Solar Wind Turbulence Outlined Through Magnetic Islands and Nonlinear Waves

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Various space missions and observations over the past decades have provided unexampled details about the nature of solar wind, the acceleration mechanism, and different nonlinear phenomena responsible for energy transfer and turbulence in the interplanetary space. This review focuses on the role of Alfvénic fluctuations—both kinetic Alfvén wave (KAW) and dispersive Alfvén wave (DAW)—in driving solar wind turbulence and magnetic reconnection at 1 AU. The process of filamentation has been studied through a nonlinear coupling system of KAW/IAW (ion acoustic wave) and relatively high-frequency pump KAW (HKAW, i.e., frequency less than ion cyclotron frequency) in the presence of LKAW (low-frequency KAW, i.e., frequency very much less then ion cyclotron frequency) perturbation by formulating their dynamical equations in the presence of ponderomotive force and using the numerical results for the same. A simplified model is presented to have a deeper insight into the evolution pattern using the results of simulation. The formation of coherent structures and current sheets using a numerical and semi-analytical approach is elaborated near the magnetic reconnection sites. In addition to this, the relevance of the generated turbulence is also depicted through the energy spectrum by examining the spectral index which is noticeable in determining the energy cascade down to smaller scales.

Keywords: turbulence, magnetic islands, energy spectrum, solar wind, Alfvén waves

INTRODUCTION

Solar wind predominantly permeates the whole heliosphere, providing an indispensable medium to study collisionless plasma. It develops a strong turbulent character on expansion, and for decades, intensive efforts have been directed to understand solar wind and its turbulent nature through various space missions like SOHO (Solar Heliospheric Observatory), Ulysses, Voyager, Helios, Yohkoh, FREJA, POLAR, FAST, Cluster, and TRACE (Transition Region and Coronal Explorer). Emanating from the solar corona and expanding outward into space (Parker, 1958), solar wind acts as a natural laboratory (Bruno and Carbone, 2013) for in situ spacecraft measurements to investigate solar wind turbulence. To understand the turbulent system and perform multipoint measurements, the NASA MIDEX mission HelioSwarm is in a Phase-A study (Klein, 2019; Spence, 2019; Hautaluoma and Fox, 2020). The latest NASA mission, the Parker Solar Probe, is designed to probe the mechanisms leading to acceleration of solar wind. At the closest approach to the Sun, the Parker Solar Probe has progressed one step toward reaching the Sun to explore the mysteries of the evolution of the Sun and provide deeper insights into the flow of energy from the solar corona to the
accelerating solar wind. The evolution of astrophysical environments is significantly affected by the turbulent cascade of energy and, so to identify with the dynamics of energy dissipation, is therefore of cardinal importance to the astrophysics communities.

In a turbulent medium like solar wind, the magnetic field lines constantly break and reconnect at some scales (Franci et al., 2017; Mallet et al., 2017; Vech et al., 2018), making magnetic reconnection an inherent part of a turbulent cascade. This process is associated with the release of large amount of energy and is responsible for the transfer of energy between different length scales, thereby leading to the acceleration of particles in solar wind (Lazarian et al., 2020). This review presents an outline of the solar wind plasma portraying the influence of the nonlinear wave–wave interactions (Kraichnan, 1965; Howes & Nielson, 2013; Iroshnikov P.S., 1964; Pezzi et al., 2017a, 2017b; Roberts et al., 2017; Narita, 2018; Roberts et al., 2022) and their role in understanding the nature of turbulence through the transfer of energy from large scales to smaller scales. The understanding of mechanisms which play a decisive part in creation of astrophysical environments through the dissipation of turbulent energy to heat has always been an issue of debate. Different schools of thought have structured this debate. In the first, Alfvénic turbulence is considered one of the processes responsible for the transfer of energy (Coleman, 1968; Isenberg and Hollweg, 1982; Tu et al., 1984; Hu et al., 2000; Isenberg, 2004). Being low-frequency magnetohydrodynamic (MHD) modes and difficult to dissipate, they are able to permeate the whole solar atmosphere and are ubiquitously present in the solar wind. An illustrative characteristic of magnetized turbulence is the propensity to form sheets of current density that are liable to magnetic reconnection (Matthaeus & Lamkin, 1986; Biskamp & Welzer, 1989; Dmitruk & Matthaeus, 2006; Retinò et al., 2007; Servidio et al., 2009; Comisso and Sironi, 2019). These reconnecting current sheets are typical sites of particle acceleration and magnetic energy dissipation (Dmitruk et al., 2004). Concurrently, it has long been known that particles can gain energy through random scattering by turbulence fluctuations (Kulsrud & Ferrari, 1971). Therefore, turbulence fluctuations and magnetic reconnection work in alliance, and for a comprehensive understanding of the physics in a turbulent surrounding we need to have a detailed examination of their interplay.

