On the Crystallinity of Silicate Dust in Evolving Protoplanetary Disks due to Magnetically Driven Disk Winds

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Abstract

We present a novel mechanism for the outward transport of crystalline dust particles: the outward radial drift of pebbles. A dust-ring structure is frequently observed in protoplanetary disks. One of the plausible mechanisms for the formation of dust rings is the accumulation of pebbles around the pressure maximum, which is formed by the mass loss due to magnetically driven disk winds. Due to magnetically driven disk winds, dust particles in evolving protoplanetary disks can migrate outwardly from the crystallization front to the pressure maximum by radial drift. We found that the outward radial drift process can transport crystalline dust particles efficiently when the radial drift timescale is shorter than the advection timescale. Our model predicts that the crystallinity of silicate dust particles could be as high as 100% inside the dust-ring position.

Unified Astronomy Thesaurus concepts: Protoplanetary disks (1300); Circumstellar disks (235); Planet formation (1241); Silicate grains (1456); Planetary system formation (1257)

1. Introduction

Protoplanetary disks are the birthplaces of planetary systems. Therefore, disk evolution is of great importance in understanding how planets formed (e.g., Testi et al. 2014). As planetesimal formation via accumulation of dust particles is the first step of planet formation (e.g., Johansen et al. 2014), the spatial distribution and migration of dust particles in evolving protoplanetary disks have been studied extensively so far.

Several pieces of evidence suggest that silicate dust undergoes significant thermal processing in protoplanetary disks. In the interstellar medium, silicate dust is thought to be completely amorphous, as suggested by broad and smooth absorption features (e.g., Kemper et al. 2004). In contrast, crystalline silicate features are often found toward disks around Herbig Ae/Be stars (e.g., Hanner et al. 1995) and T Tauri stars (e.g., Honda et al. 2003, 2006). Forsterite (Mg2SiO4) is the most abundant silicate mineral in disks (e.g., Juhasz et al. 2010; Maaskant et al. 2015). Laboratory experiments suggest that the crystallization temperature of forsterite is approximately 600–1000 K (e.g., Hallenbeck et al. 2000; Yamamoto & Tachibana 2018). However, in some disks around young stars, crystalline forsterite has been observed much farther away from the “crystallization front,” where the disk temperature is equal to the crystallization temperature (e.g., Juhasz et al. 2010; de Vries et al. 2012; Sturm et al. 2013). In addition, comets and interplanetary dust particles in our solar system contain crystalline silicate (e.g., Honda et al. 2004; Ootsubo et al. 2007; Ogliore et al. 2009). These facts strongly suggest that the crystallization and outward transport processes occurred both in extrasolar protoplanetary disks and in the early solar nebula.

Several mechanisms have been suggested for driving the outward transport of crystalline dust particles in protoplanetary disks, including turbulent diffusion (e.g., Gail 2001; Ciesla 2010), large-scale circulations associated with mass and angular momentum transfer (e.g., Keller & Gail 2004; Ciesla 2007, 2009), spiral arms in gravitationally unstable massive disks (e.g., Boss 2008), photophoresis (e.g., Mousis et al. 2007), and radiation pressure (e.g., Vinković 2009; Tazaki & Nomura 2015). Dulmondeg et al. (2006) proposed that the majority of crystalline dust particles are formed in the very early phase of disk formation via the collapse of molecular cloud cores (see also Yang & Ciesla 2012). In addition, parts of crystalline dust particles might be formed in situ in the outer region of protoplanetary disks by exothermic chemical reactions of reactive molecules (Tania et al. 2010) and/or by shock waves (e.g., Harker & Desch 2002; Miura et al. 2010).

The physical process of outward transport by turbulence is described by the diffusion equation. The key parameter for turbulent mixing in steady-state disks is the Schmidt number, which is the ratio of the kinetic viscosity and diffusion coefficient (e.g., Clarke & Pringle 1988). Pavlyuchenkov & Dulmondeg (2007) reviewed how the Schmidt number affects the crystallinity of disks. They found that the radial distribution of the crystallinity in steady-state disks is given by a power-law distribution, and the exponent depends on the Schmidt number. A small value of the Schmidt number lower than 1 is required for efficient outward diffusion in standard accretion disks (e.g., Clarke & Pringle 1988; Pavlyuchenkov & Dulmondeg 2007; Hughes & Armitage 2010).

The radial drift of dust particles due to gas drag is another important process in understanding the radial transport of dust particles. In classical views of accretion disks (e.g., Lynden-Bell & Pringle 1974; Hartmann et al. 1998), the pressure gradient at the midplane is negative throughout the disk, and dust particles migrate inward due to gas drag (e.g., Adachi et al. 1976). This inward drift offsets the outward transport of dust particles (e.g., Ciesla 2010). In addition, millimeter- to centimeter-sized large dust particles spiral into the central star within 1 Myr unless a local pressure maximum prevents the dust particles from inwardly migrating (e.g., Desch et al. 2018; Fukai & Arakawa 2021). Therefore, it is difficult for standard accretion disks to outwardly transport dust particles formed at high temperatures and to maintain high crystallinity for a long
time in the outer region. We briefly review the effect of radial drift on the radial distribution of the crystallinity in Section 3.1.

