Relevance of mathematical modelling of textile fillers’ 3-D structures for composite materials reinforcement

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Abstract. This paper discusses current and promising areas in the field of solving the fundamental problem of mathematical modelling in the design of bulk structures of textile fillers, which largely depend on the strength and weight characteristics of elements of aircraft made of polymer composite materials (PCM). Examples of mathematical modelling results of an aircraft engine blade’s textile filler for PCM are given. The study was funded by RFBR, project number 19-29-13044.

1. Introduction

Designing articles made of polymeric composite materials (PCM) with textile filler, aircrafts in particular, requires a complex approach in new technologies development with focus on performance improvement. The main tasks in the field of textile fillers development are enhancing strength characteristics and derating weight characteristics of PCM articles. These tasks could be accomplished by embedding textile fillers in the reinforcement article in desired directions according to inertial forces applied in every point of the structure [1, 2]. Developing the mechanic-mathematical basis for textile fillers structure design using mesh theory, topology and fractal geometry principles will allow to proceed to a new generation of PCM reinforcements, which will possess enhanced strength and weight efficiency compared to existing analogs.

2. Main part

Nowadays, aircraft composite constructions, which are manufactured by traditional reinforcement technologies, involve layering of textile fillers with subsequent autoclave and out-of-autoclave molding. Such composites possess a number of disadvantages during exploitation: low impact, shear and lamination resistances.

Over the last decades the number of studies in the field of examination of structural geometry, physical and mechanical properties of natural reinforcing structures and their application as prototypes...
in different industry fields has greatly increased [3, 4] both in Russia and in foreign countries. The most promising approach in automation of reinforcement pattern design is mathematical modeling methods.

2.1. Relevance of mathematical modelling in the design of woven composite fillers
During reinforcement pattern design topological and fractal factors of textile structures’ geometry are almost out of consideration. However, considering these factors will increase the strength of reinforced articles while decreasing material and labor costs by reducing the number of seams, in particular, down to zero. This was confirmed by research carried out at «Art modeling, design and technology of apparel» faculty of Russian State University named after A.N. Kosygin [5-9].

In order to determine estimated mechanical strength in any given point of 3D PCM article we propose using the main principle of mesh theory based on the mathematical construction of a 3D model of material mesh structure. It is formed by several reinforcing thread systems intersecting and is comprised of regular, irregular, correct and incorrect triangular, quadrangular, pentangular and other spatial forms of cells [5]. The proposed approach provides an opportunity to construct a 3D-model of layered or woven reinforced composite with the given arrangement of structural cells and spatial directions of their sides during the design stage. The combination of these characteristics defines strength and load resistance in each point of the reinforcing structure.

Another fundamental task of polymer composites design consists in fractality examination of a given structural element, which involves determining optimum ratio of mass per unit length, thickness and strength of used threads, size and form of structural cells, as well as size and form of the PCM structure itself. Research of these ratios will provide a possibility to create effective 3D mesh constructions with desired parameters of reinforcing threads and sizes of structural cells. In general, it will reduce the weight and increase the strength of a construction.

During the performed tests methods of mathematical modeling of woven frameworks geometry for the following PCM articles were developed:
- aircraft fairing;
- aircraft engine fan blade;
- curved woven parts, i.e. broad goods for curvilinear pultrusion, with respect to their deformation over spherical, conical and toroidal surfaces;
- window frame, curved bulkhead, ribbed panel, reinforced grid, trailing-edge flap.

The developed methods of mathematical modeling of the structural geometry of woven frameworks for PCM articles have been tested during the calculation of woven fillers and the manufacture of a number of aircraft parts together with the National Institute of aviation technologies (Moscow, RF) [6, 10-14] (figure 1). Practical testing of the manufactured samples showed that using mesh theory in mathematical modeling of woven frameworks for PCM articles allows to improve strength characteristics and attain structures with predictable behavior during exploitation [6].

2.2. Mathematical modelling of a woven structure framework of the aircraft engine fan blade
The aspects of woven frameworks’ structure mathematical modeling are considered below in the example of a fan blade.

Special properties of 3D woven frameworks, applied as textile fillers of PCM, are curvilinearity, multilayer and uneven thickness. Mathematical modeling with the use of mesh theory allows to develop novel techniques of braiding and weaving for manufacturing 3D seamless and dartless structures of aircraft engine fan blades and, therefore, enhance the strength of the finished article [6, 7, 9].
Figure 1. Examples of modeled and manufactured aircraft PCM parts: a - ribbed panel, b- curved bulkhead, c – demonstrator of the aircraft trailing-edge flap, d – window frame, e – fan blade

As a result, it was established that the forming process of 3D multilayered woven PCM fillers with uneven thickness and orthogonal arrangement of warp and weft threads could be described by Chebyshev’s meshes. The formation of single-layer or multi-layer shells according to the Chebyshev’s meshes law is based on the use of a mesh fabric structure. The 3D-form of the woven shell is achieved by the orientation of the fabric threads which is in turn achieved by changing the mesh angles while maintaining the lengths of the cells’ sides (figure 2). Due to the fact that no elongation or movement of threads occurs while putting a flat fabric with a mesh structure on the surface, the density of threads distribution changes in the shell.

The linear element of the mesh is expressed by the formulas:

Without inflection along the diagonal of the cell
\[ dS_1 = \sqrt{dx^2 + dy^2 + 2\cos\phi \cdot dx \cdot dy} \]

With inflection along the diagonal of the cell
\[ dS_1 = \sqrt{dx^2 + dy^2 - 2 \cos \phi dx dy}, \]

where \( dx, dy \) – side of mesh cell, \( \phi \) – mesh angle, \( dS_i \) – diagonal of mesh cell.

