A possible alternative route to a quantum theory of gravity is presented. The usual path is to quantize the gravitational field in order to introduce the statistical structure characteristic of quantum mechanics. The procedure followed here instead is to remove the statistical element of quantum theory by introducing final boundary conditions as well as initial. The relevant quantum formalism then becomes compatible with the non-statistical nature of general relativity and a viable theory can be constructed without difficulty. This approach also provides a simple method of avoiding the configuration space description of quantum mechanics and allows the formulation to be carried out entirely within the four dimensions of spacetime. These advantages are made possible by the inherent retrocausal nature of the final boundary conditions.

1. Introduction

This paper outlines a simple theory of quantum gravity. It seeks to bypass many of the present obstacles to such a theory by relaxing an implicit assumption which is usually taken for granted but which is not supported by any experimental evidence. This assumption is that only initial boundary conditions should be involved. The model discussed here adopts the traditional picture of general relativity wherein all of spacetime exists together in the form of a block universe laid out like a map through time, with the time dimension treated on the same footing as the spatial dimensions. In such a picture, imposing final boundary conditions as well as initial ones can be seen as a natural and indeed more symmetric possibility. This extra restriction allows an alternative approach to quantizing gravity. Instead of starting with the statistical nature of quantum mechanics and therefore attempting to make the gravitational field (or curvature) statistical as well, extra information is introduced via the final boundary conditions to make the quantum mechanical description become non-statistical so that it is compatible with the original, classical form of general relativity. This approach avoids the basic difficulty of trying to formulate a quantum theory of gravity without a pre-existing spacetime background. An obvious additional problem which exists in the many-particle case of quantum mechanics is that the wavefunction is defined in configuration space, whereas a description in 4-dimensional spacetime is needed here. This issue is also found to be easily resolvable once the backwards-in-time influence or “retrocausality” associated with final boundary conditions is taken into account. This model also has implications concerning the longstanding question of the correct interpretation for quantum mechanics since the technique
used here for eliminating probabilities is not compatible with some of the competing interpretations.

The discussion is limited to the case where the various source particles involved are not presently interacting with each other, although they will be described by a many-particle wavefunction which allows for entanglement existing from past interactions. The view will be taken that the case of continuing mutual interaction should be treated via quantum field theory using Feynman diagrams and this is intended to be the focus of a subsequent paper. Here the discussion will be limited to a wavefunction description, rather than employing the formalism of quantum field theory, because both the problems and the solutions are clearer in this format. In particular, this approach has the advantage of avoiding any ambiguity in mathematically defining the energy-momentum tensor and issues of regularisation do not arise [1]. Given the absence of experimental evidence guiding us in the area of quantum gravity, the much greater simplicity of this model speaks in its favour compared with other contenders.

The structure of the paper is as follows. In Sec. 2, the well-known approach is considered of modifying the classical field equation by replacing the classical energy-momentum tensor with a similar expression provided by quantum mechanics. Two obvious problems are then evident, namely the statistical/non-statistical mismatch of the two sides of the equation and the unwanted involvement of configuration space in the formalism. These are used to guide the approach in the rest of the paper. Sec. 3 introduces a method of imposing final boundary conditions and Sec. 4 then explains how this overcomes the first of the two problems quoted. Consistency of the new formalism with standard quantum mechanics is demonstrated in Sec. 5 and the resulting new gravitational field equation for a single-particle source is introduced. To resolve the second of the two problems raised in Sec. 2, the many-particle case is considered in Secs. 6 and 7 and it is shown how to keep the description within the four dimensions of spacetime. Finally, a simple version of the model for the special case of Klein-Gordon particles is presented in Sec. 8 by way of illustration.

2. Field Equation

In the classical theory of general relativity, the field equation has the form:

$$ G^{\mu\nu} = 8\pi T^{\mu\nu} \quad (\mu, \nu = 0,1,2,3) $$

where $G^{\mu\nu}$ is the Einstein curvature tensor and $T^{\mu\nu}$ is the energy-momentum tensor. The left hand side of this equation describes the curvature of spacetime resulting in the observed gravitational effects, whilst the right hand side describes the source inducing this curvature. The theory also contains the geodesic equation, which determines the path matter follows through the curved spacetime. This second aspect will be addressed later.

