High-sensitivity optical Faraday magnetometry with intracavity electromagnetically induced transparency

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Abstract
We suggest a multiatom cavity quantum electrodynamics system for the detection of a weak magnetic field, based on Faraday rotation with intracavity electromagnetically induced transparency. Our study demonstrates that the collective coupling between the cavity modes and the atomic ensemble can be used to improve the sensitivity. With single-probe photon input, the sensitivity is inversely proportional to the number of atoms, and a sensitivity of 2.45 nT Hz\(^{-1/2}\) could be attained. With multiphoton measurement, our numerical calculations show that the magnetic field sensitivity can be improved to 105.6 aT Hz\(^{-1/2}\) with realistic experimental conditions.

Keywords: optical magnetometry, Faraday rotation, intracavity electromagnetically induced transparency

(1 Some figures may appear in colour only in the online journal)

1. Introduction

The ability to detect a magnetic field by optical means with high sensitivity [1–3] is a key requirement for a wide range of practical applications ranging from geology and medicine to mineral exploration and defense. A particularly important application is magnetic resonance imaging. Over the past few decades, a variety of techniques including a superconducting quantum interference device (SQUID) [4], a cavity optomechanical [5], negatively charged nitrogen-vacancy (NV) centers in diamond [6, 7] and Bose–Einstein condensates (BECs) [8, 9] and so on [10, 11], have been suggested for extremely high-sensitivity optical magnetometry.

The electromagnetically induced transparency (EIT) [12, 13] technique enables one to control the absorption and dispersion by changing the refractive index with cancelled absorption. The changes in the refractive index make it is possible to sense a weak magnetic field [14–16]. When a linear polarized field enters a medium subjected to a longitudinal magnetic field, its left- and right-circularly polarized components couple different atomic transitions, and consequently accumulate different phase shifts. As a result the direction of polarization is rotated, which is the so-called Faraday rotation [17]. Large optical Faraday rotation has been observed [18–22], and it has been suggested that this can be used to detect quantum fluctuation [23] or as an atomic filter [24, 25]. The strong dependence of Faraday rotation upon magnetic fields naturally suggests it as a magnetometry technique [17, 26].

In order to enhance the sensitivity of the rotation angle to the strength of the magnetic field in Faraday rotation based on EIT, it is necessary to reduce the width of the transparency window. When an ensemble of two-level atoms is placed inside an optical cavity, the atom–cavity interaction strength can be enhanced to be \(g\sqrt{N}\), where \(g\) is the single-atom–cavity coupling strength and \(N\) is the number of atoms in the cavity mode. The resonator response is consequently...
drastically modified, resulting in substantial narrowing of the spectral features [27–32]. Due to the strong optical confinement and small mode volumes, the optical cavity provides an excellent platform for strong light–matter interactions, allowing for vacuum-induced transparency [33], all-optical transistors [34] and cavity-enhanced optical nonlinearity [35]. The cavity-enhanced Faraday rotation in NV centers in diamond can push the sensitivity of a microwave magnetometer to the aT Hz$^{-1/2}$ range [7]. Furthermore, owing to the established technology for microcavity fabrication, cavities may be an attractive choice for miniaturized systems.

To detect a weak magnetic field via Faraday rotation through a microwave cavity [7], the whole detection setup should work at low temperature. There is much interest in extending cavity-enhanced Faraday magnetometry from microwave cavities to optical cavities, not only for understanding the nature of light–matter interaction but also for the suppression of thermal noise and the feasibility to push the technique to room temperature [29–31, 36, 37]. In the present paper, we suggest a composite atom–cavity system for high-sensitivity optical Faraday magnetometry based on intracavity EIT. In doing so, we apply a linear polarized probe field to couple into the cavity, and analyze the transmissions and phase shifts of its left- and right-circularly polarized components based on intercavity EIT [27, 38–41]. The main idea is to combine cavity-enhanced Faraday rotation and intracavity EIT. Our study shows that the cavity-enhanced Faraday rotation supports the detection of weak magnetic fields, resulting in cavity-enhanced optical Faraday magnetometry. For single-photon measurement, the sensitivity is inversely proportional to the number of atoms. With multiphoton measurement, the limit of sensitivity can be improved to 105.6 aT Hz$^{-1/2}$ from 2.45 nT Hz$^{-1/2}$ with single-photon measurement with an input power $P_{in} = 1.0 \mu W$ and realistic experimental parameters. Compared with the microwave Faraday magnetometer proposed by Xia et al [7], the physical mechanism behinds our Faraday magnetometer is intracavity EIT not resonance absorption. The sensitivity limit of Faraday magnetometry can therefore be improved by controlling the driving field and adjusting the number of atoms confined in the cavity.

