Merging of transport theory with TDHF: multinucleon transfer in U+U collisions

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Multinucleon transfer mechanism in the collision of 238U + 238U system is investigated at \( E_{\text{c.m.}} = 833 \) MeV in the framework of the quantal diffusion description based on the stochastic mean-field approach (SMF). Double cross-sections \( \sigma(N,Z) \) as a function of the neutron and proton numbers, the cross-sections \( \sigma(Z) \) and \( \sigma(A) \) as a function of the atomic numbers and the mass numbers are calculated for production of the primary fragments. The calculation indicates the 238U + 238U system may be located at an unstable equilibrium state at the potential energy surface with a slightly negative curvature along the beta stability line on the \((N,Z)\)–plane. This behavior may lead to rather large diffusion along the beta stability direction.

I. INTRODUCTION

It has been recognized that multinucleon transfer in heavy-ion collisions involving massive nuclei provide a suitable mechanism for synthesizing new neutron rich heavy nuclei [1–14]. For this purpose, experimental investigations have been carried out in heavy-ion collision with actinide targets near barrier energies [15–18]. Collisions of massive systems near barrier energies predominantly lead to dissipative deep-inelastic reactions and quasi-fission reactions. In dissipative collisions the most part of the bombarding energy is converted into the internal excitations, and the multinucleon transfer occurs between the projectile and target nuclei. A number of experimental and theoretical investigations have been made of the multinucleon transfer mechanism in heavy-ion collisions near barrier energies. The multi dimensional phenomenological Langevin type dynamical approaches have been developed for describing dissipative collisions between massive nuclear systems [13,19–22]. These phenomenological models provide a qualitative and in some cases semi-quantitative description of the transfer process. Since many years, the time-dependent Hartree-Fock (TDHF) approach has been used for describing the deep-inelastic collisions and the quasi-fission reactions [11,23–28]. The TDHF provides a microscopic description in terms of Skyrme-type energy density functionals. The mean-field theory provides good a description for the most probable dynamical path of the collective motion at low energy heavy-ion-collisions including the one-body dissipation mechanism. However the mean-field theory severely underestimates the fluctuations around the most probable collective path. The particle number projection method of the TDHF indeed shows the fragment mass and charge distributions are largely underestimated for strongly damped collisions [8,29]. The fragment mass and charge distributions observed in symmetric collisions provide a good example for the shortcoming of the mean-field description. In the TDHF calculations of the symmetric collisions, the identities of the projectile and target are strictly preserved, i.e., the mass and charge numbers of the final fragments are exactly same as of those at the initial fragments. The experiments, on the other hand, exhibits broad mass and charge distributions of final fragments around their initial values. The dominant aspect of the data is a broad mass and charge distribution around the projectile and target resulting from multinucleon diffusion mechanism. The description of such large fluctuations requires an approach beyond the mean-field theory. The time-dependent RPA approach of Balian and Veneroni provides a possible approach for calculating dispersion of fragment mass and charge distributions and dispersion of other one-body observables [30–34]. However, this approach has severe technical difficulties in applications to the collisions of asymmetric systems. In this work, we employ the quantal diffusion description based on the stochastic mean-field (SMF) approach to calculate double cross-sections \( \sigma(N,Z) \), the cross-section as function of mass number \( \sigma(A) \) and cross-section as a function of the atomic number \( \sigma(Z) \) of the primary fragments in the collisions of the symmetric 238U + 238U system at \( E_{\text{c.m.}} = 833 \) MeV [35,36]. In the quantal diffusion description, the transport theoretical concepts are merged with the mean-field description of the TDHF. As a result, it is possible to calculate the transport coefficients of macroscopic variables in terms of the mean-field properties provided by the time-dependent wave functions of the TDHF, which is consistent with the fluctuation-dissipation theorem of the non-equilibrium statistical mechanics. In Sec. II, we present a brief description of the quantal nucleon diffusion description of the multinucleon exchange. In Sec. III, we present results of calculations of the cross-sections for production of the primary fragments, and conclusions are given in Sec. IV. Some calculations details are provided in the Appendices.

II. QUANTAL DIFFUSION OF MULTINUCLEON TRANSFERS

In the SMF approach, the dynamics of heavy-ion collisions is described in terms of an ensemble of mean-field events. Each event is determined by the self-consistent mean-field Hamiltonian of that event with the initial conditions specified by the thermal and quantal fluctuations at the initial state. We consider uranium-uranium collisions at bombarding ener-
gies near Coulomb barrier. During the collision, the projectile and the target form a di-nuclear complex and interact mainly by multinucleon exchanges. Because of the di-nuclear structure, rather than generating an ensemble of stochastic mean-field events, it is possible to describe the dynamics in terms of several relevant macroscopic variables, such as neutron and proton numbers of the one side of the complex and relative momentum of projectile-like and the target-like fragments. It is possible to deduce the Langevin-type transport description for the macroscopic variables \([37,38]\) and calculate transport coefficients of the macroscopic variables in terms of the TDHF solutions. In this manner, the SMF approach provides a ground for merging transport theory with the mean-field description. For the detail description of the SMF approach and the applications, we refer the reader the previous publications \([39–44]\). Here we take the neutron \(N_1^\lambda\) and the proton \(Z_1^\lambda\) numbers of the projectile-like fragments as the macroscopic variables. In each event \(\lambda\), the neutron and proton numbers are determined by integrating the nucleon density over the projectile side of the window between the colliding nuclei,

