Constraints on pulsar masses from the maximum observed glitch

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Neutron stars are unique cosmic laboratories in which fundamental physics can be probed in extreme conditions not accessible to terrestrial experiments. In particular, the precise timing of rotating magnetized neutron stars (pulsars) reveals sudden jumps in rotational frequency in these otherwise steadily spinning-down objects. These ‘glitches’ are thought to be due to the presence of a superfluid component in the star, and offer a unique glimpse into the interior physics of neutron stars. In this paper we propose an innovative method to constrain the mass of glitching pulsars, using observations of the maximum glitch observed in a star, together with state-of-the-art microphysical models of the pinning interaction between superfluid vortices and ions in the crust. We study the properties of a physically consistent angular momentum reservoir of pinned vorticity, and we find a general inverse relation between the size of the maximum glitch and the pulsar mass. We are then able to estimate the mass of all the observed glitches that have displayed at least two large events. Our procedure will allow current and future observations of glitching pulsars to constrain not only the physics of glitch models but also the superfluid properties of dense hadronic matter in neutron star interiors.

Although the exact nature of the trigger mechanism for glitches is still debated, with crustquakes, vortex avalanches and fluid instabilities likely contenders (see ref. 4 for a comprehensive review), the multifluid framework for describing the hydrodynamics of superfluid neutrons in neutron stars is well established5–14 and enables us to model the glitch itself and the subsequent relaxation15–17. In fact, recent calculations have shown that combining observational constraints from the average glitching activity of the Vela pulsar with state-of-the-art nuclear physics models of the effective mass of superfluid neutrons can lead to constraints on the mass of the star and on the equation of state (EOS) of dense matter18–23.

Here we show how the maximum glitch amplitude recorded in a given pulsar can robustly constrain its mass when coupled to state-of-the-art calculations of the pinning force between superfluid vortices and ions in the crust24. We analyse a physically consistent scenario for the reservoir of angular momentum25 and propose a method to bracket the mass values using observational data of the maximum event. After studying all known ‘large’ glitches (defined here as those pulsars for which the maximum recorded glitch is $\Delta \Omega \geq 0.5 \times 10^{-4}$ rad s$^{-1}$), and in particular those that have displayed at least two large events, we obtain a general inverse relation between the mass of frequently glitching pulsars and their largest glitch. Future observations have the potential to both verify and calibrate this relation, constraining at the same time the microphysics used as theoretical input. We note that the young Crab pulsar cannot be classified as a large glitcher: the maximum observed event is only $\Delta \Omega_{\text{Crab}} = 0.4 \times 10^{-4}$ rad s$^{-1}$. Indeed, the small glitches in the Crab are usually thought to be associated with crustquakes3, a scenario alternative to superfluidity but unable to explain Vela-like large glitches.

Pinning and maximum angular momentum reservoir

Our main assumption is that superfluid vortices can pin to the lattice of ions in the crust of a neutron star26, as widely assumed in most pulsar glitch models: pinned vortex lines cannot move out as the normal component of the star spins down, and the superfluid
lags behind, storing angular momentum which is then released during a glitch (the pinning paradigm). A consistent description of the multifluid problem must include entrainment, a non-dissipative coupling between the two components: the diminished mobility of neutrons caused by entrainment can be expressed in terms of an effective mass for the superfluid neutrons. To describe the differential rotation of the neutron superfluid in the presence of density-dependent entrainment, we adopt the formalism developed in ref. under the assumption of axial symmetry; this simplified geometry is the first natural approximation to the complex two-fluid hydrodynamical problem in which turbulence is likely to develop.

The model we adopt for the reservoir of pinned vorticity is discussed elsewhere and detailed in the Methods. It assumes parallel straight vortex lines, pinned only in the crust but threading the entire star; that is, the neutron superfluid is continuous throughout the star interior with no layer of normal neutrons separating the star interior with no layer of normal neutrons separating the neutron superfluid. The range of observational lags used in the present study is also indicated (lighter shading), corresponding to the values listed in the last two columns of Table 1.

