On the Optimal Location of Several Piezoelectric Elements on the Structure Surface

Dmitrii A Oshmarin, Maksim A Iurlov, Natalia V Sevodina, Nataliia A Iurlova
Institute of Continuous Media Mechanics UB RAS, Russia

E-mail: a yurlova@icmm.ru

Abstract. In this paper, based on the solution of the natural vibration problem an approach for determining the best options for the arrangement of piezoelectric elements on the surface of a structure allowing an essential improvement of its dynamic characteristics is proposed. The condition for a maximum value of the generalized electromechanical coupling coefficient, which characterizes the fraction of the converted mechanical energy of vibrations into the electrical energy, is adopted as an optimality criterion. The performance of the proposed approach is demonstrated by solving an example problem of locating two piezoelectric elements on the surface of a spatial structure providing damping of its vibrations at the selected frequencies. The obtained results are substantiated by the frequency response plots of the electric potential, obtained for different options of placement of piezoelectric elements on the surface of a structure under forced steady-state vibrations.

1. Introduction

Nowadays, damping of structure vibrations by converting the energy of mechanical vibrations into the electrical energy with the aid of attached piezoelectric elements, which, as the structure elements, are connected to the electrode surfaces of external electrical circuits, is an intensively developed area of research in mechanics.

To make reliable progress in this field, it is essential to find such technological solutions, in which piezoelectric elements will serve most effectively the ultimate purpose of their application in smart structures (vibration damping, control of the structure shape under the action of various external factors, flaw detection, etc.). The efficiency of piezoelectric elements depends not only on the control parameters (corresponding to the schemes of external electrical circuits), but also on the location of piezoelectric elements that act as sensors and actuators. At the same time, as practice shows, with reference to a specific purpose of using smart technology, the optimal locations of sensors / actuators can be different. This is reflected in the selection of conditions for the optimal fitting of structural elements, as well as in the algorithms for the numerical implementation of the problem under consideration.

Conventionally, devices for passive vibration damping are located at points where the deformation energy in the structure is maximal [1]. As a rule, these locations are determined from the analysis of the deformation behavior of the structure. Nevertheless, the question of choosing the location of the piezoelectric element attracts attention of many researchers, whose studies are targeted at the development of both the optimization methods and the criteria for the optimal location of piezoelectric elements [1-5]. As noted in [6-8], this is also due to the fact that the location of sensor at points, where they fail to generate a significant electric potential, leads even to a decrease in the rate of vibration damping.

Deeper insight into the state of art in this area can be obtained from the review article [9], which summarizes the approaches presented in 109 papers for identification of the optimal location of sensors or actuators in various structures. The condition for the optimal location of a piezoelectric element can be specified based on the maximization of forces, moments or strains transferred by the piezoelectric element acting as an actuator, minimization of changes in the dynamic characteristics of
the original structure, as well as on the need of obtaining the desired dynamic characteristics of the original structure, drastic reducing the control function and maximal damping of vibrations of the initial structure, etc.

Nevertheless, as it was noted in [9], in the previous studies no consideration was given to spatial thin-walled structures. Moreover, despite the differences in the construction of optimization algorithms and approaches, the methods proposed in [5-8] are linked together by the fact that they do not consider the electric potential generated on the surface of a piezoelectric element, although it is a key factor in damping the structure vibrations with piezoelectric elements and external electrical circuits.

An attempt to calculate the value of the electric potential generated on the surface of piezoelectric element with the aim to determine its optimal location was made in [1] using a finite element analysis. However, this study examined the possibility of damping only the first mode of vibrations, which in its self is quite simple. The study of the electromechanical behavior of the piezoelectric element attached to the structure under more complicated vibration modes is of much greater interest to researchers from the viewpoint of its use as a sensor or actuator (the value of the electric potential generated on its surface).

In work [10], using a beam as an example, the authors showed that one piezoelectric element would suffice only for control in cases of simple loading, when the deflection curve is convex or concave along the entire beam length. In more complicated situations it is necessary to use several piezoelectric elements to induce oppositely directed electric fields. It was demonstrated that under complex loading conditions the optimal effect can be produced by a greater number of piezoelectric elements.

