DUNKL–WILLIAMS INEQUALITY FOR OPERATORS ASSOCIATED WITH $p$-ANGULAR DISTANCE

F. DADIPOUR, M. FUJII AND M. S. MOSLEHIAN

Abstract. We present several operator versions of the Dunkl–Williams inequality with respect to the $p$-angular distance for operators. More precisely, we show that if $A, B \in \mathcal{B}(\mathcal{H})$ such that $|A|$ and $|B|$ are invertible, $\frac{1}{r} + \frac{1}{s} = 1$ ($r > 1$) and $p \in \mathbb{R}$, then

$$|A|A^{p-1} - B|B|^{p-1}|^2 \leq |A|^{p-1}(r|A - B|^2 + s||A|^{1-p}|B|^{p} - |B|^2)||A|^{p-1}.$$ 

In the case that $0 < p \leq 1$, we remove the invertibility assumption and show that if $A = U|A|$ and $B = V|B|$ are the polar decompositions of $A$ and $B$, respectively, $t > 0$, then

$$||(U|A|^p - V|B|^p)|A|^{1-p}|^2 \leq (1 + t)|A - B|^2 + (1 + \frac{1}{t})||B|^p|A|^{1-p} - |B|^2.$$ 

We obtain several equivalent conditions, when the case of equalities hold.

1. Introduction

In 1964, Dunkl and Williams [3] showed that, for any two nonzero vectors $x$ and $y$ in a normed space $(\mathcal{X}, \|\|)$,

$$\left\| \frac{x}{\|x\|} - \frac{y}{\|y\|} \right\| \leq \frac{4\|x - y\|}{\|x\| + \|y\|}. \quad (1.1)$$

In the same paper, the authors proved that the constant 4 can be replaced by 2 if $\mathcal{X}$ is an inner product space. This inequality has some applications in the study of geometry of Banach spaces. Kirk and Smiley [6] showed that inequality (1.1) with 2 instead of 4 characterizes inner product spaces. Thus, the smallest number which can replace 4 in inequality (1.1) measures “how much” this space is close (or far) to be a Hilbert space, cf. [5].

Now the inequality (1.1) is regarded as an estimation of the angular distance between given vectors $x$ and $y$. It has many interesting refinements which have obtained over the years, e.g. Maligranda [7], Mercer [8], Dragomir [2], and Pečarić and Rajić [10].

2010 Mathematics Subject Classification. Primary 47A63; Secondary 26D15.

Key words and phrases. Dunkl–Williams inequality; $p$-angular distance; operator parallelogram law.
Now we pay our attention to the following improvement of Dunkl–Williams inequality due to Pečarić and Rajić:

\[
\left\| \frac{x}{\|x\|} - \frac{y}{\|y\|} \right\| \leq \frac{2\|x - y\|^2 + 2(\|x\| - \|y\|)^2)\right\|^{1/2}}{\max\{\|x\|, \|y\|\}}
\]

Also they introduced an operator version of (1.2) by estimating \(|A|A|^{-1} - B|B|^{-1}|\), where \(A\) and \(B\) are Hilbert space operators such that \(|A|\) and \(|B|\) are invertible (see Corollary 2.4 below).

In [7], Maligranda considered the \(p\)-angular distance \((p \in \mathbb{R})\), as a generalization of the concept of angular distance (when \(p = 0\)), between nonzero elements \(x\) and \(y\) in a normed space \((\mathcal{X}, \|\cdot\|)\) as \(\alpha_p[x, y] := \|\|x\|^{p-1}x - \|y\|^{p-1}y\|\); see also [1].

In this paper, we introduce an operator version of the \(p\)-angular distance for Hilbert space operators as a generalization of the Pečarić–Rajić inequality presented in [11]. Thus we will obtain the following estimation of it: If \(|A|\) and \(|B|\) are invertible, \(\frac{1}{r} + \frac{1}{s} = 1\) \((r > 1)\) and \(p \in \mathbb{R}\), then

\[|A|A|^{p-1} - B|B|^{p-1}|^2 \leq |A|^{p-1}(r|A - B|^2 + s|A|^{1-p}|B|^p - |B|^2)|A|^{p-1}.\]

On the other hand, Saito and Tominaga [12] recently generalized Pečarić and Rajić inequality by deleting the invertibility condition on \(|A|\) and \(|B|\). We also discuss their result.

