On the Formulation of the Generic Supersymmetric Standard Model (or Supersymmetry without R parity)

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Abstract

The generic supersymmetric version of the Standard Model would have the minimal list of superfields incorporating the Standard Model particles, and a Lagrangian dictated by the Standard Model gauge symmetries. To be phenomenologically viable, soft supersymmetry breaking terms have to be included. In the most popular version of the supersymmetric Standard Model, an ad hoc discrete symmetry, called R parity, is added in by hand. While there has been a lot of various kinds of R-parity violation studies in the literature, the complete version of supersymmetry without R parity is not popularly appreciated. In this article, we present a pedagogical review of the formulation of this generic supersymmetric Standard Model and give a detailed discussion on the basic conceptual issues involved. Unfortunately, there are quite some confusing, or even plainly wrong, statements on the issues within the literature of R-parity violations. We aim at clarifying these issues here. We will first discuss our formulation, about which readers are urged to read without bias from previous acquired perspectives on the topic. Based on the formulation, we will then address the various issues. In relation to phenomenology, our review here will not go beyond tree-level mass matrices. But we will give a careful discussion of mass matrices of all the matter fields involved. Useful expressions for perturbative diagonalizations of the mass matrices at the phenomenologically interesting limit of corresponds to small neutrino masses are derived. All these expressions are given in the fully generic setting, with information on complex phases of parameters retained. Such expressions have been shown to be useful in the analyses of various phenomenological features.
I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) is no doubt the most popular candidate theory for physics beyond the Standard Model (SM). An alternative version with a discrete symmetry, called R parity, not imposed deserves no less attention. *Supersymmetry without R parity is nothing but the generic supersymmetric Standard Model* [1], *i.e., a theory built with the minimal superfield spectrum incorporating the SM particles, interactions dictated by the SM (gauge) symmetries, and the idea that supersymmetry (SUSY) is softly broken (at or below the TeV scale).*

From the theoretical point of view, R parity is simply an *ad hoc* global symmetry imposed by hand. It does simplify the Lagrangian very substantially and restores the accidental symmetries of baryon and lepton numbers of the SM, but is not otherwise well motivated. Phenomenologically, not imposing R parity would beg an alternative mechanism to protect proton decay; and may resulted in, but not necessary mandates, losing the so-called lightest supersymmetric particle (LSP) as the favorite dark matter candidate. Concerning protecting proton decay, it has been established that R parity is not the only candidate for the job; nor is it the most effective [2]. It is the most restrictive, though, in terms of what terms are admitted in the renormalizable Lagrangian or otherwise. On the other hand, giving up R parity *does* allow the neutrinos to have masses and mixings *without* the need of introducing extra superfields beyond the supersymmetric SM spectrum. At the present time, experimental results from neutrino physics [3] is actually the only data we have demanding physics beyond the SM, while signals from SUSY are still absent [4]. Hence, the case for giving up R parity is stronger than ever. The generic supersymmetric Standard Model (GSSM) is, at least conceptually, the simplest model with SUSY and neutrino masses. It also promises exciting new phenomenology, in collider machines and beyond, and a strong link between neutrino physics and the latter. Hence, we conclude that it makes sense to take the GSSM and study the experimental constraints on the various couplings without *a priori* bias. From the theoretical point of view, in relation to proton decay, baryon number is expected to be protected by some sort of symmetry, while lepton numbers have to be violated.

There are certainly no lack of studies on various “R-parity violating models” in the literature. However, such models typically involve strong assumptions on the form of R-parity violation. In most cases, no clear statement on what motivates the assumptions taken is explicitly given. In fact, there are quite some confusing, or even plainly wrong, statements on issues concerned. It is important to distinguish among the different R-parity violating (RPV) “theories”, and, especially, between such a theory and the unique GSSM. Results from studies on a particular version of RPV model would have no general validity, if they are based on naive but strict assumptions on the vanishing of a set of the generally

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1 This review is meant to be pedagogical. Only a minimal basic background of supersymmetry is indeed presumed. The rest of the discussions in this section set our case among the background literature of supersymmetric Standard Model(s). Readers not familiar with the latter or the notion of R parity may want to skip that. The statement in italics above defines the model we discuss here. In fact, some idea about what the statement means is all that is needed to appreciate the present review.
admissible RPV couplings. However, by carefully embedding such studies into the GSSM framework, and addressing explicitly the questions of how small the neglected couplings have to be in order not to upset the phenomenological features under study, we could piece together a more comprehensive story of interesting GSSM phenomenology.

Many works on RPV phenomenology give the wrong claim or lead to the wrong impression that they are studying some aspects of the GSSM. The truth is that the strong assumptions, hidden or otherwise, behind their formulations restrict the general validity of their studies. There are subtle issues involved in taking a specific RPV model as a limiting case of the GSSM. Such issues are important in the latter interpretation, but too often overlooked. We hope to clarify such issues in this review.

If one looks at the GSSM independent of the MSSM and the other RPV theories, the notion of R parity is simply not there at all. In fact, to the extent that the physical leptons contain components in the gaugino and higgsino directions, there is, strictly speaking, no way to assign the (SM) lepton number, hence (MSSM) R parity, to the list of GSSM superfields free from ambiguity. In relation to that, we would go so far as to ask the readers to give up the preconception that R-parity violation is small, for the moment. Whether it is small is actually a delicate question, at least from the theoretical point of view. Among other things, there is the question of “How small is small?” We will come back to all these below. Anyway, our formulation here is generally valid. The only expressions inside the article with limited validity are the perturbative diagonalization formulae of the mass matrices. That perturbative regime is firmly based on phenomenology — the fact that neutrino masses are substantially smaller than the electroweak scale.

If the readers are willing, however, to forget totally about the notion of R parity for the moment and just follow our discussion here seeing the subject as what it is theoretically — the GSSM, he or she will be in the best place to appreciate our discussion of the formulation part. In our opinion, that also set the best stage for looking at all the R-parity violation works — something we will come back to, in a Q & A format, in the Appendix.

In the section below we discuss the basic formulation and the issue of parametrization. Then, we will focus on the so-called single-VEV parametrization first explicitly advocated in Ref. [5]. Readers may first look at it as one possible parametrization of the GSSM. We will discuss some details of the model under the parametrization and illustrate its phenomenological merits. The formulation under the parametrization will also provide a platform to address the various issues, to be discussed below. We elaborate on the content of the model using our formulation. We discuss in section III and IV the fermion sectors and the scalar sectors, giving the explicit mass matrices, and perturbative diagonalization matrix elements in the phenomenologically interesting (small $\mu_i$) region. Such results are all given here admitting the most generally nature of the parameters involved, including possible nontrivial complex phases. Most of the results are taken from previous works of the present author and various collaborators [5–11]. Some of the expressions are however not published before in the present most general form. In particular, most explicit results on the part of the scalar mass matrices, used to some extent in Refs. [10,11], are not available elsewhere. In section V, we conclude this review with some remarks. In the Appendix section, we recapitulate on some of the important issues on the formulation aspect, and address other formulations and approaches used in the literature, in a Q & A format.
II. FORMULATION OF THE GSSM

Let us start from the beginning and look carefully at the supersymmetrization of the SM. The gauge field sector is relatively trivial. In the matter field sector, all fermions and scalars have to be promoted to chiral superfields containing both parts. It is straightforward for the quark doublets and singlets, and also for the leptonic singlets. The leptonic doublets, however, have the same quantum number as the Higgs doublet, $H_d$, that couples to the down-sector quarks. Nevertheless, one cannot simply get the Higgs, $H_d$, from the scalar partners of the leptonic doublets, $L$’s. Holomorphicity of the superpotential requires a separate superfield to contribute the Higgs which couples to the up-sector quarks. This $H_u$ superfield then contributes an extra fermionic doublet, the higgsino, with nontrivial gauge anomalies. To cancel the latter, an extra fermionic doublet with the quantum number of $H_d$ or $L$ is needed. So, the result is that we need four superfields with that quantum number. As they are a priori indistinguishable, we label them by $\hat{L}_\alpha$ with the Greek subscript being an (extended) flavor index going from 0 to 3.

A. The Superpotential and the Single-VEV Parametrization

The most general renormalizable superpotential with the spectrum of minimal superfields discussed above can be written as

$$W = \varepsilon_{ab} \mu_a \hat{H}_u \hat{L}_\alpha^a \hat{\Upsilon}_\alpha^b + h_{ik} \hat{Q}_i^a \hat{H}_u^b \hat{\Upsilon}_k^C + X_{\alpha jk} \hat{L}_{\alpha}^a \hat{Q}_j^b \hat{D}_k^C + \frac{1}{2} \lambda_{\alpha \beta k} \hat{L}_{\alpha}^a \hat{L}_{\beta}^b \hat{E}_k^C + \frac{1}{2} \lambda'_{ij k} \hat{U}_i^C \hat{D}_j^C \hat{D}_k^C , \quad (1)$$

where $(a, b)$ are $SU(2)$ indices, $(i, j, k)$ are the usual family (flavor) indices (going from 1 to 3). We have explained the origin of the 4 $\hat{L}_\alpha$’s, with the $(\alpha, \beta)$ indices as extended flavor indices going from 0 to 3. The rest of the superfield notations are obvious. Note that $\lambda$ is antisymmetric in the first two indices, as required by the $SU(2)$ product rules, shown explicitly here with $\varepsilon_{12} = -\varepsilon_{21} = 1$. Similarly, $\lambda'$ is antisymmetric in the last two indices from $SU(3)_C$, though color contents are not shown here.

Besides the superpotential, the Lagrangian contains the gauge interaction part, including kinetic terms of the matter superfields, and a soft SUSY breaking part. The former is trivial. The latter we will postpone till after we address the question of choosing a specific parametrization for the theory.

First, it is important to note that after the supersymmetrization some of the superfields lose the exact identities they have in relation to the physical particles as in the SM. The physical particles have to be mass eigenstates, which have to be worked out from the Lagrangian

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This is a constraint only when we insist on having the SM Yukawa terms as the source of generation of up-sector quark masses. If one is ready to accept a loop level mass generation, a supersymmetric SM with only the three leptonic doublets and singlets as the colorless chiral superfield is certainly an interesting option, which has apparently not been explored. However, with the not historically expected very large top mass confirmed, such a model would have severe problem with top mass generation and hence becomes very unappealing.
of the model. Assuming electroweak symmetry breaking, we have now five (color-singlet) charged fermions, for instance. Involved in their masses are 1+4 admissible VEVs consistent with the symmetry breaking, together with a SUSY breaking gaugino mass. If one writes down naively the (tree-level) mass matrix, the result is extremely complicated (see Ref. [12] for an explicit illustration), with all the $\mu_\alpha$ and $\lambda_{\alpha\beta k}$ couplings involved. Note that the only definite experimental data we have here are the three physical lepton masses as the light eigenvalues, and the overall magnitude of the electroweak symmetry breaking VEVs. The task of analyzing the model seems to be formidable.

