An extension of the standard model with a single coupling parameter

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Abstract

We show that it is possible to find an extension of the matter content of the standard model with a unification of gauge and Yukawa couplings reproducing their known values. The perturbative renormalizability of the model with a single coupling and the requirement to accommodate the known properties of the standard model fix the masses and couplings of the additional particles. The implications on the parameters of the standard model are discussed.

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There are two main guidelines going beyond the standard model. The first one is based on consistency requirements which include as a crucial ingredient the incorporation of the gravitational interaction and it leads at the end to a candidate for the theory of everything (superstring theory, M-theory,...). The problem with this approach is that, at least at present, the understanding of the details how the standard model is contained in the fundamental theory is not good enough to make testable predictions at low energies. An alternative approach is to take the presence of a large number of free parameters in the standard model as the main motivation to go beyond it considering possible extensions with a smaller number of independent parameters. The main principle that has been used in the construction of extensions along this second line is an enlargement of the symmetries of the theory. The unification of gauge couplings in grand unified theories is not accompanied by a unification of the remaining couplings and the number of parameters is not substantially reduced. In the case of supersymmetric theories the situation is even worst because the unification of couplings is accompanied by the necessity to introduce additional free parameters describing the breaking of supersymmetry and one ends up with a theory with more free parameters than the standard model.

The unification of couplings through a symmetry is an example of relations between renormalized couplings which are independent of the renormalization scale and compatible with the renormalization group equations. The possibility to have a renormalizable theory (not necessarily based on a symmetry principle) with a reduced number of parameters (method of reduction of couplings) has been studied in recent years for different purposes. The program of reduction of couplings was initiated in \[2\] by looking for massless renormalizable theories in the power counting sense with a single dimensionless coupling parameter. The same idea can be applied in the case of effective field theories. Examples of phenomenological applications of the idea of reduction are the attempts to calculate quark masses within the framework of reduction of couplings in the standard model and more recently in supersymmetric grand unified theories as well as the unification of soft supersymmetry breaking parameters.

In this paper we consider the principle of reduction of couplings as an alternative to the symmetry principle in order to go beyond the standard

\[4\] For a recent review with a list of references see \[1\].
model. The conjecture is that the theory beyond the standard model is such that its low energy limit is described by a theory with a perturbative expansion in terms of a single independent coupling. All the couplings of the theory can be expressed as a power expansion in a single coupling with scale independent coefficients, valid at any scale above a mass $M$ identified as the reduction scale (reduction principle). From the point of view of selection of solutions of the renormalization group equations the reduction principle corresponds to a complete reduction of couplings, as referred in [4], but only above a scale $M$; it should be distinguished from the more general reduction of couplings [4] which correspond to solutions of the renormalization group equations with a number of constants smaller than the number of couplings. The reduction of couplings should also be distinguished from the relations between couplings which reflect the independence of the infrared renormalization group flow on the ultraviolet physics [7].

We show that it is necessary to go beyond the matter content of the standard model in order to have a perturbatively renormalizable theory with a single coupling and, assuming that there are no additional gauge interactions, the reduction principle fixes a minimal extension of the standard model.

The perturbative reduction of couplings can be systematically studied order by order in perturbation theory. The starting point is the one-loop renormalization group equations for the gauge couplings, Yukawa couplings and scalar self couplings of the standard model. The structure of these equations allows to determine first the reduction of each gauge coupling independently of the remaining parameters. Then one can consider the reduction of the Yukawa couplings from the corresponding renormalization group equations because they depend only on the gauge couplings (already reduced at the previous step) and the Yukawa couplings. Finally one can consider the renormalization group equations and the reduction for the scalar sector. This separation in three steps of the reduction of couplings can be repeated order by order in a perturbative expansion of the reduction based on a loop expansion of the renormalization group equations.

