Doubly Robust Collaborative Targeted Learning for Debiased Recommendations

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Abstract

In recommender systems, the collected data always contains various biases and leads to the challenge of accurate predictions. To address selection bias and confounding bias, the doubly robust (DR) method and its variants show superior performance due to the double robustness property and smaller bias under inaccurate propensity and error imputation models. However, we theoretically show that the variance of the error imputation-based (EIB) method is much smaller than that of DR, although EIB may suffer from a much larger bias. In this paper, we propose a doubly robust targeted learning method that effectively combines the small-bias property of DR and the small-variance property of EIB, by leveraging the targeted maximum likelihood estimation technique. Theoretical analysis shows that the proposed targeted learning is effective in reducing the variance of DR while maintaining double robustness. To further reduce the bias and variance during the training process, we propose a novel collaborative targeted learning approach that decomposes imputed errors into parametric and nonparametric parts and updates them collaboratively, resulting in more accurate predictions. Both theoretical analysis and experiments demonstrate the superiority of the proposed methods compared with existing debiasing methods.

1 Introduction

Addressing various tasks in recommender systems (RS) with causality-based methods has become increasingly popular [4, 33]. One of the main reasons for this trend is that many practical tasks in RS are inherently causal problems, such as rating prediction [38], post-click conversion rate prediction [7], post-click conversion rate prediction [7, 44], and uplift modeling [23, 25, 26]. Causality-based recommendation has shown its great potential in both numeric experiments and theoretical analyses across extensive literature [4, 36, 43]. Generally, the basic question faced in RS is that "what would the feedback be if recommending an item to a user", requiring to estimate the causal effect of a recommendation on user feedback. To answer the question, many methods have been proposed, such as inverse propensity score (IPS) [24, 27], self-normalized inverse propensity score (SNIPS) [27, 30], error imputation based (EIB) methods [8, 28], and doubly robust (DR) methods [5, 7, 36, 37]. Among them, the DR method and its variants show superior performance.

To assess the performance of these methods, we introduce five perspectives to compare and evaluate different approaches in RS, including doubly robust [36], robust to small propensities [40], bounded-
ness \cite{16,34}, without extrapolation \cite{16,31} and low variance \cite{31}. Failing to meet any of them may lead to sub-optimal performance \cite{16,34}. We compare various estimators in these five aspects and find that the IPS, EIB, and DR methods only enjoy limited desired properties.

Generally, DR is superior to IPS in terms of both bias and variance. When comparing EIB with DR, it involves the bias-variance trade-off, since usually DR has a smaller bias \cite{36} and EIB has a smaller variance \cite{11,31}. In this article, we first propose a novel method, DR-TMLE, that can capture the merits of both DR and EIB effectively by leveraging the targeted maximum likelihood estimation (TMLE) technique \cite{34,35}. Remarkably, DR-TMLE possesses many more desired properties than the existing methods aforementioned. In addition, to further reduce the bias and variance during the training process, we propose a novel uniform-data-free collaborative targeted learning approach, called DR-TMLE-TL, that decomposes imputed errors into a parametric imputation model part and a nonparametric TMLE error part, where the latter adaptively rectifies the residual bias of the former. By updating the two parts collaboratively, DR-TMLE-TL achieves a more accurate and robust prediction. Both theoretical analysis and experiments demonstrate the superiority of DR-TMLE and DR-TMLE-TL compared with existing methods.

2 Preliminaries

2.1 Problem Statement

Many debiasing tasks in RS can be formulated using the widely adopted potential outcome framework \cite{17,19,43}. Denote $\mathcal{U} = \{u\}$, $\mathcal{I} = \{i\}$ and $\mathcal{D} = \mathcal{U} \times \mathcal{I}$ as the sets of users, items and user-item pairs, respectively. Let $x_{u,i}$, $r_{u,i}$, and $o_{u,i}$ be the feature, feedback, and exposure status of user-item pair $(u, i)$, where $o_{u,i} = 1$ or 0 represents whether the item $i$ is exposed to user $u$ or not. Define $r_{u,i}(1)$ as the potential outcome if $o_{u,i}$ had been set to 1, which is observed only when $o_{u,i} = 1$. In RS, we are often interested in answering the causal question: "if we recommend products to users, what would be the feedback?". This question can be formulated as to learn the quantity $E(r_{u,i}(1)|x_{u,i})$, i.e., it requires to predict $r_{u,i}(1)$ using feature $x_{u,i}$, where $E$ denotes the expectation with respect to the target distribution $\mathbb{P}$. Many classical tasks in RS can be defined as estimating this quantity, such as rating prediction \cite{27,36,38}, post-view click-through rate prediction \cite{7}, and post-click conversion rate prediction \cite{7,45}. More examples can be found in \cite{43}.

Let $f_{\theta}(x_{u,i})$ be a model used to predict $r_{u,i}(1)$ with parameter $\theta$. Ideally, if all $r_{u,i}(1)$ for $(u, i) \in \mathcal{D}$ were observed, $\theta$ can be trained directly by optimizing the following ideal loss

$$L_{\text{ideal}} = |\mathcal{D}|^{-1} \sum_{(u,i) \in \mathcal{D}} e_{u,i},$$

where $e_{u,i}$ is the prediction error, e.g., the squared loss $e_{u,i} = (r_{u,i}(1) - f_{\theta}(x_{u,i}))^2$. However, since $r_{u,i}(1)$ is observed only when $o_{u,i} = 1$, the ideal loss is non-computable. Restricting the analysis to non-missing data will obtain biased conclusions \cite{7,43}, as the observed data may form an unrepresentative sample of the target population. Different debiasing methods are designed to approximate and substitute the ideal loss. For example, the IPS and EIB estimators are given as

$$L_{\text{IPS}} = |\mathcal{D}|^{-1} \sum_{(u,i) \in \mathcal{D}} o_{u,i} e_{u,i} / \hat{p}_{u,i}, \quad L_{\text{EIB}} = |\mathcal{D}|^{-1} \sum_{(u,i) \in \mathcal{D}} [o_{u,i} e_{u,i} + (1 - o_{u,i}) \hat{c}_{u,i}],$$

where $\hat{p}_{u,i}$ is an estimate of propensity score $p_{u,i} := \mathbb{P}(o_{u,i} = 1|x_{u,i})$, $\hat{c}_{u,i}$ is an estimate of $g_{u,i} := E(e_{u,i}|x_{u,i})$, i.e., it fits $e_{u,i}$ using $x_{u,i}$. And the DR estimator is formulated as

$$L_{\text{DR}} = |\mathcal{D}|^{-1} \sum_{(u,i) \in \mathcal{D}} \left[ \hat{c}_{u,i} + \frac{o_{u,i}(e_{u,i} - \hat{c}_{u,i})}{\hat{p}_{u,i}} \right].$$

2.2 Related Work

Debiasing in Recommendation. Bias is a common problem inherent in RS \cite{4,39}, such as popularity bias \cite{46}, model selection bias \cite{44}, user self-selection bias \cite{21}, position bias \cite{11}, and conformity bias \cite{14}. Various methods were proposed for unbiased learning. For example, \cite{27} considered the recommendation as treatment and introduced the IPS and self-normalized IPS (SNIPS) methods
to debiasing in explicit feedback data. \cite{20,24} extended it to the implicit recommendation. \cite{36} proposed a doubly robust joint learning approach that improved the IPS method. Subsequently, a series of enhanced DR methods were developed, such as more robust doubly robust method \cite{7} (MRDR) and multi-task learning \cite{45}. In addition, \cite{2,5,13,37} designed new debiasing algorithms via using an extra small uniform dataset. \cite{4} provided a thorough discussion on the recent progress on debiasing tasks in RS. \cite{43} established a unified causal analysis framework and gave formal causal definitions of various biases in RS from the perspective of violating the assumptions adopted in causal analysis. Unlike the existing enhanced DR approaches, which pursue a better bias-variance trade-off, the proposed DR-TMLE reduces both the bias and variance and is theoretically guaranteed.

