Geometrical and Material Nonlinear Finite Analysis of Fiber Reinforced Concrete Slabs

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Abstract. Geometrical nonlinearity arises from the continuous change of shape under increasing loads on thin or long span members, and the materials nonlinearity due to the nonlinear behavior of the materials composing the structural element. In this study the finite element method is used to analyze fibrous reinforced concrete slabs, material and geometric nonlinearities were considered. Concrete is represented by degenerate quadratic thick shell element. Concrete behavior in compression is modeled by elastic perfectly plastic or strain hardening plasticity. A tensile strength criterion is used to follow crack initiation and propagation. A parabolic stress degradation function, which is based on the fracture energy concept, is used to model the tension stiffening effect of steel fibers. The reinforcement is considered as smeared within the concrete layers, and assumed as either an elastic perfectly plastic material or as an elastic-plastic material with linear strain hardening. Von Karman assumptions which is based on the total Lagrangian approach is used for the geometric nonlinearity. The numerical results using for both normal and lightweight fiber reinforced concrete slabs were predicted satisfactorily. The average predicted failure load of the investigated cases to the experimental failure load was 0.899 with a standard deviation of 0.092.

Keywords: Finite element; Geometrical nonlinearity; Reinforced concrete slabs; Steel fibers; Tension stiffening.

1. Introduction
Reinforced concrete slabs are important structural member in any reinforced concrete building, since it is the only member that takes the live loads directly. The thickness of the flat plates or flat slabs is mostly controlled by the punching shear stresses created in the vicinity of the columns. To prevent the catastrophic and brittle punching shear failure, the ACI Code, (2019) recommend the increase of the...
slab thickness or use of shear reinforcement. The shear reinforcement may be in the form of single leg, multiple legs, bent-down bars, shear studs, or shear head reinforcement.

Steel, glass, polymer and other types of short discrete fibers have been added as reinforcement to concrete, thus producing a composite material with enhanced tensile strength, ductility, well defined post cracking behavior, and strain capacity, ACI Committee 544 (2002), and ACI Committee 544.4R (2018). The ACI Code, (2019) recommends the use of deformed steel fibers complying with ASTM Specifications, (2016) as shear reinforcement with length to diameter ratio ranging between 50 and 100. Steel fibers has been used to improve the behavior and strength of reinforced concrete slabs in shear and flexure, Swamy and Ali, (1982), Theodorakopoulos and Swamy, (1993), Yaseen, (2006), Min et al., (2011), Jalate and Kalurkar, (2013), Tan and Venateshwaran, (2017), and Abdel-Rahman et al., (2018).

The finite element (FE) is used extensively for the nonlinear analysis of fiber reinforced concrete (FRC) members, Al-Taan and Ezzadeen, (1995), and Xu et al., (2008). Geometrical and material nonlinearity due to tensile cracking, bond-slip of the bars embedded in concrete, nonlinear behavior of concrete in compression have been taken into account. The (FE) is used also by many researchers for the nonlinear analysis of fiber reinforced concrete slabs, Abdel-Rahman et al., (2018), Tazaly, (2012), Mepegetis, (2012), Irani and Abadi, (2013), Alvarez, (2013), Sayhood et al., (2014), Puddicome, (2018).

Mepegetis, (2012) used nonlinear finite element to analyze interior square fiber reinforced concrete pile supported slabs. Three types of steel fibers at a dosage of 35-50kg/m² were used, 35×0.75 mm hooked fibers, 35×0.55 mm hooked fibers, and 25×0.5 mm twisted wire fibers. Three and four nodded shell elements with reduced integration were used to model the slab. In the post cracking stage, the analysis adopts multilinear stress – displacement (σ-w) relationship derived from inverse analysis on notched beams, round determine panels (supported on three equally spaced supports), and round indeterminate panels (supported on six equally spaced supports) rather than a stress – strain (σ-ε) one. Discrete and smeared cracking were used in the analysis and showed comparable results when displacement control is used.

