Genuine multipartite system-environment correlations in decoherent dynamics

Jonas Maziero and Fábio M. Zimmer
Departamento de Física, Universidade Federal de Santa Maria, 97105-900, Santa Maria, RS, Brazil

We propose relative entropy-based quantifiers for genuine multipartite total, quantum, and classical correlations. These correlation measures are applied to investigate the generation of genuine multipartite correlations in decoherent dynamics induced by the interaction of two qubits with local-independent environments. We consider amplitude- and phase-damping channels and compare their capabilities to spread information through the creation of many-body correlations. We identify changes in behavior for the genuine 4- and 3-partite total correlations and show that, contrary to amplitude environments, phase-noise channels transform the bipartite correlation initially shared between the qubits into genuine multipartite system-environment correlations.

I. INTRODUCTION

The nonlocality [1,2], nonseparability [3,4], and quantumness [5,6] of correlations in composite systems are currently recognized as important ingredients for the efficiency of protocols in quantum information science. Substantial progress has been achieved latterly regarding the characterization, identification, and quantification of total, classical, and quantum correlations [7,21]. The nonclassicality originated from local indistinguishability of quantum states is the more recent paradigm for analyzing correlations. This kind of correlation was studied mostly in the case of bipartite systems, with novel and interesting results already obtained. By its turn, the investigation of (genuine) multipartite quantum and classical correlations has been receiving its deserved attention only very recently [22,42]. In this article we are interested in this last scenario. We shall define measures for genuine n-partite correlations (Section II) and study how these correlations develop during decoherent system-environment dynamics (Section III).

A. The decoherence process

The thorough investigation of the decoherence process [44,46] is an important issue to be addressed towards large scale implementations of, for instance, quantum computers, quantum simulators, and communication protocols. The decoherence phenomenon is a result of the inevitable interaction between a quantum system and its surroundings. Let us consider a system $s$ and its environment $E$ in an initial product state $ρ_{sE} = ρ_s ⊗ |0_E⟩⟨0_E|$, where $|0_E⟩$ is the vacuum state of the environment. The dynamics of the whole closed system is unitary, i.e., $ρ_{sE}(t) = U_{sE}(t)ρ_{sE}U_{sE}^†(t)$ with $U_{sE}(t)U_{sE}^†(t) = I$, where $I$ is the identity operator. Then, by tracing out the environmental variables we write down the system’s evolved state in the Kraus or operator-sum representation: $E(ρ_s) = \sum_i K_iρ_sK_i^†$, where $K_i := ⟨i_E|U_{sE}(t)|0_E⟩$ are linear operators on the state space of the system, $H_s$, such that $\sum_i K_i^†K_i ≤ I$ and $\{|i_E⟩\}$ is a basis for the environment with $i$ specifying number of excitations that is distributed in all its modes. We observe that the quantum operation $E$ can be used to describe general transformations between quantum states [13,18].

One can verify that the dynamical map for the evolution of the system-environment state:

$$U_{sE}|ψ⟩ ⊗ |0_E⟩ = \sum_i K_i|ψ⟩ ⊗ |i_E⟩,$$

(1)

leads to the same motion equation for the system state as shown above. Thus, we can obtain the Kraus’ operators describing the noise channel acting on the system using a phenomenological approach [42,50] or via quantum process tomography [51,52] and use this information to investigate, for example, the system-environment correlations, without worry about the usually complicated structure of the environment.

B. A partial classification of quantum states

A possible, partial, classification of multipartite quantum states concerning its correlations, or with regard to the operations needed to generate such correlations, can be introduced as follows. For an n-partite system prepared in a product state vector $|ψ_{init}\rangle = |ψ_{01}\rangle···|ψ_{0n}\rangle$, any uncorrelated n-partite product state of this system,

$$ρ^n_{1···n} = ρ_1 ⊗ ··· ⊗ ρ_n,$$

(2)

can be created by means of local quantum operations (LQO). The sub-index $is$ in $|ψ_{init}\rangle$ specifies the state and subsystem, respectively, and we use throughout this article the notation $|ψ_{01}···ψ_{0n}\rangle := |ψ_{01}\rangle ⊗ ··· ⊗ |ψ_{0n}\rangle$.

Starting with the system in state $|ψ_{init}\rangle$, any n-partite classically-correlated state can be prepared via local classical operations (LCO) coordinated by the exchange of classical communication (CC) and has the form:

$$ρ^n_{1···n} = \sum_{i_1,···,i_n} p_{i_1,···,i_n}|ψ_{i_1}···ψ_{i_n}\rangle⟨ψ_{i_1}···ψ_{i_n}|,$$

(3)
where the states \( \{ |\psi_{is}\rangle \} \in \mathcal{H}_s \) form a complete \( (\sum_{is} |\psi_{is}\rangle \langle \psi_{is}| = \mathbb{I}_s) \) orthonormal \( (|\psi_{is}\rangle \langle \psi_{is}| = \delta_{ij}) \), and therefore distinguishable, basis for the subsystem \( s \).

