Exploring the dynamical evolution of minimoon 2020 CD3 in a circular restricted three-body problem framework

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Abstract. Asteroid 2020 CD3 was one of the most recently captured natural objects that became Earth’s transient satellite or minimoon. The asteroid is very likely residing in an Earth-like orbit (ELO) with 1:1 mean-motion resonance and occasionally captured by the Earth. In this study, the dynamical evolution of this object is conducted in a framework of Circular Restricted Three-Body Problem (CR3BP). Five thousand test objects are generated by distributing the orbital space with a random uniform scheme around the object’s nominal values comprising ten sigma-uncertainties of the orbital parameters and four Metonic cycles of the epoch. This numerical task has run for 1000 years with the IAS15 integrator known for its adaptive integration and high accuracy for millions of years of simulation. We found that almost all of the test particles are lying in a boundary region, hence follow chaotic and quasi-periodic trajectories and have become minimoon with an average of 15-16 times. The duration of captures of more than a year is very rare. Interestingly, there seems a preferred recapture time at $\sim 30$ years. Most of them still stay in the ELO region, although a few have either been hit the Earth or expelled into vast space.

1. Introduction
A Near-Earth Object (NEO) traveling in an Earth-like orbit and encountering an Earth-Moon system with low-velocity will eventually become a temporary satellite known as a minimoon. The object will orbit in a geocentric manner within tripled Earth’s Hill radius. Regarding the duration of capture event, the minimoon is categorized either as a drifter or a temporary captured flybys when it fails to accomplish at least one full revolution around the Earth [1, 2]. In this paper, we refer to the minimoon as all temporary captured objects, regardless of capture duration.

Despite having a steady-state population, the small size of the minimoon ($\sim 80$ cm) makes it difficult to observe as few objects are identified to date [3]. One of them is 2020 CD3 which was recently discovered by T. Oruyue and K. Wierzchoń from Mt. Lemon Survey in Arizona [4]. The object has become the second moon of Earth for $\sim 4$ years. It is longer than the previous minimoon, 2006 RH120 that has stayed as a minimoon for approximately a year [5, 6]. Nevertheless, the latter minimoon is moving in a chaotic trajectory most of the time, yet it is
expected to get multiple encounters in the past and the future as it has been caught in some mode of resonant, implying Arjuna group origin [6].

Orbital evolution of the object has been carried out in the framework of $N$-body approach by de la Fuente Marcos and de la Fuente Marcos [6]. Furthermore, it computed the orbit with solar perturbation model as radiation pressure appeared significant in astrometric data. However, a more straightforward numerical simulation approach is performed in this paper using the framework of circular restricted three-body problem (CR3BP) approach, involving the Sun and the Earth-Moon Barycentric object (EMB) as primary bodies. In addition, there is a distinct underlying motivation to do this simulation. Instead of predicting a precise orbital evolution of 2020 CD3, this simulation treats the orbit as a region of minimoons’ potential source. The objective of this study is to explore some features of the minimoons’ dynamical evolution, that may have appeared in the previous simulations or analytical inquiries [7, 8].

2. Initial distribution and methods
Granvik [1] defined a region around the Earth’s orbit containing NEOs which will immediately be captured by the Earth-Moon system. The volume of this region is constrained within semimajor axis $a$ of 0.87 – 1.155 au, eccentricity $e < 0.12$, inclination $i < 2.50^\circ$, while longitude of ascending node $\Omega$, perihelion argument $\omega$, and mean anomaly $M$ in $[0, 2\pi]$ rad. In principle, this region is treated as a source of minimoons that provides initial distributions of orbital elements of test particles. Hence, the values of the elements are generated randomly from each uniform distribution within the corresponding boundary. In addition, the epoch of objects is also generated randomly from a uniform distribution with a range within a period of a Metonic cycle to afford variations of the geometrical arrangement of the Earth, the Moon, and the Sun.

As of April 11, 2021, there are 99 objects listed in the Small-Body Database Query of Solar System Dynamics Group from Jet Propulsion Laboratory (SBDQ SSDG JPL) that belongs to this region, including minimoon 2020 CD3.

The same method is used to generate 5000 test particles in this study. However, orbital elements and epoch of 2020 CD3 which are provided in table 1 are used as the boundaries of the initial distributions instead of the previous ones. Each value is treated as a center of a uniform distribution with a range of 10 times of its uncertainty (10-$\sigma$). Furthermore, values of $\Omega$, $\omega$, and $M$ of the particles are still generated randomly from a uniform distribution in $[0, 2\pi]$ rad, while the epoch is generated randomly from a uniform distribution with a range of 2 Metonic cycles ($\sim 2 \times 19$ years).

