Study of anomalous gauge-Higgs couplings using Z boson polarization at LHC

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Abstract

We estimate model independent bounds that could be obtained on the anomalous $ZZH$ vertex using polarization parameters of the $Z$ boson produced in the Higgstrahlung process at the LHC. We calculate the eight independent polarization parameters from the spin density matrix elements of the $Z$, which can probe underlying new physics contributions to $ZH$ production. By using the approach that connects these polarization observables to the coefficients in the angular distribution of the decay products of the $Z$, we estimate the limits on the anomalous $ZZH$ coupling that can be obtained at the 14 TeV LHC.

1 Introduction

In the absence of evidence so far of any definitive beyond the Standard Model (SM) physics at the Large Hadron Collider (LHC), it becomes important to probe with high precision the properties of the 125 GeV Higgs ($H$) at the planned high luminosity phase of the LHC (HL-LHC). This requires precise measurements of the couplings of the Higgs to electroweak gauge bosons ($V = W^\pm, Z, \gamma$), its Yukawa couplings to the fermions as well as its self-couplings. Of these, the $VVH$ couplings, whose form is fixed by the $SU(2)_L \times U(1)_Y$ gauge structure of the SM have a particular importance. Although the present scenario indicates that the couplings of the Higgs boson are in good agreement with the SM predictions, one would need more accurate measurements to further constrain the couplings or to see a small deviation from the SM predictions which could be a hint towards some underlying new physics. This will require one to go beyond usual observables like cross
sections and differential rates which will be possible with higher statistics at the HL-LHC.

A large amount of work has been carried out on probing the structure of the $VVH$ couplings at the LHC and at planned $e^+e^-$ colliders [1–17]. These studies have probed the most general tensorial form of the $VVH$ coupling by using a variety of observables involving kinematic distributions of the $Z$ and the charged leptons from $Z$ decay. Study of Higgs-gauge coupling in the effective field theory framework at the LHC has been studied in [18,19] and at future $e^+e^-$ colliders in [12,20,21].

In this paper, we propose studying the $ZZH$ coupling by making use of the spin observables of the $Z$ boson. We study the $ZZH$ coupling using the associated production of the $Z$ with the Higgs at the LHC. The formalism used connects various angular asymmetries of the decay products of the $Z$ to its eight independent polarization parameters extracted from the $Z$ production spin density matrix [22,23]. With the help of these parameters, we estimate limits on the anomalous couplings. $Z$ polarization has been studied in the context of new physics at the LHC [18,22] and at an $e^-e^+$ collider [24–26]. Analogously, polarization of the $W$ boson produced in association with the Higgs at the LHC has been studied in [18,27].

The main significance of our work is that we use completely analytical expressions for the matrix element for the production and decay of polarized $Z$ at the partonic level. Hence the angular asymmetries that we calculate involve no numerical calculations at the partonic level. Only the integrations over parton distributions have to be done numerically.

Our analytical approach has some overlap with that employed in [18]. However, we have estimated limits on anomalous couplings that would be obtained at the HL-LHC, which has not been done in [18].

We consider the process $pp \to ZHX$, where the vertex $Z_{\mu}(k_1) \to Z_\nu(k_2)H$ has the Lorentz structure

$$\Gamma_{\mu\nu}^V = \frac{g}{\cos \theta_W} m_Z \left[ a_Z g_{\mu\nu} + \frac{b_Z}{m_Z^2} (k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1.k_2) + \frac{\tilde{b}_Z}{m_Z^2} \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right]$$  \hspace{1cm} (1)

where $g$ is the $SU(2)_L$ coupling and $\theta_W$ is the weak mixing angle. $a_Z$ and $b_Z$ are invariant under CP, while $\tilde{b}_Z$ corresponds to CP violating term in the Lagrangian. In the SM, at tree level, the coupling $a_Z = 1$, whereas the other two couplings $b_Z$, $\tilde{b}_Z$ vanish. These vanishing couplings are the anomalous couplings which could arise from loop corrections in the SM or in
any extension of SM with some new particles or interactions. However, we are not concerned with the predictions of any specific model here and derive the helicity amplitudes for the process of our interest in a model-independent way using the general form of the $ZZH$ vertex in Eqn. (1).

The current experimental bound on the $ZZH$ anomalous couplings is obtained by the CMS collaboration [28, 29]. Although the current data are consistent with the SM predictions, the constraints are still weak enough to allow for beyond the SM contributions to the vertex. The 68% confidence level (CL) upper bounds on the $ZZH$ couplings, assuming them to be real, in our notation translate to $|\text{Re } b_Z| < 0.058$ and $|\text{Re } \tilde{b}_Z| < 0.078$. These limits are obtained from measurements of ratios of the cross section contributions arising from the different $ZZH$ couplings. Ref [30] obtains possible bounds on the anomalous $ZZH$ coupling at CLIC. For example, the 95% CL limits obtained are $-0.118 < b_Z < 0.041$ and $-0.096 < \tilde{b}_Z < 0.096$ at 3 TeV centre of mass energy (c.m.) and 1000 fb$^{-1}$ integrated luminosity, neglecting systematic uncertainties. The possibility of a future Large Hadron electron Collider (LHeC) to probe anomalous $ZZH$ couplings has been studied in [31], where weak limits are found, viz., $-0.21 < b_Z < 0.43$ and $-0.32 < \tilde{b}_Z < 0.32$ for an electron beam energy of 60 GeV and mild improvement for a beam energy of 140 GeV, with proton beam energy of 7 TeV in either case.

