Supergiant Pulses from Extragalactic Neutron Stars

J. M. Cordes * & Ira Wasserman †

Astronomy Department, Cornell University, Ithaca, NY 14853

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ABSTRACT

We evaluate the hypothesis that extragalactic radio bursts originate from neutron stars. These could be active pulsars or dormant, slowly spinning objects, but the different population distances for these two classes require correspondingly different contributions to burst dispersion measures from any host or intervening galaxies combined with the intergalactic medium. The large, apparent burst rate $\sim 10^4$ sky$^{-1}$ day$^{-1}$ is comparable to the core-collapse supernova rate in a Hubble volume and can be accommodated by a single burst per object in the resulting large reservoir of $\sim 10^{17}$ neutron stars. A smaller population distance requires more bursts per object but the likelihood of seeing repeated bursts from any single object is extremely low on human timescales. Gravitational microlensing could play a role for high redshift sources. Extrapolation of the Crab pulsar’s giant pulses — exemplars of coherent, high brightness temperature radiation — to a rate of one per $10^3$ yr yields an amplitude detectable out to $0.3$ Gpc. Objects with spindown energy loss rates $\dot{E}$ up to $10^6$ times larger could be seen even further if allowed by coherent radiation physics. While ample energy is available in neutron-star magnetospheres, the coherent-radiation process must convert particle energies with high efficiency to produce individual, coherent shot pulses. Detailed requirements depend significantly on how coherent shot pulses with unit time-bandwidth product are incoherently summed to form millisecond-duration bursts with large time-bandwidth products. For plausible incoherent summing, the largest shot pulse seen from the Crab pulsar is similar in amplitude to those needed for extragalactic bursts. We briefly discuss triggering of neutron-star magnetospheres by internal or external agents.

Key words: neutron stars – radio sources – bursts – Crab pulsar.

1 INTRODUCTION

Over the last decade individual radio bursts have been found as a byproduct of surveys for periodic radio pulsars. Reobservations ultimately reveal that many of the sources are radio pulsars, differing only in the means used to initially discover them, but otherwise having similar pulse widths (milliseconds) and dispersion measures (DMs), the free electron column density. However, a minority of the bursts have defied redetection. Some are clearly Galactic but others have DMs much too large to be accounted for by the modeled foreground interstellar medium (ISM) in the Milky Way. In the literature, bursts found through single-pulse detection algorithms have loosely been referred to as ‘rotating radio transients’ (RRATs; McLaughlin et al. 2006) and are consistent with a Galactic population of neutron stars (NS; Spitler et al. 2014). The more recently discovered events with DMs too large to be accounted for by the Galaxy have been termed ‘fast radio bursts’ (FRBs) (Lorimer et al. 2007; Keane et al. 2011; Thornton et al. 2013; Spitler et al. 2014; Burke-Spolaor & Bannister 2014; Ravi et al. 2014) and appear to be extragalactic in origin. To avoid confusion, we use the term ‘extragalactic radio burst’ (ERB) for these events.

Some radio bursts appear to have an origin local to the Earth or its magnetosphere (‘perytons’: Burke-Spolaor et al. 2011; Saint-Hilaire et al. 2014), but have time-frequency signatures or angular distributions inconsistent with astrophysical events showing pulsar-like dispersion. We assume that most if not all of the reported ERBs are in fact astrophysical in nature.

The lack of any new events in reobservations of ERB sky directions requires that detectable bursts originate from non-repeating astrophysical cataclysms or from repetitive sources with very low burst rates. Millisecond durations require compact sources and, along with observed flux densities $\sim 1$ Jy, radiation with high spatial coherence as required by the high implied brightness temperatures. Associating the light-travel time with source size, the brightness temperature is of order

$$T_b = \frac{S_c}{2
u} \left(\frac{d}{\nu \Delta t}\right)^2 \sim 3 \times 10^{35} K S_c Jy (d Gpc / \nu 2164 \Delta t ms)^2.$$ (1)

ERBs are not unlike pulses from Galactic pulsars and RRATs so they could have similar underlying radiation physics. While arguments have been made that the high dispersion measures of ERBs originate from or are mimicked by an emission process in the sources themselves (e.g. Kulkarni et al. 2014; Loeb et al. 2014), the simplest interpretation at present is that the burst sources are extragalactic (e.g. Luan & Goldreich 2014; Dennison 2014). Extragalactic distances require larger radio luminosities, however, and one goal of this paper is to examine whether coherent processes in neutron-star magnetospheres can produce detectable bursts originating from large distances. Despite the small number found so far, the implied aggregate event rate $\sim 10^4$ day$^{-1}$ over the entire sky or somewhat smaller, Law et al. (2014), Keane & Petroff (2014) is astonishingly large, much larger than

* E-mail: jmc33@cornell.edu
† E-mail: ira@astro.cornell.edu
‡ For example, an FRB candidate event could be Galactic if the large DM is due to electrons in a Galactic HII region (e.g. Bannister & Madsen 2014), in which case the source might be termed an RRAT.

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the observed rate of gamma-ray bursts, for example (Thornton et al. 2013; Spitler et al. 2014). However, until a distance scale is established for ERBs, it is not possible to identify any particular underlying source class.

We consider populations of active pulsars, including young pulsars and millisecond pulsars, as well as old, dormant NSs. Transient, pulsar-like action may take place in dormant neutron stars whose magnetospheres are triggered by an external agent, such as interstellar debris or a high-energy cosmic ray, or internally by a crust quake or some other stimulus within the NS itself. We use giant pulses (GPs) from the Crab pulsar and a few other pulsars as prototypes for the brightest coherent emission. We extrapolate from the rates and amplitudes of GPs from the Crab pulsar to larger time scales of observation and to objects that have larger energy loss rates.

Objects residing in dense star-forming regions or in the centers of galaxies will have a significant host contribution to their measured DMs and may not require a dominant contribution from the intergalactic medium (IGM; e.g. a dominant contributor, NSs in the outer parts of galaxies or old objects unbound from their birth sites) will require the IGM to contribute significantly to DMs from sources with distances \( \gtrsim 100 \) Mpc. As with giant pulses from the Crab pulsar, reaction radiation is a significant factor (e.g. Buschauer & Benford 1976), implying that large-amplitude pulses can tap a large fraction of the free energy available in relativistic particles. We find, however, that obtaining \( \sim 1 \) Jy pulses from sources at Gpc distances is very challenging for ‘antenna’-like coherence processes because they require high number densities in small volumes. However, incoherent summing of radiation from individual coherent regions relaxes the requirements. Nonetheless, radiation reaction is likely a generic issue for understanding ERBs regardless of the class of objects that cause them, including the collapse of supramassive NSs (Falcke & Rezzolla 2014) and other sources.

The plan of the paper is as follows. Section 2 summarizes GPs seen from the Crab pulsar and their energetics and extrapolates to longer time frames and to younger, inactive pulsars; Section 3 discusses extragalactic GPs and populations needed to obtain the observed ERB rate; Section 4 summarizes limits on time scales and flux densities from coherent radiation; Section 5 analyzes detailed predictions from coherent curvature radiation; and Section 6 summarizes our results and conclusions.

2 MAXIMAL RADIO EMISSION FROM PULSARS

Rotation-driven NS dissipate their energy through emission in particles accelerated by rotationally-generated electric fields, low-frequency waves, and high-energy radiation. It is well known from early studies of pulsars that coherent radio emission causes significant energy losses of relativistic particles (Sturrock 1970; 1971; Ruderman & Sutherland 1975; Buschauer & Benford 1976). It therefore follows that the amplitudes and durations of narrow pulses are limited by reaction radiation, a point emphasized by Buschauer & Benford (1980) in their analysis of the Crab pulsar.

Here we place constraints on the beaming and efficiency of radio emission in order to provide detectable pulse amplitudes from Crab-pulsar analogs. Radio emission is typically a tiny fraction of the spindown energy loss rate, \( E = \frac{I \Omega}{\Omega} \) (\( \Omega \) = spin frequency and \( I \) = moment of inertia), particularly for the average emission from objects with large values of \( \dot{E} \), such as the Crab pulsar. However, some older, long-period pulsars have \( \dot{E} / \dot{E} \approx 1 \) or \( 10\% \) (e.g. Arzoumanian et al. 2002). Recent work has revealed a direct link between radio emission and \( \dot{E} \) in intermittent pulsars showing on-and-off intensity transitions that accompany large changes in \( \dot{E} \) (e.g. Kramer et al. 2006).

2.1 Giant Pulses from the Crab Pulsar

The brightest GP detected so far had a peak amplitude \( S_{\nu, \text{max}} = 2.2 \) MJy at 9 GHz and a pulse width \( \Delta t \) smaller than the 0.4 ns time resolution provided by the 2.5 GHz receiver bandwidth (Hankins & Eilek 2007). The implied brightness temperature \( T_b \approx 10^{14} \) K requires a coherent emission process. The upper bound on \( \Delta t \) implies a time-bandwidth product \( v \Delta t \lesssim 4 \), suggesting that it is a single shot pulse produced by an individual, coherent line-emitting column. Most of these have time-bandwidth products much larger than unity because they are incoherent superpositions of shot pulses. Relatively frequent GPs at 0.43 GHz with \( \sim 100-200 \) kJy maxima occur about once per hour (Hankins & Rickett 1975; Cordes et al. 2004; Crossley et al. 2010). They are broadened by interstellar scattering to \( \sim 100 \) \( \mu \)s from intrinsic widths of about 50 \( \mu \)s. The implied time-bandwidth product \( \approx 50 \mu s \times 0.43 \) GHz \( \sim 2 \times 10^4 \) requires a combination of coherent emission and incoherent superposition and this is consistent with the appearance of single GPs (e.g. Jandov et al. 1995; Mickaliger et al. 2012). Fits of power-law distributions \( S_{\nu} \propto \nu^{\alpha} \) yield exponents ranging from \( \alpha \approx 2.3 \) – 3.5 in observations at different frequencies (Mickaliger et al. 2013). The power-law component may be augmented with a long tail associated with additional, supergiant pulses (Cordes et al. 2004; Mickaliger et al. 2012), but as yet poorly constrained parameters for the tail.

2.2 GP Energetics

The largest GPs from the Crab pulsar are easy to accommodate within the total energy-loss budget of its magnetosphere but require significant fractions of the available particle energies to be directed into coherent emission.

We relate maximal GP emission to the spindown energy loss rate \( E \approx 10^{38.7} \) erg s\(^{-1}\) by combining the radio emission efficiency \( \epsilon_r \lesssim 1 \) with the beaming solid angle \( \Omega_c \) into a factor \( \zeta_c = (4 \pi \epsilon_r / \Omega_c) \). The peak flux density is then

\[
S_{\nu, \text{max}} = \left( \frac{4 \pi \epsilon_r}{\Omega_c} \right) \left( \frac{E}{\Delta t \nu^2 c^3 \nu_{\text{crab}}} \right) \approx 100 \text{cMJy} \left( \frac{E_\nu}{4.6} \right).
\]

The approximate equality uses parameter values for the Crab pulsar including \( d_{\text{crab}} = 2 \) kpc. Evaluating for the 2 MJy shot pulse at 9 GHz and the more frequent \( \sim 100 \) kJy pulses at 0.43 GHz, we obtain \( \zeta_c = 0.02 \) and 0.002, respectively. We may expect infrequent but very large pulses from the Crab pulsar when \( \zeta_c \gg 1 \), which may occur as a consequence of favorable relativistic beaming or large instantaneous departures from the average radio efficiency \( \epsilon_r \).

The ‘Crab twin’ pulsar B0540–69 in the Large Magellanic Cloud (\( P = 50 \) ms, \( B = 5 \times 10^{13} \) G) shows giant pulses (Johnston & Romani 2003) that have smaller \( S_{\nu, \text{max}} \) by a factor \( \gtrsim 10 \) than those of the Crab pulsar. Some of the difference could be due to less-favorable beaming for B0540–69.

