Simultaneous ground-state cooling of two mechanical modes of a levitated nanoparticle

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The quantum ground state of a massive mechanical system is a stepping stone for investigating macroscopic quantum states and building high-fidelity sensors. With the recent achievement of ground-state cooling of a single motional mode, levitated nanoparticles have entered the quantum domain. To overcome detrimental cross-coupling and decoherence effects, quantum control needs to be expanded to more system dimensions, but the effect of a decoupled dark mode has so far hindered cavity-based ground-state cooling of multiple mechanical modes. Here, we demonstrate two-dimensional ground-state cooling of an optically levitated nanoparticle. Utilizing coherent scattering into an optical cavity mode, we reduce the occupation numbers of two separate centre-of-mass modes to 0.83 and 0.81, respectively. By controlling the frequency separation and the cavity coupling strengths of the nanoparticle’s mechanical modes, we show the transition from 1D to 2D ground-state cooling. This 2D control lays the foundations for quantum-limited orbital angular momentum states for rotation sensing and, combined with ground-state cooling along the third motional axis shown previously, may allow full 3D ground-state cooling of a massive object.
A sketch of our optomechanical system is shown in Fig. 1a. Additional information can be found in the Methods. We detect and cool the COM mechanical modes of a single spherical SiO$_2$ nanoparticle of nominal diameter 143 ± 6 nm and mass 3.4 ± 0.4 fg. The nanoparticle is levitated in high vacuum (pressure of $(3 ± 0.1) \times 10^{-9}$ mbar) using optical tweezers at a wavelength of 1.550 ± 0.05 nm (frequency $\omega_0$) with optical power of $1.20 \pm 0.08$ W, focused by a high numerical aperture (NA, 0.75) lens. The polarization at the focus is defined by the tilt angle $\theta$ between the major axis of the polarization ellipse and the cavity axis and by the degree of ellipticity. We choose the polarization by tuning a set of waveplates, compensating for the birefringence of our vacuum window and trapping lens. The nanoparticle’s reference frame is defined by the tweezers’ propagation (z) and polarization (x) axes, as well as the axis orthogonal to the two (y). Strong focusing of the linearly polarized optical tweezers results in non-degenerate, bare mechanical frequencies of the COM motion of $\Omega_x/2\pi = 242 \pm 2, 268 \pm 2, 80 \pm 1$ kHz. The asymmetric cavity consists of two mirrors with different transmission separated by 6.4 ± 0.1 mm, resulting in a linewidth of $\kappa/2\pi = 330 \pm 9$ kHz. The nanoparticle scatters light into the cavity, which leaks through the higher transmission mirror, is combined with a local oscillator ($\omega_{LO}/2\pi = \omega_0/2\pi + 1.5$ MHz) and then split equally onto a balanced photodetector. Heterodyne spectra are calculated as power spectral densities (PSDs) of the balanced photodetector voltage.

