The role of observers in the measurement of the Teleparallel Gravitoelectromagnetic fields in different geometries

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Abstract

In the context of the Teleparallel Equivalent of General Relativity (TEGR) we have investigated the role of local observers, associated with tetrad fields, in description of the gravitational interaction through the concepts of the gravitoelectric (GE) and gravitomagnetic (GM) fields. We start by analyzing the gravitoelectromagnetic (GEM) fields obtained from an observer freely falling in the Schwarzschild space-time. Then, we investigated whether it is possible to distinguish between this situation and to be at rest in the Minkowski space-time. We conclude that, although there are non-zero components for the fields obtained for the case of free fall, its dynamical effect, measured by the gravitational Lorentz force, is null. Moreover, the gravitational field energy obtained from the GEM fields for an observer freely falling in the Schwarzschild space-time is zero. These results are in complete agreement with the equivalence principle.

\textbf{keywords:} Teleparallel Equivalent of General Relativity, Gravitoelectromagnetism, Free falling frame.

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1 Introduction

The study of gravitation through the GEM fields can bring some new insight [1]. To give a contemporary example, the interpretation of objects recently defined by the literature, as the vortex and tendex lines [2] can be facilitated if we use the fields to describe them [1]. Recently, at the linear regime of gravitation, we had a new confirmation of the existence of the GEM fields, through the Gravit Probe B experiment [3].

In the literature, we found several studies that address the stationary GEM fields due to the fact that the similarity between the field equations of electromagnetic theory and of gravitation is reached in this context [4]. There are still some works that deal with the time-dependent GEM and the issue about the Faraday’s law in the context of general relativity (GR) [5]. However, there is no studies about the behavior of GEM fields for cases in which the observer is moving with respect to the source.

On the other hand, with the advent of relativistic mechanics, the relativity principles were extended to the electrodynamics in analyzing the behavior of electric and magnetic fields in relation to different inertial frames. Just as in electrodynamics, we expect that, in gravitation, the GEM fields proposed assume different expressions depending on how the source is observed, i.e., different observers note different GEM fields. Thus, it is natural to study the physical consequences of these different fields upon the observer itself.

In a previous work [6], motivated by the fact that the TEGR can be described as a gauge theory, we have proposed a new way to define the gravitoelectric and the gravitomagnetic fields. These definitions, that are conceptually different from those that arise in the RG, were made in a very similar way to what is done on the Yang-Mills theory and the Electrodynamics, being based on the field strength components. On that work, in the weak field limit we have obtained the analogous Maxwell equations and for a set of tetrads which is adapted to a stationary observer relative to Schwarzschild, the gravitoelectric components calculated were in total agreement with the newtonian field.

According to [7] we can interpret the extra degrees of freedom of the tetrad field as a choice of reference system. Two sets of tetrad fields may represent the same spacetime, though they are physically different. That is, besides being the fundamental object of the theory, we can interpret them as ideal observers in spacetime. This subtleness is not present in the metric description of gravity.

In this paper, on the context of the TEGR, we discuss the issue of how different observers feel the GEM fields. For two different observers, we will analyze the behavior of GEM fields for the Schwarzschild and the Minkowski spacetime. First, we will consider a free falling observer in Schwarzschild black hole, i.e. an observer who falls radially into a black hole due to its gravitational force, then we will consider a second observer, but now, the observer is standing in the Minkowski spacetime. As expected, from the equivalence principle, we have concluded in this approach that those observers are indistinguishable from the dynamic point of view, that is, being null the gravitational Lorentz force [8] felt by each one of them, they follow the same trajectory. Moreover, their energies were calculated, being zero in both cases. It is interesting to understand how these results occur even though we have found non-zero GEM components, as can be viewed along the subsections 2.1 and 2.2.

\(^1\)This possibility is still under investigation.
Notation: According to its gauge structure, to each point of spacetime there is attached a tangent spacetime (the fiber of the correspondent tangent bundle), on which the gauge group acts, and whose metric is assumed to be $\eta_{ab} = (-1,1,1,1)$. The spacetime indices will be denoted by the Greek alphabet ($\mu, \nu, \sigma, ... = 0,1,2,3$) and the tangent space indices will be denoted by the first half of the Latin alphabet ($a, b, c, ... = 0,1,2,3$). The second half of the Latin alphabet will be used to represent space tensor components, that is, $(i, j, k, ...)$ assume the values 1,2 and 3. Indices in parentheses will also be related to tangent space. We adopt the light velocity as $c = 1$.

