Testing galaxy formation and dark matter with low surface brightness galaxies

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ABSTRACT

Galaxies are the basic structural element of the universe; galaxy formation theory seeks to explain how these structures came to be. I trace some of the foundational ideas in galaxy formation, with emphasis on the need for non-baryonic cold dark matter. Many elements of early theory did not survive contact with observations of low surface brightness galaxies, leading to the need for auxiliary hypotheses like feedback. The failure points often trace to the surprising predictive successes of an alternative to dark matter, the Modified Newtonian Dynamics (MOND). While dark matter models are flexible in accommodating observations, they do not provide the predictive capacity of MOND. If the universe is made of cold dark matter, why does MOND get any predictions right?

1. Cosmic context

Cosmology is the science of the origin and evolution of the universe: the biggest of big pictures. The modern picture of the hot big bang is underpinned by three empirical pillars: an expanding universe (Hubble expansion), Big Bang Nucleosynthesis (BBN: the formation of the light elements through nuclear reactions in the early universe), and the relic radiation field (the Cosmic Microwave Background: CMB) (Harrison, 2000; Peebles, 1993). The discussion here will take this framework for granted.

The three empirical pillars fit beautifully with General Relativity (GR). Making the simplifying assumptions of homogeneity and isotropy, Einstein’s equations can be applied to treat the entire universe as a dynamical system. As such, it is compelled either to expand or contract. Running the observed expansion backwards in time, one necessarily comes to a hot, dense, early phase. This naturally explains the CMB, which marks the transition from an opaque plasma to a transparent gas (Sunyaev and Zeldovich, 1980; Weiss, 1980). The abundances of the light elements can be explained in detail with BBN provided the universe expands in the first few minutes as predicted by GR when radiation dominates the mass-energy budget of the universe (Boesgaard & Steigman, 1985).

The marvelous consistency of these early universe results with the expectations of GR builds confidence that the hot big bang is the correct general picture for cosmology. It also builds overconfidence that GR is completely sufficient to describe the universe. Maintaining consistency with modern cosmological data is only possible with the addition of two auxiliary hypotheses: dark matter and dark energy. These invisible entities are an absolute requirement of the current version of the most-favored cosmological model, ΛCDM. The very name of this model is intended as a minimal introduction for the non-expert. In ΛCDM, there are at least two forms of unseen mass: normal matter that is thought to dominate the mass budget of the universe and play a critical role in galaxy formation. The term ‘dark matter’ is dangerously crude, as it can reasonably be used to mean anything that is not seen. In the cosmic context, there are at least two forms of unseen mass: normal matter that happens not to glow in a way that is easily seen — not all ordinary material need be associated with visible stars — and non-baryonic cold dark matter. It is the latter form of unseen mass that is thought to dominate the mass budget of the universe and play a critical role in galaxy formation.

I discuss some of the unexpected predictive successes of the alternative to dark matter, MOND (Milgrom, 1983c). The existential problem that this poses for the dark matter paradigm is...
addressed in §6.

2. Cold dark matter

Cold dark matter is some form of slow moving, non-relativistic (‘cold’) particulate mass that is not composed of normal matter (baryons). Baryons are the family of particles that include protons and neutrons. As such, they compose the bulk of the mass of normal matter, and it has become conventional to use this term to distinguish between normal, baryonic matter and the non-baryonic dark matter.

The distinction between baryonic and non-baryonic dark matter is no small thing. Non-baryonic dark matter must be a new particle that resides in a new ‘dark sector’ that is completely distinct from the usual stable of elementary particles. We do not just need some new particle, we need one (or many) that reside in some sector beyond the framework of the stubbornly successful Standard Model of particle physics. Whatever the solution to the mass discrepancy problem turns out to be, it requires new physics.

The cosmic dark matter must be non-baryonic for two basic reasons. First, the mass density of the universe measured gravitationally ($\Omega_m \approx 0.3$, e.g., Faber and Gallagher, 1979; Davis et al., 1980, 1992) clearly exceeds the mass density in baryons as constrained by BBN ($\Omega_b = 0.05$, e.g., Walker et al., 1991). There is something gravitating that is not ordinary matter: $\Omega_m > \Omega_b$.

The second reason follows from the absence of large fluctuations in the CMB (Peebles and Yu, 1970; Silk, 1968; Sunyaev and Zeldovich, 1980). The CMB is extraordinarily uniform in temperature across the sky, varying by only $\pm 1$ part in $10^5$ (Smooot et al., 1992). These small temperature variations correspond to variations in density. Gravity is an attractive force; it will make the rich grow richer. Small density excesses will tend to attract more mass, making them larger, attracting more mass, and leading to the formation of large scale structures, including galaxies. But gravity is also a weak force: this process takes a long time. In the long but finite age of the universe, gravity plus known baryonic matter does not suffice to go from the initially smooth, highly uniform state of the early universe to the highly clumpy, structured state of the local universe (Peebles, 1993). The solution is to boost the process with an additional component of mass — the cold dark matter — that gravitates without interacting with the photons, thus getting a head start on the growth of structure while not aggravating the amplitude of temperature fluctuations in the CMB.

Taken separately, one might argue away the need for dark matter. Taken together, these two distinct arguments convinced nearly everyone, including myself, of the absolute need for non-baryonic dark matter. Consequently, CDM became established as the leading paradigm during the 1980s (Peebles, 1984; Steigman and Turner, 1985). The paradigm has snowballed since that time, the common attitude among cosmologists being that CDM has to exist.

From an astronomical perspective, the CDM could be any slow-moving, massive object that does not interact with photons nor particulate in BBN. The range of possibilities is at once limitless yet highly constrained. Neutrons would suffice if they were stable in vacuum, but they are not. Primordial black holes are a logical possibility, but if made of normal matter, they must somehow form in the first second after the Big Bang to not impair BBN. At this juncture, microlensing experiments have excluded most plausible mass ranges that primordial black holes could occupy (Mediavilla et al., 2017). It is easy to invent hypothetical dark matter candidates, but difficult for them to remain viable.

From a particle physics perspective, the favored candidate is a Weakly Interacting Massive Particle (WIMP: Peebles, 1984; Steigman and Turner, 1985). WIMPs are expected to be the lightest stable supersymmetric partner particle that resides in the hypothetical supersymmetric sector (Martin, 1998). The WIMP has been the odds-on favorite for so long that it is often used synonymously with the more generic term ‘dark matter.’ It is the hypothesized particle that launched a thousand experiments. Experimental searches for WIMPs have matured over the past several decades, making extraordinary progress in not detecting dark matter (Aprile et al., 2018). Virtually all of the parameter space in which WIMPs had been predicted to reside (Trotta et al., 2008) is now excluded. Worse, the existence of the supersymmetric sector itself, once seemingly a sure thing, remains entirely hypothetical, and appears at this juncture to be a beautiful idea that nature declined to implement.

In sum, we must have cold dark matter for both galaxies and cosmology, but we have as yet no clue to what it is.

3. Galaxy properties

Cosmology entered the modern era when Hubble (1929) resolved the debate over the nature of spiral nebulae by measuring the distance to Andromeda, establishing that vast stellar systems — galaxies — exist external to and coequal with the Milky Way. Galaxies are the primary type of object observed when we look beyond the confines of our own Milky Way: they are the building blocks of the universe. Consequently, galaxies and cosmology are intertwined: it is impossible to understand one without the other.

Here I sketch a few essential facts about the properties of galaxies. This is far from a comprehensive list (see, for example Binney & Tremaine, 1987) and serves only to provide a minimum framework for the subsequent discussion. The properties of galaxies are often cast in terms of morphological type, starting with Hubble’s tuning fork diagram.

The primary distinction is between Early Type Galaxies (ETGs) and Late Type Galaxies (LTGs), which is a matter of basic structure. ETGs, also known as elliptical galaxies, are three dimensional, ellipsoidal systems that are pressure supported: there is more kinetic energy in random motions than in circular motions, a condition described as dynamically hot. The orbits of stars are generally eccentric and oriented randomly with respect to one another, filling out the ellipsoidal shape seen in projection on the sky. LTGs, including spiral and irregular galaxies, are thin, quasi-two dimensional, rotationally supported disks. The majority of their stars orbit in the same plane in the same direction on low eccentricity orbits. The lion’s share of kinetic energy is invested in circular motion, with only small random motions, a condition described as dynamically cold. Examples of early and late type galaxies are shown in Fig. 1.

Finer distinctions in morphology can be made within the broad classes of early and late type galaxies, but the basic structural and kinematic differences suffice here. The disordered motion of ETGs is a natural consequence of violent relaxation (Lynden-Bell, 1967) in which a stellar system reaches a state of dynamical equilibrium from a chaotic initial state. This can proceed relatively quickly from a number of conceivable initial conditions, and is a rather natural consequence of the hierarchical merging of sub-clumps expected from the Gaussian initial conditions indicated by observations of the CMB (White, 1996). In contrast, the orderly rotation of dynamically cold LTGs requires a gentile settling of gas into a rotationally supported disk. It is essential that disk formation occur in the gaseous phase, as gas can dissipate and settle to the preferred plane specified by the net angular momentum of the system. Once stars form, their orbits retain a memory of their initial state for a period typically much greater than the age of the universe (Binney & Tremaine, 1987).

Consequently, the bulk of the stars in the spiral disk must have formed there after the gas settled.

