Cabibbo Mixing in Superstring Derived Standard–like Models

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ABSTRACT

We examine the problem of generation mixing in realistic superstring derived standard–like models, constructed in the free fermionic formulation. We study the possible sources of family mixing in these models. In a specific model we estimate the Cabibbo angle. We argue that a Cabibbo angle of the correct order of magnitude can be obtained in these models.

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1. Introduction

One of the fundamental problems in high energy physics is the origin of the fermion masses and mixing hierarchy. The standard model uses thirteen free parameters to parameterize the observed spectrum. Possible extensions to the standard model, like Grand Unified Theories (GUTs) and supersymmetric GUTs, reduce the number of free parameters and can explain inter–family relations between some of the masses. However, GUTs and SUSY GUTs can neither explain the hierarchy among the generations nor the observed values for the family mixing. Over the past few decades many attempts have been made to understand the structure of fermion mass matrices in terms of radiative corrections and additional horizontal symmetries that constrain the allowed interactions [1]. However, all these attempts suffer from a large degree of arbitrariness. Within the context of unified theories it is conceivable that the free parameters in the fermion mass matrices are determined by a fundamental theory at the Planck scale. Superstring theories [2] are the most developed Planck scale theories to date. Therefore, it is important to examine whether realistic superstring models can lead to a qualitative understanding of the fermion mass matrices.

In Ref. [3,4,5] realistic superstring standard–like models were constructed in the four dimensional free fermionic formulation, with the following properties: 

1. Three and only three generations of chiral fermions.
2. The gauge group is $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3R}} \times U(1)^n \times hidden$. The weak hypercharge is uniquely given by $U(1)_Y = T_{3R} + \frac{1}{2}(B-L)$ and has the standard $SO(10)$ embedding. Therefore, it leads unambiguously to the prediction $\sin^2 \theta_W = \frac{3}{8}$ at the unification scale.
3. There are enough scalar doublets and singlets to break the symmetry in a realistic way and to generate a realistic fermion mass hierarchy [4,5].
4. The models are free from gauge and gravitational anomalies apart from a single “anomalous $U(1)_A$” symmetry that is broken by the Dine–Seiberg–Witten (DSW) mechanism [6].
5. The free fermionic standard–like models suggest an explanation for the fermion mass hierarchy. At the cubic level of the superpotential only the top quark gets a nonvanishing mass term. The mass terms for the lighter quarks
and leptons are obtained from nonrenormalizable terms. $SO(10)$ singlet fields in these terms obtain nonvanishing VEVs by the application of the DSW mechanism. Thus, the order $N$ nonrenormalizable terms, of the form $c f f h(\Phi/M)^{N-3}$, become effective trilinear terms, where $f, h, \Phi$ denote fermions, scalar doublets and scalar singlets, respectively. $M$ is a Planck scale mass to be defined later. The effective Yukawa couplings are given by $\lambda = c(\langle \Phi \rangle/M)^{N-3}$ where the calculable coefficients $c$ are of order one [7].

In this paper we examine the problem of generation mixing in the realistic superstring derived standard–like models. We show that the family mixing arises due to hidden sector states that are obtained from specific sectors in the massless spectrum. We contemplate two possible scenarios for generating the Cabibbo angle in these models. One is due to condensates of a non–Abelian hidden gauge group. In the second scenario the hidden sector states obtain VEVs by the application of the DSW mechanism. We demonstrate, in a specific model, that the second scenario can produce a Cabibbo angle of the correct order of magnitude, while the first scenario is marginal.

2. The superstring standard–like models

The superstring standard–like models are constructed in the four dimensional free fermionic formulation [8]. The models are generated by a basis of eight boundary condition vectors for all world–sheet fermions. The first five vectors in the basis consist of the NAHE set $\{1, S, b_1, b_2, b_3\}$ [9,5]. The gauge group after the NAHE set is $SO(10) \times E_8 \times SO(6)^3$, with $N = 1$ space–time supersymmetry, and 48 spinorial 16 of $SO(10)$. The standard–like models are constructed by adding three additional vectors to the NAHE set [10,3,4,5]. Three additional vectors are needed to reduce the number of generations to one generation from each sector $b_1, b_2$ and $b_3$. The three vectors that extend the NAHE set and the choice of generalized GSO projection coefficients for our model are given in table 1 [3]. The observable and hidden gauge groups after application of the generalized GSO projections are
respectively. The weak hypercharge is given by $U_\alpha$ and $SU(3)_C \times SU(2)_L \times U(1)_L \times U(1)_Y^6$ and $SU(5)_H \times SU(3)_R \times U(1)^2$, respectively. The weak hypercharge is given by $U(1)_Y = \frac{1}{3} U(1)_C + \frac{1}{2} U(1)_L$ and has the standard $SO(10)$ embedding. The orthogonal combination is given by $U(1)_{\alpha} = U(1)_C - U(1)_L$. The vectors $\alpha, \beta, \gamma$ break the $SO(6)_j$ horizontal symmetries to $U(1)_R \times U(1)_{\ell,j+3}$ $(j=1,2,3)$, which correspond to the right–moving world–sheet currents $\bar{\eta}^1_\alpha \eta^1_\alpha$ $(j = 1, 2, 3)$ and $\bar{y}_3 y_6, \bar{y}_1 \omega_5, \bar{\omega}_2 \omega_4$, respectively. For every right–moving $U(1)$ symmetry correspond a left–moving global $U(1)$ symmetry. The first three correspond to the charges of the supersymmetry generator $\chi^{12}, \chi^{34}$ and $\chi^{56}$. The last three, $U(1)_{\ell,j+3}$ $(j = 1, 2, 3)$, correspond to the complexified left–moving fermions $y_3^2 y_6^2, y_1^4 \omega^5$ and $\omega^2 \omega^4$. Finally, the model contains six Ising model operators that are obtained by pairing a left–moving real fermion with a right–moving real fermion, $\sigma^i_\pm = \{\omega^1 \bar{\omega}^1, y^2 y^6, \omega^3 \bar{\omega}^3, y^4 y^4, y^5 y^5, \omega^6 \bar{\omega}^6\} \pm$.

