Revisiting Vector-like Quark Model with Enhanced Top Yukawa Coupling

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Abstract

We revisit a scenario with an enhanced top yukawa coupling in vector-like quark (VLQ) models, where the top yukawa coupling is larger than the standard model value and the lightest VLQ has a negative yukawa coupling. We find that the parameter space satisfying the LHC bounds of the Higgs signal strengths consistently with the precision measurements is rather wide. Because the Lagrangian parameters of the yukawa couplings are large, such scenario can be realized in some strongly interacting theories. It also turns out that there is a noticeable relation between the contributions of the triangle and box diagrams in the $gg \to hh$ process by using the lowest order of the $1/M$ expansion where $M$ is the heavy mass running in the loops.

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I. INTRODUCTION

The LHC experiments have discovered a Higgs boson and revealed that its properties are similar to that of the Standard Model (SM) \cite{1}. It is thus essential to explore any signatures of physics beyond the SM (BSM). One of the hints is that the observed signal strength of $pp \rightarrow t\bar{t}h$ channel is deviated from the SM value about twice, $\mu_{tth} = 2.3^{+0.7}_{-0.6}$ for the Run 1 combined data \cite{1}, $\mu_{tth} = 1.8^{+0.7}_{-0.7}$ for the ATLAS Run 2 \cite{2}, and $\mu_{tth} = 1.5^{+0.5}_{-0.5}$ for the CMS Run 2 in the multilepton final states\cite{3}, although the uncertainties are still large.

These experiments provide a reason for considering models based on strongly interacting theories. In this direction\cite{2}, widely studied are vector-like quark (VLQ) models \cite{9–19}, the minimal composite Higgs models (MCHMs) \cite{20, 21}, and the Little Higgs models \cite{22, 23}. We easily find, however, the top yukawa coupling is always suppressed in the VLQ model having only one up-type quark \cite{13}. For example, introducing the VLQ $U_{L,R}$ having $+2/3$ electric charge as in the top-seesaw model \cite{24}, the top yukawa coupling is modified as $c_L^2 g_{tth}^{SM}$, where $c_L \equiv \cos \theta_L$ represents the cosine of the mixing angle between $t_L$ and $U_L$, and we defined the SM top Yukawa coupling by $g_{tth}^{SM} = m_t/v$ with $m_t$ and $v$ being the top mass and the vacuum expectation value (VEV) of the Higgs field, respectively. It is the case\cite{3} for the MCHMs such as MCHM4, MCHM5, and MCHM10 where the fermions are embedded in the spinorial, 5, and 10 representations of $SO(5)$, respectively \cite{25, 26}. Nevertheless one should not jump to a conclusion: A simple model is effective for a benchmark, but it might be misguided if simplified too much.

In this paper, we reconsider a scenario that the top yukawa coupling is larger than the SM value by $\mathcal{O}(10\%)$, and the lightest VLQ with the mass around 1 TeV has a negative yukawa coupling of the order of $-m_t/v$, introducing more than one up-type VLQ \cite{16, 17, 28}. In our scenario, owing to the cancellation among the yukawa couplings, the Higgs signal strengths can be consistent with the experiments. A similar analysis\cite{4} was performed in Ref. \cite{17}. 

1 A combined result of the CMS Run 2 has not yet been reported. For other decay channels, $\mu_{tth} = 1.91^{+1.5}_{-1.2}$ in the $h \rightarrow \gamma\gamma$ decay channel \cite{4}, $\mu_{tth} = -0.19^{+0.80}_{-0.81}$ in the $h \rightarrow b\bar{b}$ decay channel \cite{5}, and $\mu_{tth} = 0.00^{+1.19}_{-0.60}$ in the $h \rightarrow ZZ \rightarrow 4\ell$ channel \cite{6}.

2 Although the top condensate model \cite{7} and the chiral fourth generation \cite{8} directly predict large yukawa couplings, they had been severely constrained.

3 Quite recently, it is shown that the MCHMs with the fermions of the $5 + 10$ or $14$ representations can have the enhanced or suppressed $tth$ coupling \cite{27}.

4 In the framework of the two Higgs doublet model, the cancellation mechanism via the light stop was
|                  | $SU(3)_c$ | $SU(2)_W$ | $U(1)_Y$ |
|------------------|-----------|-----------|-----------|
| $q_L = (t, b)_L$ | 3         | 2         | $\frac{1}{6}$ |
| $t_R$           | 3         | 1         | $\frac{2}{3}$ |
| $b_R$           | 3         | 1         | $-\frac{1}{3}$ |
| $Q_{L,R} = (X, T)_{L,R}$ | 3         | 2         | $\frac{7}{6}$ |
| $U_{L,R}$       | 3         | 1         | $\frac{2}{3}$ |

**TABLE I: Charge assignment of the VLQ model.**

Although the allowed region looked narrow in Ref. [17], we find that our scenario is possible in a rather wide parameter space.

We numerically show that our scenario is realized, roughly speaking, when the Lagrangian parameters $y_{ij}$ of the yukawa interactions are large, say, $|y_{ij}| \gtrsim 2$. This may suggest the existence of the underlying strongly interacting models where the dynamically generated yukawa couplings are typically around $3 \sim 5$ [24]. As for the di-Higgs production process $gg \rightarrow hh$ [32–39], we find a noticeable relation between the contributions of the triangle and the box diagrams in the lowest order of the $1/M$ expansion, where $M$ is the relevant heavy mass running in the loops. The di-Higgs production process may give information on the off-diagonal yukawa couplings.

The paper is organized as follows: In Sec. II we introduce the VLQ model. In Sec. III we first describe the existence proof of our scenario in an analytical approach, and next show a numerical calculation. Sec. IV is devoted to summary. In Appendix A, the oblique parameters [40] in our model are presented. Analytical expressions of the triangle and the box contributions to the $gg \rightarrow hh$ process in the lowest order of the $1/M$ expansion are given in Appendix B.

