Optimizing Production Schedule of Coalbed Methane Wells Using a Stochastic Evolution Algorithm

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1.Introduction

Coalbed methane is an unconventional natural gas in coal seams. Global CBM in place is estimated to range from 36 to 230 × 1012 m3 [1]. The major component of CBM is methane, accounting for 80 to 95% [2]. CBM is a kind of clean energy with high calorific value. The combustion heat of 1 m³ coalbed methane is equivalent to 1 kg of fuel, while the pollution caused by coalbed methane combustion is only 1/40 of oil and 1/800 of coal [3]. Exploration and development of CBM are of great significance for energy sustainable development and has attracted a lot of attention.

During the development, CBM well first continuously produces water at a low bottom hole pressure to depressure the reservoir. When the reservoir pressure drops below the critical desorption pressure, methane is desorbed from the coal seam and transmitted to the well; the gas rate increases, and the water rate drops. The performance of CBM well is...
directly affected by its production schedule. Coal seams are generally heterogeneous with widespread cleats, fractures, and matrix pores [4, 5]. The matrix permeability and porosity are low. The pore volume compressibility of coal rock and matrix pores [4, 5]. The matrix permeability and porosity are low. The pore volume compressibility of coal rock and matrix pores [4, 5].

Some studies and field practices have demonstrated that during the initial production period of CBM wells, the gas desorption quantity is limited and the reservoir stress sensitivity effect is dominant. An incorrect pressure drop rate can lead to a serious stress sensitivity in the coal reservoir, leading to a decline in permeability and poor gas production. Moreover, a fast pressure drop will mobilize coal dust in the reservoir, blocking the gas seepage channels and further reducing gas production. A proper production schedule is conducive to gas wells having high and stable yields [6–9]. Successful development experiment show that reducing the bottom hole pressure of CBM well step by step and slowly at the early stage can make the pressure wave propagate as far as possible and improve the recovery of coalbed methane reservoir [10].

Although the above results have clearly shown the importance of determining a reasonable production schedule of CBM well, finding the optimal production schedule is still challenging. There has no one production schedule that fits all CBM wells. Many parameters, including formation and rock-liquid physical characteristics, should be fully considered in the production schedule optimization. This results in different optimal schedules for different wells. The practical experience can guide the CBM well production, and quantitative methods are still needed for fine development [6–8]. To the best of our knowledge, research on optimizing CBM well production schedule quantitatively is still limited [11].

Production schedule planning of CBM well can be formulated as an optimization problem and finding the optimal solution using optimization algorithms. This idea has not been investigated for this problem before, but it already has many successful applications in conventional oil and gas production optimization problems [12–18], CBM well placement optimization problems [19, 20], CBM EOR optimization problems [21, 22], etc. A number of algorithms have been proposed and investigated for above optimization problems. These algorithms fall into two categories: gradient-based methods [23–26], e.g., steepest ascent algorithm, conjugate gradient algorithm, etc., and derivative-free methods [12, 15–17, 27, 28], e.g., generalized pattern search (GPS), genetic algorithm (GA), particle swarm optimization (PSO), covariance matrix adaptation evolution strategy (CMA-ES), etc. The derivative-free methods only require the value of objective function and involve no explicit gradient calculations can be applied to CBM well production schedule optimization problem more easily. Among the derivative-free methods, CMA-ES performed very well in oil and gas production optimization and many other fields [15, 27, 29]. Hence we choose CMA-ES for our model.

In this paper, a step-down production process is proposed for coalbed methane reservoir development. In order to determine the optimal step-down production schedule, the optimization model is constructed and the features of the problem are analyzed. An intelligent algorithm, CMA-ES, is chosen as the solver and the optimization performance is evaluated. After that, a CBM well production schedule quantitative optimization approach is built. Three experiments including both normal well and real featured wells are used to study the effectiveness of the optimization approach. Among this, the effect of the control frequency on the production schedule optimization problem is investigated too. Figure 1 shows a general sketch of this study.

Comparing with the previously performed studies in literature, we are the first to introduce the optimal control theory to address the CBM well production schedule optimization problem, which takes the step-down well control at each time step instead of a constant production rate as the optimization variable. The constructed optimization problem is solved by CMA-ES coupling with a reservoir simulator. Though CMA-ES shows excellent performance while solving optimization problems of conventional oil and gas reservoirs, this is the first time, to the best of our knowledge, it investigates its performance for CBM optimization problems. Moreover, the effect of control frequency on the CBM well production is also considered in this paper, which has not been deeply discussed in previous studies.

This paper is structured as follows. Section 2 describes the CBM well production schedule optimization problem formulation, including the optimization model, the solving algorithm, and the solving process. Section 3 gives the experiments description, results, and discussions. In Section 4, we provide a summary and conclusions of this work.

