MASS SCALES AND STABILITY OF THE PROTON IN 
$[SU(6)]^3 \times Z_3$

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ABSTRACT

We proof that the proton is stable in the left-right symmetric gauge model \([SU(6)]^3 \times Z_3\), which unifies nongravitational forces with flavors, broken spontaneously by a minimal set of Higgs Fields and Vacuum Expectation Values down to \(SU(3)_c \otimes U(1)_{EM}\). We also compute the evolution of the gauge coupling constants and show how agreement with precision data can be obtained.
1 Introduction

Recently, we have proposed [1, 2] a grand unification model (GUM) of forces and flavors based on the gauge group $G = [SU(6)]^3 \times Z_3$. Our aim has been to provide some clues for the explanation of the intriguing fermion mass spectrum and mixing parameters (since $G$ includes the so called horizontal interactions, it has predictions for the masses of some elementary fermions). Comparing our model with related attempts made in the context of GUMs with simple groups without discrete symmetries (such as [3] SO(18) and [4] $E_8$), we find it simpler, more economical and more elegant than their competitors. Not only the model described in [1, 2] does not contain mirror fermion fields, but it has fewer gauge fields, fermion fields and Higgs fields than the other two mentioned models.

As it is clear from Refs. [1] and [2], the fermion content of this model includes in a single irreducible representation of $G$ the three families of known fermions, each family being defined by the dynamics of the $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)'_{Y(B-L)}$ gauge group. This last group is the left-right symmetric (LRS) extension of the $SU(3)_c \otimes SU(2)_L \otimes U(1)'_Y$ standard model (SM).

The rest of this communication is as follows: In section two we review the model, in section three we do the renormalization group equation analysis, and in section four we demonstrate the stability of the proton in the context of our model.

2 The model

The gauge group is given by $G = SU(6)_L \otimes SU(6)_C \otimes SU(6)_R \times Z_3$ [1]. $SU(6)_C$ is a vector like group which includes three hadronic and three leptonic colors. $SU(6)_C$ includes as a subgroup the $SU(3)_c \otimes U(1)'_{Y(B-L)}$ of the LRS model. $SU(6)_L \otimes SU(6)_R$ is the left-right symmetric flavor group which includes the $SU(2)_L \otimes SU(2)_R$ gauge group of the LRS model, and $SU(3)_{HL} \otimes SU(3)_{HR}$ is the horizontal gauge group in $G$.

2.1 The Gauge and Fermion Fields

The 105 gauge fields (GF) and the 108 Weyl Fermions in $G$ are explicitly depicted in Ref. [1, 2]. Let us describe here some of them:

The 105 GF can be divided in two sets, 70 of them belonging to $SU(6)_L \otimes SU(6)_R$ and 35 associated with $SU(6)_C$. The first set includes $W_{L}^{\pm}$ and $Z_{L}^{0}$, the GF of the known weak interactions, plus the GF associated with the postulated right weak
interaction, plus the GF of the horizontal interactions, etc. All of them are SU(3)_c singlets and have electrical charges 0 or ±1. The second set includes the 8 gluon fields of SU(3)_c; nine leptoquark GF, X_i, Y_i, and Z_i, i = 1, 2, 3 with electrical charges −2/3, 1/3, and −2/3 respectively; other 9 leptoquark GF charge conjugated to the previous ones; six diquark GF, P^±_a, P^0 and P^−_a, a = 1, 2, with electrical charges as indicated, and three GF associated with diagonal generators in SU(6)_C, where By_{(B−L)} the GF associated with the U(1)_{Y(B−L)} factor in the LRS model is one of them.

The ordinary (known) fermions for the model are included in
\[
\psi(108)_L = Z_3 \psi(6,1,6) \equiv \psi(6,1,6)_L + \psi(1,6,6)_L + \psi(6,6,1)_L, \text{ with the following particle content:}
\]

