Lattice QCD and heavy quarks
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Some interesting heavy-flavor questions are mentioned that may yield to the methods of lattice QCD. Special emphasis is placed on topics which arise in the discussion of CP violation in $B$ decays. Other subjects include quarkonium and light-quark systems and some potential applications outside QCD.

1. Introduction

Quarks are bound into hadrons by the interaction of quantum chromodynamics (QCD), some aspects of which cannot be treated by perturbation theory. As a result, non-perturbative methods have been developed, of which lattice gauge theory is at present the leading contender.

The study of hadronic properties of heavy-quark systems is valuable for at least two reasons. (1) By peeling away effects of the strong interactions, one can uncover fundamental quark properties, such as the sources of masses, flavor mixings, and CP violation as encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. (2) By drawing an analogy between heavy quarks and atomic nuclei, and between light quarks and gluons and atomic electrons and electromagnetic field quanta, one can simplify the description of hadrons. Systems with one heavy quark are like hydrogen atoms; the replacement of one heavy quark by another is analogous to an isotope change.

I wish to touch on some aspects of (mostly) heavy-quark physics for which lattice gauge theory can provide insights. The conference organizers originally called this talk “Interesting Heavy Flavor Physics That Lattice People Should Study,” a provocative and peremptory title which seems to have evoked the desired response [2].

I will begin with a brief review of flavor-changing transitions among quarks as described by the CKM matrix (Sec. 2). Some questions demanding answers from non-perturbative methods in QCD arise in the study of CP-violating decays of $B$ mesons (Sec. 3). Specific places where the lattice (or other approaches) can help are noted in Sec. 4 for CP studies and in Sec. 5 for other non-perturbative questions in heavy-quark physics.

Systems with more than one heavy quark are also worthy test-beds for methods in QCD. Bound states of heavy quarks and antiquarks (“quarkonium”) have yielded information even when studied with the simplest nonrelativistic methods, but questions remain, some of which the lattice seems uniquely poised to answer (Sec. 6). I would also like to mention a few favorite light-quark questions (Sec. 7), and some areas outside QCD where the lattice could be of help (Sec. 8). A summary is contained in Sec. 9.

2. Quarks and the CKM Matrix

We begin by updating some previous analyses in the light of results from the summer 1998 conferences. The weak charge-changing transitions between the quarks $i = (u,c,t)$ of charge $2/3$ and those $j = (d,s,b)$ of charge $-1/3$ are described by a unitary $3 \times 3$ matrix $V_{ij} (V^{-1} = V^\dagger)$ which can be parametrized [3] as

$$V = \begin{pmatrix}
1 - \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.$$  

Here $\lambda = \sin \theta_C \approx 0.22$ is determined from strange particle decays (for a recent analysis, see [8]): $A\lambda^2 = V_{cb} = 0.0392 \pm 0.0027$ is obtained from decays of hadrons containing $b$ quarks to charmed particles [9], and $|V_{ub}| = (3.56 \pm 0.22 \pm 0.28 \pm 0.43) \times 10^{-3}$ is the result of an average...
of several results for decays of hadrons containing $b$ quarks to charmless final states. This last result implies $|V_{ub}/V_{cb}| = 0.091 \pm 0.016$, or $(\rho^2 + \eta^2)^{1/2} = 0.41 \pm 0.07$. However, we still do not know $\gamma = \text{Arg}(V_{ub}^*)$ well.

Our uncertainty can be expressed as a region of allowed parameters in the complex $\rho + i\eta$ plane. The relation $V_{ub}^* + V_{td} = A\lambda^3$ is a consequence of the unitarity of the CKM matrix, so that a figure in the complex plane with vertices $(\rho, \eta)$ (interior angle $\alpha$), $(1,0)$ (interior angle $\beta$), and $(0,0)$ (interior angle $\gamma$) is often referred to as the unitarity triangle (see Fig. 1). The constraint on $|V_{ub}/V_{cb}|$ then leads to an allowed annulus centered on $(0,0)$ in the $(\rho, \eta)$ plane.

CP-violating $K$-$\bar{K}$ mixing is encoded in the parameter $\epsilon_K = (2.28 \pm 0.02) \times 10^{-3}$ [11]. In the CKM theory $\epsilon_K$ is due primarily to top-quark loops in the box diagrams governing mixing, and so should be proportional to $\text{Im}(V_{td}^2) \sim \eta(1-\rho)$. Including the contribution of charmed quarks in the loop, one can write the constraint [3] as

$$\eta(1-\rho + 0.44) = 0.51 \pm 0.18$$

This relation defines a band bounded by two hyperbolaes in the $(\rho, \eta)$ plane. The error in [4] is dominated by that in $V_{cb}$; we have used a parameter $\hat{B}_K = 0.8 \pm 0.2$ (the hat defines a particular renormalization scheme) describing the matrix element of the short-distance mixing operator between $K^0$ and $\bar{K}^0$. The error on the top quark’s mass [1] is insignificant by comparison.

The top quark dominates the loop diagrams governing $B^0-\bar{B}^0$ mixing. We have used a matrix element parameter $f_B\sqrt{B_B} = 200 \pm 40$ MeV to extract a value of $|V_{td}|$ implying

$$|1 - \rho - i\eta| = 1.01 \pm 0.21$$

This relation defines a $(\rho, \eta)$ region bounded by two circles with centers at $(1,0)$.

