NLO QCD and EW corrections to vector-boson scattering into $W^+W^-$ at the LHC

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ABSTRACT: We present the full next-to-leading-order electroweak and QCD corrections to vector-boson scattering into a pair of off-shell opposite-sign W bosons decaying into leptons of different flavour at the LHC. We include full leading-order predictions for the irreducible background. Explicitly, we investigate the process $pp \rightarrow e^+\nu e^-\bar{\nu}jj + X$ at leading orders $\mathcal{O}(\alpha^6), \mathcal{O}(\alpha\alpha^5), \mathcal{O}(\alpha_s^2\alpha^4)$, supplemented by the loop-induced $\mathcal{O}(\alpha_s^4\alpha^4)$ contribution, and at next-to-leading orders $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$ in two setups providing fiducial cross sections as well as differential distributions. We take full account of photon-induced next-to-leading-order contributions, which prove to be non negligible. With $-11.4\%$ and $-6.7\%$ in the two setups, the electroweak corrections are smaller than for other vector-boson-scattering processes. This can be traced back to the presence of the Higgs-boson resonance in the fiducial phase space, whose effects we analyse within an additional unphysical, but manifestly gauge-invariant setup. The QCD corrections amount to $-5.1\%$ and $-21.6\%$ in the two setups. The large size of the latter correction, compared to other vector-boson scattering processes, is explained by a very restrictive definition of its fiducial phase space.

KEYWORDS: Higher Order Electroweak Calculations, Higgs Properties, Higher-Order Perturbative Calculations

ArXiv ePrint: 2202.10844
1 Introduction

Vector-boson scattering (VBS) processes allow for important tests of the electroweak (EW) sector and the Standard Model (SM) as a whole. Especially the scattering of vector bosons into a pair of $W$ bosons provides the possibility to investigate triple and quartic gauge couplings and the couplings of EW gauge bosons to the scalar Higgs sector. Owing to strong gauge cancellations in the SM, VBS is very sensitive to possible deviations from the SM in the EW sector and an experimentally interesting testing ground.

In recent years, VBS processes have been observed by both ATLAS and CMS in leptonically decaying same-sign $W$ [1–5], $WZ$ [6–8] and $ZZ$ [9–11] final states (denoted for short as same-sign $W$, $WZ$ and $ZZ$ scattering). Very recently, also opposite-sign $W$ scattering could be measured by the CMS experiment [12]. Besides, opposite-sign $W$ scattering is an important part of the EW Higgs-boson production and decay, which has also been studied at the LHC [13, 14].

On the theoretical side, next-to-leading-order (NLO) QCD corrections to all VBS processes and its irreducible background and especially to opposite-sign $W$ scattering have been calculated [15–17] and matched to parton showers [18, 19] over the last two decades, whereas NLO EW corrections to massive VBS processes have only become available recently for same-sign $W$ [20, 21], $WZ$ [22] and $ZZ$ scattering [23, 24]. In this article, we continue
this series of studies and provide EW and QCD corrections to the scattering of opposite-sign
W bosons decaying into a pair of oppositely charged leptons of different generations and
two corresponding neutrinos, leading to the final state $e^{+}\nu_{\mu}\bar{\nu}_{\mu}jj$. We exclude jets with
bottom quarks in the final state, since a major experimental problem in opposite-sign W
scattering is the background from $t\bar{t}$ production, whose cross section is typically orders of
magnitude larger than VBS cross sections [25] and needs to be suppressed by a b-jet veto.

The final states of VBS scattering processes receive contributions from triple vector-
boson production where one of the vector bosons decays hadronically and two decay
leptonically. While we are not aware of existing NLO QCD and EW calculations for these
processes, there are calculations for off-shell triple W-boson production in the fully leptonic
channel [26–29].

At leading order (LO), the process $pp \rightarrow e^{+}\nu_{e}\mu^{-}\bar{\nu}_{\mu}jj + X$ receives tree-level contributions
of the orders $O(\alpha^{6})$, $O(\alpha_{s}\alpha^{5})$, and $O(\alpha_{s}^{2}\alpha^{6})$. We also include the loop-induced process of
$O(\alpha_{s}^{4}\alpha^{4})$ with two gluons in the initial and final states in our calculation. VBS scattering is
part of the EW contribution of $O(\alpha^{6})$, the order $O(\alpha_{s}^{2}\alpha^{6})$ is the QCD-induced background,
and the $O(\alpha_{s}^{5})$ consists of interferences of EW and QCD contributions and gluon-photon-
induced contributions. The QCD-induced background constitutes the largest part of the
cross section, followed by the EW contribution, the loop-induced process and the interference.

In this paper, we focus on the NLO contributions of orders $O(\alpha^{7})$ and $O(\alpha_{s}\alpha^{6})$. These
result from EW and QCD corrections to the LO EW contribution and from non-separable
EW corrections to the LO interference. For simplicity, we call the $O(\alpha_{s}\alpha^{6})$ contributions
QCD corrections for short. In our previous investigations, we found the EW corrections
to VBS processes to be typically $-15\%$, resulting from EW logarithms enhanced by high
intrinsic scales of VBS. For opposite-sign W scattering a sizeable fraction of the cross
section results from the intermediate Higgs-boson resonance at the scale of the Higgs-boson
mass, leading to smaller corrections. We fully include photon-induced corrections, which
involve new partonic channels with VBS signatures at NLO.

This paper is structured as follows: in section 2, the process and details of the validation
of our calculation are described. Section 3 summarises the numerical input parameters, the
event-selection criteria, and the integrated cross sections in two physical setups. Special
attention is devoted to the Higgs resonance, which we investigate in an additional unphysical
setup. We conclude with a presentation of differential distributions for the physical setups.
In section 4, we give a brief summary.

2 Description of the calculation

In this article we investigate the process

$$pp \rightarrow e^{+}\nu_{e}\mu^{-}\bar{\nu}_{\mu}jj + X$$

(2.1)

at the LHC. We only consider the final state with $e^{+}\mu^{-}$ and do not take into account the
charge-conjugate final state. Since we treat the leptons as massless and use the same cuts
for electrons and muons, the $e^{-}\mu^{+}$ final state yields identical results. The two jets $j$ in
the final state result from clustering of quarks, antiquarks, gluons and photons. However,
Figure 1. Examples of LO diagrams of $\mathcal{O}(g^6)$. The first row (a) – (c) shows signal diagrams containing VBS subprocesses, the second row diagrams with two (d), one (e), or no (f) resonant vector bosons, and the third row triple-vector-boson production (g), an $s$-channel diagram (h), and a photon-induced (i) diagram.

Unclustered photons are not considered as jets. As a consequence, we only have to take into account partonic processes with at least two strongly-interacting partons in the final state.

2.1 Leading order

Just like all other processes with $2j + 4\ell$ final states, the cross section of (2.1) consists of three components at LO: a purely EW one of $\mathcal{O}(\alpha^6)$ which includes VBS, one of $\mathcal{O}(\alpha_s\alpha^5)$ from interference, (anti)quark-photon-induced and gluon-photon-induced contributions, and a QCD-induced one of $\mathcal{O}(\alpha_s^2\alpha^4)$.

A sample of Feynman diagrams at order $\mathcal{O}(g^6)$ is shown in figure 1. VBS appears at LO as a subprocess in quark-induced partonic processes as scattering of a pair of electroweak gauge bosons emitted from different quark lines into a pair of opposite charged $W$ bosons that decay leptonically into two opposite charged leptons and their corresponding neutrinos (in the following simply referred to as $W^+W^-$ final state) [figure 1(a) – (c)]. Besides diagrams with triple and quartic gauge couplings, VBS includes diagrams with an $s$-channel Higgs exchange [figure 1(c)]. In each partonic channel, all of these sub-diagrams contribute.

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\[ \text{We denote the weak coupling constant by } g = e/s_w \text{ and the strong coupling constant by } g_s. \]
Figure 2. Examples of LO diagrams of $O(g_s g^5)$, a gluon-photon-induced contribution (a), $O(g_s^2 g^4)$, a $t$-channel gluon exchange (b), and $O(g_s^4 g^4)$, a loop-induced contribution (c).

In contrast to the similar process $p p \rightarrow e^+e^-\mu^+\mu^-jj + X$ (ZZ scattering), which we investigated in refs. [23, 24], the two neutrinos in the final state are only visible as missing transverse momentum in the experiment. It is hence impossible to construct a physical cut to suppress contributions of resonant Higgs production and decay, such as an invariant-mass cut on the four-lepton system. As we will discuss in the course of this paper, the presence of the Higgs resonance changes the characteristics of $W^+W^-$ scattering compared to other VBS processes.

Besides diagrams containing the VBS subprocess, the order $O(g^6)$ receives contributions from ($t$-channel) diagrams with two, one or no resonant vector bosons [figure 1(d), (e), and (f)], $s$-channel diagrams [figure 1(g) and (h)], and photon-induced contributions [figure 1(i)]. In partonic processes that involve $s$-channel sub-diagrams, in particular, triple-vector-boson production appears [figure 1(g)]. Contributions of the irreducible background emerge via gluon-photon-induced diagrams at $O(g_s g^5)$ [figure 2(a)] and diagrams of $O(g_s^2 g^4)$ [figure 2(b)], which are characterised by a $t$- or $s$-channel gluon exchange. An example diagram for the loop-induced channel is shown in figure 2(c).

