Letter

η, η′ mixing angle and η′ gluonium content extraction from the KLOE $R_{\phi}$ measurement

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1 Introduction

The $\eta'$-meson, being a pure $SU(3)$ singlet, has been considered for years the meson within which a gluon condensate contribution can show up. In this paper we try to extract the gluon condensate and the $\eta, \eta'$ mixing angle in the constituent quark model using the approach from [1] and the wave function spatial overlapping parameters introduced by ref. [2]. In particular the same method of ref. [3] will be used but in addition the $\eta' \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$ branching ratios are fitted according to the prescriptions from [4]. This method is chosen because it relates our measurement of $Br(\phi \rightarrow \eta'\gamma)/Br(\phi \rightarrow \eta\gamma)$ [5] to the $\eta'$ gluonium content and the $\eta, \eta'$ mixing angle. The $\eta$ and $\eta'$ mixing angle and the presence of a gluonium component in the $\eta'$-meson have been mostly investigated in the past, but are still without a definitive conclusion [6]. Following the approach from [1,3] the $\eta$ and $\eta'$ wave functions can be decomposed in three terms: the $u, d$ quark wave function $|qq\rangle = \frac{1}{\sqrt{2}}(|uu\rangle + |dd\rangle)$, the strange component $|\bar{s}s\rangle$ and the $|\text{glue}\rangle$. The wave functions are written as follows:

$$|\eta\rangle = \cos(\varphi_G)|q\bar{q}\rangle + \cos(\varphi_F)|\bar{s}s\rangle + \sin(\varphi_G)\langle q\bar{q}|\text{glue}\rangle,$$

$$|\eta'\rangle = \cos(\varphi_F)|q\bar{q}\rangle - \sin(\varphi_F)|\bar{s}s\rangle,$$

where $\varphi_F$ is the $\eta, \eta'$ mixing angle and $Z_G^2 = \sin^2\varphi_G$ the gluon contribution. The ratio of the two branching ratios: $R_{\phi(1020)} = Br(\phi(1020) \rightarrow \eta'\gamma)/Br(\phi(1020) \rightarrow \eta\gamma)$ is related to this decomposition by the formula

$$R_{\phi(1020)} = \cot^2(\varphi_F)\cos^2(\varphi_G) \times \left(1 - \frac{m_s}{m} \frac{Z_{NS} \tan \varphi_V}{Z_S \sin 2 \varphi_F} \right)^2 \left(\frac{p_{\eta'}}{p_{\eta}}\right)^3.$$

In this formula $p_{\eta'}$ and $p_{\eta}$ are the momenta of the $\eta'$ and $\eta$-meson, respectively, $m_s/m = 2m_s/(m_u + m_d)$ is the constituent quark masses ratio, $Z_{NS}$ describes the spatial wave function overlapping between the $q\bar{q}$ component of the $\omega$-meson and $\eta$-meson, and $Z_S$ the one between the $s\bar{s}$ component of the $\eta$ and $\phi(1020)$-meson, $\varphi_V$ is the $\omega,$
\(\phi(1020)\) mixing angle. The parameters \(Z_S, Z_{NS}, \phi_V\) and 
\(m_s/m\) are taken from [7] in which the \(Br(\phi(1020) \to \eta\gamma)\) and 
\(Br(\phi(1020) \to \eta\gamma)\) are fitted together with other \(V \to P\gamma\) 
decays (\(V\) indicates the vector mesons \(\rho, \omega, \phi(1020)\) and 
\(P\) the pseudoscalars \(\pi^0, \eta, \eta'\)).

As in the KLOE [8] paper [5] we fit the ratio \(R_{\phi(1020)}\) from 
the KLOE measurement

\[
R_{\phi(1020)} = \frac{Br(\phi(1020) \to \eta\gamma)}{Br(\phi(1020) \to \eta\gamma)} = 4.77 \pm 0.09_{\text{stat}} \pm 0.19_{\text{syst}} \times 10^{-3}
\]

together with the available data [9] on \(\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma), \Gamma(\eta' \to \rho\gamma)/\Gamma(\omega \to \pi^0\gamma)\) and 
\(\Gamma(\eta' \to \omega\gamma)/\Gamma(\omega \to \pi^0\gamma)\). The dependence of these ratios on 
the mixing angle \(\varphi_P\) and the gluonium content \(\phi_G\) is given by 
the following equations:

\[
\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{1}{9} \left( \frac{m_{\eta'}}{m_{\pi^0}} \right)^3 
\times \left( 5 \cos \varphi_G \sin \varphi_P + \sqrt{2} \frac{f_\rho}{f_\pi} \cos \phi_G \cos \varphi_P \right)^2, \quad (2)
\]

\[
\frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = 3 \frac{Z_{NS}}{\cos(\phi_V)} \left( \frac{m_{\eta'}}{m_{\rho}} - \frac{m_{\eta'}}{m_{\omega}} \right) \left( \frac{m_{\omega}}{m_{\rho}} \right)^2 X_{\eta'}, \quad (3)
\]

\[
\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \pi^0\gamma)} = \frac{1}{3} \left( \frac{m_{\eta'}}{m_{\omega}} - \frac{m_{\eta'}}{m_{\rho}} \right) \left( \frac{m_{\omega}}{m_{\rho}} \right)^3 
\times \left[ Z_{NS} X_{\eta'} + 2 \frac{m_s}{m} Z_S \tan \phi_V Y_{\eta'} \right]^2. \quad (4)
\]

