FIELD THEORY OF SCALING LATTICE MODELS.
THE POTTs ANTIFERROMAGNET

Gesualdo Delfino
SISSA
via Beirut 2-4, 34014 Trieste
Italy
delfino@sissa.it

Abstract In contrast to what happens for ferromagnets, the lattice structure participates in a crucial way to determine existence and type of critical behaviour in antiferromagnetic systems. It is an interesting question to investigate how the memory of the lattice survives in the field theory describing a scaling antiferromagnet. We discuss this issue for the square lattice three-state Potts model, whose scaling limit as $T \to 0$ is argued to be described exactly by the sine-Gordon field theory at a specific value of the coupling. The solution of the scaling ferromagnetic case is recalled for comparison. The field theory describing the crossover from antiferromagnetic to ferromagnetic behaviour is also introduced.

Keywords: Field theory, statistical mechanics, antiferromagnets, Potts model, integrability

1. Introduction

The description of statistical systems nearby their phase transition points is one of the most stimulating applications of quantum field theory. The typical problem one has to face in this context is that of non-trivial fixed points of the renormalisation group, i.e. of strongly interacting quantum field theories. It is then an important discovery of the last years that quantum field theory can actually be done non-perturbatively in two dimensions. Here, conformal [1] and integrable [2]

*Talk given at the NATO Advanced Research Workshop on Statistical Field Theories, Como 18-23 June 2001
field theories describe exactly the critical points and the scaling limits of statistical models, respectively.

This fact has been exploited to solve the scaling ferromagnetic models in two dimensions directly in the continuum limit. Since scaling ferromagnets exhibit universal behaviour independent on lattice details, the recipe is: find the simplest integrable field theory with the internal symmetry characteristic of the given universality class. It is remarkable that the basic two-dimensional ferromagnets are integrable in the continuum limit (not on the lattice, usually) and can be solved in this way.

It should not be surprising that much less is known for antiferromagnets. The spins in an antiferromagnet want to be in a state which is different from that of their nearest neighbours. Hence, the number of such neighbours, namely the lattice structure, participates in a crucial way to determine existence and type of critical behaviour. It follows that the phenomenology is much richer than in the ferromagnetic case. Nevertheless, if a critical point with infinite correlation length exists, there must be a quantum field theory describing the scaling limit. It is an intriguing question to find such a theory and to understand how it ‘remembers’ about the lattice structure. Once the lattice and internal symmetries have been disentangled in the field theory of the scaling antiferromagnet, another interesting point is that of breaking the first symmetry while preserving the second. The result should be a theory describing the crossover from antiferromagnetic to ferromagnetic behaviour.

Once again, one expects that in two dimensions these issues can be investigated in a precise, non-perturbative way. We will show that this is indeed the case for a non-trivial example, the three-state Potts model on the square lattice. In the next Section we recall the solution of the ferromagnetic case before turning to the antiferromagnet in Section 3 and discussing the crossover between the two in Section 4.

2. Scaling Limit of the Ferromagnetic Three-state Potts Model

The three-state Potts model is defined by the Hamiltonian

\[ H = -J \sum_{\langle i,j \rangle} \delta_{s_i,s_j}, \quad s_i = 1, 2, 3 \]

(1)

where \( s_i \) denotes the spin located at the \( i \)-th site of a regular lattice and the sum is taken over nearest neighbours. The model is characterised by the invariance under global permutations of the values of the spin. The permutation group \( S_3 \) can be seen as the product of the group \( Z_3 \) of cyclic permutations times a “charge conjugation” \( C \). The elementary
$Z_3$ transformation and charge conjugation act as follows on the complex spin variable $\sigma_j = e^{2i\pi s_j/3}$:

\[
Z_3 : \quad \sigma_j \rightarrow e^{2i\pi/3} \sigma_j, \quad C : \quad \sigma_j \rightarrow \sigma_j^*;
\]

of course they leave invariant the energy operator $\varepsilon_j = \sum_i \delta_{s_i, s_j}$.

