Multiband superfluidity and superfluid to band-insulator transition of strongly interacting fermionic atoms in an optical lattice

A.A. Burkov

Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

Arun Paramekanti

Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7

(Dated: February 6, 2009)

We study the multiband superfluid and the superfluid (SF) to band insulator (BI) transition of strongly interacting fermionic atoms in an optical lattice at a filling of two fermions per well. We present physical arguments to show that a consistent mean field description of this problem is obtained by retaining only intraband pairing between the fermions. Using this approach we obtain a reasonable account of the experimentally observed critical lattice depth for the SF-BI transition and the modulated components of the condensate density, and make predictions for the lattice depth dependence of the quasiparticle gap which can be tested in future experiments. We also highlight some interesting features unique to cold atom superfluids within this intraband pairing approximation; for instance, the pair field is forced to be uniform in space and the Hartree field vanishes identically. These arise as a result of the fact that while the pairing interaction is cut off at the scale of the Debye frequency in conventional superconductors, or at the lattice scale in tight binding model Hamiltonians, such a cutoff is absent for cold Fermi gases.

I. INTRODUCTION

Studies of cold atomic gases have led to significant experimental [1] and theoretical progress [2] in our understanding of the crossover regime between a weakly paired BCS superfluid and a strongly paired BEC superfluid. Recent experiments have also studied a strongly interacting fermion superfluid (SF) in an optical lattice. Upon imposing the optical lattice potential on a SF of $^6$Li atoms with near-unitary scattering, at a density of two atoms per lattice well, it was found [3] that the SF retained its coherence at weak to moderate lattice depths as deduced from the interference peaks in the momentum distribution of Feshbach molecules. Beyond a certain critical lattice depth, however, the interference peaks disappeared, signalling a possible SF to band insulator (BI) quantum phase transition. Motivated by these experiments, theoretical studies [4, 5] have focussed on the BI states of strongly interacting fermions in deep lattices, and shown that such BIs can be unstable to Cooper pair formation below a critical lattice depth. Interestingly, a toy model for such an interaction dependent SF-BI transition was proposed and studied some years ago [6] in the context of the cuprate superconductors.

In this paper, we focus on the problem of understanding the multiband SF, and SF-BI transition, of fermions in a three dimensional (3D) optical lattice, at a density of two fermions per well, using a mean field intraband pairing approximation which is equivalent to Anderson’s idea of pairing exact time-reversed eigenstates [7]. This is known to be an excellent approximation in the context of disordered solid-state superconductors [8] and in metallic nanoparticles [9], where pairing between single-particle states which are not related by time-reversal symmetry gets suppressed by quenched randomness. In the present context, we argue that the intraband pairing approximation provides an internally self-consistent mean-field approach and that interband pairing is expected to be dynamically suppressed. Using this theory we obtain several interesting results, summarized below.

On the theoretical front: (i) We point out that there exists a significant discrepancy between the earlier theoretical estimates [4, 5] of the critical lattice depth for the SF-BI transition, and we provide a physical resolution of this discrepancy. (ii) Our mean field approach provides a numerically tractable route to understanding properties of the SF phase which was not considered in earlier theoretical work [4, 5]. (iii) Finally, we show that there are a number of ways in which the cold atom multiband SF is distinct from multiband solid state superconductors. These differences arise from the fact that in solid state systems there is a cutoff in the pairing interaction, set by the Debye frequency in a conventional superconductor or by the lattice cutoff in model tight-binding Hamiltonians, which is absent in cold atom SFs. More generally, this implies that results obtained on such effective lattice models in the context of cold atom Fermi superfluids must be interpreted with caution.

On the experimental front: (i) As seen from Fig. 1, we find that the SF-BI transition, at unitarity, occurs at a critical lattice depth $V^{(\text{crit})}_L \approx 4E_R$, where $E_R = \pi^2 \hbar^2 / (2m a_L^2)$ is the atomic recoil energy, with $a_L = \lambda / 2$ being the lattice spacing ($\lambda$ being the laser wavelength) and $m_a$ being the atom mass. Our result is somewhat larger than the experimental estimate [5] of $V^{(\text{crit})}_L \approx 3E_R$, but is reasonably close, given that our approach is a mean-field theory, which neglects fluctuations (but see below). Our result for $V^{(\text{crit})}_L$ coincides with the result of Ref. [3] but follows from a set of internally consistent assumptions. We also show why the result of Ref. [5], while obtained within a valid formalism,
leads to a significant overestimate of the critical lattice depth for the SF-BI transition at mean field level. (ii) We conclude in Section IV with a theoretical discussion where we develop our mean-field theory and find a nonmonotonic dependence which can be tested with the experiment. We also compute the quasiparticle modulated components of the condensate density in the SF-BI transition, the dependence of both uniform and intraband pairing for lattice depths slightly smaller than the critical lattice depth.

