LEADING $\ln(1/x)$ AND HEAVY QUARK CORRECTIONS IN STRUCTURE FUNCTIONS

R. S. THORNE
Department of Physics, Theoretical Physics, 1 Keble Road,
Oxford OX1 3NP, England
E-mail: thorne@v2.rl.ac.uk

I present a calculation of structure functions at leading order which includes an unambiguous inclusion of the leading $\ln(1/x)$ terms for each power of $\alpha_s$, and also the correct effects due to the mass of the charm and bottom quarks. I compare the results of fits to data to those obtained using conventional NLO in $\alpha_s$ calculations, noting a clear preference for my approach, especially at small $x$. The predictions for both the charm structure function and $F_L(x, Q^2)$ using the two approaches are compared, the latter being much more discriminating.

1 Introduction

There has recently been a great deal of theoretical activity concerning the calculation of structure functions at small $x$, driven by the vast increase in data in this region obtained at HERA\cite{1,2,3}. One of the main questions is whether one should include potentially important leading $\ln(1/x)$ terms at high orders in $\alpha_s$, or simply order in powers of $\alpha_s$ alone. In a number of previous publications I showed that it is possible to include the leading $\ln(1/x)$ terms within the conventional renormalization group approach in an unambiguous manner at leading order.\cite{3} The method of calculation is based on 3 points.

1. The quantities one calculates to a given order are directly observable. Hence, I calculate in terms of the structure function at a particular scale ($Q_I^2$) and its evolution away from this scale.\cite{4}

2. The leading–order expression for each independent part of a physical quantity begins at its lowest power of $\alpha_s$, i.e. if a term $\ln^m(1/x)$ first appears at order $\alpha_s^n$ this is the leading order for this form of $x$ dependence.

3. The inputs for the structure functions are two flat nonperturbative functions for $F_L$ and $F_2$, convoluted with calculable perturbative contributions. These perturbative parts are determined by demanding that the expressions for the structure functions are invariant under changes of starting scale order by order in $\alpha_s$, and are hence determined by the perturbatively controlled evolution.

This leads to a leading–order–renormalization–scheme–consistent (LORSC) calculation of structure functions, which naturally combines the leading $\ln(1/x)$ expansion with the more conventional $\alpha_s$ expansion. When a global fit was

\footnote{This results in expressions in terms of Catani’s “physical anomalous dimensions”\cite{5}.}
Table 1: Quality of fit using the LORSC(H) and the NLO–in–\(\alpha_s\) (MRST) approaches.

| \(x\)-range | data points | LORSC(H) \(\chi^2\) | MRST \(\chi^2\) |
|-------------|-------------|----------------------|----------------|
| \(x \geq 0.1\) | 597         | 688                  | 682            |
| \(0.1 > x \geq 0.01\) | 385         | 382                  | 377            |
| \(x < 0.01\) | 278         | 219                  | 272            |
| total       | 1260        | 1289                 | 1332           |

performed the quality using the LORSC calculation was superior to that using a conventional NLO–in–\(\alpha_s\) calculation, particularly at small \(x\).

2 Implementation of Heavy Quarks

The main problem with this previous analysis was that it used a very naive prescription for heavy quarks, i.e. the charm and bottom quarks were both treated as infinitely massive below thresholds \(Q^2 = m_H^2\), and as massless above these thresholds. With the direct data on the charm structure function\(^5\),\(^6\),\(^7\), and the fact that charm comprises \(\sim 20\%\) of the total \(F_2(x, Q^2)\) at the lowest \(x\) values at HERA this is no longer sufficient. A method for including the heavy quark contributions in a manner which guarantees both smoothness at the threshold of \(W^2 \equiv Q^2(x^{-1} - 1) = 4m_H^2\) and the correct summation of \(\ln(Q^2/m_H^2)\) terms was developed for the conventional approach\(^1\). The extension to the LORSC calculation is in principle quite simple, but in practice rather involved. Essentially it involves imposing matching conditions at \(Q^2 = m_H^2\) such that the evolution is continuous at this point, but in terms of effective heavy quark coefficient functions and anomalous dimensions which determine the heavy quark structure function and its evolution in terms of the light quark structure functions, rather than parton distributions. The details of this LORSC(H) calculation will be presented in a future publication.

