Weak Phase $\gamma$ Using Isospin Analysis and Time Dependent Asymmetry in $B_d \rightarrow K_s \pi^+ \pi^-$

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Abstract

We present a method for measuring the weak phase $\gamma$ using isospin analysis of three body $B$ decays into $K\pi\pi$ channels. Differential decay widths and time dependent asymmetry in $B_d \rightarrow K_s \pi^+ \pi^-$ mode needs to be measured into even isospin $\pi\pi$ states. The method can be used to extract $\gamma$, as well as, the size of the electroweak penguin contributions. The technique is free from assumptions like SU(3) or neglect of any contributions to the decay amplitudes. By studying different regions of the Dalitz plot, it is possible to reduce the ambiguity in the value of $\gamma$. 

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Time dependent measurements of asymmetries of decay modes of $B_d$ into $CP$ eigenstates are very useful in determining angles of the Unitarity Triangle. This technique is particularly significant, since weak phases can be extracted without any theoretical uncertainty from modes whose amplitudes have a single weak phase. The time dependent $CP$ asymmetry in the golden mode $B_d \rightarrow J/\psi K_S$, thus yields information on $\sin 2\beta$. This method has proved successful in the measurement of $\sin 2\beta = 0.78 \pm 0.08$,[2], which is in good agreement with theoretical estimates.

Measurement of other angles using modes like $B_d \rightarrow \pi^+ \pi^-$, are beset with theoretical uncertainties because the amplitude gets contributions from tree and penguin diagrams which have different dependence on weak phases. Nevertheless, the theory error can in principle be removed using the method of Gronau and London (GL)[4]. This method relies on the assumption of isospin invariance and the fact that the amplitude for $B^+ \rightarrow \pi^+ \pi^0$ gets contributions only from the tree diagrams (barring a small contribution from the electroweak penguin). This method will lead to a measurement of $\sin 2\alpha$.

It is widely believed that $\gamma$ cannot be measured using the time dependent techniques developed to measure the phases $\beta$ and $\alpha$. As an alternative, several other methods have been developed[3] to measure this weak phase. While $\gamma$ can be measured cleanly using some of these techniques at a later date, techniques[3] assuming flavor SU(3), are expected to provide the first estimates of angle $\gamma$.

In this letter, we propose a method, which uses time dependent asymmetry in the three body $K\pi\pi$ decay mode of the $B_d$. Our technique is on almost as good a footing as the Gronau-London method, and relies on construction of triangles based on isospin analysis. The extra ingredient that we use is that the tree and the electroweak penguin pieces of the weak Hamiltonian responsible for $\Delta I = 1$ transition have the same strong phase because of the operator structure of the interaction in the standard model. However, the method is free from approximations such as SU(3) symmetry, neglect of annihilation or rescattering contributions. Further, our method is sensitive to the relative weak phase between the tree and penguin contribution, and as such will probe new physics. Recently, several three body non charmed decay modes of $B$ meson have been observed. In particular the branching ratios of the modes $B^0 \rightarrow K^0 \pi^+ \pi^-$ and $B^0 \rightarrow K^+ \pi^- \pi^0$ have been measured[5, 8] to be around $5 \cdot 10^{-5}$. In fact, even with limited statistics, a Dalitz plot analysis has been performed and quasi two body final states have been identified.
The three body decay modes such as $B \to K\pi\pi$ provide valuable information that can pin down the phases in the standard model. Importance of these modes was first pointed out by Lipkin, Nir, Quinn and Snyder (LNQS)\cite{9}, however, their analysis did not incorporate the large electroweak penguin effects known to be present in these decays\cite{10}. These decays are described by six independent isospin amplitudes $A(I_t, I_{\pi\pi}, I_f)$, where $I_t$ stands for the transition isospin, and describes the transformation of the weak Hamiltonian under isospin and can take only the values 0 and 1 in the standard model; $I_{\pi\pi}$ is the isospin of the pion pair and takes the value 0, 1, and 2 and $I_f$ is the final isospin and can take the values 1/2 and 3/2. Even values of $I_{\pi\pi}$ has the pair of pions in a symmetric state, and thus have even angular momenta. Similarly states with $I_{\pi\pi}$ odd must be odd under the exchange of two pions. A separation between $I_{\pi\pi} =$ even and $I_{\pi\pi} =$ odd should be possible through a study of the Dalitz plot.

