A joint analysis of Planck and BICEP2 B modes including dust polarization uncertainty

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Abstract. We analyze BICEP2 and Planck data using a model that includes CMB lensing, gravity waves, and polarized dust. Recently published Planck dust polarization maps have highlighted the difficulty of estimating the amount of dust polarization in low intensity regions, suggesting that the polarization fractions have considerable uncertainties and may be significantly higher than previous predictions. In this paper, we start by assuming nothing about the dust polarization except for the power spectrum shape, which we take to be $C_l^{BB,\text{dust}} \propto l^{-2.42}$. The resulting joint BICEP2+Planck analysis favors solutions without gravity waves, and the upper limit on the tensor-to-scalar ratio is $r < 0.11$, a slight improvement relative to the Planck analysis alone which gives $r < 0.13$ (95% c.l.). The estimated amplitude of the dust polarization power spectrum agrees with expectations for this field based on both HI column density and Planck polarization measurements at 353 GHz in the BICEP2 field. Including the latter constraint on the dust spectrum amplitude in our analysis improves the limit further to $r < 0.09$, placing strong constraints on theories of inflation (e.g., models with $r > 0.14$ are excluded with 99.5% confidence). We address the cross-correlation analysis of BICEP2 at 150 GHz with BICEP1 at 100 GHz as a test of foreground contamination. We find that the null hypothesis of dust and lensing with $r = 0$ gives $\Delta \chi^2 < 2$ relative to the hypothesis of no dust, so the frequency analysis does not strongly favor either model over the other. We also discuss how more accurate dust polarization maps may improve our constraints. If the dust polarization is measured perfectly, the limit can reach $r < 0.05$ (or the corresponding detection significance if the observed dust signal plus the expected lensing signal is below the BICEP2 observations), but this degrades quickly to almost no improvement if the dust calibration error is 20% or larger or if the dust maps are not processed through the BICEP2 pipeline, inducing sampling variance noise.

Keywords: gravitational waves and CMBR polarization, CMBR experiments, cosmological parameters from CMBR, inflation

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1 Introduction

$B$ mode polarization of cosmic microwave background (CMB) anisotropies is a powerful experimental approach to study inflationary models and is expected to improve upon the constraints from the temperature anisotropies (see [1] for a review). This is because $B$ modes can be generated by the primordial gravity waves from inflation [2–4] and not by ordinary scalar modes, meaning that the signal can be detected without the sampling variance from scalar perturbations which dominates other CMB channels such as temperature and $E$ polarization. The above statement needs to be modified to some extent, since gravitational lensing can generate $B$ polarization by lensing $E$ polarization [5]. At high $l$ this lensing signal dominates, and the small gravity wave signal is masked by the sampling variance uncertainties. Only for $l < 100$ is the lensing signal sufficiently small to be able to probe gravity waves below the current limits set by the Planck satellite [1]. Nevertheless, one expects that $B$ modes will dominate the limits or detections of primordial gravity waves in the future.