**Cooling to 2D ground state**

Our two-mode ground-state cooling experiment relies on coherent scattering$^{26-27}$, referring to light being scattered off a polarizable particle and populating an optical cavity. This method has attracted interest as, compared with other cavity cooling schemes, it offers larger optomechanical coupling strengths and reduced phase noise heating$^{28}$. Here, we exploit coherent scattering for coupling two motional modes of a single nanoparticle to an optical cavity mode$^{23,24}$. A harmonically trapped nanoparticle scatters light elastically (Rayleigh) and inelastically (Raman). The Raman processes lead to sidebands in the scattered light spectrum. Figure 1b shows a schematic of the resulting heterodyne spectrum, which illustrates the cooling mechanism of coherent scattering. The positive and negative frequencies correspond to the destruction and creation of a phonon, which are denoted by anti-Stokes and Stokes scattering, respectively. The grey line represents the spectrum of the mechanical oscillations without cavity. We introduce an optical cavity (dashed blue line is the intensity transfer function), whose resonance frequency $\omega_c$ is detuned by $\Delta = \omega_c - \omega_0$. As we choose $\Delta = (\Omega_x + \Omega_y)/2$, the cavity enhances anti-Stokes relative to Stokes scattering in the spectrum (black line), which reduces the occupation numbers $n_j (j = x, y)$ of the COM modes. The asymmetry between Stokes and anti-Stokes peaks is additionally influenced by the fact that their scattering rates are proportional to $n_j + 1$ and $n_j$, respectively. Taking into account the cavity transfer function, we use the measured asymmetry in the PSDs to extract $n_j$ through a technique called sideband thermometry$^{24,30}$ (Methods). Figure 2a shows the measured heterodyne PSDs normalized to shot noise level. The PSDs contain Stokes and anti-Stokes sidebands of both transversal modes (x and y), simultaneously coupled to the cavity at $\theta_c = 0.25\pi$, for different cavity detunings $\Delta$. The cooling by coherent scattering becomes more efficient as $\Delta$ approaches $(\Omega_x + \Omega_y)/2$, which results in a smaller amplitude and broader width of the sidebands. For each detuning and each COM mode, we fit Lorentzians (lines) of equal widths but independent amplitudes to the Stokes and anti-Stokes sidebands. The asymmetries that we use for sideband thermometry are then given by the ratio of anti-Stokes to Stokes amplitudes. Figure 2b shows the extracted occupation numbers as a function of the cavity detuning. The shaded areas represent simulations based on coupling strengths $g_x$ and heating rates $\Gamma_j$ which we extract (Methods) from our data to be $g_x/2\pi = 14.1 \pm 2.7$ kHz, $g_y/2\pi = 15.4 \pm 1.9$ kHz, $\Gamma_x/2\pi = 1.0 \pm 0.4$ kHz and $\Gamma_y/2\pi = 1.0 \pm 0.4$ kHz. We infer the heating rates to be limited by photon recoil (Methods), as they are in good agreement with values calculated from system parameters. For $\Delta/2\pi = 232$ kHz close to $(\Omega_x + \Omega_y)/2$, we reach occupation numbers of $n_x = 0.83 \pm 0.10$ and $n_y = 0.81 \pm 0.12$, cooling the COM motion into its two-dimensional (2D) quantum ground state.

**Transition from 2D to 1D ground-state cooling**

We explore the robustness of our cooling scheme to changes of the coupling rates by changing the polarization angle $\theta$ of the trapping
light. The linearized optomechanical coupling strengths $g_{x,y}$ for linear polarization have the form $g_x \propto \cos \theta$ and $g_y \propto \sin \theta$ (ref. 28). Figure 3a–d displays anti-Stokes sidebands of $x$ and $y$ modes for different $\theta$ at $\Delta/2\pi = 246 \pm 8$ kHz. By tuning $\theta$ from 0.25\pi to 0.5\pi, we observe the transition from 2D to 1D ground-state cooling. Close to $\theta = 0.5\pi$, the shrinking/rising peak amplitudes indicate the motion along $y/x$ being cooled more/less efficiently owing to larger/smaller coupling strength $g_y/g_x$. Note that, at $\theta = 0.5\pi$, the $x$ motion is still imprinted in the spectrum of the cavity field and remains cooled. We attribute this to imperfections in the polarization state and in the angular alignment between the optical axes of the trap and cavity. Additionally, small shifts in the frequencies $\Omega_j$ for different $\theta$ are caused by power drifts of the optical tweezers on the 0.5% level. The phonon occupations extracted from sideband thermometry are displayed in Fig. 3c. The simulations (shaded areas; Methods) show the increasing and decreasing occupation numbers of the $x$ and $y$ mode, respectively, in agreement with the data. This result is well aligned with the predicted decrease (increase) of $g_y \propto \cos \theta$/$g_x \propto \sin \theta$ as $0.5\pi$ is increased from 0.25\pi to 0.5\pi.

Experimentally, we find robust two-mode ground-state cooling at $\theta = 0.25\pi$ and 0.33\pi. Furthermore, we observe our lowest single-mode phonon occupation of $n_x = 0.46 \pm 0.05$ paired with a high phonon occupation of $n_y = 14 \pm 12$ at $\theta = 0.5\pi$.

Limits of 2D sideband cooling and thermometry

To efficiently cool two COM modes ($x, y$) of a levitated nanoparticle, several conditions must be met. First, the optical cavity must simultaneously resolve the anti-Stokes sidebands of the $x$ and $y$ modes, that is $|\Omega_z - \Omega_i| \leq \kappa \leq |\Omega_i|$. Further, the system needs to be in the weak coupling regime $|g| \ll \kappa$, to prevent hybridization of the cavity and mechanical modes, which hinders efficient cooling28. Finally, $\Omega_x$ and $\Omega_y$ must be sufficiently separated. The $x$ and $y$ modes are cooled by the cavity via a collective mechanical mode, while the orthogonal dark mechanical mode is only sympathetically cooled when coupling to the bright mode25. For near-degenerate $\Omega_x$ and $\Omega_y$, the dark mode decouples, and this inhibits cooling of its constituent $x$ and $y$ modes (Methods). The condition $|\Omega_x - \Omega_i| \geq |g|$ is thus necessary for 2D ground-state cooling25,29 (see Methods for details).