Recall that in the SM, the only three unit-charged fermions have a mass matrix that is essentially diagonal. In another word, one can choose to write the Lagrangian in the flavor basis corresponding to the physical charged leptons $e, \mu,$ and $\tau$. The leptonic Yukawa terms are then by definition flavor diagonal, hence involving only 3 real parameters. One would like to achieve a similar simplification here. There is some problem though. We have 4 “leptonic doublets” $L_\alpha$’s, and, added to that, a gaugino from an adjoint triplet all contributing to $e, \mu,$ and $\tau$. As the superpotential has to respect electroweak symmetry, mass eigenstate basis cannot be used here. Choosing flavor bases to write the Lagrangian is not just a matter of convenience. Doing phenomenological studies without specifying a choice of flavor bases is in fact ambiguous. A still better example is provided by thinking about doing SM quark physics with 18 complex Yukawa couplings, instead of the 10 real physical parameters, namely 6 quark masses and 4 real numbers needed to parametrize the CKM matrix. As far as the SM itself is concerned, the extra 26 (real) parameters are simply redundant, and attempts to relate the full 36 parameters to experimental data will be futile. There is simply no way to learn about the 36 real parameters of Yukawa couplings for the quarks in some generic flavor bases, so far as the SM is concerned. The best thing to do is to write the Lagrangian with a specific optimal set of parameters that also helps to simplify the analysis and make the physics more transparent. Again, this is exactly what we do with SM quark physics. For instance, one can choose to write the SM quark Yukawa couplings such that the down-quark Yukawa couplings are diagonal, while the up-quark Yukawa coupling matrix is a product of (the conjugate of) the CKM and the diagonal quark masses, and the leptonic Yukawa couplings diagonal. Doing that has imposing no constraint or assumption onto the model. On the contrary, not fixing the flavor bases makes the connection between the parameters of the model and the phenomenological observables ambiguous.

The choice of parametrization is not unique. However, a specific, consistent, choice has to be made before doing phenomenological studies — before one uses experimental data to constrain or pin down the value of any parameter. A parametrization using generic

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3The full set of 36 parameters may be of interest only when we want to model the origin of the flavor structure at a deeper level and hence higher energy scale [13]. Any meaningful attempts in the direction is likely to be possible only when we do have some good knowledge about the phenomenological value of the SM parameters (in a specific basis).

4Here, what we are doing may be a bit unconventional when compared with what is usually done in SM studies. However, our choice is well-motivated. The down-sector quarks have extra couplings, the $\lambda$’s, while the up-sector is kept simple.
flavor bases is ambiguous and redundant. In the case of the GSSM, the choice of flavor basis among the 4 $\hat{L}_\alpha$'s is a particularly subtle issue, because of the fact that they are superfields the scalar parts of which could bear VEVs. A parametrization called the single-VEV parametrization (SVP) has been advocated by the author and collaborators since Ref. [5]. The central idea is to pick a flavor basis such that only one among the $\hat{L}_\alpha$'s, designated as $\hat{L}_0$, bears a non-zero VEV. There is to say, the direction of the VEV, or the Higgs field $H_d$, is singled out in the four dimensional vector space spanned by the $\hat{L}_\alpha$'s. Explicitly, under the SVP, flavor bases are chosen such that: 1/ $\langle \hat{L}_i \rangle \equiv 0$, which implies $\hat{L}_0 \equiv H_d$; 2/ $y^c_{jk}(\equiv \lambda_{bjk} = -\lambda_{k0j}) = \sqrt{2} \nu_0 \text{diag}\{m_1, m_2, m_3\}$; 3/ $y^d_{jk}(\equiv \lambda_{bjk}) = \sqrt{2} \nu_0 \text{diag}\{m_d, m_s, m_b\}$; 4/ $y^u_{ik} = \frac{\nu_0}{v_u} V^{T}_{\text{CKM}} \text{diag}\{m_u, m_c, m_t\}$, where $v_0 \equiv \sqrt{2} \langle \hat{L}_0 \rangle$ and $v_u \equiv \sqrt{2} \langle \hat{H}_u \rangle$. 2/ to 4/ are more straightforward choices that look just like the SM case, except that such choice can be consistently implemented in the GSSM case only because of choice 1/ — an issue we will elaborate on below. The other important point to note is that the $m_i$'s above are, conceptually, not the charged lepton masses. They are some unknown real parameters, though we will see below that they might turn out to be numerically essentially the same as the charged lepton masses. The parametrization given here still contains redundant complex phases among the couplings. It is otherwise optimal. Removing the redundant phases is especially important when we want to probe details of the CP violating physics. However, for our discussion here, we simply assume all the admissible nonzero couplings within the SVP are generally complex.

Electroweak symmetry breaking is the sole source of chiral (SM) fermion masses, as well as the $LR$-mixings of their scalar superpartners. Hence, it is no surprise that the parametrization with the minimal number of nonzero VEVs gives the simplest structure for the (tree-level) mass matrices of both the fermions and the scalars. The adoption of the SVP has, hence, a strong phenomenological advantage, which we will illustrate below.

**B. The Soft SUSY Breaking Part**

Now we turn to the soft SUSY breaking part of the Lagrangian, details of which are too often overlooked. Following our notation above, the soft terms can be written as follow [7]:

$$V_{\text{soft}} = \epsilon_{ab} B_a H_u \bar{L}_a^b + \epsilon_{ab} \left[ A^c_{ij} \bar{Q}_i^c H_u^b \bar{t}_j^c + A^d_{ij} H_d^b \bar{d}_j^c + A^e_{ij} H_d^a \bar{L}_i^b \bar{E}_j^c \right] + \text{h.c.}$$

$$+ \epsilon_{ab} \left[ A'^{e}_{ijk} \bar{L}_i^a \bar{Q}_j^c \bar{H}_k^c + \frac{1}{2} A'^{d}_{ijk} \bar{L}_i^a \bar{d}_j^b \bar{E}_k^c \right] + \frac{1}{2} A''^{e}_{ijk} \bar{U}_i^a \bar{C}_j^c \bar{D}_k^c + \text{h.c.}$$

$$+ \bar{Q}^\dagger \tilde{m}_Q^2 \bar{Q} + \bar{U}^\dagger \tilde{m}_U^2 \bar{U} + \bar{D}^\dagger \tilde{m}_D^2 \bar{D} + \bar{L}^\dagger \tilde{m}_L^2 \bar{L} + \bar{E}^\dagger \tilde{m}_E^2 \bar{E} + \tilde{m}_{H_u} |H_u|^2$$

$$+ \frac{M}{2} \bar{B} \bar{B} + \frac{M}{2} \bar{W} \bar{W} + \frac{M}{2} \tilde{g} \tilde{g} + \text{h.c.} \right),$$

where we have used $H_d$ in the place of the equivalent $\hat{L}_0$ among the trilinear $A$-terms. Note that $\bar{L}^\dagger \tilde{m}_L^2 \bar{L}$, unlike the other soft mass terms, is given by a $4 \times 4$ matrix. Comparing with the MSSM case, $\tilde{m}_{h_{00}}^2$ corresponds to $\tilde{m}_{h_{ij}}^2$ while $\tilde{m}_{h_{0k}}^2$'s give new mass mixings. The other notations are obvious. The writing of the soft terms in the above form makes identification of the scalar mass terms straightforward. Recall that only the doublets $H_u$ and $H_d$ bear VEVs. The $A$-terms in the second line of Eq.(2) hence do not contribute to scalar masses.
For the sake of completeness, we include here the admissible nonholomorphic soft terms [14,15], to be given as

$$V_{\text{soft}}^{\text{NH}} = C_{ij}^u \bar{Q}_i^u (H_d^u)^* \bar{U}_j^C + C_{ij}^l (H_u^a)^* \bar{Q}_i^l \bar{D}_j^C + C_{ij}^e (H_u^a)^* \bar{L}_i^a \bar{E}_j^C + C_{ik}^l (H_u^a)^* H_u^a \bar{E}_k^C + \text{h.c.} + C_{ijk}^l \bar{L}_i^a \bar{D}_j^C \bar{U}_k^C + C_{ijk}^q \bar{Q}_i^l \bar{D}_j^C \bar{E}_k^C + \frac{1}{2} \epsilon_{abc} C_{ij}^a \bar{Q}_i^b \bar{Q}_i^c (\bar{D}_k^C)^* + \text{h.c.},$$

where we have dropped bilinear terms which could be incorporated into $\tilde{m}_l^2$ above. Here again, only terms in the first line could contribute to scalar masses.

C. Some Notation for the Component Fields

Note that though the SVP enforces the identification of $\hat{L}_0$ as the one having “Higgs” properties (of $\hat{H}_d$), it still maintains couplings similar to those of the $\hat{L}_i$’s. Put it in another way, the charged leptons in GSSM generally contain higgsino components, and the Higgs field may be partly the superpartners of the physical charged leptons.

We write the components of a $\hat{L}_\alpha$ fermion doublet as $l_0^\alpha$ and $l_-^\alpha$, and their scalar partners as $\tilde{l}_0^\alpha$ and $\tilde{l}_-^\alpha$. Apart from being better motivated theoretically, the common notation helps to trace the flavor structure. However, we will also use notations of the form $h \star^d$ (i.e. $h_0^d$ and $h_-^d$) $\tilde{h} \star^d$, as alternative notations for $\tilde{l}_0^\alpha$ and $\tilde{l}_-^\alpha$, in some places below. This is unambiguous under our formulation. We will also referred to the states $h_0^\alpha (\equiv \tilde{l}_0^\alpha)$ and $h_-^\alpha (\equiv \tilde{l}_-^\alpha)$ as Higgs and higgsino, respectively; while they are generally also included in the terms slepton and lepton.

In the left-handed lepton and slepton field notations introduced above, we have dropped the commonly used $L$-subscript, for simplicity. For the components of the three right-handed leptonic superfields, we use $l_+^i$ and $\tilde{l}_+^i$, with again the $R$-subscript dropped. The notation for the quark and squark fields will be standard, with the $L$- and $R$-subscripts. A normal quark state, such as $d_{Lk}$, denotes a mass eigenstate, while a squark state the supersymmetric partner of one. A quark or squark state with a $\prime$ denotes one with the quark state being the $SU(2)$ partner of a mass eigenstate. For instance, $\tilde{u}_{L3}^\prime$ is the up-type squark state from $\tilde{Q}_3$ which contains the exact left-handed $b$ quark according to our parametrization of the Lagrangian. The scalar and fermion states of the up-sector Higgs doublet are denoted by $h_0^\alpha$ and $h_-^\alpha$, respectively.

D. Explicit Scalar-Fermion-Fermion Couplings

We are now ready to spell out the couplings of the component fields. Of particular phenomenological interest are the scalar-fermion-fermion couplings. The gaugino couplings are, of course, standard. Coming from the gauge interaction parts, they are exactly the same as in MSSM. The couplings from the superpotential is, however, much richer in content. As an explicit illustration of our notation and for easy reference, we list them here. Firstly, we give the corresponding couplings concerning the (color-singlet) charged and neutral fermion, from the superpotential. Compared with that of MSSM, we have a modified higgsino part and a list of new terms from the trilinear couplings. We have
Here, we count the $\phi^\alpha$ of Ref. [7]. Recall that $\lambda^\alpha$ corresponds to the down-Yukawa coupling matrix, and $\lambda^\alpha_{ijk}$ corresponds to the charged lepton Yukawa coupling matrix, both of which are diagonal under the SVP; in addition, we have $u^\alpha_i = V_{\text{CKM}}^{ij} u_{ij}$ being $SU(2)$ partner of the mass eigenstate $d_{i1}$, and $u^\alpha_i$ its scalar partner. We also use below $\tilde{d}^\alpha_{ij}$, which is explicitly, $V_{\text{CKM}}^{ij} \tilde{d}_{ij}$.