The one-loop renormalization group equations for the three gauge couplings $g_3, g_2, g_1$ of a theory with the local $SU(3) \times SU(2) \times U(1)$ symmetry of the standard model are

$$16 \pi^2 \mu \frac{d g_i}{d \mu} = b_i g_i^3$$

(1)
where the coefficients $b_i$ are fixed by the matter content of the model

$$b_i = -\frac{11}{3} S_1(G_i) + \frac{4}{3} S_3(F_i) + \frac{1}{6} S_3(S_i)$$ (2)

The constants $S_1(G_i)$, $S_3(F_i)$ and $S_3(S_i)$ are defined, in terms of the structure constants $C_i^{abc}$ and the generators $T_i^a$ of the group $G_i$, by the identities

$$C_i^{ac} C_i^{bcd} = S_1(G_i) \delta^{ab},$$ (3)

$$Tr_F(T_i^a T_i^b) = S_3(F_i) \delta^{ab},$$ (4)

$$Tr_S(T_i^a T_i^b) = S_3(S_i) \delta^{ab},$$ (5)

and $F_i(S_i)$ is the representation of the fermions (scalars).

Introducing $\alpha_i = g_i^2 / 4\pi$, the reduction of the three gauge couplings to a single coupling requires that

$$\frac{\alpha_2}{\alpha_1} = \frac{b_1}{b_2}$$

$$\frac{\alpha_3}{\alpha_1} = \frac{b_1}{b_3}.$$ (6)

This is only possible if the three coefficients $b_i$ have the same sign which is not the case in the standard model. Then in order to have a perturbative reduction of couplings it is necessary to include additional matter fields such that the sign of $b_2$ and $b_3$ becomes positive\footnote{A reduction of couplings based on a more general solution of the renormalization group equations is not consistent with the value of the top quark mass \footnote{4}}.

The minimal extension of the standard model that one can consider in order to avoid the obstruction to a reduction of gauge couplings is to add vector-like fermions in a non-trivial representation of $SU(3)$ and $SU(2)$. In this way one has an extension whose mass is independent of the breaking of the $SU(2) \times U(1)$ symmetry and then can be made naturally larger than the Fermi scale (which is compatible with the absence of any signal of this extension). Another consequence of the vector-like character of the extension is that no additional couplings are introduced with the extension of the fermionic field content.

The perturbative reduction of gauge couplings leads to a constant value for the two ratios of gauge couplings at scales larger than the mass $M$ of the

\footnote{A reduction of couplings based on a more general solution of the renormalization group equations is not consistent with the value of the top quark mass \footnote{4}}.
vector-like fermions. These ratios are given in terms of the first coefficients of the $\beta$-functions by (1). At scales lower than $M$ the additional fermions decouple and the gauge couplings evolve according to the renormalization group equations of the standard model. The first nontrivial phenomenological test of the validity of the perturbative reduction of couplings is to find a representation for the fermionic extension compatible with the known values of the gauge couplings. In fig.1 the values of the ratios $\alpha_2/\alpha_1$ and $\alpha_3/\alpha_1$ are plotted for (different multiplicities of) several representations of $SU(3) \times SU(2) \times U(1)$. We have considered the lower dimensional representations of $SU(2)$ and $SU(3)$ up to the adjoint representation and integer values of the $U(1)$-hypercharge $Y = 0, 1, 2$. Also represented in the figure is the curve of values of these ratios in the standard model as a function of $\mu$ (curve starting on the right at $\mu = M_Z$). Ratios of gauge couplings for a reduction of couplings based on an extension of the standard model with additional fermions in a representation of $SU(3) \times SU(2) \times U(1)$ with a given multiplicity (points). $SU(5)$ unification (cross).
the renormalization scale $\mu$, for $M_Z \leq \mu \leq M_{Pl}$. There are points on the curve, which means that the values of the gauge couplings at low energies are compatible with the reduction of couplings, and all of them correspond to different multiplicities $2 \leq N \leq 10$ of a unique representation, $(8,3)$, of $SU(3) \times SU(2)$ with $Y = 2$. The different points on the curve lead to different values of the scale $\mu = M$ of the fermionic extension increasing with the multiplicity. For $N = 3$ one has a vector-like representation for each generation and the mass of the additional fermions is $M \approx 70$ Tev. This result for the unification of gauge couplings through a perturbative reduction can be compared with the $SU(5)$-unification which corresponds to the point $(1,1)$, close to the standard model curve at a much higher scale.

