Numerical solution of the conjugate heat transfer problem for
turbulent liquid flow in a tube using the large eddy simulation
method

V I Artemov, M V Makarov, G G Yankov and K B Minko

National Research University "MPEI",
Russia, 111250 Moscow, Krasnokazarmennaya, 14

Makarovmv2000@yandex.ru

Abstract. This article presents the results of turbulent flow and temperature fluctuations simulation in the liquid and the tube wall in a conjugate formulation using the hybrid Reynolds-averaged Navier-Stokes / large eddy simulation (RANS/LES) method. Calculations are performed for water, air and mercury. The results show that the "conjugate" LES model implemented in in-house CFD code ANES provides a good reproduction of the available experimental results on temperature fluctuations near the wall for cases of different wall thicknesses and thermal activity of the wall material and the fluid. An approximate model for the wall is proposed and can be used as a boundary condition for calculations by direct numerical simulation (DNS) and LES methods in the non-conjugate formulations.

1. Introduction

In the last decade, there has been a significant increase in interest in the study of the heat transfer processes involved in the turbulent flow of electrically conductive liquids in channels under the combined influence of gravitational and magnetic fields (MF) [1 – 5]. The relevance of these studies lies in their association with applications such as elements cooling in fusion reactors with liquid metal or molten salts, as well as with other promising developments.

Significant and inhomogeneous wall heat fluxes are heat transfer features in the cooling channels of fusion reactors. These conditions lead to strong thermo-gravitational convection (TGC) arising at any orientation of the channel relative to the gravity vector. Experimental studies [1 – 3] show that the result of the combined influence of two factors – MF and TGC – is quite complex and can lead to unexpected effects. In particular, previous work [2, 3] in a horizontal tube under inhomogeneous heating has revealed the presence of abnormally large-amplitude low-frequency fluctuations of the liquid temperature in the core of the flow at high values of the MF intensity (Hartmann number of about 300). These regimes were modelled using direct numerical simulation (DNS) methods in [4, 5], and the results qualitatively coincided with the experimental data. At the same time, the question of penetration of significant temperature fluctuations into the solid wall of the channel or tube has not been studied and remains open. Clearly, a detailed analysis of the temperature profiles at the wall is a relevant topic, with a focus on the operating parameters, the properties of the coolant, the material and the thickness of the wall.

The DNS method typically uses high-order accuracy schemes in space to provide a significant reduction in scheme diffusion; however, this inevitably leads to problems with the realization of the
conjugate formulation for the "solid wall - liquid" interface surface. Therefore, in studies of turbulent flows and heat transfer by the DNS method, a solid wall was not considered, and the thermal boundary condition was formulated on the inner surface of the wall. Note that the use of a zero dimensionless temperature [6] or a constant heat flux [5] in the DNS methods on the inner surface of the channel wall does not correspond to any of the known methods for real heating of the channel walls (such as passing electric current through the wall, or the use of overhead heaters on the external wall surface). A conjugate formulation of the problem is required to obtain realistic values of the fluctuations of the liquid temperature near the wall and in the wall. This allows one to analyze the influence of various factors on the intensity of the previously mentioned fluctuations. A similar problem was solved by Kasagi et al. [7], who modelled the hydrodynamics by setting the analytical dependencies for three instantaneous velocity components with sinusoidal distributions in the direction of the flow and in time. Despite the approximation of the hydrodynamic model, this formulation allowed determination of the effect of the wall and the thermal boundary condition on the intensity of temperature fluctuations in the most important zone $y^* = 0 – 30$, with dependence on three determining parameters: the Prandtl number $\text{Pr}$, the dimensionless wall thickness $\delta_{\text{w}}^{++}$, and the parameter $K$ characterizing the thermal activity of wall material and fluid

$$ y^* = \frac{u_r y_w}{v_f}, \quad \delta_{\text{w}}^{++} = \frac{u_r \delta_{\text{w}}}{v_f} \left( \frac{a_f}{a_s} \right)^{1/2}, \quad a = \frac{\lambda}{\rho c_p}, \quad K = \left( \frac{(\rho c_p \lambda_f)}{(\rho c_p \lambda_s)} \right)^{1/2}, \quad (1) $$

where $u_r$ – wall-shear velocity; $y_w$ – distance from the wall; $\delta_{\text{w}}$ – wall thickness; $v$ – kinematic viscosity; $\lambda$ – thermal conductivity; $c_p$ – specific heat at constant pressure; $\rho$ – density; indexes: $f$ – fluid, $s$ – solid wall.

With this in mind, there is reason to believe that new information on the structure of near-wall turbulent flows and temperature fluctuations in the near-wall zone and in the channel walls can be obtained using large eddy simulation (LES) in a conjugate formulation. This paper presents the results of verification of the "conjugate" LES method implemented using our proprietary CFD code ANES [8], which allows determination of the intensity of temperature fluctuations in the wall region and in the tube wall for various liquids and for the parameters characterizing the wall thickness and thermal activities of the wall material and the coolant.

