Reconciliation of the Rosen and Laue theories of special relativity in a linear dielectric medium

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The theory of dielectric special relativity was derived by Laue from a fundamental physical basis in Einstein’s special relativity and the relativistic velocity sum rule. The Laue theory is experimentally verified by the Fizeau water tube experiment. In contrast, the Rosen version of dielectric special relativity was derived heuristically and has no experimental validation. Consequently, the Rosen theory and its consequences are mostly ignored in the scientific literature and there is little to no discussion about the incompatibility of the two theories of relativity in a dielectric. In this article, the Laue theory is developed from boundary conditions using inertial reference frames moving at constant velocity along the interface between a simple linear dielectric medium and the vacuum. Then, the Rosen theory is derived in the context of inertial frames of reference moving at constant velocity in the interior of an arbitrarily large linear isotropic homogeneous dielectric medium. These derivations show that the Laue and Rosen theories of dielectric special relativity are both correct but have different regimes of applicability. The Rosen theory applies to physics in the interior of a simple linear dielectric and the Laue theory is used to relate these physics to a Laboratory Frame of Reference in the vacuum where measurements can be performed.

I. INTRODUCTION

In 1907, Laue [1, 2] applied the Einstein relativistic velocity sum rule to a transparent block of dielectric with macroscopic refractive index \( n \) that is located in the vacuum. When the dielectric is at rest in the Laboratory Frame of Reference the speed of light in the dielectric is \( w = c/n \). When the dielectric block is moving at velocity \( \mathbf{v} \) in the Laboratory Frame of Reference, the speed of light in the moving dielectric medium is given by the Einstein relativistic velocity sum rule as [1]

\[
w' = \sqrt{\frac{v^2}{c^2} + \frac{c^2}{n^2} + \frac{2v}{cn} \cos \theta - \frac{v^2}{n^2} \sin^2 \theta} \left( 1 + \frac{1}{\gamma} \cos \theta \right),
\]

where \( \theta \) is the angle between the direction of light propagation and the direction in which the dielectric is moving. In two simple limiting cases, we have [1]

\[
w' = \frac{c}{n} \pm \frac{v}{1 \pm \frac{c}{cn}}
\]

(1)

for light propagating in the +/- direction of the velocity of the dielectric through the vacuum and [1]

\[
w' = \sqrt{\frac{c^2}{n^2} + v^2 \left( 1 - \frac{1}{n^2} \right)} = \frac{c}{n} + \frac{v^2}{2nc} (n^2 - 1) - \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{v^4}{6 nc^3} (n^2 - 1)^2 + \ldots
\]

(2)

for transverse propagation. Dispersion is treated by using the value of \( n \) corresponding to the frequency of the light field. [1] The Lorentz factor in the dielectric medium

\[
\gamma_n = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

(3)

is the same as the vacuum Lorentz factor.

In a 1952 American Journal of Physics article, Rosen [3] considered an arbitrarily large, but finite, macroscopic Maxwellian dielectric such that an observer in the interior of the dielectric has no access to the vacuum i) microscopically, because the continuum limit precludes an interstitial vacuum, and ii) macroscopically, because the boundary of the dielectric is too far away from the interior for light to travel within the duration of an experiment. Because the vacuum is inaccessible, there is no way to determine the speed or direction of the dielectric medium with respect to the Laboratory Frame of Reference. Consequently, there is no way to apply the Einstein relativistic velocity sum rule. Then the speed of light in a Maxwellian dielectric \( w \) is independent of the motion of the source or material. The index-dependent material Lorentz factor [3–5]

\[
\gamma_m = \frac{1}{\sqrt{1 - \frac{v^2}{w^2}}} = \frac{1}{\sqrt{1 - \frac{n^2 v^2}{c^2}}}
\]

(4)

that was obtained by Rosen by replacing \( c \) in the vacuum theory with

\[
w = c/n
\]

(5)

is incommensurate with the Lorentz factor, Eq. (4), of the Laue theory.

