Calculating the fundamental frequency of power law functionally graded beam using ANSYS software

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Abstract. In this work free vibration of simply supported, clamped-clamped and clamped-free functionally graded (FG) beam with material graduation transversally through the thickness, using the power-law model, were investigated. The beam's functionally graded material (FGM) consists of aluminium (Al₂O₃) and steel (used in a typical case). Pure steel is the beam's bottom surface, whereas pure aluminium is the beam's top surface throughout the thickness change condition. Two finite element models were proposing to calculate the first non-dimensional frequency parameters of FG beam. These models are shell and solid, and they were employed using the ANSYS APDL version 17.2. The two models have been verifying with the previously published works, and a good agreement was founding. Numerical results were presented in graphical forms to study the effects of the power-law index (i.e. material distribution), length-to-thickness ratio, modulus ratio and types of support for the first non-dimensional frequency parameter of the FG beam. The above mention effects play a significant role in the free vibration of the beam. Power-law index (K) is one of the parameters affecting mainly on the frequency parameter of FG beam. The frequency parameter increase with increasing the power-law index when the modulus ratio less than one and decrease with increasing the power-law index when the modulus ratio more than one, when the modulus ratio equal one (i.e. pure material), there is no effect of the power-law index in this case. When the modulus ratio (E ratio) increases, the frequency parameter increases too, but with the change rate depending on the length-to-thickness ratio (L/h) and power-law index(K). Also, the frequency increase with increasing the length to thickness ratio at any power-law index.

1. Introduction:

The selection of material is one of the essential keys in engineering design. The smart materials with modified properties are using to reach the requirement of the machine, system, etc. Composite materials have an excellent ability to alter the properties in one dimension concerning others, and this ability has a significant effect on the design of beams, plates and shells in different composite structures like macro- and nano-scales. FGMs are a class of composite materials which have attracted a lot of attention in the last decades. It consists of two or more dissimilar materials. It can be defined "a class of composites that have a continuous variation of material properties from one surface to another and thus eliminate the stress concentration at the interface of the layers found in laminated composites [1]. FGMs initiated in Japan in 1984 during a space project [2]. It is using as structural
elements in modern industries such as aerospace structures, turbine blades, rocket engine components, aircraft, space vehicles, defence industries, electronic, nuclear engineering, aeronautics and biomedical equipment. Due to the importance of beam structures in an engineering field, several studies have been accomplishing on the bending and vibration problems of FG beams. Sankar [3] introduced an elasticity solution for simply supported FG beams sinusoidal transverse loading, assuming an exponential model of Young's modulus and sinusoidal transverse loading. He based on Euler-Bernoulli beam theory to analysis the bending of FG beams.

Chakraborty et al. [4] used the first-order shear deformation theory to develop a new beam element in order to study the thermo-elastic behavior of FG beam. They assumed that the elastic and thermal properties are varying along its thickness as a power-law and exponential models to examine different stress variations. Goupee and Senthil [5] optimized the natural frequencies of FG structures by tailoring their material distribution for three model problems using a genetic algorithm. The goal of first problem was to find the material distribution at the maximum of first three natural frequencies of FG beam. In the second problem, the goal was minimizing the mass of FG beam while constraining its natural frequencies to lie outside certain prescribed frequency bands. Finally, the third goal was minimizing the mass of FG beam by simultaneously optimizing its material distribution and thickness such that the fundamental frequency is greater than a recommended value. Aydogdu and Taskin [6] adopted Navier type solution method to study the free vibration of simply supported FG beam based on classical beam (CBT) and higher order theories. Li [7] approached a new unified for analyzing the static and dynamic behaviors of FG beams considering the effects of shear deformation and rotary inertia. He extended the Timoshenko beam theory to treat FG beam as well as layered beams. Xiang and Yang [8] utilized the differential quadrature method based on Timoshenko beam theory to calculate the free and forced vibration of a thermally pre-stressed, laminated FG beam of variable thickness. The results illustrated that natural frequencies increases and amplitude of vibration decreases when the thicker FGM layers with a smaller volume fraction index were used. Kapuria et al. [9] studied, experimentally and theoretically, the static deflection and free vibration for layered FG beams using third-order zigzag theory and modified rule of mixtures (MROM). This study confirmed the capability of the zigzag theory to model the mechanics of layered FG beams with the ceramic content.