In addition to the dichotomy in structure, ETGs and LTGs also differ in their evolutionary history. ETGs tend to be ‘red and dead,’ which is to say, dominated by old stars. They typically lack much in the way of recent star formation, and are often devoid of the cold interstellar gas from which new stars can form. Most of their star formation happened in the early universe, and may have involved the merger of multiple proto-galactic fragments. Irrespective of these details, massive ETGs appeared early in the universe (Steinhardt et al., 2016), and for the most part seem to have evolved passively since (Franck and McGaugh, 2017).

Again in contrast, LTGs have on-going star formation in interstellar media replete with cold atomic and molecular gas. They exhibit a wide range in stellar ages, from newly formed stars to ancient stars dating to...
Galaxies have a characteristic size and surface brightness. The same amount of stellar mass can be concentrated in a high surface brightness (HSB) galaxy, or spread over a much larger area in a low surface brightness (LSB) galaxy. For the purposes of this discussion, it suffices to assume that the observed luminosity is proportional to the mass of stars that produces the light. Similarly, the surface brightness measures the surface density of stars. Of the three observable quantities of luminosity, size, and surface brightness, only two are independent: the luminosity is the product of the surface brightness and the area over which it extends. The area scales as the square of the linear size.

The distribution of size and mass of galaxies is shown in Fig. 2. This figure spans the range from tiny dwarf irregular galaxies containing ‘only’ a few hundred thousand stars to giant spirals composed of hundreds of billions of stars with half-light radii ranging from hundreds of parsecs to tens of kpc. The upper boundaries represent real, physical limits on the sizes and masses of galaxies. Bright objects are easy to see; if still higher mass galaxies were common, they would be readily detected and cataloged. In contrast, the lower boundaries are set by the limits of physical galaxies during major mergers (Barnes & Hernquist, 1992). Aside from its morphology, an obvious property of a galaxy is its mass. Galaxies exist over a large range of mass, with a type-dependent characteristic stellar mass of $5 \times 10^{10} \, M_{\odot}$ for disk dominated systems (the Milky Way is very close to this mass: Bland-Hawthorn & Gerhard, 2016) and $10^{11} \, M_{\odot}$ for elliptical galaxies (Moffett et al., 2016). Above this characteristic mass, the number density of galaxies declines sharply, though individual galaxies exceeding a few $10^{11} \, M_{\odot}$ certainly exist. The number density of galaxies increases gradually to lower masses, with no known minimum. The gradual increase in numbers does not compensate for the decrease in mass: integrating over the distribution, one finds that most of the stellar mass is in bright galaxies close to the characteristic mass.

Fig. 1. Galaxy morphology. These examples shown an early type elliptical galaxy (NGC 3379, left), and two late type disk galaxies: a face-on spiral (NGC 628, top right), and an edge-on disk galaxy (NGC 891, bottom right). Elliptical galaxies are quasi-spherical, pressure supported stellar systems that tend to have predominantly old stellar populations, usually lacking young stars or much in the way of the cold interstellar gas from which they might form. In contrast, late type galaxies (spirals and irregulars) are thin, rotationally supported disks. They typically contain a mix of stellar ages and cold interstellar gas from which new stars continue to form. Interstellar dust is also present, being most obvious in the edge-on case. Images from Palomar Observatory, Caltech.

Fig. 2. Galaxy size and mass. The radius that contains half of the light is plotted against the stellar mass. Galaxies exist over many decades in mass, and exhibit a considerable variation in size at a given mass. Early and late type galaxies are demarcated with different symbols, as noted. Lines illustrate tracks of constant stellar surface density. The data for ETGs are from the compilation of Dahnringhausen and Fellhauer (2016) augmented by dwarf Spheroidal (dSph) galaxies in the Local Group compiled by Lelli et al. (2017). Ultra-diffuse galaxies (UDGs: van Dokkum et al., 2015; Mihos et al., 2015, × and +, respectively) have unsettled kinematic classifications at present, but most seem likely to be pressure supported ETGs. The bulk of the data for LTGs is from the SPARC database (Lelli et al., 2016a), augmented by cases that are noteworthy for their extremity in mass or surface brightness (Brunker et al., 2019; Dalcanton, Spergel, Gunn, et al., 1997; de Blok et al., 1995; McGaugh and Bothun, 1994; Mihos et al., 2018; Rhode et al., 2013; Schombert et al., 2011). The gas content of these star-forming systems adds a third axis, illustrated crudely here whether an LTG is made more of stars or gas (filled and open symbols, respectively).

systematically under-represented in galaxy catalogs (Allen & Shu, 1979; Disney, 1976; McGaugh et al., 1995a).

Individual galaxies can be early type or late type, high mass or low mass, large or small in linear extent, high or low surface brightness, gas poor or gas rich. No one of these properties is completely predictive of the others: the correlations that do exist tend to have lots of intrinsic scatter. The primary exception to this appears to involve the kinematics. Massive galaxies are fast rotators; low mass galaxies are slow rotators. This Tully-Fisher relation (Tully and Fisher, 1977) is one of the strongest correlations in extragalactic astronomy (Lelli et al., 2016b). It is thus necessary...
to simultaneously explain both the chaotic diversity of galaxy properties and the orderly nature of their kinematics (McGaugh et al., 2019). Galaxies do not exist in isolation. Rather than being randomly distributed throughout the universe, they tend to cluster together: the best place to find a galaxy is in the proximity of another galaxy (Rubin, 1954). A common way to quantify the clustering of galaxies is the two-point correlation function ξ(r) (Peebles, 1980). This measures the excess probability of finding a galaxy within a distance r of a reference galaxy relative to a random distribution. The observed correlation function is well approximated as a power law whose slope and normalization varies with galaxy population. ETGs are more clustered than LTGs, having a longer correlation length: r_0 ∼ 9 Mpc for red galaxies vs. ∼ 5 Mpc for blue galaxies (Zehavi et al., 2011). Here we will find this quantity to be of interest for comparing the distribution of high and low surface brightness galaxies.

4. Galaxy formation

Galaxies are gravitationally bound condensations of stars and gas in a mostly empty, expanding universe. The tens of billions of solar masses of baryonic material that comprise the stars and gas of the Milky Way now reside mostly within a radius of 20 kpc. At the average density of the universe, the equivalent mass fills a spherical volume with a comoving radius a bit in excess of 1 Mpc. This is a large factor by which a proto-galaxy must collapse, starting from the very smooth (∼ 1 part in 10⁵) initial condition at z = 1090 observed in the CMB (Planck Collaboration et al., 2018). Dark matter — in particular, non-baryonic cold dark matter — plays an essential role in speeding this process along. The mass-energy of the early universe is initially dominated by radiation. The baryons are held in thrall to the photons until the expansion of the universe turns the tables and matter becomes dominant. Exactly when this happens depends on the mass density (Peebles, 1980); for our purposes it suffices to realize that the baryonic components of galaxies can not begin to form until well after the time of the CMB. However, since CDM does not interact with photons, it is not subject to this limitation. The dark matter can begin to form structures — dark matter halos — that form the scaffolding of future structure. Essential to the ΛCDM galaxy formation paradigm is that the dark matter halos form first, seeding the subsequent formation of luminous galaxies by providing the potential wells into which baryons can condense once free from the radiation field.

The theoretical expectation for how dark matter halos form is well understood at this juncture. Numerical simulations of cold dark matter — mass that interacts only through gravity in an expanding universe — show that quasi-spherical dark matter halos form with a characteristic NFW (e.g., Navarro et al., 1997) density profile. These have a ‘cuspy’ inner density profile in which the density of dark matter increases towards the center approximately as a power law, ρ(r) ∼ r⁻¹. At larger radii, the density profile falls of as ρ(r → ∞) ∼ r⁻³. The centers of these halos are the density peaks around which galaxies can form. The galaxies that we observe are composed of stars and gas: normal baryonic matter. The theoretical expectation for how baryons behave during galaxy formation is not well understood (Scannapieco et al., 2012). This results in a tremendous and long-standing disconnect between theory and observation. We can, however, stipulate a few requirements as to what needs to happen. Dark matter halos must form first; the baryons fall into these halos afterwards. Dark matter halos are observed to extend well beyond the outer edges of visible galaxies, so baryons must condense to the centers of dark matter halos. This condensation may proceed through both the hierarchical merging of protogalactic fragments (a process that has a proclivity to form ETGs) and the more gentle accretion of gas into rotating disks (a requirement to form LTGs). In either case, some fraction of the baryons form the observed, luminous component of a galaxy at the center of a CDM halo. This condensation of baryons necessarily affects the dark matter gravitationally, with the net effect of dragging some of it towards the center (Blumenthal et al., 1986; Dubinski, 1994; Gnedin et al., 2004; Sellwood and McGaugh, 2005a), thus compressing the dark matter halo from its initial condition as indicated by dark matter-only simulations like those of Navarro et al. (1997). These processes must all occur, but do not by themselves suffice to explain real galaxies.

Galaxies formed in models that consider only the inevitable effects described above suffer many serious defects. They tend to be too massive (Abadi et al., 2003; Benson et al., 2003), too small (the angular momentum catastrophe: Katz, 1992; Steinmetz, 1999; D’Onghia et al., 2006), have systematically too large bulge-to-disc ratios (the bulgeless galaxy problem: D’Onghia and Burkert, 2004; Kormendy et al., 2010), have dark matter halos with too much mass at small radii (the cusp-core problem: Moore et al., 1999b; Kuzio de Naray et al., 2006, 2009; de Blok, 2010; Kuzio de Naray and McGaugh, 2014), and have the wrong over-all mass function (the over-cooling problem, e.g., Benson, 2010, also known locally as the missing satellite problem (Klypin et al., 1999; Moore et al., 1999a). This long list of problems have kept the field of galaxy formation a lively one: there is no risk of it becoming a victim of its own success through the appearance of one clearly-correct standard model.