2. Problem Formulation

2.1. CBM Step-Down Production Process. According to the successful experience during the development of coalbed methane, the production process of coalbed methane is generally divided into three stages [30–32]:

(i) Dewatering stage: At the beginning of production, a large amount of water is drained from the well and the coal reservoir pressure continues to decline. When the reservoir pressure drops below the critical desorption pressure, the gas begins to produce. The time required for this stage depends on coalbed methane geological and reservoir characteristics, and the drainage speed.

(ii) Stable production stage: As the drainage continues, the gas production gradually rises and stabilizes, and the water production gradually decreases. The gas production peak occurs at this stage. The duration of this stage depends on the abundance of coalbed methane resources and the permeability of the coal seam.

(iii) Production decline stage: When a large amount of gas has been produced, the desorbed gas in the coal matrix begins to decrease. Although the drainage operation continues, gas production and water production are continuously decreasing. The duration of this stage is generally long (up to 10 years).
Coalbed methane reservoirs generally have low permeability and strong stress sensitivity. If the drainage speed is too fast in the development of coalbed methane, the pressure near the wellbore will drop sharply, and the permeability near the wellbore will be further reduced, which will result in the pressure drop radius not being fully expanded. The coal seam where the pressure drops below the critical desorption pressure only exists in a small area near the wellbore, which makes the gas supply to the well severely restricted. In this case, the gas production is rapidly reduced after reaches its peak, due to insufficient supply of the gas source. Therefore, how to increase the pressure drop volume and increase the gas supply radius of the coalbed methane reservoir should be carefully considered during the CBM production.

In order to reduce reservoir damage caused by rapid pressure drop, a step-down production process is proposed. The basic idea of this production method is reducing the bottom hole pressure of CBM well step by step. The bottom hole pressure remains constant at every step, and each step keeps a certain length of time. Although the cleats and cracks also have a certain closure at each step, the degree of closure is relatively small since the pressure declines small. This makes the permeability of coal seam not decline sharply and helps the pressure drop radius expand fully. When the pressure drop radius expanded fully at one step, the bottom hole pressure is lowered, and go to the next step, and keeps loop this way.

The step-down production process, by controlling the bottom hole pressure drop stepwise, can make the pressure wave propagate as far as possible, to expand the pressure drop funnel and maximize the pressure relief volume. Hence, the desorption area of the coalbed methane can be increased, making more coalbed methane desorbed from the coal seam, which can obtain a longer stable production duration and improve the recovery of coalbed methane reservoir. At the same time. The step-down production process can reduce the damage of coal powder migration in the reservoir to the wellbore.

2.2. Optimization Model. Although the step-down production process has many advantages, determining the optimal production schedule is very challenging. Since the properties of rock and fluid around each well are different, the corresponding optimal production schedules are also different. In this work, we describe the problem of determining the step-down production schedule of coalbed methane wells as an optimization problem and build the corresponding mathematical model. The mathematical model of the optimization problem generally includes three parts: the optimization variables, the objective function, and the constraints.

2.2.1. Optimization Variables. The whole production cycle of coalbed methane wells is divided into a series of control time steps, and each control time step represents a production time, such as half a year, one month, etc. At the beginning of each time step, the production parameter is reset. The production of coalbed methane well is usually controlled by rate or bottom hole pressure [8, 9, 32]. For the step-down production process, the well is controlled by bottom hole pressure.

For a coalbed methane well with $n$ time steps, the optimization variables can be defined as the bottom hole pressure at each time step

$$\mathbf{p} = [p_1, p_2, \ldots, p_n],$$

where $\mathbf{p}$ denotes the bottom hole pressure control sequence of the well; $p_1$ to $p_n$ denote the bottom hole pressure of the well from the first to the $n$th time step, respectively.

The optimization variables can also be defined as the decline of bottom hole pressure at each time step:

$$\Delta \mathbf{p} = [\Delta p_1, \Delta p_2, \ldots, \Delta p_n],$$

where $\Delta \mathbf{p}$ denotes control sequence of the well described by the decline of bottom hole pressure; $\Delta p_1$ to $\Delta p_n$ denote the bottom hole pressure decline value of the well from the first to the $n$th time step, respectively.

The two kinds of optimization variables can be easily converted into each other by the following equations:

$$p_i = p_r - \sum_{k=1}^{i} \Delta p_k,$$

$$\Delta p_i = \begin{cases} p_r - p_1, & i = 1, \\ p_{i-1} - p_i, & i = 2, \ldots, n, \end{cases}$$

where $p_r$ denotes the initial pressure of the coalbed methane reservoir.
2.2.2. Objective Function. The purpose of CBM well production is to extract as much coalbed methane as possible from the reservoir, i.e., to maximize the cumulative gas production. Therefore, the cumulative gas production is selected as a performance index to evaluate the effectiveness of a production schedule. Correspondingly, the objective function of the optimization model is set to maximize cumulative gas production.

