On the possibility of a metallic phase in granular superconducting films

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We investigate the possibility of finding a zero-temperature metallic phase in granular superconducting films. We are able to identify the breakdown of the conventional treatment of these systems as dissipative Bose systems. We do not find a metallic state at zero temperature. At finite temperatures, we find that the system exhibit crossover behaviour which may have implications for the analysis of experimental results. We also investigate the effect of vortex dissipation in these systems.

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Recently, there has been renewed interests in the problem of superconductor-insulator (SI) transition in low-\(T_c\) thin films. These systems undergo transition from superconductor to insulator as a function of disorder, film thickness, or applied magnetic field. Theories describing this kind of superconductor-insulator transition describe a second-order quantum phase transition where a zero-temperature metallic phase exists as a critical point between the superfluid and insulating phases. However, recent experiments found that the metallic phase may be more than a point in the phase diagram in certain systems. Instead, the zero-temperature conductivity appears to be finite and non-zero in a finite region in the phase diagram. It remains controversial whether these systems remain metallic down to zero temperature. It has been observed that some of these systems become superconducting at very low temperatures.

In this paper, we re-examine our theoretical understanding of the SI transition. We will use a variational treatment which, in principle, may describe superfluid, metallic and insulator phases at zero temperature. We will discuss dissipation arising from normal resistance or vortex motion. We find that, although there may be a direct SI transition at zero temperature, finite-temperature crossover phenomena may give rise to apparently metallic behaviour in experiments. We also see that the two dissipation mechanisms affect the low-temperature behaviour on different sides of the SI critical point.

\section*{I. INTRODUCTION}

The destruction of the superconducting state at zero temperature is a result of strong Coulomb interactions. Consider a lattice model for Cooper pairs. Strong Coulomb repulsion leads to a Mott insulating state where there is an integral number of Cooper pairs at each site. However, if the system is coupled to a normal fluid, any excess charge on a site (arising the motion of Cooper pairs from site to site) can be screened to a certain extent by the normal component. This is effective when the normal fluid has low resistance, \(R_n\), because it can respond rapidly to charge fluctuations. Since the coupling to the normal fluid requires exchange of energy, the normal fluid can be regarded as a dissipative environment for the Cooper pairs. The strength of this dissipative coupling (or dynamic screening) is inversely proportional to \(R_n\).

We will also consider dissipation originating from the motion of the normal cores in vortices. In this case, a similar picture applies when we study the system in a dual representation where vortices are the elementary bosonic objects.

In principle, dissipation may lead to non-superfluid but mobile Cooper pairs (or vortices) at zero temperature. To investigate this issue, we require a formulation which can differentiate between the superfluid, metallic and insulating states. We will see below that we can do so by considering separately local phase fluctuations which, over a timescale of \(\hbar/k_B T\), are small compared to \(2\pi\) and those which are larger than \(2\pi\). Previous work has investigated either a superfluid-to-non-superfluid transition or an insulator-to-conductor transition. We want to see if these transitions are separate so that all three phases exist. Otherwise, they are different descriptions of the same critical point, in which case the Bose metal does not exist in the model at zero temperature. After establishing the ground state, we will also discuss the finite-temperature behaviour of these systems.

\section*{II. DISSIPATIVE BOSE MODEL}

For simplicity, we will consider first dissipation for the Cooper pairs. Vortex dissipation will be discussed later. We will review the conventional discussion of this problem and we extend previous treatments by a more careful consideration of large phase fluctuations.

As our starting point, we use a model of superconducting grains on a square array. We assume that well-defined Cooper pairs exist in each grain so that we can treat them as charge-2e bosons. An imaginary-time action which describes the coupling between grains is
energy between nearest-neighbor grains. (We have set the charging energy of a grain with self-capacitance $J$ where $\theta = 0$ is the Josephson coupling energy between nearest-neighbor grains. $K_b = 2e^2/C$ is the charging energy of a grain with self-capacitance $C$.)

For large $J_b/K_b$, we expect a superconductor with long-range phase coherence. When $J_b/K_b$ is small, however, the on-site repulsion dominates and we have a Mott insulator. (Our calculations below will focus on this limit.) The system becomes incompressible. (See vertical axis on Fig. 1.) The phase, $\theta^i$, of the local superconducting order parameter should fluctuate strongly at each site due to the number-phase uncertainty relation.

