Cavity Sub- and Superradiance Enhanced Ramsey Spectroscopy

Christoph Hotter, Laurin Ostermann, and Helmut Ritsch

1 Institut für Theoretische Physik, Universität Innsbruck, Technikerstr. 21a, A-6020 Innsbruck, Austria

(Dated: January 25, 2022)

Ramsey spectroscopy in large, dense ensembles of ultra-cold atoms trapped in optical lattices suffers from dipole-dipole interaction induced shifts and collective superradiance limiting its precision and accuracy. We propose a novel geometry implementing fast signal readout with minimal heating for large atom numbers at lower densities via an optical cavity operated in the weak single atom but strong collective coupling regime. The key idea is controlled collective transverse $\pi/2$-excitation of the atoms to prepare a macroscopic collective spin protected from cavity superradiance. This requires that the two halves of the atomic ensemble are coupled to the cavity mode with opposite phase, which is naturally realized for a homogeneously filled volume covering odd and even sites of the cavity mode along the cavity axis. The origin of the superior precision can be traced back to destructive interference among sub-ensembles in the complex nonlinear collective atom field dynamics.

In the same configuration we find surprising regular self-pulsing of the cavity output for suitable continuous illumination. Our simulations for large atom numbers employing a cumulant expansion are qualitatively confirmed by a full quantum treatment of smaller ensembles.

INTRODUCTION

Precise measurement of the energy difference of atomic transitions and their corresponding frequencies has been at the heart of atomic physics for more than a century. Nowadays, laser spectroscopy features incredible accuracy and precision of up to 20 digits [1]. These advances allow for precise tests of fundamental physics, from general relativity and astrophysics to effects of super-symmetry in strong interactions with nuclei or even the spectral properties of antimatter [2, 3]. Yet, there is still a strong desire to push these limits even further, while, in parallel, one also aims at technologically simpler, faster and more robust methods to reach a desired precision.

Ramsey spectroscopy, employing a carefully engineered sequence of time delayed short phase-coherent pulses, can be used to resonantly excite a particle to a long-lived and thus energetically very narrow state. It is one of the most commonly used and well proven methods in precision spectroscopy. Applied to large ensembles of ultra-cold atoms trapped in a magic wavelength optical lattice it constitutes the current state of the art [4]. In order to realize precise measurements fast, a large number of particles needs to be interrogated. They should be confined to a small volume in order to ensure a homogeneous environment. Here, the remaining weak inter-particle interactions are a source of inaccuracy and the emerging fast superradiant collective decay limits the maximal measurement time and precision [5].

The measurement readout requires a precise determination of the fraction of excited atoms for a given probing frequency. In principle, one has to measure the induced fluorescence after the Ramsey sequence. However, for very long lived clock states this is too slow, inefficient and imprecise in order to be practical and more elaborate procedures are necessary. Hence, besides methods relying on additional fast transitions [6], using cavity enhanced detection of dispersive shifts or collective decay has been suggested [2, 7]. However, while cavity superradiance is fast and precise for counting excitations [8, 9], the presence of the cavity perturbs the free evolution between the Ramsey pulses, which again limits measurement precision.

In this work we propose an improved implementation of cavity enhanced Ramsey spectroscopy combining fast collective readout with high precision. The central idea is to address special collective excited states of the atomic ensemble, which decouple from the cavity during the free evolution phase [10, 11]. The final readout is still performed via the superradiant manifold allowing for fast detection of the ensemble. Surprisingly, despite sounding challenging and complicated, there is a very natural and simple possibility to implement this scheme for a large
but diluted ensemble within a standing wave cavity.

