The galaxy–halo size relation of low–mass galaxies in FIRE

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ABSTRACT

Galaxy sizes correlate closely with the sizes of their parent dark matter haloes, suggesting a link between halo formation and galaxy growth. However, the precise nature of this relation and its scatter remains to be understood fully, especially for low-mass galaxies. We analyse the galaxy–halo size relation for low-mass ($M_* \sim 10^{-7} M_\odot$) central galaxies over the past 12.5 billion years with the help of cosmological volume simulations (FIREbox) from the Feedback in Realistic Environments (FIRE) project. We find a nearly linear relationship between the half-stellar mass galaxy size $R_{1/2}$ and the parent dark matter halo virial radius $R_{\text{vir}}$. This relation evolves only weakly since redshift $z = 5$: $R_{1/2} \propto (0.053 \pm 0.002) (R_{\text{vir}} / 35 \text{kpc})^{0.934 \pm 0.054}$, with a nearly constant scatter ($\sigma_r = 0.084$ [dex]). Whilst this ratio is similar to what is expected from models where galaxy disc sizes are set by halo angular momentum, the low-mass galaxies in our sample are not angular momentum supported, with stellar rotational to circular velocity ratios $v_\text{rot} / v_\text{circ} \sim 0.15$. Introducing redshift as another parameter to the GHSR does not decrease the scatter. Furthermore, this scatter does not correlate with any of the halo properties we investigate – including spin and concentration – suggesting that baryonic processes and feedback physics are instead critical in setting the scatter in the galaxy–halo size relation. Given the relatively small scatter and the weak dependence of the galaxy–halo size relation on redshift and halo properties for these low-mass central galaxies, we propose using galaxy sizes as an independent method from stellar masses to infer halo masses.

Key words: cosmology: theory – galaxies: evolution – galaxies: dwarf – galaxies: haloes – methods: numerical

1 INTRODUCTION

According to the standard picture of galaxy formation, galaxies form at the centres of their parent dark matter haloes (White & Rees 1978; Cole et al. 2000). Haloes tend to form bottom-up, with small haloes collapsing first and subsequently merging into larger and more massive haloes (Blumenthal et al. 1984). This gravity-driven hierarchical picture of halo collapse, however, contrasts with a much more complex picture of galaxy formation which is set by a variety of baryonic processes (e.g., Somerville & Davé 2014, and references therein). The link between galaxies and their parent haloes is therefore far from trivial, especially given that the latter are more than order of magnitude larger and more massive than the former.

Nonetheless, central galaxies and their parent haloes do appear to be linked tightly, at least in the local Universe (see Wechsler & Tinker 2018, for a recent review). For instance, the stellar-to-halo mass relation (SHMR), also called the $M_* - M_{\text{halo}}$ relation, has been widely studied empirically (via abundance matching, Kravtsov & Klypin 1999; Kravtsov et al. 2004; Conroy & Wechsler 2009; Behroozi et al. 2010), observationally via – e.g., gravitational lensing (Mandelbaum et al. 2006; Behroozi et al. 2019) or galaxy kinematics (More et al. 2011; Kravtsov et al. 2018) – and theoretically, with the help of analytic models (e.g., White & Frank 1991; Mo et al. 1998; Wechsler et al. 1998; White et al. 2007) and cosmological simulations (Pearce et al. 2001; Berlind et al. 2003; Simha et al. 2012; Hopkins et al. 2014; Khaldai et al. 2015; McAlpine et al. 2016; Feldmann et al. 2016; Chaves-Montero et al. 2016; Pillepich et al. 2018b; Engler et al. 2020). This relation forms a broken power law of increasing galaxy formation efficiency with halo mass until a peak at $M_{\text{halo}} \sim 10^{12} M_\odot$ and a sharp decrease towards higher halo masses (Wechsler & Tinker 2018). Interestingly this relation has a nearly constant scatter over five orders of magnitude in halo...
mass ($M_{\text{halo}} \sim 10^{10-15} M_\odot$), and the baryonic feedback processes – such as stellar winds, supernovae, cosmic rays and active galactic nuclei (AGN) – largely determine the shape of this relation despite the haloes dominating in size and mass (e.g., Dekel & Silk 1986; Silk & Rees 1998; Bullock et al. 2000; Croton et al. 2006; Hopkins et al. 2012b).

More recently, Kravtsov (2013) discovered a tight connection between the sizes of galaxies and haloes at $z = 0$. Unlike the SHMR, this galaxy–halo size relation (GHSR) forms an approximately linear relation with constant scatter across nearly three orders of magnitude in halo size ($R_{\text{halo}} \sim 5 - 1,500$ kpc, corresponding to $M_{\text{halo}} \sim 10^{8-15} M_\odot$). Furthermore, this relationship is largely independent of galaxy morphology and nearly identical for centrals and satellites. The observation of the GHSR at $z = 0$ thus raises a variety of intriguing questions such as: When was the GHSR established? How do its slope, normalization, and scatter evolve over cosmic time? Are the GHSR and its scatter linked to the growth histories of structure in the Universe?

However, observing this galaxy–halo connection at high redshift poses a number of challenges. For instance, measuring galaxy and halo sizes with sufficiently high spatial resolution becomes increasingly more challenging at higher redshift. Fortunately, a variety of approaches have been developed to probe the galaxy–halo link at earlier cosmic times. For instance, di Teodoro & Fraternali (2015) find in the EAGLE simulation that at a fixed stellar mass, the galaxy size weakly correlates with halo mass, but also the hydrodynamics of gas, and the complex baryonic processes (e.g., Springel 2005, 2010; Hopkins 2015) have made it increasingly more challenging at higher redshift. However, observing this galaxy–halo connection at high redshift poses a number of challenges. For instance, measuring galaxy and halo sizes with sufficiently high spatial resolution becomes increasingly more challenging at higher redshift. Fortunately, a variety of approaches have been developed to probe the galaxy–halo link at earlier cosmic times. For instance, di Teodoro & Fraternali (2015) find in the EAGLE simulation that at a fixed stellar mass, the galaxy size weakly correlates with halo mass, but also the hydrodynamics of gas, and the complex baryonic processes (e.g., Springel 2005, 2010; Hopkins 2015) have made it increasingly more challenging at higher redshift.

Another approach is to simulate galaxy formation in a cosmological context to study the galaxy–halo connection across cosmic history (see Somerville & Davé 2014, for a recent review). Over the past decade, advances in computational power and numerical techniques (e.g., Springel 2005, 2010; Hopkins 2015) have made it possible to simulate not only the gravitational collapse of dark matter, but also the hydrodynamics of gas, and the complex baryonic processes such as gas cooling, star formation, and stellar feedback (e.g., Vogelsberger et al. 2014; Schaye et al. 2015; Crain et al. 2015; Tremmel et al. 2017; Hopkins et al. 2018; Pillepich et al. 2018a; Nelson et al. 2019). Recent simulations provide a deeper understanding of, and challenge long-standing assumptions about, galaxy formation and evolution. Perhaps contrary to the expectation based on specific angular momentum conservation (Fall & Efstathiou 1980; Mo et al. 1998), Desmond et al. (2017) find in the EAGLE simulation that at a fixed stellar mass, the galaxy size weakly correlates with halo mass, concentration or spin. On top of that, Somerville et al. (2018) conclude from their sample of Galaxy And Mass Assembly (GAMA) and the Cosmic Assembly Near Infrared Deep Extragalactic Legacy (CANDELS) surveys mapped to the Bolshoi-Planck dissipationless N-body simulation that the ratio of galaxy to halo size decreases slightly with cosmic time for less massive galaxies, while the ratio of galaxy size to halo size times halo spin – $R_{1/2}(R_{200,\lambda})$ – is lower for more massive galaxies below $z \leq 3$. In the VELA (Ceverino et al. 2014; Zolotov et al. 2015) and NIHAO (Wang et al. 2015) zoom-in simulations, Jiang et al. (2019) find that the halo spin only weakly correlates with that of the galaxy, and this correlation becomes weaker with increasing redshift. Moreover, the gas that builds galaxies in cosmological simulations typically has higher specific angular momentum than that of the dark matter (e.g., Danovich et al. 2015; Stewart et al. 2017; Zjupa & Springel 2017; El-Badry et al. 2018; Kretschmer et al. 2020).

In general, these recent results point to differences between halo and galactic properties once thought to be tightly linked. It thus appears that baryonic properties are more significant in setting galaxy sizes. Using zooms of Milky-Way mass objects with the same FIRE-2 model, Garrison-Kimmel et al. (2018) find that of their studied parameters, the best predictor of galaxy size and morphology is the gas spin at the time the galaxy formed half of its $z = 0$ stars. Similarly for massive galaxies ($M_{\star} \sim 10^{9-12.5} M_\odot$) in the Sloan Digital Sky Survey Data Release 7, Zanisi et al. (2020) find in their semi-empirical models that the specific stellar angular momentum is the best mediator to the GHSR. Despite the numerous works on potential correlations between halo properties and galaxy sizes (Rodriguez et al. 2021), the GHSR has not been well studied in simulations and observations, in particular, in the regime of low stellar masses and at higher redshifts.

In this paper, we study the GHSR and its scatter for low-mass centrals, as predicted by high resolution cosmological volume simulations from the FIREbox simulation suite, which is part of the Feedback in Realistic Environments (FIRE) project. FIREbox follows the growth of galaxies and haloes in a 15 cMpc $h^{-1}$ (22 pMpc at $z = 0$) side-length cosmological box with the help of the FIRE-2 baryonic model (Hopkins et al. 2018) and the Meshless Finite Mass hydrodynamic solver gizmo$^2$ (Hopkins 2015). This simulation suite contains a large sample of galaxies ranging from isolated dwarfs to Milky-Way (MW) analogs, facilitating an in-depth analysis of the GHSR from $z = 5$ until today.

