Stars and Halos of Degenerate Relativistic Heavy-Neutrino and Neutralino Matter

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Heavy-neutrino (or neutralino) stars are studied using the general relativistic equations of hydrostatic equilibrium and the relativistic equation of state for degenerate fermionic matter. The Tolman-Oppenheimer-Volkoff equations are then generalized to include a system of degenerate neutrino and neutralino matter that is gravitationally coupled. The properties and implications of such an interacting astrophysical system are discussed in detail.

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I. INTRODUCTION

One of the most tantalizing puzzles of this universe is the issue of dark matter, the presence of which is inferred from the observed flat rotation curves in spiral galaxies \cite{1}, the diffuse emission of x-rays in elliptical galaxies and clusters of galaxies, as well as from cluster dynamics. Primordial nucleosynthesis entails that most of baryonic matter in this universe is nonluminous, and such an amount of dark matter falls suspiciously close to that required by galactic rotation curves. However, although a significant component of dark matter in galactic halos is presumably baryonic \cite{2}, the bulk part of dark matter in this universe is believed to be nonbaryonic. Many candidates have been proposed \cite{3}, both baryonic as well as nonbaryonic, to explain the dark matter paradigm, but the issue of the nature of dark matter is still far from being resolved.

One of the most conservative candidates for nonbaryonic dark matter are, of course, massive neutrinos. In this paper we are particularly interested in neutrinos with masses between 10 and 25 keV/c\(^2\), as these could form supermassive degenerate neutrino stars, which may explain, without invoking the black-hole hypothesis, some of the features observed around the supermassive compact dark objects with masses ranging from \(10^{6.5}\) \(M_\odot\) to \(10^{9.5}\) \(M_\odot\). These have been reported to exist at the center of a number of galaxies \cite{5, 6, 20} including our own \cite{1, 3} and quasistellar objects. It is interesting to note that neutrinos in this mass range can also cluster around ordinary stars, and thus these neutrinos could account for at least part of galactic dark matter. A further motivation for studying the collapsed structures of heavy neutrino matter is the recent increased interest in fermionic cold dark matter models \cite{14} in which massive neutrinos play an important role in structure formation in the early universe.

A 10 to 25 keV/c\(^2\) neutrino is in conflict neither with particle and nuclear physics experiments nor with astrophysical observations \cite{4}. On the contrary, if the conclusion of the LSND collaboration which claims to have detected \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) flavor oscillations \cite{17} is confirmed, and the quadratic see-saw mechanism involving the up, charm, and top quarks \cite{18, 19}, is the correct mechanism for neutrino mass generation, the \(\nu_\tau\) mass may be between 6 and 32 keV/c\(^2\) \cite{20}, which is well within the cosmologically forbidden range. It is well known that such a quasistable neutrino would lead to an early neutrino-matter dominated phase, which may have started as early as a couple of weeks after the big bang. Thus, a critical universe that remained neutrino-matter dominated all the time would have reached the current microwave background temperature in less than 1 Gyr, i.e., much too early to accommodate the oldest stars in globular clusters, cosmochronology, and the Hubble expansion age.

It is conceivable, however, that, in the presence of such heavy neutrinos, the early universe might have evolved quite differently than described in the homogeneous standard model of cosmology. Neutrino stars may have emerged in local condensation processes during a gravitational phase transition, shortly after the neutrino-matter dominated epoch began. The latent heat produced in such a first-order phase transition, apart from reheating the gaseous phase, might have reheated the radiation background as well. Annihilation of heavy neutrinos into light neutrinos via the \(Z^0\) would take place in the interior of neutrino stars \cite{21, 16}. Both these processes would decrease the density of heavy neutrinos, as perceived today, and also increase the time which photons need to cool down to the present microwave background temperature. Thus a quasistable neutrino in the mass range between 10 and 25 keV/c\(^2\) is presumably not in contradiction with cosmological and astrophysical observations \cite{16}.

In fact, it has recently been shown \cite{12, 20, 24} that degenerate neutrino stars \cite{16} may indeed have been formed during a gravitational phase transition in the early universe. Whereas the existence of this first-order phase transition is firmly established in the framework of the Thomas-Fermi model at finite temperature \cite{22}, the microscopic mechanism through which the latent heat is released during the phase transition and dissipated into observable and

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perhaps unobservable matter or radiation remains to be identified. At this stage, however, it is still not clear whether an efficient dissipation mechanism can be found within the minimal extension of the standard model of particle physics or whether new physics is required in the right-handed neutrino sector. We therefore have to assume in the following that such an efficient dissipation mechanism exists, in order to make sure that fermions can actually settle in the state of lowest energy in a time much shorter than the age of the universe.