**Figure 1** | Critical lag profile as a function of the cylindrical radius $x$.

The lag profile $\omega(x)$ is obtained from equation (3) for a neutron star with mass $1.4M_\odot$ using the Bsk21 EOS. The solid horizontal line indicates the increasing nominal lag $\omega^* = t |\Delta \omega|$, and the shaded area below it represents the corresponding lag $\omega(x)$ developed between the two components since corotation (see equation (6)). The distance from the rotational axis of the star is expressed in units of the neutron drip radius ($R_d$), which delimits the superfluid. The range of observational lags used in the present study is also indicated (lighter shading), corresponding to the values listed in the last two columns of Table 1.

**Figure 2** | Upper limit to the mass for a selection of pulsars.

The theoretical maximum glitch $\Delta \Omega_{\text{max}}$, given by equation (5), is plotted as a function of the stellar mass for three EOSs: Sly (yellow), Bsk20 (blue) and Bsk21 (red). Horizontal lines, labelled by pulsar names, indicate the largest glitch amplitude $\Delta \Omega_{\text{abs}}$ recorded in the corresponding pulsar. The mass values $M_{\text{abs}}$ are given by the intersection of the horizontal lines and the curves $\Delta \Omega_{\text{max}}(M)$. The upper limit for the mass defines a forbidden region, shown here for the case of Bsk21 (shaded). The curves are terminated by the maximum mass allowed by each EOS (crosses); this determines the minimum $\Delta \Omega$ that can be constrained by the corresponding EOS.

The lag profile $\omega(x)$ (red solid curve) is obtained from equation (3) for a neutron star with mass $1.4M_\odot$ using the Bsk21 EOS. The solid horizontal line indicates the increasing nominal lag $\omega^* = t |\Delta \omega|$, and the shaded area below it represents the corresponding lag $\omega(x)$ developed between the two components since corotation (see equation (6)). The distance from the rotational axis of the star is expressed in units of the neutron drip radius ($R_d$), which delimits the superfluid. The range of observational lags used in the present study is also indicated (lighter shading), corresponding to the values listed in the last two columns of Table 1.
this would lead to a lower value for the maximum mass of the star. Moreover, as shown by equation (5), the limit is entrainment-independent and uniquely determined by the density profile of the pinning force. At present, out of 127 objects that have undergone at least one glitch, there are 51 observed large glitches for which a mass limit can be obtained; the remaining pulsars with smaller observed maximum glitch are not constrained, as any mass can account for these smaller events. In some cases, this could be due to observational selection effects (such as short time of observation or slow evolution due to small spin-down), and some of these objects may be constrained in the future.

Most of these 51 objects are single glitchers — that is, pulsars that so far have displayed a single large event that is greater by at least one order of magnitude than all the other recorded glitches; thus, the typical time intervals between large glitches and the average glitching activities are as yet unidentified in these objects, until new observations improve the statistics. There are, however, 17 large glitches that have displayed at least two large events of comparable magnitude: they are listed in Table 1. For these pulsars, we can further determine a lower limit for the mass using their observed timing behaviour; we are thus able to bracket the mass within a range of values determined only by the observed parameters of the maximum event.

### Mass estimates from glitch observations

To proceed, we rely on the scenario sketched in Fig. 1: starting from corotation at \( t = 0 \), we can measure time in terms of a nominal lag defined as \( \Delta\Omega = \Omega \Delta t \); in this way we can treat all pulsars within a unified model, regardless of their specific spin-down \( \Omega \). Then, the increasing \( \Delta\Omega \) determines the amount of angular momentum that can be accumulated according to the pinning paradigm. This is indicated by the shaded region in Fig. 1: the curve that delimits it, \( \Delta\Omega_{\text{act}}(\omega^*, M) \), represents the lag built up between the two components in an interval \( \omega^* \) since corotation.

We now make the additional assumption that the maximum glitch depletes the whole available reservoir of angular momentum. This approximation is generally made for all glitches in the Vela pulsar\(^{13}\) and in the other frequent glitches that show a preferred size for the events and glitch quasi-periodically\(^{44}\): here we extend it to all large glitches, but only for their maximum-size event. Given the reservoir \( \omega(x) \), from angular momentum conservation, we can then find the glitch amplitude corresponding to total depletion of the reservoir, namely we calculate \( \Delta\Omega = \Delta\Omega(\omega^*, M) \); this expression depends on entrainment.

