Developments on the Bayesian Structural Time Series Model: Trending Growth∗

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Abstract

This paper investigates the added benefit of internet search data in the form of Google Trends for nowcasting real U.S. GDP growth in real time through the lens of the mixed frequency augmented Bayesian Structural Time Series model (BSTS) of Scott and Varian (2014). We show that a large dimensional set of search terms are able to improve nowcasts before other macro data becomes available early on the quarter. Search terms with high inclusion probability have negative correlation with GDP growth, which we reason to stem from them signalling special attention likely due to expected large troughs. We further offer several improvements on the priors: we allow to shrink state variances to zero to avoid overfitting states, extend the SSVS prior to the more flexible normal-inverse-gamma prior of Ishwaran et al. (2005) which stays agnostic about the underlying model size, as well as adapt the horseshoe prior of Carvalho et al. (2010) to the BSTS. The application to nowcasting GDP growth as well as a simulation study show that the horseshoe prior BSTS improves markedly over the SSVS and the original BSTS model, with largest gains to be expected in dense data-generating-processes.

Keywords: Global-Local Priors, Non-Centred State Space, Shrinkage, Google Trends

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1 Introduction

The objective of nowcast models is to produce ‘early’ forecasts of the target variable which exploit the real time data publication schedule of the explanatory data set. Nowcasting is particularly relevant to central banks and other policy environments who are tasked with conducting forward looking policies on the basis of key economic variables such as GDP or inflation. These, however, are published with a lag of up to 7 weeks with respect to their reference period. Since even monthly macro data arrive with considerable lag, it is now common to combine, next to traditional macroeconomic data, ever more information from Big Data sources such as internet search terms, satellite data, scanner data, etc. (Bok et al., 2018) which have the advantage of being available in near real time.

As such, a burgeoning literature has studied the utility of using Google search data in the form of Google Trends (GT), which measure the relative search volume of certain search terms entered into the Google search engine, to nowcast economic time-series. The majority of these applications study improvements of GT based nowcasts compared to simple autoregressive processes and survey indicators. The latter comparison seems natural, as search term volume can be viewed as surveys to supply and demand of products and an expression of views and sentiments (Scott and Varian, 2014). In particular, the literature established improvements in predicting the UK housing market (McLaren and Shanbhogue, 2011), unemployment related data in the UK, Israel and Germany (Askitas and Zimmermann, 2009; Suhoy et al., 2009), US private consumption (Vosen and Schmidt, 2011) and price expectations (Guzman, 2011, D’Amuri and Marcucci, 2017). A commonality within these and related studies is that search terms have predictive power especially when they are directly related to demand/supply decisions and signals of preference (Niesert et al., 2020), such as unemployment data where internet search engines provide the dominant funnel through which job seekers find jobs (Smith, 2016). While links of internet search to aggregate economic behaviour is less clear from a theoretical point of view, a growing amount of studies report that Google Trends are even useful in nowcasting there. D’Amuri and Marcucci (2017) show that hand-picked search categories related to ‘jobs’ help predict headline US unemployment, which is confirmed by Choi and Varian (2012) as well as Niesert et al. (2020) who select Google Trends in a more automatic way through

1The exact lag in publications of GDP and inflation depends as well on which vintage of data the econometrician wishes to forecast. Since early vintages of aggregate quantities such as GDP can display substantial variation between vintages, this is not a trivial issue.
Google Correlate. Niesert et al. (2020) extend this result also to other countries and Koop and Onorante (2019) comprehensively show that Google Trends improve nowcasts of a variety of monthly macro variables, with the caveat that these improvements heavily depend on the time period and the variable under investigation. Similar to Choi and Varian (2012), Koop and Onorante (2019) find that Google Trends perform particularly well in predicting turning points and large troughs in the data. The working hypothesis for this finding is that Google Trends related to the macro variable of interest pick up a form of ‘collective wisdom’ which needn’t signify positive or negative relationship, but capture attention likely related to turning points. We investigate whether this holds true for quarterly macroeconomic data, namely GDP growth. While Ferrara and Simoni (2019) show that Google Trends improve nowcasts of quarterly Euro Area growth prior to arrival of more traditional macroeconomic information, no study to the best of our knowledge, has considered using Google Trends to nowcast U.S. GDP growth. As a motivating illustration, figure [9] plots U.S. annualised real GDP growth and the end of quarter monthly search term ‘real gdp growth’.

![Figure 1: Time series plot of real annualised U.S. GDP growth (blue) and the deseasonalised Google Trends search term ‘real gdp growth’ (orange).](image)

One can clearly see how the search term evolves counter cyclically, with the largest spike in popularity right around the financial crisis. It stands to reason that search terms such as ‘real gdp growth’ are most popular when the economic information they are related to attract special attention which is probably when they are unexpectedly high or low. Similar reasoning has lead to a larger literature on analysing the link between textual information and economic
activity (Gentzkow et al. 2019; Alexopoulos and Cohen 2015; Baker et al. 2016; Manela and Moreira 2017; Shapiro et al. 2020) among many others. Given that – compared to traditional macro data – Google Trends have a much shorter publication lag, potential improvements in fit could be substantial. We contribute to this literature by investigating whether a large set of Google Trends offer improvements in nowcasting real U.S. GDP growth over and beyond more commonly used monthly macro information.

Although conceptually straightforward, using high frequency Big Data sources like Google Trends for nowcasting quarterly macro data, pose multiple econometric and data challenges. The latter are described further in section (2). The main econometric challenges relate to mismatches in sampling frequencies, asynchronous data publications (ragged edges), time-variation in model parameters and form of regularisation. While the literature on mixed frequency now-casting models (see for summary e.g. Baámbura et al. (2013)) has proposed a variety of approaches, our interest is in using the popular Bayesian structural time series model (BSTS) by Scott and Varian (2014) for it’s flexibility of addressing all econometric issues above as well as it’s intuitive appeal, and improve it in several dimensions. To set the stage, the underlying idea of this model is to combine an unobserved component model to capture long-run trends and seasonality, which can be viewed as the structural part of the time-series as proposed by Harvey (2006), with a spike-and-slab shrinkage prior to capture ‘irregular’ movements explained by a possibly large dimensional regressor set:

\[
\begin{align*}
    y_t &= \tau_t + x_t'\beta + \delta_t + \epsilon_t, \epsilon_t \sim N(0, \sigma_y^2) \\
    \tau_t &= \mu_{t-1} + \alpha_t + \epsilon_t^\tau, \epsilon_t^\tau \sim N(0, \sigma_\tau^2) \\
    \alpha_t &= \alpha_{t-1} + \epsilon_t^\alpha, \epsilon_t^\alpha \sim N(0, \sigma_\alpha^2) \\
    \delta_t &= -\sum_{s=1}^{S-1} \delta_{t-s} + \epsilon_t^\delta, \epsilon_t^\delta \sim N(0, \sigma_\delta^2)
\end{align*}
\]

In (1), any persistence in the data are modeled through a latent time-varying slope, \( \tau_t \), with drift \( \alpha_t \), originally proposed by Harvey (1990) as the local-linear-trend model, and seasonality are captured by \( S \) seasonal components. The local-linear trend model has the intuitive appeal, that the deviation from \( \tau_t \), describes deviations from a long-run trend which applied to the level of GDP can be interpreted as the output gap (Watson, 1986; Grant and Chan, 2017). The second level of the trend, \( \alpha_t \), thus allows for stochastic changes in the output gap, which, when first differencing \( y_t \), results in GDP growth following a unit-root process (Clark, 1987). While assuming that GDP growth can drift without bound, remains a topic of controversy in economics,
there is mounting evidence that U.S. GDP growth experienced multiple structural breaks (Kim
and Nelson 1999; McConnell and Perez-Quiros 2000, Jurado et al. 2015). Antolin-Diaz et al.
(2017) convincingly analysed that allowing for smooth shifts in GDP growth is preferred over
deterministic structural breaks, especially when the state variances are tightly controlled by
 priors such that the stochastic trend does not wander too wildly. In fact, Antolin-Diaz et al.
(2017) show that nowcasts of GDP growth improve markedly by modeling time-variation in
long-run growth as opposed to filtering it out via e.g. the Hodric-Prescott filter. As there is
not much theoretical basis for a local linear trend in GDP growth, our first point of improve-
ment is to generalise the state space approach (1) to a non-centred formulation which offers
more aggressive regularisation on the latent state variances so as to prevent over-fitting. We
use a prior with positive probability on zero which, importantly, allows to formally test for the
null of zero posterior variance (Frühwirth-Schnatter and Wagner 2010) and, therefore, whether
a state is constant or time varying. This would not be possible in a frequentist approach to
estimating model (1), due to boundary testing issues.

