Extended Chaplygin Gas in Horava-Lifshitz Gravity

B. Pourhassan\textsuperscript{a} E.O. Kahya\textsuperscript{b}

\textsuperscript{a}School of Physics, Damghan University, Damghan, Iran
\textsuperscript{b}Physics Department, Istanbul Technical University, Istanbul, Turkey

E-mail: bpourhassan@yahoo.com, eokahya@itu.edu.tr

Abstract: In this paper, we investigate cosmological models of the extended Chaplygin gas in a universe governed by Horava-Lifshitz gravity. The equation of state for an extended Chaplygin gas is a \((n+2)\)-variable equation determined by \(A_n\), \(\alpha\), and \(B\). In this work, we are interested to the case of second order \((n=2)\) equation of state which recovers quadratic barotropic equation of state. In that case there are four free parameters. Under some assumptions we relate free parameters to each other to have only one free independent parameter \((A_2)\). It help us to obtain explicit expression for energy density in terms of scale factor. The allowed values of the second order extended Chaplygin gas parameter is determined using the recent astrophysical and cosmological observational data.

Keywords: Cosmology; Dark Energy; Horava-Lifshitz; Chaplygin Gas.
# Contents

1 Introduction .............................................. 1

2 Horava-Lifshitz cosmology ............................... 3

3 Extended Chaplygin gas ................................. 4

4 Observational constraints ............................... 6

5 Cosmological parameters ............................... 7

6 Thermodynamics ......................................... 9

7 Conclusion ............................................... 10

## 1 Introduction

Recent cosmological observations verify the accelerated expansion of universe, and show the deceleration to the accelerated phase transition in the recent past [1–3]. Such accelerated expansion may described by dark energy models. The simplest candidate of dark energy which is consistent with current observations is the cosmological constant. Due to the lack of a good explanation of the small value of the cosmological constant, many dynamical dark energy models were explored, such as a canonical scalar field (quintessence) model [4–9], a phantom model which has the equation of state parameter $\omega < -1$ [10–16], or the combination of quintessence and phantom in a unified model, which named quintom [17–19]. Moreover, many theoretical studies performed on dark energy within some quantum gravitational principles, such as the so-called holographic dark energy proposal [20–23]. The dynamical nature of dark energy introduces a new cosmological problem, namely why are the densities of vacuum energy and dark matter nearly equal today although they scale independently during the expansion history. To solve the problem, one would require that the matter density and dark energy density always approach their current values independent of the initial conditions. The elaboration of this coincidence problem lead to the consideration of interaction between components. Thus, various forms of interacting dark energy models [24–33] have been constructed in order to fulfil the observational requirements.

There are also other interesting models to describe the dark energy such as Chaplygin gas [34, 35], which emerged initially in cosmology from string theory point of view [36, 37], that are based on Chaplygin gas (CG) equation of state and developed to the generalized Chaplygin gas (GCG) [38]. It is also possible to study possibility of viscosity in GCG [39–43]. Then, GCG was extended to the modified Chaplygin gas (MCG) [44]. Recently, viscous
MCG is also suggested and studied [45, 46]. A further extension of CG model is called modified cosmic Chaplygin gas (MCCG) which was proposed recently [47, 48]. The MCG equation of state (EoS) has two parts, the first term gives an ordinary fluid obeying a linear barotropic EoS, and the second term relates pressure to some power of the the inverse of energy density. So, one essentially dealing with a two fluid model. However, it is possible to consider barotropic fluid with quadratic EoS or even with higher orders EoS [49–51]. Therefore, it is interesting to extend MCG EoS which recovers at least barotropic fluid with quadratic EoS, and is called extended Chaplygin gas (ECG) [52–55]. On the other hand, Horava-Lifshitz (HL) gravity appears to be an attractive model to achieve a complete quantum gravitational theory [56]. It has some application in the black hole properties [57–62], the thermodynamic properties [63–67], the dark energy phenomenology [68–70], etc. Additionally, application of HL gravity as a cosmological framework gives rise to HL cosmology [71, 72].

In that case, HL cosmology with GCG has been studied by the Ref. [73]. Also, MCG in HL gravity and observational constraints has been studied by the Refs. [74, 75] with possibility to extension by consideration of Variable $G$ and $\Lambda$ [76]. Now, we would to investigate ECG in HL gravity. The equation of state for a ECG is a $n + 2$-variable equation determined by $A_n$, $\alpha$ and $B$. We assume second order EoS which recovers quadratic barotropic EoS and use the special assumptions to reduce number of free parameters to one. Therefore, we have only an independent parameter. The allowed values of this parameter of the EOS are determined using the recent astrophysical and cosmological observational data such as $H(z)$ analysis.

On the other hand, the temperature behavior and the thermodynamic stability of the generalized Chaplygin gas has been studied by the Ref. [77], and it is found that the generalized Chaplygin gas cools down through the expansion without facing any critical point or phase transition. Also, thermodynamics of the generalized Chaplygin gas has been investigated by introducing the integrability condition, and thermodynamics quantities has been derived as functions of either volume or temperature [78]. Validity of the generalized second law of gravitational thermodynamics in a non-flat Friedmann-Robertson-Walker (FRW) universe and an expanding Gdel-type universe containing the generalized Chaplygin gas confirmed by the Refs. [79] and [80] respectively. In the extension of the Ref. [77], the similar work performed for the case of the modified Chaplygin gas [81] and the same result obtained. More discussion on thermodynamical behavior of the modified Chaplygin gas found in the Ref. [82]. Also, Ref. [83] developed the Ref. [78] to the case of the modified Chaplygin gas. Validity of the generalized second law of thermodynamics in the presence of the modified Chaplygin gas investigated by the Ref. [84] and observed that the generalized second law of thermodynamics always satisfied for the modified Chaplygin gas model. The generalized second law of thermodynamics in the brane-world scenario including the modified Chaplygin gas verified on the apparent horizon in late time by the Ref. [85]. Already, the thermodynamics in HL cosmology has been investigated and validity of of the generalized second law of thermodynamics verified [67]. So, there is no thermodynamical study of the ECG in HL cosmology. This is also another subject of this paper.