While GPs can be accommodated by spin-down losses, they may saturate the typical energy available in relativistic particles. Force-free conditions \( \mathbf{E} \cdot \mathbf{B} = 0 \) in a neutron-star’s magnetosphere require a charge (number) density \( n_{\text{GJ}} \approx \frac{\rho}{B^2/2\pi e^2} \), the Goldreich-Julian (GJ) density, \( n_{\text{GJ}, \text{in}} \approx 10^{10.84} \text{cm}^{-3} B_{12} P^{-1} (R/r)^3 \), where the radial scaling applies to the open-field-line region of the magnetosphere with \( r \geq R \) where \( R \) is the NS radius, and \( r \ll r_{\text{in}} \) where the
light-cylinder radius is \( r_{lc} = cP/2\pi \). For standard \( \sim 10^{12} \) Volt potential drops in polar cap models, the particle energy loss rate (assuming an electron mass) is approximately

\[
E_p = c A_{\text{PC}} n_c \gamma^2 m_e^2 \approx 10^{30} \text{erg s}^{-1} B_{\text{B}} R_{\text{lc}}^2 P^{-2} \approx E_{1.2}^2,
\]

where \( \gamma \) is the Lorentz factor and the area of the magnetic polar cap is \( A_{\text{PC}} \approx \pi R_{\text{lc}}^2 \approx P^{-1} \). The particle loss rate scales as the square root of the spin-down loss rate in standard polar-cap models (Ruderman & Sutherland 1975; Arons & Scharlemann 1979). The Crab pulsar \((B_{\text{B}} = 3.78 \text{ and } P^{-1} = 30.2 \text{ Hz})\) yields \( E_p \approx 10^{31} \text{erg s}^{-1} \). For the 9-GHz pulse described above, highly beamed radiation with \( \Omega_r/4\pi \approx 2 \times 10^{-1} \) is required to provide \( L_r \lesssim E_p \). Since the beam solid angle is likely even smaller than this limit for \( \gamma \gtrsim 10^3 \), there is significant headroom in the possible range of GP amplitudes to the extent that particle flows and radiation physics can allow much larger values.

### 2.3 Extrapolation of Rates and Amplitudes from the Crab Pulsar

The maximum detectable rate of a Crab GP with amplitude \( S_n \) is \( \eta \) times the Crab value \( S_{\text{Crab}} \) of \( \sim 13 \) Jy referenced to the Crab pulsar’s distance are needed to be detectable from analogous objects 1 Gpc away.

We scale from Crab GPs to Gpc distances using

\[
S_{n,\text{max}} = S_{n,\text{max,Crab}} \left( \frac{\zeta_S}{\zeta_{S,\text{Crab}}} \right) \left( \frac{d_{\text{Crab}}}{d} \right)^2 \left( \frac{E_{1.2}}{4.6} \right)^{0.2} \mu Jy \frac{E_{1.2}}{\Delta v_{\text{Crab}} d_{100}^2} \frac{\zeta_S}{\zeta_{S,\text{Crab}}},
\]

(5)

Pulsars with \( S_n \sim 1 \) Jy require a combination of larger \( \zeta_S \) (i.e. larger radio efficiency or smaller beaming solid angle) with larger spin-down loss rates \( \dot{E} \) and distances smaller than 1 Gpc. To be detectable at 1 Gpc with 60 mJy threshold, \((\zeta_S/\zeta_{S,\text{Crab}})E_{1.2} \geq 3 \times 10^7 \). Objects undoubtedly exist with spin-down loss rates \( \dot{E} \propto P^{-\alpha} \) much larger than that of the Crab pulsar. This can include the Crab pulsar itself when it was born with \( P \sim 10 \) ms compared to its present-day 33 ms period. Most pulsars do not show GPs and have PDFs that are typically asymmetric but do not have long tails; many are consistent with log-normal distributions (Burke-Spolaor et al. 2012). GP amplitude PDFs, however, are ‘heavy tailed’ with a power-law type shape for at least a range of pulse amplitudes.

For a GP rate \( \eta_0 \) the average number in a time interval \( T \) is \( \eta_0 T \) and the average number \( N_p \) above a threshold \( S \) is \( N_p(>S,T) = \eta_0 T \left[ 1 - F_S(S) \right] \). In a time \( T \), the largest pulse with flux density \( S_1 \) implies \( \eta T_1 \left[ 1 - F_{S_1}(S_1) \right] = 1 \), on average, so the number above a threshold \( S \) expected in time \( T \) is

\[ N_p(>S,T) = \frac{T}{T_1} \frac{1 - F_S(S)}{1 - F_{S_1}(S)} \]

(6)

We solve Eq. 6 for the threshold flux density \( S \) above which \( N_p \) pulses are detected typically in time \( T > T_1 \). For a power law PDF \( f_S(S) \propto S^{-\alpha} \) between lower and upper cutoffs \( S_L \) and \( S_u \) and \( \alpha \neq 1 \), the maximum amplitude expected in time \( T \) is

\[
S = S_1 \left( \frac{1}{T/T_1} \right)^{1/(\alpha-1)} + \left( \frac{T}{T_1} - 1 \right) \left( \frac{S_1}{S_u} \right)^{1/\alpha} \left( \frac{T}{T_1} \right)^{1/(\alpha-1)},
\]

(7)

where the approximate equality holds for \( \alpha > 1 \). \( N_p(T/T_1)(S_c/S_u)^{\alpha-1} \ll 1 \), and \( S < S_c \). For \( \alpha = 1 \), \( S = S_1(S_c/S_u)^{N_p/T_1}/T \).

If the PDF extends to very large flux densities \( S_\alpha > S_1 \), the time \( T \) required to see pulse amplitudes that span most of the range up to the upper cutoff \( S_\alpha \) is strongly dependent on \( \alpha \). Figure 2 shows the expected \( S_\alpha \) as a function of observation time \( T \) for different values of \( \alpha \) and for a dynamic range \( S_u/S_1 = 10^6 \). Similar results are obtained for other values of this ratio. For \( T_1 = 1 \) hr, the plotted range extends to \( T \approx 10^7 \) yr, the age of the Crab pulsar. Bursts within a factor of two of the maximum \( S_u \) are seen in time spans that are modest multiples of \( T_1 \) for shallow power laws with \( \alpha \lesssim 1 \). However, for steeper power laws like those inferred for the Crab pulsar (\( \alpha \approx 2.3 - 3.5 \)) it can take very long times, comparable to the current spin-down age of the pulsar. GP-emitting pulsars at large distances may therefore produce detectable bursts only rarely.

The largest pulse expected in \( 10^7 \) yr is \( \left( \text{Equation 7} \right) S = 10^{-7} \) Jy (or \( 3.5 \) to \( 10^{19} \) Jy) for \( \alpha = 2.5 \). The corresponding maximum detection distances are \( d_{\text{max}} = 15 \text{ Mpc} \) to \( 300 \text{ Mpc} \) using \( d_{\text{max}} \approx 0.63 \text{ Mpc} \text{ (T/1 yr)}^{1/2(\alpha-1)} \).

### 3 EXTRAGALACTIC POPULATIONS OF NEUTRON STARS

Extragalactic NS formed over cosmological time are potential sources of super-strong bursts that occur only rarely per object. We associate the burst rate to the cosmological birth rate of NS and assume each object emits a small number of detectable bursts, \( N_b = n_b T_b \), at a low rate \( n_b \) at a burst phase of duration \( T_b \). The more detailed calculation can include separate birth and burst rates, but the net result is the same if maximum source distances are much less than the Hubble distance, \( d_H = c/H_0 = 3H_0^{-7} \text{ Gpc} \). We use the Galactic rate of NS formation per unit stellar mass, \( n_{\text{NS, M}} \approx n_{\odot, M} \times 10^{-12} \text{ yr}^{-1} M_\odot^{-1} \text{ (e.g. one NS every 100 yr per } M_\odot \text{ in stars) in combination with the average stellar mass density } \rho_\odot = 3H_0^2/8\pi G \text{ is the closure density and } \rho_\odot \approx 0.003 M_\odot L_\odot^{-1} \text{ (Read & Trentham 2005).} \)

About \( n_{\text{NS}} \sim 10^{17} \) NS are produced in a Hubble volume \( V_H = 4\pi d_H^3 \) over a 10 Gyr period. The aggregate NS birth rate (scaled from the local rate) \( \Gamma_{\text{ns}} \sim 4 \times 10^3 \text{ day}^{-1} \). The inferred ERB burst rate \( \Gamma_{\text{obs}} \sim 10^4 \text{ day}^{-1} \) (Thornton et al. 2013; Spitler et al. 2013), suggesting that only a few detectable

2 Assuming a bandwidth of 50 MHz and a system equivalent flux density of 4 Jy along with a matched filter with 1 ms width.

3 A pure power-law model does not always appear to be a good model for histograms of pulse amplitudes from the Crab pulsar. However, they always show long tails so we proceed by using power-law PDFs to make extrapolations while keeping in mind that the true form of the PDF may differ.
bursts per NS are needed to account for ERBs. Larger \( N_b \) would allow a nearer population with \( d_{\text{max}} < d_H \) but \( N_b \) is bounded by the fact that no repeats have been detected with net on-source times of up to \( \sim 10 \) hr.

In the following we estimate ERB rates by including beaming of radiation and a redshift-dependent star formation rate (SFR). We assume that beaming toward Earth is favorable for a fraction \( f_b = 0.1 f_b,0.1 \) of the bursts that are bright enough to be detectable throughout the relevant volume. Values of \( f_b \) for pulsars are of this order but depend on spin period and on rotation of the beam on the sky assuming steady emission. ERBs could have substantially smaller \( f_b \) if burst durations are a purely temporal phenomenon in a frame rotating with the pulsar. However, the beaming fraction for \( f_b \) can be comparable to that of pulsars if pulse widths are mostly due to beam rotation. The burst rate per unit volume is then \( d\Gamma_b/d(\text{vol}) = f_b N_b \dot{n}_{\text{ns}} M \rho_* \).

### 3.1 Low-redshift Populations

For a local (low-redshift) population, the burst rate for a maximum population distance \( d_{\text{max}} \) is

\[
\Gamma_b(d_{\text{max}}) = \Gamma_H \left( d_{\text{max}}/d_H \right)^3,
\]

where the local burst rate is normalized to a Hubble volume is

\[
\Gamma_H = f_b N_b \Gamma_{\text{ns}} = \frac{c^3 f_b N_H \dot{n}_{\text{ns}} M_0 \Omega_{b,0}}{2H_0 G} \approx 4 \times 10^3 \text{ day}^{-1} f_{b,0.1} N_b \left( \frac{\Gamma_{\text{ns}}}{4 \times 10^3 \text{ day}^{-1}} \right). \tag{9}
\]

To match \( \Gamma_b(d_{\text{max}}) \) with \( \Gamma_{\text{obs}} \) requires

\[
N_b = \frac{\Gamma_{\text{obs}}}{f_b \Gamma_{\text{ns}}} \left( \frac{d_H}{d_{\text{max}}} \right)^3 = \frac{200}{f_{b,0.1} d_{H,0}^3 G_{\text{pc}}} . \tag{10}
\]

Even for a relatively nearby population with \( d_{\text{max}} = 100 \) Mpc, the number of bursts per object, \( N_b \approx 2 \times 10^3 \), is a small fraction of the total number of turns of a NS in its lifetime, whether defined as the giant-pulse emitting time span, the total radio-emitting span, or the age of the NS itself. In time \( T \) the number of turns is \( \dot{\psi}_T \sim \Gamma_{\text{ns}}^{-1} \left( T/T_0 \right)^{(n-2)/(n-1)} \) for a starting period \( P_0 \) and spindown time \( T_0 = P_0/\Gamma (n-1) P_0 \), where \( n \) is the braking index and \( P_0 \) is the period derivative. The Crab pulsar will make \( \sim 10^{36} \) turns in the next \( 10^9 \) yr and \( 10^{34} \) turns in \( 10^9 \) yr assuming a constant braking index. If ERBs are similar to giant pulses emitted in the first \( 10^3 \) yr of a NS, the interval between ERBs per object is only tens to hundreds of hours. However, rare events occurring at any point in the lifetime of a NS (e.g. \( \gtrsim 1 \) Gyr) imply intervals of tens to hundreds of years or longer.

