TESTING GRAVITY IN THE OUTER SOLAR SYSTEM: RESULTS FROM TRANS-NEPTUNIAN OBJECTS

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ABSTRACT

The inverse square law of gravity is poorly probed by experimental tests at distances of ~10 AU. Recent analysis of the trajectory of the Pioneer 10 and 11 spacecraft have shown an unmodeled acceleration directed toward the Sun, which was not explained by any obvious spacecraft systematics, and occurred when at distances greater than 20 AU from the Sun. If this acceleration represents a departure from Newtonian gravity or is indicative of an additional mass distribution in the outer solar system, it should be detectable in the orbits of trans-Neptunian objects (TNOs). To place limits on deviations from Newtonian gravity, we have selected a well-observed sample of TNOs found orbiting between 20 and 100 AU from the Sun. By examining their orbits with modified orbital fitting software, we place tight limits on the perturbations of gravity that could exist in this region of the solar system.

Subject headings: astrometry — celestial mechanics — ephemerides — interplanetary medium — minor planets, asteroids — solar system: general

1. INTRODUCTION

The theory of general relativity (GR) has been verified with a wide variety highly sensitive of experiments. The effects of time dilation, gravitational radiation (via timing of binary pulsars), and gravitational lensing have been tested to very high precision. However, most of the experiments that test GR are in the strong limit of gravity, where the gravitational field and associated mass density are typical for stars and compact objects. Even in the solar system, we see the effects of GR on the precession of Mercury’s orbit as well as in other precision experiments. However, in the weak limit of gravity when objects are moving slowly, GR reduces to the familiar Newtonian form of the inverse square law (Will 2006). This law is used in orbital dynamics to predict the location of planets with objects more than ~1 AU from the Sun without including relativistic corrections. Although Newtonian gravity’s inverse square law shows excellent agreement with observed data throughout on scales of a few AUs, testing gravity in the outer solar system at distances greater than 20 AUs has been difficult.

Since objects (TNOs in particular) in the outer part of the solar system cannot be observed with radar, determining their orbits is done using optical astrometric observations coupled with limited spacecraft observations. The accuracy of these observations and the relatively long time span needed to observe the outer planets has led to some difficulties in matching their orbits to Newtonian gravity. Even after the discovery of Pluto, the anomalies in Neptune’s orbit were attributed to a perturbing 10th planet, until this issue was resolved with modern measurements of planetary mass obtained from spacecraft (Talmadge et al. 1988). The limited astrometric accuracy, the long orbital period, and relatively short time since most of these objects have been accurately observed has led to uncertainty their orbits and precluded using them for accurate tests of the inverse square law.

Other tests of the weak limit of gravity at distances greater than ~10 AU have generally met with limited success. The flat rotation curves of galaxies, for example, have been generally interpreted as evidence of dark matter. However, we have not yet directly detected dark matter particles by any observational or experimental technique. This has led some to interpret the flat rotation curves of galaxies as possible evidence that the Newtonian approximation breaks down in the weak field limit. Instead of invoking the existence of dark matter, modified Newtonian dynamics (MOND; Milgrom 1983) was developed to provide an alternative explanation of the observed flat rotations curves. This theory has had good success at modeling the rotation curves of many galaxies based only on the distribution of the old stellar population. Bekenstein (2004) has presented a Lorentz-covariant theory of gravity known as TeVeS that yields MOND in its weak field limit. Although MOND, TeVeS, and other alternative theories of gravity have not been verified, the idea of Newtonian gravity breaking down in its weak limit must be considered as an alternative to dark matter to explain galaxy rotation curves.

The orbits of periodic comets in our solar system also have shown deviations from Newtonian gravity. These deviations have been characterized as nongravitational forces (Królikowska 2004; Marsden 1969; Marsden et al. 1973; Milani 1999) and are generally attributed to out-gassing of the comets as they approach the Sun. Each comet that shows these deviations from Newtonian motion are fit to a set of three acceleration parameters based on astrometric observations of the orbit, not on physical models of cometary out-gassing. Because of the parametric nature of the fitting process, the orbits of long-period comets cannot confirm that Newtonian gravity is consistent with orbits in the outer part of the solar system.

