ENvironments for Magnetic Field Amplification by Cosmic Rays

Ellen G. Zweibel\textsuperscript{1,2,3} and John E. Everett\textsuperscript{1,2,3}

\textsuperscript{1} Department of Astronomy, University of Wisconsin-Madison, 475 N Charter Street, Madison, WI 53706, USA
\textsuperscript{2} Department of Physics, University of Wisconsin-Madison, 1150 University Avenue, Madison, WI 53706, USA
\textsuperscript{3} Center for Magnetic Self Organization in Laboratory and Astrophysical Plasmas, University of Wisconsin, 1150 University Avenue, Madison, WI 53706, USA

Received 2009 September 24; accepted 2009 December 17; published 2010 January 13

ABSTRACT

We consider a recently discovered class of instabilities, driven by cosmic ray streaming, in a variety of environments. We show that although these instabilities have been discussed primarily in the context of supernova-driven interstellar shocks, they can also operate in the intergalactic medium and in galaxies with weak magnetic fields, where, as a strong source of helical magnetic fluctuations, they could contribute to the overall evolution of the magnetic field. Within the Milky Way, these instabilities are strongest in warm ionized gas and appear to be weak in hot, low density gas unless the injection efficiency of cosmic rays is very high.

Key words: cosmic rays – instabilities – magnetic fields

1. INTRODUCTION

Recently, a powerful instability which couples a high flux of cosmic rays to their host medium has been discovered (Bell 2004; Blasi & Amato 2008); we refer to this as the “Bell instability.” The instability amplifies low frequency, right circularly polarized electromagnetic fluctuations with wavenumber parallel to the ambient magnetic field. In contrast to the classical cyclotron resonant streaming instability (Wentzel 1968; Kulsrud & Pearce 1969), in which cosmic rays with Lorentz factor \( \gamma \) amplify Alfvén waves with wavelength of the order of the cosmic ray gyroradius \( r_{\gamma} \simeq \gamma c / \omega_{\text{ci}} \) (where \( \omega_{\text{ci}} \) is the non-relativistic ion-cyclotron frequency), the characteristic wavelength of the Bell instability is much less than \( r_{\gamma} \). The Bell instability is thought to be an important ingredient of diffusive shock acceleration in supernova remnants. It might be responsible for amplifying the magnetic field by up to \( \approx 2 \) orders of magnitude above its interstellar value and increasing the maximum energy to which cosmic rays can be accelerated, possibly up to the “knee” at \( \approx 10^{15} \) eV (Drury 2005; Reville et al. 2008b). This is an important result, because it has been known since the work of Lagage & Cesarsky (1983) that standard models of diffusive shock acceleration in the interstellar medium fail to reach the energy of the knee.

Although the Bell instability has been applied primarily to supernova remnants, there are many other environments in which the cosmic ray flux may be large enough to excite it: galactic wind termination shocks, intergalactic shocks, and shocks in disks and jets. Cosmic rays streaming away from local sources or from their host galaxies may also constitute a sufficient flux. If the Bell instability exists in any of these environments it can amplify the magnetic field and transfer cosmic ray energy and momentum to the background plasma as well as increasing the efficiency of cosmic ray acceleration. In view of our current uncertainty as to how galactic and intergalactic magnetic fields originated and are maintained ( Widrow 2002; Kulsrud & Zweibel 2008), any mechanism for amplifying them is of interest. Because fluctuations generated by the Bell instability have a definite helicity relative to the background magnetic field, they could be significant in amplifying the field at large scales (Pouquet et al. 1976). This paper assesses the conditions under which the Bell instability, or more generally any rapidly growing, nonresonant electromagnetic streaming instability, can exist.

In order to excite the Bell instability, the cosmic ray particle flux \( n_{\gamma} v_{D} \), thermal ion density \( n_{i} \), and Alfvén speed \( v_{A} \) must satisfy the inequality (as we show in Section 2.2.1)

\[ n_{\gamma} v_{D} > n_{i} \frac{v_{A}^{2}}{\langle \gamma \rangle c}, \]  

(1)

where \( \langle \gamma \rangle \) is of the order of the mean cosmic ray Lorentz factor. This can be written in terms of the cosmic ray and magnetic energy densities \( U_{\gamma}, U_{B} \),

\[ \frac{U_{\gamma}}{U_{B}} > \frac{c}{v_{A}}. \]  

(2)

Equations (1) and (2) express the requirement that the characteristic wavenumber of the Bell instability be much greater than the reciprocal of the mean cosmic ray gyroradius \( c \langle \gamma \rangle / \omega_{\text{ci}} \).

The Bell instability was originally derived for a cold plasma, which is valid when the thermal ion and electron gyroradii \( r_{\gamma}, r_{e} \), are much less than the characteristic wavelength \( k_{\text{Bell}} \) of the instability; \( k_{\text{Bell}} r_{\gamma} \rightarrow 0 \). Reville et al. (2008a) considered \( k_{\text{Bell}} \) small but nonzero. Their condition that the instability is significantly modified by thermal effects can be written in terms of the cosmic ray flux and ion thermal velocity \( v_{i} \equiv \sqrt{2 k_{B} T / m_{i}} \) as (as we derive in Section 2.2.2)

\[ n_{\gamma} v_{D} > n_{i} \frac{v_{A}^{3}}{v_{i}^{2}}. \]  

(3)

When the inequality (3) is satisfied, the wavenumber of maximum instability, \( k_{\text{wice}} \) (warm ions, cold electrons) decreases relative to \( k_{\text{Bell}} \). And while the maximum growth rate of the Bell instability and the growth rate of the resonant streaming instability are independent of \( B \) as long as \( v_{D} \) much exceeds the Alfvén speed \( v_{A} \), the growth rate of the thermally modified instability increases linearly with \( B \).

