A version of the connection representation of Regge action

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Abstract

We define for any 4-tetrahedron (4-simplex) the simplest finite closed piecewise flat manifold consisting of this 4-tetrahedron and of one other 4-tetrahedron identical up to reflection to the present one (call it a bisimplex built on the given 4-simplex, or a two-sided 4-simplex). We consider an arbitrary piecewise flat manifold. The gravity action for it can be expressed in terms of the sum of the actions for the bisimplices built on the 4-simplices constituting this manifold. We use a representation of each bisimplex action in terms of rotation matrices (connections) and area tensors. This gives some representation of any piecewise flat gravity action in terms of connections. The action is a sum of terms each depending on the connection variables referring to a single 4-tetrahedron. Application of this representation to the path integral formalism is considered. Integrations over connections in the path integral reduce to independent integrations over finite sets of connections on separate 4-simplices. One of the consequences is exponential suppression of the result at large areas or lengths (compared to the Plank scale). This is important for the consistency of the simplicial description of spacetime.

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1. Introduction

Recently, a modification of the genuine Regge calculus (RC) [1] has often been considered, where the same edge can have different lengths depending on the 4-simplex where it is defined, e.g. the so-called area RC [2, 3]. If we invoke a description of the RC in terms of tetrad and connection [4], it is natural to study an analogous modification in the connection sector. Can connection on the 3-simplex also depend on the 4-simplex containing it? If variables are divided into independent sets referred to as separate 4-simplices, the theory is simpler.
idea is to apply connection representation separately to the (properly specified) contributions to the action of the different 4-tetrahedra.

An idea of how to specify the contribution from separate 4-simplices admitting the connection representation can be illustrated by the two-dimensional case. Consider a 2-simplex (triangle) and closed (strongly curved) surface consisting of both sides of this triangle viewed as different 2-simplices, call it a (two-dimensional) bisimplex. Evidently, the angles of the triangle can be expressed in terms of the defect angles of the bisimplex. Analogously, hyperdihedral angles of any 4-tetrahedron can be expressed in terms of the defect angles of the bisimplex consisting of only two 4-tetrahedra with mutually identified vertices. These can be viewed as different sides of the same 4-tetrahedron. The Regge action of arbitrary simplicial spacetime is a combination of actions of the bisimplices constructed by this method from separate 4-simplices. The action of the bisimplex can be written in the connection representation using some connection orthogonal rotational matrices as independent variables. This representation is considered in section 2. The representation of the full gravity action is then obtained in section 3 as a linear combination of the representations of the actions for bisimplices built on separate 4-simplices.

The connection variables are generally arbitrary orthogonal matrices, but in the equations of motion the curvature matrices formed of these should have a physical sense of rotations in the local frame of the 4-simplex around its two-dimensional faces (triangles) by hyperdihedral angles of the 4-simplex at these triangles (more accurately, by $2\pi - 2 \times \text{(hyperdihedral angle)}$). There is no need in the explicit presence of the above bisimplices in the general simplicial complex. It is sufficient that the Regge action for the general simplicial spacetime consists of terms which can be interpreted as actions for bisimplices built on separate 4-simplices. Therefore, all these terms and thus this Regge action are represented using new variables having a possible geometrical sense of rotations by hyperdihedral angles. The sense of this representation is that in the equations of motion for the rotational variables it is just the Regge action in terms of the purely edge lengths.

The use of the representation studied is shown in the path integral formalism in section 4. Integrations over connections reduce to independent integrations over finite sets of connections on separate 4-simplices and are analyzable. One of the consequences is exponential suppression of the path integral at large areas or lengths. This means suppression of the physical amplitudes with large areas/lengths. This is important for the consistency of the simplicial minisuperspace description of spacetime which should be close to the continuum one at large scales.

The connection representation includes ‘arcsin’ functions which serve to express the hyperdihedral angles which normally are not small. Therefore, it is necessary to define proper branches of the ‘arcsin’ functions. This implies division of the whole region of variation of the edge lengths into certain sectors. Section 5 considers a typical example of the simplicial structure: the region of variation of the edge lengths which contains zero curvature physical configuration as a particular case. The analytical form of the representation studied is specified and expressed in terms of a combination of the principal value of ‘arcsin’ functions together with terms which are constants different in different sectors. These terms are important for possible integration over edge lengths in the path integral and do not influence the integrations over connections of section 4.

2. Bisimplex

Given a 4-simplex $\sigma^4$, consider the above-mentioned bisimplex, the simplest simplicial complex $\Delta \gamma$ built of only two 4-simplices, the vertices of which are mutually identified,
\( \sigma^4 \) and identical to it, up to reflection w.r.t. any 3-face, 4-simplex \( \sigma^4 \). Let us write the Regge action for the bisimplex constructed of the given 4-simplex \( \sigma^4 \):

\[
S_\Sigma(\sigma^4) = \sum_{\sigma^7 \subset \sigma^4} (2\pi - 2\alpha_{\sigma^7}^{\sigma^8}) A_{\sigma^7}^{\sigma^8}.
\]

(1)

Here, \( \alpha_{\sigma^7}^{\sigma^8} \) is the hyperdihedral angle of the 4-simplex \( \sigma^4 \) at the 2-face \( \sigma^2 \), and \( A_{\sigma^7}^{\sigma^8} \) is the area (either real or imaginary) of the triangle \( \sigma^2 \) in the 4-simplex \( \sigma^4 \).

Invoking the notion of discrete tetrad and connection first considered in [4], we have suggested in [5] the representation of the minisuperspace Regge action in terms of area tensors and finite rotation \( SU(2) \times SU(2) \) (more accurately, modulo the element \((-1, -1)\)) or \( SO(3) \times SO(3) \) (more accurately, plus the same times the element \(-1\) of \( SO(4) \)). Then the \( SO(3,1) \) element is represented by two mutually complex conjugated elements of \( SU(2) \) or \( SO(3) \) with complex parameters (that is, of \( SU(2, C) \) or \( SO(3, C) \)). For definiteness, one might imply the following notations and sign conventions concerning splitting the tensors into (anti-)self-dual parts in the Minkowsky spacetime. Suppose there is an \( SO(3,1) \) matrix:

\[
\Omega = \exp(\psi^k E^a_k + \psi^k L^a_k).
\]

Its generator is expanded over the set of independent generators:

\[
E_{kab} = -\epsilon_{kab}, \quad L_{kab} = \delta_{kab} g_{0b} - \delta_{kab} g_{0b} \quad (g_{ab} = \text{diag}(-1,1,1), \epsilon_{123} = +1).
\]

