Anomalous $A$-dependence of
diffractive leptoproduction of radial
excitation $\rho'(2S)$

J.Nemchik$^{1,3}$, N.N.Nikolaev$^{2,3}$ and B.G.Zakharov$^2$

$^1$Institute of Experimental Physics, Slovak Academy of Sciences,
Watsonova 47, 043 53 Kosice, Slovak Republik
$^2$L.D.Landau Institute for Theoretical Physics, GSP-1, 117940,
ul.Kosygina 2, V-334, Moscow, Russia
$^3$IKP(Theorie), KFA Jülich, D-52425 Jülich, Germany

Abstract

We predict a strikingly different $A$ and $Q^2$ dependence of quasielastic leptoproduction of the $\rho^0(1S)$-meson and its radial excitation $\rho'(2S)$ on nuclei. Whereas for the $\rho^0$ production nuclear transparency $T_A$ decreases monotonically with $A$, for the $\rho'$ nuclear transparency $T_A$ can have the counterintuitive nonmonotonic $A$-dependence, having the minimum for light nuclei, and increasing with $A$ for medium and heavy nuclei. Strong enhancement of the $\rho'/\rho^0$ cross section ratio makes nuclear targets the $\rho'$-factory. The origin of the anomalous $A$-dependence is in the interplay of color transparency effects with the nodal structure of the $\rho'$ wave function. The predicted effects take place at moderate $Q^2 \lesssim (2-3)\text{GeV}^2$, which can be explored in high statistics experiments at CEBAF.

Submitted to Physics Letters B
E-mail: kph154@zam001.zam.kfa-juelich.de
The recent FNAL E665 experiment [1] on the exclusive virtual photoproduction of the \( \rho^0 \)-mesons on nuclei [1] confirmed the long anticipated effect of color transparency (CT): decrease of nuclear attenuation of the produced \( \rho^0 \) mesons with increasing virtuality \( Q^2 \) of photons [2,3]. Regarding the accuracy of the data, the potential of the CERN and FNAL muon scattering experiments is nearly exhausted. Fortunately, very high statistics experiments can be performed at CEBAF upgraded to the 8-12 GeV energy range [4]. (For discussion of the vector meson physics as the case for 10-20 GeV electron facility see [5], high energy aspects of leptoproduction of vector mesons are discussed in [3,6], for the recent review on CT see [7].) Apart from the much more detailed studies of the onset of CT in the \( \rho^0 \) production, the CEBAF experiments can open an entirely new window on CT, as due to high luminosity and excellent CLAS facility, an accurate measurement of production of the radially excited \( \rho' \)-meson becomes possible for the first time. As we shall demonstrate below, CT leads to a spectacular pattern of anomalous \( A \) and \( Q^2 \) dependence of the \( \rho' \) production.

The crucial observation is that the very mechanism of CT leads to a novel phenomenon of scanning the wave function of vector mesons [2,8,9]. Specifically, the amplitude of the free-nucleon reaction \( \gamma^* + N \rightarrow V + N \) can be written as

\[
M_N = \int d^2 \vec{r} \sigma(r) \int_0^1 dz \Psi_V^*(z, r) \Psi_V(z, r) .
\]

Here we use the wave functions of the \( q\bar{q} \) Fock states of the vector meson and the photon in the lightcone representation [10], where \( \vec{r} \) is the transverse separation of the quarks, and \( z \) is a fraction of the lightcone momentum carried by the quark. In Eq. (1), \( \sigma(r) \) is the cross section for interaction with the nucleon of the \( q\bar{q} \) color dipole of size \( r \). By virtue of CT, for the small-size color dipole [10,11]

\[
\sigma(r) \propto r^2 ,
\]

and the production amplitude (1) receives the dominant contribution from ([3] and see below)

\[
r \sim r_S = \frac{6}{\sqrt{m_V^2 + Q^2}} .
\]

The wave function of the radial excitation \( \rho'(2S) \) has a node. For this reason, in the \( \rho' \) production amplitude there is the nodal effect - cancellations between the contributions
from $r$ below, and above, the node, which depends on the scanning radius $r_S$. If the node effect is strong, even the slight variations of $r_S$ lead to an anomalously rapid variation of the $\rho'$ production amplitude, which must be contrasted to the smooth $Q^2$ and $r_S$ dependence of the $\rho^0(1S)$ production amplitude. The point which we wish to make in this communication is that apart from changing $Q^2$, one can also vary the effective scanning radius employing a CT property of stronger nuclear attenuation of the large-$r$ states. We predict a strikingly different, and often counterintuitive, $A$ (and $Q^2$) dependence of the $\rho'$ and $\rho^0$ production on nuclei, the experimental observation of which will shed new light on our understanding of the mechanism of CT.