Sina et al. [10] employed a new beam theory to study free vibration of FG beams. They applied a simple power-law model to describe the variation of beam properties through the thickness. Also, they used an analytical method to solve the resulting system of ordinary differential equations of the free vibration. The natural frequency result of the new theory was a little different in from the traditional first-order shear deformation beam theory while the mode shapes of the two methods were coincidental. Simsek and Kocaturk [11] adopted the Euler–Bernoulli beam theory to derive the system of equations of motion using Lagrange’s equations. They studied the dynamic behavior and free vibration characteristics of a simply-supported FG beam under a concentrated moving harmonic load. Their results showed the excitation frequency and the velocity of moving harmonic load are crucial on the dynamic behavior of FG beam. Simsek [12] implemented the Lagrange multiplier method to solve the fundamental frequency of FG beams based on different higher order beam theories. Malekzadeh et al. [13] practiced the first-order shear deformation theory (FSDT) to develop a new formulation for the out-of-plane free vibration analysis of circular curved FG beams in thermal environment. They assumed that material properties are temperature dependent and graded in the direction normal to the plane of the beam curvature. The results showed that the natural frequency parameter was significant affected by the temperature dependence of the material properties. Ke et al. [14] applied Timoshenko beam theory and von Karman geometric nonlinearity to investigate the nonlinear free vibration of nanocomposite FG beams reinforced by single-walled carbon nanotubes (SWCNTs). The properties of nano-composite FG beam (FG-CNTRCs) were graded in the thickness direction and estimated though the rule of mixture. They found that an increase in CNT volume fraction leads to higher linear and nonlinear frequencies for both uniform distribution and FG-CNTRC beams.
Alshorbagy et al. [15] presented the dynamic characteristics of FG beam with material graduation in transversally through the thickness or axially through the length based on the power law model using the finite element method. They used principle of virtual work under the assumptions of the Euler–Bernoulli beam theory to derive the system of equations of motion. Their model was more effective for replacing the non-uniform geometrical beam with transversally or axially uniform geometrical FG beam. From numerical results, the material distribution, slenderness ratios, and boundary conditions were affected significantly on the dynamic characteristics of the FG beam. Shahba et al. [16] derived the mass and stiffness matrices for buckling and free vibration behaviors of tapered axially FGM beams with elastic end supports. Wattanasakulpong et al. [17] investigated the vibration thermal buckling and of FG beams and they found that when the temperature increases towards the critical temperature the fundamental frequency decreases to zero. Thai and Vo [18] studied the bending and free vibration problems of FG beams and proposed a new analytical solution based on higher-order shear deformation theory. Anandrao et al. [19] presented free vibration analysis of exponent FG beams for various classical boundary conditions using finite element methods. They used principle of virtual work to develop two separate elements, one based on Timoshenko beam theory and the other based on Euler-Bernoulli beam theory. They investigated the effect of transverse shear on the mode shapes and natural frequencies for different aspect ratios (length / thickness), volume fraction and boundary conditions. Results showed that transverse shear significantly affects the mode shape and fundamental frequency for lower aspect ratio of FG beams.

Wattanasakulpong and Ungbhakorn [20] applied the differential transformation method to study free vibration Power – Law FG beams supported by arbitrary boundary conditions, including various types of elastically end constraints. They discussed the effects of the material volume fraction index, values of translational and rotational spring constants and boundary conditions on the mode shapes and natural frequencies. Nguyen et al. [21] derived analytically transverse shear stiffness and shear correction factor based on a first-order shear deformation theory for investigating the static and free vibration problems of axially loaded FG beams. Vo et al. [22] adopted a refined shear deformation beam theory to investigate the static and free vibration behaviors of FG beams. Wattanasakulpong et al. [23] employed the differential transformation method to study the linear and nonlinear natural frequencies of the porous FG beams with different elastic supports using the modified rule of mixture to calculate approximate material properties of FG beam.