One obvious way to measure gravity in the outer solar system is by using the high-accuracy tracking data of spacecraft leaving the solar system. When the Pioneer 10 and 11 spacecraft were about 20 AU from the Sun, their tracking data showed a systematic unmodeled acceleration of $(8.74 \pm 1.33) \times 10^{-8}$ cm s$^{-2}$ directed toward the Sun. This acceleration appears at between 10 and 20 AU, and then remains constant outside of about 20 AU. The analysis of this data is detailed in Anderson et al. (1998, 2002a, 2002b). Obvious explanations such as interactions with the solar wind, scattering of diffuse gas off a warm spacecraft, and electromagnetic effects have been considered. Thus far, there...
are no convincing physical phenomena that could cause this acceleration. Although unmodeled spacecraft systematics are the most likely explanation, it is possible that some new physical phenomenon may be responsible for this effect. There are currently preliminary plans to develop a spacecraft to investigate the Pioneer anomaly directly (Dittus et al. 2005). In addition, there is ongoing work to reanalyze the Pioneer spacecraft tracking data (e.g. Turyshev et al. 2005, 2006a, 2006b), and there is considerable debate about the meaning of the original results. Even so, it is certainly clear that separating the effects of spacecraft dynamics from gravitational deceleration is a difficult task when one is trying to measure small deviations from Newtonian gravity.

Whitmore & Matteo (2003) have looked at the orbits of Oort Cloud comets in order to independently examine the Pioneer effect. If the Pioneer effect was affecting comets, the gravitational binding energy would be higher, and Galactic tides could not play the dominate role in making these objects observable. In this paper, we use trans-Neptunian objects (TNOs) to place limits on deviations from Newtonian gravity in the outer parts of the solar system. The use of planetary orbits to measure these deviations from Keplerian orbits is not new, and similar analyses have been completed by other authors using astrometric data on the major planets (cf. Talmadge et al. 1988; Hogg et al. 1991; Sereno & Jetzer 2006; Iorio & Giudice 2006) In these papers, the authors either examine the residuals of the orbital fit or project the orbital trajectories forward in time and look for the expected deviation between existing theories and observational uncertainty. There are limitations on both of these approaches.

In this paper, we take advantage of the large body of astrometric data that has recently become available on TNOs. Our approach is to use an ensemble of objects that have been found in the outer solar system, and whose observations are archived in the Minor Planet Center Extended Computer Service (ECS). Extending the modeling technique of Page et al. (2006), we fit the orbits using a modified orbital fitting program that allows a radially directed force of arbitrary strength to be added to gravitational accelerations already calculated by the program. For each object, we calculate this anomalous acceleration along with a statistically derived error using the well-documented bootstrap technique (Wall & Jenkins 2003; Efron & Tibshirani 1993).

Even though most of these TNOs have only been recently identified, some of them have long observational arcs because of the reanalysis of archival images. The ensemble of observations used in this paper covers a combined total of 562 years of observations over 24 objects, making it a very sensitive data set for examining gravity in the outer solar system.

Beyond the results of this study, the methodology presented in this paper can be extended to new objects discovered with future large sky surveys such as Pan-STARRS and LSST. Using this technique, strict limits on the deviations from Newtonian gravity can be found, constraining the solar system dark matter distribution as well as other alternative theories of gravity such as MOND or TeVeS.

2. METHODOLOGY
2.1. The Sample

To investigate possible gravitational perturbations to the inverse square law in the outer solar system, we formed a sample of objects from the ECS. Our sample was selected based on three criteria:

1. The object must be observed at least 20 AU from the Sun, where the Pioneer anomaly was detected by Anderson et al. (2002a).
2. The object must have been observed over at least seven oppositions at a heliocentric distance greater than 20 AU.
3. There must be at least 40 archived observations of the object.

The first constraint is imposed because the Pioneer anomaly was first unambiguously detected in the spacecraft tracking data when it was more than 20 AU from the Sun. The last two constraints were derived empirically. Our analysis has shown that orbits with less than 40 observations over at least seven oppositions simply are not well enough constrained to produce accurate values of the orbital elements including the perturbing acceleration. When additional objects are included, the large errors associated with their fits make them extraneous to the final weighted average of the results.

