Abstract

Using a recent parametrization of nuclear effects in parton distribution functions we calculate the neutrino-nucleon cross section at energies relevant for ultra high energy neutrino telescopes. The modification of the cross section in comparison with the calculation using parton densities in free nucleons is of the order of few per cent for the parameter range of interest in neutrino telescopes ($A=10$ and $E=10^6$ GeV) and it reaches 20% at the highest energies ($E=10^{12}$ GeV) and for the largest nuclear size ($A=190$) considered.
1 Introduction

The high energy neutrino cross section is a crucial ingredient in the calculation of the event rate in high energy neutrino telescopes. The detectors presently in design or construction stages look for Čerenkov light from the neutrino-induced muon in charged current interactions and take advantage of both the long muon range and the rise of the neutrino cross to meet the neutrino detection challenge. The background of atmospheric muons is rejected searching for neutrinos that have travelled through the earth. For energies above a few hundred TeV the interaction length for neutrinos becomes comparable to the Earth diameter. As a result the event rate is a convolution of the differential spectrum, the cross section and the exponential attenuation of the neutrino flux. Uncertainties in the cross section get amplified into the energy spectrum and the angular distribution of the event rate and it is important that they are kept under control.

The DIS cross-sections are obtained from $x$ and $Q^2$ dependent structure functions which are calculated in perturbative QCD. In practice, structure functions are obtained from $x$ and $Q^2$ dependent parton distribution functions which follows DGLAP evolution equations. The DGLAP formalism effectively resum logarithmic terms in $Q^2$ which appear in the perturbative series.

The calculation of the neutrino-nucleon cross section involves an integration over $x$. As the energy increases, the integral becomes dominated by the interaction with low $x$ quarks and gluons while the average $Q^2$ rises to values of order the electroweak boson mass squared (for a detailed discussion see for example Ref. [3]). Then, in the process of the total cross section calculation, the parton densities extracted from present data must be extrapolated using the theoretical formulas to the region in the $x, Q^2$ plane not supported by experiments.

The uncertainty due to the extrapolation to high $Q^2$ is not expected to be large because the $x$ shape of the parton distribution functions at low $Q^2$ is narrowly constrained by HERA data while the further evolution to high $Q^2$ values seems to be well defined in DGLAP. At the highest energies, uncertainties between $\pm 20\%$ [4] and a factor $2^{\pm 1}$ [5] are typically reported.

However, DGLAP is expected to break down at low $x$ because of potentially large
logarithmic terms in $x$ which also appear in the perturbative series. These dangerous logarithms are partially considered under certain approximations in DGLAP. A different approach is provided by the BFKL formalism [6] which deals with the $Q^2$ evolution of unintegrated in transverse momentum parton densities. The BFKL predictions are nowadays under discussion: the leading order (LO) result does not agree with data and the next-to-leading order (NLO) corrections are found to be suspiciously large to be trusted, although it seems that the adequate choosing of the renormalization scale [7] can solve this problem.

We discuss in this letter a different source of uncertainty in the calculation of the high energy neutrino-nucleon DIS cross section, which comes from the fact that parton densities widely used were extracted under the assumption that partons belong to isolated nucleons when, on the contrary, the nucleons are usually bound in larger nuclei at the interaction sites. In this work we perform a more realistic calculation taking into account these effects by using the standard parton densities in free nucleons with the modifications due to nuclear effects as given by the EKS parameterizations [8].

2 Nuclear effects

It is experimentally well known that the structure function $F_2$ in deep inelastic lepton-nucleus scattering for large atomic mass, $A$, is different from that measured for hydrogen or deuterium targets (see for example [9]). If one maintains the partonic view of the nucleon, this result may indicate that parton distributions of bound nucleons are different from those of free nucleons.

Parton distributions of the nucleon in a nucleus containing $A$ nucleons have been obtained in Ref. [8] using QCD evolution at leading twist and leading order of perturbation theory together with the experimental ratios $F_2^A/F_2^D$ measured by EMC and NMC at CERN [9, 10].

