Spray Generation by Plunging Breakers - Part 2. Droplet Characteristics

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The positions, diameters ($d \geq 100 \, \mu m$), times and velocities of droplets generated by three plunging breaking waves are measured as the droplets move up across a measurement plane located 1.2 cm above the highest point reached by the crests during breaking. The three breakers are created by dispersively focused wave packets that differ primarily only through the overall amplitude of the wave maker motion used to generate them. The breakers are designated qualitatively by their intensities: weak, moderate and strong. The droplets are measured with two in-line cinematic holographic systems operating at 650 holograms per second with measurement volumes that span the width of the tank. In combination with the wave profile measurements described in part 1 of this two part paper, the droplet measurements are used to explore the mechanisms of droplet generation and the effects of these mechanisms on the measured properties of the droplets and their motion.

It is found that there are four major mechanisms for droplet production, closure of the indentation between the top surface of the plunging jet and the splash that it creates (labeled Region I-A), the bursting of large bubbles that were entrapped under the plunging jet at impact, splashing and bubble bursting in the turbulent zone of the front face of the wave (combined with the large bursting bubbles in a region labeled I-B) and the bursting of small bubbles that reach the water surface at the crest of the nonbreaking wave following the breaker (Region II). The droplet diameter distributions for the entire droplet set for each breaker contain separate small- and large-diameter regions of power law behavior that cross at a diameter, $d_i$, which increases monotonically from 820 \, \mu m to 1480 \, \mu m from the weak to the strong breaker, respectively. Similar power law behavior is found in Regions I-A and I-B.

The average droplet speed in Region I-A over the three breakers is 1.02 m/s and the average direction is only about 8.9° downstream from vertical (downstream is defined herein as the direction of wave travel) while in I-B the average speed is 1.21 m/s and the average direction is 33.9° downstream from vertical. The near vertical direction in Region I-A is thought be be due to the nature of the closure of the large single crater in the first indentation. The average speed in Region II is 0.74 m/s and is directed only 15.9° downstream from vertical. Since it is expected that there would be an omni-directional velocity distribution for droplets from a field of small bursting bubbles on a calm water surface, the downstream component of the average velocity in Region II is though to be the result of the downstream fluid particle motion at the crest of the following wave. Results of a simple calculation of the droplet motions is presented that, together with the droplet data, indicates the strong influence of the motion of the air on the droplet trajectories.

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Abstract must not spill onto p.2
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1. Introduction

As mentioned in the introduction to Part 1, sea spray is a natural phenomenon with wide ranging implications in the transfer of mass, momentum and energy between the ocean and the atmosphere, see for example Andreas (2002), Melville (1996), or Andreas (1992). These spray-droplet-augmented transfer processes can have a dramatic impact on the weather, climate (Andreas & Emanuel (2001)) and chemical reactions that take place near the water surface, see for example Galbally et al. (2000) and Prather et al. (2013). It is widely accepted that breaking waves play a key role in the production of sea spray droplets. In breaking events (spilling or plunging), droplets are thought to be produced by splashing, wind shear, and through the bursting of breaker-entrained air bubbles as they return to the water surface. The generation mechanisms of sea spray droplets have been widely studied, and Veron (2015) has given a thorough review of the state of knowledge of sea spray droplet generation and behavior.

A number of studies have focused on the vertical distributions of droplet characteristics in wind wave systems in the field and the laboratory. In the field, Wu et al. (1984) conducted measurements of the droplet size spectrum in the Delaware Bay for droplet diameters $d > 50 \, \mu m$ and found that the normalized droplet diameter distribution consisted of two linear regions when plotted on a log-log plot. de Leeuw (1986) measured droplets ($10 \leq d \leq 100 \, \mu m$) in the North Atlantic at wind speeds up to 11 m/s and at heights up to 11 m above the instantaneous sea level. From the data, the effect of wind speed on the function describing the variation of droplet concentration and diameter with vertical height was explored. Smith et al. (1993) measured droplets ($2 \leq d \leq 50 \, \mu m$) off the west coast of Scotland for wind speeds up to 30 m/s and quantified the the relationship between droplet production rate and wind speed. In the laboratory, Wu (1979) studied the problem in a wind wave tank where he found the droplet diameter distribution curve plotted in log-log coordinates drops off rapidly for $d > 200 \, \mu m$, in qualitative agreement with his above referenced field experiments in 1984. Koga (1981) studied the movement and production mechanism for droplets generated by wind waves in light wind conditions. Anguelova et al. (1999) observed sprays of spume droplets tearing off wave crests and measured droplets with diameters in the range $1.3 \leq d \leq 10.5 \, mm$. Veron et al. (2012) studied spray generation at high wind speeds (31.3 to 47.1 m/s) and that the numbers of large droplets exceeded theoretical predictions. More recently, Erinin et al. (2022) measured droplet speed and acceleration statistics of spray generated at wind speeds up to 12 m/s in a wind-wave field in a laboratory tank and reported on droplet speed and acceleration probability density functions. The authors found droplets with speeds greater than the measured wind speed.

A model accounting for the droplet production rate with wind speed was first proposed by Monahan et al. (1986) and refinements and additional models were proposed by de Leeuw (1986), Andreas (1992), Wu (1990) and Andreas et al. (1995). However, in a review by Andreas (1998) it is pointed out that estimates of the production rate of droplets can vary over six orders of magnitude.

In the review article, Veron et al. (2012) postulates that there are two primary generation mechanisms for droplet generation in the ocean. The first is due to bubbles, initially entrained by the breaking processes, rising to the free surface and popping. The popping bubble generates two different types of drops, film and jet drops. Film droplets are generated when
the bubble film fragments creating many droplets with diameters reported to range from 20 nm to 200 µm in experiments, Lhuissier & Villermaux (2012). In these experiments, jet droplets were observed to form as a result of the violent bubble cavity collapse which ejects up to six droplets ranging in size from 2-200 µm. Results of similar experiments were reported by Wu (2002) and others. More recently, revised scaling arguments for the size of droplets generated by bursting bubbles have been presented by Gañán Calvo (2017). Deike et al. (2018) studied the conditions, including the speed of upward traveling jet, under which a jet droplet is observed by using experimental and numerical results. They found that the jet ultimately controls the velocity of the resulting droplets. It is important to emphasize that these studies primarily involve single bubble bursting events in calms water and not a field of bursting bubbles as found in wave breaking events.

The second primary mechanism for droplet generation occurs when the wind speed above the breaking wave is sufficiently high to tear off water from the crest of the wave. These "spume droplets" are thought to be largest droplets, say \( d \geq 1.0 \) mm, measured in the above-mentioned wind wave systems in the laboratory and the field. Tang et al. (2017) developed a direct numerical simulation (DNS) scheme to study the generation and transportation of spume droplets by wind blowing over breaking waves. They found that droplets are generated near and/or behind the wave crest, depending on the wave age. To date, simulations are only able to resolve relatively large droplets, and often the droplets are represented by points in the computations. Veron (2015) also identifies the droplets that may be produced by the plunging jet impacting on the free surface, although it is believed that this mechanism is not as efficient at producing droplets. Lubin et al. (2019) discussed the instabilities which may be responsible for air-entrainment and droplet generation in breaking waves via numerical and experimental visualizations.