4.1. Threads of development

Entering the 1980s, options for galaxy formation were frequently portrayed as a dichotomy between monolithic galaxy formation (Eggen et al., 1962) and the merger of protogalactic fragments (Searle and Zinn, 1978). The basic idea of monolithic galaxy formation is that the initial ∼ 1 Mpc cloud of gas that would form the Milky Way experienced dissipational collapse in one smooth, adiabatic process. This is effective at forming the disk, with only a tiny bit of star formation occurring during the collapse phase to provide the stars of the ancient, metal-poor stellar halo. In contrast, the Galaxy could have been built up by the merger of smaller protogalactic fragments, each with their own life as smaller galaxies prior to merging. The latter is more natural to the emergence of structure from the initial conditions observed in the CMB, where small lumps condense more readily than large ones. Indeed, this effectively forms the basis of the modern picture of hierarchical galaxy formation (Efstathiou et al., 1988).

Hierarchical galaxy formation is effective at forming bulges and pressure-supported ETGs, but is anathema to the formation of orderly disks. Dynamically cold disks are fragile and prefer to be left alone: the high rate of merging in the hierarchical ΛCDM model tends to destroy the dynamically cold state in which most spirals are observed to exist (Abadi et al., 2003; Peebles, 2020; Toth and Ostriker, 1992). Consequently, there have been some rather different ideas about galaxy formation: if one starts from the initial conditions imposed by the CMB, hierarchical galaxy formation is inevitable. If instead one works backwards from the observed state of galaxy disks, the smooth settling of gaseous disks in relatively isolated monoliths seems more plausible.

In addition to different theoretical notions, our picture of the galaxy population was woefully incomplete. An influential study by Freeman (1970) found that 28 of three dozen spirals shared very nearly the same central surface brightness. This was generalized into a belief that all spirals had the same (high) surface brightness, and came to be known as Freeman’s Law. Ultimately this proved to be a selection effect, as pointed out early by Disney (1976) and Allen and Shu (1979). However, it was not until much later (McGaugh et al., 1995a) that this became widely recognized. In the mean time, the prevailing assumption was that Freeman’s Law held true (e.g., van der Kruit, 1987) and all spirals had practically the same surface brightness. In particular, it was the central surface brightness of the disk component of spiral galaxies that was thought to be universal, while bulges and ETGs varied in surface brightness. Variation in the disk component of LTGs was thought to be
restricted to variations in size, which led to variations in luminosity at fixed surface brightness.

Consequently, most theoretical effort was concentrated on the bright objects in the high-mass ($M > 10^{10}$ M$_\odot$) clump in Fig. 2. Some low mass dwarf galaxies were known to exist, but were considered to be insignificant because they contained little mass. Low surface brightness galaxies violated Freeman’s Law, so were widely presumed not to exist, or be at most a rare curiosity (Bosma & Freeman, 1993). A happy consequence of this unfortunate state of affairs was that as observations of diffuse LSB galaxies were made, they forced then-current ideas about galaxy formation into a regime that they had not anticipated, and which many could not accommodate.

The similarity and difference between high surface brightness (HSB) and LSB galaxies is illustrated by Fig. 3. Both are rotationally supported, late type disk galaxies. Both show spiral structure, though it is more prominent in the HSB. More importantly, both systems are of comparable linear diameter. They exist roughly at opposite ends of a horizontal line in Fig. 2. Their differing stellar masses stem from the surface density of their stars rather than their linear extent — exactly the opposite of what had been inferred from Freeman’s Law. Any model of galaxy formation and evolution must account for the distribution of size (or surface brightness) at a given mass as well as the number density of galaxies as a function of mass. Both aspects of the galaxy population remain problematic to this day.

4.2. Two hypotheses: spin and density

Here I discuss two basic hypotheses for the distribution of disk galaxy size at a given mass. These broad categories I label SH (Same Halo) and DD (Density begets Density) following McGaugh and de Blok (1998a). In both cases, galaxies of a given baryonic mass are assumed to reside in dark matter halos of a corresponding total mass. Hence, at a given halo mass, the baryonic mass is the same, and variations in galaxy size follow from one of two basic effects:

- **SH**: variations in size follow from variations in the spin of the parent dark matter halo.
- **DD**: variations in surface brightness follow from variations in the density of the dark matter halo.

Recall that at a given luminosity, size and surface brightness are not independent, so variation in one corresponds to variation in the other. Consequently, we have two distinct ideas for why galaxies of the same mass vary in size. In SH, the halo may have the same density profile $\rho(r)$, and it is only variations in angular momentum that dictate variations in the disk size. In DD, variations in the surface brightness of the luminous disk are reflections of variations in the density profile $\rho(r)$ of the dark matter halo. In principle, one could have a combination of both effects, but we will keep them separate for this discussion, and note that mixing them defeats the virtues of each without curing their ills.

The SH hypothesis traces back to at least Fall and Efstathiou (1980). The notion is simple: variations in the size of disks correspond to variations in the angular momentum of their host dark matter halos. The mass destined to become a dark matter halo initially expands with the rest of the universe, reaching some maximum radius before collapsing to form a gravitationally bound object. At the point of maximum expansion, the nascent dark matter halos torque one another, inducing a small but non-zero net spin in each, quantified by the dimensionless spin parameter $\lambda$ (Peebles, 1969). One then infers that as a disk forms within a dark matter halo, it collapses until it is centrifugally supported: $\lambda \approx 1$ from some initially small value (typically $\lambda \approx 0.05$, Barnes & Efstathiou, 1987, with some modest distribution about this median value). The spin parameter thus determines the collapse factor and the extent of the disk: low spin halos harbor compact, high surface brightness disks while high spin halos produce extended, low surface brightness disks.

The distribution of primordial spins is fairly narrow, and does not correlate with environment (Barnes & Efstathiou, 1987). The narrow distribution was invoked as an explanation for Freeman’s Law: the small variation in spins from halo to halo resulted in a narrow distribution of disk central surface brightness (van der Kruit, 1987). This association, while apparently natural, proved to be incorrect: when one goes through the mathematics to transform spin into scale length, even a narrow distribution of initial spins predicts a broad distribution in surface brightness (Dalcanton, Spergel, & Summers, 1997; McGaugh and de Blok, 1998a). Indeed, it predicts too broad a distribution: to prevent the formation of galaxies much higher in surface brightness than observed, one must invoke a stability criterion (Dalcanton, Spergel, & Summers, 1997; McGaugh and de Blok, 1998a) that precludes the existence of very high surface brightness disks. While it is physically quite reasonable that such a criterion should exist (Ostriker and Peebles, 1973), the observed surface density threshold does not emerge naturally, and must be inserted by hand. It is an auxiliary hypothesis invoked to preserve SH. Once done, size variations and the trend of average size with mass work out in reasonable quantitative detail (e.g., Mo et al., 1998).

Angular momentum conservation must hold for an isolated galaxy, but the assumption made in SH is stronger: baryons conserve their share of the angular momentum independently of the dark matter. It is considered a virtue that this simple assumption leads to disk sizes that are about right. However, this assumption is not well justified. Baryons and dark matter are free to exchange angular momentum with each other, and are seen to do so in simulations that track both components (e.g., Book et al., 2011; Combes, 2013; Klypin et al., 2002). There is no

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Fig. 3. High and low surface brightness galaxies. NGC 7757 (left) and UGC 1230 (right) are examples of high and low surface brightness galaxies, respectively. These galaxies are about the same distance away and span roughly the same physical diameter. The chief difference is in the surface brightness, which follows from the separation between stars (McGaugh et al., 1995b). Note that the intensity scale of these images is not identical; the contrast has been increased for the LSB galaxy so that it appears as more than a smudge.
guarantee that this exchange is equitable, and in general it is not: as baryons collapse to form a small galaxy within a large dark matter halo, they tend to lose angular momentum to the dark matter. This is a one-way street that runs in the wrong direction, with the final destination uncomfortably in conflict with most of the angular momentum sequestered in the unobservable dark matter. Worse still, if we impose rigorous angular momentum conservation among the baryons, the result is a disk with a completely unrealistic surface density profile (van den Bosch, 2001a). It then becomes necessary to pick and choose which baryons manage to assemble into the disk and which are expelled or otherwise excluded, thereby solving one problem by creating another.

Early work on LSB disk galaxies led to a rather different picture. Compared to the previously known population of HSB galaxies around which our theories had been built, the LSB galaxy population has a younger mean stellar age (de Blok & van der Hulst, 1998; McGaugh and Bothun, 1994), a lower content of heavy elements (McGaugh, 1994), and a systematically higher gas fraction (McGaugh and de Blok, 1997; Schombert et al., 1997). These properties suggested that LSB galaxies evolve more gradually than their higher surface brightness brethren: they convert their gas into stars over a much longer timescale (McGaugh et al., 2017). The obvious culprit for this difference is surface density: lower surface brightness galaxies have less gravity, hence less ability to gather their diffuse interstellar medium into dense clumps that could form stars (Gerritsen and de Blok, 1999; Mihos et al., 1999). It seemed reasonable to ascribe the low surface density of the baryons to a correspondingly low density of their parent dark matter halos.

