ADaPTION: Toolbox and Benchmark for Training Convolutional Neural Networks with Reduced Numerical Precision Weights and Activation

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Abstract

Deep Neural Networks (DNNs) and Convolutional Neural Networks (CNNs) are useful for many practical tasks in machine learning. Synaptic weights, as well as neuron activation functions within the deep network are typically stored with high-precision formats, e.g. 32 bit floating point. However, since storage capacity is limited and each memory access consumes power, both storage capacity and memory access are two crucial factors in these networks. Here we present a method and present the ADaPTION toolbox to extend the popular deep learning library Caffe to support training of deep CNNs with reduced numerical precision of weights and activations using fixed-point notation. ADaPTION includes tools to measure the dynamic range of weights and activations. Using the ADaPTION tools, we quantized several CNNs including VGG16 down to 16-bit weights and activations with only 0.8% drop in Top-1 accuracy. The quantization, especially of the activations, leads to increase of up to 50% of sparsity especially in early and intermediate layers, which we exploit to skip multiplications with zero, thus performing faster and computationally cheaper inference.

1 Introduction

In the last decade, machine learning applications based on Convolutional Neural Network (CNN) have gained substantial attention due their high performance on classification and localization tasks [16]. In parallel, dedicated hardware accelerators have been proposed to speed up inference of such networks after training is completed [2-4, 11, 15]. Since the target of such hardware accelerators are mobile devices, IoT devices and robots, reducing their memory consumption, memory...
Figure 2: Motivation for reduced precision in weights and activations of deep Convolutional Neural Networks. (a) Memory arrangement and comparison in terms of energy consumption, latency and capacity. (b) Target platforms for convolutional neural networks.

access and computation time are crucial (see Fig. 2). Pruning and model compression [12], quantization methods [5, 6, 9, 13, 21], as well as toolboxes [1, 10, 22] have been developed to reduce the numbers of neurons and connections. The deployment of CNNs on embedded platforms with only fixed point computation capabilities and the development of dedicated hardware accelerators has also spurred the development of toolboxes that can train CNNs for fixed point representations [1, 10, 22]. The popular toolbox Ristretto [10], for example, can be used to train CNNs with fixed-point weights, but not fixed-point activations. To adapt both weights and activations of CNNs trained on conventional GPUs using 32 bit floating-point representation to fixed-point hardware, we developed the ADaPTION toolbox [see Fig. 1]. Furthermore, our toolkit supports also to train CNNs from scratch with specified precision for both weights and activations to run on the recently developed hardware accelerator NullHop [2].

In the next section we will introduce new functionalities and parameters added to Caffe, as well as the workflow of ADaPTION. In Section 3 we will discuss the crucial components to achieve SOA classification accuracy with reduced precision weights and activations and compare ADaPTION directly to other existing toolboxes.

2 Low-precision add-on for Caffe

Caffe is a deep learning library developed by the Berkeley Vision and Learning Center [14]. It provides all state-of-the-art error backpropagation gradient descent tools such as ADAM, ADA-Grad and many different layer types, as well network architectures. To incorporate low-precision training within this framework we added three new layer types: LPInnerProduct, LPConvolution and LPAct, where the LP stands for low-precision. These layers operate the same way as their high-precision counterpart, except that during the forward pass the values are quantized to the respective fixed-point representation given a specified bit-precision and decimal point, e.g. signed 16 bit, Q1.15. To round the weights and activations we introduced three additional parameters: BD, AD and rounding_scheme, which are specified in the network configuration file. The parameter BD (Before Decimal point) specifies the maximum integer value that can be represented. It determines the number of bits of the integer part of the respective value is allowed to occupy, including the sign bit. The parameter AD (After Decimal point) specifies the precision that can be represented. It determines the number of bits the fractional part of the respective value is allowed to occupy. The rounding_scheme parameter is a flag that sets the the option to either round weights and activations deterministically or stochastically. The notation Qm.f is implemented in Caffe as QBD.AD.

