Observation of the Bloch-Siegert Shift in a Qubit-Oscillator System in the Ultrastrong Coupling Regime

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We measure the dispersive energy-level shift of an LC resonator magnetically coupled to a superconducting qubit, which clearly shows that our system operates in the ultrastrong coupling regime. The large mutual kinetic inductance provides a coupling energy of \( \approx 0.82 \text{ GHz} \), requiring the addition of counter-rotating-wave terms in the description of the Jaynes-Cummings model. We find a 50 MHz Bloch-Siegert shift when the qubit is in its symmetry point, fully consistent with our analytical model.

The study of driven two-level systems has been at the heart of important discoveries of fundamental effects, both classical and quantum mechanical. A generic example is the field of nuclear magnetic resonance where the dynamics of nuclear spins is controlled by the applicability of radio frequency pulses, resulting in coherent Rabi oscillations of the spin moments [1]. In the usual description, the applied harmonic field is decomposed into two mutually counterrotating fields. At resonance in the weak-driving limit only the rotating component interacts constructively with the spins, leading to a Rabi frequency that scales linearly with the driving strength. Thus for this single component rotating regime the rotating-wave approximation (RWA) is known to hold. If the driving is so strong that the Rabi frequency approaches the Larmor frequency, the counterrotating terms need to be taken into account. This leads to an energy shift in the level transition, denoted as the Bloch-Siegert shift. This non-RWA regime has been observed in a variety of strongly driven systems. In the field of quantum electrodynamics (QED) a quantum Bloch-Siegert shift has been considered for atoms very strongly coupled to single photons [1], although the experimental verification is difficult [5]. In the dispersive regime this shift is sometimes referred to as dynamical Stark shift [9]. For an atom that resides in a resonant cavity the interaction strength \( g \), the Rabi rate when the cavity contains a single photon, is found to be typically \( 10^{-4} \) of the atomic Larmor frequency \( \omega_0/2\pi \) and the cavity frequency \( \omega_c/2\pi \). The Jaynes Cummings (JC) model [2], that fully relies on the validity of the rotating-wave approximation, therefore yields a good description of the system [7].

In circuit QED [8] superconducting qubits play the role of artificial atoms. With energy level transitions in the microwave regime, they can be easily cooled to the ground state at standard cryogenic temperatures. These “atoms” can interact very strongly with on-chip resonant circuits and reproduce many of the physical phenomena that had been previously observed in cavities with natural atoms [9][10]. The large dipolar coupling achievable in superconducting circuits enabled exploring the strong-dispersive limit [11]. One now starts addressing the ultrastrong coupling regime \( g/\omega_0 \sim 1 \) [12–14]. In this Letter we experimentally resolve the quantum Bloch-Siegert shift in an LC resonator coupled to a flux qubit with a coupling strength \( g/\omega_r \approx 0.1 \), thus entering the ultrastrong coupling regime.

Our system consists of a four-Josephson-junction flux qubit [15], in which one junction is made smaller than the other three by a factor of approximately 0.5. The qubit is galvanically connected to a lumped-element LC resonator [Fig. 1]. In previous work the employed LC resonators were strongly coupled to the flux qubit [8][10], but since they were loaded by the impedance of the external circuit their quality factor was low. Flux qubits have also been successfully coupled to high-quality transmission line resonators [17]. In our experiment we use an interdigitated finger capacitor in series with a long superconducting wire, following the ideas from lumped-element kinetic inductance detectors [18]. In order to read out the qubit state a dc-switching SQUID magnetometer was placed on top of the qubit. The detection procedure can be found in [19].

The qubit and the resonator were fabricated in the same layer of evaporated aluminum using standard lithography techniques [19]. A second aluminum layer galvanically isolated from the first one contains the SQUID and its circuitry together with the microwave antenna to control the local frustration and to produce flux and microwave pulses in the qubit [Fig. 2]. An external coil is used to generate a magnetic field in the qubit and SQUID in order to bias them at their operating points. A second qubit with its own circuitry was also coupled to the resonator [Fig. 1 (a)], but during the experiment it was always flux biased such that it did not affect the measurements.

