TRACING MAGNETIC FIELDS BY ATOMIC ALIGNMENT IN EXTENDED RADIATION FIELDS

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ABSTRACT

Tracing magnetic field is crucial as magnetic field plays an important role in many astrophysical processes. Earlier studies have demonstrated that ground state alignment (GSA) is an effective way to detect a weak magnetic field \((1 G \lesssim B \lesssim 10^{-15} G)\) in a diffuse medium. We explore the atomic alignment in the presence of an extended radiation field for both absorption lines and emission lines. The alignment in the circumstellar medium, binary systems, disks, and the local interstellar medium are considered in order to study the alignment in the radiation field where the pumping source has a clear geometric structure. Furthermore, the multipole expansion method is adopted to study GSA induced in the radiation field with unidentified pumping sources. We study the alignment in the dominant radiation components of the general radiation field: the dipole and quadrupole radiation field. We discuss the approximation of GSA in a general radiation field by summing the contribution from the dipole and quadrupole radiation field. We conclude that GSA is a powerful tool for detecting weak magnetic fields in the diffuse medium in general radiation fields.

Key words: atomic processes – binaries: general – circumstellar matter – Galaxy: disk – ISM: magnetic fields – polarization

1. INTRODUCTION

The astrophysical magnetic field plays an essential role as it is found almost everywhere from relatively small scale systems like solar wind, to considerably large scale systems such as molecular clouds, galaxies, and clusters of galaxies. Many astrophysical processes involve magnetic fields including star formation, cosmic ray acceleration, accretion disks jets, etc. However, few techniques are available to detect magnetic fields and the applicable environment of each technique is limited. Even the directions of magnetic fields obtained from the same region of sky with different techniques differ substantially. The synergic use of different techniques is necessary (see Yan & Lazarian 2013). Therefore, it is important to explore new promising magnetic tracers.

Ground state alignment (GSA) has been demonstrated to be a powerful tool for studying magnetic fields in radiation-dominated environments (see Yan & Lazarian 2006, 2007, 2008, 2012, 2013 for details). GSA is sensitive to the weak magnetic field in a diffuse medium (see Yan & Lazarian 2006). It is worth noting that optical pumping was first proposed by Kastler (1950), and then the atomic alignment in the presence of a magnetic field was studied in the laboratory (see Hawkins 1955). This effect was then applied to aligned atoms in toy models by Varshalovich (1968, 1971). Later, the case of emission from an atom with an idealized fine structure for a particular geometry of a magnetic field and light beam was discussed in Landolfi & Landi Degl’Innocenti (1986).

The basic idea of GSA has been well illustrated: anisotropic radiation pumps atoms with different probabilities from the sublevels of the ground state to the upper levels. The decay from the upper levels has the same probability for all of the sublevels of the ground state. As a result, the occupation of the atoms in different sublevels of the ground state are changed. In the presence of a magnetic field, the angular momenta of the atoms are then redistributed among the different sublevels due to fast magnetic precession. The alignment is altered according to the angle between the magnetic field and the radiation field \(\theta\). This is referred to as magnetic realignment (see Yan & Lazarian 2012, 2013).

Calculations for GSA with fine and hyperfine structure in astrophysical environments were provided in Yan & Lazarian (2006, 2007, 2008). They demonstrated an exclusive feature of GSA, that is, that it reveals the three-dimensional (3D) direction of the magnetic field. Shangguan & Yan (2013) discussed the applicability of GSA, particularly in the interplanetary medium.

These previous works have already applied GSA to the detection of a magnetic field in a diffuse medium where the radiation field is a beam of light. However, GSA with a general radiation field can be different. For example, when the medium is close to the radiation source, the pumping source cannot be treated as a point source. The spatial distribution of the pumping source directly decides the anisotropy of the radiation field, leading to a different alignment from the case with a beam of light.

The structure of the paper is organized as follows. The physics for GSA with an extended radiation field are discussed in Section 2. The general formulae of GSA are presented in Section 3. We first illustrate the alignment in the radiation field with identified pumping sources including the circumstellar medium in Section 4, a binary system in Section 5, and the Local ISM in Section 6. The method of multipole expansion is illustrated and adopted in Section 7 to study the alignment in the radiation field with unidentified pumping sources. Discussions and summary are provided in Sections 8 and 9, respectively.

2. PHYSICS FOR GSA

2.1. Toy Model for GSA

An idealized toy model for GSA is presented in Figure 1(a) to demonstrate the basic physics (see Yan & Lazarian 2006). It is important to note that all the angles used in the paper are...
The condition where GSA can be applied to trace the magnetic field is to mix the different $M$ states, which is known as magnetic realignment. The atoms in the interstellar medium occupy the sublevels of the ground state differently influenced by the magnetic field since the direction of the incoming photon generally does not coincide with the magnetic field (see Yan & Lazarian 2006). Clearly, the randomness in this situation is not complete and the magnetic realignment reflects the direction of the magnetic field. It is important to note that magnetic realignment only occurs when the magnetic precession rate is much higher than the excitation rate from the ground state. This is especially true for most diffuse mediums, and so GSA is an excellent magnetic tracer in the diffuse medium.

In summary, for the atoms to be aligned, there must be sufficient degrees of freedom (the quantum angular momentum number should be $\geq 1$) as well as anisotropic incident radiation. It is important to note that in this study the collision excitation rate is lower than the radiative excitation rate, which is satisfied in many astrophysical environments such as the interplanetary medium, the interstellar medium, the intergalactic medium, etc. The condition where GSA can be applied to trace the magnetic field is defined as the magnetic realignment regime in Yan & Lazarian (2012). The atomic alignment in a diffuse medium with the existence of an extended radiation field can be different because the anisotropy of the extended radiation field is different from that of a point source.

