A High-Precision Dual-Frequency Magnetic Induction Through-the-Earth Positioning Method

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ABSTRACT To achieve the high-precision through-the-earth positioning in mine rescue, we propose a dual-frequency magnetic induction positioning method. The dual-frequency transmitter underground is a loop energized coil. The receiver on the ground is a three-axis magnetic core coil. We first carry on the algebraic transformation to the expression of the dual-frequency positioning signal, and obtain the transceiver distance function that is not affected by the layered earth conductivity. Then, through the tilt angle of the receiver and the rotation matrices, we correct the receiver tilt. By the direction of the geomagnetic field, we correct the direction deviation of the receiver. Finally, we obtain the three-dimensional coordinates of the transmitter by decomposition of the transceiver distance. The analysis and simulation results are as follows. The dual-frequency positioning method eliminates the positioning error caused by the path loss of the stratified earth media. The rotation matrices correct the error caused by the receiver tilt. When the receiver is close to the top of the transmitter, the positioning error can be reduced. By configuring proper transmitter parameters, we can maintain a balance between transmitter size and rescue efficiency.

INDEX TERMS High-precision, dual-frequency, magnetic induction, positioning, through-the-earth.

I. INTRODUCTION

When accidents such as landslides, leaks, and explosions occur in the mine, communication facilities such as base stations and cables will be damaged. The damages lead to the communication interruption between the ground and the mine. The trapped miners cannot provide the rescuers with the exact location of the mine accident. This situation hinders the implementation of the rescue measures such as dredging roadways and drilling holes.

The through-the-earth positioning technology is instrumental to the mine rescue [1], [2]. The positioning signal transmitter can transmit the through-the-earth positioning signal in the mine. The signal penetrates the earth and reaches the positioning signal receiver on the ground. The receiver determines the position of the transmitter by using the direction and the intensity of the positioning signal. The rescuers will carry out the rescue according to the position information.

There are several potential methods for the through-the-earth positioning technology, such as the quasi-static electric field method, the electromagnetic wave method, and the magnetic induction method.

The quasi-static electric field positioning method uses the extremely low-frequency electric field to transmit the positioning signal. The transmitter of this method is a high voltage AC power supply. The receiver is a voltmeter [3]. When the two ends of the AC power supply are inserted into the earth media separately, an alternating electric field is induced in the earth media. The strength of the electric field decreases with the increase of the transceiver distance. According to this law, the rescuers can gradually determine the transmitter position. This method is easy to deploy but is easily affected by the earth media. The conductivity of rock, soil, and water in the earth media are different. So, the attenuation rate of the alternating electric field in these media is very different [4]. The electric field strength at the receiver cannot correspond to the transceiver distance accurately. Therefore, this method has limited positioning accuracy.
The electromagnetic wave positioning method uses the electromagnetic wave to transmit the positioning signal. This method has the problems of low positioning accuracy and short transmission distance. When propagating in the earth media, the electromagnetic wave is easily reflected, refracted, and scattered [5]–[12]. These phenomena deviate the direction and amplitude of the positioning signal, which reduce the positioning accuracy of the method. The frequency of the electromagnetic wave is inversely proportional to the length of the antenna. So, only the antennas that emit the high-frequency electromagnetic wave can be deployed in the mine with limited space. Affected by the skin effect, the high-frequency electromagnetic wave attenuates rapidly in the earth media. This characteristic results in a short transmission distance of the positioning signal.

The magnetic induction positioning method uses the extremely low-frequency magnetic field to transmit the positioning signal. The transmitter of the positioning method is a loop energized coil. The receiver is one or more loop coils. When the low-frequency alternating current is turned on, the transmitter coil can induce the low-frequency alternating magnetic field. The magnetic field penetrates the earth media and reaches the ground. The receiver on the ground can detect the intensity and the direction of the magnetic field. By using the information, the magnetic induction method can determine the transmitter position. Because the low-frequency magnetic field is less affected by the earth skin effect, the positioning signal can penetrate quite deep earth. Different from the electromagnetic wave and the quasi-static electric field, the low-frequency magnetic field can not be reflected or scattered in the earth [13], [14]. In the earth media without ferromagnetic material, the low-frequency magnetic field will not refract. Compared with the former two methods, the magnetic induction method has a longer transmission distance and higher positioning accuracy. Therefore, this method has an extensive application prospect in the through-the-earth positioning.

The magnetic induction positioning technology usually includes two kinds of methods: The path method and the direction method.

The path method achieves the through-the-earth positioning by using the path loss of the positioning signal. In [15] and [16], the analytical equation between the positioning signal strength and the receiver coordinates is established. When ignoring the skin effect, the coordinates of the receiver is obtained by algebraic calculation. In [17], the strength of the positioning signals at the receivers is acquired firstly. The receiver positions are known. Then, several path loss equations between the transceiver distance and the signal strength. Finally, the coordinates of the receivers are established. The positioning methods in [15]–[17] can achieve the through-the-earth positioning. However, similar to the quasi-static electric field positioning method, the path method is also easily affected by the earth media. The conductivity of soil, rock, and water in the earth media is different. When passing through these media, the positioning signal has a different path loss rate. The signal strength at the receiver will change accordingly. The positioning results calculated by the path loss rate of the positioning signal will also have an error.

The direction method achieves the through-the-earth positioning by using the direction of the positioning signal. Wait [18] found that on the transmitter coil axis, the positioning signal is along the axis. Powell [19] found that the horizontal component of the positioning signal always points to the transmitter axis. By intersecting the positioning signal vectors at two locations, Powell determined the position of the vertical projection of the transmitter coil on the ground. Tian et al. [20] further discovered the analytical relationship between the direction of the positioning signal vector and the transmitter depth. After measuring the direction of the positioning signal vector, Tian et al. [20] obtained the transmitter depth. The direction method does not depend on the path loss of the positioning signal. Therefore, this method has higher accuracy than the path method. However, the direction method usually needs to collect positioning signals in different locations. The operation of this method is complex.