In Fig. 3, we plot the curve \( \Delta\Omega(\omega^*, M) \) as a function of the nominal lag for different values of the neutron star mass in the range 0.9–2.2\(M_\odot\) and for the Bsk21 EOS; the other EOSs produce qualitatively similar results. For large enough \( \omega^* \) (of order \( 10^{-4}\text{ rad s}^{-1} \)), the curves reach their maximum value \( \Delta\Omega_{\text{max}}(M) \) which is independent from entrainment; indeed, the time-dependent reservoir tends to its maximum allowed profile \( \omega(x) \) (when the rising horizontal line in Fig. 1 has reached the peak in the crust), so that equation (7) naturally tends towards equation (4) for the maximum allowed glitch.

In particular, for each pulsar we now consider the observed waiting time of its maximum event, namely the time \( t_{\text{max}} \) measured between the maximum observed glitch and the one preceding it. The corresponding nominal lag is \( \Delta\Omega_{\text{max}}(M_{\text{max}}) = \Delta\Omega(\omega^*, M) \); each pulsar is then characterized by two observed quantities, the amplitude \( \Delta\Omega \) and waiting lag \( \omega^* \) of its maximum event. These values allow us to locate the pulsar in the plane of Fig. 3, thus determining a corresponding mass \( M_{\text{max}} \). This amounts to inverting the relation \( \Delta\Omega(\omega^*, M_{\text{max}}) = \Delta\Omega \). The value obtained for \( M_{\text{max}} \) in this way is obviously a lower limit on the mass of the star: unless the glitch preceding the largest one has emptied the entire reservoir, thus ensuring initial corotation (which in general is not the case), the angular momentum accumulated since the previous glitch is larger than \( \Delta\Omega_{\text{max}} \), and thus a mass larger than \( M_{\text{max}} \) is enough to reproduce \( \Delta\Omega \).

Summarizing, the angular momentum transferred during the maximum glitch must lie between two extrema: the minimum amount that can have been built up since the previous glitch, and the maximum that the pinning force can sustain. We can thus estimate the mass of a pulsar by bracketing it between the corresponding values \( M_{\text{pre}} \) and \( M_{\text{max}} \).

The same procedure can be used to fit a mass value \( M_{\text{pre}} \) that can reproduce the pulsar absolute activity \( A_{\text{abs}} \) defined as the average rate

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**Table 1** | Observational parameters for the pulsars considered in this work.

| J-name | \( |\Omega| \) (10\(^{-4}\text{ rad yr}^{-1} ) \) | \( A_{\text{abs}} \) (10\(^{-4}\text{ rad yr}^{-1} ) \) | \( \Delta\Omega \) (10\(^{-4}\text{ rad s}^{-1} ) \) | \( \omega_{\text{act}}^* \) (10\(^{-4}\text{ rad s}^{-1} ) \) | \( \omega_{\text{pre}}^* \) (10\(^{-4}\text{ rad s}^{-1} ) \) |
|--------|------------------|------------------|------------------|------------------|------------------|
| J0205+6449 | 88.97 | 0.63 ± 0.11 | 3.63 ± 0.38 | 508 ± 125 | 88 ± 20 |
| J037-6910 | 394.97 | 3.41 ± 0.06 | 2.65 ± 0.25 | 106 ± 30 | - |
| J0631+1036 | 2.51 | 0.04 ± 0.01 | 0.72 | 41 ± 11 | 2.91 ± 0.03 |
| J0835–4510 | 31.07 | 0.50 ± 0.01 | 2.17 | 134 ± 2 | 101 |
| J1048–5832 | 12.49 | 0.22 ± 0.03 | 1.55 | 86 ± 11 | 28.1 ± 0.4 |
| J1105–6107 | 7.86 | 0.12 ± 0.03 | 0.97 ± 0.01 | 62 ± 10 | 21.3 ± 3.23 |
| J1341–6220 | 13.43 | 0.22 ± 0.02 | 1.00 | 59 ± 4 | 12.5 ± 1.3 |
| J1431–6414 | 8.10 | 0.13 ± 0.02 | 0.53 | 32 ± 3 | 25.8 ± 0.9 |
| J1420–6048 | 35.47 | 0.47 ± 0.03 | 1.86 ± 0.01 | 138 ± 9 | 112 ± 9 |
| J1709–4429 | 17.56 | 0.25 ± 0.05 | 1.76 ± 0.02 | 121 ± 14 | 59 ± 4 |
| J1730–3350 | 8.65 | 0.11 ± 0.02 | 1.44 | 107 ± 16 | 97.2 ± 0.7 |
| J1801–2451 | 16.25 | 0.28 ± 0.03 | 1.89 | 106 ± 8 | 62.5 ± 0.5 |
| J1803–2137 | 14.91 | 0.29 ± 0.03 | 2.25 | 116 ± 10 | 95.8 ± 0.1 |
| J1826–1334 | 14.49 | 0.20 ± 0.04 | 2.22 | 159 ± 24 | 19.0 ± 0.1 |
| J1932+2220 | 5.47 | 0.25 ± 0.05 | 1.94 | 42 ± 7 | 50 ± 1 |
| J2021+3651 | 17.63 | 0.31 ± 0.06 | 1.57 | 89 ± 17 | 24.9 |
| J2229+6114 | 58.23 | 0.30 ± 0.05 | 1.49 ± 0.01 | 282 ± 45 | 74.5 ± 0.5 |