The nonlinear finite element method is used by Irani and Abadi, (2013) for the analysis of two-way octagonal concrete slabs with a side length of 1.0 m using the program DIANA. Three different configurations of reinforcement were considered: Conventional reinforcement, steel fiber reinforcement (hooked ends, length = 60 mm, 0.9 mm diameter, and Vf = 0.45%) and a combination of both. Concrete, and steel bars were represented by two-dimensional curved shell elements (with five degrees of freedom) and embedded bar elements respectively. Concrete behavior in compression is assumed as an elastic perfectly plastic and in tension six different models were used including multi-linear stress-strain diagram that is not dependent on fracture energy. Smeared cracking model was used to model cracking of concrete. Von Mises plasticity model was used for the reinforcement and to specify the hardening. Displacement control was used in static analyses, with Secant method for iterations. Similar numerical results were obtained for the used tensile models except the constant model. Slabs with steel fiber reinforcement only, higher capacity and stiffer behavior were obtained compared to experimental results. Slabs with combined steel fiber and conventional reinforcement, showed larger cracking load while the load from yielding until failure was lower in the analysis than in the experiments.

Abdel-Rahman et al., (2018) tested fourteen reinforced concrete slabs reinforced longitudinally with 1.2% in both directions under concentric and eccentric load. Steel fibers with dimensions (50×0.52×0.72 mm) with volume fractions of 0.5 to 1.5% were used to study the influence of steel fibers on the punching shear strength of the slabs. The test results showed an increase in the punching shear strength up 24%. A nonlinear (FE) analysis is conducted using the program ANSYS R14.5 to compare the numerical with the experimental results. Concrete is represented by eight node solid elements, solid65, and three-dimensional link elements 180 were used to represent the reinforcing bars. The average ratio of the numerical-to the experimental load is 0.98 with a standard deviation of 0.14, which indicates a good agreement.

Puddicome, (2018) presented a (FE) model using the program ABAQUS for the analysis of (FRC) slabs. The nonlinear behavior of concrete is simulated using the Concrete Damaged Plasticity constitutive
model. An exponential decay expression with variables such as concrete compressive strength, flexural reinforcement ratio, steel fibers volume fraction, and the reinforcing steel yield strength is used to derive a tension – stiffening model. Eight-noded hexahedral (brick) elements were used to model concrete with reduced integration, and the reinforcement is modeled as two-node linear truss elements. The model is applied to published test results of nine slabs from the literature, and the ratio of the numerical to the experimental failure load varied from 0.78 to 1.281 with an average of ratio of 1.068 and a standard deviation of 0.195.

Al-Taan and Abdul-Razzak, (2020) used the computer program written by Hinton and Owen (1984) for the nonlinear (FE) analysis of fiber reinforced lightweight and normal weight square concrete slabs. Thick shell element was used to represent concrete and the thickness is discretized into layers. The steel reinforcement was smeared within the concrete layers. Concrete in compression assumed to follow an elastic-plastic or strain hardening behavior. In tension Tensile strength criterion is used to detect crack initiation and propagation. The descending post peak stress-strain relationship in tension is modeled using a parabolic stress degradation function that is based on the fracture energy concept of steel fiber concrete. The numerical results and the experimental results of square lightweight and normal weight fibrous reinforced concrete slabs showed good agreement.

In this paper the computer program written by Hinton and Owen, (1984) for the nonlinear analysis of reinforced concrete plates and shells is modified by including the constitutive material relationships of steel fiber concrete for the nonlinear geometrical and material (FE) analysis of (FRC) slabs under increasing monotonic loading. Modified version of the Newton-Raphson method has been used for the solution of the nonlinear problem.