\[
\psi(6,6,1)_L = \left( \begin{array}{cccc}
        d_x^{-1/3} & d_y^{-1/3} & d_z^{-1/3} & E_1^- \\
        u_x^{2/3} & u_y^{2/3} & u_z^{2/3} & E_0 \\
        s_x^{-1/3} & s_y^{-1/3} & s_z^{-1/3} & E_2^- \\
        c_x^{2/3} & c_y^{2/3} & c_z^{2/3} & E_2^0 \\
        b_x^{-1/3} & b_y^{-1/3} & b_z^{-1/3} & E_3^- \\
        t_x^{2/3} & t_y^{2/3} & t_z^{2/3} & E_3^0 \\
\end{array} \right)_{L} \equiv \psi_\alpha^A, \quad (1)
\]

where the rows (columns) represent color (flavor) degrees of freedom; E_i^{−0}, L_i^{+0}, and T_i^{−0} i = 1, 2, 3 stand for leptonic fields with electrical charges as indicated, and d,u,s,c,b,and t stand for the corresponding quark fields eigenstates of G, but not mass eigenstates \[\] the subindices x, y, z in the quark fields refers to SU(3)_c color indices. \(\psi(1,6,6)_L \equiv \psi_\alpha^A\) includes the 36 fields charge conjugated to the fields in \(\psi(6,6,1)_L\), and \(\psi(6,1,6)_L \equiv \psi_\alpha^A\) represents 36 exotic Weyl leptons, 9 with positive electric charges, 9 with negative (the charge conjugated to the positive ones) and 18 are neutrals. As it is clear, we are using a,b,... as SU(6)_L tensor indices; A,B,... as SU(6)_R tensor indices, and \(\alpha, \beta, ..\) as SU(6)_C tensor indices.

### 2.2 The Higgs Fields

The analysis done in Ref. \[\] shows that the most economical set of Higgs Fields (HF) and Vacuum Expectation Values (VEVs) which breaks the symmetry from G down to SU(3)_c ⊗ U(1)_{EM} and at the same time produces what we called the modified horizontal survival hypothesis is formed by:

\[
\phi_1 = \phi(675) = \phi_{1,[A,B]}^{[A,B]} + \phi_{1,[\alpha,\beta]}^{[\alpha,\beta]} + \phi_{1,[a,b]}^{[a,b]} \quad (2)
\]
with VEVs in the directions \([a,b]=[1,6]=-\{2,5\}=-\{3,4\}\), \([A,B]\) similar to \([a,b]\) and \([\alpha,\beta]=[5,6]\);

\[
\phi_2 = \phi(1323) = \phi_{2,[a,b]}^{[A,B]} + \phi_{2,[\alpha,\beta]}^{[a,b]}
\]
(3)

with VEVs in the directions \([a,b]=[1,4]=-\{2,3\}\), \([A,B]\) similar to \([a,b]\) and \([\alpha,\beta]=[4,5]\);

\[
\phi_3 = \phi'(675) = \phi_{3,[a,b]}^{[A,B]} + \phi_{3,[\alpha,\beta]}^{[a,b]}
\]
(4)

with VEVs such that \(\langle \phi_{3,[a,b]}^{[A,B]} \rangle = \langle \phi_{3,[\alpha,\beta]}^{[a,b]} \rangle = 0\), and \(\langle \phi_{3,[A,B]}^{[4,6]} \rangle = M_R\);

\[
\phi_4 = \phi(108) = \phi_{4,A}^A + \phi_{4,a}^a + \phi_{4,A}^A
\]
(5)

with VEVs such that \(\langle \phi_{4,A}^A \rangle = \langle \phi_{4,a}^a \rangle = 0\) and \(\langle \phi_{4,A}^A \rangle = M_L\), with values different from zero only in the directions \(\phi_{2,3}^2 = \phi_{1,1}^2 = \phi_{2,4}^3 = \phi_{1,2}^3 = \phi_{4,1}^4 = \phi_{2,3}^5 = \phi_{2,4}^6 = \phi_{2,5}^6 = \phi_{2,6}^6 \approx M_Z \sim 10^2\) GeVs.

In \((\mathbb{P})\), \((\mathbb{R})\), and \((\ldots)\) and \(\{,\ldots\}\) stand for the commutator and anticommutator respectively of the indices inside the brackets. The mass hierarchy suggested in Ref. \(\mathbb{P}\) is \(\langle \phi_3 \rangle \geq \langle \phi_1 \rangle \simeq \langle \phi_2 \rangle \gg M_Z \sim 10^2\) GeVs. Since at this stage of our analysis we are not interested in studying CP violation in the context of this model, we will assume throughout the paper that \(\langle \phi_i \rangle, i=1,2,3,4\), are real numbers.

