Mitigation Strategy of Subsynchronous Oscillation Based on Fractional-Order Sliding Mode Control for VSC-MTDC Systems With DFIG-Based Wind Farm Access

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ABSTRACT To solve the subsynchronous oscillation (SSO) caused by doubly-fed induction generator (DFIG)-based wind farm access in voltage source converter based multi-terminal direct current (VSC-MTDC) systems, the fractional-order sliding mode control (FOSMC) strategy is proposed to suppress the SSO and improve the system robustness. First, the rotor side converter (RSC) of the DFIG and the converter connected with the wind farm are linearized by the feedback linearization method. The stability of the internal system is ensured by the application of zero-dynamic theory. Then, the appropriate switching manifold surface and the exponential approach law \( v_s \) are selected for the design of FOSMC strategy. The control law of the system can be derived when the stability of the system is ensured. After that, the designed FOSMC strategy is applied to the RSC and the VSC connected with the wind farm. To verify the superior performance of the proposed control strategy, the FOSMC strategy is compared with the traditional subsynchronous damping controller (SSDC) by time-domain simulation under different operating conditions. The simulation results show that the FOSMC strategy can effectively suppress the SSO under different operating conditions, stabilize the system quickly, and enhance the robustness of the system against parameter disturbances.

INDEX TERMS Doubly-fed induction generator (DFIG)-based wind farm, voltage source converter based multi-terminal direct current (VSC-MTDC), feedback linearization, fractional-order sliding mode control (FOSMC), subsynchronous oscillation (SSO).

I. INTRODUCTION

The current research results show that the voltage source converter based high voltage direct current (VSC-HVDC) has unique advantages in renewable energy interconnection and will become one of the schemes for wind power integration [1], [2]. However, the wind farm connected to the single terminal will be out of operation when the VSC at one terminal of the VSC-HVDC fails out. VSC based multi-terminal direct current (VSC-MTDC) systems can not only solve the problem, but also has the advantages of multi-terminal system, such as economy, flexibility, and reliability. Therefore, VSC-MTDC has more technical advantages than VSC-HVDC in operation flexibility, reliability and other aspects, and can better ensure the reliable output of a wind farm [3]. However, the interaction mechanism between the wind turbine and control device of VSC-MTDC systems is more complicated with the large-scale wind power access, which may cause subsynchronous oscillation (SSO) and bring certain influence on the safe and stable operation of power grid. Therefore, the suppression of SSO needs to be solved urgently [4], [5].

To solve the above problems, a lot of research have done on it. In wind farms integrated to the grid through VSC-HVDC, references [5], [6] designed a subsynchronous damping controllers (SSDC) at the SSO of the wind turbine and VSC for
caused by the connection of a wind farm and VSC-HVDC system. In [7], [8], the SSO caused by the doubly-fed induction generator (DFIG)-based wind farm integrated into grid through MMC-HVDC system was studied. The control method of attaching SSO current was proposed in [7], and the control methods of increasing the virtual resistance in the MMC bridge arm, compensating the resonance voltage, and suppressing the harmonic circulation in the VSC connected with the wind farm were studied in [8]. In [9], the multi-frequency oscillation caused by a direct-driven wind farm interfaced with VSC-HVDC was studied and the additional SSDC was designed.

In wind farms connected to the grid via series compensation, references [10], [11] designed the damping controllers on grid-side converter (GSC) and rotor-side converter (RSC) to mitigate the unstable subsynchronous control interaction (SSCI). In [12], a supplementary damping control scheme for static synchronous compensator (STATCOM) was proposed to stabilize the oscillation caused by SSCI phenomenon in a series-compensated DFIG-based wind farm. In [13], a robust subsynchronous damping control (SSDC) was proposed in [16] based on SMC. In summary, the approximate linearization mathematical model at one operating point was adopted in [7], [9], which ignored the strong inherent nonlinearity of the wind power system. Therefore, the desired stability and dynamic quality in approximately linearized system have no practical effect for the actual running state of the system [17]. References [10]–[16] studied the oscillation caused via the wind farm through a series-complement grid was studied. A subsynchronous resonance suppression strategy based on partial feedback linearization (PFL) was proposed in [14], a sliding mode control (SMC) was proposed in [15]. Furtherly, a fractional order sliding mode control (FSMC) was proposed in [16] based on SMC. In summary, the approximate linearization mathematical model at one operating point was adopted in [7]–[9], which ignored the strong inherent nonlinearity of the wind power system. Therefore, the desired stability and dynamic quality in approximately linearized system have no practical effect for the actual running state of the system [17]. References [10]–[16] studied the oscillation caused via the wind farm through a series-complement grid. However, the causes of oscillation are different from VSC-MTDC system with DFIG-based wind farm access. Although the adaptive techniques can be used to compensate for nonlinear system with slow dynamic process and have better control performance, it is difficult to achieve the ideal effect for the wind power system with fast dynamic process [15].

