Hadron structure from $\gamma^* p$ scattering: interpreting hadronic matrix elements

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Abstract. Hadron structure from high-$Q^2 \gamma^* p$ scattering processes is often expressed in terms of hadronic matrix elements of nonlocal operators. Properly defining and interpreting these quantities is very important in light of experiments aiming to extract transverse momentum dependent parton distributions or generalized parton distributions. The current status will be reviewed, including recent developments concerning Wigner distributions.

INTRODUCTION

The hard $\gamma^* p$ scattering processes to be considered are inclusive deep inelastic scattering (DIS), semi-inclusive DIS and deeply virtual Compton scattering (DVCS). In these processes one probes parton densities, transverse momentum dependent parton distributions (TMPDs) and generalized parton distributions (GPDs), respectively. Properties of these quantities will be reviewed, with emphasis on interpretation, since having a clear interpretation is crucial for motivating experiments. The various distributions all carry different information about hadron structure, but can be viewed as different reductions of one underlying quantity, a quantum phase space (Wigner) distribution.

HARD INCLUSIVE $\gamma^* p$ PROCESSES

Inclusive DIS ($e p \rightarrow e' X$) is a process of two scales, provided by the hadron momentum $P$ and the virtual photon momentum $q$: $P^2 = M^2$ and $Q^2 = -q^2$, such that $Q^2 \gg M^2$. The hard scale $Q$ serves to separate hard (perturbative) from soft (nonperturbative) parts of the process, i.e. one can apply factorization:

$$\sigma(\gamma^* p \rightarrow X) \propto \int \phi_H(x) \Phi(x) + O(1/Q)$$

In DIS one only encounters functions of lightcone momentum fractions ($x = k^+ / P^+$ is the fraction of light-cone momentum of a quark ($k^+$) inside a hadron ($P^+$)). Here

$$\Phi_{ij}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \psi_j(0) \omega[0, \lambda] \psi_i(\lambda n_-) | P \rangle, \quad (n_-^2 = 0) \quad (1)$$

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is an operator matrix element (OME) of a nonlocal lightcone operator and

$$\mathcal{L}[0,\lambda] = \mathcal{P} \exp \left( -ig \int_{0}^{\lambda} d\eta A^+ (\eta n_-) \right) \xrightarrow{A^+ \to 0} 1 \tag{2}$$

This path-ordered exponential (link) is not inserted by hand, but derived \(^1\). For an unpolarized hadron \(\Phi(x)\) can be parametrized by one function (at leading twist):

$$\Phi(x) = \frac{1}{2} f_1(x) P. \tag{3}$$

This function \(f_1^q(x) \equiv q(x)\) can be written in terms of the good field \(\psi_+ = \frac{1}{2} \gamma^- \gamma^+ \psi\) as

$$q(x) = \frac{1}{\sqrt{2}} \sum_n \delta(P^+ + xP^+ - p_n^+) \left| \langle n | \mathcal{L}[\infty,0] \psi_+(0) | P \rangle \right|^2, \tag{4}$$

which lends \(q(x)\) the interpretation of probability of finding a quark of flavor \(q\) in the proton, with a light-cone momentum fraction \(x\). Taking Mellin moments \(\int dx x^N q(x)\), leads to local OMEs (which can be evaluated on the lattice). For example, \(N = 0\) yields the number of quarks minus antiquarks: \(f_1^q dx q(x) = \langle P | \psi_+(0) \psi_+(0) | P \rangle / (\sqrt{2} P^+)\).

Semi-inclusive DIS (SIDIS), \(e p \to e' \pi X\), is a three-scale process, where next to \(M\) and \(Q\) also the transverse momentum of the measured final-state hadron (a pion for definiteness) sets a scale: \(|P_{\perp}^\pi|\) with \(|P_{\perp}^\pi|^2 \ll Q^2\). This results in a different factorization theorem (recently discussed in Ref. \(^2\), based on methods developed in Ref. \(^3\)). One needs to include parton transverse momentum \(\Phi(x) \to \Phi(x,k_T)\) (discussed in Refs. \(^4\), \(^5\) and by many others), which leads to transverse momentum dependent parton distribution functions (TMPDs). TMPDs for unpolarized hadrons are defined as \(^6\)

$$\Phi(x,k_T) = f_1(x,k_T^2) \frac{P}{2} + h_1^T(x,k_T^2) \frac{ik_T \cdot P}{2M}. \tag{5}$$

Upon integration over transverse momentum one retrieves Eq. \(^3\). \(\Phi(x,k_T)\) is a matrix element of operators that are nonlocal off the lightcone

$$\Phi(x,k_T) = \text{F.T.} \langle P | \psi(0) \mathcal{L}[0,\xi] \psi(\xi) | P \rangle \bigg|_{\xi = (\xi^- \xi^+ \xi_T)} \tag{6}$$

Like for \(\Phi(x)\), the link \(\mathcal{L}[0,\xi]\) can be derived as discussed in Refs. \(^7\), \(^8\), \(^9\). In many respects \(\Phi(x,k_T)\) is similar to \(\Phi(x)\) (it defines momentum distributions), but the link structure leads to considerable differences, of which two will be mentioned here.