Above, by LCO we mean (complete) transformations between the pointer basis states \([44]\). It can be seen that if the probability distribution in the state \( \rho^{1\cdots n}_{s} \) factorizes, i.e., \( \rho^{1\cdots n}_{s} = \rho_1 \cdots \rho_n \), there is no correlation at all in the system, that is to say, it is an \( n \)-partite product state. An \( n \)-partite separable but quantumly correlated state needs LQC and CC to be generated from \( |\psi_{\text{init}}\rangle \). This kind of state has the general form

\[
\rho^{1\cdots n}_{s} = \sum_i p_i \rho_1 \otimes \cdots \otimes \rho_n, \tag{4}
\]

with \( \{ p_i \} \) being a joint probability distribution and \( \{ \rho_{is} \} \) being noncommuting density operators. We note that if the density operators \( \rho_{is} \) commute for different \( i \), then the state \( \rho^{1\cdots n}_{s} \) is an \( n \)-partite classically-correlated state. In addition the probability distribution \( \{ p_i \} \) factorizes, then \( \rho^{1\cdots n}_{s} \) is an \( n \)-partite product state.

The \( n \)-partite entangled, or non-separable, states \( \rho^{1\cdots n}_{s} \) cannot be prepared locally, requiring direct or mediated interaction for its generation. One famous example of entangled state is the GHZ state \([43, 50]\):

\[
2^{-1/2}(|0_1 \cdots 0_n\rangle + |1_1 \cdots 1_n\rangle), \tag{5}
\]

where \( \{|0_s\rangle, |1_s\rangle\} \) is the one-qubit computational basis.

### C. Relative entropy-based measures of correlation

Considering the operational state classification of the last subsection and with the aim of quantifying the different kinds of correlation in an unified manner, Modi et al. \([41]\) introduced measures of correlation using the relative entropy \([55]\),

\[
S(\rho || \sigma) = \text{tr}(\rho \log_2 \rho - \log_2 \sigma), \tag{6}
\]

to estimate the “distance” between two states \( \rho \) and \( \sigma \). The total correlation in an \( n \)-partite state \( \rho_{1\cdots n} \) is quantified by how distinguishable or how distant it is from an uncorrelated \( n \)-partite product state \([41]\), i.e.,

\[
I(\rho_{1\cdots n}) = \min_{\rho^{1\cdots n}_{s}} S(\rho_{1\cdots n} || \rho^{1\cdots n}_{s}) = S(\rho_{1\cdots n} || \rho_1 \otimes \cdots \otimes \rho_n). \tag{7}
\]

The last equality was established in Ref. \([41]\) and shows that the closest \( n \)-partite product state of any state \( \rho_{1\cdots n} \) is obtained from its marginal states in the product form. Recalling the state classification presented in the previous paragraph, the quantum part of the correlation in \( \rho_{1\cdots n} \) can be defined as its minimal distance from \( n \)-partite classically-correlated states \([41]\):

\[
Q(\rho_{1\cdots n}) = \min_{\rho^{1\cdots n}_{s}} S(\rho_{1\cdots n} || \rho^{1\cdots n}_{s}) = S(\rho_{1\cdots n} || \chi^{1\cdots n}_{s}), \tag{8}
\]

with \( \chi^{1\cdots n}_{s} = \sum_{i_1 \cdots i_n} p_{i_1 \cdots i_n} |\psi_{i_1 \cdots i_n}\rangle \langle \psi_{i_1 \cdots i_n}| \) and \( \rho_{1\cdots i_1 \cdots i_n} = |\psi_{i_1 \cdots i_n}\rangle \langle \psi_{i_1 \cdots i_n}| \) \([41]\). In the second equality for \( Q \), we leave implicit the minimization over local basis needed to find the closest classically-correlated state \( \chi^{1\cdots n}_{s} \). Finally, the classical part of the correlations in \( \rho_{1\cdots n} \) is given by the total correlation of \( \chi^{1\cdots n}_{s} \) \([41]\):

\[
C(\rho_{1\cdots n}) = \min_{\rho^{1\cdots n}_{c}} S(\chi^{1\cdots n}_{s} || \rho^{1\cdots n}_{c}), \tag{9}
\]

and quantifies how distant \( \chi^{1\cdots n}_{s} \) is from being an uncorrelated state.

### D. Genuine multipartite correlations

Though the state classification and the correlation quantifiers presented above are very useful in several contexts, they are restricted in a certain sense because they classify the system state without considering the possibility of a more complex distribution for the correlations. For instance, in studying multipartite spin systems, which may have a far more intricate density operator with groups or cluster of spins in different families of states, it would be desirable to generalize the results presented in the last two paragraphs in order to include and quantify such complexity. As we will discuss in the sequence, the concept of genuine multipartite correlation fits well for the study of these more general scenarios and can be utilized as the starting point to define generalizations of the correlation quantifiers discussed above.