The solar wind is inhabited by Alfvénic fluctuations spanning from fraction of a second to several hours. Their presence in the astrophysical plasma is endorsed by various observational evidences (Belcher and Davis, 1971; Cirtain et al., 2007; He et al., 2009; Okamoto et al., 2007). The restoring force and inertia for the Alfvén wave are provided by the magnetic tension and the ion mass, respectively. Ideal MHD equations administer the propagation of this non-dispersive Alfvén wave which is now called the shear Alfvén wave. The Alfvénic wave disturbance moves with no attenuation with distance along the background magnetic field, and this remarkable property has important ramifications for the transport of energy in plasma fluids. The nonlinear phenomena are of prominent interest to properly assess the acceleration and transportation of the particles in the solar wind. Alfvén waves may come across instabilities and may convert into other dispersive modes which may provide a pathway to carry large amounts of energy and then dissipate it as heat (Bogdan et al., 2003).

When the Alfvén waves attain wavelengths comparable to the ion gyroradius in the direction perpendicular to the background magnetic field, they are known as kinetic Alfvén waves (KAWs) (Podesta, 2013; Narita et al., 2020). KAWs are believed to play an indispensable role in particle heating and acceleration mechanisms (Wu, 2003; Wu and Chao, 2004). The transfer of energy from MHD scales to kinetic scales is also possible through the interaction of KAWs with large-scale MHD waves (Zhao et al., 2011). The in situ measurements in the solar wind back up the idea of an Alfvén wave turbulent cascade in the perpendicular direction at the ion or electron scales producing the KAWs (Howes et al., 2008a, Howes et al., 2008b, Howes et al., 2011; Alexandrova et al., 2013; Sahraoui et al., 2009; Zhao et al., 2013). KAWs are also dominating at proton kinetic scales (He et al., 2009; Roberts et al., 2017) and also shorter scales (Chen et al., 2010a). In addition, they can also couple with high-frequency modes (Zhao et al., 2013). The shear and kinetic Alfvén waves account for various nonlinear effects like parametric processes such as three-wave decay interactions, modulational instability, and the background plasma number density being modified by the ponderomotive force of the Alfvén wave. The nonlinear interaction of the KAW and shear Alfvén wave has been intensively studied by many authors and energy transfer processes so as to interpret the observations to have a deeper insight into the turbulent cascade of energy (Zhao et al., 2013). With the finite frequency correction, i.e., when the frequency of the Alven wave becomes comparable to but less than the ion cyclotron frequency, we get circularly polarized dispersive Alfvén waves (DAWs). This dispersion of the wave also occurs when the wavelength is around the ion inertial length, which many authors (Meyrand & Galtier, 2012; Ghosh, et al., 1996) consider the Hall term, and this Hall term contributes significantly to the transfer of energy over and across the small scales as well as has an important role in the increased reconnection rate (Shay et al., 2001; Smith et al., 2004). The interaction of these dispersive Alfvén waves undergoing filamentation with the pre-existing chain of magnetic islands may also contribute toward the turbulent cascade of energy in the solar wind.

Nonlinear processes being dominant in the transfer of energy from long wavelength magnetic fluctuations to shorter wavelengths, the study of the frequency spectrum is of utmost importance to unravel the nature of turbulent plasma. The energy spectrum depicting the turbulence scaling comprises the energy injection scales followed by the energy-containing scale, known as the forcing range or injective range, marking the energy source. Next, the “inertial range,” also called the intermediate range, marks the energy transfer process to smaller scales. This range is often described as the “dissipation range,” “dispersion range,” or the “scattering range.” In this range, the fluctuations are converted to thermal energy, thereby causing the heating of particles. For magnetic fluctuations at very large scales (i.e. $f \lesssim 10^{-3}$ Hz), the power spectra go as $k^{-4}$. This range is the energy
reservoir feeding the turbulent cascade and so called the 1/f range (Matthaeus and Goldstein 1986; Chandran 2018; Matteini et al., 2018). Various measurements account for the Kolmogorov $k^{-5/3}$ spectrum in the inertial range ($10^{-3} \text{ Hz} \leq f \leq 10^{-2} \text{ Hz}$) followed by steepening at ion kinetic scales. A spectral break appears around $k_{p}\rho_{i} > 1$ (where $\rho_{i}$ is the ion gyroradius), and the spectral index lies between -2 and -5 (Bale et al., 2005; Shaikh and Shukla, 2009; Sahraoui et al., 2009). Between the ion scales and the electron scales, a small scale turbulent cascade is established (Alexandrova 2009). As the turbulent cascade crosses the ion scales and before reaching the electron scales (i.e. $3 \leq f \leq 30 \text{ Hz}$), the magnetic spectra follow $\sim k^{-2.8}$ (Alexandrova et al., 2009; Chen et al., 2010a; Sahraoui et al., 2010). The spectral index is conspicuous of the energy transfer phenomena. Many pioneering works have been carried out to understand the dynamics and the different nonlinear processes responsible for this steepening (Leamon et al., 2000; Sahraoui et al. 2010; Rudakov et al., 2011).