According to the analysis presented in Ref. \(\mathbb{P}\), \(\langle \phi_1 \rangle + \langle \phi_2 \rangle\) breaks G down to the LRS gauge group; \(\langle \phi_3 \rangle\) alone breaks G down to SU(6)\(_L\)\(\otimes SU(4)\_C\otimes U(1)\_Y\otimes G_R\), where U(1)\(_Y\) is the same abelian factor of the SM and SU(4)\(_C\) \(\supseteq\) SU(3)\(_c\) but U(1)\(_Y\)\(_{B-L}\) is not a subgroup of SU(4)\(_C\) neither SU(2)\(_R\) of the LRS model is a subgroup of G\(_R\). Finally, \(\langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle\) breaks G down to SU(3)\(_C\)\(\otimes SU(2)\_L\otimes U(1)\_Y\), the local gauge group of the SM.

Let us emphasize that prior to symmetry breaking the model is both, left-right symmetric and iso-spin symmetric. It is the choice of VEVs for \(\langle \phi_4 \rangle\) which resulted in an spontaneous breaking of the iso-spin symmetry and in a flavor democratic mass matrix for the Up quarks. The iso-spin breaking was introduced by hand, by demanding that the only VEVs of \(\langle \phi_4 \rangle \neq 0\) are the ones stated above. This particular choice of VEVs is what produces the modified horizontal survival hypothesis \(\mathbb{R}\) \(\mathbb{P}\) (the only ordinary particle with a tree level mass different from zero is the t quark). On the other hand, the flavor democratic form of the mass matrices for the quark sectors resulted as a consequence of minimizing the Higgs
field potential for $\phi_4$ alone, and it is hoped that a careful study of the entire Higgs fields potential provides a basis for the iso-spin breaking too.

3 The Renormalization Group Equation Analysis

For the renormalization group equation (RGE) analysis which follows, we implement our model with the working conditions known as "the survival hypothesis" [5] and "the extended survival hypothesis" [6].

1-The survival hypothesis claims that [5] at each energy scale, the only fermion fields which are relevant are those belonging to chiral representations of the unbroken symmetries.

2-The extended survival hypothesis claims that [6] at each energy scale the only scalars which are relevant are those that develop VEVs at that scale and at lower mass scales.

In the context of our model we can show that both hypothesis follow mandatory by a wise selection of VEVs for the scalar fields, and the inclusion of appropriate terms in the scalar potential and Yukawa Lagrangian.

Now, for the proper implementation of the survival hypothesis, the Higgs scalars which develop VEVs must couple to $\psi(108)_L$ via Yukawa type terms. Therefore they must be of the form $Z_3\phi(n,1,m)$ where $n,m=6,\bar{6},15,\bar{15},21,\bar{21}$. (Higgs fields of the form $Z_3\phi(6,35,\bar{6})$, and Higgs fields which do not develop VEVs are excluded from our model for economic reasons [5].)

3.1 The RGE analysis with two mass scales.

For the two mass scale symmetry breaking pattern

$$G \xrightarrow{M} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{M_Z} SU(3)_c \otimes U(1)_{EM}$$

where $M=\langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle$ and $M_Z = \langle \phi_4 \rangle$, the one loop running coupling constants of the standard model satisfy the relationships

$$\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) - b_i \ln(M/M_Z) \quad (6)$$

where $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$ refers to $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively, and the beta functions $b_i$ are given by

$$b_i^\kappa = \left\{ \frac{11}{3} C_i^\kappa (\text{vectors}) - \frac{2}{3} C_i^\kappa (\text{Weyl - fermions}) - \frac{1}{6} C_i^\kappa (\text{scalars}) \right\} / 4\pi \quad (7)$$
with \( C_i^c(\ldots) \) the index of the representation to which the (\ldots) particles are assigned. For a complex field, the value \( C_i^c(scalars) \) should be doubled. With the normalization of the generators in \( G \) such that \( \alpha_1(M) = \alpha_2(M) = \alpha_3(M) \), the relationship

\[ \alpha_{EM} = \frac{1}{3} \alpha_2 \sin^2 \theta_W = \frac{3}{14} \cos^2 \theta_W, \]

where \( \theta_W \) is the weak mixing angle, is valid at all energy scales. This last equation implies also that at all energies

\[ 3\alpha_{EM}^{-1} = 14\alpha_1^{-1} + 9\alpha_2^{-1}. \]