The layout of this paper is as follows. In § 2 we outline the methodologies for the numerics of the simulation (§ 2.1), the halo finding algorithms (§ 2.2), and the sample selection (§ 2.3). § 3 compares the galaxy/halo pairs to observations and other recent works. § 4 constructs and details the GHSR from 0 $\leq z \leq 5$. § 5 analyzes the scatter in the GHSR and addresses potential halo (§ 5.1), galaxy (§ 5.2), and environment (§ 5.3) properties affecting the $R_{1/2} - R_{vir}$ relation. Specifically, § 5.1.1 investigates the effects of halo spin and concentration on the GHSR in more detail. Lastly, we summarise the major findings in § 6.

2 METHODOLOGY

2.1 FIREbox simulation suite

The galaxies and haloes analysed in this paper are extracted from the FIREbox suite of $V = (15 \text{ cMpc } h^{-1})^3$ cosmological volume simulations (Feldmann et al. in prep), which are part of the FIRE project (Hopkins et al. 2014, 2018). Unlike all previous FIRE simulations, FIREbox does not use the zoom-in set-up to study galaxy evolution – but instead, it simulates gas, stars, and dark matter in a cubic cosmological volume with periodic boundary conditions. Initial conditions at $z = 120$ were created with MUlti Scale Initial Conditions (MUSIC; Hahn & Abel 2011) using cosmological parameters consistent with Planck 2015 results (Alves et al. 2016): $\Omega_m = 0.3089, \Omega_{\Lambda} = 1 - \Omega_m, \Omega_b = 0.0486, h = 0.6774, \sigma_8 = 0.8159, n_s = 0.9667$ and a transfer function calculated with camb$^1$ (Lewis et al. 2000, 2011).

All FIREbox simulations start from the same initial conditions but they differ in particle number, numerical resolution, and whether they

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1 http://fire.northwestern.edu/
2 http://www.tapir.caltech.edu/~phopkins/Site/GIZMO.html
3 http://camb.info
are run as dark-matter-only (DMO) simulations or include baryonic physics. All simulations are run with gizmo (Hopkins 2015). Gravitational forces between particles are calculated with a heavily modified version of the parallelisation and tree gravity solver of GADGET-3 (Springel 2005) allowing for adaptive force softening, while hydrodynamics is solved with the meshless-finite-mass method introduced in Hopkins (2015). All hydrodynamical FIREbox simulations are run with the FIRE-2 model to account for gas cooling and heating, star formation, and stellar feedback (Hopkins et al. 2018). Feedback from supermassive black holes is not included. Star formation is modelled to occur in dense (n > 300 cm$^{-3}$ for the 1024$^3$ FIREbox simulation; n > 100 cm$^{-3}$ and > 10 cm$^{-3}$ for the 512$^3$ and 256$^3$ simulations respectively), molecular, self-gravitating gas, and the gas to star conversion takes place on a local free-fall time with a 100% local efficiency. Due to stellar feedback, the realized star formation efficiency is lower, consistent with Kennicutt-Schmidt relations (Schmidt 1959; Kennicutt 1998; Orr et al. 2018). Stellar feedback includes energy, momentum, mass, and metal injections from supernovae (type II and type Ia) and stellar winds (OB and AGB stars). Radiative feedback (photo-ionisation and photo-electric heating) and radiation pressure from young stars is accounted for in the locally extincted background radiation in optically thin networks (LEBROON) approximation (Hopkins et al. 2012a). The FIRE-2 model has been extensively validated in a number of publications analysing properties of galaxies across a range in stellar masses and numerical resolutions, including simulations at this FIREBox resolution (Wetzel et al. 2016; Hopkins et al. 2018; Ma et al. 2018a,b).

Most of the analysis in this paper is based on the FIREBox pathfinder hydrodynamical simulation ($N_b = 1024^3$ and $N_{DM} = 1024^3$). The mass resolution of this run is $m_b = 6.3 \times 10^4$ $M_\odot$ for baryonic (gas and star) particles and $m_{DM} = 3.3 \times 10^5$ $M_\odot$ for dark matter particles. The force softening lengths for star and dark matter particles are $h_\star = 12$ pc (physical) and $h_{DM} = 80$ pc respectively. The softening of gas particles is set to their smoothing length down to a minimum of 1.5 pc, which is reached only in the densest parts of the interstellar medium. The force resolution is set such that the highest density we formally resolve is 1,000 times the star formation threshold (see Hopkins et al. 2018, § 2.2 for more details.)

Mass and force resolution of the $N_{DM} = 512^3$ ($N_{DM} = 256^3$) FIREBox run are correspondingly lower, e.g., $m_b \sim 5 \times 10^5$ $M_\odot$ ($m_b \sim 4 \times 10^5$ $M_\odot$) and $h_{DM} = 32$ pc ($h_{DM} = 128$ pc). All FIREBox simulations examined in this paper are evolved to $z = 0$.

2.2 Halo Finding and Definitions

We employ the AMIGA Halo Finder (AHF)\(^4\) to identify and characterise the properties of dark matter haloes (Knollmann & Knebe 2009). We only consider haloes containing at least 100 particles of any type, which corresponds to a minimum halo mass of $M_{\text{vir}} \sim 10^7$ $M_\odot$ $h^{-1}$. The halo radius, $R_{\text{vir}}$, is defined based on the virial overdensity criterion

$$M_{\text{vir}} = (4/3)\pi \Delta(z) \rho_m(z) R_{\text{vir}}^3,$$

where $\rho_m(z)$ is the matter density at a given redshift $z$, and $\Delta(z)$ is defined by Bryan & Norman (1998). Halo centres and the centres of their central galaxies are identified as the halo radius with the highest total matter density using AHF’s maximum-density (MAX) setting.

\(^4\) http://popia.ft.uam.es/AHF/Download.html

| $\Delta N$ | $R_{\text{vir}}/\text{kpc}$ | $M_{\text{vir}}/\text{M}_\odot$ | $N_{\text{FB-S}}$ | $N_{\text{small}}$ | $N_{\text{FB-l}}$ |
|-----------|----------------|-----------------|---------------|----------------|----------------|
| $\Delta N$ | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ |
| 0 | $[1.70, 2.10]$ | $[9.80, 11.00]$ | 826 | 47 | 189 |
| 1 | $[1.50, 1.80]$ | $[9.90, 11.25]$ | 1092 | 88 | 284 |
| 2 | $[1.30, 1.65]$ | $[9.80, 10.90]$ | 1373 | 49 | 210 |
| 3 | $[1.20, 1.55]$ | $[9.85, 10.95]$ | 1160 | 80 | 121 |
| 4 | $[1.10, 1.45]$ | $[9.85, 10.90]$ | 801 | 59 | 57 |
| 5 | $[1.00, 1.35]$ | $[9.80, 10.85]$ | 488 | 21 | 32 |

(1) Redshift; (2) and (3) Ranges of the halo virial radius and mass respectively for the ‘FB-S’ galaxies; (4) Number of galaxy/halo pairs included in the analysis; (5) and (6) Numbers of objects excluded because their halo sizes are below and above the virial radius ranges (column 2).

2.3 Sample Selection and Galaxy Definitions

This paper only considers “central galaxies”, galaxies that form in main haloes (i.e., not satellite galaxies nor subhaloes). These galaxies are the most massive within $R_{\text{vir}}$ of the host halo and dominate the baryonic processes of the halo. Galaxy sizes and masses are calculated from the $\text{d} \Delta \text{mass}$ files provided by AHF. We consider the total galaxy radius to be 10% of the halo virial radius (Price et al. 2017). At lower redshifts Hopkins et al. (2018); Samuel et al. (2020) find that satellites can exist within $0.1 R_{\text{vir}}$ for massive galaxies, but this does not affect our sample of low mass galaxies. When including all stellar material within $0.2 R_{\text{vir}}$, the same qualitative results hold (see the last paragraph of § 4 for the results). Additionally, see Appendix A for the results using an iterative $R_{1/2}$ calculation starting from all stars within $0.15 R_{\text{vir}}$ (Hopkins et al. 2018).

We calculate the galaxy stellar mass $M_\star \equiv M_\star(<0.1 R_{\text{vir}})$ by linearly interpolating in log $-\log$ space between radii $r$ and the cumulative stellar mass $M_\star(<r)$. Subsequently the inverse interpolation of $M_\star(<r)$ at $0.5 M_\star$ yields the three-dimensional spherical half-stellar mass radius $R_{1/2}$ (see Appendix A for the cumulative stellar mass radial profiles for the 1024$^3$ ‘FB-S’ galaxies at redshifts $z = 0$, 2). The average stellar particle mass in the 1024$^3$ simulation is $\sim 3 \times 10^4$ $M_\odot$ at $z = 2$ (roughly half the baryonic mass resolution listed in § 2.1 due to stellar mass loss). We consider the galaxies resolved when $N_\star(<0.1 R_{\text{vir}}) > 300$, implying $M_\star > 10^7$ $M_\odot$ in the 1024$^3$ simulation. We maintain this $N_\star$ criterion for all resolutions, meaning that the $M_\star$ lower limit in lower resolution simulations increases by factors of 8.

