Teaching Geometry According to the Discovery Method with GeoGebra Software:
A case study in Vietnam

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Abstract—In the context of the industrial revolution 4.0, the trend of adopting active teaching methods in mathematics for high school students with the help of dynamic geometry software is popular in smart schools in Vietnam. This article focuses on studying the teaching of transformations on the plane with the help of dynamic geometrical software GeoGebra, because this software has many outstanding advantages such as highly interactive ability, allowing the creation of new geometric problems whose results are solved immediately, allowing verification of the geometry problem results, allowing image movement and illustration of theorems and the properties by animations. In the research process, we use the discovery teaching method and empirical research methods with 371 high school students in Vietnam. The results of the study have proposed the teaching process to explore the forms of transformations on the plane, illustrated with examples using the help of GeoGebra software, allows embedding in the Internet environment, integrating with other software products to create tools to support solving geometry problems.

Keywords—dynamic geometry; GeoGebra software; the discovery teaching method; smart school

I. INTRODUCTION

The discovery learning is a teaching method based on the viewpoint of focusing on students, helping students to find, discover knowledge, build new knowledge and skills based on existing knowledge, life skills, experience of the student himself.

Nowadays, the application of information technology in general and GeoGebra dynamic geometry software in particular is developing very fast in Vietnam. GeoGebra (from Geometry and Algebra) is one of the most innovative, open-source math software (GNU General Public License) which can be freely downloaded from www.geogebra.org. GeoGebra works on a wide spectrum of operating system platforms which have Java virtual machine installed on [1]. GeoGebra dynamic geometry software has many outstanding advantages that lots of other dynamic geometry software cannot have. First, this software is free of charge and can interact by drawing geometric objects or entering commands from the screen. Second, GeoGebra software has high accuracy, accuracy up to 15 numbers after the comma. Currently, we have not discovered any synthetic geometry problem using GeoGebra software to give wrong answers. Third, GeoGebra software allows to predict math problems. Fourth, GeoGebra software allows creation of new math results. Fifth, GeoGebra software allows image movement. This is the difference between building shapes on software and on blackboard with white chalk. Blackboards with white chalks cannot move shapes like GeoGebra software. Sixth, GeoGebra software can be designed to create animations that illustrate mathematical concepts, mathematical properties. Seventh, GeoGebra software allows inserting images, programming as shape calculation software, … [2].

The plane transformation is an easy-to-apply feature of GeoGebra software. GeoGebra software makes teaching better. In particular, the application of the discovery learning method of transformations on a plane with the help of GeoGebra software is an appropriate combination of teaching facilities, student-centered teaching methods and the lesson. This is because GeoGebra software needs a lot of discovery activities as well as the content of transformations on a plane that needs lots of predictability, calculation, shape movement, and animation of objects.

In this article, we would like to present some of the problems of discovery learning. Practice in discovering for oneself teaches one to acquire information in a way that makes that information more readily viable in problem solving [2,3]. In addition, we provide a way to teach some knowledge of plane transformations with the help of GeoGebra software in Vietnam.

There are many researches on Discovery method and GeoGebra software. The effectiveness of discovery learning model on mathematical problem solving [4]. Joolingen notes on the cognitive for discovery learning [5]. Resmawati and Prabowo write on the discovery learning model with a scientific approach to increase science learning achievement of students [6]. Majerek researches on the application of GeoGebra for teaching mathematics [7]. Arbaian and Shukor refers to the effects of GeoGebra on student achievement [8]. Yorganci notes on A study on the views of graduate students on the use of GeoGebra in mathematics teaching [9], etc. In
addition, there are some similar previous researches on the application ICT in teaching. Shieh and Yu refer to a study information technology integrated guided discovery instruction towards students’ learning achievement and learning retention [10]. Lyu and Wang research on the effects of the application of computer network technology to guided discovery teaching on learning achievement and outcome [11], etc. However, there is not any research on teaching transformations according to the discovery method with GeoGebra software in Vietnam. This thing is discussed below.

