Effect of thermophysical property of energy pile on heat transfer based on the dimensionless analytical solution

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Abstract. Energy pile, as the heat exchanger extracts heat from the ground, is gradually accepted by engineers. When using it, enhancing the heat transfer power is the critical factor, and there are many ways to achieve this goal. This paper analyses the effects of the energy pile's thermophysical property changing on the heat transfer via using the dimensionless mathematical model. Firstly, it transformed the mathematical model from the literature [1] to a dimensionless form. After obtained the analytical solution, the calculation results verified that the correctness of a dimensionless analytical answer. On this basis, discussed the effect of the difference of thermophysical properties between the pile and surrounding soil on heat transfer in typical heat transfer time. The results show that thermal conductivity difference affects heat transfer during the whole heat transfer period. In contrast, the diversity of volumetric heat capacity affects the heat transfer only at the initial stage of heat transfer and the pile’s thermal conductivity is small. Therefore, engineers should fully consider the thermophysical properties difference between the pile and surrounding soil. To improve the heat transfer of the energy pile, enhancing thermal conductivity is the most effective way.

1. Introduction
After World War II, with the growth of the number of human beings and the endless pursuit of a comfortable life, the burden on the earth is getting heavier, resulting in a large number of greenhouse gas emissions, global temperature rise, haze, and other negative feedback. As a result, renewable energies characterized by no fuel, no emission, and low volatility have become the hotspot of research and application, geothermal energy is one of them.

As early as 1900, genius physicist Tesla pointed out the use of heat from the earth's interior [2]. Since the 1980s, with the full cooperation of structural engineers and HVAC engineers, energy pile has implemented the function of the heat exchanger after satisfying the structural load. As a result, it saves

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the construction cost of the borehole heat exchanger and is less affected by the surface temperature than the horizontal heat exchanger. Here complementary lines of energy pile’s research can be distinguished, one focusing on thermal analysis and another focusing on thermo-geomechanical study [3]. The former improves the efficiency of geothermal energy, while the latter ensures the safety of the structure. In thermal analysis, mathematical models, numerical models, and experimental methods are chosen to explore the most efficient heat transfer method.

The mathematical model, aiming at the critical parameters, abstracts and simplifies the heat transfer process, puts forward the mathematical model and solves it. Although the solving process is complex and the parameters are limited, it is convenient to use in energy pile’s evaluating and designing. The research on the mathematical model began with Lord Kelvin's infinite line heat source model [4], which is widely used in the design of energy pile and the inversion of thermophysical parameters of soil in TRT up till now. The solid cylindrical model, which Prof. Man Yi proposed, considers the influence of the pile itself on heat transfer for the first time [5]. Prof. Li Min skillfully extended the mechanical working condition to the energy pile’s heat transfer, and put forward the moving line heat source model of the composite medium [6]. Based on the law of energy conservation, Prof. Yang Jun puts forward a cylindrical model which can consider the difference of thermophysical properties inside and outside the pile, which clearly shows the heat transfer distribution of the energy pile to itself and the surrounding soil [1]. Subsequently, models considered the pipe configuration, the pile’s geometric dimension, the soil’s vertical temperature distribution, and the seepage are also derived and solved [7]. The numerical model can simulate the whole heat transfer process of the energy pile accurately and analyze the influence of various factors on different heat transfer stages. Francesco Cecinato and Fleur A. Loveridge analyze the heat transfer of the U-shape pipe energy pile via ABAQUS. The result show that the influence is in the following order: the number of U-shaped pipes (total surface area of the pipe), the length of the pile, concrete thermal conductivity, pile diameter, cover thickness, pipe diameter and liquid flow velocity [8]. For the pipe configurations, Zhao Qiang, Zarrella et al. used COMSOL [9] and thermal resistance model (CaRM) [10] to compare the transient heat transfer of various types of pipe configuration. The result shows the energy pile with spiral pipe owned the better heat transfer performance, under the same heat flux condition, the temperature difference between the heat-carrying fluid and ground is more minor; at the same time, the study shows that the turn spacing of spiral tube has little effect on long-term heat transfer [11-13]. Daehoon Kim found that when the heat exchangers work continuously, the grouting thermal conductivity positively affects the heat transfer performance, while in indirect work, both thermal conductivity and specific heat play a role [14]. When studying the heat transfer of energy pile, the numerical model needs more parameters, and it often needs experimental research to provide parameters or comparative verification. Liu Hanlong, Li Xiangyu et al. studied the heat transfer of energy pile by 1:20 scale ratio model and field test, respectively [15, 16]. The measure model test shows that the heat transfer of the energy pile in the saturated sand is mainly along the radial direction [17].