The resonator is made of two capacitors, each containing 50 fingers of 150 \( \mu \text{m} \) length and 1.5 \( \mu \text{m} \) width, separated by 2 \( \mu \text{m} \) [Fig. 1 (a)]. The two capacitors are linked by two 500 \( \mu \text{m} \) long superconducting wires of 1 \( \mu \text{m} \) width. With these parameters we estimate a capacitance of \( C_r \approx 0.5 \text{ pF} \) and an inductance of \( L_r \approx 1.5 \text{ nH} \), corresponding to a resonance frequency
The qubit-resonator interaction can be rewritten in terms of rising and lowering operators $\sigma_\pm = (\sigma_z \pm i \sigma_y)/2$. This yields corollating terms $\sim (\sigma_+ a + \sigma_- a^\dagger)$ as well as counterrotating terms $\sim (\sigma_+ a^\dagger + \sigma_- a)$. In the regime where $g$ is comparable to $\omega_q$ and $\omega_r$, the usual RWA is not valid and the counter-rotating terms cannot be neglected. In order to evaluate their effect on the system, we perform a unitary transformation $H'_E = e^{i S} He^{-i S}$, with $S = \gamma (\sigma_+ a^\dagger - \sigma_- a) + g \sin(\theta)/(\omega_q + \omega_r)$ to eliminate the counter-rotating terms. If $|\gamma| \ll 1$ we can safely neglect off-resonant terms of order $\gamma^2$. Off-diagonal two-photon processes can be removed by similar canonical transformations [3]. For frequencies not too far from resonance, keeping terms up to second order in $\gamma$, we obtain the effective Hamiltonian

$$H'_E = \frac{\hbar \omega_q}{2} \sigma_z + \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) + \hbar g (\cos(\theta) \sigma_z - \sin(\theta) \sigma_y) (a + a^\dagger), \quad (1)$$

with $\omega_{qy} \equiv \sqrt{\omega_q^2 + \Delta^2}$ and $\tan(\theta) \equiv \Delta/\epsilon$. Note that the RWA has not been employed to obtain Eq. 1.

The qubit-resonator interaction is described by a coupling of dipolar nature $H_{int} = h g (a^\dagger + a) \sigma_z$ in the basis of the persistent current states, where $a^\dagger (a)$ is the photon creation (annihilation) operator in the basis $\{|n\}$ of Fock states of the resonator. In the basis of the eigenstates of the qubit, $\{|g\}, \{|e\}\}$, the Hamiltonian reads

$$H'_E = \frac{\hbar \omega_q}{2} \sigma_z + \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) + \hbar g (\cos(\theta) \sigma_z - \sin(\theta) \sigma_y) (a + a^\dagger), \quad (1)$$

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constant has been renormalized to \( g(\hat{n}) \equiv -g\sin(\theta)[1 - \hat{n}\omega_{BS}/(\omega_q + \omega_r)] \).

In the basis \( \{ |g, n+1\rangle, |e, n\rangle \} \), the effective Hamiltonian [Eq. 2] is box-diagonal. The box corresponding to \( n \) photons has eigenvalues \( \lambda_{n,m=0,1} = \hbar\omega_r (n + 1) + (\hbar/2)(-1)^m\sqrt{\delta^2 + 4g_n^2/\omega_r^2} \), where \( \delta = \omega_q - \omega_r \) is the detuning, and \( \delta_{n+1} = \delta + 2\omega_{BS}(n + 1) \) and \( g_{n+1} = g\sin(\theta)\sqrt{n + 1}[1 - (n + 1)\omega_{BS}/(\omega_q + \omega_r)] \). \( m = 0 \) (1) corresponds to the qubit in the ground (excited) state. In the limit \( \omega_{BS} \to 0 \), the JC result is recovered [8]. For the qubit in the ground state the oscillator resonance is shifted with respect to the JC model.

We prepare the qubit in the ground state by cooling it to 20 mK in a dilution refrigerator. Using the protocol shown in Fig. 2(a), we measure the spectrum of the qubit-resonator system [Fig. 3]. To obtain a higher resolution in the relevant region around 8.15 GHz, we repeated the spectroscopy using lower driving power in combination with the application of flux pulses in order to equalize the qubit signal by reading out far from its degeneracy point [Fig. 4(a)].

All the curves are subtracted from the JC model. We can identify the energy-level transitions on the basis of the JC ladder shown in Fig. 2(b). A large avoided crossing between states \( |g, 1\rangle \) and \( |e, 0\rangle \) is observed around a frequency of \( \approx 8 \) GHz. This is very close to the estimated resonance frequency of the oscillator. The energy splitting \( (2g/2\pi)(\Delta/\omega_r) \) [Fig. 3 (inset)] is approximately 0.9 GHz. A combined least-squares fit of the full Hamiltonian [Eq. 1] of the data from Figs. 3 and 4 leads to \( \Delta/h = (4.20 \pm 0.02) \) GHz, \( I_p = (500 \pm 10) \) nA, \( \omega_r/2\pi = (8.13 \pm 0.01) \) GHz and \( g/2\pi = (0.82 \pm 0.03) \) GHz.

