Revised electrostatic model of the LISA Pathfinder inertial sensor

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Abstract. A comprehensive electrostatic finite-element (FE) analysis of the LISA Pathfinder Inertial Sensor (IS) has been carried out at Astrium GmbH. Starting with a detailed geometrical model of the IS housing and test mass (TM) flight units, FE results were derived from multiple analyses runs applying the Maxwell 3D field simulation software. The electrostatic forces and torques on the TM in 6DoF, as well as all capacitances between the TM, the 18 electrodes, and the housing, have been extracted for different TM translations and rotations. The results of the FE analyses were expected to confirm the existing IS electrostatic model predictions for forces and torques used in performance analyses, simulations, and on-board algorithms. Major discrepancies were found, however, between the FE results and the model used so far. As expected, FE results give considerably larger capacitance values than the equivalent infinite non-parallel plate estimates. In contrast, however, the FE derived forces and torques are significantly lower compared to the analytic IS electrostatic model predictions. In this paper, these results are discussed in detail and the reasons for the deviations are elaborated. Based on these results, an adapted analytic IS electrostatic model is proposed that reflects the electrostatic forces, torques, and stiffness values in the LISA Pathfinder IS significantly more accurate.

1. Motivation
The electrostatic FE analysis of the LISA Pathfinder IS has been conducted for the following reasons: First, the results are used to verify the approximations of the current IS electrostatic modeling applied by performance analyses, simulations, and on-board algorithms. Second, operational scenarios, such as the TM release strategy, are confirmed by analyzing the impact of the retractable grabbing plunger in close vicinity of the TM. Third, caging finger and TM interface design trades are supported and patch effects due to non-coated surfaces are analyzed. In this paper, the focus is on the first topic; specifically on the electrostatic forces and torques exerted onto the TM. The FE derived results are compared with the existing infinite non-parallel plate based analytic IS electrostatic model. This model is currently applied throughout the LISA Pathfinder project [1, 2, 3, 4].

2. FE Analysis Tool, IS Model, and Settings
All FE analyses results presented in this paper have been generated with the field simulation software MAXWELL 3D [5]. In order to verify the time consuming three-dimensional simulation results, significantly faster solutions have been computed independently using the field solver of MAXWELL 2D for a simplified two-dimensional geometrical model of the IS. Both electrostatic
field simulators provide a large amount of software specific settings that influence the surface approximation of the geometrical model, the convergence of the field solution, and the solution parameters. They allow for the direct calculation of the electrostatic forces, the torques, and the capacitance matrix for all specified conductors. The forces and torques are computed independently from the capacitances via the principal of virtual work; that is, by distorting the surface mesh.

A detailed three-dimensional geometrical model of the IS housing and the TM has been build from scratch in Maxwell using current flight unit mechanical interface drawings [6]. Consequently, conversion artifacts from CAD model importing have been avoided. The model of the IS used for the 3D analyses is depicted in figure 1. It consists of the 18 electrodes, the guard ring and housing structures, the eight caging fingers, the two grabbing plungers, and the test mass with its indentions and corner features.

In order to find the best trade-off between accuracy and computation time, and to quantify the accuracy of each solution, a significant amount of time was spent on assessing the electrostatic field solution specific software settings. Maxwell is equipped with an adaptive mesh generator that refines the mesh based on user defined convergence criteria. The local field error contribution to the total field energy was chosen as one of the stopping criteria to be smaller than 0.5%. At the same time, the surface approximation by the mesh was largely improved in order to cope with the intricate TM corner features. The electrostatic force or torque on the TM, depending on the conducted simulation run, were defined as additional error criteria in order to enable monitoring their convergence for each mesh refinement pass during the field solution computation.

