Learning design for transition from arithmetic thinking to algebraic thinking

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Abstract. Existing studies show that the role of algebra in learning is currently focused in many perspectives so traditional algebra learning requires fundamental reforms and changes in thinking. Algebraic learning currently must be able to focus on the mastery and application of algebra in various contexts in everyday life. The study was conducted to design mathematics learning to support the transition of arithmetic thinking to the algebraic thinking of junior high school students, and to describe the socio-mathematical norms in mathematics learning for a transition from arithmetic thinking into algebraic thinking. The research method used is design research. The results showed that the indicator of transitions from arithmetic thinking into algebraic thinking can be achieved well, some students made mistakes when working on negative number operations because students find it was difficult to distinguish between signs of numbers and signs of operations involved in an algebraic expression. This should be anticipated in order to prepare students for a more advanced level of algebraic understanding.

1. Introduction

Arithmetic has been studied since elementary school. Arithmetic introduced to elementary school children uses numbers that range from units to thousands [1, 13, 18]. The numbers used in arithmetic operations in elementary schools are adjusted to the grade level of students, the higher the level of class students, the greater the numerical value used. Meanwhile, algebra is a way of expressing generalizations about numbers, numbers, relationships, and functions. For this reason, a good understanding of the connection between numbers, numbers, and relationships is related to the student to be successful in learning algebra. NCTM Principles and Standards for School Mathematics [15,16] explains that algebra includes everything related to symbol manipulation, more fully algebra is defined as (1) understanding patterns, relationships, and functions; (2) represent and analyze mathematical situations and mathematical structures using algebraic symbols; (3) using mathematical models to represent and understand quantitative relationships, and; (4) analyzing changes in various contexts.

Formal Algebra in learning schools are currently focused on many perspectives so learning traditional algebra requires fundamental renewal and also changes in the way of thinking [3]. Algebraic learning currently aimed at mastering algebraic concepts and apply the concept of algebra in various the context of everyday life. The mathematics teacher currently argues that arithmetic understanding is a stage early in understanding algebra [20]. Mathematical skills in the field of geometry and measurement are the most essential basis in learning algebra. At school, algebra taught
with an arithmetic view that is Arithmetic is one of the algebraic structures can be calculated. In summary, currently, Algebraic learning is understood as a generalization of arithmetic. Some understanding from knowledge arithmetic must be transformed in such a way so that could help students develop algebraic thinking skills. Ability to think arithmetic cannot directly be utilized by students for capital developing algebraic thinking skills [4]. This study aims to find out an overview of the transition of arithmetic thinking skills to algebraic thinking skills. The main reason for algebra being a problem for students at school is due to the nature of its abstractness. Algebraic ideas are connected to all fields of study in mathematics and also there are several contexts outside the field mathematics [5]. Blanton and Kaput [3] found several different categories of algebraic reasoning, that is generalized mathematics, functional relationship, properties of numbers and operations, and algebraic treatment of number. Several of these categories indicate knowledge that must be mastered by students in learning algebra. A transition of knowledge is required so students could understand algebra easily. The reason why this transition is needed because arithmetic and algebra have the same usage symbols and operations but represent things differently. Figure 1 shows the main difference from arithmetic and algebra, an equal sign can have different roles. In arithmetic, the equal sign is for showing the answer, while in algebra, it is the symbol for showing the relation between two algebra expressions.

![Arithmetic vs algebra: The role of equal sign '=?'

Arithmetic vs algebra:
The role of equal sign

Arithmetic: Calculating expressions
'=' signifies the answer

Transition from arithmetic to algebra:
Mathematics learning through developed specific tasks