Using the first criterion, we searched the 2006 May 1 Minor Planet Center’s ECS database of planetary orbits (MPCORB.DAT) and extracted an initial sample of 31 objects from the 294,488 entries. Observational data for each of these objects was then extracted from the Minor Planet Center’s observational archives (mpn.arc), and preliminary orbits were fit using the OrbFit Consortiums OrbFit (version 3.3) program. Using these fitted orbits, we rejected an additional seven objects as unsuitable for our analysis, because they failed the second and third criteria. These resulting list of 24 objects and their orbital characteristics are listed in Table 1.
The model we are fitting to these data is very simple and is applied separately to each object in our sample. We use the bootstrap technique to estimate errors in our fits and then explore the results for systematic trends based on position and orbital parameters. Finally, we combine the results to place a limit on deviations from the inverse square law in the outer solar system using the ensemble of data.

2.2. Orbital Fitting

To search for perturbations on Newtonian gravity, we used a modified version of the OrbFit program that is used to fit orbits to observations of asteroids. This code is well documented and is widely used in the field. For our study, we add an additional term to the gravitational acceleration from the Sun. The effective acceleration of gravity from the Sun becomes

\[ g_{\text{eff}} = -\left(\frac{G M_\odot}{R^2} + \kappa \right) \hat{r}, \]  

where

\[ \kappa = \begin{cases} 0, & R < 20 \text{ AU} \\ \epsilon, & R \geq 20 \text{ AU} \end{cases} \]  

where \( \epsilon \) is an arbitrary parameter we fit to the observed data for each object.

Although we realize this model is not physically realistic, we adopt it based on four considerations. First of all, the model is consistent with what was seen in the Pioneer data and other solar system constrains on the inverse square law. The anomalous acceleration is constant after approximately 20 AU. The Pioneer tracking data show this anomaly turns on between 10 and 20 AU from the Sun (cf. Anderson et al. 2002b, their Fig. 7). The particular form of the transition is not well constrained by data. We also know that the inverse square law is well characterized in the inner solar system, and more poorly constrained in the outer solar system. The use of a transition fits this behavior. Second, the model is very simple, with only one free parameter. Since we fit this equation to each object separately and then later examine its dependence on a set of orbital parameters, we are making very minimal assumptions about the any anomalous perturbation. Since most objects are found within a narrow range of distances from the Sun, fitting each object separately allows us to investigate an possible dependence of \( \epsilon \) on heliocentric distance. This is also true with variables such as ecliptic longitude and orbital parameters such as eccentricity. Third, only one object in our sample ever goes inside the 20 AU cutoff. The inner and outer orbital radii during the observational arc of each object in our sample is presented in Table 1. Including a more complicated transition would add unnecessary complexity to the fit and add no significant knowledge to gravitational perturbation in the outer solar system. Finally, the single object (42355) that does go inside 20 AU has a large error on the final fit of \( \kappa \), and does not significantly bias our final results. Thus, the final fit we are using is effectively gravitational acceleration plus a fitted radially directed acceleration. The representation of the transition region at 20 AU has no significant effect in our conclusions.

Because we have introduced the new parameter \( \epsilon \) into the code, we converge on the best value of this perturbing acceleration by using a modified bisection method to find value of \( \epsilon \) that produces the minimal residual. Although there may be exceptions, brute force examination of the residual on selected test cases has shown that there is a single value for this minimum residual, and
the values smoothly decrease toward this minimum. Using this bisection method, we are able to converge to a value for the perturbing force at a suitable accuracy with only about 20 iterations. The best fit is estimated by fitting a parabola to the three points nearest the minimum and interpolating the location of the minimum. For any given set of observations, we calculate the value of $\epsilon$ along with the minimal residual.

It is important to note that adding an extra parameter to any model will inevitably lead to nonzero values in that parameter in poorly characterized data. As we discuss in §2.3, it is critical to be able to characterize the quality of the data before making conclusions about the overall value of the parameter $\epsilon$.

2.3. Statistical Analysis and Reliability of the Results

In order to have confidence in the results from this study, we use the bootstrap method to resample the observational files. As described in Efron & Tibshirani (1993), there are two basic versions this technique that can be applied to fits of data.

The first method of “bootstrapping the observations” directly resamples the observational file. For a given observational file with $n$ entries, the bootstrap file samples the table of observations $n$ times with replacement. The resulting data table is of the same length as the original, but some entries have been duplicated and others have been dropped. This method has limited utility with orbital fitting, since the resampling can fundamentally change the character of the fit. If, for example, the resampling drops a single critical observation from 50 years ago, the overall quality and reliability of the fit will be substantially diminished.

