Generation of Magnetic Field due to Excited Q-Balls

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We investigate phase transitions due to excited Q-balls. As excited Q-balls have angular momentum, a magnetic field can be generated if one considers gauged Q-balls. Based on the course of the phase transition we estimate the strength of the magnetic field and then we find that it might be the origin of observed magnetic fields of astrophysical objects such as galaxies and clusters of galaxies.

I. INTRODUCTION

The generation of the seed of the galactic magnetic fields for galaxies and clusters is one of the most crucial problems in cosmology because one knows from magnetohydrodynamics that magnetic fields cannot exist if the initial field strength is zero. But now the coherent magnetic field over galaxies is observed as $\sim 10^{-6}$ Gauss [1].

If seed magnetic fields with $10^{-19}$ Gauss exist over the present comoving scale of the proto galaxy, $\sim 100$ kpc, one expects that the galactic magnetic fields can be amplified by the dynamo mechanism [2].

Recently, some strong candidates for the generation mechanism have been proposed in the electroweak phase transition. One of them is based on thermal fluctuations [3]. However, the coherent scale is too small and then the field strength is not enough for the onset of the dynamo mechanism. Although several studies have been attempted, the detail of the phase transition has not been clear at present [5].

So we propose a new mechanism. We consider phase transitions for a complex scalar field. The concrete particle for the complex scalar field can be called a squark. As we consider general cases in this paper, we do not assign a concrete particle.

As complex scalar fields has a conserved charge, one can expect that the phase transition differs from that of real scalar fields. In fact, there exist stable non-topological solitons: Q-balls [8]. The generation of Q-balls [8] and the effect on phase transitions [9] [10] have been actively investigated. However, these studies are focused on the ground state of the Q-ball. In the actual phase transition, excited Q-balls with surface waves might play an important role. Hence we shall reexamine the phase transition by taking account of the excitations. In such cases, the Q-ball has angular momentum and then gauged Q-balls [7] with magnetic moment will be induced if, for example, the charge of the Q-ball is baryon number. Thus one can expect that a magnetic field will be generated in general.

For simplicity, we consider the case of Q-balls which do not couple with gauge fields in this paper. Properly speaking, if one wants to consider the generation of the magnetic field, one should investigate using the gauged Q-ball. Thus, the present estimation of the magnetic field will be guessed from the non-gauged Q-balls, but it is sufficient for the order estimation.

The rest of the present paper is organised as follows. In Sec. II, we construct the total energy of excited Q-balls. In Sec. III, the phase transition will be considered. In Sec. IV, the typical strength of the generated magnetic field is estimated and then the net magnetic fields over the galaxy scale.
II. THE TOTAL ENERGY OF EXCITED Q-BALLS

In this section we examine the typical quantities of the excited Q-balls. The total energy is given by

$$E = \frac{Q^2}{2} \int \varphi^2 d^3x + \frac{1}{2} \int \nabla^2 \varphi \, d^3x + \int U(\varphi) d^3x,$$  \hspace{1cm} (2.1)$$

where we put the complex field $\psi$ as $\psi = e^{i\omega t} \varphi$ and $Q = \omega \int d^3x \varphi^2$. $\varphi$ is a time independent real function. Using the radius $R$, the wall width $\delta$, the field value at the asymmetric phase $\sigma_+$ and the field value at the top of the potential barrier $\sigma_-$, the terms on the right-hand side are given by

$$\int \varphi^2 d^3x \simeq \frac{4\pi}{3} R^3 \sigma_+^2,$$  \hspace{1cm} (2.2)$$

$$\int (\nabla^2 \varphi)^2 \simeq 4\pi R^2 \delta \left(\frac{\sigma_+}{\delta}\right)^2 \left[1 + \ell(\ell + 1)\right]$$  \hspace{1cm} (2.3)$$

and

$$\int U(\varphi) d^3x \simeq \frac{4\pi}{3} R^3 U(\sigma_+) + 4\pi R^2 \delta U(\sigma_-),$$  \hspace{1cm} (2.4)$$

respectively. Here we note that the contribution from the surface tension contains the perturbation of the mode $\ell$ in order to treat excited Q-balls. For brevity, we use the notation $U_\pm := U(\sigma_\pm)$ hereafter. Substituting these expression into eq. (2.1), one can obtain

$$E = \frac{3}{8\pi} \frac{Q^2}{R^3 \sigma_+^2} + 2\pi R^2 \delta \left(\frac{\sigma_+}{\delta}\right)^2 \left[1 + \ell(\ell + 1)\right] + 4\pi R^2 \delta U_+ + \frac{4\pi}{3} U_+ R^3.$$  \hspace{1cm} (2.5)$$

