TYPE OF ERROR IN COMPLETING MATHEMATICAL PROBLEM BASED ON NEWMAN’S ERROR ANALYSIS (NEA) AND POLYA THEORY

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Abstract

This study discusses student’s errors in completing mathematical problems based on Newman’s Error Analysis and Polya Theory. The study uses a qualitative descriptive approach. The subject of this study are 78 students of Madrasah Aliyah Negeri (MAN) 2 Cirebon. The study uses tests and interviews as data collection techniques. Students take a test to determine their mathematical ability, and the test uses sequence and series as subjects. There are 3 phases in collecting data: data reduction, data presentation, and conclusion. Outline of Error analysis is created using five types of student mathematical error of Newman’s Error Analysis and four types of student mathematical error of Polya Theory. The study results based on Newman’s Error Analysis are errors reading by 1%, error understanding by 0%, error transforms by 3%, error processing ability by 5%, and error encoding by 7%. As the result of the study based Polya theory, errors when understanding the problem by 31%, errors when devising a plan by 11%, errors when carrying out the plan by 9%, and errors when Looking back by 33%. The result of the interview shows that the error occurs when students don’t do calculations carefully, don’t learn the formula, and cannot distinguish between sequence and series. Students also have a lack of understanding when completing the problem about compound interest.

Keywords: Student Mathematical Errors; Newman’s Error Analysis; Polya Theory.

INTRODUCTION

According to Polly et al. (2015), mathematics learning is a teaching and learning activity managed by the teachers to improve student achievement and the student's creativity in thinking on mathematics. So mathematics learning is a learning process for students and teachers to develop a logical mindset using a
learning model that grows and develops optimally (Boaler, 2015). Mathematics learning is conducted from elementary to college (Winarso, 2015).

Mathematical learning in high school has higher cognitive levels so that the subject presentation and learning assessment of mathematics are increasingly diverse. Learning assessment aims to develop mathematics learning for students (Winarso, 2018). Learning evaluation is a learning process that determines the value of achievement to achieve the learning objectives (Kitchen et al., 2017).

The mathematical learning process should attract student’s attention so that learning objectives can be achieved. It is similar to the statement of Coenders & Terlouw (2015) that one of the innovations in the mathematics learning model that is relevant to the present and the future is problem-solving. Problem-solving has two functions in learning mathematics. First, problem-solving is an essential tool for learning mathematics. Many mathematical concepts that can be effectively introduced to students through problem-solving. Second, problem-solving can equip students with knowledge and tools so students can formulate, approach, and solve problems according to what they have learned in school (Schoenfeld & Sloane, 2016). As an implication, students must be allowed to develop their abilities and problem-solving strategies (Karatas & Baki, 2017).

Problem-solving improves student reasoning in learning mathematics (Toheri et al., 2020). As for problem-solving, according to Schoenfeld & Sloane (2016), there are several types of mathematical questions in problem-solving such as (1) verbal/word problems, (2) non-routine mathematics problems, and (3) real/application problems. The test form in mathematics learning is presented using story and nonstory problems. But according to Roth (1996), the precise aim of mathematics learning is to emphasize students' reasoning about the mathematical story and support their objectives. The test of story problems has been applied in junior high school, but students consider it the hardest question to solve (Silver, 1992). Jupri & Drijvers (2016), investigate student difficulties in solving word problems in algebra. Meanwhile, according to Ching (2017), mathematics anxiety has a more pronounced impact on mathematical problems that require more processing resources than simple arithmetic problems and straightforward story problems, and children who are higher in working memory are more vulnerable to its deleterious impacts. It was proven based on the results of studies conducted by other researchers at the national and international levels.

The results of the PISA survey in 2014, revealed that the average mathematics achievement was lower than science. The average math rate is 386, whereas science reaches an average of 403. Based on the 2015 PISA rating, Indonesia is ranked lowest. Grades 1-9 are ranked 386 and grades 10-12 are ranked 411 (Wijaya et al., 2014). As for the results of a mathematics survey according to the Ministry of Education and Culture at TIMMS in 2015, Indonesia ranks 45 out of 50 with a total score of 397 (Caliskan et al., 2018). This condition...
indicates that the quality of mathematics education in Indonesia is still poor. In this regard, it is necessary to look for the error in solving math problems. One way to find out where students are at fault is by giving them a description test. When students have finished solving problems, based on the analysis conducted on students' answers, various forms of errors in their completion can be identified.

