Measures of the galaxy clustering

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Abstract. A brief introduction is given to some aspects of the statistical description of the luminous matter distribution. I review the features of the redshift surveys that arise in the statistical analysis of the galaxy clustering. Special topics include intensity functions, correlation functions, correlation integrals, multifractals and multiscaling.

1. Introduction

I am sure that, in spite of the title of the School, at the end of these two weeks, we shall have a less dark picture of the three-dimensional distribution of matter in the Universe. There are still a huge amount of unsolved problems regarding the origin and evolution of the observed large scale structure in the Universe. Although important developments have occurred during the last two decades, the task has revealed so elusive, that most of the students of this School will have interesting research projects on these topics in the following years.

The statistical study of the clustering patterns formed by the three-dimensional distribution of galaxies is one of the most important observational clues to learn about the physical processes that led to the large scale structure of the Universe. A detailed statistical description of the observed distribution of matter in the Universe is needed to confront theoretical predictions of models of structure formation, such as N-body simulations involving dark matter, against observations.

The aim of this lecture is to introduce some statistical aspects of the description of the clustering in the Universe. This introduction will be followed by more detailed lectures given by Drs. Borgani and Coles.

2. Surveys of galaxy redshifts

In the 1980s different groups of astronomers started systematic observational programs to construct a true three-dimensional fair sample of the Universe. The task consists in measuring the location in space of galaxies lying in the studied region of the sky. In addition to its angular position, we need to know the distance to each object. This is usually done by measuring the redshift $z$ in the spectrum of the galaxies, which is related to the line-of-sight recession velocity, $v_{\text{rec}} = cz$. The Hubble law permits to estimate the distance $d$ to each galaxy, as $v_{\text{rec}} \simeq v_H = H d$ (but see sec. 2.3), and therefore to have the three-dimensional map of the Universe.

We denote by $\{\vec{x}_i\}_{i=1}^N$ the position of the $N$ galaxies in a portion of the universe with volume $V$. Techniques of point fields statistics may be used to describe the
statistical features of the distribution of such galaxies. Some words of caution might be given before embarking on the different techniques of the statistical analysis. Catalogues of galaxies are not simple point samples as could be many of the planar point processes usually studied in the literature of spatial statistics (like the positions of trees in forests). To handle properly galaxy samples, we need to have in mind some of the characteristics of their construction.

2.1. Galaxy obscuration

The extragalactic optical light does not reach the Earth uniformly from all directions. The plane of the Milky Way, our own Galaxy, is filled with interstellar dust which absorbs most of the light coming from extragalactic sources. Therefore catalogues are incomplete below galactic latitudes $|b| < 20^\circ - 30^\circ$. This fact implies that the band of the sky corresponding to low galactic latitudes is usually not considered in the optical samples analyzed. The geometry of the three-dimensional regions often becomes irregular because of this and other observational constraints. E.g., the analysis of the CfA-I redshift survey is usually performed in the Northern Galactic hemisphere (with $b \geq 40^\circ$) and equatorial declination $\delta > 0^\circ$. Other kind of obscurations are related to the observational devices is the mask of the IRAS satellite. The QDOT-IRAS redshift survey covers 74 % of the sky after removal of the masked regions and the band corresponding to galactic latitude $|b| \leq 10^\circ$ (IRAS looks in the infrared and safely identifies galaxies closer to the Galactic plane than optically observed galaxies).

The brightness of the galaxies is also affected by the galactic absorption. This effect is usually modelled by the cosecant law in latitude $\Delta m = A \csc b$.

2.2. Brightness and apparent magnitude limit

Galaxies in a redshift survey have different intrinsic brightness. Most of the catalogues are built by fixing an apparent magnitude limit $m_{\text{lim}}$. Therefore galaxies with $m > m_{\text{lim}}$ are not seen by the telescope or are not considered because of observational strategies. An apparent magnitude-limited sample is therefore not uniform in space, as intrinsically faint objects are only seen if they are close enough to the Earth. To analyze this kind of flux-limited samples we can follow two strategies:

(i) One can extract volume-limited samples by fixing a value of the depth $D_{\text{max}}$ (in $h^{-1}$ Mpc, $h$ being the Hubble constant in units of 100 $\text{Mpc}^{-1} \text{ km s}^{-1}$) and keeping only galaxies brighter than

$$M_{\text{lim}} = m_{\text{lim}} - 25 - 5 \log(D_{\text{max}}).$$

For example, if the catalogue has an apparent magnitude limit $m_{\text{lim}} = 14.5$ and we consider as the maximum depth of the volume to be studied $D_{\text{max}} = 100 h^{-1}$ Mpc, only galaxies with absolute magnitude $M \leq -20.5 + 5 \log h$ will remain in the volume-limited sample. With this strategy, however, we loose a part of the information provided by the redshift survey. To avoid this problem we can follow the second strategy.

