Occam’s Razor in Lepton Mass Matrices
- The Sign of Universe’s Baryon Asymmetry -

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Abstract

We discuss the neutrino mass matrix based on the Occam’s Razor approach in the framework of the seesaw mechanism. We impose four zeros in the Dirac neutrino mass matrix, which give the minimum number of parameters needed for the observed neutrino masses and lepton mixing angles, while the charged lepton mass matrix and the right-handed Majorana neutrino mass matrix are taken to be real diagonal ones. The low-energy neutrino mass matrix has only seven physical parameters. We show successful predictions for the mixing angle $\theta_{13}$ and the CP violating phase $\delta_{CP}$ with the normal mass hierarchy of neutrinos by using the experimental data on the neutrino mass squared differences, the mixing angles $\theta_{12}$ and $\theta_{23}$. The most favored region of $\sin \theta_{13}$ is around $0.13 \sim 0.15$, which is completely consistent with the observed value. The CP violating phase $\delta_{CP}$ is favored to be close to $\pm \pi/2$. We also discuss the Majorana phases as well as the effective neutrino mass for the neutrinoless double-beta decay $m_{ee}$, which is around $7 \sim 8$ meV. It is extremely remarkable that we can perform a “complete experiment” to determine the low-energy neutrino mass matrix, since we have only seven physical parameters in the neutrino mass matrix. In particular, two CP violating phases in the neutrino mass matrix are directly given by two CP violating phases at high energy. Thus, assuming the leptogenesis we can determine the sign of the cosmic baryon in the universe from the low-energy experiments for the neutrino mass matrix.

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1 Introduction

The standard model has been well established by the discovery of the Higgs boson. However, the origin and structure of quark and lepton flavors are still unknown in spite of the remarkable success of the standard model. Therefore, underlying physics for the masses and mixing of quarks and leptons is one of the fundamental problems in particle physics. Actually, a number of models have been proposed based on flavor symmetries, but there is no convincing model at present.

On the other hand, the neutrino oscillation experiments are going on a new step to reveal the CP violation in the lepton sector. The T2K experiment has confirmed the neutrino oscillation in the $\nu_{\mu} \rightarrow \nu_e$ appearance events \cite{1}, which may provide us a new information of the CP violation in the lepton sector. Recent NOνA experimental data \cite{2} also indicate the CP violation in the neutrino oscillation. Thus, various informations are now available to discuss Yukawa matrices in the lepton sector.

Recently, the Occam’s Razor approach was proposed to investigate the neutrino mass matrix \cite{3} in the case of two heavy right-handed neutrinos. Because of tight constraints it was shown that only the inverted mass hierarchy for the neutrinos is consistent with the present experimental data. The quark sector was also successfully discussed in this approach \cite{4} and we found a nice prediction of the Cabibbo angle, for instance.

In this paper, we discuss the seesaw mechanism \cite{5} with the three right-handed heavy Majorana neutrinos, predicting the normal mass hierarchy of the light neutrinos. We impose four zeros in the Dirac neutrino mass matrix, which give the minimum number of parameters needed for the observed neutrino masses and lepton mixing angles in the normal mass hierarchy of neutrinos \cite{6,7}. Here, the charged lepton mass matrix and the right-handed Majorana neutrino mass matrix are taken to be real diagonal ones. The Dirac neutrino mass matrix is given with five complex parameters. Among them, three phases are removed by the phase redefinition of the three left-handed neutrino fields. The remained two phases are removed by the field-phase rotation of the right-handed neutrinos. Instead, these two phases appear in the right-handed Majorana neutrino mass matrix. After integrating the heavy right-handed neutrinos, we obtain a mass matrix of the light neutrino, which contains five real parameters and two CP violating phases.

In the present Occam’s Razor approach with the four zeros of the Dirac neutrino mass matrix, we show the successful predictions of the mixing angle $\theta_{13}$ and the CP violating phase $\delta_{CP}$ with the normal mass hierarchy of neutrinos. We also discuss the Majorana phases and the effective neutrino mass of the neutrinoless double-beta decay.

