Search for $B^0$ meson decays into $\Lambda$ and missing energy with a hadronic tagging method at Belle

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We present a search for the decays of $B^0$ mesons into a final state containing a $\Lambda$ baryon and missing energy. These results are obtained from a $711 \text{ fb}^{-1}$ data sample that contains $772 \times 10^6 B\bar{B}$ pairs and was collected near the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider. We use events in which one $B$ meson is fully reconstructed in a hadronic decay.
According to the $B$-Mesogenesis mechanism [11][2], the $CP$-violating oscillations and subsequent decays of $B$ mesons in the early Universe can simultaneously explain the dark matter (DM) relic abundance and baryon asymmetry. A robust prediction of this mechanism is a branching fraction larger than $\mathcal{B}_M = 10^{-4}$ for $B^0$ mesons decaying into a final state containing a $\Lambda$ baryon, missing energy in the form of a GeV-scale dark sector antibaryon $\psi_{DS}$, and any number of light mesons; $\mathcal{B}(B^0 \rightarrow \Lambda \psi_{DS} + \text{mesons}) > 10^{-4}$. The limit $\mathcal{B}_M$ strongly depends on the semileptonic asymmetries in neutral $B$ meson decays [2][3]. At present, the best bound on such a process is an exclusive branching fraction of $\mathcal{B}(B^0 \rightarrow \Lambda \psi_{DS}) \lesssim 2 \times 10^{-4}$ derived from an inclusive ALEPH search for events with large missing energy arising from $b$-flavored hadron decays at the $Z$ peak [2][4]. In order for this decay to exist, a new TeV-scale bosonic colored mediator $Y$ is required. This heavy mediator can be integrated out to yield an effective four-fermion operator $\mathcal{O}_{4s} = \psi_{DS} b s$. An example diagram of the corresponding decay is shown in Fig. 1. Successful baryogenesis requires a $\psi_{DS}$ mass $\lesssim 3.5 \text{ GeV}/c^2$ as indirectly constrained by LHC searches on TeV-scale color-triplet scalars [2][5]. We report the first search for $B^0 \rightarrow \Lambda \psi_{DS}$ exclusive decays using the full Belle data sample of 711 fb$^{-1}$ collected near the $\Upsilon(4S)$ resonance. Charge-conjugate decays are implied throughout this letter.

![FIG. 1. An example diagram of the $B$ meson decay process as mediated by the heavy colored scalar $Y$ that results in DM and a visible baryon. The dark sector antibaryon $\psi_{DS}$ decays into stable DM particles: a dark sector scalar antibaryon $\phi_{DS}$ and a dark Majorana fermion $\xi_{DS}$.](image)

This measurement is based on a data sample that contains $(772 \pm 11) \times 10^6 B\bar{B}$ pairs, collected with the Belle detector [2] at the KEKB asymmetric-energy $e^+e^-$ collider [7] operated at the $\Upsilon(4S)$ resonance. In addition, we employ an 89 fb$^{-1}$ data sample recorded at a center-of-mass (CM) energy 60 MeV below the $\Upsilon(4S)$ resonance (off-resonance data) to characterize the background. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K_{L}^0$ mesons and to identify muons. Two inner detector configurations were used. A 2.0 cm radius beampipe and a 3-layer SVD were used for the first sample of $152 \times 10^6 B\bar{B}$ pairs, while a 1.5 cm radius beampipe, a 4-layer SVD, and a small-inner-cell CDC were used to record the remaining $620 \times 10^6 B\bar{B}$ pairs [8].

We study properties of signal events, identify sources of background, and optimize selection criteria using Monte Carlo (MC) simulated events. These samples are generated using the software packages EVTGEN [9] and PYTHIA [10], and final-state radiation is included via PHOTOS [11]. The detector response is simulated using GEANT3 [12]. We produce $B^0 \rightarrow K^0_S\psi_{DS}$ MC events according to a phase-space model for eight individual values of the $\psi_{DS}$ mass in the range $1.0 \text{ GeV}/c^2 \leq m_{\psi_{DS}} \leq 3.9 \text{ GeV}/c^2$ to calculate signal reconstruction efficiencies. To estimate background, we use MC samples that describe all $e^+e^- \rightarrow q\bar{q}$ processes. Events containing $e^+e^- \rightarrow B\bar{B}$ with subsequent $b \rightarrow c$ decays, and $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events, are both simulated with six times the integrated luminosity of Belle. Rare charmless $B$ meson decays are simulated with 50 times the integrated luminosity.