To reduce the positioning error and operation complexity, we propose a dual-frequency magnetic induction through-the-earth positioning method. The transmitter of the method is a loop energized coil placed horizontally in the mine. The receiver on the ground is an orthogonal three-axis magnetic core coil. There are two sinusoidal currents in the transmitter coil. The currents induce two sinusoidal magnetic induction positioning signals. The signals penetrate the earth media and reach the ground. The earth media usually does not contain ferromagnetic materials such as iron, cobalt, nickel and so on, which can alter the earth geomagnetic permeability. The receiver can measure the direction and amplitude of the positioning signals. On this basis, we first obtain the transceiver distance function by substituting the dual-frequency signal strength into the transceiver distance function, we get the value of the transceiver distance. Then, by using the receiver tilt angle and the rotation matrix, we correct the receiver tilt. Finally, we get the coordinates of the transmitter by decomposing the transceiver distance.

Specifically, to improve the positioning accuracy, we eliminate the earth conductivity in the transceiver distance function. This operation removes the positioning error caused by the earth skin effect. To reduce the positioning error caused by the tilted receiver, we use the rotation matrix to correct the receiver tilt. We determine the direction of the positioning signal by using the phase synchronization of the dual-frequency signal. This operation eliminates the non-unique positioning results caused by the alternating of the positioning signal vector.

The remainder of this paper is organized as follows. Section II introduces the composition and the basic principle of the dual-frequency positioning method. In section III, the transceiver distance is determined by using the
The positioning signal vector $\mathbf{B}_{Qi}$, $\mathbf{B}_{Qi}$, and $\mathbf{B}_{Qi}$ at the receiver vary with the transceiver azimuth. Therefore, using the directions and amplitudes of $\mathbf{B}_{Qi}$, $\mathbf{B}_{Qi}$, and $\mathbf{B}_{Qi}$, we can determine the transmitter position.

According to the electromagnetic field theory, the transmitter coil can be regarded as a magnetic dipole. The positioning signal vector can be expressed in the rectangular coordinate system and the spherical coordinate system. In Fig. 2, we first establish the rectangular coordinate system 1 with point $O$ as the origin, $X_1$ axis, $Y_1$ axis, and $Z_1$ axis as the axes. Then, we establish a spherical coordinate system with point $O$ as the origin and the $Z_1$ axis as the central axis. The transmitter is at point $O$. The receiver is at point $Q$. The rectangular coordinates and the spherical coordinates of point $Q$ are $(x, y, z)$ and $(r, \theta, \phi)$, respectively. The peak value of the through-the-earth positioning signal at receiver $Q$ is [21], [22]

$$|\mathbf{B}_{Qi}| = \frac{n\mu_0 R^2}{4\pi^3} \cdot (e_r 2 \cos \theta + e_\theta \sin \theta) \cdot e^{-\sqrt{\mu_0 \sigma}r},$$

where $e_r$ and $e_\theta$ are the unit distance vector and the unit elevation vector of $\mathbf{B}_{Qi}$ in the spherical coordinate system, respectively. $r$ is the length of $OQ$, $\theta$ is the degree of $\angle Z_1 O Q$, $R$, $n$, and $I$ represent the radius, the number of turns, and the current of the transmitter coil, respectively. $f_i$ is the frequency of the dual-frequency signal, $i = 1, 2$. In this paper, $f_1$ and $f_2$ are set as 8Hz and 12Hz, respectively. $\sigma$ is the conductivity of the earth media. $\mu_0$ is the permeability of the earth media.

To obtain the components of $\mathbf{B}_{Qi}$, we decompose $\mathbf{B}_{Qi}$ in the horizontal direction and the vertical direction. As shown in [20], the values of the horizontal component $|\mathbf{B}_{Qi}|$ and the vertical component $|\mathbf{B}_{Qi}|$ are respectively

$$|\mathbf{B}_{Qi}| = \frac{n\mu_0 R^2}{4\pi^3} \cdot 3 \cos \theta \sin \theta \cdot e^{-\sqrt{\mu_0 \sigma}r}$$

$$|\mathbf{B}_{Qi}| = \frac{n\mu_0 R^2}{4\pi^3} \cdot \left(2 \cos^2 \theta - \sin^2 \theta\right) \cdot e^{-\sqrt{\mu_0 \sigma}r}. \quad (3)$$

We decompose the $\mathbf{B}_{Qi}$ on $X_1$ axis and $Y_1$ axis. The components of $\mathbf{B}_{Qi}$ are $\mathbf{B}_{Xi}$ and $\mathbf{B}_{Yi}$, respectively. Their
values are
\[
\begin{align*}
1 B_{Xi} &= 1 B_{Hi} \cdot \cos \phi \\
1 B_{Yi} &= 1 B_{Hi} \cdot \sin \phi.
\end{align*}
\]

Therefore, the values \((1 B_{Xi}, 1 B_{Yi}, 1 B_{Zi})\) of \(1 B_{Xi}, 1 B_{Yi},\) and \(1 B_{Zi}\) in coordinate system 1 are respectively
\[
\begin{align*}
1 B_{Xi} &= \frac{n \mu_0 R_2^2}{4 r^3} \cdot 3 \cos \theta \sin \theta \cos \phi \cdot e^{-\sqrt{\mu_0 \sigma} f r} \\
1 B_{Yi} &= \frac{n \mu_0 R_2^2}{4 r^3} \cdot 3 \cos \theta \sin \theta \sin \phi \cdot e^{-\sqrt{\mu_0 \sigma} f r} \\
1 B_{Zi} &= \frac{n \mu_0 R_2^2}{4 r^3} \cdot \left(2 \cos^2 \theta - \sin^2 \theta \right) \cdot e^{-\sqrt{\mu_0 \sigma} f r}.
\end{align*}
\]

In reality, there may be an angle between the three axes of the receiver and coordinate system 1. \((1 B_{Xi}, 1 B_{Yi}, 1 B_{Zi})\) and \((B_{Xi}, B_{Yi}, B_{Zi})\) are not the same. \(B_{Xi}, B_{Yi},\) and \(B_{Zi}\) are the value of \(B_{xi}, B_{yi},\) and \(B_{zi}\), respectively. To get the values of \(B_{Hi}\) and \(B_{Zi}\), we rotate the coordinates between coordinate system 1 and the receiver. We assume that the angle between coordinate system 1 and the receiver is \(\theta_{1r}\). According to the coordinate rotation method [23], \(B_{Qi} (B_{Xi}, B_{Yi}, B_{Zi})\) at the three-axis receiver is
\[
\begin{align*}
\begin{bmatrix}
B_{Xi} \\
B_{Yi} \\
B_{Zi}
\end{bmatrix}
&= \begin{bmatrix}
\cos (-\theta_{1r}) & \sin (-\theta_{1r}) & 0 \\
-\sin (-\theta_{1r}) & \cos (-\theta_{1r}) & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 B_{Xi} \\
1 B_{Yi} \\
1 B_{Zi}
\end{bmatrix} \\
&= \begin{bmatrix}
B_{Xi} \cos (-\theta_{1r}) + B_{Yi} \sin (-\theta_{1r}) \\
-B_{Xi} \sin (-\theta_{1r}) + B_{Yi} \cos (-\theta_{1r}) \\
B_{Zi}
\end{bmatrix}.
\end{align*}
\]

