Can supermassive black hole shadows test the Kerr metric?

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The unprecedented image of the M87\textsuperscript{*} supermassive black hole has sparked some controversy over its usefulness as a test of the general relativistic Kerr metric. The criticism is mainly related to the black hole’s quasi-circular shadow and advocates that its radius depends not only on the black hole’s true spacetime properties but also on the poorly known physics of the illuminating accretion flow. In this paper we take a sober view of the problem and argue that our ability to probe gravity with a black hole shadow is only partially impaired by the matter degrees of freedom and the number of non-Kerr parameters used in the model. As we show here, a more intriguing situation arises from the mass scaling of the dimensional coupling constants that typically appear in non-GR theories of gravity. Existing limits from gravitational wave observations imply that supermassive systems like the M87\textsuperscript{*} black hole would suffer a suppression of all non-GR deviation parameters in their metric, making the spacetime and the produced shadow virtually Kerr. Therefore, a supermassive black hole shadow is likely to probe only those extensions of General Relativity which are endowed with dimensionless coupling constants or other special cases with a screening mechanism for black holes or certain types of spontaneous scalarisation.

I. INTRODUCTION

The centenary of the Eddington-Dyson 1919 observation of light deflection by the sun \cite{1} was marked by another important milestone in gravitational physics, the release of the direct image of the supermassive black hole in the nucleus of the M87 galaxy by the Event Horizon Telescope (EHT) collaboration \cite{2–4}. This millimeter-band radio image of unprecedented angular resolution, itself an example of extreme light deflection, has provided direct quantitative evidence of the presence of supermassive black holes in galactic centers and has shed some light in the inner workings of active galactic nuclei. A second image, that of our galactic SgrA\textsuperscript{*} supermassive black hole, is scheduled to be released by the EHT collaboration in the near future.

From the point of view of fundamental physics, a key element of an image like that of the M87\textsuperscript{*} black hole is the geometric shape of the shadow seen by an asymptotic observer, as superimposed in the brighter background of the luminous matter surrounding the black hole. By its very nature, and in contrast to the observation of gravitational waves from compact binary systems, the generation and observation of a black hole image is an ‘experiment’ on the geodesic motion of photons emitted by the accretion flow, and therefore probes the geometry rather than the dynamical properties of the classical spacetime.

One of the key motivations behind the conception of EHT was to use the shadow as evidence for the existence of black holes and as a probe of General Relativity (GR) \cite{5} for a review. This exciting possibility has spawn a significant amount of work over the last decade or so, mostly focused on the calculation of shadows of non-Kerr black holes beyond GR \cite{6–14} but also on improving our understanding of the image produced by garden-variety Kerr black holes \cite{15–17}.

Throughout this paper we use relativistic units $G = c = 1$.

II. PROBING GRAVITY WITH BLACK HOLE SHADOWS

In a recent paper, Psaltis et al. \cite{18} used the physical shape of the M87\textsuperscript{*} black hole shadow as a test of GR. A prerequisite for this type of test is the independent knowledge of the black hole’s mass, that is, the system’s intrinsic yardstick. In the case of M87\textsuperscript{*} the mass has been estimated to be $M = (6.6 \pm 0.4) \times 10^9 M_\odot$ by stellar kinematics with the assumption of a distance of 17.9Mpc \cite{19}. The deviation from GR was modelled with the help of the Johannsen metric \cite{20} (hereafter ‘J-metric’) which is a theory-agnostic deformation of the Kerr metric. To lowest order, the deformation is encapsulated in the four constant \textit{dimensionless} parameters \{$\alpha_{22}, \alpha_{13}, \alpha_{52}, \epsilon_3$\}. The J-metric is Kerr-like in the sense that it is separable (thus admitting a third constant of motion like the Carter constant), admits spherical photon orbits \cite{21}, and is endowed with a spherical event horizon with the same radius $r_+ = M + \sqrt{M^2 - a^2}$ as a Kerr black hole of the same mass $M$ and spin $a$. Therefore, it is not too surprising that black holes in the J-metric cast Kerr-like shadows \cite{7, 14}, namely, shadows that have a nearly constant circular radius unless the spin parameter $a$ lies close to its maximum allowed value. Based on

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this property, one can focus on a non-rotating system for which the shadow is exactly circular with its radius given by the impact parameter \( b \) associated with the unstable circular photon orbit, i.e. the black hole’s photon ring. The photon ring radius \( r_{ph} \) and the impact parameter depend on the single metric component [18, 22]

\[
g_{tt}(r) = - \left( 1 - \frac{2M}{r} \right) \frac{(1 + \varepsilon_3 M^3/r^3)}{(1 + \alpha_3 M^3/r^3)^2},
\]

and are given by

\[
b = \frac{r_{ph}}{\sqrt{-g_{tt}(r_{ph})}}, \quad r_{ph} \frac{dg_{tt}}{dr}(r_{ph}) = 2g_{tt}(r_{ph}).
\]