**Targeted Learning.** Targeted learning is a general framework in causal inference \cite{34,35} designed to protect us from wasting resources on the irrelevant part of a problem and to help us focus on what is relevant. It has been widely used in fields of survival analysis \cite{29}, genomics \cite{6}, epidemiology \cite{18} and etc.. TMLE \cite{32,33} acted as a basic methodology in targeted learning.

## 3 Doubly Robust Targeted Maximum Likelihood Estimation

In this section, we first compare the existing debiasing methods in five aspects. Then, we propose DR-TMLE and DR-TMLE-TL methods that effectively capture the merits of both the EIB and DR.

### 3.1 Motivation

There are five desired statistical properties for evaluating a debiasing method, including doubly robust, robust to small propensities \cite{40}, boundedness \cite{16,34,41}, without extrapolation \cite{16,31}, and low variance. Table 1 compares various debiasing methods in these five aspects. Note that the existing debiasing methods cannot simultaneously satisfy the desired statistical properties to a certain extent, which motivates us to develop new approaches. Next, we elaborate on these properties in detail.

| Doubly Robust | Robust to Small Propensities | Boundedness | Without Extrapolation | Low Variance |
|---------------|-------------------------------|-------------|-----------------------|-------------|
| IPS           | ×                             | ×           | ✓                     | ×           |
| EIB           | ×                             | ✓           | ✓                     | ✓           |
| DR            | ✓                             | ×           | o                     | ✓           |
| DR-TMLE (ours)| ✓                             | ✓           | ✓                     | √           |

Note: symbols ✓, o and × denotes good, medium and bad, respectively.

**Doubly robust.** It is well known that DR enjoys the property of double robustness, which means that $L_{DR}$ is an unbiased estimator of $L_{ideal}$ if either $e_{u,i}$ estimates $g_{u,i}$ accurately or $\hat{p}_{u,i}$ estimates $p_{u,i}$ accurately. In contrast, IPS and EIB do not meet the property of double robustness.

**Robust to small propensities.** Both the IPS and DR use $1/\hat{p}_{u,i}$ as the weight to recover the target distribution \cite{9}. In the presence of small propensities, the weights will become extremely large and cause instability. In contrast, EIB does not suffer from such a problem \cite{10,31,42}.

**Boundedness.** Both the IPS and DR may lie outside the range of $L_{ideal}$, i.e., they do not enjoy the property of boundedness. For example, if we set $e_{u,i} \in [0,1]$, then $L_{ideal} \in [0,1]$, while $L_{IPS}$ and $L_{DR}$ may not be within the range. The EIB can guarantee boundedness property easily if the error imputation model is chosen appropriately \cite{16,34}.

**Without extrapolation.** EIB usually has a large bias, which is a consequence of making implicitly extrapolations. Specifically, the error imputation model is trained with exposed events while using the predicted values for unexposed events. This relies heavily on extrapolation since the exposed events are sparse and there may exist a significant difference between the distributions of exposed events and unexposed events. Thus, it is hard to obtain accurate error imputation and leads to poor performance \cite{16}. In comparison, the estimation of propensity score doesn’t rely on extrapolation.

**Low variance.** EIB has the smallest variance among these methods, the associated results are presented in Theorem 1 (see Appendix A for the proof).
Theorem 1. If \( \hat{p}_{u,i} \) and \( \hat{e}_{u,i} \) are accurate estimates of \( p_{u,i} \) and \( g_{u,i} \), respectively, i.e., \( \hat{p}_{u,i} = p_{u,i} \) and \( \hat{e}_{u,i} = g_{u,i} \), then IPS, EIB and DR estimators are unbiased, and their variances satisfy

\[
\forall (\mathcal{L}_{EIB}) \leq \forall (\mathcal{L}_{DR}) \leq \forall (\mathcal{L}_{IPS}),
\]

where the equality holds if and only if \( p_{u,i} = 1 \) for all \((u, i) \in \mathcal{D}\). The variances are given as

\[
\forall (\mathcal{L}_{IPS}) = |\mathcal{D}|^{-1} \left[ \mathbb{E} \left( \frac{\sigma^2(x_{u,i}) + g_{u,i}^2}{p_{u,i}} \right) - \{\mathbb{E}(e_{u,i})\}^2 \right],
\]

\[
\forall (\mathcal{L}_{DR}) = |\mathcal{D}|^{-1} \left[ \mathbb{E} \left( \frac{\sigma^2(x_{u,i})}{p_{u,i}} + g_{u,i}^2 \right) - \{\mathbb{E}(e_{u,i})\}^2 \right],
\]

\[
\forall (\mathcal{L}_{EIB}) = |\mathcal{D}|^{-1} \left[ \left( p_{u,i} \sigma^2(x_{u,i}) + g_{u,i}^2 \right) - \{\mathbb{E}(e_{u,i})\}^2 \right],
\]

where \( \sigma^2(x_{u,i}) = \mathbb{V}(e_{u,i}|x_{u,i}) \). In addition, when \( p_{u,i} \) tends to 0, \( \forall (\mathcal{L}_{IPS}) \) and \( \forall (\mathcal{L}_{DR}) \) tends to infinity, and \( \forall (\mathcal{L}_{EIB}) \) tends to its minimum \(|\mathcal{D}|^{-1}\mathbb{V}(g_{u,i})\).

In summary, DR outperforms IPS in terms of both bias and variance. When compared with EIB, if \( \hat{e}_{u,i} \) is inaccurate but \( \hat{p}_{u,i} \) is accurate, DR tends to have a smaller bias, but if both \( \hat{e}_{u,i} \) and \( \hat{p}_{u,i} \) are accurate, then EIB has a smaller variance. Furthermore, if \( \hat{e}_{u,i} \) is accurate but \( \hat{p}_{u,i} \) is inaccurate, then EIB may be superior to DR in terms of both bias and variance. In practice, both \( \hat{p}_{u,i} \) and \( \hat{e}_{u,i} \) are likely to be at least mildly inaccurate, so choosing from EIB and DR involves the bias-variance trade-off. Ideally, it is desirable to develop a method that combines the merits of both DR and EIB.

