Assessment of stress state and dynamic characteristics of plane and spatial structure

Z. Urazmukhamedova¹, D. Juraev¹, M. Mirsaidov¹

¹Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39 Kori Niyoziy str., 100000, Tashkent, Uzbekistan;
E-mail: mirsaidov1948@mail.ru, ziyoda-itiame@mail.ru, diyorbekmuhammadamir@mail.ru
Corresponding authors: mirsaidov1948@mail.ru, theormir@mail.ru

Abstract. This study is devoted to the assessment of the stress state and dynamic characteristics of various structures. The actual task at the design stage is to determine the parameters of a structure. In this article, a mathematical model was developed for assessing the stress state and dynamic characteristics of plane and spatial structures based on the Lagrange variational equation using the d'Alembert principle. The variational problem for the structures considered by the finite element method leads to the solution of nonhomogeneous algebraic equations or to the solution of algebraic eigenvalue problems. To assess the adequacy of the model and the accuracy of the numerical results obtained, a plane and spatial test problem with an exact solution was solved. Using the proposed model, the eigenfrequencies and modes of oscillations of the gravitational and earth dams (296 m high) of the Nurek reservoir were investigated. At that, it was revealed that in the natural modes of vibration of earth dams, the greatest displacements under low frequencies are observed at the crest part or at the middle of the slopes.

Keywords: plane and spatial structures, variational setting, stress state, eigenfrequencies and vibration modes, gravity and earth dams.

1. Introduction

One of the most difficult problems in the science of mechanics is the assessment of stresses, strains and dynamic characteristics (i.e. eigenfrequencies, modes and decrement of vibrations) of various structures with complex geometry and nonhomogeneous design features. This, in turn, requires the development and use of mathematical models and computational methods that take into account these design features. At present, the methods of finite elements (FEM) and finite differences (FDM), with account for these features, and using a computer make it possible to solve such complex problems [1]–[8]. Therefore, one of the urgent problems of mechanics is the development of an adequate mathematical model and a computational algorithm that allows obtaining a solution for the assessment of static and dynamic processes occurring in plane and spatial structures of varying complexity.

To date, there are a number of scientific publications devoted to the study of the stress-strain state and dynamic behavior of earth structures in plane and spatial settings.

The static stress state and dynamic behavior of various earth dams in a plane and spatial setting are considered in [1]–[20]; they take into account the structural features of structures, moisture properties...
of soil, the interaction of structures with the water medium of the reservoir and other features of structures.

In [21], the plane stress-strain state of earth dams under the action of kinematic impact applied at the base of the structure was investigated by the method of finite differences.

In [22], using the finite element method, the seismic response of concrete gravity dams in a plane setting is investigated. The considered model takes into account the joint operation of the dam and foundation with the water medium of the reservoir. The stress state is investigated to assess the ultimate strength of the dam.

The study in [23] describes scientific achievements and the main conclusions, i.e.: the accumulated experience in the construction of high earth-and-rockfilled dams is systematically summarized; key technical issues are discussed, including control of strains, seepage, slope stability, safety assessment and other issues related to earth dams.

In [24], the finite element method is used to study the stress state of earth dams under static and dynamic influences, taking into account the elastoplastic strain of the dam material. The numerical results obtained are compared with the results of field measurements of the Wenchuan earthquake.

The bending strain of the dam was investigated in [25] since bending often led to the destruction of structures.

In [26], the use of unconventional materials (soil and stone mixtures) to ensure the stability of the slopes of earth dams was analyzed in detail.

This review of publications shows that the problem of assessing the stress-strain state and dynamic characteristics of earth dams in a plane and spatial setting was not studied enough, therefore, research in this direction is of great interest.

So, this study is devoted to the development of a mathematical model for assessing the stress state and eigenfrequencies, vibration modes of various plane and spatial structures using finite element methods. Some of the results obtained are compared with known solutions.

2. Methods.

A model of a three-dimensional deformable rigid body occupying volume V is considered (Fig. 1).

