Exactly solvable multichannel Kondo-lattice model

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In this work, a multichannel Kondo-lattice model is studied in the thermodynamic limit. The conduction band is described by a constant hopping amplitude between any pair of lattice sites. For this system we have obtained the exact thermodynamical properties and the ground-state energies. In the limit of strong interaction between the conduction electrons and the impurity spins, the wavefunctions take the Jastrow product form.
1 Introduction

The Kondo-lattice model consists of one or several electronic conduction bands (channels) interacting with some impurities through a spin exchange term [1, 2]. This model is relevant for the studies of the so-called Kondo-insulators and heavy fermion rare-earth compounds. There are considerable recent interests in the study of magnetic impurity effects in conduction electrons [3-7].

For one dimensional Kondo-lattice, various numerical and theoretical investigations were carried out [8-13]. It was found that at half-filling any arbitrarily small interaction would make the system an insulator [8-13]. One interesting open question is whether dimensionality plays a role in this Kondo-insulator, similar to the Mott-insulator due to electron-electron correlation.

In this work, we consider a Kondo-lattice model with multichannel conduction bands. In the extreme situation, where the conduction electrons have constant hopping amplitude between any pair of lattice sites, we show that one can rigorously construct the free energy of the system in the thermodynamic limit. This generalizes the previous result [14] to the multichannel case. In particular, at zero temperature, our exact solutions demonstrate interesting metal-insulator phase transitions in the system when the filling numbers of the electrons vary. In the limit of infinite interaction, the ground-states take Jastrow product form. It remains unclear how to define effective masses of electrons in this simplified model, and how to study heavy mass for
the electrons induced by the impurity spins. Nevertheless, in this simple case, one can write the exact solutions for the ground-state and the free energy, and we report these results and their derivations in the following sections.

2 The multichannel Kondo-lattice model

The hamiltonian of the \( M \)-channel Kondo model on a lattice with exactly one impurity spin at each site and an arbitrary number of orbital conduction electrons has the form

\[
H = H_0 + JV
\]

where \( H_0 \) is the kinetic energy of the electrons, \( J \) is the coupling constant, and the spin-spin interaction term \( V \) is given by

\[
V = \sum_{i=1}^{L} \; \sum_{m=1}^{M} \; \vec{S}_i^f \cdot \vec{S}_{im}^e
\]

with \( \vec{S}_i^f \) the spin operator for the impurity at the site \( i \), \( L \) the number of sites, \( m \) labels the channels, and \( \vec{S}_{im}^e \) the spin operator for the electron at the site \( i \) in the \( m \)-th band.

Introducing the creation and annihilation fermionic operators \((c_{i\sigma m}^+, c_{i\sigma m})\) for an electron at site \( i \), in the \( m \)-th band, with spin \( \sigma \), we have

\[
\vec{S}_{im}^e = \frac{1}{2} \sum_{\alpha,\beta=\uparrow,\downarrow} c_{i\alpha m}^+ \vec{\sigma}_{\alpha\beta} c_{i\beta m}
\]

where \( \vec{\sigma} \) are the Pauli matrices. Similarly we introduce creation and annihilation fermionic operators \((f_{i\sigma}^+, f_{i\sigma})\) for an impurity at site \( i \) with spin \( \sigma \) and
express the spin of the impurity at site \( i \) as

\[ \vec{S}_i = \frac{1}{2} \sum_{\alpha, \beta = \uparrow, \downarrow} f_{i\alpha}^{+} \vec{\sigma}_{\alpha \beta} f_{i\beta}. \]  

(4)

Clearly the \( c \)-operators commute with the \( f \)-operators.

Let \( \mathcal{H} \) denote the Hilbert space with an arbitrary (0, 1, or 2) number of impurities at each site. The condition that there is exactly one impurity at each site means that we restrict ourselves to the subspace \( \bar{\mathcal{H}} \) where

\[ \sum_{\sigma = \uparrow, \downarrow} f_{i\sigma}^{+} f_{i\sigma} = 1, \text{ for all } i = 1, \ldots, L. \]  

(5)

In our model we consider a kinetic energy \( H_0 \) with constant hopping amplitude \( t \), i.e.

\[ H_0 = -t \sum_{1 \leq i \neq j \leq L} \sum_{\sigma = \uparrow, \downarrow} \sum_{m=1}^{M} c_{i\sigma m}^{+} c_{j\sigma m}. \]  

(6)

Because of the special form of \( H_0 \), the dimensionality of the lattice is irrelevant and the system is basically one dimensional. In the next section we shall thus discuss the one-dimensional system in the thermodynamic limit \( L \to \infty \).

