Antihydrogen-Hydrogen annihilation at sub-kelvin temperatures

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Annihilation cross-section of ultra low-energy atomic antihydrogen \((E \leq 10^{-5}\text{ a.u.})\) on atomic hydrogen are calculated within a quantum mechanical coupled channels approach. The results differ from the extrapolations of semiclassical models to low energies. The main features of the observables are found to be determined by a family of \(HH\) nearthreshold metastable states.

I. INTRODUCTION

The projects of synthesis and spectroscopy of ultracold atomic antihydrogen in traps \([1]\) require theoretical calculations of the low energy matter-antimatter inelastic collisions rates. We present here a quantum mechanical model of atomic antihydrogen \(H\bar{H}\) and atomic hydrogen \((H)\) interaction, describing inelastic collisions \((H\bar{H} + H \rightarrow Pn^* + e^- e^+)\) at energies less than \(10^{-5}\text{ a.u.}\).

We developed a unitary coupled channel model which incorporates the main physical inputs. Our results for the annihilation cross-section will be compared to those provided by the semiclassical methods \([2,3,4]\). We will show that in our approach the low energy properties are determined by a rich spectrum of nearthreshold S-matrix singularities generated by long range van der Waals potential.

II. THE FORMALISM

We develop the effective potential approach to the low energy rearrangement collisions, used by the authors for \(H\bar{p}\) scattering \([5]\). The details of the formalism would be published elsewhere, here we give only very brief description. The wave-function is represented as a sum of two components:

\[
\Phi = \Phi_1 + \Phi_2
\]

\[
\Phi_1 = (1 - \hat{F})\Phi
\]

\[
\Phi_2 = \hat{F}\Phi
\]

Here \(\hat{F}\) is the projection operator on the subspace of opened Protonium formation channels, i.e. the Protonium states with principal quantum number \(N \leq 24\). It is easy to see, that the first component asymptotically describes elastic channel, while the second describes all the inelastic channels. The equation system, obtained for the wave-function components \(\Phi_1\) and \(\Phi_2\) is transformed into the one-channel Schrodinger equation, governing the \(H\bar{H}\) relative motion wave-function \(\chi(R)\):

\[
\left[-\frac{1}{M}\partial_R^2 + W_{eff} + V_{loc}(R) + U_{nuc}(R) - E + 2\varepsilon_B\right]\chi(R) = 0
\]

where \(M\) is the \(\bar{p}\) mass, \(E\) is the \(H\bar{H}\) center of mass energy, \(\varepsilon_B\) is the H-atom Bohr energy, \(V_{loc}\) is the local part of interaction, obtained within adiabatic approximation, \(W_{eff}\) - nonlocal, complex operator, which accounts for the coupling with the Pn+Ps formation channels, \(U_{nuc}\) is a nuclear potential of Woods-Saxon type, fitted to describe \(p\bar{p}\) nuclear scattering length. Effective interaction \(W_{eff}\) is calculated by solving the coupled equations system for propagator (Green function) of Pn+Ps formation channels. In practical calculations only the channels with Pn principal quantum number \(21 \leq N \leq 24\), Pn angular momentum \(L \leq 1\), and Ps quantum numbers \(n \leq 2, l \leq 1\) were included. It was checked that coupling with other channels is relatively small. In particular all channels with production of unbound \(e^- e^+\) were excluded due to their vanishing contribution.
The $H\bar{H}$ scattering length is found to be $a_{H\bar{H}} = (6.1 - i2.7)$ a.u. It was found to be an oscillating function of reduced mass of $H\bar{H}$ thought of as a free parameter (see Fig. 1). The immediate consequence of such a behavior of the scattering length is a strong isotope effect. In particular, the $D\bar{H}$ scattering length was found to be $a_{D\bar{H}} = (15.0 - i11.6)$ a.u. The annihilation cross-sections for $H\bar{H}$ and $D\bar{H}$ systems are presented on Fig. 2. It should be mentioned that the semiclassical models show no sign of oscillating behavior of observables. The extrapolated to low energies values of semiclassical cross-sections depend on velocity $v$ like $(1/v)^{2/3}$ in contradiction with quantum mechanical $1/v$-law and significantly smaller than the calculated quantum cross-section.