Şimşek [24, 25] described the variations of material properties in length and thickness directions by an exponent model to study the forced vibration and buckling analyses of 2-D FGM Timoshenko beams. Nguyen et al. [26] used the power-law model to describe material properties variations in both the thickness and length directions. They applied finite element method to study the forced vibration problem of 2-D FG Timoshenko beams excited by a moving load. Avcar and AlSaid Alwan [27] implemented free vibration of simply supported FG beam using Rayleigh method. They described the material properties of FG beam along the thickness by power-law model. They observed that the varying of material properties significantly effects on the dimensionless free vibration frequency parameters of FG beam. Thom and Kien [28] practiced free vibration of 2-D FG beams using the finite element method based on Timoshenko beam theory. They used a higher-order beam element and hierarchical functions to interpolate the displacements and rotation in the analysis. They described the variations of material properties in length and thickness directions by Power- Law model. Pradhan and Sarangi [29] investigated the free vibration analysis of Power-Law FG beam using ANSYS 15.0 software. They studied the effects of power law index and boundary conditions on the natural frequency of the FG beams. Thom and Kien [30] utilized a new third-order shear deformation theory to analyze free vibration problem of 2-D FGM beams in thermal environment. They described the material properties in the length and thickness directions by a power-law model with temperature-dependent effect. They derived and employed a finite element formulation to compute the vibration characteristics of the FG beams. Also, they studied the effects of temperature rise and power law index on the vibration characteristics.
In the current study, free vibration analysis of simply-supported, clamped–clamped and clamped–free FG beam was employed by using two finite element models implemented by APDL version 17.2. In the first model the shell element (SHELL281) was adopted, while in the second model solid element (SOLID186) was investigated. The modulus of elasticity, density and other properties taken in the account assumed to varying through thickness direction based on power-law model of the volume fractions of the beam material constituents. The main difference between the current study and literature review is solid model which is three dimensional model and applying this model for three types of boundary condition and making compression between these types. Finally, the effects of the effect of length to thickness ratio, modulus ratio, mode number, power law index and types of support (simply support, clamped-clamped and clamped–free) on the first non-dimensional frequency parameters of FG beam were studied.

2. Problem Description:
The dimensions of FG beam with a rectangular cross-section are length (L), width (W), thickness (h) as shown in Figure (1-a). It is combined of two materials, typically metal and ceramic. A pure ceramic locates at the top surface of the beam while the bottom surface is a pure metal. The power-law model was used to describe the distribution of mechanical and physical properties along the thickness of beam according to [31], (see Figure (1-b)):

\[ P(y) = (P_c - P_m) \left[ \frac{y}{h} \right]^k + \frac{1}{2} + P_m \]  \hspace{1cm} (1)

Where \( P(y), P_c \) and \( P_m \) represent the property at any thickness, property of the pure ceramic and property of pure metal respectively.

![Figure 1](image-url)  (a) Geometry of Functionally Graded Beam.  
(b) Material Distribution.

3. Simulation Models:
Two finite element models were used to simulate the FG beam as shown below:

(1) Shell Model:
A two-dimensions model was created by drawing the length and width of FG beam after entering the properties (modulus of elasticity, density and poison ratio) to ANSYS, while the thickness of FG beam was represented as a layer having different properties. In this work, the thickness of FG beam was divided into ten layers and the number of elements depends on length of the FG beam and the convergent criteria of numerical solution. The next step after drawing the beam is meshing the body (in this model is area) as shown in Figure 2(c). Three types of boundary conditions applied on the edges of beam, and these types are clamped both ends beam, simply-supported beam, and clamped free beam. The natural frequency of shell model determined after applying boundary condition shell element (SHELL281) was used to represent the variation in the mechanical and physical properties along the thickness of FG beam, where "SHELL281 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses.
The element accounts for follower (load stiffness) effects of distributed pressures", [32]. In this model there were about (3250 – 9300) elements and the number of nodes were about (10500-30000) nodes, (see Figure (2)).

