Estimation of evaporation residue cross section for the synthesis of superheavy nuclei.

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Abstract. To estimate the evaporation residue cross section of superheavy nuclei, we clarify the parameter dependence of the models. At the first attempt, we focus our attention on the parameter dependence of the survival probability. Then we discuss the the evaporation residue cross section for each parameter.

1 Introduction

Since 1966, according to macroscopic-microscopic calculations [1, 2], the existence of the island of stability in the nuclear chart surrounding the doubly magic superheavy nucleus containing 114 protons and 184 neutrons has been predicted. The property and structure of nuclei in the superheavy mass region have been investigated, taking into account a large multidimensional deformation for the ground state [3–5]. Recently, within the relativistic mean-field model [6] and nonrelativistic Skyrme-Hartee-Fock approach [7], some other spherical magic numbers have been found, such as $Z=120$ and $N=172$ [8].

Attempts for synthesizing heavy elements beyond the atomic number $Z \approx 100$ have become active since the 1970s by means of various developments in experimental techniques [9–12] and have recently succeeded in identifying the element till $Z=118$.

The synthesis of these superheavy elements has been carried out using heavy-ion fusion reactions between stable nuclei, in which two different types of reaction have been employed. In the cold fusion reactions, lead and bismuth targets are used [13, 14]. The element with $Z=113$ reported by RIKEN used this type of reaction [12]. The superheavy nuclei (SHN) synthesized in the cold fusion reaction produce nuclei with relatively small number of neutrons.

The other type of reaction, called the hot fusion reaction, on the other hand, uses actinide nuclei as targets. With this type of reaction, production of elements till $Z=118$ were reported by the Flerov Laboratory of Nuclear Reactions (FLNR) [15]. Recently, other laboratories than FLNR also performed experiments of hot fusion reactions and obtained results that are consistent with the data by FLNR [16–21].

For synthesis of new nuclei and elements, an accurate prediction of the production cross sections is an important issue in the superheavy elements research.

Many theoretical studies on the synthesis of superheavy elements have been published and the evaporation residue cross section corresponding to the above experiments has been estimated [22]. In the theoretical calculation, the evaporation residue cross section is obtained as the product of the fusion probability forming a compound nucleus and its survival probability in the competition with the fission process [23]. Since a substantial uncertainty is involved in each stage and model, we discuss uncertainties of parameters. Here, we focus our attention on the parameters of the statistical model.

In Sec. 2, we describe in detail the framework of the model. We discuss uncertainty of parameters in the models in Sec. 3. In Sec. 4, we investigate the parameter dependence of the survival probability. In consequence, we show the evaporation residue cross section in the reaction $^{48}$Ca$^{+244}$Pu which is calculated with the several parameters in the statistical model. In the last section, we gave also an outlook to our next step for improvements of calculations.

2 Estimation of evaporation residue cross section

To estimate the evaporation residue cross section, the whole fusion process is divided into the three stages depend on the reaction time scale $t$. The first stage is the approaching process, $t < 10^{-22} - 10^{-21}$s, and the capture probability is denoted by $T_c$. The second stage corresponds to the competition between the fusion and quasi-fission processes, $10^{-21} \leq t \leq 10^{-18}$s, and the formation probability of forming a compound nucleus is denoted by $P_{CN}$. The decay process of the compound process is presented as the third stage $t \geq 10^{-18}$s, and the survival probabil-
ity of compound nuclei during de-excitation is denoted by $W_{sur}$.

Using these probabilities, the evaporation residue cross section $\sigma_{EV}$ is estimated as

$$\sigma_{EV} = \frac{\pi \hbar^2}{2 \mu_0 V_{Inc}} \sum_{l=0}^\infty \frac{2(\lambda + 1)}{\Pi_l(E_{INC}, l) P_{CN}(E^*, l) W_{sur}(E^*, l)}$$

where $\mu_0$ denotes the reduced mass in the entrance channel, $E_{INC}$ and $E^*$ denote the incident energy in the center-of-mass frame and the excitation energy of the compound nucleus, respectively. $E^*$ is given as $E^* = E_{INC} - Q$, where the latter denotes the $Q$-value of the reaction. $\Pi_l(E_{INC}, l)$ is the capture probability of the $l$-th partial wave, which is calculated with the coupled channel model [24, 25]. To estimate $P_{CN}(E^*, l)$, we use the dynamical model and employ the Langevin equation [26, 27]. $P_{CN}(E^*, l)$ is obtained to To estimate $P_{CN}$, the definition of the fusion area in the deformation space is very important. It is reasonable to define the fusion area around the pocket near the ground state in the deformation space. Here, we define the fusion area (fusion box) as the inside of the fission saddle point in the system, which is $\{z < 0.8, \delta < 0.3, |\gamma| < 0.3\}$ [26]. $W_{sur}(E^*, l)$ is calculated using a statistical model [28, 29].

### 3 Uncertainty of parameters in the theoretical models

Many theoretical models have been developed and applied to estimate the evaporation residue cross section [22]. The results show rather good agreement with the experimental data. However, for the synthesis of unknown elements, $Z = 119, 120$ etc, the predictions from different theoretical approaches are quite different [22].