Recent astronomical observations have revealed a variety of structures in protoplanetary disks (e.g., Fukagawa et al. 2013; ALMA Partnership et al. 2015; Tsukagoshi et al. 2016; van Boekel et al. 2017; Andrews et al. 2018). The observed disk structures provide us with plenty of clues to reveal how planets formed (see Andrews 2020, and references therein). In particular, a dust-ring structure is observed in a large number of disks. To date, several mechanisms have been proposed for the origin: planets (e.g., Dong et al. 2015; Kanagawa et al. 2018), dust growth (e.g., Lambrechts & Johansen 2014; Ohashi et al. 2021), condensation fronts (e.g., Okuzumi et al. 2016; Pinilla et al. 2017), photoevaporative flows (e.g., Ercolano & Pascucci 2017), disk instabilities due to dust–gas friction and self-gravity (or turbulent gas viscosity) (e.g., Takahashi & Inutsuka 2014; Tominaga et al. 2019), and magnetically driven disk winds (e.g., Takahashi & Muto 2018).

In this study, we focus on the crystallinity of silicate dust particles in ring structures formed by the magnetically driven disk winds. Suzuki & Inutsuka (2009) found that magneto-hydrodynamic turbulence in protoplanetary disks drives disk winds. Suzuki et al. (2010) revealed that the mass-loss timescale of the magnetically driven disk winds is proportional to the local Keplerian rotation period, and disk winds disperse the gas component of disks from the inner region. In other words, magnetically driven disk winds potentially create a maximum of gas pressure around 1–10 au from the central stars (e.g., Suzuki et al. 2016), and dust particles which are dynamically decoupled with gas (referred to as “pebbles”) are accumulated around the pressure maximum (e.g., Haghighipour & Boss 2003). This is the formation mechanism for dust rings in evolving disks due to disk winds that was proposed by Takahashi & Muto (2018).

In this study, we propose a novel mechanism for outward transport of crystalline dust particles. In evolving protoplanetary disks due to magnetically driven disk winds, dust particles can migrate outwardly by radial drift. We found that the outward radial drift process can transport crystalline dust particles efficiently when the radial drift overcomes the advective flow. Our model predicts that the crystallinity of silicate dust particles could be as high as 100% inside the dust-ring position, and this is totally different from the prediction for accretion disks without disk winds (e.g., Pavlyuchenkov & Dullemond 2007).

2. Models

In Section 2, we briefly introduce the equations used to compute the evolution of protoplanetary disks. We calculate the temporal evolution of the surface densities of gas and dust using vertically integrated disk models, and we also obtain the radial distribution of the crystallinity of dust particles. The basic equations for the evolution of gas and dust disks are described in Sections 2.1 and 2.2, respectively.

2.1. Evolution of Gas Disks

We set the initial distribution of the gas surface density, \( \Sigma_{\text{gas,0}} \), as a self-similar profile, which is described as follows

\[
\Sigma_{\text{gas,0}} = \frac{(2 - \gamma) M_{\text{disk}}}{2 \pi r_0^2} \left( \frac{r}{r_0} \right)^{-\gamma} \exp \left[ -\left( \frac{r}{r_0} \right)^{2-\gamma} \right],
\]

(1)

where \( M_{\text{disk}} = 0.01 M_{\odot} \) is the total mass of the gas disk, \( r_0 = 100 \) au is the initial disk radius, and \( \gamma = 1 \) is the exponent for the gas surface density profile. Here \( r \) denotes the distance from the central star. As an example, we take a Herbig Ae/Be star with a mass of \( M_{\star} = 2.5 M_{\odot} \) (\( M_{\odot} \) is the solar mass) as assumed in Pavlyuchenkov & Dullemond (2007). For simplicity, we assume that the (midplane) temperature of the disk is given as follows:

\[
T = T_1 \left( \frac{r}{1 \text{ au}} \right)^{-q},
\]

(2)

where \( T_1 = 800 \) K is the temperature at \( r = 1 \) au, and \( q = 1/2 \) is the exponent for the temperature structure. We also set the location of the crystallization front at \( r = 1 \) au, and that all dust is crystalline for \( r \leq r_c \) (Pavlyuchenkov & Dullemond 2007).

The basic equation of the evolution of gas surface density of accretion disks with magnetically driven disk winds is

\[
\frac{\partial \Sigma_{\text{gas}}}{\partial t} = \frac{1}{2 \pi r} \frac{\partial M_{\text{gas}}}{\partial r} + \dot{\Sigma}_{\text{wind}},
\]

(3)

where

\[
M_{\text{gas}} = 6 \pi r^{1/2} \frac{\partial (r^{1/2} \Sigma_{\text{gas}} \nu)}{\partial r},
\]

(4)

is the (vertically integrated) mass flux at every location \( r \), and \( \dot{\Sigma}_{\text{wind}} \) is the mass-loss rate due to the disk wind. Then, the advection velocity, \( v_{\text{adv}} \), is

\[
v_{\text{adv}} = \frac{M_{\text{gas}}}{2 \pi r \Sigma_{\text{gas}}}. \]

(5)

The advection velocity is positive when the gas flows inwardly.

The mass flux is proportional to the kinematic viscosity,

\[
\nu = \alpha_{\text{acc}} \sigma_s \hbar_g,
\]

(6)

where \( \alpha_{\text{acc}} \) is the angular momentum transport efficiency parameter called the alpha parameter (Shakura & Sunyaev 1973), \( \sigma_s \) is the sound speed, and \( \hbar_g \) is the gas scale height. The gas scale height, \( \hbar_g \), and the midplane gas density, \( \rho_g \), are given by

\[
\hbar_g = \frac{c_s}{\Omega_K},
\]

(7)

\[
\rho_g = \frac{\Sigma_{\text{gas}}}{\sqrt{2 \pi \hbar_g}},
\]

(8)

where \( \Omega_K = \sqrt{GM_{\star}/r^3} \) is the Keplerian frequency, and \( G \) is the gravitational constant.