The value of \(B_{Hi}\) is
\[
\begin{align*}
|B_{Hi}| &= \sqrt{(B_{Xi})^2 + (B_{Yi})^2} \\
&= \sqrt{(B_{Xi})^2 + (B_{Yi})^2} \\
&= |1 B_{Hi}|.
\end{align*}
\]

Through (6)-(7), we can find that no matter what the value of \(\theta_{1r}\) is, \(B_{Zi} = B_{Zi}\) and \(|B_{Hi}| = |1 B_{Hi}|\) holds. This law means that the values of \(B_{Zi}\) and \(|B_{Hi}|\) are independent of \(\theta_{1r}\). When obtaining \(B_{Zi}\) and \(|B_{Hi}|\), we do not need to consider the direction of the horizontal two axes of the receiver. \(B_{Xi}, B_{Yi},\) and \(B_{Zi}\) can be directly used to get \(B_{Zi}\) and \(|B_{Hi}|\). By using this rule, we begin to locate the transmitter.

The workflow of the dual-frequency positioning method is shown in Fig. 3. \(\phi_1\) and \(\phi_2\) are the tilt angles of the receiver in Fig. 4. \(\alpha\) and \(\delta\) are the rotation angles of the rotation matrices \(r_{1z}\) and \(r_{1z}\) in (21) and (22). \(r_{1z}\) and \(r_{1z}\) can be used to correct the tilt of the receiver. \(\theta_{23}\) is the direction error of the positioning signal in Fig. 6. \(3 B_{Hi}\) is the positioning signal vector in coordinate system 3, \(i = 1, 2, 3\).

Next, we will implement the positioning method.

### III. DETERMINE THE TRANSEIVER DISTANCE

As shown in (5), when penetrating the earth media, the positioning signal attenuates due to the skin effect. Among the parameters that affect the attenuation rate of the positioning signal, \(\mu_0\) is usually the vacuum permeability \(\mu_0 = 4\pi \times 10^{-7}\text{H/m}\), the operating frequency of the transmitter, and the sensitivity of the receiver antenna are also stable, while \(\sigma\) varies with the soil, rock, water, and other media in the stratified earth. Affected by the change of \(\sigma\), the values of \(1 B_{Xi}, 1 B_{Yi},\) and \(1 B_{Zi}\) in (5) may be different in different earth media with the same depth. The transceiver distance determined by the path loss of the positioning signal will inevitably have an error.

To correct the positioning error, we propose a positioning method using dual-frequency signals. We first transform the expression of \(|B_{Qi}|\) and get the expression of \(\sigma\). Then, we substitute \(\sigma\) into the expression of \(|B_{Q2}|\). After the form transformation and simplification of \(|B_{Q2}|\), we get the transceiver distance \(r\) eliminating \(\sigma\). Till then, we eliminate the influence of the earth conductivity on the positioning accuracy. To prove that the dual-frequency positioning method is suitable for multi-layer earth media, we carry out the form transform to the skin effect factor \(e^{-\sqrt{\sigma} f r} \cdot \sqrt{3 \cos^2 \theta + 1} \cdot e^{-\sqrt{\mu_0 \sigma} f r}\).

We find that there is an equivalent \(\sigma\) in multi-layer earth media. The equivalent \(\sigma\) can also be eliminated. Therefore, the dual-frequency positioning method is suitable for multi-layer earth media. The steps to determine the transceiver distance are as follows.

We assume that the earth is a monolayer medium, the earth conductivity \(\sigma\) is a constant. The value of the positioning signal at point \(Q\) in Fig. 2 is obtained as
\[
|B_{Qi}| = \frac{n \mu_0 R_2^2}{4 r^3} \cdot \sqrt{3 \cos^2 \theta + 1} \cdot e^{-\sqrt{\mu_0 \sigma} f r}.
\]
In (8), both $\theta$ and $\sigma$ can affect the correspondence between $r$ and $|B_Q|$. The value of $\theta$ has been obtained in [20] as

$$\theta = \arctan \left( \frac{-3|B_z| + \sqrt{9|B_z|^2 + 8|B_H|^2}}{2|B_H|} \right)$$

$$= \arctan \left( \frac{-3(B_z) + \sqrt{9(B_z^2) + 8(B_x^2) + 8(B_y^2)}}{2\sqrt{(B_x^2) + (B_y^2)}} \right).$$

(9)

After obtaining the value of $\theta$, we continue to eliminate the influence of $\sigma$ on positioning accuracy. We substitute $i = 1$ into (8), then transform the form of (8), and get the earth conductivity as

$$\sigma = \left[ \ln \left( n\mu_0IR^2 \cdot \sqrt{3\cos^2\theta + 1} / \left( |B_{Q1}| \cdot 4r^3 \right) \right) \right]^2 \pi f_1 \mu_0 T^2.$$  

(10)

By substituting $i = 2$ and (10) into (8), we get the transceiver distance of eliminating $\sigma$ in the single-layer earth medium as

$$r = \left( \frac{n\mu_0IR^2}{4} \cdot \sqrt{3\cos^2\theta + 1} \right)^{\frac{1}{3}} \cdot \left( \frac{|B_{Q1}| \sqrt[3]{f_1}}{|B_{Q2}|} \right)^{\frac{1}{3}}.$$  

(11)

Now we extend the dual-frequency positioning method from single-layer earth medium to multi-layer earth media.