The second method of “bootstrapping the residuals” initially fits the orbit using all the original observational entries creating a model orbit. The residuals of the fit are then resampled and added to the model orbit, creating a set of synthetic observations. These synthetic observations are created at the same time intervals as the original data. For orbital data this method is preferable, since it does not introduce the systematic bias that would occur from dropping and duplicating observations.

In both methods, a new orbit is then fit to the synthetic observations. The process is repeated, and the acceleration parameter $\epsilon$ is tabulated. A mean and standard deviation for $\epsilon$ is then derived from the ensemble of runs. Details of this method and its statistical basis are discussed elsewhere (Efron & Tibshirani 1993; Wall & Jenkins 2003). As Wall & Jenkins observe, this technique, which seems to give something for nothing, is well established. In addition, the bootstrap method has been shown to provide converging estimates to the underlying statistical properties of the resampled data.

For our analysis using the bootstrap methods, we created a set of 100 simulated orbits for each object. When we bootstrapped the residuals of these orbits (method 2 from above), the runs all converged and gave us an estimate of the anomalous acceleration $\epsilon$.

Bootstrapping the observations (method one from above) was more problematic. For some objects, some of the synthetic orbits failed to converge because of the nature of the resampled observations. In some cases, entire years of observational measurements can be dropped because of the resampling being done in the method. At the same time, duplicate observations are created giving extra weight to arbitrary entries. The resampling inherent with directly bootstrapping the observations can lead to large gaps in the observational arc that make the trial data sets fall below the criteria of seven oppositions with 40 observations we set for our sample selection. Although we do not reject these runs a priori, the results can be a failure to find a robust orbit that fully converges. Nevertheless, about 75% of the runs that bootstrapped the observations did converge in our analysis.

Although we believe this lack of convergence in some of our runs will not likely lead to a significant bias in our results, we present the results from bootstrapping the observations only for completeness. As discussed above, the objects that have the highest fraction of nonconverging runs were those that have the shortest and poorest sample of observational arcs. It is likely that the values of the anomalous acceleration ($\epsilon$) and the errors ($\delta\epsilon$) are being underestimated on these objects. All eight of the objects that had convergence rates of less than 90% have error estimates of greater than 100 times the Pioneer effect. Since the best-sampled orbits have errors so much smaller than those that only marginally fit our criteria, the impact on our final results is small. We further discuss the impact of the nonconverging runs in §3.

It is important to reiterate that the convergence problems were not present when we bootstrapped the residuals. Since the time intervals and data were much more consistent with the original data fits, the fitting process was much more robust. For parametric fits, bootstrapping the residuals is generally preferred over bootstrapping the observations because of these issues (Efron & Tibshirani 1993). Although we present the results from both methods, we believe the results from the bootstrapped residuals are more reliable.

3. RESULTS AND ANALYSIS

The results of the analysis using the bootstrapped residuals method are shown in Table 2. The second column in this table represents the average acceleration ($\epsilon$ in equation [1]) and its standard deviation from the 100 run ensemble. We also calculate the average residual and standard deviation of the residual for the ensemble of runs. In the “acceleration” column we present the fitted anomalous acceleration using all the data in the observational files without using bootstrapping. We refer to this fit as our best-fit model. No formal error can be found on for this “best-fit” model, since they are derived from a single set of observations. The last column represents the residual we found by fixing $\epsilon = 0$, thus providing no perturbative force.

All the forces in the table and the text below are measured in terms of the Pioneer effect acceleration of $7.74 \times 10^{-8}$ cm s$^{-2}$. An acceleration of one in these units would be expected if the Pioneer anomaly was affecting the orbits of these objects, while an acceleration of zero would indicate consistency with standard Newtonian gravity.

To examine the consistency of our results, in Figure 1 we plot the measured error in the anomalous acceleration ($\delta\epsilon$) against the absolute value of the anomalous acceleration ($\epsilon$) for each object in our sample. In this plot, there is an obvious correlation between the error and the value of the anomalous acceleration derived from our fits. The objects with large accelerations are those with large errors in our sample, suggesting the true value of the acceleration $\epsilon$ should be near zero.

