Dynamical dephasing of optically controlled charge qubits in quantum dots

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Abstract. We describe the mechanism of phonon-induced dynamical dephasing of an exciton confined in a quantum dot and discuss methods of reducing the dephasing effect by appropriately tailored driving.

Quantum dots (QDs) are promising candidates for implementing quantum information processing schemes due to their atomic-like discrete spectra, controllable properties, feasibility of building arrays of many coupled QDs (providing for scalability) and availability of fast optical control methods. Indeed, proposals for QD-based quantum computing [1, 2] were recently followed by an experimental demonstration of a controlled-NOT gate [3]. On the other hand, the progress of coherent control over confined charge states in QDs encounters serious obstacles resulting from strong dephasing effects due to the dynamics of the surrounding crystal environment. Experimentally, this dephasing manifests itself as a partial decay (on picosecond time scale) of the optically induced coherent polarization in QDs [4]. In self-assembled dots, this effect may be attributed to carrier-phonon interactions [5]. Another experiment where the effects of dephasing may be observed are pulse area dependent Rabi oscillations [6].

An interesting feature of these dephasing processes is that they appear in a structure with strong quantization of levels under resonant coherent excitation to the ground confined exciton state and at low temperatures so that real phonon-assisted transitions and scattering on free carriers may be excluded. It turns out that the decay of coherence may be interpreted in terms of the lattice response to a change of the carrier distribution in a QD. Due to carrier-lattice interactions, the equilibrium positions of lattice ions is different in the presence of a confined exciton. An excitation of the carrier subsystem much faster than the typical phonon response times (i.e., in the sub-picosecond range) is followed by spontaneous lattice relaxation to this new equilibrium [7]. A theoretical description formulated in the limit of infinitely short pulses agrees very well with the experimental data [8] on the decay of coherent polarization.

On the other hand, in order to explain the damping of Rabi oscillations observed in experiments [6] the dynamics under driving pulses of finite duration must be described. This is also necessary for a study of coherence properties of an excitonic qubit [2] implemented on a confined exciton with the quantum logical states $|0\rangle$ and $|1\rangle$ corresponding to the absence and presence of the exciton, respectively. Here, we present a method that allows one to perturbatively include phonon perturbation into the driven dynamics of the excitonic system for arbitrary control pulses. We discuss also a few applications of this formalism.

Under resonant driving conditions the system may be treated in a two-level approximation (ground system state and a single confined exciton) with the exciton state coupled to lattice
displacement by various carrier-phonon interaction mechanisms (the deformation potential coupling to longitudinal acoustic phonons and the Fröhlich coupling to longitudinal optical phonons are most important in the InAs/GaAs system considered here). The two charge states are coupled to a pulsed laser beam which is used for driving. Thus, the system is described by the Hamiltonian (in the rotating wave approximation)

$$H = \epsilon |1\rangle\langle 1| + \frac{1}{2} \left[ f(t)e^{i\omega t}|0\rangle\langle 0| + \text{H.c.} \right] + \sum_k \hbar \omega_k b_k^\dagger b_k + |1\rangle\langle 1| \sum_k f(k)(b_k^\dagger + b_{-k}),$$

(1)

where $\epsilon$ is the exciton energy, $f(t)$ is the slowly varying envelope of the laser pulse, $\omega_k$ is the frequency of phonons with wave vector $k$, $b_k, b_k^\dagger$ are the corresponding phonon creation and annihilation operators (the branch index is included in $k$), and $f(k)$ are carrier-phonon coupling constants (see [9] for explicit formulas corresponding to various coupling mechanisms). The exciton wave function (determining the coupling constants $f(k)$) is modelled as a product of two anisotropic Gaussians with lateral extension of $l_e$ for the electron, and $l_h \approx 0.8l_e$ for the hole (based on numerical diagonalization [7]) and with a much smaller width along the growth direction of the structure.

