NUCLEON FORM FACTORS IN A RELATIVISTIC THREE-QUARK MODEL

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Abstract

We report the calculation of electromagnetic form factors of nucleons within a relativistic three-quark model with Gaussian shape for the nucleon-quark vertex. The allowed region for two adjustable parameters, the range parameter $\Lambda_N$ in the Gaussian and the constituent quark mass $m_q$, is obtained from fitting the data for magnetic moments and electromagnetic radii of nucleons. It is found that their values calculated with $m_q=420$ MeV and $\Lambda_N=1.25$ GeV agree very well with the experimental data.

It is turned out that the electric proton and magnetic nucleon form factors fall faster than the dipole fit of the experimental data for momentum transfers $0 \leq Q^2 \leq 1$ GeV$^2$.

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1 Introduction

Electromagnetic properties of nucleons are an important source of information on the internal structure of baryons. The success of the nonrelativistic quark model for the description of static characteristics (masses, magnetic moments, etc.) and the results from deeply inelastic lepton scattering are a clear indication for the three-quark structure of nucleons.

In view of the difficulties of describing the nucleon as a relativistic three quark system rigorously, on the basis of QCD, many methods and models have been developed which implement important aspects of QCD at least partially. We mention a few. Some approaches [1, 2] are formulated using classical hadronic degrees of freedom (color singlets) without explicit reference to quarks and gluons. This holds in particular for chiral perturbation theory which addresses low energy scattering of hadrons [3]. In the Skyrmion model the baryon is considered to be a solitonic excitation of classical meson fields which are described by an effective chiral Lagrangian, see [4], e.g. The electromagnetic radii and magnetic moments of nucleons as well as the behavior of electromagnetic form factors up to 1 GeV$^2$ are described in this way.

In the context of quark models the nonrelativistic approximation is problematic even for the constituent-quark picture at low energies since the effective masses and the intrinsic momenta have the same order of magnitude. An attempt to implement relativistic invariance for the description of the electromagnetic properties of the
nucleon is the covariant constituent quark models by Konen and Weber [5], and Chung and Coester [6] which use light front dynamics for the constituent quarks. In this framework the available data for $0 \leq Q^2 \leq 1.5 \text{ GeV}^2$ [5] and $0 \leq Q^2 \leq 6 \text{ GeV}^2$ [6] have been well described with two adjustable parameters (constituent quark mass and confinement scale).

For large momentum transfers, $Q^2 >> m_N^2$, perturbative QCD predicts that the magnetic form factor behaves as $G_M \sim 1/Q^4$ [7]. The use of QCD sum rules allows to extrapolate the electromagnetic form factors to low and moderate $Q^2$ by incorporating local quark-hadron duality [8]. Recall that the calculation of hadronic form factors by sum rules in their original version [9] is restricted to intermediate momentum transfers and does not work in the infrared region due to power corrections $1/Q^{2n}$. Therefore, in order to calculate magnetic moments of nucleons QCD sum rules in the external field approach have been introduced [10]. For a general method of calculating the nucleon magnetic form factors at small $Q^2$ see [11].

For applications at high energies the diquark model [12] has been proposed. In this model the proton is built from quarks and diquarks and the diquark is treated as a quasi-elementary particle. Fits to the data for $Q^2 > 3 \text{ GeV}^2$ [13] with few parameters have been obtained.

A different line of attack uses both quark and hadronic degrees of freedom. Chiral potential models (see, for example, [14] and references therein) fall into this class. They use an effective confining potential for the quarks and a quark pion interaction which preserves chiral symmetry. Incorporating the lowest-order pionic correction, the magnetic moments of the nucleon octet have been calculated [14]. Alternative approaches [14] start from effective Lagrangians which describe the transition of hadrons to their constituent quarks, combined with some assumptions on the behavior of quarks at low energies and on the shape of the hadron-quark form factor. This approach is connected fairly directly to the difficult issues of hadronization and quark confinement in QCD.