Let us introduce the Fourier transform of the electronic operators by

\[ c_{k\sigma m}^{+} = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{ikj} c_{j\sigma m}^{+} \text{ and } c_{k\sigma m} = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{-ikj} c_{j\sigma m} \text{ where } k \text{ is in the first Brillouin zone.} \]

In terms of these operators, the kinetic part \( H_0 \) of the hamiltonian is diagonal

\[ H_0 = \sum_{k \in FBZ} \sum_{\sigma = \uparrow, \downarrow} \sum_{m=1}^{M} \epsilon(k) c_{k\sigma m}^{+} c_{k\sigma m} \]  

(7)

with the dispersion relation

\[ \epsilon(k) = -t'L\delta_{0k} + t \]  

(8)
and $t' = t$. However, in the following, we will consider $t'$ and $t$ as two independent parameters.

### 3 Cluster expansion

Although we are not able to diagonalize the hamiltonian (1) for the finite system, we can, as in [14] or [15], use a cluster expansion to obtain the grand-canonical potential $\Omega/L$ in the thermodynamic limit. The potential $\Omega$ is defined by

$$\Omega = -\frac{1}{\beta} \ln Z$$

(9)

with $Z$ the grand-canonical partition function

$$Z = \text{Tr}_{\mathcal{H}} e^{-\beta K}$$

(10)

where

$$K = H - \sum_{m=1}^{M} \mu_m \hat{N}_m^e = K_0 + JV$$

$$K_0 = H_0 - \sum_{m=1}^{M} \mu_m \hat{N}_m^e.$$  

(11)

$\mu_m$ and $\hat{N}_m^e$ are the chemical potential and the number of electrons operator of the $m$-th band.

In the definition of $Z$, the trace is performed over the Hilbert space $\mathcal{H}$ where at each site there is exactly one impurity. It is however more convenient to work in the Hilbert space $\mathcal{H}$. For this, following [16], we introduce the
impurity isospin at site $i$ by

$$\tilde{\tau}_i^f = \frac{1}{2} \sum_{\alpha, \beta = \uparrow, \downarrow} \tilde{f}_{i\alpha}^* \tilde{\sigma}_{\alpha\beta} \tilde{f}_{i\beta}$$  \hspace{1cm} (12)$$

where $\tilde{f}_{i}$ is the Nambu spinor:

$$\tilde{f}_{i} = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^* \end{pmatrix}. \hspace{1cm} (13)$$

Let us consider the sum of the spin and of the isospin at site $i$: $\tilde{S}_i^f + \tilde{\tau}_i^f$. In the subspace where at site $i$ we have exactly one impurity, $\tilde{S}_i^f$ acts as a $1/2$ spin operator and $\tilde{\tau}_i^f$ as the zero operator. In the subspace where at site $i$ we have zero or two impurities, $\tilde{S}_i^f$ acts as the zero operator and $\tilde{\tau}_i^f$ as a $1/2$ spin operator (the zero impurity spin state corresponding to a down spin and the two impurities state corresponding to a up spin). Thus, if we replace in $K$ the spin by the sum of the spin and of the isospin, we have $2^L$ different subspaces in which this new operator acts similarly. This allows us to rewrite the partition function with a trace on the whole Hilbert space:

$$Z = Tr_He^{-\beta K(S^f)} = \frac{1}{2^L} Tr_He^{-\beta K(S^f + \tilde{\tau}^f)}. \hspace{1cm} (14)$$

Instead of using the expressions (4) and (12) for the definitions of the spin and of the isospin, we prefer to use Majorana fermions. For this, we decompose the $f$-spinor in real and imaginary part by:

$$\begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\eta_i^1 + i\eta_i^2 \\ \eta_i^3 + i\eta_i^0 \end{pmatrix} \hspace{1cm} (15)$$
where \( \eta^0_i, \eta^1_i, \eta^2_i \) and \( \eta^3_i \) are the Majorana fermions. These operators are hermitian and satisfy anticommutation relations: \( \{ \eta^a_i, \eta^b_j \} = \delta_{ij} \delta_{ab} \). We define their Fourier transform

\[
\eta^a_k = \frac{1}{\sqrt{L}} \sum_{j=1}^{L} e^{ikj} \eta^a_j
\]

which satisfy the anticommutation relations \( \{ \eta^a_k, \eta^b_{k'} \} = \delta_{k,-k'} \delta_{ab} \). In terms of the Majorana fermions the sum of the spin and of the isospin has a simple form:

