Lepton flavor violation via four-Fermi contact interactions at the International Linear Collider

Gi-Chol Cho\textsuperscript{a}, Yuka Fukuda\textsuperscript{b}, Takanori Kono\textsuperscript{a}

\textsuperscript{a}Department of Physics, Ochanomizu University, Tokyo 112-8610, Japan
\textsuperscript{b}Graduate school of Humanities and Sciences, Ochanomizu University, Tokyo 112-8610, Japan

Abstract

Lepton flavor violating (LFV) process $e^+e^- \to e^+\tau^-$ induced by the four-Fermi contact interactions at the International Linear Collider (ILC) is studied. Taking account of the event selection conditions, it is shown that the ILC is sensitive to smaller LFV couplings as compared to the measurement of $\tau \to 3e$ process at the B-factory experiment. The upper bounds on some of the LFV couplings are improved by several factors using polarized $e^-/e^+$ beams at $\sqrt{s} = 250$ GeV and by an order of magnitude at $\sqrt{s} = 1$ TeV.
1 Introduction

Observation of neutrino oscillation [1] tells us that neutrinos have tiny mass and the lepton flavor is no longer conserved. However, if neutrino mass is the only source of lepton flavor violation (LFV), we may not be able to observe any LFV processes since the size of phenomena is too small to measure experimentally. This is because the lepton flavor symmetry is restored in the limit of massless neutrino. For example, the branching fraction of LFV decay process $\mu \rightarrow e\gamma$ with non-zero neutrino mass is estimated as

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3}{32\pi} \alpha |U_{ei}^* U_{\mu i}|^2 \left( \frac{m_{\nu_i}}{m_W} \right)^4 < 10^{-48} \left( \frac{m_{\nu_i}}{1 \text{ eV}} \right)^4,$$

where $\alpha, U_{ij}$ and $i$ denote the fine-structure constant, lepton-flavor mixing matrix and the generation index, respectively.

On the other hand, some new physics models may have sources of LFV beyond the neutrino mass, and it might be expected to observe sizable LFV effects at future experiments. No signature of new physics, however, has been found yet at the LHC experiments, so an interpretation of data at the LHC suggests that a scale of new physics is much higher than $O(\text{TeV})$. In such a case, it is known that mediation by a heavy particle at collider processes are described by four-Fermi contact interactions in a good approximation.

We study LFV process $e^+e^- \rightarrow e^+\tau^-$ via four-Fermi contact interactions at the International Linear Collider (ILC) [2]. One of the authors (G.C.C.) has studied possibilities to search for LFV contact interactions in $e^+e^- \rightarrow e^+\ell^-$ and $e^-e^- \rightarrow e^-\ell^-$ at the ILC [3] where $\ell$ stands for $\mu$ or $\tau$. It was pointed out that present bounds on LFV contact interactions for $\ell = \mu$ from the measurement of Br($\mu \rightarrow 3e$) at SINDRUM experiment [4] are stringent so that no improvement of constraints on the LFV contact interactions is expected at the ILC. Moreover, bounds on LFV interactions for $\ell = \tau$ were obtained merely by naive evaluation of signal and background cross sections at parton level.

In this paper, we introduce a selection using the variable $m_{\ell T}$ in order to reduce the background process efficiently and investigate the sensitivity of ILC for the LFV contact interaction taking account of detector effects. Moreover, we show that appropriate use of lepton beam polarizations allows to reduce SM background and thus increases the sensitivity to some of the LFV parameters, although the polarization may have negative effect on signal events for certain parameters. We show that, even at the first stage of the ILC experiment, i.e., $\sqrt{s} = 250$ GeV with the integrated luminosity 2 ab$^{-1}$ [6], the upper bounds on some LFV contact interactions could be improved by several factors compared to the previous bounds from Br($\tau \rightarrow 3e$) at the Belle experiment [5]. The improvement of the bounds could be nearly an order of magnitude for $\sqrt{s} = 1$ TeV.

There are some previous studies on this topic, e.g., refs. [7, 8]. Those works have been done in the parton level without taking account of the hadronization/reconstruction
efficiency of the $\tau$-lepton at the detector. The effect of beam polarization has not been evaluated and the analysis has been performed on only vector-type contact interactions while scalar-type interactions are examined in our study.

