Non-Cooperative Optimization Algorithm of Charging Scheduling for Electric Vehicle

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Abstract: In this paper, we aim to propose a charging scheduling algorithm for electric vehicles on highways. While the number of electric vehicles has been increasing recently, charging stations are not becoming widespread compared to gas stations. The distance that an electric vehicle can run on one charge is only around 120 km to 400 km. Therefore, it is necessary to plan to recharge in advance when driving long distances. Problems related to planning algorithms are called charging scheduling problems of electric vehicles. In this paper, we assume that there is no difference in the power of the electric vehicle and the charging station, and consider the situation where each acts to maximize its profit. First, since the electric vehicle can select the charging station freely, it motivates us to solve the optimal allocation problem of the electric vehicle to the charging station using matching theory. Then, non-cooperative game theory is utilized to obtain the energy demand and energy price for the electric vehicles and charging stations, respectively. In addition, the convergence condition of the non-cooperative game is theoretically derived. Finally, the effectiveness of the proposed non-cooperative charging scheduling algorithm is confirmed by numerical simulation.

Key Words: charging scheduling, electric vehicle, matching theory, game theory.

1. Introduction

In recent years, the development of electric vehicles is progressing due to environmental problems. Compared to conventional cars, electric vehicles have a remarkably short distance to travel with one charge. Therefore, when traveling over long distances, it is necessary to plan the location and time of charging in advance. This planning problem is called the charging scheduling problem of electric vehicles. The recent advancement of intelligent transport system (ITS) technology makes the researches on the charging scheduling progressed actively. This is because ITS utilizes information communication technology (ICT) to integrate and cooperate with people, road infrastructures, and vehicles based on real-time traffic information obtained to solve problems related to road traffic management, advanced operation, efficiency, and other traffic-related issues [1]. Based on the information obtained by the ITS, it becomes possible to change the charging price in real time.

We briefly discuss some of the existing work on the charging scheduling problem. A study in [2] dealt with the problem of selecting the charging stations for electric vehicles by taking the price and distance into consideration. In addition, prices are decided by the charging stations using game theory. The drawbacks of this study are that electric vehicles perform a non-cooperative scheduling only between two charging stations, the mechanism of determining the energy demand of electric vehicles was not provided in the algorithm, and no theoretical analysis on the equilibrium point in the game theory is conducted. In [3], a cooperative distributed charging scheduling algorithm was proposed. The transition of the number of electric vehicles in the charging station is modeled by using queuing theory. Meanwhile, the problem of balancing the utilization of each charging station was solved by using a consensus algorithm. In [4], they dealt with the charging problem of electric vehicles on highways by assuming that the derivation of the waiting time is extremely difficult. The optimization problem of the driver’s selection was solved by using the optimized route searching algorithm, which is called the A* algorithm. In [5], they formulated assignment problems of electric vehicles and dynamic pricing of multiple stands. They solved the problem of minimizing the waiting time by introducing the queue in the model of the station, but the electric vehicle just reacts to the price and determines the behavior as well concerning the energy demand. In [6], they dealt with Stackelberg games where charging stations are the leaders and electric vehicles are the followers, but there was no equilibrium analysis provided. In [7], they dealt with the charging problem on general roads when the behavior of electric vehicles is affected by waiting time, including the case where the energy price is determined dynamically.

In this paper, our objective is to propose a hierarchical uncooperative optimal charging scheduling algorithm [8] with the aim of leveling the utilization rate of charging stations and maximizing the profit only for themselves, i.e., charging stations and electric vehicles. Note that the advantage of the hierarchical scheme is the ability to reduce the computational load in solving the charging scheduling problem (instead of relying on only one central controller) and thus increase the possibility of implementing the proposed strategy in real-time. The proposed strategy is summarized here. First, the charging station determines the number of electric vehicles to be scheduled. Second,
the electric vehicle decides which charging stations to charge their battery according to their preferences. Third, the charging station decides the preferences from the electric vehicle by considering its situation. Finally, the energy demand and the charging price are determined once the charging station and the electric vehicles are paired. The contributions of this paper are 1) to express the charging station selection of electric vehicles by using a matching theory and propose a matching algorithm; 2) to propose a charging scheduling algorithm using a repetitive game which regards the charging stations and electric vehicles as non-cooperative players; and 3) to theoretically derive equilibrium points and the conditions to achieve equilibrium in the repetitive game. This is because equilibrium in a repetitive game is generally difficult to analyze theoretically, and many of the existing studies in this area of research just focused on the numerical solution.

