CHARGED HEAVY LEPTON PRODUCTION IN
SUPERSTRING INSPIRED E6 MODELS

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Abstract

The possibility of studying superstring inspired E6 phenomenology at high energy hadron colliders is investigated. A very simple low energy rank-5 Supersymmetric (N=1) model is considered, which consists of three scalar-Higgses, $H^0_{i=1,2,3}$, two charged-Higgses, $H^\pm$, one pseudo-scalar-Higgses, $P^0$, and an extra vector boson, the $Z'$. The production of charged heavy leptons pairs, $L^+L^-$, by gluon-gluon fusion and Drell-Yan mechanisms is discussed. For gluon-gluon fusion an enhancement in the parton level cross-section is expected due to the heavy (s)fermion loops which couple to the gluons. This mechanism is expected to dominate over Drell-Yan for $L^+L^-$ invariant masses above the $Z'$ mass.
I. INTRODUCTION

In this paper the production of charged heavy leptons pairs, \( L^+L^- \), via superstring inspired \( E_6 \) models, at high energy hadron colliders will be investigated \cite{1,2}. A general overview of \( E_6 \) models will be presented in which several simplifying assumptions will be made in order to restrict the \( L^+L^- \) production computation to a manageable one. In particular, a low energy rank-5 model, arising from \( E_6 \), will be constructed, with this specific application in mind. The \( L^+L^- \) production cross-sections will then be computed followed by a discussion and then finally conclusions.

Many aspects of the rank-5 models, that will be considered here, are covered in the literature. Unfortunately, when trying to extract the particular model dependent information, needed for \( L^+L^- \) production, it appeared that the existing literature was not consistent. Therefore, it was felt that in order to avoid any ambiguities that the model should be carefully reconstructed from the ground up. When constructing the model careful attention was paid to being as consistent as possible with the literature concerning: factors of two, hypercharge conventions, signs, ambiguous notational subtleties, etc. Much of the analysis of the model was done by using Mathematica \cite{3} to generate the various couplings, mass matrices, etc., directly from the superpotential. This enabled easy comparison with various literature sources \cite{4–9}. Differing conventions and normalizations aside, the most significant problem arose with the charged-Higgs, Eq. (38), and pseudo-scalar-Higgs, Eq. (39), mass terms; a factor two was missing in front of the \( \sin \beta \cos \beta \) terms, \textit{op. cit.}. For example, in the case of the pseudo-scalar-Higgs the aforementioned authors disagree to by an overall factor of two in their mass-mixing matrices but not in their eigenvalues. As a result, the analysis of the mass constraints in the Higgs sector \cite{9} had to be re-evaluated, Figs. 6-9. In addition Appendix A contains a summary of the couplings used for \( L^+L^- \) production which, in general, could not be obtained from the literature.
II. SUPERSTRING INSPIRED E\(_6\) MODELS

The SU(3)\(_c\) \(\otimes\) SU(2)\(_L\) \(\otimes\) U(1)\(_Y\) standard model (SM) is a very successful model \([10]\). It has thus far withstood a lot of rigorous experimental testing; however, despite its success the SM has many of problems:

- no unification of the forces
- gauge hierarchical and fine tuning problems
- three generations of quarks and leptons for no particular reason
- too many parameters to be extracted from experiment

Some of the earlier attempts at unification tried to unify the strong and electroweak forces by embedding the SU(3)\(_c\) \(\otimes\) SU(2)\(_L\) \(\otimes\) U(1)\(_Y\) structure into higher groups, such as SU(5) and SO(10). These “grand unified theories” \([11]\), or GUT’s, were only partially successful. The simplest of the GUT’s was SU(5) which seemed promising at the time because it predicted the ratio of the SU(2)\(_w\) and U(1)\(_{em}\) couplings and the proton lifetime \([12]\). However, the ordinary SU(5) GUT is no longer a possibility because more refined experimental measurements are now in disagreement with its predictions for the couplings and the proton lifetime \([4]\). In addition this simple model had too many parameters and no explanation for family replication. The next likely candidate group was SO(10) \([13]\), although the three (or more) copies of the generational structure still had to be inserted by hand.

Difficulties with the SM and GUT models concerning gauge hierarchy and fine tuning problems led to theoretical remedies such as technicolour and supersymmetry (SUSY) \([14]\). The most appealing of these theories was SUSY \([15]\), which had generators that related particles of different spin in the same supermultiplet. The locality of these generators leads to supergravity models. SUSY (and its extended versions) however, did not have enough room for all of the SM particles \([12]\). To solve this problem direct product structures were
made with SUSY and Yang-Mills gauge groups. These structures are now commonly referred to as “SUSY” models [16,17]. Of course the price paid for this was a large particle spectrum (at least twice that of the SM) and the problem of family replication still remained.

In the early 1970’s some interest was sparked in E$_6$ as a GUT when it was discovered that all the then known generations of fermions could be placed in a single 27 dimensional representation. This (“topless”) model [18] was quite popular because the newly discovered $\tau$ lepton and $b$ quark could also be fitted neatly into the 27; there was no need for a third generation. However this model was quickly disallowed, as it was experimentally [18] shown that the $\tau$ and $b$ belonged to a third generation, and the idea of E$_6$ as a GUT died.

$$27 = \left\{ \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ \nu_{\mu} \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau} \end{pmatrix}_L, \begin{pmatrix} e \end{pmatrix}_L, \begin{pmatrix} d_{L}^c, d_{L}^{c'} \end{pmatrix}_L, \begin{pmatrix} \nu_{e,L}^c, \nu_{\mu,L}^c, \nu_{\tau,L}^c \end{pmatrix}_L, \begin{pmatrix} \nu_{\mu}^c \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau}^c \end{pmatrix}_L \right\}$$

$$27 = \left\{ \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} \nu_{\mu} \end{pmatrix}_L, \begin{pmatrix} \nu_{\mu}^c \end{pmatrix}_L, \begin{pmatrix} \nu_{\mu}^{c'} \end{pmatrix}_L, \begin{pmatrix} \mu_{L} \end{pmatrix}_L, \begin{pmatrix} \mu_{L}^c \end{pmatrix}_L, \begin{pmatrix} \mu_{L}^{c'} \end{pmatrix}_L, \begin{pmatrix} \mu_{L}^{c''} \end{pmatrix}_L \right\}$$

$$27 = \left\{ \begin{pmatrix} t \\ b \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau} \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau}^c \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau}^{c'} \end{pmatrix}_L, \begin{pmatrix} \nu_{\tau}^{c''} \end{pmatrix}_L, \begin{pmatrix} \tau \end{pmatrix}_L, \begin{pmatrix} \tau' \end{pmatrix}_L, \begin{pmatrix} \tau' \end{pmatrix}_L \right\}$$

FIG. 1. E$_6$ particle content. The SM particles are shown in the boxes on the left and their “exotic” counterparts outside the boxes on the right. Although the exotics are labeled in away that suggests they have the same quantum numbers as the non-exotics, in general they need not. The labeling for these particles in the literature has not been settled upon and varies quite significantly from paper to paper [4]. Here the labeling scheme was chosen to reflect a specific E$_6$ model that will be constructed in this paper. In particular, all the exotics will carry the “expected” quantum numbers as their non-exotic counter part do, with the exception being L=0 for the primed and double primed ones.

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*aNote: Embedded in the 27’s is the symmetry group SU(2)$_1$ [1] due to an ambiguity in the particle assignments $\left\{ \begin{pmatrix} \nu_{\mu}^c \\ \nu_{\mu}^{c'} \end{pmatrix}_L \right\} \leftrightarrow \left\{ \begin{pmatrix} \nu_{\mu}^c \\ \nu_{\mu}^{c'} \end{pmatrix}_L \right\}$ and $\left\{ \nu_{\mu,L}^c \right\} \leftrightarrow \left\{ \nu_{\mu,L}^{c'} \right\}$ [cf. Fig. 2(d)]. This ambiguity can easily be seen via the decomposition $27=\sum_{\oplus, (SO(10),SU(5))}$.*
In late 1984 Green and Schwarz [19] showed that 10 dimensional string theory is anomaly free if its gauge group is either \( E_8 \otimes E_8' \) or SO(32). The group that had received the most attention was \( E_8 \otimes E_8' \) as it led to chiral fermions, similar to those in the SM, whereas SO(32) did not. Furthermore, it was shown that compactification down to 4 dimensions (assuming N=1 SUSY) can lead to \( E_6 \) as an “effective” GUT group. Each family of SM particles now sits in its own 27, Fig. 1. The generational problem may be solved because it is expected that any reasonable compactification scheme should generate the appropriate number of copies of the 27. For instance in a Calabi-Yau compactification scheme [20],

\[
E_8 \otimes E_8' \rightarrow SU(3) \otimes E_6 \otimes E_8',
\]

the number of generations is related to the topology of the compactified space. A further assertion that the matter fields remain supersymmetrically degenerate ensures proper management of any gauge hierarchical and fine tuning problems. It is assumed that the hidden sector, \( E_8' \), which couples to the matter fields of \( E_6 \) by gravitational interactions will provide a mechanism for lifting the degeneracy.

So the inspiration for using \( E_6 \) is that if it proves to be a possible GUT then it opens up the possibility of finding a TOE (Theory Of Everything). However, it should be pointed out that \( E_6 \) is not the only possible stop en route to the SM, but it is the most studied [4]. It is for this reason that the low energy phenomenology resulting from \( E_6 \) will be considered.

A. \( E_6 \) Phenomenology

1. An extra \( Z_E \)

In order to produce the SM gauge structure, \( E_6 \) must be broken. Also, to handle any hierarchical and fine tuning problems, SUSY must be preserved [20]. This restriction makes the task more difficult, using most naïve breaking schemes. The solution to the problem was found by using a Wilson-loop mechanism [20] over the non-simply-connected-compactified-string-manifold to factor out the various subgroups of \( E_6 \). Fig. 2 shows some of the possible,
popular, rank 5 and rank 6 groups that can be produced by this scheme. As it can be seen,

\[(a) \ E_6 \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y_E}\]

\[(b) \ E_6 \rightarrow SO(10) \otimes U(1)_{\psi} \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_{\chi} \rightarrow SU(3) \otimes U(1)_Y \otimes U(1)_\theta \]

\[\text{ER5M} \rightarrow U(1)_{Y=V=L+R} \]

\[(c) \ E_6 \rightarrow SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R \]

\[(d) \ E_6 \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SU(2)_I \otimes U(1)' \]

FIG. 2. E_6 Wilson-loop-breaking schemes [4]. (a) shows a rank-5 model and (b) through (d) show rank-6 models. Scheme (a) gives the SM plus an extra \(U(1)_{Y_E}\). Schemes (b) through (d) can produce effective rank-5 models, ER5M, by taking a large VEV limit.

The various breaking schemes always give rise to extra vector bosons beyond the SM: in fact it is unavoidable [21–24]. Here, only the simplest of these models [Fig. 2(a)] which generates an extra vector boson, the \(Z_E\), will be considered.

2. The Supermatter Fields

The most general superpotential that is invariant under \(SU(3)_c \otimes SU(2)_L \otimes U(1)_Y\) and renormalizable for the fields given in Fig. 1 is of the form (neglecting various isospin contractions and generational indices) [4],

\[W = W_0 + W_1 + W_2 + W_3\]

\[W_0 = \lambda_1 \Phi_R \Phi_Q \Phi_{u_L} + \lambda_2 \Phi_L \Phi_Q \Phi_{d_L} + \lambda_3 \Phi_L \Phi_L \Phi_{e_L} + \lambda_4 \Phi_R \Phi_L \Phi_{\nu_{e_L}} + \lambda_5 \Phi_{d_L} \Phi_{e_L} \Phi_{\nu_{e_L}}\]

\[W_1 = \lambda_6 \Phi_{d_L} \Phi_{u_L} \Phi_{e_L} + \lambda_7 \Phi_L \Phi_{d_L} \Phi_{Q} + \lambda_8 \Phi_{\nu_{e_L}} \Phi_{d_L} \Phi_{d_L}\]

\[W_2 = \lambda_9 \Phi_{d_L} \Phi_{Q} \Phi_{Q} + \lambda_{10} \Phi_{d_L} \Phi_{u_L} \Phi_{d_L}\]

\[W_3 = \lambda_{11} \Phi_R \Phi_L \Phi_{\nu_{e_L}}\]
\( \Phi_A = \Phi(A, \tilde{A}) \) is the superfield, such that \( A = R', Q, u_L^c, \ldots \), and

\[
\begin{align*}
\Phi_Q &= \left( \Phi_u \right)_L, \\
\Phi_L &= \left( \Phi_{\nu e} \right)_L, \\
\Phi_{L'} &= \left( \Phi_{\nu' e} \right)_L, \\
\Phi_{R'} &= \left( \Phi_{\nu' e} \right)^c_L,
\end{align*}
\]

for the first generation of the 27's, and similarly for the other generations. The Yukawa couplings, \( \lambda_i \)'s, also carry generational which have been suppressed; the couplings are inter-generational as well as intra-generational. The superpotential, \( W \), summarizes the entire possible spectrum of low energy physics which can occur within the context of an E\( _6 \) framework.

Notice that \( W \) was only required to be invariant under the SM gauge group. Further constraints from E\( _6 \) model building may cause some of the \( \lambda_i \) terms to disappear. Furthermore, not all of these terms can simultaneously exist without giving rise to \( \Delta L \neq 0 \) and \( \Delta B \neq 0 \) interactions; E\( _6 \) models say nothing about the assignments of baryon (B) and lepton (L) number until they are connected to SM representations. As a result various scenarios may occur,

- Leptoquarks: \( B(q_L') = \frac{1}{3}, L(q_L') = 1 \) \( \implies \lambda_9 = \lambda_{10} = 0 \)
- Diquarks : \( B(q_L') = -\frac{2}{3}, L(q_L') = 0 \) \( \implies \lambda_6 = \lambda_7 = \lambda_8 = 0 \)
- Quarks : \( B(q_L') = \frac{1}{3}, L(q_L') = 0 \) \( \implies \lambda_6 = \lambda_7 = \lambda_8 = \lambda_9 = \lambda_{10} = 0 \)

where it has been assumed that \( L(\nu_L^c) = -1 \) (these scenarios assume that there exist only three copies of the 27; more complicated ones can be constructed by adding extra copies).

In this paper the least exotic of these models, \( i.e., \) the “Quarks,” will be investigated. Furthermore, to avoid any fine tuning problems with the neutrino masses,

\[
m_{\nu e_L} << m_e \iff \lambda_{11} << \lambda_3,
\]

it will be assumed \( \lambda_{11} = 0 \).

In this model the masses of the particles are generated by letting the role of the Higgs fields be played by

\[
\tilde{L}' = \begin{pmatrix} \tilde{\nu}_{eL}' \\ \tilde{e}_{L}' \end{pmatrix}, \quad \tilde{R}' = \begin{pmatrix} \tilde{e}_{cL}' \\ \tilde{\nu}_{eL}' \end{pmatrix}.
\]
for each generation. It is possible to work in a basis where only the third generation of Higgses acquire a VEV; the remainder become “unHiggses” \cite{4,5}. In this basis the Yukawa couplings,

$$
\lambda_{ij}^{\prime} \Phi_{R_i} \Phi_{L_j} \Phi_{\nu_k}^\prime,
$$

where \( i, j, k = 1, 2, 3 \) are generational indices, takes on a much simpler form,

$$
\lambda_{4} \in \{ \lambda_{ij}^{\prime} | \lambda_{4}^{33} = \lambda_{4}^{3i} = \lambda_{4}^{3k} = 0, \lambda_{4}^{33} = \lambda_{4}^{jk} = \lambda_{4}^{jk} = \lambda_{4}^{jk} \neq 0 \text{ s.t. } i = 1, 2 \& j, k = 1, 2, 3 \}.
$$

This basis also eliminates the potential problem of flavour changing neutral currents at the tree level. It is also assumed that the \( \lambda_i \)'s are real and that the couplings to the unHiggses are very small. The former assumption helps to further simplify the model and reduce any effects it might have in the CP violation sector \cite{4}.

**B. Heavy Lepton Production**

\( E_6 \) models are very rich in their spectrum of possible low energy phenomenological predictions. If any new particles are found that fit within this framework then perhaps it will lead the way to a more unified theory of the fundamental forces of nature. However this is no small task, for a full theory would have to be able to actually predict the mass spectrum of the particles and the relationships between various couplings, and yet require very few parameters. Superstring inspired \( E_6 \) models are far from being able to complete this task. However, proof that \( E_6 \) is an effective GUT would be a good first step. But even this would not necessarily qualify superstrings to be the next step for it is not totally inconceivable that some other theory might give rise to \( E_6 \) as an effective GUT — \textit{caveat emptor}.