Our basic tool is the generalized parallelogram law for operators:

\[|A - B|^2 + \frac{1}{t}|tA + B|^2 = (1 + t)|A|^2 + (1 + \frac{1}{t})|B|^2.\]

for any nonzero \(t \in \mathbb{R}\). We, in addition, consider several equivalent conditions when the case of equality holds in the obtained inequality. The reader is referred to [4, 9] for undefined notation and terminology related to Hilbert space operators.

2. Dunkl–Williams inequality for operators

In this section, we consider Dunkl-Williams inequality for operators as an application of generalized parallelogram law of operators (GPL):

\[|A - B|^2 + \frac{1}{t}|tA + B|^2 = (1 + t)|A|^2 + (1 + \frac{1}{t})|B|^2.\]

for any nonzero \(t \in \mathbb{R}\). This equality can be easily verified by using \(|C|^2 = C^*C\) \((C \in B(H))\).

The following lemma follows from it easily.
Lemma 2.1. Let $A, B \in \mathcal{B}(\mathcal{H})$ be operators with the polar decompositions $A = U|A|$ and $B = V|B|$. Then for each $t > 0$

$$|A - B|^2 \leq (1 + t)|A|^2 + (1 + \frac{1}{t})|B|^2.$$

The equality holds if and only if $tA + B = 0$.

We now state our main results, which are understood as an application of the above lemma.

Theorem 2.2. Let $A, B \in \mathcal{B}(\mathcal{H})$ be operators with the polar decompositions $A = U|A|$ and $B = V|B|$ and let $t > 0$ and $0 < p \leq 1$ be arbitrary. Then

$$|(U|A|^{p} - V|B|^{p})|A|^{1-p}|^2 \leq (1 + t)|A - B|^2 + (1 + \frac{1}{t})||B|^p|A|^{1-p} - |B||^2.$$

The equality holds if and only if $t(A - B) + V(|B|^p|A|^{1-p} - |B|) = 0$.

Proof. Replace $A$ and $B$ in the preceding lemma by $A - B$ and $V(|B|^p|A|^{1-p} - |B|)$ respectively. Then we have

$$|A - V|B|^p|A|^{1-p}|^2 \leq (1 + t)|A - B|^2 + (1 + \frac{1}{t})||B|^p|A|^{1-p} - |B||^2$$

$$= (1 + t)|A - B|^2 + (1 + \frac{1}{t})||B|^p|A|^{1-p} - |B||^2$$

because $V^*V$ is a projection. Hence we have the required inequality. The equality holds if and only if $t(A - B) + V(|B|^p|A|^{1-p} - |B|) = 0$. \hfill \Box

Next we have an estimation of the operator $p$-angular distance.

Theorem 2.3. Let $A, B \in \mathcal{B}(\mathcal{H})$ such that $|A|$ and $|B|$ are invertible, $\frac{1}{r} + \frac{1}{s} = 1$ ($r > 1$) and $p \in \mathbb{R}$. Then

$$|A| |A|^{p-1} - B|B|^{p-1}|^2 \leq |A|^{p-1}(r|A - B|^2 + s||B|^p|A|^{1-p} - |B||^2|A|^{p-1}.$$  

Moreover the equality holds if and only if

$$(r - 1)(A - B)|A|^{p-1} = B(|A|^{p-1} - |B|^{p-1}).$$

Proof. The proof is similar to the above, that is, put $A_1 = A - B$, $B_1 = B|B|^{p-1}|A|^{1-p} - B$ and $t = r - 1$ in Lemma 2.1. Since $r = t + 1$ and so $s = 1 + \frac{1}{t}$, we have the conclusion including the equality condition. \hfill \Box
A special case of Theorem 2.3, where \( p = 0 \) gives rise to the main result of Pečarić and Rajić [11, Theorem 2.1].

**Corollary 2.4.** Let \( A, B \in \mathbb{B}(\mathcal{H}) \) such that \( |A| \) and \( |B| \) are invertible and \( \frac{1}{r} + \frac{1}{s} = 1 \) \( (r > 1) \). Then

\[
|A|A^{-1} - B|B^{-1}|^2 \leq |A|^{-1}(r|A - B|^2 + s(|A| - |B|)^2)|A|^{-1}.
\]  

(2.1)

Further, the equality holds if and only if

\[
(r - 1)(A - B)|A|^{-1} = B(|A|^{-1} - |B|^{-1})
\]

We here give some conditions equivalent to the equality condition in Theorem 2.3.

