Operational capability evaluation based on improved generalized grey incidence analysis model

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Abstract. To solve the problem of operational capability evaluation of weapon equipment system-of-systems (WESoS), a capability-based method is proposed based on an improved generalized grey incidence analysis (GGIA) model. A hierarchy description framework is proposed, which establishes a mapping relationship between weapons and operational capabilities. Four types of capability requirement functions are proposed to assess the performance of weapons on tactical indicators. The deficiencies of the traditional GGIA model are analyzed, and the improvement is proposed to calculate the satisfaction degree of weapon equipment to a certain operational capability. Then, regardless of other subjective factors, the contribution degree of a certain operational capability to WESoS is determined combined with the maximum entropy model and genetic algorithm. Finally, the feasibility and effectiveness of the proposed algorithm are verified by experimental comparison.

1. Introduction

Weapon equipment system-of-systems (WESoS), an organic combination of multiple weapon systems, plays an increasingly important role in modern information warfare. Operational capability is a vital indicator of whether the WESoS satisfies its operational mission. Operational capability evaluation plays an important role in guiding the construction of WESoS, discovering capacity gaps, and optimizing the WESoS architecture.

In recent years, research on operational capability evaluation of WESoS has achieved some results. Aiming at the operational capability evaluation of the city system, Wei Xia et al. presented an evaluation model based on BP neural network and simulated annealing algorithm [1]. Xiaobing Song et al. introduced a model based on fuzzy sets and rough sets to assess the operational capability of WESoS, then the weight coefficient of each attribute value is studied [2]. Moreover, several researchers integrated ANP into operational capability evaluation and achieved some results [3][4], which makes the weight of each capability indicator more reasonable. Reference [5][6] used grey fuzzy comprehensive evaluation method to evaluate operational capability. Xili Huang et al. proposed two kinds of models for index aggregation and the aggregation relationship of WESoS index system is analyzed [7]. Pengcheng Luo et al. studied the methods to aggregate operational capability indices, and the concept of discount coefficient and a new framework to evaluate the operational capability of WESoS are proposed [8]. Reference [9] constructed a capability requirement structure and proposed three aggregate rules to get the top level sufficient capability degree.

Most of the above researches are directly from the operational capability or capability indicators, and the subjective methods such as expert scoring are used to study the overall operational capability of WESoS. However, the weaponry entities that ultimately determine the operational capability are
considered less. Starting from the operational mission, analyzing the operational capability requirements of the WESoS from the top down, and then transforming the abstract mission tasks into specific WESoS solutions through operational capability requirements is an inevitable requirement for adapting to the characteristic of modern warfare and development of weapons and equipment [10]. Based on this idea, an operational capability hierarchy description framework is proposed in this paper, which establishes a mapping relationship between weapons and operational capabilities. In order to assess the performance of weapons on tactical indicators, four capability requirement functions are proposed. Then, the defects of the traditional generalized grey incidence analysis method (GGIA) are introduced in detail, and the improvement is made in combination with the specific application background. The improved GGIA is used to obtain the satisfaction degree of weapons to its associated operational capability. Regardless of other subjective factors, the contribution degree of a certain operational capability to WESoS is determined combined with the maximum entropy model. Finally, the contribution degree of each operational capability is aggregated to obtain the operational capability value of the WESoS.

2. WESoS operational capability evaluation framework based on improved GGIA

2.1 Operational capability hierarchy description framework

The relationships between operational capability requirements of WESoS and weapon types are shown in figure 1, which is consistent with the capability-based idea [10]. The essence is to transform the abstract mission requirements into specific weapon equipment requirements through capacity requirements.