We first discuss the positioning method in the double-layer earth media. To make the calculation results consistent with the practical application scenario, we assume that the earth is divided into two layers with different conductivities. The depths of the earth media are $r_1$ and $r_2$, respectively. The conductivities of the earth media are $\sigma_1$ and $\sigma_2$, respectively. $r = r_1 + r_2$. When the positioning signal penetrates the two layers of the earth media and reaches the receiver $Q$, the signal strength is

$$|B_Q| = \frac{n\mu_0IR^2}{4r^3} \cdot \sqrt{3\cos^2\theta + 1} \cdot e^{-\sqrt{\pi f_1 \mu_0 \sigma_1} r_1} \cdot e^{-\sqrt{\pi f_1 \mu_0 \sigma_2} r_2}.$$  

(12)

We assume that $r_1$, $r_2$, $\sigma_1$, and $\sigma_2$ are arbitrary values. When there is an equivalent conductivity $\sigma$ in the double-layer earth media, the double-layer earth media can be simplified to a single-layer earth medium with conductivity $\sigma$. The condition for the existence of equivalent conductivity $\sigma$ is that

$$e^{-\sqrt{\pi f_1 \mu_0 \sigma_1} r_1} \cdot e^{-\sqrt{\pi f_1 \mu_0 \sigma_2} r_2} = e^{-\sqrt{\pi f_1 \mu_0 \sigma} r}$$

(13)

holds.

To verify the validity of (13), we simplify and deform (13) as

$$\sigma = \frac{\sigma_1 r_1^2 + 2\sqrt{\sigma_1 \sigma_2} r_1 r_2 + \sigma_2 r_2^2}{(r_1 + r_2)^2}.$$  

(14)

In (14), no matter what values of $r_1$, $r_2$, $\sigma_1$, and $\sigma_2$ are, the equivalent conductivity $\sigma$ always exists. The double-layer earth media can be simplified to a single-layer earth medium. Therefore, in the double-layer earth media, we can also use (11) to calculate the transceiver distance.

Multi-layer or even innumerable layers of earth media are similar to double-layer earth media. We assume that the conductivities of $n$-layer earth media are $\sigma_1$, $\sigma_2$, $\ldots$, $\sigma_n$, $n$ is infinite. As shown in (14), $\sigma_1$ and $\sigma_2$ can be replaced by an equivalent conductivity $\sigma_1.2$. After the two layers of earth media are approximated to one layer of earth medium, $\sigma_{1.2}$ and $\sigma_3$ can also be replaced by an equivalent conductivity. And so on, $\sigma_1$, $\sigma_2$, $\ldots$, $\sigma_n$ can be replaced by an equivalent conductivity. Therefore, in multi-layer earth media or even innumerable-layer earth media, we can use (11) to calculate the transceiver distance.

IV. DETERMINE THE POSITION OF THE TRANSMITTER

After obtaining the transceiver distance, we will determine the position of the receiver. We first correct the tilted receiver by using the coordinate rotation method. Then, by using the direction of the geomagnetic field, we determine the geographical orientation of the positioning position. Finally, we determine the three-dimensional coordinates of the transmitter by decomposing the positioning signal.

A. CORRECT THE TILT OF THE RECEIVER

In the through-the-earth positioning, the receiver may tilt during movement. The receiver tilt will lead to the direction deviation of the received positioning signal. The deviation will reduce the measurement accuracy of the transmitter position. Therefore, we need to correct the tilt of the receiver.

We use the coordinate rotation method to correct the tilt of the receiver. We first measure the two-dimensional tilt angle of the receiver by using the inclinometer. Then, we calculate the value of the rotation angle by using the geometric relationship between the two-dimensional inclination angle and the rotation angle. The rotation angle is the angle used in the coordinate rotation method. Finally, we establish the rotation matrix of the coordinate rotation method by using the rotation angle. After the coordinate transformation by using the rotation matrix, we can obtain the rotation angle.
After obtaining $\phi_1$ and $\phi_2$, we can correct the receiver tilt by the coordinate rotation method. The steps of tilt correction are as follows.

1. In the plane $Qx_1y_1$, rotate $Qx_1$ and $Qy_1$ counterclockwise by $\alpha$. At this point, $Qx_1$ is coincident with the $X_2$ axis. $\alpha$ is the angle between $Qx_1$ and $X_2$ axis.

2. Rotate the $Qx_1$ and $Qy_1$ counterclockwise by $\delta$ along the $X_2$ axis. At this point, the plane $Qx_1y_1$ coincides with the plane $Qx_2y_2$. $\delta$ is the angle between the plane $Qx_1y_1$ and the plane $Qx_2y_2$. After the two steps of tilt correction, $Qx_1$ and $Qy_1$ take a horizontal attitude.

However, the values of $\alpha$ and $\delta$ are unknown. To correct the receiver tilt by using the coordinate rotation method, we need to calculate the values of $\alpha$ and $\delta$ by using the angles of $\phi_1$ and $\phi_2$. In Fig. 4, $d_1$ and $d_2$ are the vertical projections of $x_1'$ and $y_1'$ on the $X_2$ axis, respectively. $\beta$ is the residual angle of $\alpha$. As shown in Fig. 4, we have

$$\begin{align*}
x_1x_1' &= Ox_1 \cdot \sin \phi_1 \\
d_1x_1 &= Ox_1 \cdot \sin \alpha.
\end{align*}$$

Therefore, $\delta$ can be shown as

$$\sin \delta = \frac{x_1x_1'}{d_1x_1} = \frac{\sin \phi_1}{\sin \alpha}. \quad (16)$$

Similarly, $\delta$ can also be shown as

$$\sin \delta = \frac{y_1y_1'}{d_2y_1} = \frac{\sin \phi_2}{\sin \beta}. \quad (17)$$

By combining (16) and (17), we get that

$$\sin \alpha = \frac{\sin \phi_1}{\sin \beta}, \quad \sin \beta = \frac{\sin \phi_2}{\sin \alpha}. \quad (18)$$

Since $\beta$ and $\alpha$ are complementary, we get that

$$\tan \alpha = \frac{\sin \phi_1}{\sin \phi_2}. \quad (19)$$

Therefore, the values of $\alpha$ and $\delta$ are respectively

$$\begin{align*}
\alpha &= \arctan \left( \frac{\sin \phi_1}{\sin \phi_2} \right) \\
\delta &= \arcsin \left( \frac{\sin \phi_1}{\sin \alpha} \right). \quad (20)
\end{align*}$$

