Optimal operation of district heat-electric system considering multi-energy flexibility

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Abstract. With increasingly integration of the renewable energy, various uncertainties bring a severe challenge to the operation of the district heat-electric system (DHES). In order to enlarge the accommodation of renewable energy, this paper proposes an optimal operation method of integrated electricity and heat system considering the temporal-coupling feature of multi-energy flexibility. Firstly, the models of multi-energy devices are established to describe the energy conversion. The time delay of heat network is considered and the model of DHES is built as well. Then, considering the uncertainty of load, the time-related upward and downward flexibility constraints are established. The objective function of the optimal economic operation problem is formulated as three parts, including the purchasing price, the operation price, and the curtailment penalty of PV. Finally, a case study based on the IEEE 33-bus power distribution grid and 6-node district heating network is conducted to validate the effectiveness of the method.

1. Introduction

In recent years, the rapid growth of energy demand has brought great challenge to the environment. In order to achieve the goal of carbon neutrality, various countries have issued a series of policy to control the emission of carbon [1]. An efficient way is to develop renewable energy and promote the integration of multi-energy complementarity. The distributed power generation, renewable energy and storage are applied in the district heat-electric system (DHES) to promote the supply and demand interaction of various energy forms, which improves the regional energy utilization efficiency and the consumption level of renewable energy [2-4].

Flexibility is the ability of a system to coordinate schedulable resources to maintain a real-time energy supply and demand balance [5]. The traditional generation side regulation resources mainly based on thermal power units are faced with lack of flexibility when dealing with renewable energy integration. In recent years, with the wide application of distributed flexible resources in distribution systems, the dependence and energy interaction of various forms of energy systems, such as electric power, natural gas, heating/cooling, are greatly strengthened. These multi-energy devices have different flexible abilities with load uncertainties. In order to meet the balance between supply and demand of various energy sources, it is necessary to explore the potential of each device in the operation optimization to improve the operation flexibility [6]. Reference [7] provides a systematic approach to evaluate the flexibility level and investigate the role of flexibility in generation planning and market operation. In [8], a plant-level operation domain model is established and the adjustable
range of heat and power loads are quantitatively obtained. Reference [9] tracks the operational changes in response to electricity time-of-use rates to exam the flexibility supplied by the combined heat and power plants (CHPs) of four types of business facilities. In [10], a novel characterization of the short-term energy flexibility was proposed, which was further utilized for the district heating capacity extension. Reference [11] compares the methods for improving the operational flexibility of CHP plants to support wind power integration. Most of the above studies are conducted to evaluate the flexibility after the optimal operation and the constraints of multi-energy devices are not considered. Besides, the formulation of the heating network is not fully modeled.

To solve the problem mentioned above, this paper aims to provide an optimal economic operation method considering multi-energy flexibility and the constraints of the heating network, including the transmission delay. The remainder is organized as follows: Section 2 describes the models of multi-energy devices and the characteristics of heat network. The optimal economic problem with flexible constraints is formulated in Section 3. Section 4 presents a case study. Conclusions are drawn in Section 5.

2. Modelling of multi-energy devices and energy network

2.1. Structure of DHES

The typical structure of DHES is depicted in Figure 1. Various of devices are integrated in the system, including the diesel generator units (DGUs), the photovoltaic power stations (PVs), the combined heat and power plants (CHPs), the gas boilers (GBs) and the battery energy storages (BESs). The distribution network and the heating network is coupled by the CHPs.

![Figure 1. Structure of DHES.](image)

2.2. Model of multi-energy devices

The model of multi-energy devices includes three varieties, namely electrical devices, heating equipment, and combined heat and power devices.