Event reconstruction for this analysis is performed entirely in the Belle II Software Framework [13] by converting Belle data structures to that of Belle II [14]. We identify signal candidates by fully reconstructing the accompanying neutral $B$ meson ($B_{\text{tag}}$) and requiring a single $\Lambda$ baryon on the signal side. We analyze the data in an unbiased manner by finalizing all selection criteria before viewing events in the signal region.

The $B_{\text{tag}}$ candidates are reconstructed in hadronic decay channels using the Full Event Interpretation algorithm [15]. The algorithm employs a hierarchical reconstruction ansatz in six stages. In the first stage, tracks and neutral clusters are identified and required to pass some basic quality criteria. In the second stage, boosted decision trees (BDTs) are trained to identify charged tracks and neutral energy depositions as detector-stable particles ($\pi^\pm, K^\pm, \mu^\pm, e^\pm, \gamma$). In the third and fourth stages, these candidate particles are combined into composite parents ($\pi^0, J/\psi, K_{S}^0, D^0, D^+, D_s^+$), and for each...
target final state, a BDT is trained to identify probable candidates. At the fifth stage, candidates for excited mesons \( (D^0, D^{*+}, D_s^{*+}) \) are formed and separate BDTs are trained to identify viable combinations. The input variables of each stage aggregate the output classifiers from all previous reconstruction stages. The final stage combines the information from all previous stages to form \( B_{\text{tag}} \) candidates. The viability of such combinations is assessed by a BDT that is trained to distinguish correctly reconstructed candidates from wrong combinations, and whose output classifier score we denote as the signal probability \( a_{\text{tag}} \). The purity of the \( B_{\text{tag}} \) candidate is improved by selecting candidates with \( a_{\text{tag}} > 10^{-3} \). We further select the \( B_{\text{tag}} \) candidates using the energy difference \( \Delta E \equiv E_B - E_{\text{beam}} \) and the beam-energy constrained mass \( M_{bc} \equiv \sqrt{E_{\text{beam}}^2/c^4 - (\vec{p}_B)^2/c^2} \), where \( E_B \) and \( \vec{p}_B \) are the reconstructed energy and momentum of the \( B_{\text{tag}} \) candidate in the \( Y(4S) \) CM frame, and \( E_{\text{beam}} \) is the beam energy in this frame. We require \( B_{\text{tag}} \) candidates to satisfy the requirements \( 5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2 \) and \( |\Delta E| < 0.06 \text{ GeV} \). We apply a calibration factor for the hadronic tagging efficiency, derived from exclusive measurements of \( B^0 \to X \ell \nu \) decay channels, to all correctly reconstructed \( B_{\text{tag}} \) candidates in MC simulation. A full description of this procedure can be found in Ref. [13].

The particles in the event not associated with the \( B_{\text{tag}} \) meson are used to reconstruct a \( B_{\text{tag}} \to \Lambda \psi(\text{DS}) \) candidate. The \( \Lambda \) candidates are reconstructed via \( \Lambda \to p \pi^- \) decays in the mass range \( 1.112 \text{ GeV}/c^2 < M_{p \pi^-} < 1.119 \text{ GeV}/c^2 \) (corresponding to a \( \sim 3\sigma \) range around the nominal \( m_{\Lambda} \) [3]) and selected using \( \Lambda \)-momentum dependent criteria based on four parameters: the distance between the two decay product tracks at their closest approach, in the direction opposite that of the \( e^+ \) beam; the minimum distance between the decay product tracks and the interaction point (IP) in the transverse plane; the angular difference between the \( \Lambda \) flight direction and the direction pointing from the IP to the \( \Lambda \) decay vertex in the transverse plane; and the flight length of the \( \Lambda \) in the transverse plane. Measurements from CDC, TOF, and ACC are combined to form the charged particle identification (PID) likelihoods \( \mathcal{L}(h) \) \( (h = p, K, \text{ or } \pi) \), where \( \mathcal{L}(h; h') \), defined as \( \mathcal{L}(h)/[\mathcal{L}(h) + \mathcal{L}(h')] \), is the discriminant between the \( h \) and \( h' \) hypotheses. We require \( \mathcal{L}(p; K) > 0.6 \) and \( \mathcal{L}(p; \pi) > 0.6 \) for the proton from the \( \Lambda \) decay. The proton PID efficiency in the produced \( B^0 \to \Lambda \psi(\text{DS}) \) MC events is 70–99\%, with the purity being 100\%.