At $O(\alpha^6)$, there are 60 quark-induced partonic channels (not counting $qq'$ and $q'q$ initial states separately and omitting bottom quarks), which we divide in 5 subclasses. Specifically, 36 partonic channels include Feynman diagrams with VBS topology, but no triple-vector-boson production, 4 include both VBS and triple-W production, 8 VBS and WWZ production, 4 triple-W production but no VBS, and 8 WWZ production but no VBS. For further details concerning the structure of the process, we refer to our paper on ZZ scattering [23]. The partonic processes for opposite-sign W scattering are obtained from those for ZZ scattering upon replacing two oppositely-charged leptons by two neutrinos, without touching the external quarks, gluons, and photons. Therefore, in particular, the counting of partonic processes is the same in both cases.

In our calculation we neglect quark mixing and use a unit CKM matrix. Similarly to ZZ scattering, this affects the suppressed quark-induced $s$-channel contributions by about 5% [24], hence its effects are negligible. Furthermore, we exclude the contributions of initial- or final-state bottom quarks. Those of initial-state bottom quarks are suppressed owing to their parton-distribution functions (PDFs). Thus, for ZZ scattering, the contributions of bottom quarks have been found to be below 3% [24]. In contrast to ZZ scattering, the contribution of final-state bottom quarks would be overwhelming for $W^+W^-$ scattering.
Figure 3. LO and NLO contributions to $W^+W^-$ scattering and its irreducible background. In this paper, we discuss both the EW and QCD NLO corrections to $O(\alpha^6)$ and the non-separable EW corrections to the interference process for several experimental setups.

owing to contributions of $t\bar{t}$ production to the identical final state. In this respect, we stress the importance of experimental bottom-jet vetoes when measuring VBS cross sections to avoid contamination from $t\bar{t}$ production. Our calculation is based on the assumption of a perfect bottom-jet veto.

Further contributions to the hadronic process (2.1) originate from photon-induced partonic processes with $\gamma\gamma$, $q\gamma$, or $g\gamma$ as initial states which are included in this computation. The process $\gamma \gamma \rightarrow qg + 4\ell$ appears at $O(\alpha^6)$, the processes $g \gamma \rightarrow qg + 4\ell$ and $q\gamma \rightarrow qg + 4\ell$ at $O(\alpha_s \alpha^5)$ as non-interference processes. Finally, at $O(\alpha_s^2 \alpha^4)$ a loop-induced channel $gg \rightarrow gg + 4\ell$ with four external gluons opens up, which has been shown to be non-negligible for ZZ scattering, despite of its higher order, owing to the enhanced gluon PDFs.

2.2 Real emission

Real EW and QCD corrections to all LO processes lead to four contributions of orders $O(\alpha^7)$, $O(\alpha_s \alpha^6)$, $O(\alpha_s^2 \alpha^5)$, and $O(\alpha_s^3 \alpha^4)$ at NLO (see figure 3). In this paper, we discuss only the first two orders, which are the NLO corrections to the VBS signal process. The order $O(\alpha^7)$ are pure EW corrections to the EW LO contribution, whereas the $O(\alpha_s \alpha^6)$ contains both QCD corrections to the EW LO process as well as EW corrections to the LO interference.

Apart from the emission of a real photon from any charged particle, which does not change the classification of the process [figure 4(a) and (b)], at $O(\alpha^7)$ additional photon-induced processes appear in which the initial-state photon splits into a quark-antiquark pair [figure 4(c)], leading to an additional jet in the final state. As a consequence, VBS topologies appear in photon-induced processes.

Real QCD corrections, on the other hand, always produce an additional jet, either from emission of a real gluon from a quark line [figure 4(d), (e), and (f)], or from a gluon splitting [figure 4(g)] giving rise to an additional quark or antiquark in the final state. Gluon splitting also leads to diagrams of $O(gg^6)$ with VBS topologies, e.g. diagram figure 4(c) with the initial-state photon replaced by a gluon, which contribute at $O(\alpha_s \alpha^6)$. figure 4(h) illustrates a contribution to the photon-gluon-induced process, and figure 4(i) a QCD-induced real-correction diagram that contributes at $O(\alpha_s \alpha^6)$ via the interference with diagram figure 4(b).
Figure 4. Examples of real-emission diagrams of $\mathcal{O}(g^7)$, $\mathcal{O}(g_s g^6)$, and $\mathcal{O}(g_s^2 g^5)$: in the first row emission of a real photon from a quark (a) or a lepton line (b) and initial-state photon splitting (c). In the second row gluon emission from signal (d) and background (e,f) diagrams. In the third row a gluon-induced process with final-state photon splitting (g), photon emission from a photon-gluon-induced diagram (h) and from a QCD-induced diagram (i). Note that diagrams of $\mathcal{O}(g_s^2 g^5)$ only contribute via interferences within of the calculations of this article.

At the orders $\mathcal{O}(\alpha_s^2 \alpha^5)$ and $\mathcal{O}(\alpha_s^3 \alpha^4)$, which we do not discuss in this article, VBS never appears as a subprocess.

To summarise, three types of partonic processes constitute the real corrections at $\mathcal{O}(\alpha^7)$: $qq \to qq\gamma + 4\ell$, $\gamma \gamma \to qq\gamma + 4\ell$ and $q\gamma \to qqq + 4\ell$. At $\mathcal{O}(\alpha_s \alpha^6)$, the partonic channels $qq \to qqg + 4\ell$, $\gamma \gamma \to qqg + 4\ell$ and $gq \to qqq + 4\ell$ contribute as QCD corrections to the EW process, $qq \to qq\gamma + 4\ell$ and $q\gamma \to qqq + 4\ell$ as EW corrections to the interference, and $q\gamma \to qq\gamma + 4\ell$ and $g\gamma \to qqq + 4\ell$ as EW corrections to the non-interference $\mathcal{O}(\alpha_s \alpha^5)$ processes $q\gamma \to qq + 4\ell$ and $g\gamma \to qq + 4\ell$.

At $\mathcal{O}(\alpha_s \alpha^6)$, partonic channels of the type shown in figure 4(g) contain both QCD and QED singularities. Therefore, strictly speaking, QCD and EW corrections cannot be separated in a physically meaningful way, and counting these contributions as QCD corrections is merely a convention.
The real emission of massless particles leads to IR singularities in the fiducial phase space. We use the Catani-Seymour dipole subtraction method [30] and its generalisation to QED [31, 32], just as in our previous works. The previously mentioned $\mathcal{O}(\alpha_s\alpha^6)$ contributions from diagrams where a photon splits into a quark-antiquark pair [figure 4(g)] lead to an additional complication. Their QED singularities would cancel against a virtual correction to a $j\gamma + 4\ell$ final state that is not part of our signal phase space requiring two final-state jets. As already in WZ and ZZ scattering, this singularity can be absorbed into a photon-to-jet conversion function, which is related to the non-perturbative hadronic vacuum polarisation [33], while treating the photon as a jet in this case.

2.3 Virtual corrections

Virtual corrections emerge from purely EW one-loop diagrams of order $\mathcal{O}(g^8)$ and from one-loop diagrams of order $\mathcal{O}(g_s^2g^6)$ involving a virtual gluon (see figure 5 for sample diagrams). Interfering these with tree-level diagrams of orders $\mathcal{O}(g^6)$ or $\mathcal{O}(g_s^2g^4)$ leads to contributions at orders $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$. As is the case for the real corrections, the virtual ones of order $\mathcal{O}(\alpha^7)$ consist only of EW corrections to the EW diagrams [figure 5 (a)], whereas the order $\mathcal{O}(\alpha_s\alpha^6)$ contains both the interference of $\mathcal{O}(g_s^2g^6)$ [figure 5 (b) and (c)] with $\mathcal{O}(g^6)$ and the one of $\mathcal{O}(g^8)$ [figure 5 (a)] with $\mathcal{O}(g_s^2g^4)$. As for all VBS processes, it is impossible to separate QCD and EW corrections at $\mathcal{O}(\alpha_s\alpha^6)$ on the basis of Feynman diagrams, because diagrams of order $\mathcal{O}(g_s^2g^6)$ with mixed EW and QCD particle content [figure 5 (c)] can be viewed as QCD corrections to the EW $\mathcal{O}(g^6)$ diagram or vice versa as EW corrections to the QCD $\mathcal{O}(g_s^2g^4)$ diagram.

The virtual corrections contain both ultraviolet and infrared (IR) singularities. The IR singularities cancel against those of the real corrections or are absorbed in the PDFs. The ultraviolet singularities are treated by renormalisation. We remark that diagrams like the one in figure 5(c) give rise to IR singularities of both QCD and EW origin.

