Kinematic Analysis and Trajectory Planning Simulation of Citrus Picking Manipulator

Lijia Xu, Yuan Liang and Zhiliang Kang*
School of Mechanical and Electrical Engineering, Sichuan Agricultural University, Yaan, China
*Corresponding author, e-mail: kangzhiliang96@163.com

Abstract. Trajectory planning is the premise of studying robot control. This study takes PUMA560 robotic arm as the research object and applies it to citrus picking. This research carried out kinematics analysis and simulation on its picking process, and then performed trajectory planning and workspace analysis. Firstly, the kinematics analysis and simulation of the picking robot arm were carried out in the forward and reverse direction, and the kinematics model was established by Denavit-Hartenberg method. Secondly, the trajectory planning of the robotic arm based on quintic polynomial interpolation is performed in the joint space, and the fitting curves of joint angular displacement, angular velocity and angular acceleration are plotted respectively. Finally, the working space of the picking manipulator is analysed and its working space diagram is drawn. The simulation results show that the trajectory planning and workspace drawing methods selected in this study are effective for the trajectory planning of citrus picking manipulator and provide theoretical support for the control research of citrus picking manipulator.

1. Introduction
In the process of agricultural production, the mechanical arm of agricultural robots usually have low picking efficiency, and they are prone to shake at the moment when they start and stop the picking action, which will not only damage the quality of the fruit, but also cause wear and tear on its components in the long run. Therefore, the trajectory of the mechanical arm needs to be planned so that it can reach the target accurately and quickly according to the needs of agricultural production, and the motion speed of the end-effector should be as stable as possible to avoid the sudden change of inertial force. At present, many domestic and foreign scholars have studied the trajectory planning of mechanical arm [1-4]. Li et al. [1] took SCARA mechanical arm as the research object, took the velocity, acceleration and jerk of each joint of the mechanical arm into consideration, and made trajectory planning in joint space. Liu et al. [2] studied a time-optimal optimization method with continuous acceleration. Lu et al. [3] proposed to calculate the inverse solution of the six-degree-of-freedom manipulator by geometric method and Euler Angle. The workspace of the manipulator determines its range of motion and plays a key role in the trajectory planning of the manipulator.

2. D-H Model of Robotic Arm
The robotic arm of the citrus picking robot is designed with six degrees of freedom. The linkage coordinate system of the robotic arm established according to denavit-hartenberg law is shown in figure 1. D-H parameters of the six joints of the robotic arm are shown in table 1. Part of the link length of the manipulator arm is zero, which is given a length in figure 1, so that the structure of the manipulator arm can be clearly shown.
Figure 1. The linkage coordinates of the manipulator

The D-H parameters of the manipulator are shown in table 1, in which $\theta_i$ represents joint rotation angle, $\alpha_{i-1}$ is the torsion angle of the connecting rod and $d_i$ is the joint offset of the prismatic joint. $a_2=0.432m$, $a_3=0.02m$, $d_2=0.149m$ and $d_4=0.433m$ in table 1. According to the joint parameters, the three-dimensional model diagram of each link of the manipulator is established, as shown in figure 1.

| Link $i$ | $\theta_i$ | $a_{i-1}$ | $a_i$ | $d_i$ | Variable scope       |
|----------|------------|-----------|-------|-------|----------------------|
| 1        | $\theta_1$ | 0         | 0     | 0     | -160°~160°          |
| 2        | $\theta_2$ | -90°      | 0     | $d_2$ | -225°~45°           |
| 3        | $\theta_3$ | 0         | $a_2$ | 0     | -45°~225°           |
| 4        | $\theta_4$ | -90°      | 0     | $d_4$ | -110°~170°          |
| 5        | $\theta_5$ | 90°       | 0     | 0     | -100°~100°          |
| 6        | $\theta_6$ | -90°      | 0     | 0     | -226°~226°          |

3. Forward Kinematic Analysis of Manipulator

According to the coordinate transformation rules between linkage of mechanical arm, the homogeneous transformation matrix of coordinate system transformation of two adjacent linkage can be expressed as:

$$
A_{i-1}^i = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & a_{i-1} \\
\sin \theta_i \cos \alpha_{i-1} & \cos \theta_i \cos \alpha_{i-1} & -\sin \alpha_{i-1} & -d_i \sin \alpha_{i-1} \\
\sin \theta_i \sin \alpha_{i-1} & \cos \theta_i \sin \alpha_{i-1} & \cos \alpha_{i-1} & d_i \cos \alpha_{i-1} \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

