A single-electron hysteretic inverter designed for enhancement of stochastic resonance

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Abstract: To improve stochastic resonance in a single-electron (SE) device, we propose an SE device having hysteretic characteristics. We first demonstrate by analyzing a mathematical model that the correlation coefficient between the subthreshold input and the inverter output is improved by introducing hysteresis into an ideal inverter (NOT gate). To realize hysteresis in an SE inverter, we have designed an SE device having hysteretic characteristics (2ID-FJI) by combining two input discretizers (IDs) and an SE four-junction inverter (FJI). Evaluations of the correlation coefficients prove that the 2ID-FJI can achieve a significant improvement in stochastic resonance.

Keywords: single-electron device, stochastic resonance

Classification: Electron devices, circuits, and systems

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1 Introduction

On the one hand, stochastic resonance has attracted numerous researchers [1, 2, 3]. It is a nonlinear phenomenon in which a subthreshold coherent signal can be detected by adding a noise source to the input of the system, making the response of the system look like a resonance curve [2]. One of systems having many applications of stochastic resonance is the two-state system [4]. The inverter is one of the simple devices that work as the two-state system including high and low states. On the other hand, these days, with the objective of scaling down device sizes and power supply, the single-electron (SE) devices are becoming prospective for replacing semiconductors [5]. Stochastic resonance has been applied for some SE devices [6, 7]. One typical structure of an SE inverter has four junctions, so it can be called as the four-junction inverter (FJI) [8]. Therefore, we suggested using SE FJI for application of stochastic resonance.

To evaluate the characteristics of stochastic resonance, we have used a correlation coefficient parameter, $CC$, between the input signal and the output signal. However, the $CC$ of the FJI does not reach $-1$ since the output state of the FJI does not keep stably when the noisy input signal fluctuates randomly. To improve $CC$, the device should have input-output characteristics which can eliminate random fluctuations in the output signal. Hysteretic characteristics are such input-output characteristics. Hence, we supposed that an improvement of stochastic resonance can be done by designing an SE hysteretic inverter with two following bases. Firstly, to determine the effect of parameters of the system on stochastic resonance as a basis for designing the new device, we have calculated stochastic resonance for an ideal hysteretic inverter which has two thresholds, $V_{S1}$ and $V_{S2}$. Secondly, since two cascaded input discretizers (IDs) can have two different switching voltages for forward tunneling and backward one, a combination of two IDs with an SE transistor results in the appearance of the hysteresis in the input-output characteristics [9]. Yet, in the previous work, the output is the current, which makes the device difficult to couple to other ones.

In this paper, we have designed the SE inverter which has hysteretic characteristics by combining the two IDs with the FJI (2ID-FJI). The new structure has four significant differences from the previous work [9]. Firstly, the main device is changed from an SE transistor to an SE FJI. Secondly, the IDs are designed for the device having output voltage instead of output current. Thirdly, the numbers of excess electrons in the center islands of two IDs are determined by simulation instead of assuming of 0 or 1 value as in the previous work. Finally, parameters of two IDs have been chosen to have the wide hysteresis. After designing the 2ID-FJI, comparisons of correlation coefficients show that the 2ID-FJI achieves performance of stochastic resonance as well as the ideal hysteretic inverter and better than the FJI with no hysteresis.

2 Stochastic resonance in SE FJI

Before estimation, we define some parameters for all calculations and simulations discussed in this paper as following: the input signal is a unipolar rectangular wave with an amplitude $V_{in \_signal}$, i.e., the low level $V_{in \_signal \_low}$ is 0 and the high level
The charging energy of a single electron on the island between SIMON program [10] with the conditions of noise discretized with a step $\Delta t = 1$ ns. It has a uniform distribution in a range from $-V_{in\text{,noise}}/2$ to $+V_{in\text{,noise}}/2$, then $V_{in\text{,noise}}$ is called as the peak-to-peak noise voltage or noise level. The noisy input signal, $V_{in}$, is the input signal with the addition of the input noise. Under $V_{in\text{,noise}} = 0$, $V_{in} = V_{in\text{,signal}}$.

