Comments on the Erhan-Schlein model of damping the pomeron flux at small x-pomeron

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ABSTRACT

We explore the theoretical and experimental consequences of a model proposed by Samim Erhan and Peter Schlein for unitarizing the diffractive amplitude by damping the pomeron flux at small x-pomeron and conclude that the model is unphysical and contradicts well established experimental data.

1 Introduction

In Regge theory, the high energy behaviour of hadronic cross sections is dominated by pomeron exchange [1, 2]. For a simple pomeron pole, the $pp$ elastic, total and single diffraction dissociation cross sections are given by

$$\frac{d\sigma_{el}}{dt} = \frac{\beta^4_{\mathbb{P}p}(t)}{16\pi} \left( \frac{s}{s_0} \right)^{2[\alpha(t)-1]} = \frac{\beta^4_{\mathbb{P}p}(t)}{16\pi} \left( \frac{s}{s_0} \right)^{2\alpha't} \left( \frac{s}{s_0} \right)^{2\epsilon} \tag{1}$$

$$\sigma_T = \beta^2_{\mathbb{P}p}(0) \left( \frac{s}{s_0} \right)^{\alpha(0)-1} = \beta^2_{\mathbb{P}p}(0) \left( \frac{s}{s_0} \right)^{\epsilon} \tag{2}$$

$$\frac{d^2\sigma_{sd}}{d\xi dt} = \frac{\beta^2_{\mathbb{P}p}(t)}{16\pi} \xi^{1-2\alpha(t)} \cdot \left[ \beta_{\mathbb{P}p}(0) g(t) \left( \frac{s'}{s_0} \right)^{\alpha(0)-1} \right] \equiv f_{\mathbb{P}p/\mathbb{P}}(\xi, t) \cdot \left[ \sigma_T(p', t) \right] \tag{3}$$

where $\alpha(t) = 1 + \epsilon + \alpha't$ is the pomeron Regge trajectory, $\beta_{\mathbb{P}p}(t)$ is the coupling of the pomeron to the proton, $g(t)$ the triple-pomeron coupling, $s'$ the $\mathbb{P} - p$ center of mass energy squared, $\xi \equiv x_{\mathbb{P}} = s'/s = M^2/s$ the fraction of the momentum of the proton carried by the pomeron, $M$ the diffractive mass and $s_0$ an energy scale not specified by the theory.

In analogy with Eq. (2), the term in brackets in (3) is identified as the $\mathbb{P} - p$ total cross section, and therefore the factor

$$f_{\mathbb{P}p/\mathbb{P}}(\xi, t) \equiv \frac{\beta^2_{\mathbb{P}p}(t)}{16\pi} \xi^{1-2\alpha(t)} = \frac{\beta^2_{\mathbb{P}p}(0)}{16\pi} \xi^{1-2\alpha(t)} F^2(t) = K \xi^{1-2\alpha(t)} F^2(t) \tag{4}$$
is interpreted as a “pomeron flux” (per proton) and used in calculating hard processes in diffraction dissociation in the Ingelman-Schlein model \[3\].

Experimentally, the triple-pomeron coupling was found not to depend on \( t \) \[1\] and therefore we will use \( g(t) = g(0) \) and \( \sigma_T^{pp}(s', t) = \sigma_T^{pp}(s') \).

The function \( F(t) \) represents the proton form factor. Donnachie and Landshoff proposed \[4\] that the appropriate form factor for \( pp \) elastic and diffractive scattering is the isoscalar form factor measured in electron-nucleon scattering, namely

\[
F_1(t) = \frac{4m^2 - 2.8t}{4m^2 - t} \left[ \frac{1}{1 - t/0.7} \right]^2
\]

where \( m \) is the mass of the proton. When using this form factor, the pomeron flux is referred to as the Donnachie-Landshoff (DL) flux.