\[
\left(\begin{array}{c}
N_1^\lambda(t) \\
Z_1^\lambda(t)
\end{array}\right) = \int d^3\mathbf{r} \Theta\left[x'(t)\right] \left(\begin{array}{c}
\rho^\lambda_n(\mathbf{r},t) \\
\rho^\lambda_p(\mathbf{r},t)
\end{array}\right),
\]

where \(x'(t) = [y - y_0(t)] \sin \theta + [x - x_0(t)] \cos \theta\). The \((x,y)\)-plane represents the reaction plane with \(x\)-axis being the beam direction in the center of mass frame (COM) of the colliding ions. The window plane is perpendicular to the symmetry axis and its orientation is specified by the condition \(x'(t) = 0\). In this expression, \(x_0(t)\) and \(y_0(t)\) denote the coordinates of the window center relative to the origin of the COM frame, \(\theta(t)\) is the smaller angle between the orientation of the symmetry axis and the beam direction. We neglect fluctuations in the orientation of the window and determine the mean evolution of the window dynamics by diagonalizing the mass quadrupole moment of the system for each impact parameter \(b\) or the initial orbital angular momentum \(\ell\), as described in Appendix A, of Ref. \([40]\). In terms of the TDHF description, it is possible to determine time evolution of the rotation angle \(\theta(t)\) of the symmetry axis. The coordinates \(x_0(t)\) and \(y_0(t)\) of the center point of the window are located at the center of the minimum density slice on the neck between the colliding ions. Since uranium is a deformed nucleus, the outcome of the collisions depends on the relative orientation of the projectile and target. In the present work, we consider two specific collision geometry: (i) the side-side collisions in which deformation axes of the both the projectile and the target are perpendicular to the beam direction and (ii) the tip-tip is collisions in which deformation axes of the both the projectile and the target are parallel to the beam direction. As an example, Fig. 1 shows the density profile in the tip-tip geometry (left panel) and in the side-side geometry (right panel) of the \(^{238}\text{U} + ^{238}\text{U}\) system at \(E_{\text{c.m.}} = 833\) MeV with the initial orbital angular momentum \(\ell = 300\hbar\) at times \(t = 300\text{ fm}/\text{c} , t = 500\text{ fm}/\text{c}\) and 
\(t = 700\text{ fm}/\text{c}\) at tip-tip geometry (left panel) and side-side geometry (right panel).

Below, we briefly discuss the derivation of the Langevin equations for the neutron and proton numbers of the projectile-like fragments, for further details we refer the reader to Refs. \([39–42]\). The rate of changes the neutron and
the proton numbers of the projectile-like fragment are given by,
\[
\frac{d}{dt} \left( \frac{N^k_i(t)}{Z^k_i(t)} \right) = - \int d^3r \Theta \left[ x'(t) \right] \left( \frac{\nabla \cdot \mathbf{J}^k_P(t)}{\mathbf{V} \cdot \mathbf{J}^k_P(t)} \right). \tag{4}
\]

In obtaining this expression we neglect a term arising from the rate of change of the position and the rotation of the window plane and employ the continuity equation, with the fluctuating neutron and proton current densities
\[
J^k_{\alpha}(\vec{r}, t) = \frac{\hbar}{m} \sum_{ij \in \alpha} \text{Im} \left( \Phi^*_{\alpha}(\vec{r}, t; \lambda) \mathbf{V} \Phi^k_{\alpha}(\vec{r}, t; \lambda) \mathbf{p}^k_{\alpha} \right). \tag{5}
\]

By carrying out a partial integration, we obtain a set of coupled Langevin equations for the macroscopic variables $N^k_i(t)$ and $Z^k_i(t)$,
\[
\frac{d}{dt} \left( \frac{N^k_i(t)}{Z^k_i(t)} \right) = \int d^3g(x') \left( \mathbf{\hat{e}} \cdot \mathbf{J}^k_P(t), \mathbf{\hat{e}} \cdot \mathbf{J}^k_{\alpha}(t) \right) = \left( v^k_{\alpha}(t), v^k_P(t) \right), \tag{6}
\]

with $\mathbf{\hat{e}}$ as the unit vector along the symmetry axis with components $\hat{e}_\alpha = \cos \theta$ and $\hat{e}_\xi = \sin \theta$. In the integrand, we replace the delta function by a smoothing function $\delta(x') \rightarrow g(x')$ in terms of a Gaussian $g(x) = \left(1/\kappa \sqrt{2\pi}\right) \exp(-x^2/2\kappa^2)$ with dispersion $\kappa$. The Gaussian behaves almost like delta function for sufficiently small $\kappa$. In the numerical calculations dispersion of the Gaussian is taken in the order of the lattice side $\kappa = 1.0$ fm. The right side of Eq. (6) defines the fluctuating drift coefficients $v^k_{\alpha}(t)$ for the neutrons and the protons. There are two different sources for fluctuations of the drift coefficients: (i) Fluctuations due to different set of wave functions in each event $\lambda$. This part of the fluctuations can be approximately described in terms of the fluctuating macroscopic variables as $v^k_{\alpha}(t) \rightarrow V_{\alpha}(N^k_i(t), Z^k_i(t))$, and (ii) fluctuations introduced by the stochastic part $\delta J^k_{\alpha} = \mathbf{p}^k_{\alpha} - \mathbf{p}_{\alpha}$ of the density matrix at the initial state. In this work, we consider small amplitude fluctuations, and linearize the Langevin Eq. (6) around the mean values of the macroscopic variables $\delta N^k_i = N^k_i - \overline{N}^k_1$ and $\delta Z^k_i = Z^k_i - \overline{Z}^k_1$. The mean values $\overline{N}^k_1 = N^k_1$ and $\overline{Z}^k_1 = Z^k_1$ are determined by the mean-field description of the TDHF approach. Table I and table II show the results of the TDHF calculations for the mean values for a set of observable quantities in the collisions of $^{238}\text{U} + ^{238}\text{U}$ system at $E_{c.m.} = 833$ MeV for the range initial orbital angular momentum $\ell = (100 - 460)\hbar$.

