Unifying flipped $SU(5)$ in five dimensions

S.M. Barr and Ilja Dorsner

Bartol Research Institute
University of Delaware
Newark, DE 19716

Abstract

It is shown that embedding a four-dimensional flipped $SU(5)$ model in a five-dimensional $SO(10)$ model, preserves the best features of both flipped $SU(5)$ and $SO(10)$. The missing partner mechanism, which naturally achieves both doublet-triplet splitting and suppression of $d = 5$ proton decay operators, is realized as in flipped $SU(5)$, while the gauge couplings are unified as in $SO(10)$. The masses of down quarks and charged leptons, which are independent in flipped $SU(5)$, are related by the $SO(10)$. Distinctive patterns of quark and lepton masses can result. The gaugino mass $M_1$ is independent of $M_3$ and $M_2$, which are predicted to be equal.

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*Electronic address: smbarr@bxclu.bartol.udel.edu
†Electronic address: dorsner@physics.udel.edu
I. INTRODUCTION

A beautiful feature of flipped \( SU(5) \) is that it provides a natural setting for the missing partner mechanism. This mechanism, when implemented in flipped \( SU(5) \), not only solves the doublet-triplet splitting problem but also allows one to avoid entirely the Higgsino-mediated proton decay that is such a difficulty for supersymmetric grand unified theories (SUSY GUTs). On the other hand, flipped \( SU(5) \) gives up one of the most attractive features of grand unification, namely unification of gauge couplings, because it is based on the group \( SU(5) \times U(1) \). Another drawback of flipped \( SU(5) \) models is that the masses of down quarks and charged leptons come from different operators, so that one does not obtain the relation \( m_b(M_{GUT}) = m_\tau(M_{GUT}) \). The unification of gauge couplings and relations between down quark masses and charged lepton masses could be recovered by embedding the group \( SU(5) \times U(1) \) in the simple group \( SO(10) \). However, in that case, the missing partner mechanism no longer works, since the partner that was missing from the \( SU(5) \) multiplets is present in the larger \( SO(10) \) multiplets.

One thus has somewhat of a quandary. The point of this paper is that a way out of this quandary is provided by unification in five dimensions. We show that if the group \( SO(10) \) in five dimensions is broken by orbifold compactification to the group \( SU(5) \times U(1) \) in four dimensions it is possible to have at the same time the good features of both flipped \( SU(5) \) and of \( SO(10) \). The essential reason is that if \( SO(10) \) is broken by the orbifold compactification then the fields of the effective four-dimensional theory need not be in complete \( SO(10) \) multiplets. This means that at the four-dimensional level the famous missing partners can still be missing and the doublet-triplet splitting can be achieved without the dangerous Higgsino-mediated proton decay. On the other hand, because there is \( SO(10) \) at the five-dimensional level, there is approximate unification of gauge couplings, and there is also the possibility of getting \( SO(10) \)-like Yukawa couplings for the quarks and leptons.

By now there are many models that use orbifold compactification of extra dimensions to break grand unified symmetries. The first such models showed that with one extra dimension it is possible to construct \( SU(5) \) models which have natural doublet-triplet splitting and no problem with the \( d = 5 \) proton decay operators that plague four-dimensional SUSY GUTs. The breaking of grand unified symmetries by orbifold compactification of a single extra dimension does not reduce the rank of the group. Thus
to break $SO(10)$ all the way to the Standard Model by orbifold compactification requires at least two extra dimensions. Interesting six-dimensional $SO(10)$ models have been constructed in several papers \[12, 13, 14\]. However, it is also possible that the breaking from the grand unified group to the Standard Model is achieved by a combination of orbifold compactification and the conventional four-dimensional Higgs mechanism. That allows the construction of realistic $SO(10)$ models with only a single extra dimension, as was shown by Dermišek and Mafi \[15\]. In their model the theory in the five-dimensional bulk has $\mathcal{N} = 1$ supersymmetry and gauge group $SO(10)$. Orbifolding breaks $SO(10)$ to the Pati-Salam \[16\] symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$. The orbifold has two inequivalent fixed points $O$ and $O'$. On $O$ there is a full $SO(10)$ symmetry, but on $O'$ only the Pati-Salam group. On the brane at $O$ the conventional Higgs mechanism breaks $SO(10)$ down to $SU(5)$. Thus the unbroken symmetry in the low-energy theory in four dimensions is the intersection of $SU(5)$ and the Pati-Salam group, i.e. the Standard Model group.

The model we shall present is similar in some ways to that of Dermišek and Mafi but differs from it in several important respects. Whereas they use orbifold compactification to break to the Pati-Salam group and Higgs fields on the brane $O$ to break to $SU(5)$, we shall use orbifold compactification to break to $SU(5) \times U(1)$ and Higgs fields in the bulk to break the rest of the way to the Standard Model. They use orbifold breaking to split the doublets from the triplets, whereas we use the four-dimensional flipped-$SU(5)$ missing partner mechanism.

II. MISSING PARTNERS IN FOUR DIMENSIONS

Before we consider higher dimensional theories we shall briefly review the missing partner mechanism in four-dimensional theories, showing why it works in flipped $SU(5)$ but not in $SO(10)$.

A. Flipped $SU(5)$

First recall what happens in ordinary (i.e. Georgi-Glashow) $SU(5)$ \[17\]. In ordinary $SU(5)$ the two Higgs doublets of the MSSM, which we shall denote $\mathbf{2}$ and $\overline{\mathbf{2}}$, have color-triplet partners, which we shall denote $\mathbf{3}$ and $\overline{\mathbf{3}}$. (We use this shorthand notation for Standard
Model representations: $\mathbf{2} \equiv (1,2,-\frac{1}{2}), \overline{\mathbf{2}} \equiv (1,2,\frac{1}{2}), \mathbf{3} \equiv (3,1,\frac{1}{3}), \overline{\mathbf{3}} \equiv (3,1,-\frac{1}{3})$. These are contained in fundamental and anti-fundamental multiplets of $SU(5)$: $\mathbf{5} = \mathbf{2} + \mathbf{3}$ and $\overline{\mathbf{5}} = \overline{\mathbf{2}} + \overline{\mathbf{3}}$. A combination of an $SU(5)$-singlet mass term and a Yukawa coupling to a Higgs in the adjoint representation, can (with suitable fine-tuning) give GUT-scale mass to the triplet partners while leaving the MSSM Higgs doublets light. This can be represented schematically as

\[
\begin{pmatrix}
\mathbf{3} \\
\mathbf{2}
\end{pmatrix}
\parallel
\begin{pmatrix}
\overline{\mathbf{3}} \\
\overline{\mathbf{2}}
\end{pmatrix}
\parallel
\mathbf{5}
\overline{\mathbf{5}}
\]

where the solid horizontal line represents a large Dirac mass $M_3$ connecting the colored Higgsinos $\mathbf{3}$ and $\overline{\mathbf{3}}$. It is well-known that the exchange of these colored Higgsinos gives a dangerous $d = 5$ proton-decay operator, as shown in Fig. 1. From the figure one sees that this proton decay amplitude is proportional to the mass connecting $\mathbf{3}$ to $\overline{\mathbf{3}}$. Suppressing this proton decay therefore requires severing this connection. This can be done by introducing an extra pair of Higgs multiplets $\mathbf{5}' + \overline{\mathbf{5}}'$, so that the triplets in the unprimed multiplets get mass not with each other but with the triplets in the primed multiplets as shown in the following diagram

\[
\begin{pmatrix}
\mathbf{3} \\
\mathbf{2}
\end{pmatrix}
\parallel
\begin{pmatrix}
\mathbf{3}' \\
\overline{\mathbf{2}}'
\end{pmatrix}
\parallel
\begin{pmatrix}
\mathbf{3}' \\
\overline{\mathbf{2}}'
\end{pmatrix}
\parallel
\mathbf{5}
\mathbf{5}'
\overline{\mathbf{5}}'
\overline{\mathbf{5}}
\]

