Transport Anomalies Associated with the Pseudogap From a Preformed Pair Perspective

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Transport studies seem to be one of the strongest lines of support for a preformed pair approach to the pseudogap. In this paper we provide a fresh, physically transparent look at two important quantities: the diamagnetic susceptibility and conductivity. We use a three dimensional preformed pair framework which has had some success in the cold Fermi gases and in the process we reconcile recently observed inconsistencies. Specifically, while the preformed pairs in our theory give a large contribution to the diamagnetic susceptibility, the imaginary part of the conductivity is suppressed to zero much closer to $T_c$, as is observed experimentally.

Our goal is to address the observed conflict between transport experiments and a variety of precursor superconductivity scenarios before reaching the definitive conclusion that the pseudogap derives from a non-superconducting order parameter. We argue here that it is necessary to investigate one more precursor superconductivity approach. Most importantly, this particular scenario, based on a stronger than BCS attractive interaction. We are thereby able to reconcile inconsistencies with cuprate experiments. Importantly, there is no more theoretical flexibility here than in standard BCS theory so that predictions are concrete and testable.

Our physical picture for the way in which transport is affected by preformed pairs is relatively simple to understand. In the presence of stronger than BCS attraction there are both fermionic and metastable Cooper pair degrees of freedom. The latter can be viewed as non-condensed pairs, or pair-correlated fermions. It can be seen from simple Boltzmann arguments that bosons provide very large transport responses, provided they are in proximity to condensation. The Bose-Einstein distribution function which is then peaked at small wavevectors, is in stark contrast to its fermionic, Pauli principle restricted counterpart; it leads to a much stronger bosonic response to external field perturbations. Importantly, if one associates the pseudogap with long lived and meta-stable pairs in three dimensional systems, these enhancements, in transport can be shown to persist to temperatures nearer to $T^* \gg T_c$, as one sees in a variety of different transport experiments. This should be distinguished from conventional fluctuation effects, which contribute in the critical regime very close to $T_c$.

In the usual BCS-like, purely fermionic Hamiltonian only fermions possess a hopping kinetic energy and and

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Figure 1: Schematic figures showing the effect for $T > T_c$ of the pseudogap on $\sigma(\omega)$ and $\chi^{\text{dia}}$. (a) The real part of the conductivity $\sigma_1$ as a function of frequency with (black curve) and without (red dashed curve) the pseudogap. Inset: The imaginary part of the conductivity $\sigma_2$ as a function of frequency with (black curve) and without (red dashed curve) the pseudogap. (b) The diamagnetism with (black curve) and without (red dashed curve) the pseudogap. (c) The conductivity and diamagnetic susceptibility as functions of $\Delta_{pg}$ at $T = 15\text{meV}$.

thereby directly contribute to transport. The contribution to transport from pair correlated fermions enters indirectly by liberating these fermions through a break-up of the pairs. Technically, we can associate this coupling to fermionic transport as via the well known Aslamazov-Larkin diagram, importantly modified to include the self consistently determined fermionic pairing gap.

A stronger than BCS attractive interaction can be accomodated by a simple extension of Gor’kov theory. This leads to non-condensed pair effects [8] above and below $T_c$. Important here is the general form of the superconducting electromagnetic response which consists of three distinct contributions: (1) superfluid acceleration, (2) quasi-particle scattering and (3) pair breaking and pair forming. These all appear in conventional BCS superconductors, but at $T = 0$ this last effect is only present when there is disorder. However, in the presence of stronger than BCS attraction, and at $T \neq 0$, non-condensed pairs provide an alternate way to decrease the superfluid density, and the pair breaking and pair forming contributions will be concomitantly more prominent [18].

Without any detailed calculations we are now in a position to predict results associated with the Thz conductivity and the diamagnetism, which will be supported by later microscopic theory. We now show how $\sigma_1(\omega = 0)$ is depressed by the presence of a pseudogap $\Delta_{pg}$, how $\sigma_2(\omega)$ over a range of $\omega$ is also depressed while $\chi$ is greatly enhanced.