1.1 Teleparallel Equivalent of General Relativity and Gravitoelectromagnetism

Let us present some of the more important expressions in TEGR that will be used in the whole paper.\(^2\)

The field strength of the theory is defined in the usual form

$$F^a_{\mu\nu} = \partial_\mu h^a_\nu - \partial_\nu h^a_\mu = h^a_\rho T^\rho_{\mu\nu}, \quad (1)$$

with $h^a_\mu$ being the components of the tetrad field. The object $T^\rho_{\mu\nu}$ is the torsion that represents alone the gravitational field, defined by $T^\rho_{\mu\nu} \equiv \Gamma^\rho_{\nu\mu} - \Gamma^\rho_{\mu\nu}$, where $\Gamma^\rho_{\nu\mu}$ is the Weitzenböck connection given by $\Gamma^\rho_{\nu\mu} \equiv h^a_\rho \partial_\mu h^a_\nu$. Therefore, torsion can also be identified as the field strength written on the tetrad base.

The dynamics of the gauge fields will be determined by the lagrangian \(^10\)

$$\mathcal{L}_G = \frac{h}{16\pi G} S^\rho_{\mu\nu} T^\rho_{\mu\nu}, \quad (2)$$

with $h = \det(h^a_\mu)$ and

$$S^\rho_{\mu\nu} = -S^\rho_{\nu\mu} \equiv \frac{1}{2} \left[ K^{\nu\rho}_{\mu} - g^{\rho\nu} T^\theta_{\mu} + g^{\rho\mu} T^\theta_{\nu} \right], \quad (3)$$

which is called superpotential, that will play an important role in theory, as we will see. The object $K^{\mu\nu\rho}$ is the contorsion tensor defined by

$$K^{\mu\nu\rho} = \frac{1}{2} T^{\mu\nu\rho} + \frac{1}{2} T^{\nu\rho\mu} - \frac{1}{2} T^{\mu\rho\nu}. \quad (4)$$

The field equations resulting from this lagrangian are

$$\partial_\sigma (h S^\sigma_{\mu\nu}) - 4\pi G (h j^\rho_{\mu\nu}) = 0 \quad (5)$$

with

$$j^\rho_{\mu\nu} \equiv \frac{\partial \mathcal{L}}{\partial h^\rho_{\mu\nu}} = h^a_\lambda (F^c_{\mu\lambda} S^\nu_{c\mu} - \frac{1}{4} \delta^c_{\rho} F^c_{\mu\nu} S^\nu_{c\mu}), \quad (6)$$

being the gauge energy-momentum current of the gravitational field \(^9\).
Let us now introduce the gravitoelectromagnetism in the teleparallel context. On one hand, in the context of TEGR, the field strength \( F_{a\mu\nu} \) can be associated to the torsion tensor, in such way that we could use it to define our fields. On the other hand, the superpotential, defined above, assumes the role of the field strength in the field equations, similarly to what occurs in the electromagnetic equations. Therefore, inspired on the electromagnetism, we define the gravitoelectric and gravitomagnetic fields in terms of the superpotential components. The gravitoelectric field (GE) is defined by
\[
S^0_i \equiv E^i_a \tag{7}
\]
and the gravitomagnetic field (GM) is as follows
\[
S_{ij} \equiv \epsilon^{ijk} B_{ak}. \tag{8}
\]
As already stated, these definitions were tested and passed on some important tests \[6\]. Calculating these fields in specific configurations will allow us to better understand the role of observers in gravitation.