To estimate the typical quantities one must seek the most probable configuration. The configuration will be decided by the variational principle. From $\partial E/\partial \delta|_{\delta = \delta_*} = 0$, the wall width has the expression

$$\delta_* = \frac{1}{\sqrt{2}} \sqrt{\frac{\sigma_+}{\sqrt{U_-}}} \left[1 + \ell(\ell + 1)\right]^{1/2}$$  \hspace{1cm} (2.6)$$

Thus, the total energy becomes

$$E = \frac{3}{8\pi} \frac{Q^2}{R^3 \sigma_+^2} + 4\sqrt{2\pi} [1 + \ell(\ell + 1)]^{1/2} \sigma_+ \sqrt{U_-} R^2 + \frac{4\pi}{3} U_+ R^3$$  \hspace{1cm} (2.7)$$

Let $T_c$ be the critical temperature at which the two vacua degenerate ($U(0) = U_+ = 0$). At $T > T_c$, as $U_+ > 0$, one can see that the Q-ball is bounded inside finite radius and a stable solution exists. On the other hand, under the critical temperature, the Q-ball can evolve to a macroscopic size. In particular, it is worth noticing that there exists a critical charge $Q_c$. If the charge exceeds the critical one, Q-balls must expand for any energy. If the charge is smaller than the critical one, there exists a range of the total energy in which the Q-ball is bounded. In the next section, we examine the evolution of the phase transition as the temperature of the universe cools down.

III. PHASE TRANSITION DUE TO EXCITED Q-BALLS

First we consider the case of the critical temperature because one can expect that the difference between the phenomenon at the critical temperature and one at the higher temperature is qualitatively similar. At $T = T_c$, as $U_+$ vanishes, the total energy becomes

$$E = \frac{3}{8\pi} \frac{Q^2}{R^3 \sigma_+^2} + 4\sqrt{2\pi} [1 + \ell(\ell + 1)]^{1/2} \sigma_+ \sqrt{U_-} R^2$$  \hspace{1cm} (3.1)$$

From $\partial E/\partial R|_{R = R_*} = 0$ one obtains the radius at which the energy is minimum,
Thus the total energy becomes

$$E_\ast = E(R_\ast) = \frac{15}{16\pi} \left( \frac{64 \sqrt{2\pi^2}}{9} \right)^{3/5} [1 + \ell(\ell + 1)]^{3/10} Q^{4/5} U^{-3/10}_- \sigma^{1/5}_+ \left[ 1 + \ell(\ell + 1) \right]^{3/10} Q^{4/5} U^{-3/8}_- \sigma^{5/4}_- \
\frac{\eta^{5/4}_-}{\eta^{9/4}_- U^{3/8}_-}. \quad (3.3)$$

On the other hand, if the net charge is given by

$$Q_\ast \simeq \left( n_\varphi \frac{4\pi}{3} R^3_\ast \right)^{1/2} \quad (3.4)$$

one can obtain the expression

$$Q_\ast = \left( \frac{4\pi}{3} \right)^{5/4} \left( \frac{9}{64 \sqrt{2\pi^2}} \right)^{3/4} \eta^{5/4}_- [1 + \ell(\ell + 1)]^{-3/8} \frac{n^{5/4}_-}{\eta^{9/4}_- U^{3/8}_-}, \quad (3.5)$$

where $n_\varphi$ is the number density of $\psi$ particles and $\eta_\varphi = n_\varphi / n_\gamma$. Using this expression, the final expression of the total energy is given by

$$E_\ast = \frac{5}{4} n_\varphi \frac{\sigma^2_+}{\sigma^2_-} \quad (3.6)$$

As $\beta E_\ast < 1$, the Q-ball is generated and then the probability will be proportional to $e^{-\beta E_\ast}$.

Next, we consider the cases in which the temperature is lower than the critical one. As we stated in the previous section, a typical charge $Q_c$ exists. Hence, we concentrate on the critical case. For this case also, various quantities are decided by the variational principle:

$$\frac{\partial E}{\partial R} \bigg|_{R=R_c, Q=Q_c} = \frac{\partial^2 E}{\partial R^2} \bigg|_{R=R_c, Q=Q_c} = 0. \quad (3.7)$$

The results are as follows;

$$Q_c \simeq 34.0 [1 + \ell(\ell + 1)]^{3/2} \frac{\sigma^4_+}{\sigma^2_- \sqrt{U_- / U_+}} \left( \frac{U_- / U_+}{U_- / U_+} \right)^{5/2}, \quad (3.8)$$

$$R_c \simeq 2.36 [1 + \ell(\ell + 1)]^{1/2} \frac{\sigma_+}{\sqrt{U_- / U_+}} \quad (3.9)$$

and

$$E_c \simeq 54.8 [1 + \ell(\ell + 1)]^{3/2} \frac{\sigma^2_+}{\sqrt{U_- / U_+}} \left( \frac{U_- / U_+}{U_- / U_+} \right)^2. \quad (3.10)$$

