Lepton masses in a supersymmetric 3-3-1 model

J. C. Montero*, V. Pleitez† and M. C. Rodriguez‡

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900– São Paulo, SP
Brazil

(March 25, 2022)

We consider the mass generation for both charginos and neutralinos in a 3-3-1 supersymmetric model. We show that $R$-parity breaking interactions leave the electron and one of the neutrinos massless at the tree level. However the same interactions induce masses for these particles at the 1-loop level. Unlike the similar situation in the minimal supersymmetric standard model the masses of the neutralinos are related to the masses of the charginos.

PACS number(s): 12.60.Jv, 14.60.-z, 14.60.Pq.

I. INTRODUCTION

The generation of neutrino masses is an important issue in any realistic extension of the standard model. In general, the values of these masses (of the order of, or less than, 1 eV) that are needed to explain all neutrino oscillation data are not enough to put strong constraints on model building. It means that several models can induce neutrino masses and mixing compatible with experimental data. So, instead of proposing models built just to explain the neutrino properties, it is more useful to consider what are the neutrino masses that are predicted in any particular model which has motivation other than the explanation of neutrino physics. For instance, the 3-3-1 model was proposed as a possible symmetry on the lightest lepton sector ($\nu_e, e^-, e^+$) \cite{4}. Once assumed that symmetry it has to be implemented in the rest of the leptons and also in the quark sector. Like in the standard model if we do not introduce right-handed neutrinos and/or violation of the total lepton number neutrinos remain massless at any order in perturbation theory. In this vain it has been done some effort to produce neutrino masses in the context of that 3-3-1 model and some of its extensions \cite{3}.

In this work we consider the generation of neutrino masses in a supersymmetric 3-3-1 model with broken $R$ parity. We show that, as an effect of the mixing among all leptons of the same charge, at the tree level only one charged lepton and one neutrino remain massless but they gain mass through radiative corrections. In order to compare this model we do the same calculations in the context of the minimal supersymmetric standard model with $R$ broken parity also. In both cases we are not assuming that sneutrinos gain non-vanishing vacuum expectation values (VEVs) i.e., the only non-zero VEVs are those of the scalars of the non-supersymmetric models.

The outline of this work is as follows. In Sec. II we review the origin of the lepton masses in the minimal supersymmetric standard model context under the same assumptions that we will use in the case of the 3-3-1 supersymmetric model. In Sec. III we consider the supersymmetric version of a 3-3-1 model which has only three triplets of Higgs scalars. We explicitly show that leptons gain mass only as a consequence of their mixing with gauginos and higgsinos. Our conclusions are found in the last section.

II. NEUTRINO MASSES IN THE MSSM

Let us consider in this section the lepton masses in the minimal supersymmetric standard model (MSSM) \cite{5}. In this model the interactions are written in terms of the left-handed (right-handed) $\bar{L} \sim (2, -1)$ ($l^c \sim (1, 2)$) leptons, left-handed (right-handed) quarks $\bar{Q} \sim (2, 1/3)$ ($u^c \sim (1, -4/3), d^c \sim (1, 2/3)$); and the Higgs doublets $\tilde{H}_1 \sim (2, -1), \tilde{H}_2 \sim (2, 1)$. With those multiplets

*E-mail address: montero@ift.unesp.br
†E-mail address: vicente@ift.unesp.br
‡E-mail address: mcr@ift.unesp.br
the superpotential that conserves $R$-parity is given by $W_{2RC} + W_{3RC} + W_{2RC} + W_{3RC}$, where

$$W_{2RC} = \mu \epsilon \hat{H}_1 \hat{H}_2,$$

$$W_{3RC} = \epsilon L_a f^a_{\mu \nu} \hat{H}_1 \hat{H}_2 + \epsilon Q_i f^i_{\mu \nu} \hat{H}_2 \hat{u}_e^c + \epsilon Q_i f^i_{\mu \nu} \hat{H}_1 \hat{d}_j^c, \quad (1)$$

while the $R$-parity violating terms are given by $W_{2RV} + W_{3RV} + W_{2RV} + W_{3RV}$, where

$$W_{2RV} = \mu_a \epsilon \hat{L}_a \hat{H}_2,$$

$$W_{3RV} = \epsilon \hat{L}_a \lambda^a_{\mu \nu} \hat{L}_b \hat{d}_c^e + \epsilon \hat{L}_a \lambda_{\mu \nu \alpha} \hat{Q}_i \hat{d}_j^c \hat{d}_k^d \hat{d}_l^e,$$  \quad (2)

and we have suppressed the coupling constants in that expression should be set to zero in order to avoid a too fast proton decay. Defining the basis $\Psi_{\text{MSSM}} = (\nu_e, \nu_\mu, \nu_\tau, -i\lambda_3, -i\lambda_B, H_1^0, H_2^0) T$, the mass term is $-(1/2)\Psi_{\text{MSSM}} \lambda_{\text{MSSM}} \epsilon \Psi_{\text{MSSM}} + H.c.$, where $\lambda_{\text{MSSM}}$ is the mass matrix

$$\lambda_{\text{MSSM}} = \left( \begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
-\mu_0 e & -\mu_0 \mu & -\mu_0 \tau & -M_Z s_\beta c_W & -M_Z c_\beta s_W & \mu \\
\end{array} \right), \quad (3)$$

where $p$ is an $SU(2)$ index and $\lambda_A, \lambda_B$ are the supersymmetric partners of the respective gauge vector bosons but we have omitted generation indices and the gluino-mass terms.

With the interactions in Eq. (1) it is possible to give mass to all charged fermions in the model (see below) but neutrinos remain massless. Hence, we must introduce $R$-parity violating term like those in Eq. (2). Some of the coupling constants in that expression should be set to zero.

$$\mathcal{L}_{\text{soft}} = \left( \frac{3}{2} m_1 \lambda^a_{\mu \nu} \lambda^a_{\mu \lambda} + m' \lambda_\beta \lambda_B + H.c. \right) - M^2 L^1 \bar{L} - M^2 e^{\dagger} \bar{e} - M^2 \hat{Q} \hat{Q} - M^2 u^{\dagger} \bar{u} - M^2 d^{\dagger} \bar{d} - M^2 H_1 \hat{H}_1 - M^2 H_2 \hat{H}_2 - \left[ A_L H_1 \tilde{L} \tilde{e} + A_U H_2 \tilde{Q} \tilde{u} + A_D H_1 \tilde{Q} \tilde{d} \right]$$

$$+ M^2 i_2 H_1 H_2 + B H_2 L + C_1 \bar{L} \tilde{L} \tilde{e} + C_2 \tilde{Q} \tilde{Q} \tilde{d} + C_3 \bar{u} \tilde{u} \tilde{d} + H.c., \quad (4)$$