The paper is organized as follows. Section III contains a theoretical discussion where we develop our mean-field theory and show that the intraband pairing ansatz gives an internally consistent description of the physics at density of two atoms per well. Section III explores the experimental consequences of our mean-field theory. Specifically, we present results for the critical lattice depth for the SF-BI transition, the dependence of both uniform and modulated components of the condensate density in the SF phase on the lattice depth and compare these results with the experiment. We also compute the quasiparticle excitation gap as a function of the lattice potential depth and find a nonmonotonic dependence which can be tested in future experiments. We conclude in Section IV with a brief summary of the results.

II. MODEL AND MEAN FIELD THEORY

Let us consider fermions moving in a 3D lattice potential

\[ V(R) = -\frac{V_L}{2} \left[ \cos\left(\frac{2\pi X}{b_L}\right) + \cos\left(\frac{2\pi Y}{b_L}\right) + \cos\left(\frac{2\pi Z}{b_L}\right) \right], \]

with lattice constant \( b_L = \lambda / 2 \), formed by three pairs of counterpropagating laser beams with wavelength \( \lambda \). Let us use units where we measure momentum (distance) in units of \( b_L^{-1} \) (\( b_L \)) and energies in units of the single atom recoil energy, \( E_R = \pi^2 \hbar^2 / 2 m a L^2 \). Solving the (dimensionless) single-particle Schrodinger equation, we find band energies \( \varepsilon_{nk} \) and the Bloch functions \( u_{nk}(r) \).

Expanding the fermion field operator in the Bloch basis as \( \psi(r) = \sum_{nk} \psi_{nk} u_{nk}(r) e^{i k r} \), where \( N \) is the total number of unit cells, and including the interactions, leads to the Hamiltonian

\[ H = \sum_{nk\sigma} \xi_{nk} \psi_{nk\sigma}^\dagger \psi_{nk\sigma} - w_S \int d^3r \psi_{\downarrow}\dagger(r) \psi_{\downarrow}(r) \psi_{\uparrow}(r) \psi_{\uparrow}\dagger(r). \]

(2)

Here \( \xi_{nk} = \varepsilon_{nk} - \mu \) (\( \mu \) is the chemical potential), \( w_S = g_S / (b_L^2 E_R) \) is the (dimensionless) contact interaction which can be tuned with a magnetic field, and \( g_S \) determines the s-wave scattering length, \( a_S \), via

\[ \frac{m_a}{4\pi \hbar^2 a_S} = -\frac{1}{g_S(\Lambda)} + \int_0^\Lambda d^3Q \frac{m_a}{(2\pi)^3 \hbar^2 Q^2}. \]

(3)

As indicated, \( g_S \) must depend on the ultraviolet cutoff \( \Lambda \) to recover the measured (low energy) scattering length \( a_S \). We ensure that \( \Lambda \) is large enough for physical properties to have converged to a cutoff-independent value in our numerical calculations. An important point to note is that \( g_S \) (and hence \( w_S \)) scales as \( \sim 1 / \Lambda \) when \( \Lambda \to \infty \).

To proceed with the many-body problem, we first follow the standard Bogoliubov-de Gennes method \[10\] and introduce mean fields \( \rho(r) = \sum_{n}\langle \psi_{n\uparrow}(r) \psi_{n\downarrow}(r) \rangle \) and \( \Delta(r) = w_S \langle \psi_{\downarrow}(r) \psi_{\uparrow}(r) \rangle \), where the expectation values are evaluated in the mean-field ground state which is to be determined self-consistently. Previous work \[4, 5\] ignored the Hartree mean field \( \rho(r) \); we first show that this is justified in the cold atom superfluid despite the inhomogeneous density induced by the optical lattice. In order to see this, we note that the inhomogeneous Hartree shift is simply a renormalization of the optical lattice potential as \( V_{eff}(r) = V_L(r) - w_S \rho(r) / 2 \). In the limit that the cutoff \( \Lambda \to \infty \), \( w_S \sim 1 / \Lambda \), while the density \( \rho(r) \) is certainly finite everywhere. This means that \( V_{eff}(r) = V_L(r) \) and that the Hartree terms play no role in the limit \( \Lambda \to \infty \). We will later see that this same argument fails for the pairing potential \( \Delta(r) \) since \( \langle \psi_{\downarrow}(r) \psi_{\uparrow}(r) \rangle \) is in fact ultraviolet divergent unlike \( \rho(r) \); this divergence is precisely compensated by \( w_S \sim 1 / \Lambda \).