Our second improvement concerns the SSVS prior. The SSVS prior as proposed in the original
BSTS model of Scott and Varian (2014), has been formulated under the assumption of a fixed
expected model size parameter. As shown by Giannone et al. (2017), however, the posterior
distribution of model sizes and therefore the resulting degree of sparsity in the model, might
be overly influenced by fixing the model size parameter a-priori. They therefore advocate to
use an ‘agnostic’ prior that allows the data to determine which model size is preferred which
we extend to the BSTS model.

While spike-and-slab priors are widely acknowledged to be the gold-standard for Bayesian vari-
able selection and model averaging, a computational bottle-neck is presented by their discrete
mixture representation which quickly becomes infeasible to calculate in very large dimensional
settings (George and McCulloch 1993). Further, spike-and-slab priors struggle with correlated
data, as the correlation causes multi-modal posteriors and therefore bad mixing (Piironen et al.
2017). While Niesert et al. (2020) have improved on mixing via Hamiltonian Monte Carlo, our
third methodological contribution is to further extend the BSTS framework to more modern
global-local priors which offer continuous shrinkage. Global-local priors such as the horseshoe
(Carvalho et al., 2010) and Dirichlet-Laplace prior (Bhattacharya et al., 2016) not only provide
computational advantages over the spike-and-slab prior, but offer favourable asymptotics as

\footnote{This is supported by findings of Sims (2012) who finds that modeling time-variation is preferred over
detrending a priori.}
well (Bhadra et al., 2019). We focus in this paper on the horseshoe prior, but the sampling algorithm provided in Appendix A.1 can be used for any prior that can be expressed as a mixture of normals. Finally, although global-local priors shrink noise variables to close to zero, the regression posterior may be hard to interpret without any further sparsifying assumptions. To bridge this gap, we adapt the scheme provided by Ray and Bhattacharya (2018), the SAVS algorithm, to the horseshoe prior augmented BSTS model. The SAVS algorithm groups posterior regression estimates into signals and noise, where the latter are thresholded to zero. We apply the SAVS algorithm on a iteration basis which allows to interpret the posterior regression coefficients akin to Bayesian model averaging posteriors (Huber et al., 2019). The results from our nowcasting application show that Google’s search indices improve nowcasts early on in the quarter as well that the horseshoe prior BSTS performs best with accuracy gains of 30-40% over the original BSTS model of Scott and Varian (2014) in point as well as in density nowcasts. These results are confirmed in a simulation study which checks robustness to a variety of data-generating processes.

In what follows, we will firstly elaborate on the data that are used for nowcasting, how we deal with mixed frequency and the data publication calendar. Secondly, we elaborate on the posteriors for our extended BSTS models and provide efficient sampling algorithms. We then present results based on our empirical application of nowcasting U.S. GDP growth as well as results from our simulations. Finally, we provide discussion and avenues for future research.

2 Data

2.1 Mixing Frequencies

In this paper, we relate monthly macro data based on Giannone et al. (2016) and internet search information via U-MIDAS skip-sampling to real quarterly U.S. GDP growth. The U-MIDAS approach to mixed frequency belongs to the broader class of ‘partial system’ models (Bańbura et al., 2013), which directly relate higher frequency information to the lower frequency target variable by vertically realigning the covariate vector. Switching notation from equation (1) to make explicit that \( x_t \) is sampled at a higher, i.e., monthly frequency, denote \( x_{t,M} = (x_{1,t,M}, \cdots, x_{K,t,M}) \) and \( \beta_m = (\beta_{1,M}, \cdots, \beta_{K,M})' \) where \( M = (1, 2, 3) \) denotes the monthly observation of the covariate within quarter, \( t \). By concatenating each monthly column, we obtain a \( T \times K * 3 \) regressor matrix \( X \) and a \( K * 3 \times 1 \) regression coefficient vector \( \beta \). This
vertical realignment is visualised for a single representative regressor below:

\[
\begin{pmatrix}
  y_{1\text{st quarter}} & x_{\text{Mar}} & x_{\text{Feb}} & x_{\text{Jan}} \\
  y_{2\text{nd quarter}} & x_{\text{Jun}} & x_{\text{May}} & x_{\text{Apr}} \\
  \vdots & \vdots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \vdots \\
  \vdots & \vdots & \vdots & \vdots 
\end{pmatrix}
\]  

(2)

2.2 Macro Data

As early data vintages of U.S. GDP can exhibit substantial variation compared to final vintages (Croushore, 2006; Sims, 2002), it is not trivial which data to use in evaluating nowcast models on historical data. Further complications can arise through changing definitions or methods of measurements of data (Carriero et al., 2015). However, as in our application, only a few explanatory variables have recorded real time vintages (see Giannone et al. (2016)) and our training window is restricted to begin with 2004 only (since this is the earliest data point for the Google Trends data base) we decided to use final vintages of our data. We therefore consider a pseudo real-time data set: we use the latest vintage of data, but, at each point of the forecast horizon, we use only the data published up to that point in time.

The target variable for this application is deseasonalised U.S. real GDP growth (GDP growth) as of downloaded from the FRED website \(^3\). We found that pre-deseasonalised data improved forecast accuracy compared to modeling it in our state space system. This might be due to the small sample size. As Google Trends are only available from 01/01/2004-01/06/2019, at the time of download, the period under investigation pertains to the same period in quarters (60 quarters). We split the data set into a training sample of 45 quarters (2004q2-2015q2) and a forecast sample of 15 quarters (2015q3-2019q1).

The macro data set pertains to an updated version of the data base of Giannone et al. (2016) (henceforth, ‘macro data’) which contains 13 time series which are closely watched by professional and institutional forecasters such as real indicators (industrial production, house starts, total construction expenditure etc.), price data (CPI, PPI, PCE inflation), financial market data (BAA-AAA spread) and credit, labour and economic uncertainty measures (volume of commercial loans, civilian unemployment, economic uncertainty index etc.) Table (1) gives an overview over all data along with FRED codes.

\(^3\)The deseasonalisation pertains here to the X-13-ARIMA method and was performed prior to download from the FRED-MD website.
2.3 Google Trends

Google Trends (GT) are indices produced by Google on the relative search term popularity of a given search term or pre-specified search categories, conditional on a given time frame and location. Our sample comprises 27 Google Trends (overview in Koop and Onorante (2019) and Bock (2018)). In general, there is no consensus on how to optimally select search terms for final estimation.

Methods which have been proposed in the previous literature fall into: (i) pre-screening through correlation with the target variable as found via Google Correlate (Scott and Varian, 2014; Niesert et al., 2020; Choi and Varian, 2012), (ii) through cross-validation (Ferrara and Simoni, 2019), (iii) use of prior economic intuition where search terms are selected through backward induction (e.g.: Smith (2016); Ettredge et al. (2005); Askitas and Zimmermann (2009)), and (iv) root terms, which similarly specify a list of search terms through backward induction, but additionally download ”suggested” search terms from the Google interface. This serves to broaden the semantic variety of search terms in a semi-automatic way. As as methodologies based on pure correlation do not preclude spurious relationships (Scott and Varian, 2014; Niesert et al., 2020; Ferrara and Simoni, 2019), we opt for the root term methodology as from the authors’ perspective, it currently provides the best guarantee of finding economically relevant Google Trends. It is also important to note that in the selection process, search terms are not tossed out based on expected performance but rather that they are simply not related to GDP.

