Gravity currents in aquatic canopies

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A lock exchange experiment is used to investigate the propagation of gravity currents through a random array of rigid, emergent cylinders which represents a canopy of aquatic plants. As canopy drag increases, the propagating front varies from the classic profile of an unobstructed gravity current to a triangular profile. Unlike the unobstructed lock exchange, the gravity current in the canopy decelerates with time as the front lengthens. Two drag-dominated regimes associated with linear and nonlinear drag laws are identified. The theoretical expression for toe velocity is supported by observed values. Empirical criteria are developed to predict the current regime from the cylinder Reynolds number and the array density.

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1. Introduction

[2] Gravity-driven convective currents in the sidearms and littoral zones of reservoirs have been studied in the field [e.g., Adams and Wells, 1984], in the laboratory [e.g., Lei and Patterson, 2002; Starman and Ivey, 1998], and through modeling [e.g., Brocard and Harleman, 1980; Farrow and Patterson, 1993; Horsch et al., 1994]. These flows are driven by spatial heterogeneity in the temperature, and thus density, that may result from spatial variability in water depth [e.g., Monismith et al., 1990; Roget and Colomer, 1996], groundwater discharge [Roget et al., 1993], light compensation depth [e.g., MacIntyre et al., 2002; Nepf and Oldham, 1997], shading due to floating macrophytes [Coates and Ferris, 1994], or sheltering from the wind [MacIntyre et al., 2002].

Aquatic gravity currents also occur in estuaries where salt water and freshwater meet [O’Donnell, 1993]. Simpson [1997] presents additional examples and a comprehensive review of gravity currents.

[3] In shallow systems, aquatic vegetation will often be present and can influence the propagation of a gravity current by providing a mechanism for energy dissipation. For example, Oldham and Sturman [2001] have demonstrated that steady, buoyancy-driven, down-slope flow decreases with decreasing permeability within a vegetated region. However, estimated timescales of circulation formation [Wells and Sherman, 2001] and response to forcing [Farrow and Patterson, 1993] strongly suggest that diurnally forced convective currents are often unsteady under typical field conditions [e.g., Farrow and Patterson, 1993; Wells and Sherman, 2001]. Here, we examine the behavior of unsteady front propagation.

[4] Lock-exchange flows have been studied extensively through both laboratory experiments and numerical modeling. These flows can easily be produced in a laboratory tank by installing a removable partition. The resulting two reservoirs are filled with fluids of different density. When the partition is removed, the horizontal density gradient generates an exchange flow. The heavier fluid propagates toward the lighter fluid reservoir along the bottom of the tank and the lighter fluid propagates along the free surface in the opposite direction [Simpson, 1997].

[5] The classic lock exchange has negligible dissipation and is inertia dominated. It exhibits a predominantly horizontal interface that curves sharply toward the free surface and the bed at the leading edges of the surface current and undercurrent, respectively [Benjamin, 1968]. In contrast, exchange flows through sand are drag dominated, and the interface is inclined to the horizontal plane, rotating about its midpoint [Keulegan, 1954]. Figure 1 illustrates these two limits.

[6] This paper examines the behavior of exchange flows between the two limits, specifically obstructed lock exchange under drag conditions representative of aquatic canopies. The goal of this paper is to document and explain the transition from the inertia-dominated regime to the drag-dominated regime. Our mathematical description of front propagation is validated with our experimental observation. The transition between regimes is classified by the cylinder Reynolds number and the dimensionless array drag.

2. Mathematical Formulation

[7] The two-dimensional Cartesian coordinate system (x, z) is defined with x = 0 at the lock and z = 0 at the bed (Figure 2). The x axis is aligned with the direction of the undercurrent. Emergent vegetation is modelled by an array of randomly distributed cylinders of diameter d that span the water column. The cylinder array contains N elements distributed over the horizontal footprint of the tank, A. The porosity, n = 1 − (π/4) ad, defines the volume fraction of...
The difference between the two layers, where \( a = N A \) is the frontal area of the cylinders per unit volume. The components of the pore velocity of the fluid \((u, w)\) are aligned with the axes \((x, z)\), respectively. When the lock is removed, the fluid of higher density \(\rho_1\) flows in the \(+x\) direction and the fluid of lower density \(\rho_2\) flows in the \(-x\) direction, forming two layers separated by the interface \(\eta(x, t)\), as shown in Figure 2. From scale analysis it can be shown that viscous stresses in the fluid are small compared to the array drag in the current experiment (Section 4). Similarly, turbulent stresses are negligible if \(ad > 0.005\) [Burke and Stolzenbach, 1983], which is satisfied in all but three of the obstructed experimental runs (Table 1).

