New Type Ia Supernova Yields and the Manganese and Nickel Problems in the Milky Way and Dwarf Spheroidal Galaxies

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Received 2019 May 29; revised 2020 April 24; accepted 2020 April 25; published 2020 June 4

Abstract

In our quest to identify the progenitors of Type Ia supernovae (SNe Ia), we first update the nucleosynthesis yields for both near-Chandrasekhar- (Ch) and sub-Ch-mass white dwarfs (WDs) for a wide range of metallicities with our 2D hydrodynamical code and the latest nuclear reaction rates. We then include the yields in our galactic chemical evolution code to predict the evolution of elemental abundances in the solar neighborhood and dwarf spheroidal (dSph) galaxies Fornax, Sculptor, Sextans, and Carina. In the observations of the solar neighborhood stars, Mn shows an opposite trend to α elements, showing an increase toward higher metallicities, which is very well reproduced by the deflagration–detonation transition of Ch-mass WDs but never by double detonations of sub-Ch-mass WDs alone. The problem of Ch-mass SNe Ia was the Ni overproduction at high metallicities. However, we found that Ni yields of Ch-mass SNe Ia are much lower with the solar-scaled initial composition than in previous works, which keeps the predicted Ni abundance within the observational scatter. From the evolutionary trends of elemental abundances in the solar neighborhood, we conclude that the contribution of sub-Ch-mass SNe Ia to chemical enrichment is up to 25%. In dSph galaxies, however, larger enrichment from sub-Ch-mass SNe Ia than in the solar neighborhood may be required, which causes a decrease in [(Mg, Cr, Mn, Ni)/Fe] at lower metallicities. The observed high [Mn/Fe] ratios in Sculptor and Carina may also require additional enrichment from pure deflagrations, possibly as SNe Iax. Future observations of dSph stars will provide more stringent constraints on the progenitor systems and explosion mechanism of SNe Ia.

Unified Astronomy Thesaurus concepts: Type Ia supernovae (1728); Supernovae (1668); Nucleosynthesis (1131); Explosive nucleosynthesis (503); Chemical abundances (224); Galaxy chemical evolution (580); Dwarf spheroidal galaxies (420); Sculptor dwarf elliptical galaxy (1436); Fornax dwarf spheroidal galaxy (548)

1. Introduction

Although Type Ia supernovae (SNe Ia) have been used as a standard candle to measure the expansion of the universe, there is a small but significant variation in their luminosities. Brighter SNe Ia show a slower decay, which shows a correlation between the peak luminosity and light-curve width (Phillips 1993). The luminosity variation is empirically corrected in the supernova cosmology (Riess et al. 1998; Perlmutter et al. 1999), although the physical origin of the relation is uncertain. The dependence of this variation on the host galaxies was first reported by Hamuy et al. (1996), where the mean peak brightness is dimmer in elliptical galaxies than in spiral galaxies. Umeda et al. (1999) provided the first theoretical explanation for this dependence, assuming that a smaller C/O ratio leads to a dimmer SN Ia (see also visualization in Figure 7 of Nomoto et al. 2000). Similar dependencies of the luminosity variation on various properties of host galaxies are found in more recent observations (e.g., Childress et al. 2013), but the origin of the variation has not been confirmed yet.

The progenitor of SNe Ia is still a matter of big debate (see Hillebrandt & Niemeyer 2000; Maoz et al. 2014; Soker 2019 for a review). It is a combined problem of the progenitor systems and the explosion mechanism. In recent works, common progenitors are (1) deflagration or delayed detonation (DDT) of a near-Chandrasekhar (Ch)-mass carbon–oxygen (C + O) white dwarf (WD) in a single degenerate system (Whelan & Iben 1973; Nomoto 1982a), (2) a sub-Ch-mass explosion in a double degenerate system (e.g., Iben & Tutukov 1984; Webbink 1984; Pakmor et al. 2012), (3) double detonations of sub-Ch-mass WDs in a single or double degenerate system (e.g., Nomoto 1982b; Iben & Tutukov 1991; Ruiter et al. 2014), (4) weak deflagration of a near-Ch- or super-Ch-mass WD with a low-mass WD remnant in a single degenerate system, which possibly corresponds to a Type Iax supernova (SN Iax; Foley et al. 2013; Fink et al. 2014; Meng & Podsiadlowski 2014; McCully et al. 2014), and (5) delayed explosion of a rotating super-Ch-mass C+O WD (Benvenuto et al. 2015), which could be formed from merging of a C+O WD with the core of a massive asymptotic giant branch star during common envelope evolution (Soker 2015).

For the nucleosynthesis yields of SNe Ia, the so-called W7 model (Nomoto et al. 1984, 1997; Thielemann et al. 1986; Iwamoto et al. 1999) has been the most favored 1D model for reproducing the observed spectra of SNe Ia (Hoeflich & Khokhlov 1996; Nugent et al. 1997). In recent works, 3D simulations of a delayed detonation in a Ch-mass WD and a violent merger of two WDs (Röpke et al. 2012) and 2D simulations of a double detonation in a sub-Ch-mass WD (Kromer et al. 2010) can also give a reasonable match with observations. The advantage of the W7 model is that it also reproduces the Galactic chemical evolution (GCE) in the solar neighborhood, namely, the observed increase of Mn/Fe with metallicity, as well as the decrease of α elements (O, Mg, Si, S, and Ca; Kobayashi et al. 2006); with the updated nucleosynthesis yields of core-collapse supernovae (with a mix of normal
supernovae with $10^{51}$ erg and hypernovae (HNe) with $\geq 10^{52}$ erg at $\geq 20 M_\odot$ stars, $[\text{Mn}/\text{Fe}]$ is about $-0.5$ at $[\text{Fe}/\text{H}] \lesssim -1$ and increases toward higher metallicities because of the delayed enrichment of SNe Ia. However, there is a remaining problem in GCE with the W7 yields; the Ni/Fe ratio is higher than observed at $[\text{Fe}/\text{H}] \gtrsim -1$, which could be solved with DDT models (e.g., Iwamoto et al. 1999). An updated GCE model with the DDT yields from Seitenzahl et al. (2013) was shown in Sneden et al. (2016), which indeed gives Ni/Fe ratios closer to the observational data.

In contrast to these Ch-mass models, sub-Ch-mass models, which have been reconsidered for SNe Ia with a number of other observational results, such as supernova rates (e.g., Maoz et al. 2014) and the lack of donors in supernova remnants (Kerzendorf et al. 2009), do not match the GCE in the solar neighborhood. The Mn production from sub-Ch-mass models is too small to explain the observations in the solar neighborhood (Seitenzahl et al. 2013). The SNe Iax could compensate for this with their large Mn production, but their rate seems to be too low for the solar neighborhood (Kobayashi et al. 2015, 2019, hereafter K19).

Recently, dwarf spheroidal galaxies (dSphs) have been used as another site for constraining nucleosynthesis yields because of their low metallicities. Using our GCE model, Kobayashi et al. (2015) showed that a mix of sub-Ch-mass SNe Ia and Iax may be able to explain the scatter in the observed abundance ratios, which was confirmed by a stochastic chemical evolution model in Cescutti & Kobayashi (2017). Recently, Kirby et al. (2019) used a large sample of observational data and concluded that sub-Ch-mass SNe Ia are the main enrichment source in dSphs.

In this paper, we test SN Ia progenitor models using updated SN Ia yields sets both for Ch- and sub-Ch-mass explosions. The yields are calculated with our new 2D explosion and nucleosynthesis code (Leung et al. 2015a). The code has been applied to various types of SN explosions, including subluminous SNe Ia (Leung et al. 2015b), near-Ch-mass SNe Ia (Nomoto & Leung 2017a; LN18), sub-Ch-mass SNe Ia (LN19), and electron-capture SNe (Nomoto & Leung 2017b; Leung & Nomoto 2019; Leung et al. 2020). The code includes the necessary physics, such as the flame-capturing scheme by the level-set method (Reinecke et al. 1999b) with reinitialization (Sussman et al. 1994), subgrid turbulence (Clement 1993; Niemeyer et al. 1995; Schmidt et al. 2006), and the three-step simplified nuclear reaction scheme (Calder et al. 2007). In contrast to Calder et al. (2007), we choose to record the chemical composition in the hydrodynamical simulations explicitly; our hydrodynamical code includes a simplified seven-isotope network of $^4\text{He}, ^{12}\text{C}, ^{16}\text{O}, ^{20}\text{Ne}, ^{24}\text{Mg}, ^{28}\text{Si}$, and $^{56}\text{Ni}$ (Equation (8) of LN18; see also Timmes et al. 2000) with their three-step scheme.