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considered in Ref. [29, 30]. See also Ref. [31].
II. VECTOR-LIKE QUARK MODEL

Let us introduce two types of the VLQ’s, \(U_{L,R}\) and \(Q_{L,R}\) = \((X,T)_{L,R}\) having the hypercharges \(\frac{2}{3}\) and \(\frac{7}{6}\), respectively. (See also Table II) Because of no mixing between the bottom quark and VLQ’s, the flavor constraints such as \(Z \to b\bar{b}\), etc. can be suppressed in this model. Assuming one Higgs doublet model, the mass terms and the yukawa interactions are

\[
\mathcal{L}_Y = -y_{13}q_L \bar{H} U_R - y_{21} Q_L H t_R - y_{23} \bar{Q}_L H U_R - y_{32} \bar{U}_L H^\dagger Q_R + (h.c),
\]

\[
\mathcal{L}_{VM} = -m_{22} \bar{Q}_L Q_R - m_{33} \bar{U}_L U_R - m_{31} \bar{U}_L t_R + (h.c),
\]

with \(q_L = (t,b)_L\), and \(\bar{H} \equiv i\tau_2 H^*\). The SM term of \(y_{11}q_L \bar{H} t_R\) was rotated away via the \(t_R - U_R\) mixing like in the top seesaw model \[24\], while \(m_{31}\) is removed in literature \[16, 17\].

We here abbreviated the SM part such as the light quark sector, gauge kinetic terms, etc.

After the electroweak symmetry breaking (EWSB), the mass matrix is then

\[
\mathcal{L}_M = -(\bar{t}_L \bar{T}_L \bar{U}_L) \mathcal{M} \begin{pmatrix} t_R \\ T_R \\ U_R \end{pmatrix} - m_{22} \bar{X}_L X_R
\]

with \(\langle H \rangle = \left(0, \frac{v}{\sqrt{2}}\right)^T\), \(v = 246\) GeV, and

\[
\mathcal{M} \equiv \frac{v}{\sqrt{2}} Y \oplus M = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 & 0 & y_{13} \\ y_{21} & 0 & y_{23} \\ 0 & y_{32} & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{22} & 0 \\ m_{31} & 0 & m_{33} \end{pmatrix},
\]

where the mass of the \(X\) quark with \(+5/3\) electric charge is not affected by the EWSB, i.e., \(M_X = m_{22}\). We diagonalize \(\mathcal{M}\) by

\[
\mathcal{M}_{\text{diag}} = V_L^\dagger \mathcal{M} V_R, \quad \mathcal{M}_{\text{diag}} = \text{diag}(m_1, m_2, m_3),
\]

with \(0 \leq m_1 \leq m_2 \leq m_3\), and

\[
\begin{pmatrix} t_{L,R} \\ T_{L,R} \\ U_{L,R} \end{pmatrix} = V_{L,R} \begin{pmatrix} t'_{L,R} \\ T'_{L,R} \\ U'_{L,R} \end{pmatrix},
\]

where \((t, T, U)_{L,R}\) and \((t', T', U')_{L,R}\) represent the gauge and mass eigenstates, respectively. Each up-type quark mass is identified by \(m_t = m_1\), \(M_T = m_2\), and \(M_U = m_3\). The yukawa
coupling matrix $G^h$ in the mass eigenstates is given by

$$\mathcal{L}_Y = -h (\bar{l}'_L \bar{L}_R \bar{U}'_L) G^h \begin{pmatrix} t'_R \\ T'_R \\ U'_R \end{pmatrix},$$  

(7)

with

$$G^h = \frac{1}{\sqrt{2}} V^\dagger_L Y V_R.$$  

(8)

### III. ANALYTICAL AND NUMERICAL STUDIES

#### A. Analytical study with crude approximation

We schematically show our scenario having $G^h_{11} > m_t/v$ and $G^h_{22} < 0$ is possible in an analytical approach. For this purpose, we employ a crude approximation in this subsection. A numerical study without such approximation will be shown in the next subsection.

Let us take the mass matrix as a symmetric one,

$$\mathcal{M} = M_X \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 1 & \xi \\ \epsilon & \xi & a \end{pmatrix},$$  

(9)

where we scaled the mass matrix by $M_X = m_{22}$. For the perturbation theory, the parameters arising from the yukawa couplings should not be so large, i.e., $\epsilon^2, \xi^2 \ll 1$. We also assume $a \geq 1$. We diagonalize $\mathcal{M}$ by the matrices of

$$V_L = (\bar{v}_t \bar{v}_T \bar{v}_U), \quad V_R = (-\bar{v}_t \bar{v}_T \bar{v}_U) = V_L \text{diag}(-1, 1, 1),$$  

(10)

where the first, the second and the third components ($v_{t1}, v_{T2}$ and $v_{U3}$) of $\bar{v}_t, \bar{v}_T$ and $\bar{v}_U$ are order of unity,

$$v_{t1} \equiv \bar{v}_t^{(1)} = \mathcal{O}(1), \quad v_{T2} \equiv \bar{v}_T^{(2)} = \mathcal{O}(1), \quad v_{U3} \equiv \bar{v}_U^{(3)} = \mathcal{O}(1),$$  

(11)

respectively, and then obtain the mass eigenvalues,

$$\frac{m_t}{M_X} \equiv \lambda_1 = \mathcal{O} \left( \frac{\epsilon^2}{a} \right), \quad \frac{M_T}{M_X} \equiv \lambda_2 < 1, \quad \frac{M_U}{M_X} \equiv \lambda_3 > a.$$  