At the noise sensitivity level this expectation has already been reached by the BICEP2 experiment: their recent results [6] have generated a great deal of excitement in the field, with an excess signal seen above the expectations from lensing. Original estimates of the foreground contribution, primarily dust polarization, suggested that it was at most a minor correction to the observed signal, of order 25%. However, recently released dust polarization maps from the Planck satellite suggest a relatively high polarization fraction in low intensity regions of the sky, around 8–10% on average but with a lot of scatter [7]. These maps do not include the BICEP2 region,\footnote{During the late stages of the review process for this paper, an analysis of Planck polarization maps that include the BICEP2 region was presented in [8]. We briefly discuss the impact of these new results in later sections.} citing as some of the reasons both noise and residual systematic uncertainties, especially contamination from the cosmic infrared background (CIB). The BICEP2 team assumed a polarization fraction of 5% for dust in their field, based on a preliminary map presented at a conference [9]. A visual comparison of this map with the new version in [7] suggests that there is imperfect agreement between the two in many regions and that the polarization fractions are significantly higher in the new maps relative to the old ones. One reason for the discrepancy is the CIB, which was not corrected for in the old maps; since CIB is not polarized correcting for it reduces intensity but not polarization, increasing the polarization fraction. We note that changing the polarization fraction from
5% to 8.5% would increase the dust polarization power spectrum by a factor of 3, which would result in a very large signal, potentially able to explain most of the observed signal as dust. In addition, the “DDM2” dust polarization estimate presented by the BICEP2 team, which is the most realistic model they consider, lacks degree-scale angular variations of the polarization direction, making it uncertain at higher \( l \). (The “DDM1” model lacks fluctuations in the polarization fraction as well, making it even more of an underestimate). Given the uncertainties in the CIB, including the monopole, the estimates of the intensity zero modes, etc., the dust power could be significantly underestimated. Furthermore, because the degree-scale angles are unreliable, it is perhaps unsurprising that the cross-correlation of BICEP2 data with such an imperfect map results in little or no correlation, making the cross-correlation an unreliable test of dust foregrounds. Overall, using the polarization fraction from a preliminary map, without a proper error analysis or understanding of its systematics, is not reliable, even more so now that we know that the map has changed considerably in the new version published in [7].

Given all these considerations, in this paper we take a step back, ignore any information about the dust polarization that may or may not be currently available in the BICEP2 field and revisit the analysis using a more conservative approach. We ask the question: what limits on \( r \) can we deduce from BICEP2 and Planck given what we currently know about dust foregrounds? One of the more robust results presented by the Planck team [11] is that the power spectrum of the dust polarization scales as \( \Delta_{BB,dust}^2 \propto l^{-0.3} \) independent of the amount of dust, with an overall amplitude that strongly depends on the amount of dust in the field. Recent Planck analyses confirm the scalings presented in [11] for dust intensity [12] and polarization [8]. For the BICEP2 dust column density, which based on the dust maps is estimated to be \( N_{HI} = 1.5 \times 10^{20} \text{cm}^{-2} \) [13], extrapolation using the scaling presented in [11] suggests \( \Delta_{BB,dust,100}^2 \sim 0.015 \mu K^2 \), a factor of 5 higher than the BICEP2 DDM2 estimate. Although this extrapolation is rather uncertain, it once again suggests that one should worry about the overall dust polarization levels. In this paper, we will retain only the power-law scaling of the dust spectrum with \( \Delta_{BB,dust}^2 \propto l^{-0.42} \), assuming a flat amplitude prior between 0 and \( \Delta_{BB,dust,100}^2 = 0.03 \mu K^2 \). We will also assume that the covariance matrix for the dust polarization component is Gaussian; this is the most optimistic assumption, as there may be additional non-Gaussian bandpower correlations present which we will neglect here.

Unless otherwise specified, throughout this paper \( r \) will denote the tensor-to-scalar ratio at \( k = 0.05 \text{Mpc}^{-1} \), upper limits are reported at 95% c.l., and constraints with both upper and lower limits are 68% c.l. ranges.

## 2 Likelihood analysis

We evaluate combined constraints from BICEP2 and Planck data by importance sampling Planck parameter chains using the BICEP2 likelihood code. Specifically, we use the

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2 Note that in the peer-reviewed, published revision of the BICEP2 paper [10], which appeared after the preprint of our paper, the BICEP2 team acknowledges many of these issues, including the larger uncertainty in the level of dust contamination suggested by the new measurements from [7]; their revised paper also omits the “DDM2” dust polarization model.

