Direct Numerical Simulations on the three-dimensional wake transition of flows over NACA0012 airfoil at Re = 1000

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Abstract
For micro air vehicles (MAV), the precise prediction of aerodynamic force plays an important role. The aerodynamic force of a comparative low Reynold number (Re) vehicle tends to be affected by the different flow modes. In this paper, the aerodynamic performance of a three-dimensional NACA0012 airfoil is studied numerically. A range of angles of attack (\(\alpha\)) 0°–25° and Reynolds number 1000 is considered. Mean and fluctuating coefficients of aerodynamic forces around NACA0012 airfoil are analyzed for different wake modes. The difference of aerodynamic forces between two and three-dimensional simulations are compared. The results show that the wake remains steady two-dimensional for lower angles of attack. At \(\alpha = 9°\), Von Karman vortex pattern is noticed. Flow transition to three-dimensional as the angle of attack increases from \(\alpha = 13°\). 3D wake is found to be stable with parallel shedding mode for 14°–17°. However, these modes become finer with the gradual increase in angle of incidence. While, wake loses its three-dimensional stability to chaotic with gradual increment in angle of attack afterwards.

Keywords
DNS, airfoil, vortex shedding, wake modes, transition

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Introduction
MAVs generally work at a comparatively low Reynold number, which suggests the viscosity of fluids play a competitive role with the inertial force. Flows over a foil have extensive application in MAV, as well as marine engineering, energy harvesting, and so on. For example, the movement of an airfoil under an incoming flow plays a significant role to wind energy harvesting for both inshore and offshore wind farm. By using aerodynamic force to drive the mechanical moving of airfoil is among the most frequency applied and reliable technologies to extract sustainable energy. Airfoil as lifting surface is widely applied to lots of aircrafts as well as marine equipment in order to balance buoyancy/gravity or manipulate the gesture of vehicle. In more and more situation, airfoil are frequently applied for small size equipment such as MAV and autonomous underwater vehicle (AUV). As the decreasing of size, turbulent motion of fluids are rarely came into play, which may generally stabilize the system. However, as the fluid viscosity plays more important role, the competitive behavior between viscous and inertial force make the flow mode complicate or lead to the abrupt transition of aerodynamic force.

Flow dynamics over airfoil are highly dependent on shape of object and freestream parameters, which are mainly parameterized as Reynolds number and angle of attack. Wake behind streamlined bodies remains stable as Reynolds number increases up to a certain critical value.

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Slight increment in critical Re alters stable configuration to unsteady. To understand the characteristics and behavior of oscillatory forces, a number of research efforts have been directed for decades.\(^1,2\) Especially subcritical region is swayed by Re where laminar separation is dominant. Low Reynolds number aerodynamics has notable importance during the transition from flow from laminar state to turbulent state. At small angle of incidence and low Re, flow remains laminar having thick shear layer over airfoil. Flow separates near leading edge (LE) in presence of high adverse pressure gradient in region. Increment in angle of attack leads flow separation near trailing edge (TE). As a result, separation point moves toward upstream of airfoil which causes increment in width of wake. Increase in width implies increase in drag coefficient and decrease in Strouhal number.\(^3,4\)

For assessment of aerodynamic characteristics: attached flow, separation vortex, TE vortex, LE vortex and blunt body effect, several experiments have conducted in water tunnel by Huang et al.\(^5\) and Mahbub Alam et al.\(^6\) Particle image velocimetry (PIV), Particle tracking velocimetry (PTV), Laser doppler velocimetry (LDA) and Laser-induced fluorescence (LIF) measurements are employed to investigate the dependence of these characteristics to angle of incidence for low and moderate Reynolds number. The flow around an airfoil at Reynolds number < \(10^5\) is sensitive to the boundary layer separation results in the lift coefficient reduction. Nonlinear variation with respect to for NACA0012 for chord Reynolds number (Re)\(_c\) is studied.\(^7\) Dramatic increase in the lift coefficient for \(Re_c = 2.3 \times 10^3\) is observed between angles of attack \(1^\circ\) and \(2^\circ\). Similarly, \(C_l\) increases abruptly between angles of attack \(2^\circ\) and \(3^\circ\) for \(Re_c = 4.8 \times 10^4\).