There are some more scalar-fermion-fermion terms besides those given in $\mathcal{L}_X$. These extra terms are slepton-quark-quark terms. With the above explicit listed terms, however, it is straight forward to see what the extra terms are like. They are given by

$$\mathcal{L}_{\text{qqqs}} = y_{\alpha} V_{\text{CKM}}^{ij} h_u^\alpha u_{\alpha i} d_{ij} + y_{\alpha} \tilde{l}_0^i d_{\alpha i} u_{\alpha i} + X_{\alpha ij} \tilde{l}_0^i d_{\alpha k} + y_{\alpha} \tilde{l}_0^i d_{\alpha i} u_{\alpha i} - X_{\alpha ij} \tilde{l}_0^i d_{\alpha k} + \text{h.c.} \ .$$

(6)

In both of the above expressions for $\mathcal{L}_X$, the terms present in MSSM can be easily identified, with the replacement of the $l^0_\alpha$ and $\tilde{l}_0^i$ states by the more familiar notation of $h_u^\alpha$ and $h_u^\alpha$. The nice feature, obtained without approximation, is a consequence of the SVP. The simple structure of the trilinear coupling contributions to the $d$-quark and charged lepton masses, is what make the analysis simple and easy to handle. We want to emphasize that the above expressions are exact tree-level results without hidden assumptions behind its validity. The only point of caution here is that the $l^*_\alpha$ states are not exactly the charged leptons and neutrinos. We will come to the mass matrices for the fermions in the next section.

E. Notes on the Scalar Potential

As the SVP, conceptually, involves identifying the direction of the VEV among the $\tilde{L}_\alpha$‘s, we will take a look at the scalar potential here and see what this really means. The discussion also serves to answer queries on whether this can be consistently done — a question that causes some confusion. The major part of the results here is first presented in the appendix of Ref. [7].

In terms of the five, plausibly electroweak symmetry breaking, neutral scalars fields $\phi_n$, the generic (tree-level) scalar potential, as constrained by SUSY, can be written as:

$$V_{\phi_n} = Y_n |\phi_n|^4 + X_{mn} |\phi_m|^2 |\phi_n|^2 + \tilde{m}_n^2 |\phi_n|^2 - (\tilde{m}_m^2 e^{i\theta_m} \phi_m^\dagger \phi_n + \text{h.c.}) \ .$$

(7)

Here, we count the $\phi_n$’s from −1 to 3 and identify a $\phi_\alpha$ (recall $\alpha = 0$ to 3) as $\tilde{l}_0^\alpha$ and $\phi_{-1}$ as $h_u^\alpha$. Parameters (all real) in the above expression for $V_{\phi_n}$ are then given by
\[ \hat{m}_{\alpha}^2 = \tilde{m}_{\alpha}^2 + |\mu_{\alpha}|^2 , \]
\[ \hat{m}_\omega^2 = \tilde{m}_\omega^2 + \mu_{\alpha}^* \mu_\alpha , \]
\[ \hat{m}_{i\alpha}^2 e^{i\theta_{i\alpha}} = -\tilde{m}_{i\alpha}^2 - \mu_{\alpha}^* \mu_\beta , \quad \text{(no sum)} , \]
\[ \hat{m}_{i\omega}^2 e^{i\theta_{i\omega}} = B_\alpha , \quad \text{(no sum)} , \]
\[ Y_n = \frac{1}{8} (g_1^2 + g_2^2) , \]
\[ X_{-\alpha} = -\frac{1}{4} (g_1^2 + g_2^2) = X_{-\alpha} . \quad (8) \]

Under the SVP, we write the VEVs as follow:

\[ v_+ (\equiv \sqrt{2} \langle \phi_+ \rangle) = u_+ , \]
\[ u_0 (\equiv \sqrt{2} \langle \phi_0 \rangle) = u e^{i\theta} , \]
\[ v_i (\equiv \sqrt{2} \langle \phi_i \rangle) = 0 , \quad (9) \]

where we have put in a complex phase in the VEV \( u_0 \), for generality.

The equations from the vanishing derivatives of \( V_{EW} \) along \( \phi_+ \) and \( \phi_0 \) give

\[ \left[ \frac{1}{8} (g_1^2 + g_2^2) (v_u^2 - v_d^2) + \hat{m}_\omega^2 \right] v_u = B_0 v_e e^{i\theta_e} , \]
\[ \left[ \frac{1}{8} (g_1^2 + g_2^2) (v_d^2 - v_u^2) + \hat{m}_\omega^2 \right] v_d = B_0 v_e e^{i\theta_e} . \quad (10) \]

Hence, \( B_0 e^{i\theta_e} \) is real. In fact, the part of \( V_{EW} \) that is relevant to obtaining the tadpole equations is no different from that of MSSM apart from the fact that \( \tilde{m}_{\alpha}^2 \) and \( \tilde{m}_0^2 \) of the latter are replaced by \( \hat{m}_{\alpha}^2 \) and \( \hat{m}_0^2 \) respectively. As in MSSM, the \( B_0 \) parameter can be taken as real. The conclusion here is therefore that \( \theta_\beta \) vanishes, or all VEVs are real, despite the existence of complex parameters in the scalar potential. The above tadpole equations could then be written as

\[ B_0 \cot \beta = \left[ \hat{m}_{\alpha}^2 + \mu_{\alpha}^* \mu_\alpha + \frac{1}{8} (g_1^2 + g_2^2) (v_u^2 - v_d^2) \right] , \]
\[ B_0 \tan \beta = \left[ \hat{m}_0^2 + |\mu_0|^2 + \frac{1}{8} (g_1^2 + g_2^2) (v_d^2 - v_u^2) \right] . \quad (11) \]

Results from the other tadpole equations, in a \( \phi_i \) direction, are quite simple. They can be written as complex equations of the form

\[ \hat{m}_{i\alpha}^2 e^{i\theta_{i\alpha}} \tan \beta = -e^{i\theta_e} \hat{m}_{i\omega}^2 e^{i\theta_{i\omega}} , \quad (12) \]

which is equivalent to

\[ B_i \tan \beta = \hat{m}_{i\omega}^2 + \mu_{\alpha}^* \mu_i , \quad (13) \]

where we have used \( v_u = v \sin \beta \) and \( v_d = v \cos \beta \). Note that our \( \tan \beta \) has the same physical meaning as that in the MSSM case. For instance, \( \tan \beta \), together with the corresponding Yukawa coupling ratio, gives the mass ratio between the top and the bottom quarks.

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The three complex equations for the $B_i$'s reflect the redundancy of parameters in a generic $\hat{L}_\alpha$ flavor basis. The equations also suggest that the $B_i$'s are expected to be suppressed, with respect to the $B_0$, as the $\mu_i$'s are, with respect to $\mu_0$. They give consistence relationships among the involved parameters (under the SVP) that should not be overlooked. The $\tilde{m}^2_{i0i}$ parameters in particular are missing in some of the relevant discussions in the literature. From a different perspective, one may tend to think that the parameters are similar to the $\tilde{m}^2_{ij}$ parameters linked to soft flavor mixings. However, fixing $\tilde{m}^2_{i0i}$ in Eq.(13) leads to definite relations between a $B_i$ and a $\mu_i$ term, which may not be satisfied a priori. The parameters $B_i$, $\mu_i$, and $\tilde{m}^2_{i0i}$ are not independent free parameters, because of the fact that freely chosen values of the set of parameters in a top-down approach, in general, do not land the model automatically into the single-VEV basis [16]. One can think about then performing a flavor basis rotation to recast the model into the SVP framework. The basis rotation would necessarily produce rotated $B_i$, $\mu_i$, and $\tilde{m}^2_{i0i}$ parameters satisfying the tadpole equations. Nonzero values of $\tilde{m}^2_{i0i}$ would be generated, for instance, even if one starts by choosing them to be zero through whatever SUSY breaking scenario consideration. Whether a more fundamental theory such as a specific SUSY breaking theory with a particular prediction on the flavor structure of the GSSM would be automatically compatible with the single-VEV basis is a very interesting problem to be explored.

III. THE (COLOR-SINGLET) FERMIONS

In this section, we start to look at the (tree-level) mass matrices of the matter fields. We discuss the fermions here, and the scalars in the next section. We are quite ignorant about the scalar sector, knowing only that if the particles exist at all, they have to be relatively heavy. For the fermions, we have some light particles observed, namely the charged leptons $e$, $\mu$, and $\tau$ and the neutrinos. In the GSSM framework, however, the neutrinos may not be exact $SU(2)$ partners of the charged leptons. The known charged leptons physical masses correspond only to mass eigenvalues of a big matrix incorporating also two heavy particles called charginos. For the neutral fermions, we know that there have to be three very light states, corresponding to the (physical neutrinos), and four heavy states called neutralinos. There is only some information on likely oscillations among the neutrinos. Nevertheless, using the popular notion that all neutrinos have masses at or below the sub-eV scale gives quite stringent constraints on parameters such as the $\mu_i$'s. This is the scenario that we will mainly focus on. Within such a scenario, we will discuss perturbative diagonalizations of the mass matrices explicitly. The range of validity of our diagonalization results will be self-illustrative. Such results are useful in various phenomenological studies. For the generic scenario in which our analytical perturbative diagonalization procedures fail, the diagonalizations and, hence, the relationship between the physical states and the states used to write the Lagrangian could only be extracted through numerical procedures. While the numerical approach could always be used, our analytical diagonalization results are very useful for a clear understanding of the various phenomenological features (see Refs. [6–11] for illustrations).

The SVP gives simple and straightforward tree-level mass matrices for the quarks in exactly the same form as in the SM, without any approximation or simplifying assumption. Hence, we do not discuss the quark mass expressions further. We would like to emphasize,
however, that this is not the case if one works in an alternative flavor basis. If the \( \hat{L}_i \) VEVs are not zero, they contribute to down-sector quark masses through the \( \lambda_{ijk} \) couplings and the Lagrangian or superpotential can no longer be within the mass eigenstate basis of the quarks. Given the small masses of the down and strange quarks, one may have to be particularly cautious on neglecting such new contributions.