The number of bursts per source is tied to the contributions of the IGM and any host galaxy to the observed DMs of ERBs. If the maximum population distance is of order \( d_H \), the IGM can and must account for nearly all the excess over the Milky Way's contribution. Conversely, a nearby \( \sim 100 \) Mpc (or closer) population requires a dominant contribution from host galaxies, in particular from dense star-forming regions or galactic centers.

### 3.2 High-redshift Populations

For populations extending to high redshifts, we relate the local NS birth rate \( \dot{n}_{\text{ns},M}(z) \) to the SFR \( \psi(z) \) (in standard units \( M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3} \); e.g. Madau & Dickinson 2014)

\[
\dot{n}_{\text{ns},M}(z) = \nu_{\text{ns}}(z) \psi(z) \tag{11}
\]

where \( \nu_{\text{ns}}(z) \) is the number of neutron stars per stellar mass formed at redshift \( z \) and the stellar mass density is a co-moving density. Using the beaming fraction, assumed independent of \( z \), and the number of bursts per NS, \( N_b(z) \), now possibly redshift dependent, the burst rate out to a maximum redshift \( z_{\text{max}} \) is

\[
\Gamma_b(z_{\text{max}}) = \frac{4\pi f_b}{\Omega} \int_0^{z_{\text{max}}} \frac{dz}{dz} \left( \frac{r(z)}{r^2(z)} \right) N_b(z) \nu_{\text{ns}}(z) \psi(z) \tag{12}
\]

where \( r(z) \) is the comoving radial coordinate and \( 4\pi dz r^2(z) dr(z)/dz \) is the comoving volume element. The \( 1 + z \) factor in the denominator accounts for the redshift of the NS birth rate.
By normalizing \( \nu_{ns}(z) \) and \( \psi(z) \) to local values,
\[
\nu_{ns}(z) = \nu_{ns}(0) \hat{\nu}_{ns}(z) \quad \text{and} \quad \psi(z) = \psi(0) \hat{\psi}(z),
\]
the rate becomes
\[
\Gamma_z \left( \leq z_{\text{max}} \right) = 4 \pi \int_0^{z_{\text{max}}} dz \frac{\hat{\nu}_{ns}(z)\hat{\psi}(z)}{1+z}.
\]

### 3.2.1 Constant \( N_b \) vs. \( z \)

One possibility is that ERBs are associated with extremely rare, triggered events. A neutron star may experience \( N_b \) such events on average during its lifetime. If bright enough intrinsically – flux densities \( \gtrsim 10^{12} \) Jy – such events could be detectable at cosmological distances, up to some cutoff \( z_c \gtrsim 1 \).

If so, then it is plausible that the average number of such events per pulsar that are detectable, \( \tilde{N}_b(z) \), is independent of \( z \) since it is determined entirely by internal (to the NS) or magnetospheric activity or to external triggers that do not involve redshift-dependent conditions. In general, \( \nu_{ns}(z) \) depends on the initial mass function (IMF), which depends on \( z \). However, at modest \( z \sim 1 \) (but not \( z \gg 1 \)) it is also plausible that it is close to the local value and therefore \( \nu_{ns}(z) = 1 \). Under these circumstances we can write the burst rate in terms of a dimensionless integral over the normalized SFR, \( \hat{\psi}(z) \),
\[
\Gamma_z \left( \leq z_{\text{max}} \right) = \Gamma_M G_b(z_{\text{max}})
\]
\[
G_b(z) = 3 \int_0^z dz' \frac{\hat{\nu}(z')}{1+z'}.
\]

The dimensionless radial coordinate is \( \hat{\nu}(z) = \int_0^z dz'/E(z') \) where \( E(z) = \sqrt{1 - \Omega_M + \Omega_M(1+z)^3} \) for a \( \Lambda \)CDM cosmology with a fractional matter density \( \Omega_M \).

To obtain \( G_b(z) \) we use the empirical fit to the cosmological SFR (Madau & Dickinson 2014, Eq. 15),
\[
\psi(z) = \frac{0.015(1+z)^{2.7}}{1 + [(1+z)/2.9]^{0.45}} \frac{M_\odot}{\text{yr}^{-1} \text{Mpc}^{-3}},
\]
and normalize it to \( z = 0 \) to get \( \hat{\psi}(z) \).

Figure 2 (left-hand panel) shows \( G_b(z) \). For low maximum redshifts, \( G_b(z) \approx z^3 \) so the ratio \( G_b(z)/z^3 \), also shown in the figure, is unity as expected. At high maximum redshifts \( G_b(z) \) flattens to a value \( \sim 10 \) times the value at \( z = 1 \), indicating that a very distant population would not require all NS to contribute to observed ERBs.

### 3.2.2 Power-law Models for Pulse Amplitudes

So far, the results have been based on a burst number per source \( N_b \) that is independent of distance, for which the observed burst rate is closely related to the birth rate of the sources themselves and not dependent on any burst luminosity distribution. We now consider detection issues.

The observed range of pulse amplitudes from ERBs is likely to be small given that most of the ERBs detected to date are near the detection limits of telescopes (e.g. within a factor of ten). First, consider pulses with a standard-candle luminosity density \( L_{\nu} = d^3 S_{\nu} / (d^2) \) for a fiducial distance \( d \ll d_M \). The maximum detection distance in Euclidean space for a flux density threshold \( S_{\nu,\text{min}} \) is \( d_{\text{max}} = \sqrt{L_{\nu}/S_{\nu,\text{min}}} \). The PDF for population flux densities above threshold is the standard Euclidean result,
\[
f_S(S_\nu | L_{\nu}, S_{\nu,\text{min}}) \propto S_{\nu}^{-5/2}, \quad S_{\nu} \geq S_{\nu,\text{min}}.
\]
Assuming the PDF extends to flux densities much larger than \( S_{\nu,\text{min}} \) for sources closer than \( d_{\text{max}} \), we find that 97% of bursts are within a factor of 10 of the minimum detectable flux density and 65% are within a factor of two. This may account for the apparent standard-candle nature of ERBs deduced by (Dolag et al. 2014) from a statistical analysis of ERBs. For burst surveys with time-solid angle coverage that is plausibly obtainable with existing

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4 Dispersion measures of ERBs indicate redshifts \( z < 1 \) if the IGM is smoothly distributed. The clumpiness of the cosmic web may allow more distant objects to be seen with the measured DMs (Lorimer et al. 2007; Thornton et al. 2013). However, scattering in the IGM or radiation physics may not allow bursts from such sources to be detectable.
For an ACDM cosmology, we calculate $N_b(z)$ for power-law luminosity distributions. The key question is detectability: requiring a flux density above $S_{\text{th}}$ implies a local flux density $S_{\ell}(d)$ at a reference distance $d$ (e.g. 1 kpc, or else the distance to the Crab pulsar, 2.2 kpc)

$$L_{\nu}(d) \equiv S_{\nu}(d) d^2 \geq S_{\nu,\text{min}} d^2_{\nu}(z) \times \frac{P_{\nu}(\hat{\mathbf{n}}, \nu)}{(1+z)P_{\nu}(\hat{\mathbf{n}}, \nu)(1+z)} \equiv L_{\nu,\text{min}}(z, \hat{\mathbf{n}}),$$

where $d_{\nu}(z) = \nu(z)(1+z)$ is the luminosity distance and the second factor is a “color correction” (see Eq. (20)), which may depend on direction to the source if it is observed slightly off-axis from optimal beaming. Eq. (19) is a condition on the local $L_{\nu}(d)$ at the observer’s frequency, not at the corresponding emitted frequency $\nu(1+z)$ for a source at redshift $z$. We make this possibly awkward definition because observations to detect the distribution of giant pulse flux densities from pulsars will generally be done at the same $\nu$(or set of $\nu$) as observations of FRBs. In obtaining Eq. (19) we assume a universal intrinsic radiation spectrum that is independent of $z$, which is an oversimplification. For example, curvature radiation has a frequency scale $\nu_c \gamma^2/c/\rho_c$ that differs from one pulsar to another. Below $\nu_c$, however, the spectrum $\propto \nu^{2/3}$ and the color correction is simply a factor of $(1+z)^{-5/3}$; above, the spectrum cuts off exponentially, amounting to a color correction factor $\sim \exp(\nu_c/\nu_c)$. Precisely, we may average the color correction over $\hat{\mathbf{n}}$, $\nu_c$ (assuming coherent curvature radiation is the emission mechanism).

Let $L_{\nu}$ be the luminosity density above which we expect a single pulse above the detection threshold for a source at fiducial distance $d$ observed at frequency $\nu$. Then if $f_{\ell}(L_{\nu})$ is the corresponding distribution of luminosity densities, and $L_{\nu,\text{min}}(z, \hat{\mathbf{n}}) \equiv \ell(z, \hat{\mathbf{n}}) L_{\nu}$ (where $\ell$ is dimensionless),

$$N_b(z, \hat{\mathbf{n}}) = \int_{L_{\nu,\text{min}}}^{L_{\nu}} dL_{\nu} f_{\ell}(L_{\nu}) = \frac{[\ell(z, \hat{\mathbf{n}})]^{1-\alpha} - \ell_u^{1-\alpha}}{1 - \ell_u^{1-\alpha}}$$

(20)

where $f_{\ell}(L_{\nu}) \propto L^{-\alpha}_{\nu}$ and $L_{\nu} \equiv \ell_u L_{\nu}$. As expected, Eq. (20) implies $N_b(z) \geq 1$ for $\ell(z, \hat{\mathbf{n}}) \leq 1$, $N_b(z) < 1$ for $\ell(z, \hat{\mathbf{n}}) > 1$, and $N_b(z) = 0$ for $\ell(z, \hat{\mathbf{n}}) \geq \ell_u$. Using Eq. (19) we get

$$\ell(z, \hat{\mathbf{n}}) = \frac{S_{\nu,\text{min}} d^2_{\nu}(z)}{L_{\nu}} \times \frac{P_{\nu}(\hat{\mathbf{n}}, \nu)}{P_{\nu}(\hat{\mathbf{n}}, \nu)(1+z)} \approx \left( \frac{1.1 \times 10^{12} \text{ Jy kpc}^2}{L_{1}} \right) \left( \frac{S_{\nu,\text{min}}/10^6 \text{ mJy}}{b_{\nu}^2} \right) \left( \frac{P_{\nu}(\hat{\mathbf{n}}, \nu)}{P_{\nu}(\hat{\mathbf{n}}, \nu)(1+z)} \right)$$

(21)

For comparison, in $\left[22\right]$ we estimated

$$L_{\nu} \approx 10^5 \text{ Jy kpc}^2 \left( \frac{T}{1 \text{ yr}} \right)^{1-\alpha}$$

(22)

where $T$ is the burst lifetime for a Crab-like pulsar. Inverting implies

$$T \approx \frac{(2.08 \times 10^8 L_1/10^{12} \text{ Jy kpc}^2)^{1-\alpha}}{8760}$$

(23)

or 18,000 years for $\alpha = 2.3$ but 715 Gyr – roughly 50 times the age of the Universe – for $\alpha = 3.5$, the two examples considered.