The remainder of this paper is structured as follows. In Section 2, we briefly describe the system model, which is the requirement of this research. Then, we discuss the traffic flow model for an electric vehicle to enter and exit a charging station, the queuing model for the electric vehicle to charge its battery, and the state of charge of the electric vehicle. In Section 3, we present our proposed non-cooperative charging scheduling and the state of charge of the electric vehicle. In Section 4, we validate the proposed algorithm by numerical solution. Finally, we provide the conclusion of the paper and future work in Section 5.

2. Problem Formulation

In this section, we briefly discuss the system model, the model of the traffic flow, the queuing model of the electric vehicle at charging station, and the utility functions of the charging station (CS) and electric vehicle (EV). Note that the charging station is regarded as a leader while the electric vehicle is viewed as a follower in this paper.

2.1 System Model

An overview of the system model used in this research is shown in Fig. 1. Each CS belongs to the same business operator and exchanges information with each other. When the CS scheduling interval is assumed to be $T$, each station presents the charging price to each vehicle at the scheduling time $k = nT$ ($n = 1, 2, \ldots$). The exchange of information between EV and CS can be achieved through vehicle-to-infrastructure (V2I) and infrastructure-to-vehicle (I2V) communications. In addition, each EV communicates with other neighboring EVs through vehicle-to-vehicle (V2V) communication to determine their desired charging demand while maintaining a non-cooperative behavior towards other EVs. Note that setting up such a communication network is reasonable considering that most of the CSs are connected to the network (e.g., customers can pay items they bought at the CS by using credit cards) and car navigation systems are installed in EVs.

2.2 Traffic Flow

The first component of the model describes the flow of EVs on the highway as shown in Fig. 2. From the figure, the interchange is represented by node $i = 0, 1, \ldots, N$, where $N$ is the total number of nodes. We consider a chain topology in which each node $i$ is connected only with nodes $i - 1$ and $i + 1$. Thus, edges represent visual links between two successive entrances and exits of the interchange.

Let $\alpha(k)$ be the average of EV flow arriving at node $i$, which is given by

$$\alpha_i(k) = \begin{cases} \gamma_i(k) & \text{if } i = 1, \\ \gamma_i(k) + \gamma_{i-1}(k - d_{i-1,i}) & \text{if } i \neq 1, \end{cases} \quad (1)$$

where $\gamma_i(k)$ is the exogenous flow entering the chain network at node $i$ at time $k$, $\gamma_{i-1}(k)$ is the average EV flow coming from node $i - 1$ to node $i$, $d_{i-1,i}$ is the time required for an EV to traverse the edge from node $i - 1$ to node $i$. The number of EVs departing from node $i$, $\gamma_i(k)$, can be written as follows:

$$\gamma_i(k) = \alpha_i(k) + g_i(k) - f_i(k), \quad (2)$$

where $g_i(k)$ represents the EV flow coming out from charging station $i$, and $f_i(k)$ represents the EV flow entering charging station $i$. In the next subsection, we will explain the relationship between $g_i(k)$, $f_i(k)$, and $x_i(k)$ from Fig. 2.

2.3 Queuing Model

We model the queue at each CS. Let $x_i(k)$ be the number of EVs currently being served or in the queue at CS $i$.

$$x_i(k + 1) = x_i(k) + f_i(k) - g_i(k), \quad (3)$$

where $f_i(k)$ is the number of vehicles flowing into the CS, which is decided by an EV allocation algorithm, and $g_i(k)$ is the number of outflow vehicles. However, even if scheduling is done, the time distribution at which the vehicle arrives cannot be determined precisely. Rather, the distribution of the vehicle arrival follows the Poisson distribution. Also, it is not possible to schedule exactly how many EV stays at CS. Based on these observations, we use the $M/M/c_i$ queuing model. In [3] and [9], the charging station and the parking lot are modeled by queuing. Kendall’s notation, i.e., $M/M/c_i$ is used to define the
where the number of chargers for CS $i$ is $c_i$.  Furthermore, let $\lambda_i$ be the average arrival rate of customers at every sampling time, and $\mu_i$ be the average service rate at every sampling time. Then, the following assumption is introduced.

**Assumption 1.** The customer’s average arrival rate is no larger than the average service rate. Hence, $\lambda_i < c_i \mu_i$.

