LOW SURFACE BRIGHTNESS GALAXIES: MASS PROFILES AS A CONSEQUENCE OF GALACTIC EVOLUTION

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ABSTRACT
This Letter presents a principal component analysis of rotation curves from a sample of low surface brightness galaxies. The physical meaning of the principal components is investigated and related to the intrinsic properties of the galaxies. The rotation curves are re-scaled using the optical disk scale length, and the resulting principal component decomposition demonstrates that the whole sample is properly approximated using two components. The ratio of the second to the first component is closely related to the inner mass profile, is correlated to the gas fraction in the galaxy, and is uncorrelated to other parameters. As a consequence, the gas fraction appears as a fundamental variable with respect to the galaxies’ rotation curves, and its correlation with the inner mass profile is especially important. The effects of noise, re-scaling biases, and systematics are investigated in detail, and the correlation between the principal components’ ratio and the gas fraction is demonstrated to be stable. Since the gas fraction is related to the degree of galaxy evolution, it is very likely that the shape of the inner mass profile is a consequence of galaxy evolution. More evolved galaxies have shallower central profiles and statistically less gas, as a consequence of more star formation and supernovae. The differences in evolution, gas fractions, and the inner mass profile of the galaxies could be due to the influence of different environments.

Key words: dark matter – galaxies: dwarf – galaxies: evolution

Online-only material: color figure

1. INTRODUCTION
Cold dark matter (CDM) predicts steep density profiles at the center of dark matter halos (Navarro et al. 1996, 1997; Moore et al. 1998). This prediction is not consistent with observations of constant core density in low surface brightness galaxies (LSBGs; Moore 1994; Burkert 1995; Navarro et al. 1996; McGaugh & de Blok 1998; de Blok & Bosma 2002; Kuzio de Naray et al. 2006). Various modifications of the CDM scenario have been proposed to account for this discrepancy, for example, collisional dark matter (Carlson et al. 1992; Spergel & Steinhardt 1999). Various modifications of the CDM scenario have been proposed to account for this discrepancy, for example, collisional dark matter (Carlson et al. 1992; Spergel & Steinhardt 1999; Moore et al. 2005; Swaters et al. 2003; Van den Bosch & Swaters 2001). An important issue in this controversy is the complexity of LSBGs’ dynamics and in particular the non-circular motions (Pizzella et al. 2008a, 2008b). This Letter will not attempt to favor one or another scenario, but will focus on analyzing RCs’ data without a preconceived hypothesis. The approach will focus on isolating the fundamental features in the RCs and will relate them to the intrinsic physical parameters of LSBGs.

2. DATA ANALYSIS
de Blok & Bosma (2002, hereafter dBB) published the RCs of 26 LSBGs. The dBB sample was supplemented by introducing RCs from the sample of de Blok et al. (2001, hereafter DMR). Note that the galaxies belong to two different original samples and photometry is available only for eight of them (see DMR for more details). The dBB and DMR samples contain galaxies of different sizes and masses requiring the re-scaling of the rotations curves. In practice, the RCs were re-scaled using the scale length $h$ inferred from the visible light (Table 1 in dBB and DMR). Provided the various RCs are related to the same universal physical process, the re-scaled curves should be reducible to a minimal set of fundamental components, which is equivalent to a principal component analysis (PCA). The PCA requires a re-sampling of the curves to a common grid, this re-sampling was performed using linear interpolation. The PCA vectors are re-constructed in the range $0 < r/h < R_{\text{max}}$. The range $R_{\text{max}}$ must include a sufficient number of points (a minimum of five points was required). An additional requirement was the availability of an estimation of the H$\text{I}$ contribution to the RC. $R_{\text{max}}$ was adjusted to be as large as possible while maximizing the number of curves. The optimal value is $R_{\text{max}} = 2.85$, which corresponds to a total of 15 curves for the dBB sample and 8 curves for the DMR sample. It was not possible to estimate the gas fraction for UGC 6614. As a consequence, the latter was rejected from the subsequent analysis, which requires the knowledge of the gas fraction. The PCA was performed on several samples, first the whole sample of 22 galaxies (Sample 1), two samples with inclination cuts for the galaxies, $0^\circ < i < 70^\circ$ (17 galaxies, Sample 2), $30^\circ < i < 70^\circ$ (13 galaxies, Sample 3), and finally the initial dBB sample of 15 galaxies (Sample 4). Since the mass scales as the square of the velocity, the PCA is performed using the square of the velocity, or equivalently on the quadratic rotation curves (QRCs). The gas contribution was subtracted from the QRCs using the corrected gas velocities provided by dBB and DMR. Finally, the curves were renormalized by dividing the velocities $V_i$ with a renormalization factor $A_N$. $A_N$ is defined using the following equation: $A_N \sum V_i^2 = 1$. The PCA requires finding the eigenvalues of the QRC vectors’ covariance matrix; numerically this is best achieved by using a singular
value decomposition of the matrix. Components are sorted by the descending order of contribution to the power spectrum (see Figure 1).

