Control and enhancement of interferometric coupling between two photonic qubits

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We theoretically investigate and experimentally demonstrate a procedure for conditional control and enhancement of an interferometric coupling between two qubits encoded into states of bosonic particles. Our procedure combines local coupling of one of the particles to an auxiliary mode and single-qubit quantum filtering. We experimentally verify the proposed procedure using a linear optical setup where qubits are encoded into quantum states of single photons and coupled at a beam splitter with a fixed transmittance. With our protocol, we implement a range of different effective transmittances, demonstrate both enhancement and reduction of the coupling strength, and observe dependence of two-photon bunching on the effective transmittance. To make our analysis complete, we also theoretically investigate a more general scheme where each particle is coupled to a separate auxiliary mode and show that this latter scheme enables to achieve higher implementation probability. We show that our approach can be extended also to other kinds of qubit-qubit interactions.

I. INTRODUCTION

The ability to design and control interactions between quantum systems represents one of the key capabilities required for quantum computing and quantum information processing [1]. During recent years, significant theoretical and experimental effort has been devoted to development and demonstration of various elementary quantum logic gates and quantum processors for many physical platforms such as trapped ions [2–4], Rydberg atoms [5], superconducting qubits [6–9], or single photons processed by linear optics [10–12]. While a number of important achievements have been reached, the engineering of quantum operations is in practice inevitably limited by various factors such as noise, decoherence, or limited interaction strength.

Recently, we have addressed the issue of limited interaction strength [13] and we have shown that a weak coupling between two qubits can be conditionally enhanced by a combination of quantum interference and partial quantum measurement [14, 15] which serves as a quantum filter. Our scheme is based on local coupling of one of the particles to an additional auxiliary internal quantum state [16], or an auxiliary mode in case of a bosonic particle. In particular, we have shown that this technique allows us to conditionally implement a maximally entangling two-qubit controlled-Z gate for two qubits which are either weakly interferometrically coupled or whose coupling is described by a controlled-phase gate with arbitrary small conditional phase shift.

In our previous work [13] we have considered an asymmetric one-sided scheme, where the coupling to an auxiliary quantum state is introduced for one of the qubits only. This configuration could be particularly suitable for hybrid architectures such as quantum networks combining photonic and matter qubits [17], where one of the quantum systems may be more difficult to control than the other. Nevertheless, it is interesting to consider also more general class of configurations where the coupling to an auxiliary quantum state is introduced for both particles, and to fully exploit the potential of this technique to control and engineer the coupling between the qubits.

This detailed in-depth analysis is the goal of the present paper. For the sake of presentation clarity we shall mainly focus on the interferometric coupling of two photons at a beam splitter with transmittance $T$. In this configuration, the transmittance provides a natural measure of the interaction strength, and the higher the transmittance the weaker the coupling. For instance, implementation of a linear optical quantum CZ gate requires $T = \frac{1}{3}$ [18–20]. Here we extend our previous analysis [13] beyond the implementation of the quantum CZ gate and we investigate how to conditionally implement a coupling of two photons at a beam splitter with an arbitrary effective transmittance $T_0$ when the two photons are coupled at a beam splitter with a given fixed transmittance $T$. We theoretically consider both one-sided and two-sided configurations, where the coupling to an auxiliary mode is introduced for one or both photons, respectively. We find that the two-sided scheme is generally more advantageous than the one-sided scheme, and the former yields higher implementation probability than the latter. Nevertheless, even with the technically simpler one-sided scheme we can achieve any $T_0 \in (0, 1)$ with a finite non-zero probability using any beam splitter coupling with $T \in (0, 1)$.

We experimentally demonstrate this general ability to control and tune the coupling strength with a linear optical setup whose core is formed by a partially polarizing beam splitter inserted inside an inherently stable interferometer formed by two calcite beam displacers. The qubits are encoded into states of correlated signal and idler photons generated in the process of spontaneous parametric down-conversion. In our experiment, $T = \frac{2}{3}$ is fixed and we demonstrate tunable effective transmittance $T_0$ which can be both higher or lower than $T$. We have performed full quantum process tomography of the resulting two-qubit operation and we have observed
dependence of two-photon Hong-Ou-Mandel interference [21] on the effective target transmittance $T_0$, with a clear dip close to $T_0 = \frac{1}{2}$.

For completeness, we also systematically investigate whether the more general class of two-sided schemes where the coupling to an auxiliary mode is introduced for both photons can be exploited to increase the success rate of conditional implementation of the quantum CZ gate. Remarkably, we find that the asymmetric one-sided scheme that we have previously proposed and experimentally demonstrated [13] is globally optimal if $T > \frac{1}{3}$ and the coupling strength needs to be increased. By contrast, if $T < \frac{1}{3}$ and the coupling strength needs to be decreased, then the optimal scheme is symmetric, with an equal-strength coupling to auxiliary modes introduced for both qubits. To demonstrate the general applicability of our technique, we also briefly consider a two-qubit interaction that results in a controlled phase gate and we show that the effective conditional phase shift can be freely tuned by our method.