Accounting for the volume fraction occupied by the cylinders and assuming a quadratic drag law, we obtain the two-dimensional momentum equation for a cylinder array

\[
\frac{D u}{D t} = -n \frac{\partial P}{\partial x} - C_D a u |u| - \frac{\partial P}{\partial x},
\]

where \(C_D\) is the array drag coefficient, \(\rho\) is the fluid density, and \(P\) is the hydrostatic pressure. Equation (1) is strictly valid only at spatial scales that encompass multiple cylinders, by definition of \(C_D\). Note that despite the anisotropy of the cylinder array, the average planar porosity is equivalent to the volumetric porosity [Brenner and Edwards, 1993, p. 188].

If the drag term is negligible, equation (1) reduces to the classic unobstructed lock exchange, for which the interface \(\eta(x, t)\) is predominantly horizontal. The leading edges, or toes, of this current propagate at the steady velocity [e.g., Shin et al., 2004]

\[
u_{\text{hoo}} = \pm \sqrt{\frac{g' H}{4}} = \pm \sqrt{\frac{g \Delta \rho H}{4}},
\]

Here, \(g'\) is the reduced gravity based on \(\Delta \rho\), the density difference between the two layers, \(g = 980 \text{ cm s}^{-2}\) is the gravitational acceleration, and \(H\) is the total water depth [Shin et al., 2004]. In this scenario \(\nu_{\text{hoo}}\) is independent of time because of the absence of energy sources and sinks.

In this study, we are interested in defining \(\nu_{\text{hoo}}\) when the drag dominates inertia. Scaling the inertial term as \(\nu_{\text{hoo}} u_{\text{hoo}} \sim u_{\text{hoo}}^2 L\), we expect from equation (1) that array drag dominates inertia when \(C_D a L/n \geq O(10)\). Under these conditions, equation (1) reduces to a balance between only array drag and buoyancy:

\[
u_{\text{hoo}}(x, t) = \frac{2n}{C_D a} \frac{\partial P}{\partial x}.
\]

The hydrostatic pressure, \(P\), in the lower \((z \leq \eta(x, t))\) and upper \((\eta(x, t) < z \leq H)\) layers is described respectively as

\[
P_1(x, z, t) = P_0(x, t) - \rho_1 g z
\]

and

\[
P_2(x, z, t) = P_0(x, t) - \rho_1 g \eta(x, t) - \rho_2 g (z - \eta).
\]
Table 1. Summary of Experimental Conditions and Model Parameters

