The Quenching of the Axial Coupling in Nuclear and Neutron-Star Matter

G. W. Carter and M. Prakash

Department of Physics and Astronomy, SUNY at Stony Brook, Stony Brook, NY 11794-3800
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Using a chirally invariant effective Lagrangian, we calculate the density and isospin dependences of the in-medium axial coupling, $g_A^*$, in spatially uniform matter present in core collapse supernovae and neutron stars. The quenching of $g_A^*$ with density in matter with different proton fractions is found to be similar. However, our results suggest that the quenching of the nucleon’s $g_A^*$ in matter with hyperons is likely to be significantly greater than in matter with nucleons only.

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The accurately measured beta decay lifetime of the neutron in vacuum, $n \rightarrow p + e^- + \bar{\nu}_e$, fixes the ratio of the axial and vector couplings of the neutron to be $|g_A/g_V| = 1.2601 \pm 0.0025$. Studies of beta decays in nuclei, however, have long suggested that a value of $|g_A/g_V| \approx 1$ better fits the observed systematics. Such a lower value also appears to be consistent with pion-nucleus optical potentials and the systematics of Gamow-Teller resonances in nuclei. Data from muon capture on nuclei, in which the relevant momentum transfer $q^2 \approx -0.9m_n^2$, have been recently analyzed including detailed nuclear structure effects with the conclusion that a quenched $g_A$ (assuming $g_V \equiv 1$) is not necessary inasmuch as the vacuum value of $g_A$ adequately accounts for the data.

The above experiments measure space-like axial transitions in nuclei. At finite density, however, space-like and time-like axial matrix elements are not necessarily equal, since Lorentz invariance is broken. In fact, there are indications from experiments with first-forbidden $\beta$ decays of light nuclei that the time-like axial charge increases by about 25% in medium. A theoretical expectation of this enhancement in terms of soft-pion exchanges has been offered in Ref.

The space-like quenching of $g_A$ in nuclei at low momentum transfers has been attributed to a combination of effects, including the partial restoration of chiral symmetry in a nuclear medium, the direct participation of the $\Delta(1232)$ in renormalizing the in-medium axial-vector current, and tensor correlations in nuclei in which shell structure effects are important. An illuminating discussion of the extent to which the quenching phenomenon is intrinsic to the basic property of the “vacuum” defined by a baryon-rich medium has been given by Rho (cf. Ref. and references therein). Later discussions of the quenching phenomenon in chiral approaches to spatially uniform matter can be found in Ref.

The issue of breaking and restoring fundamental symmetries at large baryon density is presently intractable in lattice gauge simulations. We therefore employ an effective field-theoretical approach, based upon chiral symmetry, to consider medium modifications of the nucleon’s axial coupling.

The precise value of the in-medium axial coupling, denoted by $g_A^*$ hereafter, in the dense matter encountered in astrophysical phenomena such as core collapse supernovae and neutron stars is crucially important. In these cases, weak interaction rates (that are $\propto g_A^{2*}$) drive the dynamics from beginning to end. In contrast to laboratory nuclei, in which the weak processes occur at nuclear subnuclear densities, astrophysical settings feature supernuclear densities. This highlights the need for knowledge of $g_A^*$ well beyond nuclear densities and in uniform matter with varying isospin content. In the supernova environment, $e^-\nu$-capture reactions on neutrons and nuclei begin the process of neutronization and decrease of the total lepton fraction, $Y_L = n_L/n_B$, whose value after $\nu$-trapping ($\approx 0.38 - 0.4$) determines the masses of the homologous core and initial proto-neutron star and thus the available energy for the shock and subsequent $\nu$-emission. The bremsstrahlung $(n+n \rightarrow n+n+\nu+\bar{\nu})$ and modified Urca $(n+p \rightarrow n+n+e^++\nu+\bar{\nu})$ processes dominate in the production and thermalization of $\mu$ and $\tau$ neutrinos. The $\nu$-luminosity and the time scale over which $\nu$s remain observable from a proto-neutron star are also governed by charged and neutral current interactions involving baryons at high density. The long-term cooling of a neutron star, up to a million years of age, is controlled by $\nu$-emissivities from the densest parts of the star; thereafter the star is observable through photon emissions, which may allow us to determine the star’s mass, radius, and internal constitution.