We shall consider only the $I_{\pi\pi} = 0$ and 2 channels in this paper, and these are described by the three amplitudes $A(0, 0, \frac{1}{2})$, $A(1, 0, \frac{1}{2})$, and $A(1, 2, \frac{3}{2})$. The amplitudes for the various decay modes with $I_{\pi\pi} =$ even, obey useful isospin relations. It is straightforward to derive the following \cite{9}:

\begin{align*}
A(B^+ \to K^0(\pi^0\pi^0)_{\text{even}}) &= X \\
A(B^0 \to K^+(\pi^-\pi^0)_{\text{even}}) &= -X \\
A(B^+ \to K^+(\pi^+\pi^-)_{\text{even}}) &= -\frac{1}{3}X - Y + Z \\
A(B^0 \to K^0(\pi^+\pi^-)_{\text{even}}) &= \frac{1}{3}X + Y + Z \\
A(B^+ \to K^0(\pi^0\pi^0)_{\text{even}}) &= -\frac{2}{3}X + Y - Z \\
A(B^0 \to K^0(\pi^0\pi^0)_{\text{even}}) &= \frac{2}{3}X - Y - Z ,
\end{align*}

where

$$X = \sqrt{\frac{2}{5}} A(1, 2, \frac{3}{2}) , \quad Y = \frac{1}{3} A(1, 0, \frac{1}{2}) , \quad \text{and} \quad Z = \sqrt{\frac{1}{3}} A(0, 0, \frac{1}{2}) .$$

The subscript even represents the isospin of the $\pi\pi$ system. It is easy to see that Eq. \cite{10} implies the following two isospin triangles relations:

\begin{align*}
A(B^+ \to K^0(\pi^0\pi^0)_{\text{even}}) &= A(B^0 \to K^0(\pi^+\pi^-)_{\text{even}}) + A(B^0 \to K^0(\pi^0\pi^0)_{\text{even}}) , \quad (2) \\
A(B^0 \to K^+(\pi^-\pi^0)_{\text{even}}) &= A(B^+ \to K^+(\pi^+\pi^-)_{\text{even}}) + A(B^+ \to K^+(\pi^0\pi^0)_{\text{even}}) , \quad (3)
\end{align*}
and also implies the relation,

\[ A(B^+ \to K^0(\pi^+\pi^0)_{\text{even}}) = -A(B^0 \to K^+(\pi^-\pi^0)_{\text{even}}). \]  

(4)

Decays corresponding to conjugate processes will obey similar relations. The iso-triangle represented by Eq. (2) and its conjugate are the ones that interest us.

The decay \( B(p_B) \to K(k)p(p_1)p(p_2) \), (where \( p_B, k, p_1 \) and \( p_2 \) are the four momentum of the \( B, K, \pi_1 \) and \( \pi_2 \) respectively) may be described in terms of the usual Mandelstam variables \( s = (p_1 + p_2)^2 \), \( t = (k + p_1)^2 \) and \( u = (k + p_2)^2 \). States with \( I_{\pi\pi} = \text{even} \) must be symmetric under the exchange \( t \leftrightarrow u \). In what follows we shall be concerned with differential decay rates \( d^2\Gamma/(dtdu) \). These can be extracted from the Dalitz plot of the three body decays. In particular, if states with \( t = u \) are selected, they will be automatically symmetric in \( \pi\pi \). A detailed angular analysis will of course permit extraction of even isospin \( \pi\pi \) events and offer a larger sample. Note that \( B \to K_s\pi^0\pi^0 \) mode being symmetric in pions, always has pions in isospin even state.