This paper is organized as follows. In Sec. II, we briefly review our effective Lagrangian given by LFV contact interactions and some observables at experiments. Results of numerical analysis and constraints on LFV contact interactions will be given in Sec. III. We give some discussions in Sec. IV. Sec. V is devoted to summarize our work.

2 Effective Lagrangian

The effective Lagrangian which describes the LFV process $e^+e^- \rightarrow e^+\tau^-$ via contact interactions consists of the following six operators after the Fierz rearrangement

$$L_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \left\{ g_1 (\overline{\tau_R}e_L)(\overline{e}_Re_L) + g_2 (\overline{\tau_L}e_R)(\overline{e}_Re_R) ight.$$

$$+ g_3 (\overline{\tau_R}\gamma^\mu e_R)(\overline{e}_R\gamma^\mu e_L) + g_4 (\overline{\tau_L}\gamma^\mu e_L)(\overline{e}_L\gamma^\mu e_R)$$

$$+ g_5 (\overline{\tau_R}\gamma^\mu e_R)(\overline{e}_L\gamma^\mu e_L) + g_6 (\overline{\tau_L}\gamma^\mu e_L)(\overline{e}_R\gamma^\mu e_R) \right\} + \text{h.c.},$$

(2)

where $G_F$ denotes the Fermi coupling constant, and the subscripts $L$ and $R$ represent the chirality of a fermion $f$, i.e., $f_{L(R)} \equiv \frac{1-(+)}{2} f$. The six couplings $g_i$ ($i = 1 \sim 6$) are dimensionless parameters. The first two terms in eq. (2) are the scalar-type interactions, while the others are the vector-type interactions.

In the limit of massless leptons, the spin-averaged differential cross section in the center-of-mass (CM) system for $e^+e^- \rightarrow e^+\tau^-$ is calculated from the effective Lagrangian (2) as

$$\frac{d\sigma(e^+e^- \rightarrow e^+\tau^-)}{d\cos \theta} = \frac{G_F^2s}{64\pi} \left[ (G_{12} + 16G_{34}) (1 + \cos \theta)^2 + 4G_{56} \{4 + (1 - \cos \theta)^2\} \right],$$

(3)

where the parameter $G_{ij}$ is defined as

$$G_{ij} \equiv |g_i|^2 + |g_j|^2.$$  

(4)

Integrating (3) over $\cos \theta$, the cross section is given as

$$\sigma = \frac{G_F^2s}{24\pi} \{G_{12} + 16(G_{34} + G_{56})\} \approx 4.4 \text{ fb} \left( \frac{\sqrt{s}}{250 \text{ GeV}} \right)^2 \left\{ G_{12} + 16(G_{34} + G_{56}) \right\}^{10^{-4}}$$

$$= 4.4 \text{ fb} \left( \frac{\sqrt{s}}{250 \text{ GeV}} \right)^2 \sum_{i=1}^{6} a_i \left( \frac{g_i}{10^{-2}} \right)^2,$$

(5)

where a coefficient $a_i$ is given by 1 for $i = 1, 2$ and 16 for $i = 3 \sim 6$, respectively.
The LFV process $\tau \rightarrow eee$ ($\tau \rightarrow 3e$) is also given by the same effective Lagrangian \cite{2}. The branching ratio of $\tau \rightarrow 3e$ is expressed in terms of $G_{ij}$ as

$$\text{Br}(\tau \rightarrow 3e) = \frac{\tau_\tau}{\tau_\mu} \left( \frac{m_\tau}{m_\mu} \right)^{5/3} \left( \frac{G_{12} + 16G_{34} + 8G_{56}}{8} \right),$$

\approx 0.022 \times (G_{12} + 16G_{34} + 8G_{56}), \quad (6)$$

where $\tau_\tau$ and $\tau_\mu$ are the lifetime of $\tau$ and $\mu$, respectively, and we adopt $\tau_\tau = 2.91 \times 10^{-13}$ s and $\tau_\mu = 2.20 \times 10^{-6}$ s for the numerical evaluation \cite{10}. The upper bound on $\text{Br}(\tau \rightarrow 3e)$ at 90% CL has been given by the Belle collaboration as \cite{5}

$$\text{Br}(\tau^+ \rightarrow e^- e^+ e^-) < 2.7 \times 10^{-8},$$

\quad \text{(7)}$$

and the bound (7) can be read as upper limits on the LFV couplings $G_{ij}$ as

$$\{G_{12}, G_{34}, G_{56}\} < \{1.2 \times 10^{-6}, 7.5 \times 10^{-8}, 1.5 \times 10^{-7}\}. \quad (8)$$