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4 There could be degenerate cases where the pumping source is the same as the background source (see Yan & Lazarian 2006).

2.2. Absorption and Emission

Yan & Lazarian (2006) proposed that the absorption line induced by GSA can be used to study the astrophysical magnetic field. As illustrated in Figure 1(b), the aligned atoms change the polarization of the photons from background sources. For an absorption line in the optically thin case, the Stokes parameters $S = [I, Q, U, V]$ are given by $Q = -\eta_d d l_0$, $I = l_0(1 - \eta_l d l_0)$, where the quantities $\eta_l$ are the absorption coefficients defined in Equation (51) in Appendix C, the quantity $d$ is the thickness of the medium, and the quantity $l_0$ is the intensity of the background. We find that for unpolarized incoming light from background sources, $U = V = 0$, meaning that the polarization is linear and can be only either parallel or perpendicular to the magnetic field due to fast magnetic precession. Thus, the degree of polarization $P$ per unit optical depth $\tau = \eta_0 d$ is

$$
P = \frac{Q}{I \eta_0 d} \approx \frac{1.5 \sigma_0 \sqrt{2} \sin^2 \theta \omega_{J,\ell, m}^2}{\sqrt{2} + \sigma_0^2(J_l)(1 - 1.5 \sin^2 \theta) \omega_{J,\ell, m}^2},
$$

where $\sigma_0^2$ depends on $\theta$ (the angle between the direction of the radiation field and the magnetic field), $\theta$ is the angle between the line of sight and the magnetic field shown in Figure 1(b), and the quantity $\omega_{J,\ell, m}^2$ is presented in Appendix C. $\sigma_0^2 \equiv \rho_0^2 / \rho_0^2$, the normalized dipole component of the ground state density matrix, is the actual measure of the GSA. The quantity $\rho_0^2$ is the total atomic population in the level, whereas the quantity $\rho_0^2$ is the dipole component of the density matrices. A positive value of $P$ means that the polarization is parallel to the direction of the magnetic field, whereas a negative $P$ implies a perpendicular case. Resulting from the sign reversal of $\sigma_0^2$, the polarization $P$ in the magnetic realignment regime flips between being parallel and perpendicular to the magnetic field in unpolarized background sources. In reality, this means that the detection of any polarization in the absorption line provides us with the magnetic field projected onto the plane of sky with 90° degeneracy. This is all of the information available from the Goldreich & Kylafis (1982) effect, which deals with the polarization of molecular radio lines arising from magnetic redistribution in the upper levels instead of the ground state (see Yan & Lazarian 2006, 2012 for details). The 90° degeneracy is also suggested in the grain alignment observed...
by Orion KL (see Rao et al. 1998). Additionally, if the degree of polarization is observed, then both $\theta_\parallel$ and $\theta_\perp$ can be determined, leading to the removal of the 90° degeneracy and the attainment of the 3D direction of the magnetic field.

We only consider the influence of the magnetic field on the ground state because the magnetic influence on the upper level is negligible when the magnetic precession period is comparable to the life time of the upper level. The emission line is influenced by GSA through scattering from the aligned ground state. The linear polarization degree $P$ for the optically thin case for an emission line is given by

$$P = \sqrt{Q^2 + U^2} / I = \sqrt{c_2^2 + c_1^2} / c_0, \quad (2)$$

where $c_i$ are the emission coefficients given in Appendix C.

In the paper, we mainly focus on the absorption line in the extended radiation field. Only in Section 4 is the polarization of emission line discussed for the circumstellar medium as an example since the effect of GSA is less straightforward for the emission line discussed for the circumstellar medium. The $9 - j$ symbols are represented by the matrices with “{}”, whereas the $3 - j$ symbols are indicated by the matrices with “()” (see Zare & Harter 1989 for details). The evolution of the upper state $[p^1_{j}\{j\}]$ is illustrated in Equation (3), whereas Equation (4) describes the ground state $[p^0_{j}\{j\}]$.

The second terms on the left side of Equations (3) and (4) represent magnetic realignment, which can be neglected in the upper levels. The two terms on the right side represent spontaneous emission and excitation in the ground level. The transitions to all upper states and all ground sublevels are counted by summing $J_\parallel$ and $J_\perp$. Note that the quantities $k$ and $q$ are conserved for the symmetric processes of spontaneous emission and magnetic realignment.

The excitation depends on

$$J_{\parallel}^k = \int_0^\infty \frac{d\nu}{\nu} \xi (\nu - \nu_0) \frac{d\Omega}{4\pi} \sum_{i=0}^3 J_i^k (i, \Omega) S_i (\nu, \Omega), \quad (8)$$

where $J_{\parallel}^k$ is the radiation tensor of the incoming light averaged over the whole solid angle and line profile $\xi (\nu - \nu_0)$. The nonzero elements of the radiation tensors for the incoming radiation ($\theta_\parallel$, $\phi_\parallel$) are obtained by substituting

$$J_{\parallel}^\parallel (i, \Omega) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$J_{\parallel}^\perp (i, \Omega) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 - 1.5 \sin^2 \theta \\ -3/2 \sin^2 \theta \end{bmatrix},$$

$$J_{\perp}^\parallel (i, \Omega) = \begin{bmatrix} \sin^2 \theta/4 \\ -1 \cos^2 \theta \end{bmatrix},$$

$$J_{\perp}^\perp (i, \Omega) = \begin{bmatrix} \pm \sin 2 \theta/4 \\ \pm \cos 2 \theta \end{bmatrix}. \quad (9)$$

The data of the Einstein coefficients used in the paper are taken from the Atomic Line List (http://www.pa.uky.edu/~peter/atomic/) and the NIST Atomic Spectra Database.