![Figure 1. The relationships between operational capability requirements and weapon types.](image)

As shown in figure 1, the operational capability requirement indicators and equipment tactical indexes establish the underlying relationships between operational capability and weapons. Based on this idea, an operational capability hierarchy description tree is constructed in form of “WESoS capability layer — operational capability layer — tactical index layer — weapons layer”. An example is shown in figure 2. It can not only choose weapons and equipment that are suitable for combat missions from a large library of weapons and equipment. Moreover, through operational capability evaluation, it has a clear and direct guiding role for the determination of the operational capabilities, performance indicators and technical indicators of future weapons and equipment, and proposes constructive development ideas for the evolution of weapons and equipment.

![Figure 2. Operational capability hierarchy description framework.](image)

2.2 Operational capability evaluation framework based on improved GGIA

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The method proposed in this paper is shown in figure 3. It is mainly divided into four processes.

1. **Capability requirement functions** convert weapons’ tactical data into support degree for indicators. Improved GGIA method takes the support degree and the mapping relationships in figure 2 as input, and determine the satisfaction degree of weapons to its associated operational capability. Then, through maximum entropy model and genetic algorithm (GA), the contribution of operational capability to WESoS is obtained. Finally, according to the relationships between capabilities, the operational capability value of WESoS is calculated by the aggregation method proposed in the reference [8][9].

3. **Evaluation Algorithm**

3.1 **Capability requirement functions**
Operational capability requirement indicators correspond to the weapons tactical indexes, and describe the indicator requirements for achieving a capability. Capability requirement functions depict the support degree of operational capability indicators in tactical index layer.

Tactical indicators are mainly divided into two categories: qualitative indicators and quantitative indicators. For qualitative indicators, a number of experts in relevant fields are invited to give a comprehensive evaluation within 0~1 according to the requirements of combat missions and the tactical parameters of weapons, and use it as the support degree for the indicator. For the quantitative indicator, capability requirement functions are divided into the lower limit type, the upper limit type, the center type and the intermediate type, as shown in figure 4, in which the horizontal axis $d_i (i=1,2,3,4)$ represents the limit value or the optimal value of the capability indicator. The support degree of weapons for the quantitative indicator is evaluated according to different indicator types, combined with the corresponding dimensionless function.

3.2 **Defects of GGIA and improvements**

3.2.1 **GGIA model and its defects**
Grey relation analysis (GRA) model is proposed by professor Julong Deng [11], which relies on the distance between corresponding points of the sequences to measure similarity. However, the order of sequences in this model directly affects the results, and the degree of incidence $\gamma(X_0,X_i)$ not only related to $X_0$ and $X_i$, but also related to $X_j (j \neq 0,i)$. GGIA was built upon GRA by professor Sifeng
Liu [12], which includes absolute, relative and synthetic grey incidence analysis model. Assume sequences:

\[ X_i = (x_i(1), x_i(2), \cdots, x_i(n)) \quad \text{and} \quad X_j = (x_j(1), x_j(2), \cdots, x_j(n)) \]

and their initial zero starting point images are

\[ X_i^0 = (x_i^0(1), x_i^0(2), \cdots, x_i^0(n)) \quad \text{and} \quad X_j^0 = (x_j^0(1), x_j^0(2), \cdots, x_j^0(n)) \]

where \( x_i^0(k) = x_i(k) - x_i(1) \), \( x_j^0(k) = x_j(k) - x_j(1), k = 1, 2, \cdots, n \). GGIA is formulated in formula (1), (2) and (3) respectively as follows:

\[
\epsilon_{ij} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_i - s_j|}
\]  

(1)

where \( s_i = \int_1^n x_i^0(t)dt \), \( s_j = \int_1^n x_j^0(t)dt \) and \( s_i - s_j = \int_1^n (x_i^0(t) - x_j^0(t))dt \).

\[
\gamma_{ij} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_i - s_j|}
\]  

(2)

where \( s_i' = \int_1^n x_i^0(t)/x_i(1)dt \), \( s_j' = \int_1^n x_j^0(t)/x_j(1)dt \) and

\[
s_i' - s_j' = \int_1^n (x_i^0(t)/x_i(1) - x_j^0(t)/x_j(1))dt .
\]  

(3)

where \( \theta \in [0,1] \) is called synthetic coefficient.