The Laue [1, 2] theory of special relativity in a dielectric was firmly established as a fundamental physical
principle decades before the Rosen [3] article. Further, the Laue theory has a fundamental basis in Einstein special relativity and the relativistic velocity sum rule and it is experimentally verified by the Fizeau [6] water tube experiment. In contrast, the Rosen [3] dielectric special relativity theory was derived heuristically and the material Lorentz factor, Eq. (5), is incommensurate with the Lorentz factor, Eq. (4) of Einstein’s special relativity. Moreover, there is no experimental verification of Rosen’s special relativities. Consequently, the Rosen theory and its consequences are almost completely ignored in the scientific literature and there is little to no discussion about the incompatibility of the two theories of relativity in a dielectric.

Here, we argue that the Laue and Rosen treatments of special relativity in a dielectric are both correct but have different regimes of applicability. These differences in applicability are obscured by the plug-and-play manner in which the two theories were originally derived: i) There is nothing in the velocity sum rule derivation of the Laue theory to indicate any limitations on its validity. ii) Likewise, the phenomenological substitution of \( w = c/n \) for \( c \) presents no obvious restrictions to impose on the Rosen theory.

In this article, we derive the Laue and Rosen theories in the context of inertial frames of reference moving at constant velocities in two different physical configurations: i) The Laue theory is developed from boundary conditions for inertial frames of reference moving at constant velocity along the interface between a simple linear dielectric medium and the vacuum of free space. ii) The Rosen theory is derived in the context of inertial frames of reference moving at constant velocity in the interior of an arbitrarily large linear isotropic homogeneous dielectric medium. We conclude that the Rosen theory applies to the relativistic physics in the interior of an arbitrarily large simple linear dielectric and the Laue theory is used to relate these physics to a Laboratory Frame of Reference in the vacuum where other-than-optical measurements can be performed. A Maxwellian dielectric with a macroscopic refractive index \( n \) is continuous at all length scales so that any non-optical measuring device, such as a clock (no matter how small), will always displace the dielectric. (The terrestrial atmosphere is displaced by a clock (no matter how small), will always displace the scales so that any non-optical measuring device, such as a clock (no matter how small), will always displace the dielectric. (The terrestrial atmosphere is displaced by a non-optical measuring device, such as a clock (no matter how small), will always displace the dielectric. (The terrestrial atmosphere is displaced by a measuring device, but the effect can often be neglected).

II. DERIVATION OF LAUE RESULT

We consider two inertial reference frames, \( S(x, y, z) \) and \( S'(x', y', z') \), in a standard configuration [7] in which \( x \) and \( x' \) are collinear, \( y \) is parallel to \( y' \), \( z \) stays parallel to \( z' \), and \( S' \) translates at a constant speed in the direction of the negative \( x \)-axis. The origins of the two systems coincide at some initial time \( t_0 = 0 \). At each point in each coordinate system, time is measured by an idealized clock and all the clocks in each coordinate system have been synchronized by one of the usual methods.

![FIG. 1. Coordinate frame \( S \) at the dielectric/vacuum boundary.](image)

We require that the \( x, x', z, \) and \( z' \) axes lie on the surface of a semi-infinite dielectric, Figs. 1 and 2. Then the upper half-space, \( y > 0 \) and \( y' > 0 \), is modeled as a linear isotropic homogeneous dielectric with index of refraction \( n \) in the rest-frame \( S \) where the speed of light is \( c/n \). (The dielectric can also be finite, we only require a plane surface for an interface.) The lower half-space, \( y < 0 \) and \( y' < 0 \), is vacuum in which the speed of light is \( c \).

At time \( t_d = t'_d = t_v = t'_v = 0 \), a bi-directional light pulse is emitted from the common origin \( o \) along the ±\( y' \)-axes. The pulse is reflected by a mirror in the vacuum at \( y = -D_v \) and returns to the origin at time \( \Delta t_v = 2D_v/c \). The pulse is also reflected by a mirror in the dielectric at \( y = D_d \) and returns to the origin at time \( \Delta t_d = 2nD_d/c \). The locations of the mirrors are adjusted so that both reflections return to the origin at the same time such that

\[
\Delta t_v = \Delta t_d , \tag{7}
\]

by construction.

