Apparent Lightning Return Stroke Speed and the Correction

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Abstract. A varying return-stroke speed during the propagation process along the lightning channel is extensively reported. In this paper, we found, due to the propagation time difference of the emitted light by lightning in the propagation path from the lightning channel to the optical observation point, even in the case of an assumed constant return-stroke speed, the observed speed, termed as the apparent return-stroke speed, is not constant. For conventional return-stroke engineering models that the return stroke is assumed to propagate from the ground, we investigated the influence of the observation distance, the observation altitude, and the assumed value of the return-stroke speed on the variation trend of the apparent speed. In addition, the deviation of the apparent speed from its actual value is compared for the concerned influencing factors. The need of propagation time correction in obtaining the characteristics of return stroke propagation from the time-resolved optical records is emphasized here.

Keywords: electromagnetic fields, initiation process, lightning, modeling, return stroke, return-stroke speed.

1. Introduction

The return-stroke speed is an important parameter in lightning studies. In the models of calculating the electromagnetic fields [1-3], return-stroke speed is a basic input parameter. Meanwhile, an assumption of the return-stroke speed is involved in inferring lightning currents from remotely measured electric or magnetic fields [4-7]. Return-stroke speed is also of importance in lightning protection studies. For example, analysis on the return-stroke speed in the attachment process can be used in obtaining the initiation height and accordingly, the striking distance [8, 9].

The propagation speed of the return-stroke current, although cannot be directly measured, is commonly assumed to be the same as the return-stroke speed from time-resolved optical records. An extensive effort has already been made to determine the return-stroke speed in cloud to ground flashes [8-21]. The measured return-stroke speed generally ranged from 1/3 to 2/3 of the speed of light. And, specially, variation of return-stroke speed with height has widely been reported. For instance, Idone and Orville [12], using the high-speed streaking photographic techniques, reported that the observed return-stroke speed decreased with height, and that velocities in upper channel lengths were often reduced by 25% or more relative to speeds near ground. Mach and Rust [18] found that the return-stroke speed was larger over channel segments of about 330 m from the ground than that over channel segments of about 900 m from the ground. Wang et al [19], from two dimensional speed profiles within 400 m of the ground for two return strokes in rocket-triggered lightning, also noted that return strokes exhibited a
speed decrease as they propagated upward. The return-stroke speeds within the bottom 60 m of the channel were $1.3\times10^8$ and $1.5\times10^8$ m/s. However, a different varying trend was observed by Olsen et al. [21] in the bottom channel segments between 7 m and 170 m of a rocket-triggered lightning, as the return-stroke speed initially increased with height and then decreased with increasing height.

In this study, we found, even if the return stroke has a constant optical propagation speed, the remote observer will “see” a varying return-stroke speed. We also found that the observation position (e.g., the observation altitude, the observation distance from the lightning channel) will exert great influence on the observed characteristics of lightning return stroke propagation from optical measurement.

2. Apparent Return-stroke speed

As shown in Fig. 1, in conventional return-stroke models, the return stroke is assumed to create an extending vertical channel whose lower end is fixed at the ground, and upper end is associated with the return-stroke front propagating upward from the ground with a constant speed $v_f$. The remote observer at P (assumed to be at the ground level) “sees” the return-stroke front passing a height $z'$ at the time $t_b(z')=z'/v_f+R(z')/c$. The observed height $H(t)$ by the remote observer at time $t$, is thus given by the solution of the equation

$$t = \frac{H(t)}{v_f} + \frac{[H(t)^2 + r^2/v_f^2]}{c} \tag{1}$$

Where $c$ is the speed of light. Since $H(t) \geq 0$, $t \geq r/c$. By substitution we find

$$\left(\frac{1}{v_f} - \frac{1}{c}\right) \cdot \frac{H(t)^2 - 2t}{v_f} + \frac{H(t)}{v_f} + \frac{r^2}{c^2} = 0. \tag{2}$$

Then $H(t)$ is derived as:

![Fig. 1](image) Geometry used in deriving the expressions for apparent height of return-stroke front and apparent return-stroke speed at a point P on ground a horizontal observation distance $r$ from the vertical lightning return stroke channel. The return stroke front is assumed to propagate upward from the ground with a constant speed $v_f$. 