Of the resolved galaxies, we study a selected range of halo sizes that depends on redshift and simulation resolution. At each redshift, we define a lower limit on $R_{\text{vir}}$ such that the number of haloes in each $R_{\text{vir}}$ bin decreases with increasing halo size. This lower limit on the halo size translates to a nearly constant lower limit on the halo mass of $M_{\text{vir}} \gtrsim 10^8.8$ $M_\odot$ since $z = 5$. This lower limit cuts out a small percentage ($\sim 4-8\%$, see Table 1 column 5) of additional objects at each redshift. When we instead employ a looser criterion of a minimum halo mass of $M_{\text{vir}} > 10^9$ $M_\odot$, we only exclude a few additional haloes ($< 5$ at each redshift), and the results remain qualitatively consistent. At large $R_{\text{vir}}$, the FIREBox galaxy sizes no longer increase with increasing halo size, which disagrees with observational results at these halo sizes (Krantzov 2013; Huang et al. 2017; Zanisi et al. 2020). At a fixed halo mass at $z = 0$, these galaxies also have slightly larger stellar masses than is expected from the SHMR (see Feldmann et al., in prep for more details). Moreover, these galaxies have more rotational support ($v_{\text{rot}}/v_{\text{circ}} \sim 0.4$ at $z = 0$; see § 5.2 for...
Figure 1. A convergence test of the GHSR at redshift $z = 0$ (left) and $z = 2$ (right), displaying the 256\(^3\) (blue), 512\(^3\) (red) and 1024\(^3\) (black) runs from FIREbox (FB). The open circles represent the galaxies within the selected $R_{\text{vir}}$, denoted ‘FB-S.’ and the filled squares the galaxies with larger $R_{\text{vir}}$, denoted ‘FB-L.’ Average sizes within each $lg R_{\text{vir}}$ bin are overplotted with standard errors indicated by error bars. We also plot galaxy-halo pairs from FIRE-2 zoom simulations as triangles (Wetzel et al. 2016; Hopkins et al. 2018). These high resolution (gold triangles, $m_h \approx 7 \times 10^6 M_\odot$) zoom-in simulations have an 8 times higher peak resolution than the FIREbox pathfinder (1024\(^3\)). Medium resolution zoom-in runs (grey triangles; $m_h \approx 6 \times 10^4 M_\odot$) have comparable mass resolution to the 1024\(^3\) run. FIREbox galaxies in moderately massive haloes ($M_{\text{vir}} \sim 10^{11-13} M_\odot$ at $z = 0 - 2$) have approximately constant size with increasing halo size. In this study, we focus on the ‘FB-S’ galaxies. The figure shows that the $R_{1/2} - R_{\text{vir}}$ relation appears converged at both $z = 0$ and $z = 2$ for the FIREbox pathfinder run.

more details) than the galaxies at smaller $R_{\text{vir}}$ ($v_{\text{rot}}/v_{\text{circ}} \sim 0.1 - 0.2$).

Lower-mass galaxies are more dispersion supported both for isolated dwarf galaxies in the Local Group (Wheeler et al. 2017) and in the FIRE-1 zoom simulations (Wheeler et al. 2015, 2017). Hereafter, we denote the galaxies in the large-size regime as ‘FB-L,’ and the fiducial sample within the selected halo size range as ‘FB-S.’ Due to the differences in the GHSR and amount of dispersion or rotational support between the ‘FB-L’ and ‘FB-S’ samples, we focus on the low-mass ‘FB-S’ galaxies here. We plan to study the rotation supported, higher-mass ‘FB-L’ galaxies in more detail in future work. Table 1 summarises these limits and gives the number of galaxy/halo pairs at each redshift.

Figure 1 displays the GHSR at redshifts $z = 0$ and $z = 2$ for the 256\(^3\), 512\(^3\) and 1024\(^3\) FIREbox simulations for galaxies residing in both small (‘FB-S’) and large haloes (‘FB-L’). We also compare the predictions by FIREbox with recent FIRE-2 zoom simulations (Wetzel et al. 2016; Hopkins et al. 2018). The latter reach a mass resolution up to $\sim 8x$ higher than FIREbox pathfinder run allowing us to check for resolution effects. We also show results from 8 and 64 times lower resolution re-runs of these zoom-ins. The figure shows that the $R_{1/2} - R_{\text{vir}}$ relation appears well converged at both $z = 0$ and $z = 2$ for the 1024\(^3\) FIREbox run. However, Hopkins et al. (2018) shows that galaxy properties, including size, may still change when the mass resolution changes from $m_h \approx 6 \times 10^4 M_\odot$ to $m_h \approx 7 \times 10^3 M_\odot$. Interestingly, the ‘FB-L’ galaxies shows a prominent turn-over, i.e., a decrease in galaxy size with increasing halo size for $M_{\text{vir}} \sim 10^{11 - 10^{12}} M_\odot$ haloes. They also show an increased scatter. A few of the galaxies from the highest resolution zoom simulations have smaller sizes than the ‘FB-S’ galaxies, which stems from the $M_\star$ criterion. At a fixed $R_{\text{vir}}$, the zoom simulations can resolve galaxies with $8x$ lower stellar masses than the FIREbox pathfinder simulation. However, the ‘FB-S’ sample contains many more galaxies larger than the zoom simulations, facilitating statistical analysis across a range of halo sizes.

In summary, the baryonic particles within an AHF defined dark matter halo must meet these criteria to be included in this analysis:

(i) The host halo must be a main halo, i.e. not a proper subhalo.

(ii) The galaxy must be resolved with $N_\star \geq 300$.

(iii) The halo’s virial radius must be in the selected range outlined in Table 1.

3 COMPARISONS TO OBSERVATIONS

In Figure 2, we compare our galaxy size-stellar mass relation to observations at $z = 2$ from the Cosmic Assembly Near-infrared Deep Extragalactic Legacy Survey (CANDELS) / 3D-HST (van de Wel et al. 2014) and Cosmic Evolution Survey (COSMOS) / Drift And SHIfT (DASH) (Mowla et al. 2019b) surveys. We assume that on average for a random projection, the intrinsic three-dimensional half-stellar mass radius $R_{1/2}$ is similar to the projected two-dimensional half-mass radius $R_{\text{mass}}$ (van de Ven & van der Wel 2021), see their text for how this approximation varies with intrinsic ellipsoidal axis ratios and for comparisons between $R_{1/2}$ and $R_{\text{eff}}$. We convert $R_{\text{mass}}$ to the 2D half-light radius $R_{\text{eff}}$ using the empirical fit from Suess et al. (2019a), their Table 2 fit using $M_\star$ with a completeness limit of $M_\star > 10^{10} M_\odot$ at $1 < z < 2.5$. Converting between $R_{1/2}$ and $R_{\text{eff}}$ typically introduces a smaller scatter than the intrinsic scatter of the galaxy size-stellar mass relation (Price et al. 2017; Genel et al. 2018; van de Ven & van der Wel 2021).
in the low-mass regime, while the quiescent relation flattens out for low-mass galaxies. There are no observational studies of dwarf galaxies at $z = 2$. See Appendix B for more details regarding the galaxy size-stellar mass relation, and for the figures at $z = 0$ (Figure B1). Briefly, at $z = 0$ the FIREbox galaxies follow the galaxy size-stellar mass relation for star-forming galaxies from Nedkova et al. (2021), but they are systematically larger by $\sim 0.3-0.5$ [dex] at a fixed stellar mass. The value of the discrepancy depends on the correction factor employed from Suess et al. (2019b). We emphasize that Figures 2 and B1 should be understood as illustrations rather than proper comparisons with observations; we plan to analyse synthetic images of FIREbox galaxies using radiative transfer (e.g., Liang et al. 2019, 2021) in future work.

Shibuya et al. (2015) find that a log-normal distribution well approximates the $R_{\text{eff}}$ distribution in their combined legacy dataset of 3D-HST, CANDELS, Hubble Ultra Deep Fields (HUDF) 09+12 and the Hubble Frontier Fields (HFF) surveys. While their sample of star-forming and Lyman break galaxies focuses on more massive galaxies ($M_\ast \sim 10^{8.5-11}$ M$_\odot$ since $z = 6$), our $R_{1/2}$ distribution qualitatively agrees. Figure 3 shows our best fitting Gaussians to the histograms of $\lg R_{1/2}$ in our final samples at each redshift. We create Q-Q plots for the fits to each redshift (omitted here), which indicate that the Gaussian well approximates each snapshot’s $\lg R_{1/2}$ distribution. However, the Shapiro-Wilks tests and chi-squared $p$-values for normal distributions indicate that the distributions are likely not perfectly log-normal. Further, Shibuya et al. (2015) find that the average galaxy size decreases significantly towards higher redshift whilst maintaining a roughly constant standard deviation. Figure 3 demonstrates that ‘FB-S’ galaxies display the same behaviour.

We find an approximately constant standard deviation $\sigma_{\lg R_{1/2}} = 0.12$ [dex] since $z = 5$ in qualitative agreement with Shibuya et al. (2015). However, their value for the scatter is somewhat higher at $\sigma \sim 0.2-0.3$ [dex]. Their sample of galaxies covers a broad range of luminosities, so it is not surprising that their measured scatter is slightly higher than ours. Additionally, van der Wel et al. (2014) find a scatter $\sigma \sim 0.15 - 0.22$ [dex], in closer agreement to our measured value. We note that our galaxy sample changes across the different redshifts due to the minimum stellar mass criterion. Thus we are reporting the population average and not the progenitor average, which is more similar to observations.

4 THE GALAXY-HALO SIZE RELATION

At each redshift, we construct the GHSR by fitting a power-law of the form

$$R_{1/2} \text{[kpc]} = \beta \left(\frac{R_{\text{vir}}}{35 \text{kpc}}\right)^{\alpha},$$

(2)

where 35 kpc is the midpoint of the $R_{\text{vir}}$ range between $z = 0-5$ in the ‘FB-S’ haloes. The first and fiducial power-law is to the unbinned dataset. The $\alpha$ and $\beta$ values denote the power-law index and normalisation, respectively. Then we calculate the average scatter $\langle \sigma \rangle$ as the standard deviation of the residuals $\Delta \lg R_{1/2}$ from the fiducial power-law. Lastly, we fit a power-law to this scatter as a function of $\lg R_{\text{vir}}$.

