Relationship between mathematical abstraction in learning parallel coordinates concept and performance in learning analytic geometry of pre-service mathematics teachers: an investigation

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Abstract. As one of the non-conventional mathematics concepts, Parallel Coordinates is potential to be learned by pre-service mathematics teachers in order to give them experiences in constructing richer schemes and doing abstraction process. Unfortunately, the study related to this issue is still limited. This study wants to answer a research question “to what extent the abstraction process of pre-service mathematics teachers in learning concept of Parallel Coordinates could indicate their performance in learning Analytic Geometry”. This is a case study that part of a larger study in examining mathematical abstraction of pre-service mathematics teachers in learning non-conventional mathematics concept. Descriptive statistics method is used in this study to analyze the scores from three different tests: Cartesian Coordinate, Parallel Coordinates, and Analytic Geometry. The participants in this study consist of 45 pre-service mathematics teachers. The result shows that there is a linear association between the score on Cartesian Coordinate and Parallel Coordinates. There also found that the higher levels of the abstraction process in learning Parallel Coordinates are linearly associated with higher student achievement in Analytic Geometry. The result of this study shows that the concept of Parallel Coordinates has a significant role for pre-service mathematics teachers in learning Analytic Geometry.

1. Introduction
The issue related to what mathematics concepts essential for pre-service mathematics teachers is always interesting to be discussed. Some mathematics concepts are believed can provide specific mathematics experiences such as abstraction process, proving, and problem-solving for pre-service mathematics teachers. Non-conventional mathematics concepts defined as mathematics concepts that not belong to school mathematics neither belongs to advanced mathematics concepts can be used to provide rich mathematical experiences for pre-service mathematics teachers [1]. Working with non-conventional structure not only can help learners to construct richer and more abstract schemas in mathematical abstraction process but also can be an essential tool for developing critical thinking in mathematics teacher education. In addition, non-conventional representations can provide a powerful problem-solving tool within mathematics [2].
A non-conventional mathematics concept, Parallel Coordinates, can be chosen as one of the alternatives to develop horizon knowledge of pre-service mathematics teachers [3]. Horizon knowledge is defined as an awareness of how mathematical topics are related over the span of mathematics included in curriculum and it also includes the vision useful in seeing connection to much later mathematical ideas [4]. With some adjustments, this concept is suitable for pre-service mathematics teachers at an early stage. One of the alternatives proposed by [3] is embedding the concept of Parallel Coordinates in Analytic Geometry Classroom. By doing some customization related to curriculum of Analytic Geometry and pedagogical theories for pre-service mathematics teachers, some topics in Parallel Coordinates were selected in order to investigate the role of this concept in abstraction process of pre-service mathematics teachers.

This study especially designed for investigating the role of Parallel Coordinates concept in Analytic Geometry Classroom. The main research question in this study is “To what extent the abstraction process of pre-service mathematics teachers in learning concept of Parallel Coordinates could indicate their performance in learning Analytic Geometry? To be able to answer this question, at least there are two sub-questions need to be answered: (1) Is there any relationship between participants’ prior knowledge and their performance on abstraction test in Parallel Coordinates? And (2) Is there any relationship between the performances of the participants on abstraction in Parallel Coordinates and the performance of the participants on Analytic Geometry?.

2. Methods
As this study wants to investigate a phenomenon of a group of pre-service mathematics teachers who learn concept of Parallel Coordinates, so this study belongs to a qualitative research with a case study design [5]. This study did in mathematics education department of a government university in Bandung, it involved 45 participants of pre-service mathematics teachers who attended a class of Analytic Geometry. This study was conducted for 6 months.

2.1. Data Collection
As the concept of Cartesian coordinate is basic for learning Parallel Coordinates, so the participants need to accomplish the Cartesian coordinate before learning Parallel Coordinates. In addition, there also a prior knowledge test was taken place to extend their mastery of Cartesian coordinate. After the participants finished learning concept of Parallel Coordinates then a test was held to measure their abstraction level in learning concept of Parallel Coordinates. Then they continued to learn other topics Analytic Geometry such as circle, hyperbola, parabola, ellipse, and hyperboloids. In the end of the semester there was a final test on all topics in Analytic Geometry except Cartesian dan Parallel Coordinates.