The aforementioned strategy was adopted in Ref. [18] and the results are summarised in Fig. 1 where we show \( b \) for a non-rotating black hole in the J-metric as a function of the deformation parameters \( \{\alpha_3, \varepsilon_3\} \). The reported 17% uncertainty in the observed shadow radius [18] translates into an allowed range \(-3.6 \lesssim \alpha_3 \lesssim 6 \) and \(-7 \lesssim \varepsilon_3 \lesssim 12 \) for the deformation parameters. It can be noticed that the two parameters are anti-correlated: a positive (negative) \( \alpha_3 \) (\( \varepsilon_3 \)) gives rise to a bigger shadow relative to the canonical GR radius \( b_{GR} = 3\sqrt{3}M \) (and vice versa for the opposite signs).

This, however, is not the end of the story: the apparent shadow radius is also a function of the geometry of the illuminating accretion flow [15]. Assuming GR gravity, this radius (which represents the peak of the emitted flux) is given by \( b_{GR} \) when the black hole is ‘backlit’ from a distant uniform source. The same is true for the more astrophysically relevant scenario of a spherically symmetric flow in the vicinity of the black hole [15, 23]. In contrast, illumination by a thin accretion disk would lead to a somewhat larger shadow radius \( b \approx 6.2M \) [15]. A more realistic alternative possibility for a system like M87* is that of a geometrically thick/optically thin disk; in such a case the analysis of Ref. [15] suggests a shadow radius of \( b \approx 5.8M \) that lies between the two previous values. The matter-induced deviation of the apparent shadow radius from the mathematical value \( b_{GR} \) has been the subject of more sophisticated modelling in [24]. According to this recent work the resulting ‘error’ in the radius is \( \approx 5\% \) which translates to \( b \approx 5.5M \).

The uncertainty in \( b \) caused by the unknown accretion physics of M87* is shown in Fig. 1 as a grey band and has some clear implications for the earlier constraints on the deviation from GR. A large portion of the parameter space previously associated with an enlarged black hole shadow as a result of deviations from GR is now occupied by the more prosaic accretion physics ‘error’ [25]. Only the spacetime deviations for which \( b < b_{GR} \) can be cleanly probed by the shadow measurement. Indeed, and given that it marks the peak of the geodesic potential, \( b_{GR} \) is the absolute minimum shadow radius irrespectively of the accretion physics details.

The situation could deteriorate further if both deformation parameters are non-vanishing [18, 26]. As a consequence of their anti-correlation, the shadow of a black hole with \( \alpha_3 \sim \varepsilon_3 \) could lie significantly closer to \( b_{GR} \) for the same degree of deformation. This is exemplified by the dashed curve in Fig. 1 which shows \( b \) for \( \alpha_3 = 1.2\varepsilon_3 \).

It is clear that in such a case most of the deviation away from \( b_{GR} \) overlaps with the accretion physics error and the constraints on \( \alpha_3, \varepsilon_3 \) are far less reliable. Of course, we would have drawn the exact opposite conclusion if \( \alpha_3 \sim -\varepsilon_3 \).

The upshot of this discussion is that the quality of black hole shadows as probes of GR gravity could be diluted by the system’s matter degrees of freedom and by possible (anti)correlations between the non-GR parameters of the metric. An additional complication lies in the shadow’s shape itself: nearly circular shadows are relatively ubiquitous in non-Kerr spacetimes, resembling the shape of a Kerr shadow for most of the allowed spin range. As a case in point, consider the parametrised metric of Carson & Yagi [27] which is an extension of the J-metric and represents the most general family of separable and asymptotically flat spacetimes. This is a metric that can be mapped on black hole solutions originating from various modified theories of gravity; it too can lead to Kerr-like quasi-circular shadows for a wide range of its deformation parameters, especially when the spin is not too high.
III. THE IMPORTANCE OF NON-GR COUPLING PARAMETERS

In a sense, our discussion so far was just the tip of the proverbial iceberg because even if we were to put aside the complications related to the physics of the accretion flow and the commonness of quasi-circular shadows, we would still have to face a much more serious problem related to the mass dependence of the non-Kerr deformation parameters. The key issue here is to understand to what extent the constraints placed on these parameters by the M87* image are compatible with limits placed by gravitational wave observations of merging black holes [28–30] or electromagnetic observations of astrophysical black holes in X-ray binaries [31].