To estimate rates, we simplify the model by adopting

$$\ell(z, \hat{\mathbf{n}}) = \ell_0 \nu^2(z)(1+z)^{\sigma}$$

(24)

and assume that $\ell(z, \hat{\mathbf{n}}) \to \infty$ for $z > z_c$. (Actually, $\ell(z, \hat{\mathbf{n}})$ should rise exponentially at high $z$, but we are replacing the exponential by a sharp frequency cutoff, which approximates its effect without introducing an additional parameter, the cutoff frequency, explicitly.) In the same spirit as $\left[24\right]$ we assume that $\ell_0$ is large enough to be ignored. With these simplifications

$$N_b(z) \approx \left[ \ell(z, \hat{\mathbf{n}}) \right]^{1-\alpha} \approx \frac{\Theta(z_c - z)}{(\ell_0 \nu^2(z)(1+z)^{\sigma})^{1-\alpha}}$$

(25)

and the rate of detectable bursts is

$$\Gamma_b(z) \approx 1.1 \times 10^9 \, \text{d}^{-1} \, f_0 b_{0.1} \lambda_{0.1} \frac{\Omega_{\nu}}{0.003} \left( \frac{d\nu_{\nu}(0)/dM}{10^{-13}M_{\odot}^{-1} \text{yr}^{-1}} \right) G_b(z)$$

$$G_b(z) = \int_0^{z_c} dz \, \frac{\nu(z)}{d\nu(z)} \left[ \frac{\Omega_{\nu}}{(1+z)^{\sigma}} \right]^{-\alpha}$$

(26)

For small $z$, $G_b(z) \approx z^{-5/2}(5-2\sigma)(2\sigma-3)$, and $\Gamma_b \propto z^{5/2}(5-2\sigma)^{1-\alpha}$. This limit has a straightforward interpretation: $(cz/H_0)^3$ is proportional to the spatial volume, $L_1(c\nu_0(z)/H_0)^2$ is of order the minimum value of the intrinsic luminosity density of a burst detectable to redshift $z$, and the expected number of such bursts per pulsar is $\sim \left[\ell_0(c\nu_0/H_0)^2\right]^{1-\alpha}$. At large $z$, $G_b(z) \approx 10$.

3.2.3 Role of Gravitational Lensing

The estimates above ignore gravitational lensing, which has been considered by $\left[27\right]$. Without lensing, the minimum luminosity density required for detection is

$$L_{\nu,\text{min}} = S_{\nu,\text{min}} d^2_{\nu}(z) = \frac{1.1 \times 10^{12} \text{ Jy kpc}^2 (S_{\nu,\text{min}}/10^6 \text{ mJy})}{(H_0/70 \text{ km s}^{-1} \text{Mpc}^{-1})^2} \left( \frac{H_0 d_{\nu}(z)}{c} \right)^2$$

(27)

where $d_{\nu}(z)$ is the “flux density distance” (Eq. (10) in Appendix C). $L_{\nu,\text{min}}$ is somewhat above the values the maximum flux densities estimated in $\left[23\right]$ for the Crab Pulsar, $L_{\nu,\text{min}} \approx 2 \times 10^6 \text{ Jy kpc}^2 (\alpha = 3.5)$ to $8 \times 10^{10} \text{ Jy kpc}^2 (\alpha = 2.3)$, but lensing with amplifications $A \approx 5500$ or $A \approx 14$, respectively, could lift such pulses to flux densities above $S_{\nu,\text{min}}$.
8.9 × 10^{14} \text{cm}(M/M_\odot)^{1/2}, much larger than stellar radii. Amplifications are $A(b) \sim R_E/b$ for rays passing at impact parameter $b \ll R_E$, and the maximum amplification is $A(R) \sim 1.3 \times 10^9(M/M_\odot)^{1/2}(R_E/R)$. The duration of a lensing event with amplification $A$ is $R_E/Av \sim 9.4(A/(M/M_\odot))^{1/2}(v/300 \text{ km s}^{-1})^{-1}$ years, where $v$ is a characteristic relative speed between lens and source projected onto the sky. Because we are interested in short-lived pulses, it is the lensing probability that is relevant, not the lensing rate, which is appropriate for steady light sources, such as the stars monitored in microlensing surveys. The probability density for $A$ is $p(A) dA \sim \Omega_s A^{-3} dA$ at large $A$, where $\Omega_s = f_s \Omega_0 \approx 0.04 f_s$ is the closure fraction in stars; $P(A) = A p(A) \sim \Omega_s \frac{2 \times 10^{-3} f_s}{(A/14)^2}$

(28)

is a measure of the probability of lensing amplification $\sim A$ out to $z \sim 1$. Thus, the rate of bursts that are visible because of amplification by gravitational lensing is $P(A) f_b N_\gamma N_{\nu} \sim \frac{10 N_0 f_s}{(A/14)^2} \frac{\Omega_{\nu 0.13}}{(A/14)^2}$

(29)

which is $\sim 10^3 \text{ day}^{-1}$ if $N_0 f_s \Omega_{\nu 0.13} \sim 1000(A/14)^2$. Thus, if the brightest $\sim 1000$ bursts per pulsar can be seen at $z \sim 1$ if amplified by a factor $\sim 14$ then they can account for the observed rate. This is comparable to the value of $N_0$ needed if bursts come from distances less than 250 Mpc. In this case, there may be two populations of pulsars responsible for ERBs, one nearby ($z \sim 0.1$) and another distant ($z \sim 1$). If much larger values of $A$ are needed for detection, then $N_0 \propto A^2$ soon becomes prohibitively large.

4 SHOT NOISE MODEL FOR ERBS

Radiation coherence underlies the high brightness temperatures of pulsar radiation and ERBs, as discussed previously. The fundamental unit of radio emission is a coherent shot pulse that has unit time-frequency product, so detection at radio frequencies $\nu \gtrsim 1$ GHz requires widths $W_\nu \lesssim 1$ ns. This picture is corroborated by detection of individual shot pulses from the Crab pulsar with durations $\sim \nu^{-1}$ [Hankins & Eilek 2007].

Actual pulses have large time-frequency products $\nu W_\nu \gg 1$ because they are incoherent combinations of coherent shot pulses. In this section we discuss the requirements for generating shot pulses that are sufficiently large to provide detectable amplitudes. We also briefly discuss the interplay between the number of incoherently combined shot pulses and the required shot-pulse amplitudes.

4.1 Maximal Shot Pulse Amplitudes

To account for ERB amplitudes, a model needs to provide shot pulses that are sufficiently bright and numerous to provide a detectable incoherent sum. First we consider an individual shot pulse from a single coherently-emitting entity, which might be a charge clump in the relativistic flow of a magnetosphere [Weatherall & Benford 1991; Weatherall 1998; Hankins et al. 2003; Asseo & Porzio 2006]. Clumps may be produced from a two-stream instability or a bunching instability associated with coherent synchrotron radiation [Goldreich & Keeley 1971; Buschauer & Benford 1978; Schmekel et al. 2005; Schmekel & Lovelace 2006; Schmekel 2005].

A simple estimate for maximal shot-pulse amplitudes is as follows. Consider a dense, relativistic charge clump that radiates a fraction $\epsilon_B$ of its total energy $E_\gamma = N \gamma mc^2$ ($N$ = number of particles) into a solid angle $\Delta \Omega \sim \gamma^{-2}$ in a time $W_\nu$, as seen by an observer. The shot pulse has unit time-bandwidth product (a consequence of Fourier transforms), so the spectrum extends to an upper frequency $\nu_s \sim 1/W_\nu$ with a spectral shape $P(\nu)$ normalized to unit area. For a receiver bandwidth $\Delta \nu_r \ll \nu$ at a center frequency $\nu \lesssim \nu_s$ and a source distance $d = 1 \text{ Gpc}$, the peak flux density
of the shot pulse is (c.f. Appendix A)

\[
S_v = \frac{\epsilon c E_v \Delta \nu_v P(\nu)}{\Delta t d^2} = \frac{\epsilon \gamma^7 N \eta n c^2 \Delta \nu_v P(\nu)}{(\gamma^2 \Omega)} \approx 10^{-29} \epsilon N \frac{\eta n}{t_{\text{shot}}} \Delta \nu_v (\gamma^2 \Omega)^{-1} P(\nu) \mu Jy,
\]

where the electron mass has been used to evaluate the expression and the factor \( \gamma^2 \Omega \) indicates that the relevant solid angle may differ from the assumed \( \Omega = \gamma^{-2} \). A 1-Jy shot pulse at \( d = 1 \) Gpc requires the energy in at least \( N = 10^{29} \) electrons and/or positions for nominal values of other parameters in this estimate. However, this number can be reduced substantially by adjusting \( \gamma \) and the distance and, as we argue, by incoherent aggregation of a large number of individual shot pulses.

The required number of particles can be expressed as \( N = n_\eta n c W \gamma^3 \lambda^3 \) where \( n_\eta n \) is a multiplier of the surface GJ density \( n_{\text{GJ}} \) and \( \gamma^3 \lambda^3 \) is the 'coherence' volume from which particles produce a coherent pulse measured by an observer; \( \gamma^3 \lambda^3 \) \( \eta \gamma \) \( \sim 1 \) is often assumed (e.g. [Benford & Buschauer 1977; Kinkhabwala & Thorsett 2000]), but for relativistic motion in a magnetic dipole, we find \( \eta \gamma \gg 1 \) (Appendix B). To provide the number of particles, the combined multiplier must satisfy

\[
\eta \gamma \sim 10^{14} \frac{P_{\gamma} d_{\text{GJc}}^2 n_{\gamma}}{\epsilon \gamma^7 \Delta \nu_v P(\nu) B_d G}\mu Jy.
\]

The multiplicity factor is reduced significantly if shot pulses have peak flux densities \( S_v \ll 1 \) Jy or if the spin period is small or the magnetic field and Lorentz factor large.

To calibrate the analysis, we revisit the 2 MJy shot pulse from the Crab pulsar at 9 GHz, which would become 2 \( \mu \)Jy if emitted at 1 Gpc. This shot pulse itself requires a large multiplier,

\[
(\eta \gamma)_{\text{Crab}} \approx 10^{10} \frac{d_{\text{GJc}}^2 n_{\gamma}}{\epsilon \gamma^7 \Delta \nu_v P(\nu) B_d G}\mu Jy.
\]

Pair production accounts for some of the multiplicity, typically \( 10^3 \) to \( 10^4 \) in pulsar models, and an \( e^\pm \) Lorentz factor \( > 10^3 \) could increase this further. The remaining multiplicity can come from the volume factor. Emission far from the NS surface increases the required multiplicity because the particle density scales as \( r^{-3} \), so high altitude emission regions require a volume multiplier that is comparable to that needed for ERBs.

An alternative picture is that observed pulses arise from a plasma maser or from a process involving ducted waves that couple to electromagnetic modes higher in the magnetosphere and therefore tap a much larger volume and total particle energy. These possibilities are beyond the scope of this paper. Instead, we focus on curvature radiation from multiple charge clumps in more detail.

### 4.2 Incoherent Summation of Shot Pulses

Most pulsars have shot-pulse sequences that appear as a modulated Gaussian process because the number of shot pulses contributing in a resolution time is large. These phenomena underlie the amplitude modulated noise model for pulsar emission (AMN; [Rickett 1975]) where a Gaussian noise process with characteristic time scales \( \ll \nu \) is modulated on time scales of microseconds and longer. While the AMN model is an adequate description of macroscopic pulses, a better physical description may be 'rate modulated shot noise,' a sequence of shot pulses with a time-dependent Poisson rate (Cordes 1985; Cortes et al. 2004), see also [Oslowski et al. 2011] and [Oswin et al. 2011]. In Appendix B we demonstrate that amplitude and rate modulations of shot pulses provide the same description of averaged pulses in the high-rate limit. A simulated amplitude-modulated shot pulse sequence is shown in Figure 3 where the density of shot pulses is small. Rate and amplitude modulations are indistinguishable unless individual shot pulses can be identified in high-resolution time series. Such is not yet the case for ERBs. Time resolution of order one nanosecond are needed so an ERB would have to be very bright to allow single-shot detections.

The scalar electric field selected by the receiver and feed antenna is a series of shot pulses (Appendix B).

\[
E(t, \nu) = \sum_j \eta_j A_j W_j \cdot \text{sinc}(2\pi \nu_0 (t - t_j)),
\]

where \( \text{sinc}(x) = \sin(x)/x \).

Taking the squared magnitude to get the flux density the average shape of the composite pulse is determined by the time dependence of the product of the shot-pulse rate \( \eta_j \) and average amplitude \( A_j = C_n |\eta_\nu| \) (where \( C_n \) accounts for the conversion from electric-field squared to flux density)

\[
\langle S_v(t) \rangle = \eta_\nu A_j W_j.
\]

Here we relate the shot-pulse parameters \( \eta_j, A_j, \) and \( W_j \) to the width \( W_i \) and amplitude \( S_{\nu, \text{max}} \) of the composite pulse comprising the incoherent sum of shot pulses.