A final constraint is provided by a new bound on mixing between the strange $B$ meson $B_s \equiv bs$ and its antiparticle. The mixing amplitude can be parameterized in terms of the splitting between mass eigenstates: $\Delta m_s > 12.4$ ps$^{-1}$ (95% c.l.) [12]. By comparing this value with the corresponding one for non-strange $B's$, $\Delta m_d = 0.471 \pm 0.016$ [13], and using the estimate [14] $f_{B_s}\sqrt{B_{B_s}}/f_B\sqrt{B_B} \leq 1.25$ (see other talks in this Conference), one concludes $|V_{ts}/V_{td}| > 4.0$ or $|1 - \rho - i\eta| < 1.14$.

The resulting constraints are shown in Fig. 2. [The region of parameters is slightly smaller than actually shown at the Conference as a result of improvements in bounds on several parameters.] The improved lower bound on $\Delta m_s$ has contributed to the the evidence for $\eta \neq 0$ (i.e., for a non-trivial phase in the CKM matrix) independent of that provided by CP-violating $K^0-\bar{K}^0$ mixing. The maximum allowed value of
Table 1
Range of angles of the unitarity triangle (Fig. 1) permitted by the constraints in Fig. 2.

| Angle | Deg | \(\rho\) | \(\eta\) |
|-------|-----|--------|--------|
| \(\alpha\) Min | 58 | -0.09 | 0.33 |
| Max | 113 | 0.25 | 0.27 |
| \(\beta\) Min | 17 | -0.09 | 0.33 |
| Max | 29 | 0.22 | 0.43 |
| \(\gamma\) Min | 47 | 0.25 | 0.27 |
| Max | 105 | -0.09 | 0.33 |

\(\Delta m_s\) allowed by this plot is about \((1.14/0.8)^2 \times 12.4 \text{ ps}^{-1} \simeq 25 \text{ ps}^{-1}\), or \(x_s \equiv \Delta m_s/\Gamma_s \simeq 40\).

The corresponding range of the angles \(\alpha\), \(\beta\), and \(\gamma\), which can be probed in \(B\) decays (Sec. 3), are shown in Table 1. These correspond to 
\(-0.72 \leq \sin 2\alpha \leq 0.90, 0.55 \leq \sin 2\beta \leq 0.85, 0.54 \leq \sin^2 \gamma \leq 1\). (For a slightly different analysis see \[15\]. These authors, in our opinion, underestimate the errors on several key quantities such as \(|V_{cb}|\) and obtain an allowed region which is a subset of ours.)

Many useful parameters contributing to this plot have been calculated or are being refined in lattice QCD. These include \(B_K, f_B,\) and \(B_B\). It is quite likely, for example, that with an actual measurement (rather than a bound) for \(\Delta m_s\), and a good calculation of \(f_B \sqrt{B_B}/f_B \sqrt{B_B}\), one will be able to strengthen the case, already suggestive, for nonzero \(\eta\).

As an exercise in “futurism,” one can imagine a \((\rho, \eta)\) plot as shown in Fig. 3 emerging in five years \[16\]. The potential for inconsistency among all these measurements (pointing to new physics) is of course much increased. However, the constraints in Fig. 3 (aside from that due to \(\epsilon_K\)) will largely circumvent any dependence on lattice or other nonperturbative QCD calculations. Hence, although lattice QCD has made great strides in recent years, its days for “prediction” of certain quantities such as \(f_B\) may be numbered. There will still be a need for others, such as the ratio \(f_B \sqrt{B_B}/f_B \sqrt{B_B}\) (to interpret \(\Delta m_s/\Delta m_d\)) and the quantity \(B_B\) itself (to interpret \(\Gamma(B^+ \to \tau^+\nu_\tau)/\Delta m_d\)).

3. CP-Violating \(B\) Decays

3.1. New modes and their implications

The branching ratios for decays of \(B\) mesons to several exclusive charmless final states have been pinned down in the past couple of years, improving the prospects for learning the phases of CKM matrix elements and for seeing CP violation.

The decay \(B^+ \to K^0\pi^+\) is expected to be a pure penguin process, while \(B^0 \to K^+\pi^-\) and \(B^+ \to K^+\pi^0\), though dominated by penguin amplitudes, should have other contributions at the level of some tens of percent. By comparing rates \[13\] for these processes, all of which have branching ratios between one and two parts in \(10^3\), one can learn about the relative weak and strong phases of these various contributions; in particular, one learns about the angle \(\gamma\) illustrated in Fig. 1 \[17\] \[18\].
The branching ratios for the decays $B^{+,0} \to K^{+,0}\eta^\prime$ are quite large \cite{20} (averaging to $(6.8 \pm 1.1) \times 10^{-5}$ if they are equal as one expects from penguin dominance of the decays). The $\eta^\prime$, pre-dominantly a singlet of flavor $SU(3)$, appears to couple very favorably to the rest of the system, whether due to an intrinsic gluonic or $c\bar{c}$ component or to the QCD anomaly. One consequence of this enhanced coupling is an improved prospect for seeing CP violation through unequal rates for $B^+ \to \pi^+\eta'$ and $B^- \to \pi^-\eta'$, while the amplitudes in $B^\pm \to \pi^\pm\eta$ also turn out to favor CP-violating rate differences \cite{21}.

Finally, $B$ decays to charmless final states involving one or two vector mesons, such as $B^+ \to \pi^+\omega$ and $B^+ \to K^+\omega$ \cite{22}, provide details of form factors which check specific dynamical models on which many expectations are based \cite{23}.

