Relay-Aided MIMO Cellular Network Using Opposite Directional Interference Alignment

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Abstract

In this paper, we first propose interference alignment (IA) scheme for uplink transmission of multiple-input-multiple-output (MIMO) cellular network with a help of relay which operates in half-duplex mode. The proposed scheme only requires global channel state information (CSI) knowledge at relay and no transmitter beamforming and time extension is required at user equipment (UE), which differs from the conventional IA schemes for cellular network. We derive the feasibility condition of the proposed scheme for the general network configuration and analyze the degrees-of-freedom (DoF) performance of the proposed IA scheme. Extension of proposed scheme for downlink and full-duplex network are further described in this paper. The sum rate performance of the proposed scheme is compared to that of the conventional IA scheme for cellular network through numerical analysis. It is also shown that the same advantage as the uplink case can be obtained for downlink case through relay induced interfering multiple access channel (IMAC) and interfering broadcast channel (IBC) duality. Furthermore, full-duplex network is shown to have same advantages with half-duplex cases.

Index Terms

Beamforming, cellular network, channel state information (CSI), degrees-of-freedom (DoF), downlink, full-duplex network, half-duplex relay, interference alignment (IA), multiple-input-multiple-output (MIMO), time extension, uplink.

I. INTRODUCTION

It has been known that high degrees-of-freedom (DoF) can be achieved in the multi-user interference environment by interference alignment (IA). The optimal DoF in single-input-single-
output (SISO) time varying interference channel through asymptotic IA scheme is derived in [1].

Researches of IA in multiple-input-multiple-output (MIMO) channels also have been done, and
the optimal DoF region for certain multi-user MIMO interference channel was revealed [2],
which means that IA can also increase the DoF of MIMO communication systems.

Recently, IA scheme for cellular network has been researched for DoF improvement. The
subspace IA scheme for uplink transmission of SISO cellular network was suggested in [3],
whose result states that the optimal DoF of 1 can be achieved. But, subspace IA scheme requires
more time extension as the number of users increases. Thus the transmission/reception delay of
the information can take long time when the number of users in the network becomes large,
which is not suitable for cellular network.

Conventional IA schemes cannot be applied to cellular network because the channel state
information (CSI) feedback to transmitter and time extension are required in cellular network.
In general, user equipment (UE) is practically not able to handle/process beamforming by CSI
feedback and communicating nodes requires small transmission/reception delay. In order to
reduce the CSI feedback and time extension, the relay-aided IA schemes were suggested, where
the relay handles CSI instead of transmitter. It is shown that relay can potentially reduce the time
extension requirement in SISO interference channel [4]. But it still needs to calculate beamformer
at transmitter which leads to CSI feedback to transmitter and requires one low-powered relay
per one receiver each which leads to heavy cost to network. [7] suggests two time slot using
IA scheme without CSIT feedback for symmetric MIMO interference channel with optimal
DoF performance. But it also requires more than two time extensions and even cannot achieve
optimal DoF for asymmetric MIMO environment. To overcome prementioned limits, author of
[13] suggested an IA scheme named as opposite directional interference alignment(ODIA) to
reduce the number of relay to one(only one relay serves the entire network) and to remove CSIT
feedback requirement. It is shown that ODIA in interference channel can also achieve optimal
DoF.

There had been a large interest in constituting good IA schemes on cellular network re-
cently, Paper [9] and [10] reveals achievable DoF through their proposing scheme consid-
ering semi-practical cellular network with directional antenna or omni-dirctional antenna on
BSs(basestation). But, these schemes somehow cannot be directly attached on general cellular
network. In [11], optimal degrees of freedom and its achieving scheme was revealed in certain
condition of symmetric network with help of [5]. But, it also requires asymptotic IA scheme to achieve its optimal bound in large network which is not rather practical on cellular network. Furthermore, [12] suggests one-shot linear IA scheme based on subspace alignment, but finding and feeding back optimizing value to align interferences in a good manner may lead to heavy load on UEs.

Some researchs focused on possibilities of relay on cellular network IA problem. Authors of [15] proposed IA scheme on two-cell downlink transmission using group of half-duplex relay based on DF(Decode-and-Forward) protocol. But due to the increase of precessing complexity by DF based relaying(see [8]), DF protocol is out of our interest. [16] shows that in quasi-static flat fading cellular environment with a full-duplex relay, symmetric MIMO cellular network can achieve higher DoF with finite time extension. But the scheme from [16] has its own limit which is that the time extension grows linearly with the number of users in network which requires very long coherence time in cellular network. Large requirement for coherence time is very unpractical due to the mobility of cellular user makes channel variation. And still requires simple transmit beamforming through extended symbol leading heavy load to UEs. Practically, relay echo problem due to the full-duplex operation at relay can severely harm the performance of the network.

To solve prementioned problems of constituting IA scheme for MIMO cellular network, we suggest IMAC-ODIA scheme for uplink transmission with the help of half-duplex AF relay. Only two time slots needed regardless of network size for proposed scheme to achieve interference alignment. It has its advantage of no time extension needed and able to operate in generic channel. We evaluate the DoF performance of proposed scheme and show that with certain antenna configuration, proposed scheme achieves optimal DoF of MIMO cellular network. Furthermore, for large network, feasibility condition of proposed scheme might lead to heavy load on relay. To solve prementioned problems, we consider a new cellular topology for cellular network differ from those of [9] and [10]. Feasibility condition reduction for IA is a main concern for industrial reasons. [17] suggested an user clustering algorithm to simplify IA feasibility problem on practical environment case. Just like scenarios in [17], bottleneck of the network performance is heavily controlled by cell boundary users in practical cellular network. In this paper, we apply our proposing IMAC-ODIA schemes on prementioned three cell cluster considering presence of boundary users and show feasibility conditions and DoF performance
for this topology.

For cellular downlink transmission, frequency division based transmission method is used due to simplicity of handling multiple user which cannot expect any DoF gain. Due to the requirement of high throughput in downlink transmission, downlink IA was considered. And also known to be able to achieve among the duality between uplink and downlink on cellular network. But, heavy load on constructing decorrelator on UE due to conventional IA schemes made it unprobable to apply conventional IA schemes on downlink transmission. So, in [19], a simplified IA scheme was proposed for cellular downlink transmission with only inner-cell feedback. This provides possibility for applying IA scheme on downlink transmission.

In [20], it was proven that also relay-aided cellular network has its uplink-downlink duality satisfied. With uplink-downlink duality, we also propose a downlink pointed view of IMAC-ODIA, which leads to IBC-ODIA with similar feature to that of IMAC-ODIA. No beamforming requirement on UEs. DoF performance and description of IBC-ODIA scheme are also stated in this paper.

This paper is organized as follows. In Section II, we present the preliminaries for the properties of tensor product which is used on describing our proposed scheme. In Section III, system model of cellular network with a relay is introduced and IA condition of IMAC-ODIA is stated in Section IV. Section V describes proposed schemes in detail. IBC-ODIA for cellular downlink transmission is described on Section VI and Clustering topology is shown at Section VII. Finally, Section VIII and IV are consisted with simulation result and conclusion.

II. Preliminaries

In this section, we introduce several properties of tensor product of matrices in [14]. Throughout the paper, scalars are denoted by lowercase letters \((a, b, \ldots)\), vectors are written in bolfase lowercase \((a, b, \ldots)\), and matrices correspond to boldface capitals \((A, B, \ldots)\). The entry with row index \(i\) and column index \(j\) in a matrix \(A\), i.e., \((A)_{ij}\), is symbolized by \(a_{ij}\) (also \((a)_{i} = a_{i}\)). If no confusion is possible, the \(i\)th column vector of a matrix \(A\) is denoted as \({A}\}_{i}, i.e., A = [{A}_1, {A}_2, \ldots]\ and \({A}\}_{i;j} denotes the column vector set matrix, i.e., [{A}_1, {A}_{i+1}, \ldots, {A}_j]. The rank of a matrix \(A\) will be denoted by rank\((A)\) or \(r_A\). The superscripts \(\cdot^T, \cdot^H, \cdot^\dag\) denote the transpose, complex conjugated transpose and Moore-Penrose pseudoinverse, respectively. The
The $(I \times J)$ zero matrix is denoted by $0_{I \times J}$ (also zero column vector of size $I$ is denoted by $0_I$) also, subscript can be omitted when the size of zero matrix isn’t important in our development. Similarly, identity matrix of size $I \times I$ is denoted as $I_I$ and rectangular identity matrix $I_{I \times J}$ is denoted as $I_{I \times J}$ for $I \geq J$. For field notation, we use $\mathbb{K}$ to denote $\mathbb{R}$ or $\mathbb{C}$.

