Controlling Functional Uncertainty

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Abstract. There have been two different methods for checking the satisfiability of feature descriptions that use the functional uncertainty device, namely $\text{I}$ and $\text{II}$. Although only the one in $\text{I}$ solves the satisfiability problem completely, both methods have their merits. But it may happen that in one single description, there are parts where the first method is more appropriate, and other parts where the second should be applied. In this paper, we present a common framework that allows one to combine both methods. This is done by presenting a set of rules for simplifying feature descriptions. The different methods are described as different controls on this rule set, where a control specifies in which order the different rules must be applied.

1 Introduction

This paper is concerned with an extension to feature descriptions, which has been introduced as “functional uncertainty” in $\text{I}$. This formal device plays an important role in the framework of LFG in modeling so-called long distance dependencies and constituent coordination. For a detailed linguistic motivation see $\text{III}$. Functional uncertainty consists of constraints of the form $x Ly$, where $L$ is regular expression. $x Ly$ is interpreted as $\bigvee \{xwy \mid w \in L\}$. Since this disjunction may be infinite, functional uncertainty gives additional expressivity. Let us recall an example from $\text{III}$ and consider the topicalized sentence $\text{Mary John telephoned yesterday}$. Using s as a variable denoting the whole sentence, the LFG-like clause $s \text{topic } x \land s \text{comp obj } x$ specifies that in s, $\text{Mary}$ should be interpreted as the object of the relation telephoned. The sentence could be extended by introducing additional complement predicates, as e.g. in sentences like $\text{Mary John claimed that Bill telephoned; Mary John claimed that Bill said that } \ldots \text{ Henry telephoned yesterday}$. For this family of sentences the clauses $s \text{topic } x \land s \text{comp obj } x$, $s \text{topic } x \land s \text{comp comp obj } x$ and so on would be appropriate; specifying all possibilities would yield an infinite disjunction. Using functional uncertainty, it is possible to have a finite presentation of this infinite specification, namely the clause $s \text{topic } x \land s \text{comp}^* \text{obj } x$.

It was shown in $\text{III}$ that consistency of feature descriptions is decidable, provided that a certain acyclicity condition is met. More recently, $\text{III}$ has shown that the satisfiability problem is decidable without additional conditions. Both algorithms have their merits. The one in $\text{II}$ solves the satisfiability problem using an extended syntax, which makes it possible to avoid the computational explosion that causes the undecidability in the cyclic case. But there are cases where the additional syntax causes some overhead. In these cases, one would like to switch to the method used in $\text{I}$, where this overhead is avoided. On the other hand, the algorithm in $\text{I}$, which is used in the implementation of the LFG system, cannot be extended to the cyclic case.

In this paper, we present a new algorithm that allows one to combine both methods under a common framework. We use the extended syntax as proposed in $\text{III}$ and present a new set of rewrite rules. The different methods used in $\text{II}$ and $\text{I}$ can then be described as different control on this rule set, where a control specifies the order of rule application. Thus, it is now possible to compare both algorithms and their effects.

In $\text{III}$, this was not possible since the set of rules presented there was tailored for the purpose of proving decidability. As an extension, we present a control which allows the flexibility to switch between both methods. This flexibility is needed since none of the methods is optimal for all parts of a clause. Which one is best depends on the regular languages used in the corresponding part.

In Section 2, we present some needed preliminaries. In Section 3, we introduce the input clauses and two different output clauses of our algorithm. In Section 4 and 5 we present the rule system and some of its basic properties. Equipped with these tools, we turn to the most interesting part in Section 6, where we define three different controls for the given set of rules and compare their properties.

2 Preliminaries

Our signature consists of a set of sorts $S$ ($A, B, \ldots$), first-order variables $X$ ($x, y, \ldots$), path variables $P$ ($\mu, \nu, \ldots$), and features $F$ ($f, g, \ldots$). We assume a finite set of features and infinite sets of variables and sorts. A path is a finite string of features. A path $u$ is a prefix of a path $v$ (written $u \prec v$) if there is a non-empty path $w$ such that $v = uw$. Note that $\prec$ is neither symmetric nor reflexive. Two paths $u, v$ diverge (written $u \mid v$) if there is a common, possibly empty prefix $w$ of $u$ and $v$ and paths $w_1, w_2$ such that $u = wfw_1 \land v = wgw_2$. Clearly, $\mid$ is a symmetric relation. Furthermore, for any pair of paths $u$ and $v$, then exactly one of the relations $u = v$, $u \prec v$, $u \succ v$, or $u \mid v$ holds.

