Symmetric bidirectional quantum teleportation using a six-qubit cluster state as a quantum channel

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Abstract. Bidirectional quantum teleportation is a fundamental protocol for exchanging quantum information between two quantum nodes. Till now, all bidirectional quantum teleportation protocols have achieved a maximum efficiency of 40%. Here, we propose a new scheme for a symmetric bidirectional quantum teleportation using a six-qubit cluster state as the quantum channel, for symmetric (3 ↔ 3) qubit bidirectional quantum teleportation of a special three-qubit entangled state. The novelty of our scheme lies in its generalisation for (N ↔ N) qubit bidirectional quantum teleportation employing a 2N-qubit cluster state. The efficiency of the proposed protocol is remarkably increased to 50% which is the highest till now. Interestingly, only GHZ-state measurements and four Toffoli-gate operations are necessary which is independent of the number (N) of qubits to be teleported.

Keywords. Bidirectional quantum teleportation; six-qubit cluster state; GHZ-state measurements; Toffoli-gate operations.

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1. Introduction

Quantum entanglement has been one of the most puzzling and debated features of quantum mechanics. Once considered to strike at the heart of the quantum theory, over the past few decades, entanglement theory has elevated into a resource theory [1]. Entanglement is a fundamental property of quantum systems enabling to perform the computational tasks, such as quantum teleportation (QT) [2], superdense coding [3,4], quantum key distribution (QKD) [5–7], quantum state sharing (QSTS) [8], etc., which otherwise are impossible. However, entanglement lacks a complete characterisation and classification for many qubit systems. A profound understanding of the multipartite entanglement might inspire better protocols in quantum computation and information. QT, first proposed by Bennett et al [2], is one of the most extraordinary applications of the quantum correlations, i.e. entanglement [1] present in a composite quantum system. It is a communication protocol that enables transmission of quantum information from one location to another, with the aid of preshared entanglement, local operations and classical communications. Since its inception, many key developments have been reported in the QT [9–27].

Bidirectional quantum teleportation (BQT) [28] is an advanced version of QT and is defined as the bilateral QT protocol between two quantum nodes in a quantum communication channel. This protocol finds its applications in cryptography, such as quantum secure direct communication (QSDC) [29], quantum secret sharing (QSS) [30,31], cryptographic switch [32], etc. BQT can be carried out with or without the aid of a controller. The first bidirectional quantum controlled teleportation (BQCT) protocol was introduced in [28], that employed a five-qubit cluster state for mutual transfer of an unknown single-qubit state between Alice and Bob with the help of controller Charlie. Several BQCT protocols have been proposed by utilising multi-qubit cluster states as a quantum channel [33–37]. In [35], seven-qubit cluster state is used for asymmetric BQCT where Alice sends an arbitrary single-qubit state to Bob and Bob sends a two-qubit state to Alice with the control of Charlie. Several two-party BQT protocols have been proposed [38–45]. Sang et al [38] proposed an asymmetric BQCT protocol in the absence of the controller. By utilising a five-qubit cluster state as the channel, Alice teleports a two-qubit entangled state to Bob and Bob sends an arbitrary single-qubit state to Alice. Using the eight-qubit cluster state as the quantum channel, Zadeh et al
faithfully achieved the BQT of an arbitrary two-qubit state. To achieve the BQT of multi-qubit states, different cluster states were used. Chen et al. [40] utilised a four-qubit GHZ state and two Bell states as a quantum channel to mutually teleport two arbitrary single-qubit states and an unknown three-qubit state between Alice and Bob. In [41], a new BQT protocol was proposed using a six-qubit cluster state. However, it used non-local CNOT operation for its action, resulting in a decrease of intrinsic efficiency and an increase in operational complexity. This use of non-local CNOT operation was eliminated by Verma [42] by proposing a modified BQT protocol. In order to employ \( N \)-qubit BQT, Verma [43] proposed a protocol with one of the G-state [23] as the quantum channel. In this protocol, to achieve faithful transmission of \( N \)-qubits, \( 2(2N - 1) \)-CNOT gate operations along with two Hadamard gate operations are required. This increases the operational complexity without any improvement in intrinsic efficiency. It is therefore desirable to have a new scheme that reduces the operational complexity by reducing the number of CNOT gate operations and increasing the intrinsic efficiency.