In terms of the diffractive mass, the diffraction dissociation cross section is given by

\[
d^2\sigma_{sd} = K\sigma_0^{pp} \cdot s^{2\epsilon} \int_{M_0^2}^{\infty} \frac{1}{(M^2)^{1+\epsilon}} \left( \frac{s}{M^2} \right)^{2\alpha't} F^2(t) d\xi dt \sim s^{2\epsilon}
\]

where the lower limit of the \( M^2 \) integration is the effective diffractive threshold, \( M_0^2 = 1.5 \) GeV\(^2\) \[1\], and the upper limit, \( M_{\text{max}}^2 = 0.1s \), corresponding to \( \xi = 0.1 \), is dictated by the coherence condition for diffraction \[1\] (the contribution of the \( M^2 > 0.1s \) region to the integrated cross section would be, in any case, negligibly small).

The \( \sim s^{2\epsilon} \) dependence of \( \sigma_{sd}(s) \) eventually leads to a diffractive cross section larger than the total, and therefore to violation of unitarity. The unitarity problem is a more general problem in pomeron pole dominance. It is well known that the power law \( s \)-dependence of the total cross section, \( \sim s^\epsilon \), violates the unitarity based Froissart bound, which predicts that the total cross section cannot rise faster than \( \sim \ln s \). Unitarity is also violated by the \( s \)-dependence of the ratio \( \sigma_{el}/\sigma_T \sim s^\epsilon \), which eventually exceeds the black disc bound of one half (\( \sigma_{el} \leq \frac{1}{2}\sigma_T \)). However, in both the elastic and total cross sections unitarization can be achieved by eikonalizing the elastic amplitude \[5\].

2 The renormalized pomeron flux model

Attempts to unitarize the diffractive amplitude by eikonalization \[6\] or by including cuts \[7\] have met with only moderate success. Through such efforts it became clear that these “screening corrections” affect mainly the normalization of the diffractive amplitude, but leave the \( M^2 \) dependence almost unchanged. Such a trend is clearly present in the data, as demonstrated \[8\] by comparing the CDF diffractive differential \( pp \) cross sections.

\[1\] The factor \( K \) in the DL flux is \( K_{DL} = (3\beta_{pp})^2/4\pi^2 \), where \( \beta_{pp} \) is the pomeron-quark coupling.
at $\sqrt{s} = 546$ and 1800 GeV with $pp$ cross sections at $\sqrt{s} = 20$ GeV. Motivated by these theoretical results and the trend observed in the data, a phenomenological approach to unitarizing the diffractive amplitude was proposed [9] based on “renormalizing” the pomeron flux according to

$$f_N(\xi, t) = D \cdot f_{IP/p}(\xi, t)$$  \hspace{1cm} (8)

where the factor $D$ is determined by setting

$$D \int_{\xi_{min}}^{0.1} \int_0^\infty f_{IP/p}(\xi, t) d\xi dt = 1$$ \hspace{1cm} (9)

if the value of the integrated standard flux exceeds unity (the limits $\xi_{min}$ and 0.1 of the $\xi$-integration are related to the limits $M_0^2$ and 0.1s of the $M^2$-integration in Eq. (7) by $\xi = M^2/s$). Such a normalization, which corresponds to at most one pomeron per proton, leads to interpreting the pomeron flux as a probability density simply describing the $\xi$ and $t$ distributions of the exchanged pomeron in a diffractive process.

The integrated standard flux is given by

$$N(\xi_{min}) = \int_{\xi_{min}}^{0.1} \int_0^\infty f_{IP/p}(\xi, t) d\xi dt \sim (\xi_{min})^{-2\epsilon} \sim s^{2\epsilon}$$ \hspace{1cm} (10)

and therefore $D \sim s^{-2\epsilon}$. Thus, flux renormalization approximately cancels the $s^{2\epsilon}$ dependence in Eq. (1) resulting in a slower rise of the diffractive relative to the total cross section with energy and preserving unitarity. Figure 1, which is an updated version (including more data) of a figure presented in Ref. [9], shows the total diffractive cross section as a function of $\sqrt{s}$ along with the predictions obtained using the standard (dashed curve) and renormalized (solid curve) pomeron flux (the dashed-dotted curve is the prediction of the Erhan-Schlein model, which is discussed below). Renormalized flux predictions of differential cross sections also show good agreement with data [10, 11]. Finally, using the Ingelman-Schlein model [3], the renormalized pomeron flux predicts correctly the measured rates of hard diffractive processes [3, 11].