Equations (8), (9) and (10) give straightforward

\[ \frac{3}{23} \alpha_{EM}^{-1}(M_Z) = \alpha_3^{-1}(M_Z) + (b_3^0 - \frac{14}{23} b_1^0 - \frac{9}{23} b_2^0) \ln(M/M_Z) \]

and

\[ \sin^2 \theta_W(M_Z) = 3\alpha_{EM}(M_Z) \alpha_2^{-1}(M_Z) \]

\[ = 3\alpha_{EM}(M_Z)[\alpha_3^{-1}(M_Z) + (b_3^0 - b_2^0) \ln(M/M_Z)] \]

where \( b_3^0 = (11 - 4)/2\pi \), \( b_2^0 = [\frac{22}{9} - \frac{4}{3}(3-n^0_2) - \frac{N_H^n}{15}] / 2\pi \) and \( b_1^0 = -[\frac{4}{3}(3-n^0_1) + \frac{N_H^n}{28}] / 2\pi \), \( N_H = 9 \) is the number of low energy Higgs fields doublets in \( \langle \phi_4 \rangle \), and \( n_2^0 = 2 \), \( n_1^0 = 27/14 \) are two values related to the number of fermion fields which decouple from \( \psi(108)_L \) according to the survival hypothesis and the Appelquist-Carrazzone theorem \( (n_0^1 = n_0^2 = 0 \text{ when all the fermion fields in } \psi(108)_L \text{ contribute to } b_{1,2}^0) \).

Plugging in the last two equations the experimental values \( \sin^2 \theta_W(M_Z) = 0.233, \alpha_{EM}(M_Z) = 127.9 \), and \( \alpha_3(M_Z) = 0.122 \) we get from Eq.(11) \( \ln(M/M_Z) = 6.3 \), and from Eq.(10) \( \ln(M/M_Z) = 1.1 \) which are widely incompatible and inconsistent solutions. So, the model with only two mass scales is excluded by experimental results (this same conclusion was reached in a different way in Ref. [2]).

### 3.2 The RGE analysis with three mass scales

For the three mass scale symmetry breaking pattern

\[ G^M \rightarrow G_L \otimes G_C \otimes G_R \otimes \cdots \rightarrow M_H^c \text{SU(3)}_c \otimes \text{SU(2)}_L \otimes \text{U(1)}_Y \rightarrow M_Z^c \text{SU(3)}_c \otimes \text{U(1)}_{EM} \]

where \( M >> M_H >> M_Z = \langle \phi_4 \rangle \), the one loop running coupling constants of the standard model satisfy now

\[ \alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M) - b_i^0 \ln(M_H/M_Z) - b_i^1 \ln(M/M_H), \]
Now the algebra gives

\[
\frac{3}{23}a_{EM}^{-1}(M_Z) = \alpha_3^{-1}(M_Z) + (b_3^0 - \frac{14}{23}b_1^0 - \frac{9}{23}b_2^0) \ln\left(\frac{M_H}{M_Z}\right) + (b_3^1 - \frac{14}{23}b_1^1 - \frac{9}{23}b_2^1) \ln\left(\frac{M}{M_H}\right)
\]

(13)

and

\[
\sin\theta_W(M_L) = 3a_{EM}(M_Z)[\alpha_3^{-1}(M_Z) + (b_3^0 - b_2^0) \ln\left(\frac{M_H}{M_Z}\right) + (b_3^1 - b_1^1) \ln\left(\frac{M}{M_H}\right)],
\]

(14)

where \(b_i^0, i = 1, 2, 3\) are the same as in Sec. 3.1, but \(b_i^1, i = C, Y, L\) depend upon the structure of the subgroup \(G_L \otimes G_C \otimes G_R \otimes \cdots \equiv G_H\) as mentioned in Ref. 4.