\( \Omega \), spin-down rate; \( A_{\text{abs}} \), absolute activity; \( \Delta\Omega \), maximum observed glitch; \( \omega_{\text{act}}^* \) and \( \omega_{\text{pre}}^* \), nominal lags. The observational errors on the glitch parameters are also reported; no errors are listed when they are so small that they do not significantly affect our mass estimates.
of spin-up due to all glitches and derived from observations. If the angular momentum is released in a succession of glitches of maximum size \(\Delta \Omega\), each depleting the available reservoir, the mean waiting time between glitches that reproduces the activity is \(t_{\text{act}} = \Delta \Omega / \Omega_{\text{act}}\). The corresponding nominal lag is \(\omega_{\text{act}} = \omega_{\text{pre}} / t_{\text{act}} = \Delta \Omega / \Delta \Omega_{\text{act}}\); as before, we can invert the relation \(\Delta \Omega_{\text{act}}(\omega_{\text{act}}, M_{\text{pre}}) = \Delta \Omega\) to obtain the corresponding mass \(M_{\text{act}}\), again entrainment-dependent. This is shown graphically in Fig. 3, where the observational values \(\Delta \Omega\) and \(\omega_{\text{act}}\) are indicated for a sample of pulsars, together with their reported observational errors.

### Results

The glitch data used in the analysis are given in Table 1, and the results for the three mass estimates are shown in Fig. 4 for the Bsk21 EOS; the other EOSs produce similar results. Although there are quantitative differences between EOSs (see Fig. 5), several qualitative features are evident for all models. First, it is remarkable that for most pulsars we can set tight constraints for the mass of the star, except J0537–6910, which, despite being one of the pulsars with the largest number of observed glitches, only has an upper limit on the mass, as the maximum glitch was also the first observed glitch\(^{16}\). Moreover, we can see how a tight range of masses (approximately between 1.1 and 2.2\(M_\odot\)) can explain a spread of almost an order of magnitude in glitch sizes. In particular, the results for \(M_{\text{pre}}\) (the lower bound on the mass) and \(M_{\text{act}}\) (the mass estimate constrained by the activity) again show the inverse relation between mass and maximum glitch size, noted previously for the maximum reservoir and indicated in Fig. 4 by the solid line (which provides the upper bound \(M_{\text{max}}\)). These mass values, however, correspond to a partially filled reservoir and are determined using additional independent observational constraints, so that they could have been scattered randomly. Their consistency with the maximum curve provides a test for the validity of our scenario and suggests that an inverse relation may indeed exist between pulsar mass and maximum glitch allowed; if this is the case, it indicates that mass can be a key ingredient to understand the different behaviour of glitching pulsars (in addition to age, temperature and rotational parameters).

As already observed, the mass values found here correspond to present state-of-the-art microphysical input: future theoretical advances may renormalize the masses but maintain the qualitative general relation. Direct mass measurements of glitching pulsars are of course necessary to verify the relation, but a single observation would already allow calibration of the curve and give constraints on the microphysical input.

