EQUILIBRIUM STRATEGIES IN A SUPPLY CHAIN WITH CAPITAL CONSTRAINED SUPPLIERS: THE IMPACT OF EXTERNAL FINANCING

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Abstract. In this study, we consider a two-echelon supply chain, where two capital constrained suppliers compete to sell their products through a common retailer. The retailer may provide advance payment to one or two suppliers. We show that whether the retailer considers merging with only one supplier depends upon the revenue sharing ratio and the additional administrative costs of the revenue sharing contract. Meanwhile, the supplier who drops out of the market may adopt a hybrid financing scheme by combining bank credit with equity financing to return to the market. We find that the deselected supplier can be allowed to participate in the market when the bank loans ratio is below a certain threshold. We further investigate the impact of the bank loans ratio and competition intensity on the players’ decisions and profits. In addition, we find that there exists an optimal bank loans ratio for the deselected supplier. Specifically, it is optimal for the deselected supplier to adopt pure bank credit if the production cost is sufficiently low.

1. Introduction. The financial constraints of suppliers can strongly affect supply chain performance, especially for small and medium-sized enterprises (SMEs). Due to their lack of collateral and credit history, they tend to find it difficult to obtain adequate financial supports from banks (see [10, 39, 40, 50]). In response to the suppliers’ financial pressure, many retailers such as Wal-Mart and Carrefour sometimes

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prepay the entire or a fraction of the purchasing costs to their upstream suppliers (i.e., advance payment). Moreover, as the largest online retailer in China, at the end of 2013, JD launched its first financing service for its capital-constrained suppliers. By 2015, the loans had exceeded 4 billion and served more than 2000 SMEs (see [47]). Prepayments from the buyers help the sellers avoid order cancellations and support their production. To induce the buyers to prepay, the suppliers commonly offer them a price discount. As a short-term financing scheme, advance payment has become one of the most popular financing sources for SMEs in today’s business transactions (see [28, 29, 30, 38, 49, 50]).

Advance payment is one of the most secure and riskless strategies for vendors while is the least attractive one for buyers (see [37]). In addition, in a competitive supply chain, each participant may be worse off when the competition is intensified (see [41, 45]). Therefore, when the upstream suppliers face capital constraints, a large-sized and powerful retailer may strategically provide financial services to only a portion of its suppliers and merge with them, thereby controlling the cash flow risks and decreasing costs. Mergers widely exist in today’s business transactions and are generally divided into two categories: (1) vertical strategy, i.e., merging with upstream or downstream enterprises and (2) horizontal strategy, i.e., merging with competitors. Both merger strategies are likely to improve the total profits of the merging enterprises. For example, when the retailer merges with one of its upstream suppliers, they unite as one entity. That is, the supplier and the retailer are fully integrated to maximize the total profit of the supply chain (see [26, 44]). Notice that when the suppliers have no access to financial supports from the downstream retailer, they may drop out of the market. However, according to Yang et al [44], the capital constrained enterprises have opportunities to join the supply chain by utilizing external financing. Brander and Lewis [4] show that enterprises with greater debt tend to be more aggressive and such enterprises have strategic advantages in the competitive market.

Many studies have showed that the merger of a supplier and a retailer can make both better off (see [44]). However, mergers do not always achieve the expected goals. For example, the merger of Lenovo and Google’s Motorola Mobility business led to a loss of 469 million dollars in the 2015-2016 financial year (see [26]). One of the main reasons is the operating costs after the merger. Wholesale price contracts are widely used in academic and practical research. According to Cachon [7], a wholesale price contract generally cannot coordinate a supply chain. However, it is commonly observed in practice. One of the main reasons is the operating cost. For example, wholesale price contracts are easy to administer. Therefore, a supplier may prefer a wholesale price contract to a coordinating contract if the additional administrative fee associated with the coordinating contract exceeds the supplier’s potential benefit. Motivated by the above cases, we address the following questions in this paper. (i) In a competitive market with two capital constrained suppliers and a common retailer, what are the main factors that influence the retailer’s incentive to merge? (ii) In terms of external financing, what is the impact of external financing on the players’ optimal decisions and profits? (iii) Can the deselected supplier return to the market by adopting external financing? To answer these questions, in this paper, we consider a competitive market with two capital constrained suppliers and a common retailer. The retailer can provide advance payment to both suppliers. Some research has shown that each player’s profit decrease as the competitive intensity increases (see [15, 44]). To avoid the double marginalization effect and
control the cash flow risks, the retailer may provide advance payment to only one supplier and merge with this supplier. The supplier without financial support will drop out of the market. However, the deselected supplier may resort to external financing.

Our study is distinct in that it considers a competitive supply chain where the upstream suppliers are capital constrained. The downstream retailer provides advance payment to one or two suppliers. The supplier without financial support may adopt external financing to enter the market. With this setup, we consider three different strategies: in Strategy AA, the retailer provides advance payment to both suppliers; in Strategy A, the retailer provides advance payment to only one supplier and the other supplier without financial support drops out of the market; and in Strategy AE, the retailer provides advance payment to one supplier while the abandoned supplier enters the market by using external financing including bank credit and equity financing. Our work contributes to the existing literature in three aspects. First, this study is among the first to study advance payment and external financing in a competitive supply chain with capital constrained suppliers. Second, we incorporate equity financing into our model and analyse how the impact of the bank loans ratio on the players’ decisions and profits. We find that there is an optimal bank loans ratio for the deselected supplier. Third, by comparing the equilibrium results of the three different strategies, we derive the key conditions for equilibrium strategy evolution. We show that the retailer can be better off when it merges with one of the suppliers under some circumstances. In addition, when the bank loans ratio is below a certain threshold, the entrance of the deselected supplier can break the merger between the retailer and its rival.

The rest of this paper is organized as follows. Section 2 reviews the literature. Section 3 formulates the model. Section 4 analyses the players’ optimal decisions in three strategies and examines how the bank loans ratio and competition intensity affect the players’ decisions and profits. Section 5 concludes and discusses the management insights. All proofs are presented in the Appendix.

2. Literature review. Our work incorporates supply chain financing with supply chain operations and is closely related to three aspects of research: (i) supplier competition, (ii) supply chain financing, and (iii) supply chain coordination.

2.1. Supplier competition. The first stream of the literature is on supplier competition. Supplier competition has been addressed in a variety of studies (see [6, 18, 19, 20, 46]). For example, Babich et al [2] investigate how the correlations of the supplier default risks affect the suppliers’ wholesale prices. According to Cachon and Gürhan Kök [8], under a quantity-discount contract and a two-part tariff contract, the manufacturers compete more aggressively than under a wholesale price contract. Sinha and Sarmah [35] study the channel coordination issues under three different cases, including price competition without channel coordination, price competition with channel coordination and global coordination. Some scholars also study a competitive supply chain consisting of two suppliers and a single retailer by taking into account suppliers’ prices and reliability differences (see [34]). Recently, Yang et al [47] study the optimal decisions in a supply chain consisting of an incumbent manufacturer, a capital-constrained new entering manufacturer and a common retailer.
2.2. Supply chain financing. Many scholars have incorporated supply chain financing into operational decisions (see [11, 22, 27, 36, 42]). In the literature on supply chain finance, bank credit financing (BCF) and trade credit financing (TCF) have been frequently addressed. For example, Buzacott and Zhang [5] find that the asset-based financing enables a capital constrained retailer to earn more returns. Some scholars point out that the retailer’s internal capital level has a significant influence on the participants’ optimal decisions under bank credit (see [14]). Katehakis et al. [23] propose an optimal ordering policy to maximize the capital-constrained firm’s profits under bank credit. Alan and Gaur [1] examine the bank’s optimal decisions using the asset-based lending that is given to SMEs. Further, a borrower’s investment decision can be affected by demand information asymmetry and his/her bankruptcy costs. Meanwhile, capital constrained enterprises also fund their businesses using trade credit (see [12, 13, 33]). Gupta and Wang [17] present a discrete time model of a retailer’s operations with stochastic demand. It finds that the structure of the optimal policy is not affected by the credit terms. Zhou et al. [51] derive the conditions of a retailer choosing the optimal financing channel based on his initial capital level and the interest rates of the financing services. Chod et al. [13] examine the impacts of competition among supply chain members on the supply chain’s decisions under trade credit. Finally, some scholars also focus on the trade-off between bank credit and trade credit (see [9, 11, 22, 24, 25, 48]).