Fig1(a) shows how the normal state $\sigma_1(\omega)$ (and in the inset $\sigma_2(\omega)$) behaves as a function of frequency. The red dashed curves are the results of conventional Drude theory. What happens when an above $T_c$ pseudogap is present is shown by the black curves. The curves are normalized by $\sigma_0$, the normal state value of the conductivity at $\omega = 0$ in Drude theory. Both theories (with or without the pseudogap) are consistent with the f-sum rule, and thus have the same fermionic carrier number $\frac{m}{2}$. In the dc regime, with a pseudogap present, there are fewer fermions available to contribute to transport. Their number is reduced by the pseudogap. However, once the frequency is sufficient to break the pairs into individual fermions, the conductivity rises above that of the Drude model. One can see that the effect of the pseudogap is to transfer the spectral weight from low frequencies to higher energies ($\omega \approx 2\Delta$, where $\Delta$ is the pairing gap, and $\Delta \equiv \Delta_{pg}$ above $T_c$). In this way one finds an extra "mid-infrared" contribution to the conductivity which is, as observed [19] strongly tied to the presence of a pseudogap.

The behavior of $\sigma_2(\omega)$, shown in the inset, is rather similarly constrained. On general principles, $\sigma_2$ must vanish at strictly zero frequency - as long as the system is normal. Thus both the red and black curves show that $\sigma_2(\omega = 0) = 0$. Here one can see that the low frequency behavior is also suppressed by the presence of a pseudogap because of the gap-induced decrease in the number of carriers. Similarly, the second peak (around $2\Delta$) in $\sigma_1(\omega)$ leads, via a Kramers Kronig transform to a slight depression in $\sigma_2(\omega)$ in this frequency range. Hence as shown in the inset, $\sigma_2(\omega)$ is significantly reduced relative to the Drude result and tends overall to increase with $\omega$. There is virtually no sign of a $\omega^{-1}$ upturn in $\sigma_2$ which would reflect a remnant of the superfluid density above $T_c$. This presumably is a fluctuation effect which pertains to the narrow critical regime.

In Fig1(b) we present similar comparisons of the behavior of the orbital susceptibility above $T_c$ in a non-gapped normal state (red dashed curve) and in the presence of a pseudogap (black curve). The curves are normalized by $\chi_0$, the absolute value of the diamagnetic susceptibility for $\Delta_{pg} = 0$ at $T = T_c$. One can see that in the absence of a pseudogap only a very weak Landau diamagnetism appears. However, the figure shows that in the presence of a pseudogap the diamagnetic contribution is significantly enhanced. This diamagnetism originates from the large electromagnetic response associated with bosonic degrees of freedom; the breaking of pairs allows this diamagnetism to be reflected in the fermionic response. It should be noted that Van Hove effects enhance this diamagnetism, as does $d$-wave pairing which...
leads to an excess of low energy fermionic excitations. Moreover, this diamagnetism is not restricted to two dimensional models.

All of this leads to a simple anti-correlation between the dc conductivity and the diamagnetic susceptibility in the normal state, which is shown in Fig. 1(c). Here we plot on the left and right hand axes the zero frequency conductivity as a function of varying pseudogap energy scale \( \Delta_{pg} \) and the orbital (diamagnetic) susceptibility with varying \( \Delta_{pg} \) respectively. The former is depressed as the pairing gap increases whereas the latter is enhanced.

