Individuality for Quantum-Inspired Individuals Based on Nonuniform Convergence Speed in Maximum Cut Problem

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Abstract The quantum-inspired evolutionary algorithm (QEA) and QEA with a pair-swap strategy (QEAPS) have quantum-inspired individuals, where each gene is represented by a quantum bit (qubit). QEA and QEAPS iterate the evolution using the unitary transformation of probability amplitudes in each qubit, and can automatically shift the evolution from a global search to a local search. Here, the convergence speed depends on the rotation angle of each qubit toward either the |0⟩ or |1⟩ state vector, and the probability amplitude diverges or the population is likely to fall into a local solution if the rotation angle is too large. If the rotation angle is too small, the convergence speed may become low and the search performance will degrade. In this study, we introduce nonuniform convergence speeds into the quantum-inspired individuals and regard the convergence speed as the individual feature (individuality) of each quantum-inspired individual. Introducing the proposed individuality can eliminate the cumbersome process required to design a rotation angle while ensuring the quality of the obtained solution.

Keywords: quantum bit (qubit), quantum-inspired evolutionary algorithm (QEA), QEA with pair-swap strategy (QEAPS), individuality, nonuniform convergence speed, maximum cut problem

1. Introduction

The quantum-inspired evolutionary algorithm (QEA) [1], [2] and QEA with a pair-swap strategy (QEAPS) [3], [4] are evolutionary algorithms incorporating the principles of quantum computation. They have genes, which are represented by a quantum bit (qubit), and have shown superior search performance to the classical genetic algorithm (CGA) [5] in the 0-1 knapsack problem (0-1KP). QEA and QEAPS can automatically shift the evolution from a global search to a local search, and the convergence speed depends on the rotation angle of each qubit toward either the |0⟩ or |1⟩ state vector. The rotation angle is used to update probability amplitudes, which influence the observation result of the qubit, by the unitary transformation. The probability amplitude diverges or the population is likely to fall into a local solution if the rotation angle is too large. If the rotation angle is too small, the convergence speed may become low and the search performance will degrade [1], [2]. Therefore, it is important to design the rotation angle appropriately in accordance with the application problem.

On the other hand, ant colony optimization (ACO), which was inspired by co-operative food retrieval in ants, is metaheuristic [6], [7], [8]. ACO algorithms are applied to combinational optimization problems, such as the traveling salesman problem (TSP) and the quadratic assignment problem (QAP), and their effectiveness has been confirmed. In ACO, generally, an agent called an artificial ant selects a path according to the pheromone secreted by each ant and secretes a pheromone to indicate its own path. ASrank is a sophisticated ACO algorithm and can search for superior solutions by ranking the amounts of pheromone [9]. Moreover, for ASrankR⁰, which is ASrank with a random search introduced, it has been shown that diversity can be maintained and more accurate solutions can be obtained by introducing random selection [10]. Every ant of ASrank has a uniform random selection probability, which is used for path selec-
A selection of ants, improves the search performance of cumbersome process for designing a random selection probability. Therefore, the strategy can eliminate the is not used, and an approximate range is used for theviduality) of each ant. In the individuality ant strategy, a strictly designed random selection probability is not used, and an approximate range is used for the probability. Therefore, the strategy can eliminate the cumbersome process for designing a random selection probability. Moreover, it has been shown from experimental results for TSP that introducing the strategy, that is a nonuniform probability for the random selection of ants, improves the search performance of AQESRS.

In this study, we introduce a nonuniform rotation angle that has various convergence speeds into the quantum-inspired individual in QEA and QEAPS, which is regarded as the individuality. The diverse individualities are expected to improve the search performance, and introducing the proposed individuality can eliminate the cumbersome process required to design the rotation angle, which is an important parameter for deciding the convergence speed of the solution search. From the results of our computational experiment, although we have not been able to confirm the improvement of the search performance, we have confirmed that introducing the proposed individuality can eliminate the cumbersome process required to design the rotation angle while ensuring the quality of the obtained solution. It is important for evolutionary computations including QEA and QEAPS to design the parameter according to the characteristics of the problem, and their search performances are strongly influenced by the values of the parameters. However, the parameter design takes a lot of time, and reducing the number of parameters that need to be designed is significantly advantageous in the use of QEA and QEAPS. To confirm the possibility of novel evolutionary algorithms in which each bit of a gene is not deterministically but probabilistically determined after observing the gene, we have used QEA and QEAPS as representative algorithms. In this study, we use the maximum cut problem for the computational experiment. However, the proposed methods can be applied to other combinatorial optimization problems. The maximum cut problem is an NP-complete combinatorial optimization problem with applications in clustering, network science, and statistical physics.

