Transients and turbulence pockets in thermal convection of paramagnetic fluid subjected to strong magnetic field gradients

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Abstract. We performed combined experimental and numerical studies of the flow and heat transfer of a paramagnetic fluid inside a differentially heated cubical enclosure subjected to various strong non-uniform magnetic field gradients. Two different heating scenarios are considered: unstable (heated from below) and stable (heated from above) initial thermal stratification. In contrast to the previously reported studies in literature, which observed solely laminar flow regimes, we investigated also appearance and sustenance of the periodic- and fully transient-flow motions for the very first time. This was consequence of using significantly stronger magnetic field strength (up to 10 T experimentally, and up to 15 T numerically) than those used in previous studies (up to 5 T). Detailed comparison between experiments and numerical simulations are performed and generally very good agreements were obtained.

1. Introduction

A cubical enclosure filled with a paramagnetic fluid and subjected to differential temperature gradients (heated from top, side or bottom) and exposed to a strong external non-homogeneous magnetic field of different orientations is studied experimentally and numerically. Despite its simple geometry, complex phenomena involving coupled fluid dynamics/electromagnetic interactions make this setup particularly challenging. Starting from a pioneering work of Braithwaite et al. (1991), potentials of a magnetically controlled convection in paramagnetic, or even in ordinary fluids, have been investigated both experimentally and numerically, e.g. Tagawa et al. (2002), Shigemitsu et al. (2003), Bednarz et al. (2005). A major contribution of these studies was in providing the integral heat transfer (Nusselt number) behaviour under strong magnetic fields and in reporting some basic flow visualisations. The goal of our present investigation is to significantly extend the range of working parameters towards transitional flow regimes in contrast to laminar flow regimes considered in previous studies. We will provide detailed insights into fluid flow and heat transfer changes under influence of combined effects of the thermal-buoyancy and magnetisation force over a wide range of working parameters (different values of Prandtl number of working fluids and different values of imposed magnetic field). In order to achieve that, we combine the heat transfer measurements (thermal probes...
inside the enclosure walls and inside the working fluid) with PIV for measuring the instantaneous velocity fields. The experiments in the present paper mainly focus on the integral heat transfer measurements. The experimental cubical enclosure was constructed to provide an easy controllable heating or cooling of the thermally active horizontal walls, which are made from the copper plates. The remaining four side walls are made from a plexiglass. The heating is done by a nichrome wire placed just below the heated plate. The electric voltage and current for nichrome wire were recorded by two multi-meters. The cooling was achieved by running water through a constant temperature bath. The temperatures of the copper plates were measured by the T-type sheathed thermocouples inserted into small holes inside the plate. The integral heat transfer coefficient (Nusselt number) was calculated as a ratio between the net convective heat transfer rate and the net pure conduction contribution, Wrobel et al. (2010).

In parallel to these experimental studies, a high-resolution numerical simulations with a finite volume integrated Biot-Savart’s and Navier-Stokes equations solver are also performed, Kenjereš & Hanjalić (2007), Kenjereš (2008, 2011).

2. Equations and Numerical Method

The system of equations describing the flow and the heat transfer of a paramagnetic fluid subjected to a strong external non-uniform magnetic field consists of the conservation laws of mass, momentum and heat. The extended momentum equation can be written as:

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + f^B_i + f^M_i \tag{1}
\]

where additional forces are the thermal buoyancy \((f^B_i)\) and magnetisation force \((f^M_i)\). By applying Boussinesq approximation, these forces can be expressed as:

\[
f^B_i = -\beta g_i (T - T_0), \quad f^M_i = -\left(1 + \frac{1}{\beta T_0}\right) \frac{\chi_0 \beta (T - T_0)}{2\mu_m} \frac{\partial |b_0|^2}{\partial x_i}. \tag{2}
\]