A. Kronecker product and Khatri-Rao product

In this subsection, we will briefly introduce pre-defined tensor product of matrices, Kronecker and Khatri-Rao product. Kronecker product of $A$ and $B$ is defined below

**Definition 1:** $A \otimes B$ is the Kronecker product of $A$ and $B$ which is defined as

$$
A \otimes B = \begin{pmatrix}
a_{11}B & a_{12}B & \cdots \\
a_{21}B & a_{22}B & \cdots \\
& & \ddots
\end{pmatrix}
$$

(1)

Let, $A = [A_1, ..., A_R]$ and $B = [B_1, ..., B_R]$ be two partitioned matrices with equal number of partition. Then the Khatri-Rao product of $A$ and $B$ is defined as the partitionwise Kronecker product as below.

**Definition 2:** $A \odot B$ is the Khatri-Rao product of partitioned matrices $A$ and $B$ which is defined as

$$
A \odot B = \left( A_1 \otimes B_1 \ldots A_R \otimes B_R \right)
$$

(2)

Following subsection introduces the Kruskal rank and generalized Kruskal rank for partitioned matrix.

B. Kruskal rank and generalized Kruskal rank

In this subsection, Kruskal and generalized Kruskal rank are introduced. Kruskal rank of $A$ is defined below

**Definition 3:** The Kruskal rank or $k$-rank of a matrix $A$, denoted by $\text{rank}_k(A)$ or $k_A$, is the maximal number $r$ such that any set of $r$ columns of $A$ is linearly independent.
Let, $A = [A_1, \ldots, A_R]$ be partitioned matrix. Then generalized Kruskal rank of partitioned matrix $A$ is defined below.

**Definition 4:** The generalized Kruskal rank of a partitioned matrix $A$, denoted by $\text{rank}_{k'}(A)$ or $k'_A$, is the maximal number $r$ such that any set of $r$ submatrices of $A$ yields a set of linearly independent columns.

We introduce some propositions for generalized kruskal rank from [14], which would be used in further sections.

**Proposition 1 (Generalized k-rank of uniformly partitioned generic matrix):** Let $A = [A_1, \ldots, A_R]$ be generic matrix partitioned in $R$ submatrices with $A_r \in \mathbb{K}^{I \times L}$. Then, generalized k-rank of $A$ is $\min(\lfloor \frac{I}{L} \rfloor, R)$

**Proposition 2 (Lemma 3.2, [14]):** Consider partitioned matrices $A = [A_1, \ldots, A_R]$ with $A_r \in \mathbb{K}^{I \times L_r}$ and $B = [B_1, \ldots, B_R]$ with $B_r \in \mathbb{K}^{J \times M_r}$ with $1 \leq r \leq R$. Then,

1) If $k'_A = 0$ or $k'_B = 0$, then $k'_{A \odot B} = 0$.
2) If $k'_A \geq 1$ and $k'_B \geq 1$, then $k'_{A \odot B} \geq \min(k'_A + k'_B - 1, R)$.

### III. System Model: Cellular Network with a Single Relay

In this section, we describe cellular network with a single relay serving whole network. Throughout the paper, relay transmits in half-duplex mode with its AF relaying protocol. We first focus on uplink transmission phase in cellular network which can be modeled as an IMAC(Interfering Multiple Access Channel) environment. Here, we are starting from the symmetric network.

**A. Degrees of Freedom of Cellular Network**

In this paper, the information theoretic quantity of our interest is the DoF. DoF of a user is defined as the number of sucessfully decodable data streams in desired receiver that are transmitted by the corresponding user. The term ”sucessfully decodable” means that the desired data stream are received in the directions independent of those on which the other streams received, i.e., the total number of interference free directions(also can be referred as interference free dimension).
For cellular network, DoF per cell is our interest and it is defined as interference-free dimension at the base station as receiver or transmitter. Once, the DoF per cell be calculated through the paper, it can be used as a guide to determine the number of users to be scheduled per cell in a MIMO cellular network served by half-duplex based relay.

Finally, the DoF analysis carried out in this paper assumes a isolated cluster of $C$ cells with a single relay serving whole network. In reality, the spacial coverage of relay can be limited due to the power limit and channel attenuation. So, numerous relays should be deployed for whole network, one for each cluster. Clusters are consisted with co-locating cells that can be covered by a single relay. Also interference management for cluster should consider with the presence of out-of-cluster(inter-cluster) interferences caused by cell boundary users. The IA scheme and communication protocol considering boundary users will be depicted in latter section.

**B. System Configuration**

We first focus on uplink phase only, downlink phase will be considered latter. So, consider a fully connected symmetric network with $C$ cells and $K$ users in each cell. Cell $j$ is served by a
single base station denoted as BS\(_j\) for \(j \in \{1, \ldots, C\}\). User \((k, i)\) denoted as UE\(_{(k,i)}\) where \(k \in \{1, \ldots, K\}\) is served by BS\(_i\), as shown in Figure 1. Each UE is assumed to have \(M\) antennas, each BS is assumed to have \(N\) antennas and relay is assumed to have \(N_R\) antennas. The channel from UE\(_{(k,i)}\) to BS\(_j\) is denoted as the \(N \times M\) matrix \(H_{j,(k,i)}\). UE to relay channel from UE\(_{(k,i)}\) to relay is denoted as the \(N_R \times M\) matrix \(H_{UR}^{(k,i)}\) and relay to BS channel from relay to BS\(_j\) is denoted as the \(N \times N_R\) matrix \(H_{RB}^{j}\). In our proposing scheme, only relay has access on global channel information, BS only has its relating channel, details will be depicted on next section.

The channel is Rayleigh fading and generic. Coefficients in the channel matrices are complex independent identically Gaussian distributed random variables with zero mean and unit variance. Due to the average power constraint in cellular networks, the data vector from UE\(_{(k,i)}\) is normalized such that the average transmit power of the user is limited by \(P\), i.e., \(E[||x_{k,i}||^2] \leq P\), where \(x_{k,i}\), \(E[\cdot]\) and \(||\cdot||\) are the data column vector sent by UE\(_{(k,i)}\), expectation and norm functions, respectively.

We consider two-hop half-duplexing communication scenario. \(y_{j,1}\) and \(y_R\) which are the received signal vector of BS\(_j\) and relay at 1\(^{st}\) time slot are,

\[
y_{j,1} = \sum_{i=j, k=1}^{K} H_{j,(k,i)} x_{k,i} + \sum_{i \neq j, k=1}^{K} H_{j,(k,i)} x_{k,i} + n_{j,1}, \forall j \in \{1, \ldots, C\} \tag{3}
\]

\[
y_R = \sum_{i=1}^{C} \sum_{k=1}^{K} H_{UR}^{(k,i)} x_{k,i} + n_R \tag{4}
\]

\(n_{j,1} \in \mathbb{C}^{N \times 1}\) and \(n_R \in \mathbb{C}^{N_R \times 1}\) are zero-mean unit-variance circularly symmetric additive white Gaussian noise vectors at BS\(_j\) and relay respectively. The received signal vector of BS\(_j\) at 2\(^{nd}\) time slot is,

\[
y_{j,2} = H_{j}^{RB} T y_R + n_{j,2}, \forall j \in \{1, \ldots, C\} \tag{5}
\]

\(n_{j,2} \in \mathbb{C}^{N \times 1}\) is also zero-mean unit-variance circularly symmetric additive white Gaussian noise vector at BS\(_j\). \(T \in \mathbb{C}^{N_R \times N_R}\) is the relay beamforming matrix. Relay beamformer is implemented based on global channel information.

