RECENT DEVELOPMENTS IN SOFT-COLLINEAR EFFECTIVE THEORY

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Soft-collinear effective theory provides a systematic theoretical framework to describe the factorization of short- and long-distance QCD dynamics in hard-scattering processes that contain both, soft and energetic particles/jets. I present a short guide to recent theoretical achievements and to phenomenological applications in heavy B-meson decays.

Keywords: QCD; Factorization; B-meson decays.

1. Introduction

The idea of factorization in high-energy processes is to isolate (perturbative) short-distance QCD dynamics from long-distance (non-perturbative) hadronic effects. A well-known example is the weak effective Hamiltonian which describes quark and lepton flavour transitions in terms of local operators $O_i(x)$, multiplied by Wilson coefficients $C_i(\mu)$, containing the short-distance dynamics above a scale $\mu \leq M_W$. In $B$-meson decays the theoretical task remains to calculate hadronic matrix elements $\langle X|O_i(x)|B \rangle_{\mu \sim m_b}$, with $|X\rangle$ an exclusive or inclusive final state. As the mass of the decaying $b$-quark is large compared to the QCD scale, $m_b \gg \Lambda$, QCD dynamics which involves virtualities of the order $m_b^2$ ("hard modes") can still be treated perturbatively.

In the following, we will concentrate on $B$-meson decays where the final state consist of hadronic systems with large energy but small invariant mass. Three typical examples are shown in Fig. 1. In these cases a second perturbative scale $\mu_{hc} \sim \sqrt{\Lambda m_b} \gg \Lambda$ appears. In inclusive decays, it is related to the invariant mass of the hadronic jet. In exclusive decays, these ("hard-collinear") modes arise from the interaction of soft spectators in the $B$-meson and energetic ("collinear") degrees of freedom in the final state. The remaining non-perturbative dynamics is encoded in hadronic matrix elements of non-local operators. For inclusive $B$-decays, these define so-called shape functions (SFs) which describe the momentum distribution of the $b$-quark in the $B$-meson due to soft gluon interactions. In exclusive decays, light-cone distribution amplitudes (DAs) for the $B$-meson and its decay products appear.

In the effective-theory approach, the factorization theorems can be obtained from a 2-step matching procedure, based on a simultaneous expansion in $\alpha_s$ and $\Lambda/m_b^{1,2,3,4}$. In the first step, one integrates out hard modes from QCD to derive a soft-collinear effective theory (SCET$_I$) where soft and hard-collinear degrees of freedom interact. The renormalization group for operators in SCET$_I$ is used to evolve the coefficient functions down to the hard-collinear scale, re-summing logarithms $\ln[m_b/\mu_{hc}]$. Then, in inclusive decays, one integrates out the hard-collinear (jet) modes applying quark-hadron duality, ending up with the usual heavy-quark effective theory (HQET). In exclusive decays, one is left with collinear degrees of freedom, whose interactions with soft modes is described by a so-called SCET$_{II}$.

2. Inclusive Decays (SF region)

Experimental studies of inclusive $B$ decays often require certain kinematic cuts which imply sensitivity to the shape-function region as described above$^{5,6}$. A prominent ex-
The effective-theory treatment of the partial rate leads to the factorization theorem,\(^7,8\)

\[
\frac{d\Gamma\nu}{dP_+} \propto \int_0^1 dy y^{-a} H_u(y, m_b) U(m_b, \mu_{hc}) P_+ \times \int_0^1 d\hat{\omega} J(y(P_+ - \hat{\omega}), \mu_{hc}) \hat{S}(\hat{\omega}, \mu_{hc}),
\]

valid to leading power in \(\Lambda/m_b\). Here the hard function \(H_u(y)\) can be calculated perturbatively in QCD \((y = (E_X + |P_X|)/m_b)\). The jet function \(J(u)\) is calculated in SCET\(_1\), and \(\hat{S}(\hat{\omega})\) is the leading shape function in HQET. The renormalization group functions \(U(m_b, \mu_{hc})\) and \(a = a(m_b, \mu_{hc})\) resum logarithms between the hard and hard-collinear factorization scale. An analogous factorization theorem holds for photon-energy spectra in \(B \rightarrow X_s\gamma\).

Shape-function effects also have to be considered in inclusive \(B \rightarrow X_s\ell^+\ell^-\) decays in the region of small invariant lepton-pair mass. Here the experimental studies require an additional cut on the hadronic invariant mass to eliminate combinatorial background like \(b \rightarrow c\ell\nu \rightarrow s\ell\ell\nu\nu\). The description within SCET has recently been discussed in Ref.\(^9\).

