Elements of the mathematical support for the design of an autonomous tractor

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Abstract. The article discusses the issue of substantiating the requirements for the location of passive benchmarks as part of a local complex of precision farming in its minimum configuration. As such, we consider a mobile chassis (for example, a wheeled agricultural tractor) equipped with an autonomous control system and a set of two or three passive benchmarks. References are given to works that reveal the technologies that are in demand in the design of the chassis of such a tractor and a description of the test equipment used in the design and debugging of units and assemblies of robotic machines. Within the framework of the concept under consideration, it is important that the complex combines the capabilities of autonomous, remote and manual control and is based on a mobile chassis that meets the necessary ergonomic requirements. The considered simplest system can be built into a structure that uses satellite positioning technologies and supports the principle of organizing multi-agent systems. The object of the research is a local complex of precision farming in the minimum composition: an autonomous agricultural tractor and a set of passive benchmarks. The purpose of the study is to determine the basic requirements for the selection of sites for the installation of passive benchmarks. The basic technology is the use of binary maps of permeability as a variant of the discrete working plan of an autonomous robotic complex. The technology for building maps of cross-country ability has been worked out when creating models of rovers and special vehicles. The paper illustrates the actual method of solving the problem in its mathematical interpretation, provides the main results and practical recommendations. It is shown that the proposed solutions can be extended to the areas of the forest engineering and logging industry, to the solution of logistic problems of the Far North, the territories of the Arctic and Antarctic.
1. Introduction

The variety of environmental management and nature management tasks includes the types of work that can be performed in the future by autonomous (unmanned, uninhabited) machines and groups of such machines. Also, in the future, autonomous complexes can be assigned to work on laying the road network, some other types of construction work. Currently, agriculture and forestry are the closest to the use of autonomous vehicles [1-3].

On the one hand, the analysis of publications [2-3] allows us to state that technologies of precision farming in Russia have not yet formed as a fundamental direction. However, the theoretical foundations and the experience described in [2-3] make it possible to form the structure of the system that provides such a technological approach and the primary appearance of the elements of this system. In particular, the article [4] formulates the concept of global and local complexes of precision farming, defines the specifics of the chassis of the main unit of the complex - a wheeled tractor, with a transmission that allows stabilization of rectilinear movement and maximally adapted to autonomous operation, along with maintaining the possibility of the traditional today human control (see publications [5-9] and others). An alternative solution is the creation of specialized machines oriented to autonomous operation and new units as part of their transmissions, but this way will require more significant financial investments and time costs [10-11]. For debugging and testing of such units, Russia already has specialized and tested equipment, and the theoretical basis can be traditional works for mechanical engineering [12-14] and others, in which proven methods of designing units and assemblies of chassis for transport and transport technological machines, provides the necessary information from the theory of the movement of wheeled and tracked chassis and describes the main technological developments in the transport engineering industry.

In all these cases, when it comes to the agricultural and forestry industry, the basis for building a robotic complex is an autonomous chassis operating in an environment that is close to stationary in the first approximation, collective control algorithms are based on solving the problem of controlling an individual robot, and a general approach to solving problems of group management in principle developed [15].

To organize the control of an individual chassis or a group of machines, you can use the principle of navigation through maps of cross-country ability, developed earlier for planetary rovers and spherical robots [15]. Map cells are divided into two groups: "allowed for movement" (contain a value of 0) and "forbidden" (contain a value of 1). To navigate a map tied to the area, it is enough to determine the real coordinates of the car. Traffic safety can be ensured using infrared sensors, lidars or other devices, and it is not necessary to use a computer vision system that requires significant computing resources.

The key problem is the positioning accuracy of the machine. This accuracy is determined by the specifics of the technological process. It is obvious that in most cases the accuracy value is significantly less than the characteristic size of the permeability map cell. We assume that the permissible positioning error for all coordinates is the same, is known and specified. Let's denote it by a. Then the cells of the permeability map can be divided into squares with sides less than a, and their number is integer. The resulting map will be called a positioning map. Its cells are also divided into "allowed" and "prohibited" for movement.