A natural question to ask would be, “Where to look for \( E_6 \) phenomenology?” High energy hadron colliders, such as the TEVATRON at Fermilab (1.8 TeV c.o.m., \( \mathcal{L} \sim 10^2 pb^{-1}/yr \), \( pp \)) or the LHC (14 TeV c.o.m., \( \mathcal{L} \sim 10^5 pb^{-1}/yr \), \( pp \)), offer possibilities of observing phenomena beyond the SM by looking for the production of heavy leptons through a mechanism known as gluon-gluon fusion \cite{25,26}, see Fig. 3. This is a interesting process because there are
FIG. 3. Gluon-gluon fusion to two heavy leptons, $gg \rightarrow L^+L^-$. The loop contains quarks, $q$, which couple to vector bosons, $Z_{i=1,2}$, scalar-Higgses, $H^0_{i=1,2,3}$, and a pseudo-scalar-Higgs, $P^0$, and squarks, $\tilde{q}$, which couple to scalar-Higgses.

enhancements in the cross-sections related to the heavy (s)fermions running around in the loop. The computation was done in the minimal supersymmetric standard model (MSSM) by Cieza Montalvo, et al., \cite{Montalvo} in which they predict $\mathcal{O}(10^5)$ events/yr. Therefore for $E_6$ is it expected that the production rate should in principle be higher since there are more particles running around in the loop.

FIG. 4. Drell-Yan, direct, production to two heavy leptons, $q\bar{q} \rightarrow L^+L^-$. The other contribution to $L^+L^-$ production is the Drell-Yan mechanism, see Fig. 4. In general, this is expected to be small for $L^+L^-$ invariant masses above the $Z$ resonances
Indeed, for MSSM it is the least dominant effect. However, since $E_6$ has an extra vector boson, $Z'$, that is fairly massive, it is expected that Drell-Yan production will compete with gluon-gluon fusion below the $Z'$ threshold.

These processes will be investigated in this paper.

III. A LOW ENERGY $E_6$ MODEL

In section I, a general overview of superstring inspired $E_6$ models was given. In addition, several comments were made about the type of model that would be presented. We shall now expand on these assumptions to determine their low energy consequences.

There are many ways of breaking $E_6$ down to SM energies. Invariably these breaking schemes lead to SM phenomenologies which contain extra gauge bosons. Here a rather simple model was chosen in which only an extra $Z$, the $Z'$, is produced [4,7,8]:

$$E_6 \rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y_E}$$

(cf. Fig. 2). In general the $Z'$ can mix with the SM $Z$ to produce the mixed states

$$Z_1 = \cos \phi Z + \sin \phi Z', \quad (2)$$

$$Z_2 = -\sin \phi Z + \cos \phi Z', \quad (3)$$

[cf. Eq. (46)].

Recall that in order to avoid potential problems with flavour-changing-neutral currents, at the tree level, a basis was chosen in which the third generation of primed-exotic-sleptons were assigned to play the role of the Higgses [4,5]:

$$\tilde{L}_3 = \left( \begin{array}{c} \tilde{\nu}_L' \\ \tilde{\tau}_L' \end{array} \right), \quad \tilde{R}_3 = \left( \begin{array}{c} \tilde{\tau}_c^L \\ \tilde{\nu}_c^L \end{array} \right), \quad \tilde{\nu}_c^L,$$

(i.e., $\tilde{R}_3 \equiv \tilde{L}_3^c$) or by redefining $\tilde{L}_3'$, $\tilde{R}_3'$ and $\tilde{\nu}_c^L$, in terms of the complex-isodoublet fields, $\Phi_1$ and $\Phi_2$, and the complex-isoscalar field, $\Phi_3$, respectively, Eq. (4) becomes...
\[ \Phi_1 = \begin{pmatrix} \phi_0^1 \\ \phi_1^1 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \quad \Phi_3 = \phi_3^0. \] (5)

This assignment was accomplished by setting
\[
\lambda^{33i}_4 = \lambda^{33i}_4 = \lambda^{33i}_4 = 0 \quad i = 1, 2 \\
\lambda^{3jk}_4 = \lambda^{3jk}_4 = \lambda^{3jk}_4 \neq 0 \quad j, k = 1, 2, 3
\] (6)
in the superpotential, Eq. (1), where the \( ijk \)'s are generation indices. Therefore in order to avoid lepton-number violation the lepton-numbers of all of the primed and double-primed exotic-leptons must be zero.

Further restrictions were placed on the superpotential by requiring that the baryon and lepton numbers of the exotic-quarks, \( q' \), of the \( 27' \)'s (Fig. 1), are the same as those of their non-exotic SM counterparts \[4\]:
\[
B(q'_L) = \frac{1}{3}, \quad L(q'_L) = 0 \quad \Rightarrow \quad \lambda_6 = \lambda_7 = \lambda_8 = \lambda_9 = \lambda_{10} = 0.
\]

Also \( \lambda_{11} \) was set equal to zero in order to avoid any fine tuning problems with the \( \nu_{\ell_L} \) masses \[4\].

With all the aforementioned assumptions about the Yukawa couplings, the superpotential now simplifies to
\[
W = \lambda_1 i \Phi Q \tau_2 \Phi R \Phi u_L + \lambda_2 i \Phi Q \Phi d_L \tau_2 \Phi L \Phi e_L \\
+ \lambda_3 i \Phi Q \Phi d_L \Phi e_L + \lambda_4 \Phi d_L \Phi d_L \Phi e_L \\
+ \lambda_5 \Phi d_L \Phi d_L \Phi e_L
\] (7)

where the \( \lambda \)'s were chosen to be real, plus similar terms for the other generations and their cross-terms. The \( \Phi_A = \Phi(\psi_A, A) \) are the superfields which contain a two-component-spinor field, \( \psi_A \), and a complex-scalar-singlet field, \( A \). Table 1 summarizes the particle properties of this model.

The superpotential specifies all of the couplings between the particles of the \( 27' \)'s. According to appendix B of Haber and Kane \[16\], the Yukawa interactions are given by
\[
\mathcal{L}_{\text{Yuk}} = -\frac{1}{2} \left[ \left( \frac{\partial^2 W}{\partial A_i \partial A_j} \right)_{\psi_A^s=0} \psi_i \psi_j + \left( \frac{\partial^2 W}{\partial A_i \partial A_j} \right)_{\bar{\psi}_A^s=0} \bar{\psi}_i \bar{\psi}_j \right]
\] (8)
and the scalar interactions are given by

\[ V = V_F + V_D + V_{\text{Soft}}. \] (9)

In Eq. (9), we have

\[ V_F = F_i^*F_i, \] (10)

\[ V_D = \frac{1}{2} [D^a D^a + (D')^2], \] (11)

with

\[ F_i = \left( \frac{\partial W}{\partial A_i} \right)_{\psi_i^A s = 0}, \] (12)

\[ D^a = g A_i^a T_i^a A_j, \] (13)

\[ D' = \frac{1}{2} g' Y_i A_i^a A_j, \] (14)

where \( g \) represents an SU(N) coupling constant with generators \( T^a \), and \( g' \) represents a U(1) coupling constant with hypercharge \( Y \). \( V_{\text{Soft}} \) is a soft-SUSY-breaking term that was put in by hand in order to lift the supersymmetric-mass-degeneracy between \( m_{\psi} \) and \( m_A \):

\[ V_{\text{Soft}} = V_{\tilde{q}} + V_{\tilde{t}} + V_{\text{HSoft}}, \] (15)
The minimization condition \[4\]

\[\dot{V}_H \supset \tilde{M}_Q^2 |\bar{Q}|^2 + \tilde{M}_u^2 \bar{u}_R \bar{u}_R + \tilde{M}_d^2 \bar{d}_R \bar{d}_R + \tilde{M}_L^2 \bar{\bar{d}}_L \bar{\bar{d}}_L + \tilde{M}_d^2 \bar{d}_R \bar{d}_R +
2 \text{Re} [\lambda_1 A_u i \bar{Q}^T \tau_2 \bar{d}_R + \lambda_2 A_d i \bar{\bar{Q}}^T \tau_2 \bar{d}_R + \lambda_5 A_d \bar{\bar{d}}_R \bar{\bar{d}}_R \bar{\bar{Q}}_3], \] (16)

\[V_l \supset \tilde{M}_L^2 |\bar{L}|^2 + \tilde{M}_e \bar{\bar{e}}_R \bar{\bar{e}}_R + \tilde{M}_{\nu e} \bar{\nu}_{eR} \bar{\nu}_{eR} + 2 \lambda_3 A_e \text{Re} [i \bar{\Phi}_1^T \tau_2 \bar{\bar{e}}_R], \] (17)

\[V_{H_{sof t}} = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \mu_3^2 |\Phi_3|^2 - \frac{1}{\sqrt{2}} \lambda A (i \bar{\Phi}_1^T \tau_2 \Phi_2 \Phi_3 + h.c.). \] (18)

The coefficients \(\tilde{M}_A^2, A_A, \mu_i^2,\) and \(A\) are the soft-SUSY-breaking parameters, and \(\lambda \equiv \lambda_i^{33}.\)

The Higgs potential can be extracted directly from Eq. (17) and is given by (13)

\[V_H = V_{H_{sof t}} + \lambda^2 (|\Phi_1|^2 |\Phi_2|^2 + |\Phi_1|^2 |\Phi_3|^2 + |\Phi_2|^2 |\Phi_3|^2)
+ \frac{1}{8} (g^2 + g') (|\Phi_1|^2 - |\Phi_2|^2)^2 + \frac{1}{12} g'' (|\Phi_1|^2 + 4 |\Phi_2|^2 - 5 |\Phi_3|^2)^2
+ (\frac{7}{2} g^2 - \lambda^2) |\Phi_1| |\Phi_2|^2, \] (19)

where \(g, g',\) and \(g''\) are the \(SU(2)_L, U(1)_Y,\) and \(U(1)_{Y_E}\) coupling constants, respectively.

The minimization condition (14)

\[\frac{\partial V_H}{\partial \phi_i^0} \bigg|_{VEV's} = 0, \] (20)

can be used to fix the \(\mu_i^2\) terms in \(V_{H_{sof t}}\) [Eq. (18)], where the vacuum expectation values, \(VEV's,\) are given by

\[\langle \Psi \rangle = \left\{ \begin{array}{ll}
\frac{\nu_i}{\sqrt{2}} & \text{if } \Psi = \phi_i^0 \\
0 & \text{otherwise}
\end{array} \right. \] (21)

Therefore, we have

\[\mu_i^2 = \frac{3}{72} g'' (\nu_1^2 + 4 \nu_2^2 - 5 \nu_3^2) Y_{E_i} + \frac{1}{8} (g^2 + g') (\nu_1^2 - \nu_2^2) Y_i
- \sum_{j<k} \tilde{\varepsilon}_{ijk} \left[ \frac{(\nu_j^2 + \nu_k^2)}{2} \lambda^2 - \nu_j \nu_k \lambda A \right] \] (22)

where \(\tilde{\varepsilon}_{ijk} = |\varepsilon_{ijk}|.\) The kinetic terms for the scalar fields are given by (28)

\[L_{K.E.} \supset |\mathcal{D}_\mu \Phi_i|^2 \equiv |(\partial_\mu - i \frac{1}{2} G_\mu) \Phi_i|^2, \] (23)
with (cf. [4])

\[ G_{\mu} = (g \tau_3 \sin \theta_W + g' Y \cos \theta_W) A_{\mu} + (g \tau_3 \cos \theta_W - g' Y \sin \theta_W) Z_{\mu} \]

\[ + \sqrt{2} g [\tau_+ W_{\mu}^- + \tau_- W_{\mu}^+] + g'' Y E Z'_{\mu}, \]  

(24)

where \( \tau_\pm = \frac{1}{2}(\tau_1 \pm i \tau_2), \tau_3 |\Phi_3\rangle \equiv 0 \), and \( g' \approx g'' \) [4,6]. The \( \tau_i \)'s are the Pauli matrices acting in isospin space.

The \( \Phi_i \) fields have complex components, \( \phi_i^a \), which were chosen to be of the form [5,28] (cf. [29])

\[ \phi_i^a = \frac{1}{\sqrt{2}} (\phi_{iR}^a + i \phi_{iI}^a), \]  

(25)

where \( \phi_{iR}^a/\sqrt{2} \) and \( \phi_{iI}^a/\sqrt{2} \) are the real and imaginary parts, respectively. Therefore, the \( \{\Phi_i\} \) fields have a total 10 degrees of freedom: four are “eaten” to give masses to the \( W^\pm, Z, \) and \( Z' \) bosons, and the remainder yield [4] two charged-Higgs bosons, \( H^\pm, \) one pseudo-scalar-Higgs boson, \( P^0, \) and three scalar-Higgs bosons, \( H_{i=1,2,3}^0. \) The mass terms for the Higgs fields can be obtained from the second order terms of the expansion of \( V_H(\phi_k) \) about its minimum [28],

\[ V_H(\phi_k) \supseteq \frac{1}{2} \mathcal{M}_{ij}^2 (\phi_i - \langle \phi_i \rangle)(\phi_j - \langle \phi_j \rangle), \]  

(26)

where

\[ \mathcal{M}_{ij}^2 = \frac{\partial^2 V_H}{\partial \phi_i \partial \phi_j} \bigg|_{V_{EV'}}, \]  

(27)

is the Higgs-mass-mixing matrix. Therefore the mass terms for the Higgs fields are simply

\[ \mathcal{L}_M \supseteq - (\phi_{2R}^0 \phi_{1R}^0) \mathcal{M}_{H^\pm}^2 \begin{pmatrix} \phi_2^+ \\ \phi_1^- \end{pmatrix} - \frac{1}{2} (\phi_{2R}^0 \phi_{1I}^0 \phi_{3I}^0) \mathcal{M}_{P^0}^2 \begin{pmatrix} \phi_{2I}^0 \\ \phi_{1I}^0 \\ \phi_{3I}^0 \end{pmatrix} - \frac{1}{2} (\phi_{1R}^0 - \nu_1 \phi_{2R}^0 - \nu_2 \phi_{3R}^0 - \nu_3) \mathcal{M}_{H^0}^2 \begin{pmatrix} \phi_{1R}^0 - \nu_1 \\ \phi_{2R}^0 - \nu_2 \\ \phi_{3R}^0 - \nu_3 \end{pmatrix}. \]  

(28)
The mass-mixing matrices are

\[
\mathcal{M}_{H^\pm}^2 = \frac{1}{2} \begin{pmatrix}
\frac{(\frac{1}{2} g^2 - \lambda^2)\nu_1^2}{\nu_1} + \lambda A \frac{\nu_{13}}{\nu_2} (\frac{1}{2} g^2 - \lambda^2)\nu_{12} + \lambda A \nu_3

\frac{1}{2} g^2 - \lambda^2)\nu_{12} + \lambda A \nu_3
\end{pmatrix},
\]

\[
\mathcal{M}_{\tilde{B}_o}^2 = \frac{\lambda A \nu_3}{2} \begin{pmatrix}
\nu_1 & 1 & \nu_3

\nu_2 & \nu_2 & \nu_2
\nu_3 & \nu_2 & \nu_3
\nu_3 & \nu_3 & \nu_3
\end{pmatrix},
\]

\[
\mathcal{M}_{H^0}^2 = \frac{1}{2} \begin{pmatrix}
B_1 \nu_1^2 + \lambda A \frac{\nu_{23}}{\nu_1} & B_2 \nu_{12} - \lambda A \nu_3 & B_3 \nu_1 - \lambda A \nu_2

B_2 \nu_{12} - \lambda A \nu_3 & B_4 \nu_2^2 + \lambda A \frac{\nu_{13}}{\nu_2} & B_5 \nu_{23} - \lambda A \nu_1

B_3 \nu_{31} - \lambda A \nu_2 & B_5 \nu_{32} - \lambda A \nu_1 & B_6 \nu_3^2 + \lambda A \frac{\nu_{12}}{\nu_3}
\end{pmatrix},
\]

where \(\nu_{ij} = \nu_i \nu_j\). In Eq. (31)

\[
B_1 = \frac{1}{2}(g^2 + g'^2) + \frac{1}{16} g'^2 \quad B_2 = 2 \lambda^2 + \frac{8}{3} g'^2 - \frac{1}{2}(g^2 + g'^2) \quad B_3 = 2 \lambda^2 - \frac{5}{8} g'^2
\]

\[
B_4 = \frac{1}{2}(g^2 + g'^2) + \frac{8}{3} g'^2 \quad B_5 = 2 \lambda^2 - \frac{10}{9} g'^2
\]

\[
B_6 = \frac{25}{18} g'^2
\]

