Slow down of the mean multiplicity growth at the LHC

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Abstract

We discuss the possibility of changing the energy dependence of the mean multiplicity, i.e. slow down of its growth at the LHC energies due to a gradual transition to the reflecting scattering mode.
The possibility that the elastic scattering amplitude can exceed the limitation assumed by the black disk model at very high energies was discussed a long time ago [1]. Such an energy dependence of the amplitude is a manifestation of a gradual transition to the reflective scattering mode [2]. The appearance of this mode follows, in its turn, from the fact that opening of the new inelastic channels with an energy increase, would not lead to saturation of the total probability of the inelastic collisions at small transverse distance, but instead, it would result in the self-damping of the inelastic channels [2, 3]. The natural question is how the LHC data can be interpreted in that sense, namely, are there direct or indirect indications on the presence of this reflective scattering mode at the LHC energies. To answer this question the experimental signatures of the reflective scattering mode should be considered first.

The straightforward reconstruction of the impact-parameter dependent elastic amplitude is preferable for that purpose. It will allow one to conclude on possible crossing of the black disk model limit for the elastic scattering amplitude. It requires careful analysis of the available experimental data based on the Fourier-Bessel transformation and an extra assumption on the real part of the elastic scattering amplitude.

Another way is to analyse the experimental data on elastic scattering at large transferred momenta (deep-elastic scattering) to deduce the possible consequences for the asymptotics of the elastic amplitude. Since the deep-elastic scattering probes the region of small impact parameters, it was proposed [4] to use these data for discrimination of the asymptotic modes in the hadron scattering. In this region the reflective and absorptive scattering modes have the most significant differences at high energies. These two modes are associated with the different impact parameter profiles of the inelastic overlap function. As a result, in the reflective scattering mode associated with the unitarity saturation, the elastic scattering amplitude will asymptotically decouple from the particle production [4].

At finite energies it should be observed that the deep-elastic scattering has decreasing correlations with particle production as the collision energy increases. On the other hand the saturation of the black disk limit implies strong correlation of deep-elastic scattering with the particle production processes. Respective asymptotic differential cross-section $d\sigma/dt$ is expected to be four times lower than it is in the case of the reflective scattering mechanism domination.

In this note we consider the implications of the above mentioned decoupling for the global observable related to the many-particle production dynamics, namely, the mean multiplicity of the secondary particles $\langle n \rangle(s)$ which is a most general and transparent quantity related to the particle pro-
duction processes.

Let us consider the inelastic overlap function

\[ h_{\text{inel}}(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{\text{inel}}}{db^2} \]

which enters the unitarity equation for the elastic scattering amplitude \( f(s, b) \).

In the impact parameter representation this equation takes simple form

\[ \text{Im} f(s, b) = h_{\text{el}}(s, b) + h_{\text{inel}}(s, b). \]

The function \( S(s, b) = 1 + 2if(s, b) \) is the 2 \( \rightarrow \) 2 elastic scattering matrix element. For simplicity, we consider the scattering amplitude \( f(s, b) \) to be a pure imaginary function, i.e. \( f \rightarrow if \). The function \( S(s, b) \) is a real one in this case, but it can change sign and take negative values. The maximum value of \( h_{\text{inel}}(s, b) = 1/4 \) can be reached at high energies at the positive non-zero values of the impact parameter, i.e., for instance, at \( b = R(s) \). The derivatives of \( h_{\text{inel}}(s, b) \) have the form:

\[ \frac{\partial h_{\text{inel}}(s, b)}{\partial s} = S(s, b) \frac{\partial f(s, b)}{\partial s}, \quad \frac{\partial h_{\text{inel}}(s, b)}{\partial b} = S(s, b) \frac{\partial f(s, b)}{\partial b}. \]

It is evident that

\[ \frac{\partial h_{\text{inel}}(s, b)}{\partial b} = 0 \]

at \( b = R(s) \), if \( S(s, b) = 0 \) at this value of the impact parameter. Evidently, the derivative of the inelastic overlap function has the sign opposite to the sign of \( \partial f(s, b)/\partial b \) in the region where \( S(s, b) < 0 \). It is the region of the \( s \) and \( b \) variables where the function \( S(s, b) \) is negative (the phase of \( S(s, b) \) is such that \( \cos 2\delta(s, b) = -1 \)) is responsible for the transformation of the central impact–parameter profile of the function \( f(s, b) \) into a peripheral profile of the inelastic overlap function \( h_{\text{inel}}(s, b) \) (Fig.1). It can be easily seen by expressing the function \( h_{\text{inel}}(s, b) \) as a product, i.e

\[ h_{\text{inel}}(s, b) = f(s, b)(1 - f(s, b)). \]