As mentioned above, many factors are affecting the heat transfer of the energy pile. In practical, the thermophysical property of the soil around the pile is unchangeable, and the structure load determines the geometric dimensions of the pile. When selecting the spiral pipe, the adjustment of the pile material further improves the heat transfer efficiency of the energy pile. The analytical solution
proposed in reference [1] takes the spiral pipe energy pile as the simplified object, considering
the difference of thermophysical properties between the pile and surrounding soil. The author has already
analyzed the excess temperature results with different thermophysical parameters between the pile and
soil[18]. However, the specific influence of thermophysical parameters should be further discussed.

The remainder of the paper is organized as follows. Firstly, the mathematical model from the
literature [1] is dimensionless. After obtained the analytical solution, the calculation results verify that
the dimensionless analytical solution is correct. On this basis, the effect of the difference of thermophysical
properties between the pile and surrounding soil in typical heat transfer time on heat transfer is discussed, and the opinions and suggestions for improving the pile’s heat transfer efficiency
are proposed and summarized.

2. Solution and verification of the dimensionless model

As shown in Figure 1, assuming the heat source locates at the surface of the pile, divided the heat
transfer space into two parts: the pile inside the heat source and the soil outside the heat source, and
$i=1, 2$ represent the energy pile and the surrounding soil, respectively. Define $\alpha = C_1/C_2$ and $\beta = k_1/k_2$;
$a_i = k_i/C_i$ is the volumetric heat capacity in J/(m$^3\cdot$K); $k_i$ is the thermal conductivity in W/(m$\cdot$K);
ia_i = k_i/C_i is the thermal diffusivity in m$^2$/s; the parameters as $r, T_i, t, q_i$ are the dimensionless quantity
of $r', T_i', t', q_i'$ respectively. Then $r = r'/r_0, T = k_0\theta/Q, t = a_2 t'/r_0^2, q_i = q_i'(2\pi Q). \theta_i$ is the excess temperature
in °C; $Q$ is the heating rate per length of the pile in W/m; $q_i'$ is the heat transfer rate per length to the
either side in W/m; $r'$ is the radius of the calculation position in m, $r_0$ is the radius of the energy pile in
m, and $t'$ is the calculation time in s.

![Figure 1. Heat transfer model](image)

(1) Heat transfer equation in cylindrical coordinates:

$$\frac{\partial^2 T_1(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_1(r,t)}{\partial r} = \frac{\alpha}{\beta} \frac{\partial T_1(r,t)}{\partial t}$$  \hspace{1cm} (2-1)

$$\frac{\partial^2 T_2(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T_2(r,t)}{\partial r} = \frac{\partial T_2(r,t)}{\partial t}$$  \hspace{1cm} (2-2)

(2) Boundary conditions:

$$\left. \frac{\partial T_1(r,t)}{\partial r} \right|_{r=0} = 0$$  \hspace{1cm} (2-3)

$$\left. \frac{\beta \partial T_1(r,t)}{\partial r} \right|_{r=1} = q_i$$  \hspace{1cm} (2-4)

$$\left. -\frac{\partial T_2(r,t)}{\partial r} \right|_{r=1} = q_2$$  \hspace{1cm} (2-5)
\[ q_1 + q_2 = \frac{1}{2\pi} \]  
\[ T_1(r,t) \big|_{t=1} = T_2(r,t) \big|_{t=1} \]  
\[ T_2(r,t) \big|_{t=\infty} = 0 \]

(3) Initial conditions:

\[ T_1(r,t) = T_2(r,t) = 0 \]