After obtaining the values of $\alpha$ and $\delta$, we correct the receiver tilt according to the steps of the coordinate rotation method. As shown in Fig. 5, we first rotate $Qx_1$ and $Qy_1$ counterclockwise on the $Qz_1$ axis by $\alpha$. At this point, the $Qx_1$ is coincident with the $X_2$ axis, $Qx_1$, $Qy_1$, and $Qz_1$ are rotated to $Qx_1'$, $Qy_1'$, and $Qz_1'$. The rotation matrix $r_z1$ of this step is

$$r_z1 = \begin{bmatrix}
\cos (-\alpha) & \sin (-\alpha) & 0 \\
-\sin (-\alpha) & \cos (-\alpha) & 0 \\
0 & 0 & 1
\end{bmatrix}. \quad (21)$$

Then we rotate $Qx_1'$ and $Qy_1'$ counterclockwise on the $X_2$ axis by $\delta$. At this point, the plane $Qx_1'y_1'$ coincides with the plane $Qx_2y_2$. $Qx_1'$, $Qy_1'$, and $Qz_1'$ are rotated to $Qx_1''$, $Qy_1''$, and $Qz_1''$. The rotation matrix $r_x1$ of this step is

$$r_x1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \delta & \sin \delta \\
0 & -\sin \delta & \cos \delta
\end{bmatrix}. \quad (22)$$

As shown in Fig. 5, after the correction, the receiver axes $Qx_1''$, $Qy_1''$, and $Qz_1''$ have been coincident with the axes of coordinate system 2. At this time, the $Qx_1$ and $Qy_1$ are parallel to the horizontal ground.

When the receiver tilts, the positioning signal received by the receiver will shift accordingly. To reduce the positioning error, we will correct the direction of the positioning signal. We assume that the positioning signal vector at the tilted receiver is $B_{Hi}$ ($B_{dHi}$, $B_{sHi}$, $B_{zHi}$). After the coordinate transformation is carried out by using the rotation matrices $r_z1$ and $r_x1$, $B_{Hi}$ is rotated as $2B_{Hi}$ ($2B_{dHi}$, $2B_{sHi}$, $2B_{zHi}$) in coordinate system 2. The $2B_{Hi}$ is

$$2B_{Hi} = \begin{bmatrix}
2B_{dHi} \\
2B_{sHi} \\
2B_{zHi}
\end{bmatrix} = r_x1 \cdot r_z1 \cdot B_{Hi}. \quad (23)$$

where $2B_{dHi}$, $2B_{sHi}$, and $2B_{zHi}$ are the three components of $2B_{Hi}$ in coordinate system 2, respectively. $2B_{dHi}$, $2B_{sHi}$, and $2B_{zHi}$ are the values of $2B_{dHi}$, $2B_{sHi}$, and $2B_{zHi}$, respectively.

**B. DETERMINE THE GEOGRAPHICAL DIRECTION OF THE POSITIONING SIGNAL**

As shown in (23), when correcting the receiver tilt, we need to rotate $Qx_1$ and $Qy_1$ in Fig. 4. After that, $2B_{dHi}$, $2B_{sHi}$, and $2B_{zHi}$ will deviate from the three axes of the receiver. In this case, the staff can not determine the geographical direction of the positioning signal. As a result, the staff can not determine the geographical location of the transmitter. To obtain the
The geographical direction of the positioning signal, we need the assistance of the geomagnetic direction. The geomagnetic direction can be obtained by the three-dimensional compass. We assume that the three magnetic sensors of the three-dimensional compass are parallel to the three axes of the receiver. The direction vector of the geomagnetic field received by the compass is \( \mathbf{B}_S (B_{xS}, B_{yS}, B_{zS}) \). The geographical direction of the positioning signal can be obtained by using \( \mathbf{B}_S \).

The process of obtaining the geographical direction of the positioning signal is as follows.

We first correct the tilt of \( \mathbf{B}_S \). After the receiver tilts, \( \mathbf{B}_S \) will tilt accordingly. To revise \( \mathbf{B}_S \), we transform \( \mathbf{B}_S \) into coordinate \( \mathbf{B}_S (2B_{xS}, 2B_{yS}, 2B_{zS}) \) in coordinate system 2 by using the rotation matrices \( r_1 \) and \( r_2 \).

\[
\mathbf{B}_S (2B_{xS}, 2B_{yS}, 2B_{zS}) = r_1 \cdot r_2 \cdot \mathbf{B}_S. \tag{24}
\]

Then, to obtain the geographical direction of the positioning signal, we will establish the coordinate system \( \mathcal{N}(X_3, Y_3, Z_3) \) corresponding to the geographical direction. In Fig. 6, the \( X_3 \) axis points to the geographical south pole. The \( Z_3 \) axis is straight up. To rotate the vector \( \mathbf{B}_{Hi} \) to coordinate 3, we need to determine the angle \( \theta_{23} \) between the \( X_3 \) axis and the \( X_2 \) axis. As shown in Fig. 6, \( \theta_{23} \) consists of \( \theta_{H3} \) and \( \theta_{H2} \). \( \theta_{H3} \) is the angle between the \( X_3 \) axis and \( \mathbf{B}_{HS} \). \( \mathbf{B}_{HS} \) is the horizontal component of \( \mathbf{B}_S \). \( \theta_{H2} \) is the angle between the \( X_2 \) axis and \( \mathbf{B}_{HS} \). After obtaining the \( \theta_{H3} \) and \( \theta_{H2} \), we can rotate the vector \( \mathbf{B}_{Hi} \) counterclockwise to coordinate system 3 by \( \theta_{23} \).