The paper is organized as follows. In the next section we give brief review of HL cosmology,
then in section 3 we introduce the ECG. In section 4 we use observational data to constrain
the free parameters of the model. In section 5 we investigate some cosmological parameters
and in section 6 we study thermodynamics aspect of the model and finally in section 7 we
give conclusion.

2 Horava-Lifshitz cosmology

HL gravity described by the following metric \[ ds^2 = -N^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \] (2.1)

where \( N \) and \( N^i \) are the lapse and shift functions which are used in general relativity in
order to split the space-time dimensions. Using the projectable version of HL gravity \[ 86 \]
with the detailed balanced principle \[ 87 \] one can write the gravity action of HL theory as
follow,

\[
S = \int dt dx^3 \sqrt{g}N[\tilde{L}_0 + L_0 + L_1],
\] (2.2)

where,

\[
\tilde{L}_0 = \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2),
\]

\[
L_0 = -\frac{\kappa^2}{2\omega^4}C_{ij}C^{ij} - \frac{\kappa^2\mu}{2\omega^2}R_{kl}R^{kl} - \frac{\kappa^2\mu^2}{8}R_{ij}R^{ij},
\]

\[
L_1 = \frac{\kappa^2\mu^2}{8(1 - 3\lambda)}\left(1 - \frac{4\lambda}{4}R^2 + \Lambda R - 3\Lambda^2\right),
\] (2.3)

where \( \kappa, \lambda, \mu, \omega \) and \( \Lambda \) (is a positive constant, which as usual is related to the cosmological
constant in the IR limit) are constant parameters, \( R_{ij} \) and \( R \) are Ricci tensor and Ricci
scalar respectively. Also, the Cotton tensor is defined as follow,

\[
C^{ij} = \frac{\varepsilon^{ijk}}{\sqrt{g}}\nabla_k(R^j_i - \frac{1}{4}R\delta^j_i),
\] (2.4)

also the extrinsic curvature is defined as,

\[
K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_iN_j - \nabla_jN_i).
\] (2.5)

It is usual to use FRW metric with \( N = 1 \) and \( N^i = 0 \) to obtain the following Friedmann
equations,

\[
H^2 = \frac{\kappa^2}{6(3\lambda - 1)}\rho - \frac{\kappa^4\mu^2}{8(3\lambda - 1)^2}\left(\frac{\Lambda k}{a^2} - \frac{k^2}{2a^2} - \frac{1}{2}\Lambda^2\right),
\] (2.6)

and,

\[
\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\lambda - 1)}\rho - \frac{\kappa^4\mu^2}{16(3\lambda - 1)^2}\left(\frac{\Lambda k}{a^2} + \frac{k^2}{2a^2} - \frac{3}{2}\Lambda^2\right),
\] (2.7)

where \( H = \dot{a}/a \) and \( a \) are Hubble parameter and scale factor respectively, \( k \) is curvature
constant corresponding to open \((k < 0)\), flat \((k = 0)\), and closed \((k > 0)\) universe. Also,
$p$ and $\rho$ are corresponding to total pressure and energy density which contain radiation, dark matter and dark energy. Using the usual notations,

$$G_{\text{cosmo}} = \frac{\kappa^2}{16\pi(3\lambda - 1)},$$  \hspace{1cm} (2.8)  

$$G_{\text{grav}} = \frac{\kappa^2}{32\pi},$$ \hspace{1cm} (2.9)  

and,

$$\frac{\kappa^4 \mu^2 A}{8(3\lambda - 1)^2} = 1,$$ \hspace{1cm} (2.10)  

the conservation equations are,

$$\dot{\rho}_r + 3H(p_r + \rho_r) = 0,$$ \hspace{1cm} (2.11)  

$$\dot{\rho}_b + 3H\rho_b = 0,$$ \hspace{1cm} (2.12)  

and,

$$\dot{\rho}_c + 3H(p_c + \rho_c) = 0,$$ \hspace{1cm} (2.13)  

where $p_r$ and $\rho_r$ are pressure and energy density of radiation matter, $\rho_b$ is energy density of baryonic matter which is pressureless, and $p_c$ and $\rho_c$ are pressure and energy density of the extended Chaplygin gas which is unification of the dark matter and dark energy and discuss in the next section. Tow first equations given by (2.11) and (2.12) yield to the following solutions,

$$\rho_r = \rho_{r0}a^{-4}, \hspace{1cm} \rho_b = \rho_{b0}a^{-3},$$ \hspace{1cm} (2.14)  

where we used the fact that $p_r = 1/3\rho_r$ and $p_b = 0$. Also $\rho_{r0}$ and $\rho_{b0}$ are current values of the energy densities.