The unique in situ tunability of levitated systems allows us to observe the effect of the dark mode decoupling on $n_{x,y}$. For a highly focused beam, the shape of the focus spot and thus the resulting trap frequencies are dependent on the incoming polarization24. We change $\Omega_{x,y}$ by tuning the ellipticity of the trapping beam polarization while keeping $\theta = 0.25\pi$ and $\Delta/2\pi = 257 \pm 11$ kHz. Comparing Fig. 4a and b, we observe that $\Omega_x$ and $\Omega_y$ approach each other as the polarization changes from linear to elliptical. In Fig. 4b, both modes heat up to $n_x = 2.0 \pm 0.4$ and $n_y = 2.8 \pm 0.8$ as weak coupling of the dark mode inhibits cooling. As we polarize the tweezer circularly for Fig. 4c, $x$ and $y$ peaks merge and we are unable to extract individual occupation numbers by conventional sideband thermometry.

We further theoretically test the validity of extracting phonon numbers by sideband thermometry using a full quantum model29 (Methods). We first calculate the true phonon occupation $n_f^\text{model}$ for $f = x, y$. Then, using the same model, we calculate PSDs of the heterodyne detection and perform sideband thermometry on them to extract $n_f$. We define the systematic error $\delta n_f = |(n_f^\text{model} - n_f)/n_f^\text{model}|$ and show...
the result in Fig. 4d,e. Mostly we find that \( \tilde{n}_j \) underestimates \( \tilde{n}_j^{\text{model}} \). In the weak coupling regime and for well-separated COM mode frequencies, \( \delta h \) is negligible. For stronger coupling and constant mode spacing, hybridization between optical and mechanical modes becomes more impactful and \( \delta h \) increases. At constant coupling rate, the error also increases as the mechanical frequencies approach degeneracy and the effect of the dark mode gains importance. We cannot perform 2D sideband thermometry for degenerate peaks, which occurs at small mode spacing and large coupling strength (Fig. 4d,e white areas). Finally we display the estimated errors for all measurements presented in Figs. 2–4. These errors of our sideband thermometry method are marginal, for Fig. 2 only about 1%, which confirms that we have achieved two-mode ground-state cooling.

Conclusions
We have simultaneously prepared two out of three COM modes of a levitated particle in their ground state with residual occupation numbers of \( \tilde{n}_x = 0.83 \) and \( \tilde{n}_y = 0.81 \). With respect to the optical axis of the tweezers, our cooling scheme controls the transversal degrees of freedom, resulting in two important implications. First, together with the demonstrated ground-state cooling along the tweezers’ axis\textsuperscript{12–14}, 3D COM quantum control is within experimental reach. Demonstrating 3D ground-state cooling would be an important step towards full control of large systems at the quantum limit. Second, control over transversal COM motion implies control of the orbital angular momentum along the tweezers’ axis, given by \( \hat{L}_z = x\hat{\rho}_y - y\hat{\rho}_x \), where \( \langle x, y \rangle \) and \( \langle \hat{\rho}_x, \hat{\rho}_y \rangle \) are the transverse position and momentum vector operator, respectively. As the particle’s transversal motion is in a thermal state, the variance of the corresponding angular momentum is given by \( \langle \hat{L}_z^2 \rangle^2 = (\tilde{n}_x + 1/2)(\tilde{n}_y + 1/2)/(\Omega_x/\Omega_y + \Omega_y/\Omega_x) - 1/2 \). With our occupation numbers and trap frequencies, we find \( \sqrt{\langle \hat{L}_z^2 \rangle} \approx 1.7\ h \). We have therefore prepared our system close to an angular momentum eigenstate along \( \hat{z} \) (\( \langle \hat{L}_z^2 \rangle = 0 \)) with \( \langle \hat{L}_z \rangle = 0 \). This opens the door to realizing protocols combining 2D ground-state cooling with coherently pumped orbital angular momentum\textsuperscript{3} to stabilize a state with large orbital angular momentum \( \langle \hat{L}_z \rangle \gg h \) and quantum-limited variance. Those minimally fluctuating high orbital angular momentum states (‘quantum orbits’) would be promising not only for fundamental studies of low-noise and massive high angular momentum states but also for becoming building blocks of a gyroscope with quantum-limited performance.