To suppress background with particles undetected along the beampipe, we require the cosine of the polar angle of the missing momentum in the laboratory frame [6] to lie between \(-0.86 \) and \( 0.95 \). After identifying the \( B_{\text{tag}} \) candidate and reconstructing the \( \Lambda \) baryon, we require that no additional charged tracks remain in the event. This veto and the PID requirements for protons eliminate any multiple \( \Lambda \) candidates in our samples. If there are multiple \( B_{\text{tag}} \) candidates in an event, the candidate with the highest \( a_{\text{tag}} \) value is chosen.

While the topological distribution of particles in \( B \) decay events is more isotropic, that for continuum events, the dominant background at this level, is more jet-like. To suppress the continuum background, we simultaneously optimize the requirements on two event-shape variables: the ratio of the second to zeroth Fox–Wolfram moments [10] of the event, \( R_2 \); and the cosine of the angle between the thrust axis \( \cos \theta_T \) of the \( \Lambda \) and the thrust axis of the \( B_{\text{tag}} \). This is done by maximizing a figure-of-merit \( \varepsilon/(\sqrt{\tau} + \sqrt{N_B}) \), which is independent of the assumed signal branching fraction and optimized for new decay modes [15]. Here, \( \varepsilon \) is the signal efficiency while \( N_B \) denotes the number of background events passing the requirements on the two-event-shape variables. Both values are determined from MC simulation, with the number of continuum events being corrected with an overall factor based on the off-resonance data. We choose \( \alpha = 3 \), the number of standard deviations of the desired sensitivity.

The most powerful variable to identify signal decays is the residual energy in the ECL, \( E_{\text{ECL}} \), which is the sum of the energies of ECL clusters that are not associated with the \( B_{\text{tag}} \) decay products nor with the signal-side \( \Lambda \) candidate. To suppress contributions from noise in the ECL, minimum thresholds on the cluster energy are required: 50 MeV for the barrel, 100 MeV for the forward endcap, and 150 MeV for the backward endcap region. These thresholds were determined to achieve an optimal signal-to-noise ratio in the calorimeter clusters. The decays \( B \to D^* \ell \nu \) are examined as control samples; the observed \( E_{\text{ECL}} \) distributions are found to be in good agreement with MC simulations. In a correctly reconstructed signal event, no additional activity should appear in the calorimeter, so the \( E_{\text{ECL}} \) distribution for signal events will peak at low \( E_{\text{ECL}} \) values. The \( E_{\text{ECL}} \) signal region is selected from MC simulation. It is defined by requiring that the expected number of background events corresponding to the full Belle dataset is \( \pm 3 \), thus allowing downward fluctuation of the number of observed events, \( n_{\text{obs}} \). The contamination of continuum background events in the \( E_{\text{ECL}} \) signal region is determined from MC simulation, with the scale factor being determined with the off-resonance data. In addition, a correction for possible background from rare \( B \) decays is applied to the number of \( B \bar{B} \) events. After all the selection requirements are applied, approximately half of the expected background consists of continuum events and the other half from \( B \bar{B} \) events. The observed and expected \( E_{\text{ECL}} \) distributions for \( m_{\psi(\text{DS})} = 2.5 \text{ GeV}/c^2 \) are shown in Fig. [2] together with the corresponding signal distribution.

The signal yield is obtained by counting the number of events in the signal region. The profile likelihood method [19] is used to calculate the 90% confi-
The number of background MC candidates surviving our selection criteria, $b_0$, is a free parameter of the likelihood. It has the same relative systematic uncertainty as $n_{bkg}$.