Figure 2 shows the relationship between the measured error in the anomalous acceleration and the observed observational arc in radians. As expected, the best-characterized objects have longer observational arcs. However, other factors such as frequency of sampling also play a critical role in reducing the errors in orbital determination. Regular observations over a long time period are likely to yield significant improvements of these results.

In both Figures 1 and 2, the results are what we would have expected in this experiment. Specifically, the estimates we make of our errors seem to be consistent with the behavior of the estimates of the accelerations. Based on the results, we believe error estimates we derive using the bootstrap technique provides us with a reliable measurement of the error associated with the
By doing this, we are assuming fitting the inverse of the variance as the weights (Bevington 1969). The objects. We will examine the validity of this assumption on the acceleration is small. All accelerations are measured in units of the standard deviation from 100 trial runs of the bootstrap analysis for each object.

FIG. 1.—Error of the measured anomalous acceleration $\Delta a$ vs. the magnitude of measured anomalous acceleration $|\epsilon|$. The error of the measured acceleration is correlated with the strength of the acceleration, indicating that the true value of the acceleration is small. All accelerations are measured in units of the measured Pioneer anomaly ($8.74 \times 10^{-8}$ cm s$^{-1}$). Errors were derived using one standard deviation from 100 trial runs of the bootstrap analysis for each object.

TABLE 2

| Object Name       | Bootstrap Acceleration$^a$ | Bootstrap Residual$^b$ (arcsec) | Acceleration$^c$ (arcsec) | Residual$^d$ (arcsec) |
|-------------------|---------------------------|----------------------------------|---------------------------|-----------------------|
| 15760             | 85.4 ± 54.5               | 0.67042 ± 0.039546               | 86.8                      | 0.68433               |
| 15788             | 332 ± 201                 | 0.68436 ± 0.062450               | 119                       | 0.80242               |
| 15789             | −34.1 ± 40.5              | 0.59386 ± 0.043949               | −29.4                     | 0.61228               |
| 15807             | 42.9 ± 29.4               | 0.76216 ± 0.057195               | 37.7                      | 0.81021               |
| 15809             | 37.9 ± 21.6               | 0.46411 ± 0.042229               | 38.8                      | 0.48921               |
| 15874             | −1.98 ± 4.51              | 0.48916 ± 0.028637               | −2.19                     | 0.50035               |
| 15875             | 32.9 ± 96.8               | 0.55931 ± 0.069292               | 16.8                      | 0.56625               |
| 16684             | 117 ± 42.7                | 0.44258 ± 0.023309               | 121                       | 0.48952               |
| 19521             | 1.44 ± 5.53               | 0.41250 ± 0.034758               | 0.722                     | 0.42752               |
| 20000             | 0.426 ± 0.966             | 0.47528 ± 0.046502               | 0.359                     | 0.49634               |
| 24835             | 0.801 ± 1.14              | 0.58405 ± 0.050270               | 0.642                     | 0.58942               |
| 26181             | −0.754 ± 1.22             | 0.65125 ± 0.061349               | −0.671                    | 0.67256               |
| 26308             | 1.15 ± 0.693              | 0.47548 ± 0.030127               | 1.10                      | 0.50099               |
| 26375             | −2.69 ± 1.83              | 0.39551 ± 0.044992               | −2.94                     | 0.40950               |
| 26978             | 0.432 ± 2.51              | 0.36524 ± 0.049641               | 0.485                     | 0.37840               |
| 42355             | 0.260 ± 3.10              | 0.51636 ± 0.062756               | 0.378                     | 0.53024               |
| 50000             | 1.0 ± 0.537               | 0.52599 ± 0.042784               | 1.18                      | 0.56972               |
| 55636             | 0.0102 ± 0.231            | 0.44148 ± 0.055523               | −0.0168                   | 0.45739               |
| 79360             | 3.31 ± 24.7               | 0.53585 ± 0.038037               | 6.76                      | 0.54377               |
| 90482             | −0.0403 ± 0.542           | 0.40644 ± 0.050609               | −0.0404                   | 0.41001               |
| J03F00W           | 33.8 ± 51.7               | 0.53675 ± 0.047901               | 37.5                      | 0.56737               |
| K03E61L           | 0.602 ± 0.686             | 0.41072 ± 0.043557               | 0.503                     | 0.41953               |
| K03UV3B           | −1.26 ± 1.76              | 0.4380 ± 0.029639                | −1.21                     | 0.44969               |
| K05F09Y           | −0.307 ± 0.359            | 0.23759 ± 0.013461               | −0.289                    | 0.23766               |

$^a$ The average of the best fit to the anomalous acceleration $\epsilon$ in terms of the Pioneer anomaly ($8.74 \times 10^{-8}$ cm s$^{-1}$) along with the 1 $\sigma$ error from the bootstrap analysis.