There is a need to replace the classical field equation (1) with a quantum mechanical version. The simple case where the source consists of a single particle will be considered first. A well-
known first step [2] is to replace the tensor $T^{\mu\nu}$ in Eq. (1) with its simplest counterpart in quantum mechanics, namely the following expression$^1$:

$$T^{\mu\nu}(x) = \psi^*(x) \hat{T}^{\mu\nu} \psi(x) \tag{2}$$

where $\hat{T}^{\mu\nu}$ and $\psi(x)$ are the energy-momentum operator and the wavefunction, respectively, corresponding to the particular wave equation under consideration$^2$ and the notation $x \equiv (x, t)$ is used to represent spatial position $x$ at time $t$. The expression for $T^{\mu\nu}(x)$ in Eq. (2) is always real for any wave equation. The field equation (1) then becomes:

$$G^{\mu\nu}(x) = 8\pi \psi^*(x) \hat{T}^{\mu\nu} \psi(x) \tag{3}$$

There are two obviously unsatisfactory things about this equation, namely:

1. The right hand side is statistical, whereas the left hand side is not. This can be seen from the fact that $\psi(x)$ must be updated discontinuously after a measurement because any spatial regions of the wavefunction excluded by the measurement result must cease contributing to $T^{\mu\nu}$. Furthermore, this update is meant to occur simultaneously with the measurement and instantaneously throughout space, which is not possible to define unambiguously while maintaining relativistic covariance since it requires favouring one particular space-like hyperplane in spacetime. The classical/quantum mismatch of Eq. (3) can therefore only give rise to a semi-classical theory, not full quantum gravity.

2. In going to the more general case of a many-particle entangled state, the wavefunction will be of the form $\psi(x, x', x'', ...)$ and defined in configuration space, not in spacetime. Hence it cannot then simply be inserted into the right hand side of Eq. (3) because it would be incompatible with the function $G^{\mu\nu}(x)$ defined in spacetime on the left.

It will now be shown how these two problems can be avoided by introducing final boundary conditions as well as the usual initial ones.

### 3. Introduction of final boundary conditions

The way to proceed here will be made clearer by switching to Dirac notation instead of continuing with wavefunctions. Assuming that the source particle under consideration is in an initial state $i$ at time $t$, Eq. (2) can be written in the form:

$^1$ See, e.g., [3]. Note that expression (2) is a function of position $x$. Integrating it over all space would yield the expectation value for $T^{\mu\nu}$.

$^2$ The most common choices are the Dirac equation and the Klein-Gordon equation. In this regard, the energy-momentum tensor for the Klein-Gordon case is given in Eq. (22). Note that each wave equation will need to be generalised to a form applicable in Riemannian curved spacetime.
Here the wavefunction $\psi(x)$ has been re-expressed as the amplitude $\langle x,t \mid i,t \rangle$ of the state $i$ at initial time $t_i$ onto the position state $x$ at later time $t$. The state $i$ can be thought of as the initial boundary condition imposed on the particle. Looking at the form of Eq. (4), the obvious way to include a final boundary condition as well is to employ a familiar technique [4,5,6] and replace one of the initial states with a final state $f$ relating to some later time $t_f$, as follows:

$$T^{\mu\nu}(x,t) = \langle f; t_f \mid x; t \rangle \hat{T}^{\mu\nu} \langle x; t \mid i; t_i \rangle$$  \hspace{1cm} (5)

Here $\langle x; t \mid f; t_f \rangle$ is then the amplitude of the state $f$ at final time $t_f$ onto the position state $x$ at earlier time $t$. Eq. (5) can be written more neatly by returning to the notation $x \equiv \langle x; t \rangle$ and deleting explicit mention of $t_i$ and $t_f$ to give:

$$T^{\mu\nu}(x) = \langle f \mid x \rangle \hat{T}^{\mu\nu} \langle x \mid i \rangle$$  \hspace{1cm} (6)