| Run | Frontal Area Per Unit Volume (a) (±0.5%), stems/cm² | Mass of Salt Added (mS), g | Reduced Gravity g’ (±0.3), cm s⁻² | Mean Water Depth H (±0.6), cm |
|-----|---------------------------------------------------|--------------------------|----------------------------------|-------------------------------|
| 1   | 0.12                                             | 1393.80                  | 43.2                             | 14.0                          |
| 2   | 0.12                                             | 1468.77                  | 48.0                             | 14.2                          |
| 3   | 0.12                                             | 500.00                   | 18.9                             | 13.5                          |
| 4   | 0.091                                            | 100.00                   | 3.8                              | 13.5                          |
| 5   | 0.068                                            | 100.00                   | 4.0                              | 13.7                          |
| 6   | 0.023                                            | 100.00                   | 3.9                              | 13.3                          |
| 7   | 0.12                                             | 100.00                   | 3.9                              | 13.9                          |
| 8   | 0.0046                                           | 100.00                   | 3.9                              | 13.2                          |
| 9   | 0.011                                            | 100.00                   | 3.9                              | 13.3                          |
| 10  | 0.0091                                           | 100.00                   | 3.9                              | 13.5                          |
| 11  | 0.12                                             | 10.00                    | 1.2                              | 13.9                          |
| 12  | 0.091                                            | 100.00                   | 3.9                              | 13.7                          |
| 13  | 0.12                                             | 502.43                   | 18.7                             | 13.7                          |
| 14  | 0.12                                             | 700.00                   | 24.1                             | 14.3                          |
| 15  | 0.12                                             | 1000.00                  | 33.8                             | 13.8                          |
| 16  | 0.12                                             | 500.00                   | 18.7                             | 13.9                          |
| 17  | 0.091                                            | 750.00                   | 26.2                             | 14.0                          |
| 18  | 0.12                                             | 100.00                   | 4.0                              | 14.0                          |
| 19  | 0.046                                            | 100.00                   | 3.9                              | 13.0                          |
| 20  | 0.091                                            | 100.00                   | 4.2                              | 13.4                          |
| 21  | 0.046                                            | 190.00                   | 7.1                              | 12.8                          |
| 22  | 0.023                                            | 190.00                   | 6.5                              | 12.9                          |
| 23  | 0.0046                                           | 100.00                   | 4.1                              | 12.8                          |
| 24  | 0.011                                            | 100.00                   | 4.4                              | 13.0                          |
| 25  | 0.068                                            | 10.00                    | 0.6                              | 13.6                          |
| 26  | 0.068                                            | 10.00                    | 0.5                              | 13.4                          |
| 27  | 0.068                                            | 50.00                    | 2.2                              | 13.4                          |
| 28  | 0.068                                            | 50.00                    | 2.1                              | 13.5                          |
| 29  | 0.068                                            | 50.00                    | 1.9                              | 13.3                          |
| 30  | 0.068                                            | 100.00                   | 4.0                              | 13.5                          |
| 31  | 0.068                                            | 100.00                   | 4.0                              | 13.4                          |
| 32  | 0.068                                            | 100.00                   | 4.0                              | 13.3                          |
| 33  | 0.068                                            | 300.00                   | 13.2                             | 13.1                          |
| 34  | 0.068                                            | 300.00                   | 13.2                             | 13.4                          |
| 35  | 0.068                                            | 500.00                   | 18.1                             | 13.3                          |
| 36  | 0.068                                            | 500.00                   | 18.1                             | 13.3                          |
| 37  | 0.068                                            | 500.00                   | 18.1                             | 13.3                          |
| 38  | 0.068                                            | 500.00                   | 18.1                             | 13.3                          |
| 39  | 0.068                                            | 126.81                   | 5.0                              | 14.0                          |
| 40  | 0.068                                            | 252.14                   | 9.2                              | 14.3                          |
| 41  | 0.068                                            | 380.71                   | 14.0                             | 14.1                          |
| 42  | 0.068                                            | 504.14                   | 17.8                             | 14.3                          |
| 43  | 0.068                                            | 630.14                   | 22.1                             | 14.3                          |
| 44  | 0.068                                            | 126.00                   | 4.4                              | 13.8                          |
| 45  | 0.068                                            | 126.00                   | 9.2                              | 13.8                          |
| 46  | 0.068                                            | 252.00                   | 9.2                              | 13.8                          |
| 47  | 0.068                                            | 375.55                   | 13.2                             | 13.3                          |
| 48  | 0.068                                            | 513.86                   | 18.3                             | 13.1                          |
| 49  | 0.068                                            | 675.80                   | 23.6                             | 14.0                          |

*The g’ values presented here for runs 1–44 have been corrected for temperature, and the density values used to calculate them were slightly different from the uncorrected measurements.

[11] In a lock exchange, temporal and spatial variations in $H$ are insignificant, and therefore neglected. Then, mass conservation requires zero net flux at each vertical cross section and

$$u_1 \eta = -u_2 (H - \eta),$$

where $u_1$ and $u_2$ are velocities in the lower and upper layers, respectively.