In the present paper we extend the line of thought developed in [18] and [19] which use local hadron-quark or local hadron-quark-diquark vertices together with a confined quark propagator (no poles). It has been shown for these models that the electromagnetic form factors of nucleons tend to be somewhat above the data for $Q^2 < 1 \text{ GeV}^2$. This might be a hint that the assumption of a local coupling of baryons with their constituents is not quite adequate. Introducing a nonlocal hadron-quark vertex may be a good way of effectively introducing other degrees of freedom (mesonic cloud, soft gluons, etc.). The Nambu-Jona-Lasinio models, NJL, with separable interactions are candidates for this purpose. Many successful applications of such models for low-energy pion physics exist (see, e.g., [15, 16] and references therein).

In [20, 21] the Lagrangian formulation of the NJL model with separable interaction has been given both for mesons and (for the first time) for baryons. The gauging of nonlocal interactions has been done by using the time-ordering P-exponent. The pion weak decay constant, the two-photon decay width, as well as the form factor of the $\gamma^*\pi^0 \rightarrow \gamma$ transition, the pion charge form factor, and the strong $\pi NN$ form factor have been calculated and good agreement with the data has been achieved with three parameters, the range parameters characterizing the size of mesons $\Lambda_M$ and baryons $\Lambda_B$, and the constituent quark mass $m_q$. In this talk we report the results for the electromagnetic form factors of nucleons [22] within the approach developed in [20, 21]. First, we slightly modify the gauging of the nonlocal quark-hadron vertex by using a path-independent definition for the derivative of the time-ordering
Second, we derive Feynman rules for the diagrams which describe the electromagnetic nucleon form factors. For simplicity, we use a Gaussian shape for the nucleon-quark vertex. The permissible range for the two adjustable parameters, the Gaussian range $\Lambda$ of the separable interaction and the constituent quark mass $m_q$, is obtained by fitting the experimental values of the magnetic moments and the electromagnetic radii. It is found that their values calculated with $m_q=420$ MeV and $\Lambda_N=1.25$ GeV agree very well with the experimental data.

It is turned out that the electric proton and magnetic nucleon form factors fall faster than the dipole fit of the experimental data for momentum transfers $0 \leq Q^2 \leq 1$ GeV$^2$.

2 Model

We start with a brief review of our approach. We consider the hadron as being a bound state of relativistic constituent quarks with masses $m_q$ \[20\]-\[22\]. The transition of hadrons into constituent quarks and vice versa is described by the corresponding interaction Lagrangian. The interaction Lagrangian of nucleons with quarks is written as

$$\mathcal{L}^{\text{int}}_N(x) = \bar{N}(x) \int d\xi_1 \int d\xi_2 F(\xi_1^2 + \xi_2^2) \times \sum_{I=V,T} g^I_N J^I_N(x, x + \xi_1 - \sqrt{3}\xi_2, x + \xi_1 + \sqrt{3}\xi_2) + \text{h.c.}$$

with $J^V_N$ and $J^T_N$ being the vector and tensor currents, respectively. The currents are symmetric under permutation of any two quarks

$$J^V_N(y_1, y_2, y_3) = \bar{\tau} \gamma^\mu \gamma^5 q^{a_1}(y_1) q^{a_2}(y_2) \gamma_2 \tau C \gamma^\mu q^{a_3}(y_3) \epsilon^{a_1 a_2 a_3},$$

$$J^T_N(y_1, y_2, y_3) = \bar{\tau} \sigma^{\mu\nu} \gamma^5 q^{a_1}(y_1) q^{a_2}(y_2) \gamma_2 \tau C \sigma_{\mu\nu} q^{a_3}(y_3) \epsilon^{a_1 a_2 a_3}.$$

The notation for the spin-flavour structure of the nucleon currents is the same as in \[10\]. It was shown \[13\], \[19\] that the tensor current $J^T_N$ is more suitable for the description of the data. For this reason we will use the tensor current in the approach developed in this paper. The nucleon-three-quark coupling $g^N_T$ is calculated from the compositeness condition \[20\]-\[22\] which means that the renormalization constant of the nucleon wave function is equal to zero, $Z_N = 1 - \Sigma_N(m_N) = 0$, with $\Sigma_N$ being the nucleon mass operator.