Nonlinear control theory indicates that the nonlinearity of the system can be seriously affected by coordinate transformation and nonlinear state feedback, which can ensure the stability and good dynamic performance of the system. However, feedback linearization control is extremely sensitive to the uncertainty of the system parameters and has great limitations in practical application. SMC has strong robustness to external interference and system parameter perturbation [18], which is suitable for DFIG-based wind power generation system. However, as the actual sliding mode of SMC does not occur accurately on the set switched manifold surface, chattering of the system is easy to be caused [19]. Fractional calculus operator has memory ability and genetic attenuation characteristics, which is helpful for the controller to release energy slowly in the high-frequency switching process. Therefore, fractional calculus theory is introduced into SMC to weaken chattering and improve the steady and dynamic performance of the system [16]. Besides, the oscillation convergence rate can be improved by using the added degree of freedom of calculus operator.

It can be concluded from [15], [16] that the SMC on the RSC and the FOSMC on the GSC were proposed. Both adopt the method of PFL. Based on the above points, the FOSMC strategy is designed in this paper to suppress SSO caused by DFIG-based wind farm interfaced with VSC-MTDC systems. Compared with the control method proposed in [15], [16], the contribution of this paper can be summarized as below:

First, the method of state feedback linearization is used to eliminate the inherent nonlinearity of the system. The PFL method is adopted on DFIG and the exact feedback linearization (EFL) method is adopted on VSC connected with the wind farm, which makes the system have desired stability and dynamic quality. Then, the appropriate switching manifold surface and the exponential approach law $v_s$ are selected to design FOSMC strategies on the RSC and the VSC connected with the wind farm, respectively. The effectiveness of the control method is verified via comparing the control method proposed in this paper with the traditional SSDC from the following two aspects. FOSMC has faster converge speed compared with traditional SSDC under the operation condition of variable wind speed and different numbers of wind turbines. Besides, the control performance of FOSMC strategy is almost unaffected by the change of parameters, and can improve the robustness of the system when the parameters of the system are disturbed.

The rest of this paper is organized as follows. In Section II, the mathematical model of the system is presented. In Section III, the feedback linearization of DFIG-based wind farm grid connection system is shown. In Section IV, the FOSMC suppression strategy is designed for RSC and VSC. Besides, the SSDC is designed to comprise with the FOSMC in the paper. In Section V, the simulations are carried out to demonstrate the effectiveness of the proposed method. The conclusion is shown in Section VI.

## II. STRUCTURE AND MATHEMATICAL MODEL OF THE SYSTEM

### A. STRUCTURE OF THE SYSTEM

The structure of the system studied in this paper is shown in Fig. 1. The wind farm is composed of DFIGs, and the unit adopts the form of “one machine and one transformer”. A single wind turbine is connected to a 35kV collector line via a generator-side transformer, and then connected to the VSC-MTDC system via 35/330/750kV transformers.
The paper ignores the interaction between wind turbines. Therefore, the wind farm can be equivalent to a single wind turbine.

In Fig. 1, VSC1 provides stable AC power for the point of common coupling (PCC) and controls the amplitude and frequency of AC voltage. VSC3 uses constant DC voltage control and provides voltage support for the DC system. VSC2 and VSC4 use active power control.

B. MATHEMATICAL MODEL OF THE SYSTEM

1) MATHEMATICAL MODEL OF DFIG

The mathematical model of a DFIG includes induction generator, RSC, DC bus link, GSC and shafting, which can be described as follows [20]:

\[
\begin{align*}
\frac{d}{dt} i_{sd} &= \omega_1 i_{sq} + \frac{R_s}{L_s} i_{sd} + \frac{u_{sd}}{L_s} \\
\frac{d}{dt} i_{sq} &= -\omega_1 i_{sd} + \frac{R_s}{L_s} i_{sq} + \frac{u_{sq}}{L_s}
\end{align*}
\]

(1)

\[
\begin{align*}
\frac{d}{dt} i_{id} &= \omega_1 i_{iq} - \frac{R_f}{L_f} i_{id} - \frac{u_{aid}}{L_f} + \frac{u_{ac}}{L_f} S_{sd} \\
\frac{d}{dt} i_{iq} &= -\omega_1 i_{id} - \frac{R_f}{L_f} i_{iq} - \frac{u_{aq}}{L_f} + \frac{u_{ac}}{L_f} S_{sq}
\end{align*}
\]

