Optimal non-classical correlations of light with a levitated nano-sphere

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Nonclassical correlations provide a resource for many applications in quantum technology as well as providing strong evidence that a system is indeed operating in the quantum regime. Optomechanical systems can be arranged to generate quantum entanglement between the mechanics and a mode of travelling light. Here we propose automated optimisation of the production of quantum correlations in such a system, beyond what can be achieved through analytical methods, by applying Bayesian optimisation to the control parameters. Two-mode optomechanical squeezing experiment is simulated using a detailed theoretical model of the system, while the Bayesian optimisation process modifies the controllable parameters in order to maximise the non-classical two-mode squeezing and its detection, independently of the inner workings of the model. The Bayesian optimisation treats the simulations or the experiments as a black box. This we refer to as theory-blind optimisation, and the optimisation process is designed to be unaware of whether it is working with a simulation or the actual experimental setup. We find that in the experimentally relevant thermal regimes, the ability to vary and optimise a broad array of control parameters provides access to large values of two-mode squeezing that would otherwise be difficult or intractable to discover. In particular we observe that modulation of the driving frequency around the resonant sideband, when added to the set of control parameters, produces strong nonclassical correlations greater on average than the maximum achieved by optimising over the remaining parameters. We also find that using our optimisation approach raises the upper limit to the thermal regime in which squeezing can be achieved. This extends the range of experimental setups in which non-classical correlations could be generated beyond the region of high quantum cooperativity.

I. INTRODUCTION

Nonclassical correlations are necessary to enhance the performance of a variety of quantum technological tasks, including sensing [1], communications and cryptography [2], quantum computing [3–5], quantum thermodynamics [6–8], as well as having significance for foundational questions in quantum physics [9]. Such correlations have been observed in a variety of physical platforms including optical photons [10–13], cold atoms [14–16], trapped ions [17–19], superconducting circuits [20, 21], nitrogen-vacancy centres [22] and, the platform we address here, optomechanics.

In optomechanical setups, nonclassicality has been observed through the production of squeezed states of mechanical motion in electromechanical systems [23, 24], entanglement between distant mechanical systems coupled by light [25] or microwaves [26] and entanglement between the mechanical mode and the microwave mode that leaks from a cavity [27]. This is accomplished by engineering a particular interaction between microwaves and mechanics through an external classical driving [28]. This in turn means that a certain set of experimental conditions must be satisfied in order for the nonclassical correlations to be generated, particularly against the deleterious effects of environmental noise. The determination of these parameters under the constraints of an experimental setting is a complicated optimisation problem even under a small number of tunable variables.

One theoretical method to determine the required parameter values is to develop and analyse a mathematical model of the physical system. Typically, in order to make such a model tractable, many simplifying assumptions must be made. Further, analytical solutions to the problem are often unavailable and the optimisation must proceed numerically. While a broadly applied technique, numerical simulation suffers from a structural weakness, in that the optimisation is guided by the accuracy of the mathematical model rather than the experimental data. Here we invert this viewpoint and propose to use Bayesian methods in optimising the production of non-classical correlations from an optomechanical system. In our analysis, the optimisation variables are the control parameters that drive the actual experiment and the figure of merit is taken directly from measurements of the system.

Fig. 1 outlines the process. The optimisation proceeds without any preconceived description of the behavioural response of the optomechanical setup to any changes in the control parameters \( \mathbf{x} \). This is often referred to as treating the setup as a ‘black box’, which in this case produces two-mode squeezing (idealised as \( F(\mathbf{x}) \), the output of the black box) in response to a set of control parame-

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under the name ‘learning control’ [30]. Further development and usage has continued, primarily focusing on controlling chemical reactions [31–36]. There are many implementations and ideas using a variety of optimisation algorithms in the quantum information processing (QIP) field. An early example used a hybrid approach, combining classical and theory-blind optimisation, to improve gate fidelities [37]. This approach was adopted to improve gate fidelities and reduce drift errors in single- and two-qubit gates [38, 39]. Further applications of theory-blind optimisation in QIP can be seen in Refs. [40, 41] for example. The theory-blind protocol is advantageously applicable in levitated optomechanics experiments, where a small number of precisely controlled parameters characterise the setup. This can be used to maximise nonclassical correlations and is immediately possible in an experimental setup such as that used in Ref. [42].

The possibility to achieve two-mode optomechanical squeezing in a specific levitated optomechanical experiment was shown in Ref. [43], and further details of the optomechanical theory used in this manuscript can be found there and in the references therein. Here we take the results of [43] as an initial benchmark and demonstrate that BO is capable of discovering parameter sets that generate significantly stronger two-mode optomechanical squeezing. This is achieved by efficient exploration of the parameter space, particularly in the regimes where analytical description of the optomechanical system is difficult or intractable, specifically, beyond the rotating wave approximation of the resonant-sideband driving, and outside of the resolved sideband. Despite the fact that the generation of nonclassical correlations in optomechanics via a two-mode squeezing interaction is well-investigated theoretically [28, 44–49], and has been demonstrated in a number of cryogenic setups [25, 27], it remains a challenging task for new optomechanical platforms such as levitated nanoparticles. Additionally, although some theoretical study has been made into driving correlations through frequencies off the blue sideband [47], only specific fixed frequencies were analysed. Other recent work considers driving off the blue sideband [50], but using very short pulses (less than the period of mechanical motion) and in the regime where the optical decay is much greater than frequency of the mechanical oscillator.

Experiments are expensive and time-consuming so in order to show the feasibility of this idea in the specific context of nonclassical states of nano-oscillators, we simulate the protocol. Since the optimisation algorithm does not know that the experiment is simulated rather than real, this functions as a test of the efficacy of theory-blind Bayesian methods to optimise the production of a figure of merit. Mathematical models of optomechanics are particularly robust and well-tested in the linearised regimes that our analysis and simulations focus on herein [51, 52], thus the process also provides predictions on how to maximise the generation of nonclassical correlations in an optomechanical setting.
The remainder of this manuscript is organized as follows. Section II describes the model used in the simulation of the optomechanical system for calculation of the figure of merit based on the environmental and controllable input parameters. Section III provides an overview of Bayesian optimisation and explains its suitability for this application. Section IV gives results of the simulated theory-blind optimisation procedure, demonstrating how allowing Bayesian optimisation increasing degrees of freedom enables it to discover parameter sets that improve upon the two-mode squeezing levels. The results and their implications are discussed in Section V.

II. THEORY

In this manuscript we aim at maximising nonclassical optomechanical correlations. This section contains a formal description of the optomechanical system formed by a nanoparticle levitated inside a cavity, and a pulse of travelling light. We provide the Hamiltonian of the optomechanical interaction inside the cavity and obtain the differential equations for the quadratures of the mechanical motion and the intracavity light. With the help of input-output relations we derive a Lyapunov equation for the matrix of covariances between the mechanical motion and the light pulse, and describe how to quantify the nonclassical correlations between them knowing the covariance matrix. This section provides the theory necessary to reproduce our results, more details can be found in Refs. [53, 54].

A. Gaussian Hamiltonian dynamics of opto-mechanical system

Our focus is on a levitated nanoparticle of mass \( \mu_p \) trapped in a tweezer beam within a optical cavity (see Fig. 1 (b)). In this setup, the potential for the mechanical motion of the particle is determined by the spatial intensity profile of the tweezer. The Gaussian profile can be well approximated near the origin by a quadratic potential \( V(x) = \frac{1}{2} \mu_p \Omega_m^2 x^2 \) of a harmonic oscillator parametrized by eigenfrequency \( \Omega_m \). The mechanical motion of the particle is coupled to a cavity mode that itself is a harmonic oscillator characterized by the frequency \( \omega_{\text{cav}} \).

The optomechanical coupling can be introduced in one of two ways depending on the positioning of the nanoparticle inside the cavity and the tweezer polarization. When the nanoparticle is placed in the antinode of the cavity optical mode, its displacement influences the eigenfrequency of the cavity, which induces the so-called dispersive optomechanical coupling [55]. The dispersive optomechanical coupling is inherently nonlinear in the field quadratures [51, 56]. It is, however, typically very weak so in an experiment it is routinely enhanced by a strong coherent driving in the presence of which the interaction is effectively linearized. Alternatively, when the nanoparticle is in the node of the cavity mode, given an appropriate polarization of the tweezer laser, the optomechanical coupling by coherent scattering of the tweezer photons off the nanoparticle into the cavity mode takes place. Such interaction is linear both in the field and mechanical quadratures [57]. It is this type of coupling that allowed ground-state cooling [42] of levitated nanoparticle. In both cases, the system can be described by the linearized Hamiltonian of the optomechanical interaction [51, 52]

\[
\frac{1}{\hbar} H = \frac{1}{4} \Delta(t)(X_m^2 + Y_m^2) + \frac{1}{4} \Omega_m (X_m^2 + Y_m^2) - g(t) X_c X_m,
\]

where \( X_c, Y_c (X_m, Y_m) \) are the canonical dimensionless quadratures of the cavity (mechanical) mode normalized such that \( [X_c, Y_c] = [X_m, Y_m] = 2i \), and \( \Delta(t) = \omega_{\text{cav}} - \omega_{\text{drive}}(t) \) is the time-dependent detuning of the coherent drive (or the tweezer frequency for the coherent-scattering coupling) from the cavity frequency. The coupling strength \( g(t) \) can be set by the power of the coherent drive (or by power and polarization of the tweezer). In an experiment, the detuning \( \Delta(t) \) and the drive power (and consequently, \( g(t) \)) can be controlled by a suitable modulation (e.g., electro-optical) of the laser light (symbol EOM in Fig. 1 where the case of the dispersive optomechanical coupling is pictured). As we show below, a careful optimisation of these parameters allows achieving stronger optomechanical squeezing compared with the primitive regime of constant-power resonant-sideband driving [43].

