Unmodeled search for black hole binary systems in the NINJA project

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Abstract
The gravitational-wave signature from binary black hole coalescences is an important target for ground-based interferometric detectors such as LIGO and Virgo. The Numerical INJection Analysis (NINJA) project brought together the numerical relativity and gravitational wave data analysis communities, with the goal to optimize the detectability of these events. In its first instantiation, the NINJA project produced a simulated data set with numerical waveforms from binary black hole coalescences of various morphologies (spin, mass ratio, initial conditions), superimposed to Gaussian colored noise at the design sensitivity for initial LIGO and Virgo. We analyzed the NINJA simulated data set with the Q-pipeline algorithm, designed for the all-sky detection of gravitational-wave bursts with minimal assumptions on the shape of the waveform. The algorithm filters the data with a bank of sine-Gaussians, sinusoids with Gaussian envelope, to identify significant excess power in the time-frequency domain. We compared the performance of this burst search algorithm with lalapps_ring, which match-filters data with a bank of ring-down templates to specifically target the final stage of a coalescence of black holes. A comparison of the output of the two algorithms on NINJA data in a single detector analysis yielded qualitatively consistent results; however, due to the low simulation statistics in the first NINJA project, it is premature to draw quantitative conclusions at this stage, and further studies with higher statistics and real detector noise will be needed.

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(Some figures in this article are in colour only in the electronic version)
1. The NINJA project

The Numerical INJection Analysis (NINJA) project [1, 2] is a collaboration of numerical relativists and gravitational-wave data analysts, with the goal to improve detectability of binary black hole (BBH) coalescences with numerical relativity waveforms. The first NINJA project represented an important step toward the use of numerical waveforms to enhance the performance of data analysis. Nine numerical relativity groups shared waveforms for BBH coalescences, with no restrictions on spin, eccentricity or mass ratio, and ten data analysis groups analyzed them with various methods based on both modeled (i.e., matched-filtering) and unmodeled searches. Numerical relativity can now provide complete coalescence waveforms, from inspiral to ring-down of the final remnant, through the merging of the black hole constituents of the binary system. These waveforms offer a unique opportunity for testing the data analysis search pipelines for efficiency in detection and faithfulness in source parameter estimations. A recent review of the status of binary black hole simulations is available in [3].

The NINJA project added numerical relativity BBH waveforms to Gaussian noise colored by multiplying the flat spectrum by the power spectral density of each of the Laser Interferometer Gravitational-wave Observatory (LIGO) [4, 5] and Virgo [6] detectors, at design sensitivity. This ideal, simulated Gaussian noise did not include typical features of real data, such as non-Gaussian noise transients and narrow band features like the mirrors’ oscillation normal modes or violin modes, the suspension thermal noise around the frequency of standing wave modes of the mirror suspensions. A population of simulated gravitational signals has also been produced, from numerical relativity data. Such population covers a broad range of black hole masses, distances and orientations. For the numerical waveforms that were used, see [7–22]; for a description of the numerical codes see [12, 18, 22–31]. For additional details on this data set, and a description of the parameter space covered by these waveforms, we refer to [1, 2].

The NINJA project analyzed this data set with several analysis techniques, developed and used by the LIGO Scientific Collaboration (LSC) and Virgo data analysis: (1) matched filtering with analytical waveform models of the inspiral stage, from post-Newtonian expansion [32], (2) matched filtering with hybrid models of the full coalescence, created by matching post-Newtonian perturbation templates with numerical relativity waveforms [33], (3) matched filtering to ring-down templates, to target the final phase of the coalescence, with the lalapps_ring code (section 2.1) and (4) unmodeled searches that do not rely on templates, with two algorithms, the Q-pipeline (section 2.2) and Hilbert Huang transform [34]. Parameter estimation techniques were also tested on this data set [35, 36]. Details and a comparison of the performance of all methods in NINJA are provided in [1].