1 Code available: https://github.com/NeuromorphicProcessorProject/ADaPTION
2 The notation Qm.f sets the decimal point of a fixed-point number, where m represents the integer part and f the fractional part of the number.
Our toolkit ADaPTION has the following features (see also Fig. 3):

- extraction of network structure into a net_descriminator
- dynamic, layer-wise distribution of predefined available number of bits for weights and activations independently according to the respective dynamic range
- creation of a new low-precision network based on the extracted or user-defined net_descriminator, as well as layer-wise or global bit-distribution
- fine-tuning\(^3\) of extracted high-precision weights or re-training from scratch with user-specified rounding_scheme
- exporting the network to a NullHop\(^4\) compatible file format

These changes are fully compatible with the original version of Caffe and can be merged or used as a stand-alone Caffe version.

2.1 ADaPTION workflow and method

For adapting the quantized weights and activations, we used the method called power2quant, formerly known as dual copy rounding, developed by \(^2\) and concurrently by \(^5\). ADaPTION works in the following way: it extracts the network structure and the pre-trained weights of a given CNN model. The structure is adapted to use low-precision convolutional, Rectified Linear Unit (ReLU) and fully-connected layers provided as separated layers. The low-precision activation layer is separate from the actual activation layer in Caffe. The activations are quantized before they are sent to the activation layer, i.e., the ReLU layer. The separation of quantization procedure and activation function has the potential advantage that new activation functions can be directly used in ADaPTION. The pre-trained weights are converted into the low-precision Caffe blob\(^4\) structure. An ADaPTION method measures the dynamic range of weights and activations by inferring a random set of training images. The measured dynamic ranges are used by another

\(^3\)A network subject to fine-tuning is initialized with pre-trained weights, which are subject to further training on the same data set.

\(^4\)A blob is a data storage structure that is used by Caffe to store the neuron parameters, such as the weights or biases.
Figure 4: Comparison of sparsity in activations of VGG16: Sparsity within each layer of VGG16 classifying 25,000 random test images from ImageNet. The high sparsity (up to ∼50% increase) can only be achieved if network is fine-tuned to a given fixed point bit-precision.

Method to iteratively allocate the total number of bits (specified by the user according to their needs) between the integer and fractional part of the fixed-point representation, as explained in Sec. 2.3. The low-precision blob structure, as well as the layer-wise bit distribution is then used to generate the low-precision model. The model can either be initialized using random weights or using the pre-trained high-precision weights. The latter normally results in faster convergence, as well as higher classification accuracy. In the beginning we allocate for each layer two weight and two bias blobs. One blob is used to perform inference, which will quantize the weights, biases and activations to its specified bit precision. The second blob is used to calculate the gradients during training. Once the classification accuracy is close to floating-point network level, or it does not change anymore we stop the training. The resulting Caffe model can then be converted to a specific hardware accelerator format, such as NullHop.

2.2 Effect of one-time rounding on sparsity & classification accuracy

Training large networks, such as VGG16, is time consuming. Thus, we investigated if it is possible to perform quantization in a single step, without fine-tuning or training from scratch. One-time rounding of weights to 16 bits with reasonable decimal point does not impair classification accuracy, if the activations are kept at 32-bit floating-point. If one quantizes the weights down to 16-bit fixed-point, it turns out that ∼90% of them can be represented with only 4 bits and ∼99% with 8 bits. Even reducing the maximum number of available bits for weights down to 8 bits does not severely affect classification accuracy (∼65%). This finding clearly shows that the weights are not the limiting factor when a full precision network is quantized to reduced precision.

On the other hand, one time-rounding (deterministic or stochastic; see Sec. 2.4) of weights, and reducing activations 16-bit fixed point without fine-tuning of weights reduces accuracy to chance level. The level of sparsity is also increased much less than if we fine-tune the network. These results suggest that quantizing the activations is actually a difficult problem, since the network’s performance depends more on the available dynamic range of activations than on the weights.

The average sparsity in activations of VGG16 is 57% after quantizing weights and activation down to 16 bit without fine-tuning, however the classification accuracy drops to chance level. In contrast, if we fine-tune the network with quantized weights and activations to a single global fixed
point representation (e.g. Q8.8), we achieve an average sparsity of 82% and a Top-1 classification accuracy of 59.4%. Sparsity values are obtained using 25,000 random images from the test set (Fig. 4). The increase in sparsity of activations, especially in early layers of VGG16, of up to 50% can only be achieved if the network is trained from scratch or if it is fine-tuned using reduced precision weights and activations. This sparsity can optimally be exploited by the NullHop hardware accelerator to efficiently skip computations [2].