The physical states are obtained by diagonalizing the terms in \(\mathcal{L}_M\). The eigenvectors for the charged and pseudo-scalar Higgs terms are respectively,

\[
H^\pm = \cos \beta \phi_2^\pm + \sin \beta \phi_1^\pm,
\]

\[
G^\pm = -\sin \beta \phi_2^\pm + \cos \beta \phi_1^\pm,
\]

and

\[
P^0 = \sqrt{\frac{\lambda A \nu_3}{2 \nu_3^2}} \left[ \sqrt{\frac{\nu_1}{\nu_2}} \phi_{2I}^0 + \sqrt{\frac{\nu_2}{\nu_1}} \phi_{1I}^0 + \sqrt{\frac{\nu_2}{\nu_3}} \phi_{3I}^0 \right],
\]

\[
G_{10} = \frac{\nu_2^2 \nu_3}{\sqrt{(\nu_2^2 + \nu_3^2)(\nu_2^2 + \nu_2^2 \nu_3^2)}} \left[ \frac{\nu_3}{\nu_2} \phi_{2I}^0 - \frac{\nu_1}{\nu_3} \left( 1 + \frac{\nu_2^2}{\nu_3^2} \right) \phi_{1I}^0 + \phi_{3I}^0 \right],
\]

\[
G_{20} = \frac{\nu_3}{\sqrt{\nu_2^2 + \nu_3^2}} \left[ \frac{\nu_2}{\nu_3} \phi_{2I}^0 + \phi_{3I}^0 \right],
\]

where \(\phi_i^\pm = (\phi_i^\mp)^*\), \(\nu^2 = \nu_1^2 + \nu_2^2\), and \(\tan \beta \equiv \nu_2/\nu_1\). Here, the \(H^\pm\) are the charged-Higgs states with masses
$$m^2_{H^\pm} = \frac{\lambda A \nu_3}{\sin(2\beta)} + \left(1 - 2\frac{\lambda^2}{g^2}\right)m^2_W,$$

(38)

the $P^0$ is the pseudo-scalar-Higgs state with mass

$$m^2_{P^0} = \frac{\lambda A \nu_3}{\sin(2\beta)} \left(1 + \frac{\nu^2}{4\nu_3^2} \sin^2(2\beta)\right),$$

(39)

and the $G^\pm$ and $G^0_{1,2}$ are the Goldstone-boson states with zero mass. The scalar-Higgs term can be diagonalized analytically, however the result is, in general, not very enlightening. For our purposes it suffices to resort to numerical techniques. In the unitary gauge (U-gauge) the Goldstone modes vanish, i.e., the $G'$s = 0, and the fields become physical. This allows the change of basis:

$$\phi^\pm_1 = \sin \beta H^\pm,$$

(40)

$$\phi^\pm_2 = \cos \beta H^\pm,$$

(41)

$$\phi^0_{1I} = \kappa \nu_2 P^0,$$

(42)

$$\phi^0_{2I} = \kappa \nu_3 P^0,$$

(43)

$$\phi^0_{3I} = \kappa \nu_1 P^0,$$

(44)

$$\phi^0_{iR} = \nu_i + \sum_{j=1}^{3} U_{ij} H^0_j,$$

(45)

where $\kappa = 1/\sqrt{\nu_1^2 \nu_2^2 + \nu_2^2 \nu_3^2}$. The $U_{ij}$ are the elements of the inverse of the matrix that was used in the similarity transformation to diagonalized the scalar-Higgs-mass term. With these transformations at hand it is now a straightforward matter to get all of the masses and couplings for the various particles in this model.

The mass terms for the gauge fields can be found by transforming the kinetic terms for the $\Phi_i$ fields, Eq. (23), to the U-gauge basis, Eqs. (40)-(45), yielding:

$$\mathcal{L}_{K.E.} \supset m^2_W W_\mu^+ W^-\mu + \frac{1}{2} (Z Z')_\mu \mathcal{M}_{Z-Z'}^2 \left(\begin{array}{c} Z \\ Z'\end{array}\right)_\mu.$$

(46)

As a consequence, the $W$ mass is

$$m^2_W = \frac{1}{4} g^2 \nu^2,$$

(47)
and the $Z - Z'$-mass-mixing matrix is \[4,6–8\]

$$
\mathcal{M}_{Z-Z'}^2 = \begin{pmatrix}
m_Z^2 & \delta m^2 \\
\delta m^2 & m_{Z'}^2
\end{pmatrix},
$$

(48)

with matrix elements:

$$
m_Z^2 = \frac{1}{4}(g^2 + g'^2) \nu^2, \hspace{1cm} (49)
$$

$$
m_{Z'}^2 = \frac{1}{36} g''(\nu_1^2 + 16\nu_2^2 + 25\nu_3^2), \hspace{1cm} (50)
$$

$$
\delta m^2 = \frac{1}{12} \sqrt{g^2 + g'^2 g''(4\nu_2^2 - \nu_1^2)}.
$$

(51)

Diagonalization of $\mathcal{M}_{Z-Z'}^2$ yields the mass eigenstates given by Eqs. (2) and (3) with eigenvalues

$$
m_{Z_1}^2 = m_Z^2 \cos^2 \phi + \delta m^2 \sin(2\phi) + m_{Z'}^2 \sin^2 \phi, \hspace{1cm} (52)
$$

$$
m_{Z_2}^2 = m_Z^2 \sin^2 \phi - \delta m^2 \sin(2\phi) + m_{Z'}^2 \cos^2 \phi, \hspace{1cm} (53)
$$

and mixing angle

$$
\tan(2\phi) = \frac{2\delta m^2}{m_Z^2 - m_{Z'}^2}.
$$

(54)

Notice that in the large $\nu_3$ limit, $\phi \to \pi/2$, and therefore $Z_1 \to Z$, and $Z_2 \to Z'$. In fact for the range of $V E V$'s that will be considered here, $m_{Z_1} < m_{Z_2}$. Therefore, $Z_1$ will be designated the role of the observed $Z$, at facilities such as LEP or SLC.

The mass terms for the fermions, and hence the Yukawa couplings, can be found by evaluating $\mathcal{L}_{Yuk}$, Eq. (8), in the U-gauge basis and then using Appendix A of Haber and Kane \[16\], to convert to four component spinor notation\[4\]. The result is

$$
\mathcal{L}_{Yuk} \supset -\frac{1}{\sqrt{2}} \left\{ \lambda_1 \nu_2 \bar{u} u + \lambda_2 \nu_1 \bar{d} d + \lambda_3 \nu_1 \bar{e} e + \lambda_4 \nu_2 \bar{e}' e' + \lambda_5 \nu_3 \bar{d}' d' \right\}.
$$

(55)

Therefore, the Yukawa couplings for the first generation are given by:

\[1\text{cf. Eq. (A28). For a more explicit example see section 4.2 of Gunion and Haber 29}\]
\[ \lambda_1 = \frac{g m_u}{\sqrt{2} m_W \sin \beta}, \quad (56) \]
\[ \lambda_2 = \frac{g m_d}{\sqrt{2} m_W \cos \beta}, \quad (57) \]
\[ \lambda_3 = \frac{g m_e}{\sqrt{2} m_W \cos \beta}, \quad (58) \]
\[ \lambda_4 = \frac{\sqrt{2}}{\nu_3} m_{e'}, \quad (59) \]
\[ \lambda_5 = \frac{\sqrt{2}}{\nu_3} m_{d'}, \quad (60) \]

and similarly for the other generations.

The sfermion masses are obtained by evaluating the scalar-interaction potential, \( V \) [Eq. (9)], and then transforming it to the U-gauge basis:

\[ \mathcal{L}_M \supset -V \supset -(\tilde{f}_L \tilde{f}_R)^{T} \mathcal{M}^2_{f} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \quad (61) \]

with

\[ \mathcal{M}^2_{f} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^2_{LR} & M^2_{RR} \end{pmatrix}, \quad (62) \]

being the sfermion-mass-mixing matrix. The mass-mixing matrix elements are given by:

\[ M^{(u)}_{LL} = \tilde{M}_Q^2 + m_u^2 + \frac{1}{6} (3 - 4x_W) m_Z^2 \cos(2\beta) - \frac{1}{36} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2), \quad (63) \]
\[ M^{(u)}_{RR} = \tilde{M}_u^2 + m_u^2 + \frac{2}{3} x_W m_Z^2 \cos(2\beta) - \frac{1}{36} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2), \quad (64) \]
\[ M^{(u)}_{LR} = m_u (A_u - m_{e'} \cot \beta), \quad (65) \]

for the \( \tilde{u}_{L,R} \) squarks;

\[ M^{(d)}_{LL} = \tilde{M}_Q^2 + m_d^2 - \frac{1}{6} (3 - 2x_W) m_Z^2 \cos(2\beta) - \frac{1}{36} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2), \quad (66) \]
\[ M^{(d)}_{RR} = \tilde{M}_d^2 + m_d^2 - \frac{1}{3} x_W m_Z^2 \cos(2\beta) + \frac{1}{72} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2), \quad (67) \]
\[ M^{(d)}_{LR} = m_d (A_d - m_{e'} \tan \beta), \quad (68) \]

for the \( \tilde{d}_{L,R} \) squarks;
\[ M^{(\nu_1 \nu_2)}_{LL} = \tilde{M}^2_{LL} + m^2_{\nu} + \frac{1}{6}x_W m^2_Z \cos(2\beta) + \frac{1}{18} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2) , \]  
(69)

\[ M^{(\nu_1 \nu_2)}_{RR} = \tilde{M}^2_{RR} + m^2_{\nu} - \frac{1}{3}x_W m^2_Z \cos(2\beta) + \frac{1}{72} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2) , \]  
(70)

\[ M^{(\nu_1 \nu_2)}_{LR} = m_{\nu} (A_{\nu} - m_{\nu} \frac{\nu_1 \nu_2}{\nu_3^2}) , \]  
(71)

for the \( \tilde{d}^{\nu}_{L,R} \) squarks;

\[ M^{(\nu_1 \nu_2)}_{LL} = \tilde{M}^2_{L} + m^2_{\nu} - \frac{1}{2}(1 - 2x_W) m^2_Z \cos(2\beta) + \frac{1}{72} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2) , \]  
(72)

\[ M^{(\nu_1 \nu_2)}_{RR} = \tilde{M}^2_{e} + m^2_{\nu} - x_W m^2_Z \cos(2\beta) - \frac{1}{36} g''^2 (\nu_1^2 + 4\nu_2^2 - 5\nu_3^2) , \]  
(73)

\[ M^{(\nu_1 \nu_2)}_{LR} = m_{\nu} (A_{e} - m_{\nu} \tan \beta) , \]  
(74)

for the \( \tilde{e}^{\nu}_{L,R} \) sleptons; and similarly for the other generations. The mass eigenstates are given by

\[ \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\tilde{f}} & \sin \theta_{\tilde{f}} \\ -\sin \theta_{\tilde{f}} & \cos \theta_{\tilde{f}} \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} \]  
(78)

with mass eigenvalues

\[ m^2_{\tilde{f}_1} = M^{2}_{LL} \cos^2 \theta_{\tilde{f}} + M^{2}_{LR} \sin(2\theta_{\tilde{f}}) + M^{2}_{RR} \sin^2 \theta_{\tilde{f}} , \]  
(79)

\[ m^2_{\tilde{f}_2} = M^{2}_{LL} \sin^2 \theta_{\tilde{f}} - M^{2}_{LR} \sin(2\theta_{\tilde{f}}) + M^{2}_{RR} \cos^2 \theta_{\tilde{f}} , \]  
(80)

and mixing angle

\[ \tan(2\theta_{\tilde{f}}) = \frac{M^2_{LR}}{M^2_{LL} - M^2_{RR}} . \]  
(81)

Notice that for fairly large \( \nu_3 \)

\[ \tan(2\theta_{\tilde{f}}) \sim \mathcal{O} \left( \frac{m_{\tilde{f}} A_{\tilde{f}}}{\nu_3^2} \right) , \]  
(82)
where the soft terms have been assumed to be large and degenerate. Therefore, in general, the mixing is only expected to affect the sfermions that have fairly heavy fermion partners.

The supersymmetric partner, or spartner, degrees of freedom for the neutral-Higgs fields (neutral-Higgsinos), $\nu^\prime_{\tau L}$, $\nu^\prime \tau_{L}$, and $\nu^\ddagger_{\tau L}$, along with the spartner degrees of freedom for the neutral gauge fields (neutral-gauginos), $\tilde{\gamma}$, $\tilde{Z}$, and $\tilde{Z}'$, mix to form a (6×6) neutralino, $\tilde{\chi}^0$, mass-mixing matrix.\footnote{A detailed study of the $\tilde{\chi}^0$ mass spectrum can be found in [30].} Similarly the charged-Higgsinos, $\tau^\prime_{L}$ and $\tau^\ddagger_{L}$, and the charged-gauginos, $\tilde{W}^\pm$, form a (2×2) chargino, $\tilde{\chi}^\pm$, mass-mixing matrix.\footnote{The full form of these mass matrices can be found in Ellis, et al., [5] and the details of how to obtain them can be found in appendix B of Haber and Kane [16].} By virtue of supersymmetry the neutralino and chargino mass-mixing matrices contain the same Yukawa and gauge couplings as their spartners, modulo soft terms.

The real and imaginary parts of the sfermion fields, $\tilde{\nu}^\prime_{e L}$, $\tilde{\nu}^\prime \nu_{e L}$, $\tilde{\nu}^\ddagger_{e L}$, $\tilde{\nu}^\prime_{\mu L}$, $\tilde{\nu}^\prime \mu_{L}$, and $\tilde{\nu}^\ddagger_{\mu L}$, yield two separate (6×6)-mass-mixing matrices for the neutral-unHiggses [cf. Eq. (28) for $\phi^0_{i R}$ and $\phi^0_{i I}$], which contain the Yukawa couplings given in Eq. (6). In general, these mass-mixing matrices are expected to lead to very massive unHiggs states.\footnote{The sfermion fields $\tilde{e}^\prime_{L}$, $\tilde{e}^\ddagger_{L}$, $\tilde{\mu}^\prime_{L}$, and $\tilde{\mu}^\ddagger_{L}$, yield two separate (2×2)-mass-mixing matrices for the charged-unHiggses [cf. Eq. (28) for $\phi^\pm_{1 I}$]. These matrices have a large number of unknown parameters and quite naturally acquire a very large mass [cf. (3)].}

The spartner degrees of freedom for the neutral-unHiggses, $\nu^\prime_{e L}$, $\nu^\prime \nu_{e L}$, $\nu^\ddagger_{e L}$, $\nu^\prime_{\mu L}$, $\nu^\prime \mu_{L}$, and $\nu^\ddagger_{\mu L}$, form a (6×6)-mass-mixing matrix for the neutral-unHiggsinos. Therefore, the neutral-unHiggsino mass-mixing matrix contains the same Yukawa couplings as their neutral-unHiggs partners.

Finally, the spartner degrees of freedom for the charged-unHiggses give diagonalized mass eigenstates [see Eq. (55)] which correspond to the charged heavy leptons.
IV. $L^+L^-$ PRODUCTION IN E$_6$

A. Gluon-Gluon Fusion

Fig. 5 shows the Feynman diagrams used for computing the parton level gluon-gluon fusion to heavy leptons matrix elements. It was found that the E$_6$ matrix element computations are very similar to the corresponding MSSM calculation by Cieza Montalvo, et al., [27] (cf. [31]) and can, with some care, be extracted from their paper. The matrix elements are as follows:

1. For the $Z_{1,2}$ exchange diagram shown in Fig. 5(a)

$$\hat{\sigma}^{qZ_{1,2}}_{L^\pm} = \frac{\alpha^2 \alpha_s}{128\pi^2} \frac{m_W^2}{m_W} \beta_L \left| \sum_{i=1}^2 \tilde{C}^{L\pm Z_{1,2}}_L - \tilde{C}^{L\pm Z_{1,2}}_R \right| \xi^{Z_{1,2}}(\hat{s}) \sum_q \tilde{C}^{qZ_{1,2}}_L - \tilde{C}^{qZ_{1,2}}_R |(1 + 2 \lambda_q I_q)|^2 \quad (83)$$

where the left-right Fermion, $f$, couplings are given by

$$\begin{pmatrix} \tilde{C}^{fZ}_L \\ \tilde{C}^{fZ}_R \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} C^{fZ}_L \\ C^{fZ}_R \end{pmatrix} \quad (84)$$

such that
\[ C_{fL,R}^{fZ} = T_{3L,R} - e_f x_W, \]  
\[ C_{fL,R}^{fZ'} = \frac{1}{2} \left( \frac{g''}{g} \right) y_{fL,R}^L \sqrt{1 - x_W}, \]  

where \( T_{3L} = -T_{3L}^c \) is the isospin, \( y_{fL}^L = -y_{fL}^L \) is the \( Y_E \)-hypercharge, and \( e_f \) is the electric charge.