For simplicity we define the amplitudes \( A^{+-}, A^{00} \) and \( A^{+0} \) corresponding to the modes \( B \to K_s(\pi^+\pi^-)_{\text{even}}, B \to K_s(\pi^0\pi^0)_{\text{even}}, \) and \( B \to K_s(\pi^+\pi^-)_{\text{even}} \) respectively. It may be understood that all observables, amplitudes and strong phases depend on the the two independent Mandelstam variables \( t \) and \( u \), even though we suppress explicitly stating the ‘\( t \)’ and ‘\( u \)’ dependence. These amplitudes may be expressed as follows, purely based on CKM \([11]\) contributions:

\[ A^{+-} = a^{+-}e^{i\delta_a^{+-}}e^{i\gamma} + b^{+-}e^{i\delta_b^{+-}} \]  

(5)

\[ A^{00} = a^{00}e^{i\delta_a^{00}}e^{i\gamma} + b^{00}e^{i\delta_b^{00}} \]  

(6)

\[ A^{+0} = a^{+0}e^{i\delta_a^{+0}}e^{i\gamma} + b^{+0}e^{i\delta_b^{+0}} \]  

(7)

The magnitudes \( a^{+-}, b^{+-}, a^{00}, b^{00}, a^{+0} \) and \( b^{+0} \) actually contain all possible contributions (tree, color-suppressed, annihilation, W-exchange, penguin, penguin-annihilation and electroweak-penguin amplitudes) and include the magnitudes of the CKM elements. Their explicit composition is irrelevant for this analysis, except for the fact that the isospin 3/2 amplitude \( A^{+0} \) cannot get contributions from gluonic penguins. The amplitudes \( \bar{A}^{+-}, \bar{A}^{00}, \bar{A}^{+0} \), corresponding to the conjugate process \( \bar{B} \to \bar{K}\pi\pi \) can be written similarly with the weak phase \( \gamma \) replaced by \( -\gamma \). In the presence of two contributions to the amplitude as described in Eq. \([3]\) and \([3]\), the direct asymmetry is non-vanishing. The time dependent
$CP$ asymmetry for $B^0(t) \rightarrow f$ then has the form,

$$A_{CP}^f(t) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} ,$$

$$= a^f_{dir} \cos(\Delta mt) + \frac{2 \text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \sin(\Delta mt) ,$$

where

$$a^f_{dir} = \frac{|\bar{A}^f|^2 - |A^f|^2}{|A^f|^2 + |\bar{A}^f|^2} , \quad \lambda_f = \frac{q}{p} \frac{\bar{A}^f}{A^f} \quad \text{and} \quad \frac{q}{p} = e^{-i2\beta} .$$

Fig. 1 depicts the two triangles formed by the amplitudes $A^{+0}$, $A^{00}$ and $A^{+0}$ and the corresponding conjugate amplitudes in isospin space, along with the relative orientations and defines the angles used in the derivations. The relative phase between $A^{+}$ and $\bar{A}^{+}$ (i.e. $\text{arg}((A^{+})^*\bar{A}^{+})$), defined as $2\theta^{+}$, can be obtained from the coefficient of the $\sin(\Delta mt)$ piece in the time dependent $CP$ asymmetry for the mode $B \rightarrow K_s(\pi^+\pi^-)$ even:

$$\frac{2 \text{Im}(\lambda^{+})}{1 + |\lambda^{+}|^2} = y^{+} \sin(2\theta^{+} - 2\beta) .$$

where $y^f$ is defined as $y^f = \sqrt{1 - (a^f_{dir})^2}$. Note that this measurement involves time dependent asymmetry in the partial decay rate $d^2\Gamma^{+}/dtdu$ at a fixed $t$ and $u$. Again, events symmetric in $t \leftrightarrow u$ need to be selected in the Dalitz plot.

With the knowledge of $\beta$, the angle $2\theta^{+}$ may be regarded as an observable. In addition, measurement of six partial decay rates $d^2\Gamma^{00}/dtdu$, $d^2\Gamma^{+0}/dtdu$ and $d^2\Gamma^{00}/dtdu$ as well as their conjugates at the same $t$ and $u$ as used for $\theta^{+}$ determination, now allows us to construct the two triangles in Fig. 1 with two fold ambiguity. From the figure, we see that the angle $2\bar{\gamma}$ between $A^{+0}$ and $\bar{A}^{+0}$ is related to $2\theta^{+}$ as,