The electrostatic forces and torques exerted onto the TM are generated by assigning corresponding realistic voltages on the 12 IS model actuation electrodes according to [2]. Multiple analyses runs have been carried out for varying TM translations (±200 µm) and rotations (±2 mrad). For each of these runs, the forces and torques in 6 DoF, as well as all capacitances in-between the TM, the 18 electrodes, and the housing, were computed. Least-squares polynomials have been fitted to the electrostatic force, the torque, and capacitance data in order to allow for the accurate prediction of their gradients and second order derivatives within the model accuracies.

3. Results and Discussion
Major discrepancies have been identified between the FE analyses results and the IS electrostatic model predictions used so far.
Table 1. Finite-element vs. current analytic model capacitances (for nominal TM position and orientation). Analytic capacitances are derived from electrode dimensions and gaps using the infinite non-parallel plate capacitor formula according to [1, 2]. Analytic $C_{T M,H}$ is adopted from [3]. $C_{T M}$ is computed as sum of all analytic capacitances.

| Conductors | Capacitance [pF] | Error [%] |
|------------|------------------|-----------|
| $C_{EL_{1-4},TM}$ | ($C_x$) | 1.18 | 1.09 | 7.8 |
| $C_{EL_{5-8},TM}$ | ($C_y$) | 0.95 | 0.83 | 12.5 |
| $C_{EL_{9-12},TM}$ | ($C_z$) | 0.70 | 0.61 | 13.0 |
| $C_{EL_{13-14},TM}$ | ($C_{y_{inj}}$) | 1.29 | 1.35 | 4.6 |
| $C_{EL_{15-18},TM}$ | ($C_{z_{inj}}$) | 0.41 | 0.43 | 3.8 |
| $C_{TM,H}$ | 18.67 | $\approx$ | 10.82 | 42.0 |
| $C_{TM}$ | ($C_{tot}$) | 34.21 | $\approx$ | 25.34 | 25.9 |

Table 1 compares the FE derived capacitance values with the current infinite non-parallel plate estimates for nominal (zero) TM position and orientation. For the actuation electrode capacitances ($C_{EL_{1-12},TM}$), and the capacitance between TM and housing ($C_{TM,H}$), the FE results give considerably larger values than the equivalent analytic estimates. In contrast, the capacitances between injection electrodes and test mass ($C_{EL_{13-18},TM}$) are a little lower. In sum, however, the total capacitance ($C_{TM}$) between the TM and its surrounding is more than 25% larger than its current estimate.

These differences in capacitance were expected and are attributable to the fact that the infinite non-parallel plate approximations ignore fringing field effects at the electrode edges and, thus, underestimate the capacitances by neglecting the effective area increase. The magnitude of the differences matches very well the analytic predictions of capacitance correction formulas given in literature for Kelvin guard-ring capacitors [7]. Furthermore, the results are consistent qualitatively with previous findings using the ANSYS FE software for a less mature IS geometrical model [4]. Since the injection electrodes are further recessed behind the guard rings compared to the actuation electrodes, see figure 2, the infinite non-parallel plate model overestimates their capacitance. That is, despite neglecting the fringing field, the dominating effect in this case is the effective area decrease due to field lines closing onto the neighboring guard ring rather than the TM.

In contrast to the capacitances, the FE derived forces and torques are in general significantly lower compared to the analytic IS electrostatic model predictions. This is shown in table 2, where FE derived forces and torques at nominal TM position and orientation are compared to the existing infinite non-parallel plate based analytic model estimates. For the forces along $x$ and $y$, as well as the torques around $\phi$ and $\theta$ that are generated by using the same actuation electrodes, the FE results give about 20% lower magnitudes. For the $z$-$\eta$ actuation couple, the relative error is as big as 30%.

