Gravitational collapse in spatially isotropic coordinates

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Abstract We investigate the dynamical nature of the collapse process of a spherically symmetric star undergoing dissipative collapse. The star collapses from an initial static configuration by dissipating energy in the form of a radial heat flux. The perturbation in the density and pressure profiles are such that the star is always close to hydrostatic equilibrium. The temperature profiles are studied using a causal heat transport equation.

Keywords Gravitational collapse · Exact solutions · Einstein’s field equations · Radiating star model · Extended irreversible thermodynamics

1 Introduction

Seeking exact solutions to Einstein’s field equations, capable of describing realistic astrophysical systems, has been an area of active research ever since the discovery of the Schwarzschild solution in 1916 (Shapiro and Teukolosky 1983). Various techniques and assumptions based on particle physics, hydrostatic equilibrium and physical observations have been employed to generate reasonably viable stellar models. At the same time, various ad-hoc approaches have also been used to simplify the non-linearity of the field equations so as to generate solutions which are well behaved and can be utilized to describe realistic physical processes. A comprehensive study of exact static solutions of the Einstein field equations, based on physical acceptability, picks out a very small class of solutions that satisfy all the conditions for hydrostatic equilibrium and causality (Finch and Skea 1998; Delgaty and Lake 1998). Nevertheless, in the absence of any reliable information about the physics of matter content at extremely high densities, a geometrical approach has been found to be a meaningful technique to study compact stellar objects, a ‘natural laboratory’ for understanding particle interactions under extreme conditions. For example, the Tikekar super-dense stellar model (Tikekar 1990), describing the gravitational field of a highly compact spherically symmetric star, was shown to exhibit a reasonable EOS for neutron stars. Later, this observation has prompted many investigators to look for exact solutions capable of describing a large variety of astrophysical systems where relativistic effects can not be ignored; thereby expanding the Tikekar model to include charge (Sharma et al. 2001), pressure anisotropy (Karmakar et al. 2007), quark matter (Sharma et al. 2002), scalar fields (Sharma and Mukherjee 2001) and higher dimensional analogues (Singh and Kotambkar 2005; Patel and Singh 2001). Amongst many such realistic models, the static stellar model proposed of Pant and Sah (1985) is of particular interest. The Pant and Sah model describes a spherically symmetric compact star in spatially isotropic coordinates. The solution regains the well-known Buchdahl polytrope solution (Buchdahl 1964) of index 5. The physical viability of the Pant and Sah model was recently looked at in detail by Deb et al. (2012) and it has been shown that the model can be utilized to describe a wide variety of compact stellar objects including strange stars.

In this paper, we have incorporated dynamical effects into the Pant and Sah model (Pant and Sah 1985) by allowing...
certain model parameters to evolve with time. This has allowed us to investigate the non-adiabatic collapse of a star in a spatially isotropic background space-time. In our time-dependent model, we have assumed that the star begins its collapse from an initial static configuration by dissipating energy in the form of a radial heat flux. The collapse proceeds in such a manner which ensures that the mass loss is small and that the stellar body is in quasi-static equilibrium. This epoch corresponds to the stage just before the formation of the compact object. One regains the Pant and Sah model as the static limit of the dynamical collapse proceeds.