The fluctuations evolve according to the linearized coupled Langevin equations,
\[
\frac{d}{dt} \left( \frac{\delta Z^k_i}{\delta N^k_i} \right) = \left( \frac{\delta v^k_{\alpha}(Z^k_i - \overline{Z}^k_1)}{\delta N^k_1} \right) + \left( \frac{\delta v^k_P(N^k_i - \overline{N}^k_1)}{\delta N^k_1} \right) + \left( \frac{\delta v^k_P(N^k_i - \overline{N}^k_1)}{\delta v^k_{\alpha}} \right), \tag{7}
\]

where the derivatives of drift coefficients are evaluated at the mean values $\overline{N}^k_1$ and $\overline{Z}^k_1$. The linear limit provides a good approximation for small amplitude fluctuations and it becomes even better if the driving potential energy has nearly harmonic behavior around the mean values. The stochastic part $\delta v^k_{\alpha}(t)$ of drift coefficients given by,
\[
\delta v^k_{\alpha}(t) = \frac{\hbar}{m} \sum_{ij \in \alpha} \int d^3g(x') \text{Im} \left( \Phi^*_{\alpha}(\vec{r}, t) \mathbf{\hat{e}} \cdot \mathbf{\hat{v}} \Phi^k_{\alpha}(\vec{r}, t) \delta \mathbf{p}^k_{\alpha} \right). \tag{8}
\]

According to the basic postulate of the SMF approach the stochastic elements of the initial density matrix $\delta \mathbf{p}^k_{\alpha}$ are specified in terms of uncorrelated distributions, then it follows that the stochastic part of the neutron and proton drift coefficients $\delta v^k_{\alpha}(t)$ are determined by uncorrelated Gaussian distributions with variances discussed in the following section.

III. MASS AND CHARGE DISTRIBUTIONS OF THE PRIMARY FRAGMENTS

A. Quantal diffusion coefficients of neutrons and protons

It is well known that Langevin equation for a macroscopic variable is equivalent to the Fokker-Planck equation for the distribution function of the macroscopic variable and the solution is given by a single Gaussian function [48]. When there are two coupled Langevin equations, as we have it in Eq. (7), the solution of the Fokker-Planck equation for the distribution function $P(N, Z)$ of fragments with neutron and proton num-

| & $A^f_1$ & $A^f_2$ & $Z^f_1$ & $Z^f_2$ & $\ell_f(h)$ & $\text{TKE}$ (MeV) & $E^*+\ell_h$ (MeV) & $\theta^{lab}_1$ & $\theta^{lab}_2$ |
|---|---|---|---|---|---|---|---|---|---|
| 100 & 238 & 92.0 & 238 & 92.0 & 73.4 & 527 & 306 & 158 & 48.3 & 9.55 |
| 120 & 238 & 92.0 & 238 & 92.0 & 95.4 & 514 & 319 & 154 & 49.6 & 11.5 |
| 140 & 238 & 92.0 & 238 & 92.0 & 114 & 505 & 328 & 149 & 50.4 & 13.6 |
| 160 & 238 & 92.0 & 238 & 92.0 & 132 & 521 & 312 & 149 & 51.7 & 13.7 |
| 180 & 238 & 92.0 & 238 & 92.0 & 153 & 510 & 323 & 138 & 51.3 & 18.5 |
| 200 & 238 & 92.0 & 238 & 91.7 & 172 & 515 & 317 & 132 & 50.8 & 20.9 |
| 220 & 238 & 92.0 & 238 & 92.0 & 177 & 525 & 318 & 129 & 50.4 & 22.4 |
| 240 & 238 & 92.0 & 238 & 92.0 & 182 & 552 & 281 & 126 & 51.6 & 24.1 |
| 260 & 238 & 92.0 & 238 & 92.4 & 184 & 577 & 256 & 123 & 52.2 & 25.6 |
| 280 & 238 & 92.0 & 238 & 92.0 & 189 & 595 & 238 & 120 & 51.6 & 27.4 |
| 300 & 238 & 92.0 & 238 & 92.0 & 201 & 616 & 217 & 116 & 51.2 & 29.3 |
| 320 & 238 & 92.0 & 238 & 92.0 & 225 & 625 & 208 & 113 & 50.3 & 31.0 |
| 340 & 238 & 92.0 & 238 & 92.0 & 245 & 645 & 188 & 109 & 49.5 & 32.8 |
| 360 & 238 & 92.0 & 238 & 92.0 & 271 & 654 & 179 & 106 & 48.3 & 34.6 |
| 380 & 238 & 92.0 & 238 & 92.0 & 333 & 714 & 119 & 101 & 47.8 & 37.7 |
| 400 & 238 & 92.0 & 238 & 92.0 & 374 & 751 & 82.3 & 98.4 & 47.5 & 39.5 |
| 420 & 238 & 92.0 & 238 & 92.0 & 429 & 797 & 35.9 & 96.2 & 47.4 & 41.4 |
| 440 & 238 & 92.0 & 238 & 92.0 & 439 & 785 & 48.1 & 93.4 & 45.8 & 42.5 |
| 460 & 238 & 92.0 & 238 & 92.0 & 491 & 819 & 14.1 & 91.4 & 45.5 & 44.1 |
and the mixed variances for the neutron out ensemble averaging, we obtain a couple set of equations noting the neutron, proton and mixed dispersions, respectively.

