Equilibrium spin current through the tunnelling junctions

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(Received November 15, 2018)

We study equilibrium pure spin current through tunnelling junctions at zero bias. The two leads of the junctions connected via a thin insulator barrier, can be either a ferromagnetic metal (FM) or a nonmagnetic high-mobility two-dimensional electron gas (2DEG) with Rashba spin orbital interaction (RSOI) or Dresselhaus spin orbital interaction (DSOI). As a lead of a tunnelling junction, the isotropic RSOI or DSOI in 2DEG can give rise to an average effective planar magnetic field orthogonal or parallel to the current direction. It is found by the linear response theory that equilibrium spin current \( \vec{J} \) can flow in the following three junctions, 2DEG/2DEG, 2DEG/FM, and FM/FM junctions, as a result of the exchange coupling between the magnetic moments, \( \vec{h}_l \) and \( \vec{h}_r \), in the two electrodes of the junction, i.e., \( \vec{J} \sim \vec{h}_l \times \vec{h}_r \).

An important distinction between the FM and 2DEG with RSOI (DSOI) lead is that in a strict one-dimensional case RSOI (DSOI) cannot lead to equilibrium spin current in the junction since the two spin bands are not spin-polarized as in a FM lead where Zeeman spin splitting occurs.

Pacs numbers: 73.23.-b, 72.25.Dc, 71.70. Ej

I. INTRODUCTION

Spin related transport in magnetic microstructures has drawn considerable interests in research community for the purpose of spintronics.\(^1\)–\(^3\) In order to manipulate the spin degree of freedom of an electron, one needs first to build a source of spin-polarized electrons and efficiently injects spins into a non-polarized medium. The ferromagnetic metal (FM) is an ideal spin source for spin injection into semiconductor devices; however, the injection efficiency experimentally measured is extremely low due to the much larger conductivity of FM than that of semiconductor.\(^4\)–\(^6\) This issue has not yet been resolved although it is fundamentally important. As an alternative solution to the spin injection problem, the pure spin current was proposed and has attracted much attention in recent years. The pure spin current is composed of equal spin down and spin up electron currents flowing along opposite directions without any charge current.\(^7\) Several different methods have been proposed to create pure spin current. Based on the spin pumping principle,\(^8\)–\(^9\) an alternating or inhomogeneous magnetic field as a spin-pumping force could result in a pure spin current. Experimentally, Stevens et al.\(^10\) and Hubner et al.\(^11\) have independently realized the pure spin current by using the quantum interference of two-color laser fields with cross-linear polarization in ZnSe and GaAs semiconductors. The transport of spin current by magnons have been theoretically studied by several groups,\(^12\)–\(^14\) e.g., Meier et al.\(^12\) demonstrated that by using a finite length spin chain between magnetic reservoirs, pure spin current can be generated without the transport of electrical charge. The equilibrium spin current (ESC) in a magnetic or nonmagnetic system is also very attractive. The newly unearthed spin Hall current, discovered by Murakami et al.\(^15\) in p-doped semiconductors and Sinova et al.\(^16\) in two-dimensional electron systems (2DEG), is actually a dissipationless spin current, which is a transverse response to a longitudinal external electric field \( E_x \). The spin Hall current in a 2DEG is generated by the Rashba spin orbital interaction (RSOI). In a single RSOI system the spin state of an electron is dependent on its momentum direction, so that at equilibrium the spin current could flow in the system without bias.\(^17\) As long as a system has noncollinear magnetic order, a ESC could exist in the system.\(^18\) For instance, König et al.\(^19\) found a dissipationless spin current in a thin film ferromagnet, in which a spiral magnetic order exists and the spin phase coherence can affect the electronic transport properties.