(12)
In general, by taking the trace and the determinant, we find
\[-\lambda_1 + \lambda_2 + \lambda_3 = 1 + a, \quad \lambda_1\lambda_2\lambda_3 = \epsilon^2, \quad -G_{tt}^h + G_{TT}^h + G_{UU}^h = 0, \tag{13}\]
where we defined the diagonal components of the yukawa couplings in the mass basis as
\[G_{11,22,33}^h \equiv G_{tt,TT,UU}^h.\]
More explicitly, the yukawa couplings of \(G_{tt}^h\) and \(G_{TT}^h\) are given by
\[G_{tt}^h = \frac{m_t}{v} \left[ 1 + \frac{v_{t1}^2}{(1 + \lambda_1)\epsilon^2} \left\{ (a\lambda_1 - \epsilon^2)(2 + \lambda_1) + \lambda_1^2 \right\} \right], \tag{14}\]
\[G_{TT}^h = -(1 - \lambda_2) \frac{M_X}{v} \left[ 1 + \frac{(1 - \lambda_2)(a - \lambda_2)}{\xi^2} \right] v_{T2}^2 < 0, \tag{15}\]
and the situation of \(G_{tt}^h > m_t/v\) is realized when
\[\xi^2 > \frac{1 + \lambda_0}{2} \left( \epsilon^2 - (a - 1)\lambda_0 \right), \tag{16}\]
with
\[\lambda_0 \equiv \frac{4\epsilon^2}{2a - \epsilon^2 + \sqrt{(2a - \epsilon^2)^2 + 8\epsilon^2(1 + a)}}. \tag{17}\]
An analytic solution is frequently useful. Let us take \(\xi = \frac{\epsilon}{a} \sqrt{a + \epsilon^2}\), for example. In this case, we find that the eigenvalues are
\[\lambda_1 = \frac{\epsilon^2}{a}, \quad \lambda_2 = 1 - \delta^2, \quad \lambda_3 = a + \frac{\epsilon^2}{a} + \delta^2, \tag{18}\]
with
\[\delta^2 = \frac{2\epsilon^2}{a(a - 1) + \epsilon^2 + \sqrt{(a(a - 1) + \epsilon^2)^2 + 4\epsilon^2}}. \tag{19}\]
and the corresponding eigenvectors are
\[\vec{v}_t = v_{t1} \begin{pmatrix} 1 \\ \frac{\epsilon^2}{a\sqrt{a + \epsilon^2}} \\ -\frac{\epsilon}{a} \end{pmatrix}, \quad \vec{v}_T = v_{T2} \begin{pmatrix} \frac{\lambda_3 \delta^2}{\sqrt{a + \epsilon^2}} \\ -\frac{\epsilon}{a\sqrt{a + \epsilon^2}} \\ 1 \end{pmatrix}, \quad \vec{v}_U = v_{U3} \begin{pmatrix} \frac{\epsilon(1 - \delta^2)}{a} \\ \frac{\epsilon\sqrt{a + \epsilon^2}}{a\lambda_3 - 1} \\ 1 \end{pmatrix}, \tag{20}\]
with
\[v_{t1}^{-1} = \sqrt{1 + \frac{\epsilon^2(a + 2\epsilon^2)}{a^2(a + \epsilon^2)}}, \tag{21}\]
\[v_{T2}^{-1} = \sqrt{1 + \frac{\delta^4(a^2 + \epsilon^2\lambda_3^2)}{\epsilon^2(a + \epsilon^2)}}, \tag{22}\]
\[v_{U3}^{-1} = \sqrt{1 + \frac{\epsilon^2}{a^2} \left( (1 - \delta^2)^2 + \frac{a + \epsilon^2}{(\lambda_3 - 1)^2} \right)}. \tag{23}\]
The following relations might be useful:

\[(1 - \lambda_2)(\lambda_3 - 1) = \delta^2(\lambda_3 - 1) = \frac{\epsilon^2}{a}, \quad \lambda_2\lambda_3 = \lambda_3(1 - \delta^2) = a.\] (24)

Substituting the above results for Eqs. (13)–(15), we explicitly obtain

\[G^h_{tt} = \frac{m_t}{v} \left[ 1 + \frac{ae^2}{a^3 + a(a+1)e^2 + 2e^4} \right] > \frac{m_t}{v}, \tag{25}\]
\[G^h_{TT} = -\frac{M_X}{v} \delta^2 \left( 2 - \frac{a\delta^2 + \epsilon^2}{a + \epsilon^2} \right) v^2 < 0, \tag{26}\]
\[G^h_{UU} = G^h_{tt} - G^h_{TT}. \tag{27}\]

In this way, our scenario can be realized in the framework of the VLQ model. The parameter space should be constrained by the $S$ and $T$-parameters, however.

B. Numerical study without approximation

Without assuming the symmetric mass matrix (9), we now calculate numerically the signal strengths in our model:

\[\mu^V_V \equiv \mu^V_{ggF + ttH} = \frac{(\sigma^F_{gg} + \sigma_{ttH})B^V}{(\sigma^F_{gg} + \sigma_{ttH})_{SM}B^V_{SM}} \simeq \kappa^2, \tag{28}\]
\[\mu^\gamma_\gamma \equiv \mu^\gamma_{ggF + ttH} = \frac{(\sigma^F_{gg} + \sigma_{ttH})B^{\gamma\gamma}}{\sigma^F_{gg} + \sigma_{ttH})_{SM}B^{\gamma\gamma}_{SM}} \simeq \kappa^2 \kappa^2, \tag{29}\]

where $VV = WW$ and $ZZ$, and the scaling factors $\kappa_{g,\gamma,f}$ are defined by

\[\kappa_g = \frac{\kappa A_1(x_i) + \frac{m_t}{MT} \kappa_T A_1(x_T) + \frac{m_t}{MU} \kappa_U A_1(x_U)}{A_1(x_i)}, \tag{30}\]
\[\kappa_\gamma = \frac{A_1(x_W) + \frac{4}{3}(\kappa A_1(x_i) + \frac{m_t}{MT} \kappa_T A_1(x_T) + \frac{m_t}{MU} \kappa_U A_1(x_U))}{A_1(x_W) + \frac{4}{3} A_1(x_i)}, \tag{31}\]
\[\kappa_t = G^h_{tt}/g^SM_{tt}, \quad g^SM_{tt} \equiv \frac{m_t}{v}, \tag{32}\]
\[\kappa_T = G^h_{TT}/g^SM_{TT}, \quad \kappa_U = G^h_{UU}/g^SM_{UU}, \tag{33}\]

with $x_i \equiv m^2_h/(4m^2_t)$. The loop functions for spin 1 and 1/2 are represented by $A_1(x)$ and $A_{1/2}(x)$, respectively [41, 42],

\[A_1(x) = -\frac{1}{x^2} \left[ 2x^2 + 3x + 3(2x - 1)f(x) \right], \tag{34}\]
\[A_{1/2}(x) = \frac{2}{x^2} \left[ x + (x - 1)f(x) \right], \tag{35}\]
with
\[
f(x) \equiv \begin{cases} \arcsin^2 \sqrt{x} & \text{for } x \leq 1, \\ -\frac{1}{4} \left[ \ln \frac{1 + \sqrt{1 - x^{-1}}}{1 - \sqrt{1 - x^{-1}}} - i\pi \right]^2 & \text{for } x > 1. \end{cases}
\] (36)

In our model, the scaling factor \( \kappa_V \) of the \( hWW \) and \( hZZ \) couplings is SM-like, \( \kappa_V = 1 \). Since we do not change the down quark and lepton sectors, the scaling factors of the bottom and tau are also \( \kappa_b = \kappa_\tau = 1 \).