Addressing this subject, Bennett et al. (see Ref. \([43]\)) postulated that if an \( n \)-partite state has no genuine \( n \)-partite correlation, then we cannot create genuine \( n \)-partite correlation by adding a subsystem in a product state or by performing trace non-increasing local quantum operations. Also, we cannot create genuine \((n + 1)\)-partite correlation by splitting a subsystem in two. They proved that the following definitions satisfy these requirements. An \( n \)-partite state has genuine \( n \)-partite correlation only if it is non-product under any bipartite cut of the system. For \( k \leq n \), an \( n \)-partite state has genuine \( k \)-partite correlation only if there exists a subset of \( k \) subsystems presenting genuine \( k \)-partite correlation.

Bennett and collaborators also defined the degree of correlation of an \( n \)-partite state as the maximum number \( k \) of subsystems possessing genuine \( k \)-partite correlation.

## II. QUANTIFIERS FOR GENUINE MULTIPARTITE CORRELATIONS

The investigation of the different types of correlation presented in quantum states is one of the main problems in quantum information science. Recently the identification and quantification of genuine multipartite correlations (which are those correlations we cannot account
looking only to a part of the system) has been receiving its first studies. In this section we shall introduce definitions and quantifiers for genuine multipartite correlations by using the concepts reviewed in Section ID to generalize the measures discussed in Section IC.

From the definition given in Section ID it follows that an n-partite state has genuine n-partite correlation only if it is non-product under any bipartite cut of the system, i.e., if \( \rho_{1...n} \not\approx \rho_{c_1} \otimes \rho_{c_2} \), where \( c_1 (c_2) \) indicates a group with a number \( n_1 (n_2) \) of subsystems, and \( n_1 + n_2 = n \). In other words, if \( \rho_{1...n} \) possesses genuine n-partite correlation then there does not exist two completely uncorrelated clusters of particles in the system. Thus, a measure for genuine n-partite total correlation of an n-partite state can be logically defined as

\[
I_n(\rho_{1...n}) := \min_{(c_1, c_2)} S(\rho_{c_1} \otimes \rho_{c_2}),
\]

where the minimum is taken over all possible bi-partitions \((c_1, c_2)\) of the system. Therefore the quantity \( I_n \) measures the minimum distance between \( \rho_{1...n} \) and states that do not possess genuine n-partite correlation.

Starting from a possible generalization of Shannon's mutual information for tripartite systems, the quantifier for genuine n-partite total correlation \( I_n(\rho_{1...n}) \) was also noticed by Giorgi and colleagues in Ref. [42]. They also defined measures for genuine classical and quantum correlations using a quantum discord-like approach, i.e., assuming additivity of mutual information in classical and quantum correlations. As shown in Ref. [34], the complicated alternative definitions presented in Ref. [42] for genuine n-partite quantum and classical correlation of n-partite states do not coincide in general. In sequence of this article, we shall start from a direct extension of the operational classification of states discussed in Section ID and use relative entropy-based distinguishability measures to define simple and general correlation quantifiers for genuine multipartite correlations that are valid for any number of subsystems and for any dimension of its Hilbert spaces. It is also worthwhile to mention that, although we choose the relative entropy as a measure of distance, our quantifiers for correlation can be defined in a similar manner using other distance measures.

Regarding total correlations, we can also look to the structure of the system's correlation by defining a measure for genuine k-partite total correlation of an n-partite state:

\[
I_k(\rho_{1...n}) := \max_{\rho_k} I_k(\rho_k).
\]

In the last equation, the maximum is taken over all states \( \rho_k \) comprising k subsystems and aims to identify the most correlated k-partite group of particles in the system. For example, if \( k = n-1 \), \( I_{n-1}(\rho_{1...n}) \) is the maximal genuine \((n-1)\)-partite total correlation of the states obtained by tracing out one subsystem of \( \rho_{1...n} \).

This last quantifier, \( I_k(\rho_{1...n}) \), can be applied to study the degree of correlation (as defined in Section ID) of a system and can find application, for instance, in the study of quantum phase transitions in critical systems.

Based on the discussion about classically-correlated states and of the quantum correlation present in non-classical states, addressed in the introductory section, we propose the following definition.

**Definition 1.** An n-partite state \( \rho_{1...n} \) has genuine n-partite quantum correlation only if it is not a classical state under any bipartite cut of the system, viz., \( \rho_{1...n} \not\approx \chi^p_{c_1 c_2} \), with \( \chi^p_{c_1 c_2} = \sum_{i, c_1, c_2} p_{(c_1, i; c_2)} |\psi_{i c_1} \rangle \langle \psi_{i c_2}| \), where \( \{ p_{c_1, i, c_2} = \langle \psi_{i c_1} | \psi_{i c_2}| \rho_{1...n} |\psi_{i c_1} \rangle \psi_{i c_2} \} \) is a probability distribution and \( \{ |\psi_{i c_1} \rangle \} \) is an orthonormal basis for \( \mathcal{H}_{c_2} \).