![Figure 1. Model of a three-dimensional deformable body](image.png)

The bottom surface of the body \( \Sigma_0 \) is rigidly fixed at the base, the front and rear surfaces (\( \Sigma_1, \Sigma_1^1 \)) and the two side surfaces (\( \Sigma_2, \Sigma_2^1 \)) are stress-free. The body is under the influence of mass forces \( \vec{f} \) and distributed load \( \vec{P} \) is applied to the surface \( \Sigma_3 \).

It is necessary to determine the stresses arising under the action of loads in a three-dimensional body (Fig. 1) and the dynamic characteristics of the body in the absence of a load.
To simulate the strain process in a body (Fig. 1), the Lagrange variational equation based on the d'Alembert principle is used, i.e.:

\[- \int_v \sigma_{ij} \delta \varepsilon_{ij} \, dV - \int_v \rho \, \ddot{u} \, \delta \, \ddot{u} \, dV + \int_v \tilde{f} \, \delta \, \ddot{u} \, dV + \int_{\mathcal{S}} \tilde{P} \, \delta \, \ddot{u} \, d\mathcal{S} = 0, \quad (1)\]

\[i,j=1,2,3.\]

To create a mathematical model, in addition to the variational equation (1), the generalized Hooke's law is used, which takes into account the physical properties of the body [27],

\[\sigma_{ij} = \lambda \, \varepsilon_{kk} \delta_{ij} + 2 \mu \, \varepsilon_{ij} \quad (2)\]

and the Cauchy relation [27]

\[\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3)\]

and the boundary condition

\[\dddot{x} \in \sum \theta \dddot{u} = 0 \quad (4)\]

Here \(\ddot{u}, \varepsilon_{ij}, \sigma_{ij}\) are the components of the displacement vector, strain and stress tensors, respectively; \(\delta \ddot{u}, \delta \varepsilon_{ij}\) are the isochronous variations of the components of the displacement vector and strain tensors; \(\rho\) is the density of the body material; \(\ddot{f}\) is the vector of mass forces; \(\tilde{P}\) is the externally distributed load applied to the surface \(\mathcal{S}\); \(\lambda\) and \(\mu\) are the Lamé constants; \(\theta = \varepsilon_{ii}\) is the volumetric strain; \(\{u_i,u_2,u_3\} = \{u,v,w\}\) are the components of the vector of displacements of the point of the body; when solving three-dimensional problems \(i,j,k = 1,2,3\), and when solving plane problems; \(i,j,k = 1,2\); \(\{x\} = \{x_1,x_2,x_3\} = \{x,y,z\}\).

Now, the problems to be solved using this mathematical model can be formulated as follows:

1. It is required to determine in the body (Fig. 1) the fields of displacements \(\ddot{u}(\dddot{x},t)\), strains \(\varepsilon_{ij}(\dddot{x},t)\) and stresses \(\sigma_{ij}(\dddot{x},t)\), arising under the action of mass (\(\ddot{f}\)) and surface (\(\tilde{P}\)) forces that satisfy equations (1) - (3) and boundary conditions (4) for arbitrary virtual displacements \(\delta \ddot{u}\) (virtual work of inertial forces is not taken into account in (1));

2. It is required to determine the eigenfrequencies and modes of vibration of the body (Fig. 1) occupying the volume \(V\) (in the absence of external loads), satisfying equations (1) - (3) and boundary conditions (4) for arbitrary virtual displacements \(\delta \ddot{u}\) (virtual work of mass and surface forces is not taken into account in (1)).

To solve the above tasks, the finite element method was used [28].

When solving plane problems, triangular super-bounded elements with 6 degrees of freedom were used, and when solving three-dimensional problems, super-elements in the form of parallelepips with 24 degrees of freedom were used.