5 The quantities $p^j_{\parallel}$ and $J_i^k$ both defined in Appendix D, are irreducible density matrices for the atoms and the radiation field, respectively. The 6 - $j$ and 9 - $j$ symbols are represented by the matrices with “{}”, whereas the 3 - $j$ symbols are indicated by the matrices with “()” (see Zare & Harter 1989 for details). The evolution of the upper state $[p^1_{j}\{j\}]$ is illustrated in Equation (3), whereas Equation (4) describes the ground state $[p^0_{j}\{j\}]$. The second terms on the left side of Equations (3) and (4) represent magnetic realignment, which can be neglected in the upper levels. The two terms on the right side represent spontaneous emission and excitation in the ground level. The transitions to all upper states and all ground sublevels are counted by summing $J_\parallel$ and $J_\perp$. Note that the quantities $k$ and $q$ are conserved for the symmetric processes of spontaneous emission and magnetic realignment.

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Equation (9) into Equation (8):

\[
\begin{align*}
\vec{J}^0_0 &= I_0, \\
\vec{J}^2_0 &= \frac{W_a}{2\sqrt{2}W} (2 - 3 \sin^2 \theta) I_0,
\end{align*}
\]

where the quantity \( W \), composed of an anisotropic part \( W_a \) and an isotropic part \( W_i \), is the dilution factor of the radiation field. Clearly, \( W_i \) is nonzero for an extended radiation field, while it is zero for a beam of light. Therefore, both the alignment and polarization in the extended radiation field are expected to be different compared to the case of a point source. \( I_0 \) is the intensity of the radiation source averaged over the whole solid angle. For example, the intensity of the blackbody radiation source is given by

\[
I_0 = \frac{2 h v^3}{c^2} \frac{1}{e^{\frac{hv}{kT}} - 1}.
\]

Hence, the steady state occupation of atoms in the ground state is obtained by setting the left side of Equations (3) and (4) to zero:

\[
2\pi \mu^2_q \langle J \rangle q g \nu \lambda - \sum_{J \in \lambda} p_k \left( J_0, J \right) \sum_{J'_i} \left[ \frac{[J_i]}{A'} + i\Gamma q \right] \sum_{J'} B_{J_0} [J_i] \\
\times \left[ \delta_{kk'} p_{k'} \left( J_0, J' \right) \vec{J}^0_0 + \sum_{Qq'} p_k \left( J_0, J_i, Q, q' \right) \vec{J}^0_0 \right] \\
- \sum_{J \in \lambda} \sum_{Qq'} s_{kk'} \left( J_0, J_0, J_i, Q, q', \vec{J}^0_0 \right) \rho^k_q (J) = 0,
\]

where \( \Gamma \) equals \( 2\pi \nu L R_i / 4A \).

Due to the fast magnetic precession, all of the Zeeman coherence components \((q \neq 0)\) disappear (see Yan & Lazar\-\ian 2006). Moreover, only the \( J_{0,0}^2 \) components are involved due to the selection rule of the 3\( - j \) symbols. It can then be deduced from Equation (12) that \( \sigma_0^2 \equiv \rho^2_0 / \rho_0^0 \propto J_0^2 \). Thus, the degree of polarization \( P \) is proportional to the radiation tensor \( J_0^2 \) (see Equation (1)).

The photons come from different directions in the extended radiation field. Therefore, it is necessary to integrate the radiation tensor over the entire solid angle distribution of the extended radiation field\(^6\) (see Equation (8)). Specifically, the extended radiation source can be treated as an aggregation of point sources if the radiation source is identified. Alternatively, the radiation field can be decomposed to study the alignment parameter in each multipole component in the case where the individual radiation source cannot be pinpointed, e.g., in the interstellar radiation field.

4. GSA IN THE CIRCUMSTELLAR MEDIUM

A geometric model is proposed in Figure 2 to illustrate GSA in the circumstellar region where a star, the dominant radiation source, is not distant enough to be treated as a point source. The incoming radiation from the star to the medium region A forms a cone A-BC with the symmetric axis of the line OA, as demonstrated in Figure 2(a). The cone angle \( \theta \) is dependent on the radius of the star as well as the distance between the star center and the medium A. The radiation frame (\( xyz \)) and angle coordinate \((\theta_a, \phi_a)\) are defined in Figure 2(b). However, for the sake of simplicity, the calculation is performed in the coordinate system \((x'y''z'')\), as shown in Figure 2(c). The mathematics involved in the coordinate conversion are demonstrated in Appendix A.

The alignment in this region is compared with the case of the point radiation source in Figure 3. For the absorption line, we use the elements Si and C\( \alpha \) as examples without the loss of generality. The case of the Si absorption line in a beam of light was demonstrated in Yan & Lazar\-\ian (2006). The ground state of Si is \( 4S^0_2 \) and the upper states are \( 4P_{2,2,2} \) \((J_e = 3/2)\) and the upper states are \( 4P_{2,2,2} \) \((J_e = 3/2, 5/2, 7/2)\). Since we consider the more general situation where the incoming light is unpolarized, the nonzero density matrices of the ground state are those with \( k = 0, 2 \).