We denote

\[
(\Sigma^{ab})^- = \frac{1}{2} \epsilon^{cd}_{ab} \Sigma^{cd} = \frac{i}{2} \Sigma^{ab} (\epsilon^{123} = +1), \quad \Sigma^{a b k}_{k a} \Sigma^{c i b}_{i c} = -\delta_{ab} \epsilon_{i k} + \epsilon_{i k} \Sigma^{a c}_{a c},
\]

then

\[
\Omega = \Omega^\dagger \Omega, \quad \Omega^\dagger = \exp\left(\frac{\psi^k \pm i \psi^k}{2} \Sigma^{a b}_{k k}\right) = \cos \frac{\phi^a}{2} + \Sigma^{a b} n \sin \frac{\phi^b}{2}.
\]

Here \( \phi = \sqrt{\phi^2} \), \( \phi^a = \phi^{\pm a} \). For a triangle spanned by the two 4-vectors \( l_1^2 \), \( l_2^2 \), we can define the bivector \( v^{ab} = \frac{1}{2} \epsilon^{ab} l_1^2 l_2^2 \) or, as a shorthand, \([l_1, l_2]^{ab}\). This variable splits additively:

\[
v^{ab} = v^{ab} - v^{ab}, \quad v^{ab} = \left\{ v^{ab} \pm \frac{1}{4} \epsilon^{ab} v^{cd} \right\}.
\]

In particular,

\[
2 v^a \circ v^a = v^a \pm i v^a \ast v^a.
\]

Here \( A \circ B = \frac{1}{2} A_{ab} B^{ab}, A \ast B = \frac{1}{2} \epsilon_{abcd} A^{ab} B^{cd} \) for the two matrices \( A, B \). The \( \dagger \)-parts map into three-dimensional vectors \( v^a \):

\[
\dagger v^{ab} = \frac{1}{2} v^{ab} \pm \frac{1}{4} \epsilon^{ab} v^{cd}, \quad 2 v^k = -\epsilon_{k m} v^{m l} \pm i (v_{k 0} - v_{0 k}).
\]

(9)

For a bivector \( 2 v^a \pm i v^a \times l_2 - l_1 l_0^2 + l_2 l_0^2 \). Additional overall \( i \) compared to the usual definition of the area vector (real for the space-like area) is here due to the fact that \( v^{ab} \) is the dual area tensor. Besides that

\[
\dagger v^a = 2 v^a \circ \dagger v^a.
\]

(10)
representation for $\Omega(6)$ means the $SO(3)$ rotation in the space of $\Omega$:

$$\Omega^{ab} = \Omega^{a}_{n=a} \Omega^{b}_{n=b} + (\delta^{ab} - \Omega^{a}_{n=a}) \cos \phi + \epsilon^{abc} \Omega_{c} \sin \phi. \quad (11)$$

The considered $SU(2) \times SU(2)$ representation in terms of (anti-)self-dual parts of finite rotation matrices can be written for the considered manifold as

$$\hat{\rho}^{SU(2)}(\sigma^4) = \sum_{\sigma^{+} \in \mathbb{Z}^{+}} \sqrt{\hat{\rho}^{2}_{\sigma^{+}} \pi} \arcsin \left( \frac{\hat{\rho}^{\sigma^{+}} \circ R^{\sigma^{+}}(\Omega)}{\sqrt{\hat{\rho}^{2}_{\sigma^{+}} \pi}} \right) \quad (12)$$

where $\hat{\rho}^{\sigma^{+}}$ are area vectors (9) of the triangle $\sigma^{+}$; in the Minkowsky case $\Omega_{\sigma^{+}}$ is the rotation $SO(3,1)$ matrix on the tetrahedron $\sigma^{+}$ which we call simply connection, and $R_{\sigma^{+}}$ is the curvature matrix on the triangle $\sigma^{+}$ (holonomy of $\Omega$’s). The vector/tensor indices of $\rho$, $R$ might refer to the frame of any one of the two 4-simplices $\sigma^{+}, \sigma^{-}$; for definitness we choose the original $\sigma^{+}$. Further, there are five connection matrices $\Omega_{\sigma^{+}}$, in our particular simplicial complex and ten curvature matrices $R_{\sigma^{+}}$, each $R$ being the product of certain two matrices $\Omega_{\pm}$. The ‘a’rsine’ means the proper solution for the inverse function of ‘sin’ while ‘arc’sin means the principal value whose real part at real argument lies in the region $[-\pi/2, +\pi/2]$. The superscript $SU(2)$ means that $\hat{\rho}$ can be viewed as a $2 \times 2$ antisymmetric matrix, and $\hat{\rho}$ as an $SU(2)$ matrix (to be precise, $SU(2,C)$ in the Minkowsky case). This is a fundamental $SU(2)$ representation. Also adjoin, $SO(3) \times SO(3)$ representation is of interest for us:

$$\hat{\rho}^{SO(3)}(\sigma^4) = \sum_{\sigma^{+} \in \mathbb{Z}^{+}} \frac{1}{\sqrt{2}} \sqrt{\hat{\rho}^{2}_{\sigma^{+}} \pi} \arcsin \left( \frac{\hat{\rho}^{\sigma^{+}} \circ R^{\sigma^{+}}(\Omega)}{\sqrt{\hat{\rho}^{2}_{\sigma^{+}} \pi}} \right) \quad (13)$$

Here $\hat{\rho}^{\sigma^{+}}$ is the $SO(3)$ matrix (to be precise, $SO(3,C)$ in the Minkowsky case); for a 3-vector $v$ and a $3 \times 3$ matrix $R$ we have denoted $v \cdot R \equiv 1/2 v^a R^{bc} \epsilon_{abc}$.

The sense of the considered representations is that upon excluding rotation matrices by classical equations of motion (that is, on-shell) these result in the same (half of) Regge action. Taking into account that in the Minkowsky case $S = (\sigma)^{+}$ we can write out the most general combination of $S$, $\sigma^{+}$ which (i) reduces to Regge action on-shell and (ii) is real, as $S = C^{+}S + C^{-}S$ where $C^{+} = 2$, that is $C = 1 + i \cdot (\text{real parameter})$. At the same time, in the continuum theory the Holst action which generalizes the Cartan–Weyl form of the Einstein action [6, 7] is easily seen to have the form $(1 + i/\gamma)S_{\text{cont}} + (1 - i/\gamma)^{-1}S_{\text{cont}}$ where $S_{\text{cont}}$ are (anti-)self-dual parts of the Cartan–Weyl continuum action; $\gamma$ is known as the Barbero–Immirzi parameter [8, 9]. Therefore, we can write $C = 1 + i/\gamma$ where the discrete analog of $\gamma$ is denoted by the same letter. Thus,

$$S = \left( 1 + \frac{i}{\gamma} \right)^{+} S + \left( 1 - \frac{i}{\gamma} \right)^{-} S. \quad (14)$$

In the considered bisimplex case $\hat{\rho}^{\sigma}$ implies $\hat{\rho}^{SU(2)}(\sigma^4)$ or $\hat{\rho}^{SO(3)}(\sigma^4)$.