For the FM/FM tunnelling junction, a spontaneous ESC has been verified\(^20\)–\(^23\) flowing across the interface when the two magnetic moments \( \vec{h}_l \) and \( \vec{h}_r \) in the two leads of the junction are not collinear, and their exchange interaction determines the ESC \( \vec{J}_s \), \( \vec{J}_s \sim \vec{h}_l \times \vec{h}_r \). This indicates non-collinear magnetizations in the two leads can lead to spin flip and consequently a ESC through the junction. Nogueira et al.\(^22\) and Lee et al.\(^23\) referred to the ESC through the FM/FM junction as the spin supercurrent as an analogy to the Josephson effect in superconductor junctions. The spin current in a FM/FM junction was also shown to result in magnetic moment reversal in one of the FM leads of the junction.\(^24\)

In 2DEG with RSOI or Dresselhaus spin orbital interaction (DSOI), there is a pseudomagnetic field which could lead to ESC in the non-magnetic system. This pseudomagnetic field is different from a real magnetic field as it is dependent on the direction of electron momentum and keeps the system’s time reversal symmetry. A natural question thus arises whether a spin supercurrent (or ESC) could flow through a nonmagnetic tunnelling junction in which the electrodes are 2DEGs with either RSOI or DSOI. A recent paper\(^25\) by Borkje and Susho has been dedicated to this issue and authors obtained a ESC in a 2DEG junction with RSOI by rotating one of the 2DEG electrodes along the current direction.
to change the direction of the pseudomagnetic field, so that the product of two pseudomagnetic fields in two leads is nonzero, meanwhile, the absence of ESC through the 2DEG/FM tunnelling junction was also predicted. In this paper, we will study systematically the ESC in three different junctions: 2DEG/2DEG, 2DEG/FM, and FM/FM junctions, using the linear response theory. It is found that ESC is present in all three junctions resulting from the exchange coupling between the two magnetic moments in the two electrodes of the junctions. Non-magnetic 2DEG with RSOI or DSOI as an electrode in a tunnelling junction can lead to a ESC, because only half of the electrons (those with \( k_x > 0 \), if the 2DEG is on the left of the junction) in an electrode contribute to the tunnelling current so that the isotropic RSOI or DSOI can give rise to an average planar effective magnetic field orthogonal or parallel to the current direction. We also found that in a strict one-dimensional (1D) case RSOI or DSOI cannot result in a ESC in the junction because the 1D density of states is not spin-polarized, which is very different from that of a 1D FM lead.

This paper is organized in the following way. In the second part, we give a general formula of the ESC in a tunnelling junction derived using the linear response theory. In the third part, we analyze in detail the ESC in three junctions: 2DEG/2DEG, 2DEG/FM, and FM/FM junctions. A conclusion was drawn in the last section.

II. FORMULA

We start our discussion with the derivation of a general formula of ESC through a junction, consisting of two leads of either 2DEG with RSOI (DSOI) or FM metal and an insulator barrier between them. The Hamiltonian of such a tunnelling system is given by

\[
\mathcal{H} = \mathcal{H}_L(C^\dagger \sigma; C \sigma) + \mathcal{H}_R(C^\dagger \sigma; C \sigma) + \mathcal{H}_T
\]

\[
\mathcal{H}_T = \sum_{kq\alpha\beta} (t_{kq} C_{k\alpha}^\dagger C_{q\beta} + h.c.)
\]

where \( \mathcal{H}_{L(R)} \) is the Hamiltonian of a non-interacting free-electron gas in the left (right) lead, whose specific form will be present in next section; \( \mathcal{H}_T \) is the tunnelling part connecting two leads. \( C_{k\sigma}^\dagger (C_{k\sigma}) \) and \( C_{q\sigma}^\dagger (C_{q\sigma}) \) are the fermion creation (annihilation) operators of the left and right leads, respectively. The hopping matrix \( t_{kq} \) is independent of spin, so that there is no spin-flip when electrons tunnel through the junction. In the following discussion the quantum number \( k \) and \( q \) can also denote implicitly the left and right lead.