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\(^6\) In earlier studies, the distribution of the radiation source with the point-source assumption is \( \delta \) function.
Using the formulae presented in Section 3, we obtain the alignment parameter of SII in the circumstellar medium:

$$
\sigma_0^2(J_l) = \frac{-1.7141 f_c \cos \theta_B \cos \theta_c}{21.3806 + 0.2027 f_c \cos \theta_B \cos \theta_c}, \quad (13)
$$

in which

$$
f_c(x, y) = \frac{1}{2} \left(3x^2 - 1\right) \left(-y^3 + y\right). \quad (14)
$$

By substituting Equation (13) into Equation (1), we obtain the polarization for the SII absorption line in the circumstellar medium:

$$
P = \frac{\left\{ \left( -2.571 f_c \cos \theta_B \cos \theta_c \sin^2 \theta_0 \sigma_{0, ul}^2 \right) \right\}}{\left\{ \left[ 30.2367 + 0.2867 f_c \cos \theta_B \cos \theta_c \right] - 1.7141 f_c \cos \theta_B \cos \theta_c \right\}} \left(1 - 1.5 \sin^2 \theta \right) \sigma_{0, ul}^2, \quad (15)
$$

The results of SII in a circumstellar medium with different cone angles are shown in Figure 3(a).

Atom species such as CII have more than one ground sublevel, which are different from SII. We consider absorption from the two sublevels of the ground state 2P^0_{1/2,3/2} to the upper states 2S_{1/2, 3/2}. Only the sublevel 2P^0_{3/2} in the ground state can be aligned with the two density tensors \( \rho_0^{0,2} \). The results for CII in the circumstellar medium with different cone angles are plotted in Figure 3(b).

Figure 3 indicates that the polarization of the absorption line induced by GSA flips between being parallel and perpendicular to the magnetic field when \( \theta_B \) is the Van Vleck angle 54° 7 (van Vleck 1925; House 1974), resulting from the coupling of two oscillators in the \( x-y \) plane caused by Larmor precession around the magnetic field. This switch is a general feature regardless of the specific atomic species if the background source is unpolarized. Furthermore, the alignment in the circumstellar medium approaches that in a beam of light (see Yan & Lazarian 2006) as \( \theta_L \to 0 \), as demonstrated in Figure 3.

Apparent in the radiation source can be considered as a point source when \( \theta_L \) approaches zero, i.e., when the radiation source is sufficiently distant.

Yan & Lazarian (2006) illustrated that the magnetic dipole transition has to be taken into account for the alignment of atoms with multiple ground levels in the weak pumping regime where the medium is distant enough from the radiation source, i.e., the magnetic dipole radiation rate \( A_m \) is comparable to the pumping rate \( \tau_{\text{p}}^{-1} \). Conversely, the magnetic dipole can be neglected in the strong pumping regime. The boundary of the two regimes \( r_m \) is defined as the place where

$$
A_m \sim \tau_{\text{p}}^{-1} = \left( \frac{R_\ast}{r_m} \right)^2 B I_e, \quad (16)
$$

in which \( R_\ast \) is the radius of the star. \( A_m \) is the Einstein coefficient of the magnetic dipole transition on the ground state. Inserting Equation (11) into Equation (16), we obtain the radius of the boundary sphere inside which the strong pumping regime is located:

$$
\frac{r_m}{R_\ast} = \left[ \frac{J_0}{J_1} \right] \frac{A(J_0 \to J_1)}{A_m} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}. \quad (17)
$$

The boundary for CII is illustrated in Figure 4(a). Obviously, the boundary varies with the type of the central star. The strong pumping approximation is valid for the regime noted in Figure 4(a). An example is provided in Figure 4(b) for a B-type star with a temperature of 15000 K, revealing the alignment of CII with a different cone angle \( \theta_L \) in the strong pumping approximation.

The polarization of the D2 emission line of NaI in the circumstellar medium with a different cone angle \( \theta_L \) is compared with that in a beam of light, as shown in Figures 5(a) and (b) for \( \theta = 0 \) and \( \theta = \pi/2 \), respectively. Obviously, the linear polarization of NaI’s D2 emission line approaches the case with a point source (see Yan & Lazarian 2007) as \( \theta_L \to 0 \).

In this section, we demonstrate that the radiation source of the circumstellar medium can be treated as a point source when it is far away from the medium. In addition, the switch point of the polarization between being parallel and perpendicular to the
magnetic field is the Van Vleck angle in the circumstellar medium, similar to that in a point source. Therefore, the 2D magnetic field in the circumstellar medium can be directly determined by the direction of polarization with degeneracy; if the degree of polarization is observed, then the 3D magnetic field can be identified.

5. GSA IN BINARY SYSTEMS

In this section, we consider GSA in binary systems, which are quite common in the universe. Studies since the early 19th century have suggested that a large fraction of stars are in binary systems. Moreover, binaries can be surrounded by circumbinary disks, which may be important for star formation (Mathieu et al. 2000). Examples of circumbinary disks can be found in Artymowicz et al. (1991).

As demonstrated in Section 4, pumping by a beam of light is the asymptotic limit of pumping by a narrow cone of light. Hence, for the sake of simplicity, the binaries are treated as two point sources in the case where the medium considered is sufficiently distant, as shown in Figure 6(a). It is generally believed that circumbinary gaseous disks are geometrically coplanar with binaries (see Artymowicz & Lubow 1994). The geometry of the binary system is illustrated in Figure 6(b).