Droplet production by breaking waves generated without wind in CFD and laboratory wavetanks has also been explored. In the CFD investigations, DNS calculations are performed with the domain covering one streamwise wavelength (\( \lambda \)) of a periodic uniform wavetrain and the initial wave surface and flow field in the water are taken from 3rd order Stokes wave theory. The initial wave steepness is chosen to be excessive for 3rd order theory and the wave evolves to a plunging breaker. The dynamics of breakers in uniform wavetrains with the above-described Stokes wave initial conditions has been explored by a number of authors, see for example the 2D studies by Chen et al. (1999) and Iafrati (2009). Droplet generation in breaking Stokes wavetrains is explored using 3D DNS in the work of Wang et al. (2016) and Mostert et al. (2022). These studies are important numerical counterparts to the experiments presented herein. In Wang et al. (2016), energy dissipation, air entrainment and droplet generation are explored. The authors identify droplet production mechanisms similar to those discussed herein and provide droplet diameter distributions that are fitted well with a power law function. From the dimensionless parameters given in the paper, the gravity wavelength in the calculations is 30 cm and the minimum computational grid spacing is 65 µm. For a droplet resolved with five grid points across its diameter, the minimum resolved droplet diameter would be 0.26 mm. In Mostert et al. (2022), the discussion stresses the computed droplet diameter and velocity distributions as well as the temporal histories of the droplet generation process. Scalings based on measurements of the wave profile at the moment of jet impact are explored. From the dimensionless parameters given in the paper, the gravity wavelength ranges from 24.2 cm to 54.04 cm and the grid resolutions are 0.117 mm and 0.264 mm, respectively. Thus, with a droplet spanning 5 grid points across its diameter, droplets with diameters as small as 0.468 and 1.056 mm, respectively, are resolved.

In the present paper, droplet measurements in three plunging breakers generated mechanically with dispersively focused wave packets are presented. These are the same waves for which the profile measurements were reported in Part 1. The wave maker motion profile was
only varied primarily by changing the overall wave amplitude while keeping the frequency
components and their phases and relative amplitudes constant. The average frequency of
the packets was 1.15 Hz, which by linear theory corresponds to a wavelength of 118.0 cm.
The positions, diameters, and velocities of droplets with \( d \geq 100 \mu m \) are measured with a
cinematic in-line holography technique at many streamwise measurement locations covering
approximately 0.9\( \lambda_0 \) downstream of the location of jet impact. Only droplets that pass upward
through a plane that is 1.2 cm above the highest point reached by the breaking crest during
a breaking event are measured. The temporal and spatial distributions of droplet numbers,
diameters and velocities are reported and related to features of the profile measurements in
Part 1. It should be noted that data for the weak breaker was partially presented in Erinin
et al. (2019). That data set and analysis was smaller in scope and included wave surface
profiles for only a single breaking event.

In the following, the experimental details of the cinematic in-line holography measurements
of the droplets are presented first in § 2. This is followed, in subsections § 3.1 to § 3.5, by
descriptions and discussions of the droplet measurement results. Finally, the conclusions of
this study are discussed in § 4.

2. Experimental Details

The droplet measurements were performed in the same facility and with the same three
waves as in the wave profile measurements described in Part 1 of this two-paper sequence.
The Part 1 paper includes descriptions of the wave tank, wave maker, wave maker motions,
instrument carriage, experimental procedures and the analysis of the profiles of the three
waves studied herein. For each of these three waves, the average wave packet frequency was
\( f_0 = 1.15 \) Hz (\( T_0 = 1/f_0 = 0.870 \) s). In this section, only the droplet measurement techniques
are described.

2.1. Droplet Measurements Using In-line Holography

The droplets generated by the three breaking waves are measured with two synchronized
identical cinematic in-line holographic systems, see figure 1 for details. The two systems
are attached side by side to the instrument carriage with a horizontal distance of 40.6 cm
between their optical axes. The bottom edges of the images are horizontal and located at the
same height, 1 cm above the highest height reached by the breaking crest surface for each
wave. These wave heights are (from table 3 of Part 1) 107.8, 110.6, and 111.5 mm for the
weak, moderate and strong breakers, respectively.

The laser pulses and cameras are synchronized to take holographic images at a rate of
650 pps for a duration of 1.974 s (2.270\( T_0 \)) starting at approximately the time of jet impact
in each breaking event. Holographic image sequences are taken at 28 streamwise locations
by moving the carriage to 14 fixed positions. The results are interpolated to cover regions
between measurement locations where no hologram image sequences are recorded. The
droplet measurement locations cover a streamwise region from just before the jet impact
site to approximately 1 meter downstream. At each location, 10 experimental runs were
performed for a total of 140 individual breaking events for each of the three breakers.

Each of the holographic images are processed digitally in the following manner. First,
the background image (recorded before the breaking event in each run) is subtracted from
the recorded hologram. This subtraction removes or reduces interference patterns from
imperfections in the optical system, and dirt and water droplets stuck to the plastic walls
of the tank. The resulting hologram is digitally reconstructed every 5 mm in the \( z \) (cross-
tank) direction using the Fresnel-Huygens paraxial approximation (Katz & Sheng 2010) via a
GPU-based reconstruction algorithm provided by Professor Joseph Kats from Johns Hopkins

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University. Then, an image consisting of the maximum intensity at each \( x \)-\( y \) pixel location in the stack of reconstructed images is created to detect all of droplets in the 3D space of the hologram. The resulting image is converted to a binary image via a threshold and any droplets that may be present in the image are detected. For each detected droplet, a 200-by-200-pixel window around the diameter of the droplet is reconstructed digitally every 500 \( \mu \)m in \( I \). A method similar to the one outlined in Guildenbecher et al. (2013) is used to determine the \( I \) plane of best focus of the droplet, which is then taken as the \( I \) position of the droplet center. The hologram is then reconstructed at this \( I \) and the droplet’s diameter and the \( G \)-\( H \) position of its center are measured with a custom method using an inverse hyperbolic tangent function that is fitted to the intensity profile of the droplet, see Erinin (2020). Because of difficulties in reconstructing and accurately measuring the size of droplets near the image boundaries, where portions of the droplet’s diffraction pattern are cut off, only droplets measured when they are at least 200 pixels inside the boundaries of the 2560 x 1600 pixel images are counted in the data set. The horizontal plane containing the lower boundary of this inner rectangle in the two images is referred to in the following as the measurement plane. This plane is located 2 mm (200 pixels \( \times \) 10 \( \mu \)m/pixel) above the bottom edge of image, i.e. 1.2 cm above the highest height reached by the breaking crest surface for each wave. For the strongest breaker, the measurement plane is 12.35 cm above the still water level. A schematic drawing showing the hologram measurement locations relative to a single breaking wave profile is given in figure 2.

In order to calibrate the system and ensure that even the small droplets can be reconstructed and measured accurately across the entire width of the tank, holograms of a custom target are recorded with the target positioned at various locations across the tank width. The calibration target consists of a glass slide with 14 black chrome sputter deposited circles of diameters \( d_c \) ranging from 30 to 3000 \( \mu \)m. A motorized linear traverser is used to accurately place the
target at a range of \( z \) positions where calibration holograms are recorded. The diameters and \( z \) positions of the circles are then measured from the reconstructed holograms. The errors in the hologram-based measurements are then assessed by comparing the known and measured values of \( d_c \) and \( z \). The maximum droplet diameter measurement error is determined by reconstructing the calibration target at the farthest \( z \) distance from the focal plane and is not expected to be more than 2.5 percent. Thus, each hologram provides the diameters and 3D positions of all of the droplets with diameters \( \geq 100 \) \( \mu \)m within the imaged volume at all positions across the entire width of the tank. The accuracy of the \( x \)-\( y \) position measurements is on the order of one pixel or \( \approx 10 \) \( \mu \)m.

It should be kept in mind that the LIF water surface profile images described in Part 1 are captured over a time interval of approximately 1/650 s while the holographic images of the droplets are captured over the duration of the Nd:YLF laser light pulse, approximately 30 ns. Also, the droplets are measured over the entire width of the tank while the water surface profiles are measured over the thickness of the light sheet, about 1 mm, in the centerplane of the tank.