### 3.2 Doubly Robust TMLE

DR and EIB are related via the "correction term". Specifically, it is noted that

\[
\mathcal{L}_{DR} = \frac{1}{|\mathcal{D}|} \sum_{(u,i) \in \mathcal{D}} \left[ o_{u,i} e_{u,i} + (1 - o_{u,i}) \hat{e}_{u,i} \right] + \frac{1}{|\mathcal{D}|} \sum_{(u,i) \in \mathcal{D}} o_{u,i} (e_{u,i} - \hat{e}_{u,i}) \frac{1 - \hat{p}_{u,i}}{\hat{p}_{u,i}},
\]

which indicates that the correction term uses propensity score to estimate how much \( \mathcal{L}_{EIB} \) overestimates or underestimates \( \mathcal{L}_{DR} \) and then subtracts it. As a compromise, the correction term will increase the variance of the DR estimator according to Theorem 1. Thus, if \( \hat{e}_{u,i} \) is computed in a manner that ensures that

\[
\frac{1}{|\mathcal{D}|} \sum_{(u,i) \in \mathcal{D}} o_{u,i} (e_{u,i} - \hat{e}_{u,i}) \frac{1 - \hat{p}_{u,i}}{\hat{p}_{u,i}} = 0,
\]

then the EIB would have small bias and the DR would have small variance.

One concern is that the constraint [3] may degrade the accuracy of \( \hat{e}_{u,i} \). We propose using TMLE [34][35], which provides a general solution to this problem, to obtain a \( \hat{e}_{u,i} \) that satisfies equation [2], without sacrificing the accuracy of error imputation model. Without loss of generality, assume the error imputation model can be presented as

\[
\hat{e}_{u,i} = \varphi \left( h(x_{u,i}) \right),
\]

where \( h \) is an arbitrary function, \( \varphi(\cdot) \) is a known function, such as identity, sigmoid, or exponential functions. The basic idea of TMLE consists of the following two steps.

**Step 1 (Initialization).** Pre-train an initial estimate of \( g_{u,i} \), denoted as \( \hat{g}_{u,i}^{(0)} = \varphi \left( \hat{h}^{(0)}(x_{u,i}) \right) \).

**Step 2 (Targeting).** Update \( \hat{e}_{u,i}^{(0)} \) by fitting an extended one-parameter model as follows:

\[
\hat{e}_{u,i}^{(new)}(\eta) = \varphi \left( \hat{h}^{(0)}(x_{u,i}) + \eta \left( 1/\hat{p}_{u,i} - 1 \right) \right)
\]

which includes a single variable \( 1/\hat{p}_{u,i} - 1 \) and the offset \( \hat{h}^{(0)}(x_{u,i}) \). The parameter \( \eta \) is solved by minimizing the difference (e.g., squared loss and cross entropy loss) between \( \hat{e}_{u,i}^{(new)}(\eta) \) and \( e_{u,i} \).
in the exposed events $\mathcal{O} = \{(u, i) \in D : o_{u,i} = 1\}$, more details for estimating $\eta$ are provided in Appendix B. Then the proposed DR-TMLE estimator is given as

$$L_{\text{DR-TMLE}} = |D|^{-1} \sum_{(u,i) \in D} \left[ \hat{e}_{u,i}^{\text{new}} + o_{u,i} (\hat{e}_{u,i} - e_{u,i}^{\text{new}}) / \hat{p}_{u,i} \right].$$

Instead of directly estimating $\hat{e}_{u,i}$ satisfying the constraint (2), the targeting step updates the imputation model by adding an error correction term $1/\hat{p}_{u,i} - 1$ to approximate $e_{u,i}$ better and hence does not sacrifice the accuracy of the imputation model. In addition, importantly, the TMLE provides a model-agnostic framework due to the free choice of the initial imputed errors in Step 1. Thus, the TMLE can be assembled into any DR estimator by updating their imputation model with TMLE. The following Theorem 2 shows the validity of TMLE (see Appendix B for the proof).

**Theorem 2.** The imputed error $\hat{e}_{u,i}^{\text{new}}$ obtained with TMLE satisfies the following properties:

(a) (validity) $\hat{e}_{u,i}^{\text{new}}$ satisfies equation (2), which implies DR-TMLE would have smaller bias than EIB and smaller variance than DR based on the initial imputed error $\hat{e}_{u,i}^{(0)}$.

(b) (preservation) $\hat{e}_{u,i}$ in the targeting step will converge to 0 and renders $\hat{e}_{u,i}^{\text{new}} = \hat{e}_{u,i}^{(0)}$ if the validity of $\hat{e}_{u,i}^{(0)}$ already holds.

TMLE guarantees that equation (2) always holds, regardless of the choice of the initial imputed errors. In addition, it indicates that DR-TMLE meets the boundedness property and is robust to small propensities, since equation (2) implies that the DR-TMLE estimator is also an EIB estimator. Next, we present more desired properties of DR-TMLE estimator. Define $L_{EIB}$ and $L_{DR}$ as the EIB and DR estimators with $e_{u,i}^{(0)}$, respectively.

**Theorem 3.** The proposed DR-TMLE estimator have the following properties:

(a) (unbiasedness under accurate imputed errors) $L_{\text{DR-TMLE}}$ is unbiased if $e_{u,i}^{\text{new}}$ accurately estimates $p_{u,i}$.

(b) (unbiasedness under learned propensities) Suppose that $\hat{p}_{u,i}$ is an accurate estimate of $p_{u,i}$ and the validity of $\hat{e}_{u,i}^{(0)}$ doesn’t hold, then $L_{EIB}^{(0)}$ is biased, while $L_{\text{DR-TMLE}}$ is unbiased.

The proof of Theorem 3 is given in Appendix C. Theorem 3 a)-(b) indicates the double robustness of the DR-TMLE estimator. Besides, Theorem 3 (b) suggests that TMLE could remove the bias of $L_{EIB}^{(0)}$ even though the initial imputed errors are inaccurate, provided the learned propensities are accurate.

DR-TMLE would reduce the variance of DR as shown in Theorem 2, a further question is whether the variance-reduction will come at the expense of an increase in bias? Remarkably, DR-TMLE has no sacrifice of bias. It can be shown (see Appendix C) that the bias of both $L_{EIB}^{(0)}$ and $L_{\text{DR-TMLE}}$ are composed of the product of the errors of the propensity model and imputation model weighted by $1/\hat{p}_{u,i}$. Therefore, given the same learned propensities, the more accurate the imputed errors are, the smaller the bias is. Since DR-TMLE updates $\hat{e}_{u,i}^{(0)}$ by adding an extra term $1/\hat{p}_{u,i} - 1$, so $e_{u,i}^{\text{new}}$ is expected to be more accurate than $e_{u,i}^{(0)}$, resulting in a smaller bias for $L_{\text{DR-TMLE}}$ than $L_{DR}^{(0)}$.

### 4 Collaborative Targeted Learning Approach

In this section, we propose a novel TMLE-based collaborative targeted learning approach (TMLE-TL), in which the imputed errors are decomposed into a parametric imputation model part and a nonparametric TMLE error part obtained using the targeting step in Section 3.2. The latter corrects the residuals of the imputation model. By updating both collaboratively, the bias and variance of the DR-TMLE estimator are further reduced, resulting in more accurate predictions.