The model exhibits ferromagnetic or antiferromagnetic behaviour depending on the sign of the coupling $J$.

In two dimensions, the ferromagnetic ($J > 0$) model undergoes a second order phase transition at a critical temperature $T_c$ [3]. It was shown in [4, 5] that the critical point is described by the minimal model of conformal field theory with central charge $C = 4/5$. The spin operator $\sigma(x)$ and the energy operator $\varepsilon(x)$ correspond to the primary conformal operators $\phi_{2,3}(x)$ and $\phi_{2,1}(x)$ with scaling dimensions $X_{\sigma} = 2/15$ and $X_{\varepsilon} = 4/5$, respectively. As a consequence, the scaling limit can be described by adding to the fixed point action $A_{C=4/5}$ the thermal perturbation in the form

\[
A_F = A_{C=4/5} + \tau \int d^2 x \varepsilon(x),
\]

with $\tau$ measuring the deviation from $T_c$. While the model (1) is not solvable on the lattice away from $T = T_c$ [3], the scaling limit (2) belongs to the large class of integrable quantum field theories discovered by A. Zamolodchikov [2].

Integrable quantum field theories can be solved exactly in the scattering theory framework [6]. In fact, integrability (i.e. the existence of an infinite number of quantum integrals of motion) ensures the complete elasticity and factorisation of the scattering processes and allows the determination of the scattering amplitudes. Universality requires that the scaling limit of the ferromagnetic three-state Potts model corresponds to the simplest integrable scattering theory implementing the $S_3$ symmetry. The difference between the high and low temperature phases is made by the nature of the excitations.

At $T > T_c$ there is a single ground state and the simplest realisation of the symmetry is in terms of a doublet of charge conjugated particles $A$ and $\bar{A}$ of mass $m$ transforming under $S_3$ as the spin operators $\sigma$ and $\sigma^*$, respectively. The $Z_3$ symmetry in enforced by requiring the existence of the fusion process

\[
A A \rightarrow \bar{A}.
\]

Factorisation of multiparticle processes implies that the full scattering matrix is determined by the three two-particle amplitudes depicted in Fig. 1. The last process turns out to be incompatible with factorisation.
Figure 1. The scattering amplitudes $S_1$, $S_2$ and $S_3$.

and the property (3) together, and the simplest solution with $S_3^F(\theta) = 0$ is [7, 8]

$$S_1^F(\theta) = S_2^F(i\pi - \theta) = \frac{\sinh\frac{1}{2}(\theta + \frac{2i\pi}{3})}{\sinh\frac{1}{2}(\theta - \frac{2i\pi}{3})},$$

(4)

where $\theta$ parameterises the center of mass energy $\sqrt{s} = 2mcosh(\theta/2)$. The nonvanishing amplitudes are related by crossing symmetry while unitarity follows from the fact that complex conjugation amounts to $\theta \to -\theta$. The pole at $\theta = 2\pi i/3$ in the amplitude $S_1^F(\theta)$ corresponds to the bound state (3).

At $T < T_c$ there are three degenerate ground states and the excitations are “kinks” $K_{j,j+\pm 1}$ interpolating between the ground state $j = 1, 2, 3$ and the ground state $j+1 \pmod{3}$. Their space-time trajectories draw domain walls separating regions with different magnetisation. Due to invariance under permutations, there are only three inequivalent two-kink scattering amplitudes which are readily mapped into those of Fig.1 through the identifications $K_{j,j+1} \longleftrightarrow A$, $K_{j,j-1} \longleftrightarrow \bar{A}$. As a consequence the solution for the scattering amplitudes is the same than at $T > T_c$. This is the way in which the high-low temperature duality of the Potts model emerges in this context. The computation of the correlation functions in the two phases starting from the scattering solution can be found in [9].