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Methods
Setup
A detailed view of the experimental setup is shown in Extended Data Figs. 1 and 2. To keep our cavity free of contaminants and at vacuum conditions (below $10^{-4}$ mbar) at all times, we load nanoparticles (SiO$_2$-F-L3205-23, 143 ± 6 nm nominal diameter; microParticles GmbH) in a separate loading chamber (not drawn in Extended Data Fig. 1) using a nebulizer (Omron). The loading tweezers are mounted on a movable arm, and its light is frequency shifted by acousto-optic modulator (AOM)3 by 80 MHz to avoid interference. After loading, we seal and evacuate the loading chamber to 1 × 10$^{-2}$ mbar and move the loading tweezers into the cavity chamber. We use two photodetectors and the loading tweezers’ nanopositioner (SmarAct GmbH) to align the focal points of loading and science tweezers. Initially, we measure the intensity of light passing by the particle on PD$_{\text{left}}$ to roughly align the two foci, while the science tweezers are still turned off. Afterwards, we gradually increase the power of the science tweezers and measure the light coupling into the fibre of the loading tweezers and shining on PD$_{\text{right}}$. Eventually, we turn down and up the power in the trapping and science tweezers, respectively, to transfer the nanoparticle to the science tweezers. Thereafter, the science chamber is sealed off and pumped to the cavity length. The small sidebands necessary for the error signal are generated by EOM2 at 23 MHz.

Detuning calibration
To perform accurate sideband thermometry, we need to compensate for the effect of cavity filtering of the motional sidebands. For a known detuning $\Delta$ and linewidth $\kappa$, we can readily calculate the cavity-induced asymmetry at a mode frequency $\Omega_j$ by assuming a Lorentzian filter function:

$$A_j^{(\text{cav})} = \frac{\kappa^2 + 4(\Delta + \Omega_j)^2}{\kappa^2 + 4(\Delta - \Omega_j)^2}$$

In our experiments, the particle scatters light into the TEM$_{00}$ of the optical cavity, while the cavity is locked to the TEM$_{00}$ mode. The difference of the two TEM mode frequencies is very sensitive to drifts of the cavity length between experiments. Thus, we need to calibrate the detuning of the cavity with respect to the science tweezers. As a particle-independent measure, we temporarily send a calibration laser through the cavity before performing cooling experiments. This laser has sidebands modulated by an electro-optic modulator (EOM) at 300 kHz and higher harmonics, which are filtered by the detuned cavity and then detected in our heterodyne scheme (Extended Data Fig. 3a). We extract the asymmetry of these three sidebands and the z-peak of the particle motion while tuning the cavity lock frequency with EOM1. In Extended Data Fig. 3b, we fit the Lorentzian cavity filter function to the measured sideband asymmetry to extract the linewidth and calibrate the detuning $\Delta$. The linewidth agrees with an independent measurement of $\kappa/2\pi = 330$ kHz. The s.d. values for $\Delta$ in Fig. 2b are below 3 kHz, and therefore the error bars are smaller than the markers.

Sideband thermometry
For extracting the phonon occupation from measured heterodyne signals, we rely on the different scaling of Stokes and anti-Stokes scattering processes. As the latter requires the presence of a phonon, its scattering rate scales with the average phonon number $n$, while the Stokes scattering rate scales with $n + 1$. This leads to an asymmetry of the anti-Stokes and Stokes sidebands dependent on the occupation number:

$$A_j^{(n)} = \frac{n_j}{n_j + 1}$$

The total asymmetry of the PSD at the frequency $\Omega_j$

$$A_j = \frac{S_{\text{SN}}(\Delta \Omega_j + \Omega_j)}{S_{\text{SN}}(\Delta \Omega_j - \Omega_j)} = A_j^{(0)} A_j^{(\text{cav})}$$

is the product of the thermal asymmetry $A_j^{(0)}$ and the cavity/induced asymmetry $A_j^{(\text{cav})}$. Given both, we can calculate the average occupation number

$$\bar{n}_j = \frac{A_j}{A_j^{(\text{cav})} - A_j^{(0)}}$$

To access the thermal asymmetry, we assume a Lorentzian shape of the motional sidebands