When $p$ is chosen as the optimization variable, the objective function can be described as

$$\max \int_0^{t_{\text{end}}} q(p, t) dt,$$

where $t_{\text{end}}$ denotes the end time of well production; $q$ denotes the instantaneous production rate; and $t$ denotes the production time.

When $\Delta p$ is chosen as the optimization variable, we first calculate $p$ using equation (3), then evaluate the objective function using equation (5).

$q(p, t)$ is a complex partial differential equation and is difficult to solve. In this work, we adopt a commercial reservoir numerical simulator, Eclipse [33], which is developed by Schlumberger, to solve the problem. For a coalbed methane reservoir, $q$ can be obtained with any given $p$ by calling numerical simulation.

2.2.3. Constraints. The constraints that should be considered in the step-down production schedule optimization problem include the following:

Bound constraints of bottom hole pressure
The bottom hole pressure of the well at any time step cannot be higher than a given maximum pressure, nor can it be lower than a given minimum pressure. This constraint can be described as

$$p_{\text{min}} \leq p_i \leq p_{\text{max}}, \quad i = 1, \ldots, n,$$

where $p_{\text{min}}$ and $p_{\text{max}}$ denote the lower and upper limits of the bottom hole pressure.

Constraints of bottom hole pressure decline rate
The decline of bottom hole pressure between two adjacent time steps cannot be higher than a given maximum decline rate. This constraint can be described as

$$\Delta p_i \leq \Delta p_{\text{max}}, \quad i = 1, \ldots, n,$$

where $\Delta p_{\text{max}}$ denotes the upper bound of bottom hole pressure decline.

2.2.4. Model Selection. When $p$ is chosen as the optimization variable, the established mathematical model can be described as

$$\max \int_0^{t_{\text{end}}} q(p, t) dt$$

s.t. $$p_{\text{min}} \leq p_i \leq p_{\text{max}}, \quad i = 1, \ldots, n,$$

$$\Delta p_i \leq \Delta p_{\text{max}}, \quad i = 1, \ldots, n.$$ (8)

When $\Delta p$ is used as the optimization variable, the established mathematical model can be described as

$$\max \int_0^{t_{\text{end}}} q[p(\Delta p), t] dt,$$

s.t. $$p_{\text{min}} \leq p_i \leq p_{\text{max}}, \quad i = 1, \ldots, n,$$

$$\Delta p_i \leq \Delta p_{\text{max}}, \quad i = 1, \ldots, n.$$ (9)

Model (8) and (9) are similar in form, but the solving complexity is quite different. For model (8), it includes a bound constraint and a series of linear constraints. A bound constraint and a series of nonlinear constraints are included in model (9). The linear and nonlinear constraints make the search space topology of the optimization problem very complicated and make it difficult to find the optimal solution for the optimization algorithm [34, 35]. Moreover, many excellent optimization algorithms are designed only for problems with bound constraints.

The nonlinear constraints can be removed by using the following equation to replace equation (3):

$$p_t = \max \left( \min \left( p_t - \sum_{k=1}^n \Delta p_k, p_{\text{max}} \right), p_{\text{min}} \right).$$ (10)

With equation (10), model (9) can be simplified to

$$\max \int_0^{t_{\text{end}}} q[p(\Delta p), t] dt,$$

s.t. $$\Delta p_i \leq \Delta p_{\text{max}}, \quad i = 1, \ldots, n.$$ (11)

Model (11) includes only bound constraint. More optimization algorithms can be selected for solving this problem and the solving difficulty is relatively low.

2.3. The Solving Algorithm

2.3.1. Problem Feature Analysis. Thousands of optimization algorithms have been proposed and applied in different areas. Each algorithm has its own characteristics and no one algorithm fits for all problems. In order to find a suitable optimization algorithm for the step-down production schedule optimization problem, the features of the problem are first described as follows:

(i) The objective function is highly nonlinear and it is difficult to obtain derivatives.

(ii) The solution space is rough with many local minima, local maxima, and discontinuities areas.

(iii) The number of optimization variables is large.