The full massless spectrum was presented in Ref. [3]. Here we list only the states that are relevant for the quark mass matrices. The following massless states are produced by the sectors $b_{1,2,3}$, $S + b_1 + b_2 + \alpha + \beta$, $O$ and their superpartners in the observable sector:

(a) The $b_{1,2,3}$ sectors produce three $SO(10)$ chiral generations, $G_\alpha = e^{c}_L + u^{c}_L + N^{c}_L + d^{c}_L + Q_\alpha + L_\alpha$ $(\alpha = 1, \cdots, 3)$ where

$$
e^{c}_L \equiv [(1, \frac{3}{2}); (1, 1)]; \quad u^{c}_L \equiv [(3, \frac{1}{2}); (1, -1)]; \quad Q \equiv [(3, \frac{1}{2}); (2, 0)] \quad (1a, b, c)$$

$$N^{c}_L \equiv [(1, \frac{3}{2}); (1, -1)]; \quad d^{c}_L \equiv [(3, \frac{1}{2}); (1, 1)]; \quad L \equiv [(1, -\frac{3}{2}); (2, 0)] \quad (1d, e, f)$$

of $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$, with charges under the six horizontal $U(1)$s,

\begin{align*}
& (e^{c}_L + u^{c}_L)_{\frac{1}{2}, 0, 0, 0, 0} + (d^{c}_L + N^{c}_L)_{\frac{1}{2}, 0, 0, \frac{1}{2}, 0} + (L)_{\frac{1}{2}, 0, 0, \frac{1}{2}, 0} + (Q)_{\frac{1}{2}, 0, 0, \frac{1}{2}, 0}; \quad (2a) \\
& (e^{c}_L + u^{c}_L)_{0, \frac{1}{2}, 0, 0, 0} + (N^{c}_L + d^{c}_L)_{\frac{1}{2}, 0, 0, 0, \frac{1}{2}} + (L)_{\frac{1}{2}, 0, 0, 0, \frac{1}{2}} + (Q)_{0, \frac{1}{2}, 0, 0, \frac{1}{2}}; \quad (2b) \\
& (e^{c}_L + u^{c}_L)_{0, 0, \frac{1}{2}, 0, \frac{1}{2}} + (N^{c}_L + d^{c}_L)_{0, \frac{1}{2}, 0, 0, \frac{1}{2}} + (L)_{0, \frac{1}{2}, 0, 0, \frac{1}{2}} + (Q)_{0, 0, \frac{1}{2}, 0, \frac{1}{2}}; \quad (2c)
\end{align*}

$^*$ $U(1)_C = \frac{3}{2} U(1)_{B-L}$ and $U(1)_L = 2 U(1)_{T_{3R}}.$
The vectors $b_1, b_2, b_3$ are the only vectors in the additive group $\Xi$ which give rise to spinorial 16 of $SO(10)$.

(b) The $S + b_1 + b_2 + \alpha + \beta$ sector gives

$$h_{45} \equiv [(1, 0); (2, 1)]_{\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0} \quad D_{45} \equiv [(3, -1); (1, 0)]_{-\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0} \quad (3a, b)$$

$$\Phi_{45} \equiv [(1, 0); (1, 0)]_{\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0} \quad \Phi_{1}^\pm \equiv [(1, 0); (1, 0)]_{-\frac{1}{4}, \frac{1}{4}, 0, \pm 1, 0, 0} \quad (3c, d)$$

$$\Phi_{2}^\pm \equiv [(1, 0); (1, 0)]_{\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0} \quad \Phi_{3}^\pm \equiv [(1, 0); (1, 0)]_{\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, \pm 1} \quad (3e, f)$$

(and their conjugates $\bar{h}_{45}$, etc.). The states are obtained by acting on the vacuum with the fermionic oscillators $\bar{\psi}^{45}, \bar{\psi}^{1, \ldots, 3}, \bar{\eta}^{3}, \bar{\gamma}^{3} \pm iy^{6}, \bar{\gamma}^{1} \pm i\bar{\omega}^{5}, \bar{\omega}^{2} \pm i\bar{\omega}^{4}$, respectively (and their complex conjugates for $\bar{h}_{45}$, etc.).