Specifically, the embedding of each user $u$ and item $i$ is first obtained by matrix factorization, and the stack layer gets the embedding $x_{u,i}$ by concatenation. TMLE-based learning methods require estimated propensities for all user-item pairs, thus the Naive Bayes approach is no longer applicable. To handle this problem, the pre-trained propensities are obtained by conducting logistic regression of $o_{u,i}$ on $x_{u,i}$, and the model parameters are used as the initialization of $p(x_{u,i})$ in the iterative learning process. Let $\varphi$ be an identify function corresponding to the least squared loss and denote $\eta(1/\hat{p}_{u,i} - 1)$ as $e_{u,i}^{\text{new}}$ for simplification. Given an error imputation model $g_{\phi}(x_{u,i})$, the propensity
model $p_\xi(x_{u,i})$ and the prediction model $f_\phi(x_{u,i})$ are updated simultaneously using the training loss
\[ L_{ctmle}(\theta, \xi, \phi) = L_{DR-TMLE} + |D|^{-1} \sum_{(u,i) \in D} \left[ -o_{u,i} \cdot \log \hat{p}_{u,i} - (1 - o_{u,i}) \cdot \log (1 - \hat{p}_{u,i}) \right], \]
where $\hat{p}_{u,i} = p_\xi(x_{u,i}), e_{u,i} = (f_\phi(x_{u,i}) - r_{u,i}(1))^2, \hat{e}_{u,i} = (f_\phi(x_{u,i}) - g_\phi(x_{u,i}))^2 + (f_\phi(x_{u,i}))^2$ with $\perp$ the operator that sets the gradient of the operand to zero thus $\nabla_\theta (f_\phi(x_{u,i})) = 0$ and $\nabla_\phi (f_\phi(x_{u,i})) = f_\phi(x_{u,i})$. Then, unlike traditional alternative learning algorithms that directly use a parametric model $g_\phi(x_{u,i})$ as $\hat{e}_{u,i}$, the proposed collaborative targeted learning additionally uses $\hat{e}_{u,i}^{TMLE}$ as a non-parametric correction term summed with $g_\phi(x_{u,i})$ to estimate $e_{u,i}$. Given the prediction model and the propensity model, $\hat{e}_{u,i}$ first updates its parametric part $g_\phi(x_{u,i})$ by minimizing the loss
\[ L_\phi(\theta, \xi, \phi) = |D|^{-1} \sum_{(u,i) \in D} o_{u,i} (\hat{e}_{u,i} - e_{u,i})^2 / \hat{p}_{u,i}, \]
where $e_{u,i} = r_{u,i}(1) - f_\phi(x_{u,i}), \hat{e}_{u,i} = g_\phi(x_{u,i}) + \hat{e}_{u,i}^{TMLE}$. Next, the targeting step of TMLE as described in Section 3.2 is applied to further update the imputed errors $\hat{e}_{u,i}$. Through calculating the optimal step size for line search $\epsilon^* = \arg \min \epsilon \sum_{(u,i) \in D} o_{u,i} (e_{u,i}(\theta) - \hat{e}_{u,i}(\phi) - \epsilon (1/\hat{p}_{u,i} - 1))^2$, the TMLE error term $\hat{e}_{u,i}^{TMLE}$ is updated by adding $\epsilon^*(1/\hat{p}_{u,i} - 1)$.

From Theorems 2 and 3 when $g_\phi(x_{u,i})$ is already an accurate estimate of $e_{u,i}$, the introduction of the TMLE error term satisfies no-harm property and the unbiasedness is maintained. However, when $g_\phi(x_{u,i})$ is not an accurate estimate of $e_{u,i}$, the estimation error of $\hat{e}_{u,i}$ can be further reduced by the introduced non-parametric TMLE error term $\hat{e}_{u,i}^{TMLE}$ to make corrections. Regardless of whether $\hat{p}_{u,i}$ accurately estimates $p_{u,i}$, the bias and variance of DR can be reduced by using $g_\phi(x_{u,i})$ and $\hat{e}_{u,i}^{TMLE}$ in a collaborative manner. This idea is the same with boosting, i.e., further modeling for the fitted residuals to control the bias. However, the difference in the proposed collaborative learning approach is that we collaboratively update the parametric term $g_\phi(x_{u,i})$ and the nonparametric term $\hat{e}_{u,i}^{TMLE}$ to perform residual fitting to achieve a better trade-off to estimate $e_{u,i}$. In addition, the proposed method can avoid the high bias caused by the random initialization of the imputation model, since an additional TMLE error correction term is injected as a part of $\hat{e}_{u,i}$ in each batch updating, which also makes the proposed method to be more robust (insensitive to the random initialization of the imputation model) compared to methods such as DR-JL [36] and MRDR-DL [37]. We summarized the proposed doubly robust TMLE targeted learning approach, named DR-TMLE-TL, in Alg. 1.
The standard deviation of the EIB method is significantly lower compared to the IPS and DR methods. However, the proposed DR-TMLE method combines the advantages of the EIB in terms of lower standard deviations than IPS and DR in all settings, reflecting stronger robustness. It can be concluded that the estimation accuracy and robustness of the proposed method are significantly improved compared to the existent methods.

### 5 Semi-synthetic Experiments

In this section, following the previous studies \cite{27,36,22,7}, we aim to answer the following research question (RQ) on the semi-synthetic dataset:

**RQ1.** Do the proposed TMLE estimators in estimating the ideal loss have both the statistical properties of relatively lower bias and variance in the presence of selection bias?

#### 5.1 Experimental Setup

**Dataset and Preprocessing.** MovieLens 100K (ML-100K) is a dataset of 100,000 missing-not-at-random (MNAR) ratings from 943 users and 1,682 movies collected from movie recommendation ratings. Following the standard procedure of previous studies \cite{27,36,22,7}, the data preprocessing procedure and predicted metrics are provided in Appendix D.

**Experimental Details.** For each prediction matrix $\hat{R} = \{\hat{r}_{u,i} : (u, i) \in D\}$, the proposed DR-TMLE is compared with Naive \cite{12}, EIB \cite{8,28}, IPS \cite{24,27}, and DR \cite{36,22} methods. We obtain the propensities by $1/\hat{p}_{u,i} = (1 - \beta)/\hat{p}_{u,i} + \beta/p_e$, where $p_e = |D|^{-1} \sum (u,i) \in D o_{u,i}$, and $\beta$ is randomly sampled from $[0,1]$ to introduce noises. Define $\hat{e}_{u,i} = CE(\sum (u,i) \in D \hat{r}_{u,i} w_{u,i}, \hat{r}_{u,i})$, where $w_{u,i} = (1/\hat{p}_{u,i})/(\sum (u,i) \in D 1/\hat{p}_{u,i})$, $CE$ denotes the cross entropy loss. For EIB and DR, the imputed error is computed as $\hat{e}_{u,i} = \hat{e}_{u,i}$. For DR-TMLE, $\hat{e}_{u,i} = \hat{e}_{u,i} + \epsilon^*(1/\hat{p}_{u,i} - 1)$, where $\epsilon^* = \arg \min_\epsilon \sum (u,i) \in D (\epsilon_{u,i} - \hat{e}_{u,i} - \epsilon(1/\hat{p}_{u,i} - 1))^2$. The performance of the estimators is based on the absolute relative error (RE) of the estimated and ideal loss $RE = |L(\hat{R}) - L(\hat{R})|/|L(\hat{R})|$, where $L(\hat{R})$ denotes the estimator to be compared. RE evaluates the accuracy of the estimated loss, and a smaller RE value indicates a higher estimation accuracy.