Section 2 describes the qubit representation of a gene contained in a chromosome and the diversity of binary information obtained by observing a qubit. Section 3 provides the procedure of QEA and QEAPS, and how to interpret binary information as a graph representing the maximum cut problem and evaluate the binary information. In Sect. 4, we propose individuality-introducing methods for quantum-inspired individuals using a nonuniform rotation angle. Section 5 describes the experimental results using the maximum cut problem as a benchmark problem.

2. Qubit Representation and Diversity of the Observation Result

2.1 Qubit representation of a gene

The qubit representation is used as a gene in QEA and QEAPS. In general, a qubit is described by a two-dimensional column vector in the complex vector space, where the inner product is defined. It uses the following standard bases as orthonormal base vectors.

\[
|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

(1)

As shown in Fig. 1, QEA and QEAPS with qubit genes use an individual representation of a qubit chromosome \(q_i\). Here, \(i\) denotes the index of chromosomes or individuals when we use multiple chromosomes or individuals. The qubit \(q_{ij}\) \((j = 1, 2, \ldots, M)\) is shown as a stochastic superposition state (vector sum) of the two vectors \(|0\rangle\) and \(|1\rangle\) as follows:

\[
q_{ij} = \alpha_{ij}|0\rangle + \beta_{ij}|1\rangle = \begin{bmatrix} \alpha_{ij} \\ \beta_{ij} \end{bmatrix}
\]

(2)

where \(M\) is the number of genes or qubits included in a chromosome. \(\alpha_{ij}\) and \(\beta_{ij}\) are the complex probability amplitudes required to observe the state of 0 or 1, respectively. They are normalized so that \(|\alpha_{ij}|^2 + |\beta_{ij}|^2 = 1\). \(|\alpha_{ij}|^2\) is the probability that the state of 0 is observed and \(|\beta_{ij}|^2\) is the probability that the state of 1 is observed. A chromosome composed of genes with the qubit representation is described as the tensor product of the qubits as follows:

\[
q_i = q_{i1} \otimes q_{i2} \otimes \cdots \otimes q_{iM},
\]

(3)

\[
= \begin{bmatrix} \alpha_{i1} \\ \beta_{i1} \end{bmatrix} \otimes \begin{bmatrix} \alpha_{i2} \\ \beta_{i2} \end{bmatrix} \otimes \cdots \otimes \begin{bmatrix} \alpha_{iM} \\ \beta_{iM} \end{bmatrix}
\]

(4)
The ratio of the probability amplitudes $\alpha_{ij}$ and $\beta_{ij}$ of the superposition state is changed by using a unitary transformation, and the transformation is expressed as follows:

$$
\begin{bmatrix}
\alpha'_{ij} \\
\beta'_{ij}
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta_{ij}) & -\sin(\theta_{ij}) \\
\sin(\theta_{ij}) & \cos(\theta_{ij})
\end{bmatrix}
\begin{bmatrix}
\alpha_{ij} \\
\beta_{ij}
\end{bmatrix}
$$

How to decide the rotation angle $\theta_{ij}$ is described in Sect. 3.1.

### 2.2 Diversity of observation result

If each qubit $q_{ij}$ is quantum-mechanically observed, then each binary information $p_{ij}$ of 0 or 1 can be obtained from the probability amplitudes, $\alpha_{ij}$ and $\beta_{ij}$, of each qubit in the chromosome. Figure 2 shows the state of $\alpha_{ij}$ and $\beta_{ij}$ for qubit $q_{ij}$ in polar coordinates. The annular color scale shows the diversity of the obtained observation result of a qubit. The state that 0 and 1 are observed with equal probability is shown in white and the state that 0 or 1 is observed with probability 100% is shown in black; that is, the color is white if the diversity of the obtained observation result is high and black if the diversity of the obtained observation result is low.