$$S_{\text{SN}}(\Omega) \approx \frac{a}{2\pi} \frac{\frac{\Omega}{\Omega}^2}{(\Omega - \Omega_j)^2 + \left(\frac{\Omega}{\Omega_j}\right)^2}$$

with amplitude $a_j$, width $\gamma_j$ and centre frequency $\Omega_j$. Our fitting function

$$F = S_{\text{xx}(\text{AS})} + S_{\text{yy}(\text{AS})} + S_{\text{yy}(\text{AS})} + S_{\text{SN}}$$

lets us extract four values for $a_j$, $\gamma_j$ and $\Omega_j$, one for each Stokes and anti-Stokes sideband of mode $j = x, y, z$. and gives $A_j^{(0)} = a_j/\Omega_j$. We require $\gamma_{yj} = \gamma_{jy}$ and $\Omega_{yj} = -\Omega_{jy}$. The shot noise level $S_{\text{SN}}$ is extracted for each PSD in a region far away from any spectral features to account for small drifts in the local oscillator power. We normalize all spectra to the shot noise level. For the error bars, we propagate the s.d. of the fitted amplitudes $a_j$ and the cavity parameters $\Delta$ and $\kappa$.

Quantum model
The Hamiltonian describing coherent scattering is

$$\frac{h}{\kappa} \dot{\hat{a}} = \Delta \hat{a} + \sum_{j=x,y,z} \Omega_j \hat{b}_j \hat{b}_j - \sum_{j=x,y,z} (g_j \hat{a}^\dagger + \text{h.c.}) (\hat{b}_j^\dagger + \hat{b}_j)$$

with $\hat{a}$ ($\hat{a}^\dagger$) being the photon annihilation (creation) operator, $\hat{b}_j$ ($\hat{b}_j^\dagger$) the phonon annihilation (creation) operator along motional axis $j = x, y, z$, and h.c. the Hermitian conjugate. This interaction allows to cool the COM motion along all three axes, as has been demonstrated experimentally. The linearized optomechanical coupling strengths $g_j$ for linear polarization are given by

$$[g_1, g_2] = \frac{g_0}{2} \begin{bmatrix} k_x \, x_{zpf} \sin \phi \cos \theta \\ k_y \, y_{zpf} \sin \phi \sin \theta \\ -i k_z \, z_{zpf} \cos \phi \end{bmatrix}$$

with $k = 2\pi/n\Lambda$, the cavity wavevector, $|x_{zpf}, y_{zpf}, z_{zpf}| = \sqrt{h/2\hbar m \nu_{zpf}}$ the zero-point fluctuations (zpf) along each axis and $\phi = 2\pi n_0/\Lambda_c$, with $n_0$ being the particle position along the cavity axis and $\Lambda_c = \Lambda/4$.
corresponding to an intensity minimum of the cavity standing wave. The rate \( C_0 = a|E_0|^2 \frac{\pi}{2ncd} e_\gamma \cdot e_e \) contains the particle polarizability \( \alpha = 4\pi \varepsilon_0 R^3 \frac{\Delta^2}{\Delta^2 + \frac{1}{4}} \) (with \( n \), the refractive index of the particle, \( R \) its radius and \( e_\gamma \) and \( e_e \) the vacuum permittivity), the trap electric field amplitude \( \varepsilon_0 = \sqrt{\frac{\pi}{2ncd \omega}} \) with \( \omega \) the trap waist at the focus, and the cavity parameters, namely the mode volume \( V_c = \pi d^2 L_c / 4 \), the cavity waist \( w_c \), the cavity length \( L_c \), the frequency \( \omega_c = 2\pi c / \lambda \), and the unit vector along the cavity axis \( e_e \). Note that the vector dot product \( e_\gamma \cdot e_e \) introduces an additional dependence on the trap polarization angle \( \alpha \cos \theta \), which hinders the possibility of coupling solely the \( x \) motional mode to the cavity.