We give upper bounds (8) in terms of the LFV coupling $g_i$ for later convenience:

$$g_1, g_2 < 1.1 \times 10^{-3}, \quad g_3, g_4 < 2.7 \times 10^{-4}, \quad g_5, g_6 < 3.9 \times 10^{-4}. \quad (9)$$

Throughout this paper, we adopt the single-coupling dominant hypothesis to examine the upper limit on the LFV couplings. For example, the limits (9) are obtained by allowing only one coupling among six $g_i$ is finite while other five couplings are set to zero. The size of the cross section (5) at $\sqrt{s} = 250$ GeV corresponding to the limits obtained by the Belle collaboration (8) can be estimated as $5.3 \times 10^{-2}$ fb, $4.8 \times 10^{-2}$ fb and $1.1 \times 10^{-1}$ fb, respectively. The limits of branching fraction (7) and other various LFV decay modes of $\tau$-lepton are expected to improve by one or two orders of magnitude at the super-KEKB \cite{11}.

### 3 Constraints on LFV couplings at the ILC

In this section, we investigate sensitivity on the LFV couplings in (2) at the ILC. Throughout our analysis, we use MADGRAPH5_AMC@NLO \cite{12}, PYTHIA 8 \cite{13} and DELPHES 3 \cite{14} for event generations, hadronization and detector simulation, respectively. The limits on the LFV couplings are estimated using MADANALYSIS 5 \cite{15}.

In the detector simulation, we assume a detector like the ILD (International Large Detector) at the ILC, which is expected to have good performance on the reconstruction of the $\tau$-lepton. The reconstruction efficiency in leptonic decay modes could be about 99% \cite{16} in the pseudorapidity ($\eta$) range $|\eta| < 2.4$. In the hadronic decay modes such as $\tau \rightarrow \pi \nu$, $\rho(\rightarrow 2\pi)\nu$, $a_1(\rightarrow 3\pi)\nu$, the reconstruction efficiency is estimated as 95% for $\pi$ and 90% for $\rho$ and $a_1$ modes, respectively \cite{17}.
As a background to the signal process $e^+e^- \rightarrow e^+\tau^-$, we consider the SM process $e^+e^- \rightarrow e^+\nu_\tau\tau^-\bar{\nu}_\tau$. Among the diagrams generating this final state, the largest contribution comes from the diagram where the $W^-$ boson, which is radiated off the $e^-$ in $e^+e^- \rightarrow e^+e^-$, transforms into $\tau^-\bar{\nu}_\tau$. We apply several event selection conditions to reduce contribution from this background process. As an example, we compare several event distributions of the signal and background processes at $\sqrt{s} = 1$ TeV with an integrated luminosity of $L_{\text{int}} = 2$ ab$^{-1}$ in Fig. 1. Signal events in the figure are obtained for the LFV couplings $g_1 = 10^{-3}$ and $g_i = 0$ ($i = 2 \sim 6$). The first row in Fig. 1 shows the distributions of the transverse mass $m_{\ell T}$ defined as

$$
(m_{\ell T}^f)^2 = (|p_{\ell T}^f| + |p_T|)^2 - (p_{\ell T}^f + p_T)^2,
$$

where $p_{\ell T}^f$ and $p_T$ represent the transverse momentum of $\ell^-$ (= $e^-$, $\mu^-$) from the $\tau^-$ decay and the missing transverse momentum, respectively. Since the origin of $p_T$ in the signal event is neutrinos ($\nu_\tau$, $\bar{\nu}_\ell$) from the $\tau$ decay, the $m_{\ell T}^f$ distribution has a peak below $m_{\tau}$ while the background process does not have such a peak due to the extra $\nu_e$ in addition to ($\nu_\tau$, $\bar{\nu}_\ell$) from the $\tau$ decay. In the figure, we also compare the transverse momentum, energy and pseudorapidity of $e^+$ in the final state, respectively. Taking account of differences between signal and background distributions in Fig. 1, we require the following conditions on $m_{\ell T}$ and $p_T(e^+)$:

$$
m_{\ell T} \leq 10 \text{ GeV},
$$

$$
p_T(e^+) \geq 70, \ 150, \ 300 \text{ GeV} \ (\sqrt{s} = 250, 500 \text{ GeV}, \ 1 \text{ TeV}).
$$