The magnetisation force was modelled by considering magnetic susceptibility as a function of temperature, Tagawa et al. (2002). The \(T_0 = (T_h + T_c)/2\) is referent temperature, \(\chi_0\) is the magnetic susceptibility at \(T_0\), \(\mu_m\) is the magnetic permeability, \(\nu\) is kinematic viscosity, \(\beta\) the thermal expansion coefficient and \(g_i\) is the gravitational vector. The magnitude of the imposed magnetic field is calculated as \(|b_0| = \sqrt{b_x^2 + b_y^2 + b_z^2}\). The temperature equation can be written as:

\[
\frac{\partial T}{\partial t} + u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\nu}{Pr} \frac{\partial T}{\partial x_j} \right) \tag{3}
\]

where \(Pr = \nu / \alpha\) is Prandtl number. This, together with divergency-free conditions for velocity and magnetic fields

\[
\frac{\partial u_i}{\partial x_i} = 0, \quad \frac{\partial b_i}{\partial x_i} = 0. \tag{4}
\]

makes fully closed system of equations. The magnetic field distribution is calculated from Biot-Savart’s law for a current carrying coil as:

\[
b = \frac{\mu_m I}{4\pi} \int \frac{ds \times r}{r^3}. \tag{5}
\]
where $I$ is the strength of the electric current, $r$ distance from a wire and $ds$ is the length of the differential element of the current carrying wire, Shigemitsu et al. (2003). We consider the one-way coupling between imposed magnetic field and fluid flow, i.e. imposed magnetic field does not change in time. Note that despite constant value of the imposed magnetic field, magnetisation force is not constant since it depends from the local temperature.

The system of Eqs.(1)-(5) is discretized and iteratively solved by a three-dimensional finite-volume based integrated Neviér-Stokes/Maxwell numerical solver for general non-orthogonal geometries, Kenjereš & Hanjalić (2007), Kenjereš (2008, 2011). The convective and diffusive terms of conservative equations are discretized by the second-order central-difference scheme (CDS). The time integration is performed with a fully-implicit second-order three-consecutive time-step scheme. The numerical solver is run in parallel mode using the Message Passing Interface (MPI) domain-decomposition based directives. The simulations are performed on a numerical mesh containing $122^3$ control volumes that are non-uniformly distributed in the proximity of all walls in order to properly capture development of hydrodynamical and thermal boundary layers (10-20 CV’s is located within boundary layers).

3. Results

The strong magnetic field gradients are generated by a state-of-art superconducting helium free magnet, which make it possible to obtain a maximum magnetic field in the magnetic bore up to 10 T. The fluid flow and heat transfer of the paramagnetic working fluid (80% volume aqueous solution of glycerol with 0.8 mol/kg concentration of gadolinium nitrate hexahydrate) under influence of the magnetisation and thermal buoyancy forces have been analysed. The integral heat transfer (Nusselt number) has been measured at the thermally active walls by using the thermocouples. Starting from pure thermal conductivity flow regimes (for heated from above
Figure 2. Nusselt number dependency for different strengths of the imposed magnetic field (|b₀|max) inside the vertical superconducting magnetic coil orientation bore for heated from bottom and heated from top cubical enclosure, Ra=1.5×10⁵, Pr=584. Symbols- experiments, Lines- numerical simulations.

Figure 3. Numerically simulated time evolution of the integral heat transfer coefficient transients at the hot horizontal wall of the cubical enclosure (Ra=1.5×10⁵, Pr=584) in presence of the strong-magnetic field (0≤|b₀|max≤15 T).
Figure 4. Isosurface of temperature (in °C) and streamtraces inside the cubical enclosure heated from bottom and cooled from top, $Ra=1.5 \times 10^5$, $Pr=584$, $\Delta T=50^\circ\text{C}$. (a.) neutral case, $|b_0|_{\text{max}}=0$ T, (b.) intermediate magnetic field $|b_0|_{\text{max}}=5$ T, (c.) strong magnetic field, $|b_0|_{\text{max}}=15$ T.
Figure 5. Superimposed velocity vectors \((u - w)\) and temperature isolines in the central
\((x-z)\) vertical plane at three time instants \((t = 5008, 5010, 5012)\) sec. portraying horizontal meandering and merging of the vertical thermal plumes, \(Ra=1.5\times10^5\), \(Pr=584\), \(\Delta T=5\) K, \(|b_0|_{\text{max}}=15\) T.
Figure 6. Contours of the long-term averaged turbulent kinetic energy ($k = 1/2 (\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$) (a) and of the temperature variance ($\bar{\theta}^2$) (b) in the central (x-z) vertical plane of the cubical enclosure heated from bottom and cooled from top, $Ra=1.5 \times 10^5$, $Pr=584$, strong magnetic field case, $|b_0|_{max}=15$ T. The characteristic thermal buoyancy velocity is defined as $U_0 = \sqrt{\beta g \Delta T H}$.

