Tachyon-free Orientifolds of Type 0B Strings in Various Dimensions

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Abstract

We construct non-tachyonic, non-supersymmetric orientifolds of type 0B strings in ten, six and four space-time dimensions. Typically, these models have unitary gauge groups with charged massless fermionic and bosonic matter fields. However, generically there remains an uncancelled dilaton tadpole.

04/99

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1. Introduction

Due to its possible application to the dynamics of non-supersymmetric gauge theories in four dimensions, type 0B models have received much attention during the last months. Unfortunately, these models are plagued by the appearance of a tachyonic mode in the closed string sector. However the effective gauge theory on the D-branes of type 0B models [1] is nevertheless tachyonfree and hence consistent. The couplings of the tachyon in the effective low energy field theory allow the dilaton to depend on the distance to the branes in just the right way to be consistent with the one-loop running of the gauge coupling in non-supersymmetric gauge theories.

In order to study consistent non-supersymmetric string vacua, one should of course get rid of all tachyonic modes. Tachyon-free, non-supersymmetric heterotic strings in four dimensions with chiral, massless fermions were already constructed some time ago for example in the covariant lattice formalism [2]. In the context of the type 0B string one should include some projections modding out the tachyon. In [3,4] the standard orientifold of type 0B was considered leading to a model containing still the tachyon in the closed string sector as well as new tachyonic modes coming from open strings stretched between D9 and $\overline{D}9$ branes. The latter objects needed to be introduced into the background to satisfy tadpole cancellation conditions. In [4] the D-brane content of type 0B and its orientifold type 0 was derived using the boundary state formalism. From the partition function of type 0B it is clear that all Ramond-Ramond fields are doubled with respect to type IIB leading to a doubling of the D-branes as well.

In [5] it was observed on the level of conformal field theory partition functions that there exists another Klein-bottle projection which leads to a non-tachyonic “Type I descendant” called type 0′ in the following. This model has a non-zero dilaton tadpole which however can be cancelled by the Fischler-Susskind [6] mechanism. Compactifications of type 0′ to six and four space-time dimensions were considered in [4]. However, the microscopic description in particular the D-brane contents of these models is obscure in [4,7].

In this paper we revisit the non-tachyonic orientifold of type 0B, clarify the origin of a second orientifold projection in terms of a different world-sheet parity $\Omega'$, and determine the D-brane contents of this model. Moreover, we consider type 0B orientifolds on $T^{10-d}/\{G_1, \Omega' G_2\}$, $d = 6, 4$, and derive the anomaly free massless spectra in specific models. All of these models have in common some phenomenological appealing features like a pure bosonic always unitary gauge sector with both bosonic and fermionic charged matter. As in the parent type 0B model the gravity sector is purely bosonic.
Moreover, we study the effective theory on the D3-branes in more detail finding some peculiarities reflecting the fact that D9 and D3-branes repel each other.

2. Review of type 0B model

In the construction of critical superstring theories one has to implement the GSO projection

\[ P_{NSNS} = \frac{1}{4}(1 + (-1)^{F_L})(1 + (-1)^{F_R}), \quad P_{RR} = \frac{1}{4}(1 + (-1)^{F_L})(1 \mp (-1)^{F_R}). \] (2.1)

The two possible choices for the GSO projection in the Ramond sector lead to type IIA and type IIB, respectively. However, it is known since 1986 [8] that using only the projection

\[ \overline{P}_{NSNS} = \frac{1}{2}(1 + (-1)^{F_L+F_R}), \quad \overline{P}_{RR} = \frac{1}{2}(1 \pm (-1)^{F_L+F_R}) \] (2.2)

still leads to modular invariant partition functions on the torus

\[ Z_T = \frac{1}{2} \left| \frac{f_3}{f_1} \right|^{16} + \left| \frac{f_4}{f_1} \right|^{16} + \left| \frac{f_2}{f_1} \right|^{16}, \] (2.3)

called type 0B and type 0A depending on the sign in \( \overline{P}_{RR} \). These ten dimensional string theories are pure bosonic hence non-supersymmetric and contain a tachyon in the spectrum.

The non-supersymmetric model (2.3) can also be considered as the orbifold of type II by the space-time fermion operator \((-1)^{F_S}\), where the tachyon makes its appearance in the twisted sector. At the massless level the NS-NS sector contributes a graviton \( G_{\mu\nu} \), a dilaton \( \Phi \) and an antisymmetric 2-form \( B_{\mu\nu} \) to the spectrum. For type 0B the RR sector contributes two further scalars \( \Phi_1, \Phi_2 \), two antisymmetric 2-forms \( B_{\mu\nu}^{1,2} \) and a 4-form \( D_{\mu\nu\rho\sigma} \). In type 0A models there are two 1-forms \( A_{\mu}^{1,2} \) and two antisymmetric 3-forms \( C_{\mu\nu\rho}^{1,2} \) in the RR sector. Since compared to the type II models all fields in the RR sector are doubled, one expects the D-brane content to be doubled, as well. The explicit form of the corresponding boundary states was derived in [4], which we will briefly review for later reference.

The boundary state of a Dp-brane in type II is a sum of four terms

\[ |Dp\rangle = \frac{1}{2} (|Dp, \eta = +1\rangle_{NSNS} - |Dp, \eta = -1\rangle_{NSNS} + |Dp, \eta = +1\rangle_{RR} + |Dp, \eta = -1\rangle_{RR}) \] (2.4)
and is invariant under the GSO projections of type IIA for even \( p \) and of type IIB for odd \( p \). The individual terms are given by

\[
|D_p, \eta\rangle_{NSNS} = \int \left( \prod_{\nu=p+1}^{9} dk^\nu \right) \exp \left\{ \sum_{n=1}^{\infty} \left[ \frac{1}{n} \sum_{\mu=2}^{p} \alpha_{n-\mu}^{\mu} + \frac{1}{n} \sum_{\mu=p+1}^{9} \alpha_{n-\mu}^{\mu} \right] \right\} \\
\exp \left\{ i\eta \sum_{r>0} \left[ -\sum_{\mu=2}^{p} \psi_{r-\mu}^{\mu} \tilde{\psi}_{r-\mu}^{\mu} + \sum_{\mu=p+1}^{9} \psi_{r-\mu}^{\mu} \tilde{\psi}_{r-\mu}^{\mu} \right] \right\} |\vec{k}, \eta\rangle_{NSNS}
\]

(2.5)

and analogously for the RR sector, where one has to define the RR vacuum in a consistent way. However, in the type 0A and type 0B theories each of the four terms in (2.4) is invariant by itself under the \( \mathcal{P} \) projection, leading to four possible boundary states for each \( p \)

\[
|D_p, \eta, \eta'\rangle = |D_p, \eta\rangle_{NSNS} + |D_p, \eta'\rangle_{RR}
\]