2. For the \( H_0^{0,1,2,3} \) and \( P^0 \) exchange diagrams shown in Fig. 3(b)

\[ \hat{\sigma}^{qH_0^0} = \frac{\alpha^2 \alpha_z^2 m_L^2}{512 \pi^2 m_W^2} b_4^3 | \sum_{i=1}^{3} K^{L_0^H} \zeta^{l_0^H} (\hat{s}) \sum_{q} K^{qH_0^0} m_q^2 [2 + (4 \lambda_q - 1) I_q] |^2 , \]  
\[ \hat{\sigma}^{qP^0} = \frac{\alpha^2 \alpha_z^2 m_L^2}{512 \pi^2 m_W^2} b_4^3 | \sum_{i=1}^{3} K^{L_0^H} \zeta^{l_0^H} (\hat{s}) \sum_{q} K^{qP^0} m_q^2 I_q |^2 , \]  

where the couplings \( K^{fH_0^0} \) and \( K^{fP^0} \) are given by Eqs. (A29)-(A38).

3. For the \( H_i^{0,1,2,3} \) exchange diagrams shown in Figs. 3(c) and 3(d)

\[ \hat{\sigma}^{l_0^H} = \frac{\alpha^2 \alpha_z^2 m_L^2 b_4}{512 \pi_2 m_W^2 (1 - x_W)^2} \sum_{i=1}^{3} K^{l_0^H} \zeta^{l_0^H} (\hat{s}) \sum_{q} \sum_{k=1}^{2} \tilde{K}_k^{l_0^H} (1 + 2 \lambda q I_q) |^2 , \]  

where the sfermion mass eigenstates, \( \tilde{f}_{1,2} \), couplings to the \( H_i^0 \) are given by

\[ \tilde{K}_1^{fH_0^H} = K_{LL}^L \cos^2 \theta_f + K_{LR}^L \sin 2\theta_f + K_{RR}^L \sin^2 \theta_f , \]  
\[ \tilde{K}_2^{fH_0^H} = K_{LL}^L \sin 2\theta_f - K_{LR}^L \cos 2\theta_f + K_{RR}^L \cos^2 \theta_f , \]  

where \( K_{AB}^H \) \((A = L, R) \) are the corresponding couplings for the sfermion helicity states, \( \tilde{f}_{L,R} \), with mixing angle \( \theta_f \). The \( K_{AB}^H \) couplings are given by Eqs. (A39)-(A54).

4. For the \( q(Z_{1,2}, P^0) \) interference terms, via Figs. 3(a) and 3(b),

\[ \hat{\sigma}^{q(Z_{1,2}, P^0)} = \frac{-\alpha^2 \alpha_z^2 m_L^2}{128 \pi^2 m_W^2} b_4^3 K^{L_0^Z} \Re \{ \zeta^{l_0^H} (\hat{s}) \sum_{i=1}^{2} \zeta^{Z_i} (\hat{s})^* \tilde{C}_{L_0^Z}^{l_0^H} - \tilde{C}_{R_0^Z}^{l_0^H} \} \times \sum_{q} K^{qP^0} m_q^2 I_q \sum_{q'} \tilde{C}_{L_0^Z}^{q'Z_i} - \tilde{C}_{R_0^Z}^{q'Z_i} (1 + 2 \lambda_q I_q) \} . \]  

\[ (92) \]
5. For the $(\bar{q} - q) H_{1,2,3}^0$ interference terms, via Figs. 5(b)-5(d),

$$
\sigma_{L^\pm}^{(\bar{q} - q) H_{1,2,3}^0} = \frac{-\alpha_2^2 \alpha_2^2}{256 \pi^2 x_W^2 (1 - x_W)^2} \beta_L^2 \Im \left\{ \sum_{i=1}^{3} K_{L^\pm H_{1}^0}^0 \xi^H_i (\hat{s}) \right\} \times \sum_{q} K_{q H_{1}^0}^0 m_q^2 [2 + (4 \lambda_q - 1) I_q] \sum_{j=1}^{3} K_{L^\pm H_{j}^0}^0 \xi^H_j (\hat{s})^* \times \sum_{\tilde{q}} \sum_{k=1}^{2} K_{\tilde{q}} H_{j}^0 (1 + 2 \lambda_{\tilde{q}} I_{\tilde{q}}) \right\}.
$$

(93)

In the aforementioned list of cross-section equations,

$$
\lambda_p = \frac{m_p^2}{s},
$$

(94)

$$
I_p \equiv I_p (\lambda_p) = \int_{0}^{1} \frac{dx}{x} \ln \left[ 1 - \frac{(1 - x) x}{\lambda_p} \right]
$$

$$
= \begin{cases} 
-2 \left[ \sin^{-1} \left( \frac{1}{2 \sqrt{\lambda_p}} \right) \right]^2 & \text{if } \lambda_p > \frac{1}{4} \\
\frac{1}{2} \ln^2 \left( \frac{r_+}{r_-} \right) - \frac{1}{2} + i \pi \ln \left( \frac{r_+}{r_-} \right) & \text{if } \lambda_p < \frac{1}{4},
\end{cases}
$$

(95)

with $r_\pm = 1 \pm \sqrt{1 - 4 \lambda_p}$ such that $p \in \{f, \tilde{f}\}$,

$$
\xi_{Z_i} (\hat{s}) = \frac{\hat{s} - m_{Z_i}^2}{\hat{s} - m_{Z_i}^2 + i m_{Z_i} \Gamma_{Z_i}},
$$

(96)

$$
\xi_{H_{j}^0, \rho_0} (\hat{s}) = \frac{1}{\hat{s} - m_{H_{j}^0, \rho_0}^2 + i m_{H_{j}^0, \rho_0} \Gamma_{H_{j}^0, \rho_0}},
$$

(97)

where the $\Gamma_{V,\phi}$’s computations are summarized in § A 2, and

$$
\beta_L = \sqrt{1 - \frac{4 m_L^2}{\hat{s}}}.
$$

(98)

The details of the various components that have gone into this computation can be found in appendix [4]. Before the parton level cross-section can be used to compute the heavy lepton production rates some assumptions about the parameters and masses in the model must be made.

The first thing that has to be constrained are the VEV’s. It is reasonable to assume that $v_1/v_2 \lesssim 1$, since $m_b << m_t$, for any reasonable range of Yukawa couplings [4,7]. Now the ratios $v_1/v_2$ and $v_3/v_2$ can be constrained by looking at how the variation in the $Z_1$ (i.e., the “$Z$”) mass affects $\bar{x}_W$ ($\equiv \sin^2 \theta_W$) such that [7]
FIG. 6. Plot of $m_{Z_2}$ and $\Delta$ contour lines as a function of $v_3/v_2$ and $v_1/v_2$. The $\Delta$ contour lines are shown at the $0\sigma$, $1\sigma$, and $2\sigma$ levels. The arrows point to toward the allowed regions on the plot (cf. [7]). The $m_{Z_2} = 500$ GeV line shows the CDF and $D\phi$ constraints, assuming standard couplings [32].
\[
\sin^2 \bar{\theta}_W \equiv 1 - \frac{m_W^2}{m_{Z_1}^2} < \sin^2 \theta_W \equiv \left. \frac{g'^2}{g^2 + g'^2} \right|_{\mu = m_W}.
\]  

(99)

\(x_W(m_W)\) can be found by evolving \(x_W(m_Z) \approx 0.2319 \pm 0.0005\) down to \(m_W\) [33], which gives \(x_W(m_W) \approx 0.233 \pm 0.035\), with \(\alpha^{-1}(m_Z) \approx 127.9 \pm 0.1\) [33], \(m_Z \approx (91.187 \pm 0.007)\) GeV [33], and \(m_W \approx (80.23 \pm 0.18)\) GeV [32]. Therefore given \(\bar{x}_W \approx 0.2247 \pm 0.0019\) [33] yields

\[
\Delta \equiv x_W - \bar{x}_W \approx 0.008 \pm 0.035.
\]  

(100)

Fig. 6 shows the \(\Delta\) contour line as a function of \(v_3/v_2\) and \(v_1/v_2\), along with its 1\(\sigma\) and 2\(\sigma\) level contour lines. Also shown are the \(m_{Z_2}\) contour lines. Taking a 1\(\sigma\) level constraint implies \(v_3/v_2 \gtrsim O(3.5)\) (cf. [4,7]) and \(m_{Z_2} \gtrsim O(200)\) GeV. Unfortunately these constraints are not that tight due to the large uncertainty in \(\alpha(m_Z)\). A stronger constraint can be found by using the CDF and \(D\Phi\) limits on the \(m_{Z_2}\) mass [32], assuming SM-like couplings, Fig. 6. This constraint is fairly reasonable since \(Y_{E}'s \sim O(Y)'s\) (cf. table I). With these constraints \(v_3/v_2 \gtrsim O(7.5)\) and \(m_{Z_2} \gtrsim O(500)\) GeV.

Figs. 7 through 9 show \(H_0^3\) mass contour plots as function of \(m_{P^0}\) and \(m_{H^\pm}\) for \((v_1/v_2, v_3/v_2) = (0.02, 6.7), (v_1/v_2, v_3/v_2) = (0.5, 7.7),\) and \((v_1/v_2, v_3/v_2) = (0.9, 9.1)\), respectively, such that \(m_{Z_2}\) lies roughly around the CDF and \(D\Phi\) limits. These figures are a fairly good representation of the behavior of the \(m_{H_0^3}\) contour lines as a function of \(v_1/v_2\). For fixed \(v_1/v_2\), the contour lines change very little (i.e., \(\lesssim O(5)\)% for \(O(10) \lesssim v_3/v_2 \gtrsim O(4.5)\)). This region corresponds to the \((v_1/v_2, v_3/v_2)\) parameter space for \(m_{Z_2} \gtrsim O(300)\) GeV depicted in Fig. 6. Further examination of the other scalar-Higgses shows \(m_{H_3^0}\) is fairly insensitive to variations in \(m_{P^0}\) and \(m_{H^\pm}\) for fixed \(m_{Z_2}\) and is slightly sensitive to variations in \(v_3/v_2\), whereas the behaviour of \(m_{H_3^0}\) appears to be quite sensitive to any variation. Fortunately for the range of VEV’s considered here (i.e., large \(v_3\)), the only contributions to the parton level cross-sections turn out to be the diagrams which contain the \(Z_i\) and \(H_3^0\) propagators;
the other terms are, in general, suppressed by several orders of magnitude. Therefore the heavy lepton production cross-section is insensitive to variations in $m_{P^0}$ and $m_{H^\pm}$. Here the $P^0$ mass will be set to 200 GeV. The corresponding $H^\pm$ mass was chosen to be 215 GeV for Fig. 7 and 212 GeV for Figs. 8 and 9, which lies within the allowed regions on the $m_{H^0}$ contour plots. Based on the very limited experimental constraints that do exist for supersymmetric models these appear to be very conservative choices. They also lead to fairly reasonable values for the $H^0_i$ masses.

The next parameters that need to be fixed are the soft terms. Exactly how these terms should behave at low energy is not clear. At the moment, their behaviour is very model dependent and unless supersymmetric particles are found this situation will most likely remain so. Here the soft terms will be treated parametrically as function of a single parameter $m_S$. In particular the soft terms will be assumed to be degenerate, $\tilde{M}_f \approx A_f \approx m_S$, with the exception of the $\lambda$ and $A$ terms which were fixed by selecting the $m_{P^0}$ and $m_{H^\pm}$ masses. The selection of the soft terms in this way, including $\lambda$ and $A$ for $m_{P^0, H^\pm} \gtrsim \mathcal{O}(1)$ TeV, typify the generic outcome, for the sfermion masses, of most SUSY-breaking models, cf. [33]. In these models $\mathcal{O}(0.2)$ TeV $\lesssim m_S \lesssim \mathcal{O}(10)$ TeV. How low $m_S$ can be pushed down depends upon the choice of VEV’s ($v_3$ in particular, since it is relatively large). For the VEV’s used here it was found $m_S \gtrsim \mathcal{O}(400 - 450)$ GeV. In general the sfermions with light spartners have masses $\lesssim \mathcal{O}(m_S)$, which are roughly degenerate (within $\mathcal{O}(50)$ GeV) with their mass-eigenstate partners. The stops, $\tilde{t}_1, \tilde{t}_2$, and the exotic squarks, $\tilde{q}_1, \tilde{q}_2$, have splittings $\lesssim \mathcal{O}(\frac{1}{2} m_{t,q})$, for fermion masses $\mathcal{O}(200) \lesssim m_{t,q} \lesssim \mathcal{O}(600)$ GeV, for low values of $m_S$. As $m_S$ approaches $\mathcal{O}(1)$ TeV all of the sfermion mass become degenerate and $\approx \mathcal{O}(m_S)$.

\footnote{in the large $v_3$ limit the couplings $P^0 L^+ L^- \to 0$, via Eqs. (A18) and (A38), and $H^0_i L^+ L^- \to -(m_L/v_3) \delta_{3i}$, via Eqs. (A17) and (A33), and Eq. (4.12) of Hewett and Rizzo [4] for the $U_{3i}$’s in this limit, to $\mathcal{O}(1/v_3)$.}
FIG. 7. A plot of the $H_1^0$ mass contour lines as a function of $m_{\rho_0}$ and $m_{H^\pm}$, for $v_1/v_2 = 0.02$ and $v_3/v_2 = 6.7$ ($m_{Z_2} \approx 496\ GeV$). The dashed curve in the upper left-hand corner is a plot of the zero of the Higgs potential above which it becomes positive.
FIG. 8. A plot of the $H_1^0$ mass contour lines as a function of $m_{\rho a}$ and $m_{H^\pm}$, for $v_1/v_2 = 0.5$ and $v_3/v_2 = 7.7$ ($m_{Z_2} \approx 509$ GeV). The dashed curve in the upper left-hand corner is a plot of the zero of the Higgs potential above which it becomes positive.
FIG. 9. A plot of the $H_1^0$ mass contour lines as a function of $m_{P_0}$ and $m_{H^\pm}$, for $v_1/v_2 = 0.9$ and $v_3/v_2 = 9.1$ ($m_{Z_2} \approx 499 \, \text{GeV}$). The dashed curve in the upper left-hand corner is a plot of the zero of the Higgs potential above which it becomes positive.
Finally there is the matter of fixing the heavy fermion masses. The heavy quark masses, $m_q'$, will be assumed degenerate as will the charged heavy leptons masses, $m_{l'}$, such that $m_{q',l'} \simeq \mathcal{O}(0.1)\text{TeV}$ \[33\]. The $e^{\pm}$ will be designated to play the role of the charged heavy leptons, $L^\pm$.