Predictions of hard diffraction rates using the standard/DL flux are unreliable due to a theoretical uncertainty inherent in the flux normalization. In Eq. (2) it is seen that $\beta_{IP}(0)$ can only be determined from the experimentally measured total cross section in terms of the energy scale $s_0$, which, as mentioned above, is not given by the theory ($s_0$ is usually set to 1 GeV$^2$, the hadron mass scale, but this is only a convention). Thus, the normalization of the standard flux is unknown and therefore only predictions for relative hard diffraction rates are possible, as for example for the ratio of diffractive dijet production at two different energies.

The normalization uncertainty is resolved in the flux renormalization model. The energy scale, $s_0$, can be determined [3] by setting the flux integral to unity at $\sqrt{s} \approx 20$ GeV, where the total diffractive cross section turns from rising as $\sim s^{2\epsilon}$ to assuming a rather flat $s$-dependence, presumably due to the saturation of the pomeron flux (see Fig. 1). For $\sqrt{s} > 20$ GeV, where the flux integral is unity, the normalization is self-determined from Eq. (8). Thus, predictions for hard diffraction using the renormalized pomeron flux can be made not only for relative but also for absolute rates. The recently reported
CDF diffractive $W$ and dijet production rates \cite{12,13} are in excellent agreement with the renormalized flux predictions \cite{9,10,11}.

3 The Erhan-Schlein model

Erhan and Schlein have taken a different approach to solving the unitarity problem of the triple-pomeron amplitude \cite{14,15}. They introduce a factor $D(\xi)$ in the flux to damp the small-$\xi$ values and thus slow down the $\sim s^{2\epsilon}$ dependence of the flux integral. The normalization of the flux is left unchanged. To slow down the rise with $s$ of the differential cross section at the higher $\xi$-values that are not affected by the damping factor, they introduce a $IPPR$ term, whose contribution increases at low energies due to the $\sim 1/\sqrt{s}$ dependence of the $R$-term. To fit ISR and UA8 data at $|t| \sim 1 - 2$ GeV$^2$, two more parameters are introduced. The detailed form of the proposed diffractive differential cross section is the following:

$$d^2\sigma_{sd}/d\xi dt = f_{IP}(\xi, t) \cdot \sigma_{IP}(s')$$

$$f_{IP}(\xi, t) = D(\xi) \cdot K \cdot \xi^{1-2\epsilon(t)} \cdot F_1^2(t) \cdot e^{bt}$$

$$D(\xi) = \begin{cases} 1 & 0.015 < \xi < 0.1 \\ 1 - 2700(\xi - 0.015)^2 & 10^{-4} < \xi < 0.015 \\ 0.4 - 0.4 \times 10^8(\xi - 10^{-4})^2 & 0 < \xi < 10^{-4} \end{cases}$$

$$\sigma_{IP}(s') = \sigma_0^{IP} [ (\xi s)^\Delta + r \cdot (\xi s)^{-0.45} ]$$

$$\epsilon(t) = 1 + \epsilon + 0.26t + \alpha''t^2$$

Using Eqs. (11-15) and the fixed parameters

$$\epsilon = 0.115 \quad \Delta = 0.08$$

diffractive cross section data are fitted with the following four free parameters determined from the data:

$$C \equiv K \sigma_0^{IP} = 0.73 \pm 0.09 \text{ mb GeV}^{-2}$$

$$b = 0.75 \pm 0.27 \text{ GeV}^{-2}$$

$$\alpha'' = 0.075 \pm 0.017 \text{ GeV}^{-4}$$

$$r = 5.0 \pm 0.6$$

It is claimed that good agreement is obtained with “all available data” \cite{15}. Below, we comment on the effect of the extra parameters introduced in the standard triple-pomeron amplitude, compare the predictions of the model with experimental pp and $\bar{p}p$ data and highlight some of its experimental and theoretical implications.
3.1 The parameters of the Erhan-Schlein model