Figure 4 displays these results for $z = 2$, where the top panel shows the GHSR and the bottom panel details the scatter (standard deviation of the residuals). We also fit power-laws to the binned means and medians and found similar best fitting functions. The best fitting power-law index of the GHSR at $z = 2$, $\alpha_{\text{GHSR}} = 0.939 \pm 0.025$, is close to unity. The approximately linear GHSR in FIREbox at Cosmic Noon in our target sample is qualitatively consistent with similar findings at $z = 0$ (Kravtsov 2013) and at higher redshifts (Huang et al. 2017; Zanisi et al. 2020). However, we note that our sample consists of smaller and lower-mass galaxies than in those observational studies. The scatter in the GHSR of the ‘FB-S’ galaxies exhibits a moderate increase with $R_{\text{vir}}$, as indicated by the non-vanishing power-law index $\alpha_{\lg R_{1/2}} = 0.144 \pm 0.023$. It is possible that the decreasing number of galaxies and known turnover effects at larger $R_{\text{vir}}$ cause the sub-linearities and increasing scatter.

We show the quality of our power-law fits by binning the residuals and fitting a Gaussian distribution to the resulting histogram, as Figure 5 shows for $z = 2$. The average value of the residuals $\langle \Delta \lg R_{1/2} \rangle$...
Table 2. GHSR at each and all redshifts from the FIREbox 1024^3 ‘FB-S’ galaxies

| z   | $\alpha_{\text{GHSR}}$ | $\beta_{\text{GHSR}}$ | $\langle \sigma \rangle$ | $\alpha_\sigma$ | $\beta_\sigma$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0   | 0.929 ± 0.033   | 0.051 ± 0.002   | 0.089 ± 0.041   | 0.029 ± 0.309   |
| 1   | 1.020 ± 0.032   | 0.052 ± 0.001   | 0.082 ± 0.097   | 0.024 ± 0.199   |
| 2   | 0.939 ± 0.024   | 0.055 ± 0.001   | 0.080 ± 0.144   | 0.021 ± 0.005   |
| 3   | 0.909 ± 0.027   | 0.054 ± 0.001   | 0.082 ± 0.189   | 0.019 ± 0.004   |
| 4   | 0.838 ± 0.036   | 0.051 ± 0.002   | 0.089 ± 0.262   | 0.017 ± 0.003   |
| 5   | 0.954 ± 0.047   | 0.055 ± 0.004   | 0.089 ± 0.189   | 0.021 ± 0.007   |

mean$^a$ | 0.934 ± 0.054 | 0.053 ± 0.002 | 0.084 ± 0.150 | 0.022 ± 0.004 |

all−z$^b$ | 0.894 ± 0.005 | 0.053 ± 0.001 | 0.085 ± 0.015 | 0.033 ± 0.005 |

$^a$The averages weighted by the number of objects at each redshift.

$^b$The combined sample of all redshift snapshots, treating objects from different snapshots equally.

(1) Redshift; (2) and (3) Power-law index and normalisation for the GHSR from Equation (2); (4) Average scatter in the GHSR; (5) and (6) Power-law index and normalisation for the GHSR scatter as a function of $lg R_{\text{vir}}$ bin.

is null, and the standard deviation $\sigma_{\Delta lg R_{1/2}}$ is consistent with the average scatter in the GHSR. A Shapiro-Wilk test and the chi-squared p-value for a normal distribution indicate that the GHSR residuals $\Delta lg R_{1/2}$ do not form a perfect normal distribution, but the Gaussian fit is a good approximation.

Table 2 summarises the main parameters for the GHSR and residuals for each of the analyzed redshifts. See Table A1 for the same table using the iterative galaxy definition (Hopkins et al. 2018). Given that the GHSR is consistent with being linear (column 2), the normalisation stays roughly constant (column 3), and the average scatter does not vary with redshift (column 4), we conclude that the GHSR in FIREBox is approximately constant since $z = 5$. We calculate the weighted averages of each of the redshifts considered and find that the power-law index $\alpha_{\text{GHSR}} = 0.934 \pm 0.054$ is approximately linear and the scatter $\langle \sigma \rangle = 0.084$ is constant with individual snapshots.

Figure 6 displays the probability distributions of the galaxy to halo size ratio $\langle R_{1/2}/R_{\text{vir}} \rangle$ at each redshift. We fit log-normal distributions to histograms of $lg[R_{1/2}/R_{\text{vir}}]$, but we do not show the histograms for clarity. There is little redshift evolution in either the mean or the scatter of this size ratio, and we find typical values of $R_{1/2}/R_{\text{vir}} \approx 0.05$, which is higher than Kravtsov (2013); Shibuya et al. (2015); Somerville et al. (2018) who find values closer to $\sim 0.02$. However, these studies focus on more massive galaxies than the low-mass centrals analyzed here. As Figure 1 shows, low- and high-mass FIREbox galaxies do not necessarily follow the same GHSR. Somerville et al. (2018) find a weak redshift dependence on this ratio (see their Figure 12). However, this dependence decreases with decreasing halo mass, and our sample is more than an order of magnitude less massive than their least massive bin.

We also combine all galaxy-halo pairs from each redshift and construct a GHSR from this total dataset, treating objects from different redshifts equally. This means that the definition of $R_{\text{vir}}$ is a function of redshift, because $R_{\text{vir}}$ depends on the top hat collapse factor times the background density $\Delta \rho_{\text{back}} = \Delta(z) \rho_{\text{back}}(z)$ (Bryan & Norman 1998). Figure 7 displays these results, where the top panel shows the GHSR and the bottom panel the standard deviation of the residuals. The combined sample spans over an order of magnitude in halo size, and there is a “discretely smooth” transition between the six redshift snapshots. The best fitting power-law index decreases to $\alpha = 0.894 \pm 0.005$, indicating that this combined sample is slightly sub-linear. This contrasts with the individual and averaged GHSRs, whose power-law index remains consistent with linear at $\alpha = 0.934 \pm 0.054$. At any given instant in time the GHSR is linear, but the size of a given halo typically grows slightly faster than the size of its central galaxy over much of cosmic history. This sub-linearity and the slight decrease of $\langle lg[R_{1/2}/R_{\text{vir}}] \rangle$ with cosmic time (Figure 6) are qualitatively consistent with expectations of pseudo-evolution of halo sizes (Diemer et al. 2013). However, these low-mass central galaxies remain star-forming until $z = 0$, so their galaxy sizes continue growing as well. Nonetheless, it appears that halo sizes grow slightly quicker than galaxy sizes since $z = 5$. Hence, the average $\langle lg[R_{1/2}/R_{\text{vir}}] \rangle$ in Figure 6 shift leftward with cosmic time (decreasing $z$), and the power-law fit to the all-z sample is less linear than the fits to each individual redshift in Figure 7.

As Figure 7 shows and Table 2 summarises, the GHSR at each redshift, as well as the combined GHSR, have nearly constant scatter at $\langle \sigma \rangle \approx 0.08-0.09$ [dex], suggesting that redshift is not a significant factor in explaining the scatter in the GHSR. We emphasize this point by explicitly accounting for a redshift dependence by fitting

$$lg \left[ R_{1/2}/\text{kpc} \right] = \alpha_{\text{R}_{\text{vir}}} lg \left[ R_{\text{vir}}/\text{kpc} \right] + \alpha_{1+z} lg \left[ 1 + z \right] + \beta.$$  (3)

The power-law index for redshift $\alpha_{1+z} = 0.049 \pm 0.013$ is approximately 0, and the index for virial radius increases from $\alpha_{\text{GHSR}} = 0.894 \pm 0.005$ (Equation 2; Figure 7) to $\alpha_{\text{R}_{\text{vir}}} = 0.939 \pm 0.013$. The average scatter $\langle \sigma \rangle = 0.085$ remains exactly the same as that of Figure 7. Moreover, the log-Gaussian fits to the residuals have the same standard deviation of $\sigma_{\Delta lg R_{1/2}} = 0.086$ [dex]. Hence, redshift is not a significant factor in explaining the scatter in the GHSR in these low mass galaxies.

We repeat this analysis for $R_{25}$ and $R_{80}$, i.e. the radii containing 25% and 80% of the stellar mass within 0.1$R_{\text{vir}}$ respectively (Miller et al. 2019; Mowlia et al. 2019a). The scatter changes from $\langle \sigma \rangle \approx 0.08-0.09$ [dex] with $R_{1/2}$ to $\approx 0.05$ and = 0.11 [dex] with $R_{25}$ and $R_{80}$ respectively. We also repeat this analysis for $R_{1/2}$ using all stars within 0.2$R_{\text{vir}}$, and we find that the average scatter of the GHSR increases to $\langle \sigma \rangle = 0.12-0.13$ [dex]. When including redshift in the two-parameter fit, there is a larger redshift power-law index $\alpha_{1+z}$ to $R_{1/2}$ from equation (3): $\alpha_{\text{R}_{\text{vir}}} = 0.885 \pm 0.01$, $\alpha_{1+z} = 0.128 \pm 0.018$. However the overall scatter remains constant. Lastly, we repeat this analysis for the iterative definition of galaxy size $R_{1/2}$ described in Hopkins et al. (2018, see Appendix A for more details). Again, the scatter increases to $\langle \sigma \rangle = 0.146$ [dex], and there is a larger redshift power-law index $\alpha_{1+z}$ in the two-parameter fit: $\alpha_{\text{R}_{\text{vir}}} = 0.925 \pm 0.022$, $\alpha_{1+z} = 0.175 \pm 0.023$. The scatter still does not decrease by including redshift in the two-parameter fit. For each definition of the galaxy size, the GHSRs are approximately linear, and
Figure 4. The GHSR relation for the galaxies at $z = 2$ in FIREbox. Top Panel: The open black squares represent the means ($\langle \log R_{\text{vir}}, \langle \Delta \log R_{1/2} \rangle \rangle$) and standard errors within each $\log R_{\text{vir}}$ bin of width 0.050 [dex]. The gray region represents $\pm 1$σ scatter above and below the best fitting power-law (black dashed line). The least squares best fitting equation to the data is included in the upper left, and the Pearson correlation coefficient $R_{\text{data}}$ and average scatter ($\sigma$) are shown in the bottom center. The slope of the best fitting power-law is approximately linear ($\alpha_{\text{GHSR}} = 0.939 \pm 0.025$), agreeing with Kravtsov (2013); Huang et al. (2017); Somerville et al. (2018). Bottom Panel: The standard deviation of the residuals for each $\log R_{\text{vir}}$ bin from the GHSR in the upper panel. The least squares best fitting equation is included in the upper left, and its Pearson correlation coefficient is shown in the lower right, suggesting that the scatter slightly increases with $R_{\text{vir}}$. 