Here the exponent $\ell$ of each value of the initial orbital angular momentum is determined by the TDHF calculations, and the numbers of fragments for each angular momentum determined by the autocorrelation functions of the stochastic part of the drift coefficients as

$$\int_0^\infty dt' \delta \tilde{v}_\alpha(t) \tilde{v}_\alpha(t') = D_{\alpha\alpha}(t).$$  \hspace{1cm} (14)

We can calculate the ensemble averaging by employing the basic postulate of the SMF approach given by Eq. (3). We refer reader to Refs. [39,40] in which a detailed description of the autocorrelation functions are presented. Here, for completeness of the presentation, we give the results for the quantal expression of the proton and the neutron diffusion coefficients,

$$D_{\alpha\alpha}(t) = \int_0^\infty d\tau \int d^3r \bar{g}(x) \left[ G_T(\tau) J_{\alpha\alpha}^T(\bar{r}, t - \tau/2) + G_P(\tau) J_{\alpha\alpha}^p(\bar{r}, t - \tau/2) \right]$$

$$- \int_0^\infty d\tau \text{Re} \left[ \sum_{\ell' \in \ell, \hbar \epsilon} A_{h\ell}^\alpha(t) A_{h\ell}^\alpha(t - \tau) + \sum_{\ell' \in \ell, \hbar \epsilon} A_{h\ell}^\alpha(t) A_{h\ell}^\alpha(t - \tau) \right].$$  \hspace{1cm} (15)

Here $J_{\alpha\alpha}^T(\bar{r}, t - \tau/2)$ represents the sum of the magnitude of current densities perpendicular to the window due to the hole wave functions originating from target.

$$J_{\alpha\alpha}^T(\bar{r}, t - \tau/2) = \frac{\hbar}{m} \sum_{\ell' \in \ell, \hbar \epsilon} \left| \text{Im} \left( \Phi_h^\alpha(\bar{r}, t - \tau/2) \right) \right| \times \left( \hat{\mathbf{e}} \cdot \hat{\mathbf{v}} \Phi_h^\alpha(\bar{r}, t - \tau/2) \right).$$  \hspace{1cm} (16)

and $J_{\alpha\alpha}^p(\bar{r}, t - \tau/2)$ is given by a similar expression in terms of the hole wave functions originating from the projectile. We observe that there is a close analogy between the quantal expression and the diffusion coefficient in a random walk problem [37,38]. The first line in the quantal expression gives the sum of the nucleon currents across the window from the target-like fragment to the projectile-like fragment and from the projectile-like fragment to the target-like fragment, which is integrated over the memory. This is analogous to the random walk problem, in which the diffusion coefficient is given by the sum of the rate for the forward and backward steps. The second line in the quantal diffusion expression stands for the Pauli blocking effects in nucleon transfer mechanism, which does not have a classical counterpart. The quantities in the Pauli blocking factors are determined by

$$A_{h\ell}^\alpha(t) = \frac{\hbar}{2m} \int d^3r g(x) \left( \Phi_h^\alpha(\bar{r}, t) \hat{\mathbf{e}} \cdot \hat{\mathbf{v}} \Phi_h^\alpha(\bar{r}, t) - \Phi_h^\alpha(\bar{r}, t) \hat{\mathbf{e}} \cdot \hat{\mathbf{v}} \Phi_h^\alpha(\bar{r}, t) \right).$$  \hspace{1cm} (17)

The memory kernels $G_T(\tau)$ in Eq. (15) is given by

$$G_T(\tau) = \frac{1}{\sqrt{4\pi \tau}} \exp[-(\tau/2\tau)^2]$$  \hspace{1cm} (18)
with the memory time determined by the average flow velocity \( v_T \) of the target nucleons across the window according to \( \tau_T = \frac{k}{|v_T(\ell)|} \), and \( G_P(\tau) \) is given by a similar expression. In a previous work [40], we estimated the memory time to be about \( \tau_T \approx 25 \text{ fm/c} \), which is much shorter than the contact time of about 600 fm/c. As a result the memory effect is not important in diffusion coefficients. We note that the quantal diffusion coefficients are entirely determined in terms of the occupied single-particle wave functions of the TDHF solutions. According to the non-equilibrium fluctuation-dissipation theorem, the fluctuation properties of the relevant macroscopic variables must be related to the mean properties. Consequently, the evaluation of the diffusion coefficients in terms of the mean-field properties is consistent with the fluctuation-dissipation theorem. Fig. 2 shows neutron and proton diffusion coefficients for the \( { }^{238}\text{U} + { }^{238}\text{U} \) system at \( E_{\text{c.m.}} = 833 \text{ MeV} \) with the initial orbital angular momentum \( \ell = 300\hbar \), for the tip-tip (a) and side-side (b) geometries as function of time.

\[
\begin{align*}
D_{NN} & \quad D_{ZZ} \\
238\text{U} + 238\text{U} & \\
tip-tip & \quad 0.2 \quad 0.15 \quad 0.1 \quad 0.05 \quad 0.05 \quad 0.05 \quad 0.05 \\
D_\alpha (c/\text{fm}) & \\
time (\text{fm/c}) & \\
(a)
\end{align*}
\]

\[
\begin{align*}
D_{NN} & \quad D_{ZZ} \\
238\text{U} + 238\text{U} & \\
side-side & \quad 0 \quad 0.05 \quad 0.1 \quad 0.15 \quad 0.2 \\
D_\alpha (c/\text{fm}) & \\
time (\text{fm/c}) & \\
(b)
\end{align*}
\]

FIG. 2. Neutron and proton diffusion coefficients as a function of time in the \( { }^{238}\text{U} + { }^{238}\text{U} \) collisions at \( E_{\text{c.m.}} = 833 \text{ MeV} \) with the initial orbital angular momentum \( \ell = 300\hbar \) at tip-tip geometry (a) and side-side geometry (b).