These large differences in forces and torques were not expected and are attributable to the following reasons: First, the current analytic IS electrostatic model is based on infinite non-parallel plate capacitor approximations for the capacitances (gradients) between actuation electrodes and TM. Second, the model considers only these gradients and neglects any additional capacitances. For example, the capacitance between each electrode and its surrounding housing is neglected, which is driving the identified differences as detailed in the following.
Table 2. Finite-element vs. current analytic model forces and torques (for nominal TM position and orientation). RMS voltages, which correspond to the maximum IS forces and torques, are applied to all actuation electrodes simultaneously. The TM voltage is kept at zero potential. Analytic forces and torques, and RMS actuation voltages, are derived from the analytic IS electrostatic model according to [1, 2].

| DoF | Force [N]       | Error [%] |
|-----|-----------------|-----------|
|     | FE Analysis     | Analytic Model |
| x   | $2.12 \times 10^{-9}$ | $2.55 \times 10^{-9}$ | 20.2 |
| y   | $3.59 \times 10^{-9}$ | $4.32 \times 10^{-9}$ | 20.3 |
| z   | $5.49 \times 10^{-9}$ | $7.27 \times 10^{-9}$ | 32.5 |

| Torque [Nm] |
|-------------|
| $1.55 \times 10^{-11}$ | $1.87 \times 10^{-11}$ | 20.5 |
| $1.19 \times 10^{-11}$ | $1.59 \times 10^{-11}$ | 33.2 |
| $9.14 \times 10^{-12}$ | $1.10 \times 10^{-11}$ | 20.8 |

In table 3, the capacitances and their gradients relevant for the electrostatic force along $x$ are shown. Again, FE derived results and their equivalent analytic estimates, where available, are compared for the nominal TM position and orientation. The electrostatic force is proportional to the gradient of the capacitances w.r.t. the force direction and the square of the potentials applied. Hence, the first capacitance gradient serves as a measure of each individual conductor’s contribution to the overall force. The values clearly show that the capacitance gradient between actuation electrode and TM is the dominant force contributor. However, the gradient magnitude of the capacitance between each electrode and its surrounding housing ($C_{EL_{1-4},H}$) is in the order of 20% of the former with opposite sign, which coincides with the force error along $x$.

This result can be explained by the fact that the fringing fields at the actuation electrode edges change, depending on the test mass position altering the gap. The field lines will couple to their closest neighboring conductor. That is, by approximating the electric field lines to first order, depending on the shortest field line path either they close onto the guard ring or the TM. This effect is clearly visible in the electric field line plot, depicted in figure 2, which is

Figure 2. Electric field detail between electrode and TM in the $x$-$y$ plane. Visible on top is a cut through an $x$-actuation electrode and the laser hole. To the left a cut through a $y$-injection electrode recessed behind the guard rings is shown. Clearly visible is the fringing field at the electrode edges.
Table 3. Finite-element vs. current analytic model capacitance gradients (for nominal TM position and orientation). Analytic capacitances and gradients are derived from electrode dimensions and gaps using the infinite non-parallel plate capacitor formula according to [1, 2].

| Conductors          | Capacitance [pF] | $\partial C_{i,j}/\partial x$ [pF/m] | $\partial^2 C_{i,j}/\partial x^2$ [pF/m$^2$] |
|---------------------|------------------|---------------------------------------|-----------------------------------------------|
|                     | FE Analytic FE Analytic FE Analytic FE Analytic |                                      |
| $C_{EL_{1-4},TM}$   | 1.18             | 1.09                                  | $\pm300$                                     |
|                     |                  |                                       | $\pm273$                                      |
|                     |                  |                                       | $1.5 \times 10^5$                             |
|                     |                  |                                       | $1.4 \times 10^5$                             |
| $C_{EL_{1-4},H}$    | 5.94             | -                                     | $\mp70$                                      |
|                     |                  |                                       | $-1.6 \times 10^3$                            |
| $C_{EL_{1},EL_{2}}$ | $2.18 \times 10^{-4}$ | -                                     | $\mp0.32$                                     |
|                     |                  |                                       | $3.2 \times 10^2$                             |
| $C_{TM,H}$          | 18.67            | $\approx10.82$                       | 0                                             |
|                     |                  |                                       | $8.2 \times 10^5$                             |

generated from a MAXWELL analysis run. Thus, for a decreasing gap, the capacitance between the actuation electrodes and the TM is increased (positive gradient) while the capacitance between the actuation electrodes and the housing is decreased (negative gradient). The current analytic IS electrostatic model neglects the latter contribution. This explains the significantly larger forces and torques compared to the FE results. The in-between electrode capacitance gradient magnitude ($C_{EL_{i},EL_{j}}$) is two to three orders of magnitude lower than the other two gradients; hence, it contributes only negligibly to the electrostatic force calculation.