If \( f(s, b) > 1/2 \) at high energy and small impact parameters, then the function \( h_{\text{inel}}(s, b) \) will have maximum value 1/4 at the non-zero impact parameter value. The impact parameter dependence of the function \( h_{\text{inel}}(s, b) \) would evolve then with energy from a central to a peripheral one. The quantity \( \langle n \rangle(s) \) is obtained by the integration of the corresponding impact-parameter dependent function with the weight function \( h_{\text{inel}}(s, b) \), e.g. the mean multiplicity \( \langle n \rangle(s) \) is written in the form

\[ \langle n \rangle(s) = \frac{\int_0^\infty bdb\langle n \rangle(s, b)h_{\text{inel}}(s, b)}{\int_0^\infty bdbh_{\text{inel}}(s, b)} \quad (1) \]
The mean multiplicity $\langle n \rangle(s, b)$ in the geometric approach has a central dependence on $b$ (cf. e.g. [5]). It is often considered as a folding integral

$$
\langle n \rangle(s, b) = n_0(s)D_1 \otimes D_2,
$$

where $D_i$ are the two-dimensional impact-parameter dependent matter distributions in the colliding hadrons. Therefore, when the weight function $h_{inel}(s, b)$ evolves with energy to a peripheral profile, the energy dependence of $\langle n \rangle(s)$ should start to slow down. For example, it can be expected that power-like energy dependence would gradually slow down, namely

$$
s^\beta \rightarrow s^{\beta - \Delta}
$$

starting in the energy region where the function $S(s, b)$ takes negative values. The parameters $\beta$ and $\Delta$ ($\beta > \Delta$) are determined by the parameters of the model, e.g. [6]. Proceeding from the available experimental information, one can roughly estimate the starting energy of slow down of $\langle n \rangle(s)$ in the range of $\sqrt{s} = 3 - 5$ TeV. This estimate is based on the fact that the value of $\text{Im}f(s, b = 0)$ increases from 0.36 (CERN ISR) to 0.492 $\pm$ 0.008 (Tevatron) and has a value which is very close to the black disk limitation 0.5 at $\sqrt{s} = 2$ TeV [7]. If it is so, it would testify in favor of a gradual transition to the reflective elastic scattering starting already at the LHC energies.

Unfortunately, the data for the mean multiplicity at the LHC are available for the central region of the rapidity only [8, 9, 10]. Straightforward extrapolation of this energy dependence to the whole region of rapidity at any fixed energy [11] looks oversimplified. The data in the regions not covered by the measurements could demonstrate different energy dependences. However, assuming the above extrapolation valid, one should expect the slowing mean multiplicity growth be shifted to the region of $\sqrt{s} = 10 - 15$ TeV. This conclusion is valid also in the particular model based on assumption of the
Eq. (2) for the mean multiplicity distribution \( \langle n \rangle(s, b) \). For the function \( \langle n \rangle(s) \) the following relation is valid

\[
\langle n \rangle(s, b) = n_0(s) F(s),
\]

where the function \( F(s) \) can be calculated in a similar way to the calculation of the gap survival probability in the double-pomeron exchange processes performed in [12]. The factor \( F(s) \) has a maximum at \( \sqrt{s} = 10 - 15 \) TeV and decreases beyond those energies like a negative power of energy.

Thus, one can state that the gradual transition to the reflective scattering mode (with a prominent peripheral form of the overlap function \( h_{inel}(s, b) \)) would lead to suppression of the higher multiplicity events in the region of the small transverse distances. Asymptotically, only reflective elastic scattering would survive at small values of the impact parameter. A standard assumption in many geometrical approaches is that the distribution of the mean number of the secondary particles over impact parameter is supposed to have a maximum in the region \( b \approx 0 \). Combination of these two facts is translated (due to integration over impact parameter) to slowing down energy dependence of the mean multiplicity \( \langle n \rangle(s) \) at the energies when the inelastic overlap function \( h_{inel}(s, b) \) starts to be peripheral. The experimental measurements of the mean multiplicity in the energy region of \( \sqrt{s} = 10 - 15 \) TeV would be interesting and helpful for discrimination of the different modes of hadron interaction and would provide hints for the asymptotics.

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