In this Letter, we focus on the behavior of the in-medium space-like $g_A^*$, both with increasing density and with varying isospin content, in spatially uniform matter in which shell effects characteristic of finite nuclei are absent. This enables us to expose cleanly the role played by chiral symmetry alone. Our results are directly relevant to astrophysical situations in which spatially homogeneous matter are encountered. We consider the cases of isospin symmetric matter (proton fraction $x = n_p/n_B = 1/2$), pure neutron matter ($x = 0$), and neutron-star matter in which the equilibrium value of the proton fraction, $\bar{x}$, is determined by the conditions of beta stability and charge neutrality. The latter two cases have been largely ignored in the literature, but are
essential for astrophysical modeling in which neutrinos play a dominant role.

The nucleon coupling to the axial current is directly determined through the matrix element

$$\langle N(p_2)|\vec{A}_\mu|N(p_1)\rangle = \bar{u}(p_2)\gamma_\mu g_A \gamma_5 q A(p_1),$$

where the $\bar{u}(p_2)$ and $u(p_1)$ are the spinor solutions of the Dirac equation for nucleons and $q = p_2 - p_1$. In this form, $g_A$ is the axial coupling and $h_A(q^2)$ the form factor. Taking medium effects into account does not change this definition when we consider a medium-modified $g_A$. The axial coupling and $h_A(q^2)$ can be computed from first principles. Explicitly, we find the effective mass, chemical potentials, and $\bar{\omega}$-expectation values, which we denote by $\bar{\omega}$.

with the covariant derivatives

$$\Delta_\mu \sigma = \partial_\mu \sigma + g_\rho \partial_\mu \bar{\sigma},$$

$$\Delta_\mu \bar{\sigma} = \partial_\mu \bar{\sigma} + g_\rho \partial_\mu \bar{\sigma} - g_\rho \sigma \partial_\mu.$$

The $N$ are isospinor nucleons, $\sigma$ and $\pi$ are scalar and pseudoscalar isovector chiral mesons, and $\rho$ and $\omega_1$ are isovector mesons which are the vector and pseudovector representations of a gauged chiral symmetry group and make vector meson dominance inherent in the model [18]. The $\omega$ is a chiral singlet vector field which supplies the short-ranged repulsion necessary for saturation, with a self-interaction term included to soften nuclear matter. The non-standard coupling between the nucleon and meson fields, with the coefficient $D$, serves to generate the physical value of $g_A$ at the mean-field level while respecting chiral symmetry [19]. Note that the pion-nucleon interactions in Eq. (3) are chiral invariant.

The effective chiral Lagrangian of Eq. (3), like that of the Walecka model and its variant approaches to nuclear matter [20], is to be used at the mean-field level. This model is able to describe closed-shell nuclei similar in accuracy to that obtained in Quantum Hadrodynamical models [17]. Calculations of low-energy pion-nucleon scattering also compare well with data [3]. The scaling of the physical constants, effective masses, and the quenching of $g_A$ in isospin symmetric matter are also detailed in Ref. [4]. Note also that the Goldberger-Treiman relation, $g_{\pi NN}f_\pi = g_A M$, remains valid at finite baryon density, since the model encodes chiral symmetry.

We turn now to spatially uniform matter with an unequal number of neutrons ($n$) and protons ($p$). In medium, the $\sigma$, $\omega_0$, and $\rho_0$ fields develop non-zero expectation values, which we denote by $\bar{\sigma}$, $\bar{\omega}$, and $\bar{\rho}$. These are obtained by solving the equations of motion:

$$\frac{B}{\sigma} \left(1 - \frac{\bar{\sigma}^2}{\sigma_0^2}\right) = g \langle \bar{\sigma} N \rangle = g \left(n^*_p + n^*_n\right),$$

$$m^2_\omega + 4G^4\bar{\omega}^3 = g_\omega \langle N^\dagger N \rangle = g_\omega \langle n_p + n_n\rangle,$n^*_p \bar{\rho} = g_\rho \langle N^\dagger \tau^3 N \rangle = g_\rho \langle n_p - n_n\rangle. \quad (4)$$

Above, $n^*_i$ and $n_i$ with $i = n, p$ denote the scalar and baryon number densities, and $\tau^3$ is the third component of the isospin operator.