Fig. 2 An example of actual height of return-stroke front and actual return-stroke speed, as well as the corresponding apparent height and apparent speed, for an assumed upward return-stroke speed $v_f = 1.3 \times 10^8$ m/s and observation distance $r = 200$ m at ground.

$$H(t) = v_f \cdot \left[ \frac{c^2 t - (v_f^2 c^2 t^2 - v_f^2 r^2 + c^2 r^2)^{1/2}}{(c^2 - v_f^2)} \right].$$  \hspace{1cm} (3)

$H(t)$ is the so-called apparent height of return-stroke front, to distinguish from its actual height. Accordingly, the observed return-stroke speed $v$ by the remote observer, or the apparent return-stroke speed, is the first-time derivation of $H(t)$ given by

$$v(t) = \frac{dH(t)}{dt} = v_f \cdot \left[ \frac{1 - v_f^2 t / (v_f^2 c^2 t^2 - v_f^2 r^2 + c^2 r^2)^{1/2}}{1 - (v_f/c)^2} \right].$$  \hspace{1cm} (4)

Using (3) and (4), in Fig. 2 the actual height and the apparent height of return-stroke front are plotted and shown on the left axis, while the corresponding actual and apparent return-stroke speeds are shown on the right axis, for an assumed return-stroke speed $v_f = 1.3 \times 10^8$ m/s. The horizontal distance $r$ is exemplified as 200 m. Clearly, even in the case of an assumed constant actual return-stroke speed, the apparent speed is not constant. As the time increases, or in other words, as the height of return-stroke front increases, the apparent return-stroke speed decreases from an initial value equaling to the actual speed, to a relatively constant, but smaller value than the actual speed.

3. Influence of the Observation Position

The apparent speed deviation is adopted here to denote the difference between the actual return-stroke speed and the apparent return-stroke speed. In (5), $v$ and $v_f$ are the apparent return-stroke speed and actual return-stroke speed, respectively. Note that, in all the following analysis, the actual return-stroke speeds are assumed to be constant. Note further that, if not specified otherwise, throughout this paper the apparent return-stroke speed represents the optical propagation speed, not the current wave speed.

$$\frac{v - v_f}{v_f} \times 100\%$$  \hspace{1cm} (5)

3.1. Observation at Ground

Fig. 3 shows, for the case of observation at ground (see the geometry in Fig. 1), a plot of the speed deviation at observation distance $r = 100$ m, 200 m, and 1 km, as a function of the corresponding apparent height of return-stroke front. The curves in the plot were obtained for four different actual return-stroke
speeds, i.e., $v_f = c/3$, $v_f = c/2$, $v_f = 2c/3$, and $v_f = 0.99c$ (close to light speed). As can be seen from Fig. 3, although for different observation distances, the maximum possible deviation of the apparent speed is the same (related to the actual return-stroke speed), increasing the actual return-stroke speed corresponds to an increase deviation of the apparent speed. Also, for a given value of actual return-stroke speed, closer observation points result in greater apparent speed deviation.

It is of interest to note that, the height-dependence of the apparent return-stroke speed is sensitive to the observation distance from the lightning channel. A considerable variation of the apparent speed is generally found to exist in the channel sections below the height equaling to the observation distance.

3.2. Observation above Ground

Regarding the case of observation above ground, the lightning return stroke is also assumed to flow in a straight, vertical channel. The observation point $P$ at altitude $h$ has a horizontal distance $r$ from the channel, as shown in Fig. 4.

