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Coding techniques through Fibonacci webs, difference cordial labeling and GMJ code method

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Abstract. In this paper, a technique of coding a message using GMJ code on the Fibonacci web $FW(t,n)$ by applying the difference cordial labeling is provided. A few observations and a rule for labeling on the Fibonacci Web are given. Two illustrations using one method for two messages are dealt with in detail in this paper. The catchy clue, mathematical or non mathematical, using numbers and words for finding the suitable Fibonacci Web and GMJ coding given are meant to maintain secrecy and to induce interest. Pictorial coding is also done.

AMS Subject Classification: 05C78
Key Words and Phrases: Fibonacci Web, GMJ Coding, Difference Cordial and DCDC.

1. Introduction

Looking around, one is inspired by the magnificent nature; looking into, one is impressed by the manoeuvrings of the marvelous brain; by diving into the sea of thinking, one is sure to get pearls of concepts and techniques. The entire work presented in this paper are original and expected to be extremely useful providing coding techniques meant for communicating any message personal, official, governmental, pertaining to military services providing a high level secrecy which is the most important factor for coding. GMJ code stands for: (i) Graph Message Jumble code. A coding technique to communicate a Message through Graphs Jumbling letters is named as GMJ code.

(ii) It also refers to the name of one of the researchers of this paper (Gabriel Margaret Joan) who has conceived this method of coding.

Literature review

Much work is done by many researchers on super mean labeling [1][2][3]. Uma Maheswari et al., have introduced coding through a two star and three star with super mean labeling[4],[5]. The concept of edge product cordial labeling on some cycle related graphs was introduced by Udayan et al., [6]. In 2012 edge product cordial labeling on wheel, cycle and helm were presented by a paper by Vaidya [7]. Koh et al., defined a Web graph Pouraj et al., have introduced Difference Cordial labeling of graphs[8][9]. Motivated by these, we worked on Fibonacci Web and found a few techniques of coding by applying difference cordial labeling and hence this paper.
2. Prerequisites

**Definition 2.1. Difference Cordial Labeling:**
Let $G$ be a $(p, q)$ graph. Let $f : V(G) \rightarrow \{1, 2, \ldots, p\}$. For each edge $e = uv$, assign the label $|f(u) - f(v)|$. Then $f$ is called difference cordial labeling if $f$ is one to one and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

**Definition 2.2. Wheel:** The wheel $W_n (n > 4)$ is the graph obtained by adding a new vertex joining to each of the vertices of $C_n$. The new vertex is called the apex vertex and the vertices corresponding to $C_n$ are called rim vertices of $W_n$. The edges joining rim vertices are called rim edges.

**Definition 2.3. Helm:** The helm $H_n$ is the graph obtained from a wheel $W_n$ by attaching a pendant edge to each of the rim vertices.

**Definition 2.4. Web:** The web $W(2, n)$ is the graph obtained by joining the pendant vertices of a helm $H_n$ to form a cycle and then adding a pendant edge to each of the vertices of the outer cycle. Repeating this process again and again, the web graph $W(t, n)$ containing $t$ cycles is obtained.

![Figure 1. Helm](image1.png)

![Figure 2. Web](image2.png)

**Definition 2.5. Fibonacci Web:** The web $W(t, n)$ is called a Fibonacci web $FW(t, n)$ if $t$ and $n$ are two consecutive terms of the Fibonacci sequence. The name 'Fibonacci web $FW(t, n)$' is introduced by the researchers of this paper.

**Definition 2.6. String of Vertices:** A line connecting the center $K_1$ and a pendant vertex passing through a vertex of each cycle is defined as a string of vertices. If $t$ is the number of cycles then there are $(t + 2)$ vertices along any string.

**Definition 2.7. Position of the string of vertices:** The line containing the first vertex which takes the value one is the first string of vertices denoted by $SV_1$. Then move along clockwise and count the string of vertices which are denoted by $SV_2, SV_3, SV_4, \ldots, SV_n$.

2.1. A rule for labeling

(i) Some observations on difference cordial labeling on Fibonacci web $FW(t, n)$ are listed. Here $W(t, n)$ represents the web with $n$ number of vertices in a cycle and $t$ number of such cycles. That is $W(t, n)$ is a web in general with $t$ number of $C_n$ cycles.