We start with the derivation of the scanning radius $r_S$ Eq. (3). The most important feature of the photon wave function $\Psi_{\gamma^*}(r, z)$ is an exponential decrease at large distances [2,3,8-10]

$$\Psi_{\gamma^*}(r, z) \propto \exp(-\varepsilon r), \quad (4)$$

where

$$\varepsilon^2 = m_q^2 + z(1 - z)Q^2. \quad (5)$$

In the nonrelativistic quarkonium $z \sim \frac{1}{2}$, and the relevant $q\bar{q}$ fluctuations have a size

$$r \sim r_Q = \frac{1}{\sqrt{m_q^2 + \frac{1}{4}Q^2}} \approx \frac{2}{\sqrt{m_V^2 + Q^2}}. \quad (6)$$

The wave function of the vector meson is smooth at small $r$. Then, because of CT property Eq. (2), the integrand in Eq. (1) will be peaked at $r \sim r_S \approx 3r_Q$ as it is stated in Eq. (3). The product $\sigma(r)\Psi_{\gamma^*}(z, r)$ acts as a distribution which probes the wave function of the vector meson at the scanning radius $r \sim r_S$ [2,9]. The large numerical factor in the r.h.s. of Eq. (3) stretches the large-size dominance, and the simple nonrelativistic approximation $z \sim \frac{1}{2}$ remains viable in a broad range of $Q^2 \lesssim (3 - 5)\text{GeV}^2$ of the interest at CEBAF. (The including of the relativistic effects is straightforward, does not change the principal results, and shall not be discussed here.) In Fig.1 we show qualitatively, how this $Q^2$-dependent scanning works for the $\rho^0$ and $\rho'$ mesons. In our numerical calculations we use the dipole cross section $\sigma(r)$ of Ref. [10] and the simple harmonic oscillator model with the quark mass $m_q = 0.3\text{GeV}$ and $2\hbar\omega = 0.7\text{GeV}$. The radius of the $\rho^0$-meson is essentially identical to that of the pion, and within the model $\sigma_{\text{tot}}(\rho^0 N) \approx \sigma_{\text{tot}}(\pi N) \approx 25\text{mb}$.
For the Ψ′ and Υ′ production, the node effect is perturbatively tractable even for the real photoproduction $Q^2 = 0$. The prediction [8] of $\sigma(\gamma N \to \Psi' N)/\sigma(\gamma N \to J/\Psi N) = 0.17$ is in excellent agreement with the NMC result $0.20 \pm 0.05\text{(stat)} \pm 0.07\text{(syst)}$ for this ratio [12]. In this case the node effect is relatively weak. For the light mesons the scanning radius $r_S$ is larger and, at small $Q^2$, the node effect is much stronger. With the above described harmonic oscillator wave function, we find the $Q^2$ dependence of the $\rho'/\rho^0$ production ratio shown in Fig.2. It corresponds to the overcompensation scenario when, at $Q^2 = 0$, the $\rho'$ production amplitude is dominated by the contribution from $r$ above the node. As $Q^2$ increases and the scanning radius $r_S$ decreases, we encounter the exact node effect, the $\rho'/\rho^0$ production ratio has a dip at finite $Q^2$, and with the further increase of $Q^2$ and decrease of $r_S$, we have the undercompensation regime - the free-nucleon amplitude will be dominated by the contribution from $r$ below the node. The node effect will decrease with rising $Q^2$, see the pattern of the scanning shown in Fig. 1, and the $\rho'/\rho^0$ ratio will rise with $Q^2$.

