DYNAMICS AND PHENOMENOLOGY
OF CHARMONIUM PRODUCTION OFF NUCLEI

BORIS Z. KOPELIOVICH

Max-Planck-Institut für Kenphysik
Postfach 103980, D-69029 Heidelberg, Germany
and
Joint Institute for Nuclear Research
141980 Dubna, Moscow Region, Russia

Abstract

Nuclear suppression of charmonium production in proton-nucleus interactions is poorly understood, what restrains our attempts to single out unusual effects in heavy ion collisions. We develop a phenomenological approach, based on the light-cone dynamics of charmonium production, which has much in common with deep-inelastic scattering and Drell-Yan lepton pair production. The key observation is the existence of a soft mechanism of heavy flavour production, which scales in the quark mass and dominates shadowing corrections and diffraction. It naturally explains the surprisingly strong nuclear suppression of $J/\Psi$ at large Feynman-$x_F$. The low-$x_F$ region is subject to a complicated interplay of hard and soft mechanisms. With evaluated parameters we nicely describe available data on charmonium production in proton-nucleus collisions. Using these results we predict a new process, diffractive production of charmonium on a nucleon target, which fraction in the total production rate of charmonium is evaluated at 12%.

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1 Introduction

Production of charmonium in heavy ion collisions is believed to be a sensitive probe for new physics. In practice, however, hand-waving motivations are not sufficient, and one desperately needs a reliable baseline. Unfortunately, present understanding of dynamics of hadroproduction of charmonium off protons and nuclei is far from satisfactory. Nuclear effects are poorly understood, particularly, there is still no reasonable quantitative explanation of surprisingly strong nuclear suppression of $J/\Psi$ production at high Feynman $x_F$. It is too adventurouus to predict or to interpret the observed nuclear effects in heavy-ion collisions, having no idea about what is going on in proton-nucleus collisions.

I do not pretend in this short talk to establish a reliable baseline, but only want to make another step towards it.

This talk is based mostly on the recent unpublished results obtained in collaboration with Jörg Hufner, as well as on our previous publications, cited in the text.

2 Poor man’s formula or how long does it take to produce a $c\bar{c}$ pair

There is a wide spread believe, adopted in most recent analysis of data on $J/\Psi$ production, that the $c\bar{c}$ pair is produced momentarily at the point of interaction of the projectile hadron with a bound nucleon, and then it propagates through the nucleus and eventually escapes it. This point of view is inspired by the usual perturbative treatment of $c\bar{c}$ production in the rest frame of the charmonium. However, in the rest frame of the nucleus the production time is subject to Lorentz time dilation (or the nuclear thickness is contracted in the charmonium rest frame). When the production time is longer than the mean internucleon distance in nuclei ($\sim 2 \text{ fm}$) one cannot say anymore that the $c\bar{c}$ pair is produced on one concrete nucleon. This can also be understood as coherence between the waves produced at different nucleons. For this reason the production time is usually called coherence time (length).
The time scale of charmonium production has two extremes. In the low energy limit
the coherent time \( t_c \ll d \), where \( d \) is the mean internucleon distance, one can use a simple
(usually called a conventional approach) formula for nuclear suppression,

\[
S^\Psi_{hA} = \frac{1}{A} \int d^2b \int_0^\infty dz \ \rho(b,z) \ \exp[-\sigma_{in}^\Psi T_z(b)] = \frac{\sigma_{in}^\Psi}{\sigma_{in}^\Psi N} \approx 1 - \frac{1}{2} \sigma_{in}^\Psi \langle T \rangle. \tag{1}
\]

Here the mean nuclear thickness \( \langle T \rangle = A^{-1} \int d^2b \ T^2(b) \), where \( T(b) = T_z(b)|_{z\to-\infty} \) and
\( T_z(b) = \int_z^\infty dz \ \rho(b,z) \). The nuclear density \( \rho(b,z) \) depends on impact parameter \( b \) and lon-
gitudinal coordinate \( z \). The latter approximation in (1) uses smallness of the charmonium
inelastic interaction cross section, \( \sigma_{in}^\Psi T(b) \ll 1 \).

Although a \( c\bar{c} \) pair, rather than the \( \Psi \), is produced and propagates through the nucleus,
in the low energy limit one may think that the charmonium wave function is formed
instantaneously. We come later back to this point and formulate an exact condition for
that.