Suzuki et al. (2010) investigated the mass-loss rate due to the magnetically driven disk wind. Based on their three-dimensional local magneto-hydrodynamic simulations, the mass-loss rate due to the disk wind is given by

\[
\dot{\Sigma}_{\text{wind}} = -C_w \Sigma_{\text{gas}} \Omega_K,
\]

(9)
where \( C_w \) is the efficiency parameter (see also Takahashi & Muto 2018). We set the typical value of \( C_w = 10^{-5} \) in Section 3.2.

Miyake & Suzuki (2016) found that not only gas but also small dust particles can be blown out by the disk wind. However, we do not consider this effect. The dust blowout process works when its Stokes number satisfies \( St < C_w / 1.8 \) (Taki et al. 2021). As we set \( C_w = 10^{-5} \), pebbles that can drift due to gas drag would not be blown out by the disk wind.

### 2.2. Motion of Dust Particles

We set the initial distribution of the dust surface density, \( \Sigma_{\text{dust},0} \), as follows:

\[
\Sigma_{\text{dust},0} = 0.01 \Sigma_{\text{gas},0}.
\]

(10)

In this study, we consider two types of dust particles: crystalline and amorphous particles. We define the crystallinity, \( C \), as the fraction of the crystalline dust particles:

\[
C \equiv \frac{\Sigma_c}{\Sigma_{\text{dust}}},
\]

(11)

where \( \Sigma_c \) is the surface density of the crystalline dust particles, and the dust surface density is the sum of the surface densities of crystalline and amorphous particles: \( \Sigma_{\text{dust}} = \Sigma_c + \Sigma_a \).

We compute the temporal evolution of the surface densities of crystalline and amorphous dust particles. We consider three physical processes: advection in the mean gas flow, diffusion due to concentration gradient, and the radial drift of dust particles relative to the gas (see Fukai & Arakawa 2021, and references therein). In addition, we also take into account the effect of conversion of dust to planetesimals via the streaming instability (see Section 2.2.3).

The surface densities of crystalline and amorphous dust particles evolve according to

\[
\frac{\partial \Sigma_c}{\partial t} = \frac{1}{2\pi r} \frac{\partial M_i}{\partial r} + \Sigma_{\text{plts},i},
\]

(12)

where the subscript \( i \) denotes the crystalline \((i = c)\) or amorphous \((i = a)\) dust particles. The mass flux of dust particles, \( M_i \), is given by the sum of the three physical processes:

\[
M_i = M_{i, \text{adv}} + M_{i, \text{diff}} + M_{i, \text{drift}},
\]

(13)

and \( \Sigma_{\text{plts},i} \) is the conversion rate of dust to planetesimals.

#### 2.2.1. Advection, Diffusion, and Radial Drift

Desch et al. (2017) rederived the equations for radial transport of dust particles. The advection term is given by

\[
M_{i, \text{adv}} = 2\pi r \Sigma_i v_{\text{adv}},
\]

(14)

and the diffusion term is

\[
M_{i, \text{diff}} = 2\pi r \Sigma_i v_{\text{diff}},
\]

(15)

where the diffusion velocity, \( v_{\text{diff}} \), is given by

\[
v_{\text{diff}} = D \left( \frac{\Sigma_i}{\Sigma_{\text{gas}}} \right)^{-1} \frac{\partial}{\partial r} \left( \frac{\Sigma_i}{\Sigma_{\text{gas}}} \right).
\]

(16)

Here \( D \) is the diffusion coefficient of the dust particles, which is given by

\[
D = \frac{\nu}{Sc(1 + St^2)},
\]

(17)

where \( Sc \) is the Schmidt number and \( St \) is the Stokes number of the dust particles (see Section 2.2.2). The diffusion velocity is positive when the direction of the flow is inward. We note that the Stokes number is sufficiently small \((St \ll 1)\) in our simulations, and the diffusion coefficient is approximately given by \( D \approx \nu / Sc \). The radial drift term is given by

\[
M_{i, \text{drift}} = 2\pi r \Sigma_i v_{\text{drift}},
\]

(18)

and the drift velocity, \( v_{\text{drift}} \), is given by the following equation:

\[
v_{\text{drift}} = \frac{St}{1 + St^2} \left( \eta P \Omega_K - St v_{\text{adv}} \right).
\]

(19)

Here \( \eta \) is the normalized pressure gradient, which is given by

\[
\eta = -\frac{1}{r^2} \frac{\partial P}{\partial r},
\]

(20)

and \( P = \rho_g c_s^2 \) is the gas pressure at the midplane. We note that \( v_{\text{drift}} \) is not the radial velocity of dust particles toward the central star but the radial drift velocity relative to the gas (see Desch et al. 2017). The radial velocity of dust particles toward the central star is the sum of \( v_{\text{adv}} \) and \( v_{\text{drift}} \).

#### 2.2.2. Stokes Number of Dust Particles

The Stokes number is the key parameter for the radial drift of dust particles and controls the velocities of dust particles. Assuming that fragmentation limits dust growth,4 the Stokes number of dust particles is given by the equilibrium between the mutual collision velocities and the fragmentation velocity:

\[
\Delta v = v_{\text{frag}},
\]

(21)

where \( \Delta v \) is the mutual collision velocity, which depends on \( St \), and \( v_{\text{frag}} \) is the threshold velocity for collisional fragmentation/growth. The mutual collision velocity is given by

\[
(\Delta v)^2 = (\Delta v_i)^2 + (\Delta v_0)^2,
\]

(22)

where \( \Delta v_i \) and \( \Delta v_0 \) are the contributions from radial drift (e.g., Adachi et al. 1976) and gas turbulence (e.g., Ormel & Cuzzi 2007), respectively. For the case of \( 10^{-4} \lesssim St \lesssim 1 \), Okuzumi et al. (2016) found that the radial drift term is given by

\[
\Delta v_i \simeq 0.5 St \eta P \Omega_K,
\]

(23)

and the gas turbulence term is given by

\[
\Delta v_0 \simeq \sqrt{2.3} \alpha_{\text{turb}} St c_s,
\]

(24)

where \( \alpha_{\text{turb}} \) is the dimensionless parameter for the strength of turbulence. Then, we can calculate the Stokes number from Equation (22), which is the quadratic equation for \( St \).