configuration), interesting flow reorganisation takes place when the magnetisation force starts to be of the competitive magnitude to the thermal buoyancy force. Convective cells similar to the classical Rayleigh-Bénard cells appear despite the initial strong stable stratification.

Integral heat transfer dependency on the imposed magnetic field for two generic situations where the generated magnetisation force supports or suppresses thermal convection, for a heated-from-below situation, is shown in Fig. 2. It is demonstrated that initially fully developed thermal buoyancy convection can be entirely suppressed for relatively small values of the imposed magnetic field ($|b_0|_{max} \leq 2$ T - for the coil top configuration. In contrast to that, heat transfer is enhanced for the coil bottom configuration. It can be seen that a very good agreement between numerical simulations and experiments is obtained for both configurations.

Time evolutions of the integral heat transfer at the bottom horizontal wall, for the configuration where magnetisation forcing enhances convective motion of the fluid, are shown.
Figure 7. Contours of the long-term averaged turbulent stresses (vertical component, $\overline{w^2}$, (a); horizontal component, $\overline{u^2}$, (b)) in the central (x-z) vertical plane of the cubical enclosure heated from bottom and cooled from top (identical working parameters as in previous figure).

in Fig. 3. Four distinct regimes can be distinguished. First, a diffusion dominant regime characterised by a strong decay of the Nusselt number in very initial phase of the onset of heating. This phase last until the time instant when Nusselt number reaches its minimum. By imposing magnetic fields of different strengths, the minimum value is significantly higher and the period of the initial adjusting is significantly shortened compared to its neutral state. After this initial phase of development, convection mechanism starts to be important and Nu reaches its maximum value in relatively short time interval. This interval is then followed by a transitional state and finally, the flow is fully developed. The fully developed regime reaches a steady state for values of $|b_0|_{max} \leq 9$ T. For higher values, i.e. $|b_0|_{max} \geq 10$ T, a well defined oscillatory behaviour of Nu is obtained. This behaviour can be interpreted as a magnetically generated transitional state. Note that a significant heat transfer enhancement is achieved reaching more than 400% of its neutral value for the highest magnetic field applied, i.e. $|b_0|_{max} = 15$ T.

Reorganisation of thermal plumes and of flow structures caused by imposed magnetic fields is
shown in Fig. 4. As previously indicated in Fig. 3, steady situations are shown for natural and intermediate value of the imposed magnetic field, and, the long-term averaged temperature and velocity fields are shown for the strong magnetic field case. The neutral situation is characterised by a centrally cross-diagonally located thermal plume and two dominant co-rotating coherent structures, Fig. 4a. The central thermal plume is more elongated in the vertical direction due to enhanced flow acceleration under influence of the magnetisation force for an intermediate value of the imposed magnetic field. Appearance of additional thermal plumes located half-way between side walls and the centre of the enclosure, Fig. 4b. With further increase of the magnetic field, the secondary thermal plumes are attached to the side walls and secondary corner rolls appear, Fig. 4c.