For postprocessing nucleosynthesis, we use a larger 495-isotope network for nuclear reactions containing isotopes from $^1\text{H}$ to $^{91}\text{Tc}$. We use the tracer particle scheme (Travaglio et al. 2004), which records the thermodynamic trajectory $\rho - T$ as a function of time. We also use the torch nuclear reaction network (Timmes 1999) to compute the exact nucleosynthesis yields. Nucleosynthesis yield tables are obtained after short-life radioactive isotopes have decayed. Note that $^{26}\text{Al}$ and $^{26}\text{Fe}$ yields are added to those of $^{26}\text{Mg}$ and $^{60}\text{Ni}$, respectively, in GCE calculation.

### 2.1. Methods

We use our own 2D hydrodynamics code, primarily developed to model SNe Ia (Leung et al. 2015a). The code has been applied to various types of SN explosions, including subluminous SNe Ia (Leung et al. 2015b), near-Ch-mass SNe Ia (Nomoto & Leung 2017a; LN18), sub-Ch-mass SNe Ia (LN19), and electron-capture SNe (Nomoto & Leung 2017b; Leung & Nomoto 2019; Leung et al. 2020). The code includes the necessary physics, such as the flame-capturing scheme by the level-set method (Reinecke et al. 1999b) with reinitialization (Sussman et al. 1994), subgrid turbulence (Clement 1993; Niemeyer et al. 1995; Schmidt et al. 2006), and the three-step simplified nuclear reaction scheme (Calder et al. 2007). In contrast to Calder et al. (2007), we choose to record the chemical composition in the hydrodynamical simulations explicitly; our hydrodynamical code includes a simplified seven-isotope network of $^4\text{He}, ^{12}\text{C}, ^{16}\text{O}, ^{20}\text{Ne}, ^{24}\text{Mg}, ^{28}\text{Si}$, and $^{56}\text{Ni}$ (Equation (8) of LN18; see also Timmes et al. 2000) with their three-step scheme.

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### 2.2. Near-Ch-mass WD

For near-Ch-mass models, we first construct an isothermal hydrostatic equilibrium C+O WD. In this paper, we assume the central density $\rho_c = 3 \times 10^9$ g cm$^{-3}$ with uniform temperature $10^8$ K (Nomoto 1982a). The composition is assumed to be uniform as $X(^{12}\text{C}) = X(^{16}\text{O}) = (1 - Z)/2$ for the metallicities of $Z = 0$, 0.002, 0.01, 0.02, 0.04, 0.06, and 0.10. The Z component is scaled to the solar abundances (Lodders 2010) in this paper, which gives a significant difference in the nucleosynthesis yields. With $Z = 0.02$, the benchmark model is selected by requiring three conditions: (1) it has a yield of $^{56}\text{Ni} \sim 0.6 M_{\odot}$, as found in typical SNe Ia (Li et al. 2011b; Piro et al. 2014); (2) it has comparable Mn production at the solar metallicity; and (3) it does not severely overproduce stable Ni.

In Table 1, we tabulate the fundamental stellar parameters and the resultant explosion energies of our benchmark models

### Table 1

| Model Setup for the Benchmark Models with the Initial Metallicity $Z = 0.02$ |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Near-Ch-mass Model              | DDT             | $30$            | $1.38$          | $0$             | $1900$          | $17.7$          | $12.7$          | $0.78$          | $0.63$          | $8.46 \times 10^{-4}$ | $4.42 \times 10^{-4}$ |
| Sub-Ch-mass Model               | DD              | $0.32$          | $1.0$           | $0.05$          | $6200$          | $10.2$          | $8.7$           | $0.98$          | $0.63$          | $5.68 \times 10^{-4}$ | $1.34 \times 10^{-3}$ |

Note. “Mechanism” is the explosion mechanism used in our simulations, including the DDT and double detonation (DD) models. Central densities of $\rho_c$ are in units of $10^9$ g cm$^{-3}$. The total mass of the WDs, $M_{\text{WD}}$, and helium envelope mass, $M_{\text{He}}$, are in units of solar mass. Here $R$ is the initial stellar radius in kilometers; $E_{\text{mol}}$ and $E_{\text{tot}}$ are the energy released by nuclear reactions and final total energy, respectively, both in units of $10^{50}$ erg; $t_{\text{mol}}$ is the first detonation transition time in units of seconds; and $M(^{26}\text{Ni})$, $M(\text{Mn})$, and $M(\text{Mn})$ are the masses of $^{56}\text{Ni}$, stable Mn, and $^{56}\text{Ni}$ at the end of the simulations, after short-lived radioactive isotopes have decayed.

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4 The decay time is $10^3$ yr in LN18 and LN19 but $10^6$ yr in this paper, which results in a significant difference in Co yields.
The solar metallicity was 0.02, and the other elements were not included in 54Fe, and 58Ni. 54Cr, 58Fe, and 64Ni, while higher metallicity mostly enhances isotopes related to the direct product of 22Ne, such as 50V, 50Cr, and 58Ni, for Ch- and sub-Ch-mass SNe Ia. It can be seen that the nucleosynthesis yields of the near-Ch-mass model satisfy these three criteria of the benchmark models.

In LN18, we computed 45 models of SNe Ia using near-Ch-mass C+O WDs as the progenitors. In view of the diversity of observed SNe Ia, an extended parameter space, including central densities of $5 \times 10^3$ to $5 \times 10^4$ g cm$^{-3}$ (corresponding to initial masses of 1.30–1.38 $M_\odot$), metallicities from $X^{22\text{Ne}} = 0$ to 5 $Z_\odot$ 5 C/O mass ratios from 0.3 to 1, and different ignition kernels from the centered flame to the off-centered flame, has been surveyed. We have then shown that the central density and metallicity are important parameters that strongly affect nucleosynthesis yields; a higher central density allows larger production in neutron-rich isotopes such as 50Ti, 54Cr, 58Fe, and 64Ni, while higher metallicity mostly enhances isotopes related to the direct product of 22Ne, such as 50V, 50Cr, 54Fe, and 58Ni.

For the explosion mechanism, in this paper, the turbulent deflagration model with deflagration–detonation transition (DDT; see, e.g., Khokhlov 1991; Golombek & Niemeyer 2005; Röpke & Niemeyer 2007; Seitenzahl et al. 2013) is adopted for the following two reasons. First, the multidimensional pure turbulent deflagration (PTD) model (Reinecke et al. 1999a, 2002a, 2002b; Röpke & Niemeyer 2007; Ma et al. 2013; Fink et al. 2014) is very likely to leave a remnant, and its explosion is weak. The low ejecta mass may not be important for chemical enrichment compared to other explosion models. Second, the gravitationally confined detonation model (Plewa et al. 2004; Jordan et al. 2008, 2012; Meakin et al. 2009; Seitenzahl et al. 2016) tends to produce very strong explosions with a small amount of neutron-rich isotopes, including Mn. As discussed in Seitenzahl et al. (2013), there is not yet another major site for the production of Mn. Therefore, we focus on the DDT model, which is more robust in producing iron-peak elements, although the PTD model was also investigated in LN18.

In the core of near-Ch-mass C+O WDs, we introduce an initial carbon deflagration. The flame structure is a “three-finger” structure as in Reinecke et al. (1999a). Other flame structures were also investigated in LN18, and we showed that the overall abundance pattern is less sensitive to the initial flame structure.

The deflagration starts at the center of the WD and makes the star expand slowly, so that the core is always the place of highest central density and temperature. At $t \sim 1$ s after the deflagration started, the DDT occurs. The detonation provides a strong shock for compressing the surrounding material. This causes a sharp rise in the global maximum density and temperature ($\rho_{\text{max}}$ and $T_{\text{max}}$, respectively), which leads to a “wigging” rise in the central density and temperature from 1 to 2 s. Beyond $t \sim 10$ s, the star enters homologous expansion, and observable exothermic nuclear reactions take place (see Figures 2, 3, 25, and 26 of LN18 for the density, temperature, energy, and luminosity evolution).

