Light Gluinos and Unification of Couplings

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Abstract

We analyze the implications of the light gluino scenario for the unification of
gauge and Yukawa couplings within the minimal supersymmetric standard model.
Within this scheme all fermionic supersymmetric particles are naturally light, while
the scalar partners of quarks and leptons, together with the heavy Higgs doublet
may be heavy. This implies both a bound on $\tan \beta < 2.3$, in order to fulfill the
experimental constraints on the chargino masses, and a strong correlation between
$\sin^2 \theta_W(M_Z)$ and $\alpha_3(M_Z)$, due to the suppression of the supersymmetric threshold
corrections to the low energy values of the gauge couplings. Assuming the scalar
sparticles to be lighter than 10 TeV, the physical top quark mass is constrained to
be $145 \text{ GeV} < M_t < 210 \text{ GeV}$ for $\tan \beta > 1$, while the strong gauge coupling values,
$0.122 \leq \alpha_3(M_Z) \leq 0.133$, are in good agreement with the measured LEP ones. We
also show that a relaxation of some of the conventional assumptions is necessary
in order to achieve the radiative breaking of the electroweak symmetry within the
grand unification scheme.

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Introduction. Unification of couplings might give hints about the physics at very high energy scales. The discussion of supersymmetric Grand Unified Theories (GUTs) has recently attracted much attention. In fact, an extrapolation of the measured gauge couplings within a minimal supersymmetric extension of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model leads to a unification at a scale $M_X$ of a few $\times 10^{16}$GeV, with $M_{SUSY}$, the supersymmetry (SUSY) breakdown scale, to be in the 100GeV to TeV energy range, as expected theoretically. Given the present experimental uncertainties of the gauge couplings (especially the strong gauge coupling $\alpha_3$), the model is consistent with unification for a wide range of the parameter space. Assuming that at the scale $M_X$ the known forces are contained in a single grand unified group like SU(5) gives more restrictions. There one would expect also a unification of certain Yukawa couplings (like those of the b–quark and the $\tau$–lepton in the simplest case) and this, in fact, requires rather large values for the top quark Yukawa coupling, yielding predictions for the mass of the yet undetected top quark which are remarkably close to the infrared quasi-fixed point value for this quantity.

In this paper we are going to study a special class of such supersymmetric GUT models in which one parameter, the gaugino mass $M_{1/2}$ at the unification scale, vanishes (or is negligible compared to the other parameters which represent the SUSY breakdown), implying the appearance of a light gluino in the spectrum. Although at several instances it was claimed that the light gluino window is closed, it has to be accepted that such a particle has not yet been ruled out experimentally. There has been a thorough discussion of this issue in the literature and we shall not repeat it here, since it is outside the scope of this paper [1][2]. One of the recent motivations to consider the light gluino scenario, is in relation to the study of the evolution of the gauge couplings from low energies up to the mass of the Z-boson [3][4]. A theoretical motivation can be found in string models where SUSY is broken through gaugino-condensates, although the prediction of vanishing gaugino masses within this scheme depends on the special properties of the potential considered in these works. In the context of grand unification such a light gluino scenario is very appropriate since it gives a restricted range of parameter space and therefore enhanced predictive power. In addition, the calculation performed in this work can also have more general application and stay useful even if such a light gluino would not exist.

The fact that $M_{1/2}$ is very small combined with the non-observation of charginos, neutralinos and Higgs bosons puts severe constraints on the parameter space and primarily on $\tan \beta = v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs fields. As we shall see, assuming all supersymmetric particles to have masses lower than 10 TeV, this
then leads to a very narrow range of values of $\alpha_3$ consistent with perturbative unification

$$0.122 \leq \alpha_3(M_Z) \leq 0.133,$$

(1)