(2)

\[
\begin{align*}
\frac{d}{dt} u_{dc} &= -\frac{1}{C_{dc}} i_{id} + \frac{1}{C_{dc}} i_{iq}
\end{align*}
\]

(3)

\[
\begin{align*}
\frac{d}{dt} i_{gd} &= -\omega_1 i_{gg} - \frac{R_g}{L_g} i_{gd} - \frac{u_{ag}}{L_g} + \frac{u_{ac}}{L_g} \\
\frac{d}{dt} i_{gg} &= \omega_1 i_{gd} - \frac{R_g}{L_g} i_{gg} - \frac{u_{ag}}{L_g} + \frac{u_{ac}}{L_g}
\end{align*}
\]

(4)

\[
\begin{align*}
2H_f \frac{d\theta_s}{dt} &= T_m - K_s \theta_s - B_s \frac{d\theta_s}{dt} \\
2H_g \frac{d\theta_r}{dt} &= -T_e + K_s \theta_s + B_s \frac{d\theta_r}{dt}
\end{align*}
\]

(5)

where subscripts \(d\) and \(q\) represent the \(d\)-axis and the \(q\)-axis components, subscripts \(s\) and \(r\) represent the stator and the rotor, \(S_{sd}\) and \(S_{sq}\) are the input signals of RSC, \(u_{dc}\) is the DC bus voltage, \(i_{id}\) is the current of the GSC, \(R_g\) and \(R_f\) are filter resistances of the GSC and the RSC, \(L_g\) and \(L_f\) are filter inductances of the GSC and the RSC, \(\omega_m\) is the rotor mechanical angular velocity, \(\omega_r\) is the rotor speed, \(\omega_1\) is the synchronous angular velocity of the grid, \(T_m\) is the mechanical torque of the wind turbine, \(K_s\) is the stiffness coefficient of the transmission shaft, \(\theta_s\) is the torsional vibration angle of the shafting, \(B_s\) is the torsional vibration damping coefficient of the transmission shaft, \(\omega_1\) is the inertia constant of the mass of the generator, \(T_e\) is the electromagnetic torque of the induction motor, \(\theta_t\) is the rotation angle of the rotor.

2) MATHEMATICAL MODEL OF VSC1

The mathematical model of VSC1 in \(dq\) synchronous rotation coordinate system can be described as follows [21]

\[
\begin{align*}
\frac{d}{dt} i_{d1} &= -\frac{R_1}{L_1} i_{d1} - \frac{\omega_1}{L_1} i_{q1} - \frac{u_{ac1} - u_{ad1}}{L_1} \\
\frac{d}{dt} i_{q1} &= -\frac{R_1}{L_1} i_{q1} + \frac{\omega_1}{L_1} i_{d1} - \frac{u_{aq1} - u_{ad1}}{L_1}
\end{align*}
\]

(6)

where \(R_1\) and \(L_1\) are the equivalent resistance and inductance of the transmission line and the transformer on the rectifier side, \(i_{d1}\) and \(i_{q1}\) are current components of \(d\)-axis and \(q\)-axis, \(u_{ac1}\) and \(u_{aq1}\) represent voltage components at PCC point of \(d\)-axis and \(q\)-axis.

III. FEEDBACK LINEARIZATION OF DFIG-BASED WIND FARM GRID CONNECTION SYSTEM

Feedback linearization is a method to linearize nonlinear systems by constructing appropriate coordinate transformation and state feedback. It does not ignore any higher-order nonlinear term in the linearization process. Therefore, the linearization is applicable to the whole region with defined changes [22]–[24].

A. FEEDBACK LINEARIZATION OF DFIG

1) PARTIAL FEEDBACK LINEARIZATION OF DFIG

According to the feedback linearization theory, the nonlinear function of a DFIG can be described as follows

\[
\begin{align*}
\frac{d}{dt} x &= F(x) + G(x)u \\
y &= H(x)
\end{align*}
\]

(7)

where \(x = [i_{dq}, i_{dq}, u_{dc}, i_{dq}, i_{dq}, i_{dq}, \omega_m, \omega_r, \theta_s, \theta_r, u_{dc}]^T\), \(u = [u_1, u_2]^T\), \(u_1\) and \(u_2\) are the control variables, \(u_{dc}\) is the voltage of the DC bus which connected with VSC1, \([y_1, y_2]^T = h(x) = [h_1(x), h_2(x)]^T = [i_{dq}, i_{dq}]^T\). The expressions of \(F(x)\) and \(G(x)\) are shown in the appendix.

