I. INTRODUCTION

Three-dimensional Chern-Simons (CS) theory enjoys level-rank duality, which is well established when the level $k$ and the rank $N$ of the gauge group are finite [1–4]. This duality is believed to extend to a non-perturbative duality between conformal theories that are obtained by coupling CS theory to scalars or fermions in the fundamental representation.

There is extensive and robust evidence for the duality, especially in the large $N$ ’t Hooft limit with $\lambda \equiv N/k$ fixed. It includes matching correlation functions of local operators [5–10], the spectrum of monopole/baryon operators [11, 12], thermal free-energies [13–17], S-matrices [18–20], and relating the non-supersymmetric dualities to well-established supersymmetric ones [21, 22] via RG flow [16, 23]. Since these tests are performed in the strict planar limit, they do not distinguish between different versions of the duality that differ by half-integer shifts of the Chern-Simons level $k$ and by the gauge group being $SU(N)$ or $U(N)$ [24].

In this letter, we summarize the results of our study of line operators in the large $N$ limit. They can be either closed, such as Wilson loops, or open, such as Wilson lines stretching between a fundamental and an anti-fundamental field. We denote the latter as mesonic line operators. Detailed derivations of the results presented here will appear in separate publications [25, 26].

At leading order in the large $N$ limit, the matter does not contribute to the expectation values of closed Wilson loops. On the other hand, it does contribute to the expectation values of mesonic line operators. Correspondingly, their dependence on the path is not topological and, as will become clear, is directly related to the $1/N$ correction to the closed Wilson loop expectation value. Mesonic line operators overlap with all local operators in the theory, including the single-trace and the multi-trace ones.

A generic mesonic operator would experience an RG flow on the line. Here, we focus on the fixed points of that flow, which are the conformal line operators, and study them along arbitrary smooth paths. We classify them, as well as their relevant deformations (when such exist), and speculate about the flows between them.

Since the Wilson line in CS theory is oriented, we have two families of boundary operators, right (fundamental) and left (anti-fundamental). Both sets are uniquely classified by their conformal dimension and transverse spin. They can be further divided into those that become $SL(2,\mathbb{R})$ primaries and descendants when the line is straight. For any of the conformal line operators and on either the left or the right boundary, we find that there are two towers of primary operators. Operators in the same tower have the same twist. They all have non-zero anomalous dimension and anomalous transverse spin, equal to $\pm \lambda/2$. Correspondingly, if one starts with a boundary operator of integer (half-integer) spin at $\lambda = 0$, one ends up with a boundary operator of half-integer (integer) spin at $|\lambda| = 1$.

To determine the correlation functions and expectation values of the line operators, we need to understand their dependence on the shape of the path. This dependence is subject to the evolution equation. This first-order equation relates any smooth variation of a (conformal) mesonic line operator to a factorized product of two mesonic line operators, see figure 1. Stated differently, each of the conformal line operators has a unique operator on the line of dimension two and transverse spin.
one. These adjoint displacement operators factorize into a product of left and right boundary operators. These two boundary operators have non-trivial, but opposite, anomalous dimensions and anomalous spin.

Consider, for example, the correlation function of the mesonic line operator with a single trace operator, local or not. The evolution equation implies that under a smooth deformation of the path, the correlator factorizes into the expectation value of a mesonic line operator along a part of the original path times the same correlator, but with a shorter mesonic line operator, see figure 1. Hence, the seed for any such correlator is the expectation value of the mesonic line operator, which is our main focus.

We show that the evolution equation, combined with the spectrum of boundary operators, uniquely determines the expectation value of any of the mesonic line operators. To demonstrate this, we start with a straight line and deform it smoothly and systematically, order by order in the relative magnitude of the deformation, while imposing the above properties. In particular, we bootstrap the (normalization independent) two-point function of the displacement operator [27]. We find that it is given by

\[ \frac{\langle O_L \bar{D}_1(x_1) \bar{D}_4(x_0) O_R \rangle}{\langle O_L \bar{O}_R \rangle} = \Lambda(\Delta) \left( \frac{x_{10} x_{sf}}{x_{1s} x_{0sf}} \right)^{2\Delta}, \]

with

\[ \Lambda(\Delta) = -\frac{(2\Delta - 1)(2\Delta - 2)(2\Delta - 3) \sin(2\pi \Delta)}{2\pi}. \]

The double brackets in (1) denote expectation values in the presence of the mesonic line operator lying along a straight line stretching between \( x_0 \) to \( x_1 \). Here, \( O_L / O_R \) are the left/right boundary operators of minimal (and opposite) transverse spin and \( \Delta \) is their conformal dimension. The \( \lambda \) dependence of \( \Delta \), given below, depends on which conformal line operator we consider and whether we use the fermionic or the bosonic descriptions. We have also verified (2) in perturbation theory to all loop orders [25].

We show that the conformal line operators of the bosonic and fermionic theories satisfy the same evolution equation and that their spectra of boundary operators are related to each other through the map \( \lambda_f = \lambda_b - \text{sign}(k_b) \) [7]. It follows that their expectation values are related by the same map.

### II. SETUP AND OVERVIEW

The first hint for the existence of a dynamical interplay between fermions and bosons in three dimensions comes from the study of CS theory. This topological gauge theory is governed by the action [28]

\[ S_{CS} = \frac{ik}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \text{tr} (A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho). \]

In this paper, we work in Euclidean signature and focus on the \( 't \) Hooft limit, in which the rank of the gauge group is large

\[ N \to \infty \quad \text{with} \quad \lambda \equiv \frac{N}{k} \in [-1,1] \quad \text{fixed}. \] (4)

Here we have assumed the convention where \( k \) is the renormalized level that arises, for instance, when the theory is regularized by dimensional reduction.

The theory (3) enjoys a level-rank duality under which the parameters in (4) transform as

\[ (k, \lambda) \leftrightarrow (-k, \lambda - \text{sign}(k)). \] (5)

This duality interchanges the weak and strong coupling limits.

In the pure CS theory, the only observables are Wilson loops. When defined with framing regularization, they only depend on the topology of the loops and the self linking number \( \mathfrak{l} \). The latter counts the number of times the framing vector \( n \) winds around the loop [29]. For example, the expectation value of an unknotted loop is

\[ \langle W^\mathfrak{l}_{\text{unknot}} \rangle = e^{i\pi \lambda \mathfrak{l}} \times k \frac{\sin(\pi \lambda)}{\pi}. \] (6)

We can therefore think of the Wilson loop as being a ribbon, parameterized by the framing vector. It is expected that once we attach a Wilson line to an operator in the fundamental representation, this dependence on the framing vector would lead to fractional spin and to statistics ranging between a boson (fermion) at \( \lambda = 0 \) and a fermion (boson) at \( |\lambda| = 1 \), see figure 2. Here we will prove this expectation.

Concretely, we study CS theory coupled to fermions or bosons in the fundamental representation. The action in these two cases is given by [30]

\[ S_{E}^{\text{bos}} = S_{CS} + \int d^3x (D_\mu \phi)^\dagger :D^\mu \phi + \frac{\lambda_b}{N^2} (\phi \cdot \phi)^3, \] (7)

\[ S_{E}^{\text{fer}} = S_{CS} + \int d^3x \bar{\psi} \gamma^\mu D_\mu \psi. \] (8)

Both theories are conformal (for tuned \( \lambda_b \)) and have high spin currents that are conserved at leading order.
in the large $N$ limit (4). The level-rank duality (5) was conjectured to extend to a duality between the theory of fermions (bosons) and the Legendre transform of the theory of bosons (fermions) with respect to the scalar current $J^{(0)}$ [31], [32]. The differences between these theories and their Legendre transforms will not be relevant for our primary focus, which is the the expectation values of mesonic line operators. It enters the bootstrap of the line operators’ correlators, which is briefly discussed in section VI, through the dimension of $J^{(0)}$.