In terms of advance payment, Zhang et al. [49] show that advance payment can help a supplier to avoid the risk of buyers cancelling orders. Taleizadeh [38] studies an inventory model under advance payment when the retailer may face disruptions. Tavakoli and Taleizadeh [37] consider a buyer’s inventory control system for a decaying item under a full prepayment scheme based on three different cases, including no shortage, full prepayment with deterioration and shortage, and full advanced payment with partial backordering. Zhao and Huchzermeier [50] consider two pre-shipment financing schemes: the advance payment discount (APD) and buyer-backed purchase order financing (BPOF). It shows that the retailer prefers the APD to BPOF if her internal asset level is beyond a certain threshold.

In addition to single financing schemes, some studies have also been extended to hybrid financing schemes (see [42, 48]). Yang et al. [44] show that the deselected retailer can return to the market by utilizing a hybrid financing scheme including bank credit and equity financing. Jin et al. [21] study a two-echelon supply chain in which both players are capital constrained. Three different financing schemes including bank credit only, bank credit with trade credit and bank credit with the supplier’s credit guarantee are considered. According to Yan et al. [43], it is optimal for the supplier to offer a financing portfolio instead of pure supplier financing and pure supplier investment. Recently, Li et al. [30] study a dual-channel supply chain with a capital-constrained manufacturer. Three financing schemes, namely, trade credit, bank credit, and hybrid financing (including equity financing and bank credit), are utilized to alleviate the manufacturer’s capital pressure.

2.3. Supply chain coordination. In a supply chain context, the wholesale price contract is nearly always found to be inefficient and more complicated contracts can be utilized to overcome the double marginalization effect (see [8, 32]). Numerous studies also concentrate on the supply chain coordination with a capital-constrained participant (see [16, 27, 31]). To be more specific, Dada and Hu [14] indicate that the supply chain can partially achieve coordination by designing an effective mechanism under BCF. Chen [11] shows that with a wholesale price contract, supply
chain coordination cannot be achieved with either BCF or TCF. A revenue sharing contract can fully coordinate the supply chain. Yan et al. [42] combine TCF with BCF and find that the supply chain can be coordinated when setting a suitable credit guarantee coefficient.

Notably, the above literature ignores the extra additional administrative costs of these coordinating contracts. In practice, the operating costs of the coordinating contract are important factors that influence players’ decisions. Therefore, in this paper, we extend the extant literature by considering the situation that it will incur extra costs for all players when conducting a coordinating contract.

Our work is most related to Yang et al. [44], but there are also some significant differences. First, Yang et al. [44] consider a two-echelon supply chain with a supplier and two capital constrained retailers. However, in this paper, we consider a competitive supply chain with two capital constrained suppliers and a single retailer. Second, Yang et al. [44] concentrate on how the competitive intensity and revenue sharing ratio affect a merger and supply chain coordination. They find that when the competitive intensity is large, both the supplier and the first retailer are willing to merge as long as the revenue sharing ratio is in the Pareto zone. By contrast, we examine the impacts of revenue sharing ratio and additional administrative costs of the revenue sharing contract on the merger. We observe that when the revenue sharing ratio is in an appropriate zone, as long as the players’ additional administrative costs are relatively low, both the retailer and the selected supplier are willing to merge and realize a win-win result.

3. Model setup. We consider a two-echelon supply chain with two capital constrained suppliers (denoted by $S_1$ and $S_2$) and a capital sufficient retailer (denoted by $R$). The two suppliers compete to sell their products through a common retailer. The suppliers and retailer play a Stackelberg game, where the suppliers move first and determine the wholesale prices. Given the wholesale prices charged by the suppliers, the retailer acts as the follower and chooses the order quantities. Then, the retailer sets the retail prices and sales take place. We consider the Cournot competition and the inverse demand function is given by $p_i = 1 - q_i - bq_{i-1}^3, i = 1, 2$, where $0 \leq b \leq 1$ denotes the competitive intensity between the two products. In particular, the products are perfectly substitutable if $b = 1$ and are independent if $b = 0$. A similar assumption can be seen in Baron and Berman [3] and Yang et al. [44].

Assume that the retailer is the unique buyer for two suppliers. The retailer may strategically provide advance payment to one or two suppliers. If the retailer offers advance payment to only one supplier, the supplier without financial support will drop out of the supply chain. However, the abandoned supplier may seek external financing to return to the market. Therefore, we consider the following three different strategies.

Strategy AA: The retailer provides advance payment to both suppliers. The suppliers first simultaneously announce the wholesale prices $w_{1A}^A$ and $w_{2A}^A$ to the retailer. Given the wholesale prices $w_{1A}^A$ and $w_{2A}^A$, the retailer determines the order quantities $q_{1A}^A$ and $q_{2A}^A$. Then, the retailer provides advance payment to both suppliers simultaneously to meet their production. After receiving the products, the retailer sells the products to the market at the retail prices $p_{1A}^A$ and $p_{2A}^A$.

Strategy A: Without loss of generality, we assume that the retailer only provides advance payment to $S_1$. Therefore, the supplier $S_2$ drops out of the market. In
this case, the retailer and $S_1$ implement a revenue sharing contract to coordinate the supply chain. Meanwhile, it will incur an extra operating cost for each player in contrast to the traditional wholesale price contract. As discussed in Section 1, if the additional administrative fee associated with the coordinating contract exceeds a player’s potential benefit, the player may prefer the wholesale price contract to a coordinating one.

Strategy $AE$: The supplier $S_2$ seeks external financing including bank credit and equity financing to enter the market. That is, $S_2$ borrows from a bank to settle part of its capital pressure. Assume that the bank loans ratio is $\theta$, where $\theta \in (0, 1)$. The remaining financing comes from investors by transferring a certain proportion of its shares.

Fig. 1 illustrates the evolution process for the three strategies. For clear interpretation, we list the notations in Table 1.

![Figure 1. The evolution process of the three different strategies](image)

In our model, all information is common knowledge. The suppliers produce at a constant and identical marginal cost. To avoid trivial cases, we assume $p_i^j > w_i^j(1 + r_f)$ and $aw_i^j > c$ (see [24, 37, 49]).