Suyitno & Suyitno (2015) states that some forms of student errors in mathematics learning include: students do not understand symbols, figures, mathematical evidence, and lack of reasoning in solving problems. Factors that cause this to happen are students' feeling of laziness or difficulty concentrating while learning mathematics. It prompts students to make mistakes in learning mathematics. As for other factors that cause students to be lazy, according to Meadows & Cashdan (2017), children become lazy to learn because of irregular learning patterns, always expecting cheaters from friends, and a lack of attention from parents when learning. According to Hirsch & Emerick (2006) that 70% of learning success is influenced by student ability, while the environment affects 30%. In this case, the school has a significant role in teaching students, especially in mathematics lessons. If students experience errors in learning, there is a big possibility that students or teachers make mistakes. There is a possibility that when students work on math problems, the condition of their classrooms is not conducive so that students are unable to concentrate on calculating and reasoning.

In this regard, it is necessary to look for student error in solving math problems. The theory used to clarify students' errors in completing mathematical problems is Newman’s Error Analysis and Polya Theory. Newman's Error Analysis (NEA) provides a framework for considering the underlying reasons for students' difficulties solving mathematical story problems and the process that helps teachers determine where misunderstandings occur. In addition to newman error analysis, one strategy to overcome the mistakes made students in solving story problems that can be by applying strategies troubleshooting compiled by Polya.

Newman’s error analysis categorizes student error into five types of errors, including a) error reading, b) error understanding, c) error transforms, d) error processing ability, and e) error encoding (White, 2010; Abdullah et al., 2015). As for the other perspectives regarding the types of student errors in solving problems raised by Polya theory, there are four types of student mistakes, including a) errors in understanding the problem, b) errors in devising a plan, c) errors in carrying out the plan, and d) errors in looking back (Khalo et al., 2015; Siregar et al., 2018; Son & Fatimah, 2019).

So, based on these mistakes, people are interested in reviewing the student's errors in completing mathematical problems. As mention in the study conducted by Larrain & Kaiser (2019), which categorized several aspects of students' mathematical errors in terms of language, prerequisite, and applied aspects. As for other studies analyzing students' mathematical errors based on Watson's theory, there are eight categories of errors including
inaccurate data, inaccurate procedures, missing data, missing conclusions, conflict level responses, indirect manipulation, problems of skill hierarchy, not working on problems (Watson & Mason, 2006; Sitzman, 2007).

Based on the problems that have been described, researchers are interested in studying the types of student errors in a study titled “Type Of Student Error In Completing Mathematical Problems Based On Newman’s Error Analysis (NEA) And Polya Theory.”

METHODS
Research Design
This study uses a qualitative-descriptive type (Lambert & Lambert, 2012). The approach used in this study is a case study (Merriam, 1988; Yazan, 2015). Case studies are used to look deeply into one or more phenomena in a real-world context and reflect the viewpoints of research subjects involved in these phenomena. The case study in this study is also a research method in which researchers investigate a program, event, activity, process, or group of individuals carefully.

Participants
The study was conducted in class XII science major at MAN 2 Cirebon. The subjects of this study were 78 students. In this study, the determination of research subjects using non-probability sampling with a purposive sampling technique. Using the Purposive Sampling technique is because not all samples have criteria that fit the phenomenon under study, namely students’ mathematical errors. The researcher chooses the Purposive Sampling technique that sets specific considerations or criteria that must be met by the research subjects used in this study. Therefore, students are tested for mathematics first, then for students who make mistakes in answering the math test. Then the student is the subject of research.

Data Collection
The data sources used are primary data sources (answer sheets for tests and interviews). The data collection techniques used in this study are tests and interviews. The experiment was conducted to determine students’ mathematical abilities on the subject of sequence and series.

| Newman Error Analysis | Error indicator |
|-----------------------|-----------------|
| Reading               | Students cannot read. |
|                      | Incorrectly determines what is known in the problem. |
|                      | Write a self-made symbol without explaining the meaning of the symbol. |
| Comprehension         | Write down what was asked but did not match the question. |
|                      | Write down a brief but unclear thing that was asked. |
| Transformation        | Inaccuracy in converting information into mathematical formulas. |
| Process Skill         | Error using arithmetic operations. |
| Encoding              | Incomplete procedures/steps. |
|                      | Inappropriately writing answers. |
|                      | The answers are not appropriate to the context. |
|                      | Inaccurate conclusions. |

There are some mistakes that students do base explicitly on Newman’s procedures (White, 2010; Abdullah et al., 2015), including:
1) Reading errors include student errors in reading the story problems to take the wrong information in the problem.

