Detection of non-Gaussian Fluctuations in a Quantum Point Contact

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An experimental study of current fluctuations through a tunable transmission barrier, a quantum point contact, are reported. We measure the probability distribution function of transmitted charge with precision sufficient to extract the first three cumulants. To obtain the intrinsic quantities, corresponding to voltage-biased barrier, we employ a procedure that accounts for the response of the external circuit and the amplifier. The third cumulant, obtained with a high precision, is found to agree with the prediction for the statistics of transport in the non-Poissonian regime.

Recently, measurements of current fluctuations arising from charge discreteness have become an invaluable tool in mesoscopic physics, the most noticeable achievement being the shot noise measurement of quasi-particle charge in the fractional quantum Hall effect [1, 2]. Typically, mesoscopic shot noise experiments report zero-frequency noise power, but this quantity contains only partial information about the statistics of the transmitted charge. The counting statistics (CS) \[3\] is entirely characterized by the set of cumulants (irreducible moments) \[q\] of the charge \(q(\tau)\) transmitted through a voltage-biased system during a sampling time \(\tau\). In the long time limit they are proportional to the time, \(\langle q^n \rangle \approx \langle J^n \rangle \tau\); this expression defines the current cumulants \(\langle J^n \rangle\). For example, Gaussian noise is fully determined by the first two current cumulants, the average current \(I \approx \langle J \rangle\) and the noise power \(S = \langle J^2 \rangle\). The simplest measure of the non-Gaussianity, the third current cumulant \(S_3 = \langle J^3 \rangle\) which reflects the skewness of the current distribution, is the central focus of our paper. It is linear and universal at low bias voltage, and therefore may be used as a tool for investigation of strongly correlated systems, where large bias cannot be applied without substantially affecting their properties.

However, during a typical sampling time a large number of electrons passes through the system. This fact, by virtue of the central limit theorem, makes it difficult to observe non-Gaussian effects in electron transport [6], unless electrons are counted one by one as, e.g., in Coulomb blockaded quantum dots [7, 8]. To date, \(\langle J^3 \rangle\) has been measured only in low transmission tunneling junctions either by measuring voltage on a load resistor [8, 10], or with the help of the on-chip Josephson threshold detector [11].

In a typical experiment, due to the electrons’ charge, the voltage across the sample is not constant, so the measured statistics are not trivially related to the CS of the voltage-biased system. Indeed, the original experiment on the third cumulant of a tunneling current [9] exhibited totally unexpected results, which were explained [12, 13, 14, 15] by the back-action of the measurement apparatus on the sample. That is, in addition to \(\langle J^3 \rangle\), the experimentally measured potential fluctuations also contain contributions, dubbed “cascade corrections” in Ref. [13], from the voltage dependent current \(I\) and noise power \(S\). Surprisingly, even if the load resistance is small, the cascade corrections may be of the same order as \(\langle J^3 \rangle\).

We report here the first measurement of the third cumulant of the non-Poissonian current partitioned by a variable transmission barrier. To obtain the third cumulant, we develop a procedure that allows us to separate the \(\langle J^3 \rangle\) contribution and cascade corrections in a reliable fashion. In particular, the frequency dependence of the amplifier gain is found to affect these contributions differently. The resulting cumulant \(\langle J^3 \rangle\) agrees accurately with the predictions of [3] for all transmissions, \(0 < \Gamma < 1\), without fitting parameters. As a variable transmission barrier we use a quantum point contact (QPC) – a small quasi-1D constriction formed in a 2D electron gas by negative voltage \(V_g\) applied to split gates [10]. By varying \(V_g\) one can gradually 

![Image of Conductance Step](image-url)
**FIG. 2:** Experimental setup schematic. The current source drives constant average current \( I \) through the QPC sample, while the capacitor \( C_N \) fixes the voltage across the sample. The QPC resistance \( R_N \) can be tuned by the gate voltage \( V_g \). \( R_L \) is the load resistance and \( C_s \) is the stray capacitance of the wires at the amplifier input (point N). Current fluctuations generate voltage fluctuations at the point N, which are amplified and digitized by 12 bit analog-to-digital converter (A/D). We introduce current through the capacitor \( C_C \) and measure the response with a network analyzer to calibrate the setup.