Although the issue of gravitational collapse was first taken up by Oppenheimer and Snyder (1939), the study of a more realistic collapsing scenario in the presence of dissipative processes was possible only when Vaidya (1953) provided the metric corresponding to the interior gravitational field of a radiating star. A formal treatment of the junction conditions required the smooth matching of the collapsing core to the exterior non-empty space-time and was provided by Santos (1985). These junction conditions provided the impetus for studying dissipative collapse, with much of the early work done by Herrera and co-workers (Herrera et al. 1998, 2002, 2004, 2008; Herrera and Santos 1997; Herrera and Barreto 2011; Herrera 2011). For a radiating collapsing star, the pressure at the boundary is proportional to the magnitude of the heat flux and hence gives rise to a temporal evolution equation for the metric functions. As in the case of a static model, various solutions for radiating stars have been found based on physics, dynamical stability and ad-hoc assumptions. The interior space-time of these models have been generalized to include (apart from heat flow) the anisotropic pressure, bulk viscosity, shear and the electromagnetic field (Bonnor et al. 1989; de Oliveira and Santos 1987; de Oliveira et al. 1998; Di Prisco et al. 2007, 1997; Pinheiro and Chan 2008; Chan 2000; Chan et al. 2003, 1994; Tikekar and Patel 1992; Maharaj and Govender 2000; Sharif and Iqbal 2009; Sharif and Siddiqi 2010; Sharif and Abbas 2011; Sharif and Fatima 2011; Barreto et al. 2007; Ghezzi 2005; Goswami and Joshi 2004; Thirukkanesh and Maharaj 2009; Govinder et al. 1998; Govender and Thirukkanesh 2009; Schafer and Goepper 2000; Sarwe and Tikekar 2010; Sharma and Tikekar 2012; Thirukkanesh and Govender 2013; Govinder et al. 2014). A comprehensive review of various approaches and analyses involving gravitationally collapsing systems may be found in Joshi and Malafarina (2011). An interesting approach was adopted by Kramer in which the interior Schwarzschild solution was written in isotropic coordinates and the mass parameter was allowed to become time-dependent (Kramer 1992). The radiating Schwarzschild-like solution had as its source term, a perfect fluid with heat flow. Since the interior was radiating energy, the exterior was non-empty and was described by Vaidya’s outgoing metric. Kramer provided a first integral of the boundary condition required for the matching of the interior to the Vaidya solution. Maharaj and Govender (1997) presented the full temporal behavior of the Kramer model in terms of Li integrals. The complicated form of the analytical solution for the temporal behavior did not warrant a full study of the physics of the model. The present work takes up the initiative to use the Kramer algorithm to provide a full descriptive model of dissipative gravitational collapse. As the collapse process begins in a massive star, after exhausting all its thermonuclear fuel, prediction of the final stage of the collapsing star becomes very much speculative in nature (Chandrasekhar 1934). In fact, one of the most outstanding challenges in general relativity has been the prediction of the end state of a gravitationally bounded system. In the context of the Cosmic Censorship Conjecture (CCC), the general relativistic prediction is that such a collapse must terminate in a black hole; though there are several counter examples where it has been shown that a naked singularity is more likely to be formed (Joshi and Malafarina 2011). In our dynamical model, we show that the star begins its collapse from an initial static configuration with acceptable physical conditions which are always close to hydrostatic equilibrium.

Our paper has been organized as follows: In Sect. 2, we have laid down the equations governing collapse in spherically symmetric and spatially isotropic coordinates. For a radiating star, the exterior space-time is appropriately described by the Vaidya metric (Vaidya 1953) and the junction conditions joining the interior and the exterior regions are given. By introducing a time dependent variable in the Pant and Sah model (Pant and Sah 1985), we have developed a dynamical model of the radiating collapsing star in Sect. 3. In Sect. 4, we have studied the physical behavior of the evolving star and in Sect. 5 we have investigated the thermodynamics of the collapsing star within the framework of extended irreversible thermodynamics. Some concluding remarks have been made in Sect. 6.

### 2 Interior and exterior space-times

We write the interior space-time of a spherically symmetric shear-free collapsing star in spatially isotropic coordinates as

\[ ds^2_{(1)} = -A^2(r,t)dt^2 + B^2(r,t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{1} \]

We assume that the material composition filling the interior of the collapsing object is a perfect fluid with out-flowing radiation and accordingly we express the energy-momentum tensor in the form

\[ T_{ij} = (\rho + p)u_iu_j + pg_{ij} + q_iu_j + q_ju_i, \tag{2} \]
where \( \rho \) is the energy density, \( p \) is the isotropic fluid pressure, \( u^i = (1/A)\delta^i_0 \) is the time-like 4-velocity of the fluid and \( q^i = (0, q, 0, 0) \) is the heat flux vector which is orthogonal to the velocity vector so that \( q^iu_i = 0 \). The Einstein’s field equations describing the dynamics of the system are then obtained as

\[
\rho = 3\frac{1}{A^2} \frac{B_r^2}{B^2} - \frac{1}{B^2} \left( 2 \frac{B_{rr}}{A} + \frac{2}{A} \frac{B_r}{B} + \frac{4}{r} \frac{B_r}{B} \right),
\]

\[
p = \frac{1}{A^2} \left( -\frac{2}{B^2} \frac{B_{tt}}{A} + \frac{1}{A^2} \frac{B_t^2}{B^2} + 2 \frac{B_t}{B^2} + \frac{2}{A} \frac{A_r}{B} + \frac{A_r}{r} \right),
\]

\[
q = -\frac{2}{AB} \left( -\frac{B_{tt}}{B} + \frac{B_t}{B^2} + \frac{2}{A} \frac{A_r}{B} \right).
\]