[12] The application of equations (3), (4), and (5) to equation (6) yields an expression for the pressure gradient along the bed,

$$\frac{1}{\rho_1} \frac{\partial P_0 (x, t)}{\partial x} = g' \frac{\partial \eta}{\partial x} \frac{(1 - \eta)^2}{(1 - \eta)^2 + (\eta')^2},$$

where $g' = (\rho_1 - \rho_2)/\rho_1$. For simplicity, we have used the approximation $\rho_1 = \rho_2$, valid for our experiments ($\rho_1/\rho_2 \leq 1.05$) and for most field conditions. Then, the flow velocity in the lower and upper layers, respectively, is

$$u_1^o (x, t) = \frac{2n}{C_{Da} g'} \frac{\partial \eta}{\partial x} \frac{(1 - \eta)^2}{(1 - \eta)^2 + (\eta')^2},$$

and

$$u_2 | u_2 | (x, t) = \frac{2n}{C_{Da} g'} \frac{\partial \eta}{\partial x} \frac{(\eta')^2}{(1 - \eta)^2 + (\eta')^2}.$$

Evaluating equations (8) and (9) at the toes ($\eta = 0$ and $\eta = H$, respectively) yields

$$u_{low} (t) = \sqrt{\frac{2n}{C_{Da} g'} \frac{\partial \eta}{\partial x}} |_{\eta = 0}$$

in the lower layer and

$$u_{low} (t) = - \sqrt{\frac{2n}{C_{Da} g'} \frac{\partial \eta}{\partial x}} |_{\eta = H}$$

in the upper layer. An analytical expression for $\partial \eta / \partial x$ could not be found. However, if the interface is self-similar, its gradient can be described as

$$\frac{\partial \eta}{\partial x} (x, t) = -S \frac{H}{L},$$

where $L(t)$ is the longitudinal frontal length (Figure 2) and $S(x/L)$ is the scale constant. Observations of the interface gradient at $x = 0$, discussed later, support the assumption of self-similar behavior. Then, equations (10) and (11) simplify to

$$u_{low} (t) = \sqrt{\frac{2n}{C_{Da} L g' HS_{\eta = 0}}}$$

and

$$u_{low} (t) = - \sqrt{\frac{2n}{C_{Da} L g' HS_{\eta = H}},}$$

respectively.
Figure 3. Side view of the central section of the laboratory tank and the random array of emergent cylindrical dowels. Dowels were inserted in perforated sheets placed at the bottom of the tank. The interface between the fluid and the perforated sheets is defined as \( z = 0 \). The length of the tank is \( L_{\text{tank}} = 180 \) cm.

An estimate of the array drag coefficient \( C_D \) is necessary to evaluate equations (13) and (14). \( C_D \) may be a function of the cylinder Reynolds number \( Re = |u|d/\nu \) and the dimensionless array density, \( ad \). For a smooth, isolated circular cylinder in the \( Re \) range \( 1 < Re < 10^3 \), \( C_D \) is described by the empirical expression [White, 1991, p. 183]

\[
C_D \approx 1 + 10.0 Re^{-1.5}, \quad (15)
\]

Unfortunately, a comprehensive description of cylinder drag in an array has not yet been developed. Previous studies suggest that \( C_D \) in an array is suppressed for \( Re \gtrsim 10^3 \) due to sheltering [e.g., Raupach, 1992]. However, equation (15) remains reasonable for \( ad < 0.03 \) [Nepf, 1999]. In contrast, \( C_D \) is enhanced in low \( Re \) ranges [Koch and Ladd, 1997]. For example, numerical simulations show that \( C_D \) in a random array of 5% solid volume fraction (equivalent to \( a = 0.1 \) cm\(^{-1} \) for cylinders used in the present study) and \( Re < 35 \) behaves as [Koch and Ladd, 1997, Figure 26]

\[
C_D \approx \frac{2}{Re} \left( 12 + 1.07 Re \right), \quad (16)
\]

which predicts a higher \( C_D \) than equation (15) for \( Re < 35 \). This dependence on \( Re \) and array density allows the solution to equations (8) and (9) to take on many forms and prevents the derivation of a general analytical solution.

Under low \( Re \) (<1) \( C_D \) may approach a linear drag law regime, as implied by equation (16), where \( C_D \) is inversely proportional to \( Re \). Under this condition, an analytic expression for \( \eta(x, t) \) derived by Huppert and Woods [1995] is applicable. Using \( C_D = C^*|u| \), where \( C^* \) is a constant, equation (3) becomes

\[
\frac{u(x, t)}{H} = \frac{2n}{C^* \rho \partial P}{\partial x}, \quad (17)
\]

which is the expression used by Huppert and Woods [1995], with \( 2n/(C^* \rho) \) replacing their ratio of permeability over dynamic viscosity \( k/\mu \). Following their analysis, one can arrive at the similarity solution for interface position,