(iv) It includes bound constraints.
2.3.2. CMA-ES. CMA-ES is a stochastic, derivative-free method for numerical optimization of nonlinear or non-convex optimization problems [36, 37]. It belongs to the class of evolutionary algorithms and evolutionary computation. The basic idea of CMA-ES is inspired by biological evolution. In each iteration, new candidate solutions are sampled from the current probability distribution (described by covariance matrix). Then, the covariance matrix is updated adaptively in the next iteration based on the objective function values of previous candidates. Like this, over the iteration sequence, candidates with better and better function values are generated. This evolution strategy is particularly useful if the objective function is ill-conditioned.

Adaptation of the covariance matrix can learn a second-order model of the underlying objective function. This makes CMA-ES show excellent convergence. Moreover, in contrast to most classical optimization algorithms, fewer assumptions on the nature of objective function are made. Only the ranking between candidate solutions is exploited for learning the sample distribution and no derivatives are required by the algorithm. The detailed algorithm description of CMA-ES can be seen in Appendix.

CMA-ES performs better on the benchmark multimodal functions than all other similar classes of learning algorithms [38]. It also showed its potential in oil and gas production optimization and many other engineering practical problems [27, 28].

Considering the above features, we choose the covariance matrix adaptation evolution strategy (CMA-ES) as the optimization algorithm for the model.

Table 1: Strategy parameter values used in CMA-ES from [40].

| Parameter | Value |
|-----------|-------|
| $\lambda$ | 4 + [3 ln(n)] |
| $\mu$ | [3/2] |
| $c_c$ | $4/(n+4)$ |
| $c_\sigma$ | $(\mu_{eff} + 2)/(n + \mu_{eff} + 3)$ |
| $d_{\sigma}$ | $1 + 2\max(0, (\sqrt{\mu_{eff} - 1}/(n+1)) - 1) + c_\sigma$ |
| $\mu_{cov}$ | $\mu_{eff}$ |
| $c_{cov}$ | $(1/\mu_{cov})/(2/( (n + \sqrt{2})^2 )) + (1 - (1/\mu_{cov}))\min(1, ((2\mu_{eff} - 1)/( (n + 2)^2 + \mu_{eff}))))$ |

Figure 2 is an illustration of an optimization run with CMA-ES on a two-dimensional linear function $f(x) = x_1^2 + x_2^2$ with bound constraints $x_1, x_2 \in [-800, 800]$. The dashed lines are the contour lines of the function. The initial point is $[-200, -200]$. The optimal solution is $[0, 0]$ and is marked by blue symbol “x.” The orange and gray dots denote the population distribution for the current and the last iteration, respectively. The red cross (“+”) and the red ellipsoid denote the symmetry center $m$ and the isodensity line of the distribution for the current iteration, while the black cross and the black ellipsoid are for the last iteration. (a) Iteration 1. (b) Iteration 3. (c) Iteration 5.

Figure 2: An illustration of CMA-ES optimization on a two-dimensional linear function $f(x) = x_1^2 + x_2^2$. The dashed lines are the contour lines of the function. The optimal solution is in the upper right corner. The orange and gray dots denote the population distribution for the current and the last iteration, respectively. The black cross (“+”) and the black ellipsoid denote the symmetry center and the isodensity line of the distribution for the current iteration, while the gray cross and the gray ellipsoid are for the last iteration. (a) Iteration 1. (b) Iteration 3. (c) Iteration 5.
2.3.3. Solving Process. The process of solving the step-down production optimization problem using CMA-ES is as follows:

1. Step 1: Setting the number of variables $n$, lower/upper bounds of variables $\Delta p_{\text{min}}$, $\Delta p_{\text{max}}$, and the stopping criteria of optimization algorithm.

2. Step 2: Generate $\lambda$ sets of candidates $\Delta p^1$ to $\Delta p^\lambda$ according to model constraints.

3. Step 3: For each candidate $\Delta p^i$, calculate the production schedule $p^i$ using equation (10).

4. Step 4: Generate the schedule data file for each candidate. Call numerical simulator to predict production dynamics.

5. Step 5: Read output data of numerical simulator. Calculate the objective function value of each candidate.

6. Step 6: If any stopping criterion is met, then end the optimization process. The candidate with the maximum objective function value is the optimal solution. If no, go to Step 7.
(7) Step 7: Generate new \( \lambda \) sets of candidates using CMA-ES. goto Step 3.

3. Numerical Experiments

3.1. Experiment 1, a Simple Case

3.1.1. Reservoir Model Description. This model is a single-layer coalbed methane reservoir containing one producing wells. The reservoir model is represented by a \( 51 \times 51 \times 2 \) uniform grid blocks with \( \Delta x = \Delta y = 20 \) m. The top depth of the reservoir is 416 m. The thickness is 50 m and is divided into 2 sublayers with 20 m and 30 m, respectively. In order to describe the gas flow dynamics in the coal seam, dual-porosity and dual-permeability models are adopted for this reservoir. The permeability is homogeneous and the value is \( 0.3 \times 10^{-3} \mu \text{m}^2 \). The initial reservoir pressure is 4.26 MPa and the initial gas content is \( 3 \times 10^2 \) m\(^3\)/t. The Langmuir volume is \( 5 \times 10^2 \) m\(^3\)/t and the Langmuir pressure is 2.52 MPa.