Thus, the purpose of this work is to develop an algorithm that will allow us to determine the optimal location of several piezoelectric elements providing the maximum value of the relative electric potential for spatial structures of arbitrary geometry subject to deformation at the specified vibration frequencies.

Here we restrict ourselves to the problem of locating two piezoelectric elements on the surface of a spatial structure such as to ensure the possibility of damping the specified vibration modes.

2. The object of study

The efficiency of using piezoelectric element in various structures is generally determined, along with other factors, by the value of the electric potential generated on their electrode surfaces during the dynamic processes. Therefore, a search for such variants of piezoelectric element location on the structure surface, at which the electric potential can reach the maximum possible value, is the problem that needs to be solved in each specific case of using piezoelectric elements as a tool for controlling the dynamic behavior of the object under consideration.

The solution of this problem involves the analysis of the dependence of the electromechanical coupling coefficient (EMCC) on the location of the piezoelectric element and the mode of vibrations, to which it corresponds.

The concept of a generalized electromechanical coupling coefficient (EMCC) was introduced in [11], and its physical meaning was considered in detail in [12]. This coefficient characterizes the fraction of the mechanical energy of the \(i\)-th vibration mode converted into the electrical energy for each individual piezoelectric element and is determined by the formula:

\[
K_i = \frac{(\omega_{m_{ic}}^2 - \omega_{s_{ic}}^2)}{(\omega_{s_{ic}}^2)}
\]

where \(\omega_{m_{ic}}\), \(\omega_{s_{ic}}\) are the \(i\)-th natural vibration frequencies of the structure with the attached piezoelectric element in the open circuit mode (o/c) and short circuit(s/c) mode. The o/c mode is realized in the case when one of the electrode surfaces of the piezoelectric element is grounded (the value of electric potential is equal to zero) while the other electrode surface is free of loads. In the s/c mode the zero-valued electric potential is prescribed at both electrode surfaces. The mathematical statement of the problem of natural vibrations of electro-viscoelastic bodies and the algorithm for its numerical realization are given in [13].

In [14, 15], the authors have revealed a correlation between the value of the electric potential
generated on the electrode surfaces of piezoelectric elements during dynamic processes, the amount of the strain energy in the zone of the piezoelectric element location, and the value of electromechanical coupling coefficient. This relationship was derived by solving an example problem, in which an optimal location of a piezoelectric element is searched for to provide the most effective damping of the specified vibration modes.

However, the best placement for the piezoelectric element in spatial structures cannot always be chosen based on the values of the electric potential and the strain energy in the zone of location of the piezoelectric element, which were obtained by solving the natural vibration problem. These values are of relative nature and consider meaningful only within the limits of a single simulation made to estimate the efficiency of the piezoelectric element at different modes. Moreover, it is not always correct to compare the values of the electric potential obtained by solving the problem of natural vibrations for different positions of the piezoelectric element, its various dimensions, physical and mechanical characteristics of the material, etc. At the same time, $K_i$ is an absolute quantity, which allows us to carry out a qualitative comparison of the obtained results.

As an optimality criterion for the location of the piezoelectric element, it is reasonable to use the condition of maximum value of the electromechanical coupling coefficient (EMCC). The generalized electromechanical coupling coefficient demonstrated its high efficiency in determining the optimal location of a single piezoelectric element [14] and, therefore, it seems attractive to apply this approach to the case of several piezoelectric elements. From a practical point of view, this is necessary in the case of locating several piezoelectric elements on the structure surface. In doing so, it is desirable to ensure their independent operation, that is, to minimize their reciprocal influence, which necessitates additional investigations.

The attachment of the piezoelectric element to the structure surface changes the spectrum of natural frequencies and vibration modes interacting with each other. Accordingly, each subsequently attached piezoelectric element causes a change in the already existing vibration mode and, the deformation of the previously located piezoelectric elements. This leads to a change in the patterns of distribution of the initially obtained values of $K_i$. Thus, the zones of optimal location of the second and each successive piezoelectric element may vary at different frequencies. It therefore seems incorrect to estimate the efficiency of location of several piezoelectric elements in the structure based solely on the analysis of the distribution patterns of $K_i$ obtained for one piezoelectric element. On the other hand, the arrangement of successive piezoelectric elements also influences the expected performance of the previously located piezoelectric elements. Hence, the method proposed in [14], when extended to the case of locating several piezoelectric elements, must be modified in order to allow evaluating the cross impact of piezoelectric elements on the efficiency of their performance.

