Noise signatures for determining chiral Majorana fermion modes

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The conductance measurement of a half quantized plateau in a quantum anomalous Hall insulator-superconductor structure is reported by a recent experiment [Q. L. He et al., Science 357, 294-299 (2017)], which suggests the existence of the chiral Majorana fermion modes. However, such half quantized conductance plateau may also originates from a disorder-induced metallic phase. To identify the exact mechanism, we study the transport properties of such a system in the presence of strong disorders. Our results show that the local current density distributions of these two mechanisms are different. In particular, the current noises measurement can be used to distinguish them without any further fabrication of current experimental setup.

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Introduction.— Chiral Majorana fermions, whose antiparticles are themselves [1], have attracted extensive study interest in recent years for their potential application in fault-tolerant quantum computation [2-10]. Viewed as the edge states of a topological superconductor (TSC), they were widely proposed to exist in the ν = 5/2 quantum Hall state [11,12], the intrinsic p-wave superconductor Sr2RuO4 [13], topological insulator or strong spin orbital coupled semiconductor-superconductor heterostructures [14,15], iron-based superconductors [16-18], and cold atom systems [19,20].

In 2010, Qi et al. proposed that a quantum anomalous Hall (QAH) insulator proximitied to an s-wave superconductor is also a promising candidate to realize a TSC [21]. To detect them experimentally, a transport measurement in a sandwich structure formed by QAH insulator-TSC-QAH insulator will find half quantized conductance plateaus (HQCPs) as the reversal of external magnetic fields [22,23]. Such a strategy was examined recently in a magnetic doped topological insulator thin film with the central region coated by an s-wave superconductor, and indeed the HQCPs are observed [24].

Nevertheless, since the experimental setup is strongly disordered, some works commented that these HQCPs could also come from a disorder-induced metallic phase [25-28]. As a consequence, more transport measurements are necessary to rule out this trivial mechanism. Soon afterwards, Beenakker suggested that, in a similar device with only one edge of the QAH insulator covered by a type-II superconductor, the two terminal conductance of a TSC should exhibit a distinctive $Z_2$ interferometry [29-31]. Unfortunately, despite that the experimental setup needs to be re-fabricated, the magnetic field required to induce quantum flux into the superconductor region is also magnitude larger than the coercive field. Therefore, it becomes highly desirable to propose a method to distinguish these two mechanisms.

In this Letter, we study the transport properties of a QAH insulator-superconductor structure, and suggest an experiment that is able to distinguish the two different mechanisms. Using the non-equilibrium Green’s function method, we demonstrate that, though both TSC phase and metallic phase can give rise to HQCPs, the local current density distributions of these two phases are totally different. Particularly, by analyzing the current-current correlations, we show that the related current noises of the TSC phase are zero while those of a metallic phase remain finite once the HQCP appears. This distinct feature combined with the conductance measurement provides an unambiguous evidence for determining chiral Majorana fermions without any further fabrication. Finally, the application of this method in realistic experimental setup is examined.

Theoretical model.— The low energy effective Hamiltonian of a magnetic doped topological insulator thin film, in the space $\psi_{\mathbf{k}} = (c^\dagger_{\mathbf{k} \uparrow}, c_{\mathbf{k} \downarrow}, c^\dagger_{\mathbf{k} \uparrow}, c_{\mathbf{k} \downarrow})^T$, is $H_0(\mathbf{k}) = v_F k_y \tau_z \otimes \sigma_x - v_F k_x \tau_z \otimes \sigma_y + m(\mathbf{k}) \tau_x + M_z \sigma_z$ [32]. Here, $c^\dagger_{\mathbf{k} \sigma}$ annihilates an electron of momentum $\mathbf{k}$ and spin $\sigma$ in the top (bottom) layer. $v_F$ is the Fermi velocity. $\sigma_{x,y,z}$ and $\tau_{x,y,z}$ are Pauli matrices acting on spin and layer spaces. $m(\mathbf{k}) = m_0 - m_1 k^2$ describes the coupling between the top and bottom layers with $m_0$ the hybridization gap and $m_1$ the parabolic band component. $M_z$ represents an external magnetic field along the z direction, which can drive a phase transition from a trivial insulator with the Chern number $C = 0$ ($|M_z| < |m_0|$) to a QAH state with $C = \pm 1$ ($|M_z| > |m_0|$). The detailed parameters are fixed as follows unless otherwise specified: $v_F = m_1 = 1$, $m_0 = -0.1$. 

\[
E_{\mathbf{k}} = \sqrt{v_F^2 k_x^2 + v_F^2 k_y^2 + m(\mathbf{k})^2 + M_z^2}.
\]
by tuning the chemical potential into the bulk band and setting the pairing potentials $\Delta_i = \Delta_b = 0$ at the same time. The disorder in the metallic phase is modeled by the Anderson type random onsite potential that is uniformly distributed in $[-v/2, v/2]$.

