The effect of Aharanov-Bohm phase on the magnetic-field dependence of two-pulse echos in glasses at low temperatures

A. Akbari$^1$ and A. Langari$^{1,2}$

$^1$Institute for Advanced Studies in Basic Sciences, P. O. Box 45195-1159, Zanjan, Iran
$^2$Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Str. 38, 01187 Dresden, Germany

(Dated: November 17, 2018)

The anomalous response of glasses in the echo amplitude experiment is explained in the presence of a magnetic field. We have considered the low energy excitations in terms of an effective two level system. The effective model is constructed on the flip-flop configuration of two interacting two level systems. The magnetic field affects the tunneling amplitude through the Aharanov-Bohm effect. The effective model has a lower scale of energy in addition to the new distribution of tunneling parameters which depend on the interaction. We are able to explain some features of echo amplitude versus a magnetic field, namely, the dephasing effect at low magnetic fields, dependence on the strength of the electric field, pulse separation effect and the influence of temperature. However this model fails to explain the isotope effects which essentially can be explained by the nuclear quadrupole moment.

We will finally discuss the features of our results.

PACS numbers: 61.43.Fs 64.90.+b 77.22.Ch 72.20.Ht

I. INTRODUCTION

Unusual behavior of the heat capacity and thermal conductivity of glasses at very low temperatures can be governed by a broad distribution of low-energy excitations. These could be described as two level tunneling systems whose energies and tunneling amplitudes are randomly distributed. The origin of these excitations is assumed to come from atoms or group of atoms that can be arranged in two energetically close configurations. Such a configuration can be a cluster of few hundred atoms or molecules. Although the nature of the tunneling entities is obscure, many theories are based on such a tunneling model to describe the low temperature properties of glasses. A two level system (TLS) is characterized by two parameters of a double well potential, the asymmetry energy $\Delta$, and the tunneling amplitude $\Delta_0$. The Hamiltonian of a single TLS in the pseudo-spin operators is given by

$$ H_0 = \frac{1}{2}(\Delta \sigma_z + \Delta_0 \sigma_x). $$

As a consequence of the irregular structure of glasses, these two parameters are widely distributed and are independent of each other with a uniform distribution of

$$ f^{(1)}(\Delta, \Delta_0) = \frac{f_0}{\Delta_0}, $$

where $f_0$ is a normalization factor. While many experiments can be explained satisfactorily by this model it fails to explain some results at very low temperatures, particularly in the presence of a magnetic field.

It was a general belief that the dielectric properties of nonmagnetic insulating glasses are independent of the magnetic field. Some experiments during the last few years opened the question of magnetic field dependence. The dielectric constant of several multicomponent glasses like $BaO-Al_2O_3-SiO_2$ shows a surprising dependence on a weak magnetic field ($\sim 20 \mu T$) at very low temperatures ($\sim 2 mK$). Increasing the magnetic field the dependence shows an oscillatory behavior with a period of a few Teslas. Moreover the amount of magnetic impurities present in the nonmagnetic glasses seems to be irrelevant and the observed magnetic field effect is strongly influenced by the applied electric field. On the other hand, recent polarization echo experiments show a strong and nonmonotonic dependence of echo amplitude on the applied magnetic field. These experiments give us an insight into the coupling of the tunneling centers to the magnetic field. Namely, the echo amplitude is almost independent of the frequency of the electric field which means that tunneling systems with very different energy splittings behave the same way. Moreover the effect is strongly dependent on the delay time between two pulses and also on the magnitude of applied electric field. A surprising outcome of these experiments is a novel isotope effect observed in different glasses. The recent results show the important influence of nuclear quadrupole moments on the observed magnetic field dependence.