### 2.3.1 Approximation of $g_i(k)$

In order to obtain $x_i(k)$, it is necessary to calculate $g_i(k)$ first. From the steady solution of $M/M/c_i$,

$$x_i = \frac{p_i^{g_i+1}}{c_i \epsilon_i} \left( 1 - \frac{p_i}{c_i} \right),$$

(4)

$$\varphi_{i,0} = \left[ \sum_{n=0}^{c_i} \frac{\rho_i^n}{n!} \left( 1 - \frac{\rho_i}{c_i} \right) \right]^{-1},$$

(5)

where $\rho_i = \lambda_i / \mu_i$. From (3), when a steady-state is achieved, $g_i = f_i = \lambda_i$. Substituting this solution into (4) and (5) yields

$$x_i = \frac{g_i}{\mu_i - g_i} \implies g_i = \mu_i \frac{x_i}{1 + x_i}.$$  (8)

In this paper, we approximate $g_i$ by considering $M/M/c_i$, as $c_i > M/M/1$:

$$\hat{g}_i(x_i) = c_i \mu_i \frac{x_i}{1 + x_i}.$$  (9)

### 2.4 EV Power Model

In this subsection, we model the power consumption of the EVs, which is given as

$$e_{v,i} = e_{v,i} - d_{i+1} r_v,$$  (10)

$$e_{v,i} = e_{v,i} + \frac{E_{v,i}}{\mu_v}$$  (11)

where $e_{v,i}$ is the state of charge (SOC) when the generic EV $v$ arrives at node $i+1$, and $e_{v,i}$ indicates the SOC when EV $v$ leaves the node $i$. Since the time required to move from node $i$ to node $i+1$ is $d_{i+1}$, $r_v$ is the SOC required for unit time running. Hence, $d_{i+1} r_v$ represents the amount of charge consumed to move from node $i$ to node $i+1$.

Equation (11) shows the SOC of the EV departing from node $i$. $E_{v,i}$ is the amount of energy that EV $v$ charges at node $i$, and $\mu_v$ is the battery capacity. Each EV bids the desired amount of charge to the CS, whereas the CS independently sets prices to maximize their profits.

The charging strategy of each EV satisfies the following two constraints:

$$E_{v,i} \leq E_{v,i} \leq E_{v,i}^\text{max},$$  (12)

$$e_{v,i} + \frac{E_{v,i}}{\mu_v} \leq 1.$$  (13)

Inequalities (12) are upper and lower bounds of the amount of energy that an EV can charge at CS, and (13) describes the fact that the SOC cannot exceed 100%.

### 2.5 Utility Function of Electric Vehicle

The utility function of EV $v$, $U^k_{v,i}$ represents the benefit that the EV obtains through buying energy from a charging station. Mathematically, the function is defined as follows:

$$U^k_{v,i} = \mu_i E_{v,i}(k) - \frac{1}{2} \theta_i(k) E_{v,i}(k)^2$$

$$- p_{v,i}(k) (E_{v,i}(k) - E_i(k)),$$

(14)

where $E_i(k)$ is the average energy demand of all EVs and $p_{v,i}$ is the price of energy at node $i$. In addition, $\theta_i(k)$ is

$$\theta_i(k) = \frac{1}{\Sigma_k \left( \mu_i - e_i(k) \right) \Sigma_k \left( \mu_i - e_i(k) \right)}.$$  (15)

Equation (15) shows a satisfaction parameter indicating the measure of satisfaction of EV $v$ obtained by charging a unit of energy with CS $i$. For example, if EV $v$ has a higher need for energy demand than EV $v + 1$ (e.g., it is going to travel farther, has a larger battery, etc.), then EV $v$ requires more energy than EV $v + 1$. Therefore, the satisfaction parameter becomes $\theta_i(k) \leq \theta_{v+1}(k)$.

### 2.6 Utility Function of Charging Station

The utility function of CS $i$ is given as follows:

$$Q^k_i = \mu_i E_{v,i}(k) - a_1 \left( \sum_k E_{v,i}(k) \right)^2$$

$$- a_2 \left( p_{v,i}(k) - \overline{p} \right)^2,$$

(16)

where $\overline{p}$ is the average market price for charging, and $a_1$ and $a_2$ denote the weighting factors of each term. The first term is the profit of the charging station, the second term is the energy supplied, and the third is the term related to the difference between the general market price $\overline{p}$ of the charging with its set price $p_{v,i}(k)$. Considering the situation where the EV driver selects the charging station after knowing the charging price market, the charging station determines the difference between the market price $\overline{p}$ and set the price $p_{v,i}(k)$ as small as possible. By doing so, the utility is maximized. The optimal price is larger than the market price $\overline{p}$, and if it exceeds the optimal price, the utility decreases because of the difference from $p_{v,i}(k)$ in the third term increases.
2.7 Control Objective
Each of the CSs aims to gain profits by selling energy to the EVs while operating the station efficiently. If many EVs line up at a specific CS, they cause congestion and service degradation, and neighboring stations that are underutilized are wasteful of resources. Therefore, the primary goal is to appropriately distribute EVs by changing the charging price at each CS so that the usage rate of each CS becomes uniform. The second goal is to further maximize profits by selling energy. Meanwhile, the EV aims to charge the desired amount of energy while competing with other EVs.