Once a phenomenologically valid reduction of gauge couplings has been identified one has to consider the renormalization group equations for the Yukawa couplings. A vector-like fermionic extension does not couple directly to the scalar field and then it does not introduce any modification at the one loop level on the renormalization group equations for the Yukawa couplings $y_f$ of the standard model [8]

\[ 16\pi^2 \mu \frac{dy_{u_i}}{d\mu} = \left[ \frac{3}{2} y_{u_i}^2 - \frac{3}{2} y_{d_i}^2 + \sum_j \left( y_{f_j}^2 + 3y_{u_j}^2 + 3y_{d_j}^2 \right) \right] \left( -\frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right), \tag{7} \]

\[ 16\pi^2 \mu \frac{dy_{d_i}}{d\mu} = \left[ \frac{3}{2} y_{d_i}^2 - \frac{3}{2} y_{u_i}^2 + \sum_j \left( y_{f_j}^2 + 3y_{u_j}^2 + 3y_{d_j}^2 \right) \right] \left( -\frac{1}{4} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 \right), \tag{8} \]

\[ 16\pi^2 \mu \frac{dy_{l_i}}{d\mu} = \left[ \frac{3}{2} y_{l_i}^2 + \sum_j \left( y_{f_j}^2 + 3y_{u_j}^2 + 3y_{d_j}^2 \right) \right] \left( -\frac{9}{4} g_1^2 - \frac{9}{4} g_2^2 \right). \tag{9} \]
where $u_i, d_i, l_i$ denote the quarks of charge $2/3, -1/3$ and charged lepton of each generation and the mixing between generations has been neglected.

Once more the reduction to a single coupling at the one loop level requires the ratios $C_{u_i} = y_{u_i}^2 / g_1^2, C_{d_i} = y_{d_i}^2 / g_1^2, C_{l_i} = y_{l_i}^2 / g_1^2$ to be independent of the renormalization scale. The consistency with the renormalization group equations leads to the reduction equations

\[ C_{u_i} \left[ \frac{3}{2} C_{u_i} - \frac{3}{2} C_{d_i} + \sum_j \left( C_{l_j} + 3C_{u_j} + 3C_{d_j} \right) \right. \]

\[ \left. - \left( \frac{17}{20} + b_1 + \frac{9}{4} b_1 \frac{b_2}{b_3} \right) \right] = 0, \tag{10} \]

\[ C_{d_i} \left[ \frac{3}{2} C_{d_i} - \frac{3}{2} C_{u_i} + \sum_j \left( C_{l_j} + 3C_{u_j} + 3C_{d_j} \right) \right. \]

\[ \left. - \left( \frac{1}{4} + b_1 + \frac{9}{4} b_1 \frac{b_2}{b_3} \right) \right] = 0, \tag{11} \]

\[ C_{l_i} \left[ \frac{3}{2} C_{l_i} + \sum_j \left( C_{l_j} + 3C_{u_j} + 3C_{d_j} \right) \right. \]

\[ \left. - \left( \frac{9}{4} + b_1 + \frac{9}{4} b_1 \frac{b_2}{b_3} \right) \right] = 0. \tag{12} \]

The only solution which is compatible with the hierarchy of masses in the standard model corresponds to take all the Yukawa couplings except $y_l$ equal to zero. The lowest order approximation for the mass, $m_t^2 = y_t^2 v^2 / 2$ leads to $y_t(\mu = m_t) \approx 1$ and, using the renormalization group equation of the top quark Yukawa coupling and the values of the gauge couplings at $\mu = M_Z$, one has that $16\pi^2 \mu \frac{dy_t}{d\mu} < 0$ over an energy range including the scale $M$ identified in the reduction of gauge couplings. On the other hand the reduction to a single coupling implies that, for scales $\mu \geq M$, the squared top quark Yukawa coupling should grow with the renormalization scale $\mu$ as the gauge couplings do.
Then in order to accomodate the value of the top quark mass one has to consider an extension of the standard model which changes the one loop renormalization group equation of \( y_t \) or an extension where the relation between \( y_t \) and \( m_t \) of the standard model is modified. In the first case one has to consider additional fermions with a direct coupling to the scalar doublet, i.e., additional chiral fermions. The simplest possibility is to consider a new generation which gives an additional contribution of the required sign to the \( \beta \)-function of \( y_t \) through the renormalization of the scalar field. But in this case one has a new system of reduction equations for the Yukawa couplings of the new generation. The equations (10), (11) for the quarks of charge \( \frac{2}{3} \), \(-\frac{1}{3}\) of the forth generation will now include a term proportional to \( y_t^2 \) from the contribution of the neutral lepton to the renormalization of the scalar field and the reduction equations for the Yukawa couplings of the leptons will be

