On the dual interpretation of zero-curvature Friedmann–Robertson–Walker models

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Abstract

Two possible interpretations of FRW cosmologies (perfect fluid or dissipative fluid) are considered as consecutive phases of the system. Necessary conditions are found, for the transition from a perfect fluid to a dissipative regime to occur, bringing out the conspicuous role played by a particular state of the system (the ‘critical point’).

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1. Introduction

It is already a well established fact that a variety of line elements may satisfy the Einstein equations for different (physically meaningful) stress–energy tensors (see [1] and references therein).

Particularly interesting is the situation with Friedmann–Robertson–Walker (FRW) models, which, as has been shown ([2, 3], and references therein), do not necessarily represent perfect fluid solutions, but can also be exact solutions for viscous and heat conducting fluids, with or without an electromagnetic field.

Thus, for example, it has been shown [2], that the zero-curvature FRW metric

\[ ds^2 = -dt^2 + R^2(t) \left( dr^2 + r^2 \, d\theta^2 + r^2 \sin^2 \theta \, d\phi^2 \right) \] (1)

satisfies Einstein equations, not only for perfect fluid matter distributions

\[ \tilde{T}_{\mu\nu} = (\tilde{\rho} + \tilde{\rho}) v_\mu v_\nu + \tilde{p} g_{\mu\nu}, \] (2)

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but also, for the stress–energy tensor of a magnetohydrodynamic viscous fluid with heat conduction, namely

\[ T_{\mu\nu} = E_{\mu\nu} + (\rho + p - \xi)u_\mu u_\nu + (p - \xi)\eta_{\mu\nu} - 2\eta\sigma_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu \]  

(3)

where \( E_{\mu\nu}, \rho, \ p, \ \xi, \ \eta, \ \Theta, \ \sigma_{\mu\nu} \) and \( q_\mu \) denote, respectively, the electromagnetic stress–energy tensor, the energy density, the pressure, the bulk and shear viscosity coefficients, the expansion, the shear tensor and the heat flux vector.

A fundamental difference between both interpretations of the models resides in the fact that observers moving along \( v^\mu \) (the perfect fluid case) are comoving with the fluid, whereas those moving along \( u^\mu \) (the dissipative case) detect a radial velocity of fluid particles (tilted models) \[4\]. In other words, in the dissipative models the source is necessarily non-comoving or tilting relative to the geometrically ‘preferred’ Ricci eigenvector \[3\]. According to Coley and Tupper \[2\] this tilting velocity might be related to the observed motion of our galaxy relative to the cosmic microwave background radiation (CMB), and seems to be consistent with measurements of the dipole anisotropy of the CMB temperature distribution \[3\]. A detailed analysis of many of these models shows that their observational predictions are in good agreement with the available data \[3\].

In this work, both interpretations are not considered as alternative, mutually exclusive possibilities, but rather as consecutive phases in the evolution of the model. More specifically, we shall consider the case when, initially, the system evolves as a perfect fluid without dissipation, and then at some instant starts to dissipate, tending eventually again to a perfect fluid as \( t \to \infty \). The question we want to answer here is: what could be the necessary condition for that transition to occur?

For simplicity we shall consider neither the electromagnetic field (\( E_{\mu\nu} = 0 \)) nor the bulk viscosity (\( \xi = 0 \)). Furthermore, the viscosity term will be modelled through an anisotropic fluid and only the transport equation for the heat flux will be needed. The reason for this simplification will become clear later. Also, it should be borne in mind that the approach used in this work only provides a ‘snapshot’ of the system at the moment it enters the dissipative regime, thereby giving only a hint about its tendency, and not a complete description of its evolution. All of these simplifications represent the price we pay in order to deal with mathematically tractable equations.

Two different initial situations will be considered.

(a) The system initially evolves strictly as a perfect fluid. By this we mean that there is no dissipation whatsoever and preferred observers (associated with the 4-velocity of the fluid) are (strictly) comoving with the fluid.

(b) The system initially evolves very close to the perfect fluid regime, but there is still a small (in a sense to be defined below) amount of dissipation, because of which, preferred observers are not strictly comoving with the fluid, but detect a small radial motion of the fluid, so that quadratic and higher powers, as well as time derivatives, of radial velocity are neglected during this ‘quasi-perfect fluid’ regime.

