Diffusion of fast rising strong magnetic fields into conductors

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Abstract. The basic processes occurring in a conductor exploding in a current skinning mode are the propagation of a nonlinear magnetic diffusion wave in the conductor and the formation of low-temperature plasma at its surface. An experimental study of the phenomenon of nonlinear magnetic diffusion into conductors in magnetic fields of induction rising at a rate up to $3\times10^9$ T/s was carried out on the MIG generator capable of producing a peak current up to 2.5 MA within a rise time of 100 ns. It has been found experimentally that the average velocity of a nonlinear magnetic diffusion wave in an aluminum conductor placed in a strong magnetic field (up to 300 T) rising at a high rate (on average, $3\times10^9$ T/s) is $(2.7 \pm 3.3)\times10^5$ cm/s. This is comparable to the velocity of sound in aluminum under normal conditions and reasonably agrees with predictions of numerical simulations.

1. Introduction

The electrical explosion of conductors (EEC) has been studied for a long time and has a number of practical applications. An EEC mode of interest in the generation of superstrong magnetic fields and in the energy transfer using vacuum transmission lines is the current skinning mode [1, 2, 3, 4]. For this mode, the time of energy delivery to the conductor is less than or comparable to the time of magnetic diffusion into it. The basic processes inherent in the current skinning mode are the propagation of a nonlinear magnetic diffusion wave (NMDW), the formation of low-temperature plasma at the conductor surface, and the development of thermal instabilities [5, 6].

Nonlinear magnetic diffusion features a speed of field penetration into a conductor anomalously high compared to conventional magnetic diffusion. The high diffusion speed is related to a decrease in conductivity of the metal due to its heating by electric current. Nonlinear diffusion can occur only in a rather strong magnetic field whose induction for the most used metals should be no less than 25–45 T.

The nonlinear diffusion of a magnetic field into a conductor in a current skinning mode was investigated experimentally on the MIG pulse power generator [7]. The goal of the experiment was to determine the velocity of a nonlinear magnetic diffusion wave at a magnetic field induction rising up to 300 T at a rate of $3\times10^9$ T/s. To interpret the experimental results obtained, a numerical model was used, which described an NMDW propagating in a conductor [8].
2. Numerical model

The process of nonlinear magnetic diffusion is described by Maxwell’s equations written in a quasistationary approximation (not taking into account displacement currents) and complemented with the Ohm’s and Joule–Lentz laws. In cylindrical coordinates, the equations take the form:

\[
\frac{1}{r} \frac{\partial}{\partial r} (r B) = \mu_0 j ; \quad \frac{\partial E}{\partial r} = \frac{\partial B}{\partial t}
\]

(1)

\[
E = j \delta
\]

(2)

\[
\frac{\partial \varepsilon_w}{\partial t} = j^2 \delta
\]

(3)

where \(B\) is the induction of the azimuthal magnetic field, \(E\) is the electric field strength, \(j\) is the axial current density, \(\delta\) is the resistivity of the metal, \(\varepsilon_w\) is the energy density due to Joule heat release, and \(\mu_0\) is the permeability of vacuum. The boundary conditions for the system of equations (1)–(3) are specified as \(B^\text{out} (t) = \frac{\mu_0 I(t)}{2\pi R}\) at the conductor outer boundary, where \(I(t)\) is the current flowing in the conductor, \(R\) is the external radius of the conductor; and as \(B^\text{inner} (t) = 0\), at the conductor inner boundary.

For the majority of metals, the resistivity increases with temperature \(T\) as:

\[
\delta(T) = \delta_0 \left(1 + \frac{\partial \delta}{\partial T} T\right)
\]

(4)

where \(\delta_0\) is the resistivity under normal conditions and \(\frac{\partial \delta}{\partial T}\) is the derivative of resistivity with respect to temperature. As \(\varepsilon_w = \rho c_v T\), where \(\rho\) is the density of the conductor material and \(c_v\) is the heat capacity at constant volume, the resistivity can be expressed in terms of thermal energy density as [9]:

\[
\delta(\varepsilon_w) = \delta_0 \left(1 + \beta \varepsilon_w\right)
\]

(5)

where \(\beta = \frac{1}{\rho c_v} \frac{\partial \delta}{\partial T}\).

For diffusive penetration of a magnetic field into a material, the thermal energy density is approximately equal to the magnetic energy density [9]; that is, we have \(\varepsilon_w = \frac{B^2}{2\mu_0}\). The magnetic field of induction \(B_0\) at which the resistivity of the medium is doubled, that is, the condition \(\beta \varepsilon_w \approx \frac{\beta B^2_0}{2\mu_0} = 1\) is satisfied, is generally termed the characteristic field [9, 10], whose induction is given by [8]:

\[
B_0 = \left(\frac{2\mu_0}{\beta}\right)^{1/2}
\]

(6)

Formula (6) gives a lower threshold field, such that at higher fields the effects related to nonlinear diffusion must be taken into consideration. As mentioned above, the characteristic magnetic induction is 25–45 T for the majority of metal.
It has already been noted that if a strong magnetic field is applied at the outer boundary of a conductor, a nonlinear magnetic diffusion wave propagates into the conductor. This can be seen from figure 1a, where the results of solving the system of equations (1)–(3), (5) for an aluminum tube of external diameter 3 mm and wall thickness 250 µm are presented. The calculations were performed for the case where a magnetic field corresponding to a current pulse of the MIG generator recorded in an experimental shot was applied at the outer boundary of the conductor. The current pulse waveform is given in figure 1b (curve $I$). Figure 1a shows that the current density peaks at the NMDW front and the resistivity increases behind the front. The arrival of the NMDW at the conductor inner surface is accompanied by occurrence of an electric field inside the tube (curve $U_{\text{inner}}$ in figure 2b). As the NMDW arrives at the inner surface, the current density at this surface peaks and the time derivative of voltage has a pronounced maximum (curve $(dU / dt)_{\text{inner}}$ in figure 1b).

