The Identification of Tsunami Height Correlation Model with Earthquake Parameters

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Abstract. Tsunami is an event triggered by earthquake parameters. The earthquake parameters can provide information on the probability of a tsunami even that can be used in estimating the height of a tsunami. By excluding information beyond the earthquake parameters, this research aims to derive a mathematical model that describing the correlation between the height of the tsunami and the parameters of earthquakes. The method used in this research is to apply one of the multivariate methods of time series, namely the transfer function. Based on diagnostic results of Autocorrelation Function plot, Partial Autocorrelation Function and parameter significance, for the strength and depth of the earthquake selected model are ARIMA model (0, 1, 1). Based on the results of the processes of the input-output series model, cross-correlation, and weighting response formed twelve models of possible transfer functions to be modeled relationships between variables Tsunami height \(Y\), Strength of the earthquake \(X_1\), and Depth of the earthquake \(X_2\). By looking at the minimum AIC values of the model parameters, where the \(X_1\) variable is 1679,156 and \(X_2\) of 1668,258, we get the order transfer function model is \((0,3,0), (9,0,2), (0,1))\), with the mathematical equation model as follows: \(Y_t = (3.71978 + 1.43267B)X_{1t} + (-0.03142)(1-0.64514B)X_{2t} + (1 - 0.93230 B)a_t\).

1. Introduction
The 2004 Indian Ocean earthquake occurred on 26 December with the epicentre off the west coast of Sumatra, Indonesia. The shock had a moment magnitude of 9.1 Mw. The undersea megathrust earthquake triggered a 30 meter (100 feet) tsunami waves that hit most of the land bordering the Indian Ocean. This disaster, killing 230,000 - 280,000 people in fourteen countries. One of the causes of the great loss of life is, that times there were no tsunami warning systems in the Indian Ocean to detect tsunamis or to warn the general population living around the ocean. The height of the tsunami waves depends on the parameters of earthquakes such as time of incidence, epicenter location, strength, depth, and intensity of the earthquake. The Meteorology Climatology and Geophysics Agency (BMKG Indonesia), as an authoritative organization in Indonesia that can issue tsunami warnings, assume the probability of a tsunami in the event of a major earthquake occurring at sea, has a strength above 7.0 Mw, with a depth of < 70 km (inatews.bmkg.go.id).

Satake [1] describes the height of the tsunami every hour that occurred in the 2004 Aceh earthquake based on the NGDC Tsunami Database. The black arrow indicates the direction and speed of the Indo-Australian plate, while the red line indicates the height of the tsunami occurring at that time (Figure 1). Abe [2]-[3] has published the relationship between the height of the tsunami (in meters) to the strength of the earthquake (Mw), as given:
\[ \log H_t = M_w - \log R - 5.55 + C \]  \hspace{1cm} (1)

where \( C = 0 \) for interplate thrust earthquakes and \( C = 0.2 \) for backarc events and where \( R \) is in kilometers measured from the epicenter. As for the local tsunami event, it is formulated as follows: (Abe [4]),

\[ \log H_t = 0.5 M_w - 3.30 + C \]  \hspace{1cm} (2)

Other researchers had analysed the relationship between earthquake parameters and tsunami parameters including: Eric [5], modelling the height of the tsunami based on strength parameters and earthquake location. Anna [6] conducted a study through empirical data and processed it using the Monte-Carlo approach to model the relationship between earthquake strength and tsunami parameters such as tsunami amplitude, bottom deformation waveform, water volume, and potential energy generated from the initial height. Anawat et al. [7], studied the maximum height models of the tsunami based on the strength of the earthquake using linear regression analysis. Another approach is to model the numerical stochastic numerical simulation of tsunami heights in terms of earthquake parameters. ([8],[9]). Natawidjaja et al. [10], reported the occurrence of major earthquake events accompanied by tsunami events in the Mentawai segment region. The particular report will, of course, have a direct impact on the Bengkulu Province, especially the Mukomuko Regency and surrounding areas. As an anticipative action, we need to know the relation of earthquake parameters with tsunami parameters, such relation will contribute in strengthening disaster warning system both locally and globally.

The transfer function is one of the alternative methods to solve the problem if there is more than one periodic series variable in which one variable influences the other. The transfer function model is a time series forecasting model that combines the characteristics of Autoregressive Integrated Moving Average order \((p, d, q)\) (ARIMA \((p, d, q)\)) with a causal approach ([11]). Chen [12], uses a transfer function in modeling geomagnetic changes when an earthquake occurs in the Guam islands where the parameter estimation uses the least squares method.