#### 5.2 Experiment Results (RQ1)

In Table 2, we report the means and standard deviations of the RE of the five estimators for each predicted matrix over 20 times of sampling. On the one hand, the average RE of the IPS, DR and DR-TMLE methods is significantly lower than that of the Naive method, verifying the validity of causal-based debiasing methods. The proposed DR-TMLE achieves the lowest RE in all settings, attributed to the introduced correction term for estimating $\hat{e}_{u,i}$, that further reduces the bias. The direct application of the EIB method is even worse than the Naive method, attributed to the challenge to make accurate estimates of $\hat{e}_{u,i}$. On the other hand, same as the conclusion of Theorem 1, the standard deviation of the EIB method is significantly lower compared to the IPS and DR methods. However, the proposed DR-TMLE method combines the advantages of the EIB in terms of lower standard deviations than IPS and DR in all settings, reflecting stronger robustness. It can be concluded that the estimation accuracy and robustness of the proposed method are significantly improved compared to the existent methods.

### 6 Real-world Experiments

In this section, we conduct experiments to evaluate the proposed methods on two real-world benchmark datasets containing missing-at-random (MAR) ratings. Throughout, we adopt the uniform
data-free technique to estimate the propensity, which differs from the existing studies (e.g., Naive Bayes). We aim to answer the following RQs:

**RQ2.** How do the proposed methods compare with the state-of-the-art models in terms of debiasing performance in practice?

**RQ3.** How does the design of the learning phase affect the performance of our methods?

**RQ4.** Do our methods stably perform well with different propensity clipping thresholds?

### 6.1 Experimental Setup

**Dataset and preprocessing.** MAR ratings are necessary to evaluate the performance of debiasing methods on real-world datasets. Following previous studies, we take the following two benchmark datasets:

- **Coat Shopping**: has 4,640 MAR and 6,960 MNAR ratings of 290 users to 300 coats.
- **Yahoo! R3**: has 54,000 MAR and 311,704 MNAR ratings of 15,400 users to 1,000 songs.

**Baselines.** We take the widely used Matrix Factorization (MF) as the base model \[12\], and compare the proposed methods with the following baselines: Base Model \[12\], IPS \[24, 27\], SNIPS \[30\], CVIB \[39\], DR \[22\], DR-JL \[36\], DR-TL, MRDR-JL \[7\], MRDR-TL, where DR-TL and MRDR-TL are performed using the proposed Alg. \[1\] but without the TMLE step update (line 10-12), also for comparison purpose. In addition, the proposed TMLE-based methods include DR-TMLE, DR-TMLE-JL, and MRDR-TMLE-JL executed by single-step TMLE, and DR-TMLE-TL and MRDR-TMLE-TL executed by collaborative targeted learning approach as shown in Alg. \[1\]. The real-world experimental protocols and details are provided in Appendix E.

### 6.2 Performance Comparison (RQ2)

In Table \[3\] we report the performance of various debiasing methods using MSE, AUC, NDCG@5, and NDCG@10 as evaluation metrics. The proposed TMLE estimators are implemented by single-step and collaborative targeted learning, respectively, based on DR and MRDR as initialized error imputation models. The proposed TMLE-based methods outperform the baseline methods significantly in all AUCs, NDCG@5, and NDCG@10, attributed to the effectiveness of the introduced TMLE correction term. It is noted that the collaborative version of TMLE achieves optimal performance both within DR and MRDR, which implements the proposed targeting step repeatedly, while the single-step TMLE only implements the targeting step at the final training of the prediction model. This further confirms the effectiveness of the proposed targeting approach.

### 6.3 In-depth Analysis

**Ablation Study (RQ3).** To illustrate the specific reasons for the effectiveness of the TMLE-TL algorithm, we conduct ablation studies on DR-based and MRDR-based methods, respectively. From Fig. \[1\] it is noted that in both DR and MRDR classes, although the TL method without TMLE steps is updating the propensity model in the learning phase, the overall performance is similar to that of the JL method. However, for the proposed TMLE-TL method, there is a significant improvement in the convergence rate and value compared to the TL method. This ablation study confirms that the improvement in the proposed TMLE-TL method originates from the correction term of TMLE and not from introducing additional model parameters for updating.

**Parameter Sensitivity Study (RQ4).** An important fact is that the error correction term in the TMLE estimator is based on a given propensity model. Since propensity estimated based on logistic regression may yield extremely small values, resulting in the instability of the prediction model. We conducted repeated experiments to quantify the sensitivity of the TMLE-TL method to the propensity clipping threshold. From Fig. \[2\] the proposed method outperforms the TL and JL methods in terms of AUC, NDCG@5, and NDCG@10 on all clipping thresholds. The optimal performance is reached when the clipping threshold is equal to 0.15, which is interpreted as achieving the best trade-off between information utilization and robustness.

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4https://www.cs.cornell.edu/~schnabts/mnar/
5http://webscope.sandbox.yahoo.com/
Table 3: MSE, AUC, NDCG@5, and NDCG@10 on the MAR test set of COAT and YAHOO. We bold the outperforming DR-based and MRDR-based models. The proposed TMLE methods implemented by single-step are marked with * and collaborative targeted learning are marked with †.

|       | COAT |       | YAHOO |       | YAHOO |
|-------|------|-------|-------|-------|-------|
|       | MSE  | AUC   | NDCG@5| MSE   | AUC   | NDCG@5|
|       |      |       |       |       |       |       |
| Base Model | 0.2448 | 0.7047 | 0.5912 | 0.6667 | 0.6795 | 0.6353 | 0.7644  
| + IPS  | 0.2389 | 0.7041 | 0.6170 | 0.6852 | 0.2496 | 0.6824 | 0.6409 | 0.7674  
| + SNIPS | 0.2388 | 0.7061 | 0.6145 | 0.6945 | 0.2610 | 0.6815 | 0.6454 | 0.7701  
| + CVIB | 0.2201 | 0.7234 | 0.6221 | 0.6991 | 0.2638 | 0.6823 | 0.6483 | 0.7719  
|       | + DR  | 0.2359 | 0.7031 | 0.6213 | 0.6967 | 0.2420 | 0.6867 | 0.6613 | 0.7791  
|       | + DR-JL | 0.2352 | 0.7155 | 0.6183 | 0.6925 | 0.2496 | 0.6853 | 0.6536 | 0.7738  
|       | + DR-TL | 0.2538 | 0.7183 | 0.6261 | 0.6927 | 0.2494 | 0.6808 | 0.6334 | 0.7622  
|       | + DR-TMLE * | 0.2268 | 0.7109 | 0.6300 | 0.7006 | 0.2115 | 0.7044 | 0.7008 | 0.8016  
|       | + DR-TMLE-JL * | 0.2151 | 0.7236 | 0.6388 | 0.7047 | 0.2577 | 0.7036 | 0.6786 | 0.7884  
|       | + DR-TMLE-TL † | 0.2119 | 0.7339 | 0.6526 | 0.7112 | 0.2472 | 0.7057 | 0.6758 | 0.7871  
|       | + MRDR-JL | 0.2162 | 0.7192 | 0.6360 | 0.7016 | 0.2496 | 0.6842 | 0.6487 | 0.7717  
|       | + MRDR-TL | 0.2155 | 0.7200 | 0.6427 | 0.7047 | 0.2494 | 0.6805 | 0.6345 | 0.7623  
|       | + MRDR-TMLE-JL * | 0.2114 | 0.7278 | 0.6498 | 0.7101 | 0.2557 | 0.7036 | 0.6785 | 0.7884  
|       | + MRDR-TMLE-TL † | 0.2114 | 0.7316 | 0.6428 | 0.7088 | 0.2473 | 0.7060 | 0.6803 | 0.7902  