The quality of the LORSC(H) fit to a wide variety of structure function data (references can be found in\(^9\)) is compared to that for a NLO–in–\(\alpha_s\) fit using an analogous treatment of charm, i.e. the recent MRST fit\(^9\). The quality of the fit (using \(m_c = 1.35\text{GeV}\), in different \(x\) bins is shown in table\(^8\)). The LORSC(H) is better overall than the MRST fit, being very slightly worse at high \(x\), but considerably better at small \(x\).

We can also compare the results of the predictions from the two approaches. Fig.\(^8\) shows the LORSC(H) calculation of \(F_{2,c}(x, Q^2)\) (using \(m_c = 1.35\text{GeV}\) and \(m_c = 1.5\text{GeV}\)), and also the prediction from the MRST fit (where \(m_c = 1.35\text{GeV}\)). Although in principle the two approaches could give

\(^b\)These involve the leading \(\ln(1/x)\) heavy quark coefficient functions.\(^8\)
different predictions for $F_{2,c}(x, Q^2)$, in practice they are rather similar when the same value of $m_c$ is used. However, while the MRST fit is very sensitive to $m_c$, becoming worse quickly when it increases above 1.35GeV, the LORSC(H) fit is almost unchanged in going from 1.35GeV to 1.5GeV, and provides much more freedom in $F_{2,c}(x, Q^2)$. A larger difference between the approaches is observed when comparing the predictions for $F_L(x, Q^2)$, as seen in fig. 2. We note that both predictions for $F_L(x, Q^2)$ are significantly lower than when using the previous treatment for charm, since in the correct approach the heavy quark contribution to $F_L(x, Q^2)$ is strongly suppressed until $Q^2 \gg m_c^2$.

3 Conclusion

It appears as though the inclusion of leading ln(1/x) terms in a consistent manner significantly improve the comparison to structure function data at small x, highlighting the shortcomings of a NLO–in–$\alpha_s$ calculation in this region. The calculation of the kernel for the NLO–in–ln(1/x) gluon Green’s function has recently been completed 11,12, and is very suggestive that the NLO–in–ln(1/x) corrections are extremely large. Nevertheless, further understanding seems necessary before it is known precisely how this new result relates to a consistent calculation of structure functions. It may indeed be true that a RSC expansion scheme is not really convergent. However, it is already known that this is true for the conventional $\alpha_s$ expansion: the predictions for $F_L(x, Q^2)$
being hugely different at small $x$ at LO and NLO. Hence, I propose that the success of the inclusion of leading $\ln(1/x)$ terms in the correct manner is telling us something important about the true physics at small $x$.

References

1. H1 collaboration, *Nucl. Phys. B* 470 3 (1996).
2. ZEUS collaboration, *Z. Phys. C* 69 607 (1996); ZEUS collaboration *Z. Phys. C* 72 399 (1996).
3. R.S. Thorne, *Phys. Lett. B* 392 463 (1997); R.S. Thorne *Nucl. Phys. B* 512 323 (1998).
4. S. Catani Proc. of DIS96, Rome, April, 1996, p. 165; S. Catani *Z. Phys. C* 75 665 (1997).
5. H1 collaboration: C. Adloff et al., *Z. Phys. C* 72 593 (1996).
6. ZEUS collaboration: J. Breitweg et al., *Phys. Lett. B* 407 402 (1997); Paper N-645 presented at IECHEP97, Jerusalem 1997.
7. J.J. Aubert et al., *Nucl. Phys. B* 213 31 (1983).
8. S. Catani, M. Ciafaloni and F. Hautmann, *Nucl. Phys. B* 366 135 (1991).
9. A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *hep-ph/9803445* to be published in *Eur. Jour. Phys. A*.
10. R.S. Thorne and R.G. Roberts, *hep-ph/9709442* *Phys. Rev. D* 57 (1998); R.S. Thorne and R.G. Roberts, *Phys. Lett. B* 421 303 (1998); R.S. Thorne, these proceedings.
11. V.S. Fadin and L.N. Lipatov, *hep-ph/9802290*.
12. M. Ciafaloni and G. Camici, *hep-ph/9803389*.