\[
\eta(x, t) = \frac{H}{2} \left( 1 - \frac{x}{\sqrt{\beta t}} \right), \quad (18)
\]

where \( \beta \equiv 2\sqrt{\gamma^3/(C^* \alpha)} \). The sign change from the original solution reflects the reversal in the direction of propagation of the undercurrent from the formulation of Huppert and Woods [1995]. This solution describes a linear interface that passes through \( (x, z) = (0, H/2) \) with the leading edges \( (\eta_{\text{max}} = \pm \sqrt{\beta t}) \) propagating at velocity

\[
\frac{dx_{\text{max}}}{dt} = \pm \frac{\sqrt{\beta}}{L} = \pm \sqrt{\frac{\beta}{L}}, \quad (19)
\]

where \( L = 2|\eta_{\text{max}}| \).

In summary, theory predicts that array drag will manifest itself in two ways. First, unlike the classic lock exchange, gravity currents propagating through an array decelerate as the front lengthens (equations (13), (14), and (19)). Second, the interface deviates from the classic profile described by Benjamin [1968].

3. Experimental Methods

Experiments were conducted in a 180 cm \( \times \) 15.6 cm \( \times \) 20.3 cm glass-walled laboratory flume with a horizontal metal bottom (Figure 3). The tank was separated into two reservoirs by a removable 3 \( \pm \) 0.5 mm thick vertical partition that was positioned at approximately midtank. Randomly distributed rigid maple dowels, \( d = 0.64 \) cm in diameter, were used to model aquatic vegetation which typically has a diameter \( d = 10^{-1} \) to 1 cm [e.g., Leonard and Luther, 1995]. These dowels were inserted into perforated polypropylene sheets (0.62 holes per cm\(^2 \)) placed at the bottom of the flume in both reservoirs, directly up to both sides of the partition. The dowels spanned the water column and penetrated the free surface. The range of cylinder densities examined in this paper, \( a = 0 \) to 0.16 cm\(^{-1} \), produces a dimensionless array density \( ad = 0 \) to 0.10, which falls within the range observed in natural canopies (e.g., \( ad = 0.01 \) to 0.1 [Kadlec, 1990; Katifi, 2002]).

One reservoir was filled with 20 \( \pm \) 0.2 l of well-mixed saltwater of density \( \rho_1 \) and the other with tap water of density \( \rho_2 \) \((< \rho_1) \) until the free surface in both reservoirs was aligned. For flow visualization, the saltwater was dyed with black or blue food dye in all runs except runs 33 and 34, for which the tap water was dyed instead. The water depth, measured from the top of the perforated base sheets (\( z = 0 \)), was on average \( H = 13.6 \) cm (Table 1) and varied with the solid volume fraction of the array. This coincides with the low end of typical field depths, which span the range \( H = 10 \) to 100 cm. In contrast, \( g' \) in our study \((g' = 0.5 \text{ to } 48.0 \text{ cm s}^{-2}) \) was an order of magnitude larger than that typically observed in the field \((g' = 0 \text{ to } 1.0 \text{ cm s}^{-2}) \) [e.g., Dale and Gillespie, 1976; James and Barko, 1991; Nepf and Oldham, 1997]). Consequently, the toe velocity scale \( u_{\text{max}} \sim \sqrt{g'H} \) overlaps between field and laboratory conditions.

Experimental runs 1—44 (Table 1) began with the removal of the vertical partition in the middle of the tank. As the two fluids exchanged, a series of 640 \( \times \) 480 bitmap images were captured using a Pulnix TM-9701 CCD camera mounted on a stationary tripod in front of the tank. These images were converted to binary images, using a manually selected threshold that appeared, by visual inspection, to most accurately identify the pixels corresponding to the dyed fluid as black. The interface position, \( \eta(x, t) \), was located by edge detection between the white and black
The toe velocity is estimated as the displacement of the toe at the bed between two consecutive images, divided by the difference in time between the two images. The series of images for each run generated a series $u_{\text{toe}}(t)$. Because the interface was clearer where dyed fluid was moving into clear water, we only analyze the lower layer. The $Re$ corresponding to $u_{\text{toe}}$ is defined as $Re_{\text{toe}} = u_{\text{toe}}/v$. Then, the array drag at each position, $C_{\text{PD}}$, is determined using equation (15) to estimate $C_D$.