2. Materials Constitutive Relationships

2.1 Compression

The first part of the curve in compression can be treated using the elasticity theory, Figure 1, and the second part using the theory of plasticity. The growth of yield surface during plastic loading is described using a hardening rule. A yield function depending on the mean normal stress, $I_1$ and the shear stress invariant, $J_2$ is used:

$$f(I_1, J_2) = \sqrt{\beta_f(3J_2)} + \alpha_f I_1 = \sigma_o$$

($\sigma_o$) is the biaxial stress ratio $\omega = \sigma_1/\sigma_2$.

Abdul-Razzak, (1996) used the experimental results reported by Abdul-Ahad and Abbas, (1989), Leonard and Mansur, (1991), and Yin, et. al. (1989), in a regression analysis, and the following equation is derived for $\omega$ by including the influence of the stress ratio, fiber aspect ratio and volume fraction:

$$\omega = e^x$$

$$x = \frac{1}{3.339-0.9772\ln\left(\frac{\nu_{cf}}{\nu_f}\right)}$$

($\alpha_f$) and ($\beta_f$) can be written as follow:

$$\alpha_f = \frac{1-\omega^2}{\omega^2-2\omega} \sigma_o$$

$$\beta_f = \frac{1-2\omega}{\omega^2-2\omega}$$

The stress components are used to express and the yield condition:
\[
f(\sigma) = \left[ \beta_f \left( (\sigma_x^2 + \sigma_y^2 + \sigma_{xy}) + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right) + \sigma_f (\sigma_x + \sigma_y) \right]^{\frac{1}{2}} \tag{7}
\]

To predict the crushing state of concrete, the yield criterion is converted into strain instead of stress components. The experimental results reported by Abdul-Ahad and Abbas, (1989), Leonard and Mansur, (1991), and Yin, et. al. (1989), are used by Abdul-Razzak, (1996) to estimate the ultimate compression strain of steel (FRC):

\[
\varepsilon_{cuf} = 0.003011 + 0.002295V_f \tag{8}
\]

### 2.2 Tension

As for plain concrete, the first ascending part of the stress-strain relationship in tension, up to first cracking, Figure 2 is linear. Due to the process of micro-cracks initiation and propagation, it starts deviates from linearity. After that a formation of an unstable micro-crack system will occur, which grows rapidly under the increasing tensile stress bringing the material to its post-mortem region, Bentur and Mindess, (2007). The equations derived by Sorouhian and Lee, (1989) for the tensile strength \(f_u\) and its corresponding strain \(\varepsilon_u\) for (FRC) are used in this study:

\[
f_{tf} = f_t \left(1 + 0.016N_f^{1/3} + 0.05\pi D_f L_f N_f \right) \tag{9}
\]

\[
\varepsilon_{tf} = \varepsilon_t \left(1 + 0.35D_f L_f N_f \right) \tag{10}
\]

where \(N_f\) is the number of fibers crossing a unit area [30]:

\[
N_f = \frac{1.64V_f}{\pi D_f^2} \tag{11}
\]

After the formation of a continuous crack, a stress drop occurs and the crack is arrested at a certain level by the pullout resistance of the fires. It is assumed that the fibers bridging the crack start resisting the applied stress immediately after composite cracking. This average tensile stress is defined here as \(f_m\), and can be calculated as the product of the average bond stress by the interfacial area of all fibers crossing the cracks, Bentur and Mindess, (2007):

\[
f_u = \eta_o \sigma_u V_f \frac{L_f}{D_f} \tag{12}
\]

where \(\eta_o\) is the orientation factor. Different values were proposed for this factor and the probabilistic value of 0.41 is used in this study, Hannant, (1978). \(\sigma_u\) is the average bond strength and is equal to Sorouhian and Lee, (1989):

\[
\sigma_u = 2.62 - 0.0036N_f \tag{13}
\]