Rewrite (12), (13) in more specialized notations. Denote the vertices of the $\sigma^4$ by $i = 1, 2, 3, 4$. Denote a simplex by enumerating its vertices in round brackets. Then $\sigma^{4} = (01234), \sigma^{-} = (0'1234)$ with the common 3-face $(1234)$ and vertices 0, 0’ identified. Let all the connections $\Omega$ on the 3-faces act from $(01234)$ to $(0'1234)$, that is, if a 2-face tensor $v$ is defined in $(01234)$, then $\Omega v \Omega^T$ is defined in $(0'1234)$. Denote a 3-face in the same way as the opposite vertex, $\Omega_{0} \equiv \Omega_{0'}$ where $\sigma^{3} = (01234)\backslash\{i\}$. Here $\{\ldots\}$ denotes (sub)set of the vertices 0, 1, 2, 3, 4. Denote a 2-face in the same way as the opposite edge, $v_{(ik)} \equiv v_{\sigma^{+}}, R_{(ik)} \equiv R_{\sigma^{+}}$ where $\sigma^{2} = (01234)\backslash\{ik\}$. It is convenient to define
the variables $v, R$ on the ordered pairs of vertices $ik$, $v_{ik} = - v_{ki}$, $R_{ik} = R_{ki}^T$. Then $v_{(ik)}$ is one of the two values, $v_{ik}$ or $v_{ki}$, $R_{(ik)}$ is $R_{ik}$ or $R_{ki}$. Then,

$$R_{ik} = \Omega_i^4 \Omega_k.$$

Evidently,

$$R_{ik} R_{kl} = R_{il}$$

(15)

As independent curvature matrices we can choose $R_\alpha \equiv R_0 \alpha (\alpha, \beta, \gamma, \ldots = 1, 2, 3, 4)$, that is, the curvature on the 2-faces of the tetrahedron (1 2 3 4). With the shorthand $v_\alpha \equiv v_0 \alpha$

the actions (12), (13) read

$$S_{SU(2)}(\sigma^4) = \frac{4}{2} \sum_{a=1}^{4} \sqrt{2} v_{a}^2 \arcsin \sqrt{2} v_{a} \circ \frac{1}{2} R_a + \sum_{a<\beta} v_{a} \circ v_{\alpha} \circ \frac{1}{2} R_{\alpha} \circ \frac{1}{2} R_{\beta} \sqrt{2} v_{\alpha} \circ \frac{1}{2} R_{\alpha} \circ \frac{1}{2} R_{\beta},$$

(17)

$$S_{SO(3)}(\sigma^4) = \frac{4}{2} \sum_{a=1}^{4} \sqrt{2} v_{a}^2 \arcsin \frac{1}{2} v_{a} \circ \frac{1}{2} R_a + \sum_{a<\beta} v_{a} \circ v_{\alpha} \circ \frac{1}{2} R_{\alpha} \circ \frac{1}{2} R_{\beta} \sqrt{2} v_{\alpha} \circ \frac{1}{2} R_{\alpha} \circ \frac{1}{2} R_{\beta}.$$

(18)

The matrix $R_\alpha$ solves the equations of motion for connections if it is the rotation by an angle $(2\pi - 2\sigma_{\alpha} \gamma a)$ around a 2-face $(\sigma^2 \sigma^4 \sigma^1 \sigma^3 \sigma^0)$ where $\sigma_{\alpha} \gamma a$ is the hyperdihedral angle of the considered 4-simplex at this 2-face. Apart from the four $R_\alpha$s, one of the matrices $\Omega_i$ say $\Omega_0$, can be taken as a fifth, purely gauge connection variable absorbing the rotations of the local frame.

One non-standard feature of the considered representations is their nonperturbative nature. Matrices $\Omega, R$ cannot be considered as all of these are close to unity because the bisimplex is a strongly curved manifold.

3. Representation of arbitrary Regge action from representation of bisimplex action

It is important that according to formula (1) we also have the connection representation for the following combination:

$$\sum_{\sigma^2 \subset \sigma^4} \alpha_{\sigma^2 \sigma^4} A_{\sigma^2 \sigma^4},$$

(19)

for the given 4-simplex $\sigma^4$. On the other hand, the same combinations appear in the Regge action for any collection of the 4-simplices,

$$S = \sum_{\sigma^2} \left( 2\pi - \sum_{\sigma^2 \subset \sigma^4} \alpha_{\sigma^2 \sigma^4} \right) A_{\sigma^2} = 2\pi \sum_{\sigma^2} A_{\sigma^2} - \sum_{\sigma^4} \sum_{\sigma^2 \subset \sigma^4} \alpha_{\sigma^2 \sigma^4} A_{\sigma^2 \sigma^4} = \sum_{\sigma^4} \left[ \frac{1}{2} S_{S}(\sigma^4) + \sum_{\sigma^2 \subset \sigma^4} \left( \frac{2\pi}{N_{\sigma^2}} - \pi \right) A_{\sigma^2 \sigma^4} \right],$$

(20)

where $N_{\sigma^2}$ is the number of 4-simplices meeting at $\sigma^2$. Here $S_{S}(\sigma^4)$ can be substituted by expression (14) where $\pm S_{SU(2)}^{SO(3)}(\sigma^4)$ or $\pm S_{SO(3)}^{SO(3)}(\sigma^4)$ stand for $\pm S$. This gives some representations.
for the Regge action for an arbitrary simplicial complex. Usual Regge calculus implies independence of $A_{\sigma_2^{\gamma}\sigma_4^{\delta}}$ on $\sigma_4^{\delta}$: $A_{\sigma_2^{\gamma}\sigma_4^{\delta}} \equiv A_{\sigma_2^{\gamma}}$. The unusual feature of the considered representations is their nonperturbative nature, as above for the bisimplex alone. Even in the continuum limit or near the flat background matrices $\Omega$, $R$ involved cannot be treated as close to unity.