Since search terms can display seasonality, we deseasonalise all Google Trends by the Loess filter, as recommended by Scott and Varian (2014), which is implemented with the ”stl” command in R and make sure that they are individually stationary to avoid any spurious correlation. Finally, our pseudo-real time calendar can be found in table 1 and has been constructed after the data’s real publication schedule. It comprises a total of 31 vintages which make for an equal number of information sets \( \Omega^v \) for \( v = 1, \cdots, 31 \) which are used to construct nowcasts as explained in section 4. In order for the Google Trends search indexes to represent the latest available information within a given month, we treat them as only observable at the end of the given month.

\footnote{Unfortunately, Google Correlate has suspended updating their databases past 2017.} 

\footnote{To mitigate any inaccuracy stemming from sampling error, we downloaded the set of Google Trends seven times between 01/07-08/07/2019 and took the cross-sectional average. Since we used the same IP address and google-mail account, there might still be some unaccounted measurement error which could be further mitigated.}
| Vintage | Timing | Release | Variable Name | Pub. lag | Transformation | FRED Code |
|---------|--------|---------|---------------|----------|---------------|-----------|
| 0       | First day of month 1 | No information available | - | - | - | - |
| 1       | Last day of month 1 | Fed. funds rate & credit spread | fedfunds & baa | m | 3 | FEDFUNDS & BAAY10 |
| 2       | Last day of month 1 | Google Trends | Google Trends | m | 4 | - |
| 3       | 1st bus. day of month 2 | Economic Policy Uncertainty Index | uncertainty | m-1 | 1 | USEPUINDXM |
| 4       | 1st Friday of month 2 | Employment situation | hours & unrate | m-1 | 2 | AWHNONAG & UNRATE |
| 5       | Middle of month 2 | CPI | cpi | m-1 | 2 | CPI |
| 6       | 15th-17th of month 2 | Industrial Production | indpro | m-1 | 2 | INDPRO |
| 7       | 3rd week of month 2 | Credit & M2 | loans & m2 | m-1 | 2 | LOANS & M2 |
| 8       | Later part of month 2 | Housing starts | housst | m-1 | 1 | HOUST |
| 9       | Last week of month 2 | PCE & PCEPI | pce & pce2 | m-1 | 2 | PCE & PCEPI |
| 10      | Last day of month 2 | Fed. funds rate & credit spread | fedfunds & baa | m | 3 | FEDFUNDS & BAAY10 |
| 11      | Last day of month 2 | Google Trends | - | m | 4 | - |
| 12      | 1st bus. day of month 3 | Economic Policy Uncertainty Index | uncertainty | m-1 | 1 | USEPUINDXM |
| 13      | 1st bus. day of month 3 | Construction starts | construction | m-2 | 1 | TTLCONS |
| 14      | 1st Friday of month 3 | Employment situation | hours & unrate | m-1 | 2 | AWHNONAG & UNRATE |
| 15      | Middle of month 3 | CPI | cpi | m-1 | 2 | CPI |
| 16      | 15th-17th of month 3 | Industrial Production | indpro | m-1 | 2 | INDPRO |
| 17      | 3rd week of month 3 | Credit & M2 | loans & m2 | m-1 | 2 | LOANS & M2 |
| 18      | Later part of month 3 | Housing starts | housst | m-1 | 1 | HOUST |
| 19      | Last week of month 3 | PCE & PCEPI | pce & pce2 | m-1 | 2 | PCE & PCEPI |
| 20      | Last day of month 3 | Fed. funds rate & credit spread | fedfunds & baa | m | 3 | FEDFUNDS & BAAY10 |
| 21      | Last day of month 3 | Google Trends | Google Trends | m | 4 | - |
| 22      | 1st bus. day of month 4 | Economic Policy Uncertainty Index | uncertainty | m-1 | 1 | USEPUINDXM |
| 23      | 1st bus. day of month 4 | Construction starts | construction | m-2 | 1 | TTLCONS |
| 24      | 1st Friday of month 4 | Employment situation | hours & unrate | m-1 | 2 | AWHNONAG & UNRATE |
| 25      | Middle of month 4 | CPI | cpi | m-1 | 2 | CPI |
| 26      | 15th-17th of month 4 | Industrial Production | indpro | m-1 | 2 | INDPRO |
| 27      | 3rd week of month 4 | Credit & M2 | loans & m2 | m-1 | 2 | LOANS & M2 |
| 28      | Later part of month 4 | Housing starts | housst | m-1 | 1 | HOUST |
| 29      | Last week of month 4 | PCE & PCEPI | pce & pce2 | m-1 | 2 | PCE & PCEPI |
| 30      | Later part of month 5 | Housing starts | housst | m-2 | 1 | HOUST |

Table 1: Pseudo real time calendar based on actual publication dates. Transformation: 1 = monthly change, 2 = monthly growth rate, 3 = no change, 4 = LOESS decomposition. Pub. lag: m = refers to data for the given month within the reference period, m-1 = refers to data with a months’ lag to publication in the reference period, m-2 = refers to data with 2 months’ lag to publication in the reference period.
3 Methodology

3.1 Non-Centred State Space

The original state space formulation of Scott and Varian (2014) collects states $\tau_t$, $\alpha_t$ and $\delta_t$ in (1) and estimates them jointly via a forward filtering backward sampling (FFBS) algorithm of Durbin and Koopman (2002) which is based on the Kalman filter. While very popular, the recursive structure of the algorithm is costly in terms of computation, but more importantly, relies on independent Normal-Inverse Gamma (N-IG) priors for the states and state variances which Frühwirth-Schnatter and Wagner (2010) show can lead to overfitting state spaces and therefore imprecise forecasts. This is due to the fact that in state space models, the state variances determine whether a state process is fixed or time varying (see equation (1)). Instead, Frühwirth-Schnatter and Wagner (2010) propose a non-centred representation of the state space which dissects the dynamics into a non-time varying and time varying component, where the former models the state standard deviation directly in the observation equation. A normal prior on the state standard deviation can be shown to imply a Gamma prior on the state variance which allows for far more mass on 0, therefore applying more shrinkage. Taking (1) as our centred state space (and ignoring $X$ for now), we re-write it equivalently as:

$$y_t = \tau_0 + \sigma_\tau \tilde{\tau}_t + t\alpha_0 + \sigma_\alpha \sum_{s=1}^{t} \tilde{\alpha}_t + \epsilon_t, \epsilon_t \sim N(0, \sigma^2)$$

(3)

where

$$\tilde{\tau}_t = \tilde{\tau}_{t-1} + \tilde{u}_t^\tau, \tilde{u}_t^\tau \sim N(0, 1)$$

$$\tilde{\alpha}_t = \tilde{\alpha}_{t-1} + \tilde{u}_t^\alpha, \tilde{u}_t^\alpha \sim N(0, 1)$$

(4)

with starting values $\tilde{\tau}_0 = \tilde{\alpha}_0 = 0$. To see that (3) and (4) is equivalent to (1), let:

$$\alpha_t = \alpha_0 + \sigma_\alpha \tilde{\alpha}_t$$

$$\tau_t = \tau_0 + \sigma_\tau \tilde{\tau}_t + t\alpha_0 + \sigma_\alpha \sum_{s=1}^{t} \alpha_s$$

(5)

by using web-crawling.