If the MSSM Higgs doublets are the unprimed ones, then one sees that their colored partners are not connected to each other by a mass term, so that the $d = 5$ proton-decay amplitude vanishes. Unfortunately, however, there is an extra pair of doublets that remains light, namely the primed ones. The effect of these on the renormalization group equations would destroy gauge coupling unification. To give the needed superheavy mass to these doublets one could introduce a term $M\overline{\mathbf{5}}\mathbf{5}'$; however, this would give mass terms connecting not
only 2′ to 2 but also 3′ to 3′ (indicated by dotted lines in the previous diagram) and thus indirectly (after the primed triplets were integrated out) reconnecting 3 to 3 and bringing back the dangerous $d = 5$ proton decay amplitude.

Now let us turn to flipped $SU(5)$ and see how it avoids these problems very elegantly \[^3\]. In flipped $SU(5)$ models one has Higgs fields in the following representations of $SU(5) \times U(1)$:

- $h = 5^{-2}$, $\bar{h} = 5^{-2}$, $H = 10^{1}$, and $\bar{H} = 10^{-1}$.

Under the Standard Model group these decompose as follows,

- $h = \bar{2} + 3$, $\bar{h} = 2 + \bar{3}$,
- $H = 3 + (3, 2, \frac{1}{6}) + (1, 1, 0)$, and
- $\bar{H} = 3 + (\bar{3}, 2, -\frac{1}{6}) + (1, 1, 0)$.

The Higgs superpotential contains the terms $h \bar{h} H$.

When the Standard Model singlets $(1, 1, 0)$ in $H$ and $\bar{H}$ acquire vacuum expectation values (VEVs) they break $SU(5) \times U(1)$ down to the Standard Model group and they also give mass to the triplet Higgs. Schematically,

$$
\begin{pmatrix}
3 \\
\bar{2} \\
\text{other}
\end{pmatrix} \quad \text{and} \quad
\begin{pmatrix}
3 \\
\bar{3} \\
\text{other}
\end{pmatrix}
$$

where, for simplicity, $(3, 2, \frac{1}{6}) + (1, 1, 0) \equiv \text{other}$. The triplets in $h$ and $\bar{h}$ get mass with those in $H$ and $\bar{H}$. However the doublets in $h$ and $\bar{h}$ remain massless because there are no doublets in $H$ and $\bar{H}$ for them to mate with — thus the name “missing partner mechanism”.

At first glance one might worry that the same problem arises here as in the ordinary $SU(5)$ case discussed previously. The multiplets $5'$ and $\bar{5}'$ there played the same role as the multiplets $H$ and $\bar{H}$ here. And we saw that one could not give mass to the doublets in $5'$ and $\bar{5}'$ without reintroducing the dangerous proton decay amplitude. This leads to the question whether there is not an analogous difficulty in giving mass to some of the components of $H$ and $\bar{H}$, and specifically to the $(3, 2, \frac{1}{6}) + (1, 1, 0) + (\bar{3}, 2, -\frac{1}{6}) + (1, 1, 0)$, since here also an explicit mass term $M\bar{H}H$ would reintroduce the problem of proton decay. The beautiful answer is that these “other” components of $H$ and $\bar{H}$ do not have to get mass. Indeed, they must not get mass, because they are the goldstone modes that get eaten when $SU(5) \times U(1)$ breaks to $SU(3) \times SU(2) \times U(1)$. In other words, the fact that $SU(5) \times U(1)$ breaks to the Standard Model group guarantees that there is no mass connecting $H$ and $\bar{H}$ and therefore guarantees the absence of the $d = 5$ proton decay amplitude.
B. $SO(10)$

Now let us see why embedding flipped $SU(5)$ in $SO(10)$ in four dimensions destroys the beautiful missing partner solution to the doublet-triplet splitting and proton decay problems that we have just reviewed.

In $SO(10)$ the simplest possibility is that the terms $h H H + \overline{h} \overline{H} \overline{H}$ come from the terms $10 16 16 + 10 \overline{16} \overline{16}$, where $10 = \overline{h} + h$, $16 = H + \overline{h}' + 1^5$, and $\overline{16} = \overline{H} + h' + 1^{-5}$. Here $h' = 5^3$ and $\overline{h}' = 5^{-3}$. The problem is that the doublet partners that were missing from $H$ and $\overline{H}$ are now present in $\overline{h}'$ and $h'$.

The terms $10 16 16 + 10 \overline{16} \overline{16}$ contain not only $h H \langle H \rangle + \overline{h} \overline{H} \langle \overline{H} \rangle$ but also $h h' \langle H \rangle + h' h' \langle \overline{H} \rangle$. These latter terms mate the doublet Higgs in $h$ and $\overline{h}$ with those in $h'$ and $\overline{h}'$, destroying the solution of the doublet-triplet splitting problem.

A possible remedy to this difficulty suggests itself. One can have $h$ and $\overline{h}$ come from different $10$s of $SO(10)$. Let us examine what happens in this case, since it will be directly relevant to what we shall do in five dimensions later. Suppose there are two vector Higgs representations, denoted $10_1$ and $10_2$, with couplings $10_1 16 16 + 10_2 \overline{16} \overline{16}$. We write $10_1 = h_1 + \overline{h}_1$ and $10_2 = h_2 + \overline{h}_2$. Suppose that the two light doublets of the MSSM lie in $h_1$ and $\overline{h}_2$; then the triplet partners of these light doublets will obtain mass from the terms $h_1 H \langle H \rangle + \overline{h}_2 \overline{H} \langle \overline{H} \rangle$. The terms that give superlarge mass to doublets, and which correspond to those we found troubling before, are $\overline{h}_1 h' \langle H \rangle + h_2 h' \langle \overline{H} \rangle$. These do not give superlarge mass to the MSSM doublets, but to the doublets in $\overline{h}_1$ and $h_2$. Thus, we would appear to have satisfactory doublet-triplet splitting with no dangerous $d = 5$ proton decay, just as in flipped $SU(5)$.