The model of DGUs is described as

\[
P_{g,t}^{DGU} \leq P_{g,t}^{DGU} \leq P_{g}^{DGU} \tag{1}
\]

\[
Q_{g,t}^{DGU} \leq Q_{g,t}^{DGU} \leq Q_{g}^{DGU} \tag{2}
\]

\[
-r_{g,t}^{DGU} \leq P_{g,t}^{DGU} - P_{g,t-1}^{DGU} \leq r_{g,t}^{DGU} \tag{3}
\]

where \( P_{g,t}^{DGU} \) and \( P_{g}^{DGU} \) indicate the lower and upper limits of the active power output \( P_{g,t}^{DGU} \); \( Q_{g,t}^{DGU} \) and \( Q_{g}^{DGU} \) are the lower and upper limits of the reactive power output \( Q_{g,t}^{DGU} \); and \( r_{g,t}^{DGU} \) is the ramp rate of DGUs. \( g \) is the index of DGUs.

When modeling the operating cost of DGUs, only the fuel cost is considered, and the fuel consumption can be expressed by the quadratic function of the active power output, which is given as

\[
C_{f,t}^{DGU} = a^f \left( P_{g,t}^{DGU} \right)^2 + b^f P_{g,t}^{DGU} + c^f \tag{4}
\]
where \( a^t, b^t, c^t \) are the cost coefficients of DGUs and \( C^{DGU}_t \) is the cost of operation at time \( t \).

The model of CHPs is described as

\[
P_{\text{CHP}}^t = \eta_{\text{CHP}}^{\text{CHP}} P_{\text{C}}^t, \quad P_{\text{CHP}}^t = \mu_{\text{CHP}}^{\text{CHP}} H_{\text{C}}^t
\]  \hspace{1cm} (5)

\[
-P_{\text{CHP}}^t \leq P_{\text{CHP}}^t \leq \bar{P}_{\text{CHP}}^t
\]  \hspace{1cm} (6)

\[
-\Delta_{\text{CHP}}^t \leq R_{\text{CHP}}^t \leq \Delta_{\text{CHP}}^t
\]  \hspace{1cm} (7)

where (5) represent the energy transfer functions of CHPs; (6)(7) represent the power limits and the ramp capability of CHPs respectively. \( F_{\text{C}}^t \) is the gas power. \( \eta_{\text{CHP}}^{\text{CHP}}, \mu_{\text{CHP}}^{\text{CHP}} \) are the generating and heating efficiency. \( L_{\text{CHP}}^{\text{CHP}}, \bar{P}_{\text{CHP}}^t \) are the limits of power output. \( r_{\text{CHP}}^t \) is the ramp rate of CHP. \( c \) is the index of CHP.

The model of BES is described as

\[
0 \leq P_{\text{BES},i}^{\text{BES}} \leq v_{\text{ch},i}^{\text{BES}} - v_{\text{dc},i}^{\text{BES}}, \quad 0 \leq P_{\text{BES},i}^{\text{BES}} \leq v_{\text{dc},i}^{\text{BES}} - v_{\text{ch},i}^{\text{BES}}
\]  \hspace{1cm} (8)

\[
v_{\text{ch},i}^{\text{BES}} + v_{\text{dc},i}^{\text{BES}} \leq 1, \quad P_{\text{BES},i}^{\text{BES}} P_{\text{dc},i}^{\text{BES}} = 0
\]  \hspace{1cm} (9)

\[
SOC_{i,j} = \eta_{i}^{\text{BES}} SOC_{i,j-1} + \frac{1}{Z_{i}^{\text{BES}}} (\eta_{\text{ch}}^{\text{BES}} P_{\text{BES},i}^{\text{BES}} - P_{\text{dc},i}^{\text{BES}}) \Delta t
\]  \hspace{1cm} (10)

\[
SOC_{i,j} \leq SOC_{i,j} \leq SOC_{i}
\]  \hspace{1cm} (11)

\[
SOC_{i,N} = SOC_{i,N}^{\text{DF}}
\]  \hspace{1cm} (12)