To evaluate the characteristics of stochastic resonance, we have used a correlation coefficient parameter, $CC$, between the input signal and the output signal. The correlation coefficient is calculated as following,

$$CC = \frac{\sum_{i=1}^{M} [V_{in\text{,signal}}(i) - \bar{V}_{in\text{,signal}}][V_{out}(i) - \bar{V}_{out}]}{\sqrt{\sum_{i=1}^{M} [V_{in\text{,signal}}(i) - \bar{V}_{in\text{,signal}}]^2} \sqrt{\sum_{i=1}^{M} [V_{out}(i) - \bar{V}_{out}]^2}},$$

where $V_{in\text{,signal}}(i)$ and $V_{out}(i)$ are $i^{th}$ samples of the input and output signals, respectively. $M$ is the number of signal samples. The averaged values of the input and output signals are $\bar{V}_{in\text{,signal}} = \left[\sum_{i=1}^{M} V_{in\text{,signal}}(i)\right]/M$ and $\bar{V}_{out} = \left[\sum_{i=1}^{M} V_{out}(i)\right]/M$, respectively. The ideal correlation coefficient of an inverter is $-1$ when the output signal is absolutely responsive to the input one.

Structure of the SE FJI is shown in Fig. 1 [8]. Parameters of the FJI are set to the following conditions: $C_{1} = 2C_{2}$, $C_{g} = 8C_{2}$, $C_{h} = 7C_{2}$, $C^{*} = C_{1} + C_{2} + C_{g} + C_{h}$, $V_s = 1.5e/2C^{*}$ [8]. In numerical simulation described below, we assume that $C_{1} = 2.00 \text{ aF}$, $R_{1} = 50.0 \text{ k}\Omega$, $C_{2} = 1.00 \text{ aF}$, $R_{2} = 100 \text{ k}\Omega$, $C_{g} = 8.00 \text{ aF}$, $C_{h} = 7.00 \text{ aF}$, $V_{s} = 6.70 \text{ mV}$, and $C_{L} = 1.00 \text{ fF}$. Monte-Carlo simulation was executed using the SIMON program [10] with the conditions of $T = 0$ K and no co-tunneling processes. The charging energy of a single electron on the island between $J_1$ and $J_2$ or between $J_3$ and $J_4$ (Fig. 1) is calculated as $E_c = e^2/[2(C_{1} + C_{2} + C_{g} + C_{h})] \approx 4.45 \text{ meV}$.

![Fig. 1. Schematic diagram of a single-electron four-junction inverter (SE FJI).](image)

Fig. 2(a) shows input-output characteristics of the FJI when the input voltage increases from $0 \text{ mV}$ to the source voltage ($6.70 \text{ mV}$), then decreases from the source voltage to $0 \text{ mV}$. It can be seen that the FJI has no hysteretic characteristics. Since the FJI is an inverter, the output signal is responsive to the input signal when
the correlation coefficient is negative. Then, we have defined that a strict threshold, $\theta_S$, is the minimum value of the input signal which makes the output signal respond. With the definition, the strict threshold of the FJI is $\theta_S = 2.83$ mV. Therefore, a subthreshold input signal for the FJI means a signal smaller than the strict threshold ($V_{\text{in,signal}} < \theta_S$). Fig. 2(b) illustrates the correlation coefficients between the input signal and the output signal of the FJI, $CC$, versus the normalized input noise levels, $V_{\text{in,noise}}/\theta_S$, with the different normalized input signals, $V_{\text{in,signal}}/\theta_S$. In particular, the correlation coefficients with $V_{\text{in,signal}}/\theta_S = 0.60; 0.70; 0.80; 0.90$ are respectively represented by circles, rectangles, triangles, and slashes. When the input noise level increases, the correlation coefficients of the FJI have resonance-behavior with peaks of resonance at the most negative values of the correlation coefficients. However, in all cases, the peaks of resonance of the FJI are around $-0.60$. The reason why $CC$ does not reach $-1$ is that the output state of the FJI fluctuates randomly (Fig. 2(c)). Therefore, we propose an SE hysteretic inverter to keep the output state stably.

![Figure 2](image)

Fig. 2. (a) Input-output characteristics of the FJI. (b) Correlation coefficients between the input and output signals of the FJI. (c) The output signal for $V_{\text{in,signal}}/\theta_S = 0.90$ and $V_{\text{in,noise}}/\theta_S = 1.80$, $CC \approx -0.62$ (peak of resonance curve).