In this paper, we extend Reville et al.’s (2008a) analysis to cases where \( k_{r} \) is not small, include cyclotron damping, and investigate the properties of the instability by solving the plasma dispersion relation. The results are given schematically in Figure 1 and precisely for two representative environments, for \( \langle \gamma \rangle = 1 \), in Section 3.1. Equations (1) and (3) define curves on the \((n_{\gamma} v_{D}, B)\) plane. We show that these two curves, together with the requirements \( k_{\text{wice}} r_{\gamma} > 1, k_{\text{wice}} r_{\gamma} < 1, \) and \( k_{\text{Bell}} r_{\gamma} < 1, \)
We analyze the situation considered by Bell; a singly ionized plasma with ion number density $n_i$ and temperature $T$. There is a uniform magnetic field $B = \hat{z}B$ and a population of proton cosmic rays with number density $n_{cr}$ streaming along $B$ with speed $v_D$ relative to the thermal ions. We assume from now on that the mean Lorentz factor $\langle \gamma \rangle$ of the cosmic rays is of order unity. There are applications, such as a “layered” shock precursor in which the most energetic particles have penetrated furthest upstream, where locally $\langle \gamma \rangle \gg 1$, and our results can be scaled readily to this situation; that application is important for determining the maximum energy to which particles can be accelerated. However, the instability will be excited by the bulk cosmic ray population closer to the shock, which is important for field amplification.

There are two populations of electrons, one with density $n_e$ which has no bulk velocity in the frame of the protons, and the other with density $n_{fe}$ which drifts with the cosmic rays at speed $v_D$. Thus, the system is charge neutral and current free. This is the model used in Zweibel (2003) and Bell (2004). Amato & Blasi (2009) have considered the Bell instability when all the electrons drift at speed $(n_{fe}/n_i)v_D$ and found results similar to those obtained for the two electron populations assumed here. But although the Bell instability in its original form is insensitive to the precise form of the thermal electron distribution function $f_e(v)$, it does turn out to depend on $f_e$ in a hot plasma.

We briefly consider the constraints on $f_e$ in the Appendix. Based on an assessment of the electrostatic Langmuir instability, we argue that for very large drifts and high cosmic ray densities, such as are expected in young supernova remnants expanding into diffuse interstellar gas, a separate electron beam with density $n_{fe}$ and drift velocity $v_D$ is unstable, and would tend to relax. For more moderate shocks, or in outflows where the drift speed is less than the electron thermal velocity, such a beam is stable. In general, stability considerations alone do not determine $f_e$.

The fastest growing nonresonant cosmic ray streaming instabilities are sensitive to the precise form of $f_e$ only in a plasma so hot and/or weakly magnetized that the thermal ions do not respond. Any instability present is then an instability of the electrons alone. Its physical significance is unclear, since it depends on the form of $f_e$.

2.1. Full Dispersion Relation

We are interested in right circularly polarized electromagnetic fluctuations which propagate parallel to $B$ and depend on $z$ and $t$ as $\exp(i(kz - \omega t))$; thus, instability corresponds to $\text{Im}(\omega) \equiv \omega_i > 0$. The dispersion relation for the fluctuations can be written as

$$\frac{c^2 k^2}{\omega^2} - \frac{\omega^2}{c^2} - \frac{n_{cr}}{m_i} \frac{(\omega - kv_D)^2}{\omega^2} \frac{c^2 \omega_i}{v_A^2} Z \left( \frac{\omega_i + \omega}{kv_i} \right) + \frac{2\omega_i}{\omega k v_e} Z \left( \frac{\omega_i + \omega}{kv_e} \right),$$

(4)
where we have dropped the displacement current, \( \omega_{pe,i} \equiv (4\pi n_i e^2/m_{e,i})^{1/2} \) are the thermal electron and ion plasma frequencies, \( \omega_{ce,i} \equiv \pm eB/m_{e,i}c \) are the electron and ion cyclotron frequencies, \( Z \) is the plasma dispersion function (Fried & Conte 1961)

\[
Z(z) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-s^2} ds, \tag{5}
\]

and the quantity \( \zeta \) is defined in Equation (A10) of Zweibel (2003) and is plotted in Figure 2. Although the exact behavior of \( \zeta \) depends on the cosmic ray distribution function, it has the general property that when \( k_{cr} \gg 1, \zeta \rightarrow -1 \), with a small imaginary part of \( k_{cr}^{-1} \). The physics underlying this behavior is that cosmic rays barely respond to disturbances with wavelengths much less than their gyroradius, but the electrons do respond, resulting in a large perturbed current. A fraction \( (k_{cr})^{-1} \) of cosmic rays have large enough pitch angles that they can resonate, resulting in a small imaginary part. As to the background plasma terms, the first term on the right-hand side of Equation (4) represents the response of the thermal ions, and the second term represents the cold electrons.

We have solved Equation (4) for a variety of ambient medium parameters and cosmic ray distribution functions, and have reproduced the plots of growth rate versus wavenumber in Bell (2004) and Blasi & Amato (2008). An example is shown in Figure 3, which plots growth rate versus wavenumber at fixed cosmic ray flux, ion density, and magnetic field strength for three different temperatures: \( T = 10^4 \) K, \( T = 10^6 \) K, and \( T = 10^7 \) K. The other parameters, \( B = 3 \) \( \mu \)G, \( n_i = 1 \) cm\(^{-3} \), \( n_{cr,vD} = 10^4 \) cm\(^{-2} \) s\(^{-1} \), are similar to those chosen by these authors as representative of cosmic ray acceleration in a young supernova remnant.