It is extremely remarkable that we can perform a “complete experiment” to determine the low-energy neutrino mass matrix \cite{8}, since we have only seven physical parameters in the neutrino mass matrix. In particular, two CP violating phases in the neutrino mass matrix are directly related to two CP violating phases at high energy. Thus, assuming the leptogenesis, we can determine the sign of cosmic baryon in the universe only from the low-energy experiments for the neutrino mass matrix \cite{9}.

In section 2, we show a viable Dirac neutrino mass matrix with four zeros, where we take the real diagonal basis of the charged lepton mass matrix and the right-handed Majorana neutrino mass matrix. We also present qualitative discussions of our parameters in order to
reproduce the two large mixing angles of neutrino flavors. In section 3, we show numerical results for our mass matrix. The summary is devoted in section 4. In Appendix, we show parameter relations in our mass matrix.

2 Neutrino mass matrix

On the standpoint of Occam’s Razor approach [3, 4], we discuss theneutrino mass matrix in the framework of the seesaw mechanism without assuming any symmetry. We take the real diagonal basis of the charged lepton mass matrix and the right-handed Majorana neutrino mass matrix as:

\[
M_E = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}_{LR}, \quad M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}_{RR}.
\] (1)

We reduce the number of free parameters in the Dirac neutrino mass matrix by putting zero at several elements in the matrix. The four zeros of the Dirac neutrino mass matrix give us the minimum number of parameters to reproduce the observed neutrino masses and lepton mixing angles. This is what we call the Occam’s Razor approach.

The successful Dirac neutrino mass matrix with four zeros \(^1\) is given as

\[
m_D = \begin{pmatrix} 0 & A & 0 \\ A' & 0 & B \\ 0 & B' & C \end{pmatrix}_{LR},
\] (2)

which has five complex parameters. \(^2\) The three phases can be removed by the phase rotation of the three left-handed neutrino fields. This phase redefinition does not affect the lepton mixing matrix because the charged lepton mass matrix is diagonal and the phases are absorbed in the three right-handed charged lepton fields. In order to get the real matrix for the Dirac neutrino mass matrix, the remained two phases are removed by the phase rotation of the two right-handed neutrino fields. Instead, the right-handed Majorana neutrino mass matrix becomes complex diagonal one as follows:

\[
M_R = \begin{pmatrix} M_1 e^{-i\phi_A} & 0 & 0 \\ 0 & M_2 e^{-i\phi_B} & 0 \\ 0 & 0 & M_3 \end{pmatrix}_{RR} = \begin{pmatrix} \frac{1}{k_1} e^{-i\phi_A} & 0 & 0 \\ 0 & \frac{1}{k_2} e^{-i\phi_B} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{RR},
\] (3)

where \(M_0 \equiv M_3, \ k_1 = M_3/M_1\) and \(k_2 = M_3/M_2\). We obtain the left-handed Majorana neutrino mass matrix after integrating out the heavy right-handed neutrinos,

\[
m_\nu = m_D M_R^{-1} m_D^T = \frac{1}{M_0} \begin{pmatrix} A^2 k_2 e^{i\phi_B} & 0 & A B' k_2 e^{i\phi_B} \\ 0 & A^2 k_1 e^{i\phi_A} + B^2 & B C \\ A B' k_2 e^{i\phi_B} & B C & B^2 k_2 e^{i\phi_B} + C^2 \end{pmatrix},
\] (4)

\(^1\) Other four zero textures may be available for the lepton mixing. Those will be discussed comprehensively in the future work.

\(^2\) \(A' = 0\) corresponds to the case discussed in ref. [4]. Thus, five zero textures are not excluded.
Then, the neutrino mass matrix is written as

$$A = \sqrt{M_0 k_2'} a, \quad A' = \sqrt{M_0 k_1'} a, \quad B = \sqrt{M_0 b}, \quad B' = \sqrt{M_0 k_2'} b, \quad C = \sqrt{M_0 c}. \quad (5)$$