The first theoretical attempt to explain the magnetic field dependence was based on the Aharanov-Bohm phase. In this approach the dielectric permittivity is given by a generalized TLS where the magnetic field enters in the quantum mechanical phase of a tunneling charged particle in a Mexican hat potential. It was assumed that such a generalized TLS is the result of collective effects between many tunneling centers in a mesoscopic size which carry a big amount of charge. The dipole-dipole interaction between tunneling centers is responsible for the mentioned collective (coherent) motion in a weakly interacting regime. An evidence for the Aharanov-Bohm effect can be found in the metallic glasses. Another approach which is also based on the Aharanov-Bohm phase explains the phenomena of dielectric response in terms of coupled pairs of tunneling
centers.[15] The coupling of two TLSs makes the tunneling path curved and possessing a flux dependent phase. The most recent theory is based on the nuclear quadrupole moment of tunneling entities.[17,18] In this model the interaction of nuclear quadrupole moment with the gradient of the electric field defines the new scale of energy at small magnetic fields. The different orientations of the nuclear quadrupole moment in each well lead to different level splittings which affect the dephasing mechanism and finally the echo amplitude. This model explains the echo experiments well, however, it fails to get the right order of magnitude for the magnetic field effect of dielectric response.[19] It is claimed that the observed effect in dielectric response is not a static phenomenon but of dynamical origin.[20]

The shortcoming of the isolated TLS model to explain physical phenomena at low temperatures is usually related to the interaction between tunneling centers.[21,22] Especially, interaction plays an essential role in determining the relaxation rates at low temperatures,[23] which influences the echo experiment response. In this most recent work it has been shown how the dipole-dipole interaction can lead to a relaxation rate faster than phononic counterpart at very low temperatures. The main idea is based on the delocalization process of the effective TLSs which are the result of two interacting tunneling centers.[24,25] This is in agreement with the experimental results on the magnetic field dependence of the dielectric constant and the effective model proposed in Ref.[26]. Here, the magnetic field dependence of the dielectric constant is related to the field dependence of the tunneling amplitude cutoff which is based on the theory developed earlier.[27] In this theory the localization length increases by adding the magnetic field which favors the delocalization phenomenon.

Accepting the fact that interaction plays an essential role in the low temperature physics of glasses we are going to explain the recent polarization echo experiments in the presence of a magnetic field. Our approach is based on the main idea presented in Refs.[13,23,24]. The interaction between two tunneling centers is defined in terms of flip-flop states which are considered as a new (effective) TLS. The magnetic field adds the Aharanov-Bohm phase to the tunneling process. The effective TLS responds to the electric field of two pulses by means of renormalized parameters and distribution. We are able to get most of the features of echo experiments, namely, strong dependence of the echo amplitude on low magnetic fields, pulse separation, temperature and electric field effects. Our results show how one can explain the echo experiments by using the idea of Aharanov-Bohm phase.

In the next section we define our effective model and its renormalized parameters. The response of an effective TLS to a two pulse echo is studied in Sec. IV. Then, in Sec. V we will explain the different aspects of our results to show their correspondence with experimental data. Finally we summarize our results and discuss the features of our approach.

![FIG. 1: Flip-flop configuration as an effective TLS.](image)

## II. EFFECTIVE TLS

Suppose that there are two interacting tunneling systems. The interaction between the TLSs can be considered by the Ising Hamiltonian

\[
V = \frac{1}{2} \sum_{i,j} U(R_{ij})\sigma_i^z\sigma_j^z; \quad U(R_{ij}) = \frac{U_0}{R_{ij}} \tag{3}
\]

where \(R_{ij}\) is the distance between two TLSs and \(U_0\) is the characteristic coupling constant.

Burin et al.[24] proposed that TLS-TLS coupling in glasses is dominated at low temperatures by a flip-flop configuration. While the characteristic energy of a single TLS \((E\) or \(E’\), see Fig. I) has the same order of magnitude as temperature \((T)\), the resulting spectrum of two TLSs has two energy scales. The flip-flop states define a scale of energy which is much less than \(T\) and the other remaining two represent a scale as before, i.e. \(T\). The flip-flop (pair) states correspond to the situation where one TLS is in the ground state and the other in the excited state. On the other hand the remaining part of Hilbert space shows both excited or both grounded TLSs. Concerning the relaxation of the systems which leads finally to the dephasing mechanism we only consider the flip-flop states. Taking into account the interaction between tunneling centers the Hamiltonian in the flip-flop subspace is

\[
H_p = \frac{1}{2}(\Delta_p \sigma_z + \Delta_{0p} \sigma_x)
\]

where

\[
\Delta_{0p} = \gamma \frac{\Delta_{01} \Delta_{02}}{EE’}; \quad \Delta_p = |E - E’| \tag{4}
\]