$$\zeta \pm \bar{\zeta} + 2\bar{\gamma} = 2\theta^{+} .$$

The ‘plus–minus’ sign ambiguity in the above reflects the possibility of same–side or opposite–side orientation of the triangles. Once $2\bar{\gamma}$ is known, it is possible to determine $\gamma$. The crucial additional information necessary is the observation of Neubert and Rosner\cite{12} that the electroweak penguin operators $Q_9$ and $Q_{10}$ are Fierz-equivalent to the operators $Q_1$ and $Q_2$. The isospin $3/2$ amplitude $A^{+0}$ is symmetric in the two pions ($\pi^+\pi^0$). Hence, within the Standard Model (SM) only the operator $(Q_1 + Q_2)$ with coefficient $\frac{1}{2}[\lambda_u(C_1 + C_2) - \frac{3}{2}\lambda_t(C_9 + C_{10})]$ contributes, while the operator $(Q_1 - Q_2)$ does not. The
FIG. 1: The isospin triangles formed by the $B \to K\pi\pi$ amplitudes, as represented in Eq. (2) and that for the corresponding conjugate processes. Only one orientation of the conjugate triangle is depicted; this triangle could have been flipped around the base $A^{+0}$.

The amplitude $A^{+0}$ thus has a common strong phase $\delta = \delta_a^{+0} = \delta_b^{+0}$ arising from the same quark operator. This phase $\delta$ may be set equal to zero by convention.

Thus the amplitudes $A^{+0}$ and $A^{-0} \equiv \overline{A}^{+0}$ may be written as

\begin{align*}
A^{+0} &= (e^{i\gamma} - \delta_{EW})a^{+0}, \\
A^{-0} &= (e^{-i\gamma} - \delta_{EW})a^{+0}.
\end{align*}

(12)

Here,

\begin{equation}
\delta_{EW} = \frac{-b^{+0}}{a^{+0}} \simeq -\frac{3}{2} \frac{\lambda_t}{\lambda_u} \frac{C_9 + C_{10}}{C_1 + C_2} = 0.66 \pm 0.15,
\end{equation}

(13)

where $\lambda_q = V_{q\bar{b}}^* V_{qs}$. The angle $2\tilde{\gamma}$ is then given by

\begin{equation}
\tan \tilde{\gamma} = \frac{\sin \gamma}{\cos \gamma - \delta_{EW}},
\end{equation}

(14)
Since the angle $\tilde{\gamma}$ is determined, it follows that angle $\gamma$ is now calculable from Eq. (14).

It turns out that we can determine $\gamma$ without having to use the theoretically computed value of $\delta_{EW}$, given by Eq. (13). As we will show below, $\gamma$ can be determined cleanly by relying only on the Neubert-Rosner observation that the amplitude $A^{+0}$ has a single common strong phase. We emphasize that the observation of a common strong phase $\delta$ is based on very firm grounds within the framework of the Standard Model (SM). It relies essentially, only on isospin and the operator structures contributing within the standard model. Nevertheless, an experimentally verifiable consequence of this hypothesis would be the vanishing of direct CP-violating asymmetry for the mode $A^{+0} \equiv A(K^0(\pi^+\pi^0)_{even})$.

Using the amplitudes $A^{+-}$, $\bar{A}^{+-}$, $A^{00}$ and $\bar{A}^{00}$ one can construct a maximum of seven independent observables (The amplitudes $A^{+0}$, $A^{-0}$ are not independent as they can be obtained using isospin relations). The two triangles can be completely defined in terms of seven observables, the three sides of each of the triangles and a relative angle between the two triangles. The amplitudes under consideration involve the following eleven variables: $a^{+-}$, $b^{+-}$, $a^{00}$, $b^{00}$, $a^{+0}$, $b^{+0}$, $\delta_a^{+-}$, $\delta_b^{+-}$, $\delta_a^{00}$, $\delta_b^{00}$, and $\gamma$. These variables are connected by two isospin relations (see Eq. (2) and the corresponding relation for the conjugate process), which results in four constraints, reducing the number of independent variables to seven, as we will illustrate below. Hence, all variables including $\gamma$, can be determined purely in terms of observables.

