Study of higher-order modes for waveguide grating coupler

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Abstract. The vector two-dimensional finite difference time domain method is applied to analyze the higher-order modes of waveguide grating couplers (WGCs). The phase matching function of WGC is deduced based on diffraction equation. The stable field distributions and mode patterns of $TE_1$, $TE_2$ and $TE_3$ in WGCs are obtained by using Gaussian beam as a driving source. It is found that the optical mode is determined by grating parameters and waveguide structures. According to the mode field distribution, the optimum coupling efficiency can be predicted, and thus highly-efficient WGCs which are of practical use in integrated optical circuits and optoelectronic devices can be designed conveniently.

1. Introduction

With the development of integrated optics and silicon photonics, waveguide grating couplers (WGC) have been gaining considerable interest and playing an important role in optical communication, optical compute, and optical information processing [1]. One of its applications is providing a means of coupling light into or out of waveguide between optoelectronic components, such as the coupling between lasers or detectors to waveguides [2]. This attractive coupling method greatly reduces the size and weight of optical systems, integrated waveguides and electronic components to form photoelectric systems, thus new application fields of WGC are being opened up. Some new communication apparatus, such as the optical interconnection devices, the integrated-optic disk pickup device, integrated-optic interferometer position sensor, integrated-optic scanning optical microscope and integrated-optic printer head are all employing the WGC [3].

There are two “special” problems in the use of WGC devices: cut-off frequency and higher-order mode. The widespread and thorough researches have been done on cut-off frequency in many fields, but comparatively little has been done in the work of higher-order mode presently. The main reasons for this case are follows: there are many advantages for basic-mode propagation, e.g., longer transmission distance, larger transmission capacity, bigger amplitude, wider transmission band width, stronger interaction between guided-mode and waveguide grating, and higher coupling efficiency, etc. Therefore, the research work for higher-order mode is ignored to some degree. However, in some specific situation, higher-order modes can be used to improve the optical quality for its fine performances, including bigger aperture, easier coupling, stronger capability of dispersion compensation and larger tuning range of wavelength, although the existence of intra-modal dispersion may restrict the bandwidth and increase the loss. It is very necessary to accurately know the field distributions of higher-order modes while dealing with the transition of different polarization modes or coupling of different order modes.
2. Physical model and theoretical method

A schematic diagram of grating couplers which we discuss is shown in Figure 1. The refractive indices of air, grating base, coating, film, substrate are expressed by $n_a$, $n_b$, $n_c$, $n_f$, and $n_s$, respectively. While the incident beam is upon on the grating’s surface at the incident angle $\theta$ from air, the relationship between the grating spacing and the angles of the incident and diffracted beams of light is expressed by the grating equation [4]

$$\sin \alpha_l = \sin \theta + l \frac{\lambda_0}{d} \quad l = 0, \pm 1, \pm 2, \ldots$$  (1)

Where $\alpha$, $\lambda_0$, and $d$ are the number of the diffraction angle, the order of diffraction, the wavelength of incident wave and spatial period of grating, respectively.

![Figure 1. Basic structure of waveguid grating coupler](image)

The phase displacement along the z-direction of different diffraction orders is determined by

$$\beta_l = \frac{2\pi}{\lambda_0} n_b \sin \alpha_l = \frac{2\pi}{d} n_0 + \frac{2\pi}{\lambda_0} n_b \sin \theta$$  (2)

The propagation constant of the $m$-order guided mode is named by $\beta_m$, and it must follow the condition of

$$\beta_m > \frac{2\pi}{\lambda_0} n_b$$  (3)

In order to reach the phase-matching between the grating and the waveguide mode, the following equation must be satisfied

$$\beta_l = \beta_m$$  (4)

This phase-matching condition can be satisfied in a suitable positive value of $l$ [5]. As a result, the guided-mode can be stimulated inside the waveguide.

The finite difference time domain (FDTD) [6] technique with perfectly matched layer absorbing boundary condition [7] is used. From the FDTD numerical simulation, we can get the optical field distribution and stable mode pattern along the guide direction. There are many books and papers on the topic of FDTD, for a detailed and exhaustive description, please see Ref. [8] and references therein.