We will now investigate the effect of dissipation on this bosonic Mott transition. We include dissipation phenomenologically. We assume that an action of the Caldeira-Leggett kind is necessary so that the charge currents ($\sim \Delta \nu \theta^i$) will have ohmic decay in the classical limit:

$$S_{\text{diss}} = \frac{Q^2}{2} \sum_{i\nu} \int_0^\beta d\tau \int_0^\beta d\tau' \sin^2 \left[ \frac{\Delta \nu \theta^i(\tau) - \Delta \nu \theta^i(\tau')}{2Q} \right],$$

where $\alpha(\tau) = (h/4e^2R_n)[T/\sin(\pi T\tau)]^2$ and $Q = 2$ reflecting the fact that the Cooper pairs have charge $2e$ while the dissipation is due to charge-$e$ electrons.

We will be interested in the destruction of superfluidity due to enhanced phase fluctuations. As already mentioned, we have to be careful about the compactness of the phase variables $\theta^i$. The imaginary-time evolution of the phase can be separated into a periodic part, $\theta_i$, and a non-periodic part $\bar{\theta}^i$:

$$\theta^i(\tau) = \frac{2\pi n_i \tau}{\beta} + \theta_i(\tau) + \theta_{0i},$$

where $\theta_i(\beta) = \theta_i(0) = 0$. The boson action can be written as

$$S_{\text{boson}} = \frac{2\pi^2}{\beta K_b} \sum_i n_i^2 + \frac{1}{2K_b} \sum_i \int_0^\beta d\tau \bar{\theta}^2_i d\tau$$

$$- J_b \sum_{i\nu} \int_0^\beta \cos \left( \Delta \nu \bar{\theta}_i(\tau) + \frac{2\pi \tau}{\beta} \Delta \nu n_i \right) d\tau,$$

To further simplify our calculation, we shall assume strong dissipation and keep only the $\Delta \nu \theta_i$ terms in $S_{\text{diss}}$ to second order. At low temperatures, we obtain

$$S_{\text{diss}} \rightarrow \frac{Q\pi}{4R_n} \sum_{i,\nu} [\Delta \nu n_i] + \frac{1}{8} \sum_{i\nu} \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') \times$$

$$\cos \left[ \frac{2\pi (\tau - \tau')}{Q\beta} \Delta \nu n_i \right] [\Delta \nu \bar{\theta}_i(\tau) - \Delta \nu \bar{\theta}_i(\tau')]^2.$$  

We can now discuss possible scenarios for the zero-temperature phase diagram of the system. First of all, let us concentrate on the part of the action which involves only the “winding numbers”, $n_i$. We can ignore the charging term in (4) proportional to $n_i^2$ because it vanishes as $T \to 0$. The winding numbers are controlled by the first term in (2). This is, in fact, the “absolute solid-on-solid” (ASOS) model which has a “roughening” transition of the Kosterlitz-Thouless type at $R_n = R_{c\text{ASOS}} \simeq 0.6(h/4e^2)$. For large $R_n$, the phase at each site fluctuates wildly with little correlation between different sites. This is what is expected from number-phase uncertainty) in an insulator where the local particle number does not fluctuate. For small $R_n$, the system becomes “smooth” in the sense that large excursions in the phase are suppressed. The system is now compressible and the charges are mobile. This model has been used to describe an insulator-conductor transition in normal tunneling junction networks when $R_n$ is small enough.

We have seen that the Mott insulator breaks down and charges are mobile at small $R_n$. What about superfluidity for these mobile charges? This requires long-range phase coherence in the system. In other words, in addition to a “smooth” $n$-field, the fluctuations of $\theta$ at different sites must also be coherent. Therefore, in principle, we may have a superfluid or metallic state for these mobile charges, depending on whether the phase stiffness for $\theta$ fluctuations is finite or not.

![FIG. 1. Possible scenarios at zero temperature. Ins: insulator, SF: superfluid. Solid and dashed lines denote first- and second-order transitions. ASOS line: transition for winding numbers in the absolute solid-on-solid model. $\theta$ line denotes transition for small fluctuations, $\theta$, as given in Ref. 2.](image-url)

If we ignore the coupling of the $\theta$-field to the winding numbers $n_i$, then we expect a superfluid at small $R_n$ at $T = 0$. ($R_n < h/2e^2$ in two dimensions) A primary purpose of this paper is to investigate whether the onset of a finite phase stiffness for $\theta$ coincides with the appearance of the smooth phase in the SOS model for the winding number (i.e. a direct superfluid-insulator transition, as shown in Fig. 2a). Another scenario is that...
a metal phase exists for intermediate values of $R_n$ where the ASOS model is smooth before long-range phase coherence sets in at an even lower value of $R_n$ (Fig. 3).