The lowest temperature is essentially the cold plasma result. The growth rate peaks at \( k_{Bell} \) and then plunges, but remains positive as \( k \) increases. The \( T = 10^6 \) curve is similar to the \( T = 10^8 \) K curve, except that the peak growth rate is reduced and occurs at a slightly smaller \( k \), and the positive tail is damped. The \( T = 10^7 \) K curve is markedly different. The wavelength of the fastest growing mode is longer, and the peak growth rate is lower, than at \( T = 10^6 \) K. The instability cuts off abruptly at a longer wavelength than in the \( T = 10^8 \) K and \( T = 10^6 \) K cases, then re-emerges in a short interval of \( k \) before disappearing again. We explain these features with an analytical treatment in the following two subsections.

The effect of magnetic field strength on the fastest growing mode is shown in Figures 4(a) and 4(b). Figure 4(a) plots the maximum growth rate \( \omega_{fgm} \) versus \( B \) at \( T = 10^4 \) K, with other parameters as in Figure 3. For \( B \leq 1 \) \( \mu \)G, \( \omega_{fgm} \) increases linearly with \( B \). At larger \( B \), \( \omega_{fgm} \) is independent of \( B \). Different behavior is seen for \( f_{hm} \); the wavenumber of the fastest growing mode, plotted in Figure 4(b), is independent of \( B \) at low field strength (here, for \( B \leq 1 \) \( \mu \)G) and decreases as \( B^{-1} \) at larger \( B \). These features, too, are derived in the following two subsections.

2.2. Analytical Results

2.2.1. Standard Case

We recover the essential form of the Bell instability from Equation (4) by setting \( \zeta = -1 \) and taking the zero temperature limit of the right-hand side. Using \( Z(z) \rightarrow -1/z \), \( |z| \gg 1 \), Equation (4) becomes

\[
\frac{c^2 k^2}{\omega^2} + \frac{\omega_{cr}}{n_i} \left( \frac{\omega - kv_D}{\omega_{cr}} \right) \frac{c^2}{\omega^2} = -\frac{\omega_{pe}^2}{\omega(\omega_{cr} + \omega)} - \frac{\omega_{pe}^2}{\omega(\omega_{ce} + \omega)}. \tag{6}
\]

We now assume \( \omega \ll \omega_{ce} \), approximate \( (\omega_{cr} + \omega)^{-1} \) by \( \omega_{cr}^{-1}(1 - \omega/\omega_{cr}) \), and use \( \omega_{pe}^2/\omega_{ce} = -\omega_{pe}^2/\omega_{cr} \). Multiplying the resulting dispersion relation by \( \omega^2 v_A^2/c^2 \) yields

\[
k^2 v_A^2 + \omega_n n_{cr} n_i (\omega - kv_D) = \omega^2. \tag{7}
\]

The solution to Equation (7) is

\[
\omega = \left[ \frac{\omega_{cr} n_{cr}}{2 n_i} + \left( \frac{\omega_{cr}^2 n_{cr}}{2 n_i} \right)^2 - \omega_n n_{cr} n_i k v_D + k^2 v_A^2 \right]^{1/2}. \tag{8}
\]

The fastest growing mode occurs at the Bell wavenumber \( k_{Bell} \)

\[
k_{Bell} \equiv \frac{\omega_{cr} n_{cr} v_D}{2 n_i v_A} \tag{9}
\]

and has

\[
\omega(k_{Bell}) = \omega_{Bell} = \frac{\omega_{cr} n_{cr}}{2 n_i} \left[ 1 + \frac{v_D}{v_A} \left( \frac{v_A^2}{v_D^2} - 1 \right)^{1/2} \right] \rightarrow i \frac{\omega_{cr} n_{cr} v_D}{2 n_i v_A}. \tag{10}
\]

\(^4\) At sufficiently low ISM temperatures ion-neutral damping of the fluctuations must also be considered (Zweibel & Shull 1982; Reville et al. 2008a), but that is beyond the scope of this paper.
where the limiting expression following the arrow holds for \( v_D/v_A \gg 1 \).

Equation (10) shows that the instability requires \( v_D/v_A > 1 \). This is also the threshold for the classical resonant streaming instability. When the cosmic ray flux is too low to satisfy Equation (1), the dominant instabilities are resonant instabilities of Alfvén waves. The peak growth rate occurs at wavenumbers which resonate with cosmic rays near the mean energy, \( k \sim r_{ci}^{-1} \), and is of order \( \omega_{ci}(n_{ci}/n_i)(v_D/v_A - 1) \) (Kulsrud & Cesarsky 1971). For \( v_D/v_A \gg 1 \), this expression agrees to within a factor of order unity with \( \omega_{Bell} \), but the two cannot be used simultaneously because they apply for opposite cases of the Equation (1).

At fluxes which satisfy Equation (1), the assumptions made in deriving the classical resonant growth rate—that the underlying waves are Alfvén waves and that the growth time is much longer than the wave period—are incorrect (Zweibel 1979, 2003; Achterberg 1983). At wavenumbers \( k < r_{ci}^{-1} \), the growth rate is of order \( \omega_{ci}(n_{ci}/n_i)(v_D/v_A - 1)^{1/2} \) for \( v_D \) slightly greater than \( v_A \) and peaks at \( \omega_{ci}(n_{ci}v_D/n_i)^{1/2} \) for \( v_D \gg v_A \) (Zweibel 2003). Comparing this expression to \( \omega_{Bell} \), we see that \( \omega_{Bell}/\omega_{Bell} \sim (n_{ci}v_A^2/n_i^2v_D^2)^{1/2} \). From Equation (1), we see that whenever the Bell instability operates, its growth rate exceeds the growth rate of the resonant instability, and the growth rate of the resonant instability is lower than predicted by the classical theory.