In Figure 1, we show the distribution of $T_{\text{max}}$ against $\rho_{\text{max}}$ for the near-Ch-mass benchmark model according to the thermodynamic trajectories of the tracer particles. There are two populations of tracer particles. For $\rho_{\text{max}} \geq 10^9$ g cm$^{-3}$, there is a tight relation of $T_{\text{max}}$ increasing with $\rho_{\text{max}}$. This corresponds to the particles being incinerated by the deflagration wave. Due to the subsonic nature, no shock wave is created during its propagation. The particles are burned according to their local density. On the other hand, for particles with $\rho_{\text{max}} < 10^9$ g cm$^{-3}$, $T_{\text{max}}$ spans a wider range. This corresponds to the particles being incinerated by the detonation wave. Because there is more than one C detonation triggered during the explosion, the collision of shock waves provides an observable shock heating, which creates the $T_{\text{max}}$ spectra as seen in the figure.

In Figure 2, we show the distribution of the electron fraction, $Y_{e}$, against $T_{\text{max}}$ for the tracer particles. It can be seen again that there are two populations of particles. At $T_{\text{max}} > 7 \times 10^9$ K, $Y_{e}$ drops toward higher $T_{\text{max}}$ from the initial 0.5 to ~0.46. This corresponds to the particles incinerated by the deflagration wave at high densities, where electron capture can efficiently take place. The other population corresponds to the particles burned by the detonation wave or deflagration wave with a low
density. Electron capture occurs at a much slower rate, so that $Y_e$ stays between 0.5 and 0.499 at $T_{\text{max}} < 7 \times 10^9$ K.

The nucleosynthesis yields of the benchmark model are shown in Figure 3, where mass ratios scaled to the solar ratios, $[X/Y]_{56}$, are plotted against the mass number. The two horizontal lines correspond to twice- and half-solar ratios. Due to the fast detonation wave, very small amounts of C, O, and Ne are left in the WD. The detonation wave mostly burns matter at a low density and produces intermediate-mass elements from Si to Ca close to the solar ratios. One can also see a healthy production of iron-peak elements from Cr to Ni, except mild overproduction of $^{56}$Fe and $^{58}$Ni. Heavier iron-peak elements, such as Co and Zn, are underproduced.

In Figure 4, we plot the same abundances split into the deflagration and detonation components. As noted previously, the detonation (dashed lines) mainly burns the low-density matter, and small amounts of C, O, and Ne are left. It mostly produces intermediate-mass elements, in particular $^{28}$Si, $^{32}$S, $^{36}$Ar, and $^{46}$Ca, with values even higher than the solar ratios. The detonation wave also produces some iron-peak elements very close to the solar ratios. On the other hand, the deflagration wave (solid lines) burns mainly high-density matter, and no fuel is left. It also produces very small amounts of intermediate-mass elements. However, electron capture occurs mainly in matter burned by the deflagration, where the iron-peak elements, including neutron-rich ones such as $^{54}$Fe, $^{55}$Mn, and $^{58}$Ni, are largely enhanced.

### 2.3. Sub-Ch-mass WD

For the sub-Ch-mass models, we construct a two-layer WD with carbon–oxygen in the core and pure helium in the envelope. The helium layer has to be thin (see below for a further discussion), and in this paper, we adopt $M(\text{He}) = 0.05 \, M_\odot$. Note that this value is smaller than assumed in the binary population synthesis model by Ruiter et al. (2011; 0.1 $M_\odot$) and is consistent with other previous works on explosions (Bildsten et al. 2007; Shen & Bildsten 2009; Fink et al. 2010; Kromer et al. 2010; Woosley & Kasen 2011). The total WD masses including the He layer are 0.9, 0.95, 1.0, 1.1, and 1.2 $M_\odot$. The assumption of the composition is the same as the near-Ch-mass models in Section 2.2 but with metallicities of $Z = 0, 0.001, 0.002, 0.004, 0.01, 0.02$, and 0.04. For $Z = 0.02$, the benchmark model is selected to produce a normal SN Ia of $^{56}$Ni mass $\sim 0.6 \, M_\odot$. It is known that sub-Ch-mass models cannot produce sufficient Mn for explaining the solar abundance (Seitenzahl et al. 2013). Thus, we do not impose any constraint on the Mn production. Again, we also require stable Ni to not be overproduced.

In LN19, we have computed a series of 40 models of SNe Ia using sub-Ch-mass C+O WDs as the progenitors. A wide range of models with progenitor masses from 0.9 to 1.2 $M_\odot$ has been computed for metallicities from 0 to 5 $Z_\odot$. C/O mass ratios from 0.3 to 1.0, and He envelope masses from 0.05 to 0.2 $M_\odot$. The initial mass and metallicity strongly affects nucleosynthesis yields. Unlike the near-Ch-mass models where the central density determines the occurrence of electron capture, the initial mass determines the $^{56}$Ni production, and the abundance pattern mainly depends on the scaling with $^{56}$Fe. Therefore, compared to the near-Ch-mass models, there is a smaller variety of abundance patterns for sub-Ch-mass models because of its pure detonation nature, where most matter does not have a sufficiently high density for rapid electron capture before it cools down by expansion.

For the explosion mechanism, in this paper, the double detonation model is used, where carbon detonation is triggered by helium detonation. In LN19, multiple types of detonation triggers were investigated: one bubble (a spherical shell), multiple bubbles, and a belt-shaped helium detonation at the beginning of the simulations. Although this affects the minimum helium mass required for detonation, for the models with $M(\text{Fe}) \sim 0.6 \, M_\odot$, the abundance patterns of iron-peak elements are not so different; thus, we only use the spherical one ($S^+$)-type in LN19 in this paper.

The simulation starts from a He detonation in a 100 km spherical shell just outside of the C+O core. Since it is supersonic, both the central density and temperature of the WD remain unchanged for the first 1 s, although the global maximum temperature ($T_{\max}$) gradually decreases. Once the shock wave reaches the center, the central C detonation is triggered, and the central temperature and density rapidly increase. After that, the expansion allows the matter to cool down rapidly. Both central and global maximum densities drop

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6 The initial composition and the decay time are updated in this paper, similar to near-Ch-mass models.
together, showing that the core has relaxed and starts its expansion (see Figures 5, 6, and 7 of LN19 for the temperature, energy, and luminosity evolution).

As in Figure 1, Figure 5 shows the distribution of $T_{\text{max}}$ against $\rho_{\text{max}}$ for the near-Ch-mass benchmark model. Due to the detonation nature, there is always a wide spectrum of $T_{\text{max}}$ for a given $\rho_{\text{max}}$. This means that the detonation waves inside the stars can efficiently reheat the matter, even when the matter is completely burned. Compared to the near-Ch-mass model, this model can achieve similar $T_{\text{max}}$ even with a lower $\rho_{\text{max}}$. This is because part of the tracer particles can encounter much stronger shock heating due to geometric convergence, especially near the center.

As in Figure 2, Figure 6 shows the distribution of $Y_e$ against $\rho_{\text{max}}$ for the near-Ch-mass benchmark model. Compared to the near-Ch-mass counterpart, there are many fewer tracer particles where significant electron capture takes place. Although the maximum $\rho_{\text{max}}$ can be comparable to the near-Ch-mass model, the high density is due to shock compression, and the time duration for the particle to remain in such a density is comparatively short. Therefore, the fluid elements have less time to carry out weak interactions than in the near-Ch-mass model. Therefore, only a few particles can be found at relatively low $Y_e$ as ~0.499. Note that the range of $Y_e$ is much smaller than in Figure 2.