The key idea of the method of determining the optimal locations of piezoelectric elements, which is based on the condition for a maximum value of $K_i$, can be formulated as follows. After finding an optimal location for a single piezoelectric element ensuring the most effective damping of vibrations at the specified frequency, new piezoelectric elements are added one by one to the structure, each being designed to damp vibrations at other frequencies, taking into account the effect of previously located piezoelectric elements.

In this case, along with fixing the value of the electromechanical coupling coefficient of the new piezoelectric element, it is necessary to monitor its variation in the previous piezoelectric elements, the location of which is fixed.

3. The placement of piezoelectric element on the shell surface according to the proposed approach. Numerical example

Let us demonstrate the performance of the proposed method using as an example a spatial structure in the form of a thin-walled shell schematically represented in Fig.1. The geometrical dimensions of the shell are as follows: $L = 290$ mm, $B = 150$ mm, $H = 120$ mm, $t = 0.26$ mm. The shell is made of an isotropic elastic material with the following physical and mechanical characteristics: $E = 6.5 \times 10^{10}$ Pa, $\nu = 0.28$, $\rho = 2900$ kg/m$^3$. The edges of the shell are clamped along their entire length.

It is necessary to select the locations of a few piezoelectric elements such that each of the elements can generate the maximum possible electric potential at the specified vibration frequency.
To illustrate the performance of this approach, we consider the problem of locating two piezoelectric elements in the segment S1 of the structure surface (the upper horizontal surface of the shell with the coordinate \( z=H \), 290x120 mm in size, see Fig. 1). The values of the first 5 natural frequencies of the shell are 13.63, 41.82, 47.20, 67.35, 81.63 (Hz), respectively.

For further investigations, the second and third natural vibration frequencies (41.82 and 47.20 Hz) were chosen as closest to one another as possible and, as a result, showing the maximum effect of reciprocal influence. Accordingly, the first piezoelectric element must generate the maximum electric potential at the second natural vibration frequency, and the second piezoelectric element - at the third natural vibration frequency.

The piezoelectric elements are made of PZT-19 piezoelectric ceramics, with upper and lower electrode surfaces. The lower electrode surface is grounded (the electric potential there is equal to zero), and the material is polarized in the z-axis direction. The dimensions of piezoelectric elements are as follows: 50x20x0.36 mm. Piezoceramics has the following physico-mechanical properties:

\[
C_{11} = C_{22} = 10.9 \cdot 10^{10} \text{ Pa}, \quad C_{12} = C_{23} = 5.4 \cdot 10^{10} \text{ Pa}, \quad C_{13} = 6.1 \cdot 10^{10} \text{ Pa}, \quad C_{33} = 9.3 \cdot 10^{10} \text{ Pa},
\]

\[
C_{44} = C_{55} = C_{66} = 2.4 \cdot 10^{10} \text{ Pa}, \quad \beta_{31} = \beta_{32} = -4.9 \text{ C/m}^2, \quad \beta_{33} = 14.9 \text{ C/m}^2, \quad \beta_{51} = \beta_{42} = 10.6 \text{ C/m}^2,
\]

\[
\alpha_{11} = 2 \cdot 10^{-9} \text{ F/m}, \quad \gamma_{33} = 8.4 \cdot 10^{-9} \text{ F/m}, \quad \rho = 7500 \text{ kg/m}^3.
\]

3.1. Determination of the optimal location of the first piezoelectric element

The determination of the optimal location of the piezoelectric element on the surface \( S_1 \) to obtain the maximum possible electric potential at the second natural frequency of vibrations was carried out by the direct search method. A series of calculations was carried out for different options of location of piezoelectric element, which were determined by the coordinates of its center of mass.

For convenience of presentation of the obtained results, the zone of clamp and lateral surfaces of the shell were excluded, and the results are presented in the form of a flat picture only for a part of the surface \( S_1 \). The x-coordinate is plotted on the horizontal axis, and the y-coordinate is plotted on the vertical axis. The axis of symmetry of the shell is aligned along the y-axis.