Using the finite element method procedure, the variational problem (1) under the action of static loads is reduced to a system of high-order nonhomogeneous algebraic equations, i.e.:

\[\begin{bmatrix} K \end{bmatrix} \{ u \} = \{ P \} \quad (5)\]

Here: \(\begin{bmatrix} K \end{bmatrix}\) is the stiffness matrix for the considered body (Fig. 1); \(\{ u \}\) are the sought-for components of the displacement vectors, at the nodes of the finite element (after dividing the body into finite elements); \(\{ P \}\) are the components of the external (mass and surface) force acting on the nodes of the finite element (formed by mass and external forces).
When determining the dynamic characteristics (i.e., eigenfrequencies and vibration modes) of a body, the variational problem (1) is reduced by the finite element method to a homogeneous algebraic eigenvalue problem, i.e.:

\[
\left[ [K] - \omega^2 [M] \right] \{ \ddot{u} \} = 0,
\]

(6)

Here: \([K], [M]\) are the matrices of stiffness and mass of the considered body (Fig. 1); \(\omega, \{ \ddot{u} \}\) is the eigenfrequency and mode of vibration of the body, respectively; \([K], [M]\) - these matrices are of a band structure, which makes it easier to find the eigenvalues \(\lambda = \omega^2\) and eigenvectors \(\{ \ddot{u} \}\) of the characteristic determinant of equations (6).

Equations (5) and (6) were solved using the ABAQUS program. The number of unknowns in equations (5) and (6) when solving specific problems reached 10,000.

3. Results and Discussion

Several specific problems were solved using the above mathematical model and methods.

**Problem 3.1.**

The stressed state of a long rectangular parallelepiped (Fig. 2), which is in a state of plane strain under the distributed load \(P\) is considered. The parallelepiped is supported on an absolutely rigid and sliding base, i.e.: \(y=0; u_1=0; \sigma_{12} = 0\).

**Figure 2.** Deformable parallelepiped on a sliding base

It is required to find the components of the displacement vector \((u_1, u_2)\) and the components of the stress tensor \((\sigma_{11}, \sigma_{22}, \sigma_{12})\) at any point of the body (Fig. 2). The exact solution to this problem is given in the literature [29], and this solution has the form:

\[
\begin{align*}
    u_1 &= \nu(1+\nu) \frac{E}{x_1}; \\
    u_2 &= -\left(1-\nu^2\right) \frac{E}{x_2}; \\
    \sigma_{22} &= -P; \\
    \sigma_{12} &= \sigma_{21} = \sigma_{11} = 0
\end{align*}
\]

A numerical solution to this problem was obtained using the above model and methods, the results obtained were compared with the exact solution [29].

When solving this problem, the following initial data were used: \(P=1.0; a=b=1.0; E=1.0; V=0.25\). To obtain a numerical solution to this problem, a system of algebraic equations (5) was solved.

Comparison of the numerical and exact solutions obtained (i.e., the components of the displacement vector \(-u_1, u_2\)) are given in Table 1.
Table 1. Comparison of numerical and exact solutions.