**Proposition 2.5.** Let \( p \in \mathbb{R} \), \( \frac{1}{r} + \frac{1}{s} = 1 \) \( (r > 1) \) and \( A, B \in \mathbb{B}(\mathcal{H}) \) such that \( |A| \) and \( |B| \) are invertible for the case where \( p < 1 \). Then the following conditions are mutually equivalent:

1. \( (r - 1)(A - B)|A|^{p-1} = B(|A|^{p-1} - |B|^{p-1}) \);
2. \( (s - 1)B(|A|^{p-1} - |B|^{p-1}) = (A - B)|A|^{p-1} \);
3. \( r(A - B)|A|^{p-1} + sB(|B|^{p-1} - |A|^{p-1}) = 0 \);
4. \( A|A|^{p-1} - B|B|^{p-1} = sB(|A|^{p-1} - |B|^{p-1}) \).

**Proof.** The equivalence (1) \( \Rightarrow \) (2) \( \Rightarrow \) (3) \( \Rightarrow \) (1) is easily checked.

To complete the proof, we prove (3) \( \Leftrightarrow \) (4).

Putting \( t = r - 1 \), we have \( s = \frac{t + 1}{t} \), by which (3) and (4) are written respectively as follows:

\[
t(A - B)|A|^{p-1} + (t + 1)B(|B|^{p-1} - |A|^{p-1}) = 0
\]

and

\[
tA|A|^{p-1} - B|B|^{p-1} = (t + 1)B(|A|^{p-1} - |B|^{p-1}).
\]

It is obvious that they are equivalent.

\[\square\]

Next we give some necessary conditions for the equality condition in Theorem 2.3.

**Proposition 2.6.** Let \( A, B \in \mathbb{B}(\mathcal{H}) \) such that \( |A| \) and \( |B| \) are invertible, \( \frac{1}{r} + \frac{1}{s} = 1 \) \( (r > 1) \), \( p \in \mathbb{R} \) and

\[
(r - 1)(A - B)|A|^{p-1} = B(|A|^{p-1} - |B|^{p-1}).
\]

(2.2)

Then the following statements hold:
Proof. Put \( t = r - 1 \) and then \( s = \frac{t+1}{t} \).

(1) Since \( t(A - B) = B(1 - |B|^{p-1}|A|^{1-p}) \) by the assumption, we have

\[
 tA - (t + 1)B = -B|B|^{p-1}|A|^{1-p}.
\]

Therefore it implies that

\[
|tA - (t + 1)B|^2 = |A|^{1-p}|B|^{2p}|A|^{1-p} = C.
\]

On the other hand, (1) is expressed as

\[
 t(t + 1)|A - B|^2 = C + t|A|^2 - (t + 1)|B|^2.
\]

So it suffices to check that

\[
|tA - (t + 1)B|^2 = t(t + 1)|A - B|^2 - t|A|^2 + (t + 1)|B|^2.
\]

(2) It follows from (1) and the Löwner-Heinz inequality.

(3) Since \( t(A - B) = B - B|B|^{p-1}|A|^{1-p} \) by the assumption, we have

\[
 t|A - B| = |B - B|B|^{p-1}|A|^{1-p} = ||B| - |B|^p|A|^{1-p}|,
\]

which is equivalent to (3).

\[\square\]

Remark 2.7. Assume that

\[
 (r - 1)(A - B)|A|^{-1} = B(|A|^{-1} - |B|^{-1}).
\]

This is the same equation (2.2) in the special case when \( p = 0 \). From (2) of Proposition 2.6 we have

\[
 |B| \leq \left( \frac{1}{r}|A|^2 + \frac{1}{s}|A|^2 \right)^\frac{1}{2} = |A|
\]

and so

\[
 \frac{r}{s}|A - B| = |A| - |B|, \text{ or } |A| = |B| + \frac{r}{s}|A - B|,
\]

which has been shown by Pečarić and Rajić [11].
3. Saito-Tominaga’s generalization

Very recently, Saito-Tominaga improved Pečarić and Rajić inequality without the assumption of the invertibility of the absolute value of operators.