The ratio of the probability amplitudes $\alpha_{ij}$ and $\beta_{ij}$ is updated by using the unitary transformation, and the probability that the state of 0 or 1 is observed is changed. The magnitude of the rotation angle $\theta_{ij}$ influences the convergence speed of the solution search. QEA and QEAPS can automatically shift the evolution from a global search to a local search. However, if the magnitude of the rotation angle $\theta_{ij}$ is too large, the population falls into a local solution before a sufficiently global search of the population is performed. If the magnitude of the rotation angle $\theta_{ij}$ is too small, the convergence speed of the solution search may become low, the population cannot shift the evolution to a local search, and the search performance will degrade. Therefore, it is important that a balance is achieved between a global search and a local search. This balance is designed on the bases of the magnitude of the rotation angle $\theta_{ij}$ in accordance with the application problem.

### 3. Procedure of QEA and QEAPS in the Maximum Cut Problem

This section first overviews the procedure of QEA and QEAPS, which use the qubit representation for a gene of an individual. After that, a way to interpret binary information as an observation result representing qubits as a graph of the maximum cut problem is described.

#### 3.1 Procedure of QEA and QEAPS

Figure 3 shows a flowchart of QEA and QEAPS and Fig. 4 shows the update cycle of a quantum-inspired individual to a local search, and the search performance will degrade.
inspired individual in QEA and QEAPS. Both algorithms involve the following steps.

**Step 1: Initialize individuals**
A quantum-inspired individual \( i \) \((i = 1, 2, \ldots, N)\) has a qubit chromosome \( q_i \) and best solution information \( b_i = [b_{i1}, b_{i2}, \ldots, b_{iM}] \) obtained in the past evolution, where \( N \) is the number of individuals. The chromosome \( q_i \) is formed from \( M \) qubits. The initialization is carried out by setting \( \alpha_{ij} \) and \( \beta_{ij} \) to \( 1/\sqrt{2} \) to observe states 0 and 1 with equal probability in individual \( i \).

**Step 2: Check termination condition**
QEA and QEAPS iterate the following procedures until a given termination condition is satisfied.

**Step 3: Observe each qubit**
The binary information \( p_i = [p_{i1}, p_{i2}, \ldots, p_{iM}] \) is obtained from observing chromosome \( q_i \) according to its probability amplitudes. In Fig. 4, \( p_i \) is \([0, 1, 0, 0, 1, 1]\).

**Step 4: Evaluate chromosomes**
The fitness of the chromosome \( f(p_i) \) can be evaluated from the binary information \( p_i \) in the same way as in CGA. In Fig. 4, \( f(p_i) \) is 238. How to evaluate binary information \( p_i \) is shown in Sect. 3.2.

**Step 5: Update best solution information**
If the currently obtained \( p_i \) is superior to \( b_i \), which is the personal best obtained up to the present generation, then \( b_i \) is replaced by \( p_i \). In the case in Fig. 4, \( b_i \) is not updated by \( p_i \) because the fitness of the personal best \( f(b_i) \) is 251, meaning that \( p_i \) is inferior to \( b_i \).

**Step 6: Update amplitude**
If \( p_i \) is inferior to \( b_i \), chromosome \( q_i \) is updated by the unitary transformation in Eq. (5) on the basis of the rotation angle list \( u_i \) so that it becomes closer to the binary best solution information \( b_i \). The rotation angle list \( u_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{iM}] \) is the list of rotation angles \( \theta_{ij} \) used to change the ratio of the probability amplitudes \( \alpha_{ij} \) and \( \beta_{ij} \) of the superposition state. \( u_i \) is created from each value of \( p_{ij} \), \( b_{ij} \), and the magnitude of the correlation of \( f(p_i) \) and \( f(b_i) \). How to decide the rotation angle \( \theta_{ij} \) is shown in Table 1 [1]. \( \theta_{ij} \) has not only the magnitude of the rotation angle but also the direction of rotation for the unitary formation. The maximum magnitude of the rotation angle \( \theta_{ij} \) is denoted as \( \theta_{ij} \). A unitary transformation can be used to change the ratio of the probability amplitudes \( \alpha_{ij} \) and \( \beta_{ij} \) of the superposition state. This list is used to increase and decrease the observation probabilities of 0 and 1.