### 3.2. Rescattering issues

The assumption that the decay $B^+ \to K^0\pi^+$ is purely a penguin process is called into question if rescattering from other final states, such as $K^+\pi^0$, is important \cite{17,24}. If more than one amplitude contributes to $B^+ \to K^0\pi^+$, there can be an observable rate difference between that process and its charge conjugate, and determinations of $\gamma$ by comparison with other rates \cite{17,19} are no longer so straightforward.

In the context of a flavor-$SU(3)$ analysis \cite{25}, one normally expects suppression of an “annihilation” amplitude involving the spectator quark in which $b\bar{u} \to W^{++} \to s\bar{u} \to (\bar{s}d)(\bar{d}u)$. Rescattering (e.g., through the $K^+\pi^0$ state) can imitate such effects. Present estimates of such contributions are very model-dependent. However, a flavor-$SU(3)$ relation between rescattering contributions to $B^+ \to K^0\pi^+$ and contributions to $B^+ \to \bar{K}^0\pi^+$ can be obtained by the “U-spin” interchange $s \leftrightarrow d$ \cite{17,24}, with the result that the rescattering contributions to $B^+ \to \bar{K}^0\pi^+$ should be about $|V_{ud}/V_{us}|^2 \approx 20$ times those in $B^+ \to K^0\pi^+$, and thus should be large enough to enhance the decay rate of $B^+ \to \bar{K}^0\pi^+$ by a visible amount if they are at all important in $B^+ \to K^0\pi^+$.

If a few intermediate states dominate the rescattering, one also expects a visible enhance-ment of the $B^0 \to K^+K^-$ decay rate, which would normally be expected to be due to the suppressed $\bar{b}d \to \bar{u}u \to (\bar{s}d)(\bar{d}u)$ “exchange” subprocess \cite{27}. The experimental upper bound $\mathcal{B}(B^0 \to K^+K^-) < 2.4 \times 10^{-6}$ is considerably better than that $\mathcal{B}(B^+ \to K^+\bar{K}^0) < 9.3 \times 10^{-6}$ discussed in Refs. \cite{7,20}, and is expected to be a factor of 60 below present limits if the hierarchy described in Ref. \cite{22} is correct.

### 3.3. CP asymmetry in $B \to J/\psi K_S$

The CKM theory predicts that mixing and decay amplitudes interfere to give a rate difference between $B^0 \to J/\psi K_S$ and $\bar{B}^0 \to J/\psi K_S$:

\[
\frac{\Gamma(B^0 \to J/\psi K_S) - \Gamma(\bar{B}^0 \to J/\psi K_S)}{\Gamma(B^0 \to J/\psi K_S) + \Gamma(\bar{B}^0 \to J/\psi K_S)} = \frac{x_d}{1 + x_d^2} \sin 2\beta ,
\]

where $x_d \equiv \tau(B^0) \Delta m_d$. The flavor of the neutral $B$ is that at time of production. This must be obtained either by studying the flavor of the “other” $B$ produced in association, or by a “same-side tagging” method \cite{28} in which the sign of a pion produced not far from the neutral $B$ in phase space signifies its flavor. Using this “same-side” method, both the OPAL and CDF Collaborations have reported asymmetries whose central values are larger than physically expected (probably as a result of uncertainty in estimating tagging efficiencies) but which exclude some region of negative $\sin 2\beta$:

\[
\sin 2\beta = \begin{cases} 3.2^{+1.8}_{-1.0} \pm 0.5 \quad &\text{(OPAL \cite{29})} \, , \\ 1.8 \pm 1.1 \pm 0.3 \quad &\text{(CDF \cite{30})} \, . \end{cases}
\]  

(3)

For example, the CDF result excludes values of $\sin 2\beta$ less than $-0.20$ at 95% confidence level.

The same-side tagging method uses the fact that if a $b$ quark fragments into a $B^0 = \bar{b}d$ meson, a $d$ quark must be nearby in rapidity. If this $d$ quark materializes into a charged pion, that pion must be a $\pi^-$. The correlation between $B^0$ and $\pi^-$, and between $\bar{B}^0$ and $\pi^+$, follows both from fragmentation models and from resonances. Although the $B^*$ is too light to decay to $B\pi$, there is a family of “$B^{*n}$” resonances expected to lie several hundred MeV above the $B$, so the decays
\( B^* \rightarrow \bar{B}^{(*)} 0 \pi^- \) and \( B^{**} \rightarrow \bar{B}^{(*)} 0 \pi^+ \) are permitted.

The CDF Collaboration can find a suitable pion for tagging the neutral \( B \) in 2/3 of the observed \( J/\psi K_S \) decays (a sample of about 200 in the 110 pb\(^{-1} \) accumulated during Run I). The tagging method is calibrated by measuring \( B^0 - \bar{B}^0 \) oscillations in \( B \rightarrow D^{(*)} l \bar{\nu}_l \) decays \(^{31} \). The “dilution factor” is measured to be \( D_0 \equiv 2P_0 - 1 = 0.181^{+0.036}_{-0.032} \), where \( P_0 \) is the probability that the tag correctly identifies the \( B^0 \) flavor.

4. Where the Lattice Can Help

4.1. Extraction of CKM matrix elements

One can extract the CKM matrix elements \( V_{ub} \) and \( V_{cb} \) either from exclusive \(^{32} \) or inclusive \(^{33} \) measurements. In either case one needs theoretical guidance in passing from data at the hadron level to conclusions at the quark level.