For each $m_{\psi DS}$ value we allow different optimization requirements, and different systematic uncertainties are included based on the kinematics and reconstruction efficiency of each sample. The systematic uncertainty arising from the number of $B\bar{B}$ pairs is 1.4%. The world average value of $B [\Upsilon(4S) \to B\bar{B}]$ is $48.6 \pm 0.6$% [3], leading to a systematic uncertainty of 1.8% on the number of $B\bar{B}$ pairs. The world average value of $B_f$ is $(63.9 \pm 0.5)$% [3], resulting in the systematic uncertainty of 0.8%.

The calibration factor for the hadronic tagging efficiency is studied in Ref. [15] and found to be 0.860 \pm 0.074. The uncertainty in this value is taken as a systematic uncertainty. The uncertainty due to proton PID is evaluated using an independent sample of $\Lambda \to p\pi^-$ decays. The systematic uncertainty due to charged track reconstruction is calculated using partially reconstructed $D^{*+} \to D^0\pi^+$ decays, with $D^0 \to K_S^0\pi^+\pi^-$ and $K_S^0 \to \pi^+\pi^-$. For the pion, an additional systematic uncertainty based on a study of low-momentum tracks from $B^0 \to D^{*-}\pi^+$ decays is applied. The slow pion emitted in the decays of the $D^*$ allows one to probe the low-momentum region. The difference in the charged track veto efficiency between data and MC simulation is estimated by comparing the effect of requiring no extra tracks available in the event on samples of $B^0 \to D^{(*)}l\nu$ events. The systematic uncertainty due to $\Lambda$ reconstruction is determined from a comparison of yield ratios of $B^+ \to \Lambda\bar{K}^+$ with and without the $\Lambda$ selection requirements in data and MC samples. The weighted average of the data-MC difference over the momentum range is assigned as the systematic uncertainty. Furthermore, we include the binomial error of the efficiency as a systematic uncertainty.

The statistical uncertainty of the correction on the number of continuum events in MC simulation is 21.0–26.9%, based on off-resonance data; this is assigned as a systematic uncertainty. Since none of the $B\bar{B}$ background MC decays surviving our selection criteria are from exclusively observed and measured processes in experiment, we assign a conservative 50% systematic uncertainty on their branching fractions; this is the dominant systematic uncertainty on $n_{bkg}$. The statistical uncertainty of the correction for possible background from rare $B$ decays applied to the number of $B\bar{B}$ background events in MC simulation is included as a systematic uncertainty.

The range of systematic uncertainties in the estimate of the signal efficiencies, $\epsilon_s$, and the number of expected $B\bar{B}$ background events, $n_{bkg}$, across the different values of $m_{\psi DS}$ are listed in Table I. The observed and expected
90% CL upper limits on $\mathcal{B}(B^0 \to \Lambda \psi_{DS})$ as a function of $m_{\psi_{DS}}$ are shown in Fig. [3]. A summary of these limits and the different distinct variables used in their calculation for each $m_{\psi_{DS}}$ is presented in Table [I].

TABLE I. Range of systematic uncertainties in the estimate of the signal efficiencies, δε, and the number of expected $B \bar{B}$ background events, $\delta n_{bkg}$, across the different values of $m_{\psi_{DS}}$.

| Source                      | δε (%) | $\delta n_{bkg}$ (%) |
|-----------------------------|--------|-----------------------|
| $B_{\text{tag}}$ correction | 8.6    | 8.6                   |
| Proton PID                  | 0.5–2.8| 4.3–5.7               |
| Tracking efficiency         | 0.7–1.9| 1.1–1.9               |
| Charged track veto          | 5.3–6.5| 5.3–6.5               |
| $A_0$ selection             | 2.5–3.6| 4.4–4.7               |
| Signal MC veto              | 1.2–2.0| –                     |
| Rare $B$ decays correction  | –      | 10.6–13.4             |
| Branching fractions         | –      | 50.0                  |

Fig. 3. The observed (solid line) and median expected (dashed line) 90% CL upper limits on $\mathcal{B}(B^0 \to \Lambda \psi_{DS})$ as a function of $m_{\psi_{DS}}$. The ±1σ and ±2σ expected exclusion regions are indicated in green and yellow, respectively. A linear interpolation is performed between the values obtained for the probed $m_{\psi_{DS}}$ values. The gray shaded region shows the resulting 90% CL constraints from the reinterpretation of a search at ALEPH for decays of $b$-flavored hadrons with large missing energy [24].