After the sudden drop in stress from \(f_u\) to \(f_c\), a descending nonlinear stress-strain relationship is used in this investigation to simulate the post-peak constitutive model as shown in Figure 2. The derivation of the nonlinear curve is based on the fracture energy concept, Abdul-Razzak, (1996) using experimental load-deformation curve reported by Visalvanich and Naaman, (1983). The stress-strain relation can be expressed as follows:

\[
\sigma_i = f_u \left( \frac{\varepsilon_i - \varepsilon_m}{\varepsilon_{tf} - \varepsilon_m} \right)^2 \tag{14}
\]

where \(\varepsilon_m\) is the limiting tensile strain. This value is calculated by Abdul-Razzak, (1996) assuming that the descending stress-strain curve is similar to that of a load-displacement curve of a fiber exhibiting pull-out, and the area under this curve is equal to the fracture energy \(G_f\):

\[
\varepsilon_m = \frac{3\sigma_f}{h f_u} + \varepsilon_{tf} \tag{15}
\]

\(h\) = characteristic length of the Gauss point. The fracture energy \(G_f\) which is derived by Visalvanich and Naaman, (1983) is adopted in this study:
\[ G_f = 0.171 \beta \tau_u V_f \left( \frac{2L_f^2}{L_f^2} \right)^{2/3} \]  

(16)

The tension stiffening effect is considered as indicated in Figure 1 by assuming a gradual release of the concrete stress component normal to the cracked plane. Unloading and reloading of cracked concrete is supposed to follow the linear behavior shown in Figure 2 with a fictitious secant modulus \( E^s \). In the biaxial tension-compression zone, the parabolic relationship derived by Al-Taan and Mahmood (1988) is used:

\[ \sigma_{2p} = \left[ \frac{1+4(\alpha S)^2}{2(\alpha S)^2} \right] f'_{cf} \]  

(17)

\[ S = \frac{f'_c}{f_{tu}}, \]  

and the peak tensile strength as:

\[ \sigma_{1p} = \sigma_{2p} \alpha_2 \]  

(18)

where \( \sigma_{1p} \) and \( \sigma_{2p} \) are the limiting principal tensile and compressive strengths respectively in this zone and \( \alpha_2 \) is the principal stress ratio = \( \sigma_2/\sigma_1 \).

After cracking, the concrete is assumed to be in a state of uniaxial compression parallel to the cracks and exhibiting characteristics of the strain hardening elasto-plastic model, and the relationship proposed by Vecchio and Collins, (1986) is used in this investigation:

\[ f'_{cf, max} = \frac{f'_c}{0.8+0.3 \epsilon_2} \leq f'_{cf} \]  

(19)

![Stress - Strain curve for fibrous concrete in tension](image)

Figure 1. Stress - Strain curve for fibrous concrete in tension, Abdu-Razzak, (1996)

Perpendicular to the cracks, the elastic modulus and Poisson’s ratio is reduced to zero and a reduced shear modulus is used to represent the aggregate interlock.

2.3. Shear Modulus of Cracked Concrete

Four different relationships are used for the reduced shear modulus of cracked concrete G,

(a) Approach G1, in this relationship the value of G assumed to be decreasing linearly with the current tensile strain as assumed by Cedolin and Poli, (1977), for concrete cracked in in direction 1,

\[ G_{12} = 0.25G \left( 1 - \frac{\epsilon_1}{\epsilon_m} \right) \]  

for \( \epsilon_1 < \epsilon_m \)  

(20a)

\[ G_{12} = 0 \]  

for \( \epsilon_1 > \epsilon_m \)  

(20b)

(b) Biaxial orthotropic relationship G2: This approach has been proposed by Chen, (2007), the cracked shear
modulus \( \overline{G} \) can be calculated as follows:
\[
\overline{G}_{12} = \frac{E_1 E_2}{(E_1 + E_2 + 2\nu E_2)}
\] (21)

If Poisson’s ratio \( \nu \) equal to zero Eq. (21) becomes:
\[
\overline{G}_{12} = \frac{E_1 E_2}{E_1 + E_2}
\] (22)

E1 and E2 are the elastic moduli in directions 1 and 2 respectively.