The rest of the present paper is organized as follows. In Sec. II we consider implementation of a linear optical quantum CZ gate with arbitrary interferometric coupling between the two photons and we describe the optimal configuration maximizing the success probability for a given $T$. In Sec. III we extend our analysis to universal control and tuning of the beam splitter coupling and we determine the optimal one-sided and two-sided configurations and compare their performance. The experimental setup is described in Sec. IV, where we also present the experimental results. In Sec. V we discuss application of our technique to interaction which results in a unitary two-qubit phase gate. Finally, brief conclusions are provided in Sec. VI.

II. QUANTUM CONTROLLED-Z GATE

The quantum controlled-Z gate [1] is a two-qubit quantum gate which introduces a $\pi$ phase shift (a sign flip), if and only if both qubits are in quantum state $|1\rangle$, $U_{CZ}|jk\rangle = (-1)^{jk}|jk\rangle$, $j, k \in \{0, 1\}$. A linear optical quantum CZ gate based on a two-photon interference on an unbalanced beam splitter is schematically illustrated in Fig. 1 [18, 20, 22]. The scheme exploits the widely used dual rail encoding of qubits into quantum states of light, where each qubit is represented by a state of a single photon which can propagate in two different modes. Specifically, the logical qubit states $|0\rangle$ and $|1\rangle$ of photon A (B) are associated with the presence of this photon in modes $A_0$ and $A_1$ ($B_0$ and $B_1$), respectively. The logical qubit states should not be confused with the Fock states and the former can be expressed in terms of the latter as

$$
|jk\rangle = \left|\begin{array}{c}
A_0\text{ at } B_1 \\
A_1\text{ at } B_0
\end{array}\right|
$$

follows,

$$
\begin{align}
|00\rangle_{AB} &= |1010\rangle_{A_0A_1B_0B_1}, \\
|01\rangle_{AB} &= |1001\rangle_{A_0A_1B_0B_1}, \\
|10\rangle_{AB} &= |0110\rangle_{A_0A_1B_0B_1}, \\
|11\rangle_{AB} &= |0101\rangle_{A_0A_1B_0B_1}.
\end{align}
$$

The gate operates in the coincidence basis [23] and a successful implementation of the gate is heralded by presence of a single photon in each pair of output modes $A_0$, $A_1$, and $B_0$, $B_1$. The core of the gate consists of a two-photon interference [21] at an unbalanced beam splitter BS with transmittance $T = t^2 = 1/3$, which occurs only if both qubits are in logical state $|1\rangle$. Let $t$ and $r$ denote the amplitude transmittance and reflectance of BS, with $t^2 + r^2 = 1$. In the Heisenberg picture, the beam splitter coupling is described by a linear transformation of annihilation operators $\hat{a}_1$ and $\hat{b}_1$ associated with modes $A_1$ and $B_1$,

$$
\hat{a}_{1,\text{out}} = t\hat{a}_1 + r\hat{b}_1, \quad \hat{b}_{1,\text{out}} = \tilde{t}\hat{b}_1 - r\hat{a}_1.
$$

Conditional on presence of a single photon in each pair of output modes $A_0$, $A_1$, and $B_0$, $B_1$, the coupling at the central beam splitter BS results in a transformation $\hat{B}$ which is diagonal in the computational basis,

$$
\begin{align}
\hat{B}|00\rangle &= |00\rangle, \\
\hat{B}|01\rangle &= t|01\rangle, \\
\hat{B}|10\rangle &= t|10\rangle, \\
\hat{B}|11\rangle &= (t^2 - r^2)|11\rangle.
\end{align}
$$

Note that this operation is generally not unitary and represents a purity-preserving quantum filter. The auxiliary beam splitters $\text{BS}_A$ and $\text{BS}_B$ serve as additional quantum filters that attenuate the amplitudes of qubit states $|0\rangle_A$ and $|0\rangle_B$. Assuming identical transmittances of all
The quantum CZ gate occurs only if \( T < \frac{1}{2} \).

The sign flip of the amplitude of state \(|11\rangle\) required for the quantum CZ gate occurs only if \( T < \frac{1}{2} \) and the value \( T = \frac{1}{2} \) is singled out by the condition \( 2T - 1 = -T \) which ensures unitarity of the conditional gate (4).

We will now investigate implementation of the quantum CZ gate for arbitrary interferometric coupling, i.e. for arbitrary transmittance \( T \) of the central beam splitter BS in the optical scheme in Fig. 1. As shown in our recent work [13, this can be achieved by coupling one of the qubits to an auxiliary mode C, see Fig. 2(a). This introduces an additional path that allows the photon A to partly bypass the coupling with the other photon B at the central beam splitter BS [13]. We will first briefly review this one-sided scheme and we will then consider a more general setting where the bypass is introduced for both qubits, see Fig. 2(b). The one-sided scheme in Fig. 2(a) may be advantageous in hybrid settings where the system B is more difficult to address and control than system A. Nevertheless, it is useful and instructive to analyze in depth also the more general scheme shown in Fig. 2(b), as it may potentially lead to higher implementation probability.

