MSSM-like AdS Flux Vacua with Frozen Open-string Moduli

Ching-Ming Chen, Tianjun Li, Van Eric Mayes, and D.V. Nanopoulos

1George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA

2Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China

3Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA; Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece

We construct supersymmetric Pati-Salam and MSSM-like IIA AdS flux models from intersecting D6-branes on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$. The models constructed have three generations of MSSM matter plus right-handed neutrinos. Because the cycles wrapped by the D-branes are rigid there are no extra massless fields in the adjoint representation, arising as open-string moduli. Moreover, in order to include D-brane instanton effects, the E2-branes associated with these instantons must wrap rigid cycles. In order to generate desirable superpotential couplings via these D-brane instantons, the zero-mode structure of the E2-branes is highly constrained. The inclusion of flux in the models to cancel tadpoles can then make it easier to incorporate these effects by providing some flexibility in choosing the hidden sector.
I. INTRODUCTION

With the dawn of the Large Hadron Collider (LHC) era, the prospects for the discovery of new physics may finally be arriving. In particular, whatever physics is responsible for stabilizing the electroweak scale should be discovered. Signals of the favored mechanism, broken supersymmetry, may be observed as well as the Higgs states required to break the electroweak symmetry. Together with the new windows opening up with advances in cosmology, we may soon be on the road to a detailed understanding of the universe.

In principle, it should be possible to derive all known physics in a top-down approach directly from string theory, as well as potentially predicting new and unexpected phenomena. However, at present there seems to be a tremendous degeneracy of possible vacua, and in the absence of any non-perturbative formulation of string theory, there seems to be no reason for singling out one vacuum or another. Conversely, following a bottom-up approach, one may ask if it is possible to deduce the origin of new physics given such a signal at LHC. For example, in the case of low-energy supersymmetry, it may be possible from the experimental data to deduce the structure of the fundamental theory at high energy scales which determines the supersymmetry breaking soft terms and ultimately leads to electroweak symmetry breaking (EWSB). Although still in initial stages, there has been some effort put towards this approach in recent years studying the pattern of signatures of broad classes of string vacua [1, 2, 3, 4, 5, 6].

Both top-down and bottom-up approaches have various strengths and weaknesses. However, it seems likely that progress will be made by a combination of the two approaches. Specifically, by constructing vacua which increasingly resemble our world, including the hoped for new data gleaned from LHC in the coming years, it should be possible to at least restrict the landscape of possible vacua to isolated regions. Then, from the low-energy effective action of these models it should be possible to extract experimental signatures which may be matched to those observed at LHC. Thus, for this program to succeed it is imperative that we have concrete, realistic models in hand. However, as is well known, at present there is not even one string-derived model which can be considered fully realistic. Thus, it is difficult to draw any inferences based on the statistical study of the landscape since there are no truly representative samples that match the universe we live in.

An important question is what criteria must a particular model satisfy in order to be con-
sidered fully realistic. Besides obtaining three-generations of chiral fermions which transform as bifundamental representations of $SU(3)_C \times SU(2)_L \times U(1)_Y$, such a model should be capable of reproducing the observed hierarchy of fermion masses, including the observed small neutrino masses. The values of the gauge coupling constants should also be predicted. Although it has become somewhat fashionable these days to dismiss the apparent unification of the gauge couplings in the MSSM as accidental, we still believe that this is an important and significant feature that should be present in any fully realistic model. Furthermore, it should be possible to address the issue of supersymmetry breaking within the model and extract detailed predictions for the superpartner spectrum. It should also be possible to calculate the $\mu$ and $B\mu$ terms in the effective action. Combined with a calculation of the Yukawa couplings for quarks and leptons, this then allows important parameters such as $\tan \beta$ to be determined. Mechanisms within the model for realizing inflation and dark energy would also be desirable. Finally, the importance of moduli stabilization cannot be overstressed as all physical parameters such as Yukawa and gauge couplings depend crucially upon the values taken by the moduli VEVs.

Much work towards model building has been focused on the heterotic string. Indeed, many of the phenomenologically most appealing of such models were those constructed within the free-fermionic formulation based on the NAHE set [7]. In addition, progress has also been made in recent years in constructing heterotic M-theory models [8, 9]. On the Type II side, D-branes [10] have created new approaches to constructing semi-realistic vacua. In particular, intersecting D-brane worlds, where chiral fermions are localized at the intersections between D-branes (in the Type IIA picture) [11] or in the T-dual (Type IIB) picture involving magnetized D-branes [12], have provided an exciting new avenue in model building. Much work has been done in recent years in constructing such models. For recent reviews of this topic, see [13] and [14].

Although much progress has been made in constructing intersecting D-brane worlds, none of the models to date have been completely satisfactory. Problems include extra chiral and non-chiral matter and the lack of a complete set of Yukawa couplings, which are typically forbidden by global symmetries. In most of the cases where a complete set of Yukawa couplings has been allowed by global symmetries, the models have typically suffered from a rank one problem in the Yukawa mass matrices, rendering them ineffective in giving masses and mixings to all three generations. This problem may ultimately be traced to the
fact that not all of the intersections are present on the same torus (in the case of toroidal orientifold compactifications). In addition, unlike heterotic models, the gauge couplings are not automatically unified in intersecting D-brane models. Thus, it is non-trivial to find an intersecting D-brane model which possesses these features. Nevertheless, one example is known of an intersecting D6-brane model in Type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold where these problems may be solved [15, 16]. Indeed, this model automatically realizes tree-level gauge coupling unification, and it is possible within this model to obtain correct Yukawa mass matrices for the quarks and leptons for specific values of the moduli VEVs [17, 18].

Despite the appealing properties of this model, the issue of moduli stabilization has not yet been fully worked out. Although the stabilization of the closed-string moduli has already been addressed to some extent [16, 19], there is still the problem of stabilizing the open-string moduli associated with D-brane positions in the internal space and Wilson lines. These unstabilized moduli result in adjoint matter associated with each stack of D-branes. Such light-scalars charged under the SM gauge group are not observed and would have a detrimental effect on the successful running and eventual unification of the gauge couplings. Moreover, the Yukawa couplings for quarks and leptons directly depend upon the D-brane positions in the internal space as well as other geometric moduli [20]. Thus, if one wants to calculate Yukawa couplings, and maintain the successful running of gauge couplings from the unification scale to the electroweak scale, these open string moduli must be completely frozen.

One way to do this is to construct intersecting D-brane models where the D-branes wrap rigid cycles, which was first explored [21] in the context of Type IIA compactifications on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ which is the only known toroidal background which possesses such rigid cycles, as well as the T-dual construction involving magnetized fractional D-branes [22]. The importance of rigid cycles has also been made more clear in recent years by the study of D-instanton induced superpotential couplings [23, 24, 25]. Indeed, such couplings may provide a mechanism for generating naturally small neutrino masses, solving the $\mu$-problem of the MSSM, and quite possibly for addressing the issues of supersymmetry breaking and inflation. Moreover, the absence of matter in the adjoint representation is consistent with heterotic models with a $k = 1$ Kac-Moody algebra [26] as well as recent F-theory constructions [27], some of which may be related to Type II vacua by various chains of dualities.

In this paper, we construct supersymmetric Pati-Salam and MSSM-like models in the
framework of Type IIA flux compactifications where the D-branes wrap rigid cycles. This letter is organized as follows: First, we briefly review rigid intersecting D6-brane constructions in Type IIA on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$. We then proceed to construct a supersymmetric three-generation Pati-Salam model in AdS as an example of Type IIA flux vacua. We then break the Pati-Salam gauge symmetry to the SM by displacing the stacks on one torus by requiring them to run through different fixed points. Because the cycles wrapped by the D-branes are rigid in these models, the open-string moduli are completely frozen, and so these fields will create no difficulties with asymptotic freedom or with astrophysical constraints on light scalars. Moreover, in order to include D-brane instanton effects, the E2-branes associated with these instantons must wrap rigid cycles. In order to generate desirable superpotential couplings via these D-brane instantons, the zero-mode structure of the E2-branes is highly constrained. The inclusion of flux in the models to cancel tadpoles can then make it easier to incorporate these effects by providing some flexibility in choosing the hidden sector.

II. INTERSECTING BRANES ON $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$

Here, we briefly summarize model building on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$. For a detailed discussion of model building on this background we direct the reader to \cite{21}, which we summarize in the following. In Type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ orientifold background, the $T^6$ is product of three two-tori and the two orbifold group generators $\theta, \omega$ act on the complex coordinates $(z_1, z_2, z_3)$ as

$$
\theta : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) \\
\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)
$$

(1)

while the antiholomorphic involution $R$ acts as

$$
R(z_1, z_2, z_3) \rightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3).
$$

(2)

The signs of the $\theta$ action in the $\omega$ sector and vice versa have not been specified, and the freedom to do so is referred to as the choice of discrete torsion. One choice of discrete torsion corresponds to the Hodge numbers $(h_{11}, h_{21}) = (3, 51)$ and the other corresponding to $(h_{11}, h_{21}) = (51, 3)$. These two different choices are referred to as with discrete torsion ($\mathbb{Z}_2 \times \mathbb{Z}'_2$) and without discrete torsion ($\mathbb{Z}_2 \times \mathbb{Z}_2$) respectively. For $T^6/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$ the twisted
homology contains collapsed 3-cycles. There are 16 fixed points, from which arise 16 additional 2-cycles with the topology of $P^1 \cong S^2$. As a result, there are 32 collapsed 3-cycles for each twisted sector. A $D6$-brane wrapping collapsed 3-cycles in each of the three twisted sectors will be unable to move away from a particular position on the covering space $T^6$, and thus the 3-cycle will be rigid.

