Study on the Compressive Elastic Modulus and Poisson's Ratio of Tobacco Main Veins

Jie Yu, Jinkui Chu* and Yang Li

School of Mechanical Engineering, Dalian University of Technology, Dalian, China
*Email: chujk@dlut.edu.cn

Abstract. Mechanical properties of the tobacco main veins are basic parameters for the design of tobacco leaves harvester. And the compressive elastic modulus and Poisson’s ratio of tobacco main veins are related to the contact force between the tobacco leaves and the harvesting tool, which is an important design parameter for tobacco leaves harvester. In this work, lateral compression tests were carried out on tobacco main veins. And the force-displacement equation during lateral compression was derived based on the Hertz contact theory. Then the compressive elastic modulus and Poisson’s ratio were obtained by the method of piecewise nonlinear fitting. The fitting results of Poisson's ratio are in the range of 0.33 to 0.56. The fitting results of the compressive elastic modulus are in the range of 1.30MPa to 3.07MPa. And the compressive elastic modulus increases with the increase of strain.

1. Introduction
Tobacco is an important cash crop, which is widely cultivated around the world. With the development of agricultural mechanization, mechanical harvesting of tobacco leaves has greatly improved the efficiency of tobacco leaves harvesting. However, there are few studies on the mechanical properties of tobacco leaves, which hinders the further development of tobacco leaves harvester. Especially, the compressive elastic modulus and Poisson’s ratio of tobacco main veins are important mechanical parameters for studying the contact force between the tobacco leaves and the harvesting tool. Therefore, the focus of this work is to obtain the compressive elastic modulus and Poisson's ratio of tobacco main veins.

The uniaxial compression test is usually used to obtain the compressive elastic modulus and Poisson's ratio of the material. In addition to the original loading measurement system, a measurement system for measuring the horizontal deformation of the specimen is also required [1,2], which is very complicated. In order to simplify the experimental conditions, the force-displacement equation during lateral compression was derived based on the Hertz contact theory in this work. And the compressive elastic modulus and Poisson's ratio were obtained through only one loading measurement system.

2. Measuring Principle
The schematic diagram of the tobacco main vein compression test is shown in figure 1. The tobacco main vein is considered as an elastic semi-cylinder in the case of small strain. There are two rigid plates above and below the main vein. The bottom plate is fixed and a vertical force is applied to the upper plate. That is, the cylindrical surface is in contact with the upper plate. According to the Hertz elastic contact theory, when a cylinder is in contact with a rigid plane, the half Hertzian contact width \( b \) can be expressed as [3,4]
in which $F$ is the force, $R$ is the radius of the main vein, $L$ is the length of the main vein, $\nu$ is the Poisson’s ratio of the main vein, $E$ is the compressive elastic modulus of the main vein.

**Figure 1.** Schematic diagram of the tobacco main vein compression test.

Based on the Hertz elastic contact theory, the contact surface is a narrow rectangle. And then the polar coordinate system is established with the center of the rectangle as the origin, as shown in figure 2. The displacement of the origin along the radial direction of the cylinder is defined as $w$. The rectangle is geometrically symmetrical along the $x$ and $y$ axes. And the contact stress distribution is also symmetrical. Therefore, a quarter of the rectangle is taken to calculate the relationship between force and displacement, as shown in figure 3.

**Figure 2.** Rectangular contact surface.

The infinitesimal element of origin displacement ($dw$) caused by the infinitesimal element of the load ($qrdrd\theta$) can be expressed as

$$dw = \frac{(1-\nu^2)}{\pi Er} qrdrd\theta = \frac{(1-\nu^2)}{\pi E} qdrd\theta$$

(2)

where $q$ is the stress distribution along the width of the contact surface, $r$ and $\theta$ are the polar radius and polar angle of the infinitesimal element, respectively. And the stress distribution is as follows

$$q = \sqrt{1 - \frac{y^2}{b^2}} \cdot q_0$$

(3)
\[ q_0 = \sqrt{\frac{FE}{L\pi R(1-\nu^2)}} \]  

(4)

\[ y = r\sin\theta \]  

(5)

in which \( q_0 \) is the maximum contact stress, \( y \) is the coordinate value.

Substituting equation (3-5) into equation (2), and then integrating, yields

\[ w = \frac{1-\nu^2}{\pi E}q_0\int \int \sqrt{1-\frac{r^2\sin^2\theta}{b^2}} \, dr \, d\theta \]  

(6)

Integrating the area in figure 3(a), yields

\[ w_1 = \frac{1-\nu^2}{\pi E}q_0\int_{\frac{\pi}{2}}^{\frac{b}{2}} \int_{0}^{L} \frac{1}{\cos(\frac{\pi}{2}\theta)} \sqrt{1-\frac{r^2\sin^2\theta}{b^2}} \, dr \, d\theta \]

\[ = \frac{1-\nu^2}{\pi E}q_0 \int_{\arctan(\frac{2b}{L})}^{\arctan(\frac{2b}{L})} L \cdot \frac{1}{\cos\theta} \sqrt{1-(\frac{L}{2b}\tan\theta)^2} \, d\theta + \frac{1}{2} \cdot \frac{b}{\sin\theta} \cdot \arcsin(\frac{L}{2b}\tan\theta) \]  

