than IMM in NetHEPT, DBLP, Twitter, and LiveJournal, respectively. It is faster than DSSA by 134X, 116X, 86.8X and 57.5X, respectively. Besides that, the performance of PMC is worse than that of SG. Although PMC is proposed to improve SG, it costs more time since it is sensitive to the connectivity of the graph. Unfortunately, the graph connectivity is poor with WC model, the overhead of generating DAGs in PMC is larger than its benefits.

The memory consumptions of each approach in four datasets are illustrated in Figure 13. RCELF’s memory requirement is stable with regard to \( k \) in all datasets as the space complexity of our approach is \( O(m + n) \). It requires the minimum memory space among all competitors. For example, when \( k = 5 \) DSSA, IMM and RCELF require 1,548MB, 1,625MB and 47MB memory space on NetHEPT, respectively. In particular, DSSA consumes more than 27GB memory space when \( k = 1 \) to return a solution on a 18MB dataset DBLP.

Figure 14 shows the spread values of different approaches in four datasets by varying \( k \) from 1 to 100. As expected, our proposal RCELF performs as well as other competitors (e.g., IMM, DSSA) in all tested settings. This also verified that RCELF guarantees the approximation ratio of the result as other approximate approaches.

RCELF in IC and LT: Figure 15 shows the execution time of RCELF, SG, PMC, IMM, and DSSA by varying \( k \) in IC model, where \( w(u, v) \) is the setting of [4], i.e., \( \rho = 0.1 \) in IC model, and follow the parameter setting in [7]. However, PMC performs worse than other competitors in all cases.

5. We use the author released code of PMC and follow the parameter setting in [7]. However, PMC performs worse than other competitors in all cases.
faster than IMM and DSSA in NetHEPT, DBLP among all four datasets, as shown in Figure 15 in LiveJournal dataset, IMM cannot return results as it incurs extremely large memory consumption for all $k$ values. When $k = 1, 2$ and 5, DSSA also infeasible due to huge memory consumption, it confirms that DSSA performs worse when $k$ is small (see Figure 15(d)).

SG and PMC do not work with LT [28]. Figure 16 shows the execution time of RCELF, IMM, and DSSA by varying $k$ in LT model. RCELF is better than or at least comparable with IMM and DSSA in all four datasets.

For the sake of presentation, we omit the memory consumption and result quality results as they are similar with Figure 13 and 14 respectively.

Scalability of RCELF: We verify the scalability of RCELF in the largest dataset (i.e., TwitterLarge) used in literature in WC model. Figure 17(a) shows the execution cost of RCELF, IMM, and DSSA by varying $\rho$ with $k = 5$. RCELF outperforms other competitors in all cases. We measure the execution time of RCELF, IMM, and DSSA by varying $k$ with $\rho = 0.1$ in Figure 17(b). When $k$ is small, RCELF performs better than IMM and DSSA. When $k$ is large, DSSA outperforms RCELF due to its superiority for large $k$. Besides, the memory consumption of RCELF is less than IMM and DSSA as RCELF does not incur any extra memory overhead.

Optimization Evaluation: Here we evaluate the effectiveness of our proposed optimization techniques (i.e., Lemma 1). We evaluate the effect of Lemma 1 with $k = 100$ and $\rho = 0.1$. As illustrated in Figure 18, our Lemma 1 optimization offer saving of 1.0%, 26.7%, 21.7%, 40.9% (compared to RCELF without Lemma 1) in NetHEPT, DBLP, Twitter and LiveJournal, respectively.

5 Conclusion
In this paper, we discuss existing approximation solutions for IMP, which are compromising time efficiency or memory consumption for the approximate result quality. In order to address that, we propose a residual-based algorithm RCELF for IMP, which achieves good time efficiency, low memory consumption and approximate guaranteed result quality concurrently in generalized IC and LT models. Besides, we propose several optimizations to accelerate the performance of RCELF. We demonstrate the superiority of RCELF on standard real benchmarks. We plan to extend our RCELF to Triggering model and Time Aware model in future work.

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Fig. 12: Execution time vs. $k$ with $\rho = 0.1$, WC

Fig. 13: Memory consumption vs. $k$ with $\rho = 0.1$, WC

Fig. 14: Result quality vs. $k$ with $\rho = 0.1$, WC

Fig. 15: Execution time vs. $k$ with $w = 0.001$, IC

Fig. 16: Execution time vs. $k$ with $\rho = 0.1$, LT
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