Intrinsic efficiency is a key ingredient for evaluating the quality of QT from a practical viewpoint. It is given by eq. (18) and is discussed in detail in Conclusion. Efforts are continuously being made to have better protocols with greater efficiency and less operational complexity [38–45]. In [38], the efficiency of the proposed protocol is 33.3\% and it increased to 40\% in protocols given in [42,43]. Hassanpour and Houshmand [44], utilising two three-qubit GHZ states as a quantum channel for bilateral transmission of EPR pair, predicted an efficiency of 66.6\%. However, they have not considered the six classical bits consumed by six single-qubit measurements in the proposed protocol. The actual efficiency is 33.3\%. To increase the efficiency, Zhou et al. [45] proposed a symmetric BQT (2 ↔ 2) protocol for mutual transmission of an unknown two-qubit state with 40\% efficiency and by introducing the auxiliary qubits in the same work for asymmetric BQT (2 ↔ 3) the efficiency increases to a maximum of 45\%. Motivated by the use of auxiliary qubits for enhancing the efficiency, the present protocol is proposed with the result being a remarkable increase in efficiency to 50\%. Along this direction, to have greater efficiency and minimum complexity, we propose a new symmetric BQT protocol as schematically described in figure 1, for mutual transmission of a special three-qubit entangled state using six-qubit BQT. The novelty of our scheme when compared to the previous protocols [38–45] is: (i) only four Toffoli-gate operations are required which is independent of the number \( (N) \) of the qubits to be teleported, (ii) operation complexity is greatly reduced, (iii) by introducing the auxiliary qubits, we achieved the highest intrinsic efficiency till now (50\%) in any BQT protocol.

This paper is organised as follows: In §2, the proposed BQT protocol based on six-qubit cluster state is presented in detail. In §3, we generalise our proposed protocol for \( N \)-qubit BQT. The comparison and conclusion of our work with the previous protocols is presented in §4. In Appendix, the construction of the six-qubit entangled quantum channel is described on IBM quantum composer platform.

## 2. Protocol for symmetric BQT (3 ↔ 3) using six-qubit cluster state

We propose a scheme for BQT that utilises a six-qubit cluster state as the quantum channel shared between Alice and Bob:

\[
|\phi\rangle_{123456} = \frac{1}{\sqrt{2}} (|000000\rangle + |000111\rangle + |111000\rangle + |111111\rangle),
\]

where the indices 1–6 refer to the ordering of qubits. Assume that Alice is in the possession of a special three-qubit entangled state, which is given by

\[
|\chi\rangle_{a1a2a3} = (\alpha|000\rangle + \beta|111\rangle)_{a1a2a3}
\]

with \( |\alpha|^2 + |\beta|^2 = 1 \). At the same time, Bob also possesses a similar three-qubit entangled state which can be expressed as

\[
|\chi\rangle_{b1b2b3} = (\gamma|000\rangle + \delta|111\rangle)_{b1b2b3}
\]

such that \( |\gamma|^2 + |\delta|^2 = 1 \). Now, Alice and Bob want to transmit the respective states to each other simultaneously. Let the quantum channel given by eq. (1) be distributed between Alice and Bob so that the qubits 1, 4, 5 belong to Alice and qubits 2, 3, 6 belong to Bob. The proposed scheme for implementing BQT (3 ↔ 3) is executed in the following manner:

**Step I:** Conversion of three-qubit entangled state to two-qubit entangled state and single-qubit state (Toffoli-gate operations): Toffoli-gate (also known as CCNOT-gate) is a three-qubit gate with two controls and one target. It performs an X on the target only if both controls are in the state |0\rangle.