(c) Approach G3: In this expression the value of G is decreasing parabolically with the current tensile strain, using a function similar to that used in Eq. (14), Abdul-Razzak, (1996), for concrete cracked in direction 1,
\[
\overline{G}_{12} = 0.25G\left[\frac{(\varepsilon_1 - \varepsilon_m)}{(\varepsilon_{tf} - \varepsilon_m)}\right]^2 \quad \text{for} \quad \varepsilon_1 < \varepsilon_m \quad (23a)
\]
\[
\overline{G}_{12} = 0 \quad \text{for} \quad \varepsilon_1 \geq \varepsilon_m \quad (23b)
\]

(d) Approach G4: This relationship was proposed initially by Al-Mahaidi, (1978) and modified in this study by substituting \( \varepsilon_{tf} \) for \( \varepsilon_t \). The reduced shear modulus of cracked concrete \( G \) can be calculated as follows:
\[
\overline{G}_{12} = \frac{0.4G}{\varepsilon_1 \varepsilon_{tf}}
\] (24)

3. Finite Element Formulation

The eight-node serendipity thick shell element is used in the present study. At each node, three displacements and two independent rotations of the normal are considered. The formulation of this element is mentioned by Onate et al., (1978). This element is the most popular degenerated shell element in isoparametric formulation and it is the simplest element considered in the original work of Ahmad, Onate et al., (1978), and this element requires \( C^0 \) continuity. Shear locking occurs even in moderately thin situations when using full integration. A great improvement of the results was obtained when the reduced integration was used, Hinton and Owen, (1984). Smeared approach is implemented for the steel reinforcement which is incorporated with the layered approach. Across the thickness, each element is divided into a number of concrete layers. The stresses of each layer are computed at the Gauss points. Smeared approach is implemented for the steel reinforcement which is incorporated with the layered approach.

3.1 Geometrical Nonlinearity

The nonlinearity in structural members may be classified into two types; material and geometric. The material nonlinearity, arises from the nonlinear behavior of the material composing the structural element. While the geometrical nonlinearity arises from the changes of the geometry associated with the increased deformation of the monotonically loaded member. The problem is usually solved by following the geometrical change of both the structural member and its elements in a step by step manner. In this study a specific total Lagrangian formulation is adopted in which large deflections and moderate rotations (in the sense of the Von Karman hypotheses) are accounted for. The strain-displacement matrix is calculated once during the nonlinear process and its nonlinear part is updated using the current displacement by a simple matrix product. The total strain is divided into parts, the linear part \( \{\varepsilon\}_L \) and the nonlinear part \( \{\varepsilon\}_N \)
\[
\{\varepsilon\} = \{\varepsilon\}_L + \{\varepsilon\}_N
\] (25)

and
\[
d\{\varepsilon\} = d\{\varepsilon\}_L + d\{\varepsilon\}_N
\] (26)

The strain-displacement matrix is also divided into two parts,
\[
d\{B\} = d\{B\}_L + d\{B\}_N
\] (27)
Where \( \{ B \}_o \) is the linear part of the strain-displacement matrix, and \( \{ B \}_L \) is the nonlinear part of the strain-displacement matrix. The nonlinear part of the strain vector can be written as follows, Hinton and Owen, (1984):

\[
\{ \varepsilon \}_L = \frac{1}{2} \left( \begin{array}{c}
\frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \\
\frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 \\
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y} \\
0 \\
0
\end{array} \right) = \frac{1}{2} \left( \begin{array}{c}
\frac{\partial w}{\partial x} \\
0 \\
\frac{\partial w}{\partial y} \\
0 \\
0
\end{array} \right) \left( \begin{array}{c}
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y}
\end{array} \right)
\]

(28)

or

\[
\{ \varepsilon \}_L = \frac{1}{2} [S] [R]
\]

(29)

where

\[
[S]^T = \left[ \begin{array}{ccc}
\frac{\partial w}{\partial x} & 0 & \frac{\partial w}{\partial y} \\
0 & \frac{\partial w}{\partial y} & 0
\end{array} \right]
\]