3. The Square Lattice Antiferromagnet

At $T = 0$ the antiferromagnetic ($J < 0$) three-state Potts model on the square lattice finds infinitely many ground states in which each spin has a value different from that of its nearest neighbours. After transferring the labels from sites to faces, a configuration in terms of arrows can be obtained through the following rule: if an observer in one face labelled $j$ looks across an edge to an adjacent face labelled $j+1 \pmod{3}$, then put an arrow on this edge pointing to the observer’s left; if the adjacent face is labelled $j - 1 \pmod{3}$, point the arrow to the right. All sites of the
resulting configuration satisfy the rule “two arrows in, two arrows out” which defines the six-vertex model. The latter is exactly solvable on the lattice [3] and is known to be critical and equivalent at long distance to a free massless boson.

This exact mapping of the square lattice antiferromagnet (1) at zero-temperature onto a particular case\(^1\) of the six-vertex model has been known for long time and has been used to determine the scaling dimensions of a number of relevant operators [10, 11, 12, 13]. These are the staggered magnetisation \(\Sigma_j = (-1)^{j_1+j_2} e^{2i\pi s_j/3}\) with dimension 1/6, the uniform magnetisation \(\sigma_j = e^{2i\pi s_j/3}\) with dimension 2/3, and the staggered polarisation\(^2\) \(P_j = (-1)^{j_1+j_2} \sum'_i(2\delta_{s_i,s_j} - 1)\) with dimension 3/2. Here and in the following we identify the \(j\)-th site of the square lattice through a pair of integers \((j_1, j_2)\), and call even (odd) sublattice the collection of the sites with \(j_1 + j_2\) even (odd).

The model is not solved on the lattice at non-zero temperature and the issue of the approach to criticality (which involves first of all the determination of the scaling dimension of the energy operator \(\epsilon_j = \sum_i \delta_{s_i,s_j}\)) has made the object of recent studies [14, 15, 16]. The authors of [15] exploited a mapping onto a height model to study the lattice excitations at \(T > 0\) and explain the anomalous corrections to scaling observed in simulations. Here we will follow Ref. [16] to show how the scaling limit as \(T \to 0\) is in fact described by an integrable quantum field theory.

Since the critical model is described by a gaussian fixed point, the expectation that the scaling limit corresponds to a massive integrable field theory is very natural. In fact, under the (mild) assumption that the thermal operator \(\epsilon_j\) gives in the continuum limit a single relevant operator \(\epsilon(x)\), the action one is left with for the scaling limit is that of the sine-Gordon model,

\[
A_{AF} = \int d^2x \left( \frac{1}{2} \partial_a \varphi \partial^a \varphi - \mu \cos \beta \varphi \right), \tag{5}
\]

which is integrable. Having said that, it remains to be understood how the action (5) actually describes the Potts antiferromagnet, namely where this action hides the relevant lattice and \(S_3\) symmetries, how the Potts degrees of freedom are expressed in terms of the bosonic field \(\varphi\), and which value of \(\beta\) determines the scaling dimension \(X_\epsilon = \beta^2/4\pi\) of the energy operator \(\epsilon = \cos \beta \varphi\).

To answer these questions we have to recall few facts about the operator content of the sine-Gordon model. At the gaussian fixed point

\(^1\)The one in which all vertices are equally weighted, the so called ice point.

\(^2\)The primed sum indicates summation over the next nearest neighbours of \(j\).
\( (\mu = 0) \) the bosonic field decomposes into holomorphic and antiholomorphic parts as \( \varphi(x) = \phi(z) + \bar{\phi}(\bar{z}) \), where \( z = x_1 + ix_2 \) and \( \bar{z} = x_1 - ix_2 \). The scaling operators \( V_{p,\beta}(x) = \exp[ip\phi(z) + p\bar{\phi}(\bar{z})] \) have scaling dimension \( X = (p^2 + \bar{p}^2)/8\pi \), spin \( s = (p^2 - \bar{p}^2)/8\pi \), and satisfy the gaussian operator product expansion

\[
V_{p_1,\beta_1}(x)V_{p_2,\beta_2}(0) = z^{p_1\bar{p}_2/4\pi} \bar{z}^{\bar{p}_1p_2/4\pi} V_{p_1+p_2,\beta_1+\beta_2}(0) + \ldots \quad (6)
\]