### 3 Method for improving performance of stochastic resonance

A hysteretic inverter has high and low thresholds, $V_{S1}$ and $V_{S2}$ ($V_{S1} > V_{S2}$). A model of its input-output characteristics is illustrated in Fig. 3(a).

![Figure 3](image)

Fig. 3. (a) Input-output characteristics of the ideal hysteretic inverter. (b) Model structure for calculating stochastic resonance in the ideal hysteretic inverter.
The output $V_{out}(i)$ of the ideal hysteretic inverter at the time $i$ can be described as following: $V_{out}(i) = 0$ if $V_{in}(i) \geq V_{S1}$, $V_{out}(i) = V_{out}(i-1)$ if $V_{S2} < V_{in}(i) < V_{S1}$, and $V_{out}(i) = 1$ if $V_{in}(i) \leq V_{S2}$. Thus, in the range of the hysteresis, $(V_{S2}, V_{S1})$, random fluctuations of the input do not affect the state of the output. (That is, these fluctuations disappear in the output.) As a result, the hysteretic characteristics can improve the correlation between the input and output signals through stochastic resonance.

To demonstrate this, we have analyzed stochastic resonance using the model shown in Fig. 3(b). The normalized width of the hysteresis, $(V_{S1} - V_{S2})/V_{S1}$, is variable from 0.00 to 1.00 with a step of 0.025. The input includes a subthreshold rectangular signal, $V_{in,\text{signal}}$, and a noise source having the uniform distribution from $(-V_{in,\text{noise}}/2)$ to $(+V_{in,\text{noise}}/2)$. The normalized input noise level, $V_{in,\text{noise}}/V_{S1}$, was increased from 0.00 to 5.00 with a step of 0.025. We calculated the correlation coefficients between the input signal, $V_{in,\text{signal}}$, and the output signal, $V_{out}$, in two cases of the normalized input signals, $V_{in,\text{signal}}/V_{S1} = 0.50$ and 0.90, which are illustrated in Fig. 4. Three typical characteristics can be seen from Fig. 4 as following. Firstly, behaviors of the correlation coefficients are similar to characteristics of a stochastic resonance process where a peak of resonance curve is the most negative value of the correlation coefficients. Secondly, with a large enough input signal, there is the appearance of correlation coefficients near $-1$ (Fig. 4(b)). Finally, when the normalized width of the hysteresis, $(V_{S1} - V_{S2})/V_{S1}$, is closer to 1, the range of the input noise levels for the excellent correlation coefficients ($CC \leq -0.90$) is wider. For example, in Fig. 4(b), the width of this noise range at $(V_{S1} - V_{S2})/V_{S1} = 0.80$ is larger than that at $(V_{S1} - V_{S2})/V_{S1} = 0.40$. Therefore, the hysteresis improves the correlation coefficients between the input and output signals through stochastic resonance, with two conditions of the large enough normalized input signal, $V_{in,\text{signal}}/V_{S1}$, and the large enough normalized width of the hysteresis, $(V_{S1} - V_{S2})/V_{S1}$. Based on these advantages of the hysteresis, we have proposed an SE device that has hysteretic input-output characteristics with the $(V_{S1} - V_{S2})/V_{S1}$ as large as possible.
4 An SE hysteretic inverter designed for enhancement of stochastic resonance

4.1 Design of an SE hysteretic inverter

A combination of two IDs with the FJI to make a structure of 2ID-FJI is described in Fig. 5. In Fig. 5, two IDs composed of two junctions \( J_{01} \) and \( J_{02} \) and two capacitors grounded \( (C_{01} \) and \( C_{02} \) ), are serially-cascaded and connected to two capacitors of the FJI \( (C_g) \).

![Fig. 5. Schematic diagram of the 2ID-FJI.](image)

An equivalent structure of the 2ID-FJI is shown in Fig. 6, in which an equivalent capacitance of the FJI seen from two IDs to the right side is replaced by \( C_{eq} \) as following,

\[
C_{eq} = 2\left[\frac{C_g}{(C_1 + C_2 + C_b)}\right] = 2 \frac{C_g(C_1 + C_2 + C_b)}{C_g + (C_1 + C_2 + C_b)}.
\]  (2)

The polarization charges on the individual capacitors and the junctions shown in Fig. 6 can be expressed as