\[
C_{l_i} \left[ \frac{3}{2} C_{l_i} - \frac{3}{2} C_{n_i} + \sum_j \left( C_{l_j} + C_{n_j} + 3C_{u_j} + 3C_{d_j} \right) \right]
- \left( \frac{9}{4} + b_1 + \frac{9 b_1}{4 b_2} \right) = 0 , \quad (13)
\]

\[
C_{n_i} \left[ \frac{3}{2} C_{n_i} - \frac{3}{2} C_{l_i} + \sum_j \left( C_{l_j} + C_{n_j} + 3C_{u_j} + 3C_{d_j} \right) \right]
- \left( \frac{9}{20} + b_1 + \frac{9 b_1}{4 b_2} \right) = 0 . \quad (14)
\]

The contribution due to the renormalization of the scalar field is the same for all the Yukawa couplings and, since quarks and leptons are in the same representation of \( SU(2) \), the coefficients of the terms due to the scalar-fermion vertex corrections are also the same. These two facts lead to the absence of any solution to the reduction equations with all the Yukawa couplings of one generation different from zero as it is required phenomenologically.

The other alternative to make compatible the value of the top quark mass with the reduction of couplings is to modify the scalar sector. The simplest possibility is to consider a two Higgs doublet model [9]. One still has the
electroweak \( \rho \) parameter equal to one in the Born approximation and flavour-changing neutral currents are avoided if all fermions with the same electric charge couple to the same Higgs doublet \([10]\). The renormalization group equations for the Yukawa couplings are in this case given by

\[
16\pi^2 \mu \frac{dy_{ui}}{d\mu} = y_{ui} \left[ \frac{3}{2} y_{ui}^2 + \frac{1}{2} y_{d_i}^2 + \sum_j \left( y_{nu_j}^2 + 3y_{u_j}^2 \right) \right] - \frac{17}{20} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 ,
\]

(15)

\[
16\pi^2 \mu \frac{dy_{di}}{d\mu} = y_{di} \left[ \frac{3}{2} y_{di}^2 + \frac{1}{2} y_{u_i}^2 + \sum_j \left( y_{nu_j}^2 + 3y_{d_j}^2 \right) \right] - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 ,
\]

(16)

\[
16\pi^2 \mu \frac{dy_{li}}{d\mu} = y_{li} \left[ \frac{3}{2} y_{li}^2 + \frac{1}{2} y_{n_i}^2 + \sum_j \left( y_{lu_j}^2 + 3y_{d_j}^2 \right) \right] - \frac{9}{4} g_1^2 - \frac{9}{4} g_2^2 ,
\]

(17)

\[
16\pi^2 \mu \frac{dy_{ni}}{d\mu} = y_{ni} \left[ \frac{3}{2} y_{ni}^2 + \frac{1}{2} y_{l_i}^2 + \sum_j \left( y_{nu_j}^2 + 3y_{u_j}^2 \right) \right] - \frac{9}{20} g_1^2 - \frac{9}{4} g_2^2 ,
\]