- Alice applies Toffoli-gate with \( a_1a_2 \) as the control and \( a_3 \) as the target such that

\[
|\chi\rangle_{a1a2a3} \rightarrow |\chi\rangle_{a1a2} \otimes |0\rangle_{a3}
\]

with

\[
|\chi\rangle_{a1a2} = (\alpha|00\rangle + \beta|11\rangle)_{a1a2}.
\]
Next, Bob applies Toffoli-gate with $b_1b_2$ as the control and $b_3$ as the target such that $|\chi\rangle_{b_1b_2b_3} \rightarrow |\chi\rangle_{b_1b_2} \otimes |0\rangle_{b_3}$ with

$$|\chi\rangle_{b_1b_2} = (\gamma|00\rangle + \delta|11\rangle)_{b_1b_2}. \quad (5)$$

After this step, the joint state of the qubits $a_1a_2b_1b_2123456$ becomes

$$|\psi\rangle_{a_1a_2b_1b_2123456} = |\chi\rangle_{a_1a_2} \otimes |\chi\rangle_{b_1b_2} \otimes |\phi\rangle_{123456}$$

$$= \frac{1}{2} (\alpha|00\rangle + \beta|11\rangle)_{a_1a_2}$$

$$\otimes (\gamma|00\rangle + \delta|11\rangle)_{b_1b_2}$$

$$\otimes (|000000\rangle + |000111\rangle + |111000\rangle + |111111\rangle)_{123456}. \quad (6)$$

The joint state $|\psi\rangle_{a_1a_2b_1b_2123456}$ takes the form

$$|\psi\rangle_{a_1a_2b_1b_2123456} = \frac{1}{4} \left[ |\eta_1^+\rangle |\xi_1^+\rangle |\psi_1^+\rangle + |\eta_1^+\rangle |\xi_3^+\rangle |\psi_3^+\rangle \right.$$ 

$$+ |\eta_2^+\rangle |\xi_2^+\rangle |\psi_2^+\rangle + |\eta_2^+\rangle |\xi_2^+\rangle |\psi_2^+\rangle$$ 

$$+ |\eta_2^+\rangle |\xi_3^+\rangle |\psi_3^+\rangle + |\eta_2^+\rangle |\xi_3^+\rangle |\psi_3^+\rangle \right] \quad (7)$$

where $|\eta_i^\pm\rangle$ and $|\xi_i^\pm\rangle$ are GHZ states (figure 2) given by

$$|\eta_1^\pm\rangle_{a_1a_2} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \quad (8)$$

$$|\eta_2^\pm\rangle_{a_1a_2} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \quad (9)$$

$$|\xi_1^\pm\rangle_{b_1b_2} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \quad (10)$$

$$|\xi_2^\pm\rangle_{b_1b_2} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \quad (11)$$

and $|\psi_i^\pm\rangle$ are the states of the qubits 2, 3, 4 and 5 shown in table 1.

**Step II: Measurements and classical communications**

- Alice performs a GHZ-state measurement (GSM) on her qubits $(a_1a_2)$ in the measurement basis made up of the states $|\eta_i^\pm\rangle$, $i = 1, 2$.  

![Figure 1. Schematic representation of the symmetric BQT proposed in this paper. Alice and Bob share a six-qubit cluster state and both parties send three qubit states (eqs (2) and (3)), bidirectionally. The upper dashed boxes represent three-qubit GHZ-state measurement performed by Alice and the lower dashed box represents Bob’s three-qubit GHZ-state measurement. $U_2 \otimes U_3$ refers to recovery operations performed by Bob and $U_4 \otimes U_5$ by Alice as shown in table 2. $|\chi\rangle_{a_1a_2}$ and $|\chi\rangle_{b_1b_2}$ are the two-qubit entangled states recovered by Bob and Alice, respectively. Using only two auxiliary qubits ($|0_A\rangle$, $|0_B\rangle$) along with four Toffoli-gate operations, the bidirectional teleportation is successful with the utilisation of minimum resources and maximum efficiency compared to the previous proposed protocols. $(\alpha|00\rangle + \beta|11\rangle)_{23B}$ and $(\gamma|00\rangle + \delta|11\rangle)_{45A}$ are the recovered three-qubit states.](image-url)
Bob and the corresponding unitary operations are given.