Absolute degree as in equation (1) used to analyze the relations between absolute quantities in sequences. Relative degree as in equation (2) mainly deals with the relation of changing velocity of sequence contrasted to the initial point. Synthetic degree as in equation (3) not only reflects the similarity level, but also depicts the degree of closeness of the rates of change relative to their initial points.

\( |s_i - s_j| \) depicts the absolute value of differences between the area of the parts with \( X_i^0 \) on the top of \( X_j^0 \) and that of the parts with \( X_i^0 \) beneath \( X_j^0 \). As shown in figure 5, when there are intersections between \( X_i^0 \) and \( X_j^0 \), incorrect results may be caused by area offset.

In figure 5, it is seen intuitively that \( X_i^0 \) is more similar to \( X_j^0 \) than \( X_j^0 \) in geometry. However, due to the area offset between \( X_i^0 \) and \( X_j^0 \) in the subinterval, \( \epsilon_{ij} \) is greater than \( \epsilon_{ik} \), which violates our expectations.

3.2.2 Improvements and application

The improved GGIA translates the support degree (weapons to capability indicators) into the weapons’ satisfaction degree with their associated operational capability. Here, \( X_o = (x_o(1), x_o(2), \cdots, x_o(n)) \) is
the optimal value sequence of tactical indicators, and its zero starting point image is
\( X^o_j = (x^0_0(1), x^0_0(2), \cdots, x^0_0(n)) \) as in section 3.2.1. \( X_j = (x_j(1), x_j(2), \cdots, x_j(n)) \) is the support degree sequence of weapon \( j \) to capability indicators. To obtain the satisfaction degree to the optimal value of indicators, we redefine \( X^o_j = (x^o_j(1), x^o_j(2), \cdots, x^o_j(n)) \), where \( x^o_j(k) = x_j(k) - x_0(k), k = 1, 2, \cdots, n \).

To overcome the area offset in section 3.2.1, the meaning of \( s_j \) need to be redefined. The area of all shaded parts should be summed together. Hence, the intersection point in every subinterval should be calculated firstly. The intersection point in subinterval \([k, k+1]\) is illustrated by \((x, y)\):

\[
\begin{align*}
x &= k + \frac{x^o_j(k) - x^o_0(k)}{\sigma^o_{0j}(k)} \\
y &= x^o_j(k) + \frac{[x^o_j(k+1) - x^o_0(k)][x^o_0(k) - x^o_j(k)]}{\sigma^o_{0j}(k+1) - \sigma^o_{0j}(k)} \quad \text{(4)}
\end{align*}
\]

where \( \sigma^o_{0j}(k+1) = x^o_j(k+1) - x^o_0(k+1), \sigma^o_{0j}(k) = x^o_j(k) - x^o_0(k) \). After calculation, \( \left| s_0 - s_j \right|_{\text{imp}} \) in improved GGIA is presented as follows:

\[
\left| s_0 - s_j \right|_{\text{imp}} = \begin{cases} 
\sum_{k=1}^{x-1} \frac{\sigma^o_{0j}(k+1) + \sigma^o_{0j}(k)}{2} & \text{if } (x, y) \notin [k, k+1] \\
\sum_{k=x}^{x-1} \frac{(k+1-x)\sigma^o_{0j}(k+1) + (x-k)\sigma^o_{0j}(k)}{2} & \text{others}
\end{cases} \quad \text{(5)}
\]

So far, satisfaction degree of weapons to its corresponding capability is obtained by improved GGIA in equation (3). Synthetic coefficient reflects the different emphasis of decision-makers on the amounts and rates of indicator value changes.

3.3 Contribution degree algorithm

Information entropy model depicts the un informativeness of a system [13]. Maximum entropy only obtains feature information from known facts, and does not make any assumptions about other unknown information. By choosing the distribution with the maximum entropy allowed by our information, the most reasonable distribution is chosen. Therefore, in the absence of other supporting information, the weights of all weapons associated with a certain combat capability are obtained using the maximum entropy model.