Note that $W(1, n)$ is a helm and $W(2, n)$ is a web. The following steps are to be followed for labeling Fibonacci web $FW(t, n)$ by difference cordial labeling.

(ii) $n$ is odd:
Take any string of vertices and assign the number 1 to its pendant vertex. This string is
the first string of vertices denoted by $SV_1$. The other strings are numbered as $SV_2, SV_3$ and so on by moving in clockwise direction. From the pendant vertex numbered 1 move into and number the adjacent vertex as 2. Then move along the outer cycle to the next adjacent vertex numbering it as 3 and move out to the pendant vertex, numbering it as 4. Then the pendant vertex in the next string (jump from pendant vertex to pendant vertex) is given the number 5. Move into and number the adjacent vertex and move along the outer cycle. Repeat this pattern and reach to the apex vertex $K_1$.

(iii) $n$ is even:

On the outer cycle, number any vertex as 1 and this fixes the first string of vertices $SV_1$. Moving along the clockwise directions the other strings are denoted by $SV_2, SV_3$ and so on. The pendant vertex on $SV_1$ which is got by moving out is given the number 2. The pendant vertex in the next string $SV_2$ is given the number 3. Then moving into and moving along the outer cycle and repeating the process the vertices are numbered.

2.2. GMJ coding method:

By assigning numbers to the 26 alphabets of English in a different manner, choosing a suitable labeled graph with a given clue mathematical or non-mathematical, finding the number in the graph for each letter of the given message and presenting the letter codes in a unique way in some form, writing it as a horizontal string or in any other way and creating a picture with the codes after shuffling the order of the letters in order to increase the secrecy of the coded message is named as GMJ coding method.

2.3. Procedure for encoding

Step 1: A suitable Fibonacci Web graph has to be taken. A clue, mathematical or non-mathematical is stated to find the Fibonacci Web graph $FW(t, n)$ which is to be used.

Step 2: The difference cordial labeling is done on the Fibonacci Web graph $FW(t, n)$ using the explanation given above.

Step 3: The 26 alphabets of English are divided in some way (GMJ coding) and the numbers attached to the alphabets are noted down.

Step 4: A coding method for each letter is stated.

Step 5: The message to be coded is written.

Step 6: The coding is presented in a shape desired, allowing of shuffling the order of the letters if required.

Step 7: For decoding the message, the instructions given for coding and a knowledge of difference cordial labeling on any Fibonacci Web graph $FW(t, n), n > 4$ are required.

(i) Numbering of alphabets: DCDC

Clue: Difference Cordial

Numbering is alternated cordially using THREE, the difference in the number of letters of the words DIFFERENCE and CORDIAL. Starting from the last set of three alphabets ($X, Y, Z$) which take the numbers (1, 2, 3), then to the first set of three alphabets ($A, B, C$) which takes the numbers (4, 5, 6), then to the second set of three alphabets from the back ($U, V, W$) which takes the number (7, 8, 9), the numbering is done justifying the words DIFFERENCE and CORDIAL (movement from back and front done smoothly giving equal importance and hence 'cordial'). Hence this method is named as DCDC.
The function for encoding is given below:

The rule for triplets of alphabets from the beginning is
\[ g(a_{3k+i}) = i + 3\ell, \quad i = 1, 2, 3 \quad \{(k, \ell)\/(0, 1)(3, 7)(1, 3)(2, 5)\} \]

The rule for triplets of alphabets from the back is
\[ g(a_{3(k-1)+i}) = i + 3\ell, \quad i = 1, 2, 3 \quad \{(k, \ell)\/(8, 0)(7, 2)(6, 4)(5, 6)\} \]

Two letters M and N are not included in the triplets for them the rule is
\[ g(a_{3k+i}) = i + 3\ell, \quad i = 1, 2 \quad \{(k, \ell)\/(4, 8)\} \]

(ii) **Clue for Messages:** Site or house in a perfect ratio

Here site (website) and house (spider’s web) are used to hint at the graph Web which is denoted by \( W(t, n) \) in general. Here \( t \) and \( n \) are two consecutive terms of a Fibonacci sequence. This is understood by the word ‘perfect ratio’ meaning golden ratio. So, the first Fibonacci web \( W(3, 5) \) to be used for the coding is understood.