The prime candidates contributing to the spectral properties and the steepening of the spectra are the various nonlinear processes like transverse collapse or filamentation due to KAW, its interaction with other wave modes, and/or the formation of coherent (magnetic) structures resulting from the current sheets. In the literature, the analysis of the field fluctuations from the proton to the electron scales shows the presence of current sheets or possible coherent structures of sub-proton scales which are possible sites of magnetic reconnection and energy dissipation (Perri et al., 2012). The current sheets may be formed self-consistently from the Alfvén wave-driven turbulence as discussed by Tanbarge & Howes, (2013). These coherent structures play an important role in driving the nonlinear transfer of energy to the smaller scales as they make some additional self-consistent energy injection available (Ma et al., 1995; Sturrock et al., 1999) and thus support the continuation of the turbulence over the smaller scales (Cerri & Califano, 2017). Therefore, it is required to further examine if this turbulent energy transfer across and below the ion scales takes place as a result of instabilities or some other mechanism such as magnetic reconnection that causes the formation of coherent/ localized structures or if it is both. A plethora of studies show that instabilities such as those driven by the temperature anisotropy (Gary and Lee, 1994; Verscharen et al., 2014), stream-shear-driven instabilities (Roberts et al., 1992), and parametric instabilities (Longtin and Sonnerup, 1986; Brodin and Stenflo, 1988; Viñas and Goldstein, 1991; Stenflo and Shukla, 2007; Primavera et al., 2019) such as filamentation, all may possibly lead to the dissipation of solar wind turbulence. At the same time, observations also show that reconnection can typically cause turbulence in the solar wind (Vörös et al., 2014). The spectral scaling of these fluctuations driven by the reconnection flow (obtained using the WIND spacecraft data) bears resemblance with the observations in the inertial and dispersive regime of the solar wind. Thus, these two processes, that is, turbulence and magnetic reconnection, work in alliance, and an associative study is required to be carried out.

Here, we revisit the nonlinear effects caused by dispersive Alfvén waves and KAWs to comprehend the dynamics of solar wind turbulence. Part A discusses the nonlinear effects and the turbulence due to the KAW presenting the effect of initial conditions on the spectra and the detailed explanation governing the evolution pattern through a simplified model. In part B, we study the nonlinear evolution of dispersive Alfvén waves in the vicinity of pre-existing chains of magnetic islands. The wave becomes dispersive due to the wave frequency which is finite but less than the ion cyclotron frequency.

### A. TURBULENCE DUE TO THE KINETIC ALFVÉN WAVE

Many simulations backed up with observations have led to a deeper understanding of turbulence at 1 AU (Howes and Qutaert, 2010; Sahraoui et al., 2010; Narita et al., 2020) unwinding the energy transfer processes. This section predominantly focuses on the role of KAWs in understanding the dissipation of energy through different nonlinear processes. One of the processes leading to energy transfer is filamentation or transverse collapse which has been widely investigated. The sensitivity of the nonlinear coupling of KAWs/IAWs resulting in the transverse collapse on different initial conditions is addressed. Considering the wave propagation in the x-z plane and background field $B_{0}z$ and using a two-fluid model, the following system of equations is derived in the dimensionless form (Gaur and Sharma, 2015):

$$
\begin{align*}
\frac{\partial B_{y}}{\partial t} + i \frac{\partial B_{x}}{\partial z} + 2\chi_{c} \frac{\partial B_{x}}{\partial x} + \frac{\partial^{2} B_{x}}{\partial x^{2}} + c_{s}^{2} \frac{\partial B_{y}}{\partial z} + nB_{y} &= 0, \\
\frac{\partial^{2} n}{\partial t^{2}} - \frac{\partial^{2} B_{x}}{\partial t \partial x} - \beta \frac{\partial^{2} n}{\partial z^{2}} &= \frac{\partial^{2}}{\partial t^{2}} \frac{\partial}{\partial x} B_{y} - \beta \frac{\partial^{2} |B_{y}|}{\partial z^{2}}.
\end{align*}
$$