The intention is for this new expression to give a more specific valuation for $T^{\mu\nu}(x)$ when both the initial and final boundary conditions are known. It will now be demonstrated that Eq. (6) is a suitable basis for constructing the type of model proposed here. Two preliminary adjustments are needed, namely (i) an extra factor $\langle f \mid i \rangle$ needs to be included to ensure consistency with the standard expression (2) for $T^{\mu\nu}$ when only the initial state is known, and (ii) the expression needs to be real and be fully symmetric with respect to the states $i$ and $f$, both of which can be achieved simply by taking the real part. The result is:

$$T^{\mu\nu}(x) = \Re \left( \frac{\langle f \mid x \rangle \hat{T}^{\mu\nu} \langle x \mid i \rangle}{\langle f \mid i \rangle} \right)$$  \hspace{1cm} (7)

This expression is analogous in structure to the formalism of other authors, e.g., the “weak values” of Aharonov, Albert and Vaidman [4,5], although the physical interpretation is perhaps somewhat different. Here it is being postulated that Eq. (7) represents what actually exists in terms of energy and momentum density at times between any two boundary conditions imposed via successive measurements\(^3\). Using the relevant wave equation it is easily shown that expression (7) has zero divergence, in analogy to the usual expression (2). The way in which (7) resolves the first of the two issues raised in Sec. 2 will now be explained.

\(^3\) Note that the new expression can be viewed as the conditional value of $T^{\mu\nu}$ given that the prior and subsequent measurement results are found to be $i$ and $f$, respectively. This conditional wording automatically excludes the unwelcome possibility of the denominator in (7) being zero - a zero value for the probability $|\langle f \mid i \rangle|^2$ would contradict the stipulation that $i$ and $f$ actually occur.
4. Non-statistical energy-momentum tensor

The expression in Eq. (7) is to be interpreted as the non-statistical version of the energy-momentum tensor which becomes available once both the initial and final boundary conditions are given. It therefore resolves the statistical/non-statistical mismatch associated with Eq. (3). In contrast to Eq. (7), the standard quantum mechanical expression in Eq. (2) is dependent only on the initial state and its statistical nature is assumed to arise from our lack of knowledge of the final boundary condition. The fact that Eq. (7) reduces back to Eq. (2) in such circumstances is shown in Sec. 5.

Note that the initial and final boundary conditions are specified as Hilbert space vectors $|i\rangle$ and $|f\rangle$. In standard quantum mechanics the initial state $|i\rangle$ is the result of a measurement, or of state preparation, at time $t_i$ and summarises the particle’s relevant past. By symmetry, the interpretation of state $|f\rangle$ is taken to be similar. Here it is defined for simplicity to be the result of the next measurement performed on the particle, this being carried out at some later time $t_f$. This result is taken to summarise the particle’s relevant future.

From the form of Eq. (7) it is clear that $|i\rangle$ and $|f\rangle$ determine $T^{\mu\nu}$ uniquely. One could perhaps also say that the new expression for $T^{\mu\nu}$ is no longer statistical because it contains the actual outcome $|f\rangle$ of the measurement and so other possible outcomes and their probabilities have become irrelevant. Note too that $T^{\mu\nu}$ as defined in (7) is not a directly measurable quantity because it refers to the situation existing between measurements. The only actual measurement outcomes are $|i\rangle$ and $|f\rangle$. For the purposes of the simple proof in Sec. 5, it should be noted that, since $|f\rangle$ is a measurement result, it will correspond to one of the eigenstates of a complete orthonormal set.

Since this model removes the statistics from the quantum mechanical description, it demonstrates that there is actually no need to relinquish the classical, non-statistical description of curvature in order to construct a quantum theory of gravity. For the simple case considered so far of a single source particle, the model consists of the standard field equation (1) of general relativity with the energy-momentum tensor taken to be expression (7). Specifically, the field equation takes the form:

$$G^{\mu\nu} = 8\pi \text{Re} \frac{\langle f | x \rangle \hat{T}^{\mu\nu} \langle x | i \rangle}{\langle f | i \rangle}$$

$$\tag{8}$$

---

The frequent reference to measurements in this paper is not intended to imply that measurement interactions have any special status compared with other interactions, but merely to simplify the presentation. An analysis of the measurement process itself is presented in Sec. A1 of the Appendix, where it is pointed out that the introduction of final boundary conditions also allows a “no collapse” interpretation of the wavefunction. In particular, the measurement process can be viewed as taking place by continuous evolution via the relevant wave equation, with no discontinuous or stochastic change in the wavefunction.
This equation can be written out with the various times shown explicitly as follows:

\[
G^{\mu\nu}(x,t) = 8\pi \text{Re} \left\langle \frac{\langle f, t_f | x, t \rangle \hat{T}^{\mu\nu} \langle x, t | i, t_i \rangle}{\langle f | i \rangle} \right\rangle
\]  

(9)

This form emphasises that the initial boundary conditions \( i \) are imposed at some earlier time \( t_i \), the final boundary conditions \( f \) are imposed at a later time \( t_f \) and both \( G^{\mu\nu} \) and the energy-momentum tensor \( T^{\mu\nu} \) are evaluated at some intermediate time \( t \).