For a capability associated with \( w \) weapons, according to maximum entropy model, the problem is formulated to find the maximum \( E \) in equation (6):

\[
E = -\sum_{j=1}^{w} \rho_{0j} \mu_j \cdot \ln \left( \rho_{0j} \mu_j \right) \quad \text{(6)}
\]

under the constraint:

\[
\sum_{j=1}^{w} \mu_j = 1 \quad \text{(7)}
\]

where \( \rho_{0j} \) is the satisfaction degree in equation (3), \( \mu_j (j = 1, 2, \cdots, w) \) is the weight of each weapon. The above constraint optimization problem is solved by GA in this paper. After each iteration, normalized gene correction was performed under the constraint equation (7). Finally, contribution degree of the capability to WESoS is formulated by:

\[
c = \sum_{j=1}^{w} \rho_{0j} \mu_j \quad \text{(8)}
\]
4. Experimental comparison and analysis

In this section, GRA, GGIA, and improved GGIA are compared in a case. Taking the reconnaissance discovery capability in figure 2 as an example, the corresponding capability indicators and weapons are shown in table 1. Assuming the optimal value of all indicators is 1. Reliability, anti-jamming, and stability are qualitative indicators. Location accuracy and resolution are upper limit type indicators, and support degrees are calculated with figure 4(b). Reconnaissance range and frequency bands belong to lower limit type and intermediate type indicator respectively, and their corresponding support degrees are obtained by figure 4(a) and figure 4(d) respectively.

| Location accuracy | Resolution | Reconnaissance range | Frequency bands | Reliability | Anti-jamming | Stability |
|------------------|------------|----------------------|---------------|-------------|--------------|-----------|
| Intelligence satellite | 0.85 | 0.90 | 1.00 | 1.00 | 0.95 | 0.90 | 0.98 |
| Electronic scout A | 0.73 | 0.80 | 0.90 | 1.00 | 0.87 | 0.87 | 0.92 |
| Electronic scout B | 0.95 | 0.92 | 0.95 | 1.00 | 0.95 | 0.94 | 0.98 |
| Tactical scout A | 0.84 | 0.80 | 0.82 | 0.97 | 0.90 | 0.85 | 0.75 |
| Tactical scout B | 0.74 | 0.85 | 0.86 | 1.00 | 0.88 | 0.86 | 0.70 |
| Scout helicopter A | 0.83 | 0.85 | 0.73 | 1.00 | 0.79 | 0.70 | 0.77 |
| Scout helicopter B | 0.77 | 0.83 | 0.74 | 0.78 | 0.90 | 0.73 | 0.65 |

There are seven weapons corresponding to the operational capability, and their support degrees to corresponding indicators are presented in table 1. The corresponding satisfaction degrees obtained by GRA, GGIA, and improved GGIA are shown in table 2. Here, synthetic coefficient \( \theta = 0.5 \).

| Intelligence satellite | Electronic scout A | Electronic scout B | Tactical scout A | Tactical scout B | Scout helicopter A | Scout helicopter B |
|-----------------------|--------------------|--------------------|-----------------|-----------------|-------------------|-------------------|
| GRA                   | 0.7838             | 0.6186             | 0.8088          | 0.5603          | 0.5812            | 0.5277            |
| GGIA                  | 0.7247             | 0.6633             | 0.9756          | 0.9139          | 0.6811            | 0.9025            |
| Improved GGIA         | 0.7613             | 0.6819             | 0.8672          | 0.7313          | 0.6839            | 0.6877            |

The original sequences and their zero starting point images in the absolute degree and relative degree in improved GGIA are shown in figure 6(a), (b), (c) respectively.

Figure 6. Original sequences and their zero starting images in improved GGIA.