**Conductance.**— We start by studying the half quantized two terminal conductance, which was initially believed as a fingerprint of chiral Majorana fermion modes in a sandwich structure as shown in Fig. 1(a), two QAH regions experience phase transitions directly from $\mathcal{N} = -2$ to $\mathcal{N} = 0$ then to $\mathcal{N} = 2$ as the reversal of magnetic fields. Meanwhile, the central region experiences two additional TSC phases with $\mathcal{N} = \pm 1$ near the boundaries of the phases $\mathcal{N} = \pm 2$ due to the effect of superconducting proximity [21]. In those two phases, because the normal and Andreev processes have equal probability of 1/4, the two terminal conductance is half quantized with $\sigma_{12} = e^2/2h$ [22]. However, this half quantized conductance only provides a necessary condition for the verification of chiral Majorana fermion modes. Suppose the central region is replaced with a disordered metal while the two ends remain QAH states, under a bias, an electron moving along the edge of the left QAH region is injected into the central region. After multiple scatterings by disorders inside, this electron has equal probabilities of transmission and backscattering, also resulting in a half quantized conductance [25–27].

In Fig. 1(b), the conductances $\sigma_{12}$ for both TSC phase (red line) and metallic phase (blue line) versus the external magnetic field $M_z$ are plotted. In the presence of domains, the width of the right HQCP decreases heavily while the left one remains almost unchanged. Besides, the slopes of the plateau transitions from $\mathcal{N} = \pm 1$ to $\mathcal{N} = \pm 2$ also decrease. Therefore, the conductance behavior is in high agreement with the experiment data reported in Ref. [24], which indicates that the percolation domains lead to an asymmetry of HQCPs [43]. By contrast, the conductance of a metallic phase exhibits a similar $\sigma_{12} = e^2/2h$ plateau as shown in the same figure. When $M_z$ exceeds the coercive field, the left and right regions transit into QAH states with $\mathcal{N} = 2$ while the central region remains a disordered metal. One can observe the same half quantized conductance due to the mechanism stated above. Enhance $M_z$ further, the central region also transits into a QAH state with the same Chern number, and the conductance increases to $\sigma_{12} = e^2/h$ finally. Thus, the conductance measurement is not an unambiguous evidence of chiral Majorana fermion modes.

**Local current density distributions.**— Though the conductances of both TSC phase and metallic phase show the same feature, the distributions of non-equilibrium local current density from site i to site j, $J_{i \rightarrow j}$, are quite different [34]. In Fig. 2 the typical distributions of local currents for a TSC are plotted in left panels. When $M_z = 0$, $\sigma_{12} = 0$, all regions are trivial insulators. There is no current in any site as shown in Fig. 2(a). Tune
the magnetic field strength to $M_z = 0.2$, where the half quantized conductance $\sigma_{12} = e^2/2h$ appears. In this case the left and right regions are QAH insulators while the central region is generally a TSC with $N = 1$. The current distributions at different regions are slightly different. At left region, the currents locate on both the upper and lower edges moving on opposite directions, but at central and right regions, the currents only locate on the lower edge. However the total net currents calculated by the summation of current distributions along the $y$ direction are all $e^2/2h$, which is exactly the conductance. Enlarge the magnetic field strength further to $M_z = 0.5$, all regions are QAH states with the same Chern number $N = 2$ and a quantized conductance $\sigma_{12} = e^2/h$ arises. The attached plots the current distributions along the $y$ direction at the center of three different regions. Different widths of these edge currents originate from different bulk gaps [21].

As a comparison, the typical distributions of local currents for a disordered metal are plotted in the right panels of Fig. 2. The current distributions at QAH regions are the same as left panels because they experience same phase transitions. But the current distributions at central region are completely different. Fig. 2(d) illustrates the case of $M_z = 0$. Even though the conductance is zero, there can be some localized currents circling around disorders. If the magnetic field strength increases to $M_z = 0.5$, the central region becomes a QAH state, but the edge currents are broaden by disorders as shown in Fig. 2(f).

The key difference comes when $M_z = 0.2$, where the half quantized conductance arises. Different from a TSC phase, where currents only locate on the edge, the currents here spread on the entire central region as one can see clearly in Fig. 2(e). This indicates that the currents in a metallic phase are high coupled. Notably, because of percolation domains, a TSC may also hosts some localized currents in the central region. But the emergence of the HQCPs ensures that the chiral Majorana fermion modes are still independent [28]. The direct imaging of different local currents is the most accurate way to distinguish a TSC from a metal, even though the ground superconductor covered in the central region makes it a challenge to image the localized currents [33–37]. Additional to that, these different current distributions can lead to different current-current correlations, which indicates some other signatures to distinguish these two mechanisms.

Current noises.— To analysis the current-current correlations, we study the related current noises [38, 39], which are defined as

$$S_{ab}(\omega) = \int d\tau e^{i\omega \tau} \langle \delta \hat{I}_a(t) \delta \hat{I}_b(t+\tau) + \delta \hat{I}_b(t+\tau) \delta \hat{I}_a(t) \rangle,$$

where $\delta \hat{I}(t) = \hat{I}(t) - \langle \hat{I}(t) \rangle$, and the subscripts $a, b$ can be $L$ or $R$ with the same (different) subscripts denoting the local (non-local) current noise. In particular, we consider the current noise at zero frequency, which represents the correlation of current fluctuations [40].