The GHZ-state measurement outcomes of Alice and Bob are respectively in possession of the following two-qubit entangled states:

- Alice applies Toffoli-gate with qubits 4, 5 as the control and $|0\rangle_A$ as the target qubit to recover the target three-qubit entangled state $(\gamma|000\rangle + \delta|111\rangle)_{45A}$ shown in figure 1.
- Bob also applies Toffoli-gate with qubits 2, 3 as the control and $|0\rangle_B$ as the target qubits to recover the desired state $(\alpha|000\rangle + \beta|111\rangle)_{23B}$ shown in figure 1.

Thus, symmetric $(3 \leftrightarrow 3)$ BQT protocol is observed faithfully and deterministically.

### 3. Generalisation to $N$-qubits

In this section, our proposed scheme for symmetric BQT can be generalised for $(N \leftrightarrow N)$ qubit BQT using the $2N$-qubit cluster state as the quantum channel. In our scheme, no additional quantum resources are required which makes it a novel scheme for BQT. Let Alice and Bob are respectively in a possession of the following $N$-qubit entangled states:

$$|\chi\rangle_{a_1a_2...a_N} = (\alpha|00...0\rangle + \beta|11...1\rangle)_{a_1a_2...a_N} \quad (12)$$

$$|\chi\rangle_{b_1b_2...b_N} = (\gamma|00...0\rangle + \delta|11...1\rangle)_{b_1b_2...b_N} \quad (13)$$

By utilising the $2N$-qubit cluster state and employing our scheme, Alice and Bob can faithfully achieve mutual transmission of these $N$-qubit states. The $2N$-qubit quantum channel has the form

$$|\psi\rangle_{12...2N} = \frac{1}{2}(|x_1\rangle + |x_2\rangle + |x_3\rangle + |x_4\rangle)_{12...2N}, \quad (14)$$

where

$$|x_1\rangle = \bigotimes_{i=1}^{N} |0\rangle \otimes_{i=N}^{2N} |0\rangle,$$

$$|x_2\rangle = \bigotimes_{i=1}^{N} |1\rangle \otimes_{i=N}^{2N} |1\rangle,$$

$$|x_3\rangle = \bigotimes_{i=1}^{N} |0\rangle \otimes_{i=N}^{2N} |1\rangle,$$

$$|x_4\rangle = \bigotimes_{i=1}^{N} |1\rangle \otimes_{i=N}^{2N} |0\rangle. \quad (15)$$

Let Alice has $1, N + 1, \ldots, 2N - 1$ qubits and Bob has $2, 3, \ldots, N, 2N$ qubits. The steps involved in $(N \leftrightarrow N)$ qubit symmetric BQT protocol are the same as that of

### Table 1. The collapsed states of the qubits 2, 3, 4 and 5 after GHZ-state measurements.

| $|\psi_{i}^{\pm}\rangle$ | $(\alpha|00\rangle \pm \beta|11\rangle)_{23} \otimes (\gamma|00\rangle \pm \delta|11\rangle)_{45}$ |
|-----------------|----------------------------------------------------------|
| $|\psi_{2}^{\pm}\rangle$ | $(\alpha|00\rangle \pm \beta|11\rangle)_{23} \otimes (\gamma|00\rangle \pm \delta|11\rangle)_{45}$ |
| $|\psi_{3}^{\pm}\rangle$ | $(\alpha|00\rangle \pm \beta|11\rangle)_{23} \otimes (\gamma|00\rangle \pm \delta|11\rangle)_{45}$ |
| $|\psi_{4}^{\pm}\rangle$ | $(\alpha|00\rangle \pm \beta|11\rangle)_{23} \otimes (\gamma|00\rangle \pm \delta|11\rangle)_{45}$ |
| $|\psi_{5}^{\pm}\rangle$ | $(\alpha|00\rangle \pm \beta|11\rangle)_{23} \otimes (\gamma|00\rangle \pm \delta|11\rangle)_{45}$ |

- Bob also performs a GSM on his qubits $(b_1 b_2 6)$ in measurement basis consisting of the states $|\zeta_i\rangle^{\pm}, i = 1, 2$.

Afterwards Alice and Bob convey their measurement outcomes to each other by classical communication and corresponding to each measurement outcome is a collapsed state of the qubits 2, 3, 4 and 5. In order to recover the target states, Alice and Bob apply the local operations on their qubits. The collapsed states of particles 2, 3, 4 and 5 after the GSM are given in table 1.