2 Free Falling in Schwarzschild spacetime

In this section we analyze the GEM fields obtained from an observer in free fall in Schwarzschild spacetime. Initially we consider the Schwarzschild metric which can be written as
\[
ds^2 = -\alpha^{-2} dt^2 + \alpha^2 dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{9}
\]
with
\[
\alpha^{-2} = 1 - \frac{2m}{r}. \tag{10}
\]
An observer that moves radially in free fall due to attraction of the Schwarzschild black hole must have a four-velocity like \[11\]
\[
\dot{u}^\nu = \left[ \left( 1 - \frac{2m}{r} \right)^{-1}, - \left( \frac{2m}{r} \right)^{1/2}, 0, 0 \right]. \tag{11}
\]
A set of tetrad fields that satisfies the above condition is given by \[7\]
\[
h_{a\mu} = \begin{pmatrix}
-1 & -\alpha^2 \beta & 0 & 0 \\
\beta \sin \theta \cos \phi & \alpha^2 \sin \theta \cos \phi & 0 & r \cos \theta \cos \phi - \sin \theta \sin \phi \\
\beta \sin \theta \sin \phi & \alpha^2 \sin \theta \sin \phi & r \cos \theta \sin \phi & \sin \theta \cos \phi \\
\beta \cos \theta & \alpha^2 \cos \theta & -\sin \theta & 0
\end{pmatrix}, \tag{12}
\]
where \( \beta \) is defined by
\[
\beta = \sqrt{\frac{2m}{r}}. \tag{13}
\]
Through the expression of torsion written in terms of the tetrad fields
\[
T^\sigma_{\mu\nu} = h_a^\sigma \partial_\mu h^a_\nu - h_a^\sigma \partial_\nu h^a_\mu, \tag{14}
\]
we calculate the components of $T_{\sigma\mu\nu}$, of which the non null are

\[
\begin{align*}
T_{001} &= -\beta \partial_r \beta, \\
T_{101} &= -\alpha^2 \partial_r \beta, \\
T_{202} &= -r \beta, \\
T_{303} &= -r \beta \sin^2 \theta, \\
T_{212} &= r (1 - \alpha^2), \\
T_{313} &= r (1 - \alpha^2) \sin^2 \theta.
\end{align*}
\] (15)

With these results, we can calculate the superpotential that will allow us to find the GEM fields. In this way, to get the GE fields, we need the components of $S_{b\mu\nu}$ following:

\[
S_{b0i}^{0i} = \frac{1}{4} \left[ h_b^i g_{00}^i T_{j0k} + h_b^i g_{00}^i T_{k0j} \right] + \frac{1}{2} \left[ h_b^i g_{ij}^i T_{kjl} - h_b^i g_{00}^i T_{j0k} \right].
\] (16)

The GE radial components are obtained by making $i = 1$, that is

\[
S_{b01} = E_{b1} = \frac{1}{2} \left[ h_b^0 g_{11}^0 g_{22}^0 T_{212} + h_b^0 g_{11}^0 g_{33}^0 T_{313} - h_b^1 g_{00}^0 g_{22}^0 T_{202} - h_b^1 g_{00}^0 g_{33}^0 T_{303} \right].
\] (17)

To the angular components $\theta$ we perform $i = 2$ in the above expression (16) and we find

\[
S_{b02} = E_{b2} = -\frac{1}{2} \left[ h_b^2 g_{00}^2 g_{11}^2 T_{101} + h_b^2 g_{00}^2 g_{33}^2 T_{303} \right].
\] (18)

The $\phi$ components are obtained by making $i = 3$

\[
S_{b03} = E_{b3} = -\frac{1}{2} \left[ h_b^3 g_{00}^3 g_{11}^3 T_{101} + h_b^3 g_{00}^3 g_{22}^3 T_{202} \right].
\] (19)

Let us now consider the internal index equal to zero in the above expressions, that is, $b = 0$:

\[
\begin{align*}
E_{(0)}^r &= 0, \\
E_{(0)}^\theta &= 0, \\
E_{(0)}^\phi &= 0.
\end{align*}
\] (20)

Then we calculate the spacial components for $b$. Considering (17) and attributing $b = 1, 2, 3$ we get

\[
\begin{align*}
E_{(1)}^r &= -\frac{\beta}{r} \sin \theta \cos \phi, \\
E_{(2)}^r &= -\frac{\beta}{r} \sin \theta \sin \phi, \\
E_{(3)}^r &= -\frac{\beta \cos \theta}{r}.
\end{align*}
\] (21)

In the same way, assigning the values $b = 1, 2, 3$ in (18), we obtain

\[
\begin{align*}
E_{(1)}^\theta &= -\frac{\alpha^2 \beta}{4r^2} \cos \theta \cos \phi, \\
E_{(2)}^\theta &= 0, \\
E_{(3)}^\theta &= 0.
\end{align*}
\] (22)

\[
\begin{align*}
E_{(1)}^\phi &= 0, \\
E_{(2)}^\phi &= 0, \\
E_{(3)}^\phi &= 0.
\end{align*}
\] (23)

\[
\begin{align*}
E_{(1)}^\phi &= 0, \\
E_{(2)}^\phi &= 0, \\
E_{(3)}^\phi &= 0.
\end{align*}
\] (24)
\[ E_{(2)}^\theta = -\frac{\alpha^2 \beta}{4r^2} \cos \theta \sin \phi, \]  

\[ E_{(3)}^\theta = -\frac{\alpha^2 \beta}{4r^2} \sin \theta, \]  

and finally, making \( b = 1, 2, 3 \) in (19) we get

\[ E_{(1)}^\phi = \frac{\alpha^2 \beta \sin \phi}{4r^2 \sin \theta}, \]  

\[ E_{(2)}^\phi = \frac{\alpha^2 \beta \cos \phi}{4r^2 \sin \theta}, \]  

\[ E_{(3)}^\phi = 0. \]  