We focus on the tunnelling ESC and the bulk ESC in the RSOI (DSOI) lead is not considered here because it does not contribute to the tunnelling current. The total spin in the left lead is \( S_L = (\hbar/2) \sum_{kq\beta} C_{k\alpha}^\dagger \sigma_{\alpha\beta} C_{k\beta} \) where \( \alpha \) and \( \beta \) are spin indices, and \( \sigma \) is the Pauli spin operator. The time evolution of \( S_L \) in the Heisenberg picture is given by \( S_L = (1/\hbar)[S_L, H_T] \), and thus the operator of spin current \( J_s = \partial S_L / \partial t \) reads

\[
J_s = -\frac{i}{2} \sum_{kq\alpha\beta} \sigma_{\alpha\beta}(t_{kq} C_{k\alpha}^\dagger C_{q\beta} - t_{kq} C_{q\alpha} C_{k\beta}).
\]

Here the coupling \( H_T \) is assumed to be very weak and we only need to calculate the spin current to the lowest order of the coupling. Since the spin current operator \( J_s \) itself is already linear in \( t_{kq} \), and thus according to the Kubo formula the spin current is to first order in \( H_T \) given by

\[
J_s(t) = \int_{-\infty}^{t} dt' e^{-\alpha^2(t-t')} \Pi_R(t, t'),
\]

\[
\Pi_R(t, t') = -i\theta(t-t')\langle \hat{J}_s(t), H_T(t') \rangle,
\]

where \( \langle \cdots \rangle \) is the thermal statistical average on two independent leads \( \mathcal{H}_L + \mathcal{H}_R \) (the unperturbed part of \( \mathcal{H} \)), which are assumed to be in local equilibrium. \( \Pi_R(t,t') \) is the retarded Green’s function. After some straightforward algebra and keeping the average only in lead \( \mathcal{H}_L \) or \( \mathcal{H}_R \), we could express Eq. 3 in the steady state as

\[
J_s = -\frac{1}{2} \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \sum_{kq} |t_{kq}|^2 Tr \left\{ \frac{G_k^r(\omega_1)\sigma G_k^< (\omega_2) - G_k^< (\omega_1)\sigma G_k^r (\omega_2)}{i(\omega_2 - \omega_1 - i0^+)} \right\} + \frac{G_q^r(\omega_2)\sigma G_q^< (\omega_1) - G_q^< (\omega_2)\sigma G_q^r (\omega_1)}{i(\omega_1 - \omega_2 - i0^+)}
\]

where the trace \( Tr \) is over the spin space, \( G_k^{r<}(\omega) \) are the Keldysh Green’s functions in the left (right) lead and can be readily solved for leads of free-electron model. Their definitions are \( G_k^{r<}(\omega) = i(C_k^{(r<)}(t)C_k^{(r<)}(t')) \) and \( G_q^{r<}(\omega) = -i(C_q^{(r<)}(t)C_q^{(r<)}(t')) \), respectively. The Fourier transform of the Green’s function is given by \( G_k^{r<}(\omega) = \int \frac{d\omega'}{2\pi} G_k^{r<}(\omega') e^{-i\omega'(t-t')} \).

It is noted that Eq. 4 above is a general formula of spin current through a tunnelling junction and the bias could be reflected in the local equilibrium Green’s functions \( G_k^{r<}(\omega) \) of the two leads. For zero bias on the junction considered in this article, i.e., zero charge current flow through the junction, we can further simplify Eq. 4 by putting \( 1/(x + i0^+) = P(1/x) - i\pi\delta(x) \) with \( P \) denoting the principle integral,

\[
J_s = \frac{1}{2} \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \sum_{kq} |t_{kq}|^2 P \left\{ \frac{f(\omega_2) - f(\omega_1)}{i(\omega_2 - \omega_1)} \right\} \left\{ A_k(\omega_1)\sigma A_q(\omega_2) - A_q(\omega_2)\sigma A_k(\omega_1) \right\}
\]

\[
A_k(\omega) = i(G_k^{r>}(\omega) - G_k^{r<}(\omega)),
\]