(30)

The vector \( \{ R \} \) can be defined as:

\[
\{ R \} = \left( \begin{array}{c}
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y}
\end{array} \right)^T = [G] [\sigma]
\]

(31)

\( [G] \) is a rectangular matrix having two rows and columns equal to the total number of element nodal variables. The geometrical stiffness matrix may be written as,

\[
[K]_\sigma = \int [G]^T [\sigma] [G] d\nu
\]

(32)

\( \{ \sigma \} \) are the components of the Piola-Kirchhoff stress vector.

4. Numerical Examples

The first example is a normal weight reinforced concrete square slab-column connection with a side length of 1800 mm and overall thickness of 125 mm tested by Swamy and Ali, (1982), Figure 2. The slab was designed to fail in shear. The slab was simply supported along all four edges, with span of 1690 mm. The slab was loaded through the column stub over an area of 150×150 mm. The geometry was chosen to simulate a flat plate-column connection with a column spacing of 4.0 m center to center in both directions. One quarter of the slab is analyzed due to symmetry, Figure 2. Dimensions, reinforcement details and loading are shown in Figure 2a. The materials properties are summarized in Table 1. Two steel layers at the top and two at the bottom are used to represent the reinforcement and six layers to represent concrete are found to be enough for the analysis. This slab was analyzed by Al-Taan and Abdul-Razzak, (2020) using material nonlinearity, and in the present study it is analyzed using the effect of geometrical nonlinearity. The slab is modeled by three meshes, four, nine, and eleven elements. However, the loading, geometry of the slab and the obtained results showed that using eleven elements gives a reliable simulation and results.
Table 1. Materials properties of slab S-2, Swamy and Ali, (1982)

| Property          | Value       |
|-------------------|-------------|
| $E_c$ (GPa)       | 33.29       |
| $E_s$ (GPa)       | 17          |
| $V_f$ %           | 0.5         |
| $f_{cf}$ (MPa)    | 34.67       |
| $f_c$ (MPa)       | 4.2         |
| $f_t$ (MPa)       | 425         |
| $A_c$ bars        | 7- 8        |
| $A_s$ bars        | 12- 10      |
| $\varepsilon_{cf}$  $\times 10^{-3}$ | 4.4         |
| Experimental     | 243.6       |
| failure load (kN) |             |

* Crimped steel fibers 0.5×50 mm, † calculated from Equation (8), $\nu_c = 0.15$

Figure 2. Details and finite element mesh for slab S-2, (a) Dimensions and reinforcement, (b) Finite element mesh

Figure 3 shows a comparison between the four shear moduli (G1-G4) models with elasto-perfectly plastic and strain hardening models. From this figure it is seen that G1 with elasto-perfect plastic model and G3 model with strain hardening model give very good results compared with the experimental results, while G4 gives good response but with slightly smaller failure load. The shear crack model G2 underestimates the strength and in some cases leads to premature failure.

Figure 4 below, show that there is an effect of the concrete compression model used in the analysis, if the compression model changed from elasto-perfectly plastic to stain hardening. Also, when the geometrically nonlinear analysis is used G1 with elasto-perfectly plastic model and G3 model with strain hardening model give very good results compared to experimental ultimate load.
Figure 3. Load-deflection curves for slab S-2, (a) elasto-perfectly plastic model, (b) Strain-hardening model

Figure 4. Variation of the ultimate load with the shear model, slab S-2

Initial damage of the slab occurs in the form of tension cracking in the radial direction, Figure 5. The damage zone adjacent to the column stub prior to failure was dominated by two way tension cracking action, high shear and compression stresses and strains leading to crushing of concrete under the load, Figure 5.
Figure 5. Cracking and crushing patterns, (a) cracking at load 67.5 kN, (b) cracking at load 180 kN, (c) cracking at failure load, (d) crushing at failure load, (e) Experimental cracking at failure.