| No | Coordinates of the point | Exact solution [29] | Numerical solution obtained by the authors | Difference in solutions |
|----|--------------------------|---------------------|---------------------------------------------|------------------------|
|    | x           | y           | u₁     | u₂     | u₁      | u₂      | u₁      | u₂      |               |
| 1.  | 0.25       | 0           | 0.078125 | 0.000  | 0.077500 | 0.000  | 0.8%    | 0%      |
| 2.  | 0.125      | 0           | 0.03906  | 0.000  | 0.038250 | 0.000  | 2.1%    | 0%      |
| 3.  | 0.5        | 1           | 0.15625  | -0.9375| 0.15600  | -0.92900| 0.2%    | 1%      |
| 4.  | 0.5        | 0.375       | 0.15650  | -0.35156| 0.15300  | -0.36500| 2.3%    | 2.3%    |
| 5.  | 0.5        | 0.75        | 0.15650  | -0.70312| 0.15500  | -0.71000| 1%      | 2%      |
| 6.  | -0.375     | 1           | -0.11718 | 0.9875 | -0.099750| -1.000  | 2.3%    | 2%      |
| 7.  | -0.375     | 0           | 0.1171875| 0.000  | -0.093750| 0.000  | 2.3%    | 0%      |
| 8.  | -0.391     | 0.308       | -0.122236| -0.28941| -0.117894| -0.29871| 2.4%    | 1.9%    |
| 9.  | -0.192     | 0.89        | -0.060181| -0.83504| -0.059145| -0.84071| 1.2%    | 1.3%    |
| 10. | 0.391      | 0.69        | 0.1220968| -0.64929| 0.119678 | -0.65258| 2.4%    | 1.6%    |
| 11. | 0.404      | 0.18        | 0.126315 | -0.17316| 0.12105  | -0.17471| 2.5%    | 1.1%    |
| 12. | -0.065     | 0.11        | -0.020045| -0.10144| -0.019362| -0.11143| 0.3%    | 0.69%   |
| 13. | 0.271      | 0.77        | 0.084825 | -0.72223| 0.083908 | -0.73053| 1.3%    | 2.3%    |
| 14. | 0.283      | 0.628       | 0.08863  | -0.58934| 0.087908 | -0.58863| 1.7%    | 3.9%    |
| 15. | -0.13      | 0.22        | -0.04158 | -0.2082 | -0.04097 | -0.21208| 0.8      | 1.3%    |
| 16. | -0.058     | 0.64        | -0.018436| -0.60175| -0.018474| -0.60175| 0.3%    | 4%      |
| 17. | 0.840      | 0.66        | 0.026671 | -0.61912| 0.026514 | -0.61840| 0.5%    | 1.2%    |
| 18. | -0.06      | 0.476       | -0.019276| -0.44616| -0.019321| -0.45991| 0.3%    | 2.97%   |

Problem 3.2.

Problem 3.1 shown above was considered in a three-dimensional formulation and the numerical solution obtained was compared with the results of the exact solution given in: (Table 2).

The exact solution of the three-dimensional problem, (that is, the components of the stress tensor), has the following form [29]:

\[ \sigma_{22} = -P; \quad \sigma_{11} = \sigma_{33} = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0. \]

Table 2. Comparison of numerical and exact solution of three-dimensional problems.

| No | Name | \( \sigma_{11} \) | \( \sigma_{22} \) | \( \sigma_{33} \) | \( \sigma_{12} \) | \( \sigma_{13} \) | \( \sigma_{23} \) |
|----|------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1. | Exact solution[29] | 0 | -1 | 0 | 0 | 0 | 0 |
| 2. | Numerical solution obtained by the authors | -55 E-18 | -1 | -222 E-18 | -44 E-18 | -266 E-18 | -310 E-18 |
| 3. | Difference (%) | 0 | 0 | 0 | 0 | 0 | 0 |
Comparison of the results obtained (numerical and exact solutions) given in Tables 1 and 2 shows that numerical solutions of high accuracy were obtained in both problems. Consequently, the results obtained using this model and methods make it possible to assess with sufficiently high accuracy the stress state not only of a simple structure but also of a structure of complex configuration.

**Problem 3.3.**

A model of a gravity dam (Fig. 3) with the following parameters is considered: the height of the dam $h=100$ m, the width at the base $b = 75$ m; concrete grade M-100, its characteristic are $E=2.5 \times 10^6$ tf/m$^2$; $\gamma=2.4$ tf/m$^3$. The dam operates in a plane deformed state. It is necessary to determine the eigenfrequencies of the dam vibration.

![Figure 3. Design diagram of a concrete gravity dam](image)

To determine the eigenfrequencies of the dam, the problem under consideration is reduced, using the proposed model and methods, to the system of equations (6) and these equations are solved. The numerical results obtained for this dam (i.e., the frequency of natural oscillations), are compared with the results of other authors, Table 3.