(2) Solid Model:
Similar to Shell model, the first step of applying this model is input the properties to ANSYS software. Three-dimensions model was created by dividing the thickness of FG beam into ten parts and each part represents a layer and these ten layers have different properties. The next step after drawing the beam is meshing the body (in this model is volume). Three types of boundary conditions applied on the edges of beam, and these types are simply supported beam, clamped-clamped beam, and clamped-free beam. The natural frequency of solid model determined after applying boundary condition, by entering ten numbers as the number of frequencies to be extracted then Run the model. The solid element (SOLID186) was used to represent the variation in the mechanical and physical properties along the thickness of FG beam, where "SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyper elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperplastic materials", [32]. The number of elements depends on length of the FG beam and the convergent criteria of numerical solution. In this model there were about (28000-285000) elements and the number of nodes were about (45000-450000) nodes, (see Figure (2)).
Figure (2): Properties and shapes of Shell and Solid elements used in this work.
4. Results Analysis

For **simply supported** FG Beam, Figure (4) shows the variation of the non-dimensional frequency parameter ($\lambda$) calculated by SHELL and SOLID models due to varying of power-law index and Length-to-height ratio for different modulus ratio. The comparison between SHELL and SOLID models shows that there are a very good agreement between the two models at each modulus ratio except modulus ratio =0.25. When the modulus ratio =0.5, the non-dimensional frequency parameter ($\lambda$) increases when the power-law index and length-to-height ratio increase. The increasing rate of the non-dimensional frequency parameter ($\lambda$) due to increase the power-law index increases when then the length-to-height ratio increases. When the modulus ratio = 1.0 (i.e. pure material), there is no effect of the power-law index in this case but the non-dimensional frequency parameter ($\lambda$) increases when the length-to-height ratio increases. When the modulus ratios are 2.0 and 4.0, the non-dimensional frequency parameter ($\lambda$) decreases when the power-law index increases for any length-to-height ratio. Also, the non-dimensional frequency parameter ($\lambda$) increases when the length-to-height ratio increases at any power-law index.

As the module ratio (E ratio) increases, the frequency parameter increases too, but with the change rate depending on the ratio of length, To. thickness and the index of power law. It's clear that the variation of the frequency parameter is proportion inversely with power-law index (K) when the modulus ratio (E ratio) is more than (1). While the variation of the frequency parameter is proportion directly with power-law index (K) when the modulus ratio (E ratio) is less than (1).

For **clamped-clamped** FG beam, the variation non-dimensional frequency parameter ($\lambda$) due to vary the power-law index and length-to-height ratio is illustrated in Figure (5). When the modulus ratio is 0.25, the variation of the non-dimensional frequency parameter ($\lambda$) has different form, because the variation of length-to-height ratio leads to change the vibration plane. The non-dimensional frequency parameter ($\lambda$) increases when the power law index and length-to-height ratio increase when the modulus ratio =0.5. Similar to results of simply supported FG beam, there is no effect to the power-law index on the non-dimensional frequency parameter ($\lambda$) when the modulus ratio =1.0. When modulus ratio is 2.0 and 4.0, the non-dimensional frequency parameter ($\lambda$) decreases when the power-law index increases for any length-to-height ratio. Also, the non-dimensional frequency parameter ($\lambda$) increases when the length-to-height ratio increases for any power-law index. When the modulus ratio (Eratio) increases, the frequency parameter increases too, but with change rate depending on ratio of length-to-thickness (L/h) and index of power-law. The frequency increase with increasing of power-law index when modulus ratio less than one and decrees with increasing power-law index when modulus ratio more than one.