This simple formula corresponding to the low-energy limit is nowadays widely used as
a phenomenological basis for nuclear effects in charmonium production. The absorption
cross section \( \sigma_{in}^\Psi \) is usually treated as an unknown parameter.

In the high energy limit, \( t_c \gg R_A \), a fluctuation, containing the \( c\bar{c} \) pair lives much
longer than the nuclear size. The nuclear suppression factor has in this case a form similar
to that for quasielastic scattering [1],

\[
S_{hA}^{\Psi}|_{t_c \gg R_A} = \frac{1}{A} \sigma_{free} \int d^2b \ T(b) \ \exp[-\sigma_{free} T(b)] \approx 1 - \sigma_{free} \langle T \rangle. \tag{2}
\]

I would like to draw attention to the fact that in the approximation \( \sigma_{in}^\Psi T(b) \ll 1 \) the
shadowing term in (1) has a factor \( 1/2 \) compared to that in (2). This is an explicit
manifestation of the space-time pattern of interaction: at high energy the mean length of
path of the \( c\bar{c} \) pair in the nucleus is twice as long as at low energy.

Let us postpone to the next section interpretation of the freeing cross section \( \sigma_{free} \),
but for now we concentrate on the problem of the coherence length. The lifetime of a
fluctuation of the projectile hadron containing the $c\bar{c}$ pair is given by the energy denominator,

$$t_c = \frac{2E_h}{M_{fl}^2 - m_h^2}, \quad (3)$$

where $E_h$ and $m_h$ are the energy and mass of the projectile hadron, and $M_{fl}$ is the effective mass of the fluctuation, which is to be considered on mass shell in the light-cone formalism. The kinematical formula for $M_{fl}^2$ reads,

$$M_{fl}^2 = \sum_i m_i^2 + k_i^2 = \sum_i \alpha_i \langle T \rangle \int d^2b \rho(b, z) e^{iqc z}, \quad (4)$$

where $m_i$, $k_i$ and $\alpha_i$ are the mass, the transverse momentum and the fraction of the total light-cone momentum carried by the $i$-th parton, and we sum over all the partons in the fluctuation ($\sum_i \alpha_i = 1$).

It is easy to find from (4) the minimal effective mass of a fluctuation containing a $c\bar{c}$ pair, which corresponds to a longest coherence time,

$$t_c \leq \frac{2E_\Psi}{M_{fl}^2} \quad (5)$$

This is the most general upper restriction for the lifetime of any fluctuation containing a $c\bar{c}$ pair.

It turns out that at energies corresponding to available data on $\Psi$ production we have a full coverage of possible scenarios of space-time development. For instance at $x_F \approx 0$ we have $t_c \leq \sqrt{s}/2m_NM_\Psi$. This is only about 1 fm at $E_h = 200$ GeV, however, $t_c$ becomes compatible with the radiiuses of heavy nuclei at Tevatron energies, $E_h \approx 800$ GeV, and it is much longer in the latter case if $x_F \rightarrow 1$.

It is demonstrated on different exact solutions in [2, 3, 4] that in the approximation $\sigma^{\Psi N}(T) \ll 1$ the variation of nuclear suppression as function of the coherence length always has a form of a linear function of the nuclear longitudinal formfactor squared,

$$F_A^2(q_c) = \frac{1}{\langle T \rangle} \int d^2b \left| \int_{-\infty}^{\infty} dz \rho(b, z) e^{iq_c z} \right|^2, \quad (6)$$

where the longitudinal momentum transfer $q_c = 1/t_c$. This formfactor varies from zero in the low-energy limit $t_c \ll R_A$ up to one at $t_c \gg R_A$. The formula for nuclear suppression,
which interpolates between the low- and high-energy limits, (1) and (2), reads,

$$S^\Psi_{hA} = 1 - \frac{1}{2} \sigma^N_{\text{in}} \langle T \rangle \left[ 1 - F^2_A(q_c) \right] - \sigma_{\text{free}} \langle T \rangle F^2_A(q_c)$$

(7)

In the two following sections we elaborate more with the cross sections $\sigma_{\text{free}}$ and $\sigma^N_{\text{in}}$.

3 The freeing cross section and the soft mechanism of charmonium production

The soft spectator mechanism of heavy flavour production was the subject of my talk in Hirschegg in 1995 [5]. Let me summarize the observations.

