A new method for CE-OFDM phase demodulation

Jinghui Li \(^1\) and Chao Chen\(^2\)

\(^1\)Communication University of China, Beijing 100024, China
\(^2\)Communication University of China, Beijing 100024, China
*Corresponding author’s e-mail: lijinghuijiayou@126.com

Abstract. CE-OFDM technology is used to modulate the phase of OFDM signal, and it ensures the range of signal envelope to be constant and overcomes the disadvantage of high OFDM signal peak-to-average ratio. This essay puts forward a kind of phase demodulation mode based on Costas loop for CE-OFDM system, and makes a comparison with the original demodulation modes by MATLAB simulation.

1. Introduction

OFDM (orthogonal frequency division multiplexing) technology is widely applied in communication field, and has advantages as follow: high utilization ratio of frequency spectrum, anti multi-path fading, anti inter-symbol Interference etc. However, OFDM system also has some disadvantages, such as high peak-to-average ratio, higher requirement for the power amplifier and other devices[1]. Scholars put forward a lot of solutions for this problem, for example, constellation diagram extension, selective mapping and other technologies, most of the solutions aim to reduce the peak-to-average ratio through changing OFDM baseband signal[2]. CE-OFDM technology, namely constant envelope OFDM, is applied to modulate the phase of OFDM baseband signal, so as to get the constant range of signal envelope, to make sure the peak-to-average ratio is as lower as 0dB, and the original OFDM signal can be recovered only by phase demodulating at the receiving end. CE-OFDM system not only has the advantages as the traditional OFDM system, but also overcomes the disadvantage of high peak-to-average ratio. Thus it has an important application value in wireless communication field [3]. In order to have a better demodulating of baseband OFDM signal, this essay makes some improvements for the phase demodulation mode, by using the Costas loop to demodulate the phase.

Costas loop is also called inphase orthodoxy loop, and it is a kind of special phase locked loop that can trail the frequency and phase of the signal, as well as eliminate the frequency offset and phase noise in the signal[4]. J.P. Costas once mentioned that the carrier signal can be recovered by inphase orthodoxy loop for the first time in 1956[5]. In modern engineering, Costas loop is widely applied to DPSK and carrier synchronization[6]. In addition, Costas loop also has the functions of frequency demodulation and phase demodulation. Therefore, in this essay, the phase demodulation function of Costas loop is conveniently used to demodulate CE-OFDM signal.

2. CE-OFDM System

From the comparison of figure 1 and figure 2, it indicates that the difference between CE-OFDM system and OFDM system lies in the phase modulation and phase demodulation modules.
2.1. Phase modulation
Through the phase modulating of baseband OFDM signal to obtain the modulated signal, namely CE-OFDM signal, the expression is shown as follows:

\[ S(t) = A_m(t) \cos[2\pi f c t + \varphi_m(t)] \]

\( m(t) \) means baseband OFDM signal, \( A_m(t) = |m(t)| \), \( \varphi_m(t) = 2\pi k_p m(t) \).

2.2. Phase demodulation based on orthogonal demodulation
Steve C. Thompson, who puts forward the CE-OFDM technology, demodulates the phase in his paper Constant Envelope OFDM by means of orthogonal demodulation[2]. The detailed structure is shown in figure 3, the demodulated CE-OFDM signal is respectively carried out the mixed frequency and down frequency conversion with the co-frequency orthogonal signal and inphase signal, so as to get the signal which is to be demodulated, and in the way of arc-tangent function to get the angle of signal, namely the original modulation signal. Phase unwinding module is used to the eliminate the phase jump in the signal.

2.3. Phase demodulation based on Costas loop
The structure of phase demodulation based on Costas loop is shown in figure 4, we use Costas loop to track the phase of input signal, and then recovers the original modulation signal. The phase detector of Costas loop can be operated in two ways: multiplication and arc tangent function. Multiplication phase detection is a kind of approximate algorithm, and in order to get a better demodulation performance, the phase detector of Costas loop in this essay is adopted by arc tangent function, and at the same time the phase jump in the signal is eliminated by phase unwinding module.
3. Costas Loop

3.1. Working principle of Costas loop

The structure of Costas loop is shown in figure 5. The input signal respectively multiplied by the inphase and orthogonal carrier signal when across the up and down bypass, and then the low-pass filter is used to detect the phase, and at last, the signal for controlling the local oscillator is output by loop filter to generate the local synchronous carrier [6].

Respectively multiply the input signal with the local carriers, if we suppose the loop is locked, the orthogonal carriers generated from NCO are:

\[ C_1(t) = \cos(2\pi f_c t + \theta) \]  
\[ C_2(t) = \sin(2\pi f_c t + \theta) \]

The multiplying results are:

\[ I(t) = A_m(t) \cos[2\pi f_c t + \varphi_m(t)] \cos(2\pi f_c t + \theta) \]  
\[ Q(t) = A_m(t) \cos[2\pi f_c t + \varphi_m(t)] \sin(2\pi f_c t + \theta) \]

Let the multiplying results respectively across the low-pass filter, and to filter the high frequency component, we get:

\[ I'(t) = 1/2 A_m(t) \cos(\varphi_m(t) - \theta) \]  
\[ Q'(t) = 1/2 A_m(t) \sin(\varphi_m(t) - \theta) \]

Inputting the result to the phase detector to demodulate, the phase detector is adopted by arc tangent function, and the phase demodulating result we get is shown as follow:

\[ P_d(t) = \text{actan}(\frac{Q'(t)}{I'(t)}) = \varphi_m(t) - \theta \]

The result of phase discriminator \( P_d(t) \) is fed back to NCO through loop filter, and to control NCO to generate carrier signals which shares the same frequency and phase with input signals.