### Table 1 Lookup table of the rotation angle \( \theta_{ij} \)

| \( p_{ij} \) | \( b_{ij} \) | \( f(p_i) > f(b_i) \) | \( \theta_{ij} \) (for unitary transform) |
|-------------|-------------|----------------------|----------------------------------|
| 0           | 1           | false                | \( \pm \theta_{ij} \)             |
| 1           | 0           | false                | \( \pm \theta_{ij} \)             |
| Otherwise   |             |                      | 0                                 |

In Fig. 4, the probability amplitudes \( \alpha_{i1} \) and \( \beta_{i1} \) are updated so that \( q_i \) becomes closer to \([1]\), as shown in Fig. 2, because \( p_i \) is inferior to \( b_i \), \( p_{i1} \) is 0, and \( b_{i1} \) is 1. On the other hand, \( \alpha_{i2} \) and \( \beta_{i2} \) are updated so that \( q_i \) becomes closer to \([0]\) because \( p_{i2} \) is 1 and \( b_{i2} \) is 0.

**Step 7: Transfer best solution information**
An individual’s best solution information \( b_i \) is transferred to the other individuals by the local migration process and global migration process in QEA [1], [2] or the pair-swap process in QEAPS [3], [4].

**Migration in QEA** The population is divided into several subpopulations in QEA. Migration involves local migration and global migration. The local migration is the process of distributing the local best solution information of the individual with the highest fitness in each subpopulation to all other individuals in each subpopulation, which is repeated in every generation. The global migration is the process of distributing the global best solution information of the individual with the highest fitness in all subpopulations to all other individuals in all subpopulations, which is repeated for each fixed generation.

**Pair-swap in QEAPS** QEAPS utilizes a pair-swap operation instead of local and global migration used in QEA. To begin with, two individuals are randomly selected as a pair from all individuals. Then, \( N/2 \) pairs are generated by selecting two individuals from \( N \) individuals with no overlaps. Only the best solution information is exchanged in each pair. Two parameters in QEA, which are the number of subpopulations and the timing of global migration, are unnecessary in QEAPS.

### 3.2 Interpretation method and evaluation method for the maximum cut problem

Figure 5 shows the graph cut in the case of using the binary information \( p_i \). In the maximum cut problem, an undirected graph \( G \) is constructed from vertices \( V = \{v_1, v_2, \ldots, v_M\} \) and edges \( E \), where \( M \) is
is obtained as an observation result of chromosome \( q \). The magnitude of the shift the evolution from a global search to a local search can be determined by multiplying the unitary transformation, and the amplitudes of the qubit gradually converge upon \( \theta \). However, a cumbersome process is required. It is important to design the rotation angle appropriately for each quantum-inspired individual.

4. Introducing Individuality into Quantum-Inspired Individuals

In the initial phase of the solution search, the state 0 or 1 is obtained with nearly equal probability by observing qubit \( q_{ij} \). Consequently, QEA and QEAPS behave similarly to a random search. The probability amplitudes of the qubit gradually converge upon applying the unitary transformation, and the algorithms shift the evolution from a global search to a local search. The magnitude of \( \theta_{C_{i,j}} \) affects the convergence speed and the quality of the solution obtained. Thus, it is important to design the rotation angle appropriately for each gene locus \( \theta_{C_{i,j}} \) is set to a random value.

As shown in Fig. 6(b), each subpopulation consists of various individuals with different \( \theta_{C_{i,j}} \). Therefore, in each subpopulation, the search by an individual having a large \( \theta_{C_{i,j}} \) converges rapidly, and that by an individual having a small \( \theta_{C_{i,j}} \) converges slowly. In this study, we nearly evenly divide the individuals with different \( \theta_{C_{i,j}} \) between each subpopulation.

As shown in Fig. 6(c), each subpopulation consists of individuals with similar \( \theta_{C_{i,j}} \). Therefore, the search by a subpopulation consisting of individuals with a large \( \theta_{C_{i,j}} \) converges rapidly, and that by a subpopulation consisting of individuals with a small \( \theta_{C_{i,j}} \) converges slowly. This study, we sort the individuals by \( \theta_{C_{i,j}} \) and divide them in order of the rotation angle \( \theta_{C_{i,j}} \).

The diversity concerning the individuality of the proposed individuality-introducing methods decreases in the order \( R > C-E > C-O \) in the case of QEA. This is because the rotation angles of the individuals in a
R is more diverse than varied in C.