In this paper, we focus primarily on gravitationally clustered, degenerate nonbaryonic matter consisting of two species of weakly interacting stable or quasistable fermions: one with a mass around 15 keV/c^2 which we subsequently call “neutrino”, and the other with a mass around 1 GeV/c^2 which we henceforth call “neutralino”. The chosen neutralino mass, although a little bit on the low side, offers the possibility of replacing the neutralino with a neutron, as the strong-interaction effects of the neutron in neutron star matter can be simulated by an effective mass. Of course, this substitution makes sense only as long as the binding energy of the neutron is larger than the Q value for the neutron decay, so that the neutron can be considered stable in neutron star matter.

It is interesting to note that a variety of similar scenarios can be treated within the same framework: Apart from a neutrino halo around a neutron star, one could also study a neutrino halo around a white dwarf or around an ordinary star [23], since all these baryonic stars can be approximated using similar polytropic equations of state which eventually result in the same nonlinear differential equations of the Lane-Emden type. Moreover, by varying the polytropic index of the equation of state, one can also investigate the properties of a cold neutrino star immersed in a hot radiation field, or in a hot baryonic background, or in a vacuum with nonzero energy density, which all may have played a role in the formation process of primordial neutrino stars. Thus the study of this simple interacting neutrino-neutralino system allows us to learn a great deal about the properties of gravitationally clustered baryonic and nonbaryonic matter.

This paper is organized as follows: In Sec. II in a general relativistic framework, we discuss the properties and implications of degenerate neutrino (and neutralino) stars and their Newtonian and ultrarelativistic limits. In Sec. III we generalize the Tolman-Oppenheimer-Volkoff (TOV) equations to include gravitationally clustered, degenerate nonbaryonic matter, consisting of neutrinos and neutralinos. Our results are summarized in Sec. IV.

II. DEGENERATE NEUTRINO STARS

A spherically symmetric cloud of degenerate neutrino matter can be characterized by its mass density $\rho_\nu(r)$, pressure $P_\nu(r)$, and the metric in the Schwarzschild form [25]:

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$

The pressure and the density satisfy the general relativistic TOV equations of hydrostatic equilibrium [26,27]:

$$\frac{dP_\nu}{dr} = -\frac{1}{2}(\rho_\nu c^2 + P_\nu) \frac{d\nu}{dr},$$

$$e^\lambda = \left(1 - \frac{2Gm}{c^2 r}\right)^{-1},$$

$$\frac{dP_\nu}{dr} = -G\frac{(\rho_\nu + P_\nu/c^2)(m + 4\pi r^3 P_\nu/c^2)}{r(r - 2Gm/c^2)},$$

$$\frac{dm}{dr} = 4\pi r^2 \rho_\nu(r),$$

where $m(r)$ is the mass enclosed within a radius $r$. The relevant boundary conditions are $m(0) = 0$, $P_\nu(R) = 0$, and $\rho_\nu(R) = 0$, as the pressure and the density vanish at the radius $R$ of the star. Outside the star, the functions $\nu$ and $\lambda$ are determined by the usual Schwarzschild solution

$$e^\nu = e^{-\lambda}, \quad e^\lambda = \left(1 - 2GM/c^2 r\right)^{-1},$$

$$M = \int_0^R 4\pi \rho_\nu(r)r^2 dr.$$
We now introduce the equation of state of a degenerate relativistic Fermi gas [28] which may be parameterized as

\[ P_\nu = K \left[ X(1 + X^2)^{1/2} \left( \frac{2}{3} X^2 - 1 \right) + \log \left( X + (1 + X^2)^{1/2} \right) \right], \tag{8} \]

\[ \rho_\nu = \frac{K}{c^2} \left[ X(1 + X^2)^{1/2}(2X^2 + 1) - \log \left( X + (1 + X^2)^{1/2} \right) \right], \tag{9} \]

\[ n_\nu = \frac{8KX^3}{3m_\nu c^2}. \tag{10} \]