The physical meaning of a matrix $R$ used in the considered representation is rotation by an angle $(2\pi - 2\alpha_{\sigma_2^{\gamma}\sigma_4^{\delta}})$ around a certain 2-face $\sigma_2$ where $\alpha_{\sigma_2^{\gamma}\sigma_4^{\delta}}$ is the hyperdihedral angle of the 4-simplex $\sigma_4^{\delta}$ on this face. There is no need in the explicit presence of the bisimplices in the general simplicial complex. Sufficient is that when excluding rotations via the equations of motion from (20) we get exactly Regge action for the general simplicial spacetime.

4. Application to path integral formulation

Let us write out a discretized functional integral

$$\int \exp(iS) Dq$$

$q$ are field variables (some factors of the type of Jacobians could also be present). Suppose we are interested in the result of integration over connections as a function of area tensors. Of course, different (components of) area tensors are not independent, but nothing prevents us from studying analytical properties in the extended region of varying these area tensors as if these were independent variables. This result splits into separate factors corresponding to integration over connection matrices $\Omega_{\sigma_2^{\gamma}\sigma_4^{\delta}}$ in the separate 4-simplices $\sigma_4^{\delta}$,

$$\int \exp(iS) \prod_{\sigma_2^{\gamma}\subseteq\sigma_4^{\delta}} D\Omega_{\sigma_2^{\gamma}\sigma_4^{\delta}} = \exp \left[ \sum_{\sigma_2^{\gamma} \subseteq \sigma_4^{\delta}} \left( \frac{2\pi}{N_{\sigma_2^{\gamma}}} - \pi \right) A_{\sigma_2^{\gamma}\sigma_4^{\delta}} \right] \prod_{\sigma_2^{\gamma} \subseteq \sigma_4^{\delta}} \int \exp(iS_\Delta (\sigma_4^{\delta})/2) \prod_{\sigma_3^{\epsilon} \subseteq \sigma_4^{\delta}} D\Omega_{\sigma_3^{\epsilon}\sigma_4^{\delta}}. \quad (21)$$

Each such factor is a certain function of area tensors in the given 4-simplex proportional to (in the case of $SO(3) \times SO(3)$ representation)

$$\mathcal{N}(\{v_a\}, \{v_{\alpha\beta}\}) = \int \exp \left\{ \frac{1}{4} \left( 1 + \frac{i}{\gamma} \right) \left[ \sum_{a=1}^{4} \sqrt{v_a^2} \arcsin \frac{v_a^* R_a}{v_a^2} + \sum_{\alpha < \beta} \sqrt{v_{\alpha\beta}^2} \arcsin \frac{v_{\alpha\beta}^* (R_{\alpha\beta}^T + R_{\beta\alpha})}{v_{\alpha\beta}^2} \right] + \text{complex conjugate} \right\} \prod_{a=1}^{4} DR_a, \quad (22)$$

after reducing ‘arcsin’s to the principal values ‘arcsin’s and, probably, redefining $v_{\alpha\beta} \rightarrow -v_{\alpha\beta}$ as considered in section 5. We have taken into account that

$$\prod_{\sigma_2^{\gamma} \subseteq \sigma_4^{\delta}} D\Omega_{\sigma_2^{\gamma}\sigma_4^{\delta}} = \prod_{i=0}^{4} D\Omega_i = \prod_{a=1}^{4} D\Omega_a \prod_{a=1}^{4} D\Omega_a \quad (23)$$

and have been divided by the volume of gauge group by omitting $D\Omega_0$. Here

$$DR = D^*RD = \frac{\sin^2(\phi/2) \sin^2(\phi/2)}{4\pi^2 \phi^2} \frac{d^3+ \phi}{d^3+ \phi} \quad (24)$$

$$= \left( \frac{1}{\sqrt{1 - r^2} - 1} \right) \left( \frac{1}{\sqrt{1 - r^2} - 1} \right) d^3+ \phi d^3+ \phi.$$

6
where \( R^\alpha = e^\alpha_{\beta c} \hat{R}^c \) (25) is the Fourier transform of 1 on the hyperplane of \( so(3) \) to its tangential hyperplane, \( so(3,1) \). Thus, we get \( \delta \)-functions expressing closeness of the surfaces of the tetrahedra, that is, relations of the type of Gauss law which should hold identically on a real physical system.

Expression (25) is the Fourier transform of 1 on the hyperplane of \( so(3,1) \). In reality, we have the Fourier transform on the nonlinear manifold \( so(3,1) \). This should lead to smoothed and broadened \( \delta \)-functions. To reveal the type of this broadening, consider integrals of the function of interest \( \mathcal{N}(v_a, v_{\alpha\beta}) \) with products of the components \( \pm v_a \) (monomials),

\[
\int \mathcal{N} \prod_{a,\alpha} \left( v_a^\alpha \right)^{3\alpha} \left( -v_a^\alpha \right)^{3\alpha} d^3 v_a^\alpha d^3 v_a^\alpha.
\]  

(26)

so-called moments. Let us use the calculational model with the function ‘arcsin’ in the action being linearized. To define (26), we first integrate \( \exp(is) \) over \( \prod_{a} d^6 v_a d^3 v_a \) then over \( \prod_{a} D R_a \). Upon first integration we get the product of (the derivatives of) the \( \delta \)-functions

\[
\prod_{a=1}^{4} \delta^3 \left( R_a + R^T_a \right) \delta^3 \left( -R_a - R^T_a \right).
\]  

(27)

Occurrence of the support of this at zero \( \pm r_a \) somewhat justifies the adopted calculational model with linearized ‘arcsin’ as if the \( \pm r_a \)s were small. So this model should be qualitatively correct. Subsequent integration over \( \prod_{a} D R_a \) gives a finite answer. Note that if integration over \( d^6 v_{\alpha\beta} \) (maybe with some product of the components of \( v_{\alpha\beta} \)) were additionally inserted into the definition of moment, this integration would lead to singularity of the type of \( \delta^6(0) \),

\[
\delta^3 \left( R^\alpha \phi - R^T \phi R^\alpha \right) \delta^3 \left( -R^\alpha \phi - R^T \phi R^\alpha \right) \prod_{a=1}^{4} \delta^3 \left( R_a + R^T_a \right) \delta^3 \left( -R_a - R^T_a \right).
\]  

(28)

Finiteness of the integral of a function with any product of its arguments means that this function is decreasing faster than any inverse power of arguments. The simplest such function is an exponentially decreasing one. This type of decreasing is most natural in the present case when the function of interest \( \mathcal{N} \) is itself integral of exponent (and, by proper deformation of integration contours in complex plane, is expected to be representable as an \textit{a priori} combination of increasing and decreasing monotonic exponents). So the function \( \mathcal{N} \) should
This behaves as \( \exp \) can be reduced to include integrations over new variables, which are areas integral by deforming integration contours over

can be obtained in a simple way from the formal expression of the Euclidean version of this

\( \delta \)-function is itself a limiting case of the extremely rapidly decreasing exponent.