In this manuscript we are interested in pulsed driving in the vicinity of the upper mechanical sideband of the cavity at frequency \( \omega_{\text{drive}}(t) \approx \omega_{\text{cav}} + \Omega_m \). As is known [51, 53], driving on the upper mechanical sideband produces an optomechanical interaction which approaches the parametric amplification capable of producing nonclassical correlations by scattering the drive photons to the Stokes sideband. In order to run efficiently, this process requires that the scattering into the anti-Stokes sideband is suppressed, which occurs when the mechanical frequency exceeds the cavity linewidth: \( \Omega_m \gg \kappa \).

We assume a pulsed operation, i.e. \( g(t) \) to be nonzero for \( 0 \leq t \leq \tau \) and zero otherwise. The advantages of the pulsed manipulation stem from working at shorter timescales compared to the steady states of continuous driving. Since the pulsed operation does not require the system to reach a steady state, it can use coupling strengths that are prohibitively large for the continuous drive. Indeed, driving the optomechanical cavity on the upper mechanical sideband adds into the dynamics of the mechanical mode a negative damping proportional to the driving strength [58]. This negative damping easily overwhelms the intrinsic low damping of mechanics thus making its dynamics unstable. In addition, operating at faster timescales helps to decrease the impact of the noisy thermal environment.
The optomechanical system is open, with each of its modes coupled to its corresponding environment. Whereas the optical environment has low noise, the multi-mode mechanical one is at a high temperature. We take this into account in terms of Langevin-Heisenberg equations in the form

$$\dot{v} = \lambda v + \nu,$$

(2)

where \(v = (X_c, Y_c, X_m, Y_m)\) is a vector of unknowns and \(\nu = (\sqrt{2\kappa}X^{in}, \sqrt{2\kappa}Y^{in}, 0, \sqrt{2\gamma}Y^{th})\) is a vector of input noises. In this notation, \(\kappa\) is the cavity linewidth and \(\gamma\) is the mechanical damping rate. The drift matrix reads

$$\mathcal{A}(t) = \begin{pmatrix} -\kappa & \Delta(t) & 0 & 0 \\ -\Delta(t) & -\kappa & 2g(t) & 0 \\ 0 & 0 & 0 & -\Omega_m -\gamma \\ 2g(t) & 0 & -\Omega_m & -\gamma \end{pmatrix}.$$  

(3)

The components of the noise vector \(\nu\) satisfy the standard Markovian autocorrelations

$$\langle Q^{in}(t) \circ Q^{in}(t') \rangle = \sigma_c \delta(t - t'), \quad \text{for } Q = X, Y,$$

(4)

$$\langle \xi^{th}(t) \circ \xi^{th}(t') \rangle = \sigma_c (2n_{th} + 1) \delta(t - t').$$

(5)

Here \(a \circ b = \frac{1}{2} (ab + ba)\) is the Jordan product, \(\sigma_c = 1\) is the shot-noise variance, and \(n_{th}\) is the mean occupation of the thermal environment of the nanoparticle. The nanoparticle’s environment manifests itself in a number of ways including collisions with residual gas particles, trapping photon recoil, etc [42]. Therefore, a more experimentally relevant value is the heating rate \(\Gamma \equiv \gamma n_{th}\).

An important characteristic of the input fluctuations is the so-called diffusion matrix \(\mathcal{D}\), defined as

$$\langle \nu_i(t) \circ \nu_j(t') \rangle = \mathcal{D}_{ij} \delta(t - t'),$$

so in our case

$$\mathcal{D} = \text{diag}(2\kappa, 2\kappa, 0, 4\Gamma).$$

(6)

**B. Input-output formalism**

Since we are interested in control of the nonclassical correlations between mechanics and the leaking light, that can be directed to another quantum system or detector, we have to obtain an expression for the latter. We start doing so with the input-output relations for a high-\(Q\) cavity [60]

$$\begin{pmatrix} X^{out} \\ Y^{out} \end{pmatrix} = -\begin{pmatrix} X^{in} \\ Y^{in} \end{pmatrix} + \sqrt{2\kappa} \begin{pmatrix} X_c \\ Y_c \end{pmatrix},$$

(7)

Next, we define a mode of the leaking light that is detected at the output. This mode is characterized by its temporal profile \(f^{out}(t)\) and is described by quadratures

$$\mathcal{X}^{out}, \mathcal{Y}^{out} = \int_0^\tau ds \begin{pmatrix} X^{out}(s) \\ Y^{out}(s) \end{pmatrix} f^{out}(s).$$

(8)

Because the quadratures satisfy the commutation relation

$$[\mathcal{X}^{out}, \mathcal{Y}^{out}] = 2i \int_0^\tau ds \left( f^{out}(s) \right)^2,$$

(9)

the mode profile has to satisfy the normalization condition

$$\int_0^\tau ds \left( f^{out}(s) \right)^2 = 1$$

(10)

for \(\mathcal{X}^{out}, \mathcal{Y}^{out}\) to be canonical variables. In an experiment, the choice of different mode profiles \(f^{out}(t)\), that is detection of quadratures of the modes with different temporal profiles, can be implemented in the homodyne detection by either using a local oscillator with time-dependent amplitude or by frequently sampling the instantaneous value of quadrature with a constant-amplitude local oscillator and subsequently assembling an integral sum of the form Eq. (8) from samples [61].

The choice of a certain temporal detection profile \(f^{out}(t)\) is a particularly important task in the problem of detecting the quantum correlations [54]. A simple intuition can be used in the case when the drift matrix is time-independent. In this case, an analytical solution of the dynamics exists that allows expression of the instantaneous amplitudes of the leaking field \(X^{out}(t), Y^{out}(t)\) in terms of the initial values \(u(0)\) and the input fluctuations. Such an expression contains a term proportional to \(X_m(0)\) with the coefficient \(T_m(t)\). Setting the detection profile equal to this coefficient \(f^{out}(t) = T_m(t)\) gives the temporal mode of light that has maximal contribution of \(X_m(0)\). We refer to such a detection profile as the ‘optimal’ profile in Section IV A.

Having the definitions for the output mode, considering it a function of the upper integration limit, we can extend Eq. (2) to include the output mode

$$\dot{u} = \mathcal{B} u + \mu,$$

(11)

where \(u = (X_c, Y_c, X_m, Y_m, \mathcal{X}^{out}, \mathcal{Y}^{out})\) is the extended 6-vector of unknowns and \(\mu\) is the extended vector containing noise terms.

$$\mu = ([\nu]_{1 \times 4}, \sqrt{2\kappa} f^{out}(t)X^{in}, \sqrt{2\kappa} f^{out}(t)Y^{in}).$$

(12)

The new drift matrix reads

$$\mathcal{B}(t) = \begin{pmatrix} [\mathcal{A}(t)]_{4 \times 4} & 0_{2 \times 4} \\ \sqrt{2\kappa} f^{out}(t) & 0_{2 \times 2} \end{pmatrix}.$$ 

(13)
and for the $6 \times 6$ diffusion matrix we obtain
\begin{equation}
F(t) = \begin{pmatrix}
[D]_{4 \times 4} & -f_{\text{out}}(t)\sqrt{2\kappa \sigma_v}I_2 \\
-f_{\text{out}}(t)\sqrt{2\kappa \sigma_v}I_2 & 0_{2 \times 2}
f_{\text{out}}(t)^22\kappa \sigma_v I_2
\end{pmatrix}.
\end{equation}

Above we used notation $I_n$ for an $n$-dimensional identity matrix, and $0_{m \times n}$ for a matrix of zeros of corresponding dimensions.

The dynamics of the system are linear, therefore the initial multimode zero-mean Gaussian state and multimode zero-mean Gaussian state of the noises are mapped by Eqs. (2) and (11) onto another zero-mean Gaussian state. An important feature of such states is that they are fully described by their second moments that form a covariance matrix. The latter is defined as
\begin{equation}
U_{ij} = \langle u_i \circ u_j \rangle.
\end{equation}

The covariance matrix $U(t)$ evolution is governed by the matrix Lyapunov equation:
\begin{equation}
\dot{U} = B U + U B^T + F.
\end{equation}

To analyze the nonclassical optomechanical correlations of the modes of our interest, we derive the covariance matrix of a bipartite system formed by the nanopartitions of the modes of our interest, we derive the covariance matrix. The latter is defined as
\begin{equation}
\bar{V}_{ij} = U_{i+2,j+2} \text{ with } 1 \leq i, j \leq 4.
\end{equation}

**C. Optomechanical two-mode squeezing**

From the covariance matrix $V$ of the bipartite optomechanical system we obtain the two-mode squeezing from its minimal eigenvalue $\lambda_{\text{min}}$. A squeezed state is indicated by $\lambda_{\text{min}} < \sigma_v = 1$, with squeezing increasing as $\lambda_{\text{min}}$ decreases. The two-mode optomechanical squeezing is given by
\begin{equation}
S_{\text{gen}} = \max \left\{ 0, -10 \log_{10} \lambda_{\text{min}} \right\},
\end{equation}

which is clearly maximised for minimal $\lambda_{\text{min}}$.

