Neutron star natal kicks and the long-term survival of star clusters

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ABSTRACT

We investigate the dynamical evolution of a star cluster in an external tidal field by using \textit{N}-body simulations, with focus on the effects of the presence or absence of neutron star natal velocity kicks. We show that, even if neutron stars typically represent less than 2\% of the total bound mass of a star cluster, their primordial kinematic properties may affect the lifetime of the system by up to almost a factor of four. We interpret this result in the light of two known modes of star cluster dissolution, dominated by either early stellar evolution mass loss or two-body relaxation. The competition between these effects shapes the mass loss profile of star clusters, which may either dissolve abruptly (“jumping”), in the pre-core-collapse phase, or gradually (“skipping”), after having reached core collapse.

Key words: methods: numerical, stellar dynamics – globular clusters: general

1 INTRODUCTION

The rate at which star clusters lose mass has been one of the enduring problems of stellar dynamics. In one of the earliest results, Ambartsumian (1938) already highlighted the role of relaxation. The landmark survey of Chernoff & Weinberg (1990) added a mass spectrum, stellar evolution and a tidal boundary, and also revealed the importance of the initial structure of the star cluster. But several other factors also influence the lifetime of star clusters, including the binary population (e.g. Tanikawa & Fukushige 2009), the form of the Galactic orbit (e.g. Baumgardt & Makino 2003), the form of the Galactic potential and tidal shocking (e.g. Gnedin & Ostriker 1997), and the crossing time scale (Whitehead et al. 2013).

In this Letter we add one more influence: natal kicks of neutron stars (NS). Though neutron stars may account for less than 2\% of the cluster by mass, we find, astonishingly, that the presence or absence of kicks may change the lifetime of a star cluster by almost a factor of four. Though the existence of natal kicks of neutron stars is not in doubt, their distribution and dispersion are difficult to establish (see, for example, Podsiadlowski et al. 2005).

In order to isolate the effect of this one factor we consider models from another landmark survey of the evolution of star clusters: that by Baumgardt & Makino (2003, hereafter BM03). As it happens, they imposed no natal kicks on neutron stars, and it was the attempt to reproduce some of their results that led to our discovery. Indeed their principal models, which begin with a King profile with $W_0 = 5$, evolve very differently, both qualitatively and quantitatively, if natal kicks are applied.

The particular models we considered are described in the following section, while Sect. 3 presents our results in some detail, including some information on core collapse and mass segregation.

The final section summarises our conclusions, and attempts to interpret them in the context of other recent work.

2 DESCRIPTION OF THE RUNS

We simulate the evolution of a globular cluster as in BM03, but using NBODY6 (Nitadori & Aarseth 2012). We have performed a survey of simulations in an accelerating, non-rotating frame, using a number of particles between $N = 8192$ and $N = 131072$, a Kroupa IMF (Kroupa 2001), with the mass of the stars between 0.1 $M_\odot$ and 15 $M_\odot$ (resulting in a theoretical mean mass $\langle m \rangle = 0.547 \, M_\odot$), and metallicity $Z = 0.001$. Natal kicks, when they were applied, had a Maxwellian distribution with $\sigma = 190$ km s$^{-1}$ (see eq. 3 in Hansen & Phinney 1997).

In our simulations, the cluster is in a circular orbit, or in an elliptical orbit with eccentricity $e = 0.5$, in a logarithmic Galactic potential $\phi = V_G^2 \ln (R_G)$, where $V_G$ is the circular velocity and $R_G$ is the Galactocentric distance. For the majority of our runs we have used a Roche-lobe filling King (1966) model with $W_0 = 5$ as initial condition. Additional simulations have been performed by increasing the initial concentration of the King profile ($W_0 = 7$). The clusters start at a Galactic radius of 8.5 kpc, with an initial velocity of 220 km s$^{-1}$ (in the circular case); in the elliptical case the apogalacticon is at 8.5 kpc and the initial speed there is reduced appropriately, while the size of the cluster is determined by assuming a Roche-lobe filling condition at perigalacticon. The initial conditions for all the simulations have been generated using MCLUSTER (Küpper et al. 2011).

The tidal radius of the cluster was defined as the Jacobi radius

$$r_J = \left( \frac{GM}{2V_G^2} \right)^{1/3} R_G^{2/3},$$

(1)
where \( M \) is the “bound” cluster mass. The quantities \( M \) and \( r_J \) were determined self-consistently and iteratively by first assuming that all stars are still bound and calculating the tidal radius with this formula. In a second step, we calculated the mass of all stars inside \( r_J \) relative to the density centre of all stars, and used it to obtain a new estimate for \( r_J \). This method was repeated until convergence. Escapers were not removed from the simulations.