In mathematical modeling, the accuracy of results is influenced by empirical data. Invalid and unreliable data will affect the level of model accuracy. Madlazim [13], reported that the magnitude of the earthquake, time, and depth provided by Ina-TEWS differs significantly from the global CMT catalog, while the location (latitude and longitude position) is appropriate. Based on this, this study takes empirical data from sources: www.globalcmt.org, http://iti.ic.unesco.org and http://tsun.ssc.ru/tsunami-database.

![Figure 1](image-url)  \hspace{1cm} Figure 1. Historical data of major earthquakes occurring in Sumatra and tsunami heights in some areas due to the 2004 Aceh earthquake, Satake [1].
2. Review on Autoregressive Integrated Moving Average (p, d, q) (ARIMA (p, d, q)) and Transfer Function Modeling

2.1. Autoregressive Integrated Moving Average (p, d, q) (ARIMA (p, d, q))

Suppose that \( X_t \) is an observation of the time period \( t \), the data is not stationary to either the average or the variance, one of the models that can be used to describe the pattern of data is the ARIMA \((p, d, q)\), general form as follows:

\[
\Phi_p(B)(1 - B)^dX_t = \Theta_q(B)\alpha_t
\]

(3)

\( B \) denotes the shift operator backward, \( d \) denotes the number of differencing, \( \Phi_p \) expresses the \( p \)th autoregressive order, and \( \Theta_q \) represents the moving average value \( q \)th. For the ARIMA model, the order \( d \) represents the number of differentiation performed so that the stationary conditions are satisfied, whereas the \( p \) and \( q \) orders are based on the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) patterns as shown in Table 1 [14]-[17].

The first stage performed in time series modeling is the stationary test of the data. In this paper the stationary test used the Dickey-Fuller (DF) test. Suppose we have a series of observations that have the following relationships [14]

\[
X_t = \rho X_{t-1} + \nu_t
\]

(4)

Equation (4) can be written in the form of:

\[
\Delta X_t = (\rho - 1)X_{t-1} + \nu_t = \delta X_{t-1} + \nu_t
\]

(5)

Data is said to be stationary if the data does not contain the root of the unit, the hypothesis can be written \( H_0: \delta = 0 \), test statistic used is \( t = \frac{\hat{\delta}}{\text{SE}(\hat{\delta})} \), with test criteria, reject \( H_0 \) if \( |t| \geq |t_{(n-1,\alpha)}| \). If the data is not stationary then do differencing on the original series as in the equation (6): [15]

\[
\nabla^dX_t = (1 - B)^dX_t
\]

(6)

The second stage identifies the model by looking at the characteristics of the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) pattern [16]. The third stage is the model suitability test, the residual must be independent or not correlated. The test statistic used is the Ljung-Box equation as follows:

\[
Q = n(n+2)\sum_{k=1}^{M} \frac{\rho_k^2}{n-k}
\]

(7)

\( n \) is the number of observations, \( M \) Maximum lag, and \( \rho_k \) : residual autocorrelation coefficient on lag- \( k \). Reject \( H_0 \) if \( Q > \chi^2_{\alpha,M-p-q} \), with \( p \) and \( q \) are the expected parameters in the model ARIMA [16].

In practice, it is possible to produce two or more models that satisfy equation (7). When this happens, then the best model is selected. One approach that can be done in selecting the best model is to see the value of Mean Square Error (MSE), by the formula:

\[
MSE = \frac{1}{n}\sum_{t=1}^{n} (X_t - \hat{X}_t)^2
\]

(8)

| Model        | ACF                          | PACF                              |
|--------------|------------------------------|-----------------------------------|
| AR(p)        | dies down exponentially or sinusoid | Cut off after lag p               |
| MA(q)        | Cut off after lag q          | dies down exponentially or sinusoid|
| ARMA(p, q)   | dies down after lag (q-p)    | dies down after lag (p-q)         |

Table 1. ACF and PACF criteria in determining order of p and q
2.2. Transfer Function Modelling

The transfer function model will produce a simple model that describes the relationship \( Y_t \) (output) with \( X_t \) (input) and \( N_t \) (noise), so by determining the role of the leading indicator of the input series it can be determined the estimation of the output variable. The general model of single input transfer function is as follows: [17]

\[
Y_t = \frac{\omega_x(B)}{\delta_r(B)} X_{t-b} + \frac{\theta(B)}{\Phi(B)} N_t
\]

(9)