Combining Eqs. (3) and (4), we get

\[
\frac{A_{rr}}{A} + \frac{B_{tt}}{B} - \left( \frac{2}{r} \frac{B_r}{B} + \frac{1}{r} \right) \left( \frac{A_r}{A} + \frac{B_r}{B} \right) = 0,
\]

which is the pressure isotropy equation.

The exterior space-time, in the presence of an outgoing flux of radiation around the spherically symmetric collapsing matter source, is described by the Vaidya metric (Vaidya 1953)

\[
ds^2 = -\left( 1 - \frac{2m(v)}{r} \right) dv^2 - 2dvdr + r^2 \left[ d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right],
\]

where, the mass function \( m(v) \) is a function of the retarded time \( v \). Assuming that \( \Sigma \) divides the space-times into two distinct regions, the junction conditions joining smoothly the interior space-time (1) and the exterior space-time (7) across \( \Sigma \) forming the boundary of the star are obtained as Santos (1985)

\[
\rho \Sigma = (q) \Sigma,
\]

\[
m \Sigma = \left[ \frac{r^3 B r^2}{2A^2} - r^2 B_r - \frac{r^3 B_r^2}{2B} \right] \Sigma,
\]

where \( m \Sigma \) is the total mass within a sphere of radius \( r \Sigma \).

3 Generating dynamical solutions

Note that in Eq. (1), the metric potentials \( A(r, t) \) and \( B(r, t) \) are yet to be specified. To generate a viable dynamical model, let us assume that the system begins its collapse from an initial static configuration \((A_0(r), B_0(r))\). For the initial static configuration, we choose the Pant and Sah solution (Pant and Sah 1985) which describes the interior space-time of a static spherically symmetric star in isotropic coordinates. In our construction, we generalize the Pant and Sah solution so as to develop a viable model of a collapsing star. We note that Eq. (6) admits a solution

\[
A(r, t) = \frac{a(1 - \alpha(r)k(t))}{(1 + \alpha(r)k(t))},
\]

\[
B(r, t) = \frac{(1 + \alpha(r)k(t))^2}{(1 + r^2/R^2)},
\]

for an arbitrary \( k(t) \), where

\[
\alpha(r) = \sqrt{1 + \frac{r^2}{R^2}}.
\]

Obviously, the static limit of the model is obtained by setting \( k(t) = K \), a constant (i.e., \( k = 0 \)). For an evolving system, we need to determine \( k(t) \) which can be obtained by solving the junction condition (8). The resultant ‘surface equation’ in this construction turns out to be highly non-linear in nature and extremely difficult to solve. However, it is possible to generate an approximate solution of the equation by setting \( k(t) = K + \epsilon h(t) \), with \( 0 < \epsilon \ll 1 \). Neglecting terms \( O(\epsilon^2) \) and noting that pressure at the boundary of the initial static star vanishes, the surface equation then assumes a simple form

\[
\mu \dot{h} + v \ddot{h} + \eta h = 0,
\]

where \( \mu, v \) and \( \eta \) are constants evaluated at the boundary \( \Sigma \) and are given by

\[
\mu = \left[ \frac{\alpha(1 + K\alpha)^5}{a^2(K\alpha - 1)^2} \right] \Sigma,
\]

\[
v = \left[ \frac{\alpha(1 + K\alpha)^2 \sqrt{(1 - \alpha^2)(\alpha^2\lambda - 1)}}{aR(K\alpha - 1)^2} \right] \Sigma,
\]

\[
\eta = \left[ \frac{4\alpha - 6K\alpha^2 + 2\lambda K\alpha^6(1 + 2K\alpha(K\alpha - 1))}{R^2(k^2\alpha^2 - 1)^2} \right] \Sigma.
\]

Equation (13) is easily solvable and the most general solution of the equation can be written as

\[
h(t) = C e^{\frac{k t}{2\mu}(-\nu - \sqrt{\nu^2 - 4\mu\eta})} + D e^{\frac{k t}{2\mu}(-\nu + \sqrt{\nu^2 - 4\mu\eta})},
\]

where \( C \) and \( D \) are integration constants.