Figs. 10 through 16 show the rapidity distribution at $y=0$ for $pp^{(\ast)} \rightarrow gg \rightarrow L^+L^-$ as function of $m_L$, for various scenarios. The rapidity distribution, $d\sigma/dy$, is related to the parton level cross-section, $\hat{\sigma}(gg \rightarrow L^+L^-)$, through

$$
\frac{d\sigma}{dy} = \int_{\tau_{\text{min}}}^{e^{|y|}} d\tau G(\sqrt{\tau}e^y, Q^2)G(\sqrt{\tau}e^{-y}, Q^2)\hat{\sigma}(\tau s),
$$

where $\tau = \hat{s}/s$, $\tau_{\text{min}} = 4m_L^2/s$, $\sqrt{s}$ is the center-of-mass energy, and $G(x, Q^2)$ is the gluon structure function. The values of $\sqrt{s}$ have been set to $14\text{TeV}$ (LHC) for Figs. 10-15, and $1.8\text{TeV}$ (Tevatron) for Fig. 16. In these figures the SM couplings and masses were extracted from the PDG \[33\], except for $m_t \approx 180 \pm 12\text{GeV}$ \[34\]. For $G(x, Q^2)$ the leading order Duke and Owens 1.1 (DO1.1) \[35, 36\] gluon distribution was used. The results were compared with the next to leading order MRSA \[37\] gluon distribution function, which yielded a negligible difference. Although these results include squark mixing it was found that there was no significant change if mixing is not included. Since $d\sigma/dy$ is flat about $y = 0$, the relationship between $\frac{d\sigma}{dy}|_{y=0}$ and the total cross-section is immediate. Therefore the total event rate for the $pp^{(\ast)} \rightarrow gg \rightarrow L^+L^-$ production mechanism can be estimated from $y = 0$,

$$
\sigma = \int_{\ln(\sqrt{\tau_{\text{min}}})}^{-\ln(\sqrt{\tau_{\text{min}}})} d\sigma dy \approx -\ln(\tau_{\text{min}})\sigma.
$$

Figs. 10-12 show $\frac{d\sigma}{dy}|_{y=0}$ for different VEV's ratios along the $m_{Z_2} \approx \mathcal{O}(500)\text{GeV}$ contour line of Fig. 6. Notice that as $v_1/v_2$ becomes comparable to $v_3/v_2$ the large $v_3$ limit breaks down and the generally small $qP^0$ term starts to contribute (the $q(Z_{1,2} - P^0)$ contribution also grows quite significantly but remains a negligible contribution). Therefore for relatively large values of $v_1/v_2$ variations in $m_{P^0} (\approx m_{H^\pm} \text{ up to at least } \mathcal{O}(1)\text{TeV})$ become important. However it is more natural to assume that the intragenerational Yukawa couplings are of the
FIG. 10. Rapidity distribution at $y = 0$ for charged heavy lepton production at LHC (14 TeV) as a function of heavy lepton mass, where $v_1/v_2 = 0.02$, $v_3/v_2 = 6.7$, and $m_S = 400$ GeV. The mass spectrum for the non-SM particles involved in these processes are, $m_{Z_2} \approx 496$ GeV ($\Gamma_{Z_2} \approx 20.9$ GeV), $m_{P_0} \approx 200$ GeV ($\Gamma_{P_0} \approx 16.4$ GeV), $m_{H^\pm} \approx 215$ GeV, $m_{H_1^0} \approx 94.3$ GeV ($\Gamma_{H_1^0} \approx 7.50 \times 10^{-3}$ GeV), $m_{H_2^0} \approx 200$ GeV ($\Gamma_{H_2^0} \approx 16.5$ GeV), $m_{H_3^0} \approx 495$ GeV ($\Gamma_{H_3^0} \approx 0.230$ GeV), $m_{q'} = 200$ GeV.
FIG. 11. Rapidity distribution at $y = 0$ for charged heavy lepton production at LHC (14 TeV) as a function of heavy lepton mass, where $v_1/v_2 = 0.5$, $v_3/v_2 = 7.7$, and $m_S = 400$ GeV. The mass spectrum for the non-SM particles involved in these processes are, $m_{Z_2} \approx 509$ GeV ($\Gamma_{Z_2} \approx 21.5$ GeV), $m_{P^0} \approx 200$ GeV ($\Gamma_{P^0} \approx 2.52 \times 10^{-2}$ GeV), $m_{H^\pm} \approx 212$ GeV, $m_{H^0_1} \approx 75.4$ GeV ($\Gamma_{H^0_1} \approx 3.65 \times 10^{-3}$ GeV), $m_{H^0_2} \approx 212$ GeV ($\Gamma_{H^0_2} \approx 7.49 \times 10^{-2}$ GeV), $m_{H^0_3} \approx 507$ GeV ($\Gamma_{H^0_3} \approx 0.198$ GeV), $m_{q'} = 200$ GeV.
FIG. 12. Rapidity distribution at $y = 0$ for charged heavy lepton production at $LHC$ (14 $TeV$) as a function of heavy lepton mass, where $v_1/v_2 = 0.9$, $v_3/v_2 = 9.1$, and $m_S = 400$ GeV. The mass spectrum for the non-SM particles involved in these processes are, $m_{Z_2} \approx 499$ GeV ($\Gamma_{Z_2} \approx 20.8$ GeV), $m_{P_0} \approx 200$ GeV ($\Gamma_{P_0} \approx 8.13 \times 10^{-3}$ GeV), $m_{H^\pm} \approx 212$ GeV, $m_{H^0_1} \approx 52.3$ GeV ($\Gamma_{H^0_1} \approx 1.87 \times 10^{-3}$ GeV), $m_{H^0_2} \approx 216$ GeV ($\Gamma_{H^0_2} \approx 1.37 \times 10^{-2}$ GeV), $m_{H^0_3} \approx 498$ GeV ($\Gamma_{H^0_3} \approx 0.130$ GeV), $m_{q'} = 200$ GeV.
FIG. 13. Rapidity distribution at $y = 0$ for charged heavy lepton production at LHC (14 TeV) as a function of heavy lepton mass, where $v_1/v_2 = 0.02$, $v_3/v_2 = 9.5$, and $m_S = 450$ GeV. The mass spectrum for the non-SM particles involved in these processes are, $m_{Z_2} \approx 700$ GeV ($\Gamma_{Z_2} \approx 31.9$ GeV), $m_{P_0} \approx 200$ GeV ($\Gamma_{P_0} \approx 16.5$ GeV), $m_{H^\pm} \approx 215$ GeV, $m_{H'^1_1} \approx 94.6$ GeV ($\Gamma_{H'^1_1} \approx 7.49 \times 10^{-3}$ GeV), $m_{H'^0_2} \approx 200$ GeV ($\Gamma_{H'^0_2} \approx 16.5$ GeV), $m_{H'^0_3} \approx 700$ GeV ($\Gamma_{H'^0_3} \approx 1.04$ GeV), $m_q = 200$ GeV.
FIG. 14. Rapidity distribution at $y = 0$ for charged heavy lepton production at LHC (14 TeV) as a function of heavy lepton mass, where $v_1/v_2 = 0.02$, $v_3/v_2 = 6.7$, and $m_S = 400 \text{ GeV}$. The mass spectrum is for the non-SM particles involved in these processes are, $m_{Z_2} \approx 496 \text{ GeV} (\Gamma_{Z_2} \approx 19.4 \text{ GeV})$, $m_{P_0} \approx 200 \text{ GeV} (\Gamma_{P_0} \approx 16.4 \text{ GeV})$, $m_{H^\pm} \approx 215 \text{ GeV}$, $m_{H^0_1} \approx 94.3 \text{ GeV} (\Gamma_{H^0_1} \approx 7.50 \times 10^{-3} \text{ GeV})$, $m_{H^0_2} \approx 200 \text{ GeV} (\Gamma_{H^0_2} \approx 16.5 \text{ GeV})$, $m_{H^0_3} \approx 495 \text{ GeV} (\Gamma_{H^0_3} \approx 0.138 \text{ GeV})$, $m_{q'} = 600 \text{ GeV}$. 
FIG. 15. Rapidity distribution at $y = 0$ for charged heavy lepton production at LHC ($14 \text{ TeV}$) as a function of heavy lepton mass, where $v_1/v_2 = 0.02$, $v_3/v_2 = 6.7$, and $m_S = 1\text{ TeV}$. The mass spectrum for the non-SM particles involved in these processes are, $m_{Z_2} \approx 496\text{ GeV}$ ($\Gamma_{Z_2} \approx 20.9\text{ GeV}$), $m_{p_0} \approx 200\text{ GeV}$ ($\Gamma_{p_0} \approx 16.4\text{ GeV}$), $m_{H^\pm} \approx 215\text{ GeV}$, $m_{H_1^0} \approx 94.3\text{ GeV}$ ($\Gamma_{H_1^0} \approx 7.50 \times 10^{-3}\text{ GeV}$), $m_{H_2^0} \approx 200\text{ GeV}$ ($\Gamma_{H_2^0} \approx 16.5\text{ GeV}$), $m_{H_3^0} \approx 495\text{ GeV}$ ($\Gamma_{H_3^0} \approx 0.230\text{ GeV}$), $m_{q^{'}} = 200\text{ GeV}$. 

$\frac{d\sigma}{dy} (\text{nb})$
FIG. 16. Rapidity distribution at $y = 0$ for charged heavy lepton production at the Tevatron (1.8 TeV) as a function of heavy lepton mass, where $v_1/v_2 = 0.02$, $v_3/v_2 = 6.7$, and $m_S = 400 GeV$. The mass spectrum for the non-SM particles involved in these processes are, $m_{Z_2} \approx 496 GeV$ ($\Gamma_{Z_2} \approx 20.9 GeV$), $m_{P_0} \approx 200 GeV$ ($\Gamma_{P_0} \approx 16.4 GeV$), $m_{H^\pm} \approx 215 GeV$, $m_{H_1^0} \approx 94.2 GeV$ ($\Gamma_{H_1^0} \approx 7.50 \times 10^{-3} GeV$), $m_{H_2^0} \approx 200 GeV$ ($\Gamma_{H_2^0} \approx 16.5 GeV$), $m_{H_3^0} \approx 495 GeV$ ($\Gamma_{H_3^0} \approx 0.230 GeV$), $m_{q'} = 200 GeV$. 
same order of magnitude and therefore for \( v_1/v_2 \) to be small. For the rest of these figures then, it will be assumed that \( v_1/v_2 = 0.02 \). Fig. 10 is the figure with the default values.

Fig. 13 shows what happens when a larger \( Z_2 \) mass of \( O(700 \, \text{GeV}) \) (i.e., \( v_3/v_2 = 9.5 \)) is used. For this figure \( m_s \) had to be pushed up slightly to 450 \( \text{GeV} \), in order to produce physical squark masses. The noticeable difference between this and all of the other figures is that the peak has broadened. This is expected since the \( Z_2 \) can remain on-shell for larger values of \( m_L \). Notice that the \( H^0_3 \) resonance cut off seems to follow the \( Z_2 \)'s. More precisely, \( m_{H^0_3} \approx m_{Z_2} \) for large \( v_3 \). This becomes immediately evident when taking the large \( v_3 \) limits of the \( H^0_i \) and \( Z_i \) mass mixing matrices, Eqs. (31) and (48) respectively:

\[
\lim_{v_3 \to \infty} m^2_{H^0_3} = \lim_{v_3 \to \infty} m^2_{Z_2} = \frac{25}{36} g'^2 \, v_3^2 v_2^2 \approx \frac{25}{9} \frac{(v_3/v_2)^2 \, x_W}{1 + (v_1/v_2)^2} \, m^2_Z,
\]

which is in fairly good agreement with all of the figures. Also the overall production is slightly suppressed due to the smallness of the gluon distribution function at large momentum fraction.

Fig. 14 shows what happens when the heavy quark mass was pushed up to 600 \( \text{GeV} \). The effect is quite dramatic. To see why this is so, notice the slight kink in the curve around \( m_L \approx 600 \, \text{GeV} \). There is also a much more significant kink in all of the other graphs around 200 \( \text{GeV} \), i.e., around \( m_L \approx m_q' \). Further examination of the parton level cross-section shows that kink occurs when the heavy quarks in the loops can no longer be on shell.

In Fig. 15 the scalar mass was pushed up to 1 \( \text{TeV} \). Increasing \( m_s \) has caused the terms involving the squarks to be suppressed by several orders of magnitude. The difference between the heavy and light squark cases is that, for heavy squarks, the gluon luminosity is relatively small in the kinematical region where the squarks in the loop are on shell. The \( qH^0_1 \) term now enhances \( L^+L^- \) production, below the \( m_{H^0_3} \) threshold, as the destructive interference with \( \bar{q}H^0_1 \) term, i.e., \( (\bar{q} - q)H^0_1 \), has been suppressed.

Finally Fig. 16 shows what happens at \( \sqrt{s} = 1.8 \, \text{TeV} \), the Tevatron. The overall topology is the same as depicted in Fig. 10 but the \( L^+L^- \) production rate is dramatically reduced: very little gluon luminosity is available to produce these heavy particles.
B. Drell-Yan

Fig. 17 shows the Feynman diagrams used for computing the parton level Drell-Yan contribution to heavy lepton production [4,38]. Drell-Yan production of heavy leptons occurs through an $s$-channel $\gamma$, $Z_1$ or $Z_2$. The differential cross section for this process can be expressed as follows:

$$\frac{d\sigma_{L\pm}}{d\Omega} = \frac{1}{64\pi^2 s^\beta} \left\{ |M_\gamma|^2 + |M_{Z_1}|^2 + |M_{Z_2}|^2 + 2\text{Re} (M_\gamma M_{Z_1}^\dagger + M_\gamma M_{Z_2}^\dagger + M_{Z_1} M_{Z_2}^\dagger) \right\}$$

(104)

The "direct" squared matrix elements (the first three terms in the above expression) are given by

$$|M_i|^2 = \frac{2G_i^4}{(s - M_i^2)^2 + \Gamma_i^2 M_i^2} \left\{ 4v_q^i a_q^i v_L^i a_L^i (2m_L^2 (t - \hat{u}) - (t^2 - \hat{u}^2)) \right\}$$

(105)

while the interference terms are given by

$$2\text{Re} (M_i M_j) = \frac{4G_i^2 G_j^2 ((s - M_i^2)(s - M_j^2) + \Gamma_i M_i \Gamma_j M_j)}{(s - M_i^2)^2 + \Gamma_i^2 M_i^2)((s - M_j^2)^2 + \Gamma_j^2 M_j^2)} \left\{ (v_q^i v_q^j + a_q^i a_q^j) \right\}

\times \left[ (v_L^i v_L^j + a_L^i a_L^j)(2m_L^2 (s - m_L^2) + \hat{u}^2 + \hat{t}^2) - (v_L^i L_q^j + a_L^i L_q^j)(2m_L^2 s) \right]

+ (v_q^i a_q^j + a_q^i v_q^j)(v_L^i a_L^j + a_L^i v_L^j)(2m_L^2 (t - \hat{u}) - (t^2 - \hat{u}^2)) \right\}.$$ 

(106)

In the expressions above, $i$ and $j$ can be $\gamma$, $Z_1$ or $Z_2$; $M_i$ and $\Gamma_i$ are the mass and width of the gauge boson; $m_L$ is the mass of the heavy lepton; $G_\gamma = e$ and $G_{Z_1} = G_{Z_2} = g/\sqrt{1 - x_W}$. 

39
If $i = \gamma$, $v_i^f = Q_f$ and $a_i^f = 0$ for both quarks ($f = q$) or the heavy lepton ($f = L$). If $i = Z_{1,2}$, $v_i^f = (\tilde{C}_R^{fi} + \tilde{C}_L^{fi})/2$ and $a_i^f = (\tilde{C}_R^{fi} - \tilde{C}_L^{fi})/2$, where again $f = (q, L)$ and the couplings $\tilde{C}_{R,L}^{fi}$ are given by Eqs. (85) and (86). Also, $\hat{s}$, $\hat{t}$ and $\hat{u}$ are the usual parton level process Mandelstam variables: $(m^2_L - \hat{t}) = \hat{s}(1 - \beta \cos \theta)/2$ and $(m^2_L - \hat{u}) = \hat{s}(1 + \beta \cos \theta)/2$, where $\theta$ is the angle between the outgoing $L^-$ and the incoming quark $q$.

![Drell-Yan rapidity distribution at $y = 0$, as a function of $m_L$, for $L^+L^-$ production at LHC and the Tevatron. The DO1.1 quark and anti-quark parton distribution functions were used to obtain these results.](image)

**Fig. 18.** Drell-Yan rapidity distribution at $y = 0$, as a function of $m_L$, for $L^+L^-$ production at LHC and the Tevatron. The DO1.1 quark and anti-quark parton distribution functions were used to obtain these results.

**Fig. 18** shows $d\sigma/dy|_{y=0}$, as a function of $m_L$, for Drell-Yan production of $L^+L^-$ at LHC and the Tevatron. These results are shown for the regions of the E$_6$ parameter
space that was explored for gluon-gluon fusion, in the previous section: \( i.e., m_{Z_2} = 500 \text{ GeV} \) and 700 \( \text{ GeV} \) for \( LHC \), and \( m_{Z_2} = 500 \text{ GeV} \) for the \( \text{Tevatron} \). Notice that Drell-Yan production becomes rapidly suppressed for \( 2m_L > m_{Z_2} \): the \( Z_2 \) must now go off-shell to produce the heavy lepton pairs.

In contrast to gluon-gluon fusion, the \( LHC \) results for Drell-Yan production are in general, higher by an order of magnitude. For the \( \text{Tevatron} \) results the difference is quite dramatic! At the \( \text{Tevatron} \), a \( p\bar{p} \) collider, Drell-Yan production occurs mainly via the \( q \) and \( \bar{q} \) valence partons from the \( p \) and \( \bar{p} \), respectively; at the \( LHC \), a \( pp \) collider, the \( \bar{q} \in p \) must come from the sea. The gluon distribution is more similar to sea quark distributions than valence quark distributions, which explains why gluon-gluon fusion is comparable to Drell-Yan production at the \( LHC \), but not at the \( \text{Tevatron} \).

C. Results

Fig. 19 gives a summary of the total cross-section for \( L^+ L^- \) production for the \( E_6 \) model parameter space studied in the previous sections. At the \( \text{Tevatron} \) \( \mathcal{O}(10^{1\pm 1}) \text{ events/yr} \) are expected for \( m_L \lesssim \mathcal{O}(300) \text{ GeV} \), all of it coming from Drell-Yan production (\( i.e., \) gluon-gluon fusion yields \( \lesssim \mathcal{O}(0.1) \text{ events/yr} \)). For \( LHC \), over a reasonable range of parameter space, \( \mathcal{O}(10^{4\pm 2}) \text{ events/yr} \) are expected.