At small $Q^2$ the scanning radius is large, and we are not in the domain of the perturbative QCD. What, then, are our firm predictions? Firstly, the strong node effect and the strong suppression of the real photoproduction of the $\rho'$ is not negotiable, and this prediction is consistent with the meagre experimental information (for the review see [13]). Secondly, the wave functions of the $\rho^0(1S)$ and $\rho'(2S)$ at the origin are approximately equal. Therefore, at very large $Q^2$ when $r_S$ is very small, we have a firm prediction that $d\sigma_{\rho'} \sim d\sigma_{\rho}$. In the above overcompensation scenario, the $\rho'/\rho^0$ ratio has a dip at finite $Q^2$. The position of this dip is model dependent, but the prediction of the steep rise of the $\rho'/\rho^0$ in the region of $Q^2 \lesssim (2-3)\text{GeV}^2$ is a firm consequence of the $Q^2$-dependence of the scanning radius, which is driven by CT. This is precisely the region where the high statistics data can be taken at CEBAF, and the experimental observation of such a dramatic large-distance manifestation of CT will be very important contribution to our understanding of the onset of CT.

The $\rho'$ production on nuclei is indispensable for testing the node affect, as nuclear attenuation gives still another handle on the scanning radius. For the sake of definiteness, we discuss the quasielastic (incoherent) $\rho'$ production on nuclei, extension to the coherent production is straightforward and will be presented elsewhere.
At high energy \( \nu \), the (virtual) photon forms its \( q\bar{q} \) Fock state at a distance (the coherence length) in front of the target nucleus (nucleon)

\[
l_c = \frac{2\nu}{Q^2 + m_V^2}.
\]

(7)

After interaction with the target, the \( q\bar{q} \) pair recombines into the observed vector meson \( V \) with the formation (recombination) length

\[
l_f = \frac{\nu}{m_V \Delta m},
\]

(8)

where \( \Delta m \) is the typical level splitting in the quarkonium. At low energy and \( l_f \ll R_A \), where \( R_A \) is a radius of the target nucleus, recombination of the \( q\bar{q} \) pair into the vector meson takes place well inside the nucleus, nuclear attenuation will be given by the free-nucleon \( VN \) cross section and CT effect disappears. The condition \( l_f \gtrsim R_A \) is crucial for the onset of CT, and for the \( \rho^0, \rho' \) system it requires

\[
\nu \gtrsim (3-4) \cdot A^{1/3} \text{GeV},
\]

(9)

which for light and medium nuclei is well within the reach of the 8 – 12GeV upgrade of CEBAF. (In this energy range we can also limit ourselves to the contribution from the lowest \( q\bar{q} \) Fock states of the photon and vector meson.) In the opposite to that, the condition \( l_c \gtrsim R_A \) is not imperative for the onset of CT. Nuclear attenuation effects only increase somewhat when \( l_c \) increases from \( l_c \ll R_A \) to \( l_c \gtrsim R_A \) [2,14]. The finite-energy effects can easily be incorporated using the path integral technique developed in [8,2]. For the sake of simplicity, in this paper we concentrate on the high-energy limit of \( f_f, l_c \gtrsim R_A \), when nuclear transparency \( T_A = d\sigma_A/A\sigma_N \) equals [2,9,14]

\[
T_{RA} = \frac{1}{A} \int d^2b T(b) \frac{\langle V|\sigma(r)\exp\left[-\frac{1}{2}\sigma(r)T(b)\right]|\gamma^*\rangle^2}{\langle V|\sigma(r)|\gamma^*\rangle^2}.
\]

(10)

Here \( T(b) = \int dz n_A(b,z) \) is the optical thickness of a nucleus, where \( n_A(b,z) \) is the nuclear matter density (For the compilation of the nuclear density parameters see [15]). The \( A \)-dependence of the node effect comes from the nuclear attenuation factor \( \exp[-\frac{1}{2}\sigma(r)T(b)] \) in the nuclear matrix element \( M_A(T) = \langle V|\sigma(r)\exp\left[-\frac{1}{2}\sigma(r)T(b)\right]|\gamma^*\rangle \).
Firstly, consider the value of $Q^2$, at which the cross section for the $\rho'$ production on the free nucleon takes its minimal value because of the exact node effect. Because of the $r$-dependence of the attenuation factor, in the nuclear amplitude the node effect will be incomplete. Consequently, as a function of $Q^2$, nuclear transparency $T_A$ will have a spike $T_A \gg 1$ at a finite value of $Q^2$ [2].