(c) The Neveu–Schwarz $O$ sector gives, in addition to the graviton, dilaton, antisymmetric tensor and spin 1 gauge bosons, scalar electroweak doublets and singlets:

$$h_{1} \equiv [(1, 0); (2, -1)]_{1, 0, 0, 0, 0, 0} \quad \Phi_{23} \equiv [(1, 0); (1, 0)]_{0, 1, -1, 0, 0, 0} \quad (4a, b)$$

$$h_{2} \equiv [(1, 0); (2, -1)]_{0, 1, 0, 0, 0, 0} \quad \Phi_{13} \equiv [(1, 0); (1, 0)]_{1, 0, -1, 0, 0, 0} \quad (4c, d)$$

$$h_{3} \equiv [(1, 0); (2, -1)]_{0, 0, 1, 0, 0, 0} \quad \Phi_{12} \equiv [(1, 0); (1, 0)]_{1, -1, 0, 0, 0, 0} \quad (4e, f)$$

(and their conjugates $\bar{h}_{1}$, etc.). Finally, the Neveu–Schwarz sector gives rise to three singlet states that are neutral under all the $U(1)$ symmetries. $\xi_{1,2,3} : \chi_{1}^{12} \omega_{1}^{3} \bar{\omega}_{1}^{6} |0\rangle_{0}$, $\chi_{1}^{34} \bar{\gamma}_{1}^{5} \bar{\omega}_{1}^{4} |0\rangle_{0}$, $\chi_{2}^{56} \bar{\gamma}_{2}^{5} \bar{\gamma}_{1}^{4} |0\rangle_{0}$.

The sectors $b_{i} + 2\gamma + (I) \ (i = 1, \ldots, 3)$ give vector–like representations that are $SU(3)_{c} \times SU(2)_{L} \times U(1)_{L} \times U(1)_{C}$ singlets and transform as 5, $\bar{5}$ and 3, $\bar{3}$ under the hidden $SU(5)$ and $SU(3)$ gauge groups, respectively (see table 2). As will be shown below, the states from the sectors $b_{j} + 2\gamma$ produce the mixing between the chiral generations. We would like to emphasize that the structure of the massless spectrum exhibited in Eqs. (1–4), and in table 2, is common to a large number of free fermionic standard–like models. All the standard–like models contain three
chiral generations from the sectors $b_j$, vector–like representations from the sectors $b_j + 2\gamma$, and Higgs doublets from the Neveu–Schwarz sector and from the sector with $\alpha + \beta$ plus some combination of $\{b_1, b_2, b_3\}$. Therefore, the source of the family mixing is a general characteristic of these models. It arises due to the basic set $\{1, S, b_1, b_2, b_3\}$ and the use of the $Z_4$ twist to break the symmetry from $SO(2n)$ to $SU(n) \times U(1)$.

In addition to the states above, the massless spectrum contains massless states from sectors with some combination of $\{b_1, b_2, b_3, \alpha, \beta\}$ and $\gamma + (I)$. These states are model dependent and carry either fractional electric charge or $U(1)_{Z'}$ charge. As argued in Ref. [11] the $U(1)_{Z'}$ symmetry has to be broken at an intermediate energy scale that is suppressed relative to the Planck scale. Therefore, the states from these sectors do not play a significant role in the quark mass matrices and we do not consider them in this paper.

The model contains six anomalous $U(1)$ symmetries: $\text{Tr} U_1 = 24$, $\text{Tr} U_2 = 24$, $\text{Tr} U_3 = 24$, $\text{Tr} U_4 = -12$, $\text{Tr} U_5 = -12$, $\text{Tr} U_6 = -12$. Of the six anomalous $U(1)$s, five can be rotated by an orthogonal transformation and one combination remains anomalous. The six orthogonal combinations are given by [3],

\begin{align}
U'_1 &= U_1 - U_2, & U'_2 &= U_1 + U_2 - 2U_3, & (5a, b) \\
U'_3 &= U_4 - U_5, & U'_4 &= U_4 + U_5 - 2U_6, & (5c, d) \\
U'_5 &= U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6, & (5e) \\
U_A &= 2U_1 + 2U_2 + 2U_3 - U_4 - U_5 - U_6, & (5f)
\end{align}

with $\text{Tr}(Q_A) = 180$. The anomalous $U(1)$ symmetry generates a large Fayet-Iliopoulos D–term by the VEV of the dilaton field [6]. Such a D–term would in general break supersymmetry and destabilize the string vacuum, unless there is a direction in the scalar potential $\phi = \sum_i \alpha_i \phi_i$ which is F–flat and also D–flat with respect to the non–anomalous gauge symmetries and in which $\sum_i Q_i^A|\alpha_i|^2 < 0$. If such a direction exists, it will acquire a VEV, breaking the anomalous D–term,
restoring supersymmetry and stabilizing the vacuum [12]. Since the fields corresponding to such a flat direction typically also carry charges for the nonanomalous D–terms, a nontrivial set of constraints on the possible choices of VEVs is imposed [5].