Lattice or other nonperturbative schemes can predict form factors for decays such as \( B \rightarrow D^{(*)} l \bar{\nu}_l \), \( \pi l \bar{\nu}_l \), \( p l \bar{\nu}_l \), ... and \( D \rightarrow \bar{K}^{(*)} l \bar{\nu}_l \), \( \pi l \bar{\nu}_l \), \( p l \bar{\nu}_l \), ... Now, heavy-quark methods permit one to use information in processes involving \( D \) decays to pin down hadronic effects in certain kinematic regions of \( B \) decays. Thus, one can in principle avoid having to rely on theoretical form factor estimates. However, until measurements of such rare processes as \( B \rightarrow D l \bar{\nu}_l \) and \( D \rightarrow \rho l \bar{\nu}_l \) have reached the requisite accuracy, such estimates are very useful.

Inclusive determinations of CKM matrix elements rely on comparison of data on \( B \rightarrow X_s l \bar{\nu}_l \) and \( B \rightarrow X_b l \bar{\nu}_l \) with model predictions. One has to distinguish charmed inclusive states \( X_c \) from non-charmed ones \( X_b \) by means of various kinematic variables, such as \( M(X) \), lepton spectra, and missing energy carried away by the neutrino. Calculations of these variables at the quark level give a zeroth-order approximation; for example, leptons beyond the endpoint for \( B \rightarrow X_s l \bar{\nu}_l \) are assumed to have come from \( B \rightarrow X_b l \bar{\nu}_l \). A systematic expansion of differential and integrated decay rates in inverse powers of the heavy quark mass exhibits our ignorance in terms of a few parameters, which can either be extracted from data or calculated using nonperturbative (e.g., lattice) methods.

4.2. Decay constants

We have already noted the importance of the decay constant \( f_B \) in determining \( |V_{td}| \) from \( B^{0} \rightarrow \bar{B}^{0} \) mixing, or \( |V_{ub}| \) from \( B^{+} \rightarrow \tau^{+} \nu \). Lattice calculations appear to be the front-runners in estimating these quantities. As one example, we quote published results of the MILC Collaboration \(^{34} \):\(^{34} \)
\[
\begin{align*}
    f_D & = 195 \pm 11^{+15+15}_{-8-0} \text{ MeV} \\
    f_{D_s} & = 213 \pm 9^{+23+17}_{-9-0} \text{ MeV} \\
    f_B & = 159 \pm 11^{+22+21}_{-9-0} \text{ MeV} \\
    f_{B_s} & = 175 \pm 10^{+28+25}_{-10-1} \text{ MeV}
\end{align*}
\]
where the errors are statistical, systematic, and an estimate of the effects of quenching, respectively. The JLQCD Collaboration \(^{35} \) has found \( f_D \) = 197 \pm 2 \text{ MeV}, \( f_{D_s} \) = 224 \pm 2 \text{ MeV}, \( f_B \) = 173 \pm 4 \text{ MeV}, \( f_{B_s} \) = 199 \pm 3 \text{ MeV} in a quenched calculation. The errors are statistical; additional systematic and scale errors of 5% (each) are estimated. The observation of the decays \( D_s \rightarrow \mu \nu \) and \( D_s \rightarrow \tau \nu \) has permitted the measurement \( f_{D_s} = 255 \pm 21 \pm 28 \text{ MeV} \) \(^{36} \), in accord with these predictions.

For a full review of heavy meson decay constants, see \(^{37} \). Some salient averages relevant to the physics of CKM matrix elements are \(^{37} \) \( f_B \sqrt{B_B} = 200 \pm 50 \text{ MeV} \) (a slightly more conservative error than we used in Sec. 2), and \( f_{B_s} \sqrt{B_{B_s}}/(f_B \sqrt{B_B}) = 1.17 \pm 0.06 \pm 0.12 \) (again, slightly conservative compared to our quark-model estimate of less than 1.25 for this ratio).

4.3. Spectroscopy of orbitally excited mesons

The spectroscopy of P-wave levels of a heavy quark (e.g., \( b \)) and a light quark (e.g., \( u \)) can be of interest for tagging the flavor of neutral \( B \) mesons. Moreover, these \( B^{**} \) levels (and their lighter relatives \( D^{**} \) involving charmed quarks) are intrinsically interesting as tests of theories of the spectrum.

One describes P-wave bound states of a single heavy antiquark \( \bar{Q} \) and a light quark \( q \) in
the following manner [33]. First, couple the light quark’s spin \((s = 1/2)\) to its orbital angular momentum \((l = 1)\) to form total light-quark angular momenta \(j = 1/2\) or \(j = 3/2\). Then, couple \(j\) to the heavy-quark spin to form total angular momenta \(J = 0, 1\) (twice), and 2. Labelling states by the notation \(J^P\), where the parity \(P\) is even for \(l = 1\) \(Qq\) states, we then have states \(0^+_{1/2}, 1^+_{1/2}, 1^+_{3/2}\) and \(2^+_{3/2}\). The splittings between \(0^+_{1/2}\) and \(1^+\) levels, and between \(1^+_{3/2}\) and \(2^+_{3/2}\) levels, should be of order \(1/m_Q\).

The \(j = 1/2\) states are predicted to decay to the ground \(Qq\) states and a pion in an S-wave, while the \(j = 3/2\) states should undergo these decays in a D-wave and hence should be considerably narrower. It is likely that these latter ones are the states that have been seen in the \(D^{**}\) and \(B^{**}\) systems.

Predictions of the masses and decay widths of the unseen \(j = 1/2\) states are thus of great interest. Do they contribute in a significant way to the \(\pi B^{(*)}\) correlations useful in same-side tagging? At this Conference, some encouraging lattice results have been presented [26] indicating that the \(0^+_{1/2}\) level lies about 130 MeV below the \(2^+_{3/2}\) level, and that the \(0^+_{1/2} - 1^+\) splitting is of order \(1/m_Q\) as expected.