![Fig. 6. Simplified circuit model used for analysis of the 2ID-FJI.](image)
\[ Q_{J1} = (V_{in} - V_1)C_0, \]
\[ Q_{01} = V_1C_0, \]
\[ Q_{J2} = (V_1 - V_2)C_0, \]
\[ Q_{02} = V_2C_0, \]
\[ Q_{eq} = V_2C_{eq}. \]

Here, \( Q_{J1}, Q_{J2}, Q_{01}, Q_{02}, \) and \( Q_{eq} \) are respectively charges on the junctions \( J_{01}, J_{02} \) and the capacitors \( C_{01}, C_{02}, \) and \( C_{eq} \). \( V_{in} \) is the input voltage. \( V_1 \) and \( V_2 \) are the voltages across the capacitors \( C_{01} \) and \( C_{02} \), respectively.

The quantization of charges on the first and the second center islands of the IDs can be written in the form as following,

\[ -Q_{J1} + Q_{01} + Q_{J2} = N_1e + Q_{P1}, \]
\[ -Q_{J2} + Q_{02} + Q_{eq} = N_2e + Q_{P2}, \]

in which \( N_1, N_2, e (> 0) \), and \( Q_{P1}, Q_{P2} \) are the numbers of excess electrons in the first and the second center islands, the elementary charge, and the background polarization charges, respectively. The background polarization charges are assumed to be eliminated, that is, \( Q_{P1} = Q_{P2} = 0 \). By substituting Eqs. (3)–(5) to Eq. (8) and Eqs. (5)–(7) to Eq. (9), we have

\[ -(V_{in} - V_1)C_0 + V_1C_{01} + (V_1 - V_2)C_0 = N_1e, \]
\[ -(V_1 - V_2)C_0 + V_2C_{02} + V_2C_{eq} = N_2e. \]

By eliminating \( V_2 \) from Eqs. (10) and (11), we obtain

\[ -V_{in}C_0 + V_1\left[C_0 + C_{01} + \frac{(C_{02} + C_{eq})}{C_0 + C_{02} + C_{eq}}C_0\right] = N_1e + \frac{N_2eC_0}{C_0 + C_{02} + C_{eq}}. \]

Otherwise, the condition for tunneling through the first junction of the two IDs has to satisfy

\[ V_{in} - V_1 = \pm e/(2C_1^*), \]

in which signs “+” and “−” are equivalent to the forward tunneling and backward one through the first junction. \( C_1^* \) is the total capacitance between the first center island of the IDs and its environment [8], calculated as

\[ C_1^* = C_0 + C_{01} + C_0/(C_{02} + C_{eq}) = C_0 + C_{01} + \frac{C_0(C_{02} + C_{eq})}{C_0 + (C_{02} + C_{eq})}. \]

Combining Eqs. (12)–(14), the input voltage \( V_{inS} \) for tunneling through the junctions is expressed as

\[ V_{inS} = \frac{C_0 + C_{02} + C_{eq}}{C_0C_{01} + (C_0 + C_{01})(C_{02} + C_{eq})} \left( \frac{e}{2} + \frac{N_1e}{C_0 + C_{02} + C_{eq}} \right). \]

We may design the threshold voltages, \( V_{S1} \) and \( V_{S2} \), by using Eq. (15), which are described as

\[ V_{S1} = \frac{C_0 + C_{02} + C_{eq}}{C_0C_{01} + (C_0 + C_{01})(C_{02} + C_{eq})} \left( \frac{e}{2} + \frac{N_1e}{C_0 + C_{02} + C_{eq}} \right); \]
\[ V_{S2} = \frac{C_0 + C_{02} + C_{eq}}{C_0C_{01} + (C_0 + C_{01})(C_{02} + C_{eq})} \left( -\frac{e}{2} + \frac{N_1e}{C_0 + C_{02} + C_{eq}} \right). \]
According to Eqs. (16) and (17), the threshold voltages, $V_{S1}$ and $V_{S2}$, depend on parameters of $C_{01}$ and $C_{02}$ of the IDs. To have a wide hysteresis, we choose $C_{01} = C_{02} = 72.0 \text{ aF}$. Parameters of the FJI are set as mentioned above. Parameters of the IDs are chosen as $C_0 = 1.00 \text{ aF}$, $R_0 = 100 \text{ k}\Omega$, $C_{01} = C_{02} = 72.0 \text{ aF}$. Monte-Carlo simulation was executed using the SIMON program with the conditions of $T = 0 \text{ K}$ and no co-tunneling processes.

