Radiative seesaw and degenerate neutrinos

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The radiative see-saw mechanism of Witten generates the right-handed neutrino masses in SO(10) with the spinorial 16\textsubscript{H} Higgs field. We study here analytically the 2\textsuperscript{nd} and 3\textsuperscript{rd} generations for the minimal Yukawa structure containing 10\textsubscript{H} and 120\textsubscript{H} Higgs representations. In the approximation of small 2\textsuperscript{nd} generation masses and gauge loop domination we find the following results: (1) $b - \tau$ unification, (2) natural coexistence between large $\theta_t$ and small $\theta_q$, (3) degenerate neutrinos.

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A. Introduction. SO(10) grandunified theory offers a natural and simple arena for the study of fermionic masses and mixings since it relates quark and lepton properties. Through the see-saw mechanism [1] and the resulting large mass for the right-handed neutrinos it explains the smallness of neutrino masses and offers a natural setting for leptogenesis [2]. It is tempting thus to construct the minimal realizations of the theory and see whether or not they can be realistic and predictive. The crucial point here is the minimal Yukawa structure especially if we stick to the renormalizable theory. We know that the minimal Higgs 10\textsubscript{H} does not suffice since the SM doublets in 10\textsubscript{H} are SU(4)\textsubscript{C} singlets and thus would give the same quark and lepton structure; furthermore, with one Yukawa matrix no mixings are allowed.

Two simple extensions are possible: (i) $10\textsubscript{H} + 120\textsubscript{H}$; (ii) $10\textsubscript{H} + 120\textsubscript{H}$.

(i) The first case is appealing since it simultaneously corrects bad mass relations $m_q = m_l$ through a Pati-Salam (2,2,15) field in $120\textsubscript{H}$ and provides a large mass for right-handed neutrinos through (1,3,10) in $120\textsubscript{H}$ [3,4]. It has only $3 + 12 = 15$ real Yukawa couplings and can be considered the minimal supersymmetric GUT [5]. It was studied at length in the recent few years in the supersymmetric version [6], [7], where it appears to be realistic [8]. It gives $\theta_{13} \gtrsim 0.1$ and in the context of type II see-saw, large $\theta_{atm}$ is intimately related to the $b - \tau$ unification [9]. Furthermore, in SO(10), as in any theory with gauged $B - L$, $R$-parity is a gauge symmetry [10] and it can be shown in this case to be an exact symmetry at low energies [11], leading to the stable LSP, a natural candidate for the dark matter.

(ii) This version has only 9 real Yukawa couplings (see below). The charged fermions were studied in [12], but the crucial point is the connection between neutrino and charged fermion masses that we address here. The seesaw mechanism takes the radiative form [13]: right-handed neutrino masses are generated at the two loop level utilizing a 16\textsubscript{H} Higgs with $(16\textsubscript{H}) \approx M_{GUT} \approx 10^{16}$ GeV. This proposal fell from grace due to the advent of low-energy SUSY, which inhibits radiative corrections to the superpotential. It can of course be implemented in ordinary SO(10), but there typically $M_{\nu_R}$ ends up being too small. Schematically,

$$M_{\nu_R} \approx \left(\frac{\alpha}{\pi}\right)^2 Y_{10} M_{GUT}^2 \tilde{m},$$

where $M_R$ is the scale of the breaking of SU(2)\textsubscript{R} symmetry and $\tilde{m}$ the effective susy breaking scale in the visible sector (in ordinary, nonsupersymmetric theories, the formula works with $\tilde{m} = M_{GUT}$). With low-energy supersymmetry, $\tilde{m} \approx 1$ TeV, this obviously fails, while without supersymmetry gauge coupling unification forces $M_R$ to lie much below $M_{GUT}$, which again fails unless some extra fine-tuning is done. On the other hand, as we argued in [14], this works nicely in split susy [15].