3. The superpotential

We now turn to the superpotential of the model. At the cubic level the following terms are obtained in the observable sector [3],

\[
W_3 = \{(u^c_{L_1} Q_1 \bar{h}_1 + N^c_{L_1} L_1 \bar{h}_1 + u^c_{L_2} Q_2 \bar{h}_2 + N^c_{L_2} L_2 \bar{h}_2 + u^c_{L_3} Q_3 \bar{h}_3 + N^c_{L_3} L_3 \bar{h}_3) \\
+ h_1 \bar{h}_2 \Phi_{12} + h_1 \bar{h}_3 \Phi_{13} + h_2 \bar{h}_3 \Phi_{23} + h_1 h_2 \Phi_{12} + h_1 h_3 \Phi_{13} + h_2 h_3 \Phi_{23} + \Phi_{23} \Phi_{13} \Phi_{12} \\
+ \Phi_{23} \Phi_{13} \Phi_{12} + \Phi_{12}(\Phi_1^+ \Phi_1^- + \Phi_2^+ \Phi_2^- + \Phi_3^+ \Phi_3^-) + \Phi_{12}(\Phi_1^- \Phi_1^+ + \Phi_2^- \Phi_2^+ + \Phi_3^- \Phi_3^+) \\
+ \frac{1}{2} \xi_3(\Phi_{45} \bar{\Phi}_{45} + h_{45} \bar{h}_{45} + D_{45} \bar{D}_{45} + \Phi_1^+ \Phi_1^- + \Phi_2^+ \Phi_2^- + \Phi_3^+ \Phi_3^-) \\
+ \Phi_3^- \Phi_3^+ \} + h_3 \bar{h}_{45} \Phi_{45} + \bar{h}_3 h_{45} \bar{\Phi}_{45}
\]

with a common normalization constant \(\sqrt{2}g\).

Nonrenormalizable contributions to the superpotential are obtained by calculating correlators between vertex operators [7], \(A_N \sim \langle V^f_1 V^f_2 V^h_3 \cdots V^h_N \rangle\), where \(V^f_i, (V^h_i)\) are the fermionic (scalar) components of the vertex operators. In the analysis of nonrenormalizable terms we imposed the F–flatness restriction \(\langle \Phi_{12}, \Phi_{13}, \xi_3 \rangle \equiv 0\) [11].

At the quartic order there are no potential quark mass terms. At the quintic order the following mass terms are obtained,

\[
d_1 Q_1 h_{45} \Phi_1^+ \xi_2 \\
d_2 Q_2 h_{45} \bar{\Phi}_2^- \xi_1
\]

(7a, b)

\[
u_1 Q_1 (\bar{h}_{45} \Phi_{45} \Phi_{13} + \bar{h}_2 \Phi_1^+ \Phi_1^-) \\
u_2 Q_2 (\bar{h}_{45} \Phi_{45} \Phi_{23} + \bar{h}_1 \Phi_1^+ \Phi_1^-)
\]

(7c, d)

\[
(u_1 Q_1 h_1 + u_2 Q_2 h_2) \frac{\partial W}{\partial \xi_3}.
\]

(7g)
At order \( N = 6 \) we obtain mixing terms for \(-\frac{1}{3}\) charged quarks,

\[
\begin{align*}
d_3 Q_2 h_{45} \Phi_{45} V_3 \bar{V}_2, & \quad d_2 Q_3 h_{45} \Phi_{45} V_2 \bar{V}_3, \quad (8a, b) \\
d_3 Q_1 h_{45} \Phi_{45} V_3 \bar{V}_1, & \quad d_1 Q_3 h_{45} \Phi_{45} V_1 \bar{V}_3, \quad (8c, d)
\end{align*}
\]

At order \( N = 7 \) we obtain in the down quark sector,

\[
\begin{align*}
d_2 Q_1 h_{45} \Phi_{45} (V_1 \bar{V}_2 + V_2 \bar{V}_1) \xi_i, & \quad d_1 Q_2 h_{45} \Phi_{45} (V_1 \bar{V}_2 + V_2 \bar{V}_1) \xi_i \quad (9a, b) \\
d_1 Q_3 h_{45} \Phi_{45} V_3 \bar{V}_1 \xi_2, & \quad d_3 Q_1 h_{45} \Phi_{45} V_1 \bar{V}_3 \xi_2 \quad (9c, d) \\
d_2 Q_3 h_{45} \Phi_{45} V_3 \bar{V}_2 \xi_1, & \quad d_3 Q_2 h_{45} \Phi_{45} V_2 \bar{V}_3 \xi_1, \quad (9e, f)
\end{align*}
\]