2) Comprehension error includes student's errors in understanding the concept so that students do not know what is asked in the problem.

3) Transformation errors include students experiencing errors in changing story problems into mathematical form.

4) Process abilities errors include student errors in calculations.

5) Write errors (coding errors) include student's error in concluding an answer.

Meanwhile, according to Polya theory, four steps can be done in solving problems in mathematics include (Khalo et al., 2015; Siregar et al., 2018; Son & Fatimah, 2019):

1) Understand the problem; in this stage, students are asked to understand what is known and asked in the problem, then write it on the answer sheet.

2) Devising a plan; students create strategies or examples of the problem story into a mathematical form, such as making variables and formulas.

3) Carrying out the plan; includes students displaying the ability to count, applying formulas, using appropriate strategies to answer the problems.

4) Looking back, in this stage, students looking back on the answers to the problems that have been completed so that students can provide conclusions.

Similar to this, Ariani & Kenedi (2018) suggests that Polya was a teacher of mathematicians who had a significant influence in the 20th century. The stage presented by Polya in Ariani & Kenedi (2018) consists of 4 steps, including 1) understanding of the problem, 2) problem planning, 3) Implementation of the problem plan, and 4) re-checking the answer. It is also supported by Saad & Ghani (2008), which reveals that the stage of the Polya has a clear framework so that it is widely used in the mathematical world curriculum. According to Daulay & Ruhaimah (2019), The implementation of the Polya model can improve the ability to solve mathematical problems.

From this description, it can be interpreted that Polya is a learning model that can be problem-solving with systematic solutions. So that when students do not carry out learning activities according to Polya's stages, the student makes an error. As for the mistakes that students do is 1) mistakes in understanding the problem, 2) errors in planning completion, 3) errors in implementing the completion plan, and 4) errors in the re-checking of the problem.

Meanwhile, the type of interview used is unstructured interviews. The use of unstructured interviews is that the researcher wants to explore more information on the subject of research on the factors that cause students to problem-solving mathematic. The interview topic studied was about the causes of students' mathematical errors in answering the sequence and series.

Table 2. Indicator and interview questions

| Indicator     | Question     |
|---------------|--------------|
| Instrument of Tes | Do you like math? |
Data Analysis

In this study, researchers also used instrument analysis. Item validation sheets are used to validate the suitability of indicators with questions to experts so that they can use instruments that are valid and reliable for the study.

Data analysis techniques used are the Miles and Huberman model (Lyons, 2000; Gusnardi, 2019). Miles and Huberman state that qualitative data analysis consists of three concurrent flows of activity: data reduction, data display, and conclusion drawing/verification. According to Chowdhury (2015), three stages in this model include data reduction, data presentation, and conclusion. In this study, researchers used Newman and Polya's theories to find out the types of students' errors in completing mathematical problems.

RESULTS AND DISCUSSION

After conducting the study on 78 of 12th-grade students of Mathematics and Natural Sciences at MAN 2 Cirebon, researchers analyzed student answer sheets based on the student's mathematical error type according to the perspective of NEA and Polya. Ten mathematical stories on the subject sequence and series are tested on students. The results obtained by 78 subjects were an average score of 51.62, a maximum score of 76, and a minimum score of 29.

Based on the score, it was concluded that the test results were not satisfactory, so there was a need to search for students' mathematical errors. Researchers analyzed student worksheets using students' mathematical error indicators according to the perspectives of Newman and Polya. The search results obtained according to the aspect of the NEA as figure 1.