A simple path term $(s, t, \ldots)$ is either a feature or a path variable. A path term $(p, q, \ldots)$ is either a simple path term.

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or a concatenation of two path terms poq (called a complex path term). The set of constraints is given by

\[
\begin{array}{l}
Ax \quad sort \ restriction \\
x \equiv y \quad \text{agreement} \\
x[p]y \quad \text{subterm agreement} \\
p \cdot \in L \quad \text{path restriction}
\end{array}
\]

We exclude empty paths in subterm agreement since \(x y z\) is equivalent to \(x \equiv y\), and use \(p \cdot q\) as a synonym for \(q \cdot p\). A clause is a finite set of constraints denoting their conjunction.

An interpretation \(\mathcal{I}\) is a standard first-order structure, where every feature \(f \in F\) is interpreted as a binary, functional relation \(F^2\) and where sort symbols are interpreted as unary, disjoint predicates (hence \(A^2 \cap B^2 = \emptyset\) for \(A \neq B\)). A valuation is a pair \((\alpha_0, \alpha_F)\), where \(\alpha_X\) is a standard first-order valuation of the variables in \(X\) and \(\alpha_P\) is a function \(\alpha_P : P \rightarrow F^+\). We define \(\alpha_P(f) = f\) for every feature \(f \in F\), and \(\alpha_P(p|q)\) to be the path \(\alpha_P(p) \cdot \alpha_P(q)\). Validity for sort restrictions and agreement constraints is defined as usual. The other constraints are valid in an interpretation \(\mathcal{I}\) under a valuation \((\alpha_0, \alpha_F)\) iff

\[
\begin{align*}
(\alpha_0, \alpha_F) &\models x[p]y \iff (\alpha_0(p) = f_1 \cdots f_n, \\
(\alpha_0(x), \alpha_0(y))) &\in F^2_1 \cdots F^2_n \\
(\alpha_0, \alpha_F) &\models x[p]y \iff \alpha_P(p) \in L \\
(\alpha_0, \alpha_F) &\models x[p]y \iff \alpha_P(p) \cdot \alpha_P(q) \text{ for } \phi \in \{\mathcal{I}, =, \neq\}.
\end{align*}
\]

where \(\bullet\) denotes binary concatenation of relations. Note that the validity of a path constraint depends only on the path valuation. The set of all \(X\)-solutions of a clause \(\phi\) in some interpretation \(\mathcal{I}\) is the set of all valuations \(\alpha_X\) such that there is a path valuation \(\alpha_F\) with \((\alpha_0, \alpha_F) \models \phi\).

### 3 Prime, Pre-Solved, and Solved Clauses

In this section, we define the input and output clauses for both phases of the algorithm. In the following, we consider only those clauses \(\phi\) such that for every distinct pair of variables \(x, y, x \equiv y\) is in \(\phi\) if and only if \(x \neq y\) and \(x\) occurs only once in \(\phi\). A clause \(\phi\) is called prime iff

Pr1. every path term in \(\phi\) is simple,
Pr2. for every path variable \(\mu\) used in \(\phi\) there is at most one constraint \(x[p]y \in \phi\), and
Pr3. \(\phi\) has no constraints of the forms \(s \parallel t\), \(s \preceq t\), or \(s \geq t\).

Kaplan/Maxwell \[3\] formulated the satisfiability problem for functional uncertainty in an unsorted syntax. Essentially, this syntax consists of the atomic constraints \(Az, x f y\) and \(x \equiv y\) together with the additional constraint \(x Ly\). Constraints of this form are interpreted as \(x Ly = \bigvee\{x y w \mid w \in L\}\). A clause \(\phi\) in Kaplan/Maxwell Syntax can be translated into an clause in our syntax by replacing every constraint \(x Ly\) by \(x[p]y \land \mu \in L\), where \(\mu\) is a new variable. The resulting clause will have the same \(X\)-solutions. The resulting clauses are prime clauses and hence our input clauses. A clause is called simplified iff

Si1. \(Az \in \phi\) and \(Bx \in \phi\) implies \(A = B\),
Si2. \(p \in L \in \phi\) and \(p \in L' \in \phi\) implies \(L = L'\),
Si3. \(p \in \emptyset\) is not in \(\phi\),

Si4. \(f \in L\) implies that \(f\) is an element of denotation of \(L\),
Si5. \(x[f]y \in \phi\) and \(x[f]z \in \phi\) implies \(y = z\),
Si6. \(\phi\) contains no constraint of the form \(s \preceq t\) or \(s \geq t\),
Si7. every path term in \(\phi\) is simple.