\( \theta_{H3} \) can be regarded as a constant when the time and the place are determined. \( \theta_{H2} \) needs to be solved in coordinate system 2. As shown in Fig. 7, \( -\mathbf{B}_{HS} \) is located in the \( X_2Y_2 \) plane. The components of \( -\mathbf{B}_{HS} \) on the \( X_2 \) axis and \( Y_2 \) axis are \( -B_{xS} \) and \( -B_{yS} \), respectively. Within the range of \( \theta_{H2} \in [0, 2\pi] \), \( \sin \theta_{H2} \) and \( \cos \theta_{H2} \) generally correspond to multiple values. To uniquely determine the value of \( \theta_{H2} \), we calculate the sine and cosine of \( \theta_{H2} \) at the same time. The formula is as follows:

\[
\begin{align*}
\cos \theta_{H2} &= \frac{-2B_{xS}}{\sqrt{(-2B_{xS})^2 + (-2B_{yS})^2}} \\
\sin \theta_{H2} &= \frac{-2B_{yS}}{\sqrt{(-2B_{xS})^2 + (-2B_{yS})^2}}.
\end{align*} \tag{25}
\]

The solutions are

\[
\theta_{H2} = \arccos \left( \frac{-2B_{xS}}{\sqrt{(-2B_{xS})^2 + (-2B_{yS})^2}} \right) \quad \text{and} \quad \arcsin \left( \frac{-2B_{yS}}{\sqrt{(-2B_{xS})^2 + (-2B_{yS})^2}} \right). \tag{26}
\]

Among the multiple values of \( \theta_{H2} \), the value established by (26) is the actual value of \( \theta_{H2} \).

After getting the values of \( \theta_{H3} \) and \( \theta_{H2} \), we can rotate \( \mathbf{B}_{Hi} \) counterclockwise to coordinate system 3. According to (21), the rotation matrices that rotate \( \mathbf{B}_{Hi} \) to coordinate system 3 are respectively

\[
\begin{align*}
r_{\theta H3} &= \begin{bmatrix}
\cos(\theta_{H3}) & \sin(\theta_{H3}) \\
-\sin(\theta_{H3}) & \cos(\theta_{H3})
\end{bmatrix}, \quad \text{and} \quad \begin{bmatrix}
0 & 0 \\
0 & 1
\end{bmatrix} \\
r_{\theta H2} &= \begin{bmatrix}
\cos(\theta_{H2}) & -\sin(\theta_{H2}) \\
\sin(\theta_{H2}) & \cos(\theta_{H2})
\end{bmatrix}.
\end{align*} \tag{27, 28}
\]

After the rotation, \( \mathbf{B}_{Hi} \) is changed to \( \mathbf{B}_{Hi} (3B_{xHi}, 3B_{yHi}, 3B_{zHi}) \) in coordinate system 3. The \( \mathbf{B}_{Hi} \) is

\[
3\mathbf{B}_{Hi} = r_{\theta H3} \cdot r_{\theta H2} \cdot 2\mathbf{B}_{Hi}. \tag{29}
\]

C. DETERMINE THE POSITION OF THE TRANSMITTER

After the tilt correction and direction correction of the positioning signal, we continue to determine the transmitter position. Using the corrected positioning signal, we first recalculate the transceiver distance. Then, in the right triangle containing the transceiver distance, we decompose out the depth coordinate of the transmitter by using the trigonometric
function. Finally, using the ratio between the two horizontal components of the positioning signal, we calculate the horizontal coordinates of the transmitter. The process of positioning is as follows.

As shown in Fig. 8, the transmitter position can be expressed as the coordinate \((\hat{x}_O, \hat{y}_O, \hat{z}_O)\) of point \(O\) in coordinate system 3. The receiver is located at the origin \(Q\) of the coordinate system 3. The transmitter is located at point \(O\). \(\mathbf{B}_{\text{Hi}}\) is the positioning signal vector at point \(Q\). \(\mathbf{B}_{\text{Hi}}\) is the horizontal component of \(\mathbf{B}_{\text{Hi}}\), \(\|\mathbf{B}_{\text{Hi}}\| = \sqrt{3B_{z\text{Hi}}^2 + 3B_{y\text{Hi}}^2}\). Point \(C\) is the intersection of the transmitter coil axis and the \(X_3QY_3\) plane. \(\theta\) is the angle between \(OQ\) and \(OC\), \(r\) is the length of \(OQ\). \(\theta\) and \(r\) in Fig. 6 are the same as in Fig. 3.

The \(\hat{z}_O\) coordinate of point \(O\) in coordinate system 3 can be determined by using \(\theta\) and \(r\). To reduce the positioning error, we replace \(\mathbf{B}_{\text{Hi}}\) and \(\mathbf{B}_{\text{Hi}}\), and \(\mathbf{B}_{\text{Hi}}\) in (7) with the corrected tilted \(\mathbf{B}_{\text{Hi}}\), \(\mathbf{B}_{\text{Hi}}\), and \(\mathbf{B}_{\text{Hi}}\). By recalculating (9), we get the value of \(\theta\) as

\[
\theta = \arctan \left( \frac{-3(3\mathbf{B}_{z\text{Hi}}) + \sqrt{9(3\mathbf{B}_{z\text{Hi}})^2 + 8(3\mathbf{B}_{y\text{Hi}})^2 + 8(3\mathbf{B}_{y\text{Hi}})^2}}{2\sqrt{(3\mathbf{B}_{z\text{Hi}})^2 + (3\mathbf{B}_{y\text{Hi}})^2}} \right) \tag{30}
\]

After replace \(\mathbf{B}_{Q1}\) and \(\mathbf{B}_{Q2}\) in (11) with \(\mathbf{B}_{\text{Hi}}\) and \(\mathbf{B}_{\text{Hi}}\), we get the value of \(r\) as

\[
r = \left( \frac{n\mu_0 IR^2}{4 \cdot \sqrt{3 \cos^2\theta + 1}} \right)^{\frac{1}{2}} \cdot \left( \frac{\mathbf{B}_{\text{Hi}}}{\sqrt{3\mathbf{B}_{\text{Hi}}}} \right) \cdot \frac{1}{\sqrt{3\mathbf{B}_{\text{Hi}}}} \cdot \frac{1}{\sqrt{\mathbf{B}_{\text{Hi}}}} \tag{31}
\]

In the right triangle \(OCQ\), we get the values of \(OQ\) and \(\angle COQ\). Using the trigonometric function, we can decompose the component \(CO\) of \(\mathbf{OQ}\) in the \(Z_3\) direction. The coordinate \(\mathbf{z}_O\) of point \(O\) is the opposite number of \(|\mathbf{CO}|\), which is

\[
\mathbf{z}_O = -\|\mathbf{CO}\| = -r \cdot \cos \theta \tag{32}
\]