**Theorem 3.1.** Let $A, B \in \mathcal{B}(\mathcal{H})$ be operators with the polar decompositions $A = U|A|$ and $B = V|B|$, and let $p, q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$. Then

$$|(U - V)|A||^2 \leq p|A - B|^2 + q(|A| - |B|)^2.$$ \hspace{1cm} \text{The equality holds if and only if}

$$p(A - B) = qV(|B| - |A|) \quad \text{and} \quad V^*V = U^*U.$$ \hspace{1cm} \text{We here remark that it just corresponds to the case $p = 0$ in Theorem 2.2. In this section, we consider Theorem 3.1 based on the discussion in the preceding section. For this, we rewrite it as follows:}

**Theorem 3.2.** Let $A, B \in \mathcal{B}(\mathcal{H})$ be operators with the polar decompositions $A = U|A|$ and $B = V|B|$, and $t > 0$. Then

$$|(U - V)|A||^2 \leq (t + 1)|A - B|^2 + (1 + \frac{1}{t})(|A| - |B|)^2.$$ \hspace{1cm} \text{The equality holds if and only if}

$$t(A - B) = V(|B| - |A|) \quad \text{and} \quad V^*V = U^*U.$$ \hspace{1cm} \text{Note that Theorem 3.1 is obtained by taking $t = p - 1$ in above inequality.}

Now we prepare a lemma for the equality condition in above.

**Lemma 3.3.** Let $A, B \in \mathcal{B}(\mathcal{H})$ be operators with the polar decompositions $A = U|A|$ and $B = V|B|$ and $t > 0$. If $t(A - B) + V(|A| - |B|) = 0$ is satisfied, then

$$t|A - B|^2 \leq |A|^2 - |B|^2,$$ \hspace{1cm} \text{and so} \hspace{1cm} |A| \geq |B| \hspace{1cm} \text{and} \hspace{1cm} U^*U \geq V^*V.$$ \hspace{1cm} \text{In addition, if} \hspace{1cm} U^*U = V^*V, \hspace{1cm} \text{then} \hspace{1cm} t|A - B|^2 = |A|^2 - |B|^2.$$

**Proof.** Since $tA - (t + 1)B = -V|A|$ by the assumption, we have

$$|tA - (t + 1)B|^2 = |A|V^*V|A|.$$ \hspace{1cm} \text{Adding} \hspace{1cm} t|A|^2 - (t + 1)|B|^2 \hspace{1cm} \text{to both sides, we get}

$$t(t + 1)|A - B|^2 = |A|V^*V|A| + t|A|^2 - (t + 1)|B|^2 \leq (t + 1)(|A|^2 - |B|^2).$$
so that

\[ 0 \leq t|A - B|^2 \leq |A|^2 - |B|^2. \]

Hence it follows that \(|A| \geq |B|\) and \(U^*U \geq V^*V\). Moreover, if \(U^*U = V^*V\) is assumed, then \(V^*V|A| = |A|\) and so \(t|A - B|^2 = |A|^2 - |B|^2\). \(\square\)

**Proof. of Theorem 3.2** We replace \(A\) and \(B\) in Lemma 2.1 by \(A - B\) and \(V(|A| - |B|)\) respectively. Then we have the required inequality, and the condition for which the equality holds is that

\[ t(A - B) = V(|B| - |A|) \quad \text{and} \quad V^*V = U^*U. \]

The latter in above is equivalent to \(|A|V^*V|A| = |A|^2\), or \(V^*V|A| = |A|\), that is, \(V^*V \geq U^*U\). By the help of the preceding Lemma 3.3, \(|B| \leq |A|\) and \(V^*V \leq U^*U\), so that \(V^*V = U^*U\). \(\square\)

Finally, along with the argument due to Saito and Tominaga \[12\], we investigate the equality condition in Theorem 3.2.

**Theorem 3.4.** Let \(A, B \in \mathcal{B}(\mathcal{H})\) be operators with the polar decompositions \(A = U|A|\) and \(B = V|B|\), and \(C = W|C|\) the polar decomposition of \(C = A - B\). Assume that the equality

\[ |(U - V)|A||^2 = (t + 1)|A - B|^2 + (1 + \frac{1}{t})(|A| - |B|)^2. \]

holds for some \(t > 0\).

(1) If \(t \geq 1\), then \(A = B\).

