Study of time series of meteorological parameters by wavelet analysis

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Abstract The paper presents the results of applying the continuous wavelet analysis method to assess the cyclic components of discrete time series. The work is based on an analysis of a long-term series of data on observations of air temperature in the south-west of the Valdai Upland, as well as analysis of data on observations of CO\textsubscript{2} concentration in the canopy of the tropical monsoon forest in southern Vietnam. The patterns of wavelet coefficients and integral spectra have been studied for all considered time series. Additionally, the cyclicities contained in the studied series were revealed.

1. Introduction

The development of both long-term (over 30 years) analysis methods and a series of short-term observations are critical goals for meteorological and environmental studies. The analysis of long-time series of meteorological parameters and the results of short-term field experiments are important for studying modern climate changes. Additionally, data from a weather station (an ordered set of variable values measured over a constant period of time) is presented as a series of dynamics of meteorological parameters.

Quite often, when analyzing time series, efforts are focused on the allocation of cyclic components and estimation from the period. From a formal point of view, the time series of meteorological parameters can be represented as the sum of sequences of fragments similar in shape to each other, but not identical. Moreover, over time, the shape of the repeating fragments can change significantly. As a result, well-developed Fourier analysis methods for stationary random sequences may become inapplicable. However, the transition to windowed Fourier transform does not completely solve the problem. With such a windowed Fourier transform, the time interval for the existence of a series is divided into several intervals — time windows of fixed duration — and then the Fourier transform is calculated for each interval. However, a narrow window gives us a better resolution in time, while a wide window gives a better resolution in frequency. In addition, due to the non-stationary nature of the series [10] for different sections, it may be necessary to use windows of various duration. In this case, using the wavelet analysis of the time series allows us to study the dynamic structure of the signal [1]. In fact, wavelet analysis makes it possible to study both the time and frequency structure of a series by representing signal sections of different durations in the form of a linear combination of scalable basis functions of a given shape. Moreover, it is best for the form of the terms to be adapted to the analyzed
signal. As practice shows [4, 10, 16, 17], the wavelet transform (WT) is an analysis tool that is well-used to study multiscale non-stationary processes occurring in finite spatial and temporal regions. Since its inception, WT has been widely used in various fields of science, such as seismic signal detection, image processing, optics, turbulence, quantum mechanics, chaos, fractal and medical research, etc.

Despite the advantages of WT and the features of the series of dynamics of meteorological parameters describing the Earth’s climate noted here, WT is still not frequently used in studies to detect and analyze climatic signals [10, 17]. The purpose of this work is to show the possibilities of using wavelet analysis to study cyclic changes in the series of dynamics of two meteorological parameters: specifically, surface air temperature and concentration of carbon dioxide in the atmosphere.

The paper analyzes the surface air temperature data obtained as part of basic observation at a meteorological station in Central Forest State Natural Biosphere Reserve (CFSNBR, Tver region) from 1971 to 2016. Thanks to its reserve mode, the territory of the CFSNBR has been minimally changed by anthropogenic activities, and therefore the variability of the temperature regime can be associated exclusively with modern climatic changes.

Due to the fact that carbon dioxide is the main carbon-containing greenhouse gas in the atmosphere and one of its main pollutants, the construction of mathematical models of periodic changes in its concentration in the atmosphere is extremely relevant. To analyze the cyclic changes in the time series of CO$_2$ concentration, in this work we use data of year-round measurements from the Nam Cat Tien flux station in Vietnam from 2011 to 2018 provided by the AN Severtsov Institute of Ecology and Evolution RAS. The concentration of CO$_2$ in the atmosphere is recorded every second, followed by averaging to half-hour values and expressed in millionths (ppm). The studied time series of CO$_2$ concentration are presented as ordered sequences of averaged half-hour CO$_2$ values by various heights above the Earth’s surface.

In this paper, along with WT, Fourier analysis methods are also applied to smoothed series: this is due to the fact that smoothing by a moving average, which is consistent in duration with the duration of the series, makes the series almost static in some cases. As a result of the analysis, a set of periodic components of time series that are incommensurable with the duration of daily and other seasonal changes is identified in the work.

2. Subject of Study

As noted in the introduction, to analyze the cyclical changes in the time series of air temperature, we studied the averaged time series with an averaging period of 1 week (series 1) because the analysis of this time series would reveal cyclic changes with periods of at least a year.

Meteorologists note that the spring period makes a significant contribution to the formation of annual temperature. Meteorological spring begins when the temperature is equal to or greater than zero for the duration of five consecutive days. In this regard, values of 90 days were selected from a number of dynamics, which were counted from the beginning of the meteorological spring for each year (series 2).

From the studied observations of CO$_2$ concentration data, the minimum and maximum values of daily data were selected at a height of 0.3 meters and 46 meters above the ground (series 3).