With the development in technology, several methods are introduced to achieve better accuracy regarding separation, reattachment of boundary layer and transition for simple flows. Kurtulus\(^3,4\) has conducted study on the unsteady behavior of flow around a 2D NACA0012 foil numerically. Reynolds number \(10^5\) and angle of incidence ranges from \(0^\circ\)–\(90^\circ\) were considered for simulation. Vortex splitting is observed for \(\alpha = 23^\circ\)–\(41^\circ\). Additionally, for \(\alpha \geq 50^\circ\), vortices emergence to form single Karman vortex before shedding to wake with time is noticed. Although two dimensional results are found to be worthwhile for revealing some flight mechanism,\(^8\) extensive literature on three dimensional wake transition for flow past bluff bodies through direct numerical simulation (DNS) methodology suggest that the aerodynamic force is sensitive to the transition to the three dimensionality of wake. Through various investigations, it has shown that flow in the wake of blunt bodies undergoes a transition sequence of primary wake instability to three dimensional instabilities.\(^9,12\)

During the transition from two to three dimensional wake for the secondary instabilities regime, mode A (spanwise wavelength of \(4d\)) and mode B (spanwise wavelength of < \(1d\), \(d\) is diameter of cylinder), are reported for flow over circular cylinders and square cylinders.\(^13–15\) These flow regimes appear when symmetric wake is distributed by applying asymmetric conditions. Tong et al.\(^16\) found mode A and B are present for six different angles of attacks. Spanwise wavelength of braids reveals dependence on angle of attack. Hoarau et al.\(^17\) have used DNS to analyze three-dimensional NACA0012 at angle of attack \(20^\circ\) for Reynolds numbers \(8\times10^5\)–\(10^6\). Two kinds of organized modes in (i) Von-Karman for \(Re < 2\times10^4\) (ii) shear layer mode for \(Re \geq 2\times10^4\) were reported.

Transition to chaos in the cylinder wake directly through three-dimensional instability mode C is studied recently.\(^18\) Mode C is generated by placing a small wire in the wake at \(Re = 400\). Another study has figured out that a wire of \(1/100\)th of a cylinder diameter placed at position of five diameters upstream of the cylinder, sufficiently perturbs the flow to substantially affect certain wake transitions.\(^19\) Balakumar investigated turbulent statistical quantities for \(Re_c \geq 23°\) Angle of attack \(5^\circ\) with \(M=0.4\) for \(Re_c = 50\times10^3\) and angle of attack \(15^\circ\) along with \(M=0.2\) for \(Re_c = 10^6\).

![Figure 1. Schematic 3D model of computational domain (left) C-grid mesh topology around airfoil (right).](image-url)
Figure 2. Comparison between present results and literature (a) coefficient of drag and (b) lift coefficient for 2D geometry.

Table 1. Time and grid refinement study for three-dimensional airfoil.

| mesh | $\Delta t = 0.05$ | $\Delta t = 0.025$ | $\Delta t = 0.005$ |
|------|-----------------|-----------------|-----------------|
|      | $C_d$ | $C_l$ | $St$ | $C_d$ | $C_l$ | $St$ | $C_d$ | $C_l$ | $St$ |
| M1   | 0.1655 | 0.4151 | 0.87 | 0.1656 | 0.4156 | 0.87 | 0.1661 | 0.4184 | 0.87 |
| M2   | 0.1656 | 0.4150 | 0.87 | 0.1657 | 0.4154 | 0.87 | 0.1661 | 0.4180 | 0.87 |
| M3   | 0.1656 | 0.4150 | 0.87 | 0.1657 | 0.4156 | 0.87 | 0.1661 | 0.4180 | 0.87 |

Figure 3. Grid and time-step independence study for lift coefficient (upper) and drag coefficient (lower).
respectively were chosen for investigation purpose. Based on DNS data study disclosed existence of complex flow features regarding flow over an airfoil: the laminar separation bubble on the suction side, a turbulent reattachment, and turbulent separation on trailing edge at two different angles of attack. Periodic doubling is identified at 22° which causes chaos at 27° during numerical analysis of 2D NACA0012 airfoil. The interaction of these foils shows dependent but more complicated flows behavior.21 Disruption of laminar separation bubble to turbulence is simulated in several investigations.22–24 To model the flow over oscillatory airfoil, Wu and Xu have implemented boundary element technique.12 Moreover, the effects of periodic motion aerodynamic coefficients and Strouhal number were explored.