Then it is assumed that at some instant (say \( t = \tilde{t} \)), our system starts to deviate from either of the initial regimes and enters into the dissipative regime. In the first case the system leaves the perfect fluid condition by starting to dissipate. In the second case the system abandons the quasi-perfect fluid regime by increasing the rate of dissipation.

We shall find the conditions required for the transitions described above to occur. To this end we shall evaluate field equations and transport equations immediately after \( \tilde{t} \), where immediately means on a time scale of the order of the relaxation time. We shall see that the
so-called ‘critical point’ plays a fundamental role in the occurrence of the transition to the dissipative regime.

The critical point [5–7], corresponds to the situation when (in relativistic units)

$$\frac{\kappa T}{\tau (\rho + p)} = 1$$

(4)

where $\kappa$, $T$ and $\tau$ denote the heat conduction coefficient, temperature and relaxation time, respectively.

It has been shown [5] that under this condition, the effective inertial mass of any fluid element, just after the system departs from equilibrium, vanishes. Also, it has been shown that as a self-gravitating system approaches the critical point, its active gravitational mass behaves in such a way that enhances the instability of the system [8].

It will be shown below that the transition from the perfect fluid to the dissipative regime is allowed only if the system is at the critical point. However, the transition from the quasi-perfect fluid phase to a dissipative regime is, in principle, always possible, and closer the system is to the critical point, the faster the transition.

2. Field and transport equations

2.1. Field equations

First of all, note that for any locally Minkowskian observer, comoving with the fluid, the 4-velocity takes the obvious form

$$\vec{u}^\mu = (1, 0, 0, 0),$$

(5)

then, performing a Lorentz boost to the frame with respect to which a fluid element has radial velocity $\omega$, we find (in the coordinate system of (1))

$$u^\mu = \left(\frac{1}{\sqrt{1 - \omega^2}}, \frac{\omega R^{-1}}{\sqrt{1 - \omega^2}}, 0, 0\right)$$

(6)

which is the same expression as (2.1) of [2], with their $\alpha$s and $\beta$s defined in terms of $\omega$ by

$$\alpha \equiv \frac{1}{\sqrt{1 - \omega^2}},$$

(7)

$$\beta \equiv \frac{\omega}{\sqrt{1 - \omega^2}}.$$  

(8)

Next, the heat flux vector $q^\mu$, satisfying $q^\mu u_\mu = 0$, is given by

$$q^\mu = Q \left(\frac{\omega}{\sqrt{1 - \omega^2}}, -\frac{R}{\sqrt{1 - \omega^2}}, 0, 0\right)$$

(9)

which is the same expression as (2.3) of [2], with $Q^2 = q_\mu q'^\mu$.

Then the energy–momentum tensor (3) of the source may be written as

$$T^\mu_\nu = (\rho + P_\perp) u_\mu u_\nu + P_\perp g^\mu_\nu + (P_r - P_\perp) s_\mu s_\nu + q_\mu u_\nu + u_\mu q_\nu$$

(10)

where $P_r$ and $P_\perp$ denote the two principal (anisotropic) stresses, linked to $p$ and $\eta \sigma_{\mu \nu}$ by the relations specified below, and the vector $s_\mu$ satisfies the conditions

$$s^\mu s_\mu = 1; \quad s^\mu u_\mu = 0; \quad T^\mu_\nu s^\nu s^\mu = P_r.$$  

(11)
Then, Einstein’s equations read (in relativistic units and omitting the $8\pi$ factor)

\[
\frac{3\dot{R}^2}{R^2} = \frac{1}{1 - \omega^2}[\rho + P_r \omega^2 - 2Q\omega] \quad (12)
\]

\[
-\left(\frac{2\dot{R} + \dot{R}^2}{R^2}\right) = \frac{1}{1 - \omega^2}[(\rho \omega^2 + P_r - 2Q\omega] \quad (13)
\]

\[
-\left(\frac{2\dot{R} + \dot{R}^2}{R^2}\right) = P_\perp \quad (14)
\]

\[
0 = (\rho + P_r)\omega - Q(1 + \omega^2) \quad (15)
\]

where dots denote derivatives with respect to $t$.

