Research on Criterions of Hydraulic Fracturing in Earth Core Rockfill Dams

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Abstract. Hydraulic fracturing is an important destruction mode in the earth core rockfill dam engineering. In this paper, to explicit the mechanism of hydraulic fracturing and thus to verify the existing criterions, hydraulic fracturing tests were conducted based on true triaxial apparatus. Additionally, the precision of initial fracturing water pressure calculated by existing criterions is compared respectively. It is proved that the tensile strength and fracture initiation pressure pf all increase with the increasing of dry density. pf shows linear relationship with the minor principal stress σ3. The crack of hydraulic fracturing existed perpendicular to the minor principal stress plane. In addition, for T-S criterion, the predicted values are all less than the test data. On the contrary, most of the predicted values from M-C criterion are greater than the test data. Considering the safety of ECRD engineering, T-S criterion is suggested to predict the occurrence of hydraulic fracturing.

1. Introduction
In recent years, earth core rockfill dams (ECRDs) are developing rapidly all over the world, especially in China, and a number of high ECRDs which reach 200 m or even 300 m have been planned, designed or constructed [1]. Hydraulic fracturing is a physical phenomenon that cracks occur and propagate in the core material due to the elevation of water pressure. Gravelly soils are often used as the core material to improve the modulus of the core, thus to reduce the possibility of hydraulic fracturing [2-3]. However, the modulus of the core is still much lower than that of the dam shell, and the arch effect is still significant. Hydraulic fracturing of core wall is critical to dam safety, and the consequences will be catastrophic. In general, hydraulic fracturing is still a prominent problem in the design of ECRDs, and is an urgent problem to be further studied.

Yet, some progress has been made focusing on the mechanism of hydraulic fracturing [4-5], however, the criterion of hydraulic fracturing is still immature. For low dams, the possibility of hydraulic fracturing is relatively low, and the anti-crack design is not very concerned. For high CFRDs, the arch effect is strong and the hydraulic fracturing needs to be further predicted. For this purpose, some researchers presented criterions to calculate initial fracturing water pressure so as to predict the occurrence of hydraulic fracturing. Nevertheless, it is still doubtful which criterion can meet the requirement of high CFRD design, and the reliability of the existing criterions needs to be further verified.
In this paper, to explicit the mechanism of hydraulic fracturing and thus to verify the existing criterions, hydraulic fracturing tests were conducted based on true triaxial apparatus. Additionally, the precision of initial fracturing water pressure calculated by existing criterions is compared respectively.

2. The mechanism and criterions of hydraulic fracturing

2.1. The mechanism of hydraulic fracturing
Currently, the mechanism of hydraulic fracturing in terms of ECRD has been generally recognized, and the mechanism called arching effect has been accepted by most researchers and engineers. The proposed mechanism considers that the core cannot be constructed completely homogeneous during filling period, and there should exist different size of permeable weak surface/block which is shown as region ABC in Fig. 1. The permeability of these surface/block is larger than that of the most surrounding core materials. As the upstream water level rises rapidly, the pressure water enters into the ABC weak surface/block. As a consequent, there will exist the upward water pressure on AC surface and downward water pressure on BC surface. If the water pressure is large enough and the crack resistance of the core material is poor, cracks may occur at point C and the high-pressure water will trigger crack propagation. Thus, the hydraulic fracturing happens.

![Figure 1. Sketch diagram of hydraulic fracturing in the ECRD.](image)

2.2. The criterions of hydraulic fracturing
In order to predict the occurrence of hydraulic fracturing, some researchers have studied the criterions of hydraulic fracturing based on the cavity expansion theory [6-8]. As can be seen in Fig. 2, it is assumed that there is a small circular hole with a radius of \( r_0 \) in the center of a long rectangular cylinder with a radius of \( r_a \). The upper and lower sides of the cylinder are under uniform pressure \( \sigma_3 \), the left and right sides are under uniform pressure \( \sigma_1 \). The inner wall of the circular hole is under uniform pressure \( p \).
Yu [9] has deduced the stress formula of the cylinder based on the elastic theory.