Substituting (4) for \(y_R\) in (5) leads to,
Fig. 2: Interference alignment from relay

\[ y_{j,2} = \sum_{i=j, k=1}^{K} H_{j}^{RB} H_{k,i}^{UR} x_{k,i} + \sum_{i \neq j, k=1}^{K} H_{j}^{RB} H_{k,i}^{UR} x_{k,i} + (H_{j}^{RB} T_{n} + n_{j,2}), \forall j \in \{1, \ldots, C\} \quad (6) \]

IV. INTERFERENCE ALIGNMENT CONDITION FOR PROPOSING SCHEME

We first describe the key idea of proposed scheme. Then we simplify the equations (3) and (6) using augmented channel matrices which will be used for further development. Finally, IA conditions to be satisfied for our proposing scheme will be described.

A. Key Idea: Align Interferences only at Relay

The key idea of conventional IA is to align interferences in a subspace of whole signal space that are linearly independent with desired signal space by designing beamformer at transmitter. Then, through constructing correct decorrelator at the receiver side, aligned interferences can be nulled perfectly. But, designing beamformer at transmitter requires CSIT feedback and computing complexity. In cellular system, UE are not likely to have these spec.

So key idea of our proposing scheme is this. Output interference vector of direct channel(received signal 1st time slot) and output interference vector of relay(received signal 2nd time
slot) sum to zero by designing only relay beamforming matrix which is depicted in Figure 2. It implies no CSIT feedback nor complex computing required at UE.

B. Channel Matrix and Data Vector Augmentation

Through augmentation of matrices and vectors, IA description can be simplified. The augmented channels for BS\textsubscript{j} are $\bar{H}_j$, $\bar{H}_j^{UR}$, $\hat{H}_j$ and $\hat{H}_j^{UR}$ for interfering channel set, interfering UE to relay channel set, desiring channel set and desiring UE to relay channel set respectively. Detail description of matrices are following,

$$\bar{H}_j = \begin{bmatrix} H_{j,1,1} & \ldots & H_{j,1,K_j-1} & H_{j,1,j+1} & \ldots & H_{j,1,K,C} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H_{j,K_j-1,1} & \ldots & H_{j,K_j-1,K_j-1} & H_{j,K_j,j+1} & \ldots & H_{j,K_j,K,C} \\ \end{bmatrix} \in \mathbb{K}^{N \times (C-1)KM}$$ (7)

$$\hat{H}_j = \begin{bmatrix} H_{j,1,j} & \ldots & H_{j,K,j} \end{bmatrix} \in \mathbb{K}^{N \times KM}$$ (8)

$$\bar{H}_j^{UR} = \begin{bmatrix} H_{1,1}^{UR} & \ldots & H_{1,K_j-1}^{UR} & H_{1,j+1}^{UR} & \ldots & H_{1,K,C}^{UR} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H_{K_j-1,1}^{UR} & \ldots & H_{K_j-1,K_j-1}^{UR} & H_{K_j,j+1}^{UR} & \ldots & H_{K_j,K,C}^{UR} \\ \end{bmatrix} \in \mathbb{K}^{N_R \times (C-1)KM}$$ (9)

$$\hat{H}_j^{UR} = \begin{bmatrix} H_{1,j}^{UR} & \ldots & H_{K,j}^{UR} \end{bmatrix} \in \mathbb{K}^{N_R \times KM}$$ (10)

Also, the data stream vectors from UEs should be augmented form with same manner which would be described as,

$$\bar{x}_j = \begin{bmatrix} x_{1,1} \\ \vdots \\ x_{K_j-1} \\ x_{1,j+1} \\ \vdots \\ x_{K,j} \end{bmatrix} \in \mathbb{K}^{(C-1)KM \times 1}, \quad \hat{x}_j = \begin{bmatrix} x_{1,j} \\ \vdots \\ x_{K,j} \end{bmatrix} \in \mathbb{K}^{KM \times 1}$$ (11)

$\bar{x}_j$ and $\hat{x}_j$ represents the interfering data stream and desired data stream each.

With augmented matrices and vectors, (3) and (6) can be simply described as below.

$$y_{j,1} = \hat{H}_j \hat{x}_j + \bar{H}_j \bar{x}_j + n_{j,1}, \forall j \in \{1, \ldots, C\}$$ (12)
\[
y_{j,2} = H_{RBj}^T (\hat{H}_{j}^{UR} \hat{x}_j + \hat{H}_{j}^{UR} \bar{x}_j) + (H_{RBj}^R n_j + n_{j,2}), \forall j \in \{1, \ldots, C\} \tag{13}
\]

C. Interference Alignment Condition

Ignoring the noise term and adding (12) and (13), the total received signal vector becomes,

\[
y_{j,1} + y_{j,2} = (\tilde{H}_{j} + H_{RBj}^R \tilde{H}_{j}^{UR}) \hat{x}_j + (\tilde{H}_{j} + H_{RBj}^R \tilde{H}_{j}^{UR}) \bar{x}_j, \forall j \in \{1, \ldots, C\} \tag{14}
\]

In (14), if effective channel of interference signal goes to zero, total received effective interference vanishes without any further process. So, the IMAC-ODIA condition would be described as below.

\[
\tilde{H}_{j} + H_{RBj}^R \tilde{H}_{j}^{UR} = 0, \forall j \in \{1, \ldots, C\} \tag{15}
\]

When the condition (15) is satisfied, the interference term in (14) vanishes which means no further cooperation between base stations is needed. It means that not only the global CSIT knowledge but also global CSIR knowledge are unnecessary due to the relay beamforming. This will support our motive of applying the scheme of [13] in cellular network. Now, our goal is finding relay beamformer satisfying \( C \) equations of (15). The relay beamformer \( T \) design and its existence condition would be revealed in next section.

V. PROPOSED SCHEME: CELLULAR OPPOSITE DIRECTIONAL INTERFERENCE ALIGNMENT FOR UPLINK (IMAC-ODIA)

Here, the proposed relay-aided cellular IA scheme is explained in detail. We present the detail of relay beamformer satisfying the IA conditions mentioned in previous section and analysis for achievable DoF of the cellular network through our proposing scheme.

A. Existence of Relay Beamformer and Its Design

The following theorem provides an lower bound on required number of antennas at relay to implement IMAC-ODIA.

\textit{Theorem 1 (Required number of antennas at relay):} The relay beamformer \( T \) satisfying condition (15) exists if and only if \( N_R \geq \max\{(C - 1)KM, CN\} \)
The rest of this subsection is devoted to the proof of Theorem 1. We first introduce an important property of kronecker product which simplifies the matrix equation.

**Proposition 3 (Convenient representation for matrix equation from Kronecker product):** Consider matrices $A, B, C$ and $X$ and matrix equation (16).

$$AXB = C$$ (16)

(16) can be transformed to (17) by attaching vectorization on both side which is,

$$\text{vec}(AXB) = (B^T \otimes A)\text{vec}(X) = \text{vec}(C)$$ (17)

Here, $\text{vec}(X)$ denotes the vectorization for matrix $X$ formed by stacking the columns of $X$ into a single column vector.

Then, (15) can be transformed by (17) as below,

$$\{(H_{UR}^j)^T \otimes H_{RB}^j\}\text{vec}(T) = \text{vec}(-H_j), \forall j \in \{1,\ldots,C\}$$ (18)

Total $C$ equations of (18) can be represented as a single matrix equation (19).

$$H \times \text{vec}(T) = h$$ (20)

Let $H = \begin{bmatrix} (H_{UR}^1)^T \otimes H_{RB}^1 \\ (H_{UR}^2)^T \otimes H_{RB}^2 \\ \vdots \\ (H_{UR}^C)^T \otimes H_{RB}^C \end{bmatrix}$ and $h = \begin{bmatrix} \text{vec}(-H_1) \\ \text{vec}(-H_2) \\ \vdots \\ \text{vec}(-H_C) \end{bmatrix}$ then, (19) can be represented as (20).

