A new look for good old parton dynamics

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A short review is given of the idea and of the present status of recently proposed evolution equations that respect the Gribov/Lipatov reciprocity between space-like and time-like parton dynamics in all orders.

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I. GRIBOV (LIPATOV RECIPROCITY)

Evolution equations describing dynamics of distributions of QCD partons (quarks and gluons) that determine cross sections of hard processes have a long history.

Logarithmic violation of the Bjorken scaling — dependence of DIS structure functions on the hard scale of the process — and, simultaneously, scale dependence of fragmentation functions in e+e- annihilation, were first addressed in a general QFT context by Gribov and Lipatov back in 1971 [1].

These fundamental results were explicitly presented in the language of evolution of a system of partons by Lipatov in 1974 [2]. QCD parton evolution equations followed suit in 1977 [3,4].

The universal nature of the parton dynamics goes under the name of factorisation of collinear (in assm) singularities.

Physically, it is due to the fact that quark (gluon multiplication processes happen at much larger space (time) distances than the hard interaction itself. It is this separation that makes it possible to describe quark (gluon cascades in terms of independent parton splitting processes. They succeed one another in a cleverly chosen evolution time, whose o.w. "counts" basic parton splittings that occur at well separated, strongly ordered, space (time) scales.

Perturbative structure of the cross section of a given process p characterised by the hardness scale Q^2 can be cast, symbolically, as a product (convolution) of three factors (for a review see [5]):

\[ \left( p \right) t = C \left( p \right) s \left( t \right) \exp \int_{t_0}^{t} P \left( s \left( \right) \right) \]

Here the functions C \left( s \right) (hard cross section; coefficient function) and P \left( s \right) (parton evolution; anomalous dimension matrix) are perturbative objects analysed in terms of the \[ s \] -expansion. The last factor \[ w_b \left( t_0 \right) \] embeds non-perturbative information about parton structure of the participating hadrons (s) \[ h \], be it a target hadron in the initial state (parton distribution) or a hadron triggered in the final state (fragmentation function).

Gribov and Lipatov found the splitting functions \[ P_{AB} \left( x \right) \] determining the evolution of parton distributions to be identical for space-like (DIS) and time-like (e+e- annihilation) cases:

\[ P_{AB}^{(S)} \left( x_{B j} \right) = P_{AB}^{(T)} \left( x_{Feynman} \right); \]

\[ x_B = \frac{Q^2}{2(pq)}; \]

\[ x_F = \frac{2(pq)}{Q^2}; \]

The identity of evolution Hamiltonians in the two channels is a highly non-trivial property since the energy fraction argument of the splitting functions depend on [3] are not the same but rather reciprocal.

Combined together with the Drell-Leyv-Yan relation [6] (which followed from similarity of Feynman diagram for the two crossing channels), the identity [2] translated into an internal symmetry relation for the splitting functions,

\[ P_{AB} \left( x \right) = \left( x \right) P_{AB} \left( x^{-1} \right); \]

known as the Gribov/Lipatov reciprocity (with the factor depending on the spins of participating partons A, B; for a review see, e.g. [4]).
However, the Gribov-Lipatov reciprocity (GLR) was found to be broken beyond the leading logarithm in approximation (the one-loop parton Hamiltonian).

Having observed the difference between space- and time-like two-loop splitting functions, Curci, Fumanski, and Ponzio [3] have remarked that the violation of the GLR might be of kinematical origin.

II. RECIPROCITY RESPECTING PARTON EVOLUTION EQUATION

A few years ago the following evolution equation has been suggested in [3,10]:

$$\frac{\partial D (x; Q^2)}{\partial \ln Q^2} = \frac{Z}{z} P [z; s] D \frac{x}{z} Q^2$$

(5)

where $x = 1(+1)$ for the space-like (time-like) case. Appearance of the combination $z Q^2$ in the hardness argument of the parton distribution under integral corresponds to choosing the evolution time (or evolution) in the universal (channel independent) reciprocity.

Such a choice of an evolution variable is known to be "wrong" for either of the two channels.

Indeed, it is the transverse momentum ordering, $k_t^2$ in the S channel, and the angular ordering, $(k_t^2 = z)^2$, for the T-evolution, correspondingly, that make anomalous dimensions free of series of double logarithmically enhanced terms $s \ln^2 x)$. Such terms become large in the small-x region which region is practically important both for DIS (high energy scattering) and for jet physics (soft gluon multiplication).

At the same time, the evolution time ordering variable, $k_t^2$, happens to be lying right in between the two "better choices" and thus preserves the symmetry between the two channels. Therefore, one may hope that formulating parton evolution in terms of equation (5) will result in the universal (channel independent) reciprocity respecting evolution kernel $P(x)$.