The initial stage of modeling the series of inputs and outputs, by identifying the stationary of the input data and output data. Furthermore, the prewhitening process of the input sequence and the output series is a correlation series process that correlates to uncorrelated white noise behavior. This bleaching process uses the ARIMA model for the input sequence for \( X_t \), using the following formula:

\[
\alpha_t = \frac{\phi_x(B)}{\Theta_x(B)} x_t
\]

(10)

As for the output series

\[
\beta_t = \frac{\phi_y(B)}{\Theta_y(B)} y_t
\]

(11)

\( \phi_x(B) \) is an autoregressive operator, \( \theta_x(B) \) is moving average operator, \( \alpha_t \) is prewhitening input series, \( x_t \) is stationary input series, \( \beta_t \) is prewhitening output series, and \( y_t \) is stationary output series. The second stage determines the cross-correlation function \( (r_{a\beta}(k)) \) between the prewhitening input series \( (\alpha_t) \) and prewhitening output series \( (\beta_t) \) Using the equation:

\[
r_{a\beta}(k) = \frac{C_{a\beta}(k)}{S_a S_\beta}
\]

(12)

\( C_{a\beta}(k) \) is the value of covariance between \( \alpha_t \) and \( \beta_t \) at the lag-\( k \), \( S_a \) value of standard deviation \( \alpha_t \), \( S_\beta \) value of standard deviation \( \beta_t \).

The third stage, calculate cross-correlation \( (v_k) \) or the weight of the impulse response useful for calculating the noise series, [14]:

\[
v_k = r_{a\beta}(k) \frac{S_\beta}{S_a}
\]

(13)

To know the significance of the cross-correlation then the value \( v_k \) compared with the value \( \frac{1}{\sqrt{n-k}} \). If the value \( |v_k| > \frac{1}{\sqrt{n-k}} \) then the value of impulse response weighted is significant.

The fourth stage, calculating the three parameters in the transfer function model that is a constant \((b, r, s)\), \( b \) Indicates the delay noted on \( x_{t+b} \), \( r \) is the order of \( \delta(B) \) and \( s \) is the order of \( \omega(B) \). The constants \( b, r, \) and \( s \) determined by the pattern of cross-correlation function between \( \alpha_t \) and \( \beta_t \) [17].

3. Material and Methods

Data used in this research was secondary data of earthquake and tsunami parameter with period 1990-2016. This data was compiled from websites www.globalcmt.org, http://itic.ioc-unesco.org and http://tsun.ssc.ru/tsunami-database. For simplicity in modelling, we used the definition of variables, for the height variables of the tsunami (Meters) with variables \( Y \) as the dependent variable. The strength of the earthquake (Mw) defined by \( X_1 \), and the depth of the earthquakes (Km) defined by \( X_2 \). We used software Minitab 15 and SAS 9.2 for data analysis. The flowchart of data analysis performed is as follows:
Figure 2. Flowchart of transfer function modeling in earthquake and tsunami data

4. Results and Discussion
Based on the search results of earthquake and tsunami events from 1990 to 2016 at www.globalcmt.org, http://itic.ioc-unesco.org and http://tsun.ssc.ru/tsunami-database/, not all major earthquakes are followed by a large tsunami phenomenon. There are several earthquake events that are only followed by a tsunami height below 1 meter, not even a tsunami. From the data obtained can be described, incidents that occur in Indonesia, especially on the island of Sumatra and Java. The 2004 Indian Ocean earthquake, which had a moment magnitude of 9.1-9.3, making it the deadliest tsunami. The initial surge was measured at a height of approximately 33 meters (108 ft), making it the largest earthquake-generated tsunami in recorded history. A 7.7 magnitude earthquake rocked the Indian Ocean seabed on 17 July 2006, 200 km south of Pangandaran, a famous beach to surfers for its perfect waves. This earthquake triggered tsunami which is varied from 2 meters at Cilacap to 6 meters at Cimerak beach. On 25 October 2010, 7.7 earthquake struck near South Pagai Island in Indonesia triggering a localized tsunami.

The result of the correlogram test on the three variables, the data will be stationary after the differentiation once, this is seen from the ADF value of each variable < MacKinnon Value ($\alpha = 0.05$) (Table 2). Based on the test results, d value for ARIMA parameter of each variable equal to 1.

| Variable                  | ADF value | MacKinnon critical value (5%) | Test Result |
|---------------------------|-----------|--------------------------------|-------------|
| Tsunami height ($Y_t$)   | -13,344   | -1.942069                      | Stationary  |
| Strength of earthquakes ($X_{t1}$) | -17,149   | -1.942296                      | Stationary  |
| The depth of the earthquake ($X_{t2}$) | -19,761   | -1.942059                      | Stationary  |
To obtain the ARIMA model from the input sequence, both ACF and PACF plots were identified. Plot ACF and PACF for the variable strength of the earthquake is shown in the following Figure 3 and 4.

