Dependence of stress state of monolithic lining on weight of process equipment in underground mines

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Abstract. The analytical method is developed to design monolithic lining for tunnels as function of weight of process equipment in underground mines. The method is implemented as a computer program which makes it possible to find and adjust regularities of stress state formation in mine support and adjacent rock mass.

1. Introduction

Mineral mining, processing and storage use underground structures and special process equipment of considerable weight. Mine planning, design, construction and operation requires assessment of stress of support structures in order to determine their load-bearing capacity. Different methods used in stress state assessment can be divided into experimental (in-situ or lab-scale) and computational. According to [1], support design should take into account both external effects and internal loads generated, for instance, by heavy process equipment, haulage machines or piled materials.

Widely applicable computer engineering allows using dedicated software packages implementing numerical methods, for example, the finite element method, and conventional engineering analysis based on hypotheses and assumptions of structural theory [2]. Despite some essential shortages, such methods are widely applied in practical design.

For some years, the Tula State University has been engaged in research aimed to improve the theory of modeling interaction between underground structures and surrounding rock mass as elements of a unified system exposed to deformation, as well as to develop new strict analytical methods of mine support design (tunnel lining). These methods rest upon an integrated scientific and methodological basis—continuum mechanics, in particular, elasticity theory [3], geomechanics and mechanics of underground structures [2, 4].

Developed at the Tula State University, the design methods for monolithic, multi-layered (hybrid), shotcrete plus rock bolting support types (lining) for deep and shallow underground excavations (tunnels) subject to static loading—rock weight, groundwater pressure, tectonic forces and seismic effect of earthquakes [2, 5–7]—are recommended for practical application in various purpose mine design by effective normative legal documentation both in Russia and abroad.

The lining design method for a deep tunnel of arbitrary cross section under load generated by weight of heading and haulage machines, hoisting facilities, jacks and expandable mechanism thrust against lining is proposed in [8]. It should be emphasized that down to recent time, there is yet not a strict mathematical method for design of mine support constructed in situ immediately nearby ground surface and subjected to local loads due to weight of heavy mining and transportation machines operating in mines.
The science-based methods to calculate such loads are in high demand in design and construction of special-purpose underground storages or repositories for safe keeping of radioactive waste and spent nuclear fuel. The pressure produced by lead radioactive waste containers on the floor of underground structures can reach considerable values and exert appreciable influence on stresses and strains in support (lining). Irrespective of the type of rocks and repository, it should be provided that the containers are removed from the storage, and this can be accompanied by essential loading exerted by weight of the containers and transport equipment on lining.

2. Problem statement
In connection with this, based on the mathematical modeling of interaction between circular cross-section tunnel constructed in subsurface and rock mass as elements of a unified system exposed to deformation under internal local load, the relevant problem of elasticity and creep was formulated and its rigorous solution was obtained. The analytical model of the problem is depicted in Figure 1.

Rock mass is modeled by a semi-infinite, uniform, isotropic, linearly deformable ponderable medium \( S_0 \) with a straight-line boundary \( L \). Material of the medium \( S_0 \) is characterized by the bulk density \( \gamma \), deformation modulus \( E_0 \) and Poisson’s ratio \( \nu_0 \). The circular hole with the boundary \( L_0 \) and radius \( R_0 \), located in the medium \( S_0 \) at a distance \( H \) from \( L \) and supported by rings \( S_1 \) with the inner boundary \( L_1 \) and radius \( R_1 \), models the tunnel lining. The ring \( S_1 \) is made of a material with the deformation characteristics \( E_1 \) and \( \nu_1 \).

3. Mathematical modeling results
The problem schematically shown in Figure 1 is solved as a sum of solutions of two plane elasticity problems for a ponderable medium free from the load on the boundary \( L_1 \) (problem on rock mass weight) and for an imponderous medium with its boundary \( L_1 \) subjected to the local distributed load (problem on weight of an object located in tunnel).

The weight of rocks in the domain \( S_0 \) is modeled as a field of unequal initial stresses, while the composite stress state in \( S_0 \) is presented as a sum of the initial and unknown additional stresses conditioned by the reinforced hole in the medium. The weight of the tunnel lining as against rock mass weight is neglected.