The nucleosynthesis yields, $[X/\text{Fe}]$, are shown in Figure 7 for the sub-Ch-mass benchmark model (solid lines) compared to the model with a thicker helium envelope. The star is completely burned, and only small amounts of C, O, and Ne are left. Intermediate-mass elements from Si to Ca show the ratios close to half-solar values. With $M(\text{He}) = 0.1 M_\odot$ (dashed lines), there is a large enhancement of $^{48}\text{Ti}$, $^{51}\text{V}$, and $^{52}\text{Cr}$. This is related to the helium detonation, especially during the end of He detonation. The iron-peak elements are also healthily produced, except for Mn.

In Figure 8, we plot $[X/\text{Fe}]$ for the sub-Ch-mass benchmark model with the He- and C-detonation components separately. Again, in the C-detonation component (solid lines), since low-density matter in the core is also detonated, small amounts of C, O, and Ne remain. Intermediate-mass elements are still produced. Here Sc, Ti, and Cr are underproduced.

\section*{Figure 5.} Shown is $T_{\text{max}}$ against $\rho_{\text{max}}$ for the sub-Ch-mass benchmark model according to the thermodynamic trajectories. Contours correspond to tracer particle numbers of 100 (purple), 300 (blue), 500 (green), 700 (red), and 900 (orange) for the C+O matter (dashed lines) and those with 10 times smaller numbers for the He matter (solid lines).

\section*{Figure 6.} Shown is $Y_e$ against $\rho_{\text{max}}$ for the sub-Ch-mass benchmark model according to the thermodynamic trajectories. The contours are the same as in Figure 5.

\section*{Figure 7.} Shown is the $[X/\text{Fe}]$ of stable isotopes in the sub-Ch-mass benchmark model (solid lines) after short-lived radioactive isotopes have decayed. The ratios are scaled to the solar ratios. The horizontal lines at ±0.3 correspond to 0.5 or 2.0 times the solar values. A similar model but with a thicker helium layer $M(\text{He}) = 0.1 M_\odot$ (dashed lines) is shown for comparison.

\section*{Figure 8.} Same as Figure 7 but for the particles ignited by carbon (solid lines) and helium (dashed lines) detonation.
unlike the full abundance profile in Figure 7. Among iron-peak elements, only $^{57}$Fe and $^{56}$Ni are sufficiently produced. On the other hand, the He detonation (dashed lines) produces a very different abundance pattern. Intermediate-mass elements are significantly underproduced. In contrast, there is a large enhancement of Ti, Cr, and V, with ratios to $^{56}$Fe as large as $\sim 30$ times the solar values. Iron-peak elements from Mn to Zn look enhanced, but this is due to the small production of $^{56}$Ni. Note that the mass of the He envelope is 20 times smaller than that of the C+O core.

In Figure 9, we plot the total yielded mass of Ca, Mn, Fe, and Ni in the ejecta for our sub-Ch-mass models as a function of WD mass. Clear trends can be observed for all elements. The mass yields of Fe and Ni are monotonically increasing against the WD, while that of Ca is monotonically decreasing. In contrast, the Mn mass increases and then decreases with a transition at $M_{\text{WD}} = 1.0 \, M_\odot$. These trends show how the C-detonation strength contributes to the formation and destruction of elements during nucleosynthesis. For intermediate-mass elements such as Si, S, and Ca, when the WD mass increases, the C+O fuel is more likely to undergo complete burning until nuclear statistical equilibrium (NSE); thus, the nuclear reactions do not stop at Ca but continue to form iron-peak elements. This also explains the monotonic increase in Fe and Ni with WD mass. The falling part of Mn is also a consequence of the strong C detonation, which gives more NSE burning instead of $\alpha$-rich freeze-out. The rising part of Mn is caused by suppression of the incomplete and complete Si burning at the globally low density in low-mass WDs.

2.4. Comparison between Benchmark Models

Finally, in Figure 10, we plot the total yielded masses of Cr, Mn, Fe, and Ni in the Ch- and sub-Ch-mass models as a function of initial metallicity $Z$. The metallicity dependence is significantly different from the yields in LN19, which are shown in Figure 11 for comparison. In general, Cr, Mn, and Ni are produced more in near-Ch-mass models than in sub-Ch-mass models by a factor of ~2, 10, and 6, respectively, and the metallicity dependence for Mn and Ni is stronger (i.e., $Z = 0$–0.04) in sub-Ch-mass models than in near-Ch-mass models.

The total Cr mass decreases when $Z$ increases. This trend comes from the lower-energy releases with higher $Z$. At $Z > 0.04$ in Figure 11, however, the Cr mass increases with $Z$. For these near-Ch-mass models, when $Z$ further increases, deflagration is further suppressed, leaving more matter to be burned by detonation. Note that Cr is produced not only by deflagration and but also by detonation (Figure 4).

The total Mn mass increases monotonically with $Z$ because the initial $^{22}$Ne is the seed of $^{55}$Mn. Much more Mn is produced in near-Ch-mass models than in sub-Ch-mass models. This is due to electron capture during the initial deflagration phase, where more matter can have the $Y_e$ required to form Mn. In near-Ch-mass models, Mn is mainly produced by NSE during deflagration via $^{52}$Fe($\alpha$, $p$)$^{56}$Co, and a 10 times smaller amount of Mn can also be produced by incomplete Si burning at detonation (Figure 4), depending on $Z$. In sub-Ch-mass models, Mn mostly comes from incomplete Si burning at He detonation (Figure 8), which also depends on $Z$.

The total Fe mass decreases monotonically with $Z$ because most Fe comes from $^{56}$Fe, most of which comes from the decay of $^{56}$Ni (which has $Y_e = 0.5$). This isotope is produced by the ash in detonation that enters the NSE region. Increasing metallicity lowers the original $Y_e$ of the fuel. As a result, even without significant electron capture compared to the
deflagration ash, the high metallicity automatically suppresses production of $^{56}\text{Ni}$ and hence decreases the total Fe mass.

The total $^{58}\text{Ni}$ mass increases monotonically with Z because the initial $^{22}\text{Ne}$ is connected to $^{58}\text{Ni}$ directly by an $\alpha$-chain (e.g., $^{54}\text{Fe}(\alpha, \gamma)^{58}\text{Ni}$). However, this trend becomes much weaker if we adopt the solar composition for the initial metallicity (Figure 10). Higher-metallicity models have a slightly stronger detonation, which also enhances $^{58}\text{Ni}$ production. The $^{58}\text{Ni}$ is produced in NSE by deflagration in near-Ch-mass models (Figure 4) independent of Z, as well as by incomplete Si burning at detonation in near-Ch- and sub-Ch-mass models, depending on Z (Figure 8).

3. Galactic Chemical Evolution

3.1. The GCE Code

Since nucleosynthesis yields are significantly different between Ch- and sub-Ch-mass SNe Ia, changing the relative contribution results in different elemental abundance ratios at a given metallicity. The evolutionary tracks of elemental abundance ratios depend on the star formation history. However, the star formation history can be tightly constrained from the other independent observations, namely, the metallicity distribution function (MDF) of stars in the system considered, and in the solar neighborhood, only a small variation is possible for the evolutionary tracks of elemental abundance ratios (see the Appendix). Therefore, the elemental abundance ratios in the solar neighborhood have been used as the most stringent constraint for the nucleosynthesis yields of core-collapse supernovae (e.g., Timmes et al. 1995; Kobayashi et al. 2006; Romano et al. 2010) and the progenitor models of SNe Ia (e.g., Matteucci & Greggio 1986; Kobayashi et al. 1998).

The evolutionary tracks of elemental abundance ratios are calculated with GCE models (Tinsley 1980; Pagel 1997; Matteucci 2001), and our basic equations are described in Kobayashi et al. (2000). The code follows the time evolution of elemental and isotopic abundances in a system where the interstellar medium (ISM) is instantaneously well mixed, and thus it is also called a “one-zone” model. No instantaneous recycling approximation is adopted; thus, chemical enrichment sources with long time delays, such as SNe Ia, are properly included.

