Peak Effect in Superconductors: Melting of Larkin Domains

Xinsheng Ling,\textsuperscript{(1)} Chao Tang,\textsuperscript{(1)} S. Bhattacharya,\textsuperscript{(1)} and Paul M. Chaikin\textsuperscript{(1,2)}

\textsuperscript{(1)}NEC Research Institute, 4 Independence Way, Princeton, NJ 08540
\textsuperscript{(2)}Department of Physics, Princeton University, Princeton, NJ 08544

(April 25, 1995)

Abstract

Motivated by the recent observations of the peak effect in high-$T_c$ YBCO superconductors, we reexamine the origin of this unusual phenomenon. We show that the mechanism based on the \(k\)-dependence (nonlocality) of the vortex-lattice tilt modulus \(C_{44}(k)\) cannot account for the essential feature of the peak effect. We propose a scenario in which the peak effect is related to the melting of Larkin domains. In our model, the rise of critical current with increasing temperature is a result of a crossover from the Larkin pinning length to the length scale set by thermally excited free dislocations.

74.60. Ge, 64.70.Dv

Typeset using REVTeX
About 35 years ago Le Blanc and Little [1] discovered a striking phenomenon, later known as the “peak effect”, in a conventional superconductor Nb that the sample can carry more supercurrent at a higher temperature (or field) slightly below $B_{c2}(T)$ where it becomes normal. Over the years, the peak effect was found to be ubiquitous in conventional superconductors [2–4] and it has been observed recently in the high-$T_c$ superconductor YBCO crystals [5,6]. Pippard [7] and Larkin and Ovchinnikov [8] proposed that the peak effect is a result of an anomalous softening of the vortex lattice. The basic physics is that a soft lattice can be pinned more strongly than a more rigid lattice. In fact, an infinitely rigid lattice cannot be pinned at all by random pinning. The unresolved problem, however, is the mechanism which leads to the abrupt loss of the vortex-lattice rigidity.

In this paper, we first reexamine the standard interpretations of the peak effect and show that the mechanism of an anomalous softening of the wavevector dependent tilt modulus $C_{44}(k)$ does not account for the essential features of the peak effect. We then propose a scenario in which a melting of the “Larkin domains” leads to the peak effect. The rise of the critical current with increasing temperature is a result of a crossover of two length scales: from the Larkin pinning length to the average separation between thermally excited free dislocations [9].

Let us first recall briefly the general features of the peak effect phenomenon in both conventional superconductors and high-$T_c$ superconducting YBCO single crystals. In samples with high values of the critical current density $j_c$ at a fixed magnetic field $B$, one usually finds that $j_c$ decreases to zero monotonically with increasing temperature and the peak effect is absent [2,4,5]. But in samples with low $j_c$ (weak pinning), the temperature dependence of $j_c$ can be quite different. Fig. 1 is a plot of the critical current density as a function of temperature extracted from Ref. [5] for a YBCO crystal. With increasing temperature, $j_c$ initially decreases monotonically, then suddenly rises, reaches a peak before finally dropping to zero. Experimentally, the peak effect is identified as a dip in resistance [2,4,5], a dip in the in-phase part of ac susceptibility [4], or a peak in critical current density $j_c$ (obtained with the standard voltage criterion) [2,4,5] as a function of $T$ or $B$. In Nb and other low-$T_c$
superconductors, the onset (the rise of $j_c$) temperature $T_p$ of the peak effect is very close to $T_c(B)$ and $T_c(B) - T_p \sim 0.5$ K. In YBCO, $T_c(B) - T_p \sim 5$ K is about 10 times larger. However, $(T_c(B) - T_p)/T_c(B) \sim 0.05$–0.1 is about the same for both low-$T_c$ and high-$T_c$ superconductors. The problem under consideration here is why the critical current rises with increasing temperature.