(2.6)

and their corresponding anti-branes for which the sign in front of the Ramond-Ramond boundary state is reversed. The action of the world-sheet fermion number operator \((-1)^{F_R}\) exchanges the \(|D_p, \eta, \eta'\rangle\) brane with the \(|D_p, -\eta, -\eta'\rangle\) brane. Using the boundary states (2.4) one first computes the cylinder amplitude in tree channel

\[
\tilde{\mathcal{A}} = \frac{1}{2} \int_0^{\infty} \frac{dt}{2} \langle D_p, \eta_1, \eta'_1 | e^{-tH_{cl}} | D_p, \eta_2, \eta'_2 \rangle
\]

(2.7)

and then transforms to loop channel

\[
A_{(\eta_1, \eta'_1), (\eta_2, \eta'_2)} = \int_0^{\infty} \frac{dt}{t} \text{Tr}_{open} \left[ \left(1+(-1)^{F_R}\right)^{p+1} \left(1+(-1)^{F_S}\right)^{p+1} e^{-2\pi t L_0} \right].
\]

(2.8)

For the amplitudes we will be interested in this yields the following result

\[
A_{(\eta_1, \eta'_1), (\eta_2, \eta'_2)} = \frac{V_{p+1}}{(8\pi^2 \alpha')^{p+1}} \int_0^{\infty} \frac{dt}{t^{p+3}} \frac{f_3^8(e^{-\pi t}) - f_4^8(e^{-\pi t})}{f_1^8(e^{-\pi t})}
\]

(2.9)

for open strings stretched between the same type of Dp-branes. However, for open strings stretched between opposite Dp-branes one obtains

\[
A_{(\eta_1, \eta'_1), (-\eta_2, -\eta'_2)} = \frac{V_{p+1}}{(8\pi^2 \alpha')^{p+1}} \int_0^{\infty} \frac{dt}{t^{p+3}} \frac{-f_3^8(e^{-\pi t})}{f_1^8(e^{-\pi t})},
\]

(2.10)

where the extra minus sign appears due to the action

\[
(-1)^{F_R} |D_p, \eta\rangle_{NSNS} = -|D_p, -\eta\rangle_{NSNS}.
\]

(2.11)
$V_{p+1}$ is the regularized $Dp$-brane volume. From (2.9) and (2.10) one derives that for open strings stretched between the same $Dp$-branes the world-sheet fermions have half-integer mode expansion whereas for open strings stretched between a $|Dp, \eta, \eta'|$ and a $|Dp, -\eta, -\eta'|$ brane they have integer mode expansion. In the latter case fermionic zero modes appear leading to space-time fermions.

In [4], and originally in [5], an orientifold of type $0B$ was studied, with the world-sheet parity operation acting in the closed string sector as follows

$$\begin{align*}
\Omega \alpha_n \Omega &= \tilde{\alpha}_n, \\
\Omega \psi_r \Omega &= \tilde{\psi}_r, \\
\Omega \tilde{\psi}_r \Omega &= -\psi_r, \\
\Omega |0\rangle_{NSNS} &= |0\rangle_{NSNS}.
\end{align*}$$

(2.12)

Thus, the tachyon survived the projection and the Klein-bottle only produced a NS-NS tadpole which was partially cancelled by introducing the same number of $|D9, +, +\rangle$ and anti- $|D9, +, +\rangle$ branes in the background. It was not possible to cancel the tachyon tadpole. From open strings stretched between the different 9-branes they got gauge bosons of $SO(32) \times SO(32)$ and a further tachyon transforming in the bi-fundamental $(32, 32)$ representation of the gauge group.

Moreover, in [4] it was shown that D1, D5 and D9-branes do survive the orientifold projection and that the world-volume theory on the D1-branes is in agreement with the world-sheet theory of the bosonic string compactified on an $SO(32)$ lattice to ten dimensions. The latter observation led to the conjecture that the two models might provide a string-weak dual pair. We will now show in section 3 that there exists a different orientifold projection leading to the tachyon-free model first discussed in [5].

### 3. The type $0'$ orientifold

Instead of (2.12) one can define the action of $\Omega$ in the closed string sector as follows

$$\begin{align*}
\Omega' \alpha_n \Omega' &= \tilde{\alpha}_n, \\
\Omega' \psi_r \Omega' &= \tilde{\psi}_r, \\
\Omega' \tilde{\psi}_r \Omega' &= \psi_r, \\
\Omega' |0\rangle_{NSNS} &= -|0\rangle_{NSNS}.
\end{align*}$$

(3.1)
which apparently means $\Omega' = \Omega (-1)^{F_R}$. In the type IIB theory this makes no difference for the action of $\Omega'$ and $\Omega$ on physical states is identical. On the contrary, in the non-supersymmetric type 0B theory the two actions define two completely different models as can be seen from the action of $\Omega'$ on the NS vacuum, which projects out the tachyon.

3.1. The Klein bottle amplitude

In doing an orientifold there generically appear tadpoles which ought to be cancelled by contributions from an open string sector. Therefore, we compute the Klein bottle amplitude for this model

$$K = 32 c \int_0^\infty \frac{dt}{t^6} \mathrm{Tr} \left[ \Omega' \frac{1+(-1)^{F_L+F_R}}{2} (-1)^{F_S} e^{-2\pi t(L_0+T_0)} \right]$$

$$= -32 c \int_0^\infty \frac{dt}{t^6} \frac{f_8(e^{-2\pi t})}{f_1(e^{-2\pi t})},$$

where $c = V_{10}/(8\pi^2\alpha')^5$. The transformation into tree-channel leads to

$$\tilde{K} = -2^{11} c \int_0^\infty dl \frac{f_2(e^{-2\pi l})}{f_8(e^{-2\pi l})},$$

showing that there is only a contribution from RR 10-form exchange. The appropriate branes one has to introduce to cancel this tadpole must be charged under the RR 10-form, which are of course D9-branes. However, due to the action of the right handed worldsheet fermion number operator $(-1)^{F_R}$ on D9-branes we are forced to introduce the same number $N$ of $|\bar{D}p, \eta, \eta\rangle$ and $|Dp, -\eta, -\eta\rangle$ branes. Without loss of generality, we choose $\theta = |D9, +, +\rangle$ and $\theta' = |D9, -, -\rangle$ branes in the following. The next step is to compute the cylinder and Möbius strip amplitudes for these branes.