From Figure 5(b), we know that $\alpha_1 = \Psi - \alpha_2$. Thus, the angles between the magnetic field and the two radiation lines can be represented as $(\cos \alpha_1 \cos \phi)$ and $(\cos(\Psi - \alpha_1) \cos \phi)$, respectively. We define $r$ as the intensity ratio of the radiations from $r_1$ and $r_2$:

$$r = \frac{I_1}{I_2}. \quad (18)$$

The radiation tensor is obtained by summing the contribution from the two radiations:

$$T_0^b = \frac{1}{2\sqrt{6}} \left( 3 \left( \frac{\cos^2 \alpha_1 + r \cos^2(\Psi - \alpha_1) \cos^2 \phi_b}{1 + r} \right) - 1 \right). \quad (19)$$

Obviously, once the ratio $r$ and the angle $\Psi$ are known, the degree of polarization is solely determined by the direction of the magnetic field $(\phi_b, \alpha_1)$. In addition, we define

$$t = \frac{\cos^2 \alpha_1 + r \cos^2(\Psi - \alpha_1)}{1 + r} \quad (20)$$
to measure the anisotropy of the binary system. Apparently, when \( r \rightarrow 0 \) or \( \infty \), the system falls back to the case of a beam of light.

Inserting Equation (19) into Equation (12), we obtain the alignment parameter of SII in a binary system:

\[
\sigma_0(J) = \frac{0.4199 - 1.2596 \cos^2 \phi_b}{0.1490 \cos^2 \phi_b - 5.2868}. \tag{21}
\]

Substituting Equation (21) into Equation (1), we obtain the polarization of the Si absorption line in a binary system:

\[
P \tau = \begin{cases} 
\left\{ \left( 0.6299 - 1.8894 \cos^2 \phi_b \sin \theta \omega_{L,L} \right) \right\}^2 \\
\left( 0.2107 \cos^2 \phi_b - 7.4767 \\
+ \left( 0.4199 - 1.2596 \cos^2 \phi_b \right) \\
\times \left( 1 - 1.5 \sin^2 \theta \right) \omega_{L,L} \right\}. \tag{22}
\end{cases}
\]

The results for binary systems with different ratios \( r \) are demonstrated in Figure 7(a), showing that the closer the intensity of the secondary is to that of the primary, the smaller the alignment is. This is because the binary system becomes more isotropic as the radiation from the secondary becomes comparable to that from the primary. As an example, a binary system with \( \psi = \pi/3 \) and \( r = 0.5 \) is discussed in Figure 7(b) for polarization with a varying magnetic field \((\phi_b, \alpha_1)\). The polarization is apparently dependent on the direction of the magnetic field.

The isosurfaces for polarization with a varying binary parameter \((b)\) and magnetic field inclination \((\phi_b)\) are presented in Figure 8. The polarization crosses zero when \( \phi_b = \arccos \frac{1}{\sqrt{3}} \). It is a general feature in binary systems for any atomic species that the polarization of an absorption line flips between being parallel and perpendicular to the magnetic field when the inclination angle of the magnetic field \( \phi_b \) is \( \arccos \frac{1}{\sqrt{3}} \). Therefore, the 2D magnetic field in binary systems...
can be directly determined by the direction of polarization with 90° degeneracy, whereas the 3D magnetic field can be obtained if the degree of polarization is observed.

6. GSA WITH A DISK SHAPED RADIATION FIELD

In this section, we apply GSA to the disk shaped radiation field and discuss the alignment in the Local Interstellar Medium (LISM) as an example in Section 6.1.

The disk shaped radiation field is illustrated in Figure 9(a). The intensity of the pumping source is evenly distributed, and 2α₀ = arcsin(R/d₀) is the flare angle of the radiation field. The radius of the disk is much larger than the thickness of the disk. Hence, the thickness of the radiation source can be neglected and a thin disk approximation can be applied. It is assumed that the magnetic field has two components: one along the circumference of the disk and one perpendicular to the disk, hereafter B₀ and B₁, which means that the projection of the magnetic field onto the plane of the disk is tangential to the disk, e.g., the case of the Galactic magnetic field in the Spiral Arm.

The radiation tensor can be obtained by integrating all of the radiation from the disk:

\[ J₀^2 = \frac{1}{2\sqrt{6}} \left( \frac{3}{2} \cos^2 \phi f₁(α₀) - 1 \right) \]  \hspace{1cm} (23)

where

\[ f₁(x) = 1 - \frac{\sin 2x}{2x}, \quad \phi f₁ = \arctan \frac{B₁}{B₀} \]  \hspace{1cm} (24)

The alignment parameter of SII in the disk shaped radiation field is obtained by inserting Equation (23) into Equation (12):

\[ \sigma_0^2(J₁) = \frac{6.2982 \cos^2 \phi f₁(α₀) - 4.1988}{52.8680 - 0.7448 \cos^2 \phi f₁(α₀)} \]  \hspace{1cm} (25)

Substituting Equation (25) into Equation (1), we obtain the polarization of the SII absorption line in the disk shaped
radiation induced by GSA:

\[
\sum_{l=2}^{\infty} \sum_{m=-l}^{l} C_{l}^{m}(\phi_{i}, \alpha_{0}) \cdot Y_{l}^{m}(\theta, \phi) = 6.2982 \cos^{2} \phi_{i} - 4.1988 \sin^{2} \phi_{i}.
\] (29)

and accordingly the alignment parameter of $S_{ii}$ in LISM,

\[
P = \frac{6.2982 \cos^{2} \phi_{i} - 4.1988 \sin^{2} \phi_{i}}{52.8680 - 0.7448 \cos^{2} \phi_{i}}.
\] (30)

The isocontour plot of polarization for a varying direction of the line of sight ($\theta$) and the magnetic field inclination ($\phi_{i}$) is provided in Figure 11. The polarization of the absorption line in LISM switches between being parallel and perpendicular to the direction of the magnetic field when $\phi_{i}$ is $35\degree.3$. Hence, the 2D magnetic field can be directly determined by the direction of the polarization with a $90\degree$ degeneracy. The 3D information of the magnetic field in LISM can be obtained if the degree of polarization is observed.