### A. One-sided bypass

In the one-sided bypass scheme shown in Fig. 2(a), an auxiliary mode \( C \) is introduced to partly bypass the beam splitter interaction BS [13, 16]. The beam splitters BS\(_X\) and BS\(_Y\) locally couple mode \( A_1 \) to \( C \) both before and after the beam splitter interaction between modes \( A_1 \) and \( B_1 \). Similarly to the standard linear optical quantum CZ gate scheme in Fig. 1, this generalized scheme also includes two beam splitters BS\(_A\) and BS\(_B\) that can attenuate modes \( A_0 \) and \( B_0 \), respectively. In what follows, we denote by \( t_1 \) and \( r_1 \) the amplitude transmittance and reflectance of beam splitter BS\(_Y\). If we postselect on presence of a single photon in each output port of the gate then the overall transformation \( W \) implemented by the setup shown in Fig. 2(a) is diagonal in the computational basis, \( W|jk\rangle = w_{jk}|jk\rangle \), where

\[
\begin{align*}
    w_{00} &= t_A, \\
    w_{01} &= t_A, \\
    w_{10} &= (tt_Xt_Y - r_Xr_Y)t_B, \\
    w_{11} &= (2t^2 - 1)t_Xt_Y - tr_Xr_Y.
\end{align*}
\]

The quantum CZ gate is conditionally implemented provided that \( w_{00} = w_{10} = w_{01} = -w_{11} \), which yields the conditions \( t_A = tt_Xt_Y - r_Xr_Y \), \( t_B = t \), and

\[
\frac{r_Xr_Y}{tt_Xt_Y} = \frac{3T - 1}{2t}.
\]

The probability of implementation of the gate can be expressed as \( P_I = |t_At_B|^2 \) and after some algebra we obtain

\[
P_I = \frac{1}{4}(1-T)^2t_X^2.
\]

Formula (6) describes a one-parametric class of schemes implementing the quantum CZ gate. Using Eq. (6) we can express \( t_X^2 \) in terms of \( t_X^2 \), insert the resulting expression into formula for \( P_I \), and search for its maximum over \( t_X \). This optimization can be performed analytically and we find that the implementation probability is maximized by a symmetric configuration, where

\[
t_X^2 = t_Y^2 = \frac{2t}{2t + |1 - 3T|}.
\]
For this choice of coupling to mode C we get
\[ P_I = \frac{(1-T)^2T}{(2t + |1-3T|)^2}. \]  

The quantum interference conditionally enhances the coupling of modes \( A_1 \) and \( B_1 \) although this interaction is partially bypassed by coupling mode \( A_1 \) with mode \( C \).

### B. Two-sided bypass

We will now turn our attention to the more general class of schemes with coupling to auxiliary modes introduced for both qubits, c.f. Fig. 2(b). We can see that the coupling of modes \( A_1 \) and \( B_1 \) to auxiliary modes \( C \) and \( D \), respectively, is provided by four beam splitters \( BS_{XA}, BS_{Y A}, BS_{XB}, \) and \( BS_{Y B} \). The conditional transformation \( W \) introduced above remains diagonal in the computational basis even for this extended scheme, only the expressions for the amplitudes \( w_{jk} \) become more involved,

\[
\begin{align*}
w_{00} &= t_At_B, \\
w_{01} &= t_At_B(t_X B^t Y B t - r_X B^t Y B), \\
w_{10} &= t_B(t_X A^t Y A^t - r_X A^t Y A), \\
w_{11} &= w_{01}w_{00}^{-1} - t_X A^t X_B Y A^t Y B t^2.
\end{align*}
\]

The quantum CZ gate is implemented provided that
\[
\begin{align*}
t_A &= t_X A^t Y A^t - r_X A^t Y A, \\
t_B &= t_X B^t Y B t - r_X B^t Y B,
\end{align*}
\]

and
\[
\frac{r_X A^t Y A^t}{t_X A^t Y A^t} = t - \frac{1}{2} \frac{t_X B^t Y B (1-T)}{t_X B^t Y B - r_X B^t Y B}. \tag{12}
\]

This formula generalizes Eq. (9) and describes a three-parametric class of schemes implementing the quantum CZ gate with probability \( P_I = t_A^2 t_B^2 \).