3.2. The Cylinder amplitude

The annulus amplitude is defined as

$$A = c \int_0^\infty \frac{dt}{t^6} \mathrm{Tr}_{99,9'9',9'9} \left[ \frac{1+(-1)^{F} + (-1)^{F_S}}{2} e^{-2\pi t L_0} \right]$$

where the space time fermion number projection implies that only the open string NS sector contributes. Using the results (2.9) and (2.10) it is straightforward to determine the individual contributions

$$A_{99} = A_{9'9'} = c \int_0^\infty \frac{dt}{t^6} \frac{N^2}{2} \frac{f_8(e^{-\pi t}) - f_8(e^{-\pi t})}{f_1(e^{-\pi t})}$$

$$A_{99'} = A_{9'9} = -c \int_0^\infty \frac{dt}{t^6} \frac{N^2}{2} \frac{f_8(e^{-\pi t})}{f_1(e^{-\pi t})}.$$
adding up to the complete tree channel cylinder amplitude

\[ \tilde{A} = c \int_0^\infty dl \, 2 N^2 \frac{f_3^8(e^{-2\pi l}) - f_4^8(e^{-2\pi l}) - f_2^8(e^{-2\pi l})}{f_1^8(e^{-2\pi l})}. \]  

(3.6)

Up to a factor of two this is identical to the cylinder amplitude of two D9 branes of type IIB. Therefore, even without supersymmetry the overall force between the same number of D9 and D9'-branes vanishes, hence permitting a stable configuration of these branes. Note, that the tree channel amplitude (3.6) contains a dilaton tadpole as well as a RR 10-form tadpole.

3.3. The M"obius amplitude

Finally, we have to compute the M"obius strip amplitude

\[ M = c \int_0^\infty dt \, \frac{1}{t^6} \text{Tr} \left[ \Omega' \frac{1+(-1)^F}{2} \frac{1+(-1)^F_2}{2} e^{-2\pi t L_0} \right], \]  

(3.7)

where again there is no contribution from the open string R sector. Since \( \Omega' \) exchanges D9 and D9'-branes, the only non-zero contribution in (3.7) is from open strings stretched between D9 and D9'-branes. For the loop channel M"obius amplitude one gets

\[ M = c \int_0^\infty dt \, \frac{1}{t^6} \frac{f_2^8(i e^{-2\pi t})}{f_1^8(i e^{-2\pi t})} \text{Tr}(\gamma_{\Omega'}^T \gamma_{\Omega'}^{-1}), \]  

(3.8)

where \( \gamma_{\Omega'} \) describes the action of \( \Omega' \) on the Chan-Paton factors. Since the resulting tree channel amplitude is given by

\[ \tilde{M} = c \int_0^\infty dl \, 2 N^2 \frac{f_3^8(i e^{-2\pi l}) - f_4^8(i e^{-2\pi l}) - f_2^8(i e^{-2\pi l})}{f_1^8(i e^{-2\pi l})} \text{Tr}(\gamma_{\Omega'}^T \gamma_{\Omega'}^{-1}), \]  

(3.9)

there is only a contribution to the RR exchange tadpole.

3.4. Tadpole cancellation

Adding up all the three contributions, the tadpole cancellation condition for the RR 10-form potential reads

\[ N^2 - 32 \text{Tr}(\gamma_{\Omega'}^T \gamma_{\Omega'}^{-1}) + 32^2 = 0. \]  

(3.10)

However, from NS-NS exchange we are left with an uncancelled dilaton tadpole

\[ c \int_0^\infty dl \, 2 N^2 \frac{f_3^8(e^{-2\pi l}) - f_4^8(e^{-2\pi l})}{f_1^8(e^{-2\pi l})} = \int_0^\infty dl \, (32 c N^2 + O(e^{-2\pi l})). \]  

(3.11)
However, it was shown in [6] that a NS-NS tadpole does not render the theory inconsistent for it means that we are not expanding around the true vacuum. One has to introduce a non-constant dilaton and a tree level cosmological constant to cancel such a tadpole. In section 4 we will mention that such a tadpole can possibly also be cancelled by introducing lower dimensional D-branes in the background.

The RR tadpole cancellation condition (3.10) implies that the matrix $\gamma \Omega'$ has to be symmetric. We are free to choose

$$\gamma \Omega' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{2N,2N},$$

(3.12)

where the first $N$ entries denote the D9-branes and the second $N$ entries denote the D9'-branes. The off-diagonal choice in (3.12) reflects the fact that $\Omega'$ exchanges D9 and D9'-branes. Plugging (3.12) into (3.10) one realises that the RR tadpole is cancelled for precisely 32 D9 and 32 D9'-branes.

3.5. Massless spectrum

At the massless level in the NS-NS closed string sector $\Omega'$ acts in the same way as $\Omega$ leaving the graviton and the dilaton invariant, whereas the antisymmetric 2-form is projected out. In the R-R sector before the projection one had

$$C^8 \otimes \tilde{C}^8 = 1 + 28 + 35^+, $$

$$S^8 \otimes \tilde{S}^8 = 1 + 28 + 35^-.$$  

(3.13)

Since $C^8 \otimes \tilde{C}^8$ is invariant under $(-1)^F$, these states have to be antisymmetrized under $\Omega'$ giving one 2-form field. However, $(-1)^F$ gives an extra minus sign when acting on $S^8 \otimes \tilde{S}^8$, so that these states are symmetrized leading to a scalar and an anti-selfdual 4-form potential. Thus, the closed string spectrum is the bosonic part of type IIB, in particular one expects the brane contents to be the same as in type IIB.

In the open string sector, strings stretched between two D9 or two D9'-branes, respectively, carry the massless mode

$$\psi_{-\frac{1}{4}}^{\mu} |0\rangle_{NS} \lambda_G$$

(3.14)

which leads to the condition $\lambda_G = -\gamma \Omega' \lambda_G^T \gamma^{-1} \Omega'$. The solution is

$$\lambda_G = \begin{pmatrix} A & 0 \\ 0 & -A^T \end{pmatrix}_{64,64},$$

(3.15)
with $A$ any hermitian matrix implying that the state (3.14) describes a vector boson of the gauge group $U(32)$. For open strings stretched between D9 and D9′-branes there are massless fermion states

$$|s_1, s_2, s_3, s_4\rangle \lambda_F$$

with $s_a = \pm \frac{1}{2}$ and $\sum s_a = \text{odd}$ from the GSO projection. Invariance under $\Omega'$ requires Chan-Paton factors of the form

$$\lambda_F = \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix}_{64,64}$$

with both $B^T = -B$ and $C^T = -C$. One can show that $B$ and $C$ behave differently under gauge transformations leading to a left-handed Majorana-Weyl fermion in the $496 \oplus 496$ representation of $U(32)$. It was shown in [5] that the spectrum is anomaly-free. Note, the $R^6$ term in the gravitational anomaly

$$496 \hat{I}_{\text{Weyl}} = \hat{I}_{\text{4-form}}$$

vanishes and exactly for $n = 32$ the trace in the antisymmetric representation of $U(n)$ factorizes

$$\text{Tr}_A F^6 = (n - 32) \text{tr}(F^6) + 15 \text{tr}(F^2) \text{tr}(F^4)$$

so that a generalised Green-Schwarz mechanism can take over.