5. **Consistency with the standard energy-momentum tensor of quantum mechanics**

If the conditional dependence of the energy-momentum tensor on both \( i \) and \( f \) is included explicitly in the notation, the left hand side of Eq. (7) should be written as \( T^{\mu\nu}(x | i, f) \). In contrast, the standard energy-momentum expression (2) predicted by quantum mechanics should be written as \( T^{\mu\nu}(x | i) \), since it depends only on the initial state. Using this notation, it will now be shown that expression (7) reduces to the standard one when a weighted average is taken over the unknown final state. Under this procedure, the two expressions are expected to be related via:

\[
T^{\mu\nu}(x | i) = \int_{-\infty}^{+\infty} T^{\mu\nu}(x | i, f) \rho(f) df
\]

(10)

where \( \rho(f) \) is the probability density for the subsequent measurement result \( f \). Inserting the usual expression:

\[
\rho(f) = |\langle f | i \rangle|^2
\]

(11)

together with (7) into Eq. (10) yields:

\[
T^{\mu\nu}(x | i) = \int_{-\infty}^{+\infty} \text{Re} \left\langle \frac{\langle f | x \rangle \hat{T}^{\mu\nu} \langle x | i \rangle}{\langle f | i \rangle} \right\rangle |\langle f | i \rangle|^2 df
\]

= \text{Re} \int_{-\infty}^{+\infty} \langle i | f \rangle \langle f | x \rangle \hat{T}^{\mu\nu} \langle x | i \rangle df

= \text{Re} \left[ \langle i | x \rangle \hat{T}^{\mu\nu} \langle x | i \rangle \right]
\]

(12)

This result is then seen to be equivalent to the standard energy-momentum expression (2) as required, since the factor in the square bracket is always real as mentioned in Sec. 2.

6. **Two-particle case**

The second problem raised in Sec. 2 will now be addressed. This was that a many-particle wavefunction in configuration space is not compatible with the desired 4-dimensional description associated with general relativity. Fortunately, the introduction of final boundary
conditions provides a way of overcoming this problem by allowing a separate wavefunction to be introduced for each particle.

An indication of the way to proceed can be seen by considering the well-known case of a pair of particles which have previously interacted with each other and are now far apart (such as in the usual set-up employed in discussing Bell’s theorem). The position coordinates of the 1st and 2nd particle will be represented by \( x \) and \( x' \), respectively. In the non-relativistic case, before any measurement is performed on either particle, the pair is represented by a single entangled wavefunction \( \psi(x, x'; t) \). Suppose a measurement is now performed on the 2nd particle and this yields a state represented by the wavefunction \( \psi_2(x'; t) \). The 1st particle’s state can, by means of this information, be updated to a separate wavefunction \( \psi_1(x; t) \). The actual form of this new wavefunction can be obtained by taking the scalar product of the 2nd particle’s new state \( \psi_2(x'; t) \) with the original wavefunction \( \psi(x, x'; t) \) as follows:

\[
\psi_1(x; t) = \frac{1}{N} \int_{-\infty}^{+\infty} \psi_2(x'; t) \psi(x, x'; t) \, d^3x'
\]

where \( N \) is a normalisation constant.

An important issue now arises. The collapse of the 1st particle’s wavefunction from the initial entangled state \( \psi(x, x'; t) \) to the reduced state \( \psi_1(x; t) \) is meant to occur simultaneously with the measurement on the 2nd particle. In going to the relativistic case, however, the interval between these two well-separated events is spacelike and an ambiguity arises in defining “simultaneous”. Also, at whatever time the collapse is taken to occur, the introduction of Lorentz invariance means there will be a range of reference frames available in which the collapse occurs before the measurement. This means some sort of backwards-in-time effect, or retrocausality, is unavoidable in the standard picture as long as special relativity is assumed valid. Having heeded this point, it will now be shown how the explicit inclusion of this notion in the present formulation will enable the configuration space problem to be overcome.