Our approach is based on the evolution equation for the density matrix of the total system in the second order (Born) approximation with respect to the carrier-phonon interaction

$$\rho(t) = \rho(s) + \frac{1}{i\hbar} \int_s^t d\tau [V(\tau), \rho(s)] - \frac{1}{\hbar^2} \int_s^t d\tau \int_s^\tau d\tau' [V(\tau'), [V(\tau''), \rho(s)]]$$

(2)

where $\rho(t)$ and $V(t)$ are the density matrix and the carrier-phonon interaction (the last term in Eq. (1)) in the interaction picture with respect to the first three terms of Eq. (1). Note that the effect of the driving field is included non-perturbatively.

Upon tracing out the phonon degrees of freedom, the first (zeroth order) term in (2) gives the unperturbed evolution, the second term vanishes (since it contains the thermal average of an odd number of phonons) and the third (second order) term describes the leading-order phonon correction to the dynamics of the carrier subsystem. The latter may be represented in the form of the spectral integral (see [9] for details)

$$\rho^{(2)}(t) = \int d\omega \frac{R(\omega)}{\omega^2} \tilde{S}(\omega)$$

(3)

where $\tilde{S}(\omega)$ is a certain operator depending on the initial system state and on the driving field and

$$R(\omega) = \frac{1}{\hbar^2} |n_B(\omega)|^2 + 1 \frac{1}{N} \sum_k |f(k)|^2 [\delta(\omega - \omega_k) + \delta(\omega + \omega_k)]$$

(4)

is the phonon spectral density. Physical quantities are linear functionals of the density matrix and may be expressed in the form identical to Eq. (3) with a certain scalar function $S(\omega)$.

As a first application of the presented method, let us considered the excitonic qubit driven with a Gaussian pulse by an angle $\pi/2$ in the pseudo-spin (Bloch sphere) picture (this corresponds e.g. to creating an equal superposition from the ground state). The quality of such an operation is quantified by the fidelity $F$, i.e., the probability that after the operation the qubit is found in the desired (unperturbed) state. Since, in general, the initial state of the qubit in a quantum computing process is not known, we calculate the error $\delta = 1 - F$ averaged over initial states, which is given by [10]

$$\delta = \int d\omega \frac{R(\omega)}{\omega^2} S(\omega), \quad S(\omega) = \frac{|F(\omega)|^2 + |F(-\omega)|^2}{12}, \quad F(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \exp \left[ i \int_{-\infty}^{t} f(\tau) d\tau \right]$$,
Figure 1. (a) The total error for a $\pi/2$ rotation on an excitonic qubit, resulting from pure dephasing and finite exciton lifetime [4] for two QD diameters (QD height is 20% of its diameter). (b) The spectral density of the phonon reservoir at $T = 0 \text{ K}$ and $T = 10 \text{ K}$ (solid and dashed) and the spectral function of the driving (dotted) corresponding to a $\pi/2$ rotation performed by a Gaussian pulse.

Figure 2. (a-c) The error resulting from performing a $\pi/2$ rotation on an excitonic qubit by a series of $N = 2, 3, 4$ pulses of duration $\tau_p = 100 \text{ fs}$, either identical (dashed) or optimized (solid), with the total duration of the pulse sequence $t_{tot}$; the small oscillations come from the crossing between the spectral function of the driving and LO phonon lines which are included in this calculation. (d) The error as a function of the number of pulses for two durations of the pulse sequence. Here $l_e = 4.5 \text{ nm}$.