In the case of QEA and QEAPS, the rotation angle for conventional QEA and QEAPS was set to \([0.005\pi, 0.050\pi]\) [rad] through a preliminary experiment. We varied the rotation angle \(\theta_{c,i}\) in increments of 0.005π [rad] \((\Delta \theta = 0.005\pi [\text{rad}])\). First, we used the same range and the same increment to compare the conventional methods and the proposed methods. That is, in the case of QEA and QEAPS, the rotation angle in the gene locus \(\theta_{C,i}\) was set to \(\{\theta_{C,i} | 0.005\pi, 0.010\pi, \ldots, 0.050\pi\}\) [rad]. In the case of QEA_{C,E}, QEA_{C,O} and QEAPS_{C}, the rotation angle of the quantum-inspired individual \(\theta_{C,i}\) was set to \(\{\theta_{C,i} | 0.005\pi, 0.010\pi, \ldots, 0.050\pi\}\) [rad]. In the case of conventional QEA and QEAPS, the rotation angle \(\theta_{C} = \theta_{C,1} = \theta_{C,2} = \ldots = \theta_{C,N}\) was set to \(\{\theta_{C} | 0.005\pi, 0.010\pi, \ldots, 0.050\pi\}\) [rad]. The number of nodes \(M\) was 800. The number of individuals was 100 and the upper limit of the evaluation frequency was 2,000,000 as the termination condition of the search. The number of subpopulations was five and the interval of global migration was 100 generations, which are used in QEA. We performed 100 trials for each condition.

The experimental results are depicted in Fig.8. The horizontal axis shows the rotation angle for conventional QEA or QEAPS and the vertical axis shows the sum of the edge weights, that is, the fitness. Each bar graph shows the average fitness of the best solution when the termination condition is satisfied, where the search algorithm is the conventional method (QEA,....

http://stanford.edu/~yyye/yyye/Gset/
(a) Experimental results in the case of QEA, where the problem used was G1

(b) Experimental results in the case of QEAPs, where the problem used was G1

(c) Experimental results in the case of QEA, where the problem used was G12

(d) Experimental results in the case of QEAPs, where the problem used was G12

(e) Experimental results in the case of QEA, where the problem used was G14

(f) Experimental results in the case of QEAPs, where the problem used was G14

Fig. 8 Experimental results of the average fitness of the best solution at the termination as a function of $\theta_C$
Table 2 p-Values obtained from the two-tailed t-test of the difference between the average fitness values of the conventional QEA and the proposed methods (QEA<sub>R</sub>, QEA<sub>C,E</sub>, QEA<sub>C,O</sub>): A bold number shows that the accuracy of the solution obtained by using a proposed method is significantly higher than that obtained by using a conventional method at the significance level of 0.05. An underlined number shows that a conventional method is significantly superior to a proposed method at the significance level of 0.05.

| Problem instance | Search algorithm | Rotation angle for the conventional QEA θ<sub>C</sub> [×π rad] | 0.005 | 0.010 | 0.015 | 0.020 | 0.025 | 0.030 | 0.035 | 0.040 | 0.045 | 0.050 |
|------------------|-----------------|-------------------------------------------------------------|------|------|------|------|------|------|------|------|------|------|
| G1               | QEA<sub>R</sub> | 4.10E-10, 4.25E-8, 6.02E-5                               | 3.33E-1 | 5.51E-1 | 9.59E-6 | 2.59E-10 | 3.55E-26 | 4.32E-21 | 1.67E-8 | 1.78E-1 | 0.005 | 0.010 | 0.015 | 0.020 | 0.025 | 0.030 | 0.035 | 0.040 | 0.045 | 0.050 |
| QEA<sub>C,E</sub> | 3.74E-11, 1.53E-13 | 1.01E-2 | 1.92E-1 | 4.16E-62 | 1.13E-15 | 2.81E-3 | 7.92E-4 | 1.10E-8 |
| QEA<sub>C,O</sub> | 1.07E-6, 1.99E-13 | 1.12E-2 | 1.82E-1 | 5.53E-62 | 1.05E-15 | 2.58E-3 | 2.57E-3 | 7.25E-4 | 1.40E-8 |
| G12              | QEA<sub>R</sub> | 9.62E-21, 9.99E-26, 1.03E-22, 3.44E-2 | 3.74E-19 | 7.79E-2 | 1.32E-21 | 2.40E-28 | 7.25E-25 | 9.52E-26 |
| QEA<sub>C,E</sub> | 3.16E-15, 6.26E-19 | 4.21E-17 | 4.12E-1 | 5.09E-35 | 6.01E-9 | 6.12E-15 | 4.12E-21 | 7.46E-63 |
| QEA<sub>C,O</sub> | 7.34E-10, 1.34E-13, 2.05E-11 | 1.17E-3 | 7.61E-30 | 2.13E-14 | 2.59E-10 | 5.15E-20 | 3.55E-26 | 1.18E-59 |
| G14              | QEA<sub>R</sub> | 1.63E-28, 3.12E-33, 7.08E-17 | 2.53E-1 | 1.06E-43 | 2.39E-4 | 3.58E-14 | 1.64E-4 | 1.59E-6 | 5.32E-78 |
| QEA<sub>C,E</sub> | 3.77E-23, 3.98E-28 | 5.22E-11 | 6.31E-2 | 6.26E-40 | 9.61E-11 | 1.67E-8 | 1.20E-9 | 7.04E-15 | 9.08E-77 |
| QEA<sub>C,O</sub> | 1.55E-19, 4.08E-24 | 2.36E-8 | 1.01E-2 | 3.49E-36 | 1.02E-11 | 3.06E-6 | 1.72E-10 | 1.43E-13 | 4.32E-73 |