All the tests consisted of four essay problems. Every set of the test was validated by two experts and tried out to volunteers. In this case, the set of instruments can only be tried out by volunteers because they need to learn the concept of Parallel Coordinates first before they solve the problems on this topic. The volunteers consisted of three second-year of pre-service mathematics teachers and two third-grade of pre-service mathematics teachers.

2.2. Data Analysis
This case study applied descriptive statistic method to analyze quantitative data resulted from scores tests to provide a meaningful and a convenient way of characterizing and portraying important features of data [6]. The presence of an association between two variables first and foremost can be revealed by scatter plot. From the scatter plot, it can be identified whether the data tend to be linear or not. If the data tend to be linear so the data analysis will be continued by finding the Pearson coefficient correlation to investigate how strong the linear relationship between those variables is.

The main research question in this study is demanding answer related to prediction. It indicates that correlation is not sufficient for answering the question. Correlation and prediction indeed are closely related: without a correlation between two variables, there can be no meaningful prediction from one
to the other. However, although the size of $r$ is indicative of the predictive potential, the coefficient by itself does not tell us how to make a prediction [6]. A regression line needs to be investigated further. Finally the result from descriptive statistics was interpreted to answer all the research questions.

2.3. Data Analysis
Parallel Coordinates, invented by Alfred Inselberg in the 1970s, is a system for doing and visualizing analytic and synthetic multi-dimensional Geometry and it is as a way to visualize a high-dimensional data [7]. In Parallel Coordinates, the parallelism and orthogonality are used to be the fundamental concepts. Parallel Coordinates uses $N$ copies of the real line $\mathbb{R}$ labeled $X_1, X_2, \ldots, X_N$ as the axes in Euclidean N-dimensions space $\mathbb{R}^N$. These axes are placed equidistant and perpendicular to $x$-axis in $xy$-Cartesian coordinate. A point $C \in \mathbb{R}^N$ with coordinates $(c_1, c_2, \ldots, c_N)$ is represented by the complete polygonal line $\bar{C}$. A one-to-one correspondence between points in $\mathbb{R}^N$ and planar polygonal lines with vertices on the parallel axes is established [7].

The topic of Parallel Coordinates system has become a popular elective course in applied mathematics/computer science curricula in Tel Aviv University, Israel. In many other universities, Parallel Coordinates system becomes part of some courses such as information visualization, analytic geometry, complex variables, data mining, and others fields dealing with multivariate problems [7].

Below is an example of a problem and an answer from a participant in this study on the test of Parallel Coordinates concept:

"Let $h(1, -1)$ is intersect with $g(3, 1)$ in $A(-2, 3)$. Find the values of $d$ in 2D Parallel Coordinates."

![Figure 1. The Example of A Participant Answer of The Problem Given on Parallel Coordinates Test](image)

Before the participants learn further concept about Parallel Coordinates and its application, they need to know two basic concepts that support the concept of Parallel Coordinates: they are projective Plane Model and point-line duality. Unfortunately, based on the curriculum of mathematics teacher training education, neither the topic of projective Plane Model nor the topic of point-line duality is stated in the curriculum in any subject. Based on this condition, this study will offer an alternative perspective. Parallel Coordinates system is introduced first in order to develop the concept of point-line duality.

Cartesian coordinate system becomes a basis for constructing Parallel Coordinates. The concept of duality from projective geometry will be used to generalize point-line duality. First of all, students will be introduced to concept of constructing Parallel Coordinates, and then they will explore how to
represent a graph of linear function with various slopes and analyze its representation based on the characteristic of the slopes. Finishing with the representation of linear function or lines (in Cartesian coordinate), students will be challenged to explore the representation of a point in Parallel Coordinates system. This means they will learn point-line duality concept.