The actions $\mathcal{S}$ and $\mathcal{S}_0$ form the basis of our calculations. The model cannot be solved exactly even without the dissipative term. We shall pursue a variational approach since we are only interested at the qualitative behaviour of the system — in particular, whether a zero-temperature metallic phase exists under appropriate conditions. We consider the following trial action:

$$S_0 = \sum_i \int_0^\beta dt \left[ \frac{1}{2K_b} \dot{\theta}_i^2 + \frac{J_{\text{eff}}}{} \sum_i (\Delta_\nu \theta_i)^2 \right]$$

$$+ \frac{1}{8} \sum_{i\nu} \int_0^\beta dt \int_0^\beta dt' \alpha_{\text{eff}}(\tau - \tau') \left[ \Delta_\nu \theta_i(\tau) - \Delta_\nu \theta_i(\tau') \right]^2$$

$$+ \frac{2\pi^2}{\beta K_b} \sum_i n_i^2 + \sum_{i\nu} \left[ \frac{Q_\nu}{4R_{\text{eff}}} |\Delta_\nu n_i| - \beta J_{\text{MS}} \delta_{\Delta_\nu n_i} \right], \quad (6)$$

where $\alpha_{\text{eff}}(\tau)/\alpha(\tau) = R_n/R_{\text{eff}}$. $J_{\text{eff}}$, $J_{\text{MS}}$ and $R_{\text{eff}}$ are parameters to be determined variationally. Note that the solid-on-solid part of the model has been modified by the presence of the $J_{\text{MS}}$ term. Similar to the other terms in the ASOS model, it also suppresses the spatial fluctuations in the winding number. We therefore expect this modified solid-on-solid (MSOS) model to be similar to the ASOS model with a shifted critical point $R_{\text{c,MOS}}^\text{MSOS}$.

The possibilities of superconductor, insulator, and metal phases at zero temperature are all included in $S_0$. A finite value for the phase stiffness, $J_{\text{eff}}$, indicates that we have a superconductor (marked “SF” in Fig. 1). If $J_{\text{eff}} = J_{\text{MS}} = 0$, then the system is non-superfluid. To determine whether it is an insulator or a metal, we examine the large phase fluctuations, i.e. the SOS model for the winding numbers. The system is an insulator if $R_{\text{eff}} > R_{\text{c,MSOS}}^\text{MSOS}$ so that the SOS model is in the rough phase. If $R_{\text{eff}} < R_{\text{c,MSOS}}^\text{MSOS}$, the SOS model is in the smooth phase and we have a metallic state. (See Fig. 4b. Note that the ASOS and MSOS models are the same if $J_{\text{eff}} = J_{\text{MS}} = 0$.)

The variational parameters are determined by minimizing the free energy per unit volume given approximately by $F = F_0 + \langle S_{\text{MS}} + S_{\text{SOS}} - S_0 \rangle_0/\beta L^2$, where $F_0$ is the free energy calculated using $S_0$ and $\langle \cdots \rangle_0$ denotes averages taken with respect to $S_0$. We obtain the mean-field equations:

$$R_{\text{eff}} = R_n,$$

$$J_{\text{MS}} = J_b e^{-\langle |\Delta \theta|^2 \rangle/2},$$

$$J_{\text{eff}} = J_{\text{MS}} P_{\text{SOS}}(0), \quad (7)$$

where

$$\langle |\Delta \theta|^2 \rangle = \frac{1}{2\beta L^2} \sum_{\hat{q},i\omega_n} G_{0\theta}(\hat{q},i\omega_n), \quad (8)$$

with $G_{0\theta}^{-1}(\hat{q},i\omega_n) = \omega_n^2/K_b + \gamma(\hat{q})(J_{\text{eff}} + |\omega_n|/4R_{\text{eff}})$. $\gamma(\hat{q}) = 4[\sin^2(q_x/2) + \sin^2(q_y/2)]$ is the lattice dispersion relation. $P_{\text{SOS}}(m) = \langle \delta(|\Delta_\nu n_i| - m) \rangle_{\text{MSOS}}$ is the probability that the nearest-neighbor integer difference $|\Delta_\nu n_i| = m$ in the MSOS model. Note that we can also regard our trial action as a “Hartree” decoupling of the fields $\theta_i$ and $n_i$.