Following Coleman’s estimation of surface waves the frequency is written as

$$(k_{\ell}^0)^2 = \frac{\alpha}{\rho_0 R_0^3} \ell(\ell + 2)(\ell - 1) \quad (3.11)$$

where $\alpha = \int_0^{\sigma^+} d\varphi [2U - \omega^2 \varphi^2]^{1/2}$. As $\alpha \simeq \sqrt{2U_- \sigma_+}$ and $\rho_0 R^3_0 \simeq \frac{3}{\pi^2} E^{\ell=0}_c$, the frequency becomes

$$(k_{\ell}^0)^2 \simeq 0.11 \frac{U_-}{\sigma^2_+} \left( \frac{U_+ / U_-}{U_- / U_+} \right)^2 \ell(\ell + 2)(\ell - 1) \quad (3.12)$$

and then the rotation velocity $v_{\ell}$ is given by

$$v_{\ell} \simeq R^{\ell=0}_{\ell} k_0^\ell \simeq 0.78 [\ell(\ell + 2)(\ell - 1)]^{1/2}. \quad (3.13)$$

Assuming conservation of angular momentum, one can derive the final rotation velocity as follows,
\[ v_f = \left( \frac{R_f}{R_c} \right)^{1/2} v_f \]
\[
\simeq 0.66 f_b^{1/2} g^*^{-1/4} \sqrt{\ell(\ell + 2)(\ell - 1)} \left( \frac{m_{pl}}{T_f} \right)^{1/2} \left( \frac{|U_+|}{U_-} \right)^{1/2} \left( \frac{U_+^{1/2}}{\sigma_+ T_f} \right)^{1/2}, \tag{3.14}
\]

where \( R_f \) is the final radius of the Q-ball as the phase transition finishes and given by \( \sim f_b H^{-1} \).

Even if \( Q_* < Q_c \) at \( T > T_c \), the Q-ball will attain critical charge soon by charge accretion at the temperature \( T < T_s < T_c \). Hence, when the temperature cools down such that \( (T_c - T)/T_c \ll 1 \), the Q-ball can expand. But the fraction of such Q-balls might be too small. The probability \( P(\ell) \) of a Q-ball with the surface wave of mode \( \ell \) is given by

\[ P(\ell) \sim e^{-\beta E_c} \tag{3.15} \]

because the probability is decided by \( P(\ell) \sim e^{-\beta E_*} \) at the critical temperature.

The phase transition proceeds by the expanding Q-ball with probability \( P(\ell) \). A Q-ball with surface waves has angular momentum. Thus, if one considers a gauged Q-ball, the excited gauged Q-ball has non-zero magnetic moment and a magnetic field can be generated. In the next section we estimate the order of the magnitude of the magnetic field.

**IV. GENERATION OF MAGNETIC FIELDS**

Now, we can estimate the magnetic field. As we stated in Sec. I, excited Q-balls have angular momentum and then have a magnetic field if one considers gauged Q-balls. Thus one can observe the generation of a magnetic field. As one must consider the gauged Q-ball, note that the Q-ball has an upper bound on the total charge. The maximum value \( Q_{\text{max}} \) is given by

\[ Q_{\text{max}} = \frac{1}{\sqrt{2}} \frac{\pi}{e^4} \tag{4.1} \]

for \( e < 2^{-3/2} \). In this paper, as we stated in the introduction, we estimate the typical order of the generated magnetic field only using the non-gauged excited Q-ball in the previous section, because the above value \( Q_{\text{max}} \) is not greatly different from the critical charge \( Q_c \) when \( e \sim 2^{-3/2} \) and \( \ell = 2 \). Hence, this extrapolation cannot destroy the present basic idea on the generation mechanism.

The magnetic moment is given by

\[ M_\ell \simeq \frac{Q_v}{R_f} v_f \times R_f \times R_f^3 \]
\[ = Q_c v_f R_f \tag{4.2} \]

Thus the magnetic field becomes

\[ B_\ell \sim \frac{M_\ell}{R_f^2} \sim 7.76 f_b^{3/2} g^*^{1/4} \left[ 1 + (\ell + 1)]^{3/2} \ell(\ell + 2)(\ell - 1) \right]^{1/2} T_f^2 \left( \frac{U_-}{|U_+|} \right)^{3/2} \left( \frac{T_f}{m_{pl}} \right)^{3/2} \left( \frac{\sigma_+}{U_+^{1/4}} \right)^3 \left( \frac{\sigma_+}{T_f} \right)^{1/2} \tag{4.3} \]