Pippard [7] proposed that the rise of the critical current has to result from a rapidly decreasing rigidity of the vortex lattice. The rigidity of the vortex lattice prevents the vortex lines from taking advantage of the valleys of random pinning potential. Thus a rapidly decreasing rigidity would allow the lattice to conform better to the pinning potential and enhance critical current. This idea on pinning was subsequently put forward more rigorously by Larkin and Ovchinnikov (LO) in their theory of collective pinning [8]. It was shown by Larkin [10] that in the presence of random pinning the vortex lattice loses its long-range translational order and breaks up into domains of correlated regions in which the vortex lines interact elastically. LO argued [8] that the critical current density is determined by the fluctuations of random potential in a domain and the pinning force density $j_cB = (nf^2/V_c)^{1/2}$, where $n$ and $f$ are the density and strength of the pins, $V_c$ the volume of the domain. The size of the Larkin domains can be estimated by a simple energy consideration. The vortex lattice deforms to take advantage of the random pinning potential at the cost of the elastic energy. The total unit volume energy change is [8]

$$\delta F = C_{66}(\frac{r_p}{R})^2 + C_{44}(\frac{r_p}{L})^2 - f r_p (\frac{R}{V})^{1/2},$$

(1)

where $C_{66}$ is the shear modulus of the lattice, $r_p$ the range of the pinning potential, $R$ and $L$ are the transverse (to the field) and longitudinal (along the field) dimensions of the domain, and $V = R^2L$. The minimization of Eq. (1) gives the pinning lengths $R_c$ and $L_c$:

$$R_c \sim C_{66}^{3/2} C_{44}^{1/2} r_p^2/n f^2, \quad L_c = (C_{44}/C_{66})^{1/2} R_c.$$  

In very thin samples with a perpendicular field, if the pinning is so weak that $L_c$ is greater than the sample thickness, the problem becomes two-dimensional (2D) and only $R_c \sim C_{66} r_p/n^{1/2} f$ is relevant. In the LO theory, since $j_cB = (nf^2/V_c)^{1/2}$, the peak effect can be accounted for if the volume of Larkin domain.
$V_c$ drops faster than $nf^2$ in some field or temperature range. The central question here is what mechanism does that.

It was found by Brandt [11] that, near upper critical field $B_{c2}$ the vortex fields overlap strongly and the tilt modulus $C_{44}$ becomes nonlocal: it softens substantially for short wavelength tilt deformation. Most experiments of the peak effect on low-$T_c$ materials are carried out as a function of field while keeping the temperature constant, the peak effect manifests itself as a peak in $j_c$ (or a dip in resistance) near $B_{c2}(T)$. It is thus natural to relate the $C_{44}(k)$ softening mechanism to the peak effect. LO found [8] that this mechanism leads to an exponential form for $V_c$ when $R_c$ becomes smaller than $\lambda' = \lambda/(1 - b)^{1/2}$, where $\lambda$ is the penetration depth and $b = B/B_{c2}$, and $j_c \sim \exp(-BC_{66}^3/2k_h^2/r_p/W)$, with $W = nf^2$ and $k_h = 1/\lambda'$. It was customary to assume [8] a scaling function for the field dependence of $W$, $W \sim b(1 - b)^2$, and with $C_{66} \sim b(1 - b)^2$ [12] and $k_h^2 \sim (1 - b)$, one indeed finds that $j_c$ rises exponentially with $B$ for $b \sim 1$.

The above interpretation of the peak effect has two major difficulties. The first is that if this mechanism is the relevant one, it should also account for the temperature dependence: $j_c$ rises with increasing $T$ in the peak regime. Giving the most liberal estimate for the temperature dependence of the parameters, however, the above mechanism fails to explain why $j_c$ rises with $T$. The elementary pinning force $f$ is a function of the local gradient of the amplitude of the gap function, or $f \sim a_1|\Delta(r)|^2$. For $T_c$ smearing pins, $a_1 \sim (1 - t)$, $|\Delta(r)|^2 \sim (1 - t)$, $t = T/T_c$, $f \sim (1 - t)^2$, and for pins that do not smear $T_c$, $f \sim (1 - t)$. Thus $W \sim (1 - t)^2$ or $(1 - t)^4$. Without melting, $C_{66} \sim (1 - t)$ and $k_h \sim (1 - t)^{1/2}$. Thus $j_c$ either does not change with $T$ or decreases exponentially with increasing temperature and $j_c$ never increases with increasing $T$, according to this mechanism. The second difficulty of this mechanism is that the peak effect has been observed in thin films [3] and in very thin NbSe$_2$ crystals with pinning weak enough such that $L_c$ exceeds the sample thickness [4,13], in which $C_{44}$ does not seem to play any role. Therefore we believe that this mechanism cannot be responsible for the peak effect.