A simplified clause is called pre-solved iff

Ps1. \(s \parallel t\) \in \(\phi\) if and only \(s \neq t\), either \(s\) or \(t\) is a path variable, and there is an \(x\) such that \(\{x[s]y, x[t]z\} \subseteq \phi\).

Pre-solved clauses are not consistent per se, since it might be that a divergence constraint contradicts some of the path restrictions. E.g., the pre-solved clause \(x[p]y \land x[q]z \land \mu \parallel \nu \land \mu \in f^+ \land \nu \in (f f)^+\) is inconsistent. A clause \(\phi\) is called solved if it is either \(\perp\), or it is simplified and satisfies

So1. \(\phi\) contains no constraint of form \(s \parallel t\), and
So2. if \(x[p]y\) is in \(\phi\), then there is no \(x[s]z\) with \(s \neq \mu\) in \(\phi\).

**Lemma 1** Let \(\phi\) be a pre-solved clause different from \(\perp\). Then \(\phi\) is satisfiable iff there is a path valuation \(\alpha_P\) with \(\alpha_P \models \phi_0\), where \(\phi_0\) is the set of constraints in \(\phi\) of the forms \(s \parallel t\) or \(s \in \phi\). Furthermore, every solved clause different from \(\perp\) is satisfiable.

### 4 Simplification Rules

The first set of rules, \(\mathcal{R}_{\text{Simp}}\), is displayed in Figure 4 and allows one to simplify a clause satisfying certain restrictions that will be captured under the notion of a admissible clause. Most of the rules are deterministic, i.e., replacing a clause with the result of applying one of these rules yields a clause having the same \(X\)-solutions. The rules \(\{\text{ReD}\}\) and \(\{\text{Dec}\}\) are non-deterministic rules, which implies that we have to replace a clause by the disjunction of all possible applications of the corresponding rule. Thus, applying \(\{\text{ReD}\}\) to a clause of the form \(\mu_0 \parallel \nu \lor \cdots\) yields the disjunction

\[
(\mu_0' \parallel \nu \lor \mu \parallel \nu \lor \cdots) \lor (\mu_0' \parallel \nu \lor \mu \preceq \nu \lor \cdots)
\]

The rule set is indexed by the decomposition function \(\text{DFun}\) used in \(\{\text{Dec}\}\). The simplest version of \(\text{DFun}\) just decomposes a regular language \(L\) into a set of pairs \((P, S)\) with the property that there is a state \(q\) in the minimal automaton \(A\) for \(L\) with \(P = \{w \neq \epsilon \mid \delta_A(q_0, w) = q\}\) and \(S = \{w \neq \epsilon \mid \delta_A(q, w) \in \text{Fin}_A\}\). Here, \(q_0\) is the initial state, \(\text{Fin}_A\) is the set of final states and \(\delta_A\) the transition function of \(A\). This decomposition function is sufficient for the case of non-cyclic clauses. For cyclic clauses, we have to use a different decomposition function (as will explained later). In any case, in order to preserve all solutions of a clause the decomposition function has to satisfy

\[
\forall L, \forall w_1, w_2 \neq \epsilon : \\
[w_1, w_2 \in L \Rightarrow \exists P, S \in \text{DFun}(L) : (w_1 \in P \land w_2 \in S)]
\]

The simplification does not handle arbitrary clauses. E.g., we handle only those prefix and equality constraints \(s \preceq t\) and...
s \doteq t in a clause \phi with the property that there is a variable x and variables y, z such that x[s]y and x[t]z is in \phi. Furthermore, the rules cannot reduce divergence constraints of the form sos'. I \doteq tol' with s \not\doteq t, and the control imposed on our rewrite rules carefully avoid such constraints. The reason is that for decomposing the complex path terms in sos'. I \doteq tol', we might be forced to introduce complex path terms that have a length greater than 2, which we must avoid to achieve a quasi-terminating rewrite system.

