Flexibility in solving open-ended mathematics problems based on students' thinking styles

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Abstract. Flexibility is one of the abilities needed to solve mathematical problems using a variety of different perspectives. One factor that affects flexibility is students' thinking styles. Students' thinking styles are classified into 4 types, namely Concrete Sequential (CS), Abstract Sequence (AS), Concrete Random (CR), and Abstract Random (AR). These four thinking styles have an important role in increasing students' understanding of the learning process. The aim of this research is to see the extent of students' mathematical flexibility abilities in solving open problems based on students' thinking styles. This research uses descriptive research with a qualitative approach. The subjects in this study were 4 students. The research began by giving students a thinking style questionnaire which was then continued by giving open test questions and interviews. The validity of data measured using the triangulation method. The results of this study indicate that CS students, AS students, and CR students, have not been able to achieve indicators of flexibility, whereas AR students can achieve indicators of flexibility in solving open problems.

1. Introduction

In mathematics education, students will continuously face new situations and new problems [1] which require them not only to know and implement various strategies but also to be flexible [2,3] in solving them, because one situation may not necessarily have a way of solving and the same rules as the other situations [4]. This makes flexibility an important component in mathematical ability [5]. This ability is used to solve problems with many procedures and apply them adaptively to various situations [6,7,8]. For example, mathematicians use more procedures and choose to use different procedures when trying to solve identical problems [9]. Besides, flexibility can also be defined as knowledge of various strategies and the ability to use strategies in innovative ways in various problem-solving situations [7, 8, 10].

Flexibility is one indicator of creativity [11,12]. This indicator has a strong connection with the ability to solve problems [13], so to develop these thoughts, students must be allowed to freely try to solve them using their methods [14]. One of them is by giving open-ended problems. Open-ended can be defined as a problem that has many different solutions [15]. The solution in this case, not only limited to the answers to problems that have been solved but open-ended also may have a single answer obtained in various ways [16]. The key point of open-ended is the problem of giving students opportunities and stimuli to explore various solutions, methods or strategies [17].
Unfortunately in the learning process in schools, teachers still rarely use open-ended problems and tend to use mathematical problems that use routine methods in their resolution, this is evidenced by the results of interviews conducted by several junior high school teachers in Kediri on May 21, 2019, which states that students are given mathematical problems that use routine methods in accordance with the material taught, but if there are students who use their methods, some teachers do not make an issue of it, but there are also teachers who limit students to using methods in accordance with what has been taught, so in this case the teacher is less concerned with student flexibility in problem-solving mathematical. The lack of flexible knowledge has an impact on low academic achievement which results in students experiencing great difficulties when faced with new mathematical problems.

Factors of student success in solving problems are influenced by various factors, one of which is students' thinking styles. Gregorc [18] concluded that there were two possibilities for brain domination so that when combined there would be four combinations of behavioral groups called thinking styles. This thinking style consists of concrete sequential, abstract sequential, concrete random and abstract random. If someone knows his style of thinking, then it is possible that the person can develop ways of thinking to be able to solve problems by choosing effective solutions [18].

Many studies have been conducted to discuss flexibility in mathematics, including the first research conducted by Heinzo, Star, and Verschaffel [19] which provides an overview of the use of strategies and flexible and adaptive representations in mathematics education. The second research conducted by Star and Rittle-Johnson [20] discusses several strategic directions to increase flexibility in problem-solving by using discovery learning and direct instruction. The third studies conducted by Dina, Amin, and Masriyah [13] which describe the flexibility in solving mathematical problems using AQ. The fourth studies conducted by Elia, van den Heuvel-Panhuizen, and Kolovou [4] on the use of strategy and strategy flexibility in non-routine problem solving by elementary school achievers in mathematics which show that students rarely apply heuristic strategies in their study solve problems and tend to solve problems by trial and error. From the studies that have been done, there are no reviews regarding internal factors related to characteristics of students’ thinking styles, one of which is an important factor regarding the success of individuals in solving problem-solving. Based on research conducted by Hercht and Vagi [21] the characteristics of children are also a factor in developing problem-solving skills, for this reason, it is necessary to have an internal factor to see the ability of children's flexibility in problem-solving, so in this case, the researcher is interested in analyzing student flexibility in solving open-ended problems based on students' thinking styles.

2. Method

The method in this research uses the descriptive qualitative method. This method is used to analyze flexibility in solving open problems based on students' thinking styles. The steps taken to get the data, namely first determining the research subject using purposive sampling, so that one of the 8 classes is obtained with a total of 32 students who are then given a questionnaire to get subjects with Concrete Sequential (CS), Abstract Sequential (AS), Concrete Random (CR), and Abstract Random (AR). From each style of thinking one subject was taken. Taking the subject is assisted by the subject teacher, this is with the consideration that the teacher is the party who knows the students the most so that four research subjects are obtained. This research was conducted at MTsN 1 Kediri. In the second step, the researcher gives a written mathematics test using open questions with a duration of 40 minutes, then the researcher interviews the research subject. Through this procedure, the researcher can obtain the required data from the written test answer sheet and the results of the research subject interviews. To verify the credibility of the data, researchers used the triangulation method, which then analyzed the data descriptively. Techniques in analyzing research data include data reduction, data presentation, and drawing conclusions. The validity of the data is seen from the written test data with the interview data analyzed using indicators of flexibility.
3. Results and Discussion

3.1 Results

From the questionnaire distributed, four research subjects were obtained with different thinking styles. From the four students, the results were obtained that the four subjects solved the open-ended problem with different strategies. This can be seen based on the description of the answers of the subjects accompanied by interviews that can be explained below.