The reservoir production lifetime is set to 3 years (by 360 days per year). The bottom hole pressure of well is updated every 15 days. This gives 72 control time steps in total and results in 72 optimization variables. The minimum bottom hole pressure is set to 0.2 MPa and the maximum bottom hole pressure decline is set to 0.3 MPa per step. CMA-ES is used to solve the optimization problem and the maximum iteration is chosen as a stopping criterion. The maximum number of iteration is set to 2000.
Moreover, to show the effect of the control frequency on the production schedule optimization problem for the first model, four control frequencies are considered and these constitute four variations of the optimization problem:

(i) Time step = 1 d: The well bottom hole pressure is updated every day (1080 control time steps). This gives 1080 optimization variables in total.

(ii) Time step = 7 d: The well bottom hole pressure is updated every 7 days (155 control time steps). This gives 155 optimization variables in total.

(iii) Time step = 15 d: The well bottom hole pressure is updated every 15 days (72 control time steps). This gives 72 optimization variables in total.

(iv) Time step = 30 d: The well bottom hole pressure is updated every 30 days (36 control time steps). This gives 36 optimization variables in total.

3.1.2. Results and Discussion. The optimization process of experiment 1 is shown in Figure 3. In this figure, each dot represents a candidate solution (i.e., production schedule)
and the corresponding objective function value (i.e., cumulative gas production). The solid line illustrates the best function value found before. We can see that CMA-ES converges to the optimal solution after 1161 function evaluations. The corresponding cumulative gas production is $1.72 \times 10^7 \text{ m}^3$. When the number of function evaluation reaches 199, the optimal function value reaches $1.718 \times 10^7 \text{ m}^3$. The difference from the final optimal function value is only 0.1163%. In this example, CMA-ES can converge to the vicinity of the optimal solution efficiently.

The well production performance under the optimized production schedule is compared with the performance under the steepest production schedule. The steepest production schedule is defined as for each control time step, the well bottom hole pressure declines with the upper bound of

![Graphs showing well bottom hole pressures and gas production rates for different control frequencies.]
pressure drop, until the well bottom hole pressure reaches the lower bound of bottom hole pressure. The well bottom hole pressure of the optimized schedule and steepest schedule versus time are shown in Figure 4. The well gas production rate and the cumulative gas production, for the optimized schedule and the steepest schedule, are shown in Figures 5 and 6, respectively.

From the figures, we can see that the bottom hole pressure of optimized schedule and steepest schedule are same at the early stage. And then the rate of bottom hole pressure decline of optimized schedule slows down. Compared to the steepest schedule, the optimized schedule takes longer to drop to the lower pressure bound. With the steepest schedule, the well production rate reaches its peak earlier and the rate decreases to a lower value to the stable production stage. This results in the cumulative gas production of steepest schedule is 11.76% lower comparing with the optimized schedule at the end of production.

The optimization processes of experiment 1 under different control frequencies are shown in Figure 7. Consider the optimal objective function values for problems with different control frequencies are different, we use the relative value of the objective function as the ordinate in this figure. For each problem, the relative value equals the absolute value divided by the optimal absolute value of this problem. The relative value ranges from 0 to 1. Four solid lines represent the converge of CMA-ES for problems with four different control frequencies. The line reaches 1, the algorithm converges to the optimal solution. From the figure, we can see that with the increases of control frequency, the converge rate decreases rapidly.

In detail, the number of function evaluations required CMA-ES required converge to 99% of the optimum value and 100% of optimum value for all problems are shown in

Table 2: The reservoir parameters of QW1.

| Parameter                         | Value       |
|-----------------------------------|-------------|
| Area, m²                          | 9.06 × 10⁴  |
| Thickness, m                      | 6.1         |
| Top depth, m                      | 910         |
| Matrix porosity                   | 0.004       |
| Matrix permeability, 10⁻³ μm²     | 0.1         |
| Fracture half-length, m           | 50          |
| Fracture equivalent permeability, 10⁻³ μm² | 70          |
| Initial pressure, MPa             | 6.8         |
| Desorption pressure, MPa          | 2.9         |
| Desorption time, d                | 1.0         |
| Initial water saturation          | 1.0         |
| Fissure distance, m               | 0.01        |
| Langmuir volume, m³/t             | 27.0        |
| Langmuir pressure, MPa            | 3.0         |

Figure 11: The well cumulative gas production for the problems with different control frequencies.