It is important that since the different terms in the argument of delta, \( ^8v_\alpha + \sum_{\beta=1}^{4} v_\beta \gamma_{\beta \alpha} \), enter the exact expression (22) in a different way, we expect that there should be suppression not only over such sum of these terms, but also over these terms separately at their large values.

On the other hand, defining an integral over connections as a function of area tensors from knowing its moments, i.e. integrals with products of area tensor components, is itself a physically sensible way of defining a conditionally convergent integral such as a path integral.

The moments have the physical sense of the expectation values for the products of area tensor components in a theory with independent area tensors.

In more detail, \( \mathcal{N} \) is exactly calculable for zero \( v_\alpha \)'s when the \( \mathcal{N} \) factorizes into functions of separate \( v_\alpha \) s,

\[
\mathcal{N}(\{v_\alpha\}, \{0\}) = \prod_{\alpha=1}^{4} \mathcal{N}_0(v_\alpha),
\]

(29)

\[
\mathcal{N}_0(v) = \int \exp \left( \frac{1}{4} \left( 1 + \frac{i}{\gamma} \right) \sqrt{v^2} \arcsin \left( \frac{v \ast R}{\sqrt{v^2}} \right) + \text{complex conjugate} \right) D R.
\]

(30)

In the calculational model with linearized ‘arcsin’ we get for \( \mathcal{N}_0 \), in accordance with \( \text{SO}(4) = \text{SO}(3) \times \text{SO}(3) \) (more accurately, plus the same times the element \( -1 \) of \( \text{SO}(4) \)), the square of the (analytically continued in a proper way) suppression factor for the length \( l \) in \( \text{SO}(3) \) gravity [10]. The latter was defined from knowing edge expectation values, i.e. its moments. It turns out to be proportional to \( K_1(l)/l \); \( K_1(l) \) is the modified integral Bessel function. Now

\[
\mathcal{N}_0(v) = \left[ \frac{K_1 (\frac{1}{\gamma} \sqrt{v} )}{\pi \sqrt{\left( \frac{1}{\gamma} \right)^2 v^2 - 1}} \right]^{2}, \quad K_1(l) = \int_0^{\pi/2} \exp \left( - \frac{l}{\sin \varphi} \right) d\varphi.
\]

(31)

At \( ^8v^2 = -|v|^2 \) (space-like region) \( \mathcal{N}_0(v) \) behaves as \( \exp(-|v|^2/2) \). At \( ^8v^2 = |v|^2 \) (time-like region) \( \mathcal{N}_0(v) \) behaves as \( \exp(-|v|/(2\gamma)) \).

The above exponential suppression over areas can be illustrated by the model integral

\[
\int_{-\infty}^{\infty} e^{\sqrt{-v^2} \sin \varphi} d\psi = 2K_0(\sqrt{-v^2}), \quad K_0(l) = \int_0^{\pi/2} \exp \left( - \frac{l}{\sin \varphi} \right) d\varphi.
\]

(32)

Here \( \sqrt{-v^2} \) is modeling module of the space-like area, \( \psi \) is modeling Lorentz boost angle. This behaves as \( \exp(-\sqrt{-v^2}) \) at large \( v^2 \). Nonzero \( \gamma^{-1} \) mixes space-like and time-like area components and leads to exponential suppression in the time-like region too.

The \( \mathcal{N}_0(v) \) with exact function ‘arcsin’ can be exactly computed and is exponentially suppressed at large \( |^8v^3| \) as well [11]. Due to the oddness of ‘arcsin’, a moment (26) can be reduced to include integrations over new variables, which are areas \( \sqrt{\gamma^2 v_\alpha^2} \) with sign.

Integration over these just gives (the derivatives of) the \( \delta \)-functions of both ‘arcsin’ functions in equation (22) which being further integrated allows us to define the moment.

In the case of the \( \text{SU}(2) \times \text{SU}(2) \) representation some more complicated expressions for \( \mathcal{N} \) can be written explicitly in the form of absolutely convergent exponentially suppressed integrals. Suppose we define an integral over connections as a function of area tensors from knowing its moments, i.e. integrals with products of area tensor components. Then this integral can be obtained in a simple way from the formal expression of the Euclidean version of this integral by deforming integration contours over \( DR \sim d^3\phi d^\tau \phi \) into the complex plane in a
The polar angle

There the analytically continued unit vector of the rotation

The azimuthal angle of

Namely, we write \( \hat{\phi} \) in the spherical coordinates relative to the direction of

so that

where

\( n \)

is parameterized in the spherical coordinates in the usual way,

Relative to \( n^{(0)} \), the \( \hat{n} \) is parameterized in the spherical coordinates in the usual way,

Here \( \hat{e}_1, \hat{e}_2 \) are the two unit vectors constituting with \( \hat{v}(\hat{v}^2)^{-1/2} \) an orthonormal triple.

Generally we also have in the exponential a scalar part \( \hat{m}_0 \) in \( \hat{m} \),

so that

\[ \hat{m}_0 \circ \hat{R} = \hat{m}_0 \cos \frac{\hat{\phi}}{2} + \hat{m} \hat{n} \sin \frac{\hat{\phi}}{2}. \]
So we have a combination of \(\cos(\frac{\mp \phi}{2})\) and \(\sin(\frac{\mp \phi}{2})\) in the exponential reducing to \(\sin(\frac{\mp \phi}{2} + \frac{\mp \beta}{2})\). The above deformation of integration contours over \(DR\) is applied to the shifted radial variable,

\[
\frac{1}{2} \frac{\mp \phi}{2} + \frac{i}{2} \mp \eta - \frac{\mp \beta}{2}.
\]  

(44)

Calculation shows that the result (36) gets naturally generalized by replacing \( \frac{\mp m^2}{2} \) instead of the \( \pm \)

\[
\frac{\mp m^2}{2} \Rightarrow \mp m \circ \mp m = \mp m^2 + \mp m_0.
\]