(7)

Integrating the area in figure 3(b), yields

\[ w_2 = \frac{1-\nu^2}{\pi E}q_0 \int_{\arctan(\frac{2b}{L})}^{\arctan(\frac{2b}{L})} L \cdot \frac{1}{\cos\theta} \sqrt{1-(\frac{L}{2b}\tan\theta)^2} \, d\theta \]

\[ = \frac{1-\nu^2}{\pi E}q_0 \int_{\arctan(\frac{2b}{L})}^{\arctan(\frac{2b}{L})} \left[ \frac{L}{4} - \frac{1}{\cos\theta} \sqrt{1-(\frac{L}{2b}\tan\theta)^2} \right] \, d\theta \]  

(8)

Expanding the trigonometric functions in equation (9) with power series, and then integrating

\[ w_2 = \frac{1-\nu^2}{\pi E}q_0 \left[ \frac{L}{2}t + \left( \frac{L}{12} - \frac{L^3}{144b^2} \right)t^3 + \left( \frac{L}{48} - \frac{7L^3}{1440b^2} - \frac{L^5}{6400b^4} \right)t^5 \right. \]

\[ + \left( \frac{31L}{30240} - \frac{311L^3}{120960b^2} - \frac{11L^5}{53760b^4} - \frac{L^7}{28672b^6} \right)t^7 \]  

(9)

Therefore, the radial displacement of the center of the contact surface is as follows

\[ w = 4(w_1 + w_2) = \frac{4L}{\pi} \sqrt{\frac{F(1-\nu^2)}{LR\pi E}} \cdot \left( a_1 t + a_2 t^3 + a_3 t^5 + a_4 t^7 - \frac{\pi b}{4L} \ln \frac{\sin t}{1 + \cos t} \right) \]  

(11)

\[
\begin{align*}
  a_1 &= \frac{1}{2} \\
  a_2 &= \frac{1}{12} - \frac{L^2}{144b^2} \\
  a_3 &= \frac{1}{48} - \frac{7L^2}{1440b^2} - \frac{L^4}{6400b^4} \\
  a_4 &= \frac{31}{30240} - \frac{311L^2}{120960b^2} - \frac{11L^4}{53760b^4} - \frac{L^6}{28672b^6}
\end{align*}
\]  

(12)
Therefore, the lateral compression test can be carried out on tobacco main veins to obtain the force-displacement relationship. And then equation (11-12) can be used for nonlinear fitting to obtain the compressive elastic modulus and Poisson's ratio. It should be noted that the derivation process refers to the method in Ref.[5].

3. Mechanical experiments

3.1. Experimental equipment and materials
In this work, the Instron 3345 material testing machine is used for mechanical experiments. The test force of the machine is in the range of 0-5 kN, the speed is in the range of 0.05-1000 mm/min, and the measurement accuracy of force and displacement is ±0.5%. The tobacco main vein samples are selected from the mature middle leaves of the representative tobacco variety “Zhongyan-100(CF965)”. The length of the main vein samples is cut to 15mm. And the radius values of the main vein samples are listed in table 1.

| Number | 1  | 2  | 3  | 4  | 5  | Average value |
|--------|----|----|----|----|----|---------------|
| Radius (mm) | 10.72 | 11.40 | 10.20 | 11.68 | 11.04 | 11.01 |

3.2. Lateral (radial) compression test
In order to make the main vein evenly stressed, two PMMA plates are set above and below the main vein. And the main veins need to be loaded slowly during the test. Therefore, the loading rate of the indenter is set to 2mm/min. The compression test of the main vein is shown in figure 4.

![Figure 4. Compression test of the main vein. 1. Indenter 2. Tobacco main vein 3. PMMA plates.](image)

![Figure 5. Force-displacement curves.](image)

4. Results and discussions
The force-displacement curves obtained from the compression tests are shown in figure 5. It can be found that stress relaxation occurred in the main vein during the compression process. That is, the tobacco main veins show viscoelastic properties [6]. In order to simplify the calculation model, the tobacco main vein is still regarded as an elastic semi-cylinder in this work.

It should be noted that the main vein is not a homogeneous isotropic material. The compressive elastic modulus and Poisson's ratio are not fixed values, but change with strain. Therefore, the method of piecewise fitting is applied in this work. Taking a group of 60 data points (displacement of 2mm), and the nonlinear fitting is performed for each group by using equation (11-12). Figure 6 shows the fitting curves of one of the samples. It can be seen that the fitting results are not ideal in the stress relaxation area. And when the deformation is large, the influence of plastic deformation will not be...
ignored. Therefore, only the fitting results of compressive elastic modulus and Poisson's ratio before occurring the stress relaxation are discussed, as shown in table 2 and table 3. Since the radius of the main vein samples is similar in size, the radius value $R$ is unified to 11mm when calculating the strain in table 2 and table 3.