where \( \xi_i = \{\xi_1, \xi_2\} \). In the up quark sector we obtain,

\[
\begin{align*}
u_1 Q_2 \tilde{h}_1 \Phi_{45} \{\tilde{\Phi}_2^- (T_1 \tilde{T}_2 + T_2 \tilde{T}_1) + \tilde{\Phi}_1^+ (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} & \quad (10a) \\
u_2 Q_1 \tilde{h}_1 \Phi_{45} \{\tilde{\Phi}_1^- (T_1 \tilde{T}_2 + T_2 \tilde{T}_1) + \tilde{\Phi}_2^+ (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} & \quad (10b) \\
u_1 Q_2 \tilde{h}_2 \Phi_{45} \{\tilde{\Phi}_2^+ (T_1 \tilde{T}_2 + T_2 \tilde{T}_1) + \tilde{\Phi}_1^- (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} & \quad (10c) \\
u_2 Q_1 \tilde{h}_2 \Phi_{45} \{\tilde{\Phi}_1^+ (T_1 \tilde{T}_2 + T_2 \tilde{T}_1) + \tilde{\Phi}_2^- (V_1 \bar{V}_2 + V_2 \bar{V}_1)\} & \quad (10d) \\
u_3 Q_1 \tilde{h}_1 \Phi_{45} \{\tilde{\Phi}_1^- T_1 \tilde{T}_3 + \tilde{\Phi}_3^+ V_3 \bar{V}_1\} & \quad u_1 Q_3 \tilde{h}_1 \Phi_{45} \{\tilde{\Phi}_3^- T_1 \tilde{T}_3 + \tilde{\Phi}_1^+ V_3 \bar{V}_1\} \quad (10e) \\
u_3 Q_1 \tilde{h}_2 \Phi_{45} \{\tilde{\Phi}_3^- T_1 \tilde{T}_3 + \tilde{\Phi}_1^+ V_3 \bar{V}_1\} & \quad u_1 Q_3 \tilde{h}_2 \Phi_{45} \{\tilde{\Phi}_3^- T_1 \tilde{T}_3 + \tilde{\Phi}_1^+ V_3 \bar{V}_1\} \quad (10f) \\
u_3 Q_2 \tilde{h}_1 \Phi_{45} \{\tilde{\Phi}_2^- T_2 \tilde{T}_3 + \tilde{\Phi}_3^+ V_3 \bar{V}_2\} & \quad u_2 Q_3 \tilde{h}_1 \Phi_{45} \{\tilde{\Phi}_3^- T_2 \tilde{T}_3 + \tilde{\Phi}_2^+ V_3 \bar{V}_2\} \quad (10g) \\
u_3 Q_2 \tilde{h}_2 \Phi_{45} \{\tilde{\Phi}_2^- T_2 \tilde{T}_3 + \tilde{\Phi}_3^+ V_3 \bar{V}_2\} & \quad u_2 Q_3 \tilde{h}_2 \Phi_{45} \{\tilde{\Phi}_3^- T_2 \tilde{T}_3 + \tilde{\Phi}_2^+ V_3 \bar{V}_2\} \quad (10h)
\end{align*}
\]

At order \( N = 7 \) we obtain generation mixing terms in the up and down quark sectors. The states that induce the mixing come from the sectors \( b_j + 2\gamma \). In the up quark sector, mixing is obtained by 5, 5 and 3, 3 of the hidden \( SU(5) \) and \( SU(3) \) gauge groups, respectively. In the down quark sector, the mixing is only by the 3, 3 of the hidden \( SU(3) \) gauge groups. At order \( N = 8 \) we obtain mixing in the down quark sector by the \( SU(5) \) states from the sectors \( b_j + 2\gamma \),

\[
\begin{align*}
d_3 Q_1 h_{45} \Phi_{45} \{\Phi_1^+ \Phi_3^- + \Phi_3^+ \Phi_1^-\} T_1 \tilde{T}_3, & \quad d_1 Q_3 h_{45} \Phi_{45} \{\Phi_1^+ \Phi_3^- + \Phi_3^+ \Phi_1^-\} T_3 \tilde{T}_1 \quad (11a)
\end{align*}
\]
\[ d_3Q_2h_{45}\Phi_{45}\{\Phi_2^+\Phi_3^- + \Phi_3^+\Phi_2^-\}T_2\bar{T}_3 \quad d_2Q_3h_{45}\Phi_{45}\{\Phi_2^+\Phi_3^- + \Phi_3^+\Phi_2^-\}T_3\bar{T}_2 \quad (11b) \]

The analysis of the nonrenormalizable terms up to order \( N = 8 \) shows that family mixing terms are obtained for all generations. The mixing arises due to the states from the sectors \( b_j + 2\gamma \). These sectors, and their relation to the sectors \( b_j \), is a general characteristic of the realistic free fermionic models that use a \( Z_4 \) twist. Therefore, the family mixing due to these states is a general characteristic of these models. In the next two sections we estimate the size of the off–diagonal terms, and examine whether these models can account for the observed value of the Cabibbo angle.