4. **Equilibrium analysis among three strategies.** In this section, we concentrate on the players' optimal decisions for each strategy. As Stackelberg leaders,
Table 1. Notations and explanations

| Notation | Explanation |
|----------|-------------|
| $b$      | Competitive intensity of the products, where $0 \leq b \leq 1$. |
| $c$      | Unit production cost. |
| $\theta$ | Bank loans ratio, where $0 < \theta < 1$. |
| $r_f$    | Interest rate of bank loans. |
| $\lambda$ | Revenue sharing ratio. |
| $p_j^i$  | Retailer’s retail price for product $i$ in Strategy $j$, where $i = 1$ and 2, and $j = AA, A$ and $AE$. |
| $w_j^i$  | Wholesale price charged by $S_i$ in Strategy $j$. |
| $q_j^i$  | Retailer’s order quantity from $S_i$ in Strategy $j$. |
| $k_1$    | Retailer’s additional administrative costs associated with the revenue sharing contract in Strategy $A$. |
| $k_2$    | Supplier $S_1$’s additional administrative costs associated with the revenue sharing contract in Strategy $A$. |
| $\pi_R^j$ | Retailer’s profit in Strategy $j$. |
| $\pi_{S_1}^j$ | Supplier $S_1$’s profit in Strategy $j$. |

The suppliers first determine the wholesale prices, and then the retailer chooses its order quantities. According to the backward induction approach, we first examine the retailer’s optimal order quantities and then focus on the suppliers’ decisions.

4.1. Strategy $AA$: The retailer provides advance payment to both suppliers. In this strategy, the retailer provides advance payment to both suppliers. Under the wholesale price contract, The suppliers first announce the wholesale prices $w_1^{AA}$ and $w_2^{AA}$, and then the retailer chooses the order quantities $q_1^{AA}$ and $q_2^{AA}$. The retailer pays $aw_1^{AA}q_1^{AA}$ and $aw_2^{AA}q_2^{AA}$ to the suppliers for production, where $a$ denotes the price discount from two suppliers. After receiving the products, the retailer determines the retail prices $p_1^{AA}$ and $p_2^{AA}$. The retailer’s profit function is as follows:

$$
\pi_R^{AA} = p_1^{AA}q_1^{AA} + p_2^{AA}q_2^{AA} - aw_1^{AA}q_1^{AA}(1 + r_f) - aw_2^{AA}q_2^{AA}(1 + r_f),
$$

(1)

At the beginning of the sales season, $S_1$ receives $aw_1^{AA}q_1^{AA}$ from the retailer. Meanwhile, $S_1$’s production cost is $cq_1^{AA}$. When time value is considered, at the end of the sales season, $S_1$’s profit function is as follows:

$$
\pi_{S_1}^{AA} = (aw_1^{AA}q_1^{AA} - cq_1^{AA})(1 + r_f), i = 1, 2.
$$

(2)

After solving the above problems, we have the following results in Lemma 1.

Lemma 1. In the strategy in which the retailer provides advance payment to both suppliers, the equilibrium wholesale prices are $w_1^{AA*} = w_2^{AA*} = \frac{1 + c + cr_f - b}{a(2 - b)(1 + r_f)}$, and the order quantities are $q_1^{AA*} = q_2^{AA*} = \frac{1 - c - cr_f}{2(1 + b)(2 - b)}$.

According to Lemma 1, due to symmetry, two suppliers announce equal wholesale prices and the retailer orders the same quantity from two suppliers. Therefore, two suppliers’ profits are $\pi_{S_1}^{AA*} = \pi_{S_2}^{AA*} = \frac{(1 - b)(1 - c - cr_f)^2}{2(1 + b)(2 - b)^2}$ and the retailer’s profit is $\pi_R^{AA*} = \frac{(1 - c - cr_f)^2}{2(6 + b)(2 - b)^2}$. Next, we investigate how the competition intensity $b$ affects the players’ optimal decisions and profits.
Proposition 1. (i) \( \frac{\partial w^{AA^*}}{\partial b} < 0 \); (ii) If \( 0 \leq b \leq \frac{1}{2} \), then \( \frac{\partial q^{AA^*}}{\partial b} > 0 \); (iii) \( \frac{\partial \pi^{AA^*}}{\partial b} > 0 \); (iv) \( \frac{\partial \pi^{AA^*}}{\partial b} < 0 \).

Proposition 1 is in sharp contrast to the results of Yang et al. [44]. On the one hand, according to Yang et al. [44], the wholesale price is independent of \( b \) and the retailers' order quantities decrease with \( b \). However, Proposition 1 suggests that in Strategy AA the suppliers' wholesale prices decrease with \( b \), while the retailer's order quantities from two suppliers first decrease and then increase with \( b \). The reason is as follows. First, when the supplier sells through two competitive retailers, two retailers are equivalent to an aggregate retailer. Therefore, the competition intensity has no influence on the wholesale price and the order quantities decrease with \( b \). Second, a higher wholesale price will generally lead to a lower order quantity. However, according to Proposition 1, when two competitive suppliers sell through a common retailer, two suppliers cannot be regarded as an aggregate supplier. The suppliers will set high wholesale prices if the competition intensity is relatively weak (e.g., \( b \leq \frac{1}{2} \)). The retailer's extra profit from the increased order quantities will exceed the extra ordering cost. Therefore, the retailer will order less if charged higher wholesale prices. However, when the competition intensity is strong (e.g., \( b > \frac{1}{2} \)), the wholesale prices are relatively low. The extra profit from the increased order quantities can offset the extra ordering cost. Therefore, in this case, the retailer tends to improve its order quantities as \( b \) grows.

On the other hand, according to Yang et al. [44], the wholesale price is independent of \( b \) and the order quantities decrease with \( b \). Therefore, the profits of the supplier and two retailers decrease with \( b \). By contrast, Proposition 1 shows that the retailer's profit increases with \( b \) while the suppliers' profits decrease with \( b \). The reason is as follows. First, when the competition intensity is relatively weak, the wholesale prices and order quantities decrease with \( b \). Therefore, both suppliers' profits decrease with \( b \). For the retailer, the positive effect of the decreased wholesale price dominates the negative effect of the decreased order quantity on the retailer's profit as \( b \) increases. Therefore, the retailer's profit increases with \( b \). Second, when the competition intensity is relatively strong, the wholesale prices decrease with \( b \) but the order quantities increase with \( b \). In this case, the retailer will always benefit from an increase in \( b \). For each supplier, as \( b \) grows, the negative effect of the decreased wholesale price dominates the positive effect of the increased order quantity on each supplier's profit. Therefore, both suppliers' profits decrease with \( b \).

4.2. Strategy A: The retailer merges with \( S_1 \). In this strategy, the retailer only provides advance payment to supplier \( S_1 \) while \( S_2 \) without financial support will drop out of the market. We conduct a revenue sharing contract between two players. Under the revenue sharing contract, \( S_1 \) and the retailer are integrated to maximize the overall supply chain profit. The retailer determines its order quantity, which is equal to the optimal order quantity of the supply chain. We assume that the retailer's revenue sharing ratio is \( 0 \leq \lambda \leq 1 \), which depends on the retailer's negotiation power relative to \( S_1 \). As discussed in Section 3, under the revenue sharing contract, it will incur an extra operating cost for each player in contrast to the traditional wholesale price contract. The retailer's and \( S_1 \)'s profit functions in
Strategy $A$ are as follows:

$$\pi_R^A = \lambda p_A^A q_A^A - aw_A^A q_A^A (1 + r_f) - k_1,$$

$$\pi_{S_1}^A = (1 - \lambda)p_A^A q_A^A + (aw_A^A q_A^A - cq_A^A)(1 + r_f) - k_2,$$

where $k_1$ and $k_2$ represent the retailer’s and $S_1$’s extra operating costs, respectively. To make sure that each player can obtain a positive profit, we assume $0 < k_1 < \pi_R^A$ and $0 < k_2 < \pi_{S_1}^A$. After solving the above problems, we have the following results in Lemma 2.