3.2. Design of Costas loop

The design of Costas loop is the key point in system design, and it directly determines the functions of the system. Phase locked loop has two tracking modes: carrier tracking and modulation tracking. The loop which works under the carrier tracking state has a narrow loop bandwidth, and it is only used to
track the change of carrier, rather than the change of modulation signal. The loop which works under the modulation tracking state has a wider loop bandwidth compared with the former, and it not only tracks the change of carrier, but also tracks the phase change caused by the modulation signal. The essential difference between them lies in the output of phase detector when the loop trended to be steady. When tracking carrier, the output of phase detector is the phase of input signal, and in the state of modulation tracking, the output of phase detector trends to be in a stable value. Generally, the modulation tracking loop is used to demodulating FM signal, and carrier tracking loop is used to demodulating PM signal. The tracking statement of the loop can be adjusted by selecting the coefficients.

According to the two kinds of loop tracking statement, we learns that it is impossible for phase-locked loop to get a balance between the precise tracking of phase and frequency. If we requires the loop has a larger tracking range for the frequency, meanwhile, the loop can not track the signal phase accurately. On the contrary, if we command the loop makes a better track for the signal phase, then the loop will have a smaller capture range. The purpose of this essay is to design a kind of phase demodulator with the excellent demodulation function, and in order to get a better result, we decide to reduce the capture function of the loop, and mainly focuses on the phase tracking of input signal. Based on this principle, the parameters of Costas loop are accurately designed.

(1) Generally speaking, if we want to have a better capture function of the loop, we should wide the capture range and shorten the capture time[7]. The calculating method of the capture range shows in equation (9); the calculating method of the quick capture range shows in equation (10) and the calculating formula of the maximum quick capture time is the equation (11).

\[
\Delta \omega_p = 2 \sqrt{K \omega_n \xi} \tag{9}
\]

\[
\Delta \omega_L = 2 \omega_n \xi \tag{10}
\]

\[
T_L = \frac{5}{\omega_n \xi} \tag{11}
\]

In the previous formula, \(\omega_n\) is oscillation angular frequency and \(\xi\) is damped coefficient. From the formula, we can see that we need to increase \(\omega_n\) and \(\xi\) if we want to increase the capture range, and to reduce the capture time.

(2) The noise in the loop may affects the capture performance of loop. In the formula, \(B_L\) is the single noise bandwidth of loop, shows the filtering capacity of input noise. The smaller the \(B_L\) is, the stronger filtering capacity for the noise will be. The formula is shown as follow [8]:

\[
B_L = \frac{\omega_n}{8 \xi (1 + 4 \xi^2)} \tag{12}
\]

In the formula, the unit of \(\omega_n\) is rad/s and the unit of \(B_L\) is Hz.

Figure 6 shows the relationships of the loop bandwidth and damp coefficient. We can figure out that when \(\xi=0.5\), we get the minimum value of \(B_L\) and \(B_{L\min} = 0.5 \omega_n\). Therefore, from the perspective of noise suppression, \(\xi=0.5\) is the best choice.

![Figure 6. Relationships between the loop bandwidth and damped coefficient](image-url)
Set the input signal-to-noise ratio as \((S/N)_i\) , and the pre-bandwidth of loop is \(B_i\), so the signal-to-noise ratio of loop is:

\[
(S/N)_L = (S/N)_i \frac{B_i}{B_L}
\] (13)

In it, only when the loop signal-to-noise ratio is in the range of \((S/N)_L \geq 6\text{dB}\), the loop can be locked normally. This paper required the loop can be locked normally when the loop input signal-to-noise ratio is in the range of \((S/N)_i < 1\text{dB}\), so the \(\omega_n\) value is within the range:

\[
\omega_n < \frac{8 \xi (S/N)_i B_i}{(S/N)_L} = 1767 \times 10^3 \text{rad/s}
\] (14)

According to the analysis, we know that the capture and noise performance of loop are directly related with \(\omega_n\) and \(\xi\). The capture performance of loop would be improved when increasing \(\omega_n\) and \(\xi\), but the single noise bandwidth of loop would become much larger, and the anti-noise performance of the system would be reduced as well. Therefore, when we are designing \(\omega_n\) and \(\xi\), we have to make a choice according to the requirements. This paper emphasis on the anti-noise performance of the system instead of the capture performance. Therefore, we decide to select \(\omega_n=150\text{Hz}\) and \(\xi=0.5\) to fit our requirements.