2. Methodology

Previously, the authors of [9] analysed the effectiveness of four subgrid-scale models for turbulent stresses (SGS) in the LES modelling of flows with turbulence suppression under the influence of mass forces of various natures. The simulations included the flow of an electrically conductive fluid in a channel under the influence of a transverse magnetic field, as well as the upward flow of air in a heated vertical pipe under the strong influence of buoyancy forces. Comparison of these results with the available DNS reference data showed some advantages of the hybrid RANS/LES model KDES, based on the equation for the subgrid turbulent kinetic energy.

The present work focuses on the verification of the use of KDES in the conjugate problem formulation and the simulation of temperature fluctuations in the near-wall region and in the wall for the turbulent flow of various coolants.

An approximate "0D" model of the wall was also developed, which allowed replacement of a given heat flux density on the outer wall surface or of a given volumetric heat release by non-stationary boundary conditions on the inner wall surface.

By integrating the thermal conductivity equation over the control volume of the wall with central point $S$ shown in figure 1, and neglecting the heat fluxes along the wall, we obtain equation (2):
where \( q_w \) and \( q_{\text{out}} \) – heat fluxes on the inner and outer surfaces of the wall, \( T_s \) and \( T_w \) – temperatures in the point \( S \) and in the centre of the boundary face, \( \Delta S_w \) – area of the boundary face.

The temperature \( T_w \) can be obtained using an approximation for the heat flux in the liquid in the form:

\[
T_w = \frac{\alpha_s T_s + \alpha_f T_p}{\alpha_s + \alpha_f}, \quad \alpha_f = \frac{\lambda_f}{\delta_b},
\]

Taking into account equation (3), equation (2) can be represented as follows:

\[
\frac{dT_s}{d\tau} = A - B T_s, \quad A = \frac{q_{\text{out}} + \alpha_s T_p}{(\rho c_p)_s \delta_w}, \quad B = \frac{\alpha_b}{(\rho c_p)_s \delta_w}, \quad \alpha_b = \frac{\alpha_s \alpha_f}{\alpha_s + \alpha_f}.
\]

The solution of this equation can be obtained analytically. In this case, \( T_s \) at the time \( \tau \) can be linked to the \( T_s^0 \) at the previous time \( \tau - \Delta \tau \)

\[
T_s = T_s^0 \exp(-B\Delta \tau) + \frac{A}{B} \left[1 - \exp(B\Delta \tau)\right]
\]

As a result, we obtain non-stationary boundary conditions on the inner wall surface

\[
q_w = \alpha_b (T_s - T_p)
\]

The calculations were performed using unstructured meshes with local refinement and cut cells. Time derivatives were calculated using a second-order three-layer scheme. Spatial derivatives were approximated with a second order of accuracy. The main features of the algorithms, numerical schemes and grids used are given in [9].

Further, the calculations performed without taking into account the wall are labelled "NoW". The data obtained with the approximate wall model are denoted as "W0D". The results of the exact calculation of the wall are indicated by "Wqv" at a given volumetric heat release and by "Wqw" at a given wall heat flux.

3. Simulation results
The results obtained are for water (Pr = 6.2), air (Pr = 0.71) and mercury (Pr = 0.025). Figure 2 presents the data of root-mean-square fluctuations obtained for water flow in a round tube for
Reynolds number (Re) equal to 7500. The results are compared with experimental data from [10]. The value of $T_c$ in figure 2 is defined as follows:

$$T_c = \frac{q_w}{\rho c_p \mu_c},$$

where $q_w$ – heat flux on the inner surface of the tube that is equivalent to the volumetric heat release in the solid wall.

The number of cells was 1.85 million for the variant without a wall and 2.58 million for variants with a solid wall. The steel tube wall in experiments [10] had a thickness of 0.49 mm, and is categorized as "thin" ($\delta_w^{++} = 1.35$) and "low thermal conductive" ($K = 0.2$) wall. Figure 2 clearly shows that the results obtained in the "Wqv" and "W0D" modes are in good agreement with the experimental data of [10], although the wall temperature fluctuations in the "W0D" mode are somewhat overestimated. The ratio of the root-mean-square fluctuations of the wall temperature in the "NoW" mode to the fluctuations in the "Wqv" mode is approximately 3.

![Figure 2. Comparison of temperature fluctuation distributions with data from [10] for water flow.](image)

In figure 3, the calculation results are compared with the DNS data from You et al. [10] for the upward air flow (Re = 5300, Pr = 0.7) in the heated pipe under the strong influence of free convection ($Bo = 80000Gr^{3.425}Re^{0.18}Pr^{-0.8} = 0.18$).

The dimensionless temperature in figure 3 is defined as

$$\theta = \frac{T_w - T}{T_w - T_b},$$

where $T_w$ – temperature on the inner surface of the tube, $T_b$ – bulk temperature.

The calculated grid had 1.435 million cells for the mode "NoW" and 1.92 million cells for the mode "Wqv". The pipe was assumed to be made of aluminum, which corresponds to the experimental study [11]. According to Kasagi et al. [7], for dimensionless wall parameters $\delta_w^{++} = 2.73$, $K = 2.5 \times 10^{-4}$, the temperature fluctuations on the inner wall surface should be close to zero. This pattern can be seen in figure 3 (modes "Wqv" and "W0D"). At the same time, setting the equivalent heat flux at $r = 1$ (option "NoW") leads to very noticeable temperature fluctuations on the inner surface of the pipe. The
results obtained for the "Wqw" and "W0D" variants are in good agreement with the DNS data from You et al [6], in which the temperature fluctuations on the wall were strictly equal to zero.