Here, \( n_\nu \) denotes the neutrino-number density, and \( K \) and \( X \) are given by

\[ K = \frac{g_\nu m_\nu c^5}{16\pi^2\hbar^3}, \quad X = \frac{p_\nu}{m_\nu c}, \tag{11} \]

where \( p_\nu \) stands for the local Fermi momentum of the neutrinos of mass \( m_\nu \), and \( g_\nu \) is the spin degeneracy factor of neutrinos and antineutrinos, i.e., \( g_\nu = 2 \) for Majorana and \( g_\nu = 4 \) for Dirac neutrinos and antineutrinos. Using (8) and (11), and introducing dimensionless variables \( x = r/a_\nu \) and \( \mu = m/b_\nu \) with the scales

\[ a_\nu = 2\sqrt{\frac{\pi}{g_\nu}} \left( \frac{M_{P1}}{m_\nu} \right)^2 L_{P1} = 2.88233 \times 10^{10} g_\nu^{-1/2} \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^2 \text{ km}, \tag{12} \]

\[ b_\nu = 2\sqrt{\frac{\pi}{g_\nu}} \left( \frac{M_{P1}}{m_\nu} \right)^2 M_{P1} = 1.95197 \times 10^{10} M_\odot g_\nu^{-1/2} \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^2, \tag{13} \]

where \( M_{P1} = (\hbar c/G)^{1/2} \) and \( L_{P1} = (\hbar G/c^3)^{1/2} \) denote Planck’s mass and length, respectively, the TOV equations (4) and (5) can be written as

\[ \frac{dX}{dx} = -\frac{1 + X^2}{X(x^2 - 2\mu x)} \left\{ \mu + x^3 \left( X(1 + X^2)^{1/2} \left( \frac{2}{3} X^2 - 1 \right) + \log \left( X + (1 + X^2)^{1/2} \right) \right) \right\}, \tag{14} \]

\[ \frac{d\mu}{dx} = x^2 \left[ X(1 + X^2)^{1/2}(2X^2 + 1) - \log \left( X + (1 + X^2)^{1/2} \right) \right], \tag{15} \]

subject to the boundary conditions \( X(0) = X_0 \) and \( \mu(0) = 0 \). In addition to (14) and (15), there is also an equation governing the number of particles \( N \) within a radius \( r = a_\nu x \): 

\[ \frac{d\hat{n}}{dx} = \frac{1}{3} X^3(1 - 2\mu/x)^{-1/2}, \tag{16} \]

where \( \hat{n} = n/N_0 \) is the rescaled neutrino-number density subject to the boundary condition \( \hat{n}(0) = 0 \), with

\[ N_0 = \frac{8b_\nu}{3m_\nu} = 3.3765 \times 10^{72} \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^3 g_\nu^{-1/2}. \tag{17} \]

Equations (14)-(16) may be solved numerically. Picking up a value \( X_0 \) for the Fermi momentum at the center (in units of \( m_\nu c \)), one obtains the total mass of the star \( M \), the radius \( R \), and the total number of particles \( N \), by integrating outward until \( X \) vanishes. The results are summarized in Figs. 1 and 2. In Fig. 1 the total mass is plotted against the radius of the neutrino star. The curve has a maximum, namely, the Oppenheimer-Volkov (OV) limit [27], at \( \mu_{OV} = 0.15329 \), which corresponds to a neutrino star mass of

\[ M_{OV} = 0.15329 b_\nu = 0.54195 M_{P1} m_\nu^{-2} g_\nu^{-1/2} = 2.9924 \times 10^9 M_\odot \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^2 g_\nu^{-1/2}. \tag{18} \]

Owing to their large mass, neutrino stars could serve as candidates for supermassive compact dark objects observed in the mass range.
at the centers of a number of galaxies. Assuming that the most massive and violent objects are neutrino stars at the OW limit with \( M_{\text{OW}} = (3.2 \pm 0.9) \times 10^6 M_\odot \), such as the supermassive compact dark object at the center of M87, the neutrino mass required for this scenario is

\[
12.4 \text{ keV}/c^2 \leq m_\nu \leq 16.5 \text{ keV}/c^2 \quad \text{for } g_\nu = 2, \\
10.4 \text{ keV}/c^2 \leq m_\nu \leq 13.9 \text{ keV}/c^2 \quad \text{for } g_\nu = 4. 
\]

The radius of such a neutrino star is \( R_{\text{OV}} = 4.4466 \, R_{\odot}^{\text{OV}} \), where \( R_{\text{OV}}^* = 2GM_{\text{OV}}/c^2 \) is the Schwarzschild radius of the mass \( M_{\text{OV}} \). Thus, at a distance of a few Schwarzschild radii away from the supermassive object, there is little difference between a neutrino star at the OW limit and a black hole, in particular since the last stable orbit around a black hole already has a radius of \( 3 \, R_{\text{OV}}^{\text{OV}} \). A neutrino star of mass \( M_{\text{OV}} = 3 \times 10^9 M_\odot \) would have a radius \( R_{\text{OV}} = 3.9396 \times 10^{10} \) km, or 1.52 light-days.