Algebra: Transforming

Figure 1. The key difference between arithmetic and algebra

For example, in basic school arithmetic, emphasized "calculating answer" which means that the sign is the same as on expression "5 + 3 = 8" represent the number to the right of the sign 'equals' is the answer to the left-hand expression of the sign 'together with'. Meaning equal sign will make it difficult for students in understanding the expression "2 + 5 = a + 8" because in the expression the sign 'equal' no longer means 'answer' but states relationship from left and right expression. Therefore that, there must be a transition from arithmetic to algebra which allows students to interpret the symbol 'equals' as a relation expression. There is a gap between arithmetic abilities and algebraic abilities which must be bridged with a learning mathematics and design instructional classes that can motivate the emergence of a transition ability[10, 14, 19]. This matter aligned with the purpose of this study, namely to investigate learning seen from that aspects. If indeed learning is felt already bring up ideas that support leaps knowledge and ability transitions, then need to be described how the transition occurs by explaining the students' thinking process starting from reasoning, explaining, refuting, generalize, and justify [5, 12, 15]. Then the ability transition of arithmetic thinking into algebraic thinking can be explained by investigating the emergence of transition indicators. Kieran [11] describes some difficulties in moving arithmetic to algebra, including algebraic focus is not on calculation but on relationships, and students must
understand the meaning of inverse and operations on addition and multiplication. Understanding the meaning of algebraic expression must be mastered by students to have good algebraic thinking ability.

2. Methods
The method used in this study is design research to develop instruction theory in algebra learning specifically to describe the transition from arithmetic to algebraic thinking. Design research combines theoretical thoughts of the researchers and empiric-based analysis to build the instruction theory [6, 7, 8]. Three phases of design research are preparation phase, teaching experiment, and retrospective analysis. Research subjects are students in grade 7 of Junior High School in Palembang, the students were selected based on their abilities in arithmetic. Preparation phase is conducted to obtain students’ prior knowledge and their response toward the learning, it will be useful to support the socio-mathematical norms during the learning. Also, in this phase, the Hypothetical Learning Trajectory is formulated and refined based on the results of the preliminary stage of the teaching experiment.

During the teaching experiment, the data is also used to describe the socio-mathematical norms occur in the classroom while the students learn. Social-mathematical norms relate to the assessment of students’ explanations whether they can be accepted mathematically. Cobb [6, 7, 17] introduces social mathematical norms as mathematical explanations that can be accepted and justifications include those considered as mathematical differences and mathematical sophistication. Differences in student opinions, solutions, and arguments can occur in class discussions or group work. By sharing the answers they can discuss, they will learn and pay attention to which arguments can be categorized as mathematical sophistication. Then some opinions are collected and discussed throughout the class, the students themselves are expected to determine which opinions are most acceptable, or which opinions are the most sophisticated.

3. Result and Discussion
A related study has been conducted previously to know whether arithmetic knowledge used for understanding algebra. Data collection is done in a junior high school in Palembang. Instrument used has been validated by experts in the form of questions about basic arithmetic and algebra for students Middle school. Arithmetic questions include questions the problem for operating times, adding, dividing, and subtracting as well as squaring and fractions. While for algebra problems, the subject chosen is in the early stages of the introduction of algebra about variables, coefficients, and constants as well simple story about problems related to arithmetic and algebra. As much of 30 students were involved as the subject of this study. The selected students are students who have already get a lesson about the subject algebra. Students' abilities vary from low, average, to high level. Results of students' work is collected for analysis then followed by doing interview to clarify their answers to the problem which are given. There is a transition from ability think arithmetic into thinking skills algebra seen from the following indicators: (1) distinguish between variables, coefficients, and constants, (2) differentiate signs of operation and signs of numbers, (3) use properties in operations algebra, (4) using the concept of arithmetic in algebraic expression (5) generalize in algebraic form. In doing some tasks given about arithmetics, students did not have any difficulties, they had good knowledge about operating numbers whether it was adding, subtracting, multiplying, or dividing numbers. The problem appeared when students had to think about the relationship between numbers if they were followed by some other quantities, let say variables. The situation was how many 24 times c is equal to 8 times d, and the students were asked to explain what c over d was. They struggled to transform this kind of number relation to the expected solution which was in the form of fraction. They tried to separated two group of numbers and compared. Some other students could come up to the solution, they used the sense in manipulating fraction. However, it can be seen that from the task that, the difficulties of problem increase when they have to work on the variables, because, they were not accustomed to treat the letters of variable the same way as the treat numbers.
Table 1. Indicators of each task given to students