We now define the restriction imposed on derivable clauses. Given a clause \phi, we define the outgoing edges of a first-order variable x in \phi as

\text{outgoing}_\phi(x) := \{s \mid \text{there is z with } x[s]z \in \phi\}

We say that a variable x in \phi is tagged if there is a prefix constraint s \preceq \mu in \phi with \{s, \mu\} \subseteq \text{outgoing}_\phi(x). A clause is called admissible if \phi contains no complex path terms in prefix or path equality constraints and

- Ad1. for every path variable \mu in V_\phi(\phi), there is exactly one constraint x[\mu]y \in \phi,
- Ad2. for every path constraint of the forms s\{\doteq, \prec, \parallel\}t in \phi, there exists a variable x such that \{s, t\} \subseteq \text{outgoing}_\phi(x),
- Ad3. if \phi contains a prefix constraint, then \phi contains no path equality constraint,
- Ad4. if \phi contains at most one tagged variable,
- Ad5. if \phi contains two different prefix constraints s \prec \mu and t \prec \nu, then either s = t, or s and t are different features,
- Ad6. \phi contains no trivial constraints of the form s \prec s, \prec f, f \equiv g, or f \equiv f.

The last condition just lists constraints which either are inconsistent or superfluous. We could also get rid of these constraints using some appropriate rewrite rules, but we note that it is more efficient to avoid these constraints. Note that every prime clause is admissible. A clause is called basic if is derivable using \mathcal{R}_{\text{simpl}}^\text{DFun} from an admissible \phi that contains no complex path terms.

**Proposition 2** Every basic clause is admissible.

The tedious part of the proof of this proposition are the rules \text{(Pre)}, \text{(Div1)}, and \text{(RelD)}, since one has to check whether the new introduced constraints satisfy the conditions \text{Ad2} and \text{Ad3}. For this purpose, one has to record exactly all possible effects that the introduction of complex path terms in the \text{(Pre)} rule can have on admissible clauses. E.g., it is guaranteed by the definition of \text{(Pre)} that if a basic \phi contains a complex path term \alpha_\phi_\psi in \phi, then there are variables x, y, z such that x[\mu]y and y[\nu]z are in \phi. This, together with condition \text{Ad1} implies that if \phi contains a constraint of the form \mu_\psi_\phi_\alpha in \phi, then there are variables x, y, z, z' such that \{x[\mu]y, y[\nu]z, y[\nu]'z'\} \subseteq \phi. Hence, we know for the new relation introduced in \text{(Div1)} the condition \text{Ad2} is satisfied.

**Lemma 3 (Termination,Completeness)** A basic clause is irreducible w.r.t. \mathcal{R}_{\text{simpl}}^\text{DFun} iff \phi is simplified. Furthermore, for every admissible clause \phi there are no infinite derivations starting with \phi, and \phi has the same \mathcal{X}-solutions as the set of simplified clauses derivable from \phi.

**Proof (Sketch)** We consider only the claim of termination. Here, the \text{(RelD)} rule is the most difficult part since it introduces a new relation. All other rules reduces either the number of variables, constraints or complex path terms. To show that \text{(RelD)} terminates, it is necessary to know that there is exactly one variable x in \phi such that for all constraints of the form sos' I \mu in \phi, both s and \nu are in \text{outgoing}(x). But this is an immediate consequence of the fact that there is at most one tagged variable in \phi (Condition \text{Ad4}) together with Condition \text{Ad3}. Hence, \text{(RelD)} only adds constraints between unrelated simple terms that are in \text{outgoing}(x), where
x is the tagged variable of φ. Since there are only finitely many possible relations, and since both the (Pre) rule and the (DivX) rule do not increase the number of unrelated simple terms in outgoinggₘ(x), (RelI) cannot cause non-termination. This leads to the following termination ordering. Let x be the tagged variable in φ, and let Θₐ₁ₙₚ(φ) be the quadruple

( #unrelated terms in outgoinggₘ(x), #constraints,
  #complex path terms in φ, #variables)

Then for every r ∈ R̃ₕₚ, if φ is the result of applying r to a basic clause φ, then Θₐ₁ₙₚ(φ) >ₜ Θₐ₁ₙₚ(φ'), where >ₜ is the lexicographic greater ordering on quadruples.

5 Generating pre-solved and solved clauses

As we have explained in the introduction, one of the main tools for solving prime clauses is to “guess” the different relations between path variables, and to check this relation for consistency with the rest of the clause afterwards. Clearly, one has to “guess” all possible relations, which implies that the rules for introducing this relation must be non-deterministic. We have already encountered one rule for non-deterministically introducing relations between simple path terms, namely the rule (RelI). The other two rules are listed below and form the rule set Rₚₜₚ.