### A. Charged Fermions

Under the SVP, the (color-singlet) charged fermion mass matrix is given by the simple form:

\[
\mathcal{M}_c = \begin{pmatrix}
M_d & \sqrt{2} M_w \cos \beta & 0 & 0 & 0 \\
\sqrt{2} M_w \sin \beta & \mu_0 & \mu_1 & \mu_2 & \mu_3 \\
0 & 0 & m_1 & 0 & 0 \\
0 & 0 & 0 & m_2 & 0 \\
0 & 0 & 0 & 0 & m_3
\end{pmatrix},
\]

with explicit bases for right-handed and left-handed states given by \((-iW^+, \tilde{l}_1^e, l_1^e, l_2^e, l_3^e)\) and \((-iW^-, l_0^-, l_1^-, l_2^-, l_3^-)\), respectively. Here, we allow \( M_d \) and all four \( \mu_i \) parameters to be complex, though the \( \mu_0 \)'s are mostly restricted to be real, for reasons that would become clear below. Obviously, each \( \mu_i \) parameter here characterizes directly the deviation of the \( l_i^- \) from the corresponding physical charged lepton \( (\ell_i = e, \mu, \tau) \), i.e. light mass eigenstates. For any set of other parameter inputs, the \( m_i \)'s can then be determined, through a simple numerical procedure, to guarantee that the correct mass eigenvalues of \( m_e, m_\mu, \) and \( m_\tau \) are obtained — an issue first addressed and solved in Refs. [5,6]. The latter issue is especially important when \( \mu_i \)'s not substantially smaller than \( \mu_0 \) are considered. Such an odd scenario is not definitely ruled out [6]. However, for the more popular of small-\( \mu_i \) scenario, we have \( l_i^- \approx \ell_i^- \), and deviations of the \( \ell_i^- \)'s from mass eigenstates \( \ell_i^+ \)'s and \( m_i \)'s from the (real) \( \ell_i \) masses are very negligible.

We introduce unitary matrices \( V \) and \( U \) diagonalizing the \( R \)- and \( L \)-handed states with

\[
V^\dagger \mathcal{M}_c U = \text{diag}\{M_{3n}\} \equiv \text{diag}\{M_{c1}, M_{c2}, m_e, m_\mu, m_\tau\}.
\]

Here, the mass eigenvalues \( M_{3n} \) with \( n = 1 \) and \( 2 \), i.e. \( M_{c1} \) and \( M_{c2} \), are the chargino masses. Note that notation here is different from those given in Refs. [5,6], and many others in the literature. More explicitly, we have \( R \)- and \( L \)-handed mass eigenstates given by \( (\chi_{i,n}) = V^\tau [-iW^+, \tilde{\chi}_0^\dagger, l_1^+, l_2^+, l_3^+]^T \) and \( (\chi_{-n}) = U^\dagger [-iW^-, \tilde{l}_0^-, l_1^-, l_2^-, l_3^-]^T \); which form the five Dirac fermions \( \chi_n = \begin{pmatrix} \chi_{i,n}^\dagger \\ \chi_{-i,n} \end{pmatrix} \). Consider further

\[
R_L \begin{pmatrix}
M_d \\
\sqrt{2} M_w \sin \beta \\
\mu_0
\end{pmatrix}, \ 
R_L = \text{diag}\{M_{c1}^o, M_{c2}^o\}
\]

with \( M_{c1}^o \) and \( M_{c2}^o \) being the chargino masses in the \( \mu_i = 0 \) limit. One can then write the diagonalizing matrices in the block form.
\[ V = \begin{pmatrix} R_R & -R_R V^1 \\ V & I_{3 \times 3} \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} R_L & -R_L U^1 \\ U & I_{3 \times 3} \end{pmatrix}. \tag{17} \]

Elements in the \( R_R \) and \( R_L \) matrices are all expected to be of order 1. For \( \mu_i \ll M_{\text{ca}}^a \) (\( a = 1 \) and 2), a block perturbative diagonalization can be performed directly on the matrix \( M_{\text{ca}} \) to obtain the following simple result:

\[
\begin{align*}
U_{(i+2)1} & \simeq \frac{\mu_i^*}{M_{\text{ca}}} R_{R1} , \\
U_{(i+2)2} & \simeq \frac{\mu_i^*}{M_{\text{ca}}} R_{R2} , \\
V_{(i+2)a} & \simeq \frac{m_i}{M_{\text{ca}}} U_{(i+2)a} \quad (a = 1 \text{ and } 2). \tag{18}
\end{align*}
\]

(Note: the \( m_i \)'s are assumed to be real here.) The above expressions give the strength of the given matrix elements in the \( U \) and \( V \) blocks of Eq.(17). We note that the \( L \)-handed mixings are roughly measured by the ratio of a \( \mu_i \) to the chargino mass scale, while \( R \)-handed mixings are further suppressed by a charged lepton to chargino mass ratio. The \( U_{(i+2)a} \) elements as given above show no obvious dependence on \( \tan \beta \), though some nontrivial dependence is expected through the \( R_{R2a} \) elements. An exact numerical study also confirms a weak sensitivity on the \( \tan \beta \) value (see Ref. [10] for example).

On the other hand, we have, from Eqs.(17) and (18),

\[
\begin{align*}
U_{a(i+2)} & \simeq -\mu_i \cdot \left[ R_L \left( \text{diag} \{ M_{\text{ca}}^1, M_{\text{ca}}^2 \} \right)^{-1} R_R^1 \right]_{a2} , \\
V_{a(i+2)} & \simeq -\mu_i \cdot \left[ R_R \left( \text{diag} \{ M_{\text{ca}}^1, M_{\text{ca}}^2 \} \right)^{-2} R_L^1 \right]_{a2} ; \tag{19}
\end{align*}
\]

giving the result

\[
\begin{align*}
U_{1(i+2)} & \simeq \frac{\mu_i \sqrt{2} M_W \cos \beta}{M_0^2} , \\
U_{2(i+2)} & \simeq -\frac{\mu_i M_2}{M_0^2} , \\
V_{1(i+2)} & \simeq \mu_i m_i \frac{\sqrt{2} M_W (M_{\chi^+}^* \sin \beta + \mu_0^* \cos \beta)}{|M_0|^4} , \\
V_{2(i+2)} & \simeq -\mu_i m_i \frac{(|M_0|^2 + 2 M_W^2 \cos \beta)}{|M_0|^4} , \tag{20}
\end{align*}
\]

where

\[ M_0^2 \equiv \mu_0 M_2 - M_W^2 \sin 2\beta , \]

with magnitude given by \( M_{\text{ca}}^1 M_{\text{ca}}^2 \simeq M_{\text{ca}}^1 M_{\text{ca}}^2 \). These matrix elements correspond to those given in Ref. [5,6], where only real parameters are taken. The crucial \( \cos \beta \) dependence of the non-standard \( Z^0 \)-boson couplings of the physical charged leptons \( (\ell_i \equiv \chi_{i+2}) \) through \( U_{1(i+2)} \) is emphasized in the latter study. The different \( \tan \beta \) dependence between \( U_{(i+2)a} \) and \( U_{a(i+2)} \) is hence very important.
The 7 × 7 Majorana mass matrix for the neutral fermion can be written as

\[
\mathcal{M}_N = \begin{pmatrix}
M_1 & 0 & M_Z \sin \theta_W \sin \beta & -M_Z \sin \theta_W \cos \beta & 0 & 0 & 0 \\
0 & M_2 & -M_Z \cos \theta_W \sin \beta & M_Z \cos \theta_W \cos \beta & 0 & 0 & 0 \\
M_Z \sin \theta_W \sin \beta & -M_Z \cos \theta_W \sin \beta & 0 & -\mu_0 & -\mu_1 & -\mu_2 & -\mu_3 \\
-M_Z \sin \theta_W \cos \beta & M_Z \cos \theta_W \cos \beta & -\mu_0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 \\
0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

(21)

with explicit basis \((-i \tilde{B}, -i \tilde{W}, \tilde{h}_u^C, \tilde{h}_d, l_1^0, l_2^0, l_3^0\). Note that \(\tilde{h}_d^0 \equiv l_3^0\); and, from the above discussion of the charged fermions, we have, for small \(\mu_i\)’s, \((l_1^0, l_2^0, l_3^0) \approx (\nu_e, \nu_\mu, \nu_\tau)\). The symmetric, but generally complex, matrix can be diagonalized by using unitary matrix \(X\) such that

\[
X^T \mathcal{M}_N X = \text{diag}\{M_n^0\}.
\]

(22)

Again, the first part of the mass eigenvalues, \(M_n^0\) for \(n = 1-4\) here, gives the heavy states, \textit{i.e.}, neutralinos. The last part, \(M_n^0\) for \(n = 5-7\) are hence physical neutrino masses at tree-level.

Consider the mass matrix in the form of \(4 + 3\) block submatrices:

\[
\mathcal{M}_N = \begin{pmatrix}
\mathcal{M}_n & \xi \\
\xi & m_{\nu}^0 \\
\end{pmatrix}.
\]

(23)

In the interest of small neutrino masses, a perturbative (seesaw) block diagonalization can be applied. Explicitly, the diagonalizing matrix can be written approximately as

\[
Z \simeq \begin{pmatrix}
I_{4 \times 4} & (\mathcal{M}_n^{-1} \xi^c)^t \\
-(\mathcal{M}_n^{-1} \xi^c) & I_{3 \times 3} \\
\end{pmatrix}.
\]

The tree-level effective neutrino mass matrix can be then obtained as

\[
(m_\nu) \simeq - (\mathcal{M}_n^{-1} \xi^c)^t \mathcal{M}_n (\mathcal{M}_n^{-1} \xi^c) = - \xi \mathcal{M}_n^{-1} \xi^c \\
\simeq \frac{M_Z^2 \cos^2 \beta (M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W)}{\det(\mathcal{M}_n)} (\mu_i \mu_j),
\]

(24)

where

\[
(\mathcal{M}_n^{-1} \xi^c)_{1j} = -\mu_i \frac{M_Z \cos \beta \mu_a M_3 \sin \theta_W}{\det(\mathcal{M}_n)},
\]

\[
(\mathcal{M}_n^{-1} \xi^c)_{2j} = \mu_i \frac{M_Z \cos \beta \mu_a M_3 \cos \theta_W}{\det(\mathcal{M}_n)},
\]

\[
(\mathcal{M}_n^{-1} \xi^c)_{3j} = \mu_i \frac{M_Z^2 \cos^2 \beta (M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W)}{\det(\mathcal{M}_n)},
\]

\[
(\mathcal{M}_n^{-1} \xi^c)_{4j} = -\mu_i \frac{\mu_a M_i M_j - M_Z^2 \sin \beta \cos \beta (M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W)}{\det(\mathcal{M}_n)}.\]

(25)
and
\[
\text{det}(\mathcal{M}_n) = \mu_0 \left[ -\mu_0 M_1 M_2 + M_Z^2 \sin 2\beta (M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W) \right]
\] (26)
is equivalent in expression to the determinant of the MSSM neutralino mass matrix.

