Triangle singularities in the $T^+_{cc} \to D^+ D^0 \to \pi^+ D^0 D^0$ decay width

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The values of the masses of the particles involved in the decay of $T^+_{cc} \to D^+ D^0 \to \pi^+ D^0 D^0$ suggest that due to the final state interactions in the transition vertex $T^+_{cc} \to D^+ D^0$ there may be triangle logarithmic singularities. We discuss their possible role and show that the tree approximation for calculating the decay widths $T^+_{cc} \to (D^+ D^0 + D^{*0} D^+) \to \pi^+ D^0 D^0, \pi^0 D^0 D^+, \gamma D^0 D^+$ is quite sufficient at the current level of measurement accuracy.

I. INTRODUCTION

At the end of July 2021, the LHCb Collaboration announced the discovery of the doubly charmed tetraquark $T^+_{cc}$ \cite{ref1, ref2} and then published detailed measurement results along with their theoretical processing and interpretation \cite{ref3, ref4}. Over the next days, weeks and months, a very interesting discussion is going on in the literature about the possible internal structure of the $T^+_{cc}$ state, about the mechanisms of its production and decay through intermediate states $D^+ D^0$ and $D^{*0} D^+$, on the possible values of its total decay width and partial decay widths into coupled channels $\pi^+ D^0 D^0, \pi^0 D^0 D^+, \gamma D^0 D^+$, about its line shape and the shapes of the two-particle mass spectra $DD$ and $\pi D$, as well as about the possible existence of other similar states. A detailed discussion of all these issues can be found in \cite{ref5, ref6}; see also the references cited therein.

In Sec. II of this article, we discuss the possible role of triangle singularities in the width of the decay $T^+_{cc}[3.875; I(J^P) = 0(1^+)] \to D^+(J^P = 1^-)D^0(J^P = 0^-) \to \pi^+ D^0 D^0$. In so doing, we proceed within the framework of a scalar model, i.e., we treat all particles in this decay as spinless and scalar. Such a simplification, however, seems quite reasonable. First, the decay of $T^+_{cc} \to D^+ D^0$ occurs in the near-threshold region and therefore is mainly S wave. Second, the ratio of the $D^+ \to (D \pi)$ decay width (it is $\approx 83$ keV \cite{ref18}) to the distance to the $(D \pi)^+$ threshold is $\approx 1/70$ and, consequently, the change of this width on the energy interval of the order of itself is small (i.e., in the $D^+$(2010) region, it is almost constant). The formulas [see below Eqs. \cite{ref13} and \cite{ref14}], which we use to estimate the possible role of interactions of $D^* D$ pairs in the final state, are in fact expansions of the Omn`es functions (solutions) for form factors \cite{ref15} in case of weak coupling (i.e., smallness of $D^* D$ scattering at low energies). Discussions of a number of dynamic approximations of the Omn`es functions can be found, for example, in Refs. \cite{ref20, ref21}. The performed analysis allows us to conclude that the tree approximation used in Refs. \cite{ref3, ref4} for calculating the decay widths $T^+_{cc} \to (D^+ D^0 + D^{*0} D^+) \to \pi^+ D^0 D^0, \pi^0 D^0 D^+, \gamma D^0 D^+$ is quite sufficient at the current level of measurement accuracy.

The LHCb Collaboration results \cite{ref3, ref4} obtained from the fit to the $\pi^+ D^0 D^0$ mass spectrum indicates that the Breit-Wigner mass of the $T^+_{cc}$ relative to the $D^+ D^0$ mass threshold $\delta m_{BW} = -273 \pm 61$ keV and its Breit-Wigner width $\Gamma_{BW} = 410 \pm 165$ keV (only statistical uncertainties are indicated here). The measured $\delta m_{BW}$ value corresponds to a mass of approximately 3875 MeV.

II. $T^+_{cc} \to D^+ D^0 \to \pi^+ D^0 D^0$ DECAY IN THE SCALAR MODEL

In the tree approximation, the $T^+_{cc} \to D^+ D^0 \to \pi^+ D^0 D^0$ decay is described by two diagrams shown in Fig. II which differ in the permutation of identical $D^0$ mesons. The corresponding decay width is given by

$$
\Gamma_{T^+_{cc} \to D^+ D^0 \to \pi^+ D^0 D^0}(s_1) = \frac{g^2_{T^+_{cc} D^+ D^0} g^2_{D^+ D^0 \pi^+ D^0}}{16 \pi} \frac{1}{16 \pi} \frac{1}{2 \pi s_1^{3/2}} \int ds \int dt \frac{1}{m_{D^0}^2 + m_{s_1}^2} \left( \frac{1}{D(s)} + \frac{1}{D(t)} \right)^2,
$$

where $s_1$ is the invariant mass squared of the virtual $T^+_{cc}$ state, $s$ and $t$ are the $\pi^+ D^0(1)$ and $\pi^+ D^0(2)$ invariant mass squared, respectively, and $t_{\pm}(s_1, s)$ denote the boundaries of the physical region for the variable $t$ for fixed values