Table 3 p-Values obtained from the two-tailed t-test of the difference between the average fitness values of the conventional QEAPS and the proposed methods (QEAPS<sub>R</sub>, QEAPS<sub>C</sub>): A bold number shows that the accuracy of the solution obtained by using a proposed method is significantly higher than that obtained by using a conventional method at the significance level of 0.05. An underlined number shows that a conventional method is significantly superior to a proposed method at the significance level of 0.05.

| Problem instance | Search algorithm | Rotation angle for the conventional QEAPS<sub>C</sub> [×π rad] | 0.005 | 0.010 | 0.015 | 0.020 | 0.025 | 0.030 | 0.035 | 0.040 | 0.045 | 0.050 |
|------------------|-----------------|-------------------------------------------------------------|------|------|------|------|------|------|------|------|------|------|
| G1               | QEAPS<sub>R</sub> | 3.44E-11, 3.32E-8, 3.10E-4 | 2.92E-1 | 4.42E-13 | 3.11E-1 | 1.00E-3 | 9.26E-1 | 3.45E-1 | 5.24E-32 |
| QEAPS<sub>C</sub> | 2.31E-8, 2.74E-4 | 1.72E-1 | 2.14E-1 | 2.27E-8 | 7.00E-4 | 3.13E-1 | 4.51E-4 | 1.26E-1 | 6.53E-28 |
| G12              | QEAPS<sub>R</sub> | 1.78E-1, 3.25E-3, 4.68E-1 | 9.70E-1 | 8.29E-8 | 1.78E-1 | 1.07E-4 | 6.00E-1 | 8.90E-4 | 7.54E-19 |
| QEAPS<sub>C</sub> | 3.40E-3, 2.21E-1 | 3.06E-1 | 2.62E-5 | 6.34E-2 | 2.83E-5 | 8.94E-1 | 9.89E-4 | 1.27E-4 | 1.03E-10 |
| G14              | QEAPS<sub>R</sub> | 2.39E-1, 2.21E-4, 3.50E-5, 8.33E-1, 1.50E-13, 1.86E-5, 5.47E-5, 8.27E-5 | 2.90E-1 | 3.39E-26 |
| QEAPS<sub>C</sub> | 4.27E-4, 1.15E-1 | 3.82E-1 | 2.99E-1 | 8.10E-8 | 2.27E-3 | 5.12E-2 | 3.55E-2 | 4.96E-4 | 3.98E-19 |

QEAPS). Dashed lines show the average fitness of the best solution of the proposed method (QEA<sub>R</sub>, QEAPS<sub>R</sub>, QEA<sub>C,E</sub>, QEA<sub>C,O</sub>, QEAPS<sub>C</sub>). Error bars show the standard error. The dark gray bars show the obtained solutions with higher fitness than those obtained by any proposed methods. For example, in Fig.8(a), conventional QEA with θ<sub>C</sub> = 0.005π [rad] is inferior to the three proposed methods since the bar lies below the dashed lines. The solution obtained by QEA<sub>C</sub> has the highest fitness. Here, QEA<sub>C,E</sub> has nearly equal performances to QEA<sub>C,O</sub>, i.e., the yellow and orange dashed lines appear to overlap. An another example, in Fig.8(b), conventional QEAPS with θ<sub>C</sub> = 0.005π [rad] is inferior to the two proposed methods shown as dashed lines. Moreover, the fitness of the solution obtained by QEAPS<sub>R</sub>, shown as a blue dashed line, is higher than that of QEAPS<sub>C</sub>, shown as an orange dashed line.