This paper aims to highlight the potential of unmodeled analysis in the detection of BBH coalescences, by comparing the performance of the Q-pipeline [37–39] algorithm to lalapps_ring [40, 41] on the first NINJA simulated data set. Both algorithms are applied to the output of a single LIGO simulated detector, whose data span a duration of $\sim 30$ h. Section 2 describes the analysis, section 3 presents the results, and conclusions are drawn in section 4.

2. The analysis

Vacuum solutions to the Einstein’s equation for high mass systems are scale invariant, and the total mass of the system determines the frequency scale for the waveform: the higher the mass is, the smaller is the frequency of the coalescence, and the fewer cycles are in the
sensitive band of ground-based interferometers like LIGO and Virgo. For BBH coalescences in the mass range covered by the NINJA simulations (50–350 $M_\odot$), most of the signal-to-noise ratio is in the merger and ringdown phases [42]. Matched filter to ring-down templates and unmodeled searches sensitive to the merger are useful techniques to detect these systems; here we provide some detail on these searches, with reference to two specific algorithms used by the LSC and Virgo.

2.1. Matched filtering to ring-downs with lalapps_ring

The lalapps_ring [40, 41] code was developed by the LSC for ring-down searches. The code implements the matched filtering technique [43] with a bank of damped sinusoids, to represent the quasi-normal modes of a black hole relaxation after the merger. For this study of NINJA data, we used the same version as in the LSC S4 ring-down analysis [44, 45]. The template bank is made of sinusoids with given frequency damped with an exponential, covering the parameter range 50–2000 Hz in frequency and 2–20 in $Q_{rd}$, where $Q_{rd}$ is $\pi$ times the damping time, in unit of the sinusoid period. The analysis is performed in overlapping blocks of 2176 s; the low-frequency cutoff is 45 Hz for the analysis of LIGO data. The Virgo noise curve allows a lower cutoff; however, in this paper we only report single detector results from the LIGO noise curve.

2.2. Unmodeled algorithms: Q-pipeline

A gravitational-wave burst is a short duration signal for which the waveform is not necessarily known. The LSC and Virgo have implemented searches for such signals that do not use templates but instead look for instances of significant excess power in multiple detectors. Several algorithms have been developed, each making use of a different method for the creation of time-frequency maps and different details in the statistical analysis [46–54]; in particular, the Q-pipeline [37, 38] is used in the LSC–Virgo flagship search for gravitational-wave bursts [55]. The Q-pipeline is a multi-resolution time-frequency search for statistically significant excess signal energy, equivalent to a templated matched filter search for sine-Gaussians in whitened data. The template bank is constructed to cover a finite region in the following parameters: (1) central time, or the time of maximum of the Gaussian envelope, (2) central frequency, that is the frequency of the sinusoid, and (3) quality factor $Q_q$, or the number of oscillations under the Gaussian envelope, which, up to a constant, is the ratio of central frequency to bandwidth of the signal. The mismatch between any sine-Gaussian in this signal space and the nearest basis function does not exceed a maximum of 20% in energy. For this study, as in [55], the data are analyzed in 64 s blocks, in the frequency range 48–2048 Hz, with $Q_q$ between 3.3 and 100.

2.3. Analysis strategy

Our ultimate goal is a multi-detector analysis, inclusive of a coherent followup that checks for sky location, and an event-by-event comparison of triggers from inspiral, burst and ring-down analyses, to explore the three phases of the coalescence, using Q-pipeline, lalapps_ring and inspiral matched filtering triggers produced by one of the other NINJA analysis teams [1].