Let us now calculate the GM fields for this configuration. Writing the superpotencial in terms of torsions,

\[ S_{b}^{ij} = \frac{1}{4} \left[ h_{a}^{ik} g^{jm} (T_{mk0} + T_{0km} - T_{k0m}) + h_{a}^{ik} g^{jm} (T_{mkn} + T_{nkm} - T_{kmn}) \right] + \frac{1}{2} \left[ -h_{a}^{j} g^{ik} (g^{nm}T_{mkn} - g^{00}T_{00k}) + h_{a}^{j} g^{ik} (g^{nm}T_{mln} - g^{00}T_{00l}) \right], \]

and using the definition (8) with the internal index \( b = 0 \) in the above expression we obtain:

\[ B_{(0)\phi} = 0, \]  

\[ B_{(0)\theta} = 0, \]  

\[ B_{(0)r} = 0. \]  

In the sequence we consider \( b = 1, 2, 3 \) for each spacetime coordinate. For \( \phi \) component:

\[ B_{(1)\phi} = \frac{m}{2r^3} \cos \theta \cos \phi, \]  

\[ B_{(2)\phi} = \frac{m}{2r^3} \cos \theta \sin \phi, \]  

\[ B_{(3)\phi} = -\frac{m}{2r^3} \sin \theta. \]

For \( \theta \) component:

\[ B_{(1)\theta} = \frac{m}{2r^3} \frac{\sin \phi}{\sin \theta}, \]  

\[ B_{(2)\theta} = -\frac{m}{2r^3} \frac{\cos \phi}{\sin \theta}, \]  

\[ B_{(3)\theta} = 0. \]

Finally, the remaining radial components

\[ B_{(1)r} = B_{(2)r} = B_{(3)r} = 0. \]

On the other hand, if we consider a static observer in Minkowski spacetime and perform a similar calculation we obtain all the GEM field components equal to zero. However, assuming
valid the equivalence principle, we should not be able to discern between two observers, one of them freely falling in Schwarzschild black hole, and other static in Minkowski spacetime. This apparent inconsistency should be clarified when investigating the role of the non-zero components $b = 1, 2, 3$ for dynamics.

Before doing this analysis let us make some comments. According to [12] in the linearized GEM the operational definition for the GEM fields must be in accordance with the equivalence principle, that is, for a non-rotating and free fall observer there is no gravitational forces and therefore the GEM fields are zero. Our definition is in full agreement with this since in the weak field limit

$$\frac{m}{r} << 1$$

(39)

all the above components are zero. Moreover, even in the exact case, we have shown that the $b = 0$ components also vanish. This show that the operational definitions must be related with $b = 0$ components, which is in agreement with a similar analysis in [6].

Let us now verify the effects of the non-zero components of the GE and GM fields on the dynamics of the observers.

### 2.1 Gravitational Lorentz Force

As mentioned earlier, as a consequence of the equivalence principle, an observer represented by a not spinning tetrad field and freely falling in Schwarzschild spacetime, should not be able to distinguish - at least from the dynamic point of view - if is freely falling in this spacetime or at rest with respect to the Minkowski spacetime. A way to tackle this issue is to use the equation that describes the behavior of scalar particles in the presence of gravitation: the TEGR gravitational Lorentz force [8]

$$h^a_\mu \frac{d u_a}{d s} = F^a_\mu \nu u_\nu,$$

(40)

in which the right side of the equation plays the role of force, analogous to the Lorentz force of Electromagnetism. Alternatively, this equation can be rewritten as the geodesic equation in the context of RG [8]. From this expression, we can evaluate the consequences of the non-zero GEM fields components previously obtained.