7. GSA WITH A GENERAL RADIATION FIELD

It has been discussed in the previous sections that GSA can be applied to trace the magnetic field where the specific pumping sources of the radiation field are identified. However, the sources of the radiation field may not be easily identified in some circumstances. In this case, the method of multipole expansion can be adopted, that is, to compute the alignment in multipole components and then add them up.

Any radiation field can be decomposed in terms of the irreducible representations of the rotational symmetry group, which can be represented by spherical harmonics and related sets of orthogonal functions $f(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=0}^{\infty} C_{l}^{m}(\theta, \phi) Y_{l}^{m}(\theta, \phi)$ (see Jackson 1975). The function $f(\theta, \phi)$ is the spatial distribution of the radiation ($\theta, \phi$), $Y_{l}^{m}(\theta, \phi)$ are the standard spherical harmonics, and $C_{l}^{m}(\theta, \phi)$ are the coefficients. The quantity $l$ in both terms refers to the order of the multipole component. For example, $l = 2$ denotes the
dipole component, the intensity of which is
\[
\sum_{\ell=2}^{2} C^{m}_{2} Y^{m}_{2} (\theta, \phi) \propto \sin^{2} \theta, \tag{31}
\]
and \( l = 4 \) represents quadrupole components with the intensity
\[
\sum_{m=-4}^{4} C^{m}_{4} Y^{m}_{4} (\theta, \phi) \propto \cos^{2} \theta \sin^{2} \theta. \tag{32}
\]
The overall alignment in the total radiation field could be obtained by studying the alignment in each multipole radiation component. For example, GSA in dipole and quadrupole radiation fields are discussed. As we demonstrate below, these are the dominant contributions to the alignment.

7.1. Physics for GSA in a Multipole Radiation Field

The dipole radiation field is presented in Figures 12(a) and (b) in the directions perpendicular and parallel to the symmetrical axis, respectively. We define the coordinate system in Figure 12(c) and illustrate the coordinate transform needed to simplify the computation in Appendix A. The radiation tensor can be obtained by inserting Equation (31) into Equation (8):
\[
J_{\theta}^{0} = \frac{1}{4\pi} \int_{0}^{2\pi} \int_{0}^{\pi} \left( \frac{1.5 \sin \theta_{rd} \sin \Psi_{kd} \cos \phi_{kd}}{+ \cos \theta_{rd} \cos \Psi_{kd}} - 0.5 \sin^{3} \Psi_{kd} \right) \left( \sqrt{6} \right) \sin^{2} \theta_{kd} d\theta_{kd} d\phi_{kd}. \tag{33}
\]

The irreducible radiation tensor in the dipole radiation field is
\[
J_{\theta}^{0} = \frac{\sqrt{6} \left( 1 - 3 \cos^{2} \theta_{rd} \right)}{90}. \tag{34}
\]

Inserting Equation (34) into Equation (12), we obtain the alignment parameter of \( \text{SII} \) in the dipole radiation field:
\[
\alpha^{2}_{0} (J_{l}) = \frac{1.6795 \cos^{2} \theta_{rd} - 0.5598}{34.8481 - 0.1986 \cos^{2} \theta_{rd}}. \tag{35}
\]

By substituting Equation (35) into Equation (1), the polarization of the \( \text{SII} \) absorption line in the dipole radiation...
The geometric distribution of the quadrupole radiation field is presented in Figures 13(a) and (b) in the directions perpendicular and parallel to the symmetrical axis, respectively. The coordinate system is defined in Figure 13(c). We apply a similar procedure to calculate GSA in the quadrupole radiation field:

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   J_0^2 &= \frac{\sqrt{6} (3 \cos^2 \theta_q - 1)}{630}, \\
   J_0^2 &= \frac{\left( -0.3599 \cos^2 \theta_q + 0.1200 \sin^2 \theta \right)}{630},
\end{align*}
\]

and the polarization in the quadrupole radiation field

\[
\begin{align*}
   \frac{P}{\tau} &= \left[ \left( 2.1913 \cos^2 \theta_\alpha - 0.8397 \right) \sin^2 \theta \omega^2_{\beta \gamma} \right] \\
   \frac{P}{\tau} &= \left[ \left( -0.3599 \cos^2 \theta_q + 0.1200 \sin^2 \theta \right) \omega^2_{\beta \gamma} \right].
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field

\[
\begin{align*}
   \sigma^2(J_1) &= \frac{-0.2399 \cos^2 \theta_q + 0.0800}{6.9923 - 0.0284 \cos^2 \theta_q}, \\
   \sigma^2(J_0) &= \frac{-0.3599 \cos^2 \theta_q + 0.0402 \cos 0.1200 \sin 9.8886}{6.9923 - 0.0284 \cos^2 \theta_q},
\end{align*}
\]

The alignment parameter in the quadrupole radiation field
dipole radiation field and even further reduced in quadrupole radiation field compared to that in a beam of light since the dipole and quadrupole radiation fields are more isotropic. The results suggest that an even higher order of radiation components can be neglected and a good approximation can be made by counting the radiation components up to the quadrupole component. In addition, the observed polarization reaches a maximum at $\theta = \pi/2$ with the same $\theta_r$ (see also Equation (1)). The switch of polarization between being parallel and perpendicular to the magnetic field in the dipole and quadrupole radiation fields occurs at the Van Vleck angle, similar to the case for a beam of light, regardless of the atomic species. As a result, the 2D information for the magnetic field can be directly obtained from the direction of polarization with $90^\circ$ degeneracy. If the degree of polarization is observed, then the 3D magnetic field can be obtained.