II. MATERIALS AND METHODS

A. Concept of Discovery Learning

Polya stated that good education was education that provided opportunities for students to find things, which in this case are mathematical concepts by themselves [12].

The concepts in mathematics are not given directly by the teacher to students. Students must be involved in the process of rediscovering the concept. Students are required to create ideas, look for relationships to form concepts [13].

Discovery Learning is a method of inquiry-based instruction, discovery learning believes that it is best for learners to discover facts and relationships for themselves [3]. Vygotsky believed that learning occurs when students work or learn to handle complex tasks or problems that are still within the cognitive reach of students or those tasks are in the Zone of Proximal Development [14].

Discovery learning is a learning process that emphasizes the intellectual mental of the students in solving various problems encountered, to find a concept or generalization that can be applied in the field. Discovery learning is a learning process to find something new in teaching and learning activities. In discovery learning the learning process can find something if the teacher prepares in advance a variety of material to be delivered. Students are encouraged to primarily study themselves through active involvement by finding principles in understanding concepts [15].

In discovery learning, teacher do not deliver the materials or lessons directly to the students, but students are encouraged to identify what they want to know. As guidance, teachers can use scaffolding or giving certain tasks in form of students’ worksheets.

Discovery learning is a type of learning strategy structured to allow the learner to explore and find answers. In this framework students interact with their environment by exploring and manipulating objects, wrestling with questions and controversies, or performing experiments. Discovery learning is widely used particularly in science. Furthermore, is used in corporate training and development initiatives. Concepts such as business finance and strategy, project teams, leadership, organizational performance, and change management have been thought utilizing the techniques associated with discovery learning. Discovery learning is not like traditional classroom learning; it is a constructivist based approach to education. Discovery learning consists of three main attributes: (a) Trough exploration and problem solving students create, integrate and generalize knowledge, (b) Student driven, interest-based activities which the students determine the sequence and frequency, (c) Activities to encourage integration of new knowledge into the learners own existing knowledge base [16].

Discovery is the process of working and thinking, which can include observation, analysis, identification, evaluation, hypothesis, reasoning, to give concepts, discover the properties and rules, in things, phenomena and relationships between them [17].

Discovery learning is relevant scientific approach designed so that students actively construct concepts, principles through stages of observing, formulating problems and hypotheses, collecting data, drawing conclusions and communicating concepts and principles that have been discovered. The scientific approach provides students with an understanding of various materials using a scientific approach. That information can come from anywhere does not depend on the direction of the teacher’s information. Learning encourages students to find out from various sources through observation and not just being told [6].

From these points of view, we propose the concept of discovery learning as follows: Discovery learning is the process in which students actively make efforts to discover the relationship between this thing and this phenomenon and other things and phenomena as well as internal relationships in things and phenomena. New things and phenomena are closely related to the students’ most recent cognitive development. Students can ask for help from teachers, friends, instructors or the help of information technology to carry out their discovery activities.

When considering students’ discovery activities, people focus on the level of initiative and independence of students. In order to distinguish the levels of discovery learning activity, we can base on the level of teacher intervention in the student's discovery process. Thus, discovering teaching is often divided into three types, corresponding to different levels: (1) Guided discovery learning. Problems and answers given by teachers, students find ways to explain. (2) Modified discovery learning. The problems are given by the teacher, students find answers. (3) Free discovery learning. Problems and answers are discovered by students themselves [18].

B. Information Technology Integrated with Discovery Learning

Information technology today has many applications in teaching in general and in discovery learning in particular. Information technology allows the teaching process to take place anytime, anywhere. Students can participate in learning process anywhere regardless of skin color and age. Typical of teaching takes place all the time, it is teaching by Mobile learning. Blanka said that Mobile learning is a well-established methodology thanks to its countless benefits such as accessing learning content anytime and anywhere, adjusting the content to students’ needs, and timely feedback [19].