Another modulus of the vortex lattice which enters the pinning problem is the shear
The peak effect, the rise of $j_c$, suggests to us that $C_{66}$ of the vortex lattice vanishes at the peak effect regime much faster than what was calculated \cite{12} from the Ginzburg-Landau theory. In fact, Pippard \cite{7} suggested that the peak effect could be due to the softening of the shear modulus of the vortex lattice. Here we propose that the rise of the critical current, or the peak effect itself, is a result of vortex lattice melting.

Indeed, the fact that a vortex lattice can melt has been pointed out by several authors \cite{14-16}. Most of the efforts has been focused on the possibility of melting of a perfect Abrikosov lattice and the determination of the phase diagrams. A 2D vortex lattice can melt via the mechanism of the unbinding of the dislocation pairs \cite{14,13}, similar to the melting of a 2D crystal studied by Kosterlitz and Thouless \cite{17}, and by Halperin and Nelson \cite{18}. Much less is known for the melting of a 3D lattice in general. For a perfect 3D vortex lattice both analytic considerations \cite{19} and numerical simulations \cite{20} suggest a first order transition. In particular, it has been shown that a finite density of free edge dislocations would result in a zero long wavelength shear modulus \cite{21}.

Generally speaking, the melting of a lattice (2D and 3D) in the presence of quenched disorder is not well understood. Indeed, often it is not clear what “melting” means in this situation. Naively, if the pins are dilute compared to the vortex density, the melting would be more or less the same as in a pure system and would result in a free (unpinned) vortex fluid. It is doubtful that such a regime exists in real samples. In fact, almost all weak-pinning (low $j_c$) samples show peak effect \cite{2,4,5}, which implies a high density of pins according to our scenario described below. It was argued \cite{22} that a vortex lattice, disordered due to random pinning, could be a “vortex glass” at low temperatures, and the melting transition is then a vortex-glass-to-vortex-fluid transition, which is second order at least for strong random pinning. While a vortex glass phase, if it exists, would determine the small current behavior especially for systems with strong disorder, the peak effect is a property of (relatively) large current in samples with weak pinning.

For a vortex lattice in the presence of random pinning, the lattice deforms locally to take advantage of the fluctuations of the random pins and the long range order of the lattice modulus $C_{66}$. The peak effect, the rise of $j_c$, suggests to us that $C_{66}$ of the vortex lattice vanishes at the peak effect regime much faster than what was calculated \cite{12} from the Ginzburg-Landau theory. In fact, Pippard \cite{7} suggested that the peak effect could be due to the softening of the shear modulus of the vortex lattice. Here we propose that the rise of the critical current, or the peak effect itself, is a result of vortex lattice melting.

Indeed, the fact that a vortex lattice can melt has been pointed out by several authors \cite{14-16}. Most of the efforts has been focused on the possibility of melting of a perfect Abrikosov lattice and the determination of the phase diagrams. A 2D vortex lattice can melt via the mechanism of the unbinding of the dislocation pairs \cite{14,13}, similar to the melting of a 2D crystal studied by Kosterlitz and Thouless \cite{17}, and by Halperin and Nelson \cite{18}. Much less is known for the melting of a 3D lattice in general. For a perfect 3D vortex lattice both analytic considerations \cite{19} and numerical simulations \cite{20} suggest a first order transition. In particular, it has been shown that a finite density of free edge dislocations would result in a zero long wavelength shear modulus \cite{21}.

Generally speaking, the melting of a lattice (2D and 3D) in the presence of quenched disorder is not well understood. Indeed, often it is not clear what “melting” means in this situation. Naively, if the pins are dilute compared to the vortex density, the melting would be more or less the same as in a pure system and would result in a free (unpinned) vortex fluid. It is doubtful that such a regime exists in real samples. In fact, almost all weak-pinning (low $j_c$) samples show peak effect \cite{2,4,5}, which implies a high density of pins according to our scenario described below. It was argued \cite{22} that a vortex lattice, disordered due to random pinning, could be a “vortex glass” at low temperatures, and the melting transition is then a vortex-glass-to-vortex-fluid transition, which is second order at least for strong random pinning. While a vortex glass phase, if it exists, would determine the small current behavior especially for systems with strong disorder, the peak effect is a property of (relatively) large current in samples with weak pinning.