In order to determine $\gamma$, we express all the amplitudes and strong phases, in terms of observables and $\gamma$. The variables, $a^{ij}$ and $b^{ij}$ may be solved as a function of $\gamma$ and other observables as follows:

\[
|a^{+-}|^2 = \frac{B^{+-}}{2\sin^2\gamma} \left(1 - y^{+-} \cos(2\theta^{+-})\right) \tag{15}
\]

\[
|b^{+-}|^2 = \frac{B^{+-}}{2\sin^2\gamma} \left(1 - y^{+-} \cos(2\theta^{+-} - 2\gamma)\right) \tag{16}
\]

\[
|a^{+0}|^2 = \frac{B^{+0}}{2\sin^2\gamma} \left(1 - \cos(2\tilde{\gamma})\right) \tag{17}
\]

\[
|b^{+0}|^2 = \frac{B^{+0}}{2\sin^2\gamma} \left(1 - \cos(2\tilde{\gamma} - 2\gamma)\right) \tag{18}
\]

\[
|a^{00}|^2 = \frac{B^{00}}{2\sin^2\gamma} \left(1 - y^{00} \cos(2\theta^{00})\right) \tag{19}
\]

\[
|b^{00}|^2 = \frac{B^{00}}{2\sin^2\gamma} \left(1 - y^{00} \cos(2\theta^{00} - 2\gamma)\right) \tag{20}
\]
where $2\theta^{00}$ defined in analogy to $2\theta^{+-}$, is the angle between $A^{00}$ and $\overline{A}^{00}$, and $B^{+-}$, $B^{00}$ and $B^{+0}$ are given by,

$$B^{+-} = \frac{|A^{+-}|^2 + |A^{+-}|^2}{2}, \quad B^{00} = \frac{|A^{00}|^2 + |A^{00}|^2}{2}, \quad B^{+0} = \frac{|A^{-0}|^2 + |A^{+0}|^2}{2}. \quad (21)$$

The angle $2\theta^{00}$ need not be measured but can be determined from geometry of the two triangles. It is given by

$$\cos\left(2\theta^{00} - 2\tilde{\gamma}\right) = \frac{B^{00} - B^{+-} + |A^{+-}| \cos(2\theta^{+-} - 2\tilde{\gamma})}{|A^{00}||A^{00}|} \quad (22)$$

We define $\delta^{+-} = \delta_b^{+-} - \delta_a^{+-}$ and $\delta^{00} = \delta_b^{00} - \delta_a^{00}$, which are conveniently expressed in terms of $\gamma$ and observables as:

$$\tan\delta^{+-} = \frac{a_{\text{dir}}^{+-} \tan \gamma}{1 - y^{+-} [\cos 2\theta^{+-} - \sin 2\theta^{+-} \tan \gamma]} \quad (23)$$

$$\tan\delta^{00} = \frac{a_{\text{dir}}^{00} \tan \gamma}{1 - y^{00} [\cos 2\theta^{00} - \sin 2\theta^{00} \tan \gamma]} \quad (24)$$

Our task now is to express the strong phases $\delta_a^{+-}$ and $\delta_a^{00}$ in terms of $\gamma$ and observables, just as we have done for the other variables. One finally intends to solve for $\gamma$, only in terms of observables.

The isospin triangle relation given by Eq. (2) and the similar relation for the conjugate process may be expressed as:

$$(a^{+-} e^{i\delta_a^{+-}} + a^{00} e^{i\delta_a^{00}}) e^{\pm i\gamma} + (b^{+-} e^{i\delta_b^{+-}} + b^{00} e^{i\delta_b^{00}}) = (a^{+0} e^{\pm i\gamma} + b^{+0}) \quad (25)$$

Using Eq. (25) one can derive the ‘four’ equations:

$$a^{+-} \cos(\delta_a^{+-} \pm \gamma) + a^{00} \cos(\delta_a^{00} \pm \gamma) + b^{+-} \cos(\delta_b^{+-} \pm \delta_b^{00}) = a^{+0} \cos \gamma + b^{+0} \quad (26)$$

$$a^{+-} \sin(\delta_a^{+-} \pm \gamma) + a^{00} \sin(\delta_a^{00} \pm \gamma) + b^{+-} \sin(\delta_b^{+-} \pm \delta_b^{00}) = \pm a^{+0} \sin \gamma \quad (27)$$