A nonzero function \(h_1^T\) means that the transverse polarization \(S_T^q\) of a noncollinear quark inside an unpolarized hadron in principle can have a preferred direction. This implies an intrinsic handedness, e.g. in the infinite momentum frame (IMF): \(S_T^q \sim P_{\text{hadron}} \times k_{\text{quark}}\). At first sight such handedness appears to violate time reversal invariance (following an argument of Collins \(^10\)), but a model calculation by Brodsky, Hwang and Schmidt \(^11\) implied otherwise. This is precisely due to the link structure. The proper gauge invariant definition of TMPDs in SIDIS contains a future pointing
Wilson line, whereas in Drell-Yan (DY) it is past pointing. As a consequence there is a calculable process dependence \[12\]: \((f_1)_\text{DIS} = (f_1)_\text{DY}\), but \((h_1^\perp)_\text{DIS} = -(h_1^\perp)_\text{DY}\). More complicated processes \[13\] still require further study.

Another consequence of the link structure is that sometimes one is dealing with intrinsically nonlocal lightcone OMEs, such as

\[
h_1^{\perp(1)}(x) \equiv \int d^2k_T \frac{k_T^2}{2M^2} h_1^{\perp(2)}(x, k_T^2) A^+ = 0 \quad \text{F.T.} |P| |\bar{\psi}(0)\rangle \int_{-\infty}^{\infty} d\eta^- F^+_{\alpha}(\eta^-) \Gamma\psi(\xi^-) |P\rangle,
\]

for which Mellin moments do not yield local OMEs (hampering a lattice evaluation).

**HARD EXCLUSIVE \(\gamma^* p\) PROCESSES**

To get a handle on orbital angular momentum of quarks, Ji proposed \[14\] to use Deeply Virtual Compton Scattering: \(\gamma^* p \rightarrow \gamma p'\). This involves Generalized Parton Distributions, which are off-forward, nonlocal lightcone OMEs

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}(\lambda/2) \gamma^+ \mathcal{L}[-\lambda/2, \lambda/2] \psi(\lambda/2) |P\rangle =
H_q(x, \xi, t) \bar{u}(P') \gamma^+ u(P) + E_q(x, \xi, t) \bar{u}(P') \frac{i\sigma^+ \nu \Delta \nu}{2M_N} u(P),
\]

with \(\Delta = P' - P\), \(\xi = -\Delta^+/\nu (P' + P^+)\) and \(t = \Delta^2\). GPDs encompass both parton distributions and form factors. In the forward limit: \(H_q(x, 0, 0) = q(x)\) and \(E_q(x, 0, 0) \equiv E_q(x)\) (the latter function is not accessible in DIS), which determine the total and orbital angular momentum of quarks in a proton, \(J_q(x)\) and \(L_q(x)\) \[15\],

\[
J_q(x) = \frac{1}{2} x [q(x) + E_q(x)], \quad L_q(x) = J_q(x) - \frac{1}{2} \Delta q(x),
\]

together with \(\Delta q(x)\), the quark helicity distribution (\(\text{Tr}[\Phi(x) \gamma^+ \gamma_5] \sim \lambda \Delta q(x)\)).

The reduction to form factors goes via \(x\)-integration:

\[
\int dx H(x, \xi, \Delta^2) = F_1(\Delta^2), \quad \int dx E(x, \xi, \Delta^2) = F_2(\Delta^2),
\]

where \(F_1\) and \(F_2\) are the usual Dirac and Pauli form factors. Like form factors, GPDs are best interpreted by taking Fourier transforms (F.T.). In that way GPDs yield a more complete picture of momentum and spatial distributions of partons \[16, 17, 18, 19\], albeit a frame dependent picture. Two standard choices are the IMF and the Breit frame.

In the IMF \((P_z \rightarrow \infty)\) one effectively has localization in the transverse directions, which leads to a 2-D position space interpretation (information along the \(z\)-axis is integrated over). It allows to define the charge distribution in impact parameter space

\[
\rho(b_\perp) \equiv \frac{1}{2P^+} \langle P^+, R_\perp = 0 | J^+(0^-, 0^+, b_\perp) | P^+, R_\perp = 0 \rangle,
\]
where $|P^+, R_\perp = 0 \rangle$ is the proton state localized in the $\perp$ direction. One can show that

$$\rho (b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} F_1 (-\Delta_\perp^2).$$

(12)

Hence, the Dirac form factor is the F.T. of the charge distribution in the transverse plane.