(Relate1) x[u]y ∧ x[v]z ∧ ψ µ, ν ∈ R, unrel in ψ. (RelI) not applicable

(Relate2) f{µ, ν} ∧ x[f]y ∧ x[µ]z ∧ ψ f, µ unrel in ψ, (RelI) not applicable

Using the following set of rules Rₜₚ, we can transform a pre-solved clause into an equivalent set of solved clauses.

(Inst) µ ⊢ f ∧ ψ g ⊢ µ ∧ ν ⊢ f ∧ ψ f ≠ g

(Intro) f ⊢ µ ∧ x[µ]y ∧ x[v]z ∧ ψ f ⊢ µ ∧ x[µ]y ∧ x[f]y twisted if vz : x[f]z ∉ ψ

(Solv1) µ ⊢ ν ∧ ψ f ⊢ µ ∧ g ∧ ν ⊢ ν ∧ ψ f = g

(Solv2) µ ⊢ ν ∧ x[µ]y ∧ x[v]z ∧ ψ f ⊢ µ ∧ g ∧ ν ⊢ ν ∧ x[δ]u ∧ u ∧ u[v]y ∧ u[v]y ∧ ψ subs

where ψ subs = ψ[µ ← δ ν, ν ← δ ν]}, f ≠ g and δ, u are new variables

The two rules (Solv1) and (Solv2) together will be seen as one complex, non-deterministic rule called (Solve). The (Solve) directly expands a divergence constraint into its definition, thus solving a single divergence constraint. The (Solv1) rules reflects the case that two paths diverge with an empty prefix while (Solv2) reflects the case that the common prefix is not empty. Since the valuations always associates non-empty paths to path variables, we have to distinguish these cases. Note that (Intro) is the only deterministic rule, and that all of the other rules are non-deterministic.

Proposition 4 If a simplified clause is not pre-solved, then one of (Relate1) or (Relate2) is applicable. Furthermore, a clause is pre-solved if none of the rules in Rₕₚ ∪ Rₜₚ is applicable, and solved if none of the rules in Rₕₚ ∪ Rₜₚ ∪ Rₚₜₚ is applicable.

6 Controlling rule application

In this section, we present different possible controls over the set of rules given by Rₕₚ = Rₕₚ ∪ Rₜₚ ∪ Rₚₜₚ. A control is a partial order < on Rₕₚ. A derivation φ₁ →ₜ φ₂ ... is licenced by a control < iff for every step φᵢ →ₜ φᵢ₊₁, no rule instance r with r <ₜ rᵢ is applicable. We use <ₜ-derivative and <ₜ-derivation in the obvious way.

If we would apply the rules without any control, then not only is termination not guaranteed, but we may even produce a clause that is not admissible. E.g., consider the clause x[µ]y ∧ x[v]z ∧ x[v]z'. Then applying (Relate1) twice may produce the clause µ ≺ ν ∧ ν ≺ ν' ∧ x[µ]y ∧ x[v]z ∧ x[v]z', which is not admissible since it does not fulfill condition (Ad). Hence, our minimal control <ₜ-guarantees that the simplification rules are applied before one of the rules in Rₜₚ ∪ Rₚₜₚ are applied, i.e.,

∀r ∈ Rₕₚ, ∀r' ∈ Rₜₚ ∪ Rₚₜₚ : r <ₜ r'.

Proposition 5 If φ is derivable with Rₕₚ from a prime clause using the control <ₜ, then φ is admissible.

Proof (Sketch) This follows from the fact that if φ is an admissible clause that contains no complex path terms (which prime clauses are), then it is basic and therefore admissible due to Proposition 2. Furthermore, it can be simplified due to Lemma 3. Hence, according to the control <ₜ-guarantees, we can apply a rule in Rₜₚ ∪ Rₚₜₚ if and only if the corresponding rule is simplified. And it is easy to check that applying a rule Rₜₚ ∪ Rₚₜₚ to a simplified clause yields an admissible clause that contains no complex path terms.

If for every prime clause φ there are no infinite derivations using Rₕₚ, then we know that we could transform every prime clause φ into an equivalent set of solved clauses. But this is not the case. Consider e.g. the clause

x[µ]x ∧ x[f]y ∧ µ ∈ f⁺ ∧ Ax ∨ By.

