Experimental validation and improvement of actuator line model in the large-eddy simulation of wind-turbine wakes

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Abstract. Wind-turbine wakes significantly affect the power output of downstream wind turbines. In order to improve the calculation accuracy of wind-turbine wakes in the atmospheric boundary layer, a new improved actuator line - large-eddy simulation (AL-LES) method is proposed and verified by experiments. The traditional AL-LES method is improved in three aspects. Firstly, the atmospheric turbulence is generated by a dynamic k-equation LES with a wall shear stress model and a buoyancy effect model. Secondly, the nacelle and tower are modelled based on the static actuator line method. Finally, the distribution of blade body force is improved by using an anisotropic 3D Gaussian function. Based on the results of three wind tunnel experiments conducted by Norwegian University of Science and Technology (NTNU), China Aerodynamics Research and Development Centre (CARDC) and Von Karman Institute for Fluid Dynamics (VKI), the improved AL-LES method is validated from the aspects of wind-turbine power performance, the generation of tip vortex, and the distribution of wake velocity under typical offshore and onshore conditions. In addition, a full-scale wind turbine installed in Gansu is used for the experimental and numerical research. The main conclusion is that compared with the traditional method, the new AL-LES method improves the numerical accuracy by nearly 22%, and can more accurately simulate the interaction between atmospheric turbulence and wind-turbine wakes. The results can help the research of wind-turbine wakes and the micro-location selection of wind farm.

1. Introduction

Atmospheric turbulence plays an important role in the wake development of wind turbines. Factors such as average speed, fluctuation intensity, turbulent kinetic energy, and coherent vortex structures significantly affect the power performance and wake characteristics of full-scale wind turbines. Reynolds average method cannot predict unsteady wake characteristics which plays an important role in predicting the performance of full-scale wind turbines. Therefore, this work uses the AL-LES method, which is a popular numerical method in the current research of wind-turbine wakes. However, Zheng et al. [1] has found that this method has low numerical accuracy under yaw conditions, with an error of up to 31.85% from the experiment results. Therefore, it is important to improve the numerical accuracy of the traditional AL-LES method under yaw conditions. To improve the accuracy of the algorithm, this work improves the traditional AL-LES method in three aspects. Compared with the experimental results of wind tunnel and the field measurement, it is found that the new method proposed in this work can effectively solve the problem of low numerical accuracy of the traditional method under the yaw condition.
2. Numerical methods

A two-bladed full-scale wind turbine is set in the centre of the geometry region. The yaw angle error is set to 10.6°. The aerodynamic information of the airfoil with the angle of attack at -180°~180° of the wind turbine derived from XFOIL software and Viterna-Corrigan model is used to calculate body forces of rotor based on blade element momentum theory. These body forces are transported to grid cells near the blades smoothly by 3D Gaussian functions, which reflect the effects of wind turbine. Combined with ALM, LES is suitable for simulating the wake flow field of wind turbines.

The continuity and Navier–Stokes equations for the incompressible fluid are shown below:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho \mathbf{u}}{\partial x_i} = 0
\]

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \frac{\partial \rho \mathbf{u}}{\partial x_j} \left( \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial x_j} + \nu \frac{\partial^2 \mathbf{u}}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \mathbf{f}
\]

where the terms from right to left in the Navier–Stokes equation represent the forces exerted by the rotor of turbine, subgrid scale stress, viscosity stress, pressure, convective, and transient, respectively.

In order to capture the helical tip and hub wakes in the flow field of the turbine, the subgrid scale stress is modeled by the localized dynamic kinetic energy (ksgs) model, which is proposed by Menon et al [2-4].

The actuator line model of Sørensen et al [5] is used to calculate the body force of turbine on the flow. In this actuator line model, the local relative velocity on the rotor is determined as

\[
V_{rel} = \sqrt{U_0^2 + (\Omega r - V_s)^2}
\]

The angle of attack distributed on the rotor is defined as \(\alpha = \phi - \beta\), where \(\beta\) denotes the pitch angle, and \(\phi = \arctan \left( \frac{U_0}{\Omega r - V_s} \right)\) is the angle between the rotor plane and \(V_{rel}\). For each actuator points on the actuator line, once the angle of attack and the relative local velocity are determined, the body force per spanwise length is given by:

\[
\mathbf{f}_{2D} = \frac{1}{2} \rho V_{rel}^2 c \left( C_L \mathbf{\vec{e}_L} + C_D \mathbf{\vec{e}_D} \right) \cdot \mathbf{F}_{tip}
\]

where \(C_L = C_L(\alpha, \text{Re}_c)\) is the lift coefficient, \(C_D = C_D(\alpha, \text{Re}_c)\) is the drag coefficient, and \(c\) is the chord length. The unit vectors in the lift and drag directions are represented by \(\mathbf{\vec{e}_L}\) and \(\mathbf{\vec{e}_D}\), respectively. \(F_{tip}\) is the tip loss function, given by:

\[
F_{tip} = \frac{2}{\pi} \arccos \left[ \exp \left( -\frac{N_b (R-r)}{2r \sin \phi} \right) \right]
\]

where \(N_b\) is the blade number and \(R\) is the blade tip radius. The body force \(\mathbf{f}\) exerted by the turbine is given by the following 3D Gaussian projection:

\[
\mathbf{f} = \mathbf{f}_{2D} \otimes \eta_c, \quad \eta_c = \frac{1}{\varepsilon^3} \frac{1}{\varepsilon^{3/2}} \exp \left( -\frac{d^2}{\varepsilon^2} \right)
\]