Model. – We consider \( N \) two-level atoms with a narrow transition at frequency \( \omega_a \) coupled to a single mode cavity. The atoms are coherently driven with a detuning between the laser and the atomic transition frequency of \( \delta_a = \omega_l - \omega_a \), the corresponding Rabi frequency is denoted by \( \Omega \). The cavity is detuned by \( \delta_c = \omega_l - \omega_c \) from the laser and we have an atom-cavity coupling of \( g_j \) for the \( j \)-th atom. The system is depicted in Fig. 1. Its Hamiltonian in the rotating frame of the pump laser reads

\[
H = -\delta_c a \hat{a}^\dagger + \sum_{j=1}^N \left[ -\delta_a \sigma_j^{22} + g_j(\sigma_j^1 \sigma_j^1 + \sigma_j^{21}) \right],
\]

(1)

with the cavity photon creation (annihilation) operator \( \hat{a} \) and the atomic transition operator \( \sigma_j^{kl} = |k\rangle_j \langle l|_j \) for the \( j \)-th atom. The coherent interaction is accompanied by dissipative processes, which are accounted for by the Liouvillian \( \mathcal{L}[\rho] \) in the master equation,

\[
\dot{\rho} = i[\rho, H] + \mathcal{L}[\rho].
\]

(2)

In the Born-Markov approximation [12] we can write the Liouvillian in Lindblad form as

\[
\mathcal{L}[\rho] = \sum_i R_i \left( 2 J_i \rho J_i^\dagger - J_i^\dagger J_i \rho - \rho J_i^\dagger J_i \right),
\]

(3)

with the jump operator \( J_i \) and its corresponding rate \( R_i \) for the \( i \)-th dissipative process. We list these processes in Tbl. I, including cavity photon losses as well as decay and dephasing of the atoms with rate \( \Gamma \) and \( \nu \), respectively.

As we are targeting narrow clock transitions for our atoms the system is operated in the bad and large volume cavity regime \( \kappa \gg \Gamma \) with only a small single atom cooperativity \( C_j = g_j^2 / (\kappa \Gamma) \ll 1 \) but a sufficiently large ensemble to enter the strong collective coupling regime \( NC = \sum_j C_j \gg 1 \). Typically, this parameter regime implies a very large atom number \( N \gg 1 \), which does not allow for a full quantum simulation, but we can very well treat this problem in a second order cumulant expansion [13, 14]. A comparison with a full quantum simulation for a small atomic ensemble is shown in the appendix [15]. For a qualitative illustration of our deliberations even first order cumulant equations, i.e. mean field, (see appendix) suffice for most calculations. Additionally, we neglect dipole-dipole interaction [16], as our atomic ensemble is sufficiently dilute such that neither dispersive, nor dissipative collective processes among the individual atoms will play a considerable role.

Collective Cavity mediated Super- and subradiance. – In the following, for simplicity, we assume the atoms located close to cavity mode anti-nodes with half of the atoms at the maxima and half at the minima of the mode function along the cavity axis. Hence their respective effective coupling is well approximated by \( +g \) and \( -g \). As confirmed by more involved simulations, investigating various different distributions for the atom-field coupling, this simplification already captures the essential physics discussed below.

It is well established that inverting all atoms with a short \( \pi \)-pulse induces the emission of a delayed intense light pulse due to cavity enhanced superradiant decay [17–22]. Synchronized stimulated emission in a cavity occurs even for a dilute ensemble, which does not exhibit free space superradiance. Figure 2a shows typical trajectories for the corresponding time evolution of the intra-cavity photon number \( \langle a^\dagger a \rangle \), when all atoms are initially coherently prepared at \( \langle a^\dagger a \rangle = 80\% \) (black line). Figure 2b depicts the corresponding time evolution of the excited state population. For comparison we show the behavior for all atoms equally coupled to the cavity (dashed line, \( g_j = g \)) as well.

However, for the system we consider \( (g_j = (-1)^j g) \),

\[
\begin{array}{ccc}
  i & J_i & R_i \\
 1 & a & \kappa \text{ cavity photon losses} \\
 2 & \sigma_j^{12} & \Gamma \text{ decay from } |2\rangle_j \text{ to } |1\rangle_j \\
 3 & \sigma_j^{22} & \nu \text{ dephasing of the } j\text{-th atom} \\
\end{array}
\]