The relation degree of the system can be obtained by calculating the Lie derivative

\[
\begin{align*}
\begin{bmatrix} L_{g1} L_{j1}^{-1} h_1(x) = L_{g1} h_1(x) & u_{dc} \cr L_{g2} L_{j1}^{-1} h_2(x) = L_{g2} h_2(x) & 0 \end{bmatrix} & = 0 \\
\begin{bmatrix} L_{g1} L_{j1}^{-1} h_2(x) = L_{g1} h_2(x) & 0 \cr L_{g2} L_{j1}^{-1} h_2(x) = L_{g2} h_2(x) & u_{dc} \end{bmatrix}
\end{align*}
\]

\[ (8) \]

\[ (9) \]

It can be obtained from (8) and (9) that the relative order \(r_1 = 1\) and \(r_2 = 1\). The total relative order of which is less
than the total order of the DFIG. Therefore, the PFL method is used to design the controller of RSC. According to the zero dynamic design method, the system dynamics can be divided into external dynamics and internal dynamics [25]. Therefore, the nonlinear system can be transformed into the following two subsystems.

\[
\begin{align*}
\frac{d}{dt} \tilde{z} &= \tilde{A}\tilde{z} + \tilde{B}\tilde{v} \\
y &= C\tilde{z}
\end{align*}
\]  
\tag{10}

where (10) is the linear subsystem obtained by nonlinear coordinate transformation, \( \tilde{z}, \tilde{v}, \tilde{A} \) and \( \tilde{B} \) are the state variables, control variables, system matrix and input matrix of the linear subsystems, respectively; (11) denotes the internal dynamics of the system, \( \tilde{z} \) and \( \tilde{A} \) are the state variables and the system matrix of the system, respectively.

2) COORDINATE TRANSFORMATION

The core of feedback linearization is to transform a complex nonlinear system into a linear system by using appropriate coordinate transformation [22]–[24]. The system state variable after PFL is \( z = \phi(x) = [\tilde{z} \tilde{v}]^T \). Therefore, the coordinate transformation of a DFIG is as follows

\[
\begin{align*}
\tilde{z}_1 &= \phi_1(x) = h_1(x) = i_{d1} \\
\tilde{z}_2 &= \phi_2(x) = h_2(x) = i_{q1}
\end{align*}
\]  
\tag{12}

Take the derivative of (12)

\[
\begin{align*}
\frac{d}{dt} \tilde{z}_1 &= \omega_1 i_{q} - \frac{R_f}{L_{eq}} i_{q} + \frac{u_{eq}}{L_{eq}} u_1 \\
\frac{d}{dt} \tilde{z}_2 &= -\omega_1 i_{d} - \frac{R_f}{L_{eq}} i_{d} + \frac{u_{eq}}{L_{eq}} u_2
\end{align*}
\]  
\tag{13}

Let \( \frac{d}{dt} \tilde{z}_i = \tilde{v}_i \), the partial linearization model of a DFIG can be obtained

\[
\begin{align*}
\tilde{v}_1 &= \omega_1 i_{q} - \frac{R_f}{L_{eq}} i_{q} + \frac{u_{eq}}{L_{eq}} u_1 \\
\tilde{v}_2 &= -\omega_1 i_{d} - \frac{R_f}{L_{eq}} i_{d} + \frac{u_{eq}}{L_{eq}} u_2
\end{align*}
\]  
\tag{14}

The proof of zero dynamic stability of the system is shown in [14]. Therefore, the state feedback law can be derived according to (14)

\[
\begin{align*}
u_1 = & \frac{L_{eq}}{u_{eq}} (\tilde{v}_1 - \omega_1 i_{q} + \frac{R_f}{L_{eq}} i_{q} + \frac{u_{eq}}{L_{eq}} u_1) \\
u_2 = & \frac{L_{eq}}{u_{eq}} (\tilde{v}_2 + \omega_1 i_{d} + \frac{R_f}{L_{eq}} i_{d} + \frac{u_{eq}}{L_{eq}} u_2)
\end{align*}
\]  
\tag{15}

