Density perturbations in decaying holographic dark energy models

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Abstract

We study cosmological perturbations in the context of an interacting dark energy model, where the holographic dark energy with IR cutoff decays into the cold dark matter (CDM). For this purpose, we introduce three IR cutoffs of Hubble horizon, particle horizon, and future event horizon. Here we present small perturbations under the case that effective equation of state (EOS: $\omega^{\text{eff}}$) for the holographic energy density is determined to be the same negative constant as that for the CDM. Such new matter productions tend to dilute the density perturbations of CDM (matter contrast). For a decelerating universe of $\omega^{\text{eff}} > -1/3$, the matter contrast is growing as the universe evolves, while for an accelerating universe of $\omega^{\text{eff}} < -1/3$, the matter contrast is decaying, irrespective of the choice of IR cutoff. This shows clearly that the acceleration suppresses the growing of the density perturbations at the early universe.

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I. INTRODUCTION

The cosmological constant problem has acquired a renewed importance since several independent observations have been pointing to the presence of a negative pressure component in the cosmic fluid [1]. In view of quantum field theories, the natural candidate for such a dark energy is the quantum vacuum energy. Since it has the symmetry of the background, its energy-momentum tensor has the form $T_{\mu\nu} = \Lambda g_{\mu\nu}$, where $\Lambda$ is a scalar function of coordinates. This leads to the equation of state $p_\Lambda = -\rho_\Lambda = -\Lambda$, where $\Lambda$ may be a function of time ($\Lambda(t)$) in general. In the case of a constant $\Lambda$, the vacuum contribution plays the role of a cosmological constant in Einstein’s equations. The model based on a constant $\Lambda$ and a variable $\Lambda(t)$ is called the $\Lambda$CDM model and $\Lambda(t)$CDM model, respectively.

The idea of a time-dependent cosmological term has provided different phenomenological implementations [2], being a subject of renewed interest in recent years [3, 4, 5]. A general feature of all those approaches is the production of a new kind of matter, compatible with the vacuum decay in order to assure the conservation of the total energy-momentum [6].

On the other hand, there exists dynamical cosmological constants derived from the holographic principle. Cohen et al have shown that in quantum field theory, the UV cutoff $\tilde{\Lambda}$ is related to the IR cutoff $L_\Lambda$ due to the limit set by forming a black hole [7]. In other words, if $\rho_\Lambda = \tilde{\Lambda}^4$ is the quantum vacuum energy density caused by the UV cutoff, the total energy of system with size $L_\Lambda$ should not exceed the mass of the system-size black hole: $L_\Lambda^3 \rho_\Lambda \leq 2L_\Lambda M_p^2$. If the largest size $L_\Lambda$ is chosen to be the one saturating this bound, the holographic energy density is then given by $\rho_\Lambda = 3c^2 M_p^2/8\pi L_\Lambda^2$ with a parameter $c$ [8, 9, 10], in contrast with the conventional energy density of $\rho \propto 1/L^3$. We consider $\rho_\Lambda$ as the dynamical cosmological constant.

Also, we have two different views of determining the equation of state for the holographic energy density. The first view is that its native equation of state is not changing as the universe evolves [11]. It is fixed by $\omega_{\Lambda 0} = -1$ initially. An important point to note is that the holographic energy density itself is changing as a result of decaying into the CDM. According to the total energy-momentum conservation, its change must be compensated by the corresponding change in the CDM sector [12]. The second view is that the equation of state for the holographic energy density is not fixed in general, but it is changing as the universe expands with/without the interaction [8, 9]. If an interaction is present between
two matters, one would be better to use the effective equation of state $\omega_{\Lambda}^{\text{eff}}$ than the native equation of state $\omega_{\Lambda}$. In the presence of an interaction, the first view leads to a constant EOS $\omega_{\Lambda}^{\text{eff}}$, while the second view implies a dynamical EOS $\omega_{\Lambda}^{\text{eff}}(\Omega)$ with density parameter $\Omega_i = \rho_i/\rho_c$.