Alternatively, one could use bridge methods as in Ferrara and Simoni (2019) who update monthly GT indexes on a weekly basis. We leave this for future investigation.
Hence, by setting \( y_t = \tau_t + \epsilon_t \), it is clear that

\[
\begin{align*}
\alpha_t - \alpha_{t-1} &= \sigma \alpha (\tilde{\alpha}_t - \tilde{\alpha}_{t-1}) \\
&= \sigma \alpha + \tilde{u}_t^\alpha \\
\tau_t - \tau_{t-1} &= \alpha_0 + \sigma_\alpha \tilde{\alpha}_t + \sigma_\tau (\tilde{\tau}_t - \tilde{\tau}_{t-1}) \\
&= \alpha + \sigma_\tau + \tilde{u}_t^\tau
\end{align*}
\]

which recovers (1). Since \( \sigma_{\tau, \alpha} \) are allowed to have support on the real line, they are not identified in multiplication with the states: the likelihood is invariant to changes in signs of \( \sigma_\alpha \) and \( \sigma_\tau \). Consequently, mixing of the posterior state standard deviations can be bad and their distributions are likely to be bi-modal (Frühwirth-Schnatter and Wagner, 2010). This issue is combated by randomly permuting signs in the Gibbs sampler as explained below. Similar to Frühwirth-Schnatter and Wagner (2010), we assume normal priors centred at 0 for \( \sigma_i : \sigma_i \sim N(0, V_i) \forall i \in \{\tau, \alpha\} \).

Collecting all state space parameters in \( \theta = (\tau_0, \alpha_0, \sigma_\tau, \sigma_\alpha) \), we assume an independent multivariate normal prior with diagonal covariance:

\[
\theta \sim N(\theta_0, V_\theta)
\]

While the state processes \( \{\tilde{\tau}, \tilde{\alpha}\}_{t=1}^T \) can be estimated by any state space algorithm, we opt for the precision sampler method of Chan (2017) as outlined in Appendix (A.2.1). In contrast to FFSB type algorithms, it samples the states without recursive estimation which speeds up computation significantly.

### 3.2 SSVS Prior

Variable selection in the BSTS model of Scott and Varian (2014) is done via a two component conjugate spike-and-slab prior which utilises a variant of Zeller’s g-prior and fixed expected model size. While computationally fast due to the assumption of conjugacy, many high-dimensional problems benefit from prior independence (Moran et al., 2018) and a fully hierarchical formulation to let the data decide on the most likely value of the parameters (Ishwaran et al., 2005). We therefore follow Ishwaran et al. (2005)’s extension to the SSVS prior,
the Normal-Inverse-Gamma prior:

$$
\beta_j | \gamma_j, \delta_j^2 \sim \gamma_j N(0, \delta_j^2) + (1 - \gamma_j)N(0, c \times \delta_j^2) \\
\delta_j^2 \sim \text{Gamma}(a_1, a_2) \\
\gamma_j \sim \text{Bernoulli}(\pi_0) \\
\pi_0 \sim \text{Beta}(b_1, b_2)
$$  

(8)

where \( j \in (1, \cdots, K) \). The intuition compared to the spike-and-prior of \cite{ScottVarian2014} remains the same in that the coavariate’s effect is modeled by a mixture of normals where a it is either shrunk to close to zero via a narrow distribution around zero, the spike component, or estimated freely though a relatively diffuse normal distribution, the slab component. Sorting into each component is handled through an indicator variable, \( \gamma_j \), and the hyperparameter \( c \) is chosen to be a very small number, thereby forcing shrinkage of noise variables to close to zero. While in the original BSTS model, the indicator variable, \( \gamma_j \), depends on a fixed prior \( \pi_0 \) which governs the prior inclusion probability of a variable, (8) allows for it to be estimated by the data through another level of hierarchy. We set \( b_1 = b_2 = 1 \), which effectively assumes that any expected model size is a-priori possible and thus allows for sparse but also dense model solutions as recommended by \cite{Giannoneetal2017}. Finally, also the prior variance \( \delta_j^2 \) is allowed to be hierarchical. Posterior are standard and described in the appendix (A.1). The posterior of \( \gamma_j \) is of special interest to the analyst as it gives a data informed measure of importance of a variable. Namely, \( p(\gamma|y) \) can be interpreted as the posterior inclusion probability of a variable.

### 3.3 Horseshoe Prior

The horseshoe prior, like many recently popularised shrinkage priors, belongs to the broader class of global-local priors which take the following general form:

$$
\beta_j | \lambda_j^2, \nu^2, \sigma^2 \sim N(0, \lambda_j^2 \nu^2 \sigma^2), j \in (1, \cdots, K) \\
\lambda_j^2 \sim \pi(\lambda_j^2) d\lambda_j^2, j \in (1, \cdots, K) \\
\nu^2 \sim \pi(\nu^2) d\nu^2
$$  

(9)

The idea of this family of priors is that the global scale \( \nu^2 \), controls the overall shrinkage applied to the regression, while the local scale \( \lambda_j^2 \) allows for the local possibility of regressors to escape shrinkage when they have large effects on the response. A variety of global local shrinkage
priors have been proposed (see Polson and Scott (2010)), but here we focus on arguably the most popular, the horseshoe prior of Carvalho et al. (2010) which employs two half Cauchy distributions for $\nu$ and $\lambda$:

$$
\lambda_j^2 \sim C_+(0, 1)
$$

$$
\nu^2 \sim C_+(0, 1)
$$

(10)

It can further be shown that these two fat tailed priors imply a shrinkage profile that has the spike-and-slab prior in its limit and therefore offers a continuous approximation to the SSVS (Piironen et al., 2017). Due to its special connection to frequentist shrinkage priors (Polson and Scott, 2010), it can be shown that it not only offers good finite sample performance, but also has favourable asymptotic behaviour compared to competing global priors (Bhadra et al., 2019). Posteriors are described in the appendix (A.1).

### 3.4 SAVS Algorithm

Although, the horseshoe prior will shrink noise variables to close to zero, the importance of a variable for nowcasts may not be immediately clear from posterior summary statistics of the coefficients, especially, when the posterior is multi-modal. To aid interpretability and simultaneously preserve predictive ability, we employ the SAVS algorithm of Ray and Bhattacharya (2018) to the posterior coefficients on a draw by draw basis. The algorithm uses a useful heuristic, inspired by frequentist lasso estimation, to threshold posterior regression coefficients to zero:

$$
\phi_j = \text{sign}(\hat{\beta}_j)|X_j||^{-2}max(|\hat{\beta}_j| ||X_j|| - \kappa_j, 0),
$$

(11)

where $X_j = (x_{j1}, \ldots, x_{jT})'$ is the $j^{th}$ column of the regressor matrix $X$, sign(x) returns the sign of x and $\hat{\beta}$ represents a draw from the regression posterior. The parameter $\kappa_j$ in (11) acts as a threshold for each coefficient akin to the penalty parameter in lasso regression which can be selected via cross-validation. Ray and Bhattacharya (2018) propose as simpler solution,

$$
\kappa_j = \frac{1}{||\beta_j||^2},
$$

(12)

which ranks the given coefficient inverse-squared proportionally and provides good performance compared to alternative penalty levels (Ray and Bhattacharya, 2018; Huber et al., 2019). To see the similarity to lasso style regularisation, the solution to (11) can be obtained by the following minimisation problem which reminds of Zou (2006):

$$
\bar{\phi} = \arg\min_{\phi} \left\{ \frac{1}{2}||X\hat{\beta} - X\phi|| + \sum_{j=1}^{K} \kappa_j|\phi_j| \right\}
$$

(13)
\( \bar{\phi} \) is the sparsified regression vector. The relative frequency of non-zero entries in the posterior coefficient vector can analogously to the SSVS posterior be interpreted as posterior inclusion probabilities. And integrating over the uncertainty of the parameters to obtain the predictive distribution \( p(\tilde{y}|y) \), we receive something similar to a Bayesian Model Averaged (BMA) posterior [Koop and Onorante, 2019].