A basis of twisted 3-cycles may be defined as

\[ [\alpha^\theta_{ij,n}] = 2[\epsilon^\theta_{ij}] \otimes [a^3], \quad [\alpha^\omega_{ij,n}] = 2[\epsilon^\omega_{ij}] \otimes [b^3], \quad (3) \]

\[ [\alpha^\omega_{ij,m}] = 2[\epsilon^\omega_{ij}] \otimes [a^1], \quad [\alpha^\theta_{ij,m}] = 2[\epsilon^\theta_{ij}] \otimes [b^1], \quad (4) \]

\[ [\alpha^{\theta\omega}_{ij,n}] = 2[\epsilon^{\theta\omega}_{ij}] \otimes [a^2], \quad [\alpha^{\theta\omega}_{ij,m}] = 2[\epsilon^{\theta\omega}_{ij}] \otimes [b^2]. \quad (5) \]

where $[\epsilon^\theta_{ij}]$, $[\epsilon^\omega_{ij}]$, and $[\epsilon^{\theta\omega}_{ij}]$ denote the 16 fixed points on $T^2 \times T^2$, where $i, j \in 1, 2, 3, 4$.

A fractional D-brane wrapping both a bulk cycle as well as the collapsed cycles may be written in the form

\[ \Pi^F_a = \frac{1}{4} \Pi^B_a + \frac{1}{4} \left( \sum_{i,j \in S^g_{\theta}} \epsilon^\theta_{a,ij} \Pi^\theta_{ij,a} \right) + \frac{1}{4} \left( \sum_{j,k \in S^g_{\omega}} \epsilon^\omega_{a,jk} \Pi^\omega_{jk,a} \right) + \frac{1}{4} \left( \sum_{i,k \in S^g_{\theta\omega}} \epsilon^{\theta\omega}_{a,ik} \Pi^{\theta\omega}_{ik,a} \right), \quad (6) \]

where the $D6$-brane is required to run through the four fixed points for each of the twisted sectors. The set of four fixed points may be denoted as $S^g$ for the twisted sector $g$. The constants $\epsilon^\theta_{a,ij}$, $\epsilon^\omega_{a,jk}$, and $\epsilon^{\theta\omega}_{a,ki}$ denote the sign of the charge of the fractional brane with respect to the fields which are present at the orbifold fixed points. These signs, as well as the set of fixed points, must satisfy consistency conditions. However, they may be chosen differently for each stack.

The intersection number between a brane $a$ and brane $b$ wrapping fractional cycles is given by

\[ \Pi^F_a \circ \Pi^F_b = \frac{1}{16} \Pi^B_a \circ \Pi^B_b + 4(n^3_a m^3_b - m^3_a n^3_b) \sum_{i,a} \sum_{j,b} \epsilon^\theta_{a,ja} \epsilon^\theta_{b,ib} \delta_{ia} \delta_{ib} + \]

\[ 4(n^1_a m^1_b - m^1_a n^1_b) \sum_{j,a} \sum_{k,b} \epsilon^\omega_{a,ja} \epsilon^\omega_{b,jk} \delta_{ja} \delta_{jb} + \]

\[ 4(n^2_a m^2_b - m^2_a n^2_b) \sum_{i,a} \sum_{k,b} \epsilon^{\theta\omega}_{a,ia} \epsilon^{\theta\omega}_{b,ik} \delta_{ia} \delta_{ik}. \quad (7) \]
while the 3-cycle wrapped by the O6-plane is given by

$$\Pi_{O6} = 2\eta_{KR}[a^1][a^2][a^3] - 2\eta_{KR\theta}[b^1][b^2][a^3] - 2\eta_{KR\omega}[a^1][b^2][b^3] - 2\eta_{KR\theta\omega}[b^1][a^2][b^3].$$  (8)

where the cross-cap charges $\eta_{KR}$ give the RR charge and tension of a given orientifold plane $g$, of which there are two types, $O6^{(-,-)}$ and $O6^{(+,+)}$. In this case, $\eta_{KR} = +1$ indicates an $O6^{(-,-)}$ plane, while $\eta_{KR} = -1$ indicates an $O6^{(+,+)}$ while the choice of discrete torsion is indicated by the product

$$\eta = \prod_g \eta_{KR}.\quad (9)$$

The choice of no discrete torsion is given by $\eta = 1$, while for $\eta = -1$ is the case of discrete torsion, for which an odd number of $O^{(+,+)}$ must be present.

The action of $\Omega R$ on the bulk cycles changes the signs of the wrapping numbers as $n_i^a \to n_i^a$ and $m_i^a \to -m_i^a$. However, in addition, there is an action on the twisted 3 cycle as

$$\alpha^g_{ij,n} \to -\eta_{KR}\eta_{KRg}\alpha^g_{ij,n}, \quad \alpha^g_{ij,m} \to \eta_{KR}\eta_{KRg}\alpha^g_{ij,m}.\quad (10)$$

Using these relations, the intersection number of a fractional cycle with it’s $\Omega R$ image is given by,

$$\Pi^F \circ \Pi^F_a = \eta_{KR}\left(2\eta_{KR} \prod_I n_i^am_i^a - 2\eta_{KR\theta}n_a^3m_a^3 - 2\eta_{KR\omega}n_a^1m_a^1 - 2\eta_{KR\theta\omega}n_a^2m_a^2\right)\quad (11)$$

while the intersection number with the orientifold planes is given by

$$\Pi_{O6} \circ \Pi^F_a = 2\eta_{KR} \prod_I m_i^a - 2\eta_{KR\theta}n_a^1n_a^2m_a^3 - 2\eta_{KR\omega}m_a^1n_a^2n_a^3 - 2\eta_{KR\theta\omega}n_a^1m_a^2n_a^3.\quad (12)$$

The multiplicity of states in bifundamental, symmetric, and antisymmetric representations is shown in Table I.

The fractional cycle wrapped by a D-brane is specified by several sets of topological data. Specifically, the fractional cycles are described by the bulk wrapping numbers \{(n^1, m^1)(n^2, m^2), (n^3, m^3)\}, the sets of fixed points in each of the twisted sectors \(S^\theta, S^\omega, \) and \(S^\theta\omega\), as well as the signs in each twisted sector \(\epsilon^i_{ij}, \epsilon^\omega_{jk}, \) and \(\epsilon^\theta_{ki}\). Essentially, the sets of fixed points \(S^\theta\) are specified by the position of the fractional brane on the three two-tori, while the signs \(\epsilon^g_{ij}\) are related to the choice of discrete Wilson lines for each stack of branes. The fixed point sets can in fact be determined for each fractional brane from the bulk wrapping numbers. For a 1-cycle on a \(T^2/\mathbb{Z}_2\), a fractional brane will pass
TABLE I: General spectrum for D6-branes wrapping fractional cycles and intersecting at generic angles. \( \mathcal{M} \) represents the multiplicity, and \( a_S \) and \( a_A \) denote the symmetric and anti-symmetric representations of \( U(N_a) \), respectively.

| Sector   | Representation                                      |
|----------|-----------------------------------------------------|
| \( ab + ba \) | \( \mathcal{M}(N_a, N_b) = \Pi_a^F \circ \Pi_b^F \) |
| \( a'b + ba' \) | \( \mathcal{M}(N_a, N_b) = \Pi_a^F \circ \Pi_b^F \) |
| \( a'a + aa' \) | \( \mathcal{M}(a_S) = \Pi_a^F \circ \Pi_a^F - \Pi O_6 \circ \Pi_a^F ; \quad \mathcal{M}(a_A) = \Pi_a^F \circ \Pi_a^F + \Pi O_6 \circ \Pi_a^F \) |

through a pair of fixed points, which can be determined up to a choice from the wrapping numbers of the 1-cycle:

\[
S_{T_2}^{i} = \begin{cases} 
\{1, 4\} \text{ or } \{2, 3\} & \text{for } (n^l, m^l) = (\text{odd, odd}), \\
\{1, 3\} \text{ or } \{2, 4\} & \text{for } (n^l, m^l) = (\text{odd, even}), \\
\{1, 2\} \text{ or } \{3, 4\} & \text{for } (n^l, m^l) = (\text{even, odd}), 
\end{cases}
\]  

(13)

where the two possible choices are related by a transverse translation of the 1-cycle on the torus. Let us define the variable \( \delta \) such that \( \delta = 0 \) indicates that we make the first choice, while \( \delta = 1 \) indicates the second choice. For example, \( (n^l, m^l) = (\text{odd, odd}) \) with \( \delta = 0 \) indicates \( S_{T_2}^{i} = \{1, 4\} \), while \( (n^l, m^l) = (\text{odd, odd}) \) with \( \delta = 1 \) indicates \( S_{T_2}^{i} = \{2, 3\} \).