$^b$ Average residual and its 1 $\sigma$ error from the bootstrap analysis.

$^c$ The best fit to the anomalous acceleration $\epsilon$ in terms of the Pioneer anomaly ($8.74 \times 10^{-8}$ cm s$^{-1}$). No formal error is available on this measurement, since it was derived from a single fit.

$^d$ Residual of the orbit assuming no perturbing force ($\epsilon = 0$).

By doing this, we are assuming fitting $\epsilon$ to a single value for all the objects. We will examine the validity of this assumption below.

Fig. 2.—Error of the measured anomalous acceleration $\Delta a$ vs. the observed geoecentric arc of observations measured in radians. Objects with very short observational arcs (<0.2 radians) are poorly characterized and do not have errors small enough to contribute to the analysis. A long arc length of observations by itself is not sufficient for small errors in the orbital fit. Additional factors such as the sampling rate and the quality of the astrometry play a role the reliability of the orbital fit as well. All accelerations are plotted in units of the measured Pioneer anomaly ($8.74 \times 10^{-8}$ cm s$^{-1}$). The acceleration errors $\Delta a$ were derived using one standard deviation from 100 runs of the bootstrap analysis for each object.
The weighted average for the ensemble of bootstrapped residual runs was \(0.10 \pm 0.16\) times the Pioneer acceleration, where the uncertainty is a 1 \(\sigma\) error. If we use the bootstrap errors for the weighting and use the “best-fit” models as the values for the run, the average acceleration is identical to two significant digits. These results are consistent with standard Newtonian gravity and a value of zero for \(\epsilon\). Using the “bootstrapped observation” method we find the ensemble of bootstrap runs gives us an average acceleration of \(-0.23 \pm 0.28\) times the Pioneer acceleration. If we use the bootstrap errors for the weighting and use the “best-fit” models as the values for the run, the average acceleration is \(0.03 \pm 0.28\). When we only include the objects that have had no problems with convergence in any of their runs, the final results we obtain for the acceleration is identical to three significant digits as the weighted average of the overall sample. The lack of convergence in the some runs using the bootstrapped observation method creates no significant bias in our results.

To further examine gravity in the outer solar system, we looked at how the measured acceleration is correlated with the position and orbital parameters of the objects in our sample. Figure 3 shows the relationship between the measured anomalous acceleration \(\epsilon\) and the derived heliocentric distance of the orbits of our objects. For clarity, only the objects with accelerations errors less than \(\pm 10\) times the Pioneer effect were included in this plot for clarity. All accelerations are plotted in terms of the measured Pioneer anomaly \((8.74 \times 10^{-8} \text{ cm s}^{-2})\). The acceleration errors \((\Delta\epsilon)\) were derived using one standard deviation from 100 trial runs of the bootstrap analysis for each object.

Fig. 3.—Anomalous acceleration \(\epsilon\) vs. average distance of the object from the Sun in AU over the observed arc. There is no apparent trend between the measured anomalous acceleration and the distance from the Sun. Only the objects with errors less than \(\pm 10\) times the acceleration of the Pioneer effect were included in this plot for clarity. All accelerations are plotted in terms of the measured Pioneer anomaly \((8.74 \times 10^{-8} \text{ cm s}^{-2})\). The acceleration errors \((\Delta\epsilon)\) were derived using one from 100 trial runs of the bootstrap analysis for each object.