2.2. Droplet Tracking Algorithm

The droplets from the breaking wave are tracked in time as they move through the 3D image space by using a tracking algorithm based on a modified nearest-neighbor algorithm developed for Brownian particle motion by Crocker & Grier (1996), see Erinin (2020) for details. Each trajectory is then fitted with separate second order polynomials for \( x_d(t) \), \( y_d(t) \) and \( z_d(t) \). The polynomials are then used to find the time, \( x \) and \( z \) positions, and velocity of each droplet as it crosses the measurement plane. Only droplets that are moving up through the measurement plane are included in the data set. Droplets that are moving downward through the measurement plane or into the 3D image space through its top or side surfaces are assumed to have been accounted for moving up across the measurement plane at other measurement or interpolated positions. A sample of some of the \( x \)-\( y \) coordinates of the droplet tracks computed from a single 2-second-long hologram movie are shown in figure 3. A droplet must appear three times in a sequence of images in order to be tracked successfully and since the images are wider (2,560 pixels) than they are tall (1,600 pixels), the upper limit for the velocity measurement is determined by the vertical component. Thus, with the image resolution of 10 \( \mu \)m per pixel, it is estimated that the maximum vertical velocity of a droplet must be below approximately 4 m/s. Very few droplets with vertical speeds greater than 3.5 m/s were found. The measurements of the cross stream position (\( z \) coordinate) of the droplet position is inaccurate due to the difficulties in determining the position of best
Figure 3: A selection of droplet trajectories measured from a single hologram image sequence from one breaking event. The relative droplet diameters are indicated by the diameters of the markers shown in the plot. Due to dropouts of some droplets in some image frames, the droplets shown for a given track in the figure are not always spaced equally in time. The measurement plane is shown as a solid red line on the bottom of the image. The color gradient from one marker to the next on each trajectory indicates the time that has passed since the droplet was first measured, at which time the marker color is black. Because the measurements of the droplets near the boundaries are inaccurate, only droplets in a window 2 mm inside the image field of view (the white area in the drawing) are considered in the data set.

focus of the droplet image from the hologram reconstructions, see §2.1 for details, and this leads to even greater inaccuracies in the \( z \)-component of the droplet velocity. Thus, only the streamwise (\( u \)) and vertical (\( v \)) droplet velocity components are reported and discussed below.

2.3. Humidity and Droplet Evaporation

Estimates of the droplet diameter decrease due to evaporation during the time of flight from the location of droplet generation at the free surface to the measurement plane, located approximately 1.2 cm above the maximum height of the breaker crest, were estimated via the theory of Pruppacher & Klett (1978). In order to estimate this change in diameter, the humidity in the wave tank is required. To this end, a humidity sensor (Thorlabs, model TSP01) was placed in the wave tank at the level of the measurement plane for the moderate breaker experiments. The minimum relative humidity was found to be approximately 50\%. Using this value in the theory, it was found that the decrease in droplet diameter would be less than 4\% after 1 s for a droplet with an initial diameter of \( d = 100 \mu m \) and therefore insignificant for the 70-ms maximum flight time estimated from the present wave profile and droplet velocity data, see below.

3. Results and Discussion

The presentation and discussion of the results is divided into five subsections with the spatio-temporal distributions of the numbers and diameters of the droplets measured over the breaking crests in § 3.1, the ensemble averaged spatio-temporal contour maps of the
local number of droplets over the entire measurement field in § 3.2, the droplet diameter distributions in § 3.3, the droplet velocity distributions in § 3.4 and a low-order droplet motion model in § 3.5. It should be kept in mind that, some of the weak-breaker droplet data in the plots below were previously published in Erinin et al. (2019), see figure captions for details. The results are presented in $(\tilde{x}, \tilde{t})$ coordinates, where $\tilde{x}$ is the streamwise horizontal coordinate relative to the position of the plunging jet tip impact (positive downstream, i.e., the direction of wave propagation) and $\tilde{t}$ is time relative to the time of plunging jet tip impact (See Part 1 for details). To assist with the verbal descriptions in this section, four white light movies are given as supplemental material. Movie 1 includes above and below surface views to the strong breaker and uses stop action and closeup views to identify the various droplet generation mechanisms. Movies 2, 3 and 4 consist of above and below surface views of the strong, moderate and weak breakers, respectively, and provide uninterrupted images sequences of each breaking event.

3.1. Droplet Distribution over the Breaking Crest

In order to explore the relationship between events in the breaking process and the spatio-temporal distribution of droplet generation over the breaking crests, the position, time and diameter of each droplet measurement are indicated on top of the crest profiles for the weak, moderate and strong breakers in figures 4(a), (b) and (c), respectively. The profiles are ensemble averages over 10 realizations of each breaker that are taken from figure 12 of Part 1 and plotted in the laboratory reference frame. The droplet data is from three breaker realizations at each measurement location. Plotting only a subset of the droplet data was necessary because higher numbers of droplets from additional breaker realizations made the plots difficult to interpret. Further details of the plotting scheme are given in the figure caption. Since the horizontal droplet measurement plane is located slightly above the wave crest, see § 2.1, the droplet measurement times and positions are not the same as the positions and times when the droplets were generated at the water surface. The motion of a given droplet between the time it is generated and measured is influenced by the initial droplet ejection velocity, the vertical distance from the measurement plane, and the interaction between the droplet and the airflow above the wave, which is in itself modified by the breaking wave. Thus, the droplets measured in this experiment may have been generated upstream or downstream of their plotted location and were generated at times earlier than the measured times. These issues will be discussed further in § 3.5. Finally, it should be kept in mind that the droplets are measured across the entire width of the tank while the individual wave profiles are measured only at the center plane of the tank. In spite of this limitation of the the profile measurements, the ensemble average profile and distributions of standard deviation should be independent of cross-tank position, except very near the tank walls.

As can be seen in figures 4(a), (b) and (c), the first droplets are typically measured just before or after the profiles marked (i), which corresponds to $\tilde{t} = 123$ ms. Near profile (ii), the beginning of a region where many droplets with diameters as large as approximately 1 mm cross the measurement plane. For the strong breaker, this region (measuring approximately 150 mm by 98 ms for the strong breaker) begins at approximately the last of the inverted magenta triangles that mark the trajectory of the first indentation and is located on the back face of the wave near the crest. As the breaker strength is decreased, the time (profile number) of the beginning of this high-droplet-number region decreases. As discussed in detail in Part 1, the first indentation forms the boundary between the top surface of the plunging jet and the splash that it creates. The surface profile normal standard deviation, $r_{sd}$, and the surface profile arc length standard deviation, $s_{sd}$, show only moderate values at the time and position ranges of this intensive droplet flux, see Part 1, figures 12 to 14. As discussed in Part 1, this is the region where the deep crater in the bottom of the first indentation pinches off close to its
Figure 4: Subplot (a). The spatial distributions of droplets plotted on top of the evolution of the ensemble average surface profiles for the weak, moderate and strong breakers are presented in subplots (a), (b), and (c), respectively. The ensemble average surface profiles are the same as those shown in figure 10 of Part 1; however, the profiles shown here are plotted in the laboratory reference frame. A similar version of subplot (a) was presented in Erinin et al. (2019). The coordinates $\tilde{t}$ and $\tilde{x}$ are defined in Part 1 as the time and streamwise horizontal position relative to the impact of the jet tip in each realization of the breaker in the profile measurements. The bottom-most profile in each plot is the crest shape at the moment of jet impact, $\tilde{t} = 0$, and each successive profile (the time between profiles is $\Delta \tilde{t} = 12.3$ ms) is plotted $dy = 20$ mm above the previous. The green squares are located at the highest point on each surface profile and the magenta upside-down triangles are located at the first three indentations, see the caption for figure 10 in Part 1 for more details. The droplets are plotted as filled circles on the profile recorded at the time closest to the time when the given droplet crosses the measurement plane and are located on that profile at the streamwise position of the droplet crossing. The vertical bands with no droplets appear at locations where droplet data was not collected. Because the breaker profiles and droplets were measured in separate runs, the exact location and time of jet impact is not known for the droplet measurements. In view of this difficulty, the droplets are plotted relative to the ensemble average position and time of jet impact, $\tilde{x} = 0$ and $\tilde{t} = 0$, respectively. The color of the droplets indicates their diameter as given by the logarithmically scaled color bar. The blue and red colored backgrounds between the profiles show the spatio-temporal limits of two droplet-producing regions, Region I-A (in orange) and I-B (in blue).

deepest point and retracts rapidly toward the free surface leaving a small region of bubbles in front and near the bottom of the tube of air entrapped at jet impact, see Movie 1 given as supplemental material.