where (10) - (12) are the constraints of BES’s state of charge (SOC); \( P_{\text{BES},i}^{\text{BES}} \) and \( P_{\text{dc},i}^{\text{BES}} \) are the charge and discharge power of BES at time \( t \). \( v_{\text{ch},i}^{\text{BES}}, v_{\text{dc},i}^{\text{BES}} \) are binary variables. \( \bar{P}_{\text{BES}}^{\text{BES}} \) is the limit of power output. \( SOC_{i}, SOC_{j} \) are the limits of SOC. \( \eta_{i}^{\text{BES}}, \eta_{\text{ch}}^{\text{BES}} \) and \( \eta_{\text{dc}}^{\text{BES}} \) represent the energy self-decay rate, charge and discharge efficiency, and \( Z_{i}^{\text{BES}} \) indicates its maximum capacity. \( i \) is the index of BES.

2.3. Model of power and heat network

The power distribution network is modeled as

\[
\sum_{k,j \in k} P_{jk} = \sum_{i \rightarrow j} P_{ij} - P_{j} \quad P_{ij} \leq P_{ij} \leq \bar{P}_{ij}
\]

\[
\sum_{k,j \in k} Q_{jk} = \sum_{i \rightarrow j} Q_{ij} - Q_{j} \quad Q_{ij} \leq Q_{ij} \leq \bar{Q}_{ij}
\]

\[
U_{j} = U_{i} - \frac{R_{ij} P_{ij} + X_{ij} Q_{ij}}{U_{j}} \quad \forall \leq U_{j} \leq \bar{U}_{j}
\]

\[
P_{\text{elec},i}^{\text{EXT}} \leq P_{\text{elec},i}^{\text{EXT}} \leq \bar{P}_{\text{elec},i}^{\text{EXT}}, \quad Q_{\text{elec},i}^{\text{EXT}} \leq Q_{\text{elec},i}^{\text{EXT}} \leq \bar{Q}_{\text{elec},i}^{\text{EXT}}
\]

where (13) refers to the constraints of distribution network and (14) represents the constraints of the line capacity connected to the grid. \( P_{ij} \) and \( Q_{ij} \) are the active and reactive power from node \( i \) to node \( j \). \( P_{j} \) and \( Q_{j} \) are the active and reactive lode power at node \( j \). \( R_{ij} \) and \( X_{ij} \) are the resistance and reactance of branch \( ij \). \( U_{i} \) is the voltage amplitude of node \( i \).

The district heat network model is as follows.

\[
H_{n}^{\text{in}} = C_{w} m^{a}_{w} (T_{n}^{\text{in}} - T_{n}^{\text{in}}), \quad H_{n}^{\text{out}} = C_{w} m^{a}_{w} (T_{n}^{\text{out}} - T_{n}^{\text{out}})
\]

\[
t^{\text{de}} = \min_{n \in N} \left\{ \frac{n m^{a}_{w} \Delta t}{\rho \pi D^{2}_{L}} = M_{\rho} \right\}
\]  \hspace{1cm} (15)

(16)
where (15) represents the energy exchange in station. $H^w_n$ indicates the heat absorbed from the heat sources at node $n$ and $H^m_n$ is the heat transferred to the secondary pipe network, namely to the consumer of node $n$. $C_w$ is the specific heat capacity of water. $m_i^p$ and $m_i^n$ are the mass flow of node $n$. $M_p$ is the total mass in pipe $p$. $\rho$ is the density of water. $D_p$ and $L_p$ are the diameter and length of pipe $p$. (16) is the time delay that water mass flows through the pipe. (17) is the outlet temperature of pipe $l$ and (18) is the temperature mixing equation. $T^G_D$ is the ground temperature. $K_p$ is the coefficient of heat conduction.

3. Optimal economic problem considering multi-energy flexibility
The fluctuation of electric and heating load in DHES will affect the operation of the system, which brings the flexibility demand. In order to fully utilize the adjustment ability of multi-energy supply in the system, an optimal economic operation method considering flexible constraints is proposed in this paper. The prediction error of electric and heating load will generate various up-down energy demands, so the DHES reserves a certain energy margin by adjusting the output power of each energy source at node $n$.