From the Dirac equation,

$$(i\gamma^\mu \partial_\mu - g_\omega \gamma^\alpha \partial_\alpha - \frac{1}{2}g_\rho \tau^3 \gamma^5 \bar{\rho} - g_\sigma \bar{\sigma}) N = 0, \quad (5)$$

we find the effective mass, $M^* = g_\sigma$, and the effective chemical potentials, $\mu^*_i = \mu_i - g_\omega \omega_0 - \frac{1}{2}g_\rho \tau^3 \rho_0$. The scalar and baryon densities depend self-consistently on these effective parameters:

$$n^*_i = \frac{M^*}{2\pi^2} \left[k_f E^*_f - M^{*2} \ln \left(\frac{k_f^* + E^*_f}{M^*}\right)\right],$$

$$n_i = \frac{k_f^3}{3\pi^2}, \quad (6)$$

$$\bar{G}_{\mu\nu} = \partial_\mu \bar{\rho}_\nu - \partial_\nu \bar{\rho}_\mu + g_\rho \bar{\rho}_\mu \times \bar{\rho}_\nu + g_\rho \bar{\sigma}_\mu \times \bar{\sigma}_\nu,  \quad (3)$$

$$\bar{F}_{\mu\nu} = \partial_\mu \bar{\rho}_\nu - \partial_\nu \bar{\rho}_\mu + g_\rho \bar{\rho}_\mu \times \bar{\rho}_\nu + g_\rho \bar{\sigma}_\mu \times \bar{\sigma}_\nu.$$
where \( E^*_{fi} = \sqrt{k^2_{fi} + M^{*2}} \) and \( k_{fi} = \sqrt{\mu^{*2} - M^{*2}} \). In pure neutron matter, \( n_p = 0 \) and \( n_n^* = 0 \). In beta-stable and charge-neutral neutron-star matter, the constraints

\[
\begin{align*}
\mu_n - \mu_p &= \mu_e = \mu, \\
n_p - n_e - n_\mu &= 0,
\end{align*}
\]

(7)

where \( n_e \) and \( n_\mu \) are the number densities of electrons and muons, determine the equilibrium proton fraction. With the solutions to Eqs. (3), the energy per baryon, \( E/A = \mathcal{E}/n_B - M \), is easily computed.

In order to compute the axial current, we will need only the terms in Eq. (2) which are finite for a nonzero mean field and contribute to the nucleon mean field element Eq. (1). The relevant terms in the Lagrangian, Eq. (3), will be the nucleon kinetic energy and derivative coupling to the pion, the latter since \( g^a_\pi \sigma^a \delta \sigma \). Renormalization of \( \pi - a_1 \) mixing introduces the physical fields

\[
\pi' = \left(1 - \frac{g^2_\pi \sigma^2}{m^2_\pi}\right) \pi, \quad \bar{a}_\mu' = \bar{a}_\mu - \frac{g_\rho_\sigma}{m^2_\rho} \partial_\mu \pi,
\]

(9)

with its mean-field dependence made explicit. Density dependence is thus generated by the scalar mean field, \( \sigma \), alone. We note that the functional form of Eq. (12), orginally derived in Ref. (3) for isospin symmetric matter, remains intact for isospin asymmetric matter. The numerical value of \( g^*_A \), however, is controlled by the magnitude of \( \sigma \) which depends on the proton fraction of matter. This relation and its implications to be discussed below are among the principal results of this work.

We now specify the physical parameters in Eq. (1) and discuss quantitative results in matter. Fixing the vacuum value of the nucleon mass determines the chiral coupling \( g = 9.2 \). We take \( \sigma_0 = f_\pi/\sqrt{Z}_\pi = 102 \text{ MeV} \), where \( Z_\pi \) is the pion renormalization constant necessitated by \( \pi - a_1 \) mixing. The value \( D = 1.17 \) reproduces \( g_A = 1.26 \) in vacuum. The saturation of nuclear matter at its empirical density \( n_0 \approx 0.15 \text{ fm}^{-3} \) and energy per baryon \( E/A = -16 \text{ MeV} \) require \( B = (323 \text{ MeV})^4 \) and \( g_\rho = 11.4 \). The omega self-interaction strength is \( G = 0.19g_\rho \), fixed to produce maximal softening of the EOS while not generating spurious field solutions. At saturation, the resulting effective mass is \( M^* = 0.68M \) and the compression modulus \( K \approx 320 \text{ MeV} \). The rho coupling \( g_\rho = 8.0 \) yields the empirical symmetry energy of \( \pm 30 \text{ MeV} \).