Figure 5 contains, two sequences of three images, taken from Movie 2, that show the moments before, during and after the indentation closure. The images in the top row (a, c, e) and bottom row (b, d, f) are from the camera views take from above and below the water surface, respectively. The images in each pair of below and above surface images
Figure 4: Subplots (b) and (c). See figure 4 (a) for detailed caption.
were recorded at the same time and the movies from which these images were taken are combined in a time-synchronized movie which is given as supplementary material, see Movie 2. Movies showing similar two-camera views of the moderate and weak breakers are also given as supplementary material, see Movie 3 and Movie 4, respectively. Images (a-b) show the indentation 147 ms after jet impact and just before the crater below the indentation closes. By this time the crest point is located on the splash-up generated by the plunging jet impact. In the below-surface image, the view of the roller of air entrained at the moment of jet impact is nearly obscured by the crater that extends downward from the indentation at the surface. Only 4 ms later, images (c) and (d), the indentation crater has nearly reached full retraction as shown in image (d) and leaves behind a small amount of air bubbles, which can be more clearly seen in Movies 2, 3 and 4. In images (e) and (f), taken 182 ms after jet impact, the indentation crater is completely retracted and droplets are being ejected all along the indentation. From the plots in figure 4, it can be seen that the number of droplets in this crater closure region increases dramatically with breaker strength. Also, in view of the fact that the region of high droplet number is small, well defined and close to the crater location, it is likely that the droplets’ initial velocities are nearly vertical and that the free surface in this region is close to the measurement plane. Droplet generation during the closure of the indentation is also seen in the numerical simulations of Wang et al. (2016) and Mostert et al. (2022).

Another region of intense droplet flux through the measurement plane is found over the breaking wave crest between profiles (iii) and (vi). These droplets are most likely generated by two breaking processes: splashing and bubble popping near the leading edge of the breaking zone on the upstream side of this high droplet flux region and the bursting of the large bubbles on the back face of the wave on the downstream side of the region. These large bursting bubbles on the back face are from the air entrapped under the jet at the moment of jet impact. Both of these droplet ejection processes can be seen clearly in the white-light movies 3 and 4 given as supplemental material. Also, both regions associated with these processes contain some of the highest values of \( n_{sd} \) and \( s_{sd} \), see figure 10(a) to (c) and subplots (b) and (c) in figures 12 to 14 in Part 1.

For later analysis of droplet number, diameter and velocity distributions, the droplet generation shown in figure 4 is broken into two regions. The first region, called Region I-A and marked by the orange background, includes the jet impact, formation of the first splash, and the formation and closing of the first indentation. The second region, called I-B and indicated by the blue background, covers the remaining regions of the breaking crest and includes the subsequent sequence of splash impacts and splash ups as well as the emergence and bursting of the large bubbles that were entrapped at the moment of jet impact. The spatial and temporal boundaries of Regions I-A and I-B are given in Table 1.

3.2. The Distribution of Droplets over the Entire Wave Field

As discussed above, the position (\( \tilde{x} \)), time (\( \tilde{t} \)), diameter (\( d \geq 100 \mu m \)) and velocity (\( \vec{v} = u\hat{u} + v\hat{v} \)) of each droplet is measured as it travels upward across the measurement plane. Here we define the droplet number distribution function \( N(\tilde{x}, \tilde{t}, d, u, v) \) to be the number of droplets per breaking event per meter of crest length in bins centered on the values of the five independent variables within the ranges \( 0 \leq \tilde{x} \leq 1050 \text{ mm}, 0 \leq \tilde{t} \leq 2000 \text{ ms}, d \geq 100 \mu m, -3 \leq u \leq 3 \text{ m/s} \) and \( 0 \leq v \leq 3 \text{ m/s} \). In the following, we present results from various integrations of the distribution function over one or more of these independent variables. For notation, the distribution function is always presented with the independent variables that remain after the integrations. For example, the local number of droplets of all diameters and velocities per breaking event per meter of crest length is written \( N(\tilde{x}, \tilde{t}) \) and the total number of droplets per breaking event is \( N \).
Figure 5: Three white-light image pairs in a time-sequence of the closing of the first indentation with views from above the water surface (images a, c, and e) and below the water surface (images b, d, and f). The cyan arrow shows the location of the indentation in each above-surface image and the magenta arrows point to the tip of the indentation crater in the below-surface images. The first and last images approximately correspond to profiles (ii) and (iii) in figure 4. In images (a) and (b), taken 147 ms after jet impact, the splash-up is located to the left of the indentation and the smooth upper water surface of the post-impact plunging jet is to the right of the indentation. The tube of air entrained under the jet at impact is seen in (b) to the right of the indentation, though most of the view of the tube of air is blocked by the crater of the indentation. Air-entrainment due to free-surface activity in the splash is observed to the left of the indentation. (c) and (d), which were recorded only 4 ms later, show the indent as it is closing. By comparing images (b) and (c), one can observe the very rapid rise of the tip of the indentation. The early stage of droplet ejection along the entire length of the indentation can be seen in image (e) which was recorded at $\tilde{t} = 182$ ms after jet impact. See Movie 1 given as supplementary material and the text for more details.

Contour plots of $N(\tilde{x}, \tilde{t})$ are given for the weak, moderate and strong breakers in figures 6(a), (b) and (c), respectively. See the figure caption for exact definitions and details. The most striking features of these droplet number contour plots are the two large spatio-temporal regions of droplet production tilted with a slope close to the speed of the toe of the wave shortly after jet impact, $(u_{toe})$, and with their dark blue boundaries extending approximately 0.7$\lambda_0$ horizontally and 0.6$T_0$ vertically. These two regions were previously identified for the weak breaker in figure 2 of Erinin et al. (2019). In the present paper, the rectangular regions encompassing each of the main droplet producing regions are called Region I (roughly the lower half of each plot and consisting of Regions I-A (orange
Figure 6: Contour maps of $N(\tilde{x}, \tilde{t})$, the number of droplets moving up across the measurement plane per surface area ($m^2$) per ms per breaking event are shown for the weak, moderate and strong breakers in subplots (a), (b), and (c), respectively. The data is from at least 10 breaker realizations at each droplet measurement location and from interpolation in $\tilde{x}$ intervals where no data was recorded. The contour maps are shown in the laboratory reference frame and cover the full measurement region, $\approx 1050$ mm in streamwise distance and $\approx 2000$ ms in time with a resolution of 13.02 mm by 25 ms. Only droplets with $d \geq 100 \mu m$ are counted. Spatio-temporal bins where $N(\tilde{x}, \tilde{t}) \leq 0.05$ are colored solid orange, light blue and tan in droplet producing Regions I-A, I-B and II, respectively. The horizontal and vertical orange lines indicate the locations of the maxima of $N(\tilde{t})$ and $N(\tilde{x})$, respectively, with the solid, dashed and dotted-dashed lines for the first, second and third local maxima, respectively. The solid black lines are drawn with their slopes corresponding to, $\langle D_C \rangle$, the average speed of the toe shortly after jet impact as computed from the data plotted in figure 11(c) of Part 1. For reference, the last wave profile shown in each of the three subplots of figure 4 was recorded at $\tilde{C} = 0.851 - 10^{-3}$. As can be seen in figures 7(a) - (c), the area of large droplet production in Region I is generally aligned with the breaking wave crest and in Region II with the following wave crest.
for the three breakers. From inspection of white-light movies of the breaking events and the comparisons with the contour plots of \( n_{sd} \) and \( s_{sd} \), these maxima seem to be associated with different physical surface processes in the breakers. The localized regions of high droplet production are discussed in detail below. For the weak breaker, there are three prominent local maxima in Region I, as originally identified and discussed in detail in Erinin et al. (2019). The first local maxima (identified by the white filled black circle in figure 6 (a)) occurs shortly after and upstream (to the left) of jet impact (at \((\tilde{x}, \tilde{t})\) = (0, 0)). This location is near the end of the first indentation, which is marked by the rightmost magenta line in the plot and in figure 14 (a) in Part 1. The second maximum, marked by the white-filled black triangle, is just to the right of the second indentation and occurs in a region of low surface normal and arc length standard deviations between two regions of high standard deviation which stem from the first and second splashup on the two sides of the second indent. The third local maximum (marked by the solid black square) is located close to the region of
high standard deviation associated with the bursting of large air bubbles that were initially entrained under the plunging jet. This local maximum is in on the back face of the wave crest.