Comparing (3) and (10) the following relations follow:

\[
P_r = p - \frac{2\eta}{R^2}\sigma_{11}(1 - \omega^2) \quad (16)
\]

\[
P_\perp = p + \frac{2\eta}{R^2}\sigma_{11}(1 - \omega^2) \quad (17)
\]

\[
\sigma_{00} = \frac{2}{3}(1 - \omega^2)X = \frac{\omega^2\sigma_{11}}{R^2} = -\frac{2\omega^2\sigma_{22}}{r^2R^2(1 - \omega^2)} = -\frac{\omega}{R}\sigma_{01} \quad (18)
\]

where $X$ is given by [2]

\[
X = \frac{\omega\dot{\omega} - \omega^2\omega' R^{-1} - \omega R^{-1}r^{-1}(1 - \omega^2)}{(1 - \omega^2)^{3/2}} \quad (19)
\]

and a prime denotes a derivative with respect to $r$.

Observe that from (13) and (14), one obtains

\[
(\rho + P_\perp)\omega^2 + (P_r - P_\perp) - 2Q\omega = 0 \quad (20)
\]

and using (15) and (20) it follows that

\[
-(\frac{P_r - P_\perp}{\omega}) + Q(1 - \omega^2) = 0. \quad (21)
\]

Thus, if $P_r = P_\perp$ (which implies by virtue of (16)–(18), $\eta\sigma_{\rho\phi} = 0$), then $\omega = 1$, which is obviously inadmissible. Therefore, pure heat conduction (plus a fluid) is not consistent with the FRW metric.

After some simple algebra, the field equations (12)–(15) may be written as

\[
\rho = \frac{3\dot{R}^2}{R^2} + (P_r - P_\perp) \quad (22)
\]

\[
P_\perp = -\left(\frac{2\dot{R} + \dot{R}^2}{R^2}\right) \quad (23)
\]

\[
P_r = -\left(\frac{2\dot{R} + \dot{R}^2}{R^2}\right) + Q\omega \quad (24)
\]

\[
Q(1 - \omega^2) = 2\omega \left(\frac{R^2}{R^2} - \frac{\dot{R}}{R}\right). \quad (25)
\]

It is worth noting that (23)–(25) imply that dissipation takes place only if $\omega \neq 0$ and $P_r \neq P_\perp$. 
Later it will be convenient to write $R(t)$ in the form

$$R(t) = t^n$$

(26)

where in general $n = n(t)$. Thus, $n = \frac{2}{3}$ reproduces the Einstein–de Sitter model, whereas

$$n = \frac{H_0 t}{\ln t}$$

(27)

reproduces the exponential inflationary model with $R \sim e^{H_0 t}$, $\rho + p = 0$ and $p = P_r = P_\perp$.

Using (26), field equations (22)–(25) become

$$\rho = 3 \left( \dot{n} \ln t \right)^2 + 6 n \frac{\ln t}{t} + \frac{3n^2}{t^2} + \left( P_r - P_\perp \right)$$

(28)

$$P_\perp = -3 \left( \dot{n} \ln t \right)^2 - 6 n \frac{\ln t}{t} - \frac{3n^2}{t^2} - 2 \dot{n} \ln t - \frac{4 \dot{n}}{t} + \frac{2n}{t^2}$$

(29)

$$P_r = Q \omega - 3 \left( \dot{n} \ln t \right)^2 - 6 n \frac{\ln t}{t} - \frac{3n^2}{t^2} - 2 \dot{n} \ln t - \frac{4 \dot{n}}{t} + \frac{2n}{t^2}$$

(30)

$$Q \left( 1 - \omega^2 \right) = \omega \left( \frac{2n}{t^2} - \frac{4 \dot{n}}{t} - 2 \dot{n} \ln t \right).$$

(31)