The QPC resistance \( R \) is the average QPC transmission as:

\[
\Gamma(I) = \frac{I}{g_0 V_s(I)}, \tag{1}
\]

where \( I \) is the average current and \( V_s \) is the voltage across the sample. The observed noise agrees well with the theoretical expectations \cite{17} and has only thermal and shot noise contributions. No material contribution, e.g. 1/f noise, is detected. The measurements are done at elevated temperature \( T \approx 5 \) K in order to reduce nonlinearity of the current-voltage characteristic. Nevertheless, for QPC conductance below \( 2e^2/h \), the transport is dominated by a single channel, as confirmed by the agreement of the measured noise with the single-channel expectations (see Fig. 4).

It is instructive to discuss the issues encountered in the investigation of the third cumulant prior to presenting our experimental results. We focus on the experimental setup shown in Fig. 2 however our conclusions are quite general. Current fluctuations \( J \) generated by the QPC sample and by thermal noise in the load resistor \( R_l \) give rise to the voltage \( V_N(\omega) = Z(\omega)J(\omega) \) at the amplifier input (point N in Fig. 2), where the impedance \( Z(\omega) = R_l/(1-i\omega \tau_{RC}) \) is determined by \( R_l = R_s R_l/(R_s + R_l) \approx 7 \) KOhm, and by the time constant \( \tau_{RC} = R_N C_N \) (here \( C_{\|} = C_s + C_C = 3 \) pF) which sets the high frequency cut-off of the circuit \( 1/(2\pi \tau_{RC}) \approx 7 \) MHz. The amplified voltage is:

\[
V_a(\omega) = K(\omega)V_N = A(\omega)J(\omega), \tag{2}
\]

where \( K \) is the amplifier gain and \( V_N \) is the input voltage.

In the long time limit (which is justified since the high frequency cut-off of the system is much smaller then \( \max(eV_s, k_B T)/h \)) all current cumulants are frequency-independent. The amplified voltage fluctuations \( \langle \delta V_a(\omega)^2 \rangle \) bear a simple relation to the noise power

\[
\langle \delta V_a(\omega)^2 \rangle = B_2 \frac{1}{2\pi} \int_{-\infty}^{+\infty} A(\omega)A(-\omega) d\omega. \tag{3}
\]

In contrast, the third cumulant \( \langle \delta V_a(\omega)^3 \rangle \) may be decomposed as

\[
\langle \delta V_a(\omega)^3 \rangle = B_3 \langle J^3 \rangle + B_{cn} J_{cn} + B_{nl} J_{nl}. \tag{4}
\]

That is, apart the third current cumulant \( \langle J^3 \rangle \), it also contains “environmental” cascade correction \( J_{cn} \) originating from the back-action of the voltage fluctuation across the load on the current fluctuations in the sample at later times \cite{12,13,14,15}, and the correction \( J_{nl} \) due to nonlinearity of the sample:

\[
J_{cn} = 3R_l S d S/d V_N \tag{5}
\]

\[
J_{nl} = 3R_{\|}^2 S^2 d I/d^2 V_N, \tag{6}
\]

where \( S = S_s + S_l \) is the noise power generated by the sample and the load. The coefficients \( B_3, B_{cn}, B_{nl} \) derived below, depend on the circuit only and are of the same order. The thermal noise of the load and its resistance are current-independent, and thus do not contribute to the derivative. Note that since \( R_l S \approx 2k_B T \) at \( R_l \to 0 \), reducing load resistance does not eliminate the corrections \( J_{cn} \) and \( J_{nl} \).

Our measurements are performed using a setup (see Fig. 2) similar to the one discussed in detail in \cite{10}. We improved it by increasing the total gain of the amplification chain to utilize the full 12 bit resolution of the A/D converter, thus eliminating the necessity to compensate for its nonlinearity. We also managed to reduce the nonlinearity of the cryogenic amplifier by increasing the current through the transistors. The amplified signal is computer analyzed to construct the probability distribution function (PDF) of the amplified voltage \( V_a(\omega) \).