Consider the incoherent volume \( V_i \) that contains coherent emitting regions of volume \( V_e \), as defined previously, that produce an ERB. These can be expressed in terms of light-travel distances as \( (cW_i)^3 \) and \( (cW_e)^3 \) but scaled in a model-dependent way by multipliers \( g_e \) and \( g_i \). The coherent multiplier is related to the multiplier \( g_i \) of the previous section as \( g_e = \eta_\nu/(cW_i)^3 \approx \eta_\nu \) if the coherent shot-pulse width is of order the inverse observation frequency \( \nu \). The multipliers are functions of the Lorentz factor(s) relevant to emitting particles, the radius of curvature of particle orbits (assuming transverse acceleration is important), expressed in units of the light-cylinder radius \( r_{lc} \) of a rotating object, and geometric factors. Then

\[
V_i = (cW_i)^3 g_e (\gamma, \tau_c/r_i), \quad V_e = (cW_e)^3 g_i (\gamma, \tau_c/r_i),
\]

in which geometric factors involving various angles are implicit in \( g_e \) and \( g_i \). The 'g' factors can increase or decrease the volumes significantly. The measured flux density depends strongly on the number of coherent shots that are summed. For a volume filling factor \( \zeta \), the number of coherently-emitting regions in \( V_i \) is

\[
N_e = \zeta \frac{V_i}{V_e} = \zeta \frac{g_i}{g_e} \left( \frac{W_i}{W_e} \right)^3.
\]

Observationally, the \( N_e \) shot pulses are spread over the time interval \( \sim W_i \), giving a shot-pulse rate \( \eta_j \sim N_e/W_i \). Combining all factors, we estimate the peak flux density of an ERB,

\[
S_{\nu, \text{max}} = \eta_j A_j W_i.
\]
from which we can solve for the required, typical shot-pulse amplitude,

\[ A_s = \frac{S_{\nu,\text{max}}}{\Delta \nu} \left( \frac{g_e}{g_i} \right) \left( \frac{W_s}{W_{\gamma}} \right)^2 \approx 10^{-1/2} S_{\nu,\text{max}} \left( \frac{g_e}{g_i} \right) \left( \frac{W_s}{W_{\gamma,\text{max}}} \right)^2. \]  

A sparsely-filled volume requires much larger shot pulses but increases the likelihood that individual shots may be detectable. The ratio \( g_e/g_i \) is of course model dependent. In Section 5 and Appendix D a curvature radiation model implies \( g_e \sim \gamma^3 \) if the spectrum peaks at the observation frequency; for Lorentz factors \( \gamma \sim 10^7 \) to \( 10^8 \), this can be a large factor. The incoherent factor \( g_i \) will also depend on \( \gamma \) in a model dependent way. Nominally, Eq. \( \text{[38]} \) suggests that shot pulse amplitudes can be much smaller than the observed peak ERB flux density when the shot noise has high temporal density, \( \eta \gamma W_{\gamma} \gg 1 \). When this is the case, the requirements on the properties of coherent regions in Section 4.2 see also Section 3 are relaxed significantly. The observed \( S_{\nu,\text{max}} \sim 1 \) Jy pulses can be accounted for with micro-Jy shot pulses if, conservatively by \( g_e/g_i \sim 10^{-6} \). Not coincidently, micro-Jy shot pulses are of order the amplitude of the largest Crab shot pulse observed to date (Section 4.2) if the Crab pulsed were placed at a 1-Gpc distance. Specific geometries and emission altitudes, along with the role of spin, are considered in Section 5 for estimating this factor for curvature radiation.

5 COHERENT CURVATURE RADIATION FROM A RELATIVISTIC BUNCH

Using general arguments we have shown that the largest shot pulse seen from the Crab pulsar requires extreme radiation efficiency and particle concentration; even more extreme conditions are required to produce \( \sim \) Jy shot pulses from distances \( \sim \) Gpc. However, incoherent summation of shot pulses, as noted, can allow the required shot pulse amplitudes to be comparable to the maximum seen so far from the Crab pulsar. Here we derive the requirements for the specific case of curvature radiation.

The estimate for the particle energy and number was estimated in Section 4 to produce detectable shot pulses. We assumed that both the required energy and the shot-pulse width (the reciprocal of the spectral width) could be satisfied. Curvature radiation from the required number of particles, if identically charged, would produce a shot pulse with too short a lifetime to be consistent with the assumed spectral bandwidth. The clump therefore needs to be nearly charge neutral.

5.1 Fractional Clump Charge

A simple estimate of the net fractional charge that can produce a consistent result is as follows. A single electron moving on a curved path with radius \( r_e = 10^7 r_e, G \) cm radiates power \( P_1 = 2 \pi e^2 c/3r_e^2 \) and has a long radiation lifetime,

\[ \tau_l = \frac{\gamma m_e c^2}{P_1} = \frac{3}{2 \gamma r_e^2} = 10^{11.2} \gamma^{3.2} r_e^2. \]  

(39)

The lifetime of \( N \) charges of the same sign radiating coherently at a rate \( P_N = N^2 P_1 \) is \( \tau_N = \tau_l/N \). A shot pulse with observed duration \( r_e/\gamma c \) corresponds to a time interval \( \tau_l/\gamma c \) over which the clump radiates toward the observer. However, if radiation losses cause the clump lifetime to be shorter than this time, \( \tau_l < r_e/\gamma c \), the observed pulse width will be narrower, \( \tau_N/\gamma c \), and the spectrum correspondingly wider.

For the nominal particle number \( N = 10^{17} \) calculated earlier, the lifetime \( \tau_N = 10^{-17.8} \) s is far too short to yield enough power in the radio band. The solution is for the clump of \( N \) particles to be nearly charge neutral. Letting the effective charge be \( N \gamma e \), the clump lifetime becomes \( \tau_c = N \tau_l/N^2 \) and matching it to the ‘beaming’ time \( r_e/\gamma c \) yields

\[ \frac{N \gamma}{N} = \left( \frac{\tau_l}{N \tau_c} \right)^{1/2} = \left( \frac{3 \pi}{2 \gamma r_e^2 N} \right)^{1/2} = 10^{-6.7} \gamma^{1/4} N^{-1/2} \gamma_{N/19} = 10^{-6.7} \left( \frac{\gamma c r_e \Delta \nu_{\nu} P(\nu)}{S_{\nu,\text{max}} d^2 \nu \text{pc}} \right)^{1/2}. \]  

(40)

The nearly-charge-neutral clump can radiate the total energy available in the clump but at a rate that maximizes emission in the radio band. We do not specify the mechanism that forms and holds together the clump, but it must radiate as a coherent unit long enough to produce observed \( \lesssim 1 \) ns shot pulses.

5.2 Coherent Shot Pulses from Curvature Radiation

We consider radiation from a relativistic blob of charge with Lorentz factor \( \gamma > 1 \) and containing \( N \) particles, presumed to be electrons and positrons, but a net electric charge \( Q_e \) produced in some short-lived, burst-like magnetospheric event. The blob is presumed to move along a curved path (probably a particular magnetic field line), emitting curvature radiation that is optimally beamed toward the observer at some point along the path where the curvature is \( r_c \). For notational simplicity, in this section we use \( \omega = 2 \pi \nu_r \). The observed flux density is

\[ S_0(\omega) = \frac{3}{4 \pi^2} \left( \frac{\omega r_e/c}{\sqrt{3e^2}} \right)^3 (2 m_e^2) d \nu \text{f}(\xi) \]  

(41)

where, for an observer situated at an angle \( \epsilon \) relative to the instantaneous plane of motion, \( \xi = (\omega r/e, 3e)/(2m) \) and, if \( \xi = \omega r/c (3e)^2/4 \pi^2 \),

\[ f(\xi) = \pi^{1/3} K^2_{0,3}(\xi) + \left( 1 - \frac{\epsilon^{2/3}}{\sqrt{17 \pi}} \right) K^2_{1,3}(\xi) \]  

(42)

Jackson \text{[1962]}; the effective squared charge of the blob is

\[ q^2(\omega) = N + Q E(\omega) \text{coh}. \]  

(43)

where the first term arises from incoherent radiation and the second from coherent radiation. In the language of Eq. \( \text{[2]} \) the total energy radiated over the portion of the orbit where direction is directed toward the observer is \( \sim \gamma^4 q^2(\omega)/\gamma^2 c \times \tau_c/\gamma c \sim \gamma^4 q^2(\omega)/(\tau_c/\gamma c) \) in the frequency spectrum; and \( \Delta \nu \sim (c/\omega r_e)^{2/3} \) at frequency \( \omega \lesssim \gamma^3 c/r_e \). The form factor \( F(\omega/\omega_{coh}) \) is expected to be \( \sim 1 \) for coherent emission; this will be true for observed frequency \( \omega \lesssim \omega_{coh} \). Conditions for coherence are reviewed in Appendix D. For example, for a burst that lasts a time \( t_b \) and emits charges along a single field line uniformly in time over timespan \( t_b \), \( F(\omega/\omega_{coh}) = \left[ \sin(\omega/\omega_{coh})/(\omega/\omega_{coh}) \right]^2 \), with \( \omega_{coh} = 1/2 t_b \). In Eq. \( \text{[41]} \) we assume that the detector band width \( \Delta \nu \) is narrow compared with the relatively broad band spectrum of instantaneous curvature radiation.

Eq. \( \text{[11]} \) implies pitifully small flux density unless coherence is substantial. Thus, we presume that \( F(\omega/\omega_{coh}) \gtrsim 1 \) for individual relativistic
blobs. We assume that $\omega_{coh}$ is below, but not necessarily far below, the peak frequency $\omega_{pk} = 2\pi v_{pk} = c^2/\epsilon_r$, where $\epsilon_r$ is the local curvature of the charged blob path when it is beamed toward the observer:

$$\frac{\omega_{coh}}{\omega_{pk}} = \left(\frac{\gamma_{coh}}{\gamma}\right)^2 \gamma_{coh} \omega_{coh} - \frac{\omega_{coh} r_c}{c} \epsilon_r^{1/3} = 10^3 \left(P_{vcoh,GHz}\right)^{1/3} \epsilon_r^{1/3} (r_c/r_t)^{1/3}$$

(44)

where $P$ is the pulsar period in seconds, and $v_{pk} = c/\Omega = \epsilon_pP/2\pi = 4.77 \times 10^9 P$ cm is the light cylinder radius. For comparison, the Lorentz factor required for the energy emitted in incoherent curvature radiation to be significant during a timescale $\delta t_e = \epsilon_r r_c/c$ is

$$\gamma_{incoh} = \left(\frac{3\nu}{2c^2\epsilon_r}\right)^{1/3} = \frac{2.94 \times 10^7 P^{1/3} (r_c/r_t)^{1/3}}{\epsilon_r^{1/3}} = \frac{2.94 \times 10^5 \nu_{coh}}{(\epsilon_r \nu_{coh},GHz)^{1/3}}$$

(45)

The characteristic Lorentz factor associated with pulsar voltage drops is

$$\gamma_p = \frac{\epsilon_p^2 H}{2m_e c^2} = \frac{1.29 \times 10^7 \mu_0}{P^2}$$

where $\mu_0 = 10^{-10} \mu_0$ G cm$^3$ is the pulsar magnetic moment (e.g. [Ruderman & Sutherland 1975]).

To get a rough upper bound on the coherent emission, we impose two constraints: (1) coherent radiation dominates over incoherent radiation and (2) radiation losses are relatively minor during emission. To keep the treatment simple, suppose that $r_c$ is roughly uniform along the particle path.