First of all, one should discriminate between the interaction cross section of a hadronic fluctuation, containing a heavy quark pair and the cross section of freeing (production of) this pair. The former is large, since the fluctuation as a whole is big, while the latter is usually small, of the order of the inverse heavy quark mass squared. Indeed, soft (low $k_T$) long-wave (in transverse direction) gluons, providing a large cross section, cannot resolve such a small-size structure as a heavy-quark fluctuation. The freeing amplitude can be represented as a difference between interaction amplitudes of two Fock states, which are identical, except one contains the $c\bar{c}$ fluctuation, but another one does not. This is the general principle originating from the pioneering ideas of Feinberg-Pomeranchuk and Good-Walker for diffractive production. If all the fluctuations of the projectile hadron have the same interaction amplitudes, nothing new can be produced, but the initial hadron, which coherence is not disturbed in this case.

Let us consider a concrete example of a fluctuation of a light quark, which consists of the same quark and a colorless $c\bar{c}$ pair. The freeing amplitude is equal to the difference between the amplitude of inelastic interaction of this fluctuation and that of the single quark. Although each of these amplitudes is infrared divergent, their difference is finite. Since only the light quark-spectator can interact (interaction of the colorless $c\bar{c}$ pair would lead either to open flavour production, or is a higher order $1/m_Q^2$ effect) the difference of the inelastic amplitudes comes from the difference of the impact parameters of the quark.
with and without the $c \bar{c}$ fluctuation. If the transverse separation between the quark and the center of mass of the $c \bar{c}$ pair is $r_T$, the center of gravity of the whole fluctuation (which coincides with the impact parameter of the parent light quark) is distant by $(1 - \alpha)r_T$ from the $c \bar{c}$ pair and by $\alpha r_T$ from the light quark, where $\alpha$ is the fraction of the light-cone momentum of the projectile quark. Thus, the freeing cross section equals to the total interaction cross section of a light $q \bar{q}$ pair with separation $\alpha r_T$. Therefore, at small $r_T$ color transparency leads to $\sigma_{\text{free}}(r_T) \propto r_T^2$. This was an important observation of [5] (see also [6]).

The mean transverse separation $\langle r_T \rangle$ between the $c \bar{c}$ and the $q$ can be estimated as follows [5]. The wave function of the fluctuation in the momentum representation is given by the energy denominator, $\Psi_{c \bar{c}q}(k_T) \propto 1/(M_{f\bar{f}}^2 - m_{q}^2) \propto 1/(k_T^2 + M_{c\bar{c}}^2(1 - \alpha) + \alpha^2 m_q^2)$, where (4) is used for $M_{f\bar{f}}^2$. Switching to the impact-parameter representation (only for the transverse momentum) one gets the wave function of the $qc\bar{c}$ fluctuation (the spin structure is neglected) $\Psi_{c\bar{c}q}(r_T, \alpha) \propto K_0(\tau r_T)$ [5], where $\tau^2 = (1 - \alpha)M_{\Psi}^2 + \alpha^2 m_q^2$. This distribution gives the mean separation $\langle r_T^2 \rangle \sim 1/\tau^2$, and the freeing cross section reads,

$$\sigma_{\text{free}}(\alpha) \sim \frac{\alpha^2}{(1 - \alpha)M_{\Psi}^2 + \alpha^2 m_q^2}, \quad (8)$$

As different from naive expectations, eq. (8) says that the cross section of freeing of a heavy quark fluctuation is small unless the factor $(1 - \alpha)$ is small $\sim m_q^2/M_{\Psi}^2$, i.e. the $c \bar{c}$ takes the main fraction of the initial quark momentum. In this case the freeing cross section is as large as a typical hadronic one, $\sim 1/m_q^2$ (actually, $m_q$ should be treated as an effective infra-red cut off for $r_T$ of the order of the inverse confinement radius, rather than the quark mass). This fact may have relevance to the well known problem of a substantial increase of nuclear suppression of $J/\Psi$ production at large $x_1 = p_{\Psi}^+ / p_h^+$, which corresponds to an effective absorption cross section close to that for light hadrons. We come back to this problem below.