These same conclusions (which are qualitatively compatible with experiment \[3, 18, 20\]) derive from microscopic theory. Here the linear response of the electromagnetic current \( \mathbf{J} \) to a small vector potential \( \mathbf{A} \) is characterized by the tensor \( \tilde{P} + \tilde{n} m \) through the equation \( \mathbf{J} = -\tilde{P} + \tilde{n} m \mathbf{A} \). The transverse \( f \)-sum rule is an important constraint on any theory of transport

\[
\lim_{\mathbf{q} \to 0} \int_{-\infty}^{\infty} d\omega \frac{d\omega}{\pi} \left( - \frac{\operatorname{Im} P_{xx}(\omega)}{\omega} \right) = \left( \frac{n}{m} \right)_{xx} \tag{1}
\]

where \( \tilde{n}_0/m \) is the normal fluid density and \( P_{xx} \) is the diagonal component of the paramagnetic current, \( \tilde{P} \), along the x-direction. Similarly, \( (n/m)_{xx} \) is the diagonal component of \( \tilde{n}_0/m \) along the x-direction. We stress that only the fermionic density (and mass) appears on the right hand side of Eq. (1). The sum rule establishes a strong connection between transport and the fermionic kinetic energy, so that many body interactions only serve to redistribute the spectral weight. Thus, for example, even though meta-stable pairs are present, their contribution to transport is indirect and appears when such pairs can be decomposed. This version of the \( f \)-sum rule applies to any many body Hamiltonian which contains an arbitrary two body interaction and a kinetic energy associated with fermions.

Throughout, we work in the transverse gauge. As a consequence all effects of the order parameter collective modes (which are longitudinal) do not enter. The complex conductivity is microscopically defined in terms of \( \tilde{P} \) and \( \tilde{n}_0/m \):

\[
\sigma(\omega) = -\lim_{\mathbf{q} \to 0} \frac{P_{xx}(\mathbf{q}, \omega) + (n/m)_{xx}}{i\omega} \tag{2}
\]

Above \( T_c \), the linear diamagnetic response is similarly related to \( P_{xx} \) and \( (n/m)_{xx} \). It is given by

\[
\chi^{\text{dia}}_{xx} = -\lim_{q_x \to 0} \left[ \frac{\operatorname{Re} P_{xx}^{(q_x=0)}(\mathbf{q}, \omega=0) + (n/m)_{xx}}{q_x^2} \right]_{q_x=q_z=0} \tag{3}
\]

In the superfluid phase the tensors \( \tilde{P} \) and \( \tilde{n}_0/m \) no longer cancel when \( \mathbf{q} \to 0 \), reflecting the Meissner effect. We stress that Equations (1)–(3) are completely general.

We now turn to more microscopic calculations. Previous papers \[8, 20\] have described how the parameters \( \Delta(T) \), \( \Delta_{pg}(T) \), and \( \mu \) are self consistently obtained and how one accommodates a variety of dopings, by effectively fitting the attractive interaction to match \( T^* \) and \( T_c \). Our figures correspond to moderate underdoping. A nearest neighbor tightbinding dispersion \( \xi^p = -2t(\cos(k_x a) + \cos(k_y a)) - \mu \) with \( t = 300 \text{meV} \) is used throughout, and for simplicity, we took a simple \( \tau \propto T^{-2} \) power law (associated with the Fermi arcs \[21\]) for the transport lifetime. Very few of our results depended on this assumption which was made in earlier work \[18\]. A general finding is that, while \( \Delta_{pg} \) decreases monotonically from \( T_c \) to \( T^* \), with decreasing \( T \) below \( T_c \), \( \Delta_{pg} \) decreases while \( \Delta_{sc} \) rises, reflecting the fact that finite momentum pairs are converted to the \( q = 0 \) condensate, while maintaining an overall nearly constant \( \Delta(T) \). We have previously derived microscopic representations of \( \tilde{P} \) and \( \tilde{n}_0/m \) \[18, 20, 22\]. Importantly, our gauge invariant electromagnetic response function analytically satisfies the transverse \( f \)-sum rule. One can derive these contributions in a variety of ways but the most straightforward involves inclusion of generalized Maki-Thompson and Aslamazov-Larkin diagrams. The latter are considered to be effectively equivalent to Boltzmann (or time dependent Ginsburg-Landau-like) approaches to bosonic transport. Here the stronger-than-BCS attraction enters in an important way in order to insure that the pairing gap energy scale \( \Delta \) is explicitly incorporated. The paramagnetic tensor current-current correlation function \( \tilde{P} \) is

