Inclusive Dilepton Production at RHIC: a Field Theory Approach Based on a Non-equilibrium Chiral Phase Transition

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Dedicated to the memory of Richard Slansky, Friend and Colleague
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Recently a real time picture of quantum field theory has been developed which allows one to look into the time evolution of a scattering process. We discuss two pictures for discussing non-equilibrium processes, namely the Schrodinger Picture and the Heisenberg picture and show that a Time Dependent Variational Method is equivalent to the leading order in Large-N approximation in the Heisenberg picture. We then discuss the dynamics of a non-equilibrium chiral phase transition in mean field theory in the \(O(4)\) sigma model. We show how the pion spectrum can be enhanced at low momentum because of non-equilibrium effects. We then show how to use Schwinger’s CTP formalism to calculate the inclusive dilepton spectrum coming from the pion plasma. We find that a noticeable enhancement occurs in this spectrum, but that there are large numerical uncertainties due to errors connected with the finite times used to do our numerical simulations.

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I. INTRODUCTION

In the early 70’s, the study of inclusive Hadronic Interactions was at the forefront of theoretical research because of the opening of two major accelerators–FNAL in the United States and the ISR at CERN. As data poured in many of us tried to understand from various approaches such as Regge Poles, Fireballs, and Hydrodynamics how to determine the single particle inclusive spectrum of particles. In that atmosphere in 1974, Peter Carruthers founded the Particle Physics group at Los Alamos which initially included Dick Slansky, Geoff West, David Campbell, David Sharp, Mitch Feigenbaum and myself, all of whom were working on this problem when we first arrived. At that time QCD was not yet a well developed subject, computers were relatively archaic and the idea of a first principles approach to understanding what happens after two particles collide was just a dream. Recently this dream has gotten closer to reality, and as a tribute to Dick and our early efforts I would like to review progress we have made in this direction. I will begin by first reviewing two pictures for discussing non-equilibrium processes, namely the Schrodinger and Heisenberg Pictures. In the Schrodinger Picture one can reduce the number of degrees of freedom by using a time-dependent variational method with a trial density matrix which is Gaussian. In the Heisenberg picture one can use the large N expansion of the Path Integral in the Closed Time Path Formalism to give us a controllable expansion about the mean-field approximation. We will then review our model for a non-equilibrium phase transition, namely the \(O(4)\) linear sigma model, and set up the time evolution equations for the lowest order in Large N approximation. In this model we will obtain natural cooling (quenching) of the plasma from the unbroken phase to the broken symmetry phase as a result of expansion into the vacuum with boost invariant kinematics imposed. We find that there is a growth of unstable modes when the order parameter (effective pion mass) goes negative during the evolution. This causes a distortion of the single particle distribution of pions. We also reconstruct typical classical configurations by sampling the quantum density matrix and see domains. We then obtain the dilepton spectrum from this time evolving plasma using Schwinger’s closed time path formalism.

II. STRATEGIES FOR STUDYING TIME EVOLUTION PROBLEMS IN \(\lambda\phi^4\) FIELD THEORY

A. Schrodinger Picture

In quantum mechanics, time evolution problems are usually discussed in the Schrodinger Picture. The initial state in the x representation is the wave function

\[
\Psi(x, t) = <x|\Psi> \tag{2.1}
\]

which evolves in time according to the Schrodinger equation:
\[ i \frac{\partial \Psi(x,t)}{\partial t} = H \Psi(x,t) \quad (2.2) \]

This equation can be obtained from Dirac’s Variational Principle \[ \text{II} \]. Define the action

\[ \Gamma = \int dt < \Psi | i \frac{\partial}{\partial t} - H | \Psi > \quad (2.3) \]

Minimizing the Action then yield the Schrodinger equation:

\[ \delta \Gamma = 0 \rightarrow \{ i \frac{\partial}{\partial t} - H \} | \Psi > = 0 \quad (2.4) \]

Here one thinks of \( H \) as an operator which in the \( x \) representation one has

\[ p = -i \frac{d}{dx} \quad (2.5) \]

Gaussian initial conditions for the wave function would be:

\[ \Psi(x,0) = \exp(-\alpha(x-x_0)^2). \]

In field theory, the wave function is replaced by a wave functional. For example a Gaussian Wave functional is given by:

\[ \langle \varphi | \Psi > = \psi_v[\varphi, t] \quad (2.9) \]

with \( \psi_v \) given by Eq.(2.6). The variational parameters have the following meaning:

\[ \hat{\varphi}(x,t) = \langle \Psi_v | \varphi(x,t) | \Psi_v > = \langle \Psi_v | -i \delta / \delta \varphi | \Psi_v > \]

\[ G(x,y,t) = \langle \Psi_v | \varphi(x, \varphi(y)) | \Psi_v > - \hat{\varphi}(x,t) \hat{\varphi}(y,t) \quad (2.10) \]

The Effective Action for the variational parameters is

\[ \Gamma(\hat{\varphi}, \hat{\varphi}, G, \Sigma) = \int dt < \Psi_v | i \frac{\partial}{\partial t} - H | \Psi_v > \]

\[ = \int dtdx[\pi(x,t) \hat{\varphi}(x,t) + \int dtdxy\Sigma(x,y)\hat{\varphi}(x,y,t)] \]

\[ - \int dt < H > \quad (2.11) \]

where

\[ < H > = \int dx \left\{ \pi^2/2 + 2\Sigma G \Sigma + G^{-1}/8 + 1/2(\nabla \varphi)^2 \right\} \]

\[ -1/2 \nabla^2 G + 1/2\nabla''[\varphi] G + 1/8 \nabla''''[\varphi] G^2 \quad (2.12) \]

Equations of motion that result from varying the action are:

\[ \hat{\pi}(x,t) = \nabla^2 \varphi - \partial < V > / \partial \varphi; \]

\[ \hat{\varphi}(x,t) = \pi \]

\[ \hat{G}(x,y,t) = 2 \int dz [\Sigma(x,z) G(z,y) + G(x,z) \Sigma(z,y)] \]

\[ \hat{\Sigma}(x,y,t) = -2 \int dz [\Sigma(x,z) \Sigma(z,y) + G^{-2}/8 \]

\[ + \left\{ \frac{1}{2} \nabla^2 x - \partial < V > / \partial G \right\} \delta^3(x-y) \quad (2.13) \]

In \( \phi^4 \) theory if there is translational invariance, we can simplify these equations by Fourier transforming them in three dimensional space to obtain:

\[ 2\hat{G}(k,t) G(k,t) - \hat{G}^2(k,t) + 4\Gamma(k,t) G^2(k,t) - 1 = 0 \]

\[ \Gamma(k,t) = k^2 + m^2(t) ; m^2(t) = -\mu^2 + \frac{1}{2} \lambda \int [dk] G(k,t) \quad (2.14) \]