As an important special case, let us investigate a symmetric configuration, where the coupling to the auxiliary mode is the same for both qubits,
\[
\begin{align*}
t_{XA} &= t_{XB}, & t_{YA} &= t_{YB}, \\
r_{XA} &= r_{XB}, & r_{YA} &= r_{YB}.
\end{align*}
\tag{13}
\]

In this case, Eqs. (11) and (12) yield
\[
\frac{r_X A^t Y A^t}{t_X A^t Y A^t} = t \pm \frac{r}{\sqrt{2}}, \tag{14}
\]

and
\[
t_A = t_B = \pm \frac{r}{\sqrt{2}} t_{XA^t Y A^t}.
\tag{15}
\]

The probability of implementation of the CZ gate is maximized when \( t_{XA}^2 = t_{YA}^2 = \sqrt{\frac{2}{2}}/(\sqrt{2} + |r - \sqrt{2}t|) \), and we get
\[
\tilde{P}_I = \frac{(1-T)^2}{(\sqrt{2} + |r - \sqrt{2}t|)^4}. \tag{16}
\]

### III. UNIVERSAL CONTROL OF INTERFEROMETRIC COUPLING

The realization of the linear optical CZ gate with arbitrary interferometric coupling investigated in the previous Section can be seen as an implementation of a beam splitter with effective transmittance \( T_0 = \frac{1}{2} \) using a beam splitter with a different transmittance \( T \). In this Section we extend our study beyond the quantum CZ gate and consider implementation of a beam splitter with arbitrary transmittance \( T_0 \). Specifically, we will investigate conditional implementation of the two-qubit transformation \( B \) using the schemes with one-sided and two-sided bypass as depicted in Fig. 2.

Conditional on presence of a single photon in each pair of output modes \( A_0, A_1 \) and \( B_0, B_1 \), the interferometric schemes in Fig. 2 implement the beam splitter coupling \( G \) with transmittance \( T_0 \) provided that
\[
\begin{align*}
w_{00} &= w_{01} = w_{10}, & w_{11} &= t_0^2 - r_0^2,
\end{align*}
\tag{17}
\]

where the coefficients \( w_{jk} \) are given by Eq. (10). The first two of these conditions yield the following expressions for...
transmittances \( t_A \) and \( t_B \),

\[
\begin{align*}
\frac{r_{XAR_YA}}{t_{XAR_YA}} &= t - \frac{t^2}{r_0^2} = \frac{t_{XB}t_{YB}t^2}{t_{XB}t_{YB}t^2 - r_{XB}r_{YB}}, \\
\frac{r_{XAR_YA}}{t_{XAR_YA}} &= 1 - \frac{t}{r_0} = \frac{t_{XB}t_{YB}t}{t_{XB}t_{YB}t - r_{XB}r_{YB}}.
\end{align*}
\]

(19)

Note that for certain parameter values it may happen that \( |t_A| > 1 \). In such case one should attenuate mode \( A_1 \) by factor of \( 1/t_A \) while the mode \( A_2 \) is not attenuated at all. Similarly, if \( |t_B| > 1 \) then mode \( B_1 \) should be attenuated by \( 1/t_B \). The final condition in Eq. (17) provides the following relation between the parameters of the four beam splitters that implement the two bypasses,

\[
\begin{align*}
&\frac{r_{XAR_YA}}{t_{XAR_YA}} = t - \frac{t^2}{r_0^2} = \frac{t_{XB}t_{YB}t^2}{t_{XB}t_{YB}t^2 - r_{XB}r_{YB}}, \\
&\frac{r_{XAR_YA}}{t_{XAR_YA}} = 1 - \frac{t}{r_0} = \frac{t_{XB}t_{YB}t}{t_{XB}t_{YB}t - r_{XB}r_{YB}}.
\end{align*}
\]

(19)

Choice of beam splitter parameters satisfying Eq. (17) ensures that the implemented transformation reads \( \tilde{W} = \sqrt{P_t}B \), where \( P_t \) is the implementation probability. If \( |t_A| \leq 1 \) and \( |t_B| \leq 1 \) then

\[
P_t = t^2_A t^2_B.
\]

(20)

If \( |t_A| > 1 \) according to Eq. (18) then no attenuation is applied to mode \( A_0 \) and \( P_t = t^2_B \). Similarly, if \( |t_B| > 1 \) then \( P_t = t^2_A \). Recall that the transformation \( B \) is itself non-unitary, hence only probabilistic. The implementation probability \( P_t \) of our procedure represents an additional factor, which further reduces the success probability of the resulting transformation \( W \).

We have performed numerical optimization of \( P_t \) over the whole three-parametric class of configurations specified by Eqs. (18) and (19). Based on this numerical analysis we have identified optimal configurations maximizing \( P_t \). For \( T_0 > T \), the optimal configuration is fully symmetric, with

\[
\begin{align*}
&t_{XA} = t_{XB} = t_{YA} = t_{YB}, \\
&r_{XA} = r_{XB} = -r_{YA} = -r_{YB},
\end{align*}
\]

(21)

Using Eqs. (18) and (19) we obtain

\[
\begin{align*}
&t_{XA} = t_{XB} = t_{YA} = t_{YB}, \\
&r_{XA} = r_{XB} = -r_{YA} = -r_{YB}, \\
&t_A = t_B.
\end{align*}
\]

(21)

It can be shown analytically that \( t_{XA} < 1 \) and \( t_A < 1 \) provided that \( t_0 > t \). The implementation probability in this case thus reads

\[
\hat{P}_t = \left( \frac{r}{t_0r + r_0(1-t)} \right)^4.
\]