We assume that the collapse begins in the remote past \( t \rightarrow -\infty \) from an initial static configuration as the star loses its equilibrium. This implies that we must have \( k(t \rightarrow -\infty) = K \), where \( K \) is a constant as described in the static
At the boundary, we match the static interior solution to that of the Schwarzschild exterior, and obtain

$$\frac{(1 + K\alpha)}{(1 + b^2/R^2)} = \left(1 + \frac{m_s(b)}{2b}\right)^2,$$

(24)

$$K = \frac{1}{\sqrt{\lambda \alpha_{b}^2}}.$$

(25)

$$\left[(1 + a)^4 + K^4(1 - a)^4 - 8(1 + a)^2 + 16 - 2K^2(1 - a)^2 - 8K^2(1 - a)^2\right] + \left[2\lambda(1 + a)^4 - 16\lambda(1 + a)^2\right] - 8(1 + a)^2 + 32(1 + \lambda) - 2K^2(1 + \lambda)(1 - a)^2 - 8K^2(2 + \lambda)(1 - a)^2 + 2K^4(1 - a)^4\right]y^2 + \left[\lambda^2(1 + a)^4 - 8\lambda^2(1 + a)^2 - 8\lambda K^2(1 - a)^2\right]y^6 + 16\lambda^2y^8 = 0,$$

(26)

where,

$$y = \frac{b}{R}, \quad \alpha_{b} = \sqrt{\frac{1 + y^2}{1 + \lambda y^2}}.$$

Equations (23)–(26) can be utilized to fix the model parameters for a specific choice of mass and radius of the initial static star.

### 4 Physical analysis

In order to analyze the physical behavior of the collapsing model, let us start with a static model given by Deb et al. (2012) in which the parameters were set to be \(A = 4, \lambda = 0.1211\) and \(K = 2.2\). These parameters characterize a star of mass \(M = 2.44 M_{\odot}\) with a radius of \(r_0 = 8.197\) km and \(R = 1.819\) km. This characterizes the X-ray pulsar, 4U 1700-37, as studied by Deb et al. (2012).

In our time-dependent model, we let \(C = -1\) and \(\epsilon = 0.01\). By setting \(r = 16\) km, we observe a calculated mass of \(2.44 M_{\odot}\) which varies negligibly from past to present. This is evident in Fig. 1. The radiated mass is dissipated to the exterior spacetime via a radial heat flux. Figure 2 shows that the energy dissipated throughout the collapse process can be viewed as a weak heat flux approximation. Such a collapse process was previously investigated by Lemos (1998) for a Friedmann-like dissipative sphere. In their scenario the
particles making up the stellar fluid exhibited geodesic motion. It is interesting to note that we obtain similar results even though the four-acceleration of the particles making up the stellar fluid in our model is nonzero. The corresponding dynamical nature of the pressure and energy density are depicted in Figs. 3 and 4, respectively. In the next section we turn our attention to the thermal behavior of our collapsing model.

5 Thermal evolution

Let us now investigate the thermal evolution of the collapsing system generated in Sect. 4. It is well known that the Eckart formalism of thermodynamics suffers many pathologies, some of which include super-luminal propagation velocities for the dissipative fluxes as well as the prediction of unstable equilibrium states (Anile et al. 1998). Gravitational collapse of a stellar object is dissipative in nature and is usually accompanied by heat generation via neutrino emission or free-streaming radiation. Various investigations have shown that relaxational effects predict higher core temperatures and significantly different luminosity profiles when compared to their non-causal counterparts (Govender et al. 1998, 1999; Naidu et al. 2006; Govender 2013). In order to study the impact of the relaxational effects brought about by the heat flow, we will employ the truncated causal heat transport equation (Thirukkanesh and Maharaj 2009)

\[ \tau h_{ij} \dot{q}_j + q_i = -\kappa (h_{ij} \nabla_j T + T \dot{u}_i), \]