The \(\hat{x}_O\) and \(\hat{y}_O\) coordinates of point \(O\) can be determined by \(\mathbf{B}_{\text{Hi}}\). As shown in [20], \(\mathbf{B}_{O1}\) in (2) only contains the \(e_1\) component and \(e_3\) component. The \(e_1\) component of \(\mathbf{B}_{O1}\) is 0. \(e_3\) is the unit azimuth vector of the spherical coordinate system. Therefore, the straight line where \(\mathbf{B}_{\text{Hi}}\) is located passes through the axis of the transmitter coil. \(\mathbf{B}_{\text{Hi}}\) is the horizontal component of \(\mathbf{B}_{O1}\). The \(\mathbf{B}_{O1}\) in (2) and \(\mathbf{B}_{\text{Hi}}\) in Fig. 6 are the same. Therefore, the straight line where \(\mathbf{B}_{\text{Hi}}\) is located passes through point \(C\). \(\mathbf{B}_{\text{Hi}}\) and \(\mathbf{B}_{\text{Hi}}\) are collinear. Therefore, there is a functional relationship between the coordinates of point \(C\) and the coordinates of \(\mathbf{B}_{\text{Hi}}\). That is

\[
\frac{\hat{x}_C}{\hat{y}_C} = \frac{\mathbf{B}_{z\text{Hi}}}{\mathbf{B}_{y\text{Hi}}} \tag{33}
\]

Because \(CO/\|Z_3\|, \hat{x}_C = \hat{x}_O, \hat{y}_C = \hat{y}_O\). Therefore, there is also a functional relationship between the coordinates of point \(O\) and the coordinates of \(\mathbf{B}_{\text{Hi}}\). That is

\[
\frac{\hat{x}_O}{\hat{y}_O} = \frac{\mathbf{B}_{z\text{Hi}}}{\mathbf{B}_{y\text{Hi}}} \tag{34}
\]

Using the trigonometric function, we have

\[
\sqrt{3\hat{x}_O^2 + 3\hat{y}_O^2} = r \cdot \sin \theta \tag{35}
\]

Combining (34) and (35), we solve \(\hat{x}_O\) and \(\hat{y}_O\) respectively as

\[
\begin{align*}
\hat{x}_O &= -\frac{\mathbf{B}_{z\text{Hi}} \cdot r \cdot \sin \theta}{\sqrt{(3\mathbf{B}_{z\text{Hi}})^2 + (3\mathbf{B}_{y\text{Hi}})^2}} \quad (0 \leq \theta <\frac{\pi}{2}) \tag{36} \\
\hat{y}_O &= -\frac{\mathbf{B}_{y\text{Hi}} \cdot r \cdot \sin \theta}{\sqrt{(3\mathbf{B}_{z\text{Hi}})^2 + (3\mathbf{B}_{y\text{Hi}})^2}} \\
\end{align*}
\]

\(\hat{x}_O, \hat{y}_O\) in (36) and \(\hat{z}_O\) in (32) together constitute the transmitter coordinates in coordinate system 3.

D. EXCLUDING THE NON-UNIQUENESS OF THE POSITIONING RESULTS

Since the positioning signal used in this paper is sinusoidal, the direction of the signal vector is changing. During positioning, the direction change of the positioning signal vector will lead to the non-uniqueness of the positioning results. For example, \(\mathbf{B}_{\text{Hi}}\) in Fig. 8 may point in the opposite direction. Since there are two directions in \(\mathbf{B}_{\text{Hi}}\), two calculation results will appear in (30). One of the results is wrong. To eliminate this error, we need to determine the direction of the positioning signal.

We set the initial direction at the 0 phase as the positive direction of the positioning signal. The initial phase of the positioning signal is 0. The frequencies of the dual-frequency signals are \(f_1\) and \(f_2\), respectively. \(f_2 = 1.5f_1\). As shown in Fig. 9, only when the phase of signal \(f_1\) is 0, the phases of the two positioning signals are the same. Therefore, we can use the signal direction at the 0 phase to determine the positive
direction of the signal. With this method, we rule out the non-unique positioning results.

V. SIMULATION RESULT

We will simulate the positioning accuracy and the positioning distance of the dual-frequency positioning method. In the simulation of the positioning accuracy, we will discuss the positioning errors caused by various factors. The factors include the signal-frequency signal, the dual-frequency signal, the receiver tilt, and the transceiver azimuth. After that, we will give the optimal configuration of the positioning method in terms of positioning accuracy. When simulating the positioning distance, we will analyze the positioning distance varying with the transmitter coil parameters. The parameters include the radius \( R \), the turns \( n \), and the current \( I \) of the transmitter coil. In this way, we can configure appropriate transmitter parameters under different mine depths.

The initial parameters of the positioning method are as follows. Since the underground space is limited, the maximum radius \( R \) of the transmitter coil is set as 2m. The turns \( n \) and effective current \( I \) of the transmitter coil are 3300 and 4A, respectively. In the earth media without ferromagnetic substances such as iron, cobalt, and nickel, the earth permeability can be set as the vacuum permeability \( \mu_0 = 4\pi \times 10^{-7} \) H/m. The conductivity \( \sigma \) of the earth media is set to \( 5 \times 10^{-3} \) S/m. The frequencies \( f_1 \) and \( f_2 \) of the dual-frequency signals are 8Hz and 12Hz, respectively.

A. THE SIMULATION OF THE POSITIONING ACCURACY

When simulating the positioning accuracy, we first discuss the positioning errors of the single-frequency positioning signal and dual-frequency positioning signal. In the layered earth media, the conductivity \( \sigma \) is not a fixed constant. The attenuation rate of the magnetic induction positioning signal is different in different earth media. Using the path loss law of the single-frequency positioning signal for positioning will cause an error.

The single-frequency positioning method includes two cases: considering the skin effect and ignoring the skin effect. The single-frequency positioning method considering the skin effect uses (5) to simulate the positioning error caused by the path loss law. We first use (5) to simulate the error of the single-frequency method considering the skin effect. After removing the \( e^{-\sqrt{\pi f_i \mu_0 \sigma}} \) in (5), we reuse (5) to simulate the error of the single-frequency method ignoring the skin effect. Because ignoring the skin effect further deviates from the actual situation of the earth media, the positioning method ignoring the skin effect has a higher error than the positioning method considering the skin effect.