The procedure adopted above to fix the various couplings in Eq. (1) implies that the behavior of \( g^*_A \) with increasing density, including its value at the equilibrium density \( n_0 \) of symmetric nuclear matter, is to be regarded as a prediction, albeit within the confines of the model adopted for matter. The EOS of neutron-star matter at supra-nuclear densities is subject to the constraint that it must support at least \( 1.44M_\odot \), which is the most accurately measured mass of the neutron star in the binary pulsar PSR 1913+16 (2). Presently, more severe constraints at high density are not available.

\[
\mathcal{L} = \bar{N}i\gamma^\mu \partial_\mu N \\
+ \bar{N} \gamma^\mu \gamma_5 \left( \frac{Z^*_\pi - 1}{2\sigma\sqrt{Z^*_\pi}} + \frac{D\sigma\sqrt{Z^*_\pi}}{2\sigma^2_0} \right) \bar{\pi} \cdot \partial_\mu \pi N + \ldots,
\]

(10)

where the medium-dependent renormalization constant \( Z^*_\pi = 1 - g^2_\pi \sigma^2/m^2_\pi \). Using Eq. (3), we find

\[
\bar{A}_\mu = -Z^*_\pi \left(1 + \frac{D\sigma^2}{2\sigma^2_0}\right) \bar{N} \frac{\pi}{2\gamma^5} \gamma_\mu \gamma_5 N.
\]

(11)

Inserting this into Eq. (1), we have, in terms of the sigma mean field and vacuum constants,

\[
g^*_A = \left(1 + D\frac{\sigma^2}{\sigma^2_0}\right) \left(1 - \frac{g^2_\pi \sigma^2}{m^2_\rho + (m^2_n - m^2_p) \sigma^2/\sigma^2_0}\right) \cdot (12)
\]

Above, the first factor contains the “bare” axial coupling, including the standard axial interaction and the mean-field dependent modification from the \( D \) term. The second factor is the pion renormalization constant of Eq. (1) for isospin symmetric matter.
The left panel of Fig. 1 shows how $E/A$ varies with $n_B/n_0$ as the proton fraction $x$ is varied from nuclear to beta-stable neutron-star matter. The right panel shows the concentrations of both of these quantities with increasing density at $4n_0$. The corresponding EOS yields a maximum mass of $M_{\text{max}}/M_\odot = 1.6$; for this configuration the central density $n_c/n_0 \approx 7.4$ and radius $R = 10.24$ km. These values are to be compared with $M_{\text{max}}/M_\odot = 2.2$, $n_c/n_0 \approx 5.3$, and $R = 12.74$ km for the case of pure neutron matter, which highlights the role of the isospin content in matter for this EOS.

Fig. 2 shows the density and proton fraction dependences of the scalar mean field $\bar{\sigma}/\sigma_0$ (left panel) and the axial-vector coupling $g_A^*$ (right panel). The quenching of both of these quantities with increasing $n_B$ is clearly evident, with a depreciation of 12% at $n_0$ and a 19% drop at $4n_0$. The relatively mild variation with $x$ is important insofar as guidance for the quenching of $g_A^*$ from laboratory studies (which sample a narrow range in $x$) of nuclei have the potential of being directly and immediately useful in astrophysical applications (in which a broader range of $x$ is sampled). For modeling purposes, a simple parametrization of $g_A^*$ in terms of baryon density is

$$g_A^* \simeq g_A \left(1 - \frac{n_B}{4.15 \,(n_0 + n_B)} \right). \quad (13)$$

This expression matches the results of Eq. (12) to within 1% accuracy for all $n_B \leq 4.5n_0$.

The quenching of $g_A^*$ considered in this work stems chiefly from the medium-dependent scalar field and $\pi - a_1$ mixing. The combination of these two effects has been shown to be virtually independent of the isospin content, suggesting that the in-medium behavior observed in nearly iso-symmetric matter will be present in neutron and stellar matter as well. The excitation of the $\Delta$, which is crucial in non-relativistic descriptions of quenching, has yet to be satisfactorily implemented in a relativistic field theory. Its addition would likely lead to further reduction.

It is also worthwhile to point out here that $\bar{\sigma}$ falls more rapidly with density in matter with hyperons than without hyperons (see Fig. 6 of Ref. [23]). This is mainly due to the presence of additional baryonic components with dissimilar masses. Consequently, the nucleon's $g_A^*$ would be quenched to a greater extent in the presence of hyperons. A verification of this expectation would require an extension of flavor symmetry, which has been attempted recently with only limited success [23]. Our results in this work suggest that the extension of the chiral model to incorporate strange and Delta resonances with the full effects of relativity would be worthwhile.

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