The numbering of alphabets and the clue for guessing the graph are the same for both the illustrations.

**Illustration 1:**

(i) **Message:** Between new moon and full moon contact people conduct coaching and assemble behind the Fox field.

(ii) **clue:** Site or house is in a perfect ratio.

(iii) **Graph:** The first Fibonacci web graph \( FW(3, 5) \) is considered.

(iv) **Labeling:** The difference cordial labeling done for \( FW(3, 5) \) is shown below

![Figure 3. Fibonacci web FW(3, 5)](image)

(v) **Coding a letter:**

The center gets the largest number and is denoted by \( V_{0,0} \). If the number corresponding to the center is less than 26, then the number next to it is denoted by \( V_{0,1} \) and so on. The number of vertices in the web \( W(t, n) \) is \( (t + 1)n + 1 \). If \( x > 26 \), assigned to a vertex, the letter corresponds to the number \( x \) is the same as the letter corresponding to the number \( y \) where \( x \equiv y \mod 26 \).
(i) Let \( V_{i,j} \) denote the number assigned to \( j^{th} \) vertex in the \( i^{th} \) string of vertices \( i = 1, 2, 3, \ldots, n \) and \( j = 1, 2, 3, \ldots, (n + 1) \).

(ii) Let \( E_{i,j} \) denote the number assigned to \( j^{th} \) edge along \( i^{th} \) string of vertices \( i = 1, 2, 3, \ldots, n \) and \( j = 1, 2, 3, \ldots, (n + 1) \).

(iii) Let \( C_{i,k} \) correspond to the number assigned to the \( k^{th} \) edge of the \( i^{th} \) cycle \( i = 1, 2, 3, \ldots, n \) and \( k = 1, 2, 3, \ldots, n \).

(vi) **Coding:** (word wise)

\[
\text{Between} - V_{3,1}V_{5,3}V_{4,3}V_{5,1}V_{5,3}V_{0,5} \quad \text{new} - V_{0,5}V_{5,3}V_{0,1} \\
\text{moon} - V_{0,4}V_{2,4}V_{2,4}V_{0,5} \quad \text{and} - C_{2,4}V_{0,5}E_{1,2} \quad \text{full} - V_{1,3}V_{4,2}V_{0,3}V_{0,3} \\
\text{moon} - V_{0,4}V_{2,4}V_{2,4}V_{0,5} \quad \text{contact} - E_{2,3}V_{2,4}V_{0,5}V_{4,3}C_{2,4}E_{2,3}V_{4,3} \\
\text{people} - V_{3,4}V_{5,3}V_{2,4}V_{5,4}V_{0,3}V_{0,3} \quad \text{conduct} - E_{5,3}V_{2,4}V_{0,5}V_{5,2}V_{4,2}E_{5,3}V_{4,3} \\
\text{coaching} - V_{3,2}V_{2,4}C_{3,3}E_{5,3}V_{5,4}V_{4,1}V_{0,5}V_{4,4} \quad \text{and} - C_{3,3}V_{0,5}V_{5,2} \\
\text{assemble} - C_{2,4}V_{3,3}V_{5,3}V_{0,4}V_{5,1}V_{0,3}V_{5,3} \\
\text{behind} - V_{3,1}V_{5,3}V_{5,4}V_{1,4}V_{0,5}E_{1,2} \\
\text{the} - V_{4,3}V_{5,4}V_{0,3} \quad \text{foxfield} - V_{1,3}V_{2,4}V_{1,1}V_{1,3}V_{4,1}V_{5,3}V_{0,3}V_{5,2}
\]

(vii) **Presenting the letter codes:**

The letter codes are written in accordance with the numbering of alphabets (DCDC). As there are 81 letters in the message we proceed as follows. The last three letters whose positions are 79\(^{th}\), 80\(^{th}\) and 81\(^{st}\) are written first. Then the first three letters whose positions are 1\(^{st}\), 2\(^{nd}\) and 3\(^{rd}\) are written. Maintain the same difference and moving cordially from the back and from the front, the letter codes are presented in a horizontal string. 79\(^{th}\), 80\(^{th}\), 81\(^{st}\), 1\(^{st}\), 2\(^{nd}\), 3\(^{rd}\), 76\(^{th}\), 77\(^{th}\), 78\(^{th}\), 4\(^{th}\), 5\(^{th}\), and 6\(^{th}\) etc. The decoding can be done as follows. Counting the number of codes, the number of letters in the message (here it is 81) can be found out. Drawing a diagram with 81 boxes and filling up the boxes with the letters in the above order the message gets decoded.