Let us proceed by introducing Fourier modes for the SF order parameter modulations as \( \Delta(r) = \sum_{g} \Delta(g) e^{i g \cdot r} \), as well as for the Bloch functions, \( u_{nk}(r) = \sum_{g} u_{nk}(g) e^{i g \cdot r} \), where \( g \) are reciprocal lattice vectors. \( u_{mk}(g) \) can be chosen to be real due to time reversal symmetry. We can then define matrix elements

\[ \Delta_{nn'k} \equiv \sum_{g} \Delta(g) F_{nn'k}(g), \]

(4)

where \( F_{nn'k}(g) = \sum_{g'} u_{nk}(g + g') u_{nk'}(g') \). The pairing matrix element \( \Delta_{nn'k} \) corresponds to singlet pairing of fermions at momenta \( \mathbf{k}, -\mathbf{k} \). This leads to intraband pairing for \( n = n' \) and interband pairing for \( n \neq n' \). Both types of pairing matrix elements are generally nonzero. A mean field decoupling of the interaction term in \( H \) leads to

\[ H_{mf} = \sum_{nk\sigma} \xi_{nk} \psi_{nk\sigma}^\dagger \psi_{nk\sigma} - \sum_{n'n'k} \Delta_{nn'k} \psi_{n'n'k_{n'}\uparrow}^\dagger \psi_{nk_{n'}\downarrow}^\dagger \psi_{nk_{n'}\downarrow} \psi_{n'n'k_{n'}\uparrow} \left[ \Delta_{nn'k}^* + \Delta_{nn'k}^* \right]. \]

(5)

The self-consistent mean field state can be obtained iteratively, by diagonalizing \( H_{mf} \) in Eq. (5) and using those eigenstates to evaluate the pairing field matrix elements in Eq. (5) which yields the new \( H_{mf} \). This is an extremely computationally demanding task due to a large
number of bands ($\gtrsim 100$) one needs to keep in order to converge to cutoff-independent results at unitarity. However, if one is interested in the critical lattice depth, $V_L^{(\text{crit})}$, at which the SF to BI transition occurs, the computations can be significantly simplified by studying the linearized Bogoliubov-de Gennes equations,

$$\Delta(g) = \sum_{g'} K(g, g') \Delta(g'),$$

where the pairing kernel is given by

$$K(g, g') = \frac{w_S}{N} \sum_{nn'k} \left[ 1 - n_F(\xi_{nk}) - n_F(\xi_{n'k}) \right] \xi_{nk} + \xi_{n'k} \times F_{nn'k}(g) F_{n'n'k}(g').$$

Here $n_F$ is the Fermi distribution function. (Ref.4 incorrectly assumed that this pairing kernel was diagonal in $g$.) One can calculate $V_L^{(\text{crit})}$ by diagonalizing the matrix $K(g, g') - K(g, g')$, as was done in Ref.5, and finding the value of $V_L$ at which its lowest eigenvalue becomes negative. This was shown 2 to lead to $V_L^{(\text{crit})} \approx 45 E_R$ which is more than an order of magnitude larger than the experimental estimate $V_L^{(\text{crit})} \approx 3 E_R$.

