A New Gravitational Wave Signature of Low-T/|W| Instability in Rapidly Rotating Stellar Core Collapse

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ABSTRACT

We present results from a full general relativistic three-dimensional hydrodynamics simulation of rapidly rotating core-collapse of a 70 M☉ star with three-flavor spectral neutrino transport. We find a strong gravitational wave (GW) emission that originates from the growth of the one- and two-armed spiral waves extending from the nascent proto-neutron star (PNS). The GW spectrogram shows several unique features that are produced by the non-axisymmetric instabilities. After bounce, the spectrogram first shows a transient quasi-periodic time modulation at ∼450 Hz. In the second active phase, it again shows the quasi-periodic modulation but with the peak frequency increasing with time, which continues until the final simulation time. From our detailed analysis, such features can be well explained by a combination of the so-called low-T/|W| instability and the PNS core contraction.

Key words: supernovae: general — stars: neutron — hydrodynamics — gravitational waves

1 INTRODUCTION

The LIGO and Virgo collaborations have opened a new window on the gravitational wave (GW) astronomy. The detailed analyses of the GW signal from compact binary coalescences have constrained not only the binary parameters but also physical quantities related to the neutron star (NS) physics such as the radii and nuclear equation of state (EOS) (Abbott et al. 2019).

Some of the binary black holes (BHs) detected by GWs have their masses of ~30-50 M☉, indicating their formation in low metallicity environments (e.g., Abbott et al. 2016). Recently, an astronomical observation of Liu et al. (2019) reported the detection of a ~70 M☉ BH in a binary system ("LB-1") with its companion being an ~8 M☉ solar metallicity B star. These findings motivate not only theoretical studies on the formation channels of these massive BHs (e.g., Belczynski et al. 2019, see, however, Abdul-Masih et al. 2019; Eldridge et al. 2019; Belczynski et al. 2019; Eldridge et al. 2019) , but also numerical simulation studies clarifying the hydrodynamics processes of the massive BH formation (e.g., Chan et al. 2018; Kuroda et al. 2018).

Next to the binary black hole coalescence, core collapse (CC) of massive stars and the subsequent explosions have been considered one of the most promising GW sources. Extensive studies have revealed that the candidate ingredients of GW emission from stellar CC include prompt convection, neutrino-driven convection, proto-neutron star (PNS) convection, g-mode oscillation of the PNS, the standing accretion shock instability, rotational flattening of the bouncing core, and non-axisymmetric rotational instabilities (for reviews, see Fryer & New 2011; Kotake & Kuroda 2016). Among these multiple physical elements, progenitor rotation has been long considered the primary ingredient that leads to the sizable GWs (Müller 1982).

In three-dimensional (3D) simulations of rapidly rotating CC, the growth of non-axisymmetric rotational instabilities that develop later in the postbounce phase has been reported (Ott et al. 2005, 2007; Scheidegger et al. 2008b, 2010; Kuroda et al. 2014; Takiwaki et al. 2016; Takiwaki & Kotake 2018). These studies have shown that the so-called low-T/|W| instability (Watts et al. 2005; Saijo & Yoshida 2006), where T/|W| is the ratio of the rotational to gravitational potential energy, plays a key role for the growth of the non-axisymmetric flows that extend outward in the vicinity of the PNS. Since the stability criterion depends on the compactness of the PNS, it is important to include general relativity (GR) and self-consistent neutrino transport in 3D rapidly rotating CC simulations. This was already pointed out by Ott et al. (2007, 2012); Kuroda et al. (2014) with different levels of sophistication for neutrino treatment. So far 3D simulations of rapidly rotating collapse that followed a long-term postbounce evolution with spectral neu-
trino transport have been reported in Takiwaki et al. (2016) and Obergaulinger & Aloy (2019), but with the assumption of Newtonian and post-Newtonian gravity, respectively.

In this Letter, we present a first 3D full-GR hydrodynamics simulation of rapidly rotating CC of a 70 M⊙ star with spectral neutrino transport, where we follow the longest postbounce evolution (~ 270 ms after bounce) in this context. Our results confirm that the growth of the non-axisymmetric instability leads to long-lasting quasi-periodic GW emission. As a new intriguing result, the characteristic GW frequency increases with time. Such a ramp-up feature of the GW frequency may look similar to that from the g/mode oscillation of the non-rotating PNS (Murphy et al. 2009; Müller et al. 2013; Vartanyan et al. 2019). However, in rapidly rotating CC, we show that the ramp-up feature (with much bigger GW amplitudes) is produced predominantly because of the growth of the low-T/|W| instability and the PNS contraction.