Detection of the two-mode squeezing of a bipartite system does not require full state tomography. A simple method exists that allows this detection via only one homodyne measurement of each of the two modes. The method is based on the fact that in the eigenbasis where the covariance matrix is diagonal, the smallest eigenvalue of the covariance matrix is one of its elements. This means that by a two-mode passive linear transformation it is possible to obtain a generalised quadrature $X_{\text{gen}}$ whose variance equals the smallest eigenvalue of the original covariance matrix [62]. This method can exhibit squeezing even when conditional variances do not [63]. The most general of such transformations maps the initial quadratures onto a new set, of which we are interested in the one given by
\begin{equation}
X_{\text{gen}}[\theta_c, \theta_m, \phi] = X^{\text{c}}_c \cos \phi + X^{\text{m}}_m \sin \phi,
\end{equation}

with $X^{\text{c}}_c$ and $X^{\text{m}}_m$ being the quadratures of each subsystem in a rotated basis
\begin{equation}
X^{\theta}_c = X_c \cos \theta + Y_c \sin \theta.
\end{equation}

Equation (18) thus describes an output quadrature of a virtual beamsplitter having an amplitude transmittance $\cos \phi$ with the rotated quadratures of the original modes as the two input modes. The variance of $X_{\text{gen}}$ can be computed as
\begin{equation}
\text{Var} X_{\text{gen}} = \overline{\nabla}_{11} \cos^2 \phi + \overline{\nabla}_{33} \sin^2 \phi + \overline{\nabla}_{13} \sin 2\phi,
\end{equation}

where
\begin{equation}
\overline{\nabla} = \mathbb{R}(\theta_c, \theta_m) \mathbb{V} \mathbb{R}(-\theta_c, -\theta_m),
\end{equation}

and $\mathbb{R}$ is the rotation matrix:
\begin{equation}
\mathbb{R}(\theta_c, \theta_m) = R_2(\theta_c) \otimes R_2(\theta_m), \quad R_2(\theta) = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}.
\end{equation}

For an optimal set of angles $\phi^{(c)}, \phi^{(m)}, \phi^{(o)}$ the corresponding variance assumes the value of the minimal eigenvalue of $\mathbb{V}$:
\begin{equation}
\text{Var} X_{\text{gen}}[\phi^{(c)}, \phi^{(m)}, \phi^{(o)}] = \lambda_{\text{min}},
\end{equation}

In the lab, one can directly measure $X_{\theta}^\theta$ using homodyne detection. The phase $\theta$ is set by the local oscillator. The weighting factors $\sin \phi, \cos \phi$ can be optimized off-line. The problem of detecting the two-mode squeezing is then reduced from the full Gaussian tomography of the bipartite state to the direct homodyne detection of a pair of quadratures [43].

Note that we simplify the problem of evaluation of the two-mode squeezing by assuming that we have an access directly to the mechanical part of the covariance matrix. Though technically such a direct access is impossible, the mechanical quadratures can be effectively swapped to a subsequent pulse of leaking light by driving the optomechanical cavity on the lower mechanical sideband $\omega_{\text{drive}} = \omega_{\text{cav}} - \Omega_m$. The state swap procedure via a red-detuned drive is known to be equivalent to an almost noiseless beamsplitter-like transformation from mechanics to the light [64, 65]. The problem of the optimal pulse shape is not relevant to the task of state swap as it is for the generation of squeezing. Therefore, an extension of the problem to include the verification step would only be a technical addition to the problem and would not necessarily extend the scope of the manuscript. Thereby, we analyse here an optimized upper bound on directly de-
detectable squeezing from the experimental setup with the key time-variable parameters \( g(t) \), \( \Delta(t) \), \( f_{\text{out}}(t) \) and \( \tau \).

III. BAYESIAN OPTIMISATION

With the elements of theory developed in the previous section, both the task of creating and detecting squeezed states in optomechanical systems can be turned into an optimisation problem. As ultimately these optimisations should be performed directly onto an experimental apparatus, it is desirable that the optimisation routine should converge in a small number of steps, and exhibit robustness with regards to experimental noise. Since Bayesian optimisation (BO) has been successful with these requirements, with examples in quantum optimal control problems \([66–71]\), it is deemed appropriate for the tasks at hand.

A typical optimization problem involves maximising a figure of merit \( F(\mathbf{x}) \) with respect to control parameters \( \mathbf{x} \),

\[
\mathbf{x}^{\text{opt}} = \arg \max_{\mathbf{x}} F(\mathbf{x}).
\]

In general, \( \mathbf{x} \) can be an \( N \)-dimensional vector where \( N \) is the total number of parameters. In our case, it can describe the control parameters \( g(t) \) and \( \Delta(t) \) entering the Hamiltonian in Eq. (1), the pulse duration \( \tau \), the detection output \( f_{\text{out}}(t) \) profile appearing in Eq. (8) or the detection angles \( \theta \) in Eq. (18). On the other hand the figure of merit to be maximised can be the two-mode squeezing value in Eq. (17) or, equivalently, the minimum eigenvalue \( \lambda_{\text{min}} \) of the covariance of the optomechanical state described by the matrix \( \mathbf{V} \).

This search for optimal control parameters is performed iteratively. At each iteration of the algorithm the figure of merit is evaluated for a given set of parameters, from either numerical simulation or experimental data, this information is passed to the optimisation routine, which in turn suggests a new set of parameters to be tried.

In contrast to other optimisation routines, BO relies on the construction of an internal approximation (model) of the relationship between the control parameters and the figure of merit \( F \). This model then guides the optimisation process. The choice of the next set of control parameters is turned into a Bayesian decision problem, incorporating an incentive to explore the parameter space. These two steps, of updating the model based on the full set of evaluations collected and choosing the next set of parameters, form a single iteration of BO, and are briefly explained in the rest of this section. More thorough descriptions of BO can be found in Refs. \([72–75]\).

As based on a limited number of evaluations, which are potentially non-exact due to experimental noise as a source of variance, such as infidelity in applying the controls, it is convenient to adopt a probabilistic modelling approach, that is random functions \( f \) are used to model the unknown \( F \). The prior distribution \( p(f) \) over these random functions is chosen such that it favours well-defined and regular functions. In more technical terms it means that \( f \) is taken to be a Gaussian process \([76]\). A single evaluation of the figure of merit for control parameters \( \mathbf{x}_i \) is denoted \( y(\mathbf{x}_i) \) which is allowed to deviate from the true value \( F(\mathbf{x}_i) \) due to potential noise in the acquisition of the data. Then, given a record of \( M \) evaluations of the figure of merit denoted as a vector \( \mathbf{y}_M = [y(\mathbf{x}_1), \ldots, y(\mathbf{x}_M)] \), one aims at updating the prior distribution \( p(f) \) to take into account the data collected. This is done by means of Bayes’ rule

\[
p(f|\mathbf{y}_M) = \frac{p(f)p(\mathbf{y}_M|f)}{p(\mathbf{y}_M)} ,
\]

where the term \( p(\mathbf{y}_M|f) \) denotes the likelihood of obtaining the data-set \( \mathbf{y}_M \) for a given function \( f \) and \( p(\mathbf{y}_M) \) acts as a normalisation constant. When the noise in the data is assumed to be normally distributed, and with constant strength, this conditional distribution can be obtained in closed-form \([76]\).

Rather than a single point estimate \( f(\mathbf{x}) \), this modelling approach allows to obtain the full probability distribution \( p(f(\mathbf{x})|\mathbf{y}_M) \) which can be used to select the next set of parameters \( \mathbf{x}_{M+1} \) to be evaluated. One could choose \( \mathbf{x}_{M+1} \) for which the value of \( f(\mathbf{x}_{M+1}) \) is maximal in average. However, as the internal BO model is based only on a restricted amount of data it is likely that this average value may deviate significantly from the true value of \( F \) especially far away from the parameters already evaluated. Thus it is vital to also explore other promising regions of the parameter space. These considerations can be formulated in terms of an acquisition function, which grades a set of pseudo randomly generated potential parameters, and the choice of the next parameters is taken where this acquisition function is maximised \([72]\).

The Expected Improvement (EI) acquisition function is the type predominantly used to generate the results presented in Section IV, defined as

\[
\alpha_{\text{EI}}(\mathbf{x}) = \int_{y_{\text{max}}}^{\infty} dy (y - y_{\text{max}}) p(f(\mathbf{x}) = y|\mathbf{y}_M),
\]

where \( y_{\text{max}} \) is the best evaluation recorded so far. That is, the evaluation of \( f(\mathbf{x}) \) that returned the highest figure of merit value. It effectively quantifies the expected improvement compared with the best recorded output from previous iterations, specifically in our case that is the increase in two-mode squeezing level, and encourages exploration where the width of the distribution \( p(f(\mathbf{x})|\mathbf{y}_M) \) is large. For the interested reader, a more detailed explanation of how this EI acquisition function achieves this can be found in Ref. \([72]\). This exploration feature ensures that a global search is performed making BO less prone to be trapped in local minima.
IV. OPTIMISATION RESULTS

A. Constant coupling strength

Previous (analytical, without optimisation) work has investigated what can be achieved in terms of two-mode squeezing with a square driving pulse [43, 54]. The optimisation objective is to maximise the figure of merit, which is the two-mode generalised squeezing \( S_{\text{gen}} \), with the optimal values of coupling strength \( g \) and duration \( \tau \) to be determined. Here we consider only these two parameters, which describe a square-shaped pulse, for optimisation to allow comparison with the previous work, which requires using the model with the least number of parameters possible as optimisation variables. We use the analytically derived optimal profile for the measurement function \( f_{\text{opt}}(t) \) described in Section II B. \( S_{\text{gen}} \) is calculated through simulation of the model described in Section II. Technical details of the simulation are given in Appendix B, which also gives detailed descriptions of the optimisation methods. The simulation is broadly based on the experimental setup used in Ref. [42]. The system parameters are summarised in Table I.