7.2.2. Polarization in Multipole Components

The aim of multipole expansion is to approximate the main function by taking only the first few components into account. A sufficient approximation of the radiation field can be made by using the dipole and quadrupole radiation components since higher-order radiation fields are isotropic to such a degree that the resulting alignment is very weak. Therefore, we will only discuss GSA in dipole and quadrupole radiation fields here.

We add the quadrupole radiation field to the dipole field with a varying intensity ratio ($w$) and angle between their axes ($\theta_{dq}$). The intensity ratio of the quadrupole radiation field and dipole radiation is defined as

$$ w \equiv u_q/u_d, \quad (40) $$

where the quantities $u_d$ and $u_q$ are the intensity averaged over the whole solid angle for the dipole and quadrupole radiation field, respectively. The correction due to the addition of the quadrupole field is defined as

$$ R_P \equiv \frac{\Delta P}{P} = 1 - \frac{P(\text{dipole})}{P(\text{mix})}. \quad (41) $$

Figure 17(a) presents the correction of the polarization $R_P$ with a different intensity ratio ($w$) and a different angle between their axes ($\theta_{dq}$). Evidently, the anisotropy of the dipole and the quadrupole radiation fields are opposite. The dividing line marks the place where the contributions from the dipole radiation field and the quadrupole radiation field cancel each other. It is clear that the contribution from the dipole radiation field is dominant over that from the quadrupole radiation field if $w < 7$. This is a general feature for radiation fields. As an example, Figure 17(b) is plotted to illustrate the polarization of an SiII absorption line induced by GSA when the dipole axis coincides with the quadrupole axis.

8. DISCUSSIONS

In the paper, we extend the studies of GSA to a general radiation field in a diffuse medium. We conclude that GSA can be applied to trace the magnetic field in a diffuse medium in a
general radiation field, which largely broadens the applicability of GSA in astrophysical circumstances. Calculations for both absorption and emission lines are presented, filling in the gap from earlier studies on GSA by considering the atomic alignment in different radiation fields. In addition, we adopt the method of multipole expansion in order to obtain the polarization in a general radiation field with unidentified pumping sources.

**Study of GSA for emission line.**

We have discussed the results from an absorption line in an extended radiation field. We stress that the emission line can also be used to trace the magnetic field in all of the situations discussed in the paper. As an example, we present the calculation for emission lines in the circumstellar medium. Similar approaches can be applied to study the polarization of emission lines in other circumstances.

**Tracing the 3D magnetic field with GSA.**

We conclude that the direction of the polarization provides us with the projection of the magnetic field onto the plane of sky with $90^{\circ}$ degeneracy in a general radiation field. Our results on switch angles for different radiation fields are provided in Table 1. A 3D magnetic field could be obtained if the degree of polarization is observed.

**Atomic alignment in identified radiation sources.**

We have provided a tool to trace the magnetic field in the circumstellar medium, binary systems, disks, and the Local ISM. Apparently, similar approaches can be applied to study the alignment in the diffuse medium near other radiation sources. For instance, multiple systems can be studied by considering each star as a point source and integrating all of the incoming lights to the medium.

**Strong pumping regime.**

The strong pumping approximation is specifically discussed for the alignment in the circumstellar medium in the paper. Evidently, this approximation is also applicable for alignment in similar systems where the medium is sufficiently close to the sources. The criteria are provided by Equation (17).

**The influence of collision.**

We discuss the diffuse medium where the influence of collision is negligible. Collisions redistribute atoms to different sublevels, though with reduced efficiency as disalignment of the ground state requires spin flips (see Hawkins 1955). We will evaluate the effect of collision when it cannot be neglected on GSA induced by the extended radiation field in future work.

9. SUMMARY

GSA with different radiation fields are discussed in the study. GSA has been demonstrated in earlier studies to be a powerful magnetic tracer when the radiation field is a beam of light. We extend the applicability of GSA to the medium in extended radiation fields including the circumstellar medium, binary systems, disks, and the LISM. Moreover, the method of multipole expansion is utilized for radiation fields with unidentified pumping sources. We reach the following conclusions.

1. GSA exists wherever the radiation fields are anisotropic.
2. Spectropolarimetry modulated by GSA can be applied to trace magnetic fields in the diffuse medium in a general radiation field.
3. The direction of polarization directly traces the 2D magnetic field in the plane of the sky. Similar to the case
for a beam of light, the polarization of the absorption line is either parallel or perpendicular to the magnetic field in the extended radiation field. We provide the criteria for which the polarization flips in all cases.

4. 3D mapping of the magnetic field can be determined with quantitative measurements of the polarization from GSA.

5. Alignment with unidentified radiation sources can be obtained by considering the alignment in the dipole and quadrupole radiation components of the radiation field.

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APPENDIX A
FROM MAGNETIC FRAME TO RADIATION FRAME

The symmetry axis of the radiation may not be parallel to the magnetic field in the circumstellar medium. Hence, a transformation from the magnetic frame to the extended radiation frame is needed to integrate the influence of the whole cone to the magnetic field. We illustrate the geometry for the coordinate conversion in Figure 2(c). We use \((\theta_\psi, \phi_\psi)\) for the angle coordinates in the extended frame \((x'y'z')\)-frame and \((\theta, \phi)\) for the angle coordinates in the magnetic frame. \((\theta_B, \phi_B)\) are the coordinates for the axis of the radiation cone in the \((x'y'z')\)-frame. We consider a line whose length equals 1 in the magnetic frame with the coordinates

\[
\begin{pmatrix}
\sin \theta \cos \phi \\
\sin \theta \sin \phi \\
\cos \theta
\end{pmatrix},
\]

whereas in the extended radiation frame the coordinates are

\[
\begin{pmatrix}
\sin \theta_\psi \cos \phi_\psi \\
\sin \theta_\psi \sin \phi_\psi \\
\cos \theta_\psi
\end{pmatrix}.
\]