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a similar growth of the real part of the self-energy function under the sharp increase of $\Gamma_T$ calculated in the tree approximation using Eqs. (1) and (2) is shown in Fig. 2. Note that for large values of $T$ a sharp suppression of the right and left wings of the resonance peak. A similar phenomenon takes place for the four-quark $a_0(980)$ resonance [22].
The values of the masses of the particles involved in the decay of $T_{cc}^+ \to D^{*+}D^0 \to \pi^+D^0D^0$ suggest that due to the final state interactions in the vertex transition $T_{cc}^+ \to D^{*+}D^0$ there may be triangle logarithmic singularities. Examples of corresponding “dangerous” diagrams are shown in Fig. 3. Take into account their contribution to the decay width $\Gamma_{T_{cc}^+\to D^{*+}D^0\to \pi^+D^0D^0}(s_1)$ can be done by the following substitutions in Eq. (1):

$$\frac{1}{D(s)} \to \frac{1}{D(s)} \left[ 1 + \frac{g_{D^*+D^0}^2}{16\pi} \left( F_{D^*+D^0}(s_1,s) + \frac{1}{2} F_{D^*oD^+}(s_1,s) \right) \right], \quad (3)$$

$$\frac{1}{D(t)} \to \frac{1}{D(t)} \left[ 1 + \frac{g_{D^*+D^0}^2}{16\pi} \left( F_{D^*+D^0}(s_1,t) + \frac{1}{2} F_{D^*oD^+}(s_1,t) \right) \right], \quad (4)$$

where $F_{D^*+D^0}$ and $F_{D^*oD^+}$ are the amplitudes of the triangle loops included in the diagrams in Fig. 3, the factor 1/2 at $F_{D^*oD^+}$ follows from isotopic symmetry. In our normalization

$$F_{D^*+D^0}(s_1,s) = \frac{i}{\pi^2} \int \frac{d^4k}{D_1D_2D_3}, \quad (5)$$

where $D_1 = k^2 - m^2_{D^*+} + i\varepsilon$, $D_2 = (p_1 - k)^2 - m^2_{D^0} + i\varepsilon$, and $D_3 = (k - p_3)^2 - m^2_{D^+} + i\varepsilon$ are the inverse propagators of the particles in the loop. Here and in Fig. 3, $p_1, p_2, p_3$ denote the momenta of the particles in the reaction; $p_1 = p_2 + p_3$, $p_1^2 = s_1$, $p_2^2 = s$, and $p_3^2 = m^2_{D^0}$. The amplitude $F_{D^*oD^+}(s_1,s)$ has a similar form.

If the values of the variables $\sqrt{s}$ and $\sqrt{s_1}$ are simultaneously in the intervals

$$m_1 + m_2 < \sqrt{s_1} < \sqrt{m_1^2 + m_2^2 + m_2m_3 + (m_2/m_3)(m_1^2 - m_2^2_D)}, \quad (6)$$

$$m_2 + m_3 < \sqrt{s} < \sqrt{(m_1 + m_2)(m_2m_3 + m_2^2_D)} / m_1, \quad (7)$$

where $m_1, m_2,$ and $m_3$ are the particle masses in the inverse propagators $D_1, D_2, \text{ and } D_3$, respectively [see Eq. (5)], then for each value of $s_1$ there is a unique value $s_1$ (and vice versa) for which the imaginary part of the amplitude $F_{D^*+D^0}(s_1,s)$ [and similarly that of $F_{D^*oD^+}(s_1,s)$] has a logarithmic singularity (see, for example, Refs. [24][27] and references herein). Note that the minimum value of $s_1$ in (6) herewith corresponds to the maximum value of $s$ in (7) (and vice versa). In the amplitude $F_{D^*+D^0}(s_1,s)$ (see the left diagram in Fig. 3) $m_1 = m_{D^{**}}, m_2 = m_{D^0}, m_3 = m_{\pi^+}$ and for it the numerical values of the intervals in Eqs. (6) and (7) are as follows:

$$3.8751 \text{ GeV} < \sqrt{s_1} < 3.91259 \text{ GeV} \quad (0 < \sqrt{s_1} - (m_{D^{**}} + m_{D^0}) < 37.49 \text{ MeV}), \quad (8)$$

$$2.00441 \text{ GeV} < \sqrt{s} < 2.00946 \text{ GeV} \quad (0 < \sqrt{s} - (m_{D^0} + m_{\pi^+}) < 5.05 \text{ MeV}). \quad (9)$$

Here in parentheses are the corresponding intervals in units of MeV into which the invariant masses $\sqrt{s_1}$ and $\sqrt{s}$ with the subtracted threshold values should fall. In the amplitude $F_{D^*oD^+}(s_1,s)$ (see the right diagram in Fig. 3) $m_1 = m_{D^{*o}}, m_2 = m_{D^+}, m_3 = m_{\pi^0}$ and for it, respectively, we have

$$3.8765 \text{ GeV} < \sqrt{s_1} < 3.92318 \text{ GeV} \quad (0 < \sqrt{s_1} - (m_{D^{*o}} + m_{D^+}) < 46.67 \text{ MeV}), \quad (10)$$

$$2.00464 \text{ GeV} < \sqrt{s} < 2.01073 \text{ GeV} \quad (0 < \sqrt{s} - (m_{D^+} + m_{\pi^0}) < 6.05 \text{ MeV}). \quad (11)$$

Since $m_{D^{**}} = 2.01026 \text{ GeV}$ is greater than $(\sqrt{s})_{\text{max}} = 2.00946 \text{ GeV}$ in Eq. (8), then due to the contribution of the amplitude $F_{D^*+D^0}(s_1,s) \approx m^2_{D^{*+}}$ in the vertex $T_{cc}^+D^{*+}D^0$ no triangle singularity arises. At the same time, $m_{D^{**}} =
compared to its values in the tree approximation (see Fig. 2) by about 5%, 6%, 2%, and 0.6% at $\sqrt{s} = m_{D^{*+}}$.

We hope to consider spin effects somewhere else.

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