Secondly, consider the $\rho'$ production on nuclei at a fixed value of $Q^2$ such that the free nucleon amplitude is still in the overcompensation regime. Increasing $A$ and enhancing the importance of the attenuation factor $\exp[-\frac{1}{2}\sigma(r)T(b)]$, we shall bring the nuclear amplitude to the nearly exact compensation regime. Therefore, the $\rho'/\rho^0$ production ratio, as well as nuclear transparency for the $\rho'$ production, will decrease with $A$ and take a minimum value at a certain finite $A$. With the further increase of $A$, the undercompensation regime takes over, and we encounter very counterintuitive situation: nuclear transparency for the $\rho'$ is larger for the strongly absorbing nuclei! This situation is illustrated in Fig. 3a and must be contrasted with a smooth and uneventful decrease of transparency for the $\rho^0$ production on heavy nuclei.

Evidently, the possibility of the perfect node effect in $M_A(T)$ depends on the optical thickness $T(b)$. This is shown in Fig. 4, in which we present the relative nuclear matrix element

$$R(T) = \frac{\langle \rho' | \sigma(r) \exp[-\frac{1}{2}\sigma(r)T(b)] | \gamma^* \rangle}{\langle \rho' | \sigma(r) | \gamma^* \rangle}.$$  \hspace{1cm} (11)

At large impact parameter $b$, at the periphery of the nucleus, the optical thickness of the nucleus is small, the overcompensation in the nuclear matrix element is the same as for the free nucleon, and we have $R(T) = 1$. At smaller impact parameters, one encounters the exact node effect: $R(T) = 0$. At still smaller impact parameters, the overcompensation changes for the undercompensation and $R(T) < 0$. Here the breaking of the compensation and the overall attenuation start competing. For very heavy nuclei and small impact parameters, the overall attenuation takes over and $R(T)$ starts decreasing again. For the particular case of the real photoproduction, and for the specific wave function of the $\rho'$ meson, in the undercompensation domain we have $|R(T)| \leq 1$. The resulting $A$ dependence of nuclear transparency $T_A$ is shown in Fig. 3a. It takes the minimum value for the $^7Li$ target, then
increases with $A$, flattens and starts decreasing for very heavy nuclei.

In Fig. 3 we show how nuclear transparency for the $\rho'$ varies with $Q^2$. The slight increase of $Q^2$ up to $Q^2 = 0.2\text{GeV}^2$ and the slight change of the scanning radius enhance the node effect in the free nucleon amplitude, see Fig. 2. In this case, we find almost exact node effect for the $^4\text{He}$ target, which is followed by the dominant undercompensation regime for heavier nuclei (Fig. 3b). Also, in this case the undercompensation regime for heavy nuclei is followed by $|R(T)| > 1$, which leads to significant antishadowing $T_A > 1$.

With the further increase of $Q^2$ one enters the pure undercompensation regime for all the targets. Nuclear suppression of the node effect enhances $M_A$ and nuclear transparency $T_A$, whereas the overall attenuation factor $\exp[-\frac{1}{2}\sigma(r)T(b)]$ decreases $T_A$. Of these two competing effects, the former remains stronger and we find antishadowing of the $\rho'$ production in a broad range of $A$ and $Q^2$, see Figs. 3c-e. Typically, we find a nuclear enhancement of the $\rho'/\rho^0$ production ratio on heavy targets by one order in the magnitude with respect to the free nucleon target. This makes leptoproduction on nuclei the $\rho'$ factory, and the $\rho'$ production experiments at CEBAF can contribute much to the poorly understood spectroscopy of the radially excited vector mesons. Only at a relatively large $Q^2 \gtrsim 2\text{GeV}^2$, the attenuation effect takes over, and nuclear transparency for the $\rho'$ production will start decreasing monotonically with $A$ (Fig. 3f). Still, this decrease is much weaker than for the $\rho^0$ meson. At very large $Q^2$, when the node effect disappears because of the small scanning radius $r_S$, nuclear transparency for the $\rho^0$ and the $\rho'$ production will become identical. This pattern repeats qualitatively the one studied in [2,8] for the $J/\Psi$ and $\Psi'$ mesons.