One way to think about a region in the early universe that will eventually collapse to form a galaxy is as a so-called top-hat over-density. The mass density $\Omega_m \rightarrow 1$ at early times, irrespective of its current value, so a spherical region (the top-hat) that is somewhat over-dense early on may locally exceed the critical density. We may then consider this finite region as its own little closed universe, and follow its evolution with the Friedmann equations with $\Omega > 1$. The top-hat will initially expand along with the rest of the universe, but will eventually reach a maximum radius and recollapse. When that happens depends on the density. The greater the over-density, the sooner the top-hat will recollapse. Conversely, a lesser over-density will take longer to reach maximum expansion before recollapsing.

Everything about LSB galaxies suggested that they were lower density, late-forming systems. It therefore seemed quite natural to imagine a distribution of over-densities and corresponding collapse times for top-hats of similar mass, and to associate LSB galaxy with the lesser over-densities (Dekel and Silk, 1986; McGaugh, 1992). More recently, some essential aspects of this idea have been revived under the monicker of “assembly bias” (e.g. Zehavi et al., 2018).

The work that informed the DD hypothesis was based largely on photometric and spectroscopic observations of LSB galaxies: their size and surface brightness, color, chemical abundance, and gas content. DD made two obvious predictions that had not yet been tested at that juncture. First, late-forming halos should reside preferentially in low density environments. This is a generic consequence of Gaussian initial conditions: big peaks defined on small (e.g., galaxy) scales are more likely to be found in big peaks defined on large (e.g., cluster) scales, and vice-versa. Second, the density of the dark matter halo of an LSB galaxy should be lower than that of an equal mass halo containing and HSB galaxy. This predicts a clear signature in their rotation speeds, which should be lower for lower density.

The prediction for the spatial distribution of LSB galaxies was tested by Bothun et al. (1993) and Mo et al. (1994). The test showed the expected effect: LSB galaxies were less strongly clustered than HSB galaxies. They are clustered: both galaxy populations follow the same large scale structure, but HSB galaxies adhere more strongly to it. In terms of the correlation function, the LSB sample available at the time had about half the amplitude $r_0$ as comparison HSB samples (Mo et al., 1994). The effect was even more pronounced on the smallest scales ($<2\text{ Mpc}$: Bothun et al., 1993), leading Mo et al. (1994) to construct a model that successfully explained both small and large scale aspects of the spatial distribution of LSB galaxies simply by associating them with dark matter halos that lacked close interactions with other halos. This was strong corroboration of the DD hypothesis.

One way to test the prediction of DD that LSB galaxies should rotate more slowly than HSB galaxies was to use the Tully-Fisher relation (Tully and Fisher, 1977) as a point of reference. Originally identified as an empirical relation between optical luminosity and the observed line-width of single-dish 21 cm observations, more fundamentally it turns out to be a relation between the baryonic mass of a galaxy (stars plus gas) and its flat rotation speed the Baryonic Tully-Fisher relation (BTFR: McGaugh et al., 2000). This relation is a simple power law of the form

$$ M_b = AV_f^r $$

with $A \approx 50\text{ M}_\odot \text{ km}^{-1}\text{s}^-1$ (McGaugh, 2005).

Aaronson et al. (1979) provided a straightforward interpretation for a relation of this form. A test particle orbiting a mass $M$ at a distance $R$ will have a circular speed $V$

$$ V_c^2 = GM/R $$

where $G$ is Newton’s constant. If we square this, a relation like the Tully-Fisher relation follows:

$$ V_c^4 = (GM/R)^2 \alpha M \Sigma $$

where we have introduced the surface mass density $\Sigma = M/R^2$. The Tully-Fisher relation $M \propto V$ is recovered if $\Sigma$ is constant, exactly as expected from Freeman’s Law (Freeman, 1970).

LSB galaxies, by definition, have central surface brightnesses (and corresponding stellar surface densities $\Sigma_0$) that are less than the Freeman value. Consequently, DD predicts, through equation (3), that LSB galaxies should shift systematically off the Tully-Fisher relation: lower $\Sigma$ means lower velocity. The predicted effect is not subtle (Fig. 4). For the range of surface brightness that had become available, the predicted shift should have stood out like the proverbial sore thumb. It did not (Hoffman et al., 1996; McGaugh and de Blok, 1998a; Sprayberry et al., 1995; Zwaan et al., 1995). This had an immediate impact on galaxy formation theory: compare Dalcanton et al. (1995, who predict a shift in Tully-Fisher with surface brightness) with Dalcanton et al. (1997b, who do not).

Instead of the systematic variation of velocity with surface brightness expected at fixed mass, there was none. Indeed, there is no hint of a second parameter dependence. The relation is incredibly tight by the standards of extragalactic astronomy (Lelli et al., 2016b): baryonic mass and the flat rotation speed are practically interchangeable.

The above derivation is overly simplistic. The radius at which we should make a measurement is ill-defined, and the surface density is dynamical: it includes both stars and dark matter. Moreover, galaxies are not spherical cows: one needs to solve the Poisson equation for the observed disk geometry of LTGs, and account for the varying radial contributions of luminous and dark matter. While this can be made to sound intimidating, the numerical computations are straightforward and rigorous (e.g., Begeman et al., 1991; Casertano & Shostak, 1980; Lelli et al., 2016a). It still boils down to the same sort of relation (modulo geometrical factors of order unity), but with two mass distributions: one for the baryons $M_b(R)$, and one for the dark matter $M_{DM}(R)$. Though the dark matter is more massive, it is also more extended. Consequently, both components can contribute non-negligibly to the rotation over the observed range of radii:

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2 If I had believed that we could get away with a subtle shift, I would have patched up my hypothesis accordingly. Since it was not possible, I rejected my own hypothesis.
As the disk settles, the dark matter conspiracy was that the formation of the baryonic disk has an effect on the dark matter halo, this is a half-truth: the observed velocity is a combination of increasing contribution from the dark matter halo. This decline of the baryonic contribution must be precisely balanced by an amplification of the dark matter. The baryonic contribution, the baryonic mass is encircled. It is when rotation curves persist in remaining flat past this point that we infer the need for dark matter.

A recurrent problem in testing galaxy formation theories is that they seldom make ironclad predictions; I attempt a brief summary in Table 1. SH represents a broad class of theories with many variants. By construction, the dark matter halos of galaxies of similar stellar mass are similar. If we associate the flat rotation velocity with dark mass, then galaxies of the same mass have the same circular velocity, and the problem posed by Tully-Fisher is automatically satisfied.

While it is common to associate the flat rotation speed with the dark matter halo, this is a half-truth: the observed velocity is a combination of baryonic and dark components (eq. (4)). It is thus a rather curious coincidence that rotation curves are as flat as they are: the Keplerian decline of the baryonic contribution must be precisely balanced by an increasing contribution from the dark matter halo. This fine-tuning problem was dubbed the “disk-halo conspiracy” (Bahcall & Casertano, 1985; van Albada & Sancisi, 1986). The solution offered for the disk-halo conspiracy was that the formation of the baryonic disk has an effect on the distribution of the dark matter. As the disk settles, the dark matter halo respond through a process commonly referred to as adiabatic compression that brings the peak velocities of disk and dark components into alignment (Blumenthal et al., 1986). Some rearrangement of the dark matter halo in response to the change of the gravitational potential caused by the settling of the disk is inevitable, so this seemed a plausible explanation.

The observation that LSB galaxies obey the Tully-Fisher relation greatly compounds the fine-tuning (McGaugh and de Blok, 1998a; Zwaan et al., 1995). The amount of adiabatic compression depends on the surface density of stars (Sellwood and McGaugh, 2005b): HSB galaxies experience greater compression than LSB galaxies. This should enhance the predicted shift between the two in Tully-Fisher. Instead, the amplitude of the flat rotation speed remains unperturbed.

The generic failings of dark matter models was discussed at length by McGaugh and de Blok (1998a). The same problems have been encountered by others. For example, Fig. 5 shows model galaxies formed in a dark matter halo with identical total mass and density profile but with different spin parameters (van den Bosch, 2001b). Variations in the assembly and cooling history were also considered, but these make little difference and are not relevant here. The point is that smaller (larger) spin parameters lead to more (less) compact disks that contribute more (less) to the total rotation, exactly as anticipated from variations in the term $M_b/R$ in equation (4). The nominal variation is readily detectable, and stands out prominently in the Tully-Fisher diagram (Fig. 5). This is exactly the same fine-tuning problem that was pointed out by Zwaan et al. (1995) and McGaugh and de Blok (1998a).

What I describe as a fine-tuning problem is not portrayed as such by van den Bosch (2000) and van den Bosch and Dalcanton (2000), who argued that the data could be readily accommodated in the dark matter picture. The difference is between accommodating the data once known, and predicting it a priori. The dark matter picture is extraordinarily flexible: one is free to distribute the dark matter as needed to fit any data that evinces a non-negative mass discrepancy, even data that are wrong (de Blok & McGaugh, 1998). It is another matter entirely to construct a realistic model a priori; in my experience it is quite easy to construct models with plausible-seeming parameters that bear little resemblance to real galaxies (e.g., the low-spin case in Fig. 5). A similar conundrum is encountered when constructing models that can explain the long tidal tails observed in merging and interacting galaxies: models with realistic rotation curves do not produce realistic tidal tails, and vice-versa.

\[
V^2(R) = \frac{GM}{R} = G \left( \frac{M_b(R)}{R} + \frac{M_{DM}(R)}{R} \right),
\]

where for clarity we have omitted\(^2\) geometrical factors. The only absolute requirement is that the baryonic contribution should begin to decline once the majority of baryonic mass is encompassed. It is when rotation curves persist in remaining flat past this point that we infer the need for dark matter.