2 NUMERICAL METHOD AND SETUP

The numerical schemes of our 3D-GR simulation are essentially the same as those in Kuroda et al. (2016), except the update where we adopt the directionally unsplit predictor-corrector scheme instead of the use of Strang splitting. Based on the BSSN formalism, we solve the metric evolution in fourth-order accuracy by a finite-difference scheme in space and a Runge-Kutta method in time. We consider three-flavor of neutrinos (ν e, ν μ, ν τ) with ν e denoting heavy-lepton neutrinos. Employing an M1 analytical closure scheme (Shibata et al. 2011), we solve spectral neutrino transport of the radiation energy and momentum including all the gravitational red- and Doppler-shift terms using 12 energy bins from 1 to 300 MeV. Regarding neutrino opacities, the standard weak interaction set in Bruenn (1985) plus nucleon-nucleon Bremsstrahlung is taken into account (see Kuroda et al. (2016) for more detail).

We use a 70 M⊙ zero-metallicity star of Takahashi et al. (2014). At the precollapse phase, the mass of the central iron core is ~ 4.6 M⊙ and the enclosed mass up to the helium layer is ~ 34 M⊙. The central density profile of this progenitor is similar to that of 75 M⊙ ultra metal-poor progenitor model of Woosley et al. (2002) that is used as a collapsar model in Ott et al. (2011). The central angular frequency of the original progenitor model is ~ 0.03 rad s⁻¹, by which one can roughly estimate the T/|W| at bounce as ~ 10⁻⁵ due to the angular momentum conservation. For such slow rotation, it is very unlikely to produce non-axisymmetric rotational instabilities, and the GW signature would not be significantly deviated from those from a non-rotating model. Therefore, to explore the impact of rotation, we assume a rapidly rotating iron core with the central angular velocity being 2 rad s⁻¹ (e.g., as taken in Scheidegger et al. 2008a; Ott et al. 2011). For reference, the initial rotational energy and T/|W| are ~ 1.3 × 10⁵⁵ ergs and ~ 2 × 10⁻⁷ for the original progenitor model and are ~ 2.6 × 10⁶⁹ ergs and ~ 3 × 10⁻³ for the rapidly rotating model explored in this work, respectively.

We use the EOS by Lattimer & Swesty (1991) with a bulk incompressibility modulus of K = 220 MeV (LS220). The 3D computational domain is a cubic box with 15,000 km width, and nested boxes with nine refinement levels are embedded in the Cartesian coordinates. Each box contains 64³ cells and the minimum grid size near the origin is Δx = 458 m. The PNS core surface (~ 10 km) and stalled shock (~ 110-300 km) are resolved by Δx = 458 m and 7.3 km, respectively. In the numerical analysis below, we extract GWs with a standard quadrupole formula including GR corrections (Shibata & Sekiguchi 2003; Kuroda et al. 2014). The GW spectrograms are obtained using short-term Fourier transformation with a Hann window, whose width is set as 20 ms. t pb represents the time measured after core bounce.

3 RESULT

We begin with a brief description of the post-bounce dynamical evolution in our simulation. The central density reaches ~ 3.6×10¹⁵ g cm⁻³ at core bounce. After core bounce and the subsequent ring-down oscillation, the central density monotonically increases up to ~5×10¹⁴ g cm⁻³ at the end of the simulation (t pb = 270 ms). The bounce shock propagates outward to reach ~ 200 km at t pb ~ 70 ms. As we will see later, the low-T/|W| instability takes place at this time and temporally accelerates the shock. But once the instability ceases, the shock propagation decelerates and stagnates at ~ 300 km at t pb ~ 85 ms. Afterward the shock surface starts shrinking. The shrink stops at t pb ~ 100 ms, and later the average shock radius keeps ~ 200 km until the end of our simulation.