For the moderate and strong breakers, Region I contains only two prominent local maxima which are in similar spatio-temporal locations for the two waves. The first maxima (marked by the white filled black circle) is located at approximately \((\tilde{x}, \tilde{t}) = (0.220\lambda_0, 0.259f_0^{-1})\) and \((0.212\lambda_0, 0.374f_0^{-1})\) for the moderate and strong breakers, respectively. This region seems to issue from the first indentation and droplets produced in this region are confined to a narrow spatial and temporal location. The number of droplets increases with breaker intensity. The second local maxima (marked by the filled black triangle) is located on or to the right of the second indentation, at approximately \((\tilde{x}, \tilde{t}) = (0.381\lambda_0, 0.374f_0^{-1})\) and \((0.415\lambda_0, 0.518f_0^{-1})\) for the moderate and strong breakers, respectively. As in the second maximum for the weak breaker, these maxima occur in a region of low surface normal and arc length standard deviation which is enclosed by two regions of high surface normal and arc length standard deviation directly located downstream and upstream. From visual inspection of white-light movies, this local maximum is associated with the splash region at the leading edge of the breaking zone and the sudden eruption of large air bubbles that were entrapped under the plunging jet at impact. Thus, it appears that the second maxima in the moderate and strong breakers is composed of droplets from the leading edge splashing and large bubble bursting that create the second and third maxima, respectively, in the weak breaker.

In Region II, there are no pronounced local maxima and the magnitudes of \(N(\tilde{x}, \tilde{t})\) are similar for the three waves. Observations from the white-light movies indicate that the droplets measured in Region II are primarily the result of small bubbles that burst when reaching the free surface, after the main sources of droplets on the breaking crest have ceased production. It is thought that the droplets from these small bursting bubbles are generated with low velocities. Thus, it is theorized that the reason that the droplets in Region II are measured only over the following wave crest, is that the droplets do not travel vertically more than a few centimeters and only the crest of the following wave is within this distance from the measurement plane.

In order to make better quantitative comparisons of \(N(\tilde{x}, \tilde{t})\) from one breaker to another, the distribution is integrated in \(\tilde{x}\) to obtain \(N(\tilde{t})\) and in \(\tilde{t}\) to obtain \(N(\tilde{x})\). In addition, the integration

\[
N'(\tilde{x}') = \int_0^{2000 \text{ms}} N(\tilde{x}', \tilde{t}) \, d\tilde{t}
\]  

(3.1)

is performed where \(\tilde{x}' = \tilde{x} + \langle u_c \rangle \tilde{t}\) is the streamwise coordinate of a reference frame moving with speed \(\langle u_c \rangle\), the speed of the crest point at the moment of jet impact. The results are shown in figure 8 where \(N(\tilde{t})\), \(N(\tilde{x})\) and \(N'(\tilde{x}')\) are plotted in subplots (a), (b) and (c), respectively. Each subplot contains three curves, with the results for the weak, moderate and strong breakers shown as solid green, dashed blue and dotted red lines, respectively.

The three curves of \(N(\tilde{t})\) and \(N(\tilde{x})\), in subplots (a) and (b), where jet impact occurs at the left and right ends of the horizontal axes, respectively, contain a number of local maxima. The locations of the first of these maxima (moving to the right in (a) and to the left in (b)) are marked in the figures by solid red vertical lines and also by solid red horizontal and vertical lines, for the \(N(\tilde{t})\) and \(N(\tilde{x})\) maxima, respectively, at the edges of the contour plots in figure 6. In the curve for the weak breaker in subplot (b), the overall maximum of the curve is marked as the first local maximum since the other local maxima are not significant. As can be seen in the contour plots of figure 6, the two solid red line segments would cross near the white filled black circles, indicating that these \(N(\tilde{t})\) and \(N(\tilde{x})\) local maxima are due to droplets generated during the collapse of the crater at the bottom of the first indentation.
From subplots (a) and (b), it can be seen that the peak values of $N(\tilde{t})$ and $N(\bar{x})$, increase by factors of 3.8 and 3.9, respectively, from the weak to the strong breaker. The second local maxima are marked by vertical red dashed lines and these maxima point to the white filled black triangle in figure 6, indicating that these local maxima are due to a combination of droplets from the splash region and the bursting of large bubbles entrained under the plunging jet. The peak values of $N(\tilde{t})$ and $N(\bar{x})$ at these locations increase by factors of 2.5 and 3.5, respectively, from the weak to the strong breaker. In the plot of $N(\bar{x})$, there is a small local peak at about $\tilde{t} = 0.58 f^{-1}$ in all the curves and finally a consistent small peak in all curves at $\tilde{t} = 1.5 f^{-1}$, which is the midpoint of the passage of the following wave crest. This latter peak does not appear in the plots of $N(\bar{x})$ because the integrations in time smear the effects of the breaking and following crests. In the crest-fixed coordinates of subplot (c), the computation of $N'(\bar{x}')$ results in a plot consisting of only two very clearly defined maxima for each
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Figure 9: Subplots (a) and (b) are log-log plots of the number of droplets produced per breaking event as a function of $Q^f$, the area under the plunging jet at $t = t^f$, and $v^f_C$, the vertical velocity of the crest point at $t = t^f$, respectively, as measured in Part 1. In both subplots, the total number of droplets measured are in light blue and droplets measured in Region I and II are shown in orange and green respectively. The straight lines are least square fits of power law functions to the data.

breaker. This is because in this moving coordinate system, the integrations over the breaking and following crests are entirely separated and within each crest the various peaks overlap when integrating in $\tilde{t}$. The area under the first peaks from the right are the total number of drops measured over the breaking crest (regions I-A and I-B combined) and the following peaks are the totals measured over the following crest (regions II). The magnitude of the first peak increases by a factor of approximately 3.0 while the second peak increases from the weak to the moderate breaker and then decreases for the strong breaker to a value less than that for the weak breaker. It is thought that this non-monotonic trend may be an anomaly due to the small number of droplets measured in region II and the phase of high-bubble-number regions relative to the following wave crest where the droplets in region II are measured.

The number of droplets produced in each droplet producing region (I-A, I-B, and II) and the total number of droplets measured for each breaker are given in table 1 along with the precise definitions of each region. The data is from integrations over the appropriate areas of the function $N(\tilde{x}, \tilde{t})$ in figure 6. The precise definitions of the three regions are set according to the local minima in the plots of $N(\tilde{x})$ and $N(\tilde{t})$ in figures 8(a) and (b). As indicated in the table, the droplets generated from the closing of the crater in the first indentation (Region I-A) comprise on average 32% of the total produced by each breaker. Both the number of droplets in II-B and the percent of the total increase with breaker strength, and when added to the numbers of Region I-A, comprise approximately 80% of the totals. The number of droplets produced by Regions I and II and the total are plotted versus the area under the plunging jet’s upper surface at $t = t^f$, $Q^f$, and the vertical component of the crest point velocity at $t = t^f$, $v^f_C$, in figure 9, subplots (a) and (b), respectively. The data nearly follows a straight lines on both of these log-log plots, indicating power law relationships for all regions. The fact that the two plots look similar is a result of the nearly linear relationship between $Q$ and $v^f_C$ as shown in Part 1.