2. Scheme and optical responses

As depicted in figure 1, $N$ identical four-level atoms in tripod configuration, which can be realized in the $^4$He $D_0$ line using hyperfine and Zeeman sublevels of the ground states [36, 37], are confined in the optical cavity. The three lower states $|–\rangle$, $|+\rangle$, and $|0\rangle$ correspond to the levels $2S_1/2 F = 1, m_F = 0$, $|F = 1, m_F = +1\rangle$ and $|F = 2, m_F = 0\rangle$, respectively. We choose the state $2P_0|F = 1, m_F = 0\rangle$ as the excited state $|1\rangle$. The magnetic field is applied from the outside to the inside of the page. The weak magnetic field lifts the degeneracy of the Zeeman sublevels $|–\rangle$, $|+\rangle$ and $|0\rangle$, and the energy shift is denoted by $\delta = g_a \mu_B B$ with $g_a = 2.002$ and $\mu_B = 14.0$ MHz mT$^{-1}$ being the Landé $g$-factor and Bohr magneton, respectively. A probe with vertical (V) linear polarization (perpendicular to the magnetic field) and carrier frequency $\omega_p$ couples into the cavity, and its $\sigma_+$- and $\sigma_-$-circularly polarized components drive the quantum transitions $|–\rangle \leftrightarrow |1\rangle$ and $|+\rangle \leftrightarrow |1\rangle$. We denote the detuning between the probe field with the cavity mode by $\Delta = \omega_p - \omega_c$, with $\omega_c$ being the cavity mode frequency. A classical driving field with horizontal (H) linear polarization, parallel to the magnetic field, and carrier frequency couples the transition $|0\rangle \leftrightarrow |1\rangle$. The driving field is applied from free space and through the waist of the optical cavity, such that the system works in the strong coupling regime. Thus the $A$ configuration (which is the heart of standard EIT) is formed for two probe components. The energy level $2P_0$ is above the level $2P_1$, about 29.6 GHz, and the energy separation is much larger than the Doppler width even at room temperature. Therefore, each transition is isolated, and the tripod configuration under consideration is clean [37].

In a rotating frame with the probe and driving field frequencies, the interacting Hamiltonian for the coupled multiatom–cavity system has the following form

$$H = H_a + H_{ad} + H_t,$$

in which

$$H_a = \hbar \sum_\delta (\Delta_\delta + \delta) \hat{a}^{\dagger+} + (\Delta_\delta - \delta) \hat{a}^{\dagger-} + \Delta_\delta \hat{a}^{\dagger1},$$

$$H_{ad} = -\hbar \sum_1 \left(g_a \hat{a}_+ \hat{a}_-^{\dagger} + g_\omega \hat{a}_- \hat{a}_+^{\dagger} + \Omega \hat{a}_0^{\dagger} \right) + \text{h.c.},$$

$$H_t = -\hbar \Delta_\delta \hat{a}_+ \hat{a}_- + \hbar \Delta \hat{a}_0^{\dagger} \hat{a}_0,$$

where $\Delta_\delta = \omega_p - [\omega_1 - (\omega_c + \omega_\perp)/2]$ and $\Delta_\delta = \omega_p - (\omega_1 - \omega_0)$ are one-photon detunings, $\sigma_{\alpha, \beta}$ ($\alpha, \beta = +, -, 0, 1$) is the

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**Figure 1.** Schematic of the setup and atom configuration for the detection of weak magnetic fields. The vertical linear polarized probe field couples into the optical cavity, and then transmits the cavity to the detector. The horizontally polarized driving field $\Omega$ is freely propagating, and it is much larger than the size of the atomic ensemble. We assume the cavity is symmetric such that the loss rates of the cavity fields of the right and left mirrors are equal, i.e. $\kappa_R = \kappa_L = \kappa/2$. The decay rate of the excited state $|1\rangle$ is denoted by $2\gamma$. We assume $g_+ = g_- = g$ for simplicity.
atomic operator for the \( i \)th atom, \( \hat{a}_i \) (\( \hat{a}_i^\dagger \) is the annihilation (creation) operator of the cavity photons, and the cavity–atom coupling coefficient is denoted by \( g_\pm = \mu_{\pm 0} \sqrt{\omega_i/2\hbar} V \)), As usual, we denote the Rabi frequency of the classical driving field by \( 2\Omega \). For simplicity, we assume \( g_+ = g_0 \) and the cavity is symmetric, i.e. \( \kappa_+ = \kappa_- = \kappa/2 \).