At \( k > 2k_{Bell} \), the nonresonant instability is stabilized by magnetic tension. Comparison with Figure 3 shows that although the growth rate plunges to low levels at this value of \( k \), the instability does not completely disappear, as is predicted by Equation (7). The \( \omega_i \propto k^{-1} \) tail seen in Figure 3 can be recovered by using the full \( \zeta \); for \( kr_{ci} \gg 1 \; \zeta_i \sim -1 + i/kr_{ci} \).

The Bell instability is important in shock acceleration if the growth time is shorter than the time the shock takes to travel through the layer within which the cosmic rays are confined. For acceleration at the maximum rate, the cosmic ray diffuse according to the Bohm formula with diffusion coefficient \( D \sim ckr_{ci} \), which sets the convection time across the layer as \( (\omega_{ci} v_B^2/c^2)^{-1} \). The Bell instability will be able to grow if \( (n_{ci} v_D/n_i v_A) > v_D^2/c^2 \).

2.2.2. Warm Ions, Cold Electrons

We now imagine decreasing \( B \) or increasing \( T \) such that \( k v_i/\omega_{ci} + \omega_i \) is less than unity but not infinitesimal. In this case, \( Z(cz_i) \sim -1/\zeta - 1/2\sqrt{\zeta} + i/\sqrt{4\pi} \zeta e^{-\zeta} \). The imaginary part represents ions for which the Doppler shifted wave frequency \( \omega - kv = -\omega_{ci} \); i.e. ions in cyclotron resonance. The \( -1/2\sqrt{\zeta} \) term represents the finite gyroradius of the ions, which partially decouples them from the field. Using this approximation, Equation (4) becomes

\[
\omega^2 - \omega \left( \frac{k^2 v_i^2}{2\omega_{ci}} + \omega_{ci} \frac{n_{ci}}{n_i} - i \sqrt{\frac{\pi}{\omega_{ci}}} \frac{v_D}{k v_i} \right) - k^2 v_A^2 + \omega_{ci} \frac{n_{ci}}{n_i} k v_D = 0.
\]

Comparison of Equations (7) and (11) shows that the thermal ions have two effects: cyclotron resonance, which is represented by the imaginary term, and a pressure-like effect due to the finite ion gyroradius, which increases the wave speed and is known as ion gyroviscosity. In the absence of cosmic rays, the dispersion relation agrees with Foote & Kulsrud (1979), and in the limit \( kr_{ci} \gg 1 \), with Reville et al. (2008a), except that ion cyclotron damping is neglected in both papers.

Equation (11) represents the behavior of the growth rate quite well. This is shown in Figure 5, which compares \( \omega_i \) calculated from Equations (4) and (11).

In the left panel, \( T = 10^5 \) K, thermal effects are unimportant and the differences between the two curves are entirely due to the value adopted for \( \zeta \); taking \( \zeta = -1 \) omits the effect of resonant particles at high \( k \) and underestimates the cosmic ray response at low \( k \), thereby overestimating \( \omega_i \). In the right panel, \( T = 10^7 \) K, thermal effects are important enough to modify the instability (see Figure 3 and below) but the analytical dispersion relation successfully reproduces the main features. Again, setting \( \zeta = -1 \) overestimates the growth rate at low \( k \).

It can be shown from Equation (11) that when

\[
\frac{v_A}{v_i} < \left( \frac{n_{ci} v_D}{n_i v_i} \right)^{1/3},
\]

the fastest growing mode is determined by competition between the drift term \( \omega_{ci} k v_D n_{ci}/n_i \) and the finite gyroradius term rather than the drift term and the magnetic tension term. This condition is equivalent to Equation (3). The wavenumber \( k_{wice} \) of the fastest growing mode in this regime is

\[
k_{wice} \sim \frac{\omega_{ci}}{v_i} \left( \frac{n_{ci} v_D}{n_i} \right)^{1/3}.
\]

Equation (13) neglects cyclotron damping, and thus is valid only for \( k_{wice} r_i \sim (n_{ci} v_D/n_i v_i)^{1/3} < 1 \). As \( k_{wice} r_i \to 1 \) cyclotron damping becomes large, and the instability shuts off. A further constraint is that the instability be nonresonant:

\[\text{The sign of the second term on the left-hand side of Equation (3) in Reville et al. (2008a) is incorrect.}\]
brief resurgence. This is because the resonant ion cyclotron term and temperature of the ambient medium.

\[ \frac{k_{wic}}{\omega_{ci}} \sim \frac{(c/v_i)k_{wic}}{k_{ri}} > 1. \]  

Both requirements can be written as temperature and density dependent limits on \( n_{ci}v_D \); we return to them in Section 3.1 (Equations (17), (20)).

The growth rate \( \omega_{wic} \) corresponding to \( k_{wic} \) is

\[ \omega_{wic} \sim \omega_{ci} \left( \frac{n_{ci}v_D}{n_i v_i} \right)^{2/3}, \quad (14) \]

in agreement with Reville et al. (2008a).

It can be shown from Equations (10), (12), and (14) that \( \omega_{wic}/\omega_{Bell} < 1 \). At the limits of validity of the warm ion approximation, which is \( k_{wic}v_D/\omega_{ci} = 1 \), \( \omega_{wic}/\omega_{Bell} = v_A/v_i \). The suppression of the growth rate is due not to thermal ion cyclotron damping, which is weak for \( \omega_{ci}/kvi \gg 1 \), but due to the restoring force exerted by the warm ions. Cyclotron damping does, however, come into play at shorter wavelength, obliterating the resonant tail of the instability. This happens roughly where \( (kvi/\omega_{ci})e^{-(\omega_{ci}/kvi)^2} > n_{ci}v_D/n_i c \). For the cosmic ray flux and ion density assumed in Figures 3–5, this occurs at \( kvi/\omega_{ci} \sim 0.27 \), or \( kci/vD \sim 9 \times 10^7 T^{-1/2} \). This is consistent with the behavior shown in Figure 3.