Table I. Dissipative Processes. The system features a damped cavity mode as well as atomic decay and dephasing.
we will observe such pulsed emission for an ensemble of inverted atoms only. If the excited state population is below 50% the atoms are not able to synchronize and thus will not emit a significant amount of photons into the cavity, see solid orange line in Fig. 2. Figure 3a shows the total number of emitted photons $\langle a_\text{out} \rangle$ for different values of the coherently prepared initial excited state population. We can clearly observe that almost no photons leak through the cavity mirrors until the atoms are inverted. This is due to subradiant suppression as the photons emitted by the atoms coupled to the cavity with opposite $g$ will interfere destructively [23, 24]. For a fully inverted ensemble we obtain approximately one photon for each atom. However, for an initial inversion with $\langle \sigma^{22} \rangle < 1$ we only get $N \cdot (2(\sigma^{22}) - 1)$ photons. The solid lines in Fig. 2b also indicate that the atoms retain a population of $1 - \langle \sigma^{22} \rangle$ after the photon emission into the cavity and subsequently experience free-space decay with the rate $\Gamma$ only. Again, the dashed line represents the case of all atoms coupling equally ($g_j = g$), where we see that even not fully inverted atoms will superradiantly emit photons into the cavity. Additionally we plot the peak intra-cavity photon number (blue) illustrating the same behavior. Figure 3b shows the delay time of the peak photon number as a function of the atomic excitation. For uniform coupled atoms (dashed line) a higher excitation leads to a later pulse. Whereas for alternating coupling (solid line) the delay time of the peak decreases for higher inversion, with a maximum at $\langle \sigma^{22} \rangle t = 0 = 50\%$.

Note, that a crucial requirement for cavity subradiance is that the cumulative dipole moment of the atoms projected on the cavity mode $\sum_j g_j \langle \sigma^{12} \rangle$ vanishes [10, 11]. Here, for simplicity, we chose the same laser excitation phase for all atoms, but a variable excitation phase works equally well as long as the overall relative phase disappears. Fortunately, this case is typically automatically realized for a random distribution of a sufficiently large dilute intra-cavity atomic ensemble. Note that this is also true for a ring-cavity featuring a continuous atom-cavity coupling phase along the cavity axis [25].

Cavity Enhanced Ramsey Probing. – As argued above, transverse pumping with an overall vanishing phase of the atom-cavity coupling thus allows for a $\pi/2$-pulse excitation of the atomic ensemble without an immediate rapid superradiant decay through the cavity. This intriguing feature will be an essential ingredient for a new implementation of cavity assisted Ramsey spectroscopy as discussed below, when we combine it with fast direct measurements of the number of excited atoms via the emitted cavity photons after the second Ramsey pulse. The crucial advantage of this scheme is that it can be very fast with no additional manipulation of the atoms needed for the read out, hence the signal is less perturbed. Furthermore the atoms are not significantly heated by this measurement and can therefore be reused, thus, the dead time between measurements can be significantly reduced. Another advantage in comparison with other non-demolition measurements for atomic clocks [7, 26, 27] is that the signal, i.e. the number of photons, scales linearly with the number of atoms. So, in principle, an arbitrarily large number of atoms can be employed, which drastically increases the signal to noise ratio.

Figure 4a shows the output signal of the cavity Ramsey method, the total number of photons leaking through the cavity mirrors as a function of the laser-atom detuning $\delta_a$. Similar to the conventional Ramsey method, fringes appear [28–30]. One striking difference, however, is that a non-inverted ensemble does not produce a signal, corresponding to the flat zero-photon regions. This narrows the FWHM of the cavity-Ramsey fringes slightly compared to the conventional Ramsey fringes (see Fig. 4b). Including an atomic dephasing with $\nu = 10\Gamma$ (dashed line) merely weakens the signal, yet, the shape of the curve is essentially the same. Note, that by choosing $\delta_c = \delta_a$ we have implicitly assumed that the cavity is perfectly on resonance with the atomic transition. Therefore, one might wonder if a detuned cavity impairs the signal. But, since we operate deeply in the bad cavity regime, only shifts of the cavity resonance frequency on the order of $\kappa$ are important.

Overall, this means that enhancing the Ramsey spectroscopy by adding a cavity achieves the same (or even...
the very fast cavity photon decay. Since the laser is still the photon number quickly reduces to almost zero due to the excited state population is depleted below 50% and superradiant photon pulse into the cavity. Subsequently population inversion is achieved the ensemble emits a until state, there is no significant cavity photon number at least suppressed. Therefore, with atoms initially in the ground state, there is no significant cavity photon number at least.