From this information, one can then determine the fixed-point sets for each twisted sector. This is done by taking the product of the fixed-points sets for each \( T^2 \) which is acted upon by the orbifold action \( g \), i.e.

\[
S_a^0 = S_a^{T_2} \times S_a^{T_2} = \{i_{a_1}j_{a_1}, i_{a_1}j_{a_2}, i_{a_2}j_{a_1}, i_{a_2}j_{a_2}\},
\]

\[
S_a^\omega = S_a^{T_2} \times S_a^{T_2} = \{j_{a_1}k_{a_1}, j_{a_1}k_{a_2}, j_{a_2}k_{a_1}, j_{a_2}k_{a_2}\},
\]

\[
S_a^{\omega} = S_a^{T_2} \times S_a^{T_2} = \{k_{a_1}i_{a_1}, k_{a_1}i_{a_2}, k_{a_2}i_{a_1}, k_{a_2}i_{a_2}\}.
\]

(14)

where \( i, j, k \) label the pairs of fixed points for each of the three \( T^2 \) respectively.
A. Twisted Charges and Wilson Lines

As stated in Section 2, the signs $\epsilon_{ij,a}^\theta$, $\epsilon_{jk,a}^\omega$, and $\epsilon_{ki,a}^{\theta\omega}$ are not arbitrary as they must satisfy certain consistency conditions. Specifically, the set of signs in each twisted sector for each stack of branes $a$ must satisfy

$$\sum_{i,j \in S_\theta^a} \epsilon_{a,ij}^\theta = 0 \mod 4,$$

(15)

$$\sum_{j,k \in S_\omega^a} \epsilon_{a,jk}^\omega = 0 \mod 4,$$

$$\sum_{k,i \in S_{\theta\omega}^a} \epsilon_{a,ki}^{\theta\omega} = 0 \mod 4,$$

and the signs in different twisted sectors for each stack $a$ must be related by the two conditions

$$\epsilon_{a,ij}^\theta \epsilon_{a,jk}^\omega \epsilon_{a,ki}^{\theta\omega} = 1,$$

(16)

$$\epsilon_{a,ij}^\theta \epsilon_{a,jk}^\omega = \text{constant} \forall j.$$

A trivial choice of signs which satisfies these constraints is just to have them all set to $+1$,

$$\epsilon_{a,ij}^\theta = 1 \forall ij, \quad \epsilon_{a,jk}^\omega = 1 \forall jk, \quad \epsilon_{a,ki}^{\theta\omega} = 1 \forall ki.$$

(17)

Another possible non-trivial choice of signs consistent with the constraints is given by

$$\epsilon_{a,ij}^\theta = -1 \forall ij, \quad \epsilon_{a,jk}^\omega = -1 \forall jk, \quad \epsilon_{a,ki}^{\theta\omega} = 1 \forall ki.$$

(18)

More general sets of these signs may be found \cite{21} by setting $\epsilon_{i_1j_1}^\theta = \epsilon_{j_1k_1}^\theta = \epsilon_{k_1k_1}^{\theta\omega} = 1$,

$$\epsilon_{ij}^\theta = \{1, \beta, \lambda, \lambda \cdot \beta\} \quad \text{for} \quad S_\theta = \{i_1j_1, i_1j_2, i_2j_1, i_2j_2\},$$

(19)

$$\epsilon_{jk}^\omega = \{1, \psi, \beta, \beta \cdot \psi\} \quad \text{for} \quad S_\omega = \{j_1k_1, j_1k_2, j_2k_1, j_2k_2\},$$

$$\epsilon_{ki}^{\theta\omega} = \{1, \lambda, \psi, \lambda \cdot \psi\} \quad \text{for} \quad S_{\theta\omega} = \{k_1i_1, k_1i_2, k_2i_1, k_2i_2\},$$

where $\beta$, $\lambda$, and $\psi = \pm 1$. The signs $\beta$, $\lambda$, and $\psi$ have an interpretation as the choice of discrete Wilson lines along the fractional D-brane.

B. Conditions for Preserving $\mathcal{N} = 1$ Supersymmetry

The condition to preserve $\mathcal{N} = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ \cite{11, 28}. 
Essentially, this becomes a constraint on the angles made by each stack of branes with respect to the orientifold planes, viz \( \theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \mod 2\pi \), or equivalently \( \sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0 \) and \( \cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1 \). Applying simple trigonometry, these angles may be expressed in terms of the wrapping numbers as

\[
\tan \theta_a^i = \frac{m_a^i R_2^i}{n_a^i R_1^i} = \frac{m_a^i}{n_a^i} \chi^i
\]

(20)

where \( R_2^i \) and \( R_1^i \) are the radii of the \( i \)th torus, and \( \chi^i = R_2^i / R_1^i \). We may translate these conditions into restrictions on the wrapping numbers as

\[
x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0
\]

(21)

\[
A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0
\]

(22)

where we have made the definitions

\[
\tilde{A}_a = -m_a^1 m_a^2 m_a^3, \quad \tilde{B}_a = n_a^1 n_a^2 m_a^3, \quad \tilde{C}_a = m_a^1 n_a^2 n_a^3, \quad \tilde{D}_a = n_a^1 m_a^2 n_a^3,
\]

\[
A_a = -n_a^1 n_a^2 m_a^3, \quad B_a = m_a^1 m_a^2 n_a^3, \quad C_a = n_a^1 m_a^2 n_a^3, \quad D_a = m_a^1 n_a^2 m_a^3,
\]

(23)

(24)

and the parameters \( x_A, x_B, x_C, \) and \( x_D \) are related to the complex structure parameters by

\[
x_a = \gamma, \quad x_b = \frac{\gamma}{\chi_2 \cdot \chi_3}, \quad x_c = \frac{\gamma}{\chi_1 \cdot \chi_3}, \quad x_d = \frac{\gamma}{\chi_1 \cdot \chi_2}.
\]

(25)

where \( \gamma \) is a positive, real constant.

**C. RR and Torsion Charge Cancellation**

For the present, we focus on supersymmetric AdS vacua with metric, NSNS, and RR fluxes turned on \([29, 30]\). In order to have a consistent model, all RR charges sourced by D6-branes, O6-planes, and by the fluxes must cancel. The conditions for the cancellation of RR tadpoles are then given by

\[
\sum N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2} (m h_0 + q_1 a_1 + q_2 a_2 + q_3 a_3) = 16 \eta_{1IR},
\]

(26)

\[
\sum N_a m_a^1 m_a^2 m_a^3 + \frac{1}{2} (m h_1 - q_1 b_11 - q_2 b_21 - q_3 b_31) = -16 \eta_{1IR9},
\]

\[
\sum N_a m_a^1 n_a^2 m_a^3 - \frac{1}{2} (m h_2 - q_1 b_12 - q_2 b_22 - q_3 b_32) = -16 \eta_{1IR\omega},
\]

\[
\sum N_a n_a^1 m_a^2 m_a^3 - \frac{1}{2} (m h_3 - q_1 b_13 - q_2 b_23 - q_3 b_33) = -16 \eta_{1IR\theta}.
\]
where $a_i$ and $b_{ij}$ arise due to the metric fluxes, $h_0$ and $h_i$ arise due to the NSNS fluxes, and $m$ and $q_i$ arise from the RR fluxes. We consider these fluxes to be quantized in units of eight in order to avoid subtle problems with Dirac flux quantization conditions.

For simplicity, we set all of the Kähler moduli equal to each other, $T_1 = T_2 = T_3 = T$, so that we then obtain $q_1 = q_2 = q_3 = q$ from the superpotential. In order to satisfy the Jacobi identities for the metric fluxes, we shall consider the solution $a_i = a$, $b_{ii} = -b_i$, and $b_{ji} = b_i$, where $j \neq i$.

In order to obtain supersymmetric minima, it must be required that

$$3a \text{Re}(S) = b_i \text{Re}(U^i),$$

(27)

where

$$\text{Re}(S) = \frac{e^{-\phi_4}}{\sqrt{\chi^1 \chi^2 \chi^3}}, \quad \text{Re}(U^i) = e^{-\phi_4} \sqrt{\frac{\chi^j \chi^k}{\chi^i}}, \quad i \neq j \neq k,$$

(28)

with $S$ and $U^i$ being the dilaton and complex structure moduli respectively, and where $\phi_4$ is the four-dimensional dilaton. Then, we have the relations

$$b_1 = \frac{3a}{\chi_2 \chi_3}, \quad b_2 = \frac{3a}{\chi_1 \chi_3}, \quad b_3 = \frac{3a}{\chi_1 \chi_2}.$$  

(29)

In addition, there are consistency conditions which must be satisfied

$$3h_i a + h_0 b_i = 0, \quad \text{for} \quad i = 1, 2, 3,$$

(30)

so that we then have

$$h_1 = -\frac{h_0}{\chi_2 \chi_3}, \quad h_2 = -\frac{h_0}{\chi_1 \chi_3}, \quad h_3 = -\frac{h_0}{\chi_1 \chi_2}.$$  

(31)

Thus, the RR tadpole conditions can be written in a simplified form as

$$\sum N_a n_1 a n_2 a n_3 + \frac{1}{2} (mh_0 + 3aq) = 16 \eta_{\Omega R},$$

(32)

$$\sum N_a m_1 a m_2 a m_3 - \frac{1}{2 \chi_2 \chi_3} (mh_0 + 3aq) = -16 \eta_{\Omega R \theta},$$

$$\sum N_a m_1 a m_2 a m_3 - \frac{1}{2 \chi_1 \chi_3} (mh_0 + 3aq) = -16 \eta_{\Omega R \omega},$$

$$\sum N_a n_1 a m_2 a m_3 - \frac{1}{2 \chi_1 \chi_2} (mh_0 + 3aq) = -16 \eta_{\Omega R \theta}.$$  

Since $(mh_0 + 3aq)$ can be either positive or negative, the supergravity fluxes can contribute either positive or negative D6-brane charge. Therefore, since we may also have an odd
number of $O6^{++}$ planes as well as hidden sector branes, the RR-tadpole conditions are somewhat relaxed. However, we are still constrained by the requirement of torsion charge cancellation. Finally, it can be shown \[30\] that if Eqs. (21), (27), and (30) are satisfied, then the conditions for the Freed-Witten anomalies to be cancelled,

$$h_0\tilde{A}_a + h_1\tilde{B}_a + h_2\tilde{C}_a + h_3\tilde{D}_a = 0 \quad (33)$$

are automatically satisfied.