### 4. Cabibbo mixing from hidden sector condensates

The mixing terms in the previous section contained \( 5, \overline{5} \) and \( 3, \overline{3} \) under the hidden \( SU(5) \) and \( SU(3) \) gauge groups, respectively. These bilinears may produce scalar condensates when \( \alpha_h \) becomes large. More generally, in any free fermionic standard–like model there is one or several non–Abelian hidden gauge groups. The largest non–Abelian hidden gauge group that can be obtained in the standard–like models is \( SU(7) \). Modifying the vector \( \gamma \) of Ref. [3] by \( \gamma\{\overline{5}^3\overline{5}^4\} = 1 \rightarrow \gamma\{\overline{5}^3\overline{5}^4\} = 0 \), enhances the hidden gauge group from \( SU(5)_H \times SU(3)_H \times U(1)^2 \) to \( SU(7)_H \times U(1)^2 \), for an appropriate choice of the generalized GSO projection coefficients. The observable massless spectrum, Eqs. (1–4), remains essentially the same. The states in the sectors \( b_j + 2\gamma \) form \( 7 \) and \( \overline{7} \), and singlets, of \( SU(7)_H \). The anomalous \( U(1)s \), and the anomaly free combinations are as in Eq. (5). The cubic and quintic level terms are the same as in Eqs. (6–7). The mixing terms are generated by \( 7 \) and \( \overline{7} \) of the hidden \( SU(7) \) gauge group, and by the \( SU(7) \) singlets, from the sectors \( b_j + 2\gamma \). The mixing terms are similar to the terms in Eqs. (8–10). Bilinear condensates of states from the sectors \( b_j + 2\gamma \) that transform under the hidden non–Abelian gauge groups may account for the generation mixing. To estimate the possible magnitude of mixing terms in the mass matrices we have to estimate the condensates of the hidden \( SU(7) \) gauge group.
The light Higgs representations are $\bar{h}_1$ or $\bar{h}_2$ and $h_{45}$ [11]. The mixing is dominantly in the down quark sector. An off–diagonal term in the up–quark mass matrix does not contribute much to the mixing because of the large diagonal terms. The top quark mass term is obtained at the cubic level and the top Yukawa coupling is of order one [3,4]. Eq. (7) shows that both the bottom and strange Yukawa couplings, as well as the charm quark mass term, can be obtained at the quintic order for an appropriate choice of singlet VEVs [11]. The VEVs of $\xi_1$ and $\xi_2$ are undetermined and we use them to fit the bottom and strange quark masses. We take $m_t \sim 140 \text{ GeV}$, $\tan \beta = v_1/v_2 \sim 1.5$, and therefore $\lambda_b \sim 0.01$, which fixes $\lambda_s \sim 0.001$. The sector $b_3$ produces the lightest generation states [11]. Diagonal mass terms for the states from $b_3$ can only be generated by VEVs that break $U(1)_{Z'}$. We assume that $U(1)_{Z'}$ is broken at an intermediate scale that is suppressed relative to the $SO(10)$ singlet VEVs. Otherwise, the cubic level F–flat solution is violated by higher order nonrenormalizable terms [11]. Consequently, we take the diagonal mass term for the lightest generation to be zero. The mixing term between the two lightest generations should be of the order $O(10^{-4})$ to produce a Cabibbo angle of the correct order of magnitude. Thus, we have to examine terms that mix $\{d_3; Q_3\}$ with $\{d_2; Q_2\}$ or $\{d_1; Q_1\}$.

The scale $\Lambda_7$ at which the $SU(7)$ gauge coupling constant $\alpha_7$ gets strong is given by

$$\alpha_7(\Lambda_7) = \frac{\alpha_7(M)}{C(1 - (b/2\pi)\alpha_7(M)\ln(\Lambda_7/M))} \approx 1$$

(12)

where $b = (n_f/2) - 21$. The bilinear hidden sector condensates produces a suppression factor that is given by

$$\left(\frac{\Lambda_7}{M}\right)^2 = \exp\left(\frac{64\pi}{b}\right)$$

(13)

where $\alpha_7(M) \approx 1/17$ from string gauge coupling unification [13]. The suppression factor depends on the number of $7$ and $\bar{7}$ that are massless below the Planck scale. In our model there are eight pairs of $7$ and $\bar{7}$, which gives $(\Lambda_7/M)^2 \sim 10^{-6}$, and
suppresses the mixing below the observed values. Assuming that all $7$ and $\bar{7}$ receive mass at the Planck scale gives a suppression factor $(\Lambda_7/M)^2 \sim 5 \times 10^{-5}$, which is still too small. The only way that bilinear hidden sector condensates may produce sizable mixing is if the gauge coupling at the unification scale turns out to be of the order $\alpha_U \sim \frac{1}{8} - \frac{1}{10}$. Thus, we conclude that family mixing via bilinear condensates in these models is marginal. It is possible if the hidden gauge group is large, like $SU(7)$, and for a rather large gauge coupling at the unification scale. If the hidden gauge group is $SU(5)$ or $SU(3)$, $(\Lambda_h/M)$ will be smaller, and the mixing terms will be suppressed even more. In the next section we estimate the off–diagonal terms in the case that some of the hidden sector states obtain nonvanishing VEVs. In this scenario it is possible to obtain Cabibbo angle of the correct order of magnitude, independent of the specific hidden gauge group.