We calibrate the setup by introducing a known ac signal, spanning the entire frequency band of the amplifier, through a small capacitor \( C_e \approx 2.4 \) pF. This gives us the complex-valued \( A(\omega) \) [see Eq. (2)] independently at each value \( I \) and \( V_g \), which is subsequently used to compute the coefficients in Eqs. (5) and (6).

To compensate for inaccuracy in the calibration and for the drift of amplifier parameters we multiply the measured \( A(\omega) \) by a numerical factor, which scales the measured noise \( S \) to the theoretical prediction \cite{15},

\[
S = eI \left( 1 - \Gamma \right) \coth \left( \frac{U}{2} \right) + 2\Gamma U, \quad U = \frac{eV_s}{k_B T}. \tag{7}
\]
This scale, determined individually for each value of $V_g$, is found to deviate from unity by less than 10%. This is illustrated in the upper inset of Fig. 3 which shows $S$ fitted by Eq. (7) at $V_g$ corresponding to the transmission $\Gamma \approx 0.3$ (see the lower inset of Fig. 3). We use the scaled $A(\omega)$ to find the coefficients $B_3, B_{en}, B_{nl}$, and then to obtain $\langle J^3 \rangle$ from Eqs. (10) and (8). The resulting $\langle J^3 \rangle$ is shown in the main panel of Fig. 3.

As seen in Fig. 3 the measured third current cumulant $\langle J^3 \rangle$ shows very good agreement with the prediction for noninteracting fermions [9, 10]:

$$\langle J^3 \rangle = e^2 I (1 - \Gamma) \left[ 6 \Gamma \sinh(U) / U - 1 \right].$$

Although the procedure that leads to $\langle J^3 \rangle$ involves subtraction of several terms of comparable magnitude, we believe that it does not rely on any fitting parameter other than the aforementioned scaling factor. Since we observe no systematic deviation from the prediction [9], we believe that the main sources of error in our experiment are statistical fluctuations (indicated by error bars in Fig. 3), as well as our lack of knowledge of the precise energy dependence of $\Gamma$.

The nearly linear behavior of $\langle J^3 \rangle$ at $I \leq 10 \, \text{nA}$ corresponds to the low-bias limit of Eq. (5):

$$\langle J^3 \rangle = e^2 I (1 - \Gamma), \quad |U| \ll 1.$$  (9)

The dashed line in Fig. 3 shows the prediction [9] with the measured current-dependent transmission $\Gamma(I)$. Notably, the full expression [9] provides a much better fit to our data than the low-bias limit [9]. In the data taken at larger bias (see Fig. 4), for which the quantity $U$ could be as large as 4, the agreement with the expression [9] remains very good. We also note that the large bias limit of Eq. (5), $\langle J^3 \rangle = e^2 I (1 - \Gamma) (1 - 2 \Gamma)$, exhibits a sign change at $\Gamma = 1/2$. However, the bias regime needed to observe this effect, $U \gtrsim 10$, is not accessible in our experiment because of the nonlinearity in $f(V_g)$.

In this work we present the data obtained on two different samples, QPC1 and QPC2. The sample QPC1 is measured with two amplifiers, $a$ and $b$, on the sample QPC1 for similar transmissions $\Gamma = 0.6$ and 0.7. The coefficients $B_3, B_{en}$, and $B_{nl}$ are calculated using measured response function $A(\omega)$. Solid line is the prediction [9].

FIG. 3: The current cumulant $\langle J^3 \rangle$ at $\Gamma \sim 0.3$, derived from the experimental results for $\langle V_s^3 \rangle$ and $\langle V_{en}^3 \rangle$ using Eq. (8) for the sample QPC2 measured with amplifier $b$. Solid line is the theoretical prediction for $\langle J^3 \rangle$, Eq. (5); the dashed line is the low-bias limit, Eq. (9), with current-dependent transmission $\Gamma$ (lower inset). Upper inset: Measured value of the noise power $S$ versus current; solid line is the Eq. (7).