At \( T = 10^7 \) K, something more complicated is going on; the instability growth rate decreases sharply after peaking near \( B < B_M \approx 8.7 \times 10^{-7}(n_{ci}v_D)^{1/2} \), (16)

where here and below \( n_{ci}v_D \) is given in units of cm\(^{-2}\) s\(^{-1}\) and \( B \) is in G. The condition that thermal effects modify the Bell instability such that the wavelength of the fastest growing mode is at \( k \sim k_{wic} \) (Equation (9)) rather than \( k \sim k_{Bell} \) (Equation (13)) is

\[ B < B_M \equiv 2.3 \times 10^{-9} T^{1/3} n_i^{1/6} (n_{ci}v_D)^{1/3}. \quad (17) \]

The condition for the thermally modified Bell instability to be nonresonant is \( k_{wic}k_{ri} > 1 \). At the same time, the thermal ions must be magnetized at \( k = k_{wic}; k_{wic}r_i < 1 \). These conditions limit \( n_{ci}v_D \) to the range

\[ n_i v_i^4/c^2 < n_{ci}v_D < n_i v_i, \quad (18) \]
or numerically

\[ 3 \times 10^{-16} n_i T^2 < n_{ci} v_D < 10^4 n_i T^{1/2}. \]  

(19)

Finally, the condition that the ions be magnetized at \( k = k_{Bell} \), \( k_{Bell} r_i < 1 \), is \( n_{ci} v_D < n_i v_i^2 / v_i \), or

\[ B > B_T \equiv 5 \times 10^{-10} T^{1/4} (n_{ci} v_D)^{1/2}. \]  

(20)

Equations (16)–(20) are plotted on the \((n_{ci} v_D, B)\) plane in Figures 6 and 7. Because Equations (17)–(20) depend on \( n_i \) and \( T \), we give two versions of the plot. Figure 6 represents the interstellar medium; \( n_i = 1 \text{ cm}^{-3} \), \( T = 10^4 \text{ K} \). Figure 7 represents the intracluster medium; \( n_i = 10^{-3} \text{ cm}^{-3} \), \( T = 10^7 \text{ K} \).

For \( n_{ci} v_D < n_i v_i^4 / c^3 \), \( B_S < B_M \). This is also the condition \( k_{wice} r_i < 1 \) (Equation 19) and is represented by the leftmost vertical line on the plot. To the left of this line, there can be no nonresonant instability; since \( B_M > B_S \) the instability would take the thermally modified form, but for \( B > B_M \) the field is too large and for \( B < B_S \) the flux is too low. Streaming instability exists, but it is resonant. In the cold plasma limit, the Bell instability exists for any \( B < B_S \), no matter how small.

For \( n_{ci} v_D > n_i v_i^4 / c^3 \), \( B_M < B_S \). For \( B > B_S \), the field is too large for nonresonant instability (this is the case in the interstellar medium, away from cosmic ray sources). For \( B < B_S \), the Bell instability operates in standard form as long as \( B \) exceeds \( B_S \) and \( B_T \). Although \( B_M < B_T \) is theoretically possible, it requires \( n_{ci} v_D > n_i v_i \), which is rather extreme. If we confine ourselves to \( n_{ci} v_D < n_i v_i \) (represented by the rightmost vertical line, defined by \( k_{wice} r_i = 1 \)) then the Bell instability operates for \( B_M < B < B_S \). For \( B < B_M \), there is nonresonant instability as long as \( n_{ci} v_D \) is to the left of the vertical line. To the right of this line, the ions are unmagnetized. As we have argued, in this case the instability is controlled primarily by the electrons, and for \( k r_i \sim 1 \), ion cyclotron damping is strong.

In summary, nonresonant instabilities exist in the range of cosmic ray fluxes given by Equation (19). When \( B \) is between \( B_M \), defined in Equation (17), and \( B_S \), defined in Equation (16), the maximum growth rate is independent of \( B \). When \( B < B_M \) the instability is thermally modified, occurs at longer wavelength, and grows at a rate proportional to \( B \). This is also shown in Figure 8, which is a contour plot of the maximum growth rate in Equation (4) as a function of cosmic ray flux and magnetic field. Toward the lower-right of the plot, one can see the maximum growth rate decreasing, downward, with decreasing \( B \), but relatively constant above an approximately diagonal line in \((n_{ci} v_D, B)\) space from \((10^{-8} \text{ cm}^{-2} \text{ s}^{-1}, 10^{-10} \text{ G})\) to \((10^{4.4} \text{ cm}^{-2} \text{ s}^{-1}, 10^{-3.8} \text{ G})\).

### 3.2. Astrophysical Settings

We now consider a few examples of specific astrophysical settings. In order to smoothly represent the transition from the standard Bell regime \( B_M < B < B_S \) to the thermally modified regime \( B < B_M \), we replace \( \omega_{wice} \) by \( \omega_{Bell} B / B_M \), which agrees with Equation (10) up to a factor of order unity.

