Scale Invariance in a Non-Abelian Chern-Simons-Matter Model

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Abstract

The general method of reduction in the number of coupling parameters is applied to a Chern-Simons-matter model with several independent couplings. We claim that considering the asymptotic region, and expressing all dimensionless coupling parameters as functions of the Chern-Simons coupling, it is possible to show that all β-functions vanish to any order of the perturbative series. Therefore, the model is asymptotically scale invariant.

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1 Introduction

It is well-known that quantum field theories are deeply connected to the presence of infinities (apparently insurmountable). Although the renormalization program can overcome this problem
in a mathematically proper way, there exists a general feeling to look for field models which are not plagued by infinities, and therefore allowing us the possibility of getting non-perturbative results more easily. A natural candidate field theory of interest are the so-called topological field theories.

Topological field theories\(^1\) are a class of gauge models interesting from a physical point of view. In particular, their observables are of topological nature. An important topological model that has received much attention in the last years is the Chern-Simons model in three dimensions \(^2\). In contrast to the usual Yang-Mills gauge theories, Chern-Simons theories, which include the case of three-dimensional gravity \(^3\), have some remarkable features. The main property of this class of models lies on a very interesting perturbative feature, namely, its ultraviolet finiteness \(^4\). A complete and rigorous proof of the latter property has been given in \(^5\) for \(D = 1+2\) Chern-Simons theories in the Landau gauge, and for the Chern-Simons-Yang-Mills theory in \(^6\). In the more general case where these models are coupled to matter fields, the Chern-Simons coupling constant keeps unrenormalized, the corresponding \(\beta\)-function being trivial. This has been rigorously proved in \(^7\) for the Abelian Chern-Simons theory coupled to scalar matter fields.

At the same time, for the non-Abelian theory coupled to general scalar and spinorial matter fields, an argument based on the assumed existence of an invariant regularization has been proposed by the authors of ref. \(^8\) to demonstrate the triviality of the Chern-Simons coupling. In \(^9\), a rigorous proof has been given for the vanishing of the Chern-Simons \(\beta\)-function in the presence of general scalar and spinorial matter, where one avoids the necessity of invoking a particular regularization procedure, by using the “algebraic” method of renormalization \(^10\), which only relies on general theorems of renormalization theory. The proof relies on the use of a local, non-integrated version of the Callan-Symanzik equation \(^11\).

The scale invariance of non-Abelian Chern-Simons minimally coupled to matter has already been discussed, at the two-loop level, in the paper of ref. \(^12\). It is the main goal of our present work to extend this result. Indeed, we include matter with minimal gauge coupling, but we also adjoin to the action matter-matter interaction terms that respect the power-counting renormalizability. With this rather complete approach, we investigate the possibility that such a model be completely finite, i.e. its self-coupling parameters are not renormalized, too. The idea is to use the technique of reduction of coupling parameters \(^13\), \(^14\) to unify all couplings. The nonrenormalization of the matter self-couplings would then follow from the nonrenormalization of the Chern-Simons coupling. Therefore, we are able to show the all-order asymptotic scale invariance of the model, without resorting to laborious diagrammatical analysis. By asymptotic scale invariance to all orders, we mean the absence of Callan-Symanzik coupling constants renormalization, to every order of perturbation theory. Anomalous dimensions are however allowed to be different from zero. The latter, corresponding to field redefinitions, are physically trivial and hence vanish on the mass-shell \(^15\).

\(^1\)See \(^1\) for a general review and references.

\(^2\)
The outline of this letter is as follows. In the following, we display the main purpose of this work: considering a non-Abelian Chern-Simons-matter model in the asymptotic region, using the basic construction of the reduction of coupling parameters [13, 14], we shall show that expressing all dimensionless coupling parameters as functions of a single parameter, in our case the Chern-Simons coupling, then all $\beta$-functions vanish to any order of perturbation theory. Therefore, the model is asymptotically scale invariant. In section 3, we draw our conclusions.

2 Criterion for the Scale Invariance

As mentioned in the Introduction, it has been showed in [9] that for the model (in the Landau gauge)

$$\Sigma = \int d^3x \left\{ \kappa \varepsilon^{\mu\nu\rho} (A_\mu^a \partial_\nu A_\rho^a + \frac{1}{3} \delta_{abc} A_\mu^a A_\nu^b A_\rho^c) + (i \bar{\Psi}_j \gamma^\mu D_\mu \Psi_j + \frac{1}{2} D_\mu \varphi_i^* D^\mu \varphi_i - \mathcal{V}(\varphi, \Psi)) \right. $$

$$+ (\partial_\mu b^a A_\mu^a + \partial_\mu c^a D_\mu^a c^a) + \sum_{\Phi=A_{\mu}^a, c^a, \Psi_j, \varphi_i} \Phi^* s \Phi \bigg\} ,$$