The stellar physics/empirical relations included in our GCE models are as follows. The star formation rate (SFR) is proportional to the gas fraction, which evolves with inflow and outflow to/from the system considered, as well as mass loss from dying stars and supernova explosions. The Kroupa initial mass function (IMF) is adopted (Kobayashi et al. 2011, hereafter K11). The nucleosynthesis yields of core-collapse supernovae, SNe II and HNe, are also taken from K11 but with failed supernovae at $>30 M_\odot$ (K19). The HN fraction depends on the metallicity: $\epsilon_{\text{HN}} = 0.5, 0.5, 0.4, 0.01,$ and 0.01 for $Z = 0, 0.001, 0.004, 0.02,$ and 0.05 (Kobayashi & Nakasato 2011). Then, the gas infall and star formation timescales, $\tau_1$ and $\tau_2$, are determined to match the observed MDF of the system. As shown in the Appendix, the set of $\tau_1$ and $\tau_2$ can be uniquely determined from the MDF (Figure A1), and we chose $\tau_1 = 5$ and $\tau_2 = 4.7$ for our fiducial model (K11).

Our conclusions are not affected by this choice of parameters.

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9 The yield table is identical to that in Nomoto et al. (2013).
earlier and Ch-mass SNe Ia start to occur earlier with our progenitor model; thus, the second peak caused by Ch-mass SNe Ia also appears earlier. The model with Ch-mass SNe Ia only gives the best match to the observed rate for Milky Way–type galaxies. However, note that this is not a totally fair comparison, since the observed values are for the entire galaxy, while the model is for the solar neighborhood.

It is difficult to observationally estimate the sub-Ch-mass fraction in the total SN Ia rate; it requires estimating ejecta mass from supernova light-curve modeling, as well as handling the selection bias of observed supernovae. Scalzo et al. (2014) estimated the sub-Ch-mass fraction at 25%-50% in their unbiased sample of spectroscopically normal SNe Ia. Panel (c) shows the fraction of sub-Ch-mass SNe Ia to the total SN Ia rate for our models, which evolves as a function of time. It is 100% at the beginning, while it decreases once Ch-mass SNe Ia start to occur. At present, a 50% GCE contribution of sub-Ch-mass SNe Ia results in a 25% sub-Ch-mass fraction in the SN Ia rate (blue short-dashed line), which is in reasonable agreement with the observational estimate. This fraction also depends on the evolutionary phase of the galaxy and hence the type/mass of the host galaxies. On the other hand, the observational estimate of the sub-Ch-mass fraction is for the average of various types/masses of galaxies with various stellar ages. For these reasons, we do not adopt the “observed” sub-Ch-mass fraction but instead aim to constrain the fraction using GCE modes.

Figure 15 shows the resultant [O/Fe]–[Fe/H] relations for the models with Ch-mass SNe Ia only. Without the metallicity effect on the WD winds, it is not possible to reproduce the observed evolutionary change of [O/Fe] at [Fe/H] ~ −1 (see Section 3.4 for the observational data). As noted above, changing the star formation timescale would not solve this problem while reproducing the observed MDF (see also Figure A1). Recently, the metallicity dependence of the binary fraction has been indicated from observations (Moe et al. 2019), where the binary fraction is higher at lower metallicities. If we scale our binary parameter (bMS and bRG) to the observed metallicity-dependent binary fraction, then there are many more SNe Ia at earlier epochs, which decreases the [O/Fe] ratios even further away from the observational data. Therefore, it is necessary to include our metallicity effect of Ch-mass SNe Ia in order to reproduce this most important observation of GCE.

3.4. Elemental Abundance Ratios

Not only [O/Fe] ratios but also abundance ratios among iron-peak elements are the key to constraining the fraction of sub-Ch-mass SNe Ia. For the elemental abundance ratios of individual stars, the most accurate observational data, i.e., high-resolution observations with star-by-star analysis, are available for the solar neighborhood. We take the non-LTE (NLTE) abundances for oxygen (Zhao et al. 2016), while LTE abundances are used for iron-peak elements (e.g., Reddy et al. 2003; Feltzing et al. 2007; Bensby et al. 2014; Reggiani et al. 2017). The NLTE effects of iron-peak abundances could also be important. It is worth noting, however, that the effects may not be so large with the updated atomic data (Sneden et al. 2016; but see Bergemann & Gehren 2008). The exception is for Cr, and we plot the Cr II observations (see Kobayashi et al. 2006 for the comparison between Cr I and Cr II observations).
Figure 16 shows the evolution of elemental abundances against $[\text{Fe}/\text{H}]$ for the models with various yields of Ch-mass SNe Ia. The $[\text{O}/\text{Fe}]$ shows a decrease from $[\text{Fe}/\text{H}] \sim -1$ to higher $[\text{Fe}/\text{H}]$, while $[\text{Mn}/\text{Fe}]$ shows an increase; these opposite behaviors are well reproduced by the delayed enrichment of SNe Ia. The observed $[\text{Ni}/\text{Fe}]$ ratios show a constant value of $\sim 0$ over the whole metallicity range. It has been known that the W7 yields (Nomoto et al. 1997; Iwamoto et al. 1999) overproduce Ni by $\sim 0.5$ dex (magenta long-dashed and cyan dotted–dashed lines; see also Figure 24 of Kobayashi et al. 2006). This Ni overproduction problem is mostly solved with the updated nuclear reaction rates, mainly due to the lower electron-capture rates (blue dashed lines). Here $Y_e$ becomes higher, approaching 0.5, which gives lower [(Ni, Co)/Fe] and higher [(Cr, Mn)/Fe]. Our 2D DDT yields of $1.37M_\odot$ give very similar results (red solid lines) as the updated W7 yields, but the $[\text{Mn}/\text{Fe}]$ ratio is reduced by 0.1 dex because of a slower flame speed in our more realistic 2D model, which gives a better agreement with observations. The 3D DDT yields from Seitenzahl et al. (2013) also give very similar results (green dotted lines) as our 2D DDT yields but with 0.1 dex higher [(Mn, Ni)/Fe] ratios. This is probably because their multiple ignitions result in more material to be burned by deflagration waves.

In summary, with the updated electron-capture rates, these three models (W7, 2D DDT, and 3D DDT) of Ch-mass SNe Ia give the elemental abundance ratios within the observational scatters in the solar neighborhood, and our 2D DDT gives the best fit to $[\text{Mn}/\text{Ni}]$ ratios at $-1 \lesssim [\text{Fe}/\text{H}] \lesssim 0.3$. The $[\text{Ni}/\text{Fe}]$ ratio still shows a mild increase from $-0.2$ to $+0.05$ with $[\text{Fe}/\text{H}]$. Whether this is inconsistent or not depends on the yields of core-collapse supernovae that determine the plateau value of $[\text{Ni}/\text{Fe}]$ at $[\text{Fe}/\text{H}] \lesssim -1$. Since both Ni and Fe are formed at the innermost regions of core-collapse supernovae, multidimensional effects can change the Ni/Fe ratios.

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8 The updated W7 yields were presented in Nomoto & Leung (2018) and LN18 and are recalculated with a new initial composition and decay time in this paper.
Although the star formation history is the same as in K11 (cyan dotted–dashed lines) and K19 (red solid lines), the K19 model is updated by including faint supernovae, a metal-dependent HN fraction, and different SN Ia parameters. These result in a slightly higher time-integrated SN Ia rate with a slightly later start (see Figure 14), which leads to lower [O/Fe] and higher [Mn, Ni]/Fe ratios than in the K11 model, at [Fe/H] ∼ 0. The better match of Ni is not due to the GCE modeling; the K19 model with the old W7 yields (magenta long-dashed lines) still shows the overproduction of Ni.

Figure 15. The [O/Fe]–[Fe/H] relations in the solar neighborhood for the models with Ch-mass SNe Ia only. The red solid line is our fiducial model with the metallicity effect of WD winds. The green short-dashed line does not include the metallicity effect. The blue long-dashed line does not include it either but does include the metallicity dependence of the binary fraction from Moe et al. (2019).