The complete range of derived masses for the three EOSs is displayed in Fig. 5. The errors indicated result only from observational indeterminacies in the glitch data, listed in Table 1; with the exception of two objects, these observational errors on masses are very small. Because its largest glitch is small, the mass of J1413–6141 is not constrained by the soft SLy EOS: any mass allowed by the EOS can sustain its maximum event. We note that, in general, the mass value \(M_{\text{act}}\) is higher than our lower mass estimate \(M_{\text{pre}}\) in the quasi-periodic Vela pulsar (as well as in several others that are not usually regarded as quasi-periodic), its value is fairly close to \(M_{\text{act}}\), indicating that the reservoir of angular momentum is nearly depleted during each large glitch. It has already been suggested\(^{16}\), in work with a polytropic model, that lower-mass pulsars may have a narrower distribution of glitch sizes, centered around larger events, and our current, more detailed analysis with microphysically motivated EOSs confirms that this is likely to be the case.

Our model predicts a broad distribution of masses, centered around 1.4\(M_\odot\). We note that population studies\(^{17}\) also recover a broad distribution, but one that depends strongly on the evolutionary path of the system, with masses in neutron-star–neutron-star binaries tightly distributed around 1.4\(M_\odot\) and masses in white-dwarf–neutron-star binaries much more broadly distributed around higher values. Future radio and gravitational-wave observations are likely to probe the mass distribution in more detail and thus allow us to investigate the evolutionary history of systems with glitching pulsars.

Our framework suggests a unified scenario for pulsars exhibiting large glitches, with the mass of the neutron star playing a key

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**Figure 3** | Glitch amplitude as a function of the nominal lag since corotation. The function \(\Delta \Omega (\omega_{\text{act}}, M)\), given by equation (7), is plotted (dotted curves) as a function of \(\omega_{\text{act}}\) for different values of the neutron star mass in the range 0.9–2.2\(M_\odot\) (indicated for each line) using the Bsk21 EOS. We also show the location of a sample of pulsars (red dots), used to estimate \(M_{\text{act}}\); each pulsar is characterized by its maximum observed glitch \(\Delta \Omega\) and the associated waiting lag \(\omega_{\text{act}}\), as listed in Table 1. The observational uncertainties on these quantities, also listed in Table 1, are reported as error bars or shaded regions; for Vela, the error is smaller than the symbol used and thus not reported.

**Figure 4** | Mass estimates for 17 large glitchers with the Bsk21 equation of state. The red solid curve gives \(M_{\text{act}}\) as a function of the maximum observed glitch \(\Delta \Omega\): as in Fig. 2, the shaded region indicates the forbidden region and the cross corresponds to the maximum mass (2.27\(M_\odot\)) allowed by the Bsk21 EOS. For each pulsar listed in Table 1 and characterized by its observed \(\Delta \Omega\), the mass interval \([M_{\text{pre}}, M_{\text{act}}]\) is indicated by blue vertical bars, and the estimate for \(M_{\text{act}}\) is shown as a blue circle. As explained in the text, the lower bound \(M_{\text{pre}}\) is undetermined for J0537–6910.
role; the values of the upper limit $M_{\text{up}}$ are robust and independent of entrainment, while $M_{\text{act}}$ and $M_{\text{act}}$ can be refined with the aid of hydrodynamical simulations in place of our simplified model. The approach is alternative to the methodology described previously\(^\text{36}\) that relies on the mean behaviour over many decades of pulsar evolution (that is, the activity) coupled to indirect estimates of the neutron star’s internal temperature, whereas here we use only the data associated to the largest observed event. Moreover, whereas our maximum angular momentum reservoir is determined by the profile of the pinning force and consists of neutrons paired in both the singlet and triplet channels, their reservoir is fixed by both the density and the temperature dependencies of the neutron pairing gap in the singlet channel alone. A comparison of their results with our values for $M_{\text{up}}$ is possible, as the two studies have two EOSs and eight pulsars in common. Even considering errors, and although we both interpret the Vela as a middle-mass object, our results are completely at variance with those of ref.\(^\text{21}\); their estimates and ordering of masses bear no resemblance to ours, the mass values are much more dependent on the EOS used as input, and the mass distributions are poor in low-mass objects (for example, for Bsk21, all their estimated masses are larger than $1.6M_\odot$). The difference is probably due to the additional complication introduced by using an angular momentum reservoir that depends on thermal properties as well as to the different reservoirs adopted in the two studies.