However, this is not so, for the question arises how the triplets in $\overline{h}_1$ and $h_2$ are to acquire superheavy mass. It would seem that the only way is through a mass term connecting them. But that would have to come from a term $M \overline{h}_1 h_2$, which in turn comes from $M 10_1 10_2$, and this would also give $M h_1 \overline{h}_2$ and thus superlarge mass to the MSSM doublets.

We see, then, that the missing partner mechanism does not work in four-dimensional $SO(10)$ theories. However, we shall see that it can work in five-dimensional $SO(10)$ theories. The crucial difference will be that orbifold breaking of $SO(10)$ can split the $SO(10)$ Higgs representations. In particular, in the example we just looked at the troublesome triplets in $\overline{h}_1$ and $h_2$ can be given Kaluza-Klein masses by the orbifold compactification while leaving...
the MSSM doublets in $h_1$ and $\bar{h}_2$ light.

III. AN $SO(10)$ MODEL IN FIVE DIMENSIONS

We now present an $SO(10)$ supersymmetric model in five dimensions compactified on an $S^1/(Z_2 \times Z'_2)$ orbifold that yields a realistic supersymmetric flipped $SU(5)$ model in four dimensions. The breaking of $SU(5) \times U(1)$ down to the Standard Model gauge group, the doublet-triplet splitting, and the solution to the problem of $d = 5$ proton-decay operators will all be as in conventional four-dimensional flipped $SU(5)$ models. Moreover, there will be distinctive flipped $SU(5)$ relationships among gaugino masses. However, the gauge couplings will be unified (with some threshold corrections, that can be argued to be small\[9\]) because of the underlying five-dimensional $SO(10)$ symmetry. And the Yukawa couplings of the quarks and leptons can have relationships that are similar to what is found in ordinary $SU(5)$ and $SO(10)$ models rather than in flipped $SU(5)$.

As already elaborated in Refs.\[4, 5, 6, 7, 8, 9, 10\], the fifth dimension, being the circle with coordinate $y$ and circumference $2\pi R$, is compactified through the reflection $y \rightarrow -y$ under $Z_2$ and $y' \rightarrow -y'$ under $Z'_2$ where $y' = y + \pi R / 2$. This identification procedure leaves two fixed points $O$ and $O'$ of $Z_2$ and $Z'_2$ respectively and reduces the physical region to the interval $y \in [-\pi R / 2, 0]$. Point $O$ at $y = 0$ is the “visible brane” while point $O'$ at $y' = 0$ is the “hidden brane”. The compactification scale $1/R \equiv M_C$ is assumed to be close to the scale at which the gauge couplings unify, i.e. the GUT scale $M_{GUT} \sim 10^{16}$ GeV.

The generic bulk field $\phi(x^\mu, y)$, where $\mu = 0, 1, 2, 3$, has definite parity assignment under $Z_2 \times Z'_2$ symmetry. Taking $P = \pm 1$ to be parity eigenvalue of the field $\phi(x^\mu, y)$ under $Z_2$ transformation and $P' = \pm 1$ to be parity eigenvalue under the $Z'_2$ transformation, a field with $(P, P') = (\pm, \pm)$ can be denoted $\phi^{PP'}(x^\mu, y) = \phi^{\pm \pm}(x^\mu, y)$. The Fourier series expansion
of the fields $\phi^{\pm\pm}(x^\mu, y)$ yields

$$
\phi^{++}(x^\mu, y) = \frac{1}{\sqrt{2 \sin \pi R}} \sum_{n=0}^{\infty} \phi^{++}(2n)(x^\mu) \cos \frac{2ny}{R},
$$

(1a)

$$
\phi^{+-}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi^{+-}(2n+1)(x^\mu) \cos \frac{(2n+1)y}{R},
$$

(1b)

$$
\phi^{-+}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi^{-+}(2n+1)(x^\mu) \sin \frac{(2n+1)y}{R},
$$

(1c)

$$
\phi^{--}(x^\mu, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi^{--}(2n+2)(x^\mu) \sin \frac{(2n+2)y}{R}.
$$

(1d)

In the effective theory in four dimensions all the fields in Eqs. (1) have masses of order $M_C$ except the Kaluza-Klein zero mode $\phi^{++}(0)$ of $\phi^{++}(x^\mu, y)$, which remains massless. Moreover, fields $\phi^{\pm\pm}(x^\mu, y)$ vanish on the visible brane and fields $\phi^{\pm-}(x^\mu, y)$ vanish on the hidden brane.

In our model, we assume that gauge fields and gauge-non-singlet Higgs fields exist in the five-dimensional bulk, while the quark and lepton fields and certain gauge-singlet Higgs fields exist only on the visible brane at $O$. The gauge fields in the bulk are of course in a vector supermultiplet of 5d supersymmetry that is an adjoint representation of $SO(10)$. We will denote it by $45_g$, where the subscript 'g' stands for 'gauge'. The gauge-non-singlet Higgs fields in the bulk are in hypermultiplets of 5d supersymmetry and consist of two tens of $SO(10)$, denoted $10^g_H$ and $10^{2g}_H$, and a spinor-antispinor pair of $SO(10)$ denoted $16^g_H$ and $\overline{16}_H$. The subscript 'H' indicates a Higgs field.

The vector supermultiplet $45_g$ decomposes into a vector multiplet $V$ and a chiral multiplet $\Sigma$ of $\mathcal{N} = 1$ supersymmetry in four dimensions. Each hypermultiplet splits into two left-handed chiral multiplets $\Phi$ and $\Phi^c$, having opposite gauge quantum numbers. Under the $SU(5) \times U(1)$ subgroup the $SO(10)$ representations decompose as follows: $45 \rightarrow 24^0 + 10^{-4} + \overline{10}^4 + 1^0; 10 \rightarrow 5^{-2} + 5^2; 16 \rightarrow 10^4 + 5^{-3} + 1^5$, and $\overline{16} \rightarrow \overline{10}^{-1} + 5^3 + 1^{-5}$. With these facts in mind we shall now discuss the transformation of the various fields under the $Z_2 \times Z'_2$ parity transformations.