for $1 \leq \tan \beta \leq 2$. A similar but less restrictive statement concerning $\alpha_3$ could be made in the more general case, allowing a heavy gluino, if it were possible to restrict the $\tan \beta$ range in a similar way (arguments could come from proton decay or constraints on the top quark mass but are still inconclusive). The main difference concerning the range of allowed values for $\alpha_3(M_Z)$ in the heavy gluino case however, is due to to the potentially large supersymmetric threshold corrections, which, as we shall discuss below, are much smaller in the light gluino scenario. As a second result we find a lower limit on the physical top quark mass

$$M_t > 145\text{GeV}$$

(2)

when $\tan \beta \geq 1$. It should be pointed out that (2) is mainly a consequence of b–τ Yukawa coupling unification and also holds in the general case [5][6]. We also find that, in order to achieve the unification of gauge and Yukawa couplings together with the radiative breaking of the electroweak symmetry within this scheme, the minimal supersymmetry breaking conditions at the unification scale should be modified through, for example, a non–universal scalar mass for the two Higgs doublets of the theory.

**Input Data and Constraints.** The low energy experimental data and their uncertainties are crucial in probing unification. The electroweak parameters at the $Z$–pole [7]

$M_Z = 91.187 \pm 0.007$ GeV are presently known to a very good accuracy, the remaining uncertainty stemming mainly from the unknown top quark mass. In the modified minimal subtraction scheme ($\overline{MS}$) the values that we use are

$$\sin^2 \theta_W(M_Z) = 0.2324 - 1.03 \cdot 10^{-7} GeV^{-2} \left( M_t^2 - (138 GeV)^2 \right) \pm 0.0003,$$

(3)

$$\frac{1}{\alpha(M_Z)} = 127.9 \pm 0.1.$$

(4)

The quadratic dependence of the measured value of $\sin^2 \theta_W$ on $M_t$ is explicit in eq. (3) and we incorporate this correlation in our analysis.

Unlike the electroweak couplings, the strong gauge coupling at the $Z$–pole is not so accurately known. Therefore we do not consider $\alpha_3(M_Z)$ as an input in our extrapolation, but rather as a prediction when we demand gauge and bottom to tau Yukawa coupling unification. An analysis of the strong gauge coupling from the existing experimental data assuming the presence of light gluinos, has recently been carried out [8], indicating that $\alpha_3(M_Z)$ tends to take significantly higher values.
We take the physical tau mass to be $M_\tau = 1.78\text{GeV}$ and, neglecting small QED corrections, we take the running tau mass at the physical mass $m_\tau(M_\tau)$ to be equal to the physical mass. The bottom quark mass is less accurately known. We consider for the physical bottom quark mass the range\footnote{One should be careful when using experimental lower bounds in this scenario since the analysis of the data has been carried out without taking into account decay modes allowed when gluinos are light.}

$$4.7\text{GeV} \leq M_b \leq 5.2\text{GeV} \text{,}$$

and we calculate the running mass at the physical mass through the formula

$$m_b(M_b) = \frac{M_b}{1 + \frac{4\alpha_3(M_b)}{\beta\pi} + 12.4\left(\frac{\alpha_3(M_b)}{\pi}\right)^2},$$

where two–loop QCD corrections have been taken into account\footnote{Analogous corrections are considered in the calculation of the top quark mass. The corresponding formula reads}

$$m_t(M_t) = \frac{M_t}{1 + \frac{4\alpha_3(M_t)}{3\pi} + 11\left(\frac{\alpha_3(M_t)}{\pi}\right)^2} \text{.}$$