Since the GEM fields are defined from the superpotential $S^{b\rho \mu}$ it is convenient to rewrite the above equation in terms of these quantities. For this, we should first rewrite the gravitational field strength tensor in terms of the superpotential, ie

$$F^a_\gamma \delta = h^b_\gamma g_{\rho \delta} h^a_\mu S_{b}^{\mu \rho} - h^b_\delta g_{\rho \gamma} h^a_\mu S_{b}^{\mu \rho} - \frac{1}{2} h^a_\delta g_{\rho \gamma} h^b_\theta S_{b}^{\theta \rho} + \frac{1}{2} h^a_\gamma g_{\rho \delta} h^b_\theta S_{b}^{\theta \rho}.$$  

(41)

Thus, we obtain

$$h^a_\mu \frac{d u_a}{d s} = - h^b_\nu S^{b\rho \mu} u_\rho u_\nu - \frac{1}{2} h_{b \theta} (S^{b \rho \theta} u_\rho u_\nu - S^{b \rho \theta} u_\rho u_\nu).$$  

(42)

By making use of the [11] and of the GEM fields we can calculate the right side of the equation (42) for an observer freely falling in Schwarzschild spacetime. Thus, we get a null

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3That allow a direct analogy with electromagnetism.
result for all Lorentz force components, i.e., the non-null GEM fields, obtained in earlier section, are combined so as to eliminate the force felt by the observer, and therefore do not changing its trajectory. The same result is obtained when we consider a static observer in relation to the Minkowski spacetime, since all GEM fields are null. Therefore, in some sense, we can say that the components of superpotencial with zero internal space index represent the operational definition of the GEM fields since, being equal to zero, these components were already in line with the equivalence principle.

2.2 Gravitational Field Energy

Another physical evidence that enables us to face the issue of non-zero components for the case of the freely falling reference frame in the Schwarzschild spacetime is the gravitational field energy. Again, being valid the equivalence principle, we should not be able to discern between two observers, one freely falling in Schwarzschild black hole, and another static in Minkowski spacetime. Thus, being zero the gravitational field energy associated with the second situation, an equal result should occur with the energy measured by the observer in the first situation. We can calculate the gravitational field energy as given by the zero component of (6), from the GEM fields obtained by an observer represented by the tetrad field (12), since they are defined from the superpotential which appears in the definition of energy momentum tensor. Let us consider then

\[ j^0_{(0)} = h^{0}_{(0)} \lambda^0 (F^c_{i\lambda} S^i_0 - \frac{1}{4} \delta^0_{\lambda} F^c_{\mu\nu} S^\mu_0 S^\nu_0) \].

Substituting (11) in (13) we get

\[ j^0_{(0)} = h^{0}_{(0)} \lambda^0 \left( h^b_{i\rho\lambda} h^c_{\gamma\lambda} S^\mu_0 - h^b_{\gamma\rho\lambda} h^c_{\gamma\lambda} S^\mu_0 \right) + 1 \frac{1}{4} h^0_{(0)} \left( h^b_{\mu\rho\lambda} h^c_{\gamma\lambda} S^\mu_0 - h^b_{\gamma\rho\lambda} h^c_{\gamma\lambda} S^\mu_0 \right) \].

Using the definitions (7) and (8) we can rewrite the expression above in terms of \( E^i_a \) and \( B^i_a \). Note that as it is quadratic in the superpotential, it is also quadratic in the GEM fields. After a lengthy calculation, we found out the following result for the above component

\[ j^0_{(0)} = 0 \],

ie the gravitational field energy written in terms of the GEM fields are zero for a freely falling observer in the Schwarzschild black hole. While there are non-zero components of the GEM field, they combine in such a way that do not change the expected result of the gravitational field energy, in a similar way with what happened in gravitational Lorentz force calculation. As consequence, it is not possible - at least from the dynamical point of view\(^4\) - for a local observer to distinguish between to be in free fall in the Schwarzschild black hole or to be at rest in the Minkowski spacetime.

\(^4\)Perhaps the components \( E_{(1,2,3)}^i \) and \( B_{(1,2,3)}^i \) have a measurable physical sense in a semi-classical scenario and allow a differentiation between the frames. The analysis of this issue will be presented elsewhere.
Thus, from (44) and (45), we can define an "operational energy" of the gravitational field in a manner completely analogous to that of electromagnetism, namely:

\[ P = \int \left[ (E_{(0)i})^2 + (B_{(0)i})^2 \right] d^3x. \]  

(46)

It is important to stress out that this definition was inferred based only on the case of a reference in free fall in a Schwarzschild black hole, being the extension of its validity still under investigation.