Looking at the pattern of the cut off ACF graph on the first lag while the PACF chart decreases gradually, the possible models are ARIMA (0,1,1), ARIMA (0,1,2) and ARIMA (1,1,1) (Figure 3 and 4). From Table 3, it appears that all values of model parameters are less than 1 but there are p-value values > 0.05, such as the significance value of MA parameters (2) ARIMA model (0,1,2) and AR parameters (1) ARIMA model (1,1,1) with each value is 0.950 and 0.970. With these results, we select the ARIMA model (0,1,1) as the best model with the equation: $x_{1t} = (1 - 0.9111B)x_{1t}$. With this model we get the prewhitening model of the input series $\alpha_{1t} = x_{1t} + 0.9111\alpha_{1t-1}$ and the output prewhitening model $\beta_{1t} = y_{t} + 0.9111\beta_{1t-1}$.

For the Earthquake Depth variable, the ACF graph shows that the graph is cut off in the first lag but in the second lag does not decrease drastically, the PACF graph decreases gradually then the tentative model is: ARIMA (0,1,1), ARIMA (0,1,2), ARIMA (1,1,1) ARIMA (1,1,2) (Figure 5 and 6).

| No | ARIMA   | Order | Parameter | t-count | p-value |
|----|---------|-------|-----------|---------|---------|
| 1  | (0,1,1) | MA    | 0.9111    | 35.80   | 0.000   |
| 2  | (0,1,2) | MA    | 0.9145    | 14.86   | 0.000   |
| 3  | (1,1,1) | AR    | 0.0026    | 0.04    | 0.970   |
|    |         | MA    | 0.9124    | 33.21   | 0.000   |

Table 3. Estimation of ARIMA parameters for earthquake strength variables

Figure 3. ACF plot of the series $x_{1t}$

Figure 4. PACF plot of the series $x_{1t}$

Figure 5. ACF plot of the series $x_{2t}$

Figure 6. PACF plot of the series $x_{2t}$
Table 4. Estimation of ARIMA parameters for earthquake depth variables

| No | ARIMA Models | p-value | p-value Q | MSE Value |
|----|--------------|---------|-----------|-----------|
| 1  | (0,1,1)      | MA 1 0,9865 | 0,000     | 0,665     | 374,4     |
| 2  | (0,1,2)      | MA 1 0,7708 | 0,000     | 0,293     | 380,2     |
|    |              | MA 2 0,2208 | 0,000     |           |           |
| 3  | (1,1,1)      | AR 1 0,0769 | 0,217     | 0,699     | 373,6     |
|    |              | MA1 0,9865  | 0,000     |           |           |
| 4  | (1,1,2)      | AR 1 -0,1319 | 0,038     | 0,665     | 376,0     |
|    |              | MA 1 0,7857 | 0,000     |           |           |
|    |              | MA 2 0,2086 | 0,000     |           |           |

From the four models, there are three models whose parameters are significant and pass the model conformity testing, they are: ARIMA model (0,1,1), ARIMA (0,1,2) and ARIMA (1,1,2), in determining best model We choose based on the smallest MSE value. The selected model was the ARIMA model (0,1,1) (see Table 4). Thus, we get the equation ARIMA model at earthquake depth that is: $x_{2t} = (1 - 0.9865B)a_{2t}$. By using this model, we got the prewhitening model of the input series $a_{2t} = x_{2t} + 0.9865a_{2t-1}$ and the output prewhitening model $\beta_{2t} = y_t + 0.9865\beta_{2t-1}$.

Using the result of the prewhitening of the input series and the output series, we get the result of cross-correlation between variables $\alpha_{1t}$ with $\beta_{1t}$ and $\alpha_{2t}$ with $\beta_{2t}$ (Figures 7 and 8).

The largest response weight for $\alpha_{1t}$ is in lag 0 with the value 3.65. This shows that the strength of the earthquake contributed to the triggering of a tsunami of 3.65. While the smallest response weight for $\alpha_{2t}$ is in lag 9 That is -0,02272. This shows that the depth of the earthquake reduces the tsunami heights by 0.02272.

