Optimum control of the active magnetic bearing for various parameters of the program movement

S V Majorov¹, N N Makhova¹

¹Department of Mechatronics and International Engineering, Orel State University, Naugorskoe sh. 29, Orel, Russia, 302020

E-mail: sergeyostu@yandex.ru

Abstract. We consider the mathematical model of stabilization of the situation of a rotor is given in an active magnetic bearing. Also various options of control system are considered. Investigated influence of the program movement and free controller parameters on quality of control process.

1. Introduction
One of the main elements of many machines is the rotor supported system that causes need of creation of these which would meet various requirements related by operating conditions and at the same time in reliability and quality. In cases when emphasis is placed on long term of work and high reliability at high speeds and heavy dynamic loadings, especially there is a use of active magnetic bearings. Their work is based on use of the magnetic attraction operating on a ferromagnetic body from electromagnets (Figure 1).

During the operation of active magnetic bearings resources by means of which we can exercise control are limited that does urgent use of optimum control with such parameters of the program movement which would provide the greatest energy efficiency.

Support of rotors with an electromagnetic way of creation of the bearing ability have all reasons for expansion of scope of application in different types of transport and technological machines, including main pump units. It is in many respects connected with improvement of electronic devices, manufacturing techniques of electrotechnical elements, and also with the advent of new methods and control algorithms in nonlinear systems [1 – 3].

However at operation of cars with rotors on an active magnetic suspension there are problems connected with stability of the movement of a rotor [4 – 9]. At the same time it is necessary to consider as the general questions of dynamics of rotor systems so [10 – 13], and the aspects connected with questions of calculation of forces in magnetic support [4, 7, 8, 14] and synthesis and the analysis of a control system [4, 7]. Thus, the problem of design of rotor system on an active magnetic suspension needs to be solved together with creation of a control system which would provide steady a rotor with providing operational indicators and decrease in power losses of the unit in general.

2. Mathematical model
The linear model of «rotor – active magnetic bearing» system can be presented in the form of ordinary differential equations system as follows [4].
\[
\begin{align*}
md^2 y/dt^2 - c_y y - hi &= Q; \\
L di/dt + (h_i/2) dy/dt + ry &= u,
\end{align*}
\]  

where  
m – mass of ferromagnetic body (rotor);  
y – rotor displacement reference about equilibrium position (control coordinate);  
i – coil current;  
u – voltage control;  
Q – external force (gravity);  
L – inductance;  
r – coil resistance;  
c_y – «negative» position stiffness of bearing;  
h_i – current stiffness of bearing;  
t – time.

**Figure 1.** Model of a two-sided «rotor – active magnetic bearing» system.

Position and current stiffness can be determined by the following relations:

\[c_y = \frac{2Li_i^2}{\delta^2} \]  

\[h_i = \frac{2Li_i}{\delta} \]  

where  
i_i – initial displacement current;  
\delta – bearing radial clearance.

We will note that use of linear model of the active magnetic bearing it isn't always justified in some cases it is necessary to use other approaches to determination of forces in a magnetic contour, such as in work [14].
At optimum control it is required to find such \( u = u^o \) which when translating system from any initial state in a zero state minimizes some functionality of quality [15]:

\[
\int_0^\infty \left[ y^T(t)y(t) + \rho u^T(t)u(t) \right] dt
\]

where \( \rho \) – a positive weight scalar. The size of value of management \( u \) and accuracy of positioning of system depends on this parameter. The more \( \rho \), the is less than value \( u \). Control at \( \rho \rightarrow 0 \) is called «cheap», at \( \rho \rightarrow \infty \) – «expensive».

3. Optimal current control
For the optimum PD-regulator the control law looks as follows [15]:

\[
i^o = -(k_1^o y + k_2^o \dot{y})
\]

where

\[
k_1^o = m \frac{\omega_0^2 + k^2}{h_i}, \quad k_2^o = m \sqrt{\frac{\omega_0^2 + k^2}{h_i}}
\]

Here \( k \) – module of poles of an control object,

\[
k = \frac{c_y}{m}
\]

\( \omega_0 \) – natural frequency undamped system.