This relation shows that taking \( V_{p_1,\beta_1}(x) \) around \( V_{p_2,\beta_2}(0) \) by sending \( z \rightarrow ze^{2i\pi} \) and \( \bar{z} \rightarrow \bar{z}e^{-2i\pi} \) produces a phase factor \( e^{2i\pi\gamma_{1,2}} \), where \( \gamma_{1,2} = (p_1\bar{p}_2 - \bar{p}_1p_2)/(4\pi) \) is called index of mutual locality. If \( \gamma_{1,2} \) is an integer the correlators \( \langle ..V_{p_1,\beta_1}(x)V_{p_2,\beta_2}(0) .. \rangle \) are single valued as functions of \( x \) and the two operators are said to be mutually local. Since \( \gamma_{1,1} = 2s \), the operators which are local with respect to themselves must have integer or half integer spin.

The operators of interest for the description of the statistical model are scalar \((s = 0)\) and local with respect to the energy \( \varepsilon = \cos \beta \varphi \). These requirements select

\[
V_p \equiv V_{p,p} = \exp[ip\varphi], \quad \quad \quad (7)
\]

\[
U_m \equiv V_{2\pi m/\beta,-2\pi m/\beta} = \exp[2i\pi m\tilde{\varphi}/\beta], \quad m = \pm1, \pm2, .. \quad (8)
\]

where \( \tilde{\varphi}(x) \equiv \phi(z) - \bar{\phi}(\bar{z}) \) is sometimes called the "dual" boson. A slightly more general analysis extended to the spin 1/2 operators shows that the integer \( m \) in (8) is in fact the topologic charge that in the sine-Gordon model originates from the periodicity of the potential. Hence the operators \( U_m(x) \) have charge \( m \) while the operators \( V_p(x) \) are neutral. This implies in particular that in the model (5)

\[
\langle V_p \rangle \neq 0, \quad \langle U_m \rangle = 0 . \quad (9)
\]

Another point to be remarked is that the operators \( U_m(x) \) with \(|m| > 3\) are always irrelevant \((X > 2)\) as long as the perturbation in (5) is relevant \((\beta^2 < 8\pi)\).

The lattice operators \( \Sigma_j, \sigma_j \) and \( \mathcal{P}_j \) are not invariant under the \( S_3 \) symmetry and/or the exchange of the even and odd sublattices. Hence their continuum counterparts have to be sought among the \( U_m(x) \). Comparison with the known scaling dimensions shows that the matching is complete provided we take \( \beta = \sqrt{6\pi} \), what in turn implies \( X_\varepsilon = 3/2 \). The operator identifications are summarised in Table 1 from which the following correspondences can also be read:

\[
Z_3 \text{ charge} = m \text{ (mod 3)}
\]

\[
C = \text{complex conjugation}
\]
sublattice parity = $(-1)^m$.

Hence we see that in the continuum limit both the $Z_3$ symmetry and the lattice symmetry are ruled by the topologic charge $m$.

Table 1. Relevant operators on the lattice and their continuum counterparts in the sine-Gordon model.

| Lattice definition | Continuum limit | $X$ | $m$ |
|-------------------|-----------------|-----|-----|
| $\Sigma$ $(−1)^{j_1+j_2} \exp[2\pi s_j/3]$ | $U_1 = \exp[i\sqrt{2\pi/3} \hat{\varphi}]$ | 1/6 | 1 |
| $\sigma$ $\exp[2\pi s_j/3]$ | $U_{-2} = \exp[-i\sqrt{8\pi/3} \hat{\varphi}]$ | 2/3 | −2 |
| $P$ $(−1)^{j_1+j_2} \sum_i (2\delta_i,s_j - 1)$ | $U_3 + U_{-3} = \cos \sqrt{6\pi} \hat{\varphi}$ | 3/2 | ±3 |
| $\varepsilon$ $\sum_i \delta_i,s_j$ | $V_{\sqrt{\pi}} \pm V_{-\sqrt{\pi}} = \cos \sqrt{6\pi} \hat{\varphi}$ | 3/2 | 0 |