## 3 Extended Chaplygin gas

The extended Chaplygin gas EoS given by [52, 53],

$$p_c = \sum_{n=1}^{\infty} A_n \rho_c^n - \frac{B}{\rho_c^\alpha},$$ \hspace{1cm} (3.1)  

where $A_n$, $B$, $\alpha$ and $n$ are constants, so we have generally $n + 2$ free parameters. Note that in the case $n = 1$ the above expressions recovers the standard MCG. In this paper, we are interested to the case of second order EoS ($n = 2$) which recovers quadratic barotropic EoS. In that case the EoS given by (3.1) reduced to the following expression,

$$p_c = A_1 \rho_c + A_2 \rho_c^2 - \frac{B}{\rho_c^\alpha},$$ \hspace{1cm} (3.2)  

Assuming $A_2 = 0$ gives MCG [74], while $A_2 = A_1 = 0$ gives GCG [73] where $A_1$, $B$, and $\alpha$ are positive constants with $0 < \alpha \leq 1$. Therefore, we have four free parameters in our
model. In order to simplifying calculations and reducing free parameters of the model, and also obtaining an analytical expression of the energy density in terms of the scale factor, we assume the following conditions,

\[
\alpha = 1,
\]
\[
A_1 = A_2 - 1,
\]
\[
B = 2A_2.
\]

(3.3)

Therefore, only free parameter of the model is \( A_2 \), and we can solve the conservation equation (2.14) to obtain the following relation,

\[
\rho_c = c_2 \frac{(1 + x + \sqrt{5x - 1})}{(x - 1)},
\]

(3.4)

with \( x = c_1 e^{3\pi a^{30A_2}} \), where \( c_1 \) is constant of integration and \( c_2 \) comes from the fact that the equation (3.2) is dimensionless. The density is physically defined only for \( x > 1 \) (the density is negative for \( x < 1 \)). In this model, the universe starts at \( x = 1 \) with an infinite density, then the density decreases and finally reaches an asymptotic value for \( x \rightarrow +\infty \).

In order to write solution (3.4), we used \( \tan^{-1}(\rho_c + 1) \approx \pi/2 \) approximation, which is valid when \( \rho_c \gg 1 \) corresponding to the early universe. However, our solution will be valid at all time and our approximate solution is very close to the late time behavior with \( \rho_c \ll 1 \).

This is due to the fact that \( \tan^{-1}(\rho_c + 1) \approx \pi/4 \), for \( \rho_c \ll 1 \).

The equation (3.2) is implicitly normalized by the cosmological density \( \rho_L \), i.e. by the asymptotic value of \( \rho_c \) for \( a \rightarrow +\infty \). Indeed, we can see that for \( \rho_c = 1 \) we get \( p_c = A_1 + A_2 - B = -1 \) which corresponds to the equation of state \( p = -\rho \) expected for \( a \rightarrow +\infty \). According to this normalization, we must take \( c_2 = 1 \). Then, we have to determine \( c_1 \). The present density and the cosmological density are related to each other by \( \rho_0 = 1.31\rho_L \). This means that with the previous normalization we should take \( \rho_c = 1.31 \) when \( a = 1 \). This gives \( c_1 e^{(3\pi)} = 65 \), so we can write \( x = 65a^{(30A_2)} \).

Similarly to the detailed balance case, in the IR with \( \lambda = 1 \), \( G_{cosmo} = G_{grav} \equiv G \). So, using (2.8), (2.9) and (2.10) one can rewrite Friedmann equations (2.6) and (2.7) as follows,

\[
H^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{2} - \frac{k}{a^2} + \frac{k^2}{2\Lambda a^4},
\]

(3.5)

and,

\[
\dot{H} + \frac{3}{2} H^2 = -4\pi G p + \frac{3\Lambda}{4} - \frac{k}{2a^2} - \frac{k^2}{4\Lambda a^4},
\]

(3.6)

where,

\[
\rho = \rho_r + \rho_b + \rho_c,
\]

(3.7)

is total energy density including radiation, baryonic matter, and extended Chaplygin gas respectively. Therefore,

\[
p = \frac{1}{3} \rho_r + (A_2 - 1) \rho_c + A_2 \rho_c^2 - \frac{2A_2}{\rho_c},
\]

(3.8)
is the total pressure with the fact that \( \omega_b = p_b/\rho_b = 0, \ \omega_r = p_r/\rho_r = 1/3 \) and \( \omega_c = p_c/\rho_c \). Finally it is useful to define the following relations,

\[
\Omega_i = \frac{8\pi G}{3H^2} \rho_i, \quad \Omega_k = -\frac{k}{a^2H^2}, \quad \Omega_0 = \frac{\Lambda}{2H_0^2}, \tag{3.9}
\]

with \( i = b, r, c \).