The model we are analysing is significantly more constrained than the usual supersymmetric GUTs, and therefore also more predictive and more easily falsifiable. The most constrained parameter is $\tan\beta$. The non–observation of charginos with mass below half the $Z$ mass imposes an absolute upper bound of 2.3 on $\tan\beta$ \cite{3,9}. In addition, experimental lower bounds on the neutralino mass are very likely to further constrain $\tan\beta$ from above\footnote{The model we are analysing is significantly more constrained than the usual supersymmetric GUTs, and therefore also more predictive and more easily falsifiable. The most constrained parameter is $\tan\beta$. The non–observation of charginos with mass below half the $Z$ mass imposes an absolute upper bound of 2.3 on $\tan\beta$\cite{3,9}. In addition, experimental lower bounds on the neutralino mass are very likely to further constrain $\tan\beta$ from above\cite{9}. We take as a reasonable upper bound the value 2, but we shall also discuss the implications of slightly larger values of $\tan\beta$. Concerning a lower bound on $\tan\beta$, the only firm constraint comes through the non–observation of the top quark below 108GeV. The requirement of perturbative consistency of the top quark sector at energy scales close to $M_X$ implies a lower bound on $\tan\beta \geq 0.6$ \cite{14}. However, the experimental lower bounds on the light CP–even Higgs scalar mass make values of $\tan\beta$ lower than 1 highly improbable, especially, if we constrain the sparticle masses to be below 1 TeV. Therefore we will in general consider $\tan\beta \geq 1$, a requirement naturally appearing in models with radiative electroweak symmetry breakdown, although we shall also discuss how values slightly below this limit affect our results.}

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The requirement of $b$–$\tau$ Yukawa coupling unification yields predictions for the top quark Yukawa coupling $h_t$, which are close to its infrared quasi–fixed point value. So it
may occur that the top quark Yukawa coupling becomes too large at the high energy scale. If we want to work consistently in perturbation theory, $h_t$ has to remain perturbative in the whole range of our extrapolation. To this aim, taking into account the renormalization group (RG) behaviour of the top Yukawa, it is enough to constrain $h_t$ at the unification scale $M_X$. We require that

$$\frac{h_t^2(M_X)}{4\pi} < 1,$$

which is approximately equivalent to the condition that the two–loop corrections are less than 30% of the one–loop contribution.

**Analysis and Results.** In the two–loop analysis we perform, we consider three distinct regimes. First we define $M_{scal}$ as the characteristic mass of squarks, sleptons and the heavy Higgs doublet, and we vary it within the theoretically and phenomenologically acceptable range

$$M_Z \leq M_{scal} \leq 10\, \text{TeV}.$$  

Thus, between the unification scale and $M_{scal}$ we use the Minimal Supersymmetric Standard Model (MSSM) group equations, while below that scale and down to $M_Z$ we run the Standard Model (SM) renormalization group equations with modified $\beta$–function coefficients to include the contributions from gauginos and Higgsinos. Below $M_Z$ we extrapolate using three-loop QCD including light gluinos and two-loop QED with chargino contributions. We consider in our analysis the Yukawa couplings of the third generation quarks and leptons. However, it is worth mentioning that in the range of $\tan \beta$ we are examining, the bottom and tau Yukawa coupling contributions are negligible in comparison to those associated with the top quark Yukawa coupling.

A few words are necessary about the decoupling procedure that we adopt at the various thresholds. At the unification scale we ignore possible corrections from splittings in the spectrum of the new heavy particles introduced, since they are strongly dependent on the unifying group and require a detailed analysis beyond the scope of this work. The supersymmetric particles and the heavy Higgs doublet are decoupled according to the so called $\theta$-function approximation: their contribution to the $\beta$–function coefficients is dropped as soon as we are below their physical mass and the running couplings are required to be continuous at the thresholds. Care has been taken to correctly match the

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3We decouple the charginos below $M_Z$, the exact scale being of no importance for our results.

4Given the accuracy of the input data and the two–loop analysis that we pursue, an attempt to smoothly decouple the heavy modes could have relevant effects. We choose not to follow this approach mainly because of the uncertainty concerning smooth decoupling in the renormalization schemes we are working with, but also because we want to make our results readily comparable with existing analyses.
MSSM with the SM Yukawa couplings. The top quark, the light Higgs and the $SU(2)_L$ gauge bosons are decoupled by passing from the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory to the effective $SU(3)_C \times U(1)_{em}$ theory. The transition takes place at $M_Z$ and we follow the conventions of ref. [6]. When extrapolating between $M_Z$ and the unification scale we employ the dimensional reduction ($DR$) scheme.