In this work, we investigate the vacuum decaying of holographic energy density with the IR cutoff using the first view of the constant EOS. The key of our system is an interaction between holographic energy and CDM. They are changing as a result of energy transfer from holographic energy to the CDM. We call this the H($t$)CDM model, similar to the $\Lambda$($t$)CDM model [4]. Specifically, we present small perturbations under the case of a new matter production with $\omega_{\text{m}}^{\text{eff}} = \omega_{\Lambda}^{\text{eff}}$. In general, such a matter production may tend to dilute the matter contrast. For a decelerating universe of $\omega_{\text{m}}^{\text{eff}} > -1/3$, the matter contrast is growing as the universe evolves, while for an accelerating universe of $\omega_{\text{m}}^{\text{eff}} < -1/3$, the matter contrast is decaying, irrespective of the choice of IR cutoff. This shows the connection between the background evolution and matter contrast clearly.

II. THE MODEL

For a flat universe composed of cold dark matter and holographic energy density [15], the first Friedmann equation is given by

$$H^2 = \frac{1}{3m_p^2} (\rho_{\Lambda} + \rho_m) \quad (1)$$

where $H = \dot{a}/a$, $m_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass, and the holographic dark energy takes the form

$$\rho_{\Lambda} = \frac{3c^2 m_p^2}{L_{\Lambda}^2}. \quad (2)$$

The conservation law of the total energy-momentum leads to

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0 \quad (3)$$

where the overdot() denotes the derivative with respect to the cosmic time $t$. Here $\rho_T = \rho_m + \rho_{\Lambda}$ and $p_T = p_m + p_{\Lambda}$ with $p_m = 0$ and $p_{\Lambda} = -\rho_{\Lambda}$. We are interested in an interacting case with $q \neq 0$. In this case, the conservation law is split into two equations

$$\dot{\rho}_m + 3H(\rho_m + p_m) = q, \quad \dot{\rho}_{\Lambda} = -q. \quad (4)$$
For a choice of the interaction \( q = \epsilon H \rho_m \), the solution to the above equations is given by
\[
\rho_m = \rho_{m0} a^{-3(1-\epsilon/3)}, \quad \rho_\Lambda = \frac{\epsilon}{3-\epsilon} \rho_m.
\] (5)

Then, we obtain an explicit form of the solution:
\[
a(t) = \left[ \frac{\rho_{m0} \epsilon}{4m_p^2} \right]^{\frac{3-\epsilon}{3-\epsilon}}, \quad H = \frac{\dot{a}}{a} = \frac{2}{3-\epsilon} \frac{1}{t}.
\] (6)

Their energy densities take the forms
\[
\rho_m = \left[ \frac{4m_p^2}{3-\epsilon} \right] \frac{1}{t^2}, \quad \rho_\Lambda = \left[ \frac{4m_p^2 \epsilon}{(3-\epsilon)^2} \right] \frac{1}{t^2}.
\] (7)

Finally, the constant effective EOS could be read off from Eq.(5) as
\[
\omega_{\text{m eff}} = -\frac{\epsilon}{3} = \omega_{\text{\Lambda eff}}
\] (8)
which depends on the choice of IR cutoff. As will be shown in Fig. 3, \( \omega_{\text{m eff}}(c) \) depends critically on the choice of IR cutoff \( L_\Lambda \).

On the other hand, the Newtonian equation governing the evolution of density perturbations of CDM \[17, 18\] can be generalized in order to account for matter production. It is given by \[19\]
\[
\ddot{\delta} + \left( 2H + \frac{q}{\rho_m} \right) \dot{\delta} - \left[ \frac{\rho_m}{2m_p^2} - 2H \frac{q}{\rho_m} - \left( \frac{\dot{q}}{\rho_m} \right) \right] \delta = 0,
\] (9)
where \( \delta = \delta \rho_m / \rho_m \) is the density perturbations of the cold dark matter (matter contrast) and \( q \) is the source of matter production defined by Eq.(4). In the case of a constant \( \rho_\Lambda = \bar{\Lambda}^4(q = 0) \), \[9\] reduces to the usual non-relativistic equation for the linear evolution of the matter contrast. That is, it corresponds to the matter contrast of the \( \Lambda \)CDM model. In our case of the H(\( t \))CDM model, we have \( q = -\dot{\rho}_\Lambda = \epsilon H \rho_m \). Finally, the Newtonian equation of density perturbations with the holographic dark energy is given by
\[
\ddot{\delta} + \left[ \frac{4 + 2\epsilon}{3 - \epsilon} \right] \frac{\dot{\delta}}{t} - \left[ \frac{2}{3 - \epsilon} - \frac{8\epsilon}{(3 - \epsilon)^2} + \frac{2\epsilon}{3 - \epsilon} \right] \delta = 0.
\] (10)