2.2. Transport equations

In order to avoid the important drawbacks of Eckart’s theory, we shall use transport equations derived from the Israel–Stewart approach [9–11]. However, since these equations are going to be evaluated immediately after the system leaves the perfect fluid (or the quasi-perfect fluid) regime, then the truncated version of the theory leads to the same result as the full theory, and therefore the former will be used here. For the same reason, the shear viscosity contributions result in terms which are negligible in the approximation used here (see below). Therefore, we shall deal only with the transport equation for the heat flux, which reads [11]

$$q^\mu + \tau h^{\mu\beta} u^\gamma q_{\beta;\gamma} = -\kappa \left( h^{\mu\beta} T_{,\beta} + T a^\mu \right)$$

(32)

where as usual, $h^{\mu\beta}$ denotes the projector onto the 3-space orthogonal to $u^\mu$, and $a^\mu$ denotes the 4-acceleration.

In terms of $n$, the only non-vanishing independent component of (32) is the $r$-component, which reads

$$-\frac{Q}{t^n \left( 1 - \omega^2 \right)^{1/2}} + \tau \left\{ -\frac{\dot{Q}}{t^n \left( 1 - \omega^2 \right)} - \frac{Q' \omega}{t^{2n} \left( 1 - \omega^2 \right)} \right\} = -\kappa \left\{ \frac{\omega T}{t^n \left( 1 - \omega^2 \right)} + \frac{T'}{t^{2n} \left( 1 - \omega^2 \right)} \right\}

+ \left[ 2 \omega \frac{\dot{n}}{t^n \left( 1 - \omega^2 \right)} + \frac{\omega \dot{n}}{t^{2n} \left( 1 - \omega^2 \right)^2} + \frac{\omega}{t^n \left( 1 - \omega^2 \right)} \left( \dot{\ln t} + \frac{n}{t} \right) \right]$$

(33)

where the prime denotes a derivative with respect to $r$ (the $t$-component of (32) is just (33) multiplied by $\omega$).
3. Entering into a dissipative regime

As was already mentioned, an essential feature of the dissipative regime, as implied by (25) or (31), is the non-comoving nature of observers of the congruence \( u^\mu \) \( (\omega \neq 0) \). This means that \( \omega \) may be used as a control variable to assess how far (or close) the system is from the perfect fluid regime.

So, let us now assume that for \( t < \tilde{t} \), our system is in either the perfect fluid regime or in the quasi-perfect fluid regime. Both of which are characterized as follows.

(a) Perfect fluid regime

\[ Q = \omega = 0. \] (34)

Preferred observers are strictly comoving with the fluid.

(b) Quasi-perfect fluid regime

\[ Q \approx \omega \approx O(\epsilon) \] (35)

\[ \dot{Q} \approx \dot{\omega} \approx \omega^2 \approx Q \omega \approx Q^2 \approx 0. \] (36)

Preferred observers are not strictly comoving with the fluid, but (35) and (36) hold, i.e. within this regime the system always remains ‘close’ to the perfect fluid phase. In both cases, of course, \( r \) derivatives of \( Q \) and \( \omega \), are of the same order of magnitude as these quantities.

Next, let us assume that at \( t = \tilde{t} \), the system is allowed to abandon either of regimes ((a) or (b)) and enter into a dissipative phase. We shall now find the conditions for these transitions to occur.

Let us start with case (a). Then, evaluating the transport equation immediately after the system leaves the perfect fluid regime \( (t \approx \tilde{t} + O(\tau)) \), we obtain that equation (33) yields

\[ \dot{Q} = \frac{\kappa T}{\tau} \dot{\omega} \] (37)

where (34) and the fact that \( T' \approx O(\omega) \approx O(Q) \) has been used. Observe that immediately after the system leaves the perfect fluid regime

\[ P_r - P_\perp = Q \omega = O(\omega^2) \] (38)

therefore the contribution of the shear viscosity at \( t = \tilde{t} \) is one order smaller than the pure heat conduction effects, and accordingly will not be considered here.

Next, it will be more convenient to use the ‘conservation laws’ (Bianchi identities)

\[ T^{\mu \nu}_{; \nu} = 0 \] (39)

instead of field equations.