For the small phase fluctuations, the critical point for the onset of a finite $J_{\text{eff}}$ is given by Chakravarty et al. $R_{\text{c}}^\theta = h/Qe^2$, with $J_{\text{eff}}$ becoming exponentially small as $R$ approaches $R_{\text{c}}^\theta$:

$$\left( \frac{J_{\text{eff}}}{J_b} \right) \sim \left( \frac{J_b R}{K_b} \right) R_{\text{c}}^\theta/\left( R_{\text{c}}^\theta - R_{\text{eff}} \right), \quad (9)$$

To determine the ground-state properties of the system, we also need to examine the SOS sector of the model. We see that the $J_{\text{MS}}$ term dominates the MSOS model at low temperatures, and so the SOS sector is smooth whenever $J_{\text{eff}}$ is finite. When $J_{\text{eff}}$ vanishes, we find that $R_n$ is already above the critical value for the SOS critical point (i.e., $R_{\text{c}}^\theta > R_{\text{c,MSOS}}^\text{MSOS}$). Therefore, the winding-number sector is always rough when $J_{\text{eff}} = 0$ so that the system is an insulator. This means that, at the level of this mean-field calculation, we cannot have a metallic phase at zero temperature. We see that the system has only one quantum critical point as we change $R_n$ (marked “SI” in Fig. 3): $R_c = R_{\text{c}}^\theta = R_{\text{c,MOS}}^\text{MSOS}$. This corresponds to the scenario in Fig. 4b.

III. FINITE TEMPERATURE

We will now discuss the system at finite temperature. We will see that the winding-number fluctuations have important consequences for the behaviour of the system because the MSOS model governing these fluctuations has an apparent finite-temperature phase transition. These effects show up as crossover behaviour as the system is cooled to zero temperature.

To see this, we note that the critical value $R_{\text{c,MOS}}^\text{MSOS}$ for the transition in the MSOS model depends on temperature. At high temperatures, the $\beta J_{\text{MS}}$ term in the MSOS model becomes unimportant, and so we expect $R_{\text{c,MOS}}^\text{MSOS}$ to decrease towards $R_{\text{c,MSOS}}^\text{MSOS} = 0.6(h/Qe^2)$ as the temperature increases. This is indicated by dashed line in Fig. 4. In other words, for a resistance in the region $0.6Q < Q^2 R_n/h < 2$, the system will cross a roughening transition for the winding numbers as we increase the temperature. Although this transition is probably an artefact of the variational treatment, we believe that it will manifest itself as a crossover phenomenon in the system.

More precisely, while there appears to be two correlation lengths in this formulation ($\xi_\theta$ and $\xi_{\text{SOS}}$ for the small and large fluctuations respectively), there is only one true phase correlation length, $\xi$. This should follow the shorter of $\xi_\theta$ and $\xi_{\text{SOS}}$. So, the SOS model does not
give rise to a true divergence in observable quantities, since $\xi_\theta$ is finite at all finite temperatures. Instead, the divergence will be cut off when $\xi_{\text{SOS}}$ becomes comparable to $\xi_\theta$.

More generally, we expect physical quantities, such as the conductivity, to depend on both temperature and the proximity of the resistance to the critical value, $R_n - R_c$. For instance, in the smooth phase of the SOS model (to the right of the dashed line in Fig. 2), the correlation length $\xi_{\text{SOS}}$ is finite for fluctuations of the winding numbers about a smooth background. This affects dynamic quantities such as the conductivity which should therefore depend on both $R_n - R_c$ and $T$.

On the rough side of the SOS line, we expect no long-range order in the winding number. The conductivity may not exhibit signs of superfluidity. In fact, it may appear metallic or even insulating, even at superfluid values of $R_n$, as long as we are looking at temperatures above the temperature, $T_{\text{SOS}}$, where we cross the SOS transition line. The temperature scale for this SOS crossover is given by $J_{\text{eff}}$. This can become very small close to the quantum critical point (see eq. (4)) or in strongly disordered systems. We see that the true critical behaviour of the superfluid-insulator transition is hard to access experimentally.