Figure 12: The permeability field and the fracture morphology of the reservoir of QW1.
Figure 8. From this figure, we can see that the number of function evaluations grows exponentially with the increases of control frequency. For problems with time step equal to 30 days, 15 days, and 7 days, CMA-ES converge to 99% of optimum function value efficiently. After that, the algorithm needs 3 times or even more times number of function evaluations to complete the last 1% search. For the problem with time step equal to 1 day, the number of variables is over one thousand, which makes CMA-ES more difficult to converge.

The optimized well bottom hole pressures for problems with different control frequencies are shown in Figure 9. From this figure, we can see that the optimum production schedules show the same trend for different problems. With the increase of control frequency, the bottom hole pressure curve becomes smoother.

The well gas production rate and the cumulative gas production, for the problems with different control frequencies, are shown in Figures 10 and 11, respectively. From the two figures, we can see that the peak of production rate appears earlier and the value is higher with the increases of control frequency. The cumulative gas production becomes higher with increases in control frequency. With a continued increase in the number of control steps, the cumulative production grows more and more slowly. The increase in the cumulative production is 2.96% when the control frequency increases from every 30 days to every 1 day.

Although frequent control can lead to a slight increase in production, we do not recommend control the coalbed methane well too frequently. The increase in the number of control steps will increase operating costs. Also, the problem...
with a large number of control steps is harder to optimize and the algorithm has a higher risk of falling into local optima.

3.2. Experiment 2, a Fractured Well

3.2.1. Reservoir Model Description. In this experiment, a fractured well (QW1) located in Qinshui Basin is investigated. Qinshui Basin is one of the most important regions for coal and coalbed methane production in China. The reservoir uses dual-porosity and dual-permeability models. Local mesh refinement is applied to illustrate the hydraulic fracture of the well. In detail, the reservoir parameters of QW1 is given in Table 2. Figure 12 shows the permeability field of the reservoir. The fracture morphology is clearly shown in this figure.

The well QW1 has been produced 415 days. The well bottom hole pressure and the water and gas production rate in real are shown in Figures 13 and 14.

In this experiment, we seek to optimize the well production schedule. The rationality of the real schedule can be evaluated by comparing the optimized schedule and the real schedule. The control frequency is set to 15 days. This gives 28 control time steps in total and results in 28 optimization
Figure 17: The cumulative gas production of optimized schedule and real schedule of well QW1.

Table 3: The reservoir parameters of QW2.

| Parameter                          | Value       |
|-----------------------------------|-------------|
| Area, m²                          | $9.06 \times 10^5$ |
| Thickness, m                      | 6.0         |
| Top depth, m                      | 950         |
| Matrix porosity                   | 0.008       |
| Matrix permeability, $10^{-3} \mu$m² | 0.03        |
| Fracture half-length, m           | 50          |
| Fracture equivalent permeability, $10^{-3} \mu$m² | 60          |
| Initial pressure, MPa             | 6.6         |
| Desorption pressure, MPa          | 2.1         |
| Desorption time, d                | 1.0         |
| Initial water saturation          | 1.0         |
| Fissure distance, m               | 0.01        |
| Langmuir volume, m³/t             | 28.5        |
| Langmuir pressure, MPa            | 2.5         |

Figure 18: The bottom hole pressure in reality of well QW2.
variables. The minimum bottom hole pressure is set to 0.2 MPa and the maximum bottom hole pressure decline is set to 3.0 MPa per step. CMA-ES is used to solve the optimization problem and the maximum iteration is chosen as a stopping criterion. The maximum number of iteration is set to 2000.

3.2.2. Results and Discussion. The well production performance under the optimized production schedule is compared with the performance under its real production schedule. The well bottom hole pressure of the optimized schedule and the real schedule are shown in Figure 15. The well gas production rate and the cumulative gas production, for the optimized schedule and the real schedule, are shown in Figures 16 and 17, respectively.

From the figures, we can see that the optimized production schedule and the real schedule show similar trends. This indicates the operation of the field engineer is basically reasonable. But there still have some room for improvement. With the optimized schedule, the well QW1 reaches its peak earlier and the peak is higher. When the well enters the stable period, the production rate with the optimized production still is slightly higher than the real schedule. Finally, the optimized schedule yield is 21.80% higher than the real schedule in cumulative gas production. This is a considerable amount of gas.