For **clamped-free** FG beam, the variation non-dimensional frequency parameter ($\lambda$) due to vary the power-law index and length-to-height ratio is illustrated in Figure (6). Similar to results of simply supported and clamped – clamped FG beams, there is no effect to the power-law index on the non-dimensional frequency parameter ($\lambda$) and the non-dimensional frequency parameter ($\lambda$) increases when the length-to-height ratio increases when the modulus ratio =1.0. The non-dimensional frequency parameter ($\lambda$) increases when the power law index and length-to-height ratio increase when the modulus ratio is 0.25 and 0.5. When modulus ratio are 2.0 and 0.4, the non-dimensional frequency parameter ($\lambda$) decreases when the power-law index increases for any length-to-height ratio. Also, the non-dimensional frequency parameter ($\lambda$) increases when the length-to-height ratio increases for any power-law index.

From Figures (4), (5) and (6), the variation of the non-dimensional frequency parameter ($\lambda$) of clamped- clamped and clamped-free FG beams are similar to each other and differ from that of simply supported FG beam when the modulus ratio $\neq$ 1.0. Also, the non-dimensional frequency parameter ($\lambda$) of clamped-clamped FG beam is greater than that of clamped-free FG beam. For all
supporting types, the non-dimensional frequency parameter ($\lambda$) varied with the same way when the modulus ratio =1.0 (i.e. pure material).

In Figure (7) , (8), (9), (10) and (11), the comparison between the non-dimensional frequency parameter ($\lambda$) calculating by SHELL and SOLID models for the three types of supporting when the length-to-height ratio are 20, 40, 60, 80 and 100 respectively. Generally , when the length-to-height ratio increases discrepancy percentage between the results of SHELL model and that of SOLID models decreases for all types of supporting. Also, the maximum discrepancy percentage can be seen in simply supported FG beam when the length-to-height ratio =20. In other side, the modulus ratio has no effect on the discrepancy percentage between the results of SHELL model and that of SOLID models decreases for all types of supporting. Also, the effect of supporting types on the discrepancy percentage appears in simply supporting FG beam only and these is approximately no effect in clamped-clamped and clamped-free FG beam.

5. Validation of Result :

In order to validate the two models used in this work, a comparison between the results of Alshorbagy et al. [15] and Simsek et al. [11] and the results of the current models was achieved. Alshorbagy et. al. [15] calculated the frequency parameter for simply supported FG beam. The mechanical properties of FG beam are listed in Table (1). The dimensions of FG beam were: W (width) = 0.4 m, L (length) = 20 m while the thickness of FG beam was calculated from the length-to-thickness ratio. In this work, the FG beam was considered as a simply supported beam. To apply the boundary conditions in shell model, the nodes at the two ends of FG beam were fixed in x, y and z directions. While the nodes at the two ends of neutral plane were fixed in x, y and z directions in the solid model.

| Table (1): Properties of the materials used in this work [15]. |
|-----------------|-----------------|-----------------|
| Property        | Unit           | Metal (Steel)   | Ceramic (Alumina (Al2O3)) |
| Modulus of Elasticity (E) | GPa. | 210             | 390                         |
| Density($\rho$) | Kg/m$^3$       | 7800            | 3960                        |

The dimensionless frequency parameter can be calculated from the following equation [19]:

$$\lambda^2 = \frac{\omega t^2}{\sqrt{\rho A E I}} \quad (2)$$

Where:($\lambda$) is the dimensionless frequency parameter, ($\omega$) is the frequency in (rad/sec), (A) is the cross section area of the FG beam and (I) is the moment of inertia of the cross-section of the FG beam and it is $I = bh^3/12 \quad (3)$

Table (2) shows the comparison between the first dimensionless frequency of parameters for the present work (shell and solid elements) and that calculated by Alshorbagy et al. [15] and Simsek et al. [11] when the length to thickness was (20). Table (3) shows the same comparison when the length to thickness was (100). The maximum and minimum discrepancy percentages between the first dimensionless frequency parameters $\lambda_1$ shell and solid models with respect to that of Alshorbagy et al. [15] (i.e. the summary of Tables (2) and (3)) can be seen in Figure (3) and the following points were noticed:

(1) When L/h=20, the result of shell model was closer than the result of solid model. While the results of shell and solid models were very close to each other when L/h=100.