Conclusions. – We have proposed and studied a new variant of cavity enhanced Ramsey spectroscopy which simplifies and accelerates the measurement procedure via tailored atom-field coupling. As it is particularly advantageous for long lived atoms it should be beneficial for the performance of optical atomic clocks. The underlying phenomena are, on the one hand, the well-known cavity superradiance in an inverted atomic ensemble in order to achieve a fast readout, and, on the other hand, the subradiant behavior of specific atomic ensembles with an overall vanishing collective cavity coupling. Furthermore, we found that the chosen operating conditions with weak single atom coupling but strong collective coupling also induce a striking self-pulsing instability for continuous drive at suitable Rabi frequencies. Interestingly the necessary operating conditions are within the reach of current experimental setups with only minimal adjustments required.

Some preliminary studies on the influence of imperfections in the setup as variable coupling strengths, slow atomic motion or fluctuations in the excitation procedure qualitatively yield very similar results for experimentally realistic assumptions. However, a more detailed study of these and other aspects as heating and loss is required.
for a quantitative prediction of the practical system performance.

We thank Stefan Schäffer for helpful discussions. We acknowledge funding from the European Union’s Horizon 2020 research and innovation program under Grant Agreement No. 820404 iqClock. The numerical simulations were performed with the open-source framework QuantumCumulants.jl [14] and QuantumOptics.jl [31].

Figure 5. Self-Pulsing. Time evolution of the cavity photon number (a) and excited state population (b) for a continuous drive, resulting in photon number self-pulsing. (c) and (d) Scans over $\Omega$, NC and $\delta_c$ for the peak photon number of the first pulse (red circle in (a)). The dashed white lines represent the threshold $2NCT > \Omega > |\delta_c|/(\Gamma + \nu)/2$. The parameters when kept constant are $N = 2 \times 10^5$, $g = 10\Gamma$, $\kappa = 10^4\Gamma$, $\delta_c = \delta_a = 0$, $\Omega = 100\Gamma$ and $\nu = 10$.