The first $Z_2$ symmetry (the one we denote as unprimed) is used to break supersymmetry to $\mathcal{N} = 1$ in four-dimensions. ($\mathcal{N} = 1$ in five dimensions is equivalent to $\mathcal{N} = 2$ in four dimensions; so we are breaking half the supersymmetries.) To do this we assume that under $Z_2$ the $V$ is even, $\Sigma$ is odd, $\Phi$ are even, and $\Phi^c$ are odd. The $Z'_2$ is used to break $SO(10)$ down to $SU(5) \times U(1)$. The $24^0$ and $1^0$ of $V$ are taken to be even under $Z'_2$, while the $10^{-4}$
and \( \overline{10}_4 \) are taken to be odd. In \( 10_{1H} \) the \( 5^- \) are taken to be even and the \( \overline{5}^2 \) odd, whereas in \( 10_{2H} \) the parities are taken to be the reverse, \( 5^- \) odd and \( \overline{5}^2 \) even. All told we have

\[
\begin{align*}
45_g &= V^{++}_{24} + V^{++}_{10} + V^{++}_{10} + V^{++}_{10} + \Sigma^{++}_{24} + \Sigma^{++}_{10} + \Sigma^{--}_{10} + \Sigma^{--}_{10} \\
10_{1H} &= \Phi^{++}_{5_{12}} + \Phi^{++}_{5_{1}^{2}} + \Phi^{++}_{5_{1}^{2}} + \Phi^{--}_{5_{1}^{2}} \\
10_{2H} &= \Phi^{++}_{5_{2}^{2}} + \Phi^{++}_{5_{2}^{2}} + \Phi^{++}_{5_{2}^{2}} + \Phi^{--}_{5_{2}^{2}} \\
16_H &= \Phi^{++}_{10} + \Phi^{++}_{10} + \Phi^{++}_{10} + \Phi^{++}_{10} + \Phi^{--}_{10} + \Phi^{--}_{10} \quad (2a) \\
\overline{16}_H &= \Phi^{++}_{10}, + \Phi^{++}_{10} + \Phi^{++}_{10} + \Phi^{++}_{10} + \Phi^{--}_{10} + \Phi^{--}_{10} \quad (2b)
\end{align*}
\]

Massless zero modes of the Kaluza-Klein towers exist only for fields with \( Z_2 \times Z'_2 \) parity ++. This includes \( \Phi^{++}_{5_{12}}, \Phi^{++}_{5_{2}^{2}}, \Phi^{++}_{10}, \) and \( \Phi^{++}_{10} \). We will call the zero modes of these components \( h_1, \overline{h}_2, H, \) and \( H' \), respectively, using the same notation we used in the last section. The \( h_1 \) and \( \overline{h}_2 \) contain the two Higgs doublets of the MSSM and their colored partners.

To understand these parity assignments, we observe the invariance of the action for the bulk fields \([18]\) given by

\[
S_5 = \int d^5x \left\{ \frac{1}{4k g^2} \text{Tr} \left[ \int d^2 \theta W^a W_a + h.c. \right] \right.
+ \frac{1}{k g^2} \int d^4 \theta \text{Tr} \left[ \left( \sqrt{2} \partial_5 + \Sigma \right)e^V (\sqrt{2} \partial_5 + \Sigma)e^{-V} + \partial_5 e^{-V} \partial_5 e^V \right] \\
+ \frac{4}{4} \int d^4 \theta \left[ \Phi_i^c e^V \overline{\Phi}_i^c + \overline{\Phi}_i e^{-V} \Phi_i \right] \\
+ \frac{4}{4} \left[ \int d^2 \theta \Phi_i^c (\partial_5 - \frac{1}{\sqrt{2}} \Sigma) \Phi_i + h.c. \right] \right\} \quad (3)
\]

under \( y \to -y \) reflection with the superfields transforming as

\[
\begin{align*}
V^a(x^\mu, -y)T^a &= V^a(x^\mu, y)PT^aP \quad (4a) \\
\Sigma^a(x^\mu, -y)T^a &= -\Sigma^a(x^\mu, y)PT^aP \quad (4b) \\
\Phi_i(x^\mu, -y) &= \pm P\Phi_i(x^\mu, -y) \quad (4c) \\
\Phi_i^c(x^\mu, -y) &= \mp P^T \Phi_i^c(x^\mu, -y) \quad (4d)
\end{align*}
\]

where \( V = V^a T^a \), and \( \Sigma = \Sigma^a T^a \). The \( T^a \)s are the generators of \( SO(10) \) in the appropriate representation with normalization \( \text{Tr}[T^a T^b] = k \delta^{ab} \), and \( P = P^1 \) is the parity operator. The replacement \( y \to y' \) and \( P \to P' \) in Eqs. (4) specifies the transformation of the superfields.
under $y' \rightarrow -y'$ reflection. Finally, defining $P$ and $P'$ through their action on the 10 of $SO(10)$, we associate $P = \sigma_0 \otimes I$ with the $Z_2$ and $P' = \sigma_2 \otimes I$ with $Z'_2$, where $I$ and $\sigma_0$ are $5 \times 5$ and $2 \times 2$ identity matrices and $\sigma_2$ is the usual Pauli matrix.

Having done with the parity assignment for the bulk fields we can turn our attention towards the brane physics. On the brane at $O$ we put the three families of quarks and leptons. Since the gauge symmetry on this brane is $SO(10)$, these are contained in three chiral superfields that are spinors of $SO(10)$, which we denote $16_i$, where $i = 1, 2, 3$ is the family index. Later for various reasons we shall introduce some gauge-singlet superfields on the brane at $O$, but let us first discuss the interactions of the fields introduced up to this point.

The $Z_2$ parity of fields in the $16_i$ must be positive. The $Z'_2$ parity, determined by the content of Eqs. (2), is $16 \rightarrow 10^{1+} + 5^{-3+} + 1^{5+}$, where $10_i = (Q, D, N)_i$, $5_i = (U, L)_i$, and $1_i = (E)_i$. The action for the coupling of the matter fields, residing on the visible brane, with the Higgs fields, coming from the bulk, is

$$S_{\text{matter}}^{\text{brane}} = \int d^5x \left[ \frac{1}{2} [\delta(y) + \delta(y - \pi R)] \sqrt{2\pi R} \int d^2\theta A_{ij} \Phi_{16} \Phi_{16} \Phi_{10} \right]$$

$$+ \int d^5x \left[ \frac{1}{2} [\delta(y) - \delta(y - \pi R)] \sqrt{2\pi R} \int d^2\theta B_{ij} \Phi_{16} \Phi_{16} \Phi_{10} + \text{h.c.} \right]$$

where $A_{ij}$ and $B_{ij}$ are Yukawa couplings. Integrating over the fifth dimension $y$ using the Eqs. (3), and keeping only the terms that involve the Yukawa couplings of the MSSM Higgs doublets and their triplet partners we obtain the following Lagrangian in four dimensions

$$L_{\text{matter}}^{\text{brane}} = \sum_{n=0}^{\infty} \int d^2\theta \sqrt{2\delta_{n0}} \left[ A_{ij} \Phi_{d_{1H}} \Phi_{d_{1H}} + L_i E_j \Phi_{d_{1H}} + \frac{1}{2} Q_i \Phi_{Q} \Phi_{t_{1H}} + U_i \Phi_{U} \Phi_{t_{1H}} \right]$$

$$+ B_{ij} \Phi_{d_{2H}} \Phi_{d_{2H}} + L_i N_j \Phi_{d_{2H}} + Q_i \Phi_{Q} \Phi_{t_{2H}} + U_i \Phi_{U} \Phi_{t_{2H}} + \text{h.c.}$$

where $d_{1H}^{(2n)}$ and $t_{1H}^{(2n)}$ are the doublet and triplet contained in $\Phi_{5_1}^{\pm}$ (whose zero mode is $h_1$) and $d_{2H}^{(2n)}$ and $t_{2H}^{(2n)}$ are the doublet and triplet contained in $\Phi_{5_2}^{\pm}$ (whose zero mode is $h_2$). All the remaining terms coming from Eq. (3) are found by the replacement $A_{ij} \leftrightarrow B_{ij}$, $(1H) \leftrightarrow (2H)$, $(2n) \rightarrow (2n + 1)$, and $\delta_{n0} \rightarrow 1$ in Eq. (3).