**Lemma 2.** In the strategy in which the retailer only merges with $S_1$, the optimal wholesale price is $w_1^A = \frac{\lambda}{\alpha}$ and the optimal order quantity is $q_1^A = \frac{1 - c - c r_f}{2}$.

According to Lemma 2, we further obtain that the retailer’s profit is $\pi_R^{A*} = \frac{\lambda(1 - c - c r_f)^2}{4} - k_1$ and $S_1$’s profit is $\pi_{S_1}^{A*} = \frac{(1 - \lambda)(1 - c - c r_f)^2}{4} - k_2$. Under the revenue sharing contract, the total profit of the supply chain in Strategy $A$ is greater than the sum of the retailer’s and $S_1$’s profits in Strategy $A$ if and only if $k_1 + k_2 < \frac{b(1-b)(1-c-c r_f)^2}{4(b+1)(b-6)}$. Therefore, the merger may be achieved to make both the retailer and $S_1$ better off compared with Strategy $AA$. Next, we study the condition under which both the retailer and $S_1$ are willing to merge.

**Proposition 2.** (i) If $k_1 + k_2 \geq \frac{b(1-b)(1-c-c r_f)^2}{4(b+1)(b-6)}$, then the retailer and $S_1$ cannot merge.

(ii) If $k_1 + k_2 \leq \frac{b(1-b)(1-c-c r_f)^2}{4(b+1)(b-6)}$, then we get the following.

Subcase 1: When $(\lambda, k_1, k_2) \in A_1$, then the retailer and $S_1$ are willing to merge.

Subcase 2: When $(\lambda, k_1, k_2) \in A_2 \cup A_3$, then the retailer is not willing to merge with $S_1$.

Subcase 3: When $(\lambda, k_1, k_2) \in A_4 \cup A_5$, then $S_1$ is not willing to merge with the retailer.

Where $A_1 = (\lambda, k_1, k_2) | \lambda_1 < \lambda < \lambda_2, 0 < k_1 < \tilde{k}_1$ and $0 < k_2 < \tilde{k}_2$,

$A_2 = (\lambda, k_1, k_2) | \lambda_2 \leq \lambda \leq 1, 0 < k_1 < \tilde{k}_1$ and $0 < k_2 < \tilde{k}_2$,

$A_3 = (\lambda, k_1, k_2) | \lambda_1 < \lambda < \lambda_2, 0 < k_1 < \tilde{k}_1$ and $\tilde{k}_2 \leq k_2 < \tilde{k}_2$,

$A_4 = (\lambda, k_1, k_2) | 0 \leq \lambda \leq \lambda_1, 0 < k_1 < \tilde{k}_1$ and $0 < k_2 < \tilde{k}_2$,

$A_5 = (\lambda, k_1, k_2) | \lambda_1 < \lambda < \lambda_2, \tilde{k}_1 \leq k_1 < \tilde{k}_1$ and $0 < k_2 < \tilde{k}_2$.

According to Proposition 2, whether the merger and coordination can be achieved depends on the revenue sharing ratio and additional administrative costs, which differs from the results of Yang et al. [44]. According to Yang et al. [44], when the supplier sells through two competitive retailers, whether the supplier and the selected retailer are willing to merge is related to the competition intensity and revenue sharing ratio. Further, when the competition intensity is beyond a certain threshold, the total profit of the supply chain after merger is greater than the sum of the supplier’s and the selected retailer’s profits before the merger. In this case, the merger can occur as long as the revenue sharing ratio is in the Pareto zone. By contrast, when the suppliers sell through a common retailer, we observe that the retailer and $S_1$ cannot merge when $k_1 + k_2 \geq \frac{b(1-b)(1-c-c r_f)^2}{4(b+1)(b-6)}$. This is because the
sum of the retailer’s and S1’s profits in Strategy A is lower than that in Strategy AA. However, if $k_1 + k_2 < \frac{b(1-b)(1-c-cr_f)^2}{4(6+k_1+2-k_2)}$, both the retailer and S1 may implement a revenue sharing contract to achieve supply chain coordination. According to the proof of Proposition 2, when $k_1$ and $k_2$ are equal to 0, both the retailer and S1 may implement a revenue sharing contract to achieve supply chain coordination. Therefore, in this case, when both $k_1$ and $k_2$ are relatively low (i.e., $k_1 < \overline{k}_1$ and $k_2 < \overline{k}_2$), the revenue sharing contract can realize a win-win result.

Fig. 2 depicts the impact of the revenue sharing ratio on the merger. The main parameters are as follows: $c = 0.2$, $r_f = 0.05$, $a = 0.95$, $b = 0.6$, $k_1 = 0$ and $k_2 = 0$. We observe that a larger $\lambda$ can lead to a higher profit for the retailer. Further, when the total profit of the supply chain in Strategy A is greater than the sum of the retailer’s and S1’s profits in Strategy AA, both the retailer and S1 can be better off as long as $\lambda$ is in the Pareto zone $\lambda \in (0.56, 0.67)$.

**Figure 2. The Pareto Zone**

4.3. **Strategy AE**: S2 enters the supply chain with external financing. In this strategy, the retailer also provides advance payment to S1 while S2 adopts bank credit and equity financing to fund its business. Under the wholesale price contract, the suppliers first announce the wholesale prices $w_1^{AE}$ and $w_2^{AE}$, and then the retailer determines the order quantities $q_1^{AE}$ and $q_2^{AE}$. Given $w_1^{AE}$, $w_2^{AE}$, $q_1^{AE}$ and $q_2^{AE}$, S1 receives $aw_1^{AE}q_1^{AE}$ from the retailer for production. However, S2 first borrows $\theta w_2^{AE}q_2^{AE}$ from a bank and then finances $(1 - \theta)w_2^{AE}q_2^{AE}$ from investors. After receiving the products delivered by the suppliers, the retailer sells products to the market at retail prices $p_1^{AE}$ and $p_2^{AE}$. The retailer’s profit function in Strategy AE is as follows:

$$\pi_R^{AE} = p_1^{AE}q_1^{AE} + p_2^{AE}q_2^{AE} - aw_1^{AE}q_1^{AE}(1 + r_f) - w_2^{AE}q_2^{AE}(1 + r_f),$$

(5)

Similar to Strategy AA, S1’s profit function in Strategy AE is as follows:

$$\pi_S^{AE} = (aw_1^{AE}q_1^{AE} - cq_1^{AE})(1 + r_f),$$

(6)

For S2, at the end of the sales season, S2 pays $\theta cq_2^{AE}(1 + r_f)$ including the risk-free interest to the bank and then transfers a fraction $1 - \theta$ of its profit as a dividend to the investors. Therefore, S2’s profit function in Strategy AE is as follows:
and the equilibrium order quantities are 
\[ q_{S_1}^{AE} = \theta(w_2^{AE} q_2^{AE} (1 + r_f) - \theta c q_2^{AE} (1 + r_f)), \] (7)

After solving these problems, we derive the players’ optimal decisions in Lemma 3.