To unambiguously determine the coordinates of the body on the plane, two control points are sufficient. The principle is known as the triangulation method. In the simplest case, it is enough to define the angles at which the object is "visible" from the control points. The cheapest solution seems to be placing an IR meter or a lidar on the chassis of a machine (these devices can also be used to measure angular values), and passive reference points at reference points. Passive reference points do not emit a signal, but only reflect it. If you need to define three coordinates, you need three anchor points. When the number of control points is more than necessary, it is advisable to average the coordinate values obtained when working with pairs (or triplets) of benchmarks.

To move on to the choice of equipment, determining the method of arranging benchmarks and constructing algorithms for the operation of the navigation system, it is necessary to establish a
relationship between the permissible positioning error $a$ and the error in determining the angular coordinates of the object.

2. Formulation of the problem

The object of the research is a local complex of precision farming in a minimal composition - an autonomous agricultural tractor and a set of passive benchmarks. The purpose of the study is to determine the basic requirements for the selection of sites for the installation of passive benchmarks.

Based on the analysis of the specifics of the operating conditions of agricultural tractors, the design features of the chassis of promising wheeled tractors [5-7], the experience of building control systems based on a discrete working field for mobile chassis [15], traditional and modern literary sources in the field of theory and design of transport and technological machines, tasks are formulated:

- Build a mathematical model that allows you to study the influence of the places of installation of passive benchmarks on the accuracy of determining the coordinates of an autonomous chassis;
- Conduct a study of the influence of the accuracy of setting benchmarks on the accuracy of determining the coordinates of the machine;
- Propose a set of solutions aimed at ensuring the functionality of the local complex of precision farming in the considered configuration.

3. Theoretical research results

The task. Investigate the dependence of the error in determining the Cartesian coordinates on the error in measuring angles.

Formalization of the task. Without loss of generality, it is possible to solve the problem in a square with side $L = 1$, located in the first quadrant of the Cartesian plane. The points A and B of setting the angles are located at the corners of the square on the horizontal axis (the OX axis in Figure 1). Angles $\alpha$ and $\beta$ are measured from the positive direction of the OX axis. It is necessary to find the dependence of the error in determining the coordinates of the point $T$ $x_t$ and $y_t$ on the error in measuring the angles $\alpha$ and $\beta$.

![Figure 1. Principle of triangulation: T - lidar or IR device for determining $\alpha$ and $\beta$ - angular coordinates of the chassis relative to the passive reference points A and B; L - base.](image)

Basic relationships. Point coordinates versus measured angles. Let's introduce the notation:

$$a = \tan \alpha \ ; \ b = \tan \beta$$ (1)

Then the equation of lines AT and BT, respectively, can be written in the form:

$$y = x \tan \alpha = ax \ ; \ y = x - 1 \tan \beta = -b(1-x)$$ (2)

The same equalities will serve for the transition between the Cartesian coordinates $(x, y)$ and angular $(a, b)$ or $(\alpha, \beta)$. The coordinates of point $T$ are the coordinates of the intersection of lines AT and BT:

$$x_t = \frac{b}{b-a} \ ; \ y_t = \frac{ab}{b-a}$$ (3)
Dependence of the error in the $dx$ and $dy$ coordinates on the error in measuring the angles $d\alpha$ and $d\beta$:

$$\frac{da}{d\alpha} = 1 + a^2; \quad \frac{db}{d\beta} = 1 + b^2$$  \tag{4}

We calculate the partial derivatives, which we denote, respectively, $Ax$, $Bx$, $Ay$, $By$. Then we can write that:

$$dx = Axda + Bxd\beta; \quad dy = Ayd\alpha + Byd\beta$$  \tag{5}

Behavior of the coefficients $Ay$, $By$. In the unit square, these coefficients of the formulas are bounded from above by zero, and from below by $-2$, that is:

$$-2 = Ay \ 1,1 \leq Ay \ x, y = -x^2 + y^2 \leq Ay \ 0,0 = 0;$$  \tag{6}

$$-2 = By \ 0,1 \leq By \ x, y = -(1 - x)^2 + y^2 \leq By \ 1,0 = 0;$$  \tag{7}

In this way,

$$dyt \leq \max \ dyt = 2(da + d\beta)$$  \tag{8}

Behavior of the coefficients $Ax$, $Bx$. The coefficients of formulas (2a), as before, are bounded from above by zero

$$Ax \ x, y = -x^2 + y^2 \leq Ax \ 1, y = 0; \quad Bx \ x, y = -(1 - x^2 + y^2) \leq Bx \ 0, y = 0$$  \tag{9}

But the constraint from below is no longer due to $y$ in the denominator,

$$Ax \to -\infty \text{ and } Bx \to -\infty \text{ at } y \to 0$$  \tag{10}

Let us introduce a constraint on the coefficients $Ax$ and $Bx$ and define the area where these constraints will be violated. Since the coefficients $Ax$ and $Bx$ are symmetric functions with respect to $x = 0.5$

$$Ax \ x, y = Bx(1 - x, y)$$  \tag{11}

Then it is enough to consider only one function.

Limitation on the coefficient $Ax$. Let:

$$Ax \ x, y \geq -2W$$  \tag{12}

Where $W > 0$ is a numerical parameter, which will be defined later.

The restriction will be satisfied when the value of $y$ lies between the roots of the corresponding quadratic equation:

$$y_1 \leq y \leq y_2 \text{ where } y_1 = \frac{w-k}{1-x} y_2 = \frac{w+k}{1-x} K = \frac{W^2 - x^2(1-x)^2}{1-x}; \text{ moreover } W \geq x(1-x)$$  \tag{13}

The last inequality implies that:

$$W \geq 0.5 \geq x(1-x) \text{ at } 0 \leq x \leq 1$$  \tag{14}

Right root $y_2$. It can be shown that $y_2 > 1$ with $W > 0.5$ for $0 \leq x \leq 1$. Those, the upper bound for $y$ remains 1.

Left root $y_1$. Obviously, $y_1 > 0$. Below we will consider functions on the interval $x: 0 \leq x < 1$, since the function $Ax(x, y)$ has a minimum on the right boundary. Then:

$$y_1 = y_1(x, W) = \frac{w-k}{1-x} = \frac{w}{1-x} - \frac{w}{1-x} x^2 = \frac{x}{p} (1 - \frac{1}{p^2})$$  \tag{15}
Where \( P = P \times W \) \[ P = \frac{x(1-x)}{w} \] (16)

To find out if \( y_1 \) has a maximum and, if so, what it is equal to, you need to calculate the derivative and find its root. Considering that for \( W > 0.5 \) we have \( x > 0.5 \), we get as a result:

\[ p^2 = \frac{x^2 - x^2}{w^2} = \frac{x - 3x}{1 - x^2} \quad 1 - p^2 = \frac{1 - 2x^2}{1 - x^2} \quad 2x - 1 > 0 \] (17)

\[ y_1 = \frac{x}{p} \quad 1 - \frac{2x - 1}{1 - x} = \frac{x}{1 - x^2} \quad \frac{x}{w} = \frac{Wx(2 - 3x)}{x(2 - 3x)} \] (18)

In (table 1), for various parameters \( W, (x, y) \) are calculated - the coordinates of the maximum point of the curve (8), below which the coefficient \( Ax \) takes values less than - 2W (shown in the fourth column).