Improved unified models for the superfluid properties of neutron stars and EOS-consistent calculations of the pinning forces will lead to even tighter constraints, as will further observations of glitching pulsars. A true breakthrough would, however, come from an actual measurement of the mass of a glitching pulsar, which may be possible in the near future if pulsars in binary systems are observed to glitch.\(^\text{37}\) A number of such measurements, combined with the methods illustrated above, will allow researchers to constrain neutron star interior physics further and will help to pin down properties of cold, dense matter.

**Methods**

We follow the formalism and notations of previous work\(^\text{22}\) for a consistent description of a stratified pulsar with superfluid entrainment and differential neutron rotation. Under the widespread assumption of axisymmetry of the system, we can project exactly the three-dimensional hydrodynamical problem to a one-dimensional cylindrical one. It is possible to account for the entrainment coupling by defining an average procedure for functions $\phi(x)$ of the cylindrical radius $x$

$$\langle \phi(x) \rangle = \frac{1}{l} \int_0^l \text{d}l \phi(x)$$

where $l$ is the drip radius delimiting the superfluid and $l$ is the normalization factor for the measure $\text{d}l$, representing the moment of inertia distribution of the superfluid component.

The structure of the star, namely its radial density profile $\rho = \rho(r)$, is found by integrating the Tolman–Oppenheimer–Volkoff equations with an EOS for the composition and pressure of dense matter as a function of baryonic density. We study three unified EOSs: SLy (ref.\(^\text{39}\)), Bsk20 (ref.\(^\text{40}\)) and the stiffer Bsk21 (ref.\(^\text{41}\)), with maximum allowed masses of 2.05$M_\odot$, 2.16$M_\odot$ and 2.27$M_\odot$, respectively, and thus all compatible with the recent observations of a $\sim 2M_\odot$ neutron star\(^\text{42}\). These EOSs describe a unified way both the crust and the core of the star, and they are compatible with all the constraints on nuclear matter properties around saturation obtained from experiments; moreover, they give neutron star radii that are consistent with present observational limits\(^\text{43}\).

In this first study, we evaluate the moments of inertia in the Newtonian approximation. Although describing rotations in general relativity can have non-negligible effects on the dynamics of pulsar glitches, as shown in numerical simulations\(^\text{44}\), the relativistic increase of the moments of inertia is expected to partially cancel out in the ratios of equations (4) and (7); we are currently studying this aspect. In the Newtonian approximation, the explicit form of $\text{d}l$ that encodes the entrainment corrections to the two-component dynamics is

$$\frac{\text{d}l(x)}{\text{d}x} = 4\pi\rho \int_0^{r(x)} \text{d}z \rho^2_{\text{SF}}(r) m^2(r)$$

with $\rho_{\text{SF}}$ the neutron density profile, $z(x) = \sqrt{r^2 - x^2}$ the height of vortices passing through $x$ and $r = \sqrt{r^2 - x^2}$ the spherical radius. Entrainment is introduced in terms of the adimensional neutron effective mass $m^2(r)$ (in units of the free neutron rest mass $m_n$); in this paper, we use the recent estimates of $m^2(r)$ obtained for the inner crust\(^\text{45}\) and for the core\(^\text{46}\). The density of the crust–core interface is determined by the EOS under study, whereas the drip density separating inner and outer crust and delimiting the superfluid is $\rho_{\text{SF}} = 4.3 \times 10^{13}$ g cm$^{-3}$; the drip radius $R_c$ can then be determined, once the density profile has been found for a given neutron star mass.

The normal component (comprising the crustal ions and the charged fluids) is frozen into the stellar magnetic field on Alfven timescales\(^\text{47}\); thus it rotates...
rigidly with angular velocity $\Omega$, related to the observed pulsar period $P$ by $\Omega = 2\pi/P$. On the other hand, the superfluid rotates differentially with an angular velocity $\Omega(x)$ that depends only on the cylindrical radius $x$, because axial symmetry implies vortex lines parallel to the rotation axis of the star. This quantity is related, through the standard Feynman relation, to the number of vortex lines inside the cylindrical region of radius $x$ and does not represent the kinematic velocity of superfluid neutrons. The lag between the two components, defined as $\Delta L = \Omega_3 - \Omega_1$, determines the reservoir available for a glitch. Because the excess angular momentum associated with the lag and stored in the superfluid can be expressed as $\Delta L = I \langle \dot{\omega}(x) \rangle$, where $I$ is the moment of inertia of the superfluid component corrected for entrainment. The average value of the lag is weighted by the superfluid moment of inertia of a cylindrical shell at radius $x$ and is observed in integration with the normalized measure $dL(x)/I$, (see equations (1) and (2)).