The observed height $H(t)$ by the remote observer at time $t$, is given by the solution of the equation

$$t = \frac{H(t)}{v_f} + \frac{\left(\frac{H(t)}{h} - 1\right)^2 + r^2}{c}. \quad (6)$$

The expression for $H(t)$ is obtained

$${\text{Fig. 3}}$$ The deviation of the apparent return-stroke speed versus the apparent height in Fig. 1.

$${\text{Fig. 4}}$$ Geometry used in deriving the expressions of apparent height of return-stroke front and apparent return-stroke speed at a point $P$, with an altitude of $h$ above the ground.
The first-time derivation of (7) can be obtained as

$$\frac{dH(t)}{dt} = v_f \cdot \frac{1 - (v_f^2 t - v_f h)^{1/2}}{(v_f^2 c^2 t^2 + c^2 r^2 - v_f^2 r^2 + c^2 h^2 - 2v_f c^2 h t)^{1/2}} / (c^2 - v_f^2).$$  

(8)

In (6) to (8), \(t \geq (h^2 + r^2)^{1/2}/c\).

For the observation above ground considered here, Fig. 5 plots the apparent speed deviation at observation distance \(r=100\) m, \(200\) m, and \(1\) km, versus the corresponding apparent height with two observation altitudes, \(h=20\) m (Fig. 5(a)) and \(h=50\) m (Fig. 5(b)). It can be seen that, the apparent upward return-stroke speed at a height above the observation altitude is smaller than the actual value, whereas that at a channel height below the observation altitude is larger than the actual value. At the channel height equaling to the observation altitude, the apparent return-stroke speed exactly equals to the actual value. Also, similar to the case of observation at ground, closer observation points result in more fierce variation of the apparent return-stroke speed, and the larger the actual return-stroke speed, the greater the apparent speed deviation. Note that, for different observation distances, the speed deviation in the upper channel segments approaches to the same value. In addition, a comparison of Fig. 5(a) and Fig. 5(b) indicates, as the observation altitude increases, speed deviations in the lowermost channel segment also increase.

![Graphs showing apparent speed deviation versus apparent height for different observation altitudes and distances.](attachment:image.png)

**Fig. 5** The deviation of the apparent return-stroke speed versus the apparent height in Fig. 4, with the observation altitude of (a) \(h=20\) m and (b) \(h=50\) m.

### 4. Discussion

In practical application, through determination of the time required for progression of the lightning luminosity between two channel levels, and the corresponding length of channel traversed, the obtained velocity is actually the average value of the return-stroke speed in the involved channel section. Idone and Orville [12] measured two-dimensional return stroke velocities for 63 strokes. The mean return stroke velocities near ground was found to be \(1.1 \times 108\) m/s (about one third of the light speed), and velocities in upper channel lengths (about 1 km) were often reduced by 25% or more relative to velocities near ground. It is interesting to mention that, from our results shown in Fig. 3, comparing with those
Considerable attention has been paid to the return-stroke speed within the bottom 100 m or so of the lightning channel (see, e.g., [19, 21]), due, partly, to the reason that this channel section corresponds to the time for the channel-base current to reach its peak. In addition, the return-stroke speed in the channel bottom is also involved in estimating the current peaks from remotely measured radiation fields [1]. From the results shown in Fig. 3 and Fig. 5, at close observation distances, such as 100 m or 200 m (this is always the case for the rocket-triggered lightning measurement), considerable variation of the observed apparent speed is found in the channel sections below 100 m or so. Therefore, the discrepancy between the apparent return-stroke speed and the actual speed cannot be ignored. In fact, the source of this discrepancy lies on the propagation time difference of the light signal emitted by the lightning. For example, in the case of observation at ground shown in Fig. 1, the light signal propagation path from the uppermost segment of channel observed to the observation point is longer than the propagation path from the lowermost segment. We now prove that, when measuring the time of arrival of the light, if this difference in propagation times is taken into account, the apparent return-stroke speed can be transformed into the actual return-stroke speed.