The production charmonium cross section is given by a convolution of the cross section (8) with the momentum distribution function $F_h(y)$ of the quark in the projectile hadron (more generally, the distribution of those partons, which participate in the fluctuation...
containing the $c\bar{c}$).

\[
\langle \sigma^h_{\text{free}}(x_1) \rangle \propto \int d^2r_T \int_{x_1}^1 dy \frac{1}{|\Psi_{c\bar{q}}(r_T, x_1/y)|^2} F_h(y) \sigma_{\text{free}}(r_T)
\]

Assuming the end-point behaviour for $F_h(y) \propto (1 - y)^\beta$, one arrives to the following $x_1$-dependence of the freeing cross section at large $x_1$,

\[
\langle \sigma^h_{\text{free}}(x_1) \rangle \propto \frac{1 - x_1}{M_\Psi^2} \left\{ \ln \left( \alpha_s \left[ (1 - x_1) M_\Psi^2/m_q^2 \right] + c \right) \right\}.
\]

The double-logarithmic dependence on $M_\Psi^2$ corresponds to the well known violation of the Bjorken scaling in deep-inelastic scattering given by the evolution equations. However, the hard scale in (10) is imposed by $(1 - x_1) M_\Psi^2$, rather than $M_\Psi^2$, prescribed by the conventional factorization theorem. Another violation of the factorization is due to the constant $c$ in (10), which depends on the exponent $\beta$ in the hadronic structure function.

Amazingly, the contribution of the soft asymmetric fluctuations with $1 - \alpha \sim m_q^2/M_\Psi^2$, corresponding to large freeing cross section (see (9)), does not vanish at high $M_\Psi^2$. Moreover, these soft fluctuations dominate in nuclear shadowing, which is given by the same expression (10), but with higher powers of $\sigma_{\text{free}}$. This can be explained in a simple way (like in deep-inelastic scattering [7]) if to divide all the fluctuations to two classes, the hard one with $\sigma_{\text{free}} \sim 1/M_\Psi^2$, and the soft one with $\sigma_{\text{free}} \sim 1/m_q^2$. The results for the production cross section and the first order shadowing correction are summarized in Table 1.

**Table 1. Contributions of soft and hard fluctuations to the charmonium production cross section and to nuclear shadowing**

| Fluctuation | $|\Psi_{c\bar{q}}|^2$ | $\sigma^{hN}_{\text{tot}}$ | $|\Psi_{c\bar{q}}|^2 \sigma^{hN}_{\text{tot}}$ | $|\Psi_{c\bar{q}}|^2 (\sigma^{hN}_{\text{tot}})^2$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
| Hard        | $\sim 1$        | $\sim 1/M_\Psi^2$ | $\sim 1/M_\Psi^2$ | $\sim 1/M_\Psi^2$ |
| Soft        | $\sim m_q^2/M_\Psi^2$ | $\sim 1/m_q^2$ | $\sim 1/m_q^2$ | $\sim 1/m_q^2 M_\Psi^2$ |

One can see from the Table that the soft component dominates the first-order nuclear shadowing term proportional to $\langle |\sigma^{hN}_{AN}(r_T)|^2 \rangle$, which has the same $1/M_\Psi^2$ suppression and
the impulse approximation term $\langle \sigma_{Kn}(r_T) \rangle$. This means that shadowing scales in $M_{\Psi}^2$. This is the common feature of deep-inelastic scattering, Drell-Yan pair and heavy flavour production, where the soft and hard contributions are known to be of the same order.

Although the dynamics of charmonium production in hadronic collisions is much more complicated, one may try to develop a phenomenological approach relying on the general ideas on space-time development (section 2) and on the dynamics of nuclear attenuation (this section).

Let us classify different contributions to nuclear suppression factor (7).

1. Since the dominant contribution to nuclear shadowing comes from asymmetric fluctuations $q \rightarrow c\bar{c}q$ with $\alpha \rightarrow 1$, the $J/\Psi$ has the same $x_1$-distribution as the valence quark, i.e. $\sim (1 - x_1)^3$ if the projectile is a proton. This includes also the so called color-octet mechanism which corresponds to the fluctuation $q \rightarrow q\bar{c}2g$. The exponent in the $x_1$-distribution may be even smaller if the colorless $c\bar{c}$ fluctuation is produced collectively by two or three valence quarks, as was suggested in [9].

Note that in the dominant asymmetric fluctuation $1 - \alpha \approx m_q^2/M_{\Psi}^2$ and the light quark contributes to the effective mass of the fluctuation (4) the same amount as the $c\bar{c}$ pair. Therefore, the formfactor $F_A^2(q_c)$ suppressing the soft fluctuations in (7) should be evaluated at $q_c \approx 2q_\Psi$, where $q_\Psi = M_{\Psi}^2/2E_\Psi$ is according to (3) the minimal longitudinal momentum transfer.