\[
\vec{S}^f_i + \vec{\tau}^f_i = -\frac{i}{2} \vec{\eta}_i \wedge \vec{\eta}_i
\]

where \( \vec{\eta}_i = (\eta^1_i, \eta^2_i, \eta^3_i) \). Using the commutation relations of the Pauli matrices \( (\vec{\sigma} = -(i/2)\vec{\sigma} \wedge \vec{\sigma}) \), we can rewrite the spin exchange term as

\[
V = -\frac{1}{8} \sum_{i=1}^{L} \sum_{\alpha,\beta=\uparrow,\downarrow} \sum_{m=1}^{M} c^\dagger_{\alpha m} c_{\beta m} (\vec{\sigma} \wedge \vec{\sigma})_{\alpha\beta} (\vec{\eta}_i \wedge \vec{\eta}_i).
\]

Let us now apply the cluster expansion method to the thermodynamic potential:

\[
\Omega = \Omega_0 - \frac{1}{\beta} \sum_{n=1}^{\infty} J^n W_n
\]

with \( \Omega_0 = -\frac{1}{\beta} \ln \left( \frac{1}{2\pi} Tr Ke^{-\beta K_0} \right) \) and

\[
W_n = (-1)^n \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{n-1}} d\tau_n < V(\tau_1)V(\tau_2)\cdots V(\tau_n)>_0.
\]

\( V(\tau) \) is the free Heisenberg representation of the potential, and \( < \cdots >_0 \) denotes the grand-canonical average of the system without interaction, taken only over connected diagrams:

\[
V(\tau) = e^{\tau K_0} V e^{-\tau K_0}
\]

\[
< V(\tau_1)V(\tau_2)\cdots V(\tau_n)>_0 = Tr_H(\rho_0 V(\tau_1)V(\tau_2)\cdots V(\tau_n))
\]
where $\rho_0 = e^{-\beta K_0} / \text{Tr}_H e^{-\beta K_0}$ is the density matrix of the model without interaction.

Since $K_0$ is diagonal in the Fourier space, we have for the electronic operators

$$c^+_{k\sigma m}(\tau) = e^{(\epsilon(k) - \mu_m)\tau} c^+_{k\sigma m} \tag{23}$$
$$c_{k\sigma m}(\tau) = e^{-(\epsilon(k) - \mu_m)\tau} c_{k\sigma m}. \tag{24}$$

Let us remark that $c^+_{k\sigma m}(\tau)$ and $c_{k\sigma m}(\tau)$ are no more adjoint to each other. Moreover the Majorana fermions commute with $K_0$ and thus they do not depend on $\tau$:

$$\vec{\eta}_i(\tau) = \vec{\eta}_i. \tag{25}$$

This allows us to write explicitly the interaction term $V(\tau)$ in the Fourier space:

$$V(\tau) = -\frac{1}{8L} \sum_{k_1 k_2 k_3 k_4 \in F\text{BZ}} \sum_{\alpha, \beta = \uparrow, \downarrow} \sum_{m=1}^M \delta_{k_1 - k_2 + k_3 + k_4, 0} c^+_{k_1\alpha m}(\tau) c_{k_2\beta m}(\tau) (\vec{\sigma} \land \vec{\sigma})_{\alpha\beta} (\vec{\eta}_{k_3} \land \vec{\eta}_{k_4}). \tag{26}$$

In order to calculate the grand-canonical averages, we use Wick’s theorem, which tells us that any average of product of operators can be expressed as a sum of product of averages of two operators. We can compute these averages explicitly:

$$< c^+_{k\sigma m} \eta_k^a >_0 = < c_{k\sigma m} \eta_k^a >_0 = 0$$
$$< c^+_{k\sigma m} c^{+\sigma'_{m'}} >_0 = < c_{k\sigma m} c^{\sigma'_{m'}} >_0 = 0$$
$$< c^+_{k\sigma m} c^{\sigma'_{m'}} >_0 = \delta_{kk'} \delta_{\sigma\sigma'} \delta_{mm'} \frac{e^{-\beta(\epsilon(k) - \mu_m)}}{1 + e^{-\beta(\epsilon(k) - \mu_m)}}$$
\[
\langle c_{k\sigma m}c^+_{k'\sigma' m'} \rangle_0 = \frac{1}{1 + e^{-\beta (\epsilon(k) - \mu_m)}}
\]
\[
\langle \eta^a_k \eta^b_{k'} \rangle_0 = \frac{1}{2} \delta_{k, -k'} \delta_{ab}.
\]  
(27)