5. Cabibbo mixing from hidden sector VEVs

An alternative to the scenario discussed in the previous section is that some of the hidden sector states, from the sectors $b_j + 2\gamma$, receive VEVs by the cancellation of the “anomalous” $U(1)$ D–term equation. We therefore have to find F and D flat solutions that contain nonvanishing VEVs for the states from the sectors $b_j + 2\gamma$. An explicit solution that satisfies all the F and D flatness constraints is given by the following set of nonvanishing VEVs,

$$\{V_2, \bar{V}_3, \Phi_{45}, \Phi_{23}, \bar{\Phi}_{23}, \Phi_{13}, \bar{\Phi}_{13}, \Phi^+_1, \Phi^+_2, \bar{\Phi}^+_1, \bar{\Phi}^+_2, \xi_1, \xi_2\}, \quad (14)$$

with

$$|V_2|^2 = |\bar{V}_3|^2 = \frac{1}{5} |\Phi_{45}|^2 = |\bar{\Phi}^-_1|^2 = \frac{g^2}{16\pi^2} \frac{1}{\sqrt{2} \alpha'}, \quad (15a)$$

$$3|\Phi^+_2|^2 = 3|\bar{\Phi}^+_2|^2 = |\Phi^-_2|^2 = |\bar{\Phi}^-_2|^2 = \frac{1}{4} |\Phi^+_1|^2 = \frac{1}{4} |\bar{\Phi}^+_1|^2 = |\Phi^-_1|^2, \quad (15b)$$

$$|\Phi_{23}|^2 = |\bar{\Phi}_{23}|^2 = \frac{1}{3} |\Phi_{13}|^2, \quad (15c)$$

$$|\Phi_{13}|^2 = |\bar{\Phi}_{13}|^2 = \frac{g^2}{8\pi^2} \frac{1}{\sqrt{2} \alpha'}. \quad (15d)$$
In this solution the VEVs of $\xi_1$, $\xi_2$ and $\Phi_{13}$ are undetermined and remain free parameters. The choice of VEVs, Eq. (12), does not affect the Higgs mass matrix up to order $N = 8$. Therefore, the light Higgs representations are still $\tilde{h}_1$ or $\tilde{h}_2$ and $h_{45}$ [11].

The up and down quark mass matrices are diagonalized by bi–unitary transformations,

$$U_L M_u U_R^\dagger = D_u \equiv \text{diag}(m_u, m_c, m_t), \quad (16a)$$

$$D_L M_d D_R^\dagger = D_d \equiv \text{diag}(m_d, m_s, m_b), \quad (16b)$$

with the mixing matrix given by,

$$V = U_L D_L^\dagger. \quad (17)$$

For our F and D flat solution the down quark mass matrix takes the form,

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2 \tilde{V}_3 \Phi_{45}}{M^4} & 0 \\ \frac{V_2 \tilde{V}_3 \Phi_{45}}{M^4} & \frac{\Phi_4}{M^2} & 0 \\ 0 & 0 & \Phi^+_4 \xi_2 \end{pmatrix} v_2 \quad (18)$$

where we have used [7] $\frac{1}{2}g \sqrt{2\alpha'} = \sqrt{8\pi}/M_{Pl}$, to define $M \equiv M_{Pl}/2\sqrt{8\pi} \approx 1.2 \times 10^{18} GeV$. Following Ref. [7], we assume that the coefficients of the non-renormalizable terms are of order one. The bottom and strange quark masses can be fitted by giving appropriate VEVs to $\xi_1$ and $\xi_2$. The 12 and 21 entries are then determined by our F and D flat solution. Inserting the numerical values for the VEVs of $V_2 \tilde{V}_3$ and $\Phi_{45}$ from Eq. (15a), we obtain

$$\frac{V_2 \tilde{V}_3 \Phi_{45}}{M^3} = \frac{\sqrt{5}g^6}{64\pi^3} \approx 2 - 3 \times 10^{-4}. \quad (19)$$

Note that the down quark mass matrix is not symmetric. Only the 12 entry in the down quark mass matrix has to be of the order $O(10^{-4})$ to obtain a Cabibbo
angle of the correct order of magnitude. We can use the remaining free parameter to set \( \langle \Phi^- \rangle / M \sim 0.001 \), which imposes \( \xi_1 \sim 1 \). Inserting the numerical values to the mass matrix, with \( \epsilon << 10^{-4} \) and \( g \sim 0.8 \), and performing numerical singular value decomposition, we obtain for the mixing matrix

\[
|V| \sim \begin{pmatrix}
0.98 & 0.2 & 0 \\
0.2 & 0.98 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(20)

The running from the unification scale to the weak scale does not affect the Cabibbo angle by much [14]. Thus, we conclude that a Cabibbo angle of the correct order of magnitude can be obtained in this scenario.

6. Conclusions

In this paper we discussed the problem of generation mixing in superstring derived standard–like models. These models are constructed in the free fermionic formulation. They correspond to models that are compactified on \( Z_2 \times Z_2 \) orbifold with “standard embedding”, and use a \( Z_4 \) twist to break the symmetry from \( SO(2n) \) to \( SU(n) \times U(1) \). We showed that the source of the mixing are the states from the sectors \( b_j + 2\gamma \). We believe that the source of the mixing is a general characteristic of these models, and is a consequence of the basic set \( \{1, S, b_1, b_2, b_3\} \) and the use of the \( Z_4 \) twist. We examined two possible scenarios for producing the family mixing. One is based on matter condensates of a non–Abelian hidden gauge group. The other is based on giving nonvanishing VEVs to states from the sectors \( b_j + 2\gamma \), by the cancellation of the anomalous \( U(1) \) D–term equation. We estimated the Cabibbo angle in the two scenarios and showed that in the second scenario a Cabibbo angle of the correct order of magnitude can be obtained in these models. Mixing between the two heaviest generations can be obtained by finding F and D flat solutions with a non vanishing VEV for \( V_1 \). We will expand upon the phenomenology derived from these models in future publications.