For a vortex lattice in the presence of random pinning, the lattice deforms locally to take advantage of the fluctuations of the random pins and the long range order of the lattice modulus $C_{66}$. The peak effect, the rise of $j_c$, suggests to us that $C_{66}$ of the vortex lattice vanishes at the peak effect regime much faster than what was calculated \cite{12} from the Ginzburg-Landau theory. In fact, Pippard \cite{7} suggested that the peak effect could be due to the softening of the shear modulus of the vortex lattice. Here we propose that the rise of the critical current, or the peak effect itself, is a result of vortex lattice melting.

Indeed, the fact that a vortex lattice can melt has been pointed out by several authors \cite{14-16}. Most of the efforts has been focused on the possibility of melting of a perfect Abrikosov lattice and the determination of the phase diagrams. A 2D vortex lattice can melt via the mechanism of the unbinding of the dislocation pairs \cite{14,13}, similar to the melting of a 2D crystal studied by Kosterlitz and Thouless \cite{17}, and by Halperin and Nelson \cite{18}. Much less is known for the melting of a 3D lattice in general. For a perfect 3D vortex lattice both analytic considerations \cite{19} and numerical simulations \cite{20} suggest a first order transition. In particular, it has been shown that a finite density of free edge dislocations would result in a zero long wavelength shear modulus \cite{21}.
is destroyed [10]. The lattice is ordered within Larkin domains whose size is the pinning
lengths $R_c$ and $L_c$. (We assume weak pinning by which we mean that the typical pinning
force $f$ for individual pins is small. We shall always assume that the density of pins $n$ is not
small compared with the vortex density, which seems to be the case even in samples with
extremely low $j_c$.) Note that $R_c$ and $L_c$ are also the length scales beyond which the lattice is
elastically decoupled. In other words, a small local shear deformation of the lattice would not
propagate (elastically) much farther than $R_c$. How would such a weakly pinned lattice melt?
Melting of a lattice is usually characterized by the vanishing of the long wavelength shear
modulus. For a pinned vortex lattice, however, a long wavelength shear is not sustained
by the long wavelength elasticity of the lattice. If we shear slightly the opposite sides of a
pinned vortex lattice, the shear deformation would decay (presumably exponentially) inside
the lattice with a decay length of order $R_c$, and the vortices inside would not feel the shear.
In other words, the long wavelength shear is sustained by the pinning force. It is then not
surprising that the length scale $R_c$ should play a crucial role in the melting of a pinned
lattice.

We first consider the case of a 2D lattice where much is known for the melting of a
clean (pinning free) system [17,18]. In the case of a 2D lattice, thermally excited dislocation
pairs are bound for temperatures below the melting temperature $T_m$ and, consequently, the
shear modulus is finite. At $T_m$, the largest dislocation pairs start to dissociate and the
long wavelength shear modulus drops discontinuously to zero. Above $T_m$, the density of
free dislocations rises from zero, and the mean distance between free dislocations is the
Kosterlitz-Thouless correlation length $\zeta \sim \exp[c/(T - T_m)^\nu]$ with $\nu \sim 0.37$ [18], which
diverges as $T \to T_m^+$. The correlation length $\zeta$ also sets the length scale for the $q$ (wavevector)-
dependence of the shear modulus $\mu(q,T)$: roughly speaking, $\mu$ is zero for $q < 1/\zeta$ and finite
for $q > 1/\zeta$. Now imagine that the vortex lattice is weakly pinned. For $T < T_m$, the Larkin
length $R_c$ sets the elastic length scale and the critical current density $j_c = n^{1/2}f/R_cB$.
At $T = T_m$, the vortex lattice melts with the long wavelength shear modulus dropping to
zero. However, as argued in the previous paragraph, the pinned lattice would not feel being
melted at this point since the lattice is elastically decoupled beyond the length scale of \( R_c \). The critical current density is still determined by \( R_c \). For \( T > T_m \), another length scale \( \zeta \), the Kosterlitz-Thouless correlation length, enters the system. \( \zeta \) decreases exponentially fast from the infinity as the temperature is increased and will soon become comparable to \( R_c \). For \( \zeta < R_c \), the Larkin domains melt and the relevant elastic length scale for the determination of \( j_c \) is now \( \zeta \):

\[
j_c = \frac{n^{1/2} f}{\zeta B} \approx \frac{n^{1/2} f}{B} \exp[-\frac{c}{(T-T_m)^\nu}], \quad (\zeta < R_c).
\]