In addition to the introduction of the $\mathcal{P}\mathcal{P}\mathcal{R}$ term and the parameters associated with it, several additional parameters are introduced to the standard triple-regge amplitude:

- A different pomeron intercept is used in the $\sigma^{\mathcal{P}\mathcal{P}}(s')$ term than in the flux factor (see Eq. (13)).

- Two damping factors, $D(\xi)$, are used for two different $\xi$-ranges. Figure (2a) shows $D(\xi)$ as a function of $\xi$. The roller-coaster shape of this distribution is mapped into the double resonance like distribution of the production cross section for a fixed diffractive mass at $t = 0$ as a function of energy, as shown in Fig. (2b).

- The effect of the parameter $b$ of the $e^{bt}$ term in Eq. (12) is shown by the solid line in Fig. (2c). In the region $1 < |t| < 2$ GeV$^2$ of the UA8 data, which have been presented in comparisons with the predictions of this model [14, 15], the $b$-parameter accounts for a $t$-dependent reduction of the cross section by a factor of $2^{-5}$.

- The effect of the parameter $\alpha''$, which introduces a dependence of $1/\xi^{2\alpha''t^2}$, is shown in Figs. (2c, 2d). Figure (2c) displays this factor for $\xi = 0.05$, the average UA8 $\xi$-value, as a function of $t$. Within the UA8 $t$-range, it varies from 1.5 to 6. The $\xi$-dependence as a function of $t$ for this factor is shown for various $t$-values in Fig. (2d). Again, within the UA8 range of $t$ and $\xi$, the effect is seen to be quite large.

3.2 Comparison with experimental $pp$ and $\bar{p}p$ data

In Fig. 1, which shows total $pp$ and $\bar{p}p$ diffractive cross sections as a function of $\sqrt{s}$, the dashed line represents the standard flux prediction, the solid line the renormalized flux prediction and the dashed-dotted line the Erhan-Schlein prediction. The latter exhibits two turn-overs as $\sqrt{s}$ increases, one at $\sqrt{s} \sim 10$ GeV and the other at $\sim 200$ GeV. The $\xi_{min} \sim 1/s$ values corresponding to these energies are at the interfaces of the damping factors that are used in the model. Clearly, a single damping factor would not provide a good fit to the data.

Figure (3) shows differential cross sections $\xi d^2\sigma/d\xi dt$ at $t = -0.05$ GeV$^2$ as a function of $\xi$ for fixed target $pp$ data at $\sqrt{s} = 14$ and 20 GeV [16] and for the CDF $\bar{p}p$ data at 546 and 1800 GeV [8]. The fixed target data are for masses above the resonance region and the CDF data are for $\xi$-values large enough not to be affected by the experimental resolution of the $\xi$-measurement. The dashed curves are fits to the data using the form

$$A\xi^{1-2\alpha_\mathcal{P}(t)} \cdot (\xi s)^\epsilon + B\xi^{1-2\alpha_\pi(t)} \cdot \sigma_{\mathcal{P}}(\xi s)$$

with $\alpha_\mathcal{P}(t) = 1.104 + 0.25t$ [3] and $\alpha_\pi(t) = 0.9t$. The first term in Eq. (21) is the form of the triple pomeron amplitude and the second term the form for reggeized pion exchange. Fits with $\mathcal{P}\mathcal{P}\mathcal{P}\mathcal{P}$ and $\pi$-exchange terms have been shown to represent well the
Using the renormalized flux, such fits with only one free parameter, namely the triple-pomeron coupling constant, yield differential cross sections that are in good agreement with the data not only in shape but also in normalization.