### 3.5 Sampling Algorithm

With the conditional posteriors at hand (see A.1), we sample states as well as regression parameter with the following Gibbs sampler:

1. Sample \((\tilde{\tau}, \tilde{\alpha}|y, \theta, \beta, \sigma_y^2)\)
2. Sample \((\theta|y, \beta, \tilde{\tau}, \tilde{\alpha}, \sigma_y^2)\)
3. Randomly permute signs of \((\tilde{\tau}, \tilde{\alpha})\) and \((\sigma_\tau, \sigma_\alpha)\)
4. Sample \((\beta|y, \theta, \tilde{\tau}, \tilde{\alpha}, \sigma_y^2)\)
5. Sample \((\sigma_y^2|y, \tilde{\tau}, \tilde{\alpha}, \sigma_y^2)\)

As mentioned in 3.1, states are sampled in a non-recursive fashion which exploits sparse matrix computation and precision sampling. The exact sampling algorithm is given in (A.2.1). After having sampled \( \theta \) in step 2, we randomly permute signs of \((\tilde{\tau}, \tilde{\alpha})\), \((\sigma_\tau, \sigma_\alpha)\) as alluded to in 3.1 to aid mixing. Step 4 of the sampler will depend on the prior and its respective hyperpriors. While the posterior sampling scheme for the SSVS is standard, we use the efficient posterior sampler of Bhattacharya et al. (2016) to sample the regression coefficients of the horseshoe prior. Compared to Cholesky based sampling as used for the SSVS, computation speed is markedly improved (see (A.1.1)). Note, that in step 4, we perform SAVS sparsification via (11) on an iteration basis.
Nowcasting U.S. Real GDP Growth

The predictive model used to generate in-as well as out-of-sample prediction is:

\[ y_t = \tau_0 + \sigma_t \tilde{r}_t + t \alpha_0 + \sigma_\alpha \sum_{s=1}^{t} \tilde{\alpha}_t + X_t \beta + \epsilon_t \sim N(0, \sigma_y) \]

\[ \tilde{r}_t = \tilde{r}_{t-1} + u_{t}^{\alpha}, \quad u_{t}^{\alpha} \sim N(0, 1) \]

\[ \tilde{\alpha}_t = \tilde{\alpha}_{t-1} + u_{t}^{\alpha}, \quad u_{t}^{\alpha} \sim N(0, 1) \]

where \( t = (1, \cdots, T) \) are used as a training sample. We estimate three variants of (14) based on priors (8), (10), (11), the original BSTS model of Scott and Varian (2014) as well as a simple AR(2) model for comparison. As is standard for BSTS applications, we firstly compare the in-sample cumulative absolute one-step-ahead forecast error, which is generated as a side product of the state space, as well as the inclusion probabilities of the variables so as to shed light on which variables drive fit. Out-of-sample nowcasts are generated from the posterior predictive distribution \( p(y_{T+1} | \Omega^v_T) \) for growth observation \( y_{T+1} \), conditional on the real-time information set \( \Omega^v_T \), where \( (v = 1, \cdots, 31) \) refers to vintages within the pseudo real-time calendar (1). This results in 31 different nowcasts which are generated on a rolling basis until the end of the forecast sample, \( T_{end} \). Variables that haven’t been published yet until vintage \( v \) are zeroed out as recommended by Carriero et al. (2015).

Point forecasts are computed as the mean of the posterior predictive distribution and are compared via real time root-mean-squared-forecast-error (RT-RMSFE) which are calculated for each vintage as:

\[ \text{RT-RMSFE} = \sqrt{\frac{1}{T_{end}} \sum_{j=1}^{T_{end}} (y_{T+j} - \hat{y}^v_{T+j} | \Omega^v_{T+j-1})^2}, \tag{15} \]

where \( \hat{y}^v_{T+j} | \Omega^v_{T+j} \) is the mean of the posterior predictive for vintage \( v \) using information until \( T+j-1 \).

Forecast density fit, is measured by the mean real-time log-predictive density score (RT-LPDS) and real-time continuous rank probability score (RT-CRPS):

\[ \text{RT-LPDS} = \frac{1}{T_{end}} \sum_{j=1}^{T_{end}} \log p(y_{T+j} | \Omega^v_{T+j-1}) \]

\[ = \frac{1}{T_{end}} \sum_{j=1}^{T_{end}} \log \int p(y_{T+j} | \Omega^v_{T+j-1}, \zeta_{1:T+j-1}) p(\zeta_{1:T+j-1} | \Omega^v_{T+j-1}) d\zeta_{1:T+j-1} \tag{16} \]

\[ \approx \frac{1}{T_{end}} \sum_{j=1}^{T_{end}} \log \left( \frac{1}{M} \sum_{m=1}^{M} p(y_{T+j} | \Omega^v_{T+j-1}, v^m_{1:T+j-1}) \right), \]
\[
\text{RT-LPDS} = \frac{1}{T_{\text{end}}} \sum_{j=1}^{T_{\text{end}}} \frac{1}{2} |y_{T+j} - y_{T+j}^v_{\Omega_{T+j-1}}| - \frac{1}{2} |y_{T+j}^v.A_{\Omega_{T+j-1}} - y_{T+j}^v.B_{\Omega_{T+j-1}}|,
\]

where \( \zeta_{1:T+j-1} \), for brevity of notation, collects all model parameters as defined for each model, which are estimated with expanding in-sample information until \( T + j - 1 \) and \( M \) stands for iterations of the Gibbs sampler after burn-in. Notice that in (17), \( y^v.A,B_{T+j} \) are independently drawn from the posterior predictive \( p(y_{T+1|\Omega_{T+j-1}}|y_T) \).

The LPDS, as shown by Frühwirth-Schnatter (1995), in a setting where time-varying and fixed components for a structural state space model are chosen, can be interpreted as a log-marginal likelihood based on the in-sample information and therefore makes for a model founded scoring function. The RT-CRPS can be thought of as the probabilistic generalisation of the mean-absolute-forecast-error. Similar to the log-score, it belongs to the broader class of strictly proper scoring functions (Gneiting and Raftery, 2007) which allows for comparing density forecasts in a consistent manner. To facilitate the discussion, the objective is to maximise the RT-LPDS and minimise the RT-CRPS. For all forecast metrics, the predictive distribution used for (15,16,17) is traditionally generated in state space models via the prediction equations of the Kalman filter (Harvey, 1990). We instead use the simpler approximate method of Cogley et al. (2005), which we found to make no practical difference for our sample. The method is described in A.2.2.

Finally, to test whether a state variance is equal to zero, we make use of the Dickey-Savage Density ratio test evaluated at \( \sigma_{T,\alpha} = 0 \):

\[
\text{DS} = \frac{p(\sigma_{T,\alpha} = 0)}{p(\sigma_{T,\alpha} = 0|y)}
\]

It can be shown that for nested models, the DS statistic is equivalent to the Bayes factor between the prior and the posterior distribution of the parameter of interest at zero (Verdinelli and Wasserman, 1995). The intuition for the test is simple: if the prior probability-density-function (PDF) allocates more mass at 0 than the posterior at that point, there is evidence in favour of the unrestricted model, i.e., \( \sigma_{T,\alpha} \neq 0 \). While the priors for the state variances have well known forms and thus can be evaluated analytically, we estimate the denominator for all models through Monte Carlo integration.

