Flow of He II due to an Oscillating Grid in the Low Temperature Limit

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The macroscopic flow properties of pure He II are probed in the limit of zero temperature using an oscillating grid. With increasing oscillation amplitude the initially pure superflow changes abruptly: the resonant frequency decreases and the response becomes strongly nonlinear, attributable to the growth of a boundary layer of quantized vortices that increases the effective mass of the grid. On further increase of oscillation amplitude, the flow undergoes a transition to turbulence.

Although the exotic flow properties of He II have been a subject of intensive investigation ever since the discovery of superfluidity, accumulating a vast amount of experimental data and theoretical knowledge $^1,^2$, many important features still remain to be explained. In the limit of low velocity, He II flow is very well described in terms of Landau’s two-fluid model. On increasing the flow velocity beyond a certain threshold, however, quantized vortices appear. Their presence couples the originally independent normal and superfluid velocity fields and a classical K41 Kolmogorov roll-off exponent of $-5/3$ is observed. (i) the decay of both grid-generated and counterflow turbulence displays a classical character $^3$: (ii) flow past a microsphere displays both laminar and turbulent drag $^4$, as well as (iii) a drag crisis $^5$; (iv) the energy spectrum of turbulent He II involves an inertial range with a classical $k^2$ law; (v) the energy spectrum of the grid decreases and the response becomes strongly nonlinear, attributable to the growth of a boundary layer of quantized vortices that increases the effective mass of the grid. On further increase of oscillation amplitude, the flow undergoes a transition to turbulence.

Our tool for performing such investigations is the thin electroformed nickel grid $^12$ of 70% transparency and mesh size of 127 $\mu$m shown schematically in Fig. 1. It constitutes a circular membrane $2R = 8$ cm in diameter, tightly stretched on a circular mild steel carrier, mounted horizontally, equidistant from two gold-plated plane copper electrodes, each drilled with 170 holes of 2 mm ID to connect the grid space to the rest of the experimental chamber. The electrodes are separated by $d = 2$ mm, immersed in a 1.5 liter volume of isotopically pure $^4$He in an experimental cell attached to the mixing chamber of the dilution refrigerator. The apparatus can be operated at all pressures up to the solidification pressure. A static potential, typically $V_0 = 500$ volts, is applied to the grid. A driving potential $V_1 = V_{10} \cos \omega t$ ($V_{10} \ll V_0$) applied to the upper electrode produces a driving force on the grid of form $f_d = 2\varepsilon_0 \varepsilon_r R^2 V_1 V_0 / d^2$, where $\varepsilon_0$ and $\varepsilon_r$ denote respectively the relative permittivities of free space and the liquid $^4$He. The grid can thus be considered as an oscillating membrane $^13$ under uniform tension. In this study we approximate its motion as one-dimensional and assume that the oscillation amplitude is uniform across its area. Oscillations of amplitude $\Delta D$ induce a signal of amplitude $V_2 = V \Delta D / (2d)$ on the lower electrode, which can be monitored with a lock-in amplifier (which we use to investigate the low-drive linear response of the grid) or directly with a memory oscilloscope, allowing visualization of transient processes resulting from the strongly nonlinear response at higher drives. Allowing for a reduction in the induced voltage $V_2$ by a factor of $(1 + C_r/C)^{-1} \approx 0.065$, where $C_r \approx 700$ pF is the capacitance of the connecting cable and $C \approx 47$ pF is the capacitance between the grid and the lower copper electrode, the response amplitude $|V_2|$ provides a direct
FIG. 2: Resonance curves measured at 10 bar and nominally 50 mK using the memory oscilloscope for various drive levels (in $V_{pp}$) as indicated. Doubled symbols in some places indicate where stable beatings were observed while carefully sweeping the drive frequency, typically in steps of 0.001 Hz. The solid line represents the resonant response to a 0.05 $V_{pp}$ drive in vacuum (measured using the lock-in amplifier, upper frequency axes).

measure of the amplitude and spatially averaged peak velocity $|v_g| = |\omega \Delta D|$ of the oscillating grid.

