New traveling wave structures for two higher dimensional nonlinear evolution equations with time-dependent coefficients: Horseshoe-like solitons and multiwave interaction solutions

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New traveling wave structures for two higher dimensional nonlinear evolution equations with time-dependent coefficients: Horseshoe-like solitons and multiwave interaction solutions

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Abstract In this paper, by introducing new traveling wave transformations in specific nonlinear forms, a variety of new multiwave interaction solutions for two higher dimensional nonlinear evolution equations with time-dependent coefficients are investigated. These new kinds of multiwave solutions can enrich solutions of nonlinear evolution equations with variable coefficients.

Keywords Boiti–Leon–Manna–Pempinelli equation · cylindrical Kadomtsev-Petviashvili equation · multiwave interaction solution · time-dependent coefficient · horseshoe-like soliton

1 Introduction

The study of interaction solutions for nonlinear evolution equations has become a research hotspot because they can be more applicable to many complex situations in various physical settings than single wave solutions. In recent years, there have been many interesting works with regard to multiwave interaction solutions in rich forms for nonlinear evolution equations with constant coefficients [1–14]. However, due to inhomogeneities of media, variable coefficient nonlinear evolution equations can better describe real features in scientific and engineering background, while the research on this aspect is still in progress [15–17]. In this work, by introducing new traveling wave transformations, we establish new types of multiwave and higher-order interaction solutions for two higher dimensional nonlinear evolution equations with time-dependent coefficients.

The paper is organized as follows. In Sec. 2, for a (3+1)-dimensional Boiti–Leon–Manna–Pempinelli (BLMP) equation, by introducing a specific nonlinear traveling wave transformation, new types of multiwave interaction solutions are constructed. Moreover, by considering another specific nonlinear traveling wave variable, higher-order interaction solutions among horseshoe-like solitons, breathers and lump waves for a (2+1)-dimensional cylindrical Kadomtsev-Petviashvili (cKP) equation are constructed in Sec. 3. The summary is given in Sec. 4.

2 New kinds of multiwave solutions for the (3+1)-dimensional BLMP equation with time-dependent coefficients

The (3+1)-dimensional BLMP equation

\[(u_x + u_z)_y + (u_y + u_z)_{xxx} - 3(u_x(u_y + u_z))_x = 0 \tag{1}\]

was proposed in 2012 [18]. It can be regarded as a model of the incompressible fluid. When \(z = 0\), it describes the interaction of the Riemann wave propagated along the \(y\)-axis with a long wave propagated along the \(x\)-axis. Some promising results on exact wave solutions and related properties of this equation have been reported, such as rational wave solutions, non-traveling wave solutions and other interaction solutions [19–23].
In [24], the author extended Eq. (1) to the time-dependent BLMP equation
\begin{equation}
g(t)[u_t + u_{tt}] + \alpha(u_t + u_{t})_{xxt} + \beta(u_t + u_{t})_x = 0, \tag{2}
\end{equation}
where $\alpha, \beta$ are real constants and $g(t)$ is a time-dependent coefficient, and further deduced its Painlevé integrability with arbitrary selection for $g(t)$. So far, there have been few studies on exact solutions of Eq. (2), and we intend to construct multiwave interaction solutions among horseshoe-like solitons, periodic waves and lump waves for this equation in this section.

For Eq. (2), with the Painlevé truncated expansion method, we get the transformation
\begin{equation}
u = \frac{6\alpha}{\beta}(\ln(f))_x, \tag{3}
\end{equation}
in which $f = f(x, y, z, t)$. Replace $u$ in Eq. (2) with the transformation (3) and take the numerator, the equation with respect to $f$ and its derivatives is obtained,
\begin{equation}
0 = \alpha(g(t)f_{xx} + g(t)f_{xxt} + \alpha f_{xxxx} + \alpha f_{xxxx})f^2
- (4\alpha f_{xxx} + \alpha f_{xxxx})f_x - 2\alpha f_x f_{xx} + \alpha(f_x + f_{xx}) + g(t)(f_x + f_{xx}) + \alpha(f_x + f_{xx})f_x
+ (f_x + f_{xx})(f_x + f_{xx}) + \alpha f_x + f_{xx} + 2f_x + 2f_{xx} + 2\alpha g(t)f_x + f_{xx}f_x, \tag{4}
\end{equation}