The properties of the simulations are presented in Table 1. The significance of the model label is stated in the note to the Table. Column 4 gives the orbital eccentricity; columns 5, 6 and 7 are the half-mass radius and the Jacobi radius, respectively. Column 8 gives the dissolution time, which, following BM03, is defined as the time when 95\% of the mass was lost from the cluster, while column 10 gives the core-collapse time. The corresponding quantities from BM03 are reported in columns 9 and 11, respectively. In our analysis, the moment of core collapse \( T_{cc} \) has been determined by inspecting the time evolution of the core radius and of the innermost lagrangian radius enclosing 1\% of the total mass.

### Table 1. N-body simulation properties

| Model | \( N \) | \( W_0 \) | \( e \) | \( M_0 \) | \( r_h \) | \( r_J \) | \( T_{diss} \) | \( T_{BM} \) | \( T_{cc} \) | \( T_{diss}^{OB} \) |
|-------|--------|---------|---|------|--------|--------|------------|----------|--------|-----------|
| 8kK   | 8192   | 5.0     | 0.0 | 4497.3 | 4.53   | 24.35   | 2426       | -         | 2666   | -         |
| 16kK  | 16384  | 5.0     | 0.0 | 8990.7 | 5.73   | 30.67   | 2816       | -         | -      | -         |
| 32kK  | 32768  | 5.0     | 0.0 | 18419.2 | 7.23   | 38.96   | 3669       | -         | -      | -         |
| 64kK  | 65536  | 5.0     | 0.0 | 36183.1 | 9.10   | 48.79   | 4516       | -         | -      | -         |
| 128kK*| 131072 | 5.0     | 0.0 | 71422.0 | 11.46  | 61.21   | 5927       | -         | -      | -         |
| 8kN   | 8192   | 5.0     | 0.0 | 4497.3 | 4.53   | 24.35   | 4137       | 4149     | 3142   | 3329      |
| 16kN  | 16384  | 5.0     | 0.0 | 8990.7 | 5.73   | 30.67   | 5932       | 6348     | 4810   | 5062      |
| 32kN  | 32768  | 5.0     | 0.0 | 18419.2 | 7.23   | 38.96   | 9384       | 9696     | 7788   | 8412      |
| 64kN  | 65536  | 5.0     | 0.0 | 36183.1 | 9.10   | 48.79   | 14414      | 15197    | 12375  | 13193     |
| 128kN | 131072 | 5.0     | 0.0 | 71659.0 | 11.46  | 61.27   | 24494      | 25506    | 12307  | 21339     |
| 128kKe| 131072 | 5.0     | 0.5 | 71453.0 | 5.50   | 29.43   | 5479       | -         | 6859   | -         |
| 128kNe| 131072 | 5.0     | 0.5 | 71453.0 | 5.50   | 29.43   | 11254      | 11675    | 8952   | 9332      |
| 128kK7| 131072 | 7.0     | 0.0  | 71780.9 | 7.14   | 61.31   | 18369      | -         | 18267  | -         |
| 128kN7| 131072 | 7.0     | 0.0  | 71780.9 | 7.14   | 61.31   | 24494      | 25506    | 11886  | 12620     |

Note. — The capital letter in the model label indicates if the model is characterized by the presence (K, e.g. 128kK) or the absence (N, e.g. 128kN) of NS initial kicks. The star (*) denotes a model for which two different numerical realizations have been evolved; the values are the average of those for the two simulations.

We have also considered two additional pairs of models, as representative cases of the regime of high initial concentration (\( W_0 = 7; \) models 128kK7 and 128kN7) and of the evolution of a star cluster on an elliptic orbit (\( e = 0.5; \) models 128kKe and 128kNe). Here the effects of the presence of NS kicks on the star cluster lifetime are less severe compared to those on the “reference models”, but they are still significant (see Fig. 2). Both models 128kK7 and 128kKe reach core collapse, although at a very late stage of evolution. Of the systems without NS kicks, model 128kNe reaches core collapse at a mass comparable to that of “reference models” without NS kicks, while model 128kN7 has the largest mass at \( T_{cc} \) of all the models in our survey which reach core collapse; such a result is not surprising, given its initial concentration.

Another useful diagnostic of the differences between models with and without NS kicks is provided by the mean mass of stars in the innermost lagrangian shell, enclosing 1\% of the total bound mass of a system. Its time evolution is illustrated in Fig. 3 for all models in our survey with \( N = 128k \) particles. In almost all cases, the mean mass in the innermost shell initially shows a decrease, which is due to the early evolution and escape of massive stars; as expected, this effect is more pronounced for systems with NS kicks. Nonetheless, after only a few Gyr, the value of the central mean mass starts to increase, reflecting the process of mass segregation. For models that reach core collapse, the mean mass in the final stages of evolution falls within the range \( 1.2 < \langle m \rangle < 1.4 M_\odot \), which indicates the dominance of neutron stars in the central regions of the system. Not surprisingly, the rapidly dissolving model 128kK (Fig. 3 red line) shows a final mean mass which is comparable to the initial value.
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3.2 Detailed comparison with Baumgardt & Makino (2003)

Despite our best efforts in reproducing the initial conditions and the numerical set-up described by BM03, we note that there are still non-negligible discrepancies between our models without NS natal kicks and the corresponding ones in their original investigation (see Table 1 and Figs. 1 and 2). We have attempted to identify the main reasons for these discrepancies in the intrinsic differences between the N-body codes used to perform the simulations, and in particular slightly different stellar evolution prescriptions.