Returning to (3), it is logical to take \( W = 1 \), since the \( x \) coordinate is no different from the \( y \) coordinate. The corresponding line is highlighted in the (table 1). Then

\[ Ax \ x, y \geq -2W = -2 \] (19)

The above is true for the coefficient \( Bx \) if we take \((1-x)\) instead of \( x \). The \( x \)-coordinate error obeys the inequality:

\[ dx_1 \leq \max dx_2 = 2(\alpha + \beta) \] (20)

**Table 1.** Estimating Boundary Line Coordinate Values.

| \( W \) | \( x \) for \( Ax \) | \( y \) | -2W | \( x \) for \( Bx \) |
|--------|-----------------|-----|-----|-----------------|
| 0.5    | 0.654193        | 0.156460 | -1  | 0.345807 |
| 1      | 0.663841        | 0.075016 | -2  | 0.336159 |
| 3      | 0.666361        | 0.024725 | -6  | 0.333639 |
| 10     | 0.666639        | 0.007408 | -20 | 0.333361 |
| \( \infty \) | 2/3             | 0      | \( -\infty \) | 1/3 |

Figure 2 shows a graph with \( W = 1 \). For clarity, the same graph is presented in two scales in \( y \): "dashed line" (left axis) - shows the shape of the curve; "Solid line" (right axis) - shows the share of the "bad" area of the square, which is less than 5% of its area.

In practice, it is not necessary to calculate the position of a point in coordinates \((x, y)\). You can immediately draw a conclusion about the calculation error \((x, y)\) from the measured angles \( \alpha \) and \( \beta \), since taking into account \( b <0 \) and \( a - b > 0 \):

\[ 2 \ a + b \ ^2 = -(a^2 + 1)b \ or \ 4 \sin^2(a - \beta) = -\sin(2\beta) \] (21)

A "square" in coordinates \((\alpha, \beta)\) with an upper boundary and a line (6) is shown in (figure 3).

4. Analysis of results

The reason for the appearance of a region in which the errors in the \( x \) coordinate sharply increase is, obviously, the given location of points A and B. Always on a straight line connecting the points in which the angles are measured, and around it the error in coordinates. Will catastrophically increase (figure 2 and 3).

In practice, as a rule, the number of points at which measurements are taken is more than two. Therefore, if point T approaches the corresponding straight line, when determining the coordinates, it is necessary to ignore the angles from these points.

The second way is to locate the angle calculation points at a distance of at least 0.075 relative units outside the boundary of the considered area.
5. Analysis of results

The most important in practical terms are the above recommendations to minimize the error in calculating the coordinates of the mobile chassis relative to passive benchmarks. These recommendations determine the specifics of the deployment of the benchmark system on the ground and should be used when choosing the parameters of the on-board equipment of the mobile chassis of the local precision farming complex.

The considered technology can be extended to mobile systems for monitoring soil conditions (see, for example, proposals in [15]), forestry engineering, logging industry, and also used in transport operations in the regions of the Far North, Arctic and Antarctic [11].

![Figure 2. Determination of the minimum positioning error zone.](image1.png)

![Figure 3. Working area boundaries in α-β coordinates.](image2.png)

Integration of the local system into the global one [4] will expand the functionality, but it will also require large financial investments in the development and deployment of the project.

6. Conclusion

Based on the above:

- The proposed solution to the problem of positioning an object relative to two passive reference points is a variation of the triangulation method and can be used as a basis for determining the coordinates of an object in three-dimensional space with a given accuracy;
- When an object is located on a line passing through two reference points, the error in determining the position of the object increases significantly. Therefore, the number of benchmarks must be at least three, and the control algorithm must provide for coordinate correction using data from all possible combinations of pairs of benchmarks;
- By calculation, you can determine the border of the zone within which the error in determining the coordinates will be minimal. This will allow, in most cases, to arrange benchmarks in such a way that work is carried out by a mobile chassis within the boundaries of this zone;
- The proposed mathematical model allows the error in determining the position of the vehicle on the positioning map and proceeds to the choice of on-board measuring equipment, the principle of placing passive benchmarks and the design of algorithms for automatic determination of coordinates.

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