4 Observational constraints

In this section, we use \( H(z) \) data to fix some model parameters. In order to use observational data it is useful to rewrite equations in terms of redshift. In that case, using the following relation,

\[
a = \frac{a_0}{1+z}, \tag{4.1}
\]

together with (2.14) and (3.4) in the equation (3.5) we can obtain Hubble expansion parameter in terms of redshift as follow,

\[
E^2(z) = \Omega_{r0}(1+z)^4 + \Omega_{b0}(1+z)^3 + \Omega_{c0} F(z) + \Omega_0 + \Omega_{k0}(1+z)^2 + \frac{\Omega_{k0}^2}{4\Omega_0}(1+z)^4, \tag{4.2}
\]

where,

\[
E(z) \equiv \frac{H(z)}{H_0}, \tag{4.3}
\]

and,

\[
F(z) = \frac{(1 + 65(1+z)^{-30A_2} + \sqrt{325(1+z)^{-30A_2} - 1})}{(65(1+z)^{-30A_2} - 1)}, \tag{4.4}
\]

with the current value of the Hubble expansion parameter \( H_0 \). An important free parameter is \( A_2 \) which should fixed using observational data. Moreover, present day \((z = 0)\) radiation, baryon, ECG, cosmological constant, and curvature energy densities are denoted by \( \Omega_{r0}, \Omega_{b0}, \Omega_{c0}, \Omega_0, \Omega_{k0} \) respectively which satisfy the following equation,

\[
1 = \Omega_{r0} + \Omega_{b0} + \Omega_{c0} + \Omega_0 + \Omega_{k0} + \frac{\Omega_{k0}^2}{4\Omega_0}. \tag{4.5}
\]

It is obvious that the value of \( A_2 \) is not important at present stage and, as expected, it is important at the early universe. The last term in the equation (4.5) corresponds to the dark radiation, which is a characteristic feature of the HL theory of gravity and restricted as follow [88],

\[
\frac{\Omega_{k0}^2}{4\Omega_0} = 0.135\Delta N_\nu \Omega_{r0}, \tag{4.6}
\]

where \( \Delta N_\nu \) represents the effective neutrino species with the following bound [88],

\[-1.7 \leq \Delta N_\nu \leq 2. \tag{4.7}\]

However, we restrict ourself to the case of \( 0 \leq \Delta N_\nu \leq 2 \) [74]. Using the equation (4.6) in the relation (4.5) it is easy to find [74],

\[
\Omega_0 = 1 - \Omega_{b0} - \Omega_{c0} - (1 - 0.135\Delta N_\nu)\Omega_{r0} - 0.73 \sqrt{\Delta N_\nu(\Omega_{r0} - (\Omega_{b0} + \Omega_{c0})\Omega_{r0} - \Omega_{k0}^2)}. \tag{4.8}
\]
Then, equation (4.6) gives,
\[ \Omega_{k0} = \sqrt{0.54 \Omega_0 \Delta N_{\nu} \Omega_{r0}}. \] (4.9)

So, in numerical analysis we choose \( H_0 = 71.4 \text{ Km/s/Mpc}, \) \( \Omega_{b0} = 0.04, \) \( \Omega_{r0} = 8.14 \times 10^{-5}, \)
and \( \Omega_{c0} = 0.951 \) because it is combination of the dark matter and dark energy. These yield to \( 0.0080 \leq \Omega_0 \leq 0.0089 \) and \( 0 \leq \Omega_{k0} \leq 0.00084. \) In the Fig. 1 we represent our numerical result together experimental data [89] which suggests \( 0.1 \leq A_2 < 0.15 \) for all \( 0 \leq \Delta N_{\nu} \leq 2. \)

**Figure 1.** Hubble expansion parameter in terms of redshift for \( A_2 = 0.1 \) (blue line), \( A_2 = 0.11 \) (cyan line), \( A_2 = 0.12 \) (green line), \( A_2 = 0.13 \) (red line), \( A_2 = 0.14 \) (yellow line), \( A_2 = 0.2 \) (black line), dots denote experimental results.

### 5 Cosmological parameters

Using the obtained results for model parameters we investigate behavior of some important cosmological parameters. The first quantity is the effective EoS parameter given by,

\[ \omega_{eff} = \frac{p_{eff}}{\rho_{eff}} \] (5.1)

where,

\[ p_{eff} = p + \frac{2}{k^2} \left[ \frac{k^2}{\Lambda a^4} - 3\Lambda \right], \] (5.2)

and,

\[ \rho_{eff} = \rho + \frac{2}{k^2} \left[ \frac{3k^2}{\Lambda a^4} + 3\Lambda \right], \] (5.3)

with the \( p \) and \( \rho \) given by the equations (3.7) and (3.8). In the Fig. 2 we can see behavior of the EoS parameter for open, close and flat universe. In the case of flat universe we see \( \omega_{eff} \to -1 \) while in the cases of closed or open universes, the present value of the effective EoS parameter is about -0.7 which admits accelerating universe in agreement
with observational data.
We can also investigate stability of the model using sound speed which is given by,

\[ C_s^2 = \frac{dp}{d\rho}. \]

(5.4)

So, the model is stable if \( C_s^2 \geq 0 \). In the Fig. 3 we can see that the model is completely stable for the open and closed universe while there are some instabilities in flat universe for \( 0 \leq A_2 \leq 0.2 \).

**Figure 2.** EoS in terms of redshift for \( A_2 = 0.15 \). Open and closed universe denoted by solid line and flat universe denoted by dashed line.