Without loss of generality, we assume that the level, and correspondingly, the ’t Hooft coupling in the bosonic theory is positive. Hence, the dual level and ’t Hooft coupling of the fermionic theory are negative. The other sign is related to this one by a parity transformation, $k \leftrightarrow -k$, $\lambda \leftrightarrow -\lambda$.

### III. MESONIC LINE OPERATORS IN THE BOSONIC THEORY

The most familiar line operator along any smooth path $\mathcal{C}$ is a Wilson line. However, on such a line in the bosonic theory, there is a non-zero beta-function for the coupling of the adjoint operator $\phi^d$. At the fix points of the corresponding flow, we find operators with the bi-scalar condensate,

$$W^\alpha[\mathcal{C}, n] = \left[ P e^{i f (A dx + i\alpha \frac{\partial}{\partial x} \phi^d [dz])} \right]_n, \tag{9}$$

where $\alpha = \pm 1$. We show that the operator with $\alpha = 1$ is stable, while the operator with $\alpha = -1$ has one relevant deformation that, when turned on, generates a flow that leads to the former [33]. In appendix A, we consider another non-unitary conformal line operator.

#### A. The Stable Mesonic Line Operator

First, we consider the operator (9) with $\alpha = 1$. The corresponding mesonic line operator is defined by stretching $W \equiv W^{n=1}$ between a right (fundamental) boundary operator and a left (anti-fundamental) boundary operator,

$$M = \mathcal{O}_L W \mathcal{O}_R. \tag{10}$$

It depends on the shape of the path, the framing vector, and the two boundary operators. In the planar limit, all operators on the line factorize into a product of two boundary operators $\mathcal{O}_{\text{inner}} = \mathcal{O}_R \times \mathcal{O}_L$.

In order to classify the boundary operators, it is sufficient to consider the case of a straight line along the $x^3$ direction. An infinite straight line preserves an $SL(2, \mathbb{R}) \times U(1)$ subgroup of the three-dimensional conformal symmetry. The boundary operators are uniquely characterized by two numbers, their $SL(2, \mathbb{R})$ conformal dimension $\Delta$ and their $U(1)$ spin in the transverse plane to the line. For example, at tree level, for the right operators,

$$\mathcal{O}_R^{(n,s)}(\text{tree}) = \frac{1}{\sqrt{N}} \times \left\{ \frac{\partial^s \partial^* \phi}{\partial^s \partial^* \phi} \right\} s \geq 1, \tag{11}$$

and similarly for $\mathcal{O}_L^{(n,s)}(\text{tree})$. Here, $x^\pm = (x^1 \pm ix^2)/\sqrt{2}$ parametrize the transverse plane. The operators of minimal twist, $\mathcal{O}^{(0,s)}_R$ are all $SL(2, \mathbb{R})$ primaries.

At tree-level these boundary operators have transverse spin $s$ and dimension $\Delta^{(n,s)}_R = 1/2 + n + |s|$. However, at loop level their dimensions and spins receive quantum corrections. To begin with, we computed their conformal dimensions and anomalous spin explicitly [25]. Working in lightcone gauge, we studied the expectation values of all the mesonic line operators (10) along a straight line. It was shown that terms in their perturbative expansions satisfy a recursion relation, which can be solved to give simple expressions. Resummation of these expressions yields an anomalous dimension and anomalous spin equal to $\pm \lambda/2$, as well as the two point function of the displacement operator in (2).

The set of operators with $s \geq 0$ ($s < 0$) all have the same anomalous dimension. They are related to each other by the covariant path derivatives, denoted by $\delta_\mu$, as follows, [34],

$$O_{R}^{(0,s+1)} = \delta_+ O_{R}^{(0,s)}, \quad s \geq 1, \tag{12}$$

$$O_{R}^{(0,s-1)} = \delta_- O_{R}^{(0,s)}, \quad s \geq 0, \tag{13}$$

and

$$O_{L}^{(0,s+1)} = \delta_+ O_{L}^{(0,s)}, \quad s \geq 0, \tag{14}$$

$$O_{L}^{(0,s-1)} = \delta_- O_{L}^{(0,s)}, \quad s \geq 1.$$

Here, the use of equal signs instead of a proportionality relation is a relative choice of normalization. At the bottom of these four towers of primaries we have the boundary operators

$$\{O_{L}^{(0,0)}, O_{L}^{(0,-1)}\} \text{ and } \{O_{R}^{(0,0)}, O_{R}^{(0,1)}\}. \tag{14}$$

Similarly, the descendants are obtained from the primaries by acting with the $SL(2, \mathbb{R})$ raising generator. Their form depends on the conformal frame. Since we let the endpoints vary and do not keep the line straight, we use a simpler classification of the descendants by the number of longitudinal path derivatives,

$$O_{L}^{(n+1,s)} = \delta_\mu O_{L}^{(n,s)}, \tag{15}$$

and similarly for the right operator.

The spin $s$ of the boundary operators can be computed explicitly [25] or alternatively, be determined from the anomalous dimension as we now explain. First, we notice that operators that are related to each other by a path derivative, (12), (13), and (15), must have the same anomalous spin. Second, we notice that the conformal line operator $W$ can be lifted into locally super-symmetric
Hence, the line operator (9) with ψ spins as that of the two components of the free fermions λ coupling limit, results in turning bosons into fermions and vice versa.ing dependence of a closed Wilson loop in CS theory relevant deformation. As in (17), all results we find for O, they are dual to them.

This result confirms the expectation in which the framing vector is however topological and does not contribute to the displacement operator.

We find that the displacement operator is chiral, with its two components given by [36]

\[ D_+ = -4 \pi \lambda O_R^{(0,1)} L^{(0,0)} \]
\[ D_- = -4 \pi \lambda O_R^{(0,0)} L^{(0,-1)} \]

with the understanding that the framing vector, being continuous, is the same on the right and left. Note that while the left and right boundary operator have non-zero anomalous dimensions and anomalous spins, these exactly cancel out in the combinations in (20).

This form of the displacement operator is derived by computing the Schwinger-Dyson equation for the line operator defined in (9), with no self-intersections. Alternatively, we notice that D in (20) is the unique operator on the line with exact dimension \( \Delta(D_{\pm}) = 2 \) and transverse spin equal to one. We can therefore reverse the logic and use (20) as the definition of the deformed operator.

The factorized form of the displacement operator leads to a closed equation for the mesonic line operators. We label them using the shorthand notation

\[ M^{(s_L,s_R)}_{st}(x) \equiv O^{(0,s_L)}_L W_{st}(x) O^{(0,s_R)}_R , \]

where \( x(s) = x_L(s) + x_R(s) \) is a smooth path between \( x_L = x(1) \) and \( x_R = x(0) \). In this notation, the evolution equation takes the form

\[ \delta M^{(s_L,s_R)}_{st}(x) = [\text{boundary terms}] \]
\[ -4 \pi \lambda \int_0^1 ds |\dot{x}_s|^2 \left[ v^+_s M^{(s_L,1)}_{1s} M^{(0,s_R)}_{0t} + v^-_s M^{(s_L,0)}_{1s} M^{(-1,s_R)}_{0t} \right] , \]

where \( u_s \equiv u(s) \). The boundary terms can be determined by the consistency of the equation, see section V and [26].