with $s_\beta = \sin \beta$, $s_W = \sin \theta_W$, etc are defined as $\tan \beta = v_2/v_1$ and $\theta_W$ is the weak mixing angle. The matrix in Eq. (4) is generated only by the two usual vacuum expectation values of the two scalars and by the $R$-parity breaking terms $\mu_0$. The mass matrix is similar to that in Ref. [8] but we have included the three neutrinos and we are neither assuming that sneutrinos gain nonzero vacuum expectation values nor have introduced sterile neutrinos like in Ref. [10]. The mass matrix in Eq. (4) has two zero eigenvalues: it has determinant equal to zero and its secular equation which give the eigenvalues, $x$, has the form $x^2$ times a polynomial of five degree; thus there are two neutrinos $\nu_{1,2}$, which are massless at the tree level. Using tan $\beta = 1$ and $M_Z = 91.187$ GeV, $s_W^2 = 0.223$, $\mu_0 = \mu_\beta = 0$, $\mu_\tau = 10^{-4}$ GeV (this value is consistent with that of Ref. [8]), $\mu = 100$ GeV, $m = 250$ GeV, $m' = -200$ GeV, we obtain besides the two massless neutrinos a massive one with $m_{\nu_3} = -3 \times 10^{-3}$ eV, and four heavy neutralinos with masses 267.40, -199.99, -117.40, 100.0 GeV. These zero eigenvalues are a product of the matrix structure in Eq. (4) and there is no a symmetry to protect the neutrinos to gain mass by radiative corrections. On the other hand, if $\mu_0 = 0$, $a = e, \mu, \tau$, all neutrinos remain massless at the tree level. In this case it is the $R$-parity and total lepton number conservation that protect neutrinos of gain masses. The neutralino masses above are consistent with those of Ref. [8]: two states are massless and the other ones have masses of the order $O(M_Z)$. More realistic neutrino masses require radiative corrections [11, 12, 13]. Here we will only consider the neutrino masses generated by radiative corrections arisen from the...
interactions in the above Eqs. (1) and (2) and only two VEVs. We have in this case the interactions
\[ -\frac{\lambda_{abc}}{3} (\bar{\nu}_a R_l R^c_\ell + \bar{c}_R R^c_\ell l^+_a) - \frac{\chi_{\alpha i j}}{3} (\tilde{\nu}_{\alpha R} d_i R^c_{\ell j} + \tilde{c}_R R^c_{\ell j} d_i^c) + H.c., \]
and the 1-loop diagrams like those in Ref. [6] arise. Notice however that if we introduce a discrete symmetry (called $Z'_3$ later on), $\tilde{L}_{\ell,\mu} \to -\tilde{L}_{\ell,\mu}$, and all other fields are even under this transformation, we have that
\[ \mu_{0e} = \mu_{0\mu} = 0; \]
\[ \lambda_{abc} = 0, \ (b, c = \mu, \tau); \ \lambda_{abc} = 0, \ (b, c = e, \tau); \]
\[ \chi^i_{\alpha ij} = \chi^i_{\alpha ij} = 0, \ (i, j = 1, 2, 3); \]
and the $\nu_e, \nu_\mu$ neutrinos will remain massless at all order in perturbation theory. It is also possible to choose the symmetry such as $L_{\ell,\tau} \to -L_{\ell,\tau}$ while all other fields remain invariant. In this case we have that $\nu_e$ and $\nu_\tau$ remain massless. However, if no discrete symmetry is imposed neutrinos gain mass through 1-loop effect like in Ref. [6].

Next, let us consider the charged sector. With the interactions in Eq. (3) it is possible to give mass to all charged fermions. Denoting
\[ \phi^+_{MSSM} = (e^c, \mu^c, \tau^c, -i\lambda^c_{W, H^+})^T, \]
\[ \phi_{MSSM} = (e, \mu, \tau, -i\lambda_{W, H^-})^T, \]
where all the fermionic fields are still Weyl spinors, we can define $\Psi^\pm_{MSSM} = (\phi^+_{MSSM}, \phi^-_{MSSM})^T$, and the mass term $-(1/2)[\Psi^T \Psi_{MSSM}^+ + H.c]$ where $Y^\pm$ is the mass matrix given by:
\[ Y^\pm_{MSSM} = \begin{pmatrix} 0 & X^T_{MSSM} \\ X_{MSSM} & 0 \end{pmatrix}, \]
with
\[ X_{MSSM} = \begin{pmatrix} -f^l_{\ell e} v_1 & -f^l_{\ell \mu} v_1 & -f^l_{\ell \tau} v_1 & 0 & 0 \\ -f^l_{\mu e} v_1 & -f^l_{\mu \mu} v_1 & -f^l_{\mu \tau} v_1 & 0 & 0 \\ -f^l_{\tau e} v_1 & -f^l_{\tau \mu} v_1 & -f^l_{\tau \tau} v_1 & 0 & 0 \\ \mu_{0e} & \mu_{0\mu} & \mu_{0\tau} & m_\lambda & \sqrt{2}M_W s_\beta \\ \mu_{0e} & \mu_{0\mu} & \mu_{0\tau} & \mu_{0e} & \mu_{0\mu} \end{pmatrix}. \]

With $f^l_{\ell e} = 2.7 \cdot 10^{-4}$, $f^l_{\ell \mu} = 3.9 \cdot 10^{-3}$, $f^l_{\ell \tau} = 1.6 \cdot 10^{-2}$, $f^l_{\mu e} = f^l_{\mu \mu} = f^l_{\mu \tau} = 10^{-7}$ we obtain from Eq. (1) the masses 0.0005, 0.015, 1.777 (in GeV) for the usual leptons, and 4.3 and 81 TeV for the stopinos. We see by comparing Eq. (4) with Eq. (1) that there is no relation between the charged lepton masses and the neutralino masses. Notice also that all charged leptons gain masses at the tree level. We will not consider this model (or some of its extensions) further since it has been well studied in literature [7][11][13][14][15][16].

### III. A SUPERSYMMETRIC 3-3-1 MODEL

In the nonsupersymmetric 3-3-1 model the fermionic representation content is as follows: left-handed leptons $L = (\nu_a, l_a, v_a^c)_L \sim (1, 3, 0)$, $a = e, \mu, \tau$; left-handed quarks $Q_{aL} = (u_1, d_1, J) \sim (3, 3, 2/3)$, $Q_{aL} = (d_\alpha, u_{a\beta}) \sim (3, 3^*, -1/3)$, $\alpha = 2, 3, \beta = 1, 2$; and in the right-handed components we have $u^c_R, d^c_R, l^c_R$, $i = 1, 2, 3$, that transform as in the SM, and the exotic quarks $J^c \sim (3^*, 1, -5/3), J^c_{\beta} \sim (3^*, 1, 4/3)$. The minimal scalar representation content is formed by three scalar triplets: $\eta \sim (1, 3, 0) = (\eta^c, \eta^c_1, \eta^c_2)^T$; $\rho \sim (1, 3, +1) = (\rho^+, \rho^0, \rho^0)^T$ and $\chi \sim (1, 3, -1) = (\chi^-, \chi^-, \chi^0)^T$, and one scalar sextet $S \sim (1, 6, 0)$. We can avoid the introduction of the sextet by adding a charged lepton transforming as a singlet [13][14]. Notwithstanding, here we will omit both the sextet and the exotic lepton. A seesaw-type mechanism will be implemented by the mixing with supersymmetric partners, higgsinos or gauginos. The complete set of fields in the 3-3-1 supersymmetric model has been given in Refs. [14][16]. We will denote, like in the previous section, the respective superfields as $\tilde{L}$ and so on. We recall that in the nonsupersymmetric 3-3-1 model with only the three triplets the charged lepton masses are not yet the physical ones: $0, m, -m$.