Thrust and efficiency of 2D at plate airfoil in case of low Reynolds number is numerically determined.25 CUTCCEL approach has implemented to manifest the both efficiency and thrust increase for different Reynolds number. Under low Reynolds number coupled with high angle of attack, flow is laminar and subject to separate even for low adverse pressure gradient. Vortical structure in the wake at low Re is of interest due to noise generation with variation in α. In the light of previous investigations, primary objective of this study is to provide the useful information about flow behavior over 3D NACA0012 airfoil at Reynolds number 1000, with the particular focus on the influence of asymmetric wake pattern appeared at high α. The selection of present Reynolds number 1000 is served as a represent, since the most of insects and their bionic MAV designs fly in the range of Re from 1000 to 10000. Furthermore, there are plenty of studies on the experimental and numerical study at such Re for comparison.3,4,26–28 One may expect that flow behavior with closed Reynolds number is similar but with variant critical angles of attack.

This paper is organized as follows: Section II presents the geometry and methodology used. Section III illustrates the instantaneous and mean behavior of aerodynamic forces

![Figure 4](image-url) (A) The instantaneous spanwise vorticity contours (b) time averaged spanwise vorticity contours for

\[ \omega_z = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \frac{c}{U_{\text{inf}}} \]

Moreover, the effects of periodic motion aerodynamic coefficients and Strouhal number were explored.

| α   | C_d(2D) | C_d(3D) | C_l(2D) | C_l(3D) | St(2D) | St(3D) |
|-----|---------|---------|---------|---------|--------|--------|
| 10° | 0.16608 | 0.16567 | 0.41836 | 0.41529 | 0.876  | 0.869  |
| 15° | 0.28425 | 0.2652  | 0.7042  | 0.6347  | 0.726  | 0.69   |
| 20° | 0.44595 | 0.40439 | 0.92811 | 0.82771 | 0.531  | 0.54   |
| 25° | 0.68    | 0.53    | 1.15    | 0.93    | 0.47   | 0.44   |

![Figure 5](image-url) Time-averaged streamlines for a range of low angles of attack at Re = 10^3.

![Figure 6](image-url) Comparison between 2D and 3D mean aerodynamic forces for different angles of attack.

![Table 2](image-url) Comparison between mean values of aerodynamic coefficients of C_d = \frac{F_d}{0.5 \rho U^2_{\text{inf}} A}, C_l = \frac{F_l}{0.5 \rho U^2_{\text{inf}} A} and strouhal numbers for 2D and 3D simulations at various angles of incidence.
along with the three dimensional wake flow. In the last Section IV, a brief conclusion is given.

**Methodology**

**Geometry and boundary conditions**

A symmetric airfoil NACA0012 of sharp trailing is used for computational purpose. NACA0012 with sharp trailing edge is defined by the equation combined with incompressible Navier-Stokes equations

\[ y(x) = b_1(b_2x^{1/2} + b_3x + b_4x^2 + b_5x^3 + b_6x^4) \]  

\[ b_1=0.594689180; b_2=0.298222773; b_3=-0.127125232; \]
\[ b_4=-0.357907906; b_5=0.291984971; b_6=-0.105174606 \]

\[ \frac{\partial u_i}{\partial x_i} = 0 \]  

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j^2} \]

where \( i, j = 1, 2, 3 \) refers to streamwise (x), cross-streamwise (y) and spanwise (z) directions. Both 2D and 3D simulations are carried out at Reynolds number \( 10^3 \) defined as \( Re = \frac{U_{inf}c}{\nu} \), where \( U_{inf} \) is free stream velocity, \( c \) is chord length and \( \nu \) is kinematic viscosity. Schematic diagram of computational domain and grid system are depicted in Figure 1.

C-grid topology is used around airfoil. The spanwise length of the computational domain is \( 4c \) so that the spanwise flow structure can be fully considered, where \( c \) represents the lengths along chord of the airfoil. Also, \( (L_x, L_y, L_z) = (24c, 12c, 4c) \) being domain dimensions along x, y, and z directions are adopted for simulation. Coordinates are non-dimensionalized by using chord length \( c \). Uniform flow velocity \( U_{inf} \) is applied at inlet, in x-direction. At outlet, Neumann boundary condition as well as pressure is specified as a reference value of zero. To approximate the flow past bluff bodies of an infinite spanwise length, periodic boundary conditions have been usually employed in numerical simulations.\(^2^9\) Top and bottom boundaries are considered as symmetric boundaries to avoid the thickness of no slip wall. While, no-slip boundary condition is applied on the surface of airfoil

Second order implicit discretization are utilized to discretize space and temporal domains of NS equations. Simulations are carried out at \( Re = 10^3 \) for angle of attack

![Figure 7. Dominant and subharmonic frequencies at various angles of attack (a) 10° (b) 15° (c) 20° (d) 25°.](image)
ranges zero to 20°. As angle of incidence increase, a number of phenomena can be observed (i) separation point on the suction side moves towards the leading edge (ii) the separated boundary layer is laminar. (iii) both cd and cl grow. This work aims to document the three dimensional structure in wake, lift, drag and pressure coefficients and their dependence on angle of attack and Re for a 4c depth of airfoil.