**Table 3. Comparison of eigen frequencies of the gravity dam obtained in different studies**

| Eigenfrequency, rad/s | Results obtained in [30] | Results obtained in [4] | Numerical results obtained by the authors | Difference in results, % |
|-----------------------|--------------------------|-------------------------|-------------------------------------------|-------------------------|
| $\omega_1$            | 27.5                     | 27.46                   | 26.8                                      | 3%                      |
| $\omega_2$            | 64.8                     | 64.57                   | 63.4                                      | 2%                      |
| $\omega_3$            | 73.6                     | 73.48                   | 72.8                                      | 1%                      |
| $\omega_4$            | 114.2                    | 114.6                   | 110.7                                     | 3%                      |
| $\omega_5$            | 161.4                    | 162.11                  | 159.6                                     | 3%                      |
| $\omega_6$            | 200.7                    | 172.37                  | 178.3                                     | 3%                      |
| $\omega_7$            | -                        | 202.82                  | 199.4                                     | 2%                      |
| $\omega_8$            | -                        | 219.67                  | 214.8                                     | 3%                      |

Comparison of the results obtained for gravity dams (Table 3) shows that the results obtained by the authors are of higher accuracy since the authors, solving this problem, divided the dam into a large number of finite elements. At the same time, it is shown that the use of this model and methods makes it possible to determine the dynamic characteristics of a dam of complex geometry and nonhomogeneous design features.

**Problem 3.4.**

It is necessary to determine eigenfrequencies and modes of vibration of the model of earth dam of the Nurek reservoir (Fig. 4). This dam is the highest earth dam built in a 9-point seismic zone. Generally,
the dynamic characteristic (i.e., the frequency and mode of natural vibrations) of a structure is its passport and allows us to estimate the dynamics of this structure. Therefore, in the design of a structure, the assessment of dynamic behavior begins with the determination of its dynamic characteristics. In numerical calculations, the design features, geometry and physical and mechanical characteristics of the dam materials were taken from the design documentation, i.e.: the dam height \( H = 296 \text{ m} \), the crest width \( b = 20 \text{ m} \), downstream and upstream slope ratio - \( m_1 = 2.25 \), \( m_2 = 2.2 \); average mechanical characteristics of soil \( E=306800 \text{ tf/m}^2 \); \( \gamma = 2.3 \text{ tf/m}^3 \) and \( \nu = 0.36 \). When solving this problem, the dam was considered in a plane deformed state.

When determining the eigen frequencies and vibration modes of the dam, the problem under consideration was reduced to the system of equations (6), for which non-trivial solutions were determined. The results of numerical solutions and their comparison with the results of other authors given in [4] and with the results obtained in the VNIIG named after B.E. Vedeneev are given in Table 4.

Fig. 5 shows the eigen modes of vibrations obtained by the authors under corresponding eigen frequencies.

![Figure 4. Design scheme of the Nurek dam:](image)

1 - central core, 2 - transition zones, 3 and 4 - upper and lower retaining prisms, 5,6 - upstream and downstream slopes

Analysis of the obtained numerical results (Table 4) shows some difference from the results obtained by other authors. This is due to the fact that those authors, when solving the problem, divided the dam area into a small number of finite elements. Therefore, the results obtained by the authors of this article are more accurate. The solution obtained for this earth dam makes it possible to use this technique to assess the dynamic characteristics of earth dams with more complex structural solutions and nonhomogeneous features.

| Eigenfrequencies | Results obtained in the VNIIG named after B.E. Vedeneev, Hz | Results obtained in [4], Hz | Numerical solution obtained by the authors, Hz | Difference in results, % |
|------------------|-----------------------------------------------------------|----------------------------|-----------------------------------------|--------------------------|
| \( \omega_1 \)   | 0.8136                                                   | 0.8079                    | 0.7989                                  | 2%                       |
| \( \omega_2 \)   | 1.247                                                    | 1.2405                    | 1.1637                                  | 6%                       |
| \( \omega_3 \)   | 1.4653                                                   | 1.4639                    | 1.3979                                  | 5%                       |
| \( \omega_4 \)   | 1.6733                                                   | 1.6617                    | 1.4948                                  | 10%                      |
| \( \omega_5 \)   | 1.7765                                                   | 1.7876                    | 1.7298                                  | 3%                       |
The analysis of natural vibration modes (Fig. 5) shows that under the first and second modes, the largest displacements are observed on the crest part of the structure, and under the fourth and fifth modes, the middle of the slope part of the dam receives the largest displacement.