From this definition, a quantifier of genuine n-partite quantum correlation of an n-partite state follows as

\[
Q_n(\rho_{1...n}) := \min_{(c_1, c_2)} S(\rho_{1...n} || \chi^p_{c_1 c_2}).
\]

\( Q_n(\rho_{1...n}) \) measures the minimum distance between \( \rho_{1...n} \) and the bipartite classical states \( \chi^p_{c_1 c_2} \). By definition, \( Q_n(\rho_{1...n}) \geq 0 \) with equality only in cases where \( \rho_{1...n} = \chi^p_{c_1 c_2} \).

Now a measure for genuine k-partite quantum correlation of an n-partite system is defined in an analogous manner to the genuine k-partite total correlation introduced above, namely:

\[
Q_k(\rho_{1...n}) := \max_{\rho_k} Q_k(\rho_k).
\]

Using this quantifier, we define the degree of quantumness, or degree of quantum correlation, of an n-partite system as the maximum number \( k \) of its subsystems that presents genuine k-partite quantum correlation.

One possible definition for genuine n-partite classical correlation of an n-partite state is given as follows.

**Definition 2.** An n-partite state \( \rho_{1...n} \) possesses genuine n-partite classical correlation only if its closest n-partite classical state \( \chi^c_{1...n} \) has genuine n-partite total correlation, namely, if the classical probability distribution \( p_{1...in} \) does not factorizes under any bipartite cut of \( \chi^c_{1...n} \), i.e., \( \rho_{1...n} \not\approx \rho_{c_1} \otimes \rho_{c_2} \).

Based on this definition, we propose the following quantifiers for genuine n-partite classical correlation:

\[
C_n(\rho_{1...n}) := I_n(\chi^c_{1...n}),
\]

and for genuine k-partite classical correlation:

\[
C_k(\rho_{1...n}) := \max_{\chi_k^c} I_k(\chi_k^c),
\]

of an n-partite state. The states \( \chi_k^c \) in the last equation are obtained by tracing out all but the chosen k subsystems of \( \chi^c_{1...n} \) and the maximization is made over all possibilities for \( \chi_k^c \).

Now, using \( C_k \), we can define the degree of classical correlation of an n-partite state as the maximum number \( k \) of subsystems possessing genuine k-partite classical correlation.
In the next section we will apply some of the quantifiers of genuine n-partite correlations introduced here to study the generation of genuine multipartite system-environment correlations in some decoherent dynamics.

III. SYSTEM-ENVIRONMENT CORRELATIONS IN DECOHERENT DYNAMICS

Let us consider a two-qubit system initially prepared in a Werner’s state
\[ \rho_{ab}^w = (1 - c)I_{ab}/4 + c|\psi_{ab}^\perp \rangle \langle \psi_{ab}^\perp |, \]
where \( |\psi_{ab}^\perp \rangle = (|0_s1_b\rangle - |1_s0_b\rangle)/\sqrt{2} \) and \( 0 \leq c \leq 1 \). This state has a rich structure with respect to correlations. It violates the CHSH inequality [57] for \( c \geq 1/2 \), it violates the Peres’ criterion for separability [58] when \( c > 1/3 \), and it has nonzero quantum discord [59] for all \( c \neq 0 \). So, any possible qualitative difference in the multipartite system-environment correlations due to system’s initial correlations (i.e., if the system state is nonlocal, nonseparable, discordant or classical) would be observed using the Werner’s state as the system’s initial state.

Now these two sub-systems are let to interact locally with two independent environments in the vacuum state \( |0_{Es} \rangle \), where \( s = a, b \). So, the initial state of the whole system is
\[ \rho_{abE_sE_b} = \rho_{ab}^w \otimes |0_{Es} \rangle \langle 0_{Es}| \otimes |0_{Es} \rangle \langle 0_{Es}|. \]

For this system, it was shown in Ref. [60] that, in contrast to dissipative interactions, the decoherent dynamics of the two qubits under phase-damping or Pauli channels does not generate entanglement between the systems and its respective environments or between the two environments. The bipartite quantum discord created in such a dynamics was then indicated as the mechanism for the leakage of quantum information out of the systems. Moreover, the initial quantum correlation between the two qubits was shown to be not transfered to the environments, seeming to evaporate.

Here we extend these results by studying the generation of genuine multipartite system-environment correlations. We show that the quantum correlations initially shared between the qubits do not disappear, but are transformed into genuine multipartite correlations between systems and environments. Furthermore, these correlations present an interesting dynamics with sudden changes in behavior. For more results related to the sudden-change phenomenon see Refs. [60–64]. Further works considering the dynamics of system-environment correlations can be found in Refs. [63–70].