The pulse parameters \( g \) and \( \tau \) are constrained with experimentally relevant upper bounds of \( 2\kappa \) and \( 100\kappa^{-1} \) respectively, where \( \kappa \) is the optical damping. Moreover, some combinations of pulse parameter values would lead to overheating, potentially damaging the system, and a further constraint \( \Theta_{\text{limit}} = 50 \) is applied using an adiabatic approximation of amplitude gain given by Eq. (A16).

The interactions with the environment are modelled as described in Section II. The bath temperature is characterised by the mean number of bath phonons \( n_k \). Combined with the mechanical damping \( \gamma = 2.8 \times 10^{-10}\kappa \) this gives rise to a reheating rate \( \Gamma \approx \gamma n_{\text{th}} \). The temperature of the oscillator is characterised by the mean mechanical occupation \( n_0 \). The model does not predict that cooling of the oscillator significantly increases the potential two-mode squeezing level \( S_{\text{gen}} \), except for extremely low temperatures (\( n_{\text{th}} \sim 1 \)). Therefore, for now, the oscillator is considered to be initially (prior to the driving pulse) in thermal equilibrium with the bath, that is \( n_{\text{th}} = n_0 \). Later, in Subsection IV D, we will see that cooling the oscillator is beneficial in terms of resilience to measurement imprecision.

The maximum achievable two-mode squeezing \( S_{\text{gen}} \) is dependent on the reheating [54], and the required pulse parameters to achieve this also vary with \( \Gamma \). This is illustrated in Fig. 2, which presents the squeezing level (in dB) level achieved by optimising \( g \) and \( \tau \) for specific bath temperatures over a wide range. The optimal values for \( g \) and \( \tau \) for 30 different values of \( \Gamma \) were found using Bayesian optimisation (BO). The relationship of \( S_{\text{gen}} \) to \( \Gamma \) is also plotted for three fixed \( g, \tau \) combinations, with the amplitude at maximum \( \Theta = \Theta_{\text{limit}} \) in each case. At lower temperatures the squeezing process is adiabatic and a long pulse is most effective, whereas at higher temperatures greater coupling is required to achieve maximal \( S_{\text{gen}} \).

Reheating rates where the squeezing process is adiabatically driven are not achievable in the lab [42, 77]. There is an upper bound to the reheating rate (\( \Gamma = 10.4\kappa \)), found through attempted parameter optimisation beyond which parameter values \( g, \tau \) for a square-shaped pulse cannot be determined that result in any squeezing \( S_{\text{gen}} > 0 \), indicating that it is not possible to drive two-mode squeezing using a square pulse with bath temperatures higher than this. It is clear from Fig. 2 that in the experimentally relevant heating range \( \Gamma \approx 10^{-1}\kappa \) the pulse parameter values \( g, \tau \) need to be chosen carefully in order to maximise two-mode squeezing for the particular thermal parameters, which are likely to fluctuate and may be difficult to estimate precisely. If the environmental parameters in an experiment attempting
to drive and measure two-mode squeezing differ from the ones used to determine the pulse parameters, maximum possible squeezing will not be achieved. Theory-blind optimisation will determine the correct pulse parameter values to reach maximal two-mode squeezing levels.

Despite these positive results, the square-shape is not necessarily optimal for the coupling pulse profile. The next section investigates the potential for improving on maximum squeezing by allowing for a temporally shaped pulse.

B. Time-dependent coupling

The signal generators used in state-of-the-art optomechanics experiments allow for effectively any continuous time-dependent function to be applied to temporally shape the laser pulse amplitude driving the coupling. That is, with pulse durations on the order of those used here, it is valid to consider an arbitrarily shaped coupling function \( g(t) \). Hence the search space for the optimisation can be expanded by adding variables that will allow for a time-dependent shaped coupling. The measurement function used for a square-shaped driving pulse, cannot be assumed optimal for an arbitrary shaped pulse, and so any optimisation of parameters for the driving pulse must be combined with optimisation of parameters for the measurement function \( f^{\text{out}}(t) \). In a physical experiment such as in Fig. 1 these functions \((g(t), f^{\text{out}}(t))\) are controlled by modulators (EOM).

With time-dependent coupling it is necessary to calculate the covariance matrix \( \Sigma \) by numerically solving Eq. (16). The technical details of how this is performed, including the pulse parameterisation scheme, are given in Appendix B1.

For the comparison of different optimisation variable combinations the reheating rate is fixed at the experimentally achievable value \( \Gamma = 0.063 \text{e} \) by setting specific values for the environmental parameters \( n_{th} = 2.26 \times 10^5, \gamma = 2.8 \times 10^{-10} \kappa \). Bayesian optimisation (BO) is again used to determine optimal parameters. In the time-dependent case the piecewise linear (PWL) parameters of these are optimised along with \( \tau \) and the piecewise linear (PWL) parameters of \( g(t) \) and \( f^{\text{out}}(t) \) are all optimisation variables, along with the total pulse duration \( \tau \), which is the same for both functions.

The optimisation results comparing the constant and time-dependent coupling are shown in Fig. 3. There is some variation in the value of \( S_{\text{gen}} \) found by BO, and so the results of 200 repeats of BO are shown in a histogram. The data from the different optimisation variable combinations are compared on the same axes. The results for the square and time-dependent coupling pulses are labelled ‘const. coupling’ and ‘t.d. coupling’ respectively. The rotating wave approximation (RWA) gives rise to some differences in \( S_{\text{gen}} \) for some square pulse parameters \( g, \tau \) when the mechanical frequency is in the order of the optical damping, and so the analytical solution to \( S_{\text{gen}} \) differs from the numerical solution.

FIG. 3. Optimised generalised squeezing at experimentally relevant bath temperature. Histograms showing the distribution of optimised generalised squeezing for different combinations of variables. We see in the first three result sets how average squeezing increases as extra dimensions are added to the search space, allowing the optimisation to discover regions where the non-classical correlations are greater. Each optimisation attempt produces a slightly different solution (for explanation see Appendix B3) and so 200 repetitions of Bayesian optimisation are performed for each combination of variables. The increasing number of search dimensions also causes greater variation in the squeezing value, widening the distribution. The ‘result count’ is the frequency of squeezing outcomes within a given interval. All values of \( S_{\text{gen}} \) are calculated from the covariance matrix \( \Sigma \) obtained by numerically solving the Lyapunov equation Eq. (16). Interaction with the surrounding environment is characterised by a reheating rate \( \Gamma = 0.063 \text{e} \) [42]. The colour-coded distributions are labelled in the legend. For ‘const. coupling’: the optimisation variables are coupling \( g \) and pulse duration \( \tau \) (see Section IV A). For ‘t.d. coupling’: the coupling \( g(t) \) and measurement function \( f^{\text{out}}(t) \) are time-dependent functions, and the piecewise linear (PWL) parameters of these are optimised along with \( \tau \) (see Section IV B). For ‘detuning’: \( \tau \) and the PWL parameters of \( g(t), f^{\text{out}}(t), \Delta(t) \) are all optimisation variables (see Section IV C). For ‘noisy coupling’: \( g(t), f^{\text{out}}(t) \) and \( \tau \) are optimised with white noise added to the \( g(t) \) parameters at each optimisation step (see Section IV E). For ‘\( f^{\text{out}} \) only’: \( g \) and \( \tau \) are fixed, and only the PWL parameters of the measurement function \( f^{\text{out}}(t) \) are optimised (see Section B3). Unless used as an optimisation variable, parameter values for the simulations are set as given in Table I. The table also gives the bounds for parameters when used as variables in the optimisation.

Solving method is used here for computing squeezing with the square pulse to ensure a fair comparison. In all repetitions the time-dependent pulses out-perform the square pulses. The minimum, mean and maximum squeezing are 6.20, 6.27, 6.28 dB for square and 6.73, 6.99, 7.14 dB for the PWL pulses, demonstrating that in simulation greater squeezing can be achieved with a temporarily
shaped coupling strength amplitude.

The distribution of results in terms of maximum achieved squeezing $S_{\text{gen}}$ is understood to be caused by local maxima (traps) in the optimisation variable landscape. Local maxima are observed in data for the 2-d constant coupling case when solving the dynamics numerically (without the RWA) that are not seen when the RWA is made. Further explanation of this is given in Section B3, which includes an illustration of how the RWA affects the squeezing computation and plots of the optimal pulses from this subsection.

C. Detuning frequencies

The results presented so far have been obtained for a fixed driving laser frequency detuning $\Delta = -\Omega_m$. Solving the dynamics numerically also allows for the detuning $\Delta$ to be offered as a variable for optimisation. Giving a single additional degree of freedom to Bayesian optimisation (BO), that is a fixed value for the detuning throughout the driving pulse, provides little improvement in the maximum achievable squeezing. However, allowing a time-dependent profile for the detuning, by optimising the parameters of a piecewise linear function $\Delta (t)$, enables significantly greater two-mode squeezing.