We probe unification in the following way: We impose the conditions

$$\frac{5}{3} \alpha_1(M_X) = \alpha_2(M_X) = \alpha_3(M_X) = \alpha_X ,$$

$$h_b(M_X) = 1 ,$$

where $M_X$ is the unification scale and $\alpha_X$ the gauge coupling at that scale. We then scan the five–dimensional space defined by the parameters $M_X, \alpha_X, h_t(M_X), M_{scal}$ and $\tan \beta$, taking into account the constraints mentioned previously, and extrapolate numerically to low energies. Predicting for each point in the parameter space a set of values for the low energy quantities $M_t, \sin^2 \theta_W(M_Z), \alpha_{em}(M_Z), \alpha_3(M_Z)$ and $m_b/M_t$, we confirm unification and accept the values of the above ten parameters as a solution, if eq.(3)–(5) are satisfied.

The results that we obtain are significantly constrained and highly correlated. The dominant effect comes from the fact that imposing $b$–$\tau$ Yukawa coupling unification drives the top Yukawa coupling to its infrared quasi–fixed point, thus constraining the top quark mass to high values (see fig.1). For $M_{scal}$ in the range of eq. (8) and $\tan \beta$ between 1 and 2 the physical top quark mass ranges in the interval

$$145 GeV < M_t < 210 GeV .$$

Due to the correlation between the measured value of $\sin^2 \theta_W$ and $M_t$, eq.(3), and the firm dependence of the predicted value of $\alpha_3(M_Z)$ on the former, we find solutions only in the limited range

$$0.122 \leq \alpha_3(M_Z) \leq 0.133 ,$$

and a tight correlation between $\alpha_3(M_Z)$ and $M_t$, as can be seen in fig.2. There is also a weak dependence of the predicted values of $\alpha_3(M_Z)$ on the scale $M_{scal}$ (see fig.2), which may be understood as follows: Once the unification condition is imposed, the value of $\alpha_3(M_Z)$ may be given as a function of the electroweak gauge couplings and the threshold corrections due to the presence of sparticles with masses above $M_Z$ [4, 5]:

$$\frac{1}{\alpha_3(M_Z)} = \frac{1}{\alpha_3^{SUSY}(M_Z)} + \frac{19}{28\pi} \ln \left( \frac{T_{SUSY}}{M_Z} \right) .$$

Eq. (9) is the one–loop exact matching condition in the $\overline{DR}$ renormalization scheme, if the heavy particle spectrum is taken degenerate.
There $\alpha_3^{SUSY}(M_Z)$ would be the predicted value for the strong gauge coupling if the theory were exactly supersymmetric down to $M_Z$ and $T_{SUSY}$ is an effective scale which characterizes the supersymmetric threshold corrections to the gauge couplings. In the light gluino scenario the effective scale $T_{SUSY}$ is given by

$$T_{SUSY} = M_Z \left( \frac{M_{scal}}{M_Z} \right)^{3/19} \quad (14)$$

and hence, for a fixed value of the electroweak gauge couplings, $\alpha_3(M_Z)$ decreases only slightly as $M_{scal}$ increases. In fact, for the range of $M_{scal}$ of eq.(8), $T_{SUSY}$ varies between the values

$$M_Z \leq T_{SUSY} \leq 200 GeV \quad (15)$$

According to eq.(13) this implies very small supersymmetric threshold corrections to the value of $\alpha_3(M_Z)$.