The distribution function for the parameters \(\Delta_{0p}\) and \(\Delta_p\) is defined as

\[
f^{(2)}(\Delta_p, \Delta_{0p}) = \frac{\delta(\Delta_p - |E - E’|) \times}{\delta(\Delta_{0p} - \frac{U_0 \Delta_{01} \Delta_{02}}{R^3 EE’})} > \tag{5}
\]
Therefore by averaging over the distribution parameters given by Eq. (2) of the original TLS and ensemble averaging $\Delta_p \ll E \simeq T$ we get

$$f^{(2)}(\Delta_p, \Delta_{op}) \propto \frac{T}{\Delta_{op}^2}. \quad (6)$$

Now instead of a pair of original TLSs we have an effective TLS (ETLS) which behaves like a single TLS with the new distribution function, Eq. (6). Such an effective TLS is aimed to explain the response of a glass to an electric field of the echo experiment at low temperatures when a magnetic field is present. The magnetic field dependence enters via the dependence of the original tunneling amplitude of pairs $(\Delta_{0i}(\phi) = \Delta_{0i} \cos(\pi \phi_0): i = 1, 2)$ as proposed in Ref. [13]. By a simple calculation one can show that the magnetic field dependence of $\Delta_{op}$ is similar to the original TLS one’s, i.e. $\Delta_{op}(\phi) \simeq \Delta_{op} \cos(\pi \phi_0)$, where $\phi_0$ is the renormalized quantum of flux ($\hbar/\varphi$). The Planck constant is denoted by $\hbar$ and the effective charge is $q$. We should mention that this model is valid while the temperature is not zero. At the zero temperature the distribution function of ETLSs, Eq. (6), is zero which means no accessible flip-flop states. In other words, at $T = 0$ the system does not respond to the electric field of the echo experiments and will remain in the ground state.

III. THE RESPONSE OF AN EFFECTIVE TLS TO A TWO PULSE ECHO

The wave function of an ETLS in an external time dependent electric field is a linear combination of the wave function $|\varphi_1\rangle$ and $|\varphi_2\rangle$ for each level.

$$|\psi\rangle = a_1(t)e^{-iE_1t/\hbar}|\varphi_1\rangle + a_2(t)e^{-iE_2t/\hbar}|\varphi_2\rangle, \quad (7)$$

$$|a_1(t)|^2 + |a_2(t)|^2 = 1.$$  

At any time ($t$) during the first pulse of the electric field ($\xi(t) = \xi_0 \sin \omega t$), the time variation of the probability amplitudes $a_1(t)$ and $a_2(t)$ is governed by the Schrödinger equation. In the resonant condition ($|\hbar \omega - E| \ll \hbar \omega$) it leads to the following equations:  

$$\frac{da_1}{dt} = \delta_0 e^{i\delta t} a_2(t),$$

$$\frac{da_2}{dt} = -\delta_0 e^{-i\delta t} a_1(t), \quad (8)$$

where $\delta = \omega - \frac{E}{\hbar}$, $\delta_0 = \frac{\xi_0 p_{12}}{\hbar}$ and $p_{12} = p_0 \Delta_{op}$ in which $p_0$ is the dipole moment of the ETLS. $E = E_2 - E_1$ where $E_1$ and $E_2$ are the energy of ground and the exited states of the ETLS, respectively. We can easily find that at $t = \tau_1$ the coefficients are given by,

$$a_1(\tau_1) = \frac{a_1(0)\gamma_2 - i\delta_0 a_2(0)e^{i\gamma_1\tau_1}}{\gamma_1 + \gamma_2} + \frac{a_1(0)\gamma_1 + i\delta_0 a_2(0)e^{-i\gamma_2\tau_1}}{\gamma_1 + \gamma_2},$$

$$a_2(\tau_1) = i\frac{\delta_0}{\gamma_1}a_1(0)\gamma_2 - i\delta_0 a_2(0)\gamma_1 e^{i\gamma_1\tau_1} - \frac{a_1(0)\gamma_1 + i\delta_0 a_2(0)}{\gamma_1 + \gamma_2}e^{-i\gamma_2\tau_1}, \quad (9)$$

where $\gamma_1 = \frac{\delta + \sqrt{\delta^2 + 4\beta E_1}}{2}$ and $\gamma_2 = -\frac{\delta + \sqrt{\delta^2 + 4\beta E_2}}{2}$. The initial condition is imposed by the Boltzmann weight, i.e. $|a_1(0)|^2 = \frac{1}{Z}e^{-\beta E_1}$ and $|a_2(0)|^2 = \frac{1}{Z}e^{-\beta E_2}$, where $Z = e^{-\beta E_1} + e^{-\beta E_2}$ is the partition function, $\beta = \frac{1}{K_B T}$ and $K_B$ is the Boltzmann constant.