In order to ensure torsion charge cancellation, we must satisfy

$$\sum_a N_a n^i_a (\epsilon^{i}_{a,kl} - \eta_{iR}\eta_{iR}\epsilon^{i}_{a,kl}), \quad (34)$$

$$\sum_a N_a m^i_a (\epsilon^{i}_{a,kl} + \eta_{iR}\eta_{iR}\epsilon^{i}_{a,kl}), \quad (35)$$

where the sums are over each fixed point $[\tilde{e}^{i}_{U}]$. Clearly, these conditions are non-trivial to be satisfied.

D. The Green-Schwarz Mechanism

Although the total non-Abelian anomalies cancel automatically when the RR-tadpole conditions are satisfied, additional mixed anomalies like the mixed gravitational anomalies which generate massive fields are not trivially zero \[28\]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. Integrating the G-S couplings of the untwisted RR forms to the $U(1)$ field strength $F_a$ over the untwisted cycles of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orientifold, we find

$$\int_{D_6\text{untw}} C_5 \wedge \text{tr}F_a \sim N_a \sum_i r_{ai} \int_{M_4} B^i_2 \wedge \text{tr}F_a, \quad (36)$$

where

$$B^i_2 = \int_{[\Sigma_i]} C_5, \quad [\Pi_a] = \sum_{i=1}^{b_3} r_{ai}[\Sigma_i], \quad (37)$$

and $[\Sigma_i]$ is the basis of homology 3-cycles, $b_3 = 8$. Under orientifold action only half survive. In other words, $\{r_{ai}\} = \{\tilde{B}_a, \tilde{C}_a, \tilde{D}_a, \tilde{A}_a\}$ in this definition. Thus the couplings of the four untwisted RR forms $B^i_2$ to the $U(1)$ field strength $F_a$ are \[31\]

$$N_a \tilde{B}_a \int_{M_4} B^1_2 \wedge \text{tr}F_a, \quad N_a \tilde{C}_a \int_{M_4} B^2_2 \wedge \text{tr}F_a,$$

$$N_a \tilde{D}_a \int_{M_4} B^3_2 \wedge \text{tr}F_a, \quad N_a \tilde{A}_a \int_{M_4} B^4_2 \wedge \text{tr}F_a. \quad (38)$$
Besides the contribution to G-S mechanism from untwisted 3-cycles, the contribution from the twisted cycles should be taken into account. As in the untwisted case we integrate the Chern-Simons coupling over the exceptional 3-cycles from the twisted sector. We choose the sizes of the 2-cycles on the topology of \( S^2 \) on the orbifold singularities to make the integrals on equal foot to those from the untwisted sector. Consider the twisted sector \( \theta \) as an example,

\[
\int_{D \theta} C_5 \wedge \text{tr} F_a \sim N_a \sum_{i,j \in S^2_{\theta}} \epsilon_{i,j} a \int_{M_4} B^{\theta ij}_2 \wedge \text{tr} F_a, \tag{39}
\]

where \( B^{\theta ij}_2 = \int_{[\alpha_{i,j},m]} C_5 \), with orientifold action taken again and with the choice of crosscap charges \( \eta_R = -\eta_{R\theta} = -\eta_{R\omega} = -\eta_{R\theta\omega} = -1 \). Although \( i,j \) can run through \( \{1-4\} \) for each of the four fixed points in each sector, these are constrained by the wrapping numbers from the untwisted sector so that only four possibilities remain. A similar argument may be applied for \( \omega \) and \( \theta \omega \) twisted sectors:

\[
\int_{D \omega} C_5 \wedge \text{tr} F_a \sim N_a \sum_{j,k \in S^2_{\omega}} \epsilon_{j,k} \int_{M_4} B^{\omega jk}_2 \wedge \text{tr} F_a. \tag{40}
\]

\[
\int_{D \theta \omega} C_5 \wedge \text{tr} F_a \sim N_a \sum_{i,k \in S^2_{\theta\omega}} \epsilon_{i,k} \int_{M_4} B^{\theta \omega ik}_2 \wedge \text{tr} F_a. \tag{41}
\]

In summary, there are twelve additional couplings of the Ramond-Ramond 2-forms \( B^i_2 \) to the \( U(1) \) field strength \( F_a \) from the twisted cycles, giving rise to massive \( U(1)'s \). However from the consistency condition of the \( \epsilon \)'s (see section 3.1) related to the discrete Wilson lines they may be dependent or degenerate. So even including the couplings from the untwisted sector we still have an opportunity to find a linear combination for a massless \( U(1) \) group. Let us write down these couplings of the twisted sector explicitly:

\[
N_a \epsilon_{a,ij} \int_{M_4} B^{\theta ij}_2 \wedge \text{tr} F_a, \quad N_a \epsilon^{\omega}_{a,jk} \int_{M_4} B^{\omega jk}_2 \wedge \text{tr} F_a, \quad N_a \epsilon^{\theta \omega}_{a,ik} \int_{M_4} B^{\theta \omega ik}_2 \wedge \text{tr} F_a. \tag{42}
\]

Checking the mixed cubic anomaly by introducing the dual field of \( B^i_2 \) in the diagram, we can find the contribution from both untwisted and twisted sectors having an intersection number form and which is cancelled by the RR-tadpole conditions mentioned. These couplings determine the linear combinations of \( U(1) \) gauge bosons that acquire string scale
masses via the G-S mechanism. Thus, in constructing MSSM-like models, we must ensure that the gauge boson of the hypercharge \( U(1)_Y \) group does not receive such a mass. In general, the hypercharge is a linear combination of the various \( U(1) \)s generated from each stack:
\[
U(1)_Y = \sum_a c_a U(1)_a.
\] (43)
The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand
\[
\sum_a c_a N_a \epsilon^\omega_{a,jk} m_a^1 = 0, \quad \sum_a c_a N_a \epsilon^{\theta \omega}_{a,ki} m_a^2 = 0, \quad \sum_a c_a N_a \epsilon^{\theta}_{a,ij} m_a^3 = 0,
\] (44)
for the twisted couplings as well as
\[
\sum_a c_a N_a \tilde{A}_a = 0, \quad \sum_a c_a N_a \tilde{B}_a = 0, \quad \sum_a c_a N_a \tilde{C}_a = 0, \quad \sum_a c_a N_a \tilde{D}_a = 0,
\] (45)
for the untwisted.

III. MODEL BUILDING

It is well-known that intersecting D-brane models with a Pati-Salam gauge group are the only models where it is possible to have all Yukawa couplings for quarks and leptons present at the stringy tree-level. In the case of \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2') \) where all D-branes are wrapping rigid cycles, Pati-Salam models are also favored due to both the twisted charge cancellation conditions as well as the K-theory consistency conditions.

D-brane instantons have also been greatly studied in recent years and may play an important role in generating nonperturbative contributions to the superpotential. In particular, they may in principle generate couplings which are perturbatively forbidden by global symmetries which have their origin in \( U(1) \) gauge factors which become massive via a generalized Green-Schwarz mechanism. In order to have the proper zero-mode structure, the Euclidean 2-branes giving rise to such instantons must wrap rigid-cycles. The only known toroidal orientifold where rigid cycles are available is none other than \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2') \).

Although the potential to generate perturbatively forbidden superpotential couplings via D-brane instanton effects is exciting, it is in fact non-trivial to satisfy all consistency conditions required for them to be incorporated. Besides the aforementioned restriction
that the E2-branes wrap rigid cycles, it is also necessary for the cycles wrapped by the E2-brane to be invariant under the orientifold projection which strongly restricts the possible cycles in a way which depends on the choice of cross-cap charges. In addition, the E2-brane must have the correct intersection numbers with the relevant D-branes in order to generate a potential coupling, and may not intersect any other D-branes. Clearly, this is a very difficult condition to satisfy. For the models we will construct, we will attempt to cancel the twisted charges by using stacks of branes wrapping cycles whose bulk component is invariant under the orientifold action. Our motivation in doing this is to try to make it less likely that any E2 brane that we may consider in the model will have additional intersections with these stacks, a necessary requirement to generate required couplings such as a $\mu$-term or a neutrino Majorana mass term.

A. A Three-family Pati-Salam Model

A simple way to cancel twisted tadpoles is to construct models where the stacks of branes giving rise to the observable sector all wrap bulk cycles which are homologically the same, but which differ in their twisted cycles, such as the model considered in [32]. Models with a Pati-Salam gauge group are particularly well-suited for this type of construction. Besides making it easier to cancel twisted tadpoles and satisfy K-theory constraints, the gauge couplings are automatically unified with a canonical normalization in such types of models. However, such constructions tend to result in four-family models.

A slight variation on this idea is to have stacks of branes which are not technically the same homologically, but which are related by some interchange symmetry. The wrapping numbers and twisted charge assignments for a model of this type are shown in Table II, where we have made the choice of discrete torsion $\eta_{\Omega R} = -\eta_{\Omega R\theta} = -\eta_{\Omega R\omega} = -\eta_{\Omega R\theta\omega} = -1$.

Stacks $a$, $b$, and $c$ comprise the ‘observable’ sector, which results in a three-family model with Pati-Salam gauge group. Interestingly, the bulk intersection numbers between stack $a$ and stacks $b$ and $c$ respectively are zero, which would result in a strictly non-chiral spectrum if these stacks were wrapping bulk cycles. However, one-half of each vector pair is projected out by the twisted actions, resulting in a net chiral spectrum.