The momentum distribution of the constituents in nucleons is described by an effective relativistic vertex function $F(\xi_1^2 + \xi_2^2)$. Its shape is chosen to guarantee ultraviolet convergence. At the same time the vertex function is a phenomenological description of the long distance QCD interactions between quarks and gluons. The choice of variables in the vertex function implies the use of the center of mass frame. In this work a Gaussian vertex function is used

$$F(\xi_1^2 + \xi_2^2) = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \exp(-ik_1 \xi_1 - ik_2 \xi_2) F(k_1^2 + k_2^2) \exp \left( \frac{k_1^2 + k_2^2}{\Lambda_N^4} \right)$$

where $\Lambda_N$ is the Gaussian range parameter which is related to the size of the nucleon.

The interaction of quarks with the electromagnetic field is introduced by the standard minimal substitution in the free Lagrangian. The gauging of nonlocal interactions is done by using the time-ordering P-exponent in the (1). See details.
in refs. [20]-[22]. It leads to the modification of the interaction Lagrangian which generates nonlocal vertices which couple nucleons, photons and quarks. Therefore, the electromagnetic form factors of nucleons are described by the standard triangle diagram (Fig.1a) and, additionally, by the bubble or contact diagrams (Fig.1b and Fig.1c).

We choose the standard form of quark propagator $S_q(k) = (m_q - k)^{-1}$, corresponding to a free fermionic field with mass $m_q$. For the time being we shall avoid the appearance of unphysical imaginary parts in the Feynman diagrams by restricting the calculations to $m_N < 3m_q$. Thus, there are two adjustable parameters in our model: the constituent quark mass $m_q$ and the range parameter $\Lambda_N$. We shall determine them by fitting the electromagnetic properties of nucleons.

### 3 Numerical Results and Discussion

Within the relativistic quark model described above we shall evaluate the magnetic moments, the charge radii and the behavior of the electric and magnetic form factors for $0 \leq Q^2 \leq 1$ GeV$^2$. It is convenient to separate the contributions from the triangle diagram (Fig.1a) and the bubble diagrams (Fig.1b and 1c) as follows

$$\Lambda_\mu(p, p') = \frac{q_\mu}{q^2} [\Sigma_N(p) - \Sigma_N(p')] + \Lambda_{\mu,\Delta}^+(p, p') + \Lambda_{\mu,\text{bubble}}^+(p, p')$$  \hspace{1cm} (2)

The functions $\Lambda_{\mu,\Delta}^+(p, p')$ and $\Lambda_{\mu,\text{bubble}}^+(p, p')$ are the modified triangle and bubble vertex functions which are orthogonal to the photon momentum $q^\mu \Lambda_\mu(p, p') = 0$. We shall work in the limit of isospin invariance where the masses of $u$ and $d$ quarks are equal. In a first step we set the parameters of our model $m_q$ and $\Lambda_N$ to the well measured static properties of the nucleons, to the magnetic moments and to the charge radii [26, 27]. It was found that the best fit of data is obtained for $m_q = 420$ MeV and $\Lambda_N = 1.25$ GeV. In Table 1 the best fit of the nucleon static properties is compared to other theoretical approaches.
Table 1. Static Properties Compared to Theoretical Approaches