[1] M. Giunta, M. Fischer, W. Hänsel, T. Steinmetz, M. Lessing, S. Holzberger, C. Cleff, T. W. Hänsch, M. Mei, and R. Holzwarth, IEEE Photonics Technology Letters 31, 1898 (2019), URL https://doi.org/10.1109/LPT.2019.2955096.
[2] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, Reviews of Modern Physics 87, 637 (2015), URL https://link.aps.org/doi/10.1103/RevModPhys.87.637.
[3] N. Poli, C. Oates, P. Gill, and G. Tino, La Rivista del Nuovo Cimento 36, 555 (2013), URL https://doi.org/10.1393/ncr/i2013-10095-x.
[4] M. A. Norcia, A. W. Young, W. J. Eckner, E. Oelker, J. Ye, and A. M. Kaufman, Science 366, 93 (2019), URL https://www.science.org/doi/abs/10.1126/science.aay0644.
[5] S. Krämer, L. Ostermann, and H. Ritsch, EPL (Europhysics Letters) 114, 14003 (2016), URL https://doi.org/10.1209/0295-5075/114/14003.
[6] H. G. Dehmelt, IEEE transactions on instrumentation and measurement pp. 83–87 (1982).
[7] J. Lodewyck, P. G. Westergaard, and P. Lemonde, Physical Review A 79, 061401 (2009), URL https://link.aps.org/doi/10.1103/PhysRevA.79.061401.
[8] A. T. Black, J. K. Thompson, and V. Vuletić, Phys. Rev. Lett. 95, 133601 (2005), URL https://link.aps.org/doi/10.1103/PhysRevLett.95.133601.
[9] H. Zhang, R. McConnell, S. Ćuk, Q. Lin, M. H. Schleier-Smith, I. D. Leroux, and V. Vuletić, Phys. Rev. Lett. 109, 133603 (2012), URL https://link.aps.org/doi/10.1103/PhysRevLett.109.133603.
[10] L. Ostermann, H. Ritsch, and C. Genes, Physical review letters 111, 123601 (2013), URL https://link.aps.org/doi/10.1103/PhysRevLett.111.123601.
[11] L. Ostermann, D. Plankensteiner, H. Ritsch, and C. Genes, Phys. Rev. A 90, 053823 (2014), URL https://link.aps.org/doi/10.1103/PhysRevA.90.053823.
[12] C. Gardiner, P. Zoller, and P. Zoller, Quantum noise: a handbook of Markovian and non-Markovian quantum stochastic methods with applications to quantum optics (Springer Science & Business Media, 2004).
[13] R. Kubo, Journal of the Physical Society of Japan 17, 1100 (1962), URL https://doi.org/10.1143/JPSJ.17.1100.
[14] D. Plankensteiner, C. Hotter, and H. Ritsch, Quantum 6, 617 (2022), ISSN 2521-327X, URL https://doi.org/10.22331/q-2022-01-04-617.
[15] C. Hotter, L. Ostermann, and H. Ritsch, a code example to derive and numerically solve the equations for our system is provided in the supplementary material.
[16] Z. Ficek and R. Tanaś, Physics Reports 372, 369 (2002), URL https://www.sciencedirect.com/science/article/pii/S037015730200368X.
[17] J. A. Mlynek, A. A. Abdumalikov, C. Eichler, and A. Wallraff, Nature communications 5, 1 (2014).
[18] M. A. Norcia, M. N. Winchester, J. R. Cline, and J. K. Thompson, Science advances 2, e1601231 (2016), URL https://www.science.org/doi/abs/10.1126/sciadv.1601231.
[19] M. A. Norcia, J. R. Cline, J. A. Muniz, J. M. Robinson, R. B. Hutson, A. Goban, G. E. Marti, J. Ye, and J. K. Thompson, Physical Review X 8, 021036 (2018), URL https://link.aps.org/doi/10.1103/PhysRevX.8.021036.
[20] T. Laske, H. Winter, and A. Hemmerich, Physical Review Letters 123, 103601 (2019), URL https://link.aps.org/doi/10.1103/PhysRevLett.123.103601.
[21] S. A. Schäffer, M. Tang, M. R. Henriksen, A. A. Jørgensen, B. T. Christensen, and J. W. Thomsen, Physical Review Research 3, 013819 (2020), URL https://link.aps.org/doi/10.1103/PhysRevResearch.3.013819.
[23] S. Filipp, A. F. van Loo, M. Baur, L. Steffen, and A. Wallraff, Phys. Rev. A 84, 061805 (2011), URL https://link.aps.org/doi/10.1103/PhysRevA.84.061805.

[24] R. Reimann, W. Alt, T. Kampschulte, T. Macha, L. Ratschbacher, N. Thau, S. Yoon, and D. Meschede, Phys. Rev. Lett. 114, 023601 (2015), URL https://link.aps.org/doi/10.1103/PhysRevLett.114.023601.

[25] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Rev. Mod. Phys. 85, 553 (2013), URL https://link.aps.org/doi/10.1103/RevModPhys.85.553.

[26] G. Vallet, E. Bookjans, U. Eismann, S. Bilicki, R. Le Targat, and J. Lodewyck, New Journal of Physics 19, 083002 (2017), URL https://doi.org/10.1088/1367-2630/aa7c84.

[27] R. Hobson, W. Bowden, A. Vianello, I. R. Hill, and P. Gill, Optics express 27, 37099 (2019), URL http://www.osapublishing.org/oe/abstract.cfm?URI=oe-27-26-37099.

[28] N. F. Ramsey, Physical Review 78, 695 (1950), URL https://link.aps.org/doi/10.1103/PhysRev.78.695.

[29] Y. Sortais, S. Bize, M. Abgrall, S. Zhang, C. Nicolas, C. Mandache, P. Lemonde, P. Laurent, G. Santarelli, N. Dimarco, et al., Physica Scripta 2001, 50 (2001), URL https://doi.org/10.1238/physica.topical.095a00050.

[30] S. Kramer, D. Plankensteiner, L. Ostermann, and H. Ritsch, Comput. Phys. Commun. 227, 109 (2018), URL https://www.sciencedirect.com/science/article/pii/S0010465518300328.