The additional stacks $\alpha_i$, $i = 1-3$ wrapping rigid cycles are present in order to satisfy the twisted tadpole conditions, while the stacks $1-3$ wrapping bulk cycles are present in order
TABLE II: A set of D6-brane configurations for a three-family Pati-Salam model in Type IIA on the $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orientifold, where the D6-branes are wrapping rigid cycles. This configuration preserves $\mathcal{N} = 1$ supersymmetry for $\chi_1 = 1$, $\chi_2 = 2$, and $\chi_3 = 1$. The bulk tadpole conditions Eqn. [32] are satisfied for this model by choosing $(m h_0 + 3 a q) = -64$, and all twisted tadpoles are cancelled.

| $N$ | stack | $(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$ | $(\beta, \lambda, \psi)$ | $(\delta_1, \delta_2, \delta_3)$ |
|-----|-------|-----------------------------------------------|--------------------------|-----------------------------|
| 4   | a     | $(-1, -1) \times (0, 1) \times (1, 2)$       | $(1, 1, 1)$              | $(1, 1, 1)$                 |
| 2   | b     | $(1, -2) \times (0, -1) \times (-1, 1)$      | $(1, 1, 1)$              | $(1, 1, 1)$                 |
| 4   | c     | $(-1, 2) \times (0, -1) \times (1, -1)$      | $(1, 1, 1)$              | $(1, 1, 1)$                 |
| 4   | $\alpha_1$ | $(1, 1) \times (0, 1) \times (-1, -2)$       | $(1, -1, 1)$            | $(1, 1, 1)$                 |
| 4   | $\alpha_2$ | $(0, 1) \times (0, 1) \times (-1, 0)$       | $(1, -1, -1)$           | $(1, 1, 1)$                 |
| 4   | $\alpha_3$ | $(0, 1) \times (0, 1) \times (-1, 0)$       | $(1, 1, -1)$            | $(1, 1, 1)$                 |
| 4   | $\beta_1$ | $(1, -1) \times (1, 0) \times (1, 1)$       | bulk                    | bulk                        |
| 6   | $\beta_2$ | $(1, 0) \times (2, -1) \times (1, 1)$       | bulk                    | bulk                        |
| 22  | 1     | $(1, 0) \times (0, -1) \times (1, 1)$       | bulk                    | bulk                        |
| 76  | 2     | $(0, -1) \times (1, 0) \times (0, 1)$       | bulk                    | bulk                        |
| 24  | 3     | $(0, -1) \times (0, 1) \times (1, 0)$       | bulk                    | bulk                        |

to satisfy the untwisted tadpole conditions. In order to preserve $\mathcal{N} = 1$ supersymmetry with these sets of stacks, we only need to require that the structure parameters satisfy $x_a = 2 x_c$. The parameters $x_b$ and $x_d$ are completely arbitrary due symmetries relating the stacks. In order to also fix $x_b$ and $x_c$ in terms of $x_a$, let us also introduce the stacks labeled $\beta_1$ and $\beta_2$ into the model. We should emphasize that there is some freedom in choosing these stacks so that the particular configuration that we have chosen is just one possible solution. With these additions, we must set $x_a = x_b = 2 x_c = x_d$ in order to preserve supersymmetry, from which we have

$$\chi_1 = 1/\sqrt{2}, \quad \chi_2 = \sqrt{2}, \quad \chi_3 = 1/\sqrt{2}.$$  \hspace{1cm} (46)

With this configuration, the tadpole conditions may be satisfied by setting

$$m h_0 + 3 a q = -64.$$  \hspace{1cm} (47)
TABLE III: Intersection numbers for the Pati-Salam model with the D6-brane configurations shown in Table II.

| N  | n_S | n_A | b  | b' | c  | c' | α_1 | α'_1 | α_2 | α'_2 | α_3 | α'_3 | β_1 | β'_1 | β_2 | β'_2 | 1 | 2 | 3 |
|----|-----|-----|----|----|----|----|------|------|------|------|------|------|-----|-----|-----|-----|---|---|---|
| a  | 4   | 0   | 6  | 0  | -3 | 0  | 2   | 0    | 1    | 0   | -2  | 0    | 2   | 0   | 2   | -6  | 0 | 0 | 0 |
| b  | 2   | 0   | -6 | -6 | 3  | 0  | 0   | 0    | 1    | 0   | -2  | 0    | -8  | 0   | 0   | 0   | 0 | 0 | 0 |
| c  | 2   | 0   | -6 | -6 | -3 | 0  | 0   | 0    | -1   | 0   | -2  | -8   | 0   | 0   | 0   | 0   | 0 | 0 | 0 |
| α_1| 4   | 0   | 6  | -6 | -1 | 2  | 0   | 0    | 0    | -1  | -1  | -2   | -2  | 0   | 0   | 0   | 0 | 0 | 0 |
| α_2| 4   | 0   | -6 | -6 | -3 | 0  | -1  | -1   | -1   | -2  | -2  | 0   | 0   | 0   | 0   | 0   | 0 | 0 | 0 |
| α_3| 4   | 0   | -6 | -6 | -3 | 0  | -1  | -1   | -1   | -2  | -2  | 0   | 0   | 0   | 0   | 0   | 0 | 0 | 0 |
| β_1| 4   | 0   | 0  | 0  |    |    |     |     |     |     |     |     |     |     |     |     | 1 | 2 | 3 |
| β_2| 6   | 0   | 0  | 0  |    |    |     |     |     |     |     |     |     |     |     |     | 0 | 0 | 0 |
| 1  | 22  | 0   | 0  | 0  |    |    |     |     |     |     |     |     |     |     |     | 0 | 0 | 0 |
| 2  | 76  | 0   | 0  | 0  |    |    |     |     |     |     |     |     |     |     |     | 0 | 0 | 0 |
| 3  | 24  | 0   | 0  | 0  |    |    |     |     |     |     |     |     |     |     |     | 0 | 0 | 0 |

\[ \chi_1 = 1/\sqrt{2}, \ \chi_2 = \sqrt{2}, \ \chi_3 = 1/\sqrt{2} \]

\[ mh_0 + 3aq = -64 \]

The intersection numbers between the stacks and their images respectively are shown in Table III. The resulting matter spectrum is shown in Table IV. One unsatisfactory aspect of this model is that there exist states which are charged under both the observable and hidden sector gauge groups, a generic problem in these types of models. In an ideal model, these two sectors should either be completely sequestered from one another or all states charged under both sectors should become heavy enough to evade experimental constraints. However, it is clear that there are actually many possible hidden sectors which may be compatible with the observable sector of this model. In fact, it is likely that there is a 'landscape' of hidden sectors, whereas that shown in Table II is but one solution. It is quite possible that there is at least one hidden sector somewhere in this landscape that could satisfy all phenomenological constraints. Indeed, it is possible to take the rank of the hidden sector groups to be very large and at the same time turning on a large amount of flux in such a way as to ensure both twisted and untwisted tadpole cancellation. We may eliminate the adjoint fields associated with these stacks by requiring them to wrap rigid cycles so that these gauge groups will...
have large and negative beta functions. The exotic matter charged under both observable and hidden sector gauge groups can then become quite massive since these groups can be made to become confining at high energy scales.

In order for the gauge boson of a $U(1)$ factor to remain massless, it must satisfy the Green-Schwarz anomaly cancellation conditions Eqs. (45) and (44). For this Pati-Salam model, it turns out that none of the $U(1)$ groups remain massless, and the effective gauge symmetry is

$$SU(4)_C \times SU(2)_R \times SU(2)_L \times SU(4)^4 \times SU(6) \times \prod_{i=1}^{3} Usp(N_i).$$

(48)

These $U(1)$ groups remain as global symmetries which may perturbatively forbid desirable couplings. However, these global symmetries may be broken by D-brane instanton effects.

Although the adjoint fields have been eliminated by splitting the bulk D-branes into their fractional constituents, light non-chiral matter in the bifundamental representation may still appear between pairs of fractional branes [21]. These non-chiral states smoothly connect the configuration of fractional D-branes to one consisting of non-rigid D-branes. In the present case, stacks $b$ and $c$ are wrapping bulk cycles which are homologically identical, but differ in their twisted cycles. Thus, the required states to play the role of the Higgs fields, $H_u^i, H_d^i$ are present in $bc$ sector in the form of non-chiral matter. The Higgs states arising from this sector were also present in the model discussed in [32]. Although the Higgs states required to give masses and quarks and leptons may arise in the non-chiral sector, there are additional Higgs-like states, $H_1^i, H_2^i$ in the $b'c$ sector. Although these states are charged under global $U(1)$ factors which would forbid the Yukawa couplings involving these states, it is possible that these couplings may be induced via D-brane instanton effects which can violate such global $U(1)$ symmetries.