2.4 Details and validation

Our results have been produced using the Monte Carlo program BBMC and the matrix-element generator RECOLA [34, 35]. BBMC is a multi-channel integrator which is able to compute multiparticle processes at NLO accuracy. For fast and stable integration, it uses
mappings similar to those in refs. [36–38], which map the final-state phase space according to the resonance structure of the process. RECOLA is a general tree-level and one-loop matrix-element provider which relies on the COLLIER library [39, 40], which provides one-loop scalar [41–44] and tensor integrals [45–47] numerically. The combination of RECOLA and COLLIER has been shown to be working reliably for many high-multiplicity processes and in particular for VBS.

BBMC has been checked against MoCaNLO, an independent multi-channel integrator, for same-sign W [20, 21] and ZZ scattering [23, 24]. For opposite-sign W scattering, representative channels at LO, as well as $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$ have been compared for both codes. All results were confirmed to agree within statistical uncertainties at the percent level for all VBS processes.

Both in BBMC and MoCaNLO the Catani-Seymour dipole formalism [30–32, 48] is used to subtract (and add back) IR divergences. Since opposite-sign W scattering and ZZ scattering share related partonic processes, as pointed out in section 2.1, in their calculations the same dipole structures appear. Each dipole that was not already utilised in same-sign W scattering was checked against MoCaNLO at the phase-space-point level for the real-subtracted as well as the integrated-dipole contributions in the course of our calculations for ZZ scattering. Since these dipoles are process-independent quantities, those checks remain valid for opposite-sign W scattering.

For the treatment of the vector-boson resonances we use the complex-mass scheme [37, 48–50].

3 Numerical results

3.1 Input parameters and event selection

Input parameters. Our setup is designed for an LHC run at a centre-of-mass (CM) energy of 13 TeV. We use the NLO NNPDF3.1luxQED PDF set with $\alpha_s(M_Z) = 0.118$ [51,52] via LHAPDF [53,54] in the $N_F = 5$ fixed-flavours scheme in both the LO and NLO predictions. The initial-state collinear splittings are treated by \( \overline{\text{MS}} \) redefinition of the PDFs.

As in our previous works, the central renormalisation and factorisation scales are chosen as the geometric average of the transverse momenta of the tagging jets

$$\mu_{\text{central}}^{\text{ren}} = \mu_{\text{central}}^{\text{fac}} = \sqrt{p_T_{j_1} p_T_{j_2}},$$

(3.1)

where \( j_1 \) and \( j_2 \) are the two hardest identified jets (see definition below) ordered according to transverse momentum. Based on this central scale, we perform a 7-point scale variation of both the renormalisation and factorisation scale, i.e. we calculate the observables for the pairs

$$\left(\mu_{\text{ren}}/\mu_{\text{central}}^{\text{ren}}, \mu_{\text{fac}}/\mu_{\text{central}}^{\text{fac}}\right) = \left(0.5, 0.5\right), \left(0.5, 1\right), \left(1, 0.5\right), \left(1, 1\right), \left(1, 2\right), \left(2, 1\right), \left(2, 2\right)$$

(3.2)

of renormalisation and factorisation scales and use the resulting envelope to estimate the perturbative (QCD) uncertainty.
We employ the $G_\mu$ scheme [55] to define the electromagnetic coupling, which fixes the EW coupling $\alpha$ using the Fermi constant $G_\mu$ as input parameter via

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) \quad \text{with} \quad G_\mu = 1.16638 \times 10^{-5} \text{GeV}^{-2}. \quad (3.3)$$

The masses and widths of the massive gauge bosons are taken from the PDG review 2020 [56],

$$M_{Z}^{\text{OS}} = 91.1876 \text{GeV}, \quad \Gamma_{Z}^{\text{OS}} = 2.4952 \text{GeV},$$
$$M_{W}^{\text{OS}} = 80.379 \text{GeV}, \quad \Gamma_{W}^{\text{OS}} = 2.085 \text{GeV}, \quad (3.4a)$$

and furthermore we set

$$m_t = 173.0 \text{GeV}, \quad \Gamma_t = 0 \text{GeV},$$
$$M_H = 125.0 \text{GeV}, \quad \Gamma_H = 4.07 \times 10^{-3} \text{GeV}. \quad (3.4b)$$

The bottom quark is assumed to be massless, and no partonic channels with initial-state and/or final-state bottom quarks are included. Without any resonant top quarks in the considered processes, we set the top-quark width to zero. The Higgs-boson width is taken from ref. [57]. From the measured on-shell (OS) values of the masses and widths of the weak vector bosons $V = W, Z$, we obtain the corresponding pole quantities used in the calculation via [58]

$$M_V = \frac{M_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/M_V^{\text{OS}})^2}} \quad \text{and} \quad \Gamma_V = \frac{\Gamma_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/M_V^{\text{OS}})^2}}, \quad (3.5)$$

Our calculation is performed in the 5-flavour scheme assuming a perfect bottom-jet veto. The contribution of the remaining channels with bottom quarks only in the initial state has been neglected. It is PDF-suppressed and does not involve any VBS contributions. We verified numerically that this contribution is in relative terms below $10^{-7}$ at $\mathcal{O}(\alpha^6)$ and at the level of $10^{-4}$ at $\mathcal{O}(\alpha^3\alpha^4)$ for the setups considered in this paper.

**Event selection with VBS cuts.** The event selection used for this analysis is inspired by the CMS and ATLAS measurements of opposite-sign W-boson-pair production [59, 60] and our previous works on VBS. It is similar to the one employed for the observation of this process by CMS [12]. At the LHC, different final states of opposite-sign W scattering have been probed, either with two charged leptons of opposite charge and same flavour or different flavour and at least two QCD jets. We restrict our analysis to the different-flavour channel. QCD partons (quarks, antiquarks, gluons), leptons and photons are clustered simultaneously using the anti-$k_T$ algorithm [61] with a resolution parameter $R = 0.4$ for all pairs of QCD partons, leptons and photons. Pairs of QCD partons are recombined to QCD partons, QCD partons and photons to QCD partons, and leptons and photons to (dressed) leptons. Unrecombined photons are not treated as jets. Only partons with rapidity $|y| < 5$ are considered for recombination, while particles with larger $|y|$ are assumed to be lost in the beam pipe. The rapidity $y$ and the transverse momentum $p_T$ of a particle are defined as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}, \quad p_T = \sqrt{p_x^2 + p_y^2}, \quad (3.6)$$

$$\sqrt{p_x^2 + p_y^2}, \quad (3.6)$$
where $E$ is the energy of the particle, $p_z$ the component of its momentum along the beam axis, and $p_x, p_y$ the components perpendicular to the beam axis. The result of the clustering are jets, dressed leptons, and photons. All cuts are applied to these objects and the distributions shown below are based on them. In the following, leptons have to be understood as dressed leptons throughout.

Each of the two charged leptons $\ell$ has to fulfil

$$p_{T,\ell} > 25 \text{ GeV}, \quad |y_\ell| < 2.4,$$

and together they must satisfy

$$p_{T,\ell+\ell-} > 30 \text{ GeV}, \quad M_{\ell+\ell-} > 20 \text{ GeV},$$

where $p_{T,\ell+\ell-}$ is the (vectorial) sum of the transverse momenta of the charged leptons and $M_{\ell+\ell-}$ is the invariant mass of the charged-lepton pair. The missing transverse momentum is required to fulfil

$$p_{T,\text{miss}} > 20 \text{ GeV}$$

and is computed as the transverse part of the sum of the two neutrino momenta. After jet clustering, jets that fulfil the conditions

$$p_{T,j} > 30 \text{ GeV}, \quad |y_j| < 4.5, \quad \Delta R_{j\ell} > 0.4$$

are called identified jets, where the distance $\Delta R_{ij}$ is defined as

$$\Delta R_{ij} = \sqrt{(\Delta \phi_{ij})^2 + (\Delta y_{ij})^2}$$

with the azimuthal-angle difference $\Delta \phi_{ij} = \min(|\phi_i - \phi_j|, 2\pi - |\phi_i - \phi_j|)$ and the rapidity difference $\Delta y_{ij} = y_i - y_j$. Note that the condition (3.10) demands a minimal distance between the jet and any of the charged leptons. The two identified jets with highest transverse momenta, called hardest, leading, or tagging jets, must obey

$$M_{j_1j_2} > 500 \text{ GeV}, \quad |\Delta y_{j_1j_2}| > 2.5.$$

**Event selection with Higgs-search cuts.** Since the VBS channels contain contributions consisting of a Higgs production and decay subprocess, we consider a second set of event-selection criteria which is inspired by the different-flavour event selection in the CMS Higgs search [13].