2  

$Z$ Polarization as a Probe

We consider the process $pp \to ZHX$ at the LHC, which at the partonic level proceeds via the process

\[ q(p_1) + \bar{q}(p_2) \to Z^\alpha(p) + H(k) \quad (2) \]

through $s$-channel $Z$ exchange. Here $q$ stands for both up type and down type quarks of any generation, in the massless limit of the initial particles, with the $ZZH$ vertex given in Eqn.(1). We first compute the helicity amplitudes for this process considering the following representations for the transverse and longitudinal polarization vectors of the $Z$:

\[ e^\mu(p, \pm) = \frac{1}{\sqrt{2}}(0, \mp \cos \theta, -i, \pm \sin \theta), \quad (3) \]

\[ e^\mu(p, 0) = \frac{1}{m_Z}(|\vec{p}_Z|, E_Z \sin \theta, 0, E_Z \cos \theta), \quad (4) \]
where $E_Z$ and $\vec{p}_Z$ are the energy and momentum of the $Z$ respectively, with $\theta$ being the polar angle made by the $Z$ with respect to the quark momentum taken to be along the positive $z$ axis.

The non-zero helicity amplitudes in the limit of massless initial states and assuming the SM value $a_Z = 1$ are

$$M(-,+,+) = \frac{g^2 m_Z \sqrt{\hat{s}} (c_V + c_A)}{2 \sqrt{2} \cos^2 \theta_W (\hat{s} - m_Z^2)} \left[ 1 - \frac{\sqrt{\hat{s}}}{m_Z^2} (E_Z b_Z + i \tilde{b}_Z |\vec{p}_Z|) \right]$$

$$M(-,+-,-) = \frac{g^2 m_Z \sqrt{\hat{s}} (c_V + c_A)}{2 \sqrt{2} \cos^2 \theta_W (\hat{s} - m_Z^2)} \left[ 1 - \frac{\sqrt{\hat{s}}}{m_Z^2} (E_Z b_Z - i \tilde{b}_Z |\vec{p}_Z|) \right]$$

$$M(-,0+) = \frac{g^2 \sqrt{\hat{s}} (c_V + c_A)}{2 \cos^2 \theta_W (\hat{s} - m_Z^2)} \left[ E_Z - \sqrt{\hat{s}} b_Z \right] \sin \theta$$

$$M(+,-,+) = -\frac{g^2 m_Z \sqrt{\hat{s}} (c_V - c_A)}{2 \sqrt{2} \cos^2 \theta_W (\hat{s} - m_Z^2)} \left[ 1 - \frac{\sqrt{\hat{s}}}{m_Z^2} (E_Z b_Z + i \tilde{b}_Z |\vec{p}_Z|) \right]$$

$$M(+,-,-) = -\frac{g^2 m_Z \sqrt{\hat{s}} (c_V - c_A)}{2 \sqrt{2} \cos^2 \theta_W (\hat{s} - m_Z^2)} \left[ 1 - \frac{\sqrt{\hat{s}}}{m_Z^2} (E_Z b_Z - i \tilde{b}_Z |\vec{p}_Z|) \right]$$

$$M(+,0-) = \frac{g^2 \sqrt{\hat{s}} (c_V - c_A)}{2 \cos^2 \theta_W (\hat{s} - m_Z^2)} \left[ E_Z - \sqrt{\hat{s}} b_Z \right] \sin \theta$$

Here the first two entries in $M$ denote the signs of the helicities of the quark and antiquark respectively and the third entry is the $Z$ helicity. $\sqrt{\hat{s}}$ is the partonic c.m. energy, and $c_V$ and $c_A$ are the respective vector and axial vector couplings of the relevant quark to the $Z$.

The $a_Z$ dependence can be easily recovered by multiplying the helicity amplitude expressions by $a_Z$, and then replacing $b_Z$ and $\tilde{b}_Z$ by $b_Z/a_Z$ and $\tilde{b}_Z/a_Z$, respectively.