At the lowest oscillation amplitudes, the grid displays linear behaviour in that its frequency response to the drive is a Lorentzian of narrow width caused predominantly by nuisance damping, with a quality $Q$ factor typically exceeding 5000. The resonant frequency $f_{res}$ can be altered temporarily by increasing the pressure, which we attribute to quantized vortices generated e.g. by the jet from the filling capillary: they probably become pinned to inhomogeneities on the grid surface, and between the grid and surrounding electrodes. Moving the grid violently at high drive amplitude, however, is found to stabilize $f_{res}$ typically to within ±0.1 Hz, presumably by shaking off most of this pinned vorticity.

Fig. 2 shows our central experimental observation, which for the “cleaned” grid has been repeated for several pressures spanning $0.3 \leq p \leq 24.8$ bar without appreciable change. As the drive increases and the grid amplitude reaches the first threshold (typically 10–20 mV$_{pp}$, corresponding to a mean grid velocity of $0.3 < v_g^{(1)} < 0.6 \text{ cm/s}$), the oscillation amplitude at resonance continues to rise in proportion to the drive (Fig. 3), but the resonant frequency suddenly starts decreasing and the resonance curves acquire highly nonlinear features. If one follows e.g. the resonance curve for the 0.05 $V_{pp}$ drive down from 1091 Hz (Fig. 2) by slowly sweeping the drive frequency (in practice, digitally, in steps of 0.001 Hz), the system displays the usual nearly Lorentzian stationary response while the amplitude remains below the first threshold. On further decreasing the drive frequency, the response amplitude increases above this first threshold until, shortly below 1086 Hz the amplitude suddenly collapses down to a lower stable branch. On increasing the drive frequency again from this point the system stays on the lower branch until about 1087.15 Hz, where a transition to the stable upper branch occurs. These hysteretic loops are robust to temperature increase, at least up to our maximum of 130 mK.

Within the hysteretic parameter range, beat phenomena are occasionally seen between two apparently stable amplitude levels, with envelopes of period ~1 s (see, e.g., Fig. 2 drive 0.05 V$_{pp}$ around 1086.5 Hz); once established, they are stable on the scale of hours. Small
changes of frequency do not kill them, but modify the upper and lower amplitude levels between which beating occurs. Only with further change of the driving frequency does the beating disappear. On restoring the original frequency the response is not completely reproducible; within some small frequency range beating might not appear at all; and sometimes it reappeared only later, or not at all. Despite considerable effort, we did not succeed in establishing any fully repeatable pattern or well-defined conditions for the appearance of beats.

The fall in frequency with increasing drive (Fig. 2) reaches typically 2 Hz for all investigated pressures (see Fig. 3), ceasing at a second threshold amplitude (typically 200 mV$_{pp}$ corresponding to a mean grid velocity of about $v_g^{(2)} \approx 6$ cm/s). Observation of the two well-defined resonant frequencies, at all investigated pressures shifted by $\approx 2$ Hz, is a remarkable feature of the superflow that, to our knowledge, has not previously been reported. With further increase in drive, the oscillation amplitude at resonance initially remains almost constant, while the widths of the resonance curves increase rapidly (see Fig. 2). Only for drive levels exceeding by about an order of magnitude that for the second threshold does the amplitude at resonance grow again; this time approximately in proportion to the square-root of the drive, as shown in Fig. 3. In this high-drive regime the linewidth increases rapidly while the resonant frequency decreases gradually, qualitatively in the manner expected for increasing damping. Pronounced beatings were never observed in this high-drive regime. If the oscillation amplitude is measured as a function of drive level at fixed frequency, one observes clear hysteresis within the frequency range containing the two stable branches of the grid response (Fig. 3). Outside this range, the drive dependences are nonlinear, but without any hysteresis.

Measurements of the grid response at low temperature in vacuum (smooth curve in Fig. 2) demonstrate that the presence of the He II has two effects. First, it down-shifts the low-drive resonant frequency by about 30 Hz from its vacuum value of $f_0 = (1117.2 \pm 0.05)$ Hz, depending on the pressure $p$ in the cell (see Fig. 3). This classical effect arises from hydrodynamic enhancement of the mass $M$ of the grid by $\Delta M_H = \beta \rho_{He} (p) M/G$. The measured $f_0$ and low drive resonant frequencies yield $\beta = 3.01 \pm 0.05$. Secondly, it is indeed the He II that is responsible for the observed nonlinearities.

