Quantum Gates Between Distant Qubits via Spin-Independent Scattering

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We show how the spin independent scattering of two initially distant qubits, say, in distinct traps or in remote sites of a lattice, can be used to implement an entangling quantum gate between them. The scattering takes place under 1D confinement for which we consider two different scenarios: a 1D wave-guide and a tight-binding lattice. We consider models with a delta interaction between two fermionic or two bosonic particles. A qubit is encoded in two distinct spin (or other internal) states of each particle. Our scheme, by providing a gate between qubits which are initially too far to interact directly, provides an alternative to photonic mediators for the scaling of quantum computers. Fundamentally, an interesting feature is that “identical particles” (e.g., two atoms of the same species) and the 1D confinement, are both necessary for the action of the gate. Effects of the departure from 1D confinement, the degree of control required on the initial motional states of particles, and how to achieve the same, are discussed, as well as the feasibility of atomic implementations.

Recent progress in the control of the motion of neutral atoms in restricted geometries such as traps \(^1\)\(^2\), 1D optical lattices \(^3\)\(^4\) and wave-guides \(^5\) has been astounding. Naturally, the question arises as to whether they can be used in a similar manner as photons are used, i.e., as “flying qubits” for logic as well as for connecting well separated registers in quantum information processing. Quantum logic between flying qubits exploits their indistinguishability and assumes them to be mutually non-interacting – hence the names “linear optics” \(^6\) and “free electron” \(^7\) quantum computation. In fact, for such an approach to be viable one has to engineer circumstances so that the effect of the inter-qubit interactions can be ignored \(^8\). On the other hand, in the context of photonic qubits, it is known that effective interactions, engineered using atomic or other media, may enhance the efficacy of processing information \(^9\)\(^13\). One is thereby motivated to seek similarly efficient quantum information processing (QIP) with material flying qubits which have the advantage of naturally interacting with each other. Further motivation stems from the fact that for non-interacting mobile fermions, additional “which-way” detection is necessary for quantum computation \(^7\) and even for generating entanglement \(^14\), which are not necessarily easy. Thus, if interactions do exist between flying qubits of a given species, one should aim to exploit these for QIP.

While it is known that both spin-dependent \(^13\) and spin-independent \(^16\)\(^17\) scattering can entangle, it is highly non-trivial to obtain a useful quantum gate. The amplitudes of reflection and transmission in scattering generally depend on the internal states of the particles involved which makes it difficult to ensure that a unitary operation i.e., a quantum gate acts exclusively on the limited logical (e.g. internal/spin) space that encodes the qubits. Only recently, for non-identical (one static and one mobile) particles, it has been shown that a quantum gate can be engineered from a spin dependent scattering combined with an extra potential \(^18\). We will show here that one can accomplish a quantum gate merely from the spin independent elastic scattering of two identical particles. This crucially exploits quantum indistinguishability to label qubits by their momenta, as well as the equality of the incoming pair and outgoing pair of momenta in one dimension (1D). In our scheme the quantum gate is only dictated by the Scattering matrix or S-matrix acting on the initial state of the two free moving qubits. This is thus an example of minimal control QIP where nothing other than the initial momenta of the qubits is controlled. Not only will it enable QIP beyond the paradigm of linear optics with material flying qubits, but also potentially connect well separated registers of static qubits. One static qubit from each register should be out-coupled to momenta states in matter wave-guides and made to scatter from each other. The resulting quantum gate will connect separated quantum registers. This may be simpler than interfacing static qubits with photons.

While quantum gates exploiting the mutual interactions of two material flying qubits has not been considered yet, the corresponding situation for static qubits has been widely studied (e.g., Refs.\(^19\)\(^25\)). However, these methods typically require a precise control of the interaction time of the qubits or between them and a mediating bus (e.g., Refs.\(^26\)\(^28\)). On the other hand, our method exploits a much lower control process, namely the scattering of flying qubits. Still static qubits offer the natural candidate for information storage. Motivated by this, and also by the high degree of control reached in current optical lattice experiments \(^3\)\(^4\), as a second result of this Letter we consider a lattice implementation of our gate. This experimental proposal is particularly compelling also because the qubit can be made either static of mobile depending on the tunable potential barrier on different lattice sites, thus avoiding to seek some mechanism to couple static and mobile particles and allowing
for both storage and computation with the same physical setup.