The relationship between the number of excess electrons in the center islands of the IDs, $N_1$ and $N_2$, and the output voltage of the 2ID-FJI, $V_{out}$, are plotted as functions of the input voltage $V_{in}$, in Fig. 7. As the input voltage increases from 0 mV to the source voltage, the numbers of excess electrons, $N_1$ and $N_2$ change as shown in Figs. 7(a) and 7(b). As the input voltage decreases from the source voltage to 0 mV, on the other hand, $N_1$ and $N_2$ change as shown in Figs. 7(c) and 7(d). In Fig. 7, the numbers of excess electrons in the first and the second center islands of the IDs, $N_1$ & $N_2$, are represented by closed triangles and rectangles, respectively; the output voltage, $V_{out}$, is shown by a solid line. The dash line presents the change of the output state jumping between low and high levels. $V_{S1}$ and $V_{S2}$ are respectively again the input voltages when the output switches from high to low level and vice versa. On the one hand, the output switches from high to low level when $N_1 = 2$ (Fig. 7(a)) and $N_2 = 1$ (Fig. 7(b)). Substituting these values to Eq. (16), the switching input voltage, $V_{S1}$, is calculated as 5.51 mV. On the other hand, in Figs. 7(c) and (d), the output voltage changes from low to high level with $N_1 = 1$ and $N_2 = 2$. Substituting these values to Eq. (17), the calculated switching input voltage, $V_{S2}$, is 1.15 mV.
Input-output characteristics of the 2ID-FJI with $C_{01} = C_{02} = 72.0 \, \text{aF}$ are illustrated in Fig. 8. In Fig. 8, the 2ID-FJI has hysteretic characteristics with two thresholds $V_{S1}$ and $V_{S2}$. According to the Fig. 8, the switching input voltages $V_{S1}$ and $V_{S2}$ are 5.53 mV and 1.15 mV, respectively. Thus, the calculation values are close to the simulation ones.

![Fig. 8. Hysteretic input-output characteristics of the 2ID-FJI simulated with $C_{01} = C_{02} = 72.0 \, \text{aF.}$](image)

**4.2 Evaluations of stochastic resonance in the 2ID-FJI**

**4.2.1 Equivalent performance of stochastic resonance of the 2ID-FJI in comparison with the ideal hysteretic inverter**

We evaluate stochastic resonance for the 2ID-FJI with $C_{01} = C_{02} = 72.0 \, \text{aF}$. This structure has the normalized width of the hysteresis $(V_{S1} - V_{S2})/V_{S1}$ of 0.792. To compare stochastic resonance of the 2ID-FJI with that of the ideal hysteretic inverter, we have chosen the ideal hysteretic inverter having the same normalized width of the hysteresis $(V_{S1} - V_{S2})/V_{S1} = 0.792$.

Correlation coefficients between the input signal and the output one, $CC$, versus the normalized input noise level, $V_{in\_noise}/V_{S1}$, are calculated for the ideal hysteretic inverter with $(V_{S1} - V_{S2})/V_{S1} = 0.792$ (Fig. 9(a)) and simulated for the 2ID-FJI with $C_{01} = C_{02} = 72.0 \, \text{aF}$ (Fig. 9(b)). In each figure, there are four curves presenting four different amplitudes of the normalized input signal $V_{in\_signal}/V_{S1}$. In particular, $V_{in\_signal}/V_{S1} = 0.60; 0.70; 0.80; 0.90$ are represented by circles, rectangles, triangles, and slashes. It can be seen that correlation coefficients of the 2ID-FJI are similar to those of the ideal hysteretic inverter. Namely, changes of the correlation coefficients following the increases of the normalized input noise level look like resonance shapes with the peaks of resonance at the most negative values of the correlation coefficients. Besides, with the normalized input signal, $V_{in\_signal}/V_{S1} = 0.60$, both of the ideal hysteretic inverter and the 2ID-FJI have the peaks of resonance around $-0.60$. However, when the normalized input signal is larger than or equal to 0.70 ($V_{in\_signal}/V_{S1} \geq 0.70$), there are two typical characteristics of stochastic resonance as following. First, the peaks of resonance of the correlation coefficients are close to $-1$. Second, in the case of having the excellent correlation coefficients, the width of the input noise range becomes wider as the input signal amplitude increases. Thus, the 2ID-FJI can achieve the excellent correlation coefficients over the wide range of the input noise levels.