We believe that the behaviour of the oscillating grid ought to be understandable in terms of quantized vortices generated in He II by its motion. As one possibility, we may speculate that, on exceeding the first threshold, a “boundary layer” of vortex loops (perhaps of horseshoe form) grows on the grid, thus increasing its effective mass. Correspondingly, the resonant frequency shifts down and the resonance curves acquire strongly nonlinear features. Provided the vortex loops remain pinned and that they do not reconnect to create free vorticity, the mass enhancement would be non-dissipative, as observed (see below). We note that small mass enhancements were observed earlier for a sphere vibrating in He II above 1 K, but without associated critical thresholds.

It is interesting to characterize the first threshold by a superfluid Reynolds number $Re_s = U_{max} G / \kappa$, where $U_{max}$ stands for a peak critical flow velocity through the grid window, $G$ is its linear size and $\kappa$ denotes the circulation quantum. Observed values of $Re_s \sim 10$ compare well with the critical $Re_s = UD / \kappa \approx 20$ ($U$ is the transport velocity and $D$ the diameter of the pipe) found as a temperature independent threshold in pipe flow of He II at much higher temperature when the flow of the

FIG. 4: Frequency as a function of He II density at low drive levels (circles) and for the second critical threshold (triangles) The straight line extrapolates through the zero-density (vacuum) resonant frequency of $f_0=1117.2$ Hz.
normal component was inhibited by superleaks at both ends of the pipe. The generation of quantized vortex loops on the surface of the grid by macroscopic superflow around it probably involves growth from remanent vortex lines, or from a “plasma” of half vortex rings. This idea is supported by the following observation. While measuring the response dependence on increasing the drive amplitude at the resonant frequency, the system encounters a nucleation problem when passing the first threshold: on some occasions the resonance response stopped growing with increasing the drive level, and jumped on the response/drive curve only later. With decreasing drives this feature disappears, and the response remains proportional to the drive level.

The frequency down-shift $\Delta f(p) = f_{\text{res}} - f_{\text{sh}} \sim 2$ Hz between the two critical thresholds might be considered in terms of a boundary layer of thickness $\lambda$ that enhances the hydrodynamic effective mass of the grid by $M_g' = A \rho_{He} \nu$, where $A$ denotes the surface area of the grid. Requiring that the downward shift of the resonance frequency corresponds to those observed experimentally leads to

$$\lambda = \frac{M + \Delta M_g^2}{A \rho_{He}} \left( \frac{f_{\text{res}}^2}{f_{\text{sh}}^2} - 1 \right) = \sqrt{\frac{2 \nu_{\text{eff}}}{\omega}} = 0.53 \pm 0.05 \, \mu m$$

which would correspond to an effective kinematic viscosity $\nu_{\text{eff}} \approx 10^{-5}$ cm$^2$/s. Extending the analogy to classical fluids, we may estimate the expected resonant linewidth due to the drag of such a hypothetical viscous fluid. A straightforward calculation based on estimating the flow velocity gradient in the direction inside the fluid by $\langle v_y \rangle / \lambda$ leads to a linewidth $\sim 1$ Hz, consistent with resonances for (e.g. Fig. 2) 0.2 and 0.5 V$_{pp}$ data near the second critical threshold. However, this picture seems unable to account for the re-entrant character of the non-linear resonances because the resultant viscous broadening would tend to smear out the effect in question.

We infer that, beyond the second critical threshold (Fig. 2), the vortex loops start to reconnect. Free vorticity is then shed into the liquid, corresponding to dissipation of the vibrational energy and leading to the observed increase in linewidth. Under these flow conditions, He II behaves in close analogy with a classical Navier-Stokes fluid: the square-root behaviour (Fig. 3) of the resonant response as a function of drive amplitude in this range is typical of classical turbulent drag scaling, and it is therefore likely that this threshold marks the onset of turbulence. After leaving the grid, the quantized vorticity probably “evaporates” as recently proposed in connection with turbulence in superfluid $^3$He-B.

The particular challenge posed by these results is to develop a quantitative description of vortex dynamics in the macroscopic flow around the moving grid, showing how it can give rise to the observed amplitude-dependent mass enhancements and re-entrant resonance curves, without dissipation. But the problem also carries wider significance, and will repay careful study, because the dynamical processes in question are probably fundamental to the generation of quantum turbulence.

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