where we assume that the initial state is defined before the driving pulse is switched on and the final state is taken after it has been switched off. For a family of pulses of the form $f(t) = \tilde{f}(t/\tau_p)$ ($\tilde{f}$ defines the pulse shape and $\tau_p$ is the pulse duration), the nonlinear spectral transform has the scaling property $F(\omega) = \tilde{F}(\tau_p \omega)$, where $\tilde{F}(x)$ is fixed for a given pulse shape. It turns out that the error induced by the carrier-phonon interaction decreases with growing pulse duration $\tau_p$, as seen in Fig. 1a for short durations. This may be understood by recalling that the dephasing is due to spontaneous lattice relaxation after a fast change of carrier distribution. For slow driving, high-frequency modes follow the charge evolution adiabatically and reversibly so that only the decreasing fraction of slow, low-frequency modes contributes to dephasing. This is formally confirmed by Fig. 1b: for long pulses the scaling rule causes the function $S(\omega)$ to shrink around $\omega = 0$ and the overlap integral decreases. On the other hand, extending the switching time is restricted by other dephasing processes (e.g., related to the finite exciton lifetime) which accumulate over the driving time, as becomes clear in Fig. 1a for longer times. Thus, the contrary requirements imposed by the dynamical dephasing (slow dynamics) and by the accumulation decoherence resulting from finite lifetime (short operation) leads to a trade-off situation, reflected by a minimum of $\delta$ for a certain $\tau_p$, and limits the possibility of reducing dephasing.
In view of the limited possibility of reducing decoherence by using long pulses, it is interesting to try to improve coherence by driving the system with shaped pulses. Our approach is to perform the same $\pi/2$ qubit rotation using a series of equally spaced narrow Gaussian pulses with either equal or optimized amplitudes. In Fig. 2a-c we plot the error (calculated as above) for such pulsed driving as a function of the total duration of the pulse sequence $t_{\text{tot}}$ (see [11] for details). As previously, the error drops down with growing duration of the control sequence. Although this reduction of decoherence is larger for a larger number of pulses most of the coherence gain is achieved already with 3-4 pulses, while further increase of the number of pulses pulse leads only to minor improvement. Moreover, it turns out that the overall duration of the pulse sequence is often shorter than the duration of a Gaussian pulse necessary for achieving the same level of coherence [11]. The fact that a considerable reduction of decoherence may be obtained with modest experimental resources (without the need to generate arbitrarily shaped pulses [12]) is important from the point of view of practical realization of the proposed control scheme.

Finally, let us discuss the effect of phonon-induced pure dephasing on pulse-area-dependent Rabi oscillations on a single QD [13]. In the experiment [6], a pulse with the area $\int f(t)dt = \alpha$ and duration $\tau_p \sim 1$ ps is applied to a single quantum dot placed in a photo diode structure. The photo-generated carriers then tunnel out of the dot, contributing to the photo-current, which is thus proportional to the average number of photo-excited excitons. This average number of excitons is then plotted as a function of the pulse area $\alpha$ and, in the ideal case, should show sinusoidal oscillations, corresponding to coherently driving the system from the ground state to the state with an exciton and then back to the ground state, when the pulse area exceeds $\pi$. In reality, one observes oscillations which are damped for larger pulse areas (note that the duration of the process is always the same). The perturbative method yields the final exciton occupation in the form

$$n = \sin^2 \frac{\alpha}{2} + \int d\omega \frac{R(\omega)}{\omega^2} S(\omega), \quad S(\omega) = \frac{\cos \alpha}{16} |F(\omega) - F^*(-\omega)|^2 - \frac{\sin \alpha}{16} \text{Im}[F^2(\omega) - F^*2(-\omega)],$$

where the first term in $n$ is the unperturbed result and the second term describes the phonon perturbation. The resulting Rabi oscillations are plotted in Fig. 3a-c. The most striking effect is the improvement of the oscillations both for short and for long pulses. Again, this may be explained using the spectral representation (Fig. 3d). For large pulse areas (many pseudo-spin rotations) the spectral function $S(\omega)$ takes the form of a series of maxima. Taking into account the scaling behavior of this spectral function it is clear that a reduction of the overlap with the phonon spectral density $R(\omega)$ is indeed achieved in both limits, with the spectral function either stretched beyond the phonon frequencies or squeezed around $\omega = 0$. Thus, phonon damping of
the Rabi oscillations has a resonant character and becomes very strong when the frequency of modulation of the charge distribution (corresponding to the strongest maximum of $S(\omega)$) lies in the region of strongly coupled phonon modes. This semiclassical idea of resonance provides a ‘rule of thumb’ for optimizing the driving conditions towards high-fidelity coherent driving.

To summarize, we have analyzed the physical nature of phonon-induced dephasing of excitons on QDs and proposed a perturbative method for the description of these dephasing effects under arbitrary driving. This allows us to explain the experimentally observable dephasing effects and to propose means to reduce the decoherence by using appropriate driving conditions.

Acknowledgments
Supported by the Polish Ministry of Scientific Research and Information Technology (Grant No. 2 P03B 02 424). P.M. is grateful to Alexander von Humboldt Foundation for support.

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