The charged heavy lepton in the \( MSSM \) model \cite{27} is a member of a sequential 4\textsuperscript{th} generation added arbitrarily to the model; the lepton masses were chosen larger that 50 \( \text{GeV} \) and the quarks larger than 150 \( \text{GeV} \). For \( m_L \lesssim \mathcal{O}(250) \text{ GeV} \) the \( LHC \) results are comparable in order of magnitude to the \( MSSM \) predictions, obtained by Cieza Montalvo et al. \cite{27}, which predicts \( \mathcal{O}(10^5) \text{ events/yr} \). However, the dominant mechanism in the \( MSSM \) model is gluon-gluon fusion and for \( E_6 \) this contribution yields \( \mathcal{O}(10^{4\pm 1}) \text{ events/yr} \), which is a factor of at least 10 less than \( MSSM \) results. This is a rather surprising result since it was expected that the \( E_6 \) event rate would be enhanced due to the greater number of heavy particles running around in the loop. The parameters in each model were varied to study
FIG. 19. Summary plot of results for the total $L^+L^-$ production cross-section at LHC ($\sqrt{s} = 14\, TeV$, $\mathcal{L} \sim 10^5\, pb^{-1}/yr$) and Tevatron ($\sqrt{s} = 1.8\, TeV$, $\mathcal{L} \sim 10^2\, pb^{-1}/yr$) energies as a function of $m_L$. The hatched regions are the LHC results for gluon-gluon fusion, from Figs. 10-15, and Drell-Yan production, from Fig. 18. The dash-dot lines are the Tevatron results for gluon-gluon fusion, from Fig. 16, and Drell-Yan production, from Fig. 18.
this non-intuitive result. Unfortunately, it turns out that the E₆ parameters suppress $L^+L^-$ production, since $v_3$ is fairly large. This restriction causes the production to occur mainly through the $Z_{1,2}$ and $H^0_3$ terms. The MSSM has two neutral Higgses and one pseudo-scalar Higgs that are allowed to contribute to the processes. A very simple test on the E₆ model was done by varying $v_3/v_2$, about $v_1/v_2 = 0.2$, that showed for $v_3/v_2 = 2.8$ and $m_L \lesssim 100 \text{GeV}$ a factor of 10 increase was obtained. However, this region of E₆ parameter space is forbidden, see Fig. 6.

It appears that the Drell-Yan mechanism, used in [27], includes only the s-channel photon diagram. We find that the inclusion of the Z leads to a factor of 10 increase in the $L^+L^-$ rate, which puts Drell-Yan on equal footing with gluon-gluon fusion for $m_L \lesssim \mathcal{O}(100) \text{GeV}$, in the MSSM with a fourth generation.

The $L^\pm \rightarrow \nu_L W^\pm$, $\nu_L H^\pm$ decay modes are expected to be similar for both models, as these are SM-like decays. These modes depend upon the mass difference $\Delta = m_L - m_{\nu_L}$. For $\Delta < m_W << m_{H^\pm}$ the decays modes will be by virtual W’s, $W^* \rightarrow f \bar{f}$, and on shell for $\Delta > m_W$. Leptonic decays of the W’s offer the possibility of $L^\pm$ detection by measuring $\ell^+\ell^-$ production with missing transverse momentum, $\not{p}_T$ [27]. The competing SM backgrounds with these processes are $pp \rightarrow \tau^+\tau^-, W^+W^-, Z^0Z^0$. Studies have shown, using SM couplings, that it is possible to pull the $L^+L^-$ signals from background for $\Delta > m_W$ given sufficiently large event rates; it is much more difficult for $\Delta < m_W$ [27, 39, 40]. In general the MSSM event rate is higher than E₆, and therefore detection would more likely indicate a MSSM candidate. If $m_{H^\pm} \approx \mathcal{O}(m_W)$, then $H^\pm \rightarrow f_i\bar{f}_j$ dominates, for naturally large values of tan $\beta$. Since the Higgs likes to couple to massive particles, this would lead multiple heavy jet events which in general would be very difficult to pull out of background in either the MSSM or E₆. Similar processes are expected to occur for the cases $m_W < \Delta < m_{H^\pm}$ and $m_W, m_{H^\pm} < \Delta$. For large enough $m_{H^\pm}$, the sfermion channels also open, i.e., $H^\pm \rightarrow \hat{f}_i\bar{\hat{f}}_j$ (e.g., $\hat{u}\bar{\hat{d}}^*$). The sfermions would eventually decay out leaving only the lightest supersymmetric particles (LSP’s), which will escape undetected along with the $\nu_L$’s leaving lots of $\not{p}_T$. In fact, all of the aforementioned process will lead to events
with $\not{p}_T$, as the $\nu_L$'s will pass through the detector.

In certain regions of the MSSM and E$_6$ model parameter spaces it may be possible to distinguished between the two models. If $L^+L^-$ event rates are larger than those predicted by E$_6$, then the likely candidate is the MSSM. Unlike the MSSM, it is possible that $m_{H^\pm} < m_W$ in E$_6$, and therefore if $H^\pm$'s are found in this mass range the more likely candidate would be E$_6$. Another possible way of telling the models apart is to look for sfermion production, $L^\pm \rightarrow f\bar{f}^*, f\bar{f}$, which is unique to E$_6$, since $L^\pm$ has opposite $R$ parity to the other SM-like fermions in the 27's (Fig. 1). The sfermion would eventually decay to an LSP which is stable (assuming $R$ parity conservation), yielding jets+$\not{p}_T$, in general. Whether or not it is possible to distinguish them from the MSSM and the SM-like backgrounds would require a much more detailed study, as the allowable parameter space for sfermions masses and Yukawa couplings is quite large. Finally, the MSSM does have fairly stringent unitarity constraints on the heavy lepton and heavy quark masses as a function of $\tan \beta$, in particular $m_L \lesssim (1200 \text{ GeV}) \cos \beta$. Therefore, it should be possible to eliminate $(m_L, \tan \beta)$ regions in the MSSM $L^-L^+$ production cross-section plots, as a function of $m_L$, such that only E$_6$ models are allowed. For example, assuming $m_{H_1^0} \gtrsim \mathcal{O}(600) \text{ GeV}$ rules out the MSSM for $m_L \gtrsim 242 \text{ GeV}$ and $\tan \beta \gtrsim 5$. In the allowed MSSM region this gives an upper limit on the $L^+L^-$ production cross-section of $\mathcal{O}(10) \text{ pb}$, at LHC. Also in the MSSM there are phenomenological constraints on $\tan \beta$ which could allow for further restrictions. A more detailed study of these constraints has not been carried out.

In closing, it should be pointed out that only a simple model of E$_6$ has been considered. It is possible for other E$_6$ models to produce results similar to the model studied here or to the MSSM. Therefore, in general, $L^+L^-$ production by gluon-gluon fusion should not be considered a definitive means of separating out the different models; several experiments would be required.

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V. CONCLUSIONS

The $pp \to gg \to L^+L^-$ production cross-section was computed for a simple rank-5 $E_6$ model. For a fairly conservative survey of the various parameters in the model we expect $\mathcal{O}(10^{4\pm 1})$ events/yr at $LHC$, and $\mathcal{O}(10^{1\pm 1})$ events/yr at the $Tevatron$. For $LHC$ and the $Tevatron$ it was found that Drell-Yan production dominated over gluon-gluon fusion for $2m_L \leq m_{Z_1}$. For the $Tevatron$ events are only expected to be seen for $2m_L \lesssim \mathcal{O}(m_{Z_2})$, as the Drell-Yan and gluon-gluon fusion rates drop rapidly beyond this point. The $LHC$ results were compared to the $MSSM$’s ($\mathcal{O}(10^5)$ events/yr [27]), in which gluon-gluon fusion is the dominant production mode. The gluon-gluon fusion contribution to $L^+L^-$ production at $LHC$ ($\mathcal{O}(10^{4\pm 1})$) was found to be at least a factor of 10 less than the event rates predicted for the $MSSM$, due the $CDF$ and $D\phi$ soft limits (i.e., assuming SM couplings) placed on $m_{Z_2}$ [32]. These soft constraints resulted in the $H_{1,2}^0$ and $P^0$ contributions to the $L^+L^-$ production rate to be suppressed leaving only the $H_3^0$ and $Z_{1,2}$ to contribute. For certain regions in the $MSSM$ and $E_6$ parameter spaces it was demonstrated that it is possible to distinguish between the two models, in principle. However, it should be pointed out that there are many candidate $E_6$ models which could yield overlapping results.

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APPENDIX A: COUPLINGS AND WIDTHS FOR $\hat{\sigma}(GG \to L^+L^-)$

This appendix gives a summary of the calculations that were used to obtain the couplings and the widths for the $\hat{\sigma}(gg \to L^+L^-)$ matrix elements given in § IV.
1. The Couplings

In this section the calculations of the vertex factors used to obtain the $gg \rightarrow L^+L^-$ matrix elements, given in § [IV], are summarized.

For the $Z_1, 2$ exchange diagrams shown in Fig. 3(a) the following vertex factors were used

\[
\begin{align*}
Z_i^\mu & \sim \bar f \gamma^\mu \left[ \hat C_L^f Z_i P_L + \hat C_R^f Z_i P_R \right], \\
\end{align*}
\]

(A1)

where $i = 1, 2,$

\[
\begin{align*}
P_L &= \frac{1}{2}(1 - \gamma_5), \\
P_R &= \frac{1}{2}(1 + \gamma_5),
\end{align*}
\]

(A2) (A3)

and $C_Q^{a Z_i}$ and $C_R^{a Z_i}$ were the couplings used in Eq. (83). The gauge-fermion interaction Lagrangian for $SU(2)_L \otimes U(1)_Y \otimes U(1)_E$ is given by

\[
\begin{align*}
\mathcal{L}_{\text{int}} \supset & - \frac{1}{2} (g_L \hat T^a_{ij} L^a_{\mu} + g_Y \delta_{ij} \hat Y_{Y\mu} + g_E \hat Y_{E\mu} E_{\mu}) \bar \psi_i \sigma^\mu \psi_j, \\
\end{align*}
\]

(A4)

where the $\psi_i$'s are two-component spinors, see Eq. (B.2) of Haber and Kane (HK) [16]. Defining $g = g_L, g' = g_Y, g'' = g_E$, and $\hat Y = \hat Y_i (= 2 \hat Q - \hat \tau_3)$, and using the identities

\[
\begin{align*}
a L^3_{\mu} + b Y_{\mu} &= (a \cos \theta_W - b \sin \theta_W) Z_{\mu} + (a \sin \theta_W + b \cos \theta_W) A_{\mu}, \\
E_{\mu} &= Z'_{\mu},
\end{align*}
\]

(A5) (A6)

then Eq. (A4) becomes

\[
\begin{align*}
\mathcal{L}_{\text{int}} \supset & - \left\{ \frac{g}{\cos \theta_W} (\hat T_3 - \hat Q x_W) Z_{\mu} + \frac{1}{2} g'' \hat Y_{E\mu} Z'_{\mu} \right\} \left[ \bar \psi(f_L), \sigma^\mu \psi(f_L) \right] + \left[ \bar \psi(f_{\bar L}), \sigma^\mu \psi(f_{\bar L}) \right], \\
\end{align*}
\]

(A7)

where $\hat T_3 = \hat \tau_3/2$, $\tau_i = \sigma_i$, and $x_W = g'/g$, and $\sin^2 \theta_W$. Noting that
\[
(\hat{T}_3 - \hat{Q} x_W) f^c_L > = -(\hat{T}_3 - \hat{Q} x_W) f_R > ,
\]

(A8)

\[
\hat{Y}_E | f^c_L > = -\hat{Y}_E | f_R > ,
\]

(A9)
yields

\[
\mathcal{L}_{\text{int}} \supseteq - \left\{ \frac{g}{\cos \theta_W} \left( \hat{T}_3 - \hat{Q} x_W \right) Z_\mu + \frac{1}{2} g'' \hat{Y}_E Z'_\mu \right\}_{ij} \left[ \bar{\psi}_{(f_L)^i, \bar{\sigma}^\mu \psi_{(f_L)^j}} - \bar{\psi}_{(f_R)^i, \bar{\sigma}^\mu \psi_{(f_R)^j}} \right].
\]

(A10)

Using the following identities

\[
\bar{\psi}_{(f_L)^i, \bar{\sigma}^\mu \psi_{(f_L)^j}} = \bar{f}_i \gamma^\mu P_L f_j ,
\]

(A11)

\[
-\bar{\psi}_{(f_R)^i, \bar{\sigma}^\mu \psi_{(f_R)^j}} = \bar{f}_i \gamma^\mu P_R f_j ,
\]

(A12)
to convert from two-component to four-component spinor notation yields

\[
\mathcal{L}_{\text{int}} \supseteq -\frac{g}{\sqrt{1 - x_W}} \sum_{A=L,R} \bar{f}_i \left\{ (T_{3A} - e_f x_W) Z + \frac{1}{2} \left( \frac{g''}{g} \right) y'_{fA} \sqrt{1 - x_W} Z' \right\} P_A f ,
\]

(A13)

see Eqs. (A.28) of HK [16]. Then the \( Z-Z' \)-vertex factor is

\[
\begin{aligned}
\overrightarrow{f}
\end{aligned}
\]

\[
\begin{aligned}
\downarrow \overrightarrow{Z''}, Z'''
\end{aligned}
\]

\[
\begin{aligned}
\frac{g}{\sqrt{1 - x_W}} \gamma^\mu \left[ C_L^{iZ,fZ'} P_L + C_R^{iZ,fZ'} P_R \right],
\end{aligned}
\]

(A14)

where the \( C_{L,R}^{iZ,fZ'} \)'s are defined by Eqs. (85) and (86). Using the inverse of transformations (2) and (3),

\[
\begin{pmatrix}
\tilde{Z}' \\
Z
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix},
\]

(A15)
yields the desired result

\[
\mathcal{L}_{\text{int}} \supseteq \frac{g}{\sqrt{1 - x_W}} \sum_{i=1}^{2} \sum_{A=L,R} \bar{f}_i \tilde{Z}_i \tilde{C}_A^{iZ} P_A f ,
\]

(A16)
i.e., vertex factor [A1].
For the $H_{1,2,3}^0$ and $P^0$ exchange diagrams shown in Fig. 5(b) the following vertex factors were used

$$H_{i}^0 \rightarrow -g \frac{m_f}{2m_W} K^{fH_{i}^0},$$

(A17)

and

$$P^0 \rightarrow ig \frac{m_f}{2m_W} \gamma_5 K^{fP^0},$$

(A18)

respectively, where $i = 1, 2, 3$. The $K^{fH_{i}^0}$ and $K^{fP^0}$ couplings are obtained from the Yukawa interaction part of the Lagrangian given by Eq. (8): noting that $\varepsilon_{ij} = (i\tau_3)_{ij}$ and plugging $W$, Eq. (7), into Eq. (8) yields

$$L_{Yuk} \supset -\frac{1}{2}\varepsilon_{ij} \{ -\lambda_1[\Phi_{12}(\bar{\psi}_Q, \bar{\psi}_u_L + \psi_u_L \psi_Q) + \Phi_{21}^*(\bar{\psi}_Q, \bar{\psi}_u_L + \psi_u_L \bar{\psi}_Q)] + \lambda_2[\Phi_{11}(\bar{\psi}_Q, \bar{\psi}_d_L + \psi_d_L \psi_Q) + \Phi_{11}^*(\bar{\psi}_Q, \bar{\psi}_d_L + \psi_d_L \bar{\psi}_Q)] + \lambda_3[\Phi_{12}(\bar{\psi}_d_L, \bar{\psi}_e_L + \psi_e_L \psi_d_L) + \Phi_{12}^*(\bar{\psi}_d_L, \bar{\psi}_e_L + \psi_e_L \bar{\psi}_d_L)] + \lambda_4[\Phi_{3}(\bar{\psi}_L, \bar{\psi}_{\ell}' + \psi_{\ell}' \psi_L) + \Phi_{3}^*(\bar{\psi}_L, \bar{\psi}_{\ell}' + \psi_{\ell}' \bar{\psi}_L)] + \lambda_5[\Phi_{3}(\bar{\psi}_L, \bar{\psi}_{e_L} + \psi_{e_L} \psi_L) + \Phi_{3}^*(\bar{\psi}_L, \bar{\psi}_{e_L} + \psi_{e_L} \bar{\psi}_L)] \} , \quad (A19)

$$\supset -\frac{1}{2}\{ \lambda_1[\phi_{12}^0(\bar{\psi}_u_L, \bar{\psi}_u_L + \psi_u_L \bar{\psi}_u_L) + \phi_{12}^{0*}(\bar{\psi}_u_L, \bar{\psi}_u_L + \psi_u_L \bar{\psi}_u_L)] + \lambda_2[\phi_{12}^0(\bar{\psi}_d_L, \bar{\psi}_d_L + \psi_d_L \bar{\psi}_d_L) + \phi_{12}^{0*}(\bar{\psi}_d_L, \bar{\psi}_d_L + \psi_d_L \bar{\psi}_d_L)] + \lambda_3[\phi_{12}^0(\bar{\psi}_e_L, \bar{\psi}_e_L + \psi_e_L \bar{\psi}_e_L) + \phi_{12}^{0*}(\bar{\psi}_e_L, \bar{\psi}_e_L + \psi_e_L \bar{\psi}_e_L)] + \lambda_4[\phi_{3}(\bar{\psi}_L, \bar{\psi}_{\ell}' + \psi_{\ell}' \psi_L) + \phi_{3}^{0*}(\bar{\psi}_L, \bar{\psi}_{\ell}' + \psi_{\ell}' \bar{\psi}_L)] + \lambda_5[\phi_{3}(\bar{\psi}_L, \bar{\psi}_{e_L} + \psi_{e_L} \psi_L) + \phi_{3}^{0*}(\bar{\psi}_L, \bar{\psi}_{e_L} + \psi_{e_L} \bar{\psi}_L)] \} , \quad (A20)$$
and similarly for the other generations. Defining

\[ f = \begin{pmatrix} \psi_{f_L} \\ \bar{\psi}_{f_L} \end{pmatrix} \]  

