Probing $CP$ Violation via Top Polarization at NLC†

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ABSTRACT

Possible $CP$-violation in top-quark couplings is discussed. It is shown that the lepton-energy distributions in $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^+\ell^- X / \ell^\pm X$ at next linear colliders (NLC) could give us useful information for this study. The statistical significance of $CP$-violating-parameter determination is estimated.

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1. Introduction

By the discovery of the top quark, the fermion spectrum required by the standard EW theory (SM) has been completed. Still, it is to be seen if the third generation is a copy of the first and second ones or any new interactions exist in top-quark couplings. Studying $CP$ violation via top-quark polarization is an interesting approach to this problem. This is because (i) the $CP$ violation in the top-quark couplings induced within the SM is far negligible and (ii) a lot of information on the top quark is to be transferred to the secondary leptons without getting obscured by the hadronization effects thanks to huge $m_t$.

Here we would like to show our recent work on this topic \[1\]. First we describe why $t\bar{t}$ productions are useful for our study in a little more detail. Next we show how the signal of $CP$ non-conservation that might occur there would be affected by another possible $CP$-violating interaction in the decay process. We then introduce a new asymmetry to catch their combined signal effectively, and discuss the expected statistical uncertainties in the corresponding-parameter determination.

2. Polarized Top Production and $CP$ Violation

Since $t\bar{t}$ pairs are produced mainly through the vector-boson exchange, the handedness of $t$ and $\bar{t}$ must be the same. Consequently, the helicities of $t\bar{t}$ would be $(+−)$ or $(-+)$. If the top mass were much smaller than $\sqrt{s}$. However, since the observed $m_t$ is by no means negligible, we will also face copious production of $(++)$ and $(−−)$ states. For example, $\sigma_{tot}(e^+e^- \rightarrow t\bar{t})$ is estimated to be 0.60 pb for $\sqrt{s} = 500$ GeV (and $m_t = 180$ GeV) within the SM, in which $N(−+) : N(++) : N(−−) : N(++)$ is 4.8 : 3.4 : 0.9 : 0.9, where $N(\cdot \cdot \cdot)$ denotes the number of the $t\bar{t}$ pairs with indicated helicities.

We can take advantage of this fact to explore $CP$ properties of the $t\bar{t}$ state. That is, $|−−\rangle$ and $|++\rangle$ are transformed into each other by $CP$ operation, and consequently the difference between $N(−−)$ and $N(++)$ could be a useful measure of $CP$ violation. Although what we can observe in experiments are not
the top quarks but products of their subsequent decays, the energy spectrum of $\ell^+$ and $\ell^-$ in $e^+e^- \rightarrow t\bar{t} \rightarrow \ell^+\ell^-X/\ell^\pm X$ can be a good measure of $N(--) - N(++)$, as we will see.

The leptonic energy spectrum has been studied in the existing literature\textsuperscript{1,2}. However, CP-violating interactions were assumed only in the $t\bar{t}\gamma/Z$ vertices and the standard-model vertex was used for the $t \rightarrow bW$ decay in those articles. In order to perform a consistent analysis of the top-quark couplings, we computed the spectrum assuming that both the $t\bar{t}\gamma/Z$ vertices and the $tbW$ vertex include non-standard CP-violating form factors.

3. Effects of Non-Standard Top Decay

As already mentioned, $\delta \equiv [N(--) - N(++)]/N(all)$ is a good measure of CP violation in $t\bar{t}$ productions. If there is no CP violation in the $tbW$ vertex, the energy-spectrum asymmetry $a(x)$ defined as

$$a(x) \equiv \frac{d\sigma^- / dx - d\sigma^+ / dx}{d\sigma^- / dx + d\sigma^+ / dx},$$

is known to be proportional to $\delta$\textsuperscript{2}, where $d\sigma^\pm / dx$ are the $\ell^\pm$ distributions in the reduced lepton energy $x \equiv 2E\sqrt{(1 - \beta)/(1 + \beta)/m_t}$ with $E$ being the energy of $\ell^\pm$ in the $e^+e^-$ c.m. system and $\beta \equiv \sqrt{1 - 4m_t^2/s}$. In such a case, $a(x)$ would serve as a useful observable to measure CP violation.