Figure 5. The Gaussian fit to the GHSR residuals $\Delta \log R_{1/2}$ at $z = 2$. The best fitting parameters are in the upper right corner, and the binwidth is 0.050 in $\log$ space. The Q-Q plot in the lower left plot indicates that a Gaussian well approximates the $\Delta \log R_{\text{vir}}$ distribution.

5 WHAT SETS THE SCATTER IN THE GHSR?

The scatter in the GHSR is approximately constant since $z = 5$ in the low-mass central galaxies in FIREbox (‘FB-S’). Because including redshift as another parameter to the GHSR does not decrease this scatter, we now investigate halo (§ 5.1), galaxy (§ 5.2), and environment (§ 5.3) properties that could determine this scatter. Specifically, we correlate the GHSR residuals with these physical properties. If there is a strong correlation and the average scatter decreases, then this property is important in setting the GHSR scatter.

Figure 8 exemplifies six of these scatter plots for the ‘FB-S’ galaxies from all redshifts. The vertical axis shows the residuals – i.e., the difference in log space of the true galaxy size $R_{1/2}$ and that predicted from the GHSR and the halo’s $R_{\text{vir}}$ – as a function of various properties. We employ a linear regression between the log residuals and functions of the properties. For physical properties – e.g., halo spin $\lambda$, stellar mass $M_*$, and local density $\rho$ – we fit the residuals $\Delta \log R_{1/2}$ with the $\log(x)$ of the physical property $x$; for fractional properties – e.g., shape parameters $c/a$ and mass fractions $M_*/M_{\text{bar}}$ – we use the logit ($\log(x)/(1-x)$) of the property. These functional choices ($\log(x)$ for physical properties $x \in (0, \infty)$; logit ($x$) for fractional properties $x \in (0, 1]$) map their respective properties to all real numbers ($-\infty, \infty$). For each of these residual-parameter figures, we study the statistical significance $\sigma_{m,0}$ of the slope $m$ from 0 and the percent reduction in scatter $\% \Delta \sigma$. We also calculate the vertical offset $b$ of the fit and the Pearson correlation coefficient $R$. Table 3 details these results for the ‘FB-S’ galaxies for the halo (top), galaxy (middle), and environment (bottom) properties respectively. We also repeat this analysis at each redshift and find that all results are consistent with the combined ‘all-z’ sample.
Table 3. Correlations of the ‘FB-S’ GHSR’s scatter with halo (top), galaxy (middle) and environment (bottom) properties at all considered redshifts.

| Parameter | $m$ | $\sigma_{m,0}$ | $b$ | $R$ | $\langle \sigma \rangle$ | $\% \Delta \sigma$ |
|-----------|-----|----------------|-----|-----|--------------------------|-----------------|
| $\logit [b/a \ (\text{halo}, R_{\text{vir}})]$ | 0.013 ± 0.003 | 4.14 | −0.012 ± 0.003 | 0.055 | 0.085 ± 0.001 | 0.1 |
| $\logit [c/a \ (\text{halo}, R_{\text{vir}})]$ | 0.022 ± 0.005 | 4.43 | −0.012 ± 0.003 | 0.058 | 0.085 ± 0.001 | 0.2 |
| $\logit [c/b \ (\text{halo}, R_{\text{vir}})]$ | 0.004 ± 0.004 | 0.99 | −0.003 ± 0.004 | 0.013 | 0.085 ± 0.001 | 0.0 |
| $\logit [E \equiv 1 - (b/a)^2 \ (\text{halo}, R_{\text{vir}})]$ | −0.018 ± 0.004 | 4.37 | −0.001 ± 0.001 | −0.058 | 0.085 ± 0.001 | 0.2 |
| $\logit [F \equiv 1 - (c/b)^2 \ (\text{halo}, R_{\text{vir}})]$ | −0.005 ± 0.005 | 0.97 | −0.000 ± 0.001 | −0.013 | 0.085 ± 0.001 | 0.0 |
| $\logit [T \equiv \left( 1 - (b/a)^2 \right) / \left( 1 - (c/a)^2 \right) \ (\text{halo}, R_{\text{vir}})]$ | −0.007 ± 0.002 | 2.78 | 0.001 ± 0.001 | −0.037 | 0.085 ± 0.001 | 0.1 |

(1) Parameter used as the horizontal axis – $\logit x \equiv \log (x/(1-x))$ for $x \in (0, 1)$; (2) Slope of the fit; (3) $\sigma_r$ from the slope being 0 using the statistical error of $m$; (4) Vertical offset in the fit; (5) Pearson correlation coefficient; (6) Scatter in regression in the residuals versus parameter; (7) Percentage difference in scatter between the GHSR and the residual-parameter relation. From top to bottom the sections are the halo, galaxy, and environment properties. § 5 describes what each parameter is and how it is calculated. Parameters that explain at least five percent of the scatter (column (7)) − %$\Delta \sigma > 5$ – are in bold.

5.1 Halo Properties

The top set of properties correlates the GHSR residuals with the halo properties, employing the AHF values of the properties at the virial radius $R_{\text{vir}}$ using all – dark matter and baryonic – interior particles.

The first six halo properties describe the shape using the principal axes of the moment of inertia tensor, such that $a > b > c$. Namely, $b/a$, $c/a$, and $c/b$ are the ratios of these axes, and the elongation $E$, flattening $F$, and triaxiality $T$ are derived from these ratios. In general as a halo becomes less spherical/more triaxial, the axis ratios decrease while the elongation, flattening, and triaxiality increase. Each of these six values is between 0 and 1, so we correlate the logit of each parameter with the GHSR residuals. While logit $b/a$, logit $c/a$, and logit $E$ have slopes $> 4 \sigma$ from null, they and the other four properties do not decrease the scatter. Consequently, we conclude that the halo shape is not significant in setting the scatter in the GHSR.

Many theoretical and empirical works suggest that the halo spin and concentration are significant in setting the galaxy sizes, especially in rotationally supported disk galaxies (Fall & Efstathiou 1980; Mo et al. 1998; Somerville et al. 2008; Diemer & Kravtsov 2015; Desmond & Wechsler 2015; Desmond et al. 2017; Somerville et al. 2018). We study the halo spin $\lambda_g$, $\lambda_\Delta$, $\lambda_{\Delta R_{\text{vir}}}$ (top left panel of Figure 8) employing the definition of Bullock et al. (2001):

$$\lambda = \frac{\sqrt{2} M_{\text{vir}} V_{\text{circ}} R_{\text{vir}}}{J},$$

and the classical definition of Peebles (1969):

$$\lambda_c (R_{\text{vir}}) = \frac{J |E|^{1/2}}{G M_{\text{vir}}^{5/2}},$$

where $J = J(< r)$ is the angular momentum, $V_{\text{circ}}$ is the circular velocity of all particles within the halo, $E$ is the total energy of the
The scatter in the GHSR in our FIREbox sample of low-mass central galaxies over the past 12.5 billion years.

In Appendix C, we also correlate the GHSR scatter with the difference between a property’s value and the average value of all similar-massed haloes, the value of the cross-matched halo in the dark matter only (DMO) simulation, and the difference between the cross-matched halo value and the average value of all similar-mass DMO haloes. There are no significant reductions in the GHSR scatter for any of the studied properties. Thus all null-correlations between the halo properties and GHSR scatter are consistent, and we conclude that the halo properties are not important in setting the GHSR scatter in the ‘FB-S’ galaxies.

5.1.1 Halo Concentration and Spin

In Figure 9, we construct the stellar-halo mass (top row) and galaxy-halo size (bottom row) relations for the ‘FB-S’ galaxies at $z = 2$ (the same results hold at the other redshifts), where the sample is divided into four equal quartiles by halo concentration (left column) and spin (right column). For the SHMR, at a fixed halo mass, haloes with a higher concentration have higher stellar masses (top left panel). The four SHMRs follow the same shape, but the high concentration curves have larger normalisations, appearing as vertical shifts. These results for the halo concentration agree with FIRE-1 results from Feldmann et al. (2019). These vertical translations in the SHMR with concentration are equivalent to varying normalisations of the stellar density profiles (Lilly & Carollo 2016). The trend reverses for the halo spin; at a fixed halo mass, haloes with smaller spin have larger stellar masses (top right panel). For the ‘FB-S’ galaxies, the concentration and the spin are significant properties in setting the SHMR.

However, there is no such distinction in the GHSR relations when separated into quartiles by either concentration or spin. The null result with concentration agrees with the semi-empirical model from Zanisi et al. (2021b) and disagrees with results from zoom-ins from Jiang et al. (2019), although both studies focused on more massive galaxies. The null correlation with halo spin agrees with Jiang et al. (2019). For the largest, most massive haloes in the ‘FB-S’ galaxies, there may be a slight dependence on the halo spin (bottom right panel). This could be due to the transition from dispersion to rotationally supported galaxies, where the formation of discs becomes increasingly significant (Fall & Efstathiou 1980; Mo et al. 1998). However, in the MW halo mass regime ($M_{\text{halo}} \sim 10^{12} M_\odot$) in FIRE-2 zoom simulations, Garrison-Kimmel et al. (2018) found only weak correlations between the galaxy size and halo spin. Similarly, Zanisi et al. (2020) found in their semi-empirical model using galaxies ($M_* \sim 10^9-12 M_\odot$) from the Sloan Digital Sky Survey that the halo spin may not be significant in the GHSR. Including the halo spin in the ‘FB-S’ GHSR does not reduce its scatter.