\(^2\) Strictly speaking, eq. (4) only holds for spherical mass distributions. I make this simplification here to emphasize the fact that both mass and radius matter.

### Table 1

Predictions of DD and SH for LSB galaxies.

| Observation            | DD | SH |
|------------------------|----|----|
| Evolutionary rate      | +  | +  |
| Size distribution      | +  | +  |
| Clustering             | +  | X  |
| Tully-Fisher relation  | X  | ?  |
| Central density relation | + | X  |
The data occupy a very narrow sliver of the enormous volume of parameter space available to dark matter models, a situation that seems rather contrived.

Both DD and SH predict residuals from Tully-Fisher that are not observed. I consider this to be an unrecoverable failure for DD, which was my hypothesis (McGaugh, 1992), so I worked hard to salvage it. I could not. For SH, Tully-Fisher might be recovered in the limit of dark matter domination, which requires further consideration.

4.3. Squeezing the toothpaste tube

Our efforts to evade one fine-tuning problem often lead to another. This has been my general experience in many efforts to construct viable dark matter models. It is like squeezing a tube of toothpaste: every time we smooth out the problems in one part of the tube, we simply squeeze them into a different part. There are many published claims to solve this problem or that, but they frequently fail to acknowledge (or notice) that the purported solution to one problem creates another.

One example is provided by Courteau and Rix (1999). They invoke dark matter domination to explain the lack of residuals in the Tully-Fisher relation. In this limit, \( M_b/R < M_{DM}/R \) and the baryons leave no mark on the rotation curve. This can reconcile the model with the Tully-Fisher relation, but it makes a strong prediction. It is not just the flat rotation speed that is the same for galaxies of the same mass, but the entirety of the rotation curve, \( V(R) \) at all radii. The stars are just convenient tracers of the dark matter halo in this limit; the dynamics are entirely dominated by the dark matter. The hypothesized solution fixes the problem that is addressed, but creates another problem that is not addressed, in this case the observed variation in rotation curve shape.

The limit of complete dark matter domination is not consistent with the shapes of rotation curves. Galaxies of the same baryonic mass have the same flat outer velocity (Tully-Fisher), but the shapes of their rotation curves vary systematically with surface brightness (de Blok & McGaugh, 1996; Tully and Verheijen, 1997; McGaugh and de Blok, 1998a,b;
Swaters et al., 2009, 2012; Lelli et al., 2013, 2016c). High surface brightness galaxies have steeply rising rotation curves while LSB galaxies have slowly rising rotation curves (Fig. 6). This systematic dependence of the inner rotation curve shape on the baryon distribution excludes the SH hypothesis in the limit of dark matter domination: the distribution of the baryons clearly has an impact on the dynamics.

A more recent example of this toothpaste tube problem for SH-type models is provided by the EAGLE simulations (Schaye et al., 2015). These are claimed (Ludlow et al., 2017) to explain one aspect of the observations, the radial acceleration relation (McGaugh et al., 2016), but fail to explain another, the central density relation (Lelli et al., 2016c) seen in Fig. 6. This was called the ‘diversity’ problem by Oman et al. (2015), who note that the rotation velocity at a specific, small radius (2 kpc) varies considerably from galaxy to galaxy observationally (Fig. 6), while simulated galaxies show essentially no variation, with only a small amount of scatter. This diversity problem is exactly the same problem that was pointed out before [compare Fig. 5 of Oman et al. (2015) to Fig. 14 of McGaugh and de Blok (1998a)].

There is no single, universally accepted standard galaxy formation model, but a common touchstone is provided by Mo et al. (1998). Their base model has a constant ratio of luminous to dark mass $\frac{\text{m}_\text{L}}{\text{m}_\text{D}}$ (their assumption (i)), which provides a reasonable description of the sizes of galaxies as a function of mass or rotation speed (Fig. 7). However, this model predicts the wrong slope (3 rather than 4) for the Tully-Fisher relation. This is easily remedied by making the luminous mass fraction proportional to the rotation speed $(\text{m}_\text{L} \propto \sqrt{V})$, which then provides an adequate fit to the Tully-Fisher relation. This has the undesirable effect of destroying the consistency of the size-mass relation. We can have one or the other, but not both.

This failure of the Mo et al. (1998) model provides another example of the toothpaste tube problem. By fixing one problem, we create another. The only way forward is to consider more complex models with additional degrees of freedom.

### 4.4. Feedback

It has become conventional to invoke ‘feedback’ to address the various problems that afflict galaxy formation theory (Bullock & Boylan-Kolchin, 2017; De Baerdemaker and Boyd, 2020). It goes by other monikers as well, variously being called ‘astrophysics in a box’ for gas phase astrophysics, or simply ‘baryonic physics’ for any process that might intervene between the relatively simple (and calculable) physics of collisionless cold dark matter and messy observational reality (which is entirely illuminated by the baryons). This proliferation of terminology obfuscates the boundaries of the subject and precludes a comprehensive discussion.

Feedback is not a single process, but rather a family of distinct processes. The common feature of different forms of feedback is the deposition of energy from compact sources into the surrounding gas of the interstellar medium. This can, at least in principle, heat gas and drive large-scale winds, either preventing gas from cooling and forming too many stars, or ejecting it from a galaxy outright. This in turn might affect the distribution of dark matter, though the effect is weak: one must move a lot of baryons for their gravity to impact the dark matter distribution.

There are many kinds of feedback, and many devils in the details. Massive, short-lived stars produce copious amounts of ultraviolet radiation that heats and ionizes the surrounding gas and erodes interstellar dust. These stars also produce strong winds through much of their short (~ 10 Myr) lives, and ultimately explode as Type II supernovae. These three mechanisms each act in a distinct way on different time scales. That’s just the feedback associated with massive stars; there are many other mechanisms (e.g., Type Ia supernovae are distinct from Type II supernovae, and Active Galactic Nuclei are a completely different beast entirely). The situation is extremely complicated. While the various forms of stellar feedback are readily apparent on the small scales of stars, it is far from obvious that they have the desired impact on the much larger scales of entire galaxies.

For any one kind of feedback, there can be many substantially different implementations in galaxy formation simulations. Independent numerical codes do not generally return compatible results for identical initial conditions (Scannapieco et al., 2012): there is no consensus on how feedback works. Among the many different computational implementations of feedback, at most one can be correct.

Most galaxy formation codes do not resolve the scale of single stars where stellar feedback occurs. They rely on some empirically calibrated, analytic approximation to model this ‘sub-grid physics’ — which is to say, they don’t simulate feedback at all. Rather, they simulate the accumulation of gas in one resolution element, then follow some prescription for what happens inside that unresolved box. This provides ample opportunity for disputes over the implementation and effects of feedback. For example, feedback is often cited as a way to address the cusp-core problem — or not, depending on the implementation (e.g., Benitez-Llambay et al., 2019; Bose et al., 2019; Di Cintio et al., 2014; Governato et al., 2012; Madau et al., 2014; Read et al., 2019). High resolution simulations (Bland-Hawthorn et al., 2015) indicate that the gas of the interstellar medium is less affected by feedback effects than assumed by typical sub-grid prescriptions: most of the energy is funneled through the lowest density gas — the course of least resistance — and is lost to the intergalactic medium without much impacting the galaxy in which it originates.

From the perspective of the philosophy of science, feedback is an auxiliary hypothesis invoked to patch up theories of galaxy formation. Indeed, since there are many distinct flavors of feedback that are invoked to carry out a variety of different tasks, feedback is really a suite of auxiliary hypotheses. This violates parsimony to an extreme and brutal degree.

This concern for parsimony is not specific to any particular feedback scheme; it is not just a matter of which feedback prescription is best. The entire approach is to invoke as many free parameters as necessary to solve any and all problems that might be encountered. There is little doubt that such models can be constructed to match the data, even data that bear little resemblance to the obvious predictions of the paradigm (McGaugh and de Blok, 1998b; Mo et al., 1998). So the concern is not whether ΛCDM galaxy formation models can explain the data; it is that they can’t not.

### 5. Modified Newtonian Dynamics

There is only one theory that predicted in advance the observations described above: the Modified Newtonian Dynamics (MOND) introduced by Milgrom (1983a,b,c). MOND is an extension of Newtonian theory (Milgrom, 2020). It is not a generally covariant theory, so is not, by itself, a complete replacement for General Relativity. Nevertheless, it makes unique, testable predictions within its regime of applicability (McGaugh, 2020).