The clustering of QCD partons, leptons, and photons is done in exactly the same way as for the VBS setup above, i.e. using the anti-$k_T$ algorithm with a resolution parameter of $R = 0.4$ for partons within $|y| < 5$. The two tagging jets are required to fulfil

$$p_{T,j_{1,2}} > 30 \text{ GeV}, \quad |y_{j_{1,2}}| < 4.7, \quad \Delta R_{j_1,j_2} > 0.4$$

as well as the typical VBS topology cuts

$$|\Delta y_{j_1,j_2}| > 3.5, \quad M_{j_1j_2} > 400 \text{ GeV}.$$
Additionally, we introduce a jet veto for any third jet resulting from the clustering and fulfilling the conditions \(3.13\), i.e. events are only kept if the third jet obeys

\[ p_{T,j_3} \leq 30 \text{ GeV}. \]  

The (dressed) charged leptons have to fulfil

\[ p_{T,\ell}^{\text{lead}} > 25 \text{ GeV}, \quad p_{T,\ell}^{\text{trail}} > 10 \text{ GeV}, \]  

where \( p_{T,\ell}^{\text{lead}} \) and \( p_{T,\ell}^{\text{trail}} \) are the transverse momenta of the leading (harder) and trailing (softer) charged lepton. The minimum distance and the rapidity of the charged leptons are chosen to be

\[ \Delta R_{\ell^+\ell^-} > 0.4, \quad |y_\ell| < 2.4, \]  

their pair invariant mass is required to exceed

\[ M_{\ell^+\ell^-} > 12 \text{ GeV}, \]  

and the sum of their transverse momenta must respect

\[ p_{T,\ell^+\ell^-} > 30 \text{ GeV}. \]  

The missing transverse momentum has to fulfil

\[ p_{T,\text{miss}} > 20 \text{ GeV}, \]  
as in the VBS setup. Additionally, we require the transverse mass of the lepton system \( M_T \) to be within a range below the Higgs mass,

\[ 60 \text{ GeV} < M_T < 125 \text{ GeV}, \]  

where \( M_T \) is defined as

\[ M_T = \sqrt{2p_{T,\ell^+\ell^-}p_{T,\text{miss}}[1 - \cos \Delta \phi(\ell^+\ell^-, \nu\bar{\nu})]} \]  

and \( \Delta \phi(\ell^+\ell^+, \nu\bar{\nu}) \) is the azimuthal angle between the sum of the charged-lepton momenta and the sum of the neutrino momenta.

Furthermore the rapidities of the charged leptons are bounded by the rapidities of the two tagging jets. To this end, we use the Zeppenfeld variable \( z_{\ell_1\ell_2} \), also called centrality, defined as

\[ z_{\ell_1\ell_2} = \frac{y_\ell - \frac{y_1 + y_2}{2}}{\Delta y_{12}}, \]  

and require

\[ -0.5 < z_{\ell_1\ell_2} < 0.5. \]
3.2 Cross sections

We start with presenting LO results in table 1. For the VBS setup, the EW cross section of \( O(\alpha^6) \) is 2.7 fb and thus the largest cross section among all VBS processes at the LHC. Whilst the interference is almost negligible with 0.065 fb, the QCD-induced background is very large with 6.9 fb, due to processes with two gluons in the initial state that contribute for opposite-sign W scattering as opposed to same-sign W scattering. The EW and QCD cross-section fractions are 27.3% and 70.0% and thus comparable to the situation of ZZ scattering, which features, up to the leptons, the same partonic processes as W+W− scattering. There we found 32.2% versus 59.5% with strict VBS cuts [23]. The contribution of the loop-induced gluon channel amounts to 0.19 fb or 2%. Including the 7-point scale variations, the full LO cross section is obtained as

\[
\sigma_{VBS}^{\text{LO}} = 9.871(1)^{+30\%}_{-21\%} \text{ fb}. \tag{3.25}
\]

The sizeable scale dependence results from the dominant contribution of order \( O(\alpha_s^2\alpha^4) \).

For the Higgs setup the overall picture does not change drastically at LO. Owing to stricter cuts, the EW cross section is reduced to 1.5 fb which is almost of the same size as the QCD cross section with 1.7 fb, i.e. the relative contribution of VBS is increased. While the fraction of the interference contribution drops, the one of the loop-induced gluon channel grows. Together with the 7-point scale variation for this setup the LO result is

\[
\sigma_{\text{Higgs}}^{\text{LO}} = 3.3392(8)^{+25\%}_{-17\%} \text{ fb}. \tag{3.26}
\]

The scale dependence is somewhat reduced since the relative contribution of the QCD-induced process is smaller.

To investigate the contribution of VBS to the \( O(\alpha^6) \) cross section and to compare this with its corresponding background channels of the same initial and final states, we split up the cross sections into further sub-contributions in table 2. In contrast to ZZ scattering, there is no remarkable difference between processes with internal ZZ → WW scattering.
Table 2. Division of LO cross sections into contributions of partonic channels with specific subprocesses. Note that no non-VBS processes containing two quarks in both the initial and final state contribute at order $O(\alpha_s\alpha^5)$.

| Contribution | $O(\alpha^6)$ | $O(\alpha_s\alpha^5)$ | $O(\alpha_s^2\alpha^4)$ | sum |
|--------------|--------------|----------------|----------------|------|
| VBS setup    |              |                |                |      |
| $\sigma(4q, VBS)$ [fb] | 2.6988(3) | 0.05439(2) | 2.2315(3) | 4.9846(4) |
| $\sigma(4q, \text{non-VBS})$ [fb] | $1.4734(9) \times 10^{-4}$ | — | 0.008641(3) | 0.008788(3) |
| $\sigma(\gamma\gamma/gg)$ [fb] | 6.832(2) $\times 10^{-6}$ | 0.010605(2) | 4.6820(8) | 4.6926(8) |
| $\sigma(\text{total})$ [fb] | 2.6988(3) | 0.06500(2) | 6.9221(9) | 9.6860(9) |
| Higgs setup  |              |                |                |      |
| $\sigma(4q, VBS)$ [fb] | 1.5322(2) | 0.007490(5) | 0.39866(7) | 1.9384(2) |
| $\sigma(4q, \text{non-VBS})$ [fb] | $1.850(2) \times 10^{-5}$ | — | 0.0012729(6) | 0.0012913(6) |
| $\sigma(\gamma\gamma/gg)$ [fb] | $7.764(4) \times 10^{-7}$ | 0.0015062(4) | 1.2923(3) | 1.2938(3) |
| $\sigma(\text{total})$ [fb] | 1.5322(2) | 0.008996(5) | 1.6923(3) | 3.2335(3) |

We move to the discussion of NLO contributions starting with the scale dependence. The LO scale dependence of the EW contribution and the NLO scale dependence of the orders $O(\alpha^7)$ and $O(\alpha_s\alpha^6)$ is shown in table 3. We emphasise that the scale uncertainties for the EW contribution within the VBS setup are significantly reduced when taking the NLO QCD contributions into account, whereas this reduction is not observed within the Higgs setup. This is due to the comparably large QCD corrections in this setup and their scale dependence. The large QCD corrections are related to the jet veto, as discussed below.

We present the NLO cross sections in table 4, dividing the partonic channels with four quarks into a group that contains only VBS as a subprocess, two in which triple-vector-boson production, either WWW or WWZ, can occur, and another two in which both triple-vector-boson production and VBS appear as subprocesses. Comparing the NLO corrections to $W^+W^-$ scattering to other VBS processes, we notice some important differences. At first, the overall EW corrections are with $-11.4\%$ in the VBS setup only three quarters and with $-6.7\%$ in the Higgs setup only half as large as the EW corrections for the VBS processes in same-sign WW, WZ, or ZZ scattering. We recapitulate this briefly in table 5, where we list the EW corrections of $O(\alpha^7)$ compared with the EW LO processes of $O(\alpha^6)$ for all massive VBS processes. Next, the partonic channels without VBS receive
| Order | \( \mathcal{O}(\alpha^6) \) | \( \mathcal{O}(\alpha^6) + \mathcal{O}(\alpha^7) \) | \( \mathcal{O}(\alpha^6) + \mathcal{O}(\alpha_s\alpha^6) \) | \( \mathcal{O}(\alpha^6) + \mathcal{O}(\alpha^7) + \mathcal{O}(\alpha_s\alpha^6) \) |
|-------|-----------------|-----------------|-----------------|-----------------|
| VBS setup | | | | |
| \( \sigma_{\text{central}} [\text{fb}] \) | 2.6988(3) | 2.391(1) | 2.563(3) | 2.255(4) |
| \( \sigma_{\text{min}} [\text{fb}] \) | 2.5069(3) | 2.232(1) | 2.545(4) | 2.200(4) |
| \( \delta \sigma_{\text{min}} [%] \) | -7.1 | -6.7 | -0.7 | -2.5 |
| \( \sigma_{\text{max}} [\text{fb}] \) | 2.9187(3) | 2.573(1) | 2.581(4) | 2.285(3) |
| \( \delta \sigma_{\text{max}} [%] \) | 8.2 | 7.6 | 0.7 | 1.3 |

| Higgs setup | | | | |
| \( \sigma_{\text{central}} [\text{fb}] \) | 1.5322(2) | 1.429(1) | 1.202(2) | 1.099(2) |
| \( \sigma_{\text{min}} [\text{fb}] \) | 1.4418(2) | 1.349(1) | 1.130(2) | 1.014(2) |
| \( \delta \sigma_{\text{min}} [%] \) | -5.9 | -5.6 | -6.0 | -7.7 |
| \( \sigma_{\text{max}} [\text{fb}] \) | 1.6324(2) | 1.517(1) | 1.248(2) | 1.155(2) |
| \( \delta \sigma_{\text{max}} [%] \) | 6.5 | 6.1 | 3.8 | 5.1 |