The fact that the line operators satisfy a first-order equation follows from the fact that the Chern-Simons equation of motion is first order (as opposed to second order for Yang-Mills theory). Due to the non-linear nature of this equation, the prefactor of \( 4\pi \lambda \) can be changed at will by changing the relative normalization of the operators. The normalization-independent factor that controls the strength of the deformation is the two-point function of the displacement operator (1). It is equal to the square of the prefactor in the evolution equation in the normalization where \( \langle M^{(0,0)}_{10}\text{[straight line]} \rangle = 1/|x_1|^{1+\lambda} \) and \( \langle M^{(-1,1)}_{10}\text{[straight line]} \rangle = 1/|x_1|^{3-\lambda} \). In section V, we fix it to be given by (2) using the form of the evolution equations and the spectrum.

C. The Boundary Equation

Similar to the line evolution equation, the boundary operators also satisfy a Schwinger-Dyson-type equation.
It relates $SL(2, \mathbb{R})$ primaries from the same tower as
\[
\delta_+ \mathcal{O}^{(0,-s-1)}_R = \beta \mathcal{O}^{(2,-s)}_R, \quad s \geq 0, \quad (23)
\]
\[
\delta_- \mathcal{O}^{(0,s+1)}_R = \bar{\beta} \mathcal{O}^{(2,s)}_R, \quad s \geq 1,
\]
and similarly for the left operators. On the right-hand side we have the unique boundary operator with the correct dimension and transverse spin. In section V we bootstrap the proportionality coefficients to be given by
\[
\beta = \bar{\beta} = -\frac{1}{2}. \quad (24)
\]
These values are also derived in [25] by a careful analysis of the loop corrections to the boundary equation of motion with point splitting regularization. Note that there is no analogous relation between operators in different towers.

**D. The Unstable Mesonic Line Operator**

The spectrum of the operator (9) with $\alpha = -1$ is the same as that of the operator with $\alpha = 1$, except for the flipped anomalous dimensions of $\bar{O}^{(0,0)}_L$ and $\bar{O}^{(0,0)}_R$. It is derived by repeating the resummation of perturbation theory.

The relation between the anomalous spin (17) and the anomalous dimensions is also confirmed by lifting the line operator (9) with $\alpha = -1$ to a different supersymmetric line operator in the $N = 2$ theory and repeating the analysis of its boundary operators. To summarize, the spin is given by (17) and
\[
\Delta_{\alpha L}^{(n,s)} = \begin{cases} 
\frac{1-\lambda}{2} - s_L + n & s_L \leq 0 \\
\frac{1+\lambda}{2} + s_L + n & s_L \geq 1
\end{cases}, \quad (25)
\]
and
\[
\Delta_{\alpha R}^{(n,s)} = \begin{cases} 
\frac{1-\lambda}{2} + s_R + n & s_R \geq 0 \\
\frac{1+\lambda}{2} - s_R + n & s_R \leq -1
\end{cases}, \quad (26)
\]
where the tilde is added to distinguish from the $\alpha = 1$ line.

As for the $\alpha = 1$ case, here there are also four towers of $SL(2, \mathbb{R})$ primaries that are related by path derivatives. At the bottom we have the operators
\[
\{ \bar{O}^{(0,0)}_L, \bar{O}^{(0,1)}_L \} \quad \text{and} \quad \{ \bar{O}^{(0,0)}_R, \bar{O}^{(0,-1)}_R \}. \quad (27)
\]

The operator on the line with the minimal dimension, $\bar{O}^{(0,0)}_L \times \bar{O}^{(0,0)}_R$, now has conformal dimension $\Delta_{\text{min}} = 1 - \lambda$. It is the unique relevant deformation of the $\alpha = -1$ line operator. In perturbation theory, turning this deformation on is equivalent to changing the coefficient in front of the bi-scalar condensate in (9). Doing so with a positive coefficient generates a flow between the $\alpha = -1$ and the $\alpha = 1$ line operators. Deforming by it with the opposite sign generates a flow to an (almost) trivial line. The dual picture of this flow is discussed in section IV A.

Finally, the displacement operator now takes the form
\[
\bar{\mathcal{D}}_+ = +4\pi \lambda \bar{O}^{(0,0)}_R \bar{O}^{(0,1)}_L \quad \text{for} \quad \alpha = -1, \quad (28)
\]
and the evolution equation is modified accordingly.

**IV. MESONIC LINE OPERATORS IN THE FERMIONIC THEORY**

In the fermionic theory, the Wilson line operator that only couples to the gauge field
\[
W[x(\cdot)] = \mathcal{P} e^{i A_{\mu} x^\mu ds} \quad (29)
\]
is a conformal line operator. The corresponding mesonic line operators take the form (10) with
\[
\mathcal{O}_{\alpha R}^{(n,s)} = \frac{1}{\sqrt{N}} \times \begin{cases} 
D^\mu_D^{|s|-\frac{1}{2}} \psi_+(x_R) & s \geq +\frac{1}{2} \\
D^\mu_D^{|s|-\frac{1}{2}} \psi_-(x_R) & s \leq -\frac{1}{2}
\end{cases}, \quad (30)
\]
and similarly for the left boundary. Here, the tree-level spin $s$ takes half-integer values and $D_\mu$ is the covariant derivative.

We have repeated the explicit computation of the anomalous dimensions, the anomalous spins, the lift to a conformal line operator. The corresponding mesonic line operators take the form (10) with
\[
\mathcal{O}_{\alpha R}^{(n,s)} = \frac{1}{\sqrt{N}} \times \begin{cases} 
D^\mu_D^{|s|-\frac{1}{2}} \psi_+(x_R) & s \geq +\frac{1}{2} \\
D^\mu_D^{|s|-\frac{1}{2}} \psi_-(x_R) & s \leq -\frac{1}{2}
\end{cases}, \quad (30)
\]
and the evolution equation is modified accordingly.

The form of the evolution equation in the two theories also matches. The only difference is in the coefficient, being $4\pi \lambda b_\alpha$ in (22) and $4\pi \lambda f_\alpha$ in the fermionic theory. However, as discussed above, this factor depends on the normalization of the mesonic line operators. The fact that the two-point function of the displacement operator
\[ \tilde{M}(\lambda, -\lambda) = \sum_{n} \psi_n \psi_{n+1} \psi_{n+2} \psi_{n+3} \]

Empty line with spin transport \( P e^{i f A dx} \)

FIG. 3. The condensed fermionic line operator is defined perturbatively by integrating fundamental and anti-fundamental fermions along the path, (32). Neighboring ordered pairs of fundamental and anti-fundamental are connected by Wilson lines. In between these pairs, we have a topological transport of the transverse spin with the connection (34). We call these segments “empty” because they do not have fields inserted on them.

in (2) is only a function of \( \Delta = (1 + \lambda_b)/2 = 1 + \lambda_f/2 \) means that once we match our conventional normalization of the line operators, the coefficient in the corresponding evolution equations also matches.