Table 2

Parameters of the GCE Models for the Solar Neighborhood (SN) and dSphs: Timescales of Infall (τ_i), Star Formation (τ_s), Outflow (τ_o), and the Galactic Wind Epoch τ_w, All in Gyr

|            | τ_i | τ_s | τ_o | τ_w |
|------------|-----|-----|-----|-----|
| SN, Ch only| 5   | 4.7 | ... | ... |
| SN, 75% sub-Ch| 5   | 4.0 | ... | ... |
| SN, 50% sub-Ch| 5   | 3.2 | ... | ... |
| SN, sub-Ch only| 5   | 1.0 | ... | ... |
| Fornax, no sub-Ch| 10 (τ < 9) | 25 | 5 | 12 |
| Sculptor, no sub-Ch| 1   | 40 | 1 | 9 |
| Sextans, no sub-Ch| 0.5 | 100 | 1.4 | 7 |
| Carina, 0.1 (τ < 6.5) | 0.1 (τ = 6.5–9)| 200 | 5 | 12 |
| Carina, 0.1 (τ < 6.5) | 0.1 (τ = 6.5–9)| 100 | 5 | 12 |
| Carina, Iax’50 | 0.1 (τ < 6.5) | 100 | 5 | 12 |

Figure 14. Same as Figure 13 but for supernova rate histories: the time evolution of core-collapse supernova rates (panel (a)), total SN Ia rate (panel (b)), and ratio of sub-Ch-mass SNe Ia to total SNe Ia (panel (c)). The open circles indicate the observed SN Ia rate in a Milky Way–type galaxy taken from Li et al. (2011a). The cross shows the observational estimate of the sub-Ch-mass fraction by Scalzo et al. (2014).
Figure 18 shows the evolution of elemental abundance ratios with varying the fraction of sub-Ch-mass SNe Ia. If 50% of the delay-time distribution comes from sub-Ch-mass SNe Ia, the \([\text{O}/\text{Fe}]\) slope with \([\text{Fe}/\text{H}]\) is too shallow, although the \([\text{Mn, Ni}/\text{Fe}]\) ratios are within the scatters of observational data. With 25% sub-Ch-mass SNe Ia and 75% Ch-mass SNe Ia, it is possible to reasonably reproduce all observational constraints.

4. dSph Galaxies

Detailed elemental abundances are also obtained for the stars in dSph galaxies (e.g., Tolstoy et al. 2009), and from the observed abundance patterns, it has been debated whether dSph galaxies are the building blocks of the Galactic halo or not. The very different abundance patterns of the stars in “classical” dSphs (with relatively large stellar masses) suggest that dSphs are not the building blocks but instead provide an independent constraint on stellar physics at a different environment.

The dSph galaxies are not a homogeneous population but have formed with a variety of star formation histories, and various chemical evolution models have been presented (e.g., Carigi et al. 2002; Lanfranchi et al. 2006; Cescutti et al. 2008; Vincenzo et al. 2014). Because of the shallow potential well, the ISM can be easily blown away due to supernova feedback after the initial starburst. In addition to the description in Section 3.1, outflow is also included, proportional to the SFR, i.e., the gas fraction of the system, with a timescale \(\tau_\text{w}\). If supernova energies are accumulated, star formation can be totally quenched. This is called galactic winds, and the epoch is defined with \(\tau_\text{w}\). In order to constrain GCE model parameters, it is necessary to have a number of observational constraints, such as MDFs; thus, we model four dSph galaxies, Fornax, Sculptors, Sextans, and Carina, which have stellar masses of 20, 2.3, 0.44, and \(3.8 \times 10^6 M_\odot\) (McConnachie 2012). Stellar age distributions have also been estimated comparing photometric data to stellar evolutionary tracks, which are also used for constraining model parameters. The adopted parameters are summarized in Table 2.
Figure 19 shows the adopted observational constraints of SFRs and MDFs, as well as the model predictions of age–metallicity relations, for the fiducial models of these four dSph galaxies. The resultant formation histories can be summarized as follows. The models for Sculptor and Sextans are very similar; both are formed by a rapid infall and star formation with a strong outflow. Since the star formation efficiency in Sextans is lower than in Sculptor, the average \([\text{Fe}/\text{H}]\) of the MDF is \(\sim 0.3\) dex lower, which is probably due to the mass difference of the systems. The models for Fornax and Carina are also similar; both have extended star formation histories with longer infall timescales. Since the star formation efficiency in Carina is lower than in Fornax, the peak \([\text{Fe}/\text{H}]\) of the MDF is \(\sim 0.6\) dex lower, which is also due to the mass difference of the systems. There is also an outflow, but this is weaker than in Sculptor and Sextans. Through dynamical interaction, it is possible to have multiple gas infalls in dSph galaxies. The observed age distribution of Carina is well reproduced by two infalls, one with a short timescale and another with a much longer timescale at time \(t = 6.5\) Gyr.

For the reasons described in the next section, in these fiducial models, the 100% contribution of sub-Ch-mass SNe Ia is added on top of the contributions from core-collapse supernovae and Ch-mass SNe Ia \(^{10}\) (or SNe Iax for Sculptor). In the predicted iron abundance evolutions (panel (b)), chemical enrichment timescales are shorter for more massive systems. In observational data, the age–metallicity relations should have steeper slopes at \(t \lesssim 3\) Gyr, although for such old stars, it is very difficult to estimate age and metallicity independently. The three dSph galaxies (except for Carina) reached \([\text{Fe}/\text{H}] \sim -1\) at \(t = 5–10\) Gyr, and after that, the iron abundance evolution is sped up because of Ch-mass SNe Ia. In Carina, however, \([\text{Fe}/\text{H}]\) never reaches \(\sim -1\), as in the observed MDF (panel (c)); thus, there is no enrichment from Ch-mass SNe Ia. \(^{11}\)

### 4.1. Elemental Abundance Ratios

As in Section 3.4, abundance ratios among iron-peak elements are the key to constraining the contribution of various types of SNe Ia in dSph galaxies. However, it is much harder to estimate these abundance ratios because of the distance of the

\(^{10}\) For the solar neighborhood models in Section 3, the total SN Ia rate is fixed, and only the relative contribution from Ch- and sub-Ch-mass SNe Ia is varied.

\(^{11}\) However, note that in more realistic hydrodynamical simulations (e.g., Kobayashi & Nakasato 2011), Ch-mass SNe Ia can occur at \([\text{Fe}/\text{H}] \lesssim -1\) due to inhomogeneous enrichment, and \([\alpha/\text{Fe}]\) can show a decrease in the case of strong supernova feedback.
systems (the only observed stars are RGs) and the limited samples of each system. In particular, an NLTE analysis has been made only for a small number of stars. For constraining models, Mg NLTE abundances and Ni LTE abundances are taken from Mashonkina et al. (2017), which uses the same NLTE model as in Zhao et al. (2016) for the solar neighborhood. The Mn and Cr data are taken from Jablonka et al. (2015), also with their LTE analysis. The Cr abundances are obtained from the Cr I lines, which are known to underestimate Cr abundances, and a +0.4 dex shift is applied as in Kobayashi et al. (2006). The LTE Mn abundances are also taken from the data compilation by Venn et al. (2012), and a large sample comes from medium-resolution spectra by Kirby et al. (2019).

Figures 20–23 show the evolution of elemental abundances for these four dSph models, with variations in the contributions of various types of SNe Ia. As for the solar neighborhood models, the elemental abundance ratios at [Fe/H] ≤ −3 are determined from the IMF-weighted yields of core-collapse supernovae. The predicted ratios cannot be rejected by this small sample of observations, but lower [Mg/Fe] ratios might be preferred, which could be produced with an IMF truncated at ~20M_⊙ (Kobayashi et al. 2006; Nomoto et al. 2013). Around [Fe/H] ~ −3, [X/Fe] starts to decrease in the models with sub-Ch-mass SNe Ia, while [X/Fe] stays constant without sub-Ch-mass SNe Ia. This transition is caused by the shortest lifetime, which is set at 0.04 Gyr as in the “observed” delay-time distribution (Maoz et al. 2014) for sub-Ch-mass SNe Ia in our models (Section 3.2). Around [Fe/H] ~ −1, [(Mn, Ni)/Fe] rapidly increases in the models with Ch-mass SNe Ia, while [X/Fe] stays constant without Ch-mass SNe Ia. This transition is caused by the metallicity effect of Ch-mass SNe Ia in our models.