This represents a minimal set of Yukawa terms, and would lead to the following relations among the quark and lepton mass matrices: $M_L = M_D \propto A$ and $M_D^{\text{Dirac}} = M_U \propto B$, with $A$ and $B$ being completely independent symmetric matrices. This is different from the relations that arise with a minimal set of Yukawa terms in four-dimensional models
based on $SO(10)$ or flipped $SU(5)$. In four-dimensional flipped $SU(5)$, the minimal Yukawa terms give $M_{\nu}^{\text{Dirac}} = M_{U}^{T}$, where these matrices are not predicted to be symmetric, and no relation for $M_{L}$ and $M_{D}$. In four-dimensional $SO(10)$, the minimal Yukawa terms give $M_{L} = M_{D} \propto M_{\nu}^{\text{Dirac}} = M_{U}$, with these matrices predicted to be symmetric.

The Higgs fields, though defined in the bulk, will also couple to each other on the branes. We assume that the Higgs coupling on the visible brane is of the form

$$S_{5}^{higgs} = \int d^{5}x \frac{1}{2} [\delta(y) + \delta(y - \pi R)] \sqrt{2\pi R} \int d^{2}\theta \ |10_{1H}16_{H}16_{H}| + \text{h.c.}$$

(7)

There could also be terms of the form $10_{1H}10_{jH}$, which would directly produce a GUT-scale $\mu$ term and destroy the gauge hierarchy. These must be forbidden by a symmetry. This is not a novel requirement introduced by the fact that there are extra dimensions. Terms that would directly produce a GUT-scale $\mu$ term must also be forbidden in four-dimensional unified theories. For example, in four-dimensional $SU(5)$ theories as well as four-dimensional flipped $SU(5)$ theories, there are Higgs multiplets in $\bar{5}$ and $5$, and these must be prevented from obtaining a superheavy mass term together. Similarly, in four-dimensional $SO(10)$ theories the light Higgs doublets are typically in a $10$ of Higgs, which must be prevented from acquiring a superheavy self-mass term $[19]$. The same problem arises also in GUTs in higher dimensions. Generally, some symmetry must be imposed to protect the gauge hierarchy from such dangerous terms. We shall assume here that there is a $U(1)'$ of the Peccei-Quinn type under which the quark and lepton spinors $16_{i}$ have charge $+1$, the Higgs fields $10_{1H}$ and $10_{2H}$ have charge $-2$, and the Higgs fields $16_{H}$ and $\bar{16}_{H}$ have charge $+1$.

This approach of using a vector-like symmetry to prevent a large direct $\mu$ term is used in Ref. $[15]$. A drawback of using that method here, as will be seen later, is that to generate large Majorana mass terms for the neutrinos without too large a $\mu$ term being generated by higher-dimension operators, it will be necessary to assume a hierarchy of $10^{-4}$ between the $U(1)'$ breaking scale and $M_{\text{GUT}}$.

Another way of suppressing direct GUT-scale $\mu$ terms is by means of a continuous $U(1)_{R}$ symmetry as in Ref. $[1]$. In that paper it was found that $\mu$ and $\mu B$ parameters of the order of the weak scale could be generated, without any fine-tuning, through the Giudice-Masiero mechanism $[21]$. We do not pursue other approaches such as that here.

The most general effective action of our theory should also include brane-localized kinetic
terms for the modes of the bulk fields that have non-vanishing wavefunction on the branes. Since the symmetry that survives on the hidden brane differs from the symmetry that governs the interactions on the visible brane and in the bulk, one might worry that the hidden-brane kinetic terms with the arbitrary coefficients for the gauge fields would spoil the gauge coupling unification, and that the hidden-brane kinetic terms for the Higgs fields could affect the mass matrix prediction that stems from Eq. (5).

As it turns out, the gauge kinetic terms on the hidden brane do not spoil the gauge coupling unification if the volume of the extra dimension is large enough [9]. In that case the arbitrary coefficients of the gauge kinetic terms on the hidden and the visible brane get diluted and their contribution to the gauge couplings of the Standard Model can be neglected. The dominant contribution comes from the universal coefficient that belongs to the gauge kinetic term in the bulk obeying the full symmetry of the theory.

The hidden brane kinetic terms for the Higgs fields do not affect the mass relations $M_L = M_D \propto A$ and $M_\nu^{\text{Dirac}} = M_U \propto B$. These hidden-brane terms violate $SO(10)$ but respect $SU(5) \times U(1)$, and so will have the effect of changing the relative normalization of the $\mathbf{5}$ and $\mathbf{5}$ of Higgs that are inside the same $\mathbf{10}$ of $SO(10)$. However, the $\mathbf{5}$ of Higgs and the $\mathbf{5}$ of Higgs that contribute to quark and lepton masses in this model come from different $\mathbf{10}$’s of Higgs anyway. The former comes from $\mathbf{10}_{1H}$, while the latter comes from $\mathbf{10}_{2H}$. While the matrices $A$ and $B$ will be differently affected by the hidden-brane kinetic terms, the predictions that $M_L = M_D \propto A$ and $M_\nu^{\text{Dirac}} = M_U \propto B$ are not affected by that. The essential point is that these predictions depend only on the $SU(5)$ that is respected by the hidden-brane kinetic terms and not on the full $SO(10)$.

As noted earlier, the only massless modes of the Higgs fields are $h_1 \subset \Phi_{5,2}^{++} \subset \mathbf{10}_{1H}$, $\bar{h}_2 \subset \Phi_{8}^{++} \subset \mathbf{10}_{2H}$, $H \subset \Phi_{101}^{++} \subset \mathbf{16}_H$, and $\bar{H} \subset \Phi_{10}^{++} \subset \mathbf{16}_{H}$. Therefore, after integrating over the fifth dimension, the terms in Eq. (7) yield in the superpotential of the low-energy effective theory the terms $h_1 \ H \ H + \bar{h}_2 \ H \ H$. These are just the same terms that are present in conventional four-dimensional flipped $SU(5)$ models to do the doublet-triplet splitting.

We assume that the $H$ and $\bar{H}$ acquire superlarge vacuum expectation values that break $SU(5) \times U(1)$ down to the Standard Model group. The tree-level scalar potential generated by the terms $h_1 H H + \bar{h}_2 \bar{H} \ H$ is flat in this direction. However, as noted in [3], this flatness can be lifted by radiative effects after supersymmetry is broken. It is also possible that additional terms in the Higgs superpotential on the visible brane can lead to a tree-level
superpotential that produces the required symmetry breaking, as we shall see later.