Equation (14) is very restrictive because its first entry of the right-hand side has an experimental value of 0.192 which excludes the possibility of having \(G_H = \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R \otimes \text{U}(1)_{Y(B-L)}\) (which shows up for \(M = \langle \phi_1 + \phi_2 \rangle\)) because if so Eq. (14) with \(b_3^C = b_3^0\) and \(b_3^L = b_3^1\) will imply \(M_H \sim 3M_Z\), a very small value for the right-handed weak current. Other gauge structures for \(G_H\) which do not contain flavor changing neutral currents such as \(\text{SU}(3)_C \otimes \text{SU}(2)_L \otimes [\text{U}(1)_{y}]^n\) are not allowed by the set of Higgs fields described in Sec. 2; so, with the minimal set of Higgs fields presented, \(G_H\) will contain flavor changing neutral currents and \(M_H \geq 100\) TeVs and the first two terms of the right-hand side of Eq. (14) will have a lower bound of 0.36. Then if we want to reproduce the experimental value for \(\sin^2\theta_W(M_Z)\), \((b_3^C - b_3^L) < 0\). As mentioned in Ref. 4 this is a very stringent constraint which is not satisfied by the set of Higgs fields and VeVs presented in Sec. 2.2 (in combination with the extended survival hypothesis). So, up to this point there is a demand for changing the minimum set of HF and/or VEVs presented.

Now, what is the minimum change in the set of HF and/or VEVs which properly breaks the symmetry, respect the survival hypothesis, produces appropriate values for \(\sin^2\theta_W(M_Z)\), and holds the mass hierarchy \(M \gg M_H \gg M_Z \sim 10^2\) GeVs? The study of table I shows that the three stages gauge hierarchy with \(G_H = \text{SU}(6)_L \otimes \text{SU}(4)_C \otimes \text{U}(1)_Y \otimes \cdots\) produces consistent results as far as we do the following two things:

1-Add a new set of Higgs Fields

\[
\phi'_2 = \phi'(1323) = \phi'_{2,\{A,B\}} + \phi'_{2,\{a,b\}} + \phi'_{2,\{\alpha,\beta\}}
\]

(15)

with VEVs in the directions \(\{a,b\} = \{3,6\} = \{-4,5\}\), \(\{A,B\}\) similar to \(\{a,b\}\) and \(\{\alpha, \beta\} = \{5,5\}\):

2-Orient the VEVs such that \(\phi'_{2,\{a,b\}}(5,5) = \phi'_{2,\{A,B\}} = \phi'_{2,\{a,b\}} = 0\)
For this particular choice of VEVs we have that
\[ b_1^C = \left( \frac{88}{3} - \frac{2 \times 12}{3} - \frac{148}{3} \right)/4\pi, \]
\[ b_1^L = \left( \frac{132}{3} - \frac{2 \times 12}{3} - \frac{107}{3} \right)/4\pi \]
and
\[ b_1^Y = \left(-\left(\frac{2 \times 12}{3} + \frac{9}{13}\right)\right)/4\pi, \]
where the extended survival hypothesis [5] was taken into account for the contribution of the HF.

As can be seen, the Higgs fields play a fundamental role in equations (13) and (14) (the same is true for other models [9] such as SU(15) and SU(16)). Notice also that we achieve the stated hierarchy for \( M = \langle \phi_3 \rangle \), \( M_H = \langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi'_2 \rangle \), \( M_Z = \langle \phi_4 \rangle \), and that \( \phi_3 \) plays no role in the evolution of the gauge coupling constants.

Plugging in the different beta functions in (13) and (14) and using the experimental values for \( \sin^2 \theta_W(M_Z) \), \( \alpha_{EM}(M_Z) \), \( \alpha_3(M_Z) \) we get the equations
\[ 1.10 = 1.02 \ln\left(\frac{M_H}{M_Z}\right) - 2.26 \ln\left(\frac{M}{M_H}\right) \]
\[ 7.83 = 1.25 \ln\left(\frac{M_H}{M_Z}\right) - 1.82 \ln\left(\frac{M}{M_H}\right), \]
which for \( M_Z = 91 \) GeVs have the solutions \( M_H \sim 10^9 \) GeVs and \( M \sim 10^{12} \) GeVs. These results are in good agreement with the values calculated from the analysis of the generational see-saw mechanism done in the context of this model [10].