The soft freeing cross section $\sigma_S$ can be borrowed from the data on nuclear shadowing in deep-inelastic scattering. In accordance with the modified factorization relation (10) we use the result of analysis of the data on deep-inelastic scattering at $Q^2 = (1 - x_1)M_{\Psi}^2$ and fix, $\sigma_S \approx 12 \text{ mb}$, assuming $x_1 \approx 0.5$. However, at $x_1 > 0.9$ (where no data are still available) the corresponding value of $Q^2$ is so small that one gets into the domain of the vector dominance model, and $\sigma_{s,free}^S$ should be about 20 – 30 mb

2. Hard mechanisms of charmonium production include direct interaction of the color-octet $c\bar{c}$ fluctuation with the target (e.g. the color-singlet mechanism). Due to color
screening the freeing cross section is \( \propto \langle r_{cc}^2 \rangle \approx 4/m_c^2 \approx 0.07 \text{ fm}^2 \) (the factor 4 is due to color screening suppressing the fluctuations of small size \([1]\)). Estimated perturbatively \( \sigma_H \approx 2 \text{ mb} \), substantially smaller than one expects for \( \sigma_{\Psi N}^{\Psi N} \).

Since the effective mass of the hard fluctuation is \( \sim M_{\Psi} \), the formfactor \( F_A^2(q_c) \) in (7) should be taken at \( q_c \approx q_{\Psi} \)

The produced charmonia have the same \( \propto (1-x_1)^5 \) distribution as the parent gluon. Thus, the hard contribution to the freeing cross section is suppressed by \( (1-x_1)^n \) compared with the soft mechanism, where \( n \geq 2 \).

3. The inelastic \( J/\Psi \)-nucleon cross section is known from experiment (see in \([3]\)) with a large uncertainty and ranges from 2 to 7 \text{ mb}. The lowest-order, two gluon graph provides \( \sigma_{\Psi N}^{\Psi N} \approx 5 \div 6 \text{ mb} \).

We expect quite a steep energy dependence of \( \sigma_{\Psi N}^{\Psi N} \). Indeed, \( J/\Psi \) has a smaller radius than the light hadrons, and perturbative QCD predicts a growth of the effective Pomeron intercept with decreasing radius. The recent measurements of energy dependence of the cross section of elastic photoproduction of \( J/\Psi \) at HERA \([11]\) found that it grows \( \propto s^{0.4} \). Since this cross section is \( \propto \sigma_{el}^{\Psi N} \approx (\sigma_{tot}^{\Psi N})^2/16\pi B \), one can find the energy dependence of \( \sigma_{tot}^{\Psi N} \propto s^{0.25} \), taking into account the energy dependence of the slope parameter \( B \). Note that such a high effective Pomeron intercept perfectly agrees with the measured at HERA \( x \)-dependence of the proton structure function \( F_{2p}(x, Q^2) \) at the corresponding virtuality \( Q^2 \approx M_{\Psi}^2 \).

The steep energy dependence of \( \sigma_{\Psi N}^{\Psi N} \) results in a substantial variation of nuclear suppression, provided by the first term in eq. (7).

We expect the same energy-dependence for the hard component, while the soft contribution is supposed to grow slowly, \( \propto s^{0.1} \).

4  Evolution of a \( c\bar{c} \) wave packet in a medium. The \( \Psi/\Psi' \) puzzle

The low-energy limit (3) was written assuming that the charmonium is produced momentarily at the point of interaction with a bound nucleon. However, a \( c\bar{c} \) pair is produced,
rather than the charmonium, and it takes some time to become the final charmonium, even if $t_c \ll R_A$. The evolution of the $c\bar{c}$ wave packet is controlled by a different parameter, called formation time/length,

$$t_f \geq \frac{2E_{\Psi}}{M_{\Psi,}^2 - M_{\Psi}^2},$$

(11)

which is about five times longer than $l_c$.

The evolution can be solved using either the path integral technique in quark representation [1] or, what is equivalent, in hadronic representation using the coupled-channel approach [12]. In the latter case the two channel approximation, $J/\Psi$ and $\Psi'$, turns out to have a pretty good, about 10 % accuracy. In this case the effective cross section of absorption of $J/\Psi$ in (1) and (7) should be replaced by

$$\sigma_{\Psi N}^{in} \Rightarrow \sigma_{\Psi N}^{\Psi N} \left[ 1 + \epsilon R F_A^2(q_f) \right],$$

(12)

where $q_f = 1/l_f$, and $F_A^2(q)$ is defined in [8]. Other parameters are defined in [12], $R^2 \approx 0.25$ is the experimentally known ratio of $\Psi'$ to $J/\Psi$ production rates. $\epsilon = -\sqrt{2/3}$ is the ratio of the off-diagonal to the diagonal diffractive $\Psi - N$ amplitudes evaluated in [12].