An oscillating behavior of scattering length as a function of reduced mass is explained by existence of nearthreshold quasibound states [6], generated by van der Waals long range attraction. Appearance of the new nearthreshold states with increasing of the reduced mass of the system results in a strong enhancement of annihilation cross-section. The energies, inelastic widths and mean radius of several such states are shown in Table I.

Taking in mind, that main properties of observables are governed by $-C_6/R^6$ tail of local potential, while the short range interaction, including effective potential, only modifies the spectrum of the nearthreshold states, we can get the analytical expression for the $H\bar{H}$ scattering length:

$$a_{H\bar{H}} = (2MC_6)^{1/4} \frac{\Gamma(3/4)}{2\sqrt{2\Gamma(5/4)}} (1 + \cot(\pi/8 + \delta + \frac{\sqrt{2MC_6}}{2r_0^{-2}}))$$

where $C_6$ is van der Waals constant, $r_0$ is the range of short range interaction ($r_0 \approx 1$ a.u.), $\delta$ is a complex phase shift, produced by short range complex part of the potential. It is clear from the above expression, that if Im$\delta \gg 1$ (so called black sphere limit) the scattering length becomes smooth function of reduced mass $M$ and van der Waals constant $C_6$ and insensitive to any details of short range interaction:

$$a_{H\bar{H}} = (2MC_6)^{1/4} \frac{\Gamma(3/4)}{2\Gamma(5/4)} \exp(-i\pi/4)$$

Our numerical calculations show that we are far from the black sphere limit, and the scattering length is indeed an oscillating function of $M$ and $C_6$. We have also find it to be very sensitive to short range interaction details. In particular, taking into account the nuclear potential, which describes the nuclear absorption, significantly changes the value of scattering length from $a_{H\bar{H}} = 5.2 - i1.8$ a.u. to its final value of $a_{H\bar{H}} = (6.1 - i2.7)$ a.u.

IV. CONCLUSION

A coupled channels model describing the $H\bar{H}$ system at energies less than $10^{-5}$ a.u. has been developed. The results thus obtained for annihilation cross-section substantially differ from the low energy extrapolations of the black sphere model and other classical or semiclassical approaches.

The reaction dynamics is found to be determined by the existence of several nearthreshold states. Such states are produced by van der Waals potential and have inelastic widths due to the coupling with Protonium formation channels. We predict strong isotope effect in $H\bar{H}$ and $D\bar{H}$ scattering. Our results seem to be in a qualitative agreement with recently published results [7] by P. Froelich et al.

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[6] J. Carbonell et al.: Few-Body Systems Suppl. 8, 428 (1995)
[7] P.Froelich et al., Phys. Rev. Lett., 84, 4577 (2000)
|   | $E_I$       | $E_{II}$       | $\bar{r}_{II}$ |
|---|-------------|---------------|---------------|
| 1 | $-8.1 \times 10^{-6}$ | $-6.3 \times 10^{-6} + i \, 1.8 \times 10^{-9}$ |  |
| 2 | $-1.9 \times 10^{-4}$ | $-4.3 \times 10^{-4} - i \, 2.2 \times 10^{-4}$ | 4.6 |
| 3 | $-2.9 \times 10^{-3}$ | $-5.2 \times 10^{-3} - i \, 1.4 \times 10^{-3}$ | 3.2 |
| 4 | $-1.1 \times 10^{-2}$ | $-2.8 \times 10^{-2} - i \, 8.2 \times 10^{-3}$ | 1.6 |

TABLE I. Energies, Auger widths and mean radii (a.u) of $L=0$ $H\bar{H}$ states. We denote by index I the results in van der Waals potential alone and by index II those obtained in full interaction.
FIG. 1. $H\bar{H}$ annihilation cross-section ($E=10^{-8}$ a.u.) versus reduced mass
FIG. 2. $H\bar{H}$ and $D\bar{H}$ annihilation cross-sections