3.6. D-branes in type 0′

As we have seen in the last subsection, the closed spectrum of type 0′ contains the same RR $(p+1)$-forms as type IIB. Thus, one expects that all D$p$-branes, for odd $p$, are present, as well. This can also be shown using the boundary state approach. In [4] the action of $\Omega$ on the boundary states $|Dp, \eta\rangle_{\text{NSNS, RR}}$ was determined

$$\Omega |Dp, \eta\rangle_{\text{NSNS}} = |Dp, \eta\rangle_{\text{NSNS}},$$

$$\Omega |Dp, \eta\rangle_{\text{RR}} = -(-i\eta)^{7-p} |Dp, \eta\rangle_{\text{RR}}$$

which in the case of the tachyonic type 0 orientifold implied that only D1,D5 and D9-branes survived the projection. In our case $\Omega$ is equipped with the right moving worldsheet fermion number operation $(-1)^F_R$ which has the following action on the boundary states

$$(-1)^F_R |Dp, \eta\rangle_{\text{NSNS}} = -|Dp, -\eta\rangle_{\text{NSNS}},$$

$$(-1)^F_R |Dp, \eta\rangle_{\text{RR}} = |Dp, -\eta\rangle_{\text{RR}}.$$
Combining these two actions one can show that for every odd \( p \) the \( Dp \)-brane

\[
\frac{1}{2} \left( |Dp, +1\rangle_{NSNS} - |Dp, -1\rangle_{NSNS} + |Dp, +1\rangle_{RR} + (-1)^{\frac{p-1}{2}} |Dp, -1\rangle_{RR} \right) \tag{3.22}
\]

and the corresponding anti-brane are invariant under \( \Omega' \). In particular, type 0' contains a self-dual D3-brane, which we will study a bit more in the next section.

4. Massless modes on the D3-brane

Since we have seen that type 0' contains D3-branes, due to its possible application to non-supersymmetric gauge theories we investigate further the effective theory on a number of parallel such branes.

From open strings stretched between 9-branes one gets simply the dimensional reduction of the massless spectrum discussed in section 3.5 to four dimensions. From open strings stretching between two 3-branes or two 3'-branes there are massless modes of the form (3.14). For \( \mu \) running over space-time values these states provide \( U(M) \) gauge vectors. For \( \mu \) running over transverse coordinates we obtain six scalars transforming in the adjoint representation. Strings stretching between a 3 and a 3'-brane have massless fermionic modes of the form (3.16). The four states with \( s_1 = -\frac{1}{2} \) give left-handed fermions transforming in the \( + \) representation. Strings stretching between 9 and 3-branes only carry massive modes, whereas strings between 9 and 3'-branes respectively 9' and 3-branes, have massless modes of the form

\[
|s_1\rangle \lambda_M \tag{4.1}
\]

with \( s_1 = \pm \frac{1}{2} \). The GSO projection leaves only one state \( s_1 = +\frac{1}{2} \) transforming in the \( (32, M) + (\overline{32}, \overline{M}) \) representation of the gauge group. However as a strange fact we observe that the CPT conjugates of these states are absent. As listed in Table 1, one could also make the spectrum CPT invariant but then the states would formally transform in half gauge multiplets\(^1\). It is interesting to note that the 1-loop \( \beta \)-function for the \( U(M) \) theory is independent of \( M \).

\(^1\) A similar effect happens for the effective theory on a D5 brane in Type I [3], where naively one gets half-hypermultiplets in the 95 open string sector. However, in that case the gauge group was \( SO(32) \times Sp(2N) \) and the matter in the 95 sector transformed in the \( (32, 2N) \) representation of the gauge group. This representation is pseudo-real and thus there is no inconsistency.
sector | fields | $U(32) \times U(M)$
---|---|---
99, 9′9′ | vectors | $(\text{Adj}, 1)$
| scalars | $6 \times (\text{Adj}, 1)$
| L-fermions | $4 \times \{(496, 1) + (496, 1)\}$
99′, 9′9′ | vectors | $(1, \text{Adj})$
| scalars | $6 \times (1, \text{Adj})$
| L-fermions | $4 \times \{(1, \square) + (1, \square)\}$
33, 3′3′ | vectors | $(\text{Adj}, 1)$
| scalars | $6 \times (\text{Adj}, 1)$
| L-fermions | $4 \times \{(32, \square) + (32, \square)\}$
33′, 3′3′ | vectors | $(1, \text{Adj})$
| scalars | $6 \times (1, \text{Adj})$
| L-fermions | $\frac{1}{2} \times \{(32, \square) + (32, \square)\}$

Table 1: Effective theory field content

We think that the inconsistency in the fermionic spectrum simply reflects the fact that due to the cylinder amplitude for D9, 9′- and D3, 3′-branes (open strings stretching between branes of different dimensionality)

$$A_{93} = \frac{V_4}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^{3} 2MN} \frac{f_2^2(e^{-\pi t}) f_2^6(e^{-\pi t}) - f_2^2(e^{-\pi t}) f_2^6(e^{-\pi t})}{f_2^2(e^{-\pi t}) f_2^6(e^{-\pi t})}$$

(4.2)

there exists a repelling force between these two branes. Thus a D9- and a D3-brane are not allowed to form neither a static configuration nor a bound state.

Note that $A_{93}$ contributes to the NS-NS dilaton tadpole. This observation opens at least in principle the possibility that the dilaton tadpole from the 9,9′-branes is cancelled by the D3-branes. In fact, combining the $l \to \infty$ limit of $A_{93}$ and (3.11) we find that the NS-NS tadpole is cancelled provided that

$$8N \frac{V_6}{(8\pi^2\alpha')^3} = M,$$

(4.3)

where we have formally used $V_{10} = V_4V_6$. Since the space transverse to the 3, 3′-branes is non-compact, we conclude that the dilaton tadpole only cancels in the limit $M \to \infty$ where the density of D3-branes $M/V_6$ is constant.