### 4 Stability of the Proton

#### 4.1 Baryon number for the particles

The elementary particles in the model are the ones associated to the 105 GF, the 108 Weyl fields in \( \psi(108)_L \) and the 4104 HF in \( \phi_i, i = 1-4 \) and \( \phi'_2 \). Now, all the elementary particles in our model have well defined Baryon numbers. Let us see:

1-The GF.

The 70 GF associated with SU(6)_L \( \otimes \) SU(6)_R have BN zero (all of them are color singlets).

For SU(6)_C we have that the 9 leptoquarks have BN equal to 1/3, the 9 leptoquarks charge conjugated to the previous ones have BN equal to \(-1/3\) and the other 17 GF have BN equal to zero (including the 8 gluon fields).

2-The Weyl Fermion Fields.

All the quark fields in \( \psi(6, 6, 1)_L \) have BN equal to 1/3, the quark fields in \( \psi(1, 6, 6)_L \) have BN equal to \(-1/3\) and all the other fields in \( \psi(108)_L \) have BN equal to zero.

3-The HF.

The BN for the 4104 HF of the model are given in Table 2.
4.2 Baryon number as a symmetry of the model

In the subspace defined by SU(6)$_C$ the BN can be associated with the 6 $\times$ 6 diagonal matrix $B = Dg.(1/3, 1/3, 1/3, 0, 0, 0)$. This matrix does not correspond to a generator of SU(6)$_C$ neither of G.

Now, the full Lagrangian $\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$ has a U(θ) global symmetry given by

$$U(\theta) = e^{i\theta \chi_h}$$

where $\chi_h$ is a constant value related to the irreducible representation of SU(6)$_C$ under which U(θ) acts (for example $\chi = 1$ for $\psi(6, 6, 1)$, $\chi = 0$ for G(1,35,1), $\chi = -2$ for $\phi(1, \overline{15}, 15)$, etc.).

Associated to U(θ) there is a global U(1) generator which we may write in the subspace of SU(6)$_C$ as:

$$U(1) = Dg.(1, 1, 1, 1, 1, 1)/\sqrt{12}$$

which is not an element of the Lie algebra of G either (the value $\sqrt{12}$ is introduced just for convenience).

On the other hand, in the Lie algebra of G there is a generator, element of the SU(6)$_C$ subalgebra, of the form

$$B' = Dg.(1, 1, 1, -1, -1, -1)/\sqrt{12}$$

which distinguish between quarks and leptons in the context of our model. Therefore B can be written as

$$B = [U(1) + B']/\sqrt{3}$$

4.3 Stability of the proton

Since the elementary particles of this model have well-defined values of BN it is obvious that in the unbroken theory the exchange of particles cannot break BN. This statement is also true after breaking the symmetry due to the following two facts:

- Baryon number is not gauged (there is not gauge boson associated to B).
\( \phi_i, i=1-4 \) and \( \phi'_2 \) with the VEVs as stated do not break spontaneously B. That is \( B\langle \phi_i \rangle = B\langle \phi'_2 \rangle = 0, \ i=1,2,3,4. \)

So, B is conserved in our model [11]. Once B is conserved, the proton will be stable against all decays, except possible topological effects.

Now, the single Goldstone boson associated with the broken orthogonal combination is eaten by the massive gauge field associated with B', so there are no physical Goldstone bosons and no long range force. This mechanism in which a global symmetry emerges from the simultaneous breaking of a gauge and global symmetry is due to t’Hooft [12] and was implemented in the context of GUMs in Ref. [13].

Finally we want to mention that even though the baryon number is conserved in the context of this model, the lepton number is violated due to the fact that the GF associated with \( U(1)_Y(B-L) \) generator of the SU(6)_{CE} subalgebra, and given by [2]:

\[
U(1)_{Y(B-L)} = \sqrt{\frac{3}{20}} Dg.(1/3, 1/3, 1/3, -1, 1, -1)
\]

is gauged in the context of our model. So, neither \( L, (B-L) \) or \( (B+L) \) are conserved quantities in the context of the model presented here.