(3) In phase locked loop, the design of loop filter is very important, because it directly relates to the performance of the whole loop. The essence of loop filter is a low-pass filter, while suppressing the high frequency noise, it outputs the control signal of NCO according to the feedback signal of phase detector. The formulas of loop filter coefficient are shown as follows [9]:

\[
C_1 = \frac{4\omega_n T^2 + 8\xi \omega_n T}{4 + 4\xi \omega_n T + (\omega_n T)^2 K}
\]

\[
C_2 = \frac{4(\omega_n T)^2}{4 + 4\xi \omega_n T + (\omega_n T)^2 K}
\]

In the formula, \(T\) is sampling period, and \(K\) is total gain of loop (it is usually set as 1). The sampling frequency in this essay is \(F_s = 64 \times 10^4\), and we get \(\omega_n T < 1\), to further simplify the formula as:

\[
C_1 = \frac{2\xi \omega_n T}{K}
\]

\[
C_2 = \frac{(\omega_n T)^2}{K}
\]

According to the previous designed parameter \(\omega_n=150\text{Hz}\), \(\xi=0.5\), by calculation, the loop filter coefficient is \(C_1 = 2.344 \times 10^{-4}\), \(C_2 = 5.552 \times 10^{-8}\).

4. Simulation Results and Analysis

4.1. Simulation parameters

Table 1. Simulation parameters setting
### System parameters

| Parameter                              | Numerical value |
|----------------------------------------|-----------------|
| Sampling rate                          | 640k            |
| Sampling period                        | 1/640k          |
| FFT points                             | 40960           |
| Modulation mode                        | 4QAM            |
| Numbers of subcarrier/symbols          | 256             |
| Oversampling factor                    | 10              |
| Carrier frequency                      | 150kHz          |
| Modulation coefficient                 | 0.2,0.4,0.6,0.8,1.1,4,1.8,2 |
| Cut-off frequency of low-pass filter   | 300 kHz         |
| Order of low-pass filter               | 3               |
| Bandstop attenuation of low-pass filter| 60dB            |
| AWGN channel signal-to-noise ratio     | 0-25dB          |
| Oscillation angular frequency $\omega_n$ | 150Hz     |
| Damped coefficient $\xi$               | 0.5             |
| Loop filter coefficient                | $C_1=2.344\times10^{-4}$ |
|                                       | $C_2=5.552\times10^{-8}$ |

### 4.2. Simulation results

Taking the modulation coefficient as 0.2, signal-to-noise ratio as 10dB to carry out the simulation. Figure 7 is the demodulation signal and frequency domain of original Signal. By comparison we can tell that the frequency of the demodulation signal is basically consistent with the original Signal. Figure 8 is the demodulation signal constellation diagram, and it indicates that the signal is more concentrated in distribution. Thus, we can have a better judgment of the data.

![Demodulation signal and original signal frequency domain](image1.png)

![Demodulation signal constellation diagram](image2.png)

Figure 7. Demodulation signal and original signal frequency domain

Figure 8. Demodulation signal constellation diagram

Figure 9 shows the relationship between the bit error rate and signal-to-noise ratio based on Costas loop demodulation system with different modulation coefficients (0 to 2). We can tell that when the modulation coefficient is in the range of 0<kp<1, the bigger the modulation coefficient are, the better the system performance will be. When the modulation coefficient is in the range of 1≤kp≤2, the system performance will be reduced with the increasing of demodulation coefficient, and the system performance is less than the former when the modulation coefficient is smaller than 1. The reason is that when the modulation coefficient is bigger than 1, the loop is no longer in the linear analysis, and with the increasing of modulation coefficient, the frequency spectrum of modulation signal becomes sparse and the increased bandwidth will increase the demodulation difficulty of signal[10].
Figure 9. System performance based on Costas loop demodulation

Figure 10 is the relationships between the bit error ratio and signal-to-noise ratio based on orthogonal demodulation with the different modulation coefficients (0 to 2). The system performance based on orthogonal demodulation is also affected by the modulation coefficient. When the modulation coefficient is in the condition of kp<1.4, the system performance becomes better with the increasing of modulation coefficient. When the modulation coefficient is too large to exceed 1.4, the system performance becomes much worse.

Figure 11 shows the comparison of the system performance based on orthogonal demodulation and Costas loop demodulation under the condition of modulation coefficient kp<1. When the signal-to-noise ratio and modulation coefficient are constant, the system bit error ratio adopted by Costas loop demodulation is lower than the orthogonal demodulation. At the same time, if the modulation coefficient is fixed, with the increasing of signal-to-noise ratio, the system bit error ratio based on Costas loop demodulation system would be reduced faster. According to the comparison, we draw a conclusion that the system functions adopted by Costas loop demodulation is obviously better than the orthogonal demodulation.
5. Conclusions
From the simulation results, it can be concluded that the method based on Costas loop demodulation has better anti-noise performance than that of the quadrature demodulation, and the design of its parameter possesses the characteristic of flexibility—the performance of the system can be changed according to the alternate of the parameter. Nevertheless, the Costas loop demodulation does have a few drawbacks, for example the structure of the Costas loop is more complicated and the simulation will cost much more time.

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