One can also linearize these equations by recognizing that the mean field approximation for the homogeneous problem is equivalent to a field theory with a time dependent mass which is self consistently determined. That is if we assume a quantum field obeying:

\[ (\Box + m^2(t)) \phi(x,t) = 0 \quad (2.15) \]

then one can satisfy the equation for \( G(x,y,t) \) by choosing:

\[ G(x,y,t) \equiv < \phi(x,t) \phi(y,t) > \quad (2.16) \]
In the Heisenberg Picture, the operators are time dependent, and the expectation value of the fields in an initial state are the infinite number of c-number variables. The infinite hierarchy of coupled Green’s functions need to be truncated by an approximation scheme. The large $N$ approximation orders the connected Green’s functions in powers of $1/N$, with the connected four point function going as $1/N$, 6 point function $1/N^2$ etc. The formalism for preserving causality in initial value problems was invented by Schwinger\cite{3} and was later cast in the form of path integrals \cite{4}. The Generating Functional for initial value problem Green’s functions is

\[
\int \prod [\Phi] e^{iS[\Phi]} = e^{iW[\Phi]}
\]

where $\Phi$ is the density matrix defining the initial state. We use the matrix notation:

\[
\Phi^a = \begin{pmatrix} \Phi_+ \\ \Phi_-
\end{pmatrix} ; \quad a = 1, 2
\]

with a corresponding two component source vector,

\[
J^a = \begin{pmatrix} J_+ \\ J_-
\end{pmatrix} ; \quad a = 1, 2
\]

On this matrix space there is an indefinite metric

\[
e_{ab} = \text{diag} (+1, -1) = e^{ab}
\]

so that, for example

\[
J^a e_{ab} \Phi^b = J_+ \Phi_+ - J_- \Phi_-
\]

From the Path Integral we get the following Matrix Green’s Function:

\[
G^{ab}(t,t') = \frac{\delta^2 W}{\delta J_a(t) \delta J_b(t')}|_{J=0} . \tag{2.23}
\]

We notice that $G_F = G^{11}(t,t')$ and $G_{F*} = G^{22}(t,t')$.

We also will need the relationships:

\[
G_{ret}(t,t') = i\Theta(t-t')|\Phi(t), \bar{\Phi}(t')|_\uparrow
\]

\[
= \Theta(t-t')[G_>(t,t') - G_<(t,t')]. \tag{2.25}
\]

\[
G_{adv}(t,t') = -i\Theta(t'-t)|\Phi(t), \bar{\Phi}(t')|_\uparrow
\]

\[
= \Theta(t'-t)[G_<(t,t') - G_>(t,t')]. \tag{2.26}
\]

as well as the relations between the Green’s functions:

\[
G_{ret}(t,t') = G_F(t,t') - G_<(t,t') = -G_{F*}(t,t') + G_>(t,t') \tag{2.27}
\]

1. Large-$N$ Approximation

The method for reducing the number of degrees of freedom in the Heisenberg picture is the large-$N$ approximation\cite{3}. If we have an $N$-component scalar field with Lagrangian:

\[
\bar{L}_{el}[\Phi] = \frac{1}{2} (\partial_\mu \Phi_i)(\partial^\mu \Phi_i) - \frac{\lambda}{8N} \left( \Phi_i \Phi_i - \frac{2N \mu^2}{\lambda} \right)^2 \tag{2.28}
\]

we can rewrite this as:

\[
\bar{L}_{el}[\Phi, \chi] = -\frac{1}{2} \Phi_i (\square + \chi) \Phi_i + \frac{N}{\lambda} \chi \left( \frac{\chi}{2} + \mu^2 \right) \tag{2.29}
\]

where $i = 1, \ldots, N$ and

\[
\chi = -\mu^2 + \frac{\lambda}{2N} \Phi_i \Phi_i \tag{2.30}
\]

If $\mu^2 > 0$, spontaneous symmetry breaking at the classical level. At this minimum the $O(N)$ symmetry is spontaneously broken, $\chi = 0$ and there are $N-1$ massless modes. Small oscillations in the remaining $i = N$ (radial) direction describe a massive mode with bare mass
equal to $\sqrt{2}\mu = \sqrt{\lambda} m$. The Generating functional for all
Graphs is given by  
\[
Z[j, K] = \int d\phi d\chi \exp\{iS[\phi, \chi] + i \int [j\phi + K\chi]\} \tag{2.31}
\]
Perform the Gaussian integral over the field $\phi$
\[
Z[j, K] = \int d\chi \exp\{iNS_{\text{eff}}[\chi, j, K]\} \tag{2.32}
\]
where
\[
S_{\text{eff}} = \int dx \left\{ \frac{1}{2} jG^{-1}[\chi]j + K\chi + \frac{1}{\chi} \left( \frac{\chi}{2} + \mu^2 \right) + \frac{i}{2} \text{Tr} \ln G^{-1}[\chi] \right\}
\]
\[
G^{-1}[\chi](x, y) \equiv \{ \Box + \chi \} \delta(x - y), \tag{2.33}
\]
Because of the $N$ in the exponent one is allowed to perform the integral over $\chi$ by stationary phase. This leads to an expansion of $Z$ in powers of $1/N$ the lowest term (stationary phase point) is related to the previous Gaussian (Hartree) approximation. The effective action of the leading order is:
\[
S_{\text{eff}}[\phi, \chi] = S_{\text{cl}}[\phi, \chi] + \frac{i\hbar}{2} \text{Tr} \ln G^{-1}[\chi]. \tag{2.34}
\]
Varying the action leads to the mean field equations:
\[
\{ \Box + \chi \} \phi = 0 \tag{2.35}
\]
\[
\chi = -\mu^2 + \frac{\lambda}{2N} (\phi^2 + \frac{i}{4} G(x, x; \chi))
\]
We notice that this is the same equation found in the Gaussian approximation with $m^2(t)$ being identified with $\chi$.

III. DYNAMICAL EVOLUTION OF A NON-EQUILIBRIUM CHIRAL PHASE TRANSITION

One important question for RHIC Experiments is can one produce disoriented chiral condensates (DCC’s) in a relativistic heavy ion collision? Recently, Bjorken, Rajagopal and Wilczek and others proposed that a nonequilibrium chiral phase transition such as a quench might lead to regions of DCC’s. The model Rajagopal and Wilczek considered was the $O(4)$ linear sigma model in a tree-level approximation, where a quench was assumed. Two deficiencies of that model were its classical nature (it could not describe $\pi$-$\pi$ scattering), and the quench was put in by hand. Our approach instead was to look at the quantum theory in an approximation that captures the phase structure as well as the low energy pion dynamics. We also used the natural expansion of an expanding plasma to cool the plasma and built into our approximation boost invariant kinematics which result from a hydrodynamic picture where the original plasma is highly Lorentz contracted. In the linear sigma model treated in leading order in the $1/N$ expansion the theory has a chiral phase transition at around 160 Mev and we choose the parameters of this theory to give a reasonable fit to the correct low energy scattering data. We obtain natural quenching for certain initial conditions as a result of the expansion process.