The results for repeated optimisations maximising $S_{\text{gen}}$ including the detuning variables can be seen in Fig. 3. The results for all the optimisation variable combinations are compared on the same axes. The distribution of 200 repetitions of the optimisation of $g(t)$, $f^\text{out}(t)$, $\Delta (t)$ is labelled ‘detuning’. The duration $\tau$ of the pulse is also an optimisation variable. There is a greater spread in the results than when optimising just $g(t)$ and $f^\text{out}(t)$, but the mean final squeezing (7.44 dB) is greater than the maximum (7.14 dB) achieved without optimising the detuning, and the maximum achieved with optimised detuning is 8.23 dB.

For the high temperature bath ($\Gamma = 10.4\kappa$), which, as seen in Subsection IV A, is the upper limit for driving two-mode squeezing with a square-shaped pulse, BO finds time-dependent functions that produce squeezing $S_{\text{gen}}$ of up to 6 dB with this greater degree of freedom. That is, through shaping the temporal profiles of driving pulse parameters $g(t)$ and $\Delta(t)$, and the corresponding measurement function $f^\text{out}(t)$, it is possible to achieve significant squeezing at this high-temperature, at which it is impossible to achieve squeezing with a square pulse. We emphasise that both types of pulse are bound by 5 times the standard error in the coupling $\kappa$.

Fig. 4 shows the time-dependent profiles of the optimal $g(t)$, $f^\text{out}(t)$ and $\Delta (t)$. The plots give a representation of the average pulse (see figure caption for definition). There is actually a wide variety of pulse profiles that produce high squeezing, which is explained in Section B3. The average pulses give an indication of common features. For the driving pulse we see a stronger coupling initially, with a peak before the middle, then tailing off towards the end. For the measurement function we see this starts around zero, increasing up to a peak after the middle, and then tailing off partially. The detuning functions have much greater variation, but typically start towards the middle, and finishing around $\Omega_m$.

The particular result for the detuning temporal profile is interesting, as most analytical studies have assumed fixed detuning at the blue sideband for driving two-mode squeezing. Possibly, the time-dependent detuning helps counteracting the noise, and hence leads to greater squeezing. Some evidence for this is observed when setting the mechanical oscillator frequency much greater than the optical damping ($\Omega_m = 50\kappa$). The optimised time-dependent profile for the detuning $\Delta (t)$ is then at the blue sideband when the coupling is at its strongest.

D. Detection angles

In an experimental setting, estimating $V$ would require full state tomography. There is a potentially more effi-
cient method of measuring $S_{\text{gen}}$, described in Section II C, based on the equivalence of $\text{Var} X_{\text{gen}} [\theta_c, \theta_m, \phi]$ to $\lambda_{\text{min}}$. This method requires three parameters to be determined: $\theta_c, \theta_m, \phi$. The latter is a weighting that can be determined in post processing. The homodyne measurement angles $\theta_c, \theta_m$ are experimental settings that could be determined through theory-blind optimisation. To test this idea in simulation, in which the full covariance matrix $\mathcal{V}$ is computed by numerically solving Eq. (16), $\text{Var} X_{\text{gen}}$ is calculated as per Eq. (20). The optimisation algorithm is given only $\theta_c, \theta_m$ as variables, and the figure of merit is calculated from a fixed $\mathcal{V}$. In an experiment this is equivalent to running with identical driving parameters.

For covariance matrices computed numerically, without the rotating wave approximation, with the experimentally achievable reheating rate $\Gamma = 0.063 \kappa$ and the mechanical oscillator initially in thermal equilibrium with the bath, optimisation algorithms are unable to navigate the parameter landscape to reliably find the $\text{Var} X_{\text{gen}}$ equivalent to $\lambda_{\text{min}}$, as the $\text{Var} X_{\text{gen}}$ minima is located in a narrow trough. This is explained further in Appendix B4. This would be further compounded in an experimental setting, as the $\text{Var} X_{\text{gen}}$ measurements would be intrinsically noisy.

When the initial temperature of the mechanical oscillator $n_0$ is much lower than the environment temperature $n_{\text{th}}$, for $\mathcal{V}$ computed through simulations, the upper bound of $\text{Var} X_{\text{gen}}$ is much lower, which also reduces the sharp changes in gradient. The optimisation algorithm is able to reliably find $\text{Var} X_{\text{gen}}$ equivalent to $\lambda_{\text{min}}$, so long as the oscillator is sufficiently cooled, for example phonon number $n_0 = 100$. Much lower $n_0$ values than this are now achievable in experiment [42]. Therefore this method for estimating the two-mode squeezing by determining homodyne measurement angles through theory-blind optimisation remains a viable option. Note that this only helps the measurement process, and that the squeezing is only significantly increased for much lower oscillator temperatures of $n_{\text{th}} \sim 1$.

### E. Control noise

In an experimental setting the precision to which controls can be applied will be limited. Also, how the system will respond to the controls may not be fully predictable. In this specific example of controlling the coupling of the oscillator to the light field by modulating the amplitude of the laser, it is likely that the actual coupling may have some random variation in its response. This is referred to as control noise. The optimisation algorithm is guided by the outcome of trying specific sets of parameters. Variation in the outcome will lead to reduced performance of the algorithm. One method to overcome this would be to repeat the experiment with the same parameters multiple times and take the mean outcome, however this would greatly increase the total number of times the experiment would need to be run. Bayesian optimisation (BO) can refine its model of the control landscape, taking into account that the figure of merit function value for some set of parameters may not be exact.

To replicate this scenario that one would encounter in the lab and to verify the stability of optimisation the control noise is modelled by adding some pseudo-random Gaussian distributed value to the pulse parameters. Further details are given in Appendix B5. The results for BO maximising $S_{\text{gen}}$ by optimising $g(t)$ and $f^\text{out}(t)$ when noise (standard deviation 10% of amplitude) is added to the piecewise linear parameters of $g(t)$ are shown in Fig. 3. These results are compared with others on the same axes, the distribution of squeezing achieved with 200 repeats of BO with noisy coupling parameters is labelled ‘noisy coupling’. These can be compared with the distribution when $g(t)$ and $f^\text{out}(t)$ are optimised without control noise (labelled ‘t.d. coupling’), as the detuning is fixed at the blue sideband in both cases. Although there is greater spread in the distribution, with minimum, mean, maximum $S_{\text{gen}}$ being 6.06, 6.70, 7.06 dB for the noisy controls, compared with 6.73, 6.99, 7.14 dB in the noiseless case, BO still performs very well.

### V. SUMMARY AND OUTLOOK

We have optimised the production of nonclassical optomechanical correlations, as measured by the two-mode squeezing $S_{\text{gen}}$. This is accomplished by adding a layer of Bayesian optimisation to the control variables, such as coupling rate $g(t)$, pulsed interaction duration $\tau$, drive detuning $\Delta(t)$ and the detection profile $f^\text{out}(t)$, so that repeated simulations of the optomechanics experiment are directed towards increased nonclassical correlations. Such control variables must be optimised against the uncontrolled parameters such as the heating rate $\Gamma$ that negatively affect the production of nonclassical correlations.

The optimisation shows that this can be accomplished in a way not easily replicable by analytical studies of the mathematical model.

For example, in the case of a pulse of constant interaction strength $g(t) = g$ the optimisation distinguishes between various reheating regimes and pulse lengths $\tau$ in order to maximise the optomechanical squeezing $S_{\text{gen}}$, thus granting access to higher two-mode squeezing in experimentally relevant regions of the reheating variable. Adding more variables to the optimisation procedure, including time-dependent couplings $g(t)$ and measurement functions $f^\text{out}(t)$, only adds to the power of the optimisation procedure. While requiring more resources to optimise, plainly having a greater parameter landscape to explore provides more opportunity to increase the optomechanical squeezing. We have assumed a certain fixed complexity of this time-dependence in the form of piecewise linear functions, however it seems reasonable to conjecture that increasing the detail of such functions, and therefore the number of control variables, will produce more finely tuned optimisations with greater nonclassi-
Curiously, the optimal coupling profile \( g(t) \), measurement function \( f^\text{out}(t) \) and detuning profile \( \Delta(t) \) must be used as a set, in the sense that taking an average of the optimised functions does not produce high levels of squeezing (when compared with the maximum achieved). We find that detuning away from the blue-sideband, in combination with the other optimised variables, produces noticeably greater squeezing than otherwise predicted [43]. The blue detuned drive produces nonclassical correlations perfectly in a unitary system, and we deviate from this by including noise effects from the thermal environment. Allowing the control variables to vary around this unitary ideal gives the optimisation an opportunity to locate the deviated maximum squeezing. For an estimate, while the non-optimised case predicts generation of approximately 6 dB of the optomechanical squeezing for the heating rate \( \Gamma = 0.06\kappa \), after the optimisation over all experimentally controllable parameters \( (\tau, g(t), f^\text{out}(t), \Delta(t)) \) the squeezing can reach values over 8 dB.

The optimisation not only increases the magnitude of the optomechanical squeezing but, compared to the non-optimised case [43], allows to achieve squeezing in the case of significantly higher temperatures of the environment. We have not specifically addressed the problem of finding the maximal heating rate \( \Gamma \) allowing generation of the optomechanical squeezing, because the solution of such a problem is a set of optimal parameters that yields squeezing approaching zero at a high temperature. On the contrary, we have focused on the maximisation of squeezing at a fixed, experimentally reasonable, value of the temperature of the environment corresponding to a certain value of \( \Gamma \). For instance, it happens to be impossible to achieve two-mode squeezing by driving the system with a top-hat pulse if the temperature of the mechanical environment exceeds the value corresponding to the heating rate \( \Gamma = 10.4\kappa \). The pulse with all the parameters \( (\tau, g(t), f^\text{out}(t), \Delta(t)) \) optimised can exhibit squeezing as high as 6 dB at this value of the heating rate. Note that from the point of view of the quantum cooperativity [51], which compares the optomechanical coupling rate with the decoherence rates of the system \( C_g = g^2/(\kappa \Gamma) \lesssim 0.4 \), at this high temperature, our system is clearly outside the regime of high cooperativity.