Averaging over the ensemble one can obtain the order

\[ B \sim \sqrt{\langle B^2 \rangle} = \sqrt{\frac{\sum B_\ell^2 \exp[-\beta E_c]}{\sum \exp[-\beta E_c]}} \]
\[ \simeq 4.1 \times 10^2 f_b^{-3/2} g^*^{3/4} T_f^2 \left( \frac{U_-}{|U_+|} \right)^{3/2} \left( \frac{T_f}{m_{pl}} \right)^{3/2} \left( \frac{\sigma_+}{U_+^{1/4}} \right)^3 \left( \frac{\sigma_+}{T_f} \right)^{1/2} \exp \left[ -\frac{\beta}{2} (E_\ell=2 - E_\ell=0) \right] \]
\[ \simeq 4.1 \times 10^2 f_b^{-3/2} g^*^{3/4} T_f^2 \left( \frac{T_f}{\sigma_+} \right)^{1/2} \left( \frac{\sigma_+}{U_+^{1/4}} \right) \left( \frac{T_f}{m_{pl}} \right)^{3/2} \gamma e^{-480 \gamma}, \tag{4.4} \]

*In the analysis using gauged Q-balls, one must consider the contribution from the gauge field. However, the order of the magnitude is the same as that from the \( \ell = 2 \) surface wave of the scalar field. Thus, one can expect that the probability \( P(\ell) \) does not have drastic changes from that of the gauged Q-ball. The detailed discussion will be appear in Ref. [1].
where we defined

$$\beta_2 [E_{e}^{\ell=2} - E_{e}^{\ell=0}] \simeq 480 \beta \sigma_+ \frac{\sigma_+^2}{\sqrt{U_-}} \left( \frac{U_-}{|U_+|} \right)^2$$

$$= 480 \gamma \quad (4.5)$$

The above mean value is maximum at $\gamma_* \sim 2.1 \times 10^{-3}$ and the maximum value is $B_{\text{max}} \sim 4.5 \times 10^{2} (T_f/100\text{GeV})^{1/2} \text{Gauss}$ for $f_b \sim 10^{-3}$, $g_* \sim 100$ and $T_f \sim U_+^{1/4} \sim \sigma_+$. As the Reynolds number will becomes $\text{Re} \sim 10^{12}$ at the endpoint of the phase transition, turbulence for the electroweak plasma occurs over the scale of the domain radius and then the energy of the magnetic field is equipartitioned with the energy of the fluid $\dagger$. Hence the final field strength becomes

$$B(R_f) \sim \rho^{1/2} v \sim g_*^{1/2} v T_f^2 \sim 10^{24} \left( \frac{T_f}{100\text{GeV}} \right)^2 \text{Gauss}, \quad (4.6)$$

where $\rho$ and $v$ are the density and the velocity of the fluid, respectively.

If the coherent scale is comoving, the number of magnetic domains inside the galaxy scale is $N \sim 10^{10} (10^{-2}/f_b)$. Thus, averaging over the galaxy scale $\dagger$, one can obtain the present mean value

$$\langle B_{\text{now}} \rangle_{\text{galaxy}} \sim \frac{1}{N^{3/2}} \left( \frac{a_f}{a_{\text{now}}} \right)^2 B(R_f)$$

$$\sim 10^{-21} \left( \frac{f_b}{10^{-2}} \right)^{3/2} \left( \frac{T_f}{100\text{GeV}} \right)^{-3} \text{Gauss}. \quad (4.7)$$

The above value satisfies the onset condition of the dynamo mechanism.

Finally we give some concluding remarks. First, we estimated the magnetic field using non-gauged Q-ball. However, properly speaking, one should do it using a gauged Q-ball. But, before performing a rough analysis using the gauged excited Q-ball, one should consider the fundamentals of the non-gauged and gauged excited Q-ball. The perturbation analysis will be reported in the next study $\dagger$. In the analysis, we find that the model $\ell = 1$ perturbation exists.

Second, we did not study deeply the dynamics of the phase transition due to Q-balls. One needs the information of the final stage in the phase transition. The analysis will be done by numerical simulations.

Third, we did not take account of the details of the evolution such as diffusion and reconnection of the magnetic field after recombination, and damping $\dagger$ due to the heat conductivity and shear viscosity before recombination. Although there does not exist a complete work for such kinds of physical processes, some progress exists. For example, Olsen et al showed that an inverse cascade happens if the power of the initial spectral is larger than $-3 \dagger$ and this means that the coherent scale of the magnetic field extends.

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$\dagger$ Although the amplification due to the turbulence is not clear at present, it has been used in the generation mechanism of the primordial magnetic field during the first order phase transition $\dagger$. 

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