2.3 Layer-wise quantization

The activations in the original VGG16 typically span 9 orders of magnitude from $10^{-4}$ up to $10^5$ (see Fig. 5), whereas the dynamic range of the activations, achieved with training the network with reduced precision spans only 4 orders of magnitude ($10^{-3}$ to $10^1$). Thus, constraining the weights and activations to a given fixed-point representation has the effect that the dynamic range requirement is reduced by 5 orders of magnitude. However, even if we choose two different decimal point locations for weights and activations, i.e. Q2.14 for weights and Q14.2 for the activations, globally for all layers, the Top-1 classification accuracy drops to 8.7% after quantizing the network once. After fine-tuning, we could achieve 59.4% Top-1 classification accuracy, which is merely 9% below its high-precision counterpart. The observed drop in classification accuracy suggests that the original VGG16 needs the entire dynamic range in each layer of its activation and is not able to produce state-of-the-art classification accuracy if the activations are bounded by a single given fixed-point representation. We investigated the dynamic range for each layer separately. We found that the layer-by-layer dynamic range especially of the activations differs significantly. Therefore, we implemented a per-layer decimal point. A similar approach to compress the activation functions precision per-layer, has been proposed earlier [27]. However, while [27] perform rigorous mathematical assumptions on how to distribute the available number of bits for a given layer, we propose a simple, computationally cheap iterative scheme. In our method, we keep the overall number of available bits fixed and check iteratively how many weights/activations cannot be represented if we reduce the available number of bits for the integer part. In a second step we check if the percentage of lost weights/activations is below a user defined threshold (usually below 1%). We are looking at the integer part of the weight/activation, since we can check against the maximum number present

![Figure 5: Dynamic range of activations in high (orange) and low (blue) precision setting after training. Quantization acts as regularizer to keep the activations in a resolvable range and also prevents the activations from saturating.](image-url)
in a given layer. If we would look at the smallest number we could not directly link this value to the required number of bits, since we can not easily access the precision needed to represent small values. By locating the decimal point individually for each layer according to our proposed scheme, we achieve a Top-1 classification accuracy of 64.5%. Even though in this study we used 16-bits with per-layer decimal point, ADaPTION can be used to choose any precision and any decimal point for any layer separately.

2.4 Deterministic vs. Stochastic rounding

With the aforementioned additions to ADaPTION we were able to increase the Top-1 classification accuracy from 59.4% up to 64.5% by fine-tuning the network using deterministic rounding. However, training with reduced precision and the dual-copy scheme introduces inescapable fixed points in parameters space (see Sec. 2.5 for more details).

To counteract these fixed points and to increase classification accuracy we investigated the effect of using stochasticity during training. Stochastic rounding, in contrast to deterministic, has the advantage that each step during training has pseudo-randomness. Stochastically rounded values have the correct expectation value but are drawn from a binomial distribution of the two nearest fixed point values. Using any kind of stochasticity usually yields networks that have better generalization properties, tend to have higher classification accuracies, and prevent overfitting [25]. In the context of low-precision quantization, stochastic rounding not only provides higher classification accuracies, but it is helpful, if not crucial, to successfully avoid the inescapable local fixed points introduced by the low-precision training. The training convergence time stayed the same as in the deterministic case.

Using the same number of training examples, we finally achieved a Top-1 classification accuracy of 67.5%, which is only 0.8% below its full-precision counterpart. We added the option to ADaPTION to control the rounding scheme, and added the necessary function in the quantization algorithm. To use stochastic rounding for a given layer the `rounding_scheme` option must be set to `STOACHASTIC`.