**Lemma 3.** In the strategy in which the retailer provides advance payment to \( S_1 \) and \( S_2 \) utilizes external financing to enter the market, the equilibrium wholesale prices are
\[ w_1^{AE} = \frac{2c_1 - b_1 - 2c r_f - b_1^2 + c_1 \theta^2 + b_1 c \theta + b_1^2 \theta_1 + 2}{a_1(2-b_1)(b_1+2)(1+r_f)}, \]
and the equilibrium order quantities are
\[ q_1^{AE} = \frac{2c_1 - b_1 - 2c r_f - b_1^2 + c_1 \theta^2 + b_1 c \theta + b_1^2 \theta_1 + 2}{2(2-b_1)(b_1+2)(1+r_f)}. \]

According to Lemma 3, we further derive the following players’ profits:
\[ \pi_{S_1}^{AE} = \frac{(b_1^2 c_1 - 2c_1 r_f - b_1^2 + c_1 \theta^2 + b_1 c \theta + b_1^2 \theta_1 + 2)^2}{2(1-b_1)(b_1+1)(b_1-2)(b_1+2)^2}, \]
\[ \pi_{S_2}^{AE} = \frac{\theta b_1 c_1 - 2c_1 r_f - 2c_1 \theta_1 + b_1 c \theta + b_1^2 \theta_1 + 2)^2}{2(1-b_1)(b_1+1)(b_1-2)(b_1+2)^2}, \]
\[ \pi_{R}^{AE} = \frac{4b_1^2 r_f^2 - 8c_1 \theta_1 r_f - 4c_1^2 + 8c_1 \theta_1 r_f + 8c_1 \theta_1 + 8c_1 r_f + 8c_1 - 8}{(1-b_1)(b_1+1)(b_1-2)(b_1+2)^2}. \]

Next, we investigate the impact of the bank loans ratio on the players’ optimal decisions in Strategy \( AE \), which is shown in Proposition 3.

**Proposition 3.** (i) \( \frac{\partial w_1^{AE}}{\partial \theta} > 0 \) and \( \frac{\partial w_2^{AE}}{\partial \theta} > 0 \); (ii) \( \frac{\partial q_1^{AE}}{\partial \theta} > 0 \) and \( \frac{\partial q_2^{AE}}{\partial \theta} < 0 \);

According to Proposition 3, as \( \theta \) increases, two suppliers’ wholesale prices increase, the retailer’s order quantity from \( S_1 \) increases and the order quantity from \( S_2 \) decreases. From Eq. (7), a higher value of \( \theta \) indicates that \( S_2 \)’s margin cost increases. In this case, as \( \theta \) grows, \( S_2 \) becomes less competitive in terms of costs. Therefore, \( S_2 \) will improve its wholesale price. Because of the strategic interaction, \( S_1 \) will announce a higher wholesale price to improve its profit. From the retailer’s perspective, the retailer has no cost advantages when ordering from \( S_2 \), and he will order less from \( S_2 \) as \( \theta \) grows. Because of the strategic interaction, the retailer orders more from \( S_1 \) as \( \theta \) increases. Next, we compare the players’ equilibrium wholesale prices and order quantities among the three strategies.

**Proposition 4.** (i) \( w_1^{AA} > w_1^{AE} > w_1^{AA} \) and \( w_2^{AA} > w_2^{AE} > w_2^{AA} \); (ii) \( q_1^{AA} > q_1^{AE} > q_1^{AA} \) and \( q_2^{AE} > q_2^{AA} \).

In Strategy \( A \), under the revenue sharing contract the retailer and \( S_1 \) are integrated to maximize the overall supply chain’s profit, which requires a wholesale price below the production cost. Therefore, \( S_1 \)’s wholesale price \( w_1^{AA} \) is the lowest among the three strategies. We find that each supplier’s wholesale price in Strategy \( AA \) is larger than that in Strategy \( AE \). This is because in Strategy \( AE \), both suppliers have cost advantages over Strategy \( AA \). Therefore, both suppliers will announce lower wholesale prices in Strategy \( AE \).

Similarly, in Strategy \( A \), two players implement a revenue sharing contract to achieve the supply chain coordination. Therefore, the retailer’s order quantity from \( S_1 \) is the largest among the three strategies. When \( S_2 \) adopts external financing to enter the market, \( S_2 \) becomes more competitive in terms of costs. Therefore,
comparing with Strategy \( AA \), the retailer will order more from \( S_2 \) in Strategy \( AE \). In contrast, the retailer will strategically order less from \( S_1 \) in Strategy \( AE \), i.e., \( q_1^{AA*} > q_1^{AE*} \).

Figs. 3 and 4 illustrate the impact of the bank loans ratio on the equilibrium wholesale prices and order quantities. The main parameters are as follows: \( c = 0.2 \), \( r_f = 0.05 \), \( a = 0.95 \), \( b = 0.6 \), \( \lambda = 0.7 \), \( k_1 = 0 \), \( k_2 = 0 \). Fig. 3 shows that in Strategy \( AE \), the wholesale prices increase with \( \theta \). Further, \( S_1 \)’s wholesale price in Strategy \( AA \) is the largest among the three strategies while the wholesale price in Strategy \( A \) is the lowest. For \( S_2 \), the wholesale price in Strategy \( AA \) is larger than that in Strategy \( AE \). As shown in Fig. 4, the retailer’s order quantity from \( S_1 \) in Strategy \( AE \) increases with \( \theta \) while the order quantity from \( S_2 \) in Strategy \( AE \) decreases with \( \theta \). Further, under the revenue sharing contract, the retailer and \( S_1 \) are integrated to maximize the overall supply chain’s profit, and the retailer’s order quantity from \( S_1 \) in Strategy \( A \) is the highest among the three strategies. Meanwhile, the order quantity from \( S_1 \) in Strategy \( AE \) is the lowest. The order quantity from \( S_2 \) in Strategy \( AE \) is higher than that in Strategy \( AA \).

**Figure 3.** Wholesale price changes with \( \theta 

**Figure 4.** Order quantity changes with \( \theta 

The above numerical examples examine the impact of the bank loans ratio on the equilibrium wholesale prices and order quantities. We now analyse how the bank loans ratio affects each player’s profit. We also study the condition under which the supplier \( S_2 \) will be allowed to participate in the supply chain.

**Proposition 5.** (i) \( \frac{\partial \pi_{AE*}^{S_2}}{\partial \theta} < 0 \); (ii) \( \frac{\partial \pi_{AE*}^{S_1}}{\partial \theta} > 0 \); (iii) If \( c \leq c_1 \), then \( \frac{\partial \pi_{AE*}^{S_2}}{\partial \theta} \geq 0 \); if \( c > c_1 \) and \( \theta < \theta_1 \), then \( \frac{\partial \pi_{AE*}^{S_2}}{\partial \theta} > 0 \); and if \( c > c_1 \) and \( \theta \geq \theta_1 \), then \( \frac{\partial \pi_{AE*}^{S_2}}{\partial \theta} \geq 0 \). (iv) If \( \theta \leq \theta_2 \), then \( \pi_{AE*}^{S_2} \geq \pi_{AE*}^{S_1} \); if \( \theta > \theta_2 \), then \( \pi_{AE*}^{S_2} < \pi_{AE*}^{S_1} \).

According to Proposition 3, two suppliers’ wholesale prices increase with \( \theta \). Therefore, the retailer’s profit decreases with \( \theta \). However, both \( w_{AE*}^{S_2} \) and \( q_1^{AE*} \) increase with \( \theta \). Therefore, \( S_1 \)’s profit increases with \( \theta \). We observe that the relationship between \( \pi_{AE*}^{S_2} \) and \( \theta \) depends on the production cost. When the production cost \( c \) is relatively low, it is cheap for \( S_2 \) to borrow from banks. Therefore, it is
optimal for $S_2$ to adopt pure bank credit. However, with the increase of $c$, it is gradually more costly to utilize bank credit. Therefore, there is an optimal bank loans ratio $\theta_1 > 0$ such that $\pi_{AE}^*$ first increases with $\theta$ for $\theta \in (0, \theta_1)$ and then decreases with $\theta$ for $\theta \in (\theta_1, 1)$.