A. The Condensed Fermionic Line Operator

The bosonic theory also has the conformal line operator with \( \alpha = -1 \) and we can ask what its dual fermionic description is. A hint comes from the spectrum of boundary operator at \( \lambda_b = 1 \). According to (25)-(26), and (17), in the free fermionic theory, we expect to have right and left boundary operators of dimension zero and spins \( s_L = -s_R = \frac{1}{2} \) respectively. Another clue comes from the form of the displacement operator (28) that factorizes at tree level to a dimension zero times a dimension two boundary operators.

The corresponding conformal line operator is somewhat unusual, having integrated fundamental fields in the exponent that interpolate between regions of the line with and without a Wilson line, see figure 3. To write it in a compact form, we introduce a two-dimensional space on the line [37]. Its upper (lower) component stands for the regions with (without) a Wilson line. Using this convention, the mesonic line operator takes the form

\[ \tilde{M}(\lambda, -\lambda)[x(\cdot)] = \left[ P e^{i f A_{\mu} x^\mu dx} \right]_{x_2} \]

(31)

where \( \tilde{A}(s) \) is the \( 2 \times 2 \) matrix

\[ \tilde{A}_{\mu} = \begin{pmatrix} \frac{A_x^0 + A_y^0}{2} & i P^- \nu^{-1} \\ -i \frac{A_x^0 - A_y^0}{2} \psi P_{\mu}^- \lambda_\mu + \Gamma_{\mu} + \frac{1}{2} \partial_\mu \log n^+ \end{pmatrix} \]

(32)

and in (31) we have taken the \( 2 \rightarrow 2 \) component of the matrix. Here,

\[ P^\pm_{\mu}(s) = \frac{1}{2}(e_{\mu}(s) \pm \gamma_{\mu}) \text{, with } e = \dot{x}/|\dot{x}|, \]

(33)

is a projector to the spinor \( \pm (\mp) \) component on the right (left). The term proportional to \( 1/\epsilon \) is a counter term for subtracting a power divergence, with \( \epsilon \) being a point-splitting regulator on the line. Finally, \( \Gamma_{\mu} \dot{x}^\mu \) is a spinor connection that is responsible for a topological transporting of the \( \pm \) spinorial component along the empty regions. It is given by,

\[ \Gamma_{\mu} \dot{x}^\mu = -\frac{i}{2} \epsilon_{\mu \nu \rho} (e^{\mu} e^{\nu} \gamma^\rho - i n^\nu n^\rho e^{\nu} e \cdot \gamma) \]

(34)

The four towers of boundary operators are obtained by taking path derivatives of the four operators

\[ \{ \tilde{O}_{L, R}^{(0, \frac{1}{2})}, \tilde{O}_{L, R}^{(0, \frac{1}{2})} \} \text{ and } \{ \tilde{O}_{L, R}^{(0, \frac{1}{2})}, \tilde{O}_{L, R}^{(0, \frac{1}{2})} \}, \]

(35)

with \( \tilde{M}(\lambda, -\lambda) \) defined as

\[ \tilde{M}(\lambda, -\lambda)[x(\cdot)] = \tilde{O}_{L}^{(0, \frac{1}{2})} \left[ P e^{i f A_{\mu} x^\mu dx} \right]_{x_2} \tilde{O}_{R}^{0, \frac{1}{2}} \]

(36)

and

\[ \tilde{O}_{L}^{0, \frac{1}{2}} = D_+ (\psi_{\mu} P_{\mu}^- e_\nu)/\sqrt{N}, \]

\[ \tilde{O}_{R}^{0, \frac{1}{2}} = D_- (e_\nu P_{\mu}^- \psi)/\sqrt{N}. \]

(37)

We have repeated the derivation of the boundary dimensions and the evolution equation for this condensed fermion operator. The results match those of the \( \alpha = -1 \) operator, with the replacement of \( \lambda_b \rightarrow 1 + \lambda_f \) in the spectrum (25)-(26), and \( \lambda_b \rightarrow \lambda_f \) in the displacement operator, (28). The operator in (31) also has a lift into a locally supersymmetric line operator in the \( \mathcal{N} = 2 \) theory. The corresponding circular 1/2 BPS operator was considered in the context of quiver gauge theories, with the fermion in the bi-fundamental representation [38–40]. The lift of the operator in (31) is obtained by taking the rank of one of the gauge groups to one. This limit was discussed previously in [41]. The resulting anomalous spin is given by (17), with the tree-level spin being shifted by one-half with respect to the bosonic theory. The operators (35) match with the ones in (27) and the rest are related to these by path derivatives. In the table below we summarize the map between the boundary operators at the bottom of the four towers.

| Fermionic Tree | Bosonic Tree | 5 | \( \Delta \) |
|----------------|--------------|---|----------|
| \( \tilde{O}_{R}^{0, \frac{1}{2}} \) | 1 | \( \tilde{O}_{R}^{0, \frac{1}{2}} \) | \( \phi \) | \( -\lambda_b \) | \( 1 - \lambda_b \) |
| \( \tilde{O}_{R}^{0, \frac{1}{2}} \) | \( \partial_+ \psi_+ \) | \( \tilde{O}_{R}^{0, \frac{1}{2}} \) | \( \partial_\phi \) | \( 2 + \lambda_b \) | \( 2 + \lambda_b \) |
| \( \tilde{O}_{L}^{0, \frac{1}{2}} \) | 1 | \( \tilde{O}_{L}^{0, \frac{1}{2}} \) | \( \phi_\dagger \) | \( \lambda_b \) | \( 1 - \lambda_b \) |
| \( \tilde{O}_{L}^{0, \frac{1}{2}} \) | \( \partial_+ \psi_\dagger \) | \( \tilde{O}_{L}^{0, \frac{1}{2}} \) | \( \partial_\phi_\dagger \) | \( 2 + \lambda_b \) | \( 2 + \lambda_b \) |

Deforming the condensed fermion line operator in (31) by the relevant deformation \( \tilde{O}_{L}^{0, \frac{1}{2}} \times \tilde{O}_{R}^{0, \frac{1}{2}} \) with the appropriate sign generates a flow to the Wilson line operator (29). Namely, it causes the regions of the line with a Wilson line to dominate the IR. This is the dual description of the flow between the line operator with \( \alpha = -1 \) to the line operator with \( \alpha = 1 \) in the bosonic
theory. Deforming by it with the opposite sign generates a flow to an empty line with the topological connection (34), projected to the minus component. Its coefficient is shifted by $\lambda_f$, in accordance with the anomalous spin of the Wilson line.

The condensed fermion operator itself can also be understood as the fix-point of a different RG flow on the line. This flow starts from a combination of the Wilson line (29) with a trivial line, described by the connection $a_\mu = \text{diag}(A_\mu, \Gamma_\mu + a 2 \partial_\mu \log n^+)$, where $\Gamma_\mu$ is the Euclidean gauge field. It is triggered by turning on the relevant operator, [42]

$$
\delta a_\mu = \begin{pmatrix}
0 \\
-C_L^{(0, \frac{1}{2})} & 0
\end{pmatrix} . \tag{38}
$$

V. THE LINE BOOTSTRAP

In this section, we explain how the evolution equation and the spectrum of boundary operators can be used to evaluate the expectation value of mesonic line operators. The details of this bootstrap program will be presented in a separate publication [26]. Here, we summarize our results and explain the main ideas that lead to them. For concreteness, we use the labeling of the operators in the bosonic theory with $\alpha = 1$. The construction, however, does not depend on this.