(2.1)

the Chern-Simons coupling keeps unrenormalized to all orders. Here, the gauge field $A_{\mu}^a(x)$ lies in the adjoint representation of the gauge group $SU(N)$, with Lie algebra $[X_a, X_b] = if_{abc}X_c$. The scalar matter fields $\varphi_i(x)$ and the spinor matter fields $\Psi_j(x)$ are in the fundamental representation of $SU(N)$, the generators being represented by the matrices $T_a^\varphi$ and $T_a^\psi$, respectively. $c^a$, $\bar{c}^a$ and $b^a$ are the ghost, the antighost and the Lagrange multiplier fields, respectively. $A_{\mu}^a$, $c^a$, $\Psi_j^\dagger$, $\varphi_i^\dagger$ are the “antifields” coupled to the nonlinear variations under BRS transformations. The generalized covariant derivative is defined by

$$D_\mu \Phi(x) = (\partial_\mu - i A_{\mu}^a(x)T_a^\Phi)\Phi(x) .$$

(2.2)

The function $\mathcal{V}(\varphi, \Psi)$ defines the self-interactions of the matter fields and their masses:

$$\mathcal{V}(\varphi, \Psi) = \frac{1}{2} \lambda_1 \bar{\Psi}_j \gamma^\mu D_\mu \Psi_j \varphi_k^\dagger \varphi_k + \frac{1}{2} \lambda_2 \bar{\Psi}_j \gamma^\mu \gamma^\nu \Psi_k^\dagger \varphi_j \varphi_j^\dagger + \frac{1}{6} \lambda_3 (\varphi_i^\dagger \varphi_i)^3$$

$$+ \text{mass terms} + \text{dimensionful couplings} .$$

(2.3)

We refer to [9] for more details.

We are interested in showing that, under a particular circumstance, the model (2.1) may be scale invariant. Scale invariance means here the vanishing of the Callan-Symanzik $\beta$-functions. We retain the possibility of wave function renormalization, so that the anomalous dimensions are allowed to be different from zero. It means the scale invariance of physical quantities still holds since the anomalous dimensions, corresponding to field redefinitions, are physically trivial.

Let us next outline the criterion for the model (2.1) to be scale invariant. The starting point for our analysis is the Callan-Symanzik equation for the model (2.1) [9]

$$(D + \beta_\kappa \partial_\kappa + \beta_{\lambda_1} \partial_{\lambda_1} + \beta_{\lambda_2} \partial_{\lambda_2} + \beta_{\lambda_3} \partial_{\lambda_3} - \gamma_A N_A - \gamma_\Psi N_\Psi - \gamma_\varphi N_\varphi) \Gamma \sim 0 ,$$

(2.4)
where

$$D\Gamma = \sum_{\text{all dimensionful parameters } \mu} \mu \frac{\partial \Gamma}{\partial \mu}.$$ 

The signal \( \sim \) means equality up to mass terms and dimensionful couplings. Note that, for a moment, we are considering that \( \beta_\kappa \) is not vanishing.

In (2.1) the independent dimensionless parameters are \( \kappa, \lambda_1, \lambda_2, \lambda_3 \). We call \( \kappa \) the primary coupling. Demanding that the coupling constants \( \lambda_1, \lambda_2, \lambda_3 \) have to be functions of the coupling constant \( \kappa \):

$$\lambda_r = \kappa f_r(\kappa) \quad r = 1, 2, 3,$$  \hspace{1em} (2.5)

where

$$f_r(\kappa) = f_r^{(0)} + \sum_{m=1}^{\infty} \chi_r^{(m)} \kappa^m,$$  \hspace{1em} (2.6)

is a formal power series solution (special solution of ref. [14]), then any relation among couplings of this type can be expressed by reduction equations [13, 14]

$$\beta_\lambda_r = \beta_\kappa \left( \kappa \frac{df_r}{d\kappa} + f_r \right).$$  \hspace{1em} (2.7)

Since \( \beta_\kappa \) is identically zero, then (2.7) implies that \( \beta_\lambda_r = 0 \) to all orders of perturbation theory. The reduction equations (2.7), in general, are necessary and sufficient conditions insuring that the renormalizable model (2.1), with several independent couplings, can be reduced to a renormalizable model where all couplings are functions which depend only upon the Chern-Simons coupling.