2.1. Effect of Disk Subtraction on the PCA

In the renormalized coordinates adopted for the PCA analysis, the disk scale is constant and equal to unity. The RC of a thin exponential disk with scale length unity, $u_d^2$, is approximated with a linear combination of the first two components to a relative accuracy of a few percent, which is close to the noise expectation (see the next section). As a consequence, the disk RC is contained in the PCA basis, and subtracting or adding a disk component will not change the PCA.

2.2. Effect of Noise on the PCA Analysis

Numerical experiments were performed by adding Gaussian noise (as provided by dBB and DMR) to the velocities of the QRCs. The effect of this noise is to change the amplitude of the QRCs' projections on each principal component. The variance with respect to a given PCA vector is obtained by summing the square of the corresponding amplitudes for the QRCs sample. By normalizing this variance by the total QRCs' variance we obtain the relative amplitude of the PCA component in the power spectrum. This relative amplitude is affected by the simulated noise, and the amplitude of the corresponding fluctuation on the power spectrum for each component is about $(0.035)^2$.

As a consequence, only the first two components are significant (see Figures 1 and 2). The PCA for the four samples has been plotted in Figures 1 and 2. The PCA variations are small; for instance, Figure 1 demonstrates that the differences between the PCA samples are of the order of the noise amplitude. In the former experiment, the PCA vectors were kept fixed. Let us now investigate the stability of these two components with respect to the noise fluctuations using Monte Carlo simulations, on the velocities (MCV) and the re-scaling errors (MC S). The simulations MCV were performed by adding Gaussian noise with variance taken from dBB and DMR to the sample of QRCs. The singular value decomposition was re-computed for each simulated sample to estimate the corresponding variance on the principal components. The result of 1000 simulations shows that the noise accounts for about 0.5% of the total variance on the first component and 15% of the total variance on the second component.

The PCA fluctuations due to the re-scaling errors (MC S) are estimated by introducing uniform random fluctuations of the scale length. Each QRC of the sample was re-scaled using the simulated scale length and the PCA was re-conducted for each simulated sample. The results show that for uniform deviations of the scale length with respective amplitudes of 10%, 20%, and 40%, the corresponding variances of the noise as a fraction of the total variance of the second component are 2%, 4%, and 9%, respectively. In addition to these results, note that the mean correlation coefficients between the simulated second
component and the original one is 0.9 when noise fluctuations are considered and higher for scale length fluctuations. The first component is much less affected and has a mean correlation close to unity. In addition to these numerical experiments, the stability of the PCA with respect to the scaling errors can be demonstrated analytically. Let us consider a re-scaled QRC, $F(r)$ with the following PCA decomposition: $F(r) = au_1(r) + \beta u_2(r)$, with $u_1(r)$ and $u_2(r)$ being the first two PCA vectors. Since $\beta$ is small with respect to $\alpha$ (see Figure 1), $\beta = \eta \alpha$, and $\eta = \epsilon \eta$ with $\epsilon \ll 1$. Let us now introduce a re-scaling error with scale length $1 + h = 1 + \epsilon \tilde{h}$, then to first order in $\epsilon$, $F_{1 - h} = \alpha(u_1(r) - \left(\frac{d}{dr}\right)\epsilon h + \eta u_2(r))$. Since $u_1$ is very close to a power law, $(\frac{d}{dr})r \simeq \gamma u_1$, and finally

$$F \left( \frac{r}{1+h} \right) \simeq \alpha((1 - \gamma h)u_1(r) + \eta u_2(r)). \quad (1)$$