![Figure 19: The gas and water production rate in reality of well QW2.](image1)

![Figure 20: The bottom hole pressure of optimized schedule and real schedule of well QW2.](image2)
3.3. Experiment 3, Another Fractured Well

3.3.1. Reservoir Model Description. Similar to experiment 2, the well (QW2) in this experiment is a fractured well and is located in Qinshui Basin, China. Since the coal seam in Qinshui Basin is heterogeneous, the reservoir parameters of QW2 are different from QW1 in experiment 2. The details of reservoir parameters of QW2 are given in Table 3.

The well QW2 has been produced 178 days. The well bottom hole pressure and the water and gas production rate in reality are shown in Figures 18 and 19.

Same as experiment 2, we optimize the well production schedule and evaluate the reality schedule for experiment 3. The optimization parameters are the same as experiment 2. Since well QW2 only produced 178 days, the number of variables is 12 for this optimization problem.

3.3.2. Results and Discussion. The well production performance under the optimized production schedule is compared with the performance under its reality production schedule. The well bottom hole pressure of the optimized schedule and the real schedule are shown in Figure 20. The well gas production rate and the cumulative gas production, for the optimized schedule and the real schedule, are shown in Figures 21 and 22, respectively.

From the figures, we can see that the bottom hole pressure of the optimized schedule declines slower than the
real schedule at the early stage. The bottom hole pressure drops too fast in reality and the reservoir near the wellbore was damaged. That makes the production rate does not show an obvious peak. With the optimized schedule, the pressure drop funnel expanded sufficiently. The production peak of the optimized schedule is 3 times higher than the reality. Finally, the optimized schedule yield is 27.74% higher than the real schedule in cumulative gas production.

4. Concluding Remarks

In this paper, an optimization model is constructed to determine the optimal step-down production schedule of CBM wells. The objective of the model is to maximize cumulative gas production. The decline of bottom hole pressure at each time step is used as the optimization variable. Bottom hole pressure and its decline rate are limited to certain ranges as model constraints. Considering the features of the optimization problem, an intelligent algorithm, CMA-ES, is chosen as the solver. The process of solving the step-down production optimization problem using CMA-ES is given.

Three experiments including both normal well and real featured wells are studied. Based on thorough numerical experiments, the following conclusions can be drawn:

(i) CMA-ES, with its stochastic evolution mechanism, can solve the CBM well production schedule optimization problem efficiently. The converge speed is highly affected by the number of optimization variables. With the increases of variables, the converge rate decreases rapidly. During the optimization, CMA-ES needs 3 or even more times number of function evaluations to converge to 100% of the optimum value comparing to 99%.

(ii) The optimized schedule can better fit the heterogeneity and complex dynamic changes of CBM reservoir, which results a higher production rate peak and a higher stable period production rate. The cumulative production under the optimized schedule can increase by 20% or even more.

(iii) With the increases of control frequency, the converge rate decreases rapidly and the production performance increases slightly. The increase in the number of control steps will increase operating costs. Setting a reasonable control frequency is important since the problem with a large number of control steps is harder to optimize and the algorithm has a higher risk of falling into local optima.

The proposed CBM well production schedule optimization method can also be used to evaluate the rationality of the realistic schedule. The findings in the experiment results are also instructive for understanding the relationship between control strategy and CBM well production performance. Although the proposed method has shown its potential in this work, there are still many possible avenues for the future. Currently, the size of control time step is given before the optimization, and each time step is exactly the same. Optimizing the control value of each time step and the time step size simultaneously will be considered in the future work. Joint optimization of well locations and production schedule of CBM reservoir also needs additional study.

Nomenclature

\( p \): Bottom hole pressure control sequence
\( p_i \): Bottom hole pressure at the \( i \)th time step
\( n \): Number of time steps
\( \Delta p \): Control sequence of the well described by the decline of bottom hole pressure
\( \Delta p_i \): Bottom hole pressure decline value at the \( i \)th time step
\( p_r \): Initial pressure of the coalbed methane reservoir
\( t_{end} \): End time of well production
\( q \): Instantaneous production rate
\( t \): Production time
\( p_{min} \): Lower limits of the bottom hole pressure
\( p_{max} \): Upper limits of the bottom hole pressure
\( \Delta p_{max} \): Upper bound of bottom hole pressure decline
\( \lambda_1 \): Individual at generation \( k \)
\( \lambda \): Random vector from a multivariate normal distribution
\( m_k \): Mean value of the search distribution at generation \( k \)
\( s_k \): Step-size at generation \( k \)
\( C_k \): Covariance matrix at generation \( k \)
\( \omega_i \): Recombination weights
\( \mu \): Number of best individuals
\( x_{\mu} \): \( \mu \)th best individual at generation \( k \)
\( c_{cov} \): Learning rate for the rank-\( \mu \) update of the covariance matrix update
\( \mu_{cov} \): Variance effective selection mass for the mean
\( p_k^* \): Evolution path at generation \( k \)
\( c_k \): Learning rate for cumulation for the rank-one update of the covariance matrix
\( \mu_{eff} \): Variance effective selection mass
\( p_k^c \): Conjugate evolution path at generation \( k \)
\( c_k \): Learning rate for the cumulation for the step-size control
\( d_k \): Damping parameter for step-size update
\( E \): Expectation value.

