Aharonov-Bohm oscillations of four-probe resistance in clean systems with topologically protected channels in silicene and bilayer graphene

Bartłomiej Rzeszotarski, Alina Mreńca-Kolasińska, and Bartłomiej Szafran
AGH University of Science and Technology,
Faculty of Physics and Applied Computer Science,
al. Mickiewicza 30, 30-059 Kraków, Poland

We consider observation of Aharonov-Bohm oscillations in system without disorder based on flow of topologically protected currents in silicene and bilayer graphene gated nanodevices. The chiral channels in these materials are defined by flips of the vertical electric field. The flip confines states at the junction with the direction of the current flow that is determined by the valley. We present an electric field profile that forms a crossed ring channels with four terminals and find that the conductance matrix elements oscillate in the perpendicular magnetic field in spite of the absence of backscattering of the valley protected channels in the system. We propose a four-probe resistance measurement setup and demonstrate that the resistance oscillations have large visibility provided that the system is prepared in such a way that a direct transfer of the valley current between the current probes is forbidden.

I. INTRODUCTION

In graphene [1] nanoribbons with zigzag edges [2] the Fermi wave vectors corresponding to the current flow in one or opposite direction appear near different Dirac points e.g. in opposite valley states [3,5]. In this sense the flow of current is chiral [6]. The chiral valley current within a quasi one-dimensional channel is protected against backscattering by smooth potential variation. Only potential defects that are short range on the scale of the lattice constant can induce intervalley transition that implies backscattering [3,5].

In staggered monolayer graphene [7], in buckled silicene lattice [8-11] or other 2D Xene materials [12,14], similar chiral channels for the electron flow can be tailored within the interior of the crystal by a symmetry breaking potential along its zero lines [7,15,16]. For buckled silicene [8-11] a hexagonal crystal with sublattices placed on parallel planes – the symmetry breaking potential is introduced by perpendiccular electric field [15,16]. Similar chiral channels appear in bilayer graphene [17,21] along the flip of the vertical electric field or in bilayer graphene with a line dislocation [20,22] or twist of the layers [23,24] that induces appearance of AB/BA stacking interface. These interfaces in twisted bilayer graphene form a triangular lattice with the direction of the current flow opposite for both the valleys [21,25].

Recently, a ring-like system with splittings of the chiral zero-line channels, was proposed for both silicene and bilayer graphene [16]. The electron passage time across this system is an periodic function of the external magnetic field due to the Aharonov-Bohm phase difference accumulated from the vector potential [16]. However, the two-terminal Landauer conductance of these systems is independent of the external magnetic field, since the backscattering is absent due to the valley protection. In order to observe the Aharonov-Bohm oscillations in two-terminal conductance one should rely on atomic-scale disorder and presence point defects. The point disorder is hard to control and electron interferometers should be difficult to construct in this way.

The message of this paper is that in spite of the absence of backscattering one can design a interferometer device for observation of the Aharonov-Bohm oscillations of conductance for clean chiral channels defined in both silicene and bilayer graphene. We study the chiral current flow in channels formed by the electric potential flip that define 4-terminal crossed-quantum ring in silicene and bilayer graphene. A simulation of two nonequivalent 4-point resistance measurement setups is given. We find a distinct Aharonov-Bohm periodicity in the resistance amplitude that is associated with the interference on 1/4 of the circle area. We discuss the interference paths that stand behind this periodicity.

II. THEORY

A. Silicene

In this work we use the atomistic tight-binding Hamiltonian spanned by $p_z$ orbitals [10,11,26] $H = -t \sum_{\langle n,m \rangle} (p_{nm} c^\dagger_n c_m + h.c.) + \sum_n V(r_n) c^\dagger_n c_n \ (1)$

where $\langle n,m \rangle$ stands for the nearest neighbor ions. The $c^\dagger_n$ ($c_n$) is the creation (annihilation) operator for an electron on site $n$, and the hopping parameter $t = 1.6 \text{ eV}$ [10,26]. We introduce the magnetic field via the Peierls phase in the hopping elements $p_{nm}$ term, where $p_{nm} = e^{i \mathbf{\AA} \cdot \mathbf{r}_n}$. We define positions $\mathbf{r}_n^A = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2$ of the ions of the $A$ sublattice, where the silicene lattice constant $a = 3.89 \ \text{Å}$, with integers $n_1$, $n_2$, and the ions of the $B$ sublattice are shifted by the basis vector $(0, d, \tau)$, where $d = 2.248$
Å is the nearest neighbor in-plane distance and τ = 0.46 Å is the vertical shift between the sublattice planes.