After the Laplace and inverse Laplace transformation, the dimensional analytical solutions of excess temperature are obtained as (2-10) \sim (2-13).

\[ T_1(t,r) = \frac{1}{2\pi^2} \int_0^\infty \frac{J_0(\sqrt{\lambda})}{\lambda} \frac{P}{\sqrt{\lambda}} d\lambda \]  
\[ T_2(t,r) = \frac{1}{2\pi^2} \int_0^\infty \frac{J_0(\sqrt{\lambda})}{\lambda} \frac{P}{\sqrt{\lambda}} d\lambda \]

\[ \sqrt{\alpha\beta}J_1(\sqrt{\lambda})J_0(\sqrt{\lambda}) = J_0(\sqrt{\lambda})J_1(\sqrt{\lambda}) = P \]  
\[ \sqrt{\alpha\beta}J_1(\sqrt{\lambda})Y_0(\sqrt{\lambda}) = J_0(\sqrt{\lambda})Y_1(\sqrt{\lambda}) = Q \]

To verify the correctness of the analytical solution, the thermophysical properties of concrete and dry clay are shown in Table 1. The value of \( \alpha = C_1/C_2 = 5.35 \), \( \beta = k_1/k_2 = 4 \), picking the heat transfer time are 1~10 day, the calculation results are shown in Figure 2. The results calculated by the analytical solution and the dimensionless analytical solution are the same, verifies the correctness of the dimensionless analytical solution from the calculation result.

**Table 1.** Thermal parameters of typical materials

| Material     | \( k \) (W m\(^{-1}\)K\(^{-1}\)) | \( c \) (J kg\(^{-1}\)K\(^{-1}\)) | \( \rho \) (kg m\(^{-3}\)) | \( a \) (m\(^2\) s\(^{-1}\)) | \( C \) (J m\(^{-3}\)K\(^{-1}\)) |
|--------------|-------------------------------|---------------------------------|-----------------|-----------------|-------------------|
| Concrete     | 2                             | 1000                            | 2500            | 8\times10\(^{-1}\) | 2425000           |
| Dry clay     | 0.50                          | 270                             | 1680            | 1.10\times10\(^{-6}\) | 453600           |

![Figure 2. Calculation results by analytical solution and dimensionless analytical solution](image-url)
Meanwhile, from the temperature distribution, the value of excess temperature raised with the heat transfer time, and the value at the heat source is the largest, which means the effects of different factors on heat transfer are the most obviously at this point. So, the temperature variation of this point is followed as the analytical objects.

3. Analysis of the influence of thermophysical parameters on heat transfer

Chosen the typical dimensionless heat transfer time $t=1, 5, 10$, convert to actual heat transfer time with parameters shown in Table 1 is about 2.6 day, 13.2 day, and 26.3 day. When heat is transferred at the equal power, the study focuses on the variation of $T$ when the values of $\alpha$ and $\beta$ change as $[0.2:0.1:5]$, where the value of $T$ is reduced, that is, the temperature enriched at the surface of the pile, has the better the heat transfer effect. When $t=1$, the effect of different value of $\alpha$ and $\beta$ on heat transfer is shown in Figure 3. The values of $T$ are reduced with the increase of the value of $\beta$, and the larger the value of $\alpha$ at all, the smaller the value of $T$, which means when the thermal conductivity of the pile is gradually increased, the volumetric heat capacity increases, the heat transfer efficiency is enhanced. To further illustrate the effect of $\alpha$ changing on heat transfer, picking the typical value of $\beta=0.2, 1, 2, 3, 4, 5$, the changing of $\alpha$ affects the heat transfer as shown in Figure 4. When $\beta=1$, the curve of $\alpha-T$ shows that when the value of $\alpha$ in the range of $0.2-1$, the smaller the $\alpha$, the bigger the $T$, which means the smaller the volumetric heat capacity of pile, the less power transferred from the pile. When the value of $\beta$ increased, the variation range of $\alpha$ is getting larger. In order to quantify this, define two parameters, for the step $dl$, $\Delta T_{\beta}=T(\beta,\alpha)-T(\beta+dl,\alpha)$, $\Delta T_{\alpha}=T(\beta,\alpha)-T(\beta,\alpha+dl)$. If $\Delta T_{\beta}>\Delta T_{\alpha}$, the thermal conductivity has greater effect on heat transfer, when $\Delta T_{\beta}>5\Delta T_{\alpha}$, the effect of volumetric heat capacity can be ignored. As shown in Figure 5, as the value of $\beta$ increased, the volumetric heat capacity’s effect increases gradually, the range of negligible the volumetric heat capacity is reduced. But the thermal conductivity is continuously dominating the pile’s heat transfer.