We evaluate the elements of the spin-density matrix for $Z$ production, which can be expressed in terms of the helicity amplitudes as follows

$$\rho(i,j) = \sum_{\lambda,\lambda'} M(\lambda,\lambda',i) M^*(\lambda,\lambda',j)$$

4
the average being over the initial helicities $\lambda, \lambda'$ of the quark and antiquark respectively and also over the initial color states. The $Z$ helicity indices $i, j$ can take values $\pm, 0$ and with $i = j$ corresponding to the diagonal elements of Eqn. (11) which are the squared matrix elements for $Z$ production with definite polarization. It is known that a complete information of the state of polarization is encoded in all the density matrix elements. So to attain maximum possible information, it is necessary to study the full density matrix description, which also includes the off diagonal elements. The density matrix elements, for $q\bar{q} \rightarrow ZH$, derived from the helicity amplitudes are given by

$$\rho(\pm, \pm) = \frac{g^4 m_Z^2 s}{8 \cos^4 \theta_W (s - m_Z^2)^2} \left[ (c_V + c_A)^2 (1 \mp \cos \theta)^2 \right.$$  
$$+ (c_V - c_A)^2 (1 \pm \cos \theta)^2 \left] 1 - 2 (\Re \, b_Z \mp \beta Z \, \Im \, \bar{b}_Z) \frac{E_Z \sqrt{s}}{m_Z^2} \right.$$  
$$+ \frac{E_Z^2 s}{m_Z^4} |b_z|^2 + \frac{2 E_Z P_Z \hat{s}}{m_Z^4} (\Im \, \bar{b}_Z \, \Re \, b_Z - \Im \, b_Z \, \Re \, \bar{b}_Z)$$  
$$+ \frac{P_Z^2 \hat{s}}{m_Z^4} |\bar{b}_Z|^2 \right]$$  

(12)

$$\rho(0, 0) = \frac{g^4 E_Z^2 s}{2 \cos^4 \theta_W (s - m_Z^2)^2} \sin^2 \theta \left( c_V^2 + c_A^2 \right) \left[ 1 - 2 \Re \, b_Z \frac{\sqrt{s}}{E_Z} \right.$$  
$$+ \frac{\hat{s}}{E_Z} |b_z|^2 \right]$$  

(13)

$$\rho(\pm, \mp) = \frac{g^4 m_Z^2 s}{4 \cos^4 \theta_W (s - m_Z^2)^2} \sin^2 \theta \left( c_V^2 + c_A^2 \right) \times \left[ 1 - 2 (\Re \, b_Z \pm i \beta Z \Re \, \bar{b}_Z) \frac{E_Z \sqrt{s}}{m_Z^2} + \frac{E_Z^2 \hat{s}}{m_Z^4} |b_z|^2 \right.$$  
$$\mp \frac{2 E_Z P_Z \hat{s}}{m_Z^4}$$  
$$\left( \Im \, \bar{b}_Z \, \Im \, b_Z + \Re \, b_Z \, \Re \, \bar{b}_Z \right) - \frac{2 P_Z^2 \hat{s}}{m_Z^4} |\bar{b}_Z|^2 \right]$$  

(14)

$$\rho(\pm, 0) = \frac{g^4 m_Z E_Z s}{4 \sqrt{2} \cos^4 \theta_W (s - m_Z^2)^2} \sin \theta \times \left[ (c_V + c_A)^2 (1 \mp \cos \theta) - (c_V - c_A)^2 (1 \pm \cos \theta) \right] \times \left[ 1 - \Re \, b_Z \sqrt{s} \left( \frac{E_Z^2 + m_Z^2}{E_Z m_Z^2} \right) - i \sqrt{s} \frac{E_Z}{m_Z^2} \left( \Im \, b_Z \beta_Z^2 \pm \bar{b}_Z \beta_Z \right) \right.$$  

(15)
± \frac{s}{m_Z^2} |b_Z|^2 ± \frac{sP_Z}{m_Z^2 E_Z} (\text{Im } b_Z + i \text{Re } b_Z)(\text{Re } \tilde{b}_Z + i \text{Im } \tilde{b}_Z) \right] \tag{15}

where $\beta_Z = |\vec{p}_Z|/E_Z$ is the velocity of the $Z$ in the c.m frame. The analytical manipulation software FORM [35] has been used to verify these expressions. We have kept the finite $Z$ width in our numerical calculations later.

The full density matrix Eqn.(11) on integrating over an appropriate kinematic range, can be parametrized in terms of the 3 components of the vector polarization $\vec{P}$ and 5 components of the tensor polarization $T$ of the $Z$ boson [36]. Defining this as $\sigma(i, j)$ we have

$$\sigma(i, j) \equiv \sigma \left( \begin{array}{ccc} \frac{1}{3} + \frac{P_x}{2} + \frac{T_{xx}}{\sqrt{6}} & \frac{P_x - i P_y}{2\sqrt{2}} + \frac{T_{xx} - iT_{xy}}{\sqrt{3}} & \frac{T_{xx} - iT_{yy} - 2iT_{xy}}{\sqrt{6}} \\ \frac{2P_x + T_{xx}}{2\sqrt{2} T_{yy} + 2iT_{xy}} & \frac{P_x + i P_y}{2\sqrt{2}} - \frac{iT_{zz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{P_x - P_y}{2} + \frac{T_{xx}}{\sqrt{6}} \\ \frac{T_{xy}}{\sqrt{6}} & \frac{T_{xy}}{\sqrt{6}} & \frac{T_{xx} - iT_{yy} - 2iT_{xy}}{\sqrt{6}} \end{array} \right) \right) \tag{16}