B. FEEDBACK LINEARIZATION OF VSC1

According to the dynamic model of VSC1, the nonlinear model of VSC1 can be obtained

\[
\begin{align*}
\dot{x} &= \tilde{F}(\tilde{x}) + \tilde{G}(\tilde{x})\tilde{u} \\
y &= \tilde{H}(\tilde{x})
\end{align*}
\]  
\tag{16}

where \( \tilde{x} = [i_{d1}, i_{q1}]^T \), \( \tilde{u} = [\tilde{u}_1, \tilde{u}_2]^T = [u_{eq1} - u_{eq1}, u_{eq1} - u_{eq1}, \tilde{y}_1, \tilde{y}_2]^T = [\tilde{h}_1(x), \tilde{h}_2(x)]^T = [i_{d1}, i_{q1}] \), \( \tilde{F}(x) = \left[ \tilde{F}_1(x) \tilde{F}_2(x) \right]^T = \left[ -\frac{R_f}{L_f} i_{d1} - \omega_1 i_{q1} - \omega_1 \frac{R_f}{L_1} i_{q1} + \frac{u_{eq1}}{L_1} \right]^T \), \( \tilde{G}(x) = \left[ \tilde{G}_1(x) \tilde{G}_2(x) \right]^T = \left[ -u_{eq1} + \frac{u_{eq1}}{L_1}, 0, 0 - u_{eq1} + \frac{u_{eq1}}{L_1} \right]^T \).

The relation degree of the system can be obtained by calculating the Lie derivative

\[
\begin{align*}
L_{x1} L_1^{-1} \tilde{h}_1(x) &= L_{x1} \tilde{h}_1(x) = -\frac{u_{eq1}}{L_1} + \frac{u_{eq1}}{L_1} \\
L_{x2} L_1^{-1} \tilde{h}_2(x) &= L_{x2} \tilde{h}_2(x) = 0
\end{align*}
\]  
\tag{17}

\[
\begin{align*}
L_{x1} L_1^{-1} \tilde{h}_2(x) &= L_{x1} \tilde{h}_2(x) = 0 \\
L_{x2} L_1^{-1} \tilde{h}_2(x) &= L_{x2} \tilde{h}_2(x) = -\frac{u_{eq1}}{L_1} + \frac{u_{eq1}}{L_1}
\end{align*}
\]  
\tag{18}

According to (17) and (18), the relative order \( r_3 = 1 \) and \( r_4 = 1 \). The total relative order is equal to the total order of the VSC1, so the given output function can be linearized via the EFL method.

Therefore, the coordinate transformation of VSC1 is as follows

\[
\begin{align*}
z_1 &= \phi_1(x) = \tilde{h}_1(x) = i_{d1} \\
z_2 &= \phi_2(x) = \tilde{h}_2(x) = i_{q1}
\end{align*}
\]  
\tag{19}

Take the derivative of (19)

\[
\begin{align*}
\frac{d}{dt} z_1 &= -\frac{R_1}{L_1} i_{d1} - \frac{\omega_1}{L_1} i_{q1} + \frac{1}{L_1} \tilde{u}_1 \\
\frac{d}{dt} z_2 &= -\frac{R_1}{L_1} i_{q1} - \frac{\omega_1}{L_1} i_{d1} + \frac{1}{L_1} \tilde{u}_2
\end{align*}
\]  
\tag{20}

Let \( \frac{d}{dt} z_i = v_i \), the EFL model of VSC1 can be obtained

\[
\begin{align*}
v_1 &= -\frac{R_1}{L_1} i_{d1} - \frac{\omega_1}{L_1} i_{q1} + \frac{1}{L_1} \tilde{u}_1 \\
v_2 &= -\frac{R_1}{L_1} i_{q1} - \frac{\omega_1}{L_1} i_{d1} + \frac{1}{L_1} \tilde{u}_2
\end{align*}
\]  
\tag{21}

IV. DESIGN OF FOSMC SUPPRESSION STRATEGY

FOSMC strategies are designed for the RSC and the VSC1 when the stability of the internal dynamics is ensured.

A. FRACTIONAL CALCULUS THEORY

In this paper, Riemann-Liouville (RL) type calculus definition is adopted [26], and its integral and differential expressions are as follows

\[
\begin{align*}
\alpha D_t^{-\lambda} f(t) &= \frac{1}{\Gamma(\lambda)} \int_a^t (t - \xi)^{\lambda - 1} f(\xi) d\xi \\
\alpha D_t^{-\lambda} f(t) &= \frac{d^n}{d\xi^n} \int_a^t (t - \xi)^{\alpha - n} f(\xi) d\xi
\end{align*}
\]  
\tag{22}

where \( \alpha D_t^{-\lambda} \) is the calculus operator, \( a \) and \( t \) are upper and lower limits of \( \alpha D_t^{-\lambda} \), \( \lambda > 0 \) is its order, \( \sigma = n - \lambda \), \( n \) is the smallest integer, it is greater than \( \lambda \), \( \Lambda(\lambda) \) is the Gamma function, and its expression is \( \Gamma(\lambda) = \int_0^\infty e^{-t} t^{\lambda - 1} dt \).
**Theorem 1:** The Laplace transform of fractional calculus of type RL is
\[
L_0\left(D^{-\alpha}_{t}f(t)\right) = s^\alpha F(s) - \sum_{j=0}^{n-1} (D^{-\alpha}_{t}f(t))_{t=0} s^j \quad (24)
\]
\[
L_0\left(D^{-\lambda}_{t}f(t)\right) = s^\lambda F(s) \quad (25)
\]