This discussion warns us that, unless we work at extremely low temperatures, the critical behaviour of the system may not follow a simple one-parameter scaling scheme (when $R_n$ is close to $R_c$ so that the system is to the left of the SOS line in Fig. 2). We believe that this may be an important source of difficulties for the scaling analysis of experimental data, and may be responsible for the observation of an apparent metallic phase in some experiments.

To be cautious, we should stress that this result depends on the observation that $R^0_c > R_{\text{SOS}}^n$ (see discussion below eq. (4)) so that it is sensitive to our estimates of $R^0_c$ and $R_{\text{eff}}$. For instance, we note that $R_c$ is unrenormalized in our variational equations (6). A more careful treatment of the dissipative term might renormalize this quantity and therefore shift the relative positions of the critical points of the $\theta$ and SOS sectors. We will assume that these estimates are correct in the next section.

The above analysis is based on a treatment which treats the coupling between the small and large phase fluctuations ($\theta$ and $n$) in a Hartree-like manner. In the next section, we will check that this is reasonable by considering higher-order fluctuations. We will see that the crossover effect mentioned above shows up as the breakdown of our Hartree-like decoupling of the small and large phase fluctuations.

### IV. BEYOND GAUSSIAN FLUCTUATIONS

To consider higher-order fluctuations, let us examine the free energy density $f$. This can be written as:

$$f = f_0 - \ln(\exp[-(S - S_0)])_0/\beta L^2$$

$$\simeq (S - S_0)_0 - ((S - S_0)^2)_{\text{c0}}/2/\beta L^2 + \cdots$$  \hspace{1cm} (10)

where averages are taken with respect to the trial action $S_0$ and $\langle AB\rangle_c = \langle AB \rangle - \langle A \rangle \langle B \rangle$ denotes the connected part of the correlation function. Minimizing the first term in this expansion gives the variational treatment in the previous section. To consider the validity of this approach, we should check that higher-order terms do not diverge. These correspond to fluctuations beyond the Hartree-like treatment in the previous section. We restrict our attention to the first correction.

We can separate the Josephson and dissipative parts of $S - S_0$ as $\delta S^J + \delta S^D$ where

$$\delta S^J = \sum_{\nu} \int_0^\beta d\tau \left[ -J_0 \cos \left( \Delta_\nu \theta_1 + \frac{2\pi \nu}{\beta} \Delta_\nu n_1 \right) - \frac{J_{\text{eff}}}{2} (\Delta_\nu \theta_1)^2 + J_{\text{MS}} \delta_\nu \theta_1 \right]$$  \hspace{1cm} (11)

$$\delta S^D = \sum_{\nu, \nu'} \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') \times$$

$$\left[ \cos \left( \frac{2\pi (\tau - \tau')}{\beta} \Delta_\nu n_1 \right) - 1 \right] \left[ \Delta_\nu \theta_1(\tau) - \Delta_\nu \theta_1(\tau') \right]^2$$  \hspace{1cm} (12)

As we approach the SI transition ($J_{\text{eff}} \to 0$, $R_{\text{eff}} \to R_c$) at zero temperature, we find that the singular part of $\langle (\delta S^J)^2 \rangle/\beta L^2$ comes from fluctuations in $\theta$, scaling as $J_{\text{eff}}^2 R_{\text{eff}}/R_c - 1$. We see that $\langle (\delta S^J)^2 \rangle$ does not diverge even at the critical point. In the non-superfluid phase
(\(J_{\text{eff}} = 0\)), these fluctuations are proportional to \(T\) as \(T \to 0\).

The contributions to \(\langle \delta S^I \delta S^D \rangle / \beta L^2\) are also finite, scaling as \(T\) when \(T \to 0\) at finite \(J_{\text{eff}}\), and scaling as \(J_{\text{eff}}\) when \(J_{\text{eff}} \to 0\) at finite \(T\).