[22] The slope of the interface, $\partial h/\partial x$, at $x = 0$ in each image is estimated for runs 1–44 by performing a linear regression on the data points that fall within the range $1.25H < x < x_{\text{toe}} - 1.25H$ for runs with a stem density of $a = 0.068 \text{ cm}^{-1}$, $a = 0.091 \text{ cm}^{-1}$, or $a = 0.12 \text{ cm}^{-1}$, and the range $0 < x < x_{\text{toe}} - 1.25H$ for the less densely vegetated runs. The respective ranges were selected to capture as much of the interface as possible without capturing the head of the current at the free surface and the bed. The linear regression was restricted to $x > 0$ for the less densely vegetated runs because of turbulence, which blurs the interface.

[23] The linear drag constant $C$ was computed from pairs of consecutive images in each experimental run. The two images in the pair are referred to as $i_{\text{previous}}$ and $i_{\text{current}}$. Because the interface rotates about the toe, fitted data were restricted to $\eta < H/3$. Because data points were not necessarily available at the same depths in both images, a polynomial of degree two was fitted to the data points in each image in the range $H/10 < \eta < H/3$ and evaluated at ten evenly spaced depths in the range $0 \leq \eta \leq H/3$. Equation (18) can be rewritten to describe the position of the interface at a given depth,

$$\frac{x_{\text{interface}}}{L} = \frac{1}{2} \left[ \frac{z}{H} \right].$$

Starting from the interface position in $i_{\text{previous}}$, the interface in $i_{\text{current}}$ was predicted from

$$\frac{dx_{\text{interface}}}{dt} = \frac{v_{\text{PD}}}{L} \left[ \frac{1}{2} \left( \frac{1}{H} \right) \frac{z}{H} \right]$$

using 60 time steps in between. The prediction was carried out for a range of $C'$, until a minimum difference, in the least squares sense, was found between the predicted and observed interface in $i_{\text{current}}$. In this manner, a best fit $C'$ was determined for every pair of images analyzed in a run.

4. Experimental Results

[24] Toe velocities observed in this study ranged from 0.07 to 7.9 cm s$^{-1}$. These values are similar to those reported in field studies of temperature-driven exchange flows [e.g., Kalff, 2002; MacIntyre et al., 2002; Stefan et al., 1989]. The data show that even in the least obstructed scenario (run 11), the viscous stress, which varied between $nu_{\text{toe}}(H/2)^2 = 5 \times 10^{-4}$ and $7 \times 10^{-4}$ cm$^2$ s$^{-2}$, was consistently one order of magnitude smaller than the array drag, which ranged from $C_{\text{PD}}u_{\text{toe}}^2/2 = 4 \times 10^{-3}$ to $7 \times 10^{-3}$ cm s$^{-2}$. This confirms the omission of viscous stress from equation (1).
The toe velocity of the undercurrent is compared to theory in Figure 4 by normalizing the observed velocity by the classic inertial solution, \( \sqrt{gH/4} \), and by the drag-dominated scale, \( \sqrt{gH/C_{pDL}} \) (equation (13)). \( C_D \) is estimated from the relation for isolated cylinders (equation (15)), which is appropriate for \( ad < 0.03 \) [Nepf, 1999]. The two theories are compared across a range of array drag, characterized by \( C_{pDL} \). Under low drag conditions, the front propagation is described reasonably well by the classic solution. Overprediction by the classic solution (solid circles) at low \( C_{pDL} \) is attributed to turbulence or the neglected bed drag. Indeed, some images indicate shear turbulence near the toe. In contrast, the toe velocity under high drag conditions is better described by the drag-dominated scale (open circles). That is, normalization by the drag-dominated scale yields ratios of approximately unity for \( C_{pDL} > 10 \), but normalization by the inertial solution yields ratios that decline rapidly as \( C_{pDL} \) increases.

The presence of drag will suppress \( \tilde{u} \) from the classic flow scenario of \( \tilde{u} = 1 \). For discussion, when \( \tilde{u}/C_{pDL} > 0.65 \), the flow is classified as inertia dominated. With this classification, 80% of flows with \( C_{pDL} < 10 \) and 97% of flows with \( C_{pDL} > 10 \) in Figure 4 are appropriately classified as inertia dominated and drag dominated, respectively.