Another point of interest is the nature of \(D^{**}\) resonances that contribute to the roughly 1/3 of all semileptonic \(B \to X_c l\nu\) decays not involving \(X_c = D\) or \(D^*\) [10]. Such resonances decay to \(D\pi\) (for \(J^P = 0^+, 2^+\)) or \(D^{(*)}\pi\) (for \(J^P = 1^+, 2^+\)). If the semileptonic decay \(B \to D^{(*)} l\nu\) also involves another “secondary” pion, that pion can be confused with the “same-side” tagging pion if vertex resolution is poor. The sign of the secondary pion is opposite to that of the same-side tagging pion for a given flavor of \(B\), so good understanding of secondary pion production in \(B\) semileptonic decay is highly desirable [31,11].

### 4.4. Rescattering questions

One may be asking too much for the lattice to estimate final-state interaction effects in low-multiplicity decays of \(D\) or \(B\) mesons. These effects, in \(B\) decays, are crucial in interpreting certain varieties of CP-violating asymmetries, should they arise in future data, in terms of fundamental CKM phases. As two examples, we pose the following.

1) Naive estimates indicate that the decay rate for \(B^0 \to K^+ K^-\) should be very small in comparison with some related modes (such as \(B^+ \to K^+ \bar{K}^0\)). This is because \(B^0 \to K^+ K^-\) either requires the \(b\) and \(d\) in the \(B^0\) to exchange a \(W\) and materialize into \(\bar{u}u\), or to proceed via rescattering from some less-suppressed intermediate state. Can the lattice say anything about this process?

2) Final-state phase differences in \(B \to K\pi\) scattering have been estimated in some quarters to be small [24]. This is a pity as otherwise conditions could be favorable for a large CP-violating difference between the rates for such processes as \(B^0 \to K^+ \pi^-\) or \(B^+ \to K^+ \pi^0\) and their charge conjugates. If there is a source of large final-state phases, it could be the charm-anticharm intermediate state [12]. Can the lattice say anything about this?

### 5. Other Nonperturbative Questions

#### 5.1. The \(D^* D\pi\) coupling constant

It is possible to calculate strong coupling constants in lattice gauge theories [13]. The \(D^* D\pi\) coupling is of interest for a couple of reasons.

1) The hadronic widths of \(D^*\) states are too small to be measured directly. All that exists is an upper limit [14] \(\Gamma_{tot}(D^{*+}) < 130\) keV. However, the \(D^{(*)}\) branching ratios for hadronic and electromagnetic decays are comparable to one another. Now, the \(D^{(*)}\to D^{0}\gamma\) width depends on the magnetic transition moment of the charmed quark (which is calculable) and that of the \(\bar{u}\) quark (which is much harder to estimate). These two contributions interfere constructively in the matrix element.

Recently the CLEO Collaboration [15] has measured the ratio \(B(D^{*+} \to D^{+}\gamma) = (1.68 \pm 0.42 \pm 0.29 \pm 0.03)\%\), where the first error is statistical, the second is systematic, and the third is associated with uncertainty in a kinematic factor describing the ratio of the \(D^0\pi^+\) and \(D^+\pi^0\) decays. This is to be contrasted with the much larger value [11] of \(B(D^{*0} \to D^{0}\gamma) = (38.1 \pm 6.8\)\%\).
The branching ratio of \( D^{*+} \rightarrow D^+ \gamma \) is so small because the contributions of the charmed and light quark interfere destructively, almost cancelling one another. By combining this information with hadronic branching ratios, including \( B(D^{*+} \rightarrow D^{+}\pi^0) = (30.73 \pm 0.13 \pm 0.09 \pm 0.61)\% \) and \( B(D^{*+} \rightarrow D^0\pi^+) = (67.59 \pm 0.29 \pm 0.20 \pm 0.61)\% \), one can solve for the light-quark transition form factor \( F(1/m_c^2) \) corrections to the Isgur-Wise function \( B \rightarrow D^* \) at the zero-recoil point, which affects the determination of light-quark transition terms in chiral perturbation theory \([47]\). One finds \( \Gamma_{\text{tot}}(D^{*+}) = 90^{+50}_{-30} \) keV, in accord with the ACCMOR limit.  

2. The \( D^*D\pi \) coupling is relevant to some estimates of hadronic effects in semileptonic \( B \) decays. The coupling constant \( g \) enters into part of the nonperturbative \( \mathcal{O}(1/m_c^2) \) corrections to the Isgur-Wise function \( B \rightarrow D^* \) at the zero-recoil point, which affects the determination of \( V_{cb} \) \([22]\).

### 5.2. Lifetime hierarchies

Although the lifetimes of charmed hadrons vary by factors of more than 10, from less than 0.1 ps for the \( \Omega_c \) to greater than 1 ps for the \( D^+ \), conventional wisdom \([48]\) predicts less than a 10% variation among hadrons containing \( b \) quarks. The fact that the lifetime of the \( \Lambda_b \) is only about 0.8 times that of the \( B^{+}\bar{b} \) and \( B_0 \) mesons is not understood at present (see, e.g., \([18,19]\)). Some non-perturbative effects, not accounted for by the usual estimates, are apparently at work. The lattice may be able to shed some light on this question.