The system relaxes for a time duration of $t_0$ before the second pulse arrives. The second pulse of the electric field tries to recover the initial configuration during $\tau_2 = 2\tau_1$ seconds. Finally the measurement on the amplitude of the electric dipole is done at $t' = t_0$ after switching off the second pulse. This can be expressed by the following equation,

$$A(2t_0) = <\psi(2t_0) | p | \psi(2t_0)> \simeq 2Re[a_1(\tau_2)a_2(\tau_2)p_{12}e^{-i\varphi_0}], \quad (10)$$

where $p = p_0 \sigma_z$ and the details of the other coefficients can be found in the appendix.
FIG. 3: The echo amplitude versus the magnetic flux ratio for different electric field amplitudes, by averaging over the effective distribution function for $0 < \frac{\Delta \phi}{\phi_0} < 10^9 H z$ and $10^9 < \frac{\Delta \phi}{\phi_0} < 10^9 + 10^6 H z$, ($\omega = 2 \times 10^9 H z$, $t_0 = 2 \mu s$ and $T = 15 m K$). The vertical axis has been shifted.

FIG. 4: The echo amplitude versus electric field amplitude, for different values of the magnetic field flux, by averaging over the effective distribution function for $0 < \frac{\Delta \phi}{\phi_0} < 10^9 H z$ and $10^9 < \frac{\Delta \phi}{\phi_0} < 10^9 + 10^6 H z$, ($\omega = 2 \times 10^9 H z$, $t_0 = 1 \mu s$ and $T = 15 m K$).

As mentioned before we will consider the effect of the magnetic field via a Mexican hat model. In this model for describing the non-monotonic magnetic field dependence of the dielectric susceptibility of multi-component glasses, Kettemann et al. investigated the properties of tunneling systems exhibiting the peculiarity that the tunneling particle can move along different paths to go from one potential minimum to the other, thus forming a closed tunneling loop. As a result in the three dimensional Mexican hat model, using the Aharonov-Bohm effect and the assumption that the tunneling through both paths occurs with equal probability, the tunnel splitting becomes a periodic function of the magnetic flux threading through the loop,

$$\Delta_{0p} \rightarrow \Delta_{0p}(\phi) = \Delta_{0p} \cos(\frac{\phi}{\phi_0})$$ \hspace{1cm} (11)

where $\phi$ is the total magnetic flux passing through a closed tunneling path, $\phi_0$ is the renormalized quantum flux defined by $\phi_0 = \frac{\hbar}{q}$ in the previous section.

IV. RESULTS ON ECHO AMPLITUDE

In this section we are going to present our results on the amplitude of the echo response (Eq.11) as a function of the applied magnetic and electric field in addition to the pulse separation time ($t_0$) and temperature.

In the absence of a magnetic field the echo amplitude measured for small pulse separations in an ideal double-pulse experiment, ignoring relaxation, has a maximum when the amplitude of the driving electric field is given by:

$$\frac{p12\xi_0\tau_1}{2\hbar} = \frac{\pi}{4}$$ \hspace{1cm} (12)

and at $T = 10 m K$ the tunneling matrix element is $\Delta_{0p} \sim 10^9 H z$. If we consider the typical value for $\xi_0 = 1 kV/m$ and $\tau_1 = 100 ns$ then $\frac{p12\xi_0\tau_1}{2\hbar} \sim 10^9 - 10^7 H z$. To compare our results with recent experiments the following frequency $\frac{p12\xi_0\tau_1}{2\hbar} \sim 10^9 - 10^7 H z$ and pulse separation $t = t_o \sim 10^{-6} s$ are assumed.