Transient features of velocity and temperature in the central vertical (x-z) plane, are shown in Fig. 5 (for the highest value of the imposed magnetic field $|b_0|_{max} = 15$ T). The different time instant clearly portray a horizontal meandering and merging of the primary and secondary thermal plumes in the central part of the cavity. This process of the plumes merging is determined by transient vertical ejections from the horizontal boundary layers. Two clearly visible centrally located thermal plumes are present for particular time instant shown in Fig. 5a. The origin of the left de-attached thermal plume is nicely visible - it is characterised by characteristic bump in the horizontal boundary layer (approximately at x=0.01 m location). For next time instant, these two thermal plumes are merged and followed by creation of the smaller thermal plume on the right-side from the primary central plume, Fig. 5b. For next time instant, only centrally located thermal plume is present, Fig. 5c. Note that majority of the velocity vectors are oriented vertically, indicating full three-dimensionality of the phenomena. This dominant up-draft motion in the central vertical plane is compensated with dominant down-draft motion in planes closer to the side walls (not shown here).

The long-term averaged second moments are shown in Figs. 6 and 7. The contours of the turbulent kinetic energy clearly show highly turbulent core - with two distinct peaks in respect to the central vertical line, Fig. 6a. In contrast to that, temperature variance reach its peak in the centre of the horizontal thermal boundary layer, Fig. 6b. By plotting particular components of the turbulent stresses, it can be seen that a strong turbulence anisotropy is present, Fig. 7. While the vertical turbulent fluctuations are located in the enclosure centre, the horizontal components are significant in the centre of the horizontal boundary layers. These are locations where strong horizontal meandering of thermal plumes was taking place (shown in Fig. 5).

4. Conclusions

We studied natural convection of a paramagnetic fluid in a cubical enclosure heated from below and cooled from top subjected to strong magnetic field gradients over a range of working parameters ($0 \leq |b_0|_{max} \leq 15$). Generally, very good qualitative agreement in predicting integral heat transfer are observed between numerical simulations and experiments. Numerical simulations also provided detailed insights into three-dimensional spatial and temporal distributions of temperature and velocity fields. It is demonstrated that strong magnetic field gradients can enhance convective motion and bring initially stable laminar flow regime into its transient mode. Also significant heat-transfer enhancements are obtained for applied magnetic field strengths (up to 400%) compared to its neutral state. The long-term averaged flow and temperature fields statistics revealed pockets of turbulence primarily located in the central part of enclosure (for vertical velocity fluctuations) and at the centre of the horizontal boundary layer (for horizontal velocity fluctuations).

References

Bednarz T., Fornalik E., Tagawa T., Ozoe H. & Szmyd J. S. 2005 Experimental and
numerical analysis of magnetic convection of paramagnetic fluid in a cube heated and cooled from opposing vertical walls. *Int. J. Thermal Sciences* **44**, 933.

Braithwaite D., Beaugnon E. & Tourner R. 1991 Magnetically Controlled Convection in a Paramagnetic Fluid. *Nature* **354**, 134–136.

Kenjereš, S. & Hanjalić, K. 2007 Numerical simulation of a turbulent magnetic dynamo. *Phys. Rev. Lett.* **98**(10), 104501.

Kenjereš, S. 2008 Electromagnetic enhancement of turbulent heat transfer. *Physical Review E* **78**(6), 066309.

Kenjereš, S. 2011 Electromagnetically driven dwarf tornados in turbulent convection. *Physics of Fluids* **23**(1), 015103.

Shigemitsu R., Tagawa T. & Ozoe H. 2003 Numerical computation for natural convection or air in a cubic enclosure under combination of magnetizing and gravitational forces. *Numerical Heat Transfer, Part A* **43**, 449.

Tagawa T., Shigemitsu R. & Ozoe H. 2002 Magnetization force modeled and numerically solved for natural convection of air in a cubic enclosure: effect of the direction of the magnetic field. *Int. J. Heat and Mass Transfer* **45**, 267.

Wrobel W., Fornalik-Wajs E. & Szmyd J. S. 2010 Experimental and numerical analysis of thermo-magnetic convection in a vertical annular enclosure. *Int. J. Heat and Fluid Flow* **31**, 1019.