Simplex based MUSIC for solving coherent signal

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Abstract: An optimized MUSIC (MUltiple SIgnal Classification) is proposed, basing on Simplex search method. It can largely improve the scan speed of the conventional MUSIC and avoid the singularity of the Hessian matrix of the gradient based optimization methods. It also provides the possibility to distinguish the coherent signal with the acceptable time cost, in this case, we proposed an enumeration based coherence solver. Both simulations and experiments are carried out for RF localization purposes. The results shows that the proposed methods has faster computational speed with same numerical precision as the conventional MUSIC method does, which also confirms the feasibility of the proposed coherence solver.

Keywords: MUSIC, Simplex method, DoA
Classification: Wireless communication technology

References

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1 Introduction

MUSIC (MUltiple SIgnal Classification) is one of the most popular DoA (Direction of Arrival) methods for various purposes. By using the orthogonality between noise subspaces of the signal and the steering matrices, the position information can easily be evaluated by a scan through the relative parameters [1][2]. Many studies on optimizing the MUSIC processing have been made since it was invented. Variations such as the root mean MUSIC, applying the Taylor expansion on the steering matrix and modifying the covariance matrix are widely used and well-developed. This paper introduces an optimized MUSIC method, using the polynomial expansion generated from the steering matrices and the covariance matrix. Then rather than solving the roots of the equation, the proposed method directly performs optimization to acquire the minimum value. A faster optimized MUSIC provides the feasibility for solving the coherence by a single fixed position measurement by simply running an enumeration process. Both simulations and experiments are carried out in different conditions, including a single source and two coherent sources, to validate the performance of the proposed MUSIC.

2 Methodology

The proposed method is all basing on the conventional MUSIC, which assumes that the signal has a form

\[ S(t) = X(t)A(\theta, r) + N \]  

where \( X(t) \) is the received signal of the reference element in time domain, \( A(\theta, r) \) is a steering matrix containing two unknown parameters range \( r \) and angle \( \theta \), \( N \) is a noise matrix and \( S(t) \) is the received signal matrix for all channels. According to that, \( P \) is a norm indicating the result of the orthogonality and can be considered as a function of two variables \( r \) and \( \theta \) which indicate the target location, given as

\[ P(\theta, r) = \frac{1}{A^*A} \]  

where \( E_N \) is the noise subspace of \( X(t) \), and * denotes the conjugate transpose. However, such a two-dimensional search is time-consuming. To tackle this
problem, the 2D MUSIC was optimized with the Simplex method [3]. From Eq.(2), since \( E_N, E_N \) is defined from the acquired data, for instance, with a uniform linear array configuration, we can define the denominator as a function

\[
F(r, \theta) = A(\theta, r) E_N^H E_N A^H (\theta, r)
\]

which has an explicit form as shown

\[
F(r, \theta) = \left[ a_{11} \cdots a_{M} \right] \left[ \begin{array}{c} 1 \\ \vdots \\ a_{M} \end{array} \right] e^{j2\pi ur}.
\]

(4)

In a polynomial form, it donates

\[
F(r, \theta) = a_{11} + a_{12} e^{-j2\pi u} + \cdots + a_{MM} e^{-j2\pi u}
\]

where \( M \) is the number of antenna elements, and \( r \) can be expressed as

\[
\tau_i = \sqrt{r^2 + (i - 1)^2 d^2 - 2r(i - 1)d \cos \theta} - r .
\]

(6)

Since the explicit form of \( F \) is known and \( F : X \rightarrow \mathbb{R} \) where \( X = \{ (\theta, r) \in \mathbb{R}^2 : 0 \leq \theta \leq \pi \} \), many kinds of optimization techniques can be applied on a surface as shown in Fig.1, which generated by all possible combination of \( r \) and \( \theta \) in \( F \). In this work, we apply the Simplex method[4][5], which is trying to minimize function \( F \) according to the values at the vertices:

\[
F(r_1, \theta_1) \leq F(r_2, \theta_2) \leq F(r_3, \theta_3)
\]

(7)

There are four movements changing the search step length and direction: reflection, expansion, contraction and shrink. Every movement generates a new vertices which can be also considered as an iteration. Practically, parameters to be manually adjusted are tolerances or fixed iteration times, initial step length and initial guess. The convergence happens when the iteration times reaches the preset or the area of vertices is smaller than the tolerance. In the case of Eq.(5), calculation of the area might be unnecessary, calculating the difference of the function differences or the variable differences of the latest iterations is a better choice rather than area calculation.