As an example, we show the number of events in signal and background (i) before applying any cuts, (ii) requiring $m_{\ell T} \leq 10 \text{ GeV}$, (iii) further requiring $p_T(e^+) \geq 300 \text{ GeV}$, in case of $g_1 = 10^{-3}$, $\sqrt{s} = 1$ TeV and $L_{\text{int}} = 2$ ab$^{-1}$ in Table 1. With these selections, the contribution of the background process is reduced by a factor of 200 while keeping almost 60% of signal events.

|     | signal | background |
|-----|--------|------------|
| (i) | 1368   | 171120     |
| (ii)| 1167   | 9976       |
| (iii)| 779   | 801        |

Table 1: Number of events in signal and background events for $g_1 = 10^{-3}$, $\sqrt{s} = 1$ TeV and $L_{\text{int}} = 2$ ab$^{-1}$. Conditions (i)∼(iii) are: (i) no cut, (ii) $m_{\ell T} \leq 10 \text{ GeV}$, (iii) $p_T(e^+) \geq 300 \text{ GeV}$ and $m_{\ell T} \leq 10 \text{ GeV}$.

We define the statistical significance by $S/\sqrt{B}$, where $S$ and $B$ denote the number of signal and background events, respectively. We set the upper bounds on the LFV cou-
Figure 1: Event distributions of the transverse mass $m_T^\ell$, and the energy $E$, the transverse momentum $p_T$ and the pseudo rapidity $\eta$ for $e^+$. The unit of horizontal axis in $m_T^\ell$, $E(e^+)$ and $p_T(e^+)$ is GeV. Plots in the left- and the right-sides are signal and background processes, respectively. Both events are evaluated for $\sqrt{s} = 1$ TeV and $L_{\text{int}} = 2$ ab$^{-1}$. The LFV couplings in the signal events are set for $g_1 = 10^{-3}$ and $g_2 \sim g_6 = 0$.

Figure 2: $S/\sqrt{B}$ as a function of the LFV couplings $g_i$. Since, as shown in eq. (3), there are three pairs of six LFV couplings as independent parameters in the differential cross section, we show three plots for $(g_1, g_2)$, $(g_3, g_4)$ and $(g_5, g_6)$ separately. Three colored lines stand for $\sqrt{s} = 250$ GeV (green), 500 GeV (red) and 1 TeV (blue), respectively. Shaded region in three plots represent excluded range of the LFV coupling $g_i$ from the measurement of $\text{Br}(\tau \to 3 e)$ by the Belle collaboration [5]. Since the cross section of the signal process is proportional to the center-of-mass (CM) energy as shown in eq. (5), the upper bounds on the LFV couplings are much severe for larger $\sqrt{s}$. We find that better bounds can be obtained at the ILC with $\sqrt{s} = 500$ GeV and 1 TeV for all six parameters, compared to those from the Belle.
Figure 2: Statistical significance $S/\sqrt{B}$ as function of the LFV couplings. Three colored lines correspond to $\sqrt{s} = 250$ GeV (green), 500 GeV (red) and 1 TeV (blue), respectively. The integrated luminosity is fixed at $L_{\text{int}} = 2 \text{ ab}^{-1}$. The shaded region stands for the excluded range of $g_i$ from $\text{Br}(\tau \rightarrow 3e)$ at Belle [5].

The integrated luminosity is scaled by a factor $(2 \text{ ab}^{-1}/L_{\text{int}})^{1/4}$ since the significance $S/\sqrt{B}$ is proportional to $\sqrt{L_{\text{int}}}$ while the number of signal events $S$ is proportional to the LFV coupling $g_i^2$.

Next we discuss the possibility to use polarized initial beams at the ILC. In some of the SM background processes, the initial $e^-$ couples to the $W$-boson. Such processes could be suppressed efficiently by polarizing the $e^-$ beam to be right-handed since $W$-boson couples only to left-handed fermions. As the SM background process is dominated by the initial helicity state $e^-_L + e^+_R$ (97% at $\sqrt{s} = 250$ GeV and 92% at $\sqrt{s} = 500$ GeV or 1 TeV), the ILC has a stronger sensitivity to couplings with $e^-_R$ and $e^+_L$ where SM contribution is highly suppressed.