At this stage, we point out the difference between our model and decaying vacuum cosmology \[20\]. The latter case has considered the \( \Lambda(t) \)CDM model with a different interaction \( q = \sigma H \).
III. NON-INTERACTING CASE: ΛCDM MODEL

First of all, we discuss the non-interacting holographic dark energy model with $q = 0$ in Eq.(4), which is identical to the ΛCDM model. In this case, we have the EOS of $\omega_\Lambda = -1$ for the holographic energy density like the cosmological constant and $\omega_m = 0$ for the CDM.

The evolution equation for the density perturbation is given by

$$\delta'' + \left[\frac{1}{2} - \frac{3}{2}\Omega_\Lambda(x)\right]\delta' - \frac{3}{2}\left[1 - \Omega_\Lambda(x)\right]\delta = 0,$$

(11)

where $'$ denotes derivative with respect to $x = \ln a$. The density parameter $\Omega_\Lambda(x)$ satisfies the background evolution equation,

$$\Omega_\Lambda' = 3\Omega_\Lambda(1 - \Omega_\Lambda),$$

(12)

while the first Friedmann equation is $\Omega_\Lambda + \Omega_m = 1$. The solution to this equation is given by

$$\Omega_\Lambda(x) = \frac{18}{7}\frac{e^{3x}}{1 + \frac{18}{7}e^{3x}},$$

(13)

where the numerical factor $\frac{18}{7}$ is chosen to fit the present condition of $\Omega_\Lambda(0) = 0.72$ and $\Omega_m(0) = 0.28$.

If there is no cosmological constant term, then Eq.(11) becomes

$$\delta'' + \frac{1}{2}\delta' - \frac{3}{2}\delta = 0,$$

(14)

FIG. 1: The density contrasts as a function of the redshift $z$ for ΛCDM and SCDM. The left panel represents for growing modes, while the right one denotes the decaying modes.
which describes the matter contrast of the standard cold dark matter (SCDM) with $\omega_m = 0$. The solution for this equation is given by

$$\delta(x) = C_1 e^x + C_2 e^{-3x/2} \tag{15}$$

with two constants $C_1$ and $C_2$. Obviously, the first term is a growing mode and the latter is a decaying one. This could be easily conjectured because of the purely decelerating phase of the CDM.

On the other hand, the general solution of $\Lambda$CDM to Eq.(11) is given by

$$\delta(x) = C_1 \frac{1}{\sqrt{\Omega_\Lambda(x)}} \int_{-\infty}^{x} \Omega_\Lambda^{3/2}(y) e^{-2y} dy + C_2 \frac{1}{\sqrt{\Omega_\Lambda(x)}}, \tag{16}$$

where $\Omega_\Lambda(x)$ is given by Eq.(13). The connection to the redshift $z$ is given by $x = -\ln(1+z)$ with $a = 1/(1+z)$. The first term corresponds to a growing solution, while the second is a decaying one. In this case, we obtain a growing mode even the universe is composed of the cosmological constant with $\omega_\Lambda = -1$ and the CDM with $\omega_m = 0$. This is possible because the early universe is the CDM-dominated phase. However, as is shown in Fig. 1, the growing rates for SCDM and $\Lambda$CDM are different. The growing rate for SCDM is greater than that of $\Lambda$CDM since the CDM-nature of $\Lambda$CDM decreases as the universe evolves (the universe becomes the dark energy-dominated phase).