An alternative way of understanding the restricted range of $\alpha_3$ solutions is to note that the matrix $R$ defined in Ref.[11], which relates $\alpha_i^{-1}(M_Z)$ to $\alpha_X^{-1}$, $\ln (M_{scal}/M_Z)$ and $\ln (M_X/M_Z)$ becomes approximately singular if only the heavy Higgs, squarks and sleptons are above the $Z$-boson mass scale, as assumed here. It would be exactly singular at the one–loop level if the heavy Higgs mass were also below $M_Z$. In the singular case, $\alpha_3(M_Z)$ would become independent of $\alpha_X$ and $M_{scal}$, depending then primarily on $\sin^2 \theta_W(M_Z)$. In the language of $T_{SUSY}$ this singular case corresponds to an effective supersymmetric threshold scale $T_{SUSY} = M_Z$.

We would also like to remark that although there is a weak dependence of $\alpha_3(M_Z)$ on $M_{scal}$, this dependence affects the predicted range for $\alpha_3(M_Z)$ only if we go to values of $M_{scal}$ above 1TeV. This behaviour is the result of two counteracting effects: Larger $M_{scal}$ tends to reduce $\alpha_3(M_Z)$, but at the same time for large $M_{scal}$ the predicted values of $M_t$ are higher, thus tending to raise $\alpha_3(M_Z)$. The two effects practically cancel when $M_{scal} \leq 1$TeV allowing for an almost $M_{scal}$–independent range of $\alpha_3(M_Z)$. If we go though to $M_{scal}$ above 1TeV, we can no longer compensate the decrease in $\alpha_3(M_Z)$ with the higher values of $M_t$, because for these high values the top Yukawa coupling develops a Landau singularity making such potential solutions unacceptable. As a result the upper bound on the range of $\alpha_3(M_Z)$ decreases when $M_{scal}$ becomes larger than 1TeV. It is worth mentioning that when $M_{scal} \leq 1$TeV $M_t$ is always found to be smaller than 200GeV.

From our analysis $M_X$ and $\alpha_X$ are predicted to lie in the ranges

$$2.3 \cdot 10^{16} GeV < M_X < 4.3 \cdot 10^{16} GeV \quad , (16)$$
23.2 \leq \frac{1}{\alpha_X} \leq 25.2 . \quad (17)

If we now allow \( \tan \beta \) to vary in the extended range

\[ 0.6 \leq \tan \beta \leq 2.3 , \quad (18) \]

(keeping in mind that the values below 1 and above 2 are very likely to be inconsistent with various experimental lower bounds), the above limits in our results loosen up. The most dramatic effect exhibits itself in the range predicted for the top quark mass:

\[ 104 \text{ GeV} \leq M_t \leq 212 \text{ GeV} , \quad (19) \]

where the lower limit has significantly decreased responding to the low values allowed for \( \tan \beta \) (see fig. 1). The predicted range of values for the strong gauge coupling is however only slightly enlarged:

\[ 0.118 \leq \alpha_3(M_Z) \leq 0.134 . \quad (20) \]

Otherwise the correlations pointed out in the previous restricted range for \( \tan \beta \) also apply in this case. The lower bound on the unification scale \( M_X \) in eq.(16) goes to the slightly smaller value \( 1.9 \cdot 10^{16} \text{ GeV} \), while the range of \( \alpha_X^{-1} \) is still the one given in eq.(17).

**Radiative Electroweak Symmetry Breaking.** In a recent work [12], it has been examined how the requirement of radiative breaking of the \( SU(2)_L \times U(1)_Y \) gauge group, when combined with the current LEP data, constrains the parameter space of the minimal supergravity–inspired model in the presence of a light gluino. From this analysis it turns out that it is hardly possible to achieve radiative electroweak breaking in such a model, and the remaining allowed region of the parameter space might be experimentally excluded in the near future. Although we qualitatively agree with this analysis, we find less stringent limits on the values of the top quark mass consistent with radiative breakdown. In fact, the range of allowed values is increased once the uncertainties on \( \alpha_3(M_Z) \) are taken into account. In addition, the value of the top quark mass given in Ref.[12] should be associated with the running and not the physical mass. Thus, the physical top quark mass values consistent with radiative electroweak breaking turn out to be around \( 125 \text{ GeV} \), with \( \tan \beta \) between 1.8 and 2. Even these values of \( M_t \) are however inconsistent with unification, since, as shown in fig.1, for this range of \( \tan \beta \) the predicted top quark mass in a unified theory is always larger than \( 180 \text{ GeV} \).