In equation (1), parameters $a$, $d$ and $\alpha$ are the parameters of the manipulator in table 1. PUMA560 is a 6-DOF manipulator, so the pose of the end-effector relative to the base coordinate system can be obtained by the following transformation:

$$
{}^0_A = {}^0A_2^{-1}A_3^{-1}A_4^{-1}A_5^{-1}A_6^{-1}A = \begin{bmatrix} n_x & a_x & a_z & p_z \\
n_y & a_y & a_z & p_z \\
n_z & a_z & a_z & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

Substituting the joint rotation angle $\theta_i$ into equation (2) can solve the pose transformation matrix of the end-effector coordinate system relative to the base coordinate system. Beyond that, in equation (2), $n_x$, $n_y$, $n_z$, $a_x$, $a_y$, $a_z$, $p_x$, $p_y$, $p_z$ and $p_z$ represent the calculated results of the end-effector pose matrix.
The simulation of the manipulator can directly show its motion process, and the spatial position of the end-effector and other parameters can be displayed in real time [4]. The robot toolbox is used to simulate the picking robot arm, and the 3D model and visual control interface of the robot arm are obtained. Changing the angle of each joint on the control interface to control the movement of the robot arm can obtain the posture of the robot arm, as shown in Figure 2.

![Figure 2. The pose control interface of the manipulator](image)

The values of the above two sets of joint variables were substituted into equation (2) to calculate the spatial position of the end-effector. The calculated results are shown in Table 2. According to the data in Table 2, under the condition of the same joint Angle, the result of forward kinematics equation and the simulation result are basically the same. The results verify the accuracy of the forward kinematics model of the manipulator.

| Group | \( \theta_1 \) | \( \theta_2 \) | \( \theta_3 \) | \( \theta_4 \) | \( \theta_5 \) | \( \theta_6 \) | \( p_x \) (mm) | \( p_y \) (mm) | \( p_z \) (mm) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|--------------|--------------|
| 1     | 0              | 0              | 0              | 0              | 0              | 0              | 0.452        | 0.149        | -0.433       |
| 2     | 25.3           | -35.1          | 24.3           | 22.4           | -70            | 90.4           | 0.347        | 0.3288       | -0.1732      |
| 3     | 112            | -48.6          | -45            | 92.4           | 80             | -67.8          | -0.4066      | 0.6086       | 0.3712       |

### 4. Inverse Kinematic Analysis of Manipulator

The inverse kinematic analysis is based on the known D-H parameters of the manipulator, and the angular displacement of the six joint axes that enable the manipulator to reach the pose of the end-effector is obtained according to the given pose of the end-effector. The PUMA560 manipulator meets Pieper criterion [5], so the inverse kinematics solution steps are as follows: (1) The 4th, 5th and 6th joint axes of the manipulator intersect, and the origin of the corresponding link coordinate system is located at this intersection point. The first three joint angles \( \theta_1, \theta_2 \) and \( \theta_3 \) can be solved according to the intersection point position. (2) According to the given end-effector posture and the first three joint angles, the remaining joint angles \( \theta_4, \theta_5 \) and \( \theta_6 \) are calculated. According to the fourth column of equation (1), the position of the intersection point of the 4th, 5th and 6th joint axes in the basic coordinate system is:
Where,
\[
\begin{align*}
g_1 &= \cos \theta_2 f_1(\theta_1) - \sin \theta_2 f_2(\theta_1) + a_1, \\
g_2 &= \sin \theta_2 \cos \alpha_1 f_1(\theta_1) + \cos \theta_2 \cos \alpha_2 f_2(\theta_1) - \sin \alpha_1 f_1(\theta_1) - d_4 \sin \alpha_1, \\
g_3 &= \sin \theta_2 \sin \alpha_1 f_1(\theta_1) + \cos \theta_2 f_2(\theta_1) + \cos \alpha_1 f_1(\theta_1) + d_4 \cos \alpha_1, \\
f_1(\theta_1) &= a_3 \cos \theta_3 + d_4 \sin \alpha_3 \sin \theta_3 + a_2, \\
f_2(\theta_1) &= a_3 \cos \alpha_3 \sin \theta_3 - d_4 \sin \alpha_3 \cos \alpha_3 \cos \theta_3 - d_4 \sin \alpha_3, \\
f_3(\theta_1) &= a_3 \sin \alpha_3 \sin \theta_3 - d_4 \sin \alpha_3 \cos \alpha_3 \sin \theta_3 + d_4 \cos \alpha_3.
\end{align*}
\]  