The solid curves in Fig. (3) were calculated using the Erhan-Schlein model. Three features of these curves are immediately apparent:

- At low energies (Figs. 3a, 3b), the Erhan-Schlein cross sections are falling sharply with energy. This behaviour is due to the $\xi^{-0.45}$ dependence of the $\sigma_{P^{IP}}(s')$ term in the $P^{IP}R$ amplitude (second term in Eq. [14]), which dominates the low energy cross sections in this model due to the $s^{-0.45}$ factor, as compared to the $\xi^\Delta$ dependence of the $P^{IP}P$ term (first term in Eq. [14]).

- At the high energies (Figs. 3c, 3d), the curves initially rise as $\xi$ decreases and then bend over and fall as $\xi$ decreases further. The bending point is at $\xi = 0.015$, which is the $\xi$-value where the $D(\xi)$ damping takes effect.

- At $\xi = 0.035$ the predictions are in relatively good agreement with the data. However, this agreement is not very meaningful as the parameters of the model were determined by fitting data of cross sections as a function of energy at this particular value of $\xi$ [14, 15]. The argument offered for using only data at $\xi = 0.035$ in the fits is that this $\xi$-value is low enough for the data to be background-free, but also high enough so that the $\xi$-distributions are not distorted by the experimental resolution in measuring $\xi$. As mentioned above, however, none of the data points in Fig. (3) are affected by resolution and therefore the high energy data in the region of $\xi < 0.035$ provide a good testing ground for the model. Figures (3c, 3d) show that the model is not very successful in this region.

### 3.3 Experimental and theoretical implications

In addition to the experimental issues already discussed in connection with $pp/\bar{p}p$ cross sections, damping the pomeron flux at small-$\xi$ has serious implications for the HERA deep inelastic scattering (DIS) and photoproduction results. These results are in the range $10^{-4} < \xi < 10^{-2}$, within which the damping factor $D(\xi) = 1 - 2700(\xi - 0.015)^2$ is in effect. The DIS H1 [17] and ZEUS [18] results and the H1 photoproduction results [19] are compatible with a pomeron flux $\sim 1/\xi^{1+2\epsilon}$ with $\epsilon \approx 0.11$. From $\xi = 10^{-4}$ to $\xi = 10^{-2}$ the damping factor $D(\xi)$ increases by a factor of 2.2 while $\xi^{-2\epsilon}$ with $\epsilon$ from Eq. [16] decreases by a factor of 2.8. Therefore, if $\xi$-damping were indeed in effect, the value of $\epsilon$ expected at HERA would be $\epsilon \approx 0.02$. Thus, the HERA data contradict the small-$\xi$ damping hypothesis.

On the theoretical side, there is no reason to expect that screening corrections should damp the cross section preferentially at small diffractive masses. In fact, as already mentioned, eikonalization leaves the $M^2$ distribution largely unchanged [1]. As for the other parameters introduced in the model, there is no obvious reason why $\Delta$ should be different from $\epsilon$, or, if it were different, that it should have the value 0.08; neither is
there a reason why the term $e^{bt}$ should be needed in diffraction, since a corresponding term $(e^{bt})^2 = e^{1.5t}$ is not needed in the form factor for elastic scattering. Finally, the introduction of the term $\alpha''t^2$ in the pomeron trajectory makes both the diffractive and the elastic cross sections blow up at large values of $t$.

