Performance evaluation of the Bühlmann credibility approach in predicting mortality rates

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Abstract. Modeling mortality is an important thing for insurance company in determining the appropriate premium for a policy holder. This paper explains the Bühlmann credibility approach for modeling mortality. Bühlmann’s credibility model is generally used to predict the value of a random variable in the next future period. In predicting the value of a random variable in the next two or more future periods, an approach is needed. In this paper, the Bühlmann credibility approach is done through 2 strategies: the expanding window strategy and the moving window strategy. The expanding window strategy is done by adding the predicted value to the data to produce a predicted value in the following year while the moving window strategy is done by adding the predicted value to the data and discarding the oldest data to produce a predicted value in the following year. Mortality prediction performances from the Bühlmann credibility approach are being analysed by comparing the values of AAMAPE and the reduction ratio of the Bühlmann credibility approach to the Lee-Carter model. From this paper, it was found that the Bühlmann credibility approach produced better mortality predictions compared to the Lee-Carter model in case of Australian mortality data.

Keywords: Expanding window, Lee-Carter model, moving window

1. Introduction
Every person in the world is exposed to risk. The risk referred to here is the uncertainty that an adverse event will occur in the future. Buying insurance is a way to transfer a risk to the insurance companies. Insurance is an agreement between two parties, namely the insurance company and the policyholder. On the policyholder's side, the policyholder pays a premium to the insurance company and receives a reward, that is, the coverage for losses that can occur in the future in accordance with the provisions in the policy. On the insurance company’s side, the insurance company receives premiums from policyholders and has a responsibility to pay benefits if a loss occurs in the future in accordance with the provisions in the policy. Most insurance products have benefits related to mortality. Therefore, it is important for insurance companies predict the future mortality rate in order to determine the amount of premiums that are consistent with the risks they bear.

The first stochastic mortality model, which is the Lee-Carter model, was introduced in 1992 by Ronald Lee and Lawrence Carter [1]. Lee-Carter model models the natural logarithm of the central death rate with a function of two age parameters and one time parameter. The stochastic mortality model was widely developed after that. The CBD model, which is designed to model mortality at old age, was introduced in Cairns et al. [2]. Renshaw et al. [3] modified the Lee-Carter model by including the cohort
effect, Cairns et al. [4] modified the CBD model by considering the cohort effects, and Plat [5] introduced a mortality model which is a combination of the Lee-Carter and CBD model.

On the other hand, mortality can also be predicted using credibility theory. Tsai et al. [3] introduce a new approach to modeling mortality, namely the Bühlmann credibility approach. Tsai et al. [3] use Bühlmann’s credibility theory to predict the rate of change of \[ \ln(m_{x,t}) \] for next year and introduce two strategies: the expanding window (EW) and moving window (MW) strategies to predict the rate of change of \[ \ln(m_{x,t}) \] for two or more years in the future. In their research, Tsai et al. found that the predicted mortality rate using the Bühlmann credibility approach provided better predictive results compared to the Lee-Carter model based on the value of the mean absolute percentage error for US, UK and Japan mortality data [6].

Therefore, in this paper, the authors intend to present the Bühlmann credibility approach to modeling mortality. By adopting the Bühlmann credibility approach, it is hoped that a better mortality rate prediction can be obtained so that insurance companies can determine their premium consistent with the risk they bear. In this paper, we used the Australian mortality data to compare the performance of the Bühlmann credibility approach against the Lee-Carter model.

2. Experimental
This section contains a discussion about materials and methods that are used in this paper. The concepts to be discussed include the Lee-Carter model and the Bühlmann credibility approach. Before we begin the discussion, we define \[ m_{x,t} \] as the central death rate for an individual aged \[ x \] in year \[ t \]. In this section, we will assume that we use \[ \ln(m_{x,t}) \] data for \[ x = x_L, ..., x_U \] and \[ t = t_L, ..., t_U \] where \[ x_U - x_L + 1 = m \] and \[ t_U - t_L + 1 = n \] to model the mortality. The following is the discussion about the Lee-Carter model.