Our study interfaces QIP and quantum indistinguishability with two other areas, namely the Bethe-Ansatz exact solution of many-body models [29] and the 1D confinement of atoms achieved in recent experiments [30–32]. Quantum gate between flying qubits—A two qubit entangling gate is important as it enables universal quantum computation when combined with arbitrary one qubit rotations [39]. We consider the spin independent interaction to be a contact interaction between point-like non-relativistic particles. For two two spinless bosons on a line (1D) the Hamiltonian with a delta-function interaction is [29]

\[ H = -\frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} + 2\delta(x_1 - x_2), \]  

where \( x_1 \) and \( x_2 \) are the coordinates of the two particles. The above model is called the Lieb-Liniger model and has an interesting feature which we will actively exploit. This is the fact that the momenta are individually conserved during scattering. If the incoming particles have momenta \( p_1 \) and \( p_2 \), then the outgoing particles also have momenta \( p_1 \) and \( p_2 \), as shown in Fig. 1(a). Thus the scattering matrix is diagonal in the basis of momenta pairs and, is, in fact, only a phase which accumulates on scattering. The scattering matrix extracted from these wavefunctions is given, for incident particles with momenta \( p_2 > p_1 \), by [29]

\[ S(p_2, p_1) = \frac{p_2 - p_1 + ic}{p_2 - p_1 - ic}. \]

The phase accumulated on scattering is \(-i\ln S(p_2, p_1)\). Note that, as expected, for non-interacting bosons (\( c \to 0 \)), their exchange causes no phase change, while when \( c \to \infty \) (impenetrable bosons equivalent to free fermions) and have a \(-1\) factor multiplying on exchange.

We consider the case of colliding particles having some internal degrees of freedom in which a qubit can be encoded (Fig. 1(a)). The collision is assumed to have the form of a spin independent contact (delta) interaction of point-like particles as in Eq. (1). We first consider bosons with two relevant states \(|\uparrow\rangle\) and \(|\downarrow\rangle\) of some internal degree of freedom (could be any two spin states of a spin-1 boson, for example). For symmetric states of the internal degrees of freedom, the external degrees of freedom also have to be symmetric and have the same scattering matrix as spinless bosons. On the other hand, for antisymmetric spin states, the spatial wave function of the two particles is fermionic so that the amplitude for \( x_1 = x_2 \) (the chance of a contact delta interaction) is zero implying that they do not scatter from each other. The above observations lead to the S-matrix (for \( p_2 > p_1 \))

\[ S^B(p_2, p_1) = \frac{(p_2 - p_1) - ic\Pi_{12}}{p_2 - p_1 + ic}, \]

where \( \Pi_{12} \) is the SWAP (permutation) operator acting on the internal (spin) degree of freedom (\( \Pi_{12}(|u\rangle_1|v\rangle_2) = |v\rangle_1|u\rangle_2 \), where \(|u\rangle_1 \) and \(|v\rangle_2 \) are arbitrary spin states of the particles, so that \( \Pi_{12} \) is a \( 4 \times 4 \) matrix). We also consider the case where qubit states are spin states of a spin-1/2 particle (say, electrons or fermionic atoms). This is the conventional encoding in many quantum computation schemes. In this case the S–matrix was computed by C. N. Yang [41] to be (for \( p_2 > p_1 \))

\[ S^F(p_2, p_1) = \frac{ic\Pi_{12} + p_2 - p_1}{ic + p_2 - p_1}. \]

We consider a frame in which two qubits are moving towards each other so that they eventually collide. Let us call the qubit with momentum towards the right as qubit \( A \), while the qubit with momentum towards the left is called qubit \( B \). Each qubit is in a definite momenta state, whose magnitudes are \( p_A \) and \( p_B \) respectively [42]. Thus \( p_2 = p_A \) and \( p_1 = -p_B \). The evolution of the 4 possible qubit states due to the scattering is thereby given by

\[ S^B/F|\uparrow\rangle_A|\uparrow\rangle_B = e^{i\phi_F/F}|\uparrow\rangle_A|\uparrow\rangle_B \]

\[ S^B/F|\downarrow\rangle_A|\downarrow\rangle_B = e^{i\phi_B/F}|\downarrow\rangle_A|\downarrow\rangle_B \]

\[ S^B/F|\uparrow\rangle_A|\downarrow\rangle_B = \frac{p_{A+B}|\uparrow\rangle_A|\downarrow\rangle_B + ic|\downarrow\rangle_A|\uparrow\rangle_B}{p_{A+B} + ic} \]