Of course, neutrino stars that are well below the OW limit will have a size much larger than black holes of the same mass, although they will still be dark and much more compact than any known baryonic object of the same mass. As the gravitational potential of such an extended neutrino star is much shallower, significantly less energy will be dissipated through accreting matter than in the case of a black hole of the same mass. In fact, there is compact dark matter at the center of our galaxy with \((2.45 \pm 0.40) \times 10^6 M_\odot\) concentrated within a radius smaller than 0.0254 pc or 30.3 light-days, determined from the motion of stars in the vicinity of Sgr A*.

To save the black-hole idea, however, the neutrino-star model also fits the enigmatic radio-emission spectrum of Sgr A*. Interpreting this supermassive compact dark object in terms of a degenerate neutrino star of \( 2.5 \times 10^6 M_\odot \), the upper limit for the size of the object provides us with a lower limit for the neutrino mass, i.e.,

\[
m_\nu \geq 14.3 \text{ keV}/c^2 \quad \text{for } g_\nu = 2, \\
m_\nu \geq 12.0 \text{ keV}/c^2 \quad \text{for } g_\nu = 4. 
\]

In this context, it is important to note that if Sgr A* is a matter-accreting neutrino star, one can, in a natural way, explain the so-called “blackness problem” of Sgr A*, i.e., the fact that Sgr A* does not seem to emit detectable X-rays of a few tens of keV, which would be emitted by baryonic matter falling towards a black hole. As this unmistakable black-hole signature is missing, the concept of a “black hole on starvation” has been created in order to save the black-hole idea. However, the neutrino-star model also fits the enigmatic radio-emission spectrum of Sgr A* much better than the “black hole on starvation” model.

The total mass of the neutrino star \( M \) is plotted against the total number of particles \( N \) in Fig. 3. For masses much smaller than the OW limit, the relation between \( M \) and \( N \) is unique. However, as \( M \) approaches the OW limit, \( M \) becomes a multivalued function of \( N \). The part of the curve on the left side of the maximum in Fig. 1, which corresponds to the upper part of the curve in Fig. 2, represents unstable configurations for which the relative mass defect

\[
\Delta = \frac{Nm_\nu - M}{Nm_\nu} 
\]

eventually becomes negative, as seen in Fig. 3. Thus, for \( \Delta < 0 \), the system can gain energy by disintegrating. The maximal relative mass defect, or the strongest binding, is obtained at the OW limit with \( \Delta_{\text{OW}} = 3.5807 \times 10^{-2} \).

For completeness, we note that in the Newtonian limit \( X_0 \ll 1 \), the TOV equations (14) and (15) reduce to

\[
\frac{dX}{dx} = -\frac{\mu}{x^2 X}, \\
\frac{d\mu}{dx} = \frac{8}{3} x^2 X^3, 
\]

which, using the substitution \( \Theta = X^2 \) and \( \xi = 4x/\sqrt{3} \), can be cast into the nonlinear Lanel-Emden differential equation with the polytropic index 3/2

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\Theta}{d\xi} \right) = -\Theta^{3/2}. 
\]

Owing to the scaling property of the Lanel-Emden equation, the mass and radius of a nonrelativistic neutrino star scale as
\[ MR^3 = \frac{91.869 \hbar^5}{G^3 m_{\nu}^8} \left( \frac{2}{g_{\nu}} \right)^2. \]  

(26)

In the limit \( X_0 \ll 1 \), (3) and (4) yield the equation of state of a nonrelativistic degenerate Fermi gas, i.e.,

\[ P_{\nu} = \left( \frac{6}{g_{\nu}} \right)^{2/3} \rho_{\nu}^{5/3} \frac{\bar{\hbar}^2}{5m_{\nu}^{8/3}}. \]  

(27)

as expected.