However, for the first NINJA release, we forewent the coherent followup step and instead focused on single-detector results: we established a nominal signal-to-noise ratio (SNR) threshold of 5.5 and compared the parameters estimated by the two algorithms to source
parameters: the time of the waveform maximum $T_{\text{peak}}$, and the innermost stable circular orbit (ISCO) and ring-down frequencies $f_{\text{ISCO}}$ and $f_{\text{ring}}$ [56–59],

\begin{align}
    f_{\text{ISCO}} &= \frac{c^3}{6\sqrt{6}\pi G(m_1 + m_2)}, \\
    f_{\text{ring}} &= \frac{c^3}{2\pi GM}[1 - 0.63(1 - a)^{0.3}]
\end{align}

where $G$ is the Newton constant, $c$ is the speed of light, $m_{1,2}$ are the individual constituent masses and $a, M$ are the final black hole dimension-less spin and mass calculated as in [60, 61] and [62], respectively. The injected SNR is computed from the signal before injection and the detector noise spectrum with starting frequency as specified in section 2.1. Thresholds are consistent with the most recent published searches by the LSC–Virgo for ring-down [45] and bursts [55]. The dependence of the detection efficiency for this set of simulations is shown in figure 2, for both algorithms; a more accurate threshold selection requires a study of accidentals, coincidence between multiple detectors and a fine tuning that goes beyond the scope of this initial study.

3. Results

Unless otherwise stated, all the results discussed in the following sections are restricted to the 4 km Hanford detector (H1): given the limited statistics (94 signals injected with SNR $\geq 5.5$), a discussion of the response of all interferometers and a coincidence analysis would not reliably provide further insight in addition to the information presented below.

3.1. Detection performance

The NINJA project paper [1] provides a comparative analysis of the different methods employed (inspiral, hybrid and ring-down templates for matched filtering, $Q$-pipeline and Hilbert Huang transform for burst searches). Here we focus on the analysis performed by the $Q$-pipeline unmodeled search and by the lalapps_ring matched-filtering to ring-down templates, both at the single interferometer level, with the same nominal threshold of SNR$_{\text{measured}} \geq 5.5$. The statistics of this sample is too small to make inferences on which pipeline performs better in which parameter region; a more systematic study is needed to assess the power of the methods. Also, this analysis does not take into account the effect of background noise transients and accidental coincidences, which are very different in real data than in Gaussian noise, so this comparison is not complete. Nevertheless we have an indication that all pipelines have comparable chances to find these signals.

Figure 1 shows plots of all the injections used in the NINJA project as a function of injected SNR and total mass, identifying which were missed and found by $Q$-pipeline and lalapps_ring; circles are injections found with SNR $\geq 5.5$, the stars are missed. Out of 94 signals with injected SNR $\geq 5.5$, 88 (88) were found by $Q$-pipeline (lalapps_ring) and 89 (87) were found by $Q$-pipeline or (and) lalapps_ring with measured SNR $\geq 5.5$. Of the two signals that were not found by both, one was very close to the threshold, and the other (stronger) was missed due to the deadtime built in the filter for training purposes. A comparison of the difference between injected and measured SNRs is in figure 3, where we plot the SNR detected by the $Q$-pipeline and lalapps_ring as a function of the injected SNR. In both cases, the detected SNR is smaller than the injected SNR (for SNRs above threshold), which is consistent with the two algorithms only detecting a portion of the coalescence. There are few events at the detection threshold where the detected SNR is larger than the injected SNR,
due to noise fluctuations near the threshold. For stronger signals, the discrepancy between measured and injected SNRs is roughly within a factor of $\sqrt{2}$, indicated by the dashed lines in figure 3. This first comparison needs to be pursued with higher statistics, to understand how the signal-to-noise ratio is affected by the different whitening procedures used by the two algorithms, and by what fraction of the signal is detected.