At this point it is more appropriate to rewrite Eq. (13) in Lorentz invariant form and switch to Dirac notation. Since the aim here is to consider the physical reality existing between measurements, it will be assumed that a measurement of some sort is subsequently performed on each particle with the corresponding final outcomes being written as \( |f \rangle \) and \( |f' \rangle \), respectively. Then, using the notation \( x \equiv (x; t) \) and starting with an entangled state of the form \( \psi(x, x') = \langle x, x' | i \rangle \), Eq. (13) is replaced by:

\[
\langle x | i \rangle = \frac{1}{N} \int_{-\infty}^{+\infty} \langle f' | x' \rangle \gamma^0 \langle x, x' | i \rangle \, d^3x'
\]  

Further discussion concerning relativistic entangled states is provided in Sec. A2 of the Appendix. Note that the operator \( \gamma^0 \) representing the zeroth component of the relevant 4-current density needs to be included in the transition to the relativistic case. In Eq. (14) it acts on the primed coordinates.
As before, this equation provides the updated wavefunction \(\langle x | i \rangle\) for the 1\textsuperscript{st} particle once a measurement is performed on the 2\textsuperscript{nd} particle and the result is known. Also as before, the time at which this wavefunction should become applicable is unclear because of the relativity of simultaneity. Identifying a particular spacetime hyperplane on which it occurs is not consistent with maintaining Lorentz invariance. This being the case, it is natural to take the following extra step. It will be assumed that the updated wavefunction can be applied validly from the original time of separation of the two particles, rather than assuming it becomes applicable at some random later time. This provides the separate wavefunction needed.

Hence there is now a separate initial wavefunction \(\langle x | i \rangle\) for the 1\textsuperscript{st} particle, taken to be valid from the moment when the two particles move apart. Eq. (14) shows clearly that this wavefunction is dependent on the final boundary condition \(f'\) of the other particle, indicating a retrocausal effect. The measurement eventually performed on the 1\textsuperscript{st} particle will provide a final wavefunction \(\langle x | f \rangle\) for this particle as well. These two wavefunctions will form the basis for extending the definition of the energy-momentum tensor to the many-particle case\(^6\).

### 7. Separate energy-momentum tensors

Having introduced individual wavefunctions \(\langle x | i \rangle\) and \(\langle x | f \rangle\) for the 1\textsuperscript{st} particle in the previous section, a separate energy-momentum tensor for this particle can now been obtained, this being defined in 4-dimensional spacetime rather than in configuration space. This energy-momentum tensor is simply given by the expression in Eq. (7):

\[
T_{i}^{\mu\nu}(x) = \text{Re} \frac{\langle f | x \rangle \hat{T}^{\mu\nu} \langle x | i \rangle}{\langle f | i \rangle} \tag{15}
\]

i.e., the same expression as when there is only one particle present rather than a pair. By changing to wavefunction notation:

\[
\psi_{i}(x) \equiv \langle x | i \rangle \tag{16}
\]

\[
\psi_{f}(x) \equiv \langle x | f \rangle \tag{17}
\]

the 1\textsuperscript{st} particle’s energy-momentum tensor can also be written in the form:

\[
T_{i}^{\mu\nu}(x) = \text{Re} \frac{\psi_{f}^{\ast}(x) \hat{T}^{\mu\nu} \psi_{i}(x)}{\langle f | i \rangle} \tag{18}
\]

Here, in accordance with the discussion in previous sections, \(\psi_{i}(x)\) is the particle’s “initial wavefunction” at time \(t\), summarising the initial boundary conditions imposed at an earlier

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\(^6\) This procedure also has the additional advantage of avoiding any need for nonlocal communication between the particles at the time of measurement. The retrocausal effect has now become timelike and is conveyed back via the 2\textsuperscript{nd} particle’s final wavefunction.
time \( t_i \), and \( \psi_t(x) \) is its “final wavefunction” at the same time \( t \), summarising the final boundary conditions imposed at a later time \( t_f \).