For the ‘FB-S’ galaxies, the SHMR depends on the halo concentration (top left panel) and spin (top right panel); the GHSR does not depend on either of these halo properties (bottom panels). Thus we suggest that for these low-mass central galaxies, the GHSR is just as or more fundamental than the SHMR, agreeing with semi-empirical results for more massive galaxies (Zanisi et al. 2021b,a).

5.2 Galaxy Properties

The middle section of Table 3 summarises the residual-parameter correlations for the galaxies from all redshifts. Here, the rows in bold represent parameters that decrease the scatter by $> 5\%$. Each such power-law index also has $> 25\sigma$ difference from 0.
Figure 8. Six example figures graphing the residuals $\Delta \log R_{1/2}$ (i.e., the scatter) in the galaxy-halo size relation (GHSR) versus two halo (top panels), galaxy (middle), and environment properties (bottom). The best fitting power-laws are shown with black dashed lines, and their respective equations, Pearson correlation coefficients and new scatter are given in each panel. These values are summarised for all properties in Table 3. The average scatter of the GHSR is $\sigma_{\Delta \log R_{1/2}} = 0.085$ [dex]. Top Left: The halo spin. While the best fitting slope is 8.5σ from 0, the scatter does not decrease. Top Right: The halo concentration does not correlate with the GHSR scatter. Middle Left: The shape of the stars correlates with GHSR scatter and decreases the overall scatter. At a fixed halo size, galaxies with stars that are less spherical (lower $c/a$) have larger sizes. This property significant in setting the GHSR scatter. Middle Right: The ratio of stellar rotational velocity to the circular velocity at the half-mass radius. At a fixed halo size, galaxies with more rotational support (higher $v_{\text{rot}}/v_{\text{circ}}$) have larger sizes. This property anti-correlated with the stellar shape $c/a$. Bottom Left: The average total density of all central neighbors within 2 cMpc does not correlate with the GHSR scatter. Bottom Right: The average GHSR residuals of all central neighbors (size conformity) within 2 cMpc. The average size conformity correlates weakly with the residuals, but the GHSR scatter does not decrease.
We define the galaxy shape parameters using only the stellar particles within $R_{1/2}$. Many of these properties have significant correlations with the scatter. The middle left panel of Figure 8 displays the figure for the most significant shape property, logit $[c/a]$. The three axis ratios all have negative correlations, while the elongation, flattening, and triaxiality have positive correlations. Hence, at a fixed halo size, galaxies that are less spherical have larger $R_{1/2}$. This can be expected based on our spherical definition of $R_{1/2}$. For example, let a triaxial galaxy have axes $a > b > c$ and total stellar mass $M_\star(<a)$. As we integrate the interior stellar mass $M_\star(<r)$ with increasing radius $r$, we reach the edge of the shortest axis first at $r = c$. If $M_\star(<r = c) < 0.5M_\star(<a)$, then we must continue to further radii to determine $R_{1/2}$. Thus in the direction of the $c$ axis, we do not accumulate any more stellar material. The same argument holds for the $b$ axis, when $M_\star(<r = b) < 0.5M_\star(<a)$. Therefore at a fixed $R_{1/2}$, we expect galaxy sizes to be negatively correlated with the axis ratios and positively correlated with the elongation, flattening, and triaxiality.

Additionally the stellar-to-baryonic mass ratio logit $[M_\star/M_\text{bar}]$ or similarly the gas-to-baryonic mass ratio logit $[M_\text{gas}/M_\text{bar}] = \logit[1 - M_\star/M_\text{bar}]$ and the stellar-to-total mass ratio logit $[M_\star/M_\text{tot}]$ have significant power-law indices and slightly decrease the scatter. These properties are related to the star formation rates and histories of the galaxies, and these properties warrant future study.

We calculate the rotational velocity $v_\text{rot}$ of the stars,

$$v_\text{rot} = \frac{L_\star(<R_{1/2})}{M_\star(<R_{1/2})R_{1/2}},$$

and the circular velocity $v_\text{circ}$ at the half-mass radius,

$$v_\text{circ} = \sqrt{\frac{G M_\text{tot}(<R_{1/2})}{R_{1/2}}},$$

where $L_\star(R_{1/2})$ is the angular momentum of all stars within $R_{1/2}$. This ratio $v_\text{rot}/v_\text{circ}$ determines the amount of rotational support for the galaxy, where $v_\text{rot}/v_\text{circ} \sim 1$ is rotationally supported and $\sim 0$ is dispersion supported. The middle right panel of Figure 8 shows this ratio. We find that at fixed halo size, more discy galaxies (higher $v_\text{rot}/v_\text{circ}$) have larger sizes. This amount of rotational support is anti-correlated with $c/a$, meaning that at a fixed $R_{1/2}$, more discy galaxies (lower $c/a$) have more rotational support (higher $v_\text{rot}/v_\text{circ}$) and larger sizes. Importantly, most of the low-mass galaxies in our...
sample are not rotationally supported, with \( v_{\text{rot}}/v_{\text{circ}} \sim 0.0 - 0.2 \). This suggests that our sample is largely dispersion-supported, rather than angular-momentum supported, which helps explain why there is no observed correlation between halo spin and GHSR scatter. However, it remains to be understood why the sizes of low mass galaxies track their halo virial radii so closely.

5.3 Environment Properties

Lastly, we examine the environmental properties by constructing a comoving sphere of radius \( R = 2 \text{ cMpc} \) and counting each main (central) galaxy within this sphere (we varied the sphere’s radius and found similar results). For each object’s catalogue of neighbours, we calculate the distance \( \lg \min (d) \) to the nearest main galaxy; the minimum tidal disruption (Hill) radius \( \lg \min (R_{\text{Hill}}) \); the mean number \( \lg (n) \) and mass \( \lg (\rho) \) (bottom left panel of Figure 8) densities and find no significant trends. Lastly we test if galaxies that form spatially near each other systematically lie above or below the GHSR. That is, does galaxy size conformity exist? We correlate the average GHSR offset \( \langle \Delta \lg R_{1/2} \rangle \) of all neighbours (bottom right panel of Figure 8), again finding no decrease in scatter for any of the environmental properties.

6 CONCLUSIONS

In this work, we investigate the link between galaxy size and halo size based on a high resolution, cosmological volume simulation suite from the FIRE collaboration. We focus on low-mass centrals with stellar masses \( M_\star \sim 10^7 - 10^8 M_\odot \). Our main results and conclusions are as follows:

- Our galaxy sizes appear consistent with observations, lying along the extrapolated star forming \( M_\star - R_{\text{eff}} \) relation at \( z = 2 \) (Figure 2, van der Wel et al. 2014; Mowla et al. 2019b; Suess et al. 2019a; Nedkova et al. 2021) and approximating log-normal distributions at \( 0 \leq z \leq 5 \) (Figure 3; Shibuya et al. 2015). At \( z = 0 \), the FIREbox galaxies follow the galaxy size-stellar mass relation for star-forming galaxies from Nedkova et al. (2021), but they are systematically larger by \( \sim 0.3 - 0.5 \) [dex] at a fixed stellar mass (Figure B1).
- The Galaxy-Halo Size Relations (GHSRs) for the low mass objects at each redshift \( 0 \leq z \leq 5 \) are consistent with being linear, agreeing with previous studies (Kravtsov 2013; Huang et al. 2017; Somerville et al. 2018; Jiang et al. 2019; Zanisi et al. 2020). The power-law index \( \alpha = 0.934 \pm 0.054 \) and scatter \( \langle \sigma \rangle = 0.084 \) [dex] from the weighted average from all redshifts are consistent with the individual redshifts, suggesting that the GHSR is constant for low mass galaxies since \( z = 5 \). In general, we find \( R_{1/2}/R_{\text{vir}} \sim 0.05 \), which is similar to expectations from spin-based models (Mo et al. 1998).
- Whilst the GHSR at each redshift is roughly linear, the power-law index \( \alpha_{\text{all-z}} = 0.894 \pm 0.005 \) of the combined sample across redshifts suggests that individual objects may trace out paths that are sub-linear (Figure 7). This result and the leftward shift of \( \lg[R_{1/2}/R_{\text{vir}}] \) distribution with cosmic time (shown in Figure 6) are qualitatively consistent with expectations of pseudo-evolution of haloes (Diemer et al. 2013). The power-law fit to the all-z sample still details a smooth transition since \( z = 5 \), and the scatter does not decrease when accounting for redshift.
- The halo properties we explore – including spin and concentration – do not reduce the scatter in the GHSR in our sample of low-mass, dispersion supported galaxies in FIREbox. Our weak dependence on spin disagrees with classical theoretical ideas of galaxy formation (Mo et al. 1998), but agrees with recent numerical works of more massive galaxies (Desmond et al. 2017; Garrison-Kimmel et al. 2018; Jiang et al. 2019). At a fixed halo size, the weak correlation of galaxy sizes with halo spin and concentration agree with FIRE-2 zoom simulations (Garrison-Kimmel et al. 2018) and semi-empirical models (Zanisi et al. 2020, 2021b) of more massive, rotationally supported galaxies.

- The galaxy shape, amount of stellar rotational support, and potentially the stellar or gas fractions correlate with GHSR residuals. This suggests that baryonic feedback processes of galaxy evolution as well as observable galactic structure/kinematics may be significant in setting the scatter in the GHSR.