We performed our simulations by using the GPU version of NBODY6 (Nitadori & Aarseth 2012), while BM03 used the public GRAPE-6 version of NBODY4 (Aarseth 1999). The latter treats components of binaries as single stars, without collisions or exchange of mass, and the resulting differences might partially explain the increasing discrepancy after core collapse for the models depicted in Fig. 2 because of the increase in the number of binaries at this time. Moreover, BM03 used a prescription for the properties of stellar remnants by Hurley et al. (2000), while in NBODY6 the Eldridge & Tout (2004) recipe is now used. To test this, we carried out a simulation of model 128kN with the Hurley et al. (2000) prescription for stellar remnants, but we obtained a dissolution time of $T_{\text{diss}} = 23.0$ Gyr, which reduces the discrepancy by only about 30% (see data for model 128kN in Table 1).

To assess stochastic effects (such as run-to-run variations) we also performed additional simulations of models 128kN and 64kN by evolving different numerical realizations of the same initial conditions, and by evolving the same realization in several independent simulations (as in BM03). Finally, we performed a simulation of model 128kN in which the escapers were progressively removed (as in BM03), but again without any significant difference ($T_{\text{diss}} = 22.9$ Gyr).

None of these effects was able, individually, to account for the observed discrepancy. Therefore, we believe that the small but systematic discrepancy between our models without NS kicks and the corresponding ones in BM03 results from a combination of all the effects mentioned above, and others which we have not studied, including possible differences in the way in which models are virialized and scaled in different codes. As we shall show later (Sect. 4.1), the sensitivity of these runs to small effects is such that apparently trivial differences could have significant effects.
4 DISCUSSION

4.1 Two modes of star cluster dissolution

We have found that the presence or absence of neutron star kicks, in the models we have studied, can change the lifetime of a star cluster by a large factor. We shall now try to interpret our results in the context of previous studies of star cluster dissolution mechanisms, with the aim of understanding why it is that a process which affects such a small fraction of the mass can have such a dramatic effect.

We consider initially tidally filling, multi-mass models with stellar evolution. Over the years, several numerical investigations have shown that the dissolution time is strongly affected by two factors: the initial relaxation time and the initial concentration (represented by the King parameter $W_0$). In particular Chernoff & Weinberg (1990) showed that, for a Salpeter-like IMF, their models with $W_0 = 1$ or 3 dissolved quickly, in less than a Gyr, and without core collapse, while models with $W_0 = 7$ all entered core collapse, after about 10 Gyr or longer. Clusters with $W_0 = 3$ and a steeper IMF (and hence a longer time scale for mass loss by stellar evolution) could enter core collapse before dissolution, provided that relaxation was fast enough. Thus there is a tension between the time scales of stellar evolution and relaxation, which plays out differently depending on the concentration.

Recently Whitehead et al. (2013) noted that the clusters which dissolve by the effects of stellar evolution lose their mass in a qualitatively different way from those dominated by relaxation.

The former, as they approach dissolution, lose the last fraction of their mass (which may be substantial) extremely rapidly, whereas the latter lose mass at a rate which is steady, and sometimes even declining. Whitehead et al. also noted that the dividing line between the two modes of dissolution is quite sharp. For that reason it would not be surprising if a very small effect, such as the loss or retention of NS, were to place a cluster in one mode of dissolution or the other.

The two kinds of behaviour described by Whitehead et al. (2013) are plainly visible in several previous studies of star cluster evolution, such as Takahashi & Portegies Zwart (2000), and they are visible in Fig. 1 of the present Letter, where all models with kicks end their evolution by losing mass precipitately (except for the case $N = 8k$), whereas the others lose mass at a more moderate rate. We refer to these two cases as “jumping” and “skiing”, respectively. Fig. 1 also illustrates the point made by Chernoff & Weinberg (1990), i.e. the two modes of dissolution are characterised by the presence or absence of core collapse before dissolution. Indeed we see that the clusters with and without natal kicks (except for the case $N = 8k$) lie on either side of the divide between the two modes.