For the primary bodies, the Sun and EMB are used. We assign the orbital parameters of the primaries based on the particle’s epoch. The framework of CR3BP is based on the fact that the eccentricity of EMB’s orbit is nearly zero, so it follows a circular orbit as in CR3BP. We also assign the mass of the particles to be zero, even though the simulation program will automatically assign the minimum mass to them. All data have been obtained from Horizons System provided by SSDG JPL.

The simulation of the dynamical evolution of these particles is integrated using Implicit integrator with Adaptive time-Stepping, 15th order (IAS15) from REBOUND N-body simulator package available in Python library [9, 10]. With this modified, adaptive Runge-Kutta integrator, there are no constant timesteps. Instead, timesteps of a close encounter in a minimoon capture scenario would be automatically adjusted and the integration becomes more accurate. The simulation runs for 1000 years with an output of time resolution of 1 month using SPICA computer with the following specifications: SPICA 2x Intel(R) Xeon(R) Gold 5118 CPU @ 2.30 GHz, 24 cores 48 threads, RAM 64 GB, and Storage 1.1 TB (https://www.as.itb.ac.id/id/fasilitas-komputasi/).

Figure 1 shows the process of analysis following the numerical simulations. From the records of orbital parameter evolution, the particles that have become minimoon and the ones that have
Table 1. The dynamical properties of 2020 CD3. Source: SBDQ SSDG JPL (April 11, 2021).

| Property | Value (and uncertainty) |
|----------|-------------------------|
| Epoch    | 2459200.5               |
| $a \ [\text{au}]$ | $1.028407563 \pm 8.37 \times 10^{-7}$ |
| $e$      | $0.012281975 \pm 1.83 \times 10^{-6}$ |
| $i \ [^\circ]$ | $0.634245082 \pm 1.66 \times 10^{-6}$ |
| $\Omega \ [^\circ]$ | $82.22793332 \pm 3.218 \times 10^{-3}$ |
| $\omega \ [^\circ]$ | $49.67661495 \pm 7.0595 \times 10^{-3}$ |
| $M \ [^\circ]$ | $304.6694086 \pm 3.9085 \times 10^{-3}$ |

Extreme orbital properties are identified. To get an equal representation of the properties of all objects in a group, the epoch equalization proceeds. In this step, the properties of a group are represented in a time frame when the epoch of every object is equal to a “reference” epoch, or in this case, the latest initial epoch. Figure 4 is the result of the epoch equation that shows the locations of all particles in a time frame.

3. Results and Discussion

3.1. Stability of the region

In CR3BP, the change in a value of an orbital element is constrained by the Tisserand criterion but does not limit the trajectory of an object [11]. The test particles can take periodic or chaotic paths during the epoch equalization and change their dynamical group over the period. They also have a chance to experience any disturbance that may alter their initial orbits significantly, specifically when they are getting a close encounter with EMB.

This can be known by an examination of orbital stability around EMB’s orbit, especially

![Flowchart of the CR3BP simulation analysis](image-url)
the ones that are close to the equatorial plane of EMB, using the MEGNO indicator and the Lyapunov time scale [12]. The stable orbit that has a MEGNO value close to two is equivalent to a long Lyapunov time scale. The simulation for generating these two indicators uses the standard symplectic integrator WHFast to obtain a consistent time interval, but with the same CR3BP scheme as the main simulation—except that the mass of the third object is a typical minimoon mass.

The result is shown in figure 2. The test particles tend to inhabit a chaotic region near the boundary of the stable and the chaotic region. This means that the particles follow chaotic and quasi-periodic behaviors.

Based on the dynamical properties, most of them are categorized as Apollo (∼58.4%) and Aten (∼20.4%) objects when represented in the time of epoch equalization. Note that all of the test particles are orbital clones of 2020 CD3 with the same properties if we just account for $a$, $e$, and $i$. The difference of the particle’s initial values is negligible, even though the range of the variants is 10-$\sigma$. This is because the measurement and calculation of 2020 CD3’s orbit are so precise to determine values up to nine digits and the uncertainties are lower than a factor of five.

Keep in mind that this is only a representation of all particles in a time frame. Changes in orbital parameters or even dynamical groups are likely to occur, including the capture of an object that becomes a minimoon. This is supported by figure 3 which shows the dynamics of orbital parameter evolution of some test particles, particularly the ones that have ever reached extreme orbital properties.