Thus, as consequence of Equation (1), the re-scaled QRC is re-mapped to the initial set of PCA vectors, which explains the stability of the decomposition with respect to the choice of the scale length.

2.3. Interpretation of the Components

The first component represents approximately the mean of all curves. The ratio of the second component to the first component $\eta$ is strongly correlated to mass concentration parameter $M_{1/3}$ (Pearson correlation is 0.75). $M_{1/3}$ is the ratio of the dynamical mass at one-third of $R_{\text{max}}$ to the dynamical mass at $R_{\text{max}}$. The dynamical masses were derived under the assumption of a spherically symmetric mass distribution.

3. RELATION BETWEEN ROTATION CURVE AND GALAXY PARAMETERS

The QRCs of the sample are properly approximated with two PCA components, and the ratio of these components $\eta$ is associated with the concentration of mass at the center. Let us now investigate the relation between $\eta$ and the other galaxy parameters: absolute magnitude $M_g$ in R band, central surface brightness $\mu_R$ in R band, scale length $h_R$ in R band, mass of gas $M_g$ at $r = R_{\text{max}}$, and dynamical mass $M$ at $r = R_{\text{max}}$. $M_R$, $\mu_R$, and $h_R$ were derived from dBB (Table 1) and DMR (Table 1). $M_g$ and $M$ are derived from the RCs under the assumption of a spherically symmetric mass distribution. We also derive the stellar luminosity, $S \propto 10^{-M_g / 2.5}$. A significant correlation is found between the gas fraction $F_g = M_g / M$ and $\eta$ (see Figure 3). No significant correlation is found between $\eta$ and other available variables (see Table 1). Note that the main results obtained in Table 1 were derived using Sample 3, with the following inclination cuts, $30^\circ < i < 70^\circ$. However, the correlations do not change by any significant amount if other samples are used as also illustrated in Table 1. Another interesting point is the consistency between the two data sets. Taking Sample 4 alone, the probability of the association between the gas fraction and $\eta$ is high: 99%. By introducing cuts in inclination in the dBB sample ($i < 70^\circ$ or $30^\circ < i < 70^\circ$) the number of QRCs is low, and the significance falls below 90%. However by supplementing the dBB sample with the curves from the DMR sample, in both cases ($i < 70^\circ$ and $30^\circ < i < 70^\circ$) the significance of the association rises back to 98%. In addition to the Spearman correlation coefficient, the significance of the correlation is also analyzed by linear least-square fitting (with errors in both directions). The

![Figure 3](image_url)

**Figure 3.** Ratio of the second to the first principal component as a function of the gas fraction for Sample 1 (all symbols), Sample 2 (green+red symbols), and Sample 3 (green symbols).

(A color version of this figure is available in the online journal.)

| Variable | $\log(M_g / M)$ | $M_R$ | $\mu_R$ | $\log(h_R)$ | $\log(S / M)$ |
|----------|----------------|-------|---------|-------------|---------------|
| $S_C$    | -0.63          | 0.38  | 0.01    | -0.29       | -0.5          |
| $S_S$    | 0.02           | 0.3   | 0.97    | 0.32        | 0.09          |
| $s$      | -0.24          | 0.05  | 0.015   | -0.08       | -0.14         |
| $|s|/\sigma$ | 3.3           | 0.62  | 0.17    | 1.06        | 1.64          |