The p-values obtained from a two-tailed t-test of the difference between the average fitness values for the conventional methods and the proposed methods are shown in Tables 2 and 3. A bold number shows that the accuracy of the solution obtained by using a proposed method is significantly higher than that obtained by using a conventional method at the significance level of 0.05. An underlined number shows that a conventional method is significantly superior to a proposed method at a significance level of 0.05.

According to the experimental results in Fig.8, method R has the best individuality, followed by C-E and C-O in all cases. Moreover, the accuracy of the obtained solution is high when the diversity of the individuality increases.

The average fitnesses of conventional QEA when θ<sub>C</sub> = 0.030π, 0.040π, 0.045π [rad] in Figs.8(a), 8(c) and 8(e) and conventional QEAPS when θ<sub>C</sub> = 0.030π, 0.040π [rad] in Figs.8(b) and 8(f) and θ<sub>C</sub> = 0.020π [rad] in Fig.8(d) are better than the average fitness of QEA and QEAPS with introduced individuality method R. However, as shown in Table 2, there is no significant difference at the significance level of 0.05 between the conventional QEA with the optimized rotation angle θ<sub>C</sub> and the proposed QEA<sub>R</sub> when the problem used was G1. Although QEAPS with the optimized rotation angle θ<sub>C</sub> tends to obtain a solution with higher accuracy than QEAPS<sub>R</sub>, there is no significant difference between them at the significance level of 0.05 regardless of the instance of the maximum cut problem, as shown in Table 3. Moreover, introducing the proposed individuality to the quantum-inspired individual does not require the strict optimization of the rotation angle and the cumbersome process for design-
(a) Experimental results in the case of QEA, where the problem used was G1

(b) Experimental results in the case of QEAPS, where the problem used was G1

(c) Experimental results in the case of QEA, where the problem used was G12

(d) Experimental results in the case of QEAPS, where the problem used was G12

(e) Experimental results in the case of QEA, where the problem used was G14

(f) Experimental results in the case of QEAPS, where the problem used was G14

Fig. 9  Transition of the average convergence rate
ing the rotation angle can be eliminated.

Figure 9 shows the transition of the convergence rate [12] of the population. The convergence rate is a measure for confirming the state of the probability amplitudes in a qubit, which is close to 0.0 if the diversity of the qubit observation result is high and close to 1.0 if the diversity is low. The solid lines show the average convergence rate of all trials for each method. Dashed lines show the rank of the proposed individualities according to the average convergence rate in ascending order. In the case of QEA, the convergence rate increases in the order: QEAS, the diversity of QEAPS, the global search to a local search, where the search algorithm is QEA. On the other hand, in the case of QEAPS, the diversity of QEA is higher than that of QEAPS, from the initial phase to the final phase. It is important for evolutionary computations including QEA and QEAPS to create various solutions and search globally in the initial phase. From these experimental results, we have confirmed that the individuality of method R is genetically the most diverse of the proposed methods and promotes the diversification of the search in the initial phase. Moreover, in the case of QEA, it promotes the intensification of the search in the final phase.

5.2 Investigation of the magnitude of the rotation angle

To investigate the search performance as a function of the increment of the rotation angle $\Delta \theta$, we used QEA and QEAPS, in which the proposed individuality-introducing method R was implemented. Since this method obtained the best-quality solutions among the proposed individuality-introducing methods as shown in Sect. 5.1. The range of the rotation angle $\theta_{ij}$ was set to $[0.005\pi, 0.050\pi]$ [rad], and the magnitude of the increment $\Delta \theta$ was set to $10^{-5}\pi$, $10 \times 10^{-5}\pi$ and $100 \times 10^{-5}\pi$ [rad]. The number of nodes $M$ was 800. The number of individuals was 100 and the upper limit of the evaluation frequency was 2,000,000 as the termination condition of the search. The number of subpopulations was five and the interval of global migration was 100 generations, which are used in QEA. We performed 100 trials for each condition.