The strength of turbulence should be associated with the strength of mass diffusion. Therefore, the two alpha parameters, \( \alpha_{\text{turb}} \) and \( \alpha_{\text{acc}} \), might be related with the Schmidt

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4 We note that radial drift toward the central star may limit dust growth when the Stokes number exceeds \( 10^{-1} \), even if we do not consider fragmentation (e.g., Okuzumi et al. 2012; Okuzumi & Tazaki 2019).
number, which is the ratio of the kinematic viscosity to the mass diffusion coefficient. We simply assume the following equation:

\[ \alpha_{\text{turb}} = \frac{\alpha_{\text{acc}}}{Sc}, \]

although we set \( Sc = 1 \) in the main part of this study unless noted. Hence, \( \alpha_{\text{turb}} = \alpha_{\text{acc}} \) is assumed.

We assume that vertical settling of dust particles balances with turbulent diffusion. The dust scale height, \( h_d \), and the midplane dust density, \( \rho_d \), are given by Youdin & Lithwick (2007):

\[ h_d = h_d \left( 1 + \frac{St \left( 1 + 2St \right)}{\alpha_{\text{turb}} \left( 1 + St \right)} \right)^{-1/2}, \]

\[ \rho_d = \frac{\Sigma_{\text{dust}}}{\sqrt{2\pi}h_d}. \]

2.2.3. Conversion of Dust to Planetesimals

When the dust-to-gas mass ratio at the midplane was sufficiently high, hydrodynamic simulations revealed that some of the pebbles would be converted into planetesimals via the streaming instability (e.g., Carrera et al. 2015; Yang et al. 2017; Sekiya & Onishi 2018). Following the approach of Drążkowska et al. (2016), we take into account the effect of planetesimal formation. When the midplane dust density is higher than the gas density, \( \rho_d > \rho_g \), we convert part of the dust into planetesimals as follows (Drążkowska et al. 2016; Ueda et al. 2019):

\[ \Sigma_{\text{plts}} = \begin{cases} \frac{\zeta}{2\pi} \Sigma_{\text{dust}} \Omega_K \quad (\rho_d > \rho_g), \\ 0 \quad (\rho_d \leq \rho_g), \end{cases} \]

where \( \zeta = 10^{-4} \) is the planetesimal formation efficiency.

In this study, we consider two types of dust particles. As the mass-loss rate of crystalline dust particles should be proportional to the crystallinity, the mass-loss rate of dust particles via conversion of dust to planetesimals is given by

\[ \Sigma_{\text{plts},i} = -\frac{\zeta}{2\pi} \Sigma_i \Omega_K. \]

3. Results

In Section 3, we show the results of the disk evolution and radial distribution of the crystallinity of silicate dust particles. The results for disks without disk winds are shown in Section 3.1, and the results for evolving disks with disk winds are shown in Section 3.2.

3.1. Accretion Disks without Disk Winds

We performed the evolution of protoplanetary disks that evolve without disk winds. Figure 1 shows the time evolution of the radial distribution of the crystallinity of silicate dust particles. We set \( \alpha_{\text{acc}} = 10^{-3}, C_w = 0, \) and \( Sc = 1 \) in Section 3.1, and we changed the value of \( v_{\text{frag}} \) as a parameter.

Figure 1(a) shows the radial distribution of the crystallinity for the case of \( v_{\text{frag}} = 0 \) m s\(^{-1} \) (i.e., \( St = 0 \)). (b) For the case of \( v_{\text{frag}} = 1 \) m s\(^{-1} \). (c) For the case of \( v_{\text{frag}} = 3 \) m s\(^{-1} \). We set \( \alpha_{\text{acc}} = 10^{-3}, C_w = 0, \) and \( Sc = 1 \). The magenta dashed line shows the analytical solution of the radial distribution of the crystallinity in steady-state disks (Equation (30)).

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5 We acknowledge that hydrodynamic simulations of planetesimal formation via the streaming instability usually assumed protoplanetary disks with a negative pressure gradient (e.g., Bai & Stone 2010a, 2010b; Carrera et al. 2015; Yang et al. 2017), and whether planetesimals can also be formed via the streaming instability in disks with a positive pressure gradient is unclear. Although a high value of \( \rho_d/\rho_g \geq 1 \) should be beneficial for making planetesimals via instabilities, further studies on the condition for planetesimal formation are needed.
crystallinity in steady-state disks is given by

$$C = \begin{cases} 1 & (r \leq r_c), \\ \left(\frac{r}{r_c}\right)^{(3/2)S_c} & (r > r_c), \end{cases}$$

(30)

where $r_c = 1$ au is the location of the crystallization front. Here we assume that the background value of the crystallinity at $r = \infty$ is zero.

In contrast, the radial distribution of the crystallinity is different from the analytical solution when $v_{\text{frag}} \neq 0$ m s$^{-1}$ (i.e., $St \neq 0$). Figures 1(b) and (c) show the radial distribution of the crystallinity for the cases of $v_{\text{frag}} = 1$ m s$^{-1}$ and $v_{\text{frag}} = 3$ m s$^{-1}$, respectively. We found that the crystallinity decreases with increasing $v_{\text{frag}}$. The radial distribution of the Stokes number at $t = 2$ Myr is shown in Figure 2. The Stokes number increases with $r$ in most parts of the disk. This is because the mutual collision velocity is given by $\Delta v \approx \Delta v_{\text{t}}$ and therefore $\sqrt{St_c}$ is approximately constant (see Equations (21)–(24)).

