Ultra-high Energy Predictions of Proton-Air Cross Sections from Accelerator Data: an Update

M. M. Block

Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208

At the pp center of mass energy \( \sqrt{s} = 57 \pm 7 \text{ TeV} \), the Pierre Auger Observatory (PAO) collaboration has recently measured the proton-air inelastic production cross section \( \sigma_{p\text{-air}} \), using a cosmic ray beam consisting mainly of protons, with some helium contamination. Assuming a helium contamination of 25%, they subtracted 30 mb from their measured \( \sigma_{p\text{-air}} \), resulting in a p-air inelastic production cross section, \( \sigma_{p\text{-air}} = 475 \pm 22 \text{ (stat.)} \pm 20 \text{ (syst.)} \text{ mb} \), where (stat.) is the statistical error and (syst.) is the the systematic error, exclusive of helium contamination. Using this result in a Glauber calculation to obtain the pp inelastic cross section, at 57 TeV they found the inelastic pp cross section \( \sigma_{\text{inel}} = 90 \pm 7 \text{ (stat.)} \pm 11 \text{ (syst.)} \text{ mb} \) (Glaub.) mb, where (syst.) is the systematic and (Glaub.) is the error associated with the Glauber calculation needed to convert \( \sigma_{p\text{-air}} \) to pp \( \sigma_{\text{inel}} \).

Parameterization of the pp and pp cross sections incorporating analyticity constraints and unitarity has allowed us to make accurate extrapolations to ultra-high energies, and, using Glauber calculations, also accurately predict cosmic ray results for \( \sigma_{p\text{-air}} \). In this update for 57 TeV, we predict i) a pp total cross section, \( \sigma_{\text{tot}} = 133.4 \pm 1.6 \text{ mb} \), using high energy predictions from a saturated Froissart bound parameterization of accelerator data on forward pp and pp scattering amplitudes and ii) a p-air inelastic production cross section, \( \sigma_{p\text{-air}} = 483 \pm 3 \text{ mb} \), by using \( \sigma_{\text{tot}} \) together with Glauber theory. Using the POA estimates of the variation of their measured \( \sigma_{p\text{-air}} \) with helium contamination, we were able to determine independently that the helium contamination was 19%, in reasonable agreement with their estimate of 25%. Our predictions agree with all available cosmic ray extensive air shower measurements, both in magnitude and in energy dependence. Further, by using our value for the pp total cross section at 57 TeV, Block and Halzen [4] have predicted that the pp inelastic cross section is \( \sigma_{\text{inel}} = 92.9 \pm 1.6 \text{ mb} \), in agreement with the measured POA value.

PACS numbers: 13.60.Hb, 12.38.-t, 12.38.Qk
where the upper sign is for $pp$ and the lower for $\bar{p}p$ scattering. Here $\nu$ is the laboratory energy, $m$ the proton mass, $\mu = 0.5$, and $f_+(0)$ is a dispersion relation subtraction constant. The 7 real constants $c_0, c_1, c_2, \beta_1, \delta, \alpha$ and $f_+(0)$ are parameters of the fit. At high energies, $s$, the square of the cms energy, approaches $2m\nu$; hence, we see from Eq. (1) that the cross sections behave as $\ln^2 s$ at high energies, thus saturating the Froissart bound [3]. From Eq. (2), we see that $\rho \to 0$ as $1/\ln s$ as $s \to \infty$.