Then applying (Relate2) to introduce a constraint f ≺ µ followed by an application of (Pre) and (DecI) yields the same clause again. The reason for the loop is that we have a cyclic description of the form x[µ]x. But we can show that, similar to Kaplan/Maxwell’s Algorithm, Rₕₚ is terminating under <ₜ if no cyclic descriptions are encountered.
Theorem 6 Let $\phi$ be a prime clause such that no $<^{\text{basic}}$-derivative of $\phi$ contains a cycle. Then there is no infinite $<^{\text{basic}}$-derivation. Furthermore, $\phi$ has the same $\chi$-solutions as the set of solved clauses derivable from $\phi$.

Hence, the control $<^{\text{basic}}$ can be used if one does not want to handle cyclic structures. Note that one can easily recognize whether the algorithm runs in a loop using an occurrence check (i.e., by checking whether one visits some variable twice). In this case, one can either stop (without knowing anything about the satisfiability), or switch to the more complex control $<^{\text{quasi}}$ that at least guarantees quasi-termination. A rewrite system is quasi-terminating, if it may loop, but produces only finitely many different clauses. Given a quasi-terminating rewrite system, an algorithm using this system must record the previously calculated clauses and stop, if one clause is produced for the second time. This is expensive, but necessary if you want to handle cyclic structures. $<^{\text{quasi}}$ is the control extending $<^{\text{basic}}$ with the property that

$$\forall r \in R_{\text{pre}}, \forall r' \in R_{\text{solve}} : r <^{\text{quasi}} r'.$$

Since in $<^{\text{quasi}}$ the rules in $R_{\text{pre}}$ are applied first, we know that every clause is first transformed into a set of pre-solved clauses, which are then solved using $R_{\text{solve}}$. By an adaptation of the we get the following theorem.

A necessary condition for this theorem is that for every prime clause $\phi$, the set of all regular languages introduced in some $<^{\text{quasi}}$-derivative of $\phi$ by $(\text{DecF eat})$, $(\text{DecDF un})$, or $(\text{Join})$ is finite. Clearly, there are only finitely many different regular languages produced by $(\text{DecF eat})$ or $(\text{Join})$, but $(\text{DecDF un})$ may be a problem. shows how an appropriate decomposition function can be found for a given prime clause $\phi$.

Theorem 7 There exists a decomposition function $\text{DFun}$ such that for every prime clause $\phi$ there are only finitely many $<^{\text{quasi}}$-derivatives. Furthermore, $\phi$ has the same $\chi$-solutions as the set of $<^{\text{quasi}}$-derivatives that are solved.

Now what’s left? This are the $<^{\text{basic}}$-derivations that are neither $<^{\text{quasi}}$-derivations nor $<^{\text{KM}}$-derivations. The question arise whether there is any use for such derivations, and there are. The reason simply is that it depends on the used regular languages whether for a specific divergence constraint, it is more useful to solve this divergence constraint immediately (as it is done under the $<^{\text{KM}}$ control), or whether it is better to delay this solving (as in the $<^{\text{quasi}}$ control) hoping that this might be superfluous since other rules may detect a simple inconsistency. Consider a generalization of the example given in the introduction using regular languages of the form $\text{comp}^+ \{\text{grel}_1, \ldots, \text{grel}_n\}$, where $\text{grel}_1, \ldots, \text{grel}_n$ are grammatical relations such as direct object, indirect object and so on. Now let $\phi$ be a clause of the form

$$x[y]z \land x[v]z \land \mu \Pi \nu \mu \in \text{comp}^+ \{\text{grel}_1, \ldots, \text{grel}_n\} \land \nu \in \text{comp}^+ \{\text{grel}_1, \ldots, \text{grel}_n\}$$

Then we know that $\mu$ is of the form $\delta f^\nu$ and $\nu$ is of the form $\delta g^\nu$ such that the common prefix $\delta$ is in $\text{comp}^+$, $f \in \{\text{grel}_1, \ldots, \text{grel}_n\}$ and $g \in \{\text{comp}, \text{grel}_1, \ldots, \text{grel}_n\} - \{f\}$. Hence, there are $(n+1) \times n$ different possibilities that $\mu$ and $\nu$ diverge. Solving the divergence constraint immediately as forced by the $<^{\text{KM}}$ control, would produce a disjunction of $(n+1) \times n$ clauses. This makes sense for $n = 1$ (as in the case of $\text{comp}^+ \text{subj}$) since it reduces the overhead for keeping the divergence constraint. But should be delayed in the case where $n$ is greater than 1. Using the control $<^{\text{basic}}$, one has the flexibility to do so, and Theorem guarantees that the algorithm terminates in the case of non-cyclic descriptions.

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