The reason for this enormous discrepancy can be understood if we look at the real-space structure of $\Delta(r)$, corresponding to the lowest eigenvalue of the pairing kernel $K(g, g')$ near $V_L^{(\text{crit})}$, by Fourier transforming the corresponding eigenvector. We find that $\Delta(r)$ near $V_L^{(\text{crit})}$ is very strongly modulated; it is large near the center of each well of the optical lattice but is nearly zero in the region between adjacent wells. For a filling of two particles per well ($\bar{\rho} = 2$), this means that $\Delta(r)$ varies rapidly on the scale of the interparticle spacing $\sim 1/k_F \sim b_L$ which is comparable to the coherence length at unitarity. Amplitude fluctuations of the order parameter will therefore be very important for such rapidly varying components of $\Delta(r)$ and will tend to suppress such fast modulations. Since interband pairing matrix elements only arise from $\Delta(g \neq 0)$, this also means that the amplitude fluctuations will predominantly suppress interband pairing amplitudes. In short, a mean-field treatment of interband pairing is internally inconsistent at the atom density of two atoms per well: it predicts a significant variation of the pairing mean field on short length scales, where amplitude fluctuations are expected to be significant. We do expect such a mean field theory to provide a better starting point in the case where there is a large number of atoms in each well so that amplitude fluctuations on the scale of $b_L$ can be expected to be small. In this case, the dominant fluctuations will be quantum phase fluctuations which can drive a SF-BI transition analogous to the superconductor-insulator transition in Josephson junction arrays 14. We will confine our attention here to a density of two atoms per unit cell.

In contrast to interband pairing, the intraband pairing is stronger when the mean-field potentials are more homogeneous, as seen from Eq. 4. Thus one can obtain a self-consistent mean-field description of the SF-BI transition in our system by just retaining intraband matrix elements in Eq. 4. This corresponds to Anderson’s idea of pairing time-reversed eigenstates 3. This is known to be an excellent approximation in the context of the ordinary solid-state-based inhomogeneous, in particularly disordered, superconductors 4 and in metallic nanoparticles 4. In this case pairing between non-time-reversed eigenstates is suppressed by the randomness of the corresponding matrix elements. In our case the suppression is dynamical leading, however, to the same final result.

Under this intraband approximation, the mean-field Hamiltonian is obtained by setting $\Delta_{nn'k} = \Delta_{nk} \delta_{nn'}$ in Eq. 5. Defining the Bogoliubov quasiparticle dispersion as $\mathcal{E}_{nk} = \sqrt{\xi_{nk}^2 + \Delta_{nk}^2}$, we are led to the mean-field equations

$$\Delta(g) = \frac{w_S}{N} \sum_{nk} \frac{\mathcal{E}_{nk}}{2 \mathcal{E}_{nk} \tanh \left( \frac{\mathcal{E}_{nk}}{2T} \right)} F_{nnk}(g),$$

$$\bar{\rho} = \frac{1}{N} \sum_{nk} \left[ 1 - \frac{\xi_{nk}}{\mathcal{E}_{nk} \tanh \left( \frac{\mathcal{E}_{nk}}{2T} \right)} \right],$$

where $\bar{\rho} = 2$ is the atom density per well. The pairing matrix elements are in turn determined via $\Delta_{nk} = \sum_{g} \Delta(g) F_{nnk}(g)$. In order to solve these equations we need to restrict the number of bands. This is done by assuming $|g| < 2\pi \Lambda$, where $\Lambda$ is the cutoff in Eq. 3.

### III. RESULTS

We now turn to some of the key results obtained from solving the intraband mean-field equations that we motivated in the previous section. We begin by pointing out some general qualitative ways in which cold atom superfluids differ from ordinary solid state systems although the mean field gap and number equations appear to be similar for these two systems. We then turn to results obtained from numerically solving the mean field equations for some experimentally relevant observables.