Appendix

Algorithm Description of CMA-ES

CMA-ES samples a population of \( \lambda \) candidate solutions at iteration \( k \) according to
\[ x_i^k = \mathcal{N}\left( m_i^k, (\sigma^i)^2 C^k \right), \quad \text{for } i = 1, \ldots, \lambda, \]  
(A.1)

where \( \mathcal{N}(\ldots, \ldots) \) is a random vector from a multivariate normal distribution.

The mean vector \( m^k \) represents the favorite solution or best estimate of the optimum, and the covariance matrix \( C^k \) is a symmetric positive definite matrix which characterizes the geometric shape of the distribution and defines where new candidate solutions are sampled. The step-size \( \sigma \) is used as a global scaling factor for the covariance matrix. It aims at achieving fast convergence and preventing premature convergence. These three parameters are updated as the iteration proceeds.

The \( \lambda \) individuals generated by equation (A.1) are evaluated and ranked by objective function value. The mean \( m^k \) is then updated by taking the weighted mean of the best \( \mu \) individuals:

\[ m^{k+1} = \sum_{i=1}^{\mu} \omega_i x_i^{k}, \]  
(A.2)

where \( x_i^{k} \) is the \( i^{\text{th}} \) best individual.

The default weights [41] are chosen as

\[ \omega_i = \frac{\ln (\mu + 1) - \ln (i)}{\ln (\mu + 1) - \ln (\mu)}, \quad \text{for } i = 1, \ldots, \mu. \]  
(A.3)

Typically \( \mu \) is chosen as \( \mu = \lceil \lambda/2 \rceil \), where \( \lceil \cdot \rceil \) is the floor function, and \( \omega_i \) is strictly positive and normalized weight.

The covariance matrix \( C^k \) is then updated as

\[ C^{k+1} = (1 - c_{\text{cov}}) C^k + \frac{c_{\text{cov}}}{\mu_{\text{cov}}} \bar{p}_i^{k+1}, \]  
(A.4)

where quantity \( \bar{p}_i^k \) is called the evolution path. It gives a direction where we expect to see good solutions. The evolution path is given iteratively as

\[ \bar{p}_i^{k+1} = (1 - c_c) \bar{p}_i^k + \sqrt{c_c (2 - c_c) \mu_{\text{eff}} \frac{m^{k+1} - m^k}{\sigma}}. \]  
(A.5)

where \( c_c \) is a constant in \((0, 1]\). The quantity \( \mu_{\text{eff}} = 1/\sum_{i=1}^{\mu} \omega_i^2 \) denotes the variance effective selection mass. It is a measure characterizing the recombination. From equation (A.5), we can see that the new search direction \( \bar{p}_i^{k+1} \) is based on the old direction \( \bar{p}_i^k \) and the descent direction \((m^{k+1} - m^k)/\sigma\).

The adaptation of the global step size \( \sigma^{k+1} \) is given by

\[ \sigma^{k+1} = \sigma \exp \left[ \frac{c_c}{\sigma} \left( \frac{\| \bar{p}_i^{k+1} \|}{\mathbb{E}[\| \bar{p}_i^0 \|]} - 1 \right) \right], \]  
(A.6)

which depends on the conjugate evolution path \( \bar{p}_i^{k+1} \) given by

\[ \bar{p}_i^{k+1} = (1 - c_c) \bar{p}_i^k + \sqrt{c_c (2 - c_c) \mu_{\text{eff}}} \sigma^{k-1} C^{-1/2} (m^{k+1} - m^k). \]  
(A.7)

In combination with covariance matrix adaptation, step-size adaptation enables linear convergence on a wide range of, even ill-conditioned, functions [27].

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Disclosure**

The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

**Conflicts of Interest**

The authors declare no conflicts of interest.

**Authors’ Contributions**

Qiujia Hu and Xianmin Zhang conceptualized the study. Xiang Wang performed methodology. Xiang Wang provided software. Bin Fan performed validation. Bin Fan performed formal analysis. Xianmin Zhang investigated the study. Qiujia Hu provided resources. Huimin Jia performed data curation. Xiang Wang wrote the original draft. Xianmin Zhang reviewed and edited the article. Xiang Wang performed visualization. Qiujia Hu performed project administration. All authors have read and agreed to the published version of the manuscript.

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