The retailer’s incentive to switch from Strategy A to Strategy AE depends on the condition that the retailer obtains a higher profit in Strategy AE compared with Strategy A. We find that this condition is satisfied when the bank loans ratio is below a threshold $\theta_2$. This outcome implies that $S_2$’s cost advantage coming from a large portion of its finance from the bank will hurt the retailer. Therefore, when $\theta$ is low, the retailer can earn more in Strategy AE than that in Strategy A, which is similar to the results of Yang et al. [44]. According to Yang et al. [44], when the abandoned retailer adopts bank credit and equity financing, the supplier’s profit increases with equity financing ratio and decreases with bank loans ratio. Therefore, the supplier will allow the abandoned retailer to return to the market if the equity financing ratio is beyond a certain threshold (i.e., the bank loans ratio is relatively low).

Fig. 5 describes how the bank loans ratio affects the retailer’s profit. The main parameters are as follows: $c = 0.2$, $r_f = 0.05$, $a = 0.95$, $b = 0.6$, $\lambda = 0.7$, $k_1 = 0$ and $k_2 = 0$. Fig. 5 suggests that the retailer’s profit always decreases with $\theta$ in Strategy AE. This is because with the increase of $\theta$, both suppliers announce higher wholesale prices. When $\theta < 0.699$, the retailer’s profit in Strategy AE is greater than that in Strategy A, which means that the retailer is willing to allow $S_2$ to enter the market.

**Figure 5. Retailer’s profit changes with $\theta$**

Fig. 6 illustrates how the bank loans ratio affects the suppliers’ profits. The main parameters are as follows: $r_f = 0.05$, $a = 0.95$, $b = 0.6$, $\lambda = 0.7$, $k_1 = 0$ and $k_2 = 0$, where $c = 0.2$ in Fig. 6(a) and $c = 0.7$ in Fig. 6(b). We observe that $S_1$’s profit increases with $\theta$. As shown in Fig. 6(a), when the production cost $c$ is relatively low (e.g., $c = 0.2$), $S_1$’s profit increases with $\theta$. This outcome implies that it is optimal for $S_2$ to adopt pure bank credit. When production cost $c$ is relatively high (e.g., $c = 0.6$), it gradually becomes more costly to utilize bank credit. Therefore, as shown in Fig. 6(b), $S_2$’s profit first increases with $\theta$ for $\theta \in (0, 0.467]$ and then decreases with $\theta$ for $\theta \in (0.467, 1]$. 
Proposition 6. (i) \( \frac{\partial w_{AE}^*}{\partial b} < 0 \) and \( \frac{\partial q_{AE}^*}{\partial b} < 0 \); (ii) \( \frac{\partial w_{AE}^*}{\partial b} < 0 \); (iii) \( \frac{\partial \pi_{AE}^R}{\partial b} > 0 \), \( \frac{\partial \pi_{AE}^S_1}{\partial b} < 0 \), \( \frac{\partial \pi_{AE}^S_2}{\partial b} < 0 \).

Similar to Proposition 1, the suppliers’ wholesale prices decrease with \( b \) in Strategy \( AE \). According to Proposition 4, when \( S_2 \) adopts external financing, then he has a cost advantage over \( S_1 \). Therefore, in the competitive market, the retailer will decrease the order quantity from \( S_1 \) with an increased \( b \). However, when \( b \) is relatively high, the retailer’s order quantity from \( S_2 \) increases with \( b \). On the contrary, the order quantity from \( S_2 \) decreases with \( b \) when \( b \) is relatively low. Since \( \frac{\partial w_{AE}^*}{\partial b} < 0 \) and \( \frac{\partial q_{AE}^*}{\partial b} < 0 \), \( S_1 \)’s profit will naturally decrease with \( b \). For the retailer and \( S_2 \), similar to the analysis process of Proposition 1, we obtain that the retailer’s profit increases with \( b \) while \( S_2 \)’s profit decreases with \( b \).

Figs. 7-8 illustrate the impact of the competition intensity on the players’ optimal decisions. The main parameters are as follows: \( c = 0.2, r_f = 0.05, a = 0.95, \theta = 0.6, \lambda = 0.7, k_1 = 0 \) and \( k_2 = 0 \). As shown in Fig. 7, we observe that the wholesale prices in Strategies \( AA \) and \( AE \) decrease with \( b \) while the wholesale price in Strategy \( A \) is independent of \( b \). In addition, for \( S_1 \), the wholesale price in Strategy \( AA \) is the largest among the three strategies. However, for \( S_2 \), the wholesale price in Strategy \( AE \) is lower than that in Strategy \( AA \). From Fig. 8, in Strategy \( AA \), the retailer’s order quantities first decrease with \( b \) for \( b \in [0, 1/2] \) and then increase with \( b \) for \( b \in (1/2, 1] \). In Strategy \( AA \), there is no competition in the supply chain. Therefore, the wholesale price is independent of \( b \). In Strategy \( AE \), the retailer’s order quantity from \( S_1 \) decreases with \( b \). However, the retailer’s order quantity from \( S_2 \) first decreases with \( b \) for \( b \in [0, 0.394] \) and then increases with \( b \) for \( b \in (0.394, 1] \). In addition, the retailer’s order quantity from \( S_1 \) in Strategy \( AE \) is the lowest among the three strategies. Meanwhile, the retailer’s order quantity from \( S_1 \) in Strategy \( A \) is the highest.

Figs. 9-10 describe the impact of the competition intensity on the players’ profits. The main parameters are as follows: \( c = 0.2, r_f = 0.05, a = 0.95, \theta = 0.6, \theta = 0.7, k_1 = 0 \) and \( k_2 = 0 \). From Fig. 9, the retailer’s profits in Strategies \( AA \) and \( AE \) increase with \( b \) while the retailer’s profit in Strategy \( A \) is independent of \( b \). Given \( \theta, \lambda, k_1 \) and \( k_2 \), the retailer tends to merge with \( S_1 \) but refuses to allow \( S_2 \) to enter...
the market when $b \in [0, 0.584)$. Meanwhile, the retailer will allow $S_2$ to enter the market when $b \in [0.584, 0.706)$. Further, when $b \in (0.706, 1]$, the retailer is not willing to merge with $S_1$. According to Fig. 10, the suppliers’ profits in Strategies $AA$ and $AE$ decrease with $b$ while $S_1$’s profit in Strategies $A$ is independent of $b$. Further, when $b \in [0, 0.489]$, $S_1$ is willing to merge with the retailer.

5. Conclusion and management insights. In this paper, we study a competitive supply chain with two capital constrained suppliers and a common retailer. The retailer may strategically provide advance payment to one or two suppliers. We consider three different strategies: In Strategy $AA$, the retailer provides advance payment to both suppliers. In Strategy $A$, the retailer provides advance payment to only one supplier. The other supplier without financial support drops out of the market. In this scenario, the retailer and one of the suppliers conduct a revenue sharing contract to coordinate the overall supply chain. In Strategy $AE$,
the retailer provides advance payment to one supplier while the deselected supplier adopts external financing (including bank credit and equity financing). Our research differs from the existing literature in that we consider a competitive market with capital constrained suppliers and incorporate equity financing into our model. We further investigate the impact of the bank loans ratio on the players’ optimal decisions and profits.