The quantum state of the cavity–nanoparticle system is given by its density matrix \( \rho \), which obeys the dynamical equation\(^{28}\)

\[
\frac{d\rho}{dt} = -i \left( H_{\text{CS}} + H_{\text{Laser}}, \rho \right) = \frac{\alpha g}{\hbar} \left[ \hat{a} \hat{a}^\dagger - \left( \hat{a}^\dagger \hat{a} \right) / 2 \right]
\]

\[+ \sum_{j=x,y} \frac{\alpha g}{2} \left[ \hat{b}_j + \hat{b}_j^\dagger, \left[ \hat{b}_j + \hat{b}_j^\dagger, \rho \right] \right]
\]

\[+ \frac{\gamma_x}{8} \sum_{j=x,y} \left[ \hat{b}_j + \hat{b}_j^\dagger, \left[ \hat{b}_j - \hat{b}_j^\dagger, \rho \right] \right],
\]

where \( \{ \cdot \} \) denotes the anticommutator, \( \gamma \) is the friction rate due to gas damping and the heating rates \( \gamma = \gamma^{(0)} + \gamma^{(1)} \) contain a contribution from gas molecules

\[
\gamma^{(0)} = \frac{\hbar \kappa}{2} T_j
\]

and a contribution from laser recoil heating

\[
\gamma^{(1)} = \frac{n (\alpha g E_0^2 / 2\pi)}{15\hbar E_0} \left[ \frac{\hbar^2}{2g^4} \right]^{2} \frac{\hbar^4}{72\pi^2}.
\]

To obtain analytical expressions, we simplify the model by making two assumptions. First, friction due to gas molecules is negligible, which at the pressures used for this work can be checked to be a good approximation\(^{29,30}\). This amounts to neglecting the friction term in the master equation, namely the last line in equation (2). Note that the associated heating rate \( \gamma^{(1)} \gg \gamma \) is neglected. Second, the particle equilibrium position is at the intensity minimum of the cavity mode (\( \phi = \pi / 2 \)). This is the case for all measurements in this work within an accuracy of 1 nm. At this position, the couplings \( g_{x,y} \) are simultaneously maximized while the \( z \) mode becomes uncoupled. Under these approximations, the system is reduced to a three-mode system including only the cavity mode and the \( x \) and \( y \) motional modes, whose steady-state properties can be computed analytically (see below).

Within this three-mode approximation, we can explain the dark-mode effect. The Hamiltonian can be cast in the form

\[
\frac{\hbar}{\pi} \Delta \hat{a}^\dagger \hat{a} + \sum_{j=x,y} \omega_j \hat{b}_j + G_{x,y} \left( \hat{b}_x \hat{b}_y + \text{h.c.} \right) - (g_x \hat{a}^\dagger + \text{h.c.}) \hat{b}_y,
\]

where \( G_{x,y} \) are simultaneously maximized while the \( z \) mode becomes uncoupled. Under these approximations, the system is reduced to a three-mode system including only the cavity mode and the \( x \) and \( y \) motional modes, whose steady-state properties can be computed analytically (see below).

According to the Hamiltonian in equation (5), the cavity couples directly only to the bright mode and can thus only cool this mode. The dark mode can be only sympathetically cooled through its coupling to the bright mode, which has a rate

\[
G_{x,y} = \frac{\hbar^2 g_{x,y}}{2} (\Omega_{x,y} - \Omega_x).
\]

In the optimal 2D cooling configuration, \( \Omega_x = \Omega_y = \Omega \) and \( g_x = g_y = g \), so that the above rates simplify to \( G_x = G_y = g \Omega / 2 \). For 2D ground-state mechanical cooling, both the bright and dark modes must be cooled at a rate higher than their respective heating rate. For the bright state, this reduces to the standard optomechanical condition \( g \Omega > \Gamma_c > \Gamma \). As the dark state is cooled by the bright state and not by the cavity, for the dark state the condition reads

\[
\frac{4G_x^2}{(8g^2/k)} = \frac{k(\omega_x - \omega_c)^2}{8g^2} > \Gamma_c.
\]

Combining both conditions, we arrive to the inequality

\[
\sqrt{\frac{k}{8}} < \frac{g}{\omega_c - \omega_x} \sqrt{k/(8\Gamma_c)}.
\]

which defines a ‘Goldilocks zone’ for 2D cooling\(^{39}\). In particular, if the mechanical modes are close to degeneracy, the cooling of the dark mode is not efficient enough for it to reach the ground state. This in turn limits the steady-state occupations of the original modes \( x \) and \( y \), which are now limited by the thermal dark-mode occupation, \( \langle \hat{b}_x^\dagger \hat{b}_y - \langle \hat{b}_y^\dagger \hat{b}_x \rangle \). Note that, in typical coherent scattering experiments, \( \sqrt{k/(8\Gamma_c)} \) is on the order of 1–10, so that the right-hand side of the Goldilocks condition reduces to the condition \( |\omega_x - \omega_c| \geq |g| \) given in the main text.