The quantum ring with the chiral channels is formed by the electric field induced by split top and bottom gates (Fig. 1(a)). The inversion of the field creates the topologically protected conducting channel. We consider ring of radius \( R = 100 \) nm with the center at the origin formed by model potential

\[
V_A = \frac{8V_G}{\pi^2} \arctan \left( \frac{x}{\lambda} \right) \arctan \left( \frac{y}{\lambda} \right) \arctan \left( \frac{R - r}{\lambda} \right),
\]

where \( r = \sqrt{x^2 + y^2} \) is the distance from the origin and the parameter responsible for inversion length \( \lambda \) is set to 4 nm. For symmetric gating the potential on the B sublattice is opposite \( V_B(r) = -V_A(r) \). In the following for the silicene system we use \( V_G = 200 \) meV.

In Fig. 1(b) we plot the direction of the current that are open for the \( K' \) valley electron flow. The \( K' \) valley electrons move along the zero line of the potential given by Eq. (2) leaving the region of negative potential on the \( A \) sublattice on the left hand-side. Note, that the current injected from terminal zero-line 1 can be directed to either terminal 2 or terminal 4. In terminal 3 there is no \( K' \) valley state that carries the electron flow up, away from the ring (Fig. 2). In every channel the direction of the current flow for the \( K \) valley is opposite.

FIG. 1. (a) Sketch of the silicene monolayer between top and bottom gates. Positive potential \( V_G \) is put on the blue gates, and negative \( -V_G \) on the red gates. (b) Top view of the system with leads numeration. Black (white) arrows denote for the directions of \( K' \) (\( K \)) valley protected charge currents within each gap-channel defined by gate interfaces.

The wave function of the states confined laterally at the zero line near \( x = 0 \) is plotted in Fig. 3(a). The confined states correspond to linear bands that appear within the energy gap (Fig. 3(b)).

B. Bilayer graphene

For bilayer graphene the inversion-symmetry-breaking potential can be introduced by an electric field perpendicular to the sheet. We consider a bilayer-graphene-based system analogous to the one described in Sec. 1(I.A)

\[
H = - \sum_{(n,m)} \left( t_{nm} \delta_{nm} c_n^\dagger c_m + \text{h.c.} \right) + \sum_n V(r_n) c_n^\dagger c_n, \tag{3}
\]

with the difference that due to the presence of two layers, two topological states occur instead of one as in silicene. We consider the tight-binding Hamiltonian similar as in Eq. (1)

with graphene lattice constant \( a_{CC} = 1.42 \) Å, and the parametrization of bilayer with Bernal stacking [27], where \( t_{nm} = -3.12 \) eV for the nearest neighbors within the same layer, and for the interlayer coupling, \( t_{nm} = -0.377 \) eV for the A-B dimers, \( t_{nm} = -0.29 \) eV for skew hoppings between atoms of the same sublattice, and \( t_{nm} = 0.12 \) eV — between atoms of different sublattices.
The model potential in the lower layer is described by a formula analogous as in Eq. (2), and in the upper layer it has the opposite polarization, but the sign is the same in both sublattices within the same layer. We use \( \lambda = 4 \) nm, \( R = 100 \) nm, and \( V_G = 0.1 \) eV.

### C. Landauer approach

Solution of the electron scattering problem is performed in the tight-binding model with wave-function matching (WFM) technique. The details of the method were described in Refs. [28, 29]. The electron transfer probability is calculated as

\[
T_{\xi\eta}^w = \sum_v |t_{\xi\eta}^{vw}|^2 ,
\]

where \( t_{\xi\eta}^{vw} \) denote the probability amplitude for the transfer from incoming mode \( v \) in the input lead \( \eta \) to outgoing mode \( w \) in the output lead \( \xi \). Thus, the Landauer conductance formula for the transfer from lead \( \eta \) to \( \xi \) can be written as

\[
G_{\xi\eta} = G_0 \sum_w T_{\xi\eta}^w ,
\]

where \( G_0 = e^2/h \) is the flux quantum.

We focus our attention on Fermi level \( E_F \in \{0 : 0.1\} \) eV with spin-degree of freedom so all that assumptions provide \( G = \max(G_{\xi\eta}) = 2G_0 \) for silicene, and \( G = \max(G_{\xi\eta}) = 4G_0 \) for bilayer graphene.