We start by considering the maximum amount of angular momentum that can be stored in the superfluid for a given model of the pinning force $f(x)$. This scalar quantity describes the strength of the mesoscopic interaction between a unit length of vortex line and the lattice at a given density $\rho$ in the crust: its value is the threshold above which the segment of vortex line is unpinned. This vortex–lattice force can be derived from the microscopic vortex–nucleus interaction\(^44\) by counting the effective number of pinning sites intersected by a unit length of vortex. Realistic values of $f(x)$ at the mesoscopic scale have been recently obtained\(^37\) by taking into account the finite vortex tension, the lattice Coulomb energy and the relative orientation of the lines with respect to the lattice principal axes. The mesoscopic pinning force turns out to depend very little on whether the microscopic force is attractive or repulsive in a given region of the star, which compensates for the present lack of consensus on the sign of the vortex–nucleus interaction as a function of density\(^38\). In our calculations, we use the results of ref. \(^37\) for $f(x)$ in the neutron star crust: in particular, we use the pinning forces corresponding to in-medium suppressed pairing gap (the case $\beta = 3$ and $L = 5000$); incidentally, this crustal gap is similar to the SFB model for single neutron superfluidity adopted in the study of ref. \(^35\).

The total pinning force is then derived by integration of $f(x)$ along the straight vortex lines. In most of the existing literature, the neutron superfluids in the core and the crust of the neutron star have been assumed to be separated, with the core P-wave superfluid strongly coupled to the normal component and only the S-wave crust superfluid accumulating angular momentum for the glitch. The strong entrainment found in the crust, however, challenges this model for the reservoir: the crust is not enough to explain large glitches\(^30,39\). Moreover, consistent microscopic calculations of the neutron pairing gap so far do not show any shell of normal matter that could physically separate the two superfluids and disconnect the respective vortices. Indeed, the absence of normal neutrons requires that the matter temperature in the outer core is lower than the critical temperature $T_c$ for P-wave superfluidity. On the one hand, microscopic calculations of neutron pairing gaps in the triplet channel\(^32\) give $T_c > 5 \times 10^9 K$ for densities $n > 0.08$ nucleons fm\(^3\); on the other hand, simulations of cooling constrained by observations\(^9\) predict isothermal outer cores with temperatures always smaller than $2.2 \times 10^9 K$ for all the pulsars considered (for Vela, the estimated temperature is $T = 1.2 \times 10^9 K$). The constraints on superfluid properties in neutron star cores obtained from observations of fast cooling in the central compact object in Cassiopeia A\(^34\) are still not conclusive, because different physical scenarios are able to explain the observations\(^35\); moreover, even the presence of the fast cooling itself is questioned, although not firmly excluded\(^41\). Therefore we will follow the other alternative, first outlined in ref. \(^48\) but not implemented until the approach of ref. \(^49\): we assume a continuous superfluid in the star interior, described by vortex lines that stretch across the whole neutron star.

The protons in the core are also expected to be superconducting, with quantized flux tubes carrying the magnetic flux. Owing to their mutual interaction, vortices can pin to these flux tubes, which opens interesting pinning scenarios like that of ref. \(^51\). Existing microscopic calculations of the force per unit length in the core obtain strong pinning, comparable to that in the crust\(^52\). These calculations, however, are performed in highly symmetric configurations of vortices and flux tubes, which maximize the interaction: they provide only an upper limit to core pinning. As no calculation currently exists for realistic configurations, and given the observational uncertainty on the presence of core pinning\(^53\), in this work we assume negligible $f(x)$ in the neutron star core. This is a point to be kept in mind for future developments, but a realistic profile for vortex–flux-tube pinning can easily be added to the one that we use for crustal pinning, and incorporated in our method.