| Indicators                                                                 | Example Tasks                                                                                                                                 |
|---------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------|
| T1. distinguish between variables, coefficients, and constants            | cx24 = dx8, “hasil kali dua bilangan asli 24, berapa jumlahnya? (the product of two natural numbers is 24, what is the sum?)”                  |
| T2. Differentiate signs of operation and signs of numbers                 | y − 5 = 2(10 − y), find the value of y                                                                                                      |
| T3. Use properties in operations algebra                                  | \[ \frac{3}{5} \times \left[ \frac{9}{10} \right] = 1, \text{ find } a/b \]                                                                       |
| T4. Using the concept of arithmetic in algebraic expression               | ![Algebraic Expression](image1)                                                                                                                                 |
| T5. Generalize in algebraic form                                           | The age of mother and Andi add up to 54 years old, mother and Ina is 51 years old, Andi and Ina is 21 years old, determine each of their ages. |

These indicators is derived from the ability of students to retrieve their arithmetic thinking skill to do algebra tasks. Some examples problem related to the indicator are shown in the table above, the students were expected to do these tasks based on their existing knowledge. Most of them have studied algebra. The example tasks (T1) is the task to see whether students understand the relationship of two expression between the equal sign that they can explain the equal sign is not necessarily the term to show the result. Also, the task is given to test students’ understanding in differentiating variables, coefficients, and constants. The task (T2) check how students distinguish the sign of number and operation. The task (T3) simply check students ability to use the properties of algebra. The task (T4) is more about the fluency of students to apply the concept of arithmetic to algebra, meanwhile the task (T5) ask students to make generalization about real-life situation to algebraic expression.

Figure 2 Students do operation procedure in algebraic expression
The above problem shows ability students in using algebraic operations on word problems. Students can give an overview of using algebra in certain situations. Students can also choose which operations should be used so that they have a correct calculation. From several examples, student answers above can be concluded students can reach the transition indicator the ability to think arithmetic to abilities algebraic thinking, especially on indicators use the properties of algebraic operations. But students still have difficulties in distinguishing marks operations and number marks, especially inside operate negative numbers. Students, in this case, do not experience problems in distinguishing coefficients, variables, and constants. Based on the results of the interview with students, most students can mention coefficient definitions, variables, and constants correctly. Coefficients are numbers that contain variables from a term in algebraic expression, variables is a symbol of a substitute for a number its value is unknown and symbolized in the form of letters, while constants are terms in algebra that doesn't contain variables in the form of a number.

The most difficult indicator for most students is that recognizing signs of operations and signs of numbers, especially for subtraction and negative number sign. From the analysis of student test results, most students have difficulty in calculating the results of subtraction for negative numbers. This is due to students cannot distinguish and interpret numbers and number operations, so when doing calculations students often make mistake and do the wrong calculation. Sign numbers (+ a or -a) function for determining the type of number that is a positive number or negative number, whereas number operations (a + b or a - b) are operations carried out on two numbers [18-20]. Other task were posted to students, this task asked student to guess the age of people based on the information given. The answer include some basic operation in arithmetic. However, students still need to do some trial and error without looking at the relationship.

![Figure 3. An example task to start algebra](image)

The task is intended to give the students insight about early algebra from arithmetic. The task include some arithmetic operation. However, from the picture above, it is clear that the students could not see the relationship from the information given. He did some trial and error, although it was acceptable but the transition indicators were not appeared. It is expected that the idea of early algebra can be emerged through this sort of activities. Students may find the relationship and the mathematical structures which can be represented in mathematical symbol and algebraic expression. Also, students are expected to give meaning to the mathematical situation with models. They do need some more similar activities to stimulate their algebraic thinking considering their prior knowledge in arithmetic.
In this task, students were also expected to make generalization about real life situation to algebraic expression and solve it using their knowledge.

4. Conclusion
Classroom learning should accommodate algebraic thinking ability of students properly because of algebraic abilities students really need them to understand learning other math topics and also the subject matter in other subjects. Students must be given some situated problems so that they can develop ability to give meaning to algebraic expression and make generalizations.

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