Since \( \epsilon(k) = -t' L \delta_{0k} + t \), these averages do not depend on \( t' \) except those with momentum zero. However, in the thermodynamic limit, and for \( t' > 0 \), we have

\[
\lim_{L \to \infty} \langle c^+_{0\sigma l}(\tau_i) c_{0\sigma l}(\tau_j) \rangle_0 = \begin{cases} 
1 & \tau_i = \tau_j \\
0 & \tau_i > \tau_j 
\end{cases}
\]

(28)

On the other hand, for \( t' < 0 \),

\[
\lim_{L \to \infty} \langle c_{0\sigma l}(\tau_i) c^+_{0\sigma l}(\tau_j) \rangle_0 = \begin{cases} 
1 & \tau_i - \tau_j = \beta \\
0 & \tau_i - \tau_j < \beta 
\end{cases}
\]

(29)

Therefore from Eq. (20), (28), (29), the coefficients \( W_n, n \geq 1 \), become independent of \( t' \) in the thermodynamic limit. Assuming we can permute the limit \( L \to \infty \) and the sum over \( n \), we are left with the corresponding expression for the model, with \( t' = 0 \):

\[
\Omega = -2MLt' \theta(t') - \frac{L}{\beta} \ln \left( 2 \prod_{m=1}^{M} (1 + e^{\beta(\mu_m - t)})^2 \right) - \frac{1}{\beta} \sum_{n=1}^{\infty} J^n W_n(t' = 0) + O(1)
\]

\[
= -2MLt' \theta(t') + \Omega(t' = 0) - \frac{1}{\beta} \sum_{n=1}^{\infty} J^n W_n(t' = 0) + O(1)
\]

\[
= -2MLt' \theta(t') + \Omega(t' = 0) + O(1)
\]

(30)
where $\theta(t') = 1$ if $t' > 0$ and $\theta(t') = 0$ if $t' < 0$. $\Omega(t' = 0)$ is the thermodynamic potential for the system with $t' = 0$.

We have thus shown that in the thermodynamic limit the potential is made of two contributions in which the dependence in $t'$ and $J$ separate. The first contribution is trivial and we can compute explicitly the second one, since it is site by site diagonal. In the next section, this fact will be used to obtain the ground-state energy.

## 4 Ground-state energy and phase transitions

In this section, we study the zero temperature properties of the system. The ground-state energies will be obtained as functions of electron filling numbers. As the filling numbers vary, one finds metal-insulator phase transitions.

From Eq.(30), the number of electrons in each band and the energy are given by

$$N_m = \left( - \frac{\partial \Omega}{\partial \mu_m} \right)_\beta = \left( - \frac{\partial \Omega(t' = 0)}{\partial \mu_m} \right)_{\beta \mu_1 \ldots \mu_{m-1} \mu_{m+1} \ldots \mu_M} = N_m(t' = 0) \quad (31)$$

$$E = \sum_{m=1}^{M} \mu_m N_m + \left( \frac{\partial \beta \Omega}{\partial \beta} \right)_{\mu_1 \ldots \mu_M} = -2t' M L \theta(t') + t \sum_{m=1}^{M} N_m + E^{int}, \quad (32)$$

where

$$E^{int}(\beta, \mu_1, \ldots, \mu_M) = J \sum_{i=1}^{L} \sum_{m=1}^{M} < \vec{S}_{im} \cdot \vec{S}_{f i} >_i \quad (33)$$

and

$$< A >_i = Tr A \rho_i$$
\[ \rho_i = \frac{e^{-\beta H_i}}{\text{Tr}e^{-\beta H_i}} \]
\[ H_i = \sum_{m=1}^{M} \left[ J \hat{S}_{eim} \cdot \hat{S}_{fi} + (t - \mu_m) \hat{N}_{im} \right]. \tag{34} \]

We are thus left with a system of spins with only on-site interaction.

At this point several definitions of the ground-state can be introduced. In the grand-canonical ensemble, taking \( \beta \to \infty \) with \((\mu_1, \ldots, \mu_M)\) fixed, we obtain only some specific values for the electron density \( n \). For example for a single band and \( J > 0 \), we have

\[
\begin{align*}
n = 0 & \quad \text{for } \mu < t - \frac{3J}{4} \\
n = \frac{1}{3} & \quad \text{for } \mu = t - \frac{3J}{4} \\
n = 1 & \quad \text{for } \mu \in [t - \frac{3J}{4}, t + \frac{3J}{4}] \\
n = \frac{5}{3} & \quad \text{for } \mu = t + \frac{3J}{4} \\
n = 2 & \quad \text{for } \mu > t + \frac{3J}{4}.
\end{align*}
\tag{35} \]