\footnote{We do not report calibrations tests, as there are too little out-of-sample observations to meaningfully determine calibration.}

\footnote{Although the CRPS is a symmetric scoring function, it penalises outliers less aggressive than the log-score which is of advantage in small forecast samples such as ours.}
4.1 In-Sample Results

Figure 2 shows the in-sample cumulative-one-step ahead prediction errors for the proposed priors where the information set pertains to the whole in-sample period without ragged edges. From figure (2), it is clear that the horseshoe prior BSTS (HS-BSTS) provides the best in-sample predictions at all time periods. The HS-BSTS-SAVS and SSVS-BSTS initially provide similar fit, however diverge in performance around the financial crisis. It is striking that compared to the former two, the HS-BSTS provides very stable performance as indicated by a nearly linear increase in errors throughout the entire estimation sample. It is also apparent that the SAVS algorithm is not able to retain the fit of the HS prior alone, which, as we show in the next subsection, is in contrast to the out-of-sample results.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Cumulative one-step-ahead forecast errors in-sample from 3 different models: (1) SSVS-BSTS, (2) HS-BSTS and (3) HS-SAVS-BSTS}
\end{figure}

The posterior marginal inclusion probabilities between the SSVS and HS-SAVS prior indicate which variables drive fit and are plotted for the top ten most drawn variables in figure (3). The colors of the bars indicate the sign on a continuous scale of white (positive relationship) to black (negative relationship) of the variable when included in the model, and the prefix ‘GT’ indicates whether the variable is a Google Trend. The number \([0,1,2]\) appended to a variable indicates the position in a given quarter, with 0 being the latest month. It is clear that the SAVS extended HS prior allows for larger models compared to the SSVS prior, as there are more variables with larger posterior inclusion probability. This is confirmed by the posterior
distribution of model sizes (see figure (4)). Next to differences in model selection, the inclusion probabilities of a given selected skip-sampled variable within a given quarter also indicate a difference in how they deal with correlated data. The SSVS prior tends to select only the most dominant of the skip-sampled information, while the SAVS extended HS prior allocates significant inclusion probability to all months within a quarter. For example, while the SSVS prior selects the variable ‘pce2’, i.e., PCE inflation for the first month in a given quarter, the HS-SAVS prior allocates nearly the same inclusion probability to all construction start variables (‘const0’-’const2’). Hence, it allows for greater model uncertainty which is what we would expect from correlated macro data as discussed in Cross et al. (2020); Giannone et al. (2017).

In contrast to Giannone et al. (2017), the posterior model size distributions of both the SAVS and SSVS algorithms are relatively sparse compared to the overall size of the data set.

Further, of the Google Trends, both priors select the root-term ‘real GDP growth’ as the most important one. Although, a-priori, it is not clear whether the root term refers to positive or negative news about GDP growth, the posterior sign as indicated by the color of the bar in figure (3) is clearly negative, which corroborates the graphical investigation from figure (9). This gives tentative evidence that this search term acts as a warning signal to downturns of GDP growth as alluded to in the introduction.

Lastly, we see from figure (5) that there is support in the data for local trend, but not a local linear trend model: the posterior for $\sigma^2$ is clearly bi-modal with less mass on zero than the prior, while the posterior for $\sigma_\alpha$ has substantially more mass on zero than the prior. The Bayes factors are 3.74 and 0.29 for the state standard deviations respectively.
4.2 Nowcast Evaluation

We now turn to out-of-sample nowcasting performance, where nowcasts are produced following the real time data publication calendar as explained in section 2. We first evaluate point- and then density fit. RT-RMSFE are plotted in figure (6) for the competing non-centred BSTS estimators, as well as the AR(2) benchmark and the original BSTS model. Notice that in all nowcast figures, we represent vintages in which Google Trends are published by grey vertical bars and plot the results for the original BSTS model on the right axis for readability. The following points emerge from figure (6): firstly, it is clear that all proposed BSTS models based on the non-centred state space offer large performance gains over the original BSTS model. Sec-
ondly, all models almost monotonically increase in precision as more data are released, where, as expected, the BSTS models eventually outperform the AR(2) benchmark. Thirdly, among non-centred BSTS models, the HS-BSTS does best, however, is closely followed by the HS-SAVS-BSTS. Hence, the HS-SAVS-BSTS is largely able to preserve fit, which is expected as the SAVS algorithm thresholds noise variables to zero that the horseshoe prior already shrinks to close to zero. Compared to the SSVS-BSTS, the horseshoe prior based BSTS models offer 15-20% improvements in terms of RT-RMSFE. Finally, Google Trends only provide modest improvements in point nowcast accuracy where only in the first vintage in which Google Trends are published, do the estimators show slight improvements in fit.

Figure 6: Real-Time RMSFE of all competing models. The RT-RMSFE for the BSTS are plotted on the right axis. Grey vertical bars indicate vintages in which GT are published.

Similar to the real-time point forecasts, we plot real-time LPDS (RT-LPDS) and CRPS (RT-CRPS) in figures (7) and (8). The RT-LPDS and RT-CRPS mostly confirm the findings found for the point nowcasts. As more information comes in, density nowcasts become more accurate, as can be seen from the increasing lines in (7) and decreasing lines in (8). Similar to the RT-RMSFE, the non-centred BSTS models offer large improvements over the original BSTS model of Scott and Varian (2014) and the horseshoe prior models provide the best fit, with however a slight advantage for the HS-BSTS model. Improvements of the HS based models over the SSVS-BSTS are of similar magnitude to the point nowcasts. In contrast to the point
nowcasts, all non-centred BSTS models quickly outperform the benchmark AR(2) model.

Figure 7: Real-Time log-predictive density scores (RT-LPDS) for all competing models. The RT-LPDS for the BSTS are plotted on the right axis. Grey vertical bars indicate vintages in which GT are published.

Figure 8: Real-Time CRPS scores (RT-CRPS) for all competing models. The RT-CRPS for the BSTS are plotted on the right axis. Grey vertical bars indicate vintages in which GT are published.
5 Simulation Study

The empirical application showed that the proposed BSTS models perform better in point as well as density forecasts compared to the original model of Scott and Varian (2014) and that both the SAVS augmented horseshoe prior as well as the SSVS-BSTS choose models which are relatively sparse compared to the dimensionality of the regressor set. This finding is in contrast to previous studies using macroeconomic data such as Giannone et al. (2017) and Cross et al. (2020) who find that priors able to accommodate dense models generally outperform sparsity favouring priors. As the innovation in this paper compared to previous work is the estimation of a latent local-linear trend which might filter out the co-movement with the macro data, we compare the ability of the proposed priors to the original BSTS model by Scott and Varian (2014) in capturing both sparse and dense environments. Further, to make the simulations closer to our empirical application, we additionally test the priors’ ability to detect insignificant state variances. Specifically, we simulate local-linear-trend models as (14) which either have the trend variance, the local trend variance, both, or none equal to zero.

We generate 20 fictitious samples for \((\sigma^\tau, \sigma^\alpha) = \{(0.5, 0), (0, 0.5), (0, 0), (.5, 0.5)\}\) for a dense and sparse DGP, where the sparse coefficient vector is defined as:

\[
\beta_{\text{sparse}} = (1, 1/2, 1/3, 1/4, 1/5, 0_{K-6})'
\]  

and the dense coefficient vector is defined as:

\[
\beta_{\text{dense}} = \begin{cases} 
1/3 & \text{with probability } p_d \\
0 & \text{with probability } 1 - p_d 
\end{cases}
\]  

where \(p_d\) is set to 2/3. For both coefficient vectors, the dimensionality, \(K\), is set to 300 which is high dimensional compared to the number of observations \(T = 150\). We account explicitly for mixed frequencies by first generating the covariate matrix according to a multivariate normal distribution with mean 0 and a covariance matrix with its \((i,j)\)th element defined as \(0.5|\text{i-j}|\) and then skip-sample each covariate individually after the U-MIDAS methodology as in (2). Since in the simulations, we know the true regression coefficient values as well as state variances, we compare the performance of the different priors via coefficient bias for the regression coefficients and Dickey Savage Density ratios evaluated at zero state variances. Bias is calculated as

\[
\text{Root Mean Coefficient Bias} = \sqrt{\frac{1}{20} ||\hat{\beta} - \beta||^2_2},
\]  

where \(\hat{\beta}\) refers to the mean of the posterior distribution. We estimate the original BSTS model with the expected model size, \(\pi_0\), equal to the true number of non-zero coefficients.
As can be seen from table (2), both the non-centred BSTS models as well as the original BSTS model of [Scott and Varian (2014)] do better in sparse than in dense DGPs which is similar to the finding of [Cross et al. (2020)]. The largest gains in estimation accuracy however of the proposed BSTS models over [Scott and Varian (2014)] can be found for dense DGPs where the proposed estimators offer gains in accuracy well in excess of 50%. In sparse designs, however, the latter slightly outperforms the former. This was to be expected given that the spike-and-slab prior uses a point mass prior on zero and that the true expected model size is used. It is encouraging that the differences in accuracy are very small.