**Mesh independence study**

2D mesh is constructed at first and airfoil surface is discretized with 250 nodes. Height of the first layer mesh is kept at 0.0002c next to the airfoil surface, which corresponds to small enough non-dimensionalized wall distance of $y^+ = 0.02$. A total of 0.198 million quadrilateral cells are used for two-dimensional mesh. Close up view of 2D grid system in vicinity of airfoil is shown in Figure 1(b). For benchmark, present results of mean coefficients of drag ($Cd$) and lift ($Cl$) with data presented in previous works are illustrated in Figure 2(a) and 2(b) respectively and found to be in good agreement with literature.\textsuperscript{3,26–28}

Three-dimensional mesh is constructed by extruding 2D geometry in z-direction, resulting in identical mesh of same resolution in all the planes normal to the spanwise direction. Cell size ($\Delta z$) = 0.08c is considered in spanwise direction. Schematic 3D model of computational domain and boundary conditions are shown in Figure 1(a). Three different meshes, M1, M2 and M3 with 7227990, 9133110 and 9761535 hexahedral cells respectively, are used for grid refinement study.

Table 1 shows the results of averaged aerodynamic force and unsteady flow feature under the time-step and grid refinement for $\alpha = 10^\circ$. Figure 3 shows the instantaneous lift and drag coefficients for varying time-step and grid setup. Error is found to be less than 1% in the results for mean value of aerodynamic coefficients of M1, M2 and M3. It is also found that the errors between $\Delta t = 0.05$ and $\Delta t = 0.025$ is small enough.

Furthermore, results are trivial in case of Strouhal number. Hence, M2 is considered sufficient fine for simulation. In addition, for chaotic flow appeared at $\alpha > 17^\circ$, the statistical time range for calculating aerodynamic forces is as long as 500 non-dimensional time units. For asymmetrical and three-dimensional irregular flows, peak frequency

Figure 8. Pressure distribution along spanwise position z/c = 2 for different time periods.
of time-history of Cl derived from fast Fourier transform, is chosen to determine shedding frequency $f_s$ to calculate Strouhal number. Mathematically, Strouhal number is defined by $St = \frac{f}{U_{inf}c}$.

Results

Globe flow behavior and the difference between 2d and 3d simulations

In the present work, both 2D and 3D simulations for flow over NACA0012 are studied for angles of attack between 0°-25°. However, flows remain to be two-dimensional for lower angles of attack. Mean aerodynamic forces as well as instantaneous pressure distribution along upper and lower surfaces of airfoil are reported. As angle of incidence increased, the three dimensional instability of mode C is observed for a range of angles of attack with growing wavelengths. In case of three-dimensional flows, periodic doubling is observed for the two-dimensional simulations.

Firstly, computations are carried out at very low angle of attack for $Re = 1000$. Figure 4 clearly illustrates the instantaneous and time-averaged vorticity contours for flow over 3D configuration. Meanwhile, associated characteristic mean streamlines pattern for flow over 3D airfoil is presented in Figure 5. It is obvious that attached flows is observed for $\alpha = 0°$ and flows are steady for the cases with very low angles of attack, which are suggested by the exactly same views for the instantaneous and averaged flow field. Counter-rotating vortices grow and become more visible till $\alpha = 8°$. With slight increment in angle of attack, transverse oscillations develop at the rear of NACA0012. Alternative vortex pair of equal magnitude starts to shed from upper and lower part of airfoil. 9° angle of incidence is considered as one of the critical angles where fluctuation in aerodynamic coefficients takes place. Regular Karman Vortex shedding is also observed at $\alpha = 8°$ for 2D case.
The mean streamlines also suggest that the mean separation point moves toward leading edge with the increment in angle of incidence, and separation bubble covers half of the upper surface of airfoil.