\[
A_q(\omega) = i(G_q^{r>}(\omega) - G_q^{r<}(\omega)),
\]
Here $A_{k(q)}(\omega)$ is the spectral function of the left (right) lead with $G^{(r)}(\omega)$ denoting the retard (advanced) Green’s function and $f(\omega)$ denoting the Fermi distribution function. The ESC in Eq. 5 is mainly determined by the difference between two Hermitian-conjugate matrices in spin space, so that the off-diagonal term in the spectral function $A_{k(q)}$ is utmost important to the formation of a ESC in a tunneling junction. For FM or 2DEG with RSOI (DSOI), this off-diagonal term in spin space exists when two magnetizations (pseudo-magnetic fields in 2DEG) are not collinear in the two leads, as a result, a nonzero ESC can flow through the junction according to Eq. 5. In the following section we will discuss the ESC in three specific junctions: 2DEG/2DEG, 2DEG/FM, and FM/FM junctions.

III. RESULTS AND DISCUSSION

A. 2DEG/2DEG tunnelling junction

In this section we study the ESC through a 2DEG/2DEG junction with spin orbital interaction. First we review briefly the characteristics of a 2DEG with RSOI, which stems from the structure inversion asymmetry of the confining potential of a quantum well.26 The magnitude of the RSOI strength could be modulated by mechanical strain to the lead. Although the pseudomagnetic field in 2DEG is isotropic, only half of the electrons (electrons with $k_x > 0$ on the left and $x$ is the current direction in Fig. 1) in the 2DEG contribute to the tunnelling current, as a result, the average pseudomagnetic field of the tunnelling electrons is nonzero and along the $y$ direction ($B\hat{y}$). If we could change the direction of the pseudomagnetic field in one of the 2DEG, a ESC would flow through a 2DEG/2DEG junction. A celebrated example is the RSOI/DSOI junction, where one 2DEG lead possesses RSOI while the other has DSOI. By exchanging the spin axis $\sigma_x \leftrightarrow \sigma_y$, RSOI can be transformed into DSOI as

$$H_{\text{dsoi}} = \frac{\lambda}{\hbar}(\sigma_x p_x - \sigma_y p_y).$$

This coupling is due to the lack of the bulk inversion symmetry in the material.28 Thus in the RSOI/DSOI tunnelling junction, the average pseudomagnetic field in the DSOI lead is along the $x$ direction, which is different from that in the RSOI lead, so that a nonzero ESC $\sim B_{\text{rsoi}}\hat{y} \times B_{\text{dsoi}}\hat{x}$ exists as shown below in Eq. 11. In Ref. 25, Borkje and Sudbo attempted to rotate one 2DEG lead along the current direction ($x$-direction in Fig. 1) so as to rotate the orientation of the pseudomagnetic field. However this method may not create a ESC through the RSOI/RSOI junction as discussed below. The rotation of the average pseudomagnetic field could be also obtained by rotating physically a quasi-one-dimensional 2DEG along the $z$-axis by an azimuthal angle $\alpha$, which is schematically shown in Fig. 1. Since the pseudomagnetic field $B \sim \hat{z} \times \mathbf{p}$ is dependent on the direction of the electron momentum, an oblique incident electron will feel pseudomagnetic fields with different orientations when it tunnels through a RSOI/RSOI junction where the energy band edges of the two 2DEG leads are different. This change in pseudomagnetic field direction could lead to a ESC in the junction. The energy band shift in one 2DEG lead may be obtained by applied a gate voltage or mechanical strain to the lead.

To understand the relation of a rotated pseudomagnetic field and ESC, we consider a rotation of the spin $x - y$ axes by an angle $\alpha$ with respect to the spatial $x - y$ axes. The rotated angle $\alpha$ of the pseudomagnetic field could be incorporated into the Pauli matrices $\sigma_x(\alpha) = \begin{pmatrix} 0 & e^{i\alpha} \\ e^{-i\alpha} & 0 \end{pmatrix}$ and $\sigma_y(\alpha) = \begin{pmatrix} 0 & -ie^{i\alpha} \\ ie^{-i\alpha} & 0 \end{pmatrix}$. Here $\alpha$ is macroscopic in that it applies to all the electrons and may denote the rotation angle of the average pseudomagnetic field.