Following the direct algebraic method (DAM), by assuming the required solution as a specific form, the problem of solving a nonlinear differential equation is transformed into the problem of solving nonlinear algebraic equations. However, for higher-order wave solutions or complicated interaction solutions, the obtained nonlinear algebraic equations are usually too large to be solved directly. In this case, we present a step-by-step strategy to solve such (super) large-scale nonlinear algebraic equations, and we call it the inheritance solving strategy. It means to further build a higher-order solution on the basis of a known lower-order solution named as the initial solution. In this way, the large-scale nonlinear algebraic equations usually can be simplified sharply. It is to be noted that in the following, $g(t)$ is taken as a specific form, that is, $g(t) = 1/t$, in order to simplify calculation. For other forms of $g(t)$, it can be calculated in the similar way as well.

We first calculate 1-soliton solution for Eq. (2) by the DAM, when the auxiliary function $f$ is assumed as
\begin{equation}
f = q_1 + q_2 \exp(\xi), \tag{5}
\end{equation}
where
\begin{equation}
\xi = \delta_1(k_1x - (s_1y)^2 - r_1z^2 + 2r_1yz + p_1y - p_1z + \omega_1)t + c_1, \tag{6}
\end{equation}
is the nonlinear traveling wave variable and $\delta_1$, $k_1$, $r_1$, $s_1$, $p_1$, $\omega_1$, $c_1$, $q_i$ are unknown parameters.

Substitute the assumption (5) into Eq. (4), collect coefficients of the same power and let coefficient of each term be zero, and we obtain a nonlinear algebraic equations with 16 equations and 9 variables. Solve the obtained algebraic equations directly, a set of solution is obtained,
\begin{equation}
\{r_1 = -s_1\}. \tag{7}
\end{equation}

Substitute the above solution into the auxiliary function (5) and the transformation (3) in turn, 1-soliton solution of Eq. (2) is obtained as
\begin{equation}
u = \frac{-6q_2 \delta_1 k_1 \exp(\sigma) + q_1}{2q_2 + q_2 \exp(\sigma)}, \tag{8}
\end{equation}
in which
\begin{equation}
\sigma = \delta_1(k_1x - (s_1y)^2 + s_1z^2 - 2s_1yz + p_1y - p_1z + \omega_1)t + c_1. \tag{9}
\end{equation}

Similarly, we further calculate 2-soliton and 3-soliton solutions of Eq. (2), and the corresponding assumptions of the auxiliary function $f$ are
\begin{equation}
f = q_1 + q_2 \exp(\xi_1) + q_3 \exp(\xi_2) + q_4 \exp(\xi_1 + \xi_2), \tag{10}
\end{equation}
and
\begin{equation}
f = q_1 + q_2 \exp(\xi_1) + q_3 \exp(\xi_2) + q_4 \exp(\xi_3)
+ q_5 \exp(\xi_1 + \xi_2) + q_6 \exp(\xi_1 + \xi_3) + q_7 \exp(\xi_2 + \xi_3)
+ q_8 \exp(\xi_1 + \xi_2 + \xi_3), \tag{11}
\end{equation}
respectively, where $\xi_i$ has the same form as that in (5). These three solitons are shown in Fig. 1(a)-1(c), and 1(d)-1(f) are density graphs corresponding to them, respectively. It can be seen that the solitons are horseshoe shaped.