The radial distribution of the crystalline silicate dust particles is given by the balance among three physical processes, that is, advection, radial drift, and diffusion of crystalline dust particles. We define the timescales of these processes as follows:

$$t_{\text{adv}} = \frac{r}{v_{\text{adv}}},$$

(31)

$$t_{\text{drift}} = \frac{r}{v_{\text{drift}}},$$

(32)

$$t_{c,\text{diff}} = \frac{r}{v_{c,\text{diff}}}.\quad (33)$$

Figure 3 shows the timescales of advection, radial drift, and diffusion at $t = 2$ Myr. We found that the diffusion timescale is negative in the region beyond the crystallization front ($r > r_c$). Therefore, the direction of the diffusion is outward. The diffusion timescale is balanced with the advection or radial drift timescales. In particular, the radial drift and diffusion timescales are balanced when the radial drift timescale is shorter than the advection timescale (see Figure 3(c)).

For the case of $v_{\text{frag}} = 0$ m s$^{-1}$, crystalline dust particles do not drift relative to the gas but diffuse due to the gradient of $\Sigma_c/\Sigma_{\text{gas}}$. The equilibrium of the radial distribution of the crystallinity is given by the balance between diffusion and advection. The radial distribution of the crystallinity at $t = 2$ Myr is approximately consistent with the steady-state solution (Equation (30)) at $r \ll r_0$.

In contrast, for the case of $v_{\text{frag}} = 3$ m s$^{-1}$, crystalline dust particles drift inwardly, and the effect of the inward advection is negligibly smaller than that of radial drift (Figure 3(c)).
equilibrium of the radial distribution of the crystallinity is given by the balance between diffusion and strong radial drift. Thus, outward transport of crystalline dust particles are suppressed compared to the case of $v_{\text{frag}} = 0 \text{ m s}^{-1}$. Figure 1(c) shows that the radial distribution of the crystallinity already reaches the steady state at $t = 2 \text{ Myr}$. As the radial drift timescale is inversely proportional to the Stokes number, calculations with a large value of $v_{\text{frag}}$ lead to the depletion of the crystallinity beyond the crystallization front for the case of disks that evolve without disk winds. This result is qualitatively inconsistent with the observed findings that crystalline dust particles are found farther away from the crystallization front.

3.2. Evolving Disks due to Disk Winds

We performed the evolution of protoplanetary disks that evolve due to disk winds. Figure 4 shows the radial distributions of the gas pressure at the midplane and the gas surface density. We set $\alpha_{\text{acc}} = 10^{-4}$, $C_w = 10^{-5}$, and $Sc = 1$ in Section 3.2.

Figure 4(a) shows the evolution of the radial distributions of the gas pressure at the midplane. As shown in previous studies (e.g., Suzuki et al. 2016; Takahashi & Muto 2018), magnetically driven disk winds create a maximum of gas pressure. Takahashi & Muto (2018) revealed that the location of the pressure maximum moves outward with time because the timescale of wind mass loss is longer for a larger orbital radius. Our result is consistent with that of Takahashi & Muto (2018).

The location of the pressure maximum at $t = 2 \text{ Myr}$ is $r = 12 \text{ au}$, and this is approximately consistent with that obtained from the analytical solution for steady-state disks with viscous accretion and magnetically driven disk winds (see Appendix).

Figure 4(b) also shows the evolution of the gas surface density. It is clear that the locations of the maxima of the gas pressure and gas surface density are different: the location of the pressure maximum is inner than that of the gas surface density. This relation is also explained by the analytical solution for steady-state disks (see Appendix).

Figure 5 shows the radial distribution of the dust surface density. We set $\alpha_{\text{acc}} = 10^{-4}$, $C_w = 10^{-5}$, and $Sc = 1$.

Figure 5 shows the radial distribution of the dust surface density for the case of $v_{\text{frag}} = 3 \text{ m s}^{-1}$. We found that a narrow dust ring is formed in the disk, and the location is approximately identical to that of the pressure maximum. This is because large pebbles are accumulated around the pressure maximum (e.g., Haghighipour & Boss 2003; Takahashi & Muto 2018). The dust surface density is significantly depleted beyond the dust ring due to the inward radial drift of large pebbles. Inside the dust ring, the dust surface density is controlled by the conversion of dust to planetesimals (see Equation (28)).
The radial distribution of the Stokes number at \( t = 2 \) Myr is shown in Figure 7. We set \( \alpha_{\text{acc}} = 10^{-4} \) instead of \( \alpha_{\text{acc}} = 10^{-3} \) in Section 3.2, and the Stokes number shown in Figure 7 is larger than that shown in Figure 2. For the case of \( v_{\text{frag}} = 3 \) m s\(^{-1}\), the Stokes number is in the range of \( 10^{-2} \lesssim \text{St} \lesssim 10^{-1} \) throughout the disk. This Stokes number provides the values of the dust-to-gas mass ratio under \( \rho_d = \rho_g \). The dust scale height is given by Equation (26), and the midplane dust density is inversely proportional to the dust scale height. Therefore, \( \Sigma_{\text{dust}}/\Sigma_{\text{gas}} \) is given by

\[
\frac{\Sigma_{\text{dust}}}{\Sigma_{\text{gas}}} = \left( 1 + \frac{\text{St}}{\alpha_{\text{turb}}^2} \right)^{-1/2},
\]

when \( \rho_d = \rho_g \) is achieved. This estimation explains the radial distribution of \( \Sigma_{\text{dust}}/\Sigma_{\text{gas}} \) shown in Figure 6.