We will show how, in the present model supersymmetry and the R-violating interactions give the correct masses to $e, \mu$ and $\tau$, even without a sextet or the charged lepton singlet. We have the higgsinos $\tilde{\eta}, \tilde{\rho}, \tilde{\chi}$ and their respective primed fields which have the same charge assignment of the triplets $\eta, \rho$ and $\chi$, for details see Ref. [16].

Due to the fact that in the supersymmetric model we have the gauginos and higgsinos, (for details on the lagrangian of the model see [14]), when the R-parity is broken we have in analogy with the MSSM, but with important differences, a mixture between the usual leptons and the gauginos and higgsinos.

One part of the superpotential is given by $W_2 + \bar{W}_2$ where
\[ W_2 = \mu_{0e} \tilde{L}_a \tilde{\eta}_1 + \mu_\rho \tilde{\rho} \tilde{\eta}_1 + \mu_\tau \tilde{\tau} \tilde{\chi}_1 \]
and
\[ \bar{W}_2 = \phi^+_{MSSM} + \phi^-_{MSSM} + H.c. \]

3
\(a = e, \mu, \tau\); and \(W_3 + \bar{W}_3\) where
\[
W_3 = \lambda_{1ab} \epsilon L_a \bar{\ell}_b \bar{L}_c + \lambda_{2ab} \epsilon L_a \bar{\ell}_b \bar{\nu}_c + \lambda_{3a} \lambda_a \bar{L}_a \tilde{\nu} \chi + f_1 \epsilon \tilde{\nu} \tilde{\phi} \chi + f'_1 \epsilon \tilde{\nu} \tilde{\phi} \tilde{\chi} + \lambda_{1ai} \epsilon \bar{Q}_a \bar{L}_a \tilde{e}_c + \\
+ \lambda_{ijk} \tilde{u}_i \tilde{d}_j \tilde{d}_k + \lambda_{ij3} \tilde{u}_i \tilde{d}_j \tilde{d}_3 + \lambda_{i3j} \tilde{u}_i \tilde{d}_3 \tilde{d}_j + \lambda_{3ij} \tilde{u}_3 \tilde{d}_i \tilde{d}_j + \\
+ \lambda_{ij3} \tilde{d}_i \tilde{d}_j \tilde{d}_3 \tilde{d}_3 + \kappa_{1i} \tilde{Q}_1 \tilde{d}_i + \kappa_{2j} \tilde{Q}_1 \tilde{d}_j + \kappa_{3j} \tilde{Q}_1 \tilde{d}_j \tilde{d}_3 + \kappa_{4a} \tilde{Q}_a \tilde{d}_i \tilde{d}_3 + \\
+ \kappa_{5ai} \tilde{Q}_a \tilde{d}_i + \kappa_{6a3} \tilde{Q}_a \tilde{d}_i \tilde{d}_3 \tilde{d}_3 ,
\]
(11)
with \(\epsilon\) the completely antisymmetric tensor of \(SU(3)\) but we have omitted the respective indices; the generation indices are as follows: \(a, b, c = e, \mu, \tau\) and \(i, j, k = 1, 2, 3\).

The gaugino masses come from the soft-terms shown in the Appendix A Eq. (4.4). The \(\mu_0, \lambda_1, \lambda_2, \lambda'\) and \(\lambda''\) terms break the \(R\)-parity defined in this model as \(R = (-1)^{3F + 2S}\) where \(F = B + L, B(L)\) is the baryon (total lepton) number; \(S\) is the spin. The \(\lambda_2\) term of the superpotential \(W_3\) implies interactions like (see Eq. (4.4) below) \(\tilde{\nu}_a L \tilde{\nu}_b L_b \tilde{\nu}_a L \tilde{\nu}_b L_b\) and we have also the interactions
\[
\mathcal{L}_\eta = \int d^4 \tilde{\eta}_j \epsilon^{2\sigma \tilde{\eta}_j},
\]
(12)
where \(\tilde{V}\) is the superfield related to the \(V^a\) gauge boson of \(SU(3)_L\). This interaction mixes higgsinos with gauginos as shown in Ref. [17].

The parameters \(\mu_0\) and \(\mu_2\) are the equivalent of the \(\mu\) parameter in the MSSM [2]. The terms proportional to \(\lambda_2\) and \(\mu_\chi\) have no equivalent in the MSSM. The \(\lambda'\) and \(\lambda''\) coupling constants are constrained by the proton decay such that [18]
\[
\lambda''_{1ij} \lambda_{12j} < 10^{-24},
\]
(13)
assuming the superpartner masses in the range of 1 TeV.