By using the results of the LHC Run 1 via the six-parameter fit shown in Ref. [1],
\[
\mu_{VV}/\mu_F = 1.09^{+0.36}_{-0.28},
\]
(37)
\[
\mu_{\gamma\gamma}^{\gamma\gamma} = 1.10^{+0.23}_{-0.21},
\]
(38)
\[
\mu_{ZZ}^{ZZ} = 1.27^{+0.28}_{-0.24},
\]
(39)
\[
\mu_{WW}^{WW} = 1.06^{+0.21}_{-0.18},
\]
(40)
\[
\mu_{TT}^{TT} = 1.05^{+0.33}_{-0.27},
\]
(41)
\[
\mu_{bb}^{bb} = 0.64^{+0.37}_{-0.28},
\]
(42)
we read the 2\( \sigma \) constraints as
\[
0.79 < \mu_{VV}^F < 1.48, \quad 0.68 < \mu_{\gamma\gamma}^F < 1.56,
\] (43)
because of \( \mu_{WW}^F = \mu_{ZZ}^F \equiv \mu_{VV}^F \) in our model. On the other hand, the best fit values of \( (\sigma \cdot B)^{ZZ}_{ggF} \), \( \sigma_{VV/F}/\sigma_{ggF} \) and \( B^{\gamma\gamma}/B^{ZZ} \) yield \( \mu_{ZZ}^{ggF} = 1.42^{+0.35}_{-0.31} \) and \( \mu_{\gamma\gamma}^{ggF} = 0.67^{+0.25}_{-0.21} \) in the ATLAS Run 2 [43]. The signal strengths in the CMS Run 2 are \( \mu_{ggF}^{ZZ} = 1.20^{+0.22}_{-0.21} \) [6] and \( \mu_{\gamma\gamma}^{ggF} = 0.77^{+0.25}_{-0.23} \) [4]. One should keep in mind that both of the Run 2 results for \( \mu_{\gamma\gamma}^{ggF} \) are much smaller than that of the LHC Run 1.

The parameter space of the mass matrix [44] is constrained by the precision measurements [40]. Especially, owing to the mixing among \( t, T \) and \( U \), the \( T \)-parameter is potentially large. We explicitly show the expression of the \( S \) and \( T \)-parameters in our model in Appendix A [9, 10]. Fixing \( U = 0 \), we impose the constrains [44],
\[
\Delta S = 0.07 \pm 0.08, \quad \Delta T = 0.10 \pm 0.07,
\]
(44)
where \( m_t = 173.3 \) GeV, \( m_b = 4.2 \) GeV, and \( m_h = 125.1 \) GeV [44].

We now describe the numerical results. In the following analysis, we take the Higgs mass, the pole mass of the top, the \( \overline{\text{MS}} \) mass of the bottom, and the CKM matrix element for \( t \) and
$b$ as $m_b = 125.1$ GeV, $m_t^{\text{pole}} = 173.2$ GeV, $m_t^{\text{MS}} = 4.2$ GeV, and $|V_{tb}| = 0.95$, respectively. The relation $m_1 = m_t$ must hold. Although strong couplings are acceptable in our scenario, we may impose $|y_{ij}| < 5$ for the Lagrangian parameters in Eq. (4). Even in this case, there is still wide parameter space, as we will see below. Considering the lower mass bound for the $T$ quark $^{45}$, we fix $M_T = 1.2$ TeV and take the mass range for the heavier VLQ to $1.5 \leq M_U \leq 3.5$ TeV.

The signal strengths of $\mu V V F$ and $\mu \gamma \gamma F$ are depicted in Figs. 1 and 2. For the red points, the $S,T$ constraints and the $2\sigma$ bounds (43) of the Higgs signal strengths are satisfied, while the green points are outside of the $2\sigma$ bounds (43). For the blue points in Figs. 1 and 2, $G_{tt}^h > g_{tt}^{\text{SM}}$ and $G_{TT}^h > 0$. We did not plot the data with $G_{tt}^h < g_{tt}^{\text{SM}}$ in our model, although they exist. We also show the results for MCHM4 and MCHM5, where the scaling parameters are $\kappa_V = \sqrt{1-\xi}$ for both and $\kappa_f = \sqrt{1-\xi}$ and $\kappa_f = (1-2\xi)/\sqrt{1-\xi}$ for MCHM4 and MCHM5, respectively, with $\xi = v^2/f^2$ and $f$ being the typical scale of the MCHMs $^{25, 27, 46, 47}$. The $2\sigma$ constraint of the top yukawa coupling from the Run 1 combined data $^{1}$, which reads $1.05 < \kappa_t < 1.92$, is also shown in Figs. 1 and 2 with the proviso that the Run 2 data do not restrict the top yukawa so much yet, within the $2\sigma$ bounds, $0.63 < \kappa_t < 1.79$ (ATLAS Run2 $^{2}$) and $0.71 < \kappa_t < 1.58$ (CMS Run2 $^{3}$). In passing, we comment that there is a parameter space inside of the $2\sigma$ bounds (43), even if we take $M_T = 2.0$ TeV. Although the window is closed at $M_T = 2.4$ TeV under the condition $|y_{ij}| < 5$, the parameter space still exists even for $M_T = 3.0$ TeV, if we allow $|y_{ij}| > 5$.

In our scenario, we require $G_{tt}^h > g_{tt}^{\text{SM}}$ and $G_{TT}^h < 0$ in order for the Higgs signal strengths to be consistent with the experiments. For this cancellation mechanism among the diagonal yukawa couplings in the gluon fusion process, we show the normalized yukawa couplings of $\kappa_{T,U} = G_{TT,UU}^h/g_{tt}^{\text{SM}}$ vs $\kappa_{t} = G_{tt}^h/g_{tt}^{\text{SM}}$ in Fig. 3. At the red points, the conditions of $G_{tt}^h > g_{tt}^{\text{SM}}$ and $G_{TT}^h < 0$, the $2\sigma$ constraints (43), and the $S,T$-constraints (44) are satisfied. On the other hand, the green points are outside of the $2\sigma$ constraints (43). We find that the cancellation mechanism works up to $\kappa_t \lesssim 1.4$.

The Lagrangian parameters are important for the model-building. We depict them in Fig. 4. The entry of $y_{21}$ can be either positive or negative. The vanishing $y_{21}$ is also possible. This is consistent with the analytical approach in the previous subsection. Although $y_{23}$ barely takes negative or small positive values, $y_{ij}$ except for $y_{21}$ are positive and large, say, $y_{ij} \gtrsim 2$, in the wide parameter space. Thus the VLQ model discussed here might be provided
from some underlying theories based on strongly interacting systems.