(1)

The localized structures of KAWs with the magnetic field $B_{y}$ in the transverse direction are studied at 1 AU under different initial conditions by performing numerical simulation of this set of equations. Here, $c_{1} = k_{0} \rho_{i}$, $c_{2} = k_{0} \rho_{i}^{2}/4$, and $\beta = c_{i}^{2}/V_{A}^{2}$. The equation is normalized using the parameters $x_{n} = \rho_{i}$, $z_{n} = 2/k_{0}$, $t_{n} = (2\omega/\Omega V_{A}^{2} k_{0}^{2})$, $n_{0} = n_{0}$, and $B_{0} = [\{1 - \eta(1 + \delta)|V_{A}^{2} k_{0}^{2}/16\pi n_{0} T_{i} \omega^{2} \}^{1/2}$. Here, $\eta = \omega_{c}^{2} r_{0}$ and $\delta = m_{2}^{2} c_{s}^{2}/$ and $T_{e} + T_{i} = T_{0}$, $V_{A}$ is the perpendicular wave vector component and $k_{0}$ is the parallel wave vector component to $zB_{0}$, where $\omega$ refers to the KAW frequency, $c_{s}$ is the acoustic speed, $V_{A}$ is the Alfvén speed, and $\rho_{i}$ is the ion acoustic gyroradius. Three different perturbations imposed on a uniform plane KAW are periodic perturbation (IC-1), Gaussian perturbation (IC-2), and the third random perturbation (IC-3):

$$
\begin{align*}
\text{IC-1} & \quad B_{y}(x, z, 0) = B_{0} y (1 + \epsilon \cos(\alpha_{x} x))(1 + \epsilon \cos(\alpha_{z} z)) n(x, z, 0) \\
& = |B_{y}(x, z, 0)|^{2} \\
\text{IC-2} & \quad B_{y}(x, z, 0) = B_{0} y (1 + \epsilon \exp(-x^{2}/r_{01}^{2}))(1 + \epsilon \exp(-z^{2}/r_{01}^{2})) \\
\text{IC-3} & \quad B_{y}(x, z, 0) = B_{0} y (1 + \epsilon \exp(2\pi i \theta(x)))(1 + \epsilon \exp(2\pi i \theta(z)))
\end{align*}
$$
Here, the initial amplitude of the main KAW is $B_{yk} = 1$, $\varepsilon = 0.1$ is the magnitude of the perturbation, $r_{01}$ (normalized by $x_0$) is the transverse scale size of the perturbation, $r_{02}$ (normalized by $z_0$) is the longitudinal scale size of the perturbation, and $\theta(x)$ and $\theta(z)$ are the random variables uniformly distributed on [0,1] (Sharma et al., 1996). The random value for theta is attributed at each grid point in the x–z plane. A seed value was initially provided, and random variables were generated and uniformly distributed on [0,1] in the x–z plane.

Figures 1A–3A,C present the simulation results indicating the splitting of pump KAW at different instances of time with periodic, Gaussian, and random perturbation forms, respectively, in real space alongside contour plots in the Fourier space, i.e., Figures 1B–3B,D. The figures demonstrate that there is no regularity in the filament formation with the formation of the most intense structures at the early time and an increase in complexity at later times. The contour plot shows the dependence of $|B_k|^2$ on Fourier modes. As is noticeable in these figures, the filamentary structures obtained have different intensities and patterns. Also, there is a varied scale size of the structures under the three initial conditions ($\approx 0.3 \rho_a$ with periodic, $\approx 1.6 \rho_a$ with Gaussian, and $\approx 0.5 \rho_a$ with random perturbation). For $\rho > 1$, the transverse size of Alfvén vortex tubes is of the order of $\rho_a$ (Chmyrev et al., 1988).

Figure 4 illustrates the effect of formation of localized structures on the wave number spectrum. In the power spectra obtained by plotting $|B_k|^2$ against $k$, $k_z = 0$, the scaling law approaches $k^{-3/2}$ for $k < 1$ followed by a spectral break at around $k \approx 1$. The scaling index for $k > 1$ shows dependence on the initial conditions being $k^{-5/6}$ for periodic, $k^{-2.2}$ for Gaussian, and $k^{-2.6}$ for the random forms of perturbation.