Optimization of this cyber-physical-human system, which is called a charging scheduling algorithm, is proposed using a Stackelberg game for this system. In the Stackelberg game, EVs and CSs are treated as players, and mutual exchanges are expressed as games. The result of the non-cooperative charging scheduling algorithm using Stackelberg games must satisfy the goal of the CS and the purpose of the EV.

3. Non-Cooperative Charging Scheduling Algorithm
In this paper, we divide the scheduling problem into two categories. The first is to determine which charging station the EV needs to enter, and the second is to determine how much energy demand and price are determined by using the Stackelberg game. The information flow of steps 1 to 4 is provided in Fig. 4 shown later.

1. Based on the number of inflow EVs, each CS determines the number of EVs to be scheduled in order to make the waiting time uniform (uniform utilization rate).

2. The waiting time information is distributed to all EVs.

3. Each EV determines the preference for charging stations in terms of waiting time and its SOC. The SOC information of the EV is sent to the CS. Then, each CS determines the preference for the EV according to its current situation.

4. The information, i.e., the preferences from the CSs and EVs are sent to the business operator (centralized controller). From this information, the business operator can perform matching.

5. Once the CS and EV pairs are decided, energy demand and price are determined by using the Stackelberg game. The information flow of steps 1 to 4 is provided in Fig. 4 shown later.

3.1 EV Allocation Problem
3.1.1 Uniformity of waiting time
At the steady-state condition of $M/M/c_i$, the probability of $n$ customers existing in the system of traffic intensity $\rho_i = \frac{\lambda_i}{\mu_i}$ is

$$P_n = \begin{cases} P_{0n} = \frac{1}{n!} \left(1 + \rho_i\right)_{c_i}, & (0 \leq n \leq c_i), \\ P_{ln} = \frac{1}{c_i!} \left(1 + \frac{\rho_i}{c_i}ight)_{l}, & (n \geq c_i), \end{cases}$$

(17)

$$P_0 = \frac{\sum_{n=0}^{c_i-1} \frac{\rho_i^n}{n!} + \frac{\rho_i}{c_i} \frac{1}{c_i!} \left(1 - \frac{\rho_i}{c_i}\right)}{1 + \rho_i + \frac{\rho_i^2}{2!} + \cdots + \frac{\rho_i^{c_i-1}}{(c_i-1)!} + \frac{\rho_i^c}{c_i! c_i!}}$$

(18)

When a customer arrives at a CS, the probability that the customer will wait into the queue is equal to the probability of all $c_i$ contacts that are occupied, that is, $E_{2,c_i}(\rho_i) = \frac{\sum_{n=c_i}^{\infty} P_n}{\rho_i(c_i - \rho_i)}$.

This equation is called the Erlang C formula. Using the equation, the average waiting time in this system $W_i$ can be written by

$$W_i = E_{2,c_i} \frac{1}{\mu_i(c_i - \rho_i)}.$$  

(19)

In order to equalize the waiting time at each CS, we can control $\lambda_i$ so that (19) is equal at each station. In order to solve for the optimal value of $\lambda_i$, it is obvious that $\sum_k f_i(k) = \sum_k f_i$ when infinite intervals are considered. Using this concept, we consider how many EVs should be charged at a scheduling time in order to equalize the waiting time.

Let the assumed time horizon be $H_p$. At this time, the CS solves the following problem and determines the optimum number of permitted charging $f_i^*(k)$ at the $k$th step:

$$f_i^*(k) = \arg \min_{f_i(k) \in \mathbb{N}} \max_{\mu_i(k) \in \mathbb{N}} W_i,$$  

(20)

s.t. $\lambda_i = \frac{1}{H_p \sum_{k=0}^{H_p} f_i(k - l)},$  

(21)

$$W_i = \frac{\rho_i^c}{\mu_i(c_i - \rho_i)} \left(1 + \rho_i + \frac{\rho_i^2}{2!} + \cdots + \frac{\rho_i^{c_i-1}}{(c_i-1)!} + \frac{\rho_i^c}{c_i! c_i!} \right) \times \frac{1}{\mu_i(c_i - \rho_i)},$$  