### 3.3. Droplet Diameter Distributions

The droplet number distributions integrated over $\tilde{x}, u$ and $v$ ($N(\tilde{t}, d)$) and the droplet diameter distributions ($N(d)$) are discussed in this subsection. Contour plots of $N(\tilde{t}, d)$ on the $\tilde{t} - d$ plane, for the weak, moderate and strong breakers are shown in figure 10(a), (b) and (c), respectively. In each subplot the red/yellow/blue background colors represent the three droplet
between the previous values of \( \beta \) does not fall within a specific tolerance (in this case \( \Delta t = 28.8 \text{ ms} \)), with a temporal resolution of \( \Delta t = 28.8 \text{ ms} \). The scale of the vertical axis is logarithmic. The colored red/yellow/blue backgrounds in each subplot show the temporal limits of the three droplet producing regions as identified in § 3.2.

The droplet diameter distributions, \( N(d) \), are presented in the four log-log subplots in figure 11 where the subplots in (a) to (d) contain the data from the entire measurement plane, Region I-A, Region I-B and Region II, respectively. Each subplot contains three data sets, one for each breaker. The diameter bins are logarithmically spaced from \( d = 100 \) to 4000 \( \mu \text{m} \) with 32 bins. In order to decrease the noise in these plots, all bins containing less than five measured droplets (all found at large \( d \)) are removed. The three data sets in figure 11 (a) are a bit noisy but generally indicate an increasing droplet number with increasing breaker strength at all diameters. In each of the three data sets in this plot, separate straight lines were fit by least square error minimization separately to the smaller diameter and larger diameter droplet data. The boundary between the larger and smaller diameter groups was determined by an iterative bisection-like routine outlined as follows, see also the supplemental material in Erinin et al. (2019): First, an initial guess for the break in slope, \( d_0 \), is estimated, the data is split into two distinct sets and a power law, of the form described above, is fitted to each set. The diameter where the two lines intersect, \( d_i \), is found. If the difference between \( d_0 \) and \( d_i \) does not fall within a specific tolerance (in this case \( d_{tol} = 1 \mu\text{m} \)), a new guess for \( d_0 \) half way between the previous values of \( d_0 \) and \( d_i \) is assigned and the processes is repeated until the tolerance is reached. This break in slope follows the idea of the Hinze scale in distributions of bubble diameter in a turbulent two-phase flow. It should be noted that this determination of the diameter where the break in slope occurs is inherently inaccurate because of the small number of droplets in each bin at the larger diameter. This is a classic problem in bubble and droplet measurements and to emphasize this inadequacy, the power laws fitted to the large...
diameter droplet data are drawn as dashed lines. In a laboratory wind wave system, Wu et al. (1984) measured the droplet size distribution at several heights above the mean water level. The measurements were made over long periods of time and no correlation with breaking events was attempted. In that study, the distribution has a shape that is qualitatively similar to the one shown here. The values of \( \alpha \), \( \beta \) and \( d_i \) for the three breakers studied herein are given in table 2. The slope for the small diameters, \( \alpha \), increases monotonically with increasing breaker strength while the slope for the large droplets first decreases and then increases. The intersection diameter increases monotonically with values ranging from 820 to 1480 \( \mu \)m. In subplot (b), the data from subplot (a) is plotted with the \( d/d_i \) as the independent variable. This normalization nearly collapses the data to a single two-slope curve.

The droplet diameter distributions for droplet producing regions I-A, I-B and II are given in subplots (b), (c) and (d) of figure 11, respectively. Each plot is created from many fewer droplet measurements than in subplot (a) but still displays interesting features. In Region I-A, the two linear regions of the curves for the moderate and strong breakers are still evident; however, in view of the noisy data, the fitting was not performed. The vertical separation between the curves is larger than in the distribution for the full data set, but the trend with breaker strength is not monotonic. For the data in Region I-B, the curves are less noisy and closer together. The two-slope nature of the previously discussed distributions has been replaced by smooth arcs in which the local slope magnitude increases with increasing \( d \).
| Breaker Type | Weak | Moderate | Strong |
|--------------|------|----------|--------|
| All Regions  |      |          |        |
| $\alpha$     | -2.4 | -2.1     | -1.9   |
| $\beta$      | -5.4 | -6.1     | -5.8   |
| $d_i$ (µm)   | 820  | 1140     | 1480   |

Table 2: Parameters for the straight lines fitted by least squares error minimization to the distributions of droplet diameter in figure 11(a). The variables $\alpha$ and $\beta$ are the slopes of the straight lines for the small and large diameter droplet data, respectively, while $d_i$ is the diameter at which the two lines intersect.

This trend continues in Region II where the data forms smooth curves that are nearly on top of one another. The structure of these data sets is thought to be a result of the different droplet generation mechanisms in the three regions. The very different curves in Region I-A are thought to be a result of variations in droplet production during the crater collapse at the bottom of the first indent as was discussed in § 3.2 and as seen in Movie 1 given as supplementary material. This hypothesis is also supported by the droplet velocity data reported in the following subsection. In Region II, droplets are generated as small bubbles come to the surface and burst. It is thought that the probability distribution in this region should be relatively independent of breaker intensity since the main difference between the three cases is probably the number of bubbles bursting. In Region I-B, where droplets are generated by splashing and bubbles with a large range of diameters, the trends created by varying breaker intensity are in between those found in Regions I-A and II. Admittedly, these ideas are primarily speculation from observations of high-speed movies of breaking events.

### 3.4. Droplet Velocity Distributions

Contour plots of the droplet number distributions $N(V, d)$, $N(u, d)$, and $N(v, d)$ are given in the nine subplots of figure 12. In this figure, the top, middle and bottom rows of subplots consist of the three distributions for the weak, moderate and strong breakers, with $N(V, d)$, $N(u, d)$, and $N(v, d)$ in the left, middle and right columns, respectively, and each distribution includes all droplets measured for the given breaker. See the figure caption for additional details. From these contour plots, one can see that as the breaking intensity increases (moving down from plot to plot in each column), the number of droplets with large diameter and the range of the velocities measured increase in all three columns. The peaks of the distributions are located at small diameters and at small values of $V$, $u$ and $v$. The distributions $N(u, d)$ (center column) indicate that there are droplets with positive (in the direction of wave travel) and negative horizontal velocities at all diameters and that the distributions are centered vertically at slightly positive values of $u$. The distributions $N(v, d)$ are located, of course, entirely in the region of positive $v$ with a few droplets having speeds of nearly 4 m/s and the number of these fast moving droplets increasing with breaker intensity. In considering these droplet speeds, it should be kept in mind that for these breakers the plunging jet speed ($\langle V_j \rangle$) and crest speed ($\langle u_c \rangle$) at jet impact are roughly, 2.0 m/s and 1.3 m/s, respectively, (see table 3 in Part 1 for details). The droplet velocity estimates divided by the crest speeds from the numerical calculations reported in Mostert et al. (2022), where the breaker wavelengths of approximately 30 and 50 cm, are similar in range to the present results where the nominal wavelength is 118 cm.

Probability distributions of the droplet speeds and the streamwise horizontal and vertical velocity components are presented in separate plots with data for all of the droplets and for
Figure 12: Contour maps of $N(V, d)$, the number of droplets per breaking event per meter of crest width per bin widths as a function of droplet speed ($V = (u^2 + v^2)^{0.5}$) and diameter ($d$), (left column), $N(u, d)$ (middle column), and $N(v, d)$ (right column), for the weak (top row), moderate (middle row), and strong (bottom row) plunging breakers. In all subplots, the vertical axis has equally spaced bins of height 0.1 m/s and the horizontal axis has 80 logarithmically spaced bins with bin edges from $d = 100$ on the left to 2844 µm on the right. The horizontal solid red and yellow lines in the $N(V, d)$ and $N(u, d)$ subplots, respectively, are located at the values of $\langle V_j \rangle$ (the plunging jet speed at impact) and $\langle u_j \rangle$ (the crest point speed at jet impact), respectively, from table 3 of Part 1.