Then the total energy radiated breaks into two pieces: the coherent part, which is from $\omega \lesssim \omega_{coh}$, where $P(\omega/\omega_{coh}) \sim 1$, and the incoherent part that arises from $\omega \sim \omega_{pk}$:

$$E_{coh} \sim \epsilon_r N e^2 r_c^4$$

$$E_{incoh} \sim \epsilon_r N e^2 (\omega_{coh} r_c/c)^{4/3}$$

We assume that $\delta t_e \gtrsim r_c/(2c^2) r_c \sim (r_c/c)^2 \omega^{-1/3}$, which is the characteristic timescale at the emitter for radiation toward a particular observer at frequency $\omega$, but allow for the possibility $\delta t_e \ll r_c/c$, in which case observation of the blob is highly fortuitous; Consequently, with $\nu = \omega/2\pi = 10^9 \nu_{GHz}$,

$$1 \gtrsim \epsilon_r \gtrsim \frac{[10^{-3} (\nu_{GHz})^{1/3} (r_c/r_t)^{1/3}]^{1/3}}{\epsilon_r^{1/3}}$$

(48)

Near the lower bound, from the point of view of the observer the blob only exists for the time $\sim \omega^{-1}$ during which it emits coherently toward the observer, a doubly lucky circumstance. Eq. (47) shows that the coherent contribution dominates if

$$\frac{\gamma_{coh}}{\gamma_{incoh}} \gtrsim \frac{\omega_{coh}}{\omega_{pk}}$$

(49)

Since the total energy contained in the burst is $E_b = N \gamma_m c^2$, then if we assume that coherent radiation dominates, radiation reaction does not severely limit the lifetime of a coherently emitting clump as long as

$$\frac{Q_{coh}^2}{N} \lesssim \gamma_m c^2 r_c^4$$

$$= \gamma_{incoh} \epsilon_r N e^2 (\omega_{coh} r_c/c)^{4/3} = \frac{c e_r \nu_{coh}}{\omega_{coh}} = \frac{\omega_{pk}}{\omega_{coh}}$$

(50)

Eqs. (49) and (50) are consistent with one another provided that $\gamma \lesssim \gamma_{incoh}$.

In order to understand the physical requirements for this model, we investigate the conditions necessary for Eqs. (41) and (50) to account for the brightest giant pulse from the Crab pulsar. Eq. (41) implies a flux density

$$S_0(\omega) \approx 2.95 \times 10^{24} \frac{Q_{coh}^2}{Q_{coh},GHz} [\nu_{coh}/(c/e_r)]^{1/3} \times 10^{-9} \times 10^{-5} \times 10^{-4}$$

(51)

for coherent curvature radiation, where $Q_{coh} = 10^{20} Q_{coh,21}$ and $C(\omega/\omega_{coh}) = (\omega/\omega_{coh})^{2/3} F(\omega/\omega_{coh})$; Eq. (50) implies

$$N \gtrsim \frac{5.92 \times 10^{29} Q_{coh}^2}{\nu_{coh}/(c/e_r)^{1/3}}$$

(52)

Thus, the blob must be nearly neutral, with a concentrated charge $\gtrsim 10^{22}$ to account for the brightest Crab giant pulse; at $d \sim 1$ Gpc, such a blob would be detected with a flux density $\sim 10^3$ Jy. A flux density $\sim 1$ Jy at $d \sim 1$ Gpc requires $Q_{coh} \sim 10^{23}$. Incoherent summation of individually coherent shots could lower the required charge per coherently emitting clump dramatically; if each shot pulse contributes $f\gamma$, then $Q_{coh} \sim 10^{24} \sqrt{f} \sim 10^{21} \sqrt{f}$, which may be comparable to the charge required for the brightest shot pulse from the Crab pulsar. Just how neutral the blob must be depends on

$$1 \gtrsim \epsilon_r (\omega_{coh}/\omega_{pk})^{1/3} \gtrsim \gamma^{-1} (\omega_{coh}/\omega_{pk})^{1/3}$$

$$\left[\frac{c}{\nu_{coh}}\right]^{1/3}$$

(53)

The charge per lepton in a nearly neutral blob is $Q_{coh}/N$, so we expect acceleration by an electric field to result in $\gamma \lesssim (Q_{coh}/N)_{\gamma_p}$, or

$$Q_{coh}/N \approx \frac{\gamma_{coh}}{\gamma_p} \left(\frac{\omega_{pk}}{\omega_{coh}}\right)^{1/3} = \frac{7.75 \times 10^{-5} P^{1/3} \mu_0^{-3/4} r_c^{1/3} (r_c/r_t)^{1/3} (\omega_{pk}/\omega_{coh})^{1/3}}{\epsilon_r^{1/3}}$$

(54)

consistency with Eq. (52) implies

$$Q_{coh,21} \epsilon_r \lesssim 2.18 \times 10^{-4} P^{1/3} \mu_0^{-3/4} \left[\frac{c}{\nu_{coh}/(c/e_r)^{1/3}}\right]^{1/3} \equiv Q_{coh,21} \epsilon_r \lesssim 0.218 (\omega_{coh}/\omega_{pk})^{1/3}$$

(55)

Eq. (55) favors fortuitously short lives for the brightest bursts, $\epsilon_r \sim (c/\omega r_c)^{1/3}$, moreover, it suggests that $Q_{coh} \sim 10^{24}$ is only possible for $P \lesssim 0.03$ s, but incoherent summation of shot pulses mitigates this spin period restriction. Note that Eq. (53) implies a total lepton number $N \sim 10^{29}$ for $Q_{coh} \sim 10^{24}$, consistent with our rougher estimates in [44,1]. Incoherent summation of shot pulses would lower these values to $N \sim 10^{25} \sqrt{f}$ and $Q_{coh} \sim 10^{21} \sqrt{f}$.

Now that we have an idea of the physical attributes of the blob must have, we can consider other constraints. The blob experiences drag by ambient radiation in addition to the "radiation reaction drag" it experiences by interactions with its own radiation field. The drag force depends on how
opaque the blob is, which requires us to model its shape, but at a given \( N \) an upper bound is found by assuming that the blob is optically thin, which implies that the blob loses energy via drag at a rate \( -\dot{E}_{\text{drag}} \lesssim N^2\sigma v f(k)U_{\text{rad}} \), where \( U_{\text{rad}} \) is the ambient radiation energy density in the inertial frame where the blob moves relativistically. The factor \( f(k) \) depends on the characteristic photon energy \( k r c^2 \) in the rest frames of scattering leptons in the blob; \( f(k) < 1 \) in general, \( f(0) = 1 \) and \( f(k) \approx 3 \ln(2k)/8k \) for \( k \gg 1 \) (e.g. Jauch & Rohrlich 1976). Since the blob has energy \( N^2 \gamma m c^2 \), the drag lifetime is \( \tau_{\text{drag}} \), where

\[
\tau_{\text{drag}} = \frac{mk c^2}{\sigma v c f(k)U_{\text{rad}}} = \frac{5.61 \times 10^4 \omega_{\text{coh}} / \omega_{\text{ph}}}{\nu_{\text{coh}} / \nu_{\text{ph}}} (r_c/r_s)^{1/3} \int k_2 f(k)U_{\text{rad}} / \int c \xi \frac{f(k)}{U_{\text{rad}} / \int c} \quad ;
\]

we estimated \( U_{\text{rad}} \) by \( U_0 = 1.3 \times 10^{51} \text{erg} / \text{cm}^3 \), \( \rho = 3 \times 10^{-3} \text{erg} / \text{cm}^3 \), \( P = 5 \times 10^{34} \text{erg} / \text{s} \) is the spin-down rate. Although Eq. (55) suggests that drag is unimportant for nominal parameter values, this may be deceptive in specific cases: for Crab pulsar parameters, Eq. (55) implies

\[
\tau_{\text{drag}} \geq 5 \times 10^{-4} \omega_{\text{coh}} / \omega_{\text{ph}} c^{-2/3} \quad \text{(Crab)}
\]

suggesting a short lifetime: for comparison, \( \tau_d \gtrsim 3 \times 10^{-3} \nu_{\text{Ghz}} / (r_c/r_s) \)^{-1} \( \text{yr} \) for the Crab according to Eq. (48), comparable to but larger than Eq. (56). The mild discrepancy may be resolved if the blob is moderately opaque even if \( k \ll 1 \); and the drag timescale is considerably longer — perhaps even \( \tau / c - 1 \) if \( k \gg 1 \).

Since the blob consists of \( N \) electrons and positrons with a relatively small net excess of one charge \( Q \approx N \gamma \) its lifetime may also be limited by \( e^2 \) annihilation. If \( n_e^0 \) is the pair density in the rest frame of the blob, then the annihilation rate per volume, a Lorentz scalar, is \( \sim n_e^2 = n_e^0 (\sigma v f(k)U_{\text{rad}} / \int c) \); then the annihilation rate coefficient for relative motion with Lorentz factors \( \gamma \), where \( A(1) = 1 \) and \( A(\gamma) \approx \ln \gamma / \gamma_0 \), for \( \gamma \gg 1 \) (Jauch & Rohrlich 1976). (We assume that the blob is hot enough in its rest frame that we can neglect annihilation via the formation of positronium.) Lorentz transform to the inertial frame of the magnetosphere, where the density is \( n_s = n_e \gamma \gamma_0 \); then the annihilation rate per lepton in this frame is \( n_s (\sigma v f(k)U_{\text{rad}} / \int c) / \gamma^2 \gamma_0 \) in this frame, implying a lifetime \( \tau_{\text{ann}} \), where

\[
\tau_{\text{ann}} = \frac{\gamma_n c}{n_s \sigma v f(k)U_{\text{rad}}} \sim 8 \omega_{\text{coh}}^2 (r_c/r_s)^{-2/3} \nu_{\text{coh}} c^{-2/3} \quad \text{(57)}
\]

where \( \gamma_n \) is the Thomson optical depth of the blob and the length of the blob along its direction of motion is \( \sim c / \omega_{\text{coh}} \). The lifetime of the blob against pair annihilation is long compared with its drag and radiation reaction lifetimes.

A possible mechanism for the formation of the relativistic blobs responsible for the sub-bursts is the bunching instability associated with coherent synchrotron radiation. There has not yet been a treatment of this instability that made specific reference to a nearly neutral system with \( e^2 \) pairs: analyses done to date (Goldreich & Keeley 1971; Schmekel et al. 2005; Schmekel & Lovelace 2006) considered configurations of relativistic electrons only. Linear growth rates found in these treatments appear to be fast enough to initiate the instability on timescales much shorter than \( t_\text{dr} \) on small length scales. However, we have seen that coherent emission requires a large number of leptons confined to a small volume, leading to extremely large density contrast relative to the GI density; thus a nonlinear analysis is needed to establish that this mechanism is viable. Another possible mechanism for generating dense clumps is the two stream instability (e.g. Ruderman & Sutherland 1975; Benford & Buschauer 1977; Buschauer & Benford 1978).

Magnetospheric reconnection events conceivably may play a role by producing relativistic charge streams or could trigger secondary activity (e.g. pair production) that leads to giant bursts.

### 6 SUMMARY & CONCLUSIONS

In this paper, we have examined the hypothesis that fast radio bursts are associated with rare bright pulses from extragalactic neutron stars. If sources are at cosmological distances, only a few pulses per NS can account for the estimated rate of ERBs. The number of pulses required per NS scales inversely as the population volume, but even for substantially closer populations, the pulse rate is still too low to expect any repeats. Population distance interplays significantly with roles of host galaxies and the IGM in accounting for the dispersion measures of ERBs.

From a phenomenological standpoint, the largest single giant pulse observed to date from the Crab pulsar (Hanks & Eilek 2007) could only be detected within the Local Group at a flux density \( \gtrsim 1 \text{ Jy} \). However, in Section 4.3 we used the distribution of flux densities for Crab giant pulses from observations lasting \( \approx 10 \text{ hours} \) to conclude that the brightest single giant pulse emitted during the entire lifetime of the pulsar could have been bright enough to have been visible at distances \( \approx 15 - 300 \text{ Mpc} \). Moreover, in Section 3 we argued that the inferred fast radio burst rate \( \approx 10^3 \text{ d}^{-1} \) is consistent with the idea that they originate with neutron stars out to cosmological distances. Strong gravitational lensing by stars may contribute to the rate of detectable bursts if the source population extends to \( z \gtrsim 1 \). ERBs from neutron stars will repeat, but only very rarely, so that from an observational standpoint, no repeats are needed over time scales of years or decades in order to account for the inferred ERB rate. The NS birth rate is approximately equal to the rate of core-collapse supernovae (CCSNae).