Due to the integrability of the sine-Gordon model, also the scaling antiferromagnet admits an exact scattering description. This time the elementary excitations are the soliton $A$ and antisoliton $\bar{A}$ interpolating between adjacent sine-Gordon vacua. They carry topologic charge 1 and $-1$, respectively, and then transform under the symmetries as the staggered magnetisation $\Sigma$ and $\Sigma^*$. This is what makes the difference with the ferromagnetic case at the level of the scattering theory: the sublattice parity (which plays no role in the ferromagnet) would now be violated by the fusion process (3), which is therefore forbidden. One consequence is that the last amplitude in Fig. 1 is no longer forced to vanish. The three amplitudes are the sine-Gordon ones [6],

$$S_{1AF}^A(\theta) = S_{2AF}^A(i\pi - \theta) = -\exp \left\{ \int_0^\infty dx \frac{\sinh x}{x} \frac{\sin \frac{\theta x}{\pi}}{\sinh \frac{x}{\xi}} \right\}$$

$$S_{3AF}^A(\theta) = -\frac{\sinh \frac{\pi^2}{\xi} \sinh \frac{\pi}{\xi}(\theta - i\pi)}{\sinh \frac{\pi}{\xi}(\theta - i\pi)} S_{1AF}^A(\theta),$$

evaluated at the value $\xi = 3\pi$ corresponding to $\beta = \sqrt{6\pi}$. This value falls in the sine-Gordon repulsive region in which the solitons do not form any bound state. In particular no asymptotic particle corresponding to the field $\varphi$ in (5) is present in the spectrum.

Correlation functions can be computed starting from the scattering theory through the form factor approach (see [16]). Here we only mention few straightforward predictions for the antiferromagnet dictated by the topologic charge of the operators. Defining the ‘exponential’ correlation length $\xi_\Phi$ associated to an operator $\Phi(x)$ as

$$\langle \Phi(x)\Phi^*(0) \rangle_{\text{connected}} \sim \exp(-|x|/\xi_\Phi), \quad |x| \rightarrow \infty,$$
then the following universal ratios should be observed in simulations as $T \to 0$: $\xi_\sigma / \xi_\Sigma = \xi_\xi / \xi_\Sigma = 1/2$, $\xi_\varphi / \xi_\Sigma = 1/3$.

4. Crossover from Antiferromagnetic to Ferromagnetic Behaviour

We see from Table 1 that the field theory of the scaling antiferromagnet contains a single relevant operator with the symmetry properties required to break the sublattice symmetry while keeping the $S_3$ symmetry. Therefore the action that should describe the crossover to ferromagnetic behaviour reads

$$A_{\text{cross}} = A_{AF} - \tilde{\mu} \int d^2x \mathcal{P}(x) = A_{C=1} - \int d^2x \left[ \mu \cos \sqrt{6\pi} \varphi + \tilde{\mu} \cos \sqrt{6\pi} \tilde{\varphi} \right], \quad (13)$$

and defines a one-parameter family of renormalisation group trajectories (labelled by $\mu/\tilde{\mu}$) flowing out of the gaussian ($C = 1$) fixed point. Since $\mathcal{P}(x)$ has topologic charge $\pm 3$, it reintroduces in the theory the three-particle vertex (3), so that the $Z_3$ symmetry is again manifest.