The remarkably tight GHSR with nearly constant scatter and linear power-law indices since \( z = 5 \) allow for estimating halo masses from the sizes of galaxies. This technique is independent from, and as accurate as, other commonly-employed mass-based methods, namely abundance matching, and has the potential to be a better environmental indicator in low-mass galaxy surveys (Yang et al. 2007, 2009). Especially with upcoming low-mass galaxy surveys at high redshifts, with, for example, the James Webb Space Telescope, we suggest in-ferring halo masses and sizes from the galaxy sizes. Given a measurement of a galaxy’s stellar mass \( M_\star \), one can estimate the halo mass \( M_{\text{vir}} \) using the SHMR and associated scatter \( \sigma \approx 0.25 \) [dex]. That is, a halo mass \( M_{\text{vir}}([M_\star]/M_\odot) \) inferred from the stellar mass \( M_\star \) has an associated error of roughly \( \pm 0.25 \) [dex]. Now with the galaxy size \( R_{1/2} \), one can estimate the halo size \( R_{\text{vir}} \) via the GHSR using the scatter \( \sigma \approx 0.08 - 0.09 \) [dex] for the ‘FB-S’ galaxies. Then a halo size \( \lg[R_{\text{vir}}(R_{1/2})/\text{kpc}] \) inferred from the galaxy size \( R_{1/2} \) has an associated error \( \pm 0.08 - 0.09 \) [dex]. Because \( M_{\text{vir}} \) scales exactly as \( M_{\text{vir}} \propto R_{\text{vir}}^3 \); then the error on estimating \( \lg[M_{\text{vir}}/M_\odot] \) is three times that of estimating the halo size, given the galaxy size. That is, a halo mass \( M_{\text{vir}}([R_{1/2}]/[M_\star]/M_\odot) \) estimated from the galaxy size \( R_{1/2} \) has an associated error of approximately \( \pm 0.24 - 0.27 \) [dex], comparable to the error from using \( M_\star \). However, using galaxy sizes could be more beneficial than using stellar masses because the GHSR depends less on the unobservable dark matter halo properties, namely spin and concentration. Thus for dispersion supported, low-mass, central galaxies, we suggest that the galaxy-halo size relation is just as or even more fundamental than the SHMR.

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DATA AVAILABILITY AND SOFTWARE USED

The data underlying this article will be shared on reasonable request to the corresponding author.

Software used: PYTHON (Van Der Walt et al. 2011); IPYTHON (Pérez & Granger 2007); NUMPY (Van Der Walt et al. 2011; Harris et al. 2020); SCIPY (Virtanen et al. 2020); MATPLOTLIB (Hunter 2007); JUPYTER (Kluyver et al. 2016).

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We check our definition of galaxy size – 

\[ M(< R_{\text{vir}}/2) = 0.5 M_\star(< 0.1 R_{\text{vir}}), \]

using an iterative approach. Hopkins et al. (2018) calculates an initial three-dimensional half-stellar mass radius \( R_{1/2} \).
Table A1. GHSR at Each and All Redshifts for the Iterative Definition of Galaxy Size from Hopkins et al. (2018) for the FIREbox 1024$^3$ ‘FB-S’ galaxies.

| $z$  | $\alpha_{\text{GHSR}}$ | $\beta_{\text{GHSR}}$ | $\langle \sigma \rangle$ | $\sigma_{\text{GHSR}}$ | $\alpha_{\sigma}$ | $\beta_{\sigma}$ |
|------|------------------------|------------------------|--------------------------|-------------------------|-----------------|-----------------|
| 0    | $0.858 \pm 0.049$      | $0.061 \pm 0.003$    | $0.131$                  | $-0.020 \pm 0.055$     | $0.042 \pm 0.027$ |
| 1    | $1.046 \pm 0.051$      | $0.063 \pm 0.001$    | $0.134$                  | $0.134 \pm 0.051$     | $0.023 \pm 0.022$ |
| 2    | $0.917 \pm 0.043$      | $0.070 \pm 0.001$    | $0.137$                  | $0.259 \pm 0.045$     | $0.016 \pm 0.004$ |
| 3    | $0.917 \pm 0.050$      | $0.074 \pm 0.002$    | $0.151$                  | $0.368 \pm 0.023$     | $0.012 \pm 0.001$ |
| 4    | $0.826 \pm 0.068$      | $0.070 \pm 0.004$    | $0.166$                  | $0.473 \pm 0.035$     | $0.010 \pm 0.001$ |
| 5    | $1.029 \pm 0.093$      | $0.086 \pm 0.007$    | $0.174$                  | $0.461 \pm 0.041$     | $0.013 \pm 0.002$ |

\text{mean}$^a$ $0.930 \pm 0.076$ $0.070 \pm 0.018$ $0.146$ $0.26 \pm 0.20$ $0.013 \pm 0.011$

\text{all}$^b$ $0.765 \pm 0.008$ $0.067 \pm 0.001$ $0.147$ $-0.017 \pm 0.011$ $0.042 \pm 0.005$

$^a$The averages using the number of objects at each redshift as weights.
$^b$The combined sample of all redshift snapshots, treating objects from different snapshots equally.

The same as Table 2, here using the iterative definition of galaxy size (Hopkins et al. 2018). (1) The redshift; (2) and (3) the power-law index and normalisation for the GHSR from Equation (2); (4) the average scatter in the GHSR; (5) and (6) the power-law index and normalisation for the GHSR scatter as a function of $\lg R_{\text{vir}}$ bin.
within a large cutoff at 0.15 \( R_{\text{vir}} \). Then they define an intermediate total radius \( R_{\text{tot}} = 3R_{1/2,0} \) and recalculate the half-stellar mass radius: \( M_* (< R_{1/2,1} ) = 0.5M_* (< R_{\text{tot}} ) = 0.5M_* (< 3R_{1/2,0} ) \). They repeat this process until convergence

\[
\frac{R_{1/2,n} - R_{1/2,n+1}}{R_{1/2,n+1}} < \epsilon
\]

for some tolerance \( \epsilon \), where \( n \) is the number of iterations. We choose \( \epsilon = 10^{-3} \), which typically takes \( n \lesssim 10 \) iterations. We require convergence within \( n_{\text{max}} = 100 \) iterations, within which every galaxy size estimate converges.

Figure A1 details the similarities between these definitions, where the left panels compare the galaxy sizes \( R_{1/2} \) directly and the right panels the stellar masses \( M_* \) at \( z = 0 \) (top) and \( z = 2 \) (bottom). In general, there is good agreement between the definitions, except at the smallest and largest galaxy sizes. Hopkins et al. (2018) warns at lower redshifts our definition fails because satellites can exist within 0.1\( R_{\text{vir}} \), but this typically affects more massive galaxies. We also check our results using all stellar material within 0.2\( R_{\text{vir}} \) and find qualitatively similar results. The top panels at \( z = 0 \) show good agreement between the two galaxy size definitions.

We repeat Table 2 and Figure 7 using the iterative galaxy definition in Table A1 and Figure A2. The same qualitative results hold, except there is larger overall scatter \( \langle \sigma \rangle = 0.146 \) [dex] compared to that in Figure 7 \( \langle \sigma \rangle = 0.085 \) [dex]. At any given instant in time the GHSR is roughly linear, but the size of a given halo typically grows faster than the size of its central galaxy over much of cosmic history.

As a final check for our definition of \( R_{1/2} \), we plot the stellar mass radial profiles in Figure A3 for the galaxies in the 1024\(^3\) run at redshift \( z = 0 \) (left) and \( z = 2 \) (right). The median stellar mass curves flatten to \( M_* \approx M_* (< R_{\text{vir}} ) \) (horizontal dashed line) by the total galaxy size \( R \approx 0.1R_{\text{vir}} \) (vertical dashed line), affirming that this is a robust value of the total galaxy size.

### APPENDIX B: GALAXY SIZE-STELLAR MASS RELATIONS

In Figure 2, we use a correction from Suess et al. (2019a) to convert our 3D intrinsic half-mass radii \( R_{1/2} \) to 2D projected effective half-light radii \( R_{\text{eff}} \) at \( z = 2 \), assuming that the 3D \( R_{1/2} \) has on average a similar value to the 2D half-mass radius \( R_{\text{mass}} \) (van de Ven & van der Wel 2021). The completeness limit for Suess et al. (2019a) is \( M_* \geq 10^{10} \) \( M_\odot \), and it is unclear if this holds in the stellar mass regime of the ‘FB-S’ galaxies (\( M_* \sim 10^{7−9} M_\odot \)). However, the correction does not significantly affect the results at \( z = 2 \), where \( (R_{1/2}/R_{\text{eff}}) \approx 0.9 \) for both star-forming and quiescent galaxies. Without this correction, the ‘FB-S’ galaxies are still within the scatter of the star-forming relation.

In Figure B1, we compare our galaxy size-stellar mass relations to those observed at \( z = 0 \) (Nedkova et al. 2021) in the Hubble Frontier Fields (HFF) and COSMOS Assembly Near-infrared Deep Extragalactic Survey (CANDELS). In the left panel, we approximate the effective radius \( R_{\text{eff}} \) as our half-mass radius \( R_{1/2} \); in the right panel, we convert \( R_{1/2} \) to \( R_{\text{eff}} \) using the correction factor from Suess et al. (2019b), assuming \( R_{1/2} \sim R_{\text{mass}} \). Specifically, Suess et al. (2019b) find for both quiescent and star-forming galaxies of stellar mass \( M_* > 10^{10.1} M_\odot \) at redshift \( z \approx 1 \) that \( (R_{\text{mass}}/R_{\text{eff}}) \approx 0.7 \). It is unclear if this correction holds at the stellar masses of the ‘FB-S’ galaxies (\( M_* \sim 10^{7−9} M_\odot \)) at \( z = 0 \). At a fixed stellar mass, the ‘FB-S’ galaxies systematically have a half-mass radius \( R_{1/2} \) larger than the star-forming galaxies from Nedkova et al. (2021). At stellar masses \( M_* \sim 10^{9−10} M_\odot \), the ‘FB-S’ and ‘FB-L’ galaxy sizes flatten out. Then the ‘FB-L’ galaxy sizes turn over at \( M_* \sim 10^{10} M_\odot \), where \( R_{1/2} \) starts to decrease with increasing stellar mass. Without the correction from Suess et al. (2019b), left panel), the ‘FB-S’ galaxy sizes are systematically \( \sim 0.3 \) [dex] larger than observations. Then the ‘FB-L’ galaxies lie partly between the galaxy size-stellar mass relations for star-forming and quiescent galaxies. With the correction (right panel), the galaxy size offset is \( \sim 0.5 \) [dex], and the ‘FB-L’ galaxies are consistent with the size-mass relation for star-forming galaxies.