For large central densities \( X_0 \gg 1 \), \( \mu \) oscillates around \( \mu_{\infty} = 0.09196 \), which corresponds to a neutrino star mass \( M_{\infty} = 1.795 \times 10^8 M_{\odot} \). For a gas of neutralinos of a mass precisely equal to the neutron mass \( m_n = 0.93955 \text{ GeV}/c^2 \) and a degeneracy factor \( g_n = 2 \), the infinite density limit is \( M_{\infty} = 0.4164 M_{\odot} \), whereas the OV limit is \( M_{OV} = 0.7091 M_{\odot} \) and \( R_{OV} = 9.1816 \text{ km} \). Thus, owing to their compactness, neutralino stars could mimic the properties of “machos” which have been detected in the dark halo of our galaxy, and which are usually assumed to be baryonic brown dwarfs. For large \( X \), the solutions of the TOV equations (14) and (15) tend to

\[ \mu = \frac{3}{14} x \quad \text{and} \quad X = \left( \frac{3}{28} \right)^{1/4} x^{-1/2}. \]  

(28)

The pressure and the density thus become

\[ P_{\nu} = \frac{c^4}{56\pi} \frac{1}{r^2} \quad \text{and} \quad \rho_{\nu} = \frac{3c^2}{56\pi} \frac{1}{r^2}, \]  

(29)

yielding the equation of state of radiation

\[ P_{\nu} = \frac{1}{3} c^2 \rho_{\nu}, \]  

(30)

as expected.

III. DEGENERATE NEUTRINO AND NEUTRALINO MATTER

We now turn to the discussion of an astrophysical system consisting of degenerate heavy-neutrino and neutralino matter that is gravitationally coupled. As each component satisfies the equation of hydrostatic equilibrium separately, i.e., Eq. (2) and

\[ \frac{dP_n}{dr} = -\frac{1}{2} (\rho_n c^2 + P_n) \frac{d\nu}{dr}, \]  

(31)

the total pressure \( P = P_n + P_\nu \) and the total mass density \( \rho = \rho_n + \rho_\nu \) will also obey the same equation

\[ \frac{dP}{dr} = -\frac{1}{2} (\rho c^2 + P) \frac{d\nu}{dr}. \]  

(32)

In addition to the equation of state for neutrino matter, (3) and (4), we now have the equation of state for neutralino matter:

\[ P_n = K \frac{g_n}{g_{\nu}} \left( \frac{m_n}{m_{\nu}} \right)^4 Y^2 \left( \frac{2}{3} Y - 1 \right) \log \left( Y + (1 + Y^2)^{1/2} \right), \]  

(33)

\[ \rho_n = K \frac{g_n}{c^2 g_{\nu}} \left( \frac{m_n}{m_{\nu}} \right)^4 Y^2 \left( 2Y^2 + 1 \right) \log \left( Y + (1 + Y^2)^{1/2} \right), \]  

(34)

where \( g_n \) is the spin-degeneracy factor for neutralinos and antineutralinos, and \( Y \) is the local Fermi momentum of neutralino matter (in units of \( m_n c \)). Inserting (33) and (34) into (31), after integration we arrive at

\[ Y = \left( (1 + Y^2)^2 \right)^{1/2}, \]  

(35)
with $Y_0 = Y(0)$. Using (35), (36), and the equation of hydrostatic equilibrium (2), a similar relation for the Fermi momentum of neutrinos (in units of $m_\nu c$) is obtained:

$$X = [(1 + X_0^2)e^{\nu(0) - \nu(r)} - 1]^{1/2}. \quad (36)$$

Combining (35) and (36), the two local Fermi momenta are related by

$$X^2 = \frac{(X_0^2 + 1)Y^2 + X_0^2 - Y_0^2}{1 + Y_0^2}. \quad (37)$$

The condition $X^2 \geq 0$ restricts the range of allowed values of $Y$ to

$$Y^2 \geq \frac{Y_0^2 - X_0^2}{1 + X_0^2}. \quad (38)$$

The total pressure and mass density is given by

$$P(Y) = P_n(Y) + P_\nu(X(Y)) \quad (39)$$

and

$$\rho(Y) = \rho_n(Y) + \rho_\nu(X(Y)), \quad (40)$$

respectively.