the droplets in Regions I-A, I-B and II in figure 13, subplots (a) through (l). See the figure caption for additional details. Average values of the droplet speeds and velocity components corresponding to these plots are given in table 3. The PDFs of $V$ (the four subplots in the left column) are peaked at values of $V$ near 0.8 m/s in all but Regions II where the peak occurs at approximately 0.2 m/s. The probabilities of the highest and lowest speeds are one to two orders of magnitude lower than the probability at the peak. The average values of $V$, see table 3, range from 0.67 to 1.38 m/s, are lowest in Region II and are highest in Region I-B. The data for the three breakers nearly fall on single curves for Regions I-A and II, but in Region I-B, the PDF for the weakest breaker is higher than that for the moderate and strong breakers near the peak PDF value, and lower than the PDF for the moderate and strong breakers at higher $V$ values. The PDFs of $u$ (the four subplots in the middle column) are approximately symmetric with peaks located between approximately 0.1 and 0.3 m/s and average values between 0.05 and 0.75 m/s. Among the three sub-regions, the average values of $u$ in Region I-B (0.58 m/s averaged over the three breakers) are the highest while the
average $u$ in Regions I-A and II are only 0.18 and 0.21 m/s, respectively. Since the bursting of small bubbles like those found in Region II would produce a $u$ PDF distribution centered on $u = 0$ and with zero mean if the bubbles were bursting on an otherwise still water surface, the small positive mean and most probable $u$ values of approximately $u = 0.14 \langle u'_x \rangle$ are probably an indication of horizontal fluid motion due to the fluid particle velocity in the crest of the nonbreaking following wave. The PDFs of $v$ (the four subplots in the right column) are qualitatively similar to the corresponding plots of $V$. Both the value of $v$ at the peak PDF
and the range of \( v \) values decrease monotonically in going down in the column of plots from Regions I-A to I-B to II. These differences and trends in the \( V, u \) and \( v \) PDFs between and in, respectively, the three droplet producing regions are consistent with the picture of the droplet producing mechanisms in the three regions: crater collapse in the first indentation in Region I-A, splashing and large and small bubble bursting in Region I-B and small bubble bursting in Region II.

The differences in the droplet velocities and their relationship to the droplet production mechanisms is further demonstrated by the examination of the velocity direction, as measured by the angle \( \theta \) between the horizontal direction and the velocity vector. Positive \( \theta \) is taken as clockwise and \( \theta = 0^\circ \) is in the direction of wave motion. The distributions of \( N(d, \theta) \) and \( N(\theta) \) are presented in seven subplots in figure 14. Subplots (a), (b) and (c) are contour plots of \( N(d, \theta) \) for all droplets in the weak, moderate and strong breakers, respectively. The plots are in polar coordinates where the radial and azimuthal coordinates are \( d \) and \( \theta \), respectively. All three distributions have an isolated region of high \( N(d, \theta) \) at small \( 100 < d \leq 300 \) \( \mu \)m and \( \theta \approx 60^\circ \). To better compare the three waves, the distribution \( N(\theta) \) is plotted in polar coordinates with \( N(\theta) \) and \( \theta \) as the radial and azimuthal coordinates, respectively. There is one curve for each breaker intensity. The curves are somewhat irregular but one can see that the value of \( \theta \) at the highest values of \( N(\theta) \) increase with increasing breaker intensity. This increase in \( \theta \) with breaker intensity is reflected in the average values of \( \bar{\theta} \) for the all droplets as given in the bottom row of table 3. Subplots (h), (i) and (j) contain polar plots of \( N(\theta) \) for Regions I-A, I-B and II, respectively. These plots and the \( \bar{\theta}_m \) data in table 3 demonstrate clear differences in droplet production between the three regions. In the three subplots, one can see that \( \bar{\theta}_m \) is only about 20° from vertical in the direction of wave propagation in Regions I-A and II, while it is on the order for 40° downstream from vertical in Region I-B. This trend is reflected in the average values of \( \bar{\theta} \) in the three regions: 81.1°, 56.2° and 74.1° in Regions I-A, I-B and II, respectively, where each value is the average over the three waves. The narrow distribution of nearly vertical motion of the droplets in Region I-A is clearly observed in Movie 1 given as supplementary material and is thought to be related to the generation during the closure of the crater at the bottom of the first indentation. The narrow nearly vertical distribution in Region II is probably due to a combination of the distribution of droplet velocity in an individual small bubble bursting event and the fact that measuring the droplets as they pass through the measurement plane above the generation point favors droplets generated with velocity vectors pointing up.

| Run | All regions | Region I-A | Region I-B | Region II |
|-----|-------------|------------|------------|-----------|
| \( N \) | 657 | 899 | 1122 | 228 | 369 | 565 | 146 | 235 | 169 |
| \( \bar{V} \) (m/s) | 0.91 | 1.17 | 1.09 | 0.94 | 1.16 | 0.96 | 1.00 | 1.38 | 1.25 |
| \( \bar{u} \) (m/s) | 0.31 | 0.34 | 0.22 | 0.25 | 0.23 | 0.05 | 0.48 | 0.75 | 0.50 |
| \( \bar{v} \) (m/s) | 0.63 | 0.87 | 0.82 | 0.74 | 1.05 | 0.85 | 0.63 | 0.84 | 0.81 |
| \( \bar{\theta} \) (deg.) | 65.6 | 66.5 | 75.0 | 78.4 | 75.4 | 89.5 | 54.3 | 51.0 | 63.3 |

Table 3: The average speed (\( \bar{V} \)), horizontal velocity component (\( \bar{u} \)), and vertical velocity component (\( \bar{v} \)) of droplets as they pass through the measurement plane is given for all regions and separately for Regions I-A, I-B, and II. The average angle of droplet motion is given by \( \bar{\theta} \). The values \( N \) are the number of droplets per breaking event per meter of crest width in each region, the same numbers are reported in table 1.
3.5. Low-order Predictions of Droplet Generation Sites and Time of Flight

As discussed above, the droplet data presented herein is recorded as the droplets move upward through the measurement plane located 1.2 cm above the wave crest. In an effort to determine an approximate time and position of generation of a given droplet at the water surface, the velocity and location of each droplet at the measurement plane was used with a model of the droplet motion to simulate the trajectories of the droplets backward in time until they intersect with the time evolving ensemble average profile history. This model also yields an estimate of the droplet time of flight ($\Delta t_f$) as it travels from the water surface to the measurement plane. This travel time is essential for estimating the reduction in droplet diameter due to evaporation before each droplet is measured. In this model, the forces on the particles are assumed to be those due to gravity and drag relative to still air. Since, the air motion is very likely an important factor in determining the droplet motion, particularly for the smaller droplets, the degree to which the computed generation sites seem plausible
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is used to indicate the importance of the air velocity in the calculation and thus motivate additional research.

In this simplified model, the equations of motion for the droplet trajectory are

\[
m_d \ddot{X}_d = C_D \frac{1}{2} \rho_a (-\dot{X}_d) \sqrt{\dot{X}_d^2 + \dot{Y}_d^2} \quad (3.2)
\]

\[
m_d \ddot{Y}_d = C_D \frac{1}{2} \rho_a (-\dot{Y}_d) \sqrt{\dot{X}_d^2 + \dot{Y}_d^2} + m_d g, \quad (3.3)
\]

where \( X_d \) and \( Y_d \) are the horizontal vertical positions, respectively, of a given droplet and the dots indicate derivatives with respect to time, \( m_d \) is the droplet mass, \( \rho_a \) is the density of air and \( C_D \) is the drag coefficient. For the drag coefficient, the correlation for a spherical particle given by Cheng (2009) is used:

\[
C_D = \frac{24}{Re} (1 + 0.27 Re)^{0.43} + 0.47 [1 - \exp(-0.04 Re^{0.38})], \quad (3.4)
\]

where \( Re \) is the Reynolds number of the particle in air given by

\[
Re = \frac{d (\dot{X}_d^2 + \dot{Y}_d^2)^{0.5}}{\nu_a}, \quad (3.5)
\]

where \( \nu_a \) is the kinematic viscosity of air.