The expectation value of the mesonic line operators along a straight line, $\mathcal{M}(s,s') \equiv \langle \mathcal{M}(s,s') \rangle$, is fixed by conformal symmetry to take the form

$$
\mathcal{M}(s,s') = \frac{c_s(\lambda) \delta_{s, s'}}{4\pi |x_L - x_R|^{1+|2s+\lambda|}} . \tag{39}
$$

In our normalization of the boundary operators (12)-(13), the normalization constants $c_s$ are not independent [43]. Using the chiral form of the evolution equation (22) and demanding that the expectation values are invariant under constant translation in the transverse plane, we find that

$$
c_{s+1} = -\beta(s + 1 + \lambda)(s + 2 + \lambda) c_s , \quad s \geq 0 , \tag{40}
c_{s-1} = -\tilde{\beta}(s + 1 - \lambda)(s + 2 - \lambda) c_{s} , \quad s \geq 1 ,
$$

where $\beta$ and $\tilde{\beta}$ are defined in (23). Demanding that the expectation values are also invariant under rigid rotations fixes them to be given by (24).

Note that the normalization independent two-point function of the displacement operator (1) is the yet to be fixed combination

$$
\Lambda((1 + \lambda)/2) = \lambda^2 c_0(\lambda) c_{-1}(\lambda) . \tag{41}
$$

So far, the results (40) and (24) were obtained using only deformations for which the contribution of the displacement operator (20) drops out. To proceed, we must include deformations to which they do contribute. This is done using a form of conformal perturbation theory on the straight line, as we next describe.

We deform away from the straight line as $x(\cdot) \mapsto x(\cdot) + v(\cdot)$. A longitudinal variation of the path endpoints acts trivially. Therefore, without loss of generality, we can assume that it is absent and fix the parametrization of the deformation such that $v$ is transverse at any point along the line, $v \cdot \dot{x} = 0$.

At any order in $v$, we add all operators of the corresponding dimension and spin to the local action of the straight line, as well as to its boundaries. This includes scheme dependent counterterms that cancel power divergences arising from the integration of the displacement operator. We then fix their coefficients systematically, imposing the correct spectrum of boundary operators, the conformal symmetry of the line, and the evolution equation.

Fixing the counter terms can be shown to be equivalent to an integration prescription. For example, at first order, the linear variation of the operator $M_{0,1}^{(0,1)}$ along the path $x(s) = s \dot{x}$ with $s \in [0,1]$ takes the form

$$
\delta M_{10}^{(0,1)} = -\frac{4\pi i \lambda}{\sin(\pi \lambda)} \oint_{[0,1]} ds v_{s} M_{1s}^{(0,0)} M_{s0}^{(-1,1)} , \tag{42}
$$

where the integral goes around the cut of $M_{1s}^{(0,0)} M_{s0}^{(-1,1)}$ between $s = 0$ and $s = 1$. At higher orders, similar, but scheme dependent prescriptions can be specified to make the integrals finite.

Demanding that the straight line transforms covariantly under conformal transformations is sufficient to fix all the coefficients at order $v$, but not at order $v^2$. We then demand that if we first preform an arbitrary smooth deformation, and, on top of that, we apply a conformal transformation then the deformed line transforms covariantly. These conditions turn out to fix all the second-order coefficients. We then evaluate the two-point function of the displacement operator and find that it is given by (2) with $\Delta = (1 + \lambda_0)/2$. The analysis in the fermionic theory is manifestly identical, with the only difference being in the normalization convention.

Going to higher orders in the deformation is tedious, but systematic. It can be used to unambiguously evaluate the expectation value order-by-order in the deformation from the straight line. It follows that the expectation values of the line operators in the bosonic and fermionic theories are related to each other by the duality map $\lambda_f = \lambda_b - \text{sign}(k_b)$. This is because their spectrum of boundary operators are related to each other by this map, and the forms of their evolution equations are the same.

Another conclusion from the derivation above is that a match of the $1/N$ corrections to the expectation values of closed line operators between the bosonic and fermionic theories. That is a direct outcome of the fact that their
deformations are governed by the same local displacement operators. In other words, smooth deformations of a closed loop (and circular in particular) are equal to \(1/N\) times factorized expectation values of mesonic line operators.

VI. DISCUSSION

In this paper, we have classified the conformal line operators of large \(N\) Chern-Simons theory coupled to fermions or bosons in the fundamental representation. We have computed the spectrum of conformal dimensions and transverse spins of their boundary operators at finite \(\ 't \) Hooft coupling. In particular, their displacement operators factorize into a product of fundamental and anti-fundamental boundary operators. Together, the spectrum and the form of the displacement operator were shown to fix the expectation value of the mesonic line operators uniquely. We have found that the line operators of the theory coupled to bosons and the ones of the theory coupled to fermions are related to each other through the strong-weak duality map \(\lambda_f = \lambda_b - \text{sign}(k_b)\).

To complete the derivation of the duality at the planar level, one should also match the connected piece of the correlation functions between mesonic line operators. The path dependence of this piece is controlled by the expectation values studied here. To see this, recall that the evolution equation (22) expresses a deformation of the expectation value of the mesonic line operator (32) by flipping the tree level spins as well as the anomalous dimensions

\[
\hat{\Delta}_\mu^{(n,s)} = \begin{cases} 
|\frac{1}{2} - |s| + n + \lambda/2 \ s \leq \frac{1}{2} \\
|\frac{1}{2} + |s| + n - \lambda/2 \ s \geq \frac{3}{2}
\end{cases}, \tag{A2}
\]

and

\[
\hat{\Delta}_\mu^{(n,s)} = \begin{cases} 
|\frac{1}{2} + |s| + n + \lambda/2 \ s \geq -\frac{1}{2} \\
|\frac{1}{2} - |s| + n - \lambda/2 \ s \leq -\frac{3}{2}
\end{cases}. \tag{A3}
\]

The anomalous spin is unchanged and is given by (17). At the bottom of these four towers we now have the operators

\[
\{\hat{O}_{\hat{L}}^{(0,\frac{1}{2})}, \hat{O}_{\hat{L}}^{(0,\frac{3}{2})}\} \quad \text{and} \quad \{\hat{O}_{\hat{R}}^{(0,\frac{1}{2})}, \hat{O}_{\hat{R}}^{(0,\frac{3}{2})}\}. \tag{A4}
\]

The corresponding displacement operator is

\[
\hat{D}_+ = -4\pi\lambda \hat{O}_{\hat{R}}^{(0,\frac{1}{2})} \hat{O}_{\hat{L}}^{(0,\frac{3}{2})}, \quad \hat{D}_- = -4\pi\lambda \hat{O}_{\hat{R}}^{(0,\frac{3}{2})} \hat{O}_{\hat{L}}^{(0,\frac{1}{2})}. \tag{A5}
\]

If we quantize the theory radially in the presence of a straight conformal line operator, then reflection positivity restricts the dimensions of boundary operators to be positive [50]. The dimensions of the operators \(\hat{O}_{\hat{R}}^{(0,\frac{1}{2})}\) and \(\hat{O}_{\hat{R}}^{(0,\frac{3}{2})}\) are however negative, (given by \(\lambda/2\) of the fermionic theory). This is because conjugation in radial quantization relate \(\psi_-\) to \(\bar{\psi}_+\) and \(\psi_+\) to \(-\bar{\psi}_-\). As a result,

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Appendix A: Non-unitary Conformal Line Operators