For the sake of simplicity, we shall consider the action (2.1) based on group the \( SU(2) \). We get the following \( \beta \)-functions [17]:

$$\beta_\kappa = 0,$$

$$\beta_{\lambda_1} = \frac{9}{16} \kappa^3 - \frac{9}{4} \lambda_2 \kappa^2 - \frac{39}{8} \lambda_1 \kappa^2 + \frac{1}{4} \lambda_1 \kappa - \frac{1}{4} \lambda_2 \kappa^2 + \frac{1}{3} \lambda_1 \lambda_2 \kappa + \frac{1}{2} \lambda_1 \lambda_3 \kappa^2 + \frac{16}{3} \lambda_1 \lambda_2 \kappa + \frac{22}{3} \lambda_1 \kappa$$

$$\beta_{\lambda_2} = -\frac{3}{8} \lambda_2 \kappa^2 + \lambda_2 \kappa^2 + 2 \lambda_1 \lambda_2 \kappa + \frac{7}{3} \lambda_3 \kappa^2 + \frac{34}{3} \lambda_1 \lambda_2 \kappa + \frac{34}{3} \lambda_1 \lambda_3 \kappa,$$

$$\beta_{\lambda_3} = \frac{3}{64} \kappa^4 + 204 \lambda_3 \kappa^2 - \frac{27}{2} \lambda_3 \kappa^2 + (32 \lambda_1^2 + 32 \lambda_1 \lambda_2 + 20 \lambda_2) \lambda_3 - (2 \lambda_1 \lambda_2^2 + \lambda_2^3) \kappa +$$

$$+ \frac{1}{4} (6 \lambda_1^2 + 6 \lambda_1 \lambda_2^2 + \lambda_2^3) \kappa^2 + \frac{1}{32} (24 \lambda_1 + 12 \lambda_2) \kappa^3 - 8 \lambda_1^2 + 16 \lambda_2^2 - 28 \lambda_1 \lambda_2 \kappa^2 -$$

$$- 20 \lambda_1 \lambda_2 \kappa - 5 \lambda_2^2 \kappa^2.$$  \hspace{1em} (2.8)

A few comments about the \( \beta \)-functions which we are using are now in order. In fact, in [17] they represent the \( \beta \)-functions of the renormalization group, and were evaluated by using the
minimal subtraction (MS) scheme. The \( \beta \)-functions occurring in the Callan-Symanzik equation (which describes the breaking of dilatations) are mass-independent, while those of the renormalization group (which describes the invariance of the model under variations of the normalization point) will in general depend on the mass ratios. However, a major progress in the study of Callan-Symanzik and renormalization group equations was initiated by E. Kraus [18]. In her work, Kraus has observed that for an asymptotic normalization point (where mass effects are neglected) the \( \beta \)-functions and the anomalous dimensions of the Callan-Symanzik and renormalization group equations are the same to all orders of perturbation theory, and mass-independent if a regularization scheme like MS is used.

In practice, we usually know the \( \beta \)-functions only as asymptotic expansions in the small coupling limit. Within this framework, it is convenient to introduce the notation [13]:

\[
\beta_{\lambda_i}(\kappa, \lambda_r) = C^{(0)}_r \kappa^3 + C^{(0)}_{r_1} \kappa^2 \lambda_1 + C^{(0)}_{r_1 r_2} \kappa \lambda_1 \lambda_2 + C^{(0)}_{r_1 r_2 r_3} \lambda_1 \lambda_2 \lambda_3 +
+ \sum_{n=1}^{\infty} \sum_{m=0}^{n} C^{(n-m)}_{r_1 \ldots r_m} \lambda_{r_1} \ldots \lambda_{r_m} \kappa^{n-m},
\]

(2.9)

where \( r, r_1, \ldots, r_m = 1, 2 \), and

\[
\beta_{\lambda_3}(\kappa, \lambda_r) = C^{(0)}_3 \kappa^4 + C^{(0)}_{3, r_1} \kappa^3 \lambda_1 + C^{(0)}_{3, r_1 r_2} \kappa^2 \lambda_1 \lambda_2 + C^{(0)}_{3, r_1 r_2 r_3} \kappa \lambda_1 \lambda_2 \lambda_3 +
+ C^{(0)}_{3, r_1 r_2 r_3 r_4} \lambda_1 \lambda_2 \lambda_3 \lambda_4 + \sum_{n=5}^{\infty} \sum_{m=0}^{n} C^{(n-4)}_{3, r_1 \ldots r_m} \lambda_{r_1} \ldots \lambda_{r_m} \kappa^{n-m},
\]

(2.10)

where \( r_1, \ldots, r_m = 1, 2, 3 \).