[31] R. Dum, P. Zoller, and H. Ritsch, Physical Review A 45, 4879 (1992), URL https://link.aps.org/doi/10.1103/PhysRevA.45.4879.

[32] K. Mølmer, Y. Castin, and J. Dalibard, JOSA B 10, 524 (1993), URL http://www.osapublishing.org/josab/abstract.cfm?URI=josab-10-3-524.
Mean-field equations

Throughout the paper we calculate the dynamics with a second order cumulant expansion [14]. Nevertheless, the mean-field equations also contain the key physics, therefore we show these much simpler equations for a qualitative description of the system:

\[
\frac{d}{dt} \langle a \rangle = - \left( i\delta_c + \frac{\kappa}{2} \right) \langle a \rangle - i \sum_{j=1}^{N} g_j \langle \sigma_j^{12} \rangle \tag{4}
\]

\[
\frac{d}{dt} \langle \sigma_j^{22} \rangle = - \Gamma \langle \sigma_j^{22} \rangle + \frac{\Omega}{2} \left[ \langle \sigma_j^{12} \rangle - \langle \sigma_j^{21} \rangle \right] + ig_j \left[ \langle a \rangle \langle \sigma_j^{12} \rangle - \langle \sigma_j^{21} \rangle \right] \tag{5}
\]

\[
\frac{d}{dt} \langle \sigma_j^{12} \rangle = \left( i\delta_a - \frac{\Gamma + \nu}{2} \right) \langle \sigma_j^{12} \rangle + i \left( \frac{\Omega}{2} + g_j \langle a \rangle \right) \left[ 2 \langle \sigma_j^{22} \rangle - 1 \right] \tag{6}
\]

In equation (4) we see that for a vanishing coupling phase and equal coherence for all atoms, the cavity field does not grow. We also observe this in Fig. 6. On some point, however, the atoms will gain a sufficient inversion to emit a superradiant pulse into the cavity. Note that there is an initial inaccuracy (e.g. in the cavity field) needed in the mean-field description, otherwise the cavity photon number will never be unequal zero. Due to this initial inaccuracy the time evolution for the two ensembles is striking different. For example we see that for the second pulse the excited state population is drastically decreased for one ensemble, but increased for the other. This is a feature of the mean-field description, which does not occur in that way in a higher order cumulant expansion.

Adiabatically eliminating the cavity field shows that the two frequencies \( \Omega \) and \( N g^2 / \kappa \) compete with each other. Furthermore we see that for a small cavity field the atoms basically do independent Rabi-oscillations with frequency \( \Omega \).

Comparison with full quantum model

To ensure the qualitative validity of our second order cumulant expansion we compare the results with a full quantum model. Of course this is only possible for a relative small number of atoms. To push the number of atoms as far as possible we use the Monte Carlo wave-function method [32, 33], and describe the atoms in the Dicke basis which means that only collective atomic effects can be investigated. Thus we neglect individual atomic decay and dephasing. Fig. 7 shows the comparison between the second order cumulant expansion and the full quantum model for the cavity subradiance (a)-(c), cavity Ramsey method (d) and self-pulsing (e)-(f). Overall we find a good qualitative agreement. A perfect quantitative correspondence is not to be expected for such small atom numbers, due to the needed strong single atom cooperativity.

Figure 6. Mean-field self-pulsing. The time evolution of the photon number (a) excited state population (b) and coherence (c) for a continuous drive. Due to the initial seed the behavior of the atoms in the two ensembles \( g_j = +g \) and \( g_j = -g \) is striking different. The parameters are the same as in Fig. 5 (a).
Figure 7. Full quantum model. The 2nd order cumulant expansion (solid line) is compared with a full quantum model (dashed line) for the cavity subradiance in (a)-(c), the cavity Ramsey method in (d) and the self-pulsing in (e) and (f). In all plots we used $\kappa = 200, \Gamma = \nu = \eta = 0$ and $\delta_c = \delta_a$. For (a)-(c) the remaining parameters are $N = 20, g = 10$ and $\delta_a = 0$, for (d) $N = 20, g = 10, \Omega = 100$ and $T = \pi/10$ and for (e)-(f) $N = 2 \cdot 50, g = 4, \delta_a = 0$ and $\Omega = 4$. 