Based on Figure 9, the ACF plot is cut off on the lag to 1, the PACF plot decreases gradually (Figure 10), the ARMA model (0,1), ARMA (0,2), ARMA (1,1), ARMA (1,2) are obtained. There are 4 models that are suitable for use as a model. Based on the results of testing the significance of parameters (Table 5), selected model

\[ n_{1t} = (1 - 0.9508B)a_{1t} \quad (15) \]
While for earthquake depth variable,

\[ n_{2t} = y_t + 0.017x_t + 0.023x_{t-1} + 0.014x_{t-2} - \cdots + 0.008x_{t-27} \]  

(16)

Based on Figure 11, the ACF plot is cut off on the lag to 1, and Figure 12, the PACF plot decreases gradually, the ARMA model (0,1), ARMA (0,2), ARMA (1,1), ARMA (1,2) are obtained. There are 4 models that are suitable for use as a model. Raised test results of parameter significance (Table 6), selected

\[ n_{2t} = (1 - 0.9209 B)\alpha_{2t} \]  

(17)
Table 6. Estimation of ARMA parameters of noise series $x_{zt}$

| No | Model     | Parameter | t-value | p-value | Significant |
|----|-----------|-----------|---------|---------|-------------|
| 1  | ARMA(0,1) | MA 1      | 0.9209  | 36.14   | 0.000       | Yes         |
|    |           |           |         |         |             |             |
| 2  | ARMA(0,2) | MA 1      | 0.9362  | 14.63   | 0.000       | Yes         |
|    |           | MA 2      | -0.0203 | -0.31   | 0.755       | No          |
| 3  | ARMA(1,1) | AR 1      | -0.0240 | -0.34   | 0.736       | No          |
|    |           | MA 1      | 0.9124  | 31.48   | 0.000       | Yes         |
|    |           | MA 2      | 0.8930  | 17.70   | 0.000       | Yes         |

Table 7. Estimation of multivariate transfer function parameters

| Variabel | Parameter | Value       | p-value | Result of Significant |
|----------|-----------|-------------|---------|-----------------------|
| $Y_t$    | $\theta_1$| 0.93230     | 0.0001  | Significant           |
| $X_1$    | $\omega_0$| 3.71978     | 0.0001  | Significant           |
|          | $\omega_1$| -1.43267    | 0.0317  |                       |
| $X_2$    | $\omega_0$| -0.03142    | 0.0869  |                       |
|          | $\delta_1$| 0.64514     | 0.0353  |                       |

Based on Table 7, the parameters $X_1, X_2$ are significant for p-value values smaller than 0.09. So the model equation of the transfer function to predict the height of the tsunami

$$y_t = (3.71978 + 1.43267B) x_{1t} + \dfrac{(-0.03142)}{(1 - 0.64514B)} x_{2t} + (1 - 0.93230 B) a_t$$

Equation 18, is a mathematical equation that describes the relationship between the estimated height of the tsunami with the strength and depth of the earthquake. Assuming that the event index is based on each tsunami event, then there is an influence of the differentiation variables in the model. Regardless of the differentiation factor of the model, when a large earthquake occurs and generates tsunami waves, the maximum estimated tsunami heights are affected by earthquake strength of 3.71978, meaning that the strength of the earthquake contributes positively to the height of the tsunami by 3.72 times. While the variable earthquake depth, affecting -0.03142, suggesting that the depth of the earthquake contributes negatively or can reduce the height of the tsunami of 0.03142.

5. Conclusion

From the calculation of the weight of the response, the largest response weight for $\alpha_{1t}$ is in lag 0 with a value of 3.65. This indicates that the strength of the earthquake directly contributes to the triggering of a tsunami of 3.65. While the largest response weight for $\alpha_{2t}$ is in lag 9, ie -0.02272. This shows that the depth of the earthquake reduces the tsunami heights by 0.02272. But this quantity is corrected when we get the model of inter-relationship between variables as seen in equation 18, where the influence of earthquake strength increased to 3.72 while the earthquake depth variables decreased to -0.03142. This difference has not been tested statistically so that we can use both in estimating tsunami heights.

The results of this study can be combined with the elevation information in each coastal area, tsunami height simulation, and the thematic map of Bengkulu Province, to obtain a map of tsunami-prone areas. This research is expected to contribute directly in raising public awareness of tsunami disaster. This will be realized if there are further activities in the form of socialization and dissemination to parties directly related to the handling of natural disasters, especially earthquakes and tsunamis.
Acknowledgments
This research has been supported by Research Ministry, Technology, and High Education which Number: 061/SP2H/LT/DRPM/IV/2017 and have been done based on by Letter of Assignment Implementation Competitive Research Grant Fiscal Year 2017 Number: 947/UN30.15/LT/2017.

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