We find that the most singular term comes from \(\langle (\delta S^D)^2 \rangle\). Let \(A_{\tau,\tau'} = \sum_{\mu\nu} [\Delta \theta_i(\tau) - \Delta \theta_i(\tau')]^2\) and \(B_{\tau,\tau'} = \sum_{\nu} \alpha(\tau - \tau') |\cos(2\pi(\tau - \tau')\Delta \varphi_n_i / (\beta \beta)) - 1|^2 / 2\). Then, the contribution from

\[
I = \frac{1}{\beta L^2} \int \langle A_{\tau_1,\tau_2} \rangle \langle B_{\tau_1,\tau_2} \rangle d\tau_1 d\tau_2 \sim \frac{1}{\beta} \sum_{\omega, \omega'} \frac{\sum_{\mu\nu} (g_{\mu}(r, \omega) g_{\nu}(0, \omega')) c}{(4J_{\text{eff}} R_{\text{eff}} + |\omega|)(4J_{\text{eff}} R_{\text{eff}} + |\omega'|)}
\]

(13)

\[
= \sum_{\omega, \omega'} \frac{\sum_{\mu\nu} \langle |\Delta \varphi n_i |^2 \rangle_c}{(4J_{\text{eff}} R_{\text{eff}} + |\omega|)(4J_{\text{eff}} R_{\text{eff}} + |\omega'|)}
\]

(14)

where \(g_{\mu}(r, \omega) = \min(\omega, 2\pi Q^{-1}T|\Delta \varphi n_i|)\). The numerator is a connected correlation function for the MSOS model. We expect it to have exponential decay with correlation length \(\xi_{\text{SOS}}\) in the smooth phase (and power-law decay in the rough phase).

In the smooth phase of the MSOS model where \(J_{\text{eff}}\) is also finite, we see that \(I \sim T \xi_{\text{SOS}}^2 \ln(R_b / J_{\text{eff}})\) as \(T \to 0\). On the other hand, we expect the quantity \(\sum_{\mu\nu} \langle |\Delta \varphi n_i |^2 \rangle_c\) to have the same critical behaviour as the energy fluctuations — it diverges as we cross the line of SOS critical points. As discussed in the previous section, this will not be a true divergence, but only a crossover. Nevertheless, this means that this contribution from \(\langle (\delta S^D)^2 \rangle\) will be large if we cross the SOS transition line as we raise the temperature in the superfluid phase.

This marks the breakdown of our treatment of the phase fluctuations in this model (in the model to the left of the dashed line in Fig. 2). However, since no divergences occur if we work at zero temperature, the conclusion of the superfluid-insulator transition appears robust (subject to the remark at the end of the previous section about the accuracy of our estimations for the critical points for the two sectors of the model.)

V. VORTEX DISSIPATION

We will now discuss dissipation by vortex motion. Microscopically, this is due to the motion of the normal vortex core. We will, however, follow a phenomenological approach here.

For this purpose, it is convenient to study the system in a vortex representation. Fluctuations can be described by vortex loops in Euclidean space-time. In particular, the superfluid state for the Cooper pairs corresponds to a vortex representation. There is a gap to the addition of a vortex — the Meissner effect. Conversely, the duality transformation shows that the Meissner phase of the vortices corresponds to an insulating state for Cooper pairs (i.e., there is a gap to density excitations.)

To obtain the vortex representation, a duality transformation can be applied to the action \(\mathcal{A}\) to obtain the dual action \(S_{\text{dual}} = S_A + S_v\) for vortices, where

\[
S_A = \sum_i \int_0^\beta d\tau \left[ \frac{1}{2J_b} \sum_{\mu} |(\nabla \times \vec{A})_i^\mu|^2 + (K_0/2) \sum_i (d_\mu^i \theta_v^\mu)^2 - J_v \sum_i \cos D_v \theta_v^v \right]
\]

(15)

where \(D_v \theta_i = \Delta_v \theta_i - \Delta_v^c\) is a covariant derivative. The internal gauge field, \(A_i\), is defined so that \(\nabla \times \vec{A}\) is the boson 3-current. Its action, \(S_A\), describes the phonons in the (original) boson superfluid. (The superscripts \(s\) and \(\tau\) denote the spatial and temporal components respectively.) The action \(S_v\) describes vortices in the system: \(\theta_v^\mu\) is the phase of the vortex wavefunction on site \(i\) of the dual lattice. We have introduced the terms \(K_v = J_v\) and \(J_v \sim 2e/\sqrt{\hbar}\) to characterize the core energy and the hopping integral of the vortices respectively.

The coupling of the vortex phase to the gauge field expresses the fact that vortices are advected by the current of the original bosons.