A. review of the linear $\sigma$ model

The Lagrangian for the $O(4)$ $\sigma$ model is:
\[
L = \frac{1}{2} \partial \Phi \cdot \partial \Phi - \frac{1}{4} \lambda (\Phi \cdot \Phi - v^2)^2 + H\sigma. \tag{3.1}
\]
The mesons form an $O(4)$ vector
\[
\Phi = (\sigma, \pi_i)
\]
As we discussed earlier in our discussion of the large-$N$ approximation we introduce:
\[
\chi = \lambda (\Phi \cdot \Phi - v^2)
\]
and use the equivalent Lagrangian:
\[
L_2 = -\frac{1}{2} \partial_i (\Box + \chi) \partial^i + \frac{\chi^2}{4\lambda} + \frac{1}{2} \lambda v^2 + H\sigma \tag{3.2}
\]
The leading order in $1/N$ effective action which we obtain by integrating out the $\phi$ field and keeping the stationary phase contribution to the $\chi$ integration is
\[
\Gamma[\Phi, \chi] = \int d^4x [L_2(\Phi, \chi, H) + \frac{i}{2} N \text{tr} \ln G_0^{-1}] \tag{3.3}
\]
\[
G_0^{-1}(x, y) = i[\Box + \chi(x)] \delta^4(x - y)
\]
This results in the equations of motion:
\[
[\Box + \chi(x)] \pi_i = 0 \quad [\Box + \chi(x)] \sigma = H, \tag{3.4}
\]
and the constraint or gap equation:
\[
\chi = -\lambda \sigma^2 + \lambda (\sigma^2 + \pi \cdot \pi) + \lambda N G_0(x, x). \tag{3.5}
\]
We will introduce fluid proper time and rapidity variables to implement the kinematic constraint of boost invariance.
\[
\tau \equiv (t^2 - z^2)^{1/2}, \quad \eta \equiv \frac{1}{2} \log \frac{t - z}{t + z}.
\]
To implement boost invariance we assume that mean (expectation) values of the fields $\Phi$ and $\chi$ are functions of $\tau$ only.
\[ \tau^{-1} \partial_\tau \phi \partial_\tau \Phi_i(\tau) + \chi(\tau) \Phi_i(\tau) = H \delta_{i1} \]
\[ \chi(\tau) = \lambda \left( -v^2 + \Phi_i^2(\tau) + N G_0(x, \tau, \tau) \right), \]  \hspace{1cm} (3.6)

To calculate the Green’s function \( G_0(x, y; \tau, \tau') \) we first determine the auxiliary quantum field \( \phi(x, \tau) \)
\[ \left( \tau^{-1} \partial_\tau \phi \partial_\tau - \tau^{-2} \partial^2_\eta - \partial^2_\perp + \chi(x) \right) \phi(x, \tau) = 0. \]  \hspace{1cm} (3.7)
\[ G_0(x, y; \tau, \tau') \equiv T\{ \phi(x, \tau) \phi(y, \tau') \} > . \]

We expand the quantum field in an orthonormal basis:
\[ \phi(\eta, x, \tau) = \frac{1}{\pi} \int d^3k \left( e^{i k \cdot x} + h.c. \right) \phi_k(\tau) \chi_{k}(\tau, \eta) \]
where \( k \equiv \omega - k \cdot \vec{x} \), \( [d^3k] \equiv dk d^2k_\perp / (2\pi)^3 \). The mode functions and \( \chi \) obey:
\[ \chi_k(\tau) = \lambda \left( -v^2 + \Phi_i^2(\tau) + \frac{1}{\tau} N \int [d^3k]|f_k(\tau)|^2 \frac{1}{2 + 2 m_e} \right) = \frac{1}{\sqrt{2 \tau^2}} \chi_0 \]
\[ \chi(\tau) = \lambda \left( -v^2 + \Phi_i^2(\tau) + \frac{1}{\tau} N \int [d^3k]|f_k(\tau)|^2 \frac{1}{2 + 2 m_e} \right) \]
\hspace{1cm} (3.9)

when \( \chi \) goes negative, the low momentum modes with
\[ \frac{k^2}{\tau^2} + \frac{1}{4} + k^2_\perp < |\chi| \]
grow exponentially. These growing modes then feed back into the \( \chi \) equation and get damped. Low momentum growing modes lead to the possibility of DCC’s as well as a modification of the low momentum distribution of particles. To fix the parameters of this mode we use the PCAC relation:
\[ \partial_\mu A_\mu(x) = f_\pi m^2_\pi \phi(x), \]  \hspace{1cm} (3.10)
and the definition of the broken symmetry vacuum.
\[ \chi_0 = m^2_\pi \sigma_0 = H \]
\[ \sigma_0 = f_\pi = 92.5 \text{MeV} \]
\[ m^2_\pi = -\lambda v^2 + \lambda f^2_\pi + \lambda N [\int [d^3k]|f_k(\tau)|^2 (1 + 2 n_k) - \frac{1}{2 + 2 m_e}] \]
The mass renormalized gap equation is
\[ \chi(\tau) - m^2_\pi = -\lambda f^2_\pi + \lambda \Phi^2_i(\tau) + \frac{\lambda}{\tau} N \int [d^3k]|f_k(\tau)|^2 (1 + 2 n_k) - \frac{1}{2 + 2 m_e} \]  \hspace{1cm} (3.11)

\( \lambda \) is chosen to fit low energy scattering data
We choose our initial data (at \( \tau_0 = 1 \)) so that the system is in local thermal equilibrium in a comoving frame
\[ n_k = \frac{1}{e^{\beta_0 E_k^0} - 1} \]  \hspace{1cm} (3.12)
where \( \beta_0 = 1/T_0 \) and \( E_k^0 = \sqrt{k^2 \omega^2 + k^2_\perp + \chi(\tau_0)} \).

The initial value of \( \chi \) is determined by the equilibrium gap equation for an initial temperature of 200 MeV and is \( .7 \text{fm}^{-2} \) and the initial value of \( \sigma \) is just \( \frac{H}{\chi_0} \). The phase transition in this model occurs at a critical temperature of 160 MeV.

To get into the unstable domain, we then introduce fluctuations in the time derivative of the classical field.

For \( \tau_0 = 1 \text{fm} \) there is a narrow range of initial values that lead to the growth of instabilities \( 25 < |\bar{\sigma}| < 1.3 \).

The results of numerical simulations described in [8] for the order parameter \( \chi \) are shown in figure 1.

\[ \text{FIG. 1. Proper time evolution of the } \chi \text{ field for two different initial values of } \sigma. \]

Fig.1 displays the results of the numerical simulation for the evolution of \( \chi \) \( \{3.3\} \). We display the auxiliary field \( \chi \) in units of \( \text{fm}^{-2} \), the classical fields \( \Phi \) in units of \( \text{fm}^{-1} \) and the proper time in units of \( \text{fm} \) (\( 1 \text{fm}^{-1} = 197 \text{MeV} \)) for two simulations, one with an instability (\( \bar{\sigma}|_{\tau_0} = -1 \) and one without (\( \bar{\sigma}|_{\tau_0} = 0 \)).. We notice that for both initial conditions, the system eventually settles down to the broken symmetry vacuum result as a result of the expansion. We also considered a radial expansion and obtained similar results \[8\]. In the radial case, the outstate was reached earlier, but the number of oscillations where \( \chi \) became negative was similar. To determine the single particle inclusive pion spectrum we go to an adiabatic basis and introduce an interpolating number operator which interpolates from the initial number operator to the out number operator. Introduce mode functions \( f_k \) which are first order in an adiabatic expansion of the mode equation.
\[ f_0^k = e^{-ikz} \frac{1}{\sqrt{2\omega_k}}, \quad dy_k/dt = \omega_k, \]  
\[ \phi(\eta, x_\perp, \tau) \equiv \frac{1}{\tau^{1/2}} \int [d^3k] (\exp(ikx)J_k^\mu(\tau) a_k(\tau) + h.c.) \]  
\[ \text{(3.14)} \]