### APPENDIX C: ADDITIONAL HALO PROPERTIES AND THE GHSR SCATTER

Similarly to Table 3, Table C1 summarises the correlation fits between the GHSR scatter and additional halo properties at \( z = 2 \).

The top set of parameters correlates the GHSR residuals with the difference between the property of a given halo and the mean property of all haloes within the \( \lg R_{\text{vir}} \) bin of width 0.050 [dex]. Similarly to the direct value of the halo property, only the center-of-mass offset has a slope significantly different from 0, but no offset parameters decrease the scatter by > 5%. So far, the halo properties are always from the hydrodynamic simulation, but perhaps the baryonic properties within the halo affect...
Figure A3. The stellar mass profiles for the galaxies in FIREbox pathfinder at redshift \( z = 0 \) (left) and \( z = 2 \) (right). The thick black curves show the median values within \( \frac{R}{R_{\text{vir}}} \) bins of width 0.1 [dex]. We normalize the stellar masses \( M_\star(<R) \) by the total stellar mass within the halo \( M_\star(<R_{\text{vir}}) \), and the radius \( R \) by virial radius \( R_{\text{vir}} \). We mark where \( M_\star(<R) = M_\star(<R_{\text{vir}}) \) and \( R = 0.1R_{\text{vir}} \) with horizontal and vertical dashed lines, respectively. The median curves flatten to \( M_\star \approx M_\star(<R_{\text{vir}}) \) by 0.1\( R_{\text{vir}} \), affirming that this cutoff radius contains most of the haloes’ stellar mass.

Figure B1. Similar to Figure 2. A comparison to the galaxy size-stellar mass relation obtained from the Hubble Frontier Fields (HFF) and Cosmic Assembly Near-infrared Deep Extragalactic Survey (CANDELS) at \( z = 0 \) (Nedkova et al. 2021). In the left panel, we approximate the effective radius \( R_{\text{eff}} \) from Nedkova et al. (2021) with our half-mass radius \( R_{1/2} \); in the right panel, we convert \( R_{1/2} \) to \( R_{\text{eff}} \) using the correction factor from Suess et al. (2019b). Specifically, Suess et al. (2019b) find for both quiescent and star-forming galaxies of stellar mass \( M_\star > 10^{10.1} M_\odot \) at redshift \( z \leq 1 \) that the 2D half-mass radius \( R_{\text{max}} \sim 0.7R_{\text{eff}} \). The ‘FB-S’ galaxies follow a similar galaxy size-stellar mass relation to the star-forming galaxies from Nedkova et al. (2021), but the ‘FB-S’ galaxies are systematically larger by ~ 0.5 [dex].

Figure C1. A comparison between the halo centers in the hydrodynamical (horizontal axis) and DMO simulation (vertical axis). There is strong agreement in the halo centers (shown above) and the masses (not shown).
the dark matter properties, specifically the spin and concentration. We run FIREbox with dark matter only (DMO) and with all particles (ALL-hydro), and we create a mapping between haloes in the hydro and DMO simulations.

First, we create two AHF catalogues from the hydro simulation: one using only dark matter particles (DMO-hydro), one using all particles (ALL). We correlate haloes between these catalogues using the spatial coordinates of the halo centers. Specifically, we create a sphere of radius $R_{\text{vir}}$ centered on the ALL-halo center’s coordinates in the DMO-hydro catalogue and tabulate every DMO-hydro halo within this sphere. Then we choose the halo that is most similar in mass to the ALL-halo as the best match. This completes the mapping between the two AHF catalogues generated from the hydro simulation.

Next, we map the AHF catalogue from the DMO simulation with that of the DMO-hydro using the dark matter particle IDs. We use AHF\^\text{mergetree} to link DMO-hydro haloes to those of the DMO. The likelihood of connection is

$$\max_j M_{ij} = \frac{N_{ij}^2}{N_i N_j},\quad (C1)$$

where $M_{ij}$ is the merit function, $N_{ij}$ is the number of shared particles between a DMO and its corresponding DMO-hydro halo, while $N_i$ and $N_j$ are the total number of particles in the DMO and DMO-hydro haloes respectively. This completes the mapping between the DMO-hydro and DMO halo catalogues.

Lastly, we combine the two maps to correlate the ALL-hydro and DMO haloes. We require that the DMO halo is a main halo, since the ALL-hydro halo is by a main halo by definition. Then we require the DMO virial mass be within a factor of 2 of that of the ALL-hydro halo. There are no explicit requirements on the halo centers between these catalogues, and we use this as a final check of accurate mapping. Figure C1 details the positions, and we find that the halo centers are in great agreement. We successfully map 1361/1373 haloes at $z = 2$, where each halo is central, and has similar size, mass and coordinates. The same results hold at the other redshifts.

Thus, we correlate the GHSR residuals with the pristine dark matter properties of the matched haloes in the DMO simulation (middle set of Table C1). Nevertheless, the DMO halo properties do not decrease the GHSR scatter, and the COM offset has a weaker significance. We expect that the lack of satellite galaxies causes the typical COM values to be smaller, causing the weaker correlation.

The bottom set combines the the techniques of the second and third sets of correlations by correlating the GHSR residuals with the difference between the DMO property of a given halo and the mean property of all DMO haloes in the same $R_{\text{vir}}$ bin. Agreeing with our previous results, we find that no studied halo property significantly explains the scatter in the GHSR in the ‘FB-S’ galaxies since $z = 5$.
Table C1. Correlations of the ‘FB-S’ GHSR’s Scatter with Additional Halo Properties at all redshifts.

| Parameter | $m$ | $\sigma_{m,0}$ | $b$ | $R$ | $\langle \sigma \rangle$ | $\% \Delta \sigma$ |
|-----------|-----|----------------|-----|-----|-----------------|-----------------|
| $\Delta \logit \left[ b/a \ (\text{halo, } R_{\text{vir}}) \right]$ | $0.016 \pm 0.003$ | $4.76$ | $-0.001 \pm 0.001$ | $0.055$ | $0.085 \pm 0.001$ | $0.2$ |
| $\Delta \logit \left[ c/a \ (\text{halo, } R_{\text{vir}}) \right]$ | $0.027 \pm 0.005$ | $5.17$ | $-0.001 \pm 0.001$ | $0.058$ | $0.085 \pm 0.001$ | $0.2$ |
| $\Delta \logit \left[ c/b \ (\text{halo, } R_{\text{vir}}) \right]$ | $0.004 \pm 0.004$ | $1.09$ | $-0.000 \pm 0.001$ | $0.013$ | $0.085 \pm 0.001$ | $0.0$ |
| $\Delta \logit \left[ E \equiv \sqrt{1 - (b/a)^2} \ (\text{halo, } R_{\text{vir}}) \right]$ | $-0.021 \pm 0.004$ | $4.99$ | $-0.001 \pm 0.001$ | $-0.058$ | $0.085 \pm 0.001$ | $0.2$ |
| $\Delta \logit \left[ F \equiv \sqrt{1 - (c/b)^2} \ (\text{halo, } R_{\text{vir}}) \right]$ | $-0.005 \pm 0.005$ | $1.08$ | $-0.000 \pm 0.001$ | $-0.013$ | $0.085 \pm 0.001$ | $0.0$ |
| $\Delta \logit \left[ T \equiv \left( 1 - (b/a)^2 \right) \left/ \left( 1 - (c/a)^2 \right) \ (\text{halo, } R_{\text{vir}}) \right. \right]$ | $-0.007 \pm 0.002$ | $3.08$ | $-0.000 \pm 0.001$ | $-0.037$ | $0.085 \pm 0.001$ | $0.1$ |
| $\Delta \logit \left[ \log t \right]$ | $0.030 \pm 0.004$ | $6.93$ | $0.002 \pm 0.001$ | $0.103$ | $0.084 \pm 0.001$ | $0.4$ |
| $\Delta \logit \left[ \log \chi \right]$ | $0.028 \pm 0.005$ | $6.11$ | $0.002 \pm 0.001$ | $0.086$ | $0.084 \pm 0.001$ | $0.3$ |
| $\Delta \logit \left[ \log c_{\text{NFW}} \right]$ | $0.000 \pm 0.008$ | $0.06$ | $0.000 \pm 0.001$ | $-0.013$ | $0.085 \pm 0.001$ | $0.0$ |
| $\Delta \logit \left[ \sigma_r / (\text{km s}^{-1}) \right]$ | $0.010 \pm 0.011$ | $0.99$ | $0.000 \pm 0.001$ | $0.030$ | $0.085 \pm 0.001$ | $0.0$ |
| $\Delta \logit \left[ \Delta \text{COM/kpc} \right]$ | $0.063 \pm 0.004$ | $18.02$ | $0.007 \pm 0.001$ | $0.247$ | $0.082 \pm 0.001$ | $2.7$ |

The same as Table 3, except only for the alternative halo properties. (1) The parameter used as the horizontal axis; (2) the slope of the fit; (3) the $\sigma$ from the slope being 0 using the statistical error of $m$; (4) the vertical offset in the fit; (5) the Pearson correlation coefficient; (6) the scatter in regression in the residuals versus parameter; (7) the percentage difference in scatter between the GHSR and the residual-parameter relation. From top to bottom the sections are the difference between the value and the average value of similar-mass haloes, the value of the cross-matched halo in the dark matter only (DMO) simulation, and the difference between the cross-matched halo value and the average value of all similar-mass DMO haloes. § 5.1 describes what each parameter is.