While we have emphasised experimental values from a certain levitated setup [42] in the text, the present simulations can be applied to other levitated experiments [78–81]. Moreover, any optomechanical [82–84] or electromechanical [85, 86] system capable of functioning in the pulsed regime of linearized optomechanical interaction can be analysed using exactly the same tools. The latter also allow consideration of multiple mechanical oscillators for investigation of mechanical-mechanical correlations generated by the pulsed interaction. Furthermore, the problem can be extended to include two- and three-dimensional motion of the levitated nanoparticles, which allows consideration of non-classical correlations between the motion of one or multiple nanoparticles in orthogonal directions [87, 88].

The theory-blind process can be interfacised directly with an experimental setup, in which one can imagine an autonomous process that repeatedly inputs new control variables in response to a Bayesian update from the optimisation, producing large values of squeezing. Bayesian optimisation improved upon the theoretically predicted optimal parameters for driving two-mode squeezing in the numerical model. The efficient method for measuring the squeezing by determining the optimal homodyne measurement angles through optimisation has also been demonstrated viable through simulation. The theory-blind optimisation process in this application has therefore been thoroughly tested in simulation and demonstrated to be valuable. The technological requirements for shaping the frequency and amplitude of the driving pulses, and the coordinated measurement of the output, all controlled by some automated process, are available in most labs experimenting with optomechanical setups. A parameter set found to be optimal in simulation is unlikely to be the optimal solution in the lab, due unexpected couplings, parameter drift, and other sources of noise. Theory-blind optimisation would find the true optimal solution for the specific set, in the specific moment, as this is based on response and measurements from the actual system. Further improvement upon the maximum two-mode squeezing level found in simulation could potentially be achieved through this full theory-blind optimisation, for instance taking advantage of attributes of the setup that were not modelled in the simulation, such as nonlinearities in the coupling [89, 90] and non-Markovianity [91] in the environmental interactions.

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Appendix A: Approximations in the theory of cavity optomechanics

1. Dynamics of slow opto-mechanical amplitudes

To investigate the interaction between the optical and mechanical modes, it is instructive to switch to the rotating frame defined by the free evolution of the modes (the first two terms in Eq. (1)). This is equivalent to a transition from the instantaneous values of the quadratures $\upsilon$ to their slowly varying envelopes $\upsilon_{\text{env}}$ following the rule

$$\upsilon = R \upsilon_{\text{env}}$$  \hfill (A1)

with $R = \mathbb{R}_2(\Delta t) \oplus \mathbb{R}_2(\Omega_{m} t)$, where $\mathbb{R}_2$ is the matrix of unitary rotation

$$\mathbb{R}_2(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}.$$  \hfill (A2)

Substituting Eq. (A1) into Eq. (2) yields the equations of motion for the envelopes $\upsilon_{\text{env}}$:

$$\dot{\upsilon}_{\text{env}} = \mathcal{A}_{\text{env}} \upsilon_{\text{env}} + \nu_{\text{env}},$$  \hfill (A3)

completely analogous to Eq. (2) with notations

$$\mathcal{A} = \mathbb{R}^{-1}(\mathcal{A} \mathbb{R} - \dot{\mathbb{R}}), \quad \nu = \mathbb{R}^{-1} \nu.$$

(A4)

In particular, the full expression for $\mathcal{A}_{\text{env}}$ reads

$$\mathcal{A}_{\text{env}} = \begin{pmatrix} -\kappa \cdot 12 & g \cdot \Theta(\Delta, \Omega_{m}) \\ g \cdot \Theta(\Omega_{m}, \Delta) & -\frac{1}{2} \cdot (12 + \Phi(\Omega_{m})) \end{pmatrix},$$  \hfill (A5)

with notation

$$\Theta(\Delta, \Omega_{m}) = \begin{pmatrix} -2 \cos(\Omega_{m} t) \sin(\Delta t), & 2 \sin(\Omega_{m} t) \sin(\Delta t) \\ 2 \cos(\Omega_{m} t) \cos(\Delta t), & 2 \cos(\Omega_{m} t) \sin(\Delta t) \end{pmatrix},$$

$$\Phi(\Omega_{m}) = \begin{pmatrix} -\cos(2\Omega_{m} t) \sin(2m_{t}) & \sin(2\Omega_{m} t) \cos(2m_{t}) \\ \sin(2\Omega_{m} t) \cos(2m_{t}), & \cos(2\Omega_{m} t) \sin(2m_{t}) \end{pmatrix}.$$  \hfill (A6)

(A7)

Also for the noises one can write

$$\nu_{\text{env}} = \begin{pmatrix} 2\kappa \cdot 12 & 0_{2 \times 2} \\ 0_{2 \times 2}, & 2\Gamma \cdot \psi \end{pmatrix}, \quad \psi = \begin{pmatrix} \cos 2\Omega_{m} t & \sin 2\Omega_{m} t \\ \sin 2\Omega_{m} t, & \cos 2\Omega_{m} t \end{pmatrix}.$$  \hfill (A8)

A Lyapunov equation with the matrices substituted according to the rule $\bullet \rightarrow \mathcal{A}_{\text{env}}$ can be written for the equations of motion Eq. (A3). It is important to note that this equation is exact and valid for an arbitrary detuning.

2. Rotating wave approximation

To simplify the further analysis, we assume that the system is operated in the resolved-sideband regime, where the mechanical frequency significantly exceeds the linewidth of the cavity, and the opto-mechanical coupling is weak: $\Omega_{m} \gg \kappa, g$, that the drive tone is tuned to the upper (blue) mechanical sideband of the cavity: $\Delta = -\Omega_{m} + \delta$, where $\delta \ll \Omega_{m}$. After substitution of the detuning we apply the rotating wave approximation (RWA) which amounts to ignoring all the rapidly oscillating terms in the equation of motion Eq. (A3). As a result, the equations are greatly simplified. In particular, we immediately see that the matrices $\Phi$ and $\Theta$ vanish as they are comprised of rapid terms only. In the interesting case of driving exactly on resonance ($\delta = 0$),

$$\Theta(-\Omega_{m}, \Omega_{m}) \overset{\text{RWA}}{=} \sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$  \hfill (A9)

so the drift and diffusion matrices take the simple time-independent form

$$\mathcal{A}_{\text{env}} = \begin{pmatrix} -\kappa \cdot 12 & g \cdot \sigma_1 \\ g \cdot \sigma_1 & -\frac{1}{2} \cdot 12 \end{pmatrix}, \quad \nu_{\text{env}} = 2 \text{diag}[\kappa, \kappa, \Gamma, \Gamma].$$

(A10)

These matrices can as well be used to compute a covariance matrix using a Lyapunov equation analogous to Eq. (16).

Importantly, since the coefficients in the equations of motion in RWA are time-independent, these equations can be solved analytically:

$$\upsilon_{\text{env}}(t) = e^{\mathcal{A}t} \upsilon_{\text{env}}(0) + \int_{0}^{t} dt' e^{\mathcal{A}(t-t')} \nu_{\text{env}}(t').$$  \hfill (A11)

One can then proceed substituting this solution into the definition of the covariance matrix to compute the latter.

In the particular case of RWA an analytical solution for the quadratures can be obtained by substituting solution Eq. (A11) into the input-output relations Eq. (7) and the definition of the pulse quadratures Eq. (8). Substitution of this solution into definition of the covariance matrix yields an analytical expression for $\mathcal{V}$. If, in addition to the condition of resolved sideband, the condition of weak coupling $g \ll \kappa$ and long pulses $\tau \gg \kappa^{-1}$ is satisfied, the intracavity optical mode can be adiabatically eliminated and the dynamics of the quadratures approaches the pure two-mode squeezing described by the transformations [28]

$$X_{m}(\tau) = \sqrt{\mathcal{V}} X_{m}(0) + \sqrt{\mathcal{V} - 1} X_{m}^{\text{in}},$$

$$Y_{m}(\tau) = \sqrt{\mathcal{V}} Y_{m}(0) - \sqrt{\mathcal{V} - 1} Y_{m}^{\text{in}},$$

$$X_{\text{out}} = \sqrt{\mathcal{V}} X_{m}(0) + \sqrt{\mathcal{V} - 1} X_{m}(0),$$

$$Y_{\text{out}} = \sqrt{\mathcal{V}} Y_{m}(0) - \sqrt{\mathcal{V} - 1} Y_{m}(0).$$  \hfill (A12) \hfill (A13) \hfill (A14) \hfill (A15)
Here we define the amplification gain $\mathcal{G}$ and the measurement weight function $f^\text{out}$ which in the adiabatic regime have simple expressions:

$$\mathcal{G} = \exp \left[ \frac{2g^2t}{\kappa} \right], \quad f^\text{out}(t) \propto e^{g^2t/\kappa}. \quad (A16)$$

The quantities $\mathcal{F}^\text{in}$ and $\mathcal{F}^\text{in}$ are the canonical quadratures of the input light defined in an analogy with Eq. (8) with the mode profile $f^\text{in} \propto e^{-g^2t/\kappa}$.