\[
\sigma_r = \frac{r_u^2}{r^2} u_a + \frac{\sigma_2 + \sigma_3}{2} \left(1 - \frac{r_u^2}{r^2}\right) + \frac{\sigma_2 - \sigma_3}{2} \cdot \cos 2\theta \left(1 - \frac{r_u^2}{r^2}\right) \left(1 + 3 \frac{r_u^2}{r^2}\right)
\]

\[
\sigma_\theta = -\frac{r_u^2}{r^2} u_a + \frac{\sigma_2 + \sigma_3}{2} \left(1 + \frac{r_u^2}{r^2}\right) - \frac{\sigma_2 - \sigma_3}{2} \cdot \cos 2\theta \left(1 + 3 \frac{r_u^2}{r^2}\right)
\]

\[
\tau_{r\theta} = \tau_{\theta r} = -\frac{\sigma_2 - \sigma_3}{2} \cdot \sin 2\theta \left(1 - \frac{r_u^2}{r^2}\right) \left(1 + 3 \frac{r_u^2}{r^2}\right)
\]

(1)

where \(u_a\) is the water pressure applying on the internal surface of the cavity, \(\sigma_r\) is radial stress, \(\sigma_\theta\) is circumference stress, and \(\tau_{r\theta}\) is shear stress.

Based on the formula, Zhang [10] deduced fracture initiation pressure by assuming that the hydraulic fracturing failure meets the tensile failure criterion (T-S criterion). The formula is given by:

\[
p_f = 3\sigma_3 - \sigma_1 + \sigma_r
\]

(2)

Where \(p_f\) is the fracture initiation pressure, and \(\sigma_3\) is the tensile failure of the core material.

According to the failure mechanism of the soil, tensile failure and/or shear failure may happen when the soil is under loading. Eq. (2) is presented based on tensile failure criterion. On the other hand, shear failure criterion may be applicable, and it is valuable to predict the occurrence of hydraulic fracturing based on shear failure criterion. Wang [11] deduced an equation to calculate the fracture initiation pressure by assuming that Mohr-Coulomb shear failure (M-C criterion) happens when hydraulic fracturing occurs. The equation is given by

\[
p_f = \frac{(3\sigma_3 - \sigma_2)}{2} (1 + \sin \varphi) + c \cos \varphi
\]

(3)

3. Test and verification of the criterions

3.1. Apparatus and test procedure

To verify the T-S and M-C criterions deduced by Zhang and Wang respectively. Hydraulic fracturing tests were conducted based on the true triaxial apparatus. The specimen is a cuboid with the size of 70mm×70mm×35mm and it was compacted inside a mould in five layers. There was a borehole in \(\sigma_1\) direction (axial direction) with the diameter of 8mm and the depth of 50mm. Coarse sand was filled into the borehole through the copper tube and was compacted, as is shown in Fig.3. Pressured water was injected into the sand of specimen to initiate hydraulic fracturing.
The soil used in the test was from the core of an actual ECRD which refers to a clay-gravel mixture (gravelly soil). The water content of the samples was 17.3%, four different dry densities were chosen: $\rho_d = 1.60$ g/cm$^3$, $\rho_d = 1.65$ g/cm$^3$, $\rho_d = 1.72$ g/cm$^3$, $\rho_d = 1.76$ g/cm$^3$.

Each specimen was tested under four stress states, i.e. $\sigma_1$, $\sigma_2$ and $\sigma_3$ are (100 kPa, 75 kPa, 50 kPa), (200 kPa, 150 kPa, 100 kPa), (300 kPa, 225 kPa, 150 kPa), (400 kPa, 300 kPa, 200 kPa) respectively. The specimen was first consolidated under those stress states. After consolidation, water pressure was increased until the fracture occurred in the specimen. During the test, the water pressure was increased by a rate of 10 kPa/min. When the water pressure reaches a maximum value and then drops rapidly, hydraulic fracturing is believed to occur. The water pressure is called as fracture initiation pressure $p_f$.