In the following, we focus our attention on the transmission and the Faraday rotation angle of the probe field. The responses of the two circularly polarized components of the probe field can be characterized by intensity transmission coefficients \( t_\pm \) and the phase shifts \( \phi_\pm \) of the corresponding components at the output. Following the standard processes \([40, 42]\), we solve the Heisenberg equations in steady state and the weak-field limit \( (g \ll \Omega) \). The steady-state solutions read

\[
\frac{a_\pm^\dagger}{a_\pm} = t_\pm e^{i\phi_\pm} = \frac{\kappa}{\kappa - i\Delta_\pm - i\chi_\pm},
\]

where \( \chi_\pm \) are the atomic susceptibilities, and they are given by

\[
\chi_\pm = -i \frac{g^2 N}{2(d_{\pm 0} + |\Omega|^2/d_{\pm 0})}, \quad j = +, -,
\]

in which \( d_{\pm 0} = i(\Delta_0 \pm \delta) - \gamma \) and \( d_{\pm 1} = i(\Delta_1 - \Delta_0 \pm \delta) - \gamma' \). \( 2\gamma \) is decay rate of the excited state \( |0\) and \( 2\gamma' \) is the dephasing rate between the three lower states.

In order to provide a detailed picture of the detection of weak magnetic fields we present the intensity ratio of the cavity transmitted probe fields \( t_+^2 \) and phase shifts \( \phi_+ \) of the (two circularly polarized probe components; they are shown in figures 2(a) and (b) as functions of the probe detuning \( \Delta_p \). All parameters are normalized by \( \gamma \), and they are taken as \( g\sqrt{N} = 10.0, \Omega = 1.0, \Delta_0 = \Delta = 0, \kappa = 2.0, \gamma' = 10^{-3} \) and \( \delta = 10^{-2} \). As presented in figure 2(a), the probe transmissions of the two probe components are Lorentzian shaped around \( \Delta_p = \pm \delta \). In the regime of standard intracavity EIT \( (\gamma' \ll \Omega^2) \), and combining with the relation \( g\sqrt{N} \gg \Omega \) \([13]\), the effective half widths of the transparency peaks of the two circular components are \([27, 39]\)

\[
w_1 \approx \gamma' + 2\kappa \frac{\Omega^2}{g^2 N}.
\]

The effective half widths are proportional to \( (\Omega/g\sqrt{N})^2 \), and they are much smaller than the bare cavity half width \( 2\gamma \) when \( g\sqrt{N} \gg \Omega \). The weak magnetic field lifts the degeneracy of Zeeman level \( (\pm \ell) \), and thus the positions of the transparency peaks are totally determined by the strength of the magnetic field, i.e. \( \Delta_p = \delta \) \( (\Delta_p = -\delta) \) for the \( \sigma_+ \) \( (\sigma_-) \) component. The vanishing absorptions ensure the application of the composite system for weak magnetic field detection at a low light level.

Within the narrow transparency peaks, the phase shifts of the two components of the probe field \( \phi_\pm \) are shown in figure 2(b). Owing to intracavity EIT, the dispersion curves are narrowed \([31]\). As a result, the phase shifts \( \phi_+ \) (solid curve) and \( \phi_- \) (dashed curve) vary rapidly, leading to enhancement of the Faraday rotation angle \( \phi = (\phi_+ - \phi_-)/2 \). This is the so-called cavity-enhanced Faraday rotation \([7]\). The Faraday rotation angle \( \phi \) versus probe detuning \( \Delta_p \) is illustrated in figure 2(b) with the long-dashed curve. For the sake of clarity, we amplify \( \phi \) four times. With the set of parameters given above, the effective half widths of the transparency peaks and Faraday rotation angle are, respectively, \( w_1 \approx 0.04 \) and \( \phi \approx 0.26 \) rad with respect to its initial direction. We assume the atomic ensemble is cold, i.e. taking the dephasing rates of the three lower states as \( \gamma' = 5 \times 10^{-4} \) such that the transparency condition \( \gamma' \ll \Omega^2 \) can be satisfied \([13]\). Changing the parameters as \( \Omega = 0.5, g^2 N = 200.0 \) and keeping the other parameters unchanged, one immediately obtains \( \phi \approx 0.18 \) rad with \( \delta = 10^{-3} \) together with \( w_1 \approx 5 \times 10^{-3} \). With this set of parameters, the dispersion within the transparency peaks becomes more sensitive to the magnetic field. This is consistent with the experimental observations in \([31]\). A large value of \( g^2 N/\Omega^2 \) leads to rapid changes in dispersion \([31]\) around the two-photon resonance, and results in enhancement of the sensitivity of the Faraday rotation angle to the strength of the weak magnetic field (details will be shown in section 3). Owing to the cavity-
enhanced Faraday rotation, it is feasible to realize cavity-enhanced optical Faraday magnetometry.