(ii) The second procedure is based on the knowledge of the selection function: $\varphi(x)$. The function $\varphi(x)$ gives an estimate of the probability that a galaxy more brilliant
than a given luminosity cutoff, at a distance \( x \) is included in the sample. If the sample is complete up to a distance \( R \), \( \varphi(x) = 1 \) for \( x \leq R \). For example, in Fig. 1, we show the selection function of one QDOT-IRAS subsample for galaxies with \( L \geq 10^{9.19} L_\odot \). With this luminosity limit, the sample is complete up to \( R = 40 h^{-1} \) Mpc \(^5\). Beyond this distance the selection function \( \varphi(x) \) falls down and attains values close to zero for \( R \approx 200 h^{-1} \) Mpc.

The selection function is derived from the luminosity function \( \phi(L) \). The luminosity function is defined by requiring that the mean number of galaxies, per unit volume, with luminosity in the range \( L \) to \( L + dL \), is \( \phi(L)dL \). The empirical luminosity function is often fitted by the analytical expression \(^6\)

\[
\phi(L)dL = \phi_* \left( \frac{L}{L_*} \right)^\alpha \exp \left( -\frac{L}{L_*} \right) d \left( \frac{L}{L_*} \right),
\]

(2)

where \( L_* \) and \( \alpha \) are the fitting parameters, while \( \phi_* \) is related with the number density of galaxies. The previous expression in terms of magnitudes is

\[
\phi(M)dM = A \phi_* \left( 10^{0.4(M_*-M)} \right)^{\alpha+1} \exp \left( -10^{0.4(M_*-M)} \right) dM.
\]

(3)

where \( A = \frac{2}{5} \ln(10) \). Therefore, the selection function is just the ratio

\[
\varphi(x) = \frac{\int_{-\infty}^{M(x)} \phi(M)dM}{\int_{-\infty}^{M_{\text{max}}} \phi(M)dM} = \frac{\Gamma(\alpha + 1, 10^{0.4(M_*-M(x))})}{\Gamma(\alpha + 1, 10^{0.4(M_*-M_{\text{max}}}))},
\]

(4)

where \( M(x) = m_{\text{lim}} - 25 - 5 \log(x) \), \( \Gamma \) being now the incomplete Gamma function, \( M_{\text{max}} = \max(M(x), M_{\text{com}}) \) and \( M_{\text{com}} \) is the absolute magnitude for which the catalogue is complete. The parameters of the luminosity function depend on the sample \(^7\). Typical values are \( \alpha = -1.1 \) and \( M_* = -19.3 + 5 \log h \).

Within this strategy, we can assign to each galaxy a weight \( w = 1/\varphi(x) \) depending on its distance \( x \) to us.

### 2.3. Redshift-space distortions

The observed recession velocity \( v_{\text{rec}} \) is not only due to the Hubble expansion. Other components have to be added to \( v_H \) to obtain \( v_{\text{pec}} \). Bulk flows or streaming motions on large scale or local velocities within clusters on small scales might not be negligible. The peculiar velocity is the velocity of a galaxy with respect to the Hubble flow. Let us indicate its component along the line-of-sight by \( v_{\text{pec}} \); then the observed recession velocity is

\[
v_{\text{rec}} = cz = H_0d + v_{\text{pec}},
\]

(5)

and therefore we have to distinguish between ‘redshift space’ and ‘real space’; the first one is artificially produced by setting each galaxy at the distance \( d \) obtained by considering \( v_{\text{pec}} = 0 \), and is therefore a distorted representation of the second one. The effect of this radial distortion is clearly illustrated when dense clusters of galaxies, almost spherical in real space, appear as structures elongated along the line of sight, in redshift space. These structures are known as ‘fingers of God’.
2.4. Segregation

The point field formed by the galaxies in the surveyed volume is clearly a marked point field, in the sense that qualitative marks like morphological type and quantitative marks like intrinsic luminosity, distinguish different objects within the same catalogue. The statistical properties of the spatial distributions of different kinds of objects can be different. In fact, it is well established that elliptical galaxies are more frequent in denser regions such as rich clusters, while spirals are more often found in low-density environments [8]. Statistical descriptors such as the two-point correlation function or multifractal measures might provide us with different results if they are applied on different categories of galaxies. Although less evident, it seems also established that a certain degree of luminosity segregation exists [9], at least for galaxies with absolute magnitude $M_B \leq -20 + 5 \log h$. Bright galaxies are stronger correlated than faint galaxies. It seems that both kinds of segregation exist but are independent effects. Mass segregation will be reviewed in Dr Campos lecture during this School. The segregation mechanisms are to be understood on the basis of a convincing structure formation theory.