Fig. 4.—Measured anomalous acceleration \(\epsilon\) vs. orbital inclination, ecliptic longitude, semimajor axis, and orbital eccentricity. Only the objects with errors less than \(\pm 10\) times the acceleration of the Pioneer effect were included in this plot for clarity. There is no evidence of any statistically significant correlation between these four parameters and the corresponding acceleration. All accelerations are plotted in terms of the measured Pioneer anomaly \((8.74 \times 10^{-8} \text{ cm s}^{-2})\). The acceleration errors \((\Delta\epsilon)\) were derived using one standard deviation from 100 trial runs of the bootstrap analysis for each object.
less than 10 times the *Pioneer* anomaly were included in the plot. There is no clear correlation between the heliocentric distance and the anomalous acceleration. However, the error bars on our measurements are too large to completely rule out such a correlation.

In Figure 4 we compare the anomalous accelerations for these TNOs with the eccentricity of the orbit, semimajor axis of the orbit, average heliocentric distance during the observations, and ecliptic longitude. No statistically significant correlations were found in the data between mean acceleration $\epsilon$ and any of these parameters. There is a weak correlation seen between semimajor axis and $\epsilon$ as well as orbital eccentricity and $\epsilon$. Although this should be examined further with future observations, it is well within the noise and not considered significant. Because of the large uncertainty in some of these measurements, detecting small trends in the data is not possible. With that caveat, there is no evidence to support any systematic deviations from the inverse square law as a function of these variables. In general, we find no correlation between the orbital elements or positions of our objects and the results we find for the anomalous acceleration.

4. CONCLUSIONS

In this paper, we have presented a new method using orbital measurements of an ensemble of TNOs to measure deviations from the inverse square law of gravity in the outer solar system. The method relies on doing separate orbital fits for each object, and then characterizing the accuracy of each fit using the bootstrap technique. Since no significant systematic trends were detected in our sample, we combined the data from all the objects using a weighted average to place limits on deviations from the gravitational inverse square law in the outer solar system. Using existing data, we have measured the deviation from the inverse square law to be $ba = 8.7 \times 10^{-9} \pm 1.6 \times 10^{-8}$ cm s$^{-2}$ directed outward from the Sun for objects at heliocentric distances of 20–100 AU. This result is consistent with zero at the 1 $\sigma$ limit.

Based on our analysis of the observational data of TNOs, we find that the gravitational acceleration in the outer solar system is inconsistent with the *Pioneer* anomaly at the $\sim$5 $\sigma$ level using both variations of the bootstrap analysis. All of our results were consistent (within 1 $\sigma$) with Newtonian gravity without any additional radial perturbative forces. This suggests that the deceleration seen in the *Pioneer* tracking data was probably the result of spacecraft systematics rather than exotic physics. Even so, we cannot rule out the possibility that exotic physics is affecting the *Pioneer* spacecraft trajectories. Our work only shows that the trajectory data from the *Pioneer* spacecraft is inconsistent to what we see in large, slowly moving rocks in the outer solar system (Page et al. 2006).

These results were derived using existing astrometric data on minor planets in the outer part of the solar system. The use of the bootstrap method in this analysis has allowed us to provide a test of the reliability of each orbital fits. Although bootstrapping the observations directly is not held to be as reliable as bootstrapping the residuals, both results are consistent with our conclusions. The combination of these two methods provide a cross check, albeit a weak one, on our results.

We found no evidence of any correlation between the measured values of $\epsilon$ and the object’s position in the solar system or its orbital characteristics. However, we would have only detected such a trend if it were strongly present in the data with an amplitude much greater than the *Pioneer* effect acceleration.

Overall, the results we present confirm the veracity of the Newtonian gravitational potential in the outer parts of the solar system. However, future analysis of astrometric data from Pan-STARRS and LSST will provide a much more sensitive test of gravity. If the number of TNOs in our sample is expanded by a factor of 100, and these objects have a long arc of regularly observed positions, we will be able to increase the sensitivity of our results by a factor of about 10, depending on the arc lengths and rate of observations. The data sets from Pan-STARRS and LSST that will be created within the next 10 years will provide strong limits on alternative gravitational theories such as MOND.

Additional future work will focus on how well this technique works for finding accelerations using ensembles of synthesized observations. Of specific interest is the role that orbital eccentricity and the length and completeness of the observational arc plays in the results we obtain from fitting acceleration parameters to observational data.

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4 See http://cfa-www.harvard.edu/iau/services/ECS.html
5 See http://newton.dm.unipi.it/orbitfit
6 See http://ssd.jpl.nasa.gov/eph_info.html