2.1. Lee-Carter Model
The Lee-Carter model is a stochastic mortality model which is used to predict future values of \[ \ln(m_{x,t}) \] using \[ \ln(m_{x,t}) \] data with a fitting age span \[ [x_L, x_U] \] and a fitting year span \[ [t_L, t_U] \]. The Lee-Carter model models the value of \[ \ln(m_{x,t}) \] because there is a linear relationship between \[ \ln(m_{x,t}) \] with respect to time. This model was introduced by Lee and Carter in 1992, and has been widely analyzed and developed since then. The Lee-Carter model (1992) is given as follows [1]:

\[
\ln(m_{x,t}) = \alpha_x + \beta_x \cdot k_t + \epsilon_{x,t}, \quad x = x_L, ..., x_U, \quad t = t_L, ..., t_U.
\] (1)

Note that \( \alpha_x \) is an age-specific parameter, \( k_t \) is a time related parameter, and \( \beta_x \) is an age specific parameter that represents the sensitivity of \( \ln(m_{x,t}) \) against the time related parameter \( k_t \). The error of the model is represented by \( \epsilon_{x,t} \) and is assumed to be independent and identically distributed over time. Lee-Carter model has two constraints, which are

\[
\sum_{x=x_L}^{x_U} \beta_x = 0 \quad \text{and} \quad \sum_{t=t_L}^{t_U} k_t = 0.
\] (2)

2.1.1. Parameter estimation. There are several ways to estimate the parameter of Lee-Carter model. In this paper, the parameter is estimated using a close approximation to the singular value decomposition (SVD). For the close approximation to SVD, two constraints are added, which are

\[
\sum_{t=t_L}^{t_U} \epsilon_{x,t} = 0 \quad \text{and} \quad \sum_{x=x_L}^{x_U} \epsilon_{x,t} = 0.
\] (3)

By using the added constraints, the \( \alpha_x \) and \( k_t \) parameter can be estimated. The estimator of \( \alpha_x \) and \( k_t \) are given below.
Then $\hat{\alpha}_x$ is estimated by regressing $[\ln(m_{x,t}) - \hat{\alpha}_x]$ on $\hat{\beta}_t$ which yields the formula below.

$$\hat{\beta}_x = \sum_{t=t_{\text{U}}}^{t_{\text{L}}} \frac{[\ln(m_{x,t}) - \hat{\alpha}_x]}{(k_t)^2} \hat{k}_t$$

(6)

In predicting the value of $\ln(m_{x,t})$ in the future, the Lee-Carter model models the parameters that depend on time $k_t$ using a random walk model with drift $\hat{\beta}_t$ which can be written as,

$$k_t = k_{t-1} + \theta + \varepsilon_t$$

(7)

where $\varepsilon_t$ is assumed to be independent and identically distributed over time. Besides that, $\varepsilon_t$ is also assumed to be uncorrelated with the error of the model, which is $\varepsilon_{x,t}$. The estimator of $\theta$ is given below.

$$\hat{\theta} = \frac{k_{t_{\text{U}}} - k_{t_{\text{L}}}}{t_{\text{U}} - t_{\text{L}}}$$

(8)

Before assuming that the time related parameter $k_t$ follows a random walk with drift, it is found that the predicted value of $\ln(m_{x,t})$ is given below.

$$\ln(m_{x,t_{U}+s}) = \hat{\alpha}_x + \hat{\beta}_x (\hat{k}_{t_{U}+s} + \hat{\theta})$$

(9)

The equation 9 above implies that the Lee-Carter model models the value of $\ln(m_{x,t})$ as a linear function of time.

2.2. The Bühlmann credibility approach

Based on the previous discussion about the Lee-Carter model, it is known that the Lee-Carter model models the value of $\ln(m_{x,t})$ as a linear function of time, i.e. the rate of decline in the value of $\ln(m_{x,t})$ is constant from year to year. Empirical mortality data for Australian males between 1956 and 2015 also shows a downward trend over time with an average rate of decline that tends to be constant. This can be seen in figure 1.

Based on figure 1, it can be seen that $Y_{x,t}$ has an average that tends to be constant over time. Therefore, we will define a new random variable, which is $Y_{x,t}$ (the rate of change of $\ln(m_{x,t})$ for 1 year) that will be predicted using the Bühlmann credibility approach by assuming that the risk parameter is age $x$. We know that $\ln(m_{x,t_{U}+s})$ expresses the natural logarithm of the central death rate for age $x$ in year $t_{U} + s$. It is known that for $\tau \in \mathbb{Z}^+$, the value of $\ln(m_{x,t_{U}+\tau})$ can be written in relation to $Y_{x,t}$ as:
Therefore, in predicting the value of \( \ln(m_{x,t+\tau}) \) we need the values of \( \ln(m_{x,t}) \) and \( Y_{x,t} \) for \( t = t_0, \ldots, t_0 + \tau \). The value of \( \ln(m_{x,t}) \) is known while the value of \( Y_{x,t} \) for \( t = t_0 + 1, \ldots, t_0 + \tau \) is unknown. Therefore, in order to predict the value of \( \ln(m_{x,t+\tau}) \) we must predict the value of \( Y_{x,t} \) for \( t = t_0, \ldots, t_0 + \tau \). In predicting the value of \( \hat{P}_{x,t+\tau} \) we will use the Bühlmann’s credibility model and estimate the parameter using a nonparametric approach. Therefore the estimate of \( \hat{P}_{x,t+\tau} \) is obtained by the equation below.