Figure 2 shows the patterns of distribution of the EMCC values at the second natural frequency of vibrations in the case of vertical (Fig. 2a) and horizontal (Fig. 2b) orientation. The obtained results demonstrate that the values of EMCC obtained at the horizontal orientation of the piezoelectric element are 1.5 times higher than the values obtained at the vertical orientation. Therefore, further calculations were made on the assumption of horizontal orientation of the first piezoelectric element.

The analysis of the distribution patterns of the electromechanical coupling coefficient shown in Fig. 2b, allowed us to determine the coordinates of the center of mass of the piezoelectric element, which are optimal for obtaining the maximal value of the EMCC at vibration with the frequency corresponding to the natural vibration frequency of the second mode (\( K_2 = 0.037 \)): \( x=0 \text{ mm}, \ y=145 \text{ mm} \). For such arrangement of the piezoelectric element, the value of the EMCC obtained at the
frequency corresponding to the natural vibration frequency of the third mode is small \((K_3 = 0.002)\). It means that the above mentioned position of the piezoelectric element at this frequency is not optimal.

Consider the problem of forced steady-state vibrations of the shell under the impact of the force \(F_z = 1\) N, applied to the surface \(S_1\) of the structure at the point with coordinates \(x = 0\) mm, \(y = 0\) mm (Fig. 3), for three options of the piezoelectric element location. The coordinates of the center of mass of the piezoelectric element in the optimal option of location \((x=0\) mm, \(y=145\) mm) were determined by the method described above, whereas for the intermediate \((x=20\) mm, \(y=210\) mm) and non-optimal locations \((x=25\) mm, \(y=280\) mm) they were chosen arbitrarily (table 1).

Figure 4 shows the amplitude-frequency response plots of the electric potential generated by the piezoelectric element at different locations. It can be seen from the plots that when the location of the piezoelectric element is optimal for its effective performance at the second natural frequency, the electric potential is maximum and equal to 120.1 V. In this case, the EMCC also takes the maximum value.

It should also be noted that in the intermediate option of piezoelectric element location, a sufficiently high level of the electric potential (~ 100 V) is generated on its surface at the frequencies corresponding to the second and third natural vibration frequencies, which indicates its ability to work equally effectively at two frequencies.
Table 1. The values of the second natural frequency of vibrations and the electromechanical coupling coefficient depending on the location of the first piezoelectric element

| Options of piezoelectric element location | Optimal | Intermediate | Nonoptimal |
|------------------------------------------|---------|--------------|------------|
| Coordinates of the center of mass of piezoelectric element, (mm) | x=0 | x=20 | x=−25 |
| y=145 | y=210 | y=280 |
| Second natural frequency of the structure with piezoelectric element $f_{oc}$ (Hz) | 42.17 | 41.95 | 42.03 |
| Electromechanical coupling coefficient $K_2$ | 0.037 | 0.032 | 0.025 |

Figure 4. The amplitude-frequency response plots of the electric potential generated by the piezoelectric element at different locations: optimal (black line), intermediate (blue line), non-optimal (red line).

3.2. The effect of electrical boundary conditions on the value of the coefficient of the electromechanical coupling coefficient

As it was noted in [3], the attachment of the piezoelectric element to an external electric circuit can produce changes in its stiffness properties in the range determined by the two limiting cases: short circuit (s / c) and open circuit (o / c) condition. The changes in the stiffness properties give rise to the changes in the natural vibration frequencies and vibration modes of the structure. In this regard, it is important to know what effect these variations can exert on the electromechanical coupling coefficient.

Table 2. Natural vibration frequencies and electromechanical coupling coefficients for the first option of location of the second piezoelectric element with the coordinates of the center of mass: x = 15 mm, y = 70 mm