Information technology allows unlimited training of students. Students can explore and discover knowledge through
the Internet. Teachers can control students' computers to help students form and develop knowledge. Teachers designed learning doses to match each student's level of awareness. Good, fair, average or weak students are assigned the appropriate learning dose of the students themselves. Today, when artificial intelligence develops, remote control, voice control, thinking-controlled technologies will be very helpful in teaching. We can even grade essay or test for students just by computer. This approach helps students get immediate feedback. Students know where they are wrong. That saves effort, time and money for both teachers and students. Li reported the importance of ICT in promoting student-centered education [20].

Chen and other authors describe information technology integrated into teaching such as integrating information technology into programs, content, and teaching so that science and technology become essential tools for teachers' teaching and student learning, that is, the application of science and technology becomes part of everyday life and sometimes see information technology as a way to approach or a program to find answers at any time or any location [21].

According to Wouter, discovery learning is considered a promising way of learning for many reasons, mainly because students actively participate in the field of learning leading to a better knowledge compared to traditional learning where knowledge is merely transmitted from teacher to student [22]. Of course, there is always a continuum from "pure discovery" to "pure interpretation" of learning style and in fact, both extremes will not exist in educational practice. Similar continuity can be found in a computer-aided learning environment, from a learning environment (explanation) giving students information to an open-ended environment such as using a computer or a website on discovery learning platform helps students find knowledge by themselves. This continuity is meaningful at the time when we want to support students to reap knowledge from the learning environment. In the storage and control system in a discovery learning environment, the question is, how can we support students to choose and use information? The change in control of the learning system forces us to think about different means of learning support than traditional learning. Therefore, we must think differently about the role and position of the smart system as an aspect of the learning environment.

Dikovic wrote, GeoGebra is a dynamic geometry software that supports the construction of shapes of points, lines and conic cross sections [1]. It also provides users with specific features for calculating algebraic systems such as finding key points of the function (solution, local extremes and inflection points of functions), directly entering functions and coordinates, find the derivative and integral of the function entered. That's why, GeoGebra is one of the best software when working with mathematical objects.

Ruthven and Hennessy stated that the basic idea of the GeoGebra interface is to provide two frameworks for working with mathematical objects in its Algebra and Graphics windows [23]. If you change an object of one of these windows, the remaining working frame will immediately be updated and changed accordingly. Algebraic calculation systems (such as Mathematica, Maple software and etc.) and dynamic geometry software (such as Geometer's Sketchpad, Cabri Geometry software, and so on) are powerful technology tools for teaching math. Many studies have shown that this packaged software can be used to encourage discovery, experimentation and visualization in traditional mathematics teaching. However, many studies also suggest that, for the majority of teachers, the main problem is how to provide the technology needed for technology integration with successful teaching.

According to Dikovic [1], the advantages of using GeoGebra software are:

- When compared with graphic calculators, GeoGebra software is more user-friendly, easy-to-use interface, multilingual menu, multiple commands and easy-to-manipulate help.
- GeoGebra software encourages students' mathematical projects, presentations, experiments, and discoveries.
- Students personalize their creativity by adapting to the interface (such as font size, language, graphics quality, colors, coordinates, thick strokes, line styles and other features).
- GeoGebra is designed to help students receive better mathematical knowledge. Students manipulate variables easily by dragging "free" objects on the drawing plane or using sliders. Students can make changes using techniques that work with free objects, and then students can learn how to get their dependent objects affected. In this way, students have the opportunity to solve problems by examining mathematical relationships using dynamic geometry.
- Cooperative teaching is the right context for mathematics courses. Lectures should be replaced by oriented interactive tasks in the classroom. The first role of teaching is not only to preach, explain or other efforts to "convey" mathematical knowledge but also create a situation for students to encourage their way of constructing spiritual values needed. In that sense, GeoGebra provides a good opportunity for collaborative teaching, that is, a good opportunity for the solution of collaborative problems in small groups, or whole-class teaching, or personal representation/student group.
- Algebra command input box allows users to create new objects or define existing objects with commands on the command input box. Worksheet files can be easily embedded into web pages.
- GeoGebra stimulates teachers to use and evaluate technology: visualizing mathematics, mathematical surveys, interacting with classrooms through websites or over distances away from classrooms, math and applications, etc.
III. FINDINGS AND RESULTS