2.5 Training VGG16 with reduced-precision on weights and activations

Insights Like in other studies [7], we found that hyperparameters such as the learning rate must be 10 to 100 times smaller during training compared to high-precision network to ensure convergence: Smaller jumps further ensure that fixed points in parameter space caused by quantization are avoided. These fixed points represent parameter combinations that lead to sudden tremendous decreases in accuracy. For example, we observed single-iteration jumps from 50% accuracy down to chance level. After these large jumps, the network has to start training again from scratch. The lower learning rate results in longer training time [19], especially when training low precision networks and also using low-precision gradients [5,7,28].

During training, the gradient is calculated based on the full precision 32-bit floating-point weights and activations, that are only quantized during inference, i.e. per batch. This training scheme is different from low-precision training in which the gradient is calculated based on the quantized weights and activations and the gradient itself is constrained to a low-precision fixed-point number. Low- or even extreme low-precision training has been investigated by [22,28], but so far the resulting accuracies are not competitive. Furthermore, weights must be initialized taking the respective fan-in and fan-out of each unit into consideration [8]. In order to keep the activation introduced by the stimulus (image) in a range that can be resolved by the first low-precision convolution layer, we use scaling. The scaling parameter normalizes pixel intensities so that the highest value does not saturate the integer bit range precision. Scaling uses a single scalar value that multiplies each pixel value. Without scaling the input, the activations saturate at the maximal possible value and the images are harder to classify. Hence, with scaling the full dynamic range of possible values can be used, which speeds up training and leads to higher classification accuracies. These insights and factors are necessary to reach state-of-the-art classification accuracy.

Stochastic rounding enabled the low-precision network to reach state-of-the-art classification accuracy (Top-1 67.5% vs 68.3% in high-precision network [24]).
3 Discussion

3.1 Benefits & Limitations of ADaPTION

The direct effect of quantizing the activation to an intermediate precision, such as 16 bit fixed-point, is that the average sparsity, i.e. the number of zero elements divided by the total number of elements, is at 82%, whereas in the high-precision case the average sparsity is only 57%. Especially early and early-intermediate layers show a much higher sparsity when quantized. In these layers the sparsity increased by up to 50%, while the average increase was 25%. This suggests that features in early-intermediate layers are not as crucial as late layers to perform a correct image classification. This sparsity can be exploited, for a hardware accelerator that skips multiplication if the includes zeros. The NullHop accelerator is an example [2].

The secondary effect of quantization is that it acts as a regularizer to keep the activations and the weights in resolvable range, without saturation effects (Fig. 6).

Sparsity in weights is not as strongly affected as activations. The smaller weight sparsity is because weights tend to be quite small and centered around zero in 32 bit floating-point. Thus, weight quantization has only an effect if the available number of bits drops below 8 bits.

ADaPTION supports any bit-distribution down to a single bit. However, detailed analysis of extreme low-precision quantization is beyond the scope of this paper and has been analyzed in [6, 17, 19, 23]. We can propose a close-to-optimal decimal point location per layer, if we can check the dynamic range, using for example a pre-trained high-precision model. Without this check, for example with a complete new architecture directly trained with reduced precision, it is not a straightforward process to allocate the available number of bits. One way to allocate the available number of bits is to set a fixed-global distribution, e.g. Q8.8 for weights and activations. For smaller networks this was sufficient [18, 20]. Another way of doing this is to first train a model without quantizing the weights and activations and fine-tune it afterwards to the desired bit precision. It has the drawback of long training time and high computational cost.

A trend we observe is that weights tend to be very small and always centered around zero. Reserving just 2 bits (including the sign) for the integer part is usually enough. Activations, in contrast, tend to be quite large in early layers, thus requiring more bits for the integer part compared to later layers. Activations decrease significantly with depth in the network and thus require more bits for the fractional part.

As a direct consequence of reduced bit precision, we achieve a reduction of the overall memory consumption: this reduced memory footprint can improve the performance of generic hardware but we expect it to be particularly significant for custom hardware architectures. Similarly, quantization has the useful side effect that sparsity, i.e. the percentage of zero-valued elements, is increased, which can be exploited by hardware supporting sparse convolution operations to save power and reduce the computation time.