In the canonical ensemble we can consider the ground-states either for fixed \((N_1, \ldots, N_M)\), or for fixed \( N = \sum_{m=1}^{M} N_m \), with respect to the hamiltonian

\[
H = -2t'L M \theta(t') + t \sum_{m=1}^{M} N_m + E^{\text{int}}
\]
\[
E^{\text{int}} = J \sum_{i=1}^{L} \sum_{m=1}^{M} \hat{S}_{eim} \cdot \hat{S}_{fi}. \tag{36} \]

Therefore we only need to compute the ground-state energy of the spin-exchange interaction, which we shall now obtain explicitly for one and two bands.
For a single band, we have to diagonalize the on-site exchange operator $\vec{S}_i \cdot \vec{S}_i^f$. For a given site $i$, the size of the Hilbert space is 8. The four basis states with zero or two electrons on this site have energy zero. For the four states with one electron we have a triplet with energy $J/4$ and a singlet with energy $-3J/4$. Using these results, we recover immediately the ground-state energy obtained in [14], both for the antiferromagnetic ($J > 0$) and the ferromagnetic ($J < 0$) case, i.e. for the antiferromagnetic case:

$$E_{GS}^{int} = \begin{cases} 
-\frac{3J}{4}N & N \leq L \\
-\frac{3J}{4}(2L - N) & N \geq L,
\end{cases}$$

(37)

for the ferromagnetic case:

$$E_{GS}^{int} = \begin{cases} 
\frac{J}{4}N & N \leq L \\
\frac{J}{4}(2L - N) & N \geq L.
\end{cases}$$

(38)

For the two bands case, we have to diagonalize the on-site operator $\vec{S}_i^e(1) \cdot \vec{S}_i^f + \vec{S}_i^e(2) \cdot \vec{S}_i^f$. The size of the Hilbert space for the site $i$ is 32. The 8 states with either zero or two electrons in each band at site $i$ have energy zero since the electronic spins are zero. For the 16 states with one band occupied by zero or two electrons and the other band occupied by one electron, we have again a triplet with energy $J/4$ and a singlet with energy $-3J/4$, since the energy for the band with zero or two electrons is zero. Finally, for the 8 states with exactly one electron in each band, we have a quadruplet with energy $J/2$, a doublet with energy $-J$ and a doublet with energy zero. The ground-state energy $E_{GS}^{int}$ for the system with $N_1$ electrons in band 1 and $N_2$
electrons in band 2 can thus be computed explicitly and is given by $E_\alpha$ for $(N_1, N_2)$ in the domain $D_\alpha$, $\alpha = 1, \ldots, 8$, represented in figure 1 below.

Figure 1: Ground-state energy for $J > 0$ (a) and for $J < 0$ (b). Solid lines indicate filling numbers where phase transitions occur.

For the antiferromagnetic case ($J > 0$), the ground-state energies are given by

\begin{align*}
E_1 &= -\frac{3}{4}J(N_1 + N_2) \\
E_2 &= -\frac{1}{4}J(2L + N_1 + N_2) \\
E_3 &= -\frac{1}{4}J(6L - N_1 - N_2) \\
E_4 &= -\frac{3}{4}J(4L - N_1 - N_2) \\
E_5 &= -\frac{3}{4}J(2L - N_1 + N_2) \\
E_6 &= -\frac{1}{4}J(4L - N_1 + N_2) \\
E_7 &= -\frac{1}{4}J(4L + N_1 - N_2) \\
E_8 &= -\frac{3}{4}J(2L + N_1 - N_2)
\end{align*}

(39)
and for the ferromagnetic case \((J < 0)\)

\[
\begin{align*}
E_1 &= E_2 = \frac{1}{4}J(N_1 + N_2) \\
E_3 &= E_4 = \frac{1}{4}J(4L - N_1 - N_2) \\
E_5 &= E_6 = \frac{1}{4}J(2L - N_1 + N_2) \\
E_7 &= E_8 = \frac{1}{4}J(2L + N_1 - N_2).
\end{align*}
\]

(40)