Using the value of \(Z_{NS}\) and \(Z_S\) from [7], we obtain 
\(\varphi_P = (39.7 \pm 0.7)\) and \(\phi_G = (32.1 \pm 0.7)\). Imposing \(\varphi_G = 0\) the \(\chi^2\) probability of the 
fit decreases to 1%. The ratio of the \(\Gamma's\) is obtained using 
the branching fractions of the decay and the total decay 
widths \(\Gamma_{\pi^0}, \Gamma_\omega\) from the PDG 2006 [9]. All the correlations 
amongst the measurements of the several branching ratios 
are taken into account. The correlations are due to the 
choice of normalising all decay widths to the \(\Gamma(\omega \to \pi^0\gamma)\) 
and to the use of a constrained fit technique in the PDG 
2006 in order to obtain more accurate estimates.

The parameter \(m_s/m\) is determined mainly by \(K^{*+} \to K^+\gamma\) 
while \(\phi_V\) is given by the \(V \to \pi^0\gamma\) transitions, 
giving negligible correlations to the \(\varphi_P\) and \(Z_G^2\) parameters.

As in ref. [3] a similar procedure to the one of [7] was 
followed taking into account also the possibility of having 
a gluonium content. They find \(Z_G^2 = 0.04 \pm 0.09\) that 
deviates of 1 s from our result but with a larger error.

In [3] and [10] this difference was attributed to the use 
of overlapping parameters obtained by a fit which assumes 
no gluonium content. In order to check out this possibility, 
we have performed several tests on the fit procedure. We 
first performed a new fit using the overlapping parameter 
\(Z_S\) and \(Z_{NS}\) extracted by the fit in ref. [3], where the 
gluonium content was left free, together with all the other 
parameters: \(Z_{NS} = 0.86 \pm 0.03, Z_S = 0.79 \pm 0.05, \phi_V = 
(3.2 \pm 0.7)^\circ, \frac{m_s}{m} = 1.24 \pm 0.07\). We obtained a result 
in perfect agreement with our previous determination: the 
errors remain unchanged while the central values move to 
\(\varphi_P = 40.1, Z_G^2 = 0.12\). The value of the fit has been also 
repeated for different values of \(Z_{NS}\) and \(Z_S\) in the range 0.5-1.3, 
and the resulting \(Z_G^2\) varied between 0.07 and 0.18, showing 
small sensitivity to the used parameters \(Z_{NS}\) and \(Z_S\) that 
cannot cause the different result obtained by ref. [3].

Excluding the \(P \to \gamma\gamma\) constraint from the fit we obtain 
\(\varphi_P = (40.4 \pm 0.9)^\circ\) and \(Z_G^2 = 0.12 \pm 0.05\), showing 
that this constraint improves the sensitivity for the gluonium 
content. All results are summarized in table 1.

A global fit to all the \(V \to P\gamma\) ratios of the branching 
fractions is in progress. This will allow the overlapping 
parameters to be left free as in the approach of ref. [3], 
that is quite different than ours in both fit procedure and 
input values.

### 2 New value using KLOE \(\Gamma(\omega \to \pi^0\gamma)\) measurement

We have recently published [11] a new preliminary measurement 
of the \(Br(\omega \to \pi^0\gamma) = (8.40 \pm 0.19)\%\). Using this

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**Table 1.** Summary of the results obtained using new values for 
\(Z_{NS}\) and \(Z_S\) from [3].

| Used inputs | \(\varphi_P\) | \(Z_G^2\) |
|-------------|-------------|-------------|
| 1, 2, 3, 4  | (40.1 \(\pm 0.7\)) | 0.12 \(\pm 0.04\) |
| 1, 3, 4     | (40.4 \(\pm 0.9\)) | 0.12 \(\pm 0.05\) |
| 1, 2, 3, 4  | (40.0 \(\pm 0.7\)) | 0.13 \(\pm 0.04\) |
| \(\Gamma(\omega \to \pi^0\gamma)\) from PDG 2006 [9] | \(\Gamma(\omega \to \pi^0\gamma)\) from KLOE [11] |
value we obtain $Z_G^2 = 0.13 \pm 0.04$ and $\varphi_P = (40.0 \pm 0.7)^\circ$.

The allowed regions in the plane $(X_{\eta'} = \cos(\varphi_G)\sin(\varphi_P), Y_{\eta'} = \cos(\varphi_G)\cos(\varphi_P))$, corresponding to the constraints in eqs. (2)–(4) are shown in fig. 1. Theoretical parameters are taken from [7]. All the allowed regions do not overlap on the no-gluonium line $X_{\eta'}^2 + Y_{\eta'}^2 = 1$, suggesting that the no-gluonium picture is wrong.

References

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