3.2. Verification of the criterions

A total of 16 specimens were conducted by a true triaxial apparatus under different three-dimensional stress states. During the process of gradually increasing the water pressure, it can be seen that the water level of the water column was decreasing slowly. When the water pressure in the inner chamber increased to a certain value, the water level of the column in the inner tube dropped rapidly. At the same time, a large amount of turbid water was discharged from the drain pipe communicating with the bottom of the sample, which indicates that the sample is cracked. It can be seen that there was an obvious crack in the position along the minor principal stress surface of the sample, and this part of the soil is soft relatively. From the experiment phenomenon, it is proved that the hydraulic crack was generated on the surface perpendicular to the direction of the minor principal stress.

The fracture initiation pressure as well as the tensile strength of specimens under different dry densities are listed in Table 1. As can be seen in Table 1, it can be seen that the fracture initiation pressure increases with the increasing dry density, as well as the tensile strength.

| Fracture initiation pressure (kPa) | Maximum, intermediate and minor principal stress (kPa) |
|-----------------------------------|------------------------------------------------------|
|                                   | (100, 75, 50) | 130 | 120 | 155 | 175 |
|                                   | (200, 150, 100)| 180 | 220 | 235 | 285 |
|                                   | (300, 225, 150)| 270 | 285 | 320 | 370 |
|                                   | (400, 300, 200)| 355 | 365 | 415 | 455 |
| Dry density/g·cm$^3$              | 1.60          | 1.65 | 1.72 | 1.76 |
| Tensile strength/kPa              | 26.1          | 33.0 | 56.8 | 72.2 |

To verify the criterions of Eq. (2) and Eq. (3), Fig. 4 gives the relationship between fracture initiation pressure $p_f$ and minor principal stress $\sigma_3$. It can be seen that the calculating results of T-S and M-C criterions exist different errors compared with test values. The calculating values of M-C criterion are closer to the test results than that of T-S criterion under low stress state.
To further investigate the difference of criterions, the predicted relative error of absolute value $R_a$ are given in Table 2. The relative error is defined by

$$R_a = \frac{\sum_{i=1}^{n} |v_i - v_i^0|}{nv}$$  \hspace{1cm} (4)$$

where $n=4$ for four stress states, $v_i$ is the predicted value of the two criterions, $v_i^0$ is the test data, $R_a$ is the average value of $R_a$ corresponds to different densities of a given criterion.

| Dry density $/\text{g}\cdot\text{cm}^{-3}$ | 1.60 | 1.65 | 1.72 | 1.76 | $\bar{R}_a/%$ |
|----------------------------------------|------|------|------|------|-------------|
| $R_a/%$ of T-S1                         | -9.9 | -11.3| -13.2| -18.9| -13.3       |
| $R_a/%$ of M-C                         | 5.3  | 14.4 | 38.6 | 43.3 | 25.4        |

Form Table 2, it can be seen that the prediction error $R_a$ from T-S criterion are in the range of -9.9–18.9%, and the calculating values of T-S are all smaller than the test values. The average prediction error $\bar{R}_a$ for different dry densities from T-S criterion is -13.3%. On the other hand, $R_a$ from M-C criterion is merely 5.3% under the dry density of 1.60g/cm$^3$, and the prediction is fairly exact apparently. However, the prediction errors from M-C criterion are relatively higher under high densities, and most of the calculating values of M-C are greater than the test values. Overall, considering the safety of ECRD engineering, T-S criterion corresponding to Eq.(2) is suggested to predict the occurrence of hydraulic fracturing.
4. Conclusions
For the gravelly soil tested in this paper, the tensile strength and fracture initiation pressure $p_f$ all increase with the increasing of dry density. $p_f$ shows linear relationship with the minor principal stress $\sigma_3$. The crack of hydraulic fracturing existed perpendicular to the minor principal stress plane. In addition, for T-S criterion, the predicted values are all less than the test data. On the contrary, most of the predicted values from M-C criterion are greater than the test data. Considering the safety of ECRD engineering, T-S criterion is suggested to predict the occurrence of hydraulic fracturing.

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