Thus the onset of peak effect occurs when the two length scales \( R_c \) and \( \zeta \) cross each other (Fig. 2). The exponential increase of \( j_c \) with \( T \) (Eq. (4)) would continue until \( \zeta \) is of the order of the lattice constant. However, the temperature dependence of the pinning force \( f \) and thermally activated vortex motion \[24\] would presumably dominate the behavior of \( j_c \) at even higher \( T \) and cause \( j_c \) to vanish.

In the 3D case, melting of a pure lattice is much less understood. If the melting transition is mediated by generation of the free edge dislocations, one would expect a similar mechanism for the peak effect as in 2D, with \( \zeta \) now being the mean distance between dislocation lines. To make a qualitative or semi-quantitative estimate for \( j_c \) in the peak effect regime, we take the Landau-Ginzburg-like free energy often used in 3D dislocation systems \[25\]:

\[
F(\rho) = -F_1 \rho \ln C \rho + F_2 \rho + F_3 \rho^2,
\]

where \( \rho \) is the areal density of dislocation lines, \( F_1 \) and \( F_3 \) are positive constants, \( C \) is a constant of the order \( a^2 \) with \( a \) being the lattice constant, \( F_2 > 0 \) at low temperatures and \( F_2 < 0 \) at high temperatures. It is easy to see that Eq. (3) implies a first order transition:

\[
\rho = \begin{cases} 
0, & (T < T_m) \\
\rho_c \exp[A(T - T_m)^\nu], & (T \geq T_m)
\end{cases}
\]

where \( \rho_c = F_1/2F_3 \) (\( \approx \mu a/32\pi^2 k_B T_m \)), with \( \mu \) being the shear modulus, if we use the values in Ref. \[25\] for \( F_1 \) and \( F_3 \), \( A = \sqrt{-F'_2/F_3 \rho_c} \), and \( \nu = 1/2 \). The mean distance between dislocation lines, \( \zeta = \rho^{-1/2} = \rho_c^{-1/2} \exp[-A(T - T_m)^\nu/2] \), is the length scale to be compared
with the Larkin length $R_c$. In the region where $\zeta < R_c$, the critical current density is determined by $\zeta$:

$$
    j_c = \frac{n^{1/2} f}{V_c^{1/2} B} = \begin{cases} 
    \frac{n^{1/2} f}{L_c^{1/2} R_c B} \approx \frac{n^2 f^4}{C_{66}^2 C_{44}^4 4^{1/2} V_p^2}, & (\zeta > R_c) \\
    \frac{n^{1/2} f}{L_c^{1/2} \zeta B} \approx \frac{n^{2/3} f^{4/3} \rho^{2/3}}{C_{44}^{1/3} \gamma^{1/3} B} \exp\left[\frac{2}{3} A(T - T_m)^\nu\right], & (\zeta < R_c)
    \end{cases}
$$

(5)

where in the region of $\zeta < R_c$ Eq. (1) is minimized, with $R_c$ replaced by $\zeta$, to determine $L_c$. Since $\zeta$ has a discontinuous jump at $T_m$ (if the transition is first order), it is possible that $j_c$ will have a jump at the onset of the peak effect which may occur in samples with “very weak” pinning (Fig. 2). In fact, a jump in $j_c$ was observed experimentally [26]. Note that our argument does not depend on the detailed nature of the melting transition, e.g. first vs. second order [27], although we have used Eq. (3) to obtain some estimates. The key point in our scenario is that some other (elastically relevant) length scale enters the system around the melting transition and it crosses the Larkin length. In 2D, we believe this length scale is the Kosterlitz-Thouless correlation length. While in 3D, it is most likely that it is the length scale set by thermally excited edge dislocations.