4 Conclusions

In an effort to solve the unitarity problem inherent to the triple-pomeron description of the diffractive cross section, Erhan and Schlein introduce a $\xi$-damping factor, $D(\xi)$, that decreases the pomeron flux for $\xi < 0.015$. Since for $\xi > 0.015$ the flux is left unchanged, the differential cross section $d^2\sigma/d\xi dt$ given by the $\pi\pi\pi\pi$ amplitude still rises as $\sim s^{\epsilon}$ in this region, while experimentally it is found to decrease as $\sim s^{-\epsilon}$. To balance the $s$-dependence, a $\pi\pi\pi\pi R$ term that varies with $s$ as $\sim s^{-0.45}$ is introduced in the model. However, as this term has a $\xi$-dependence sharper than that of the $\pi\pi\pi\pi$ term (a factor of $\xi^{\epsilon}$ is replaced by $\xi^{-0.45}$), it is now more difficult to fit the $\xi$-distributions of the data. To obtain better fits, more parameters are introduced. In addition to choosing $\Delta \neq \epsilon$, four free parameters are used. These do not include the three parameters needed for the damping factors (two $\xi$-values and the value $D = 0.4$ at $\xi = 10^{-4}$), which were chosen to make the model agree with the $s$-dependence of the total diffractive cross section. Including these three parameters and the parameter $\Delta$ in the list of free parameters, a total of 8 free parameters are used, not taking into account the shape of the damping factors, which was also chosen to optimize the fits. Furthermore, Erhan and Schlein point out that their model does not agree with data at $|t| > \sim 0.5$ GeV$^2$, which includes the UA8 data, unless the $\xi$-damping factor is not used in this region [13]. However, no prescription is offered as to what happens at the interface between $|t| < 0.5$, where damping is needed, and $|t| > 0.5$, where it is not. Presumably more free parameters, which are currently hidden, would be required to take this effect into account. Despite the large number of explicit and hidden free parameters, the predictions of the model are contradicted by both the $pp/\bar{p}p$ and the HERA data. The model is also theoretically unsound as it predicts the vanishing of the $t = 0$ cross section for small and increasingly larger diffractive masses as the energy increases.

In conclusion, the proposed model is ill-defined (low versus high $|t|$ behaviour), theoretically inconsistent and unphysical, contradicts $pp/\bar{p}p$ and HERA experimental data, and has no predictive power for hard diffraction rates within the framework of the Ingelman-Schlein model.

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Figure 1: The total $pp/\bar{p}p$ single diffraction dissociation cross section for $\xi < 0.05$ as a function of center of mass energy. The dashed curve is the triple-pomeron prediction, the solid curve the renormalized flux prediction and the dashed-dotted curve the prediction of the Erhan-Schlein model of damping the pomeron flux at small-$\xi$. The latter exhibits two “turn-overs” as $\sqrt{s}$ increases, one at $\sqrt{s} \sim 10$ GeV and the other at $\sim 200$ GeV. The $\xi_{\text{min}} \sim 1/s$ values corresponding to these energies are at the interfaces of the damping factors that are used in the model.
Figure 2: The parameters of the Erhan-Schlein model: (a) the damping factor, \( D(\xi) \), as a function of \( \xi \); (b) \( d^2\sigma/dM^2 dt \big|_{t=0} \) versus \( \sqrt{s} \) for \( M^2 = 5 \text{ GeV}^2 \) (solid) and \( 10 \times d^2\sigma/dM^2 dt \big|_{t=0} \) for \( M^2 = 50 \text{ GeV}^2 \) (dashed); (c) the factors \( e^{bt} \) (solid) and \( \xi^{-2\alpha' t^2} \) at \( \xi = 0.05 \) (dashed) versus \( t \); (d) the factor \( \xi^{-2\alpha' t^2} \) versus \( \xi \) for \( t = 0, 1, 1.5 \) and \( 2 \text{ GeV}^2 \).
Figure 3: Cross sections $\xi d^2\sigma/dtd\xi$ versus $\xi$ for $pp/\bar{p}p$ data compared to theoretical predictions. The dashed lines are fits with a triple-pomeron term and a pion exchange term (see Eq. [21]). The solid lines represent the predictions of the Erhan-Schlein model. At the two highest energies, as $\xi$ decreases the solid curves are seen to “bend-over” and fall at $\xi \approx 0.015$, where the $\xi$-damping factor becomes effective.