Figure 5(b) shows the radial distribution of the crystallinity of silicate dust particles for the case of \( v_{\text{frag}} = 3 \) m s\(^{-1}\). We found that the crystallinity is almost 100% around and inside the location of the dust ring. The radial distribution of the crystallinity shown in Figure 5(b) is completely different from that for disks without disk winds. In Section 3.3, we unveil the mechanism for the radial transport of crystalline dust particles in evolving disk due to disk wind. The key physics of the efficient radial transport is the outward radial drift of pebbles.

Figures 8 and 9 show the radial distributions of \( \Sigma_{\text{dust}} \) and \( C \) for the cases of \( v_{\text{frag}} = 1 \) m s\(^{-1}\) and \( v_{\text{frag}} = 0.3 \) m s\(^{-1}\), respectively. As shown in Figure 8(a), the radial distribution of the dust surface density for the case of \( v_{\text{frag}} = 1 \) m s\(^{-1}\) is similar to that for the case of \( v_{\text{frag}} = 3 \) m s\(^{-1}\), although the width and the maximum value of \( \Sigma_{\text{dust}} \) of the dust ring are different. The radial distribution of the crystallinity shown in Figure 8(b) is also similar to that shown in Figure 5(b).

In contrast, the radial distributions of \( \Sigma_{\text{dust}} \) and \( C \) for the cases of \( v_{\text{frag}} = 0.3 \) m s\(^{-1}\) are completely different from those for \( v_{\text{frag}} = 3 \) m s\(^{-1}\). As shown in Figure 9(a), the dust surface density hardly changes with time beyond the maximum of the dust surface density. On the other hand, inside the maximum of the dust surface density, the dust-to-gas mass ratio is controlled by Equation (34). Then, the dust surface density is approximately given by the following equation for the case of small pebbles: \( \Sigma_{\text{dust}} \approx \min \left( \sqrt{\frac{\alpha_{\text{turb}}}{\text{St}} \Sigma_{\text{gas}}, \Sigma_{\text{dust},0}} \right) \). The location of the dust ring is therefore not necessarily identical to that of the pressure maximum.

The radial distribution of the crystallinity of silicate dust particles with \( v_{\text{frag}} = 0.3 \) m s\(^{-1}\) is shown in Figure 9(b). In contrast to the radial distribution shown in Figures 5(b) and 8(b), the crystallinity is \( C < 1 \) outside the crystallization front, and the crystallinity decreases with increasing \( r \). The
crystallinity around the dust ring is $C \ll 1$ for the case of $v_{\text{frag}} = 0.3 \text{ m s}^{-1}$.

### 3.3. Outward Radial Drift as a New Mechanism for Radial Transport of Crystalline Dust Particles

As shown in Figures 5(b) and 8(b), the crystallinity is almost 100% around and inside the location of the dust ring when the threshold velocity for collisional fragmentation/growth is $v_{\text{frag}} \geq 1 \text{ m s}^{-1}$. In Section 3.3, we show the condition for driving efficient radial transport.

Figure 10 shows the timescales of advection, radial drift, and diffusion at $t = 2 \text{ Myr}$. Inside the pressure maximum, the radial drift timescale is negative while the advection timescale is positive. For the case of $v_{\text{frag}} \geq 1 \text{ m s}^{-1}$, the Stokes number of pebbles is large and the radial drift timescale is shorter than the advection timescale: $|t_{\text{drift}}| < |t_{\text{adv}}|$ (see Figures 10(a) and (b)). In this case, the outward radial drift of pebbles can transport the crystalline dust particles from the crystallization front to the pressure maximum. Then, the crystallinity reaches almost 100% around and inside the location of the dust ring.

In contrast, for the case of $v_{\text{frag}} = 0.3 \text{ m s}^{-1}$, the Stokes number of pebbles is small and the radial drift timescale is longer than the advection timescale: $|t_{\text{drift}}| > |t_{\text{adv}}|$ (see Figure 10(c)). In this case, the outward radial drift of pebbles cannot transport the crystalline dust particles efficiently. Then, the diffusion timescale is balanced with the advection timescale, and the crystallinity decreases with increasing $r$.

Our novel mechanism for the radial transport of crystalline dust particles is illustrated in Figure 11. The condition for driving efficient radial transport by the outward radial drift is $|t_{\text{drift}}| < |t_{\text{adv}}|$. As the radial drift timescale is inversely proportional to the Stokes number, calculations with a large value of $v_{\text{frag}}$ lead to the efficient radial transport. Therefore, we expect that the crystallinity around and inside the dust ring reflects the size of pebbles and the threshold velocity for collisional fragmentation/growth.

By comparing our calculations with the observational findings that the crystalline silicate dust particles exist in the cold regions of the protoplanetary disks and the solar nebula, we suggest that this outward radial drift would be the key mechanism to transport crystalline silicate dust particles. This idea is supported by the fact that the recent high-spatial-resolution observations revealed that the ring structures are relatively common among protoplanetary disks. As the radial structures of gas disks strongly affect the dynamics of pebbles, further observational studies on the link between disk structure and dust composition are required.