\[
\hat{P}_{x,t+\tau} = \hat{Y}_x + (1 - Z) \hat{\mu}
\]

where

\[
Z = (n - 1)/[(n - 1) + \hat{\delta}/\hat{\alpha}]
\]

and

\[
\hat{Y}_x = \frac{1}{n-1} \sum_{t=t_0+1}^{t_0+\tau} Y_{x,t}.
\]

**Figure 1.** \( \ln(m_{x,t}) \) and \( Y_{x,t} \) against \( t \) for Australian Males.
By using the nonparametric estimation, we found the estimator of $\hat{\mu}$, $\hat{\nu}$ and $\hat{\sigma}$ which are given below.

$$\hat{\mu} = \bar{y} = \frac{1}{m} \sum_{x=x_L}^{x_U} \bar{y}_x$$  \hspace{1cm} (14)

$$\hat{\nu} = \frac{1}{m(m-2)} \sum_{x=x_L}^{x_U} \sum_{t=t_L+1}^{t_U} (y_{x,t} - \bar{y}_x)^2$$  \hspace{1cm} (15)

$$\hat{\sigma} = \frac{1}{m-1} \sum_{x=x_L}^{x_U} \frac{(\bar{y}_x - \bar{y})^2}{m(m-1)(n-2)} - \frac{1}{m} \sum_{x=x_L}^{x_U} \sum_{t=t_L+1}^{t_U} (y_{x,t} - \bar{y}_x)^2$$  \hspace{1cm} (16)

There are 2 strategies to predict the value of $\bar{y}_{x,t_U+s}$ for $s \geq 2$. That 2 strategies are the expanding window strategy and the moving window strategy. The expanding window strategy is carried out by adding the predicted value to the data to produce predictive values in the following year while the moving window strategy is done by adding the predictive value to the data and discarding the oldest data to produce the predicted value in the following year. By using one of these strategies, the data used to generate the predicted value $\bar{P}_{x,t_U+s}$ will be different for each different $s$. Therefore, we need a general form of the predicted value $\bar{P}_{x,t_U+s}$. Following is the general form of the predicted value of $\bar{P}_{x,t_U+s}$.

$$\bar{P}_{x,t_U+s} = Z(t_U + s), \bar{P}_x(t_U + s) + [1 - Z(t_U + s)] \bar{P}(t_U + s)$$  \hspace{1cm} (17)

2.2.1. Expanding Window Strategy. First, the expanding window (EW) strategy is used to predict the value of $\bar{y}_{x,t_U+2}$. The expanding window strategy is done by adding the predicted value of $\bar{P}_{x,t_U+1}$ to the data which is used to predict $\bar{P}_{x,t_U+1}$, which is $\{y_{x,t_U+1}, y_{x,t_U}, y_{x,t_U+1}\}$. By doing that, we get new data, that is $\{y_{x,t_U+1}, y_{x,t_U}, \bar{P}_x(t_U + 2)\}$. Therefore, the predictive value of $\bar{y}_{x,t_U+2}$ is obtained using equation 17 for $s = 2$.