For completeness we also present the Möbius strip amplitude for the 3, 3′-brane system

$$M_3 = \frac{V_4}{(8\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^{3} 2M} \frac{f_2^8(i e^{-\pi t})}{2f_2^8(i e^{-\pi t})}.$$  

(4.4)

This amplitude is divergence-free both in the limits $t \to 0$ and $t \to \infty$. 

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5. Compactifications of type $0'$ to six dimensions

Non-tachyonic compactifications of type $0'$ were considered in the abstract conformal field theory setting in \[7\]. In the present and the following section we consider orientifolds $T^{10-d}/\{G_1, \Omega'G_2\}$, $d = 6, 4$, with $G_1, G_2$ internal Abelian symmetries. We will provide a microscopic description of these models in terms of open strings ending on D-branes. The computational technique is very similar to the one presented in section 3, so that we restrict ourselves to the main issues. We still want to get models without tachyons, which highly restricts the number of possible internal symmetries.

In the supersymmetric case there are two possible kinds of orientifolds of a $\mathbb{Z}_n$ orbifold, which involve different actions of world-sheet parity on the twisted sector ground states. In the mostly studied kind one actually divides out by $\Omega'J'$ where $J'$ exchanges the $g$ twisted sector with the $g^{-1}$ twisted sector. In six dimensions this leads to models with additional tensor multiplets, cancelled tadpoles and anomaly-free perturbative massless spectra. In generalising such orientifolds to our non-supersymmetric case one realizes that generically there appear tachyons in twisted sectors, which are mapped under $\Omega'J'$ to tachyons in different twisted sectors. Thus, linear combinations of twisted sector tachyons survive the projection. As was already observed in \[7\], the only $\Omega'J'$ orientifolds in six dimensions where this does not happen must have only twisted sectors of order two. This leaves the two possibilities

\[
\mathbb{Z}_2 : \{(1 + R) \times (1 + \Omega'J')\}, \\
\mathbb{Z}_{4B} : \{(1 + R) \times (1 + \Omega'J'\omega)\},
\]

(5.1)

where $\omega^2 = R$ and $R$ is the reflection $z_i \rightarrow -z_i$ of both internal coordinates.

However as pointed out in \[10\], one can also divide out just by $\Omega$ without exchanging twisted sectors. Unfortunately, this introduces new tadpoles from the now non-zero twisted sector Klein-bottle contributions for which we do not know how to cancel them. Consistently, one does not get an anomaly free perturbative massless spectrum. In the supersymmetric case Type I-heterotic duality was the guiding principle in determining what extra non-perturbative states from the Type I side are massless, as well. In the type $0'$ case these pure $\Omega'$ orientifolds turn out to be important, for in these models apparently the twisted sector tachyons are divided out. Of course, one faces the problem of additional non-perturbative massless states which luckily in six-dimensions might be detected by imposing anomaly freedom. We will discuss one such example namely the $\mathbb{Z}_3$ orientifold in subsection 5.3.
5.1. The $T^4/\mathbb{Z}_2$ orientifold

Computing the Klein bottle amplitude for this model shows that one gets a tadpole for both untwisted RR 10-form and RR 6-form exchange, whereas similar to type $0'$ in ten dimensions there is no NS-NS tadpole. Thus one is led to introduce D9 and D5-branes and of course the corresponding D9' and D5'-branes. The computation of the cylinder and Möbius amplitude is straightforward and very similar to the computation done in [11]. One finally ends up with three RR tadpoles to be cancelled. Analogous to the Gimon-Polchinski model [11] the twisted RR 6-form tadpoles are cancelled by choosing $\gamma_R$ traceless for both 9 and 5-branes. The remaining two untwisted tadpole conditions are

$$N_9^2 - 32 \text{Tr}(\gamma_{\Omega^{'},9}\gamma_{\Omega^{'},9}^{-1}) + 32^2 = 0,$$

$$N_5^2 - 32 \text{Tr}(\gamma_{\Omega^{R,5}}\gamma_{\Omega^{R,5}}^{-1}) + 32^2 = 0.$$  \hspace{1cm} (5.2)

Similar to the type $0'$ model there arises an uncancelled dilaton tadpole in the annulus amplitude. The tadpole cancellation (5.2) implies that the two matrices $\gamma_{\Omega^{'},9}$ and $\gamma_{\Omega^{'},5}$ are symmetric. A consistent choice is

$$\gamma_{\Omega^{'},9} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{2N_9,2N_9}, \quad \gamma_{\Omega^{'},5} = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}_{2N_5,2N_5},$$

$$\gamma_{R,9} = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}_{2N_9,2N_9}, \quad \gamma_{R,5} = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}_{2N_5,2N_5}. \hspace{1cm} (5.3)$$

The traceless matrix $M$ is given by

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{N_9,5,N_9,5}. \hspace{1cm} (5.4)$$

The tadpole cancellation condition (5.2) implies that the number of 9 and 5-branes must be 32. Next we compute the massless spectrum. The pure bosonic massless spectrum from the closed string sector is presented in Table 2, where we classified the states according to their Lorentz group $SO(4) = SU(2) \times SU(2)$ representation.

| sector            | spin $SU(2) \times SU(2)$                                                                 |
|-------------------|------------------------------------------------------------------------------------------|
| untwisted NS-NS   | $(3,3) + 11 \times (1,1)$                                                                |
| untwisted R-R     | $4 \times (3,1) + 4 \times (1,3) + 8 \times (1,1)$                                       |
| twisted NS-NS     | $64 \times (1,1)$                                                                        |
| twisted R-R       | $16 \times (3,1) + 16 \times (1,1)$                                                     |

Table 2: Closed string spectrum of $T^4/\mathbb{Z}_2$
Thus at the massless level one gets the graviton, the dilaton, 20 self dual 2-forms, 4 anti-self dual 2-forms and 98 further scalars. This spectrum is anomalous and must be extended by massless states from the open string sector. In Table 3 we list the spectrum with maximally enhanced gauge symmetry, where we have stuck together all D5 branes on the same fixed point of $R$ and where we have not turned on any Wilson lines in the D9 brane gauge group.

| sector       | spin | gauge              |
|--------------|------|--------------------|
| 99, 55       | (2,2) | $(U(16) \times U(16) \times U(16) \times U(16))$ adjoint |
| 9′9′, 5′5′    | (1,1) | $4 \times \{(16, \overline{16}; 1, 1) + (\overline{16}, 16; 1, 1) + (1, 1; 16, \overline{16}) + (1, 1; \overline{16}, 16)\}$ |
| 95, 9′5′     | (1,1) | $2 \times \{(16, 1; \overline{16}, 1) + (\overline{16}, 1; 16, 1) + (1, 16; 1, \overline{16}) + (1, \overline{16}; 1, 16)\}$ |
| 99′, 55′     | (1,2) | $2 \times \{(120 \oplus \overline{120}; 1, 1) + (1, 120 \oplus \overline{120}; 1, 1) + (1; 1; 120 \oplus \overline{120})\}$ |
| 95′, 59′     | (2,1) | $2 \times \{(16, 16; 1, 1) + (\overline{16}, \overline{16}; 1, 1) + (1, 1; 16, 16) + (1, 1; \overline{16}, \overline{16})\}$ |