Notice, that for the $\rho^0$-production, the nuclear transparency is a very slow function of $Q^2$. For instance, for the lead target $T_{Pb}(Q^2 = 0) \approx 0.1$ and $T_{Pb}(Q^2 = 2\text{GeV}^2) \approx 0.23$. The reason for this slow onset of color transparency is that because of large numerical factor in the r.h.s of Eq. (3), even at $Q^2 = 2\text{GeV}^2$ the scanning radius is still large, $r_S \sim 0.8f$. Furthermore, the more detailed analysis [3] has shown that nuclear shadowing is controlled by a still larger $r \sim 5r_Q$. Our numerical predictions [2,3] for the $Q^2$ dependence of the incoherent and coherent $\rho^0$ production on nuclei are in excellent agreement with the E665 data.
The above presented results refer to the production of the transversely polarized $\rho^0$ and $\rho'$ mesons. Accurate separation of production of the transverse and longitudinal cross section can easily be done in the high statistics CEBAF experiments. Here we only wish to mention the interesting possibility that for the longitudinally polarized $\rho'$ mesons, the exact node effect can take place at a value of $Q^2$ different from that for the transversely polarized $\rho'$ mesons, and polarization of the produced $\rho'$ can exhibit very rapid change with $Q^2$.

Discussion of the results and conclusions:

We presented the strong case for the anomalous $Q^2$ and $A$ dependence of incoherent production of the $\rho'$ meson on nuclear targets. The origin of the effect is in the $Q^2$ dependence of the scanning radius $r_S$, which follows from color transparency property in QCD. At the relatively small values of $Q^2$ discussed in this paper, the scanning radius is rather large, of the order of the size of the $\rho^0$ meson. For this reason, the $\rho'$ production amplitude proves to be extremely sensitive to the nodal structure of the $\rho'$ wave function. Specifically, the node effect leads to a strong suppression of the $\rho'/\rho^0$ ratio in the real photoproduction, which is consistent with the meagre experimental information on the production of radially excited vector mesons.

In the overcompensation scenario suggested by the nonrelativistic oscillator model, the most striking effect is the nonmonotonic $A$-dependence of nuclear transparency shown in Fig. 3. The numerical predictions are very sensitive to the position of the node in the wave function of $2S$ states. It is quite possible that the dip of nuclear transparency $T_A$ will take place for targets much heavier than in Figs. 3a,b, and disappearance of the node effect and the onset of the more conventional nuclear shadowing $T_A < 1$ for the $\rho'$ production like in Fig. 3f only will take place at much larger $Q^2$. Also, the possibility of the undercompensation at $Q^2 = 0$ can not be excluded. However, the strikingly different $A$-dependence of the incoherent $\rho^0$ and $\rho'$ production on nuclei persists in such a broad range of $Q^2$ and of the scanning radius $r_S$, that the existence of the phenomenon of anomalous $A$ and $Q^2$ dependence of the $\rho'$ production is not negotiable. It is a direct manifestation of the color-transparency driven $Q^2$ dependence of the scanning radius and, as such, it deserves a
dedicated experimental study.

For the sake of simplicity, here we concentrated on the high energy limit in which the theoretical treatment of nuclear production greatly simplifies. One can easily go beyond the frozen-size approximation employing the path-integral technique [2,8]. The detailed analysis performed in [2] shows that at energies of the virtual photon $\sim 6 - 10\text{GeV}$, the subasymptotic corrections do not change much the predictions for nuclear transparency. This is the energy range which can be reached at CEBAF after the energy upgrade.

Few more comments about the possibilities of CEBAF are worth while. Because of the strong suppression of the $\rho'/\rho^0$ production ratio, the high luminosity of CEBAF is absolutely crucial for high-statistics experiments on the $\rho'$ production. Notice, that the most interesting anomalies in the $A$ and $Q^2$ dependence take place near the minimum of the $\rho'$ production cross section. Furthermore, the observation of the $\rho'$ production requires detection of its 4-pion decays, and here one can take advantage of the CLAS multiparticle spectrometer available at CEBAF. Finally, similar effects must persist also for the $\omega'$ and $\phi'$ production, and also in the coherent production of the radially excited mesons.