Within the proposed estimators in dense designs, the HS prior BSTS versions are 30-40% more accurate compared to the SSVS-BSTS which is in line with the empirical application. Hence, these results offer the conclusion that continuous shrinkage priors are clearly preferred over spike-and-slab models in dense DGPs with a latent local-linear trend component.

The Dickey-Savage density ratio tests confirm that the non-centred state space models are able to correctly identify which of the state variances are significant and which are not, even in high dimensional regression settings. It is interesting to note that the Dickey-Savage tests are, however, sensitive to correctly pinning down the regression coefficient vector: in dense designs, where the SSVS prior does worse than the horseshoe prior, the DS tests in cases (0,0.5) and (0,0) show false support for significant $\sigma^2$.  

6 Conclusion

In this paper, we investigated the added benefit of a host of Google Trends search terms in nowcasts of U.S. real GDP growth through the lens of improved Bayesian structural time series (BSTS) models. We have extended the BSTS of [Scott and Varian (2014)] to a non-centred formulation which allows to shrink state variances to zero in order to avoid overfitting states and therefore let the data speak about the latent structure. We have further extended and compared priors used for the regression part which allow for agnosticity of the underlying model dimensions to accommodate both sparse and dense solutions, as well as the widely successful horseshoe prior of [Carvalho et al. (2010)].

We find that Google Trends improve point as well as density nowcasts in real time within

9Note that we do not report DS tests for the original BSTS model. This is due to the fact that the prior on the state variance has no mass on zero and therefore is not testable.
Table 2: Average Dickey-Savage Density ratio and bias results the simulations. Since the SAVS algorithm is performed on an iteration basis after inference, the posterior of $\sigma^\tau, \alpha$ remains unaffected, hence receives the same results as the HS-BSTS model.

![Table 2](image)

...
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**A Appendix**

**A.1 Posterioris**

In this section of the appendix, we provide the conditional posterior distribution for the regression parameters.
A.1.1 Horseshoe Prior

Starting from model 3 and assuming that the states and state variances have already been drawn in steps 1.-3. in 3.5 which is further described in (reference posterior of the states in appendix). We subtract off $\tau$ such that $y - \tau = y^\ast = X\beta + \epsilon$, $N(0, \sigma^2 I_T)$. Printing the prior here again for convenience:

$$
\beta_j | \lambda_j, \nu, \sigma \sim N(0, \lambda_j^2 \nu^2 \sigma^2), \ j \in 1, \cdots, K
$$

$$
\lambda_j \sim C_+(0, 1)
$$

$$
\nu \sim C_+(0, 1)
$$

$$
\sigma^2 \sim f
$$

(22)

Then, by standard calculations (see Bhattacharya et al. (2016)):

$$
\beta | y^\ast, \lambda, \nu, \sigma \sim N(A^{-1}X' y^\ast, \sigma^2 A^{-1})
$$

$$
A = (X'X + \Lambda^{-1}_*)
$$

$$
\Lambda_* = \nu^2 \text{diag}(\lambda_1^2, \ldots, \lambda_K^2)
$$

(23)

Instead of computing the large dimensional inverse $A^{-1}$, we rely on a data augmentation technique introduced by Bhattacharya et al. (2016). This reduces the computational complexity from $O(K^3)$ to $O(T^2 K)$. Suppose the posterior is normal $N_K(\mu, \Sigma)$ with

$$
\Sigma = (\phi' \phi + D^{-1})^{-1}, \ \mu = \Sigma \phi' \alpha,
$$

(24)

where $\alpha \in R^{T \times 1}$, $\phi \in R^{T \times K}$ and $D \in R^{K \times K}$ is symmetric positive definite. Bhattacharya et al. (2016) show that an exact sampling algorithm is given by:

**Algorithm 1** Fast Horseshoe Sampler

1: Sample independently $u \sim N(0, D)$ and $\delta \sim N(0, I_T)$
2: Set $\xi = \Phi u + \delta$
3: Solve $(\Phi D \Phi' + I_T)w = (\alpha - \xi)$
4: Set $\theta = u + D \Phi' w$

Notice that $\phi = X/\sigma$, $D = \sigma^2 \Lambda_*$ and $\alpha = y^\ast/\sigma$. 

30
A.1.2 SSVS Prior

Conditioning on the states as in A.1.1 we apply the prior:
\[
\beta_j | \gamma_j, \delta_j^2 \sim \gamma_j N(0, \delta_j^2) + (1 - \gamma_j) N(0, c \times \delta_j^2) \\
\delta_j^2 \sim \mathcal{G}^{-1}(a_1, a_2) \\
\gamma_j \sim \text{Bernoulli}(\pi_0) \\
\pi_0 \sim \mathcal{B}(b_1, b_2),
\]

where \( \mathcal{G}^{-1} \) and \( \mathcal{B} \) stand for the inverse gamma and beta distribution respectively. The conditional posteriors are standard and derived for example in George and McCulloch (1993) and Ishwaran et al. (2005). The difference to the prior of George and McCulloch (1993) lies in the additional prior for \( \delta_j^2 \) which is assumed to be inverse gamma. It can be shown that this implies a mixture of student-t distributions for \( \beta_j \) marginally (Konrath et al., 2008). We sample from the conditional posteriors in the following way:

**Algorithm 2 SSVS Sampler**

1: For \( j \in \{1, \cdots, K\} \), sample each \( \gamma_j | \beta_j, \delta_j^2, \pi_0, y \sim (1 - \pi) N(\beta_j | 0, c \times \delta_j^2) I_{\gamma_j=0} + \pi_0 N(\beta_j | \beta_j | 0, \delta_j^2) I_{\gamma_j=1} \)
2: Sample \( \pi_0 \sim \mathcal{B}(b_1 + n_1, b_2 + K - n_1) \), where \( n_1 = \sum_j I_{\gamma_j=1} \)
3: Sample \( \beta | \gamma, \delta^2, \sigma^2, y \sim N(A^{-1} X'y^*/\sigma^2, A) \), where \( A^{-1} = X'X/\sigma^2 + D^{-1}, D = \text{diag}(\delta_j \gamma_j) \)
4: Sample \( \sigma^2 \sim \mathcal{G}^{-1}(\bar{c}, \bar{C}) \), where \( \bar{c} = c + \frac{T}{2}, \bar{C} = C + \frac{1}{2}((y^* - X\beta)'(y^* - X\beta)) \) and \( p(\sigma^2) \sim \mathcal{G}(c, C) \)

A.2 State Space Estimation and Forecasting

A.2.1 Estimation

Assume analogously to A.1.1 and A.1.2 that all regression parameters have been sampled such that conditionally on \( \beta \), we estimate \( y - X\beta = \hat{y}_t = \tau_0 + \sigma_\tau \tilde{\tau}_t + t\alpha_0 + \sigma_\alpha \sum_{s=1}^{t} \tilde{\alpha}_s + \epsilon_t \sim N(0, \sigma_y) \) and \( \tilde{\tau}_t = \tau_{t-1} + u_t^\tau, u_t^\tau \sim N(0, 1), \tilde{\alpha}_t = \alpha_{t-1} + u_t^\alpha, u_t^\alpha \sim N(0, 1) \). Since the state processes \( \{\tilde{\tau}, \tilde{\alpha}\}_{t=1}^T \) are independent of the other parameters in the non-centred formulation, we proceed by first estimating the states and then \( \theta = \{\tilde{\tau}_0, \tilde{\alpha}, \sigma_\tau, \sigma_\alpha\} \).