### 5.3. Branching ratios of \( \Lambda_c \)

There exist no direct measurements of \( \Lambda_c \) branching ratios. The absence of this information has far-reaching consequences on estimates of \( \Lambda_c \) production and other “engineering” quantities \([50]\). Branching ratios are calibrated by assuming that the exclusive semileptonic decay \( \Lambda_c \rightarrow \Lambda \nu \) saturates the inclusive semileptonic rate, which is then calculated perturbatively. It would be helpful if the lattice or some other nonperturbative scheme could provide form factors for \( \Lambda_c \rightarrow \Lambda \) transitions, and guidance about what other states (if any) are likely to be excited.

### 5.4. \( \Sigma_b^{(*)} \) spectra

A couple of years ago the DELPHI Collaboration \([51]\) claimed a large splitting \( M(\Sigma_b^0) - M(\Sigma_b^\ast) = 56 \pm 16 \) MeV. This value is hard to understand on the basis of heavy-quark physics \([72]\). In the states \( \Sigma_Q \) and \( \Sigma_Q^\ast \), where \( Q \) is a heavy quark, the light quarks \( q \) are coupled to a state of isospin \( I_Q = 1 \) and spin \( S_Q = 1 \). This spin is then coupled to the spin \( S_Q \) of the heavy quark \( Q \) to give total angular momentum \( J = 1/2 \) for \( \Sigma_Q \) or \( J = 3/2 \) for \( \Sigma_Q^\ast \). The hyperfine splitting between these two states should be inversely proportional to the heavy quark mass \( m_Q \), so that one expects \( M(\Sigma_b^0) - M(\Sigma_b^\ast) = (m_c/m_b)[M(\Sigma_b^0) - M(\Sigma_b^\ast)] \approx (1/3)[M(\Sigma_b^0) - M(\Sigma_b^\ast)] \).

Since the DELPHI report first appeared, the CLEO Collaboration has presented convincing evidence for the \( \Sigma_b^\ast \) at a mass about 65 MeV above the \( \Sigma_c \) \([53]\). One would then expect \( M(\Sigma_b^0) - M(\Sigma_b^\ast) \) to be around 20 MeV, lower than the DELPHI result and in accord with a lattice estimate presented at this Conference \([39]\).

### 6. Quarkonium Issues

The bound states of a heavy quark \( Q \) and the corresponding antiquark \( \bar{Q} \), known as quarkonium, have contributed much to our understanding of QCD. The lattice has used these systems to extract remarkably precise values of the strong coupling constant and to study the behavior of light degrees of freedom (quark-antiquark pairs and gluons) surrounding the nonrelativistic \( QQ \) system \([57]\). Some other possible topics of interest are mentioned below.

#### 6.1. Universal separation corresponding to flavor threshold

The number of \( QQ \) bound states below flavor threshold can be shown to increase as \( m_Q^{-1/2} \) \([55,56]\). This result is easily seen using a WKB estimate \([70]\) and the assumption of a universal \( QQ \) separation corresponding to flavor threshold.
If \( M(Q\bar{q}) \) denotes the mass of the lightest flavored meson, the \( Q\bar{q} \) potential at threshold separation \( r_{\text{th}} \) should satisfy \( V(r_{\text{th}}) = 2M(Q\bar{q}) - 2m_Q \), and as \( m_Q \to \infty \) this quantity should approach a constant. An estimate of the corresponding value of \( r_{\text{th}} \) is 1.4 to 1.5 fm. This appears to be in the range of lattice estimates, and studies are continuing.

6.2. Mixing of S and D states

Firm predictions exist for the masses of D-wave charmonium states \([59]\) (if they are not perturbed by nearby thresholds). Such states are difficult to observe. They can be produced via electromagnetic transitions from P-wave levels, or in \( e^+e^- \) collisions via mixing with S-wave states. Estimates of this mixing vary. Intermediate states consisting of pairs of flavored mesons probably play a key role, especially for the D-wave charmonium states, all but two of which are almost certain to lie above flavor threshold. The exceptions, the lowest-lying \( ^1D_2 \) and \( ^3D_2 \) levels, may lie low enough in mass to forbid their strong decays, which cannot occur to \( DD \) and must at least involve \( DD^* \) or \( \bar{D}D^* \). Searches for these last two levels are part of the program of charmonium studies in the Fermilab Accumulator Ring \([51]\). Searches for the D-wave \( bb \) levels are possible at CESR if the energies for running below the \( \Upsilon(4S) \) – a key component of background studies for \( B \) production – are chosen appropriately.

Lattice estimates of S–D mixing thus would be very helpful. Such estimates probably must await a more thorough understanding of the role of light quark pairs, which undoubtedly are a key feature of this mixing.

6.3. Masses and transition matrix elements of \( \eta_b \) states

The \( ^1S_0 \) levels of the \( bb \) system – the \( \eta_b, \eta_b', \eta_b'' \ldots \) states – have not yet been seen. Their masses influence determinations of \( \alpha_s \) from charmonium spectroscopy \([54]\) because one would like to use spin-averaged levels spacings (for example, in the comparison of 1P – 1S and 2S – 1S spacings) but this is not possible as long as spin-singlet levels have not been seen. Thus, one must either work with the observed spin-triplet levels or make a theoretical correction for the spin-splittings.

Estimates of hyperfine splittings between \( ^3S_1 \) and \( ^1S_0 \) based on perturbation theory \([52]\) indicate that the next-to-leading-order corrections are very important. Moreover, the square \( |\Psi(0)|^2 \) of the S-wave wave function at zero interquark separation – intrinsically a nonperturbative quantity – enters such calculations. Leptonic widths are sensitive to \( |\Psi(0)|^2 \), but with important relativistic corrections – again indicating the importance of methods transcending perturbation theory. It appears that lattice methods have some difficulty in estimating heavy-quark hyperfine splittings, but it is worth thinking whether some lattice insight might nonetheless complement the more usual methods.