**Figure 3.** Squared sound speed in terms of redshift for \( A_2 = 0.15 \) (red line), \( A_2 = 0.2 \) (blue line). Open and closed universe denoted by solid line and flat universe denoted by dashed line.
6 Thermodynamics

It is important to investigate thermodynamic properties of the model which may be lead to study of the generalized second law of thermodynamic. In order to study thermodynamics of the model we consider the universe as a thermodynamical system with the apparent horizon surface being its boundary. The apparent horizon given by [67],

\[ r_A = \left( H^2 + \frac{k}{a^2} \right)^{-\frac{1}{2}}. \]  

(6.1)

Then, the temperature and the entropy given by the following expressions respectively,

\[ T = \frac{1}{2\pi r_A}, \]  

(6.2)

and,

\[ S = \frac{\kappa^2}{32\Lambda G^2} \left[ \Lambda r_A^2 + 2k \ln (\sqrt{\Lambda} r_A) \right]. \]  

(6.3)

We should mentioned that the first term corresponds to the general relativity, while the second term is arising from HL gravity. Using the equation (3.5) we can obtain,

\[ T = \sqrt{\frac{8\pi G}{3} \rho + \frac{\Lambda}{2} + \frac{k^2}{2\Lambda a^4}}, \]  

(6.4)

and,

\[ S = \frac{\kappa^2}{32\Lambda G^2} \left[ \frac{\Lambda}{8\pi G} \rho + \frac{\Lambda}{2} + \frac{k^2}{2\Lambda a^4} + 2k \ln \left( \sqrt{\frac{8\pi G}{3} \rho + \frac{\Lambda}{2} + \frac{k^2}{2\Lambda a^4}} \right) \right]. \]  

(6.5)

In that case the first law of thermodynamics [67] given by,

\[ T dS = dE + pdV, \]  

(6.6)

where \( V = \frac{4}{3}pr_A^3 \) is the volume of the system bounded by the apparent horizon, so \( dV \) denotes volume-change and \( dE = pdV \) is energy. We can rewrite the equation (6.6) as follow,

\[ \frac{dS_2}{dt} = \frac{4\pi}{T} (p + \rho)r_A^2 \left( \frac{dr_A}{dt} - Hr_A \right). \]  

(6.7)

where \( S_2 \) is the entropy given by the first law of thermodynamics (6.6). So, using \( p = \omega \rho \) and differentiating the Friedmann equation (3.5) we can obtain,

\[ \frac{dS_2}{dt} = \frac{4\pi}{T} (1 + \omega)\rho r_A^2 \left( \frac{dr_A}{dt} - Hr_A \right). \]  

(6.8)

Using the relation (6.2) gives,

\[ \frac{dS_2}{dt} = 2(1 + \omega)\rho r_A \left( \frac{dr_A}{dt} - Hr_A \right). \]  

(6.9)

These are in agreement with the equation (6.3) which can verified easily. Differentiating (6.3) we have,

\[ \frac{dS_1}{dt} = \frac{\kappa^2}{16\Lambda G^2} \left[ \Lambda r_A + \frac{k}{r_A} \right] \frac{dr_A}{dt}, \]  

(6.10)
where \( S_1 \) is the entropy given by the apparent horizon. Now, we can calculate total entropy change,

\[
\frac{dS_{\text{tot}}}{dt} = \frac{dS_1}{dt} + \frac{dS_2}{dt}.
\]

It is easy to check that \( \frac{dS_{\text{tot}}}{dt} > 0 \), at least for flat and closed universe, in agreement with [67].

7 Conclusion

In this paper, we considered the extended Chaplygin gas in Horava-Lifshitz gravity. First of all we reviewed the HL cosmology and then solved conservation equation for the special case of the ECG. We obtained energy density in terms of scale factor which allow us to investigate Hubble expansion parameter in terms of redshift. We considered some assumptions which reduced free parameters of the ECG to one. We used \( H(z) \) data to constraint this parameter in HL cosmology. \( H(z) \) observations suggest \( 0.13 \leq A_2 \leq 0.25 \), which means that \( 0.26 \leq B \leq 0.5 \) and \( -0.87 \leq A_1 \leq -0.75 \). Using the obtained model parameters we studied evolution of EoS parameter and found that, for the cases of \( k < 0 \) and \( k > 0 \), the present value of the effective EoS parameter is about -0.7 while it yields to 1/3 at high redshift. On the other hand, for the flat universe \( (k = 0) \) we found \( \omega_{\text{eff}} \rightarrow -1 \). Stability of the model also investigated using squared sound speed and found that the model is completely stable for the open and closed universe while there are some instabilities in flat universe for the case of \( 0.1 \leq A_2 \leq 0.2 \). It seems \( A_2 = 0.15 \) is the best fitted value. Then, we investigated thermodynamic properties of the model. We found that the generalized second law of thermodynamics is valid for a flat or closed universe.

References

[1] A. G. Riess et al. [Supernova Search Team Collaboration], Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. 116, 1009 (1998), [arXiv:astro-ph/9805201].

[2] A.G. Riess et al. [Supernova Search Team Collaboration], Type Ia Supernova Discoveries at \( z > 1 \) From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Astron. J. 607, 665 (2004), [arXiv:astro-ph/0402512].

[3] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Measurements of Omega and Lambda from 42 high redshift supernovae, Astrophys. J. 517, 565 (1999), [arXiv:astro-ph/9812133].

[4] B. Ratra and P. J. E. Peebles, Cosmological Consequences of a Rolling Homogeneous Scalar Field, Phys. Rev. D 37, 3406 (1988).

[5] C. Wetterich, Cosmology and the Fate of Dilatation Symmetry, Nucl. Phys. B 302, 668 (1988).

[6] A. R. Liddle and R. J. Scherrer, A Classification of scalar field potentials with cosmological scaling solutions, Phys. Rev. D 59, 023509 (1999), [arXiv:astro-ph/9809272].