The form of error performed by students in completing mathematical problems on the subject sequence and series can be seen in the following table 3.
Figure 1. Analysis results of the student’s mathematical error in newmans’ perspective

Table 3. Types of mathematical errors in Newmans’ perspective

| Types of mathematical errors in Newman's perspective | Proof of performance | Type of error |
|------------------------------------------------------|----------------------|---------------|
| 1. Reading Error                                     | Subject E-004         | Subject E-004 incorrectly enters a number. It should be 12, but the subject enter the number 2 |
|                                                      | Subject E-007         | Subject E-007 doesn’t know symbols typically used in series and sequence formulas. Then subject uses a self-created, but unclear, new symbol as a substitute. |
|                                                      | Subject E-014         | The subject incorrectly uses the symbol in series dan sequence formulas and incorrectly enters the number. In the case of subject E-014, it should be written \( b=2 \), but the subject wrote it \( U_2=20 \). So the value of \( b \) changes \( b=U_2-U_1=20-10=10 \). In the case of Subject E-060, it should be written \( r = 2 \), but the subject wrote it \( r = \frac{2}{6} = \frac{1}{3} \). |
|                                                      | Subject E-060         |                                                                        |
| Types of mathematical errors in Newman’s perspective | Proof of performance | Type of error |
|--------------------------------------------------|----------------------|--------------|
| Subject E-064                                     | ![Image](image1.png) | Subject E-064 incorrectly chose the symbol. The value 4 should be for \( U_2 \), but he writes \( a = 4 \). |
| Subject E-056                                     | ![Image](image2.png) | The subject incorrectly determines the ratio (r) of a geometric series. In the case of Subject E-056, the result of \( r \) should be 2. In the case of Subject E-018, the result of \( r \) should be 0.91. |
| Subject E-18                                      | ![Image](image3.png) | |
| Subject E-049                                     | ![Image](image4.png) | Subjects E-049 incorrectly see the result of calculations and enter it into the formula. The value of \( n \) should be 4, but the subject inputs 9 into the formula. |
| Types of mathematical errors in Newman’s perspective | Proof of performance | Type of error |
|---------------------------------------------------|---------------------|---------------|
| **Subject E-054**                                 | ![Subject E-054](image) | Subject E-054 incorrectly wrote down the correct formula. It should be written \( r = 1 + i \), but the subject wrote it \( r = 1 - i \) |

2. Comprehension Error

| **Subject E-055**                                 | ![Subject E-055](image) | Subject E-055 did not understand the test questions. In this case, the subject does not master the concept of compound interest on the subject sequence and series, so incorrectly write the correct formula. |

3. Transformation Error

| **Subject E-008**                                 | ![Subject E-008](image) | The subject incorrectly chose the formula. On the case of Subject E-008, the formula of the arithmetic sequence should be \( S_n = \frac{n}{2} (2a + (n - 1)b) \). |

| **Subject E-015**                                 | ![Subject E-015](image) | On the case of Subject E-015, the formula of the geometric sequence should be \( S_n = \frac{a(r^n-1)}{r-1} \). |
### Types of mathematical errors in Newman’s perspective

| Type of error                                  | Proof of performance |
|------------------------------------------------|----------------------|
| 4. Processed skill Error                      | Subject E-017        |
|                                                | ![Image](image1)     |
| The subject E-017 incorrectly calculates the result in the formula. The result of n-1 should be 7, but the subject answered it by 8. |
| 5. Encoding Error                             | Subject E-035        |
|                                                | ![Image](image2)     |
| Subject E-035 made a wrong operation at the end of the answer. |

As for the results of error analysis of students in solving the mathematical problems of the ranks of lines and series according to the perspective of Polya consists of 4 criteria as follow:

Table 4. Types of mathematical errors perspective polya

| Types of mathematical error in Polya perspective | Proof of work |
|------------------------------------------------|---------------|
| 1. Understanding Problem                       | Subject E-046 |
|                                                | ![Image](image3) |
| Subject E-046 does not understand what is asked and the information contained in the problem |
| Types of mathematical error in Polya perspective | Proof of work | Error form |
|------------------------------------------------|--------------|-----------|
| Subject E-074                                  | Subject E-074 does not write the formula |
| 2. Devising a Plan                             |              |           |
| Subject E-078                                  | Subject E-078 don’t know the formula and how to solve the given problem |
| 3. Carrying out the plan                       |              |           |
| Subject E-015                                  | Subject E-015 doesn’t write down the steps of work, but the answer is correct |
| 4. Looking Back                                | Subject E-077 did not write down conclusions from the answers |