2.5. Ergodic hypothesis

Obviously all the statistical measures will be applied to a portion of the Universe. Let $D$ be the size of a portion and $L$ be the scale to which the measure refers. If $L$ is not much smaller than $D$, and we apply the same measure to another portion of size $D$, we expect to find different results. This is sometimes referred to as ‘cosmic variance’. If $D \gg L$, we shall have many realizations of the probability distribution within our sample, and therefore we expect that the results do not depend too much upon the studied region. Statisticians would say that we are assuming ergodicity [10, 11], in the sense that our sample is enough to obtain statistically reliable results [12], as it contains an adequate number of independent realizations.

3. Statistical measures

We have summarized in the previous section how the observation of the Universe at large scale provide us with obscured, truncated, distorted and segregated samples of galaxies. In spite of all their shortcomings, they are of extraordinary interest in Cosmology. From the detailed analysis of these samples we learn about the past and future of the Universe.

In this section, we will introduce some of the mathematical techniques often used to statistically describe the distribution of galaxies.

3.1. Second-order characteristics

I will start this section by using the terminology and notation often employed by spatial statisticians and I will relate it with that used by cosmologists.

In a point process, the galaxy distribution in our case, we can define [13] the second order intensity function $\lambda_2(\vec{x}_1, \vec{x}_2)$ as follows: Let us consider two infinitesimally small spheres centered in $\vec{x}_1$ and $\vec{x}_2$ with volumes $dV_1$ and $dV_2$. The joint probability that in each of the spheres lies a point of the point process is approximately

$$dP = \lambda_2(\vec{x}_1, \vec{x}_2)dV_1dV_2. \tag{6}$$
(See [5, 13] for an exact definition). If the point field is homogeneous (sometimes called stationary), \( \lambda_2(x_1, x_2) \) depends only on the distance \( r = |x_1 - x_2| \) and the direction of the line passing through \( x_1 \) and \( x_2 \), \( 0 \leq \beta < \pi \): \( \lambda_2(r, \beta) \). If, in addition, the process is isotropic, the angle \( \beta \) becomes unimportant and the function depends only on \( r \), \( \lambda_2(r) \). In the following, we will assume the Cosmological Principle: the Universe at large scales is homogeneous and isotropic. Let \( n \) be the mean number density of galaxies in a huge volume, assumed to be a fair sample of the Universe. The two-point correlation function commonly used in Cosmology is [10]

\[
\xi(r) = \frac{\lambda_2(r)}{n^2} - 1.
\]

The expected number of points within a distance \( r \) from an arbitrary given galaxy is

\[
\langle N \rangle_r = \int_0^r 4\pi ns^2(1 + \xi(s))ds = \frac{4\pi}{n} \int_0^r s^2\lambda_2(s)ds.
\]

The last expression may also be referred to as a correlation integral \( C(r) \) [4]. \( K(r) = C(r)/n \) is called the Ripley’s \( K \)-function [15] and is extensively used in the literature of point fields. In this context \( n \) is the first-order intensity function \( \lambda \) of the point field, which is constant for homogeneous processes.

3.2. Estimators of \( \xi(r) \)

Different estimators may be used to evaluate \( \xi(r) \). For volume-limited samples all the galaxies have the same weights \( w = 1 \), while when selection functions are used to account for the incompleteness, each galaxy is counted with a weight \( w \geq 1 \). Davis & Peebles [16] use the estimator

\[
1 + \xi_{DP}(r) = \frac{DD(r) N_R}{DR(r) N_D},
\]

where \( DD(r) \) is the number of pairs with separation \( r \) in the galaxy catalogue with \( N_D \) galaxies and \( DR(r) \) is the number of pairs with separation \( r \) between the data and a random distributed sample with \( N_R \) points. Equivalently we can use [5, 17] the following estimator by averaging over the \( N \) galaxies of the sample

\[
1 + \xi_R(r) = \frac{1}{N} \sum_{i=1}^{N} \frac{N_i(r)}{nV_i(r)},
\]

where \( N_i(r) \) is the number of galaxies lying in a shell of thickness \( dr \) at distance \( r \) from galaxy \( i \) and \( V_i(r) \) is the volume of the shell lying within the sample boundaries.