#### A. What is special about atomic Fermi superfluids?

We begin by pointing out an interesting and unexpected consequence of the intraband pairing ansatz — namely, the pairing field in real space is completely uniform when we take the limit $\Lambda \to \infty$ as we should. This can be seen by inspection of the expression for the Fourier component of the pairing field at wavevector $g$ in Eq. 8. As seen from Eq. (3), the coupling constant $w_S$ vanishes as $\sim 1/\Lambda$ as the cutoff is taken to infinity. This vanishing of the coupling constant is cancelled by the expansion of phase space for pairing, i.e. the number of bands at high energy which contribute to the sum in Eq. 8, provided the matrix element $F_{nnk}(g)$ allows it. However, high energy ($\xi_{nk} \gg V_L$) states are simply free-particle
states for which $F_{nnk}(g) \approx \delta_{k,0}$. This means that only $\Delta(g = 0)$ component of the pairing field survives in the limit $\Lambda \to \infty$. Note that this result is independent of the form of the lattice potential and can be expected to hold, for example, in the case of the random optical lattice potential, which can be created by laser speckle \[12\]. This has potentially interesting implications for the disorder-driven SF to insulator transition in cold atom SFs. Also note that these observations apply only to the cold atom SFs; in conventional superconductors, the finite cutoff (at the Debye frequency in conventional superconductors, or at the lattice cutoff in model tight binding Hamiltonians) does allow for spatial variations of $\Delta(r)$ within the approximation of pairing time-reversed eigenstates \[8\]. In terms of a Landau theory, if we have a uniform superfluid order parameter and a periodic density modulation, then a symmetry allowed term in the Landau functional such as $\int d^3r \rho(r)|\Delta(r)|^2$ will certainly lead to an order parameter modulation with the same period as the density modulation. Our results indicate that the coefficient of this term must vanish, in the intraband pairing approximation, as the cutoff $\Lambda \to \infty$. One final peculiarity of cold atom SFs is that the pairing field (and the pairing gap) is completely rigid, i.e., it does not depend on the band index $n$ or the momentum $k$, unlike in multiband solid state superconductors. To the extent that interband pairing is suppressed, as we have argued is the case for two atoms per well, our above observations imply that any attempt to model this multiband cold atom SF by imposing a finite band cutoff, as is customarily done in deriving effective tight-binding lattice models, will generally lead to incorrect results. We now turn to results obtained from a numerical solution of the intraband mean-field equations for observables of experimental interest.

B. Critical lattice depth for the SF-BI transition

We begin by investigating the critical lattice depth for the SF-BI transition at atom density $\bar{\rho} = 2$. For fermions with unitary scattering, the SF order parameter vanishes at $V_{L}^{\text{crit}} \approx 4E_{R}$, close to the value given in Ref. \[4\] obtained by simply assuming, in effect, that the order parameter is uniform. The results and arguments presented in this paper justify such an assumption. As we already mentioned, experimentally it appears that $V_{L}^{\text{crit}} \approx 3E_{R}$ (note that Chin et al. \[3\] quote their results in units of the molecular recoil $E_{R}/2$). Part of the reason for the remaining relatively mild discrepancy with the experiment might be quantum and thermal fluctuations beyond mean-field theory, which are known to be somewhat important at unitarity even in the absence of the lattice \[13\]. One must also keep in mind, however, that there is no good experimental estimate of the temperature after the lattice is turned on \[3\].

C. Condensate density

It is important to realize that uniformity of the pairing field does not imply uniformity of the condensate density, which is what was measured in the experiment of Ref. \[3\]. The condensate density is obtained from the two particle density matrix \[14\]. In a lattice environment, this condensate density has Fourier components

$$n_{c}(g) = \frac{1}{N} \sum_{nnk} F_{nnk}(g) \left| \langle \psi^{\dagger}_{nk} \psi_{n-k} \rangle \right|^2.$$  \hspace{1cm} (9)

The sum over the band index $n$ in Eq. (9) is well defined in the limit $\Lambda \to \infty$ and all Fourier components of $n_{c}$ are in general nonzero. As shown in Fig. 1, the uniform component $n_{c}(g = 0)$ decreases monotonically with increasing lattice depth, while the modulated components have a nonmonotonic dependence on $V_{L}$ as observed in the experiment. The ratio of the leading modulated component to the uniform condensate density is in a good agreement with the data.

D. Quasiparticle gap

The SF state as well as the insulating state have a gap to quasiparticle excitations. On the SF side, the quasiparticles are Bogoliubov quasiparticles while the quasiparticles in the insulating phase correspond to the original fermions. As shown in Fig. 2, the minimum excitation gap, $2E_{g}$, in the SF phase decreases with $V_{L}$ (tracking $2\Delta(0)$ over a wide range of lattice depths) whereas it increases with $V_{L}$ on the insulating side. This leads to a minimum in the excitation gap for lattice depths close to but below $V_{L}^{\text{crit}}$. This nonmonotonic variation of the gap, shown in Fig. 2, could be tested in experiment. Note that there is a small window of lattice depths, close to but below $V_{L}^{\text{crit}}$, where the quasiparticle gap is slightly less than the gap of the underlying BI. This arises be-
IV. CONCLUSIONS

In this paper, we have discussed the multiband SF phase of strongly interacting fermions in an optical lattice. Our results for the critical lattice depth of the SF-BI transition and the dependence of the Fourier components of the condensate density on the lattice depth are in a good agreement with experimental data of Ref. [3]. Our prediction for the quasiparticle gap in the SF and insulating phases could be tested in future experiments. We have also obtained new results of general relevance to cold atom SFs. In particular, the uniformity of the pairing amplitude under time-reversed-eigenstate pairing conditions is an interesting and surprising result which can be expected to have a strong effect, for example, on the nature of the disorder-driven SF to insulator transition in cold atom SFs. This will be explored in future work.