#### 3.2.1. Supernova Remnants and Superbubbles

First, we consider cosmic ray acceleration by a supernova-driven shock traveling at \( 10^4 \text{ km s}^{-1} \) through the interstellar medium. As in Figure 6(a) we take \( n_i = 1 \text{ cm}^{-3} \), \( T = 10^4 \text{ K} \). According to Equation (19), nonresonant instability exists for

\[ 3 \times 10^{-8} \text{ cm}^{-2} \text{ s}^{-1} < n_{ci} v_D < 10^6 \text{ cm}^{-2} \text{ s}^{-1}. \]

Taking \( n_{ci} v_D = 10^4 \text{ cm}^{-2} \text{ s}^{-1} \), as assumed in Figure 3, we find from Equations (16) and (17) that the standard Bell instability operates for \( B \) between 1.1 and 87 \( \mu \text{G} \), which encompasses
most of the field strengths measured in the diffuse interstellar medium. The growth rate in the Bell regime is $2.5 \times 10^{-4} \text{s}^{-1}$ and $2.5 \times 10^{-4}(B/1.1 \mu G) \text{s}^{-1}$ in the thermally modified regime. The condition for efficient field amplification at shocks, $\omega > \omega_{\text{c}}(v_D/c)^2$, is $\omega > 1.1 \times 10^{-1} B$. Growth is amply fast in the Bell regime and also in the thermally modified Bell regime, since the growth rate and advection time scale in the same way with $B$. Of course, if $B$ is too small, the cosmic ray acceleration time becomes long compared to the shock evolution time. But even if $B \sim 10^{-9} \mu G$, about an order of magnitude less than the disordered field estimated by Rees (1987) to arise from a superposition of plerion supernova remnants in the early galaxy, the characteristic growth time of the instability is less than a year, much faster than the timescale on which the remnant evolves.

On the other hand, if the shock propagates in a hot, low density medium such as the superbubbles modeled by MacLow & McCray (1988), with $n_i = 3 \times 10^{-3} \text{cm}^{-3}$, $T = 3 \times 10^6 \text{K}$, then according to Equation (19), the nonresonant instability could exist for $8.1 \times 10^{-4} \text{cm}^{-3} \text{s}^{-1} < n_{\text{cr}} v_D < 5.2 \times 10^{-8} \text{cm}^{-3} \text{s}^{-1}$. If the cosmic ray injection efficiency were the same as at higher densities, $n_{\text{cr}}/n_i = 10^{-5}$, Equations (16) and (17) show that the standard Bell instability operates between 0.39 and 4.7 $\mu G$. The upper limit is only slightly less than the rms Galactic field.

The growth rate in the Bell regime, $1.4 \times 10^{-5} \text{s}^{-1}$, exceeds the advection rate through the cosmic ray scattering layer only for $B > 1.3 \mu G$, suggesting that nonresonant instability is less important for shock acceleration in a low density medium than in a high density medium. The injection efficiency, rather than thermal effects, is the deciding factor: increasing $n_{\text{cr}}/n_i$ above $10^{-5}$ would enhance the growth rate in proportion. These results imply that nonresonant instabilities might not occur for cosmic ray acceleration in superbubbles, where the combined effects of many hot star winds and explosions make the density low.

Denser superbubbles have been observed (Dunne et al. 2001). Increasing $n_i$ to $3 \times 10^{-2} \text{cm}^{-3}$ while leaving the other parameters the same would increase $\omega_{\text{nwell}}$ to $3.8 \times 10^{-5} \text{s}^{-1}$ and change the range in which the standard Bell Instability operates to $1.2 \mu G < B < 15 \mu G$. The rms Galactic field is estimated to be $\sim 5.5 \mu G$ (Ferrière 2001), so the conditions for instability should be satisfied. The instability growth rate exceeds the advection rate through the acceleration layer as long as $B < 4.1 \mu G$.

3.2.2. Shocks in Galaxy Clusters

Next, we consider acceleration at shocks in the intracluster medium. We take $n_i \sim 10^{-3} \text{cm}^{-3}$ and $T = 10^7 \text{K}$. According to Equation (19), nonresonant instability exists for $3 \times 10^{-5} \text{cm}^{-2} \text{s}^{-1} < n_{\text{cr}} v_D < 3.1 \times 10^{-8} \text{cm}^{-2} \text{s}^{-1}$. If acceleration occurs at the same efficiency assumed for galactic supernova remnants then $n_{\text{cr}} = 10^{-5} \text{cm}^{-3}$; assuming $v_D \sim v_S \sim 1000 \text{km s}^{-1}$, then $B_S$, the maximum field strength for nonresonant instability, is $8.7 \times 10^{-7} \mu G$. This is slightly less than the fields inferred in galaxy clusters (e.g., Govoni & Feretti 2004), but the parameters are too uncertain to rule out resonant instability. According to Equation (10), the growth rate in the Bell regime is $7.5 \times 10^{-7} \text{s}^{-1}$. With these same parameters, $B_M = 1.6 \times 10^{-7} \mu G$; below this value the instability is thermally modified and grows at the rate $7.5 \times 10^{-7} (B/1.6 \mu G) \text{s}^{-1}$. Due to the high temperature, the range in which the standard Bell instability operates without thermal effects is quite small, but the growth rate of the thermally modified instability is still fast. The condition for efficient field amplification at shocks, $\omega > \omega_{\text{c}}(v_D/c)^2$, is $\omega > 1.1 \times 10^{-11} B$. Although the instability growth rates are lower than for supernova remnants, they are still large enough to satisfy this condition in both the standard and thermally modified nonresonant regimes. Thus, it appears that nonresonant instabilities could play a role in shock acceleration and could amplify magnetic fields in galaxy clusters.

This could make intergalactic shocks a favorable environment for acceleration of ultra-high-energy cosmic rays.