**Extracting coupling and heating rates**

Within the two above assumptions (negligible friction and \( z \) motion uncoupled), the heterodyne power spectral density can be analytically calculated. The heterodyne PSD, after subtraction of the noise floor and normalization to it, can be written as\(^{40}\)

\[
S_{\text{het}}(\omega) = k \left[ S_x(\omega_{\text{lo}} - \omega) + S_y(\omega + \omega_{\text{lo}}) \right]
\]

with \( \omega_{\text{lo}} = \omega_{\text{lo}} - \omega_0 \). It is expressed in terms of the cavity PSD

\[
S_c(\omega) = \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{-i\omega s} \langle \hat{a}^\dagger(0) \hat{a}(s) \rangle_{ss},
\]

where the subscript ‘ss’ indicates the steady state. We compute the two-time correlator using the quantum regression theorem\(^{40}\), obtaining the following analytical expression:

\[
S_c(\omega) = 16 \left[ 4g_x^2 x \Omega_x^2 \left( \omega^2 - \Omega_x^2 \right)^2 
\right.
\]

\[
+ g_x^2 x \Omega_x \left( \omega^2 - \Omega_x^2 \right) \left( \Omega_x \left( \omega^2 - \Omega_x^2 \right) \left( 4(\Delta - \omega^2) + k^2 \right) 
\right.
\]

\[
+ 8g_x^2 x \Omega_x \left( \omega^2 - \Omega_x^2 \right) \left( 4(\Delta - \omega^2) + k^2 \right)
\left.
\right) \left( \Omega_x \left( \omega^2 - \Omega_x^2 \right) \left( 4(\Delta - \omega^2) + k^2 \right)
\right.
\]

\[
+ 16 \Delta \left( g_x^2 \Omega_x \left( \omega^2 - \Omega_x^2 \right) + g_x^2 \Omega_y \left( \omega^2 - \Omega_y^2 \right) \right)
\]

\[
+ (k^2 - 4\Delta^2) \left( \omega^2 - \Omega_x^2 \right) \left( \omega^2 - \Omega_y^2 \right) \right]
\]

\[
+ 16 \Delta \left( \omega^2 - \Omega_x^2 \right)^2 \left( \omega^2 - \Omega_y^2 \right) \right] .
\]
To extract from our data the experimental values for $g_{x,y}$ and $\Gamma_{x,y}$, we fit equation (11) to our time traces while setting $\kappa$ and $\Delta$ to the values from our calibration.

**Calculated heating rates**
Using equations (3) and (4), we calculate the heating rates of $x$ and $y$ motion using the parameters given in the main text and $T = 300$ K, $n_i = 1.439$, $\omega_i = 1.023$ $\mu$m, $w_i = 0.856$ $\mu$m and $\gamma = 4.812$ Hz $\times p_{gas,mbar}$ with $p_{gas,mbar}$ the gas pressure in mbar. At $p_{gas,mbar} = 5 \times 10^{-7}$, we obtain $\Gamma_{x}^{(0)} = 2\pi \times 0.535$ kHz, $\Gamma_{y}^{(0)} = 2\pi \times 0.445$ kHz, $\Gamma_{x}^{(1)} = 2\pi \times 0.825$ kHz and $\Gamma_{y}^{(1)} = 2\pi \times 1.379$ kHz. The total heating rate is thus dominated by photon recoil, $\Gamma_{xy} = \Gamma_{x}^{(0)} + \Gamma_{y}^{(0)}$ and $\Gamma_{xy}^{(0)}$ agrees with the values extracted from our measured PSDs. We heavily suppress heating due to phase noise, as we position the particle in the cavity node and use an ultra-low phase noise laser source. Note that the stated pressure reading of $p_{gas} = 5 \times 10^{-9}$ mbar is measured close to the ion-getter pump. The actual pressure at the position of the nanoparticle could be larger. $\Gamma_{xy}$ becomes comparable to $\Gamma_{xy}^{(0)}$ at $p_{gas,mbar} = 10^{-7}$, suggesting that the gas pressure near the nanoparticle is lower than this value.