Then, the neutrino mass matrix is expressed as

$$m_\nu = \begin{pmatrix}
a^2K_2e^{i\phi_B} & 0 & abK_2'e^{i\phi_B} \\
0 & a^2K_1'e^{i\phi_A} + b^2 & bc \\
abk_2e^{i\phi_B} & bc & b^2K_2'e^{i\phi_B} + c^2
\end{pmatrix}, \quad (6)$$

where

$$K_1 = k_1'k_1 = \left(\frac{A'B'}{AB}\right)^2 \frac{M_3}{M_1}, \quad K_2 = k_2'k_2 = \left(\frac{B'}{B}\right)^2 \frac{M_3}{M_2}. \quad (7)$$

Finally, the neutrino mass matrix is expressed by five real parameters, $a, b, c, K_1, K_2$ and two phases $\phi_A, \phi_B$. Since we can input five experimental data of neutrinos, the mass squared differences $\Delta m^2_{\text{atm}}, \Delta m^2_{\text{sol}}$ and three lepton mixing angles $\theta_{23}, \theta_{12}$ and $\theta_{13}$, there remains two free parameters. Those two parameters are determined by the Dirac CP violating phase $\delta_{CP}$ and the effective neutrino mass $m_{\nu e}$ for the neutrinoless double-beta decay $[8]$. Here we comment on the concern with the texture zero analysis of the left-handed neutrino mass matrix $[10]$. Actually, some two zero textures of the left-handed neutrino mass matrix are consistent with the recent data $[11]$. On the other hand, our neutrino mass matrix of Eq.(6) is a zero one texture. The two zero textures are never realized without the tuning among parameters as seen in Eq.(1) since we start with the seesaw mechanism of the neutrino masses, in which we take the right-handed Majorana neutrino mass matrix to be diagonal $[12]$. Although there are seven parameters in the neutrino mass matrix in Eq.(6), we can give clear predictions at the large $K_1$ and $K_2$, which corresponds to the large mass hierarchy among the right-handed Majorana neutrinos.

We can obtain the eigenvectors by solving the eigenvalue equation of Eq.(5). The mass eigenvalues are expressed by $a, b, c, K_1, K_2$ and $\phi_A, \phi_B$ as seen in Appendix. And then, we get the lepton mixing matrix, so called the Maki-Nakagawa-Sakata (MNS) matrix $U_{\text{MNS}}$ $[13, 14]$. It is expressed in terms of three mixing angles $\theta_{ij}$ ($i, j = 1, 2, 3; \ i < j$), the CP violating Dirac phase $\delta_{CP}$ and two Majorana phases $\alpha$ and $\beta$ as

$$U_{\text{MNS}} \equiv \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\
s_{12}s_{23} - -c_{12}s_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13}
\end{pmatrix} \begin{pmatrix}
e^{i\alpha} & 0 & 0 \\
0 & e^{i\beta} & 0 \\
0 & 0 & 1
\end{pmatrix}, (8)$$

where $c_{ij}$ and $s_{ij}$ denote $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively.

There is a CP violating observable, the Jarlskog invariant $J_{CP}$ $[15]$, which is derived from the following relation:

$$i\mathcal{C} \equiv [M_\nu M_\nu^T, M_E M_E^T],$$

$$\det \mathcal{C} = -2J_{CP}(m_3^2 - m_2^2)(m_2^2 - m_1^2)(m_1^2 - m_3^2)(m_2^2 - m_\mu^2)(m_\mu^2 - m_\tau^2)(m_e^2 - m_\tau^2), \quad (9)$$
where \( m_1, m_2 \) and \( m_3 \) are neutrino masses with real numbers. The predicted one is expressed in terms of the parameters of the mass matrix elements as:

\[
J_{CP} \simeq \frac{1}{2} \frac{F}{(\Delta m_{atm}^2)^2 \Delta m_{sol}^2},
\]

where

\[
F = 2a^2b^4c^2K_2^2 \left\{ b^4K_2 \sin \phi_B + a^4K_1K_2 \sin(\phi_A - \phi_B) + a^2c^2(K_1 \sin \phi_A - K_2 \sin \phi_B) + a^2b^2K_2(K_1 \sin(\phi_A + \phi_B) - K_2 \sin 2\phi_B - \sin \phi_B) \right\}.
\]