A. Amplitude-damping channels

We begin by studying the situation in which the two qubits, \( a \) and \( b \), evolve under the influence of local-independent amplitude-damping channels. The Kraus’ operators for a dissipative reservoir at zero temperature are \( K_0 = |0_s\rangle \langle 0_s| + \sqrt{1 - p_s}|1_s\rangle \langle 1_s| \) and \( K_1 = \sqrt{p_s}|0_s\rangle \langle 1_s| + \sqrt{p_s}|1_s\rangle \langle 0_s| \), where \( p_s \) is a parametrization of time for the subsystem \( s \), with \( p = 0 \) corresponding to \( t = 0 \) and \( p = 1 \) being equivalent to \( t \rightarrow \infty \) [71]. Throughout this article we consider identical environments and consequently \( p_s = p_b := p \). Thus, by using the Kraus operators shown above, we obtain the dynamical map for the system-environment evolution:
\[ U_{sE_s}|0_s0_{Es}\rangle = |0_s0_{Es}\rangle, \]
\[ U_{sE_s}|1_s0_{Es}\rangle = \sqrt{1 - p_s}|1_s0_{Es}\rangle + \sqrt{p_s}|0_s1_{Es}\rangle. \]

Utilizing this map, we find the evolved state for the whole system:
\[ \rho_{abE_aE_b}^{ad}(p) = (1 - c)\xi_{ad}(p) + c|\Upsilon_{ad}(p)\rangle \langle \Upsilon_{ad}(p)|, \]
with \( \xi_{ad}(p) \) and \( |\Upsilon_{ad}(p)\rangle \) given in Appendix A.

A detailed analysis of the bipartite correlations can be found in Ref. [59].

We observe that, due to the assumed symmetries of the systems and environments, the following partitions are equivalent in what refer to correlations: \( aE_a \equiv bE_b, \ aE_b \equiv bE_a, \ abE_a \equiv abE_b, \) and \( aE_aE_b \equiv bE_bE_a \). Considering these symmetries we have, from the definition of genuine n-partite total correlation introduced in Section II, that
\[ I_d(\rho_{aE_a,bE_b}) = \min_{(c_1,c_2)} (S(\rho_{c_1}) + S(\rho_{c_2})) - S(\rho_{ad_{aE_a,bE_b}}), \]
with the following possible bi-partitions of the system: \( (c_1,c_2) = (a,E_a,bE_b), (E_a,abE_b),(ab,E_aE_b),(aE_a,bE_b) \).
qubits and its respective reservoirs up to a certain instant. We see that, in general, multipartite correlations are genuine total correlations discussed above are shown in Fig. 2. Multipartite system-environment quantum correlations for local amplitude-damping channels. For \( p = 1 \) the systems state is \( |0_s\rangle \) and its initial correlations were altogether transferred to the environments.

Figure 2: Multipartite system-environment quantum correlations for local amplitude-damping channels. For \( p = 1 \) the systems state is \( |0_s\rangle \) and its initial correlations were altogether transferred to the environments.

Figure 3: Genuine multipartite system-environment classical correlation for local amplitude-damping channels. The generic behavior of these correlations is similar to that of genuine multipartite total correlation.

\((aE_a, bE_a)\). In the last equation \( S(\rho) = -\text{tr}(\rho \log_2 \rho) \) is the von Neumann entropy.

The 3-partite genuine total correlation of the whole system is given by

\[
I_3(\rho_{aE_a,bE_a}^{ad}) = \max(I_3(\rho_{aE_a,b}^{ad}), I_3(\rho_{aE_a}^{ad}))
\]

with analogous bi-partitions used to compute the genuine 3-partite correlation of the 3-partite states. All genuine total correlations discussed above are shown in Fig. 1. We see that, in general, multipartite correlations are generated during the dissipative interaction between the qubits and its respective reservoirs up to a certain instant of time from which such correlations suddenly begin to decrease going to zero in the asymptotic time \( p = 1 \).

We also calculated the 3- and 4-partite quantum correlations and genuine classical correlations of the system, which are presented in Figs. 2 and 3 respectively. As expected, the asymptotic behavior of the genuine 3- and 4-partite classical correlations is similar to that of genuine multipartite total correlation. We observe that the quantum correlation remaining at \( p = 1 \) is due solely to the environments, once the systems state in this limit is \( |0_s\rangle \).

It is worthwhile mentioning that, although explicit parametrizations for states and unitary operators of systems with dimension greater than two are possible in principle [72], all the important aspects we want to emphasize here can be addressed without computing the genuine multipartite quantum correlations. We leave related issues for future investigations.

B. Phase-damping channels

Let us consider the dynamics of two qubits under local phase-damping channels. This kind of noise environment causes loss of phase relations in the system without exchange of energy. The Kraus’ operators for these channels are given by: \( K_0 = |0_s\rangle \langle 0_s| + \sqrt{1-p}|1_s\rangle \langle 1_s| \) and \( K_1 = \sqrt{p}|1_s\rangle \langle 1_s| \). Thus, the following map for the system-environment evolution is obtained

\[
U_{sE}|0_s0_{E_s}\rangle = |0_s, 0_{E_s}\rangle,
\]

\[
U_{sE}|1_s0_{E_s}\rangle = |1_s\rangle \otimes \left( \sqrt{1-p} |0_{E_s}\rangle + \sqrt{p} |1_{E_s}\rangle \right).
\]

In a correspondent manner as we did for amplitude-damping channels, we use the map shown in the last equation to compute the global evolved state and then calculate its correlations. In this case, \( \rho_{aE_a,bE_a}^{pd}(p) \) is given as in Eq. (19) but with \( t_{pd}(p) \) and \( |\Upsilon_{pd}(p)| \) presented in Appendix B. The genuine total correlations generated in the evolution under local phase environments are shown in Fig. 1. These correlations also exhibit the sudden-change phenomenon.
there exists multipartite quantum correlation involving the whole system in this asymptotic limit. However, as can be observed in Fig. 5, there is no or feeble quantumness in the genuine tripartite correlations in the asymptotic limit while there still exists multipartite quantum correlation involving the whole system during the system-environment interaction.