**Table 3.** Cross sections at LO and NLO with 7-point scale variation of \( \mu_{\text{fact}} \) and \( \mu_{\text{ren}} \) around the central value (3.1). We list the result for the central scale and those for the scales that show the largest deviation from the central value, both in absolute and relative quantities.

| Contribution | \( \sigma_{\text{LO}}^6 [\text{fb}] \) | \( \Delta \sigma_{\text{NLO}}^6 [\text{fb}] \) | \( \delta \sigma^7 [%] \) | \( \Delta \sigma_{\text{NLO}}^{\alpha_s\alpha^6} [\text{fb}] \) | \( \delta \alpha_s\alpha^6 [%] \) |
|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| VBS setup | | | | | |
| VBS only | 2.1695(3) | -0.2812(8) | -13.0 | -0.146(3) | -6.7 |
| VBS + WWW | 0.13783(3) | -0.0164(2) | -11.9 | 0.0071(4) | 5.2 |
| VBS + WWZ | 0.39140(6) | -0.0427(3) | -10.9 | -0.013(1) | -3.3 |
| WWW only | 5.319(8) \times 10^{-5} | -1.49(5) \times 10^{-5} | -28.0 | 0.01169(1) | 2.2 \times 10^4 |
| WWZ only | 9.415(3) \times 10^{-5} | -2.72(3) \times 10^{-5} | -28.8 | 0.003907(2) | 4.1 \times 10^3 |
| \( \gamma\gamma/\gamma g \) | 6.832(2) \times 10^{-6} | 0.03292(1) | 4.8 \times 10^5 | -0.0002(6) | -2.9 \times 10^3 |
| total | 2.6988(3) | -0.3074(9) | -11.4 | -0.136(3) | -5.1 |

| Higgs setup | | | | | |
| VBS only | 1.1958(2) | -0.0913(12) | -7.6 | -0.2400(13) | -20.0 |
| VBS + WWW | 0.06603(1) | -0.0052(2) | -7.8 | -0.0093(3) | -14.1 |
| VBS + WWZ | 0.27030(4) | -0.0160(5) | -5.9 | -0.0451(7) | -16.7 |
| WWW only | 6.28(2) \times 10^{-6} | -1.8(1) \times 10^{-7} | -28.9 | 0.002508(5) | 4.0 \times 10^4 |
| WWZ only | 1.223(2) \times 10^{-5} | -3.30(8) \times 10^{-6} | -27.0 | 0.006770(7) | 5.5 \times 10^3 |
| \( \gamma\gamma/\gamma g \) | 7.764(4) \times 10^{-7} | 0.00916(2) | 1.2 \times 10^6 | -0.039(1) | -5.0 \times 10^5 |
| total | 1.5322(2) | -0.1033(13) | -6.7 | -0.330(2) | -21.6 |

**Table 4.** NLO corrections at \( \mathcal{O}(\alpha^7) \) and \( \mathcal{O}(\alpha_s\alpha^6) \) in relation to their LO counterparts of \( \mathcal{O}(\alpha^6) \), classified after appearing subprocesses, and corresponding relative corrections \( \delta \sigma^7 = \Delta \sigma_{\text{NLO}}^7/\sigma_{\text{LO}}^6 \) and \( \delta \alpha_s\alpha^6 = \Delta \sigma_{\text{NLO}}^{\alpha_s\alpha^6}/\sigma_{\text{LO}}^6 \).
Table 5. NLO cross section of $\mathcal{O}(\alpha^7)$ and LO cross section of $\mathcal{O}(\alpha^6)$ in fb and as relative correction in percent for different VBS processes $W^+W^+$ (pp $\rightarrow e^+\nu_e\mu^+\nu_\mu jj + X$) [21], $WZ$ (pp $\rightarrow e^+\nu_e\mu^+\nu_\mu jj + X$) [22], ZZ (pp $\rightarrow e^+e^-\mu^+\mu^- jj + X$) [23] and $W^+W^-$ (pp $\rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu jj + X$) normalized to the EW LO cross section, $\delta^{\alpha^7} = \Delta \sigma_{NLO}^{\alpha^7}/\sigma_{LO}^{\alpha^6}$.

| Process | $W^+W^+$ | $W^+Z$ | ZZ | $W^+W^-$ (VBS setup) | $W^+W^-$ (Higgs setup) |
|---------|-----------|---------|-----|----------------------|------------------------|
| $\Delta \sigma_{LO}^{\alpha^7}[fb]$ | $-0.2169(3)$ | $-0.04091(2)$ | $-0.015573(5)$ | $-0.307(1)$ | $-0.103(1)$ |
| $\sigma_{LO}^{\alpha^6}[fb]$ | $1.4178(2)$ | $0.25511(1)$ | $0.097683(2)$ | $2.6988(3)$ | $1.5322(2)$ |
| $\delta^{\alpha^7} [%]$ | $-15.3$ | $-16.0$ | $-15.9$ | $-11.4$ | $-6.7$ |

significantly higher negative EW corrections than those with VBS subprocesses in both setups, but because of their small absolute size this does not lead to a sizeable effect in the complete cross section.

The relatively small value of the EW corrections for the total fiducial cross section of opposite-sign W scattering can be traced back to the presence of the Higgs resonance in VBS subprocesses. As noted in ref. [20], the large EW corrections emerge from EW logarithms that become large at high invariant masses of the four-lepton system. While typically a large di-jet invariant mass and/or a large rapidity separation is required for the tagging jets in VBS, the large EW corrections are not a result of this event selection but an intrinsic feature of VBS processes at the LHC [20]. In fact, the di-jet invariant-mass cut varies between $100$ GeV and $500$ GeV for the results for $W^+W^-$, $W^+Z$, and ZZ listed in table 5. In contrast, the Higgs resonance drives a considerable fraction of the cross section towards low four-lepton invariant masses around $M_H$. This can be seen from the distribution in the four-lepton invariant mass shown in section 3.4.1. In the Higgs setup, with cuts tailored to enhance the Higgs contribution, this effect is even more pronounced. As a result, the EW corrections are almost halved, from $-11.4\%$ in the VBS setup to $-6.7\%$ in the Higgs setup. Note that the Higgs resonance is eliminated by the cuts in the setup for VBS into ZZ in table 5.

Another interesting effect at $\mathcal{O}(\alpha^7)$ are the NLO contributions of photon-induced processes. While their absolute contribution remains at the level of one percent with respect to the complete $\mathcal{O}(\alpha^6)$ contribution, they are extremely large when compared to the LO contributions of the photon-induced channels. This is due to the appearance of new partonic channels with a photon and a quark or antiquark in the initial state with internal VBS topology (see figure 4(c)) compared to partonic channels with two photons in the initial state at LO.

At $\mathcal{O}(\alpha_s\alpha^6)$, we notice an even larger quantitative difference between the two setups, this time with larger corrections in the Higgs setup. This effect can be traced back to the jet veto (3.15), which suppresses the (positive) real corrections of an emitted final-state gluon, while it leaves the (negative) virtual corrections unaffected. Furthermore, there are very large relative corrections to non-VBS processes in both setups. These can be explained as follows: in non-VBS processes at LO, both final-state jets can only be produced via an
\(s\)-channel vector boson, which does not lead to the required back-to-back jets with high invariant mass. At NLO, however, one of the incoming partons can emit a hard gluon in forward direction (see figure 4(e)), providing one of the tagging jets. The \(s\)-channel vector boson can become resonant leading to an enhancement of the cross section. The enhancement of the \(\mathcal{O}(\alpha_s \alpha^6)\) contributions owing to these radiative triple-vector-boson-production diagrams is also seen in the VBS + triple-gauge-boson-production channels, where the relative QCD corrections are more positive than in the pure VBS channels. The same effect was already observed in other VBS processes [23, 62]. The photon-gluon-induced corrections at \(\mathcal{O}(\alpha_s \alpha^6)\) are about \(-2.5\%\) of the complete \(\mathcal{O}(\alpha^6)\) in the Higgs setup and compatible with zero in the VBS setup. The apparently large relative corrections are again due to the smallness of the corresponding LO contributions.

### 3.3 The role of Higgs VBF in opposite-sign VBS

We previously argued that the size of the EW corrections for opposite-sign W-boson scattering is smaller than for all other VBS processes, because the presence of an \(s\)-channel Higgs propagator shifts a large fraction of events towards its resonance. Therefore it lowers the effective four-fermion invariant mass \(\langle M_{4\ell} \rangle\) from high values, typical for genuine VBS, towards lower values near \(M_H\), which drives the size of the EW correction. Roughly speaking, our signal is not only opposite-sign W-boson scattering, but also VBF Higgs-boson production with subsequent decay into four leptons. In the Higgs setup the EW corrections are found to be even smaller, which is consistent with this picture, because in this setup the cuts increase the fraction of VBF into a Higgs boson.