With these parameters one concludes that if $l_f \gg R_A$ the effective absorption cross section (12) is only about 0.6 of that for $J/\Psi$.

Another interesting observation of [12] concerns nuclear suppression of $\Psi'$. Naively, using the poor man’s formula (1) one would expect much stronger suppression of the $\Psi'$ than $J/\Psi$. However, the corresponding effective absorption cross section turns out to be quite different from $\sigma_{\Psi N}^{\Psi N}$,

$$\sigma_{\Psi N}^{\Psi N} \Rightarrow \sigma_{\Psi N}^{\Psi N} \left[ 1 + \frac{\epsilon}{r R} F_A^2(q_f) \right],$$

(13)

where $r = \sigma_{\Psi N}^{\Psi N}/\sigma_{\Psi N}^{\Psi N} \approx 7/3$ as evaluated in [12]. It is interesting that with these parameters the effective absorption cross sections (12) and (13) are approximately equal, provided that $l_f \gg R_A$, i.e. $F_A^2(q_f) = 1$. This observation naturally explains the surprising experimental result of nearly the same nuclear suppression of $J/\Psi$ and $\Psi$. Note that
in the case of the soft mechanism only the light projectile quark interacts with the target, and the structure of the $c\bar{c}$ pair also does not affect the nuclear attenuation.

5 Phenomenology of $J/\Psi$ production off nuclei

Summarizing the results of previous sections, eq. (7) can be presented in the unitary form,

$$S_{hA}^{\Psi} = \exp \left[ -\sigma_{\text{eff}}(E_h, x_1) \langle T \rangle \right],$$

(14)

where

$$\sigma_{\text{eff}}(E_h, x_1) = \frac{1}{2} \sigma_{\text{in}}^{\Psi N} \left[ 1 + \epsilon R \ F_A^2(q_\Psi) \right] \left[ 1 - F_A^2(q_\Psi) \right] - \frac{(1 - x_1)^n F_A^2(q_\Psi) \sigma_H^2 + \gamma F_A^2(2q_\Psi) \sigma_S^2}{(1 - x_1)^n \sigma_H + \gamma \sigma_S}.$$  

(15)

All the notations here were introduced before, except $\gamma$, which is the ratio of the weight factors of the soft to the hard components. According to Table 1 this factor is suppressed by the $J/\Psi$ mass, $\gamma \sim m_q^2/M_{\Psi}^2 \approx 0.004$, if to treat $m_q$ as a cut off, which is the inverse confinement radius $\sim \Lambda_{QCD}$. Other parameters are also pretty well known. $\sigma_{\text{in}}^{\Psi N} = \sigma_0^{\Psi} (x_1 E_h)^{\Delta}$, where $\Delta = 0.25$ and the parameter $\sigma_0^{\Psi}$ should provide the cross section about $5 \div 6 \, mb$ at the energy few tens GeV. Then we expect $\sigma_0^{\Psi} \sim 2 \, mb$. The hard cross section $\sigma_H = \sigma_0^{H} (x_1 E_h)^{\Delta}$, as was mentioned, is expected to be smaller than $\sigma_{\text{in}}^{\Psi N}$, of the order of $2 \, mb$. Then we expect $\sigma_0^{H} \sim 0.7 \, mb$. The exponent $n \geq 2$ is not very important, we compare with the data at $n = 3$.

To elaborate with low-energy data on $J/\Psi$ production one should take care of possible effects of induced energy loss, which are due to rescattering of the projectile partons in the nucleus [13]. This is unimportant at the energy $E_p = 800 \, GeV$, where the best data exist, but may produce an additional suppression at lower energies. We include this effect in a rough way through the extra factor $K(E_h, x_1)$ to the expression [14]

$$K(E_h, x_1) = \left( \frac{1 - \bar{x}_1}{1 - x_1} \right)^m.$$  

(16)

Here $\bar{x}_1 = \Delta E/E_h$ and $\Delta E = \kappa \langle L \rangle$, where $\langle L \rangle \approx 3R_A/4[1 + F_A^2(q_\Psi)]$ is the mean free path of a parton in the nucleus, where we took into account the Lorentz stretching of the path.
of a fluctuation. The density of energy loss $dE/dz = -\kappa$ was fixed at the color-triplet string tension $\kappa = 1 \text{ GeV}/\text{fm}$. The exponent $m = 5$ in accordance with the measured $x_1$ dependence of $J/\Psi$ production rate in $pp$ collisions.