(22)

$$\sum_{l} f_i(k) = \sum_{l} \gamma_i(k).$$  

(23)

3.1.2 EV preference
In this paper, we will consider the optimum allocation of EVs to the CS using matching theory. First, in the matching theory, preferences are defined as follows.
Definition 1. On a set $X$, the preference of player $i$, $\preceq_i$ is a binary relation on $X$ that satisfies the following conditions:
1. $x \preceq_i x \forall x \in X$
2. $[x \preceq_i y$ and $y \preceq_i z] \Rightarrow x \preceq_i z \forall x, y, z \in X$
3. $x \preceq_i y$ or $y \preceq_i x \forall x, y \in X$

where $x \prec y$ indicates that player $i$ prefers $y$ more than $x$ under the preference of individual $i$.

Each EV considers choosing a CS under the following factors:
- SOC at the arrival of the CS (lower is preferable, but it is not an option if it becomes 0 or less)
- Estimated waiting time (smaller is preferable)
- Other factors such as personal preference and tolerance level, etc.

Let us set the current SOC of EV $v$ as $E_v^0$. At this time, in determining the preference, the evaluation function of EV $v$ to CS $i$ can be written as follows:

$$J'_i = \begin{cases} 
\omega_1 \tilde{e}^i_{v,j} + \omega_2 W_i + \omega_3 \hat{Y}_i^0, & \text{if } \tilde{e}^i_{v,j} > 0, \\
(\text{otherwise}), & \end{cases} \tag{24}$$

$$\tilde{e}^i_{v,j} = e^0_{i,p} - \sum_{k=0}^{i} d_{j,k+1} r^i_{v,j}. \tag{25}$$

The first term in (24) indicates the estimated SOC when arriving at CS $i$, the second term is related to the waiting time, and the third term is other factors. On the other hand, $\omega_1, \omega_2, \omega_3$ are appropriate weights where $\omega_1 \gg 0$ is a penalty to CS $i$, which cannot be reached without charging. In this paper, by setting $\omega_k$ to be the preference for itself of EV $v$, we set $0 < \omega_k < \omega_d$ to avoid matching with a CS that cannot be reached without charging.

Since EV $v$ prefers CS $i$ in descending order of the evaluation function in (24) and (25), we calculate the evaluation function for each CS $i$ and decide the preference vector $P'_v$ by obtaining the index $i$ in ascending order. Here, we introduce Assumption 2.

Assumption 2. All EVs have initial SOC to reach at least the nearest CSs when entering the highway.

3.1.3 CS preference
Charging stations have no requirement for EVs in particular, but they must permit charging for EVs that cannot reach the next CS without charging. Let the next available CS of CS $j$ be $i'$. Then the evaluation function for EV $v$ can be written with the following equation:

$$J'_i = \begin{cases} 
\chi_1 & \text{if } \tilde{e}^i_{v,i'} < 0, \\
\chi_2 & \text{if } \tilde{e}^i_{v,i'} < 0, \\
\chi_3 & \text{otherwise}, 
\end{cases} \tag{26}$$

where $\chi_1 < 0, \chi_2 \gg 0, \chi_1 < \chi_3 < \chi_2$. The first condition of (26) indicates that the EV cannot reach the next CS without charging and the second condition means that the EV cannot reach CS $i$. For CS $i$, EV $v$ not subject to scheduling means CS $i$ is not matching with that EV. We set the evaluation function value of CS $i$ to itself as $\chi_i$. By setting the condition of $\chi_i$ to $\chi_3 < \chi_1 < \chi_2$, it is possible to exclude matching with the EV that is not subject to scheduling.

Since the smaller evaluation function $J'_i$ is better for EVs, so we calculate the function to obtain the index $i$ and decide the CS preference vector $P'_v$. Here, we introduce the following assumption to discuss the stability of matching.

Assumption 3. At scheduling step $k$, the number of EVs with $\tilde{e}^i_{v,i'} < 0$ in CS $i$ is always equal or smaller than $f'_i(k)$.  

3.1.4 EV allocation algorithm based on matching theory
Consider allocating CSs for each EV using matching theory based on the determined preference. After preferences are decided, there are several algorithms to decide actual matching. In general, matching theory has a problem where the number of iterations increases exponentially as the scale of the problem gets bigger. Even if the number of iterations is small, the stability of matching may not be guaranteed.