(23)

If \( T_0 < T \), then the optimal configuration is specified by conditions

\[
\begin{align*}
&t_{XA} = t_{YA}, \\
&t_{XB} = t_{YB}, \\
&t_B = 1,
\end{align*}
\]

(24)

and \( r_{YA} = r_{XA}, r_{YB} = r_{XB} \). After some algebra, we get

\[
\begin{align*}
&t^2_{XA} = \frac{(1 + t)r_0^2}{(1 + t)^2 r_0^2 - (1 + t_0)r^2 t_0}, \\
&t^2_{XB} = \frac{1 + t_0}{1 + t},
\end{align*}
\]

(25)

and

\[
\hat{P}_t = \frac{r^2}{(1 + t)^2 (1 - t_0 - r^2 t_0)^2}.
\]

(26)

Let us now consider implementation of an arbitrary beam splitter coupling with the one-sided bypass scheme shown in Fig. 2(a). Since \( t_{XB} = t_{YB} = 1 \) in this case, we have

\[
t_B = \frac{t}{t_0}.
\]

(27)

If \( T_0 < T \), then \( t_B > 1 \), hence mode \( B_1 \) has to be attenuated by factor of \( t_0/t \), and the optimal configuration maximizing the implementation probability \( P_t \) is symmetric, with

\[
\begin{align*}
&t_{XA} = t_{YA} = \frac{t_0 r}{t^2_0 + t^2 - t^2_0},
\end{align*}
\]

(28)

and \( r_{XA} = r_{YA} \). On inserting these expressions into the formula (18) for \( t_A \), we get

\[
\hat{P}_t = \left( \frac{t_0 r^2}{t^2_0 + r^2 - t^2_0} \right)^2.
\]

(29)

If \( T_0 > T \), then \( t_B < 1 \) and one can choose the amplitude transmittances \( t_{XA} \) and \( t_{YA} \) such that \( t_A = 1 \). This is achieved for

\[
\begin{align*}
&t^2_{XA} = \frac{1}{2} \left( 1 + x - y + \sqrt{(1 + x - y)^2 - 4x} \right), \\
&t^2_{YA} = \frac{1}{2} \left( 1 + x - y - \sqrt{(1 + x - y)^2 - 4x} \right),
\end{align*}
\]

(30)
where
\[
x = \frac{t^2 r_0^4}{t_0^2 r^4}, \quad y = \left( t_0 - \frac{t^2 r^2}{t_0^2 r^2} \right)^2.
\] (31)

Since \( t_A = 1 \), the success probability of this optimal configuration with one-sided bypass reads \( t_B^2 \), hence
\[
P_I = \frac{T}{T_0}.
\] (32)

The implementation probability \( \tilde{P}_I \) achieved by the optimal two-sided bypass scheme as well as implementation probability \( P_I \) achieved by the optimal scheme with one-sided bypass are plotted in Fig. 4 for two different values of \( T \). We can see that the two-sided scheme generally outperforms the one-sided scheme. In particular, a totally reflecting beam splitter with \( T_0 = 0 \) can be implemented with a non-zero probability \( \tilde{P}_I = r^4/(1 + t)^4 \) with the two-sided bypass scheme, while this probability is equal to zero for the one-sided scheme. Also, \( \tilde{P}_I = 1 \) in the limit \( T_0 = 1 \), because with the two-sided bypass it is trivial to deterministically switch off the qubit coupling at the beam splitter BS. By contrast, with a single-sided bypass such switching off of the coupling can be performed only probabilistically, and the corresponding implementation probability reads \( P_I = T \).

IV. EXPERIMENT

We have experimentally tested the control of interferometric coupling between two photonic qubits with the experimental setup depicted in Fig. 5. Time-correlated orthogonally polarized photon pairs were generated in the process of frequency-degenerate collinear type II spontaneous parametric downconversion in a nonlinear crystal pumped by a laser diode with central wavelength of 405 nm (not shown in Fig. 5). The pump beam was removed by a dichroic mirror and the downconverted signal and idler photons at 810 nm were spatially separated on a polarizing beam splitter, coupled into single-mode fibers and guided to the two input ports of the bulk interferometer shown in Fig. 5, where they were released into free space. Qubit A was encoded into path encoding. The second calcite beam displacer BD2 converts the path encoding back into polarization which ensures that the output state of qubit A can be analyzed with the use of a standard single-photon polarization detection block DB which consists of a HWP, QWP, PBS and single-photon detectors APD.