An analysis of the data at low altitudes will characterize the local effects observed in the territory where the measurements were performed; at high altitudes, data characterizes regional phenomena. When examining data of this type, the goal is to determine seasonal cycles with periods lasting from tens of days to one and a half years.

To identify cycles of high-frequency periodicities, we also consider a number of dynamics of every second concentration measurements averaged over a period of 30 minutes (series 4). Our work uses data for a year and a half time period (10.15.12 – 04.15.14). The choice of the length of the studied series of dynamics is explained by the presence of edge effects on the spectrum when processing the
corresponding series of dynamics. Also, the choice was influenced by our desire to obtain the current spectrum of this series throughout the year.

The wavelet transform method was applied to the following series of different length and time resolution:

1. The average daily temperature, averaged by week;
2. The average daily temperature related to the spring;
3. Data (maxima and minima) of daily CO₂ concentration at different altitudes (0.3 m and 46 m);
4. CO₂ concentration data for a year and a half averaged over a period of 30 minutes.

The studied time series were checked for stationarity using the Augmented Dickey-Fuller test. The results turned out to be predictable: series 1, 2, 4 were non-stationary, thereby the use of wavelet analysis was justified. Time series 3, on the contrary, turned out to be stationary: this, generally speaking, does not exclude the possibility of using the wavelet transform as one of the research methods. In this regard, the aforementioned series of dynamics were also used as analyzed time series.

3. Continuous Wavelet Transform

The essence of continuous wavelet transform is as follows. Using a suitable mother wavelet \( \psi(t) \), wavelet functions \( \psi_{a,b}(t) \) are computed as:

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right). \tag{1}
\]

The parameter \( a \) is called the scale parameter of the wavelet transform: it takes strictly positive values and is responsible for the width of the wavelet. Additionally, \( b \) is the shift parameter that determines the position of the wavelet on the \( t \) axis. These parameters perform functions similar to the parameters of shear and stretches in a discrete wavelet transform, but unlike these, they can take any values - not just integer values.

Next, the wavelet coefficients \( W(a, b) \) are determined:

\[
W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} s(t) \psi_{a,b}^* dt. \tag{2}
\]

where \( s(t) \) -- analyzed time series and \( * \) means complex pairing.

After that, a qualitative analysis of the picture of wavelet coefficients and the construction of the integrated spectrum are carried out:

\[
S(a) = \int_{a_1}^{a_2} |W(a, b)|^2 db. \tag{3}
\]

This procedure shows the existence of cycles in the original time series. Scale \( a \) is related to the coordinates of the time axis \( t \) as:

\[
t = \frac{a}{F_c},
\]

where \( F_c \) is a center wavelet frequency.

In connection with the type and features of the studied series [4, 7, 10], the following mother wavelets were chosen for continuous wavelet transform:

The Morlet wavelet (Fig. 1):

\[
\psi(t) = e^{-\frac{x^2}{2}} \cos(5x)
\]

And the seventh-order Hermitian wavelet - seventh-order derivative of a Gaussian distribution (Fig. 1):

\[
F(x) = e^{-x^2}
\]

The central frequency of these mother wavelets is 0.8125 and 0.6.
The analyzed time series are functions of time given in a finite number of points. To calculate the integrals (2) and (3), the Simpson formulas are applied in a Matlab programming environment. The time step is consistent with the smoothness of the integrable functions so as to ensure sufficient accuracy of the calculations.

4. Time Series Analysis
The pictures of wavelet coefficients allow us to closely study the structure of the cycles contained in the representative series of dynamics (in contrast, for example, to the Fourier analysis method). A scale that has maximal coefficients (that is, these make the maximum contribution to the time series) is characterized by a certain period of cyclicity. In other words, the bright horizontal lines of the spectrum correspond to the desired cycles. The picture of wavelet coefficients allows us to assess the magnitude of the cyclicities and point to the time when they are observed qualitatively. It is also important to note the edge effects and to disregard the behavior of the left and right edges of the spectrum.

The integrated spectrum, in contrast to the wavelet coefficient picture, allows us to calculate the value of the cycle period. The local extrema of the graph correspond to the maximum coefficients in a given scale, illustrating the values of the cycle period.

In the course of our research, we obtained pictures of the wavelet coefficients and the integrated spectra of all the studied series of dynamics.

4.1. Weekly averaging data
Let's consider a series of temperature dynamics with averaging over weeks. On the spectrum of the complete time series averaged over weeks (Figure 2a) and obtained using the Morlet wavelet, one can see that the cycle of about 3 years is not always observed: at some points, it is a superposition of cycles of 4 and 2 years (estimated).

Note that a bright light line at the bottom of the spectrum indicates a cycle of a year. It does not make sense to consider the spectrum of this series on a large scale, since the nearest visible cycle (not indicated on the spectrum) is about 19 years, which is slightly less than half of the studied series. In this regard, this cyclicity should not be noted.