**B. D-brane Instanton Induced Superpotential Couplings**

Non-perturbative D-brane instanton induced superpotential couplings have been receiving a great deal of attention of late. In particular, superpotential couplings such as a Majorana mass term for right-handed neutrinos as well as the $\mu$-term for the Higgs states are typically forbidden perturbatively due to selection rules which arise from global symmetries. These global symmetries arise due to the $U(1)$ gauge groups associated which each stack of D-branes
TABLE IV: The chiral and vector-like superfields, and their quantum numbers under the gauge symmetry $SU(4)_C \times SU(2)_R \times SU(2)_L \times SU(4)^4 \times SU(6) \times \prod_{i=1}^{3} Usp(N_i)$.

|   | Quantum Number | $Q_4$ | $Q_{2R}$ | $Q_{2L}$ | Field                      |
|---|---------------|-------|---------|---------|----------------------------|
| $ab$ | $3 \times (1, 2, 1, 1, 1, 1, 1, 1, 1, 1)$ | -1     | 1       | 0       | $F_R(Q_R, L_R)$             |
| $ac$ | $3 \times (4, 1, \overline{3}, 1, 1, 1, 1, 1, 1, 1)$ | 1       | 0       | -1      | $F_L(Q_L, L_L)$             |
| $a_A$ | $6 \times (6, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ | 2       | 0       | 0       | $D_1(D^c_1, D_1)$          |
| $b_A$ | $6 \times (1, \overline{3}, 1, 1, 1, 1, 1, 1, 1, 1)$ | 0       | -2      | 0       | $S^i_R$                    |
| $c_A$ | $6 \times (1, 1, \overline{3}, 1, 1, 1, 1, 1, 1, 1)$ | 0       | 0       | -2      | $S^i_L$                    |
| $bc$ | $6 \times (1, \overline{3}, 2, 1, 1, 1, 1, 1, 1, 1)$ | 0       | -1      | 1       | $H^i_u, H^i_d$              |
|     | $6 \times (1, 2, \overline{3}, 1, 1, 1, 1, 1, 1, 1)$ | 0       | 1       | -1      | $\overline{T}^i_u, \overline{T}^i_d$ |
| $b'c$ | $6 \times (1, 2, 2, 1, 1, 1, 1, 1, 1, 1)$ | 0       | 1       | 1       | $H_1(H^i_1, H^i_2)$        |
| $\alpha_{1A}$ | $6 \times (1, 1, 1, 6, 1, 1, 1, 1, 1, 1)$ | 0       | 0       | 0       | -                         |
| $a'\alpha_1$ | $2 \times (1, 1, 1, \overline{3}, 1, 1, 1, 1, 1, 1)$ | -1      | 0       | 0       | -                         |
| $a'\alpha_2$ | $1 \times (\overline{4}, 1, 1, 1, \overline{3}, 1, 1, 1, 1, 1)$ | -1      | 0       | 0       | -                         |
| $a'\alpha_3$ | $2 \times (\overline{4}, 1, 1, 1, 1, \overline{3}, 1, 1, 1, 1)$ | -1      | 0       | 0       | -                         |
| $ba_1$ | $3 \times (1, \overline{3}, 1, 4, 1, 1, 1, 1, 1, 1)$ | 0       | -1      | 0       | -                         |
| $ba_3$ | $1 \times (1, \overline{3}, 1, 1, 1, 4, 1, 1, 1, 1)$ | 0       | -1      | 0       | -                         |
| $ca_1$ | $1 \times (1, 1, 1, \overline{4}, 1, 1, 1, 1, 1, 1)$ | 0       | 0       | 1       | -                         |
| $ca_3$ | $1 \times (1, 1, 1, 2, 1, 1, \overline{4}, 1, 1, 1, 1)$ | 0       | 0       | 1       | -                         |
| $a_1\alpha_2$ | $1 \times (1, 1, 1, 1, 4, 1, 1, 1, 1, 1)$ | 0       | 0       | 0       | -                         |
| $a'\alpha_2$ | $1 \times (1, 1, 1, 1, \overline{4}, 1, 1, 1, 1, 1)$ | 0       | 0       | 0       | -                         |

whose gauge bosons pick up string-scale masses via a generalized Green-Schwarz mechanism. Under suitable conditions, Euclidean D2-brane (E2) instantons may break these global $U(1)$ symmetries and generate $U(1)$ violating interactions.

An E2-brane wrapping a cycle $\Xi$ which intersects a stack of D6-branes $a$ wrapping a cycle $\Pi_a$ will result in zero-modes present at the intersection, which results in the E2-brane being charged under the $U(1)_a$ gauge group,

$$Q_a = N_a \Xi \circ (\Pi_a - \Pi'_a) .$$  \hspace{1cm} (49)
TABLE V: Cycles wrapped by an E2 instanton.

| E2 | N (n^1, t^1) × (n^2, t^2) × (n^3, t^3) | (β, λ, ψ) | (δ_1, δ_2, δ_3) |
|---|---|---|---|
| E2_1 | (−1, 0) × (−1, 0) × (1, 0) | (1, 1, 1) | (0, 0, 0) |

TABLE VI: Intersection numbers of E2 branes with the D6-brane configurations shown in Table II.

| E2 | a | b | c | α_1 | α_2 | α_3 | β_1 | β_1' | β_2 | β_2' | 1 | 2 | 3 |
|---|---|---|---|-----|-----|-----|-----|-----|-----|-----|---|---|---|
| E2_1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Perturbatively forbidden superpotential couplings may by then be generated by an E2-instanton provided it has a suitable zero-mode structure. In particular, the cycle Ξ must be rigid to ensure that there exist no reparametrization zero modes. In addition, the cycle wrapped by the instanton must not have an intersection with its own image, Ξ ∩ Ξ' = 0, or equivalently the cycle wrapped by the instanton must be invariant under the orientifold projection and carry gauge group O(1), in which case Eq. (49) is modified to

\[ Q_a = -N_a \Xi \circ \Pi_a . \]  

(50)

Let us consider the E2-branes wrapping the fractional cycles shown in Table V with the intersection numbers shown in Table VI. With the zero mode structure of E2_1, we may induce a Yukawa term in the superpotential of the form

\[ W_Y = cF_L F_R H_1 , \]  

(51)

which can provide masses for the quarks and leptons. Thus, it may be possible to obtain the mass hierarchies for quarks and leptons from these instanton induced couplings. We defer a deeper analysis of this question to future work.

IV. A THREE-GENERATION MSSM-LIKE MODEL

The Pati-Salam model of the previous section may be broken to the Standard Model explicitly by splitting the stacks on one torus such that

\[ a \rightarrow a_1 + a_2 \quad \text{and} \quad b \rightarrow b_1 + b_2 \]  

(52)
where \( N_{a1} = 3, N_{a2} = 1, N_{b1} = 1 \) and \( N_{b2} = 1 \) as shown in Table VII. For D6-branes wrapping non-rigid cycles, this would be accomplished by giving a VEV to an adjoint scalar associated with each stack. However, for the present model, the adjoint scalars are not present since all D6-branes are wrapping rigid cycles. Thus, to split the stacks, we simply require that stacks \( a1 \) and \( a2 \), as well as \( c1 \) and \( c2 \), must pass through different fixed points on the third torus recalling from before that for a given set of wrapping numbers, there are two possible choices for the fixed points in which a particular one-cycle will pass. These stacks will then wrap different fractional cycles, thus breaking the gauge symmetry. Since the D6-brane configuration has been changed slightly from the previous Pati-Salam model, the twisted charge cancellation will also be modified. Indeed, we must add an additional stack(s) in order to cancel the twisted tadpoles, as can be seen in Table VII where stacks \( d_i, i = 1 - 4 \) and \( e_j, j = 1 - 8 \) have been added to the previous Pati-Salam model. With this configuration of D6-branes, all consistency conditions are satisfied and \( \mathcal{N} = 1 \) supersymmetry is preserved. In general, one would not expect to have the same intersection numbers for two stacks which have been split in this way. Remarkably, for the present model, the number of Standard Model fermions remains three even after splitting the stacks. The resulting spectrum is then that of a three-family MSSM model with an extended gauge group where the hypercharge \( U(1)_Y \) is defined by Eq. (55). The intersection numbers are shown in Table VIII and the corresponding MSSM matter spectrum is shown in Table IX. For brevity, we do not show the exotic states charged under both observable and hidden sector gauge groups.

The gauge bosons for almost all \( U(1) \) groups will become massive as can be seen from the GS cancellation conditions Eqs. (44) and (45). After splitting the stacks, the \( U(1) \) factors

\[
U(1)_{B-L} = \frac{1}{6}(U(1)_{a1} - 3U(1)_{a2}),
\]

\[
U(1)_{I3R} = \frac{1}{2}(U(1)_{b1} - U(1)_{b2}),
\]

survive the GS conditions Eq. (45). If the relevant D6-branes were wrapping bulk cycles, this would guarantee that the gauge bosons associated with these groups would remain massless. However, since stack \( a1 \) and \( a2 \) do not pass through the same fixed points on the third torus, \( U(1)_{B-L} \) is anomalous due to the additional constraints Eq. (44). Similar considerations apply to \( U(1)_{I3R} \). For phenomenological reasons, this might actually desirable. In particular, it is not possible to generate couplings such as a Majorana neutrino mass term \( W_M = \lambda_i N N \) via D-brane instantons if \( U(1)_{B-L} \) remains gauged at the string scale.
TABLE VII: A set of D6-brane configurations for a three-generation MSSM-like model in Type IIA on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$ orientifold, where the D6-branes are wrapping rigid cycles. This configuration preserves $\mathcal{N} = 1$ supersymmetry for $\chi_1 = 1$, $\chi_2 = 2$, and $\chi_3 = 1$. The bulk tadpole conditions Eq. (32) are satisfied for this model by choosing $(m h_0 + 3 a q) = -256$, and all twisted tadpoles are cancelled.