An analogous process for the 2\textsuperscript{nd} particle yields a separate initial wavefunction \( \langle x'|i \rangle = \psi_i(x') \) and a separate final wavefunction \( \langle x'|f' \rangle = \psi_{f'}(x') \), the latter describing the outcome \( f' \) of the measurement on this particle. A corresponding energy-momentum tensor \( T_2^{\mu\nu}(x) \) for the 2\textsuperscript{nd} particle can then be constructed from these two wavefunctions.

In Sec. A3 of the Appendix, the procedure explained in Sec. 6 is extended from two particles to \( n \) particles. This makes it possible to assign a separate pair of wavefunctions and a separate energy-momentum tensor to each of the \( n \) particles. The total energy-momentum tensor will then simply be the sum of the tensors for the individual particles:

\[
T_{\text{total}}^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} + \ldots
\]

and each term will separately have zero divergence. The quantum gravity model being formulated here is then completed simply by inserting the total energy-momentum tensor of Eq. (19) into the classical field equation (1) to give the new field equation for a many-particle system:

\[
G^{\mu\nu} = 8\pi T_{\text{total}}^{\mu\nu}
\]

This equation, which looks largely unchanged in the new model, determines the curvature of spacetime in response to the presence of matter.

The other aspect of the classical theory that needs to be taken into account is the geodesic equation. It is replaced in this model by appropriate wave equations for the initial and final wavefunctions, as will be illustrated in the next section.

8. Klein-Gordon particles

A specific example of the quantum gravity model proposed here will now be presented, namely the Klein-Gordon case. Choosing units such that \( \hbar = c = 1 \), the Klein-Gordon equation in curved spacetime has the following form:

\[
g^{\mu\nu} \nabla_\mu \nabla_\nu \psi - m^2 \psi = 0
\]

where \( \nabla_\mu \) is the covariant derivative and \( g^{\mu\nu} \) is the metric tensor, the signature being \(-+++\). The corresponding energy-momentum tensor can be written in the form [2]:

\[
T^{\mu\nu} = \psi^* \hat{T}^{\mu\nu} \psi
= \psi^* \frac{1}{2m} \left[ \hat{\nabla}^\mu \hat{\nabla}_\mu + \hat{\nabla}^\nu \hat{\nabla}_\nu - \hat{\nabla}^\alpha g_{\alpha\beta}^{\mu\nu} \hat{\nabla}_\beta + m^2 g^{\mu\nu} \right] \psi
\]
Transferring the operator $\hat{T}^{\mu\nu}$ in Eq. (22) into the proposed new energy-momentum tensor (18) then yields:

$$T^{\mu\nu}(x) = \text{Re} \frac{\psi_i^*(x) \frac{1}{2m} \left[ \tilde{\nabla}^\mu \tilde{\nabla}^\nu + \tilde{\nabla}^\nu \tilde{\nabla}^\mu - \tilde{\nabla}^\alpha g_{\alpha\beta} g^{\mu\nu} \tilde{\nabla}^\beta + m^2 g^{\mu\nu} \right] \psi_i(x)}{\langle f \mid i \rangle}$$

(23)

This expression is for a one-particle source. As before, the total energy-momentum tensor $T^{\mu\nu}_{\text{total}}$ for the system is then simply the sum of all the separate $T^{\mu\nu}$'s for the individual particles, in accordance with Eq. (19). Finally, inserting $T^{\mu\nu}_{\text{total}}$ into the classical field equation (1) will yield the field equation (20) for the many-particle system, as already explained.

There is also a need for an equation to determine how matter will move in the curved geometry. In the classical theory, this is supplied by the geodesic equation. In the present model, the analogous task is performed by Eq. (21), which provides the following wave equations for the initial and final wavefunctions:

$$\left( g^{\mu\nu} \nabla_\mu \nabla_\nu - m^2 \right) \psi_i(x) = 0$$

(24)

$$\left( g^{\mu\nu} \nabla_\mu \nabla_\nu - m^2 \right) \psi_f(x) = 0$$

(25)

In summary, Eq. (20) describes the influence of the wavefunctions on the geometry, whilst Eqs. (24) and (25) describe the influence of the geometry on the wavefunctions. Within the context of the Klein-Gordon case, these three equations comprise the theory of quantum gravity being proposed.