3 Final remarks

As obtained in a previous work \[6\], for a set of tetrads which is adapted to a stationary observer relative to Schwarzschild spacetime, it has been showed that in the weak field limit the gravitoelectric components are in total agreement with the Newtonian field and, in addition, all GM components are zero. The conceptual definitions of what we expect to be analogous to electromagnetic fields were identified as the zero internal components of GEM fields, that is \( b = 0 \), what we have called "operational definitions". Here in this work, when we consider a freely falling observer in the Schwarzschild black hole we obtained a null result for all the GE and GM components with zero internal index, which corroborate the idea of "operationality" for \( b = 0 \) component fields. We would like to emphasize that the choice of coordinate systems is the same in both cases above mentioned, through the use of appropriate tetrad fields.

We have obtained as main conclusion in this work that through the use of GEM fields it is not possible for a local observer to distinguish between free falling in the Schwarzschild black hole or at resting in the Minkowski spacetime. Although this idea seems to be natural, due to the equivalence principle, it emerged in this approach after a deeper analysis, since we found out that non-null fields arise in the free fall case. One possibility would be to consider only the operational components \( b = 0 \), since they are all equal to zero and simply to discard the other non null components that came from spacial internal indices. Then it would be straightforward to postulate the equivalence between the references. But these non null components could store some important information that would violate the central idea.

To investigate the role of non null components of GEM fields in dynamics we have used the gravitational Lorentz force written in terms of them and we concluded that their contributions cancel each other resulting in a null total force measured by the free falling observer in the Schwarzschild geometry, in the same way it were placed at rest in Minkowski spacetime. Thus, any experiment which make use of dynamical effects from the gravitational field will not be able to distinguish between those two references. Moreover, in order to support the results, we showed that the gravitational field energy measured by the reference in free fall is zero, as expected if compared with the field energy associated with the flat spacetime. We also should like to stress out that all the calculations were done using the GEM fields and outside the weak field limit, that is, we have obtained exact results that can also be applicable to treat intense fields like, for example, jet formations in supermassive black holes.
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References

[1] B. Mashhoon Int. J. Mod. Phys. D. 14 (2005) 12; Maxwell J.C. Phil. Trans. 155, 1865) 492; G. Holzmuller, Z. Moth. Phys. 15, 69 (1870); F. Tisserand, Compt. Rend. 75, 760 (1872); 110, 313 (1890); B. Mashhoon, F. W. Hehl, D. S. Theiss, GRG 16, 8 (1984); I. Ciufolini and J. A. Wheeler, Gravitation and Inertia (Princeton University Press, Princeton, 1951).

[2] R. Owen et al, Phys. Rev. Lett. 106, 151101 (2011).

[3] C. W. F. Everitt et al, Phys. Rev. Lett. 106, 221101 (2011).

[4] B. Mashhoon Gravitoelectromagnetism: A Brief review in Iorio, L. (Ed.), Measuring Gravitomagnetism: A Challenging Enterprise, (Nova Publishers, Hauppauge NY, 2007) pp. 29-39, arxiv:gr-qc/0311030. More references: Phys.Lett.A 292 (2001) 49; Phys.Rev.D 65 (2002) 064025; Int. J. Mod. Phys. D. 14, 12 (2005).

[5] D. Bini, C. Cherubini, C. Chicone and B. Mashhoon Gravitational Induction, arxiv:gr-qc/0803.0390v2.

[6] Spaniol, E. P.; Andrade, V. C.. International Journal of Modern Physics D 19, 489; (2010).

[7] J.W. Maluf, F.F. Faria and S.C. Ulhoa, Classical and Quantum Gravity 34, 2743 (2007). F. H. Hehl, J. Lemke and E. W. Mielke, Two Lectures on Fermions and Gravity, in Geometry and Theoretical Physics, edited by J. Debrus and A. C. Hirshfeld (Springer, Berlin Heidelberg, 1991).

[8] V. C. de Andrade and J. G. Pereira, Phys. Rev. D 56, 4689 (1997).

[9] V. C. de Andrade, L. C. T. Guillen and J. G. Pereira, Phys. Rev. Lett. 84, 4533 (2000).

[10] J. W. Maluf, J. Math. Phys. 35, 335 (1994).

[11] J. B. Hartle, Gravity: An Introduction to Einsteins General Relativity (Addison-Wesley, San Francisco, 2003), p. 198.

[12] C. Schmid, Phys.Rev.D 74, 044031 (2006).