\[
P_{xx}(\mathbf{q}, \omega) = \sum_{\mathbf{p}} \frac{\partial \xi^p}{\partial p_x} \frac{\partial \xi^p}{\partial p_x} \left[ \frac{E^+ + E^- E^+ E^- - \xi^+ \xi^- - \Delta^2_{sc} + \Delta^2_{pg}}{E^+ E^- - (E^+ + E^-)^2} \left( 1 - f(E^+) - f(E^-) \right) \right. \]

\[
- \frac{E^+ - E^-}{E^+ E^-} \frac{E^+ E^- + \xi^+ \xi^- + \Delta^2_{sc} - \Delta^2_{pg}}{\omega^2 - (E^+ - E^-)^2} \left( f(E^+) - f(E^-) \right) \right] \tag{4}
\]

where \( \omega \) has a small imaginary part and \( f \) the Fermi function. Here \( E_p \equiv \sqrt{\xi^2_p + \Delta^2(T)} \), where \( \xi_p = \epsilon_p - \mu \), and
\[ \Delta_{sc} (\Delta_{pg}) \] is the gap component of the condensed (non-condensed) pairs, with \( \Delta = \sqrt{\Delta_{sc}^2 + \Delta_{pg}^2} \). All transport expressions in this paper reduce to those of strict BCS theory when the attraction is weak and \( \Delta_{pg} = 0 \). Here we define \( E_{p\pm q/2}^\pm = E_{p\pm q/2} \) and \( \xi_{p\pm q/2}^\pm = \xi_{p\pm q/2} \). Importantly, the terms on the first line in Eq. (4) represent the pair breaking and pair forming contributions. The second line is associated with fermionic scattering. Also important to the electromagnetic response is the number density \( \tilde{n}/m \)

which can be rewritten as

\[ 2 \sum_{p} \frac{\partial \xi_{p}}{\partial p} \frac{\partial \xi_{p}}{\partial p} \left[ \frac{\Delta_{sc}^2 + \Delta_{pg}^2}{2 E_{p}} \left( 1 - 2 f(E_p) \right) - f(E_p) \right]. \tag{5} \]

Eqs. (2) and (3) yield analytic expressions for \( \sigma(\omega) \) and \( \chi_{\text{dia}} \). Fig. 2 displays our more quantitative results for \( \sigma_1 \) and \( \sigma_2 \) as both functions of \( \omega \) and \( T \). The layout is designed to duplicate figures from Ref. 3 and the general trends are similar. Thus one sees from Fig. 2(a) and its inset to (c). One sees that well above \( T_\text{c} \), the real part of the conductivity is almost frequency independent. The imaginary part is small in this regime. At the lowest temperatures \( \sigma_1 \) contains much reduced spectral weight while the frequency dependence of \( \sigma_2 \propto \omega^{-1} \); both of these reflect the characteristic behavior of a superfluid.

Here as in the experimental studies, we focus primarily on the temperature dependent plots in Figs. 2(b), (d) and the inset to (c). One sees that \( \sigma_1 \) shows a slow decrease as the temperature is raised above \( T_\text{c} \). Somewhat below \( T_\text{c} \), \( \sigma_1 \) exhibits a peak which occurs at progressively lower temperatures as the probe frequency is decreased. Roughly at \( T_\text{c} \) we find that \( \sigma_2 \) shows a sharp upturn at low \( \omega \). The region of finite \( \sigma_2 \) above the transition can be seen from the inset in Fig. 2(d) and it is clearly very small in the pseudogap state. The inset of Fig. 2(d) shows an expanded view of \( \sigma_2(T) \) near \( T_\text{c} \). In agreement with experiment, the nesting of the \( \sigma_2 \) versus \( T \) curves switches orders above \( T_\text{c} \). This important point reflects the fact that \( \sigma_2(\omega) \) is generally increasing with increasing \( \omega \) above \( T_\text{c} \) as seen in the inset in Fig. 2(a) and in exper-