FIG. 5. (Color online) Experimental setup. HWP - half-wave plate, QWP - quarter-wave plate, PPBS - partially polarizing beam splitter with transmittances \( T_V = \frac{2}{3} \) and \( T_H = 1 \) for vertical and horizontal polarizations, respectively, PBS - polarizing beam splitter, BD - calcite beam displacer. The inset shows the single-photon polarization detection block DB which consists of a HWP, QWP, PBS and single-photon detectors APD.
FIG. 6. (Color online) Quantum process matrices of conditional two-qubit operations induced by interferometric coupling. Results are shown for three different values of target beam-splitter transmittance $T_0 = 0.3$ (a), $T_0 = 0.5$ (b), and $T_0 = 0.8$ (c). The nominal transmittance of the beam splitter reads $T = 2/3$. The first two columns contain real and imaginary parts of process matrices $\tilde{\chi}$ reconstructed from the experimental data, while the third and fourth column display real and imaginary parts of the theoretical matrices $\chi_B$ for comparison.

make the interferometer symmetric with identical wave plates and glass plates inserted into both of its arms. By tilting one of the glass plates GP we can control the relative phase shift between the two interferometer arms and set it to zero.

Our protocol also requires tunable attenuation of output modes $B_0$ and $B_1$. We accomplish this with the use of another interferometer formed by a pair of calcite beam displacers BD$_3$ and BD$_4$ with a HWP inserted in each arm of the interferometer. This configuration allows us to selectively attenuate mode $B_0$ or $B_1$ by suitable rotations of either HWP$_{B0}$ or HWP$_{B1}$, respectively. After filtering, polarization state of qubit B is analyzed with the help of a second polarization detection block DB. The scheme operates in the coincidence basis and its successful operation is indicated by coincidence detection of two photons, one by each detection block DB. For additional details about the experimental setup, see Refs. [13, 24].

Following the optimal one-sided bypass protocol theoretically described in Sec. III, we have used our setup to implement a beam splitter coupling $\hat{B}$ with 11 different effective transmittances $T_0 = 0.3 + 0.05j$, where $j = 0, 1, \ldots, 10$. We have performed full quantum process tomography of the implemented two-qubit operations. We have probed the operation with 36 different product two-qubit states, where each of the qubit is chosen to be in one of the six states $|0\rangle$, $|1\rangle$, $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, or $\frac{1}{\sqrt{2}}(|0\rangle \pm i|1\rangle)$. We label these 36 two-qubit input states by an integer $m$. For each input state $m$, the number of coincidence detections $C_{mn}$ corresponding to projection of the output photons onto a two-qubit product state $n$ was measured for a fixed time interval of 1 s. Utilizing the Choi-Jamiolkowski isomorphism [25, 26], we can represent a two-qubit quantum operation by a positive semidefinite operator $\chi$ acting on Hilbert space of four qubits (two input qubits and two output qubits). The quantum process matrix $\chi$ was reconstructed from the measured coincidences $C_{mn}$ using the maximum likelihood estimation procedure [27].

The process matrix $\chi_B$ representing the target operation (3) is proportional to a density matrix of a pure entangled four-qubit state, $\chi_B = |\Phi_B\rangle\langle\Phi_B|$, where $|\Phi_B\rangle$ is obtained by applying the operation $\hat{B}$ to one part of a
Explicitly, we have
\[ |\Phi_B\rangle = \hat{I} \otimes \hat{B} \sum_{j,k=0}^{1} |jk\rangle |jk\rangle. \]  

(33)

For ease of visual comparison between theory and experiment, we introduce normalized process matrices
\[ \tilde{\chi} = \frac{\chi}{\langle 0000 | 0000 \rangle}, \]  

(35)

This normalization ensures that \( \langle 0000 | 0000 \rangle = 1 \) which holds for the matrices \( \chi_B \) of the ideal target operations \( \hat{B} \). In Fig. 6 we plot the reconstructed quantum process matrices \( \tilde{\chi} \) for three target transmittances \( T_0 = 0.3, T_0 = 0.5, \) and \( T_0 = 0.8 \), together with the corresponding ideal process matrices \( \chi_B \). Since \( T = \frac{2}{\pi} \), the cases \( T_0 = 0.3 \) and \( T_0 = 0.5 \) correspond to enhancement of the interferometric coupling, while the case \( T_0 = 0.8 \) illustrates reduction of the coupling strength. We observe a good agreement between the experimental and theoretical process matrices. Since \( \chi_B \) is proportional to a density matrix of a pure state, we can quantify the similarity between \( \chi \) and \( \chi_B \) by a normalized overlap of process matrices \( [23] [23] \).

\[ F = \frac{\text{Tr}[\chi \chi_B]}{\text{Tr}[\chi] \text{Tr}[\chi_B]}, \]  

(36)

This quantum process fidelity satisfies \( 0 \leq F \leq 1 \) and if \( F = 1 \), then the implemented operation \( \hat{W} \) is a purity-preserving quantum filter which coincides with the target operation \( \hat{B} \) up to a constant prefactor, \( W = \sqrt{T_0} \hat{B} \). The quantum process fidelity \( F \) is plotted in Fig. 7(a), where the red dots represent experimental data and the solid line indicates prediction of a theoretical model of the experimental setup, which accounts for imperfect two photon interference with visibility \( V = 0.94 \) and imperfections of the partially polarizing beam splitter PPBS, whose measured transmittances \( T_V = 0.687 \) and \( T_H = 0.981 \) slightly differ from the nominal transmittances \( T_V = \frac{2}{\pi} \) and \( T_H = 1 \). This model is similar to the model presented in the Appendix of Ref. [13], where we refer the reader for more details. We can see in Fig. 7(a) that the theoretical model correctly predicts the qualitative dependence of fidelity on \( T_0 \), but the quantitative agreement with the experiment is not exact. This remaining discrepancy is likely caused by other effects that may reduce the fidelity, such as phase fluctuations and imperfections of the various wave plates and other optical components.