However, if CP non-conservation occurs also in the $tbW$ vertex, it becomes

$$a(x) = -2(\delta/\beta) g(x) + \text{Re}(f^R_2 - f^L_2)[\delta f(x) + \eta \delta g(x)]$$

$$+ \text{Re}(\bar{f}_2^L - f_2^R)[\delta g(x) + \eta \delta f(x)]$$

$$\times \left[ f(x) + \eta g(x) \right],$$

where the functions $f(x)$, $g(x)$, $\delta f(x)$ and $\delta g(x)$, and the parameter $\eta$ are defined in\textsuperscript{3}. The top couplings with $\gamma/Z$ and $W$ we assumed for computing the above $a(x)$ are

$$\Gamma^\mu = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu (A_v - B_v\gamma_5) + \frac{(p_t - p_i)^\mu}{2m_t} (C_v - D_v\gamma_5) \right] v(p_t),$$

\textsuperscript{51}We did not aim to give a complete reference list owing to limited space. See the lists of our articles\textsuperscript{1,2}. 
\[ \Gamma^\mu = -\frac{g}{\sqrt{2}} V_{tb} \bar{u}(p_b) \left[ \gamma^\mu (f_1^L P_L + f_1^R P_R) - \frac{i\sigma^{\mu\nu} k_\nu}{M_W} (f_2^L P_L + f_2^R P_R) \right] u(p_t), \quad (3) \]

\[ \bar{\Gamma}^\mu = -\frac{g}{\sqrt{2}} V_{tb}^* \bar{v}(p_t) \left[ \gamma^\mu (\bar{f}_2^L P_L + \bar{f}_2^R P_R) - \frac{i\sigma^{\mu\nu} k_\nu}{M_W} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] v(p_b), \quad (4) \]

where \( g \) is the SU(2) gauge-coupling constant, \( v = \gamma/Z \), \( P_L/R \equiv (1 \mp \gamma^5)/2 \), \( V_{tb} \) is the \((tb)\) element of the Kobayashi-Maskawa matrix and \( k \) is \( W \)'s momentum.

Measuring asymmetries like \( a(x) \) under such a circumstance is a challenging task since it is differential and therefore the expected statistics cannot be high. That is why we looked for another more effective quantity.

4. CP-Violating Asymmetry

We introduce the following CP-violating asymmetry:

\[ A_{\ell\ell} \equiv \left( \iint_{x<\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} - \iint_{x>\bar{x}} dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}} \right) / \iint dx d\bar{x} \frac{d^2\sigma}{dx d\bar{x}}, \quad (5) \]

where \( x \) and \( \bar{x} \) are the reduced energies of \( \ell^+ \) and \( \ell^- \) respectively.

For the SM parameters \( \sin^2 \theta_W = 0.2325 \), \( M_W = 80.26 \) GeV, \( M_Z = 91.1884 \) GeV, \( \Gamma_Z = 2.4963 \) GeV and \( m_t = 180 \) GeV, it becomes

\[
A_{\ell\ell} = 0.3089 \text{ Re}(f_2^R - \bar{f}_2^L) + 0.3638 \text{ Re}(D_\gamma) + 0.0609 \text{ Re}(D_Z)
\]

\[
= 0.3089 \text{ Re}(f_2^R - \bar{f}_2^L) - 0.3441 \xi,
\]

where \( \xi \) is given by \( \xi = -\delta/\beta \) and therefore characterizes the CP violation in the production process. For \( \text{Re}(f_2^R) = -\text{Re}(\bar{f}_2^L) = \text{Re}(D_\gamma) = \text{Re}(D_Z) = 0.2 \), e.g., we have \( A_{\ell\ell} = 0.2085 \). Its statistical error for \( N_{\ell\ell} \) events is estimated thereby to be

\[
\Delta A_{\ell\ell} = \sqrt{(1 - A_{\ell\ell}^2)/N_{\ell\ell}} = 0.9780/\sqrt{N_{\ell\ell}}.
\]

Since \( \sigma_{\ell\ell\rightarrow tt} = 0.60 \) pb for \( \sqrt{s} = 500 \) GeV, the expected number of events is \( N_{\ell\ell} = 600 \epsilon_{\ell\ell} L B_{\ell\ell}^2 \), where \( \epsilon_{\ell\ell} \) stands for the \( \ell^+\ell^- \) tagging efficiency \( (= \epsilon_\ell^2 ; \epsilon_\ell \) is the single-lepton-detection efficiency), \( L \) is the integrated luminosity in fb\(^{-1}\) unit, and \( B_\ell (\simeq 0.22) \) is the leptonic branching ratio for \( t \). Consequently we
obtain $\Delta A_{\ell\ell} = 0.1815/\sqrt{\epsilon_{\ell\ell} L}$, and thereby we are able to compute the statistical significance of the asymmetry observation $N_{SD} = |A_{\ell\ell}|/\Delta A_{\ell\ell}$. For $L = 50$ fb$^{-1}$ and $\epsilon_{\ell\ell} = 0.5$, for example, we get $N_{SD} = 5.7$. This means we can confirm $A_{\ell\ell}$ to be non-zero at 5.7$\sigma$ level concerning the statistical uncertainty.