In finding the predictive value of $\bar{P}_{x,t_U+s}$ where $s > 2$, do the same procedure to get the general form of the formula for $\bar{P}_x(t_U + s), Z(t_U + s), \bar{P}(t_U + s)$ for $s = 2, 3, \ldots, \tau$ under the expanding window strategy, which are:

$$\bar{P}_{x}(t_U + s) = \frac{1}{n + s - 2} \sum_{t=t_U+1}^{t_U+s-1} Y_{x,t} + \sum_{t=x_U+1}^{t_U+s-1} \bar{P}_{x,t}$$  \hspace{1cm} (18)

$$\bar{P}(t_U + s) = \bar{\mu}(t_U + s) = \frac{1}{m} \sum_{x=x_L}^{x_U} \bar{P}_x(t_U + s)$$  \hspace{1cm} (19)

and

$$Z(t_U + s) = \frac{n + s - 2}{(n + s - 2 + \frac{\bar{\nu}}{\bar{\sigma}})}$$  \hspace{1cm} (20)
By using equation 17, we will get the value of $\hat{Y}_{x,t_U+s}$ for $s = 2, 3, \ldots, \tau$. The values of $\hat{v}$ and $\hat{a}$ on $Z(t_U + s)$ are same as the values of $\hat{v}$ and $\hat{a}$ on $Z(t_U + 1)$ given in equation 15 and equation 16, respectively. Therefore, $Z(t_U + s)$ in equation 20 is an increasing function of $s$ under the EW strategy.

2.2.2. Moving window strategy. First, the moving window (MW) strategy is used to predict the value of $\bar{Y}_{x,t_U+2}$. The moving window strategy is done by adding the predicted value of $\bar{Y}_{x,t_U+1}$ to the data which is used to predict $\bar{Y}_{x,t_U+1}$, which is $\{Y_{x,t_U+2}, \ldots, Y_{x(t_U+1)}\}$ and removes the oldest data, in this case $Y_{x,t_U+1}$. By doing that, we get new data, that is $\{Y_{x,t_U+2}, \ldots, Y_{x(t_U+1)}, \bar{Y}_{x(t_U+1)}\}$. By applying the Bühlmann’s credibility model for the new data, we will get that $\bar{Y}_{s(t_U + 2)} = \bar{Y}(t_U + 2) = \bar{Y}(t_U + 2) = \frac{1}{m} \sum_{x=L_x}^{x_U} \bar{Y}_x(t_U + 2)$, and $Z(t_U + 2) = \frac{(n-1)}{(n-1 + \bar{v}/\bar{a})}$. Therefore, the predictive value of $\bar{Y}_{x(t_U+1)}$ is obtained using equation 17 for $s = 2$. In finding the predictive value of $\bar{Y}_{x(t_U+s)}$ where $s > 2$, do the same procedure to get the general form of the formula for $\bar{Y}_s(t_U + s)$, $Z(t_U + s)$, and $\bar{Y}(t_U + s)$ for $s = 2, 3, \ldots, \tau$ under the moving window strategy, which are:

$$\bar{Y}_s(t_U + s) = \frac{1}{n-1} \sum_{t=t_L+s}^{t_U+s-1} \bar{Y}_x(t)$$

(21)

$$\bar{Y}(t_U + s) = \bar{Y}(t_U + s) = \frac{1}{m} \sum_{x=x_L}^{x_U} \bar{Y}_x(t_U + s)$$

(22)

and

$$Z(t_U + s) = \frac{n-1}{(n-1 + \bar{v}/\bar{a})}$$

(23)

By using equation 17, we will get the value of $\bar{Y}_{x(t_U+s)}$ for $s = 2, 3, \ldots, \tau$. The values of $\bar{v}$ and $\bar{a}$ on $Z(t_U + s)$ are same as the values of $\bar{v}$ and $\bar{a}$ on $Z(t_U + 1)$ given in equation 15 and equation 16, respectively. Therefore, $Z(t_U + s)$ in equation 23 is an increasing function of $s$ under the EW strategy.

3. Results and discussion

In practice, it is often assumed that $\mu_{x,t}$ is constant within each integer age $x$ and year $t$, that is $\mu_{x+r,t+s} = \mu_{x,t}$ for $r, s \in [0, 1)$ where $x,t \in \mathbb{Z}$. Based on these assumptions, it was found that $q_{x,t} = 1 - \exp[-\exp(ln(m_{x,t})]]$. This equation is used to convert the predicted value of $ln(m_{x,t})$ under the Bühlmann credibility approach and the Lee-Carter model into a mortality rate $\bar{q}_{x,t}$.