3 This scaling matches the best estimate of the power spectrum shape from recent Planck data over large regions of the sky [8]. We have also performed our analysis with the \( l^{-0.3} \) scaling from [11], and find that this small change in the exponent has little impact on our results, affecting the upper limits on \( r \) at the level of a few percent.
"Planck+WP" chains from the Planck Legacy Archive (PLA),\footnote{http://www.sciops.esa.int/index.php?page=Planck_Legacy_Archive&project=planck} which include WMAP polarization constraints on the reionization optical depth in addition to Planck temperature data, analyzed assuming the \( \text{"base}_r \) flat \( \Lambda \text{CDM} + r \) model. To speed up computations for some tests, we “thin” the chains by a factor of two, using every other model from the PLA chains in our analysis; we have verified that this does not significantly change the constraint on \( r \) from Planck+WP data alone \((r < 0.13)\). Note that we only use the Planck team’s analysis without high \( l \) experiments (ACT and SPT), hence our limits on \( r \) are somewhat weaker than those presented in \cite{14} and somewhat stronger than the Planck reanalysis presented in \cite{15}.

We consider two analyses. The main one for the purpose of deriving current constraints assumes there is sampling variance on the dust polarization amplitude in the BICEP2 observing field, as expected if the amplitude prior is based on observations over a larger region of the sky. We then also ask how much better one will be able to do with future data, such as Planck, if the dust prior is based on measurements in the BICEP2 field and therefore has no sampling variance, but may still have some overall error that could arise from noise or systematics.

For the first analysis, we use CAMB \cite{16} to compute the predicted \( B \) mode power spectrum, including both lensing and gravity wave components, for each sample in the thinned PLA chains. Assuming that the dust foreground scales as \( \Delta_{BB,\text{dust},l} \propto l^{-0.42} \) but that its amplitude is uncertain, we draw a random dust polarization amplitude from a flat prior, \( 0 \leq \Delta_{BB,\text{dust},100} \leq 0.03 \mu K^2 \), and add the dust spectrum with that amplitude to the \( B \) mode spectrum. To ensure that the dust amplitude prior is sampled reasonably well, we draw several amplitudes for each model in the chains. We use the public BICEP2 likelihood code,\footnote{http://bicepkeck.org/} including the 9 bandpowers from \( l \sim 45 \) to \( l \sim 300 \), to evaluate the likelihood of the total lensing+gravity wave+dust \( B \) mode spectrum. We then use this likelihood to importance sample the PLA chains (see, e.g., appendix B of \cite{17}) by multiplying the original weight of each model by the BICEP2 likelihood; the new weights can then be used to compute marginalized parameter estimates that include constraints from both Planck and BICEP2. While for most of the analyses we use all 9 bandpowers of BICEP2, we also explore the effects of only using the first 5 bandpowers.

In the second case, where the dust spectrum in the BICEP2 field is assumed to be known without sampling variance, we subtract the dust spectrum from the data points rather than adding it to each model. To do so, we have to assume a specific realization of the dust spectrum in the BICEP2 field, which in turn determines what level of gravity waves will best fit the data. The objective in this case is to determine how uncertainty in the measured dust polarization spectrum propagates into uncertainty in \( r \). For the purposes of this test, we assume that the BICEP2 measurements are equal to the sum of a lensing \( B \) mode spectrum, a gravity wave component with a particular tensor-to-scalar ratio,\footnote{Specifically, for lensing we use the expected bandpowers from http://bicepkeck.org/B2_3yr_cl_expected_lensed_20140314.txt, and for gravity waves we rescale the \( r = 0.1 \) spectrum from http://bicepkeck.org/B2_3yr_cl_expected_withB_20140314.txt.} a dust polarization spectrum, and instrumental noise bias \((N_l)\), and that the same dust spectrum is independently measured in the BICEP2 field with some uncertainty in its normalization at 150 GHz:

\[
\Delta_{BB,\text{BICEP2},l}^2 = \Delta_{BB,\text{lens},l}^2 + \Delta_{BB,\text{GW},l}^2 + \Delta_{BB,\text{dust},l}^2 + N_l^2, \tag{2.1}
\]