One of the trajectories described by (13) should flow into the ferromagnetic ($C = 4/5$) fixed point in the infrared limit. This trajectory marks a phase boundary across which a continuous ordering phase transition takes place. The existence of such a transition can be argued as follows. When $\tilde{\mu} = 0$ we are in the sine-Gordon model (5) in which the vacuum expectation value of the spin operator $\sigma = \exp[-i\sqrt{8\pi/3} \tilde{\varphi}]$ (i.e. the spontaneous magnetisation) vanishes according to (9). It can be argued in the spirit of Ref. [17] that, since $\mathcal{P}(x)$ is local with respect to the solitons of the theory (5), no phase transition takes place as soon as the perturbation is switched on, namely $\langle \sigma \rangle = 0$ for $\tilde{\mu} \simeq 0$. On the other hand, we can perform similar considerations for the opposite limit ($\mu \simeq 0$) of the action (13), where we are perturbing around a sine-Gordon model of the dual boson $\tilde{\varphi}(x)$. The operator $\sigma(x)$ has zero topologic charge with respect to this model and its vacuum expectation value no longer vanishes, namely $\langle \sigma \rangle \neq 0$ for $\mu \simeq 0$. These conclusions for the two limits are compatible if a phase transition takes place in between giving rise to the phase diagram of Fig. 2.

Generically, the theory (13) will not be integrable when both perturbations are switched on. There are reasons to expect, however, that the massless trajectory connecting the two fixed points is integrable. As a matter of fact, some years ago Fateev and Al. Zamolodchikov used the thermodynamic Bethe ansatz to provide evidence for an infinite series of integrable flows between the $Z_N$ parafermionic conformal field theories.
Figure 2. Phase diagram associated to the action (13).

(With central charge $2(N - 1)/(N + 2)$) and the unitary minimal models $\mathcal{M}_{N+1}$ (central charge $1 - 6/[(N + 1)(N + 2)]$) [18]. The case $N = 4$ gives a flow between the fixed points with central charges $C = 1$ and $C = 4/5$ which should correspond to the one we are discussing in connection with the Potts model and the action (13).

A relation between $Z_4$ symmetry and the Potts antiferromagnetic fixed point was suggested in Ref. [19]. It should be stressed, however, that this symmetry corresponds to an universality class which differs from that of the Potts antiferromagnet\footnote{The scaling dimensions of the spin and energy operators in the $Z_4$ model are $1/8$ and $2/3$, respectively [5]. The possibility that the “same” quantum field theory accounts for different universality classes is well known (see e.g. Ref. [20], Section 6, for a discussion and references on the argument).}, and that it is explicitly broken in the off-critical actions (5) and (13). According to the discussion of this section, the flow from $C = 1$ to $C = 4/5$ should be described by a factorised massless $S$-matrix exhibiting the $S_3$ symmetry characteristic of the 3-state Potts model. To the best of our knowledge this $S$-matrix did not appear in the literature so far.

5. Conclusion

We have seen that both the ferromagnetic and (square lattice) antiferromagnetic scaling limits of the three-state Potts model correspond to
integrable quantum field theories, and that the massless crossover trajectory should also be integrable. It would be interesting to confirm this latter point by determining the exact massless $S$-matrix. Of course, one can think of extending the investigation of this paper to other antiferromagnets possessing a critical point.

We conclude by mentioning that actions of the type (13) containing both kinds of scaling operators (7) and (8) are not uncommon in the description of scaling lattice models. For example, the Ashkin-Teller model consists of two Ising models coupled through their energy terms (four spin interaction). As long as the two Ising models are kept at the same temperature the scaling limit is described by the action (5) with $\beta$ parameterising this time the line of fixed points characteristic of the Ashkin-Teller model (see [21]). A temperature difference corresponds to the addition of the operator $\cos[4\pi\tilde{\varphi}/\beta]$. It can be argued that also this action admits a massless flow which in this case ends in the infrared limit into an Ising fixed point with $C = 1/2$.

**Acknowledgments:** I thank J. Cardy and V. Fateev for interesting discussions.

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