### A. Charged lepton masses

In this model there are interactions like
\[
- \frac{\lambda_{4a0}}{3} \left[ \omega \left( L_3 \tilde{H}_3^+ + \tilde{L}_3 \tilde{H}_3^+ \right) + u \left( L_3 \chi^+ + \tilde{L}_3 \chi^+ \right) \right] + \frac{1}{2} \mu_0 \left( L_3 \tilde{H}_3^+ + \tilde{L}_3 \tilde{H}_3^+ + \tilde{L}_3 \tilde{T}_2 \right) ,
\]
(14)
which imply a general mixture in both neutral and charged sectors. Let us first considered the charged lepton masses. Denoting
\[
\phi^+ = (e^c, \mu^c, \tau^c, -i \lambda_1^c W_r, -i \lambda_3^c N_r, \tilde{\eta}_1^c, \tilde{\eta}_3^c, \tilde{\eta}_3^c, \tilde{\chi}^+)^T ,
\]
\[
\phi^- = (e, \mu, \tau, -i \lambda_1 W_r, -i \lambda_3 N_r, \tilde{\eta}_1, \tilde{\eta}_3, \tilde{\eta}_3, -\tilde{\chi}^-)^T ,
\]
(15)
where all the fermionic fields are still Weyl spinors, we can also, as before, define \(\Psi^\pm = (\phi^+ \phi^-)^T\) and the mass term \(-(1/2)\left| \Psi^\pm Y^\pm \Psi^\pm + H.c. \right|\) where \(Y^\pm\) is given by:
\[
Y^\pm = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} ,
\]
(16)
with
\[
X = \begin{pmatrix}
0 & -\frac{\lambda_{420}}{3} & -\frac{\lambda_{430}}{3} & 0 & 0 & -\frac{\mu_0}{2} & 0 & -\frac{\lambda_{420}}{3} & \mu_0 \\
\frac{\lambda_{420}}{3} & \frac{\lambda_{420}}{3} & 0 & 0 & 0 & -\frac{\mu_0}{2} & 0 & 0 & \frac{\mu_0}{3} \\
\frac{\lambda_{430}}{3} & 0 & \frac{\lambda_{430}}{3} & 0 & 0 & 0 & -\frac{\mu_0}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\mu_0}{2} & -\frac{\mu_0}{2} & -\frac{\mu_0}{2} & 0 & -\frac{\mu_0}{2} & -\frac{\mu_0}{2} & 0 & -\frac{\mu_0}{2} & 0 \\
0 & 0 & 0 & -\frac{\mu_0}{2} & 0 & -\frac{\mu_0}{2} & 0 & 0 & 0 \\
-\frac{\lambda_{420}}{3} & -\frac{\lambda_{430}}{3} & -\frac{\lambda_{430}}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
,\]
(17)
where we have defined
\[
v = \frac{v_\eta}{\sqrt{2}} \quad u = \frac{v_\eta}{\sqrt{2}} \quad w = \frac{v_\eta}{\sqrt{2}} \quad v' = \frac{v_\eta}{\sqrt{2}} \quad u' = \frac{v_\eta}{\sqrt{2}} \quad w' = \frac{v_\eta}{\sqrt{2}}
\]
(18)
The chargino mass matrix \(Y^\pm\) is diagonalized using two unitary matrices, \(D\) and \(E\), defined by
\[
\tilde{\chi}^{\pm}_i = D_{ij} \Psi^{\pm}_j , \quad \tilde{\chi}^{\pm}_i = E_{ij} \Psi^{\pm}_j , \quad i, j = 1, \ldots, 9,
\]
(19)
\[
(D\text{ and } E \text{ sometimes are denoted, in non-supersymmetric theories, by } U^T_i \text{ and } U^T_i \text{, respectively}). \text{ Then we can write the diagonal mass matrix as}
\[
M_{SCM} = E^T X D^{-1}.
\]
(20)
To determine \(E\) and \(D\), we note that
\[
M^2_{SCM} = DX^T \cdot XD^{-1} = E^* X \cdot X^T (E^*)^{-1}.
\]
(21)
and define the following Dirac spinors:

\[ \Psi(\tilde{\chi}_i^\pm) = \begin{pmatrix} \chi_i^+ \chi_i^- \end{pmatrix}, \quad \Psi^c(\chi_i^-) = \begin{pmatrix} \chi_i^- \chi_i^+ \end{pmatrix}^T, \]

where \( \chi_i^+ \) is the particle and \( \chi_i^- \) is the anti-particle. We have obtained the following masses (in GeV) for the charged sector:

\[ 3186.05, 3001.12, 584.85, 282.30, 204.55, 149.41, \]

and the masses for the usual leptons (in GeV) \( m_\mu = 0.1052 \) and \( m_\tau = 1.777 \). These values have been obtained by using the following values for the dimensionless parameters

\[ \lambda_{2e\mu} = 0.001, \; \lambda_{2e\tau} = 0.001, \; \lambda_{2\mu\tau} = 0.393, \]
\[ \lambda_{3e} = 0.0001, \; \lambda_{3\mu} = 1.0, \; \lambda_{3\tau} = 1.0, \]

and for the mass dimension parameters (in GeV) we have used:

\[ f_1 = 0.254, \; f'_1 = 1.0, \]

and the trilinear interactions generate the low vertices in Fig. 1. The interactions of the leptons with the sleptons written in term of Dirac fermions (although we are using the same notation) are given by (and the respective hermitian conjugate)

\[ \mathcal{L}' = \frac{\lambda_{2ab}}{3} \left[ \bar{\eta}_2 R (l_a l_b \tilde{\nu}_a - l_a \tilde{\nu}_b) + \bar{\eta}_3 R (l_a l_b \tilde{\nu}_a - l_a \tilde{\nu}_b) \right] + \frac{\lambda_{3a}}{3} \left[ \bar{\rho}_R l_a \chi^+ + \tilde{\rho}_R (l_a L \chi^0 - l_a L \chi^-) - \bar{\chi}_R (l_a L \rho^+ - l_a L \rho^-) - \bar{\chi}_R (l_a L \rho^+ - l_a L \rho^-) \right] + \frac{\lambda_{0ai}}{2} \left[ \bar{u}_R l_a + \tilde{\bar{u}}_R (l_a L \tilde{\nu}_a + l_a L \tilde{\nu}_a) \right]. \]

(25)

The \( \lambda' \) interactions generate the low vertices in Fig. 1. On the other hand, the interactions between the squarks, sleptons and scalars, see the Appendix A, are given by the scalar potential. The soft part contributes only through the trilinear interactions

\[ V_{soft} = \epsilon_{1ab} (\tilde{t}_a \tilde{t}_b - \tilde{t}_a \tilde{t}_b) \eta^0, \]

while the \( D \)-terms have only quartic interactions

\[ V_D = \frac{g^2}{4} \sum_i (X_i^0 X_i^0 + X_i^0 X_i^0) \sum_a \left( \tilde{t}_a \tilde{t}_a + \frac{1}{2} \tilde{\nu}_a \tilde{\nu}_a \right), \]

(26)

\[ V_{4F} = \left( \frac{\lambda_{3d} \lambda_{1ab}}{3} + \frac{f_1 \lambda_{2ab}}{9} \right) (\tilde{t}_a \tilde{t}_b - \tilde{t}_a \tilde{t}_b) \rho^0 \chi^0 + \frac{4 \lambda_{2ad} \lambda_{2ab}}{9} (\tilde{t}_a \tilde{t}_b + \tilde{t}_a \tilde{t}_b) \eta^0 \eta^0 \]

(27)

and the quartic ones

\[ \mu \neq 0, \; \mu \rho = 0, \; \mu \tau = 0, \; \mu \chi = 0, \; m = 0, \]

(28)

We also use the constraint \( V^2 - V^2 = (246 \text{ GeV})^2 \) coming from \( M_W \), where, we have defined \( V^2 = v^2 + v^2 \) and \( V^2 = v^2 + v^2 \). Assuming that \( v_\eta = 20 \text{ GeV}, v'_\eta = v'_\rho = 1 \text{ GeV}, \) and \( 2 v_\chi = v_\chi = 2 \text{ TeV} \), the value of \( v_\rho \) is fixed by the constraint above.