We should mention that the result of Eq.11 is valid for low magnetic field amplitudes. At high magnetic fields the effective tunneling amplitude $\Delta_{0p} \cos(\frac{\phi}{\phi_0})$ becomes small and the resonance condition $|\delta| \ll \omega$ is not fulfilled. Therefore, the resonance condition $|\omega - \sqrt{\Delta_{0p}^2 + \Delta_{0p}^2(\phi)^2}| \ll \omega$ is satisfied whenever $1 - \cos(\frac{\pi \phi}{\phi_0}) \ll 1$, having assumed that $\frac{\Delta_{0p}}{\hbar} \sim \omega$. In other words the resonance condition is guaranteed for $|\pi \frac{\phi}{\phi_0}| \ll 1$.

A. Magnetic field dependence

We have plotted in Fig. 2, the square of the normalized echo amplitude $|A(B)/A(B=0)|^2$ versus the magnetic field. The magnetic field ($B$) in the horizontal axis is presented as flux ($\phi = SB$, $S$ is the effective area of ETLS) divided by the quantum flux ($\phi_0$) to simplify data analysis. To arrive at this plot we have used Eq.11.
with $t_0 = 2\mu s$. We have also averaged over the tunneling parameters $(\Delta_p, \Delta_0)$. Note that, in this process we have assumed $\Delta_p < \Delta_0$, to have reasonable flip-flop configuration. Moreover the resonance condition of the ETLS limits the range of $\Delta_0$ to be very close to the frequency of the electric field $10^9 < \frac{\Delta_0}{2\hbar} < 10^9 + 10^6\, Hz$. The distribution of the ETLS parameters is given by Eq.1. The echo amplitude has a maximum at $B = 0$ as presented in Fig.2. It then decreases to a minimum before rising up. This is in agreement with the experimental observation presented in Ref.10,11,12.

### B. Dependence on the electric field

We have examined the dependence of our results on the amplitude of the electric field. The echo amplitude versus the magnetic field for different strengths of the electric field is plotted in Fig.5. The data for each value of the electric field have been shifted by a constant for clarity. We will see that the height of the central peak is enhanced by increasing the electric field. This feature also coincides with the experimental data on the defected crystal $KBr:CN$ (Fig.1 in Ref.10).

We have then plotted the echo amplitude versus the amplitude of the electric field in Fig.4 for different magnetic fields. The echo amplitude rises and has a maximum for an electric field around $700 V/m$, it then decreases for larger fields. The general feature is in agreement with the experiments on multicomponent glasses (Fig.2 in Ref.4).

### C. Effect of pulse separation

We have presented in Fig.5 the same quantity as Fig.2, but for different pulse separations $(t_0 = 1, 1.5 \mu s)$. Although the global behavior is similar to Fig.2 there is a slight difference at small magnetic fields. We have observed that for short pulse separations, $t_0 = 1 \mu s$, the echo amplitude gets a minimum at $B = 0$. The effect is similar to the experiments on crystals with point defects (Fig.2 of Ref.10). For short pulses the magnetic field does not have a dephasing effect. However the central peak broadening does not accompany the decreasing of pulse separation as in Fig.3 of Ref.10. In contrast, the broadening of the central peak decreases with $t_0$. This can be seen in Fig.6 where we have scaled the horizontal axis by a factor of $t_0$. The width of the central peak diminishes by applying shorter pulse separations.

### D. Temperature dependence

The temperature dependence of our results enters via the Boltzmann weight as the initial probability of the states in ETLS. This dependence is plotted in Fig.7. We have plotted the amplitude of the echo response versus magnetic field for different temperatures. To have a clear plot for different curves we have added a constant value to each curve. Thus they are well represented instead of falling onto each other. We have seen that the intensity of the central peak is reduced by increasing the temperature and becomes almost flat at high tem-
temperatures. Apart from that the general behavior is unchanged. This can also be compared to the experimental data on the multicomponent glasses (Fig. 8 of Ref. [11]). Although the profile of the echo amplitude versus magnetic field does not fit completely, the reduction of central peak is in agreement.