Figure 1: Nuclear suppression of $J/\Psi$ production rate as measured in [14, 15] and calculated with eq. (14). See explanations in the text.

Fitting the data depicted in Fig. 1 on nuclear suppression from the experiments E772 at 800 GeV [14] and NA3 at 200 GeV [15] in $pA$ collisions, as well as the data of the NA38/50 experiments at 200 ($\bar{x}_1 \approx 0.25$) [16] and 450 GeV ($\bar{x}_1 \approx 0.1$) [17] we arrived at the following best values of the free parameters in (15), $\gamma = 0.0037 \pm 0.0015$, $\sigma^0_{\Psi} = 1.95 \pm 0.018 \text{ mb}$ and $\sigma^0_{H} = 0.64 \pm 0.05 \text{ mb}$, which are in a perfect agreement with our pre-evaluations. Comparison with the data, in Fig. 1 and Table 2 demonstrates a nice agreement.

Table 2. Data on $pA$ collisions from the NA50 experiment at 450 GeV [17] and from the NA38 experiment at 200 GeV [14] compared to our results (bottom row)

|        | 450, W | 450, Cu | 450, Al | 450, C | 200, U | 200, W | 200 Cu |
|--------|--------|--------|--------|--------|--------|--------|--------|
| 450, W | 0.67 ± 0.08 | 0.75 ± 0.08 | 0.76 ± 0.09 | 0.85 ± 0.1 | 0.63 ± 0.13 | 0.65 ± 0.05 | 0.75 ± 0.17 |
| 200, U | 0.74 | 0.82 | 0.87 | 0.90 | 0.69 | 0.71 | 0.80 |
Although our results agree with the NA38/50 data within error bars, they are yet systematically higher than the nominal experimental values. This is mostly a result of a possible inconsistency between the data from the NA3 and NA38/50 experiments. The former are higher and have quite small error bars (see Fig. 1). This region of low $x_1$ is controlled by the values of $\sigma^0_\Psi$ and $\sigma^0_H$, which are the free parameters and do not need to vary much in order to fit better either of the data.

6 Diffractive production of charmonia

Using the data on nuclear shadowing one can estimate the cross section of diffractive production of charmonium in hadron-nucleon interaction, i.e. diffractive excitation of the projectile hadron to a state containing the charmonium and light hadrons, while the target nucleon is intact.

It is well known since Gribov’s paper [18] that nuclear shadowing of the total cross section is closely related to diffraction. It is a direct consequence of unitarity that the forward diffractive cross section can be represented as

$$\int dM^2 \frac{d\sigma_{DD}}{dp_T^2dM^2}\bigg|_{p_T=0} = \frac{1}{16\pi} \langle \sigma^2 \rangle$$

one averages here over the eigen states of interaction, which may be treated as the projectile fluctuations. The cross section (17) includes also the elastic channel.

We are interested only in that part of diffraction in (17), which contains a colorless $c\bar{c}$ pair in final state. In this case one should replace the interaction cross sections in (17) by the freeing cross sections. Then one arrives at the same expression, which we had for nuclear shadowing term in charmonium production (see section 3) when the nuclear formfactor saturates ($q_c R_A \ll 1$),

$$1 - S^\Psi_{hA} = \frac{16\pi}{\sigma(hp \rightarrow \Psi X)} \langle T \rangle \int \frac{M^2_{m}}{(m_N+m_\Psi)^2} \int dM^2 \frac{d\sigma^{\Psi}_{DD}}{dM^2dp_T^2}\bigg|_{p_T=0},$$

where $d\sigma^{\Psi}_{DD}/dM^2dp_T^2$ is the differential cross section of diffractive production of charmonium in $hN$ collisions, i.e. the process $hN \rightarrow X_\Psi N$, where the multiparticle state $X_\Psi$
consists of $\Psi$ and light hadrons. $M$ is the effective mass of the $X_\Psi$, which is restricted by the nuclear formfactor at $M^2 < M_m^2 \sim E_h/R_A$.