(27)

where \( \kappa \) is the thermal conductivity, \( \dot{u}_j = u_j;_i u^i \) and \( h_{ij} = g_{ij} + u_i u_j \) is the projection tensor and \( \tau \) is the relaxation time. We obtain the Eckart temperature by setting \( \tau = 0 \) in (27). We assume that the neutrinos are thermally generated within the stellar core with energies of the order of \( k_B T \). At neutron star densities, neutrino trapping takes place via electron-neutrino scattering and nucleon absorption. The mean collision time for thermally generated neutrinos is given by

\[ \tau_c \propto T^{-3/2}, \]

(28)

to good approximation (Martinez 1996). Following (28) we adopt a power-law dependence for the thermal conductivity and relaxation time:

\[ \kappa = \gamma T^3 \tau_c, \quad \tau_c = \left( \frac{\alpha}{\gamma} \right) T^{-\sigma}, \]

(29)
where $\alpha \geq 0$, $\gamma \geq 0$ and $\sigma \geq 0$ are constants. We further assume that the relaxation time is directly proportional to the mean collision time

$$\tau = \left( \frac{\beta \gamma}{\alpha} \right) \tau_c$$

(30)

where $\beta \geq 0$ is a constant. The causal transport equation (27) together with the above assumptions reduces to

$$\beta(qB)\dot{T}^{-\sigma} + A(qB) = -\alpha \frac{T^{3-\sigma}(AT)'}{B},$$

(31)

where the Eckart temperature $T_0$ is obtained by setting $\beta = 0$ in (31). In the case of constant collision time ($\sigma = 0$) we are in a position to write down the solution to (31) as

$$(AT)^4 = -\frac{4}{\alpha} \left[ \beta \int A^3 B(qB) \, dr + \int A^4 qB^2 \, dr \right] + F(t),$$

(32)

where $F(t)$ is an arbitrary function of integration. The function $F(t)$ can be determined from the effective surface temperature of a star as given by

$$(T^4)_{\Sigma} = \left( \frac{1}{r^2 B^2} \right) \Sigma \left( \frac{L_\infty}{4\pi \delta} \right),$$

(33)

where $L_\infty$ is the total luminosity at infinity and $\delta > 0$ is a constant.

Making use of the solution generated in Sect. 4 and for the particular case considered in Sect. 5, we have plotted the temperature within the collapsing stellar core as a function of time at $r = 1$ km in Fig. 5. The plots show an increase in temperature with time as is complemented with increases in pressure and energy density as already shown. The plot $\beta = 0$ represents the non-causal case. We see that a causal temperature, $\beta > 0$, is always greater than a non-causal temperature. It is clear that relaxational effects contribute to the enhancement of the temperature for a collapsing star system as described in our work. It is interesting to note that even though our model is based on a weak heat flux approximation ($0 < \epsilon < 1$), relaxational effects lead to very different outcomes for the temperature. Our results confirm earlier findings by Govender and Govinder (2002).

6 Discussions

In our work, we have generated a dynamical solution from the static stellar model of Pant and Sah (1985) to investigate the nature of dissipative collapse. This has been achieved by allowing a constant parameter in the static model to evolve with time. The resulting dynamical model is a radiating collapsing star with heat conduction enveloped by a radiation atmosphere. Though in our construction, the star begins its collapse from an initial static configuration described by the Pant and Sah model, unlike many previous models describing collapse from an initial static configuration, the usual method of assuming metric separability in their variables $r$ and $t$ has not been adopted in our approach. Secondly, the background space-time has been couched in spatially isotropic coordinates. The static model of Pant and Sah, as analyzed by Deb et al. (2012), has the following key features: (1) $\rho > 0$, $p > 0$; (2) $\rho \rho' < 0$, $p \rho' < 0$; (3) $dp/d\rho < 1$. This implies that the collapse begins from a physically acceptable initial configuration which includes the fulfillment of (at least) the weak energy condition. The collapse is found to proceed without formation of an event horizon, with heat generated mostly due to the collapse process as the mass has shown to be largely conserved. We have also studied the thermodynamics of the collapsing star within the framework of extended irreversible thermodynamics. Our results confirm earlier findings (through various different approaches) that relaxational effects can significantly alter the physical characteristics such as the temperature of the collapsing system. It must be pointed out that the spheroidal parameter, $k$ (in the static case) is usually chosen on an ad-hoc basis to fit theoretical data to observed data related to neutron stars and strange stars. Our approach in allowing the spheroidal parameter to dynamically evolve with time gives us snapshots...
of the collapse process, particularly during the latter stages, just before the formation of the remnant.

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