The critical lag for depinning can next be found as\(^55\)

$$\omega_c(x) = \int_0^{\rho_c(x)} f(x) \rho(x) \, dz \int_0^{\rho_c(x)} 4\pi g_p(r) \, m^2(r)$$

(3)

where $\kappa = \pi \hbar /m_s$ is the quantum of circulation of the neutron superfluid. The maximum reservoir of angular momentum is $\Delta L_{\text{max}} = I \langle \dot{\omega}_c(x) \rangle$ and simple angular momentum conservation during a glitch (angular momentum losses due to radiation proceed over much longer timescales) then gives the size of the maximum permitted glitch (the change in $\Omega_3$ before and after the event) as\(^57\)

$$\Delta \Omega_{\text{max}} = I \langle \dot{\omega}_c(x) \rangle$$

(4)

where $I$ is the total moment of inertia of the star. In general, the scaling of the maximum glitch size with mass seen in Fig. 2 is the same that can be expected for the average glitching activity of a pulsar, and is related to the fact that both quantities are roughly proportional to the ratio between the moment of inertia of the reservoir and the total moment of inertia of the star.

Although both $I_c$ and $\langle \omega(x) \rangle$ have an explicit dependence on the neutron effective mass $m^*$, it turns out analytically that these cancel out, so that the maximum glitch is independent from entrainment; indeed, from equations (3) and (4), we can derive the following expression

$$\Delta \Omega_{\text{max}} = \frac{4\pi}{R} \int_0^{R} dx \int_0^{\rho_c(x)} dz f(x)$$

(5)

which shows that the maximum glitch is independent of $m^*$ and, for a given stellar structure $\rho(r)$, is determined by the pinning force $f$. This is to be expected: entrainment affects the rate at which the reservoir is filled and the dynamical times for exchange of angular momentum through dissipative mutual friction, but has no effect on the maximum allowed amount of stored angular momentum, which is determined only by the strength of the pinning force. We also note that the maximum glitch of equation (5) does not depend on whether the vortex lines stretch across the entire neutron star interior (both S- and P-wave superfluidity reservoir, as we assume here) or are limited to the crustal zone (only S-wave superfluidity reservoir). Since S-wave superfluidity reservoir, the option usually studied in the literature, this implies that the upper limit obtained for the mass would have the same value $M_{\text{max}}$ in both scenarios for the reservoir, which further strengthens the robustness of this constraint.

We finally consider the partial filling of the reservoir in a time $t$ since corotation; at nominal lag $\omega^* = t \Omega_3$, the accumulated lag is (see Fig. 1)

$$\omega(x) = \min(\omega_c(x), \omega^*)$$

(6)

From this reservoir we can derive the angular momentum $\Delta I_c = I \langle \omega(x) \rangle$ accumulated after a time $t$ since corotation, where the average is again calculated with normalized measure $dL(x)/L$. Note that this quantity includes the effect of entrainment, because terms depending on $m^*$ do not cancel out as they did in equation (4); $\Delta I_c$ is reduced by strong entrainment, as expected. From angular momentum conservation, we can then find the glitch corresponding to total depletion of the reservoir after a time $t$ since corotation as

$$\Delta \Omega_t = \frac{I}{I} \langle \omega(x) \rangle$$

(7)

Once the microphysical input has been fixed, this expression depends only on the nominal lag and on the neutron star mass: that is, $\Delta \Omega_t = \Delta \Omega_t(\omega^*, M)$.

The absolute activity is defined as $A_t = \sum \Delta \Omega_t / \tau$, where $\Delta \Omega_t$ are the observed glitch sizes during the observation time $\tau$. We find it from the data, with a least-squares fit of the cumulative spin-up due to glitches as a function of time.

The errors in the mass estimates reflect only the observational uncertainties of some glitch parameters, listed in Table 1; they were calculated by standard error propagation.

The glitch parameters and their observational uncertainties were extracted from the up-to-date database that is maintained by the Jodrell Bank Observatory; they are reported in Table 1, where we list the relevant data used in our method (Jodrell Bank Observatory database: http://www.jb.man.ac.uk/pulsar/glitches.html).

Data availability. The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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P.M.P. led the research, contributed to developing the model and wrote the final manuscript. M.A. contributed to developing the model, selected the observational data, performed the calculations and contributed to the initial manuscript. B.H. contributed to the model and to the initial manuscript. S.S. contributed to the model and selected the observational data.

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