### D. Conductance matrix

The scattering problem for the four-terminal system was solved for each lead as input channel and the results were collected in conductance matrix \( G \) with general form

\[
G = \begin{pmatrix}
g_{11} & -g_{12} & -g_{13} & -g_{14} \\
-g_{21} & g_{22} & -g_{23} & -g_{24} \\
-g_{31} & -g_{32} & g_{33} & -g_{34} \\
-g_{41} & -g_{42} & -g_{43} & g_{44}
\end{pmatrix},
\]

with \( g_{ij} = \sum_{\delta \neq i} G_{ij} \). Due to the rotational symmetry (\( C_4 \) in terms of channel shape) the conductance matrix \( G \) can be put in the form

\[
G = \begin{pmatrix}
G & -B & 0 & -A \\
-A & G & -B & 0 \\
0 & -A & G & -B \\
-B & 0 & -A & G
\end{pmatrix},
\]

where coefficients

\[
A = G_{14} = G_{21} = G_{32} = G_{43} \quad (8) \\
B = G_{41} = G_{12} = G_{23} = G_{34} \quad (9) \\
0 = G_{13} = G_{24} = G_{31} = G_{42} \quad (10)
\]

and \( B = G - A \).

Assuming the \( V_3 = 0 \) we can truncate 3rd column \( G \) and calculate resistance matrix \( R = G^{-1} \) that can be written as follows

\[
R = \left( \begin{array}{ccc}
R_{11} & R_{12} & R_{14} \\
R_{21} & R_{22} & R_{24} \\
R_{41} & R_{42} & R_{44}
\end{array} \right) = \frac{1}{W} \left( \begin{array}{cccc}
G^2 & B G & A G & 0 \\
A G & G^2 - A B & A^2 & 0 \\
B G & A G & G^2 - A B & 0 \\
0 & 0 & 0 & G^2 - A B
\end{array} \right)
\]

with matrix determinant \( W = G(B^2 + A^2) \) that is always positive.

### E. 4-point Resistance measurement

We consider two configurations of resistance measurement (Fig. 4) in the system with varied voltage and current terminals.

For the first configuration from Fig. 4(a) the resistance is calculated as

\[
R' = \frac{V'}{I'} = \left[ \frac{V_2}{I_1} \right]_{I'_2 = 0} = R_{21} - R_{24} = \frac{AB}{W} \quad (12)
\]

and for the other [Fig. 4(b)],

\[
R = \frac{V}{I} = \left[ \frac{V_4 - V_2}{I_1} \right]_{I_2 = I_4 = 0} = R_{41} - R_{21} = \frac{G(B - A)}{W} \quad (13)
\]

## III. RESULTS AND DISCUSSION

### A. Silicene

In Fig. 3(b) and 3(b) we plotted the results for the conductance matrix elements (upper plots) and the resistances \( R \) and \( R' \) (lower plots) for \( E_F = 20 \) meV.
For magnetic fields such that $B = A = G_{41} - G_{14}$, the resistance changes sign (Fig. 5(b) and Fig. 6(b)). Hence, the changes of sign of $R$ appear when the electron transfer probability from terminal 1 to 4 crosses the electron transfer probability in the opposite direction. The directions become non-equivalent from the point of view of the electron transfer when the external magnetic field is introduced.

The current probe terminals for configuration $R'$ correspond to an open direct current path. The $R'$ resistance has a constant sign since the numerator in Eq. (12) is always nonnegative $AB \geq 0$.

By taking the Fourier transform of the resistance $R$ and $R'$ (from Fig. 5(b) and 6(b), respectively) for magnetic field $B$ range from 0 to 40 T we can distinguish 4 characteristic peaks (Fig. 8) $f_B = \{11.9, 23.8, 35.7, 47.6\} \frac{B}{h}$ associated to periods $\Delta B = 2\pi/f_B$ $\Delta B = \{528 \text{ mT}, 264 \text{ mT}, 176 \text{ mT}, 132 \text{ mT}\}$ respectively. For each period the area $\Lambda$ can be calculated as $\Lambda = \pi R^2$, and putting to the Aharanov-Bohm formula for period $\Delta B = \frac{h}{e\Delta B}$ we obtain