In Fig. 3 both the local (a) and non-local (b) current noises versus the magnetic field $M_z$ are plotted. The most interesting feature is that, when the conductance is half quantized, the current noises vanish if the central region is a TSC, while they remain finite otherwise. This difference can be understood in the following way.

![FIG. 2:](color online) Configurations of the local-current-flow vector for a TSC phase (left) and a metallic phase (right) at various $M_z = 0.0(a), (d), 0.2(b), (e), 0.5(c), (f)$. The plots one-to-one correspond to three different conductance plateaus of $\sigma_{12} = 0, 0.5e^2/h, e^2/h$ as pointed out in Fig. 1(b). The attached plots show the magnitude of the local current along the cross section at middle of three different region as labeled out by arrows of same color in (a).
For a metal, because electrons in the central region are randomly scattered by disorders, the current fluctuations both in the same lead and in different leads are highly coupled. Thus the noises should be visible. However, for a TSC, since the HQCPs come from four independent Majorana fermions hosted at four edges of the central region, both local and non-local noises must vanish. Most significantly, as long as the HQCPs are not destroyed, these vanishing noises are visible as shown in Fig. 3 even in the presence of strong disorders.

Experimental setup.— Up to now, we have shown that the current noises combined with the two terminal conductances can be used as a smoking-gun evidence for the experimental confirmation of chiral Majorana fermion modes. To apply this method to the realistic experiment, the central TSC in Fig. 1(a) is replaced with a magnetic doped topological insulator thin film and a coated s-wave superconductor [24]. In the Nambu space \( \Phi_k = (\phi_k, \phi_{-k}^\dagger)^T \) with \( \phi_k = (c^a_{k\uparrow}, c^b_{k\downarrow}, c^a_{k\downarrow}, c^b_{k\uparrow})^T \), this system can be simulated by the Hamiltonian

\[
\mathcal{H} = \begin{pmatrix}
    H_c(k) - \mu & \Delta_c \\
    -\Delta_c & -H_c^*(-k) + \mu
\end{pmatrix}.
\]

(3)

Here, \( H_c(k) = \begin{pmatrix} H_s(k) & t \\ t^\dagger & H_0(k) \end{pmatrix} \) is the central Hamiltonian. \( H_s = \epsilon_0 k^2 - \mu_0 + \lambda \sigma_z \) is the Hamiltonian of a metal, where \( \epsilon_0 \) is the onsite energy, \( \mu_0 \) is the chemical potential, \( \lambda \) represents the magnetic field strength. \( \Delta_c = \begin{pmatrix} i \Delta_s \sigma_y & 0 \\ 0 & 0 \end{pmatrix} \) is the pairing potential, which only act in the metal layer. The metal only couples the top layer of the topological insulator film, which has the form \( t = \begin{pmatrix} t_c & 0 \\ 0 & t_c \end{pmatrix} \). As the increase of the magnetic field strength \( M_z \), the superconducting pairing potential and the magnetic field strength satisfy the relations \[ \lambda = 0, \Delta_s = \Delta \left( 1 - M_z^2/M_c^2 \right) \text{ when } |M_z| \leq M_c \]; \[ \lambda = g M_z, \Delta_s = 0 \text{, otherwise.} \] Here, \( g \) is the effective landé g-factor, which is taken as \( g = 1 \) [42]. \( M_c > 0 \) is the critical magnetic field.

Figure 4 plots the conductance and related current noise responses to a high magnetic field. Here, \( \Delta_s = 0.3, t_c = 0.4, M_c = 0.8 \). The parameters in the metal layer are \( \epsilon_0 = 1, \mu_0 = 0.5 \). The disorder strength is \( v = 1.6 \), and the disorder in the metal layer is taken as \( \lambda v \) due to the Meissner effect. All datas are calculated under 1000 times average.

Figure 4 plots the conductance and related current noise responses to a high magnetic field. At low magnetic field, the conductance is zero. When the magnetic field exceeds the coercive field, the conductance is half quantized. After the magnetic field eliminates the superconducting phase, the central region becomes a disordered metal, and the conductance reaches \( e^2/2h \) again. Such conductance behavior is in perfect agreement with experimental findings [24]. Therefore, the experimental setup can be simulated by this model. To better understand the mechanisms of the two HQCPs, we perform the current noise study of this system. When HQCP1 appears, the current noise is zero. However, after the superconductor is eliminated by high magnetic fields, the existence of HQCP2 can only be attributed to a metallic phase. In this case, the current noise remains finite. This demonstrates again that the current noises can be used to determine the chiral Majorana fermion modes.

Summary.— In summary, we have studied the transport properties of a QAH insulator-superconductor hybrid structure. We demonstrated that both the TSC phase and the metallic phase can give rise to HQCPs.
but the local current distributions are different. In particular, based on the effective model study and a realistic experiment simulation, we showed that the current noises combined with the conductance measurements can be used to distinguish these two mechanisms even though the system is strongly disordered. Our work provides an unambiguous method for determining the chiral Majorana fermion modes.

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