The conformal line operator defined in (31)-(32) has the components of the fermion \(\psi_-\) and \(\psi_+\) condensed in the exponent. We can instead use the components \(\psi_+\) and \(\psi_-\) as

\[
\hat{A}_\mu = \left( \begin{array}{c}
A_\mu \\
-\frac{i}{\kappa} \gamma_\mu \bar{\psi} \psi P_\mu - \frac{4\pi}{\kappa} 1 \gamma_\mu + \Gamma_\mu + \frac{\lambda}{2} \partial_\mu \log n^+ \end{array} \right), \tag{A1}
\]

The resulting line operator is also conformal. The spectrum of boundary operators is related to the ones of the line operator (32) by flipping the tree level spins as well as the anomalous dimensions

\[
\hat{\Delta}_R^{(n,s)} = \begin{cases} 
|\frac{1}{2} - |s| + n + \lambda/2 \ s \leq \frac{1}{2} \\
|\frac{1}{2} + |s| + n - \lambda/2 \ s \geq \frac{3}{2}
\end{cases}, \tag{A2}
\]

and

\[
\hat{\Delta}_L^{(n,s)} = \begin{cases} 
|\frac{1}{2} + |s| + n + \lambda/2 \ s \geq -\frac{1}{2} \\
|\frac{1}{2} - |s| + n - \lambda/2 \ s \leq -\frac{3}{2}
\end{cases}. \tag{A3}
\]
the line operator with (32) is unitary while the one with (A1) is not.

We should be able to construct the dual operator in the bosonic theory. One way is to start with the combination of the operator with $\alpha = 1$ and a trivial line, as described above equation (38). Then, turn on the deformation analogous to (38), but with the operators that have $O^{(0,-1)}_L$ or $O^{(0,1)}_R$ in the off-diagonal, and flow to the fixed-point.

Another way of relating the two directly at the fixed-point is by taking a longitudinal path derivative. Let $M_\Delta$ be a mesonic line operator such as $\bar{\psi}^+ W(x) |\psi_\perp\rangle$, with end-point dimensions $\Delta_L = \Delta_R = \Delta$ and opposite spins. For a straight line with a constant framing vector, its expectation value takes the form

$$\langle M_\Delta | x \rangle = \frac{\delta^{2\Delta - 2}}{x^2 (1 - \Delta)} , \quad (A6)$$

where $x = |x_L - x_R|$. Here, $\delta$ is a point-splitting regulator on the line and the factor of $\delta^{2\Delta - 2}$ is a wave-function normalization choice. We can define the condensed operator $\tilde{M}_{1-\Delta}$ with endpoints dimensions $\Delta_L = \Delta_R = 1 - \Delta$ through the equation

$$\frac{\partial}{\partial \Delta} \tilde{M}_{1-\Delta} = (2\Delta - 1) \frac{x^{-\Delta}}{2 - 2\Delta} \langle M_\Delta | x \rangle + \delta \left( M_\Delta | x \rangle, \tilde{M}_{1-\Delta} \right) . \quad (A7)$$

where the second line subtract the leading divergence at the two boundaries. Using large $N$ factorization, $\langle M_\Delta | x \rangle \tilde{M}_{1-\Delta} | y \rangle = \langle M_\Delta | x \rangle \langle M_\Delta | y \rangle + O(1/N)$, it then follows that as long as $\Delta > 1/2$, $\langle \tilde{M}_{1-\Delta} | x \rangle = \frac{\delta^{2\Delta - 2}}{x^{2(1 - \Delta)}} . \quad (A8)$

Equation (A7) relates the operator $M(\frac{1}{2},-\frac{1}{2}) | x \rangle$ in the fermionic theory to the unitary line operator with condensed fermion (31). Similarly, it relates $M(\frac{1}{2},\frac{1}{2})$ to the 1-1 component of the non-unitary line operator with the connection (A1). The bosonic dual of $M(-\frac{1}{2},\frac{1}{2})$ is the operator $M(-1,1)$ in (21). For a straight line, we can therefore define the bosonic dual of the non-unitary operator using (A7) with $M_{\Delta} \propto M(-1,1)$. The displacement operator (A5) can then be used to deform it.

Appendix B: SUSY and spectrum

In this appendix, we study BPS boundary operators of a straight supersymmetric Wilson line in the $\mathcal{N} = 2$ CS-matter theory [51–54]. We then relate these operators to ones in the theory of bosons and fermions that are discussed in the main text.

For definiteness, we consider a straight Wilson line along a segment of the $x^3$ axis. The bosonic subgroup of the three-dimensional conformal group $so(1,4)$ that preserves the $x^3$ axis is $so(1,2) \simeq sl(2)$. This subgroup consists of the dilatation operator $D$, the rotation $M_{12}$ around the line, the translation $P_3$ and the boost $K_3$ along the line. A 1/2-BPS infinite Wilson line also preserves $so(2) \simeq u(1)$-symmetry generated by $R$ and four superconformal charges. Together, these form the symmetry group $osp(2|2)$. It contains two out of the four $\mathcal{N} = 2$ supercharges, denoted by $Q_\alpha, Q_\dot{\alpha}$, and their Hermitian conjugates, the superconformal charges $S^a$ and $S^\dot{a}$. The subset of the preserved supercharges depends on the line operator. The fundamental matter fields discussed in the main text, including $\phi, \bar{\phi}, (\phi^\dagger, \bar{\phi}^\dagger)$, are identified with the component fields of the chiral (anti-chiral) multiplet.

1. Fermion Components

We follow the spinor convention of [55]. All spinor indices are raised/lowered by the antisymmetric $\varepsilon$ tensor from the left. In components, we have $\varepsilon_{21} = \varepsilon^21 = 1$.

The classical transverse spin for matter fields are given by

$$s(\psi^1) = s(\bar{\psi}^1) = +\frac{1}{2}, \quad s(\psi^2) = s(\bar{\psi}^2) = -\frac{1}{2} . \quad (B1)$$

The + (−) path derivative increases (decreases) the transverse spin by one unit. For definiteness, the spin 1/2 matrix generator is taken to be $M_{12}^{(1/2)} = \gamma^3/2$.

2. SUSY Transformations

A general SUSY transformation is parameterized by two independent constant spinors, $\delta_\xi = e^Q + \bar{e}Q$ [56]. It generates the field transformation [55, 57],

$$\delta A_\mu = -\frac{i}{2} \left( \bar{\epsilon}_\gamma \mu \lambda - \bar{\lambda} \gamma_\mu \epsilon \right) , \quad \delta \lambda = +\frac{1}{2} \left( \epsilon \lambda - \bar{\lambda} \epsilon \right) . \quad (B2)$$

where $\lambda$ is a two-component complex spinor, and $\sigma$ is an auxiliary scalar in the $\mathcal{N} = 2$ gauge multiplet. It is related to the scalar adjoint by the equation of motion of the auxiliary $D$ field, $\sigma = -\frac{2\pi}{T} \phi \phi^\dagger$.