\[
\Delta_{BB,\text{dust},l}^2 = (1 + \epsilon_l) \Delta_{BB,\text{dust},l}^2, \tag{2.2}
\]
where hats indicate observed quantities. In the first 4 bandpowers ($l \leq 160$), we take the error on the dust spectrum to be a constant $\epsilon$ for each model, drawn from a Gaussian with mean zero and width $\sigma_{\text{dust}}$. To avoid getting artificially strong constraints related to the excess power above the lensing spectrum in BICEP2 bandpowers 5-7, we set $\epsilon_l = 0$ in the 5 highest-$l$ bandpowers. We then study the expected upper limits or constraints on $r$ as a function of the uncertainty in the dust polarization amplitude, $\sigma_{\text{dust}}$, by drawing several values of $\epsilon$ for each sample in the PLA $\Lambda$CDM+$r$ chains, subtracting the resulting $\Delta^2_{BB,\text{dust},l}$ from the BICEP2 bandpowers, and importance sampling the chains using the BICEP2 likelihood as described above. We note that of course we do not know what the actual realization of dust polarization will be, so our analysis is only meant to give an approximate idea of what one can expect from external dust polarization maps and at what level the results depend on residual noise and systematics in these maps.

Since the PLA chains contain relatively few samples with $r \sim 0.2$, importance sampling is not expected to be accurate for such large values of $r$ and combined constraints with large $r$ should be interpreted with caution. However, we find that most of our analyses limit $r$ to values $\lesssim 0.1$ where importance sampling should be more reliable.

3 Impact of polarized dust on BICEP2 inflation constraints

We start with the most conservative analysis, where we assume only a weak prior on the possible range of dust polarization amplitudes and no specific knowledge of the dust polarization in the BICEP2 field. The left panel of figure 1 shows that there is a clear anticorrelation in the resulting constraints on $r$ and the dust polarization amplitude $\Delta^2_{BB,\text{dust},100}$. The joint constraints favor models with small $r$ and $\Delta^2_{BB,\text{dust},100} \approx 0.01 \mu K^2$. This amplitude is consistent with the information about dust polarization in the BICEP2 field that is currently available.

At 2$\sigma$ in this 2D parameter space, the contours do not extend to dust-free $r \approx 0.2$ models. In principle, the preference for small $r$ could be driven by the Planck+WP constraints which by themselves disfavor $r = 0.2$ at almost 3$\sigma$. However, we find that even when considering the BICEP2 likelihood alone, models with $r = 0$ and a polarized dust component fit the BICEP2 data better than models with $r = 0.2$. In fact, despite marginalizing over the dust polarization amplitude, the joint constraint from Planck+WP+BICEP2 still imposes a slightly stronger limit on the tensor-to-scalar ratio ($r < 0.11$) than Planck+WP alone ($r < 0.13$), if we use the first 9 bandpowers of BICEP2. If we only use the first 5 bandpowers then the constraints are unchanged relative to Planck.

The Planck collaboration recently released a new analysis of their data that included estimates of 150 GHz dust polarization power spectra in the BICEP2 field, inferred by extrapolating from constraints at 353 GHz [8]. These results show that the amplitude of B mode power from dust is $(0.013 \pm 0.004) \mu K^2$ in the band $40 < l < 120$, where the error includes noise and uncertainty from frequency extrapolation, added in quadrature. A full assessment of the impact of these new measurements on the interpretation of the BICEP2 data will have to wait until the completion of the joint analysis that the BICEP2 and Planck teams are currently carrying out, but for now we can estimate the approximate effect on the constraint on $r$ (see also [18] which analyzes the same combination of data and finds similar constraints).

We do this by retaining our assumption of a power-law dust polarization spectrum with index $-0.42$ (consistent with the measurements of [8] over larger regions of the sky) and simply replacing our assumption of a flat prior on the dust amplitude at $l = 100$ with a Gaussian
constraint with mean $0.012 \mu K^2$ (adjusted downwards from the power at the midpoint of the measured band, $l = 80$, using the power law scaling) and width $0.004 \mu K^2$. We still include sampling variance on the dust contribution in this case since the shape of the dust spectrum in the BICEP2 field is poorly constrained. This is a somewhat conservative choice since the overall amplitude of the spectrum in the BICEP2 field is measured without sampling variance, but the extra variance is negligible relative to the $\sim 30\%$ uncertainty in the measured amplitude. The left panel of figure 1 shows that this constraint is fully consistent with our initial result that assumed a flat prior on the dust spectrum amplitude. Since the new Planck constraint favors models with relatively large contributions from dust, combining the constraints tightens the limit on the gravity wave component to $r < 0.09$.