As in Kobayashi et al. (2015), we call pure deflagrations of hybrid C+O+Ne WDs “SNe Iax” and use the nucleosynthesis yields from Fink et al. (2014), which can produce very high [Mn/Fe] ratios at [Fe/H] ≤ −1. In the fiducial models of this paper, the SN Iax rate is determined from the calculated mass range of hybrid WDs (K19), which is much narrower and gives a lower SN Iax rate than in Kobayashi et al. (2015). The normalization is given by the same binary parameters for Ch-mass C+O WDs: \( b_{RG} = 0.02 \) and \( b_{MS} = 0.04 \).

In Figure 20 for Fornax, the observed [Mg/Fe] ratios seem to decrease from [Fe/H] ~ −2 toward higher metallicities, which cannot be reproduced without sub-Ch-mass SNe Ia (blue long-dashed lines). The model with only sub-Ch-mass SNe Ia

![Figure 18](image-url)
cannot reproduce the monotonic increase of $[\text{Mn}/\text{Fe}]$ from $[\text{Fe}/\text{H}] \sim -1$ to $-0.5$, and thus it is necessary to include Ch-mass SNe Ia as well. The model including all three SN Ia channels (red solid lines) is in good agreement not only with the $[\text{Mg}/\text{Fe}]$ but also with the $[\text{Mn}/\text{Fe}]$ ratios. Some of the stars with very high $[\text{Mn}/\text{Fe}]$ ratios at $[\text{Fe}/\text{H}] \lesssim -2$ might be locally enriched by SNe Iax. The errors of the other iron-peak elements are too large to place further constraints.

Also in Figure 21 for Sculptor, the observed $[\text{Mg}/\text{Fe}]$ decreases at $[\text{Fe}/\text{H}] \gtrsim -2$, which cannot be reproduced without sub-Ch-mass SNe Ia. The observed $[\text{Mn}/\text{Fe}]$ ratios are highest around $[\text{Fe}/\text{H}] \sim -1.5$ and then sharply decrease until $[\text{Fe}/\text{H}] \sim -1$. This is not reproduced with the model including all three SN Ia channels (red solid lines). Obviously, without Ch-mass SNe Ia, $[\text{Mn}/\text{Fe}]$ monotonically decreases (green dotted lines). The SN Iax is a potentially good source to reproduce this $[\text{Mn}/\text{Fe}]$ evolution, but a higher rate is required.

If we multiply the SN Iax rate by a factor of 50, then it is possible to reproduce the rapid increase of $[\text{Mn}/\text{Fe}]$ from $[\text{Fe}/\text{H}] \sim -2$ to $-1.5$ (blue short-dashed lines). Then, in order to reproduce the $[\text{Mn}/\text{Fe}]$ decrease from $[\text{Fe}/\text{H}] \sim -1.5$ to $-1$, it is better to exclude normal Ch-mass SNe Ia (magenta long-dashed lines).

In Figure 22 for Sextans, the observed $[\text{Mg}/\text{Fe}]$ ratios cannot be reproduced without sub-Ch-mass SNe Ia (green short-dashed lines) but are in reasonably good agreement with the model including all three SN Ia channels (red solid lines). The observed $[\text{Mn}/\text{Fe}]$ can be better explained if the SN Iax rate is boosted by 10 times (blue long-dashed lines). In Figure 23 for Carina, the observed scatters are larger than the ranges of $[\text{X}/\text{Fe}]$ evolution, and the inhomogeneous enrichment should be important (Venn et al. 2012). Nonetheless, the model including all three SN Ia channels (red solid lines) is closer to the observed $[\text{Mg}/\text{Fe}]$ ratios, while the model without sub-Ch-mass SNe Ia is better for the observed $[\text{Mn}/\text{Fe}]$ ratios.
Similar to Sculptor, with a 50× boosted SN Iax rate (blue long-dashed lines), the model is in good agreement with the observations for both \([\text{Mg}]/\text{Fe}\) and \([\text{Mn}]/\text{Fe}\). In summary, for all four dSphs we have modeled, the \([\text{Mg}]/\text{Fe}\) ratios can be well reproduced with larger enrichment from sub-Ch-mass SNe Ia than in the solar neighborhood. However, with sub-Ch-mass SNe Ia, the \([\text{Mn}]/\text{Fe}\) ratios become too low, which can be solved with additional enrichment from SNe Iax.

4.2. The Mn/Fe–Ni/Fe Diagram

The key elements to constrain the enrichment sources in dSphs are Mn and Ni, and we present a useful diagram in Figure 24. We show our nucleosynthesis yields of near-Ch- and sub-Ch-mass models for various WD masses and initial metallicities in the diagram of \([\text{Mn}]/\text{Fe}\) versus \([\text{Ni}/\text{Fe}\). The near-Ch-mass models with different WD masses are calculated with changing central densities of WDs (LN18). The solid lines...
are for the new yields in this paper, while the dashed lines are for the LN18 and LN19 yields with only $^{22}\text{Ne}$ for the Z component of the initial composition. There is an almost linear trend where both $\frac{\text{Mn}}{\text{Fe}}$ and $\frac{\text{Ni}}{\text{Fe}}$ increase with higher metallicity (see Figure 10 for the reasons). At a given metallicity, $\frac{\text{Mn}}{\text{Fe}}$ and $\frac{\text{Ni}}{\text{Fe}}$ are higher for less massive WDs in sub-Ch-mass models but more massive WDs in near-Ch-mass models.

The dotted lines denote the solar ratios, and the large open circle indicates the average SN Ia yields in the solar neighborhood at $\frac{\text{Fe}}{\text{H}} \lesssim -1$. The stars with error bars show the empirical SN Ia yields obtained from the observed abundances of stars in dSphs from Kirby et al. (2019). The low Mn/Fe and Ni/Fe can be better explained with sub-Ch-mass models. However, note that the initial composition of the nucleosynthesis calculation is crucial for this argument; with simplified models with only $^{22}\text{Ne}$, the observational data of dSphs could be well reproduced with low-metallicity sub-Ch-mass SNe Ia, while with a more realistic solar-scaled initial composition, the dSph data can be better reproduced with metal-rich sub-Ch-mass SNe Ia. Normal Ch-mass SNe Ia (1.37 $M_\odot$; red solid line) clearly cannot reproduce the dSph data, and this finding is consistent with our GCE results in Figures 20–23.

5. Conclusions

In our quest to identify the progenitors of SNe Ia, we first update the nucleosynthesis yields for both Ch- and sub-Ch-mass C+O WDs for a wide range of metallicities with our 2D hydrodynamical code (Leung et al. 2015a) and the latest nuclear reaction rates. In particular, new electron-capture rates even change the W7 yields significantly for Cr, Mn, and Ni. For the explosion mechanism, DDT is used for Ch-mass SNe Ia (LN18), while the double detonation model with carbon...
detonation triggered by helium detonation is used for sub-Ch-mass SNe Ia (LN19). The helium envelope has to be as thin as \( M(\text{He}) = 0.05 \, M_\odot \), otherwise Ti, V, and Cr will be overproduced at \([\text{Fe}/\text{H}] \gtrsim -1.5\) (Figure 7).

We then include the nucleosynthesis yields in our GCE code (Kobayashi et al. 2000) to predict the evolution of elemental abundances in the solar neighborhood and dSph galaxies. For Ch-mass SNe Ia, the timescale of supernovae is mainly determined from the metallicity-dependent secondary mass range of our single degenerate model (Kobayashi et al. 1998; KN09). For sub-Ch-mass SNe Ia, we use the delay-time distribution estimated from observed supernova rates (Maoz et al. 2014). Including failed supernovae, the star formation histories are assumed in order to reproduce other observational constraints, such as the MDFs (K19).

In the observations of the solar neighborhood stars, Mn shows an opposite trend to \( \alpha \) elements, showing an increase toward higher metallicities, which is very well reproduced by Ch-mass SNe Ia but never by sub-Ch-mass SNe Ia alone. The Mn is mainly produced by NSE during deflagrations in Ch-mass WDs where electron captures lower the electron fraction of the incinerated matter, and the double detonation models for sub-Ch-mass WDs do not have enough material with such a low electron fraction. A small amount of Mn can also be produced by incomplete Si burning during detonations, depending on the initial metallicity.