To construct more complicated multiwave interaction solutions among 1-lump, 1-soliton and the Jacobi elliptic cosine function for Eq. (2), we suppose the auxiliary function as
\begin{equation}
f = q_1 + q_2 \exp(\xi_1) + \xi_2^2 + \xi_3^2 + q_3 \text{JacobiCN}(\xi, R), \tag{12}
\end{equation}
in which $\text{JacobiCN}$ is the Jacobi elliptic cosine function, and
\begin{equation}
\xi_i = \delta_i(k_ix - (s_iy)^2 - r_i z^2 + 2r_iyz + p_iy - p_iz + \omega_i)t + c_i, \tag{13}
\end{equation}
where $\delta_i$, $k_i$, $r_i$, $s_i$, $p_i$, $\omega_i$, $c_i$, $q_i$ are unknown parameters and the modulus $R$ must meet $R^2 \leq 1$. Substitute the assumption (10) into Eq. (4), collect the same power terms and let coefficients of different power terms be zero, we have a nonlinear algebraic equations with 5342 equations and 32 variables, whose scale is too large to be solved directly. Therefore, we apply the inheritance
New traveling wave structures and multiwave interaction solutions

solving strategy and treat solution (6) as an initial solution. Solve the simplified nonlinear algebraic equations, we get one set of solution for the parameters,

\[ \{r_1 = -s_1, r_2 = -s_2, r_3 = -s_3, r_4 = -s_4\}. \]  (11)

Substitute the solution into (10) and (3) in order, a multiwave interaction solution of 1-lump, 1-soliton and the Jacobi elliptic cosine function for Eq. (2) is obtained. Evolution graphs of this solution at different times are shown in Fig. 2. It can be seen that the direction of soliton and the degree that soliton curls up change over time. Especially, the soliton and lump wave both are line shaped at \( t = 0 \).

We can further construct higher-order interaction solutions of 1-lump, 3-soliton and the Jacobi elliptic cosine function for Eq. (2) by assuming the auxiliary function as

\[
\begin{align*}
f = q_1 + q_2 \exp(\xi_1) + q_3 \exp(\xi_2) &+ q_4 \exp(\xi_3) \\
+ q_5 \exp(\xi_1 + \xi_2) + q_6 \exp(\xi_1 + \xi_3) &+ q_7 \exp(\xi_2 + \xi_3) \\
+ \xi_1 &+ q_8 \exp(\xi_1 + \xi_2 + \xi_3) + \xi_2^2 + \xi_3^2 \\
+ q_9 \text{JacobiCN}(\xi_6, R).
\end{align*}
\]  (12)
where
\[
\xi_i = \delta_i (k_i x - (s_i y)^2 - r_i z^2 + 2r_i y z + p_i y - p_i z + \omega_i t + c_i), \quad i = 1, 2, \ldots, 6.
\]

Take the solution (6) as an initial solution, one set of solution for parameters can be gotten,
\[
\{ r_1 = -s_1, r_2 = -s_2, r_3 = -s_3, r_4 = -s_4, r_5 = -s_5, r_6 = -s_6 \}.
\]

Substitute (13) into the auxiliary function (12) and the transformation (3) in turn, we obtain a three wave interaction solution among 1-lump, 3-soliton and the Jacobi elliptic cosine function for Eq. (2). Furthermore, a 2-soliton can be converted into a 1-breather by conjugated parameters assignment [25], with which technique this three wave interaction solution can be converted into a four wave interaction solution among 1-lump, 1-breather, 1-soliton and the Jacobi elliptic cosine function. These two kinds of multwave interaction solutions are shown in Fig. 3.

3 Higher-order interaction solutions among horseshoe-like solitons, breathers and lump waves for the (2+1)-dimensional cKP equation

The Hirota method is an effective method to construct soliton solutions for nonlinear evolution equations. Once soliton solutions were obtained, one can further calculate breather solutions by the conjugated parameters assignment. Meanwhile, lump wave solutions could also be established based on solitons by the long wave limit technique [25]. In this section, we construct new types of multwave interaction solutions by decomposing soliton solutions for the following cKP equation

\[
(u_t + 6uu_x + u_{xxx} + \frac{u}{2t})_x + 3\gamma \frac{u_x y}{t^3} = 0,
\]

where \( \gamma \) is a real constant. Eq. (14) was first proposed in 1978 and was deduced for internal waves in a stratified medium and for magnetized plasmas with pressure effects and transverse perturbation [26]. Its 1-, 2-, 3-solitons, rational wave solutions and interaction solutions between 1-lump and solitons have been reported [27]. In this paper, we calculate higher-order interaction solutions among horseshoe-like solitons, breathers and lump waves for Eq. (14) by introducing a new specific nonlinear traveling wave transformation.