In terms of the initial distribution of particles \( n_0(k) \) and \( \beta \) we have:
\[ n_\perp(\tau) = f(k, k_\perp, \tau) \equiv \langle \phi(k, k_\perp, \tau) \rangle \]
\[ = n_0(k) + \beta k, k_\perp, \tau \]  
\[ = n_0(k) + |\beta(k, \tau)|^2(1 + 2n_0(k)). \]  
\[ \text{(3.15)} \]

where \( \beta(k, \tau) = i(\int f_0^k \partial f_0^k - \partial f_0^k f_0^k), n_\perp(\tau) \) is the interpolating number density. The distribution of particles is
\[ f(k, k_\perp, \tau) = \frac{d^3N}{\pi^2dx_\perp^2dk_\perp^2d\eta dk_\eta}. \]  
\[ \text{(3.16)} \]

Changing variables from \((\eta, k_\eta)\) to \((z, y)\) at a fixed \( \tau \) we have
\[ E \frac{d^3N}{d^3k} = \frac{d^3N}{\pi dy dk_\perp} = \int \pi dz \, dx_\perp \, J \, f(k, k_\perp, \tau) \]
\[ = A_\perp \int dk_\eta f(k, k_\perp, \tau) = \int f(k, k_\perp, \tau) k^\mu d\sigma_\mu. \]  
\[ \text{(3.17)} \]

To compare our field theory calculation with some standard phenomenological approach, we considered a hydrodynamic calculation with boost invariant kinematics and determined the spectrum assuming that at hadronization the pions where at the breakup temperature \( T = m_\pi \) (as well as \( T = 1.4m_\pi \)), with the distribution given by the Cooper-Frye-Schonberg Formula [11]
\[ E \frac{d^3N}{d^3k} = \frac{d^3N}{\pi dk_\perp^2 dy} = \int g(x, k) k^\mu d\sigma_\mu \]  
\[ \text{(3.18)} \]

Here \( g(x, k) \) is the single particle relativistic phase space distribution function. When there is local thermal equilibrium of pions at a comoving temperature \( T_c(\tau) \) one has
\[ g(x, k) = g_\pi \{ \exp[k^\mu u_\mu/T_c(\tau)] - 1 \}^{-1}. \]  
\[ \text{(3.19)} \]

The comparison is shown in Figures 2 and 3.

**FIG. 2.** Single particle transverse momentum distribution for \( \sigma = -1 \) initial conditions compared to a local equilibrium Hydrodynamical calculation with boost invariance.

**FIG. 3.** Single particle transverse momentum distribution for \( \sigma = 0 \) initial conditions compared to a local equilibrium Hydrodynamical calculation with boost invariance.

We therefore find that a non-equilibrium phase transition taking place during a time evolving quark-gluon or hadronic plasma can lead to an enhancement of the low momentum distribution of pions.

**B. Determination of the Effective Equation of State**

Equation of state is obtained in the frame where the energy momentum tensor is diagonal-- we are already in that boost invariant frame:
\[ T_{\mu\nu} = \text{diag}(\epsilon, p_\eta, p_\perp) \]

When we have massless goldstone pions in the \( \sigma \) model \( (H = 0) \) then \( \chi \) goes to zero at large times. In the spatially homogenous case:
\[ <T_{00}> = \epsilon \quad <T_{ij}> = p\delta_{ij} \]

The equation of state becomes \( p = \epsilon/3 \) at late times even though the final particle spectrum is far from thermal equilibrium.
C. Dephasing and looking for DCC’s

As we have shown in [10], dephasing justifies the replacement of the exact Gaussian $\rho$ by its diagonal elements. At large-N or in mean field theory the density matrix is a product of Gaussians in $\phi_k$ space:

$$\langle \varphi_k' | \rho_{eff} | \varphi_k \rangle = \frac{1}{(2\pi \xi_k^2)^{\frac{3}{2}}} \exp \left\{ - \frac{\xi_k^2}{8 \sigma_k^2} (\varphi_k' - \varphi_k)^2 \right\} \cdot \left\{ 1 - \frac{1}{8 \sigma_k^2} (\varphi_k' + \varphi_k)^2 \right\},$$

(3.20)

After a short while because of dephasing, the Gaussian distribution off the diagonal $\varphi_k' = \varphi_k$ is strongly suppressed

$$\frac{\xi_k}{\sigma_k} \approx \frac{\hbar}{2kn(k)} << \xi_k$$

This is shown in Fig. 5. We find no support for “Schrödinger cat” states in which quantum interference effects between the two classically allowed macroscopic states at $v$ and $-v$ can be observed.

An ensemble may be regarded as a classical probability distribution over classically distinct outcomes. The particle creation effects in the time dependent mean field give rise to strong suppression of quantum interference effects and mediate the quantum to classical transition of the ensemble.

If we project the density matrix onto an adiabatic number basis, we can reconstruct classical field configurations from the diagonal density by replacing the field operator $a(k)$ by

$$a(k) \rightarrow [(n(k))]^{1/2} e^{i\phi(k)}$$

with $n(k)$ obtained by throwing dice on the density matrix and $\phi$ being randomly chosen between $0 < \phi < 2\pi$. Typical field configurations as a function of $r$ (averaging over angles) are shown in Fig. 6.

FIG. 5. The Gaussian $\rho_{eff}$ for $k = \frac{4}{k}$ from Ref. [10] illustrating the strong suppression of off-diagonal components due to dephasing.

FIG. 6. Four typical field configurations drawn from the same classical distribution of probabilities.

IV. INCLUSIVE DILEPTON PRODUCTION AND SCHWINGER’S CLOSED TIME PATH FORMALISM

Schwinger’s CTP Formalism is designed to allow one to calculate expectation values of operators in the initial
density matrix. One quantity we are interested in for obtaining an effective hydrodynamics is the expectation value of the energy momentum tensor:

\[ <\text{in}|T^\mu\nu(x)|\text{in}> = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \]  

(4.1)

where \( T^\mu\nu(x) \) is the Field Theory energy momentum tensor. Also by Fourier transforming this energy momentum tensor and looking in a comoving frame, we can ask how much energy is in the “free” part of various components and define an equivalent number of quanta by dividing by \( h\omega_k \) for each species.