One possible way to make radiative electroweak breaking compatible with unification would be to relax the requirement of exact bottom to tau Yukawa unification at \( M_X \). In
view of the still unknown high scale threshold effects and the unclear situation concerning
the generation of fermion masses, this could be considered as a mild compromise. How-
ever it should not escape one’s attention that the difficulty in breaking radiatively the
electroweak symmetry is intrinsic to the vanishing $M_{1/2}$ scenario within the minimal su-
persymmetry breaking scheme, independently of unification. Hence, an alternative way of
making the radiative breaking of $SU(2)_L \times U(1)_Y$ compatible with unification of couplings
would be to relax the universality of the soft supersymmetry breaking scalar masses at
the high energy scale, through, for example, non-universal soft supersymmetry breaking
Higgs masses at $M_X$.

In order to understand the above let us review the properties of the renormalized Higgs
mass parameters $m_i^2$ appearing in the potential. We assume that at the grand unification
scale, the squarks and sleptons acquire a common soft supersymmetry breaking mass $m_0$
and the Higgs doublets $H_1$ a breaking mass $m_{H_1}$, while the trilinear term $A_0$ vanishes. In
the one–loop approximation [13], the renormalized values of the Higgs mass parameters
$m_1^2$ and $m_2^2$ appearing in the potential are then given by:

$$
m_1^2 \approx m_{H_1}^2 + \mu^2
$$

$$
m_2^2 \approx m_{H_2}^2 + \mu^2 - \frac{1}{2} \left( \frac{h_t}{h'_t} \right)^2 \left( 2m_0^2 + m_{H_2}^2 \right),
$$

(21)

where $\mu$ is the renormalized SUSY mass parameter appearing in the superpotential, and
$h_t$ and $h'_t = \sqrt{32\pi\alpha_3/9}$ are the top quark Yukawa coupling and its infrared quasi-fixed
point value respectively. The minimization condition for the potential yields

$$
\tan^2 \beta = \frac{m_1^2 + M_Z^2/2}{m_2^2 + M_Z^2/2},
$$

(22)

where we have neglected the radiative correction contributions which for squarks in the
TeV range are of order $M_Z^2$.

From our results it follows that unification of gauge and $b-\tau$ Yukawa couplings force
the top quark Yukawa coupling to be at most 10% away from its infrared quasi-fixed point
values. Moreover, within the light gluino scenario there is an upper bound on the mass
parameter $\mu$ of about $M_Z$. Hence, in the case of universal soft supersymmetry breaking
mass parameters $m_{H_2} = m_{H_1} = m_0$, and due to the fact that the top quark Yukawa
coupling is close to its infrared fixed point value, the requirement of radiative breaking
necessarily implies low values of $m_0 \leq M_Z$, as can be easily verified from eqs.(21) and
(22). In addition, it is straightforward to show that both the CP–odd Higgs mass and
the soft SUSY breaking squark mass terms are also of the order of or lower than $M_Z$. 

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Thus, the universality of the SUSY breaking Higgs mass parameter at $M_X$ would imply values for the lightest CP–even Higgs mass which are below its present experimental limit and hence the model would be ruled out. The above conclusion is not preserved if we relax the assumption of universality of soft SUSY breaking scalar mass terms at the unification scale. Indeed, if for example we assume $h_t/h_f^t \simeq 0.9$ (corresponding to a physical top quark mass of about 180GeV for $\tan \beta \simeq 2$), and $m^2_{H_2} = 3/2m^2_0$, while in addition $m_0 \gg M_Z$ in order to avoid problems in the Higgs sector, we obtain

$$\tan^2 \beta \simeq \frac{m^2_{H_1}}{0.1m^2_{H_2}}. \quad (23)$$

From the above equation, it is easy to verify that, while varying $m_{H_1}$ from $m_{H_2}/3$ up to $m_{H_2}/\sqrt{2}$, we can cover values of $\tan \beta$ from 1 up to 2.3. In addition, the SUSY breaking mass parameters of the supersymmetric partners of the right and left–handed top quarks are given by $m^2_u \simeq 0.1m^2_0$ and $m^2_q \simeq 0.5m^2_0$ respectively.