Notice, from Eq. (23), that the electron is massless at the tree level. This is again a result of the structure of the mass matrix in Eq. (4) and there is not a symmetry that protects the electron to get a mass by loop corrections. Hence, it can gain mass trough radiative corrections like that shown in Fig. 1. The interactions of the leptons with the sleptons written in term of Dirac fermions (although we are using the same notation) are given by (and the respective hermitian conjugate)
where $X^0_i = \chi^0_i, \rho^0$ which will contribute to the upper quartic vertex in Fig. 1. Due to the interactions given in Eq. (23)–(24), we can generate the appropriate mass to the electron. The dominant contributions, assuming the mass hierarchy $m_{\text{fermion}} \ll m_{\text{scalar}}$ where fermion means a fermion different from $j_{1,2}$ and scalar means $\bar{\nu}, \bar{l}, H$ ($H$ denote the heaviest Higgs scalar) and using the values of the masses and the parameters given in Eqs. (23), (24a) and (24d) we obtain that the dominant contribution to the electron mass is, up to logarithmic corrections,

$$m_e \propto X'_{\alpha \epsilon i} N^\epsilon (e^2 + \epsilon^2) \frac{m_{\lambda_i}}{g^2 m_\rho^2},$$ (30)

and with all the indices fixed, $V_j$ denotes mixing matrix elements in the two dimension $j_{1,2}$ space, $V_b$ means the same but in the d-like squark sector. We obtain $m_e = 0.0005$ GeV if $\epsilon_0$ and $\epsilon'_0$ have the values already given above, and with $X'_{\alpha \epsilon i} N^\epsilon (e^2 + \epsilon^2) \approx 10^{-6}$, which imposes

$$\frac{m_{\lambda_i}}{m_\rho} \approx 9 \times 10^{-4},$$

or $m_\rho = 3.33 \times m_j V_j V_b$ GeV. Using $m_j \sim 250(320)$ GeV [19], we have $m_\rho \sim 526(596) V_j V_b$ GeV. With $V_j V_b \sim 0.14(0.12)$ we obtain squarks masses of the order of $m_\rho \sim 75$ GeV [20].

\[ Y^0 = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_\mu}{2} & \frac{\lambda_3}{3} w & 0 & \frac{\lambda_3}{3} u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_\mu}{2} & \frac{\lambda_3}{3} u & 0 & \frac{\lambda_3}{3} u & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{g_\mu}{\sqrt{2}} & \frac{g_\mu}{\sqrt{2}} & 0 & -\frac{\mu_\mu}{2} & \frac{\lambda_3}{3} u & 0 & \frac{\lambda_3}{3} u & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g_\mu}{\sqrt{2}} & \frac{g_\mu}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{g_\mu}{\sqrt{2}} & \frac{g_\mu}{\sqrt{2}} & 0 & 0 & \frac{\mu_\mu}{2} & \frac{\lambda_3}{3} u & 0 & \frac{\lambda_3}{3} u & 0 \\
\frac{\lambda_3}{3} w & \frac{\lambda_3}{3} w & \frac{\lambda_3}{3} w & \frac{\lambda_3}{3} w & 0 & \frac{g_\mu}{\sqrt{2}} & \frac{g_\mu}{\sqrt{2}} & 0 & 0 & 0 & \frac{f_1 u'}{3} & 0 & \frac{f_1 w'}{3} & 0 \\
\frac{\lambda_3}{3} u & \frac{\lambda_3}{3} u & \frac{\lambda_3}{3} u & \frac{\lambda_3}{3} u & 0 & 0 & \frac{g_\mu}{\sqrt{2}} & \frac{g_\mu}{\sqrt{2}} & 0 & 0 & 0 & \frac{f_1 u'}{3} & 0 & \frac{f_1 w'}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{2 g_\mu}{\sqrt{2}} & \frac{2 g_\mu}{\sqrt{2}} & 0 & 0 & 0 & \frac{f_1 u'}{3} & \frac{f_1 w'}{3} & 0 & \frac{f_1 u'}{3} & 0 & \frac{f_1 w'}{3} & 0 \\
\end{pmatrix} \] (35)

All parameters in Eq. (33), but $m_i'$, are defined in Eqs. (18), (24a) and (24d); $g$ and $g'$ denote the gauge coupling constant of $SU(3)_L$ and $U(1)_N$, respectively.

The neutralino mass matrix is diagonalized by a 12 × 12 rotation unitary matrix $N$, satisfying

$$M_{NMD} = N^* Y^0 N^{-1},$$ (36)

and the mass eigenstates are

$$\chi_i^0 = N_{ij} \Psi_j^0, j = 1, \cdots, 12.$$ (37)

\[ \chi_i^0 = \begin{pmatrix}
\chi^0_\tau \\
\chi^0_\mu \\
\chi^0_e \\
\end{pmatrix} \] (38)

As above the subindices $a, b, c$ run over the lepton generations $e, \mu, \tau$.

With the mass matrix in Eq. (35), at the tree level we obtain the eigenvalues (in GeV),

$$\frac{1}{2} \mu_{\alpha \beta} \left( \eta^0 e^0 + \bar{\eta}^0 \bar{e}^0 \right) + \frac{1}{2} \lambda_{\alpha \beta} \left( \eta^0 \eta^0 + \bar{\eta}^0 \bar{\eta}^0 \right),$$ (32)

and from Eq. (11) we have also the interactions

$$\frac{\lambda_{3a}}{3} \left[ w(\mu_u \bar{\rho}_0 + \bar{\nu}_3 \bar{\rho}_0) + u(\mu_u \bar{\chi}^0 + \bar{\nu}_3 \bar{\chi}^0) \right].$$ (33)

These interactions imply a mass term for the neutrinos and neutralinos. The mass term in the basis

$$\Psi^0 = \left( \nu_e \nu_e \nu_e - i \lambda_A^3 - i \lambda_A^3 \eta^0 \eta^0 + \bar{\rho}_0 \rho^0 \right)^T,$$ (34)

is given by $-(1/2)\left( \Psi^0 \right)^T Y^0 \Psi^0 + H.c.$ where
we obtain an electron neutrino mass of the order of 10−10, too small, in fact, the charged lepton which gives the main contribution is the heavy chargino and neutralino and that one neutrino remains massless at the tree level. We can always rotate the neutral fields in such a way that the electron neutrino is the one which remains massless; or we can also assume µ0τ = λ3e = 0, so that the electron neutrino decouples from the other neutrinos and neutralinos. In this case, diagonal and non-diagonal mass terms in Eq. (35) will be induced by loop corrections like that in Fig. 2. Thus, a 3 × 3 non-orthogonal mixing matrix will appear in Eq. (32). Here we will only consider the order of magnitude of a mass generated by this process.

The massless neutrino can get a mass from the loop correction like that in Fig. 2 as a consequence of the Majorana mass term of the neutral lepton in the triplet. This is equivalent to the mechanism of Ref. 22, but now with a triplet of leptons instead of a neutral singlet. For instance, the λ2 interactions will contribute in the left- and right vertices in Fig. 2.