2.1. Theoretical status

Besides the hard matching coefficients (known to NLO for \(b \rightarrow u\ell\nu\) and \(b \rightarrow s\gamma\)), there has been recent progress in the evaluation of the jet function which is now known to NNLO accuracy\(^10\) for massless quarks. The two-loop evolution kernel for the leading-power \(B\)-meson shape function has been derived in Ref.\(^11\). Sub-leading shape functions, which enter at order \(\Lambda/m_b\) in SCET, have been classified in Refs.\(^{12,13,14,15}\) for the massless case, and in Ref.\(^{16}\) for the massive case.

2.2. SF-independent relations

From the factorization formulas for \(B \rightarrow X_s\gamma\) and \(B \rightarrow X_s\ell\nu\) one can derive a SF-independent relation between partially integrated spectra,\(^17\) (see also Refs.\(^{18,19,20}\))

\[
\Gamma_u(\Delta) \equiv \int_0^\Delta dP_+ \frac{d\Gamma_u}{dP_+} = |V_{ub}|^2 \int_0^\Delta dP_+ W(\Delta, P_+) \frac{1}{\Gamma_s} \frac{d\Gamma_s}{dP_+},
\]

where, to leading power in \(\Lambda/m_b\), the weight function \(W(\Delta, P_+)\) is perturbatively calculable. Choosing \(\Delta \sim 650\) MeV, and considering the two experimental spectra, one can determine the CKM element \(|V_{ub}|\) in the SM with rather good accuracy. The analysis in Ref.\(^{17}\) based on the two-loop result for \(W(\Delta, P_+)\) predicts \(\Gamma_u(0.65\text{ GeV}) = (46.5 \pm 4.1)|V_{ub}|^2\text{ ps}^{-1}\), where the quoted number
also includes an estimate of power corrections. Situations with more complicated cuts have been studied in Ref.\textsuperscript{21}.

Similar relations can be derived from the $B \to X_c \ell \nu$ decay\textsuperscript{22}, if one treats the charm-quark as a massive quark in SCET\textsubscript{I} (assuming the power counting $m_c^2 \sim m_b \Lambda$). Here one considers the spectral variable $U = P_+ - m_c^2/y m_b^2$ in the region of phase space where $U \sim \Lambda$. The comparison of $d \Gamma_c/dU$ and $d \Gamma_u/dP_+$ gives an estimate of $|V_{ub}/V_{cb}|$. The one-loop weight function for this case has been calculated in Ref.\textsuperscript{23}. Power corrections are potentially large. The experimental analysis of the $U$-spectrum in $B \to X_c \ell \nu$ is still to be performed.

3. Exclusive Decays (large recoil)

Factorization theorems also exist for exclusive $B$-meson decays with large energy transfer to the final-state hadrons. For instance, the amplitudes for charmless non-leptonic two-body $B$ decays can be written as\textsuperscript{24}

$$A_i(B \to MM') = \xi_M \cdot C_i \otimes \phi_{M'} + T_i \otimes \phi_B \otimes \phi_M \otimes \phi_{M'},$$

up to power corrections of order $\Lambda/m_b$. Here the distribution amplitudes $\phi_{B,M,M'}$ parameterize the non-perturbative dynamics of the two-quark Fock state in the respective hadron. The short-distance coefficient $C_i$ stems from the perturbative matching between QCD and SCET\textsubscript{I}. The spectator term $T_i$ further factorizes into a hard function $C_i$ and a jet function in SCET\textsubscript{II}. A peculiarity of exclusive $B$ decays is the appearance of “non-factorizable” input functions, in the above case the universal form factor $\xi_M$ for $B \to M$ transitions. In these objects the soft and collinear degrees of freedom in SCET\textsubscript{II} cannot be completely separated (at least not with standard perturbative methods). Leading-power factorization proofs exist for the somewhat simpler cases of $B \to \gamma \ell \nu$\textsuperscript{25,26}, $B \to \pi(\rho)\ell \nu$\textsuperscript{27,28}, and $B \to K^*(\rho)\gamma$\textsuperscript{29}. Applications of QCD factorization for $B \to K^*(\rho)\ell^+\ell^-$ decay observables can be found in Refs.\textsuperscript{30,31,32}. For the status of perturbative calculations of the short-distance coefficient functions\textsuperscript{33,34,35,36} see Ref.\textsuperscript{37} in these proceedings.

Factorization theorems have also been formulated\textsuperscript{38} for the exclusive semi-leptonic radiative decay $B \to \pi\gamma \ell \nu$. In the phase-space region where the pion is soft and the photon is energetic, the leading contribution to the decay amplitude factorizes as $A = H \cdot J \otimes S(B \to \pi)$. Here, the soft function $S$ is the generalized parton distribution for $B \to \pi$ transitions\textsuperscript{39}. Factorization-based numerical predictions for photon spectra and angular distributions differ from popular Monte Carlo models for real-photon corrections to $B \to \pi \ell \nu$.