We now formulate the coupled differential equations describing a gravitationally interacting system of degenerate heavy-neutrino and neutralino matter. We first keep the mass of the neutrino halo constant while varying the mass of the neutralino star. Introducing the dimensionless variables $x = r/a_n$ and $\mu = m/b_n$ with the scales

$$a_n = \frac{2}{m_\nu^2} \sqrt{\frac{\pi h^3}{g_n G}} \quad \text{and} \quad b_n = \frac{2}{m_\nu^2} \sqrt{\frac{\pi h^3 c^3}{g_n G^3}}, \quad (41)$$

the relevant TOV equations can be written in the form

$$\frac{dY}{dx} = -\frac{1 + Y^2}{Y(x^2 - 2 \mu x)} \left\{ \mu + x^3 \left[ Y(1 + Y^2)^{1/2} \left( \frac{2}{3} Y^2 - 1 \right) + \log \left( Y + (1 + Y^2)^{1/2} \right) \right. \\
+ \left. \left( \frac{m_\nu}{m_n} \right)^4 \frac{g_\nu}{g_n} \left( X(1 + X^2)^{1/2} \left( \frac{2}{3} X^2 - 1 \right) + \log \left( X + (1 + X^2)^{1/2} \right) \right) \right]\right\}, \quad (42)$$

$$\frac{d\mu}{dx} = x^2 \left\{ Y(1 + Y^2)^{1/2}(2Y^2 + 1) + \log \left( Y + (1 + Y^2)^{1/2} \right) \right. \\
+ \left. \left( \frac{m_\nu}{m_n} \right)^4 \frac{g_\nu}{g_n} \left( X(1 + X^2)^{1/2}(2X^2 + 1) + \log \left( X + (1 + X^2)^{1/2} \right) \right) \right\}, \quad (43)$$

where $X$ is related to $Y$ through (37). If the condition (38) is not fulfilled, i.e., the neutrino pressure and density have already vanished, the system is solved with the $Y$ terms describing the neutralinos only.

In order to solve Eqs. (42) and (43) numerically, we fix the Fermi momentum of neutrinos (in units of $m_\nu c$) at the center and vary the central values of the corresponding quantity $Y_0$ for neutralinos. The total mass (including neutrinos and neutralinos) enclosed within the radius $R_n$ of the neutralino star is shown in Fig. 4. Here, the neutrino mass and the degeneracy factor are taken to be $m_\nu = 17.2$ keV/c² and $g_\nu = 2$, respectively, while for the neutralino mass we have chosen $m_n = 939.55$ MeV/c² and $g_n = 2$, with the scales $a_n = 6.8304$ km and $b_n = 4.6257 \ M_\odot$.

For small neutralino-star masses, the total mass enclosed in $R_n$ scales as $R_n^3$, corresponding to a constant density governed by the gravitational potential of the surrounding supermassive neutrino halo. However, as the radius of the neutralino star approaches that of a “free” neutralino star, the gravitational potential of the neutralino star becomes dominant and the mass now scales as $R_n^{-3}$ up to the OV limit. Thus there is always a maximal radius of a neutralino star within a neutrino halo of a given mass. Substituting neutralinos by neutrons, we must take care of the fact that (i) the neutron interacts strongly in the nuclear medium (simulated, e.g., by an effective mass) and (ii) the neutron decays through weak interactions. Thus, stable neutron stars can exist only in the range from 0.2 $M_\odot$ to 2 $M_\odot$ [31], where the binding energy is larger than the Q value for the neutron decay.
It is instructive to study the properties of a degenerate gas of neutralinos and neutrinos in the nonrelativistic approximation. In the limits \( X \ll 1 \) and \( Y \ll 1 \), (42) and (43) simplify to

\[
\frac{dY}{dx} = -\frac{\mu}{x^2 Y},
\]

\[
\frac{d\mu}{dx} = \frac{8}{3} x^2 [Y^3 + g_n \left( \frac{m_n}{m_\nu} \right)^4 (Y^2 + X_0^2 - Y_0^2)^{3/2}],
\]

with the boundary conditions

\[
\mu(0) = 0; \quad Y^2 \geq Y_0^2 - X_0^2; \quad Y(0) = Y_0.
\]

This system of equations can be rewritten in the form of a Landé-Emden type equation by introducing \( \Theta_n = Y^2 \), \( \Theta_\nu = X^2 \), and a new radial dimensionless radial variable \( \xi = 4x/\sqrt{3} \)

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\Theta_n}{d\xi} \right) = -[\Theta_n^{3/2} + g_n \left( \frac{m_n}{m_\nu} \right)^4 (\Theta_n + \Theta_\nu - \Theta_\nu)\ell^{3/2}],
\]

where \( \Theta_\nu \) and \( \Theta_{\nu_0} \) are the central values of the neutralino and neutrino densities, respectively. For very small neutralino densities, i.e., \( Y \ll 1 \) and \( Y_0 \ll 1 \), the mass equation (45) can be integrated to give

\[
\mu(x) = \frac{8}{9} \left( \frac{m_n}{m_\nu} \right)^4 x^3 Y_0^3 X_0^3,
\]

which confirms the conclusion drawn in the context of Fig. 4.