3.1. Measures of the forecasting error

In this section, we will discuss more about measures of the forecasting error that will be used in this paper. The measure of forecasting error that are used are AAMAPE and reduction ratio. In order to understand about AAMAPE and reduction ratio, we must first understand about MAPE and AMAPE. In calculating the forecasting error it is assumed that $x_L$ is the age lower bound of the data that we use to predict, $x_U$ is the age upper bound of the data that we use to predict, $t_L$ is the year lower bound of the data that we use to predict, $t_U$ is the year upper bound of the data that we use to predict, and $T_2$ is the upper bound of our forecasting period. The formulas of MAPE and AMAPE are given below.

$$MAPE_{x,t_U+t} = \left| \frac{\hat{q}_{x,t_U+t} - q_{x,t_U+t}}{q_{x,t_U+t}} \right|$$

(24)
Based on equation 25, it can be seen that $AMAPE_{[t_0,T_2]}^{[t_0,T_2]}$ is the average of $MAPE_{x,t+1}^{[t_0,T_2]}$ throughout the range of prediction years $[t_0 + 1, T_2]$ and the age span $[x_l, x_u]$. After having an understanding about MAPE and AMAPE, we will define the formula of AAMAPE and reduction ratio below.

\[
AMAPE_{[t_0,T_2]}^{[t_0,T_2]} = \frac{1}{(T_2 - t_0) \times (x_u - x_l + 1)} \sum_{t=t_0}^{T_2} \sum_{x=x_l}^{x_u} MAPE_{x,t+1}^{[t_0,T_2]} \tag{25}
\]

\[
AAMAPE_{[t_0+1,T_2]}^{[t_0+1,T_2]} = \frac{1}{(T_2 - 4 - T_1 + 1)} \sum_{t=T_1}^{t_0-4} AMAPE_{t+1,T_2}^{[t_0+1,T_2]} \tag{26}
\]

\[
RR(AAMAPE)_{S,LC}^{[t_0+1,T_2]} = 1 - \frac{AAMAPE_{S}^{[t_0+1,T_2]}}{AAMAPE_{LC}^{[t_0+1,T_2]}} \tag{27}
\]

where S = EW or MW.

### 3.2. Comparisons of the forecasting error

This section contains a discussion about the comparisons of the forecasting error between the Bühlmann credibility approach and the Lee-Carter model. In this paper, we are using mortality data for Australia in both sexes with age span [21,85] and year span [1956,2015] to compare the forecasting error. First, we calculate $AAMAPE_{[t_0+1,2015]}$ and reduction ratios for age span [21,85] by using mortality data with age span [21,85] and year span [1956, $t_U$] for $t_U = 1985, 1995, 2005$ to compare the forecasting error of the Bühlmann credibility approach between different year spans.

From the table 1 it was found that:

- When $t_U = 2005$, which is a prediction period of 10 years, the average value of $AAMAPE_{[t_0+1,2015]}$ for both sexes is 12.62 % for the Lee-Carter model, 9.06 % for the EW strategy, and 9.19 % for the MW strategy. The average RR value of the Lee-Carter model of both sexes was 26.66 % for the EW strategy and 25.54 % for the MW strategy.

- When $t_U = 1995$, which is a prediction period of 20 years, the average value of $AAMAPE_{[t_0+1,2015]}$ of both sexes is 17.10 % for the Lee-Carter model, 12.73 % for the EW strategy, and 12.51 % for the MW strategy. The average RR value for the Lee-Carter model of both sexes was 25.23 % for the EW strategy and 26.48 % for the MW strategy.

- When $t_U = 1985$, which is a prediction period of 30 years, the average value of $AAMAPE_{[t_0+1,2015]}$ of both sexes is 22.60 % for the Lee-Carter model, 17.14 % for the EW strategy, and 16.58 % for the MW strategy. The average RR value for the Lee-Carter model of both sexes was 24.30 % for the EW strategy and 26.19 % for the MW strategy.

Based on the information obtained from table 1, it can be seen that the average RR against LC of both sexes for the EW strategy and the MW strategy at $t_U = 2005, 1995, 1985$ is positive and large. Therefore, the Bühlmann credibility approach is better than Lee-Carter model in predicting mortality rates for fitting age span [21,85]. Second, we calculate $AAMAPE_{[t_0+1,2015]}$ and Reduction Ratios for higher age span [56,85] by using mortality data with age span [21,85] and year span [1956, $t_U$] for $t_U = 1985, 1995, 2005$ to compare the forecasting error of the Bühlmann credibility approach between different length of year span.