**A Simplified Model: A Deeper Insight Into the Evolution Pattern of Kinetic Alfvén Wave**

Implementing the conditions $\partial_x B_y > k_{0x} B_y$ and $\partial_z B_y < k_{0z} B_y$ on the aforementioned system of equations in the un-normalized form (Sharma et al., 2011a), the dynamics of the evolution are studied in an extended paraxial regime semi-analytically. As elaborately discussed by Sharma et al. (2014), the equation for dimensionless beam width parameter $f_0$ is obtained as (using normalization distance $\xi = R_d$, where $R_d = k_0 \tau_0^2$):

\[
\frac{\partial^2 f_0}{\partial k^2} = \frac{1}{a^2 f_0} (1 - 2a_0^2 - 2a_0 + 6a_1) - \frac{R_{dx}^2 R_{dy}^2}{a_0^2 f_0^2} (1 + a_0) - \frac{R_{dx}^2 a_0^2}{2!} \sum_{m=1}^{4} n_m m^2.
\]

(a)

In the RHS of the aforementioned equation, the finite transverse size of the KAW accounted by the first term causes diffraction, and the nonlinearity is marked by the last three terms. The equation represents the interplay of diffraction, and
FIGURE 2 | With IC-2. (A) Normalized intensity profile of the KAW magnetic field \( |B_x^2|/B_0^2 \) at normalized \( t = 6 \) (\( t \) normalized by \( t_n = 2\omega/\sqrt{V_2/A^2 k_0 z} \)). (B) Contours of normalized \( |B_x^2| \) against Fourier modes of KAW at \( t = 6 \). (C) Normalized magnetic field intensity profile of KAW at \( t = 13 \). (D) Contours of normalized \( |B_x^2| \) against Fourier modes of KAW at \( t = 13 \). "Reproduced from [Gaur and Sharma (2014), Astrophys Space Sci (2014) 350, https://doi.org/10.1007/s10509-013-17701], with the permission of Springer Publishing.

FIGURE 3 | With IC-3. (A) Normalized intensity profile of the KAW magnetic field \( |B_x^2|/B_0^2 \) at normalized \( t = 6 \) (\( t \) normalized by \( t_n = 2\omega/\sqrt{V_2/A^2 k_0 z} \)). (B) Contours of normalized \( |B_x^2| \) against Fourier modes of KAW at \( t = 6 \). (C) Normalized magnetic field intensity profile of KAW at \( t = 13 \). (D) Contours of normalized \( |B_x^2| \) against Fourier modes of KAW at \( t = 13 \). "Reproduced from [Gaur et al. (2014), Astrophys Space Sci (2014) 350, https://doi.org/10.1007/s10509-013-17701], with the permission of Springer Publishing."
nonlinear terms indicate the divergence less travel of the wave when these terms balance each other. The equation for $S_{02}$ is obtained as:

\[
\frac{\partial S_{02}}{\partial \zeta} = -\frac{\rho_r^2}{2 f_0^2 f_0^2} (11 a_0 a_1 - 2 a_0^3 - 2 a_0^2 + 4 a_1) + \frac{y B_{00}^2}{4 f_0^2} + \frac{a_1 y B_{00}^3}{2 f_0^2} - \frac{a_0 y B_{00}^2}{2 f_0^2} + \frac{r_0^2 a_0^4 n_1}{2 (4!) \sum_{n} n_m m^4}.
\]

The Equations for $a_0$ and $a_1$ are expressed as:

\[
\frac{\partial a_0}{\partial \zeta} = \frac{12 S_{02} f_0^2}{ar_0^2},
\]

\[
\frac{\partial a_1}{\partial \zeta} = (8 - 20 a_0) \frac{S_{02} f_0^2}{ar_0^2}.
\]

Here, $B_{00}$ is the initial wave field at $z = 0$, $a_0$ and $a_1$ are the coefficients of $x^2$ and $x^4$, respectively, $S_{00}$ is the slowly varying functions of $x$ and $z$, $r_0$ is the transverse scale size of the wave, and $y = 1/B_{00}^2$ is the normalization factor. The system of coupled equations (a)-(d) is numerically solved using the fourth-order Runge–Kutta method for a plane wave front under initial conditions being $f_0 = 1$ and $d f_0 / dz = 0$ at $z = 0$ and $a_0 = a_1 = 0$ at $z = 0$.