In fact, flow exhibits two-dimensional characteristics and no difference in flow characteristics for 2D and 3D simulations for angle of attack lower than 14°. The transition of symmetric 2D wake to asymmetric wake pattern under adverse pressure gradient occurs at high angle of attack for \( Re = 10^3 \). Table 2 demonstrates the difference of mean drag and lift forces between 2D and 3D simulations occurs at high angles of incidence. In two-dimensional simulation, airfoil experiences greater lift and drag forces. For larger angles of attack, difference becomes more dominant. For instance, at \( \alpha = 20° \), the 2D drag and lift forces acting on airfoil are 10.3% and 12.1% larger than its 3D values respectively. And flow is transitioned to chaotic state in 3D simulation. However, periodic doubling was only observed at \( \alpha = 22° \) in literature for 2D case, which leads to chaos at increasing angles of attack.\(^{21}\)

In Figure 6, it is obvious that \( C_d \) and \( C_l \) start to decrease at \( \alpha = 14° \) where three-dimensional instability occurs. \( \alpha = 14° \) is referred as second critical angle in 3D study of airfoil. Results listed in table 2 verify present simulation with previous studies conducted on three-dimensional configurations. In this case, \( C_l \) drops rapidly than \( C_d \) (Figure 6). This drop is relative small in contrast to other blunt geometries.\(^{30}\) However, lower drag and lift forces are also reported for flow over 3D airfoil with Gurney flap.\(^{31}\) Meanwhile, the ratios of lift to drag at different angles of attack remain the approximately equivalent values from 2D to 3D results.

Additionally, comparison of shedding frequencies is also listed in table 2. Fast Fourier transform is applied at streamwise velocity component to analyze dominant and subharmonic frequencies for different angles of attack (Figure 7). \( f_1 \) is main shedding frequency of dominant vortices in case of \( \alpha = 15° \). Other frequencies can be determined in terms of \( f_1 \). Numerous small-scale vortices appear upstream of separation during boundary-layer transition process. Shedding frequency decreases with the increase in angle of attack. Therefore, Strouhal number is found to be lower for three-dimensional computations.

Figure 10. Mean \( C_p \) profiles for various angles of attack.
with the increase in angle of attack. This behavior is similar to a plate is subject to angle of attack $20^\circ - 30^\circ$.\(^{30}\)

Pressure profile near the airfoil

To further discover the aerodynamics force, the instantaneous pressure profile $C_p = \frac{p - P_{\text{inf}}}{0.5 \rho U_{\text{inf}}^2}$ along the foil are plotted for $\alpha = 10^\circ, 13^\circ$ and $15^\circ$ four different periodic times T/4, T/2, 3T/4 and T in Figures 8 and 9. Figure 9 depicts the magnitude of suction pressure on upper surface for both leading edge and trailing edge increases with the variation in time period as well as for growing angle of incidence from $10^\circ$ to $15^\circ$. Additionally, for $[T/4, T]$ with $\Delta T = T/4$, suction pressure remains same at leading edge, and slight variation occurs at trailing edge about each specific $\Delta T$ which gives rise to higher lift experienced by airfoil (Figure 8). Therefore, significant difference in pressure distribution at trailing edge is evident. At every interval of time period magnitude of suction pressure at leading edge is increased with the increment of $\alpha$.

Pressure distribution for mean $C_p = \frac{P - P_{\text{inf}}}{0.5 \rho U_{\text{inf}}^2}$ curves are also plotted in Figure 10. Mean $C_p$ is calculated through taking averaged pressure coefficients at various non-

Figure 11. Mode C with time period 2T at angle of incidence $15^\circ$ (right). Perspective view of spanwise vorticity component to corresponding time interval (left).
dimensional time in an time interval together with different spanwise average. And the mean pressure coefficient is calculated by taking mean of \(C_p\) at long enough time so that the results is statistically convergent. Remarkable pressure difference among various angles of attack can be observed for the cases between the 2D flow at relatively low angles of attack and those 3D flows at higher angles of attack. In general, it is clear that the profiles of \(C_p\) and \(C_{p}'\) behave slightly different as the flow transition from 2D to 3D wake flows with the increasing \(\alpha\).

\[\text{Figure 12. Top view of parallel modes C (a) } \alpha = 14^\circ \text{ (b) } \alpha = 15^\circ \text{ (c) } \alpha = 17^\circ \text{ with growing wavelength. Perspective view of } \omega_z = \pm 0.5 \text{ (d) } \alpha = 14^\circ \text{ (e) } \alpha = 15^\circ \text{ (f) } \alpha = 17^\circ.\]