In the right panel of figure 1, we show the constraints in the $r-n_s$ plane from this analysis compared with the constraints from Planck+WP alone. While the upper limit on $r$ only improves slightly with the addition of BICEP2 data fit with a polarized dust component, the joint constraints place increasing pressure on large-$r$ inflation models such as quadratic potentials. This is even more true with the addition of the dust amplitude constraint from Planck 353 GHz data, which excludes quadratic inflation models with $\sim 60$ $e$-folds of inflation at more than $2\sigma$ in the $r-n_s$ plane.

The 1D projection of the posterior probability for $r$ is shown in figure 2 for both Planck+WP and Planck+WP+BICEP2, marginalized over the dust polarization amplitude with either a flat prior or the Planck 353 GHz constraint. We also plot the BICEP2 likelihood; this is only intended as a qualitative comparison since, as noted by [19], the likelihood analysis presented in [6] differs in several ways from the public likelihood code, leading to small shifts in $r$. Despite these caveats, it is clear that the combined constraint from Planck+WP+BICEP2, allowing a free dust polarization amplitude, is completely different from the main constraint from BICEP2 alone which did not include a dust component. We see that the combined analysis improves the limits at high values of $r$. For example, taking
the prediction of a $V = M^2 \phi^2/2$ model with about 55 e-folds of inflation, $r = 0.14$, as a representative example, we find that only 1.7% of the posterior distribution has $r > 0.14$ in the combined analysis (and only 0.5% with the Planck 353 GHz constraint), compared to 3.3% for Planck+WP alone.

To illustrate how models with polarized dust and $r = 0$ are able to fit the BICEP2 data as well as or better than models with negligible dust polarization and $r = 0$, we compare the $B$ mode spectra for these models in figure 3. Although the first bandpower of BICEP2 is low relative to the $r = 0$ model with dust, the fit nevertheless remains acceptable due to considerable uncertainty from sampling variance in the BICEP2 field, which has an effective area of about 380 deg$^2$ or approximately 0.9% of the sky.

We next consider how accurate measurements of the dust polarization pattern in the BICEP2 field could further tighten limits on $r$. First, we assume that $r = 0$ so that the measured BICEP2 bandpowers are the sum of the lensing $B$ mode power spectrum and a dust component, which is independently measured with a fractional uncertainty $\sigma_{\text{dust}}$. In the most optimistic scenario where the dust polarization in the BICEP2 field is measured perfectly, the joint constraint on $r$ from Planck+WP+BICEP2 improves significantly to $r < 0.05$ (left panel of figure 4). If the dust polarization can be measured with 10% uncertainty, the limit degrades to $r < 0.07$; with 20% uncertainty, $r < 0.09$; and with 30% uncertainty, $r < 0.10$, in which case the direct measurement of the dust polarization provides almost no additional information about the tensor-to-scalar ratio.

If we perform the same analysis assuming that the true tensor-to-scalar ratio is $r = 0.1$ (so that approximately half of the $B$ mode power measured by BICEP2 is attributed to dust polarization, after subtracting lensing), the Planck+WP+BICEP2 constraint is $r = 0.08 \pm 0.03$ (right panel of figure 4). This is biased low relative to the assumed value of $r$ because of the upper limit on $r$ from Planck data. In this case, the BICEP2 data by themselves disfavor a model without gravity waves by $\Delta \chi^2 \approx 11$ relative to the hypothesis of $r = 0.1$ plus dust. Including uncertainty on the measured dust spectrum increases the uncertainty on $r$ to some degree and also increases the bias in the combined constraints; e.g., with 20% uncertainty on dust polarization, $r = 0.06^{+0.04}_{-0.03}$ from Planck+WP+BICEP2 and