This is the scenario we follow here. We focus on the minimal Yukawa structure

$$W_Y = 16^T_F (Y_{10}10\textsubscript{H} + Y_{120}120\textsubscript{H}) 16_F$$

with $Y_{10} = Y_{10}^T$ and $Y_{120} = -Y_{120}^T$. We can diagonalize $Y_{10}$, thus we have 3 real parameters, which together with the 3 complex parameters of $Y_{120}$ add to 9 in total. Still, the full 3 generation case is rather involved and messy, needing numerical studies. In order to get some physical insight and simple analytical results, we focus here on the heaviest two generations. In the limit of small $m_s/m_b$, $m_{\mu}/m_{\tau}$ and $m_{c}/m_{t}$ we find

(a) $b - \tau$ unification of $10\textsubscript{H}$ remains valid;
(b) naturally small quark mixing angle $\theta_{bc}$;
(c) large $\theta_{atm}$ implies degenerate neutrinos.

In other words, up to corrections $m_2/m_3$, $b - \tau$ unification is still a prediction of the theory, in spite of the fact that the $120\textsubscript{H}$ adds a (2,2,15) field. The prediction of degenerate neutrinos is remarkable, since this theory has only a canonical, type I, see-saw and it is often argued that degenerate neutrinos are in contradiction with type I see-saw.

In arriving at the above results two assumptions were made: 1) the renormalizable interactions provide the complete picture, which amounts to neglecting all possible higher-dimensional operators (for an approach with higher dimensional operators see e.g. [16] and references therein); 2) as in the original work of Witten, the radiative seesaw is dominated by the gauge loop effects.
Furthermore, we allow for no singlets or textures (for an opposite approach see e.g. [17] and references therein).

B. Charged fermions and Dirac neutrinos. With this strategy we can write for the fermionic mass matrices

\[ M_D = M_0 + M_2 , \quad M_U = c_0 M_0 + c_2 M_2 , \]
\[ M_E = M_0 + c_3 M_2 , \quad M_{\nu_D} = c_0 M_0 + c_4 M_2 , \]

(3)

where

\[ M_0 = Y_{10} \left( (2, 2, 1)_{10}^d \right) , \]
\[ M_2 = Y_{120} \left( (2, 2, 1)_{120}^d + (2, 2, 15)_{120}^d \right) , \]
\[ c_0 M_0 = Y_{10} \left( (2, 2, 1)_{10}^u \right) , \]
\[ c_2 M_2 = Y_{120} \left( (2, 2, 1)_{120}^u + (2, 2, 15)_{120}^u \right) , \]

(4)

(5)

(6)

(7)

and

\[ c_3 = \frac{(2, 2, 1)_{120}^d - 3(2, 2, 15)_{120}^d}{(2, 2, 1)_{120}^d + (2, 2, 15)_{120}^d} , \]
\[ c_4 = \frac{(2, 2, 1)_{120}^d - 3(2, 2, 15)_{120}^d}{(2, 2, 1)_{120}^d + (2, 2, 15)_{120}^d} . \]

(8)

(9)

We diagonalize \( M_0 \) (\( Y_{10} \)), which preserves the antisymmetry of \( M_2 \), and thus

\[ M_0 = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} , \quad M_2 = \begin{pmatrix} 0 & i\alpha \\ -i\alpha & 0 \end{pmatrix} . \]

(10)

Notice that \( a, b \) can be chosen to be positive real numbers. Similarly, \( c_0 \) can be taken real, whereas the other parameters, \( c_{2,3,4} \) and \( \alpha \) are in general complex numbers. In the limit \( m_s = 0 \) one gets

\[ ab = \alpha^2 , \]

(11)

which shows that \( \alpha \) must be real in this approximation. Similarly, \( m_u = 0 \) \( (m_c = 0) \) implies \( c_3 = \pm 1 \) \( (c_2^2 = c_3^2) \). In other words,

\[ M_D = \begin{pmatrix} a & i\alpha \\ -i\alpha & b \end{pmatrix} , \quad M_U = c_0 \begin{pmatrix} a & \pm i\alpha \\ \mp i\alpha & b \end{pmatrix} . \]

(12)

Obviously \( \theta_U = \pm \theta_D \), while from

\[ M_E = \begin{pmatrix} a & \pm i\alpha \\ \mp i\alpha & b \end{pmatrix} , \quad M_{\nu_D} = \begin{pmatrix} c_0 a & i c_4 \alpha \\ -i c_4 \alpha & c_0 b \end{pmatrix} . \]

(13)

one has \( \theta_E = \pm \theta_D \), whereas \( \theta_{\nu_D} \) is at this point undetermined.