Coefficients in expression (6) are received by equating of a characteristic polynom of the optimum closed system with a polynom of a program system. At the same time control parameters \( \rho \) and \( \zeta \) are from relationships:

\[
\rho = \frac{1}{(\omega_0^2 - k^2)}; \quad \zeta = \frac{1}{2} \sqrt{2 \left( 1 + \frac{k^2}{\omega_0^2} \right)}
\]

Equations (6) contain the varied \( \omega_0 \) parameter (natural frequency undamped system). As \( \rho \) shall be the positive, this parameter shall meet the following condition:

\[
\omega_0 \geq k
\]

Therefore we will set \( \omega_0 \) on formula:

\[
\omega_0 = k_o k
\]

where \( k_o \) – free varied parameter.

We receive the most «expensive» control at \( k_o = 1 \) (Figure 2 (a)) when \( \omega_0 = k \).

In Figures 2 and 3 dependence of rotor displacement and control current on time is shown respectively at various \( k \) and \( \omega_0 \) values.

Apparenty from schedules, we receive the smallest expenses on current (Figure 3 (a)) at expensive management where \( k_o = 1 \) and at the set parameters of the \( \omega_0 = 118.9 \, \text{rad/s} \), however to the same case there corresponds the longest transient process (Figure 3(a)). With increase in \( \omega_0 \) time of transient process is reduced, but at the same time the size of the current necessary for bearing control increases. In work [4] it is specified that in practice of \( \omega_0 \) value 20 – 500 Hz usually make, however It should be noted that this size substantially depends on such parameters as inductance of \( L \), current of shift of \( i_c \).
and size of a gap between rotor and electromagnets poles $\delta$. In particular, reduction of $\delta$ leads to increase in the minimum $\omega_0$ value which is necessary to provide stability of system.

4. **Optimal voltage control**

Optimum control of $u'$ is set as follows [15]:

$$ u' = -(g_1^o y + g_2^o \dot{y} + g_3^o i + g_4^o \sigma) $$

(11)

where

$\sigma$ – integral of rotor displacement by time;

$g_1^o$, $g_2^o$, $g_3^o$, $g_4^o$ – optimal values of feedback strengthening coefficients respectively on rotor displacement, speed, control current and time domain integral from rotor displacement.

We will consider a problem of optimum voltage control at $\rho \to 0$ («cheap» control).

For definition of optimum values of feedback strengthening coefficients of it is necessary to solve the characteristic equation of system:

Figure 2. Dependence of rotor displacement on control time at
(a) $k_o=1$; (b) $k_o=5$; (c) $k_o=20$; (d) $k_o=100$. 


\[ s^3 + \frac{r}{L} s^2 - \frac{c_r}{mL} = 0 \]  

Therefore we find poles of the control object of \( \pi_1, \pi_2 \text{ and } \pi_3 \) which can be expressed:

\[ \pi_1 = \alpha_1; \quad \pi_2 = -\alpha_2 + j\beta_2; \quad \pi_3 = -\alpha_2 - j\beta_2 \]  

where
\[ \alpha_0, \alpha_1, \alpha_2, \beta_2 \text{ – positive numeric values}; \]
\[ j \text{ – imaginary unit}. \]

At \( \rho \to 0 \) \( \pi_1=s_1, \pi_2=s_2, \pi_3=s_3 \). To consider an integrated component we enter an additional pole of \( s_4=\alpha_0 \), small on the module.

We receive the following coefficients of optimum voltage control:

\[
\begin{align*}
g_1^* &= \left\{ c_r L (\alpha_0 + \alpha_1 + 2\alpha_2) + mL\left[ (\alpha_0 + \alpha_1)(\alpha_2^2 + \beta_2^2) + 2\alpha_0\alpha_1\alpha_2 \right] \right\} / h; \\
g_2^* &= mL\left[ (\alpha_0^2 + 2\alpha_0\alpha_1 + \beta_2^2 + \alpha_0(\alpha_1 + 2\alpha_2) \right] / h; \\
g_3^* &= L(\alpha_0 + \alpha_1 + 2\alpha_2) - r; \\
g_4^* &= mL\alpha_0\alpha_1(\alpha_2^2 + \beta_2^2).
\end{align*}
\]  

\[ (12) \]

\[ (13) \]

\[ (14) \]

**Figure 3.** Dependence of control current on control time at
(a) \( k_w=1 \); (b) \( k_w=5 \); (c) \( k_w=20 \); (d) \( k_w=100 \).
In this case the varied parameter is $\alpha_0$ which we will set as the smallest of the available coefficients of $\alpha_1$ and $\alpha_2$ increased by some coefficient. Transient time depended characteristics at various $\alpha_0$ values are provided on Figures 4 and 5.