Besides breaking the gauge symmetry from $SU(5) \times U(1)$ down to $SU(3) \times SU(2) \times U(1)$, the vacuum expectation values of the fields $H \subset 16_H$ and $\overline{H} \subset \overline{16}_H$ allow masses for the right-handed neutrinos. Such masses come from effective operators of the form $16_i16_j\overline{16}_H\overline{16}_H$. However, this product of fields has charge $+4$ under the symmetry $U(1)'$. Consequently, this symmetry must be spontaneously broken. It must be broken in such a way as to permit sufficiently large right-handed neutrino masses without at the same time allowing too large a $\mu$ parameter (which is the coefficient of the term $10_1 H 10_2 H$). This can be done in the following way (which we do not claim to be unique). Suppose that there are fields $S$ and $\overline{S}$ living on the brane at $O$ that are singlets under $SO(10)$ and that have $U(1)'$ charges $+1$ and $-1$ respectively. In the superpotential on the brane at $O$ there can be terms of the form $(\overline{S} S - M^2)X$, where $M = \epsilon M_{GUT}$, with $\epsilon \ll 1$. These terms force $\langle S \rangle = \langle \overline{S} \rangle = M$. Let us suppose that on the brane at $O$ there are, in addition to the quark and lepton families in $16_i$, some leptons $1_i$ ($i = 1, 2, 3$) that are $SO(10)$ singlets but have charge $-1$ under $U(1)'$. Then the following terms in the superpotential at $O$ are possible: $C_{ij}16_i16_j\overline{16}_H\overline{16}_H/M_s + F_{ij}1_i1_jS^2/M_s$, where the dimensionless coefficients $C_{ij}$ and $F_{ij}$ are assumed to be of order one. The mass $M_s$ is an ultraviolet cutoff that specifies the scale at which new physics (e.g., other dimensions beyond five, strings) become important. We take $M_s$ to be close to $M_{GUT}$ but, of course, somewhat larger. These terms give a mass matrix for the neutrinos that has the form

$$
\begin{pmatrix}
(v_i & N^c_i & 1_i) \\
(M^{Dirac})_{ij} & 0 & C_{ij}\epsilon\overline{M} \\
0 & C_{ji}\epsilon\overline{M} & F_{ij}\epsilon^2\overline{M}
\end{pmatrix}
\begin{pmatrix}

\nu_j \\
N^c_j \\
1_j
\end{pmatrix},
$$

where $\overline{M} \equiv M^2_{GUT}/M_s$. (Note that we have taken $\langle 16_H \rangle = M_{GUT}$.) It is clear that the six superheavy neutrinos have masses of order $\epsilon\overline{M}$, whereas the three light (left-handed) neutrinos have masses of order $(M^{Dirac})^2/\overline{M}$. Taking the largest neutrino mass $m_3$ to be about $6 \times 10^{-2}$ eV, as suggested by the atmospheric neutrino data, and its Dirac mass to be $m_\nu \approx 174$ GeV, as suggested by the relation $M^{Dirac} = M_U$ (which would hold in a minimal $SO(10)$ model), one has that $\overline{M} \sim 10^{15}$ GeV. This accords well with the assumption that $M_s$ is slightly larger than the GUT scale $M_{GUT} \sim 10^{16}$ GeV.

The reason that we have assumed that the parameter $\epsilon \equiv \langle S \rangle/M_{GUT}$ is much smaller than one is that it suppresses certain dangerous operators. For example, $U(1)'$ allows the
effective operator $\mathbf{16}_i \mathbf{16}_j \mathbf{16}_k \mathbf{16}_\ell S^i / M_*^2$. This gives a $d = 5$ proton decay operator with coefficient of order $\epsilon^4(1/M_*)$. Sufficient suppression of proton decay requires that $\epsilon \sim 10^{-3}$ to $10^{-4}$. Similarly, $U(1)'$ allows the operator $\mathbf{10}_{1_H} \mathbf{10}_{2_H} S^i / M_*^3$. This gives a $\mu$ parameter for the MSSM doublet Higgs fields that is of order $\epsilon^4 M_*$. Requiring that this be no larger than the weak scale requires that $\epsilon$ be less than about $3 \times 10^{-4}$. This corresponds to right-handed neutrino masses of order $3 \times 10^{11}$ GeV. Such intermediate mass scales for $M_R$ are good from the point of view of leptogenesis [21].

The same singlet Higgs field $S$ can play a role in generating the vacuum expectation value for the spinor Higgs fields $\mathbf{16}_H$ and $\mathbf{16}_H$. Such VEVs, as we have already noted, can arise due to radiative effects after SUSY breaking. But they can also arise at tree level from a term in the superpotential on the brane at $O$ of the form $(\lambda \mathbf{16}_H \mathbf{16}_H - S^2) Y$, where $Y$ is a singlet superfield with $U(1)'$ charge of $-2$, and $\lambda \sim \epsilon^2$. Note that the $F$-terms of the fields $\mathbf{16}_H$ and $\mathbf{16}_H$ force $\langle Y \rangle = 0$, meaning that there is no mass term linking $\mathbf{16}_H$ to $\mathbf{16}_H$ and thus $\mathbf{16}_H$ to $H$. The $U(1)'$ charge assignments allow the higher dimensional term $S^2 \mathbf{16}_H \mathbf{16}_H / M_*$. This will shift the VEV of $Y$, but the $F$-terms of the fields $\mathbf{16}_H$ and $\mathbf{16}_H$ still enforce the condition that there is no mass term linking $\mathbf{16}_H$ to $\mathbf{16}_H$.

Let us now examine the doublet-triplet splitting and proton decay problems. The terms $h_1 H \langle H \rangle + \bar{h}_2 \bar{H} \langle \bar{H} \rangle$ will couple the triplets in $h_1$ and $\bar{h}_2$ to those in $H$ and $\bar{H}$. The doublets in $h_1$ and $\bar{h}_2$ remain light and are the two doublets of the MSSM. There is no problem with $d = 5$ proton decay, because the triplet partners of the MSSM Higgs doublets are not connected to each other. The triplets in $h_1$ and $H$ have no mass terms with the triplets in $\bar{h}_2$ and $\bar{H}$. Moreover, there are no unwanted light states contained in the Higgs multiplets $\mathbf{10}_{1_H}, \mathbf{10}_{2_H}, \mathbf{16}_H, \mathbf{16}_H$. In the zero modes ($h_1, \bar{h}_2, H$, and $\bar{H}$), the doublets remain light, the triplets become superheavy by coupling to the VEVs of $H$ and $\bar{H}$, and the other gauge-non-singlet fields get eaten by the Higgs mechanism when $SU(5) \times U(1)$ breaks to the Standard Model group. All the non-zero modes, of course, have superheavy Kaluza-Klein masses. This is the crucial difference with four-dimensional theories in which flipped $SU(5)$ is embedded in $SO(10)$. In four dimensions, as we saw in the last section, the $SO(10)$ Higgs multiplets $\mathbf{10}_{1_H}$ and $\mathbf{10}_{2_H}$ when decomposed under $SU(5) \times U(1)$ contain not only $h_1$ and $\bar{h}_2$ but also $\bar{h}_1$ and $h_2$; and these multiplets have triplets that cannot be given mass without destroying the gauge hierarchy. Here, however, these extra pieces are all made heavy by the orbifold compactification, since they do not have parity $++$. Thus it is the fact that the unification of
SU(5) \times U(1) \text{ into } SO(10) \text{ occurs only in higher dimensions that allows the missing partner mechanism to be implemented.}