Figure 2. The Polygonal Line $\overline{C}$ Represent the Point $C = (c_1, c_2, c_3, c_4)$

In this study, topic of Parallel Coordinates is embedded in Analytic Geometry curriculum, it focuses on geometric representation. The concepts of Parallel Coordinates that constructed by the participants in this study was limited in 2-dimensions only. Parallel coordinates in 2D consist of two parallel axes, labeled as $X_1$ and $X_2$. A point $A(x_1, x_2)$ in 2D-Cartesian coordinate corresponds to a line $\overline{A}(x_1, x_2)$ that intersect $X_1$ at $x_1$ and $X_2$ at $x_2$, $(x_1, x_2 \in \mathbb{R})$ in 2D-Parallel coordinates as illustrated in Figure 2 and Figure 3.

Figure 3. Representation of point $A(x_1, x_2)$ in 2D Cartesian coordinate

Figure 4. Representation of point $\overline{A}(x_1, x_2)$ in 2D Parallel Coordinates

3. Results and Discussion

There are three variables resulted from scores tests. First, score from prior knowledge test (CC), second, score from test of abstraction in solving problem on Parallel Coordinates (PC), and third, score from final test on Analytic Geometry (AG). The summary of mean scores and standard deviation of the participants in CC, PC, and AG test are described in Table 1.

| Variables | Minimum | Maximum | Mean    | Std. Deviation |
|-----------|---------|---------|---------|----------------|
| CC        | 6.25    | 100.00  | 70.1944 | 23.00253       |
| PC        | 10.00   | 100.00  | 47.0000 | 23.35594       |
| AG        | 11.625  | 97.50   | 64.0630 | 22.77878       |

It can be identified that the variability of the scores in CC and PC relatively similar while the scores in AG has more variability. The means score of CC is the highest, it can be understood that the topic of Cartesian coordinate has been learning by the participants since there were still in secondary and high school. Some topics in Analytic Geometry such as Circle and Parabola are also familiar for the participants because these topics are part of high school curriculum also.
As suggested by [6], the presence of an association between two variables first and foremost can be revealed by scatter plot, so in Figure 5 below is a scatter plot depicting the relationship between the score of prior knowledge and the score of participants’ performance on Parallel Coordinates test and Figure 6 is a scatter plot depicting the relationship between score test on Analytic Geometry and score of participants’ performance on Parallel Coordinates.

As can be seen from Figure 4 the relationship between CC and PC tend to be linear, as well as in Figure 5 linear relationship with a positive direction between PC and AG also identified. Based on these result, in order to investigate how strong the relationship between CC and PC as well as PC and AG the Pearson correlation coefficient need to be found. In this study, the Pearson correlation coefficients were analyzed by utilizing Microsoft Excel. The results are: (1) Pearson correlation coefficient between CC and PC is 0.621 and (2) Pearson correlation coefficient between PC and AG is 0.643. In the behavior science, correlation greater than ±0.70 are rare [6], so it can be stated that both correlations, in this study, are relatively high.

Although the size of $r$ is indicative of the predictive potential, the coefficient by itself does not tell us how to make a prediction so that a line of “best fit” should be found and used for purposes of predicting values of Analytic Geometry from Parallel Coordinates test to be able to answer the main research question.

Based on the regression analysis result, it is obtained the equation of “best fit” line $Y’ = 0.643X + 33.43$ and $r^2 = 0.42$. It means that we can predict the score of the conventional topic on analytic geometry using this linear equation that resulted from a score of Parallel Coordinates Test as a non-conventional concept and a full 42% of the variance in a test of Analytic Geometry Scores is explained by variation in the test of Parallel Coordinates topic.
4. Conclusion
As this study investigated the role of Parallel Coordinates concept in Analytic Geometry, it can be concluded that this concept has significant role in Analytic Geometry, especially related to the abstraction process in learning Analytic Geometry for pre-service mathematics teachers. Beside in Analytic Geometry, the concept of Parallel Coordinates also potential to be embedded in Calculus. In further study, it also interesting to investigate the role of this concept in improving horizon knowledge of pre-service mathematics teachers.

5. References
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