We also get some meaningful findings. First, when the additional operating costs associated with the revenue sharing contract are relatively low, both the supplier $S_1$ and the retailer can choose an appropriate revenue sharing ratio to realize a win-win result. Second, by utilizing external financing including, the deselected supplier can break the merger of its rival and the retailer. Third, there is an optimal bank loans ratio for the deselected supplier. Interestingly, it is optimal to adopt pure bank credit if the production cost is sufficiently low.

Our work points to two important management insights. First, under a revenue sharing contract, the retailer and the supplier should take efficient actions to decrease additional operating costs. Further, each player should try its best to increase bargaining power to obtain a greater revenue sharing ratio. Second, for the deselected supplier, he may choose a suitable bank loans ratio to derive an optimal capital structure.

The paper can be extended along the following possible directions. In this paper, we assume that the market demand is deterministic. However, the retailer generally faces stochastic demand. Therefore, it is worthwhile to investigate the impact of demand uncertainty on the supply chain performance. In addition, in our model, the retailer is endowed with sufficient capital. In practice, however, the retailer commonly faces capital shortages and also resorts to other financial services. Therefore, considering the supply chain with all capital constrained players is another interesting topic.

Appendixes

Appendix (Proof of Lemma 1)

Substituting $p_1^{AA}$ and $p_2^{AA}$ into Eq.(1), we have \[ \frac{\partial \pi_{AA}^*}{\partial q_1} = 1 - 2bq_2^{AA} - aw_1^{AA}(1 + r_f) - 2q_2^{AA} \] and \[ \frac{\partial \pi_{AA}^*}{\partial q_2} = 1 - 2bq_2^{AA} - aw_2^{AA}(1 + r_f) - 2q_2^{AA}. \] According to the first order condition, we obtain $q_1^{AA} = \frac{b + aw_1^{AA} - abw_1^{AA} + aw_2^{AA}r_f - abw_2^{AA}r_f - 1}{2(b - 1)(b + 1)}$ and $q_2^{AA} = \frac{b + aw_1^{AA} - abw_1^{AA} + aw_2^{AA}r_f - abw_2^{AA}r_f - 1}{2(b - 1)(b + 1)}$. Since \[ \frac{\partial^2 \pi_{AA}^*}{\partial q_1^{AA}^2} = -2 < 0 \] and \[ \frac{\partial^2 \pi_{AA}^*}{\partial q_1^{AA}^2} = \frac{\partial^2 \pi_{AA}^*}{\partial q_2^{AA}^2} = 4(1 - b^2) > 0, \] therefore, there exists a unique ($q_1^{AA*}$, $q_2^{AA*}$) that maximizes $\pi_{AA}^*$. Substituting $q_1^{AA*}$ and $q_2^{AA*}$ into $\pi_{AA}^*$, we obtain $\pi_{AA}^* = -\frac{a(b - 1 + r_f) - 2aw_1^{AA} + abw_1^{AA}r_f + abw_1^{AA}r_f - 1}{2(b - 1)(b + 1)}$. According to the first order condition, we derive $w_1^{AA*} = w_2^{AA*} = \frac{1 + c - cr_f - b}{a(2 - b)(1 + r_f)}$. Since \[ \frac{\partial^2 w_1^{AA}}{\partial q_1^{AA}^2} = \frac{a^2(1 + r_f)^2}{(b - 1)(b + 1)} < 0, \] therefore, $w_1^{AA*}$ is the unique optimal wholesale price for $S_1$. Further, the optimal order quantities are $q_1^{AA*} = q_2^{AA*} = \frac{1 - c - cr_f}{2(b + 1)(2 - b)}$.

Appendix (Proof of proposition 1)

(i) From Lemma 1, we have \[ \frac{\partial w_1^{AA*}}{\partial b} = \frac{\partial w_2^{AA*}}{\partial b} = \frac{-1 - c - cr_f}{a(2 - b)(1 + r_f)} < 0. \]
(ii) From Lemma 1, we have \( \frac{\partial q_{AA}^*}{\partial b} = \frac{\partial q_{AA}^*}{\partial b} = \frac{(1-c-cr)(2b-1)}{2(6+1)^2} \). Therefore, when 
0 \leq b \leq 1/2, then \( \frac{\partial q_{AA}^*}{\partial b} \geq 0 \); when \( 1/2 < b \leq 1 \), then \( \frac{\partial q_{AA}^*}{\partial b} \leq \frac{\partial q_{AA}^*}{\partial b} < 0 \).

(iii) From Lemma 1, we have \( \frac{\partial q_{AA}^*}{\partial b} = \frac{-(3b(1-c-cr))^2}{(2b+1)(6+2)^2} > 0 \).

(iv) From Lemma 1, we have \( \frac{\partial q_{AA}^*}{\partial b} = \frac{(1-c-cr)^2(b^2-b+1)}{(6+1)^2(6-2)^3} < 0 \).

\( \square \)

**Appendix (Proof of Lemma 2)**

Proof of Lemma 2 is similar to that of Lemma 1, and hence is omitted. \( \square \)

**Appendix (Proof of Proposition 2)**

Comparing the sum of the retailer’s and \( S_1 \)’s profits between Strategy \( AA \) and Strategy \( A \), we have \( \pi_{AA}^* + \pi_{R}^* - \pi_{AA}^* - \pi_{R}^* = \frac{(1-b)(1-c-cr)^2}{4(6+1)(2-b)} - k_1 - k_2 \). We consider the following cases: (i) When \( k_1 + k_2 \geq \frac{(1-b)(1-c-cr)^2}{4(6+1)(2-b)} \), then the total profit of the supply chain in Strategy \( A \) is less than the sum of the retailer’s and \( S_1 \)’s profits in Strategy \( AA \). Therefore, the retailer and \( S_1 \) cannot merge. Case (ii) When \( k_1 + k_2 < \frac{(1-b)(1-c-cr)^2}{4(6+1)(2-b)} \), i.e., \( \pi_{AA}^* + \pi_{R}^* > \pi_{AA}^* + \pi_{R}^* \), then the merger may be achieved. We consider the following three subcases: (a) If \( \pi_{AA}^* > \pi_{AA}^* \) and \( \pi_{AA}^* > \pi_{AA}^* \), i.e., both players are willing to merge. When \( \lambda < \frac{2 - 3b^2 + 2b + 2}{(6+1)(6-2)^2} \) and \( k_2 < \frac{2 - 3b^2 + 2b + 2}{(6+1)(6-2)^2} \), then \( \pi_{AA}^* > \pi_{AA}^* \). To ensure \( \pi_{AA}^* > \pi_{AA}^* \) and \( \pi_{AA}^* > \pi_{AA}^* \), then \( \lambda, k_1 \) and \( k_2 \) hold \( \lambda \geq \frac{2 - 3b^2 + 2b + 2}{(6+1)(6-2)^2} \) and \( k_2 \leq \frac{2 - 3b^2 + 2b + 2}{(6+1)(6-2)^2} \).