(viii) **Horizontal string:**

\[
V_{5,3}V_{0,3}V_{5,2} \quad V_{3,1}V_{5,3}V_{4,3} \quad V_{1,1}V_{1,3}V_{4,1} \quad V_{5,1}V_{5,3}V_{5,3} \quad V_{5,3}V_{1,3}V_{2,4} \quad V_{0,5}V_{0,5}V_{5,3} \\
E_{1,2}V_{4,3}V_{5,4} \quad V_{3,1}V_{0,4}V_{2,4} \quad V_{5,4}V_{1,4}V_{0,5} \quad V_{2,4}V_{0,5}C_{2,4} \quad V_{5,3}V_{3,1}V_{5,3} \quad V_{0,5}E_{1,2}V_{1,3} \\
V_{0,4}V_{3,1}V_{0,3} \quad V_{4,2}V_{0,3}V_{0,3} \quad V_{3,3}V_{3,3}V_{5,3} \quad V_{0,4}V_{2,4}V_{2,4} \quad V_{0,5}V_{5,2}C_{2,4} \quad V_{0,5}E_{2,3}V_{2,4} \\
V_{0,5}V_{4,3}C_{3,3} \quad V_{0,5}V_{3,3}C_{2,4} \quad E_{5,3}V_{5,3}V_{4,1} \quad E_{2,3}V_{4,3}V_{3,4} \quad V_{3,2}V_{2,4}C_{3,3} \quad V_{5,3}V_{2,4}V_{3,4} \\
V_{4,2}E_{5,3}V_{4,3} \quad V_{0,5}V_{5,3}E_{5,3} \quad V_{2,4}V_{0,5}V_{5,2}
\]

(ix) **About the picture coding:** The letter codes as given above are shuffled and are arranged forming a suitable picture, here in accordance with the message. The letter codes are represented by the picture depicting New Moon and Full Moon (As New Moon and Full Moon occur in the message).
Illustration 2:

(i) **Message:** Stay there do not move till Fourth.
(ii) **clue:** Site or house in perfect ratio.
(iii) **Graph:** The second Fibonacci web graph $FW(t, n), n > 4$.
(iv) **Labeling:** The difference cordial labeling done on $FW(5, 8)$ is shown below.
(v) **Coding a letter:**
As difference cordimal labeling counts the number of edge values with one and non-one’s we use $\epsilon_{i,1}$ and $\epsilon_{i,0}$ to denote the number of one’s and non-one’s as the edge values in the $i^{th}$ string of vertices. In order to get the numbers 1 to 26, we use a combination of $\epsilon_{i,1}$ and $\epsilon_{i,0}$ using the four arithmetic operations. If $\epsilon_{i,0} = 3$ and $\epsilon_{i,1} = 2$ then the number 13 can be expressed as $3\epsilon_{i,0} + 2\epsilon_{i,1}$. Accordingly every alphabet is denoted. This expression is not unique as $\epsilon_{i,1}$ or $\epsilon_{i,0}$ can also be used directly.