There is enough confidence in the power-law behaviour of the observed galaxy two-point correlation function in the range of scales \( 0.1 < r < 10 \, h^{-1} \text{ Mpc} \) [3, 8, 18].

\[
\xi_{gg}(r) = \left( \frac{r}{r_g} \right)^{-\gamma},
\]

where the exponent \( \gamma \approx 1.8 \) and the correlation length \( r_g \approx 5 \, h^{-1} \text{ Mpc} \). It is interesting to note that the correlation function of clusters of galaxies is compatible with (11) once we replace \( r_g \) by \( r_c \approx 15 - 30 \, h^{-1} \text{ Mpc} \). The value of \( r_c \) is rather controversial due to possible selection effects in the compilation of the catalogues of galaxy clusters [19], but however exceeds \( r_g \) by a significant factor.
3.3. Moments of the cell-counts and scaling

Let us center a sphere of radius $r$ on a galaxy (labeled by $i$) and call $n_i(r)$ the number of galaxies in it excluding the central one. Averaging over the $N$ galaxies of the sample we get the mean count

$$
\langle N \rangle_r = \frac{1}{N} \sum_{i=1}^{N} n_i(r),
$$

which provides the correlation integral (8). We shall say that there is scaling for the first moment of the counts of neighbors if

$$
\langle N \rangle_r \propto r^{D_2},
$$

and the constant exponent $D_2$ is known as correlation dimension [20].

In the range of scales where $\xi(r) \gg 1$, equations (8), (11), (13) allow us to derive a relation between the exponent of the two-point correlation function $\gamma$ and $D_2$

$$
D_2 \simeq 3 - \gamma.
$$

Obviously, in the regime where $\xi(r)$ is of the order of unity, the previous relation does not hold.

Scaling may be generalized to moments of any order if

$$
Z(q, r) = \frac{1}{N} \sum_{i=1}^{N} r_i(n)^{q-1} \propto r^{\tau(q)},
$$

with scaling indices $\tau(q)$ independent of $r$ in a suitable interval. There we define the generalized dimensions $D_q = \tau(q)/(q-1)$. For $q = 2$, we recover the scaling of (13), where $\tau(2) = D_2$. When the scaling relation (13) holds, we will say that the point distribution has multifractal character. In a simple fractal $D_q = \text{const}$ for all $q$ values, while for a multifractal set $D_q$ is a decreasing function of $q$. The meaning of $D_q$ is clear: when $q$ is positive and large, the denser parts of the point distribution dominate the sums in (13), while for negative values of $q$ the sums are dominated by the rarefied regions of the point set.

For $q < 2$, it is usually more convenient to obtain $D_q$ through a different algorithm. Let us call $r_i(n)$ the radius of the smallest sphere centered at point $i$ and enclosing $n$ neighbors, in other words $r_i(n)$ is the distance of point $i$ to its $n$th neighbor. The exponents $\tau(q)$ are obtained through the relation

$$
W(\tau, n) = \frac{1}{N} \sum_{i=1}^{N} r_i(n)^{-\tau} \propto n^{1-q}.
$$

For $q = 0$, the previous equation provides a way for estimating the Hausdorff dimension $D_0$. Galaxy samples such as the CfA-I provide a value of $D_0 \simeq 2.1$, while the correlation dimension of the same sample is $D_2 \simeq 1.3$ [21] in the range of scales $1 \leq r \leq 10 \, h^{-1} \, \text{Mpc}$. The whole $D_q$ function of a volume-limited limited subsample of the CfA-I catalogue is illustrated in Fig. 2 (solid line). Equation (15) has been used for $q \geq 2$, while Equation (16) has been used for $q < 2$. 
3.4. Multiscaling

Beyond \(10 \, h^{-1} \) Mpc it is much more difficult to estimate the galaxy-galaxy two-point correlation function. Nevertheless, integral quantities such as \(C(r)\) can still be estimated with enough reliability. For \(r \leq 10 \, h^{-1} \) Mpc, there is also information about the statistics of the distribution of clusters of galaxies. In this section, we will provide an explanation work of the clustering of different objects such as galaxies or clusters within the same theoretical framework.