4. Conclusions
1. To assess the stress state, the dynamic characteristics of plane and spatial structures, a mathematical model was developed based on the Lagrange variational equation using the d'Alembert principle.
2. The variational problems considered for plane and spatial structures using finite element methods were reduced to a higher order nonhomogeneous algebraic problem (when determining the stress state) and to a higher order algebraic eigenvalue problem (when determining dynamic characteristics).
3. The adequacy of the mathematical model and the accuracy of the results obtained are verified by comparing the obtained solutions of plane and spatial test problems with known exact solutions.
4. The study of eigenfrequencies and modes of vibration of gravitational and earth dams showed that the proposed methods make it possible to assess the dynamic characteristics of various dams with rather complex geometry and design features.

5. It was found that in the natural modes of vibration of earth dams the greatest displacement under low frequencies is observed at the crest part or in the middle of the dam slopes.

References

[1] Zaretsky Yu.K., Lombardo V.N., “Statics and dynamics of earth dams,” in Moscow: Energoizdat, p256, Moscow: Energoizdat, 1983, p. 256.

[2] Krasnikov N.D., “Seismic resistance of hydraulic structures made of earth materials,” in Moscow: Energoizdat, p256, Moscow: Energoizdat, 1981, p. 240.

[3] I. I. N. Lyakhter V.M., “Seismic resistance of earth dams,” in Moscow: Nauka, 1986, p. 233.

[4] Mirsaidov M.M., Sultanov T.Z., “Theory and methods of strength assessment of earth dams,” in Lambert Academic Publishing. Saarbrucken/Germany, 2015, p. 341.

[5] M. Mirsaidov, T. Sultanov, J. Yarashov, and E. Toshmatov, “Assessment of dynamic behaviour of earth dams taking into account large strains,” in E3S Web of Conferences, 2019, vol. 97, doi: 10.1051/e3sconf/20199705019.

[6] M. Usarov, A. Salokhiddinov, D. M. Usarov, I. Khazratkulov, and N. Dremova, “To the theory of bending and oscillations of three-layered plates with a compressible filler,” in IOP Conference Series: Materials Science and Engineering, 2020, vol. 869, no. 5, doi: 10.1088/1757-899X/869/5/052037.

[7] et al Abdikarimov R., “Free oscillations of three-layered plates,” 2020, doi: 10.1088/1757-899X/883/1/012058.

[8] M. Mirsaidov and M. Usarov, “Bimoment theory construction to assess the stress state of thick orthotropic plates,” in IOP Conference Series: Earth and Environmental Science, 2020, vol. 614, no. 1, doi: 10.1088/1757-1315/614/1/012090.

[9] M. M. Mirsaidov and T. Z. Sultanov, “Assessment of stress-strain state of earth dams with allowance for non-linear strain of material and large strains,” Mag. Civ. Eng., vol. 49, no. 5, pp. 73-82+136-137, 2014, doi: 10.5862/MCE.49.8.

[10] T. Z. Sultanov, D. A. Khodzhaev, and M. M. Mirsaidov, “The assessment of dynamic behavior of heterogeneous systems taking into account non-linear viscoelastic properties of soil,” Mag. Civ. Eng., vol. 45, no. 1, 2014, doi: 10.5862/MCE.45.9.

[11] Pinyol N.M., Alonso E.E., “Earth dam, spatial model, stress-strain state, dynamic characteristic, natural frequency, modes of oscillations,” Int. J. Civ. Eng., vol. 17, pp. 501–513, 2019.