IV. CONCLUSIONS

In the nonsupersymmetric 3−3−1 model with only three scalar triplets η, ρχ it is not possible to generate the observed charged lepton masses. Then, it is necessary to introduce a scalar sextet in order to get the appropriate masses. When we supersymmetrized the model and allow R-parity breaking interactions we can give to all known charged leptons and neutrinos the appropriate masses even without the introduction of a scalar sextet. Of course, in order to cancel anomalies we have to introduce another set of three triplets η′, ρ′χ′. In this case...
although the correct values for the lepton masses can still be obtained, if the new VEVs $u', v'$ and $w'$ are zero, it was shown in Ref. [11] that in order to give mass to all the quarks in the model all these VEVs have to be different from zero. Hence, we have considered that the six neutral scalar components got a nonzero VEV.

As can be seen from Eq. (24), the charged lepton masses arise from a sort of seesaw mechanism since there are small mass parameters, as in Eq. (24c), related with $R$-parity breaking interactions, and large ones as in Eq. (24d), related with the mass scale of the supersymmetry breaking, this can be better appreciated in Eqs. (31).

The same happens in the neutrino sector, see Eq. (32). In a supersymmetric version of the model in which we add the sextet $(6,0)$, there is a fermionic non-hermitian triplet under $SU(2) \otimes U(1)_Y$ that is part of a sextet under $SU(3) \otimes U(1)_X$. This can also implement a seesaw mechanism for neutrino masses as it was pointed out in Ref. (23). The case of a hermitian fermion triplet was considered in the context of the standard model in Refs. [24].

It is interesting to note that in the context of MSSM a $Z_2$ symmetry [22],

$$M \rightarrow -M, \quad V \rightarrow V, \quad X \rightarrow X,$$

where $M, V, X$ is a matter, vector and scalar superfields, respectively, forbids the $R$-parity breaking terms in Eq. (2). In the present model it happens the same: the $R$-parity breaking terms in Eqs. (10) and (11) are forbidden. Notwithstanding the $Z_3$ symmetry [23],

$$\tilde{L}, \tilde{c} \rightarrow \tilde{L}, \tilde{c}; \quad \tilde{H}_1 \rightarrow \tilde{H}_1, \quad \tilde{H}_2 \rightarrow \tilde{H}_2;$$

$$\tilde{Q} \rightarrow \omega \tilde{Q}, \quad \tilde{u} \rightarrow \omega^{-1} \tilde{u}, \tilde{d} \rightarrow \omega^{-1} \tilde{d};$$

where $\omega = e^{2i\pi/3}$, forbids the $B$ violating terms but allow the $L$ violating ones. This also happens in the present model. However, if we introduce an extra discrete $Z_3'$-symmetry, such that $L_e \rightarrow -L_e$, and all other fields being even under this transformation, we have that $\mu_L = \lambda_{2e} = \lambda_{3e} = \lambda'_{rei} = 0$, at all orders in perturbation theory. This does not modify the mass matrix in the charged sector in Eqs. (10) and (11), but forbids the electron neutrino to get a mass, at all orders in perturbation theory.

The present model will induce processes contributing to $\mu \rightarrow e\gamma$, $\tau \rightarrow e(\mu)\gamma$, $(g-2)_\mu$, and other exotic decays. However these processes can be suppressed mainly by the scalar masses since these scalars do not enter explicitly in the mass matrix at the tree level. Some contributions to those processes are suppressed by the coupling constants themselves, like $\lambda_2'$s in Eq. (24a), other ones which involve $\lambda_{3\mu}, \lambda_{3\tau}$ which are of the order unity can be suppressed by combining the mixing angles and masses of scalars or charginos sectors. A more detailed study of this issue will be done elsewhere [24].

In summary, we have analyzed the charged lepton and neutrino masses in a $R$-parity breaking supersymmetric 3-3-1 model. Unlike the MSSM model the electron and its neutrino remain massless at the tree level but gain masses at the one loop level. The resulting leptonic mixing matrix $V_{MNS}$ is non-orthogonal.

**ACKNOWLEDGMENTS**

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Conselho Nacional de Ciência e Tecnologia (CNPq) and by Programa de Apoio a Núcleos de Excelência (PRONEX).

---

**APPENDIX A: THE SCALAR POTENTIAL**

The interactions between the scalars of the theory is given by the scalar potential that is written as

$$V_{331} = V_D + V_F + V_{\text{soft}},$$

(A1)

where the $V_D$ term is given by

$$V_D = -\mathcal{L}_D = \frac{1}{2} (D^a D^a + DD)$$

$$= \frac{g^2}{2} \left( \frac{2}{3} \tilde{Q}_1^i \tilde{Q}_1 - \frac{2}{3} \tilde{Q}_a \tilde{Q}_a - \frac{2}{3} \tilde{u}_i^c \tilde{d}_i + \frac{1}{3} \tilde{q}_i^c \tilde{d}_i + \rho^i \rho - \chi^i \chi - \rho^i \rho' + \chi^i \chi' \right)^2$$

$$+ \frac{g^2}{8} \sum_{i, j} (\tilde{L}_i^1 \lambda_{ij} \tilde{L}_j + \tilde{Q}_i^1 \lambda_{ij} \tilde{Q}_j + \eta_i \lambda_{ij} \eta_j + \rho_i \lambda_{ij} \rho_j + \chi_i \lambda_{ij} \chi_j - \tilde{Q}_i^a \lambda_{ij} \tilde{Q}_a - \eta_i \lambda_{ij} \eta_j - \rho_i \lambda_{ij} \rho_j - \chi_i \lambda_{ij} \chi_j),$$

(A2)

the $F$ term is
\[ V_F = -\mathcal{L}_F = \sum_m F_m^* F_m \]

\[
= \sum_{i,j,k} \left[ \frac{\mu_0}{2} \eta_i^i + \lambda_1 \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{2\lambda_2}{3} \epsilon_{ijk} \eta_j \tilde{L}_k + \frac{\lambda_3}{3} \epsilon_{ijk} \chi_j \rho_k \right]^2 + \left[ \frac{\mu_0}{2} \eta_i' + \frac{\lambda_2}{3} \epsilon_{ijk} \chi_j \rho_k \right]^2 + \left[ \frac{\mu_0}{2} \eta_i'' + \frac{\lambda_2}{3} \epsilon_{ijk} \chi_j \rho_k \right]^2 + \frac{\mu_0}{2} \eta_i''' + \frac{\lambda_2}{3} \epsilon_{ijk} \rho_j \eta_k + \frac{\lambda_3}{3} \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{40ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\mu_0}{2} \rho_i + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{50ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\mu_0}{2} \eta_i + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{60ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\kappa_{7ij}}{3} \tilde{Q}_1 \eta_i' + \frac{\kappa_{80ij}}{3} \tilde{Q}_a \rho_i + \frac{\chi''_{ij}}{3} \tilde{d}_i \tilde{d}_k + \frac{\lambda''_{ij}}{3} \tilde{u}_i \tilde{u}_k \right]^2 \]

\[
+ \frac{\kappa_{9ij}}{3} \tilde{Q}_1 \rho_i' + \frac{\kappa_{10ij}}{3} \tilde{Q}_a \eta_i + \frac{\chi''_{ij}}{3} \tilde{Q}_a \tilde{L}_i + \frac{2\lambda''_{ij}}{3} \tilde{d}_i \tilde{u}_k + \frac{\lambda''_{ij}}{3} \tilde{j}_i \tilde{j}_j \right]^2 \]