The qualitative behaviour of the system should not depend on details of the vortex interaction as long as it is short-ranged. We therefore choose the lattice spacing, \(d\), for the dual model to be of the order of the penetration depth (of the original Cooper pairs), and include only on-site repulsion for vortices. For simplicity, we choose a square lattice.

As with the boson model discussed above, we expect the system to have a superfluid-insulator transition as we increase \(K_v / J_v\). To include dissipation phenomenologically, we again assume that the action of the Caldeira-Leggett kind so that the vortex currents (\(\sim D_v \theta_v^v\)) will decay with a decay rate proportional to the current:

\[
S_{\text{diss},v} = \frac{1}{2} \sum_i \int_0^\beta \int_0^\beta d\tau d\tau' \alpha(\tau - \tau') \times \sin^2 \left[ \frac{D_v \theta_v^v(\tau) - D_v \theta_v^v(\tau')}{2} \right]
\]

(16)

where \(\alpha(\tau) = (h/4e^2R_v)[T / \sin(\pi T \tau)]^2\) with \(4e^2R_v / h \sim (1 - t)/t\) and \(t \sim e^{-4\xi_{\text{SOS}}} / h\) is the tunneling resistance of vortices from one grain to another. Note that we have set \(Q = 1\) in this case because we do not have a microscopic reason for the dissipative mechanism to involve objects with a charge that is different from the bosons. The vortex viscous drag coefficient \(\eta_{\text{diss}}\) is given by \(\Phi_0H_{c2}/R_v c^2\) where \(R_v\) is the normal-state resistance of the superconductor, \(H_{c2}\) is the upper critical field, and \(\Phi_0 = hc/2e\) is the flux quantum. The details of the relationship between \(R_v\) and \(R_n\) are not important here. It suffices to
note that they are inversely related to each other and comparable when both are of the order of $\hbar/e^2$. The coupling to the internal gauge field is required by gauge invariance.

We see that this model is similar to the one discussed in the previous sections, except that the bosons are now coupled to an internal gauge field. We can again separate the imaginary-time evolution of the phase into a periodic part, $\theta_i(\tau)$, and a non-periodic part, $2\pi n_i/\beta$, to obtain:

$$S_{\text{dual}} = S_A + \frac{2\pi^2}{\beta K_v} \sum_i n_i^2 + \frac{1}{2K_v} \sum_i \int_0^\beta \theta_i^2 \, d\tau$$

$$- J_v \sum_{i\nu} \int_0^\beta \cos \left(D_v \theta_i(\tau) + \frac{2\pi \tau}{\beta} \Delta \nu n_i \right) \, d\tau,$$

Repeating the treatment in the previous sections (with $Q = 1$), we have the trial action:

$$S_0 = \sum_i \int_0^\beta \left[ \frac{1}{2K_v} \theta_i^2 + \frac{J_{\text{eff}}}{2} \sum_{\nu} (D_v \theta_i)^2 \right]$$

$$+ \frac{1}{8} \sum_{i\nu} \int_0^\beta \int_0^\beta \alpha_{\text{eff}} (\tau - \tau') |D_v \theta_i(\tau) - D_v \theta_i(\tau')|^2$$

$$+ \frac{2\pi^2}{\beta K_v} \sum_i n_i^2 + \sum_{i\nu} \left[ \frac{\pi}{4R_{\text{eff}}} |\Delta \nu n_i| - \beta J_{\text{MS}} \delta_{\Delta \nu n_i} \right].$$

The variational parameters are now given by $R_{\text{eff}} = R_v$, $J_{\text{eff}} = J_{\text{MS}} P_{\text{SOS}}(0)$ and

$$J_{\text{MS}} = J_v \exp \left[- \langle |\Delta \theta|^2 \rangle + \langle A \rangle / 2 \right],$$

$$\langle |\Delta \theta|^2 \rangle = \frac{1}{2\beta L^2} \sum_{\bar{q},i\omega_n} \gamma(\bar{q}) G_{00}(\bar{q},i\omega_n),$$

$$\langle |A|^2 \rangle = \frac{1}{2\beta L^2} \sum_{\bar{q},i\omega_n} G_{0A}(\bar{q},i\omega_n),$$

where $G^{-1}_{0A}(\bar{q},i\omega_n) = \omega_n^2/J_b + 2K_v \gamma(\bar{q}) + |\omega_n|/4R_{\text{eff}} + J_{\text{eff}}$ and $G_{00}(\bar{q},i\omega_n) = \omega_n^2/K_v + \gamma(\bar{q})(J_{\text{eff}} + |\omega_n|/4R_{\text{eff}})$.