**Figure 2.** Marginalized constraints on $r$ from Planck+WP (dashed curve), Planck+WP+BICEP2 with free dust polarization amplitude or including the constraint from Planck 353 GHz data (solid and dotted curves), and the BICEP2 likelihood alone (dot-dashed curve).
4 Frequency dependence

One can ask whether there is other evidence in the current data sets that would favor $r > 0$. One argument presented by the BICEP2 team is that the spectral color of the signal is inconsistent with either a pure dust or pure synchrotron signal at about $2.2 \sigma$. However, the

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analysis was done using the first 5 bandpowers, and in all but the lowest of these bandpowers the contribution of the lensing spectrum, which has the spectral color of the CMB, is also important; this complication was not discussed in the BICEP2 papers.\footnote{After we posted the preprint of our paper, the BICEP2 team revised their frequency analysis to account for the lensing component \cite{BICEP2updated}.} Moreover, there is likely to be both synchrotron and dust in this field and, according to recent Planck results, the two are strongly correlated in polarization \cite{Planck13}. While in typical regions the synchrotron and dust components are roughly equal around 60 GHz, for this field where the dust levels are particularly low this point could be pushed to higher frequencies. Because the two foregrounds are correlated and may both be present in this regime, the two can combine into a spectral color that can be close to that of the pure CMB, even though separately they each have a very different spectral slope.

To make these arguments more quantitative, we perform an $l$-dependent analysis. We combine the the BICEP2 auto-correlation errors with the BICEP1 100 GHz — BICEP2 150 GHz cross-correlation errors, and first assume that auto-correlation measures exactly the same field as the BICEP1-BICEP2 cross-correlation so that there is no additional sampling variance. We test two models: first, the null hypothesis with $r = 0$, assuming that the signal consists of dust scaling as $\nu^{2.3}$ relative to the CMB,\footnote{For BICEP2, the spectral index of the CMB is $\beta = -0.7$ \cite{BICEP2updated}, and Planck data give a spectral index of $\beta \approx 1.6$ for dust \cite{Planck13}, so the relative index is $\approx 2.3$.} and the expected lensing contribution (where we ignore sampling variance fluctuations). We could also add synchrotron at the expected level \cite{Planck13}, partially correlated with dust \cite{Planck13}, although we note that this does not change the results much as we will show below. The second model is the no dust, pure CMB, hypothesis, which has the CMB color. The $l = 45$ point measures $0.0086 \mu K^2$ in BICEP2 auto-power, which the null hypothesis predicts should be $0.0038 \mu K^2$ for the cross-correlation with 100 GHz BICEP1, while the pure CMB no dust hypothesis predicts that this value is unchanged. The BICEP2-BICEP1 100 GHz cross-power measures $(-0.0034 \pm 0.0056) \mu K^2$, so the null hypothesis is favored relative to the no dust hypothesis by $\Delta \chi^2 \sim -3$. For the $l = 75$ point, the BICEP2 auto-power is $0.013 \mu K^2$, which the null hypothesis predicts should be $0.0062 \mu K^2$ for the cross-correlation with 100 GHz BICEP1. The measurement is $(0.025 \pm 0.0065) \mu K^2$, so this point favors the CMB hypothesis against the null hypothesis by $\Delta \chi^2 \sim 5$. We can repeat the same at the higher $l$ points, where the differences between the two hypotheses are smaller due to increasing lensing contribution and the errors are larger, finding that overall the difference in total $\chi^2$ is 2 in favor of the no dust model. We find that the first two bandpowers have the most discriminatory power between the two hypotheses, but they nearly cancel each other out given the observed values. Since adding synchrotron only makes the spectral dependence of dust+synchrotron closer to the CMB, this difference in $\chi^2$ is reduced if we include a small amount of correlated synchrotron polarization in our analysis.