| $N$ | stack | $(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$ | $(\beta, \lambda, \psi)$ | $(\delta_1, \delta_2, \delta_3)$ |
|-----|-------|---------------------------------|---------------------|---------------------|
| 3   | a1    | $(-1, -1) \times (0, 1) \times (1, 2)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 1   | a2    | $(-1, -1) \times (0, 1) \times (1, 2)$ | $(1, 1, 1)$ | $(1, 1, 0)$ |
| 1   | b1    | $(1, -2) \times (0, -1) \times (-1, 1)$ | $(1, 1, 1)$ | $(1, 1, 0)$ |
| 1   | b2    | $(1, -2) \times (0, -1) \times (-1, 1)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 2   | c     | $(-1, 2) \times (0, -1) \times (1, 1)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 9   | d1    | $(1, 1) \times (0, -1) \times (1, 2)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 3   | d2    | $(1, 1) \times (0, -1) \times (1, 2)$ | $(1, 1, 1)$ | $(1, 1, 0)$ |
| 3   | d3    | $(1, 2) \times (0, 1) \times (-1, 1)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 3   | d4    | $(1, 2) \times (0, 1) \times (-1, 1)$ | $(1, 1, 1)$ | $(1, 1, 0)$ |
| 3   | e1    | $(1, 0) \times (0, -1) \times (0, 1)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 1   | e2    | $(1, 0) \times (0, -1) \times (0, 1)$ | $(1, 1, 1)$ | $(1, 1, 0)$ |
| 7   | e3    | $(-1, 0) \times (0, 1) \times (0, 1)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 7   | e4    | $(-1, 0) \times (0, 1) \times (0, 1)$ | $(1, 1, 1)$ | $(1, 1, 0)$ |
| 5   | e5    | $(0, 1) \times (0, 1) \times (-1, 0)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 5   | e6    | $(0, 1) \times (0, 1) \times (-1, 0)$ | $(1, 1, 1)$ | $(1, 1, 1)$ |
| 5   | e7    | $(0, 1) \times (0, 1) \times (-1, 0)$ | $(1, 1, 1)$ | $(0, 1, 1)$ |
| 5   | e8    | $(0, -1) \times (0, 1) \times (1, 0)$ | $(1, 1, 1)$ | $(0, 1, 1)$ |
| 4   | $\beta_1$ | $(1, -1) \times (1, 0) \times (1, 1)$ | bulk | bulk |
| 6   | $\beta_2$ | $(1, 0) \times (2, -1) \times (1, 1)$ | bulk | bulk |
| 96  | 1     | $(1, 0) \times (1, 0) \times (1, 0)$ | bulk | bulk |
| 14  | 1     | $(1, 0) \times (0, -1) \times (1, 1)$ | bulk | bulk |
| 76  | 2     | $(0, -1) \times (1, 0) \times (0, 1)$ | bulk | bulk |
| 24  | 3     | $(0, -1) \times (0, 1) \times (1, 0)$ | bulk | bulk |
TABLE VIII: Intersection numbers for the MSSM sector of the model with the D6-brane configurations shown in Table VII

|   | $SU(3)_C \times SU(2)_L \times U(1)_Y$ |
|---|-------------------------------------|
|   | $N$ | $n_S$ | $n_A$ | $a_2$ | $a_2'$ | $b_1$ | $b_1'$ | $b_2$ | $b_2'$ | $c$ | $c'$ |
| $a_1$ | 3   | 0     | 6     | 0     | -4     | 3     | 0     | 3     | 0     | -3  | 0   |
| $a_2$ | 1   | 0     | 6     | -     | -     | 3     | 0     | 3     | 0     | -3  | 0   |
| $b_1$ | 1   | 0     | -6    | -     | -     | 2     | 0     | 0     | -6    |     |     |
| $b_2$ | 1   | 0     | -6    | -     | -     | -     | -     | -     | -     | 0   | -2  |
| $c$   | 2   | 0     | -6    | -     | -     | -     | -     | -     | -     | -   | -   |

Although it may be desirable for $U(1)_{B-L}$ to become massive, care must be taken to ensure that the SM hypercharge, given by

$$U(1)_{Y_0} = \frac{1}{6} (U(1)_{a_1} - 3U(1)_{a_2} + 3U(1)_{b_1} - 3U(1)_{b_2})$$

(54)
do not also. Although this linear combination for the hypercharge will obviously satisfy Eq. (45), one may easily see that $U(1)_Y$ will remain massless only for very special conditions due to the additional constraints. Indeed, Eq. (54) does in fact become massive for the present model. In general, the only chance for the combination given by Eq. (54) to remain massless is if the wrapping numbers and twisted charge assignments contrive in such a way that the GS conditions can be satisfied. Alternatively, the definition of the hypercharge may be extended to include $U(1)$ groups from other stacks of branes. Ideal candidates for such additional stacks are those wrapping bulk cycles which are invariant under the orientifold action since these will automatically satisfy Eq. (45). Clearly the complete fractional cycles of such stacks must not be invariant under the orientifold action so that a stack of $N$ of these D6-branes has a gauge group $U(N) = SU(N) \times U(1)$ in its worldvolume. After splitting the stacks SM hypercharge may remain massless provided that we redefine the SM hypercharge to be

$$U(1)_Y = \frac{1}{6} (U(1)_{a_1} - 3U(1)_{a_2} + 3U(1)_{b_1} - 3U(1)_{b_2} +$$

$$\frac{1}{3} U(1)_{d_1} - U(1)_{d_2} - U(1)_{d_3} + U(1)_{d_4}).$$

(55)
TABLE IX: The chiral and vector-like superfields for states charged MSSM gauge groups, and their quantum numbers under the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(4)^4 \times SU(6) \times \prod_{i=1}^{3} Usp(N_i)$.

| Quantum Number | $Q_{a1}$ | $Q_{a2}$ | $Q_{b1}$ | $Q_{b2}$ | $Q_Y$ | Field |
|----------------|---------|---------|---------|---------|-------|-------|
| $a1b1$         | 3       | 0       | 0       | 0       | 0     | $1/3$ | $D^c$ |
| $a1b2$         | 3       | 0       | 0       | 1       | $-2/3$| $U^c$ |
| $a1c$          | 3       | 0       | 0       | 0       | 1/6   | $Q_L$ |
| $a2b1$         | 3       | 1       | 0       | 0       | 0     | 1     | $E^c$ |
| $a2b2$         | 3       | 1       | 0       | 0       | 0     | $N$   |
| $a2c$          | 3       | 1       | 0       | 0       | -1/2  | $L$   |
| $b1c$          | 6       | 0       | -1      | 0       | 1     | -1/2  | $H^i_d$|
| $b1'c$         | 6       | 0       | 0       | 0       | -1/2  | $\overline{H^i_d}$|
| $b2c$          | 6       | 0       | 0       | 1       | 1     | 1/2   | $H^i_u$|
| $b2'c$         | 6       | 0       | 0       | 0       | 1     | -1/2  | $\overline{H^i_u}$|

Obviously, this is not especially desirable for phenomenological reasons. However, for the present we consider this as a possible solution and defer further consideration of this issue.

A. The Question of Gauge Coupling Unification

The MSSM predicts the unification of the three gauge couplings at an energy $\sim 2.4 \times 10^{16}$ GeV. In intersecting D-brane models, the gauge groups arise from different stacks of branes, and so they will not generally have the same volume in the compactified space. Thus, the gauge couplings are not automatically unified, in contrast to heterotic models. For this reason, MSSM-like models where the gauge couplings happen to be unified appear to be a coincidence; there is no apparent deep reason for them to be unified is these types of models, it just works out this way. On the other hand, Pati-Salam models where the D-branes
of the observable sector all wrap bulk cycles which are homologically the same automatically results in tree-level gauge coupling unification at the string scale. For this class of models, the apparent unification seems less coincidental since it is possible to understand from where it emerges.

The holomorphic gauge kinetic function for a D6-brane wrapping a calibrated three-cyce is given by [14]

$$f_P = \frac{1}{2\pi f^3_s} \left[ e^{-\phi} \int_{\Pi_P} \text{Re}(e^{-i\theta_P} \Omega_3) - i \int_{\Pi_P} C_3 \right].$$

(56)

In terms of the three-cycle wrapped by the stack of branes, we have

$$\int_{\Pi_a} \Omega_3 = \frac{1}{4} \prod_{i=1}^{3} (n^i a_1 m^i a_2 + i m^i a_3).$$

(57)

from which it follows that

$$f_P = \frac{1}{4\kappa_P} (n^1 a_1 n^2 a_2 n^3 a_3 - n^1 a_2 n^2 a_1 n^3 a_3 - n^1 a_3 n^2 a_2 n^3 a_1),$$

(58)

where $\kappa_P = 1$ for $SU(N_P)$ and $\kappa_P = 2$ for $USp(2N_P)$ or $SO(2N_P)$ gauge groups and where we use the $s$ and $u$ moduli in the supergravity basis. In the string theory basis, we have the dilaton $S$, three Kähler moduli $T^i$, and three complex structure moduli $U^i$ [34]. These are related to the corresponding moduli in the supergravity basis by

$$\text{Re} \left( s \right) = \frac{e^{-\phi_4}}{2\pi} \left( \sqrt{\text{Im} U^1 \text{Im} U^2 \text{Im} U^3} \right) \left( \frac{\text{Im} U^k U^l}{|U^k U^l|^2} \right) \left( j, k, l \right) = (1, 2, 3)$$

$$\text{Re} \left( u^j \right) = \frac{e^{-\phi_4}}{2\pi} \left( \sqrt{\text{Im} U^j} \text{Im} U^k \text{Im} U^l \right) \left( \frac{U^k U^l}{U^j} \right)$$

$$\text{Re} \left( t^j \right) = \frac{i \alpha'}{T^j}$$

(59)

and $\phi_4$ is the four-dimensional dilaton.