From the view of width of the hysteresis, we evaluate the relationship between it and the width of the input noise range for excellent correlation coefficients which are defined as equal to or smaller than $-0.90$, as shown in Fig. 10.
In Fig. 10(a), the range for the excellent correlation coefficients is limited by the input noise range from $V_{\text{in\_noise1}}/V_{S1}$ to $V_{\text{in\_noise2}}/V_{S1}$, so the normalized width of the input noise range is $(V_{\text{in\_noise1}} - V_{\text{in\_noise2}})/V_{S1}$. Fig. 10(b) illustrates the relationship between the width of this noise range, $(V_{\text{in\_noise1}} - V_{\text{in\_noise2}})/V_{S1}$, and the normalized width of the hysteresis, $(V_{S1} - V_{S2})/V_{S1}$, in the case of the normalized input signal $V_{\text{in\_signal}}/V_{S1} = 0.90$. The solid line represents the results of calculation for stochastic resonance of the ideal hysteretic inverter whereas the rectangle points show the results from simulation for the 2ID-FJI with different widths of the hysteresis. As seen in Fig. 10(b), the results of the Monte-Carlo simulation approach those of the model calculation. It can be seen that the width of the input noise range is proportional to the width of the hysteresis. Besides, in Fig. 10(b), the input noise range appears $(V_{\text{in\_noise1}} - V_{\text{in\_noise2}})/V_{S1} > 0$ when the normalized width of the hysteresis $(V_{S1} - V_{S2})/V_{S1}$ is greater than a certain value $\alpha$. ($\alpha$ is evaluated as 0.20 in Fig. 10(b)). The value of $\alpha$ depends on the value of the $V_{\text{in\_signal}}/V_{S1}$.

Fig. 9. Correlation coefficients between input and output signals. (a) Calculation for the ideal hysteretic inverter with $(V_{S1} - V_{S2})/V_{S1} = 0.792$; (b) Simulation for the 2ID-FJI with $C_{01} = C_{02} = 72.0$ aF.

Fig. 10. (a) Schematic drawing for determining the range of the input noise levels having the excellent correlation coefficients $(CC \leq -0.90)$. (b) Normalized width of the input noise range for the excellent correlation coefficients versus the normalized width of the hysteresis; $V_{\text{in\_signal}}/V_{S1} = 0.90$. 
Indeed, the conditions for eliminating fluctuations in the output are expressed as follows. For the high level of the noisy input signal, its maximum value is larger than the high threshold voltage, $V_{S1}$, and its minimum value is also larger than the low threshold voltage, $V_{S2}$, as

\[
\begin{align*}
V_{\text{in\_signal\_high}} + (1/2)V_{\text{in\_noise}} & \geq V_{S1} \\
V_{\text{in\_signal\_high}} - (1/2)V_{\text{in\_noise}} & > V_{S2},
\end{align*}
\]

then, the output stays at the low level. For the low level of the noisy input signal, its minimum value is smaller than the low threshold voltage, $V_{S2}$, and its maximum value is still smaller than the high threshold voltage, $V_{S1}$, as

\[
\begin{align*}
V_{\text{in\_signal\_low}} - (1/2)V_{\text{in\_noise}} & \leq V_{S2} \\
V_{\text{in\_signal\_low}} + (1/2)V_{\text{in\_noise}} & < V_{S1},
\end{align*}
\]

then, the output keeps the high level. Resulting from Eq. (18) and ($V_{\text{in\_signal\_high}} = V_{\text{in\_signal}}$), we have $(V_{S1} - V_{S2})/V_{S1} > 2(V_{S1} - V_{\text{in\_signal}})/V_{S1}$ or $\alpha = 2(V_{S1} - V_{\text{in\_signal}})/V_{S1}$. In this case, $V_{\text{in\_signal}}/V_{S1} = 0.90$, so $\alpha = 2(V_{S1} - V_{\text{in\_signal}})/V_{S1} = 0.20$. From this calculation, an increase of the input signal results in a decrease of $\alpha$.