Figure 1: Comparisons of MSE (top) and AUC (bottom) on TMLE-TL, TL and JL, with DR (left) and MRDR (right) as the imputation model, where TL skips the targeting steps in TMLE-TL.

7 Conclusion

In this paper, we propose a TMLE estimator for debiased recommendation that enjoys the properties of double robustness, boundedness, low variance, and robustness to small propensities simultaneously. We theoretically prove the validity of the targeting step, irrespective of the accuracy of the estimated propensities and imputed error initialization. In addition, we further propose a novel uniform-data-free TMLE collaborative targeted learning approach that adaptively executes the targeting step of TMLE, thus making the prediction model more robust. We conducted experiments on both semi-synthetic and real-world data. The superiority of the proposed method is demonstrated when compared with existing debiasing methods. Throughout, we adopt $1/\hat{p}_{u,i} - 1$ as a key choice of targeting step for TMLE to satisfy equation (2), which is essentially a first-order TMLE \[3\]. In future work, we will explore higher-order TMLE and more effective feature selection in the targeting step.

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Figure 2: Learning performance on MAR test set of AUC (left), NDCG@5 (middle), and NDCG@10 (right) with varying levels of propensity clipping threshold.

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A Proof of Theorem 1

Recall that \( p_{u,i} = \mathbb{P}(o_{a,i} = 1|x_{u,i}) = \mathbb{E}[o_{a,i}|x_{u,i}] \) and \( g_{u,i} = \mathbb{E}[e_{u,i}|x_{u,i}] \), both of them are functions of \( x_{u,i} \). Throughout, we maintain the common unconfoundedness assumption (i.e. \( r_{u,i}(1) \perp \perp o_{a,i} | x_{u,i} \)) and the consistency assumption, (i.e., \( r_{u,i}(1) = r_{u,i} \) if \( o_{a,i} = 1 \)). All the lower-case letters denote random variables for simplification.

**Proof of Theorem 1.** The property of unbiasedness is obvious. Next, we focus on analysing the variance. Define

\[
\sigma^2(x_{u,i}) = \mathbb{V}(e_{u,i}|x_{u,i}) = \mathbb{E}[(e_{u,i} - g_{u,i})^2 | x_{u,i}],
\]

then \( \mathbb{E}[e_{u,i}^2|x_{u,i}] = \sigma^2(x_{u,i}) + g_{u,i}^2 \). The variance of IPS estimator is given by

\[
\mathbb{V}(\mathcal{L}_{IPS}) = |D|^{-1} \cdot \mathbb{V}(\frac{g_{u,i} - \mathbb{E}[o_{a,i}|x_{u,i}] g_{u,i}}{p_{u,i}})
\]

\[
= |D|^{-1} \cdot \left[ \mathbb{E}\left[\frac{\sigma^2(x_{u,i})}{p_{u,i}}\right] - \left\{ \mathbb{E}\left(\frac{\sigma(x_{u,i})}{p_{u,i}}\right) \right\}^2 \right]
\]

\[
= |D|^{-1} \cdot \left[ \mathbb{E}\left[\frac{\sigma^2(x_{u,i})}{p_{u,i}}\right] - \left\{ \mathbb{E}(\sigma(x_{u,i})) \right\}^2 \right]
\]

\[
= |D|^{-1} \cdot \left[ \mathbb{E}\left[\sigma^2(x_{u,i}) + g_{u,i}^2\right] - \left\{ \mathbb{E}(\sigma(x_{u,i})) \right\}^2 \right],
\]

where the third equation follows by the law of iterated expectations and the unconfoundedness assumption. The variance of DR estimator is derived by

\[
|D| \cdot \mathbb{V}(\mathcal{L}_{DR}) = \mathbb{V}\left(\frac{g_{u,i} - \mathbb{E}[o_{a,i}|x_{u,i}] g_{u,i}}{p_{u,i}}\right)
\]

\[
= \mathbb{V}\left(\frac{o_{a,i} - p_{u,i}}{p_{u,i}} (e_{u,i} - g_{u,i})\right)
\]

\[
= \mathbb{V}(e_{u,i}) + \mathbb{V}\left(\frac{o_{a,i} - p_{u,i}}{p_{u,i}} (e_{u,i} - g_{u,i})\right)
\]

\[
= \mathbb{E}[\sigma^2(x_{u,i}) + g_{u,i}^2] - [\mathbb{E}(\sigma(x_{u,i}))]^2 + \mathbb{V}\left(\frac{o_{a,i} - p_{u,i}}{p_{u,i}} (e_{u,i} - g_{u,i})\right)
\]

\[
= \mathbb{E}[\sigma^2(x_{u,i}) + g_{u,i}^2] - [\mathbb{E}(\sigma(x_{u,i}))]^2 + \mathbb{E}\left(\frac{(o_{a,i} - p_{u,i})^2}{p_{u,i}^2} (e_{u,i} - g_{u,i})^2\right)
\]

\[
= \mathbb{E}[\sigma^2(x_{u,i}) + g_{u,i}^2] - [\mathbb{E}(\sigma(x_{u,i}))]^2 + \mathbb{E}\left(\frac{(o_{a,i} - p_{u,i})^2}{p_{u,i}^2} (e_{u,i} - g_{u,i})^2\right) \mathbb{E}\{x_{u,i}\}
\]

\[
= \mathbb{E}[\sigma^2(x_{u,i}) + g_{u,i}^2] - [\mathbb{E}(\sigma(x_{u,i}))]^2 + \mathbb{E}\left(\frac{p_{u,i}(1- p_{u,i}) \sigma^2(x_{u,i})}{p_{u,i}^2}\right)
\]

\[
= \mathbb{E}\left[\frac{\sigma^2(x_{u,i})}{p_{u,i}} + g_{u,i}^2\right] - [\mathbb{E}(\sigma(x_{u,i}))]^2,
\]

where the fifth equation holds by noting that

\[
\mathbb{E}\left[\frac{o_{a,i} - p_{u,i}}{p_{u,i}} (e_{u,i} - g_{u,i})\right] = \mathbb{E}\left[\frac{(o_{a,i} - p_{u,i}) x_{u,i}}{p_{u,i}} \cdot \mathbb{E}(e_{u,i} | x_{u,i})\right] = 0.
\]
B Details of the targeting step in TMLE and the proof of Theorem 2