A. Discovery Learning of Transformations on a Plane with the Help of GeoGebra Software

The discovery learning process of proven problems consists of the following steps: (1) Use the software GeoGebra to construct the objects. Discover ways to prove problems by finding a parallel, perpendicular, straight, concurrent relationship... (2) Solve the proven problem. (3) Draw the conclusion of the problem.

The discovery learning process of computational problems consists of the following steps: (1) Use the software GeoGebra to construct the objects. Discover the computational problem by measuring angle, length, area. (2) Solve the computational problem. (3) Draw the conclusion of the problem.

The discovery learning process of locus problems consists of the following steps: (1) Use the software GeoGebra to construct the objects. Moving the point to find the locus to different positions helps us "predict" the shape of the locus, and based on the visual symbol, determine the relationship between the shape of the locus and the given image. (2) Solve the locus problem. (3) Draw the conclusion of the problem.

The discovery learning of Geometry construction problems: (1) Use the software GeoGebra to construct the objects. Moving images to different positions helps us "predict" the picture solution and argue the possible cases. (2) Solve the problem of Geometry construction. (3) Draw the conclusion of the problem.

The discovery learning of problems of geometric extremes: (1) Use the software GeoGebra to construct the objects. Consider the changeable positions of the figure with a survey of the graph function of the variable quantity that needs to find extremes, thereby "predicting" the point of the extreme value thanks to its correspondence with the extreme of graph. (2) Solve the problem of geometric extremes. (3) Draw the conclusion of the problem.

An example of the discovery learning of the problem solving of transformations on a plane with the help of GeoGebra software:

Problem 1: The two villages are located in two places A and B, separated by one river (assuming the two banks of the river are two parallel lines) (Figure). It is planned to build a CD bridge across the river (the bridge must be perpendicular to the river bank) and make two straight roads from A to C and from B to D. Determine the location of the CD bridge so that AC+CD+DB is the shortest [24].

Teacher: We construct the Geometry shape on the electronic draft as follows:

Step 1. Geometry construction

- Draw the river as shown.
- Draw two points A, B apart from a river.
- Draw point C on the river.
- Draw a CD bridge perpendicular to the river.
- Connect AC, CD, DB together.
- Draw perpendicular coordinate system Oxy’ so that CD is perpendicular to Ox’.
- Draw Y point on Oy’ so that OY=AC+CD+DB.
- The lines perpendicular to Oy at Y and Ox at C meet at L.

![Fig. 1. The sum of distances is the shortest.](image)

Student: Can you prove the problem?

Teacher: Use the method to prove this problem.

Student: Suppose that, if the river is so narrow, so narrow that the banks of rivers a and b overlap. Moving point C, we find that the position of D is the intersection of river a and section AB (we know this is a familiar problem: CA+CB<sub>3</sub> AB+CA+CB. AB is the shortest when C is the intersection of a and line AB) . Thereby leading to using the translation $T_{CD}$ which allows "to go ahead" through the bridge and transfer to the above problem. Given $\nu = \overrightarrow{CD}$ (vector $\nu$ is perpendicular to the river bank and the length is equal to the width of the river). We have $T_{\nu}: A \rightarrow A', B \rightarrow B', C \rightarrow C, D \rightarrow D$. Therefore (AC+CD+DB) is the shortest when $U$ (A'D+ DB) is the shortest. $U$ A', D, B is in line.

How to draw C, D :

- Draw A' so that $AA' = \nu$.
- Draw D which is the intersection point of $A'B$ and $\nu$.
- Draw C so that $DC = -\nu$ and C,D are the positions to be searched.
Teacher: We have found a solution to a river problem. What if the problem for two points is separated by two river branches? Please give a math statement?