### Step III: Local operations

- Alice applies appropriate unitary operation $(U_4 \otimes U_5)$ to recover the two-qubit state depending on the classical information received from Bob’s measurement.
- Bob also applies appropriate unitary operation $(U_2 \otimes U_3)$ to recover the two-qubit state depending on Alice’s measurement outcome.

The GHZ-state measurement outcomes of Alice and Bob and the corresponding unitary operations are given in table 2. In order to recover the three-qubit target entangled states, once two-qubit entangled states are recovered by Alice and Bob, we introduce two auxiliary qubits $|0\rangle_A$ and $|0\rangle_B$.

### Step IV: Toffoli-gate operations

- Alice applies Toffoli-gate with qubits 4, 5 as the control and $|0\rangle_A$ as the target qubit to recover the target three-qubit entangled state $(\gamma|000\rangle + \delta|111\rangle)_{45A}$ shown in figure 1.
- Bob also applies Toffoli-gate with qubits 2, 3 as the control and $|0\rangle_B$ as the target qubits to recover the desired state $(\alpha|000\rangle + \beta|111\rangle)_{23B}$ shown in figure 1.

Thus, symmetric $(3 \leftrightarrow 3)$ BQT protocol is observed faithfully and deterministically.
the (3 ↔ 3) qubit BQT protocol. The implementation of our scheme for the general case is described in the following steps:

**First step:** Alice applies a Toffoli-gate operation with \((a_{N-2}, a_{N-1})\) as the control qubits and \(a_N\) as the target qubit to convert the \(N\)-qubit state to be teleported into an \(N - 1\) qubit-entangled state and a single qubit. Bob also applies Toffoli-gate with \((b_{N-2}, b_{N-1})\) as the control qubits and \(b_N\) as the target qubit. After this step, the joint state of the \((4N - 2)\) qubits \((a_1a_2 \ldots a_{N-1}b_1b_2 \ldots b_{N-1}12 \ldots 2N)\) becomes

\[
|\psi\rangle = |\chi\rangle_{a_1a_2 \ldots a_{N-1}} \otimes |\chi\rangle_{b_1b_2 \ldots b_{N-1} \otimes \phi} 12 \ldots 2N. \quad (17)
\]

**Second step:** Alice performs \(N\)-qubit GHZ-state measurement (NGSM) on \((a_1a_2 \ldots a_{N-1}, 1)\) qubits and convey the measurement outcomes to Bob. Bob also performs NGSM on \((b_1b_2 \ldots b_{N-1}, 2N)\) qubits and convey the outcomes to Alice.

**Third step:** Alice and Bob apply the unitary operations on their qubits \((N + 1, \ldots, 2N - 1)\) and \((2, 3, \ldots, N)\), respectively depending on their measurement outcomes enabling them to recover the corresponding \((N - 1)\)-qubit entangled states.

**Fourth step:** To recover the target \(N\)-qubit entangled states, two auxiliary qubits \(|0\rangle_A\) and \(|0\rangle_B\) are introduced as in §2. Alice applies the Toffoli-gate with \((2N - 2, 2N - 1)\) as the control qubits and \(|0\rangle_A\) as the target qubit. Bob also applies Toffoli-gate with \((N - 1, N)\) as the control qubits and \(|0\rangle_B\) as the target qubit.

In this manner, our scheme for the symmetric BQT can be generalised for \((N ↔ N)\) qubit BQT successfully. Only four Toffoli-gate operations are required for its implementation, which is interestingly the same for the
general case, i.e, independent of the number of qubits to be teleported.