where $\sigma$ is the production cross section,

$$\sigma = \sigma(+, +) + \sigma(-, -) + \sigma(0, 0). \tag{17}$$

The eight independent vector and tensor polarization observables of the $Z$ can then be constructed using appropriate linear combinations of the integrated density matrix elements of Eqn.(16):

$$P_x = \frac{\{\sigma(+, 0) + \sigma(0, +)\} + \{\sigma(0, -) + \sigma(-, 0)\}}{\sqrt{2}\sigma} \tag{18}$$

$$P_y = -i\left[[\sigma(0, +) - \sigma(+, 0)] + [\sigma(-, 0) - \sigma(0, -)]\right] \frac{1}{\sqrt{2}\sigma} \tag{19}$$

$$P_Z = \frac{[\sigma(+, +)] - [\sigma(-, -)]}{\sigma} \tag{20}$$

$$T_{xy} = \frac{-i\sqrt{6}[\sigma(-, +) - \sigma(+, -)]}{4\sigma} \tag{21}$$

$$T_{xx} = \frac{\sqrt{3}\{[\sigma(+, 0) + \sigma(0, +)] - [\sigma(0, -) + \sigma(-, 0)]\}}{4\sigma} \tag{22}$$

$$T_{yz} = \frac{-i\sqrt{3}\{[\sigma(0, +) - \sigma(+, 0)] - [\sigma(-, 0) - \sigma(0, -)]\}}{4\sigma} \tag{23}$$

$$T_{xx} - T_{yy} = \frac{\sqrt{6}[\sigma(-, +) + \sigma(+, -)]}{2\sigma} \tag{24}$$

$$T_{zz} = \frac{\sqrt{6}}{2} \left\{ \frac{[\sigma(+, +)] + [\sigma(-, -)]}{\sigma} - \frac{2}{3} \right\}$$

$\text{where } \sigma$ is the production cross section,
\[ \frac{\sqrt{6}}{2} \left[ \frac{1}{3} \frac{\sigma(0,0)}{\sigma} \right] \]  

(25)

Of these \( P_x, P_y \) and \( P_z \) are the vector polarizations, whereas the \( T \)'s are the tensor polarizations, with the constraint that the tensor is traceless. In real experiments where the \( Z \) boson decays to two leptons, these polarization observables can be extracted from kinematic distributions of its decay products. Angular asymmetries can be obtained by combining the relevant production-level density matrix elements with appropriate decay density matrix elements and integrating over the appropriate phase space. For example, \( P_x \) can be calculated from the asymmetry \( A_x \) defined by-

\[ A_x = \frac{3\alpha P_x}{4} \equiv \frac{\sigma(\cos \phi^* > 0) - \sigma(\cos \phi^* < 0)}{\sigma(\cos \phi^* > 0) + \sigma(\cos \phi^* < 0)} \]  

(26)

where, \( \alpha \) is the \( Z \) boson polarization analyzer, given in terms of its vector and axial vector couplings to charged leptons \( \ell, c_{V}^\ell \) and \( c_{A}^\ell \) respectively, as

\[ \alpha = -\frac{2c_{V}^\ell c_{A}^\ell}{c_{V}^{2} + c_{A}^{2}} \]  

(27)

The angles \( \theta^* \) and \( \phi^* \) are polar and azimuthal angles of the lepton in the rest frame of the \( Z \). The \( Z \) rest frame is reached by a combination of boosts and rotations from the laboratory frame. In the laboratory frame, the quark momentum defines the positive \( z \) axis, and the production plane of \( Z \) is defined as the \( xz \) plane. While boosting to the \( Z \) rest frame, the \( xz \) plane is kept unchanged. Then, the angles \( \theta^* \) and \( \phi^* \) are measured with respect to the would-be momentum of the \( Z \). Similarly, expressions for other asymmetries, \( \text{viz.}, A_y, A_z, A_{xy}, A_{yz}, A_{xz}, A_{x^2-y^2}, A_{zz} \) corresponding to the 2 vector polarizations \( P_y, P_z \) and 5 tensor polarizations \( T_{ij}(i,j = x,y,z) \) can be obtained and are listed in [25,26].

It is observed that the density matrix elements \( \sigma(\pm,0) \) and \( \sigma(0,\pm) \) and the asymmetries involving these elements \( A_x, A_y, A_{xz}, A_{yz} \) vanish due to the symmetric nature of the LHC, which does not allow a unique definition of a positive \( z \)-axis. Therefore to make them non-zero, we define the direction of the reconstructed momentum of the \( ZH \) combination as the positive \( z \)-axis.