**Table 3:** *Open string spectrum of $T^4/\mathbb{Z}_2$*

For the spectrum in Table 2 and Table 3 both the $R^4$ and the $F^4$ anomaly cancels \(^1\). There are two ways to reduce this spectrum. One way is by moving some of the D5-branes away from the fixed point or turning on some Wilson lines in the D9-brane gauge symmetry, respectively. The other way is by turning on a discrete background NS-NS two-form flux. Using the methods developed in [13] we have computed some of the resulting spectra that are presented in the appendix.

### 5.2. The $T^4/\mathbb{Z}_{4B}$ orientifold

This is analogous to the type IIB orientifold discussed in [14]. Just as in that case, the change $\Omega' \rightarrow \Omega' \omega$ implies that the Klein bottle amplitude by itself is divergence free. Hence, there are no branes and all matter comes from closed string sectors. The resulting massless spectrum is shown in Table 4.

---

\(^1\) Note that our spectrum does not agree with the spectrum computed in [7] where instead of the antisymmetric representation $120$ there appeared the symmetric representation $136$. It was explained to us by A. Sagnotti, that both models are consistent. The freedom appears in the definition of the $P$ modular matrix connecting the loop and tree channel Möbius amplitude [12].
sector & $SU(2) \times SU(2)$ \\
untwisted NS-NS & $(3, 3) + 7 \times (1, 1)$ \\
untwisted R-R & $2 \times (3, 1) + 6 \times (1, 3) + 8 \times (1, 1)$ \\
twisted NS-NS & $72 \times (1, 1)$ \\
twisted R-R & $10 \times (3, 1) + 6 \times (1, 3) + 16 \times (1, 1)$ \\

| sector | spin $SU(2) \times SU(2)$ |
|--------|-----------------------------|
| untwisted NS-NS | $(3, 3) + 5 \times (1, 1)$ |
| untwisted R-R | $4 \times (3, 1) + 2 \times (1, 3) + 6 \times (1, 1)$ |
| twisted NS-NS | $54 \times (1, 1)$ |
| twisted R-R | $18 \times (3, 1) + 18 \times (1, 1)$ |

**Table 4:** Spectrum of $T^4/\mathbb{Z}_{4B}$

This matter content is anomaly-free since there are no fermions and the number of self-dual and anti-self dual tensors is the same.

### 5.3. The $T^4/\mathbb{Z}_3$ orientifold

In this section we will discuss the $Z_3$ orientifold where $\Omega'$ does not exchange the two twisted sectors. It is straightforward to compute the massless spectrum in the closed string sector. The result is presented in Table 5

| sector | spin $SU(2) \times SU(2)$ |
|--------|-----------------------------|
| untwisted NS-NS | $(3, 3) + 5 \times (1, 1)$ |
| untwisted R-R | $4 \times (3, 1) + 2 \times (1, 3) + 6 \times (1, 1)$ |
| twisted NS-NS | $54 \times (1, 1)$ |
| twisted R-R | $18 \times (3, 1) + 18 \times (1, 1)$ |

**Table 5:** Closed string spectrum of $T^4/\mathbb{Z}_3$

Thus at the massless level one gets the graviton, the dilaton, 22 self dual 2-forms, 2 anti-self dual 2-forms and 82 further scalars. This spectrum is anomalous and must be extended by massless states from the open string sector and as we will see from the non-perturbative sector. To cancel the tadpole arising in the untwisted Klein-bottle amplitude, one has to introduce the same open string sector as in the $\Omega J'$ orientifolds. Thus, there are 32 D9-and 32 D9'-branes with the following action of the symmetries on the CP-factors

$$\begin{align*}
\gamma_{\Omega', 9} &= \begin{pmatrix} 0 & \Xi \\ \Xi & 0 \end{pmatrix}_{64, 64}, \\
\gamma_{\theta, 9} &= \begin{pmatrix} \Theta & 0 \\ 0 & \Theta \end{pmatrix}_{64, 64},
\end{align*}$$

(5.5)

with

$$\Xi = \begin{pmatrix} 0 & I_8 & 0 \\ I_8 & 0 & 0 \\ 0 & 0 & I_{16} \end{pmatrix}_{32, 32}, \quad \Theta = \begin{pmatrix} e^{\frac{2\pi i}{3}} I_8 & 0 & 0 \\ 0 & e^{-\frac{2\pi i}{3}} I_8 & 0 \\ 0 & 0 & I_{16} \end{pmatrix}_{32, 32}. \quad (5.6)$$
The computation of the open string spectrum yields

| sector | spin  | gauge \( U(8) \times U(8) \times U(16) \) |
|--------|-------|-----------------------------------------|
| 99, 9’9’ | (2,2) | adjoint \( 2 \times \{(8, \bar{8}; 1) + (\bar{8}, 8; 1) + (8, 1; 16) + (\bar{8}, 1; 16) + (1, 8; 16) + (1, \bar{8}; 16) \} \) |
| 99’ | (1,1) | \( 2 \times \{(8, 8; 1) + (\bar{8}, \bar{8}; 1) + (1, 1; 120 \oplus 120) \} \) |
|      | (2,1) | \( \{(28 \oplus 28, 1; 1) + (1, 28 \oplus 28; 1) \} \) |

Table 6: Open string spectrum of \( T^4/\mathbb{Z}_3 \)

It is encouraging that the \( F^4 \) gauge anomalies do all cancel. However, the \( R^4 \) anomaly is non-zero meaning that the open string spectrum can not be the complete spectrum. Unfortunately, since we do not know of any dual description we might use to determine the missing states, we are restricted to make a guess guided by anomaly cancellation and the analogy to the Type I model discussed in [10]. The simplest guess for additional fermionic matter is

\[
9 \times \{(28 \oplus 28, 1; 1) + (1, 28 \oplus 28; 1)\}_{(1,2)}. \tag{5.7}
\]

Probably, there will also be further bosonic non-perturbative states but we do not know how to detect them.