| Sample | $S_C$ | $S_S$ | $s$ | $|s|/\sigma$ |
|--------|-------|-------|-----|-------------|
| Sample 1 | -0.53 | 0.011 | -0.13 | 2.85       |
| Sample 2 | -0.55 | 0.021 | -0.26 | 3.17       |
| Sample 3 | -0.63 | 0.021 | -0.24 | 3.34       |

| Sample 1 -0.625 | Sample 2 -0.632 | Sample 3 -0.635 | Sample 4 -0.633 |

**Notes.** The effects of changing the inclination cuts and the PCA samples are shown in the central and lower parts of the table, respectively.
significance of the correlation is evaluated by estimating the probability that the Spearman correlation coefficient is different from zero, however this evaluation is complicated by the fact that each point of a given curve has a different variance. The Spearman correlation analysis should be considered as an evaluation of the real correlation between the variables. The straight line fit analysis has the advantage of taking into account the differences in variance and provides a direct estimate of the correlation by evaluating the probability that the fitted slope is different from zero. The statistical errors on the different variables and $\eta$ are computed using the initial errors on the light curve data points. A least-square minimization with errors in both variables is conducted (Press et al. 2007). The fit $\chi^2$ is $\gtrsim 3$, which is large and indicates that other errors, probably due to an intrinsic noise in the relation between the variables, are present. It is assumed that the intrinsic noise amplitude $\sigma_0$ is constant for each galaxy. The value of $\sigma_0$ is unknown and the full space of possible realizations has to be explored by running a large number of numerical simulations, for all possible values of $\sigma_0$. Among the full space of possible realizations, one has to select the subset of simulations that is consistent with the data. A simple measure of the consistency is that when fitting a straight line, the reduced chi-square of the simulation, $\chi^2_0$, is equal to the reduced chi-square of fitting a straight line to the data $\chi^2_d$. In practice, the subset of simulations consistent with the data is selected using the criteria, $|\chi^2_d - \chi^2_0| < 0.01$. The statistical distribution of the slope for the subset is well approximated by a Gaussian, with a standard deviation $\alpha$. The statistical significance of the slope $b$ is evaluated by computing the ratio, $b/\alpha$ (see Table 1). The association between $\eta$ and the gas fraction is very significant, both in the Spearman correlation coefficient (2% chance of no association between $\eta$ and the gas fraction) and the straight line fit (3.5$\sigma$). This second method confirms that there is no significant association of other variables with $\eta$. One possibility would be that the association between $\eta$ and the gas fraction is due to a systematic effect and reflects an association of each variable with another variable. However, since there is no correlation between the second variable and any other variables except $\eta$, this hypothesis can be rejected. A final point is the possible influence of the re-scaling errors on the association between $\eta$ and the gas fraction. The introduction of a scaling error with length scale $1+h$ modifies the ratio $\eta$, the new value of $\eta$, $\tilde{\eta}$, is given by Equation (1):

$$\tilde{\eta} = (1 + \gamma h)\eta.$$  

Let us now assume that the re-scaling error depends on the gas fraction $F_G$, $h = h(F_G)$, and let us examine the systematic effect on the correlation between $\eta$ and $F_G$. The variation of $\eta$ due to the re-scaling errors in Equation (2) is $\delta\eta = \eta \gamma h(F_G)$. If there are no correlation between $\eta$ and $F_G$, the additional correlation introduced by $\delta\eta$ is null too. This calculation demonstrates that the association between $\eta$ and $F_G$ cannot be due to a relation between the re-scaling error and the gas fraction.