The experimental results are depicted in Fig. 10. The horizontal axis shows the increment of the rotation angle $\Delta \theta$, and the vertical axis shows the fitness of the best solution. The solid lines show the increment of the rotation angle $\Delta \theta$.
tion angle $\Delta \theta$ and the vertical axis shows the sum of the edge weights, that is, the fitness. Here, the experimental results with the increment $\Delta \theta$ set $500 \times 10^{-5} \pi$ [rad] are those of QEA$_{\theta}$ or QEAPS$_{\theta}$ in Sect. 5.1. Each bar shows the average fitness of the best solution when the termination condition is satisfied. Error bars show the standard error. Here, these differences in fitness are significantly different on the basis of the two-tailed t-test with a p-value of less than 0.05 in the following cases: between $\Delta \theta = 10^{-5} \pi$ [rad] and $\Delta \theta = 10 \times 10^{-5} \pi$ [rad] in the case of G12 when the search algorithm used was QEAPS; between $\Delta \theta = 10^{-5} \pi$ [rad] and $\Delta \theta = 100 \times 10^{-5} \pi$ [rad] and between $\Delta \theta = 10^{-5} \pi$ [rad] and $\Delta \theta = 500 \times 10^{-5} \pi$ [rad] in the case of G14 when the search algorithm used was QEA. That is, the average fitness of the best solutions does not decrease or is higher when $\Delta \theta$ is small. In particular, in the cases of G12 and G14 when the search algorithm used was QEA, the average fitness of the best solution is the highest when $\Delta \theta$ is smallest and tends to be high when $\Delta \theta$ is small. Therefore, it is not necessary to set the increment $\Delta \theta$ to a discrete value and the nonuniform rotation angle $\theta_{Cij}$ can be selected from a continuous space. That is, we only have to set the range of the rotation angle $\theta_{Cij}$.

6. Conclusions

In this study, we have shown that the proposed individuality can eliminate the cumbersome process required to design a rotation angle while ensuring the quality of the obtained solution. We only have to decide the range of the rotation angle, and it is not necessary to decide a strict rotation angle upon introducing the proposed individuality into the quantum-inspired individual. In further research, we plan to investigate the robustness of the proposed individuality using combinatorial optimization problems other than the maximum cut problem.

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References

[1] K.-H. Han and J.-H. Kim: Quantum-inspired evolutionary algorithm for a class of combinatorial optimization, IEEE Trans. Evol. Comput., Vol. 6, No. 6, pp. 580-593, 2002.
[2] K.-H. Han and J.-H. Kim: On setting the parameters of QEA for practical applications: Some guidelines based on empirical evidence, Genetic and Evolutionary Computation — GECCO 2003, Lecture Notes in Computer Science, Springer, Vol. 2723, pp. 427-428, 2003.
[3] S. Nakayama, T. Imabeppu and S. Ono: Pair swap strategy in quantum-inspired evolutionary algorithm, Genetic and Evolutionary Computation — GECCO 2006, Late Breaking Paper.
[4] S. Nakayama, T. Imabeppu, S. Ono and I. Iimura: Consideration on pair swap strategy in quantum-inspired evolutionary algorithm, IEICE Trans. Inf. Syst. (Japanese ed.), Vol. J89-D, No. 9, pp. 2134-2139, 2006.
[5] D. E. Goldberg: Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, 1989.
[6] M. Dorigo: Optimization, learning and natural algorithms, PhD Thesis, Politecnico di Milano, 1992.
[7] M. Dorigo, G. Di Caro and L. M. Gambardella: Ant algorithms for discrete optimization, Artif. Life, Vol. 5, No. 2, pp. 137-172, 1999.
[8] E. Bonabeau, M. Dorigo and G. Theraulaz: Inspiration for optimization from social insect behaviour, Nature, Vol. 406, pp. 39-42, 2000.
[9] B. Bullnheimer, R. F. Hartl and C. Strauß: A new rank based version of the ant system - A computational study, Cent. Eur. J. Oper. Res. Econ., Vol. 7, pp. 25-38, 1997.
[10] Y. Nakamichi and T. Arita: The effects of diversity control based on random selection in ant colony optimization, IPSJ J. (Japanese ed.), Vol. 43, No. 9, pp. 2939-2947, 2002.
[11] I. Iimura, T. Matsudome, T. Nakamichi and S. Nakayama: Consideration on individuality ant strategy by ant colony optimization in traveling salesman problem, IEICE J. D-I(Japanese ed.), Vol. J88-D-I, No. 4, pp. 900-905, 2005.
[12] Y. Moriyama, I. Iimura and S. Nakayama: Investigation on introducing qubit convergence measure to QEA in maximum cut problem, Proc. IEEE 10th Int. Workshop Computational Intelligence and Applications (IEEE IWCIA2017), CFP1761U, pp. 73-78, 2017.

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