Investigation of the freeze-out parameters in B–B, O–O, Ca–Ca and Au–Au collisions at 39 GeV

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Abstract We analyzed the transverse momentum spectra of proton, deuteron and triton in Boron–Boron, Oxygen–Oxygen (O–O), and Calcium–Calcium central collisions, as well as in several centrality bins in Gold–Gold (Au–Au) collisions at 39 GeV by using the blast wave model with Tsallis statistics. The bulk properties in terms of kinetic freeze-out temperature, transverse flow velocity and kinetic freeze-out volume are extracted from the model by the least square method. We observed that with increasing the rest mass of the particle, the kinetic freeze-out temperature becomes larger, while transverse flow velocity and the kinetic freeze-out volume reduces. These parameters are also found to depend on the size of the system. Larger the size of the system, the larger they are. Furthermore, the kinetic freeze-out temperature in peripheral Au–Au collisions is close to the central O–O collisions. We also observed that the above parameters depend on the centrality, and they decrease from central to peripheral collisions. Besides, we also extracted the entropy-index parameter $q$, and the parameter $N_0$ which shows the multiplicity. Both of them depend on the size of interacting the system, rest mass of the particle and centrality. Both $q$ and $N_0$ are larger for lighter particles, and the former is smaller for large systems while the latter is larger, and the former decrease with increasing centrality while the latter increase.

1 Introduction

The investigation of Quantum Chromodynamics (QCD) phase diagram is one of the indispensable objective of high energy heavy ion collision experiments [1–4]. Normally, the QCD phase diagram is charted as temperature against baryon chemical potential ($\mu_B$). Let us consider the creation of a thermalized system in heavy-ion collisions, both temperature and $\mu_B$ may change with changing the collisions energy [5–7] and the size of the interacting system [8, 9]. Theoretically, the phase diagram undergoes a possible change from a high temperature and high-density phase which is called Quark-Gluon Plasma (QGP) phase. The QGP phase is influenced by the partonic degrees of freedom to a phase where the relevant degrees of freedom are hadronic [10–12]. There are many observations that have been connected with the actuality and existence of a phase with partonic degrees of freedom in the early stages of heavy-ion collisions [13–18]. The well-known examples of such observations are the suppression of high transverse momentum ($p_T$) particles production in nucleus-nucleus (AA) collisions relative to scaled proton-proton collisions [1–4, 13–15], large elliptic flow for the particles with light as well as heavy (strange) valence quarks and the dissimilarities between baryons and mesons at intermediate $p_T$ in AA collisions [19].

As an important quantity in the physics of high energy collisions, the temperature is widely used in different theoretical and experimental studies. There are different kinds of temperatures in literature, namely initial temperature, chemical freeze-out temperature, effective temperature and kinetic freeze-out temperature. These temperatures occur at different stages of the evolution system and they are discussed in detail in previous works [20–23]. Besides temperature, volume also has an important role in the high-energy collision processes. Like temperature, different freeze-out volumes occur at different stages in system evolution but we will study the final state particles in this work, therefore we will be limited to the kinetic freeze-out volume. More details of kinetic
Fig. 1 Transverse momentum spectra of proton, deuteron and triton produced in the most central (0–10%) B–B, O–O and Ca–Ca collision at 39 GeV. The symbols indicate the experimental data of STAR Collaboration [53] and the curves are our fit results by the blast wave model with Tsallis statistics with \( n_0 = 0 \) and \( n_0 = 1 \). The solid and dashed curves are the fit results with \( n_0 = 0 \) and \( n_0 = 1 \) respectively. The lower panel of the figure represents the data by fit ratios

freeze-out volume can be found in our previous studies [24–27]. It is very important to extract the freeze-out parameters because they help give some crucial information about the final state particles.

The \( p_T \) spectra of hadrons are important tools to understand the dynamics of the particles production in high energy collisions, and we can extract the freeze-out parameters such as chemical freeze-out temperature (\( T_{ch} \)), kinetic freeze-out temperature (\( T_0 \)), effective temperature (\( T \)), transverse flow velocity (\( \beta T \)) and kinetic freeze-out volume (\( V \)) from the \( p_T \) spectra of the particles by using different hydrodynamical models such as Blastwave model with Boltzmann Gibbs statistics [28–30], blast wave model with Tsallis statistics [31], Hagedorn thermal model [32], Standard distribution [33], modified hagedron model [34] and other thermodynamic models [35–39].

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Fig. 2 Transverse momentum spectra of proton, deuteron, anti-deuteron and triton are produced in the different centrality intervals in Au–Au collision at 39 GeV. The symbols indicate the experimental data [54–56] of STAR Collaboration and the curves are our fit results by Eq. (1) with $n_0=0$ and $n_0=1$. The solid and dashed curves are the fit results with $n_0=0$ and $n_0=1$ respectively. The lower panel of the figure represents the data by fit ratios.