It is obvious that the $3 \times 3$ matrix $(\mu_i \mu_j)$ has only one nonzero eigenvalue given by
\[
\mu_5^2 = |\mu_1|^2 + |\mu_2|^2 + |\mu_3|^2.
\] (27)

We can define
\[
R_5 = \begin{pmatrix}
\frac{\mu_1^*}{|\mu_1|^2} & 0 & \frac{\sqrt{|\mu_2|^2 + |\mu_3|^2}}{|\mu_1|^2} \\
\frac{\mu_2^*}{|\mu_2|^2} & \frac{\mu_3}{|\mu_3|^2} & -\frac{\sqrt{|\mu_2|^2 + |\mu_3|^2}}{|\mu_3|^2} \\
\frac{\mu_5^*}{|\mu_5|^2} & -\frac{\mu_5}{|\mu_5|^2} & \frac{\sqrt{|\mu_2|^2 + |\mu_3|^2}}{|\mu_5|^2}
\end{pmatrix}
\] (28)

Then, we have
\[
R_5^T (\mu_i \mu_j) R_5 = \text{diag}\{\mu_5^2, 0, 0\}. \quad \text{Here, } \mu_5 \text{ and } \sqrt{|\mu_2|^2 + |\mu_3|^2} \text{ are taken as real and positive.}
\]

With this result, we can write the overall diagonalizing matrix $X$ in the form
\[
X \simeq \begin{pmatrix}
I_{4 \times 4} & (\mathcal{M}_n^{\dagger} \xi^T) \\
-(\mathcal{M}_n^{-1} \xi^T) & I_{3 \times 3}
\end{pmatrix}
\begin{pmatrix}
R_n & 0_{4 \times 3} \\
0_{3 \times 4} & e^{i\xi} R_5
\end{pmatrix}
\begin{pmatrix}
R_n & e^{i\xi} (\mathcal{M}_n^{\dagger} \xi^T) R_5 \\
-(\mathcal{M}_n^{-1} \xi^T) & e^{i\xi} R_5
\end{pmatrix},
\] (29)

where $R_n$ is a $4 \times 4$ matrix with elements all expected to be of order 1, basically the diagonalizing matrix for the $\mathcal{M}_n$ block and $e^{i\xi}$ is a constant phase factor put in to absorb the overall phase in the constant factor in the expression of Eq.(24) so that the resulted neutrino mass eigenvalue would be real and positive. The matrix $X$ contains the important information of the gaugino and higgsino contents of the physical neutrinos. This is given by the mixing elements in the off-diagonal blocks. The $Z$ matrix in itself gives similar information for the effective SM neutrinos (flavor states). The latter matrix may be more useful in the analysis of neutrino phenomenology.

**IV. THE SCALAR SECTORS**

The SVP also simplifies much the otherwise extremely complicated expressions for the (tree-level) mass-squared matrices of the scalar sectors. Scalars with the same color and electric charges may, of course, mix with one another. The story gets a bit complicated with, again, the color-singlet scalars. Here, there are nine physical neutral scalars and seven physical charged scalars, together with a unphysical Goldstone state in each case. In the MSSM, these are separated into the Higgses and the sleptons. The separation is no longer valid in the GSSM. Recall that under the SVP we can still identify the Higgses as states that come from the superfield multiplets $\hat{H}_u$ and $\hat{L}_0$. The physical states, however, are expected to be mixture of the Higgses and the sleptons. Similar to the case for the fermions discussed in the previous section, we will be particularly interested in mixings between the Higgses and the other sleptons. We will also give explicit perturbative formulae for such mixings in the small neutrino mass scenario. Another thing we will address is the LR squark and
slepton mixings, which also have interesting contributions beyond those in the MSSM. We start first with the squarks sectors. Note that in all the expressions given in this section, we neglect contributions from the nonholomorphic soft terms [cf. Eq.(3)]. Such contributions go only into the LR-mixing part, in exactly the same way as they do in the MSSM. Their explicit incorporation is hence straightforward.

A. The squarks

The up-squark mass-squared matrix looks exactly as the one in the MSSM, hence we skip here. The down-squark sector, however, has interesting result. We have the mass-squared matrix as follows:

\[
\mathcal{M}^2_D = (\mathcal{M}^2_{2L} \mathcal{M}^2_{2R}^\dagger \mathcal{M}^2_{2R}\mathcal{M}^2_{2L}^\dagger),
\]

where

\[
\mathcal{M}^2_{LL} = \tilde{m}^2_Q + m_D^\dagger m_D + M_Z^2 \cos 2\beta \left[-\frac{1}{2} + \frac{1}{3} \sin 2\theta_W\right] I_{3 \times 3},
\]

\[
\mathcal{M}^2_{RR} = \tilde{m}^2_D + m_D m_D^\dagger + M_Z^2 \cos 2\beta \left[-\frac{1}{3} \sin 2\theta_W\right] I_{3 \times 3},
\]

and

\[
(\mathcal{M}^2_{RL})^\dagger = A_D \frac{v_u}{\sqrt{2}} - (\mu^*_\alpha X^\dagger_{\alpha j k}) \frac{v_u}{\sqrt{2}}
\]

\[
= [A_d - \mu^*_a \tan \beta] m_D + \frac{\sqrt{2} M_W \cos \beta}{g_2} \delta A^D - \frac{\sqrt{2} M_W \sin \beta}{g_2} (\mu^*_i X^\dagger_{ijk}).
\]

Here, \(m_D\) is the down-quark mass matrix, which is diagonal under the parametrization adopted; \(A_d\) is a constant (mass) parameter representing the “proportional” part of the \(A\)-term and the matrix \(\delta A^D\) is the “proportionality” violating part; \((\mu^*_a X^\dagger_{\alpha j k})\), and similarly \((\mu^*_i X^\dagger_{ijk})\), denotes the \(3 \times 3\) matrix \(\alpha j k\) with elements listed. \(^5\) The \((\mu^*_a X^\dagger_{\alpha j k})\) term is the full \(F\)-term contribution, while the \((\mu^*_i X^\dagger_{ijk})\) part separated out in the last expression gives the new contributions beyond that of the MSSM. The term actually gives new contributions to the quark electric dipole moment, for example, as discussed in Refs [8,9].

B. The neutral scalars

We have five neutral complex scalar fields, all from electroweak doublets. They are the \(\hat{H}_u\) and the four \(\hat{L}_a\)’s. Explicitly, we write the \((1 + 4)\) complex field, \(\phi_n\)’s, in the order \((\hat{h}_1^0, \hat{h}_1^0, \hat{i}_1^0, \hat{i}_1^0, \hat{i}_3^0)\). Using this basis, all the neutral scalar mass terms can be written in two parts — a simple \((\mathcal{M}^2_{\phi \phi})_{mn} \phi^\dagger_m \phi_n\) part, and a Majorana-like part in the form

\^5Note that we use this kind of bracket notations for matrices extensively here. In this case, the repeated index \(i\) is to be summed over as usual, and hence dummy.
\( \frac{1}{2} (M_{\phi\phi})_{mn} \phi_m \phi_n + \text{h.c.} \). As the neutral scalars are originated from chiral doublet superfields, the existence of the Majorana-like part is a direct consequence of the electroweak symmetry breaking VEVs, hence restricted to the scalars playing the Higgs role only. They come from the quartic terms of the Higgs fields in the scalar potential. We have, explicitly,

\[
M_{\phi\phi}^2 = \frac{1}{2} M_Z^2 \begin{pmatrix}
\sin^2 \beta & -\cos \beta \sin \beta & 0_{1 \times 3} \\
-\cos \beta \sin \beta & \cos^2 \beta & 0_{1 \times 3} \\
0_{3 \times 1} & 0_{3 \times 1} & 0_{4 \times 3}
\end{pmatrix};
\]

(33)

and

\[
M_{\phi\phi}^{2\dagger} = M_\phi^2 + M_{\phi\phi},
\]

(34)

where

\[
M_\phi^2 = \begin{pmatrix}
\tilde{m}_{h_\alpha}^2 + \mu_\alpha \mu_\alpha + M_Z^2 \cos 2\beta \left[ -\frac{1}{2} \right] \\
-(B_\alpha^*) & \tilde{m}_L^2 + (\mu_\alpha^* \mu_\beta) + M_Z^2 \cos 2\beta \left[ \frac{1}{2} \right] I_{4 \times 4}
\end{pmatrix}.
\]

(35)

Note that \( M_{\phi\phi}^2 \) here is real, due to results from Sec. II D above; while \( M_{\phi\phi}^{2\dagger} \) does have complex entries. Writing the five \( \phi_n \)'s in terms of their scalar and pseudoscalar parts, the full \( 10 \times 10 \) (real and symmetric) mass-squared matrix for the real scalars is then given by

\[
M_s^2 = \begin{pmatrix}
M_{ss}^2 & M_{sp}^2 \\
(M_{sp}^2)^T & M_{pp}^2
\end{pmatrix},
\]

(36)

where the scalar, pseudoscalar, and mixing parts are

\[
M_{ss}^2 = \text{Re}(M_{\phi\phi}^2) + M_{\phi\phi}^2 = \text{Re}(M_\phi^2) + 2 M_{\phi\phi}^2,
\]

\[
M_{pp}^2 = \text{Re}(M_{\phi\phi}^{2\dagger}) - M_{\phi\phi}^2 = \text{Re}(M_\phi^2),
\]

\[
M_{sp}^2 = -\text{Im}(M_{\phi\phi}^2) = -\text{Im}(M_\phi^2),
\]

(37)

respectively. If \( \text{Im}(M_\phi^2) \) vanishes, the scalars and pseudoscalars decouple from one another and the unphysical Goldstone mode would be found among the latter. Note that our expansion of the \( \phi_n \)'s into scalar and pseudoscalar parts here takes the universal form \( \phi_n = \frac{1}{\sqrt{2}} (\phi_n^s + i \phi_n^a) \), hence we actually have \( h_u^s = \frac{1}{\sqrt{2}} (h_u^s - i h_u^a) \).

As a real scalar mass matrix, \( M_s^2 \) could be diagonalized by an orthogonal matrix \( D^s \). However, it is sometimes useful to consider \( D^s \) as if it is just an unitary matrix. Thinking about the neutral scalars as complex scalars instead of in terms of the scalar and pseudoscalar constituents also helps to illustrate some theoretical features. These considerations are especially valid for the three \( h_u^s \)'s, which are usually called “sneutrino”.\(^6\) Hence, we write \( D^{s\dagger} M_s^2 D^s = \text{diag} \{ M_{sm}^2, m = 1 \text{ to } 10 \} \). It is useful to consider the form of \( D^s \) closest to the identity matrix, \( i.e., \), with all diagonal entries being order one. The unphysical Goldstone mode has, of course, to be found then among the first two pseudoscalars. The mode is

\(^6\)They are not exactly the scalar partners of the physical neutrinos.
naturally label as the \( m = 6 \) mass eigenstate here. All the off-diagonal entries except those related to mixing of the Higgses (\( i.e. \) the 12-, 21-, 67-, and 76-entries) are expected to be relatively small.