Figure 1. Concrete sequential student answers

Figure 1 students solve the problem by writing 40% of the mineral water they drink and for example the remaining mineral water with 450 ml. In the process, students calculate the amount of mineral water that Father has drunk by multiplying 40% by 450 to produce 180 ml, but in the process, students do not cross the results with 60% (remaining mineral water in bottles), so that in solving the problem students answer with inappropriate results. This, according to the student's statement, "I multiply only what is known, because I think if multiplied will produce an answer of mineral water that my father had drunk and add it to the remaining mineral water in a bottle, so that the original water is 630 ml." This case students have not been able to solve these problems correctly.

Figure 2. Abstract sequential student answers

Original water in bottles= 100%
Drunk water = 40%
Remaining water in a bottle = y ml
How much water was originally in the bottle?

100% ⇒ water remaining = 100%-40% = 60%
e.g. y = 50 ml maka
water remaining = 60% x 50 = 120 ml
water that has been drunk = 40% x 50 = 80 ml
lots of original water = 80 + 120 = 200 ml
Figure 2 students solve the problem by writing by what is known in the problem. In the process students describe the bottle in a whole condition by 100%, if 40% has been drunk then there will be 60% left. Then students assume that y = 50 ml, and multiply the percentage of water that is drunk and the remaining 50 ml, so that the results obtained 200 ml but the student strategy used in solving the problem is not right, students should not multiply the remaining percentage of mineral water by 50 ml, and dividing the yield of mineral water drunk by 60%, this is in accordance with the statement of the students, "I am having trouble solving this problem, finally I suppose the remaining mineral water by 50 ml, then I multiply one by one with the existing percentage so I get 200 results. ml ". Students still have difficulty solving the problems given so in this case students have not been able to solve these problems correctly.

Figure 3 students solve problems by writing by what is known in the problem, in the process students use two solving strategies. The first strategy, students take the example of 240 ml of drinking water, and look for the value of tenth percent, if 40% = 240 ml, then 10% = 60 ml, then students multiply the results by 60%, so that the results obtained 360 ml, and then for To get the original amount of water in a bottle, students add up the mineral water they have drunk with the rest of the water in the bottle so that a 600 ml result is obtained. The second strategy students use a comparative value to determine the amount of water left in the bottle and for example drinking 120 ml of water, so that the results obtained are 180 ml then students add up the results with the amount of water they drink and get a 300 ml result. Both strategies used by students are appropriate. Students can also use different points of view in problem-solving.
Figure 4. Concrete random student answers

Figure 4, students solve problems using a comparison to get a percentage of 1%. Students suppose that the remaining water in a bottle as much as 60% is 600 ml, then by using a comparison of the percentage amount with the amount of water, the result is 1:10 but in the process, students do not write a percent sign in their completion, so it looks less precise. From these results, if 1% = 10, then 100% = 100 times 10 so that the result of 1000 ml is obtained. This is consistent with students' statements: “I used a comparison to get the amount of water, in one bottle. I forgot to not write the percent sign, I think 60% is equivalent to 60”. In the process, concrete random students can solve problems in one way, although in writing they are less precise in using existing procedures.

3.2 Discussion

According to Star and Rittle-Johnson [20] problem solving is said to be flexible, if the solution can do more than one way of solving and involve knowledge of strategy efficiency. In the learning process, if students develop flexibility, students will have a greater understanding of solving problems, problems by using or adapting existing strategies [7, 22, 23, 24]. From the results of the analysis that has been done, it is known that CS students solve open problems regularly, and write information [18] that is known to the problem, to facilitate them in solving problems. But in the process, CS students have difficulty in solving problems, resulting in incorrect answers. According to research that has been done, students' difficulties in solving problems, one of which is influenced by the concept of weak knowledge that affects the knowledge procedures [21, 25, 26], in this case CS students have not been able to solve these problems correctly, and have not been able to achieve indicators of flexibility. Not much different from CS students, AS students also solve problems regularly [18] by using a strategy, but the workmanship procedure is done incorrectly, resulting in inaccurate answers. From this statement, AS students have not been able to achieve the flexibility indicator.

The next style of thinking is that AR students, in their completion AR students connect information that has been previously obtained, with the information needed in problem-solving [18]. In the process, AR uses two different strategies. Besides, AR is also able to provide reasons related to the strategy used. This is in line with Baroody's opinion, that students who are able to achieve flexibility can understand the use of the strategies used and modify them in different contexts [6], so that in this case, AR can achieve indicators of flexibility because they are able to solve these problems with more than one way of solving and use efficient strategies [19] whereas for CR students, they are able to solve problems using one strategy, even if writing is incorrect, and when asked with another strategy, CR students answer using the same strategy, and are able to solve the problem correctly and can explain it correctly about solving these problems. This is in line with research conducted by Elia, Heuvel-Panhuizen, and Kolovou that students rarely show flexibility when associated with non-routine problems [4], and these findings are suitable to support previous findings, those flexible students can be linked with students who have high mathematical competence [27]. From the results of the analysis, CR students are only able to solve the problem from one perspective, so that in this case CR students have not been able to achieve the flexibility indicator.
4. Conclusion

CS students and AS students cannot solve open-ended problems appropriately, so in this case they have not been able to demonstrate the ability of flexibility in solving open-ended problems. In contrast to AR students, they are able to solve open-ended problems using two different strategies, so as to demonstrate the ability of flexibility in problem-solving while CR students are only able to solve open-ended problems using one strategy so that in this case students have not been able to achieve the ability of flexibility.

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