We have seen that with what may be called the minimal Yukawa couplings $16_1 16_j (A_{ij} 10_{1H} + B_{ij} 10_{2H})$ this model gives distinctive relations among mass matrices that are different from those that result in four dimensional models with minimal Yukawa couplings in either $SO(10)$ or flipped $SU(5)$. In particular, $M_L = M_D$, and $M_{\nu}^{Dirac} = M_{\nu}$, with all these matrices being symmetric. This does give the desired relation $m_b = m_\tau$ at the unification scale, a result of the fact that flipped $SU(5)$ is embedded in $SO(10)$. However, this minimal set of Yukawa terms is clearly not enough to give a realistic model of quark and lepton masses.

Recently it has been found that realistic and simple models of quark and lepton masses can be constructed using so-called “lopsided” mass matrices [22, 23, 24, 25]. The essential feature of such models is that the mass matrices of the down quarks and charged leptons are highly asymmetric and that $M_L \sim M_D^T$. Such a relationship between $M_L$ and $M_D^T$ is typical of models with an ordinary $SU(5)$, not flipped $SU(5)$. However, as we shall now see, because the flipped $SU(5)$ is here embedded in $SO(10)$ at the five-dimensional level, it is possible to obtain such a lopsided structure.

Suppose that one introduces on the visible brane not only spinors of quarks and leptons, but $SO(10)$ vectors as well. And suppose that there is in the bulk a spinor Higgs field $16_H'$ that has a weak-doublet component that contributes to the breaking of the electroweak symmetry. Then a diagram like that shown in Fig. 2(a) may be possible. When decomposed under the $SU(5) \times U(1)$ subgroup, this diagram contains the two diagrams shown in Figs. 2(b) and 2(c). It is easy to see that these give contributions to $M_L$ and $M_D$ that are asymmetric and that are transposes of each other, just as needed to build “lopsided” models. The reason for this is that the diagram in Fig. 2(a) directly depends only on the GUT-scale breaking done by the $16_H$ and not on that coming from orbifold compactification. The point is that the $16_H$ VEV by itself would only break $SO(10)$ down to the Georgi-Glashow $SU(5)$. (It is the orbifold compactification that breaks $SO(10)$ to the flipped $SU(5) \times U(1)$ group.) That is why this diagram leads to the kind of mass contributions that one expects from ordinary Georgi-Glashow $SU(5)$. This reasoning also shows that in order to introduce into the mass matrices contributions that break Georgi-Glashow $SU(5)$ it is necessary that the mass-splittings produced by the orbifold compactification be involved. For example, by
mixing quarks and leptons that are on the visible brane with quarks and leptons in the bulk, it should be possible to break the (bad) minimal $SU(5)$ relations $m_s = m_\mu$ and $m_d = m_e$.

IV. GAUGINO MEDIATED SUPERSYMMETRY BREAKING

In this section we address the issue of how to break $\mathcal{N} = 1$ supersymmetry of our model below the compactification scale $M_C$. As it turns out, the solution allows the construction of viable SUSY breaking model that can easily satisfy present experimental constraints.

It is well known that the models with visible and hidden branes separated by extra dimension(s) naturally accommodate breaking of supersymmetry via gaugino mediation \[26, 27\]. The basic idea behind gaugino mediation in the models based on the orbifold compactification is as follows. The source of the SUSY breaking is localized at the hidden brane. It couples directly to the gauginos at that brane providing them with nonzero masses. If the gauge symmetry at the hidden brane is reduced with respect to the bulk gauge symmetry this coupling induces non-universal gaugino masses. For example, if the bulk symmetry is $SO(10)$ and hidden brane symmetry is flipped $SU(5)$ one obtains $M_3 = M_2 \neq M_1$. Here, $M_1$, $M_2$, and $M_3$ represent gaugino masses of the MSSM.

Following in the footsteps of \[15\], we take the source of the SUSY breaking to be a flipped $SU(5)$ singlet chiral superfield $Z$ localized on the hidden brane with the VEV

$$\langle Z \rangle = \theta^2 F_Z.$$  

(9)

The gaugino masses originate from the non-renormalizable operators at the hidden brane of the form

$$\mathcal{L}_5^Z = \frac{1}{2} [\delta(y - \pi R/2) + \delta(y + \pi R/2)] \int d^2 \theta \left( \lambda'_5 \frac{Z}{M^2} W^{i\alpha} W_{i\alpha} + \lambda'_1 \frac{Z}{M^2} W^\alpha W_\alpha + \text{h.c.} \right),$$  

(10)

where the first and the second term under the integral represent the $SU(5)$ and $U(1)$ part of the gauge group respectively. Corresponding gaugino masses generated in this way are

$$M_{SU(5)} = \frac{\lambda'_5 F_Z M_c}{M^2}, \quad M_{U(1)} = \frac{\lambda'_1 F_Z M_c}{M^2},$$  

(11)

which translates into the following MSSM gaugino masses (we normalize the generators of $SO(10)$ demanding that $k = 1/2$)

$$\frac{M_1}{g_1^2} = \frac{1}{25} \frac{M_{SU(5)}}{g_{SU(5)}^2} + \frac{24}{25} \frac{M_{U(1)}}{g_{U(1)}^2}, \quad M_2 = M_{SU(5)}, \quad M_3 = M_{SU(5)}.$$  

(12)
Here $g_{SU(5)}$ and $g_{U(1)}$ are gauge coupling constants of the $SU(5)$ and $U(1)$ gauge groups respectively, while $g_1$ represents the $U(1)_Y$ gauge coupling constant of the Standard Model (normalized as in GUTs). The relations of Eq. (12), which is valid at the compactification scale $M_C$, show that the gaugino mass $M_1$ can in general be completely different from the mass $M_2 = M_3$ due to their different origins. Namely, the mass $M_1$ is dominated by the $U(1)$ sector of the theory as oppose to the masses $M_2$ and $M_3$ that have their origin in the $SU(5)$ part of the theory. We will later see that this feature of non-universality of gaugino masses allows the construction of the theory of SUSY breaking that leads to the realistic mass spectrum.

At this point we note that the natural scale for $\sqrt{F_Z}$ is the cutoff scale $M_*$. (For the reasons that have to do with gauge coupling unification we take $(M_C \sim 10^{16}$ GeV) < $(M_{GUT} = 1.2 \times 10^{16}$ GeV) < $(M_* \sim 10M_C)$ [13].) This implies that masses in Eq. (11) are close to the compactification scale $M_C$ if the dimensionless coefficients $\lambda'_1$ and $\lambda'_5$ are taken to be of order one. To obtain SUSY breaking masses that are in the TeV range we need to decrease the value of $F_Z$ in a way that does not involve any fine-tuning. To do that we propose to use the shining mechanism [28, 29] which can reduce the natural scale of $F_Z$ by an exponential factor.