From the table 2 it was found that:

- When $t_U = 2005$, which is a prediction period of 10 years, the average value of $AAMAPE_{[t_0+1,2015]}$ of both sexes is 6.64 % for the Lee-Carter model, 6.00 % for the EW strategy, and 6.29 % for the MW strategy. The average RR value for the Lee-Carter model of both sexes was 8.36 % for the EW strategy and 3.93 % for the MW strategy.
When \( t_U = 1995 \), which is a prediction period of 20 years, the average value of \( AAMAPE_{[t_U+1,2015]} \) of both sexes is 10.10 % for the Lee-Carter model, 8.96 % for the EW strategy, and 7.71 % for the MW strategy. The average RR value for the Lee-Carter model of both sexes is 11.08 % for the EW strategy and 22.94 % for the MW strategy.

When \( t_U = 1985 \), which is a prediction period of 30 years, the average value of \( AAMAPE_{[t_U+1,2015]} \) of both sexes is 18.82 % for the Lee-Carter model, 18.83 % for the EW strategy, and 17.68 % for the MW strategy. The average RR value for the Lee-Carter model of both sexes is 0.41 % for the EW strategy and 4.45 % for the MW strategy.

Based on the information obtained from table 2, it can be seen that the average RR to LC of both sexes for the EW strategy and the MW strategy at \( t_U = 2005, 1995, 1985 \) is positive. Therefore, Bühlmann’s credibility approach is superior to the Lee-Carter model in predicting mortality rates for fitting age span [56,85].

### Table 1. \( AAMAPE_{[t_U+1,2015]} \) and reduction ratios for age span [21,85]

| \( t_U \) | Gender | LC  | EW  | MW  | RR against LC (%) |
|---------|--------|-----|-----|-----|-------------------|
|         |        | AAMAPE for model (%) |        | EMAPE for model (%) |        |
| 2005    | Male   | 13.93 | 8.15 | 8.17 | 41.49  | 41.33 |
|         | Female | 11.31 | 9.97 | 10.21 | 11.82  | 9.75  |
|         | Average| 12.62 | 9.06 | 9.19  | 26.66  | 25.54 |
| 1995    | Male   | 19.93 | 14.50| 14.18 | 27.23  | 28.88 |
|         | Female | 14.27 | 10.96| 10.83 | 23.23  | 24.09 |
|         | Average| 17.10 | 12.73| 12.51 | 25.23  | 26.48 |
| 1985    | Male   | 25.23 | 19.46| 17.63 | 22.85  | 30.11 |
|         | Female | 19.97 | 14.83| 15.52 | 25.74  | 22.27 |
|         | Average| 22.6  | 17.14| 16.58 | 24.3   | 26.19 |

### Table 2. \( AAMAPE_{[t_U+1,2015]} \) and reduction ratios for age span [56,85]

| \( t_U \) | Gender | LC  | EW  | MW  | RR against LC (%) |
|---------|--------|-----|-----|-----|-------------------|
|         |        | AAMAPE for model (%) |        | EMAPE for model (%) |        |
| 2005    | Male   | 7.25 | 5.64| 5.92 | 22.13  | 18.24 |
|         | Female | 6.03 | 6.36| 6.66 | -5.41  | -10.38 |
|         | Average| 6.64 | 6.00| 6.29 | 8.36   | 3.93  |
| 1995    | Male   | 12.13| 10.66| 8.90 | 12.06  | 26.58 |
|         | Female | 8.06 | 7.25 | 6.51 | 10.10  | 19.29 |
|         | Average| 10.10| 8.96 | 7.71 | 11.08  | 22.94 |
| 1985    | Male   | 22.99| 23.35| 20.28| -1.57  | 11.81 |
|         | Female | 14.65| 14.30| 15.08| 2.38   | -2.91 |
|         | Average| 18.82| 18.83| 17.68| 0.41   | 4.45  |
4. Conclusion
The Bühlmann credibility approach in modeling mortality is done by defining a random variable $Y_{x,t} = \ln(m_{x,t}) - \ln(m_{x,t-1})$, which is the rate of change of $\ln(m_{x,t-1})$ for 1 year, as a random variable that will be predicted using Bühlmann's credibility approach. The Bühlmann credibility approach can be done through 2 strategies: the expanding window and moving window strategies. The expanding window strategy is carried out by adding the predicted value to the data to produce predictive values in the following year while the moving window strategy is done by adding the predictive value to the data and discarding the oldest data to produce the predicted value in the following year. Mortality data for Australia in both sexes shows that the Bühlmann credibility approach through an expanding window and moving window strategy results in a better mortality prediction compared to the Lee-Carter model based on the positive average reduction ratio of both sexes, which implies that the forecasting error is lower for the Bühlmann credibility approach.

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