3.1. BBNS vs. BPRS approach

The prescription of non-factorizable power corrections to (3) requires additional hadronic parameters which, at present, cannot be estimated in a systematic way. An important example are strong phases from final-state rescattering. The factorization formula predicts these phases to be either perturbative (and calculable) or (formally) power-suppressed. Different assumptions about non-factorizable effects thus lead to different theoretical predictions for exclusive $B$-decays. Two popular examples are the “BBNS approach”\textsuperscript{40} and the “BPRS approach”\textsuperscript{41}. A qualitative comparison is given in Table 1. A (controversial) discussion can be found in Refs.\textsuperscript{42,43}.

3.2. Enhanced electroweak penguins in $B \to VV$

An advantage of the effective-theory framework is the definite power counting assigned to fields and operators appearing in the effective Lagrangian. An interesting application for $B$ decays to two light vector mesons
Table 1. Comparison of different phenomenological assumptions in BBNS and BPRS approaches.

|                      | BBNS                                      | BPRS                                      |
|----------------------|-------------------------------------------|-------------------------------------------|
| charm penguins       | included in hard functions                | complex fit parameter $\Delta_P$          |
| spectator term       | perturbative factorization                 | fit to data                                |
| ext. hadronic input  | form factor and LCDAs (different scenarios)| LCDA for light meson, only                |
| power corrections    | model-dependent estimate                   | $\rightarrow$ systematic uncertainties    |
|                      | (complex functions $X_A$ and $X_H$)        | (unspecified)                              |

has recently been pointed out in Ref.\textsuperscript{44} (see also Ref.\textsuperscript{45} for more examples). From the $(V - A)$ structure of weak interactions one would naively expect the helicity amplitudes to scale as $A_0 : A_- : A_+ = 1 : \Lambda/m_b : \Lambda^2/m_b^2$. The inclusion of the electromagnetic penguin operator $O_7^\gamma$ via QED corrections to $T_i^V$ is shown to enhance the transverse helicity amplitudes by a factor $m_b^2/\Lambda^2$, which can compensate for the electromagnetic suppression factor $\alpha_{em}$. Among others, this implies a higher sensitivity to $(V + A)$ structures in certain new-physics models than naively anticipated.

3.3. SCET sum rules

The non-factorizable form factor $\xi_M$ can be estimated from sum rules in the effective theory SCET\textsubscript{1}. Here, one considers a correlation function where the energetic meson in the final state is replaced by an appropriate current\textsuperscript{46} (see also Ref.\textsuperscript{47}). The correlation function in the Euclidean region does factorize into a hard-collinear short-distance kernel (known to NLO\textsuperscript{46}) and a soft $B$-meson DA.

$$\Pi(P_+) = \int_0^\infty d\omega T(\omega, P_+) \phi^B(\omega) + \ldots \quad (4)$$

Inserting the result into a dispersion relation, one obtains a sum rule for the form factor $\xi_M$, which depends on the parameters used to describe the continuum contribution to the spectral function. The dependence on the sum-rule parameters and the (at present) not so well-known $B$-meson DA dominate the theoretical uncertainty.

4. Summary and Outlook

Factorization of short- and long-distance QCD effects via effective-theory methods in SCET provides a systematic and well-defined framework to describe $B$-decays into energetic hadrons. For inclusive decays precise theoretical predictions can be obtained on the basis of NNLO calculations. These can be used to determine the CKM element $|V_{ub}|$ and to constrain new-physics contributions to $B \to X_s \gamma$ and $B \to X_s \ell^+\ell^-$ (see also Ref.\textsuperscript{48} in these proceedings).

Theoretical predictions for exclusive observables are plagued by non-factorizable contributions which are difficult to estimate, in particular concerning the strong rescattering phases in non-leptonic $B$-decays. Sum rules in SCET provide a promising tool to estimate simple objects, like the soft $B \to \pi$ form factor $\xi_\pi$. There are also attempts to reduce non-factorizable matrix elements to more fundamental objects in SCET\textsubscript{11} by introducing an additional factorization prescription in rapidity space.\textsuperscript{49} How this procedure can be applied beyond fixed-order perturbation theory is still to be worked out.

SCET techniques have also been used to describe other high-energy observables, like jet distributions, or DIS and Drell-Yan near the end point.\textsuperscript{50,51,52,53,54,55} Finally, SCET has been combined with non-relativistic QCD to improve the description of quarkonium production and decay spectra.\textsuperscript{56,57}
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