**Discussion on the three-dimensional flow structures**

Present simulations exposed the existence of three-dimensional instabilities in case of flow over 3D airfoil. Development of three-dimensional instabilities from two-dimensional flow configuration are presented in this section. Vortical structures formed in wake of airfoil highly depends on angle of incidence. Angle of attack 14\(^\circ\) is observed as critical angle due to flow transition to three-dimensional state. Till \(\alpha = 10^\circ\), coefficients of drag
and lift are essentially similar to two-dimensional. Additionally, wake pattern remains two-dimensional symmetric until $\alpha = 13^\circ$. Flow loses its spanwise symmetry beyond this angle of attack and von-Karman vortex shedding is perturbed by spanwise involvement of velocity along with vertical and spanwise vorticity components.

To visualize the three-dimensional wake structure, here we define three non-dimensional vorticity values as

$$\omega_x = \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \left( \frac{c}{U_{inf}} \right) = \pm 0.5, \ \omega_y$$

$$= \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \left( \frac{c}{U_{inf}} \right) = \pm 0.5 \text{ and } \omega_z$$

$$= \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \left( \frac{c}{U_{inf}} \right) = \pm 0.5$$

Where, $\omega_x$, $\omega_y$, and $\omega_z$ are components of vorticity normalized by chord length $c$ and upstream velocity $U_{inf}$. Since

Figures 8 to 10 reveal growing adverse pressure gradient at trailing edge of airfoil, vortices are weakly shed from lower surface of trailing edge of airfoil and subharmonic frequencies become more dominant in the flow regime. This diversion in pressure gives rise to three-dimensional instabilities. As a result, spanwise Karman vortex rows (parallel modes) are developed at this fixed $Re$. With increase in angle of attack, these modes become more stable and grow in direction of flow. Shear layer 3D instability has only been not predicted theoretically but also measured experimentally and is always found to be stable and continue to grow for an instance.

Figure 11 reveals the parallel large rows of Von-Karman vortices with growing spanwise wavelength for various angles of incidence. It is observed that the wake structure after the time interval $T$ is antisymmetric and return to its original structure after $2T$. This is a signature of mode C instability, which was firstly observed and analyzed by

![Figure 13. (A, b) symmetric pattern of global parameters at angle of incidence 15°. (c, d) asymmetric pattern of global parameters at angle of incidence 17°.](image-url)
Zhang both experimentally and numerically. Detailed numerical study on mechanism and development of three-dimensional structures and its wake transition characteristics in the wake of inclined plate at $\alpha = 25^\circ$ were studied by Yang et al. for a range of $Re$ from 350–500. Results showed that spanwise wavelength of organized modes depends on $Re$ as spanwise wavelength of 0.708, 0.67 and 0.75 obtained for $Re = 350$, $Re = 400$ and $Re = 500$ respectively.

In present study, braid like structures with small wave-length as shown in Figure 12 are visualized for $\alpha = 14^\circ$, 15° and 17°. Spanwise wavelength of 0.45 for $\alpha = 14^\circ$ and 0.5 for $\alpha = 15^\circ$ are obtained. With the increment in angle of incidence, spanwise wavelength of parallel modes grows. Under present simulation, maximum spanwise wavelength $\lambda_{b/c} = 0.66$ is observed for $\alpha = 16^\circ$ and 17°. This value is in good agreement with the other analogue blunt bodies wakes. Expected wavelength for equivalent blunt bodies, wake falls in range of 0.60–0.70. As the angle of attack of the airfoil grows, an analogy to blunt-body can be made by contemplating an equivalent blunt-body configuration possessing a characteristic projective length $c Sin(\alpha)$ in the upcoming velocity direction. Where, $\alpha$ is angle of incidence and $c$ represents the length of chord. Thus, the effective Reynolds number is $Re Sin(16^\circ) = 275$ in the case of airfoil analogue to vertical plate. Results of current research are in good agreement with the data present in literature. 