Figure 2: Results for \( \sigma_1 \) and \( \sigma_2 \). (a) \( \sigma_1 \) as a function of frequency. Inset: \( \sigma_2 \) as a function of frequency. (b) \( \sigma_1 \) as a function of temperature. (c) \( \omega \sigma_2 \) as a function of frequency. Inset: \( \omega \sigma_2 \) as a function of temperature. (d) \( \sigma_2 \) as a function of temperature. Inset: \( \omega \sigma_2 \) as a function of temperature near \( T_\text{c} \).

Figure 3: The diamagnetism as a function of temperature for different "hole concentrations" as parameterized via the indicated \( T/T_\text{c} \).

Increasing doping
ment. This is in contrast to the behavior expected of a fluctuation contribution where a $\omega^{-1}$ dependence would occur. However, in slightly different plots, the counterpart experimental studies reveal a small 10-15K range where this fluctuation contribution is visible. This effect would not be present in a mean field approach. As speculated in Ref. 3, one should distinguish these near $T_c$ critical fluctuations from preformed pairs which persist to much higher temperatures.

These effects are made clearer by plotting the “phase stiffness” which is proportional to the quantity $\omega\sigma_2$ and is shown in Fig. 2(c). Deep in the superconducting state there is no $\omega$ dependence to $\omega\sigma_2(\omega)$, while at higher $T$ this dependence becomes apparent. In the inset to (c), the temperature dependence of $\omega\sigma_3(T)$ is displayed. We see that above $T_c$, $\omega\sigma_2$ is never strictly constant, as would be expected from fluctuation contributions. In experiment the onset of finite frequency spreading of the curves at $T \leq T_c$ has been attributed to Kosterlitz-Thouless physics [23].

Finally, we turn to the diamagnetic response. Figure 3 shows $\chi_{\text{dia}}$ as a function of temperature for four different dopings. Independently of the particular parameters that are used, it is seen that the magnitude of $\chi_{\text{dia}}$ is enhanced even at temperatures well above $T_c$. We should not associate this diamagnetism with short range Meissner currents, as might be appropriate to alternative phase fluctuation or normal state vortex scenarios [4]. Rather here, the diamagnetism arises from the large contribution of non-condensed pairs which are in proximity to condensation [12, 14]. This has a similarity to low d fluctuation effects, but arises in the 3d systems here from stronger than BCS attraction, which stabilizes these pair degrees of freedom. Since the kinetic energy ultimately resides in the fermionic system, it is not surprising that we find it is the pair breaking terms which provide the conduit for communicating enhanced bosonic transport contributions to the fermionic transport channel. Because we are working at effectively zero magnetic field, we have not addressed diamagnetism associated with non-linear response, although this appears to be very anomalous experimentally [17]. At the leading order level we have shown here that there is a profound connection between the complex conductivity and this orbital magnetism. Importantly, while the preformed pairs in our theory give a large contribution to the diamagnetic susceptibility, as is observed experimentally, the imaginary part of the conductivity is suppressed to zero much closer to $T_c$, as observed.

We end with the following observations. Our theoretical approach, has virtually no flexibility; it was set up before there was much experimental interest in these transport measurements. In accord with experiment, we find: (i) that pseudogap effects lead to an enhanced diamagnetism above $T_c$, (ii) that the imaginary conductivity $\sigma_2(\omega)$ is reduced to zero in a very narrow range of $T$ above $T_c$, and (iii) that the real conductivity $\sigma_1(\omega \approx 0)$ is suppressed as the pseudogap becomes larger. This last point is in turn associated with a transfer of low $\omega$ conductivity spectral weight into the mid-infrared region.

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