We can extract \( \sin \delta_{CP} \) from \( J_{CP} \) by using the following relation among mixing angles, the Dirac phase and \( J_{CP} \):

\[
\sin \delta_{CP} = \frac{J_{CP}}{s_{23}c_{23}s_{12}c_{12}s_{13}c_{13}^2}. \tag{12}
\]

The Majorana phases \( \alpha \) and \( \beta \) are obtained after diagonalizing the neutrino mass matrix of Eq.(6) as follows:

\[
U_{\text{MNS}}^\dagger m_\nu U_{\text{MNS}}^* = \text{diag} \{ m_1, m_2, m_3 \}. \tag{13}
\]

Then, we can estimate the effective mass which appears in the neutrinoless double-beta decay as

\[
m_{ee} = c_{13}^2c_{12}^2e^{2i\alpha}m_1 + c_{13}^2s_{12}^2e^{2i\beta}m_2 + s_{13}^2e^{-2i\delta_{CP}}m_3. \tag{14}
\]

The neutrino mass matrix of Eq.(6) becomes a simple one at the \( K_1 \) and \( K_2 \) large limit with \( b^2K_2 \) being finite. This case corresponds to the large hierarchy of the right-handed neutrino mass ratios \( M_3/M_1 \) and \( M_3/M_2 \). Then, the magnitudes of our parameters are estimated qualitatively to reproduce the two large mixing angles \( \theta_{23} \) and \( \theta_{12} \). At first, impose the maximal mixing of \( \theta_{23} \). Then, the \( (2,3) \) element of Eq.(6) should be comparable to the \( (3,3) \) one, so that the cancellation must be realized between two terms in the \( (3,3) \) element, and then we have:

\[
K_2 \sim \frac{c^2}{b^2}, \quad \phi_B \sim \pm \pi. \tag{15}
\]

The \( (2,3) \) element of Eq.(6) is also comparable to the \( (2,2) \) one, which is dominated by the first term \( a^2K_1 \exp(i\phi_A) \) at the large \( K_1 \). So, we get

\[
K_1 \sim \frac{bc}{a^2}. \tag{16}
\]

At the next step, we impose the large \( \theta_{12} \), which requires the \( (1,3) \) element of Eq.(6) to be comparable to \( (2,2) \) within a few factor, therefore, we get

\[
aK_1 \sim bK_2r, \quad (r = 2 \sim 3). \tag{17}
\]

By combining Eqs.(15), (16), (17), we obtain

\[
acr \sim b^2, \quad K_1 \sim \left( \frac{c}{b} \right)^3, \quad K_2 \sim \left( \frac{c}{b} \right)^2, \quad K_1^2 \sim K_2^3 r^4, \quad (r = 2 \sim 3). \tag{18}
\]

Actually, those relations are well satisfied in the numerical result at the large \( K_1 \). Then, \( \theta_{13} \) becomes rather large, roughly, order of \( \sin \theta_{12}/r \) since the \( (1,3) \) element of Eq.(6) is
comparable to (2, 3) within a factor of two or three. Thus, the seizable mixing angle $\theta_{13}$ is essentially derived in this textures when the observed mixing angles $\theta_{23}$ and $\theta_{12}$ are input. This situation is well reproduced in our numerical result.

Furthermore, we expect the large CP violating phase $\delta_{CP}$ in this discussion. As shown in Eq.(15), the real part of the (3, 3) element of Eq.(6) is significantly suppressed in order to reproduce the almost maximal mixing of $\theta_{23}$. Then, the imaginary part of the (3, 3) element is relatively enhanced even if $\phi_B$ is close to $\pm 180^\circ$. Actually, $\phi_B \simeq \pm 175^\circ$ leads to the $\delta_{CP} \simeq \pm 90^\circ$ in the numerical analysis of the next section.

3 Numerical analysis

Let us discuss the numerical result with the normal mass hierarchy of neutrinos. At the first step, we constrain the real parameters $a, b, c, K_1, K_2$ and two phases $\phi_A, \phi_B$ by inputting the experimental data of $\Delta m_{\text{atm}}^2$ and $\Delta m_{\text{sol}}^2$ with 90% C.L. into the relations of Eq.(21) in Appendix. By removing $c, \phi_A$ and $\phi_B$ for a fixed $m_1$, which is varied in the region of $m_1 = 0 \sim \sqrt{\Delta m_{\text{sol}}^2}$, there remains four parameters $a, b, K_1$ and $K_2$.