Via the transformation
\[
u = 2(\ln(f))_{xx},
\]

where \( f = f(x,y,t) \), Eq. (14) can be converted into the equation with respect to \( f \) and its derivatives,
\[
0 = (6\gamma^2 f_{xyy} + 2r^2 f_{xxxx} + 2r^2 f_{xxx} + t f_{xxx}) f^3 + (-12r^2 f_{xxxx} - 6r^2 f_{xxx} + 4r^2 f_{xx} - 2r^2 f_{xxx} - 6r^2 f_{xxx} - 12\gamma^2 f_{xyy} f_x - 12r^2 f_{yyy} f_x - 3(2r^2 f_{xt} + 2f_{xy} f_{xt} - 12f_{xy}^2 f_{xt}) f_x^2 + 36r^2 f_{xxxx}^2)
\]
\[
-12r^2 f_{xx} + 12(\gamma^2 f_y^2 + t^2 f_{xx} f_x + 12f_{xx}^2 f_x + 48\gamma^2 f_{xx} f_{xxx} + 2f_{xx}^2 (6\gamma^2 f_{xy} + t f_x)) f - 48r^2 f_{xxx}^2 + 36r^2 f_{xx} f_x^2 + 12f_{xx}^2 (3\gamma^2 f_y^2 + t^2 f_{xx} f_x)
\]

We first construct a new type of soliton solutions for Eq. (14) by introducing a new nonlinear traveling wave transformation
\[
\xi_i = \delta_i (s_i y^2 x + r_i y - k_i x + \omega_i t + c_i), \quad 1 \leq i \leq N,
\]

where \( \delta_i, s_i, r_i, k_i, \omega_i, c_i \) are real parameters. Following the simplified Hirota method, soliton solutions could be expressed as combinations of exponential functions, such as
\[
\begin{align*}
f_{1x} &= 1 + \exp(\xi_1), \\
f_{2x} &= 1 + \exp(\xi_1) + \exp(\xi_2) + h_{12} \exp(\xi_1 + \xi_2).
\end{align*}
\]
with which the traveling wave structure changes to
\[
\xi_i = \delta_i \left( - \frac{k_i t^2}{12L_i^2} + r_i t y + k_i x - \delta_i^2 k_i^3 t - \frac{3}{2} \gamma_i^2 r_i^2 t + c_i \right), \quad 1 \leq i \leq N,
\]
(19)
and the interaction coefficient \( h_{ij} \) is gotten as
\[
h_{ij} = \frac{k_i^2 k_j^2 (- \delta_i^2 k_i^2 + 2 \delta_i \delta_j k_i k_j - \delta_j^2 k_j^2) + \gamma_i^2 (k_i r_j - k_j r_i)^2}{k_i^2 k_j^2 (- \delta_i^2 k_i^2 - 2 \delta_i \delta_j k_i k_j - \delta_j^2 k_j^2) + \gamma_i^2 (k_i r_j - k_j r_i)^2},
\]
(20)

With the above results, the formula of \( N \)-soliton solution has been proposed [28]. For the convenience of calculation, we rewritten it as below
\[
f_N = \sum_{T \subseteq \{1,2,\ldots,N\}} \left( \prod_{(i,j) \in T} h_{ij} \right) \exp \left( \sum_{k \in T} \xi_k \right).
\]
(21)
As we known, by the conjugated parameters assignment, \( 2m \) solitons can be converted into \( m \) breathers. In addition, by the long wave limit method together with the conjugated parameters assignment, \( 2m \) solitons could be converted into \( m \) lump waves. Therefore, by taking \( N = 2m + 2n + l \), where \( m, n \) and \( l \) are natural numbers, we transform the \( 2m \) soliton waves into \( m \) lump waves. Meanwhile, we convert the middle \( 2n \) solitons into \( n \) breathers and remain the latter \( l \) solitons unchanged. In this way, \( N \)-soliton solutions can be converted into three wave interaction solutions of \( m \)-lump, \( n \)-breather and \( l \)-soliton. For the sake of simplicity, in what follows we abbreviate this kind of solution as \( mL - nB - lS \). It should be noted that \( m, n \) or \( l \) could be zero, which means that the two wave interaction solutions, or even new single wave solutions, can also be obtained based on the above ideas.