The basic idea of MOND is that the force law is modified at an acceleration scale, $a_0$. For large accelerations, $g \gg a_0$, everything is normal and Newtonian: $g = g_N$, where $g_N$ is the acceleration predicted by the observed luminous mass distribution obtained by solving the Poisson equation. At low accelerations, the effective acceleration tends towards the limit

$$g \rightarrow \sqrt{g_N a_0} \quad \text{for} \quad g \ll a_0 \quad (5)$$
The Baryonic Tully-Fisher relation (BTFR) is a relation between the baryonic mass of a galaxy and its rotation speed. There is no dark matter involved: 

\[ M_p \sim V_f \]

This relation gives a steep Tully-Fisher relation similar to that which is observed (Aaronson et al., 1979). A test particle in a circular orbit around a point mass \( M_p \) in the deep MOND regime (eq. (5)) will experience a centripetal acceleration

\[ \frac{V^2}{R} = \frac{GM_p}{R^2} \]  
(6)

Note that the term for the radius \( R \) cancels out, so eq. (6) reduces to

\[ V^4 = a_0 GM_p \]  
(7)

which the reader will recognize as the Baryonic Tully-Fisher relation

\[ M_b = AV^4 \]  
(8)

with \( A = \zeta/(a_0G) \).

This simple math explains the flatness of rotation curves. This is not a prediction; it was an input that motivated the theory, as it motivated dark matter. Unlike dark matter, in which rotation curves might rise or fall, the rotation curves of isolated galaxies must tend towards asymptotic flatness.

MOND also explains the Tully-Fisher relation. Indeed, there are several distinct aspects to this prediction. That the relation exists at all is a strong prediction. Fundamentally, the Baryonic Tully-Fisher Relation (BTFR) is a relation between the baryonic mass of a galaxy and its flat rotation speed. There is no dark matter involved: \( V_f \) is not a property of a dark matter halo, but of the galaxy itself.

One MOND prediction is the slope of the BTFR: the power law scaling \( M \sim V^4 \) has \( x = 4 \) exactly. While the infrared data of Aaronson et al. (1979) suggested such a slope, the exact value was not well constrained at that time. It was not until later that Tully-Fisher was empirically recognized as a relation driven by baryonic mass (McGaugh et al., 2000), as anticipated by MOND. Moreover, the slope is only four when a good measurement of the flat rotation velocity is available (Verheijen, 2001; McGaugh, 2005, 2012); common proxies like the line-width only crudely approximate the result and typically return shallower slopes (e.g., Zaritsky et al., 2014), as do samples of limited dynamic range (e.g., Pizagno et al., 2007). The latter are common in the literature: selection effects strongly favor bright galaxies, and the majority of published Tully-Fisher relations are dominated by high mass galaxies (\( M > 10^{10} M_\odot \)). Consequently, the behavior of the Baryonic Tully-Fisher relation remains somewhat controversial to this day (e.g., Mancera Piña et al., 2019; Ogle et al., 2019). This appears to be entirely a matter of data quality (McGaugh et al., 2019). The slope of the relation is indistinguishable from 4 when a modicum of quality control is imposed (Lelli et al., 2016b; McGaugh, 2005, 2012; Schombert et al., 2020; Stark et al., 2009; Trachternach et al., 2009). Indeed, only a slope of four successfully predicted the rotation speeds of low mass galaxies (Giovanelli et al., 2013; McGaugh, 2011).

Another aspect of the Tully-Fisher relation is its normalization. This is set by fundamental constants: Newton’s constant, \( G \), and the acceleration scale of MOND, \( a_0 \). For \( \zeta = 0.8, A = 50 \ M_\odot \ km^{-3} s^4 \). However, there is no theory that predicts the value of \( a_0 \), which has to be set by the data. Moreover, this scale is distance-dependent, so the precise value of \( a_0 \) varies with adjustments to the distance scale. For this reason, in part, the initial estimate of \( a_0 = 2 \times 10^{-10} \ m \ s^{-2} \) (of Milgrom, 1983a) was a bit high. Begeman et al. (1991) used the best data then available to obtain \( a_0 = 1.2 \times 10^{-10} \ m \ s^{-2} \). The value of Milgrom’s acceleration constant has not varied meaningfully since then (Famaey and McGaugh, 2012; Li et al., 2018; McGaugh, 2011; McGaugh et al., 2016; Sanders and McGaugh, 2002). This is a consistency check, but not a genuine prediction.

An important consequence of MOND is that the Tully-Fisher relation is absolute: it should have no dependence on size or surface brightness (Milgrom, 1983a). The mass of baryons is the only thing that sets the flat amplitude of the rotation speed. It matters not at all how those baryons are distributed. MOND was the only theory to correctly predict this in advance of the observation (McGaugh and de Blok, 1998b). The fine-tuning problem that we face conventionally is imposed by this otherwise unanticipated result.

The absolute nature of the Tully-Fisher relation in MOND further predicts that it has no physical residuals whatsoever. That is to say, scatter around the relation can only be caused by observational errors and scatter in the mass-to-light ratios of the stars. The latter is an irreducible unknown: we measure the luminosity produced by the stars in a galaxy, but what we need to know is the mass of those stars. The conversion between them can never be perfect, and inevitably introduces some scatter into the relation. Nevertheless, we can make our best effort to account for known sources of scatter. Between scatter expected from...
observational uncertainties and that induced by variations in the mass-to-light ratio, the best data are consistent with the prediction of zero intrinsic scatter (McGaugh, 2005, 2012; Lelli et al., 2016b, 2019). Of course, it is impossible to measure zero, but it is possible to set an upper limit on the intrinsic scatter that is very tight by extragalactic standards (~6% Lelli et al., 2019). This leaves very little room for variations beyond the inevitable impact of the stellar mass-to-light ratio. The scatter is no longer entirely accounted for when lower quality data are considered (McGaugh, 2012), but this is expected in astronomy: lower quality data inevitably admit systematic uncertainties that are not readily accounted for in the error budget.

Milgrom (1983a) made a number of other specific predictions. In MOND, the acceleration expected for kinematics follows from the surface density of baryons. Consequently, low surface brightness means low acceleration. Interpreted in terms of conventional dynamics, the prediction is that the ratio of dynamical mass to light, $M_{\text{dyn}}/L$, should increase as surface brightness decreases. This happens both globally — LSB galaxies appear to be more dark matter dominated than HSB galaxies (see Fig. 4(b) of McGaugh and de Blok, 1998a), and locally — the need for dark matter sets in at smaller radii in LSB galaxies than in HSB galaxies (Figs. 3 and 14 of McGaugh and de Blok, 1998b; Famaey and McGaugh, 2012, respectively).

One may also test this prediction by plotting the rotation curves of galaxies binned by surface brightness: acceleration should scale with surface brightness. It does (Figs. 4 and 16 of McGaugh and de Blok, 1998b; Famaey and McGaugh, 2012, respectively). This observation has been confirmed by near-infrared data. The systematic variation of color coded surface brightness is already obvious with optical data, as in Fig. 15 of Famaey and McGaugh (2012), but these suffer some scatter from variations in the stellar mass-to-light ratio. These practically vanish with near-infrared data, which provide such a good tracer of the surface mass density of stars that the equivalent plot is a near-perfect rainbow (Fig. 3 of both McGaugh et al., 2019; McGaugh, 2020). The data strongly corroborate the prediction of MOND that acceleration follows from baryonic surface density.

The central density relation (Fig. 6, Lelli et al., 2016c) was also predicted by MOND (Milgrom, 2016). Both the shape and the amplitude of the correlation are correct. Moreover, the surface density $\Sigma$ at which the data bend follows directly from the acceleration scale of MOND: $a_0 = G\Sigma$. This surface density also corresponds to the stability limit for disks (Brada & Milgrom, 1999; Milgrom, 1989). The scale we had to insert by hand in dark matter models is a consequence of MOND.

Since MOND is a force law, the entirety of the rotation curve should follow from the baryonic mass distribution. The stellar mass-to-light ratio can modulate the amplitude of the stellar contribution to the rotation curve, but not its shape, which is specified by the observed distribution of light. Consequently, there is rather limited freedom in fitting rotation curves.

Example fits are shown in Fig. 8. The procedure is to construct Newtonian mass models by numerically solving the Poisson equation to determine the gravitational potential that corresponds to the observed baryonic mass distribution. Indeed, it is important to make a rigorous solution of the Poisson equation in order to capture details in the shape of the mass distribution (e.g., the wiggles in Fig. 8). Common analytic approximations like the exponential disk assume these features out of existence. Building proper mass models involves separate observations for the stars, conducted at optical or near-infrared wavelengths, and the gas of the interstellar medium, which is traced by radio wavelength observations. It is sometimes necessary to consider separate mass-to-light ratios for the stellar bulge and disk components, as there can be astrophysical differences between these distinct stellar populations (Baade, 1944). This distinction applies in any theory.

The gravitational potential of each baryonic component is represented by the circular velocity of a test particle in Fig. 8. The amplitude of the rotation curve of the mass model for each stellar component scales as the square root of its mass-to-light ratio. There is no corresponding mass-to-light ratio for the gas of the interstellar medium as there is a well-understood relation between the observed flux at 21 cm and the mass of hydrogen atoms that emit it (Draine, 2011). Consequently, the line for the gas components in Fig. 8 is practically fixed.

In addition to the mass-to-light ratio, there are two "nuisance" parameters that are sometimes considered in MOND fits: distance and inclination. These are known from independent observations, but of course these have some uncertainty. Consequently, the best MOND fit sometimes occurs for slightly different values of the distance and inclination, within their observational uncertainties (Begeman et al., 1991; de Blok & McGaugh, 1998; Sanders, 1996).