**Problem 2:** Two hamlets A and B are separated by two rivers. Find the CD bridging location on the river branch opposite point A and the EG bridging location on the river branch opposite point B so that the total distance from A to C to D to E to G and B is minimal. Suppose that the angle created by the two branches is acute [25,26].

Teacher: Can you use the method to prove problem 5 for this problem?

**Student:** Considering the translations:

\[ T_{CD}^{\text{sym}} : A \rightarrow A'. \]
\[ T_{GE}^{\text{sym}} : B \rightarrow B'. \]

Connect A'B' to cut the branches at D' and E' as shown.

![Fig. 2. Two hamlets.](image)

From D' and E' we deduce C' and G' by perpendicular construction.

We will prove that C'D' and E'G' are the locations to draw.

Indeed \( AC = AC' + C'D' + D'E' + E'G' + G'B = AA' + A'D' + D'E' + E'B' + BB' = AA' + A'B' + B'B \leq AA' + A'D + DE + EB' + B'B = 1 = AC + CD + DE + EG + GB. \)

Teacher: Consider a point as a point circle (the circle has a zero radius). We have a general problem for the circle to be stated

**Problem 3:** Given two circles with the centers, respectively \( O_1 \) and \( O_2 \) are on the same side as the line \( d \). Take turns on the circle with the center \( O_1 \) and the circle with the center \( O_2 \) of points A and B.A line CD slides over \( d \) of a constant length. Find the position of A,B,C,D so that the quadrilateral ABCD is the smallest.

Teacher: Please use the method of proof for problem 5 to prove this problem?

**Student:** If the perimeter of \( ABCD \) is the smallest, \( AD + BC \) must be the smallest.

![Fig. 3. The sum of distances is the shortest.](image)

Take the circular symmetry with center \( O_2 \) through \( d \), we get a circle with center \( O_1 \). Point B turns into \( B' \).

Consider translating: \( T_{DC}^{\text{sym}} : A \rightarrow A' \); \( (O_1) \text{®} (O_1) \)

We have \( AD = A'C \)

\( AD + CB = A'C + CB = A'C + CB' \geq A'B' \geq EF \) (\( EF \) are the intersections of \( O_1O_2 \) with circle with center \( O_1 \) and circle with center \( O_2 \)).

Call \( C' \) intersection of \( EF \) with \( d \).

Consider translating: \( T_{CD}^{\text{sym}} : E \rightarrow A_i \); \( C' \rightarrow D' \).

Through the axial symmetry of \( d \), we have: \( F a B_i \)

We have \( A_i, B_i, C', D' \) as the points to draw.

Indeed \( AD + CB \geq EF = EC' + C'F = A_iD' + C'B_i \) (Q.E.D)

### B. Experimental Pedagogy

The experiment was conducted in high schools in Thanh Hoa and Lam Dong provinces, Vietnam. The content is some lessons about the transformations on the plane of the geometry subject in the high school math program. We conduct pedagogical experiment a single round. Students are selected for experimentation and control in mass classes at high schools. In each school we randomly selected two classes: experimental group and control group. Experimental group is taught with the help of GeoGebra software and the control group is taught under normal conditions. The content of teaching is written according to the orientation of discovery learning and following the standards of knowledge and skills of Vietnam.

The experiment was conducted in the school year 2018-2019 in 03 schools: Sam Son, Trieu Son 5 and Duc Trong high schools to affirm the superiority and effectiveness of GeoGebra software in teaching. We conducted 4 experimental lessons: Introduction to transformations, translation and Isometry, axial symmetry (lesson 1), axial symmetry (lesson 2) [24]. For the experimental group, the students were assigned to go home
with self-study with the help of GeoGebra software, answering questions of the teacher assigned in the previous lesson. At school, students submit answers to teachers. If students want to ask questions about lessons, teachers will explain and answer them. Next, teachers implement all three forms of teaching organization: Organizing activities for the whole class, organizing into small groups and distributing personal questionnaires for each student. After that, the teacher assigns the task of self-studying at home to students in the next lesson. In the experimental teaching process, we watched a number of self-studying students with the help of GeoGebra software to have qualitative analysis.