As illustrated in Figure 5. The wave intensity becomes high when the parameter $f_0$ takes a minimum value and vice versa. The time dependence in the model is consolidated through the dependency of $n$ on time (using simulation results). As time progresses, the intensity and localization pattern of the KAW changes through the change in density harmonics.

**Nonlinear Effects due to KAW**

Various wave modes interact with KAW to comprehend the turbulence in the space plasma. Other wave modes are excited by the ponderomotive force of KAW which leads to modification in density resulting in the nonlinear dynamics of KAW. Low-frequency KAW (LKAW) is also one of the wave modes present in the plasma, and its excitation by the ponderomotive force of relatively high-frequency, high-power pump KAW is analyzed to study its effect on the solar wind turbulence. With cold plasma assumption and two fluid models (separate ion and electron motions) in the solar wind regime, the dynamical equations are formulated and solved numerically to study the KAW evolution and the power spectra at 1 AU [Gaur, N and R.P Sharma, 2014]:

\[
\begin{align*}
&i \frac{\partial B_x}{\partial t} + i \frac{\partial B_y}{\partial z} + 2i c_1 \frac{\partial B_x}{\partial x} + \frac{\partial^2 B_x}{\partial x^2} + c_2 \frac{\partial^2 B_y}{\partial z^2} + n B_y = 0, \\
&\frac{\partial^2 n}{\partial t^2} - c_1 \frac{\partial^2 n}{\partial x^2} + c_2 \frac{\partial^2 n}{\partial z^2} = -c_3 \frac{\partial^4}{\partial x^2 \partial z^2} |B_y|^2 \frac{\partial^4 |B_y|^2}{\partial x^2 \partial z^2}.
\end{align*}
\]

This system of equations is solved under the following initial conditions (ICs) for the magnetic field and density perturbation:
The initial amplitude of the pump wave is \( B_{y0} = 1, a_1 = 0.1 \) for density perturbation at \( t = 0 \), and \( \varepsilon \) represents the magnitude of the perturbation. The normalized transverse and longitudinal scale size of the perturbation is \( r_{01} \) and \( r_{02} \), respectively. Here, \( r_x \) and \( r_z \) are the random variables uniformly distributed on [0,1].

The filamentation process giving rise to strong magnetic filaments parallel to the ambient field is demonstrated under three initial conditions. The effect of the perturbation wave number is shown in the Figures 6–8 under different initial conditions. It exemplifies the dependence of nonlinear evolution patterns on the values of \( \alpha_x, \alpha_z \), and as shown, the structures become highly irregular and random with increased values of the perturbation wave number.

\[
B_y(x, z, 0) = B_{y0}(1 + \varepsilon \cos(\alpha_x x))(1 + \varepsilon \cos(\alpha_z z))n(x, z, 0), \quad \text{(IC - 1)}
\]

\[
B_y(x, z, 0) = B_{y0}(1 + \varepsilon \cos(\alpha_x x))(1 + \varepsilon \cos(\alpha_z z))n(x, z, 0), \quad \text{(IC - 2)}
\]

\[
B_y(x, z, 0) = B_{y0}(1 + \varepsilon \cos(\alpha_x x))(1 + \varepsilon \cos(\alpha_z z))n(x, z, 0), \quad \text{(IC - 3)}
\]
The energy distribution is further illustrated in the contour plots of $B_{yk}$ in the $(k_x,k_z)$ Fourier space in Figure 9 which clearly highlight the confinement of energy to low $k_x,k_z$ wave numbers at initial times and its distribution to higher wave numbers at later times (energy flow from larger scales to smaller scales).

Finally, the averaged power spectra for all the three initial conditions are also shown to analyze the energy transfer through the localization process in Figure 10A with IC-1 and Figure 10B with IC-2 and IC-3. For $k < 1$, the power spectrum nearly follows the Kolmogorov scaling, and for $k > 1$, the power spectrum shows steepening (for reference, the green line with a spectral index of nearly -2.6 is shown, indicating this nonlinear interaction may lead to the distribution of energy at $k > 1$ (Saharoui, 2012).

The role of kinetic Alfvén turbulence in collisionless high beta-plasmas is well documented, and the present model may be a step further to understand the turbulence in the solar wind (Boldyrev and Perez, 2012; Howes et al., 2008b).