(2) If \(0 < t < 1\), then

\[ A = B(I - \frac{2}{1-t} W^*W) \quad \text{and} \quad |A| = |B|(I + \frac{2t}{1-t} W^*W), \]

and the converse is true.

We here prepare the following two lemmas.

**Lemma 3.5.** Let \(A, B \in \mathcal{B}(\mathcal{H})\) be operators with the polar decompositions \(A = U|A|\) and \(B = V|B|\), and \(t > 0\). Suppose that \(V^*V = U^*U\). Then

\[ t(A - B) = V(|B| - |A|) \]

if and only if

\[ |A| = |B| + t|A - B| \quad \text{and} \quad A - B = -V|A - B|. \]
Proof. Since \( t(A - B) = -V(|A| - |B|) \), it follows from Lemma 3.3 that
\[
t|A - B| = ||A| - |B|| = |A| - |B|
\]
and moreover
\[
A - B = \frac{1}{t}V(|B| - |A|) = -\frac{1}{t} tV|A - B| = -V|A - B|.
\]
Conversely, since \(|A| - |B| = t|A - B|\), we have
\[
t(A - B) + V(|A| - |B|) = -tV|A - B| + tV|A - B| = 0.
\]
□

Lemma 3.6. Let \( A, B \in \mathbb{B}(\mathcal{H}) \) be operators with the polar decompositions \( A = U|A| \) and \( B = V|B| \), and \( t > 0 \). Suppose that \( V^*V = U^*U \). If \( t(A - B) = V(|B| - |A|) \), then
\[
|B||A - B| + |A - B||B| = (1 - t)|A - B|^2.
\]
Proof. Put \( C = A - B \). The preceding lemma ensures that
\[
t|C| = |B + C| - |B| \text{ and } C = -V|C|.
\]
Then it follows that
\[
|B + C| = |B| + t|C|,
\]
and that
\[
B^*C = -B^*V|C| = -(|B|V^*V)|C| = -|B||C|.
\]
Hence we have
\[
|B + C|^2 = (|B| - |C|)^2 \text{ and } |B + C|^2 = (|B| + t|C|)^2,
\]
so that
\[
(t + 1)(|B||C| + |C||B|) = (1 - t^2)|C|^2,
\]
which is equivalent to the conclusion. □

Concluding this paper, we give a proof

Proof. of Theorem 3.4 The preceding lemma leads us the fact that if positive operators \( S \) and \( T \) satisfy \( ST + TS = rS^2 \) for some \( r \in \mathbb{R} \), then (i) \( S = 0 \) if \( r < 0 \), and (ii) \( S \) and \( T \) commute if \( r \geq 0 \). (Since \( S^2T = STS - tS^3 \) is selfadjoint, \( S^2 \) commutes with \( T \) and so does \( S \).) Thus we apply it for \( S = |A - B|, T = |B| \) and \( r = 1 - t \).
(1) Since \( r = 1 - t \leq 0 \), we first suppose that \( r < 0 \). Then \( S = |A - B| = 0 \), that is, \( A = B \), as desired. Next we suppose \( r = 0 \). Then \( S = |C| \) commutes with \( T = |B| \) and so \( ST = 0 \). Hence we have \( |C|V^*V = 0 \). Moreover, since \( C = -V|C| \) by Lemma 3.5, it follows that \( |C|^2 = |C|V^*V|C| = 0 \), i.e., \( C = 0 \).

(2) We apply (ii). Namely we have
\[
|B||C| = |C||B| = \frac{1 - t}{2}|C|^2,
\]
so that
\[
B|C| = V|B||C| = \frac{1 - t}{2}V|C|^2 = \frac{t - 1}{2}C|C| = \frac{t - 1}{2}A|C| - \frac{t - 1}{2}B|C|.
\]
It implies that
\[
A|C| = \frac{2}{t - 1}(1 + \frac{t - 1}{2})B|C| = \frac{t + 1}{t - 1}B|C|,
\]
and so
\[
AW^*W = \frac{t + 1}{t - 1}BW^*W.
\]
Therefore we have
\[
A = AW^*W + A(I - W^*W) = \frac{t + 1}{t - 1}BW^*W + B(I - W^*W) = B(I + \frac{2}{t - 1}W^*W).
\]