We take \( N = 5 \) as an example to show how to calculate two or three wave interaction solutions by decomposing soliton solutions. Based on the traveling wave variables and interaction coefficients as shown in (19) and (20), together with the formula (21), the 5-soliton solution for Eq. (14) could be obtained.

Firstly, we divide \( N = 5 \) into the sum of two natural numbers, such as \( N = 2 + 3 \), to calculate the two wave interaction solution between 1-lump and 3-soliton. Following the long wave limit method, when \( \delta_i \to 0 \), the expanded key parameters are obtained as below:
\[
\theta_i = 12 \gamma_i^2 \left( k_i^3 t + k_i t r_j + k_j t c_i - 3 \gamma_i^2 r_i^2 t - k_i^2 t^2 \right), \quad 1 \leq i \leq 5,
\]
(22)
\[
\Psi_{ij} = 4 \delta_i \delta_j k_i^2 k_j^3 \left( - \delta_i^2 k_i^2 + \gamma_i^2 (k_i r_j - k_j r_i)^2 \right), \quad 1 \leq i < j \leq 5,
\]
\[
b_{12} = \frac{4 k_1^3 k_2^3}{\gamma_1 k_1 t r_2 - \gamma_2 k_2 t r_1},
\]
(23)
in which
\[
\frac{\partial \xi_i}{\partial \delta_i} = \frac{\partial \xi_j}{\partial \delta_j} = \frac{\partial h_{ij}}{\partial \delta_i} = \frac{\partial h_{ij}}{\partial \delta_j} = \frac{\partial^2 h_{ij}}{\partial \delta_i \partial \delta_j} = \delta_i = 0, \quad \delta_j = 0.
\]

Based on the parameters in (22), the auxiliary function of \( 1L - 3S \) is gotten as
\[
f_{1L3S} = \theta_1 \theta_i^* + b_{12} + e^{\xi_1} \left( [\Psi_{23} + \theta_i^*] (\Psi_{13} + \theta_1) + b_{12} \right) + e^{\xi_2} \left( [\Psi_{24} + \theta_i^*] (\Psi_{14} + \theta_1 + b_{12} \right) + e^{\xi_3} \left( [\Psi_{25} + \theta_i^*] (\Psi_{15} + \theta_1) + b_{12} \right)
\]
\[
+ h_{34 e^{\xi_1} + \xi_4} \left( [\Psi_{23} + \Psi_{24} + \theta_i^*] (\Psi_{13} + \Psi_{14} + \theta_1) + b_{12} \right) + h_{35 e^{\xi_1} + \xi_5} \left( [\Psi_{23} + \Psi_{25} + \theta_i^*] (\Psi_{13} + \Psi_{15} + \theta_1) + b_{12} \right)
\]
\[
+ h_{45 e^{\xi_1} + \xi_5} \left( [\Psi_{24} + \Psi_{25} + \theta_i^*] (\Psi_{14} + \Psi_{15} + \theta_1) + b_{12} \right) + h_{34 h_{35 e^{\xi_1} + \xi_6} + \xi_7} \left( [\Psi_{23} + \Psi_{24} + \Psi_{25} + \theta_i^*] (\Psi_{13} + \Psi_{14} + \Psi_{15} + \theta_1) + b_{12} \right)
\]
\[
\times (\Psi_{13} + \Psi_{14} + \Psi_{15} + \theta_1) + b_{12} \right),
\]
(24)
It is notable that the superscript * indicates the conjugation of parameters. Substitute (24) into the transformation (15), we can get the two wave interaction solution of \( 1L - 3S \) for Eq. (14).