Algorithm 1 Charging station allocation algorithm

Require: $f'_i(k), P'_v, P'_i$, and a strongly connected communication topology between EVs and CSs.
1: Initialization: Set initial $X'_i = \{\}$
2: for $i = 1, 2, \ldots, |\gamma(k)|$ do
3: EV $v$ applies for charging to CS $i$ with the most preference from $P'_v$
4: if $|X'_i| < f'_i(k)$ then
5: CS $i$ accepts EV $v$ and forms a temporary pair $X'_i$.
6: Update $X'_i$.
7: end if
8: if $|X'_i| = f'_i(k)$ and $v > X'_i$ then
9: CS $i$ resolves pair with $v'$ such that $v' = X'_i$ and accepts the application of EV $v$.
10: Update $X'_i$.
11: Exclude CS $i$ from $v'$ preference and update $P'_v$.
12: Set $v \rightarrow v'$ and restart from step 3.
13: end if
14: if $|X'_i| = f'_i(k)$ and $v < X'_i$ then
15: Exclude CS $i$ from $v$ preference and update $P'_v$.
16: Restart from step 3.
17: end if
18: end for
19: $X_i = X'_i(k)$

In this paper, the scale of the problem is not so large. Therefore, the Gao-Sharpley algorithm, which is superior in terms of stability, is used even though the number of times of searching is larger than the other algorithms [8],[10]. We denote $X_i$ as the final matching set in step $k$ and $X'_i$ as the set of temporary matching in Algorithm 1.

3.2 Energy Demand and Price Decision Problem
In this section, we consider the problem of determining the energy price and the energy demand by interchanging the allocated CS and EV.

At time $k$, EVs that move toward CS $i$, which has $c_j$ charger, is expressed as $\mathcal{F}(k) := \{1, \ldots, f_i(k)\}$. The EVs can determine the optimal energy demand $E_{v,i}(k)$, and the CS can achieve a reasonable price. The set of energy demand of the target EV is expressed as $E(k) = [E_{1,i}(k), \ldots, E_{N,i}(k)]$.

In this regard, when the EV informs the CS of the energy demand, the CS presents the charging price to the EV to maximize
its benefit. The electric vehicle provides the charging price and again declares the energy demand that will maximize its utility. Therefore, this problem is formulated as a repeated game of one leader and multiple followers, i.e., a non-cooperative game. An illustration is shown in Fig. 4. The leader here simply refers to the player who acts ahead.

3.2.1 Follower action

According to the utility function of the EV (follower) discussed previously, the behavior of EV v that corresponds to the presented price \( p_i(k) \) can be written as follows:

\[
\begin{align*}
\max_{k,v} & \quad U^k_{v,i}(k) \\
\text{s.t.} & \quad E^\min_{v,i} \leq E_{v,i} \leq E^\max_{v,i}.
\end{align*}
\] (27)

To solve this problem, the Lagrangian function for EV v at time \( k \) is defined as follows:

\[
L^k_{v,i} = v_i E_{v,i}(k) - \frac{1}{2} b_i \theta_i(k) (E_{v,i}(k))^2 - b_2 p_i(k) E_{v,i}(k)
\]

\[
- b_1 E_{v,i}(k) \left( \frac{1}{v_i} - \frac{1}{v_f} \right) - \kappa_i(k) (E^\min_{v,i} - E_{v,i}) - \lambda_i(k) (E^\max_{v,i} - E_{v,i}).
\] (29)

where \( \kappa_i(k), \lambda_i(k) \) are non-negative Lagrange multipliers for the constrains (12) and (13), where they are updated by using the gradient method. At the optimum, \( \frac{\partial L}{\partial p_i(k)} = 0 \) is satisfied. Then, we get optimal energy demand as

\[
E^*_v(k) = \frac{1}{b_1 \theta_i} \left[ v_i - b_2 p_i - b_1 \left( \frac{1}{v_f} - \frac{1}{v_i} \right) + (\kappa_i(k) - \lambda_i(k)) \right].
\] (30)

3.2.2 Leader action

From the utility function of the CS i discussed previously, the optimization problem for the CS can be written below:

\[
\max_{p_i(k)} Q^k_i(k).
\] (31)

In order to maximize its utility, the CS presents the optimal price \( p_i^*(k) \) to the EV based on the demand response of \( E^*(v) \) from all EVs, i.e.,

\[
\frac{\partial Q_i}{\partial p_i(k)} = \sum_v E_{v,i}(k) - a_2 (p_i(k) - \bar{p}).
\] (32)

By setting \( \frac{\partial Q_i}{\partial p_i(k)} = 0 \), the optimal energy price is

\[
p_i^*(k) = \frac{\sum_v E_{v,i}(k)}{a_2} + \bar{p}.
\] (33)

Here, we can show the following theorem for a Stackelberg solution of the game dealt in this paper.