Calculations for mercury were performed in relation to experimental studies at the facility used in the work [3]. The horizontal pipe was made of steel with an internal diameter $D = 19$ mm and a wall thickness $d_w = 0.5$ mm, dimensionless parameters were defined as $\delta^+ = 11.2$, $K = 0.51$, and heating was carried out on the outer surface of the pipe wall. For comparison, we use unpublished experimental data from N. G. Razuvanov at $Re = 7000$, $q_w = 5500$ Wm$^{-2}$. Temperature measurements were performed for fully developed turbulent flow along the horizontal ($X$) and vertical ($Y$) axes of the pipe.

![Figure 3. Comparison of dimensionless temperature and intensity of temperature fluctuations with data of [6] for air flow (Bo=0.18).](image1)

![Figure 4. Dimensionless temperature distributions for mercury flow.](image2)
For these calculations, the number of cells was chosen to be equal to 3.7 million without taking into account the wall, and 6 million for the conjugate formulation.

The results of calculations in comparison with experimental data are shown in figures 4 and 5. The figures show good agreement between the calculated and experimental data.

![Figure 5. Dimensionless temperature fluctuations distributions for mercury flow.](image)

It should also be noted that the calculated temperature fluctuations of the inner wall surface in the "Wqw" and "W0D" modes are about half as low as in the "NoW" mode.

4. Conclusion
A "conjugate" KDES model was developed and implemented in our propriety CFD code ANES. The calculation results for mercury, air and water provide a good reproduction of the available experimental and DNS data on temperature fluctuations near the tube wall, and they correctly reflect the influence of dimensionless parameters (Prandtl number, wall thickness, as well as the thermal activities of the wall material and coolant) on temperature fluctuations in the wall itself.

The proposed approximate model for taking into account the real wall quite satisfactorily predicts the temperature fluctuations in the liquid near the wall and can be used as a boundary condition for modelling turbulent flows in pipes by the DNS method.

The "conjugate" KDES model can be used to analyze temperature fluctuations in the liquid and in the wall during turbulent flows in channels.

Acknowledgments
All calculations were performed using the JIHT RAS and MVS 10P MSC RAS Fisher supercomputers. The work was supported by the Russian Foundation for Basic Research (project № 20-08-00683-a).

References
[1] Genin L G Listratov Y I Sviridov V G Zhilin V G Ivochkin Y P and Razuvanov N G 2003 Experimental studies of hydrodynamics and heat transfer of liquid metals in magnetic fields Voprosi atomnoy nauki i tekhniki Series: thermonuclear fusion Moscow 4 35-44 [in Russian]
[2] Genin L G, Zhilin V G, Ivochkin Y P, Razuvanov N G, Belyaev I A, Listratov Y I, and Sviridov V G 2011 Temperature fluctuations in a heated horizontal tube affected by transverse magnetic field. In Proc. Fundamental and Applied MHD 8th International Pamir Conference Borgo Corsica, 37–41

[3] Belyaev I A, Ivochkin Y P, Listratov Y I, Razuvanov N G, and Sviridov V G 2015 Temperature fluctuations in a liquid metal MHD-flow in a horizontal inhomogeneously heated tube. High Temperature 53 (5) 734-41

[4] Zikanov O, Listratov Y I, and Sviridov V G 2013 Natural convection in horizontal pipe flow with a strong transverse magnetic field. J. Fluid Mech. 720 486-516

[5] Listratov Y, Ognerubov D, Zikanov O, and Sviridov V 2018 Numerical simulations of mixed convection in liquid metal flow within a horizontal pipe with transverse magnetic field. Fluid Dyn. Research 50 051407

[6] You J Y, Oo J Y, and Choi H 2003 Direct numerical simulation of heated vertical air flows in fully developed turbulent mixed convection. Int. J. Heat Mass Transf. 46 1613–1627

[7] Kasagi N, Kuroda A, and Hirata M 1989 Numerical investigation of near-wall turbulent heat transfer taking into account the unsteady heat conduction in the solid wall. Int. J. Heat Mass Transf. 111 385-392

[8] CFD code ANES http://anes.ch12655.tmweb.ru

[9] Artemov V, Makarov M, Minko K, and Yankov G 2020 Assessment of performance of subgrid stress models for a LES technique for predicting suppression of turbulence and heat transfer in channel flows under the influence of body forces. Int. J. Heat and Mass Transf. 146 118822

[10] Sukomel L A 1984 Development of the method and results of experimental study of heat transfer and temperature fields for water flow in the tube. PhD thesis. MPEI Moscow Russia [in Russian]

[11] Carr A, Connor M, and Buhr H 1973 Velocity, temperature, and turbulence measurements in air for pipe flow with combined free and forced convection. ASME J. Heat Transf. 95 445–452