Based on the students' error analysis results to solve the perspective math problem, NEA and Polya have different results. Therefore,
researchers compare the analysis of student math errors in the perspectives of NEA and Polya. Thus, the data obtained by the students' Math error analysis:

![Figure 2. Results of the analysis of mathematical errors in perspective NEA and polya](image)

Based on figure 2 shows that students have more errors according to Polya's perspective. This condition indicates that students are accustomed to solving math problems by being systematic. In addition to analyzing the student's mathematical errors through the answer sheet, researchers interviewed several students to obtain the cause of error information. Based on the results of the interview known that students are not thorough in the calculation, students do not know the formula, students can not distinguish about the sequence or series, and lack of understanding in completing the problem about compound interest and students feel uncomfortable when studying in class.

To improve the quality of students understanding, teachers can use problem-solving problems (Buschman, 2004). As stated by Surya & Putri (2017), one of the innovations in the mathematics learning model that is following the present and the future is problem-solving. Questions using problem-solving can train students to reason and make students' thinking more systematic (Barros-Castro et al., 2014). However, what happened at the MAN 2 Cirebon school, the teacher felt that the problem description could be solved only by writing down the formula, method, and final answer. While systematic mathematical problem solving includes students writing what is known, what is asked, the formula used, the method of arithmetic operation, the final answer, and the conclusion drawn. Systematically, solving problems is useful as learning material for students who do not understand so they can repeat the subject. Students can also find out the mistakes they made while working on math problems.

There are some errors explicitly made by students based on Newman's procedures, among others: 1) reading errors, 2) comprehension errors, 3) transformation errors, 4) process skill error and 5) encoding error (White, 2010; Abdullah et al., 2015). Indicators of students' mathematical errors in Newman's perspective with reading error criteria include students writing incorrect numbers, giving unclear information, not using mathematical symbols, incorrectly determining symbols. Comprehension error criteria include mistaking what was asked and writing imprecise formulas. Transformation error criteria include the use of incorrect formulas. Processed Skill errors include errors in substituting numbers and errors in using formulas so that they are wrong in carrying out procedures or steps in progress. Encoding error criteria include errors in calculating the final answer and wrong in determining the formula, then the steps to work on and learn the definitive answer is wrong.

According to Polya's perspective, indicators of students'
mathematical errors with the Understanding Problem criteria include not writing what is known and asked in the problem (Maulyda et al., 2019). Devising a Plan criterion includes not writing the formula used; carrying out the plan consists of students not writing the steps, but the final answer is correct. The criteria for Looking Back includes students not writing conclusions from the answers obtained (Carifio, 2015).

Based on interview, the cause of math errors is because teachers often provide additional assignments in the form of short descriptions and multiple choice. For the student to solve the story based problem, they felt the teacher did not teach how to answer the question by writing down what was known, asked, and wrote a conclusion.

As for other causes revealed by students. The students are not careful in doing calculations, do not know the formula, can not distinguish the problem of stories that include sequence or series, lack of understanding of solving problems about compound interest, and lack of repetition of learning at home. This condition shows the real existence stated by Entwistle & Peterson (2004) that students' ability in school influences learning success while the environment influences it.

CONCLUSION

After conducting the study on 78 of 12th-grade students of Mathematics and Natural Sciences at MAN 2 Cirebon, the results obtained by 78 students were an average score of 51.62. Based on results of the study according to the student mathematical error type of NEA perspective, acquired error reading by 1%, error understanding by 0%, error transforms by 3%, error processing ability by 5%, and error encoding by 7%. As for the type of mathematical errors in the perspective of Polya, we obtained an error in understanding the problem by 31%, devising a plan by 11%, carrying out the procedure by 9%, and Looking back by 33%. So it can be concluded that the lack of mastery of the subject of sequence and series.

Thus, the teacher should always remind students to be careful in reading and understanding math problems. Mathematics learning requires a lot of vocabulary in mathematical symbols, so students do not feel familiar with mathematical symbols. Teachers can also familiarize students with math problems that involve daily life. So students have become accustomed to transforming life problems into mathematical models. As for students, students should solve math problems systematically, calmly, and thoroughly. So students can solve problems correctly through the teacher's guidance in learning mathematics in class.

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