In the end of this subsection, we comment on the di-Higgs production via the gluon fusion. The off-diagonal yukawa couplings can be extracted from the decay channels such as $T \rightarrow th$. Also, they contribute to the box diagram of the di-Higgs production, so that the $gg \rightarrow hh$ process may give us further information on the model parameters. In the lowest order of the $1/M$ expansion (LET), the triangle and box contributions normalized by the SM values are [32–34, 36–38]

$$R^{\text{tri}}_{gg\rightarrow h} = \frac{A_{gg\rightarrow h}}{A_{SM_{gg\rightarrow h}}} = v \text{Tr}(G^h M^{-1}_{\text{diag}}),$$

(45)

$$R^{\text{box}}_{gg\rightarrow hh} = \frac{A_{box_{gg\rightarrow hh}}}{A_{SM_{box_{gg\rightarrow hh}}}} = v^2 \text{Tr}(G^h M^{-1}_{\text{diag}} G^h M^{-1}_{\text{diag}}),$$

(46)

respectively. Note that $R^{\text{tri}}_{gg\rightarrow h} \approx \kappa_g$, because of $A_{2^2}(x_t) \simeq A_{2^2}(x_T) \simeq A_{2^2}(x_U) \approx A_{2^2}(0) = 4/3$. The numerical result is depicted in Fig. [5]. Also, we can analytically obtain the expressions of $R^{\text{tri}}_{gg\rightarrow h}$ and $R^{\text{box}}_{gg\rightarrow hh}$ in the general case (4) as

$$R^{\text{tri}}_{gg\rightarrow h} = 3 - 2b_{22}, \quad R^{\text{box}}_{gg\rightarrow hh} = 3 - 6b_{22} + 4b_{22}^2 = \left(R^{\text{tri}}_{gg\rightarrow h}\right)^2 - 3\left(R^{\text{tri}}_{gg\rightarrow h} - 1\right),$$

(47)

where $b_{22}$ denotes the $(2, 2)$ element of the dimensionless mass matrix inverse $M_X M^{-1}$. See also Appendix [B]. This analytic result is shown as the blue curve in Fig. [5]. Under the symmetric mass matrix assumption (9) in the previous subsection, we find $R^{\text{tri}}_{gg\rightarrow h} = R^{\text{box}}_{gg\rightarrow hh} = 1$ owing to $b_{22} = 1$. It then turns out that the box contribution $R^{\text{box}}_{gg\rightarrow hh}$ is decreasing with respect to the increasing triangle contribution $R^{\text{tri}}_{gg\rightarrow h}$ under the $2\sigma$ constraints [43]. Since the box is destructive in $gg \rightarrow hh$, it means that the di-Higgs production is either much enhanced or suppressed. In Fig. [6] we show the ratio $\sigma_{\text{LET}}/\sigma_{\text{LET}}^{SM}$ of the total cross section of the Higgs pair production through $gg$ in pp collisions normalized by the SM one under the LET approximation [32–34, 36–38]. We took the LHC center of mass energy as $\sqrt{s} = 14$ TeV and used the CT14 LO PDF set [48]. The renormalization scale ($\mu$) and the factorization scale ($Q$) are chosen equal to the invariant mass ($M_{hh} = \sqrt{s}$) of the Higgs pair, $\mu = Q = M_{hh}$. We find that the ratio can be increasing/decreasing about 40%, depending on the values of $R^{\text{tri}}_{gg\rightarrow h} \approx \kappa_g$. This might be striking, although one should take notice of inaccuracy of the LET approximation. A detailed analysis will be performed elsewhere.
FIG. 1: $\mu_{ggF+ttH}^{VV}$ vs $G_{tth}^{h}/g_{tth}^{SM}$. We fixed $M_T = 1.2$ TeV and took the mass range, $1.5 \leq M_U \leq 3.5$ TeV. The upper and lower shaded regions are outside of the 2σ constraints \cite{13}. The red points are inside of the 2σ constraints of the LHC Run 1. The green points satisfy only the conditions of $G_{tth}^{h}/g_{tth}^{SM} > 1$ and $G_{TT}^{h} < 0$, and the $S,T$-constraints, while in the blue ones, $G_{tth}^{h}/g_{tth}^{SM} > 1$ and $G_{TT}^{h} > 0$. We do not show the results with $G_{tth}^{h}/g_{tth}^{SM} < 1$ in our model, although they exist. We also show the results for MCHM4 and MCHM5.

IV. SUMMARY AND DISCUSSIONS

We revisited the scenario with the enhanced top yukawa coupling in the framework of the VLQ model. We found that the scenario can be realized in the rather wide parameter space. Since the Lagrangian parameters of the yukawa couplings except for $y_{21}$ are positive and large, such VLQ model can be obtained from some underlying strong dynamics. We also calculated the ratios of the triangle and box diagrams to the SM values in the $gg \rightarrow hh$ process and found the noticeable relation. The detailed studies will be done in future.
FIG. 2: $\mu_{ggF+ttH}^{\gamma\gamma}$ vs $G_{tt}^{h}/g_{tth}^{SM}$. We fixed $M_T = 1.2$ TeV and took the mass range, $1.5 \leq M_U \leq 3.5$ TeV. The upper and lower shaded regions are outside of the $2\sigma$ constraints. The red points are inside of the $2\sigma$ constraints of the LHC Run 1. The green points satisfy only the conditions of $G_{tt}^{h}/g_{tth}^{SM} > 1$ and $G_{T}^{h} < 0$, and the $S,T$-constraints, while in the blue ones, $G_{tt}^{h}/g_{tth}^{SM} > 1$ and $G_{T}^{h} > 0$. We do not show the results with $G_{tt}^{h}/g_{tth}^{SM} < 1$ in our model, although they exist. We also show the results for MCHM4 and MCHM5.
FIG. 3: The diagonal components of the physical yukawa couplings. The red points are inside of the $2\sigma$ constraints \([43]\), while the green points satisfy only the conditions of $G^h_{tt}/g_{tt}^{SM} > 1$ and $G^h_{TT} < 0$, and the $S,T$-constraints.

FIG. 4: The Lagrangian parameters of the yukawa couplings in Eq. \([4]\). The red points are inside of the $2\sigma$ constraints \([43]\), while the green points satisfy only the conditions of $G^h_{tt}/g_{tt}^{SM} > 1$ and $G^h_{TT} < 0$, and the $S,T$-constraints.
FIG. 5: $R_{gg\rightarrow h}^{\text{tri}}$ vs $R_{gg\rightarrow hh}^{\text{box}}$. The red points are inside of the $2\sigma$ constraints \[43\]. The blue curve corresponds to the analytical relation, $R_{gg\rightarrow hh}^{\text{box}} = \left(R_{gg\rightarrow h}^{\text{tri}}\right)^2 - 3\left(R_{gg\rightarrow h}^{\text{tri}} - 1\right)$, shown in Eq. \[47\].