As soon as nuclear shadowing of $\Psi$ production is known, we can predict the fraction of diffraction in the total cross section of $\Psi$ production, which reads

$$\delta^{\Psi}_{DD}(M_m) \equiv \frac{1}{\sigma(\overline{h}p \rightarrow \Psi X)} \int dp_T^2 \int_{(m_N+M_\Psi)^2}^{M_m^2} dM^2 \frac{d\sigma_{DD}}{dM^2 dp_T^2} = \frac{\sigma_S}{16 \pi B}. \quad (19)$$

Here $B \approx 5 \text{ GeV}^{-2}$ is the slope parameter of the differential diffractive cross section $hN \rightarrow X_\Psi N$. Its value should be about a half of the slope of elastic $NN$ scattering, since only the form factor of the target nucleon contributes to the slope in diffraction to large masses.

The parameter of the parameter $\sigma_S$ was fixed (using information from deep-inelastic scattering) at $\sigma_S = 12 \text{ mb}$ in our analysis in the previous section. Since this value well describes the data on nuclear suppression at high energy of $J/\Psi$, we can safely use it on the same footing as the data.

Thus, we arrive at the estimate,

$$\delta^{\Psi}_{DD}(M_m) \approx 0.12, \quad (20)$$

which is close to the fraction of the diffractive large rapidity gap events observed at HERA \[19\]. This is not unexpected result, since, as we emphasized, there is much in common between deep-inelastic scattering, Drell-Yan reaction and heavy flavor production, especially when it concerns the role of soft interactions, which dominate nuclear shadowing and diffraction.

One should not be confused with the fact that the left-hand sides of relations (19) and (20) depend on $M_m$, while the right-hand sides are constant. The high-mass tail $\propto 1/M_m^2$ of the diffractive $M^2$-distribution leads to a logarithmic $q_c$-dependence of the nuclear formfactor, which we neglected and eliminated the formfactor in (19) and (20).
7 Heavy ion collisions

In this case either the fluctuations, or the formed charmonium propagate through the both colliding nucleus. This may sound puzzling, indeed, we considered above propagation of the fluctuations of the projectile proton through the target nucleus. How such a fluctuation can propagate through the projectile nucleus, which has the same velocity as the projectile nucleon? This is a typical puzzle of the parton model. It is impossible to prescribe a fluctuation either to the beam, or to the target in a Lorentz-invariant way. It may be done only in a concrete reference frame.

Thus, nuclear suppression in $AB$ collision is a product of that in $pA$ and $pB$ interactions. Of course the kinematics should be treated properly, what is easier to do with Feynman variable $x_F = x_1 - M_{q\bar{q}}^2/x_1 s$. Then the nuclear suppression in $AB$ collision reads

$$S_{AB}^\Psi(x_F) = S_{pA}^\Psi(x_F) \times S_{pB}^\Psi(-x_F)$$

Using expression (14) - (15) it easy to calculate the nuclear suppression factor $S_{AB}^\Psi(x_F)$ for $J/\Psi$, or applying eq. (13), for $\Psi'$. It is interesting that due to inverse kinematics and the strong $x_F$-dependence of $S_{pA}^\Psi(x_F)$ it turns out that $S_{AB}^{\Psi'}(x_F) < S_{AB}^\Psi(x_F)$, in accordance with experimental observation. However, the observed relative suppression of $\Psi'$ is even stronger than given by (21). This is possible due to final state interaction with the produced hadrons (coomovers), which happens at long times when $\Psi'$ is formed and interacts with a large cross section.

The same coomovers are a plausible source of the additional nuclear suppression of $J/\Psi$ production rate in $AB$-collisions, which was found to be especially strong in lead-lead. This is too complicate and uncertain problem to be discussed in this note.

Summarising, we developed for the first time a phenomenological approach to the problem of nuclear suppression of charmonium production, based on the same dynamics, which is responsible for nuclear shadowing in deep-inelastic scattering and Drell-Yan reaction. The key point is an admixture of soft mechanism of heavy flavour production, which scales in $M_{Q\bar{Q}}^2$, dominates at high $x_F$ and naturally explains the observed strong
nuclear suppression. Many other features observed experimentally are understood as well. Our numerical estimates are in a good agreement with the fit to available data.

The $J/\Psi - N$ interaction cross section steeply grows with energy due to increasing radius of the gluon cloud. As a result, the $J/\Psi$ production rate is expected to be very much suppressed due to interaction with nucleons at very high energies. We expect $\sigma_{tot}^\Psi N \approx 10$ mb at RHIC and $\sigma_{tot}^\Psi N \approx 26$ mb at LHC, if $J/\Psi$ is at rest in the c.m.

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