From an astrophysical standpoint, general arguments in Section 4.1, as well as considerations of a specific, curvature-radiation model, in Section 4.2 indicate that large bursts detectable to cosmological distances must involve large numbers of coherently radiating particles that emit efficiently at radio frequencies. Radiation reaction must be important, limiting the lifetimes of the coherently emitting clumps. Requirements on particle numbers are reduced significantly by taking into account the incoherent summation of coherent slot pulses (Section 4.3). Energetic requirements are, however, modest (\( \approx 10^{52} \text{ ergs} \)), but point to clumps that are nearly neutral electrically, with a net excess of charge that is small fractionally. Thus, the required physical parameters, the brightest individual slot pulse observed to date from the Crab pulsar (\( \approx 2 \text{ MJy} \)) is comparable to that needed for an ensemble of slot pulses that comprise an ERB originating from 0.1 to 1 Gpc distances.

While we have focussed on strong radio bursts from active pulsars as exemplars, dormant NSs are much more numerous. If a dormant NS can be revived intermittently — or even just once during its entire \( \approx 10 \text{ Gyr} \) “dead time” — then it might produce ERBs. Resurrecting old pulsars would generally require some sort of triggering mechanism, either internal (e.g. accretion of an asteroid) or external (e.g. a massive starquake). Another possibility may be triggering of NS magnetospheres by jet particles from supermassive black holes (SMBHs) in the centers of galaxies. Triggering mechanisms tied to the local environment may involve proximity of a NS to its host galaxy or to the SMBH in its host galaxy. In the latter case, there may be evolution of the ERB population with redshift unless the ERB population is at low redshifts, which may be the case if a significant amount of the observed dispersion measures are from galactic centers.

Given the similarity of the CCSN rate on cosmological distances to the apparent ERB rate, it is possible that ERBs are more-directly connected.
For example, coherent emission might be produced when a proto-NS forms during the immediate aftermath of core collapse. If this coherent emission can either punch through the supernova ejecta or escape through pathways through the ejecta, it might correspond to an ERB. Alternatively, CCSNae might be able to resuscitate nearby, dormant NSs if they happen in regions of high NS density, such as in galactic centers. Triggering might consist of an enhancement of magnetic reconnection in the magnetotails of NS that injects particles into their magnetospheres that in turn stimulate pair production and coherent radio emission.

Polarization of ERBs deserves mention even though we have not addressed it directly in this paper. It is well known that curvature radiation alone cannot account for observed pulsar polarization, which is generally highly elliptically polarized, especially when measured with high time resolution. Individual pulses from the same pulsar show substantial variability in the degree and type of polarization and there are wide differences between pulsars. The broad conclusion has been that the coherence mechanism along with propagation effects in magnetospheres account for measured polarization (Cheng & Ruderman 1979, Barnard & Arons 1986). We anticipate that the same degree and variability of ERB polarization will be seen in observations of the full Stokes parameters of ERBs. In that sense, we will learn about the emission process of ERBs to the same degree as we have from pulsars.

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APPENDIX A: FLUX DENSITY FROM SHOT PULSES
Consider a pulse \( E(t) \) whose Fourier transform \( \hat{E}(\nu) \) is sampled with a receiver covering a bandwidth \( \Delta \nu_r \) centered on a frequency \( \nu_0 \). The Fourier transform is defined as \( \hat{E}(\nu) = \int dt \, E(t)e^{-2\pi i \nu t} \). The selected electric field for a rectangular bandpass shape is (showing only positive frequencies)

\[
E(t, \nu_0; \Delta \nu_r) = \int_{-\nu_0 + \Delta \nu_r/2}^{\nu_0 + \Delta \nu_r/2} \frac{d\nu}{\Delta \nu_r} \hat{E}(\nu)e^{2\pi i \nu t} = \Delta \nu_r \int_{-\infty}^{\infty} dt' E(t')e^{2\pi i \nu_0(t-t')} \sin \Delta \nu_r(t-t'),
\]

where \( \sin x = (\sin \pi x)/\pi x \). For a pulse occurring at time \( t_s \) with an intrinsic bandwidth much wider than the receiver bandwidth \( \Delta \nu_r \) (i.e. the pulse is unresolved with the receiver system),

\[
E(t, \nu_0; \Delta \nu_r) \approx \Delta \nu_r \hat{E}(\nu_0)e^{2\pi i \nu_0(t-t_s)} \sin \Delta \nu_r(t-t_s).
\]

To account for negative frequencies, we would take twice the real part of this expression.

The power collected in an area \( A \) is \( P_{\nu_0}(t) = (cA/2\pi)|\hat{E}(t, \nu_0; \Delta \nu_r)|^2 \) so the measured flux density is

\[
S_{\nu_0}(t) = \frac{c|\Delta \nu_r|\hat{E}(\nu_0)^2}{2\pi} \sin^2 \Delta \nu_r(t-t_s).
\]

The integrated flux density (fluence) is independent of the bandwidth and is equal to

\[
\int dt \, S_{\nu_0}(t) = \frac{c|\Delta \nu_r|\hat{E}(\nu_0)^2}{2\pi} \frac{c\hat{E}(\nu_0)^2}{2\pi}.
\]

APPENDIX B: RATE AND AMPLITUDE-MODULATED SHOT NOISE
An incoherent superposition of coherent shot pulses is required to account for the electromagnetic and statistical properties of observed pulses, as described in the main text. A sum of shot pulses of the form of Eq. (A2), each having a width \( \Delta \nu_r \), forms a composite pulse with large time-bandwidth product through the time dependence of the amplitudes \( \alpha_s \) as the power emitted per unit frequency into a direction

\[
\langle I(t) \rangle = \int dt' \, \eta_s(t')A_s(t') \sin \Delta \nu_r(t-t')
\]

where \( \eta_s(t) = \langle |\alpha_s(t)|^2 \rangle \). For rates and amplitudes that vary slowly compared to the inverse bandwidth, we have

\[
\langle I(t) \rangle = \eta_s(t)A_s(t)W_s,
\]

indicating that the average pulse shape is determined by the shot-rate pulse or the amplitude modulation or by a combination of the two. Also, the intensity in a time interval that resolves the pulse is linear in the shot-pulse rate.

APPENDIX C: ELECTRIC FIELD AND FLUX DENSITY IN COSMOLOGY
Work in conformal coordinates, \( ds^2 = a^2(t) \eta_{\alpha\beta} dx^\alpha dx^\beta \), which coincide with physical coordinates at the present day, when \( a(\tilde{t}) \equiv 1 \); \( \tilde{t} \) is the comoving time coordinate. These coordinates are especially convenient for electrodynamics, and the vacuum solution for outoging waves far from a source near \( r = 0 \) and \( \tilde{t} = \tilde{t}_s \) is

\[
F_{\alpha\beta} = \frac{1}{r} \int_{-\infty}^{\infty} d\tilde{t} \, \hat{E}_\alpha(\tilde{t}, \tilde{\nu}) \exp[2\pi i \tilde{\nu}(\tilde{t} - \tilde{t}_s - r)]
\]

where \( \hat{E}_\alpha(\tilde{t}, \tilde{\nu}) \) is a constant vector perpendicular to \( \hat{\mathbf{e}} \). The Poynting flux is therefore

\[
S(\hat{\mathbf{e}}, \alpha) = \frac{c^2}{4\pi e^2} \int_{-\infty}^{\infty} d\tilde{t} \, \hat{E}_\alpha(\tilde{t}, \tilde{\nu}) \hat{E}_\alpha(\tilde{t}, \tilde{\nu}) = \frac{c^2}{4\pi e^2} \int_{-\infty}^{\infty} d\tilde{t} \, |\hat{E}(\tilde{t}, \tilde{\nu})|^2.
\]

The total luminosity near the source when the scale factor is \( a_s \) is

\[
L_s = \frac{2c}{\alpha_s} \int_0^{d_L} d\tilde{r} d\tilde{\nu} |\hat{E}(\tilde{t}, \tilde{\nu})|^2,
\]

and the present-day observed flux in direction \( \hat{\mathbf{e}} \) is

\[
\hat{\mathbf{e}} \cdot S(\hat{\mathbf{e}}, 1) = \frac{L_s F(\hat{\mathbf{e}})}{4 \pi d_L^2} \quad \text{with} \quad F(\hat{\mathbf{e}}) = \frac{1}{4} \int_{-\infty}^{\infty} d\tilde{t} d\tilde{\nu} |\hat{E}(\tilde{t}, \tilde{\nu})|^2.
\]

where \( d_L = r/\alpha_s \) is the luminosity distance. If we define \( P_s(\hat{\mathbf{e}}, \nu_s) \) as the power emitted per unit frequency into a direction \( \hat{\mathbf{e}} \), then

\[
L_s = \int_0^{\infty} d\nu_s 4\pi d^2 P_s(\hat{\mathbf{e}}, \nu_s)
\]
where \( \nu_c = \tilde{\nu}/a_z \) is the local frequency at the source, and \( P_c(\hat{n}, \nu_c) = 2\pi \nu_c^{-1}|\tilde{\mathbf{E}}(\hat{n}, \nu_c, a_z)|^2 \).

At scale factor \( a \), the local electric field is

\[
\mathbf{E}(a) = \frac{1}{a^2r} \int_{-\infty}^{\infty} d\nu \tilde{\mathbf{E}}(\hat{n}, \nu) \exp[(2\pi i (\tilde{\nu} - \tilde{\nu}_0) - r)]
\]

\[
= \frac{1}{a^2} \int_{-\infty}^{\infty} dv \tilde{E}(\hat{n}, \nu a) \exp[2\pi i (\nu - a\tilde{\nu} - ar)]
\]

so the Fourier transform of the local electric field is \( \tilde{\mathbf{E}}(\hat{n}, \nu a)/a^2r \) (discarding a constant phase). From Appendix A the flux density in a narrow band centered on an observed frequency \( \nu \) is

\[
S_\nu(a) = \frac{c\Delta \nu a P_c(\hat{n}, \nu a/a_z)}{2\pi (ar)^2} \sin^2 \Delta \nu (t - t_a)
\]

\[
= \frac{\Delta \nu a P_c(\hat{n}, \nu a/a_z) \sin^2 \Delta \nu (t - t_a)}{4\pi (ar)^2}
\]

To get the flux density at frequency \( \nu \) here and now put \( a = 1 \) in Eq. (C7), the result is

\[
S_\nu(1) = \frac{\Delta \nu a P_c(\hat{n}, \nu/a_z) \sin^2 \Delta \nu (t - t_a)}{4\pi a^2}
\]

and therefore

\[
S_\nu(1) = \frac{d^2S_\nu(1)}{d\nu d\nu} = \frac{d^2S_\nu(1)}{d\nu d\nu} = \frac{d^2S_\nu(1)}{d\nu d\nu} \cdot
\]

which defines the “flux density distance,” \( d_L \).