3.2 ADaPTION vs. Ristretto

During the development of ADaPTION, Gysel and colleagues developed an add-on for Caffe called Ristretto [10]. Ristretto positions itself in a similar role as ADaPTION, but lacks key-features which makes quantization quite difficult and at the same time interesting for hardware accelerators, i.e. fixed-point activations to skip multiplication with zero. Furthermore, this key feature is crucial for our Nullhop. Ristretto does not support a pre-defined number of bits to distribute between integer and fractional part of a fixed-point number. Furthermore, the number of bits, which is provided by Ristretto, does not stay constant across the network, which is crucial for hardware accelerators, such as Nullhop. Probably most important, Ristretto also does not support fixed-point rounding of activations, which, as we showed, contributes the most to the sparsity in activations and is the hardest to constrain to a given bit-precision and still provide state-of-the-art classification accuracy.

4 Conclusion

We presented ADaPTION, a new toolbox to quantize existing high-precision CNNs to be efficiently implemented on dedicated mobile-orientated hardware accelerators. The toolbox adapts weights and activations to globally fixed or layer-wise fixed-point notation.
Table 1: Giga1Net inference parameters

| Layer | Input feature maps | Output feature maps | Kernel Size | Input Width/Height | Pooling | ReLU | Stride |
|-------|-------------------|---------------------|-------------|--------------------|---------|------|--------|
| 1 - conv | 3 | 16 | 1 | 224x224 | Yes | Yes | 1 |
| 2 - conv | 16 | 16 | 7 | 112x112 | Yes | Yes | 1 |
| 3 - conv | 16 | 32 | 7 | 54x54 | Yes | Yes | 1 |
| 4 - conv | 32 | 64 | 5 | 24x24 | No | Yes | 1 |
| 5 - conv | 64 | 64 | 5 | 22x22 | No | Yes | 1 |
| 6 - conv | 64 | 64 | 5 | 20x20 | No | Yes | 1 |
| 7 - conv | 64 | 128 | 3 | 18x18 | No | Yes | 1 |
| 8 - conv | 128 | 128 | 3 | 18x18 | No | Yes | 1 |
| 9 - conv | 128 | 128 | 3 | 18x18 | No | Yes | 1 |
| 10 - conv | 128 | 128 | 3 | 18x18 | No | Yes | 1 |
| 11 - conv | 128 | 128 | 3 | 18x18 | Yes | Yes | 1 |
| 12 - FC | 128 | 4096 | - | - | No | Yes | - |
| 13 - FC | 4096 | 1000 | - | - | - | - | - |

Quantization of weights and activations has the advantage that the overall sparsity in the network increases while preserving state of the art Top-1 ImageNet classification accuracy.

New Benchmark Networks

A major problem while comparing CNN accelerators is that many works use custom networks, making the comparison between different architectures hard. Even if some architectures (e.g. VGG16, GoogLeNet, ResNet) are more popular than others, they cover a limited range of hyperparameters and computational costs and are not ideally-suited for realistic hardware benchmarking. In order to address this issue we used our software framework to train a new CNN architecture characterized by a variety of kernel sizes and number of channels, useful to verify hardware computational capabilities in multiple scenarios. Due to the fast-changing CNNs and other DNN accelerators landscape, we will update this paper, adding new networks suited for benchmarking new emerging hardware designs.

Giga1Net

Giga1Net, defined in Table 1, requires 1 GOP/frame to classify an image from the ImageNet dataset. The prototxt necessary for the training is available in the ADaPTION repository; Giga1Net achieves 38% ImageNet Top-1 accuracy after 36h of training on a GTX980 Ti.

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References

[1] Martín Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, Greg S. Corrado, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Ian Goodfellow, Andrew Harp, Geoffrey Irving, Michael Isard, Yangqing Jia, Rafal Jozefowicz, Lukasz Kaiser, Manjunath Kudlur, Josh Levenberg, Dan Mané, Rajat Monga, Sherry Moore, Derek Murray, Chris Olah, Mike Schuster, Jonathon Shlens, Benoit Steiner, Ilya Sutskever, Kunal Talwar, Paul Tucker, Vincent Vanhoucke, Vijay Vasudevan, Fernanda Viégas, Oriol Vinyals, Pete Warden, Martin Wattenberg, Martin Wicke, Yuan Yu, and Xiaoqiang Zheng. TensorFlow: Large-scale machine learning on heterogeneous systems, 2015. Software available from tensorflow.org.