### 4. Dependence on the Schmidt Number

In Section 3, we set $Sc = 1$ for simplicity. However, the Schmidt number of protoplanetary disks does not necessarily
4.1. Accretion Disks without Disk Winds

Here we show the results for accretion disks without disk winds in Section 4.1. Figure 12 shows the time evolution of the radial distribution of the crystallinity of silicate dust particles. Here we set $\alpha_{\text{acc}} = 10^{-4}$, $C_w = 10^{-2}$, and $v_{\text{frag}} = 0$ m s$^{-1}$, and we changed the value of $Sc$ as a parameter.

We confirmed that the radial distribution at $t = 2$ Myr is approximately identical to that obtained from the analytical solution for steady-state accretion disks: $C = (r/r_c)^{3/2}Sc$ (Pavlyuchenkov & Dullemond 2007). Therefore, the radial distribution of the crystallinity is a sensitive function of $Sc$ for the case of classical accretion disks without disk winds as shown in previous studies (e.g., Clarke & Pringle 1988; Pavlyuchenkov & Dullemond 2007). It should be noted that these radial distributions of the crystallinity are derived under $v_{\text{frag}} = 0$ m s$^{-1}$. The radial distribution of the crystallinity is determined by the balance among the advection, diffusion, and radial drift when $v_{\text{frag}} = 0$ m s$^{-1}$ (Section 3.1).

4.2. Evolving Disks due to Disk Winds

In contrast, the radial distribution of the crystallinity is not a sensitive function of $Sc$ for the case of evolving disks due to disk winds. We set $\alpha_{\text{acc}} = 10^{-4}$, $C_w = 10^{-2}$, $v_{\text{frag}} = 1$ m s$^{-1}$, and $Sc = 2$ in Section 4.2. Figure 13(a) shows the radial distribution of the dust surface density. A dust ring is formed around the pressure maximum as in the case of Figures 5(a) and 8(a). Figure 13(b) shows the radial distribution of the crystallinity of silicate dust particles. The crystallinity is almost 100% around and inside the location of the dust ring as in the case of Figures 5(b) and 8(b).

Figure 14 shows the timescales of advection, radial drift, and diffusion at $t = 2$ Myr. In this case, the Stokes number of pebbles is large enough to satisfy the following condition: $|t_{\text{drift}}| < |t_{\text{adv}}|$. Then, the outward radial drift of pebbles can transport the crystalline dust particles from the crystallization front to the pressure maximum, and the crystallinity reaches almost 100% around and inside the location of the dust ring.

For the case of classical accretion disks without disk winds, the radial distribution of the crystallinity is given by the balance of the diffusion and advection (or radial drift) timescales. In contrast, for the case of evolving disks due to disk winds, the radial distribution of the crystallinity is almost 100% if the outward radial drift overcomes the inward advection. As the diffusion is not the main mechanism, the radial distribution of the crystallinity hardly depends on $Sc$ as long as the condition for the outward radial drift (i.e., $|t_{\text{drift}}| < |t_{\text{adv}}|$) is satisfied (see Figure 11).

5. Summary

Several pieces of evidence suggest that silicate dust particles undergo significant thermal processing in protoplanetary disks, and that crystalline dust particles should transport outwardly as they are found in the outer region of protoplanetary disks and the solar system. Several mechanisms have been proposed for the outward transport of crystalline dust particles (e.g., Gail 2001; Keller & Gail 2004; Dullemond et al. 2006; Ciesla 2010; Yang & Ciesla 2012).

Recent astronomical observations have revealed a variety of structures in protoplanetary disks. In particular, dust-ring structures are observed in a large number of disks, and the accumulation of pebbles around the pressure maximum created have to be $Sc = 1$. Three-dimensional magnetohydrodynamic simulations indicated that the turbulent diffusion due to magnetorotational instability is expressed by the Schmidt number with $0.85 \leq Sc \leq 10$ (Carballido et al. 2005; Johansen & Klahr 2005). Based on analytic arguments, Pavlyuchenkov & Dullemond (2007) also derived the theoretical minimum value of $Sc = 1/3$. In Section 4, we briefly review the dependence of the radial distribution of the crystalline dust particles on the Schmidt number.
by mass loss due to magnetically driven disk winds is one of the possible origins of the observed dust-ring structures (e.g., Takahashi & Muto 2018).

In this study, we proposed a novel mechanism for the outward transport of crystalline dust particles. In evolving protoplanetary disks due to magnetically driven disk winds, dust particles can migrate outwardly by radial drift. We found that the outward radial drift process can transport crystalline dust particles efficiently when the radial drift overcomes the advective flow. Our findings are summarized as follows.

1. In Section 3.1, we performed the evolution of protoplanetary disks that evolve without disk winds. The diffusion timescale is balanced with the advection or inward radial drift timescales (see Figure 3); the crystallinity is well expressed by the analytical estimation by Pavlyuchenkov & Dullemond (2007) when the advection balances with the diffusion. It should be noted that the inward radial drift significantly suppresses the outward transport of the crystalline dust particles, which is inconsistent with the observational evidence.

2. In Section 3.2, we performed the evolution of protoplanetary disks that evolve due to disk winds. As shown in previous studies (e.g., Suzuki et al. 2016; Takahashi & Muto 2018), magnetically driven disk winds create a maximum of gas pressure at a certain radius (see Figure 4). We found that the location of the pressure maximum at $t = 2$ Myr is approximately consistent with that obtained from the analytical solution for steady-state disks with viscous accretion and magnetically driven disk winds (see Appendix).

3. Figure 5(a) shows the radial distribution of the dust surface density. For the case of $v_{\text{frag}} = 3$ m s$^{-1}$, a narrow dust ring is formed in the disk, and the location is approximately identical to that of the pressure maximum. This is because large pebbles are accumulated around the pressure maximum (e.g., Haghighipour & Boss 2003; Takahashi & Muto 2018).