This section provides more details about the estimation of \( \eta \) in the targeting step of TMLE. Without loss of generality, we assume error imputation can be presented as

\[
\mathbb{E}[e_{u,i}|x_{u,i}] = \varphi(h_\phi(x_{u,i})) \tag{5}
\]

where \( h(\cdot) \) is an arbitrary function, \( \varphi(\cdot) \) is the inverse canonical link function. The implication of "canonical" means that the density function of \( e_{u,i} \) given \( x_{u,i} \) has the following form (similar to generalized linear model)

\[
f(e_{u,i} | x_{u,i}, \phi, \psi) = c(e_{u,i}, \psi) \exp\left(\frac{e_{u,i} \cdot h_\phi(x_{u,i}) - b(h_\phi(x_{u,i}))}{\psi}\right),
\]

where \( \psi \) is the scale parameter (a nuisance parameter), \( \phi \) is the parameter of interest. Different density functions have different functions of \( b(\cdot) \) and \( c(\cdot) \). For example,

- **Gaussian distribution:** \( \varphi(t) = t \),

\[
f(e_{u,i} | x_{u,i}, \phi, \psi) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(e_{u,i} - h_\phi(x_{u,i}))^2}{2\sigma^2}\right)
\]

\[
= \exp(-e_{u,i}^2/(2\sigma^2)) \cdot \exp\left(\frac{2e_{u,i}h_\phi(x_{u,i}) - \{h_\phi(x_{u,i})\}^2}{2\sigma^2}\right).
\]

Thus, \( \psi = \sigma^2 \), \( b(t) = t^2/2 \),

\[
c(t, \psi) = \frac{\exp(-t^2/(2\sigma^2))}{\sqrt{2\pi}\sigma}.
\]

Then the maximum likelihood estimation of \( \phi \) is equivalent to the least squares estimation of \( \phi \).

- **Binomial distribution:** \( \varphi(t) = \exp(t)/(1 + \exp(t)) \),

\[
f(e_{u,i} | x_{u,i}, \phi, \psi) = \left(\frac{\exp(h_\phi(x_{u,i}))}{1 + \exp(h_\phi(x_{u,i}))}\right)^{e_{u,i}} \cdot \left(\frac{1}{1 + \exp(h_\phi(x_{u,i}))}\right)^{1-e_{u,i}}
\]

\[
= \exp(e_{u,i} \cdot h_\phi(x_{u,i})) \exp(-\log(1 + \exp(h_\phi(x_{u,i}))))
\]

\[
= \exp\{e_{u,i} \cdot h_\phi(x_{u,i}) - \log(1 + \exp(h_\phi(x_{u,i}))\}.
\]

Thus, \( \psi = 1 \), \( b(t) = \log(1 + \exp(t)) \), \( c(t, \psi) = 1 \). Then the maximum likelihood estimation of \( \phi \) is equivalent to the estimation of \( \phi \) trained with cross entropy loss.

**Lemma 1.** For model \( \text{G} \), the score function (the first derivative of log likelihood function) of \( \phi \) based on the observed events is given as

\[
\sum_{(u,i) \in \mathcal{D}} o_{u,i} \left\{ e_{u,i} - \varphi(h_\phi(x_{u,i})) \right\} \frac{\partial h_\phi(x_{u,i})}{\partial \phi} = 0. \tag{6}
\]
Similarly, for model (4), the score function of $\eta$ based on the observed events is given as
\[
\sum_{(u,i) \in D} o_{u,i} \cdot \left\{ e_{u,i} - \varphi\{ \hat{h}^{(0)}(x_{u,i}) + \eta\left( \frac{1}{\hat{p}_{u,i}} - 1 \right) \} \right\} \cdot (1/\hat{p}_{u,i} - 1) = 0. \tag{7}
\]

**Proof.** It suffices to show equation (6). By taking the first-derivative with respect to $h_\phi(x_{u,i})$ of the following equation
\[
\int f(e_{u,i} | x_{u,i}, \phi, \psi) de_{u,i} = 1,
\]
it follows that
\[
E[e_{u,i} | x_{u,i}] = \left. \frac{\partial b(t)}{\partial t} \right|_{t=h_\phi(x_{u,i})} := b'(h_\phi(x_{u,i})).
\]
The likelihood function is given as
\[
L(\phi, \psi) = \prod_{(u,i) \in O} f(e_{u,i} | x_{u,i}, \phi, \psi) = \exp \left( \sum_{(u,i) \in O} \frac{e_{u,i} \cdot h_\theta(x_{u,i}) - b(h_\theta(x_{u,i}))}{\psi} \right) \cdot \prod_{(u,i) \in O} c(e_{u,i}, \psi)
\]
Letting $\partial \log L(\phi, \psi)/\partial \phi = 0$ yields that
\[
\sum_{(u,i) \in O} \{ e_{u,i} - b'(h_\phi(x_{u,i})) \} \frac{\partial h_\phi(x_{u,i})}{\partial \phi} = 0,
\]
which implies the equation (6).

**Proof of Theorem 2.** According to Lemma 1, taking the first derivative of the loss function of $\eta$ leads to that
\[
\sum_{(u,i) \in D} o_{u,i} \cdot \left\{ e_{u,i} - \varphi\{ \hat{h}^{(0)}(x_{u,i}) + \eta\left( \frac{1}{\hat{p}_{u,i}} - 1 \right) \} \right\} \cdot (1/\hat{p}_{u,i} - 1) = 0,
\]
which implies that
\[
\sum_{(u,i) \in D} o_{u,i} \cdot \left\{ e_{u,i} - \hat{e}^{new}_{u,i} \right\} \cdot (1/\hat{p}_{u,i} - 1) = 0,
\]
and implies the equation (2) holds. This finishes the proof of Theorem 2(a). If $\hat{e}^{(0)}_{u,i}$ already satisfies equation 2, then $\eta = 0$ is a solution of equation (7). Let $\hat{\eta}$ is another solution of equation (7). Since the solution of equation (7) is unique, then $\hat{\eta}$ will converges to 0. This proves the conclusion of Theorem 2(b).