Appendix A: $S,T$-parameters in the VLQ model

The parameter space of the VLQ models is severely restricted by the oblique corrections \[40\]. In particular, the $T$-parameter is essential. In our model, it reads \[9, 10\],

$$T = \frac{N_c}{16\pi s_W^2 c_W^2} \left[ \sum_{k=1}^{3} \left( |L_{1k}|^2 \theta_+(y_k, y_b) + (|L_{2k}|^2 + |R_{2k}|^2) \theta_+(y_X, y_k) + 2\text{Re}(L_{2k}^* R_{2k}) \theta_-(y_X, y_k) \right) \\
- \frac{1}{2} \theta_+(y_b, y_b) - \theta_+(y_X, y_X) - \theta_-(y_X, y_X) \\
- \sum_{k=1}^{3} \left\{ \frac{1}{2} \left( (|L_{1k}|^2 - |L_{2k}|^2)^2 + |R_{2k}|^4 \right) \theta_+(y_k, y_k) - (|L_{1k}|^2 - |L_{2k}|^2) |R_{2k}|^2 \theta_-(y_k, y_k) \right\} \\
- \sum_{i \neq j} \left\{ \frac{1}{2} \left( |L_{1i}^* L_{1j} - L_{2i}^* L_{2j}|^2 + |R_{2i}^* R_{2j}|^2 \right) \theta_+(y_i, y_j) \\
- \text{Re}\left( (L_{1i}^* L_{1j} - L_{2i}^* L_{2j}) R_{2i} R_{2j}^* \right) \theta_-(y_i, y_j) \right\} \right], \quad (A1)$$
FIG. 6: The ratio $\sigma_{\text{LET}}/\sigma_{\text{SM}}$ of the total $pp \to hh$ cross section at $\sqrt{s} = 14$ TeV normalized by the SM value under the LET approximation. We used the CT14 LO PDF set [48] and took the renormalization and factorization scales equal to the invariant mass of the Higgs pair, $\mu = Q = M_{hh} = \sqrt{s}$.

where $N_c = 3$, and we defined $s_W \equiv \sin \theta_W$ and $c_W \equiv \cos \theta_W$ with $\theta_W$ being the weak mixing angle, and also

\[
y_i \equiv \frac{m_i^2}{m_Z^2}, \quad y_b \equiv \frac{m_b^2}{m_Z^2}, \quad y_X \equiv \frac{M_X^2}{m_Z^2}, \quad (A2)
\]

\[
\theta_+(y_i, y_j) \equiv y_i + y_j - \frac{2y_i y_j}{y_i - y_j} \log \frac{y_i}{y_j} - 2(y_i \log y_i + y_j \log y_j) + \frac{y_i + y_j}{2} \Delta, \quad (A3)
\]

and

\[
\theta_-(y_i, y_j) \equiv 2\sqrt{y_i y_j} \left( \frac{y_i + y_j}{y_i - y_j} \log \frac{y_i}{y_j} - 2 + \log(y_i y_j) - \frac{\Delta}{2} \right), \quad (A4)
\]

with $\Delta$ being the divergent term in the dimensional regularization. The mass eigenvalues of the up-type quarks are $m_1 = m_t$, $m_2 = M_T$ and $m_3 = M_U$. The rotation matrices are
defined by

\[
\begin{pmatrix}
t_L \\
T_L \\
U_L
\end{pmatrix} = \begin{pmatrix}
L_{11} & L_{12} & L_{13} \\
L_{21} & L_{22} & L_{23} \\
L_{31} & L_{32} & L_{33}
\end{pmatrix} \begin{pmatrix}
t'_L \\
T'_L \\
U'_L
\end{pmatrix},
\begin{pmatrix}
t_R \\
T_R \\
U_R
\end{pmatrix} = \begin{pmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{pmatrix} \begin{pmatrix}
t'_R \\
T'_R \\
U'_R
\end{pmatrix},
\tag{A5}
\]

where \((t, T, U)_{L,R}\) and \((t', T', U')_{L,R}\) are the gauge and mass eigenstates, respectively.

We have left the divergent term \(\Delta\) for checking of the calculations \cite{10}. By using the unitarity and the mass relations

\[M_X = M_{22} = (V_L \text{diag}(m_1, m_2, m_3)V_R^\dagger)_{22} = m_1 L_{21} R_{21}^* + m_2 L_{22} R_{22}^* + m_3 L_{23} R_{23}^*, \tag{A6}\]

and also

\[0 = M_{12} = (V_L \text{diag}(m_1, m_2, m_3)V_R^\dagger)_{12} = m_1 L_{11} R_{21}^* + m_2 L_{12} R_{22}^* + m_3 L_{13} R_{23}^*, \tag{A7}\]

we can confirm that the divergent term \(\Delta\) is exactly canceled out, as it must be.

The deviation from the SM is given by

\[\Delta T = T - T_{\text{SM}}, \tag{A8}\]

with

\[T_{\text{SM}} = \frac{N_c}{16\pi s_W c_W m_Z^2} \left[ m_t^2 + m_b^2 - 2 \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \log \frac{m_t^2}{m_b^2} \right]. \tag{A9}\]

Throughout the paper, we take the 1\(\sigma\) constraint, \(\Delta T = 0.10 \pm 0.07\) \cite{44}.