**Error in sideband thermometry**
The effects of optical–mechanical mode hybridization and the decoupling of the dark mode modify the spectra measured through the cavity. As conventional sideband thermometry does not take these effects into account when estimating occupation numbers, the method is affected by a systematic error. To obtain this error, we theoretically calculate the occupation $n_j$ estimated via sideband thermometry by numerically computing the maxima of equation (13) and compensating for the cavity asymmetry as detailed above. We then compare this result with the exact phonon occupations $n_j^{\text{model}} = \langle b_j^\dagger b_j \rangle$, within our approximations (negligible friction and $z$ motion uncoupled). For the results in Fig. 4d,e, we fix $\Delta = 2\pi \times 240$ kHz, $g_x = g_y = g$, $\Omega_x = \Delta - \delta$ and $\Omega_y = \Delta + \delta$, and sweep over the parameters $\delta$ and $g$. At each point (that is, for each value of $\Omega_x$), the corresponding heating rates $\Gamma_{x,y}$ are calculated from equations (3) and (4).

**Data availability**
Source data for Figs. 2–4 and Extended Data Fig. 3 are available in the ETH Zurich Research Collection (https://doi.org/10.3929/ethz-b-000591807). All other data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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**Author contributions**
J.P. and D.W. conducted the experiments with equal contribution, supported by J.V. C.G.-B. and O.R.-I. performed the theoretical modelling, A.d.l.R.S., N.M. and R.Q. conceptualized the setup with R.R. and L.N., who directed the project.

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**Competing interests**
The authors declare no competing interests.

**Additional information**
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Extended Data Fig. 1 | Core setup for particle trapping, transfer and detection. To simplify the sketch, we show components of the detection setup and cavity lock on a separate sketch in Extended Data Fig. 2. We link the ports as indicated by the letter. All beam splitters with a black (red) outline are polarising (non-polarising) beam splitters and components labeled FI are Faraday isolators. All beams are derived from a NKT Photonics E15 1550 nm laser. In the sketched configuration (particle loaded in science tweezers), the half wave plate in the loading tweezers' section is set to dump all power at the input of a FI. Initially, while loading the particle, the loading tweezers are positioned in a separate vacuum chamber (not shown) and the full power is used to trap a particle. The photodetector PDL is used to monitor the trapping process. After aligning the loading tweezers with the science tweezers we rotate the half wave plate in front of the FI in the science tweezers' section to turn on the science tweezers. At the same time, we rotate the half-wave plate in the loading tweezers section, to turn off the loading tweezers and transfer the particle. From port B we feed in a beam to lock the cavity length by using the signal of the photodetector PD_{PDH}. To reduce noise on the detector we cross polarise the beam with respect to the tweezers and use a polariser to filter out the particle scattered light. On the opposite side of the cavity we use a photodiode to monitor the lock quality PD_{c} and an infrared camera to image the cavity mode. The mirror on the right is the high finesse mirror, therefore most of the particle scattered light leaks through the left mirror. We feed the light that leaks out of the cavity to port C to detect it with the heterodyne setup.
Extended Data Fig. 2 | Constituent setup for particle detection, cavity locking and calibration. All components are sketched as described in Extended Data Fig. 1. Light from the core setup enters from the top right through port A. We drive AOM1 (+ 1st order) and AOM2 (− 1st order) at $2\pi \times 80\text{ MHz}$ and $2\pi \times 78.5\text{ MHz}$, resulting in a local oscillator detuning $\Delta_{\text{lo}} = 1.5\text{ MHz}$. As we lock to the TEM$_{10}$ mode of the cavity, we use AOM1 and EOM1 to derive a beam at frequency close to $\omega_{10} - \omega_{00}$ (the difference of the resonance frequencies of TEM$_{10}$ and TEM$_{00}$). On the opposite side of the cavity input section, we modulate sidebands on the calibration beam before combining it with the lock beam. We implement a flip mirror to prevent the calibration beam from entering port B and consequently the cavity, while doing measurements.
Extended Data Fig. 3 | Detuning calibration by known sidebands. 

a, EOM-modulated sidebands of the calibration laser at 900 kHz (red), 600 kHz (green) and 300 kHz (blue), as well as the uncooled z-peak at 80 kHz (black) are filtered by the cavity transfer function (grey dashed line). b, We fit (lines) the detuning-dependent, expected cavity-induced asymmetry $A_{\text{cav}}^j$ to the measured asymmetries at of all four sidebands. The asymmetry of the x-peak is multiplied by 50 for visibility.