In what follows, only essential steps are outlined.

In the passive and lossless region, the curl equations of Maxwell’s equations are as follows

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} = -\mu_0 H_v \frac{\partial H}{\partial t}$$  (5)

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$$  (6)
Where \( \nabla \) is Laplace operator, \( \mu \) is magnetic permeability, \( \mu_r \) is relative permeability, \( \mu_0 \) is free-space permeability \( (4\pi \times 10^7 \text{ henrys/meter} ) \), \( \varepsilon = \varepsilon_r \varepsilon_0 \) is electrical permittivity \( (\text{farads/meter}) \), \( \varepsilon_r \) is relative permittivity, and \( \varepsilon_0 \) is free-space permittivity \( (8.854 \times 10^{-12} \text{ farads/meter} ) \).

It is obvious that the wave equations can be decomposed into six scalar equations for six vector field components. Only TE is considered in this paper. While waveguide structure has nothing to do with variable \( y \), it is easily proved that there is only three field components \( (E_y, H_x, H_z) \), where \( H_x \) and \( H_z \) can be expressed by \( E_y \). Thus, in our calculation, we choose vector FDTD to research the field distribution of \( E_y \) which can be used to represent the total field distribution. In rectangular coordinate system, the above expressions can be expanded as

\[
\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \tag{7}
\]

\[
\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_y}{\partial z} \right) \tag{8}
\]

\[
\frac{\partial H_z}{\partial t} = -\frac{1}{\mu} \left( \frac{\partial E_y}{\partial x} \right) \tag{9}
\]

Using the central-difference formula, Equality (7) becomes

\[
E_{y}^{N+1}(i,k) = E_{y}^{N}(i,k) + \frac{\Delta t}{\varepsilon} \left[ H_{x}^{N+\frac{1}{2}}(i,k+1) - H_{x}^{N+\frac{1}{2}}(i,k) \right] - \frac{\Delta x}{\varepsilon} \left[ H_{z}^{N+\frac{1}{2}}(i+1,k) - H_{z}^{N+\frac{1}{2}}(i,k) \right] \tag{10}
\]

Where \( \Delta x \) stands for space step in both directions of the \( x \) and \( z \), \( \Delta t \) stands for time step, \( N \) stands for mesh number, and \( E_{y}^{N}(i,k) = E_{y}^{N}(i,i,k,N,\Delta x,\Delta z,\Delta t) \). In general, \( \Delta x = \Delta z = \Delta s \) is chosen.

In order to satisfy the stability condition, the following relationship is usually taken

\[
\Delta t = \Delta s/(2c_{\text{max}}) \tag{11}
\]

Where \( c_{\text{max}} \) is the maximum velocity of light waves in the interaction space. To avoid numerical dispersion, \( \Delta s = 0.05\Delta \lambda_0 \) is selected.

When the above conditions are satisfied, the Eq.10 becomes

\[
E_{y}^{N+1}(i,k) = E_{y}^{N}(i,k) + A(i,k) \left[ H_{x}^{N+\frac{1}{2}}(i,k+1) - H_{x}^{N+\frac{1}{2}}(i,k) \right] - B \left[ E_{y}^{N}(i,k+1) - E_{y}^{N}(i,k) \right] \tag{12}
\]

Where \( A(i,k) = \frac{n_c}{4c_{\text{max}}} \n^2(i,k) \)

Similarly, the difference equations of Eqs.8 and 9 are as follows

\[
H_{x}^{N+\frac{1}{2}}(i+1,k) = H_{x}^{N-\frac{1}{2}}(i+1,k) + B \left[ E_{y}^{N}(i,k+1) - E_{y}^{N}(i,k) \right] \tag{13}
\]

\[
H_{z}^{N+\frac{1}{2}}(i+1,k) = H_{z}^{N-\frac{1}{2}}(i+1,k) + B \left[ E_{y}^{N}(i,k) - E_{y}^{N}(i+2,k) \right] \tag{14}
\]