In order to see the role of the cavity more clearly, we apply the resonant conditions \((\Delta_p = \Delta_d = \Delta = 0)\) and and neglect the dephasing rates \((\gamma' = 0)\), and the transmissions \(t_\pm\) and phase shifts \(\phi_\pm\) of the probe components can be immediately simplified as

\[
I_\pm = \frac{2\kappa(\delta'^2 + \gamma^2)^{1/2}}{[(2\kappa\gamma - 2\delta\beta' + g^2)N]^2 + 4(k\beta' + \delta\gamma)^2]^{1/2}},
\]

\[
\phi_\pm = \pm \frac{g^2N\beta' - 2\delta(\delta'^2 + \gamma^2)}{g^2\gamma + 2\kappa(\delta'^2 + \gamma^2)},
\]

in which \(\delta' = \delta - \Omega^2/\delta\). At the ‘resonant point’ \((\Delta_p = 0)\), the phase shifts of the two circular components of the probe field are exactly equal and opposite in sign due to the symmetry. In the limit of small magnetic field, the polarization rotation angle can be reduced to \(\phi = (\phi_+ - \phi_-)/2 = \phi_0 \approx (\delta/2\kappa)(g^2N/\Omega^2)\), which indicates clearly that the Faraday rotation can be enhanced by confining the atomic ensemble in optical cavity.

### 3. Optical magnetometry and its sensitivity

In this section, we are interested in the sensitivity limit—which is the most important characteristic of magnetometry—of the present cavity QED system to perform measurement of the static magnetic field. Assuming the input linear probe field is vertically polarized, the Faraday rotation at the output can be measured by detecting the intensity difference of two linearly polarized components. For each probe, the output has three results: vertical polarization, horizontal polarization and no photons. Combing the relation between the \(\hat{\sigma}_+\) and \(\hat{\sigma}_-\), polarized basis and the \(\hat{H}\) and \(\hat{V}\) polarized basis, i.e. \(\hat{H} = (\hat{\sigma}_+ + \hat{\sigma}_-)/\sqrt{2}\) and \(\hat{V} = i(\hat{\sigma}_+ - \hat{\sigma}_-)/\sqrt{2}\), the probabilities of three outcomes \(p(\hat{H}\alpha)\) (polarized along the \(H\) direction), \(p(\hat{V}\delta)\) (polarized along the \(V\) direction) and \(p(0\delta)\) (no photons) are, respectively, given by

\[
p(\hat{H}\delta) = |t_e e^{i\phi_+} - t_h e^{i\phi_-}|^2/4,
\]

\[
p(\hat{V}\delta) = |t_e e^{i\phi_+} + t_h e^{i\phi_-}|^2/4,
\]

\[
p(0\delta) = 1 - p(\hat{H}\delta) - p(\hat{V}\delta).
\]

The three outputs as functions of magnetic field-induced level shift \(\delta\) with \(\Delta_p = 0\) are shown in figure 3(a). All other relevant parameters are the same as those in figure 2. Without the magnetic field, the \(\sigma_+\) and \(\sigma_-\) components of the probe field experience the same phase shift, and the polarization rotation angle \(\phi = 0\). Thus no \(H\)-polarized photon can be detected (see the dashed curve in figure 3(a)). With increasing magnetic field, \(p(\hat{H}\delta)\) increases, and reaches its maximum value \(p(\hat{H}\delta)_{\text{max}} = 0.27\) at \(\delta = \pm 0.043\gamma\). On increasing \(\delta\) much more, no transmitted photons can be detected. When the two-photon resonance condition is broken, e.g. \(\Delta_p = 0.01\), the variation of the absorption and dispersion of the atomic ensemble leads to the increase of \(p(0\delta)\) associated with increase of \(p(\hat{V}\delta)\) (see figure 3(b)).