Dispersions are determined from the solutions of the coupled differential equations (11-13) in which the diffusion coefficients provide source for development of the fluctuations. In addition to the diffusion coefficients, we also need to determine the derivatives of the drift coefficients with respect to the macroscopic variables \( (N_1, Z_1) \). In order to determine these derivatives, the Einstein’s relations in the over-damped limit provide a possible approach. According to the Einstein relation, drift coefficients are determined by the derivatives of the potential energy surface in the \( (N, Z) \)-plane,

\[
\begin{align*}
\nu_n(t) &= -\frac{D_{NN}}{T^*} \frac{\partial}{\partial N_1} U(N_1, N_1) \\
\nu_p(t) &= -\frac{D_{ZZ}}{T^*} \frac{\partial}{\partial Z_1} U(N_1, Z_1),
\end{align*}
\]

where \( T^* \) indicates effective temperature of the system. Because of the analytical structure, we can immediately take derivatives of the drift coefficients. Since \( { }^{238}\text{U} + { }^{238}\text{U} \) is a symmetric system, the equilibrium state in the potential energy surface is located at the initial position with \( N_1 \rightarrow N_0 = 146 \) and \( Z_1 \rightarrow Z_0 = 92 \). When fluctuations are not too far from the equilibrium point, we can parameterize the potential energy around the equilibrium in terms of two parabolic forms as given by Eq. (A1) in the App. A [50]. One of the parabolic forms extend along the bottom of the beta stability line, which is referred to as the iso-scalar path. The second parabolic form extends towards the perpendicular direction to the iso-scalar path, which is referred to as the iso-vector path. In order to specify the derivatives of the drift coefficients, we need to determine the reduced curvature parameters \( \alpha \) and \( \beta \) of these parabolic potential energy surfaces. Since the symmetric collisions do not exhibit drift in neutron or proton numbers, it is not possible to specify the reduced curvature parameters from the mean trajectory information of the symmetric collisions. As discussed in App. A, we can estimate the iso-vector curvature \( \alpha \) parameter from the central collision of the neighboring \( { }^{88}\text{Ra} + { }^{96}\text{Cm} \) system at \( E_{\text{c.m.}} = 833 \text{ MeV} \). As seen from the drift path of this system in Fig. 7, the system follows the iso-vector path closely and reaches the charge equilibrium rather rapidly during a time interval of \( \Delta \tau \approx 150 \text{ fm/c} \). The iso-vector drift path is suitable to estimate the average value of the iso-vector curvature parameters and we find \( \alpha \approx 0.13 \). After reaching the equilibrium in charge asymmetry rather rapidly, the system spends a long time in the vicinity of \( { }^{238}\text{U} + { }^{238}\text{U} \) by following a curvy path due to complex quantal effect due to shell structure. Eventually, the system has a tendency to evolve toward asymmetry direction along the iso-scalar path, i.e along the beta stability line. It appears that the \( { }^{238}\text{U} + { }^{238}\text{U} \) system is located at an unstable state on the beta stability line with a small and negative curvature parameter \( \beta \) in the iso-scalar direction. It is not possible to provide reasonable estimation for this parameter from the drift path of the \( { }^{88}\text{Ra} + { }^{96}\text{Cm} \) system in Fig. 8 beyond the equilibrium state at \( (N_1 = 146, Z_1 = 92) \). With a negative curvature parameter in the iso-scalar direction, the system may exhibit broad diffusion along the beta stability line. In order obtain a reasonable value for \( \beta \), we employ the cross-section data for production of gold isotopes from a previous investigation of the \( { }^{238}\text{U} + { }^{238}\text{U} \) system at about the same energy [16]. As discussed in Appendix B, we determine a small negative value of \( \beta = -0.02 \) for the reduced iso-scalar curvature parameter. Using these values for the reduced curvature parameters, we can determine the derivative of the drift coefficients as given in Eqs. (A3)-(A6) and calculate the neu-
tron, the proton and the mixed dispersions from the solution of the differential Eqs. (11)-(13). As an example Fig. 3 shows the neutron, the proton and the mixed dispersions as a function of time in the $^{238}\text{U} + ^{238}\text{U}$ collisions at $E_{c.m.} = 833$ MeV with the initial orbital angular momentum $\ell = 300\hbar$ at tip-tip geometry and side-side geometry. The asymptotic values of these dispersions for a range of the initial orbital angular momentum $\ell = (100 - 460)\hbar$ in tip-tip and side-side geometries are given in Table III.

![Image of dispersions](image.png)

**FIG. 3.** Neutron, proton and mixed variance as a function of time in the $^{238}\text{U} + ^{238}\text{U}$ collisions at $E_{c.m.} = 833$ MeV with the initial orbital angular momentum $\ell = 300\hbar$ at tip-tip geometry (a) and side-side geometry (b).