### Appendix B: Simulation and optimisation methods

#### 1. Simulation of the optomechanics

The model for the optomechanical system under study is described in Section II. The objective throughout this manuscript is to maximise two-mode squeezing, which is calculated from the covariance matrix $\mathcal{V}$ using Eq. (17). The system model is characterised by the parameters that are summarised in Table I. Given these parameters, an analytical solution for the covariance matrix, for a constant coupling $g$ pulse applied for a duration $\tau$, was derived for the study reported in Ref. [54]. An implementation of this developed in Python is used in this study, utilising functions from the NumpPy library. The calculation includes the rotating wave approximation (RWA) described in Appendix A 2. In all cases where $\mathcal{V}$ is computed for a constant (or square) pulse, then the measurement function $f^\text{out}$ used is that referred to as ‘optimal’ in Ref. [54]. A variation of this code, developed for the study, that gives an exact solution for $\mathcal{V}$ using the RWA, where $g(t)$ and $f^\text{out}(t)$ are piecewise constant functions in three timeslots, was used to validate the numerical solutions for $\mathcal{V}$.

For arbitrary time-dependent functions $g(t), f^\text{out}(t), \Delta(t)$ the covariance matrix is computed by numerically solving Eq. (16), which does not require the RWA. The initial value problem solver in SciPy (scipy.integrate.solve_ivp) is used to solve the differential equation. The absolute and relative tolerance need to be set much lower than the default for sufficiently accurate computation (atol=1e-20; rtol=1e-10), as very small differences in the smallest eigenvalue have a significant effect on the squeezing. Where any direct comparisons are made between the square and time-dependent coupling pulse are made, the numerical solver is used in both cases, as the RWA does induce significant differences in $S_{\text{gen}}$ for mechanical oscillator frequencies in the order of the optical damping ($\Omega_m \lesssim 10\kappa$) with some values of $g$ (see Fig. 5).

It is beneficial for the optimisation algorithm to describe the time dependence using fewer parameters. A piecewise linear (PWL) parameterisation is used for the time-dependent function results in this work. The PWL functions vary only linearly between some specific points within the duration $\tau$, and at these points the gradient may change, but the function remains continuous. Therefore the time-dependent function can then be described over a number of timeslots $N_{\text{ts}}$ by values at the start of the timeslot (and end for the final timeslot). That is, for some PWL function $f(t), f_i$ with $i = 1, \ldots, N_{\text{ts}} + 1$. All results reported in this manuscript with time-dependent $g(t), f^\text{out}(t), \Delta(t)$ use PWL parameterisation: $N_{\text{ts}} = 5$ with all timeslots of equal duration. All time-dependent profiles in any one computation of $\mathcal{V}$ have the same total duration $\tau$.

#### 2. Optimisation of parameters

In Section IV many results are reported where numerical optimisation is used to maximise generalised squeezing $S_{\text{gen}}$, which is calculated from the simulation of the optomechanical system as described in Appendix B1. Primarily, a Bayesian optimisation algorithm is used (described in Section III). The variables made available to the algorithm are explained in each of subsections of Section IV. These variables, along with the otherwise fixed parameter values from Table I, are used to calculate the covariance matrix $\mathcal{V}$. The objective of the algorithm is to minimise the minimum eigenvalue $\lambda_{\text{min}}$ of the matrix $\mathcal{V}$, which maximises $S_{\text{gen}}$ (see Eq. (17)).

As described in Section III, BO generates and updates a model of the objective function on the multi-dimension parameter space, which is often referred to as the surrogate model. The model is initially constructed by taking pseudo-random variable sets in the parameter space. BO then uses an acquisition function to determine the variable sets used to update the surrogate model in iterative steps, guided by evaluations of the objective function. The acquisition function also uses some pseudo-

| sym | desc | default | min | max |
|-----|------|---------|-----|-----|
| $\kappa$ | optical damping | 1 | n/a | n/a |
| $\gamma$ | mechanical damping | $2.8 \times 10^{-10}$ | n/a | n/a |
| $g$ | coupling | 0.1 | 0.01 | 2.0 |
| $\tau$ | pulse duration | 30 | 1 | 100 |
| $n_{\text{th}}$ | initial bath phonons | $2.26 \times 10^8$ | 1 | $10^{10}$ |
| $n_0$ | initial mech. phonons | $2.26 \times 10^8$ | 1 | $10^{10}$ |
| $\Gamma$ | reheating rate | 0.063 | $6.3 \times 10^{-10}$ | 6.3 |
| $\mathcal{G}$ | amplitude gain | n/a | n/a | 50 |
| $\Omega_m$ | mechanical frequency | n/a | n/a | 2 |
| $\Delta$ | frequency detuning | $-\Omega_m$ | $-\frac{1}{2} \Omega_m$ | $-\frac{3}{4} \Omega_m$ |
to build the initial surrogate model, the number of objective functions were used, referred to as 'initial random', 'exploration', and 'exploitation'. The number of objective functions evaluations at each stage (see-Section III) N_initial steps to build the initial surrogate model, N_explore steps with parameter values suggested by the acquisition function exploring wide regions of the search space, and N_exploit steps to refine the model local to solutions found during the exploration phase.

TABLE II. Bayesian optimisation steps. Summary of the number of optimisation steps used for the specific variable combinations. The time-dependent variables are piecewise-linear parameterised (see Section B.1) with 6 values each. The total number of dimensions in the parameter search space N_d is the sum of 6 per time-dependent function plus 1 for each constant value. The Bayesian optimisation (BO) is performed in three phases, with a number of steps (figure-of-merit function evaluations) at each stage (see-Section III). N_initial steps to build the initial surrogate model, N_explore steps with parameter values suggested by the acquisition function exploring wide regions of the search space, and N_exploit steps to refine the model local to solutions found during the exploration phase.

| variables          | N_d | N_initial | N_explore | N_exploit |
|--------------------|-----|-----------|-----------|-----------|
| g, τ               | 2   | 40        | 40        | 20        |
| g(t), τ, f_out(t)  | 13  | 200       | 200       | 80        |
| f_out(t)           | 6   | 120       | 120       | 50        |
| g(t), τ, f_out(t), δ(t) | 19  | 300       | 300       | 100       |

The optimisation process should ideally reliably result in the same optimal value for the figure of merit with repeated runs. That is, the maximum possible value of S_gen should always be found. It is possible that there are degenerate solutions, meaning that different parameter sets may be equally optimal, but the global maximum of S_gen should always be returned by the algorithm. The results in Section IV clearly show that this is not the case. Failure to reliably find the global maximum implies the algorithm has found a local maximum or trap.

The existence of local traps in “almost all” quantum control parameter landscapes has been disproved. However, this is caveated to exclude constrained spaces. In practice, most quantum control optimisation studies encounter local traps, especially in high-dimensional parameter spaces. A typical gradient-based algorithm encounters local traps, especially in high-dimensional parameter spaces. A typical gradient-based algorithm overcomes this only through repeated attempts. The Bayesian optimisation (BO) algorithm includes the expected improvement (EI) type acquisition function δ(t) in the optimisation. The number of BO steps used for different variable combinations are summarised in Table II.

3. Local traps in optimisation landscape

The excessive amplitude gain Θ that would lead to overheating in an experiment manifests itself as ‘float value overflow’ in simulations when calculating the covariance matrix V. Hence V cannot be computed for all g and τ within the region defined by the bounds given in Table I. Therefore, the pulse parameters are further constrained using an adiabatic approximation of amplitude gain given by Eq. (A16). The same gain limit constraint Θ_limit = 50 is used for all optimisations. The constraint Θ < Θ_limit is enforced by choosing g, p_Θ as optimisation variables, with the amplitude gain proportion defined as p_Θ = Θ/Θ_limit, 0.0 < p_Θ ≤ 1. From these the pulse duration for an optimisation step can be calculated as

$$\tau = \min \left\{ \frac{\ln [p_Θ \Theta_{\text{limit}}]}{2g^2}, \tau_{\text{max}} \right\} . \quad (B1)$$

The variables are constrained as −Ω_m/2 < δ < Ω_m/2. Due to the increased number of variables, 700 (rather than 480) optimisation steps are used when including δ(t) in the optimisation. The number of BO steps used for different variable combinations are summarised in Table II.

The optimisation algorithm explores the parameter space, proposing values for g and p_Θ within the specified bounds, with τ calculated as per Eq. (B1). A variant of this is used when g is time dependent during the pulse.

The driving laser frequency detuning Δ enters the Lyapunov equation Eq. (16) in elements of the drift matrix A. Consequently, when solving numerically, this can have an arbitrary time-dependent form. Where the detuning is optimised for maximising S_gen (Section IV C), the PWL parameters δ_i of a detuning offset function δ(t) are optimisation variables, such that the detuning is

$$\Delta(t) = -\Omega_m + \delta(t) . \quad (B2)$$

The existence of local traps in the 2-d landscape when optimising g and τ when computing the covariance matrix V using the rotating wave approximation (RWA). However, traps are present when computing V without the RWA. These are shown clearly in Fig. 5, where local peaks can be seen the 2-d landscape, and also
FIG. 5. Optimal coupling and pulse duration for constant coupling pulse. The main plot shows the generalised squeezing $S_{\text{gen}}$ for different coupling strengths $g$. The value of $\tau$ is chosen such that the amplitude gain is at its upper limit $\Phi = \Phi_{\text{lim}}$, using Eq. (B1) with gain proportion at maximum $p_g = 1$. The value of $S_{\text{gen}}$ differs depending on whether the rotating wave approximation (RWA) is used in the computation of the covariance matrix $\mathcal{V}$. The blue dots indicate $S_{\text{gen}}$ with the RWA and the orange crosses without the RWA. A single peak (at $g \approx 0.55\kappa$) is seen when using the RWA, whereas there are multiple maxima when not using the RWA. The inset shows the values $g, \tau$ resulting from 200 repetitions of optimising for maximal $S_{\text{gen}}$ without using the RWA. The grouping of points correspond with the near-degenerate maxima around specific coupling strengths $g/\kappa \approx 0.6, 0.75$.

separate groupings of $g$ and $\tau$ solutions found through optimisation.