Distance matters because it sets the absolute scale. The further a galaxy, the greater its mass for the same observed flux. The distances to individual galaxies are notoriously difficult to measure. Though usually not important, small changes to the distance can occasionally have powerful effects, especially in gas rich galaxies. Compare, for example, the fit to DDO 154 by Li et al. (2018) to that of Ren et al. (2019).

Inclinations matter because we must correct the observed velocities for the inclination of each galaxy as projected on the sky. The inclination correction is $V = V_{\text{obs}}/\sin(i)$, so is small at large inclinations (edge-on) but large at small inclinations (face-on). For this reason, dynamical analyses often impose an inclination limit. This is an issue in any theory, but MOND is particularly sensitive since $M \propto V^4$ so any errors in the inclination are amplified to the fourth power (see Fig. 2 of de Blok & McGaugh, 1998). Worse, inclination estimates can suffer systematic errors (de Blok & McGaugh, 1998; McGaugh, 2012; Verheijen, 2001): a galaxy seen face-on may have an oval distortion that makes it look more inclined than it is, but it can't be more face-on than face-on.

MOND fits will fail if either the distance or inclination is wrong. Such problems cannot be discerned in fits with dark matter halos, which have ample flexibility to absorb the imparted variance (see Fig. 6 of de Blok & McGaugh, 1998). Consequently, a fit with a dark matter halo will not fail...
The expert cosmologist may object that there is a great deal more data that must be satisfied. These have been reviewed elsewhere (Bekenstein, 2006; Famaey and McGaugh, 2012; Mcgaugh, 2015; Sanders and McGaugh, 2002) and are beyond the scope of this discussion. Here I note only that my experience has been that reports of MOND’s falsification are greatly exaggerated. Indeed, it has a great deal more explanatory power for a wider variety of phenomena than is generally appreciated (McGaugh and de Blok, 1998a,b).

The most serious, though certainly not the only, outstanding challenge to MOND is the dynamics of clusters of galaxies (Angus et al, 2008; Sanders and McGaugh, 2002). Contrary to the case in most individual galaxies and some groups of galaxies (Milgrom, 2018, 2019), MOND typically falls short of correcting the mass discrepancy in rich clusters by a factor of ~ 2 in mass. This can be taken as completely fatal, or as a being remarkably close by the standards of astrophysics. Which option one chooses seems to be mostly a matter of confirmation bias: those who are quick to dismiss MOND are happy to spot their own models a factor of two in mass, and even to assert that it is natural to do so (e.g., Ludlow et al., 2017). MOND is hardly alone in suffering problems with clusters of galaxies, which also present problems for ΛCDM (e.g., Angus & McGaugh, 2008; Asencio et al., 2021; Kent, 1987).

Table 2 lists the successful predictions of MOND that are discussed here. A more comprehensive list is given by Famaey and McGaugh (2012) and McGaugh (2020) who also discuss some of the problems posed for dark matter. MOND has had many predictive successes beyond rotation curves (e.g., Famaey and McGaugh, 2013a,b; Sanders, 2016) and has inspired successful predictions in cosmology (e.g., Sanders, 1998; McGaugh, 1999, 2000; Sanders, 2001; McGaugh, 2015, 2018). In this context, it makes sense to associate LSB galaxies with low density fluctuations in the initial conditions, thereby recovering the success of DD while its ills are cured by the modified force law. Galaxy formation in general is likely to proceed hierarchically but much more rapidly than in ΛCDM (Sanders, 2001; Stachniwicz and Kutschera, 2001), providing a natural explanation for both the age of stars in elliptical galaxies and allowing for a subsequent settling time for the disks of spiral galaxies (Wittenburg et al., 2020).
gets any prediction right, let alone so many.

The situation now is analogous to that in the time of Copernicus. We are piling complication upon complication to explain what at root is a simple phenomenon. His words from nearly five centuries ago can be paraphrased for our current predicament:

“Those who devised the [eccentric feedback prescriptions] seem thereby in large measure to have solved the problem of the apparent motions with appropriate calculations. But meanwhile they introduced a good many ideas which apparently contradict the first principles of [uniform motion parsimony]. Nor could they elicit or deduce from [the eccentric feedback effects] the principal consideration, that is, the [structure of the universe and the true symmetry of its parts/ability to predict the kinematics of galaxies from their observed mass distribution]. On the contrary, their experience was just like someone taking from various places hands, feet, a head, and other pieces, very well depicted, it may be, but not for the representation of a single person; since these fragments would not belong to one another at all, a monster rather than a man would be put together from them.” — Nicolaus Copernicus, De Revolutionibus (as translated by Rosen, 1992), along with my paraphrase (bold face) paralleling his original words (italicized).

In the persistent absence of laboratory evidence to the contrary, it remains possible that ‘dark matter’ is a proxy for some deeper phenomenon, and our present conception of it is nothing more than a hypothetical entity convenient to cosmic calculations. Like ather in the 19th century, cold dark matter is a substance that simply must exist given our present understanding of physics. But does it?

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Babac, B. (1944). The resolution of messier 32, NGC 205, and the central region of the Andromeda nebula. The Astrophysical Journal, 100, 137. https://doi.org/10.1086/144650.

Babcock, J. N., & Casertano, S. (1985). Some possible regularities in the missing mass problem. The Astrophysical Journal, 293, L7-L10. https://doi.org/10.1086/184480.

Barnes, J., & Efstathiou, G. (1988). Angular momentum from tidal torques. The Astrophysical Journal, 319, 575. https://doi.org/10.1086/165480.

Barnes, J. E., & Hernquist, L. (1992). Dynamics of interacting galaxies. Annual Review of Astronomy and Astrophysics, 30, 705-742. https://doi.org/10.1146.

Begeman, K. G., Broeils, A. H., & Sanders, R. H. (1991). Extended rotation curves of spiral galaxies - dark halos and modified dynamics. Monthly Notices of the Royal Astronomical Society, 249, 523-537.

Bekenstein, J. D. (2004). Relativistic gravitation theory for the modified Newton dynamics paradigm. Phys. Rev. D, 70, 083509. https://doi.org/10.1103/PhysRevD.70.083509.

Bekenstein, J. (2006). The modified Newtonian dynamics - MOND and its implications for new physics. Contemporary Physics, 47, 387-403. https://doi.org/10.1080/00107510701445055. arXiv:astro-ph/0701848.

Bekenstein, J., & Milgrom, M. (1984). Does the missing mass problem signal the breakdown of Newtonian gravity? The Astrophysical Journal, 286, 7-14. https://doi.org/10.1086/162570.

Benitez-Llambay, A., Frenk, C. S., Ludlow, A. D., & Navarro, J. F. (2019). Baryon-induced feedback prescriptions and integrated properties. Monthly Notices of the Royal Astronomical Society, 488, 2387-2404. https://doi.org/10.1093/mnras/stz1890. arXiv:1806.09146.

Benson, A. J. (2010). Galaxy formation theory. Physics Reports, 495, 33-86. https://doi.org/10.1016/j.physrep.2009.10.001. arXiv:0909.5394.

Benson, A. J., Bower, R. G., Frenk, C. S., Lacey, C. G., Cole, S. (2003). What shapes the luminosity function of galaxies? The Astrophysical Journal, 599, 48-49. https://doi.org/10.1086/379160. arXiv:astro-ph/0302450.

Bereziani, L., & Khoury, J. (2015). Theory of dark matter superfluidity. Physical Review D, 92, 103510. https://doi.org/10.1103/PhysRevD.92.103510. arXiv:1507.01019.

van den Bergh, S. (1962). The frequency of stars with different metal abundances. The Astronomical Journal, 67, 486-490. https://doi.org/10.1086/108757.

Binney, J., & Merrifield, M. (1998). Galactic astronomy. Princeton, NJ: Princeton University Press.

Binney, J., & Tremaine, S. (1987). Galactic dynamics. Princeton, NJ: Princeton University Press.

Blanchet, L. (2007). Gravitational polarization and the phenomenology of MOND. Classical and Quantum Gravity, 24, 3529-3539. https://doi.org/10.1088/0264-9381/24/11/001. arXiv:astro-ph/0612240.

Bland-Hawthorn, J., & Gerhard, O. (2016). The galaxy in context: Structural, kinematic, and integrated properties. Annual Review of Astronomy and Astrophysics, 54, 529-596. https://doi.org/10.1146/annurev-astro-081915-023441. arXiv:1602.07702.

Bland-Hawthorn, J., Sutherland, R., & Webster, D. (2015). Ultralight dwarf galaxies—the lowest-mass relics from before reionization. The Astrophysical Journal, 807, 154. https://doi.org/10.1088/0004-637X/807/2/154. arXiv:1505.06209.

de Blok, W. J. G. (2010). The core-cusp problem. Advances in Astronomy, 2010, 789293. https://doi.org/10.1155/2010/789293. arXiv:0910.5032.

de Blok, W. J. G., Bosma, A., & McGaugh, S. (2003). Simulating observations of dark matter dominated galaxies: Towards the optimal halo profile. Monthly Notices of the Royal Astronomical Society, 336, 657-678. https://doi.org/10.1046/j.1365-8711.2003.06030.x. arXiv:astro-ph/0212102.

de Blok, W. J. G., & McGaugh, S. S. (1996). Does low surface brightness mean low density? The Astrophysical Journal, 469, L89. https://doi.org/10.1086/310266. arXiv:astro-ph/9607042.
gravitational microlensing. The Astrophysical Journal, 836, L18. doi:
http://doi.org/10.3847/2041-8213/aaab5a. arXiv:1702.00947.