### 4.2.2 Improvement in stochastic resonance of the 2ID-FJI in comparison with the SE FJI

First of all, under consideration for response speed of a system, we define switching times as following. The fall time, $t_f$, is the time for the output voltage to fall from 90% to 10% of its steady-state value. The rise time, $t_r$, is the time for the output voltage to rise from 10% to 90% of its steady-state value. We performed 1000 transient simulations for the FJI and the 2ID-FJI in the conditions of $V_{\text{in\_signal}} = V s = 6.70$ mV and $V_{\text{in\_noise}} = 0$ mV. As a result, the averaged switching times of the FJI are $t_f = 0.36$ ns and $t_r = 0.37$ ns; the averaged switching times of the 2ID-FJI are $t_f = 0.55$ ns and $t_r = 0.60$ ns. Otherwise, the discretized time for the input noise is $\Delta t = 1$ ns. It means that we ignored switching times of the systems.

To see the improvement of stochastic resonance in a hysteresis structure in comparison with a non-hysteresis, a comparison between stochastic resonance in the FJI (Fig. 2(b)) and that in the 2ID-FJI (Fig. 9(b)) shows two typical improvements as following. Firstly, with the small normalized input signal, the peak of the correlation coefficients of the 2ID-FJI ($V_{\text{in\_signal}}/V_{S1} = 0.60$, $CC \approx -0.67$) is slightly better than that of the FJI ($V_{\text{in\_signal}}/\theta_{S} = 0.60$, $CC \approx -0.52$). Secondly, when the normalized input signals are equal to or larger than 0.70, the peaks of the correlation coefficients of the 2ID-FJI ($V_{\text{in\_signal}}/V_{S1} \geq 0.70$) are close to 1 whereas those of the FJI ($V_{\text{in\_signal}}/\theta_{S} \geq 0.70$) are approximately 0.60. In other words, with a large enough normalized input signal, $V_{\text{in\_signal}}/V_{S1}$, stochastic resonance of the 2ID-FJI with the hysteresis achieves the significant improvement over that of the FJI without the hysteresis. This advantage of the 2ID-FJI is based on the hysteretic characteristics which help the output signal keep its states stably while the noisy input signal fluctuates randomly. It can be seen clearly by illustrations of input and output signals of the 2ID-FJI in Fig. 11.

Fig. 11 illustrates the good detection ability of the subthreshold input signal of the 2ID-FJI with $C_{01} = C_{02} = 72.0$ aF through stochastic resonance visually. The
input signal having the normalized input signal $V_{\text{in,signal}}/V_{S1} = 0.90$ is shown in Fig. 11(a). The noisy input signal described in Fig. 11(b) consists of the input signal with $V_{\text{in,signal}}/V_{S1} = 0.90$ and an addition of one of optimal input noise levels such as $V_{\text{in,noise}}/V_{S1} = 0.58$ which has correlation coefficient $CC \approx -0.99$. The output signal in the case of the noisy input signal is illustrated in Fig. 11(c). It can be seen that the output signal responds to the input signal well, namely the output keeps its states stably without fluctuation, resulting in the correlation coefficient close to $-1$. This is explained as following. In Fig. 11(b), at the high level of the noisy input signal, its maximum value is $V_{\text{in,signal,high}} + (1/2)V_{\text{in,noise}} = 1.19V_{S1} > V_{S1}$ while its minimum value is $V_{\text{in,signal,high}} - (1/2)V_{\text{in,noise}} = 0.61V_{S1} > V_{S2}$ since $V_{S2}/V_{S1} = 0.208$. Simultaneously, at the low level of the noisy input signal, its minimum value is $V_{\text{in,signal,low}} - (1/2)V_{\text{in,noise}} = -0.29V_{S1} < V_{S2}$ while its maximum value is $V_{\text{in,signal,low}} + (1/2)V_{\text{in,noise}} = 0.29V_{S1} < V_{S1}$. Thus, in Fig. 11(b), the noisy input signal satisfies both Eqs. (18) and (19). As a result, although the high/low level of the noisy input signal fluctuates, the amplitude of the output signal still stays at low/high level.

![Fig. 11.](image)

5 Conclusion

We proposed the SE hysteretic inverter by combining two IDs with the FJI. The new device achieves good performance of stochastic resonance as similarly as the calculation results for the ideal hysteretic inverter and better than the FJI. Furthermore, the 2ID-FJI can improve the detection ability of the subthreshold input signal through stochastic resonance over the wide range of input noise levels.