The \(S\)-parameter constraint is not so severe, compared with the \(T\)-parameter. The expression for the \(S\)-parameter in our model is as follows \cite{8}:

\[S = \frac{N_c}{2\pi} \left[ \sum_{k=1}^3 \left( |L_{1k}|^2 \psi_+(y_k, y_b) + |L_{2k}|^2 + |R_{2k}|^2 \right) \psi_+(y_X, y_k) + 2 \text{Re}(L_{2k} R_{2k}^*) \psi_-(y_X, y_k) \right] - \sum_{i \neq j} \left\{ \frac{1}{2} \left( |L_{1i}^* L_{1j} - L_{2i}^* L_{2j}|^2 + |R_{2i}^* R_{2j}|^2 \right) \chi_+(y_i, y_j) \right. \\
\left. - \text{Re} \left( (L_{1i}^* L_{1j} - L_{2i}^* L_{2j}) R_{2i} R_{2j}^* \right) \chi_-(y_i, y_j) \right\}, \tag{A10}\]

where we defined

\[\psi_+(y_i, y_j) \equiv \frac{1}{3} - \frac{1}{9} \log \frac{y_i}{y_j}, \tag{A11}\]

\[\psi_-(y_i, y_j) \equiv \frac{y_i + y_j}{6\sqrt{y_i y_j}}, \tag{A12}\]

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\[ \chi^+(y_i, y_j) \equiv \frac{5(y_i^2 + y_j^2) - 22y_i y_j}{9(y_i - y_j)^2} + \frac{3y_i y_j(y_i + y_j) - y_i^3 - y_j^3}{3(y_i - y_j)^3} \log \frac{y_i}{y_j}, \]  
(A13)

and

\[ \chi^-(y_i, y_j) \equiv -\sqrt{y_i y_j} \left( \frac{y_i + y_j}{6y_i y_j} - \frac{y_i + y_j}{(y_i - y_j)^2} + \frac{2y_i y_j}{(y_i - y_j)^3} \log \frac{y_i}{y_j} \right). \]  
(A14)

Note that \( \chi^+(y_i, y_i) = \chi^-(y_i, y_i) = 0. \)

The deviation from the SM is given by

\[ \Delta S = S - S_{\text{SM}}, \]  
(A15)

with

\[ S_{\text{SM}} = \frac{N_c}{2\pi} \left[ \frac{1}{3} - \frac{1}{9} \log \frac{m_t^2}{m_h^2} \right]. \]  
(A16)

Throughout the paper, we take the 1\(\sigma\) constraint, \(\Delta S = 0.07 \pm 0.08 \) [44].

Appendix B: Analytical expression of \( R_{gg \rightarrow h}^{\text{tri}} \) and \( R_{gg \rightarrow hh}^{\text{box}} \)

For the general mass matrix \(4\) in our model, we define the dimensionless matrices as follows:

\[ \tilde{\mathcal{M}} \equiv \frac{1}{M_X} \mathcal{M} = \begin{pmatrix} 0 & 0 & a_{13} \\ a_{21} & 1 & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad \tilde{\mathcal{Y}} \equiv \frac{1}{\sqrt{2} M_X} \mathcal{Y} = \begin{pmatrix} 0 & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ 0 & a_{32} & 0 \end{pmatrix}, \]  
(B1)

where we scaled by \(M_X = m_{22}\). We assume \(a_{13} \neq 0\) for getting \(m_t \neq 0\). We then read Eqs. \(45\) and \(46\) as

\[ R_{gg \rightarrow h}^{\text{tri}} = \text{tr}(\tilde{\mathcal{Y}} \tilde{\mathcal{M}}^{-1}), \quad R_{gg \rightarrow hh}^{\text{box}} = \text{tr}(\tilde{\mathcal{Y}} \tilde{\mathcal{M}}^{-1} \tilde{\mathcal{Y}} \tilde{\mathcal{M}}^{-1}). \]  
(B2)

Let us determine the dimensionless mass matrix inverse \(\tilde{\mathcal{M}}^{-1}\). The definition of the inverse matrix, \(\tilde{\mathcal{M}} \tilde{\mathcal{M}}^{-1} = \tilde{\mathcal{M}}^{-1} \tilde{\mathcal{M}} = \text{diag}(1, 1, 1)\), yields the expression for \(\tilde{\mathcal{M}}^{-1}\) as

\[ \tilde{\mathcal{M}}^{-1} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ \frac{1}{a_{13}} & 0 & 0 \end{pmatrix}, \]  
(B3)

and the matrix elements \(b_{ij}\) satisfy

\[ \begin{pmatrix} a_{21} & 1 \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} b_{12} & b_{13} \\ b_{22} & b_{23} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} a_{21} & 1 \\ a_{31} & a_{32} \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix} = \frac{1}{a_{13}} \begin{pmatrix} a_{23} \\ a_{33} \end{pmatrix}. \]  
(B4)
Therefore we analytically obtain $b_{ij}$,

\[
\begin{pmatrix}
    b_{12} & b_{13} \\
    b_{22} & b_{23}
\end{pmatrix} = \frac{1}{a_{21}a_{32} - a_{31}} \begin{pmatrix}
    a_{32} & -1 \\
    -a_{31} & a_{21}
\end{pmatrix},
\]

(B5)

and

\[
\begin{pmatrix}
    b_{11} \\
    b_{21}
\end{pmatrix} = -\frac{1}{a_{13}(a_{21}a_{32} - a_{31})} \begin{pmatrix}
    a_{32}a_{23} - a_{33} \\
    -a_{31}a_{23} + a_{21}a_{33}
\end{pmatrix}.
\]

(B6)

By using $a_{21}b_{12} = a_{32}b_{23} = 1 - b_{22}$, we find

\[
R_{gg\rightarrow hh}^{\text{tri}} = 3 - 2b_{22},
\]

(B7)

and

\[
R_{gg\rightarrow hh}^{\text{box}} = 1 + 2(1 - b_{22})^2 - 2b_{22}(1 - b_{22}) = 3 - 6b_{22} + 4b_{22}^2.
\]

(B8)

Eliminating $b_{22}$ from the above equations, we obtain Eq. (B7),

\[
R_{gg\rightarrow hh}^{\text{box}} = \left( R_{gg\rightarrow h}^{\text{tri}} \right)^2 - 3\left( R_{gg\rightarrow h}^{\text{tri}} - 1 \right).
\]

(B9)

When $a_{21} = 0$ as in Sec. IIIA, we immediately find $b_{22} = 1$ and thereby obtain $R_{gg\rightarrow h}^{\text{tri}} = R_{gg\rightarrow hh}^{\text{box}} = 1$.

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[1] G. Aad et al. [ATLAS and CMS Collaborations], JHEP 1608, 045 (2016) [arXiv:1606.02266 [hep-ex]].

[2] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-068.

[3] CMS Collaboration [CMS Collaboration], CMS-PAS-HIG-17-004.

[4] CMS Collaboration [CMS Collaboration], CMS-PAS-HIG-16-020.

[5] CMS Collaboration [CMS Collaboration], CMS-PAS-HIG-16-038.
[6] CMS Collaboration [CMS Collaboration], CMS-PAS-HIG-16-041.