In Fig. 3, we give the significance $S/\sqrt{B}$ for each LFV couplings with polarized electron beam. We set the polarization of the $e^-$ beam to 80% (hereafter we denote it as $P(e^-) = 0.8$). With this condition, the cross section of the background process is reduced to 17 fb from 86 fb in the unpolarized case. The upper bounds on some of LFV couplings at $S/\sqrt{B} = 2$ are much improved as compared to the unpolarized case (Fig. 2) due to the suppression of the background processes mediated by the $W$-boson as we expected. For example, the upper limits on $g_2$, $g_3$ and $g_8$ are better than those from the Belle experiment even for $\sqrt{s} = 250$ GeV. Those improvements could be an order of magnitude for $\sqrt{s} = 1$ TeV. On the other hand, the upper limits on the LFV couplings $g_1$ and $g_4$ are worse than the unpolarized case. Since the chirality of initial electron is left-handed in the operators with the coupling $g_1$ or $g_4$, these operators do not contribute to the LFV processes when the electron is polarized to be right-handed, thus causing suppression of
the event rate. We also show the results where both $e^-$ and $e^+$ beams are polarized as $P(e^-) = 0.8$, $P(e^+) = -0.3$ in Fig. 4. The limits on $g_1 g_3$ and $g_6$ are marginally better than the previous case ($P(e^-) = 0.8$) while $g_2$, $g_4$ and $g_5$ are worse. The upper limits of the LFV couplings at 95% CL in all cases (unpolarized, polarized beams) are summarized in Table 2.

|                  | $g_1$      | $g_2$      | $g_3$      | $g_4$      | $g_5$      | $g_6$      |
|------------------|------------|------------|------------|------------|------------|------------|
| Belle [5]        | $1.1 \times 10^{-3}$ | $2.7 \times 10^{-4}$ | $3.9 \times 10^{-4}$ |           |            |            |
| Belle II [18]    | $1.3 \times 10^{-4}$ | $3.2 \times 10^{-5}$ | $4.7 \times 10^{-5}$ |           |            |            |

Table 2: Summary of the upper limits on the LFV couplings at 95% CL. The limits for $\sqrt{s} = 1$ TeV in ref. [3] are shown for comparison in the row with the symbol $\star$.

4 Discussions

Results of our study are summarized in Fig. 5. In the figure, we compare the upper limits on the couplings from $\tau \rightarrow 3e$ with three cases in the ILC experiment; (i) unpolarized beam, (ii) polarized $e^-$ beam with $P(e^-) = 0.8$, and (iii) polarized $e^-$ and $e^+$ beams with $P(e^-) = 0.8$ and $P(e^+) = -0.3$. The CM energy dependence ($\sqrt{s} = 250$ GeV, 500 GeV and 1 TeV) of the upper limits is also shown in each plot of $g_i$.

Fig. 5 tells us that the ILC with unpolarized beam can reach small LFV couplings for $\sqrt{s} = 1$ TeV compared to the limits from the Belle experiment. Even for lower energy such as $\sqrt{s} = 250$ GeV, as planned as a first stage of the ILC experiment, requiring the initial $e^-$ beam to be right-handed makes the experiment sensitive to smaller LFV couplings for $g_2$, $g_3$, $g_5$ and $g_6$ in the effective Lagrangian [2]. The sensitivity of the ILC for the LFV contact interactions might be competitive with the LFV search at the super-KEKB.
Figure 3: $S/\sqrt{B}$ as a function of the LFV coupling $g_i$ with the $e^-$ beam polarization $P(e^-) = 0.8$. The description on the colored lines and shaded region are the same with those in Fig. 2.