The differences between the two cases \(J > 0\) and \(J < 0\) can be seen as follows. In the domain \(D_1\) where \(N_1 + N_2 \leq L\), the lowest energy states are obtained with zero or one electron at each site in any of the two bands, which gives \(E_1 = -\frac{3}{4}JN\) if \(J > 0\) and \(E_1 = \frac{1}{4}N\) if \(J < 0\). On the other hand in the domain \(D_2\) we have \((2L - N)\) sites with one electron and \((N - L)\) sites which are doubly occupied with one electron in each band and total electron spin 1. In this domain \(D_2\), if \(J > 0\) the total electrons plus impurity spin is 1/2, the three spins form a doublet, and the energy is \(E_2 = -\frac{J}{4}(2L + N)\); in this case \(dE_{GS}^{int}/dN\) is discontinuous at \(N = L\). However if \(J < 0\), the total electrons plus impurity spin is 3/2, the three spins form a quadruplet, and the energy is \(E_2 = \frac{1}{4}JN\); in this case there is no discontinuity of \(dE_{GS}^{int}/dN\) at \(N = L\). It is interesting to note that even though the two electronic spins are not directly coupled in the hamiltonian, they do interact indirectly via the impurity spin. For a system with \(N_1 + N_2 = N\) fixed, it is easily seen that the ground-state is obtained for \((N_1, N_2)\) in the domains \(D_\alpha, \alpha = 1, 2, 3, 4\).

The exact expressions for the ground-state energies allow us to study the conducting properties of the system at zero temperature. For the single band case, following an idea due to Mattis [17, 18], we define \(\mu^+ = E_{GS}(N + 1) - E_{GS}(N)\), \(\mu^- = E_{GS}(N) - E_{GS}(N - 1)\) and \(\Delta \mu = \mu^+ - \mu^-\). If \(\Delta \mu = 0\),
the system is conducting; if $\Delta \mu > 0$, the system is insulating due to the apparition of a gap $\Delta \mu$. For a single band, it follows from (37) and (38) that $\Delta \mu = 0$ except for $N = L$ where $\Delta \mu = 3J/2$ if $J > 0$ and $\Delta \mu = -J/2$ if $J < 0$; therefore, as shown in [14], the system is always conducting except at the point $N = L$ where it is insulating. For the two bands case, if we consider the ground-states with $N_1$ and $N_2$ fixed, we have to introduce two parameters $\Delta \mu_1$ and $\Delta \mu_2$ referring to the two bands,

$$\Delta \mu_1 = E_{GS}(N_1 + 1, N_2) - 2E_{GS}(N_1, N_2) + E_{GS}(N_1 - 1, N_2)$$
$$\Delta \mu_2 = E_{GS}(N_1, N_2 + 1) - 2E_{GS}(N_1, N_2) + E_{GS}(N_1, N_2 - 1).$$

The system is insulating if and only if a gap appears in the two bands, i.e. if $\Delta \mu_1 > 0$ and $\Delta \mu_2 > 0$.

For the antiferromagnetic case, the system is insulating at the point $N_1 = N_2 = L$ where $\Delta \mu_1 = \Delta \mu_2 = \frac{J}{2}$ and on the four lines $N_1 + N_2 = L$, $N_1 - N_2 = L$, $N_2 - N_1 = L$, $N_1 + N_2 = 3L$ where $\Delta \mu_1 = \Delta \mu_2 = \frac{J}{2}$. We can thus conclude that a metal-insulator transition occurs on the four heavy lines of figure 1(a) and at the point $N_1 = N_2 = L$, as we turn on the antiferromagnetic interaction between electrons and impurities. For the ferromagnetic case, the system is insulating only at the point $N_1 = N_2 = L$ where $\Delta \mu_1 = \Delta \mu_2 = -\frac{J}{2}$ and thus at this point a metal-insulator transition occurs as the ferromagnetic interaction is introduced. Similarly, if we consider the ground-states with $N_1 + N_2 = N$ fixed, then for $J > 0$ the system is insulating if $N = L, 2L, 3L$, while for $J < 0$ it is insulating only for $N = 2L$. 

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5 Ground-state wavefunctions at $J = \infty$

In this section, we consider a finite system $\Lambda$ with $L$ sites and an arbitrary number $M$ of electronic bands. We want to obtain the ground-state wavefunctions for the case $t < 0$ and in the limit $J = \infty$.

If the total number of electrons is smaller than the number of sites, i.e.

$$ N = \sum_{m=1}^{M} N_m < L, \quad (42) $$

then each electron will attempt to form a singlet of energy $-3J/4$ with an impurity. Therefore in the limit $J = \infty$ we can reduce the Hilbert space to the subspace where at each site there is either a singlet or an unpaired impurity spin. A general basis for this subspace is given by the following vectors

$$ | \alpha > = | X_1, \ldots, X_M; \sigma > = \prod_{m=1}^{M} \prod_{x \in X_m} \frac{1}{\sqrt{2}} (c^{+}_{x\uparrow m} f^{+}_{x\downarrow m} - c^{+}_{x\downarrow m} f^{+}_{x\uparrow m}) \prod_{n=1}^{L-N} f^{+}_{y_n, \sigma_n} | 0 > \quad (43) $$

where the $N_m$ singlets with electron in the band $m$ are located at the positions $X_m = (x_{1m}, \ldots, x_{Nm})$, $X_m \cap X_{m'} = \emptyset$, and the unpaired impurity spins $\sigma = (\sigma_1, \ldots, \sigma_{L-N})$ are located at the positions $Y = \Lambda \setminus \cup_m X_m = (y_1, \ldots, y_{L-N})$ with $y_1 < y_2 < \ldots < y_{L-N}$.