Figure 1. Mother wavelets a) Morlet wavelet. b) seventh-order Hermitian wavelet
Figure 2. a) The picture of wavelet coefficients of the series of dynamics 1; dot-dashed lines indicate the frequency evolution of the dominant periodicities.
   b) Integral spectrum of the series of dynamics 1

The peaks in the graph of the integrated spectrum of average daily temperature indicators as averaged over the weeks were obtained by using the Morlet wavelet (Figure 2b) mean cycles of 1 year (52.2 weeks), 2.7 years, 8.4 years, 19.5 years (the last value is almost half of the series, insufficient data), from left to right, respectively. seventh-order Hermitian gives very close results (all results are presented in Table 1). It can be seen that a cycle with a periodicity of 8 years is always observed, and a cycle of about 3 years at specific time intervals is a superposition of more complex phenomena.

4.2. Meteorological spring

The spectrum of a number of dynamics of the indicators of the average daily meteorological spring temperature represents a more complex picture of cyclicities. The bright light line at the bottom of the spectrum indicates a one-year cycle. A cycle of three years (also for some period of time) is a superposition of cycles of estimated 2 and 4 years (Figure 3a). The cycle of 8 years may be a superposition of more complex phenomena: yet, as the integrated spectrum has shown, it stands out as an unclear point in spring data.

Peaks in the graph of the integrated spectrum of meteorological spring indicators have been obtained using Morlet wavelet (Figure 3b): cycles of 1 year, 3.5 years, 6.4 (weak peak) and about 12 years (almost half of the series are insufficient data), as seen from left to right respectively. It can be observed that the cyclicity of about 3 years at some time intervals also represents a superposition of the cycles of 4 and 2 years.

Figure 3. a) The picture of wavelet coefficients of the series of dynamics 2; dot-dashed lines indicate the frequency evolution of the dominant periodicities.
   b) Integral spectrum of the series of dynamics 2.
The 7th order wavelet of Gauss gives very close results (all results are presented in Table 1).

The results of the wavelet analysis of cyclic changes in the fluctuations in the daily average temperature of the surface layer of the atmosphere are compared with the results obtained using the methods of regression [1]. After approximations by using the simple moving average method with a period of 1 year in certain fragments of smoothed temperature series (less than 40 years), the stationarity was confirmed, allowing Fourier analysis of this series and of the various fragments of the 40-year-old temperature series. It was shown that the periods of the main harmonics are somewhat different, ranging from 7.5 to 8.1 years and from 3.3 to 3.9 years. It should be noted that the formation of meteorological indicators is influenced by a large number of factors: changes in the gas composition of the atmosphere, variation in solar cycles, volcanic eruptions, and etc. [14, 15]. With the combined effect of a large number of different factors, we can speak about the stochastic nature of the time series, but it is necessary to take into account some violations of the homogeneity of the spectral composition of the entire studied range.

It was shown that the duration of the cycles, as determined by various methods [1,9,10], is practically the same (Table 1).

Table 1. Comparison of Results

| Mother wavelet / method | Weekly averaging | Meteorological spring |
|-------------------------|------------------|-----------------------|
| Morlet wavelet          | 2.7 years and 8.4 years | 3.5 years, 6.4 (weak peak) and about 12 years |
| Hermitian wavelet 7     | 2.7 years and 8.6 years | 3.6 years, about 12 years |
| Discrete Wavelet Transform | 3.5 years and 8.7 years | No data was collected |

Along with cycles that are common to various methods of analyzing periodic changes (such as regression analysis, Fourier analysis and continuous wavelet analysis), there are also cycles detected only by a certain analysis method as a result of Fourier analysis. This means that these cycles are not detected by other methods that use continuous wavelet-analysis. In this connection, further study of these time series remains relevant.

4.3. Maxima and minima of CO₂ concentration

We have also obtained wavelet coefficient patterns and integrated spectra for various indicators of carbon dioxide concentration in the atmosphere.

Figure 4. a) The picture of wavelet coefficients of the series of dynamics 3; dot-dashed lines indicate the frequency evolution of the dominant periodicities. b) Integral spectrum of the series of dynamics 3.
The results obtained have a more complex structure than in the case of temperature. In Figure 4a, for the concentration maxima at a height of 0.3 meters as obtained using the Morlet wavelet, periods of different cycles are visible (note that not all are indicated). The spectrum of concentration minima (not indicated) is also very blurred, and therefore it is difficult to identify long pronounced periodicities at small time scales.

The corresponding integral spectrum (Figure 4b) is also quite blurry. Peaks can be observed at 23 days, 45 days, and about 80 days for a minimum concentration; additionally, general peaks of 1.04 years and 1.5 years can be observed at all altitudes as well.