3.2.3. Unconfined Galactic Cosmic Rays

As our final example, we consider leakage of cosmic rays from galaxies into the intergalactic medium, which we take to have density and temperature $n_i = 10^{-6} \text{cm}^{-3}$, $T = 10^6 \text{K}$ (Richter et al. 2008). From Equation (19), nonresonant instabilities can be excited by cosmic ray fluxes between $3 \times 10^{-10}$ and $10 \text{cm}^{-2} \text{s}^{-1}$. In the local interstellar medium, $n_{\text{cr}} \sim 10^{-9} \text{cm}^{-3}$, while $v_D$ is roughly the scale height of cosmic rays divided by their confinement time in the galaxy, or about 100 km s$^{-1}$. This gives a galactic flux $n_{\text{cr}} v_D \sim 10^{-2} \text{cm}^{-2} \text{s}^{-1}$. Assuming cosmic rays emanate isotropically from a characteristic galaxy size $R_g$, we write $n_{\text{cr}} v_D \sim 10^{-2} (R_g/R)/(L_{\text{g}}/L_{\text{MW}})$. From Equations (16) and (17), we find $B_S = 8.7 \times 10^{-8} (R_g/R)/(L_{\text{g}}/L_{\text{MW}})1/2$ while $B_M = 7.3 \times 10^{-8} (R_g/R)/3(L_{\text{g}}/L_{\text{MW}})$1/3. These values suggest that nonresonant instabilities could be excited in the intergalactic medium even if the fields are weaker than the $10^{-8} - 10^{-10} \mu G$ range often cited as upper limits (Kulsrud & Zweibel 2008). The growth rates, however, are rather slow. For example, if $R/g = 10$, $\omega_{\text{nwell}} = 2.5 \times 10^{-9} (L_{\text{g}}/L_{\text{MW}}) \text{s}^{-1}$. In this case, $B_M = 1.6 \times 10^{-9} \mu G$; for $B = 10^{-10} \mu G$, the maximum growth rate is about $4.1 \times 10^{-10} (L_{\text{g}}/L_{\text{MW}}) \text{s}^{-1}$. Still, although the growth time exceeds 10$^3$ years, this is much shorter than any reasonable cosmic ray convection time. Therefore, nonresonant instabilities could be excited by cosmic rays from ordinary galaxies in the intergalactic medium at large. They could amplify intergalactic magnetic fields and could heat the plasma.

4. SUMMARY

Nonresonant instability driven by cosmic ray streaming has emerged as a strong candidate for amplification of magnetic fields in environments such as strong shock waves, where the cosmic ray flux is large (Bell 2004). When the flux is high enough and/or the magnetic field is low enough, that Equation (1) is violated, the nonresonant instability replaces the classical resonant streaming instability as the dominant electromagnetic instability generated by cosmic rays. Although the instability scale length predicted by linear theory is small even compared to the cosmic ray gyroradius $r_{\text{cr}}$, nonlinear simulations suggest that as the amplitude of the instability grows it generates fluctuations at larger scales. This can increase the energy to which particles are accelerated in shocks.

Cosmic ray acceleration and magnetic field growth are both of interest in a variety of environments, including young galaxies which may be actively forming stars but have not yet built up magnetic fields, shocks in galaxy clusters, and the intergalactic medium at large. In this paper, we have carried out a parameter study of nonresonant instabilities including ion thermal effects. We solved the full dispersion relation (4) numerically and verified that a simple analytical approximation, Equation (11), is quite accurate in the wavenumber regime of interest. We corroborated the criterion of Reville et al. (2008a) for when ion gyrosviscosity reduces the instability growth rate and shifts it to longer wavelength. We showed that ion cyclotron damping.
cuts off the instability at short wavelengths and argued that at wavelengths short enough that the ions are unmagnetized the instability depends only on the electron distribution function, the prediction of which is beyond the scope of this paper.

The joint requirements that the instability wavelength be much less than the cosmic ray gyroradius but much more than the thermal ion gyroradius limits the range of fluxes which excite nonresonant instability to \( n_i v_i^3 / c^3 < n_e v_D < n_i v_i \). In practical terms, this range is large and accommodates most cases of interest. Within the unstable range, there is a “strong field” regime in which all streaming instabilities are resonant, an “intermediate” regime in which nonresonant instability in the form derived by Bell dominates, and a “weak field” regime in which the instability is thermally modified. In the Bell regime, the maximum growth rate is independent of \( B \) but in the thermally modified regime it depends linearly on \( B \). Young galactic supernova remnants are generally in the intermediate regime unless the ambient medium is hot and rarefied (like the interior of a superbubble), in which case the instability is weakened. Generally, if \( B \) is in the nanogauss range, the growth rates are fast enough for nonresonant instability to be a potential source of magnetic field amplification in weakly magnetized interstellar and intergalactic gas. At much lower field strengths, the instability is too slow to be of interest, but other instabilities, such as Weibel modes, could be an important ingredient in magnetogenesis (Medvedev et al. 2006).

Although nonresonant instabilities amplify magnetic fields on rather small scales—much smaller than the eddy scales characteristic of interstellar and intergalactic turbulence—they should not be ignored in discussions of magnetogenesis. Because only the right circularly polarized modes are unstable, nonresonant instabilities are a source of magnetic helicity on scales at which the background magnetic field is coherent. Magnetic helicity is thought to be a key ingredient in the growth of large-scale magnetic fields from small-scale fluctuations (Pouquet et al. 1976). Cosmic ray generated fluctuations could be important in driving an inverse cascade of magnetic power to longer wavelengths and could prevent the pileup of power at short wavelengths that currently confounds interstellar and intergalactic dynamo theories.

We acknowledge useful discussions with P. Blasi, J. Kirk, and B. Reville, and comments by the referee. Support was provided by NSF grants AST 0507367, PHY 0821899, and AST 0907837 to the University of Wisconsin.

APPENDIX

ELECTRON DISTRIBUTION FUNCTION

Here we briefly consider the constraints on the electron distribution function \( f_e \).

One way or another, the cosmic ray current must be canceled: an uncompensated cosmic ray current \( e_n e v_D \) flowing in a channel of width \( L_{pc} \) measured in parsecs generates a magnetic field \( B \sim 0.5 n_e e v_D L_{pc} \). Even the galactic flux of \( 10^{-2} \text{ cm}^{-2} \text{ s}^{-1} \) with \( L_{pc} = 1 \) would generate a 5 mG field. Since cosmic rays are ion dominated, thermal electrons must cancel their flux.