6. Compactifications of type 0’ to four dimensions

We now consider orientifolds of type 0B on \( T^6/\{G_1, \Omega’G_2\} \) with Abelian \( G_1, G_2 \). To be more specific, acting on internal complex coordinates \( z_i, i = 1, 2, 3 \), a \( G_1 \) or \( G_2 \) element \( g \) acts as \( g z_i = e^{2i\pi v_i} z_i \). Absence of tachyons in closed twisted sectors limit the allowed \( \vec{v} \)'s that are also constrained by crystallographic action on the torus lattice. However, now the condition \( \pm v_1 \pm v_2 \pm v_3 = 0 \) can be relaxed because our models are non-supersymmetric from the start.

One allowed model is the \( \mathbb{Z}_3 \) orientifold with

\[
\mathbb{Z}_3 : \{(1 + \theta + \theta^2) \times (1 + \Omega’J')\}, \tag{6.1}
\]

where the generator \( \theta \) has \( \vec{v} = (1, 1, 1) \). This example is studied in more detail in the next section. There is also a T-dual version of the above, namely

\[
\mathbb{Z}_{6B} : \{(1 + \theta + \theta^2) \times (1 + \Omega’J'\beta^3)\}, \tag{6.2}
\]

where \( \beta^2 = \theta \). Another possibility is a \( \mathbb{Z}_2 \) model with \( G_1 = G_2 \) generator having \( \vec{v} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \).
6.1. The $T^6/\mathbb{Z}_3$ orientifold

Computing the Klein bottle amplitude we find that there are only tadpoles from the untwisted RR 10-form and from twisted RR 4-forms. There are only 9 and 9'-branes and tadpole cancellation again requires $N_9 = N_9' = 32$. Twisted tadpoles cancel provided that

$$\text{Tr} \, \gamma_{\theta,9} = \text{Tr} \, \gamma_{\theta,9'} = -4. \quad (6.3)$$

This is satisfied by

$$\gamma_{\theta,9} = \text{diag} (\alpha I_{12}, \alpha^2 I_{12}, I_8),$$
$$\gamma_{\theta,9'} = \text{diag} (\alpha^2 I_{12}, \alpha I_{12}, I_8), \quad (6.4)$$

where $\alpha = e^{2i\pi/3}$. As we will see, necessarily $\gamma_{\theta,9'} = \gamma_{\theta,9}^\ast$. In order to study the transformation of open string states it is convenient to define the full $\theta$ embedding

$$\gamma_{\theta} = \begin{pmatrix} \gamma_{\theta,9} & 0 \\ 0 & \gamma_{\theta,9'} \end{pmatrix}_{64,64}. \quad (6.5)$$

For the $\Omega'$ embedding we take

$$\gamma_{\Omega',J'} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{64,64}. \quad (6.6)$$

This is consistent with tadpole cancellation and group multiplication law.

Let us now describe the massless spectrum. Open strings stretching between two D9-branes or two D9'-branes give gauge vector states of the form (3.14), with $\mu$ a four-dimensional index. The gauge Chan-Paton factor must be of the form (3.15) as required by invariance under $\Omega'J'$. Invariance under $\theta$ also requires

$$\lambda_G = \gamma_{\theta} \lambda_G \gamma_{\theta}^{-1}, \quad (6.7)$$

which implies $A = \gamma_{\theta,9} A_{\theta,9}^{-1}$ and necessarily $\gamma_{\theta,9'} = \gamma_{\theta,9}^\ast$. Given $\gamma_{\theta,9}$ in (6.4) we then find that the gauge group is $U(12) \times U(12) \times U(8)$. From 99 and 99' open strings there are also scalar states

$$\psi_{i \frac{1}{2}} |0\rangle_{NS} \lambda_S, \quad (6.8)$$

where $i = 1, 2, 3$ refers to the internal complex coordinates. Invariance under $\Omega'J'$ gives

$$\lambda_S = \begin{pmatrix} D & 0 \\ 0 & -D^T \end{pmatrix}_{64,64}. \quad (6.9)$$
Since the world-sheet piece picks up a phase $\alpha$ under $\theta$, $\lambda_S$ must satisfy

$$\lambda_S = \alpha \gamma_\theta \lambda_S \gamma_\theta^{-1}.$$  \hfill (6.10)

Thus, $D = \alpha \gamma_{\theta,9} D \gamma_{\theta,9}^{-1}$. This determines the representations. We thus find scalar states with multiplicities and representations given by

$$3 \times \{(\mathbf{12}, \mathbf{12}, 1) + (1, 12, \overline{8}) + (\overline{12}, 1, 8)\}.$$  \hfill (6.11)

Scalar states with $\psi_{-\frac{i}{2}}|0\rangle_{NS} \lambda_S$ transform in the complex conjugate of the above.

Open strings stretched between D9 and D9$'$-branes carry massless fermions of the form (3.16) with Chan-Paton factor $\lambda_F$ as given in (3.17), where $B^T = -B$ and $C^T = -C$. The world-sheet piece acquires a phase $e^{2i\pi(s_1 + s_2 - 2s_3)/3}$ under $\theta$ so that $\lambda_F$ must satisfy

$$\lambda_F = e^{2i\pi(s_1 + s_2 - 2s_3)/3} \gamma_\theta \lambda_F \gamma_\theta^{-1}. \hfill (6.12)$$

This further constrains the matrices $B, C$ and gives specific group representations for the states. We thus find that states with $s_1 = -\frac{1}{2}$ are massless left-handed fermions in the following representations and multiplicities

$$\{(\mathbf{12}, \mathbf{12}, 1) + (1, 1, 28) + (\overline{12}, \overline{12}, 1) + (1, 1, \overline{28})\} +
3 \times \{(\mathbf{66}, 1, 1) + (1, \mathbf{12}, 8) + (1, 66, 1) + (\overline{12}, 1, 8)\}.$$ \hfill (6.13)

States with $s_1 = +\frac{1}{2}$ are the CPT conjugates of the above. The open string spectrum is summarised in Table 5

| sector | spin | gauge $U(12) \times U(12) \times U(8)$ |
|--------|------|--------------------------------------|
| 99, 9'9' | vector, scalar | adjoint |
|        | 3 $\times \{(\mathbf{12}, \overline{12}, 1) + (1, 12, \overline{8}) + (\overline{12}, 1, 8) + c.c.\}$ |
| 99' | fermion$_L$ | $\{(\mathbf{12}, \mathbf{12}, 1) + (1, 1, 28) + (\overline{12}, \overline{12}, 1) + (1, 1, \overline{28})\} +
3 \times \{(\mathbf{66}, 1, 1) + (1, \mathbf{12}, 8) + (1, 66, 1) + (\overline{12}, 1, 8)\}$ |

| Table 7: Open string spectrum of $T^6/\mathbb{Z}_3$ |

This spectrum is chiral and free of non-Abelian gauge anomalies and agrees with the results in [7]. Concerning the $U(1)$ factors, there is one non-anomalous and two anomalous combinations whose anomaly could presumably be cancelled by a generalised Green-Schwarz mechanism.
The closed string massless spectrum is computed in the standard manner. In the untwisted sector the NS-NS states are the graviton plus 10 scalars, including the dilaton and internal metric moduli, whereas the R-R states are 20 scalars and one vector that arises from the 4-form. In the twisted sector the potentially tachyonic states are instead massive but there are 27 NS-NS massless scalars and 54 R-R massless scalars.