experiment. The expected upper limits on the branching fractions of the $\tau$ LFV decays at the Belle II can be found in [18], which are extrapolated from Belle results assuming $L_{\text{int}} = 50 \text{ ab}^{-1}$. The upper limit of $\text{Br}(\tau \to 3e)$ in [7] at the Belle experiment, which has been obtained using $L_{\text{int}} = 782 \text{ fb}^{-1}$ [5], will be improved to be $\text{Br}(\tau \to 3e) \sim 4.2 \times 10^{-8}$. Then, the limits on the LFV couplings $g_i$ are $\sqrt{4.2 \times 10^{-10}/2.7 \times 10^{-8}} \sim 0.12$ times smaller than those from the Belle experiment. We give the expected upper limits of $g_i$ at the Belle II experiment in Table 2. We find that, for $\sqrt{s} = 1 \text{ TeV}$ and $P(e^-) = 0.8$, the upper limits of $g_5$ at the ILC is slightly better than that of Belle II while $g_2$ and $g_3$ are competitive. If the $e^+$ beam is polarized in addition to $e^-$, i.e., $P(e^-) = 0.8$ and $P(e^+) = -0.3$, the limits on $g_2, g_3$ and $g_5$ at the ILC with $\sqrt{s} = 1 \text{ TeV}$ are competitive with the Belle II at 50 $\text{ ab}^{-1}$.

We briefly discuss the advantage to use the polarization beam for the LFV search at the ILC. Throughout our analysis, we have been based on so called the single-coupling dominant hypothesis, in which only one of six LFV couplings is finite and the rest are assumed to be zero. However, to be realistic, it is necessary to study the case where multiple LFV couplings exist. For example, let us assume that a scalar particle mediates the LFV processes so that $g_1, g_2 \neq 0$ and $g_i = 0 (i = 3 \sim 6)$ in the effective Lagrangian [2].
Figure 4: $S/\sqrt{B}$ as a function of the LFV coupling $g_i$ with the beam polarization $P(e^-) = 0.8$ and $P(e^+) = -0.3$. The description on the colored lines and shaded region are the same with those in Fig. 2.

We compare in Fig. 3 the upper limits at 95% CL on $(g_1, g_2)$ plane from (i) unpolarized beam, (ii) $P(e^-) = 0.8$, $P(e^+) = 0$, and (iii) $P(e^-) = 0.8$, $P(e^+) = -0.3$ for $\sqrt{s} = 250$ GeV. As shown in the figure, the unpolarized beam constrains the combination $g_1^2 + g_2^2$. The polarized beam, however, constrains the LFV coupling $g_2$ much severe than the unpolarized case. This demonstrates that the beam polarization at the ILC can achieve different sensitivities to $g_1$ and $g_2$ by a cross-section measurement.

We have so far discussed constraints on the LFV processes at the ILC without considering specific new physics models. It is easy to read our constraints on the LFV couplings as those on new physics models with scalar or vector mediator. Suppose the following interactions of a flavored scalar $S$ or $Z'$:

$$\mathcal{L} \supset y_\ell^\alpha \bar{\ell} P_\alpha e S, \quad q_\ell^a g_{Z'} \bar{e} \gamma^\mu P_\alpha e Z'^\mu, \quad (13)$$

where $\ell$ represents $e$ or $\tau$, and $\alpha = L, R$ denotes the chirality of the electron. The gauge coupling of $Z'$ boson is denoted by $g_{Z'}$, and $y_\ell^a$ and $q_\ell^a$ are dimensionless couplings. In terms of couplings in eq. (13), the LFV couplings in the effective Lagrangian (2) can be
Figure 5: Summary of limits on the LFV couplings $g_i$ for $\sqrt{s} = 250$ GeV, 500 GeV and 1 TeV. The bounds from the Belle experiment are shown in blue for comparison. The limits on $g_i$ are shown for three cases: unpolarized beam (red), polarized $e^-$ beam with $\mathcal{P}(e^-) = 0.8$ (orange), polarized both $e^-$ and $e^+$ beams with $(\mathcal{P}(e^-), \mathcal{P}(e^+)) = (0.8, -0.3)$ (green).

expressed as

$$g_i \approx 0.031 \times \frac{y^\alpha y^\beta}{(m_S/1\text{ TeV})^2} \quad (i = 1, 2)$$

$$\approx 0.031 \times g^2_{Z'} \frac{q^\alpha q^\beta}{(m_{Z'}/1\text{ TeV})^2} \quad (i = 3 \sim 6),$$

where $m_S$ and $m_{Z'}$ denote the scalar and $Z'$ boson mass, respectively.