### Table 2. Fitting results of Poisson's ratio.

| Strain      | 1  | 2  | 3  | 4  | 5  |
|-------------|----|----|----|----|----|
| 0-0.018     | 0.44| 0.55| 0.55| 0.48| 0.39|
| 0.018-0.036 | 0.56| 0.34| 0.34| 0.35| 0.39|
| 0.036-0.055 | 0.52| 0.35| 0.53| 0.33| 0.37|
| 0.055-0.073 | 0.36| 0.35| 0.34| 0.35| 0.37|
| 0.073-0.091 | 0.34| 0.37| 0.37| 0.37| 0.36|
| 0.091-0.109 | 0.38| 0.38| 0.41| 0.39| 0.35|
| 0.109-0.127 | 0.41| 0.36| 0.41| 0.40| 0.33|
| 0.127-0.145 | 0.40| 0.34| 0.38| 0.42| 0.34|
| 0.145-0.164 | 0.39| 0.36| 0.42| 0.43| 0.46|
| 0.164-0.182 | 0.38| 0.37| 0.45| 0.43| 0.50|
| 0.182-0.200 | 0.54| 0.39| 0.45| 0.43| 0.35|
| 0.200-0.218 | 0.43| 0.41| 0.42| 0.40| 0.34|

### Table 3. Fitting results of compressive elastic modulus.

| Strain (average value) | Strain | 1   | 2   | 3   | 4   | 5   | Average value (MPa) |
|------------------------|--------|-----|-----|-----|-----|-----|---------------------|
| 0-0.018                | 0.0090 | 1.30| 1.44| 1.61| 1.52| 1.33| 1.44               |
| 0.018-0.036            | 0.0270 | 1.54| 1.95| 1.78| 1.69| 1.34| 1.66               |
| 0.036-0.055            | 0.0455 | 1.61| 2.03| 1.85| 1.73| 1.73| 1.79               |
| 0.055-0.073            | 0.0640 | 1.77| 2.10| 1.97| 1.87| 2.19| 1.98               |
| 0.073-0.091            | 0.0820 | 2.12| 2.37| 2.24| 2.41| 2.26| 2.28               |
| 0.091-0.109            | 0.1000 | 2.45| 2.44| 2.62| 2.61| 2.43| 2.51               |
| 0.109-0.127            | 0.1180 | 2.68| 2.64| 2.64| 2.62| 2.77| 2.67               |
| 0.127-0.145            | 0.1360 | 2.78| 2.66| 2.63| 2.48| 2.85| 2.68               |
| 0.145-0.164            | 0.1545 | 2.72| 2.69| 2.72| 2.67| 2.80| 2.72               |
| 0.164-0.182            | 0.1730 | 2.85| 2.80| 2.92| 2.78| 2.75| 2.82               |
| 0.182-0.200            | 0.1910 | 2.97| 2.92| 2.94| 2.79| 2.98| 2.94               |
| 0.200-0.218            | 0.2090 | 3.01| 3.07| 2.98| 2.85| 2.94| 2.97               |

Table 2 shows the fitting values of Poisson's ratio under different strain conditions. And the values of Poisson's ratio are in the range of 0.33 to 0.56. Table 3 shows the fitting values of compressive elastic modulus under different strain conditions. The values of compressive elastic modulus are in the range of 1.30MPa to 3.07MPa. It can be found that as the value of strain increases, the value of compressive elastic modulus increases. To show the relationship between the compressive elastic modulus and the strain, a third-order polynomial fit is applied to the experimental data in table 3, as shown in figure 7. It should be noted that the abscissa of each data point is the average value of each strain segment. The relationship between the compressive elastic modulus and the strain is as follows

$$E = -32.2462\varepsilon^2 + 14.8423\varepsilon + 1.2634$$

(13)

in which $E$ represents the compressive elastic modulus, $\varepsilon$ represents the strain. And the fitting error is very small, within the range of ±0.1MPa.
5. Conclusions
In order to obtain the compressive elastic modulus and Poisson's ratio of the tobacco main veins, lateral compression tests were carried out on tobacco main veins in this work. Specifically, the Hertz contact theory was used to derive the force-displacement equation during lateral compression, and then the force-displacement results obtained from the lateral compression tests were nonlinearly fitted using the equation. Since the main vein is not a homogeneous isotropic material, the compressive elastic modulus and Poisson's ratio are not fixed values. Therefore, a piecewise fitting method was adopted in this work to obtain the compressive elastic modulus and Poisson's ratio under different strain conditions. The fitting results of Poisson's ratio are in the range of 0.33 to 0.56. The fitting results of the compressive elastic modulus are in the range of 1.30MPa to 3.07MPa. For commonly used metal materials, the elastic modulus is a fixed value. However, for plant materials such as tobacco main veins, the elastic modulus varies with strain due to its anisotropy. And it can be obtained from this work that the compressive elastic modulus value of tobacco main veins increases with the increase of strain. Meanwhile, the specific relationship between strain and compressive elastic modulus is obtained in this work. And the study on tobacco main veins in this work can provide a reference for similar biological materials.

Acknowledgments
This research was funded by the National Natural Science Foundation of China (No.51675076) and the Fundamental Research Funds for the Central Universities (DUT20LAB303).

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