Fractal analysis of the influence of the distribution of road networks on the traffic

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Abstract. Urban population growth, together with the industrial advances, leads to consider time as one of the most precious goods and therefore, an indicator of quality of life. Larger cities usually have mobility challenges due to inefficient or insufficient transport means or inadequate design of the road layout. Therefore, people may spend large portions of their daytime moving around places. It is precisely in this type of cities, where it is more complicated to find solutions that really solve the problem because it would be harder to change the already existing infrastructure. Two roads compositions, from the cities of Barcelona, Spain, and La Plata, Argentina, were selected to incorporate a fractal analysis in order to relate it with homogeneity. This was performed using a box counting methodology which determines the fractal dimension. As a complement, fractal lagunarity was calculated to get a value of the existing voids in the magnetic resonance images. The results show that the images have similar fractal dimension regarding the differences in road structure. However, the lagunarity is not the same, La Plata, Argentina, has a lower value than Barcelona, Spain, which indicates more coherence in the road structure.

1. Introduction

With the continued growth of the world's population, it becomes evident the increase in the industrial sector, both in the construction of houses, and in the construction of roads that allow the transit to the places of study or work in the home. With this increase in population, it is also possible to observe the growing number of vehicles in the vehicle fleet that are moving through the different cities. In the absence of construction of alternate roads for traffic, the increase in vehicles generates chaos in mobility, which in turn affects the travel time, generating stress for drivers and passengers due to the delay in arriving at the desired location.

The growth of towns, notably in terms of people and buildings, have given rise to an abundance of literature. Geographers are faced with three distributions, which appear contradictory at first glance. The densities of urban populations were first measured using the clark model, an exponential decrease in distribution from urban centers to their outskirts. This model, which has often been verified, was applied
to diverse variables, for example property prices or the density of buildings. This distribution remains the subject of multiple interpretations [1]. For example, for many authors, it indicates an equilibrium state for a diffusion process [2]. Several articles by M Batty and P Frankhauser [3,4], which have inspired a few studies on very different towns in Europa and Asia, have confirmed the fractal characteristic of this decrease. According to all the authors of these studies, buildings are organized according to a power law. Fractal geometry is based on drawing as a tool, which allows us to anticipate and reveal shapes, and respond to the variety of dimensions and scales, in which today, with the help of development and technology, we move on earth. It gives us mathematical quantitative data, to describe the quality of things and to make predictions, later convertible into physical realities [5]. From all the evidence, this fractal dimension varies by town quarter, as well as indirectly varying from the center to the periphery. It does, however, remain the signature of a power law [3,4]. Given that the fractal dimension varies according to the place, we want to know if said value of the dimension could characterize the cities with respect to their distribution depending on the vehicular chaos, however, if we look at a map of the city, images that are different may have the same fractal dimension, but may differ in the homogeneity of their objects in the image. The word fractal refers to a form that can be irregular, interrupted or fragmented, and remains at any scale that occurs, being recognized due to self-similarity, that is, the whole is made up of several copies of itself [6-8].

2. Mathematical method

From one of the most important cities in the world, it was chosen a European city and one in South America to calculate the fractal dimension of these cities using the box counting method. When realizing the fractal dimensions of some of them, regarding its road distribution, it was evident that the city of Barcelona, Spain, and the city of La Plata, Argentina, have very close fractal dimensions, however, when observing the road diagram. In each of these cities, the great difference in road distribution is notorious, see Figure 1. Mandelbrot says that in nature there are many phenomena of a fractal nature, in fact, much of what we can imagine, natural landscape fractal characteristics [9].

![Figure 1. (a) La Plata, Argentina. (b) Barcelona, Spain.](image)

The fractal analysis was done through software FracLac, which holds an average error of $p = 1.02917$ and being the most effective at present to perform this type of analysis [10], thus, the fractal dimension is a fractional dimension determined by a rational number, widely implemented for the measurement of terrestrial lengths, therefore, the fractal dimension allows the description of the rules that govern an event, given that the vast majority are composed of smaller subcomponents articulated
with each other, being determined by self-similarity. To obtain the fractal dimension, different procedures have been implemented, however, the Box-counting method, counting by boxes or the entropy of Kolmogorov, is the most used technique to calculate the fractal dimension of certain objects that are represented on a plane, due to its ease of mathematical calculation and empirical estimation [11].

The Box-counting method is based on the slope of the linear fit between the points and the size of the grid with the relation where there is the finite number of subfigures that meets congruent with the numerical value, where there is a factor of contraction. Thus, in order to have a figure or set, it is covered with regular grid (all squares are equal) of a given length, which are called boxes. With this length, a specific number of boxes are needed to cover the set, then the length of the boxes is reduced and again the counting of boxes necessary to cover is repeated. Repeating this process as the length decreases, it will be necessary much more boxes to cover the set, this process of subdividing and then counting is added to generate a table with these associated data, calculating the limit when the length of the boxes that tends to zero of the quotient between the natural logarithm of the number of boxes on the natural logarithm of the reciprocal of length [12]. In order to complement the fractal analysis, the fractal lagunarity was calculated, determining the value of the existing voids in the magnetic resonance images, for this, the average of pixels per box was calculated. Subsequently, the standard deviation of pixels per box was established (σ) and with this the coefficient of variation squared per box was calculated then the calculation of the average of all possible λ and an analysis is carried out decreasing in ε, redoing the initial procedure. Finally, the average of the averages obtained in all possible λ, thus estimating the lagunarity.

3. Results and discussion

Some studies show, from cartographies, that as the urban morphology of a city changes, its fractal dimension increases. Applied to a city, the dimension will tend to be close to 2 when the urban space is homogeneously full of buildings, or close to 1, when the constructions fill this space in a dispersed and non-compact manner [13]. The range of fractal dimension values varying between 1453 and 1624 behaves the same as in many other studies around the world [14]. The main results showed that the fractal dimension of the two images were very close, however the distribution of the roads, both in Barcelona, Spain, and La Plata, Argentina, do not have much similarity. From the above, using the Fraclac software, the lagunarity was calculated in each image to observe the degree of homogeneity and distribution of the objects in the images (see Table 1).

| City      | Barcelona | La Plata |
|-----------|-----------|----------|
| Df-BxC    | 1.202     | 1.217    |
| L         | 1.627     | 1.301    |

Lagunarity is a measure of an image's inhomogeneity. It evaluates the significance of holes in a binary object, which is represented in black on a white background. The larger number of holes in terms of quantity, size and irregularity, the higher the lagunarity. Compared with the fractal dimension, which measures average irregularity, lagunarity equates to a variation coefficient. For all forms, high lagunarity is synonymous with low coherence [1].

4. Conclusions

The Figure 1(a) and Figure 1(b) above were used to represent a part of the cities of Barcelona, Spain, and La Plata, Argentina. Since the Figure 1 has fractal dimension very close to each other, it was necessary to use the concept of lagunarity. The figure of Barcelona, Spain, has a Lagunarity of 1.627 while that of La Plata, Argentina, has a lagunarity of 1.301, which indicates that the city of Barcelona, Spain, shows less coherence, that is to say that the city of La Plata, Argentina, shows a more homogeneous structure than Barcelona, Spain, in terms of Lagunarity.
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