(b) If \( \pi_{AA}^* > \pi_{AA}^* \) and \( \pi_{AA}^* > \pi_{AA}^* \), i.e., only the retailer is willing to merge. When \( \lambda \geq \frac{2 - 3b^2 + 2b + 2}{(6+1)(6-2)^2} \), then \( \pi_{AA}^* > \pi_{AA}^* \). To ensure \( \pi_{AA}^* > \pi_{AA}^* \) and \( \pi_{AA}^* > \pi_{AA}^* \), then \( \lambda, k_1 \) and \( k_2 \) hold \( \lambda \leq \frac{2 - 2b^2 + 2b + 2}{(6+1)(6-2)^2} \) and \( k_2 \geq \frac{2 - 2b^2 + 2b + 2}{(6+1)(6-2)^2} \).

(c) If \( \pi_{AA}^* > \pi_{AA}^* \) and \( \pi_{AA}^* > \pi_{AA}^* \), i.e., only the supplier \( S_1 \) is willing to merge. When \( \lambda \leq \frac{2 - 2b^2 + 2b + 2}{(6+1)(6-2)^2} \) and \( k_2 \leq \frac{2 - 2b^2 + 2b + 2}{(6+1)(6-2)^2} \), then \( \pi_{AA}^* > \pi_{AA}^* \). When \( \lambda \leq \frac{(6b^3 - 3b^2 + 2b + 2)(1-c-cr)^2}{(6+1)(6-2)^2} \), we have \( \pi_{AA}^* \leq \pi_{AA}^* \). To ensure \( \pi_{AA}^* > \pi_{AA}^* \) and \( \pi_{AA}^* \), then \( \lambda, k_2 \) and \( k_1 \) hold \( \lambda \geq \frac{(6b^3 - 3b^2 + 2b + 2)(1-c-cr)^2}{(6+1)(6-2)^2} \) and \( k_2 \leq \frac{2 - 2b^2 + 2b + 2}{(6+1)(6-2)^2} \).

\( \square \)

**Appendix (Proof of Lemma 3)**

Proof of Lemma 3 is similar to that of Lemma 1, and hence is omitted. \( \square \)
Appendix (Proof of Proposition 3)

(i) From Lemma 3, we have \( \frac{\partial w_1^{AE}}{\partial \theta} = \frac{bc}{2(2b-1)(b+2)} > 0 \) and \( \frac{\partial q_1^{AE}}{\partial \theta} = -\frac{2c}{(2b-1)(b+2)} > 0 \).

(ii) From Lemma 3, we have \( \frac{\partial q_1^{AE}}{\partial \theta} = \frac{bc(1+\theta)}{2(b-1)(b+1)(b-2)(b+2)} > 0 \) and \( \frac{\partial q_1^{AE}}{\partial \theta} = \frac{c(b^2-2)(1+r_f)}{2(b-1)(b+1)(b-2)(b+2)} < 0 \).

Appendix (Proof of Proposition 4)

(i) Under a revenue sharing contract, two players are fully integrated to maximize the overall profit of the supply chain, which requires a lower wholesale price than the production cost. Therefore, we have \( w_1^{AA} > w_1^{As} \) and \( w_1^{AE} > w_1^{As} \). From Lemmas 1 and 3, we have \( w_1^{AA} - w_1^{As} = \frac{bc(1-\theta)}{2b-1(1-b)} < 0 \). Therefore, \( w_1^{AA} > w_1^{AE} > w_1^{As} \).

We further have \( w_2^{AA} - w_2^{As} = \frac{2c-2b-2c+2bc+2c^2r_f+ab^2-b^2-abc-2ac\theta+bcr_f-2ac\theta r_f-a\theta r_f + 2}{2(b-1)(b+1)(b-2)(b+2)} < 0 \).

Since \( \frac{\partial (w_2^{AE} - w_2^{AA})}{\partial \theta} > 0 \) and \( (w_2^{AE} - w_2^{AA})|_{\theta=1} < 0 \), then \( w_2^{AE} < w_2^{AA} \).

(ii) From Lemmas 1-3, we have \( q_1^{As} - q_1^{AA} = \frac{-(b^2-2b-1)(c+c\theta r_f-1)}{2(b-1)(b+2)} > 0 \) and \( q_1^{AE} - q_1^{AA} = \frac{bc(\theta-1)(1+r_f)}{2(b-1)(b+2)(b+1)} > 0 \). Therefore, \( q_1^{As} > q_1^{AA} > q_1^{AE} \). We further have \( q_2^{AA} - q_2^{AE} = \frac{(b^2-2)(1+r)}{2b-1(b-2)(b+2)(b+1)} > 0 \).

Appendix (Proof of Proposition 5)

(i) From Lemma 3, we have \( \frac{\partial_\theta \pi_1^{AE}}{\partial \theta} = \frac{c(b^2-4c\theta r_f-3b^2-b^2-bc+6\theta-c+3b^2r_f+3b^2\theta r_f)}{2(b-1)(b+1)(b-2)(b+2)^2} > 0 \).

(ii) From Lemma 3, we have \( \frac{\partial_\theta \pi_1^{AE}}{\partial \theta} = \frac{bc(aw_1^{AE}-c)(1+r_f)}{2(b-1)(b+1)(b-2)(b+2)^2} > 0 \).

(iii) From Lemma 3, we have \( \frac{\partial_\theta \pi_1^{AE}}{\partial \theta} = \frac{m_1(bc-b-6\theta-b^2+bcr_f-6\theta r_f+3b^2\theta+c+3b^2\theta r_f)}{2(b-1)(b+1)(b-2)(b+2)^2} > 0 \), where \( m_1 = bc-b-6\theta-b^2+bcr_f-6\theta r_f+3b^2\theta+c+3b^2\theta r_f \). We note that \( bc-b-2c\theta-b^2+bcr_f-b^2\theta+2b^2 \theta r_f+2 > 0 \), \( \frac{\partial m_1}{\partial \theta} < 0 \), and \( m_1|_{\theta=0} > 0 \). Since \( m_1|_{\theta=1} < 0 \) and \( m_1|_{\theta=1} = 2(b^2-2) < 0 \) if \( c = 1/(1+r_f) \). Therefore, there exists a unique solution \( c_1 = \frac{(b^2-2)(b-1)}{2(b-1)(b+1)(b-2)(b+2)} \) such that \( m_1|_{c=c_1} > 0 \) if \( c \leq c_1 \), and \( m_1|_{c=c_1} < 0 \) if \( c > c_1 \).

Therefore, when \( c \leq c_1 \), then \( \frac{\partial_\theta \pi_1^{AE}}{\partial \theta} > 0 \); when \( c \geq c_1 \), then there exists a unique solution \( \theta_1 = \frac{b+bc-bcr_f-2}{3c(b-2)(1+r_f)} \) such that \( \frac{\partial_\theta \pi_1^{AE}}{\partial \theta} > 0 \) for \( \theta \in (0, \theta_1) \), and \( \frac{\partial_\theta \pi_1^{AE}}{\partial \theta} < 0 \) for \( \theta \in (\theta_1, 1) \).

(iv) Since \( \frac{\partial (\pi_1^{AE}-\pi_1^{AA})}{\partial \theta} < 0 \), let \( \pi_1^{AE} > \pi_1^{AA} \), then \( \theta \leq \theta_2 \), where \( \theta_2 \) satisfies the equation

\[
\frac{2b_1^3c^2\theta r_f^2 + 4b_1^3c^2\theta r_f - 2b_1^3c^2\theta - 2b_1^3c^2\theta r_f - 2b_1^3c^2\theta}{4(b-1)(b+1)(b-2)(b+2)^2} = \frac{\lambda(1-c cr_f)^2}{4} + k_1 = 0.
\]

Appendix (Proof of Proposition 6)

Proof of Proposition 6 is similar to that of Proposition 1, and hence is omitted.
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