If we consider the inclusive production of electron positron pairs the probability amplitude is

\[ <e^-(k,s)e^+(k',s')|X|i> = <X|b_{k,s}^{\text{out}} d_{k',s'}^{\text{out}}|i> \]

The inclusive distribution function for dilepton production:

\[ \frac{E_k}{m} \int \frac{d^6N}{[d^3k][d^3k']} \equiv \frac{i}{\hbar} \left| \left< \bar{b}_{k,s}^{\text{out}} b_{k',s'}^{\text{out}} \right| \right| \]

(4.2)

Using the relations between \( b, d \) to \( \Psi \) and the Free “out” fields we obtain

\[ \int d^3x_1 d^3x_2 d^3x_3 d^3x_4 e^{ik(x_2-x_4)}\{u_{k,s}^{\text{out}}(x_2)\} \{\bar{\Psi}(x_4)\} \]

(4.3)

\[ e^{ik'(x_1-x_3)}\{v_{k',s'}^{\text{out}}(x_3)\} \{\bar{\Psi}(x_1)\} \]

Now using the weak asymptotic condition \[ \Psi |_{t \to \infty} = Z^{1/2}\bar{\Psi}(\text{out}) \]

(4.4)

inside of matrix elements as well as the equation of motion of the spinors and the identity:

\[ \int_{t_1}^{t_2} \frac{dF}{dt} = F(t_2) - F(t_1), \]

(4.5)

we obtain:

\[ \text{out} <e^-(k,s)e^+(k',s')X|P_1P_2 > \text{in} = \]

\[ t^2Z^{-1} \int d^4x_1 d^4x_2 e^{ik(x_2-x_1)} \bar{u}_{k,s} D \text{out} X |\bar{T}\{\bar{\Psi}(x_2)\bar{\Psi}(x_1)\}|P_1P_2 > \text{in} \]

(4.6)

Squaring this amplitude and summing over \( X \) we obtain:

\[ \frac{E_k E_{k'}}{m} \int \frac{d^6N}{[d^3k][d^3k']} \left< (k',s';s',s') \right> = \]

\[ \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 e^{ik(x_2-x_4)}e^{ik'(x_1-x_3)} \bar{u}_{k,s} D_{x_3} \bar{u}_{k,s} D_{x_2} \times \]

\[ \text{in} <P_1P_2|\bar{T}^* \{\bar{\Psi}(x_3)\bar{\Psi}(x_4)\} \bar{T} \{\bar{\Psi}(x_2)\bar{\Psi}(x_1)\}|P_1P_2 > \text{in} \times \]

\[ D_{x_4} u_{k,x} D_{x_1} v_{k',s'} \]

(4.7)

The matrix element involved here,

\[ in <P_1P_2|T^*\{\bar{\Psi}(x_3)\bar{\Psi}(x_4)\} T \{\bar{\Psi}(x_2)\bar{\Psi}(x_1)\}|P_1P_2 > \text{in} \]

is precisely the type of Green’s function that is obtained from the generating functional of Schwinger’s CTP formalism.

The Lagrangian we will use to determine this 4 point function is the \( O(4) \) linear \( \sigma \) model + Electrodynamics.

This Lagrangian has 3 pieces: The mesons form an \( O(4) \) vector \( \Phi = (\pi_i, \sigma) \). This strongly interacting Lagrangian is given by

\[ L_{\text{strong}} = -\frac{1}{2}\partial_i(1 + \chi)\partial^i + \frac{\lambda^2}{4\lambda} + \frac{1}{2}\lambda v^2 + H\sigma \]

(4.8)

To this we add the free lepton and Photon Lagrangian:

\[ L_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \]

The interaction of the Photons with the pion plasma and the leptons is given by

\[ L_{\text{int}}[\phi_1, A_\mu, \Psi, \bar{\Psi}] = \frac{e^2}{2}(\phi_1^2 + \phi_2^2)A_\mu A^\mu + e\Phi_1 \partial_\mu \phi_2 - \Phi_2 \partial_\mu \phi_1 A^\mu \]

(4.9)

\[ -e\bar{\Psi}\gamma^\mu A_\mu + \mathcal{L}_A. \]

If we treat the electromagnetic interactions perturbatively in \( e^2 \) and the pions in the mean field approximation we obtain the graph shown in the Figure 7

FIG. 7. Leading contribution from the plasma to dilepton production. The 4 fermion Graph is to be evluated using the Matrix CTP Green’s functions.

The inverse propagators in the LSZ representation allow the external legs to be put the leptons on mass shell. One is left with:

\[ \frac{E_k E_{k'}}{m} \int \frac{d^6N}{[d^3k][d^3k']} = M_{\mu\nu}(k,s;k',s') W^{\mu\nu}(k,k') \]

(4.10)
\[ M^{\mu \nu}(k, s; k', s') = \bar{v}(k', s') \gamma^\mu u(k, s) \bar{u}(k, s) \gamma^\nu v(k', s') \]

\[ W_{\mu \nu}(k, k') = W^1_{\mu \nu} + W^2_{\mu \nu} + W^3_{\mu \nu} \]

\[ = i e^4 \int d^4y_1 \int d^4y_2 \int d^4z_1 \int d^4z_2 e^{i(k+k')(z_1-y_1)} \]

\[ \times D^{\alpha \nu}(y_1, y_2) \Pi^{\alpha \lambda 2}(y_2, z_2) D_{\lambda \nu} \]

\[ + D^{21}(y_1, y_2) \Pi^{\alpha \lambda \nu}(y_2, z_2) D_{\lambda \alpha} \]

\[ + D^{21 2}(y_1, y_2) \Pi^{\alpha \lambda \nu}(y_2, z_2) D_{\lambda \alpha} \]

If we want the invariant Mass distribution function when

\[ M^2 = q^2; \quad q = k + k' \]

we obtain

\[ \frac{dN}{dM^2 d^3q} = \frac{q^0 dN}{dM^2 d^3q} = R_{\mu \nu}(q) W^{\mu \nu}(q). \]

Let us first look at the case of a thermal plasma where we can calculate everything analytically. The vacuum polarization graph can be found using the following expansion of the pion field to calculate the Finite Temperature pion Green’s functions:

\[ \phi(x, t) = \int \frac{d^3k}{2\omega_k} \left[ \exp(ikx) a_k + \exp(-ikx) b_k^\dagger \right] \]

The creation and annihilation operators obey the commutation relations:

\[ [a_k, a_{k'}^\dagger] = [b_k, b_{k'}^\dagger] = (2\pi)^3 \delta^3(k - k') \]

And the phase space number densities \( n_k^+ \) and \( n_k^- \) are defined by

\[ \langle a_k a_{k'}^\dagger \rangle = (2\pi)^3 n_k^+ \delta^3(k - k') \]

\[ \langle b_k b_{k'}^\dagger \rangle = (2\pi)^3 n_k^- \delta^3(k - k') \]

so that the total number of positively charged particles is given by:

\[ N^+ = \int d^3k \langle a_k a_{k}^\dagger \rangle = \int d^3x d^3k n_k^+ \]

For the case of neutral plasma in thermal equilibrium at inverse temperature \( \beta \) we have

\[ n_k^+ = n_k^- = 1/(e^{\beta \omega} - 1) \]