(A21)

and using the following identities

\[ \psi_{f^c_1L} \psi_{f^c_2L} = \psi_{f^c_2L} \psi_{f^c_1L} = \bar{f}_1 P_L f_2, \]  

(A22)

\[ \bar{\psi}_{f^c_1L} \bar{\psi}_{f^c_2L} = \bar{\psi}_{f^c_2L} \bar{\psi}_{f^c_1L} = \bar{f}_2 P_R f_1, \]  

(A23)

see Eqs. (A.24), (A.25), and (A.28) of HK [16], implies

\[ L_{\text{Yuk}} \sim -\lambda_i (\phi^0_i \psi_{f^c_L} \psi_{f^c_L} + \phi^{0\ast}_i \bar{\psi}_{f^c_L} \bar{\psi}_{f^c_L}), \]  

(A24)

\[ = -\frac{1}{2} \lambda_i [\phi^0_i \bar{f}(1 - \gamma_5)f + \phi^{0\ast}_i \bar{f}(1 + \gamma_5)f], \]  

(A25)

\[ = -\lambda_i [\text{Re}(\phi^0_i)\bar{f}f - i \text{Im}(\phi^0_i)\bar{f}\gamma_5f], \]  

(A26)

\[ = -\frac{1}{\sqrt{2}} \lambda_i (\phi^0_{jR} \bar{f}f - i \phi^0_{jI} \bar{f}\gamma_5f). \]  

(A27)

Expanding the \( \phi^0_i \)'s in terms of their physical fields, Eqs. (40)-(45), yields

\[ L_{\text{Yuk}} \supseteq -\frac{1}{\sqrt{2}} \left\{ \lambda_1 \nu_2 \bar{u}u + \lambda_2 \nu_1 \bar{d}d + \lambda_3 \nu_1 \bar{e}e + \lambda_4 \nu_3 \bar{e}'e' + \lambda_5 \nu_3 \bar{d}'d' \right\} \]  

\[ -\frac{1}{\sqrt{2}} \sum_{j=1}^{3} \left\{ \lambda_1 U_{2j} \bar{u}u + U_{1j} (\lambda_2 \bar{d}d + \lambda_3 \bar{e}e) + U_{3j} (\lambda_4 \bar{e}'e' + \lambda_5 \bar{d}'d') \right\} H^0_{j} \]

\[ + \frac{ik}{\sqrt{2}} \left\{ \nu_{13} \bar{u}\gamma_5 u + \nu_{23} (\lambda_2 \bar{d}\gamma_5 d + \lambda_3 \bar{e}\gamma_5 e) + v_{12} (\lambda_4 \bar{e}'\gamma_5 e' + \lambda_5 \bar{d}'\gamma_5 d') \right\} P^0, \]  

(A28)

The couplings can now be read directly and give, via Eqs. (56)-(60),

\[ K^uH_i^0 = \frac{1}{\sin \beta} U_{2i}, \]  

(A29)

\[ K^dH_i^0 = \frac{1}{\cos \beta} U_{1i}, \]  

(A30)
\[
K^d H_i^0 = \frac{2m_W}{g\nu_3} U_{3i}, \quad (A31)
\]
\[
K^e H_i^0 = \frac{1}{\cos \beta} U_{1i}, \quad (A32)
\]
\[
K^e' H_i^0 = \frac{2m_W}{g\nu_3} U_{3i}, \quad (A33)
\]

for the scalar Higgs fields, \( H_i^0 \), and
\[
K^u P_0 = \frac{1}{\sin \beta} \kappa_{v13}, \quad (A34)
\]
\[
K^d P_0 = \frac{1}{\cos \beta} \kappa_{v23}, \quad (A35)
\]
\[
K^d' P_0 = \frac{2m_W}{g\nu_3} \kappa_{v12}, \quad (A36)
\]
\[
K^e P_0 = \frac{1}{\cos \beta} \kappa_{v23}, \quad (A37)
\]
\[
K^e' P_0 = \frac{2m_W}{g\nu_3} \kappa_{v12}, \quad (A38)
\]

for pseudo-scalar Higgs fields, \( P_0 \).

For the \( H_{1,2,3}^0 \) exchange diagrams shown in Figs. 3(c) and 3(d) the following vertex factors were used

\[
\tilde{f}_B \gamma^\nu \tilde{f}_A \kappa_{AB} = -\frac{g_{MZ}}{\sqrt{1-x_W}} K^\nu H_i^0, \quad (A39)
\]

where \( A, B = L, R \). The \( \kappa_{AB} \) couplings, which were obtained from Eq. (1), are as follows:
\( \kappa_{LL}^{H_0} = \left( \frac{1}{18} g^{\prime 2} - \frac{1}{4} g + \frac{1}{12} g^2 \right) U_{1i} \nu_1 + ( \frac{2}{9} g^{\prime 2} + \frac{1}{4} g - \frac{1}{12} g^2 - \lambda_3^2 ) U_{2i} \nu_2 \)
\[ - \frac{5}{18} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{RR}^{H_0} = \left( \frac{1}{18} g^{\prime 2} - \frac{1}{3} g^2 \right) U_{1i} \nu_1 + ( \frac{2}{9} g^{\prime 2} + \frac{1}{3} g^2 - \lambda_1^2 ) U_{2i} \nu_2 \)
\[ - \frac{5}{18} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{LR}^{H_0} = \frac{1}{2} [(U_{3i} \nu_1 + U_{1i} \nu_3) \lambda - \sqrt{2} U_{2i} A_d] \lambda_1 , \]  
\( \kappa_{LL}^{H_0} = \left( \frac{1}{18} g^{\prime 2} + \frac{1}{4} g + \frac{1}{12} g^2 - \lambda_2^2 \right) U_{1i} \nu_1 + ( \frac{2}{9} g^{\prime 2} - \frac{1}{4} g - \frac{1}{12} g^2 ) U_{2i} \nu_2 \)
\[ - \frac{5}{18} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{RR}^{H_0} = - \left( \frac{1}{36} g^{\prime 2} - \frac{1}{6} g^2 + \lambda_2^2 \right) U_{1i} \nu_1 - ( \frac{1}{9} g^{\prime 2} + \frac{1}{6} g^2 ) U_{2i} \nu_2 \)
\[ + \frac{5}{36} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{LR}^{H_0} = \frac{1}{2} [(U_{2i} \nu_1 + U_{1i} \nu_2) \lambda - \sqrt{2} U_{3i} A_d] \lambda_2 , \]  
\( \kappa_{LL}^{H_0} = - \left( \frac{1}{9} g^{\prime 2} + \frac{1}{6} g^2 \right) U_{1i} \nu_1 - ( \frac{1}{9} g^{\prime 2} - \frac{1}{6} g^2 ) U_{2i} \nu_2 \)
\[ + ( \frac{5}{9} g^{\prime 2} - \lambda_3^2 ) U_{3i} \nu_3 , \]  
\( \kappa_{RR}^{H_0} = - \left( \frac{1}{36} g^{\prime 2} - \frac{1}{6} g^2 \right) U_{1i} \nu_1 - ( \frac{1}{9} g^{\prime 2} + \frac{1}{6} g^2 ) U_{2i} \nu_2 \)
\[ + ( \frac{5}{36} g^{\prime 2} - \lambda_3^2 ) U_{3i} \nu_3 , \]  
\( \kappa_{LR}^{H_0} = \frac{1}{2} [(U_{2i} \nu_1 + U_{1i} \nu_2) \lambda - \sqrt{2} U_{3i} A_d] \lambda_3 , \)

for the squark-Higgs couplings, and

\( \kappa_{LL}^{H_0} = - \left( \frac{1}{18} g^{\prime 2} - \frac{1}{4} g + \frac{1}{12} g^2 + \lambda_3^2 \right) U_{1i} \nu_1 - ( \frac{2}{9} g^{\prime 2} + \frac{1}{4} g - \frac{1}{12} g^2 ) U_{2i} \nu_2 \)
\[ + \frac{5}{36} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{RR}^{H_0} = \left( \frac{1}{18} g^{\prime 2} + \frac{1}{2} g^2 - \lambda_3^2 \right) U_{1i} \nu_1 + ( \frac{2}{9} g^{\prime 2} - \frac{1}{2} g^2 ) U_{2i} \nu_2 \)
\[ - \frac{5}{18} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{LR}^{H_0} = \frac{1}{2} [(U_{3i} \nu_2 + U_{2i} \nu_3) \lambda - \sqrt{2} U_{1i} A_e] \lambda_2 , \)  
\( \kappa_{LL}^{H_0} = - \left( \frac{1}{36} g^{\prime 2} + \frac{1}{9} g + \frac{1}{12} g^2 \right) U_{1i} \nu_1 - ( \frac{2}{9} g^{\prime 2} - \frac{1}{9} g - \frac{1}{12} g^2 ) U_{2i} \nu_2 \)
\[ + \frac{5}{36} g^{\prime 2} U_{3i} \nu_3 , \]  
\( \kappa_{RR}^{H_0} = \left( \frac{5}{36} g^{\prime 2} (U_{1i} \nu_1 + 4 U_{2i} \nu_2 - 5 U_{3i} \nu_3 ) , \right) \)
\[ \kappa_{LR} \tilde{f}_H = 0, \]  
(A54)

for the slepton-Higgs couplings. The mass eigenstate couplings \( \tilde{f}_{1,2} \tilde{H}_i \), given by Eqs. (90) and (91), were obtained by inserting

\[
\begin{pmatrix}
\tilde{f}_L \\
\tilde{f}_R
\end{pmatrix} = \begin{pmatrix}
\cos \theta_f & -\sin \theta_f \\
\sin \theta_f & \cos \theta_f
\end{pmatrix} \begin{pmatrix}
\tilde{f}_1 \\
\tilde{f}_2
\end{pmatrix},
\]  
(A55)

which is just the inverse of Eq. (78), into the scalar potential.

The corresponding pseudo-scalar-Higgses couplings

\[ i \kappa_{AB} \tilde{f} \tilde{P}^0 = -i \frac{g_{M_Z}}{\sqrt{1 - \sin W}} K_{AB} \tilde{P}^0 \]  
(A56)

are obtained in a similar fashion as above: i.e.,

\[ \kappa_{AB} \tilde{f} \tilde{P}^0 = -\epsilon_{AB} \frac{\nu_2}{2 \sqrt{\nu_1^2 + \nu_2^2 \nu_3^2}} \left[(\nu_1^2 + \nu_3^2) \lambda + \sqrt{2} A_{u\nu_3} \cot \beta \right] \lambda_1, \]  
(A57)

\[ \kappa_{AB} \tilde{d} \tilde{P}^0 = -\epsilon_{AB} \frac{\nu_1}{2 \sqrt{\nu_1^2 + \nu_2^2 \nu_3^2}} \left[(\nu_2^2 + \nu_3^2) \lambda + \sqrt{2} A_{d\nu_3} \tan \beta \right] \lambda_2, \]  
(A58)

\[ \kappa_{AB} \tilde{E} \tilde{P}^0 = -\epsilon_{AB} \frac{\nu_3}{2 \sqrt{\nu_1^2 + \nu_2^2 \nu_3^2}} \left[\nu_2 \lambda + \sqrt{2} A_{d\nu_3} \frac{\nu_1}{\nu_3} \right] \lambda_5, \]  
(A59)

for squark-pseudo-Higgs couplings, and

\[ \kappa_{AB} \tilde{u} \tilde{P}^0 = -\epsilon_{AB} \frac{\nu_1}{2 \sqrt{\nu_1^2 + \nu_2^2 \nu_3^2}} \left[(\nu_2^2 + \nu_3^2) \lambda + \sqrt{2} A_{u\nu_3} \tan \beta \right] \lambda_3, \]  
(A60)

\[ \kappa_{AB} \tilde{d} \tilde{P}^0 = 0, \]  
(A61)

for the slepton-pseudo-Higgs couplings, where

\[ \epsilon_{AB} = \begin{cases} 
1 & \text{if } A = L, \ B = R \\
0 & \text{if } A = B \\
-1 & \text{if } A = R, \ B = L
\end{cases} \]  
(A62)

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In general, these couplings will also mix to give the mass eigenstate couplings \( \tilde{K}_{1,2} \); which are defined in a similar fashion to Eqs. (90) and (91).

2. The Widths

In this section all of the tree level two-body decay widths for \( Z_2 \), \( H^0_i \), and \( P_0 \) are computed. Therefore the generic two body decay formula is given by [4]

\[
\Gamma = \frac{S|\mathcal{M}_{ab}|^2}{16\pi m_0} \beta_{ab},
\]

(A63)

where \( m_i, i=0,a,b \), are the masses of the particles, \( p_i \), in the decay process \( p_0 \rightarrow p_ap_b \),

\[
\beta_{ab} = \sqrt{1 - \frac{2(m_a^2 + m_b^2)}{m_0^2} + \frac{(m_a^2 - m_b^2)^2}{m_0^4}},
\]

(A64)
such that \( \beta_{ab} \equiv \beta_a \) if \( a = b \), \( S \) is a symmetry factor for the out going particles, \( p_a \) and \( p_b \), and \( \mathcal{M}_{ab} \) is the amplitude for the process.

a. \( \Gamma_{Z_2} \)

For the \( Z_2 \) width, the following processes need to be computed:

\( Z_2 \rightarrow W^+W^-, Z_1H_i^0, W^\pm H^\mp, q_i\bar{q}_i, l_i\bar{l}_i, \tilde{\chi}^0_i\tilde{\chi}^0_j, \tilde{\chi}^+_i\tilde{\chi}^-_j, \tilde{\phi}_i\tilde{\phi}_j, \tilde{l}_i\tilde{l}_j, H^0_iH^0_j, H^+H^-, P^0H^0 \).

The \( Z_2 \rightarrow W^+W^- \) width, which can be found in Hewett and Rizzo [4], is given by

\[
\Gamma(Z_2 \rightarrow W^+W^-) = \frac{g^2m_{Z_2}\sin^2\phi}{192\pi(1 - x_W)} \left( \frac{m_{Z_2}}{m_Z} \right)^4 \beta_W^3 \left[ 1 + 20 \left( \frac{m_W}{m_Z} \right)^2 + 12 \left( \frac{m_W}{m_Z} \right)^4 \right].
\]

(A65)

The \( Z_2 \rightarrow q_i\bar{q}_i, l_i\bar{l}_i \) vertex factors are given by

\[
\varepsilon_{\mu}(q,\lambda) \rightarrow \gamma_{\mu}(v_f - a_f\gamma_5),
\]

(A66)
which were obtained from Eq. (A16) by converting to the $V - A$ basis: i.e.,

$$a P_L + b P_R = v_f - a_f \gamma_5,$$

where

$$v_f = \frac{1}{2} (a + b),$$

$$a_f = \frac{1}{2} (a - b),$$

which yields

$$v_f = \frac{1}{2 \sqrt{1 - x_W}} (\tilde{C}_L^{f Z_2} + \tilde{C}_R^{f Z_2}),$$

$$a_f = \frac{1}{2 \sqrt{1 - x_W}} (\tilde{C}_L^{f Z_2} - \tilde{C}_R^{f Z_2}).$$

Therefore

$$|\mathcal{M}_{ff}|^2 = \frac{1}{3} g^2 \sum_{\lambda} \sum_{\text{spin}} |\bar{v}(p')\gamma_\mu(v_f - a_f \gamma_5)u(p)\varepsilon^\mu(q, \lambda)|^2,$$

$$= \frac{4}{3} g^2 [v_f^2 (m_{Z_2}^2 + 2m_f^2) + a_f^2 (m_{Z_2}^2 - 4m_f^2)].$$

Plugging this into Eq. (A63) gives

$$\Gamma(Z_2 \to f \bar{f}) = c_f g^2 \frac{1}{12\pi} m_{Z_2} \beta_f \left[ v_f^2 \left( 1 + \frac{2m_f^2}{m_{Z_2}^2} \right) + a_f^2 \left( 1 - \frac{4m_f^2}{m_{Z_2}^2} \right) \right],$$

where $c_f$ is a colour factor which is 3 for quarks and 1 for leptons.