Inevitably, a substantial uncertainty is involved in each stage. In the first stage, there are few parameters in the coupled channel model. Though the parameters of the potential (potential depth, diffuseness) of the compound nuclei are defined to reproduce the capture cross section in experimental data, these parameters are not clear for unknown nuclei which have not been measured. In the second stage, the Langevin calculation includes parameters: potential energy (parameters of liquid drop model and shell correction energy), nuclear shape parametrization and the number of the dynamical variables (shape parameters), the transport coefficients (using the macroscopic or microscopic models). The definition of the fusion region is also unclear. In the third stage, the statistical model includes the uncertainty of the fission barrier height of the compound nucleus, the friction parameter, the level density parameter ($a_f/a_n$), etc. [28, 29]. Moreover, the reaction $Q$-value has also an uncertainty, because the masses of superheavy nuclei are not determined experimentally yet.

We should define the values of unknown parameters in each stage, or at least we would like to show explicitly and specify the values of parameters which are used in the calculation. As the first step, we have to know the parameter dependence of the evaporation residue cross section within the model calculation.

In the next section, we demonstrate the dependence using the present model.

### 4 Results

To know the parameter dependence of our model, first we focus our attention on the survival probability in the third stage.

#### 4.1 Survival probability

We calculate the survival probability using the statistical code NASADEC, which is developed by M. Ohta [29] based on the idea in the reference [28]. In the code, we change the values of parameters and investigate the influence of the survival probability. The fission barrier height of compound nucleus $B_f$, the friction parameter $\gamma$, and the level density parameter $a_f/a_n$ are changed for the compound nucleus $^{252}$Fm with the angular momentum $\ell = 30$. Figure 1 shows the results for each parameter. In the model, we calculate the fission barrier height $B_f$ [29] based on the table [30]. We introduce the adjustment parameter of $B_f$, and the results are shown in Fig. 1(a). We can see that the survival probability changes by one order of magnitude as the $B_f$ changes by 1 MeV. It seems the survival probability is moved in parallel in this case.

Fig. 1(b) shows the dependence of $\gamma$ parameters. When the $\gamma$ is large, the fission is surprised. In this case, as changing the $\gamma$ values, the slope of the survival probability changes. The dependence of the parameter $a_f/a_n$ is the same as $\gamma$, that causes to change the slope of the survival probability, which is shown in Fig. 1(c). In the figure, $a_f/a_n$ is represented by fact.

Due to the calculation results, we can see the tendency of the parameter dependence of the survival probability and the statistical model.

#### 4.2 Evaporation residue cross section

Under the parameter dependence in the survival probability, we obtain the evaporation residue cross section. Figure 2 shows the evaporation residue cross section for the reaction $^{48}$Ca + $^{244}$Pu. The dependence of $B_f$ and the $\gamma$ parameter in the statistical model are shown in Fig. 2(a) and (b), respectively. The evaporation residue cross section changes significantly depending on the values of parameters, and we can clearly understand the dependence.

By the systematical calculation and comparison with the experimental data, we try to find the reasonable parameter sets. As the first attempt, to reproduce the excitation function of the evaporation residue cross section for the reactions with $^{48}$Ca-projectile leading the heavy elements from $Z = 102$ to 118, we change the $B_f$ and $\gamma$ in the statistical model. In the systematical investigation, we found the reasonable values to reproduce the experimental data. To reproduce the experimental data, we introduce the additional value of $B_f$, which is denoted as ADDS. When we plot the correlation between the $Z$ number of the compound nuclei and ADDS, we just find the linear correlation.
Many theoretical models have been developed and applied to investigate the fission problem of compound nuclei during de-excitation. Here, we define the fusion section as the inside of the fission saddle point state in the deformation space. Here, we define the fusion area as the fusion box.

Using these probabilities, the evaporation residue cross section is calculated with the coupled channel model \[24, 25\]. Though the parameters of the statistical model are not clear for all nuclei which have not been measured. In the systematical investigation, we have found the reasonable values to reproduce the experimental data.

Figure 1. Parameter dependence of survival probability of \(^{292}\)Fl calculated by the statistical code MasaDEC. (a) Fission barrier height \(B_f\), (b) Friction parameter \(\gamma\), (c) Level density parameter \(\alpha/\alpha_c\).

Figure 2. Evaporation residue cross section for the reaction \(^{48}\)Ca + \(^{244}\)Pu. The dependence of \(B_f\) and \(\gamma\) in the statistical model are shown in (a) and (b), respectively.
It is rough estimation and seems to be the model dependence. Also, we could show that FRIC has the exponential correlation of $Z$. These correlations show as following.

$$\text{ADDS} = 0.218 \times Z - 23.2 \quad (2)$$

$$\text{FRIC} = 8.84 \times \exp\{0.169 \times Z\} \quad (3)$$

Though they are a rough estimation and we have to explain the reasons, it seems important to find the correlation among the parameters. For the future work, using the more accurate mathematical methods, for example, the covariance function, we try to predict the possibility of synthesis of new elements more accurately.

The author is grateful to Prof. K. Hagino (Tohoku University), Dr. K. Nishio (JAEA) and Mr. T. Tanaka (Kyushu University, RIKEN), Dr. Y. Kim (IBS) and J.Y. Lee (IBS) for their helpful suggestions and valuable discussion throughout the present work. This work was supported by the Sasakawa Scientific Research Grant from The Japan Science Society.

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