| Number of frequency | $f_{oc}$ (Hz) | $f_{sc}$ (Hz) | $K_2$ |
|---------------------|---------------|---------------|-------|
| EMCC for the first piezoelectric element, the second piezoelectric element is in the o/c mode | 2 | 41.89801 | 41.87122 | 0.035777 |
| | 3 | 47.18087 | 47.18073 | 0.002464 |
| EMCC for the first piezoelectric element, the second piezoelectric element is in the s/c mode | 2 | 41.87619 | 41.84944 | 0.035761 |
| | 3 | 47.16129 | 47.16114 | 0.002525 |
| EMCC for the second piezoelectric element, the first piezoelectric element is in the o/c mode | 2 | 41.89801 | 41.87619 | 0.032285 |
| | 3 | 47.18087 | 47.16129 | 0.028821 |
| EMCC for the second piezoelectric element, the first piezoelectric element is in the s/c mode | 2 | 41.87122 | 41.84944 | 0.032267 |
| | 3 | 47.18073 | 47.16114 | 0.028826 |
Table 3. Natural vibration frequencies and electromechanical coupling coefficients for the second option of location of the second piezoelectric element with the coordinates of the center of mass: x = 15 mm, y = 110 mm

| Number of frequency | \( f_{0/c} \) (Hz) | \( f_{s/c} \) (Hz) | \( K_i \) |
|---------------------|---------------------|---------------------|---------|
| **EMCC for the first piezoelectric element, the second piezoelectric element is in the o/c mode** | | | |
| 2                   | 41.80375            | 41.77910            | 0.034357|
| 3                   | 47.48214            | 47.48213            | 0.000358|
| **EMCC for the first piezoelectric element, the second piezoelectric element is in the s/c mode** | | | |
| 2                   | 41.78501            | 41.76023            | 0.034455|
| 3                   | 47.47708            | 47.47707            | 0.000375|
| **EMCC for the second piezoelectric element, the first piezoelectric element is in the o/c mode** | | | |
| 2                   | 41.80375            | 41.78501            | 0.029953|
| 3                   | 47.48214            | 47.47708            | 0.014610|
| **EMCC for the second piezoelectric element, the first piezoelectric element is in the s/c mode** | | | |
| 2                   | 41.77910            | 41.76023            | 0.030066|
| 3                   | 47.48213            | 47.47707            | 0.014600|

To estimate the influence of the electric boundary conditions on the value of the EMCC, a series of computational experiments were carried out, in which the EMCC was determined for the first piezoelectric element at the time when the second piezoelectric element was operating in the open- and short circuit modes and in the reverse order. In this case, the first piezoelectric element was held in its optimal position, whereas for the second piezoelectric element we considered two options of piezoelectric element location differing in the coordinates of the center of mass of the element.

In the first case, x = 15 mm, y = 220 mm, which means that it was located at a distance from the first piezoelectric element. In the second case x = 15 mm, y = 180 mm, which means that it was located in the immediate vicinity of the first piezoelectric element. The results obtained are presented in tables 2 and 3.

From the results given in Tables 2 and 3, it follows that a change in the boundary conditions with respect to the electrical component has inessential effect both on the natural vibration frequencies and on the electromechanical coupling coefficient (a change is less than 2%). Thus, if there are several piezoelectric elements in the structure, the electromechanical coupling coefficient for each piezoelectric element is determined by formula (1) regardless of the boundary conditions for the electrical component specified on the other piezoelectric elements. Therefore, in all subsequent calculations, the evaluation of the EMCC for one piezoelectric element is carried out under the condition of short-circuit operation specified for the other piezoelectric elements (zero potential on both electrode surfaces).

3.3. Determination of the optimal location of the second piezoelectric element

Now, let us choose for the second piezoelectric element such position that the generation of the maximum electric potential on its surface occurs at the third natural vibration frequency. In this case, it is essential that the electric potential of the first piezoelectric element remains maximum (or close to the maximum) at the second frequency.
Figure 5. Patterns of the distribution of electromechanical coupling coefficient for the first piezoelectric element at the second natural vibration frequency (a) and for the second piezoelectric element at the third natural vibration frequency (b) at different locations of the second piezoelectric element.

Using the direct search method described above, we obtained a series of the distribution parts of the electromechanical coupling coefficient for the first and second piezoelectric elements according to the location of the second piezoelectric element. Figure 5 shows the distribution patterns of the electromechanical coupling coefficient for the first piezoelectric element at the second natural vibration frequency (a) and for the second piezoelectric element at the third natural vibration frequency (b). Each point on these graphs corresponds to the position of the center of mass of the second piezoelectric element. The white rectangle shows the location of the first piezoelectric element.

As is evident from the above figures, a change in the location of the second piezoelectric element significantly affects the value of EMCC of the first piezoelectric element (the maximum change in its value is about 20% compared to the value obtained for a single piezoelectric element, which is equal to 0.037).