The $Z_2 \to \tilde{q}_i \tilde{q}_j^*, \tilde{l}_i \tilde{l}_j^*$ vertex factors are given by

$$\varepsilon_\mu(q, \lambda) \leftrightarrow -ig\kappa_{ij}(p_j^\prime - p_i)^\mu,$$

where $\tilde{f}_{k=1,2}$ are sfermion mass eigenstates, and
\[ \kappa_{ij} = \begin{cases} 
 v_f + a_f \cos 2\theta_f & \text{if } i = j = 1 \\
 v_f - a_f \cos 2\theta_f & \text{if } i = j = 2 \\
 -a_f \sin 2\theta_f & \text{if } i \neq j 
\end{cases} \]  
(A75)

The vertex factor is obtained by follow steps similar to Eqs. (A4)-(A16),

\[ \mathcal{L}_{\text{int}} \supseteq -\frac{i}{2} (g_L \tilde{\tau}^a_L L^a + g_Y \delta_{ij} \tilde{Y}_i Y_j + g_E \tilde{Y}_E E_\mu) \tilde{f}^*_i \gamma^\mu \tilde{f}_j 
(A76) \]

followed by transforming the sfermions to their mass eigenstates by using Eq. (A55),

\[ \mathcal{L}_{\text{int}} \supseteq -\frac{ig}{\sqrt{1 - x_W}} \sum_{i=1}^{2} \sum_{A=L,R} \tilde{C}_A^{f_i Z_i} \tilde{f}^*_A \gamma^\mu \tilde{f}_i Z^\mu, \]  
(A77)

and then changing to the \( V - A \) basis, via Eqs. (A67)-(A71), to get

\[ \mathcal{L}_{\text{int}} \supseteq -ig \sum_{i,j,k=1}^{2} \kappa_{ij} \tilde{f}^*_i \gamma^\mu \tilde{f}_j Z^\mu, \]  
(A79)

Therefore

\[ |M_{\tilde{f}_i \tilde{f}_j}|^2 = \frac{1}{3} g^2 \sum_\lambda |\varepsilon_\mu(q,\lambda)\kappa_{ij}(p'_j - p_i)\mu|^2, \]

\[ = \frac{1}{3} g^2 m_{Z_2}^2 \kappa_{ij}^2 \beta_{\tilde{f}_i \tilde{f}_j}^2. \]  
(A80)

Plugging this into Eq. (A63) gives

\[ \Gamma(Z_2 \rightarrow \tilde{f}_i \tilde{f}^*_j) = c_f \frac{g^2 m_{Z_2}}{48\pi} \kappa_{ij}^2 \beta_{\tilde{f}_i \tilde{f}_j}^2. \]  
(A81)

For the range of VEV’s that will be consider here (i.e., large \( v_3 \) in particular) the \( Z_2 \rightarrow Z_1 H^0, W^\pm H^\mp, H^0 H^0_1, H^+ H^-, P^0 H^0_1 \) widths can be approximated by

\[ \Gamma(Z_2 \rightarrow V + S) \approx \frac{17g^2 x_W}{864\pi(1 - x_W)} m_{Z_2}, \]  
(A82)

where the \( H^0 H^0_1 \) contributions are kinematically forbidden or suppressed \( [4] \).
The $Z_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$, $\tilde{\chi}_i^+ \tilde{\chi}_j^-$ widths are quite difficult to compute, due to the complex nature of the mass matrices, and can contribute as much as 10-20% to the total width, neglecting phase space suppression [4]. Here its contribution will be taken as 15%; this approximation proved to have no noticeable impact on $L^+L^−$ production.

\[ b. \Gamma_{H_i^0} \]

For the $H_i^0$ widths the following processes need to be computed:

\[ H_i^0 \to Z_jZ_k, W^+W^−, q_j\bar{q}_j, l_j\bar{l}_j, \tilde{\chi}_j^0\tilde{\chi}_k^0, \tilde{\chi}_j^+\tilde{\chi}_k^−, \tilde{\chi}_j^−\tilde{\chi}_k^+, H_i^0H_i^0, H^+H^−, P_0^0P_0^0. \]

The $H_i^0 \to Z_jZ_k, W^+W^−$ vertex factors are given by

\[ \Gamma_{H_i^0, H_i^0} \to \bar{q} \gamma^\mu \gamma^5 q, \]

\[ iC_{V_aV_b} \mu, \nu, \]

\[ (A83) \]

where

\[ C_{ZZ_1}^0 = C_{ZZ_1}^0 \cos^2 \phi - 2C_{ZZ_1}^0 \sin 2\phi + C_{ZZ_1}^0 \sin^2 \phi, \]

\[ C_{ZZ_2}^0 = C_{ZZ_2}^0 \cos 2\phi - (C_{ZZ_2}^0 - C_{ZZ_2}^0) \sin 2\phi, \]

\[ C_{ZZ_3}^0 = C_{ZZ_3}^0 \sin^2 \phi + 2C_{ZZ_3}^0 \sin 2\phi + C_{ZZ_3}^0 \cos^2 \phi, \]

\[ (A84, A85, A86) \]

for the $Z_i$’s, with

\[ C_{ZZ}^0 = g \cos \theta_W + g' \sin \theta_W)^2 (U_{11}v_1 + U_{21}v_2), \]

\[ C_{ZZ}^0 = g \cos \theta_W + g' \sin \theta_W)^2 (U_{11}v_1 - 4U_{21}v_2), \]

\[ C_{ZZ}^0 = g \cos \theta_W + g' \sin \theta_W)^2 (U_{11}v_1 + 16U_{21}v_2 + 25U_{31}v_3), \]

\[ (A87, A88, A89) \]

and

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C_{W^+W^-}^{H_0^i} = \frac{g^2}{2} (U_1 v_1 + U_2 v_2), \quad (A90)

for the W's. The vertex factors, $C_{V_a V_b}^{H_0^i}$, were obtained by plugging Eq. (A15) and Eqs. (40)-(45) into the kinetic terms for the scalar-Higgs fields, Eq. (23). Therefore

$$|M_{ab}|^2 = \sum_{\lambda_a \lambda_b} |\epsilon_{\mu}(p_a, \lambda_a) C_{V_a V_b}^{H_0^i} \epsilon^\nu(p_b, \lambda_b) g^{\mu\nu}|^2 = \frac{m_{H_0^i}^4 C_{V_a V_b}^{H_0^i}}{4(m_a m_b)^2} \left[ 1 - \frac{2(m_a^2 + m_b^2)}{m_{H_0^i}^2} + \frac{(m_a^2 + m_b^2)^2 + 8(m_a m_b)^2}{m_{H_0^i}^4} \right], \quad (A91)$$

which yields, via Eq. (A63),

$$\Gamma(H_0^i \to V_a V_b) = \frac{SC_{V_a V_b}^{H_0^i} m_{H_0^i}^2 \beta_{ab}}{64\pi (m_a m_b)^2} \left[ 1 - \frac{2(m_a^2 + m_b^2)}{m_{H_0^i}^2} + \frac{(m_a^2 + m_b^2)^2 + 8(m_a m_b)^2}{m_{H_0^i}^4} \right], \quad (A92)$$

where $S = \frac{1}{2}$ for identical $Z_i$'s, otherwise $S = 1$.

The $H_0^i \to q_i \bar{q}_i, l_i \bar{l}_i$ decay width is

$$\Gamma(H_0^i \to f \bar{f}) = \frac{c_f g^2}{32\pi} \left( \frac{m_f}{m_W} \right)^2 K_f H_0^i \beta_{H_0^i}^2 m_{H_0^i}, \quad (A93)$$

via Eq. (A63) with amplitude

$$|M_{t \bar{f}}|^2 = \frac{g^2}{2} \left( \frac{m_f}{m_W} \right)^2 K_f H_0^i \beta_{H_0^i}^2 \left[ m_{H_0^i}^2 - 4m_f^2 \right], \quad (A94)$$

where the $K_f H_0^i$ couplings are defined by Eq. (A17).

For the scalar processes $H_0^i \to \phi \phi^*$ the vertex factor is

$$H_0^i \quad C_{\phi_a \phi_b}^{H_0^i} \quad \phi_a^* \phi_b \quad \phi_a \phi_b, \quad (A95)$$

which yields the decay width

$$\Gamma(H_0^i \to \phi_a \phi_b^*) = \frac{c_f}{16\pi m_{H_0^i}} |C_{\phi_a \phi_b}^{H_0^i}|^2 \beta_{ab}, \quad (A96)$$

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via Eq. (A63), with amplitude

\[ |\mathcal{M}_{\phi_a\phi_b}|^2 = |C_{\phi_a\phi_b}^{H_i^0}|^2. \]  

(A97)

For \( H_i^0 \rightarrow \bar{q}_j q_k, \overline{\ell}_j \ell_k^* \) the vertex factors are

\[ C_{f_j f_k}^{H_i^0} = \frac{g m_Z}{\sqrt{1 - x_W}} K_{f_j f_k}^{H_i^0}, \]  

(A98)

where the \( K_{f_j f_k}^{H_i^0} \) couplings are given by Eqs. (A29)-(A33). For \( H_i^0 \rightarrow H_j^0 H_k^0, H^+ H^-, P^0 P^0 \) the vertex factors are:

\[ C_{H_i^0 H_j^0 H_k^0}^{H_i^0} = \frac{1}{2} \lambda_A \left( U_{12} U_{21} U_{31} + U_{11} U_{22} U_{31} + U_{11} U_{21} U_{32} \right) \]
\[ + \left\{ U_{22} \left[ \left( -\frac{1}{18} g''^2 + \frac{1}{2} \lambda^2 U_{21}^2 + U_{31}^2 \right) U_{11}^2 - \frac{1}{24} \left( 16 g''^2 + 9 g^2 + 9 g^2 \right) U_{21}^2 \right. \right. \]
\[ + \left. \left. \frac{5}{72} g''^2 U_{31}^2 - \frac{5}{18} \lambda^2 (U_{21}^2 + U_{31}^2) \right\} v_1 \]
\[ + \left\{ U_{31} \left[ \frac{5}{36} g''^2 (U_{11} U_{12} + 4 U_{21} U_{22}) - \lambda^2 (U_{11} U_{12} + U_{21} U_{22}) \right] \right. \]
\[ + \left. U_{32} \left[ \frac{5}{72} g''^2 (U_{11}^2 + 4 U_{21}^2 - 15 U_{31}^2) - \frac{1}{2} (U_{11}^2 + U_{21}^2) \right]\right. \}
\[ + \left. v_3 \right\} \]  

(A99)
\[C_{H_3^0}^{H_3^0} = \frac{1}{2} \lambda A \left( U_{13} U_{22} U_{31} + U_{12} U_{23} U_{31} + U_{13} U_{21} U_{32} \right)
+ U_{11} U_{23} U_{32} + U_{12} U_{21} U_{33} + U_{11} U_{22} U_{33} \]
\[\left\{ \left( \frac{1}{9} g'^2 - \frac{1}{4} g^2 - \frac{1}{4} g'' \right) \right\} v_1 \]
\[\left\{ \left( \frac{1}{9} g'^2 - \frac{1}{4} g^2 - \frac{1}{4} g'' \right) \right\} v_2 \]
\[\left\{ \left( \frac{5}{36} g''(U_{12} U_{13} + 4 U_{22} U_{23}) - \lambda^2(U_{12} U_{13} + U_{22} U_{23}) \right) U_{31} \right\} \]
\[\left\{ \left( \frac{1}{9} g''(U_{11} U_{13} + 4 U_{21} U_{23}) - \lambda^2(U_{11} U_{13} + U_{21} U_{23}) \right) U_{32} \right\} \]
\[\left\{ \left( \frac{5}{36} g''(U_{11} U_{12} + 4 U_{21} U_{22} - 15 U_{31} U_{32}) - \lambda^2(U_{11} U_{12} + U_{21} U_{22}) \right) U_{33} \right\} v_3. \]
for the neutral-scalar-Higgses,

\[ C_{H^+H^-}^H = \frac{-1}{4(1 + \cot^2 \beta)} \left\{ v_1(3g^2 + g'^2 - 4\lambda^2)(U_{1i} + U_{2i}) \cot \beta \right. \\
\left. + \left( g^2 - g'^2 + \frac{4}{9}g'^2 \right)(v_1 U_{1i} \cot^2 \beta + v_2 U_{2i}) \\
+ \frac{1}{9}g'^2 [v_1(v_1 U_{1i} + 16U_{2i}) \cot \beta - 5v_3(1 + 4\cot^2 \beta)U_{3i}] \right. \\
\left. + 4(\lambda^2 v_3(1 + \cot^2 \beta) + \lambda \cot \beta)U_{3i} \right\}, \tag{A103} \]

for the charged-scalar-Higgses,

\[ C_{p^0p^0}^H = \frac{v_3^2}{2(v_1^2v_2^2 + v_2^2v_3^2)} \left\{ \left[ m_Z^2 \cos 2\beta - \frac{1}{9} \left( \frac{g'^2}{g^2} \right) m_W^2 - \lambda^2 v_2^2 v_3^2 \right] (v_1 U_{1i} + v_2 U_{2i}) \right. \\
- \lambda(v_1^2 U_{1i} + v_2^2 U_{2i}) - \lambda^2 v_2 (v_1^2 + v_2^2) U_{3i} - \lambda A v_1 v_2 (v_1 U_{1i} + v_2 U_{2i} + v_3 U_{3i}) \right. \\
+ \frac{5}{36}g'^2 \left[ v_1^2 v_2^2 \left( v_1 U_{1i} + 4v_2 U_{2i} - 5v_3 U_{3i} \right) + v_3(4v_1^2 + v_2^2) U_{3i} \right], \tag{A104} \]

for the pseudo-scalar-Higgses, which were all extracted by plugging Eqs. (10)-(14) for the physical Higgs fields into the Higgs potential, Eq. (19).

The \( H_i^0 \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_k^0, \tilde{\chi}_j^+ \tilde{\chi}_k^- \) decay processes are quite complicated to compute. Here a simple approximation was made in which for \( m_{p^0} \lesssim O(500) \) GeV its contribution to the width was 15%, otherwise 50%. This addition had a negligible affect on \( L^+L^- \) production, since \( m_{p^0} = 200 \) GeV.

c. \( \Gamma_{p^0} \)

For the \( p^0 \) width the following processes need to be computed:

\[ p^0 \rightarrow Z_i Z_j, W^\pm H^\mp, q_i \bar{q}_i, l_i \bar{l}_i, \tilde{\chi}_i^0 \tilde{\chi}_j^0, \tilde{\chi}_i^+ \tilde{\chi}_j^-, \tilde{q}_i \tilde{q}_j^*, \tilde{l}_i \tilde{\chi}_j. \]

For \( p^0 \rightarrow Z_i Z_j, W^\pm H^\mp \) the widths are zero since here \( m_{p^0} < m_{Z^0} \) and \( m_{p^0} \approx m_{H^\pm} \); see Figs. 13 and discussion therein.

The \( p^0 \rightarrow q_i \bar{q}_i, l_i \bar{l}_i \) decay widths are
\[ \Gamma(P^0 \rightarrow f \bar{f}) = \frac{c_f g^2}{32\pi} \left( \frac{m_f}{m_W} \right)^2 K_f P_0^2 \beta_{P_0} m_{P_0}, \]  

(A105)

via Eq. (A63), with amplitudes

\[ |\mathcal{M}_{ff}|^2 = \frac{g^2}{2} \left( \frac{m_f}{m_W} \right)^2 K_f P_0^2 m_{P_0}^2, \]  

(A106)

where the \( K_f P_0 \) couplings defined by Eq. (A18).

The \( P^0 \rightarrow \tilde{q}_j \tilde{q}_k^*, \tilde{l}_j \tilde{l}_k^* \) decay widths are

\[ \Gamma(P^0 \rightarrow \tilde{f}_j \tilde{f}_k^*) = \frac{c_f g^2 m_{Z}^2}{16\pi(1-x_W)m_{P_0}} K^{P_0}_{jk} \beta_{\tilde{f}_j \tilde{f}_k}, \]  

(A107)

via Eq. (A96), with vertex factors

\[ C^{P_0}_{\tilde{f}_j \tilde{f}_k^*} = \frac{g m_Z}{\sqrt{1-x_W}} K^{P_0}_{jk}, \]  

(A108)

where the \( K^{P_0}_{jk} \) couplings are given by Eqs. (A34)-(A38).

In this work \( m_{P_0} \) was fixed at 200 GeV. At this mass \( P^0 \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_j^0, \tilde{\chi}_j^+ \tilde{\chi}_j^- \) decays are suppressed [9].
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