The trajectory of the light pulse in the \( S' \) frame of reference is shown in Fig. 2. The translation of the \( S' \) frame is transverse to the \( y \)-axis so the distance from the mirror at \( m'_c \) to the \( x' \)-axis is \( D_v \), the same as the distance from the mirror at \( m_v \) to the \( x \)-axis. Viewed from the
from the vacuum theory.

Regrouping terms in the Pythagorean theorem, Eq. (9), one obtains

\[(\Delta t'_d)^2 (c'_d - v_d^2) = c_d^2 \Delta t'_d^2.\]  

(11)

Substituting Eqs. (8) and (10) into the previous equation results in

\[\frac{v_d^2}{c_d^2} \Delta t_v^2 (c'_d - v_d^2) = c_d^2 \Delta t'_d.\]  

(12)

Applying Eqs. (4) and (7), we obtain

\[c'_d v_d^2 = c_d^2 \left(1 - \frac{v_d^2}{c_d^2} + \frac{n^2 v_d^2}{c^2}\right),\]  

(13)

where \(c_d = c/n\) is the speed of light in the rest frame of the dielectric. Then

\[\frac{c'_d \Delta t'_d}{c \Delta t'_v} = \frac{1}{c} \sqrt{\frac{c^2}{n^2} + v_d^2 \left(1 - \frac{1}{n^2}\right)}\]  

(14)

is the ratio of the distance traveled by light along the hypotenuse in the dielectric compared to the distance in the vacuum in Fig. 2. We multiply Eq. (14) by \(c\) and find that this analysis, Eq. (15), reproduces the Laue result, Eq. (3), where

\[w'_d = c'_d \frac{\Delta t'_d}{\Delta t'_v} = \sqrt{\frac{c^2}{n^2} + v_d^2 \left(1 - \frac{1}{n^2}\right)}\]  

(15)

is the speed of light in the dielectric that is measured by an observer in the vacuum-based Laboratory Frame of Reference. Note that Eq. (15) displays the characteristic transverse Fresnel drag formula, Eq. (3), that modifies the rest-frame speed of light in a dielectric, \(c/n\). There is, however, a new interpretation in terms of a model that matches physical quantities at the boundary between a dielectric and the vacuum instead applying the Einstein relativistic velocity sum rule to a dielectric block in inertial motion in a Laboratory Frame of Reference.

### III. DERIVATION OF ROSEN RESULT

In order to derive Rosen’s result, we now consider a different physical setting. Two inertial frames of reference, \(S(x, y, z)\) and \(S'(x', y', z')\), in a standard configuration [7] are located in the interior of an arbitrarily large linear isotropic homogeneous dielectric medium, Fig. 3. As before, the origins of the two systems coincide at time \(t_0 = 0\) and all clocks are synchronized. At time \(t_d = t'_d = 0\), a light pulse is emitted from the common origin along the positive \(y\) and \(y'\)-axes. In the \(S\) frame of reference, the pulse is reflected by a mirror in the dielectric at \(y = D_d\) and returns to the origin at time \(\Delta t_d = 2nD_d/c\). The trajectory of the light pulse in the \(S'\) frame of reference
FIG. 3. Coordinate frame $S$ in a dielectric.

is shown in Fig. 4. The translation of the $S'$ frame is transverse to the $y$-axis so the distance from the mirror at $m'_d$ to the $x'$-axis is $D_d$, the same as the distance from the mirror at $m_d$ to the $x$-axis. Viewed from the $S'$ frame, the light pulse is emitted from the point $o$ at time $t'_d = 0$, is reflected from the mirror at point $m'_d$, and is detected at the point $d'_d$ at time $t'_d = \Delta t'_d$. During that time, the point of emission/detection has moved a distance $v_d \Delta t'_d$.