### B. Cross-section of production of primary fragments

We calculate the cross-section for production of a primary fragment with neutron and proton numbers $(N, Z)$ using the standard expression,

$$
\sigma(N, Z) = \frac{\pi \hbar^2}{2\mu E_{c.m.}} \sum_{\ell = \min}^{\ell_{\max}} (2\ell + 1)P_{\ell}(N, Z).
$$

(Eq. 20)

Here, $P_{\ell}(N, Z) = \langle P_{\ell-\ell}^{+}(N, Z) + P_{\ell-\ell}^{-}(N, Z) \rangle / 2$ denotes the mean value of the probability of producing a primary fragment with neutron and proton numbers $(N, Z)$ in the tip-tip and the side-side collisions with the initial angular momentum $\ell$. These probabilities are presented in Eqs. (9)-(10) with the asymptotic values of dispersions given in Table III for tip-tip and side-side collisions. The mean values are equal to their initial values $\mathcal{N} = 146$, $\mathcal{Z} = 92$. The range of the summation over the initial angular momentum is taken as $\ell_{\min} = 300$ and $\ell_{\max} = 460$. This angular momentum range corresponds the experimental set up in which the detector is placed at an angular range $\theta = 35^\circ - 5^\circ$ in the laboratory frame. Fig. 4 shows the double cross-sections $\sigma(N, Z)$ in the $(N, Z)$-plane. We observe the cross-section distribution extends along the bottom of the beta stability and exhibits large dispersion in this direction as a result of the slight negative curvature of the potential energy along the iso-scalar direction. We note that the nucleon diffusion along the beta stability line is rather sensi-

| $\ell$ (\hbar) | $\sigma_{NN}$ | $\sigma_{ZZ}$ | $\sigma_{NZ}$ |
|----------------|-------------|-------------|-------------|
| 400            | 3.02        | 3.24        | 3.13        |
| 420            | 3.12        | 3.34        | 3.23        |
| 440            | 3.22        | 3.46        | 3.35        |
| 460            | 3.32        | 3.58        | 3.47        |

**TABLE III.** Asymptotic values of the neutron, the proton and the mixed dispersions in the $^{238}\text{U} + ^{238}\text{U}$ collisions at $E_{c.m.} = 833$ MeV with the range of orbital angular momentum $\ell = (100 - 460)\hbar$ at tip-tip geometry (left panel) and side-side geometry (right panel).
tive to the magnitude of the reduced iso-scalar curvature parameter $\beta$. Decreasing the magnitude of this parameter, the dispersion of the double cross-section along the beta stability direction is reduced. The cross-sections $\sigma(A)$ as a function of the mass numbers of the primary fragments are given by,

$$\sigma(A) = \frac{\pi h^2}{2\mu E_{c.m.}} \sum_{\ell_{\min}}^{\ell_{\max}} (2\ell + 1) P_\ell(A).$$

Here $P_\ell(A) = (P_{\ell-\ell}(A) + P_{\ell-\ell}(A))/2$ denotes the mean value of the probability of producing a primary fragment with mass numbers $A$ in the tip-tip and the side-side collisions with the initial angular momentum $\ell$. These probabilities are determined by a simple Gaussian functions,

$$P(A) = \frac{1}{\sigma_{AA} \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{A - \bar{A}}{\sigma_{AA}} \right)^2 \right],$$

where the mass dispersion is determined by $\sigma_{AA}^2 = \sigma_N^2 + \sigma_Z^2 + 2\sigma_{NZ}$ and the mean mass number as $\bar{A} = 238$. Fig. 5 shows the cross-sections as a function of the mass numbers of the primary fragments in the tip-tip and the side-side geometries and their mean values. We can calculate the cross-sections $\sigma(Z)$ of production of the primary fragments as a function of the atomic number using an expression similar to Eq. (21) by employing the Gaussian probability with the dispersion and the mean values as given by $\sigma_{ZZ}(Z)$ and $Z = 92$, respectively. Figure 6 shows the cross-sections as a function of the atomic numbers of the primary fragments in the tip-tip and the side-side geometries and their mean values.

![Cross-sections $\sigma(A)$](image1)

**FIG. 5.** Cross-sections $\sigma(A)$ for production of primary fragments as a function of mass number in the collisions of $^{238}\text{U} + ^{238}\text{U}$ at $E_{c.m.} = 833$ MeV in tip-tip, side-side geometries and mean values by dashed, dotted and solid blue lines, respectively.

![Cross-sections $\sigma(Z)$](image2)

**FIG. 6.** Cross-sections $\sigma(Z)$ for production of primary fragments as a function of mass number in the collisions of $^{238}\text{U} + ^{238}\text{U}$ at $E_{c.m.} = 833$ MeV in tip-tip, side-side geometries and mean values by dashed, dotted and solid blue lines, respectively.