To compare the detection efficiency of \textit{Q-pipeline} with \textit{lalapps\_ring}, in figure 4 we also plot the SNR recovered by \textit{lalapps\_ring} against that recovered by \textit{Q-pipeline}. Note that \textit{Q-pipeline} finds injections with a slightly larger SNR; however, the detection performance of the two algorithms is relatively consistent. Here, the same detection threshold $\text{SNR} \geq 5.5$ was used for both searches; in a real search, different thresholds may be needed for the two algorithms, depending on the false alarm rate.
3.2. Parameter estimation

The $Q$-pipeline identifies a central time for the detected event, corresponding to the peak time for the sine-Gaussian waveform with the largest SNR, while *lalapps* reports as event time the beginning of the ring-down template with best match to the data. Figure 5 shows the deviation of these two time measurements from the peak time of the injected waveform. The two algorithms are in agreement, with a single outlier, visible in the plot. This event corresponds to a weak injection, close to the threshold; in this case, *lalapps* triggered on a startup transient of the signal, 90 ms before the merger.

The $Q$-pipeline algorithm returns a frequency corresponding to the central frequency of the most significant tile in the time-frequency domain. This frequency, as well as the ring-down frequency from *lalapps*, is plotted in figure 6, along with $f_{ISCO}$ and $f_{ring}$, calculated from the injected parameters according to equation (1), for comparison. Note that for both algorithms the recovered frequency tends to be from the portion of the coalescence waveform in the most sensitive region of the detector (50–200 Hz): the inspiral (triangles) for lower masses, and the ring-down (stars) for higher masses. This can be well explained as ISCO and ring-down frequencies are inversely proportional to the total mass of the binary system so, as
Figure 5. Timing accuracy of detected injections by *lalapps_ring* versus *Q-pipeline*. Here $\Delta T = T_{\text{meas}} - T_{\text{peak}}$, where $T_{\text{meas}}$ is the recovered time, which for *Q-pipeline* is the central time of the Gaussian envelope, and for *lalapps_ring* is the beginning of the ring-down waveform template. $T_{\text{peak}}$ is the time of the waveform maximum, as described in [1].

Figure 6. Frequency as a function of mass. The squares are the frequency returned by *Q-pipeline* (left) and *lalapps_ring* (right). In both figures, triangles are $f_{\text{ISCO}}$; stars are $f_{\text{ring}}$, computed from equation (1). The discretization of frequencies in the *Q-pipeline* plot is inherited from the sine-Gaussian template bank.

Figure 7. Missed (stars) and found (circles) injections for *Q-pipeline* versus modulus of the total spin $a$ measured in dimension-less spin unit $a = \frac{c}{GM} |\vec{S}_1 + \vec{S}_2|$, $\vec{S}_i$ being the individual spin vectors. *lalapps_ring* yields the same result.

Injection masses increase and frequencies decrease, the portion of the signal falling into the best sensitivity region of the detectors moves from the inspiral to the ring-down part of the
signal, the largest frequency for any given pair of masses and spins. For the parameters tested in this study, both algorithms detect signals between $f_{\text{ISCO}}$ and $f_{\text{ring}}$.

Finally, in figure 7 we show the SNR of missed and found injections for Q-pipeline as a function of the sum of the spins of the constituents of the binary system; lalapps_ring yields the same result. This plot does not show any dependence on the spin of the black holes, additional simulations with a broader variety of spin magnitudes and orientations will be needed for a more conclusive statement.

4. Conclusions

In the context of the Numerical INJection Analysis (NINJA) project, the Q-pipeline burst search algorithm successfully analyzed numerical relativity BBH coalescence waveforms for a variety of masses and spins in simulated colored Gaussian noise. The Q-pipeline single interferometer performance is comparable to matched filtering to ring-down templates, and yields similar arrival time and frequency, and a slightly better SNR. In particular, depending on the total mass of the BBH system, both algorithms trigger between $f_{\text{ISCO}}$ and $f_{\text{ring}}$ over the broad parameter space covered by NINJA. We emphasize that this is a qualitative statement: the statistics of the NINJA data set is too small for a quantitative, systematic comparison. Moreover the absence of the non-Gaussian noise transients typical of real detectors does not allow a realistic estimation of the false alarm rate and threshold settings. Systematic studies will be subjects of future NINJA projects.

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9