(2) In the solid model, the rang of maximum discrepancy percentage were (9.5 –14) % when L/h= 20
and (-1.5 – 0) % when L/h= 100. While in the shell model, the rang of maximum discrepancy percentage are (2 –4) % when L/h= 20 and (0 – 1.75) % when L/h= 100.
(3) In the solid model, the range of minimum discrepancy percentage were (5.5–9) % when $L/h=20$
and (-9.5 – 0) % when $L/h=100$. While in the shell model, the range of minimum discrepancy
percentage were (-1.5 –0.5) % when $L/h= 20$ and (-6.5 –0) % when $L/h= 100$.

(4) When the modulus ratio increased, the maximum discrepancy percentage also increased and
the minimum discrepancy percentage decreased.

**Table (2):** The First Dimensionless Frequency Parameters $\lambda_i$ for Different Material
Distribution and Different Modulus Ratio (Ec/Em) When (L/h=20) and (pc/pm= 1).

| Ec/Em | Author                | 0.1   | 0.2   | 0.5   | 1     | 2     | 5     | 10    |
|-------|-----------------------|-------|-------|-------|-------|-------|-------|-------|
| 0.25  | SHELL281              | 2.3356| 2.4127| 2.5695| 2.7131| 2.8443| 2.9487| 3.0067|
|       | SOLID186              | 2.1571| 2.2321| 2.3863| 2.5283| 2.6571| 2.7654| 2.8027|
| 0.5   | Alshorbagy et. al.[19] | 2.3746| 2.4614| 2.5979| 2.7041| 2.8057| 2.9302| 3.0085|
|       | Simsek et. al.[12]    | 2.3379| 2.4606|--      | 2.7035| 2.8053|--    | 3.0084|
|       | SHELL281              | 2.6509| 2.6877| 2.7685| 2.8484| 2.9257| 2.9977| 3.0269|
|       | SOLID186              | 2.4544| 2.4894| 2.5678| 2.6470| 2.7243| 2.7928| 2.8169|
| 1     | Alshorbagy et. al.[19] | 2.7107| 2.7576| 2.8363| 2.8946| 2.9461| 3.0110| 3.0563|
|       | Simsek et. al.[12]    | 2.7104| 2.7573|--      | 2.8944| 2.9459|--    | 3.0562|
|       | SHELL281              | 3.0647| 3.0647| 3.0647| 3.0647| 3.0647| 3.0647| 3.0647|
|       | SOLID186              | 2.8442| 2.8442| 2.8442| 2.8442| 2.8442| 2.8442| 2.8442|
| 2     | Alshorbagy et. al.[19] | 3.1400| 3.1400| 3.1400| 3.1400| 3.1400| 3.1400| 3.1400|
|       | Simsek et. al.[12]    | 3.1399| 3.1399|--      | 3.1399| 3.1399|--    | 3.1399|
|       | SHELL281              | 3.587 | 3.556 | 3.477 | 3.387 | 3.287 | 3.181 | 3.133 |
|       | SOLID186              | 3.335 | 3.306 | 3.233 | 3.146 | 3.045 | 2.936 | 2.894 |
| 4     | Alshorbagy et. al.[19] | 3.6775| 3.6300| 3.5296| 3.4423| 3.3768| 3.3196| 3.2726|
|       | Simsek et. al.[12]    | 3.6775| 3.6301|--      | 3.4421| 3.3765|--    | 3.2725|
|       | SHELL281              | 4.2298| 4.1710| 4.0207| 3.8371| 3.6179| 3.3684| 3.2497|
|       | SOLID186              | 3.9361| 3.8827| 3.7451| 3.5726| 3.3536| 3.0896| 2.9819|
| 4     | Alshorbagy et. al.[19] | 4.3365| 4.2455| 4.0346| 3.8241| 3.6496| 3.5326| 3.4549|
|       | Simsek et. al.[12]    | 4.337 | 4.2459|--      | 3.8234| 3.6485|--    | 3.4543|
Figure (3): The Maximum and Minimum Discrepancy Percentages Between the First Dimensionless Frequency Parameters $\lambda_1$ Shell and Solid Models with Respect to Alshorbagy et. al.[15].