We now turn to the case of a neutralino star of constant mass surrounded by a neutrino halo of variable mass. The TOV equations written in terms of the functions \( X \) and \( \mu \) may be obtained from (42) and (43), in which we make the replacements \( X \leftrightarrow Y \), \( g_n \leftrightarrow g_\nu \), and \( m_\nu \leftrightarrow m_n \). Thus, we find

\[
\frac{dX}{dx} = \frac{1 + X^2}{X(x^2 - 2\mu x)} \left\{ \mu + x^3 \left[ X(1 + X^2)^{1/2} \left( \frac{2}{3} X^2 - 1 \right) + \log \left( X + (1 + X^2)^{1/2} \right) \right] \right\}
\]

\[
+ \left( \frac{m_n}{m_\nu} \right)^4 \left( \frac{g_n}{g_\nu} \right) \left[ Y(1 + Y^2)^{1/2} \left( \frac{2}{3} Y^2 - 1 \right) + \log \left( Y + (1 + Y^2)^{1/2} \right) \right] \}
\]

\[
\frac{d\mu}{dx} = x^2 \left[ X(1 + X^2)^{1/2}(2X^2 + 1) + \log \left( X + (1 + X^2)^{1/2} \right) \right]
\]

\[
+ \left( \frac{m_n}{m_\nu} \right)^4 \left( \frac{g_n}{g_\nu} \right) \left[ Y(1 + Y^2)^{1/2}(2Y^2 + 1) + \log \left( Y + (1 + Y^2)^{1/2} \right) \right] \}
\]

with \( X \) and \( Y \) subject to the condition

\[
X^2 \geq \frac{X_0^2 - Y_0^2}{1 + Y_0^2}.
\]

If this condition is not satisfied, i.e., the pressure and density of neutralinos have already vanished, (45) and (50) are solved without the \( Y \) terms, i.e., for neutrinos only. Choosing the OV limit as the mass of the neutralino star, i.e., \( M_{\text{OV}}^n = 0.7091 \, M_\odot \) for \( m_n = 0.93955 \, \text{GeV}/c^2 \) and \( g_n = 2 \), and varying the central Fermi momentum \( X_0 \), one can find the total mass (including neutralinos and neutrinos) as a function of the radius \( R_\nu \) of the neutrino halo. This scenario is reflected in Fig. 5 where the length and mass scales are \( a_\nu = 2.0381 \times 10^{10} \, \text{km} \) and \( b_\nu = 1.3803 \times 10^{10} \, M_\odot \), respectively. Here the neutrino mass and the degeneracy factor have been chosen as \( m_\nu = 17.2 \, \text{keV}/c^2 \) and \( g_\nu = 2 \), respectively. At the turning point \( A \), the total mass enclosed within the radius \( R_A = R_{\text{OV}}^n = 9.1816 \, \text{km} \) of the neutrino halo is \( M_A = M_{\text{OV}}^n = 0.7091 \, M_\odot \). At the turning point \( B \), the total mass enclosed within the radius \( R_B = 0.9912 \, \text{pc} \) of the neutrino halo is \( M_B = 3.3453 \, M_\odot \). It is interesting to note that, also in this case, there is a maximal radius \( R_B \) of the neutrino halo, for a given mass of the neutralino star.

Replacing the neutralino star by a baryonic star, such as a neutron star, a white dwarf, or an ordinary star, the only thing that will change in Fig. 5 is the point \( A \) at which the enclosed mass starts deviating from the constant
value, which depends, of course, on the mass $M_n$ of the central object. Thus for $M_n \gtrsim M_\odot$, the halo will have a size of a few light-years and a mass of a few times that of the central baryonic or nonbaryonic star. In this context, it is important to note that if every baryonic star is surrounded by such a neutrino halo, the degeneracy pressure of the neutrino halo would prevent stars from approaching each other closer than a distance of a few light-years. In such a scenario, a large fraction of galactic dark matter would be nonbaryonic. A further attractive feature of this scenario is that a neutrino mass of the order of 14 or 15 keV/$c^2$, could, at the same time, set the mass scale of the supermassive compact dark objects at the centers of galaxies and the scale of interstellar distances in galaxies.