The fraction of decays not expected to contain hadrons other than $\Lambda$ in the final state as a function of $m_{\psi_{DS}}$ is calculated in Ref. [2] using phase-space considerations. This fraction multiplied with $B_{M1}$ provides the lower bounds on $\mathcal{B}(B^0 \to \Lambda \psi_{DS})$ for $B$-Mesogenesis. Those bounds together with the observed 90% CL upper limits on $\mathcal{B}(B^0 \to \Lambda \psi_{DS})$ as a function of $m_{\psi_{DS}}$ are presented in Fig. 4. The region $m_{\psi_{DS}} \geq 3.0\text{GeV}/c^2$ is excluded for the $O_{us}^1$ and $O_{us}^4$ operator cases.

In summary, we have reported the results of a search for the decays of $B^0$ mesons into a final state containing a $\Lambda$ baryon and missing energy with a fully reconstructed $B_{\text{tag}}$ using a data sample of $772 \times 10^6$ $B \bar{B}$ pairs collected at the $T(4S)$ resonance with the Belle detector. No significant signal is observed and we set upper limits on the branching fractions at 90% CL, which are the most stringent constraints to date. Our analysis yields significant improvements, and partially excludes the $B$-Mesogenesis mechanism. We expect that the Belle II experiment [20] will be able to fully test this mechanism.

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TABLE II. Summary of the requirements on the event-shape variables $R_2$ and $\cos \theta_T$, the $E_{\text{ECL}}$ signal region, the signal efficiency $\epsilon$ and its systematic uncertainty $\delta \epsilon$, the systematic uncertainty on the expected number of background MC candidates surviving our selection criteria $\delta n_{\text{bkg}}$, the expected 90% CL upper limit, the number of observed events $n_{\text{obs}}$, and the observed 90% CL upper limit, for each $m_{\text{SDS}}$.

| $m_{\text{SDS}}$ (GeV/$c^2$) | $R_2$ | $\cos \theta_T$ | $E_{\text{ECL}}$ (GeV) | $\epsilon \times 10^{-4}$ | $\delta \epsilon$ (%) | $\delta n_{\text{bkg}}$ (%) | Expected limit (10$^{-5}$) | $n_{\text{obs}}$ | Observed limit (10$^{-5}$) |
|-----------------------------|------|-----------------|------------------------|--------------------------|----------------------|--------------------------|------------------------|----------|------------------------|
| 1.0                         | < 0.31 | < 0.66 | < 0.74 | 3.92 | 11.0 | 30.3 | 2.2 | 3 | 2.2 |
| 1.5                         | < 0.30 | < 0.66 | < 0.74 | 3.94 | 11.0 | 30.3 | 2.2 | 3 | 2.2 |
| 2.0                         | < 0.31 | < 0.70 | < 0.74 | 4.05 | 10.9 | 30.6 | 2.1 | 3 | 2.1 |
| 2.5                         | < 0.33 | < 0.67 | < 0.71 | 4.01 | 10.9 | 30.7 | 2.1 | 3 | 2.2 |
| 3.0                         | < 0.33 | < 0.70 | < 0.71 | 3.69 | 11.0 | 30.8 | 2.3 | 3 | 2.3 |
| 3.3                         | < 0.35 | < 0.70 | < 0.68 | 3.32 | 11.1 | 28.4 | 2.6 | 3 | 2.6 |
| 3.6                         | < 0.44 | < 0.70 | < 0.63 | 2.88 | 11.7 | 27.7 | 3.0 | 3 | 3.0 |
| 3.9                         | < 0.42 | < 0.79 | < 0.57 | 1.56 | 11.3 | 30.2 | 5.5 | 2 | 3.8 |

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