In second quantization formalism, Eq. 6 can be rewritten as

$$H = \sum_k \left( C_{k \uparrow}^\dagger C_{k \uparrow} \right) \begin{pmatrix} \tilde{\epsilon}_k & -i\lambda ke^{i\theta} \\ i\lambda ke^{-i\theta} & \tilde{\epsilon}_k \end{pmatrix} \begin{pmatrix} C_{k \uparrow} \\ C_{k \downarrow} \end{pmatrix},$$

where $\tilde{\epsilon}_k = \hbar^2 k^2 / 2m_s - \mu$ with $\mu$ the chemical potential, and $e^{i\theta} = e^{i(\alpha - \theta_\sigma)}$. With $\theta_r$ replaced by $\theta_d$ (tan $\theta_d =$
\( k_x/k_y \) as well as \( \lambda \) by \( -\lambda \), this Hamiltonian can represent a free 2DEG with DSOI in Eq. 7. The retard (advanced) Green’s function of this free electron model with RSOI is easily given by

\[
G^{(a)}(\omega) = \begin{pmatrix} \omega \pm i0^+ - \varepsilon_k & i\lambda ke^{i\theta} \\ -i\lambda ke^{-i\theta} & \omega \pm i0^+ - \varepsilon_k \end{pmatrix}^{-1}.
\]  

(9)

Before we present the final expression of ESC of 2DEG/2DEG junction by substituting \( G^{(a)} \) above into Eq. 5, we first focus on the spectral function \( A_k(\omega) \) of 2DEG with RSOI

\[
A_k = \begin{pmatrix} \pi[\delta_+ + \delta_-] & -i\pi[\delta_- - \delta_+] e^{i\theta} \\ i\pi[\delta_- - \delta_+] e^{-i\theta} & \pi[\delta_+ + \delta_-] \end{pmatrix},
\]  

(10)

where \( \delta_+ = \delta(\omega - \varepsilon_k + \lambda k) \) and \( \delta_- = \delta(\omega - \varepsilon_k - \lambda k) \) with \( \delta \) denoting the delta function. In the formula of ESC (Eq. 5), the summation term \( \sum_k A_k(12) = \sum_k A_k(21) = 0 \); thus, there is no ESC through the junction as discussed earlier (Eq. 5). This result stems from the fact that the 1D density of states of the two spin bands of an electron gas with RSOI or DSOF is not spin-polarized, which is very different from the 2DEG case. In the 1D case the density of states is inversely proportional to the electron velocity, and the velocities of electrons in the two spin bands with the same energy under RSOI or DSOF are identical so that the summation of the off-diagonal terms in Eq. 10 over \( k \) vanishes. In other words, the perpendicular incidence of electrons from one RSOI (DSOF) lead into another RSOI (DSOF) lead will conserve their spins whereas this is not the case for the obliquely incident electrons. Therefore, the rotation of a 2DEG along the current direction (the \( x \)-axis in Fig. 1) in the 2DEG/2DEG tunnelling junction will give rise to two 2DEG leads not in the same plane, and if the middle insulator barrier is very thin, the electron tunnelling in such a junction may be a 1D transport, i.e., only electrons with velocity along the \( x \)-direction could tunnel through the junction due to the momentum mismatch. Thus the ESC may vanish in this scheme.