In sharp contrast to the case of amplitude-damping channels, we see that the dynamics induced by phase environments does generate genuine multipartite correlations in the asymptotic limit $p = 1$. In this limit, the whole system will be correlated in a degree that is proportional to the purity of the two-qubit initial state.

We also calculated the multipartite quantum correlations and the genuine 3- and 4-partite classical correlations, which are presented in Figs. 6 and 7 respectively. It is interesting that the genuine 3-partite classical and total correlations range from zero to one for this kind of environment. This fact indicates that the system presents no or small genuine 3-partite quantum correlation. As can be seen in Figs. 6 and 7, this situation changes in the case of 4-partite correlations. In fact, we observe in Fig. 7 that tripartite quantum correlations are created during the system’s evolution but disappear in the asymptotic limit while there are 4-partite quantum correlation remnant at $p = 1$.

IV. CONCLUDING REMARKS

Summing up, we introduced operational definitions and quantifiers for genuine $n$- and $k$-partite total, quantum, and classical correlations of an $n$-partite state. We also used our correlation quantifiers to define the degree of correlation, the degree of quantumness, and the degree of classical correlation of a physical system. Using these correlation measures, we showed that, in contrast to amplitude-damping channels, for which the initial correlations between the qubits are simply transferred to the environments, phase noise channels turn such bipartite correlations into genuine multipartite system-environment total, quantum, and classical correlations.

Now, in order to get a better grasp of these results, let us look at the kind of state generated during the evolution of the system under different kinds of noise environment. It is straightforward to verify that if the system initial state is $|\psi_{GHZ}\rangle$ (i.e., if $c = 1$), the whole system state for amplitude-damping channels and $p = 1/2$ is

$$|\Upsilon_{ad}(1/2)\rangle = (|0001\rangle + |0010\rangle - |0100\rangle - |1000\rangle)/2$$
$$:= |\psi_{W}\rangle,$$

which is equivalent, modulo local rotations, to the well known four-qubit W state $|W\rangle$. For phase noise environments and $p = 1$, it follows that

$$|\Upsilon_{pd}(1)\rangle = (|0011\rangle - |1100\rangle)/\sqrt{2}$$
$$:= |\psi_{GHZ}\rangle,$$

which is, also modulo local unitaries, equivalent to a GHZ state (see Section 1).

For any $c$ and $p$, we obtain the following expressions for the fidelity,

$$F(|\psi\rangle, \rho) := \sqrt{\langle \psi | \rho | \psi \rangle},$$
between these states and its corresponding evolved density operators:

\[ F(\psi_W, \rho_{ad}^{\text{GHZ}}(p)) = \sqrt{\frac{(1 + 3c)(1 + 2\sqrt{p(1-p)})}{8}} \]

and

\[ F(\psi_{\text{GHZ}}, \rho_{ad}^{\text{GHZ}}(p)) = \sqrt{(1 + 3c)p/2}. \]

These fidelities are shown in Fig. 7. We also observe that the mutual information is invariant under local unitary transformations and, as one can easily verify, the genuine total correlations for the \( W \) and \( \text{GHZ} \) states are given by: \( I_4(|W\rangle) = I_4(|\text{GHZ}\rangle) = 2 \), \( I_3(|W\rangle) = 0.81 \), and \( I_3(|\text{GHZ}\rangle) = 1 \). Moreover, if we trace out a subsystem of a \( \text{GHZ} \) state, the obtained tripartite density operator has zero quantum correlations. The same action in a \( W \) state produces a 3-partite system possessing quantumness in its correlations. Thus, the values of the fidelities (shown in Fig. 7) and the structure of \( W \) and \( \text{GHZ} \) states with respect to its correlations help us to partially explain the general behavior of the correlations that we presented and discussed in the last section. In reality, both kinds of decoherent dynamics generated genuine multipartite correlations. The main difference is that for the amplitude-damping channel these correlations are null for \( p = 1 \) while for phase environments they generally reach its maximum value in this limit.