5. Precision of Parameter Measurements

By using $A_{\ell\ell}$, we will be able to observe a combined signal of CP violation in the productions and decays. In order to study the new interactions in more detail, however, it is indispensable to separate the parameters in the production and the decay, i.e., $\xi$ and $\text{Re}(f_R^2 - \bar{f}_L^2)$. For this purpose we applied the optimal procedure of Ref. [3] both to the $\ell^+\ell^-$ distribution and the $\ell^\pm$ distributions.

Here we show only the numerical results. Through the double distribution, we can determine the parameters with the following statistical uncertainties:

$$\Delta\text{Re}(f_R^2 - \bar{f}_L^2) = 8.3824/\sqrt{N_{\ell\ell}}, \quad \Delta\xi = 7.1651/\sqrt{N_{\ell\ell}}.$$  \hspace{1cm} (7)

On the other hand, we get $\Delta\text{Re}(f_R^2) = 7.1136/\sqrt{N_\ell}$ and $\Delta\xi = 12.3285/\sqrt{N_\ell}$ from the $\ell^+$ distribution, and analogous for $\Delta\text{Re}(\bar{f}_L^2)$ and $\Delta\xi$ from the $\ell^-$ distribution. Since these two distributions are statistically independent, we can combine them as

$$\Delta\text{Re}(f_R^2 - \bar{f}_L^2) = 10.0601/\sqrt{N_\ell}, \quad \Delta\xi = 8.7176/\sqrt{N_\ell}.$$ \hspace{1cm} (8)

It is premature to conclude from Eqs.(7) and (8) that we get a better precision in the analysis with the double distribution. As it could be observed in the numerators in Eqs.(7, 8), we lose some information when integrating the double distribution on one variable. However, the size of the expected uncertainty depends also on the number of events. That is, $N_{\ell\ell}$ is suppressed by the extra factor $\epsilon_{\ell\ell}B_{\ell\ell}$ comparing to $N_\ell$. This suppression is crucial even if we could achieve $\epsilon_{\ell\ell} = 1$. For $N$ pairs of $t\bar{t}$ and $\epsilon_{\ell\ell} = 1$ we obtain

$$\Delta\text{Re}(f_R^2 - \bar{f}_L^2) = 38.1018/\sqrt{N}, \quad \Delta\xi = 32.5686/\sqrt{N}.$$
from the double distribution, while

$$\Delta \text{Re}(f_2^R - \bar{f}_2^L) = 21.4484/\sqrt{N}, \quad \Delta \xi = 18.5859/\sqrt{N}$$

from the single distribution. Therefore we may say that the single-lepton distribution analysis is more advantageous for measuring the parameters individually.

6. Summary

Next-generation linear colliders of $e^+e^-$ will provide a cleanest environment for studying top-quark interactions. There, we shall be able to perform detailed tests of the top-quark couplings to the vector bosons and either confirm the SM simple generation-repetition pattern or discover some non-standard interactions.

What we discussed here are the non-standard $CP$-violating interactions in the $t\bar{t}$ productions and their subsequent decays. If the top-quark decay was described by the SM interactions, then we would have a useful compact formula for a measurement of $CP$ violation in the $t\bar{t}\gamma/Z$ vertices via the final-lepton-energy asymmetry \cite{footnote1}. However, in general, $CP$ violation may also enter through the top-decay process at the same strength as it does for the production. Therefore, we have assumed the most general $CP$-violating interactions both in the production and in the decay vertices in order to perform a consistent analysis.

We introduced a new asymmetry $A_{\ell\ell}$ in Eq.(5), which was shown to work quite effectively. Then, applying the optimal procedure \cite{footnote2}, we studied the statistical significances of $CP$-violation-parameter determination in analyses with the double- and single-lepton energy distributions. Taking into account the size of the leptonic branching ratio of the top quark and its detection efficiency, we conclude that the use of the single-lepton distribution is more advantageous to determine each $CP$-violation parameter separately.

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