**Theorem 2:** Let $\alpha > 0$ and $\beta > 0$, the Laplace transform of mittag-leffler function $E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \left(\frac{x^k}{\Gamma(\alpha k + \beta)}\right)$ can be described as
\[
\int_{0}^{\infty} e^{-st} t^{\alpha k+\beta-1} E_{\alpha,\beta}(\pm at^\alpha) dt = \frac{k t^{\alpha-\beta}}{(\alpha^\alpha - \alpha)^{k+1}} \quad (26)
\]

**B. DESIGN OF FOSMC STRATEGY FOR RSC**

Select the switching manifold surface
\[
S(t) = \begin{bmatrix} S_1(t) \\ S_2(t) \end{bmatrix} = \begin{bmatrix} e_1(t) + k_{11}\cdot D^k_{t} e_1(t) \\ e_2(t) + k_{21}\cdot D^k_{t} e_2(t) \end{bmatrix} \quad (27)
\]
where $e_1 = i_{d_r} - i_{d_r, ref}$, $e_2 = i_{q_r} - i_{q_r, ref}$, $\lambda_1$ and $\lambda_2$ are calculus operator orders, $k_{11}$ and $k_{21}$ are switching gains. All three variables $(e_1, e_2, k_{11}, k_{21})$ are normal numbers.

The exponential approach law $\dot{v}_s$ is selected for design
\[
\begin{align*}
\dot{v}_1 &= -k_{11}\cdot D^k_{t} e_1(t) - k_{12} S_1(t) - k_{13} sgn S_1(t) \\
\dot{v}_2 &= -k_{21}\cdot D^k_{t} e_2(t) - k_{22} S_2(t) - k_{23} sgn S_2(t)
\end{align*}
\]

where $k_{12}, k_{13}, k_{22}, k_{23}$ are normal numbers, sgn is a sign function.

Take the Lyapunov function as $V = \frac{1}{2} S^T S$, then
\[
\dot{V} = S^T \dot{S} = S_1(\dot{e}_1 + k_{11}\cdot D^k_{t} e_1(t)) + S_2(\dot{e}_2 + k_{21}\cdot D^k_{t} e_2(t)) = S_1(-k_{11}\cdot D^k_{t} e_1(t) - k_{12} S_1 - k_{13} sgn S_1) + S_2(-k_{21}\cdot D^k_{t} e_2(t) - k_{22} S_2 - k_{23} sgn S_2)
\]

It can be obtained from (29) that the system is asymptotically stable. Substituting (28) into (15), the system control law can be obtained
\[
\begin{cases}
u_1 = L_{dc} (e_1 - k_{11}\cdot D^k_{t} e_1(t) - k_{12} S_1(t) - k_{13} sgn S_1(t)) \\
-\omega_0 i_{q} + \frac{R_{ref}}{L_{ref}} i_{dc} + \frac{u_{id}}{L_{ref}} \\
u_2 = L_{dc} (e_2 - k_{21}\cdot D^k_{t} e_2(t) - k_{22} S_2(t) - k_{23} sgn S_2(t)) \\
+\omega_0 i_{d} + \frac{R_{ref}}{L_{ref}} i_{q} + \frac{u_{iq}}{L_{ref}}
\end{cases}
\]

The control principle of the RSC is shown in Fig. 2. First, the feedback linearization is used to eliminate the strong non-linearity inherent in the system. Then, the FOSMC strategy is designed and the expression of the input signal is obtained when the stability of the internal dynamics is guaranteed. The control strategy designed in the paper is applied to the RSC to realize rapid suppression of SSO.

**C. DESIGN OF FOSMC STRATEGY FOR VSC1**

The design method of the FOSMC strategy in VSC1 is similar to that of RSC. Therefore, it will not be repeated here. Select the exponential approach law $v_s$ for design
\[
\begin{align*}
\dot{v}_1 &= -\tilde{k}_{11}\cdot D^k_{t} e_1(t) - \tilde{k}_{12} S_1(t) - \tilde{k}_{13} sgn S_1(t) \\
\dot{v}_2 &= -\tilde{k}_{21}\cdot D^k_{t} e_2(t) - \tilde{k}_{22} S_2(t) - \tilde{k}_{23} sgn S_2(t)
\end{align*}
\]

where $\tilde{k}_{11}$ and $\tilde{k}_{21}$ are switching gains, $\tilde{k}_{12}, \tilde{k}_{13}, \tilde{k}_{22},$ and $\tilde{k}_{23}$ are all normal numbers.