The blast wave model has been extensively used for the description of the particle spectra in AA and proton-nucleus ($p−A$) collisions [40–45]. The present study is performed under the assumption that the blast wave model with Tsallis statistics (TBW) to be the theoretical framework to analyze $p_T$ spectra of protons, deuterons and tritons in Boron–Boron (B–B), Oxygen–Oxygen (O–O) and Calcium–Calcium (Ca–Ca) central collisions at 39 GeV. We have analyzed the transverse momentum spectra of the above particles as well as anti-deuteron at 39 GeV in different centrality intervals in Gold–Gold (Au–Au) collisions. Different collision systems at the same center-of-mass-energy are taken in order to investigate the dependence of the freeze-out parameters on the size of the system. We used TBW model with linear as well as constant flow profile to scrutinize the difference of the two cases.

It is conjectured that the system experiences a hydrodynamic evolution. However, the fluctuation occurs for the hydrodynamic evolution event by event [46] which will leave footprint on the spectra of the particles in low and intermediate $p_T$ region because of incomplete set back by the preceding interaction either at hadronic phase or QGP phase [47–49]. The emission source distribution
of the particle has been changed to Tsallis from Boltzmann distribution to take into consideration the effect of the fluctuations responsible for wide applications of the Tsallis-type of non-extensivity [50].

The remainder of the paper consists of the method and formalism in Sect. 1, followed by the results and discussion in Sect. 3. In Sect. 3.2, we summarized our main observations and conclusions.

2 The method and formalism

With the TBW model [31], the invariant momentum distribution is expressed as

$$f_1(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C \frac{gV}{(2\pi)^2} p_T m_T \int_{-\pi}^{\pi} d\phi \int_0^R r dr \times \left\{ 1 + \frac{q-1}{T_0} \left[ m_T \cosh(\rho) - p_T \sinh(\rho) \times \cos(\phi) \right] \right\}^{q-1}$$  (1)

where $C$, $g$ and $V$ are the normalized constant, degeneracy factor and kinetic freeze-out volume respectively. $m_T$ is the transverse mass and is given as $m_T = \sqrt{p_T^2 + m_0^2}$ and $m_0$ is the rest mass. $\phi$ is the azimuthal angle, $R$ denotes the maximum $r$ while $r$ is the...
We analyzed the double differential $p_T$ spectra invariant yield of $p$, $d$ and $t$ in B–B, O–O and Ca–Ca central collisions at the center of mass energy per nucleon $\sqrt{s_{NN}} = 39$ GeV. The experimental data are indicated by the symbols and different symbols indicate different particles. The curves over the data are the numerical result of the fit by the blast wave model with Tsallis statistics i.e Eq. (1). The solid curves are the fit results for which the flow is constant ($n_0=0$), and the dashed curves are the results with linear flow ($n_0=1$). It is found that the model results can well describe the experimental data of STAR Collaboration [53]. The corresponding
Fig. 3 Dependence of (a) \( T_0 \) on \( m_0 \) (b) \( T_0 \) on \( m_0 \) as well as on centrality.

Data by fit ratios are given in the lower panel of the figure. The filled and open symbols in the data/fit ratio indicate the deviation of the curves from the data with \( n_0=0 \) and \( n_0=1 \) respectively. The extracted values of the relative parameters are listed in Table 1 and 2 along with the values of \( \chi^2/dof \), where \( dof \) denoted the number of degrees of freedom. Large deviation of the fit line from the data is seen, especially at the last points in some cases. This is caused by two reasons. On the one hand, the statistics is low at the last points. If the statistics is high, the situation is expected to be changeable. On the other hand, a two-component function is needed. However, the second component from the high \( p_T \) region contributes slightly to the parameters. In fact, we do not needed to consider the second component in the present work.
Figure 2 is similar to Fig. 1, but it shows the event centrality dependent double differential pT spectra of p, d, d̅ and t produced in Au-Au collisions at 39 GeV. The experimental data [54–56] of the STAR Collaboration is indicated by the symbols and the curves over the data are our fit results by Eq. (1). Different symbols in each panel indicate different centrality intervals. The solid and dashed lines over the data are the fit results with n0=0 and n0=1 respectively. One can find that the model can fit the data well. In the lower part of each panel, the corresponding data by fit ratios are given in order to show the deviation of the curve from the experimental data. The filled and open symbols in the lower part of each panel show the results of data/fit ratios by n0=0 and n0=1 respectively.