[7] V. A. Miransky, M. Tanabashi and K. Yamawaki, Phys. Lett. B 221, 177 (1989); Mod. Phys. Lett. A 4, 1043 (1989); Y. Nambu, Enrico Fermi Institute Report No. 89-08, 1989; in Proceedings of the 1988 Kazimierz Workshop, eds. Z. Ajduk et al. (World Scientific Publishing Co., Singapore, 1989); W. A. Bardeen, C. T. Hill and M. Lindner, Phys. Rev. D 41, 1647 (1990).

[8] B. Holdom, Phys. Rev. Lett. 57, 2496 (1986) [Erratum-ibid. 58, 177 (1987)]; G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D 76, 075016 (2007); M. Hashimoto, Phys. Rev. D 81, 075023 (2010); For a review, see, P. H. Frampton, P. Q. Hung and M. Sher, Phys. Rept. 330, 263 (2000).

[9] L. Lavoura and J. P. Silva, Phys. Rev. D 47, 2046 (1993).

[10] C. Anastasiou, E. Furlan and J. Santiago, Phys. Rev. D 79, 075003 (2009) [arXiv:0901.2117 [hep-ph]].

[11] G. Cacciapaglia, A. Deandrea, D. Harada and Y. Okada, JHEP 1011, 159 (2010) [arXiv:1007.2933 [hep-ph]].

[12] G. Cacciapaglia, A. Deandrea, L. Panizzi, N. Gaur, D. Harada and Y. Okada, JHEP 1203, 070 (2012) [arXiv:1108.6329 [hep-ph]].

[13] J. A. Aguilar-Saavedra, R. Benbrik, S. Heinemeyer and M. Pérez-Victoria, Phys. Rev. D 88, no. 9, 094010 (2013) [arXiv:1306.0572 [hep-ph]].

[14] A. K. Alok, S. Banerjee, D. Kumar, S. U. Sankar and D. London, Phys. Rev. D 92, no. 1, 013002 (2015) [arXiv:1504.00517 [hep-ph]].

[15] A. K. Alok, S. Banerjee, D. Kumar and S. Uma Sankar, Nucl. Phys. B 906, 321 (2016) doi:10.1016/j.nuclphysb.2016.03.012 [arXiv:1402.1023 [hep-ph]].

[16] G. Cacciapaglia, A. Deandrea, N. Gaur, D. Harada, Y. Okada and L. Panizzi, JHEP 1509, 012 (2015) [arXiv:1502.00370 [hep-ph]].

[17] A. Angelescu, A. Djouadi and G. Moreau, Eur. Phys. J. C 76, no. 2, 99 (2016) [arXiv:1510.07527 [hep-ph]].

[18] A. Biekötter, J. L. Hewett, J. S. Kim, M. Krämer, T. G. Rizzo, K. Rolbiecki, J. Tattersall and T. Weber, Int. J. Mod. Phys. A 32, no. 05, 1750032 (2017) [arXiv:1608.01312 [hep-ph]].

[19] C. Y. Chen, S. Dawson and E. Furlan, arXiv:1703.06134 [hep-ph].

[20] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B 719, 165 (2005) [hep-ph/0412089].
[21] R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D 75, 055014 (2007) [hep-ph/0612048].
[22] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002) [hep-ph/0206021].
[23] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55, 229 (2005) [hep-ph/0502182].
[24] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. 81, 2634 (1998) [hep-ph/9712319];
R. S. Chivukula, B. A. Dobrescu, H. Georgi and C. T. Hill, Phys. Rev. D 59, 075003 (1999) [hep-ph/9809470];
For a review see, e.g., C. T. Hill and E. H. Simmons, Phys. Rept. 381, 235 (2003) Erratum: [Phys. Rept. 390, 553 (2004)] [hep-ph/0203079].
[25] J. R. Espinosa, C. Grojean and M. Muhlleitner, JHEP 1005, 065 (2010) [arXiv:1003.3251 [hep-ph]].
[26] M. Carena, L. Da Rold and E. Pontón, JHEP 1406, 159 (2014) [arXiv:1402.2987 [hep-ph]].
[27] D. Liu, I. Low and C. E. M. Wagner, arXiv:1703.07791 [hep-ph].
[28] H. C. Cheng and J. Gu, JHEP 1410, 002 (2014) [arXiv:1406.6689 [hep-ph]].
[29] M. Badziak and C. E. M. Wagner, JHEP 1605, 123 (2016) [arXiv:1602.06198 [hep-ph]].
[30] M. Badziak and C. E. M. Wagner, JHEP 1702, 050 (2017) [arXiv:1611.02353 [hep-ph]].
[31] A. Das, N. Maru and N. Okada, arXiv:1704.01353 [hep-ph].
[32] B. A. Kniehl and M. Spira, Z. Phys. C 69, 77 (1995) [hep-ph/9505225].
[33] A. Falkowski, Phys. Rev. D 77, 055018 (2008) arXiv:0711.0828 [hep-ph].
[34] I. Low, R. Rattazzi and A. Vichi, JHEP 1004, 126 (2010) arXiv:0907.5413 [hep-ph].
[35] R. Gröber and M. Mühlleitner, JHEP 1106, 020 (2011) arXiv:1012.1562 [hep-ph].
[36] M. Gillioz, R. Gröber, C. Grojean, M. Mühlleitner and E. Salvioni, JHEP 1210, 004 (2012)
arXiv:1206.7120 [hep-ph].
[37] S. Dawson, E. Furlan and I. Lewis, Phys. Rev. D 87, no. 1, 014007 (2013) arXiv:1210.6663 [hep-ph].
[38] C. Y. Chen, S. Dawson and I. M. Lewis, Phys. Rev. D 90, no. 3, 035016 (2014) arXiv:1406.3349 [hep-ph].
[39] R. Gröber, M. Mühlleitner and M. Spira, JHEP 1606, 080 (2016) arXiv:1602.05851 [hep-ph].
[40] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990); Phys. Rev. D 46, 381 (1992).
[41] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, Front. Phys. 80, 1 (2000).
[42] A. Djouadi, Phys. Rept. 457, 1 (2008) [hep-ph/0503172].
[43] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-081.

[44] C. Patrignani et al. [Particle Data Group], Chin. Phys. C 40, no. 10, 100001 (2016).

[45] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-104.

[46] S. Kanemura, K. Kaneta, N. Machida, S. Odori and T. Shindou, Phys. Rev. D 94, no. 1, 015028 (2016) [arXiv:1603.05588 [hep-ph]].

[47] V. Sanz and J. Setford, arXiv:1703.10190 [hep-ph].

[48] S. Dulat et al., Phys. Rev. D 93, no. 3, 033006 (2016) [arXiv:1506.07443 [hep-ph]].