With $P$ the projector onto this $J = \infty$ subspace, the projected hamiltonian has the form

$$ PHP = PH_0 P - \frac{3J}{4} N \quad (44) $$
where \( H_0 \) is the kinetic term. Moreover, we can then forget the (infinite) constant and consider only the term \( PH_0P \). To find the ground-state wavefunctions, we identify the singlets as spinless bosons and the unpaired impurities as spin 1/2 fermions. Let us then introduce the hamiltonian

\[
h = -\frac{t}{2} \sum_{i \neq j=1}^{L} \sum_{\sigma=\uparrow,\downarrow} \sum_{m=1}^{M} P_G F_{i\sigma} B_{im}^+ F_{j\sigma}^+ B_{jm} P_G
\]  

(45)

where the \( B \) fields are bosonic, and the \( F \) fields are fermionic. The \( B \) fields commute with the \( F \) fields and \( \sum_{i=1}^{L} B_{im}^+ B_{im} = N_m \). The Gutzwiller projector \( P_G \) projects on the subspace where at each site there is exactly one particle, either one fermion or one boson, i.e.

\[
\sum_{m=1}^{M} B_{im}^+ B_{im} + \sum_{\sigma} F_{i\sigma}^+ F_{i\sigma} = 1.
\]  

(46)

For this system, the basis vectors can be taken as follows

\[
|\bar{\alpha}\rangle = \prod_{m=1}^{M} \left[ \prod_{x \in X_m} B_{xm}^+ \right] \prod_{n=1}^{L-N} \prod_{\sigma_{yn}} F_{yn,\sigma_{yn}}^+ |0\rangle.
\]  

(47)

The two systems defined respectively by the hamiltonians \( PH_0P \) and \( h \) are isomorphic, since there is a one to one correspondence \( |\alpha\rangle \rightarrow |\bar{\alpha}\rangle \) between the basis vectors and the matrix elements of the two hamiltonians are the same

\[
<\beta|PH_0P|\alpha> = <\bar{\beta}|h|\bar{\alpha}>=0.
\]  

(48)

As for the Hubbard model \([19, 20]\), we try to write the hamiltonian as a positive definite form to determine the ground-state energy. Using the
relations

\[ P_G F_{i\sigma} B_{im} F_{j\sigma} B_{jm} P_G = -F_{j\sigma} B_{jm}^+ P_G F_{i\sigma} B_{im} \]  
\[ P_G B_{im}^+ B_{im} P_G = F_{i\sigma} B_{im}^+ P_G F_{j\sigma} B_{jm} \] \hspace{0.5cm} (49)

we can rewrite the Hamiltonian \( h \) in the following way:

\[ h = -\frac{t}{2} \sum_{i,j=1}^{L} \sum_{\sigma=\uparrow,\downarrow} F_{i\sigma} B_{im}^+ P_G F_{j\sigma}^+ B_{jm} + tP_G \sum_{m=1}^{M} \hat{N}_m P_G. \] \hspace{0.5cm} (50)

The first part of \( h \) is positive definite since \( t < 0 \), and the second part is a constant.

If \( N = \sum_m N_m \leq L - 2 \), the ground-state wavefunctions are labelled by the positions \( \vec{Y} = (y_1, \ldots, y_Q) \) and the spins \( \vec{\sigma} = (\sigma_1, \ldots, \sigma_Q) \) of \( L-N-2 = Q \) fermions. These wavefunctions are given by

\[ |\Psi_{GS}(\vec{Y}, \vec{\sigma})> = \sum_{z_1, z_2} \sum_{X_1, \ldots, X_M} F_{z_1\uparrow}^+ F_{z_2\downarrow}^+ \prod_{m=1}^{M} \left[ \prod_{x_m \in X_m} B_{x_m m}^+ \right] \prod_{n=1}^{Q} F_{y_n \sigma_{y_n}}^+ |0> \] \hspace{0.5cm} (51)

where in Eq.\((51)\) we sum over the positions \( z_1, z_2 \) of the two last fermions with opposite spin, and over the positions \( X_1, \ldots, X_M \) of the bosons in each band:

\[ X_m \cap X_{m'} = \emptyset \] if \( m \neq m' \)
\[ X_m \cap \{z_1, z_2\} = \emptyset \]
\[ z_1 \neq z_2. \]