4. Comparison and conclusions

To show the novelty of our BQT scheme, let us compare our protocol with the previous protocols as shown in table 3. The comparison is based on the following aspects: the quantum bits transmitted \( q_t \), the quantum resources consumed (number of qubits of the quantum channel \( q_s \)), the classical resources transmitted \( b_t \), the number of auxiliary qubits \( q_a \), the required operations (NO) and the intrinsic efficiency \( (\eta) \). The intrinsic efficiency \( (\eta) \) is defined as

\[
\eta = \frac{q_t}{q_s + q_a + b_t}.
\]  

(18)

It is clear that as the number of qubits to be teleported increases, both the number of quantum bits of the channel and the classical resource consumption increase, resulting in the decrease of intrinsic efficiency. However, in our scheme, by introducing the auxiliary qubits, no additional quantum resources are required for implementation to the general case and the efficiency is also remarkably increased to 50%.

In conclusion, we have proposed a novel scheme for symmetric BQT via six-qubit cluster state as the quantum channel. Furthermore, we generalised our scheme to implement \( (N \leftrightarrow N) \) qubit BQT using the \( 2^N \)-qubit cluster state as the quantum channel. The proposed protocol is based on the GHZ-state measurements (GSM), local unitary operations and Toffoli-gate operations. In our scheme, Alice and Bob are interested in mutual transmission of three-qubit entangled states without a controller. We employ Toffoli-gate operations to transform a three-qubit entangled state into a two-qubit entangled state and a single-qubit state. At the end of the protocol, we again perform Toffoli-gate to restore the target three-qubit states. The previous protocol for \( N \)-qubit BQT [43] employs \( 2(2N - 1) \) CNOT-gate operations, Hadamard gate operations for its implementation. In our scheme, only four Toffoli-gate operations are necessary for \( N \)-qubit BQT, thereby reducing the operational complexity and quantum resources involved in the BQT scheme. From table 3, its clear that our scheme has the prominent advantage of more qubits transmitted and highest intrinsic efficiency. The ideal quantum teleportation protocol requires a noiseless quantum channel, resulting in decoherence of states and accordingly it causes imperfect teleportation. Briegel and Raussendorf [48] and others [49,50] introduced a novel type of entangled states – cluster states with their remarkable properties such as maximal connectedness and persistency of entanglement, opening a window for using them for quantum communication. Since then, many key protocols have been proposed using different classes of non-maximally entangled cluster states through noisy channels for QT [51–54] as well as BQT [55–57]. In this paper, we have only discussed the ideal QT over maximally entangled six-qubit state. The extension of our proposed protocol for BQT in noisy channels and influence of auxiliary qubits on efficiency will be presented elsewhere [58].

Appendix A. Construction of six-qubit entangled channel on IBM quantum composer platform

In our proposed BQT scheme, utilising the six-qubit cluster state as the quantum channel, bilateral transmission of a special three-qubit state is achieved. Here, the construction of the six-qubit entangled quantum channel is presented using two H gates and four CNOT gates from single-qubit states. The corresponding quantum circuit as shown in figure 3 is developed on IBM quantum composer making it practically realisable.

Figure 3. Construction of six-qubit cluster state on IBM quantum composer from single-qubit state \( |0\rangle \). Here, \( (q[i], i = 0–5) \) represents the quantum registers and \( c1 \) represents the classical register.
The six-qubit channel is constructed from the joint state of six single-qubit states as an input in (3).

\[ |\psi\rangle_0 = |0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3 \otimes |0\rangle_4 \otimes |0\rangle_5 \otimes |0\rangle_6. \quad (A.1) \]

The construction of the quantum channel is as follows:

1. Hadamard gate (H) is applied on qubits 1 and 4, and the corresponding state is

\[ |\psi'\rangle = \frac{1}{2} ((|0\rangle + |1\rangle) \otimes |0\rangle \otimes |0\rangle_3 \otimes (|0\rangle + |1\rangle)_4 \otimes |0\rangle_5 \otimes |0\rangle_6. \quad (A.2) \]

2. Four CNOT gates are applied on qubit pairs (1,2), (2,3), (4,5) and (5,6) with the first qubit being the control and the second target to get the required six qubit entangled state.

\[ |\phi\rangle_{123456} = \frac{1}{2} ((|000000\rangle + |000111\rangle) + |111000\rangle + |111111\rangle). \quad (A.3) \]

This six-qubit quantum channel constructed on the IBM quantum composer platform (eq. (A.3)) can also be represented by the amplitude of the corresponding computational basis states as shown in figure 4.

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