[2] Alessandro Aimar, Hesham Mostafa, Enrico Calabrese, Antonio Rios-Navarro, Ricardo Tapiador-Morales, Iulia-Alexandra Lungu, Moritz B Milde, Federico Corradi, Alejandro Linares-Barranco, Shih-Chii Liu, et al. Nullhop: A flexible convolutional neural network accelerator based on sparse representations of feature maps. arXiv preprint arXiv:1706.01406, 2017.

[3] Yu-Hsin Chen, Tushar Krishna, Joel Emer, and Vivienne Sze. Eyeriss: An Energy-Efficient Reconfigurable Accelerator for Deep Convolutional Neural Networks. 52(1):127–138, 2016.

[4] Francesco Conti, Robert Schilling, Pasquale D Schiavone, Antonio Pullini, Davide Rossi, Frank K Gürkaynak, Michael Muehlberghuber, Michael Gautschi, Igor Loi, Germain Hau-gou, Stefan Mangard, and Luca Benini. An IoT Endpoint System-on-Chip for Secure and Energy-Efficient Near-Sensor Analytics. 2017.

[5] Matthieu Courbariaux, Yoshua Bengio, and Jean-Pierre David. Training deep neural networks with low precision multiplications. arXiv preprint [arXiv:1411.7022], 2014.

[6] Matthieu Courbariaux, Yoshua Bengio, and Jean-Pierre David. Binaryconnect: Training deep neural networks with binary weights during propagations. In Advances in Neural Information Processing Systems, pages 3123–3131, 2015.

[7] Matthieu Courbariaux, Itay Hubara, Daniel Soudry, Ran El-Yaniv, and Yoshua Bengio. Binarized neural networks: Training deep neural networks with weights and activations constrained to+ 1 or-1. arXiv preprint [arXiv:1602.02830], 2016.

[8] Xavier Glorot and Yoshua Bengio. Understanding the difficulty of training deep feedforward neural networks. In Proceedings of the Thirteenth International Conference on Artificial Intel-ligence and Statistics, pages 249–256, 2010.

[9] Suyog Gupta, Ankur Agrawal, Kailash Gopalakrishnan, and Pritish Narayanan. Deep learning with limited numerical precision. In Proceedings of the 32nd International Conference on Machine Learning (ICML-15), pages 1737–1746, 2015.

[10] Philipp Gysel. Ristretto: Hardware-oriented approximation of convolutional neural networks. arXiv preprint [arXiv:1605.06402], 2016.

[11] Song Han, Xingyu Liu, Huizi Mao, Jing Pu, Ardavan Pedram, Mark A Horowitz, and William J Dally. EIE: Efficient Inference Engine on Compressed Deep Neural Network. 2016.

[12] Song Han, Huizi Mao, and William J Dally. Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding. arXiv preprint [arXiv:1510.00149], 2015.

[13] Itay Hubara, Matthieu Courbariaux, Daniel Soudry, Ran El-Yaniv, and Yoshua Bengio. Quantized neural networks: Training neural networks with low precision weights and activations. arXiv preprint [arXiv:1609.07061], 2016.