Table 4. The values of the second and third natural frequencies of vibrations and the corresponding electromechanical coupling coefficients depending on the location of the second piezoelectric element

| Options of piezoelectric element location | Optimal | Intermediate | Nonoptimal |
|------------------------------------------|---------|--------------|------------|
| Coordinates of the center of mass of the piezoelectric element, (mm) | x=0 | x=0 | x= -25 |
| x=y=260 | y=210 | y=170 |
| Second natural frequency of the structure with the attached piezoelectric element, \( f_{alc} \) (Hz) | 42.72 | 42.28 | 42.35 |
| Third natural frequency of the structure with the attached piezoelectric element, \( f_{alc} \) (Hz) | 46.55 | 47.14 | 47.08 |
| Electromechanical coupling coefficient \( K_2 \) | 0.037 | 0.035 | 0.039 |
| Electromechanical coupling coefficient \( K_1 \) | 0.041 | 0.027 | 0.011 |

As it follows from Fig.5a, the second piezoelectric element can be located in such a way that the EMCC of the first piezoelectric element will increase. The obtained results demonstrate that the
patterns of distribution of the EMCC values for a single piezoelectric element cannot be used as a criterion for locating several piezoelectric elements.

Figure 5.b shows that, for the third natural vibration frequency the optimal location of the second piezoelectric element is such that the center of its mass coincides with the points with coordinates \( x = 0 \text{ mm}, y = 30 \text{ mm}, \text{or} x = 0, y = 260 \text{ mm} \). In this case, for the second piezoelectric element, the value of the electromechanical coupling coefficient is \( K_3 = 0.041 \), whereas the value of the EMCC for the first piezoelectric element at the second frequency is \( K_2 = 0.037 \) (the value of the EMCC without the second piezoelectric element), which indicates that the reciprocal influence of the piezoelectric elements at this particular locations is minimal. With this location of the second piezoelectric element, its electromechanical coupling coefficient at the frequency of the second vibration mode is \( K_2 = 0.035 \) and is close in the magnitude to the EMCC of the first piezoelectric element.

Based on the results obtained, it can be concluded that the locations found for both piezoelectric elements are indeed optimal from the viewpoint of the maximum value of the electromechanical coupling coefficient, characterizing the effective operation of the piezoelectric element.

Consider the problem of forced steady-state vibrations of the shell under the impact of the force \( F_z = 1 \text{ N} \) applied to the surface \( S_1 \) of the structure at the point with coordinates \( x = 0 \text{ mm}, y = 0 \text{ mm} \) (Fig.6). The location of the first piezoelectric element is considered to be fixed, and the center of mass of the second piezoelectric element will be located as follows: at the point with coordinates \( x = 0 \text{ mm}, y = 260 \text{ mm} \) (the optimal location), \( x = 0 \text{ mm}, y = 210 \text{ mm} \) (intermediate location) and \( x = 25 \text{ mm}, y = 170 \text{ mm} \) (non-optimal location) (table 4).

Figure 7 shows the amplitude-frequency response plots of the electric potential measured on the first (Fig. 7a) and second (Fig. 7b) piezoelectric elements for the three options of location of the second piezoelectric element. The curves presented in Fig. 7b show that the electric potential of the second piezoelectric element under the vibrations with a frequency corresponding to the third natural vibration frequency reaches a maximum value of 192 V when its center of mass is located at the point with the coordinates corresponding to the optimal location (see Fig. 7).

![Figure 6. Computational model for the problem of forced steady-state vibrations.](image)

However in this case, the electric potential of the first piezoelectric element is reduced to 113.5 V (Fig. 7a), which is also reflected in the value of the electromechanical coupling coefficient.

When the second piezoelectric element occupies a non-optimal location (see Fig. 7), the value of the electric potential generated by the first piezoelectric element at the frequency of the structure vibrations, corresponding to the natural vibration frequency of the second mode, increases reaching 128.4 V, which is also reflected in the value of the electromechanical coupling coefficient.
Figure 7. The amplitude-frequency response plots of the electric potential generated by the first (a) and second (b) piezoelectric elements depending on location of the second piezoelectric element: optimal location $x = 0$ mm, $y = 260$ mm (black line), intermediate location $x = 0$ mm, $y = 210$ mm (blue line), non-optimal location $x = 25$ mm, $y = 170$ mm (red line).