After straightforward algebra, the ESC for the two-dimensional RSOI/DSOF junction (two 2DEGs in the same plane) is given by

\[
J^x_s = 0,
\]

(11a)

\[
J^y_s = 0,
\]

(11b)

and

\[
J^z_s = \frac{\hbar}{2\pi e^2} G_N E_r f(-E_r) \sin(\alpha_L - \alpha_R - \pi/2).
\]  

(11c)

Here \( \alpha \) is the rotation angle of the spin axes of the left (right) lead that changes the direction of the pseudomagnetic field. \( E_r = \hbar^2 k_r^2/2m_s \) is the Rashba energy with \( k_r = \lambda n_s/\hbar^2, f(-E_r) \) is the Fermi distribution function. \( G_N = e^2/2\pi r^2 \rho_{L(R)} \) is the conductance of the normal 2DEG junction without spin-orbital interaction. \( \rho_{L(R)} \) is a constant density of state of 2DEG. \( \pi/2 \) phase difference in Eq. 11c comes from the exchange of spin axes when DSOF is compared with RSOI. Here we have assumed for simplicity the spin-orbital coupling constants of two leads are the same \( \lambda_L = \lambda_R = \lambda \) and \( t_{kg} = t \) is independent of energy.

From Eq. 11, only ESC polarized along the \( z \) direction is nonzero since the pseudomagnetic fields in RSOI/DSOF junction lie in the \( xy \) plane, their exchange interaction will lead to spin current polarized along the \( z \) direction. The phase (rotation angle) \( \alpha \) of the pseudomagnetic field is canonically conjugate to the spin \( s_z \) so that they satisfy the commutation relation \( [s_z, \alpha] = -i \), which means the spin component \( s_z \) is not a conserved quantity as the RSOI (DSOF) can change \( s_z \) by rotating the spin, and ESC could hence flow in these junctions. The physics is similar to the Josephson current in a superconductor junction, in which the macroscopic phase \( \phi \) resulted from the condensation of Cooper pairs is conjugate to the particle number \( N \) and \( [N, \phi] = -i \). The continuity equation of the spin current in 2DEG with RSOI/DSOF is

\[
\frac{\partial s}{\partial t} + \nabla \cdot \mathbf{J}_s = S,
\]

where \( J_s = \text{Re} \left[ \Psi^\dagger \tilde{v} s \Psi \right] \) is the spin current density with \( \tilde{v} \) being the velocity operator, and \( S = \text{Re} \left[ \Psi^\dagger (\tilde{B} \times \sigma) \Psi \right] \) is a source term from the pseudomagnetic field by rotating the spin component \( s_z \). A similar source term appears also in the continuity equation of the current of superconductor.

B. 2DEG/PM junction

We turn to discuss ESC in an 2DEG/PM junction. In the last subsection it was stated that RSOI (DSOF) in 2DEG can be equivalent to a pseudomagnetic field along the \( y \) (\( x \)) direction with \( \alpha = 0 \) when electrons tunnel through the junction. When one of the 2DEG leads is substituted by a PM, a ESC could also flow through the 2DEG/PM junction from the point of view of the magnetic field. The Hamiltonian of the PM in the right side of the junction is given in a simple framework of the Stoner model by

\[
H_{f\alpha} = \sum_q \begin{pmatrix} C_{q\uparrow}^\dagger & C_{q\downarrow}^\dagger \end{pmatrix} \begin{pmatrix} \varepsilon_q + \mu + \hbar \omega & 0 \\ 0 & \varepsilon_q - \hbar \omega \end{pmatrix} \begin{pmatrix} C_{q\uparrow} \\ C_{q\downarrow} \end{pmatrix}.
\]  

(12)

Here the spin quantum axis is set to be along the local magnetic moment of PM, \( \varepsilon_q = \hbar^2 q^2/2m - \mu \) and \( \hbar \) (use energy as the unit here) is a molecular field in the PM. Then the local retarded Green’s function is

\[
g_{r(\omega)}(\omega) = \frac{1}{\omega + i0^+ - \varepsilon_q + \hbar}. 
\]  