where \(d\) is the local distance, and \(\varepsilon = 2 \Delta r\) [1, 6] controls the Gaussian width. The nacelle is modelled using the static actuator line model, which only considers the contribution of drag to the body force. The body force per unit length, \(f_{nac}\), is:

\[
f_{nac} = \frac{1}{2} \rho V_{nac}^2 C_{D,nac} A_{nac}
\]

where \(V_{nac}\) is the axial velocity in front of the nacelle, \(C_{D,nac}\) is nacelle’s drag coefficient, which is set to 1.2, and \(A_{nac}\) is the projected area in the normal plane of nacelle axis. The anisotropic two-dimensional Gaussian function, \(\eta\) is defined by:
\( \eta(r, \theta, x) = \begin{cases} (\epsilon, \epsilon, \pi)^i \exp \left[ -\left( \frac{(x-x_c)}{\epsilon_c} \right)^2 \right], & r \leq r_{ncx} \\ (\epsilon, \epsilon, \pi)^i \exp \left[ -\left( \frac{(x-x_c)}{\epsilon_c} \right)^2 - \left( \frac{(r-r_{ncx})}{\epsilon_c^2} \right)^2 \right], & r > r_{ncx} \end{cases} \) \tag{8}

where \( x, r, \) and \( \theta \) are the axial, radial, azimuthal cylindrical coordinate directions, respectively, centered upon the nacelle axis.

3. Validation

In order to verify the simulation accuracy of the tip vortex wake calculated by the ALM, the PIV experiment of CARDC was used as a comparison, and the results are shown in Figure 1. After introducing the blade body force model, the streamwise positions of the tip vortex are consistent with the PIV experiment. Under the condition of experimental tip speed ratio, the tip vortex showed a linear expansion trend.

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Figure 1. Comparison of improved ALM results and CARDC experiment data
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Figure 2 shows the velocity distribution of the NTNU wind turbine at different streamwise locations \((x/D = 1, 3)\) after the rotor. Influenced by the tower and nacelle, the velocity field below the hub height is slightly lower than the velocity field upper the hub height. After introducing the nacelle effect model and tower effect model, the numerical results are in agreement with the results of experiment and immersed boundary method.

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Figure 2. Comparison of improved ALM results and NTNU experiment data.
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Figure 3 shows the comparison of the numerical results with the experimental results obtained in the VKI atmospheric boundary layer (ABL) wind tunnel. The experiment was carried out in the ABL wind tunnel, so the influencing factors such as the atmospheric characteristics, the tower and nacelle were considered in the experiment. Therefore, not only the effect of nacelle and tower should be introduced in the simulation, but also the atmospheric turbulence generation method should be introduced to improve the ALM. It can be seen from the comparison between the numerical results of the wake field
and the experimental values, that the numerical method used in this work can accurately generate the atmospheric turbulence that satisfies the target atmospheric condition. The numerical results of the velocity field in the wake are also consistent with the experimental results.

![Figure 3. Comparison of improved ALM results and VKI experiment data.](image)

4. Results and Discussion

A wake measurement was carried out in Gansu, China to verify the accuracy of the simulation method. The wake measurement instrument consists of 3 CSAT3 three-dimensional ultrasonic anemometers. The sampling frequency is 20 Hz. The initial grid points in the three directions of the computational domain are 130, 40, and 40, respectively, and then the mesh is refined by 5 layers, as shown in the Figure 4. After mesh refinement, the mesh step size of the nacelle region is 0.0469 m. Finally, the total number of grids is about 15 million.

![Figure 4. Computational domain and mesh conditions.](image)

Figure 5 shows the time history of the TKE and velocity at the three monitoring points during the sampling time of 100 s after reaching the quasi-equilibrium state. The dotted line indicates the experimental value, and the solid line indicates the numerical result. During the sampling time, the average velocity, and average turbulent kinetic energy (TKE) fall within 8% error range of the experimental value. In the work of Zheng et al. [1], the numerical error of the traditional AL-LES method is about 31.85%. The new AL-LES method proposed in this work improves the numerical accuracy by 22%.
Figure 5. Comparison of improved ALM results and the field experiment data.

Figure 6 shows the turbulent kinetic energy distribution at different streamwise positions. Before the position of $x/D=4$, there is no significant interaction between the tip vortex and the nacelle vortex. The turbine wake shows a linear expansion trend, and the TKE is dispersed into the surrounding flow field. In the wake transition zone ($x/D=4$–7), the tip vortex begins to oscillate due to the wake dispersion of nacelle wake, and the momentum exchange between wake and outflow is enhanced. In the far wake region (after $x/D=7$), the TKE distribution is relatively uniform, and the wake velocity gradually recovers.

Figure 6. Distribution of turbulent kinetic energy at different streamwise positions ($x/D=1$, 4, 7, 10).

5. Results and Discussion
Due to the large numerical error (about 31.85%) of the traditional AL-LES method under yaw conditions, this work modifies the tradition method from three aspects including the generation of atmospheric turbulence, the nacelle and tower model, and the body force distribution model. Then, the numerical accuracy of the new improved AL-LES method is validated against three wind tunnel experiments.
conducted in VKI, NTNU and VKI, respectively. Finally, a full-scale 33 kW wind turbine installed in Gansu is studied, and it is found that the new method improves the numerical accuracy by nearly 22%. The new AL-LES method greatly improves the numerical accuracy of the wind-turbine wakes, and plays an important role in micro-location selection of wind farms.

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