We can further test this hypothesis by cutting out the Higgs resonance. This should increase the size of the EW corrections to a similar magnitude as found in other VBS processes. To this end, we introduce a third setup based on the VBS setup, but additionally imposing an unphysical invariant-mass cut on the four-lepton system, i.e. requiring

\[
|M_{4\ell} - M_H| > N \Gamma_H,
\]

where \(\Gamma_H\) is the Higgs decay width and \(N = 20\). This method has, in contrast to the diagram-removal techniques proposed in refs. [63, 64], the advantage to remain manifestly gauge invariant and not spoil the important role of the Higgs diagrams for the preservation of unitarity in the high-energy limit. For those LO QCD and LO EW contributions in which the Higgs resonance is absent (WWW only + WWZ only + \(\gamma/g\)) this additional cut is almost negligible, changing their contribution to the fiducial cross section only at the per-mille level from \(1.5417(9) \times 10^{-4}\) fb to \(1.5401(3) \times 10^{-4}\) fb at \(\mathcal{O}(\alpha^6)\), which is compatible with zero within the 2\(\sigma\) integration-error range. Hence we expect this cut to mainly remove the influence of the Higgs resonance on VBS contributions while leaving contributions from other Feynman diagrams mostly unaffected. Taking a Breit-Wigner (BW) resonance peak as a basis for an estimate, the cut (3.27) results in an effective removal of a fraction

\[
\frac{\sigma_{\text{cut}}}{\sigma_{\text{BW}}} \approx \frac{2}{\pi} \arctan(2N)
\]

of the resonance contribution leading to a removal of 98.4\% of the Higgs contribution for \(N = 20\).
Table 6. LO cross section of $\mathcal{O}(\alpha^6)$ and NLO cross section of $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$ in fb and relative corrections in percent with an unphysical invariant-mass cut on the Higgs boson resonance in the VBS setup, classified after appearing subprocesses and corresponding relative corrections $\delta^\alpha = \Delta\sigma^\alpha_{\text{NLO}}/\sigma^\alpha_{\text{LO}}$ and $\delta^{\alpha_s\alpha^6} = \Delta\sigma^{\alpha_s\alpha^6}_{\text{NLO}}/\sigma^{\alpha_s\alpha^6}_{\text{LO}}$.

| Contribution          | $\sigma^\alpha_{\text{LO}}$ [fb] | $\Delta\sigma^\alpha_{\text{NLO}}$ [fb] | $\delta^\alpha$ [%] | $\Delta\sigma^{\alpha_s\alpha^6}_{\text{NLO}}$ [fb] | $\delta^{\alpha_s\alpha^6}$ [%] |
|-----------------------|----------------------------------|----------------------------------------|---------------------|--------------------------------------------------|-------------------------|
| VBS setup, Higgs-resonance contribution |
| VBS only              | 1.6117(2)                        | -0.239(2)                              | -14.8              | -0.043(3)                                        | -2.7                    |
| VBS + WWW             | 0.11398(2)                       | -0.0143(2)                             | -12.5              | 0.0080(5)                                        | 7.1                     |
| VBS + WWZ             | 0.24916(4)                       | -0.0324(3)                             | -13.0              | 0.0018(11)                                       | 0.1                     |
| WWW only              | 5.303(2) × 10^{-5}               | -1.43(2) × 10^{-5}                     | -27.0              | 0.01110(2)                                       | 2.1 × 10^4             |
| WWZ only              | 9.415(2) × 10^{-5}               | -2.80(2) × 10^{-5}                     | -29.7              | 0.004021(3)                                      | 4.3 × 10^3             |
| $\gamma\gamma/\gamma g$ | 6.832(4) × 10^{-6}               | 0.02575(3)                             | 3.8 × 10^5         | 0.0108(2)                                        | 1.6 × 10^5             |
| total                 | 1.9750(2)                        | -0.260(2)                              | -13.2              | -0.007(3)                                        | -0.4                    |

Table 7. LO cross section at $\mathcal{O}(\alpha^6)$ and NLO corrections at $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$ in fb and relative corrections in percent, classified after appearing subprocesses, for the contribution of the Higgs resonance to the cross section $\sigma^{\text{Higgs}} = \sigma^{\text{VBS setup}} - \sigma^{\text{VBS setup, Higgs cut}}$ and the fraction of the Higgs-resonance contribution to the fiducial cross section $\sigma^{\text{Higgs}}/\sigma^{\text{VBS setup}}$ at LO. We note the cancellations and correspondingly large relative combined integration errors in the subtraction for the contributions of the Higgs resonance to non-VBS processes, which lead to results that are compatible with zero.

| Contribution          | $\sigma^\alpha_{\text{LO}}$ [fb] | fraction [%] | $\Delta\sigma^\alpha_{\text{NLO}}$ [fb] | $\delta^\alpha$ [%] | $\Delta\sigma^{\alpha_s\alpha^6}_{\text{NLO}}$ [fb] |
|-----------------------|----------------------------------|-------------|----------------------------------------|---------------------|--------------------------------------------------|
| VBS setup, Higgs-resonance contribution |
| VBS only              | 0.5577(4)                        | 25.7        | -0.042(2)                              | -7.5                | -0.103(5)                                        |
| VBS + WWW             | 0.02385(4)                       | 17.3        | -0.0023(2)                             | -9.5                | -0.0007(6)                                       |
| VBS + WWZ             | 0.14224(7)                       | 36.3        | -0.0103(4)                             | -7.2                | -0.0148(15)                                      |
| WWW only              | 1.6(9) × 10^{-7}                 | -6(5) × 10^{-7} | 5.9(3) × 10^{-4}                        |                     |
| WWZ only              | 3(37) × 10^{-9}                  | 8(4) × 10^{-7} | -1.14(3) × 10^{-4}                     |                     |
| $\gamma\gamma/\gamma g$ | -5(25) × 10^{-10}               | 0.00717(3)  | 0.0110(6)                             |                     |
| total                 | 0.7238(4)                        | 26.8        | -0.047(2)                              | -6.5                | -0.129(5)                                        |

In Table 6 we present results for the VBS setup modified by the cut (3.27). We note that the relative EW corrections to VBS processes with $-12.5\%$ to $-14.8\%$ for the VBS channels, giving an average of $-14.5\%$, are comparable with those for same-sign WW, WZ and ZZ scattering. The other effects that have already been discussed in the previous section hold analogously for this setup. Besides, we remark the accidental cancellation in the total $\mathcal{O}(\alpha_s\alpha^6)$ corrections between the various contributions.

As a last step, we present the contributions of the Higgs resonance in Table 7, which are obtained by subtracting the results of Tables 4 and 6. For contributions involving VBS, its fraction ranges between 17% and 36%. We note that the cut does not affect
the non-VBS channels at LO, leading to results that are compatible with zero within the integration errors.

There are, however, effects at NLO to discuss for these channels. Non-negligible corrections show up at $O(\alpha s^6)$ for the triple-vector-boson production channels. These can be explained by the fact that also for these channels, Feynman diagrams with Higgs bosons in the $s$ channel, emitted from an $s$-channel W or Z boson, are present both at LO and NLO (see figure 4(f)). Within the QCD corrections, those channels get enhanced by emission of a gluon from an initial-state quark, allowing the event to pass the VBS cuts, which leads to a tiny, but sizeable change of the cross section when cutting out the resonance. In contrast, this enhancement is absent for the EW corrections and so is a difference in the cross section. The relatively large difference for photon- and gluon-induced channels at both $O(\alpha^7)$ and $O(\alpha s^6)$ is due to the opening up of new channels at NLO, in which VBS subprocesses appear (see figure 4(c)). The cut on the Higgs resonance evidently affects those new channels while leaving the LO cross section without VBS subprocesses untouched. While the total $O(\alpha^7)$ corrections are only $-6.5\%$ and thus very close to the result in the Higgs setup, the $O(\alpha s^6)$ corrections are with $-18\%$ almost as large as in the Higgs setup.

3.4 Differential distributions

3.4.1 LO distributions

We present distributions for the VBS and the Higgs setup in figures 6, 7, and 8 for all leading orders $O(\alpha^6)$, $O(\alpha s^5)$, $O(\alpha^2 s^4)$, and $O(\alpha^4 s^4)$ and their sum as absolute predictions in the upper panels and their relative contribution to the sum in the lower ones. The upper two diagrams always show the results for the VBS and the lower ones those for the Higgs setup. We note that the interference contribution of order $O(\alpha s^5)$ is not only negligible in the total cross section but also in most of the distributions for both setups.

In the left panels of figure 6 we show the distribution in the invariant mass of the two jets. While the QCD contribution is dominant at low invariant masses, the EW contribution starts to exceed its QCD background at $M_{j_1 j_2} \approx 1500$ GeV in the VBS and at $M_{j_1 j_2} \approx 1000$ GeV in the Higgs setup. For $M_{j_1 j_2} > 3000$ GeV, the EW signal contributes more than 80% and 90% in the VBS and Higgs setups, respectively. The loop-induced contribution is at the level of 2.5% at low invariant masses for the VBS and 5% for the Higgs setup, but drops steeply even below the interference contribution for $M_{j_1 j_2} > 1500$ GeV in the VBS setup and $M_{j_1 j_2} > 2000$ GeV in the Higgs setup.