**IV. H(t)CDM WITH HUBBLE HORIZON**

We choose the IR cutoff as Hubble horizon with $L_\Lambda = R_{HH} = 1/H$. In this case, one has $\epsilon = 3c^2 \frac{\dot{a}}{a}$. The evolution equation takes the form

$$2\dot{H} + 3(1-c^2)H^2 = 0. \tag{17}$$

The corresponding solution is given by

$$H = \frac{2}{C_1 + 3(1-c^2)t}, \quad a(t) = C_2 \left[ C_1 + 3(1-c^2)t \right]^{\frac{2}{3(1-c^2)}}, \tag{18}$$

where $C_1$ and $C_2$ are integration constants. Assuming the initial condition of $a = 0$ at $t = 0$, we obtain

$$a(t) = C_2 \left[ 3(1-c^2)t \right]^{\frac{2}{3(1-c^2)}}, \quad H = \frac{2}{3(1-c^2)} \frac{1}{t}. \tag{19}$$
The energy densities are given by
\[ \rho_\Lambda = 3c^2 m_p^2 H^2 = \frac{4c^2 m_p^2}{3(1 - c^2)} \frac{1}{t^2}, \quad \rho_m = \frac{4m_p^2}{3} \frac{1}{t^2}. \] (20)

We check that the ratio of two energy densities is fixed as
\[ \frac{\rho_m}{\rho_\Lambda} = \frac{1 - c^2}{c^2}. \] (21)

Finally, the effective EOS is given by
\[ \omega_m^{\text{eff}} = -\frac{\epsilon}{3} = -c^2 = \omega_\Lambda^{\text{eff}}. \] (22)

An important point to note is that the interaction between two matters is a mechanism to generate the matter production. In the case of a homogeneous production, the new matter tends to dilute the density perturbations, leading to a suppression of the density contrast. Plugging \( \epsilon = 3c^2 \) into Eq. (10), the Newtonian equation for matter contrast is
\[ \ddot{\delta} + \left[ \frac{4 + 6c^2}{3(1 - c^2)} \right] \frac{\delta}{t} - \left[ \frac{2}{3(1 - c^2)} - \frac{8c^2}{3(1 - c^2)^2} + \frac{2c^2}{(1 - c^2)^2} \right] \frac{\dot{\delta}}{t^2} = 0 \] (23)

In order to solve the above equation, we assume that \( \delta(t) = t^\alpha \). Then its exponents are given by
\[ \alpha_{\pm}(c) = \frac{1}{2} \left[ \frac{3(1 + c^2) + 4}{3(1 - c^2)} \right] \pm \sqrt{\left( \frac{4 + 3(9c^2 - 1)}{6(1 - c^2)} \right)^2 + \frac{2 - 2c^2(3c^2 + 1)}{3(1 - c^2)^2}}. \] (24)

![Graphs showing matter contrast as a function of redshift z for H(t)CDM with Hubble horizon. The left panel denotes the exponents \( \alpha_{\pm} \) and the right one denotes the matter contrasts for \( \alpha_+ \) with different \( c = 0.6, 1/\sqrt{3}, 0.3 \) from top to bottom.](image)
Fig. 2 shows that $\alpha_+$ is positive, zero, and negative for $c <, =, > 1/\sqrt{3}$, respectively, while $\alpha_-$ is always negative. Hence we find a growing mode of the matter contrast only for $0 < c < 1/\sqrt{3}$. This means that the decelerating phase leads to a growing mode, while the accelerating phase of $c > 1/\sqrt{3}$ implies a decaying mode. Finally, $c = 1/\sqrt{3}$ provides a constant mode.