[12] Nirimian N.A., Lahmer T., Karampour P., “Uncertainty quantification of stability and damage detection parameters of coupled hydrodynamic-ground motion in concrete gravity dams,” Front. Struct. Civ. Eng., vol. 13, pp. 303–323, 2019.

[13] Li Y., Li K., Wen L., Li B., Liu Y., “Safety standard for slopes of ultrahigh earth and rock-fill dams in China based on reliability analysis,” Int. J. Civ. Eng., vol. 17, pp. 1–16, 2019.

[14] G. L. Z. Fu, S. Chen, “Hydrodynamic pressure on concrete face rockfill dams subjected to earthquakes,” J. Hydrodyn., vol. 31, pp. 152–168, 2019.

[15] X. W. Wang M., Chen J., “Experimental and numerical comparative study on gravity dam-reservoir coupling system,” KSCE J. Civ. Eng., vol. 22, pp. 3980–3987, 2018.

[16] M. M. Mirsaidov, T. Z. Sultanov, and A. Sadullaev, “Determination of the stress-strain state of earth dams with account of elastic-plastic and moist properties of soil and large strains,” Mag. Civ. Eng., vol. 40, no. 5, pp. 59–68, 2013, doi: 10.5862/MCE.40.7.

[17] M. Mirsaidov, “An account of the foundation in assessment of earth structure dynamics,” in E3S Web of Conferences, 2019, vol. 97, doi: 10.1051/e3sconf/20199704015.

[18] M. M. Mirsaidov and E. S. Toshmatov, “Spatial stress state and dynamic characteristics of earth dams,” Mag. Civ. Eng., vol. 89, no. 5, pp. 3–15, 2019, doi: 10.18720/MCE.89.1.
et al Sultanov T., “Strength assessment of earth dams,” Web Conf., vol. 04015, p. 256, 2019, doi: 10.1051.

M. M. Mirsaidov, T. Z. Sultanov, and D. F. Rumi, “An assessment of dynamic behavior of the system ‘structure - Foundation’ with account of wave removal of energy,” Mag. Civ. Eng., vol. 39, no. 4, pp. 94–105, 2013, doi: 10.5862/MCE.39.10.

K. O. Khusanov B., “Stress-strain state of earth dam under harmonic effect,” E3S Web Conf., 2019, doi: 10.1051/e3sconf/20199705043.

A. M. Ufuk Sen, “Effect of biaxial stress state on seismic fragility of concrete gravity dams,” Earthquakes Struct., vol. 18, pp. 285–296, 2020, doi: 10.12989/eas.2020.18.3.285.

Hongqi MaFudon Chi, “Major Technologies for Safe Construction of High Earth-RockfillDams,” Engineering., vol. 2, pp. 498–509, 2016, doi: 10.1016/J.ENG.2016.04.001.

D. Z. X.Kong, J. Liu, “Kong X., Liu J., Zou D. 2016 Numerical simulation of the separation between concrete face slabs and cushion layer of Zipingpu dam during the Wenchuan earthquake,” Sci. China Technol. Sci., vol. 59, pp. 531–539, 2016.

Esmaeilzadeh M., Talkhablou M., Ganjalipour K, “Arching parametric study on earth dams by numerical modeling,” Indian Geotech. J., vol. 48, pp. 728–745, 2018.

C. R. Alonso E.E., “Behavior of materials for earth and rockfill dams,” Front. Struct. Civ. Eng., vol. 4, pp. 1–39., 2010.

P. V.D. Alexandrov A.V., “Foundations of the theory of elasticity and plasticity,” in Moscow. Higher school, 1990, p. 400.

W. E. Bate K., “Numerical methods of analysis and FEM.,” in Moscow: Stroyizdat, 1982, p. 448.

Rekach V.G., “A guide to solving problems in the theory of elasticity.,” in Moscow. Higher school, 1977, p. 215.

Konstantinov I.A, “Dynamics of hydro-technical structures.,” in Part 2. L.: LPI, 1976, p. 196.