V. SUMMARY AND DISCUSSION

We have presented a model to explain the anomalous behavior of the echo response in glasses at low temperatures. This model is essentially based on an effective TLS which is the result of interaction between the original TLSs in glasses or defected crystals. Most of the experimental features at low magnetic fields can be explained by our approach. In this respect, the dephasing effect which reduces the echo amplitude at low fields (Fig. 2), the dependence on the strength of the electric field (Figs. 3, 4) and also the effect of various temperatures (Fig. 7) have been addressed. However, in the response of different pulse separations ($t_0$) although it shows the broadening of the central peak by reduction of $t_0$ the product of $t_0\phi$ does not seem to be universal which deviates from the experimental data.

As we have already mentioned our model is valid for the low magnetic field regime. At high magnetic fields the resonance condition is lost, thus the flip-flop basis of ETLS does not contribute effectively. Because, if a TLS is initially in the ground state, the probability to find the system in the second state is given by

$$W_{12}(t) = \frac{\Delta_0(\phi)^2}{\Delta^2 + \Delta_0(\phi)^2} \sin^2(\sqrt{\frac{\Delta^2 + \Delta_0(\phi)^2}{2\hbar}}t). \quad (13)$$

For high magnetic fields this probability vanishes since $\Delta_0(\phi)$ become very small. So the particles will be frozen in the lowest well and do not respond to the external electric field. Thus we expect the echo amplitude for high magnetic fields to be constant.

The other aspect of echo response which can be explained by this model is the response to the frequency of the electric field. It has been shown that the echo amplitude of BK7 is almost independent of electric field frequency (see Fig. (7) in Ref. [11]). A raw description is given by taking into account the interaction between ETLS defined in Sec III. The resonance condition can take place for different frequencies and the final response will be the same. Note that the distribution of the effective ETLS is given by Eq. (10) which is given in the appendix.

However, there is a debate on the interpretation of the results which comes from a Mexican hat model. Let us reduce the discussion to two points. The first point is related to the route where the magnetic field enters the problem. In our model (Mexican hat type) the magnetic field effect appears in the tunneling amplitude (Eq. (11)) via the Aharonov-Bohm phenomenon. This by itself can not be ruled out. Recently, Coche, et al. [26] reported experiments on the dielectric constant of a structural glass and explained their results by considering the variation of the tunneling amplitude versus magnetic field. This is based on a microscopic theory of transport in disordered networks which can be interpreted as the tunneling between two wells. The effect of the magnetic field due to quantum phases influences the tunneling amplitude [26].

The second point is related to the interpretation of a large effective charge $q$, (or small quantum flux $\phi_0$) which scales the magnetic field to the right value of the experimental data. This is the most controversial point and we are going to add some comments in this respect. Firstly, the large value of $q$ has been taken as an indication that there is a coherent motion of tunneling particles taking place as the result of their mutual interaction [13, 14]. As far as the low temperature physics of glasses is concerned the interaction between TLSs is an important issue which has been recently addressed as a dephasing mechanism in the relaxation of cold glasses [26]. The relaxation can be enhanced by interaction through flip-flop configurations leading to delocalized state. Thus the interaction should also have a considerable effect in the dephasing mechanism in the presence of a magnetic field. And this is taken into account in the Mexican hat type model effectively. Secondly, the nature of a tunneling system (TLS) is not clearly known. The general belief is that it represents the low energy excitations of glasses at low temperatures. It has been recently argued that such types of excitation come from two energetically close configurations of a cluster of atoms or molecules [4, 30]. The cluster contains...
roughly $N = 200$ atoms or molecules contributing in a collective tunneling. Thus the flux passing through such a cluster is much larger than the case in a single atom. This by itself can improve the necessary flux (φ) in the Mexican hat model by two orders of magnitude. Taking into account the interaction between the mentioned tunneling systems gives the right order of the magnetic field.

It should be cited that due to the new experiments on glasses with different isotopes the effect of the nuclear quadrupole moment (NQM) is of significant importance which is lacking in our model. The theory which is based on NQM works with a single molecule as a tunneling center and takes its NQM into account. However, the next step is to introduce an effective nuclear quadrupole moment for a tunneling system which is a cluster of molecules. The theory explains very well the low and high magnetic field behavior of the echo response. But the intermediate regime is missing. Besides, in this approach the static electric susceptibility is six orders of magnitude smaller than the experimental one. We should mention that our model gives the temperature dependence of the echo amplitude in the presence of a magnetic field which is absent in the NQM model proposed in Ref. [17].