The pose parameters of the end-effector, \(x, y\) and \(z\) are known values. If \(r = x^2 + y^2 + z^2\), then the value of \(r\) is also known value. According to equation (3), it can be obtained:
\[
r = f_1^2 + f_2^2 + f_3^2 + a_1^2 + a_2^2 + 2d_4 f_1 + 2a_4 (\cos \theta_2 f_1 - \sin \theta_2 f_2)
\]  

Now write down the \(z\) component equation in equation (3), and form a system of equations with equation (5) to obtain:
\[
\begin{align*}
r &= (k_1 \cos \theta_2 + k_2 \sin \theta_2) 2\alpha_1 + k_3 \\
z &= (k_1 \sin \theta_2 - k_2 \cos \theta_2) \sin \alpha_1 + k_4
\end{align*}
\]  

Where, \(k_1 = f_1, k_2 = -f_2, k_3 = f_3^2 + f_2^2 + f_3^2 + a_1^2 + a_2^2 + 2d_4 f_1 + 2a_4 (\cos \theta_2 f_1 - \sin \theta_2 f_2)\) and \(k_4 = f_2 \cos \alpha_3 + d_4 \cos \alpha_3\).

In equation (6), the variable \(\theta_1\) has been eliminated, and the formula of the variable \(\theta_2\) has been simplified. As can be seen from formula (6) \(r\) equals \(k_3\). It can be known from the above that the length of linkage 1 of the manipulator is \(a_1 = 0\), \(r\) is a known quantity, and \(k_3\) is only a function of \(\theta_3\), and \(\theta_3\) can be obtained from equation (4). \(\theta_2\) can be solved according to equations (6) and \(\theta_3\), and \(\theta_1\) can be solved according to equation (3).

The solution of joint \(\theta_4\), \(\theta_5\) and \(\theta_6\) is based on the method of Z-Y-Z euler Angle transformation. According to the \(\theta_1\), \(\theta_2\) and \(\theta_3\), the rotation matrix of linkage coordinates \(\{4\}\) relative to the base coordinate system can be calculated when \(\theta_4 = 0\):
5. Simulation of Joint Space Trajectory Planning of Manipulator

In this study, the trajectory planning of the manipulator is carried out by means of quintic polynomial interpolation. Given a series of intermediate nodes of the trajectory, the continuity of angular displacement, angular velocity and angular acceleration can be satisfied simultaneously. The quintic polynomial is constructed as follows:

\[
\begin{align*}
\theta_5 &= A \tan 2\left( \sqrt{r_{31}^2 + r_{32}^2}, r_{33} \right) \\
\theta_4 &= A \tan 2\left( r_{33} / \sin \theta_3, r_{33} / \sin \theta_4 \right) \\
\theta_6 &= A \tan 2\left( r_{32} / \sin \theta_3, -r_{31} / \sin \theta_6 \right) 
\end{align*}
\]  
\hspace{1cm} (10)

The six boundary conditions are given as:

\[
\begin{align*}
q(t_0) &= q_0, \\
\dot{q}(t_0) &= \dot{q}_0, \\
v_0 &= \frac{\dot{q}(t_1) - \dot{q}(t_0)}{t_1 - t_0}, \\
\ddot{q}(t_1) &= a_1. \\
\end{align*}
\]  
\hspace{1cm} (11)

In the above boundary conditions, \( t_0 \) and \( t_1 \) are the start and end time of the manipulator, \( v_0 \) and \( v_f \) are its start and end velocity, \( a_0 \) and \( a_f \) are its start and end acceleration. In this study, the intermediate nodes of a series of joint motions are specified, and the start-stop velocity and acceleration are initialized to zero [6]. In order to get a smooth fitting curve, the intermediate node of motion is specified, but the velocity of the node is not specified, and the intermediate point is allowed to determine the appropriate velocity value according to the intuitionistic judgment. The velocity at the middle point of the trajectory is determined according to equation (12), where

\[
v_k = \begin{cases} 
0, & \text{sign}(d_k) \neq \text{sign}(d_{k+1}) \\
\frac{1}{2}(d_k + d_{k+1}), & \text{sign}(d_k) = \text{sign}(d_{k+1}) 
\end{cases}
\]

\hspace{1cm} (12)

The joint motion nodes [7] are shown in table 3. In this study, only the first two joints of the robotic arm are selected for trajectory planning simulation, while the similarity between the remaining joints and these two joints is not verified one by one.