Table (3): The First Dimensionless Frequency Parameters $\lambda_1$ for Different Material Distribution and Different Modulus Ratio(Ec/Em) When (L/h=100) and (pc/pm= 1).

| Ec/Em | Author | Power – Law Index (K) |
|-------|--------|-----------------------|
|       |        | 0.1 | 0.2 | 0.5 | 1   | 2   | 5   | 10  |
| 0.25  | SHELL281 | 2.4832 | 2.5480 | 2.6729 | 2.7765 | 2.8627 | 2.9540 | 3.0126 |
|       | SOLID186 | 2.5040 | 2.5697 | 2.6976 | 2.8059 | 2.8987 | 2.9953 | 3.0533 |
|       | Alshorbagy et. al.[19] | 2.3798 | 2.4683 | 2.6074 | 2.7159 | 2.8071 | 2.9317 | 3.0100 |
|       | Simsek et. al.[12] | 2.3752 | 2.4621 | -- | 2.7053 | 2.8071 | -- | 3.0100 |
| 0.5   | SHELL281 | 2.7604 | 2.7936 | 2.8610 | 2.9192 | 2.9682 | 3.0220 | 3.0582 |
|       | SOLID186 | 2.7766 | 2.8101 | 2.8782 | 2.9376 | 2.9890 | 3.0446 | 3.0807 |
|       | Alshorbagy et. al.[19] | 2.7121 | 2.7590 | 2.8377 | 2.8961 | 2.9476 | 3.0125 | 3.0578 |
|       | Simsek et. al.[12] | 2.7117 | 2.7587 | -- | 2.8960 | 2.9475 | -- | 3.0578 |
| 1     | SHELL281 | 3.1424 | 3.1424 | 3.1424 | 3.1424 | 3.1424 | 3.1424 | 3.1424 |
|       | SOLID186 | 3.1632 | 3.1632 | 3.1632 | 3.1632 | 3.1632 | 3.1632 | 3.1632 |
|       | Alshorbagy et. al.[19] | 3.1415 | 3.1415 | 3.1415 | 3.1415 | 3.1415 | 3.1415 | 3.1415 |
|       | Simsek et. al.[12] | 3.1415 | 3.1415 | -- | 3.1415 | 3.1415 | -- | 3.1415 |
| 2     | SHELL281 | 3.6422 | 3.6109 | 3.5403 | 3.4714 | 3.4098 | 3.3409 | 3.2892 |
|       | SOLID186 | 3.6791 | 3.6364 | 3.5644 | 3.4935 | 3.4568 | 3.3830 | 3.3324 |
|       | Alshorbagy et. al.[19] | 3.6791 | 3.6317 | 3.5313 | 3.4440 | 3.3784 | 3.3213 | 3.2743 |
|       | Simsek et. al.[12] | 3.6793 | 3.6320 | -- | 3.4440 | 3.3784 | -- | 3.2742 |
| 4     | SHELL281 | 4.2656 | 4.2114 | 4.0717 | 3.9268 | 3.7894 | 3.6396 | 3.5254 |
|       | SOLID186 | 4.3450 | 4.2844 | 4.1394 | 3.9842 | 3.8312 | 3.6692 | 3.5577 |
|       | Alshorbagy et. al.[19] | 4.3388 | 4.2476 | 4.0366 | 3.8260 | 3.5614 | 3.5343 | 3.4566 |
|       | Simsek et. al.[12] | 4.3392 | 4.2481 | 3.8259 | 3.6513 | -- | -- | 3.4565 |
Figure (4): The Non-dimensional Frequency Parameter ($\lambda$) Results of Simply Supported FG Beam Calculating by SHELL and SOLID Models for Different Modulus Ratio, Power-Law Model and Length-to-Height Ratio.
Modulus Ratio = 4.0

**Figure (5):** The Non-dimensional Frequency Parameter ($\lambda$) Results of Clamped-Clamped FG Beam Calculating by SHELL and SOLID Models for Different Modulus Ratio, Power-Law Model and Length-to-Height Ratio.
Figure (6): The Non-dimensional Frequency Parameter ($\lambda$) Results of Clamped-Free FG Beam Calculating by SHELL and SOLID Models for Different Modulus Ratio, Power-Law Model and Length-to-Height Ratio.