Another variable to separate EW contributions from QCD contributions is the rapidity difference between the jets. We show the corresponding distribution in the right panels of figure 6. Jets with high rapidity separation are a typical VBS signature, and we observe the dominance of the EW contribution in the expected phase-space region with a rapidity difference larger than $\Delta y_{j_1 j_2} \approx 6$ in the VBS and $\Delta y_{j_1 j_2} \approx 5$ in the Higgs setup. The loop-induced contribution shows a maximum with about 4.5% at $\Delta y_{j_1 j_2} \approx 6$ and $\Delta y_{j_1 j_2} \approx 5.5$ in the VBS setup and Higgs setup, respectively.

A third distribution that is regularly used in VBS physics is shown in the left panels of figure 7. It uses the Zeppenfeld variable of the positron, $z_{e+j_1 j_2}$, defined in eq. (3.23),
Figure 6. Differential distributions at LO in the invariant mass of the tagging jets (left) and the rapidity difference between the these jets (right) in the VBS (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $O(\alpha^6)$, the interference at $O(\alpha\alpha^5)$, the QCD contribution at $O(\alpha^2\alpha^4)$, the loop-induced contribution at $O(\alpha_\text{S}^2\alpha^4)$, and the sum of all contributions. The lower panels show the relative contributions normalised to the sum of all contributions. Shaded bands denote integration errors.

which gives information about the rapidity of the leptons compared to those of the jets. In agreement with the expectation that the lepton rapidity is between the jet rapidities in the EW contribution, we recognise a maximum in this contribution at $z_{e+1j12} \approx 0$, which corresponds to $y_{e^+} \approx (y_{j1} + y_{j2})/2$, i.e. the rapidity of the lepton being the arithmetical mean of the two jet rapidities. The QCD contribution, on the other hand, exhibits a maximum near $z_{e+1j12} \approx 0.3$.

The distribution in the $\Delta R_{j1\mu^-}$ separation of the hardest jet and the muon, in which the difference between the VBS and the Higgs setup becomes visible as well, is presented in the right panels of figure 7. In the Higgs setup, the relative EW contribution grows constantly and exceeds the QCD contribution at $\Delta R_{j1\mu^-} \approx 3.5$, whereas in the VBS setup, the relative EW contribution has a maximum at $\Delta R_{j1\mu^-} \approx 4$ and decreases again. The enhancement of the EW contribution leads to a more symmetric distribution about $\pi$ in the
Figure 7. Differential distributions at LO in the Zeppenfeld variable (left) and the $\Delta R$ separation of the hardest jet and the muon (right) in the VBS (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $\mathcal{O}(\alpha^6)$, the interference at $\mathcal{O}(\alpha_s\alpha^5)$, the QCD contribution at $\mathcal{O}(\alpha^2\alpha^4)$, the loop-induced contribution at $\mathcal{O}(\alpha^4\alpha^4)$, and the sum of all contributions. The lower panels show the relative contributions normalised to the sum of all contributions. Shaded bands denote integration errors.

Higgs setup. We verified that the main difference in the shape of the $\mathcal{O}(\alpha_s^2\alpha^4)$ contributions between the VBS and Higgs setups is due to the cut (3.24) on the Zeppenfeld variable. Since the QCD contributions tend to cause more central jets, they favour events with small rapidity differences between the leptons and the jets, which explains the enhancement of these contributions for small $\Delta R_{j_1\mu^-}$.

In figure 8 (left panels) we present the distributions in the (non-measurable) four-lepton invariant mass $M_{4\ell}$ at LO for the VBS (top) and the Higgs setup (bottom) to stress the impact of the Higgs resonance. We find a single bin at $100 \text{ GeV} < M_{4\ell} < 200 \text{ GeV}$, i.e. the one containing the Higgs resonance, in which the EW contribution is comparable to the QCD one (in the VBS setup) or exceeds the QCD one (in the Higgs setup). A similar but
Figure 8. Differential distributions at LO in the invariant mass of the four final-state leptons (left) and the invariant mass of charged leptons (right) in the VBS setup (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $\mathcal{O}(\alpha^6)$, the interference at $\mathcal{O}(\alpha_s\alpha^5)$, the QCD contribution at $\mathcal{O}(\alpha^2\alpha^4)$, the loop-induced contribution at $\mathcal{O}(\alpha^4\alpha^4)$, and the sum of all contributions. The lower panels show the relative contribution normalised to the sum. Shaded bands denote integration errors.

smaller effect of the Higgs resonance can also be seen in the measurable invariant mass of the two-charged-lepton system (right panels), where we find two bins at very low two-lepton invariant masses in which the EW contribution is with 50% almost comparable to the QCD contribution in the VBS setup and becomes even dominant with approximately 75% in the Higgs setup, until it drops very quickly to 20% above $M_{e^+\mu^-} = 80\text{ GeV}$, from which it rises again to a very flat second maximum near $M_{e^+\mu^-} \approx 1000\text{ GeV}$. We note that the Higgs resonance does not have a visible impact on the loop-induced contribution, although it is present therein.

3.4.2 NLO distributions

Next, we present NLO distributions for the VBS setup and for the Higgs setup in figures 9 and 10. In those plots, we use $\mathcal{O}(\alpha^6)$ as LO baseline and show results for $\mathcal{O}(\alpha^6) + \mathcal{O}(\alpha^7)$,
Figure 9. Differential distributions at NLO in the transverse momentum of the hardest jet (left) and the two charged leptons (right) in the VBS (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $\mathcal{O}(\alpha^6)$, the sum of the EW contribution and the EW corrections of $\mathcal{O}(\alpha^7)$, the sum of the EW contribution and the QCD corrections of $\mathcal{O}(\alpha_s \alpha^6)$, and the sum of all three contributions. The lower panels show the relative EW and QCD corrections normalised to the LO EW contribution. Shaded bands denote integration errors.

$\mathcal{O}(\alpha^6) + \mathcal{O}(\alpha_s \alpha^6)$, and $\mathcal{O}(\alpha^6) + \mathcal{O}(\alpha^7) + \mathcal{O}(\alpha_s \alpha^6)$ as absolute quantities in the upper panels and as corrections relative to the $\mathcal{O}(\alpha^6)$ in the lower ones. We note that the $\mathcal{O}(\alpha^6)$ contribution is not dominant in most regions of phase space. Consequently, the impact of the relative corrections shown in this section on the distributions for the complete processes $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu j j + X$ should be gauged by the relative fraction presented in section 3.4.1. On the other hand, the relative corrections shown below are directly relevant for the EW production processes typically isolated in experimental measurements.

We begin our discussion by considering distributions in energy-dependent variables. Specifically, we present distributions in the transverse momenta of the hardest jet and the two-charged-lepton system in figure 9, as well as distributions in the invariant mass of the two hardest jets and the charged-lepton pair in figure 10. All of these distributions in both setups show the typical behaviour of EW corrections that grow with the energy scale and
reach $-40\%$ to $-50\%$ in the tails of the distributions in our depicted regions. We again emphasise that the EW LO cross section peaks at two-lepton invariant masses below $M_H$, where the EW corrections contribute only a few percent. This becomes especially clear in the Higgs setup, being in accordance with our findings for the Higgs-resonance contribution to the fiducial cross section in table 7, where we also found small EW corrections in this phase-space region. While the distribution in the invariant mass of the positron-muon system drops by almost an order of magnitude above the Higgs resonance, the relative corrections behave smoothly there.

The behaviour of the QCD corrections differs in the VBS and the Higgs setup. In the VBS setup, the QCD corrections for the distribution in the transverse momentum of the hardest jet (figure 9 left) are positive at low values $p_{T,j_1} < 100$ GeV, reach a minimum at $p_{T,j_1} \approx 440$ GeV, and start growing again towards zero for large $p_{T,j_1}$. In contrast, the QCD
corrections in the Higgs setup are monotonically falling very fast and stay roughly constant above $p_{T,j_1} \approx 400$ GeV at a level of $-40\%$. This can be explained by the presence of the jet veto, which allows only for radiation of gluons that are soft or parallel to the beam axis. If already the hardest jet is very soft, the jet veto does not apply to any other jet. On the other hand, if the hardest jet is very hard, other final-state particles are also likely to have high transverse momenta and the jet veto cuts away a fraction of events independently of the actual transverse momentum of the hardest jet.

The QCD corrections to the distribution in the transverse momentum of the two leptons (figure 9 right) show a different behaviour for the VBS and the Higgs setup. While in the VBS setup, the corrections are increasing and even turn positive with higher values of $p_{T,e+\mu^-}$, in the Higgs setup they grow negatively and fluctuate around $-40\%$ for large $p_{T,e+\mu^-}$. Again, the behaviour in the Higgs setup can be explained with the jet veto, but the transverse momenta of the leptons are not that tightly correlated to the one of the softest jet as the one of the hardest jet. NLO corrections to other transverse-momentum distributions of jets and leptons are qualitatively similar.