The main effect of the gauge fields is to reduce the vortex repulsion $K$. (For weak boson repulsion, $K_v \rightarrow K^* = (K_v/4\pi^2)(J_b/2K_v)(h/4e^2 R_v)$. Since this is essentially a high-energy cutoff for the physical effects we are considering, it should not affect the critical point for the onset of a finite $J_{\text{eff}}$ for small vortex phase fluctuations. We therefore conclude that this dissipative mechanism will also give rise to a direct superfluid-to-insulator transition in the vortex liquid as $R_v$ is increased. Note that the vortex resistance, $R_v$, is large when the resistance, $R_n$, of the normal fluid is low. Therefore, we have qualitatively the same zero-temperature behaviour here as in the previous model (for direct dissipation from Cooper pair motion) in that the system, in terms of electrical transport by the original Cooper pairs, is superfluid for small $R_n$ and insulating for large $R_n$. The exact value of the critical resistance is more difficult to extract as it depends in detail on the dependence of $R_v$ on $R_n$. However, it can be verified that the critical point occurs when $e^2 R_n/h$ is of the order of unity.

At finite temperatures, we again expect crossover behaviour. This applies to the insulating side of the (original) SI transition, whereas the crossover behaviour of the previous Cooper-pair model affects the superfluid side of the transition. In terms of $R_n$ (instead of $R_v$), this model predicts that, for insulating values of $R_n$ (i.e. vortex superfluidity at $T = 0$), the finite-temperature conductivity for $R_n$ may appear metallic or even exhibit signs of (charge) superfluidity above the crossover temperature.

VI. FINITE MAGNETIC FIELD

Finally, we discuss the effect of a finite magnetic field, $B$. This gives rise to a non-zero chemical potential, $\mu_v$, for vortices so that there are a finite density of vortices in the ground state: $\rho_v = B/\Phi_0$. Again, the movement of the MSOS crossover as a function of vortex density (at fixed $R_n$ or $R_v$) will affect the finite-temperature behaviour of the system, and hence the analysis of the experimental data.

Experimentally, we see that the system goes from superconducting to insulating as we increase $B$. Some experiments indicate that there may be a metallic phase between the superconducting and insulating phases. However, there is also evidence at low applied fields that the metallic behaviour only occurs at intermediate temperatures, and that the true zero-temperature phase may be a superconductor after all.
where the bosons are Cooper pairs. (This can be viewed as the bosonic analogue of positive magnetoresistance.) This moves the the SOS transition/crossover line to lower values of $R_n$ (to the right in Fig. 4). This is illustrated in Fig. 3. We see that, if the system is near the SI critical point, the crossover temperature, $T_{SOS}$, decreases rapidly with increasing applied field $B$, as sketched in Fig. 3. This may explain why the crossover from metal to superconductor is only observed experimentally at low applied fields.4

VII. CONCLUSIONS

In this paper, we have revisited a model of dissipative Bose systems where conventional theory predicts a direct superfluid-insulator transition. By treating the phase fluctuations more carefully, we developed a variational approach which can distinguish between superfluid, normal and insulating phases. We can confirm that a Bose-metal phase does not exist at zero temperature, in agreement with conventional treatment. We are also able to establish the regime of validity for the conventional treatment by studying higher-order effects which couple the small and large phase fluctuations.

We have argued that single-parameter scaling might break down because of the existence of large phase fluctuations (the imaginary-time “winding numbers” of the order-parameter phase). There is a window around the true critical point where strong winding-number fluctuations persist down to exponentially low temperatures. This means that superconductivity may not be observable in this regime at experimentally accessible temperatures. The width of this window of crossover behaviour appears to be quite large in our mean-field analysis. We expect that it would be renormalized in a more detailed calculation, and that it would depend on details of the system (such as the degree of disorder).

This window of crossover behaviour may be responsible for an apparent metallic phase in some experiments. The recent observation of an apparently metallic phase becoming superconducting at very low temperatures appear to supports our crossover picture near the superfluid-insulator critical point.

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