Our goal is to find $\text{vec}(T)$ satisfying (20), which is equivalent to the fact that (20) has solution. Due to the condition for existence of solution for linear equation, following should be satisfied.
\[
\begin{align*}
\mathbf{H}^R = \begin{bmatrix}
\mathbf{H}_{1,1}^{UR} & \mathbf{H}_{2,1}^{UR} & \cdots & \mathbf{H}_{K,1}^{UR} \\
\mathbf{H}_{2,1}^{UR} & \mathbf{H}_{1,2}^{UR} & \cdots & \mathbf{H}_{K,2}^{UR} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{H}_{K,1}^{UR} & \mathbf{H}_{K,2}^{UR} & \cdots & \mathbf{H}_{C,K}^{UR}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathbf{H}^F = \begin{bmatrix}
\mathbf{H}_{1,1}^{UR} & \mathbf{H}_{2,1}^{UR} & \cdots & \mathbf{H}_{K,1}^{UR} \\
\mathbf{H}_{2,1}^{UR} & \mathbf{H}_{1,2}^{UR} & \cdots & \mathbf{H}_{K,2}^{UR} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{H}_{K,1}^{UR} & \mathbf{H}_{K,2}^{UR} & \cdots & \mathbf{H}_{C,K}^{UR}
\end{bmatrix}
\end{align*}
\]

Fig. 3: Matrix structure of \( \mathbf{H}^R \) and \( \mathbf{H}^F \) for \( \forall i \neq j \).

\[
\text{rank}(\begin{bmatrix}
\mathbf{H} \\
\mathbf{h}
\end{bmatrix}) = \text{rank}(\mathbf{H}) \quad (21)
\]

Since every elements of \( \mathbf{h} \) are driven from i.i.d. distribution, (21) will be satisfied if and only if \( \mathbf{H} \) has full row rank, which means \( \mathbf{H}^T \) has full column rank. To find column rank of \( \mathbf{H}^T \), we introduce another property of Kronecker product.

**Proposition 4 (Transpose of Kronecker product):** For matrices \( \mathbf{A} \) and \( \mathbf{B} \), following property holds,

\[
(\mathbf{A} \otimes \mathbf{B})^T = \mathbf{A}^T \otimes \mathbf{B}^T \quad (22)
\]

Then, \( \mathbf{H}^T \) can be stated as below by *Proposition 4*.

\[
\mathbf{H}^T = \begin{bmatrix}
\mathbf{H}_1^{UR} \otimes (\mathbf{H}_1^{RB})^T, & \mathbf{H}_2^{UR} \otimes (\mathbf{H}_2^{RB})^T, & \cdots, & \mathbf{H}_C^{UR} \otimes (\mathbf{H}_C^{RB})^T
\end{bmatrix} \quad (23)
\]

By (23), \( \mathbf{H}^T \) is Khatri-Rao product of two matrices \( \mathbf{H}^{UR} \) and \( \mathbf{H}^{RB} \) which are defined as,

\[
\mathbf{H}^{UR} = \begin{bmatrix}
\mathbf{H}_1^{UR} & \mathbf{H}_2^{UR} & \cdots & \mathbf{H}_C^{UR}
\end{bmatrix}, \quad \mathbf{H}^{RB} = \begin{bmatrix}
(\mathbf{H}_1^{RB})^T & (\mathbf{H}_2^{RB})^T & \cdots & (\mathbf{H}_C^{RB})^T
\end{bmatrix} \quad (24)
\]

Note that, \( \mathbf{H}^{UR} \in \mathbb{K}^{N_R \times C(C-1)KM} \) and \( \mathbf{H}^{RB} \in \mathbb{K}^{N_R \times CN} \).

By Figure 3, there are at least \((C-2)K\) identical submatrices for any subset of \( \{\mathbf{H}_j^{UR}\} \) with cardinality larger than one. It implies that the generalized Kruskal rank of \( \mathbf{H}^{UR} \) denoted as \( k^{'}_{\mathbf{H}^{UR}} \),
has maximum value of one. By Proposition 1, \( k_{H^R} \) can have two different values as following.

\[
k_{H^R} = \min \left\{ \left\lfloor \frac{N_R}{(C-1)KM} \right\rfloor, C, 1 \right\} =
\begin{cases} 
1, & \text{if } N_R \geq (C-1)KM \\
0, & \text{if } N_R < (C-1)KM.
\end{cases}
\] (25)

Furthermore, since \( H^{RB} \) has independent generic submatrices, \( k_{H^{RB}} \) is given as,

\[
k_{H^{RB}} = \min \left\{ \left\lfloor \frac{N_R}{N_T} \right\rfloor, C \right\} =
\begin{cases} 
C, & \text{if } N_R \geq CN \\
0, & \text{if } N_R < CN.
\end{cases}
\] (26)

By Proposition 2, we can find lower bound of generalized Kruskal rank of \( H^{UR} \otimes H^{RB} (= H^T) \). As long as \( k_{H^R} \) and \( k_{H^{RB}} \) are not zeros, lower bound of \( k_{H^R \otimes H^{RB}} (= k_{H^T}) \) given as below.

\[
k_{H^T} = k_{H^R \otimes H^{RB}} \geq \min \{1 + C - 1, C\} = C, \text{ if } N_R \geq \max \{(C-1)KM, CN\}
\] (27)

Consider the structure of \( H^T \) which is partitioned in \( C \) submatrices. It implies \( \max k_{H^T} = C \). Finally, the value of \( k_{H^T} \) is given as \( C \) which concludes that \( H^T \) has full column rank and (20) has solution \( \text{vec}(T) \). Furthermore, relay beamformer \( T \) is given below.

\[
T = \text{vec}_{N_R}^{-1} \begin{bmatrix} 
(\bar{H}_{1}^{UR})^T \otimes H_1^{RB} \hspace{1cm} \text{vec}(\bar{H}_1) \\
(\bar{H}_{2}^{UR})^T \otimes H_2^{RB} \hspace{1cm} \text{vec}(\bar{H}_2) \\
\vdots \\
(\bar{H}_{C}^{UR})^T \otimes H_C^{RB} \hspace{1cm} \text{vec}(\bar{H}_C) 
\end{bmatrix}^{\dagger}
\] (28)

Note that, \( \text{vec}_{N_R}^{-1}(\cdot) \) is defined as devectorization function of column size \( N_R \) which transforms vector into matrix of column size \( N_R \).

B. Achievable DoF of cellular network through IMAC-ODIA

We first present the result of this subsection which describes achievable DoF per cell of cellular network through IMAC-ODIA.

\textbf{Theorem 2 (Achievable DoF per cell of cellular network):} The achievable DoF per cell of \( (C, K, M, N) \)-cellular uplink network through IMAC-ODIA is given as (29).
The rest of this subsection is devoted to the proof of Theorem 2.