Now we want to decouple the unphysical pseudoscalar explicitly. Note that \( \mathcal{M}_\psi^2 \) can be rewritten, through using the tadpole equations (11) and (13) as

\[
\mathcal{M}_\psi^2 = \begin{pmatrix}
B_0 \cot \beta & -B_0 & -(B_i) \\
-B_0 & B_0 \tan \beta & (B_i) \tan \beta \\
-(B_i) \ast (B_i) \ast & (B_i) \ast & (\ast)
\end{pmatrix}, \tag{38}
\]

with \( B_0 \) taken as real; and the \( (\ast) \) denotes the last \( 3 \times 3 \) block in the original form. The matrix can be diagonalized by a simple rotation among the first two states given by

\[
R_\beta = \begin{pmatrix}
\sin \beta & -\cos \beta \\
\cos \beta & \sin \beta
\end{pmatrix}.
\]

Explicitly, we have

\[
\begin{pmatrix}
R^e_\beta & 0_{2 \times 3} \\
0_{3 \times 2} & I_{3 \times 3}
\end{pmatrix} \mathcal{M}_\psi^2 \begin{pmatrix}
R_\beta & 0_{2 \times 3} \\
0_{3 \times 2} & I_{3 \times 3}
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 \\
0 & \frac{2B_0}{\sin 2\beta} & \frac{1}{\cos \beta} (B_i) \\
0 & \frac{1}{\cos \beta} (B_i) \ast (\ast)
\end{pmatrix}, \tag{40}
\]

where the \( 3 \times 3 \) block denotes by \( (\ast) \) is left untouched. The extended rotation given by \( \text{diag} \{ I_{5 \times 5}, R_\beta, I_{5 \times 3} \} \) then obviously decouples the resulted 6-th state as the massless unphysical mode.

Next, we introduce

\[
R_\alpha = \begin{pmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{pmatrix}, \tag{41}
\]

as the diagonalizing matrix for the first \( 2 \times 2 \) block of \( \mathcal{M}_{S_S}^2 \). Define \( R_{\alpha \beta} = \text{diag} \{ R_\alpha, I_{3 \times 3}, R_\beta, I_{3 \times 3} \} \). Then, the matrix \( R^e_{\alpha \beta} \mathcal{M}_{S_S}^2 R_{\alpha \beta} \) has only small off-diagonal entries arising from two sources. One of the latter is the set of lepton-flavor mixing soft masses familiar in the MSSM, namely, the \( \tilde{m}_{\nu_{ij}} \)’s in our notation; the other is the set of terms dependent on the \( \mu_i \)’s, the \( B_i \)’s, and the \( \tilde{m}_{\nu_{ij}} \)’s. In fact, using Eq.(38), we can see that \( R^e_{\alpha \beta} \mathcal{M}_{S_S}^2 R_{\alpha \beta} \) can be written as

\[
\begin{pmatrix}
M_{S_S}^2 & 0 & 0 \left( \text{Re}[B_i] \right) [\tan \beta \sin \alpha - \cos \alpha] \\
0 & M_{S_S}^2 & 0 \left( \text{Re}[B_i] \right) [\tan \beta \cos \alpha + \sin \alpha] \\
0 \left( \text{Re}[\tilde{m}_{\nu_{ij}}^2 + \mu_i \mu_j] \right) + \frac{1}{2} M_2 \cos 2\beta I_{3 \times 3} & 0 \left( \text{Im}[B_i] \right) & 0
\end{pmatrix} = \begin{pmatrix}
\text{Re}[\tilde{m}_{\nu_{ij}}^2 + \mu_i \mu_j] + \frac{1}{2} M_2 \cos 2\beta I_{3 \times 3} & 0 & 0 \\
0 & \text{Im}[\tilde{m}_{\nu_{ij}}^2 + \mu_i \mu_j] & 0
\end{pmatrix}, \tag{42}
\]

where we have explicitly written out only the elements in the upper triangular part (of the symmetric matrix), and introduced \( M_{S_S}^2 \) and \( M_{S_S}^2 \) to denote the 11- and 22- elements. The latter notation is based on the fact that the two diagonal entries would be approximately
the mass eigenvalues $M^2_{s_1}$ and $M^2_{s_2}$, respectively, for small $B_i$’s. Again, the 6-th state is the decoupled unphysical Goldstone mode.

We are now ready to introduce the diagonalizing matrix $D^s$ to the neutral scalars $\mathcal{D}^s \mathcal{M}^2_s \mathcal{D}^s = \text{diag}\{M^2_{s_m}, m = 1 \text{ to } 10\}$ written as

$$D^s \equiv R_{\alpha \beta} R^s.$$ 

In the phenomenologically interesting case with all the off-diagonal entries to the matrix $R_{\alpha \beta}^s \mathcal{M}^2_s R_{\alpha \beta}$ being small, one can easily obtain useful expressions for the interesting diagonalizing matrix elements. In particular, we have

$$\begin{align*}
D^s_{(i+2)m} &= R^s_{(i+2)m}, \\
D^s_{(i+7)m} &= R^s_{(i+7)m},
\end{align*}$$

which can be read out from implementing a perturbative diagonalizing formula on the mass matrix $R_{\alpha \beta}^s \mathcal{M}^2_s R_{\alpha \beta}$ as explicitly given above. Furthermore, we have

$$\begin{align*}
D^s_{1(i+2)} &= \frac{-\text{Re}[B_i]}{M^2_s}, \\
D^s_{1(i+7)} &= \frac{\text{Im}[B_i]}{M^2_s},
\end{align*}$$

and

$$\begin{align*}
D^s_{2(i+2)} &= \frac{\text{Re}[B_i] \tan \beta}{M^2_s}, \\
D^s_{2(i+7)} &= \frac{-\text{Im}[B_i] \tan \beta}{M^2_s},
\end{align*}$$

where we introduce $M^2_s$ to denote a generic mass-squared parameter at the scalar mass-squared (or $\tilde{m}^2_{Lii}$) scale; in particular, here it represents the quantities $\left[\tilde{m}^2_{Lii} + |\mu_i|^2 + \frac{1}{2} M^2_Z \cos 2\beta - M^2_{s_{i'}}\right]$ and $\left[\tilde{m}^2_{Lii} + |\mu_i|^2 + \frac{1}{2} M^2_Z \cos 2\beta - M^2_{s_{i'}}\right]$ respectively. Similarly, we have

$$\begin{align*}
D^s_{6(i+2)} &= \frac{-\text{Im}[B_i]}{M^2_s}, \\
D^s_{6(i+7)} &= \frac{\text{Re}[B_i]}{M^2_s}, \\
D^s_{7(i+2)} &= \frac{\text{Im}[B_i] \tan \beta}{M^2_s}, \\
D^s_{7(i+7)} &= \frac{\text{Re}[B_i] \tan \beta}{M^2_s},
\end{align*}$$

with $M^2_s$ representing $\left[\tilde{m}^2_{Lii} + |\mu_i|^2 + \frac{1}{2} M^2_Z \cos 2\beta - \frac{2B_0}{\sin 2\beta}\right]$ and $\left[\tilde{m}^2_{Lii} + |\mu_i|^2 + \frac{1}{2} M^2_Z \cos 2\beta - \frac{2B_0}{\sin 2\beta}\right]$ respectively. Note that one would like to write the $D^s_{(i+2)m}$ and $D^s_{(i+7)m}$ matrix elements in a similar form, e.g. we write

$$D^s_{(i+2)1} \approx \frac{-\text{Re}[B_i]}{M^2_s} (\tan \beta \sin \alpha - \cos \alpha).$$
C. The charged scalars

From Eq.(2) above, we can see that the charged Higgses should be considered on the same footing together with the sleptons. We have hence an $8 \times 8$ mass-squared matrix. We use the basis \{ $h_u^\pm$, $\tilde{l}_0^\pm$, $\tilde{l}_1^\pm$, $\tilde{l}_3^\pm$, $\tilde{h}_0^\pm$, $\tilde{h}_3^\pm$ \} to write the mass-squared matrix in the following $1 + 4 + 3$ form:

\[
\mathcal{M}_E^2 = \begin{pmatrix}
\tilde{M}_{d_u}^2 & \tilde{M}_{l_l}^2 & \tilde{M}_{l_R}^2 \\
\tilde{M}_{l_l}^2 & \tilde{M}_{l_R}^2 & \tilde{M}_{l_R}^2 \\
\tilde{M}_{l_R}^2 & \tilde{M}_{l_R}^2 & \tilde{M}_{l_R}^2
\end{pmatrix} ;
\]

(47)

where

\[
\tilde{M}_{d_u}^2 = m_{d_u}^2 + \mu_u^* \mu_u + M_Z^2 \cos 2\beta \left[ \frac{1}{2} - \sin^2 \theta_W \right] + M_Z^2 \sin^2 \beta \left[ 1 - \sin^2 \theta_W \right] ,
\]

\[
\tilde{M}_{l_l}^2 = m_{l_l}^2 + m_{l_l}^2 m_l + (\mu_u^* \mu_\nu) + M_Z^2 \cos 2\beta \left[ -\frac{1}{2} + \sin^2 \theta_W \right] I_{4 \times 4} ,
\]

\[
+ \left( M_Z^2 \cos^2 \beta \left[ 1 - \sin^2 \theta_W \right] 0_{1 \times 3} \right),
\]

\[
\tilde{M}_{l_R}^2 = m_{E}^2 + m_{E} m_{E}^\dagger + M_Z^2 \cos 2\beta \left[ \sin^2 \theta_W \right] I_{3 \times 3} ;
\]

(48)

and

\[
(\tilde{M}_{l_R}^2) = \begin{pmatrix}
B_a^* \\
\frac{1}{2} M_Z^2 \sin 2\beta \left[ 1 - \sin^2 \theta_W \right] \\
0_{3 \times 1}
\end{pmatrix},
\]

\[
(\tilde{M}_{l_R}^2) = - (\mu_i^* \delta_{ij}) \frac{v_0}{\sqrt{2}} = (\mu_k^* m_k) \quad (\text{no sum over } k) ,
\]

\[
(\tilde{M}_{l_R}^2) = \begin{pmatrix}
0 & \frac{v_0}{\sqrt{2}} - (\mu_\nu^* \lambda_{ijk}) \frac{v_0}{\sqrt{2}} \\
A_e^* & \frac{\sqrt{2} M_0 \cos \beta}{g_2} (0) & - (\mu_k^* m_k \tan \beta) \frac{\sqrt{2} M_0 \sin \beta}{g_2} (\mu_i^* \lambda_{ijk})
\end{pmatrix} .
\]

(49)

Notations and results here are similar to the squark case above, with some difference. We have $A_e$ and $\delta A_e$, or the extended matrices \((\cdot)^*\) incorporating them, denote the splitting of the $A$-term, with proportionality defined with respect to $m_e$; $m_l$ = diag\{0, $m_e$\} = diag\{0, $m_\nu$, $m_\tau$, $m_\tau$\}. Recall that the $m_i$'s are approximately the charged lepton masses. A $4 \times 3$ matrix \((\mu_i^* \lambda_{ijk})\) gives the new contributions to \((\tilde{M}_{l_R}^2)\) beyond that of the MSSM. In the above expression, we separate explicitly the first row of the former, which corresponds to mass-squared terms of the type $\tilde{l}^\dagger h_u^+$ type ($h_u^+ \equiv h_b^+$). The nonzeros $\tilde{M}_{l_R}^2$ and the $B_i$'s in $\tilde{M}_{l_R}^2$ are also interesting new contributions. The former is a $\tilde{l}^\dagger (h_u^+)\dagger$ type, while the latter is a $\tilde{l}^\dagger h_u^+$ term. Note that the parts with the $[1 - \sin^2 \theta_W]$ factor are singled out as they are extra contributions to the masses of the “charged-Higgses” (i.e. $\tilde{l}_0^\pm \equiv h_u^+$ and $h_u^+$). The latter is the result of the quartic terms in the scalar potential and the fact that the Higgs doublets bear VEVs.
Introducing the diagonalizing matrix $\mathcal{D}^l$, we have $\mathcal{D}^l \mathcal{M}_E^2 \mathcal{D}^l = \text{diag}\{ M_{\tilde{\ell}m}^2, m = 1 \text{ to } 8 \}$. We label the unphysical Goldstone mode by $m = 1$. In the small neutrino mass scenario we are particularly interested in, we expected the $\mathcal{M}_E^2$ to be dominantly diagonal, apart from the mixing between the Higgses (i.e., $h_u$ and $h_d \equiv \tilde{l}_0$) to give the $m = 1$ mode. The matrix $\mathcal{D}^l$ may then be naturally chosen to be close to identity, i.e. with all diagonal entries being order 1 and only the 12- and 21-entries being possibly large (order 1) among the off-diagonal ones.