![Figure 3. The value of $T$ calculated by different $\alpha$ and $\beta$ when $t=1$](image-url)
Figure 4. The value of $T$ changed with $\alpha$ when $t=1$

With the increase of heat transfer time, as $t=5, 10$, the trend that the value of $T$ increases with the value of $\beta$ decreases does not change, when $\beta=0.2$, $T$ still does not change with $\alpha$. For other values of $\beta$, the effect of $\alpha$ on $T$ is smaller than that of $t=1$, as shown in Figure 6 and Figure 8. However, in the range of $\alpha$’s effect, with the increased value of $\beta$ and decreased value of $\alpha$, the value of $T$ decreases at beginning and then increases, that is, with the increase of thermal conductivity of energy pile and the decreases of volumetric heat capacity, the effect on heat transfer from positive to negative, but does not change the trend that the thermal conductivity increases and the heat transfer efficiency increases. Moreover, with the increase of heat transfer time, the change of $\beta$ is dominant, and the change of $\alpha$ can be ignored at $t=10$.

At the same time, through the comparative analysis of Figure 4, Figure 6, and Figure 8, the calculated values of $T$ are all about 0.5 when $t=1$, $\beta=0.2$; $t=5$, $\beta=1$; $t=10$, $\beta=5$. It can be seen that increasing the thermal conductivity of the pile can effectively improve the heat transfer capacity of the energy pile.

It is worth noting that when $\alpha=\beta=1$, is dimensionless analytical solution of the solid cylindrical model, according to the Figure 5, Figure 7, and Figure 9, ignoring the volumetric heat capacity difference between the pile and surrounding soil hardly affect the calculation result. However, neglecting the thermal conductivity difference may result in a significant error in the calculation result.
Figure 6. The value of $T$ changed with $\alpha$ when $t=5$

Figure 7. $\alpha$ and $\beta$ effect on the heat transfer when $t=5$

Figure 8. The value of $T$ changed with $\alpha$ when $t=10$

Figure 9. $\alpha$ and $\beta$ effect on the heat transfer when $t=10$

4. Conclusions
In this study, the dimensionless analytical solution of the heat transfer model is obtained and used to analyze the effect of thermal parameters on heat transfer of energy pile. The conclusions are as follows:

1. When the single energy pile transfers heat with equal power, the difference in thermophysical properties between the pile and surrounding soil cannot be ignored, especially when the thermal conductivity of the pile is greater than that of the soil. Thoroughly consider the thermophysical property difference, the calculated excess temperature is small. In design, the heat transfer capacity of the energy pile is conservative due to ignoring the difference of thermophysical properties, which cannot give full play to the actual heat transfer performance of the energy pile.
2. With the increase of heat transfer time, the effect of thermal conductivity on heat transfer increases gradually; the effect of volume heat capacity difference can be ignored gradually. Thus, it can be seen that the change of the thermal conductivity of the pile has a significant effect on the heat transfer, while the volumetric heat capacity of the pile has a limited effect on the heat transfer.

3. In the heat transfer process, enhancing the thermal conductivity of the pile can effectively improve the heat transfer efficiency of energy pile. Therefore, to improve the heat transfer capacity, the method should focus on enhancing the thermal conductivity.

In this study, the length of the energy pile is ignored during the heat transfer time. Moreover, in practical engineering, energy piles often appear as pile group, the volume of the soil around the pile is limited. This paper fails to combine the above factors for comprehensive analysis, further analysis needs to be done in the follow-up research.

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