It is verified that the system is asymptotically stable. The process of proof is the same as the RSC stability verification process, which will not be repeated here.

Therefore, the control law of VSC1 can be obtained in (32), as shown at the bottom of the next page. The control block diagram of VSC1 is shown in Fig. 3.

First, the EFL of VSC1 is carried out. Then, the FOSMC strategy is designed. The control variables $S_{d1}$ and $S_{q1}$ can be calculated. After coordinate transformation, pulse width modulation (PWM) is triggered to generate an impulse to control the VSC1.
D. DESIGN OF SSDC

To verify the effectiveness of the control strategy proposed in the paper, the traditional SSDC, which is composed of filter, proportion link, phase compensation link, and amplitude gain segment, is employed here. The control block diagram of SSDC is shown in Fig. 4 [28].

where $K$ is gain, $T_1$ and $T_2$ are the lead/lag time constants.

Select the generator speed deviation as the input signal, and the control signal of the oscillation frequency is generated through the band-pass filter and the proportional phase-shifting link, thus generating additional sub-synchronous electromagnetic torque, making the electrical damping of the oscillation frequency positive and achieving the purpose of suppressing SSO.

To minimize the impact of the DC blocking link on the phase compensation: set the limiting link is $\pm 0.1pu$, the lead/lag time constants $T_1$ and $T_2$ of the phase compensation link can be obtained via (29).

\[
\left\{ \begin{array}{l}
\alpha = \frac{1 - \sin \theta_{SSDC}}{1 + \sin \theta_{SSDC}} \\
T_1 = \frac{1}{\omega \sqrt{\alpha}} \\
T_2 = \alpha T_1 
\end{array} \right. \tag{33}
\]

where, $\omega$ is the angular frequency of the subsynchronous resonance point, $\theta_{SSDC}$ is the lag angle which is needed to be compensated.

According to the eigenvalue analysis based on small signal modal, there are 11 SSO modes, whose oscillation frequency are 2.22Hz, 3.27Hz, 3.56Hz, 3.62Hz, 11.01Hz, 22.66Hz, 27.39Hz, 27.7Hz, 29.97Hz, 34.54Hz, 49.97Hz, respectively. Mode 6 is mainly influenced by the interaction between DFIG flux, DFIG rotor side inner loop controller, VSC1 controller, and PLL of VSC1. Therefore, the design of parameters is mainly considered Mode 6.

Based on the test signal method [30], the corresponding phase compensation parameters can be obtained when the active output power of DFIG-based wind farm changes. The control parameters can be obtained via (33), which are shown in Table 1.

It can be obtained from Table 1 that $T_1$ and $T_2$ are roughly linear over the range of the output power of the wind farm. Therefore, this linear relationship can be used to design the time constants $T_1$ and $T_2$. The compensation parameters of the phase compensation link can be adjusted in real time with the change of the output power of the wind farm.

### TABLE 1. Parameters of SSDC.

| TIME CONSTANTS | $P=0.65pu.$ | $P=0.75pu.$ | $P=0.85pu.$ |
|----------------|-------------|-------------|-------------|
| $T_1$          | 0.03337     | 0.03338     | 0.03340     |
| $T_2$          | 0.01622     | 0.01633     | 0.01644     |

### TABLE 2. Parameters of the system.

| PARAMETERS | VALUE |
|------------|-------|
| Output voltage of DFIG /kV | 0.69 |
| Rated wind speed/(m/s) | 12 |
| Rated DC voltage of VSC/kV | 800 |
| Rated Capacity of VSC1 /MW | 500 |
| Proportional / integral gain ($K_{p0}$/$K_{i0}$) of outer loop controller of VSC | 0.8/10 |
| Proportional / integral gain ($K_{p1}$/$K_{i1}$) of inner loop controller of VSC1 | 0.5/10 |
| Proportional / integral gain ($K_{PLL \_ VSC}$/$K_{PLL \_ VSC}$) of PLL of VSC1 | 2/200 |

V. SIMULATION

According to the system structure described in Section II, the set capacity of the wind farm is 500MW, the wind speed and the rotate speed are 8.5m/s and 0.8pu, the rated rotor speed is 1.2pu. Other parameters are shown in Table 2.