Figure 2. Stability of the region around EMB represented in $a$ vs $e$ plane with MEGNO and Lyapunov Timescale methods. For the upper panel, the darker the region, the more stable the particles (in black), and on the contrary for the lower one. We can see that these particles lie in a chaotic region near the boundary of chaotic and stable region. These diagrams also present dynamical groups of the particles: Atira (□), Aten (▵), Apollo (▽), and Amor (○).
3.2. Dynamical characteristics of the minimoons

Despite the different initial conditions, the particles in figure 3 are roughly following the same pattern, especially around the first 200 years. Considering this fact, it is not surprising that almost all of the test particles (99.98%) are experienced at least an episode of minimoon with an average of 15-16 times (figure 5). This result indicates that the orbit of 2020 CD3 is indeed a region of minimoon source.

Furthermore, this result leads to the assumption of repeated capture as mentioned before, but we do not have a clue that this happens periodically or follows some patterns. We only know that there are $\sim 40\%$ captures, not considering only once, that seem to prefer the recapture time of $\sim 30$ years (figure 6).

However, there is no information about the revolution of the particles around EMB, so we cannot distinguish TCO (Temporarily Captured Object) and TCF (Temporarily Captured Flyby). But, there is a record about the capture duration and it seems that a small portion of the particles has a typical duration of minimoon. There are 2.5\% of test particles have an encounter duration of more than a year, while $\sim 30\%$ of them have a duration of less than a
month as shown in figure 7. The particles that become minimoons in such a brief period (< 1 month) only contribute to 1% of all captures (not shown in figure 7) which are likely TCFs. But this claim needs to be justified by further analysis.

Regardless of the minimoon type, all particles can be considered as Potentially Hazardous Asteroids (PHAs) because the criterion of the minimoon’s distance is less than 0.05 au. Moreover, there are 30% of particles reach the region between Earth and Moon (∼ 0.003 au from EMB) as presented in figure 8. Even though these objects are hypothetical, this result implies that there is a small probability that minimoon, especially with properties like 2020 CD3, can exhibit potential risk in the future.

3.3. Interesting objects
Each particle in this simulation has unique orbital parameter evolution, although some of them follow a rather similar pattern. However, some of them exhibit notable dynamics as they reach extreme properties as listed in table 2 that are quite interesting to be interpreted.

3.3.1. Most frequent captured objects
Particle ID 4156 has experienced 50 encounters over a duration of one thousand years. If we take a look into the detail of the capture trajectory in
Table 2. Summary of extreme orbital properties that the test particles have ever reach

| Parameter                              | Maximum Value       | ID     | Minimum Value       | ID     |
|----------------------------------------|---------------------|--------|---------------------|--------|
| Number of encounter                    | 50                  | 4156   | 10                  | 0068   |
| Encounter duration [day]               | \(4.05 \times 10^3\) | 4797   | —                   | —      |
| Recapture time [day]                   | \(3.36 \times 10^5\) | 2337   | \(6.09 \times 10^1\) | 3620   |
| \(a\) [au]                            | \(1.64 \times 10^0\) | 3070   | \(-4.83 \times 10^{-1}\) | 3983   |
| \(e\)                                  | \(9.00 \times 10^1\) | 0373   | \(1.02 \times 10^{-5}\) | 4354   |
| \(i\) [°]                             | \(2.71 \times 10^0\) | 0373   | \(4.60 \times 10^{-5}\) | 3011   |
| Distance to EMB obj. [au]              | \(5.02 \times 10^4\) | 0373   | \(5.26 \times 10^{-5}\) | 4090   |
| Relative planetocentric energy per unit mass \([\text{au}^2\text{year}^{-2}]\) | \(2.21 \times 10^3\) | 0373   | \(-4.33 \times 10^4\) | 4090   |

Figure 9. Trajectory of object ID 4156 as it becomes a minimoon in a narrow region when entering minimoon-quasi-satellite phase. The transparent regions sorted by thickness are the Earth Hill sphere and the minimum moon conditional sphere. The periodic orbit outside the Hill sphere indicates the phase when this object is liberating at an angle of 0° around EMB or becomes a quasi-satellite coorbital object.

In figure 9, this object has been switching mode between quasi-satellite and minimoon on numerous occasions. This phenomenon was already mentioned in previous studies about 1991 VG and 2020 CD3 that become recurrent minimoons and Earth’s co-orbital objects [13, 14, 6]. These objects are coming from a population that is frequently trapped in 1:1 mean-motion resonance with Earth and affected by Kozai-Lidov mechanism. Their trajectories are expected to be recurrently captured by Earth, chaotic in its brief orbital evolution, and short in its minimum orbit intersection distance (MOID).

3.3.2. Longest capture duration  Analytical study of satellite capture problem in CR3BP had already predicted that the capture is unlikely to happen and if it does, it would not last long [7]. However, there is no information about specific time constrain of how long is the maximum duration. The previous minimoons, 2006 RH120 and 2020 CD3, are having a minimoon episode for less than five years and a test particle with ID 4797 can last longer for nearly 11 years (table 2).