When an electron beam drifts with respect to the bulk plasma, it can excite rapidly growing electrostatic instabilities which tend to re-distribute electron momentum and bring the system to a state of marginal stability. Langmuir waves (also called plasma oscillations) with wavenumber \( \omega_{pe} / v \) (\( \omega_{pe} \) is the electron plasma frequency \( (4\pi n_e e^2 / m_e)^{1/2} \)) are destabilized if \( \partial f_e / \partial v > 0 \). If the beam and bulk electrons have the same temperature \( T_e \) and the beam velocity \( v_b \) much exceeds the electron thermal velocity \( v_e \equiv \sqrt{2 k_B T_e / m_e} \) (which is necessary for instability if the beam density \( n_b \) is much less than the bulk density \( n_e \), the case of interest here), then the requirement for stability is approximately

\[
\frac{v_b - v_e}{v_e} e^{-v_b^2 / v_e^2} > \frac{n_b}{n_e} \quad (A1)
\]

(e.g., Krall & Trivelpiece 1973). In shock acceleration, it is sometimes assumed \( n_b / n_e \sim 10^{-3} \); according to Equation (A1), stability then requires \( v_b / v_e < 3.5 \). Assuming \( T_e = T \) in the upstream plasma, beams associated with shocks of Mach number \( M < 3.5 \sqrt{m_b / m_e} \) are stable while shocks at higher \( M \) are unstable. In a \( 10^4 \) K gas, the stability boundary is at about 1500 km s\(^{-1} \). Thus, while the Langmuir instability is a constraint for very fast shocks, it is probably irrelevant for older supernova remnants, and in galaxy cluster accretion shocks or galactic wind termination shocks, where the background gas is hot and the Mach numbers are expected to be moderate. The fluxes associated with cosmic ray escape from galaxies are probably also electrostatically stable.

Therefore, it appears that \( f_e \) is not determined by stability considerations alone, but depends on other factors such as the history of the system and the source of cosmic rays.

REFERENCES

Achterberg, A. 1983, A&A, 119, 274
Amato, E. A., & Blasi, P. 2009, MNRAS, 392, 1591
Bell, A. R. 2004, MNRAS, 353, 550
Bell, A. R. 2005, MNRAS, 358, 181
Blasi, P., & Amato, E. 2006, Proc. 30th ICRC (Mexico City), 2, 235
Drury, L. O’C. 2005, in AIP Conf. Proc. 801, Astrophysical Sources of High Energy Particles and Radiation, ed. T. Bulik, B. Rudak, & G. Madejski (Melville, NY: AIP), 337
Dunne, B. C., Points, S. D., & Chu, Y.-K. 2001, ApJS, 136, 119
Ferrière, K. 2001, Rev. Mod. Phys., 73, 1031
Foote, E. A., & Kulsrud, R. M. 1979, ApJ, 233, 302
Fried, B. D., & Conte, S. 1961, The Plasma Dispersion Function (New York: Academic)
Govoni, F., & Feretti, L. 2004, Int. J. Mod. Phys. D, 13, 1549
Kraus, N. A., & Trivelpiece, A. W. 1973, Principles of Plasma Physics (McGraw-Hill)
Kulsrud, R. M., & Cesarsky, C. J. 1971, Astrophys. Lett., 8, 189
Kulsrud, R. M., & Pearce, W. P. 1969, ApJ, 156, 445
Kulsrud, R. M., & Zweibel, E. G. 2008, Rep. Prog. Phys., 71, 046901
Lagage, P. O., & Cesarsky, C. J. 1983, A&A, 125, 249
Luo, Q., & Melrose, D. B. 2009, MNRAS, 397, 1402
MacLow, M.-M., & McCray, R. A. 1988, ApJ, 324, 776
Medvedev, M. V., Silva, L. O., & Kamionkowski, M. 2006, ApJ, 642, L1
Niemiec, J., Pohl, M., Stroman, T., & Nishikawa, K. 2008, ApJ, 684, 1189
Ohyra, Y., Reville, B., Kirk, J. G., & Takahara, F. 2009, ApJ, 698, 445
Pelletier, G., Lemoinne, M., & Marcowith, A. 2006, A&A, 453, 181
Pouquet, A., Frisch, U., & Leorat, J. 1976, J. Fluid Mech., 77, 321
Rees, M. J. 1987, QJRAS, 28, 197
Reville, B., Kirk, J. G., Duffy, P., & O'Sullivan, S. 2008a, Int. J. Mod. Phys. D, 17, 1795
Reville, B., O'Sullivan, S., Duffy, P., & Kirk, J. G. 2008b, MNRAS, 386, 509
Richter, P., Paerels, F. B. S., & Kaastra, J. S. 2008, Science Rev., 134, 25
Riquelme, M. A., & Spitkovsky, A. 2009, ApJ, 694, 626
Vladimirov, A. E., Bykov, A. M., & Ellison, D. C. 2009, ApJ, 703, L29
Wentzel, D. G. 1968, ApJ, 152, 987
Widrow, L. M. 2002, Rev. Mod. Phys., 74, 775
Zirakashvili, V. N., Ptuskin, V. S., & Völk, H. J. 2008, ApJ, 678, 255
Zweibel, E. G. 1979, in Particle Acceleration Mechanisms in Astrophysics, ed. J. Arons, C. Max, & C. McKee (New York: AIP), 319
Zweibel, E. G. 2003, ApJ, 587, 625
Zweibel, E. G., & Shull, J. M. 1982, ApJ, 259, 859