Fig. 1 shows the GW strain h (top panels) and the spectrograms of the characteristic strain (h har, bottom panels, see the definition of equation (44) in Kuroda et al. 2014) for a source distance of D = 10 kpc. For convenience, we denote h + (red solid line) and h × (blue dashed line) with subscripts I and II, corresponding to the (reference) viewing direction either along the rotational axis (the positive z-axis, right panels) or equatorial plane (the positive x-axis, left panels), respectively. After core bounce and the subsequent ring-down phase, the waveforms show a quasi-periodic time modulation around t pb ~ 70 ms. This can be seen as a clear excess in the spectrograms with the peak frequency of f ~ 450 Hz. The GW amplitude is more strongly emitted toward the pole than the equator. The waveforms show that this lasts only for the duration of ~ 20 ms followed by a quiescent phase until t pb ~ 110 ms, after which the quasi-periodic GW emission becomes active again.

In the second active phase, the GW emission does not subside quickly as seen in the first phase. The spectrograms clearly show that the peak GW frequency, either seen from the equator or pole, increases with time from f ~ 450 Hz at t pb ~ 120 ms to ~ 800 Hz at the end of our simulation time (t pb ~ 270 ms). Seen from the pole, the phase difference between h + and h × is π/2. Furthermore, the GW amplitudes seen from the pole are approximately two times bigger than those from the equator, i.e., h +/h × ~ 2h +/II and the cross mode of the GWs seen from the equator (h ×/I I) is significantly smaller than the plus mode (h ×/II). These GW features are consistent with previous 3D studies where the growth of non-axisymmetric instabilities were observed (Ott et al. 2005; Scheidegger et al. 2008b, 2010; Kuroda et al. 2014; Takiwaki & Kotake 2018). We note that the GW amplitudes are bigger than those from a 75 M⊙ model.
in Ott et al. (2011) with the same initial angular momentum, where the growth of the low-$T/|W|$ instability was not fully captured by enforcing the octant symmetry in their simulation.

Now we move on to investigate the emission mechanism of the quasi-periodic GW signal seen after $t_{pb} \sim 60$ ms in our model. In the top panels of Fig. 2, we show the snapshots of the normalized density deviation from the angle averaged value, $(\rho - \langle \rho \rangle)/\langle \rho \rangle$, where $\langle \rangle$ represents an angular average at a certain radius on the equatorial plane, at $t_{pb} = 70$ ms (left) and 140 ms (right). During the first phase of the quasi-periodic GW emission ($60 \lesssim t_{pb} \lesssim 80$ ms), this one-armed flow is actually kept visible, whereas the two-armed spiral arms develop in the second active phase ($t_{pb} \gtrsim 120$ ms) as seen in the top panels of Fig. 2. In both of the phases, $T/|W|$ reaches $\sim 5\%$. Indeed, this value is close to the onset condition of the low-$T/|W|$ instability as previously identified in the literature (Ott et al. 2005; Scheidegger et al. 2010; Takiwaki et al. 2016).

In order to clarify how the growth of the one- and two-armed spiral flows lead to the quasi-periodic GW emission, we perform an azimuthal Fourier decomposition of the density on the equatorial plane (at a radius of $\sigma = (x^2 + y^2)^{1/2} \approx 100$ km) as $C_m(t) = \int_0^{2\pi} \rho e^{i m \phi} d\phi/\int_0^{2\pi} \rho d\phi$, and obtain the spectrograms of $C_m(t, f)$ (using the real part). In the bottom panels of Fig. 2, we show $C_m(t, f)$ for $m = 1$ (left panel) and $m = 2$ (right panel), respectively. In the first active phase ($60 \lesssim t_{pb} \lesssim 80$ ms), one can see that both the $m = 1$ and 2 modes grow, but the $m = 1$ mode amplitude is bigger than the $m = 2$ mode. In the second active phase ($t_{pb} \gtrsim 110$ ms), the dominant mode is $m = 2$ as clearly seen from the bottom right panel of Fig. 2. Note that likewise the GW spectrogram (bottom right panel of Fig. 1), the peak frequency of $C_m(t, f)$ (bottom right panel of Fig. 2) also increases with time.