B. Magnetic Island-Based Dispersive Alfvén Wave Model

Since turbulence and magnetic reconnection work in alliance, therefore for a comprehensive study, it is important to examine their interplay. For this, we consider the existence of a fully developed pre-existing chain of magnetic islands in the background of the parallel propagating dispersive Alfvén wave. The dispersion in the wave is considered due to the wave frequency which is finite but less than the ion cyclotron frequency. For the study, a uniform background magnetic field ($B_{0z}$) is considered in the $z$ direction, and the magnetic field ($\delta B_y$) as a result of the magnetic islands is assumed along the $y$ direction. Therefore, our wave (DAW) subjected to a transverse instability or filamentation is propagating in the vicinity of the total magnetic field, $\tilde{B}_y = B_{0z}z + \delta B_y(x,y)y$. The wave dynamical equation in terms of dimensionless flux
function is obtained using a two-fluid model as follows (Sharma et al., 2020):

\[
\frac{\partial \tilde{A}_1}{\partial z} + c_1 \left( \frac{\partial^2 \tilde{A}_1}{\partial x^2} + \frac{\partial^2 \tilde{A}_1}{\partial y^2} \right) - 2 \left( \frac{(x + x_{01})^2}{2} - \frac{(x - x_{02})^2}{2} + b_{01} \cos(k_{1y}') + b_{02} \cos(k_{2y}') \right) \tilde{A}_1 \\
+ |\tilde{A}_1|^2 \tilde{A}_1 = 0.
\]

In the aforementioned equation, \( \tilde{A}_1 \) is the right circularly polarized DAW amplitude, \( c_1 = \frac{c}{\omega_{pi}} \) (1 + \( \frac{\omega}{\omega_{ci}} \)) is constant, \( \lambda_i = c/\omega_{pi} \) is the ion inertial length, \( \rho_s = c/\omega_{ci} \) is the ion gyroradius, \( k_0 = \frac{\omega}{\omega_{ce}} \) is the wave number of propagation, \( \varepsilon_{\|0} = \frac{\omega_{pe}^2}{\omega_{ce}^2} \) is the linear part of dielectric, and \( \omega_0 \) is the frequency of DAW. Here, \( x_{01} = x_{02} = 0.1, k_1 = k_2 = 0.2, b_{01} = 0.5 \) and \( b_{02} = 0.3 \) are the magnetic island parameters with \( b_{01}, b_{02} \) as the magnitude of the perturbation and \( k_1, k_2 \) is the wave number of perturbation of the magnetic island. The normalization parameters are: \( x_n = y_n = \lambda_i, z_n = 2/k_0, n_n = n_0, A_n = 0.1/\sqrt{\alpha_0} \), where \( \alpha_0 = \frac{\omega_{pe}^2/\omega_{ci}^2 (1 - \frac{\omega}{\omega_{ce}})}{2\sigma \rho_{ci}^2 T (\omega_{ce}^2 - \omega_0^2)} \). The last two terms of the aforementioned equation are the fluctuations attributable to the
existence of magnetic islands and density perturbations, respectively.

For typical solar wind parameters, $B_0 \approx 6.2 \times 10^{-5}$ G, $T_e \approx 1.4 \times 10^5$ K, $T_i \approx 5.8 \times 10^5$ K, and $n_0 \approx 3$ cm$^{-3}$ (Wilson et al., 2018), the dynamical equation is solved numerically under the following initial condition:

$$\tilde{A}_1 (x, y, z = 0) = \cos (2x + 2.3) + \cos (y + 4.1).$$

The numerical value of the wave parameters is calculated as $\omega_{ci} \approx 0.594$ Hz, $V_A \approx 7.815 \times 10^6$ cm/s, $c_s \approx 7.713 \times 10^6$ cm/s, $\omega_{pe} \approx 9.767 \times 10^4$ Hz, $\rho_s = 1.298 \times 10^7$ cm, and $\lambda_i = 1.315 \times 10^7$ cm. For the wave with frequency, $\omega_0 = 0.8 \omega_{ci}$, the wave number is $k_\perp = 4.531 \times 10^{-6}$ cm$^{-1}$.

The results depict that the fluctuations in the field occurring due to the existence of the chain of magnetic islands may induce localization, and the amplitude of these localized structures increases with z as shown in Figure 11. As clearly visible from the contour plot in Figure 11, the symmetry in the patterns is maintained only at the early stages, and as we evolve along z, this symmetry is broken, and highly irregular structures are obtained. The characteristic scale size of these coherent/localized structures is of the order of ion inertial length scales. For DAW propagating under the influence of both the factors, that is, density fluctuations as well as magnetic islands, these localized structures seem to be more intense and well evolved (Figure 12) compared to the case when DAW was propagating under the influence of magnetic islands only. Thus, these results clearly depict that nonlinearity
supports the formation of localized structures, thereby aiding the generation of turbulence.