[7] Z. -K. Guo, N. Ohta and Y. -Z. Zhang, Parametrizations of the dark energy density and scalar potentials, Mod. Phys. Lett. A 22, 883 (2007), [arXiv:astro-ph/0603109].
[8] M. Khurshudyan, E. Chubaryan, B. Pourhassan, *Interacting Quintessence Models of Dark Energy*, Int. J. Theor. Phys. **53**, 2370 (2014), [arXiv:1402.2385 [gr-qc]].

[9] S. Dutta, E. N. Saridakis and R. J. Scherrer, *Dark energy from a quintessence (phantom) field rolling near potential minimum (maximum)*, Phys. Rev. D **79**, 103005 (2009), [arXiv:0903.3412].

[10] R. R. Caldwell, *A Phantom menace?*, Phys. Lett. B **545**, 23 (2002), [arXiv:astro-ph/9908168].

[11] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, *Phantom energy and cosmic doomsday*, Phys. Rev. Lett. **91**, 071301 (2003), [arXiv:astro-ph/0302506].

[12] S. Nojiri and S. D. Odintsov, *Quantum de Sitter cosmology and phantom matter*, Phys. Lett. B **562**, 147 (2003), [arXiv:hep-th/0303117].

[13] V. K. Onemli and R. P. Woodard, *Quantum effects can render $w < -1$ on cosmological scales*, Phys. Rev. D **70**, 107301 (2004), [arXiv:gr-qc/0406098].

[14] E. N. Saridakis, *Theoretical Limits on the Equation-of-State Parameter of Phantom Cosmology*, Phys. Lett. B **676**, 7 (2009); [arXiv:0811.1333].

[15] E. N. Saridakis, *Phantom evolution in power-law potentials*, Nucl. Phys. B **819**, 116 (2009), [arXiv:0902.3978].

[16] G. Gupta, E. N. Saridakis and A. A. Sen, *Non-minimal quintessence and phantom with nearly flat potentials*, Phys. Rev. D **79**, 123013 (2009), [arXiv:0905.2348].

[17] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, *Cosmological evolution of a quintom model of dark energy*, Phys. Lett. B **608**, 177 (2005), [arXiv:astro-ph/0410654].

[18] W. Zhao, *Quintom models with an equation of state crossing -1*, Phys. Rev. D **73**, 123509 (2006), [arXiv:astro-ph/0604460].

[19] Y. F. Cai, E. N. Saridakis, M. R. Setare and J. Q. Xia, *Quintom Cosmology: Theoretical implications and observations*, Phys. Rept. **493**, 1 (2010), [arXiv:0909.2776].

[20] H. Li, Z. K. Guo and Y. Z. Zhang, *A Tracker Solution for a Holographic Dark Energy Model*, Int. J. Mod. Phys. D **15**, 869 (2006), [arXiv:astro-ph/0602521].

[21] J. Sadeghi, B. Pourhassan, Z. A. Moghaddam, *Interacting Entropy-Corrected Holographic Dark Energy and IR Cut-Off Length*, Int. J. Theor. Phys. **53**, 125 (2014), [arXiv:1306.2055].

[22] M. R. Setare, J. Zhang and X. Zhang, *Statefinder diagnosis in a non-flat universe and the holographic model of dark energy*, JCAP **0703**, 007 (2007), [arXiv:gr-qc/0611084].

[23] E. N. Saridakis, *Holographic Dark Energy in Braneworld Models with Moving Branes and the \( w=-1 \) Crossing*, JCAP **0804**, 020 (2008), [arXiv:0712.2672].

[24] A. P. Billyard and A. A. Coley, *Interactions in scalar field cosmology*, Phys. Rev. D **61**, 083503 (2000), [arXiv:gr-qc/astro-ph/9908224].

[25] A. Nunes, J.P. Mimoso and T.C. Charters, *Scaling solutions from interacting fluids*, Phys. Rev. D **63** (2001) 083506, [arXiv:gr-qc/0011073].

[26] G. R. Farrar and P. J. E. Peebles, *Interacting dark matter and dark energy*, Astrophys. J. **604**, 1 (2004), [arXiv:gr-qc/astro-ph/0307316].

[27] J.D. Barrow and T. Clifton, *Cosmologies with energy exchange*, Phys. Rev. D **73** (2006) 103520, [arXiv:gr-qc/0604063].
[28] T. Gonzalez, G. Leon and I. Quiros, Dynamics of quintessence models of dark energy with exponential coupling to dark matter, Class. Quant. Grav. 23 (2006) 3165, [arXiv:gr-qc/astro-ph/0702227].

[29] H. Garcia-Compean, G. Garcia-Jimenez, O. Obregon and C. Ramirez, Crossing the phantom divide in an interacting generalized Chaplygin gas, JCAP 0807 (2008) 016, [arXiv:0710.4283].

[30] C. G. Boehmer, G. Caldera-Cabral, R. Lazkoz and R. Maartens, Dynamics of dark energy with a coupling to dark matter, Phys. Rev. D 78, 023505 (2008), [arXiv:0710.4283].

[31] M. Jamil and M.A. Rashid, Constraining the coupling constant between dark energy and dark matter, Eur. Phys. J. C 60 (2009) 141, [arXiv:0802.1144].

[32] X.-m. Chen, Y.-g. Gong and E. N. Saridakis, Phase-space analysis of interacting phantom cosmology, JCAP 0904, 001 (2009), [arXiv:0812.1117].