A. IMPACT OF WIND SPEED ON SSO

Keep the wind farm capacity unchanged. The suppression effect of the FOSMC under different wind speeds is analyzed.

Fig. 5 shows that the output power of the wind farm will be critical oscillation when the wind speed is 8.5m/s; it will gradually diverge when the wind speed is 7.5m/s and gradually converge when the wind speed is 9.5m/s. SSO can be suppressed by the SSDC and the FOSMC. However, compared with the SSDC, the convergence rate of the FOSMC is 0.4 seconds faster when the wind speed is 7.5m/s, and it is 0.6 seconds and 0.7 seconds faster when the wind speed is 8.5m/s and 9.5m/s. Therefore, the FOSMC strategy can make the system restore stability in the shortest time.

B. IMPACT OF CHANGES IN THE NUMBER OF WIND TURBINES IN OPERATION ON SSO

To further verify the suppression effect of the FOSMC strategy on SSO, the following simulations are performed when the number of DFIGs in operation changes. Set the wind speed to be 7.5m/s.
Fig. 6 shows that with the increase of the number of DFIGs in operation, the corresponding relationship between the oscillation trend and the wind speed remains unchanged, but the stability of the system becomes worse. SSDC and FOSMC can restore the stability of the system. The convergence rate of FOSMC strategy is 0.7 seconds faster than SSDC when there are 200 DFIGs in operation, and it is 0.3 seconds faster when there are 220 DFIGs in operation. Therefore, FOSMC has a better suppression effect.

It can be concluded from Fig. 6 that when the wind speed and the number of DFIGs in operation change, FOSMC can effectively suppress SSO and accelerate the system to restore stability.

C. IMPACT OF PARAMETER PERTURBATION

In the actual operation of a DFIG-based wind farm, the intrinsic parameters of a DFIG will perturb under the influence of current change, system heating, etc., which will have a great impact on the control performance of the system [12].

To investigate the system’s robust performance of parameter perturbation under FOSMC, the wind farm capacity is set to 500MW, and the wind speed is changed. When the resistance of the converter filter on the rotor side is increased by 20% at 2.0 seconds, the simulation results are shown in Fig. 7.
It can be seen from Fig. 5 and Fig.7 that parameter perturbation has a greater impact on SSO. Although SSDC can suppress the system resonance, the amplitude of oscillation and the convergence time of SSO significantly increase. However, the parameter perturbation has little effect on the performance of FOSMC. When the parameter of the system changes, it can still maintain a fast convergence speed. The system will return to stability in the shortest time under different wind speeds.

VI. CONCLUSION
To solve the SSO problem caused by the DFIG-based wind farm access in VSC-MTDC systems, this paper adopts the FOSMC strategy based on feedback linearization in the RSC and the VSC1 to suppress the SSO. The proposed control method is simulated and compared with the traditional SSDC. The following conclusions can be obtained: (1) When the wind speed and the number of DFIGs in operation change, both the SSDC and the FOSMC can suppress SSO. (2) Through the comparative analysis of FOSMC and SSDC, it can be seen that the FOSMC strategy has a faster convergence speed under a variety of operating conditions. It can make the system stable in the shorter time and reduce the risk of SSO. (3) The performance of the FOSMC hardly changes compared with SSDC when the system parameters change. Therefore, the robustness of the system against parameter perturbation is improved and the stability of the system is enhanced.

APPENDIX

\[
F(x) = \begin{bmatrix}
\omega_1 i_{dq} - \frac{R_f}{L_f} i_{dq} - \frac{u_{td}}{L_f} \\
-\omega_1 i_{dq} + \frac{1}{C_{dc}} i_{dc} \\
\omega_1 s_{dq} + \frac{R_s}{L_s} s_{dq} + \frac{u_{sd}}{L_s} \\
-\omega_1 s_{dq} + \frac{R_s}{L_s} s_{dq} + \frac{u_{sd}}{L_s} \\
\frac{1}{2H_f} [T_m - K_s \dot{\theta}_s - B_s (\omega_t - \omega_f)] \\
\frac{1}{2H_g} [-T_e + K_s \dot{\theta}_s + B_s (\omega_t - \omega_f)] \\
\omega_t - \omega_f \\
\frac{3(u_{dc} i_{sd1} + u_{c1} i_{sq1})}{2CU_{d1}} - I_{d1}
\end{bmatrix}
\]

\[
G(x) = \begin{bmatrix}
u_{dc} \\
0 \\
0 \\
\frac{u_{dc}}{L_{tf}} \\
0 \\
0 \\
\frac{1}{C_{dc}} i_{sd1} \\
\frac{1}{C_{dc}} i_{sq1} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

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