At the second step, we scan them in the following regions by generating random numbers in the liner scale as follows:

$$K_1 = [1 \sim 10^6], \quad K_2 = [1 \sim 10^4], \quad a = [0 \sim 0.03] \text{ eV}^{1/2}, \quad b = [0 \sim 0.2] \text{ eV}^{1/2}. \quad (19)$$

They are constrained by the experimental data of the lepton mixing angles. And then, we predict $\delta_{CP}$, $m_{ee}$, Majorana phases $\alpha$ and $\beta$. The input data are given as follows [16]:

$$\Delta m_{\text{atm}}^2 = 2.457 \pm 0.047 \times 10^{-3} \text{ eV}^2, \quad \Delta m_{\text{sol}}^2 = 7.50^{+0.19}_{-0.17} \times 10^{-5} \text{ eV}^2,$$

$$\sin^2 \theta_{12} = 0.304^{+0.013}_{-0.012}, \quad \sin^2 \theta_{23} = 0.452^{+0.052}_{-0.028}, \quad \sin^2 \theta_{13} = 0.0218 \pm 0.0010, \quad (20)$$

![Figure 1: The frequency distribution of the predicted $\sin \theta_{13}$ at $K_1 = 1 - 5000$ by inputting the data of $\theta_{12}$ and $\theta_{23}$. Here the vertical red lines denote the experimental data with $3\sigma$.](image1)

![Figure 2: The frequency distribution of the predicted $\delta_{CP}$ at $K_1 = 1 - 5000$ by inputting the data of $\theta_{12}$ and $\theta_{23}$. Here the vertical red lines denote the NO$\nu$A allowed region with $1\sigma$.](image2)
where we adopt these data with the error-bar of 90% C.L in our calculations. We assume
the normal mass hierarchy of neutrinos. Actually, we have not found the inverted mass
hierarchy, in which the three lepton mixing angles are consistent with the observed values in
our numerical calculations. Thus, we consider that the normal mass hierarchy is a prediction
in the present model as long as there is no extreme fine tuning of the parameters.

Let us show the result for $K_1 = 1 - 5000$. By inputting the data of the two mixing angles
$\theta_{12}$ and $\theta_{23}$, we present the frequency distribution of the predicted $\sin \theta_{13}$ in Fig.1, where
the vertical red lines denote the experimental data of Eq.(20) with $3\sigma$ range. The peak is
within the experimental data for $3\sigma$ range. It is remarked that $\sin \theta_{13} \approx 0.14$ is most favored.
This prediction is understandable as discussed below Eq.(18). We also present the frequency
distribution of the predicted value of $\delta_{CP}$ in Fig.2, where the vertical red lines denote the
NO$\nu$A experimental allowed region with $1\sigma$ range, which is obtained by the method of Library
Event Matching (LEM) [2]. We see that $\delta_{CP}$ is favored to be around $\pm 2$ radian, which is
consistent with the T2K [1] and NO$\nu$A data for $1\sigma$ range.

If we add the constraint of the experimental data of $\theta_{13}$, the predictions become rather
clear. By input of the experimental data of $\theta_{13}$, we obtain the allowed region on the $K_1 - K_2$
plane in Fig.3. As $K_1$ increases, the $K_2$ also increases gradually. This behavior is expected
in Eq.(18). We present the frequency distribution of the predicted value of $\delta_{CP}$ in Fig.4. The
peak of the distribution is still around $\pm 2$ radian, but the distribution becomes rather sharp
compared with the one in Fig.2.

Let us discuss the $K_1$ dependence of $\delta_{CP}$, which is shown in Fig.5. In the region of $K_1 = O(1 - 100)$, the predicted $\delta_{CP}$ is distributed broader. As $K_1$ increases, the predicted region
becomes narrow gradually. And then, it becomes consistent with the NO$\nu$A experimental
allowed region with $1\sigma$ range at the high $K_1$.