Fan blade shell sample, modeled according to Chebyshev’s mesh law, is showed in figure 3.
Figure 2. Chebyshev’s mesh: a – mesh on the blade surface, b – structural cell, c – structural cell with an inflection along the diagonal of the cell; d – development of the blade surface.

Figure 3. Variations of “dressing” by Chebyshev’s mesh of the aircraft engine fan blade surface with the location of the threads at an angle of 90 and 45 degrees to the blade axis.

It was established that the setting of desired conditions during modeling provides an opportunity to get following alternative designs of 3D curved meshes with uneven thickness:
- 3D Chebyshev’s meshes which consist of cells with equal opposing sides;
- 3D meshes of geodesic parallels which consist of cells with equidistant opposing sides;
- 3D meshes of longitudinal and transversal sections which consist of cells with regular and irregular structures.

In contrast to traditional multilayer shells, three-dimensional woven shells, including woven, braided, layered and reinforced with tufting frameworks for parts of the PCM, have a more complex
geometric structure. It is not advisable to describe these shells by flat quadrilaterals. They should be described by 3D-cells of cubic or parallelepiped forms. It was suggested to use 3D structural cells in the design of 3D seamless frameworks of fan blades with uneven thickness. Such cells have twelve sides, six edges with two-dimensional four-sided cell contours, and twenty-four grid corners (figure 4) [7].

![Figure 4](image)

**Figure 4.** 3D seamless filler of an aircraft engine fan blade with uneven thickness: a – non-formed shell, b – deformed shell, c – shell’s structural cell

During practical experiments it was established that there are only three phases of mesh angles changing (figure 5). The first phase involves changing of mesh angles of two opposite edge outlines, depicted as 1265 and 4378 (figure 5, a). The second phase involves changes in mesh angles of four opposite edge outlines: 1265, 4378, 1234, 5678 (figure 5, b). The third phase involves changes of six opposite edge outlines: 1265, 4378, 1234, 5678, 1485, 2376 (figure 5, c).

![Figure 5](image)

**Figure 5.** Transformation of spatial cell structure by mesh angle changing: a – at two opposing edges, b – at four edges, c – at six edges.

In a cell each face is connected to four adjacent faces by four edges. Each of the six faces of the cell (figure 6) can be deformed into a flat linear element with the diagonals $d_{xy}$, $d_{xz}$, $d_{yz}$ and the sum
of the cycle angles 360°, or into a spatial element when bending along one of the diagonals with an angle \( \Phi \) and the sum of the mesh angles in the cell face of less than 360°.

![Figure 6](image)

Figure 6. Forming of a 3D cell of a woven shell: a – without bending along the diagonal of the cell, b – with bending along the diagonal of the cell.

The mathematical modeling of the fan blade woven framework structure allowed to obtain generalized structural matrix elements, as well as to develop a scheme for geometric parameters determination of a three-dimensional twelve-line structural cell (figure 7).

![Figure 7](image)

Figure 7. Algorithm for defining the geometry of a 3D twelve-ribbed structural cell.

The results of theoretical and practical investigations by means of mathematical modeling of textile fillers’ 3D structures for reinforcement of composite structures are as follows:
- modeling methods of 3D multilayered seamless fan blade frameworks with uneven thickness were developed with a focus on their geometrical structure;
- practicability and possibility of manufacturing 3D seamless frameworks with equal thickness with braiding and weaving techniques were justified;
- structural features and the forming process of 3D seamless frameworks with uneven thickness were discovered;
- efficient seamless structures of frameworks made of straps with various forms and elasticity were proposed;
- a manufacturing method of a 3D fan blade seamless preform with uneven thickness made of straps was developed, which simplifies the process of manufacture and enhances the productivity;
- a manufacturing method of a 3D fan blade with uneven thickness and zonal distribution of straps of different widths was developed. The method will allow to replicate curvilinear areas of 3D fan blade outlines with high precision;
- fabrication methods of 3D multilayered and seamless frameworks with uneven thickness were developed. The novelty of developed methods was proven by a number of RF patents [10-14].

Theoretical results of this paper, namely, methods for manufacturing a multi-layer shell of a fan blade preform with a layout for RTM molding, were tested in working conditions of the National Institute of Aviation Technologies. During the testing, samples of an aircraft engine fan blade were modeled and manufactured. Then they were tested by non-destructive testing methods and received positive recommendations.

3. Conclusion

Performed tests showed that applying structural mesh theory allows to create few-seam and seamless textile reinforcing frameworks for PCM articles with effective spatial structure. Mathematical modeling provides an opportunity to determine spatial position of structural cells’ sides and density changes of reinforcing threads in structural cells. This will allow to set and determine strength properties in every point of a reinforcing framework and, therefore, gain highly precise cut details.

It is important to conduct further research in this area. The research should be focused on the development of mathematical models of 3D woven, braided and multilayered reinforcing composite structures, including articles of single and double curvature with spatial construction of textile fillers’ structural cells, with constant and variable density of warp and weft thread arrangement, and with interlamellar bonding formation.

Proposed approaches to designing effective structural patterns of 3D reinforcement by textile fillers allow to reveal geometric features of reinforcing structures and open possibilities of practical application of gained results when designing PCM aircraft articles with enhanced strength and weight properties.

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