Using 4 analyticity constraints [7], resulting from finite energy sum rules that used very high accuracy low-energy cross section measurements ($2 \leq \sqrt{s} \leq 4$ GeV), they anchored both the cross sections $\sigma_{pp}$ and $\sigma_{\bar{p}p}$ and their laboratory energy derivatives to data at $\sqrt{s} = 4$ GeV, thus reducing the number of parameters to from 7 to 4. The fit was to data with $6 \leq \sqrt{s} \leq 1800$ GeV. This use of analyticity constraints resulted in an excellent fit that, in turn, constrained $pp$ cross sections at cosmic ray energies to an accuracy $\sim 1 - 2\%$, even though (conflicting) Tevatron data provided the highest energy input. The Block and Halzen fits [3] to the $pp$ and $\bar{p}p$ cross sections are shown in Fig. 1. The fit is the dotted line crossing the fit. Our predicted PAO energy of 57 TeV is indicated by the dashed curve from Eq. (1), in mb vs. $\sqrt{s}$, in GeV. The $\sqrt{s}$ at which the upper sign is for $pp$ and the lower for $\bar{p}p$ scattering. Here $\nu$ is the laboratory energy, $m$ the proton mass, $\mu = 0.5$, and $f_+(0)$ is a dispersion relation subtraction constant. The 7 real constants $c_0, c_1, c_2, \beta_1, \delta, \alpha$ and $f_+(0)$ are parameters of the fit. At high energies, $s$, the square of the cms energy, approaches $2m\nu$; hence, we see from Eq. (1) that the cross sections behave as $\ln^2 s$ at high energies, thus saturating the Froissart bound [3]. From Eq. (2), we see that $\rho \to 0$ as $1/\ln s$ as $s \to \infty$.

Using 4 analyticity constraints [7], resulting from finite energy sum rules that used very high accuracy low-energy cross section measurements ($2 \leq \sqrt{s} \leq 4$ GeV), they anchored both the cross sections $\sigma_{pp}$ and $\sigma_{\bar{p}p}$ and their laboratory energy derivatives to data at $\sqrt{s} = 4$ GeV, thus reducing the number of parameters to from 7 to 4. The fit was to data with $6 \leq \sqrt{s} \leq 1800$ GeV. This use of analyticity constraints resulted in an excellent fit that, in turn, constrained $pp$ cross sections at cosmic ray energies to an accuracy $\sim 1 - 2\%$, even though (conflicting) Tevatron data provided the highest energy input. The Block and Halzen fits [3] to the $pp$ and $\bar{p}p$ cross sections are shown in Fig. 1. The fit is the dotted line crossing the fit. Our predicted $pp$ total cross section at 57 TeV is $\sigma_{tot} = 133.4 \pm 1.6$ mb. For brevity, we have not shown the fit to $\rho$; see Block [4].

FIG. 1: The fitted total cross section, $\sigma_{tot}$, for $pp$ (dashed curve) and $\bar{p}p$ (dot-dashed curve) from Eq. (1), in mb vs. $\sqrt{s}$, the cms energy in GeV, taken from BH [6]. The $\bar{p}p$ data used in the fit are the (red) circles and the $pp$ data are the (blue) squares. The fitted data were anchored by values of $\sigma_{tot}$ and $\sigma_{pp}$ together with the energy derivatives $d\sigma_{tot}/d\nu$ and $d\sigma_{pp}/d\nu$ at 6 GeV using FESR, as described in Ref. [3]. The vertical dotted line at $\sqrt{s} = 57000$ GeV that intercepts the fit indicates the $pp$ cms of the PAO cosmic ray experiment [2].

Comparison of $\sigma_{p-\text{air}}$, with cosmic ray data. In Fig. 2, we have plotted all available cosmic ray data for $\sigma_{p-\text{air}}^{\text{prod}}$, the proton-air inelastic production cross section, in mb, as a function of $\sqrt{s}$, in GeV. The $\sigma_{p-\text{air}}^{\text{prod}}$ curve, derived from the total cross section fit of Eq. (1) and utilizing the elastic slope parameter $B \equiv d|\ln d\sigma_{el}/dt|/d\nu|_{\nu=0}$ in a Glauber calculation, is taken from Block [4]. All of the cosmic ray data but the HiRes point were renormalized using a k-factor ($k=1.264$). For brevity, we omit the discussion of this renormalization procedure as well as detailed references to the cosmic data used in the Figure; for complete information, see Block [4].