It is noteworthy that the values in Tables 1 and 2 are extracted from Eq. (1). The values of $\chi^2$ show the deviation of the fit line from the data points and dof is the number of degrees of freedom which is the dof-number of free parameters. $N_0$ is the normalization constant. The normalization constant C used in Eq. (1) and $N_0$ are not the same. The normalization constant C is used to let the integral of Eq. (1) be unity, while $N_0$ is the normalization constant which is used to compare the fit function $f_S(p_T)$ and the experimental spectra.
Fig. 5 Dependence of (a) $V$ on $m_0$ (b) $V$ on $m_0$ as well as on centrality.

3.2 Parameters trend

To investigate the trend of parameters with the rest mass $m_0$ of the particle and with centrality, we present Figure 3. Panel (a) presents the dependence of $T_0$ on $m_0$, while panel (b) shows the rest mass as well as centrality dependence of $T_0$. Panel (a) exhibits the result of $T_0$ in B–B, O–O and Ca–Ca collisions. The legends of each panel are shown in their corresponding right panel. Each collision system is represented by different symbols and the solid and open symbols represent the results from the TBW model with $n_0=0$ and $n_0=1$ respectively in panel (a), however in panel (b) each symbol represents different particles and the filled (upper half-filled symbol) and the empty (lower half-filled symbol) show the results for $n_0=0$ and $n_0=1$ respectively. We noticed that $T_0$ increased with $m_0$ which shows the mass differential freeze-out scenario where the heavier particles freeze-out early, and it is inconsistent with our previous results [20, 22, 24–26]. Furthermore, $T_0$ in Ca–Ca collisions is larger than in O–O collisions and in the latter, it is larger than in B–B collisions. In panel (b) the dependence of $T_0$ on the centrality of collision is shown. Larger $T_0$ in a central collision is observed due to the involvement of large number of participants in the central collisions where the collision is more harsh and more energy is deposited per nucleon, however as we go towards the periphery, the participants become less which leads to less violent collision and that results in a transfer of less amount of energy per nucleon and consequently $T_0$ decreases. It is noteworthy that the...
temperature refers to the final state observable, and it might also be lower if the particles decouple later in central collisions. That is to say, whether the temperature is higher or lower in central collisions, we have a suitable explanation. The present work shows that the temperature is higher in central collisions. We explain the higher temperature as a larger energy release.

Like in panel (a), $T_0$ increases with $m_0$ in Au-Au collisions too. $T_0$ in Au-Au collision is larger than in B–B, O–O and Ca–Ca collisions which demonstrate its dependence on the size of the system. The larger the system size, the larger is the kinetic freeze-out temperature because a large system contains a large number of nucleons and the collision among large system is very harsh which highly increase the degree of excitation of the system compared to the small systems. We further noticed that the values of $T_0$ in the most peripheral Au-Au collisions are close to the central O–O collisions. For instance in Table 1, the values $T_0$ for $p$, $d$, and $t$ in Au-Au collisions are $0.042 \pm 0.005$, $0.060 \pm 0.005$ and $0.068 \pm 0.004$ respectively, while in case of O–O most central collisions, their values are $0.044 \pm 0.004$, $0.056 \pm 0.006$ and $0.062 \pm 0.004$ respectively. This indicates that the central O–O collisions and peripheral Au-Au collisions at the same center-of-mass energy have similar thermodynamic nature. Similarly in Table 2, $T_0$ for $p$, $d$ and $t$ emissions in Au-Au peripheral collision is $0.045 \pm 0.005$, $0.083 \pm 0.005$ and $0.090 \pm 0.005$ respectively and for O–O central collisions they are $0.048 \pm 0.005$, $0.061 \pm 0.005$ and $0.069 \pm 0.005$ respectively. In this case, $p$ in central O–O and peripheral Au-Au

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig6}
\caption{Dependence of (a) $q$ on $m_0$ (b) $q$ on $m_0$ as well as on centrality.}
\end{figure}
show similar thermodynamic nature, and deuteron and triton show close value. In addition, \( T_0 \) is observed to be larger at \( n_0 = 1 \) than \( n_0 = 0 \).

Figure 4 is similar to Fig. 3, but it demonstrates the dependence of \( \beta_T \) on \( m_0 \) in panel (a) and on centrality and \( m_0 \) in panel (b). It is investigated that \( \beta_T \) depends on \( m_0 \). The larger the mass of the particle, the smaller are \( \beta_T \) values. In panel (b), \( \beta_T \) is dependent on the size of the interacting system. The larger the size of the system is, the larger the \( \beta_T \). The collisions of large systems are pretty violent which deposits more energy in the system and it expands quickly. In panel (b) \( \beta_T \) gives an increasing trend with increasing centrality due to the fact that the collisions become more violent as the system goes toward centrality and the deposition of energy is large which results in the quick expansion of the fireball. Likewise \( T_0 \), \( \beta_T \) is also larger with \( n_0=1 \) in TBW model.