The localized structures unveil the turbulent behavior that may further result in the development of current sheets. The current density plot of DAW as shown in Figure 13 explains that in the beginning, there is a symmetrical distribution of current in the x–y plane, but for larger z, it grows into several asymmetric and irregular structures. The size of these current sheets is found to be of the order of sub-ion scale length.

The power spectrum of DAW (Figure 14) shows that the energy distribution takes place from larger length scales to smaller length scales. For \( k \lambda_i \approx 1 \), the fluctuations start deviating from the typical Kolmogorov’s scaling, and the spectra go steeper beyond the ion inertial length (\( k \lambda_i > 1 \)).
observed scaling exponents bear resemblance with the observations in the inertial and dispersive regime in the solar wind (Alexandrova et al., 2008; Voros et al., 2014). Although the study is restricted to limited dispersion of the wave and nonlinear evolution of the perturbation, it does provide an assessment of the turbulent energy transfer and coherent structures/current sheet formation in the vicinity of magnetic islands and the ponderomotive nonlinearity which is useful for future studies.

**Semi-Analytical Method**

For a better insight into the development of coherent structures and to determine their scale size, a semi-analytical approach is adopted for the aforementioned model equation as discussed by Sharma et al., 2020. Within the paraxial limit \( (x < < r_{01} f_1 \text{ and } y < < r_{02} f_2) \), where \( r_{01} \) and \( r_{02} \) are the transverse scale size of the DAW, following Akhmanov et al. (1967), the dimensionless beam width parameters \( f_1 \) and \( f_2 \) can be obtained as:
where $R_{d1} = k \cdot r_0^2$ and $R_{d2} = k \cdot r_2^2$ and $E_{00}$ is the amplitude of the wave. The aforementioned equations are solved using the Runge–Kutta method under the initial conditions $\frac{d f_1}{d z} = 0 \text{ at } z = 0 \text{ and } f_1 = 1 \text{ at } z = 0$.

On the right-hand side of the aforementioned equations, the opposite sign between the first and last two terms indicates that they behave contrary. The first term is the diffraction due to DAW, and the last two terms show convergence due to nonlinearity and magnetic islands. A competition between them goes on until the converging effects dominate the divergence leading to the localization of DAW. The result in Figure 15 shows that the localization of the wave occurs in both the planes, but the distribution of these structures is uneven due to different rates of diffraction and nonlinearity.

To calculate the critical scale size, we equate the diffraction and convergence term and when only magnetic islands are present, it is found to be: $r_{01} = (\rho_2^2 (1 + \varepsilon_{\text{so}}/\epsilon_{022})/4k_2^2)^{1/4} = 1.195 \times 10^7 \text{ cm}$; $r_{02} = (\rho_2^2 (1 + \varepsilon_{\text{so}}/\epsilon_{022})/4 \times 0.04 (b_01 + b_{02})k_2^2)^{1/4} = 2.827 \times 10^7 \text{ cm}$. If we plot the critical scale size versus DAW field strength, we see that with the DAW field, the scale size also varies. Thus, the coherent structures ranging from few proton scales to sub-proton scales may be formulated by changing the strength of DAW. In a similar manner, we can find out the critical scale size when ponderomotive nonlinearity as well as magnetic islands are present.

**SUMMARY AND CONCLUSION**

In this review, we have discussed the role of kinetic Alfvén waves and dispersive Alfvén waves interacting with the surrounding, nonlinearities, other low-frequency modes, and pre-existing chain of magnetic islands in the evolution of solar wind turbulence. Amongst the different validated processes for explaining the small-scale physics, emphasis is given on the en route generated coherent or localized structures which lead to the turbulent behavior and transfer of energy along and across the ion scales. The background vicinity and initial conditions affect the evolution of these structures, but the energy transfer continues to occur from larger length scales to smaller length scales. Although some variation in the spectral index is observed due to these effects, it lies within the observed spectral range of the solar wind turbulence, that is, ~1.4 for large scales and ~2.6 for small scales. As indicated by the observations, a reconnection process can also generate turbulence in the solar wind, and the magnetic reconnection (magnetic island sites) may customize the plasma conditions such that several dissipation mechanisms may contribute to the evolution of the solar wind (Voros et al., 2014). Thus, solar wind turbulence is a result of various nonlinear processes like transverse collapse or localization due to KAW, its interaction with other wave modes, and/or the formation of coherent (magnetic) structures resulting from the current sheets, and all these processes gives solar wind turbulence the structure it is found in.
AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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