The matrices \( \chi_B \) representing the ideal operations \( \hat{B} \) are real and their imaginary parts exactly vanish. Due to various experimental imperfections, we observe small nonzero imaginary parts of the experimentally determined matrices \( \chi \), which increase with decreasing \( T_0 \). It can be seen from Fig. 6 that the dominant imaginary components correspond to a residual phase shift of state \( |1111\rangle \), which cannot be compensated by local single-qubit unitary transformations on qubits A and B.

If the actually implemented operation reads \( \hat{W} = \sqrt{T_0} \hat{B} \), then the implementation probability \( P_I \) can be determined as a ratio of traces of \( \chi \) and \( \chi_B \). We can generalize this to imperfect implementations and define a quantity

\[ Q_I = \frac{\text{Tr}[\chi]}{\text{Tr}[\chi_B]}, \]  

(37)

where \( \text{Tr}[\chi_B] = 2 - 2T_0 + 4T_0^2 \). Generally, \( Q_I \) can be larger than 1. Nevertheless, in case of a high fidelity between \( \chi \) and \( \chi_B \) the parameter \( Q_I \) provides a suitable quantification of the implementation probability of the target operation \( \hat{B} \). In order to experimentally determine \( Q_I \) we have measured 36 additional reference coincidences \( R_n \), one for each two-qubit input state. These reference coincidences were recorded with qubits prepared in state \( |00\rangle \) and all half wave plates set to full transmittance. The parameter \( Q_I \) was estimated from the experimental data as follows,

\[ Q_I = \frac{4 \bar{C}}{\text{Tr}[\chi_B] R}, \]  

(38)
dependence of theoretical curves are almost identical and the measured predictions.

The experimentally determined implementation probability $Q_I$ is plotted in Fig. 7(b) together with the prediction of our theoretical model which accounts for imperfect two-photon interference and imperfections of PPBS. For comparison, the figure also shows the implementation probability $P_I$ for perfect error-free realization of the protocol, as given by Eqs. (29) and (32). The two theoretical curves are almost identical and the measured dependence of $Q_I$ on $T_0$ closely follows the theoretical predictions.

Bunching of two photons interfering at a balanced beam splitter is a fundamental nonclassical phenomenon that is exploited in countless quantum optics and quantum information processing schemes and experiments [12, 30, 31]. We can observe the presence of photon bunching and the Hong-Ou-Mandel effect in Fig. 6, where we can see that the matrix element

$$S_{11} = \langle 1111 | \hat{X} | 1111 \rangle = \langle 1111 | 1111 \rangle_{0000} - \langle 1111 | 0000 \rangle_{0000}$$

(40)

practically vanishes for $T_0 = 0.5$. It follows from Eq. (40) that $S_{11}$ can be interpreted as a ratio of probability of coincidence detection of two photons in output modes $A_1$ and $B_1$ when they are injected in modes $A_1$ and $B_1$, and probability of coincidence detection of photons in output modes $A_0$ and $B_0$ when they are injected in modes $A_0$ and $B_0$. For a perfect scheme implementing operation $W = \sqrt{P_I}B$ we have

$$S_{11} = (r_0^2 - r_0^2)^2 = (1 - 2T_0)^2,$$

(41)

irrespective of the value of the implementation probability $P_I$. The experimentally determined $S_{11}$ is plotted in Fig. 8. We can see that the experimental data follow a quadratic dependence on $T_0$ and the minimum is located close to $T_0 = \frac{1}{2}$. Remarkably, the HOM dip is formed by destructive quantum interference of three alternatives, instead of two as in the ordinary two-photon interference at a beam splitter. Specifically, the photons injected in modes $A_1$ and $B_1$ can reach the output modes $A_1$ and $B_1$ as follows: (i) the signal photon in mode $A_1$ is transmitted through $BS_X$ and $BS_Y$ and both photons are transmitted through $BS$; (ii) the signal photon in mode $A_1$ is transmitted through $BS_X$ and $BS_Y$ and both photons are reflected at $BS$; and (iii) the signal photon avoids the central beam splitter BS by being reflected at both $BS_X$ and $BS_Y$ and the idler photon is transmitted through BS to the output mode $B_1$. The data plotted in Fig. 8 thus represent an elementary example of a multi-photon interference in a multiport interferometer, a phenomenon that has been recently intensively investigated in the context of boson sampling [32–36].