We also predict the effective neutrino mass $m_{ee}$, which appears in the amplitude of the
neutrinoless double-beta decay. In Fig. 6, we present the frequency distribution of $m_{ee}$. The

Figure 3: The allowed region on the $K_1 - K_2$ plane at $K_1 = 1 - 5000$ by inputting
the data of three mixing angles.

Figure 4: The frequency distribution of the predicted $\delta_{CP}$ at $K_1 = 1 - 5000$ by
inputting the data of three mixing angles. Here the vertical red lines denote
the NO$\nu$A allowed region with $1\sigma$.
Figure 5: The $K_1$ dependence of the predicted $\delta_{CP}$ at $K_1 = 1 - 5000$ by inputting the data of three mixing angles. Here the horizontal red lines denote the NOνA experimental allowed region with $1\sigma$.

Figure 6: The frequency distribution of the predicted $m_{ee}$ at $K_1 = 1 - 5000$ by inputting the data of three mixing angles.

Favored $m_{ee}$ is around 7 meV.

As shown in Fig.5, our result depends on the $K_1$. Actually the predicted region becomes narrow as $K_1$ increases significantly. Let us discuss the result at $K_1 = 10^4 - 10^6$. We show the $K_1$ dependence of the predicted $\sin \theta_{13}$ at $K_1 = 10^4 - 10^6$ by inputting the data of $\theta_{12}$ and $\theta_{23}$ in Fig.7. The mixing angle $\sin \theta_{13}$ is larger than 0.1 in all region of $K_1$, but the large mixing angle 0.5 is allowed below $K_1 = 10^5$. However, it is remarked that $\sin \theta_{13}$ decreases gradually and converges on the experimental allowed value.

In Fig.8, we present the frequency distribution of the predicted $\sin \theta_{13}$ by inputting the

Figure 7: The $K_1$ dependence of the predicted $\sin \theta_{13}$ at $K_1 = 10^4 - 10^6$ by inputting the data of $\theta_{12}$ and $\theta_{23}$. Here the horizontal red lines denote the experimental data with $3\sigma$.

Figure 8: The frequency distribution of the predicted $\sin \theta_{13}$ at $K_1 = 10^4 - 10^6$ by inputting the data of $\theta_{12}$ and $\theta_{23}$. Here the vertical red lines denote the experimental data with $3\sigma$. 

Figure 9: The frequency distribution of the predicted $\delta_{CP}$ at $K_1 = 10^4 - 10^6$ by inputting the data of three mixing angles. Here the vertical red lines denote the NO$\nu$A allowed region with 1$\sigma$.

data of the two mixing angles $\theta_{12}$ and $\theta_{23}$. The distribution becomes rather sharp compared with the case of $K_1 = 1 - 5000$. The most favored region of $\sin \theta_{13}$ is around 0.13 $-$ 0.15, which is completely consistent with the experimental data.

In Fig. 9, we show the frequency distribution of the predicted value of $\delta_{CP}$ by inputting the data of the three mixing angles. It is remarked that the peak of the frequency distributions of $\delta_{CP}$ becomes close to $\pm \pi/2$. Moreover, the region of $\delta_{CP} = -1 \sim 1$ radian is almost excluded. Our result is consistent with the data of the T2K [1] and the NO$\nu$A [2] experiments.

The predicted $m_{ee}$ of the neutrinoless double-beta decay is not so changed compared with the case of $K_1 = 1 - 5000$. The favored value of $m_{ee}$ is around 7 $-$ 8 meV. Here, we show

Figure 11: The predicted Dirac phase $\delta_{CP}$ versus the predicted Majorana phase $\alpha$ at $K_1 = 10^4 - 10^6$ by inputting the data of three mixing angles.

Figure 12: The predicted Dirac phase $\delta_{CP}$ versus the predicted Majorana phase $\beta$ at $K_1 = 10^4 - 10^6$ by inputting the data of three mixing angles.
the predicted $\delta_{CP}$ versus $m_{ee}$ by inputting the data of three mixing angles in Fig. 10. They are rather correlated as seen in Eq.(22) of Appendix. If $\delta_{CP}$ is restricted around $-\pi/2$ in the neutrino experiment, the allowed region is restricted. Then, the predicted $m_{ee}$ is $6.5 \sim 8$ meV.