(18)

where we have included a right-handed neutrino and a Yukawa coupling for the neutrino. The result for the chiral fermion content of the minimal standard model is recovered by considering three generations, \( i = 1, 2, 3 \) and making \( y_{ni} = 0 \). Once more the hierarchy of fermion masses requires to consider a solution with all Yukawa couplings except \( y_t \) equal to zero. The consistency of the reduction of couplings fixes the ratio \( C_i = y_t^2 / g_1^2 \).
\[ C_t = \frac{17}{90} + \frac{2}{9} b_1 + \frac{1}{2} b_2 + \frac{16}{9} b_3, \tag{19} \]

for all the scales above the scale \( M \) of the reduction of couplings which we have already identified at the level of gauge couplings. Using the values of the first \( \beta \)-function coefficients \( b_1, b_2 \) and \( b_3 \) with one vector-like representation \((8,3)_2\) for each generation and the renormalization group equation of the standard model for \( y_t \) for \( \mu < M \) one finds \( y_t(m_t) = 1.86 \). In order to reproduce the top quark mass it is necessary to have

\[ y_t^2(m_t) \frac{v_1^2}{v_1^2 + v_2^2} \approx 1, \tag{20} \]

which in this case gives a condition for the v.e.v. of the two doublets, \( v_1 \) and \( v_2 \), fixing the ratio \( \tan \beta = \frac{v_1}{v_2} \approx 0.62 \). Then it is possible to accommodate the fermion masses (eleven massless fermions and a massive top quark) in an extension of the standard model with a single coupling parameter, based on the addition of a second Higgs doublet and a vector-like fermion representation \((8,3)_2\) of \( SU(3) \times SU(2) \times U(1) \) for each generation with a mass \( M \approx 70 \text{ TeV} \).

It is possible but not required to go beyond this minimal extension. For example one can consider additional generations which now due to the presence of a second Higgs doublet are compatible with a reduction of couplings. As an example of the predictive power of the reduction principle the masses of all the fermions of additional generations are fixed and turn out to be marginally compatible with present experiments due to the small values obtained for the mass of the neutrino of the additional generations. Another example corresponds to an extension of the standard model with only \( n \) generations of chiral fermions \( (n > 6 \text{ in order to have a reduction of gauge couplings}) \) and two Higgs doublets. It can be excluded because the values of the gauge couplings at \( \mu = M_Z \) are not consistent with the reduction of couplings. These examples make manifest the difficulty to find an extension of the standard model compatible with the reduction of couplings.

The last step in the renormalization group equation analysis leads us to the one-loop equations for the scalar sector. Here one has seven dimensionless couplings and three parameters corresponding to the quadratic terms which appear in the more general potential with two Higgs doublets. The one loop
$\beta$-functions for the couplings fix their reduction in terms of $g_i$ from the consistency of the relations $\lambda_i = C_i g_1^2$, $i = 1, \ldots, 7$, with the renormalization group equations leading to a set of algebraic reduction equations for the constant coefficients $C_i$. Two combinations of the three parameters corresponding to the quadratic terms are fixed by the required values of the two v.e.v. and as a consequence of the reduction to a single coupling it will be possible to determine the four masses and the mixing angle which characterize a two Higgs doublet model in terms of one free parameter. A detailed discussion of the predictions for the scalar sector together with a study of the possibility to go further in the reduction principle considering a reduction of the three mass parameters in the potential to a single one will be presented in a future work.

To summarize we have presented the reduction principle as an alternative to an enlargement of symmetries as a way to try to go beyond the standard model. The assumption behind the reduction of couplings is that the theory beyond the standard model is such that its low energy limit is as independent on the details of the theory as possible. Since the details of the theory appear at the level of the low energy limit through the values of the renormalized parameters it is the number of independent couplings which have to be minimized. The arbitrariness of the extension based on the symmetry principle which appears through the choice of symmetry group and representation of matter fields has an analog in the case of reduction of couplings in the choice of the matter field content of the extension once it is assumed that there are no additional gauge interactions. The mass scale of the extension, identified through the reduction to a single coupling, is many orders of magnitude lower than the scale of grand unified theories and there is no desert. As for the parameters of the standard model associated to the fermion masses (CKM matrix and quark and lepton masses) they are in principle determined by the reduction. At the one loop level only one fermion has a non vanishing mass and there is no CKM matrix but the reduction of couplings can be systematically extended order by order in perturbation theory and the remaining masses and mixing of generations could be a consequence of the reduction of couplings beyond one loop. This possibility as well as the extension of the reduction of couplings to the effective theory including terms of dimension greater than four in the Lagrangian which can also be done along the same lines deserve further investigation.
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