In (11), we use the dual-frequency positioning signal to eliminate the earth conductivity \( \sigma \) in (5). Since the change of \( \sigma \) cannot affect the result of (11), the dual-frequency positioning method has no error in Fig. 10.

The tilt of the receiver will also affect the positioning accuracy. After the receiver tilts, the positioning signal vector received by the receiver will deviate. The positioning result will variate accordingly. As shown in Fig. 11, the greater the receiver tilt, the greater the positioning error. After correcting the receiver tilt, the positioning error caused by the tilted receiver disappears.

We continue to evaluate the influence of the transceiver azimuth on the positioning accuracy. To simulate the environment of the mine accident site, we add magnetic noise to the positioning signal. The noise is collected by the magnetometer LEMI-30. The noise level of LEMI-30 is less than 0.1pT/\( \sqrt{\text{Hz}} \). By using the noise reduction method in [20], the noise amplitude can be reduced to \( 1.5 \times 10^{-5} \) nT. The noise will lead to the direction deviation and amplitude error of the positioning signal. The positioning result will deviate accordingly.
FIGURE 12. The positioning error varies with the distance and angle of the transceivers.

As shown in Fig. 12, with the increase of the transceiver distance, the positioning error increases accordingly. With $\theta$ increases in Fig. 8, the positioning error increases, too. Therefore, reducing the transceiver distance can effectively reduce the positioning error. Similarly, by bringing the receiver close to the top of the transmitter, we can also reduce the positioning error.

B. THE SIMULATION OF THE POSITIONING DISTANCE

In addition to the positioning accuracy, we will analyze the positioning distance. The farther the positioning distance is, the higher the search and rescue efficiency is. Therefore, the positioning distance is also worth considering. The positioning distance includes the signal discovery distance and the precise positioning distance. The signal discovery distance is the farthest transceiver distance at which the positioning signal can be found. The precise positioning distance is the transceiver distance that can achieve high-precision positioning. We will analyze the positioning distance varies with the parameters of the transmitter coil. Based on this, we can configure the appropriate transmitter coil parameters in mines with different depths. In this way, we can keep a balance between the search and rescue efficiency, the positioning accuracy, and the deployment difficulty.

We first discuss the signal discovery distance. With the increase of the transceiver distance, the positioning signal strength at the receiver decreases gradually. When the transceiver distance is too far, the positioning signal will be submerged in the magnetic noise. At this time, the receiver can not find the positioning signal. To distinguish the positioning signal, the positioning signal strength should be higher than the noise strength. In Figs. 13-15, we perform Discrete Fourier Transform on the noise and noise-doped signals respectively and get their spectrograms. As shown in Fig. 13, the noise amplitude is less than $3.5 \times 10^{-4} \text{nT}$ in the range of 7Hz-9Hz. In Fig. 14, the receiver can detect a positioning signal of three times the noise amplitude. We define the transceiver distance currently as the signal discovery distance.

After finding the positioning signal, we can gradually approach the transmitter according to the signal strength. As shown in Fig. 15, when the signal strength exceeds 130 times the noise, the positioning signal is significant in the spectrum. According to the simulation results in Fig. 12, the positioning error in Fig. 15 is less than 5m. At this time, the transceiver distance is the precise positioning distance.

The signal strength of the precise positioning distance is much higher than the signal discovery distance. So, the two positioning distances are very different. To adapt the mines with different depths, we analyze the variation of the two positioning distances with the transmitter coil parameters in Fig. 16. The parameters include radius $R$, turns $n$, and effective current $I$. We use the Scale to reduce the initial parameters of the transmitter coil proportionally. That is, the real parameter of the transmitter coil is the product of the initial parameter and the Scale. In one curve, one of $R$, $n$, and $I$ changes with Scale, the other two remain at their initial
values. The Scale changes from 1 to 0.1. The initial radius $R$ of the transmitter coil is 2m, the initial turns $n$ are 3300, and the effective value of the initial current $I$ is 4A.

In Fig. 16, when Scale $= 1$, the precise positioning distance(PPD) is 1000m, the signal discovery distance(SDD) is 3736m. The signal discovery distance is much greater than the precise positioning distance. When designing the positioning method, the accurate positioning distance only needs to be slightly larger than the mine depth. At this time, the rescuers can still find the positioning signal in a distant place. In this way, we can not only ensure the search and rescue efficiency and positioning accuracy, but also reduce the volume and power consumption of the transmitter, which reduces the difficulty of equipment deployment.

In reality, the depth of the mine is generally less than 1000m. In the layered mines, the distance between roadways may be only a few hundred meters. In this case, we can appropriately reduce the specification of the transmitter coil. As shown in Fig. 16, both the signal discovery distance and the precise positioning distance decrease with the Scale. When Scale $= 0.1$, $R = 0.2m$, $n = 330$, $I = 0.4A$. The corresponding precise positioning distances of $R$, $n$, and $I$ are 216m, 465m, and 465m. If the mine depth is less than 200m, the reduced transmitter coil can still achieve accurate positioning. In this case, the size and power consumption of the transmitter is greatly reduced. The difficulty of deployment is also greatly reduced. As shown in Fig. 16, $R$ has a greater influence on the positioning distance than $n$ and $I$. By giving priority to reducing $n$ and $I$, we can ensure the through-the-earth positioning distance of the positioning method. In different mine depths and underground environments, we can also flexibly configure the transmitter coil parameters. In this way, we can keep a balance between the search and rescue efficiency, the positioning accuracy, and the deployment difficulty.

VI. CONCLUSION
We propose a dual-frequency magnetic induction through-the-earth positioning method. By the dual-frequency positioning signal, we obtain the transceiver distance that is not affected by the layered earth media. Through the two-dimensional tilt angle and the rotation matrix, we correct the receiver tilt. After the decomposition of the transceiver distance, we get the three-dimensional coordinates of the transmitter. The findings of this paper are as follows. Using the dual-frequency positioning signal, we can eliminate the positioning error caused by the path loss in the layered earth media. By correcting the receiver tilt, we can also reduce the positioning error. Keeping the receiver close to the top of the transmitter can improve the positioning accuracy. Reducing the transceiver distance can also improve the positioning accuracy. By making full use of the signal discovery distance and the precise positioning distance of the positioning signal, we can achieve high-precision positioning with limited transmitter size. When reducing the transmitter size, we should try our best to keep the transmitter coil radius.

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