The relation between these two coordinates can be demonstrated by

\[
\begin{pmatrix}
\cos \theta \cos \phi \\
\cos \theta \sin \phi \\
\sin \theta
\end{pmatrix} = \begin{pmatrix}
\cos \phi_B \\
\sin \phi_B \\
0
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_B & \sin \theta_B \\
0 & -\sin \theta_B & \cos \theta_B
\end{pmatrix} \begin{pmatrix}
\sin \phi_B \\
\cos \phi_B \\
0
\end{pmatrix}.
\]

Thus, for an emission line,

\[
\theta_\psi = \arccos(\cos \phi_B \sin \theta_B \theta \cos \phi_\psi + \sin \phi_B \sin \theta_B \theta \sin \phi_\psi + \cos \theta_B \cos \theta_\psi),
\]

\[
\phi_\psi = -\arctan\left(\frac{\sin \theta_B \sin \theta_\psi \cos \phi_\psi - \cos \phi_B \sin \theta_B \theta_\psi \sin \phi_\psi}{\sin \phi_B \sin \theta_\psi \cos \phi_\psi + \cos \theta_B \sin \theta_\psi \theta_\psi \cos \phi_\psi}\right).
\]

APPENDIX B
SUMMARY OF ANGLES USED IN THE PAPER

| Angle between the line of sight and the magnetic field in Figure 1(a) | \(\theta\) |
| Angle between the radiation direction and the magnetic field in Figure 2(a) | \(\theta_r\) |
| Angle between the radiation direction and the magnetic field in Figure 2(b) | \(\theta_B\) |
| Angle between the cone axis and the magnetic field in Figure 2(b) | \(\theta_\psi\) |
| Angle between the cone axis and the quadrupole axis in Figure 2(c) | \(\theta_\theta\) |
| Angle between the magnetic field and the plane of radiation in a binary in Figure 6(b) | \(\psi_\psi\) |
| Angle between the two radiation lines in a binary in Figure 6(b) | \(\alpha_\psi\) |
| Angle between the magnetic field and the plane of a pumping disk in Figure 9(a) | \(\phi_\psi\) |
| Angle between the dipole axis and the magnetic field in Figure 12(c) | \(\theta_d\) |
| Angle between the radiation direction and the dipole axis in Figure 12(c) | \(\Psi_d\) |
| Angle between the quadrupole axis and the magnetic field in Figure 13(c) | \(\theta_q\) |
| Angle between the radiation direction and the quadrupole axis in Figure 13(c) | \(\Psi_q\) |
| Angle between the dipole axis and the quadrupole axis in Figure 13(c) | \(\phi_q\) |

APPENDIX C
DEFINITION OF SOME PHYSICAL PARAMETERS

The basic formulae for GSA have been well illustrated in earlier studies (see, e.g., Yan & Lazarian 2006, 2007, 2008). Only a few key terms are presented here for the sake of completeness.

The Einstein coefficients illustrating the simulated transitions \(B\) are related to the Einstein spontaneous emission rate \(A\) by

\[
[J_n]A(J_n \rightarrow J_l) = \frac{2 \hbar^3}{c^2} [J_l]B_{nl} = \frac{2 \hbar^3}{c^2} [J_n]B(J_n \rightarrow J_l).
\]

The absorption coefficients \(\eta\) to illustrate the polarized absorption from the ground level are defined as (Landi
Degl’Innocenti 1984 see also Yan & Lazarian 2006):

\[ \eta_l(\nu, \Omega) = \frac{h \nu_0}{4\pi} \frac{B_l(n)}{\Psi(\nu - \nu_0)} \times \sum_{KQ} (-1)^K \omega_j^{K_l} \sigma_q^K (J_l, \theta_l) J_Q^K (i, \Omega), (51) \]

where the total atomic population \( n_l(J_l) \) on the lower level \( J_l \) is defined as \( n_L^0 \rho_0^0 (J_l) \). Additionally,

\[ \omega_l = \begin{cases} 1 & \text{if } l = 1 \\ 0 & \text{if } l = 0 \end{cases} \times \begin{cases} 1 & \text{if } l = 1 \\ 1 & \text{if } l = 0 \end{cases} \] (52)

Moreover, the emission coefficients \( \epsilon_i \) demonstrating the polarized emission from the upper level are (see Yan & Lazarian 2007)

\[ \epsilon_i(\nu, \Omega) = \frac{h \nu_0}{4\pi} \frac{\Delta n(J_u, \theta_u)}{\Psi(\nu - \nu_0)} \times \sum_{KQ} \omega_l^K \sigma_q^K (J_u, \theta_u) J_Q^K (i, \Omega). (53) \]

APPENDIX D
DENSITY MATRIX

The irreducible tensorial formulae adopted in this paper can be represented by the standard density matrix of atoms:

\[ \rho^K_Q (J, J') = \sum_{MM'} (-1)^{J-M} (2K + 1)^{1/2} \]

\[ \times \left( \begin{array}{ccc} J & K & J' \\ -M & Q & M' \end{array} \right) \langle JM | \rho | J'M' \rangle. (54) \]

Additionally, the irreducible spherical tensor for photons is

\[ J^K_Q (J, J') = \sum_{qq'} (-1)^{q' + q} [3(2K + 1)]^{1/2} \]

\[ \times \left( \begin{array}{ccc} 1 & 1 & K \\ q & -q' & -Q \end{array} \right) J_{qq'}^{K_Q}. (55) \]

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