\[ S^B/F|\downarrow\rangle_A|\uparrow\rangle_B = \frac{p_{A+B}|\downarrow\rangle_A|\uparrow\rangle_B + ic|\uparrow\rangle_A|\downarrow\rangle_B}{p_{A+B} + ic} \]

where \( p_{A+B} = p_A + p_B \), \( e^{i\phi_B} = \frac{p_{A+B} - ic}{p_{A+B} + ic} \), and \( e^{i\phi_F} = 1 \). Unless either \( p_{A+B} \) or \( c \) vanishes, the above is manifestly an entangling gate, as is evident from the fact that the right hand sides of the last two lines of Eq. (4) is an...
entangled state. This gate is most entangling (i.e., the most useful in context of quantum computation, equivalent in usefulness to the well known Controlled NOT or CNOT gate) when $p_{A+B} \approx c$, as then the right hand sides of the last two lines of Eq. (4) correspond to maximally entangled states $\frac{e^{-ix}}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B \mp i |\downarrow\rangle_A |\uparrow\rangle_B)$ and $\frac{e^{-ix}}{\sqrt{2}} (|\uparrow\rangle_A |\downarrow\rangle_B \mp i |\downarrow\rangle_A |\uparrow\rangle_B)$ respectively. The above gates would aid universal quantum computation by means of scattering with both bosonic and fermionic qubits. The gates of Eqs. (4) are easiest to exploit as the only other requirement, namely local rotations of the qubit states are accomplishable by means of laser induced transitions between different atomic internal levels or electronic spin rotations by magnetic fields.

Error estimates:- The amplitudes in Eqs. (4) depend only on the ratio of $p_{A+B}/c$ and thereby any spread $\delta p_{A+B}$ of the relative momenta of the incoming particles only affects the amplitudes as $\delta p_{A+B}/c$. As a relevant example we consider two Gaussian wavepackets in the internal state $|\uparrow\rangle_A |\downarrow\rangle_B$ where $p_{A+B}$ is centered around $c(1+\delta)$ with a relative variance $\eta c$. After the scattering the generated entanglement between the internal degrees of freedom is measured by the concurrence $C$. It is $C=2|\langle z|f(z)\rangle|$, being $z=\frac{1-\sqrt{1+\delta}}{\sqrt{2}\eta}$ and $f(z)=\sqrt{\pi}e^{z^2} \text{erfc}(z)$. From the asymptotic expansion $zf(z)\approx 1-z^{-2}/2$ one obtains that $C$ slowly decays as a function of $(\delta, \eta)$, and that the case $\delta \gtrsim 0$ is less prone to errors when $\eta$ increases. Errors can thereby be arbitrarily reduced in principle by choosing particles with higher $c$. This is opposite to the usual paradigm of gates based on “timed” interactions, where for a given timing error $\delta t$, stronger interactions enhance the error (while weaker interactions make gates both slower and susceptible to decoherence).

Implementation via flying qubits:- One of the most promising implementation of our gates is with neutral bosonic/fermionic atoms. The delta function in- and $\delta t$ in the electronic de Broglie wavefunctions lead to working distances of the system with a discrete variant of the system with a harmonic potential to acquire their momenta (note that our gate scheme is completely different from Ref. [33], where the atomic motion is guided by internal states). The initial position of the particles $x_B = -x_A = x_0$ can be tailored so that their relative momentum has minimum variance $\Delta p_{A+B}$ when the particles reach the collision point ($x=0$). As $\omega_c \ll \omega_\perp$ (being $\omega_c$ the frequency of the longitudinal harmonic confinement) the collision does not feel the longitudinal potential, so it is approximated by Eq. (4). As shown previously, the generated entanglement is very high ($C \approx 1$) provided that $\eta \sim \Delta p_{A+B}/p_{A+B} \sim \Delta x_0/x_0 \ll 1$. A different approach (depicted in Fig. 1b) consists in suddenly moving the local trapping potentials so that the particles in $A$ and $B$ move towards their new potential minima. As in the previous case, wave-packets with well-defined and tunable momenta are created by switching off the potential when they reach the minima where their momentum uncertainties are minimal. Lastly, static atomic qubits (c.f. [23, 50]) in well separated traps may be Raman outcoupled [24] imparting them momenta towards each other in a 1D waveguide – this will link distinct registers.