At last, we show the correlation among the Dirac phase $\delta_{CP}$ and the Majorana phases $\alpha$, $\beta$ in Figs. 11, 12 and 13. There appears the tight correlation among them because we have only two phase parameters in the neutrino mass matrix of Eq.(6).

4 Summary

We have presented the neutrino mass matrix based on the Occam’s Razor approach [3, 4]. In the framework of the seesaw mechanism, we impose four zeros in the Dirac neutrino mass matrix, which give the minimum number of parameters needed for the observed neutrino masses and lepton mixing angles without assuming any flavor symmetry. Here, the charged lepton mass matrix and the right-handed Majorana neutrino mass matrix are taken to be real diagonal ones. Therefore, the neutrino mass matrix is given with seven parameters after absorbing the three phases into the left-handed neutrino fields.

Then, we obtain the successful predictions of the mixing angle $\theta_{13}$ and the CP violating phase $\delta_{CP}$ with the normal mass hierarchy of neutrinos. We also discuss the Majorana phases $\alpha$ and $\beta$ as well as the effective neutrino mass of the neutrinoless double-beta decay $m_{ee}$. Especially, as $K_1$ increases to $10^4 \sim 10^6$, the predictions become more sharp. The most favored region of $\sin \theta_{13}$ is around $0.13 \sim 0.15$, which is completely consistent with the experimental data. The $\delta_{CP}$ is favored to be close to $\pm \pi/2$, and the effective mass $m_{ee}$ is around $7 \sim 8$ meV. The reduction of the experimental error-bar of the two mixing angles of $\theta_{12}$ and $\theta_{23}$ will provide more precise predictions in our mass matrix of neutrinos.

Finally, it is emphasized that we can perform a “complete experiment” to determine the
low-energy neutrino mass matrix, since we have only seven physical parameters in the mass matrix (see Eq.(6)). In particular, two CP violating phases $\phi_A$ and $\phi_B$ in the neutrino mass matrix are directly related to two CP violating phases at high energy. Thus, assuming the leptogenesis we can determine the sign of the cosmic baryon in the universe from the low-energy experiments for the neutrino mass matrix. In fact the sign of baryon is given by the sign of $\sin \phi_A$ for the normal mass hierarchy $M_1 < M_2 < M_3$ which is suggested from the predicted hierarchy $K_1 > K_2 > 1$ shown in Fig.3. Unfortunately, the present experimental data show both sign allowed as shown in Fig.14. We expect precise measurements of three mixing angles and CP violating phases at low energy experiments.

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**Appendix**

By solving the eigenvalue equation in Eq.(6), the mass eigenvalues are expressed by $a, b, c, K_1, K_2$ and $\phi_A, \phi_B$. We have three equations among them as follows:

\[
m_1^2 + m_2^2 + m_3^2 = a^4 + b^4(1 + K_2^2) + a^2(1 + K_1^2 + K_2^2) + 2a^2b^2(\sin \phi_A + c^2(1 + K_2 \cos \phi_B) + c^2(K_2 + K_1 \cos \phi_B + K_1 \sin \phi_A \sin \phi_B) + 2a^2b^2K_2^2(1 + K_1 \cos \phi_A + 2b^2c^2K_2(K_2 + K_1 \cos \phi_B) + 2a^2b^4K_2^2(1 + K_1 \cos \phi_A + 2b^2c^2(K_2 + K_1 \cos \phi_B) + c^2(K_2 + K_1 \cos \phi_A \cos \phi_B + K_1 \sin \phi_A \sin \phi_B) \right)
\]

Since the neutrino mass matrix in Eq.(6) has one zero, it constrains the observed values. Among three mixing angles, the three phases and the neutrino masses, there is one relation:

\[
0 = c_{12}c_{13}(-s_{12}c_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}})e^{2i\alpha}m_1 + s_{12}c_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}})e^{2i\beta}m_2 + s_{13}s_{23}c_{13}e^{-i\delta_{CP}}m_3.
\]

The effect of quantum corrections of the lepton mixing matrix is neglected in the evolution from the GUT scale to the electroweak scale for the normal mass hierarchy [17].

The detailed discussion on this issue will be given in the coming paper.
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