Non-locality of the new equation (5) in longitudinal ($z$) and transverse variables (hardness scale $Q^2$) breaks identification of splitting functions with anomalous dimensions. What it offers instead is a link between the two channels by means of universal evolution kernel matrix $P$, one and the same for $T$ and $S$ evolution. In spite of the fact that the new "splitting functions" $P$ do not correspond to any clever choice of the evolution variable, in either $T$ or $S$ channel (explosive $s \ln^2 x$ terms being present in both cases), this universality can be exploited for relating DIS and e$^+$e$^-$ anomalous dimensions.

One can expect that by separating the notions of splitting functions and anomalous dimensions by means of the Reciprocity Respecting Evolution Equation (5) the Gribov-Lipatov wisdom can be rescued in all orders.

This guess was motivated by a remark made by Curci, Fumanski, and Ponzio [3] who observed that the GLR violation in the second loop non-singlet quark anomalous dimensions amount to a "quasi-Abelian" term $/ C_F^2 \times$ with a suggestive structure

$$\int \frac{1}{z} P(x) \frac{dz}{z} = \frac{Z}{z} P_{zz}^{(1)}(x) + \frac{Z}{z} P_{zz}^{(2)}(x)$$

(6)

Their observation hinted that the GLR violation was not a dynamical higher order effect but was inherited from the previous loop via a non-linear relation [3].

In the Mellin space the convolution (6) translates into

$$P_N \frac{d}{dN} P_N = P_N P_N$$

Let us check that it is this structure of the GLR breaking that emerges from (5).

Taking Mellin moments of both sides of the equation we obtain

$$(N, D_N (Q^2)) = \frac{Z}{z} \int_0^1 dz \, D_N (Q^2) \theta_{\ln Q^2} D_N (Q^2);$$

where we have used the Taylor expansion trick. The integral formally equals

$$\int (N) = (D_N) \frac{1}{2} P (N + \theta_{\ln Q^2}) D_N$$

expressing the anomalous dimension through the Mellin moments of the evolution kernel with the differential operator for the argument:

$$N \neq N + \theta_{\ln Q^2}.$$
The derivative acts upon $D_N (Q^2)$ producing, by definition, $(N)D_N$. In high orders it will also act on the running coupling the anomalous dimension depends on, $\gamma = (N; s)$. The latter action gives rise to terms proportional to the function. Such terms are scheme dependent as they can be rephrased between the anomalous dimension and the coefficient function $C [ s]$ in the expression (1).

Neglecting for the time being such contributions by treating $s$ as constant, (7) reduces to a functional equation

$$ (N) = P (N + (N)) :$$  \( (8) \)

Since $= O (s)$, we can expand the argument of the evolution kernel perturbatively,

$$ = P + P - \frac{1}{2} P^2 + O (s) + O (s^4): \quad (9a)$$

Solving (9a) iteratively we get

$$ = P + P^2 + P^2 + \frac{1}{2} P^2 P + \cdots : \quad (9b)$$

Restricting ourselves to the first loop, $P = P^{(1)}$, with $P^{(1)}$ the Mellin image of) good old LLA functions, gives

$$ = P^{(1)} + 2 P^{(1)} + P^{(1)} + \cdots : \quad (10)$$

The second term on the r.h.s. of (10) generates the two-loop Cuizi(Fumanchi,Petronzio) relation [5] all right.

Knowing the n-loop anomalous dimension matrix in the S channel, the RREE predicts the anomalous dimensions in the T channel (and vice versa).

Based on the existing three-loop results for the space-like evolution [11], the corresponding prediction of (5) for the time-like channel was verified in the case of non-singlet anomalous dimensions by Mitov,Moch and Vermaseren in [12].

Basso and Korchemsky have traced the origin of the relation (5) to the underlying conformal properties of the theory and described how to embed into the equation of non-zero function [13].

Validity of the Gribov(Lipatov) reciprocity for the kernel $P$ of the new evolution equation (5) has been checked not only for three-loop non-singlet QCD anomalous dimensions.

Basso and Korchemsky (in collaboration with Moch) have revisited virtually all known multi-loop QFT results and found GLR to hold for three-loop unpolarized singlet and two-loop polarized QCD distributions (quark transversity, linearly polarized gluon, quark singlet polarized), QFT at four loops, QCD in the $\sqrt{s} = 4$ YM theory. In the latter model the GLR was found to hold even in the strong coupling limit, $\sqrt{s} \rightarrow \infty$ (accessible through the AdS/CFT correspondence).