$$\Lambda = \frac{h}{e\Delta B}. \quad (14)$$
In our calculations the channel ring has radius equal to \( R_0 = 100 \) nm and area \( \Delta_0 = \pi R_0^2 \), hence
\[
\Lambda = \frac{\Lambda}{\Lambda_0} = \frac{\hbar}{e\Delta B \pi R_0^2}
\]
is the fraction of ring area responsible for AB interference. Thus, taking \( \{ \Delta B \} \) list from Fourier transform we obtain
\[
\Lambda = \{ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \}
\]
for peaks 1 – 4 from left to right in Fig. 8 respectively. The most pronounced is the peak first to the left that corresponds to interference paths that encircle a quarter of the ring. In the current distribution paths of Fig. 6 one can indicate the paths that encircle more quarters, but the fundamental period corresponds to 1/4 of the ring.

FIG. 8. Fourier transform on \( \mathcal{R}(B) \) and \( \mathcal{R}'(B) \) data for \( E_F \) 20 meV and 6.43 meV. Input magnetic field range \( B \) was set to [0:40] T. Inset icons describe Aharonov-Bohm interference area of the circle: full with \( T_a = 132 \) mT, \( \frac{1}{2} \) with \( T_b = 176 \) mT, half with \( T_c = 264 \) mT and quarter with \( T_d = 528 \) mT.

IV. BILAYER-GRAPHENE-BASED SYSTEM

For bilayer graphene system the results are qualitatively similar as for silicene, with the difference that for the Fermi energy within the energy gap we have \( \max(G_{\parallel}) = 4G_0 \), as the number of topological states is doubled due to two layers. This can be seen in the band structure of the armchair input leads in Fig. 9(d). For the zigzag leads, Fig. 9(a, b, c), within the energy gap also the edge states occur that however do not contribute to the inter-lead conductance. \( G_{11} \) and \( G_{22} \) is always equal to 2, with the edge modes being completely backscattered, and only the flip-modes leaving the zigzag leads.

FIG. 9. (a) Band structure for lead 1 (with zigzag edges) (b) and its zoom in the vicinity of K’ valley, and (c) in the vicinity of K valley, and (d) for lead 2 (with armchair edges) with electric field defined as described in Fig. 1. The green circles in (b) and (c) indicate the modes outgoing from lead 1.

The conductances in Fig. 10(a), and the resistances in Fig. 11(b) manifest oscillations that are consistent with the silicene system. In the Fourier transform of the resistances in Fig. 12(a) and (b) we find peaks at the frequencies \( f_B = \{ 12.7, 23.8, 36.4, 49 \} \) associated with the periods \( \Delta B = 2\pi/f_B \) \( \Delta B = \{ 495 \) mT, 263 mT, 173 mT, 128 mT \}, respectively. These correspond roughly to the area of one, two, three or four quarters of the ring, respectively.

V. SUMMARY AND CONCLUSIONS

We have considered Aharonov-Bohm interferometers based on valley protected channels induced by inhomogeneous electric field of a crossed ring profile in silicene and bilayer graphene in clean conditions i.e. without the backscattering due to the intervalley transitions within the ring. We have shown Aharonov-Bohm oscillations of electrical properties of the system in external magnetic field can be observed in spite of the absence of backscattering when a four-probe measurement setups is applied. We found that oscillations of resistance have large visibility for the choice of the terminals as the current probes.
FIG. 10. Current distribution maps in bilayer graphene ring at $E_F = 0.05$ eV for different magnetic field magnitudes: (a) 0 mT (b) 155 mT (c) 230 mT (d) 300 mT. For each map the color indicates for the averaged current amplitude $I$ and black arrows presents the direction of this current.

FIG. 11. (a) Conductance plot for simplified conductance matrix elements (eq. 7) for the bilayer graphene system in external magnetic field at fixed Fermi level $E_F = 50$ meV. (b) Resistance $R'$ for the case from Fig. 11(a) and $R$ (case Fig. 4(b)) in units of the Von Klitzing constant $R_K$.

between which a direct transfer is prohibited due to the orientation of the valley currents in the terminals. The fundamental period of the resistance oscillations corresponds to a quarter of the ring or the smallest loop that a chiral current encircles within the structure.

FIG. 12. Fourier transform output of (a) $R$ and (b) $R'$ signal of Fig. 11 in the bilayer graphene system.