**C Proof of Theorem 3**

**Proof of Theorem 3.** The result of Theorem 3(a) is obvious. To show Theorem 3(b). We first claim that if $\hat{e}^{(0)}_{u,i}$ is an accurate estimate of $g_{u,i}$, i.e., $\hat{e}^{(0)}_{u,i} = g_{u,i}$, then it will satisfy equation (2). It holds immediately from the following calculations
\[
\frac{1}{|D|} \sum_{(u,i) \in D} o_{u,i} \{ e_{u,i} - \hat{e}^{(0)}_{u,i} \} \cdot (1/\hat{p}_{u,i} - 1)
\]
\[
= \frac{1}{|D|} \sum_{(u,i) \in D} o_{u,i} \{ e_{u,i} - g_{u,i} \} \cdot (1/\hat{p}_{u,i} - 1)
\]
\[
= E\left[o_{u,i} \{ e_{u,i} - g_{u,i} \} \cdot (1/\hat{p}_{u,i} - 1)\right]
\]
\[
= E\left[E(o_{u,i} | x_{u,i}) \cdot E\{ e_{u,i} - g_{u,i} | x_{u,i} \} \cdot (1/\hat{p}_{u,i} - 1)\right]
\]
\[
= 0.
\]
Thus, if $e_{u,i}^{(0)}$ not satisfy equation (2), then $\hat{e}_{u,i}^{(0)} \neq g_{u,i}$. Given $e_{u,i}^{(0)}$ and $\hat{e}_{u,i}^{new}$, the bias of $L_{EIB}^{(0)}$ is

$$\text{Bias}(L_{EIB}^{(0)}) = E[e_{u,i} + (1 - o_{u,i})e_{u,i}^{(0)}] - E[e_{u,i}]$$

$$= E[(1 - o_{u,i})(e_{u,i}^{(0)} - e_{u,i})]$$

$$= E[(1 - p_{u,i})(e_{u,i}^{new} - g_{u,i})],$$

and the bias of $L_{DR-TMLE}$ is

$$\text{Bias}(L_{DR-TMLE}) = E\left(e_{u,i} + \frac{(o_{u,i} - p_{u,i})}{p_{u,i}}(e_{u,i}^{new} - \hat{e}_{u,i}^{new}) - E[e_{u,i}]\right)$$

$$= E\left(E]\frac{o_{u,i} - p_{u,i}}{p_{u,i}}|x_{u,i}\right)E\{e_{u,i}^{new} - \hat{e}_{u,i}^{new}|x_{u,i}\}$$

$$= E\left(\frac{0}{p_{u,i}} \cdot (g_{u,i} - \hat{e}_{u,i}^{new})\right) = 0.$$

This proves the result of Theorem 3(b).

**Biases of DR and DR-TMLE.** Given $\hat{p}_{u,i}$ and $\hat{e}_{u,i}^{new}$ for all $(u, i) \in D$, the bias of DR-TMLE is

$$\text{Bias}(L_{DR-TMLE}) = E\left(e_{u,i} + \frac{(o_{u,i} - \hat{p}_{u,i})}{\hat{p}_{u,i}}(e_{u,i}^{new} - \hat{e}_{u,i}^{new}) - E[e_{u,i}]\right)$$

$$= E\left(E]\frac{o_{u,i} - \hat{p}_{u,i}}{\hat{p}_{u,i}}|x_{u,i}\right)E\{e_{u,i}^{new} - \hat{e}_{u,i}^{new}|x_{u,i}\}$$

$$= E\left(\frac{p_{u,i} - \hat{p}_{u,i}}{\hat{p}_{u,i}} \cdot (g_{u,i} - \hat{e}_{u,i}^{new})\right).$$

Similarly, given $\hat{p}_{u,i}$ and $\hat{e}_{u,i}^{(0)}$ for all $(u, i) \in D$, the bias of DR is

$$\text{Bias}(L_{DR}^{(0)}) = E\left(\frac{p_{u,i} - \hat{p}_{u,i}}{\hat{p}_{u,i}} \cdot (g_{u,i} - \hat{e}_{u,i}^{(0)})\right).$$

### D Semi-synthetic Experimental Setup Details

**Data Preprocessing.** (1) Use matrix factorization [12] to complete the rating matrix, then all ratings are sorted in ascending order and truncated based on a more realistic rating distribution $[p_1, p_2, p_3, p_4, p_5]$ [15], setting the first bottom $p_1$ fraction to $R_{u,i} = 1$, the next $p_2$ fraction to $R_{u,i} = 2$, and so on.

(2) For each predicted ratings $R_{u,i} \in \{1, 2, 3, 4, 5\}$, assign the CTR $p_{u,i} \in (0, 1)$ with $p_{u,i} = po\times\max(1.5 - R_{u,i})$, where $p$ is set to 1 and $\alpha$ is set to 0.5 in our experiments.

(3) Transform the predicted ratings $R_{u,i}$ into true CVR $r_{u,i}^{true}$ by correspondingly replacing the rating $\{1, 2, 3, 4, 5\}$ with the conversion rate $\{0.1, 0.3, 0.5, 0.7, 0.9\}$, and sample the binary click indicator and conversion label with the Bernoulli sampling $o_{u,i} \sim \text{Bern}(p_{u,i}), r_{u,i} \sim \text{Bern}(r_{u,i}^{true}), \forall (u, i) \in D$, where Bern($\cdot$) denotes the Bernoulli distribution. Thereafter, we can derive a fully-observed conversion label matrix $R$ and a click indicator matrix $O$.

**Predicted Metrics.** The following prediction metrics are used to evaluate the debiasing performance under different scenarios.

- **ONE:** $\hat{r}_{u,i}$ is identical to the true CVR $r_{u,i}^{true}$, except that $|\{(u, i) | r_{u,i}^{true} = 0.9\}|$ randomly selected true CVR of 0.1 are flipped to 0.9.
- **THREE:** Same as ONE, but flipping true CVR of 0.3 instead.
• **FIVE**: Same as ONE, but flipping true CVR of 0.5 instead.

• **ROTATE**: \( \hat{r}_{u,i} = r_{u,i} - 0.2 \) when \( r_{u,i} \geq 0.3 \), and \( \hat{r}_{u,i} = 0.9 \) when \( r_{u,i} = 0.1 \).

• **SKEW**: \( \hat{r}_{u,i} \) is sampled from the Gaussian distribution \( \mathcal{N} \left( \mu = r_{u,i}^{true} , \sigma = (1 - r_{u,i}^{true})/2 \right) \), and clipped to \([0.1, 0.9]\).

• **CRS**: \( \hat{r}_{u,i} = 0.2 \) if the true CVR \( r_{u,i}^{true} \leq 0.6 \). Otherwise, \( \hat{r}_{u,i} = 0.6 \).

### E Real-world Experimental Protocols and Details.

**Experimental protocols and details.** For real-world experiments, the following four metrics were considered as the evaluation metrics: \( MSE, AUC, NDCG@5, \) and \( NDCG@10 \). The embedding size of both users and items is set to 4. For fast convergence in the learning phase, Adam is utilized as the optimizer for all models. We tune the learning rate in \([0.001, 0.005, 0.01, 0.05, 0.1] \), weight decay in \([1e^{-6}, 1e^{-2}] \) at 10x multiplicative ratio, and batch size in \( \{128, 256, 512, 1024, 2048\} \) for Coat and \( \{1024, 2048, 4096, 8192, 16384\} \) for Yahoo! R3. Specifically for the propensity training, we tune the clipping threshold in \( \{0.05, 0.10, 0.15, 0.20\} \). After finding out the best configuration on the validation set, we evaluate the trained models on the MAR test set. Experiments are conducted using NVIDIA GeForce RTX 2060 as the compute resources.