The unphysical Goldstone mode is, of course, to be found among the Higgs fields $h_u^+$ and $h_d^-$ ($\equiv \tilde{l}_0$). In fact, using the corresponding tadpole equations [cf. Eq.(11)], the first $2 \times 2$ (Higgs) block of the matrix $\mathcal{M}_E^2$ can be written simply as

$$
\begin{bmatrix}
2 B_0 \sin 2 \beta + M_Z^2 \cos^2 \theta_W \\
\sin \beta \cos \beta \\
\sin \beta \cos \beta \\
\sin^2 \beta 
\end{bmatrix}
\begin{bmatrix}
\cos^2 \beta & \sin \beta \cos \beta \\
\sin \beta \cos \beta & \sin^2 \beta 
\end{bmatrix}.
$$

Further using the other tadpole equations [cf. Eq.(13)], we obtain

$$
\begin{bmatrix}
R_\beta & 0_{2 \times 6} \\
0_{4 \times 2} & I_{6 \times 6}
\end{bmatrix}
\mathcal{M}_E^2
\begin{bmatrix}
R^\prime_\beta & 0_{2 \times 6} \\
0_{4 \times 2} & I_{6 \times 6}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{2 B_0}{\sin 2 \beta} + M_Z^2 \cos^2 \theta_W & \frac{1}{\cos \beta} (B_i) & \frac{1}{\cos \beta} (\mu_k m_k) \\
0 & \frac{1}{\cos \beta} (B_i^*) & * & * \\
0 & \frac{1}{\cos \beta} (\mu_k^* m_k) & * & *
\end{bmatrix},
$$

(50)

where the *’s denote $3 \times 3$ blocks exactly the same as those in the original $\mathcal{M}_E^2$, details of which skipped here. Note that there is no sum over $k$ in $(\mu_k m_k)$ or $(\mu_k^* m_k)$.

We are again interested in useful approximate expressions for off-diagonal elements of the diagonalizing matrix $\mathcal{D}^l$ responsible for mixing between the Higgses and the other sleptons. From the above, it is easy to obtain

$$
\mathcal{D}^l_{1(i+2)} \simeq \frac{B_i}{M_Z^2},
$$

$$
\mathcal{D}^l_{2(i+2)} \simeq \frac{B_i \tan \beta}{M_Z^2},
$$

(51)

where the more exact expression substituted by $M_S^2$ here is $[\tilde{m}_{E_{ii}}^2 + M_Z^2 \cos 2 \beta \left( -\frac{1}{2} + \sin^2 \theta_W \right) - \left( \frac{2 B_0}{\sin 2 \beta} + M_Z^2 \cos^2 \theta_W \right)]$. Similarly, we have

$$
\mathcal{D}^l_{1(i+5)} \simeq \frac{\mu_i m_i}{M_S^2},
$$

$$
\mathcal{D}^l_{2(i+5)} \simeq \frac{\mu_i m_i \tan \beta}{M_S^2},
$$

(52)

with $M_S^2$ representing $[\tilde{m}_{E_{ii}}^2 + M_Z^2 \cos 2 \beta \left( -\sin^2 \theta_W \right) - \left( \frac{2 B_0}{\sin 2 \beta} + M_Z^2 \cos^2 \theta_W \right)]$. Note that the elements of the type $\mathcal{D}^l_{(i+2)(j+5)}$ are standard slepton LR mixing terms with extra contributions [cf. $\tilde{M}_{\tilde{\ell}L}^2$ from Eq.(49)].
V. REMARKS

We have try to present clearly the formulation of the GSSM, or the (complete) phenomenological theory of SUSY without R parity in some details above. We emphasize that the notion of R parity and its violation is a perspective that looks at the model as an extension of the (R-parity conserving) MSSM. MAY be particle physicists are too familiar with the MSSM. The idea of studying an extension of the MSSM by adding some R-parity violating terms sounds easy and straightforward. The present approach takes a bit different perspective. We emphasize studying the GSSM as what it is which we recapitulate again — a theory built with the (minimal) superfield spectrum incorporating the SM particles, interactions dictated by the SM (gauge) symmetries, and the idea that SUSY is softly broken. The perspective helps to clarify some confusing issues within the literature of R-parity violation.

We advocate strongly the adoption of a specific parametrization, the SVP, which we elaborate on in this review. The parametrization issue has not been addressed directly and clearly often enough in the literature. Naively, when the model is considered as limited versions of extensions of the MSSM, there is no need to readdress the parametrization beyond the latter framework. Such a perspective, however, hinders a comprehensive study of the plausible interesting phenomenology. Our use of the SVP has, for instance, led to the identification of new (RPV) tree-level contributions to the superpartner mass matrices and their phenomenological implications [7–11,17]. Such features have been largely overlooked in the literature. The contributions typical involved products of bilinear and trilinear (RPV) parameters. The two group of parameters are simply seldom considered together previously. In our opinion, it is the SVP framework that renders such features transparent. Moreover, we are only beginning to study the detailed phenomenological features of the type. A lot of work still await our effort.

Among the earlier studies that address the parametrization issue with some care, the more notable ones are given by Refs. [18,19]. There has been some phenomenological use of basically the SVP in Refs. [20,21], before we advocated explicitly adoption of it as the basic formulation for studying GSSM (or R-parity violation). Other authors have come to appreciate using the formulation since then. These include Refs. [22–25].

The complete expressions for the mass matrices of fermions and scalars, together with the perturbative diagonalization expression listed above are very useful in various phenomenological studies. We again refer interested readers to our papers on the studies of various phenomenological features for illustrations [7–11].

ACKNOWLEDGMENTS

The author wants to thank M. Bisset, K. Cheung, S.-K. Kang, Y.-Y. Keum, C. Macesanu, L.H. Orr, for the enjoyable collaborations on the subject area, and H.Y. Cheng, P.H. Frampton, J. Feng, K. Hagiwara, S.C. Lee, H.-N. Li, H. Murayama, P. Nath, S. Pakvasa, and X. Tata for encouragement. Supports from colleagues at University of Rochester, and Academia Sinica and National Central University of Taiwan is also to be acknowledged. He has also benefited from activities under the SUSY sub-program of the Particle Physics program, National Center for Theoretical Sciences, and the hospitality of the center
APPENDIX A: Q & A :-

In this section, we recapitulate on some aspects of our formulation in relation to potential confusion from related presentations in the literature where, in many cases, only partial considerations of R-parity violation are addressed.

Why use 4 $\hat{L}_\alpha$’s instead of 3 $\hat{L}_i$’s and one $\hat{H}_d$?
- because we do not know \textit{a priori} to what extend the Higgs is a superpartner of the charged leptons. In the MSSM, there is a clearly enforced distinction between the superfields containing the leptons and the one containing the Higgs (scalar) doublet responsible for the masses of the leptons and down-sector quarks. This does not come out naturally from the minimal superfield spectrum containing the SM. In particular, the distinction is set by the arbitrarily imposed global symmetry of lepton number. In fact, we have nowadays, from the neutrino oscillation experiments, strongly suggestive evidence on the violation of lepton number symmetry. Without the lepton number distinction, we have the four ($\hat{L}_\alpha$) doublet superfields of the same quantum number which one should not distinguish \textit{a priori}. We have illustrated, in the discussion of the SVP, that there is a special advantage to identify the Higgs direction among the four doublets, hence the $\hat{H}_d$ notation. One should bear in mind though that the $\hat{H}_d$ ($\equiv \hat{L}_0$) superfield may contain partly the charged lepton states. Besides, keeping the four $\hat{L}_\alpha$ notation helps to keep track of the common “flavor” structure among the four doublets. The latter is well illustrated by our discussion of the scalar mass terms, with pieces that are otherwise easily overlooked.

Why should we choose a fixed parametrization (or fixed set of flavor bases) before doing anything else?
- because that is the right way to do physics; it gives an unambiguous connection between the parameters and the experimental data. Again, we do not do SM physics with quark masses or Yukawa couplings in a generic flavor basis. Fixing a parametrization removes redundancy of parameters. The clearly defined set of parameters then would have a definite relation to observable physical phenomena. Only then will discussions about the magnitude of the parameters be sensible. In fact, sets of parameters from two different parametrizations, though usually called the same names, are not quite the same quantities. They may have different phenomenological roles.

How about parametrization invariant quantities?
- only in very limited specific cases could one find quantities of the type that may be useful. This is particularly the case when there is a high degree of redundancy among naively defined generic set of parameters with no single parametrization having any specific advantage. A good example is on the admissible complex phases of a model Lagrangian. Among all the parameters admitting complex phases, in some case, only a small number (combinations of) such phases are physical. Others could be removed by an optimal parametrization. In the case of SM quark masses, or the CKM matrix, there is only one
physical phase. There, the Jarkslog invariant used to characterize the resultant CP violating effect, gives the only major example of such parametrization invariant quantities in the literature. Even in that case, the usage is limited. The standard parametrization of the CKM matrix (given by the Particle Data Group) does use a single particular (arbitrarily chosen) phase.

What about parametrization of complex phases under the discussion formulation?
— that still have to be performed. Our formulation presented here is admittedly incomplete in this sense. The issue has basically not been explicitly addressed for any R-parity violation studies in the literature. However, one does not expect much redundancy among the extra admissible complex phases. The complex phases are of interest only in CP violating physics. In most of the other phenomenological studies, only real parameters are taken. We only started to address interesting new contributions to CP violating physics (of fermion electric dipole moments) recently [8,9]. Even in that case, our ignorance of possible redundant phases does not hurt much. Nevertheless, the issue of fixing an optimal parametrization of the physical phases certainly have to be looked into carefully when we want to study all the CP violating features of the model in good details.