We have evaluated these asymmetries upto quadratic order in the anomalous couplings. It is observed that out of eight polarization asymmetries only three, \( \text{viz.}, A_x, A_{x^2-y^2} \) and \( A_{zz} \) are non-zero in the SM, which, along with the
total cross section, would be proportional to the real part of the anomalous couplings (upto linear order) or absolute square of the couplings to satisfy the CPT theorem. This will be seen in the following section.

3 Limits on the Anomalous Couplings

Here we present numerical values for the integrated density matrix elements, the corresponding asymmetries and the sensitivities of the asymmetries to the various anomalous couplings. We consider c.m. energy \( \sqrt{s} = 14 \) TeV, with integrated luminosity \( \int L dt = 1000 \) fb\(^{-1}\). In our numerical calculations, we employ MMHT2014 parton distribution functions [37] with factorization scale chosen as the square root of the partonic c.m. energy. The integrated production density matrix elements in the units of fb are

\[
\sigma(\pm, \pm) = 161.95 - 1495.62 \Re b_Z \pm 1036.98 \Im \tilde{b}_Z + 5391.21 |b_Z|^2 \\
+ 3753.23 |\tilde{b}_Z|^2 \mp 8811.36 (\Im \tilde{b}_Z \Re b_Z - \Im b_Z \Re \tilde{b}_Z) 
\]

(28)

\[
\sigma(0, 0) = 341.976 - 1495.62 \Re b_Z + 1637.98 |b_Z|^2 
\]

(29)

\[
\sigma(\pm, \mp) = 80.97 - 747.81 \Re b_Z \mp i518.49 \Re \tilde{b}_Z + 2695.6|b_Z|^2 \\
-1876.61 |\tilde{b}_Z|^2 \pm i4405.67 (\Im \tilde{b}_Z \Im b_Z + \Re \tilde{b}_Z \Re b_Z) 
\]

(30)

\[
\sigma(\pm, 0) = 59.59 - 474.46 \Re b_Z - i211.22 \Im b_Z \mp 261.88 (i \Re \tilde{b}_Z - \Im \tilde{b}_Z) \\
+738.07 |b_Z|^2 \pm 558.15 \tilde{b}_Z (\Im b_Z + i \Re b_Z) 
\]

(31)

\[
\sigma(0, \pm) = 59.59 - 474.46 \Re b_Z + i211.22 \Im b_Z \pm 261.88 (i \Re \tilde{b}_Z + \Im \tilde{b}_Z) \\
+738.07 |b_Z|^2 \pm 558.15 \tilde{b}_Z (\Im b_Z + i \Re b_Z) 
\]

(32)

The leptonic asymmetries corresponding to different polarizations calculated upto linear order in the anomalous couplings are given by

\[
A_x = 0.035 \Re b_Z - 0.028 
\]

(33)

\[
A_y = -0.125 \Re \tilde{b}_Z 
\]

(34)
\[ A_z = -0.349 \text{ Im } \tilde{b}_Z \]  
(35)

\[ A_{xy} = 0.496 \text{ Re } \tilde{b}_Z \]  
(36)

\[ A_{xz} = -0.354 \text{ Im } \tilde{b}_Z \]  
(37)

\[ A_{yz} = 0.286 \text{ Im } b_Z \]  
(38)

\[ A_{x^2-y^2} = -0.193 \text{ Re } b_Z + 0.077 \]  
(39)

\[ A_{zz} = -0.683 \text{ Re } b_Z - 0.101 \]  
(40)

It is observed that all asymmetries except \( A_x \), \( A_{x^2-y^2} \) and \( A_{zz} \) vanish in the SM and the reason for this is the CP even and T even nature of the asymmetries \( A_x, A_{x^2-y^2} \) and \( A_{zz} \), because of which they can occur at tree level in the SM. The remaining asymmetries vanish in the SM because they are either CP even and T odd or CP odd, and hence depend on the CP violating parameters which are absent in the SM at tree level.

| Observable | Coupling | Limit \((\times 10^{-3})\) |
|------------|----------|-----------------|
| \( \sigma \) | Re \( b_Z \) | 0.70 |
| \( A_x \) | Re \( b_Z \) | 136 |
| \( A_y \) | Re \( \tilde{b}_Z \) | 37.9 |
| \( A_z \) | Im \( \tilde{b}_Z \) | 13.5 |
| \( A_{xy} \) | Re \( \tilde{b}_Z \) | 9.53 |
| \( A_{yz} \) | Im \( b_Z \) | 16.5 |
| \( A_{xz} \) | Im \( \tilde{b}_Z \) | 13.3 |
| \( A_{x^2-y^2} \) | Re \( b_Z \) | 24.4 |
| \( A_{zz} \) | Re \( b_Z \) | 6.88 |

