The Equilibrium Structure of Cosmological Halos

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Abstract. We have derived an analytical model for the postcollapse equilibrium structure of cosmological halos as nonsingular truncated isothermal spheres (TIS) and compared this model with observations and simulations of cosmological halos on all scales. Our model is in good agreement with the observations of the internal structure of dark-matter-dominated halos from dwarf galaxies to X-ray clusters. It reproduces many of the average properties of halos in CDM simulations to good accuracy, including the density profiles outside the central region, while avoiding the possible discrepancy at small radii between observed galaxy and cluster density profiles and the singular density profiles predicted by N-body simulations of the CDM model. While much attention has been focused lately on this possible discrepancy, we show that the observed galaxy rotation curves and correlations of halo properties nevertheless contain valuable additional information with which to test the theory, despite this uncertainty at small radii. The available data allows us to constrain the fundamental cosmological parameters and also to put a unique constraint on the primordial density fluctuation power spectrum at large wavenumbers (i.e. small mass scale).

The TIS Model. Our model is described in detail in [14] for an EdS universe and generalized to a low-density universe, either matter-dominated or flat with \( \Lambda > 0 \) in [7]. An initial top-hat density perturbation collapses and virializes, which leads to a nonsingular TIS in hydrostatic equilibrium, a solution of the Lane-Emden equation (appropriately modified for \( \Lambda \neq 0 \)). Using the anzatz that the resulting TIS sphere is the one with the minimum-energy, out of the family of possible solutions, we find that a top-hat perturbation collapse leads to a unique, nonsingular TIS, yielding a universal, self-similar density profile for the postcollapse equilibrium of cosmic halos. Our solution has a unique length scale and amplitude set by the top-hat mass and collapse epoch, with a density proportional to the background density at that epoch. The density profiles for gas and dark matter are assumed to be the same.

Rotation Curves of Dark-Matter Dominated Galactic Halos. The TIS profile matches the observed mass profiles of dark-matter-dominated dwarf galaxies, which are well-fit by the empirical density profile of [1], with a finite density core.
The TIS profile gives a nearly perfect fit to the Burkert profile, providing it with theoretical underpinning and a cosmological context [6].

We have also combined the TIS halo model with the Press-Schechter (PS) formalism, which predicts the typical collapse epoch for objects of a given mass in the CDM model, to explain the observed correlation of \( v_{\text{max}} \) and \( r_{\text{max}} \) for dwarf spiral and LSB galaxies. The observational data indicates preference for the currently-favored, flat \( \Lambda \)CDM universe \((\Omega_0 = 1 - \lambda_0 = 0.3, \, h = 0.7)\). For more details on our methods and results, see [6].

**The Correlations of Halo Core and Maximum Phase-Space Densities with Velocity Dispersion.** A comparison of observed halo properties with other correlations predicted by this TIS+PS approach can test the CDM model while constraining the fundamental cosmological parameters and the power-spectrum shape at small mass scales (i.e. large wavenumbers \( k \)). The core densities \( \rho_0 \) and maximum phase-space densities \( Q \equiv \rho_0/\sigma_V^3 \) for dark-matter dominated halos are
predicted to be correlated with their velocity dispersions $\sigma_V$ as shown in Figure 1. For cold, collisionless DM, $Q$ is expected to be almost independent of the effects of baryonic dissipation [12]. The data on halos from dwarf spheroidal to clusters is consistent with these predictions, with preference for the flat, $\Lambda$CDM model. There have been recent claims that $\rho_0 = \text{const}$ for all cosmological halos, independent of their mass, and that such behavior is expected for certain types of SIDM [4,8]. This claim, however, does not seem to be supported by the current data (Figure 1).

**Galaxy Clusters: TIS vs. CDM Simulations** We have shown previously [5,13,14] that the TIS halo model predicts to great accuracy the internal structure of X-ray clusters found by gas-dynamical/N-body simulations of cluster formation in the CDM model at $z = 0$. The TIS prediction for the redshift evolution of the halo mass-temperature and mass-velocity dispersion relations for galaxy clusters also matches to high accuracy ($\sim$ few percent) the empirical relations derived in [11] from CDM gas/N-body simulations and by the Virgo Consortium from their Hubble volume N-body simulations [Evrard, private communication].

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