V. H(t)CDM WITH PARTICLE HORIZON AND FUTURE EVENT HORIZONS

Choosing the particle horizon (PH) and future event horizon (FH) leads to

$$\epsilon_{PH/FH} = 1 + \frac{2}{3c^2} + \frac{2\sqrt{3e^2 + 1}}{3c^2}. \quad (25)$$

The effective EOS are given by

$$\omega_m^{\text{eff}} = \epsilon_{PH/FH} = -\frac{\epsilon_{PH/FH}}{3}. \quad (26)$$

The Newtonian equations take the forms

$$\ddot{\delta}_{PH/FH} + A_{PH/FH} \frac{\dot{\delta}_{PH/FH}}{t} - B_{PH/FH} \frac{\delta_{PH/FH}}{t^2} = 0 \quad (27)$$

with

$$A_{PH/FH} = \frac{4 + 2\epsilon_{PH/FH}}{3 - \epsilon_{PH/FH}}, \quad B_{PH/FH} = \frac{2}{3 - \epsilon_{PH/FH}} - \frac{8\epsilon_{PH/FH}}{(3 - \epsilon_{PH/FH})^2} + \frac{2\epsilon_{PH/FH}}{3 - \epsilon_{PH/FH}}. \quad (28)$$

Assuming that $\delta_{PH/FH} = t^{\alpha_{PH/FH}^{\pm}}$, we could obtain its exponents

$$\alpha_{PH/FH}^{\pm} = \frac{1 - A_{PH/FH}}{2} \pm \frac{\sqrt{(A_{PH/FH} + 1)^2 + B_{PH/FH}}}{2}. \quad (29)$$

We show its behavior in graphically because its forms are very complicated. As is shown in Fig. 3, the effective EOS with the particle horizon has a bound of $-1/3 \leq \omega_m^{\text{eff}} \leq 0$, which means that this case always provides the decelerating universe for any $c$. Hence we find from Fig. 4 that the growing modes $\delta_{PH}$ appears for any $c$ when choosing $\alpha_+$. On the other hand, as is shown in Fig. 3, the effective EOS with the future event horizon has a bound of $-1 \leq \omega_m^{\text{eff}} \leq -1/3$, which means that this case always provides the accelerating universe for $c \geq 1$. Hence we find from Fig. 5 that there is no growing mode whenever choosing $\alpha_\pm$. All of $\delta_{FH}$ belong to decaying modes. This is consistent with our
FIG. 3: The three effective EOS $\omega_m^{\text{eff}}$ as a function of the parameter $c$ for H(t)CDM with Hubble horizon (HH), particle horizon (PH), and future event horizon (FH), respectively. In the case with FH, we have a region of $0 < c < 1$, which may represent the phantom phase of $\omega_m^{\text{eff}} < -1$.

FIG. 4: The matter contrast as a function of the redshift for H(t)CDM with particle horizon (PH). The left panel denotes the exponents $\alpha_{\pm}$ and the right one denotes the matter contrasts for $\alpha_+$ of growing modes with different $c = 1, 2$.

conjecture. At this stage, we point out that the region of $0 < c < 1$ with the future event horizon may lead to the phantom phase (see Fig. 3). However, the Newtonian equations for matter contrast do not work for $0 < c < 1$. Hence we could not discuss the evolution of
FIG. 5: The matter contrast as a function of the redshift for H(t)CDM with future event horizon (FH). The left panel denotes the exponents $\alpha_{\pm}$ and the right one denotes the matter contrasts for $\alpha_+$ of decaying modes with different $c = 2, 3$. In the left panel, we have a forbidden region of $0 < c < 1$, which may represents the phantom phase $\omega_{m}^{\text{eff}} < -1$.

density perturbation in the background of phantom phase.

VI. CONCLUSIONS

For the fixed-ratio of energy densities, we have the constant effective EOS for the H(t)CDM models with Hubble, particle, and future event horizons. These indicate the different matter productions, as a result of decaying of holographic dark energy into CDM. In these cases, we have definite connections for the matter contrasts: For $\omega_{m}^{\text{eff}} \geq -1/3$, we have a growing mode, while for $-1 \leq \omega_{m}^{\text{eff}} < -1/3$, we have no growing mode. These show clearly how the matter contrasts evolve differently under the different matter productions. On the other hand, for the non-interacting case of variable ratio of energy densities, we have growing modes which reflects the CDM-dominated universe in the very early universe.

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