Ballistic electrons inside carbon nano-tubes or semiconductor nanowires, or edge states may be another implementation [51, 52]. As electrons are typically screened in a solid, a contact interaction should describe their scattering. Note that the Hubbard model with solely on site interactions models a variety of electronic systems. In a low energy limit, in which the electronic de Broglie wavelength is far larger than the lattice spacing of the Hubbard model, the Hamiltonian reduces to that of Eq. (4).

Quantum gates between distant stationary qubits:– A discrete variant of the system with a $\delta$-interaction is the Bose-Hubbard Hamiltonian [29, 53]:

$$H = \sum_{j,\alpha} \frac{J_j}{2} \left[ a_{j,\alpha}^\dagger a_{j+1,\alpha} + \text{h.c.} \right] + \sum_{j,\alpha,\beta} \frac{U^{\alpha\beta}}{2} n_{j,\alpha} n_{j,\beta}$$

where $\alpha = \{\uparrow, \downarrow\}$ labels two internal degrees of freedom of the particles. We call $N$ the length of the chain. In a lattice setup, free-space evolution is replaced with particle hopping. Particle collisions lead to a scattering matrix which, for uniform couplings $J_j=J$, $U^{\alpha\beta} = U^{\alpha\beta}$, is given

$$H = \sum_{j,\alpha} \frac{J_j}{2} \left[ a_{j,\alpha}^\dagger a_{j+1,\alpha} + \text{h.c.} \right] + \sum_{j,\alpha,\beta} \frac{U^{\alpha\beta}}{2} n_{j,\alpha} n_{j,\beta}$$
by Eq. 19 with the substitutions 53, 54
\[ p_j \rightarrow \sin p_j, \quad c \rightarrow U^{\alpha \beta}/J. \] (6)

We consider different initial internal degrees of freedom and set \( U[J]^{\alpha \alpha} = 0 \). A maximally entangling gate is therefore realized when \( p_1 \sin \pi p_2 \approx 2U \), being \( U = (J/2)J \). In particular, \( p_1 = \sin^{-1} U \) when \( p_1 = -p_2 \).

The Bose-Hubbard Hamiltonian 1 naturally models cold bosonic atoms in optical lattice 8. Owing to single atom addressing techniques 56, 87 Rb atoms in different lattice sites can be initialized in either two distinguishable hyperfine internal states \( \left| \alpha \right\rangle = \left| F = 1, M_F = -1 \right\rangle \) and \( \left| \beta \right\rangle = \left| F = 2, M_F = -2 \right\rangle \). The coupling constants \( U[J]^{\alpha \beta} \) depend on the strength \( g[J]^{\alpha \beta} \) of the effective interaction between cold atoms 57. These parameters are usually experimentally measured 58, 59 and can be tuned by Feshbach resonances 60. Spin-exchange collisions are highly suppressed due to the little difference (less than 5%) between singlet and triplet scattering length of \( 87 \text{ Rb} \) 55. The one-dimensional regime is obtained by increasing the harmonic lattice transverse confinement (\( \omega_L/2 \pi \approx 18 \text{ kHz} \)) see 37, 61 for typical values.

We obtain the 1D pseudo-potential coupling constants \( g[J]^{\alpha \beta} \) from the 3D measured values 59 following 45, respectively \( g[J]^{\alpha \beta} = 1.14 \times 10^{-37} \text{ Jm} \), \( g[J]^{\alpha \beta} = 1.12 \times 10^{-37} \text{ Jm} \), \( g[J]^{\alpha \beta} = 1.09 \times 10^{-37} \text{ Jm} \). The internal spin state and the position of particles are detected by fluorescence microscopy techniques 58, 62. The parameters \( U[J]^{\alpha \beta} \) and \( J[J]^{\alpha} \) can be physically controlled in optical lattice systems locally varying the depth of the optical potential 63.

Arbitrary optical potential landscapes are generated directly projecting a light pattern by using holographic masks or micromirror device 4. In particular, \( U[J]^{\alpha \beta} \propto \sqrt{2\pi} (g[J]^{\alpha \beta}/\lambda) (V_0/E_R)^{1/4} \) and \( J[J]^{\alpha} \propto (4/\sqrt{\pi}) E_R (V/E_R)^{3/4} \exp \left[ -2 (V/E_R)^{1/2} \right] \) where \( \lambda \) is the laser wavelength, \( V_0 \) is the lattice depth and \( E_R \) is the recoil energy 57.