We conduct student quality testing after each lesson to analyze and process data.

For control group, the lessons are still going on normally, teachers have applied some active teaching methods but this is the part of “Isometry and similarity” in the plane, so it is quite abstract and has many formulas, and students find it difficult to absorb and apply to the problem solving. Through observation, we find that many students do not focus on learning at home to study gradually increased through each experimental lesson.

We conduct training for teachers and students before conducting pedagogical experiment for nearly a month so that students have conditions to familiarize themselves with discovery learning with the help of GeoGebra software about transformations on the plane of grade 11. In the process of organizing for students to learn, we received many feedbacks from students and many results of students participating in discovery learning with the help of GeoGebra software and observing lessons of pedagogical experiment and monitoring the learning of a group of students also obtained positive results as follows:

1) Qualitative analysis: For experimental groups, we designed the teaching process towards mixed teaching. Teacher assigns students to self-study at home with the help of GeoGebra software, answering questions of teachers. In class time, some contents are proactively presented by students, some contents are given for group discussions and reporting results. In addition, teachers also organize common activities and distribute learning questionnaires in accordance with the capacity of each student.

For the first lesson “Introduction to transformations” and the first part of “Translation and Isometry”, teachers ask students to submit answers to questions that have been assigned to students at home before. Teachers conduct organizing teaching activities in the direction of discovery. Thanks to the activities of exploring, discovering, leading and guiding, students are interested and active in the process of learning new lessons. After teaching the lesson “Translation and Isometry”, teachers ask students to do exercises and inform students about the knowledge sections which students will have to learn with the help of GeoGebra software in the next two lessons of the chapter “Isometry and similarity in the plane”. Watching 4 lessons "Introduction to transformations", “Translation and Isometry”, "Axis symmetry (lesson 1)”, "Axis symmetry (lesson 2)”, we see at the pedagogical experiment round, students have actively participated in activities given by teachers. Students who learn with traditional methods often have difficulty in applying transformations into specific exercises. Because of the high level of knowledge, if studying with the traditional method, the time in class is almost exclusively for receiving new knowledge without time to practice. But because students are self-learning with GeoGebra software, they have more time to master the concept and apply the theory to the exercises. The percentage of students who actively completed self-study gradually increased through each experimental lesson.

For control group, the lessons are still going on normally, teachers have applied some active teaching methods but this is the part of “Isometry and similarity” in the plane, so it is quite abstract and has many formulas, and students find it difficult to absorb and apply to the problem solving. Through observation, we find that many students do not focus on lectures of teachers, some students do not answer the questions of teachers.

2) Quantitative analysis: For a month, we organized pedagogical experiments with direct teaching that combined instruction for students to learn the knowledge of the chapter of Isometry and similarity in the plane by doing exercises and tests. At the end of the pedagogical experiment, we conducted a test for both groups. The test results are marked and processed according to statistical theory.

In order to confirm the quality of the experiment, we conduct the processing of mathematical statistics, obtaining the following results:

The statistics are obtained as follows:

### TABLE I. NUMBER OF STUDENTS IN EXPERIMENTAL AND CONTROL CLASSES

| High Schools | Group   | Grade | Number of students | Total students |
|--------------|---------|-------|--------------------|----------------|
| Sam Son      | Experimental | 11A1  | 37                 | 75             |
|              |         | 11A2  | 38                 |                |
|              |         | 11A5  | 38                 | 77             |
|              |         | 11A6  | 39                 |                |
|              | Control | 11A1  | 37                 | 77             |
|              |         | 11A3  | 39                 |                |
| Trieu Son 5  | Experimental | 11A1  | 36                 | 75             |
|              |         | 11A2  | 39                 |                |
|              | Control | 11A3  | 39                 |                |
|              |         | 11A4  | 38                 | 76             |
| Duc Trong    | Experimental | 11A1  | 36                 |                |
|              |         | 11A2  | 39                 |                |
|              | Control | 11A3  | 39                 |                |