The kinetic freeze-out volume is presented in Fig. 5. Panel (a) exhibits its dependence on \( m_0 \) while panel (b) exhibits its dependence on \( m_0 \) as well as centrality. \( V \) shows dependency on \( m_0 \) and it decreases for heavier particles which claim a volume differential freeze-out scenario and shows the early freeze-out of heavier particles. From the above discussion, one can assume different freeze-out surfaces for different particles. From panel (a) and (b) one can see that \( V \) is larger in large systems. In addition, in panel (b) \( V \) is noticed to have a decreasing trend from central collisions towards the periphery because hadrons involve in interaction decrease from central to peripheral collisions depending on their interaction volume. The system with a large number of participants reaches
an equilibrium state quickly since there are a larger number of binary collisions by the re-scattering of partons in central collisions. The large volume and a large number of participant nucleons in central collisions could be a hint for the occupation of super hadronic dense matter. Of course, the present study is not enough for the study of complete information about the local energy density of super-hadronic matter and the possible phase transition of QGP. However, we will study this in a future project. \( V \) is observed to be larger in the case of \( n_0=1 \) like \( T_0 \) and \( \beta_T \). We would like to clarify that the resultant data points for \( n_0=0 \) are not visible because the values of \( n_0=0 \) and \( n_0=1 \) are the same, and they overlap each other.

Fig. 6 is similar to Fig. 5, but it shows the dependence of the entropy parameter \((q)\) on \( m_0 \) and centrality. \( q \) in panel (b) shows less dependence on \( m_0 \) in central collisions, but in panel (a) it is larger for proton and becomes smaller for deuteron and then triton, even for \( Au–Au \) peripheral collisions. \( q \) for proton is the largest. This shows that the production of a proton is more polygenic than deuteron and triton. In addition, in panel (b) one can see that \( q \) increases from central to a peripheral collision which indicates a quick approach to an equilibrium state in central collisions. Although \( q \) has a non-monotonic increment from central to peripheral collision in the case of \( d \), but as a whole, it shows an increasing trend from central to peripheral collisions.

\( q \) is slightly larger in B–B collisions than O–O collisions and is obviously smaller in Ca–Ca collisions and Au–Au central collisions which show the dependence of \( q \) on the size of the system. A larger system has a smaller entropy parameter \( q \) which indicates that larger systems have a quick approach to equilibrium state compared to smaller systems.

The parameter \( N_0 \) dependence on \( m_0 \) and centrality is presented. One can see that the parameter \( N_0 \) is dependent on \( m_0 \). The heavier particle has a smaller \( N_0 \). The parameter \( N_0 \) is the normalization constant and it shows the multiplicity. The parameter \( N_0 \) is larger for a larger system which indicates large multiplicity in larger systems, and it decreases from central to peripheral collisions. These results and natural and understandable (Fig. 7).

4 Conclusions

The main observations and conclusions are summarized here.

a) The transverse momentum spectra of protons, deuterons and tritons are analyzed in Boron-Boron, Oxygen–Oxygen and Calcium–Calcium central collisions and in different centrality intervals in Gold–Gold collisions at 39 GeV using blast wave model with Tsallis statistics with flow profile fixed and linear flow by extracted the kinetic freeze-out temperature \((T_0)\), transverse flow velocity \((\beta_T)\) and kinetic freeze-out volume \((V)\).

b) \( T_0, \beta_T \) and \( V \) are slightly larger in a linear flow. \( T_0, \beta_T, \) \( V \), entropy parameter \((q)\) and the normalization \((N_0)\) are mass-dependent. \( T_0 \), \( q \) and \( N_0 \) increase with increasing the mass of the particle, while \( \beta_T \) and \( V \) decrease with \( m_0 \). Furthermore, \( T_0 \) in central \( O–O \) collisions and peripheral \( Au–Au \) collisions are close to each other which shows their similar thermodynamic nature.

c) \( T_0, \beta_T, \) \( V \) and \( N_0 \) are dependent on the size of the system. They are all larger in large systems except \( q \) which is smaller in large systems and indicates a quick approach of the system to equilibrium.

d) \( T_0 \) and \( \beta_T \) are larger in central collisions and they decrease from central to peripheral collisions due to the decrease of participant nucleons towards the periphery which results in less deposition of energy in the system due to less violent collisions in the periphery. \( V \) also decreases from central to peripheral collisions due to decreasing the binary collisions by the re-scattering of partons towards the periphery.

e) \( q \) and \( N_0 \) are centrality dependent. The former increase from the central to the periphery which indicates a quick equilibrium approach of the system in central collisions, while the latter is related to the multiplicity and it decreases from central to peripheral collisions.

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Declarations

Data availability The data used to support the findings of this study are included within the article and are cited at relevant places within the text as references.

Compliance with Ethical Standards The authors declare that they are in compliance with ethical standards regarding the content of this paper.

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