Table 1: 1\( \sigma \) limit obtained on the anomalous couplings from cross section and various leptonic asymmetries calculated upto linear order in the couplings at \( \sqrt{s} = 14 \text{ TeV} \).
Next we present the expressions for the total cross section and angular asymmetries including quadratic terms of couplings at $\sqrt{s} = 14$ TeV.

$$\sigma = 0.067294 \, (7506.45 \, |\tilde{b}_Z|^2 + 12420.4 \, |b_Z|^2 - 4486.85 \, \Re b_Z + 665.87) \, \text{fb} \, (41)$$

$$A_x = \frac{0.012 \, \Re b_Z - 0.019 \, |b_Z|^2 - 0.002}{0.604 \, |\tilde{b}_Z|^2 + |b_Z|^2 - 0.361 \, \Re b_Z + 0.054} \, (42)$$

$$A_y = \frac{0.024 \, \Im b_Z \, \Im b_Z + \Re b_Z \, (0.024 \, \Re b_Z - 0.011)}{|\tilde{b}_Z|^2 + 1.655 \, |b_Z|^2 + (\Re \tilde{b}_Z)^2 - 0.598 \, \Re b_Z + 0.089} \, (43)$$

$$A_z = \frac{\Im \tilde{b}_Z \, (1976.66 \, \Re b_Z - 232.627) - 1976.66 \, \Im b_Z \, \Re \tilde{b}_Z}{7506.45 \, |b_Z|^2 + 12420.4 \, |b_Z|^2 - 4486.85 \, \Re b_Z + 665.87} \, (44)$$

$$A_{xy} = \frac{\Re \tilde{b}_Z \, (0.044 - 0.374 \, \Re b_Z) - 0.374 \, \Im b_Z \, \Im b_Z}{|\tilde{b}_Z|^2 + 1.655 \, |b_Z|^2 - 0.598 \, \Re b_Z + 0.089} \, (45)$$

$$A_{yz} = \frac{-190.164 \, \Im b_Z}{7506.45 \, |b_Z|^2 + 12420.4 \, |b_Z|^2 - 4486.85 \, \Re b_Z + 665.87} \, (46)$$

$$A_{xz} = \frac{\Im \tilde{b}_Z \, (0.0404 \, \Re b_Z - 0.019) - 0.0404 \, \Im b_Z \, \Re \tilde{b}_Z}{0.604 \, |\tilde{b}_Z|^2 + |b_Z|^2 - 0.361 \, \Re b_Z + 0.054} \, (47)$$

$$A_{x^2-y^2} = \frac{-0.297 \, |\tilde{b}_Z|^2 + 0.019 \, \Re b_Z - 0.005}{|\tilde{b}_Z|^2 + 1.655 \, |b_Z|^2 - 0.598 \, \Re b_Z + 0.089} + 0.138 \, (48)$$

$$A_{zz} = \frac{0.074 \, |\tilde{b}_Z|^2 + 0.068 \, \Re b_Z - 0.019}{|\tilde{b}_Z|^2 + 1.655 \, |b_Z|^2 - 0.598 \, \Re b_Z + 0.089} + 0.113 \, (49)$$

We obtain the sensitivity of an observable $\mathcal{O}$ which depends on a parameter $f$ from the definition

$$S(\mathcal{O}(f)) = \frac{|\mathcal{O}(f) - \mathcal{O}(f = 0)|}{\delta \mathcal{O}} \, (50)$$

where $\delta \mathcal{O}$ is the estimated error on the observable. For an asymmetry, the estimated error takes the form

$$\delta A = \frac{1 - A_{SM}^2}{\sqrt{\sigma_{SM} L}} \, (51)$$
with $\sigma_{SM}$ being the SM cross section for the process $pp \rightarrow Z^*H \rightarrow \ell\bar{\ell}H$ ($\ell = e, \mu$) at the LHC with integrated luminosity $L$ and $A_{SM}$ is the corresponding value of asymmetry in the SM. Similarly, for the cross section, the error is given by

$$\delta\sigma = \sqrt{\frac{\sigma_{SM}}{L}}$$ (52)

We estimate the $1\sigma$ limits calculated up to linear order and list it in Table 1. We note that, among the four observables $\sigma$, $A_x$, $A_{zz}$ and $A_{x^2-y^2}$, which are sensitive to $\text{Re} \, b_Z$, the total cross section provides the best limits on the coupling. However, it is not sufficient to consider the total cross sections as the only probe as it is sensitive to just one coupling, $\text{Re} \, b_Z$. So to explore the couplings which do not appear in the total cross section, one will require the other angular asymmetries. The better limit on $\text{Im} \, \tilde{b}_Z$ comes from $A_x$ and $A_{xz}$, both being equally sensitive to the coupling. For the coupling $\text{Im} \, b_Z$, the best bound comes from $A_{yz}$ whereas on $\text{Re} \, \tilde{b}_Z$ the best limit of $9.53 \times 10^{-3}$ is achieved from the observable $A_{xy}$. 