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[1] F. A.H. Castro Neto, Guinea, N. M. R. Peres, K. Novoselov, and A. K. Geim, Rev. Mod. Phys. 81, 109 (2009).
[2] Y. K. Wakabayashi, Takane, M. Yamamoto, and M. Sigrist, New. J. Phys 11, 095016 (2009).
[3] K. Nakada, M. Fujita, G. Dresselhaus, and M. S. Dresselhaus, Phys. Rev. B 54, 17954 (1996).
[4] K. Wakabayashi, Phys. Rev. B 64, 125428 (2001).
[5] N. M. R. Peres, A. H. Castro Neto, and F. Guinea, Phys. Rev. B 73, 195411 (2006).
[6] M. Katsnelson, K. Novoselov, and A. K. Geim, Nature Phys. 2, 620 (2006).
[7] W. Yao, S. Yang, and Q. Niu, Phys. Rev. Lett. 102, 096801 (2009).
[8] B. Aufray, A. Kara, S. Vizzini, H. Oughaddou, C. Léandri, B. Ealet, and G. L. Lay, Applied Physics Letters 96, 183102 (2010).
[9] C.-C. Liu, W. Feng, and Y. Yao, Phys. Rev. Lett. 107, 076802 (2011).
[10] C.-C. Liu, H. Jiang, and Y. Yao, Phys. Rev. B 84, 195430 (2011).
[11] S. Chowdhury and D. Jana, Reports on Progress in Physics 79, 126501 (2016).
[12] A. Molle, J. Goldberger, M. Houssa, Y. Xu, S.-C. Zhang, and D. Akinwande, Nature Materials 16, 163 (2017).
[13] S. Cahangirov, M. Topsakal, E. Aktürk, H. Şahin, and S. Ciraci, Phys. Rev. Lett. 102, 236804 (2009).
[14] Y. Xu, B. Yan, H.-J. Zhang, J. Wang, G. Xu, P. Tang, W. Duan, and S.-C. Zhang, Phys. Rev. Lett. 111, 136804 (2013).
[15] M. Ezawa, New Journal of Physics 14, 033003 (2012).

[16] B. Szafran, B. Rzeszotarski, and A. Mrečič-Kolasińska, Phys. Rev. B 100, 085306 (2019).
[17] I. Martin, Y. M. Blanter, and A. F. Morpurgo, Phys. Rev. Lett. 100, 036806 (2008).
[18] Z. Qiao, J. Jung, Q. Niu, and A. MacDonald, Nano Lett. 11, 3453 (2011).
[19] M. Zarenia, J. M. Pereira Jr., G. Farias, and F. M. Peeters, Phys. Rev. B 84, 125451 (2011).
[20] L. Ju, Z. Shi, N. Nair, Y. Lv, C. Jin, J. Velasco Jr, C. Ojeda-Aristizabal, H. Bechtel, M. Martin, A. Zettl, J. Analytis, and F. Wang, Nature 520, 650 (2015).
[21] J. Li, K. Wang, K. McFaul, Z. Zern, Y. Ren, K. Watanabe, T. Taniguchi, Z. Qiao, and J. Zhu, Nature Nano. 11, 1060 (2016).
[22] A. Vaezi, Y. Liang, D. Ngai, L. Yang, and E.-A. Kim, Phys. Rev. X 3, 021018 (2013).
[23] R. Bistritzer and A. H. MacDonald, Proc. Natl. Acad. Sci. 108, 12233 (2011).
[24] S. G. Xu, A. I. Berdyugin, P. Kumaravadivel, F. Guinea, R. Krishna Kumar, D. A. Bandurin, S. Morozov, W. Kuang, B. Tsim, S. Liu, J. Edgar, I. Grigorieva, V. I. Fal’ko, M. Kim, and A. K. Geim, Nature Comm. 10, 4008 (2019).
[25] D. K. Efimkin and A. H. MacDonald, Phys. Rev. B 98, 035404 (2018).
[26] M. Ezawa, Phys. Rev. Lett. 109, 055502 (2012).
[27] B. Partoens and F. M. Peeters, Phys. Rev. B 74, 075404 (2006).
[28] K. Kolasinski, B. Szafran, B. Brun, and H. Sellier, Phys. Rev. B 94, 075301 (2016).
[29] B. Rzeszotarski and B. Szafran, Phys. Rev. B 98, 075417 (2018).
[30] S. Datta, Electronic Transport in Mesoscopic Systems (Cambridge University Press, 1995).