The value of \( g \) obtained is in good agreement with \( I_p\mu_{ms}L_E/h = (0.83 \pm 0.08) \) GHz. Thus we find \( g/\omega_r \approx 0.1 \). This large value brings us into the ultrastrong coupling regime, and below we will demonstrate that the system indeed shows ultrastrong coupling characteristics.

The spectral line of the resonator can be resolved when it is detuned several GHz away from the qubit [Fig. 5]. This could be caused by the external driving when it is resonant with the oscillator. By loading photons in it, the oscillator can drive the qubit off-resonantly by their large coupling. Another possibility is an adiabatic shift during state readout through the anticrossing of the qubit and resonator energies. The qubit readout pulse produces a negative shift of \( -2 \) m\( \Phi_0 \) in magnetic flux, making the spectral amplitude asymmetric with respect to the qubit symmetry point [Fig. 3]. For our parameters, this shift is coincidental with the avoided level crossing with the oscillator. Then, a state containing one photon in the resonator \( |e, 0\rangle \) can be converted into an excited state of the qubit with very high probability, as the Landau-Zener tunneling rate is very low. Both effects, off-resonant driving and adiabatic shifting, would explain that the sign of the spectral line of the resonator coincides with the one of the qubit on both sides of the symmetry point. Irrespective of the mechanism, the spectral features of Fig. 3 allow us to give a low bound for the quality factor of the resonator \( Q > 10^3 \).

In Figs. 4(a), (b) a marked difference in the resonator frequency between the fit of Eq. 1 (solid black line) and the JC model, Eq. 1 with the counterrotating terms re-
moved, (dashed green line) can be clearly resolved. The difference is largest (50 MHz) at the symmetry point of the qubit. This is the Bloch-Siegert shift \( \omega_{BS} \) associated with the counterrotating terms [Eq. 2]. The maximum difference occurs at the symmetry point as outside of it the effective coupling \( g \sin(\theta) \) decreases with increasing \( \epsilon \). Figure 4 (b) shows in a dashed red curve a plot of \( \lambda_{1,g} - \lambda_{0,g} \) subtracted from the JC model. The agreement between the measured spectral peaks of the resonator and the calculated values using \( \lambda_{1,g} - \lambda_{0,g} \) is very good. Concerning the qubit, according to \( \lambda_{0,e} - \lambda_{0,g} \) it should experience the same shift \( \omega_{BS} \) as the resonator, but with opposite sign. Since the qubit line width at the symmetry point around 4 GHz is very large (\( \approx 80 \) MHz), the Bloch-Siegert shift cannot be clearly resolved there.

In conclusion, we have measured the Bloch-Siegert shift in an LC resonator strongly coupled to a flux qubit. This demonstrates the failure of the rotating-wave approximation in this ultrastrong coupling regime of circuit QED. The large coupling of 0.82 GHz is achieved using the kinetic inductance of the wire that is shared by the two systems. The coupling could easily be further enhanced by increasing the kinetic inductance or by inclusion of a Josephson junction [14, 22]. This will allow the exploration of the system deeply into the ultrastrong coupling regime where \( g \) is comparable with \( \omega_r \).

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[1] C. Cohen-Tannoudji, Bernard Diu, and Franck Laloe, *Quantum Mechanics*, John Wiley & Sons, Inc., New York (1977).
[2] F. Bloch, A. Siegert, *Phys. Rev.* 57, 522 (1940).
[3] A. B. Klimov and S. M. Chumakov, *A Group-Theoretical Approach to Quantum Optics*, WILEY-VCH, Weinheim (2009).
[4] J. H. Shirley, *Phys. Rev.* 138, B979 (1965).
[5] E. T. Jaynes, F. W. Cummings, Proc. IEEE 51, 89 (1963).
[6] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. 73, 565 (2001).
[7] A. Blais *et al*., Phys. Rev. A 69, 062320 (2004).
[8] I. Chiorescu *et al*., Nature 431, 159 (2004). J. Johansson *et al*., Phys. Rev. Lett. 96 127006 (2006).
[9] M. Baur *et al*., Phys. Rev. Lett. 102, 243602 (2009).
[10] M. Hofheinz *et al*., Nature 454, 310 (2008).
[11] D. I. Schuster *et al*., Nature (London) 445, 515 (2007).
[12] S. Ashhab, and F. Nori, Phys. Rev. A, 81, 042311 (2010).
[13] B. Peropadre, P. Forn-Díaz, E. Solano, and J. J. García-Ripoll, Phys. Rev. Lett. 105 023601 (2010).
[14] T. Niemczyk *et al*., Nature Physics 6, 772 (2010).
The system of a flux qubit coupled to an LC resonator can be modeled using the Hamiltonian
\[ \hat{H} = \frac{\hbar \omega_q}{2} \sigma_z + \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) \]