APPENDIX D: CONDITIONS FOR COHERENCE

Let the observer be in direction \( \hat{n} \) relative to the the source, and let \( t_{e,c} \) be the time when emission is optimally beamed toward the observer, for a given accelerated charge moving on a path with radius of curvature \( r_c \). This is the time when \( \kappa = 1 - \hat{n} \cdot \mathbf{v}(t_{e,c}) \) is smallest; for strong beaming, this happens when \( \hat{\mathbf{n}} \cdot \mathbf{a}(t_{e,c}) = 0 \) (i.e. when the acceleration is perpendicular to the line of sight), which is not guaranteed to occur but is required for super-bright emission. At this time, the charge is at \( \mathbf{r}(t_{e,c}) \). Then define a null four-vector

\[
n_{\mu} = (-1, \hat{n})
\]

in the inertial frame of the observer; contracting with the four-vector \( x^\mu(t_{e,c}) = (t_{e,c}, \mathbf{r}(t_{e,c})) \) we get

\[
n_{\mu} x^{\mu}(t_{e,c}) = -t_{e,c} + \hat{n} \cdot \mathbf{r}(t_{e,c})
\]

Then the condition that a collection of particles radiates coherently is

\[
|\Delta (n_{\mu} x^{\mu}(t_{e,c}))| \lesssim r_c (2\epsilon_{coh})^{3/2} \lesssim \frac{c}{\omega_{coh}}
\]

where at optimal beaming \( \kappa = \kappa_0 = 1 - \hat{n} \cdot \mathbf{v}(t_{e,c}) \equiv 1 - v \cos \theta = 1 - v \cos \frac{1}{2}(\epsilon^2 + 1/\gamma^2) \), and we assume coherent radiation limited to \( \omega \lesssim \omega_{coh} \), for which \( \kappa = \kappa_{coh} \). Geometrically, the observer is at angle \( \epsilon \) above the plane of the (instantaneously circular) orbit of the emitting particle.

Eq. (D2) is derived by considering how long a pulse from a given charge is bright as seen by the observer. Eq. (D3) has a simple interpretation: for an electromagnetic wave of frequency \( \omega \) in the inertial frame of the observer, the wavevector is \( \mathbf{k}_\mu = \omega \gamma_\mu \) and \( k_\mu x^{\mu}(t_{e,c}) \) is the phase associated with “launching” at time \( t_{e,c} \), so the phase difference associated with a set of waves launched at different times and places, each of which can achieve optimal beaming. Note that the volumes constrained by Eq. (D3) represent upper bounds: actual volumes that emit coherently will depend on the emission mechanism. For example, for charges that are all launched at the same place but at different times and travel along identical trajectories Eq. (D3) implies that the range of launch times is \( \Delta t_{e,c} \lesssim r_c (2\epsilon_{coh})^{3/2} \).

As an example, we consider motion along magnetic dipole field lines, which are along the curves \( r = R_e \sin^2 \theta \), where \( R_e = R_e \sin^2(\theta_0) \) is a constant along a given field line, where \( R_e \) is the stellar radius and \( \theta_0 \) is the polar angle of the foot of the field line at \( r = R_e \). For a dipole field with symmetry axis along \( \hat{z} \) the unit vector tangent to a field line is

\[
\hat{l} = \frac{3 \cos \hat{\theta} - \hat{z}}{\sqrt{3 \cos^2 \hat{\theta} + 1}} \quad \frac{3 \cos \hat{\theta} \sin \hat{\phi} + \hat{y} \sin \hat{\phi} + (3 \cos^2 \hat{\theta} - 1) \hat{z}}{\sqrt{3 \cos^2 \hat{\theta} + 1}}
\]

for angles \( \hat{\theta}, \hat{\phi} \) referenced to the dipole axis and the unit vector in the curvature direction is

\[
\hat{c} = \hat{\phi} \times \hat{l} = \frac{(3 \cos^2 \hat{\theta} - 1)(\hat{\phi} \cos \hat{\phi} + \hat{y} \sin \hat{\phi}) - 3 \cos \hat{\theta} \sin \hat{\theta} \hat{z}}{\sqrt{3 \cos^2 \hat{\theta} + 1}}
\]

Using \( d\hat{l}/ds = \hat{c}/R_e \), the radius of curvature of a field line is

\[
r_e = \frac{r(3 \cos^2 \hat{\theta} + 1)^{3/2}}{3 \sin \hat{\theta}(\cos^2 \hat{\theta} + 1)}
\]

The direction to the observer is \( \hat{n} = \hat{x} \sin \alpha + \hat{z} \cos \alpha \). At the time of optimal beaming from a particular field line \( \hat{n} \cdot \mathbf{a} = 0 = \hat{n} \cdot \hat{c} \), since the acceleration \( \alpha \) is along \( \hat{c} \), so

\[
0 = -3 \cos \hat{\theta} \sin \hat{\theta} \cos \alpha + (3 \cos^2 \hat{\theta} - 1) \cos \hat{\phi} \sin \hat{\phi} \alpha.
\]
Strong beaming requires that $\epsilon \ll 1$; in this limit, Eq. (D7) implies
\[
\cos^2 \theta(\epsilon) = \cos^2 \theta(0) + \frac{\epsilon^2 \cos \alpha (\sqrt{8 + \cos^2 \alpha + \cos \alpha})^2}{12 \sqrt{8 + \cos^2 \alpha}}
\]
\[
\phi(\epsilon) = \pm \frac{\epsilon}{\sin \alpha}
\]  
(D8)

to lowest order in $\epsilon$. The displacement $\Delta r$ from the point $r(0) = R_i \sin^2 \theta(0)(\hat{x} \sin \theta(0) + \hat{z} \cos \theta(0))$ where emission is beamed directly at the observer has two components:

(i) Change field lines, remaining in the $x - z$ plane; the associated displacement is
\[
\frac{\Delta R_i r(0)}{R_i} = \Delta R_i \sin^2 \theta(0)(\hat{x} \sin \theta(0) + \hat{z} \cos \theta(0))
\]
\[
= \Delta R_i \sin \theta(0) \left[ \frac{2\Delta \sin \alpha}{3} - \frac{\epsilon(0)(\sqrt{8 + \cos^2 \alpha - 3 \cos \alpha})}{6} \right].
\]  
(D9)

(ii) Rotate an angle $\phi = \pm \epsilon/\sin \alpha$ about $\hat{z}$ and slide a distance $\Delta s = -\sqrt{3 \cos^2 \theta(0)} / 2 + \Delta (\cos \theta)$ along the field line; the associated displacement is
\[
R_i \phi \sin^3 \theta(0) \hat{y} - \Delta \sin \alpha
\]
\[
= \frac{R_i \sin \theta(0)}{\sin \alpha} \left[ \frac{\pm \epsilon \sin^2 \theta(0) \hat{y}}{3\sqrt{8 + \cos^2 \alpha}} \right] \frac{\epsilon^2 \cos \alpha (\sqrt{8 + \cos^2 \alpha + \cos \alpha})^2}{8 \sin \sqrt{8 + \cos^2 \alpha}}.
\]  
(D10)

Eq. (D3) implies that $|\hat{n} \cdot \Delta r| \leq r_e (2\kappa_{coh} \gamma^{3/2})$ for charges that reach optimal beaming at the same $t_{e,c}$; using Eqs. (D9) and (D10), this condition is
\[
\left| \frac{2 \Delta R_i \sin \alpha}{3} - \frac{\epsilon^2 \cos \alpha (\sqrt{8 + \cos^2 \alpha + \cos \alpha})^2}{8 \sin \sqrt{8 + \cos^2 \alpha}} \right| \leq \frac{R_i (\sqrt{8 + \cos^2 \alpha + \cos \alpha})^2 (2\kappa_{coh})^{3/2}}{4\sqrt{8 + \cos^2 \alpha}}.
\]  
(D11)

Unless $\epsilon^2 \ll \kappa_{coh} \sim \max(\epsilon_{coh}, \gamma^{3/2})$, Eq. (D11) requires
\[
\Delta R_l \approx \frac{3 e^2 R_i \cos \alpha (\sqrt{8 + \cos^2 \alpha + \cos \alpha})^2}{16 \sin^3 \alpha \sqrt{8 + \cos^2 \alpha}}
\]  
(D12)

and therefore
\[
\Delta r \hat{e}(0) \approx \frac{\epsilon^2 r_e \cos \alpha}{\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha}}.
\]  
(D13)

Combined with
\[
|\Delta r \hat{g}| \approx \frac{8 \epsilon r_e \sin \alpha \sqrt{8 + \cos^2 \alpha}}{3(\sqrt{8 + \cos^2 \alpha + \cos \alpha})(\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha})},
\]  
(D14)

we get the emitting area perpendicular to the velocity (including positive and negative $\epsilon$)
\[
\delta \lesssim \frac{32 e^2 r_e \epsilon_\alpha \cos \alpha \sqrt{8 + \cos^2 \alpha}}{3 \sqrt{\omega_{coh} (\sqrt{8 + \cos^2 \alpha + \cos \alpha})(\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha})}},
\]  
(D15)

and an emitting volume
\[
\gtrsim \frac{64 e^2 r_e \epsilon \alpha \cos \alpha \sqrt{8 + \cos^2 \alpha}}{3 \sqrt{\omega_{coh} (\sqrt{8 + \cos^2 \alpha + \cos \alpha})(\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha})}}
\]  
(D16)

where Eq. (D3) implies $|\epsilon| \lesssim (\epsilon_\alpha / \omega_{coh}) r_e^{1/2}$ and $|\hat{n} \cdot \Delta r| \lesssim \epsilon \omega_{coh}$. Numerically, the characteristic upper limits on the cross-sectional area and volume for a set of charges emitting coherently with simultaneous $t_{e,c}$ are
\[
\epsilon_{\alpha} = \frac{2.28 \times 10^{19} \text{ cm}^2 (r_e / r_i) P}{\nu_{\text{coh}, \text{GHz}}},
\]
\[
\epsilon_\alpha = \frac{1.09 \times 10^{11} \text{ cm}^3 (r_e / r_i) P}{\nu_{\text{coh}, \text{GHz}}},
\]  
(D17)

where the curvature radius has been expressed in units of $r_e = c P / 2 \pi$, the light cylinder radius for spin period $P$.

For a range of times of optimal beaming $\Delta t_{e,c}$, Eq. (D3) is $|\Delta t_{e,c} - \hat{n} \cdot \Delta r| \lesssim r_e (2\kappa_{coh})^{1/2}$; Eq. (D11) remains valid when
\[
\Delta R_l \approx \frac{3 e^2 R_i \cos \alpha}{2 \sin \theta(0) \sin \alpha}
\]  
(D19)

which is larger than Eq. (D12), but does not depend on $\epsilon$. Eq. (D19) means that if the burst event lasts a time $\Delta t_{e,c} \gtrsim \epsilon^2 r_e / \epsilon$ the point where emission is beamed right at the observer shifts through a succession of values of $\Delta R_l$. Around each of these points, Eq. (D11) applies, leading to Eqs. (D15) and (D16). We then find a larger region in the $\hat{n} \cdot \hat{e}(0)$ plane
\[
\Delta r \cdot \hat{n} \approx \frac{c \Delta t_{e,c} \sin \alpha}{\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha}}
\]
\[
\Delta r \cdot \hat{e}(0) \approx \frac{2 c \Delta t_{e,c} \sin \alpha}{\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha}}
\]  
(D20)
and consequently larger areas and volumes

\[ \alpha \lesssim 4(c/\omega_{coh} r_c)^{1/3} c \Delta t_e \sqrt{8 + \cos^2 \alpha (\sqrt{8 + \cos^2 \alpha - 3 \cos \alpha})} \]

\[ \gamma \lesssim 4(c/\omega_{coh} r_c)^{1/3} c \Delta t_e \sqrt{8 + \cos^2 \alpha (\sqrt{8 + \cos^2 \alpha - 3 \cos \alpha})} \]

\[ \frac{1}{3(\sqrt{8 + \cos^2 \alpha + \cos \alpha})} (\sqrt{8 + \cos^2 \alpha + 3 \cos \alpha}) \]

We have focussed on dipole field geometry in this section, but we expect many of the scalings to remain valid if \( r = R_L F(\sin \theta) \); that is, although the dependences on \( \alpha \) will differ, if the field line shapes only depend on a single length scale, then some version of Eqs. (D9) and (D10) ought to be true. Realistically, though, the shapes of pulsar magnetic field lines will depend on at least two lengths, \( R_L \) and the light cylinder radius \( r_c \), and the situation is more complicated. However, to the extent that only localized regions are involved in coherent emission, and assuming that field lines may be approximated as circular locally, the basic results we found for the extent of coherent regions ought to be similar to what we found here for dipole geometry.
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