| Approach       | $\mu_p$ | $\mu_n$ | $r_E^p$, fm | $<r^2>_E^n$, fm$^2$ | $r_M^p$, fm | $r_M^n$, fm |
|----------------|---------|---------|-------------|---------------------|-------------|-------------|
| Our            | 2.79    | -1.86   | 0.92        | -0.132              | 0.84        | 0.84        |
| Ref. [10]      | 2.96    | -1.93   |             |                     |             |             |
| Ref. [28]      | 2.80    | -1.95   |             |                     |             |             |
| Ref. [4]       | 2.77    | -1.84   | 0.97        | -0.25               | 0.94        | 0.94        |
| Ref. [29]      | 2.811   | -1.848  | 0.811       | -0.094              | 0.825       | 0.781       |
| Ref. [14]      | 2.73    | -1.975  | 0.85        |                     |             |             |
| Exp.           | 2.79    | -1.91   | 0.86±0.01   | -0.119±0.004        | 0.86±0.06   | 0.88±0.07   |

In a second step we calculate the nucleon electromagnetic form factors for the range $Q^2 \leq 1 \text{ GeV}^2$. The shape of the electromagnetic nucleon form factors in the region $0 \leq Q^2 \leq 1 \text{ GeV}^2$ for the same optimal values of the parameters, $m_q=420 \text{ MeV}$ and $\Lambda_N=1.25 \text{ GeV}$, is shown in Figs.2-5. The normalized magnetic form factors of nucleons $G_N^M(Q^2)/G_N^M(0)$ are plotted in Fig.2 and Fig.3.

The short dashed line corresponds to the contribution of the triangle vertex function $\Lambda_{\mu,\Delta}(p,p')$ whereas the medium-long dashed line gives the contribution of the bubble or contact term $\Lambda_{\mu,\text{bubble}}(p,p')$. This contribution is roughly constant and not large. It matters however for the details of the fitting procedure. The total contribution is marked by the solid line. The long-dashed line is the dipole approximation $D(Q^2) = [1 + Q^2/0.71\text{GeV}^2]^{-2}$ which fits the data taken from [30] quite well.

The electric form factors of the proton and the neutron are shown in Fig.4 and 5, respectively. It is seen that the electric proton and magnetic nucleon form factors fall faster than the dipole fit of the experimental data for momentum transfers $0 \leq Q^2 \leq 1 \text{ GeV}^2$. (The behavior of our form factors scaled by the dipole fit is shown in Fig.6).

First, one has to remark that the Dirac and Pauli form factors have similar behavior in an approach based on calculation of the Feynman quark digram like in Fig. 1 disregarding to the choice of the vertex function because of they are defined by the $Q^2$-dependence of the vector and tensor functions, respectively. It gives that the electric proton form factor goes to zero at $Q^2 \approx 1.5 \text{ GeV}^2$. Second, both the Dirac and Pauli form factors are turned out to fall faster than the dipole approximation. This may be seen as the result either of using the Gaussian shape for vertex function used here or neglecting the contribution of the pionic cloud.

4 Conclusion

The electromagnetic form factors of nucleons have been calculated within a relativistic three quark model with Gaussian shape for the nucleon-quark vertex, and standard (non-confined) quark propagators. Gauge invariance of the nonlocal hadron-quark interaction has been implemented by the path-independent definition for the derivative of the time-ordering P-exponent. The allowed region for the two adjustable parameters, the range parameter $\Lambda_N$ appearing in the Gaussian and the constituent quark mass $m_q$, has been obtained by fitting the data for the magnetic moments and the electromagnetic radii of the nucleons. It is found that their
Fig. 2. Magnetic form factor of proton.

Fig. 3. Magnetic form factor of neutron.

Fig. 4. Electric form factor of proton.

Fig. 5. Electric form factor of neutron.
values calculated with $m_q=420$ MeV and $\Lambda_N=1.25$ GeV agree very well with the experimental data.

It is turned out that the electric proton and magnetic nucleon form factors fall faster than the dipole fit of the experimental data for momentum transfers $0 \leq Q^2 \leq 1$ GeV$^2$. This may be seen as the result either of using the Gaussian shape for vertex function used here or neglecting the contribution of the pionic cloud.

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