**Theorem 1.** In the game, let \( Q^k_{1,N} \) and \( Q^k_{1,S} \) denote the benefits of CS i in a Nash equilibrium and a Stackelberg equilibrium, respectively [11]. Then,

\[
Q^k_{1,N} \leq Q^k_{1,S}.
\] (34)

**Proof.** Let each player’s strategy in a Nash equilibrium be \( \{ p_i^N(k), E^N_i(k) \} \). At this time, we assume

\[
Q^k_{1,N} > Q^k_{1,S}.
\] (35)

First, consider the uniqueness of actions taken by EVs. When the charging station presents prices, each EV chooses a strategy to maximize its utility. At this time, the problem that each EV individually maximizes its utility and the problem combining them are equivalent due to its convexity.

From [12], \( Z \) is the existence solution space and \( F(E) = - (\forall_E u^k_{v,i}(E_i)) \) where each solution of the variational inequalities VI \( (Z, F) \) is a solution of a joint convex generalized Nash equilibrium problem (GNEP). In general, it is known that in the variational inequality problem [13], there exists a potential function in the vector field \( F \), and the solution uniquely exists when the Jacobi matrix is positive definite. By definition [14],

\[
F = \begin{bmatrix}
v_1 - b_1 \theta_i E_{v,1}(k) - b_2 p_i(k) \\
\vdots \\
v_f(\theta_f(k)) - b_1 \theta_f(\theta_f(k)) E_{f,1}(k) - b_2 p_i(k)
\end{bmatrix}.
\] (36)

Then, the Jacobi matrix is

\[
J = \frac{\partial F(E)}{E} = \begin{bmatrix}
b_1 \theta_i & 0 & \cdots & 0 \\
0 & b_2 \theta_i & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & b_f \theta_f
\end{bmatrix}.
\] (37)

Since the Jacobi matrix is a diagonal matrix in the separable vector field, it can be confirmed that the potential function satisfies the symmetry. Due to the positive-definiteness of the Jacobi matrix, \( F \) monotonically increases on the solution set \( Z \). That is, the variational inequality solution uniquely exists. Thus, when a certain price \( p_i(k) \) is given, the optimal reaction strategy of EVs in the game \( R^*(p_i(k)) \) is a single set (singleton).

Since the optimal reaction strategy \( R^2(p_i(k)) \) is a simple set, assuming that each strategy space is \( \Gamma^v \), \( \Gamma^w \), the mapping \( T^v \) is defined as \( \Gamma^v \rightarrow \Gamma^w \). We use this mapping to compute the strategy \( E_i(k) = T^v \) which is represented as \( p_i(k) \in Rv(p,k) \). Furthermore, the Stackelberg cost is

\[
\max_{p_i(k) \in R^v} Q^k_i(p_i(k), T^v p_i(k)) := Q^k_{1,S}.
\] (38)

Each player’s strategy in a Nash equilibrium can also be written
as \( p(k), T^p p(k) \). At this time, the following is obtained from the assumption:

\[
\max_{p(k) \in \Gamma} Q^k(p(k), T^p p(k)) = Q^{k_S} < Q^k(p(k), T^p p(k)).
\] (40)

This is a contradiction, and the theorem was shown because the assumption does not hold.

Before proceeding to the next section, we would like to clarify the meaning of Theorem 1, where it shows that the proposed Stackelberg equilibrium solution of the game is more beneficial than the Nash equilibrium solution.

4. Numerical Simulation

A numerical simulation was performed to confirm the effectiveness of the proposed algorithm. A simulation area was created between Japanese New Tomei Expressway Ohi Matsuda Interchange and Suruga Bay Numazu Interchange, and a simulation map was created as in Fig. 5. The circles indicate interchanges and the squares indicate service areas. The numbers on the arrows between nodes mean traveling time and the numbers near the boxes show the number of charging stations in each service area. The number of vehicles inflow from outside of model is shown in Fig. 6 and the battery capacity of all vehicles and the initial SOC when inflowing are shown in Fig. 7.