(13)
In the common spin quantum axis set along the z-direction for the 2DEG/FM junction, i.e. the normal of the 2DEG plane, the Green’s functions of the FM lead above should be transformed according to

\[
G^{r(a)} = U \left( \begin{array}{cc} \theta(r) & 0 \\ 0 & \phi(r) \end{array} \right) U^\dagger, \tag{14a}
\]

\[
U = \left( \begin{array}{cc} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} e^{-i\phi} \\ \sin \frac{\theta}{2} e^{i\phi} & \cos \frac{\theta}{2} \end{array} \right), \tag{14b}
\]

where \( U \) is the unitary transformation matrix, \((\theta, \phi)\) denotes the direction of the magnetic moment of the FM lead, which in terms of the unit vectors of the cartesian coordinates should be written as \((\sin \theta \cos \phi \hat{x}, \sin \theta \sin \phi \hat{y}, \cos \theta \hat{z})\). For the specific RSOI/FM junction we consider, the rotation angle in the left lead is set as \(\alpha_L = 0\). Substituting Eq. 14 and Eq. 9 into Eq. 5, we obtain the final result as

\[
J_s^z = \int \frac{d\omega_1 d\omega_2 |P|^2}{P} \frac{f(\omega_2) - f(\omega_1)}{\omega_2 - \omega_1} \chi_{fm}(\omega_2) \chi_{2deg}(\omega_1) \cos \theta \tag{15a}
\]

\[
J_s^y = 0 \tag{15b}
\]

\[
J_s^x = \int \frac{d\omega_1 d\omega_2 |P|^2}{P} \frac{f(\omega_2) - f(\omega_1)}{\omega_2 - \omega_1} \chi_{fm}(\omega_2) \chi_{2deg}(\omega_1) \sin \theta \sin \phi \tag{15c}
\]

with

\[
\chi_{fm}(\omega) = \sum_q \{ \delta(\omega - \varepsilon_q - h) - \delta(\omega - \varepsilon_q + h) \} \tag{16a}
\]

\[
\chi_{2deg}(\omega) = \sum_k \{ \delta(\omega - \varepsilon_k - \lambda k) - \delta(\omega - \varepsilon_k + \lambda k) \}. \tag{16b}
\]

Here the prime \(\prime\) on the summation in Eq. 16b means that the incident angle \(\theta_r\) of electron from 2DEG into FM has been integrated out from \(-\pi/2\) to \(\pi/2\), which denotes only \(k_x > 0\) electrons contribute to the tunnelling current in our calculation. \(\chi(\omega)\) is the difference of the density of states of two spin bands in FM and 2DEG. For the 1D case \(\chi_{2deg} = 0\) as discussed earlier; whereas for the FM cases with any dimensions, \(\chi_{fm} \neq 0\) if the molecular field \(h\) is nonzero. Thus in 1D RSOI/DSOI/FM junction, ESC does not exist.

Eq. 15 indicates that in a RSOI/FM junction at zero bias a ESC can also flow through the junction and RSOI in the left lead acts as a magnetic field with direction along the \(y\)-direction. According to the exchange coupling between the two magnetic moments in the two sides of the junction, \(J_s^y\) is zero while \(J_s^x \sim \cos \theta\) and \(J_s^z \sim \sin \theta \cos \phi\) are nonzero. If RSOI in the left lead is replaced by DSOI to form a DSOI/FM junction, the ESC of Eq. 15 is different, \(J_s^x = 0\), \(J_s^y \sim \cos \theta\), and \(J_s^z \sim \sin \theta \sin \phi\) because in this case DSOI gives rise to an average pseudomagnetic field along the \(x\)-direction.

The results we obtained here indicate again the RSOI (DSOI) in the tunnelling junction can play the same role as in a FM metal.