Using this, consider the 4-simplex with one of the edges, say (40), small so that we can take \( v_{23} = 0, v_1 = v_{01} = -v_{41}, \ldots \) cycle perm (1, 2, 3) \ldots . In the calculational model with linearized ‘arcsin’ the \( \mathcal{N} \) takes the form

\[
\mathcal{N}_0(v) = \int \exp \left[ \frac{1}{2} \left( \frac{1}{\gamma^2} \right) \Theta \right] \left( v_1 \circ v_1 + v_2 \circ v_2 + v_3 \circ v_3 + v_4 \circ v_4 \right)
\]

+ complex conjugate \( \prod_{a=1}^{4} DR_a \),

(45)

where \( \mp v_4 \equiv \mp v_4 - \mp R_1 \mp v_1 - \mp R_2 \mp v_2 - \mp R_3 \mp v_3 \). Applying the above considered deformation of integration contours we get

\[
\mathcal{N} = \mathcal{N}^\epsilon \mathcal{N},
\]

(46)

\[
\mp \mathcal{N} = \int K_1 \prod_{a=1}^{3} \exp \left[ \frac{1}{2} \left( \frac{1}{\gamma^2} \right) \Theta \right] \left[ \pm v_a^2 \pm \eta_a \pm \xi_a \pm \chi_a \right] \int_{\pm} \eta_a d\pm \eta_a d\pm \xi_a d\pm \chi_a.
\]

(47)

where \( \pm v_a \) depends on \( \pm \eta_a, \pm \xi_a, \pm \chi_a \) through \( R_a \) parameterized by these as above considered,

\[
\pm R_a = -ish \pm \eta_a + \pm \Sigma \pm \eta_a \pm \xi_a \pm \chi_a.
\]

(48)

\[
\pm \eta_a = \frac{-i(\pm \gamma)}{\sqrt{-((\pm \gamma)^2 \pm v_a^2)}} \pm \xi_a + i(\pm \gamma) \pm \chi_a \pm \pm \eta_a \pm \gamma \pm \pm \xi_a (\pm \pm \chi_a),
\]

(49)

Note that everywhere \( \text{Re} \sqrt{\xi} \geq 0 \) for our choice of the branch of function \( \sqrt{\xi} \) with the cut along the negative real half-axis in the complex plane of \( \xi \) such that \( \sqrt{1} = 1 \).

Equation (47) illustrates the above-mentioned broadening of the \( \delta \)-functions expressing closure conditions for the tetrahedra. In this simplified configuration with zero edge (04) this condition is \( \pm v_1 + \pm v_2 + \pm v_3 - \pm v_4 = 0 \). In equation (47) we have exponentially dumped \( K \) instead of the \( \delta \)-function and, besides, \( \mp v_a \)'s enter its argument not exactly in this combination, but with \( \mp R_a \) matrix coefficients which are in no way close to unity. Besides, there are also decreasing exponents showing suppression over \( \pm v_a \)'s separately.

In the case of the \( SO(3) \times SO(3) \) representation the unit vector \( \pm n \) enters exponential bilinearly, see (11), and explicitly reducing \( \mathcal{N} \) to the absolutely convergent exponentially suppressed integrals is a more difficult problem.

To see what suppression of areas means for suppression of the lengths, introduce a tetrad taken, say, at a vertex four, \( l_0, l_1, l_2, l_3 \). The \( l_k \) means a vector directed from the vertex \( i \) to \( k \). Then \( v_1 = [l_{12}, l_{13}], v_{12} = [l_{0}, l_{13}], \ldots \) cycle perm (1, 2, 3) \ldots . The other four bivectors are defined from closure of the appropriate 3-simplices, \( v_{41} = [l_{41} - l_{42}, l_{21} - l_{41}], \ldots \) cycle perm
(1, 2, 3) ... \( v_4 = |l_{42} - l_{43}, l_{41} - l_{43} | \). Standard quantum gravity settings suggest a choice of one of these vectors \( l_{40}^\alpha \) as a lapse-shift one and the Schwinger ‘time gauge’ \([13]\) for the three others, \( l_{40}^\alpha = 0, \alpha = 1, 2, 3 \). Then \( 2^{\alpha} v_1 = \pm l_{42} \times l_{43} = 2 v_1, 2^{\alpha} v_{12} = \pm l_{41} \times l_{43} + l_{40}^\alpha l_{43} \), ... cycle perm (1, 2, 3) ... . Suppression of large \( v_\alpha \)s mean suppression of large \( l_{40} \)s since small \( v_\alpha \)s mean small \( l_{40} \)s with exception of the region where \( |v_1 \times v_2 \cdot v_3| \to 0 \). The latter region can be suppressed by the positive power of \(- \det \| g_{\mu \nu} \| = 8 |l_{40}^\alpha|^2 |v_1 \times v_2 \cdot v_3| \) in (the edge vector part of) the measure. The measure is usually considered as determinable up to a power of \(- \det \| g_{\mu \nu} \| \). (In fact, the first principles allow us to fix the measure up to a power of \(- \det \| g_{\mu \nu} \| \) \([14, 15]\).)

5. Specifying the analytical form of the representation

Generally, when defining the quantum amplitude of transition between the two three-dimensional geometries in the simplicial framework, we should sum over all simplicial four-dimensional geometries interpolating between these two as over paths in the path integral. Of these simplicial geometries those ones with arbitrarily small edge lengths can arbitrarily and accurately approximate the usual smooth geometries. So we do not lose essential continuum degrees of freedom and such properties as the diffeomorphism symmetry typical for the continuum should be restored in the exact analysis. In practical calculations summation over all simplicial structures is recently technically unachievable, and we need to specify a simplicial structure to work with.

Here we express representations considered in the present paper for action in terms of principal values of the ‘arcsin’ function. Assuming a simple periodic simplicial structure as a basic example, the whole region of variation of edge lengths gets divided into different regions in which the action is a combination of the ‘arcsin’ functions together with terms which are constants different in different regions. These terms are important for possible integration over edge lengths in the path integral. These regions are particularly specified by performing triangulation consistent with the standard canonical quantization scheme using the Schwinger time gauge.

Consider the simplest periodic simplicial structure used in \([17]\) when the spacetime is divided into 4-cubes. To each vertex of this 4-cubic lattice one of the sixteen 4-cubes containing this vertex is assigned located in the directions of the four main cubic axes defined as the positive direction. The set of links of the simplicial complex is a union over vertices of the sets of all the edges and 2-face, 3-face and 4-cube diagonals of the corresponding 4-cube emanating from the vertex. Each 4-cube is thereby divided into 24 4-simplices. Each 4-simplex contains four edges directed along all four main 4-cube axes. Further we consider one such 4-simplex.