For flying qubits we considered a fixed \( c \) and we tuned \( p_j \) to obtain the desired gate. In a lattice, on the other hand, \( U[J]^{\alpha \beta} \) can be controlled precisely, while the creation of a wave-packet requires the control and initialization of many-sites. This kind of control can be avoided by initially placing two particles at the two distant boundaries of the lattice (particle \( A \) on the left and particle \( B \) on the right) and locally tuning the coupling \( J_0 \) between the boundaries and the rest of the chain 61 (all the other couplings are uniform \( J[J]^{\alpha} = J \)). An optimal choice of \( J_0/J \) has a twofold effect: firstly it generates two wave-packets whose momentum distribution is Lorentzian, narrow around \( p_4 = - p_B \approx \pm \pi/2 \), respectively, and with a width dependent on \( J_0 \); secondly it generates a quasi-dispersionless evolution, allowing an almost perfect reconstruction of the wave-packets after the transmission (occurring in a time \( \approx N/j \)) to the opposite end. In this scheme (Fig. 1c), the particles starts from opposite locations, interact close to the center of the chain and then reach the opposite end where the wave-function is again almost completely localized, allowing a proper particle addressing. Since \( p[A/B] \) is fixed, a high amount of entanglement is generated when \( U = \sin p[A/B] = 1 \). For \( 87 \text{ Rb} \) we found that the latter condition is satisfied, e.g., when \( V_0/E_R \approx 2.2 \), giving also \( J/h \approx 240 \text{ Hz} \).

In this scheme there are two error sources. The first is due to the transmission quality, though it is above 85% even for long chains 54. The second one is due to the finite width of the Lorentzian momentum profile around \( p_0 = \pi/2 \) which, in turn, yields slightly different gates for different momentum components. To quantify the amount of those errors we evaluate numerically the join probability amplitude \( A[J]^{\alpha \beta} (\tilde{t}) \) to have respectively particle \( A \) in sites \( i \) and particle \( B \) in \( j \) as function of the inter-particle interactions \( U \). The indices \( \alpha, \beta \) refer to the initial internal state of particles \( A \) and \( B \), \( \tilde{t} \) is the transfer time, and the initial condition is \( A[J]^{\alpha \beta} (0) = 1 \). We find that for distinguishable particles \( A[J]^{\alpha \beta} (\tilde{t}) = -iU/U_{\text{opt}} \) in agreement with the prediction, apart from \( U_{\text{opt}} \) which models finite size effects: e.g. \( U_{\text{opt}} = 0.95 \) (\( U_{\text{opt}} = 0.97 \)) for \( N = 25 \) (\( N = 51 \)) and \( U_{\text{opt}} \rightarrow 1 \) for long chains. For indistinguishable particles we obtain that \( A[J]^{\alpha \beta} (\tilde{t}) = 0 \) for \( \alpha = \uparrow, \downarrow \) and for any value of \( U_1 \). Therefore, apart from a global damping factor due to the non-perfect wave-packet reconstruction, the resulting transformation is in agreement with the gate 4.

The entanglement generation between the boundaries is evaluated via the concurrence 13, 14, 15, \( C_{1N} = 2 |A[J]^{\alpha \beta} (\tilde{t}) A[J]^{\alpha \beta} (\tilde{t})^*| \). From the asymptotic analysis 64 we find that \( C_{1N} = \frac{\pi}{\gamma} \left( rac{V_{1N}}{U_{\text{opt}}} \right)^{1/2} \) where \( f_{1N} \) is the transmission probability from site 1 to site \( N \) at the transmission time. Surprisingly, \( C_{1N} \) does not directly depend on the width of the wave-packet in momentum space: its maximum value depends only on the transfer quality \( f_{1N} \). For example, for \( N = 25 \), \( f_{1N} = 0.97 \) so \( C_{1N} = 0.88 \); for \( N = 51 \), \( f_{1N} = 0.95 \) so \( C_{1N} = 0.81 \).

Concluding remarks: – We have described a mechanism for useful gates between flying qubits and hopping qubits in a lattice using the low control method of scattering. 1D matter waveguides, identical particles and a contact interaction are key requirements for the gate. Exactly solvable models enable the study 29, while experiments and results on 1D gases 1, 2, 30, 32, 34, 35, 37, 38 and optical lattices 13, 55 suggest implementations with flying and static atomic qubits. The scheme may both enhance linear optics-like QIP and connect separated atomic qubit registers.

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