11
Figure 1: Sensitivities of cross section and asymmetries to anomalous couplings, including quadratic order at $\sqrt{s} = 14$ TeV. Plots are obtained by varying one coupling at a time.

| Observable | Coupling | Limit ($\times 10^{-3}$) |
|------------|----------|--------------------------|
| $\sigma$   | Re $b_Z$ | 0.70                     |
| $\sigma$   | Im $b_Z$ | 15.9                     |
| $A_{xy}$   | Re $\tilde{b}_Z$ | 9.54           |
| $A_{xx}, A_Z$ | Im $\tilde{b}_Z$ | 13.3           |

Table 2: The best 1$\sigma$ limit on couplings and the corresponding observables at $\sqrt{s} = 14$ TeV, obtained from Figure 1.

In Figure 1, we plot the one parameter sensitivity $i.e$ $S = 1(\text{or } \Delta \chi^2 = 1)$ for the cross section and the 8 asymmetries, considered upto quadratic order in the anomalous couplings. It is observed from Figure 1 that the tightest
limit on the coupling Re $b_Z$ can be obtained from total cross section. On the
coupling Im $b_Z$, both cross section and $A_{yz}$ place comparable limits.
The observables $A_x$ and $A_{xz}$ are found to be almost equally sensitive to the
coupling Im $b_Z$. The best limit on Re $b_Z$ can be obtained from the observable
$A_{xy}$. In Table 2, we list the tightest 1σ level limit on the couplings, obtained
from Figure 1.

So far, we obtained a limit on each coupling assuming all the other couplings
to be zero. Ideally, we would like to place a limit on each coupling without making any assumptions on the remaining couplings. In practice
this would involve making a simultaneous fit to several observables varying
all the couplings. This is not only cumbersome, it would also require a large
data set. We therefore now consider simultaneous limits which may be ob-
tained by selecting a pair of couplings non-vanishing, taking the remaining
to be zero.

We vary two couplings at a time and obtain the 1σ sensitivity i.e $S = 2.3$
(or $\Delta \chi^2 = 2.3$) contours shown in Figure 2 for each observable. The black
dot in the middle of the plots represents the SM value.

A first general observation in the context of deriving limits from the
contours is that the total cross section $\sigma$, which makes use of all the events,
tends to be the most sensitive observable for measurement of all couplings.
While at linear order it depends only on Re $b_Z$, at the quadratic order it
depends on all the couplings. Thus, in most cases, the best limit for all
couplings is obtained from $\sigma$.

Another observation is that as Re $\tilde{b}_Z$ and Im $\tilde{b}_Z$ are CP-odd couplings,
you would occur linearly in CP-odd observables. Thus, even though these
CP-odd couplings could be constrained by the cross sections or any of the
asymmetries, they would get strongest limits from the CP-odd asymmetries
$A_y$, $A_{xy}$, $A_z$ and $A_{xz}$. Of these the first two are CPT even and would therefore
constrain Re $\tilde{b}_Z$, whereas the last two being CPT odd would constrain Im $\tilde{b}_Z$.

Coming to simultaneous limits on two couplings which can be read off
from the contour plots, the best limits on the combination Re $b_Z$ and Im $b_Z$
come from $\sigma$ and $A_{yz}$, and are, respectively, $[-0.002, 0.005]$ and $[-0.035, 0.035]$.
If, however, $A_{zz}$ is used in place of $\sigma$, the limit on Im $b_Z$ is very similar, but
the limit on Re $b_Z$ becomes weaker. All other asymmetries give weaker limits.

Taking the sensitivity contour plots of Re $b_Z$ versus Re $\tilde{b}_Z$, the best limits
are from $\sigma$ and $A_{xy}$, the latter being linear in the CP-odd coupling Re $\tilde{b}_Z$.
These are, respectively, $[-0.002, 0.003]$ and $[-0.022, 0.022]$. Again if $A_{zz}$ is
used instead of $\sigma$, the limit on Re $b_Z$ is weaker, viz., $[-0.02, 0.02]$. 

13
Figure 2: 1σ sensitivity contours for cross-section and asymmetries obtained by varying two parameters simultaneously. The black dot in the middle of the plots represents the SM value.
In case of the contour plots of $\text{Re} \ b_Z$ and $\text{Im} \ \tilde{b}_Z$, the best limits on $\text{Re} \ b_Z$ are as in the previous case, whereas the best limits on $\text{Im} \ \tilde{b}_Z$ are $[-0.035, 0.035]$, come from $\sigma$ and $A_z$, $A_{xz}$, the latter two being numerically very close. A similar situation holds in the case of the contour plots of $\text{Re} \ b_Z$ versus $\text{Im} \ \tilde{b}_Z$, where the best limits on $\text{Im} \ \tilde{b}_Z$, viz., $[-0.03, 0.03]$ come equally from $A_z$ and $A_{xz}$, the best limit on $\text{Re} \ b_Z$, obtained from $A_{xy}$ is $[-0.025, 0.025]$. The best limits on $\text{Re} \ b_Z$ and $\text{Im} \ b_Z$ are $[-0.02, 0.02]$ and $[-0.027, 0.027]$ respectively, obtained from the combination of $A_{xy}$ and $\sigma$. Lastly in the case of $\text{Im} \ b_Z$ versus $\text{Im} \ \tilde{b}_Z$ contour, best limit on $\text{Im} \ b_Z$ is $[-0.025, 0.025]$, comes from $\sigma$, whereas on $\text{Im} \ b_Z$, $A_z$ and $A_{xz}$ which contribute almost equally, provide a stringent limit of $[-0.031, 0.031]$ on it.