Modulus Ratio=2.0

Modulus Ratio=4.0
Figure (7): Variation of Non-dimensional Frequency Parameter ($\lambda$) of SHELL and SOLID Element Due to Increase the Power-Law Index at Different Modulus Ratio and Different Supporting Condition When Length-to-Height Ratio is (20).
Figure (8): Variation of Non-dimensional Frequency Parameter ($\lambda$) of SHELL and SOLID Element Due to Increase the Power-Law Index at Different Modulus Ratio and Different Supporting Condition When Length-to-Height Ratio is (40).
Figure (9): Variation of Non-dimensional Frequency Parameter ($\lambda$) of SHELL and SOLID Element Due to Increase the Power-Law Index at Different Modulus Ratio and Different Supporting Condition When Length-to-Height Ratio is (60).
Figure (10): Variation of Non-dimensional Frequency Parameter ($\lambda$) of SHELL and SOLID Element Due to Increase the Power-Law Index at Different Modulus Ratio and Different Supporting Condition When Length-to-Height Ratio is (80).
**Figure (11):** Variation of Non-dimensional Frequency Parameter ($\lambda$) of SHELL and SOLID Element Due to Increase the Power-Law Index at Different Modulus Ratio and Different Supporting Condition When Length-to-Height Ratio is (100).
6. Conclusion

From the obtained results the following points can concluded:

(1) The non-dimensional frequency parameter ($\lambda$) decreases when the power-law index increases if the modulus ratio is more than one. While the non-dimensional frequency parameter ($\lambda$) increase when the power-law index increases if the modulus ratio is less than one for any type of support.

(2) When the length-to-thickness ratio increase, the non-dimensional frequency parameter ($\lambda$) tends to be more regular and for any type of support.

(3) There is a significant effect of modulus ratio on the non-dimensional frequency parameter ($\lambda$) when power-law index is high and length-to-thickness ratio is low.

(4) For simply supported FG beam, the SHELL model is closer than SOLID model when they compared with model of Alshorbagy et. al. [15].

(5) There is a very good agreement between the results of SHELL and SOLID models at clamped-clamped and clamped-free FG beam at any length-to-height ratio and any power-law index. While, the maximum discrepancy percentage between the two models appears in simply supported FG beam when the length-to-height ratio is 20.

7. Recommendations

1- Calculate the fundamental frequency when FG beam properties vary in length direction.

2- Studying the effects of varying the properties of FG beam in both length and thickness direction at same time (i.e. 2D-FG Beam).

3- Studying the results of free FG beam vibration with crack

4- Analysis of FG beam force vibration in length and/or direction of thickness

List of Symbols

| Main symbols | Meaning                                      | Units        |
|--------------|----------------------------------------------|--------------|
| $k$          | Power-law index                              | ---          |
| $L/h$        | length-to-thickness ratio                    | ---          |
| $P(y)$       | property at any thickness                    | ---          |
| $P_c$        | property of the pure ceramic                 | ---          |
| $P_m$        | property of pure metal                        | ---          |
| $\lambda$    | frequency parameter                          | ---          |
| $\omega$     | frequency                                    | rad/sec      |
| $I$          | the moment of inertia of the cross-section of the FG beam | m$^4$        |
| $Ec$         | Modulus of elasticity of ceramic             | GPa          |
| $Em$         | Modulus of elasticity of metal               | GPa          |
| $\rho_c$     | Density of ceramic                            | Kg/m$^3$     |
| $\rho_m$     | Density of metal                              | Kg/m$^3$     |
| $A$          | the cross section area of the FG beam         | m$^2$        |
| $L$          | length                                       | m            |
| $W$          | width                                        | m            |
| $h$          | thickness                                    | m            |
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