The quark mixing angle \( \theta_q = \theta_U - \theta_D \) can thus take the following values: \( \theta_q = 0 \) or \( \pm 2\theta_E \).

The situation in the leptonic sector is of course more complex, especially since we utilize the radiative seesaw mechanism.

In the approximation of vanishing second generation masses the matrices \( M_D \) and \( M_E \) in (12) and (13) are Hermitian, and thus our first important prediction follows:

\[ m_b = m_c = a + b . \]

(14)

The \( b - \tau \) unification associate with \( 10_H \) Higgs field continues to be valid. For \( c_3 = 1 \) this is expected, since \( \langle (2, 2, 15)_{120}^d \rangle \approx 0 \), so that the SU(4) colour Pati-Salam (quark-lepton) symmetry would not be broken in the down quark and charged lepton sector. Surprisingly \( c_3 = -1 \) also works, in spite of \( \langle (2, 2, 15)_{120}^d \rangle \approx \langle (2, 2, 1)_{120}^d \rangle \).

Before we proceed to discuss this at length, an important question can be posed: do we really need the seesaw mechanism? Could neutrinos be Dirac particles? It is often imagined that in this case the leptonic mixing angles could not be so different from the quark ones. Notice that here this would not be true. First of all, since \( \nu_2 \) and \( \nu_3 \) are not very hierarchical, \( c_4/c_2 \) is not fixed and \( \theta_{\nu_D} \) is arbitrary. If they were very hierarchical, then clearly \( \theta_{\nu_D} = \pm \theta_E \) as in the quark case. Still there is a completely consistent solution \( \theta_1 = 2\theta_E \), which could obviously be any number and even very large. After all, \( \theta_E \) between 20 and 25 degrees is definitely a natural value, or at least as natural as any other.

What goes wrong of course are the values of neutrino masses; they are of the same order of magnitude of the charged fermions. This is why the seesaw mechanism is a must in a well defined, predictive, SO(10) theory. On the other hand the seesaw mechanism without grandunification to set the scale of the righthanded neutrino masses is of little use in the quantitative determination of light neutrino masses.

C. Not to forget the seesaw. Witten’s two loop diagram [13] is proportional to the Yukawa \( Y_{10} \), i.e.

\[ M_{\nu_R} \propto M_0 . \]

(15)

In turn, the light neutrino mass matrix \( M_N \) takes the form

\[ M_N = M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} \propto c_0^2 M_0 - c_4^2 M_2 M_0^{-1} M_2 . \]

(16)

After some elementary algebra

\[ M_N \propto (c_0^2 - c_4^2) M_0 . \]

(17)

Since we are working in the basis of \( M_0 \) being diagonal, the first immediate consequence is that \( \theta_N = 0 \), and thus the weak current lepton mixing angle \( \theta_l = \theta_{\text{atm}} = \theta_E \). From \( \theta_{\text{atm}} \approx \pi/4 \), only the solution \( \theta_{bc} = \theta_q = 0 \) found before is the physical one (the other one \( \theta_{bc} \approx \pi/2 \) does not work). This cancellation between \( \theta_U \) and \( \theta_D \) is similar in spirit to [18]. In other words, the small quark and the large leptonic mixing angle can coexist naturally as the result of the spontaneous symmetry breaking of the quark-lepton symmetry valid at \( M_{\text{GUT}} \).

Even more interesting is the fact that eq. (17) gives degenerate neutrinos. Namely, \( \theta_E \approx \pi/4 \) implies necessarily \( a \approx b \) in view of (11), or in other words, \( M_0 \propto I \).

One can ask the question as why we chose the type I see-saw as the only source of neutrino masses. As is well known, in SO(10) one generically obtains also a type II
see-saw generated through the vacuum expectation value of the SU(2)$_L$ triplet [19,3,20]. In this case the triplet is the effective operator made of the product of two doublets in $16_H$. However it will be suppressed by the same two loop effect that enhances the type I through the suppression of the right-handed neutrino masses. In short, the ratio of type II over type I contributions is suppressed by $(\alpha/\pi)^4$, which implies that type II can be safely neglected.