Meneghetti, M., Davolino, G., Bergamin, F., Rosati, P., Natarajan, P., Giocoli, C., Caminha, G. B., Metcalf, R. B., Rasia, E., Borgani, S., Calura, F., Grilli, C., Mercurio, A., & Vanzella, E. (2020). An excess of small-scale gravitational lenses observed in galaxy clusters. Science, 369, 1347–1351. doi:10.1126/science.aaz9034. arXiv:1909.00980.

Merritt, D. (2017). Cosmology and convention. Studies in the History and Philosophy of Modern Physics, 57, 41–52. https://doi.org/10.1016/j.shpsb.2016.12.002. arXiv:1702.08489.

Merritt, D. (2020). A philosophical approach to MOND: Assessing the milligravitation research program in cosmology.

Milgrom, M. (1982a). A modification of the Newtonian dynamics - implications for galaxies. The Astrophysical Journal, 270, 371–389. doi:10.1086/161131.

Milgrom, M. (1982b). A modification of the Newtonian dynamics: Implications for galaxy systems. The Astrophysical Journal, 270, 384–389. doi:10.1086/161132.

Milgrom, M. (1983c). A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. The Astrophysical Journal, 270, 365–370. doi:10.1086/161130.

Milgrom, M. (1988a). On the case of galaxy rotation curves to test the modified dynamics. The Astrophysical Journal, 332, 689. doi:10.1086/166777.

Milgrom, M. (1989). On stability of galactic disc in the modified dynamics and the distribution of their mean surface-brightness. The Astrophysical Journal, 338, 121. doi:10.1086/164184.

Milgrom, M. (2006). MOND as modified inertia. In G. A. Mamon, F. Combes, C. Defayett, & B. Fort (Eds.), EAS publications series (pp. 217–224). doi:10.1007/978-0-387-30598-6_22. arXiv:astro-ph/0510117.

Milgrom, M. (2009). Hectic MOND gravity. Physical Review D, 80, 123536. doi:10.1103/PhysRevD.80.123536. arXiv:0912.0790.

Milgrom, M. (2016). Universal modified Newtonian dynamics relation between the baryonic and ‘dynamical’ central surface densities of disc galaxies. Physical Review Letters, 117, 241102. doi:10.1103/PhysRevLett.117.241102. arXiv:1607.02568.

Milgrom, M. (2018). MOND in galaxy groups. Physical Review D, 98, 104036. doi:10.1103/PhysRevD.98.104036. arXiv:1801.00089.

Milgrom, M. (2019). MOND in galaxy groups: A superior sample. Physical Review D, 99, Article 044041. doi:10.1103/PhysRevD.99.044041. arXiv:1811.12233.

Milgrom, M. (2020). MOND vs. dark matter in light of historical paradoxes. Studies in the History and Philosophy of Modern Physics, 71, 170–195. doi:10.1016/j.shpsb.2020.02.004. arXiv:1910.04368.

Moffett, A. J., Ingarfield, S. A., Driver, S. P., Robinson, A. S. H., Kelvin, L. S., Lange, R., Metisic, U., Alpaslan, M., Baldry, I. K., Bland-Hawthorn, J., Brough, S., Cluver, M. E., Drinkwater, M. J., Driver, S. P., Dunlop, J. S., Gunn, J. E., Hambly, N. C., Hargreaves, D. A., Heymans, C., Hlozek, R. I., James, D. E., Jones, P. A., Jones, N., Koribalski, B., Liske, J., Loveday, J., Mah扁e, J., Merson, B. A. J., Norberg, P., Owers, M. S., Pajdos, C., Parkinson, B. J., Prunet, S., Reader, A., Robotham, A. S. G., Rose, J., van der Burg, S. W. R., Wray, M. A., & Zehavi, I. (2015). The SDSS-IV MaNGA survey: Overview of the galaxy and dark matter program in cosmology

Milgrom, M. (1982b). The dark core catastrophe. The Astrophysical Journal, 333, 1–13. doi:10.1086/164184. arXiv:1910.04368.
Schaye, J., Crain, R. A., Bower, R. G., Furlong, M., Schaller, M., Theuns, T., Dalla Vecchia, C., Frenk, C. S., McCarthy, I. G., Helly, J. C., Jenkins, A., Rosas-Guevara, Y. M., White, S. D. M., Baes, M., Booth, C. M., Camps, P., Navarro, J. F., Qu, Y., Rahmati, A., … Trayford, J. (2015). The EAGLE project: Simulating the evolution and assembly of galaxies and their environments. Monthly Notices of the Royal Astronomical Society, 446, 521–554. https://doi.org/10.1093/mnras/stu2058, arXiv:1407.7040.

Schombert, J., Maciel, T., & McGaugh, S. S. (2011). Stellar populations and the star formation histories of LSB galaxies—Part I: Optical and his imaging. Advances in Astronomy, 2011, 43698. https://doi.org/10.1155/2011/43698, arXiv:1109.2360.

Schombert, J., McGaugh, S., & Lelli, F. (2020). Using the baryonic tully-fisher relation to measure H0. The Astronomical Journal, 160, 71. https://doi.org/10.3847/1538-3881/ab9488, arXiv:2006.08615.

Schombert, J. M., Pildis, R. A., & Eder, J. A. (1997). Gas-rich dwarfs from the second palomar sky Survey. I. Catalog and characteristics. The Astrophysical Journal, 111, 233. https://doi.org/10.1086/313016, arXiv:astro-ph/9612130.

Searle, L., & Zinn, R. (1978). Composition of halo clusters and the formation of the galactic halo. The Astrophysical Journal, 225, 357–379. https://doi.org/10.1086/156499.

Sellwood, J. A., & McGaugh, S. S. (2005a). The compression of dark matter halos by baryonic infall. The Astrophysical Journal, 634, 70–76. https://doi.org/10.1086/491731, arXiv:astro-ph/0507389.

Sellwood, J. A., & McGaugh, S. S. (2005b). The compression of dark matter halos by baryonic infall. The Astrophysical Journal, 634, 70–76. https://doi.org/10.1086/491731, arXiv:astro-ph/0507589.

Silk, J. (1968). Cosmic black-body radiation and galaxy formation. The Astrophysical Journal, 151, 459. https://doi.org/10.1086/149449.

Skordis, C., Mota, D. F., Ferreira, P. G., & Behm, C. (2006). Large scale structure in bekenstein’s theory of relativistic modified Newtonian dynamics. Physical Review Letters, 96, Article 011301. https://doi.org/10.1103/PhysRevLett.96.011301, arXiv:astro-ph/050519.

Skordis, C., & Zloum, T. (2020). A new relativistic theory for Modified Newtonian Dynamics. arXiv e-prints, arXiv:2007.00082.

Skordis, C., & Zloum, T. (2019). Gravitational alternatives to dark matter with tensor mode speed equaling the speed of light. Physical Review D, 100, 104013. https://doi.org/10.1103/PhysRevD.100.104013, arXiv:1905.09465.

Smoot, G. F., Bennett, C. L., Kogut, A., Wright, E. L., Aymon, J., Boggess, N. W., Cheng, E. S., Stachniewicz, S., & Kutschera, M. (2001). The Wilkinson Microwave Anisotropy Probe (WMAP) observations: First results and implications for cosmology and particle physics. The Astrophysical Journal, 543, L1. https://doi.org/10.1086/320257, arXiv:astro-ph/000349.

Steigman, G., & Turner, M. S. (1985). Cosmological constraints on the properties of baryonic infall. The Astrophysical Journal, 304, 457–467. https://doi.org/10.1086/163866.

Stern, D. J. (2006). Baryonic effects on galaxy evolution. The Astronomical Journal, 131, 2333–2361. https://doi.org/10.1086/502253, arXiv:astro-ph/060222.

White, S. D. M. (1996). Violent relaxation in hierarchical clustering. In O. Lahav, J. Primack, & M. Steinhardt (Eds.), The large scale structure of the universe. Cambridge University Press, pp. 401–420. arXiv:astro-ph/960402.

Witten, K. N., Kroupa, P., & Famaey, B. (2020). The formation of exponential disk galaxies in MOND. The Astronomical Journal, 890, 173. https://doi.org/10.3847/1538-3881/2020/01/01, arXiv:2002.01941.

Yoshii, Y., & Peterson, B. A. (1995). Interpretation of the faint galaxy number counts in the K band. The Astrophysical Journal, 444, 15. https://doi.org/10.1086/175579.

Zaritsky, D., Courteau, S., Munoz-Mateos, J. C., & Sorce, J. G. (1999). Baryonic baryonic infall galaxy formation. The Astrophysical Journal, 522, 49–55. https://doi.org/10.1086/306830, arXiv:astro-ph/980357.

Zwaan, M. A., van der Hulst, J. M., de Blok, W. J. G., & McGaugh, S. S. (2011). Testing modified Newtonian dynamics with rotation curves of dwarf and low surface brightness galaxies. The Astrophysical Journal, 718, 380–391. https://doi.org/10.1088/0004-637X/718/1/380, arXiv:1005.5456.

Zwaan, M. A., van der Hulst, J. M., de Blok, W. J. G., & McGaugh, S. S. (2011). The tully-fisher relation for low surface brightness galaxies: Implications for galaxy evolution. Monthly Notices of the Royal Astronomical Society, 273, L35-L38. arXiv:astro-ph/0950110.