V. CONTROLLED PHASE GATE

So far we have considered the interferometric coupling defined by Eq. (3). In the present Section we show that the proposed procedure is applicable to a wider class of qubit-qubit interactions. In particular, we shall consider a deterministic interaction governed by a Hamiltonian $H = h\kappa|11\rangle\langle11|$. The resulting two-qubit unitary controlled phase gate $\hat{U}(\phi) = e^{-i\phi}\hat{I}$ introduces a phase shift $\phi = \kappa t$ if and only if both qubits are in the state $|1\rangle$,

$$\hat{U}(\phi) = |00\rangle\langle00| + |01\rangle\langle01| + |10\rangle\langle10| + e^{i\phi}|11\rangle\langle11|. \quad (42)$$

The controlled-Z gate is just a special case of a controlled phase gate, with $\phi = \pi$. For ease of notation, we use in this Section the ordinary symbols $|0\rangle$ and $|1\rangle$ to denote the computational basis states of a qubit, since there is no risk of confusion with Fock states.

We assume that the phase shift $\phi$ is fixed and we would like to convert $\hat{U}(\phi)$ to a modified interaction $\hat{U}(\theta)$ with a different phase shift $\theta$. This can be conditionally accomplished by a scheme similar to that shown in Fig. 2(a).

The procedure requires an auxiliary state $|2\rangle_A$ of particle A, to which the qubit state $|1\rangle_A$ is coupled before the inter-qubit interaction as follows,

$$|1\rangle_A \rightarrow t_X|1\rangle_A + r_X|2\rangle_A, \quad |2\rangle_A \rightarrow t_X|2\rangle_A - r_X|1\rangle_A. \quad (43)$$

Here $t_X$ and $r_X$ are generally complex coefficients satisfying $|t_X|^2 + |r_X|^2 = 1$. After interaction between the qubits, states $|1\rangle_A$ and $|2\rangle_A$ are coupled again, this time with coupling parameters $t_Y$, $r_Y$. Particle A is then projected onto the qubit subspace and the amplitude of state $|0\rangle_A$ is attenuated according to $|0\rangle_A \rightarrow t_A|0\rangle_A$. In Ref. [13] we have shown that this procedure allows to conditionally implement a CZ gate between the qubits for any $0 < \phi < \pi$. Here we extend this concept and explicitly
FIG. 9. (Color online) Dependence of the implementation probability $P_I$ of a two-qubit controlled phase gate $\hat{U}(\theta)$ on the target phase shift $\theta$ and the actual phase shift $\phi$.

show that the resulting conditional phase shift $\theta$ can be arbitrarily tuned. Note that, in contrast to the case of the interferometric coupling considered in the previous sections, here the quantum filtering needs to be applied only to the particle $A$.

After some algebra similar to that reported in Sec. IIA we find that the controlled phase gate $\hat{U}(\theta)$ is conditionally implemented provided that the following conditions are satisfied,

$$t_A = t_X t_Y - r_X r_Y,$$

(44)

and

$$\frac{r_X r_Y}{t_X t_Y} = \frac{e^{i\phi} - e^{i\theta}}{1 - e^{i\theta}}.$$

(45)

The implementation probability $P_I = |t_A|^2$ is maximized when

$$|t_X|^2 = |t_Y|^2 = \frac{|\sin \frac{\theta}{2}|}{|\sin \frac{\phi}{2} + |\sin \frac{\theta - \phi}{2}|}.$$

(46)

and the phases of the amplitude transmittances and reflectances should be chosen such that Eq. (45) holds. For this optimal configuration, we get

$$P_I = \left(\frac{|\sin \frac{\phi}{2}|}{|\sin \frac{\theta}{2} + |\sin \frac{\theta - \phi}{2}|}\right)^2.$$

(47)

The implementation probability $P_I$ is plotted in Fig. 9 in dependence on $\phi$ and $\theta$. Note that $P_I$ is nonzero for any $0 < \phi < \pi$, hence the above procedure enables complete conditional tuning and control of the effective conditional phase shift $\theta$.

VI. CONCLUSIONS

In summary, we have investigated in detail the ability to conditionally control and enhance interaction between two qubits by coupling one or both particles carrying the qubits to an auxiliary quantum state or mode. We have seen that quantum filtering is an essential part of our procedure, and its probabilistic nature is the price to pay for the enhancement of the interaction. The method is not limited to the interferometric coupling embodied in our work by two-photon interference at a beam splitter. As we have illustrated in the final part of our paper, the proposed concept of conditional interaction enhancement can be applied also to other qubit-qubit interactions such as that resulting in a controlled phase gate between the qubits. We have utilized the linear optics platform as a suitable test bed for demonstration and verification of the feasibility and robustness of the proposed scheme, which is mainly intended for configurations where the interaction is limited e.g. due to inherently small coupling strength, or due to noise or decoherence that puts a limit on the total achievable interaction time. We hope that our scheme may find applications for instance in heterogeneous quantum networks [17] or in quantum optomechanics where photons are coupled to phononic excitations of a mechanical oscillator [37].

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