(vi) **Coding:** (word wise)

```
Stay - (3\epsilon_{5,0} + \epsilon_{5,1})(4\epsilon_{8,0} - \epsilon_{1,1})(\epsilon_{4,1} + \epsilon_{5,1})
there - (4\epsilon_{8,0} - \epsilon_{1,1})(4\epsilon_{7,0} + \epsilon_{3,1})(2\epsilon_{0,0} + \epsilon_{2,1})(3\epsilon_{4,0} + \epsilon_{2,1})(2\epsilon_{3,0} + \epsilon_{3,1})
do - (\epsilon_{1,0} + \epsilon_{2,0})(4\epsilon_{4,0} - \epsilon_{3,1})
not - (5\epsilon_{2,0} + \epsilon_{2,1})(4\epsilon_{3,0} - \epsilon_{3,1})
move - (2\epsilon_{1,0} + 3\epsilon_{2,0}) + (4\epsilon_{3,0} - \epsilon_{3,1})(\epsilon_{7,0} + \epsilon_{8,0})(2\epsilon_{2,0} + \epsilon_{2,1})
till - (4\epsilon_{8,0} - \epsilon_{1,1}) + (5\epsilon_{4,0} - \epsilon_{4,1})(5\epsilon_{3,0} - \epsilon_{3,1})(5\epsilon_{2,0} - \epsilon_{2,1})
fourth - (2\epsilon_{3,0} + \epsilon_{4,1}) + (3\epsilon_{1,0} + 2\epsilon_{5,1})(\epsilon_{2,0} + \epsilon_{7,1})(3\epsilon_{2,0} - \epsilon_{8,1})
(2\epsilon_{3,0} + \epsilon_{2,0})(4\epsilon_{4,0} + \epsilon_{3,1})
```

(vii) **Presenting the letter codes:**
Matrices are used for presenting the letter codes. If there are $n$ words with $m$ letters, they are written as a $m \times n$ matrix. In this message, there is only one word with 2, 3, 5 and 6 letters, 3 words with 4 letters. The letter codes are presented using $(2 \times 1), (3 \times 1), (4 \times 3), (5 \times 1)$ and $(6 \times 1)$ matrices. They are given below.

(viii) **Matrix Representation:**

```
\begin{pmatrix}
\epsilon_{1,0} + \epsilon_{2,0} \\
4\epsilon_{3,0} - \epsilon_{3,1}
\end{pmatrix}
\begin{pmatrix}
5\epsilon_{2,0} + \epsilon_{2,1} \\
4\epsilon_{3,0} - \epsilon_{3,1}
\end{pmatrix}
\begin{pmatrix}
3\epsilon_{5,0} + \epsilon_{5,1} \\
4\epsilon_{8,0} - \epsilon_{1,1}
\end{pmatrix}
\begin{pmatrix}
2\epsilon_{1,0} + 3\epsilon_{2,0} \\
4\epsilon_{3,0} - \epsilon_{3,1}
\end{pmatrix}
\begin{pmatrix}
4\epsilon_{8,0} - \epsilon_{1,1} \\
4\epsilon_{7,0} + \epsilon_{3,1}
\end{pmatrix}
\begin{pmatrix}
2\epsilon_{3,0} + \epsilon_{4,1} \\
3\epsilon_{1,0} + 2\epsilon_{5,1}
\end{pmatrix}
\begin{pmatrix}
\epsilon_{2,0} + \epsilon_{7,1} \\
\epsilon_{2,0} + \epsilon_{7,1}
\end{pmatrix}
\begin{pmatrix}
\epsilon_{2,0} + \epsilon_{7,1} \\
\epsilon_{2,0} + \epsilon_{7,1}
\end{pmatrix}
\begin{pmatrix}
3\epsilon_{2,0} - \epsilon_{8,1} \\
3\epsilon_{2,0} - \epsilon_{8,1}
\end{pmatrix}
\begin{pmatrix}
2\epsilon_{3,0} + \epsilon_{2,1} \\
2\epsilon_{3,0} + \epsilon_{2,1}
\end{pmatrix}
\begin{pmatrix}
4\epsilon_{2,0} - \epsilon_{2,1} \\
4\epsilon_{2,0} - \epsilon_{2,1}
\end{pmatrix}
```

(ix) **About the picture coding:**
The letter codes are shuffled and arranged forming a suitable picture. Here the picture stands for the second Fibonacci Web.
Figure 6. Coded Message (DCDC)

The sender has to forward to the receiver the following

- clue to guess the graph
- The method used for numbering the alphabets without explanation
- Letter codes along a pattern (horizontal string or picture coding)

3. Conclusion and future work:

Difference Cordial labeling on the Fibonacci web for communicating some messages using a single method of numbering of alphabets (DCDC) and two methods for letter codings and a suitable picture coding are given in this paper. The researchers believe that in order to code a highly secretive message (official or pertaining to military services) the techniques discussed in this paper can be made use of. In the future, more coding techniques using different labelings on a Fibonacci web are planned to be done.

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