With the assumption that the return stroke propagates with constant velocity $v_f$ up a straight, vertical channel from ground (see Fig. 1), for the light signal in two apparent heights $H_1$ and $H_2$, the corresponding arriving time $t_1$ and $t_2$ at the observation point are given by

$$ t_1 = \frac{H_1}{v_f} + \frac{[H_1^2 + r^2]^{1/2}}{c} \quad (9) $$

$$ t_2 = \frac{H_2}{v_f} + \frac{[H_2^2 + r^2]^{1/2}}{c} \quad (10) $$

The time difference $\Delta t$ in the light signal propagation path is

$$ \Delta t = \frac{[H_2^2 + r^2]^{1/2}}{c} - \frac{[H_1^2 + r^2]^{1/2}}{c} \quad (11) $$

If $v'$ denotes the computed return-stroke speed in which the propagation time difference $\Delta t$ is accounted, then

$$ v' = \frac{H_2 - H_1}{t_2 - t_1 - \Delta t} \quad (12) $$

Introducing (9), (10), and (11) to (12) gives $v' = v_f$. Thus, the obtained return-stroke speed equals to the actual value. The proving processes for other cases are similar. It should be note that, in obtaining the return-stroke propagation speed, the time difference in the light signal propagation path was taken into account by Olsen et al. [21], who reported different lightning return-stroke propagation characteristics from others, as mentioned in Section I.

As to the return stroke modeling, the vertical electric field ($E_z$) and the azimuthal magnetic field ($B_\phi$) of a lightning return stroke at ground level at a horizontal distance $r$ from the bottom of a vertical lightning channel over a perfectly conducting ground (see Fig. 1) are [22]
Where \( \varepsilon_0 \) is the permittivity and \( \mu_0 \) is the permeability of free space. The last term in (13) and (14) is an additional radiation field component accounting for any discontinuity at the upward propagation return stroke front. The expressions of apparent height \( H(t) \) and apparent return-stroke speed \( \frac{dH(t)}{dt} \) derived in this paper can also be used in return stroke modeling.

Summary

We have shown in this paper that even if the optical return-stroke speed is constant, the remote observer will see a varying apparent speed. The discrepancy between the apparent return-stroke speed and the actual speed is attributed to the propagation time difference of the light signal emitted by the lightning in the traversed path to the observation point. A larger actual return-stroke speed corresponds to a greater deviation of the apparent speed.

The propagation time correction should be conducted in obtaining the characteristics of return stroke propagation from the time-resolved optical records. The derived analytical expressions for the apparent height and the apparent return-stroke speed in this paper can also be used in the calculation of the electric and the magnetic fields from lightning return-stroke engineering models, especially for the case when a discontinuity at the upward propagation return stroke front is associated.

References

[1] V. A. Rakov and M. A. Uman, “Review and evaluation of lightning return stroke models including some aspects of their application,” IEEE Trans. Electromagn. Compat., vol. 40, no. 4, pp. 403–426, Nov. 1998.
[2] C. Gomes and V. Cooray, “Concepts of lightning return stroke models,” IEEE Trans. Electromagn. Compat., vol. 42, no. 1, pp. 82–96, Feb. 2000.
[3] V. A. Rakov and F. Rachidi, “Overview of Recent Progress in Lightning Research and Lightning Protection,” IEEE Trans. Electromagn. Compat., vol. 51, no. 3, pp. 428–442, Aug. 2009.
[4] S. Mallick, V. A. Rakov, D. Tsalikis, A. Nag, C. Biagi, D. Hill, D. M. Jordan, M. A. Uman, and J. A. Cramer, “On remote measurements of lightning return stroke peak currents,” Atmos. Res., vol. 135–136, no. 0, pp. 306–313, 2014.
[5] A. Nag, V. A. Rakov, and K. L. Cummins, “Positive lightning peak currents reported by the U.S. National Lightning Detection Network,” IEEE Trans. Electromagn. Compat., vol. 56, no. 2, pp. 404–412, Apr. 2014.
[6] K. L. Cummins and M. J. Murphy, “An overview of lightning locating systems: history, techniques, and data uses, with an in-depth look at the U.S. NLDN,” IEEE Trans. Electromagn.
Compat., vol. 51, no. 3, pp. 499–518, Aug. 2009.

[7] K. L. Cummins, E. P. Krider, and M. D. Malone, “The US National Lightning Detection NetworkTM and applications of cloud-to-ground lightning data by electric power utilities,” IEEE Trans. Electromagn. Compat., vol. 40, no. 4, pp. 465–480, Nov. 1998.