The previous results are derived within a one–loop approximation, where potentially important effects such as squark decoupling have been neglected. A more detailed analysis, including two–loop corrections to the mass parameters, is necessary if one wants to make a conclusive statement on the consistency of gauge and Yukawa coupling unification with radiative electroweak breaking within this modified soft SUSY breaking scheme.

Observe that, since the Higgs doublets belong to different representations of $SU(5)$ than the squarks and sleptons, the relaxation of scalar masses we proposed above is completely consistent with the $SU(5)$ symmetry. A relaxation of gaugino mass universality at the grand unification scale would on the other hand break the $SU(5)$ symmetry of the theory at $M_X$. One should mention that if this were the case and, for example, only the partners of the massless gauge bosons were below $M_Z$ as was assumed in ref.[3], significantly lower strong gauge couplings at $M_Z$ could be obtained.

**Conclusions.** We have found that SUSY unification with light gluinos (below the bottom quark mass) constrains the value of the strong gauge coupling at $M_Z$ to within $\pm 5\%$. It is interesting to note that the central value as well as the allowed range is in good agreement with the current experimental values from LEP. However, this result as well as that of eq.(11) for grand unification in the light gluino scenario is strongly dependent on the following three assumptions: 1. Minimal SUSY particle content. 2. A common soft supersymmetry breaking gaugino mass $M_{1/2}$ at the grand unification scale. 3. Exact unification of the couplings at a single scale $M_X$ (see eq.(9) and (10)). One should also note that all the calculations have been done with $\theta$–function decoupling of the heavy
modes. A refinement of this procedure might have an effect on our results.

Moreover the additional assumption of radiative breaking of the $SU(2)_L \times U(1)_Y$ symmetry has been considered. We have shown that a relaxation of the universality of the soft supersymmetry scalar masses associated to the Higgs fields is a possible way to achieve unification of gauge and $b-\tau$ Yukawa couplings together with a proper radiative breaking of the $SU(2)_L \times U(1)_Y$ symmetry within this scheme.

Finally, we remark that the main difference between the predictions of the light and the heavy gluino scenarios is related to the size of the supersymmetric threshold corrections to the values of $\alpha_3(M_Z)$. Whereas in the heavy gluino case these corrections could be as large as 10\%, in the light gluino case, they can not exceed 2\% of the values which would be obtained if the theory were exactly supersymmetric down to $M_Z$. As a consequence, the presence of light gluinos poses stringent constraints on the allowed range of values for $\alpha_3(M_Z)$.

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FIGURE CAPTIONS

Fig. 1. Top quark mass as a function of $\tan \beta$, for $M_{scal} = M_Z$ (solid line) and $M_{scal} = 2$ TeV (dashed line). The top quark mass dependence on $\tan \beta$ does not vary for $\sin^2 \theta_W$ varying within its experimental error, eq.(3).

Fig. 2. Top quark mass as a function of the strong gauge coupling for $M_{scal} = M_Z$ (solid lines) and $M_{scal} = 2$ TeV (dashed lines), $1 \leq \tan \beta \leq 2$ and $\sin^2 \theta_W$ taking its central value (center), and its upper (left) and lower (right) experimentally allowed values at the one-$\sigma$ level, eq.(3). Observe that the $M_S = 2$ TeV curves are cut at $\alpha_3(M_Z) \simeq 0.13$, due to the loss of perturbative consistency of the top Yukawa sector of the theory at $M_X$. 