For the distributions in the invariant mass of the two tagging jets (figure 10 left), the relative QCD corrections in the VBS and the Higgs setup are only quantitatively different; they are almost constant over a wide range of $M_{j_1j_2}$, before statistical fluctuations in the tails do not allow any further statement in these regions. The jet veto leads merely to a constant shift of the corrections, since transverse momentum and invariant mass are uncorrelated observables.

For invariant masses above 100 GeV, the NLO corrections to the distributions in the invariant mass of the charged-lepton pair (figure 10 right) show a similar behaviour to those in the corresponding transverse-momentum distribution. While the NLO QCD corrections reach $-10\%$ for $M_{e+\mu^-} < 100$ GeV, they are close to zero for higher invariant masses in the VBS setup.

Next we turn to distributions in energy-independent variables. Specifically, we discuss the distributions in the rapidity of the hardest jet and the rapidity difference between the two tagging jets in figure 11. The EW corrections to the rapidity distribution of the leading jet (figure 11 left) exhibit a flat (negative) maximum at moderate rapidities around $|y_{j_1}| \approx 1.5$ and $|y_{j_1}| \approx 2$ in the VBS and Higgs setup, respectively. The QCD corrections show qualitatively the same variation in both setups: they peak at $y_{j_1} \approx 0$ and become smaller at high rapidities in the Higgs setup or even change sign in the VBS setup.

The distribution in the rapidity difference of the tagging jets (figure 11 right), a typical VBS observable, shows a qualitative difference in the two setups for both EW and QCD corrections. While the EW corrections are nearly flat in the VBS setup and decrease slightly with large rapidity difference, they increase for the Higgs setup. The QCD corrections also decrease in the VBS setup and become compatible with zero for high rapidity differences, while they have a minimum at $|\Delta y_{j_1j_2}| \approx 7.5$ in the Higgs setup. This behaviour points to a correlation of large rapidity differences with high energies.

We note that the EW and QCD corrections to the distributions in the rapidities of the leptons are flat, i.e. the corresponding corrections are roughly given by those to the fiducial cross section. This holds also for the EW corrections to the distribution in the Zeppenfeld variable.
Figure 11. Differential distributions at NLO in the rapidity of the hardest jet (left) and the rapidity difference between the two tagging jets (right) in the VBS (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $O(\alpha_6)$, the sum of the EW contribution and the EW corrections of $O(\alpha^7)$, the sum of the EW contribution and the QCD corrections of $O(\alpha_s\alpha_6)$, and the sum of all three contributions. The lower panels show the relative EW and QCD corrections normalised to the LO EW contribution. Shaded bands denote integration errors.

3.4.3 Scale dependence

Finally, we investigate the 7-point scale dependence of some distributions. In figure 12, we present those in the transverse momentum of the hardest jet (left) and in the invariant mass of the charged-lepton pair (right) in the VBS setup (top) and the Higgs setup (bottom), and in figure 13 those in the invariant mass of the two hardest jets (left) and the rapidity of the hardest jet (right). We show the absolute LO, i.e. $O(\alpha_6)$, and NLO, i.e. $O(\alpha^6) + O(\alpha^7) + O(\alpha_s\alpha_6)$, predictions including their scale dependence in the upper panels. The lower panels display the scale dependence, defined as the ratio of the envelope (shown as the band) of scale-varied LO and NLO cross sections and the corresponding LO cross section at the central scale. We stress that the NLO corrections are outside the LO scale-uncertainty band in essentially all cases.
Figure 12. Differential distributions at LO and NLO in the transverse momentum of the leading jet (left) and the invariant mass of the charged-lepton pair (right) in the VBS setup (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $O(\alpha^6)$ and the total NLO contribution $O(\alpha^6) + O(\alpha^7) + O(\alpha_s\alpha^6)$, the lower panels the relative corrections. Shaded bands indicate the 7-point scale uncertainty.

In the VBS setup, the NLO scale dependence does not show peculiarities and stays at a modest level for all values of transverse momenta or invariant masses. In the Higgs setup, we observe a qualitatively different behaviour in the scale dependence of variables that are strongly affected by the jet veto, e.g. $p_{T,j_1}$ and $M_{e^+\mu^-}$, and those that are not, e.g. $M_{j_1j_2}$ and $y_{j_1}$. The scale variation in the Higgs setup grows significantly with transverse momenta or invariant masses correlated to the third jet transverse momentum. This is related to the fact that the relative NLO QCD corrections become rather large and even exceed $-100\%$ in the tails, leading to unphysical results. This behaviour can be attributed to the jet veto and indicates that some resummation is required to obtain decent predictions in this part of phase space. The scale dependence of the distributions in variables that are not correlated to the transverse momentum of the third jet, as the jet-pair invariant mass or rapidities, is larger in the Higgs setup than in the VBS setup. However, for those variables, varying the scale does not lead to corrections larger than $50\%$. 
Figure 13. Differential distributions at LO and NLO in the invariant mass of the two tagging jets (left) and the invariant mass of the charged-lepton pair (right) in the VBS setup (top) and the Higgs setup (bottom). The upper panels show the absolute EW contribution at $\mathcal{O}(\alpha^6)$ and the total NLO contribution $\mathcal{O}(\alpha^6) + \mathcal{O}(\alpha^7) + \mathcal{O}(\alpha_s\alpha^6)$, the lower panels the relative corrections. Shaded bands indicate the 7-point scale uncertainty.

This brief analysis of the scale dependence demonstrates that the use of a jet veto requires additional efforts to provide stable NLO QCD predictions already at $\mathcal{O}(\alpha_s\alpha^6)$, i.e. for the QCD corrections to a purely EW process. This will become an even more pressing issue if the QCD corrections to the QCD background at $\mathcal{O}(\alpha_s^3\alpha^4)$ are included.

4 Conclusion

In this article, we have presented results for the NLO EW and QCD corrections of the orders $\mathcal{O}(\alpha^7)$ and $\mathcal{O}(\alpha_s\alpha^6)$ for the process $pp \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu jj + X$ excluding external bottom quarks in two different setups. We have taken the full matrix elements into account without relying on any approximations. At orders $\mathcal{O}(\alpha^6), \mathcal{O}(\alpha^7),$ and $\mathcal{O}(\alpha_s\alpha^6)$ within the considered setups, the process is dominated by vector-boson scattering (VBS) into a pair of W bosons, which contains vector-boson fusion into a Higgs boson as a subprocess, but receives also contributions of triple-vector-boson production. We have included all partonic channels in
the considered orders apart from those involving bottom quarks, which are dominated by contributions from top-pair production.

VBS into oppositely-charged W bosons offers the largest cross section for VBS at the LHC in the range of several femto-barns. Within VBS cuts, about 70% of the cross section is due to the QCD-induced background, while in a dedicated Higgs setup the fraction is reduced to 50%. The contribution of interferences is below one percent and the one of the loop-induced gluon channels below few percent.

Compared to other VBS processes, the EW corrections to the cross section are relatively small owing to the presence of the Higgs resonance in the fiducial phase space, which leads to a sizeable contribution to the EW cross section around a four-lepton invariant mass of $M_4 \ell \approx M_H$. In the tails of the distributions, the EW NLO corrections reach values of $-40\%$ to $-50\%$ in accordance with findings for other VBS processes.

The overall QCD corrections in the VBS setup are, as for other VBS processes, relatively small, whereas they are large and negative in the Higgs setup owing to the veto on a third jet. An investigation of the scale dependence of the cross section shows that jet vetoes should be used with care in VBS processes. The EW and QCD corrections depend on the fiducial volume and on the distribution. While both QCD and EW corrections exhibit a considerable energy dependence, also for energy-independent distributions the QCD and EW corrections show variations of 20% and 5%, respectively.

A further investigation in a setup in which we cut out the Higgs resonance validated our assumptions on its impact on the cross section. We found that the Higgs resonance contributes a large fraction of the cross section for VBS processes. After removing it from the fiducial phase space, NLO EW corrections are comparable to those of other VBS processes at the level of $-15\%$.

After a detailed investigation of the corrections to partonic channels that have been characterised by their subprocesses, we found a correlation between the size of the corrections and the type of processes: the relative EW corrections to non-VBS processes turned out to be large, since they are not affected by the Higgs resonance, but these contributions are negligible due to their small absolute size. As already known from other VBS processes, the relative QCD corrections for non-VBS processes are also large, since the additional emission of a gluon from an initial-state quark changes the action of the VBS cuts on the process, when not all tagging jets have to be produced via a decay of a vector boson in the $s$ channel. The full calculation of all photon-induced corrections leads to a contribution at the percent level at NLO, because new partonic channels with VBS content open up.

This article concludes the calculation of EW corrections to massive VBS processes. In the future, it will be relevant to calculate the full NLO corrections to the corresponding irreducible backgrounds and the EW corrections to polarised VBS.

**Acknowledgments**

We are grateful to Jean-Nicolas Lang and Sandro Uccirati for continuously supporting and improving RECOLA. Many thanks go to Mathieu Pellen for his support of MoCANLO. We acknowledge financial support by the German Federal Ministry for Education and Research.
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