The crucial importance of the mass scaling of a gravity theory’s non-GR parameters was first discussed in [32] and was recently emphasized in [33] in the context of gravitational waves from extreme mass ratio inspirals (EMRIs) in supermassive black holes. In order to understand the impact of the mass, we follow the reasoning of this recent work and consider modified theories of gravity, as extensions of GR, described by an action of the following general form,

$$S = S_{GR}(g_{\mu \nu}, \phi) + \alpha S_c(g_{\mu \nu}, \phi) + S_m(g_{\mu \nu}, \phi, \Psi), \quad (3)$$

where $g_{\mu \nu}$ is the metric, $\phi$ is the scalar field degree of freedom and $\Psi$ stands for the matter fields. The first term represents the GR part of the action,

$$S_{GR} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left( R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right), \quad (4)$$

where $R$ is the Ricci scalar and $g$ is the metric determinant. The last term is the matter part of the action and can be set to zero for the purposes of this paper. The theory’s non-GR physics is encapsulated in the term $S_c$ which describes non-minimal couplings between $g_{\mu \nu}$ and $\phi$; the factor in front of this term is the theory’s coupling constant.$^1$

First we need to distinguish between two different scenarios: (i) the Kerr metric is an admitted black hole solution of a non-GR theory; (a) this could be the only possibility (as in $f(R)$ gravity [34]), (b) or just a solution branch among other non-Kerr solutions, (ii) the more generic scenario of genuine non-Kerr solutions. Given that a black hole shadow is essentially the result of photons moving along the geodesics of the hole’s spacetime, it is clear that the first scenario is associated with shadows identical to the GR Kerr shadows and is therefore untenable with this method. The family of theories with non-Kerr solutions (including spin-scalarised Kerr black holes [35–37]) should generically lead to non-Kerr shadows and hereafter we focus on them.

In their vast majority, these theories (and as a consequence their black hole solutions) are endowed with a dimensional coupling constant that scales as $\alpha \sim M^n$ with $n \geq 1$. Examples of such theories include scalar Gauss-Bonnet gravity, generalised scalar-tensor theories and dynamical Chern-Simons gravity [38]. Black hole metrics in these theories depend on $M$, a spin parameter $a/M$ and a dimensionless coupling constant

$$\zeta \equiv \frac{\alpha}{M^n}. \quad (5)$$

The hole’s scalar field is expanded in $\zeta$ around its constant asymptotic value and enters the metric through a dimensionless function of order unity that multiplies $\zeta$ (e.g., see Ref. [39]). The appearance of $\zeta$ should not come as a surprise since, as we have pointed out, the mass is the system’s only available dimensional scale. Theory-agnostic deformed Kerr metrics can be typically mapped onto specific ‘genuine’ theories with dimensional constants. Examples are provided by the general Carson-Yagi metric mentioned earlier [27], and the J-metric used in this paper. As it turns out, in all cases the deformation parameters are simply related to the coupling constant of the corresponding gravity theory (e.g. [20, 27]). For the case of the J-metric we typically have $\alpha_{13} \sim \zeta^k$ with $k \geq 1$, and similarly for the other parameters.

At this point we may return to the analysis of the M87* shadow and examine what are the implications of using non-GR models with dimensional constants for its description. As we have seen in the previous section the constraint on $\alpha_{13}$, when expressed in terms of the coupling constant $\zeta$ of a given theory, amounts to

$$|\alpha| \lesssim sM^n, \quad (6)$$

where $s \sim 10$ approximately. We imagine that the same non-GR model is also used in the study of the gravitational wave-driven inspiral and merger of a black hole binary system of typical mass $M_b$, resulting in a similar constraint $|\zeta_b| \lesssim s_b$, where

$$\zeta_b = \frac{\alpha}{M_b^n} = \zeta \left( \frac{M}{M_b} \right)^n. \quad (7)$$

Existing limits from astrophysical observations (see e.g. [40, 41]) suggest $s_b \lesssim 1$. Using this as a fiducial limit for our J-metric model, we find

$$|\alpha_{13}| \sim |\zeta_b|^k \left( \frac{M_b}{M} \right)^{kn} \lesssim 10^{-8kn} \left( \frac{M_b}{M_\odot} \right)^{kn}, \quad (8)$$

where the masses have been normalised to their typical values, $M_b = M/10^9 M_\odot$, $M_{\odot} = M_b/10 M_\odot$. Thus we have shown that $|\alpha_{13}| \ll 1$: as a consequence of the mass scaling of $\zeta$, a similar result should hold for the rest of the parameters since all of them are comparable to $\zeta$.

The same argument can be turned around: a typical deformation $\alpha_{13} \sim O(1)$ coming from the shadow of

1 The conclusions of this paper should be equally applicable to actions more general than (3), including additional scalar fields and coupling constants.
M87* would be stretched by a factor $\sim (M/M_b)^{kn}$ when the J-metric is used to model the celestial mechanics of a merging binary system. This would cause an enormous deviation from the GR black hole metric which would have easily been seen in the system’s GW signal.