[33] M. Khurshudyan, B. Pourhassan, E.O. Kahya, Interacting two-component fluid models with varying EoS parameter, Int. J. Geom. Meth. Mod. Phys. 11 (2014) 1450061, [arXiv:gr-qc/1312.1162].

[34] A. Y. Kamenshchik, U. Moschella and V. Pasquier, An alternative to quintessence, Phys. Lett. B 511, 265 (2001), [arXiv:gr-qc/0103004].

[35] M. C. Bento, O. Bertolami and A. A. Sen, Generalized Chaplygin gas, accelerated expansion and dark energy matter unification, Phys. Rev. D 66, 043507 (2002), [arXiv:gr-qc/0202064].

[36] J. D. Barrow, The deflationary universe: An instability of the de Sitter universe, Phys. Lett. B 180, 335 (1986).

[37] J. D. Barrow, String-driven inflationary and deflationary cosmological models, Nucl. Phys. B 310, 743 (1988).

[38] N. Bilic, G.B. Tupper, and R.D. Viollier, Unification of dark matter and dark energy: the inhomogeneous Chaplygin gas, Phys. Lett. B 535 (2002) 17, [arXiv:astro-ph/0111325].

[39] H. Saadat and B. Pourhassan, Effect of Varying Bulk Viscosity on Generalized Chaplygin Gas, Int. J. Theor. Phys. 53, 1168 (2014) [arXiv:1305.6054].

[40] X.-h. Zhai, Y.-d. Xu, X.-z. Li, Viscous generalized Chaplygin gas, Int. J. Mod. Phys. D 15, 1151 (2006) [arXiv:astro-ph/0511814].

[41] Y. D. Xu et al. Generalized Chaplygin gas model with or without viscosity in the uu' plane, Astrophys. Space Sci. 337, 493 (2012).

[42] A.R. Amani and B. Pourhassan, Viscous Generalized Chaplygin gas with Arbitrary α, Int. J. Theor. Phys. 52, 1309 (2013).

[43] H. Saadat and B. Pourhassan, FRW Bulk Viscous Cosmology with Modified Chaplygin Gas in Flat Space, Astrophys. Space Sci. 343, 783 (2013).

[44] U. Debnath, A. Banerjee, and S. Chakraborty, Role of modified Chaplygin gas in accelerated universe, Class. Quant. Grav. 21, 5609 (2004), [arXiv:gr-qc/0411015].

[45] H. Saadat and B. Pourhassan, FRW bulk viscous cosmology with modified cosmic Chaplygin gas, Astrophys. Space Sci. 344, 237 (2013).

[46] J. Naji, B. Pourhassan, A.R. Amani, Effect of shear and bulk viscosities on interacting...
modified Chaplygin gas cosmology, Int. J. Mod. Phys. D 23, 1450020 (2013).

[47] B. Pourhassan, Viscous Modified Cosmic Chaplygin Gas Cosmology, Int. J. Mod. Phys. D 22, 1350061 (2013) [arXiv:1301.2788].

[48] J. Sadeghi, B. Pourhassan, M. Khurshudyan, H. Farahani, Time-Dependent Density of Modified Cosmic Chaplygin Gas with Cosmological Constant in Non-Flat Universe Int. J. Theor. Phys. 53, 911 (2014).

[49] E. V. Linder, R. J. Scherrer, Aetherizing Lambda: Barotropic fluids as dark energy, Phys. Rev. D 80, 023008 (2009) [arXiv:0811.2797].

[50] K. N. Ananda and M. Bruni, Cosmological dynamics and dark energy with a nonlinear equation of state: A quadratic model, Phys. Rev. D 74, 023523 (2006) [arXiv:astro-ph/0512224].

[51] P. H. Chavanis, A cosmological model describing the early inflation, the intermediate decelerating expansion, and the late accelerating expansion by a quadratic equation of state, [arXiv:1309.5784].

[52] B. Pourhassan, E.O. Kahya, FRW cosmology with the extended Chaplygin gas, Advances in High Energy Physics 2014, 231452 (2014) [arXiv:1405.0667].

[53] M. Khurshudyan, E.O. Kahya, B. Pourhassan, R. Myrzakulov, A. Pasqua, Higher order corrections of the extended Chaplygin gas cosmology with varying G and Λ, [arXiv:1402.2592].

[54] B. Pourhassan, E.O. Kahya, Extended Chaplygin gas model, Results in Physics 4, 101 (2014).

[55] E. O. Kahya, B. Pourhassan, Observational constraints on the extended Chaplygin gas inflation, Astrophys. Space Sci. 353, 677 (2014)

[56] P. Horava, Membranes at Quantum Criticality, JHEP 0903, 020 (2009) [arXiv:0812.4287].

[57] U. H. Danielsson, L. Thorlacius, Black holes in asymptotically Lifshitz spacetime, JHEP 0903, 070 (2009) [arXiv:0812.5088].

[58] R. G. Cai, L. M. Cao, N. Ohta, Topological Black Holes in Horava-Lifshitz Gravity, Phys. Rev. D 80, 024003 (2009) [arXiv:0904.3670].

[59] J. Sadeghi, B. Pourhassan, Particle acceleration in HoravaLifshitz black holes, Eur. Phys. J. C 72, 1984 (2012) [arXiv:1108.4530].

[60] M. I. Park, The Black Hole and Cosmological Solutions in IR modified Horava Gravity, JHEP 0909, 123 (2009) [arXiv:0905.4480].