According to the notations at the end of section 4, we have a tetrad taken at a vertex four of a 4-simplex of the considered simplicial complex, \( l_{40}^\mu, l_{41}^\mu, l_{42}^\mu, l_{43}^\mu \). Standard quantum gravity settings suggest a choice of one of these vectors \( l_{40} \) as a lapse-shift one and the Schwinger time gauge for the other three, \( l_{40}^\alpha = 0, \alpha = 1, 2, 3 \) (that is, \( l_{40} \), \( \alpha = 1, 2, 3 \) are space-like). Regarding lapse-shift as a parameter for choice, we are free to choose \( l_{40} \)

(i) time-like and (ii) small (by the value of components) compared to the typical scale of \( |l_{4\alpha}|, \alpha = 1, 2, 3 \), so that \( (l_{4\alpha} - l_{40})^2 \) are space-like as \( l_{4\alpha} \) are, \( \alpha = 1, 2, 3 \). Thus, the only time-like link in the simplex (0 1 2 3 4) is (0 4), the others are space-like ones. The lapse-shift (0 4) and analogous ones in the other 4-simplices are supposed to be directed along the same one of the four main 4-cube axes. The other three main cubic axes define passing through the vertex zero or four the three-dimensional section, see figure 1. This section is
the three-dimensional version of the considered simplicial complex. There we have denoted by a 3-simplex \((4\ 1\ 2\ 3)\) that one being a projection of the 4-simplex \((0\ 1\ 2\ 3\ 4)\) onto the 3-section. (More accurately, some of the vertices \(4, 1, 2, 3\) in the notation \((4\ 1\ 2\ 3)\) might be substituted by their projections \(4', 1', 2', 3'\) onto the 3-section, that is, \((1'\ 1), (2'\ 2), (3'\ 3)\) are the lapse-shift edges.) Each such 3-simplex in the unperturbed flat symmetrical (w.r.t. the permutations of \((1, 2, 3)\)) uniform such three-dimensional lattice is defined by three basic vectors, \(e_1 \equiv \vec{41}, e_2 \equiv \vec{12}, e_3 \equiv \vec{23}\) (for the particular notations of figure 1). Up to an overall scale factor, these can be regarded as normalized, \(e_1^2 = e_2^2 = e_3^2 = 1\), but generally nonorthogonal, \(e_1 e_2 = e_2 e_3 = e_3 e_1 = \lambda\) (all three directions are treated symmetrically). The values of the dihedral angles in the 3-simplex are

\[
\alpha_{2(43)} = \pi/3, \quad \alpha_{1(423)} = \alpha_{3(412)} = \pi/2, \quad \alpha_{4(23)} = \alpha_{3(412)} - 2\alpha_{4(23)}, \quad \cos \alpha_{4(23)} = \sqrt{1 + 2\lambda}/(2 + 2\lambda).
\]

(50)

It is known that as the minimal dihedral angle of a simplex is better bounded away from zero, the conditions for closeness between the continuum and discrete manifolds are better [16]. It is seen that the minimal dihedral angle is maximum at \(\lambda = -1/3\), when this angle is \(\pi/3\) (the 3-cubic lattice is shrunk along the body diagonal, so that we have rather parallelopipeds than cubes).

Specifying above the simplicial geometry we define ranges for possible values of the dihedral angles in the typical 4-simplex. Denote by \(\alpha_{0(123)}\) the angle \(\alpha_{\sigma(1\ 2\ 3)}\) on \(\sigma^5 = (1\ 2\ 3)\) in \(\sigma^4 = (0\ 1\ 2\ 3\ 4)\) and analogously for others. It is not difficult to conclude that the angles in certain triangles are as follows:

\[
\begin{align*}
\alpha_{0(234)} &= \frac{\pi}{2} + i\eta_{0(234)} + \frac{\pi}{2} + i\eta_{0(123)} + \frac{\pi}{2} + i\eta_{0(234)} = \pi/2 + i\eta_{0(123)} \\
\alpha_{1(023)} &= \pi/2 + i\eta_{1(023)} \quad \ldots 2 \text{ cycle perm } (1, 2, 3) \ldots \\
\alpha_{0(123)} &= i\eta_{0(123)}
\end{align*}
\]

(51)

\(\eta\) is everywhere real. Knowing possible ranges for the angles, it is not difficult to write identities relating angles to the values of ‘arcsin’ which can arise in the \(SO(3) \times SO(3)\) representation (in the absence of torsion),

\[
\begin{align*}
\alpha_{0(234)} &= \frac{\pi}{2} + \frac{1}{2} \arcsin[2\pi - 2\alpha_{0(234)}] \\
\alpha_{1(023)} &= \frac{\pi}{2} + \frac{1}{2} \arcsin[2\pi - 2\alpha_{1(023)}] \\
\alpha_{0(123)} &= -\frac{1}{2} \arcsin[2\pi - 2\alpha_{0(123)}].
\end{align*}
\]

(53)
The angles (55) are real ones in the triangles \((014), (024), (034)\). In the three-dimensional section, these rotations correspond to the rotations by the usual Euclidean angles around the edges. Identity (55) is valid in the region 
\[
\frac{\pi}{4} < \alpha_{1(024)} < 3\pi/4, \ldots 2 \text{ cycle perm } (1, 2, 3) \ldots .
\]
In the neighborhood of the point in the configuration superspace where the lapse-shift is orthogonal to the flat three-dimensional section, the considered hyperdihedral angles are close to the corresponding dihedral angles in the three-dimensional section (50), 
\[
\alpha_{2(014)} \approx \alpha_{2(14)} \ldots.
\]
The neighborhood of the pseudo-cubic (shrunk along the main cube diagonal) three-dimensional section at the parameter \(\lambda = -1/3\) and the dihedral angles \(\pi/3\) and \(\pi/2\) just fall into the region \((\pi/4, 3\pi/4)\).

Now we can form the defect angles using the values of angles (53), (54), (55) in the considered and neighboring 4-simplices. Each angle (53) is in the triangle in which there are three other angles of such type from neighboring 4-simplices, and for the given simplicial structure, there are also two or null angles of type (54), figure 2. As a result, the constant real (proportional to \(\pi\)) part in the resulting defect angle is 
\[
2\pi - \frac{4}{\pi} \arcsin \sin(\pi - \alpha)
\]
(55) per 4-simplex and per each part of the action, self- and anti-self-dual ones.