\[ + \hbar g \left( \cos(\theta) \sigma_z - \sin(\theta) \sigma_x \right) \left( a + a^\dagger \right), \]  

with \( \hbar \omega_q \equiv \sqrt{\epsilon^2 + \Delta^2} \), \( \epsilon = 2I_p(\Phi - \Phi_0/2) \) and \( \tan(\theta) \equiv \Delta/\epsilon \). If the rotating-wave approximation is applied, Eq. 3 becomes
\[ \hat{H}_{JC} = \frac{\hbar \omega_q}{2} \sigma_z + \hbar \omega_r \left( a^\dagger a + \frac{1}{2} \right) \]

\[ - \hbar g \sin(\theta) \left( \sigma_z a + \sigma_y a^\dagger \right), \]  

known as the Jaynes-Cummings [JC] model.

A least-squares fit of the full spectrum of the system using Eq. 3 can be seen in Fig. 3 (solid black line), with fitted parameters \( g/2\pi = 0.82 \pm 0.03 \) GHz, \( \Delta/h = 4.25 \pm 0.02 \) GHz, \( I_p = 500 \pm 10 \) nA, \( \omega_r/2\pi = 8.13 \pm 0.01 \) GHz. Eq. 4 is also plotted using these fitted parameters (dashed blue line). No significant difference can be observed between the two curves, except a small deviation at the symmetry point of the qubit.

A fit of the same spectrum using the JC model (Eq. 4) can be performed. This can be seen in Fig. 6 (dashed blue line), with fitted parameters \( g/2\pi = 0.72 \pm 0.02 \) GHz, \( \Delta/h = 4.21 \pm 0.02 \) GHz, \( I_p = 500 \pm 10 \) nA, \( \omega_r/2\pi = 8.13 \pm 0.01 \) GHz. Eq. 3 is plotted using these fitted parameters (solid black line). The difference between the two curves is similar to Fig. 6 with a small deviation at the symmetry point of the qubit.

To observe more clearly the deviations between the exact model and the JC model, a zoom in is made of the spectrum at the region near 8 GHz. Fig. 7 is a zoom in of Fig. 5 showing the fit to Eq. 3 (solid black line). Also the JC solution of Eq. 4 is plotted (dashed blue line) using the same fitting parameters values to Eq. 3. Equation 3 fits the data in all points (open circles represent Lorentzian fits to each data trace), while Eq. 4 deviates at the symmetry point of the qubit. The deviation is attributed to the counter-rotating terms that were neglected by applying the rotating-wave approximation in Eq. 3.

Figure 8 shows a zoom in of Fig. 6 with the dashed line representing the fit to the JC solution Eq. 4. Also Eq. 3 is plotted (solid black line) using the fit parameter values of Fig. 6.
FIG. 7. Zoom in of Fig. 5 around 8 GHz with the spectrum fitted using Eq. \(3\) (solid black line). In dashed blue is Eq. \(4\) using the fitted parameters.

FIG. 8. Zoom in of Fig. 6 around 8 GHz with the spectrum fitted using Eq. \(4\) the JC model (dashed blue line). In solid blue is Eq. \(3\) using the fitted parameters.

In this case the best fit of the JC model (dashed line) does not fit all data points, in particular it does not fit the ones around \(\Phi = \Phi_0/2\). On the other hand, Eq. \(3\) using the fitted parameters from Fig. 6 leads to lower values of the transition near 8.22 GHz than Fig. 7.

If the JC model was valid the fits in Fig. 5 (and Fig. 7) and 6 (and Fig. 8) should lead to the same result. This is not the case, indicating that the rotating-wave approximation is not applicable. This is most clearly seen in Fig. 8 where the Jaynes-Cummings model fails to fit all data points, in particular in the range where the counter-rotating terms included in Eq. 3 have their largest contribution providing maximum Bloch-Siegert shift.