The \( r \)-component of (39) reads

\[
T^{\mu \nu}_{; r} = 0 = \frac{(\rho + P_r)\omega}{t^n(1 - \omega^2)} - \frac{Q(1 + \omega^2)}{t^n(1 - \omega^2)^2} + \frac{(\rho + P_r) \dot{\omega}(1 + \omega^2)}{t^n(1 - \omega^2)^2} - Q \frac{4\omega \dot{\omega}}{t^n(1 - \omega^2)^2} \\
+ \frac{4}{t^n} \left( n \ln t + \frac{n}{\tilde{t}} \right) \left[ \frac{(\rho + P_r) \omega}{1 - \omega^2} - \frac{Q(1 + \omega^2)}{1 - \omega^2} \right] \\
+ \frac{1}{t^{2n}} \left[ \frac{(\rho + P_r) 2\omega^3}{(1 - \omega^2)^2} - \frac{2Q \omega}{1 - \omega^2} - \frac{2Q \omega(1 + \omega^2)}{(1 - \omega^2)^2} \right] \\
+ \frac{2}{t^{2n}} \left[ \frac{(\rho + P_\perp) \omega^3}{1 - \omega^2} - \frac{Q \omega}{1 - \omega^2} \right]
\] (40)
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which evaluated immediately after \( \tilde{t} \), yields

\[
\dot{Q} = (\rho + P_r) \dot{\omega}.
\]

(41)

Combining (37) and (41), we finally obtain

\[
\dot{\omega} (\rho + P_r) \left( 1 - \frac{kT}{\tau (\rho + P_r)} \right) = 0.
\]

(42)

Observe that in (41) and (42) we may replace \( P_r \) by \( p \), since they differ by terms of the order of \( O(\omega^2) \).

Now, since initially \((t < \tilde{t})\) the system was evolving as a perfect fluid \((\omega = Q = 0)\), then a necessary condition for leaving that regime at \( t = \tilde{t} \) is

\[
\frac{kT}{\tau (\rho + p)} = 1
\]

(43)

since otherwise \( \dot{\omega} = 0 \), and, as can be easily checked by taking consecutive time derivatives of (33) and (40), all higher time derivatives of \( \omega \) (and \( Q \)) will also vanish. As mentioned before, equation (43) defines the critical point.

Let us now consider case (b). In this case the system (initially) is evolving as a quasi-perfect fluid, leaving that regime at \( t = \tilde{t} \). Evaluating (33) at \( t < \tilde{t} \), we obtain (observe that with the choice of (9), \( Q > 0 \) implies that \( q^r \) points inward)

\[
Q = \kappa \left[ \frac{T'}{t^n} + T \omega \left( \dot{n} \ln t + \frac{n}{t} \right) + \omega T \right]
\]

(44)

then evaluating the same equation (33) immediately after \( \tilde{t} \), and using (44) we obtain

\[
\dot{Q} = \frac{kT}{\tau} \dot{\omega}
\]

(45)

as in the previous case (equation (37)).

Next, evaluating (40) just after \( \tilde{t} \) one obtains

\[
\frac{\omega (\rho + P_r)}{t^n} = - \frac{\dot{Q}}{t^n} + \frac{\dot{\omega} (\rho + P_r)}{t^n} + \frac{4}{t^n} \left( \dot{n} \ln t + \frac{n}{t} \right) (\rho + P_r) \omega - Q = 0
\]

(46)

or, using (28)–(31) and (45)

\[
\dot{\omega} (\rho + P_r) \left[ 1 - \frac{kT}{\tau (\rho + P_r)} \right] = -\omega (\rho + P_r).
\]

(47)

In the case \( n = \text{constant} \), equation (47) becomes (using (28) and (30) and replacing \( P_r \) by \( p \))

\[
\dot{\omega} = \frac{2\omega}{(1 - [kT/\tau (\rho + p)]) t}.
\]

(48)

Therefore, \( \dot{\omega} \neq 0 \), and is larger, the closer the system is to the critical point.
4. Conclusions

We have assumed in this paper that the zero-curvature FRW metric represents consecutively, first a perfect (or quasi-perfect) fluid solution, and then a dissipative solution of the Einstein equations.

Necessary conditions for the transition from one state to another have been found.