Where \( B = \frac{n_c}{4c_{\text{max}}} \)

The perfectly matched layer absorbing boundary condition (derived by Mur) is used. The condition for \( E_y \) at \( i = 1 \) (lower boundary), for example, can be written as

\[
E_{y}^{N+1}(i,k) = -0.6 \left[ E_{y}^{N+1}(i+2,k) + 1.55 \left[ E_{y}^{N}(i,k) + E_{y}^{N}(i+2,k) \right] + 0.025 \left[ E_{y}^{N}(i+2,k-2) + E_{y}^{N}(i,k-2) + E_{y}^{N}(i,k+2) \right] - E_{y}^{N-1}(i+2,k) - 0.6 \cdot E_{y}^{N-1}(i,k) \right] \tag{15}
\]
3. Numerical results

The parameter values chosen in this paper are as follows: the thickness of waveguide is 1 μm, the incident wavelength is 1.3 μm, and the grating spatial period is 0.4 μm, and which could obtain stable optical field distributions of TE₁, TE₂ and TE₃ are listed in Table 1. All these values are based on the reference [4] which involves the complex physical principle and strict mathematical deduction. For a detailed deduction process, please see for it and do the needful.

The Gaussian beam is used as incident beam. The light field distribution profiles of TE₁, TE₂ and TE₃ are shown in Figure 2 (a), Figure 3 (a), and Figure 4 (a), respectively. It is obvious that, at the beginning of grating coupler, the stable mode is not formed, and therefore the case of field distribution is far from the real distribution of corresponding mode. With the increasing of propagation distance, the light field distributions stabilize gradually for each mode. It is shown that, by using grating coupler, the guided modes can be stimulated by radiation modes in waveguide. It is also found that the optical field distribution is related to the diffraction angle of each order. While the l-order is lacked, the coupling function is vanished. According to the mode field distribution profile, the optimum coupling efficiency can be predicted. It is worth mentioning that, the waveguide grating coupler could not only be used as the input coupler, but also the output coupler whose coupling effect could only occur in phase-matched l-order harmonic.

Table 1. refractive index distributions, space step and time step for different modes

| mode value | TE₁ | TE₂ | TE₃ |
|------------|-----|-----|-----|
| n₁        | 1.4 | 1.25| 1.25|
| n₂        | 1.563| 1.85| 2.35|
| n₃        | 1.4 | 1.25| 1.25|
| Δs(μm)    | 0.0416| 0.0351| 0.0277|
| Δt(ns)    | 0.097| 0.0732| 0.0576|

The stable mode patterns of TE₁, TE₂ and TE₃ are illustrated in Figure 2 (b), Figure 3 (b), and Figure 4 (b), respectively. All of these can be seen that, while each mode propagates along the longitudinal direction, the mode pattern of cross direction maintains invariant. These results are in good agreement with analytic solutions. Thus the reliability and the accuracy of our simulation results by FDTD are guaranteed. The above results indicate that a stable light field mode at different order of l (l=1, 2, 3, …) can be coming into being as long as the grating parameters and waveguide structures and the incident wavelength are correctly chosen.

4. Conclusion

In this paper, the phase matching function of WGC is obtained based on diffraction equation, the difference equations and absorbing boundary conditions are deduced based on the vector FDTD according to Maxwell equations, and the field distributions and mode patterns of different modes in the WGC are calculated. The calculated results indicate that the different optical field distributions and stable mode patterns of different order modes can be obtained with suitable grating parameters and waveguide structures. According to the mode field distribution, the optimum coupling efficiency can be predicted. In this way, we are able to design highly-efficient waveguide grating couplers, which is
of practical use in integrated optical circuits and optoelectronic devices. To the best authors’
knowledge, the analysis of higher-order mode field distribution of WGC using the vector FDTD
method has not been reported to date.

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Figure 2  (a) Field distribution of $TE_1$  (b) Mode pattern of $TE_1$

Figure 3  (a) Field distribution of $TE_2$  (b) Mode pattern of $TE_2$

Figure 4  (a) Field distribution of $TE_3$  (b) Mode pattern of $TE_3$