The boundary conditions in the formulated plane elasticity problem govern the absence of the load at the straight line boundary \( L \) of the medium \( S_0 \), fulfillment of continuity conditions by vectors of total stresses and displacement at the line \( L_1 \) of the interface between the domains \( S_0 \) and \( S_1 \) having different deformation characteristics, and the absence of external load at \( L_1 \) in the problem on rock weight.
mass weight, or the presence of the vertical load $P$ on the part $[\theta_1, \theta_2]$ of the inner boundary $L_1$ in the problem on weight of equipment inside the tunnel [9]. In the reduced formulation of the plane elasticity problem, distributed load along the long axis of the tunnel is assumed as infinite; the influence of the finite value of this load on the stress state of the lining can be taken into account using adjusting empirical coefficients.

The boundary-value problems of the theory of complex variable function, consistent with the problems on rock mass weight and locally distributed internal load, are solved using the mathematical apparatus provided by the theory of complex variable functions—complex potentials introduced by Kolosov–Muskhelishvili [3], analytical extension of complex potentials, properties of complex Fourier and Laurent series [10, 6].

The computational technique implemented as a software package for personal computers, makes it possible to determine stresses and forces (bending moment and longitudinal forces) in the tunnel lining, as well as stress state of surrounding rock mass. The computation results allow finding and adjusting regularities of stress state formed in the monolithic lining of the circular tunnel under local vertical load on the inner surface of the lining.

Below in this article, the authors offer the results obtained in the computer modeling of stress state of the monolithic concrete lining under weight of rocks and heavy process equipment situated inside the tunnel at the following input data: occurrence depth of the tunnel is $H = 10.4$ m, outer and inner radii of the lining are $R_0 = 5.2$ and $R_1 = 4.7$ m, respectively (thickness of the concrete lining is $\Delta = 0.5$ m); deformation characteristics of rocks are $E_0 = 20$ MPa, $\nu_0 = 0.35$; concrete—$E_1 = 36000$ MPa, $\nu_1 = 0.2$; the load application range is from $\theta_1 = 205^\circ$ to $\theta_2 = 260^\circ$ (Figure 1).

Figure 2 shows contour lines of relative normal shear stresses $\sigma_{\theta}/P$ at the points of the outer (Figure 2a) and inner (Figure 2b) boundaries of the lining under the action of the locally distributed inner load.

It follows from Figure 2 that the internal locally distributed load at the points of the outer and inner boundaries of the tunnel lining induces both compressive (negative) and tensile (positive) stresses. The maximum values of the tensile stresses at the outer boundary and compressive stresses at the inner boundary appear in the radial sections in the middle of the load application segments at the level $m - m$ ($\theta \approx 240^\circ$), see Figure 1.
Figure 3 presents contour line of the normal shear stresses $\sigma_\theta$ at the boundary points under the action of rock weight.

![Contour lines of stress $\sigma_\theta$, MPa, at the points of the lining boundary due to weight of rocks: (a) outer boundary; (b) inner boundary.](image)

It is evident in Figure 3 that the maximum compressive stresses under rock weight are not higher than 9.21 and 9.23 MPa at the outer and inner boundaries, respectively. The maximum tensile stresses are 5.30 and 7.07 MPa at the outer and inner boundaries of the lining in radial sections at $\theta = 270^\circ$, which conforms with the lower point of the vertical diameter.

Figure 4 depicts the dependence of the tensile and compressive stresses $\sigma_\theta^{\text{max}} / P$ on the deformation modulus $E_1$ of concrete in case of various thickness of the lining. The input data are: $H = 10.4$ m, $R_0 = 5.2$ m; $E_0 = 20$ MPa, $\nu_0 = 0.35$; the load application range is from $\theta_1 = 205^\circ$ to $\theta_2 = 260^\circ$, which agrees with the width $b = 1.6$ m at the level $m-m$ in Figure 1. The range of the deformation modulus corresponds to the concrete grades B15 to B60; Poisson’s ratio of concrete is $\nu_1 = 0.2$.

![Dependence of $\sigma_\theta^{\text{max}} / P$ on $E_1$ at the (a) outer and (b) inner boundaries of the lining. Lines 1, 2 and 3 stand for the lining thickness $\Delta = 0.2$ m, $\Delta = 0.5$ m and $\Delta = 1.0$ m, respectively.](image)

It follows from the dependences in Figure 4 that maximum stresses in the lining grow gradually with the concrete modulus of deformation by the linear law. The most intense growth of the stresses is observed in the thinner lining—lines 1 and 2 in Figure 4. The increase in the deformation modulus of concrete at the lining thickness $\Delta = 1.0$ m almost has no effect on the value of the stresses (lines 3).
The authors believe that the developed rigorous analytical method implemented as a computer program is a convenient tool for the support design in various purpose underground excavations, including lining of traffic tunnels.

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