If the system is initially evolving as a pure perfect fluid, then transition to the dissipative regime demands that the fluid satisfies the critical point condition. One important question related to this case is what could be the physical reasons for the system to abandon the equilibrium state? One possible scenario might be the appearance of a dissipative process such as particle creation, which as is known, is formally equivalent to the introduction of viscous terms [12]. Another possibility might be the decreasing of opacity of the fluid from very high values, preventing the propagation of null mass particles such as photons and neutrinos (trapping), to smaller values allowing for radiative heat conduction and viscosity. A similar situation occurs during the gravitational collapse of stars (see [13] and references therein) and the Kelvin–Helmholtz phase of neutron star formation (see [14] and references therein).

Of course the full implementation of any of these, or any other, scenarios would require a much more elaborate set-up than that presented here. The use of FRW with a single dissipative fluid is obviously very limiting, however, as we have seen, it leads to mathematically tractable equations.

In the same order of ideas, observe that if the system is, in the initial phase, inflating exponentially with $\rho + P_r = 0$, then as follows from (41) and (45), it will not leave that regime as long as $\rho + P_r = 0$. This is an obvious consequence from (28)–(31) which implies $Q = 0$ if $\rho + P_r = 0$, parenthetically it is worth noticing that the exit from inflation is usually explained in terms of a dissipative process such as particle production. From the above, it follows that though the obtained results are independent of the equation of state, some specific cases are not allowed since they exclude the possibility of dissipation (e.g. $\rho + P_r = 0$).

If the system is allowed to evolve initially as a quasi-perfect fluid, then transition to dissipative regime may always occur, without requiring the critical point to be attained, however, such a transition will be ‘faster’, the closer the system is to the critical point.

Another important question to ask is whether real physical systems may reach the critical point. Indeed, causality and stability requirements obtained from a linear perturbative scheme are violated, close to, but below the critical point (in the absence of viscosity) [6]. On the other hand, however, examples of fluids attaining the critical point and exhibiting reasonable physical properties, have been presented elsewhere [7, 15].

The possible solution to this apparent contradiction being the non-validity of the linear scheme used to obtain causality and stability conditions, close to the critical point (see a discussion on this point in [6]). At any rate, in the presence of both, heat conduction and viscosity, the corresponding critical point is beyond causality and stability conditions [7].

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References

[1] Tabensky R and Taub A 1973 Commun. Math. Phys. 29 61
Tupper B O J 1981 J. Math. Phys. 22 2666
Tupper B O J 1983 Gen. Rel. Grav. 15 849
Raychaudhuri A K and Saha S K 1981 J. Math. Phys. 22 2237
Raychaudhuri A K and Saha S K 1982 J. Math. Phys. 23 2554
Carot J and Ibáñez J 1985 J. Math. Phys. 26 2282
[2] Coley A A and Tupper B O J 1983 Astrophys. J. 271 1
[3] Coley A A 1987 Astrophys. J. 318 487
[4] Triginer J and Pavón D 1995 Class. Quantum Grav. 12 199
[5] Herrera L, Di Prisco A, Hernández-Pastora J L, Martín J and Martínez J 1997 Class. Quantum Grav. 14 2239
[6] Herrera L and Martínez J 1997 Class. Quantum Grav. 14 2697
[7] Herrera L and Martínez J 1998 Class. Quantum Grav. 15 407
[8] Herrera L and Di Prisco A 1999 Gen. Rel. Grav. 31 301
[9] Israel W 1976 Ann. Phys., NY 100 310
Israel W and Stewart J 1979 Ann. Phys., NY 118 341
[10] Jou D, Casas-Vázquez J and Lebon G 1999 Rep. Prog. Phys. 62 1035
[11] Maartens R 1996 Preprint astro-ph/9609119
[12] Zeldovich Ya B 1970 Sov. Phys.–JETP Lett. 12 307
Hu B L 1982 Phys. Lett. A 90 375
Barrow J D 1988 Nucl. Phys. B 310 743
[13] Arnett W D 1977 Astrophys. J. 218 815
[14] Burrow A and Lattimer J M 1986 Astrophys. J. 307 178
[15] Herrera L and Martínez J 1998 Astrophys. Space Sci. 259 235