After designing the relay beamformer \( T \) as (28), inter-cell interferences are aligned at relay and fully nulled at every receiver node. So the DoF of BS\(_j\) for two use of transmission time slot can be simply calculated as rank of effective channel of desired data vector \( \hat{x}_j \) and divided by two. \( \hat{H}_{\text{effective},j} \), the effective channel for desired signal and DoF are given as,

\[
\hat{H}_{\text{effective},j} = \hat{H}_j + H_j^{RB} T \hat{H}_j^{UR} \quad (30)
\]

\[
\text{DoF}_j = \frac{r_{\hat{h}_{\text{effective},j}}}{2} \quad (31)
\]

After designing \( T \) as (28), (15) can be transformed as,

\[
\bar{H}_j + H_j^{RB} T \bar{H}_j^{UR} = 0 \Rightarrow H_j^{RB} T = -\bar{H}_j (\bar{H}_j^{UR})^\dagger \quad (32)
\]

Substituting (32) for (30) leads to,

\[
\hat{H}_{\text{effective},j} = \hat{H}_j + H_j^{RB} T \hat{H}_j^{UR} = \hat{H}_j - \bar{H}_j (\bar{H}_j^{UR})^\dagger \hat{H}_j^{UR} \quad \in \mathbb{C}^{N \times KM} \quad (33)
\]

Since every submatrix in augmented matrices \( \hat{H}_j, \bar{H}_j, (\bar{H}_j^{UR})^\dagger, \hat{H}_j^{UR} \) is generic, rank of (33) can be calculated as,

\[
r_{\hat{h}_{\text{effective},j}} = \min (N, KM) \quad (34)
\]

Finally, DoF per cell and the total DoF achievable for symmetric cellular network through IMAC-ODIA are,

\[
\text{DoF}_{\text{cell}} = \frac{\min (N, KM)}{2} \quad (35)
\]

\[
\text{DoF}_{\text{total}} = \frac{C \min (N, KM)}{2} \quad (36)
\]
C. IMAC-ODIA for asymmetric cellular network

Also, the IMAC-ODIA for asymmetric network is also described in this paper. Consider the cellular uplink transmission network with \( C \) cells and \( K_j \) users are served by a base station \( \text{BS}_j \) with \( N_j \) antennas for \( j \in \{1, \ldots, C\} \). User \((k, i)\) denoted as \( \text{UE}_{(k, i)} \) where \( k \in \{1, \ldots, K_i\} \) is served by \( \text{BS}_i \). \( \text{UE}_{(k, i)} \) is assumed to have \( M_{k, i} \) antennas. Similarly, relay is assumed to have \( N_R \) antennas. All channel parameters and matrices are defined in same manner including the augmented matrices (7) through (10). Following theorem summarizes our result.

**Theorem 3 (IMAC-ODIA for asymmetric cellular network):** The relay beamformer of IMAC-ODIA for asymmetric network exists if and only (37) is satisfied

\[
N_R \geq \max \left\{ \left( \max_{j \in \{1, \ldots, C\}} \sum_{i \neq j} K_i \sum_{k=1}^{K_i} M_{k, i} \right), \left( \sum_{j=1}^{C} N_j \right) \right\} \tag{37}
\]

and achievable DoF for \( \text{BS}_j \) is shown as (38)

\[
\text{DoF}_j = \frac{1}{2} \min \left( N_j, \sum_{k=1}^{K_j} M_{k, j} \right) \tag{38}
\]

Throughout this section, details of IMAC-ODIA for uplink network are shown. Many features of IMAC-ODIA satisfy the practical requirement which prevented cellular network from applying interference alignment which were mentioned in previous contents. Also, some might have questions about downlink transmission. Therefore, we extend our idea to downlink network. With help of uplink-downlink duality for relay-aided cellular network.

VI. IBC-ODIA FOR DOWNLINK TRANSMISSION

In this section, we extend our idea to downlink transmission which can be modeled as an IBC(Interfering Broadcast Channel) environment. Here, we only consider symmetric network. Asymmetric IBC-ODIA can be described in same manner to IMAC-ODIA for asymmetric cellular network. The term, channel reciprocity is the relation between uplink and downlink channel which means that downlink channel matrix for a pair of nodes is conjugate-transpose of uplink channel matrix with same node pair.
A. Uplink-downlink duality in ODIA-based cellular network

We first extend the idea in [20] to our network which gives us the motivation for IBC-ODIA. For uplink phase, the desired data stream $\hat{x}_j$ for BS $j$ can be obtained through following process.

$$\hat{x}_j = \hat{H}_{effective,j}^{\dagger}(y_{j,1} + y_{j,2})$$  \hspace{1cm} (39)

Note that the $\hat{H}_{effective,j}^{\dagger}$ in (39) plays role as decorrelator handling intra-cell interference. In Figure 4(a) the uplink transmission process is shown with block diagram. If we assume channel reciprocity on this system and attach the conjugate-transpose operation, block diagram in Figure 4(a) is transformed into Figure 4(b) without considering data stream block. This process provides the intuition for IBC-ODIA where the transmitter(BS) beamforming might be necessary for IBC-ODIA scheme but, no decorrelator construction needed at receiver(UE). Therefore, IBC-ODIA requirement for UE is significantly low in practical manner compared to BS. It supports our point about cellular network again.

B. IBC-ODIA: System model and inter-cell interference alignment

First, we will describe the network, downlink network is organized as same as uplink network. $C$ cells and $K$ users in each cell. Cell $j$ is served by a single base station denoted as BS$_j$ for $j \in \{1, \ldots, C\}$. User $(k, i)$ denoted as UE$_{(k, i)}$ where $k \in \{1, \ldots, K\}$ is served by BS$_i$, as shown
in Figure 5. Each UE is assumed to have $M$ antennas, each BS is assumed to have $N$ antennas and relay is assumed to have $N_R$ antennas. The channel from BS$_j$ to UE$_{(k,i)}$ is denoted as the $M \times N$ matrix $H_{(k,i),j}$, BS to relay channel from BS$_j$ to relay is denoted as the $N_R \times N$ matrix $H_{j}^{BR}$ and relay to UE channel from relay to UE$_{(k,i)}$ is denoted as the $M \times N_R$ matrix $H_{k,i}^{RU}$. $x_i$ denotes the transmitted data vector at BS$_i$. With same manner to IMAC-ODIA scheme description, augmented matrices and vectors are defined as below.

$$\bar{H}_{k,i} = \begin{bmatrix} H_{(k,i),1} & \cdots & H_{(k,i),i-1} & H_{(k,i),i+1} & \cdots & H_{(k,i),C} \end{bmatrix} \in \mathbb{C}^{M \times (C-1)N}$$ \hspace{1cm} (40)

$$\bar{H}_{i}^{BR} = \begin{bmatrix} H_{i}^{BR} & \cdots & H_{i-1}^{BR} & H_{i+1}^{BR} & \cdots & H_{C}^{BR} \end{bmatrix} \in \mathbb{C}^{N_R \times (C-1)N}$$ \hspace{1cm} (41)
\[
\mathbf{x}_i = \begin{bmatrix}
    x_1 \\
    \vdots \\
    x_{i-1} \\
    x_i \\
    x_{i+1} \\
    \vdots \\
    x_C
\end{bmatrix} \in \mathbb{K}^{(C-1)N \times 1}
\] (42)

Ignoring the noise term, the total sum received vector for two time slot is following, \( \mathbf{T}_{DL} \) denotes the relay beamforming matrix for downlink transmission.

\[
y_{(k,i),1} + y_{(k,i),2} = (\mathbf{H}_{(k,i),i} + \mathbf{H}_{k,i}^{RU} \mathbf{T}_{DL} \mathbf{H}_{i}^{BR}) \mathbf{x}_i + (\mathbf{H}_{k,i} + \mathbf{H}_{k,i}^{RU} \mathbf{T}_{DL} \mathbf{H}_{i}^{BR}) \mathbf{x}_i
\] (43)

Note that, \( \mathbf{T}_{DL} \) only aligns inter-cell interferences and remaining intra-cell interferences will be nulled by beamforming at transmitter(BS) which will be described later. BS beamforming for intra-cell interference nulling is originated from intuition for uplink-downlink duality as mentioned before.

Interference alignment condition should be satisfied with \( \forall k \in \{1, \ldots, K\} \) and \( \forall i \in \{1, \ldots, C\} \),

\[
\mathbf{H}_{k,i} + \mathbf{H}_{k,i}^{RU} \mathbf{T}_{DL} \mathbf{H}_{i}^{BR} = 0
\] (44)

\( \mathbf{T}_{DL} \) satisfying above condition is given as,

\[
\mathbf{T}_{DL} = \text{vec}^{-1}_{N_R} \begin{pmatrix}
    (\mathbf{H}_{1}^{BR})^T \otimes \mathbf{H}_{1}^{RU} \\
    (\mathbf{H}_{1}^{BR})^T \otimes \mathbf{H}_{2}^{RU} \\
    \vdots \\
    (\mathbf{H}_{1}^{BR})^T \otimes \mathbf{H}_{K,1}^{RU} \\
    (\mathbf{H}_{2}^{BR})^T \otimes \mathbf{H}_{1,2}^{RU} \\
    \vdots \\
    (\mathbf{H}_{C}^{BR})^T \otimes \mathbf{H}_{K,C}^{RU}
\end{pmatrix}^\dagger \begin{pmatrix}
    \text{vec}(\mathbf{H}_{1,1}) \\
    \text{vec}(\mathbf{H}_{2,1}) \\
    \vdots \\
    \text{vec}(\mathbf{H}_{K,1}) \\
    \text{vec}(\mathbf{H}_{1,2}) \\
    \vdots \\
    \text{vec}(\mathbf{H}_{K,C})
\end{pmatrix}
\] (45)
Following theorem gives the number of antenna required at relay for IBC-ODIA, proof is similar to that of IMAC-ODIA.