In conclusion, a quantitative analysis suggests that the non-local effect of $C_{44}$ is not the cause for $j_c$ to rise with increasing temperature at the peak effect regime. We propose a scenario based on the melting of the vortex lattice in the presence of weak pinning. The onset of the peak effect is the crossover of the two elastically relevant length scales. We believe this captures the basic physical picture, at least for weak enough pinning, although ideally the analysis should be put on a more rigorous footing. On the other hand, if the pinning is so strong that the Larkin length is of the order of the lattice constant, the sample should not, according to our scenario, show the peak effect. In this paper we have focused only on the behavior of $j_c$. The dynamics above $j_c$ can be quite different in the peak regime, where a significant enhancement of plastic flow was recently observed [28] in 2H-NbSe$_2$. This indicates that the dynamically generated defects proliferate in the peak regime as well. We emphasize the difference between the scenario concerning melting described in this paper and that suggested in literature [29] in which the onset of dissipation in the presence of a
driving force or the vanishing of $j_c$ has been interpreted as the evidence for the vortex-lattice melting transition. The rationale behind this suggestion is based upon the assumption that when the vortex lattice melts, the vortex lines will flow. This scenario may be correct if the sample is so pure that the vortex line density is much greater than the impurity density. Unfortunately, this cannot be achieved even in the cleanest crystals of 2H-NbSe$_2$ where the critical current density is many orders of magnitude smaller than that of YBCO. Instead, the clean, high quality crystals of 2H-NbSe$_2$ always exhibit a pronounced peak effect and high-quality YBCO crystals are also found recently to show a peak effect.

Note added: In the course of writing this manuscript, we received a preprint by Larkin, Marchetti, and Vinokur, in which they attribute the peak effect to the softening of $C_{66}$ just below the melting transition.
REFERENCES

[1] M. A. R. Le Blanc and W. A. Little, in *Proceedings of the Seventh International Conference on Low Temperature Physics*, 1960 (University of Toronto Press, Toronto, 1960), p. 198.

[2] T. G. Berlincourt, R. R. Hake, and D. H. Leslie, Phys. Rev. Lett. 6, 671, (1961); S. H. Autler, E. S. Rosenblum, K. Gooen, Phys. Rev. Lett. 9, 489 (1962); W. DeSorbo, Rev. Mod. Phys. 36, 90 (1964).

[3] P. H. Kes and C. C. Tsuei, Phys. Rev. B 28, 5126 (1983).

[4] For a recent extensive study of the peak effect in 2H-NbSe$_2$, see M. J. Higgins, S. Bhattacharya, to be published.

[5] X. S. Ling and J. I. Budnick, in *Magnetic Susceptibility of Superconductors and Other Spin Systems*, edited by R. A. Hein et al., (Plenum Press, New York, 1991), p. 377; X. S. Ling, Ph.D. Thesis, U. of Connecticut (1992), UMI (Microfilms), Ann Arbor, Michigan 48106; X. S. Ling, J. I. Budnick, B. W. Veal, D. Shi, and J-Z. Liu, Phys. Rev. B (to be published).

[6] W. K. Kwok, J. A. Fendrich, C. J. van der Beek, and G. W. Crabtree, Phys. Rev. Lett. 73, 2614 (1994).

[7] A. B. Pippard, Phil. Mag. 19, 217 (1969).

[8] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. 34, 409 (1979).

[9] Part of this work was reported earlier [X. S. Ling and C. Tang, Bull. Am. Phys. Soc. 40, 386 (1995)].

[10] A. I. Larkin, Zh. Eksp. Teor. Fiz. 58, 1466 (1970) [Sov. Phys. JETP 31, 784].

[11] E. H. Brandt, J. Low Temp. Phys. 26, 709 and 735 (1977); *ibid.* 28, 263 and 291 (1977); Phys. Rev. B 48, 6699 (1993).
[12] E. H. Brandt, Phys. Rev. B 34, 6514 (1986).

[13] Peak effect in thin 2H-NbSe$_2$ crystals was interpreted as a dimensional crossover by P. Koorevaar et al., Phys. Rev. B 42, 1004 (1990).

[14] B. A. Huberman and S. Doniach, Phys. Rev. Lett. 43, 950 (1979).