Using the fact that for a free pion gas:

\[ \langle \phi(x) \phi(y) \rangle_{th} = \frac{\int d^3k}{2\omega_k(2\pi)^3} \left[ n_k^+ e^{ik(x-y)} + (1 + n_k^-) e^{-ik(x-y)} \right] \]

one finds that the vacuum polarization tensor is given by

\[ -i \Pi^\mu \nu_{\nu}(q^2) = \int \frac{d^3k_1}{2\omega_1(2\pi)^3} \frac{d^3k_2}{2\omega_2(2\pi)^3} \]

\[ \frac{d^4k}{(2\pi)^4} \{(k_1 - k_2)_{\mu} (k_1 - k_2)_{\nu}, \]

\[ n_{k_1}^+ n_{k_2}^+ \delta^4(q - k_1 - k_2) \]

\[ + (1 + n_{k_1}^-)(1 + n_{k_2}^+ \delta^4(q + k_1 + k_2)) \]

\[ (k_1 + k_2)_{\mu} (k_1 + k_2)_{\nu}, \]

\[ [n_{k_1}^- (1 + n_{k_2}^+) + n_{k_1}^+ (1 + n_{k_2}^-)] \delta^4(q - k_1 + k_2) \}

The three terms in the above relation correspond to pion pair annihilation, creation and bremsstrahlung, respectively. The delta functions show that only the annihilation process survives for \( M^2 \) above the dilepton threshold. We therefore obtain:
and define phase space number densities $N^\pm$ with a homogenous thermal plasma of pions is given by:

$$\frac{1}{VT} \frac{dN^\pm}{dM^3 dq} = \frac{\alpha^2 B M}{48\pi^4} \frac{q}{q_0} (1 - \frac{4m_p^2}{M^2}) \int_{-\omega^+}^{\omega^+} \frac{d\omega}{q} n^+_kn^-k.$$  \hfill (4.24)

A. Finite Time Effects

For our numerical simulation of the interacting plasma we can only follow the time evolution of the plasma for a fixed time $2T$ which is typically about 100 fermis. Therefore we need to make sure we have the causal formulation so that dileptons at time $T$ only get contributions from processes occurring at times $t < T$. So we should not use Feynman propagators but instead use the CTP matrix propagators when we investigate the theory at finite times. Also the interacting plasma is not time translationally invariant so we must use a non-covariant formalism. In the mean field approximation (as well as for the pion gas) there is factorization of four point functions so that the current-current correlation function takes the form

$$< J^\mu(x) J^\nu(y) > = \Phi^\dagger(x) \Phi(y) > < \partial^\mu \Phi(x) \partial^\nu \Phi^\dagger(y) >$$

$$+ < \partial^\mu \Phi^\dagger(x) \Phi(y) > < \partial^\nu \Phi(x) \Phi^\dagger(y) >$$

$$+ < \partial^\mu \Phi^\dagger(x) \partial^\nu \Phi(y) > < \Phi(x) \Phi^\dagger(y) > .$$  \hfill (4.25)

If we insert the mode expansion of the charged pion fields:

$$\Phi(x,t) = \int [d^3k] \left[ \exp(ikx) f_k^+(t) a_k + \exp(-ikx) f_k^-(t) b_k \right]$$  \hfill (4.26)

and define phase space number densities $N^+_k$ and $N^-_k$ by

$$\langle a_k^\dagger a_{k'} \rangle = (2\pi)^3 N^+_k \delta^3(k - k')$$

$$\langle b_k^\dagger b_{k'} \rangle = (2\pi)^3 N^-_k \delta^3(k - k')$$  \hfill (4.27)

we obtain:

$$< J_\mu(x) J_\nu(y) > =$$

$$e^2 \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \delta^3(p-\vec{k}) \frac{1}{i} G_{\mu\nu}(\vec{k}, \vec{p}; t_x, t_y).$$  \hfill (4.28)

where

$$\frac{1}{i} G_{\mu\nu}(\vec{k}, \vec{p}; t_x, t_y) = A(\vec{k}) K^+(\vec{p}) + K^-(\vec{p}) A(\vec{p})$$

$$- N^+(\vec{k}) M^{+\mu}(\vec{p}) - M^{-\mu}(\vec{k}) N^-(\vec{p}),$$  \hfill (4.29)

and

$$K^{\pm}(\vec{k}) \equiv \begin{cases} 
B(\vec{k}) & \mu = 0 \quad \nu = 0 \\
\pm i k_j D(\vec{k}) & \mu = 0 \quad \nu = j \\
- (+) i k_i A(\vec{k}) & \mu = i \quad \nu = 0 \\
k_i k_j A(\vec{k}) & \mu = i \quad \nu = j 
\end{cases} , \hfill (4.30)

M^{\pm}(\vec{k}) \equiv \begin{cases} 
D(\vec{k}) & \mu = 0 \\
- (+) i k_j A(\vec{k}) & \nu = 0 \\
- (+) i k_i A(\vec{k}) & \nu = j 
\end{cases} , \hfill (4.31)

and

$$A(\vec{k}; t_x, t_y) = (1 + N_k)[f^+_k(t_x) f^+_k(t_y)]^* + N_k f^-_k(t_x) f^-_k(t_y)$$

$$B(\vec{k}; t_x, t_y) = (1 + N_k)[f^+_k(t_x) f^-_k(t_y)]^* + N_k f^-_k(t_x) f^-_k(t_y)$$

$$C(\vec{k}; t_x, t_y) = (1 + N_k)[f^-_k(t_x) f^-_k(t_y)]^* + N_k f^+_k(t_x) f^-_k(t_y)$$

$$D(\vec{k}; t_x, t_y) = (1 + N_k)[f^-_k(t_x) f^-_k(t_y)]^* + N_k f^-_k(t_x) f^-_k(t_y).$$  \hfill (4.33)

Contracting $G_{\mu\nu}(\vec{k}, \vec{p})$ with $\bar{L}^{\mu\nu} = q^\mu q^\nu - g^{\mu\nu} q^2$ we obtain:

$$\bar{L}^{\mu\nu} G_{\mu\nu}(\vec{k}, \vec{p}; t_x, t_y) =$$

$$\{ (\vec{q} \cdot (\vec{p} + \vec{k}) )^2 + (q_0^2 - \vec{q} \cdot \vec{q})(\vec{p} + \vec{k}) \cdot (\vec{p} + \vec{k}) \} A(\vec{p}) A(\vec{k})$$

$$- i q_0 (\vec{p} + \vec{k}) \cdot \vec{q} [ A(\vec{k}) B(\vec{p}) + B(\vec{k}) A(\vec{p}) - C(\vec{k}) D(\vec{p}) - D(\vec{k}) C(\vec{p}) ]$$  \hfill (4.34)

where

$$\vec{q} = \vec{k} - \vec{p}$$

At the special case where $\vec{q} = 0$ we obtain:

$$\bar{L}^{\mu\nu} G_{\mu\nu}(\vec{k}, \vec{k}; t_x, t_y) = 4 q_0^2 \vec{k} \cdot \vec{k} A(\vec{k}) A(\vec{k})$$  \hfill (4.35)

Here $\vec{k} \cdot \vec{k} \to \frac{q_0^2 - 4m^2}{4}$ in the infinite time limit.