The parameter \(\delta\) can be estimated from the outputs. The maximum information about a weak magnetic field that can be extracted from the measurement is given by the Fisher information [43], which determines the limit of the sensitivity. With single-photon measurement, the sensitivity limit is given by [7, 44]

\[
S \geq \frac{1}{\sqrt{F(\delta)}},
\]

where \(\varsigma\) is the number of times the procedure is repeated, and the Fisher information \(F(\delta)\) is defined by

\[
F(\delta) = (g_L\mu_B)^2 \sum_{x \in \{H,V\}} \frac{1}{p(x|\delta)} \left[ \frac{\partial p(x|\delta)}{\partial \delta} \right]^2.
\]

The Fisher information is crucially dependent on the conditional probability derivations. Recalling the limit of small magnetic fields, and combining the relation \(g\sqrt{N} \gg \Omega\), the Fisher information \(F(\delta)\) can be simplified and expanded as

\[
F(\delta) \simeq (g_L\mu_B)^2 \left[ \frac{2}{\kappa^2} \left( g\frac{\sqrt{N}}{\Omega} \right)^4 + O(\delta^2) \right].
\]

The first dominating term demonstrates that the Fisher information is proportional to \(g^2N/\Omega^4\). Recalling the relation (13), one immediately obtains \(S \sim 1/N\), which indicates that the sensitivity is inversely proportional to the number of atoms confined in the cavity. This is the main result of the collective coupling between the cavity modes and the atomic ensemble. It is therefore feasible to improve the sensitivity by increasing the value of \(g^2N/\Omega^2\).

In practice, the atoms may move with velocity \(v\), giving the 1/e temperature-related Doppler width of the atomic
ensemble as \( \Delta \omega_D = \sqrt{2k_B T \omega^2 / (mc^2)} \) [30]. With \( T = 1.0 \) mK, we immediately have \( \Delta \omega_D \approx 1.88 \) MHz. Considering the influence of Doppler broadening, the Fisher information can be rewritten as

\[
F(\delta) = \frac{1}{\sqrt{\pi \Delta \omega_D^2}} \int_{-\infty}^{\infty} F(\delta, k) e^{-\frac{1}{2}(\delta - \langle \delta \rangle)^2} d(\delta),
\]

(16)
in which \( k \) is the wave vector of the probe field. The Fisher information \( F(\gamma, \mu_B, \gamma') \) without (a) and with (b) Doppler broadening versus the level shift \( \delta \) is depicted in figure 4. In the calculation, all parameters are the same as those in figure 3. In the ideal case, i.e. two-photon resonance and without Doppler broadening, the Fisher information has a deep drop at \( \delta = 0 \) with the value \( F \approx 2256(\gamma, \mu_B, \gamma')^2 \). Changing the probe detuning to \( \Delta p = 0.01 \), the Fisher information around \( \delta = 0 \) decreases. Taking the residual Doppler broadening into account (as shown in figure 4(b)), the Fisher information is broadened, and becomes much smaller, \( F \approx 424(\gamma, \mu_B, \gamma')^2 \), than that without Doppler broadening. The influence of two-photon detuning can be ignored. Therefore, we can say, with a single-photon probe, the present optical Faraday magnetometer is robust to the frequency fluctuation of the probe and driving fields. With the realistic experimental parameters (\( ^4\text{He D}_0 \) line) [36, 37], \( \hbar \omega_p = 1.14 \text{eV}, \gamma = 5 \) MHz, \( \gamma' = 10^{-3} \gamma = 5 \times 10^{-3} \) MHz, \( g_N = 10.0 \gamma = 50 \) MHz [40], \( \Omega = 1.0 \gamma = 5 \) MHz and \( \kappa = 2.0 \gamma = 10 \) MHz, the sensitivity can be calculated to be 2.45 nT Hz\(^{-1/2}\).

For vertically polarized multiphoton measurement, we choose \( \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}^\dagger \) as the measurement operator, and the sensitivity limits can be obtained from [7, 16]

\[
\Delta B = \frac{\Delta n_{\text{out}}}{\partial \langle \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}^\dagger \rangle / \partial B},
\]

(17)
in which \( \Delta n_{\text{out}} = \sqrt{\langle \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}^\dagger \rangle - \langle \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}^\dagger \rangle^2} \) is the variance of the measurement operator \( \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}^\dagger \). Denoting the \( H \)- and \( V \)-polarized noise coupled into the cavity by \( \hat{b}_H \hat{H} \) and \( \hat{b}_V \hat{V} \), and including them in the quantum Langevin equations for the cavity modes, one immediately has

\[
\hat{a}_{H} = -\frac{i}{2}(t_x \hat{e}^{i\phi} - t_y \hat{e}^{i\phi}) (\hat{a}_{V}^\dagger + \hat{b}_V) + \frac{1}{2}(t_x \hat{e}^{i\phi} + t_y \hat{e}^{i\phi}) \hat{a}_{V}.
\]