If the matter distribution is considered a continuous density field, we could think of galaxies as being the peaks of the field above some given threshold. A larger threshold will correspond to clusters of galaxies. The higher the threshold the richer the galaxy cluster. We can use the stochastic model shown in Fig. 3(a) to illustrate this behaviour (for details see \cite{22,23}). By applying different density thresholds, which are quite naturally defined in this model, we obtain the distributions shown in Figs. 3(b), (c). The parameters of the model were chosen to provide values for \(D_0 = 2.0\) and for \(D_2 = 1.3\). The whole \(D_q\) function of this model is illustrated in Fig. 2 (dashed line).

The value of \(D_2\) is approximately the slope of the log-log plot \(Z(2, r)\) vs. \(r\) shown as a solid line in Fig. 4. After applying the threshold, \(Z(2, r)\) stills follows a power law, but with different slope (see dotted and dashed lines in the figure). In the plot we can see that the higher the density threshold, the lower is \(D_2\). Multiscaling is a scaling law where the exponent is slowly varying with the length scale due to the presence of a threshold density defining the objects \cite{24}.

We have seen that the observed matter distribution in the Universe follows some sort of multiscaling behaviour \cite{14}. If galaxies and clusters of galaxies with increasing richness are considered as different realizations of the selection of a density threshold in the mass distribution, the multiscaling argument implies that the corresponding values of the correlation dimension \(D_2\) must decrease with increasing density.

We shall show the correlation integral for galaxy samples and for cluster samples in the range \([10, 50] \, h^{-1} \) Mpc. In this range of scales \(\xi_{gg}(r)\) does not follow a power-law shape, while \(C(r)\) is nicely fitted to a power-law shape. For galaxies we have analyzed the CfA-I sample, the Pisces-Perseus sample \cite{25} and the QDOT-IRAS redshift survey \cite{4}. The cluster samples are the Abell and ACO catalogues \cite{26}, the Edinburgh-Durham redshift survey \cite{27}, the ROSAT X-ray-selected cluster sample \cite{28} and the APM cluster catalogue \cite{29}.

In Fig. 5 we see that three straight lines fit reasonably well the eight samples analyzed. All the cluster samples have a correlation integral well fitted by a power-law with exponent \(D_2 \approx 2.1\). A value of \(D_2 \approx 2.5\) appears for the optical galaxy catalogues: the CfA volume-limited sample and the Pisces-Perseus survey. Finally a value of \(D_2 \approx 2.8\) is obtained for the QDOT-IRAS galaxies. These results probe the multiscaling behaviour of the matter distribution in the Universe.

The fact that \(D_2\) for IRAS galaxies is larger than for optical samples indicates that IRAS galaxies are less correlated than optical galaxies; this is nicely interpreted if optical galaxies correspond to higher peaks of the density field. Clusters of galaxies have stronger correlations than galaxies, corresponding to the highest peaks of the background matter density.
4. Conclusions

We have given a short introduction to different aspects of the characterization of the observed surveys of galaxy clustering by means of different statistical techniques. Redshift surveys, when considered as point processes, have peculiar features which can be expressed through statistical tools. Obscuration by dust in our own Galaxy, truncation in luminosity and the use of selection functions for flux-limited samples have been discussed in some detail. The analysis of clumpiness is often done in redshift space, which has important distortions when compared to real space. Morphological and luminosity segregation is an important clue for testing galaxy formation theories. It is interesting to consider the galaxy distribution as a marked point process. We have illustrated the relationship of the two-point correlation function to other statistical quantities such as the intensity functions or cumulant quantities such as the correlation integral. The multifractal nature of the matter distribution comes from the scaling of the moments of the cell-counts. We have introduced the concept of multiscaling to provide a neat scheme for the explanation of the clustering of galaxies of different kinds and clusters with different richness. In this context, we have shown how the correlation dimension $D_2$ attains specific values for each kind of cosmic object, being a clear measure of their clustering.

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Figure captions

**Figure 1.** The selection function of the QDOT-IRAS redshift survey.

**Figure 2.** The generalized dimensions $D_q$ for a volume-limited subsample of the CfA-I catalog (solid line) and for the multifractal model shown in Fig. 3 (dashed line).

**Figure 3.** A multifractal stochastic model for the galaxy distribution (a). In (b) and (c) we see the same model after applying increasing density thresholds.

**Figure 4.** The function $Z(2, r) = C(r)/n$ for the model shown in Fig. 3. The slope $D_2$ is lower for samples with higher density threshold.

**Figure 5.** The correlation integral for different galaxy and cluster samples (reproduced with permission from Martinez et al. *Science* 269, 1245. Copyright 1995 American Association for the Advancement of Science).