**IV. CONCLUSIONS**

We have carried out an investigation of mass and charge distributions of the primary fragments produced in the collisions of the $^{238}\text{U} + ^{238}\text{U}$ system at $E_{c.m.} = 833$ MeV. We calculate the probability distributions of the primary fragments by employing the quanta diffusion description. In the quanta diffusion approach, the concepts of the transport theory are merged with the mean-field description of the TDHF with the help of the SMF approach. It is then possible to express the diffusion coefficients of the relevant macroscopic variables in terms of the occupied single-particle wave functions of the TDHF. Since the Langevin equations of the macroscopic variables are equivalent to the Fokker-Planck description for the distribution of the macroscopic variables, under certain conditions, it is possible give nearly analytical description for the distribution functions of the macroscopic variables and the cross-sections. In the calculations of the cross-sections of production of the primary fragment for each initial angular momentum or equivalently for each impact parameter, we need to determine the mean values of the neutron and proton numbers of the fragments and the neutron, the proton and the mixed dispersions of the distribution functions. The mean values are determined by the TDHF descriptions. The variances are calculated from the solutions of three coupled differential equations in which diffusion coefficients of neutron and protons act as the source terms. The behavior of the potential energy surface of the di-nuclear complex makes an important effect on the neutron and proton diffusion mechanism. It is possible to determine the curvature parameters of the potential energy in the collisions of asymmetric systems from the drift information with the help of the Einstein’s relation in the over-damped limit. Since collisions of the symmetric systems, such as the collisions of $^{238}\text{U} + ^{238}\text{U}$, do not exhibit drift of the neutron and proton degrees of freedom, we need to employ other methods to specify the curvature parameters of the potential energy. In this work, we employ the central collision of a neighboring system $^{236}\text{Ra} + ^{240}\text{Cm}$ at the same bombarding energy. The system initially drifts nearly along the iso-vector direction and reach the charge equilibrium state rather rapidly. From the iso-vector drift information, we can estimate the re-
duced curvature parameter of the potential energy as $\alpha = 0.13$. After reaching the charge equilibration, the system spends a long time in the vicinity of $^{238}\text{U} + ^{238}\text{U}$ state and eventually has a tendency drift along the iso-scalar path away from the symmetric state. This behavior indicates the symmetric $^{238}\text{U} + ^{238}\text{U}$ is located at an unstable equilibrium position with a small negative curvature toward the iso-scalar direction. However, from the drift information it is not possible to estimate the iso-scalar reduced curvature parameter $\beta$. Since the negative curvature may lead to broad diffusion along the beta stability line, it is important to determine this curvature parameter accurately. Therefore, we regard the reduced curvature in the iso-scalar direction as a parameter and estimate its value with the help of the isotopic cross-section data of gold nucleus from a previous investigation of the $^{238}\text{U} + ^{238}\text{U}$ collisions at about the same energy. In this work, we present calculations for production of the primary fragments with the curvature parameters $\alpha = 0.13$ and $\beta = -0.02$. The primary fragments are excited and cool down by the de-excitation processes of particle emission, mostly neutrons and by sequential fission of the heavy fragments. Calculations of the secondary cross-sections exceed the scope of the present work. We plan to investigate the de-excitation process of the primary fragments in the collisions of $^{238}\text{U} + ^{238}\text{U}$ and calculate the secondary cross-sections in a subsequent study.

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Appendix A: CURVATURE PARAMETERS OF THE POTENTIAL ENERGY

The charge asymmetry of uranium $^{238}\text{U}_{146}$ is $(146 - 92)/(146 + 92) = 0.227$. The dashed green line in Fig. 7 represents the nuclei with nearly equal charge asymmetry $(N - Z)/(N + Z) = 0.22 - 0.23$. We refer to this line as the iso-scalar line which extends nearly parallel to the lower part of the beta stability valley in this region. We refer to the dashed red line as the iso-vector line which is perpendicular to the iso-scalar path. We parameterize the potential energy surface in the vicinity of the equilibrium $(N_0 = 146, Z_0 = 92)$ in terms of two parabolic forms along the iso-scalar and iso-vector paths as,

$$U(N_1, Z_1) = \frac{1}{2} a R_S^2(N_1, Z_1) + \frac{1}{2} b R_V^2(N_1, Z_1).$$  \hspace{1cm} (A1)

The vertical distances $R_S$ and $R_V$ of a point $(N_1, Z_1)$ representing a fragment from the iso-scalar and the iso-vector lines, respectively are given by,

$$R_S = (Z_0 - Z_1) \cos \phi + (N_1 - N_0) \sin \phi,$$

$$R_V = (Z_0 - Z_1) \sin \phi - (N_1 - N_0) \cos \phi. \hspace{1cm} (A2)$$

According to the Einstein relation in the over-damped limit neutron and proton drift coefficients are related to the driving potential as,

$$v_n = -\frac{D_{NN} T}{\partial U} \frac{\partial U}{\partial N_1} = -\alpha D_{NN} R_S \sin \phi + \beta D_{NN} R_V \cos \phi$$

$$v_z = -\frac{D_{ZZ} T}{\partial U} \frac{\partial U}{\partial Z_1} = +\alpha D_{ZZ} R_S \cos \phi + \beta D_{ZZ} R_V \sin \phi.$$

Here, the temperature is absorbed in the reduced curvature parameters as $\alpha = a/T$ and $\beta = b/T$. Because of the analytical form, we can readily calculate the derivatives of the drift coefficients to obtain,

$$\frac{\partial v_n(t)}{\partial N_1} = -D_{NN} \left( \beta \cos^2 \phi + \alpha \sin^2 \phi \right)$$

$$\frac{\partial v_n(t)}{\partial Z_1} = +D_{NN} \left( \alpha - \beta \right) \cos \phi \sin \phi \hspace{1cm} (A3)$$

$$\frac{\partial v_p(t)}{\partial Z_1} = -D_{ZZ} \left( \beta \sin^2 \phi + \alpha \cos^2 \phi \right) \hspace{1cm} (A4)$$

$$\frac{\partial v_p(t)}{\partial N_1} = +D_{ZZ} \left( \alpha - \beta \right) \cos \phi \sin \phi. \hspace{1cm} (A5)$$

$$\alpha R_S(t) = \frac{v_z(t)}{D_{ZZ}(t)} \cos \phi - \frac{v_n(t)}{D_{NN}(t)} \sin \phi \hspace{1cm} (A6)$$

The reduced curvature parameters are determined by the drift and the diffusion coefficients as,
and

\[
\beta R_V(t) = \frac{v_c(t)}{D_{ZZ}(t)} \sin \phi + \frac{v_n(t)}{D_{NN}(t)} \cos \phi. \tag{A8}
\]

In collisions of symmetric systems, the drift coefficients vanish and the mean values of the neutron and proton numbers of the fragments are equal to the equilibrium values of the colliding nuclei \( N_1 = N_0, Z_1 = Z_0 \). As a result, it is not possible to determine the reduced curvature parameters from the Eq. (A7) and Eq. (A8).