Acknowledgments

We would like to express our deep gratitude to P. Fulde for valuable comments and useful discussions. We would also like to thank D. Bodea and I. Ya. Polishchuk for fruitful discussions.

APPENDIX A: THE ECHO AMPLITUDE AFTER THE SECOND PULSE

As mentioned in the text the Boltzmann weight is used to define the initial condition, $| a_1(0) |^2 = \frac{\beta}{Z} e^{-\beta E_1}$ and $| a_2(0) |^2 = \frac{\beta}{Z} e^{-\beta E_2}$, where $Z = e^{-\beta E_1} + e^{-\beta E_2}$ is the partition function, $\beta = \frac{1}{k_B T}$ and $K_B$ is the Boltzmann constant. So the wave function after the relaxation time $t_0$ is

$$
| \psi(t_0) \rangle = a_1(t_1) e^{-\frac{i E_1 t_0}{\hbar}} | \varphi_1 \rangle + a_2(t_1) e^{-\frac{i E_2 t_0}{\hbar}} | \varphi_2 \rangle.
$$

During the second pulse the probability amplitudes must satisfy Eq. (3), so in a time $t'$ after the second pulse we have

$$
| \psi(t_0 + t') \rangle = a_1(t_2) e^{-\frac{i E_1 t'}{\hbar}} | \varphi_1 \rangle + a_2(t_2) e^{-\frac{i E_2 t'}{\hbar}} | \varphi_2 \rangle.
$$

where

$$
a_1(t_2) = a_1(t_1) e^{-i\frac{\delta E_1}{\hbar} t_2} + a_1(t_1) e^{i\frac{\delta E_1}{\hbar} t_2},
$$

$$
a_2(t_2) = i a_1(t_1) e^{-i\frac{\delta E_1}{\hbar} t_2} e^{i\gamma_t t_2} + a_1(t_1) e^{i\frac{\delta E_1}{\hbar} t_2} e^{-i\gamma_t t_2},
$$

$$
a_1(t_1) = a_1(t_1) e^{\frac{i \delta E_1 t_1}{\hbar}} ,
$$

$$
a_2(t_1) = a_2(t_1) e^{\frac{i \delta E_1 t_1}{\hbar}} . \tag{A1}
$$

By these assumptions, the amplitude of the two pulse echo from an effective TLS at time $2t_0$ measured from the first pulse is given by

$$
A(2t_0) = \langle \psi(2t_0) | p | \psi(2t_0) \rangle 
\simeq 2 Re [a_1^*(t_2) a_2^*(t_2) p_{12} e^{\frac{i E_1 t_1}{\hbar}}], \tag{A2}
$$

where $p = p_0 \sigma_z$ and $t_2 = 2t_1$.

APPENDIX B: THE DISTRIBUTION OF TUNNELING PARAMETERS IN COUPLED INTERACTING ETLS

If we consider that the interaction between two ETLS is the same as the interaction between the original TLSs then we can use the same procedure used previously for a pair of TLSs. From Eq. (3) we have

$$
f^{(3)}(\Delta_p, \Delta_{op}) \propto T^2 \int d\Delta_{o1} \int \frac{dE_1}{\Delta_{o1}} \frac{dE_2}{\Delta_{o2}} n(E_1) \times
\int \frac{d\Delta_{o2}}{\Delta_{o2}} \frac{dE_2}{\sqrt{E_2^2 - \Delta_{o2}^2}} \left[1 - n(E_2)\right] \times
\int d^3R \delta(\Delta_p - |E_1 - E_2|) \delta(\Delta_{op} - \frac{U_0 \Delta_{o1} \Delta_{o2}}{R^3 E_1 E_2}),
$$

where $n(E_1) = [1 + \exp(\beta E_1)]^{-1}$ is the probability of finding an ETLS in the exited state. By a little manipulation we can show that

$$
f^{(3)}(\Delta_p, \Delta_{op}) \propto T^2 \frac{2T}{\Delta_{op}} \ln \left[ \frac{2T}{\Delta_{o_{min}}} \right]^2. \tag{B1}
$$