The simulation was performed by using parameters in Table 1. From Figs. 8 and 9, it can be confirmed that the inflowing EVs are allocated to each charging station without leakage. Also, as a result of the allocation of EVs, the graphs of Figs. 10 and 11, which indicate the estimated waiting time, are almost in agreement, and therefore, the waiting time equalization problem is correctly explained. From Figs. 12 and 13, it can be confirmed that the energy demand of each EV and the energy price presented by the CS converge repeatedly in the game.

5. Conclusion

In this paper, we proposed a non-cooperative optimal charging scheduling algorithm using matching theory and a Stackelberg game. First, we proposed a matching algorithm by expressing the electric vehicle allocation problem. The objective of the matching algorithm is to equalize the utilization ratio at the charging station and assist the electric vehicle in making the selection of the charging station. Furthermore, we described energy demand and energy price decision problems using the Stackelberg game and provided its solution. It was shown from the theoretical analysis and numerical simulation that the energy price converged to the final equilibrium price. There are two limitations of this work that worth to be noted. First, we assumed that each CS belongs to the same business operator and exchanges information with each other. Second, we also
assumed that each EV communicates with other neighboring EVs through vehicle-to-vehicle (V2V) communication to determine their desired charging demand while maintaining a non-cooperative behavior towards other EVs. If the numbers of CSs and EVs increase, the computational burden also increases exponentially.

Future work includes removing the above limitations and dealing with issues that consider prices as factors of the EV allocation problem. In this case, the Stackelberg game and the matching problem are solved alternately, and it is expected that it is difficult to analyze the convergence. In addition, it is necessary to propose a problem generalizing the Stackelberg game that requires a strong assumption.

References

[1] Ministry of Land, Infrastructure, Transport and Tourism Road Bureau of Japan website: Traffic management, https://www.mlit.go.jp/en/road/index.html, last accessed 20 March 2019.

[2] W. Yuan, J. Huang, and Y.J. Zhang: Competitive charging station pricing for plug-in electric vehicle, *IEEE Transaction on Smart Grid*, Vol. 8, No. 2, pp. 627–639, 2017.

[3] A. Gusrialdi, Z. Qu, and M.A. Simaan: Distributed scheduling and cooperative control for charging of electric vehicles at highway service stations, *IEEE Transaction on Intelligent Transportation Systems*, Vol. 18, No. 10, pp. 2713–2727, 2017.

[4] V.D. Razo and H.A. Jacobsen: Smart charging schedules for highway travel with electric vehicle, *IEEE Transaction on Transportation Electrification*, Vol. 2, No. 2, pp. 160–173, 2016.

[5] D. Ban, G. Michailidis, and M. Devetsikiotis: Demand response control for PHEV charging stations by dynamic price adjustments, *Proc. Innovative Smart Grid Technologies*, pp. 1–8, 2012.

[6] I.S. Bayram, G. Michailidis, and M. Devetsikiotis: Unsplittable load balancing in a network of charging stations under QoS guarantees, *IEEE Transaction on Smart Grid*, Vol. 6, No. 3, pp. 1292–1302, 2015.

[7] J. Tan and L. Wan: Real-time charging navigation of electric vehicle to fast charging stations: A hierarchical game approach, *IEEE Transaction on Smart Grid*, Vol. 8, No. 2, pp. 846–856, 2017.

[8] H. Yang, X. Xie, and A.V. Vasilakos: Noncooperative and cooperative optimization of electric vehicle charging under demand uncertainty: A robust Stackelberg game, *IEEE Transaction on Vehicular Technology*, Vol. 65, No. 3, pp. 1043–1057, 2016.

[9] L.J. Ratliff, C. Dowling, E. Mazumdar, and B. Zhang: To observe or not to observe: queuing game framework for urban parking, *IEEE Conference on Decision and Control*, pp. 5286–5291, 2017.

[10] D. Gale and L.S. Shapley: College admissions and the stability of marriage, *The American Mathematical Monthly*, Vol. 69, No. 1, pp. 9–15 1962.

[11] D. Bauso: *Game Theory with Engineering Applications*, SIAM, 2016.
[12] F. Facchinei and C. Kanzow: Generalized Nash equilibrium problems and Newton methods, *Mathematical Programming*, Vol. 117, No. 1–2, pp. 163–194, 2007.

[13] M.V. Solodov and B.F. Svaiter: A new projection method for variational inequality problems, *SIAM Journal on Control and Optimization*, Vol. 37, No. 3, pp. 765–776, 1999.

[14] D. Ardagna, B. Panicucci, and M. Passacantando: A game theoretic formulation of the service provisioning problem in cloud systems, *Proc. World Wide Web Conference*, pp. 177–186, 2011.

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