In order to visualise the transition between skiing and jumping models, it has been particularly instructive for us to take on the point of view first suggested by Weinberg (1993), and to explore the evolution of our models in the plane defined by the concentration (parameterized by $c = \log(r_c/r_c)$, where $r_c$ is the core radius) and the mass which remains bound to the system. In this representation, a system which experiences exclusively stellar evolution effects would gradually lose mass, while reducing its concentration due to the progressive expansion, giving rise to a track moving down in the plane and to the left (see Fig. 4 in Weinberg 1993). The tracks qualitatively resemble those of some of our models, as shown in Fig. 4. These are four of the models with kicks, shown in red; and one of these (128kK) is also shown in Fig. 5.

In Weinberg’s treatment, dealing with the slow evolution of spherical equilibrium models, the tracks end when equilibrium is no longer possible; the tracks end at points along a curve, which is shown as a dashed near-vertical curve in these figures. N-body models can cross this curve, but then lose mass on a dynamical time scale, explaining the jumping profile of the corresponding curves in Figs. 4 and 5. Though its precise position may differ slightly when the simplifying assumptions of Weinberg’s models are relaxed, we refer to this curve as “Weinberg’s cliff”.

In Weinberg’s models, mass-loss is driven by stellar evolution only, and his results should be applicable when this process dominates. When the effects of two-body relaxation are dominant, one of the natural consequences is the progressive increase of the central concentration, leading to core collapse. This results in a track oriented to the right-hand-side of the plane, behaviour which can be immediately recognised in the remaining models in Fig. 4 and 5. It should not come as a surprise now that all long-lived, “skiing” models show signatures of core collapse, in contrast with short-lived, “jumping” models.

These figures strongly suggest the existence of trajectories in which the two processes, stellar evolution and relaxation, are in a delicate balance overall, even though stellar evolution dominates early on and relaxation dominates thereafter. We have been particularly fortunate to have included in our survey two models whose evolution almost perfectly delimits a “separatrix” between “skiing” and “jumping” systems (see the innermost pair of red lines in Fig. 4, which correspond to models 8kK and 16kK). Even more strikingly, the model 128kK, despite the oscillations generated by the time-dependent tide, offers an excellent representation of the “separatrix” (see the green line in Fig. 5).

Evidently, the models that we have studied lie close to the separatrix dividing jumping models (which are dominated by stellar evolution, lose mass rapidly at the end of their lives, and do not reach core collapse) and skiing models (which are dominated by two-body relaxation, lose mass gently towards the end of their lives, and reach core collapse). If neutron stars are given no natal kick, as in model 128kN, or the models of Baumgardt & Makino (2003), the trend to mass segregation and core collapse is accentuated, and the model moves across the separatrix into the domain of relaxation-dominated evolution. But we warn the reader against interpreting this as a general rule. Kicks were applied to both model 8kK and model 16kK (the innermost pair of red lines in Fig. 4), and they lie on opposite sides of the separatrix. The 8kK model, because of the low particle number and consequently smaller relaxation time, is sufficiently dominated by relaxation to lie in the skiing regime.

These considerations do not immediately explain, however, why the lifetime should be so different as a factor of nearly four. But the example of models 32kK and 8kK, which lose mass in almost the same way until core collapse in the latter model (Fig. 4), shows that the effects of skiing and jumping lead to different lifetimes. Though the difference is only a factor 1.13 in this case, it seems plausible that the effect could be much bigger if the event which determines the mode of dissolution occurred very early in the lifetime of a model, e.g. the ejection of neutron stars. Furthermore, because our models lie so close to the separatrix between the two modes, it would not be surprising if very minor system-
atic differences in the initial conditions were to lead to significant systematic differences in the lifetime, as discussed in Sect. 3.2.

While this Letter has focused on kicks by neutron stars, the lesson to learn is that apparently minor changes can have very large effects, especially for clusters close to the transition between different modes of dissolution. Other factors which should be taken into account include the presence and properties of primordial binaries, variations in the high-mass end of the IMF, and the degree of primordial mass segregation, which influences both the importance of mass loss by stellar evolution and the role of remnants, not only NS but also stellar-mass black holes. The importance of these factors depends on the location of the dividing line between the two modes of dissolution that we have discussed, which can be assessed only by means of appropriate numerical experiments.

### 4.2 Conclusions

We have presented evidence, based on $N$-body simulations of the evolution of initially tidally filling King models with stellar evolution, that the presence or absence of NS natal velocity kicks can play a crucial role in the long-term survival of model star clusters. In particular we show that some of the basic models in the landmark study of Baumgardt & Makino (2003) are especially sensitive to this effect, which can change their lifetime by almost a factor of four. We explain this finding by showing that the models lie close to a dividing line between (i) models which are dominated by the effects of mass-loss from stellar evolution, and whose evolution ends with a steepening rate of mass loss, and (ii) models whose dynamical evolution is dominated by two-body relaxation, which reach core collapse before dissolving, and do so with a gently decreasing rate of mass loss.

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