[14] Yangqing Jia, Evan Shelhamer, Jeff Donahue, Sergey Karayev, Jonathan Long, Ross Girshick, Sergio Guadarrama, and Trevor Darrell. Caffe: Convolutional architecture for fast feature embedding. In Proceedings of the 22nd ACM international conference on Multimedia, pages 675–678. ACM, 2014.
[15] Norman P Jouppi, Al Borchers, Rick Boyle, Pierre-luc Cantin, Clifford Chao, Chris Clark, Jeremy Coriell, Mike Daley, Matt Dau, Jeffrey Dean, Ben Gelb, Cliff Young, Tara Vazir Ghaemmaghami, Rajendra Gottipati, William Gulland, Robert Hagmann, C Richard Ho, Doug Hogberg, John Hu, Robert Hurd, Dan Hurt, Julian Ibarz, Nishant Patil, Aaron Jaffey, Alek Jaworski, Alexander Kaplan, Harshit Khaitan, Daniel Killebrew, Andy Koch, Naveen Kumar, Steve Lacy, James Laudon, James Law, David Patterson, Dienth Le, Chris Leary, Zhuyuan Liu, Kyle Lucke, Alan Lundin, Gordon MacKean, Adriana Maggiore, Maire Mahony, Kieran Miller, Rahul Nagarajan, Gaurav Agrawal, Ravi Narayanaswami, Ray Ni, Kathy Nix, Thomas Norrie, Mark Omernick, Narayana Penukonda, Andy Phelps, Jonathan Ross, Matt Ross, Amir Salek, Raminder Bajwa, Emad Samadiani, Chris Severn, Gregory Sizikov, Matthew Snellham, Jed Souter, Dan Steinberg, Andy Swing, Mercedes Tan, Gregory Thorson, Bo Tian, Sarah Bates, Horia Toma, Erick Tuttle, Vijay Vasudevan, Richard Walter, Walter Wang, Eric Wilcox, Doe Hyun Yoon, Suresh Bhatia, and Nan Boden. In-Datacenter Performance Analysis of a Tensor Processing Unit. In *Proceedings of the 44th Annual International Symposium on Computer Architecture* - *ISCA ’17*, pages 1–12, 2017.

[16] Yann LeCun, Yoshua Bengio, and Geoffrey Hinton. Deep learning. *Nature*, 521(7553):436–444, May 2015.

[17] Zhouhan Lin, Matthieu Courbariaux, Roland Memisevic, and Yoshua Bengio. Neural networks with few multiplications. *arXiv preprint arXiv:1510.03009*, 2015.

[18] Iulia-Alexandra Lungu, Federico Corradi, and Tobi Delbrück. Live demonstration: Convolutional neural network driven by dynamic vision sensor playing roshambo. In *Circuits and Systems (ISCAS), 2017 IEEE International Symposium on*, pages 1–1. IEEE, 2017.

[19] Asit Mishra, Jeffrey J Cook, Eriko Nurvitadhi, and Debbie Marr. Wrpn: Training and inference using wide reduced-precision networks. *arXiv preprint arXiv:1704.03079*, 2017.

[20] Fabian Muller. Bachelor thesis: Spike-based classification of event-based visual data. 2016.

[21] Lorenz K Muller and Giacomo Indiveri. Rounding methods for neural networks with low resolution synaptic weights. *arXiv preprint arXiv:1504.05767*, 2015.

[22] Adam Paszke, Sam Gross, Soumith Chintala, and Gregory Chanan. pyTorch: Tensors and Dynamic neural networks in Python with strong GPU acceleration. [http://pytorch.org/](http://pytorch.org/) 2017.

[23] Mohammad Rastegari, Vicente Ordonez, Joseph Redmon, and Ali Farhadi. Xnor-net: Imagenet classification using binary convolutional neural networks. In *European Conference on Computer Vision*, pages 325–542. Springer, 2016.

[24] Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*, 2014.

[25] Nitish Srivastava, Geoffrey E Hinton, Alex Krizhevsky, Ilya Sutskever, and Ruslan Salakhutdinov. Dropout: a simple way to prevent neural networks from overfitting. *Journal of machine learning research*, 15(1):1929–1958, 2014.

[26] Evangelos Stromatias, Daniel Neil, Michael Pfeiffer, Francesco Galluppi, Steve B Furber, and Shih-Chii Liu. Robustness of spiking deep belief networks to noise and reduced bit precision of neuro-inspired hardware platforms. *Frontiers in neuroscience*, 9, 2015.

[27] Yu Wang, Lixue Xia, Tianqi Tang, Boxun Li, Song Yao, Ming Cheng, and Huazhong Yang. Low power convolutional neural networks on a chip. In *Circuits and Systems (ISCAS), 2016 IEEE International Symposium on*, pages 129–132. IEEE, 2016.

[28] Shuchang Zhou, Yuxin Wu, Zekun Ni, Xinyu Zhou, He Wen, and Yuheng Zou. Dorefa-net: Training low bitwidth convolutional neural networks with low bitwidth gradients. *arXiv preprint arXiv:1606.06160*, 2016.