One can determine the eigenfrequency of the spiral-wave pattern (namely, the $m$-th mode of $f_{\text{mode}, m}$) from the peak frequency of $C_m(t, f)$. Note that the frequency with respect to the pattern speed of the $m$-th mode is determined by $f_{\text{pat}, m} = f_{\text{mode}, m}/m$ (Watts et al. 2005). So $f_{\text{mode}, 1}$ and $f_{\text{pat}, 1}$ are identical for $m = 1$. In the first active phase ($60 \lesssim t_{pb} \lesssim 80$ ms), the $m = 1$ eigenfrequency is $f_{\text{mode}, 1} = f_{\text{pat}, 1} \sim 200$ Hz (bottom left panel of Fig. 2). Similarly, in the second active phase, the $m = 2$ eigenfrequency at $f_{\text{mode}, 2} \sim 200$ Hz (bottom right panel of Fig. 2) is, for instance, translated into $f_{\text{pat}, 2} \sim 200$ Hz.

In both the first and second active phases, it is a natural
consequence that the $m = 2$ mode frequency of $f_{\text{mode},2} = 2f_{\text{pole},2}$ (i.e., bar-mode deformation of the spiral flows) leads to the dominant quadrupole GW emission with the same frequency. In fact, one can see a nice match of the ramp-up frequency feature between the bar-mode amplitude ($C_2(t, f)$) of the spiral flows (bottom right panel of Fig. 2) and the GW spectrogram (seen as a red band from $\sim 400$ to $800$ Hz in the bottom right panel of Fig. 1).

Fig. 3 shows contribution of different spherical shells to the GW spectrogram (seen from the pole). One can see that the ramp-up signature is generated almost all in the layers between $10 \lesssim r \lesssim 100$ km, whereas the dominant contribution from each shell differs with time. Note that there is also a contribution from behind the shock ($100 < r < 200$ km). Our results clearly show that both the non-axisymmetric flows that develop in the vicinity of the PNS core surface ($r \sim 10$ km) and the spiral arms extending above coordinateately give rise to the ramp-up GW emission.

Watts et al. (2005) firstly suggested that a necessary condition for the low-$T/|W|$ instability is the existence of the corotation radius where the angular velocity is equal to the pattern speed of an unstable mode (Saijo & Yoshida 2006). Following this idea, we attempt to interpret how the ramp-up feature seen in the second active phase ($t_{\text{pb}} \gtrsim 110$ ms) is produced. In order to clarify how the development of the $m = 2$ unstable mode is related to the corotation radius, we show in Fig. 4 the time evolution of spatial profile of normalized amplitude of the density perturbation with $m = 2$ mode ($\bar{C}_2(t, r)$) and the corotation radius ($\bar{\sigma}_{\text{cor}}$). Note that there is a finite-width range ($\sim 10\%$ level) in estimating the pattern speed (corresponding to the vertical width of the green stripe in the bottom right panel of Fig. 2). Given this, the location of the corotation radius is also defined with a $10\%$ error bar as $(1.0 \pm 0.1)\bar{\sigma}_{\text{cor}}$, which is shown by the red hatched regions in Fig. 4. As one can see from Fig. 4 and Fig. 3, the region above the corotation radius ($20$ km $\lesssim r \lesssim 100$ km) contributes to the GW spectrogram more largely than the corotation radius ($10$ km $\lesssim r \lesssim 20$ km) does. One of the reasons is that the highly deformed region (the reddish region in Fig. 4), leading to stronger GW emission, locates above the corotation radius. Seen from the hatched region of Fig. 4, the corotation radius gradually shrinks from $\sim 22$ km ($t_{\text{pb}} \sim 110$ ms) to $\sim 16$ km at the final simulation time. This closely coincides with the PNS core contraction. As being dragged by the shrink of the corotation region, the inner edge of the highly deformed region with $m = 2$ mode also moves inward. This also supports the idea that the $m = 2$ PNS distortion may be generated via resonance at the corotation radius, leading to the formation of the two-armed spiral waves. The gradual recession of the corotation point leads to the spinning up the two-armed spiral waves (bottom right panel of Fig. 2), which is reconciled with the ramp-up GW feature as seen in the bottom right panel of Fig. 1.

In the end, we discuss detectability and observational

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**Figure 3.** Contributions from each spherical shell of radius $r$ to the GW spectrogram (seen from the pole) in a logarithmic scale of $h$. The range of the plotted layers is denoted in the upper right corner of each panel. As a reference, the bottom right panel shows the total GW spectrogram.

**Figure 4.** Color map of $m = 2$ mode amplitude for density variation $C_2$ as a function of time and cylindrical radius (see text for definition). The red hatched band indicates $(1.0 \pm 0.1)\bar{\sigma}_{\text{cor}}$, where $\bar{\sigma}_{\text{cor}}$ is the corotation radius.