(18)

For the coherent \( H \)-polarized input field, the output field is a noisy coherent state. Thus the variance of the measurement operator \( \hat{m}_{\text{out}}^\dagger \hat{m}_{\text{out}}^\dagger \) can be calculated, and the limit of the sensitivity reads

\[
S \geq \frac{t}{\sqrt{\pi \Delta \omega_D}} \int F_H(\delta) e^{-\frac{1}{2}(\delta - \langle \delta \rangle)^2} d(\delta),
\]

(19)
in which \( F_H = (g_{\mu_B})^2 \langle \hat{a}_{\text{out}}^\dagger \hat{a}_{\text{out}}^\dagger \rangle \langle p(\hat{H}) \rangle^2 / \langle p(\hat{H}) \rangle \) is the normal Fisher information of the horizontally polarized output, \( k_B \) is the Boltzmann constant and \( P_{in} \) denotes the input probe power. In the above derivation, the noise due to the internal loss channels is included. Equation (19) indicates that the sensitivity is inversely proportional to \( \sqrt{P_{in}} \), and thus can be improved by increasing the probe power. Taking Doppler broadening into account, the normal Fisher information of the horizontally polarized output is given by

\[
F_H = \frac{1}{\sqrt{\pi \Delta \omega_D^2}} \int_{-\infty}^{\infty} F_H(\delta) e^{-\frac{1}{2}(\delta - \langle \delta \rangle)^2} d(\delta).
\]

(20)

The evolution of \( F_H \) versus \( \delta \) and probe detuning \( \Delta p \) without (a) and with (b) Doppler broadening is shown in figure 5. All parameters used in the calculation are same as those in figure 4. Owing to the Doppler effect, the normal Fisher information of the horizontally polarized output is broadened. The maximum value of \( F_H \) decreases from 2256 to 423 \( \times (\gamma, \mu_B, \gamma')^2 \) (\( \Delta p = 0 \)) and 1993 \( \times (\gamma, \mu_B, \gamma')^2 \) (\( \Delta p = 0.01 \)) to 423 \( \times (\gamma, \mu_B, \gamma')^2 \). With \( P_{\text{in}} = 1 \mu W \) and \( T = 1 \) mK the sensitivity can be immediately calculated to be 8.63 \( f T \) Hz\(^{-1/2}\) (with Doppler broadening). Changing the set of parameters to \( g_N = 10.0 \gamma = 50 \) MHz, \( \Omega = 0.5 \gamma = 2.5 \) MHz and increasing the input power to \( P_{\text{in}} = 1 \mu W \), our numerical calculations immediately show that the sensitivity limit can be improved into 105.6 nT Hz\(^{-1/2}\). Owing to Doppler broadening.
\( \Delta D \simeq 1.88 \text{ MHz} \), around the resonant regime, the influence of probe detuning on \( F_H \) can be safely neglected. Consequently, the present optical Faraday magnetometer is robust to the frequency fluctuations of the probe and driving fields.

4. Conclusion

We have suggested a cavity QED system for detection of a weak magnetic field using cavity-enhanced Faraday rotation-based intracavity EIT. A four-level atomic ensemble in tripod configuration is confined in the optical cavity. Owing to the collective coupling between the cavity modes and the atomic ensemble, the Faraday rotation can be enhanced dramatically. Our numerical calculations show that the cavity-enhanced optical Faraday rotation supports cavity-enhanced optical Faraday magnetometry, which is robust to frequency fluctuations of the probe and driving fields. With single-photon measurement, the sensitivity is inversely proportional to the number of atoms, and a sensitivity of 2.45 \( \text{nT Hz}^{-1/2} \) can be achieved. The sensitivity can be improved by increasing the probe power with multiphoton measurement, and can be improved to 105.6 \( \text{aT Hz}^{-1/2} \) under realistic experimental conditions. Cavity-enhanced Faraday rotation can be of interest in atomic information processing methods.

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Figure 5. Fisher information of the horizontally polarized output \( F_H/(g_B^2 p_B^2) \) as a function of \( \delta \) and probe detuning \( \Delta_p \) without (a) and with (b) residual Doppler broadening. All parameters used in the calculation are same as those in figure 2.
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