The best prospects for producing the \( \eta_b \) – the \( 1^1S_0 \) \( bb \) level – are probably through the transition \( \Upsilon(2S) \to \gamma\eta_b \). Although the wave functions of 2S and 1S states are orthogonal to one another, two effects combine to give a non-zero transition amplitude. First, the matrix element must be taken of the spherical Bessel function \( j_0(kr/2) \), where \( k \) is the photon energy and \( r \) is an interaction radius \([53]\). When \( kr \) is non-negligible, this matrix element will not vanish. Second, hyperfine interactions can distort the \( ^3S_1 \) and \( ^1S_0 \) wave functions in different ways. An estimate of the first effect leads to the estimate \( B(\Upsilon(2S) \to \gamma\eta_b) \simeq 10^{-4} \)[64]. It is probably worth updating this estimate in the light of all the progress on charmonium in the past 15 years. The photon in this transition, of energy around 600 MeV, should be detectable with enough \( e^+e^- \) collisions at the c.m. energy of the \( \Upsilon(2S) \).

6.4. Hybrid states

The lattice has been able to predict the masses of hybrid states composed of both quarks and gluons \([67]\). Hybrid states \( QQg \) have been suggested recently \([50]\) as a possible solution to the “missing charm” problem in \( b \) decays. One is looking for a mechanism whereby the decay \( b \to c\bar{c}g \) is enhanced but does not lead to final states with visible charm. The known states below charm threshold apparently do not suffice. If hybrid states \( c\bar{c}g \) above \( D\bar{D} \) threshold are produced with
an enhanced rate (deemed unlikely in one calculation, however [67]) and then decay primarily via cc annihilation to gluonic and/or light-quark systems, one may be able to understand why fewer charmed particles than predicted are seen in b decays, and why the branching ratio of b quarks to nonleptonic final states is slightly higher than expected. To check this hypothesis requires estimates of production, masses, and decay rates of hybrid mesons.

6.5. Exotic systems

Although all known hadrons so far consist of a quark-antiquark pair or three quarks, other stable color singlets may exist. These are conventionally known as “exotic states.” The presence of a heavy quark has been predicted to stabilize such states. Thus, for example, it has been predicted that there are stable “pentaquark” states of the form qqqs̄c and qqs̄bar[b] where q stands for u or d. A recent quark-model estimate [69] predicts that the lowest charmed states decay strongly, while those with a b are stable except for weak decays. What does the lattice say?

7. Some Light-Quark Questions

7.1. Exotic light-quark states

Exotic states of light quarks have been predicted over the years in various forms. There is no consensus on their properties, 35 years after the introduction of the quark model. Open questions include the following:

1. Is a ΛΛ dibaryon (the “H”) a bound state? Quark-model calculations [70] indicate a gain in binding energy when quark spins are recoupled to gain the maximum possible hyperfine attraction. The two Λ’s still bind in the presence of SU(3) breaking [71]. However, instanton effects [72] may invalidate this conclusion.

2. Are there KK “molecules,” such as f0 and a0, near threshold? How about KKπ molecules, e.g., to account for the state η(1410) [73]?

3. States of two quarks and two antiquarks were discussed at this Conference [74]. Are there such states with strong couplings to baryon-antibaryon pairs [75] or to pairs of vector mesons [76]? One possible explanation of the fact that σ(γγ → ρ0ρ0) > σ(γγ → ρ+ρ−) near threshold is the cooperation of direct-channel resonances with I = 0 and I = 2 [77].

7.2. Λπ phase shifts

Can lattice QCD say anything about low-energy Λπ scattering? The difference δ between S-wave and P-wave phase shifts at c.m. energy equal to M(Ξ) governs the size of observable CP-violating effects in comparison of Ξ− → Λπ− and Ξ− → Λπ+ decays, the subject of an experimental search at Fermilab [78]. Recent calculations based on chiral perturbation theory [79] find values of δ ≃ δS − δP of order a few degrees, implying small CP asymmetries.

7.3. Weak decay matrix elements

The application of lattice gauge theory to certain weak processes (for instance, those involving kaons) has a long history [80]. Lattice methods might also be tried in radiative hyperon decays: Σ → ργ, Λ → nγ, Ξ → Λγ, Ξ → Σγ, and Ω → Ξγ. One seeks predictions of rates and of parity-violating asymmetries. Experimental upper limits for another process, the |ΔS| = 2 decay Ξ0 → ππ−, have recently been lowered to B < 1.7 × 10−5 [81]. This is far above the standard model range [82], which is not well known at present. Perhaps the lattice can help here.