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TABLE 1.
Contribution of the HF to the index value for SU(4)\(_C\) and SU(6)\(_L\).

| \(\langle \phi \rangle\) | \(\text{SU}(4)\_C\) | \(C^\text{I}_{\text{C}}(\text{scalars})\) | \(\text{SU}(6)\_L\) | \(C^\text{I}_{\text{L}}(\text{scalars})\) |
|----------------|----------------|-----------------|-----------------|-----------------|
| \(\langle \phi^1_{\{A,B\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | fourteen 4 | 14 | 0 |
| \(\langle \phi^1_{\{a,b\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | fifteen 4 | 15 | four 15 | 16 |
| \(\langle \phi^1_{\{A,B\}} \rangle\) | \(\langle \phi^1_{\{A,B\}} \rangle\) | 0 | fourteen 21 | 112 |
| \(\langle \phi^1_{\{a,b\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | fourteen 4 | 14 | 0 |
| \(\langle \phi^1_{\{a,b\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | twenty-one 4 | 21 | four 21 | 32 |
| \(\langle \phi^1_{\{A,B\}} \rangle\) | \(\langle \phi^1_{\{A,B\}} \rangle\) | 0 | fourteen 21 | 112 |
| \(\langle \phi^1_{\{a,b\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | fourteen 10 | 84 | 0 |
| \(\langle \phi^1_{\{a,b\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | twenty-one 10 | 126 | ten 21 | 80 |
| \(\langle \phi^1_{\{a,b\}} \rangle\) | \(\langle \phi^1_{\{a,b\}} \rangle\) | 0 | three 6 | 3 |
| $\phi$ | $\alpha, \beta$ | $a, b$ | $A, B$ | BN |
|-------|----------------|--------|--------|----|
| $\phi_{1(3)}^{[A,B]}$ | $\alpha, \beta = 1,2,3$ | $a, b = 1, \ldots, 6$ | $A, B = 1, \ldots, 6$ | 0 |
| | $\alpha, \beta = 4,5,6$ | | | $\frac{1}{3}$ |
| | $\alpha = 1, 2, 3; \beta = 4,5,6$ | | | |
| $\phi_{1(3)}^{[\alpha,\beta]}$ | $\alpha, \beta = 1,2,3$ | $a, b = 1, \ldots, 6$ | $A, B = 1, \ldots, 6$ | $-\frac{1}{3}$ |
| | $\alpha, \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | 0 |
| | $\alpha = 1, 2, 3; \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | 1/3 |
| $\phi_{1(3)}^{[a,b]}$ | $\alpha, \beta = 1,2,3$ | $a, b = 1, \ldots, 6$ | $A, B = 1, \ldots, 6$ | 1/3 |
| | $\alpha, \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | 0 |
| | $\alpha = 1, 2, 3; \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | $-\frac{1}{3}$ |
| $\phi_{2,\{\alpha,\beta\}}^{(1)}$ | $\alpha, \beta = 1,2,3$ | $a, b = 1, \ldots, 6$ | $A, B = 1, \ldots, 6$ | 0 |
| hline $\phi_{2,\{A,B\}}^{(1)}$ | | | | $\frac{2}{3}$ |
| | $\alpha, \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | 0 |
| | $\alpha = 1, 2, 3; \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | 1/3 |
| $\phi_{2,\{a,b\}}^{(1)}$ | $\alpha, \beta = 1,2,3$ | $a, b = 1, \ldots, 6$ | $A, B = 1, \ldots, 6$ | $-\frac{2}{3}$ |
| | $\alpha, \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | 0 |
| | $\alpha = 1, 2, 3; \beta = 4,5,6$ | | $A, B = 1, \ldots, 6$ | $-\frac{1}{3}$ |
| $\phi_{4,A}^{\alpha}$ | $\alpha = 1, 2, 3$ | $a = 1, \ldots, 6$ | $A = 1, \ldots, 6$ | 0 |
| $\phi_{4,a}^{\alpha}$ | | $\alpha = 1, 2, 3$ | $a = 1, \ldots, 6$ | 1/3 |
| | | $\alpha = 4, 5, 6$ | $a = 1, \ldots, 6$ | 0 |
| $\phi_{4,A}^{\alpha}$ | $\alpha = 1, 2, 3$ | $a = 1, \ldots, 6$ | $A = 1, \ldots, 6$ | $-\frac{1}{3}$ |
| | | $\alpha = 4, 5, 6$ | $A = 1, \ldots, 6$ | 0 |
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