For the second equality, it suffices to show that \( W^*W \) commutes with \( |B| \) because
\[
|I - \frac{2}{1-t}W^*W| = I + \frac{2t}{1-t}W^*W
\]
is easily seen. For this commutativity, we note that \( C = A - B = \frac{2}{t - 1}BW^*W \) by the first equality, \( C = -V|C| \) by Lemma 3.5, and \( V^*V \geq W^*W \) by \( W^*W \leq \sup\{V^*V, U^*U\} \) and \( V^*V = U^*U \). So we prove that
\[
|B|W^*W = V^*BW^*W = -\frac{1 - t}{2}V^*C = \frac{1 - t}{2}V^*V|C| = \frac{1 - t}{2}|C|.
\]

Incidentally the converse implication in (2) is as follows: We first note that the second equality assures the commutativity of \( |B| \) and \( W^*W \). Next it follows that
\[
|A| - |B| = -\frac{2t}{1-t}|B|W^*W
\]
and
\[
V|A| - B = V(|A| - |B|) = -\frac{2t}{1-t}BW^*W = -t(A - B)
\]
by the first equality. Hence we have
\[
(U - V)|A| = A - V|A| = A + t(A - B) - B = (1 + t)(A - B);
\]
\[
|(U - V)|A||^2 = (1 + t)^2|A - B|^2.
\]
On the other hand, since
\[
(|A| - |B|)^2 = \left(\frac{2t}{1-t}\right)^2 B^* BW^* W = t^2 |A - B|^2
\]
we have
\[
(1 + t)|A - B|^2 + (1 + \frac{1}{t})(|A| - |B|)^2
\]
\[
= ((1 + t) + ((1 + \frac{1}{t})t^2)|A - B|^2 = (1 + t^2)|A - B|^2.
\]

□

References

1. S.S. Dragomir, *Inequalities for the p-angular distance in normed linear spaces*, Math. Inequal. Appl. 12 (2009), no. 2, 391–401.
2. S.S. Dragomir, *Generalization of the Pečarić-Rajić inequality in normed linear spaces*, Math. Inequal. Appl. 12 (2009), no. 1, 53–65.
3. C.F. Dunkl and K.S. Williams, *A Simple Norm Inequality*, Amer. Math. Monthly 71 (1964), no. 1, 53–54.
4. T. Furuta, *Invitation to linear operators. From matrices to bounded linear operators on a Hilbert space*, Taylor & Francis, Ltd., London, 2001.
5. A. Jiménez-Melado, E. Llorens-Fuster and E.M. Mazcuñán-Navarro, *The Dunkl–Williams constant, convexity, smoothness and normal structure*, J. Math. Anal. Appl. 342 (2008), no. 1, 298–310.
6. W.A. Kirk and M.F. Smiley, *Mathematical Notes: Another characterization of inner product spaces*, Amer. Math. Monthly 71 (1964), no. 8, 890–891.
7. L. Maligranda, *Simple norm inequalities*, Amer. Math. Monthly 113 (2006), no. 3, 256–260.
8. P.P. Mercer, *The Dunkl–Williams inequality in an inner product space*, Math. Inequal. Appl. 10 (2007), no. 2, 447–450.
9. J.G. Murphy, *C∗-Algebras and Operator Theory*, Academic Press, San Diego, 1990.
10. J.E. Pečarić and R. Rajić, *The Dunkl–Williams inequality with n-elements in normed linear spaces*, Math. Inequal. Appl. 10 (2007), no. 2, 461–470.
11. J.E. Pečarić and R. Rajić, *Inequalities of the Dunkl–Williams type for absolute value operators*, J. Math. Inequal., to appear.
12. K.-S. Saito and M. Tominaga, *The Dunkl-Williams type inequality for absolute value operators*, preprint.

1, 3Department of Pure Mathematics, Center of Excellence in Analysis on Algebraic Structures (CEAAS), Ferdowsi University of Mashhad, P. O. Box 1159, Mashhad 91775, Iran.

E-mail address: dadipoor@yahoo.com
E-mail address: moslehian@ferdowsi.um.ac.ir and moslehian@ams.org
URL: http://profsite.um.ac.ir/~moslehian/
2Department of Mathematics, Osaka Kyoiku University, Kashiwara, Osaka 582-8582, Japan.

E-mail address: mfujii@cc.osaka-kyoiku.ac.jp