In the thus defined region of edge lengths in the configuration superspace the action attributed to the given 4-simplex \((01234)\) takes the form 
\[
\pm \mathcal{S}_{O(3)} = \left( \frac{\pi}{N_{(041)}} - \frac{\pi}{4} \right) \sqrt{\pm \nu_{23}^2 - \frac{1}{4} \nu_{23}^2 \arcsin \frac{\pm \nu_{23} \pm R_{23}}{\sqrt{\pm \nu_{23}^2}}} 
\]
\[- \frac{1}{4} \sqrt{\pm \nu_{14}^2 \arcsin \frac{\pm \nu_{14} \pm R_{14}}{\sqrt{\pm \nu_{14}^2}}} - \frac{1}{4} \sqrt{\pm \nu_{01}^2 \arcsin \frac{\pm \nu_{01} \pm R_{01}}{\sqrt{\pm \nu_{01}^2}}} 
\]
\[+ 2 \text{ cycle perm } (1, 2, 3) + \frac{1}{4} \sqrt{\pm \nu_{04}^2 \arcsin \frac{\pm \nu_{04} \pm R_{04}}{\sqrt{\pm \nu_{04}^2}}} .
\]
(57)

The number of the 4-simplices sharing the triangles, e.g. for the 4-simplex \((01234)\) built of the tetrahedron \((1234)\) of figure 1 and of lapse-shift (40) is \(N_{(041)} = 6 = N_{(043)}\), 
\[N_{(042)} = 4.
\]

Relevant to the \(SU(2) \times SU(2)\) representation identities for all the considered types of the angles can be written in the form 
\[
\alpha = \frac{\pi}{2} + \frac{\pi}{2} - \arcsin \sin(\pi - \alpha) \right] \text{sgn } \text{Re } \left( \alpha - \frac{\pi}{2} \right).
\]
(58)
The natural region $0 < \text{Re} \alpha < \pi$ is implied. For the angle of type (54), this gives
\[
\alpha_{(0,1,2,3,4)} = \arcsin (\pi - \alpha_{(0,1,2,3,4)})
\] (59)
like (54) without the constant terms; in others the constant terms $\pi/2$ remain as in the expressions for these (53), (55) as well. Therefore, we get analogously
\[
\pm \text{SSU}(2) = \left(\frac{\pi}{N_{(0,1,2,3,4)}} - \frac{\pi}{4}\right) \sqrt{\frac{h v_{2,3}^2}{v_{2,3}^2}} - \frac{1}{2} \sqrt{h v_{2,3}^2} \left[\arcsin \left(\frac{\pm v_{2,3} \circ R_{2,3}}{\sqrt{h v_{2,3}^2}}\right)\right] \text{sgn} \text{Re} (\alpha_{2,3} - \frac{\pi}{2})
\]
\[- \frac{1}{2} \sqrt{\frac{h v_{1,4}^2}{v_{1,4}^2}} \left[\arcsin \left(\frac{\pm v_{1,4} \circ R_{1,4}}{\sqrt{h v_{1,4}^2}}\right)\right] \text{sgn} \text{Re} (\alpha_{1,4} - \frac{\pi}{2})
\]
\[- \frac{1}{2} \sqrt{\frac{h v_{0,1}^2}{v_{0,1}^2}} \left[\arcsin \left(\frac{\pm v_{0,1} \circ R_{0,1}}{\sqrt{h v_{0,1}^2}}\right)\right] \text{sgn} \text{Re} (\alpha_{0,1} - \frac{\pi}{2})
\]
\[+ 2 \text{ cycle perm (1, 2, 3)} - \frac{1}{2} \sqrt{\frac{h v_{2,3}^2}{v_{2,3}^2}} \arcsin \left(\frac{\pm v_{0,4} \circ R_{0,4}}{\sqrt{h v_{0,4}^2}}\right) \text{sgn} \text{Re} (\alpha_{0,4} - \frac{\pi}{2})
\] (60)

Here $\alpha_{2,3}$ is shorthand for $\alpha_{(2,0,1,4,3)}$ etc. $\alpha_{i,4}$ in the rhs are implied to be functions of the edge lengths, and the whole region of variation of the edge lengths is divided into sectors in which sign functions keep their values. One new feature of the $SU(2) \times SU(2)$ representation is the ambiguity of the signs $\text{sgn} \text{Re}(\alpha - \pi/2)$ in $\pm \text{SSU}(2)$ for the angles (51) of the type $\pi/2 + i\eta$. Ambiguity arises because $\arcsin z$ is here on the cut $\text{Im} z = 0, z^2 > 1$, where it undergoes discontinuity. A way to fix these signs consistently might be to add to the lengths some infinitely small imaginary parts $\pm i0$.

As far as certain expressions for area tensors in terms of edge vectors are not written yet, we can redefine area tensors in (60) including an overall sign at ‘arcsin’ in their definition, e.g. $v_{2,3} \text{sgn} \text{Re} (\alpha_{2,3} - \pi/2) \Rightarrow v_{2,3}$ etc. The same can be done in $\pm \text{SSU}(3)$, and, as far as the integration over connections is concerned, we deal with the standard forms of the action used in section 4 with all the ‘arcsin’s entering with positive sign. The piecewise constant terms added to ‘arcsin’s, different in different sectors of edge length variation, lead to the combination of areas added to action and should be taken into account when considering path integration over lengths. These terms reflect the nonperturbative nature of the considered representations.

6. Conclusion

An attractive feature of the considered representations is that these allow us to deal with comparatively simple simplicial gravity action instead of that in terms of edge lengths only, with complicated trigonometric expressions. This is achieved by introducing additional rotational variables, but the dependence on these is simple and splits over separate terms in the action referred to as separate 4-simplices.

In the path integral formalism, this leads to additional integrations over rotations, but these factorize over separate 4-simplices into ordinary integrals. Upon performing a fixed finite number of integrations, we reduce the Minkowsky path integral to the form of absolutely convergent (as in the Euclidean version) exponentially suppressed (at large areas/lengths) integrals. This means suppression of the physical amplitudes with large areas/lengths (in the Plank scale) and is important for the consistency of the simplicial minisuperspace system.

Qualitatively, this consideration can be repeated for the representation with the usual connections relating neighboring local frames of the 4-simplices, and analogous conclusions...
concerning suppressing large areas/lengths can be made. To estimate the latter exactly, we should, strictly speaking, make an arbitrarily large number of integrations over connections entering the definition of path integral. Besides, passing to the variables with a clearer physical sense, independent curvatures, is achieved via arbitrarily lengthy expressions which should express other curvatures in terms of independent ones (that is, resolve Bianchi identities).

Thus, simplifying points in the representation of the present paper are (i) a fixed finite number of additional rotational variables referred to as separate 4-simplices; (ii) the action is simple in terms of independent rotations since Bianchi identities are simple (16).

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