Our results are valid for a mediator mass $M$ much larger than the momentum transfer $q$, i.e. $q^2 \ll M^2$. We adopted this condition because no signature of new physics has been found at the LHC yet. However, if the mediator particle is baryophobic, it could not be produced at the LHC and its mass can be as small as the momentum transfer at the ILC. Then properties of the mediator particle could be studied at the ILC using, for example, energy and/or angular distributions of the final states. Such a scenario might be another possibility of the LFV processes at the ILC but beyond the scope of our paper.

5 Summary

We have studied the possibility of the ILC to search for the LFV process $e^+e^- \rightarrow e^+\tau^-$ via the four-Fermi contact interactions. Taking account of event selection conditions and
Figure 6: Constraints on the scalar LFV couplings $g_1$ and $g_2$ for $\sqrt{s} = 250$ GeV. The solid, dotted and dashed lines correspond to the unpolarized beam, $P(e^-) = 0.8$, $P(e^+) = 0$, and $P(e^-) = 0.8$, $P(e^+) = -0.3$, respectively.

polarization effects of the initial $e^-$ beam, we found that the ILC could improve the upper bounds on the LFV couplings from the the measurement of $\text{Br}(\tau \rightarrow 3e)$ by the Belle collaboration. In these couplings, several factors of improvement is expected for $\sqrt{s} = 250$ GeV, and the improvement becomes an order of magnitude for $\sqrt{s} = 1$ TeV with beam polarization.

It is important to discriminate the dominant LFV couplings $g_1 \sim g_6$ once the LFV process is found in some experiments. This can be partly achieved by investigating the signal strength with different beam polarization as we demonstrated for the scalar couplings $g_1$ and $g_2$. We note that there are other proposals for discrimination on the LFV contact interactions using the polarization of $\tau$-lepton in the $\tau^+\tau^-$ production process [19, 20].

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**References**

[1] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81**, 1562 (1998) [hep-ex/9807003].
[2] H. Baer et al., “The International Linear Collider Technical Design Report - Volume 2: Physics,” arXiv:1306.6352 [hep-ph].

[3] G. C. Cho and H. Shimo, Mod. Phys. A 32, no. 24, 1750127 (2017) arXiv:1612.07476 [hep-ph].

[4] U. Bellgardt et al. [SINDRUM Collaboration], Nucl. Phys. B 299, 1 (1988).

[5] K. Hayasaka et al., Phys. Lett. B 687, 139 (2010) arXiv:1001.3221 [hep-ex].

[6] K. Fujii et al., arXiv:1710.07621 [hep-ex].

[7] P. M. Ferreira, R. B. Guedes and R. Santos, Phys. Rev. D 75, 055015 (2007) hep-ph/0611222.

[8] B. Murakami and T. M. P. Tait, Phys. Rev. D 91, 015002 (2015) arXiv:1410.1485 [hep-ph].

[9] Y. Kuno and Y. Okada, Rev. Mod. Phys. 73, 151 (2001) hep-ph/9909265.

[10] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).

[11] A. J. Bevan et al. [BaBar and Belle Collaborations], Eur. Phys. J. C 74, 3026 (2014) arXiv:1406.6311 [hep-ex].

[12] J. Alwall et al., JHEP 1407, 079 (2014) arXiv:1405.0301 [hep-ph].

[13] T. Sjostrand, S. Mrenna and P. Z. Skands, Comput. Phys. Commun. 178, 852 (2008) arXiv:0710.3820 [hep-ph].

[14] J. de Favereau et al. [DELPHES 3 Collaboration], JHEP 1402, 057 (2014) arXiv:1307.6346 [hep-ex].

[15] E. Conte, B. Fuks and G. Serret, Comput. Phys. Commun. 184, 222 (2013) arXiv:1206.1599 [hep-ph].

[16] T. Behnke et al., arXiv:1306.6329 [physics.ins-det].

[17] T. H. Tran, V. Balagura, V. Boudry, J. C. Brient and H. Videau, Eur. Phys. J. C 76, no. 8, 468 (2016) arXiv:1510.05224 [physics.ins-det].

[18] E. Kou et al. [Belle II Collaboration], “The Belle II Physics Book,” arXiv:1808.10567 [hep-ex].
[19] R. Kitano and Y. Okada, Phys. Rev. D 63, 113003 (2001) [hep-ph/0012040].

[20] T. Goto, Y. Okada and Y. Yamamoto, Phys. Rev. D 83, 053011 (2011) [arXiv:1012.4385 [hep-ph]].