With the quark ratios as defined in Ref. [8]:

$$R_V^A(x, Q^2) \equiv \frac{u_V^A(x, Q^2) + d_V^A(x, Q^2)}{u_V(x, Q^2) + d_V(x, Q^2)} \quad \text{and} \quad (1)$$

$$R_S^A(x, Q^2) \equiv \frac{\bar{u}^A(x, Q^2) + \bar{d}^A(x, Q^2) + \bar{s}^A(x, Q^2)}{\bar{u}(x, Q^2) + \bar{d}(x, Q^2) + \bar{s}(x, Q^2)} \quad (2)$$
we have generated the correction due to nuclear effects to the parton distribution functions in free nucleons. The ratios of valence and sea quarks given by Eq. (1) and (2) are plotted in Fig. 1. From low to high $x$ one can easily identify in Fig. 1 the regions of shadowing ($R < 1$), anti-shadowing ($R > 1$), EMC ($R < 1$) and Fermi motion ($R >> 1$) which are experimentally observed in the ratio $F_2^A/F_2^D$. We show below that the shadowing region (small $x$) gives the main correction to the neutrino-nucleon DIS cross section at high energy. The smooth $Q^2$ dependence of the ratio, which was difficult to observe experimentally, is also apparent in Fig. 1.

We have calculated the neutrino-nucleon DIS cross-sections following Ref. [3] but considering valence and sea parton distributions of the nucleus as corrected by Eqs. (1) and (2). In the calculation we use parton densities from the group MRST [11] at the LO approximation in $\alpha_s(Q^2)$. At the LO approximation we are consistent with the EKS parameterization of nuclear effects [8] which has been extracted with the help of LO evolution equations equations. However, the effect of higher order corrections should be small [12] and it would not modify the conclusions of the present work. In addition, in the cross section calculation we have also neglected the contribution from the longitudinal structure function, which is proportional to $\alpha_s(Q^2)$, and terms suppressed by the ratio of the nucleon mass to $Q^2$.

Let us comment some technical details of the calculation: In the integration we take the minimum value of $Q^2=4$ GeV$^2$ although the result is not sensitive to variations around this value. We have also extrapolated the $x$ dependence of the quark distribution functions beyond the limits of applicability given by the authors using the $x$ behavior at the lowest $x$ value of the parametrization For the MRST98 set, we assume the Regge type shape $\sim x^{-\lambda}$ below $x = 10^{-5}$ with $\lambda$ the slope obtained at $x = 10^{-5}$. This simple phenomenological approach agrees with the extrapolations based on perturbative QCD (for example the double-logarithmic-approximation) which explain HERA data. For the EKS parameterizations, we take below $x = 10^{-6}$ the constant behavior shown in Fig. 1 at low $x$.

In this work we are also interested in the modification due to nuclear effects of the
average value of the inelasticity $y$, which is the fraction of the neutrino energy which flows to the hadronic part of the interaction in the laboratory frame.

This parameter fixes the relative rates of the two main types of detections in neutrino telescopes, using muons in charged current interactions and using the showers. It is also responsible for the relative sizes of the electromagnetic and hadronic showers induced in a charged current electron neutrino interaction. The knowledge of $y$ is also necessary for extracting the neutrino energy in high energy neutrino telescopes from the detected muon or hadronic shower. On the other hand it has been found that $\langle y \rangle$ is directly related to the $x$ slope of the parton distribution functions at small $x$ and $Q^2$ around $M_W^2$ [3]. If $\langle y \rangle$ could be extracted from events observed in high energy neutrino telescopes [13], it should provide a unique opportunity to study the low $x$ physics at high $Q^2$, well beyond the reach of present and near future accelerators.

It is very interesting to study the stability of both, the total cross-section and $\langle y \rangle$, to the consideration of nuclear effects in parton distributions. The average value of $y$ is obtained by integrating the differential cross-section: $\langle y \rangle = 1/\sigma \int_0^1 dy \, y \, d\sigma/dy$. The results for the total cross-section and the average $y$ are compared to the non-corrected ones in Figures 2 and 3. From Fig. 2 one sees that the departure from the total cross-section for free nucleons increases with energy and $A$ at the highest energies. For example, for $E_\nu = 10^{10}$ GeV, CC interaction and $A = 60$, the modified parton distribution functions result in a 10 % reduction of the total cross section. The regions of EMC effect, anti-shadowing and shadowing are apparent in Fig. 2.