Thorough investigations about the decoherence process are essential for us to obtain a better understanding of the phenomenon per se and also for the development of methods to circumvent it in the path for large scale implementations of protocols in quantum information science. From the fundamental point of view, before 2010 one believed that the flow of coherent information from the system to the environment was caused by the creation of entanglement between the two. Nevertheless, considering bipartite correlations, one of us and colleagues showed, in Ref. [57], that this is not the case in general. For some composite systems interacting with local-independent phase-damping channels (or Pauli channels), it was shown that the systems lose its initial coherent phase relations but only bipartite non-classical correlation of separable states are generated during such decoherent dynamics. Another interesting finding reported in this article was the fact that the initial bipartite quantum correlation between the systems (non-local, non-separable, and discordant) evaporated, i.e., only bipartite classical correlations was present at the asymptotic time of evolution. In the present manuscript, besides proposing definitions and introducing quantifiers for genuine multipartite correlations, we studied the many-body correlations generated for some important decoherent processes. Our investigation extended the previous ones in several directions, helping us to understand the global structure of the states generated during these evolutions and also explaining why the bipartite quantum correlations disappeared in the asymptotic time, by showing that they are transformed into genuine multipartite correlations. Moreover, we showed that the genuine multipartite total correlation may also exhibits a sudden change in its evolution rate.

As continuation of the present work, besides the actual calculation of the genuine multipartite quantum correlations, other interesting topic for future research is considering the dynamics of such correlations for the composition of both phase and (generalized) amplitude channels, which is a common situation in nature [54, 61]. In this case, the global state generated during the evolution under local environments will be a mixture involving \( W \) and \( \text{GHZ} \) components. The extension of these results for global and correlated environments [74] and to continuous variables systems [77] is also worth pursuing.

It is also important to mention that the first experiment with complete tomography of the environment’s state has been successfully performed recently using an optical system [76]. Therefore, all the theoretical results presented here can be verified experimentally with current technology.

Acknowledgments

The authors acknowledge financial support from the Brazilian funding agencies Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Conselho Nacional de Desenvolvimento Científico e Tecnológico, and Fundação de Amparo à Pesquisa do Estado do Rio Grande do Sul. J.M. thanks Marcelo S. Sarandy and Gabriel H. Aguilar for helpful discussions. We acknowledge a referee for his (her) constructive comments.

Appendix A: Evolved State for Local Amplitude-Damping Channels

For this channels, the global evolved state is given by Eq. (19) with

\[ 4\psi_{ad}(p) = |0\rangle\langle 0| + (1 - p)(|2\rangle\langle 2| + |8\rangle\langle 8|) + p^2|5\rangle\langle 5| + (1 - p)^2|10\rangle\langle 10| + p(1 - p)(|6\rangle\langle 6| + |9\rangle\langle 9| + p|4\rangle\langle 4| + |1\rangle\langle 1| + \sqrt{p(1 - p)}(|8\rangle\langle 1| + |1\rangle\langle 2| + h.c.) + \sqrt{p^2(1 - p)^2(|6\rangle\langle 9| + |9\rangle\langle 6| + h.c.) + p(1 - p)(|5\rangle\langle 10| + + |6\rangle\langle 9| + h.c.) + \sqrt{p^2(1 - p)(|5\rangle\langle 6| + |9\rangle\langle 9| + h.c.)} \]

and

\[ \psi_{ad}(p) = \frac{1 - p}{2}(|2\rangle - |8\rangle) + \sqrt{\frac{p}{2}}(|1\rangle - |4\rangle). \]

Above, and in the next appendix, h.c. refers to the Hermitian conjugate and we use the decimal representation for the indexes of the computational bases states.
Appendix B: Evolved State for Local Phase-Damping Channels

For phase environments, the global evolved state is given as in Eq. (19) but with

$$4t_{pd}(p) = |0⟩⟨0| + (1-p)(|2⟩⟨2| + |8⟩⟨8|)
+ p(|3⟩⟨3| + |12⟩⟨12|) + (1-p^2)|10⟩⟨10|
+ p(1-p)(|11⟩⟨11| + |14⟩⟨14|) + p^2|15⟩⟨15|
+ √p(1-p)|2⟩⟨2| + |3⟩⟨3| + |12⟩⟨12| + h.c.)

and

$$|Ψ_{pd}(p)⟩ = \sqrt{\frac{1-p}{2}}(|2⟩ - |8⟩) + \sqrt{\frac{p}{2}}(|3⟩ - |12⟩).$$