3.1. Influence of the Disk on the Results

3.1.1. Relation between the Stellar and Gas Fraction

The first step is to relate the disk mass fraction $F_D$ to the gas fraction $F_G$. Galaz et al. (2002) proposed that the total gas and stellar mass is a constant fraction of the total mass. The current data set confirms this proposition, by fitting the log of the total gas+stellar mass at $R_{\text{max}}$ as a function of the log of the total dynamical mass at $R_{\text{max}}$ we find that the slope is very close to unity (0.94). Thus,

$$F_D + F_G \simeq C_0.$$  

(3)

Considering an RC $v_2^2 = \alpha_0 u_1 + \beta_0 u_2$, with corresponding mass $M_0$ at $R_{\text{max}}$ and adding a disk component with square velocity $v_d^2$ and mass $M_d = F_D M_0$, the initial coefficient $\alpha_0 = \frac{d_0}{\sigma_0}$ becomes

$$\eta = \frac{\beta_0 + \langle v_d^2, u_2 \rangle}{\alpha_0 + \langle v_d^2, u_1 \rangle}.$$  

(4)

where $\langle v_d^2, u_n \rangle$ is the scalar product with PCA component number $n$. Assuming that $\eta$, $F_G$, and $F_D$ are small quantities, it is straightforward to estimate the relative correction to the slope of the $\eta$ versus $F_G$ relation. $S_D = 1 - \frac{\beta_0}{\alpha_0}$, with $S_0$ being the unperturbed slope. Using Equations (3) and (4), the relative correction reads

$$\eta_D = \frac{\alpha_0 (R_{\text{max}})}{v_d^2 (R_{\text{max}})} \left( \frac{\langle v_d^2, u_2 \rangle}{\sigma_0} + \langle v_d^2, u_1 \rangle C_0 \right).$$  

(5)

$\eta_D = \simeq 0.15$. Note that since $\langle v_d^2, u_2 \rangle < 0$ and $S_0 < 0$, the correction is always positive, thus the effect of the disk correction is to increase the significance of the relation between $\eta$ and $F_G$.

4. DISCUSSION

The mass concentration at the center is related to $\eta$. Furthermore, $\eta$ is related to the gas fraction. The parameter $\eta$ is also related to mass model fitting of the QRCS (see Figure 4). Negative values of $\eta$ are associated with a more concentrated mass profile, shorter Burkert scale length, and better consistency with an NFW profile when compared with positive values of $\eta$. As a consequence, gas-rich galaxies will also be associated with shorter Burkert scale and will be closer to an NFW profile. Kim (2007) showed that the gas fraction is related to the evolution of galaxies, gas-poor galaxies had more stellar formation in the past than gas-rich galaxies with respect to current star formation. The results by Kim (2007) are consistent with Galaz et al. (2002), who found that gas-rich galaxies are less evolved than
gas-poor galaxies and that the enrichment is still going on in the gas-rich galaxies. The correlation between gas content and star formation is also confirmed by Wyder et al. (2009), who found that the star formation rate surface density is correlated with the gas density divided by the orbital time in the disk. Zhong et al. (2008, 2010) note also some systematic differences in evolution between blue and red LSBGs. Statistically, the gas-rich galaxies are less evolved, have lower stellar fractions, and have a better consistency with NFW profiles. Provided that the initial dark matter profile is close to an NFW profile, the effect of galaxy evolution via supernovae explosions is to form shallower profiles with nearly constant density cores (Larson 1974; Read & Gilmore 2005). The effect of tidal evolution via supernovae explosions affects both the stellar and the dark matter distributions (Read & Gilmore 2005).

Star formation, galaxy interactions, and other effects related to galaxy evolution flatten the inner mass profile, and at the same time reduce the gas fraction in the galaxy. The relation between mass profile and gas fraction is thus most likely a correlation between galaxy evolution and mass profile. It is likely that gas-poor galaxies were subject to more interactions with other galaxies, boosting stellar formation and supernovae, while at the same time the tidal interaction had also some influence on the mass profile. An analysis of SDSS data by Rosenbaum & Bomans (2004) shows that gas-rich LSBGs form in the voids and that some migrated to the edge of filaments while others remain in the voids. This scenario implies that LSBG galaxies had less tidal interaction than other brighter galaxies, although they still have some neighbors and furthermore, the level of tidal interaction may have increased recently. This would explain the differences of evolution in this sample, and the corresponding differences in the QRCs and mass profiles. To conclude, it would be interesting to extend the present analysis to high surface brightness galaxies (e.g., Gentile et al. 2004; Salucci et al. 2007).

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