**Acknowledgements:**

One of the authors (J.N.) is grateful to J.Speth for the hospitality at Institut f. Kernphysik, KFA Jülich, where this work was completed. N.N.N. thanks R.Holt and N.Isgur for discussions during the workshop on CEBAF at Higher Energies.
Figure captions:

• Fig.1. - The qualitative pattern of the the $Q^2$-dependent scanning of the wave functions of the ground state $V(1S)$ and the radial excitation $V'(2S)$ of the vector meson. The scanning distributions $\sigma(r)\Psi_\gamma^*(r)$ shown by the solid and dashed curve have the scanning radii $r_S$ differing by a factor 3. The wave function and radius $r$ are in arbitrary units.

• Fig.2. - The $Q^2$ dependence of the $\rho'(2S)/\rho^0(1S)$ ratio of forward production cross sections.

• Fig.3. - The $Q^2$ and $A$ dependence of nuclear transparency for the $\rho^0(1S)$ and $\rho'(2S)$ electroproduction on nuclei.

• Fig.4. - The impact parameter dependence of the reduced nuclear matrix element $R(T)$ for the real photoproduction of the $\rho'(2S)$.
References

[1] E665 Collaboration: G.Fang, Talk at the PANIC, Perugia, July 1993, and private communication.

[2] B.Z.Kopeliovich, J.Nemchik, N.N.Nikolaev and B.G.Zakharov, Phys. Lett. B309 (1993) 179.

[3] B.Z.Kopeliovich, J.Nemchik, N.N.Nikolaev and B.G.Zakharov, Phys. Lett. B324 (1993) 469.

[4] Workshop on CEBAF at Higher Energies, 14-16 April 1994, CEBAF, Talks by G.Fang, M.Kossov, S.Brodsky, A.Mueller and N.N.Nikolaev.

[5] N.N.Nikolaev and J.Speth, The ELFE Project: an Electron Laboratory for Europe, edited by J.Arvieux and E.De Sanctis, Editrice Compositori, Bologna (1993) pp.179-212.

[6] S.J.Brodsky et al., SLAC-PUB-6412R (1994).

[7] N.N.Nikolaev and B.G.Zakharov, Color transparency after the NE18 and E665 experiments, Juelich preprint KFA-IKP(TH)-93-30, in : Proceedings of the Workshop on Electron Nucleus Scattering, Marciana Marina, Elba, 5-10 July 1993, edited by A.Fabrocini, World Scientific (1994).

[8] B.Z.Kopeliovich and B.G.Zakharov: Phys. Rev. D44 (1991) 3466.

[9] N.N.Nikolaev, Quantum Mechanics of Color Transparency. Comments on Nuclear and Particle Physics, 21 (1992) 41.

[10] N.N.Nikolaev and B.G.Zakharov, Z. Phys. C49 (1991) 607; C53 (1992) 331.

[11] A.B.Zamolodchikov, B.Z.Kopeliovich and L.I.Lapidus, JETP Lett. 33 (1981) 595; G.Bertsch, S.J.Brodsky, A.S.Goldhaber and J.R.Gunion, Phys. Rev. Lett. 47 (1981) 267.

[12] P.Amaudruz et al., Nucl. Phys. B371 (1992) 3.
[13] T.H.Bauer et al., Rev. Mod. Phys. 50 (1978) 261; A.Donnachie and H.Mirzaie, Z. Phys. C33 (1987) 407; A.Donnachie and A.B.Clegg, Z. Phys. C34 (1987) 257; C40 (1980) 313; C42 (1989) 663; C45 (1990) 677.

[14] O. Benhar, B.Z. Kopeliovich, Ch. Mariotti, N.N. Nikolaev and B.G. Zakharov, Phys. Rev. Lett. 69 (1992) 1156.

[15] H.de Vries, C.W.de Jager and C.de Vries, Atomic Data and Nuclear Data Tables 36 (1987) 495.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9405025v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9405025v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9405025v1
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9405025v1