The shining mechanism requires the existence of a source $J$ that is localized on the visible brane and a massive hypermultiplet in the bulk. The hypermultiplet of mass $m$ is taken to be a gauge singlet and has couplings with both the source and the superfield $Z$. These couplings can be arranged in a manner that leaves the superfield $Z$ with the nonzero F-term $F_Z \sim J \exp(-\pi mR/2)$ after the massive hypermultiplet is integrated out [28]. If the mass $m$ is taken to be of order $M_*$ the $\sqrt{F_Z}$ will be of order $10^{12}$ GeV which gives desired TeV scale masses for gauginos in Eq. (11).

The matter fields in our model reside on the visible brane. Thus, due to the spatial separation between the branes the soft SUSY breaking scalar masses and trilinear couplings are negligible at the compactification scale. This is good because the number of the soft SUSY breaking parameters one has to consider is reduced with respect to the usual set.

There are two additional positive features of the gaugino mediated supersymmetry breaking models with the non-universal gaugino masses. Firstly, the renormalization group running of scalar masses and trilinear couplings between $M_C$ and electroweak scale is significantly affected by gaugino masses but these contributions, being flavor blind, do not cause
any disastrous flavor violating effects. Secondly, non-universality opens up the possibility for the deviation from the experimentally disfavored prediction of the models with universal gaugino mass of stau being the lightest supersymmetric particle (LSP). (The last statement holds for $M_C < M_{GUT}$ which is exactly the case we have.)

The class of models with non-universal gaugino mediated supersymmetry breaking has been studied in more details by Baer et al. [30]. Their numerical study of the allowed region of SUSY parameter space shows that viable models with acceptable mass spectrum and neutral LSP particle can be obtained. The study includes the case of completely independent $M_3$, $M_2$, and $M_1$, as well as the case where $M_1$ is a definite linear combination (determined by group theory) of $M_2$ and $M_3$. (The former case can be seen as a consequence of orbifold reduction of $SU(5)$ down to the Standard Model group on the hidden brane as in Ref. [3] and the latter one follows from the reduction of $SO(10)$ down to the Pati-Salam group as in Ref. [15].) We have an intermediate scenario where $M_1$ is independent of $M_2$ and $M_3$ which are made equal due to the $SU(5)$ part of the flipped $SU(5)$. (This possibility was considered in Ref. [13] in the context of a six dimensional $SO(10)$ model.)

It is not difficult to adapt the analysis of Baer et al. to our model to conclude that for large enough $M_1$ (i.e. $|M_1| > |M_2|, M_2 = M_3$) at the compactification scale $M_C$ a viable region of parameter space opens up regardless of $\tan \beta$ value yielding realistic mass spectrum with the LSP being wino-like or a mixture of higgsino and bino. An example of this behavior is shown in Fig. [3].

At the end we observe that if we had the case of $SO(10)$ being reduced on the hidden brane to the Georgi-Glashow $SU(5)$ with an extra $U(1)$ symmetry we would not only be prevented from using the simple form of the missing partner mechanism but would also obtain universal gaugino masses $M_1 = M_2 = M_3$.

V. CONCLUSIONS

We have seen that by embedding a four-dimensional flipped $SU(5)$ model into a five-dimensional $SO(10)$ model the advantages of flipped $SU(5)$ can be maintained while avoiding its well-known drawbacks. The two main drawbacks are the loss of unification of gauge couplings and the loss of the possibility of relating down quark masses to charged lepton masses, and therefore of obtaining desirable predictions such as $m_b = m_\tau$ and realistic
quark and lepton mass schemes such as those based on “lopsided” mass matrices. By embedding $SU(5) \times U(1)$ in $SO(10)$, the unification of gauge couplings is restored. There are corrections to this unification, coming for example from gauge kinetic terms on the hidden brane; however, these have been argued to be small \cite{Antoniadis:1987dx}. The embedding in $SO(10)$ also yields relationships between the charged lepton and down quark mass matrices. We have also found that interesting patterns of quark and lepton masses are possible that are different from those encountered in four-dimensional grand unified theories, for example $M_L = M_D \neq M_D^{\text{Dirac}} = M_U$.

Embedding flipped $SU(5)$ in $SO(10)$ in four dimensions is well known to destroy the missing partner mechanism for doublet-triplet splitting, which is one of the most elegant features of flipped $SU(5)$. However, when the unification in $SO(10)$ takes place in higher dimensions and the breaking to $SU(5) \times U(1)$ is achieved through orbifold compactification, then the missing partner mechanism can still operate, as we have shown. One of the advantages of the missing partner mechanism in flipped $SU(5)$ is that it kills the dangerous $d = 5$ proton decay operators that plague supersymmetric grand unified theories.

Thus in extra dimensions it is possible to have the best of both worlds, the best of $SO(10)$ combined with the best of flipped $SU(5)$. One of the distinctive predictions of the flipped $SU(5)$ scheme that we have presented is that the gaugino masses will have the pattern $M_3 = M_2 \neq M_1$. The fact that $M_1$ is independent of $M_2$ and $M_3$ allows a much larger viable region of parameter space for the MSSM.

VI. ACKNOWLEDGMENTS

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FIG. 1: The kind of graph that gives rise to $d = 5$ proton decay operators.

\[
d_{i} = \begin{pmatrix} 10 & 10 & 10 \end{pmatrix} \begin{pmatrix} 16 \end{pmatrix} = d_{j} = \begin{pmatrix} 5 & 5 & 5 \end{pmatrix} \begin{pmatrix} 16 & 16' \end{pmatrix} \times \begin{pmatrix} 16 \end{pmatrix} \begin{pmatrix} 16' \end{pmatrix}
\]

FIG. 2: (a) A diagram that can give operators producing “lopsided” contributions to $M_D$ and $M_L$. (b) A term in its $SU(5) \times U(1)$ decomposition that contributes to $M_D$. (c) A term in its $SU(5) \times U(1)$ decomposition that contributes to $M_L$. 

21
FIG. 3: This diagram represents the results of numerical analysis of Baer et al. [30] for the case of gaugino mediated SUSY breaking scenario in the flipped SU(5) setting ($M_2 = M_3 \neq M_1$) for $\tan \beta = 30$ and $\mu > 0$. The allowed region in $M_1$ vs. $M_2 = M_3$ plane is shown in dotted light gray. The excluded regions are white (due to presence of tachyonic particles in mass spectrum), light gray (due to lack of radiative breakdown of EW symmetry), gray (due to LEP constraint), dark gray (due to LEP2 constraint), and crossed gray (due to the fact that charged particle is LSP). Vertical black line is where $M_H = 114$ GeV. For a full discussion of numerical methods and assumptions used in the analysis see Ref. [30].