The line operator (9) in the bosonic theory is also a good line operator in the $\mathcal{N} = 2$ theory. For $\alpha = \pm 1$, it is preserved by $\{ Q_2, Q_1, S^1, S^2 \}$ and $\{ Q_1, Q_2, S^2, S^1 \}$, respectively. The 1/2-BPS operator with $\alpha = 1$ was first introduced in [58] while the one with $\alpha = -1$ seems to have not been considered before. The matter fields correspondingly split into $osp(2|2)$ superconformal primaries and descendents, depending on the supercharges that are preserved. For example, for $\alpha = 1$, the superconformal transformation rules for matter fields are given by [55, 57]

$$\delta \phi = -\bar{e}^1 \psi^2 , \quad \delta \phi^\dagger = -e^2 \bar{\psi}^1 , \quad (B3)$$

$$\delta \psi^1 = i\sqrt{2} \epsilon^2 D_+ \phi , \quad \delta \psi^2 = i\sqrt{2} \epsilon^1 D_3 \phi^\dagger , \quad \delta \bar{\psi}^1 = i\sqrt{2} \bar{\epsilon}^1 D_- \phi^\dagger ,$$
where $\mathbb{D}_3 = \partial_3 - i(A_3 - i\sigma)$ is the full longitudinal covariant derivative with respect to the connection $A - i\sigma$.

3. 1/2 BPS Conditions

A careful study of (B3) will put stricter constraints on the operator dimensions and spins. In particular, different component of fermions behaves differently under the line superconformal group. For $\alpha = 1$ case, $\bar{\psi}^2, \psi^1$ are SUSY descendants of $\phi, \phi^1$, respectively, as for the three-dimensional case. However, $\psi^1, \bar{\psi}^2$ are now SUSY primaries, since the three-dimensional supercharges that relate them to the bosons are not preserved by the line. The superconformal primary fields listed above are annihilated by the two superconformal generators $S, \bar{S}$, while the non-primary fermions are only annihilated by one of the two superconformal generators.

The $\mathfrak{osp}(2|2)$ 1/2-BPS conditions relate the scaling dimensions of the primaries to their transverse spin $s$, defined as the eigenvalue of $M_{12}$, and their $R$-charge $r$ [59]. Explicitly, $\phi, \psi^1$ are annihilated by $Q_2$, since their transformation does not depend on $\epsilon^2$. Similarly, $\phi^1, \bar{\psi}^2$ are annihilated by $Q_1$. Using the following anticommutation relations,

$$\{Q_a, \bar{S}^b\} = -\delta^b_a (iD - R) - \frac{1}{2} \varepsilon_{\mu\nu\rho} M^{\mu\nu}(\gamma^\rho)^b_a, \quad \{S^a, \bar{Q}_b\} = -\delta^a_b (iD + R) - \frac{1}{2} \varepsilon_{\mu\nu\rho} M^{\mu\nu}(\gamma^\rho)^a_b,$$

we find, for the primary fields,

$$\Delta_\phi = \frac{1}{2} - s_R(\phi), \quad \Delta_{\phi^1} = \frac{1}{2} + s_L(\phi^1),$$

$$\Delta_{\psi^1} = \frac{1}{2} + s_R(\psi^1), \quad \Delta_{\bar{\psi}^2} = \frac{1}{2} - s_L(\bar{\psi}^2).$$

Here we used that the action of bosonic generators on an operator $O_\Delta(0)$ sitting at the origin is

$$[iD, O_\Delta(0)] = +\Delta O_\Delta(0),$$

$$[R, O_\Delta(0)] = +r O_\Delta(0),$$

$$[M_{12}, O_\Delta(0)] = -s O_\Delta(0).$$

From (B3), it follows that

$$\Delta_\phi = \Delta_{\psi^2} - 1/2, \quad \Delta_{\phi^1} = \Delta_{D_{+}\phi} - 1/2,$$

$$\Delta_{\psi^1} = \Delta_{\bar{\psi}^1} - 1/2, \quad \Delta_{\bar{\psi}^2} = \Delta_{D_{-}\phi^1} - 1/2,$$

where we used that the scaling dimensions of SUSY variation parameter $\epsilon, \tilde{\epsilon}$ is equal to $-1/2$. Similarly, we have

$$s_R(\phi) = s_R(\psi^2) + \frac{1}{2}, \quad s_R(\psi^1) = s_R(D_{+}\phi) - \frac{1}{2},$$