9. Discussion and conclusions

A simple quantum gravity model has been presented in which the approach has been to make the energy-momentum tensor $T^{\mu\nu}$ of quantum mechanics become non-statistical and therefore compatible with the classical expression for the Einstein tensor $G^{\mu\nu}$. The tensors $G^{\mu\nu}$ and $T^{\mu\nu}$ can then be combined consistently in the field equation of general relativity. This is the opposite of the usual approach, which requires gravity to be quantized to make it statistical. One notable consequence of this model is that gravity can be incorporated into quantum theory without the need to introduce gravitons. Formulating such a model has been made possible by the inclusion of the less conventional notion of final boundary conditions. This notion has also provided a means of reducing the description in the many-particle case from configuration space to 4-dimensional spacetime, in keeping with classical general relativity. Finally note that, due to the paucity of experimental evidence available in the field of quantum gravity, a model of this type is just as viable as the more well-known approaches while being much simpler.
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Appendix

A1. The measurement process

The aim here is to demonstrate that the introduction of final boundary conditions allows a “no collapse” interpretation of the wavefunction, whereby the measurement process can be viewed as taking place by continuous evolution via the relevant wave equation. For the purposes of the discussion here, the measuring apparatus will initially be taken as a macroscopic entity which can be treated classically for simplicity. The more general case will be outlined in footnote 8. Also, the observable quantities discussed here will be assumed to have a discrete spectrum of eigenvalues (the arguments being easily generalised to the case of a continuous spectrum).

In standard quantum mechanics, an initial boundary condition can be summarised by a wavefunction and this information can then be evolved forward to the present time in readiness for a measurement to be performed. As discussed in this paper, a final boundary condition can also be imposed and this can be summarised by means of a separate (and generally different) wavefunction \( \psi_f \) which can then be evolved backwards to the present time. Now an essential feature of any measurement is that it must allow us to distinguish between the possible outcomes and identify the result. As frequently noted by previous authors, this means that the possible eigenstates of the observed system (or of something with which it interacts) must become separate in space. Hence, in the simplest case, the “initial” wavefunction \( \psi_i \) will gradually spread into a collection of spatially non-overlapping wave packets in passing through the region of the measurement interaction. This will take place by continuous evolution via the relevant wave equation. Turning to the “final” wavefunction \( \psi_f \) evolving back from the final boundary condition, it will also be split into separate eigenstates in passing through the measurement region, but this splitting will be in the opposite time direction. Perhaps a useful way of viewing the measurement process is that the initial wavefunction will spread forwards in time like separate fingers from a hand, while the final wavefunction will spread into fingers backwards in time.

Now, using the above notion that the final wavefunction will be spatially split before the measurement (from our viewpoint) and not after, one can argue as follows. The branches of the initial wavefunction must become permanently separated after the measurement and not recombine, as otherwise the measurement would be “undone”. Therefore it can be expected that the final wavefunction will be non-zero and evolving back in only one of these branches. Otherwise, unlike the initial wavefunction, it would be split both before and after the measurement, briefly merging only in the measurement region\(^7\). Furthermore, in spatially

\(^7\) This point can be justified in more detail, as discussed in the Appendix of [6].
overlapping only one branch of \( \psi_i \), it will thereby automatically determine the measurement result. In this regard, a key point to note here is that the new energy-momentum tensor (7) essentially contains a product of the initial wavefunction \( \langle x | i \rangle = \psi_i \) and the final wavefunction’s conjugate \( \langle \bar{f} | x \rangle = \psi_i^* \). This means that it will be non-zero only in regions where both the initial and final wavefunctions are non-zero. Putting this another way, a branch of the initial wavefunction will make no contribution to \( T^{\mu \nu} \) unless it is at least partially overlapped in spacetime by the final wavefunction. Consequently, even though the non-overlapped branches of \( \psi_i \) evolve on continuously after the measurement, they can be ignored as irrelevant because they contain no energy or momentum. Hence, at this point we can choose to delete from our mathematical description those branches of the initial wavefunction which will no longer contribute to \( T^{\mu \nu} \). Since there is no actual physical collapse involved in “deletion by choice”, this picture is therefore consistent with a no-collapse view of wavefunction evolution\(^8\).

Note that the zero 4-divergence of the energy momentum tensor (7) also implies the following. If there is some overlap between the initial and final wavefunctions at a particular time, the mathematics ensures they must continue to overlap at other times in order to conserve energy and momentum. Conversely, if a branch is not overlapped at some particular time, it must continue to be not overlapped for the same reason.