Other directions for future research include a study of quantum fluctuations in the multiband SF, dynamical instabilities of such fermion SFs [15], and the study of SF-insulator transitions in other lattice geometries [16].

Acknowledgments: We thank V. Galitski, H.R. Krishnamurthy, H. Moritz, P. Nikolic and M. Randeria for useful comments and discussions. We acknowledge support from the Sloan Foundation (A.P.), the Connaught Fund (A.P.), and NSERC of Canada (A.A.B., A.P.).

[1] C. A. Regal, M. Greiner, and D. Jin, Phys. Rev. Lett. 92, 040403 (2004); C. Chin, M. Bartenstein, A. Altmeyer, S. Reidl, S. Jochim, J. H. Denschlag, and R. Grimm, Science 305, 1128 (2004); T. Bourdel, L. Khaykovich, J. Cubizolles, J. Zhang, F. Chevy, M. Teichmann, L. Tarruell, S. J. Kokkelmans, and C. Salomon, Phys. Rev. Lett. 93, 050401 (2004); J. Kinast, A. Turlapov, J. E. Thomas, Q. Chen, J. Stajic, and K. Levin, Science 307, 1296 (2005); G. B. Partridge, K. E. Strecker, R. I. Kamar, M. W. Jack and R. G. Hulet, Phys. Rev. Lett. 95, 020404 (2005); M. Köhl, H. Moritz, T. Stöferle, K. Günter, and T. Esslinger, Phys. Rev. Lett. 94, 080403 (2005); M. W. Zwierlein, C. H. Schunck, A. Schirotzek, and W. Ketterle, Nature 442, 54 (2006).

[2] R. Sensarma, M. Randeria, and T.-L. Ho, Phys. Rev. Lett. 96, 090403 (2006); H. Hu, X.-J. Liu, and P. Drummond, Europhys. Lett. 74, 574 (2006); Y. Nishida and D. T. Son, Phys. Rev. Lett. 97, 050403 (2006); P. Nikolic and S. Sachdev, Phys. Rev. A 75, 033608 (2007); M. Y. Veillette, D. E. Sheehy, and L. Radzihovsky, Phys. Rev. A 75, 043614 (2007); R. Haussman, W. Rantl, S. Cerri and W. Zwierlein, Phys. Rev. A 75, 023610 (2007).

[3] J. K. Chin, D. E. Miller, Y. Liu, C. Stan, W. Setiawan, C. Sanner, K. Xu, and W. Ketterle, Nature 443, 961 (2006).

[4] H. Zhai and T.-L. Ho, Phys. Rev. Lett. 99, 100402 (2007).

[5] E.-G. Moon, P. Nikolic, and S. Sachdev, Phys. Rev. Lett. 99, 230403 (2007).

[6] P. Nozieres and F. Pistolesi, Eur. Phys. J. B 10, 649 (1999).

[7] P. W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).

[8] See e.g. A. Ghosal, M. Randeria, and N. Trivedi, Phys. Rev. B 65, 014501 (2001) and references therein.

[9] F. Braun and J. von Delft, Phys. Rev. B 59, 9527 (1999).

[10] P.G. de Gennes, *Superconductivity of Metals and Alloys* (Perseus Books, 1999).

[11] R. Fazio and H. van der Zant, Phys. Rep. 355, 235 (2001).

[12] J.E. Lye, L. Fallani, M. Modugno, D.S. Wiersma, C. Fort, and M. Inguscio, Phys. Rev. Lett. 95, 070401 (2005).

[13] R. Diener, R. Sensarma, and M. Randeria, Phys. Rev. A 77, 023626 (2008).

[14] W. Ketterle and M. W. Zwierlein, *arXiv:0801.2500* (unpublished).

[15] A. A. Burkov and A. Paramekanti, Phys. Rev. Lett. 100, 255301 (2008).

[16] E. Zhao and A. Paramekanti, Phys. Rev. Lett. 97, 230404 (2006).