**Theorem 4 (Required number of relay antenna for downlink):** The relay beamformer for downlink $T_{DL}$ satisfying condition (44) exists if and only if $N_R \geq \max\{(C - 1)N, CKM\}$

After designing $T_{DL}$ properly, inter-cell interferences(which are transmitted data from other cells) are fully nulled at receiver(UE) side. As mentioned above, intra-cell interferences will be nulled by designing transmitter(BS) beamformer. Next section will be devoted to transmitter beamformer design.

### C. Intra-cell interference nulling through BS beamformer design

As we mentioned, beamformer at BS for intra-cell interference nulling will be designed jointly with relay beamformer. For simplicity, each UE symmetrically desires $d$ independent data stream from its serving BS. Transmitted signal from BS $i$ will be designed as,

$$x_i = V_{1,i} s_{1,i} + V_{2,i} s_{2,i} + \ldots + V_{K,i} s_{K,i}$$

$s_{k,i} \in \mathbb{K}^{d \times 1}$ denotes the data stream for UE $k,i$ from BS $i$ and $V_{k,i} \in \mathbb{K}^{N \times d}$ is the beamforming matrix designed at BS $i$ for UE $k,i$.

If following condition (47) is satisfied, intra-cell interferences at each UE are fully nulled by BS beamforming.

$$
\begin{bmatrix}
H_{(1,i),i} + H_{RU,1,i} T_{DL} H_{BR}^{1,i} \\
\vdots \\
H_{(K,i),i} + H_{RU,K,i} T_{DL} H_{BR}^{K,i}
\end{bmatrix}
\begin{bmatrix}
V_{1,i} & \ldots & V_{K,i}
\end{bmatrix} = A_i V_i =
\begin{bmatrix}
I_{M \times d} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & I_{M \times d}
\end{bmatrix}
$$

Since $H_{(k,i),i}$ is generic, $A_i$ is full row matrix and has right inverse matrix if $N \geq KM$. Note that, if $N < KM$, we can turn off $M - \frac{N}{K}$ antennas at each UE to ensure that $A_i$ is full row rank matrix. Define right inverse of $A_i$ as $A_i^{right}$. Then, $V_i$ is given as,
\[ V_i = A_i^{right} \begin{bmatrix} I_{M \times d} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_{M \times d} \end{bmatrix} = \left\{ A_i^{right} \right\}_{1:d} \left\{ A_i^{right} \right\}_{M+1:M+d} \cdots \left\{ A_i^{right} \right\}_{(K-1)M+1:(K-1)M+d} \]

(48)

By (48), we can design \( V_i \). And \( A_i^{right} \) is calculated as,

\[ A_i^{right} = A_i^T \left( A_iA_i^T \right)^{-1} \]

(49)

From (47), by selecting \( Kd \) appropriate row vectors from \( A_i \), we can generate the left inverse matrix for \( V_i \). Since \( V_i \) has its left inverse, \( V_i \) also has full column rank. Note that, \( N \geq Kd \) should be guaranteed for successful data transmission. So, \( V_{k,i} \) can deliver message for \( UE_{k,i} \) successfully with DoF \( d \) and \( d \leq M \) should be satisfied.

D. Achievable DoF for downlink transmission through IBC-ODIA

After designing relay beamformer and BS beamformer, inter-cell and intra-cell interferences are totally removed for each receiver. We present a simple theorem to summarize achievable DoF result for this section.

**Theorem 5 (Achievable DoF for downlink transmission):** The achievable DoF per cell and per UE of \((C, K, M, N)\)-cellular downlink network through IBC-ODIA is given as (50)&(51)

\[ \text{DoF}_{DL,cell} = \min \left( \frac{Kd, N}{2} \right) \leq \min \left( \frac{KM, N}{2} \right) \]

(50)

\[ \text{DoF}_{DL,UE} = \min \left( \frac{KM, N}{2K} \right) \]

(51)

Since proof for **Theorem 5** is similar to that of **Theorem 2**, it is omitted.

VII. Extension to full duplex cellular network

In this section, we are focusing on cellular network where full-duplex operation is available. Due to the complexity of full-duplexing, it was not main concern of researchers before. But,
it is true that availability of full-duplexing will lead to the increase of network throughput definitely. Also, the needs of management scheme for additional interferences caused by full-duplex operation arises. Which motivates the development of our scheme for full-duplex cellular network.

A. Intuition: Full-duplex network with self-interference cancellation

There were some research about interference alignment on full-duplex network recently, like [21], [22] and [23] due to the higher throughput achievable by full-duplex mode. In [23], it is proven that allowing full-duplexing on every node achieves exactly double of DoF performance compared to that of half-duplexing multi-user interference channel. The DoF derivation in [23] was based on network decomposition and self-interference cancellation availability at every full-duplex node. It is also likely to apply similar methodology to cellular network. In [21], if every node operates in full-duplex mode, then a single full-duplex cell can be decomposed into two half-duplex cell, one on uplink transmission and the other on downlink transmission.

Combining full-duplex network decomposition with the fact that proposed scheme is flexible
which operates on either uplink or downlink, we get intuition about applying our proposed scheme on full-duplex cellular network with a little modification. Even though the relay operates in half-duplex mode, total system DoF performance is doubled as described in [23]. We first start with full-duplex network model description. In this paper, we only consider symmetric network configuration.

**B. System Model: Full-duplex nodes served by half-duplex relay**

Consider a symmetric full-duplex network with $C$ cells and $K$ users in each cell. Cell $j$ is served by a single base station operates in full-duplex mode denoted as $\text{BS}_j$ for $j \in \{1, ..., C\}$. User $(k, i)$ also operates in full-duplex mode denoted as $\text{UE}_{(k,i)}$ where $k \in \{1, ..., K\}$ is served by $\text{BS}_i$ as Figure 7. Note that, UEs only receives its desired signal from its local BS which means there is no device-to-device communication. Each UE is assumed to have $M$ antennas, each BS is assumed to have $N$ antennas and relay is assumed to have $N_R$ antennas. Note that, relay operates
in half-duplex mode and full-duplex relay will be discussed later. Allowing full-duplex operation at every source and destination node requires additional channel matrix description as follows.

The channel from UE \( (k, i) \) to BS \( j \) is denoted as the \( N \times M \) matrix \( H_{UB,j,(k,i)}^{(k,i)} \), channel from UE \( (k_1, i_1) \) to UE \( (k_2, i_2) \) is denoted as the \( M \times M \) matrix \( H_{UU,(k_2,i_2),(k_1,i_1)}^{(k_1,i)} \), UE to relay channel from UE \( (k,i) \) to relay is denoted as the \( N_R \times M \) matrix \( H_{UR,(k,i)}^{(k,i)} \), relay to UE channel from relay to UE \( (k,i) \) is denoted as the \( M \times N_R \) matrix \( H_{RU,(k,i)}^{(k,i)} \), relay to BS channel from relay to BS \( j \) is denoted as the \( N \times N_R \) matrix \( H_{RB,j}^{(k,i)} \), channel from BS \( j_1 \) to BS \( j_2 \) is denoted as the \( N \times N \) matrix \( H_{BB,j_2,j_1}^{(k,i)} \), channel from BS \( j \) to UE \( (k,i) \) is denoted as the \( M \times N \) matrix \( H_{BU,j}^{(k,i)} \), and BS to relay channel from BS \( j \) to relay is denoted as the \( N_R \times N \) matrix \( H_{BR,j}^{(k,i)} \).