The mass-suppression effect could be evaded if a black hole is exactly described by the Kerr metric within a non-GR theory but could undergo a spin-induced scalarisation above a spin threshold $[35]$ (i.e. this is the previously mentioned scenario (ib)). An example is provided by Gauss-Bonnet gravity itself which for a vanishing derivative of the scalar field coupling function does indeed admit the Kerr solution. Recent work $[35-37]$ suggests that such black holes can become non-Kerr by spontaneous scalarisation if they reside in a wedge-shaped region bounded by $a \gtrsim 0.5M$ and a negative coupling $\alpha/M^2 \sim -(0.1 - 10)$ . Although this region represents a small fraction of the parameter space it is possible to imagine a scenario in which stellar-mass black holes probed by GW observations lie outside the scalarisation wedge (and therefore are Kerr) whereas rapidly spinning supermassive black holes (a viable possibility for M87*) are scalarised and non-Kerr.

The remarkable conclusion of this section is that one should typically expect (at least for most of the straight forward extensions of GR) the black hole spacetime of M87* to be described by the Kerr metric to a very high precision, with all non-Kerr deviations suppressed by the system’s enormous mass. Once the metric is rendered Kerr for all practical purposes, it follows that all geodesic motion and the shadow itself must necessarily be also Kerr $[32, 33]$.

IV. CONCLUDING REMARKS

The take home message of this paper is rather clear: the shadow appearing in a black hole image like that of M87* could be a viable probe of GR gravity (and more specifically of the Kerr spacetime) but with some important caveats attached. This standpoint lies somewhere in between the recent opposing claims made in Refs. $[15, 18]$ but at the same time it extends to a completely orthogonal direction.

It is certainly true that the shadow radius is primarily a function of the black hole’s spacetime but also of the (largely unknown) accretion flow physics. However, if GR gravity is assumed, the radius cannot be pushed below $b_{GR}$ and therefore a $b < b_{GR}$ ought to be a clean probe of the black hole’s spacetime metric (provided it is observationally allowed in the first place). Moreover, the constraints placed on the deformation parameters of the non-Kerr model also depend on their actual number $[26]$. The quasi-circular shape of the M87* shadow is another complicating factor because similarly shaped shadows commonly emerge in non-GR gravity theories and deformed black hole spacetimes alike. The very recent work of $[42]$ represents a detailed study of the degeneracy between Kerr and non-Kerr black holes in the strict sense of exactly matching shadows; of course a more empirical approach is also possible taking into consideration that observational errors can easily accommodate a small mismatch between shadows.

Our discussion of non-GR theories with coupling constants has revealed a perhaps unexpected dichotomy: the shadow cast by a supermassive black hole is intrinsically insensitive to deviations from GR when the underlying gravity theory contains dimensional coupling constants (in which case Eq. (5) shows that the deviation parameters are mass-suppressed). The majority of known extensions of GR do indeed fall into this category. Nevertheless, there exist notable exceptions like Einstein-aether theory $[43]$ where the non-GR parameters are dimensionless quantities. The non-Kerr character of black holes in such theories is equally prominent regardless of their mass, and therefore their shadow could be used as a test of GR (see $[44]$ for a calculation along these lines in the context of Einstein-aether theory) albeit subject to the influence of the factors discussed in this paper. Even within the class of theories with dimensional coupling constants there are two mechanisms that could invalidate the mass-suppression effect but, as we argue, these are likely to be the exceptions to the general rule. The first (and most interesting) one is the spin-induced scalarisation discussed in the previous section. The second mechanism is that of screening: many theories rely on screening mechanisms in order to survive as viable extensions of GR ‘across the spectrum’ from solar system tests and compact object binaries out to cosmological scales (for a review see $[45]$). However, most of these mechanisms (such as chameleons and symmetrons) act through the coupling of scalar fields with matter and as a result black holes can become impervious to them in vacuum models (with $S_m = 0$). Screening could also operate via the presence of non-linear interactions in the Lagrangian (as in the commonly used Vainshtein mechanism $[45, 46]$) but in that scenario it would take some fine-tuning to screen the solar-mass black holes probed by GW observations while leaving supermassive black holes unscreened.

The suppression of the non-GR coupling constants in the metric of massive systems is likely to have much wider repercussions than what discussed here. Apart from its impact on EMRIIs $[33]$ and black hole spectroscopy by LISA $[47]$, we would also expect that electromagnetic radiation (such as the observed X-ray iron lines, continuum emission, or quasi-periodic oscillations) from accretion disks in active galactic nuclei, being emitted or reflected by matter moving on geodesics, to be almost completely oblivious to deviations from GR gravity $[48, 49]$. The same should be true for any astrometric observations of bodies in orbit around SgrA*’s supermassive black hole $[50]$. We plan to explore some of these issues in the near future.
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