As can be seen from figure 3, the curves of angular displacement, angular velocity and angular acceleration of each joint obtained through simulation are continuous and smooth, and there is no mutation in each curve, indicating that there is no singularity in the trajectory of the robot arm planned through simulation, and the trajectory planned in this study is reasonable.

**Table 3.** Mechanical arm joint node table

| Joint | Node | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
|-------|------|----|----|----|----|----|----|----|----|
| 1     |      | 10 | 60 | 75 | 130| 110| 100| -10| -50|
| 2     |      | 15 | 25 | 30 | -45| -55| -70| -10|  10|
| 3     |      | 45 | 180| 200| 120|  15| -10| 100|  50|
6. Simulation and Analysis of Manipulator Workspace
The workspace of the end-effector refers to its range of operations such as grasping and placing objects, which is one of the factors to measure the working capacity of the manipulator. Monte carlo method was used to solve the working space of citrus picking manipulator. Within the set range of joint angles of the robotic arm, the values of 6 joint angles of robotic arm were randomly selected to form 10,000 combinations of angles. Then these combinations are calculated by using formula (2) to obtain the spatial position coordinates of the end-effector reference point in the base coordinate system. Finally, these reference points are plotted in the base coordinate system to obtain the workspace diagram of the manipulator as shown in figure 4.

Monte carlo method was used to calculate the spatial position coordinates of 10,000 desired points, which took 44.4749s, while another method, the Superposition Step Value Method, only selected 1536 desired points and the running time reached 98.5668s, the workspace diagram is shown in figure 5. The advantage of Monte Carlo Method over ordinary numerical method is that it takes less time, but it has the disadvantage that it can only approach the real result and cannot get the real value completely. But after testing, as long as the data volume is large enough to obtain more accurate results.

7. Conclusion
In this study, the trajectory planning of the citrus picking robotic arm is studied with the PUMA560 robotic arm as the research object. First, the kinematics model of the manipulator was established, and its forward and inverse kinematics analysis and simulation verification were performed. Then, the trajectory planning of the robotic arm was initially explored, and the trajectory planning and simulation verification of the joint space were performed using the quintic polynomial interpolation method, and the simulation results reached the expected goals. Finally, the Monte Carlo Method was used to draw the cloud map of the working space of the robotic arm, which verified that the trajectory planning and workspace drawing methods used in this study for the trajectory planning of the citrus
picking robotic arm were effective, and provided the theoretical support for control study of the citrus picking robotic arm.

8. References

[1] Li Dongjie, Qiu Jiangyan, You Bo, et al. An optimization algorithm for robot trajectory planning[J]. Journal of Motor and Control, 2009, 13: 123-127 (in Chinese).

[2] Liu Huashan, Lai Xiaobo, Wu Wenxiang. Time-optimal and jerk-continuous trajectory planning for robot manipulators with kinematic constraints[J]. Robotics and Computer-Integrated Manufacturing, 2013, 29: 309-317

[3] Lu Yunhao, Yao Yuan. Kinematic analysis and verification of six-degree-of-freedom robotic arm[J]. Coal Technology, 2018, 37: 308-310 (in Chinese).

[4] Li Shirong, Mao Wenhua, Hu Xiaoaon. Kinematic analysis and trajectory planning simulation of grape bagging robot manipulator[J]. Robot Technique and Application, 2013, 34-38 (in Chinese).

[5] Jiang Hongchao, Liu Shirong, Zhang Botao. Inverse kinematics analysis of six-degree-of-freedom modular manipulator[J]. Journal of Zhejiang University (Engineering Science), 2010, 44: 1348-1354 (in Chinese).

[6] Fu Xiangxue, Han Shunjie, Zhang Dongdong, et al. Kinematic analysis and trajectory planning of packaging sorting manipulator based on D-H method[J]. Packaging and Food Machinery, 2008, 36: 47-50+41 (in Chinese).

[7] Cheng Zhengzhi. Time-optimal trajectory planning of six-joint industrial robot based on an adaptive genetic algorithm[D]. Anhui University of Technology, 2017 (in Chinese).