Figure 5 displays the progression of the interface under the three regimes: inertial (Figure 5a), nonlinear array drag (Figure 5b), and linear array drag (Figure 5c). Recall that \( Re \) dictates the nature of the drag law. First, Figure 5a depicts the inertial regime. The interface is horizontal at \( \eta = H/2 \). Near the leading edge, the interface bends sharply to the bed \( (\eta = 0) \) over a longitudinal distance of approximately one water depth. The curved head exhibits good agreement with Benjamin’s [1968] solution, shown as a dashed line. Also, there is evidence of turbulent undulations at the interface. These observations are consistent with others reported for the classic, unobstructed lock exchange (see Simpson [1997] for a description).

Following from Figure 4, we classify a flow condition as inertial when the observed toe velocity is consistent with the classic solution (equation (2)). Specifically, we define the dimensionless parameter \( \tilde{u} \):

\[
\tilde{u} = \frac{\text{observed } u_{toe}}{\sqrt{\frac{gh}{4}}}
\]  

\text{(22)}

Figure 5. Progression of the interface with time. The profiles are separated by the time step indicated at the top of Figures 5a–5c. The horizontal axes span \( 0 < x < L_{tank}/2 \). (a) Run 9: \( Re = 190–200 \) and \( C_{pDL} = 0.88–1.9 \). (b) Run 42: \( Re = 190–260 \) and \( C_{pDL} = 6.9–12 \). (c) Run 31: \( Re = 8.4–53 \) and \( C_{pDL} = 11–27 \). Dashed curves represent Benjamin’s [1968] solution for energy-conserving gravity currents.
The interface is now at a slight angle to the bed. The difference from the inertial regime is highlighted by the poor agreement of the experimental data illustrated in Figure 5b with Benjamin’s [1968] solution. Also, note that the toe velocity declines as $L$ increases (equation (13)).

Finally, Figure 5c represents the linear drag law regime with an essentially linear interface, as described by equation (18). The head, while still identifiable, is much less prominent than in Figures 5a and 5b. Consequently, for the same interface length, the interface gradient at $x = 0$ is greater in magnitude than in Figures 5a and 5b which have lower drag. This trend is similar to that observed for steady gravity currents propagating through screens. As the screen drag increased, the interface gradient at the screen increased [Rottman et al., 1985]. Also, note that the toe advances nonlinearly with time, reflecting the decline in $u_{toe}$ as $L$ increases, as predicted by equation (19). Finally, observe that each interface in Figure 5 rotates approximately about middepth, which is consistent with the present theory.

Because all fronts are initiated at $C_{D}a/L = 0$, all flows begin in the inertial regime. A flow may transition from inertial to drag dominated when $C_{D}a/L$ becomes sufficiently large. This regime transition was captured for $g = 1.0$ cm s$^{-2}$ and an array of density $a = 0.068$ cm$^{-1}$ (Figure 6). Initially, $x_{toe}$ varies linearly with time ($t^{2} = 1.0$, $n = 16$), as expected in the inertial regime, i.e., $u_{toe}$ is constant in time (equation (2)). At $C_{D}a/L \approx 10$ the toe clearly begins to decelerate and at $C_{D}a/L \approx 20$ it has transitioned to a $\sqrt{t}$ dependence, consistent with the linear drag regime (equation (19)). In this case, the transition begins when the interface spanned half of the tank length.

Next we consider the interface gradient. As illustrated in Figure 1, the energy-conserving inertial flow (i) has zero interface slope except near the toe, while the triangular profile in the linear drag regime (ii) exhibits a spatially constant interface gradient of $\partial H/\partial x = -H/L(t)$. Therefore the scale constant $S$ (as defined in equation (12)) at $x = 0$ is $S_{0} \equiv S(x = 0) = 0$ and 1 for the inertial and linear drag regime, respectively. The parameter $S_{0}$ is computed from the observed interface slope at $x = 0$ as

$$S_{0} = \left| \frac{H}{L(t)} \right|_{x=0}$$

(23)

for all images with sufficient data points to define the gradient and from which the position of the toe can be extracted.