The modification of $\langle y \rangle$ due to nuclear effects is presented in Fig. 3 for the case of neutrino-nucleon CC interaction. The correction to the results with partons in a free nucleon is less than 3 % for the largest nucleus and neutrino energies around $10^7$ GeV.

In conclusion, we have calculated the UHE neutrino-nucleon DIS cross section with parton distributions in bound nucleons to take into account nuclear effects. In comparison with the calculation using partons in free nucleons we found that the correction, which is mainly due to shadowing at small $x$, is of the order of few per cent for the parameter range of interest in neutrino telescopes ($A=10$ and $E=10^6$ GeV) and it reaches 20 % at the highest energies ($E=10^{12}$ GeV) and for the largest nuclear size ($A=190$) considered.
Acknowledgements

This work was supported by Xunta de Galicia under grant PGIDT00-PXI20615PR and CICYT grant AEN99-0589-C02-02.

References

[1] F. Halzen et al, Proc. of the 24th Int. Cosmic Ray Conference (ICRC), Rome 1995.

[2] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* 18 (1972) 438, 675;
L.N. Lipatov, *Sov. J. Nucl. Phys.* 20 (1975) 93;
G. Altarelli and G. Parisi, *Nucl.Phys.* B126, 298 (1977);
Yu.L. Dokshitzer, *Sov. Phys. JETP* 46 (1977) 641.

[3] J.A. Castro Pena, G.Parente, E. Zas, Preprint US-FT/9-00 hep-ph/0011043.

[4] M. Glück, S. Kretzer, E. Reya Astropart. Phys. 11, 327 (1999).

[5] R. Gandhi, C. Quigg, M.H. Reno, I. Sarcevic, Astropart. Phys. 5, 81 (1996); Phys. Rev. D 58, 093009 (1998).

[6] L.N. Lipatov, *Sov. J. Nucl. Phys.* 23 (1976) 642;
E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov. Phys. JETP* 44 (1976) 45; 45 (1977) 199;
Ya.Ya. Balitzki and L.N. Lipatov, *Sov. J. Nucl. Phys.* 28 (1978) 822;
L.N. Lipatov, *Sov. Phys. JETP* 63 (1986) 904.

[7] S. J. Brodsky, V. S. Fadin, V. T. Kim, L. N. Lipatov and G. B. Pivovarov, JETP Lett. 70 (1999) 155.

[8] K. J. Eskola, V. J. Kolhinen, C. A. Salgado, Eur.Phys.J. C9 (1999) 61-68.

[9] M. Arneodo, Phys. Rep. 240 (1994) 301.

[10] NMC collaboration, M. Arneodo et al., Nucl. Phys. B481 (1996) 23.

[11] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S Thorne, Eur. Phys. J. C4 (1998) 463.
[12] G. Parente and E. Zas, Proc. of the 24th Int. Cosmic Ray Conference (ICRC), Rome 1995, Vol. 7, p. 109-112.

[13] J. Alvarez-Muñiz, R. A. Vázquez, E. Zas Phys.Rev. D61 (2000) 023001

Figure captions

Figure 1. The ratio of nuclear parton (valence and sea quarks) distribution functions over free parton distributions as a function of the $x$-Bjorken variable for different values of $Q^2$.

Figure 2. The ratio of nuclear effects corrected over non corrected total neutrino-nucleon cross sections as a function of neutrino energy in the lab frame for different nuclei A.

Figure 3. The ratio of nuclear effects corrected over non corrected average inelasticity in $\nu N$ interaction for different nuclei A as a function of neutrino energy in the lab frame.
Corrections for the valence quarks

$R^A_{V}(x,Q^2)$

$Q^2=10^6 \text{ GeV}^2$

$q^2=4$

$A=60$

Log $x$

$R^A_{S}(x,Q^2)$

Corrections for the sea quarks

$Q^2=10^6 \text{ GeV}^2$

$q^2=10^4$

$q^2=4$

$A=60$

Fig. 1
Fig. 2