4. Conclusions
The method, which is generally used to determine the location of a single piezoelectric element on the surface of the structure which will ensure the maximum possible value of the electric potential generated on its electrode surface under vibration at the specified frequency, has been extended to the case of several piezoelectric elements.

A numerical investigation has shown that the electrical boundary conditions specified for other piezoelectric elements do not affect the value of the electromechanical coupling coefficient of the examined piezoelectric element.

It has been found that in the case of using several piezoelectric elements, they can be so arranged that their efficiency can be essentially improved. It is also possible to find such options of location, at which the piezoelectric elements can generate a sufficient electrical potential at several vibration frequencies, which will allow control over the dynamic characteristics of the structure at these frequencies.

Acknowledgements
The work is supported by the RFBR (projects No 18-31-00080-mol_a).

References
[1] Venna S, Lin Y-J 2013 An Effective Approach for Optimal PZT Vibration Absorber Placement on Composite Structures Modern Mechanical Engineering No 3 21-26.
[2] Han J Y, Lee I 1999 Optimal placement of piezoelectric Sensors and actuators for vibration control of a composite plate using genetic algorithms Smart Materials and Structures 8, No 2 257-267.
[3] Lammering R, Jia J, Rogers C A 1994 Optimal Placement of Piezoelectric Actuators in Adaptive Truss Structures Journal of Sound and Vibration 171, No 1 67-85.
[4] Barbini R, Mannini A, Fantini E, Gaudenzi P 2000 Optimal placement of PZT actuators for the control of beam dynamics Smart Mater. Struct. No 9 110–120.
[5] Onoda Y, Li J, Minesugi K 2002 Simultaneous optimization of piezoelectric actuator placement and feedback for vibration suppression Acta Astronautica 50, No 6 335–341.
[6] Holman R E, Spencer S M, Austin E M, Johnson C D (1997) Passive damping technology demonstration SPIE 3045, No 1 51-59.
[7] Wu S, Turner T L, Rizzi S A 2010 Piezoelectric shunt vibration damping of F-iS panel under high acoustic excitation SPIE 3989, No 2 276-287.
[8] Hollkamp J J, Gordon R W 1996 An experimental comparison of piezoelectric and constrained layer damping Smart Mater. Struct. 5 715–722.
[9] Gupta V, Sharma M, Thakur N 2010 A Technical Review Optimization Criteria for Optimal Placement of Piezoelectric Sensors and Actuators on a Smart Structure *J. Intelligent Material Systems and Structures* **21** 1227-1243.

[10] Bruch Jr J C, Sloss J M, Adali S, Sadek I S 2000 Optimal piezo-actuator locations/lengths and applied voltage for shape control of beams *Smart Mater. Struct.* **9** 205–211.

[11] Hagood N W., von Flotow A 1991 Damping of structural vibrations with piezoelectric materials and passive electrical networks *Journal of Sound and Vibration* **146**, No 2 p. 243-268.

[12] Trindade M A, Benjeddou A 2009 Effective electromechanical coupling coefficients of piezoelectric adaptive structures: critical evaluation and optimization *Mechanics of Advanced Materials and Structures* **16**, No 3 210-223.

[13] Matveenko, V.P., Oshmarin, D.A., Sevodina, N.V. and Iurlova, N.A. (2016). Natural vibration problem for electroviscoelastic body with external electric circuits and finite-element relations for its numerical implementation *Computational Continuum Mechanics* **9**, No 4 476-485.

[14] Iurlova N A, Matveenko V P, Oshmarin D A, Sevodina N V, Yurlov M A. 2016 Layout optimization of piezoelectric elements with external electric circuits in smart constructions based on solution of the natural vibrations problem *ECCOMAS Congress 2016 VII European Congress on Computational Methods in Applied Sciences and Engineering Proceedings*, (Crete Island, Greece, 5–10 June 2016) **1** 1920-1929.

[15] Sevodina N V, Yurlova N A, Oshmarin D A 2015 The optimal placement of the piezoelectric element in a structure based on the solution of the problem of natural vibrations *Solid State Phenomena* **243** 67-74.