In the higher dimensional spaces, 13-d for piecewise linear (PWL) coupling $g(t)$ and measurement $f_{\text{out}}(t)$ functions, and 19-d when the detuning is also optimised, BO has greater difficulty exploring the space, possibly simply because of the vastness, but also indicating that at least some of these dimensions contain local maxima. This is manifested in the wider spread of $S_{\text{gen}}$ seen in Fig. 3 for the higher dimensional search spaces. This is further illustrated in Fig. 6, which shows the solutions for $g(t)$ and $f_{\text{out}}(t)$ found by BO. The ‘Average’ plots indicate the common features of the optimised pulses, however, there is clearly significant variation in the solutions. The fact that they all produce high levels of squeezing indicates near degeneracy, as seen in the 2-d space.

Typically, the number of steps (function evaluations) allowed to BO must scale with the dimension of the space. However, the topography is also a factor – more traps means more steps are needed – so determining the ideal number of steps for some optimisation is non-trivial. A summary of the number of BO phase steps used is given in Table II. BO produces very similar results using half as many steps, but with a wider distribution of results. Tests with many more steps did not improve the consistency of the outcome significantly. In comparison, using the gradient-based algorithm (with approximated gradients) the (two-variable) optimisation of the constant pulse performed similarly well with the same number of function evaluations. Whereas, for the PWL cases, the results were less consistent, and required over 2000 function evaluations on average. This supports the idea of local traps in the optimisation landscape that BO negotiates with some success.

The traps seem to exist in the coupling parameter dimensions. When optimising only the measurement function $f_{\text{out}}(t)$ in repeated attempts, the maximal $S_{\text{gen}}$ has a very narrow distribution – see Fig. 3, ‘$f_{\text{out}}$ only’ dataset. The minimum, mean, maximum are are 6.543, 6.545, 6.546 dB. Correspondingly we find a distinct solution for $f_{\text{out}}(t)$ illustrated in Fig. 7.
4. Optimisation of detection angles

Section IV D reports that numerical optimisation can be used to determine the homodyne measurement angles $\theta_c, \theta_m$ that would allow direct measurement of $\text{Var} X_{\text{gen}}$ equivalent to $\lambda_{\text{min}}$, and hence be used to calculate the two-mode squeezing $S_{\text{gen}}$. The simulations find that the occupation of the mechanical oscillator must be much lower than environmental equilibrium ($n_0 = n_{\text{th}} = 2.26 \times 10^8$) for the optimisation algorithm to be able to effectively navigate the optimisation landscape and reliably find the $\theta_c, \theta_m$ values that give $\text{Var} X_{\text{gen}}$ equivalent to $\lambda_{\text{min}}$ (found through eigendecomposition).

The simulations compute the covariance matrix $\mathcal{V}$, either analytically or numerically, as described in Appendix B 1, and so $\text{Var} X_{\text{gen}}$ is calculated using Eq. (20). As $\frac{d}{d\phi} \left( \text{Var} X_{\text{gen}} \right)$ and $\frac{d^2}{d\phi^2} \left( \text{Var} X_{\text{gen}} \right)$ can be derived, then a Newton-Raphson based method is efficient for determining the optimal $\phi$. In this case the ‘Newton conjugate gradient’ method in scipy.optimize is used. Hence the optimisation landscape can be reduced to two dimensions and $\text{Var} X_{\text{gen}} [\theta_c, \theta_m]$ can be visualised in the contour plots of Fig. 8.

For covariance matrices computed using the analytical method, which uses the rotating wave approximation (RWA), the $\text{Var} X_{\text{gen}}$ landscape with respect to the detection angles appears uniform, such as in example Fig. 8.a), in that it is found invariant (within numerical error) for $\theta_c' = \theta_c + \chi, \theta_m' = \theta_m - \chi \ \forall \chi$, which is confirmed by optimisation finding the same $\text{Var} X_{\text{gen}}$ when optimising over 1 angle ($\theta_c = \theta_m$) or both independently. The covariance matrices generated by numerically solving the

**FIG. 8.** The squeezing indicator $\text{Var} X_{\text{gen}}$ for detection angle ranges, as shown for example covariance matrices. In panel a.) the covariance matrix is computed by solving analytically using the RWA, and hence based on a square coupling pulse. In b.) and c.) the covariance matrix is computed by numerically solving the Lyapunov equation Eq. (16). The particular covariance matrix is that computed based on the best parameters found while optimising piecewise linear profiles for coupling $g(t)$, measurement $f_{\text{out}}(t)$ and detuning $\Delta(t)$. The parameters for c.) differ only from those for b.) in that $n_0 = 100$, as opposed to $n_0 = n_{\text{th}} = 2.26 \times 10^8$. 
Lyapunov equation Eq. (16) are not found to have a uniform $\text{Var} X_{\text{gen}}$, such as in the example Fig. 8b). This non-uniformity is most likely also to appear in covariance matrices measured through experiment, as the numerical solution is a more accurate model of the physical system, because it does not use the RWA.

The form of Eq. (18) implies that solutions to a minimisation of Eq. (18) would be periodic in $\theta$, $\theta_m$. This is illustrated in the contour plots of Fig. 8. There is nothing obvious in the mathematics that shows the quadrature detection angles to be bounded other than as $0 \leq \theta < 2\pi$, however the physical interpretation implies that the upper bound should be $\pi$. The periodicity visible in Fig. 8 confirms this physical interpretation.

In the non-uniform cases the dark trenches in Fig. 8, indicating the lowest values of $\text{Var} X_{\text{gen}}$, are not uniformly deep, and the optimal solution is only found at specific locations. In the thermal equilibrium case (panel b.), due to the narrowness of the trench and the relative height of the surrounding landscape, neither gradient-based nor Bayesian type optimisation is reliable in locating the minimum value of $\text{Var} X_{\text{gen}}$ corresponding to the value of $\lambda_{\text{min}}$. Physically this means that the sensitivity of the squeezing measure to small variations in $\theta$, $\theta_m$ would make it practically impossible to locate values for the angles that would indicate the representative value of $S_{\text{gen}}$.

The sensitivity of $\text{Var} X_{\text{gen}}$ to variations in the detection angles is reduced by cooling the oscillator prior to application of the coupling pulse. The example Fig. 8c) is from a covariance matrix generated using the same parameters as used in b.), except that the initial occupation of the oscillator is set lower $n_0 = 100$. It can clearly be seen from the plot that the trenches are much wider. In this case the optimisation algorithm is able to reliably find the angles $\theta$, $\theta_m$ which give the smallest eigenvalue, that is $\text{Var} X_{\text{gen}} = \lambda_{\text{min}}$. Cooling of the oscillator to these levels and beyond is possible in some experimental setups, and would be necessary in order to use this method to measure the two-mode squeezing.

Small changes in $\theta$, $\theta_m$ produce large variations in $\text{Var} X_{\text{gen}}$, and hence we find they cannot be optimised in the same space as the driving pulse parameter variables, and so optimisation of $\theta$, $\theta_m$ is performed in a separate process. In tests optimising $S_{\text{gen}}$, using both the square and PWL coupling pulse, in all but a few isolated cases, the optimal detection angles could be determined for the covariance matrix at each step of the algorithm. For these tests a gradient-based algorithm was used to determine $\theta$, $\theta_m$, as it is more efficient to compute. Approximately 200 function evaluations gives a solution to satisfactory precision. In an experimental set up, Bayesian optimisation may perform better if there are limits on the precision to which the angles can be set, or there is significant variance in the measurement output.

![Graph](image_url)

**FIG. 9. Generalised squeezing for optimal coupling pulse parameters with added noise.** The histogram illustrates the distribution of generalised two-mode squeezing $S_{\text{gen}}$ when noise (Gaussian with standard deviation at 10%) is added to the piecewise linear parameters of the coupling function $g(t)$. The specific mean parameters for $g(t)$ are those found when optimising with control noise included the simulation (see Section IV E). Details of how the noise is modelled are given in Section III B 5.

5. **Optimisation with noisy controls**

The results for optimisation when the coupling pulse controls are noisy are given in Section IV E. The coupling $g(t)$ is piecewise linear (PWL) parameterised. To model the potential noise in attempting to drive this coupling, each of the PWL parameters $g_i$ have some Gaussian distributed value added to them. For the results presented, the Gaussian parameters are mean zero and standard deviation $g_i/10$. The noise is truncated at 3 standard deviation to exclude the possibility of negative coupling.

The noise is added at each step of the optimisation algorithm. That is, each set of $g_i$ parameters suggested by BO has noise added to them before the figure of merit is calculated. The final squeezing value however is calculated without noise, as this would obscure the result. This is illustrated in Fig. 9, which shows the distribution of $S_{\text{gen}}$ for 2000 repetitions using the best parameter set found through optimisation with noise added as described above. As would be expected, $S_{\text{gen}}$ results are distributed around 7.14 dB, which is the value of $S_{\text{gen}}$ with $\Psi$ computed without control noise.

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