Eqns. (26) and (27) may be recast as follows:

$$a^{+-} \sin \delta_a^{+-} + a^{00} \sin \delta_a^{00} = 0 \quad (28)$$

$$a^{+-} \cos \delta_a^{+-} + a^{00} \cos \delta_a^{00} = a^{+0} \quad (29)$$

$$b^{+-} \cos \delta_b^{+-} + b^{00} \cos \delta_b^{00} = b^{+0} \quad (30)$$

$$b^{+-} \sin \delta_b^{+-} + b^{00} \sin \delta_b^{00} = 0 \quad (31)$$
Eq. (28) and (29) may be used to solve for \( \cos \delta_a^+ \) and \( \cos \delta_a^{00} \):

\[
\cos \delta_a^+ = \frac{|a^{+0}|^2 + |a^+|^2 - |a^{00}|^2}{2|a^{+0}||a^+|} \tag{32}
\]

\[
\cos \delta_a^{00} = \frac{|a^{+0}|^2 + |a^{00}|^2 - |a^+|^2}{2|a^{+0}||a^{00}|} \tag{33}
\]

Squaring and adding Eqns. (30) and (31) we get,

\[
|b^+|^2 + |b^{00}|^2 + 2b^+b^{00}\cos(\delta_b^+ - \delta_b^{00}) = |b^{00}|^2 \tag{34}
\]

Now \( \delta_b^+ = \delta^+ + \delta_a^+ \) and \( \delta_b^{00} = \delta^{00} + \delta_a^{00} \). Hence, Eq. (34) is expressed completely in terms of observables and \( \gamma \). \( \gamma \) can thus be determined cleanly, in terms of observables.

The CKM phase \( \gamma \) can be determined simultaneously for several regions of the Dalitz plot. The ambiguities in the solution of \( \gamma \) may thereby be removed. Having measured \( \gamma \) one can use Eq. (14) to estimate the value of \( \delta_{\text{EW}} \) in terms of observables. We can thus verify our understanding of electroweak penguin contributions.

One may ask if it is possible to determine \( \gamma \) using \( B \to K\pi\pi \) without resorting to the Neubert-Rosner hypothesis. If one includes in the analysis \( B \to K_s(\pi^+\pi^-)_{\text{odd}} \) with the two pions in an isospin odd state, one adds four new variables corresponding to the amplitudes and strong phases of the two parts with different weak phases. However, one can at best obtain four new independent observables. Three of which, arise from time dependent measurement for this mode, and one results from the interference between states with pions in isospin even and isospin odd. We hence conclude that it is not possible to determine \( \gamma \) without at-least one theoretical observation, even if one uses all the information possible from \( B \to K\pi\pi \) decays.

Current experimental data [7, 8] indicates that at least two sides of the triangles in Fig. 1 are readily measurable. While \( B^+ \to K_s\pi^+\pi^0 \) has not yet been observed, the mode \( B^0 \to K^+\pi^-\pi^0 \) has been seen. Hence, using Eq. (14) the isospin triangle in Eq. (2) can still be constructed. In fact, in future, data from both \( B^+ \to K_s\pi^+\pi^0 \) and \( B^0 \to K^+\pi^-\pi^0 \) modes could be combined to improve statistics.

To conclude, the weak phase \( \gamma \) can be measured using a time dependent asymmetry measurement in the three body decay, \( B \to K\pi\pi \). A detailed study of the Dalitz plot can be used to extract the \( \pi\pi \) even isospin states. These states obey certain isospin relations which allow us to not only obtain \( \gamma \), but also determine the size of the electroweak penguin contribution. In contrast to methods of determination of \( \gamma \) using the two body decay modes
\( B \to K\pi \), this technique does not require any theoretical assumptions like SU(3) or neglect of any contributions to the decay amplitudes. By studying different regions of the Dalitz plot it is possible to reduce the ambiguity in the value of \( \gamma \).

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