D. Discussion and outlook. There are a number of important issues that ought to be addressed in order to have a full realistic theory of three generations, analogous to what has been achieved in the $10 + \overline{126}$ Yukawa case. We go through some central ones.

(i) What about a radiative $\nu_R$ mass in the presence of a $120_H$ field? It can affect the simple, predictive result of Witten only if $120_H$ mixes through a $45_H$, which happens through a superpotential coupling

$$\lambda 10_H 45_H 120_H.$$ 

If $\lambda \sim g$, the gauge coupling, there will be new diagrams exchanging both $10_H$ and $120_H$ which would spoil the prediction $\theta_N \sim 0$. Of course, a large $\theta_{\text{atm}}$ solution remains possible; if anything, there would be more freedom and more chance that the full 3 generation case works out.

A small $\lambda$ is rather appealing and can be made natural by imposing a discrete symmetry

$$(16, \overline{16}) \rightarrow i(16, \overline{16}), \quad (10, 120, 45) \rightarrow -(10, 120, 45)$$

outside of SO(10). This would ensure the stability of small $\lambda$ even in a strongly broken (split) supersymmetry or in a non-susy theory.

Furthermore, small $\lambda$ leads to the formation of domain walls generated by a nonvanishing $(45_H) \simeq M_{\text{GUT}}$. The subsequent evolution (disappearance) of these unstable domain walls has a remarkable consequence: they sweep away the magnetic monopoles [21]. This provides the simplest and most elegant field theoretic explanation of the small monopole density of the universe (for a cosmological explanation see e.g. [22]), a solution which needs no new fields or interactions and works independently of when inflation took place or how big the reheating temperature was. It is especially appealing in view of the fact that the other simple solution [23], based on the non-restoration of symmetries [24] or the $U(1)_{\text{em}}$ breaking [25] at high $T$ faces trouble in gauge theories [26], especially the supersymmetric ones [27].

(ii) Which theory can give gauge coupling and $b-\tau$ unification consistent with symmetry breaking and phenomenology? The simplest solution, split supersymmetry with superheavy sfermions, works nicely for neutrino masses and gauge coupling unification, and $b-\tau$ unification favours small $\tan \beta$ [28].

In strongly split supersymmetry there is an issue of stable gluinos though [29]. Notice however that for $c_0^2 \simeq c_1^2$ neutrino masses get further suppressed, beyond the generic see-saw. This means that sfermions may lie below the unification scale, i.e. $\tilde{m} \ll M_{\text{GUT}}$, which somewhat alleviates the above problem.

Another possibility is ordinary nonsupersymmetric SO(10), but again some extra fine-tuning would be mandatory. It has to be noticed however, that in order to continue having only two Yukawa matrices, a Peccei-Quinn-type symmetry must be imposed in this case.

(iii) What about the impact of running from $M_{\text{GUT}}$ down to $M_Z$? In this analytic approach we only wanted to get a qualitative insight into the pattern of neutrino masses and mixings. One should definitely run, though, when a 3 generation numerical study is built up around our solution.

A serious challenge appears to be the prediction of all three neutrinos being degenerate. The full three generations numerical study can invalidate this, since eq. (15) does not follow automatically even with the assumption of gauge loop domination. A way out in such a case could be allowing for larger $\lambda$.

(iv) Is it really necessary to neglect the higher dimensional operators? After all, the second generation masses remain small, compatible with zero, even in this case. The only problem is represented by a possible direct contribution to the right-handed neutrino mass squared of the order $M_{\text{GUT}}^2/M_{\text{Planck}}$. This is of the same order of magnitude as the radiatively generated term. However, there is no need for it to be proportional to $Y_{10}$. It is for this reason that we need to assume that at least this operator is suppressed.

In summary, a radiative see-saw mechanism offers an alternative simple SO(10) theory if one sticks to the minimal Yukawa structure with $10_H$ and $120_H$.

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