Another important consideration is sampling variance, which we have ignored so far; we assumed that the BICEP1-BICEP2 cross-correlation analysis samples exactly the same modes as the BICEP2 auto-correlation, but in reality this is unlikely to be the case given the differences in sky coverage and noise properties of the two experiments \cite{Planck13, BICEP1}, and the two could in fact be measuring different realizations of dust and CMB $B$ modes. The large $\chi^2$ we found in the comparison between the two measurements above for either of the two models, especially for the first two bandpowers, suggests that there may be additional sampling variance errors. This would not be surprising: the sampling variance errors for
BICEP2 auto-power are very large, about 60% for the first bandpower and 35% for the second bandpower [6] (see figure 3). Suppose, for example, that we need to add an additional error on the order of 30% (corresponding to about 0.004 $\mu$K$^2$ for the dust+lensing model) because of the sampling variance errors. Then the difference in $\chi^2$ between the two hypotheses is further reduced to 1.5, and presumably to even less if synchrotron is included. This is a very different conclusion than the one presented in the BICEP2 paper, where it was stated that dust is disfavored at $\Delta\chi^2 \sim 5$ (2.2 $\sigma$). Most of the difference can be accounted for with the lensing contribution, which we included in our analysis and which was not included in the BICEP2 analysis. The current frequency information is thus unlikely to be a strong discriminator between the dust and no dust hypotheses.

Future Keck Array data with much lower noise could be very powerful in discriminating between these hypotheses. There are a few lessons from the exercise above that should be generally applicable. Because of large sampling variance, care should be taken to minimize its error by using the same weights for the two frequency channels. Because of the lensing effect, and the sampling variance associated with it, only the lowest $l$ bandpowers contain useful information for this test. A proper statistical analysis, including lensing, synchrotron, and dust, and accounting for both measurement and sampling variance errors, is needed to assess the statistical significance of the frequency information.

5 Discussion

In this work we have performed a joint analysis of Planck and BICEP2, assuming lensing, primordial gravity waves, and dust polarization as the components contributing to the BICEP2 $B$ mode data. We have assumed that we do not have a reliable prior on the dust polarization in the BICEP2 field, so we perform a pure power spectrum analysis. We find that under the assumption of a dust polarization power spectrum with free amplitude but known scaling with $l$, the BICEP2 data prefer $r = 0$. The limits on $r$ change from $r < 0.13$ with Planck to $r < 0.11$ with Planck+BICEP2 (95% c.l.) when using all 9 bandpowers, but remain at $r < 0.13$ when only the first 5 bandpowers are used. Including the latest Planck constraints on dust polarization in the BICEP2 field from [8] reduces the limit to $r < 0.09$. With our analysis, we find that $V = M^2\phi^2/2$ inflation predicting $r = 0.14$ is disfavored at about 2.5 $\sigma$ when the dust amplitude is unconstrained, and at nearly 3 $\sigma$ when we apply the Planck dust polarization constraint. This result does not automatically mean that BICEP2 has no evidence for primordial gravitational waves in its data. It does, however, mean that the case strongly relies on careful characterization of the actual dust polarization contribution in the BICEP2 field, which appears to be higher than the various estimates presented by the BICEP2 team. It is thus too early to celebrate the BICEP2 results as a definitive proof of inflation.

Moving forward, Planck dust polarization data in the BICEP2 field has already begun to clarify the situation. However, using the data without processing them through the BICEP2 pipeline induces sampling variance errors; due to various projections of BICEP2 data, variable noise, and potentially highly variable dust polarization in the field, one needs to process Planck dust data through the BICEP2 pipeline to minimize the sampling variance. We note that the sampling variance errors in the first 3 BICEP2 bins, which have the greatest

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There is also some possible evidence of sampling variance at work in the comparison of the BICEP2 $BB$ auto-correlation and the BICEP2-Keck Array cross-correlation presented in [6], which appear to be several $\sigma$ apart for the lowest $l$ bandpowers.
sensitivity to gravity wave $B$ modes, are around 60%, 35%, and 30%. Even if the Planck analysis had measured low dust polarization on average in the BICEP2 field, given the potentially large sampling fluctuations within the field there would be no guarantee that dust contamination was actually low for the specific mode and area weighting used by BICEP2, unless the whole field has low dust polarization.