What other parametrization(s) have been used in the literature?
— the “single-$\mu$ parametrization”, to some extent. The parametrization issue for RPV physics is either not explicitly addressed or confusingly neglected in many studies in the literature. This is especially true before our first advocate of the SVP [5]. In the case that it is addressed or a particular parametrization explicitly adopted, it is usually the single-$\mu$ parametrization introduced almost twenty years ago [18]. Interpret under the present notation, the parametrization chooses to identify as $\hat{L}_i$, or rather denoted by $\hat{H}_d$, the direction in the space of the four $\hat{L}_\alpha$’s that characterizes the direction of the $\mu_\alpha$ couplings. The common way to put it is that “the three $\mu_i$’s can be rotated away without loss of generality”. However, the statement is also a common source of confusion. The first thing we want to emphasize here is that the $\mu_i$’s under our formulation cannot be set to zero. We have an optimal parametrization (apart from a possible minor redundancy in complex phases) within which no parameter (generally complex) can be set to vanish without enforcing an extra assumption and hence changing the model. All the effects involving the $\mu_i$’s we discussed in the various papers, with collaborators, are physical. Our formulation, in our opinion, simply provides the most transparent way to see the phenomenological implications. Such physical effects will be described in terms of different combinations of parameters when a different parametrization is used to study the model.

There are a few confusing aspects concerning the explicit or implicit use of the single-$\mu$ parametrization that we want to clarify. The parametrization, or the idea to rotated away the $\mu_i$’s, was first introduced only to study a limited version of RPV model [18]. Extending the framework to include other RPV terms, as many authors did, is a bit less than trivial. When one rotates away the $\mu_i$’s, one is forced to admit generally nonzero VEVs for the $\hat{L}_i$’s, often called the sneutrino VEVs. Our formulation here, the SVP, chooses to “rotates away” the latter, keeping rather nonzero $\mu_i$’s. One certainly cannot do both at the same time. In the complete model (GSSM) under the single-$\mu$ parametrization, the nonzero $\hat{L}_i$
VEVs contribute to masses and mixings, not only of the neutrinos, but also that of the down-sector quarks and charged leptons. Together with these VEVs, the \( \chi \)- and \( \lambda \)- type couplings also enter the latter mass matrices. Then, at least at the conceptually level, one cannot write the Lagrangian with even the down-quark superfields, and hence the \( \chi \)-type couplings involving them, in the corresponding mass eigenstate bases. We have illustrated the misalignment of the charged leptons \( \ell_i \) with the \( \hat{L}_i \) superfields under the SVP. In our case, the story is simple. Each \( \mu_i \) characterizes directly such misalignment between an \( \ell_i \)– \( \hat{L}_i \) pair. It should be obvious that under the single-\( \mu \) parametrization, the situation is far more complicated, as quite a number of parameters are involved for the fermion mass terms of within each \( \hat{L}_i \). And a similar story goes for the down-quark sector. To recapitulate once more, the SVP allows us the use the down-quark mass eigenstate basis and keeps the complication within the leptonic sector, with the whole deviation from the SM or MSSM setting characterizes only by the three \( \mu_i \)'s.

Within the use of single-\( \mu \) parametrization, there is also the statement that one can rotate away two of the three “sneutrino VEVs”. This is true. However, doing that is equivalent to identifying one the \( \hat{L}_i \)'s directions, say \( \hat{L}_3 \), with the direction of the VEV in the \( \hat{L}_i \) space. The catch then is that the chosen \( \hat{L}_3 \) is generally an arbitrary linear combination of what may be approximately the \( \hat{L}_e, \hat{L}_\mu, \) and \( \hat{L}_\tau \).

Furthermore, to the extent that the single-\( \mu \) parametrization has to admit nonzero VEVs for the \( \hat{L}_i \)'s, these superfields have scalar components that resume some Higgs character though effectively only the \( \hat{L}_0 \) is named a Higgs doublet. The fermion part of the latter doublet contains partly the physical charged leptons. The physical neutrinos are the light mass eigenstates of a complicated \( 7 \times 7 \) neutral fermion mass matrix, in any case not just linear combinations of the flavor neutrinos \( \nu_e, \nu_\mu, \) and \( \nu_\tau \). Calling the scalars within the \( \hat{L}_i \)'s sneutrinos is not quite right either.

What are the issues involved in going from one parametrization to another?  
— it is mainly a basis rotation among the superfields; when the VEVs are involved, it gets a bit complicated.  
For instance, we can certainly take the SVP as a starting point and perform a \( SU(4) \) rotation among the \( \hat{L}_\alpha \)'s to a new basis, say, denoted by \( \hat{L}'_\alpha \)'s requiring then the new \( \mu_i \)'s (couplings of the \( \hat{H}_u \hat{L}'_i \) terms) be zero. To progress beyond writing down the Lagrangian in the new basis, one will have to solve for the scalar potential to find all the nonzero VEVs. The rotation involving only a basis change among the \( \hat{L}_\alpha \) superfield now has an effect also on the interpretation of couplings in the other sectors. The down-quark superfields are no longer in the mass eigenstate basis (of the tree-level mass matrix), as the extra VEVs give off-diagonal contributions discussed above.

The bilinear RPV couplings can be rotated away, right?  
— depends. It should be clear from our answers to the above two questions.

Should we worry much about doing a basis rotation?  
— generally speaking, we do not have to. Physics should be about formulating a theoretical model to be checked versus experiments. Connecting different formulations of the same model is not very interesting, unless more than one formulation have special advantages in specific studies or have been commonly used (correctly) within the community
What may be a good parametrization? — one that simplifies analyses and enables more direct identification of the major role of the parameters. So long as low energy phenomenology is concerned, that is all we should care about. We hope that our discussion above has illustrated the merits of the parametrization presented and advocated here. All in all, we urge authors on the subject area to state explicitly the parametrization adopted and be careful with any extra assumptions used. As parameters under the same notation are commonly used under different parametrization, explicitly stated or otherwise, such assumptions on some of the parameters have a very different meaning when interpreted under a different parametrization. This is quite a source of confusion. Finally, for one who agrees with our opinion on the merits of the SVP advocated here, we certainly suggest adopting the formulation.

From the GSSM perspective, what is R-parity violation? Which terms are R-parity violating? — the definition of lepton number, and hence R parity, is actually ambiguous; though there is a clear MSSM limit. Naively, one can compare the Lagrangian of the model (mainly the superpotential and the soft SUSY breaking terms) with that of the MSSM and call all the extra terms RPV terms. This is the commonly adopted terminology. However, while baryon number is still a clearly definite concept within the GSSM, lepton number is not. This fact may help to appreciate why may be baryon number is still be conserved while lepton number is not.

Within the SM, we have lepton flavor numbers $L_e$, $L_\mu$, and $L_\tau$ unambiguously defined. Lepton number is then given by $L = L_e + L_\mu + L_\tau$. For instance, the electron carries one unit of $L_e$, and only particles within the same multiplet carry the $L_e$ number. The definition carries over to MSSM. In fact, the MSSM might better be interpreted as a supersymmetric version of the SM with the global symmetries of lepton number(s) and baryon number assumed as a fundamental part of the latter rather than “accidental” consequence of the gauge symmetries. The GSSM is more like a natural supersymmetric version of the SM of gauge interactions. We have seen that in the GSSM, the electron, for example, is not contained totally inside any $\hat{L}_i$ superfield multiplet. In fact, it is not contained totally inside the $\hat{L}_0$ superfield multiplet. The naive interpretation of R-parity violation mentioned amounts to assigning (opposite) $L_e$ numbers only to $\hat{L}_1$ and $\hat{E}_1^C$, and $L_\mu$ and $L_\tau$ numbers only to $\hat{L}_2$ and $\hat{E}_2^C$ and $\hat{L}_3$ and $\hat{E}_3^C$, respectively. So, the $\hat{L}_i$’s carry lepton number but not the $\hat{L}_0$. This is only under the SVP. Under a different parametrization, the $\hat{L}_i$ and $\hat{E}_i^C$ pairs each represent even less directly the exact superfields containing the corresponding charged leptons, as off-diagonal contributions would be everywhere over the fermions mass matrices (in the superfield basis). Hence, it is even more inappropriate to assign the lepton numbers in the same way. The terminology of lepton number, or R-parity, violation is mainly used for comparing against MSSM features. For the theory model in itself, under whatever parametrization, such a definition of lepton number(s) certainly sounds too arbitrary.

Of course from the experimental point of view, lepton number or even lepton flavor numbers can still be unambiguously defined. This is just like the use of strangeness. Lepton flavor numbers cease to be theoretically exact concepts in a model with leptonic or neutrino...
flavor mixings. Likewise, lepton number ceases to be theoretically exact concept in GSSM. Just like we do not talk about baryon (quark) flavor numbers in the theoretical discussion of the SM. When one insists on assigning such lepton number(s) to the GSSM theoretical ingredients (the superfield multiplets), the exact meaning of the assigned lepton number(s) then differs among different parametrizations and differs from the experimental notion.

**What exactly is “MSSM + RPV trilinear superpotential parameters”?**

— In the literature, a model of R-parity violation that received a lot of attention is described as ‘MSSM + RPV trilinear superpotential parameters’. That is to say, authors assumed the model Lagrangian is given by that of the MSSM amended by the addition of the $\lambda$, $\lambda'$, and $\lambda''$- type couplings terms to the superpotential. In some cases, only one or two of the three type of couplings are assumed. In the worst case, that is claimed as the most general Lagrangian or superpotential obtainable from the supersymmetrized SM particle spectrum. Our discussions above should have illustrated clearly the fallacy of such a claim, or similar ones. The “model” mostly received attention, apparently, because it is simple and very similar to the MSSM. One does not have to worry much about changes in the familiar identity of the superfields as there is no new contributions to the tree-level mass matrices for the fermions. Restricting from the GSSM to such a model really means assuming all the other “RPV” terms vanish. Under the SVP, the vanishing of the $\mu_i$’s would be enough to keep the meaning of the Lagrangian terms for such a model the way it was desired. The other “RPV” terms may then be neglected in some phenomenological studies without enforcing the vanishing assumption strictly. Under a parametrization with nonzero $\hat{L}_i$ VEVs, the VEVs must also be assumed to vanish, which means assumption on the soft SUSY breaking parameters involved in the scalar potential as well.

**What exactly is “MSSM + RPV bilinear (superpotential) parameters”?**

— Another relative popular version of RPV model is given by admitting only the bilinear couplings. A theoretical better motivated and clearly defined option is to obtain such a model from integrating out the heavier superfield(s) that give rise to a spontaneous breaking of an otherwise present lepton number symmetry. Without a background lepton number symmetry to begin with, the adding of the bilinear terms to MSSM is less of a clear conceptual issue. Looking at it from the perspective of our formulation here, one may of course take the assumption that the trilinear couplings, namely the $\lambda$, $\lambda'$, and $\lambda''$- type couplings as well as their soft SUSY breaking counterparts all vanish. The further vanishing of the $B_i$ parameters could be a further assumption. However, if the parametrization is fixed as the SVP, the clear division between the class of “RPV” parameters of the $\lambda$, $\chi$, and $\chi'$-type couplings, and their “R-parity” conserving counterparts is lost. The latter are called the MSSM superpotential terms, and are supposed to be terms giving the SM Yukawa couplings. As we have discussed above, the $\lambda$- and $\chi$- type couplings have no contribution to the diagonal SM (or physical) Yukawa couplings only when they are parameters defined under the SVP flavor bases. Hence, special care may be needed to handle such a RPV model.
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