### TABLE II. THE PROCESSING OF MATHEMATICAL STATISTICS

| High School | Group   | GPA    | Variance | Standard deviation |
|-------------|---------|--------|----------|--------------------|
| Sam Son     | Experimental | 6.29   | 2.64     | 1.62               |
|             | Control  | 5.80   | 1.94     | 1.48               |
| Trieu Son 5 | Experimental | 6.41   | 1.80     | 1.35               |
|             | Control  | 5.88   | 1.50     | 1.29               |
| Duc Trong   | Experimental | 6.09   | 3.36     | 1.84               |
|             | Control  | 5.47   | 3.13     | 1.95               |

### TABLE III. THE STATISTICS

| High School | Degrees of freedom | Quantity | t | t cr | Compare t and t cr |
|-------------|--------------------|----------|---|-----|-------------------|
| Sam Son     | 75                 | 1.97     | 1.67 | t > t cr         |
| Trieu Son 5 | 37                 | 2.18     | 1.69 | t > t cr         |
| Duc Trong   | 75                 | 1.82     | 1.67 | t > t cr         |
Thus, the pedagogical experiment in all three schools is remarkably effective.

Testing hypothesis $E_0$:

| High School | Degrees of freedom | Quantity $F = \frac{S_E^2}{S_C^2}$ | $F_\alpha$ | Compare $F$ and $F_\alpha$ |
|-------------|--------------------|----------------------------------|-----------|---------------------------|
| Sam Son     | 75                 | 1.36                             | 1.53      | $F < F_\alpha$            |
| Trieu Son 5 | 37                 | 1.20                             | 1.84      | $F < F_\alpha$            |
| Duc Trong   | 75                 | 1.07                             | 1.53      | $F < F_\alpha$            |

All of the above three cases produce results that accept the hypothesis $E_0$, i.e. the difference between the variance in the experimental group and the control group at each school is insignificant.

Testing hypothesis $H_0$:

| High School | Degrees of freedom ($N_c - N_e$) | Quantity $t = \frac{x_i - x_j}{\sqrt{\frac{1}{N_i - 1} + \frac{1}{N_j - 1}}}$ | $t_{\alpha}$ | Compare $t$ and $t_{\alpha}$ |
|-------------|---------------------------------|-------------------------------------------------|-------------|-----------------------------|
| Sam Son     | 147                             | 1.98                                            | 1.66        | $t > t_{\alpha}$            |
| Trieu Son 5 | 69                              | 1.71                                            | 1.67        | $t > t_{\alpha}$            |
| Duc Trong   | 149                             | 2.14                                            | 1.66        | $t > t_{\alpha}$            |

The above results confirm that the statistical results reject the $H_0$ hypothesis. So the difference of the results between the experimental group and the control group in the three schools is significant. It shows that pedagogical experiment results in 3 schools of experimental group are higher than those of the control group.

IV. CONCLUSION

Discovery learning with the help of GeoGebra software has outstanding advantages that conventional traditional teaching cannot have such as stimulating exploration, active discovery of students, interacting with drawings to find the answer of the problem, verifying the problem, moving the shape, predicting the result of the problem, creating a new problem whose result can be solved immediately. Students in different regions and countries can join GeoGebra software as it is free software. You only need a computer with an Internet connection to use this software. Its usage is relatively simple, user-friendly interface, meeting the various choices of usage. Users can interact as formality software by entering commands or interacting like dynamic geometry software by drawing pictures or moving images. Thanks to outstanding advantages, when combined with discovery learning methods, the way of teaching becomes better and more effective.

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