4. Figure 5(b) shows the radial distribution of the crystallinity of silicate dust particles. For the case of $v_{\text{frag}} = 3$ m s$^{-1}$, the crystallinity is almost 100% around and inside the location of the dust ring. We proposed that the key physics of the efficient outward radial transport is the outward radial drift of pebbles.

5. The mechanism for the radial transport of crystalline dust particles proposed in this study is illustrated in Figure 11. The condition for driving efficient radial transport by the outward radial drift is $|t_{\text{drift}}| < |t_{\text{adv}}|$. As the radial drift timescale is inversely proportional to the Stokes number, calculations with a large value of $v_{\text{frag}}$ lead to the efficient radial transport. Therefore, we expect that the crystallinity around and inside the dust ring reflects the size of pebbles and the threshold velocity for collisional fragmentation/growth.

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Appendix
Stationary Solution for Gas Disk

We found a stationary solution for the gas surface density of disk with viscous accretion and magnetically driven disk winds. We set the radial distribution of the midplane temperature and the sound velocity as follows:

\[ \tilde{T}(r) = T_0 \tilde{r}^{-q}, \quad (A1) \]

\[ c_s = c_{s,1} \tilde{r}^{-q/2}, \quad (A2) \]

where \( \tilde{r} \equiv r/(1 \text{ au}) \) is the normalized distance from the central star, and \( c_{s,1} = (k_B T_0/m_p)^{1/2} \) is the sound speed at \( r = 1 \text{ au} \).
Here $k_B$ is the Boltzmann constant, and $m_g = 2.34 m_H$ is the mean molecular mass, where $m_H$ is the mass of a hydrogen atom.

Assuming that a gas disk is in a steady state, the left-hand side of Equation (3) is zero:

$$0 = \frac{1}{2\pi r} \frac{\partial \dot{M}_{\text{gas}}}{\partial r} - C_w \Sigma_{\text{gas}} \Omega_K. \quad (A3)$$

We can rewrite the above equation as follows:

$$p^{3-q} \frac{\partial^2 \Sigma_{\text{gas}}}{\partial r^2} + \left( \frac{9}{2} - 2q \right) p^{2-q} \frac{\partial \Sigma_{\text{gas}}}{\partial r} + \left[ (2 - q) \frac{3}{2} - q \right] p^{1-q} - A \right] \Sigma_{\text{gas}} = 0, \quad (A4)$$

where the dimensionless parameter $A$ is

$$A = \frac{C_w^{1/2} K_1}{3 \alpha_{\text{acc}} c_s^2} = 26.1 \left( \frac{C_w / \alpha_{\text{acc}}}{10^{-1}} \right) \left( \frac{T_1}{800 \text{ K}} \right)^{-1} \left( \frac{M_*}{2.5 M_\odot} \right), \quad (A5)$$

where $v_K = \sqrt{GM_*/(1 \text{ au})}$ is the Kepler velocity at $r = 1 \text{ au}$.

We found that the solution of Equation (A4) is given as follows:

$$\Sigma_{\text{gas}} = \frac{\Sigma_0}{\pi} r^{-7/4} \frac{2(2p A_{\text{I}}^4)\pi^p}{\Gamma(p)} K_p(4p A_{\text{I}}^4 r^{-1}/a^p), \quad (A6)$$

where $K_p(x)$ is the modified Bessel function of the second kind, and the exponent, $p$, is given by

$$p = \frac{1}{2(1-q)}. \quad (A7)$$

For the special case of $q = 1/2$ and $p = 1$, the stationary solution of $\Sigma_{\text{gas}}$ for a gas disk with radial mass accretion and wind-driven mass loss is given by

$$\Sigma_{\text{gas}} = \frac{\Sigma_0}{\pi} r^{-5/4} \cdot 4A_{\text{I}} \cdot K_1(4A_{\text{I}}^4 r^{-1/4}), \quad (A8)$$

where $\Sigma_0$ is a parameter. In this case, $\Sigma_{\text{gas}}$ takes the maximum at

$$r = 0.66 A_{\text{I}}^2 \text{ au}. \quad (A9)$$

We can also calculate the radial profile of the gas pressure at the midplane. The gas pressure is given by

$$P = \frac{\Sigma_{\text{gas}} c_s^2 \Omega_K}{\sqrt{2\pi}} = P_0 r^{5/2 - 13/8} \frac{2(2p A_{\text{I}}^4)\pi^p}{\Gamma(p)} K_p(4p A_{\text{I}}^4 r^{-1}/a^p), \quad (A10)$$

where $P_0$ is a constant. For the case of $q = 1/2$ and $p = 1$, we obtain the following equation:

$$P = P_0 r^{-3} \cdot 4A_{\text{I}}^2 K_1(4A_{\text{I}}^4 r^{-1}/a^p). \quad (A11)$$

In this case, $P$ takes the maximum at

$$r = 0.015 A_{\text{I}}^2 \text{ au}. \quad (A12)$$

Assuming $A = 26.1$ (see Equation (A5)), the location of the pressure maximum in the steady-state disk is estimated to be $r = 10.1 \text{ au}$. Our numerical simulation shows good agreement with this analytical prediction; the location of the pressure maximum is around $r \approx 12 \text{ au}$ at $t = 2 \text{ Myr}$ (see Figure 4). Thus the structure of the gas disk would already approach the steady-state solution at $t = 2 \text{ Myr}$. We note, however, that the location of the maximum for the gas density is not consistent with the analytical prediction. This is because an exponential cutoff for the outer edge of the gas disk exists at $r \approx 100 \text{ au}$ in our numerical simulation (see Equation (1)), which is not taken into account in the analytical model for the steady-state disk.