It is also possible to include discrete or continuous Wilson lines to obtain new models. As explained in [15], the Wilson lines must satisfy tadpole cancellation conditions and they imply further projections on allowed states. In the $\mathbb{Z}_3$ orientifold at hand we can for example introduce a discrete Wilson line that gives a model with gauge group $U(4)^8$.

7. Conclusions

In this paper we have investigated a constrained set of orientifolds of type 0B in which all tachyons are projected out. In detail we have discussed the ten-dimensional case, determined the D-brane contents of this model and computed the massless spectrum. All tachyon-free orientifolds have in common that they have unitary gauge groups only, admit massless fermionic matter and that there remains an uncancelled dilaton tadpole. We have studied the effective gauge theory on parallel D3-branes with the negative result that there appears an inconsistent spectrum reflecting that D9 and D3-branes repel each other. We briefly discussed the possibility that the dilaton tadpole could be cancelled by the D3-branes.

Moreover, we have considered various tachyon-free compactifications to six and four space-time dimensions and derived the massless anomaly-free spectra.

Acknowledgements

A.F. thanks CDCH-UCV for a research grant 03.173.98, as well as the Quantum Field Theory group at Humboldt-Universität for financial support and very kind hospitality. R.B. thanks the CERN Theory division for hospitality.
Appendix

i.)

Table A1 and Table A2 list the massless closed and open string spectra of the $T^4/\mathbb{Z}_2$ orientifold with a rank two background NS-NS 2-form field turned on. In the open string sector the rank of the gauge group is reduced by a factor of two, whereas the sector of strings stretched between a D9 and a D5 brane is doubled.

| sector          | spin $SU(2) \times SU(2)$  |
|-----------------|-----------------------------|
| untwisted NS-NS | $(3, 3) + 11 \times (1, 1)$ |
| untwisted R-R   | $4 \times (3, 1) + 4 \times (1, 3) + 8 \times (1, 1)$ |
| twisted NS-NS   | $64 \times (1, 1)$         |
| twisted R-R     | $12 \times (3, 1) + 4 \times (1, 3) + 16 \times (1, 1)$ |

**Table A.1:** Closed string spectrum of $T^4/\mathbb{Z}_2$ with $rk(B)=2$

| sector | spin | gauge $U(8) \times U(8) \times U(8) \times U(8)$ |
|--------|------|--------------------------------------------------|
| 99, 55 | (2,2) adjoint | $\left\{(8, \overline{8}; 1, 1) + (\overline{8}, 8; 1, 1) + (1, 1; 8, \overline{8}) + (1, 1; \overline{8}, 8)\right\}$ |
| 99', 55' | (1,1) | $4 \times \left\{(8, 1; \overline{8}, 1) + (\overline{8}, 1; 8, 1) + (1, 8; 1, \overline{8}) + (1, \overline{8}; 1, 8)\right\}$ |
| 95, 95' | (1,1) | $4 \times \left\{(8, 1; \overline{8}, 1) + (\overline{8}, 1; 8, 1) + (1, 8; 1, \overline{8}) + (1, \overline{8}; 1, 8)\right\}$ |
| 99', 55' | (1,2) | $2 \times \left\{(28 \oplus \overline{28}, 1; 1, 1) + (1, 28 \oplus \overline{28}; 1, 1) + (1, 1; 28 \oplus \overline{28}, 1) + (1, 1; 1, 28 \oplus \overline{28})\right\}$ |
|         | (2,1) | $2 \times \left\{(8, 8; 1, 1) + (\overline{8}, \overline{8}; 1, 1) + (1, 1; 8, 8) + (1, 1; \overline{8}, \overline{8})\right\}$ |
| 95', 59' | (1,2) | $2 \times \left\{(8, 1; 8, 1) + (\overline{8}, 1; \overline{8}, 1) + (1, 8; 1, 8) + (1, \overline{8}; 1, \overline{8})\right\}$ |

**Table A.2:** Open string spectrum of $T^4/\mathbb{Z}_2$ with $rk(B)=2$

Both the $R^4$ and the $F^4$ anomaly cancels for this spectrum.
Table A2 and Table A4 list the massless closed and open string spectra of the \( T^4/\mathbb{Z}_2 \) orientifold with a rank four background NS-NS 2-form field turned on.

| sector      | spin SU(2) × SU(2) |
|-------------|--------------------|
| untwisted NS-NS | (3, 3) + 11 × (1, 1) |
| untwisted R-R | 4 × (3, 1) + 4 × (1, 3) + 8 × (1, 1) |
| twisted NS-NS  | 64 × (1, 1) |
| twisted R-R   | 10 × (3, 1) + 6 × (1, 3) + 16 × (1, 1) |

**Table A.3:** Closed string spectrum of \( T^4/\mathbb{Z}_2 \) with \( \text{rk}(B)=4 \)

| sector      | spin      | gauge U(4) × U(4) × U(4) × U(4) |
|-------------|-----------|----------------------------------|
| 99, 55      | (2,2) adjoint |                                   |
| 99', 55'    | (1,1)     |                                   |
| 95, 95'     | (1,1)     |                                   |
| 99', 55'    | (1,2)     |                                   |
| 95', 59'    | (1,2)     |                                   |

**Table A.4:** Open string spectrum of \( T^4/\mathbb{Z}_2 \) with \( \text{rk}(B)=4 \)

In Table A.5 we list the open string spectrum one gets when all 32 D5 branes are moved away from the fixed point but 16 of them still coincide and moreover appropriate 9-brane Wilson lines have been turned on to make the spectrum T-invariant.

| sector      | spin      | gauge U(16) × U(16) |
|-------------|-----------|---------------------|
| 99, 55      | (2,2) adjoint |                      |
| 99', 55'    | (1,1)     |                     |
| 95, 95'     | (1,1)     |                     |
| 99', 55'    | (1,2)     |                     |
| 95', 59'    | (1,2)     |                     |

**Table A.5:** Open string spectrum of \( T^4/\mathbb{Z}_2 \) with 9 and 5-branes not at fixed point
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