[15] D. S. Fisher, Phys. Rev. B 22, 1190 (1980).

[16] D. R. Nelson, Phys. Rev. Lett. 60, 1973 (1988); D. R. Nelson and S. Seung, Phys. Rev. B 39, 9153 (1989).

[17] J. M. Kosterlitz and D. J. Thouless, J. Phys. C 6, 1181 (1973); in Progress in Low Temperature Physics, Vol. VII-B, edited by D. F. Brewer (North-Holland, Amsterdam, 1978).

[18] B. I. Halperin and D. R. Nelson, Phys. Rev. Lett. 41, 121 (1978); 41, 519(E) (1978); and D. R. Nelson, and R. I. Halperin, Phys. Rev. B 19, 2457 (1979).

[19] E. Brezin, D. R. Nelson, and A. Thiaville, Phys. Rev. B 31, 7124 (1985).

[20] R. E. Hetzel, A. Sudbo, and D. A. Huse, Phys. Rev. Lett. 69, 518 (1992).

[21] M. C. Marchetti and D. R. Nelson, Phys. Rev. B 41, 1910 (1990).

[22] M. P. A. Fisher, Phys. Rev. Lett. 62, 1415 (1989); D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B 43, 130 (1991).

[23] We use the symbol $\zeta$, instead of the standard $\xi$, for the Kosterlitz-Thouless correlation length to avoid confusion with the superconducting coherence length.

[24] Thermal activation lowers the measured $j_c$ [P. W. Anderson, Phys. Rev. Lett. 9, 309 (1962)]. Applying Anderson’s argument to LO’s estimate for $j_c$, one obtains the measured $j_c$ as: $j_cB = (nf^2/V_c)^{1/2} - \alpha k_BT/r_p V_c$, where $\alpha$ is a sensitivity-dependent parameter and is of the order of 20 for a typical experiment. This expression implies that in
weak-pinning samples with large Larkin volume $V_c$ the correction to $j_c$ due to thermal activation is insignificant. However, thermal activation could play an important role on the high temperature side of the peak effect where $j_c$ drops rapidly to zero.

[25] T. Yamamoto and T. Izuyama, J. Phys. Soc. Japan 57, 3742 (1988).

[26] R. Wordenweber and P. H. Kes, Phys. Rev. B 34, 494 (1986).

[27] G. I. Menon and C. Dasgupta, Phys. Rev. Lett. 73, 1023 (1994).

[28] S. Bhattacharya and M. J. Higgins, Phys. Rev. Lett. 70, 2617 (1993); Phys. Rev. B 49, 10005 (1994).

[29] P. L. Gammel, L. F. Schneemeyer, J. V. Waszczak, and D. J. Bishop, Phys. Rev. Lett. 61, 1666 (1988); R. G. Beck, D. E. Farrell, J. P. Rice, D. M. Ginsberg, and V. G. Kogan, *ibid.* 68, 1594 (1992); H. Safar, P. L. Gammel, D. A. Huse, D. J. Bishop, J. P. Rice, and D. M. Ginsberg, *ibid.* 69, 824 (1992); W. K. Kwok, S. Fleshler, U. Welp, V. M. Vinokur, J. Downey, G. W. Crabtree, and M. M. Miller, *ibid.* 69, 3370 (1992); M. Charalambous, J. Chaussy, P. Lejay, and V. Vinokur, *ibid.* 71, 4366 (1993).

[30] A. I. Larkin, M. C. Marchetti, and V. M. Vinokur, (to be published).
FIGURES

FIG. 1. The temperature dependence of critical current density for a YBCO crystal in a magnetic field, extracted from Ref. [5]. We define $T_p$ as the onset temperature at which $j_c$ starts to increase.

FIG. 2. Schematic behavior of the two length scales $R_c$ and $\zeta$, as functions of temperature: (a) two dimensions; (b) three dimensions with weak pinning; and (c) three dimensions with “very weak” pinning. The temperature for the onset of the peak effect is $T_p$, where $\zeta$ becomes smaller than $R_c$. In (c), $T_p = T_m$. The solid part of the lines determines the critical current density.
Figure 1 (Ling, Tang, Bhattacharya, Chaikin)
Figure 2
(Ling, Tang, Bhattacharya, Chaikin)