To prove that \((51)\) is indeed a ground-state, one shows that it is annihilated by the first part of \( h \) and therefore \((51)\) is a ground-state wavefunction with energy \( tN \).
If \( N = \sum_m N_m = L - 1 \), the ground-state wavefunctions have only one delocalized fermion and are parametrized by the spin \( \sigma \) of this fermion. They are given by

\[
| \Psi_{\text{GS}}^{\sigma} > = \sum_{z} \sum_{X_1, \ldots, X_M} \prod_{m=1}^{M} F_{z \uparrow}^{+} \prod_{x_m \in X_m} B_{x_m m}^{+} | 0 >
\]

with

\[
X_m \cap X_{m'} = \emptyset \text{ if } m \neq m'
\]

and the energy is \( \frac{1}{2}(L - 1) \).

As for the single band case [14], if we impose periodic boundary conditions, we can write the ground-state wavefunctions in a Jastrow product form. Let us consider a state with \( N_+ \) fermions with up spin localized at \( Y^+ = (y_1^+, \ldots, y_{N_+}^+) \), \( N_- \) fermions with down spin localized at \( Y^- = (y_1^-, \ldots, y_{N_-}^-) \) and \( Q = L - N_+ - N_- \) holes localized at \( X = (x_1, \ldots, x_Q) \). The ground-state wavefunctions are given by

\[
| \Psi_{\text{GS}} > = \sum_{X,Y^-} \Psi(X,Y^-) \prod_{m=1}^{M} \left( \prod_{j=\sum_{k=1}^{m-1} N_m}^{\sum_{k=1}^{m} N_m} B_{x_j m}^{+} F_{x_j \uparrow} \right) \cdot \prod_{j=1}^{B} F_{Y_j \downarrow}^{+} F_{Y_j \uparrow} \prod_{n=1}^{L} F_{n \uparrow}^{+} | 0 >
\]

with amplitude

\[
\Psi(X, Y^-) = e^{\frac{1}{\hbar}(m_h \sum_i x_i + m_s \sum_j y_j)} \prod_{i < j} d(x_i - x_j) \prod_{i,j} d(y_i^- - x_j) \prod_{i < j} d^2(y_i^- - y_j^-)
\]

(54)
where the function \( d(n) = \sin(n\pi/L) \) and the quantum numbers \( m_h \) and \( m_s \) are integers or half-integers which make sure of the periodic boundary conditions and satisfy the following inequalities

\[
\frac{1}{2}(N_- + Q + 1) \leq m_h \leq L - \frac{1}{2}(N_- + Q + 1) \tag{55}
\]

\[
\frac{1}{2}(N_+ + Q + 1) \leq m_h - m_s + \frac{L}{2} \leq L - \frac{1}{2}(N_+ + Q + 1). \tag{56}
\]

To see that such states are ground-states, one has to verify that they are eigenstates of the hamiltonian (45) with eigenvalues \( tN \). To establish this result, one has first to apply the up-spin part of this hamiltonian and see that \( |\Psi_{GS}\rangle \) is an eigenvector with eigenvalue \( \frac{t}{2}N \). To apply the down-spin part of the hamiltonian, it is more convenient to write the ground-state in terms of holes and up-spins [21]. In these terms, the amplitude \( \Psi(X, Y^+) \) has the same form as \( \Psi(X, Y^-) \) except that \( m_h \) is replaced by \( m_h - m_s + L/2 \) and \( m_s \) by \( L - m_s \). In this manner, it gives an energy \( \frac{t}{2}N \) and the total energy is \( tN \).

6 Summary

The ground-state energy of our Kondo-lattice model was obtained explicitly in the thermodynamic limit for one and two electronic bands. From this solution the insulating or conducting properties of the system were established as a function of the number of electrons in each bands. For the ferromagnetic system with two electronic bands a metal-insulator phase transition appears.
at $N_1 = N_2 = L$, as the interaction between the electrons and the impurities is switched on at zero temperature. For the antiferromagnetic system, new metal-insulator phase transitions also appear for other values of $N_1$ and $N_2$.

For the finite size system, we have shown that the ground-state wavefunctions can be written in the well-known Jastrow product form. It remains unclear how to define the effective masses for electrons hopping with an unconstrained hopping amplitude and whether the impurity spins induce heavy masses for the conduction electrons.

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