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\[\psi^\mu \{ \chi^{12}; \chi^{34}; \chi^{56}\} \psi^1, \bar{\psi}^2, \bar{\psi}^3, \bar{\psi}^4, \bar{\psi}^5, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3 \]

\[\bar{\phi}^1, \bar{\phi}^2, \bar{\phi}^3, \bar{\phi}^4, \bar{\phi}^5, \bar{\phi}^6, \bar{\phi}^7, \bar{\phi}^8\]

\begin{table}
\begin{tabular}{|c|c|c|c|c|}
\hline
 & \(y^3y^6\), \(y^4y^4\), \(y^5\bar{y}^5\), \(\bar{y}^3\bar{y}^6\) & \(y^1\omega^6\), \(y^2\bar{y}^2\), \(\omega^5\bar{y}^5\), \(\bar{y}^1\omega^6\) & \(\omega^1\omega^3\), \(\omega^2\omega^2\), \(\omega^4\omega^4\), \(\bar{\omega}^1\bar{\omega}^3\) \\
\hline
\(\alpha\) & 1, 0, 0, 0, 0 & 0, 0, 1, 1 & 0, 0, 1, 1 \\
\(\beta\) & 0, 0, 1, 1 & 1, 0, 0, 0 & 0, 1, 0, 1 \\
\(\gamma\) & 0, 1, 0, 1 & 0, 1, 0, 1 & 1, 0, 0, 0 \\
\hline
\end{tabular}
\end{table}

Table 1. A three generations \(SU(3) \times SU(2) \times U(1)^2\) model. The choice of generalized GSO coefficients is: 
\[c\left(\frac{b_j}{\alpha, \beta, \gamma}\right) = -c\left(\frac{\alpha}{1}\right) = c\left(\frac{\beta}{\gamma}\right) = -c\left(\frac{\gamma}{1, \alpha}\right) = -c\left(\frac{\gamma}{\beta}\right) = -1\]

\((j=1,2,3)\), with the others specified by modular invariance and space–time supersymmetry. Trilevel Yukawa couplings are obtained only for \(+\frac{3}{2}\) charged quarks. The 16 right–moving internal fermionic states \(\{\bar{\psi}^1, \cdots, \bar{\psi}^5, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^1, \cdots, \bar{\phi}^8\}\) correspond to the 16 dimensional compactified torus of the ten dimensional heterotic string. The 12 left–moving and 12 right–moving real internal fermionic states correspond to the six left and six right compactified dimensions in the bosonic language. \(\psi^\mu\) are the two space–time external fermions in the light–cone gauge and \(\chi^{12}, \chi^{34}, \chi^{56}\) correspond to the spin connection in the bosonic constructions.
| F  | SEC       | $SU(3)_C \times SU(2)_L$ | $Q_C$ | $Q_L$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $SU(5) \times SU(3)$ | $Q_7$ | $Q_8$ |
|----|-----------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-----------------------|-------|-------|
| $V_1$ | $b_1 + 2\gamma + (I)$ | (1,1) | 0 | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | (1,3) | $-\frac{1}{2}$ | $\frac{1}{3}$ |
| $\bar{V}_1$ | (1,1) | 0 | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | (1,3) | $\frac{1}{2}$ | $-\frac{1}{3}$ |
| $T_1$ | (1,1) | 0 | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | (5,1) | $-\frac{1}{2}$ | $-\frac{1}{3}$ |
| $\bar{T}_1$ | (1,1) | 0 | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | (5,1) | $\frac{1}{2}$ | $\frac{1}{3}$ |
| $V_2$ | $b_2 + 2\gamma + (I)$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | (1,3) | $-\frac{1}{2}$ | $\frac{1}{3}$ |
| $\bar{V}_2$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | (1,3) | $\frac{1}{2}$ | $-\frac{1}{3}$ |
| $T_2$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | (5,1) | $-\frac{1}{2}$ | $-\frac{1}{3}$ |
| $\bar{T}_2$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | (5,1) | $\frac{1}{2}$ | $\frac{1}{3}$ |
| $V_3$ | $b_3 + 2\gamma + (I)$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | (1,3) | $-\frac{1}{2}$ | $\frac{1}{3}$ |
| $\bar{V}_3$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $\frac{1}{2}$ | (1,3) | $\frac{1}{2}$ | $-\frac{1}{3}$ |
| $T_3$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $-\frac{1}{2}$ | (5,1) | $-\frac{1}{2}$ | $-\frac{1}{3}$ |
| $\bar{T}_3$ | (1,1) | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 | $-\frac{1}{2}$ | (5,1) | $\frac{1}{2}$ | $\frac{1}{3}$ |

*Table 2.* Massless states from the sectors $b_j + 2\gamma$, and their quantum numbers.