In order to estimate the reduced curvature parameters, we consider the central collision of a neighboring system of \(^{236}\text{Ra}\) + \(^{240}\text{Cm}\) at the same bombarding energy \( E_{\text{c.m.}} = 833 \text{ MeV} \). We consider \(^{236}\text{Ra}\) as the projectile. Figure 8 shows the neutron number \( N_1(t) \) and the proton number \( Z_1(t) \) as a function of time. Blue line in Fig. 7 shows the drift path of the projectile-like fragments in the \((N,Z)\)-plane. We observe that the system rapidly evolves toward the equilibrium charge asymmetry of the \(^{238}\text{U} + ^{238}\text{U}\) system nearly along the iso-vector direction from the initial state at point A toward the state at point B. This segment of the drift path is suitable to determine the average value of the reduced iso-vector curvature as,

\[
\alpha \int_{t_A}^{t_B} dt [R_5(t)] = \int_{t_A}^{t_B} dt \left[ \frac{v_c(t)}{D_{ZZ}(t)} \cos \phi - \frac{v_n(t)}{D_{NN}(t)} \sin \phi \right]. \tag{A9}
\]

where \( t_A = 250 \text{ fm/c} \) and \( t_B = 400 \text{ fm/c} \) as indicated in Fig. 8. We find the reduced iso-vector curvature parameter as \( \alpha = 0.13 \). In Fig. 7, after the symmetric state \(^{238}\text{U} + ^{238}\text{U}\), because of quantal effects due to shell structure, the TDHF drift path follows a complex pattern for a long time and subsequently appears to drift toward asymmetry along the iso-scalar direction. This behavior indicates that the symmetric state is an unstable equilibrium point in the iso-scalar direction, i.e. along the beta stability line, and the average potential energy has an inverted parabolic shape with a negative curvature parameter. Such potential shape may lead to relative large diffusion along the beta stability direction. Unfortunately, the drift segment after the symmetric state until the time which the fragment separates is not suitable to estimate the average value of the reduced iso-scalar curvature parameter \( \beta \).

**Appendix B: CURVATURE PARAMETERS ALONG THE BETA STABILITY**

We consider the reduced iso-scalar curvature \( \beta \) as a parameter. In order obtain a reasonable value for \( \beta \), we employ the cross-section data for production of gold isotopes from a previous investigation of the \(^{238}\text{U} + ^{238}\text{U}\) system at about the same energy. We calculate the distribution of the cross-sections \( \sigma(N = A-Z, Z) \) primary gold isotopes with the atomic number \( Z = 79 \) and mass numbers \( A \) using Eq. (20) for the double \( \sigma(N,Z) \) cross-sections. In order to cover the angular range of the experimental set up in Ref. [16], the range of the angular momentum summation in Eq. (20) is taken as

![FIG. 8. Neutron \( N_1(t) \) and proton \( Z_1(t) \) numbers of radium-like fragments as function of time in the central collisions of the \(^{236}\text{Ra} + ^{240}\text{Cm}\) system at \( E_{\text{c.m.}} = 833 \text{ MeV} \) (solid blue line) at tip-tip geometry.](image)

![FIG. 9. Neutron and proton diffusion coefficients as function of time in the central collisions of the \(^{236}\text{Ra} + ^{240}\text{Cm}\) system at \( E_{\text{c.m.}} = 833 \text{ MeV} \) at tip-tip geometry.](image)
\( \ell_{\text{min}} = 100 \) and \( \ell_{\text{max}} = 460 \). Fig. 10 shows the cross-sections for production of the primary gold isotopes which are calculated with the reduced iso-vector curvature \( \alpha = 0.13 \) and the reduced iso-scalar curvature \( \beta = -0.02 \). The primary gold isotopes are excited and cool down mainly by neutron emissions. In determining the average number of the emitted neutrons, we need to estimate the average excitation energy of these isotopes. The TDHF calculations presented in Table I and Table II, do not give accurate information for the total kinetic energy loss (TKEL) in these channels. However for a rough estimate we can take the results for the initial angular momentum \( L = 300h \), which is about the waited mean value of the angular momentum range. For this angular momentum, the TKEL in the tip-tip and the side-side geometries are 217 MeV and 135 MeV, respectively. For the gold channel \( U+U \rightarrow Au(195,79)+Db(281,105) \) the \( Q_{gg} \) value is 24.1 MeV. Sharing the TKEL and the \( Q_{gg} \) value in proportion to the masses, we find the average excitation energy of the gold isotopes to be 86.8 MeV and 54.0 MeV, in the tip-tip and the side-side geometries, respectively. Assuming one neutron emitted per 10.0 MeV, on the average about 9, 5 and 7 neutrons are emitted in the tip-tip, in the side-side and in the mean geometry, respectively. In Fig. 10, if we shift the mean gold isotope distribution by 7 units to the left, the peak value of the distribution of \( Z \) is a reasonable estimate for the reduced \( \ell = 100-460 \), \( \alpha = 0.13 \), and \( \beta = -0.02 \). Solid dots indicate data taken from [16].

![FIG. 10. Cross-section of gold Z = 79 isotopes averaged over tip-tip and side-side geometries as a function of the mass numbers in the collisions of \(^{238}\text{U} + ^{238}\text{U}\) system at \( E_{\text{c.m.}} = 833\text{ MeV} \) calculated with curvature parameters \( \alpha = 0.13 \) and \( \beta = -0.02 \). Solid dots indicate data taken from [16].](image)

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