**Figure 5.** Characteristic GW spectral amplitudes of our model seen along the pole (red lines) and along the equator (blue lines) and of the non-rotating model from Kuroda et al. (2018, thin gray line) measured up to $t_{\text{pb}} \lesssim 270$ ms as a source distance of $10$ kpc (solid lines) and $1$ Mpc (dotted lines) relative to the noise amplitudes of advanced LIGO (aLIGO; cyan), advanced VIRGO (AdV; green), KAGRA (magenta) from Abbott et al. (2018), Einstein Telescope (ET; orange; Hild et al. 2011), and Cosmic Explorer (CE; navy; Abbott et al. 2017). The detector noise amplitudes are indicated by dash-dotted lines.
rate of the GW signals. Fig. 5 shows the GW spectral amplitudes seen from the polar (red lines) and the equatorial (blue lines) observer at a distance of 10 kpc (solid lines) and 1 Mpc (dotted lines) relative to the sensitivity curves of the advanced LIGO, advanced VIRGO, and KAGRA (Abbott et al. 2018); and the third-generation GW detectors of Einstein Telescope (Hild et al. 2011) and Cosmic Explorer (Abbott et al. 2017). The peak GW spectral amplitudes of our model are much larger than the non-rotating one of Kuroda et al. (2018, thin gray line). In accordance with the spectrogram analysis of Fig. 1 and 2, the peaks of the GW spectra are located around \( f \lesssim 900 \) Hz, with the GW emission stronger toward the rotational axis (red line). These GW signals can be a target of LIGO, Virgo, and KAGRA for a Galactic event. But more interestingly, these signals, the peak frequency of which is close to the best sensitivity range (\( \lesssim 1 \) kHz) of ET and CE, could be detectable out to Mpc distance scale by the third-generation detectors.

To roughly estimate the observational rate of this kind of events, we make a bold assumption that a rapidly rotating massive star considered in this work would be associated with long gamma-ray bursts (lGRBs). About 1% of massive stars would rotate at sufficient speeds for lGRB emission to be visible. These stars are isotropic, giving a rate of \( \approx 1 \) event per year. The rate for lGRBs could be higher if, for example, the progenitor mass, metallicity, rotation and magnetic fields is mandatory. This estimate neglects the fact that only the strongest GW signals (so far predicted in the context of full GR neutrino-radiation hydrodynamics simulations) could provide a unique opportunity to probe into rapid rotation and the associated non-axisymmetric instabilities, otherwise obscured deep inside the massive stellar core.

4 DISCUSSIONS

Finally we shall discuss several limitations in this work. First our simulation does not take into account magnetic fields. If magnetorotational instability (MRI) (e.g., Akiyama et al. 2003; Obergaulinger et al. 2009; Masada et al. 2012; Rembiasz et al. 2016a,b) develops in a sufficiently short timescale, MRI could transfer angular momentum of the PNS outward (e.g., Mosta et al. 2015; Masada et al. 2015) and may prevent the growth of the non-axisymmetric instabilities. We have to tackle with this by developing full 3D GR-MHD code with spectral neutrino transport, however, this is beyond the scope of this work. Second, Saijo (2018) recently pointed out (in the context of the isolated NSs) that the use of different EOSs significantly impact the growth of the low-\( T/|W| \) instability (e.g., Scheidegger et al. 2010)). Not only the impact of EOS but also of updating the neutrino opacities (e.g., Bollig et al. 2017) both of which could significantly affect the explosibility and the PNS contraction remain to be investigated. Our full GR simulations can follow evolution in the vicinity of rapidly rotating PNSs more properly than simplified GR treatments like effective GR potentials (e.g., Marek et al. 2006), which is often constructed on the basis of a spherically symmetric space. We thus believe that our full GR treatment exerts its potential to derive quantitatively, or perhaps even qualitatively, correct GWS as they can be sensitive to the connection between the PNS core contraction and the low \( T/|W| \) instability. Finally a major limitation is that we have shown results of only one simulation sample. Systematic study changing the progenitor mass, metallicity, rotation and magnetic fields is mandatory in order to draw a robust conclusion to clarify under which condition the strong GW emission found in this work can be obtained. In the decade to come, we speculate that these issues are going to be tackled by utilizing next-generation (Exa-scale) supercomputing resources.

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