7.4. Glue content of η′

Before the large branching ratio for the decays B → Kη′ was discovered [20], it was proposed [83] that the large flavor-singlet component of the η′ could lead to a significant amplitude for this process as a result of the two-gluon intermediate state in penguin processes. A related question is the glueball content of the η′. The decay η′ → ργ and the recently observed process [84] φ → η′γ with branching ratio B = (1.35±0.05) × 10−4 proceed at rates consistent with a “normal” q̅q̅ content of η′ [85]. (See, however, the arguments advanced in [86].) A good approximation to the η and η′ wavefunctions corresponds to mixing between flavor octet and singlet states η8 ≡ (u̅u + d̅d − 2s̅s)/√6 and η1 ≡ (u̅u + d̅d + s̅s)/√3 with an angle of θ = sin−1(1/3) ≃ 19.5°: η ≃ (u̅u + d̅d − s̅s)/√3, η′ ≃ (u̅u + d̅d + 2s̅s)/√6 [82,87].
8. Non-QCD Axes to Grind

8.1. Dynamical electroweak symmetry breaking

Most effort in lattice gauge theory has been devoted to QCD, for which there is overwhelming evidence from perturbative approaches. However, lattice methods may also guide searches for theories which are not yet well-established, such as dynamical electroweak symmetry breaking (“technicolor”) theories. Non-perturbative methods based on low-energy theorems, crossing symmetry, and unitarity have been used to argue that whereas some features of these schemes may be expected for a wide class of theories, others may be more sensitive to details of models. As one example, the minimal technicolor model based on fermions $F$ with charges $Q = \pm 1/2$ leads to an anomaly-free gauge sector. Higgs and Nambu-Goldstone bosons in this model are $\bar{F}F$ pairs. The lattice should be able to provide insights about the $\bar{F}F$ spectrum, depending on the underlying interaction.

8.2. Composite models

Composite models of quarks and leptons require a large mass hierarchy in which the properties of very light states appear to be determined at a much higher mass scale (since no evidence for deviations from pointlike structure has appeared up to scales of several TeV), and the need to describe chiral fermions. This last feature is a particular obstacle to the application of lattice methods, at least for the moment.

An example of a simple model in search of a theory is to imagine that the fermions $F$ mentioned above are the only fermionic constituents of matter, with quarks and leptons made of $F\bar{S}$ pairs, where $S$ are scalars which are either color singlets with charges $Q = \pm 1/2$ (leading to leptons) or color triplets with $Q = \pm 1/6$ (leading to quarks). A question in this picture (which will depend on the underlying dynamics) is the relative masses of $F\bar{F}$, $F\bar{S}$, $SS$, and baryonic (multi-$F$) states. Do we want light $SS$ states? If not, are there theories where they are naturally heavy? Do $F\bar{F}$ condensates form, leading to the desired $\Delta I = 1/2$ masses? What about multi-$F$ condensates, which could lead to large Majorana masses for right-handed neutrinos?

8.3. A phase transition to supersymmetry?

The model just described has the potential for supersymmetry. It involves an isodoublet of Dirac fermions $F_\pm$, each with four components, a quadruplet of scalars $S$ with charges $Q = -1/2, 1/6, 1/6, 1/6$ (the last three values referring to the three colors), and a corresponding quadruplet of antiscalars $\bar{S}$ with charges $1/2, -1/6, -1/6, -1/6$. There is an equal number of fermionic and bosonic degrees of freedom but no $N = 1$ supersymmetry since the charges are different. However, suppose there were a transition at high temperature to a phase where the charges of the scalars were $-1/2, -1/2, 1/2, 1/2$ and those of the antiscalars were $1/2, 1/2, -1/2, -1/2$. If so, color and electromagnetism would no longer commute; quarks would have integer charges as in the Han-Nambu model. (A related transition at high density was suggested at this conference by M. Alford.) Supersymmetry might be a desirable feature of a composite model by explaining the presence of light fermions in the spectrum.

9. Summary

The lattice has been shown to be a useful tool for peering beneath the complexities of hadron physics to learn about fundamental properties of quarks. It has permitted us to extend the usefulness of QCD beyond perturbation theory, and to explore the strong-coupling behavior of quantum field theories other than QCD.

In systems containing heavy quarks the lattice has given us a number of results, but time is running out for “predictions” rather than merely “postdictions.” For example:

1. There has been substantial progress on heavy meson decay constants such as $f_B$. As a result, one hopes to be able to extract better limits on the CKM matrix element $|V_{td}|$ from the observed strength of $B^0 - \bar{B}^0$ mixing. At present $f_B/\sqrt{B_B}$ and hence $|V_{td}|$ are known to about 20%. The measurement of other quantities such as the $B_s - \bar{B}_s$ mixing amplitude and the $B^+ \rightarrow \tau^+ \nu$, $\tau^+ \nu$, $B^0 - \bar{B}^0$ mixing amplitudes.
branching ratio may lead to more precise information on $|V_{td}|$ before lattice calculations achieve much greater accuracy.

2. Masses and widths of resonances containing a single heavy quark are starting to be predicted by the lattice. One is particularly interested in those orbitally excited mesons which have not yet been discovered, since they may play a role in the identification of neutral $B$ meson flavor at the time of production. However, experimental study of these mesons (and also of the corresponding baryons) is proceeding apace. Other questions about $D$ and $B$ mesons include the final states populated by their semileptonic decays, the corresponding form factors for such decays, and the thorny problem of non-leptonic decays.

3. Quarkonium systems generally present the lattice theorist with a set of given data from which interesting quantities (such as $\alpha_s$) can be extracted. It would be interesting also to see some predictions for as yet unseen states, such as the $^{1S_0}b\bar{b}$ mesons (whose masses influence those extractions of $\alpha_s$) and the D-wave $b\bar{b}$ systems (whose masses we think we know well, but perhaps not their production cross sections in $e^+e^-$ annihilations). Do heavy quarkonium systems really dissociate into pairs of flavored mesons at a universal separation of 1.4 to 1.5 fm? Both data and the lattice seem to indicate so.

Altogether it looks as if a wide range of problems may be accessible by lattice methods. Although lattice theorists have been attacking many of them, I hope this talk has indicated at least some others worth trying.

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