Since $\Delta_{o_{min}} \sim T$, we find that the distribution function for the coupled ETLS parameters is similar to the second distribution function (Eq. (B1)).
1. R. C. Zeller and R. O. Phol, Phys. Rev. B 4, 2029 (1971).
2. W. A. Phillips, J. Low Temp. Phys. 7, 351 (1972).
3. P. W. Anderson, B. I. Halperin and C. M. Varma, Philos. Mag. 25, 1 (1972).
4. V. Lubchenko and P. G. Wolynes, Phys. Rev. Lett. 87, 195901 (2001); and references therein; V. Lubchenko and P. G. Wolynes, PNAS, 100, 1515 (2003).
5. C. Enss, Physica (Amsterdam) 316B-317B, 12 (2002).
6. P. Strehlow, C. Enss, and S. Hunklinger, Phys. Rev. Lett. 80, 5361 (1998).
7. P. Strehlow, M. Wohlfahrt, A. G. M. Jansen, R. Haueisen, G. Weiss, C. Enss, and S. Hunklinger, Phys. Rev. Lett. 84, 1938 (2000).
8. M. Wohlfahrt, P. Strehlow, C. Enss, and S. Hunklinger, Europhys. Lett. 56, 690 (2001).
9. S. Ludwig, C. Enss, S. Hunklinger, and P. Strehlow, Phys. Rev. Lett. 88, 075501 (2002).
10. C. Enss and S. Ludwig, Phys. Rev. Lett. 89, 075501 (2002).
11. S. Ludwig, P. Nagel, S. Hunklinger, and C. Enss, J. Low Temp. Phys. 131, 89 (2003).
12. P. Nagel, A. Fleishmann and S. Hunklinger Phys. Rev. Lett. 92, 245511 (2004).
13. S. Kettemann, P. Fulde, and P. Strehlow, Phys. Rev. Lett. 83, 4325 (1999).
14. A. Langari, Phys. Rev. B. 65, 104201 (2002).
15. N. M. Zimmerman, B. Golding and W. H. Haemmerle, Phys. Rev. Lett. 67, 1322 (1991).
16. A. Würger, Phys. Rev. Lett. 88, 75502 (2002).
17. A. Würger, A. Fleishmann and C. Enss, Phys. Rev. Lett. 89, 237601 (2002), A. Würger, J. Low Temp. Phys. 137, 143 (2004).
18. D. A. Parshin, J. Low Temp. Phys. 137, 233 (2004).
19. D. Bodea and A. Würger, J. Low Temp. Phys. 136, 39 (2004).
20. D. Bodea, private communication.
21. C. Enss and S. Hunklinger, Phys. Rev. Lett. 79, 2831 (1997).
22. S. Hunklinger, C. Enss and P. Strehlow, Physica B, 280, 271 (2000).
23. A. L. Burin, Yu. Kagan, L. A. Maksimov and I. Ya. Polishchuk, Phys. Rev. B. 69, 220201(R) (2004); A detail description can be found in: A. L. Burin and I. Ya. Polishchuk, condmat/0407377.
24. A. L. Burin, Yu. Kagan, L. A. Maksimov and I. Ya. Polishchuk, Phys. Rev. Lett. 80, 2945 (1998).
25. A. L. Burin, L. A. Maksimov and I. Ya. Polishchuk, JETP Lett. 49, 784 (1989); I. Ya. Polishchuk, L. A. Maksimov and A. L. Burin, JETP 79, 634 (1994); A. L. Burin, Yu. Kagan, JETP 80, 761 (1995); A. L. Burin, Yu. Kagan, L. A. Maksimov and I. Ya. Polishchuk, Phys. Rev. Lett. 86, 5616 (2001).
26. J. Le Coche, F. Ladieu and P. Pari, Phys. Rev. B. 66, 64203 (2002).
27. E. Medina and M. Kardar, Phys. Rev. B. 46, 9984 (1992).
28. A. B. Pippard, The Physics of vibration vol. 2 (Cambrige: Cambridge University Press)(1989).
29. W. A. Phillips, Rep. Prog. Phys. 50, 1657 (1987).
30. V. Lubchenko and P. G. Wolynes, condmat/0407581.