The new point added in Fig. 2 is the large (red) square, whose central value is our prediction of $\sigma_{p-\text{air}}^{\text{prod}}$ for 57 TeV and whose error is the total PAO experimental error excluding beam contamination uncertainty. As we will show in the next Section, this value, $\sigma_{p-\text{air}}^{\text{prod}} = 482 \pm 30$ mb, has a central value that corresponds to a 19% helium contamination.

**Determination of the helium contamination in the cosmic ray ‘proton’ air showers.** The PAO collaboration [2] notes that “We recognise (sic) and identify the unknown mass composition of cosmic rays as the major source of systematic uncertainty for the proton-air cross-section analysis and we evaluate its impact on the final result.” They estimated that their best value for helium contamination was 25%. In this Section, we obtain an independent confirmation of this estimate.

We plot in Fig. 3 the helium fraction of the cosmic ray beam, in %, vs. the PAO collaboration’s [2] corrected measurements for $\sigma_{p-\text{air}}^{\text{prod}}$ in mb. The large cross marks the point on the curve that is our prediction for the p-air inelastic production cross section, $\sigma_{p-\text{air}}^{\text{prod}} = 482$ mb, which corresponds to a 19±1% helium contamination, in qualitative agreement with the Auger estimate of 25%.

**Prediction of the pp inelastic cross section, $\sigma_{\text{inel}}^{\text{pp}}$, at $\sqrt{s} = 57$ TeV.** As mentioned earlier, the PAO collab-
FIG. 3: Helium contamination of the cosmic ray beam, in %, vs. $\sigma_{\text{prod}}^{p_{-}a_{ir}}$, the inelastic $p_{-}a_{ir}$ cross section, in mb, for the PAO experiment [2]. The large cross is our prediction for $\sigma_{\text{prod}}^{p_{-}a_{ir}}$, corresponding to a 19% helium contamination (see text).

The Pierre Auger Collaboration, R. Ulrich, part2, "Estimate of the proton-air cross section", arXiv:1107.4804 [astro-phys.HE], (2011).

[1] M. M. Block and F. Halzen, arXiv:1109:2041, 2011.
[2] The Pierre Auger Collaboration, M. Mostafá, XXXI Physics in Collision Conference, Vancouver, Sept. 1, 2011.
[3] The Pierre Auger Collaboration, M. Mostafá, XXXI Physics in Collision Conference, Vancouver, Sept. 1, 2011.
[4] M. M. Block, Phys. Rev D 76, 111503, 2007.
[5] M. M. Block and F. Halzen, Phys. Rev. D72, 036006, 2005.
[6] M. M. Block, Nucl. Instrum. Methods A 556, 308, 2006.
[7] M. M. Block, Eur. Phys J. C47, 697, 2006.
[8] M. M. Block, F. Halzen and T. Stanev, Phys. Rev. Lett. 83, 4926, 1999; Phys. Rev. D62 77501, 2000.
[9] M. Froissart, Phys. Rev. 123, 1053, 1961.

Conclusions. At $\sqrt{s} = 57$ TeV, we conclude that: i) the total $pp$ cross section is $\sigma_{\text{tot}}^{pp} = 133.4 \pm 1.6$ mb, ii) $\sigma_{\text{prod}}^{p_{-}a_{ir}}$, the PAO p-air inelastic production cross section, after correction for a 19% helium contamination, is given by $\sigma_{\text{prod}}^{p_{-}a_{ir}} = 482 \pm 30$ mb ii) our prediction for the $pp$ total cross section, $\sigma_{\text{tot}}^{pp}$, taken from Ref. [1], yields a $pp$ inelastic cross section $\sigma_{\text{inel}}^{pp} = 92.9 \pm 1.6$ mb, which is consistent with the PAO [3] result of $\sigma_{\text{inel}}^{pp} = 90 \pm 7$ (stat.)$^{9}_{11}$ (syst.)$^{1.5}$ (Glaub.) mb.

Acknowledgments. M.M.B. would like to thank the Aspen Center for Physics, supported in part by NSF Grant No. 1066293, for its hospitality during the writing of this manuscript. He would like to thank his colleague Francis Halzen for invaluable discussions and aid during the preparation of this manuscript.