The flexibility of the DHES is analyzed on both sides of supply and demand. The constraints of flexibility supply are as follows.

$$P_{PV}^{up} = \max \{0, P_{PV,j} - P_{PV,j-1}\}, \quad P_{PV}^{dn} = \max \{0, P_{PV,j-1} - P_{PV,j}\}$$

$$P_{PV}^{up,RES,j} = \min \{(SOC - SOC_{j-1})Z^{RES} \eta_d, P_{PV,j}^{RES}\}, \quad P_{PV}^{dn,RES,j} = \min \{(SOC_{j-1} - SOC)Z^{RES} \eta_d, P_{PV,j}^{RES}\}$$

$$P_{CHA,up,j} = \min \{P_{CHA}^{up,j}, r_{CHA}^{up,j} \delta t\}, \quad P_{CHA}^{dn,j} = \min \{P_{CHA}^{dn,j}, r_{CHA}^{dn,j} \delta t\}$$

$$P_{DGU,up,j} = \min \{P_{DGU}^{up,j}, r_{DGU}^{up,j} \delta t\}, \quad P_{DGU}^{dn,j} = \min \{P_{DGU}^{dn,j}, r_{DGU}^{dn,j} \delta t\}$$

$$H_{GB,up,j} = \min \{H_{GB}^{up,j}, r_{GB}^{up,j} \delta t\}, \quad H_{GB}^{dn,j} = \min \{H_{GB}^{dn,j}, r_{GB}^{dn,j} \delta t\}$$

$$H_{CHA,up,j} = \frac{1}{\mu_{CHA}} P_{CHA}^{up,j}, \quad H_{CHA}^{dn,j} = \frac{1}{\mu_{CHA}} P_{CHA}^{dn,j}$$

where the variables $P_{PV}^{up,j}$, $P_{PV}^{dn,j}$, $P_{PV}^{up,RES,j}$, $P_{PV}^{dn,RES,j}$, $P_{CHA}^{up,j}$, $P_{CHA}^{dn,j}$, $P_{DGU}^{up,j}$, $P_{DGU}^{dn,j}$, $H_{GB,up,j}$, $H_{GB}^{dn,j}$, $H_{CHA,up,j}$, $H_{CHA}^{dn,j}$ mean upward flexible supply of PV, RES, CHP, DGU respectively at each time and with superscript $dn$ means downward supply. The constraints of flexibility demand are as follows.

$$P_{load,up,j}^{up} = \max \{0, (1 + \lambda_1) P_{load,j} - P_{load,j-1}\}, \quad P_{load,j}^{dn} = \max \{0, P_{load,j-1} - (1 - \lambda_1) P_{load,j}\}$$

$$H_{load,up,j}^{up} = \max \{0, (1 + \lambda_2) H_{load,j} - H_{load,j-1}\}, \quad H_{load,j}^{dn} = \max \{0, H_{load,j-1} - (1 - \lambda_2) H_{load,j}\}$$

where $\lambda_1$, $\lambda_2$ represent the fluctuation coefficient, which takes the maximum load prediction error rate of all nodes. In the following operation strategy, the use of the maximum error rate allows for a real-time operation with greater room for adjustment and better response to load changes. The constraints of the relationship between flexible supply and demand are as follows.

$$P_{PV}^{up,j} + P_{PV}^{dn,j} + P_{PV}^{up,RES,j} + P_{PV}^{dn,RES,j} \geq P_{load,up,j}^{up}, \quad P_{PV}^{up,j} + P_{PV}^{dn,j} + P_{PV}^{up,RES,j} + P_{PV}^{dn,RES,j} \geq P_{load,j}^{dn}$$

$$H_{CHA,up,j}^{up} \geq H_{load,up,j}^{up}, \quad H_{CHA}^{dn,j} \geq H_{load,j}^{dn}$$
The optimal economic problem is formulated as
\[
\min C = \sum_i \Delta t (C^{\text{b}}_{i} + C^{\text{op}}_{i} + C^{\text{aban}}_{i})
\]  
(22)
s.t. (1) - (21)