[61] M. Botta-Cantcheff, N. Grandi, M. Sturla, Wormhole solutions to Horava gravity, Phys. Rev. D 82, 124034 (2010) [arXiv:0906.0582].

[62] H. W. Lee, Y. W. Kim, Y. S. Myung, Extremal black holes in the Horava-Lifshitz gravity, Eur. Phys. J. C 68, 255 (2010) [arXiv:0907.3568].

[63] A. Wang, Y. Wu, Thermodynamics and classification of cosmological models in the Horava-Lifshitz theory of gravity, JCAP 0907, 012 (2009) [arXiv:0905.4117].

[64] R. G. Cai, L. M. Cao, N. Ohta, Thermodynamics of Black Holes in Horava-Lifshitz Gravity, Phys. Lett. B 679, 504 (2009) [arXiv:0905.0751].

[65] R. G. Cai, N. Ohta, Horizon Thermodynamics and Gravitational Field Equations in Horava-Lifshitz Gravity, Phys. Rev. D 81, 084061 (2010) [arXiv:0910.2307].
[66] J. Sadeghi, K. Jafarzade B. Pourhassan, Thermodynamical Quantities of Horava-Lifshitz Black Hole, Int. J. Theor. Phys. 51, 3891 (2012)

[67] M. Jamil, E. N. Saridakis, M. R. Setare, The generalized second law of thermodynamics in Horava-Lifshitz cosmology, JCAP 1011, 032 (2010) [arXiv:1003.0876].

[68] E. N. Saridakis, Horava-Lifshitz Dark Energy, Eur. Phys. J. C 67, 229 (2010) [arXiv:0905.3532].

[69] M. I. Park, A Test of Horava Gravity: The Dark Energy, JCAP 1001, 001 (2010) [arXiv:0906.4275].

[70] M. Jamil, E. N. Saridakis, New agegraphic dark energy in Horava-Lifshitz cosmology, JCAP 1007, 028 (2010) [arXiv:1003.5637].

[71] G. Calcagni, Cosmology of the Lifshitz universe, JHEP 0909, 112 (2009) [arXiv:0904.0829].

[72] E. Kiritsis, G. Kofinas, Horava-Lifshitz Cosmology, Nucl. Phys. B 821, 467 (2009) [arXiv:0904.1334].

[73] A. Ali, S. Dutta, E. N. Saridakis, A. A. Sen, Horava-Lifshitz cosmology with generalized Chaplygin gas, Gen. Relativ. Gravit. 44, 657 (2012) [arXiv:1004.2474].

[74] B. C. Paul, P. Thakur, A. Saha, Modified Chaplygin gas in Horava-Lifshitz gravity and constraints on its B parameter, Phys. Rev. D 85, 024039 (2012)

[75] B. C. Paul, P. Thakur, Observational Constraints on Modified Chaplygin Gas in Horava-Lifshitz Gravity, Pramana 81, 691 (2012) [arXiv:1205.2796].

[76] M. Khodadi, M. Naderi, Chaplygin Gas Model with Variable $G, \Lambda$ in Horava-Lifshitz Cosmology, Int. J. Theor. Phys. 53 (2014) DOI: 10.1007/s10773-014-2150-5.

[77] F. C. Santos, M. L. Bedran, V. Soares, On the thermodynamic stability of the generalized Chaplygin gas, Phys. Lett. B 636, 86 (2006)

[78] Y. S. Myung, Thermodynamics of Chaplygin gas, Astrophys. Space Sci. 335, 561 (2011)

[79] K. Karami, S. Ghaffari, M. M. Soltanzadeh, The generalized second law for the interacting generalized Chaplygin gas model, Astrophys. Space Sci. 331, 309 (2011)

[80] M. Salti, Thermodynamics of Chaplygin Gas Interacting with Cold Dark Matter, Int. J. Theor. Phys. 52, 4583 (2013)

[81] F. C. Santos, M. L. Bedran, V. Soares, On the thermodynamic stability of the modified Chaplygin gas, Phys. Lett. B 646, 215 (2007)

[82] B. Kr. Dev Choudhury, J. Saikia, Some Discussion on Thermodynamical Behaviour of Modified Chaplygin Gas, [arXiv:1006.1461].

[83] S. Bhattacharya, U. Deb Nath, Thermodynamics of Modified Chaplygin Gas and Tachyonic Field, Int. J. Theor. Phys. 51, 565 (2012)

[84] U. Deb Nath, M. Jamil, Correspondence between DBI-essence and Modified Chaplygin Gas and the Generalized Second Law of Thermodynamics, Astrophys. Space Sci. 335, 545 (2011)

[85] T. Bandyopadhyay, Thermodynamics of Gauss-Bonnet brane with modified Chaplygin gas, Astrophys. Space Sci. 341, 689 (2012)

[86] C. Charmousis, G. Niz, A. Padilla, P. M. Saffin, Strong coupling in Horava gravity, JHEP 0908, 070 (2009) [arXiv:0905.2579].
[87] P. Horava, *Membranes at Quantum Criticality*, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775].

[88] S. Dutta, E. N. Saridakis, *Observational constraints on Horava-Lifshitz cosmology*, JCAP **1001**, 013 (2010) [arXiv:0911.1435].

[89] P. Thakur, S. Ghose, B. C. Paul, *Modified Chaplygin gas and constraints on its B parameter from cold dark matter and unified dark matter energy cosmological models*, Mon. Not. R. Astron. Soc. **397**, 1935 (2009) [arXiv:0905.2281].