States \( \{\tilde{\tau}, \tilde{\alpha}\}_{t=1}^T \) can be sampled by any state space algorithm, e.g. Durbin and Koopman.
We instead opt for the precision sampler by Chan (2017) which exploits the joint distribution of the states which paired with sparse matrix operations yields significant increases in statistical as well as computational efficiency (Grant and Chan 2017). Since $\tilde{\alpha}_s$ enters in the observation equation as a sum, we define $\tilde{A}_t = \sum_{s=1}^{t} \tilde{\alpha}_s$. Notice that equation (4) implies that $\mathbf{H} \tilde{\alpha} = \tilde{\mathbf{u}}^\alpha$, where $\mathbf{H}$ is the first difference matrix\footnote{[Put in here the first difference matrix as defines on p.81, Chan (2017)]}. Notice that $\tilde{A}_t \sim N(\mathbf{0}, \mathbf{I}_T)$. Notice that $\tilde{A}_t = \tilde{\alpha}_1$ which implies that $\tilde{A}_t - \tilde{A}_{t-1} = \tilde{\alpha}_t$. Hence, this gives us back the desired $\mathbf{H} \tilde{\alpha} = \tilde{\mathbf{u}}^\alpha$. Solving $\tilde{\alpha} = \mathbf{H}^{-1} \tilde{\mathbf{u}}^\alpha = \mathbf{H}^{-2} \tilde{\mathbf{u}}^\alpha$. Therefore $\tilde{\alpha} \sim N(\mathbf{0}, (\mathbf{H}^2)' \mathbf{H}^2)^{-1})$ (26)

To sample the states jointly, define $\xi = (\tilde{\tau}', \tilde{A}')'$. Then $\hat{y}$ can be re-written as:

$$\hat{y} = \tau_0 \mathbf{1}_T + \alpha_0 \mathbf{1}_{1:T} + X_\xi \xi + \epsilon,$$

where $\mathbf{1}_{1:T}$ is defined as $(1, 2, \cdots, T)'$ and $X_\xi = (\sigma_\tau \mathbf{I}_T, \sigma_\alpha \mathbf{I}_T)$. Since $X_\xi$ is a sparse matrix, manipulations in programs which utilise sparse matrix operations will be very fast.

Similar calculations result in the implicit prior $\tilde{\tau} \sim N(\mathbf{0}, (\mathbf{H}' \mathbf{H})^{-1})$. Now, since by assumption $\tau$ and $\tilde{A}$ are independent, the combined for $\xi$ is:

$$\xi \sim N(\mathbf{0}, \mathbf{P}_\xi^{-1})$$

where $\mathbf{P}_\xi = \text{diag}(\mathbf{H}' \mathbf{H}, \mathbf{H}'^2 \mathbf{H}^2)$. The posterior is thus standard:

$$p(\xi|\mathbf{y}, \sigma_y^2) \sim N(\mathbf{K}_\xi, \mathbf{A}_\xi^{-1})$$

where $K_\xi = \mathbf{P}_\xi + \frac{1}{\sigma_y^2} X_{\xi}' X_\xi$ and $\mathbf{K}_\xi^{-1}(\mathbf{P}_\xi + \frac{1}{\sigma_y^2} X_{\xi}' (\mathbf{y} - \tau_0 \mathbf{1}_T - \alpha_0 \mathbf{1}_{1:T}))$.

Conditionally on $\xi$, the starting values $\beta_0 = (\tau_0, \alpha_0)$ are drawn by simple linear regression results, where we specify a generic prior covariance as $V_\beta = \text{diag}(0.1, 0.1)$.

### A.2.2 Forecasting

Taking equation (1) as our starting point, it is well known that the predictive density $p(y_t|y^{t-1}, \beta, \theta, \sigma_y^2)$, where $y^{t-1} = (y_1, \cdots, y_{t-1})$, can be generated by the Kalman filter. Since equation (14) is instead estimated by precision sampling, and hence, without Kalman recursions, the literature has proposed (1) conditionally optimal Kalman mixture approximations (Bitto and Frühwirth-Schnatter, 2019), (2) pure simulation based methods to approximate (1) (Belmonte et al., 2014),
and (3) what Bitto and Frühwirth-Schnatter (2019) call naive Gaussian mixture approximation (see A.1.2.2 of Bitto and Frühwirth-Schnatter (2019)). In simulations as well as the empirical example we found that results are very similar independent of the sampling technique. For computationally simplicity we present here method (2).

The predictive on-step-ahead distribution \(p(y_t \mid y_{t-1})\) can be generated by first drawing from the non-centred states which with the draws of the other model parameters yield draws from the predictive. More specifically, for posterior draw \(m = 1, \ldots, M\):

1. Draw \((\tilde{\tau}_t^{(m)}, \tilde{\alpha}_t^{(m)})\) from \(N(\tilde{\tau}_{t-1}^{(m)}, 1)\) and \(N(\tilde{\alpha}_{t-1}^{(m)}, 1)\) respectively
2. Generate \(\alpha_t = a_0^{(m)} + \sigma_\alpha^{(m)} \tilde{\alpha}_t^{(m)}\) and \(\tau_t = \tau_0^{(m)} + \sigma_\tau^{(m)} \tilde{\tau}_t^{(m)} + t\alpha_0^{(m)} + \sigma_\alpha^{(m)} \sum_{s=1}^{t} \alpha_s^{(m)}\)
3. Generate \(x_t^t \beta^{(m)} + \tau^{(m)} + \sigma_y^{(m)} u\), where \(u \sim N(0, 1)\)

To obtain an approximation to the continuous approximation to \(p(y_t \mid y_{t-1})\), one can then use a kernel density smoother such as ”kdensity” in Matlab.

### B Graphs

#### B.1 In-Sample Results

![Figure 9: Posterior inclusion probabilities for the original BSTS model of Scott and Varian (2014).](image-url)
| Google Trend               | Mean | Min  | Max  | Std  |
|---------------------------|------|------|------|------|
| bankdefault               | 0.03 | -21.50 | 38.61 | 8.94 |
| bankruptcy                | 0.03 | -15.23 | 27.60 | 5.10 |
| default                   | 0.07 | -36.96 | 40.32 | 12.44|
| derivatives               | -0.01| -13.64 | 22.48 | 4.55 |
| dow jones                 | -0.16| -17.03 | 45.58 | 7.13 |
| economic condition        | -0.01| -39.17 | 46.98 | 11.82|
| fed loans                 | 0.02 | -13.63 | 23.25 | 7.52 |
| fed                       | 0.03 | -21.50 | 38.61 | 8.94 |
| federal reserve system    | 0.01 | -8.13  | 19.99 | 4.12 |
| financial crisis          | 0.03 | -20.66 | 33.55 | 6.32 |
| foreclosure               | 0.00 | -20.69 | 60.62 | 6.73 |
| real gdp growth           | 6.87 | -20.69 | 60.62 | 7.89 |
| house prices              | 0.01 | -11.82 | 44.03 | 5.29 |
| industrial production     | 0.02 | -8.04  | 15.60 | 3.41 |
| insolvency                | 0.15 | -6.5   | 22.78 | 6.34 |
| jobless                   | 0.29 | -7.9   | 13.01 | 5.34 |
| jobless claims            | 0.59 | -8.99  | 5.97  | 2.74 |
| jobs                      | 0.81 | -4.99  | 14.46 | 6.91 |
| loans                     | 0.01 | -14.59 | 20.97 | 5.92 |
| real gdp growth           | -0.03| -17.31 | 31.40 | 8.22 |
| stockmarket               | -0.05| -15.66 | 8.41  | 7.69 |
| us economy                | -0.09| -36.90 | 26.58 | 8.74 |
| us default                | -38.39| 0.00   | 19.17 | 5.22 |
| recession                 | 0.00 | -12.29 | 24.01 | 3.10 |
| stock market              | 0.01 | -6.48  | 27.50 | 6.14 |
| unemployment              | 0.01 | -34.60 | 29.98 | 7.38 |
| US default                | -0.01| -20.09 | 35.51 | 8.02 |
| US growth                 | -0.06| -23.86 | 61.20 | 7.90 |

Table 3: Google Trends Summary Statistics - GDP