\[
\begin{align*}
C^{\text{b}}_{i} &= C_{i}^{\text{elec}} P^{\text{EXT}}_{i} + C_{i}^{\text{gas}} P^{\text{EXT}}_{i} \\
C^{\text{op}}_{i} &= \delta_{\text{CHP}} \sum_{t \in S_{i}} P_{t}^{\text{CHP}} + \sum_{t \in S_{i}} C_{i}^{\delta \text{DGU}} \\
 & \quad + \delta_{\text{GB}} \sum_{t \in S_{i}} H_{i,t}^{\text{GB}} + \delta_{\text{PV}} \sum_{t \in S_{i}} P_{t}^{\text{PV}} + \delta_{\text{BES}} \sum_{t \in S_{i}} (F_{i}^{\text{BES}} + F_{i}^{\text{BES}}) \\
C^{\text{aban}}_{i} &= C^{\text{aban}} (\bar{P}_{i}^{\text{PV}} - P_{i}^{\text{PV}})
\end{align*}
\]  
(23)
where \(C^{\text{b}}_{i}\), \(C^{\text{op}}_{i}\) and \(C^{\text{aban}}_{i}\) represent the cost of buying electricity and gas, operation, and the penalty of PV curtailment respectively. \(\delta\) is the price of each energy source. The problem is MINLP and it can be solved by commercial solver CPLEX. In the Non-flexible operation, the Constraints (19) - (21) are not considered.

4. Case study
In this case study, the DHES is composed of IEEE 33-bus power distribution grid and 6-node district heating network. The diagram of the system is shown in Figure 2. The DHES contains 3 battery units (BUs), 3PVs, 3DGUs, 1GB and 1CHP. The BUs, PVs and DGUs are located at same nodes. The related parameters refer to Literature [2, 3]. The following numerical experiments are conducted with MATLAB on a PC with Intel Core i7 2.9GHz and 16GB RAM.

![Diagram of the test DHES.](image)

Figure 2. The diagram of the test DHES.

The electricity and heat balance of two operation methods is shown in Figure 3. (a) is the result of method considering the flexible constraints. It can be seen that the power outputs of CHP and DGUs in (b) are more than (a) while the power bought from the grid is less than (a). Meanwhile, the BUs in (b) operate more frequently than (a).
Figure 3. Power and heat balance of DHES.

The supply and demand curve of flexibility is depicted in Figure 4. It shows that between T=22 to T=95 in (b), the upward flexibility is insufficient. Here, the flexible index is proposed to represent the deficit of the flexibility supply, which is calculated by the deficit time dividing total time. When the flexibility constraints are considered, the upward flexibility index of DHES climbs to 100%.

Figure 4. Flexibility of supply and demand.

Table 1 shows the comparison of economy and flexibility between the two methods. It can be seen that although the total cost of the secondary method is about 10% more than the first one, the flexible index improves 67%, which indicates that it can obviously tackle the uncertainties of the load at a relatively economic cost.

Table 1. Economic and flexible comparison of two operation methods.

| Method     | Gas cost ($) | Grid power cost ($) | Operation cost ($) | Total ($) | Flexible index |
|------------|--------------|---------------------|--------------------|-----------|----------------|
| Non-Flexible | 350.19       | 638.46              | 606.81             | 1595.46   | 33%            |
| Flexible    | 353.34       | 872.74              | 562.65             | 1788.73   | 100%           |
5. Conclusions
This paper proposes a method for optimizing the operation of DHES considering temporal-coupling multi-energy flexibility and heat network constraints. The simulation results based on the 33-bus power distribution network and the 6-node district heating network show that the proposed method can significantly improve the operation flexibility of the DHES while minimizing the cost of operation. It can better face the uncertainty of the load during the dispatch period.

Acknowledgement
The research is supported by science and technology project of State Grid Zhejiang Electric Power Research Institute (Research on flexibility analysis and Cooperative optimization operation of district heat-electric system, 5211DS200083).

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