Previously, the problem with Ch-mass SNe Ia was the overproduction of Ni at high metallicities, which is not observed. In this paper, however, we find that Ni yields of Ch-mass SNe Ia are much lower than in previous works when we use a more realistic initial composition of WDs (i.e., not \(^{22}\text{Ne}\) but the solar-scaled composition), which keeps the predicted Ni abundance within the observational scatter. Among Ch-mass models, W7, 2D DDT, and 3D DDT give the elemental abundance ratios within the observational scatter in the solar neighborhood, and our 2D DDT gives the best fit to the \([\text{Mn}/\text{Ni}]\) ratios at \(-1 \lesssim [\text{Fe}/\text{H}] \lesssim 0.3\). We also find that for both Ch- and sub-Ch-mass SNe Ia, the metallicity dependence of Mn and Ni is much weaker than in previous works (Figure 10).

From the evolutionary trends of elemental abundance ratios in the solar neighborhood, we conclude that the contribution of sub-Ch-mass SNe Ia in chemical enrichment is up to 25%. In dSph galaxies, however, the contribution of sub-Ch-mass SNe Ia seems to be higher than in the solar neighborhood, which is consistent with the low-metallicity inhibition of our single degenerate scenario for Ch-mass SNe Ia. In dSphs, sub-Ch-mass SNe Ia cause a decrease of \([\alpha, \text{Cr}, \text{Mn}, \text{Ni}/\text{Fe}]\), while so-called SNe Iax can increase Mn and Ni abundances if they are pure deflagrations. Among dSphs, all galaxies we model in this paper (Fornax, Sculptor, Sextans, and Carina) seem to require larger enrichment from sub-Ch-mass SNe Ia than in the solar neighborhood. The observed \([\text{Mn}/\text{Fe}]\) ratios in Sculptor and Carina may also require additional enrichment from SNe Iax. Future observations of a large number of stars in dSphs would provide more stringent constraints on the progenitor systems and explosion mechanism of SNe Ia.

Within the one-zone GCE framework, it is not possible to reproduce the observed elemental abundance ratios of dSph stars (Figures 20–23) only by variations of the IMF and SFR among dSph galaxies. Different SFRs could change the relative contribution between core-collapse supernovae and SNe Ia and thus the \([\text{Mg}/\text{Fe}]\) ratios at a given time (or \([\text{Fe}/\text{H}]\)). At the same time (or \([\text{Fe}/\text{H}]\)), \([\text{Cr}, \text{Mn}, \text{Ni}/\text{Fe}]\) ratios should also be similarly affected by SNe Ia. Variation of the IMF mostly appears as the mass dependence of nucleosynthesis yields of core-collapse supernovae and thus could change the normalization of abundance ratios. During core-collapse supernova explosions, Cr and Mn are synthesized in incomplete Si-burning regions, while Ni and Fe are synthesized in complete Si-burning regions; thus, \([\text{Cr}/\text{Mn}]\) and \([\text{Ni}/\text{Fe}]\) do not vary more than \(-0.2\) dex (Kobayashi et al. 2006). In this paper, we show that the contributions from different subtypes of SNe Ia could explain the variations among \([\text{Mg}, \text{Cr}, \text{Mn}, \text{Ni}/\text{Fe}]\) ratios. However, note that in more realistic chemodynamical simulations, selective metal loss could be caused by supernova feedback in a shallow potential well, which might explain some of these variations in elemental abundances of dSph stars.

We thank K. Shen and I. Seitenzahl for providing nucleosynthesis data and A. Ritter for binary population synthesis data. We are grateful to E. Kirby, M. de los Reyes, K. Hayashi, and A. Bunker for fruitful discussion. C.K. acknowledges funding from the UK Science and Technology Facility Council (STFC) through grants ST/M000958/1 and ST/R000905/1. This work used the DiRAC Data Centric system at Durham University, operated by the Institute for Computational Cosmology on behalf of the STFC DiRAC HPC Facility (www.dirac.ac.uk). This equipment was funded by BIS National E-infrastructure capital grant ST/H008519/1, STFC capital grant ST/K00087X/1, DiRAC Operations grant ST/K003267/1, and Durham University. DiRAC is part of the National E-Infrastructure. Numerical computations were also carried out in part on the PC cluster at the Center for Computational Astrophysics, National Astronomical Observatory of Japan. S.C.L. acknowledges support by HST-AR-15021.001-A. This work has been supported by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan, and JSPS KAKENHI grant Nos. JP17K05382 and JP20K04024.

**Appendix**

**GCE Parameter Dependence**

In our solar neighborhood models, there are only two free parameters: the timescales of infall \( \tau_i \) and star formation \( \tau_s \) in Gyr (Section 3.1). Figure A1 shows our chemical evolution models varying the two timescales, including 50% sub-Ch-mass SNe Ia. We choose the set of these timescales in order to match the observed MDF (panel (b)). The peak \([\text{Fe}/\text{H}]\) of the MDF provides “net” yields, which correspond to the average stellar metallicity and depend only on nucleosynthesis yields, outflow (metal loss), and the IMF (Tinsley 1980). Therefore, \( \tau_s \) can be determined uniquely with a given \( \tau_i \). A shorter \( \tau_i \) value is required for a longer \( \tau_s \), which results in more rapid star formation (panel (a)). However, the evolution of \([\text{O}/\text{Fe}]\) as a function of \([\text{Fe}/\text{H}]\) becomes almost identical, independent of the choice of these parameters. This is because the MDF tells how quickly the star formation and chemical enrichment from core-collapse supernovae (which produce O) take place relative to the timescale of Ch- and sub-Ch-mass SNe Ia (which produce Fe). This is why our conclusions using GCE models constrained with MDFs are robust. In this paper, in order to
compare the models varying sub-Ch-mass SNe Ia, we choose the models with \( t = 5 \text{ Gyr} \) in Table 2.

In our Ch-mass SN Ia model, the total number of SNe Ia are given by two binary parameters, \( b_{\text{MS}} \) and \( b_{\text{RG}} \), respectively, for MS+WD and RG+WD systems (Section 3.2), and the ratio is theoretically uncertain. Figure A2 shows our chemical evolution models varying the two binary parameters, including Ch-mass SNe Ia only. We first choose the set of these binary parameters in order to reproduce the slope of \([\text{O}/\text{Fe}] = \text{[Fe/H]}\) ratios against \([\text{Fe/H}]\) (panel (c)). A smaller \( b_{\text{MS}} \) value (blue long-dashed lines) gives a slightly shallower curve of the \([\text{O}/\text{Fe}] = \text{[Fe/H]}\) relation. Note that since the time delay is different for MS+WD and RG+WD systems, the total number of SNe Ia exploded by the present is not simply the summation of the two numbers. We then choose the fiducial values from the shape of the MDF; a smaller \( b_{\text{MS}} \) value gives a larger number of metal-poor stars because the iron production becomes slower, on average. The model with \( b_{\text{RG}} = 2\% \) and \( b_{\text{MS}} = 4\% \) gives the best match with the observed MDF in the solar neighborhood; thus, we use this set for the fiducial model.

For dSph galaxies, the peak \([\text{Fe/H}]\) of the MDFs is lower than in the solar neighborhood, which requires outflows, provided the same nucleosynthesis yields and IMF. We consider two forms of outflows: one is the outflow proportional to the SFR, described by a timescale \( \tau_o \), and the other is the galactic wind set by \( t_w \) (Section 4). The value of \( \tau_o \) is chosen to match the peak \([\text{Fe/H}]\) of the MDF (Figure 19(c)), while the value of \( t_w \) is chosen to reproduce the lack of young stars in the observations (Figure 19(a)). Although there are more free parameters, because there are more observational constraints for dSphs than in the solar neighborhood, it is possible to choose the best parameter set. In dSphs, SFRs are estimated not only at present but for the entire history, and from the shape, \( \tau_i \) and \( \tau_s \) are constrained. In general, slower star formation (with longer \( \tau_i \) and/or \( \tau_s \)) leads to too-low SFRs at \( t \lesssim 4 \text{ Gyr} \), while faster star formation (with shorter \( \tau_i \) and/or \( \tau_s \)) leads to too-low SFRs at \( t \gtrsim 4 \text{ Gyr} \).
Figure A2. Same as Figure A1 with a different set of binary parameters. Only Ch-mass SNe Ia are included.

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