To investigate the consequences of this idea in more detail, let us assume that the sun is surrounded by a degenerate neutrino halo. In the vicinity of the sun, in the region of the size of the planetary system, the neutrino density is governed by the gravitational potential of the sun. In fact, the mass due to neutrinos contained within a radius $r$ is, in the vicinity of a baryonic or a nonbaryonic star of mass $M_n$, given in the nonrelativistic approximation [16,23] by

$$\frac{M_\nu}{M_\odot} = 1.34 \times 10^8 g_\nu \left( \frac{M_\nu}{M_\odot} \right)^{3/2} \left( \frac{m_\nu c^2}{17.2 \text{keV}} \right)^4 \left( \frac{r}{\text{AU}} \right)^{3/2}, \quad (52)$$

where AU = 1.496 × 10$^8$ km is the astronomical unit. This means that for $m_\nu c^2 = 17.2$ keV, $g_\nu = 2$, and $M_n = M_\odot$, the mass of the neutrino (and antineutrino) halo contained within the earth’s orbit would be $M_\nu = 2.68 \times 10^{-8} M_\odot$.

From the Pioneer 10 and 11 and the Voyager 1 and 2 ranging data [33] we know that the dark mass contained within Jupiter’s orbit is $M_d = (0.12 \pm 0.027) \times 10^{-8} M_\odot$ and within Neptun’s orbit $M_d \leq 3 \times 10^{-8} M_\odot$. Of course, the Jupiter data should be taken only as a lower limit, as Jupiter tends to eject almost any matter within its orbit [33]. Nevertheless, taking the Jupiter data at face value, and interpreting dark matter as degenerate neutrino matter, the neutrino mass limits are

$$12.6 \text{keV}/c^2 \leq m_\nu \leq 14.2 \text{keV}/c^2 \quad \text{for } g_\nu = 2,$$

$$10.6 \text{keV}/c^2 \leq m_\nu \leq 12.0 \text{keV}/c^2 \quad \text{for } g_\nu = 4. \quad (53)$$

For dark matter within Neptun’s orbit, the neutrino mass limits are

$$m_\nu \leq 15.6 \text{keV}/c^2 \quad \text{for } g_\nu = 2,$$

$$m_\nu \leq 13.1 \text{keV}/c^2 \quad \text{for } g_\nu = 4. \quad (54)$$

In summary, considering (20), (21), and (54), a neutrino mass-range

$$14.3 \text{keV}/c^2 \leq m_\nu \leq 15.6 \text{keV}/c^2 \quad \text{for } g_\nu = 2,$$

$$12.0 \text{keV}/c^2 \leq m_\nu \leq 13.1 \text{keV}/c^2 \quad \text{for } g_\nu = 4. \quad (55)$$

seems to be consistent with all reliable data.

IV. CONCLUSIONS

We have studied degenerate fermion stars, consisting of massive neutrinos or neutralinos, or both. We have shown that the existence of such objects may have important astrophysical implications.

For neutrino masses in the range of several keV, neutrino stars are natural candidates for the supermassive dark objects at the centers of galaxies. Assuming that the most massive object, such as the compact dark object at the center of M87, is a neutrino star at the OV limit, the neutrino mass required for this scenario should be between 10 keV/$c^2$ and 16 keV/$c^2$, depending on the degeneracy factor $g_\nu$.

Furthermore, interpreting the supermassive dark object at the center of our galaxy as a neutrino star, we obtain from the upper limit of the size of this object, a lower bound on the neutrino mass which overlaps with the range mentioned above. In addition, our interpretation explains the so-called “blackness problem” of Sgr A* in a natural way.

By studying a two-component system consisting of neutralinos in the GeV mass range and neutrinos in the keV mass range, we have found that there is always a maximal mass and radius of a neutralino star within a neutrino halo of a given mass. Owing to their compactness, neutralino stars could mimic the properties of “machos”.

Finally, assuming that ordinary stars are surrounded by degenerate neutrino halos of maximal size, a neutrino mass of the order of 15 keV/$c^2$ sets the scale of interstellar distances to a few light-years.
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FIG. 1. The total mass $M$ of a neutrino star as a function of its radius $R$.

FIG. 2. The total mass $M$ of a neutrino star as a function of its total number of particles $N$.

FIG. 3. The relative mass defect $\Delta$ as a function of the radius $R$ of the neutrino star.

FIG. 4. The total mass (including neutralinos and neutrinos) enclosed within the radius $R_n$ of the neutralino star for various masses $M_\nu$ of the neutrino halo.

FIG. 5. The total mass of neutralinos and neutrinos contained within the radius $R_\nu$ of the neutrino halo around a neutralino star.