In figure 6(a), it is clear that scout helicopter B sequence is far from the optimal value sequence. However, the GGIA algorithm yields a satisfaction degree of 0.9308, which is obviously unreasonable. GRA only finds the relative satisfaction degree relative to other weapons. Satisfaction degrees obtained by GRA are not only directly affected by other sequences, but the order of \( X_0 \) and \( X_j \). Improved GGIA accurately reflects the satisfaction degree of weapons to their associated capability, and solves the defects of GGIA. Absolute degree in improved GGIA depicts the amount of the satisfaction gap between the values of weapon tactical indexes and the optimal value of capability indicators. Relative degree in improved GGIA reflects the rate of changes with respect to initial indicator value, and the synthetic degree comprehensively reflects the satisfaction degree of weapons to the corresponding capability.

After calculation in equation (6), (7) and (8), the weight vector of all weapons is \( [0.1608, 0.1117, 0.1510, 0.1428, 0.1398, 0.1448, 0.1490] \). And the contribution degree of reconnaissance discovery
capability to WESoS is 0.7338. According to the relationships between capabilities, the operational capability value of the WESoS is calculated by the aggregation method.

5. Conclusion
A capability-based method is proposed based on improved GGIA to evaluate the operational capability of WESoS. The evaluation framework is proposed, and the role of each module that makes up the framework is studied in detail. In order to assess the performance of weapons on tactical indicators, four capability requirement functions are proposed. The defects of GGIA are analyzed, and the improvement measures are elaborated. Finally, the feasibility and reasonability of the improved GGIA are verified by experimental comparison with GRA and classical GGIA.

References
[1] Xia W, Liu X X, Fan Y Z, Fan J L. (2017) Combat capability evaluation of city system based on mix-genetic algorithm BP neural network[J]. System Engineering and Electronics.
[2] Xiao B S, Fang Y W, Yun-Shan X U, et al. (2010) Campaign capability evaluation of weapon systems based on fuzzy sets and rough sets[J]. Systems Engineering & Electronics, 32(6):1263-1265.
[3] Zhang D, Guo Q S, Zhi-Guo L I. (2015) Capability limited-hierarchy evaluation of weapon equipment system based on ANP[J]. Systems Engineering & Electronics.
[4] Shi F L, Yang F, Yong-Ping X U, et al. (2011) Power index method for operational capability evaluation of weapon equipment based on ANP and simulation[J]. Systems Engineering-Theory & Practice, 31(6):1086-1094.
[5] Wang J S, Zou L, Sun X F. (2013) Research on Gray Fuzzy Comprehensive of Net War Capability[J]. Modern Defence Technology.
[6] Shu Y, Tan Y J, Liao L C. (2009) Combat Capability Evaluation of Weapon Equipment System Based on Capability Requirement[J]. Ordnance Industry Automation.
[7] Huang X L, Hong-Mei D U. (2010) Evaluation of Combat Capability for Integrated Electronic Warfare System[J]. Ordnance Industry Automation.
[8] Luo P C, Fu P F, Zhou J L. (2005) Framework to evaluate the combat capability of weapons SoS[J]. Systems Engineering & Electronics.
[9] Cheng B, Tan Y J, Huang W, et al. (2011) Weapon system-of-systems evaluation based on a capability requirement perspective[J]. Systems Engineering & Electronics, 33(2):320-323.
[10] Li Q L, Guo Q S, Yang X Y. (2009) A New Angle of View About the Analysis of Weapon Equipment Requirements[J]. Journal of the Academy of Equipment Command & Technology.
[11] Deng J L. (1985) The relational space in Grey System Theory, Fuzzy Mathematics, no.2, pp. 1-10.
[12] Liu S F, Lin Y. (2006) Grey Information: Theory and Practical Applications. London: Springer-Verlag London Ltd.
[13] Jaynes E T. (1957) Information Theory and Statistical Mechanics[J]. Physical Review, 106(4):620-630.