Taking this IMF point of view for GPDs leads to ($f dz \to \xi = 0$):

$$H_q(x, 0, -\Delta_\perp^2) = 2\text{-D F.T. } q(x, b_\perp),$$

(13)

$b_\perp$ is measured w.r.t. $R^{CM}_\perp \equiv \sum_i x_i r_i$; the ‘transverse center of longitudinal momentum’.

This $q(x, b_\perp)$ has an interpretation as a density, just like $q(x)$ \[16, 24\]

$$q(x, b_\perp) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P^+, R_\perp = 0 | \mathcal{W}(-\lambda/2, b_\perp) \gamma^+ \mathcal{L} \psi(\lambda/2, b_\perp) | P^+, R_\perp = 0 \rangle.$$  

(14)

Note: $\int dx q(x, b_\perp) = \rho (b_\perp), \int d^2 b_\perp q(x, b_\perp) = q(x).$  

(15)

Alternatively, in the Breit frame ($\vec{P}' = -\vec{P}$ and $t = -\vec{A}^2$) the 3-D proton charge distribution is the F.T. of the Sachs electric form factor $G_E$ \[20, 21\]

$$3\text{-D F.T. } \rho (\vec{r}) = \langle \vec{A}/2 | J^0(0) | -\vec{A}/2 \rangle / (2M) \propto G_E(t) = F_1(t) + \frac{t}{4M^2} F_2(t).$$

(16)

This Breit frame point of view for GPDs leads to ($\xi = \Delta^z/(2P^0), t = -\vec{A}^2$) \[22, 23\]:

$$f_{\gamma^0}(\vec{r}, x) \propto 3\text{-D F.T. } \left\{ H_q(x, \xi, -\vec{A}^2) - \frac{\vec{A}^2}{4M^2} E_q(x, \xi, -\vec{A}^2) \right\},$$

(17)

where the expression in brackets could be called the Sachs electric GPD $G_E(x, \xi, -\vec{A}^2)$.

The function $f_{\gamma^0}(\vec{r}, x)$ is interpreted as a 3-D density in the ‘rest frame’ of the proton for quarks with a selected value of $x$ and can be defined as a reduction of a so-called quantum phase-space or Wigner distribution \[22, 23, 24\], in the Breit ‘frame’ defined as

$$W_\Gamma (\vec{r}, k) \equiv \frac{1}{2M} \int \frac{d^3 \Delta}{(2\pi)^3} \langle \Delta/2 | \mathcal{W}_\Gamma (\vec{r}, k) | -\Delta/2 \rangle,$$

(18)

$$\mathcal{W}_\Gamma (\vec{r}, k) \equiv \int d^4 \eta \ e^{i\eta \cdot k} \bar{\psi}(\vec{r} - \eta/2) \mathcal{L} \gamma^+ \mathcal{L} \psi(\vec{r} + \eta/2).$$

(19)

Fourier transforms of GPDs are obtained as follows (considering $\Gamma = \gamma^+$ now):

$$f_{\gamma^+}(\vec{r}, x) \equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \left[ \int \frac{dk^-}{2\pi} W_{\gamma^+} (\vec{r}, k) \right]$$

$$\propto \text{F.T.} \left\{ H_q(x, \xi, t) \bar{u}(\Delta/2) \gamma^+ u(-\Delta/2) + E_q(x, \xi, t) \bar{u}(\Delta/2) \frac{i\sigma^+ \cdot \Delta}{2M} u(-\Delta/2) \right\}.$$ 

(19)
Integrating $f_{\gamma'}(\vec{r},x)$ over the $z$ coordinate yields $q(x,\mathbf{b}_\perp)$ and $\int dx f_{\gamma'}(\vec{r},x)$ is the F.T. of the Sachs electric and magnetic form factors. But TMPDs can also be seen as reductions of Wigner distributions

$$q(x,k_T) = \int \frac{d^3r}{(2\pi)^3} \left[ \int \frac{dk_-}{2\pi} W_{\gamma'}(\vec{r},k) \right].$$

Hence, both GPDs and TMPDs can be viewed as different reductions of one underlying quantity. Different choices of frame lead to complementary physical pictures of these quantities (or their Fourier transforms), as momentum and/or spatial distributions.

Finally, a suggestion concerning the 5-D phase space quantity (which has a straightforward IMF density interpretation)

$$q(x,k_T,\mathbf{b}_\perp) \equiv \int \frac{dz}{2\pi} \left[ \int \frac{dk_-}{2\pi} W_{\gamma'}(\vec{r},k) \right].$$

Perhaps $q(x,k_T,\mathbf{b}_\perp)$ can be measured in hard ‘semi’-exclusive processes, such as $\gamma^* p \rightarrow V_1 V_2 p'$, with $V_i$ either $\gamma$ or a vector meson and $V_1$ and $V_2$ have a small relative transverse momentum. Note that $k_T$ and $\mathbf{b}_\perp$ are not each other’s Fourier conjugates.

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