$$s_L(\phi^1) = s_L(\bar{\psi}^1) - \frac{1}{2}, \quad s_L(\bar{\psi}^2) = s_L(D_{-}\phi^1) + \frac{1}{2}.$$
tor Fermion Matter, Eur. Phys. J. C 72, 2112 (2012), arXiv:1110.4386 [hep-th].
[14] S. Jain, S. P. Trivedi, S. R. Wadia, and S. Yokoyama, Supersymmetric Chern-Simons Theories with Vector Matter, JHEP 10, 194, arXiv:1207.4750 [hep-th].
[15] O. Aharony, S. Giombi, G. Gur-Ari, J. Maldacena, and R. Yacoby, The Thermal Free Energy in Large N Chern-Simons-Matter Theories, JHEP 03, 121, arXiv:1211.4843 [hep-th].
[16] S. Jain, S. Minwalla, T. Sharma, T. Takimi, S. R. Wadia, and S. Yokoyama, Phases of large N vector Chern-Simons theories on S^2 x S^1, JHEP 09, 009, arXiv:1301.6169 [hep-th].
[17] T. Takimi, Duality and higher temperature phases of large N Chern-Simons matter theories on S^2 x S^1, JHEP 07, 177, arXiv:1304.3725 [hep-th].
[18] S. Jain, M. Mandlik, S. Minwalla, T. Takimi, S. R. Wadia, and S. Yokoyama, Unitarity, Crossing Symmetry and Duality of the S-matrix in large N Chern-Simons theories with fundamental matter, JHEP 04, 129, arXiv:1404.6373 [hep-th].
[19] K. Inbasekar, S. Jain, S. Mazumdar, S. Minwalla, V. Umez, and S. Yokoyama, Unitarity, crossing symmetry and duality in the scattering of N = 1 susy matter Chern-Simons theories, JHEP 10, 176, arXiv:1505.06571 [hep-th].
[20] S. Yokoyama, Scattering Amplitude and Bosonization Duality in General Chern-Simons Vector Models, JHEP 09, 105, arXiv:1604.01897 [hep-th].
[21] A. Giveon and D. Kutasov, Seiberg Duality in Chern-Simons Theory, Nucl. Phys. B 812, 1 (2009), arXiv:0808.0360 [hep-th].
[22] F. Bennin, C. Closset, and S. Cremonesi, Comments on 3d Seiberg-like dualities, JHEP 10, 075, arXiv:1108.5373 [hep-th].
[23] G. Gur-Ari and R. Yacoby, Three Dimensional Bosonization From Supersymmetry, JHEP 11, 013, arXiv:1507.04378 [hep-th].
[24] For a complete list of the dualities, see e.g., [1, 11, 60–64]. In this letter, we shall restrict to the SU(N)/U(N) version.
[25] B. Gabai, A. Sever, and D.-l. Zhong, Line operators in Chern-Simons-Matter theories and Bosonization in Three Dimensions II - Perturbative Analysis and All-loop Resummation, (2022), arXiv:2212.02518 [hep-th].
[26] B. Gabai, A. Sever, and D.-l. Zhong, Line operators in Chern-Simons-Matter theories and Bosonization in Three Dimensions III - The Line Bootstrap, To appear (2022).
[27] The displacement operator displaces the contour in an orthogonal direction. It is defined in equation (19).
[28] Here, A_{\mu} = A_{\mu}^{T} T^{T}, with T^{T} generators of the gauge group in the fundamental representation, with tr (T^{T} T^{T}) = \frac{1}{2} \delta^{ij}, and \epsilon^{123} = 1 is the anti-symmetric tensor.
[29] E. Witten, Quantum Field Theory and the Jones Polynomial, Commun. Math. Phys. 121, 351 (1989).
[30] Here, we use the convention where \psi^{a} = \psi^{a}_{\sigma}, g^{a} = \sigma^{a} and D_{\mu} \phi^{a} = \partial_{\mu} \phi^{a} - i A_{\mu}^{a}(T^{T})^{i}_{j} \phi^{j}, D_{\mu} \psi^{a} = \partial_{\mu} \psi^{a} - i A_{\mu}^{a}(T^{T})^{i}_{j} \psi^{j}.
[31] Legendre transform here represents a double trace deformation triggered by (J^{(K)})^{2}, where J^{(K)} is given by \phi^{i} \phi^{i} in the bosonic case, and by \psi^{i} \psi^{i} in the fermionic case. Details can be found in e.g. [65].
[32] The boson theory (7) also has a triple-trace (J^{(K)})^{3}/N^{2} term in the action. At order 1/N the coupling of this operator has to be tuned to the fix-point, see [8, 45] for details.
[33] A similar picture was found before for line operators with large quantum numbers in free scalar triplet and in the Wilson-Fisher O(3) model [66].
[34] The path derivatives pick the operator that multiplies the boundary value of a smooth deformation parameter, with the framing vector kept constant and perpendicular to the direction of the deformation.
[35] Here we have suppressed a relative overall phase factor that depends on the number of times the framing vector winds around the path.
[36] The chirality of the displacement operator follows from the fact that the Chern-Simons term breaks parity.
[37] It can also be realized using a worldline fermion.
[38] N. Drukker and D. Trancanelli, A Supermatrix model for N=6 super Chern-Simons-matter theory, JHEP 02, 058, arXiv:0912.3006 [hep-th].
[39] H. Ouyang, J.-B. Wu, and J.-j. Zhang, Novel BPS Wilson loops in three-dimensional quiver Chern–Simons-matter theories, Phys. Lett. B 753, 215 (2016), arXiv:1510.05475 [hep-th].
[40] K.-M. Lee and S. Lee, 1/2-BPS Wilson Loops and Vortices in ABJM Model, JHEP 09, 004, arXiv:1006.5589 [hep-th].
[41] N. Drukker, BPS Wilson loops and quiver varieties, J. Phys. A 53, 385402 (2020), arXiv:2004.11303 [hep-th].
[42] It would be interesting to check that both of these flows satisfy the general constraints derived in [67].
[43] If the framing is non-trivial, there is an additional overall phase factor of exp (i \lambda (\phi - \phi_{fix})/2), with \phi_{fix} been the total rotation angles in the plus direction at the left and right endpoints, with respect to the trivial framing.
[44] M. A. Vasilev, More on equations of motion for interacting massless fields of all spins in (3+1)-dimensions, Phys. Lett. B 285, 225 (1992).
[45] O. Aharony, G. Gur-Ari, and R. Yacoby, d=3 Bosonic Vector Models Coupled to Chern-Simons Gauge Theories, JHEP 03, 037, arXiv:1110.4382 [hep-th].
[46] I. R. Klebanov and A. M. Polyakov, AdS dual of the critical O(N) vector model, Phys. Lett. B 550, 213 (2002), arXiv:hep-th/0210114.
[47] E. Sezgin and P. Sundell, Holography in 4D (super) higher spin theories and a test via cubic scalar couplings, JHEP 07, 044, arXiv:hep-th/0305040.
[48] C.-M. Chang, S. Minwalla, T. Sharma, and X. Yin, ABJ Triality: From Higher Spin Fields to Strings, J. Phys. A 46, 214009 (2013), arXiv:1207.4485 [hep-th].
[49] R. G. Leigh and A. C. Petkou, Holography of the N=1 higher spin theory on AdS(4), JHEP 06, 011, arXiv:hep-th/0304217.
[50] It also restrict the two-point function of the displacement operator to be positive. This is indeed the case as can seen in (2).
[51] B. M. Zupnik and D. G. Pak, Superfield Formulation of the Simplest Three-dimensional Gauge Theories and Conformal Supergravities, Theor. Math. Phys. 77, 1070 (1988).
[52] E. A. Ivanov, Chern-Simons matter systems with manifest N=2 supersymmetry, Phys. Lett. B 268, 203 (1991).
[53] L. V. Aavlee, G. V. Grigorev, and D. I. Kazakov, Renormalizations in Abelian Chern-Simons field theories with matter, Nucl. Phys. B 382, 561 (1992).
[54] L. V. Avdeev, D. I. Kazakov, and I. N. Kondrashuk, Renormalizations in supersymmetric and nonsupersymmetric non-Abelian Chern-Simons field theories with matter, Nucl. Phys. B 391, 333 (1993).
[55] N. Hama, K. Hosomichi, and S. Lee, SUSY Gauge Theories on Squashed Three-Spheres, JHEP 05, 014, arXiv:1102.4716 [hep-th].
[56] The SUSY variation parameter solves the conformal Killing equation and can be spacetime dependent. The most general flat space solution is \( \epsilon = \epsilon_c + x^\mu \gamma_\mu \epsilon_s \), with \( \epsilon_c, \epsilon_s \) being constant spinors. The SUSY transformation is identified with conformal generators as \( \delta_\epsilon = \epsilon_c Q + \epsilon_s S \).
[57] A. Kapustin, B. Willett, and I. Yaakov, Exact Results for Wilson Loops in Superconformal Chern-Simons Theories with Matter, JHEP 03, 089, arXiv:0909.4559 [hep-th].
[58] D. Gaiotto and X. Yin, Notes on superconformal Chern-Simons-Matter theories, JHEP 08, 056, arXiv:0704.3740 [hep-th].
[59] We use the convention \( [Q]_R = -1 \). It implies that \( [\phi]_R = [\bar{\psi}]_R = 1/2, [\phi^\dagger]_R = [\bar{\psi}]_R = -1/2 \).
[60] O. Aharony, F. Benini, P.-S. Hsin, and N. Seiberg, Chern-Simons-matter dualities with SO and USp gauge groups, JHEP 02, 072, arXiv:1611.07874 [cond-mat.str-el].
[61] Z. Komargodski and N. Seiberg, A symmetry breaking scenario for QCD_3, JHEP 01, 109, arXiv:1706.08755 [hep-th].
[62] N. Seiberg, T. Senthil, C. Wang, and E. Witten, A Duality Web in 2+1 Dimensions and Condensed Matter Physics, Annals Phys. 374, 395 (2016), arXiv:1606.01989 [hep-th].
[63] A. Karch and D. Tong, Particle-Vortex Duality from 3d Bosonization, Phys. Rev. X 6, 031043 (2016), arXiv:1606.01893 [hep-th].
[64] J. Murugan and H. Nastase, Particle-vortex duality in topological insulators and superconductors, JHEP 05, 159, arXiv:1606.01912 [hep-th].
[65] S. Giombi, Higher Spin — CFT Duality, in Theoretical Advanced Study Institute in Elementary Particle Physics: New Frontiers in Fields and Strings (2017) pp. 137–214, arXiv:1607.02967 [hep-th].
[66] G. Cuomo, Z. Komargodski, M. Mezei, and A. Raviv-Moshe, Spin Impurities, Wilson Lines and Semiclassics, (2022), arXiv:2202.00040 [hep-th].
[67] G. Cuomo, Z. Komargodski, and A. Raviv-Moshe, Renormalization Group Flows on Line Defects, Phys. Rev. Lett. 128, 021603 (2022), arXiv:2108.01117 [hep-th].