\[V_{soft} = -\mathcal{L}_{soft} = \frac{1}{2} \left( \sum_{i=1}^{m} \frac{\mu_0}{2} \eta_i^i + \lambda_1 \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{2\lambda_2}{3} \epsilon_{ijk} \eta_j \tilde{L}_k + \frac{\lambda_3}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 + \left( \frac{\mu_0}{2} \eta_i' + \frac{\lambda_2}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 + \left( \frac{\mu_0}{2} \eta_i'' + \frac{\lambda_2}{3} \epsilon_{ijk} \rho_j \eta_k + \frac{\lambda_3}{3} \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 \]

\[
+ \frac{\mu_0}{2} \eta_i''' + \frac{\lambda_2}{3} \epsilon_{ijk} \rho_j \eta_k + \frac{\lambda_3}{3} \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{40ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\mu_0}{2} \rho_i + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{50ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\mu_0}{2} \eta_i + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{60ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\kappa_{7ij}}{3} \tilde{Q}_1 \eta_i' + \frac{\kappa_{80ij}}{3} \tilde{Q}_a \rho_i + \frac{\chi''_{ij}}{3} \tilde{d}_i \tilde{d}_k + \frac{\lambda''_{ij}}{3} \tilde{u}_i \tilde{u}_k \right]^2 \]

\[+ \frac{\kappa_{9ij}}{3} \tilde{Q}_1 \rho_i' + \frac{\kappa_{10ij}}{3} \tilde{Q}_a \eta_i + \frac{\chi''_{ij}}{3} \tilde{Q}_a \tilde{L}_i + \frac{2\lambda''_{ij}}{3} \tilde{d}_i \tilde{u}_k + \frac{\lambda''_{ij}}{3} \tilde{j}_i \tilde{j}_j \right]^2 \]

Finally, the soft term is (the following soft-terms do not include the exotic quarks)

\[V_{soft} = -\mathcal{L}_{soft} = \frac{1}{2} \left( \sum_{i=1}^{m} \frac{\mu_0}{2} \eta_i^i + \lambda_1 \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{2\lambda_2}{3} \epsilon_{ijk} \eta_j \tilde{L}_k + \frac{\lambda_3}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 + \left( \frac{\mu_0}{2} \eta_i' + \frac{\lambda_2}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 + \left( \frac{\mu_0}{2} \eta_i'' + \frac{\lambda_2}{3} \epsilon_{ijk} \rho_j \eta_k + \frac{\lambda_3}{3} \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k \right)^2 \]

\[
+ \frac{\mu_0}{2} \eta_i''' + \frac{\lambda_2}{3} \epsilon_{ijk} \rho_j \eta_k + \frac{\lambda_3}{3} \epsilon_{ijk} \tilde{L}_j \tilde{L}_k + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{40ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\mu_0}{2} \rho_i + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{50ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\mu_0}{2} \eta_i + \frac{f_1}{3} \epsilon_{ijk} \chi_j \rho_k + \frac{\kappa_{60ij}}{3} \tilde{Q}_a \tilde{Q}_j^c \right]^2 \]

\[
+ \frac{\kappa_{7ij}}{3} \tilde{Q}_1 \eta_i' + \frac{\kappa_{80ij}}{3} \tilde{Q}_a \rho_i + \frac{\chi''_{ij}}{3} \tilde{d}_i \tilde{d}_k + \frac{\lambda''_{ij}}{3} \tilde{u}_i \tilde{u}_k \right]^2 \]

\[+ \frac{\kappa_{9ij}}{3} \tilde{Q}_1 \rho_i' + \frac{\kappa_{10ij}}{3} \tilde{Q}_a \eta_i + \frac{\chi''_{ij}}{3} \tilde{Q}_a \tilde{L}_i + \frac{2\lambda''_{ij}}{3} \tilde{d}_i \tilde{u}_k + \frac{\lambda''_{ij}}{3} \tilde{j}_i \tilde{j}_j \right]^2 \]

\[ \text{APPENDIX B: NUMERICAL ANALYSIS OF MASS MATRICES} \]

Here we show explicitly the numerical values of each entry of the mass matrices in Eqs. (17) and (33) using the parameters given in Eq. (23) and \( m' = -3780.4159 \) GeV. For the charged sector we have
Here we show that the relevant parameters for the leptons masses are $\lambda_{2,3}$. We note that there are four type of parameters in the mass matrices in Eqs. (24). Firstly we have the dimensionless Yukawa couplings in the usual leptons, $\lambda_{2,3}$ and in the supersymmetric partners, $f_1, f'_1$. We also have the mass dimension parameters $\mu_{0a}$ and $\mu_{n,\rho,\chi}$ and $m_{\chi}$ and $m'$ which are soft terms in Eq. (34). Of all these parameters we expect that the relevant ones in the charged lepton and neutrinos are $\lambda_{2,3}$. To show this we consider several choices of the parameters as follows: (Below all masses are in GeV)

**case 1:**

\[
\begin{align*}
\lambda_{2e\mu} &= 0.0, \quad \lambda_{2e\tau} = 0.0, \quad \lambda_{2\mu\tau} = 0.0, \\
\lambda_{3e} &= 0.0, \quad \lambda_{3\mu} = 0.0, \quad \lambda_{3\tau} = 0.0, \\
\mu_{0e} &= \mu_{0\mu} = 0.0; \mu_{0\tau} = 0.0, \quad \text{(in GeV). (B3)}
\end{align*}
\]

Charged sector masses: 3186.03, 3001.10, 557.17, 196.55, 149.30, 16.85, 0, 0, 0.

Neutral sector masses: -4162.22, 3260.47, 3001.10, 557.79, -557.17, 450.14, -330.68, 17.18, -17.01, 0, 0, 0.

**case 2:**

\[
\begin{align*}
\lambda_{2e\mu} &= 0.0, \quad \lambda_{2e\tau} = 0.0, \quad \lambda_{2\mu\tau} = 0.0, \\
\lambda_{3e} &= 0.0, \quad \lambda_{3\mu} = 0.0, \quad \lambda_{3\tau} = 0.0, \\
\mu_{0e} &= \mu_{0\mu} = 0.0; \mu_{0\tau} = 2 \times 10^{-8}, \quad \text{(in GeV) (B4)}
\end{align*}
\]

Charged sector masses: 3186.03, 3001.10, 557.17, 196.55, 149.30, 16.85, 0.92 \times 10^{-12}, 0, 0.

Neutral sector masses: -4162.22, 3260.47, 3001.10, 557.79, -557.17, 450.14, -330.68, 17.18, -17.01, 2.80 \times 10^{-21}, 0, 0.