III. QUASI-ELASTIC LIMIT

The RREE's first demonstration of force was the derivation of all-order predictions for the structure of subleading singular term in the expansion of the quark non-singlet and gluon anomalous dimensions in the large-$x$ limit [14].

Behaviour of anomalous dimensions in the quasi-elastic ($x \approx 1$) kinematics can be cast in the following form:

$$ (x) = \frac{A x}{(1 - x)_+} + B (1 - x) + C \ln(1 - x) + D + O ((1 - x)\log(1 - x)) ; \quad (11)$$

where the coefficients $A$ and $B$ are the same in the two channels.

Specific structure of the first term is dictated by the celebrated Low (Burnett, Kroll, LBK) theorem [14]. It is a consequence of the fact that soft radiation
at the level of \( d / d! \) (! \( 1 + \text{const} \)) has classical nature.

The coefficient \( A \) in front of this structure has a meaning of the "physical coupling" as measured by the intensity of relatively soft gluon emission. This coefficient (calculated in three loops in the MS-bar scheme) is known to universally appear in all observables sensitive to soft gluon radiation: quark and gluon Sudakov form factors and Regge trajectories, threshold resummations, singular part of the \( D \)rellis factor, distributions of jet event shapes in the near-to-two-jet kinematics, heavy quark fragmentation functions, etc. The structure (11) applies to the large-\( x \) behaviour of the \( g \! / \! g \) anomalous dimension as well, with \( A_{gA} = A_{gq} = C_A = C_F, \) in all orders.

Quantum effects show up only at the level of \( d / d! \) ! that is at the level of contributions that were neglected in (11). At one loop, subleading term \( SC \ln(1 - x) \) and the constant \( D \) in (11) are absent. This suggests that in higher loops they should emerge as "inherited" rather than non-trivial entries.

Indeed, to keep under control all the terms in (11) it suffices to use the \( x \! / \! 1 \) asymptote of the one-loop evolution kernel \( P^{(1)} = P^{(1)} \) to derive from RREE (9) all-order relations [10]

\[
C = A^2; \quad (12a)
\]

\[
D = AB; \quad (12b)
\]

The relation (12a) is "conformal" while (12b) acquires correction due to running of the coupling (13).

The ideas of the universal evolution equation have recently found an interesting application in the work by Laenen, Magnea and Stavenga who have employed the RREE as means of in proving threshold resummations [13].

IV. \( N = 4 \) SYM

QCD shares the gluon sector with supersymmetric Yang-Mills models (SYM). This suggests to explore supersymmetric partners of QCD in order to shed light on the subtle structure of the perturbative quark-gluon dynamics.

QCD is not an integrable quantum field theory. In spite of this, in certain sectors of the chromodynamics the integrability does emerge [16]. This happens, markedly, in the problem of high energy Regge behaviour of scattering amplitudes in the large-\( N_c \) approximation (planar *t* Hooft limit), in the spin 3/2 baryon wave function, for the scale dependence of trace \( \langle \text{tr} \rangle \) (maximal helicity) quasi-partonic operators (for review see [17]). What all these problems have in common, is the irrelevance of quark degrees of freedom and the dominance of the classical part of gluon dynamics, in the sense of the LBK theorem [14].

The higher the symmetry, the deeper integrability. The maximally supersymmetric \( N = 4 \) SYM theory is exceptional in this respect. A string of recent theoretical developments [17, 18, 19] hinted at an intriguing possibility that this QFT, super-conformally invariant at the quantum level (\( x \! / \! 0 \)), may admit an all loop solution for anomalous dimensions of its composite operators.

The \( N = 4 \) SYM being an integrable model, there exists a powerful technology based on the Bethe Ansatz Equations well suited for perturbative calculation of anomalous dimensions of composite operators to multi-loop accuracy. The so-called "universal anomalous dimension" of the \( N = 4 \) SYM theory is given by the "maximal transcendentality" Euler-Zagier harmonic sum s [20, 21].

Applied to this theory, the kernel of (9) was found to respect GLR in four loops for the leading twist two [22], as well as in four [23] and five loops [24] for twist three operators.

The RREE was found to significantly simplify the structure of high order terms in the "universal anomalous dimension" of the theory [24].

Given that in the leading order evolution kernel of the \( N = 4 \) SYM is purely classical in the LBK sense,

\[
P^{(1)}(x) = \frac{x}{1 - x} + \text{no quantum corrections}; \quad (13)
\]

one may hope to derive one day a one-line-all-loops expression for the anomalous dimension of this theory, in which higher order terms are dynamically "inherited" from the rst loop.

QCD would greatly benefit from such a solution, since this \( N = 4 \) SYM thing would put under full theoretical control the domain part of the perturbative QCD gluon dynamics.
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