**Figure 14.** Top view of instantaneous iso-surfaces of streamwise vorticity (red and blue) and spanwise vorticity (green and yellow) along with irregular pattern of $Cd$ and $Cl$ (a-c) 20° (d-f) 25°. 
Further analysis demonstrates that the parallel modes appeared are mode C with a time period $2\, T$. $2\, T$ is a most striking feature of mode C (where $T$ being vortex shedding time period). Mode C is observed with iso-surfaces of $\omega_x = \pm 0.5$. Mode structures are uniformly distributed along spanwise direction of the airfoil (Figure 11).

Figure 12 demonstrates the instability mode with growing wavelength with respect to the growing angle of attack. With the increment in angle of attack, these parallel modes start to be perturbed (Figure 12(c)). Global parameters lose its harmonic pattern as angle of incidence approaches to $17^\circ$ as compared to $15^\circ$ (Figure 13(a) to (d)). On further increment in angle of incidence, flow transitioned to chaotic.

Figure 11 shows the uniformly distributed braid like structures of instability mode C for angle of attach $15^\circ$. As angle of attack is increased further, vorticity components become stronger and start to perturb smoothly arranged braid like structures. Spanwise wavelength reached its maximum as shown in Figure 12(c). The variation of wake instability could sensitively affect the corresponding aerodynamic force and noise, which may lead to severe impact on the working performance of aerodynamic equipment. For the airfoils in MAV, it is therefore should notice that the angle of incidence could results in chaotic wake flow and rapidly varying aerodynamic force as the variation of its angle of attack. As angle of attack increases further, wake transition process becomes more complicated.

Figure 14 demonstrates the irregular pattern of $C_d$ and $C_l$ for angle of attack $20^\circ$ and $25^\circ$. Aerodynamic forces show random behavior. With increment in angle of incidence shear layer encounters instability after separation and became chaotic.

As seeing the mean Cp profiles plotted in Figure 10, magnitude of pressure distribution at $\alpha = 20^\circ$ is significantly larger than lower angles of attack. It shows at trailing edge pressure contours are non-uniform as flow transitioned to
chaotic due to other components of velocity also being involved. From Figure 14 it is clear that wake flow has no regular periodicity property. Consequently, it demonstrates well-known turbulence-like wake structure with dissipative, random and multiple scaled characteristics in time and space. When non-linearity of flow becomes dominant, it has infinite degree of freedom. Adverse Pressure gradient over airfoil and neighboring area gives rise to non-uniform flow.

For $\alpha = 25^\circ$, Periodic doubling 2P (two distinct vortex pairs above and below to the center of wake) with highest height of wake regime is observed in 2D flow as shown in Figure 15. 2P regime is also reported not only for flow over sharp trailing edge 2D NACA0012 but also for NACA0012 with Gurney flap.\(^3,30\) In case of 3D flow, spanwise vorticity also disturbed in the 2P regime. However, flow field discover the considerable reduction of wake thickness as compared to 2D flow (Figure 15(d) and (h)). Hence, minimized the drag force experienced by airfoil during 2D flow. Altered dynamics leads to reducing lift force compared to 2P regime of 2D flow.

Concluding remarks

In this study, direct numerical simulations of moderate Reynolds number are carried out over 3D NACA0012 airfoil. Various instantaneous and time-averaged aerodynamic parameters including coefficients of lift, drag and pressure are calculated. Development of 3D transitions of wake and robustness of parallel Von-Karman vortices are analyzed with varying angle of attack at fixed Re. Braid like structures are observed which becomes more stable and finer with increment in $\alpha$. Spanwise wave-lengths of periodic modes are quantified. Parallel modes are figured out as Mode C.

For two- and three-dimensional studies over the airfoil, results are found to be similar for global parameters in case of lower angles of attack. Whereas, notable difference is noted for higher angles of incidence. Airfoil experiences lower lift and drag for 14°–25°. Additionally, opposite results are obtained for Strouhal number in context to airfoil as compared to other bluff configuration, however, similar results as streamline configurations. Strouhal number depends on angle of attack at fixed Re. In three-dimensional study, for higher St decreases at first and increases later for further increment in $\alpha$.

Non-uniform Pressure distribution causes disruption in laminar two-dimensional wake. Three dimensional instabilities lead flow to move in the third dimension and wake pattern loses its stability which amplified its irregularity. Eventually, the three-dimensional structure of flow leads significant difference for overall aerodynamic performance of airfoil. Cd and Cl becomes highly irregular at $\alpha = 25^\circ$ and flow transitioned to chaotic. 2P regime of 2D flow is suppressed by the spanwise instabilities appeared in 3D flow reducing to Karman vortex street like wake structure.

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