Now, replacing the expansions (2.9), (2.10) and (2.6) into the reduction equations (2.7), one finds that \( f_r^{(0)} \), to the lowest order, must be a solution of the equations

\[
H_r(f^{(0)}) = C^{(0)}_r + C^{(0)}_{r, r_1} f^{(0)}_{r_1} + C^{(0)}_{r, r_1 r_2} f^{(0)}_{r_1 r_2} + C^{(0)}_{r, r_1 r_2 r_3} f^{(0)}_{r_1 r_2 r_3} f^{(0)}_{r_1} f^{(0)}_{r_2} f^{(0)}_{r_3} = 0,
\]

(2.11)

where \( r, r_1, \ldots, r_m = 1, 2 \), and

\[
H_3(f^{(0)}) = C^{(0)}_3 + C^{(0)}_{3, r_1} f^{(0)}_{r_1} + C^{(0)}_{3, r_1 r_2} f^{(0)}_{r_1 r_2} + C^{(0)}_{3, r_1 r_2 r_3} f^{(0)}_{r_1 r_2 r_3} f^{(0)}_{r_1} f^{(0)}_{r_2} f^{(0)}_{r_3} +
+ C^{(0)}_{3, r_1 r_2 r_3 r_4} f^{(0)}_{r_1 r_2 r_3} f^{(0)}_{r_1} f^{(0)}_{r_2} f^{(0)}_{r_3} = 0,
\]

(2.12)

where \( r_1, \ldots, r_m = 1, 2, 3 \).

Since we wish the Lagrangian to be bounded from below, we take into account only sets of real solutions which guarantee that the coefficient of the \((\varphi^* \varphi)^3\)-term be positive. Therefore,
the reduction equations (2.7) have the solutions:

(I) \[ f_r^{(0)} \] = −0.82524 \[ f_r^{(0)} \] = 0.91585 \[ f_r^{(0)} \] = 0.092398 ,

(II) \[ f_r^{(0)} \] = 0.09062 \[ f_r^{(0)} \] = −0.91585 \[ f_r^{(0)} \] = 0.092398 .

Given a solution \( f_r^{(0)} \) of the equations (2.11) and (2.12), we obtain for the expansion coefficients \( \chi^{(m)} \) the relations

\[ M_{rr'}(f^{(0)})\chi^{(m)}_{r'} = \vartheta_r . \]  

(2.13)

\( \vartheta_r \) depends only on the coefficients \( \chi^{(p)} \), with \( p \leq m - 1 \) and on the \( \beta \)-function coefficients, evaluated em \( f_r^{(0)} \), for order \( \leq m - 1 \). The matrix \( M \) depends only \( f^{(0)} \) and is given by

\[ M_{rr'}(f^{(0)}) = \frac{\partial H_r}{\partial f_r} \bigg|_{f_r=f_r^{(0)}} . \]  

(2.14)

The lowest-order criterium to insure that all coefficients \( \chi^{(m)} \) in (2.6) are fixed is given by:

\[ \det M_{rr'}(f^{(0)}) \neq 0 . \]  

(2.15)

With the coefficients \( \chi^{(m)} \) fixed, one can use, then, the reparametrization invariance of the theory in order to bring the \( \lambda_1, \lambda_2, \lambda_3 \) couplings into a simple form:

\[ \lambda_r = \kappa f_r^{(0)} \quad r = 1, 2, 3 . \]  

(2.16)

Our matrices are:

\[ M_{(I)} = \begin{bmatrix} 17,153 & 8,2733 & 0 \\ -22,034 & 7,3282 & 0 \\ -4,0109 & -1,6834 & -8,2072 \end{bmatrix} , \quad \det M_{(I)} \neq 0 , \]

and

\[ M_{(II)} = \begin{bmatrix} 16,779 & 7,3574 & 0 \\ -6,4852 & 3,6648 & 0 \\ -6,7584 & -2,5993 & -8,2072 \end{bmatrix} , \quad \det M_{(II)} \neq 0 . \]

Hence, both solutions are uniquely determined to all orders.

To end up, we would like to emphasize that our result could indicate to even stronger phenomenon: the possibility of getting the asymptotical conformal invariance. The correspondent effect in Abelian Chern-Simons-matter model has been discovered in [20]. We hope to report our conclusions on this in a short time.
3 Conclusions

To summarize, we conclude that the general method of reduction in the number of coupling parameters, when applied to the non-Abelian Chern-Simons-matter model with several independent couplings as considered here, allows us (taking into account the asymptotic region) to show that all $\beta$-functions vanish to any order of the perturbative series. Therefore, the model is asymptotically scale invariant. In forthcoming papers, we intend to extend this analysis to other types of topological theories, e.g. the BF model [1, 21], or its extension, recently proposed, the BFK model [22] coupled with matter, too. Our aim is to obtain results valid to all orders of perturbation theory and to classify the theories which can be made finite by means of the technique of reduction of couplings.

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