Figure 4. Dependence of rotor displacement on control time at
(a) $\alpha_0=9.2$; (b) $\alpha_0=0.9$; (c) $\alpha_0=0.009$; (d) $\alpha_0=0.00009$. 

(a)  
(b)  
(c)  
(d)
Figure 5. Dependence of control voltage on control time at a) \( \alpha_0=9.2 \); b) \( \alpha_0=0.9 \); c) \( \alpha_0=0.009 \); d) \( \alpha_0=0.00009 \).

At \( \alpha_0 \) is 10 times less, than coefficients of \( \alpha_1 \) and \( \alpha_2 \), process it is unstable (Figure 4 (a) and 5(a)). At further reduction of \( \alpha_0 \) we observe improvement of quality of transient process (Figure 4 (b-d) and 5 (b-d)).

5. Conclusion

The model of an electromagnetic suspension with optimum current and voltage control is realized in the environment of Matlab, varying parameters of the program movement. Need of the correct selection of parameters for stability of system and its energy efficiency, and also for receiving transient processes of the required quality is obvious. Further it is planned to realize use of the studying systems for the purpose of simplification of selection of parameters of the program movement for each separate system.

References

[1] Abduragimov A S and Vereshchagin V P 2010 Features of the digital equipment of control of electromagnetic bearings of gas-distributing units Electromechanics questions (Moscow VNIIEIM) 115
[2] Vereshchagin V P and Rogoza A V 2008 Features of design of magnetic bearings for large machines Electromechanics questions (Moscow VNIIEIM) 106
[3] Sarychev A P 2009. II. Features and experience of creation of electromagnetic bearings for a series of compressors of gas-distributing units Electromechanics questions (Moscow VNIIEIM) 106
[4] Zhuravlev Y N 2003 Active magnetic bearings: Theory, design, application (St. Petersburg: Polytechnica) p 206
[5] Sarychev A P 2009 Development of electromagnetic bearings for a series of compressors of gas-distributing units Electromechanics questions (Moscow VNIIEIM) 110
[6] Sarychev A P 2008 Mathematical model of a rotor for the analysis of control of magnetic bearings Electromechanics questions (Moscow VNIIEIM) 107
[7] Schweitzer G and Maslen E H 2009 Magnetic bearings. Theory, design, and application to rotating machinery (Springer-Verlag Berlin Heidelberg) p 523
[8] Maslen E H 2000 Magnetic bearings (University of Virginia) p 245
[9] Schweitzer G 2002 Active Magnetic bearings – chances and limitations Proc. 6th Internat. IFToMM Conf. on Rotor Dynamics (Sydney, Sept. 30 – Oct. 3)
[10] Lee C W 1993 Rotating Vibration analysis of rotors (Springer) p 312
[11] Genta G 2005 Dynamics of rotating systems (NY: Springer) p 660
[12] Lalanne M and Ferraris G 1998 Rotordynamics prediction in engineering (J Wiley&Sons) p 266
[13] Friswell M I 2010 Dynamics of rotating machines (Cambridge University Press) p 512
[14] Solomin O V Dorofeev L V and Majorov S V 2008 Calculation of magnetic field in the radial active magnetic bearing / Proc. 6th Internat. Conf. Vibration machines and technologies (Kursk: KSTU) pp 779 –784
[15] Nogin V D 2008 Introduction to optimum control (St. Petersburg: UTAS) p 92

Acknowledgment
This work was partly supported by an RSF grant, project 16-19-00186 “Planning of optimal energy efficient rotor trajectories in mechatronic modules with complex rheology medium”.