We see from the above a significant feature that using $\sigma$ as one of the observables gives a stringent limit for all the couplings involved. We also see that the best limits on $\text{Re} \ b_Z$ is of the order of $2 - 5 \times 10^{-3}$ in magnitude from all relevant pairs of observables. This may be compared to the limit $0.7 \times 10^{-3}$ obtained when only $\text{Re} \ b_Z$ is taken as non-zero, as seen from Tables 1 and 2. Similarly, the best limits on $\text{Im} \ b_Z$ from simultaneous measurement of two observables varies between $25 \times 10^{-3}$ and $35 \times 10^{-3}$, as compared to the best individual limit of around $16 \times 10^{-3}$. For $\text{Re} \ b_Z$ the best simultaneous limits are $20 - 27 \times 10^{-3}$, the best individual limit being $9.5 \times 10^{-3}$. Likewise, the best simultaneous limits on $\text{Im} \ \tilde{b}_Z$ vary between $30 \times 10^{-3}$ and $35 \times 10^{-3}$, whereas the best individual limit is around $13 \times 10^{-3}$.

4 Conclusions and discussion

The measurement of couplings of the Higgs Boson to all other SM particles is an essential test of the SM. In this work, we study the form and magnitude of the tensor structure of the couplings of the Higgs boson to a pair of $Z$ bosons at the LHC with the help of the polarization observables of the $Z$. We estimate sensitivities of these polarization observables by adopting the formalism which connects angular asymmetries of charged leptons from $Z$ decay to the polarization parameters of the $Z$. We first calculate the $Z$ polarization parameters using the spin density matrix elements evaluated at production level and then obtain various angular asymmetries corresponding to these parameters.

We have restricted ourselves to tree-level calculations. To our knowledge, non-leading order (NLO) contributions to the process with polarized $Z$ have
not been calculated so far. However, we expect that asymmetries which we make use of, being ratios of cross sections, will be less sensitive to NLO corrections.

We see that the $1\sigma$ limits obtained on the real parts of the couplings are of the order of a few times $10^{-3}$ and an order of magnitude higher for the imaginary parts. We show that the LHC at c.m energy $\sqrt{s} = 14$ TeV with integrated luminosity $\int L dt = 1000$ fb$^{-1}$ could provide a limit on the CP conserving couplings $Re b_Z$ in the interval $[-0.7, 0.7] \times 10^{-3}$ and $Im b_Z$ in the interval $[-15.9, 15.9] \times 10^{-3}$. Similarly the CP violating couplings, $Re \tilde{b}_Z$ and $Im \tilde{b}_Z$ get a best bound of $|Re \tilde{b}_Z| \leq 9.54 \times 10^{-3}$ and $|Im \tilde{b}_Z| \leq 13.3 \times 10^{-3}$ respectively. These limits are obtained by varying one coupling at a time. With two non-zero couplings, we observe a slight weakening of bounds on all the anomalous couplings as can be expected.

We have not considered Higgs decays, which do not affect the polarization parameters and asymmetries of the $Z$. The effect of Higgs decay on the sensitivities can be estimated by multiplying the SM cross section by the Higgs branching ratio and detection efficiencies in Eqns. (51) and (52).

Associated Higgs production with $V = W, Z$ and with $H$ decaying into $b\bar{b}$ and $V$ decaying to 0, 1 and 2 leptons has been observed by both ATLAS [32] and CMS collaborations [33] at close to $5\sigma$ CL. Also, as shown in [34], a measurement of (SM) $Z$ polarization parameters themselves can help suppress backgrounds and enhance the signal sensitivity in the $Z(\ell^+\ell^-)H(b\bar{b})$ decay mode. A full scale analysis using an event generator coupled with all appropriate cuts and detection efficiencies relevant to the decay channels of the $Z$ and Higgs, as used in [32–34] with be able to refine the actual sensitivities that we have obtained for all the eight BSM polarization asymmetries.

Acknowledgement: SDR acknowledges partial support from the Department of Science and Technology, India, under the J.C. Bose National Fellowship programme, Grant No. SR/SB/JCB-42/2009, and from the Indian National Science Academy, New Delhi, under the Senior Scientist Programme.

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