The gauge coupling constant associated with a stack $P$ is given by

$$g_{D6_P}^{-2} = |\text{Re} (f_P)|.$$  

(60)

For the model under study the $SU(3)$ holomorphic gauge function is identified with stack $a1$ and the $SU(2)$ holomorphic gauge function with stack $c$. The $U(1)_Y$ holomorphic gauge function is then given by taking a linear combination of the holomorphic gauge functions from all the stacks. If we consider the definition of the hypercharge given by Eq. (54), then
we find that each of the stacks in the observable sector turns out to have the same gauge coupling. Thus, the couplings will be automatically unified at the string scale with canonical normalization on the hypercharge.

\[ g_s^2 = g_w^2 = \frac{5}{3} g_Y^2. \]  

(61)

However, we recall the combination given in Eq. (54) becomes massive due to the twisted Green-Schwarz conditions, and was redefined in Eq. (55) to include the additional stacks \( d_i, \ i = 1 - 4 \), thus we then actually have

\[ g_s^2 = g_w^2 = \left( \frac{5}{3} + \Delta \right) g_Y^2, \]  

(62)

where \( \Delta \) is an effective threshold correction which expresses the effect of the stacks \( d_i \) on the normalization of the hypercharge. For the present model, we have \( \Delta = 5/9 \) which disrupts the unification of \( g_s^2 \) and \( g_w^2 \) with \( g_Y^2 \). However, it may be possible to find an alternative combination of stacks which may be added to the hypercharge in such a way as to ameliorate this problem. Since we do not require the actual unification, only the apparent unification, we only need \( \Delta \) to be a small value. Although the model is a Pati-Salam ‘GUT’ at the string scale, the gauge unification need not occur in such models. It is desirable only because this seems to be what is observed; gauge unification appears to be a coincidence.

V. CONCLUSION

In this paper, we have constructed three-family Pati-Salam and MSSM-like models in AdS as Type IIA flux vacua on the \( T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2') \) orientifold. For each of these models, the D-branes are wrapping rigid cycles, which freezes the open-string moduli which correspond to the D-brane positions and Wilson lines. In intersecting D-brane models where the D-branes wrap only bulk cycles, there arises matter in the adjoint representation which results from unstabilized open-string moduli. As a result, there are light-scalars present in such models which are charged under the MSSM gauge groups. Besides being phenomenologically undesirable since such scalars are not experimentally observed, such fields can have a negative effect on the running of the gauge couplings. In particular, the asymptotic freedom of \( SU(3)_C \) can be destroyed, as well as the asymptotic freedom of hidden sector gauge groups. Typically, there is exotic matter present in intersecting D-brane models which are charged under both
the MSSM and hidden sector gauge groups, and one way of generating large masses for such states is if the hidden sectors groups become confining at some high scale which requires that these groups have negative $\beta$ functions. One of the effects of unstabilized open-string moduli is therefore to ruin this possibility.

For the MSSM-like model we considered, we found that the tree-level gauge couplings associated with $SU(3)_C$ and $SU(2)_L$ may be unified at the string scale. However, we found that in order for the hypercharge $U(1)_Y$ to remain massless, it is necessary to extend to the definition of the hypercharge to include $U(1)$ factors from other stacks of D6-branes. Besides being undesirable since this increases the likelihood that there will be extra matter charged under all three MSSM gauge group, it also has the effect of shifting the hypercharge away from a canonical normalization in order to be unified with the other two gauge groups. This shift, which is like an effective threshold correction, depends on the details of what is added to the hypercharge in order to keep it massless, in which there is some freedom.

Besides the reasons given above which motivate constructing models with frozen open-string moduli, it is also the case that the Yukawa couplings which must be present in the superpotential to generate masses for the quarks and leptons depend directly on the open-string moduli. Previously, for the specific three-generation intersecting D-brane model discussed in [17] and [18] it is possible to obtain correct masses and mixings for both the up and down-type quarks, as well as the tau lepton considering only the trilinear couplings. In addition, it is possible to obtain the correct electron and muon masses, in general, by considering contributions to the Yukawa couplings from four-point functions [35]. Despite the impressive successes of this model, the open-string moduli were not stabilized and so it was not possible to obtain unique calculations of the Yukawa couplings. Essentially, the open-string moduli VEVs were treated as free parameters. For the models we considered in this paper, the mass hierarchies can arise in principle from D-brane instanton induced Yukawa couplings. Since all of the D6-branes and the E2-brane associated with the instanton are wrapping rigid cycles, all of the open-string moduli are fixed and there is limited freedom to ‘tune’ the couplings to give the desired hierarchies. It would be very interesting to see if the observed mass hierarchies for quarks and leptons can be obtained for this model. We plan to explore this fully in future work.
VI. ACKNOWLEDGEMENTS

The work of C.M. is supported by the Mitchell-Heep Chair in High Energy Physics. The work of D.V. Nanopoulos and V.E. Mayes is supported by DOE grant DE-FG03-95-Er-40917. The work of T. Li was supported in part by the Cambridge-Mitchell Collaboration in Theoretical Cosmology and by the Natural Science Foundation of China under grant number 10821504.
[1] N. Arkani-Hamed, G. L. Kane, J. Thaler and L. T. Wang, JHEP 0608, 070 (2006) [arXiv:hep-ph/0512190].

[2] G. L. Kane, P. Kumar and J. Shao, J. Phys. G 34, 1993 (2007) [arXiv:hep-ph/0610038].

[3] G. L. Kane, P. Kumar and J. Shao, [arXiv:0709.4259 [hep-ph]].

[4] D. Feldman, Z. Liu and P. Nath, Phys. Rev. Lett. 99, 251802 (2007) [arXiv:0707.1873 [hep-ph]].

[5] D. Feldman, Z. Liu and P. Nath, JHEP 0804, 054 (2008) [arXiv:0802.4085 [hep-ph]].

[6] J. A. Maxin, V. E. Mayes and D. V. Nanopoulos, [arXiv:0809.3200 [hep-ph]].

[7] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B 194 (1987) 231; Phys. Lett. B 205 (1988) 459; Phys. Lett. B 208 (1988) 209 [Addendum-ibid. B 213 (1988) 562]; Phys. Lett. B 231 (1989) 65.

[8] V. Braun, Y. H. He, B. A. Ovrut and T. Pantev, JHEP 0605, 043 (2006) [arXiv:hep-th/0512177].

[9] V. Bouchard and R. Donagi, Phys. Lett. B 633, 783 (2006) [arXiv:hep-th/0512149].

[10] J. Polchinski and E. Witten, Nucl. Phys. B 460, 525 (1996) [arXiv:hep-th/9510169].

[11] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B 480, 265 (1996) [arXiv:hep-th/9606139].

[12] C. Bachas, [arXiv:hep-th/9503030].

[13] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, Ann. Rev. Nucl. Part. Sci. 55, 71 (2005) [arXiv:hep-th/0502005].

[14] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, [arXiv:hep-th/0610327].

[15] M. Cvetic, T. Li and T. Liu, Nucl. Phys. B 698, 163 (2004) [arXiv:hep-th/0403061].

[16] C. M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B 740, 79 (2006) [arXiv:hep-th/0601064].

[17] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Lett. B 665, 267 (2008) [arXiv:hep-th/0703280].

[18] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Rev. D 77, 125023 (2008) [arXiv:0711.0396 [hep-ph]].

[19] C. M. Chen, T. Li, Y. Liu and D. V. Nanopoulos, Phys. Lett. B 668, 63 (2008)
[20] D. Cremades, L. E. Ibanez and F. Marchesano, JHEP **0307**, 038 (2003) [arXiv:hep-th/0302105].

[21] R. Blumenhagen, M. Cvetic, F. Marchesano and G. Shiu, JHEP **0503**, 050 (2005) [arXiv:hep-th/0502095].

[22] R. Blumenhagen, M. Cvetic, F. Marchesano and G. Shiu, JHEP **0502**, 085 (2005) [arXiv:hep-th/0502085].

[23] L. E. Ibanez and A. M. Uranga, [arXiv:hep-th/0609213].

[24] R. Blumenhagen, M. Cvetic and T. Weigand, [arXiv:hep-th/0609191].

[25] B. Florea, S. Kachru, J. McGreevy and N. Saulina, JHEP **0705**, 024 (2007) [arXiv:hep-th/0610003].

[26] J. R. Ellis, J. L. Lopez and D. V. Nanopoulos, Phys. Lett. B **245**, 375 (1990).

[27] C. Beasley, J. J. Heckman and C. Vafa, JHEP **0901**, 059 (2009) [arXiv:0806.0102 [hep-th]].

[28] M. Cvetic, G. Shiu and A. M. Uranga, Phys. Rev. Lett. **87**, 201801 (2001) [arXiv:hep-th/0107143].

[29] G. Villadoro and F. Zwirner, Phys. Rev. Lett. **95**, 231602 (2005) [arXiv:hep-th/0508167].

[30] P. G. Camara, A. Font and L. E. Ibanez, JHEP **0509**, 013 (2005) [arXiv:hep-th/0506066].

[31] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, J. Math. Phys. **42**, 3103 (2001) [arXiv:hep-th/0011073]; JHEP **0102**, 047 (2001) [arXiv:hep-ph/0011132].

[32] C. M. Chen, V. E. Mayes and D. V. Nanopoulos, Phys. Lett. B **648**, 301 (2007) [arXiv:hep-th/0612087].

[33] R. Blumenhagen, M. Cvetic, D. Lust, R. Richter and T. Weigand, Phys. Rev. Lett. **100**, 061602 (2008) [arXiv:0707.1871 [hep-th]].

[34] D. Lust, P. Mayr, R. Richter and S. Stieberger, Nucl. Phys. B **696**, 205 (2004) [arXiv:hep-th/0404134].

[35] C. M. Chen, T. Li, V. E. Mayes and D. V. Nanopoulos, Phys. Rev. D **78**, 105015 (2008) [arXiv:0807.4216 [hep-th]].