The second example is a lightweight concrete slab-column connection tested by Theodorakopoulos and Swamy, (1993) of the same geometry as the previous example. The steel reinforcement used in the tested slab was cold-worked high-tensile deformed bars with 8 bars 10 mm diameter placed in the tension zone in both directions. Due to symmetry of the slab, only quarter of the slab is analyzed. Table 2, shows the material properties of the tested slab. Dimensions detail and loading are as shown in Figure 2 above. As for the previous example the slab is modeled in the present study by many elements namely four, nine, and eleven elements. However, the obtained results showed that using eleven elements which is shown in Figure 2 above give a reliable simulation and results. Two steel layers at the bottom are used to represent the tension reinforcement and eight layers to represent concrete are found to be enough for the analysis. This slab was analyzed also by Al-Taan and Abdul-Razzak, (2020) using material nonlinearity, and in the present study the slab is analyzed using the effect of geometrical nonlinearity.
Table 2. Materials properties of slab FS-20, Theodorakopoulos and Swamy, (1993)

| $E_c$ (GPa) | $E_s$ (GPa) | $V_f$ % | $f_{cd}$ (MPa) | $f'_c$ (MPa) | $f_y$ (MPa) | $A_s$ bars | $\epsilon_{cuf} \times 10^{-3}$ | Experimental failure load (kN) |
|-------------|-------------|---------|----------------|-------------|-------------|------------|----------------|-----------------------------|
| 19.4        | 17          | 1.0     | 46.3           | 4.38        | 425         | 8-ϕ 10     | 5.3            | 211                         |

* Crimped steel fibers 0.5×50 (mm), † calculated from Eq. (8), $\nu_c = 0.15, E_s = 204$ GPa.

It is shown in Figure 6 that the geometrically nonlinear analysis underestimates the ultimate load for all the investigated reduced shear moduli due to the relatively large deformations prior to failure (25% of the thickness) and probably due to the relatively low elastic modulus of the lightweight concrete. In Figure 6b the strain hardening model with the geometrically nonlinear analysis shows that the results are similar to those of Figure 6a, this means that changing the compression model has no influence on the results.

**Figure 6.** Load-deflection curves for slab FS-20, (a) elasto-perfectly plastic model, (b) Strain-hardening model

Figure 7, shows that when the material nonlinearity (geometrical linear analysis) is used, G4 model gives very good results compared to the experimental results presented by Al-Taan and Abdul-Razzak, (2020). But when geometrical nonlinearity is used in the analysis G1 and G3 models give good results for both elasto-perfectly...
plastic and stain hardening models compared to experimental results. Also, it is seen that there is small effect of the compression models on the results of the ultimate load because the failure of the slab is punching shear failure and may be due to the relatively low elastic modulus of the lightweight concrete. The shear crack model G2 underestimates the strength and in some cases leads to premature failure.

5. Conclusions

The presented numerical together with the experimental results showed that the used eight nodes (Serendipity) general shear deformable shell elements with five degrees of freedom proved to be efficient for the nonlinear geometrical and material analysis of the studied fiber reinforced concrete slabs. The layered approach is suitable for modeling these types of slabs for both the structural response and ultimate load. The adopted constitutive models for fiber reinforced concrete concerning the strain hardening, tension stiffening, and reduced shear crack models proved to give satisfactory results (such as; load-deflection curves, cracks initiation and propagation, and ultimate loads) for the analysis of normal weight and lightweight fibrous reinforced concrete slabs subjected to monotonic loading up to failure.

Geometric nonlinearity incorporating the present models plays an important role in determining the response of fibrous reinforced concrete slabs. For the investigated fibrous reinforced concrete slabs, it is generally shown that, when geometrical nonlinearity is used G1 and G3 models give good results compared to the experimental results. The shear crack model G2 underestimate the strength for the examples investigated and, in some cases, leads to premature failure.

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