C. Achievable DoF in full-duplex cellular network

Here, we present the result of this section by following theorem which provides the achievable DoF and number of antennas required at relay in full-duplex cellular network.

**Theorem 6 (Achievable DoF for full-duplex cellular network):** The achievable DoF per cell of \( (C,K,M,N) \)-full duplex cellular network is given as \( (52) \) with DoF per BS and UE as \( (53) \) and \( (54) \) respectively when \( N_R \geq C(KM+N) \) is satisfied.

\[
\text{DoF}_{FD,Cell} = \text{DoF}_{FD,BS} + K \times \text{DoF}_{FD,UE} = \min(KM,N) 
\]

\[
\text{DoF}_{FD,BS} = \frac{\min(KM,N)}{2} 
\]

\[
\text{DoF}_{FD,UE} = \frac{\min(KM,N)}{2K} 
\]

Note that, total achievable DoF performance is approximately doubled compared to half-duplex case which is straightforward to the result of \cite{23}. Also, It is interesting that, above scheme requires only half-duplexing AF relaying to achieve full-duplexing gain in full-duplex network. This also provides advantage of backward compatibility. In fact, asymptotic IA scheme is not able to achieve full-duplexing gain through half-duplex relaying.

From previous works, it is shown that causal MIMO full-duplex relay based on DF protocol cannot increase DoF performance of network and only non-causal and instantaneous DF relaying
can increase the performance. It is also straightforward to our result, which leads to following corollary.

**Corollary 1 (Achievable DoF with instantaneous relaying):** The achievable DoF per cell of same full-duplex cellular network with instantaneous AF relaying is

\[
\text{DoF}_{FD,\text{Cell},\text{inst}} = \text{DoF}_{FD,\text{BS},\text{inst}} + K \times \text{DoF}_{FD,\text{UE},\text{inst}} = 2 \min(KM, N)
\] (55)

\[
\text{DoF}_{FD,\text{BS},\text{inst}} = \min(KM, N)
\] (56)

\[
\text{DoF}_{FD,\text{UE},\text{inst}} = \frac{\min(KM, N)}{K}
\] (57)

Note that relay antenna requirement doesn’t change with the use of instantaneous relay.

**D. Achievability in full-duplex cellular network**

This section briefly describes the proof of Theorem 6 based on combining the concept IMAC-ODIA and IBC-ODIA. As we mentioned before, with successful self interference cancellation, full-duplex cellular network can be decompose into disjoint uplink and downlink network sharing relay. Note that, additional interferences occur due to full-duplexing. For example, UE to UE channel now becomes an interfering channel.

Since, IBC-ODIA requires additional beamforming at BS(Tx at downlink transmission) while IMAC-ODIA does not. It is natural that FD-ODIA should include BS beamforming. To describe FD-ODIA, we first start with matrix augmentation with same manner. Main point is this, relay only aligns inter-cell interferences.

\[
\overline{H}^{BS}_j = \begin{bmatrix}
  H^{BB}_{j,1} & \cdots & H^{BB}_{j,j-1} & H^{BB}_{j,j+1} & \cdots & H^{UB}_{j,(1,1)} & \cdots & H^{UB}_{j,(K,j-1)} & H^{UB}_{j,(1,j+1)} & \cdots & H^{UB}_{j,(K,C)}
\end{bmatrix} \in \mathbb{C}^{N \times (C-1)(K+1)}
\] (58)

\[
\overline{H}^{UE}_{(k,i)} = \begin{bmatrix}
  H^{BU}_{(k,i),1} & \cdots & H^{BU}_{(k,i),(i-1)} & H^{BU}_{(k,i),(i+1)} & \cdots & H^{UU}_{(k,i),(1,1)} & \cdots & H^{UU}_{(k,i),(k-1,i)} & H^{UU}_{(k,i),(k+1,i)} & \cdots & H^{UU}_{(k,i),(1,(K-1))}
\end{bmatrix} \in \mathbb{C}^{K \times \left\{M \times (C-1) + (C-1)N \right\}}
\] (59)
Also, the interference vectors should be redefined due to additional interferences which would be described as,

\[
\tilde{x}_{BS}^j = \begin{bmatrix}
x_1^{BS} \\ \vdots \\ x_{j-1}^{BS} \\ x_j^{BS} \\ \vdots \\ x_C^{BS} \\
x_1^{UE} \\ \vdots \\ x_{(k,j-1)}^{UE} \\ x_{(1,j+1)}^{UE} \\ \vdots \\ x_{(K,C)}^{UE}
\end{bmatrix} \in \mathbb{K}^{(C-1)(KM+N) \times 1}, \quad \tilde{x}_{UE}^{(k,i)} = \begin{bmatrix}
x_1^{BS} \\ \vdots \\ x_{(k-1,j)}^{BS} \\ x_{(k+1,j)}^{BS} \\ \vdots \\ x_{(K,C)}^{BS} \\
x_1^{UE} \\ \vdots \\ x_{(k-1,1)}^{UE} \\ x_{(k+1,1)}^{UE} \\ \vdots \\ x_{(K,C)}^{UE}
\end{bmatrix} \in \mathbb{K}^{((C-1)M+(C-1)N) \times 1}
\]

note that, \(x_j^{BS}\) and \(x_{(k,i)}^{UE}\) are transmitted vector from BS \(j\) and UE \((k,i)\) respectively. Definitions of desired signal channel and desired signal vectors(group of \(\hat{H}\) and \(\tilde{x}\)) are exactly same to that of half-duplex uplink or downlink case which are omitted here.

As we mentioned above, augmented interference matrices only contain inter-cell interferences. So, additional BS beamforming will align intra-cell interferences at UEs which will be described later.

IA condition is similar to IMAC/IBC case except for the fact that it should be satisfied both on BSs and UEs which are,
Existence of relay beamformer $T_{FD}$ is proved with similar manner to IMAC/IBC-ODIA case except for choosing appropriate number of relay antenna. In fact, in full-duplex network case, number of relay antenna $N_R$ should satisfy

$$N_R \geq \max\{(C-1)(KM+N), (C-1)N + (CK-1)M, C(KM+N)\} = C(KM+N)$$

which is total number of antennas in the network.

Then $T_{FD}$ satisfying (63) and (64) is derived as,

$$T_{FD} = \text{vec}^{-1}_{N_R} \begin{bmatrix} \begin{bmatrix} (H_{1}^{BR})^T \otimes H_{1,1}^{RB} \\ \vdots \\ (H_{C}^{BR})^T \otimes H_{C,1}^{RB} \end{bmatrix} & \begin{bmatrix} \text{vec}(-\bar{H}_{1}^{BS}) \\ \vdots \\ \text{vec}(-\bar{H}_{C}^{BS}) \end{bmatrix} \\ \begin{bmatrix} (H_{(1,1)}^{UR})^T \otimes H_{(1,1)}^{RU} \\ \vdots \\ (H_{(K,C)}^{UR})^T \otimes H_{(K,C)}^{RU} \end{bmatrix} & \begin{bmatrix} \text{vec}(-\bar{H}_{(1,1)}^{UE}) \\ \vdots \\ \text{vec}(-\bar{H}_{(K,C)}^{UE}) \end{bmatrix} \end{bmatrix}$$

Furthermore, we should design $V_{i}^{FD}$, the beamformer at BS$i$ with exactly same manner to IBC-ODIA which should satisfy following intra-cell interference nulling condition.

$$\begin{bmatrix} H_{(1,i)}^{BU} + H_{(1,i)}^{RU} T_{FD} H_{i}^{BR} \\ \vdots \\ H_{(K,i)}^{BU} + H_{(K,i)}^{RU} T_{FD} H_{i}^{BR} \end{bmatrix} \begin{bmatrix} V_{1,i}^{FD} \\ \vdots \\ V_{K,i}^{FD} \end{bmatrix} = A_{i}^{FD} V_{i}^{FD} = \begin{bmatrix} I_{M \times d} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & I_{M \times d} \end{bmatrix}$$