[8] D. Wang, W. R. Gamerota, M. A. Uman, N. Takagi, J. D. Hill, J. Pilkey, T. Ngin, D. M. Jordan, S. Mallick, and V. A. Rakov, “Lightning attachment processes of an “anomalous” triggered lightning discharge,” J. Geophys. Res. Atmos., vol. 119, no. 3, pp. 2013J–20787J, 2014.

[9] D. Wang, N. Takagi, W. R. Gamerota, M. A. Uman, J. D. Hill, and D. M. Jordan, “Initiation processes of return strokes in rocket-triggered lightning,” J. Geophys. Res. Atmos., vol. 118, no. 17, pp. 9880–9888, 2013.

[10] J. S. Boyle and R. E. Orville, “Return stroke velocity measurements in multistroke lightning flashes,” J. Geophys. Res., vol. 81, no. 24, pp. 4461–4466, 1976.

[11] P. Hubert and G. Mouget, “Return stroke velocity measurements in two triggered lightning flashes,” J. Geophys. Res., vol. 86, no. C6, pp. 5253–5261, 1981.

[12] V. P. Idone and R. E. Orville, “Lightning return stroke velocities in the thunderstorm research international program (TRIP),” J. Geophys. Res., vol. 87, no. C7, pp. 4903–4916, 1982.

[13] V. P. Idone, R. E. Orville, P. Hubert, L. Barret, and A. Eybert-Berard, “Correlated observations of three triggered lightning flashes,” J. Geophys. Res., vol. 89, no. D1, pp. 1385–1394, 1984.

[14] V. P. Idone, R. E. Orville, D. M. Mach, and W. D. Rust, “The propagation speed of a positive lightning return stroke,” Geophys. Res. Lett., vol. 14, no. 11, pp. 1150–1153, 1987.

[15] D. M. Mach and W. D. Rust, “Photoelectric return-stroke velocity and peak current estimates in natural and triggered lightning,” J. Geophys. Res., vol. 94, no. D1, pp. 13237–13247, 1989.

[16] J. C. Willett, J. C. Bailey, V. P. Idone, A. Eybert-Berard, and L. Barret, “Submicrosecond intercomparison of radiation fields and currents in triggered lightning return strokes based on the transmission-line model,” J. Geophys. Res., vol. 94, no. D11, pp. 13275–13286, 1989.

[17] J. C. Willett, V. P. Idone, R. E. Orville, C. Leteinturier, A. Eybert-Berard, L. Barret, and E. P. Krider, “An experimental test of the “transmission-line model” of electromagnetic radiation from triggered lightning return strokes,” J. Geophys. Res., vol. 93, no. D4, pp. 3867–3878, 1988.

[18] D. M. Mach and W. D. Rust, “Two-dimensional velocity, optical risetime, and peak current estimates for natural positive lightning return strokes,” J. Geophys. Res., vol. 98, no. D2, pp. 2635–2638, 1993.

[19] D. Wang, N. Takagi, T. Watanabe, V. A. Rakov, and M. A. Uman, “Observed leader and return-stroke propagation characteristics in the bottom 400 m of a rocket-triggered lightning channel,” J. Geophys. Res., vol. 104, no. D12, pp. 14369–14376, 1999.

[20] D. Wang, V. A. Rakov, M. A. Uman, N. Takagi, T. Watanabe, D. E. Crawford, K. J. Rambo, G. H. Schnetzer, R. J. Fisher, and Z. I. Kawasaki, “Attachment process in rocket-triggered lightning strokes,” J. Geophys. Res., vol. 104, no. D2, pp. 2143–2150, 1999.

[21] R. C. Olsen, D. M. Jordan, V. A. Rakov, M. A. Uman, and N. Grimes, “Observed one-dimensional return stroke propagation speeds in the bottom 170 m of a rocket-triggered lightning channel,” Geophys. Res. Lett., vol. 31, no. 16, pp. L16107, 2004.

[22] R. Thottappillil, V. A. Rakov, and M. A. Uman, “Distribution of charge along the lightning channel: Relation to remote electric and magnetic fields and to return-stroke models,” J. Geophys. Res., vol. 102, no. D6, pp. 6987–7006, 1997.