The behavior of $S_{0}$ with changing $Re_{toe}$ is plotted in Figure 7 for all drag-dominated conditions ($C_{D}a/L > 10$). $S_{0}$ is nearly constant for $Re_{toe} > 300$ at $S_{0} = 0.60 \pm 0.06$ (standard deviation). As stated earlier, a constant $S_{0}$ implies a self-similar evolution of the interface profile. $S_{0}$ exhibits greater scatter in the range $60 \leq Re_{toe} \leq 300$ at $S_{0} = 0.64 \pm 0.08$. Below $Re_{toe} \approx 60$, $S_{0}$ progressively increases as $Re_{toe}$ decreases. Note that the perfectly linear interface corresponds to $S_{0} = 1$. In the current laboratory study this theoretical maximum was not reached (Figure 7). This is due to the persistence of a slight deviation of the profile at the bed, even when the core region of the interface is linear (Figure 5c). Because a small toe persists, the vertical extent of the core region, $H^{*}$, is less than $H$, making the observed interface slope at $x = 0$ smaller than the theoretical slope i.e., $\left| -H^{*}/L \right| < \left| H/L \right|$, and $S_{0}(-H^{*}/H)$ less than 1. This deviation may be a result of $Re$ near the bed not being sufficiently small for $C_{D}$ to be inversely proportional to $Re$ or the presence of unaccounted momentum sinks, such as the bed drag from the perforated sheets lining the bed. On the basis of Figure 7, interfaces with $S_{0} < 0.75$ and $S_{0} \geq 0.75$ are classified as nonlinear and linear, respectively. The transition occurs at $Re_{toe} \approx 60$.

It is convenient to classify the exchange flows in a schematic, as shown in Figure 8. Although some overlap exists, the flows are largely segregated in the $C_{D}a/L - Re_{toe}$ plane, and approximate boundaries can be defined at $C_{D}a/L = 7$ and $Re_{toe} = 60$. These threshold values correctly...
classify all but 17 out of 133 measurements into inertial and nonlinear and linear drag regimes (Figure 8). Furthermore, these threshold values are consistent with those predicted from scaling the momentum equation (\(C_{drag}L = O(10)\)) and from known transitions in \(C_D\) due to cylinder wake behavior, which changes rapidly at the onset of vortex shedding at \(Re = O(100)\).

[34] The regime classification presented in Figure 8 can assist in the prediction of toe velocity and flux in the field based on easily measurable parameters. Given \(a\), \(d\), \(v\), \(H\), and \(g'\), we first predict the inertial toe velocity and the corresponding \(C_{drag}\) at a given \(L\) from equations (2) and (15). If canopy drag is significant (\(C_{drag}L > 7\)), we can predict the toe velocity from \(u_{toe} = \sqrt{\frac{ng'\dot{H}}{C_{drag}L}}\) and equation (15) iteratively. Then, on the basis of Figure 8 we can anticipate if the vertical profile of the velocity is nonlinear or linear. This classification is important in estimating the net flux, because the velocity profile dictates the flux associated with each layer. As an example, predictions are made for a current propagating in a canopy of \(d = 0.5\) cm reeds by applying field temperature measurements by James and Barko [1991] in a 1 m deep littoral zone. Our model predicts that the current was inertial, regardless of \(g'\). When \(C_{drag}L > 7\), the array drag noticeably decelerated the current. As shown in Figure 6, this transition can occur within the evolution of a single current. Furthermore, linear interfaces were observed in flows where \(Re_{toe} < 60\) and \(C_{drag}L > 7\). The present study provides a template for understanding flow regimes that arise from the presence of vegetation and insight into ways in which hydrodynamic effects of rigid vegetation may be incorporated into numerical models of convective circulation [e.g., Horsch and Stefan, 1988].

5. Conclusion

[36] A mathematical description of lock exchange in a random array of rigid emergent cylinders was derived from the conservation of momentum. When \(C_{drag}L < 7\), the gravity current was inertial, regardless of \(g'\). When \(C_{drag}L > 7\), the array drag noticeably decelerated the current. As shown in Figure 6, this transition can occur within the evolution of a single current. Furthermore, linear interfaces were observed in flows where \(Re_{toe} < 60\) and \(C_{drag}L > 7\). The present study provides a template for understanding flow regimes that arise from the presence of vegetation and insight into ways in which hydrodynamic effects of rigid vegetation may be incorporated into numerical models of convective circulation [e.g., Horsch and Stefan, 1988].

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