As shown in figure 10, we can see that the object’s capture trajectory is neatly residing in a Hill radius region. It means that this object is very well bounded with EMB’s gravitational influence and relatively stable for a typical encounter duration of minimoon.
Figure 10. Trajectory of object ID 4797 as it becomes a minimoon for around 11 years. The description of the transparent region follows figure 9.

Figure 11. Trajectory of object ID 0373 as it becomes a minimoon and then being ejected from the system.

Figure 12. Trajectory of object ID 3983 as it becomes a minimoon and then being ejected from the system.

3.3.3. Ejected and nearly-impacting objects

Other extreme objects that can be found in table 2 are the ones that are ejected from the system. We can suspect that objects ID 0373 and ID 3983 have been ejected by their unusual extreme properties. The object ID 0373 has a very hyperbolic orbit and reaches a very far maximum distance from EMB after experiencing an encounter with EMB as shown in figure 11. Meanwhile, object ID 3983 has a negative value of $a$ so it passed very close to EMB before being thrown as shown in figure 12.

In addition, there is a test particle with ID 4090 that is nearly impacting EMB. The closest distance to EMB is only about 1.2 Earth radius. Object ID 4090 had a fairly stable orbit and does not change much until at the eleventh time encounter, a drastic change occurred which caused the particle to almost touch the center of EMB. In figure 13, the object’s trajectory appears to strike the surface of EMB (with the assumption that the diameter of EMB is an
Earth radius). However, this is not the case if the conversion of inertial coordinates to rotating coordinates can be performed more thoroughly.

4. Conclusion

In this paper, we have explored some scenarios that may lead to the capture of the test particles which are based on 2020 CD3 orbital properties to be minimoons within the framework of CR3BP. It turns out that these particles are mostly lying in the boundary region, so they follow the chaotic or quasi-periodic trajectory and change their orbital parameters over time.

Almost all of them are experiencing at least an episode of minymoon with an average of 15-16 times within one thousand years period. The ones with stable behavior or captured more than a year are very rare (∼ 2.5%). Nevertheless, there seems a preferred recapture time at ∼ 30 years by 40% of captures, although we cannot impose some kind of periodicity on them without further analysis. Regardless of the dynamical groups the particles belong to, these particles possess a potential threat to Earth that 30% of them have entered the Earth-Moon region, even though they are just hypothetical objects.

We also find some interesting objects with extreme orbital parameters. Two of them are considered stable; the first one has 50 encounters and switching mode between minymoon and quasi-satellite, and the second one has the longest encounter duration (∼ 11 years). On the other hand, we also get the most chaotic particles, either ejected from the system or nearly impacting EMB.

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References

[1] Granvik M, Vaubaillon J and Jedicke R 2012 *Icarus* **218** 262–277
[2] Kary D M and Dones L 1996 *Icarus* **121** 207–224

Figure 13. Trajectory of object ID 4090 as it becomes a minymoon in a narrow region and has a very close encounter with EMB. The description of the transparent region follows figure 9. Note that even the trajectory appears to graze the surface of EMB, the two did not collide.
[3] Fedorets G, Granvik M and Jedicke R 2017 Icarus 285 83–94
[4] 2020 MPEC 2020-D104: 2020 CD3: Temporarily Captured Object [Internet] Cambridge MA: Minor Planet Center; 2020 [updated 2020 Feb 25; cited 2021 Nov 19] available from: https://minorplanetcenter.net/mpec/K20/K20DA4.html
[5] Kwiatkowski, T et al. 2009 A&A 495 967–974
[6] de la Fuente Marcos C and de la Fuente Marcos R 2020 Mon. Not. R. Astron. Soc. 494 1089–1094
[7] Egorov V 1960 The Capture Problem in the Three-body Restricted Orbital Problem NASA technical translation (National Aeronautics and Space Administration)
[8] Bailey J M 1972 ASTRON J 77 177
[9] Welcome to REBOUND! [place unknown]; 2015 [updated 2021 Nov 19; cited 2021 Nov 19] available from https://rebound.readthedocs.io/en/latest/index.html
[10] Rein H and Spiegel D S 2015 Mon. Not. R. Astron. Soc. 446 1424–1437
[11] Fitzpatrick R 2012 Three-body problem (Cambridge University Press) p 147–171
[12] Musielak Z and Quarles B 2014 Rep. Prog. Phys. 77 065901
[13] Tancredi G 1997 Celestial Mechanics and Dynamical Astronomy 69 119–132
[14] de la Fuente Marcos C and de la Fuente Marcos R 2017 Mon. Not. R. Astron. Soc. 473 2939–2948