FIG. 4: The contributions used to evaluate $\langle J^3 \rangle$ (●) from Eqs. (10): $\langle V_s^3 \rangle/B_3$ (■), $J_{en} B_{en}/B_3$ (*), and $J_{en} B_{nl}/B_3$ (♦). Panels (a) and (b) show the results obtained with two different amplifiers, $a$ and $b$, on the sample QPC1 for similar transmissions $\Gamma = 0.6$ and 0.7. The coefficients $B_3, B_{en}$, and $B_{nl}$ are calculated using measured response function $A(\omega)$. Solid line is the prediction [9].

Solving linearized equation (10) yields an exponential relaxation of fluctuations:

$$V(t) = \int dt Z(t - t_1) J(t_1),$$

where $Z(t) = C^{-1}_r e^{-t/\tau_{RC}}$. The amplified voltage $V_a$ is related to $V(t)$ as:

$$V_a(t) = \int dt' K(t - t') V(t') = \int dt_1 A(t - t_1) J(t_1).$$
We next ensemble-average the value \([V_n(t) - \langle V_n \rangle]^3\). The intrinsic contribution arises from \(\langle J(t_1)J(t_2)J(t_3) \rangle\), where \(J(t)\) can be treated as \(\delta\)-correlated in time. This leads to the first term in Eq. (4) with \(B_3\) given by (10):

\[
B_3 = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} A(\omega)A(\omega') A(-\omega - \omega') d\omega d\omega' \tag{12}
\]

The voltage across the sample fluctuates in time, being a function of current fluctuations at preceding times, as described by (11). As a result, the first two terms in Eq. (4) cancel at small \(R\). The first term in Eq. (4) with \(B_3\) leads to the first term in Eq. (4) with \(B\). Thus, our results correspond to the equal time correlator \(\langle J(t_1)J(t_2) \rangle = \delta(t_1 - t_2)S(V)\) (13) on the voltage \(V\), which in turn depends on the current \(J\) at an earlier time \(t'\). Linear in \(V\) expansion of Eq. (13) gives the environmental contribution to Eq. (4):

\[
3 \int dt_1 A^2(-t_1) \frac{dS}{dN} \int dt' Z(t_1 - t') \times \langle J(t') \int dt_3 A(-t_3)J(t_3) \rangle = 3R\|S \frac{dS}{dN} B_{en},
\]

\[
B_{en} = R\|^{-1} \int dt_1 A^2(-t_1) \int dt_3 Z(t_1 - t_3) A(-t_3), \tag{14}
\]

where the factor 3 accounts for the three possibilities to choose a later time. Finally, the nonlinear contribution to Eq. (4) comes from the \(V^2\) term in Eq. (10) with the coefficient

\[
B_{nl} = R\|^2 \int dt A(-t) \left( \int dt_1 A(-t_1) Z(t-t_1) \right)^2. \tag{15}
\]

For the frequency independent amplification \(K\), we find:

\[
B_3 = 2B_{en} = 4B_{nl} = K^3 \tau_{RC} / 3C^3. \tag{16}
\]

In this case, the first two terms in Eq. (4) cancel at small \(I\) and \(R\). Our amplifier \(a\), having \(\nu_c \approx 300\) KHz, operates close to this regime (see Fig. 4).

It is instructive to note that the ratio of the intrinsic and environmental coefficients in Eq. (16) is twice as large as that of Refs. [12, 13, 14, 15]. This difference can be traced to different assumptions about frequency dependence of the system gain. Refs. [12, 13, 14, 15] focus on the limit of the high frequency cutoff set by the amplifier, while in Eq. (16), as well as in our experiment, it is determined by \(Z(\omega)\) with the roll-off set by \(1/\tau_{RC}\). Therefore, our results correspond to the equal time correlator \(\langle V^3 \rangle = \langle \delta V(t) \rangle^3\).

In summary, we have measured the third cumulant of shot noise in variable transmission QPCs in the essentially non-Poissonian regime. In order to extract the “intrinsic” third current cumulant we developed a technique which allows reliable subtraction of the environmental and nonlinear circuit-dependent contributions. Good agreement between the experimental results and the expectations, Ref. [3], opens a venue for using high order cumulants as an experimental tool in mesoscopic physics.

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