**A2. Relativistic entangled state for a pair of particles**

The initial state (i.e., the initial boundary condition) in any physical theory is commonly expressed on a preferred hyperplane of constant time \( t_i \). Hence, for the entangled pair of particles considered in Sec. 6, the initial state is expected to be of the form \( \langle x_i, x_i'; t_i | i \rangle \). For later times, a state of the form \( \langle x, x' | i \rangle = \langle x, t; x', t' | i \rangle \) can then be obtained from this initial one via the relationship:

\[
\langle x, t; x', t' | i \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(x, t; x, t_i) \psi_i^* \left( x_i', t_i \right) \psi_i \left( x_i, t_i \right) d^3 x_i d^3 x_i' \tag{A1}
\]

where the K’s are propagators which are solutions of the relevant wave equation and carry the state forwards in time from \( t_i \) to \( t \) and \( t' \). The resulting multi-time entangled state provides the joint probability density \( |\langle x, x' | i \rangle|^2 \) for the 1\(^{st}\) particle to be at position \( x \) at time \( t \) and the 2\(^{nd}\) particle to be at \( x' \) at time \( t' \). The single-particle state \( \langle x | i \rangle \) defined in Eq. (14)

\(^8\) As mentioned earlier in this section, the measuring apparatus is being treated classically here. A more sophisticated formulation would, of course, include the initial and final wavefunctions of the apparatus as well. For example, the required spatial separation might then be related to the apparatus instead of the measured particle, e.g., pointer readings on a dial. Similarly, the final wavefunction of the overall particle-apparatus system can be expected to overlap only one branch of this system’s overall initial wavefunction in configuration space. Other branches can then be ignored, as discussed above.
is then seen to be the projection generated by the measurement on the second particle and can be identified as the 1st particle’s resulting new state.

Note that by inserting Eq. (A1) into Eq. (14) and using the standard relationship:

$$\int_{-\infty}^{+\infty} \langle f' | x', t' \rangle \hat{K}(x', t'; x_i, t_i) d^3x' = \langle f' | x_i, t_i \rangle$$

(A2)

(which propagates the state $\langle f' | x', t' \rangle$ back to time $t_i$) the following result is obtained:

$$\langle x | i \rangle = \frac{1}{N} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{K}(x, t; x_i, t_i) \langle f'| x', t' \rangle \langle x_i, x'; t_i | i \rangle d^3x_i d^3x'$$

(A3)

which shows that the updated state $\langle x | i \rangle$ for the 1st particle is not a function of the time coordinate $t'$ of the other particle.

A3. The n-particle case

The formalism presented in Sec. 6 will here be extended from 2 to n particles. Consider a configuration space wavefunction $\langle x, x', x'', \ldots | i \rangle$ which allows for entanglement existing from past interactions. It will be assumed that measurements are eventually performed on all the particles, with the outcomes being described by the separate wavefunctions $\langle x | f' \rangle$, $\langle x' | f'' \rangle$, $\langle x'' | f'' \rangle$, … . Taking the 1st particle as an example and working by analogy with Eq. (14), the updated wavefunction which becomes applicable as a result of the measurements performed on all the other particles is given by:

$$\langle x | i \rangle = \frac{1}{N} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \langle f' | x' \rangle \langle f'' | x'' \rangle \cdots \langle x, x', x'', \ldots | i \rangle d^3x_i d^3x' d^3x'' \cdots$$

(A4)

where $N$ is an appropriate normalisation constant. Note that the right hand side of this equation does not involve the 1st particle’s measurement result $\langle x | f \rangle$ (all other results being included). Using the argument presented in Sec. 6, the wavefunction given by Eq. (A4) is assumed to be already valid at earlier times. Hence the 1st particle can now be assigned this separate initial wavefunction $\langle x | i \rangle$, together with the final wavefunction $\langle x | f \rangle$ generated by the subsequent measurement on this particle. These two wavefunctions can then be combined to provide an individual energy-momentum tensor via Eq. (15). Assigning each particle a separate pair of wavefunctions and a separate energy-momentum tensor in this way, it is possible to use the same formalism developed for the single-particle case to describe the n-particle case as well.

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