The second derivatives in table 3 can be viewed as a measure for the stiffness (force gradient) contributions of each conductor. In contrast to the electrostatic force, the contribution of the electrode to housing capacitance to the electrostatic stiffness is small; it is about two orders of magnitude lower than the electrode to TM capacitance. The in-between electrode capacitance contribution, again, is three orders of magnitude smaller; thus, it can be safely neglected. The capacitance between TM and housing ($C_{TM,H}$), however, is significantly impacting the electrostatic stiffness. Its second derivative is about a factor of five larger than the electrode to TM capacitance contribution.

4. Improved Analytic IS Electrostatic Model

The inertial sensor electrostatic system can be described fundamentally by the schematic configuration of conductors and capacitances depicted in figure 3. As discussed above, in the current analytic IS electrostatic model the forces and torques exerted on the TM by the actuation electrodes are computed taking into account only the gradient of the electrode to test mass capacitance ($C_{EL_{i},TM}$) w.r.t. the force direction. This results in a force and torque error of about 20–30% when compared to FE electrostatic analyses results. The main reason for this effect was shown to be the capacitance (gradient) between each electrode and its surrounding housing that has been neglected so far. As a consequence of this, an improved electrostatic force and torque model taking into account all capacitive gradients in the inertial sensor system is derived. Such a mathematical model in its most general form, derived from standard energetic considerations of the electrostatic field of conductors, is defined as

$$F_q = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{\partial C_{i,j}}{\partial q} (V_i - V_j)^2 \quad (1)$$

Here, $q$ is the generalized coordinate, $V_i$ and $V_j$ are any two conductor potentials in a system of $N$ conductors, and $C_{i,j}$ is the corresponding capacitance between them.
Using the nomenclature of figure 3, the improved IS specific force and torque model can be derived to

\[ F_q = \frac{1}{2} \sum_{i=1}^{18} \frac{\partial C_{EL_i,TM}}{\partial q} (V_i - V_{TM})^2 + \frac{1}{2} \frac{\partial C_{TM,H}}{\partial q} V_{TM}^2 \]  

(2a)

\[ + \frac{1}{2} \sum_{i=1}^{18} \frac{\partial C_{EL_i,H}}{\partial q} V_i^2 \]  

(2b)

\[ + \frac{1}{2} \sum_{i=1}^{18} \sum_{j=i+1}^{18} \frac{\partial C_{EL_i,EL_j}}{\partial q} (V_i - V_j)^2 \]  

(2c)

Here, the term (2a) represents the existing IS electrostatic model, the term (2b) accounts for the non-negligible capacitance gradient between each electrode and its surrounding housing, and the last term (2c) represents the negligible contribution of the in-between electrode capacitance gradient. The test mass potential \( V_{TM} \) used in above equation is defined by

\[ V_{TM} = \frac{\sum_{k=1}^{18} C_{EL_k,TM} V_k + Q_{TM}}{C_{tot}} \]  

(3)

with \( Q_{TM} \) as the accumulated charge on the TM and \( C_{tot} = \sum_{k=1}^{18} C_{EL_k,TM} + C_{TM,H} \).

Without taking into account the term (2b), using FE derived results for the electrode to TM capacitances (gradients), the force and torque error is about 30–50% when compared to FE electrostatic analyses results. The improved analytic IS electrostatic model, however, including the term (2b), reflects the FE derived forces and torques to better than 1% when using FE derived results for both the electrode to TM and electrode to housing capacitances (gradients).

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