Then, we further convert the middle 2 solitons into a breather through the conjugated parameters assignment, namely, divide \( N = 2 + 2 + 1 \). In this way, the three wave interaction solution of \( 1L - 1B - 1S \) for Eq. (14) can be calculated, and the obtained auxiliary function reads
\[
f_{1L1B1S} = \theta_1 \theta_i^* + b_{12} + e^{\xi_1} \left( [\Psi_{23} + \Psi_{25} + \theta_i^*] (\Psi_{13} + \theta_1) + b_{12} \right)
\]
\[
+ e^{\xi_2} \left( [\Psi_{24} + \theta_i^*] (\Psi_{14} + \theta_1) + b_{12} \right) + e^{\xi_3} \left( [\Psi_{25} + \theta_i^*] (\Psi_{15} + \theta_1) + b_{12} \right)
\]
\[
+ h_{34 e^{\xi_1} + \xi_4} \left( [\Psi_{23} + \Psi_{24} + \theta_i^*] (\Psi_{13} + \Psi_{14} + \theta_1) + b_{12} \right) + h_{35 e^{\xi_1} + \xi_5} \left( [\Psi_{23} + \Psi_{25} + \theta_i^*] (\Psi_{13} + \Psi_{15} + \theta_1) + b_{12} \right)
\]
\[
+ h_{45 e^{\xi_1} + \xi_5} \left( [\Psi_{24} + \Psi_{25} + \theta_i^*] (\Psi_{14} + \Psi_{15} + \theta_1) + b_{12} \right) + h_{34 h_{35 e^{\xi_1} + \xi_6} + \xi_7} \left( [\Psi_{23} + \Psi_{24} + \Psi_{25} + \theta_i^*] \right)
\]
\[
\times (\Psi_{13} + \Psi_{14} + \Psi_{15} + \theta_1) + b_{12} \right),
\]
(25)
where \( \xi_i^R \) is the real part of \( \xi_i \). Substitute the auxiliary function \( f_{1L1B1S} \) into (15), we finally get a \( 1L - 1B - 1S \) solution for Eq. (14). The images based on the above solutions are shown in Fig. 4.

Theoretically, based on the above idea of partitioning soliton solutions, we can calculate all kinds of interaction solutions of any order among horseshoe-like solitons, breathers and lump waves for Eq. (14). Several higher-order solutions are shown in Fig. 5, and the
calculation process is omitted due to the limited space. The corresponding values of parameters are given in Table 1.

4 Summary

In this paper, two higher dimensional nonlinear evolution equations with time-dependent coefficients are investigated. By applying different approaches and skills, new types of solitons and novel interaction solutions are constructed. It should be emphasized that algorithms and skills applied in this paper are universal and can also be applied to other higher dimensional nonlinear evolution equations, including constant coefficient equations.

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Data Availability Statement Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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| Solution | Values of parameters |
|----------|----------------------|
| 0L-0B-7S | \( k_1 = 1/10, r_1 = 1/10, c_1 = 0, k_2 = 1/10, r_2 = 3/10, c_2 = 0, k_3 = 1/10, r_3 = 5/10, c_3 = 0, k_4 = 1/10, r_4 = 1/10,\) |
| 0L-0B-1S | \( k_1 = 0, k_2 = 1/7, r_1 = 0, c_1 = 0, k_5 = 1/7, r_5 = 3/10, c_5 = 0, k_6 = 1/7, r_6 = 3/10, c_6 = 0, k_7 = 1/10,\) |
| 0L-2B-3S | \( k_1 = 0, k_2 = 1/7, r_1 = 7/4, r_2 = 0, c_1 = 0, c_2 = 0,\) |
| 0L-2B-1S | \( k_1 = 0, k_2 = 1/7, r_1 = 7/4, r_2 = 0, c_1 = 0, c_2 = 0,\) |
| 0L-2B-1S | \( k_1 = 0, k_2 = 1/7, r_1 = 7/4, r_2 = 0, c_1 = 0, c_2 = 0,\) |
| 0L-2B-1S | \( k_1 = 0, k_2 = 1/7, r_1 = 7/4, r_2 = 0, c_1 = 0, c_2 = 0,\) |

Note: The rest of parameters not given are appointed to \( L \).

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