Similar to the IBC-ODIA case, $A_{i}^{FD}$ should be full row matrix and has right inverse matrix $A_{i}^{FD,right}$. Then, $V_{i}^{FD}$ can be derivated as,
\[ A_{i}^{FD,\text{right}} = (A_{i}^{FD})^T (A_{i}^{FD} (A_{i}^{FD})^T)^{-1} \]  \hspace{1cm} (68)

\[ V_{i}^{FD} = \left[ \{ A_{i}^{FD,\text{right}} \}_{1:d} \quad \{ A_{i}^{FD,\text{right}} \}_{M+1:M+d} \quad \cdots \quad \{ A_{i}^{FD,\text{right}} \}_{(K-1)M+1:(K-1)M+d} \right] \]  \hspace{1cm} (69)

Designing every beamformers as above, we can achieve DoF for BSs. Since interferences are fully aligned and nulled at BS, DoF of BS\(_j\) is,

\[ \text{DoF}_{FD,BS}^{j} = \frac{\min (K, M, N)}{2} \]  \hspace{1cm} (70)

Similarly, DoF of UE\(_{(k,i)}\) is,

\[ \text{DoF}_{FD,UE}^{(k,i)} = \frac{\min (K, M, N)}{2K} \]  \hspace{1cm} (71)

So, total DoF of a cell is given as (52).

Following our development of proposed scheme, you may notice that our scheme is very flexible. Which means, very simple modification (for example: Modifying channel assignment protocol in the relay) can make our scheme work in various transmission phase in cellular. We may claim that this is also an advantage of our scheme, deployment of a single half-duplex AF-based relay can serve uplink, downlink and even full-duplex network depending on network’s needs.

VIII. Discussion and Simulation Result

With the result of previous section, we provide some and brief discussion about our result and simulation.

A. Degrees of freedom improvement without time extension

With our proposed scheme, total achievable degrees of freedom of cellular uplink network \( \text{DoF}_{ODIA,\text{uplink}} \) is given as \( \frac{C \min (N, KM)}{2} \). We suggest that, given \( \text{DoF}_{ODIA,\text{uplink}} \) is huge improvement for \((C, K, M, N)\)-cellular uplink network compared to \( \text{DoF}_{linear,\text{uplink}} \) which is achievable DoF in cellular uplink network without time extension and UE beamforming. We claim the
DoF outerbound of cellular network without time extension by transmitter cooperation allowed in cellular network. Which means data sharing and processing among UEs available in uplink transmission.

First, by Figure 8, 
\((C, K, M, N)\)-IMAC transforms to 
\((C, KM, N)\)-IC, meaning a group of UE transmits its data jointly as a single super-UE. Denote the total degrees of freedom of transformed network through linear IA as \(\text{DoF}_{\text{linear,coop}}\). Then, following inequality will be satisfied.

\[
\text{DoF}_{\text{linear,uplink}} \leq \text{DoF}_{\text{linear,coop}}
\]  

Consider the transformed network with proposed scheme. In proposed scheme, second time slot is available only for relay transmission, which might have performance degradation from no source transmission. Enabling the source transmission or disabling the relay transmission will not degrade the DoF performance for sure. Because, when possibility of DoF degradation arises, source or relay simply don’t need to change its state. Result of [18] concludes that even
full-duplex DF-based relaying cannot increase the degrees of freedom of interference network. Consequently, $\text{DoF}_{\text{linear,coop}}$ is bounded as (73) from the result of [6].

$$\text{DoF}_{\text{linear,uplink}} \leq \text{DoF}_{\text{linear,coop}} \leq KM + N$$ (73)

Our result provides DoF improvement for cellular network without time extension. Where total performance grows proportional to number of cells which was not able to achieved by simple linear IA (see (73)). And from the result of [24], $\text{DoF}_{\text{ODIA,uplink}}$ is indeed maximum achievable DoF in cellular network with a half-duplex relay which can be proven with same process.

Note that, information theoretic optimal bound of MIMO IMAC($\approx \frac{KMN}{KM+N}$, see [11]) is approximately two times larger than the bound of our proposing scheme in extreme cases (i.e. $N \ll KM$ or $N \gg KM$), It is not able to be achieved through non time-extending linear IA scheme.

Also, in the antenna regime where BSs and group of UEs have similar number of antennas (i.e. $N \approx KM$), our proposing scheme achieves optimal DoF of MIMO IMAC/IBC. Which were believed to be achieved only through asymptotic IA or full-duplex relay aiding.

B. Possibility of gain from additional relay antenna

Suppose that the relay setup is extra-sufficient to serve whole network, like having more antennas than its requirement we discussed before. Then the question is that is there any gain from additional relay antennas existing by additional beam design. We suggest a single simple answer to this question in this section.

Consider uplink transmission, IA-applied data stream through decorrelator at BS is given as (39) in previous section. Our goal is design additional beam to following statement can be satisfied. Let $T_+$ denote the relay beamformer with additional gain.

$$\tilde{H}_{\text{effective},j} = \tilde{H}_j + H_j^{RB} T_+ \tilde{H}_j^{UR} = (1 + \alpha) \tilde{H}_j \Rightarrow H_j^{RB} T_+ \tilde{H}_j^{UR} = \alpha \tilde{H}_j$$ (74)

If (74) is satisfied, it can be interpreted as boosting the received signal power level by $\alpha$. Note that, DoF performance still remains same.
We skip the details since the proof is exactly same to that in Section V. Existence of $T_+$ satisfying (74) can be guaranteed when $N_R \geq \max\{CKM, CN\}$ and the design of $T_+$ is following.

\[
T_+ = \text{vec}_{NR}^{-1} \begin{bmatrix}
(H^{UR}_1)^T \otimes H^{RB}_1 \\
(H^{UR}_2)^T \otimes H^{RB}_2 \\
\vdots \\
(H^{UR}_C)^T \otimes H^{RB}_C
\end{bmatrix}^\dagger \begin{bmatrix}
\text{vec}(H_1) \\
\text{vec}(H_2) \\
\vdots \\
\text{vec}(H_C)
\end{bmatrix}
\] (75)

Where $H_j \in \mathbb{K}^{N \times CKM}$ and $H_{j}^{UR} \in \mathbb{K}^{NR \times CKM}$ are defined as below.

\[
H_j = \begin{bmatrix}
H_{j,(1,1)} & \cdots & -H_{j,(K,j-1)} & \alpha H_{j,(1,j)} & \cdots & \alpha H_{j,(K,j)} & -H_{1,(1,j+1)} & \cdots & -H_{j,(K,C)}
\end{bmatrix}
\]

Then, the relay beamformer for IMAC-ODIA can align interferences and also boost the desired signal power level with use of at most $KM$ additional antennas at relay. This is that, by extra relay antenna gain, we are further able to achieve better rate performance not only DoF performance compared to the tight antenna setup case implying the additional potential gain can also exist from extra relay antenna setup.

IX. Conclusions

In this paper, we suggest an IA scheme for cellular network based on the help of half-duplex relay with no presence of UE processing which was the fundamental limit of applying IA on cellular network. We derived the achievable degrees of freedom bound for uplink cellular network for symmetric/asymmetric system configuration with relay antenna requirement. And based on the idea that IMAC-ODIA is based on linear beamforming at relay which uplink-downlink duality can be hold, we extend our solution through BS beamformer design to downlink networks where UE also does no complex works. Furthermore, with recent works about full-duplexing networks, our proposing scheme can also be extended for full-duplexing cellular network achieving doubled
performance of half-duplexing network, only with help of half-duplex relay. This implies the flexibility and backward compatibility of the scheme on cellular network which also can be an advantage in the view of practical issue.

Although it has some DoF gap between optimal upper bound, we proved that our scheme has network gain with the network size growth which the linear IA without time extension does not have. And for some antenna regime, it is able to achieve optimal DoF performance.

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