In the aspect of numerical analysis, the Newton method needs to calculate and restore the Hessian matrix that leads to a high complexity and likely to be singular. It also applied to advanced Newton method based optimization, though the Hessian matrix can be avoided or simplified, the complexity of the algorithm is higher than
In term of the feasibility, comparing to some other popular optimizations, the Simplex method only uses the value of the function, in other words, under the circumstances of optimizing a function with 2 variables, the calculation on the derivatives will be vastly reduced. It is important when the amount of array elements is large, as a result, Eq.(5) can be large; hence modification on the polynomial should be avoided. Hence, some of the optimization methods such as DFP (Davidon–Fletcher–Powell Method) method and BGFS (Broyden–Fletcher–Goldfarb–Shanno) method can hardly be applied, because the calculation of the first order derivatives and the second order derivatives may take unacceptable long time. Note that many researches applied approximation approaches to Eq.(6) to reduce the load of calculation[6][7]; however, it highly depends on the location of the source, as the results shown in the next chapter, MUSIC with 2 variables will adaptively change its way of visualization while the sources located far away from the sensor array.

For coherent signals, especially for multiple targets located close to the reference channel, enumeration approach will be applied. According to (1), the received signal of two signal sources can be written as

$$S(t) = X(t)(p_1 A_1(\theta, r) + p_2 A_2(\theta, r)) + N$$  \hspace{1cm} (6)

where $p_1$ and $p_2$ are amplitudes of the received signal from corresponding signal sources. In this case, steering matrices $A_1$ and $A_2$ can be constructed by using the form in Eq.(4) and Eq.(5), also $X(t)$ with arbitrary initial phase can be generated. Due to the orthogonality between the noise subspace and the signal, the process can be turned into an optimization

$$\arg \min_{\theta, r} A(\theta, r)\overline{E}_n^{\mu} \overline{E}_n A^\mu(\theta, r)$$  \hspace{1cm} (7)

where $\overline{E}_n$ is the subspace of expected signal only from source one.
\[ S(t) = S(t) - p_X(t)A_2(\theta, r) \]. After solving for the first source, the coherency can be solved and the received signal matrix of the second source is \[ S(t) = S(t) - S(t) \]. Such a multiple parameters search requires heavy calculation, which is also the motivation that we develop the proposed method for reducing the computation of the nested loops of searching.

### 3 Simulations and experiments

In this section, we carried out both simulations and experiments to verify the feasibility of the proposed algorithms. All of the simulations are based on 4 elements uniform linear array. For a single target, we use the 2D MUSIC as shown in Fig.2. In this simulation, the targets are located at \((r=0.3\, \text{m}, \theta=120^\circ)\), antenna is transmitting signal at 2.45GHz and the SNR is set to be 5dB. The optimized MUSIC is also applied to this simulation, the iteration results are marked as red circles and the initial guess is at \((r=0.2\, \text{m}, \theta=100^\circ)\). As the figure shows, 2D MUSIC has two peaks, it can be reduced by adding more antennas on the array. In this simulation, only the simplex method only takes less than 20 iterations to acquire the focus point while conventional 2D MUSIC takes 18000 iterations (180 for range and 100 for angle). Meanwhile, it gives almost the same target location as the 2D MUSIC does.

![Fig.2. Simulation result.](image)

Since feasibility of the algorithm is confirmed, we conducted laboratory experiments to validate the stability of the proposed method for measured data as shown in Fig.3. The transmitters of sources are dipole antenna, transmitting unmodified CW signal in 2.4GHz. We are using 4 elements uniform linear array and an oscilloscope for data acquisition. Note that only 3 channels are used to perform the further processing for reducing the computation. The reference antenna is set to be the very left one in Fig.3. Fig.4 shows the result of a 2 sources measurement where the target are located at \((r_1=0.455\, \text{m}, \theta_1=40^\circ)\) and \((r_2=0.785\, \text{m}, \theta_2=107^\circ)\). In this case, the known parameters are a rough area where source #1 located and the amplitude of source #1. When the known parameters decreases, the accuracy of the result will be lowered and the computational time will get much longer. Due to the feature of the MUSIC, when the source is far, range will
contribute very little comparing to the angle in Eq.(5), so the result is focusing along an angle but not on a point.

Fig.3. 2 sources measurement. (a) Experiment setup. (b) Experiment result

8 Conclusion
An optimized MUSIC is proposed and used as the core for coherence solving. The proposed method can largely reduce the scan time of the conventional MUSIC and offers precise and accurate result. The imaging is adaptive depends on the target location; when the target is close to reference channel, the result will directly indicate the location of target. One more measurement is required when the target is further and each of the measurements will indicate only the azimuth angle of the target location to the reference element unit. When the multiple sources are transmitting signal in the same frequency, the coherency will be solved by the proposed enumeration approach with the optimized 2D MUSIC for time-saving. Both simulations and experiments show that the coherence solver requests known parameters to avoid redundancy of the enumeration. However, the known parameters can be roughly given, such as a certain range or angle or the possible location of the targets. Above all, solve the coherence by using a single measurement is challenging; nonetheless, the proposed method still has the great potential to solve the problem.