---

[1] N. D. Mermin, Rev. Mod. Phys. 65, 803 (1993).
[2] A. Aspect, arXiv:quant-ph/0402001v1.
[3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. 81, 865 (2009).
[4] O. Gühne and G. Tóth, Phys. Rep. 474, 1 (2009).
[5] L. C. Céleri, J. Maziero, and R. M. Serra, Int. J. Quant. Inf. 9, 1837 (2011).
[6] K. Modi, A. Brodutch, H. Cable, T. Paterek, and V. Vedral, arXiv:1112.6238v1.
[7] B. Groisman, S. Popescu, and A. Winter, Phys. Rev. A 72, 032317 (2005).
[8] L. Henderson and V. Vedral, J. Phys. A: Math. Gen. 34, 6899 (2001).
[9] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001).
[10] M. Horodecki, P. Horodecki, R. Horodecki, J. Oppenheim, A. Sen(De), U. Sen, and B. Synak-Radtke, Phys. Rev. A 71, 062307 (2005).
[11] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. 100, 090502 (2008).
[12] D. Cavalcanti, L. Aolita, S. Boixo, K. Modi, M. Piani, and A. Winter, Phys. Rev. A 83, 032324 (2011).
[13] V. Madhok and A. Datta, Phys. Rev. A 83, 032323 (2011).
[14] A. Ferraro, L. Aolita, D. Cavalcanti, F. M. Cucchietti, and A. Acín, Phys. Rev. A 81, 052318 (2010).
[15] W. H. Zurek, Phys. Rev. A 67, 012320 (2003).
[16] G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010).
[17] P. Giorda and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010).
[18] M. F. Cornelio, M. C. de Oliveira, and F. F. Fanchini, Phys. Rev. Lett. 107, 020502 (2011).
[19] A. Streitsoy, H. Kampermann, and D. Bruss, Phys. Rev. Lett. 106, 160401 (2011).
[20] J. Maziero and R. M. Serra, Int. J. Quant. Inf. 10, 1250028 (2012).
[21] G. H. Aguilar, O. Jiménez Farias, J. Maziero, R. M. Serra, P. H. Souto Ribeiro, and S. P. Walborn, Phys. Rev. Lett. 108, 063601 (2012).
[22] D. Kaszikowski, A. Sen(De), U. Sen, V. Vedral, and A. Winter, Phys. Rev. Lett. 101, 070502 (2008).
[23] Z. Walczak, Phys. Lett. A 374, 3999 (2010).
[24] A. Grudka, M. Horodecki, P. Horodecki, and R. Horodecki, arXiv:0802.1633v1.
[25] A. SaiToh, R. Rahimi, and M. Nakahara, Int. J. Quant. Inf. 6, 787 (2008).
[26] C. C. Rulli and M. S. Sarandy, Phys. Rev. A 84, 042109 (2011).
[27] I. Chakraborty, P. Agrawal, and A. K. Pati, Eur. Phys. J. D 65, 605 (2011).
[28] M. Okrasa and Z. Walczak, EPL 96, 60003 (2011).
[29] P. Parashar and S. Rana, Phys. Rev. A 83, 032301 (2011).
[30] S. Campbell, L. Mazzola, and M. Paternostro, Int. J. Quant. Inf. 9, 1685 (2011).
[31] G. L. Giorgi and T. Busch, arXiv:1206.1726v1.
[32] B. Li, L. C. Kwek, and H. Fan, arXiv:1205.6016v1.
[33] A. Saguia, C. C. Rulli, Thiago R. de Oliveira, and M. S. Sarandy, Phys. Rev. A 84, 042123 (2011).
[34] K. Modi, T. Paterek, W. Son, V. Vedral, and M. Williamson, Phys. Rev. Lett. 104, 080501 (2010).
[35] G. L. Giorgi, B. Bellomo, F. Galve, and R. Zambrini, Phys. Rev. Lett. 107, 190501 (2011).
[36] C. H. Bennett, A. Grudka, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. A 83, 012312 (2011).
[37] W. H. Zurek, Rev. Mod. Phys. 75, 715 (2003).
[38] H. P. Breuer and F. Petruccione, The Theory of Open Quantum Systems (Oxford University Press, Oxford, 2002).
[39] M. Schlosshauer, Decoherence and the Quantum-to-Classical Transition (Springer, Berlin, 2007).
[40] M. A. Nielsen and I. L. Chuang, Quantum Information and Quantum Computation (Cambridge University Press, Cambridge, 2000).
[41] J. Preskill, Quantum Information and Computation, [http://theory.caltech.edu/people/preskill/ph229/].
D. M. Greenberger, M. A. Horne, and A. Zeilinger, arXiv:0712.0921v1.

D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 82, 1345 (1999).

V. Vedral, Rev. Mod. Phys. 74, 197 (2002).

S. Sachdev, Quantum Phase Transitions (Cambridge University Press, New York, 2011).

J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).

J. Maziero, T. Werlang, F. F. Fanchini, L. C. Céleri, and R. M. Serra, Phys. Rev. A 81, 022116 (2010).

J. Maziero, L. C. Céleri, R. M. Serra, and V. Vedral, Phys. Rev. A 80, 044102 (2009).

R. Auccaise, L. C. Céleri, D. O. Soares-Pinto, E. R. deAzevedo, J. Maziero, A. M. Souza, T. J. Bonagamba, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, Phys. Rev. Lett. 107, 140403 (2011).

J.-S. Xu, X.-Y. Xu, C.-F. Li, C.-J. Zhang, X.-B. Zou, and G.-C. Guo, Nat. Commun. 1, 7 (2010).

L. Mazzola, J. Piilo, and S. Maniscalco, Phys. Rev. Lett. 104, 200401 (2010).

M. F. Cornelio, O. Jiménez Fariás, F. F. Fanchini, I. Frerot, G. H. Aguilar, M. O. Hor-Meyll, M. C. de Oliveira, S. P. Walborn, A. O. Caldeira, and P. H. Souto Ribeiro, arXiv:1203.5068v1.