These time series were also analyzed using a Hermitian wavelet. The patterns of wavelet coefficients for this mother wavelet are similar, and, while the integrated spectra are blurred even more, some weak peaks coincide with the results obtained using the Morlet wavelet.

As noted above, the integrated spectrum is blurry, and even visible peaks do not have great reliability. Using the picture of wavelet coefficients, one can see the non-stationary structure of the signal. Specifically, there is a large number of cyclic patterns (bright horizontal bands) that are “turned on” for short periods of time, which makes it difficult to distinguish the main periodic components.

To determine the cyclic changes in the maximum and minimum daily values of carbon dioxide concentration, methods of regression analysis and Fourier analysis were used [8]. The periods of the resulting daily and monthly cycles are presented in Table 2. The empty sections in the table signify an absence of a cycle, which can be compared with other cycle data in the table.

| Daily cycles, days | Max 0.3 m | 7 | 10 | 11 | 15 | 20 | 23.5 | 27.5 | 30 | 33 | 40 | 45 | 48 | 52 |
|-------------------|-----------|---|----|----|----|----|------|------|----|----|----|----|----|----|
|                   | Min 0.3 m | 10 | 11 | 15 |   |    | 30   | 33   | 37 | 41 |    |    |    |    |
| Monthly cycles, months | Max 0.3 m |         | 6 | 6.5 | 7.5 | 12 |    |      |     |     |     |     |     |     |
|                   | Min 0.3 m | 3.5 | 6.5 | 7.5 | 9 | 12 | 15   |     |     |     |     |     |     |
|                   | Max 46 m  | 2 | 2.5 | 2.85 | 3 | 3.5 | 4 | 6 | 6.5 | 7.5 | 9 | 12 | 15 |     |
|                   | Min 46 m. | 2 | 3 | 3.5 | 4 | 6 | 6.5 | 7.5 | 9 | 12 | 15 |     |     |     |

4.4. CO₂ concentration data for a year and a half averaged over a period of 30 minutes

When analyzing the data on carbon dioxide concentration for over a year and a half, a similar problem was found as for a number of temperatures. Specifically, in the picture of wavelet coefficients, obtained by Morlet wavelet, many short-duration cycles were revealed (Figure 5a). Short duration cycles are those that contribute to a number of dynamics throughout only part of the study area.

The integrated spectrum (Figure 5b, c) is quite blurry, but it allows the observer to determine the cycle of 24 hours, which confirms the correctness of the methodology.

The integrated spectrum also exhibits strong double emissions that are similar in structure. The difference between the peaks of one outlier and the difference between outliers for the scale in which these outliers are observed are shown in Table 3.

Upon further investigation, it was determined that such double outliers occur both for different time intervals and for different maternal wavelets. Moreover, all characteristic values are in good agreement with each other.
Currently, the question of the nature of the observed outliers remains open. They may appear due to the various climatic, biological or environmental phenomena.

![Image](image.png)

**Figure 5.** A) The picture of wavelet coefficients of the series of dynamics 4; dot-dashed lines indicate the frequency evolution of the dominant periodicities. B) Integral spectrum of the series of dynamics 4. C) Integral spectrum of the series of dynamics 4 logarithmic scale.

**Table 3.** Scales of double splashes on the integrated data spectrum for 1.5 years.

| Double-peak center, in days | 26  | 39  | 52.2 | 78.8 | 105 | 118.2 | 131.7 | 158 |
|----------------------------|-----|-----|------|------|-----|--------|-------|-----|
| The difference between the peaks, in hours | 16  | 24  | 16   | 48   | 16  | 24     | 16    | 48  |
| Time after the last outlier, in days | -   | 13  | 13.2 | 26.6 | 26.2| 13.2   | 13.5  | 26.3 |

**5. Conclusion**

The possibilities of using wavelet analysis to study cyclic changes are shown on the example of the series of dynamics of two meteorological parameters: surface air temperature and carbon dioxide concentration in the atmosphere. The complex structure of the cycles is revealed, and qualitative pictures of the temporal positions of cyclicities are obtained. Integral spectra, in addition to the picture of wavelet coefficients, allow numerically estimating the magnitude of the cycle period: the local extrema of the integral spectrum graph correspond to the maximum wavelet coefficients on this scale, showing the values of the cycle period.
The analysis of the complete time series of temperature dynamics and the data related to the meteorological spring containing clearly expressed seasonal components made it possible to consistently distinguish an eight-year cycle. Moreover, it became possible to observe a cycle with a period of about 3 years, which may be a superposition of more complex phenomena.

Despite the fact that the analyzed series of CO2 concentrations contain many short-lived weak cycles, wavelet analysis allows us to distinguish their location on the time axis. Although the maximums of the integrated spectra are weakly expressed, one can judge the presence in the time series of cycles whose periods are consistent with the periods obtained by linear regression methods and Fourier analysis.

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