The results of these droplet trajectory calculations are shown in figure 15 where two plots of the ensemble averaged profiles from the strong breaker are shown along with the same set of droplets measured from three realizations of the strong breaker. In subplot (a), the droplets are shown on the profiles measured at the time the droplets crossed the measurement plane and at the corresponding streamwise positions. The color of each droplet marker indicates the droplet’s diameter. This plot is identical to the one shown in figure 4(c). In subplot figure 15(b), the water surface profiles in (a) are repeated but the droplets are plotted on the profiles and at streamwise positions computed by the backward tracking model. As can be seen by comparing the two plots, many of the droplets that were measured over the back face of the wave in Region I-B are predicted to have been generated on the turbulent breaking region created by the jet impact and propagating from right to left. Also, the area where many droplets were measured in Region I-A between profiles (ii) and (iii) and thought to be due to the collapse of the crater at the bottom of the first indentation has tightened to a smaller range of time. The number of droplets in this region of (b) is only a little less than in the corresponding region of (a); this result is not visible in the plots because many of the droplets are plotted on top of one another. An additional important feature of the plots is that many of the smaller droplets, and some larger ones, are predicted to have been generated on the smooth water surface upstream (to the left) of the leading edge of the breaking zone. Though examination of the white-light movies does indicate that a few secondary droplets are generated in this upstream region due to primary droplet impacts, the large number of droplets predicted to be generated is thought to be nonphysical and an indication of the need for including the temporally evolving air flow field in the calculation of the droplet motion.

The calculated values of \( \Delta t_f \) for the droplets considered in figure 15 are presented as a function of \( d \) in figure 16. From the plot, it can be seen that the smallest droplets that were measured, \( d = 100 \mu m \), have the largest \( \Delta t_f \), approximately 70 ms. Between \( r = 100 \) and \( r = 300 \mu m \), \( \Delta t_f \) falls to approximately 40 ms and thereafter remains more or less constant. The large fluctuations in the data in the large \( d \) region of the plot are thought to be due to the effect of the small number of larger droplets on the averaging process. These times of flight were used in § 2.3 to estimate the effect of evaporation on the measured diameters of the droplets in this study.
Figure 15: (a) The spatio-temporal distribution of droplets from three realizations of the strong breaker plotted on the corresponding ensemble averaged wave profiles (from 10 realizations). Each droplet is plotted on the profile when and the streamwise position where it crossed the measurement plane. This is the same plot as given in figure 4(c) (see that figure for more plotting details). (b) The spatio-temporal positions of the droplets (a) projected back to their generation sites as determined by the simplified droplet motion model described in the text. Figure 4(c) is shown here to facilitate the comparison between the measured droplet locations in (a) and the estimates of the locations where they were generated in (b).
Droplet Characteristics

Figure 16: Estimates of the time of flight of droplets as a function of droplet diameter for the weak, moderate, and strong breakers. These estimates are based on the time, position and velocity of the droplets measured as they move upward through the measurement plane, the ensemble average profile sequence and the simplified droplet motion model given in § 3.5. The standard deviation of $\Delta t_f \approx 30$ ms for $d \approx 100 \mu m$ and $\approx 50$ ms for $d \approx 500 \mu m$.

4. Conclusions

Measurements of the droplets produced by three plunging breaking waves are presented and discussed. The breakers are created from mechanically generated dispersively focused wave packets that differ primarily by only modest changes in overall amplitude. The average frequency of the wave packet is $f_0 = 1.15$ Hz (wave period $T_0 = 1/f_0 = 0.870$ s) for all breakers and this corresponds to a wavelength $\lambda_0 = 1.18$ m by linear wave theory. The breakers, which are described in detail in Part 1 of this two-part paper, are designated as weak, moderate and strong. By using two cinematic in-line holography systems (operating at 650 fps), the droplet diameters ($d \geq 100 \mu m$), their positions, and the streamwise and vertical components of their velocity are measured as the upward moving droplets cross a measurement plane located 1.2 cm above the highest point reached by the wave crest during the breaking process. The prismatic measurement volumes of the holograms are 16 mm high, 15 mm wide and extend across the entire width of the tank. The droplet measurements were recorded for a time of $2.270T_0$ and over a streamwise distance of $0.9\lambda_0$ after the time and location of plunging jet impact, respectively. The data set for each wave was obtained from approximately 140 breaking events.

It is found that for all three waves the droplets cross the measurement plane in two regions, the first over the breaking crest (Region I) and the second over the following wave crest (Region II). (This result was first reported for the weak breaker in Erinin et al. (2019).) Approximately, 77.8, 72.0 and 84.0 percent of the droplets are measured over the breaking crest in the weak, moderate and strong breakers, respectively, while the total number of droplets measured per breaking event is $N = 657, 839$ and 1122, respectively. It is found that $N$ increases linearly with $Q_i$, the area under the upper surface of the plunging jet as defined in Part 1 where it is theorized that $Q_i$ is roughly proportional to the cross sectional area of the tube of air entrapped under the plunging jet at the moment of impact. In Region I, the primary droplet generation mechanisms are the closure of the indentation that forms just upstream of the upper surface of the plunging jet (labeled Region I-A and contributing approximately 32% of the total number of droplets per breaking event), the bursting of large bubbles entrained at moment of jet impact as they come to the surface on the back face of
the wave, and splashing and small bubble bursting in the turbulent zone created on the front face of the wave by the jet impact. The droplets generated by the latter two mechanisms appear in one area at the measurement plane in the moderate and strong breakers and are designated together as Region I-B in all three breakers. In Part 1, it was shown that the two droplet producing sub regions within Region I-B have large standard deviations of the surface shape while the standard deviations of surface shape in Region I-A are relatively small. The remainder of droplets are measured in Region II and are generated by small bubbles that rise to the surface and burst in the wake of the breaking crest.

The droplet diameter distributions exhibit power law behavior in separate regions of small and large diameter. On a log-log plot, the straight lines fitted to the distributions cross at intermediate diameters, $d_i$, which range from 820 µm for the weak breaker to 1480 µm for the strong breaker. Plots of the droplet number distribution as a function of $d/d_i$ result in a moderate collapse of the data from the three breakers. The exponents in the power laws fitted to the ranges of small and large diameter droplets for the three breakers ranged from -2.4 to -1.9 and -6.1 to -5.4, respectively. Power laws fitted to the diameter distributions in Region I-A, exhibit a clear change in slope for the three breakers, while the change in slope of the distributions for Regions I-B and II are more gradual.

Analysis of the droplet velocity data indicates that the average speeds over all droplets are all less that 1.4 m/s while the jet tip impact speeds are, as given in Part 1, approximately 2.0 m/s for the three breakers. The values of the horizontal component of the droplet velocity, $u$, are nearly evenly distributed about $u = 0$ in Region I-A with an average value of 0.15 m/s over the three breakers while $v$ in the same region reaches its highest values of nearly 4 m/s with an average of 0.85 m/s over the three breakers. These velocities are typically directed just 10° downstream from vertical. These results are consistent with the motion of the larger droplets generated by the closing of the first indentation and which can be seen in the white-light movies included as supplementary material. The $u$ component of the droplet velocities in Region I-B have an average of approximately 0.6 m/s in the downstream direction and $v$ has an average of 0.75 m/s, while the average direction of the velocity vectors is approximately 55° downstream from vertical. In Region II, the average $u$ and $v$ velocity components are small, 0.21 m/s (downstream) and 0.6 m/s, respectively. It is theorized that since the small bubble bursting on a quiescent water surface would produce droplet velocity components that are evenly distributed between upstream and downstream, the non-zero mean of $u$ is a result of the downstream fluid particle motion at the crest of the non breaking wave following the breaking crest. The droplet velocity time and positions data was used with the ensemble average profile history and a low-order particle trajectory model to predict the locations where and when the droplets were generated at the free surface and the time of flight to the measurement plane. The model predicted that a good portion of the droplets measured over Region I-B were generated on the smooth water surface downstream (in the direction of wave travel) of the breaking front. It is believed that this unrealistic result is due to omission of the motion of the air as induced by the breaker. The time of flight of the droplets was estimated at approximately 40 ms for the smallest droplets up to 160 ms for the largest droplets. Using these times of flight and a droplet evaporation model it is estimated that the reduction of droplet diameter due to the effect of evaporation during the time of flight is at most 5% in the present data set.

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