By the Pythagorean theorem, we have

$$ (c'_d \Delta t'_d)^2 = (c_d \Delta t_d)^2 + (v_d \Delta t'_d)^2, \quad (16) $$

where we have again used reflection symmetry about the midpoint. We write the previous equation as

$$ \Delta t'_d = \frac{\Delta t_d}{\sqrt{c'_d^2/c_d^2 - v_d^2/c_d^2}}, \quad (17) $$

and define the Lorentz factor $\gamma_d$ by

$$ \Delta t'_d = \gamma_d \Delta t_d \quad (18) $$

such that

$$ \gamma_d = \frac{1}{\sqrt{1 - v_d^2/c_d^2}}. \quad (19) $$

At this point, there are more unknowns than equations and we can proceed no further without some additional condition. When Einstein faced the equivalent problem for free space, he postulated that light travels at a uniform speed $c$ in the vacuum, regardless of the motion of the source. Here, the isotropy of an arbitrarily large homogeneous continuous dielectric medium leads us to postulate that light travels at a uniform speed $c_d$ in the dielectric, basically the same reasoning that led to the Einstein postulate. Then, we can substitute

$$ c'_d = c_d \quad (20) $$

into Eq. (19) to obtain

$$ \gamma_d = \frac{1}{\sqrt{1 - v_d^2/c_d^2}}, \quad (21) $$

It can be argued that the Lorentz factor is always the vacuum Lorentz factor, Eq. (3), because the dielectric can always be modeled as particles and interactions in the vacuum where Einstein’s special relativity is valid. However, we are working in the continuum limit in which the material is treated as being continuous at all length scales. Specifically, an Einsteinian material is constructed by adding fundamental physical entities to the vacuum, not by deconstructing a continuous medium. Then, in the limit of continuum electrodynamics, the macroscopic Lorentz factor in an arbitrarily large simple linear dielectric is given by Eq. (21). Now, the speed of light $c_d$ will be different in different dielectrics and we are considering only simple linear dielectric materials in which the speed of light is inversely proportional to some constant $n$. For each linear isotropic homogeneous dielectric with index $n_i$, there is a different material Lorentz factor

$$ \gamma_{m_i} = \frac{1}{\sqrt{1 - n_i^2 v_d^2/c_d^2}}, \quad (22) $$

that corresponds to a different theory of relativity for each material with a speed of light $c/n_i$ as suggested by Rosen. [3] Boundary conditions are used to relate dielectric special relativities to each other and to the vacuum as
the vacuum theory can be considered to be a special case of a dielectric special relativity with \( n = 1 \). This was demonstrated above where we derived the Laue theory using boundary conditions at a dielectric/vacuum interface.

IV. RECONCILIATION OF THE LAUE AND ROSEN ANALYSES

The Einstein theory of special relativity uses transformations between different inertial reference frames moving at constant velocities in vacuum. \([7, 8]\) Laue \([1]\) showed that the Einstein relativistic velocity sum rule explained Fizeau’s \([6]\) 19th-century experiments of light dragging by a moving dielectric significantly contributing to the acceptance and development of Einstein’s theory of special relativity. \([2, 7, 9]\) According to the Laue theory, the speed of light in a moving dielectric, Eq. (1), depends on the speed and direction of movement of the dielectric material in the Laboratory Reference Frame. In contradiction with this, the Rosen theory asserts that the speed of light in a linear isotropic homogeneous dielectric is independent of the motion of the source. Noting that the light path through the dielectric that is shown in Fig. 4 is the same as the light path through the dielectric that is shown in Fig. 2, we substitute Eq. (20) into Eq. (15) and use \( c_d = c/n \) for a simple linear dielectric to obtain

\[
\frac{\Delta t_d'}{\Delta t_v'} = \sqrt{1 + \frac{v^2}{c^2} (n^2 - 1)}.
\]

Then the measurement of an increment of time is affected by the refractive index and the motion of the dielectric in the Laboratory Frame of Reference in which the measurement is made. While the fundamental speed of light in the moving dielectric is always \( c_d' = c_d = c/n \), the specifics of the measurement dictate what velocity is observed. The apparent velocity of the light in the medium, as measured in the vacuum-based Laboratory Frame of reference

\[
w' = \frac{c}{n} \frac{\Delta t_d'}{\Delta t_v'},
\]

depends on the refractive index of the dielectric and the motion of the dielectric block in the frame of reference in which the measurement is being performed.

V. CONCLUSION

The significance of this article is that it rehabilitates the Rosen theory so that it can be used in situations in which physical theory is being applied inside a dielectric as opposed to situations in which measurements are being made in a vacuum-based (terrestrial atmosphere-based) Laboratory Frame of Reference.

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