### C. FM/FM

For completeness we present in this subsection the results of the ESC through a FM/FM junction, which are same as those found in literatures.\(^{21-23}\) The spin quantum axis is taken as the magnetic moment direction of the left FM lead \(\hat{n}_L(0,0,1)\) and the magnetic moment in the right FM lead is same as that in the last subsection \(\hat{n}_R(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)\). Using the Green’s function (Eq. 14) of the FM lead, we can obtain the formula of ESC as

\[
J_s^x = -\frac{1}{2} \int d\omega_1 d\omega_2 |P|^2 \frac{f(\omega_2) - f(\omega_1)}{\omega_2 - \omega_1} \chi_{fm}^{L,R}(\omega_2) \chi_{fm}^{L,R}(\omega_1) \sin \theta \sin \phi, \tag{17a}
\]

\[
J_s^y = \frac{1}{2} \int d\omega_1 d\omega_2 |P|^2 \frac{f(\omega_2) - f(\omega_1)}{\omega_2 - \omega_1} \chi_{fm}^{L,R}(\omega_2) \chi_{fm}^{L,R}(\omega_1) \sin \theta \cos \phi, \tag{17b}
\]

\[
J_s^z = 0. \tag{17c}
\]

Here \(\chi_{fm}^{L,R}\) is the difference between the density of states of the two spin bands of the left (right) FM lead as shown in Eq. 16a. In a simpler form, Eq. 17 can be written as \(J_s \sim \hat{n}_L \times \hat{n}_R\), which means the ESC comes from the exchange coupling between the two magnetizations in the left and right FM leads. Only the spin component along \(\hat{n}_L \times \hat{n}_R\) is not a conserved quantity since both magnetizations in the two sides of the junction can flip the spin. The ESC Eq. 11 and Eq. 15 for the 2DEG/2DEG and 2DEG/FM junction can also be expressed as the form of vector product of (pseudo)magnetic moments.

In earlier works,\(^{24}\) this spin current in the FM/FM junction was found to transport spin torque and result in magnetization reversal in an unfixed FM lead. The spin current can also induce an electric field, which may be used to experimentally detect the ESC through magnetic junctions.\(^{12,13}\) We wish to point out here the ESC stemming from the exchange coupling of two magnetic moments in two leads is two leads is achieved based on the approximation of the linear response, beyond which it is unclear whether the above results are still valid, thus it is worthwhile to study the behavior of ESC at the limit of the strong coupling.
IV. SUMMARY

We have presented a detailed analysis of the dissipationless ESC through three tunnelling junctions, 2DEG/2DEG, 2DEG/FS, and FS/FS. For these three junctions, the ESC comes from the exchange coupling between two magnetic (or pseudomagnetic) moments in the two leads in the linear approximation. Although in 2DEG with RSOI or DSOI, the pseudomagnetic field is isotropic and keeps the time reversal symmetry, only electrons with $k_x > 0$ contribute to the tunnelling current when 2DEG with RSOI or DSOI is used as the left lead of the junction as in the 2DEG/2DEG and 2DEG/FS junctions, so that the tunnelling electrons would feel an average nonzero pseudomagnetic field along the $y$-direction for RSOI or the $x$-direction for DSOI. The average pseudomagnetic fields have the same effect as a real magnetic moment and the ESC could flow in both 2DEG/2DEG and 2DEG/FS junctions. For the RSOI/DSOI junction, a spontaneous ESC could form since the pseudomagnetic fields associated with RSOI or DSOI are inherently different. While for RSOI/RSOI or DSOI/DSOI junctions, one possibility is to rotate one of the leads to change the orientation of the pseudomagnetic field, or we could shift the energy band of one of the 2DEG because the pseudomagnetic field is dependent on the direction of the electron momentum $\mathbf{P}$, which in turn is determined by the band edge. This is different from the magnetic moment in a FM electrode. Another distinction between the 2DEG with RSOI (DSOI) and FS lead is that in strict 1D case RSOI (DSOI) cannot result in an ESC through the junction, while this is not the case for 1D FM leads.

ACKNOWLEDGEMENT This work is supported by Hong Kong Research Grant Council, Project No: CityU 100303.

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FIG. 1. Schematic of a tunnelling junction with two quasi-1D electrodes with RSOI (DSOI) connected by an insulator barrier (I.B.). The left electrode could be physically rotated along $z$-direction by angle $\alpha$, which is equivalent to rotating the direction of the average pseudo-magnetic moment in RSOI (DSOI). The current flows along $x$-axis.
Figure 1 Wang and Chan