Thus, the calculation results show that to determine experimentally the time at which the NMDW arrives at the conductor inner surface (to measure the velocity of the NMDW), it is necessary to record the waveform of the voltage at the inner surface and to determine the time at which the derivative $(dU / dt)_{\text{inner}}$ reaches a maximum.

3. Experimental results
The experiment was carried out on the MIG pulse power generator [7] at a current up to 2.5 MA with 100 ns rise time. Aluminum tubes of external diameter 3 mm (see figure 2a) and wall thickness from 100 to 400 µm were used for the generator load. To record the voltage waveform at the inner surface, a resistive voltage divider was used. The location of the divider connection to the load unit is shown schematically in figure 2. As the load unit was assembled with the tube, the divider cable was inserted through the open end of the tube on the low-voltage side, so that the cable conductor was in contact with the tube farther ("high-voltage") end and the cable braid was in contact with the tube inner surface on the low-voltage side. The measured voltage amplitude at the tube inner surface was up to 20 kV.
Figure 2. Photo of the generator load (a) and a schematic image of the voltage divider connection to the load unit of the MIG generator: 1 – conical cathode fitting piece to which the test tube is fastened, 2 – return current rods, 3 – anode disk, 4 – tube, 5 – divider cable conductor, and 6 – polythene insulator of the divider cable (b).

For the above generator parameters, the thickness of the skin layer at normal conductivity was, for all tubes, substantially smaller than the tube wall thickness, which was varied in the range 100–400 µm. Thus, a current skinning mode, which should necessarily be realized to investigate magnetic diffusion into a conductor, was certainly provided. According to the experimental data, the characteristic field for aluminum (37 T) at the tube surface was reached within 18±25 ns after the onset of current flow in the tube. Thus, for the given current waveform, magnetic diffusion into the conductor became nonlinear at the 18th–25th nanosecond.

The experimental waveforms of the current through a tube \( I \), of the voltage at the tube inner surface, \( U_{\text{inner}} \), and of the derivative \( (dU/dt)_{\text{inner}} \) obtained in one of the shots are presented in figure 3.

Figure 3. Experimental waveforms of current \( I \), voltage \( U_{\text{inner}} \), and derivative \( (dU/dt)_{\text{inner}} \) at the inner surface of an aluminum tube of external diameter 3 mm and wall thickness 250 µm.

These experimental waveforms were obtained for an aluminum tube of wall thickness 250 µm, the same as was used in the calculations discussed in the previous section. Comparison of the experimental and calculated curves of voltage and its derivative shows that the curves are in rather good agreement at the beginning of the pulse and then disagree. The disagreement seems to be related
to that the plasma formation at the conductor surface observed in the experiment was not considered in
the calculations. Both the experimental and the calculated curve of \( (dU/dt)_{inner} \) have a maximum
associated with the arrival of the NMDW at the inner surface of the tube.

Signals of the \( U_{inner} \) sensor were similarly processed for all tube wall thicknesses, and for each
case the time of arrival of the NMDW at the tube inner surface was determined. Figure 4 presents the
results of the processing: the measured time of NMDW arrival at the tube inner surface and the
NMDW velocity are presented as functions of the tube wall thickness in figure 4a and figure 4b,
respectively. The NMDW velocity was determined as the ratio of the tube wall thickness to the
difference between the time of NMDW arrival at the tube inner surface and the time at which the
magnetic field at the outer surface became equal to \( B_0 \).

![Figure 4](image.png)

**Figure 4.** Experimental values of the time of NMDW arrival at the inner surface of an
aluminum tube (a) and of the NMDW velocity at this time (b) versus tube wall thickness \( h \).

As can be seen from figure 4b, for aluminum conductors placed in magnetic fields of induction up
to 300 T rising at an average rate of \( 3\times10^9 \) T/s, the NMDW velocity is \((2.7\pm3.3)\times10^5\) cm/s, which is
comparable to the velocity of sound in aluminum under normal conditions, and the NMDW velocity
weakly depends on the conductor thickness.

4. Conclusion
According to the predictions of numerical simulations, as the NMDW arrives at the inner surface of an
aluminum tube and the current density at this surface peaks, the time derivative of the electric field
strength has a pronounced maximum. Measurements of the voltage at the tube inner surface also show
the presence of a pronounced maximum in the voltage derivative. This makes it possible to measure
the velocity of propagation of the NMDW more precisely, as the voltage increases rather smoothly.

Experimentally, for aluminum conductors placed in strong magnetic fields (up to 300 T with an
average induction rise rate of \( 3\times10^9 \) T/s) it has been found that the average velocity of the NMDW is
\((2.7\pm3.3)\times10^5\) cm/s. This is comparable to the velocity of sound in aluminum under normal conditions
and reasonably agrees with the predictions of the numerical model.

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