B. pion gas

To see what the effects of finite $T$ might be, let us look at the case where everything is analytically known, namely the pion gas we discussed before. So we use the
known values of $A, B, C, D$ appropriate to the pion gas where

$$f_k(t) = \frac{e^{-\omega_k t}}{\sqrt{2\omega_k}}.$$ 

First let us look at the effect of just putting a finite time cutoff into the McLerran formula for $W$. Using the covariant form of the photon propagator and just cutting off the internal integrations to run from $-T$ to $T$ one obtains:

$$W_{\mu\nu}^{\text{cut-off}}(k, k') = i e^4 \int \frac{d^4q}{q^2} \int d^2y_2 \int d^2z_2 \int_{-T}^{T} dy_{20} \int_{-T}^{T} dz_{20} \times e^{-iq(y_2 - z_2)} \Pi_{\mu\nu}^{00}(y_2, z_2)$$

$$= \frac{iV^3}{q^4} \int_{-T}^{T} dy_{20} \int_{-T}^{T} dz_{20} e^{-iq(y_2 - z_2)} \times \int [dK] G^{\mu\nu}(\vec{K}, \vec{K} - \vec{q}; y_{20}, z_{20}) \tag{4.36}$$

Again looking only at the place $\vec{q} = 0$ and keeping only the pion annihilation contributions:

$$\tilde{L}^{\mu\nu} G_{\mu\nu}(\vec{k}; y_{20}, z_{20}) = \frac{n_\pi^2}{\omega_k^2} \tilde{K} e^{2i\omega_k (y_{20} - z_{20})} \tag{4.37}$$

we can perform the integration over $T$ and obtain the factor:

$$4 (\sin[q_0 - 2\omega_k T]/q_0 - 2\omega_k)^2$$

which in the limit $T \to \infty$ becomes the factor

$$2T \times 2\pi \delta(q_0 - 2\omega_k)$$

Thus we see that in the infinite time limit, the back to back dileptons get contributions only from pion pairs in the plasma with zero combined three momentum, each carrying energy $q_0/2$. In what follows we want to see how doing a causal calculation changes the way in which we go on mass-shell from this simple replacement of the delta function by a representation of the delta function. We will find extra terms which only slowly go to zero with the time $T$.

We now use the 3 dimensional form for the propagators:

$$D^\epsilon_{\mu\nu}(x, y) = \frac{\omega_k^2}{2q_0} \left[ e^{i\vec{q}(\vec{x} - \vec{y})} - e^{-i\vec{q}(\vec{x} - \vec{y})} \right]$$

$$\int [dq] \int \frac{d^3q}{2q_0} e^{i\vec{q}(\vec{x} - \vec{y})} [e^{-iq_0(x_0 - y_0)} - e^{iq_0(x_0 - y_0)}]$$

$$= \frac{i \Theta(x_0 - y_0) e^{-\epsilon(x_0 - y_0)} \Phi_{\mu\nu} \int [dq] e^{i\vec{q}(\vec{x} - \vec{y})} \delta[\Phi(0)]] \tag{4.38}$$

In our numerical simulations we assumed spatial homogeneity so that the vacuum polarization has the form:

$$\Pi^{\mu\nu}(x) = i \langle J^\mu(x) J^\nu(y) \rangle$$

$$= \int [dK] \int [dP] e^{-i(\vec{K} - \vec{P})(\vec{x} - \vec{y})} G^{\mu\nu}([\vec{K}, \vec{P}; x_0, y_0] \tag{4.39}$$

Inserting these into the expression for $W_{\mu\nu}(k, k')$ we then obtain

$$W_{\mu\nu}^{(1)}(k, k') = i e^4 V \int_{-T}^{T} dy_{10} \int_{-T}^{T} dz_{10} \int_{-T}^{T} dy_{20} \int_{-T}^{T} dz_{20} \left[ \frac{dK}{dP} e^{-i(\vec{y}_{10} - \vec{y}_{20})} e^{-i(\vec{z}_{10} - \vec{z}_{20})} e^{i\Phi_0(\vec{z}_{10} - \vec{y}_{10})} \right]$$

$$\Delta[\Phi(\vec{y}_{10} - \vec{y}_{20})] G^{\mu\nu}([\vec{K}, \vec{K} - \vec{q}; \vec{y}_{20}, \vec{z}_{20}] \Delta[\Phi(\vec{z}_{20} - \vec{z}_{10})] \tag{4.40}$$

where

$$\vec{q} = \vec{K} + \vec{P}$$

It is this contribution which persists when the time cutoff $T \to \infty$.

For the other two contributions we obtain:

$$W_{\mu\nu}^{(2+3)}(k, k') = -2 e^4 V \int_{-T}^{T} dy_{10} \int_{-T}^{T} dz_{10} \int_{-T}^{T} dy_{20} \int_{-T}^{T} dz_{20} \left[ G^{\mu\nu}([\vec{K}, \vec{K} - \vec{q}; \vec{y}_{20}, \vec{z}_{20}] - G^{\mu\nu}([\vec{K}, \vec{K} - \vec{q}; \vec{z}_{20}, \vec{y}_{20}]) \right]$$

$$\times \frac{e^{-i[\Phi(\vec{y}_{10} - \vec{y}_{20})]} \Delta[\Phi(\vec{z}_{20} - \vec{z}_{10})]}{2[\Phi]} \tag{4.41}$$

Things simplify dramatically at the place where $\vec{q} = 0$. At that point the second and third contributions vanish and we have

$$\frac{dN}{V \sqrt{T} \, d^{n+1} q} = \frac{4q_0^3}{(2\pi)^n} \frac{2B_n}{3} \Omega_n \int k^{n+1} dk \, F[k, \vec{q} = 0, q_0]$$

$$F[k, \vec{q} = 0, q_0] = i e^{4V} \int_{-T}^{T} dy_{10} \int_{-T}^{T} dz_{10} \int_{-T}^{T} dy_{20} \int_{-T}^{T} dz_{20} e^{-iq_0(z_{10} - y_{10})} \Delta_0(y_{10} - y_{20}) [A(k, y_{20}, z_{20})]^2 \Delta_0(z_{20} - z_{10}) \tag{4.42}$$

where

$$A(k, t, t') = (1 + n_k e^{-i\omega_k(t - t')} + n_k e^{i\omega_k(t - t')})$$

and

$$\Delta_0(t - t') = -i \frac{\sin m_\gamma(t - t')}{m_\gamma}; \quad \Omega_n = \frac{\Gamma(n/2)}{2\pi^{n/2}} \tag{4.43}$$

where $n$ is the number of spatial dimensions.