For a more reliable answer and improved constraints, a combined analysis of BICEP2 and Planck is thus required. We find that in the most optimistic case where the dust foreground is known perfectly and assumed to be consistent with the observed signal, such a combined analysis could improve the limits to $r < 0.05$ (95% c.l.), or detect a gravity wave signal with $r \sim 0.1$ with an uncertainty of about 0.03 (68% c.l.). However, various systematics such as bandpass and calibration leakage may actually make this more difficult than expected. In addition, there is some uncertainty in the scaling of the signal from high to low frequencies, since the slope can vary by about 10% [12], and there is also the possibility of the dust signal decorrelating between frequencies due to incoherent mixing of different dust sources. We show that the limits quickly degrade as we move away from perfect knowledge of the dust foreground. If the dust polarization can be subtracted off with 10% residual error, then the limit becomes $r < 0.07$; a 20% error gives $r < 0.09$ and a 30% error gives $r < 0.10$, only a minor improvement over the current limits presented here. This does not mean that such an analysis will not be useful even if the errors cannot be reduced below 20%; while we have framed our tests conservatively in terms of upper limits on $r$, it is of course possible that the measured dust polarization will be below the BICEP2 observations, in which case there will be a residual signal in the data that can be attributed to primordial gravity waves. If so, the limits become detection levels and the importance of the dust foreground is reduced. We note that the lower detector noise error expected from the Keck Array in itself does little to improve the constraints in the absence of a better understanding of the dust contribution. Another possible approach is to weight the data by the dust polarization and eliminate patches with a high dust polarization signal. The BICEP/Keck Array field is relatively large and there may be regions within it that have lower dust polarization than average. Focusing on these could be more beneficial than estimating the dust contamination level for the whole observing field, especially for Keck Array data with its low detector noise, but the price one has to pay is increased $E$ to $B$ mixing.

We analyzed the frequency dependence of the signal and showed that the current data do not strongly favor the hypothesis of no foregrounds against the null hypothesis of dust and lensing; we find that the difference in $\chi^2$ between the two hypotheses is less than 2 (i.e. less than 1.4σ) and that the first bandpower, which has the largest discriminating power since it has the lowest lensing contribution, actually favors the dust interpretation, while the second bandpower suggests just the opposite. Adding a possible correlated synchrotron component, which makes the spectral color closer to the CMB, results in even less difference between the no foreground and foreground+lensing models. Future data, such as 100 GHz measurements from the Keck Array, will greatly improve upon this test. Synchrotron may complicate the situation to some extent and the level of its correlation with dust in the low intensity regions needs to be quantified better, but overall synchrotron does not appear to be a major issue. The multi-frequency information may thus be the best path forward, but future analyses must attempt to minimize the sampling variance errors, which can quickly dominate the error budget otherwise.

What are the prospects for improving limits on $r$ from $B$ mode polarization in the future? The high level of dust, even in the cleanest patches of the sky presented in [11], as well as
an average polarization fraction of 8-10% in patches of low dust intensity [7], is relatively bad news for the field: it means more work will be needed to separate the CMB from the polarized dust. Multi-frequency observations will be required to make any such separation possible and future analyses should attempt to minimize sampling variance errors, which can quickly degrade the constraints. On the positive side, we have no evidence that we need to model more than three components in polarization, and possibly only two will be needed, so with good frequency coverage and low noise there are no obvious obstacles. There may also be small patches of the sky which are quite clean, and future surveys should choose carefully the regions of the sky they observe. Overall, we remain optimistic that future $B$ mode polarization searches will provide powerful constraints on inflationary models.

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