**case 3:**

\[
\begin{align*}
\lambda_{2e\mu} &= 0.0, \quad \lambda_{2e\tau} = 0.0, \quad \lambda_{2\mu\tau} = 0.0, \\
\lambda_{3e} &= 0.0001, \quad \lambda_{3\mu} = 1.0, \quad \lambda_{3\tau} = 1.0, \\
\mu_{0e} &= \mu_{0\mu} = 0.0; \mu_{0\tau} = 0.0, \quad \text{(in GeV). (B5)}
\end{align*}
\]

Charged sector masses: 3186.05, 3001.11, 584.85, 282.30, 149.41, 204.55, 2.10 \times 10^{-10}, 0, 0.

Neutral sector masses: -4162.22, 3260.47, 3001.10, 585.18, -585.18, 453.22, -344.14, 283.14, -271.99, 1.23 \times 10^{-11}, 0, 0.

**case 4:**

\[
\begin{align*}
\lambda_{2e\mu} &= 0.001, \quad \lambda_{2e\tau} = 0.001, \quad \lambda_{2\mu\tau} = 0.393, \\
\lambda_{3e} &= 0.0, \quad \lambda_{3\mu} = 0.0, \quad \lambda_{3\tau} = 0.0, \\
\mu_{0e} &= \mu_{0\mu} = 0.0; \mu_{0\tau} = 0.0, \quad \text{(in GeV). (B6)}
\end{align*}
\]

Charged sector masses: 3186.03, 3001.10, 557.17, 196.55, 149.30, 16.85, 1.85, 1.85, 0.

Neutral sector masses: -4162.22, 3260.47, 3001.10, 557.79, -557.17, 450.14, -330.68, 17.18, -17.01, 0, 0, 0.

**case 5:**

\[
\begin{align*}
\lambda_{2e\mu} &= 0.001, \quad \lambda_{2e\tau} = 0.001, \quad \lambda_{2\mu\tau} = 0.393, \\
\lambda_{3e} &= 0.0001, \quad \lambda_{3\mu} = 1.0, \quad \lambda_{3\tau} = 1.0, \\
\mu_{0e} &= \mu_{0\mu} = 0.0; \mu_{0\tau} = 0.0, \quad \text{(in GeV) (B7)}
\end{align*}
\]

Charged sector masses: 3186.03, 3001.11, 584.85, 282.30, 204.55, 149.41, 1.78, 0.105, 0.

Neutral sector masses: -4162.22, 3260.47, 3001.10, 585.19, -585.19, 453.22, -344.14, 283.14, -271.99, 1.23 \times 10^{-11}, 0, 0.

Notice that the values of the masses in the charged sector are not significantly affected by the values of $\mu_{0a}$.

[1] Y. Fukuda et al. (SuperKamiokande Collaboration), Phys. Rev. Lett. 81, 1562 (1998); ibid. 82, 2644 (1999).
[2] B. T. Cleveland et al. (Homestake Collaboration), Astrophys. J. 496, 505 (1998); K. S. Hirata et al. (Kamiokande Collaboration), Phys. Rev. Lett. 77, 1683 (1996); W. Hampel et al. (GALLEX Collaboration), Phys. Lett. B477, 127 (1999); J. N. Abdurashitov et al. (SAGE Collaboration), Phys. Rev. Lett. 77, 4708 (1996); Phys. Rev. C 60, 055801 (1999); Q. R. Ahmad et al. (SNO Collaboration), nucl-ex/0106015.
[3] C. Athanassopoulos et al. (LSND Collaboration), Phys. Rev. Lett. 77, 3082 (1996); ibid 81, 1774 (1998).
[4] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992); P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47, 4158 (1993).
[5] U. Okamoto and M. Yasue, Phys. Lett. B466, 267 (1999); T. Kitabayashi and M. Yasue, Phys. Rev. D63, 095002 (2001); T. Kitabayashi and M. Yasue, Phys. Lett. B508, 85 (2001); T. Kitabayashi and M. Yasue, Nucl. Phys. B609, 61 (2001); T. Kitabayashi and M. Yasue, Phys. Rev. D63, 095006 (2001); J. C. Montero, C. A. de S. Pires and V. Pleitez, Phys. Lett. 502B, 167 (2001). hep-ph/0112246.
[6] H. E. Haber and G. L. Kane, Phys. Rep. 117, 75 (1985).
[7] L. J. Hall and M. Suzuki, Nucl. Phys. B231, 419 (1984).
[8] T. Banks, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. D 52, 5319 (1995).
[9] M. A. Diaz, J. C. Romão and J. W. F. Valle, Nucl. Phys. B524, 25 (1998).
[10] F. Borzumati and Y. Nomura, Phys. Rev. D 64, 053005 (2001); F. Borzumati, K. Hamaguchi and T. Yanagida, Phys. Lett. B497, 259 (2001); F. Borzumati, K. Hamaguchi, Y. Nomura and T. Yanagida, hep-ph/0012113.
[11] R. N. Mohapatra, Phys. Rev. D 34, 3457 (1986).
[12] J. C. Romão and J. W. F. Valle, Nucl. Phys. B381, 87 (1992).
[13] S. Davison and M. Losada, hep-ph/0010323.
[14] T. V. Duong and E. Ma, Phys. Lett. B316, 307 (1993).
[15] J. C. Montero, C. A. de S. Pires and V. Pleitez, hep-ph/0011296; hep-ph/0103096; hep-ph/0112246.
[16] J. C. Montero, V. Pleitez and M. C. Rodriguez, Phys. Rev. D 65, 035006 (2002), hep-ph/0012118.
[17] M. Capdequi-Peyranere and M.C. Rodriguez, Phys. Rev. D 65, 035006 (2002), hep-ph/0103013.
[18] H. N. Long and P. B. Pal, Mod. Phys. Lett. A13, 2355 (1998).
[19] P. Das, P. Jain and D. W. McKay, Phys. Rev. D 59, 055011 (1999).
[20] D. E. Groom et al. (Particle Data Group), Eur. Phys. J. C 15, 1 (2000).
[21] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theo. Phys. 28, 246 (1962).
[22] K. S. Babu and E. Ma, Phys. Rev. Lett. 61, 674 (1988).
[23] J. C. Montero, C. A. de S. Pires and V. Pleitez, to be published in Phys. Rev., hep-ph/0112246.
[24] R. Foot, H. Lew, X.-G. He and G. C. Joshi, Z. Phys. C 44, 1989); E. Ma, Phys. Rev. Lett. 81, 1171 (1998).
[25] H. E. Haber, hep-ph/0103095.
FIG. 1. Diagram generating the electron mass. There are also a contribution with $v_\chi \rightarrow v_\chi'$. The left- and right- side vertices are proportional to $\lambda_{\alpha \varepsilon i}/3$ and $\lambda'_{\alpha' \varepsilon j}/3$, respectively.

FIG. 2. Diagram generating the mass for the lightest neutrino. There is another dominant contribution with $v_\chi \rightarrow v_\chi'$. Each vertex on the left- and right-side are proportional to $\lambda_{2 \varepsilon \tau}/3$ and $\lambda'_{2 \varepsilon \tau}/3$, respectively.