In the limit $m_\gamma \to 0$ we have:

$$\Delta_0(t - t') = -i(t - t') e^{-|t - t'|}$$

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The result for the annihilation part for massless photons is:

\[
F_{\text{ann}} = - \frac{e^4 N_c^2 (k^2)}{64 q_0^2 \omega_k^2 (q_0 - 2\omega_k)^2 T} \times [16 \omega_k^4 + q_0^4 - 8\omega_k^2 q_0^2 \cos(2(\omega_k - q_0)T)]
\]  

(4.43)

with

\[
\omega_k^2 = k^2 + m^2
\]

We can rewrite this as:

\[
F_{\text{ann}} = - \frac{e^4 N_c^2}{4q_0^2\omega_k^2} \times \left\{ \frac{\sin^2((2\omega_k - q_0)T)}{(q_0 - 2\omega_k)^2 T^2} + \frac{(q_0 + 2\omega_k)^2}{16q_0^2 \omega_k^2 T^2} \right\}
\]

(4.44)

We would like to compare the representation of the delta function squared found here (renormalized to one at the delta function) to the result of just naively putting a cutoff into the covariant calculation which gave:

\[
\sin^2\left\{\frac{(q_0 - 2\omega_k)T}{(q_0 - 2\omega_k)^2 T^2}\right\}
\]

(4.45)

The CTP formalism which preserves causality instead gives:

\[
\left\{ \frac{\sin^2((2\omega_k - q_0)T)}{(q_0 - 2\omega_k)^2 T^2} + \frac{(q_0 + 2\omega_k)^2}{16q_0^2 \omega_k^2 T^2} \right\}
\]

The last term makes a very small difference at large \( T \). In obtaining this result we assumed that

\[
\varepsilon T \to \infty \quad \text{as} \quad \varepsilon \to 0.
\]

For the brehmstrahlung contribution, we find

\[
F_{\text{breihm}} = - \frac{e^4 N_c k(1 + n_k)}{\pi T (m_\gamma^2 + \varepsilon^2) \omega_k^2 (q_0^2 - m_\gamma^2)^2} \times
\]

\[
(-2 m_\gamma^4 + 2 m_\gamma^2 q_0^2 - q_0^4 - 4 m_\gamma^2 \varepsilon^2 - 2 q_0^2 \varepsilon^2 - 2 \varepsilon^4 + 2 (m_\gamma^2 + \varepsilon^2) (m_\gamma^2 - q_0^2 + \varepsilon^2) \cos(2q_0 T)
\]

\[
+ 4 q_0 \varepsilon (m_\gamma^2 + \varepsilon^2) \sin(2q_0 T))
\]

(4.46)

so that the renormalized delta function squared for this case becomes:

\[
- \frac{1}{4q_0^2 T^2 (m_\gamma^2 + \varepsilon^2) \omega_k^2}
\]

\[
(-2 m_\gamma^4 + 2 m_\gamma^2 q_0^2 - q_0^4 - 4 m_\gamma^2 \varepsilon^2 - 2 q_0^2 \varepsilon^2 - 2 \varepsilon^4 + 2 (m_\gamma^2 + \varepsilon^2) (m_\gamma^2 - q_0^2 + \varepsilon^2) \cos(2q_0 T)
\]

\[
+ 4 q_0 \varepsilon (m_\gamma^2 + \varepsilon^2) \sin(2q_0 T))
\]

(4.47)

Here we have kept both a small photon mass (to regulate the infrared) as well as \( \varepsilon \). For the brehmstrahlung contribution one cannot set \( \varepsilon \) to zero.

At \( m_\gamma \to 0 \) this becomes:

\[
\frac{\sin^2(q_0 T)}{q_0^2 T^2} \left[ 1 - \frac{q_0^2}{4 \varepsilon^2} \right] + \frac{q_0^2}{4 \varepsilon^2} \frac{1}{\varepsilon^2 T^2}
\]

(4.48)

In order to obtain our previous results we used \( \varepsilon T - > \infty \). We now see to also have the unwanted terms going to zero we also need \( \varepsilon^4 T - > \infty \). The actual rate of production of dileptons gets from this expression a contribution which goes to zero as a constant divided by the total time. This constant is about the size of the entire dilepton production gets from this expression a contribution which goes to zero as a constant divided by the total time.

The result for the annihilation part for massless photons is:

\[
\epsilon \to (T/T_0)^{-\delta}; \quad 0 < \delta << 1,
\]

in order to smoothly go to the covariant cutoff result that is

\[
\frac{\sin^2(q_0 T)}{q_0^2 T^2}
\]

(4.49)

With this form for the \( \epsilon \), the finite \( q_0 \) dependent contribution to the cross section goes to zero as \( 1/T^{1-\delta} \). Thus in doing numerical simulations, we find that in order to avoid contamination from brehmstrahlung processes we need to go to quite large hadronic time scales \( T > 1000 \) to be in the asymptotic regime for the production of dileptons which is an electromagnetic process.

For the brehmstrahlung process, the effective \( \delta \) function is independent of \( k \) so that one can do the integration over \( k \) for any \( q_0 \) to obtain:

\[
C_n = \Omega_n \int \frac{dk}{(2\pi)^n} k^{n+1} \frac{(1 + n_k)n_k}{\omega_k^2}
\]

(4.50)

For \( \beta^{-1} = m_\pi \) we find \( C_1 = 0.127718 \), and \( C_3 = 0.0792387 \).

For the creation contribution we get a result similar to the annihilation but with \( q_0 \to -q_0 \), that is:

\[
F_{\text{creation}} = - \frac{e^4 N_c^2}{4q_0^2 \omega_k^2} \times
\]

\[
\left\{ \frac{\sin^2((2\omega_k + q_0)T)}{(q_0 + 2\omega_k)^2 T^2} + \frac{(-q_0 + 2\omega_k)^2}{16q_0^2 \omega_k^2 T^2} \right\}
\]

(4.51)

so that the renormalized square of the delta function is:

\[
\frac{\sin^2((2\omega_k + q_0)T)}{(q_0 + 2\omega_k)^2 T} + \frac{(q_0 - 2\omega_k)^2}{16q_0^2 \omega_k^2 T^2}
\]

vs. the cutoff McLerran formula result:

\[
\frac{\sin^2(\{q_0 + 2\omega_k\})}{(q_0 + 2\omega_k)^2 T^2}
\]

(4.52)
Using these formula, we have evaluated the dilepton production at fixed time $T$ for both a pion gas and for the interacting field theory described by the $\sigma$ model described above. When the effective pion mass goes negative, there is significant enhancement of the signal, however one can see the finite time effects are still not controlled in our present simulations. In figure 8 we show the finite time effects for a pure pion gas where one can determine the infinite time limit analytically. The time here is 100 fermis. In figure 9 we see that quench conditions enhances significantly the production of low mass dileptons over what one would find for a pure pion gas.

![Figure 8](image1.png)

**FIG. 8.** Finite Time effects for a pure pion gas. Here $t_f = 100f$

![Figure 9](image2.png)

**FIG. 9.** Comparison of quench (negative effective mass regime) vs. non-quench initial conditions for an interacting field theory described by the sigma model. The large rise for small effective dilepton mass is the artifact of the finite time for the simulation. This effect slowly goes away as $1/t_f$.

V. CONCLUSIONS

We have shown how to use the CTP formalism to calculate the dilepton spectrum arising from a time evolving or a thermal plasma. For a plasma undergoing a chiral phase transition we expect a strong signal for existence of DCC-states in $e^+e^-$ - channel

$$M_{inv} \sim 2m_\pi; q_\perp < 300\text{ MeV}$$

This would be visible by CERES if

$$k_{cut}^\perp = 60\text{ MeV}$$

In our calculations we have ignored possible important effects of direct two body scattering in the plasma which arise only in next order in the $1/N$ approach. A similar enhancement seen by D. Boyanovsky et. al. [15] in photon spectrum. Another problem for us is that our result is influenced by large finite Time corrections which are apparent for the free pion gas.
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