Magnetostatics of magnetic skyrmion crystals

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Abstract
Magnetic skyrmion crystals are topological magnetic textures arising in the chiral ferromagnetic materials with Dzyaloshinskii–Moriya interaction. The magnetostatic fields generated by magnetic skyrmion crystals are first studied by micromagnetic simulations. For Néel-type skyrmion crystals, the fields will vanish on one side of the crystal plane, which depend on the helicity; while for Bloch-type skyrmion crystals, the fields will distribute over both sides, and are identical for the two helicities. These features and the symmetry relations of the magnetostatic fields are understood from the magnetic scalar potential and magnetic vector potential of the hybridized triple-Q state. The possibility to construct magnetostatic field at nanoscale by stacking chiral ferromagnetic layers with magnetic skyrmion crystals is also discussed, which may have potential applications to trap and manipulate neutral atoms with magnetic moments.

1. Introduction

The lack of spatial inversion symmetry in chiral ferromagnets can give rise to the anisotropic exchange interaction between the neighbouring magnetic moments, i.e. the Dzyaloshinskii–Moriya (DM) mechanism [1, 2]. In contrast to the Heisenberg exchange interaction, which stabilizes the collinear magnetic structure with minimized free energy, DM interaction prefers non-collinear magnetic structures and enables the chiral ferromagnets to host the topological-protected magnetic skyrmions [3–9]. Magnetic skyrmions are particle-like topological defects in the magnetization configuration, and their swirling structures are characterized by topological skyrmion numbers [10, 11]. The exchange coupling between the magnetic skyrmions and the conduction electrons can further result in the exotic dynamics of emergent electromagnetic field [10, 12], such as topological Hall effect [13–16] and skyrmion Hall effect [17, 18]. The attractive properties of magnetic skyrmions have been intensively utilized to design and develop skyrmion-based topological electronics devices [19–21].

Current studies mainly focus on discovering new materials or revealing novel mechanisms to host and control the magnetic skyrmions. The magnetostatic field distribution generated by magnetic skyrmions, as governed by the Maxwell equations, deserves more explorations since it is interesting and important to generate and utilize the magnetic field at nanoscale. In fact, one important way to observe the magnetic skyrmions or other magnetised microstructures of magnetization is to detect their magnetic field profiles with various sensing techniques, including Lorentz transmission electron microscopy [7], magnetic force microscopy [22, 23], nitrogen-vacancy magnetometry [24–26], etc. Understanding the magnetostatic field distribution of the magnetic skyrmions will also be helpful to design skyrmion-based electronics devices [19–21]. Furthermore, magnetic skyrmions have the potential applications to design magnetic microtraps, which are used to trap and manipulate ultracold atoms [27, 28]. In this paper, we will investigate the magnetostatic fields generated by magnetic skyrmion crystals (SkXs) with different helicities, and show the possibility to construct the field distributions at nanoscale through stacking the chiral ferromagnet films for further applications.
2. Magnetostatic field of single magnetic skyrmion crystal

We consider a two-dimensional chiral ferromagnetic film placed in the external magnetic field, which can host magnetic skyrmion crystals [10, 29]. Its energy functional in terms of the normalized magnetic moments \( \{ \mathbf{m}_i \} \) on the discretized square lattice is given as [29]

\[
E(\{ \mathbf{m}_i \}) = -J \sum_{\langle i,j \rangle} \mathbf{m}_i \cdot \mathbf{m}_j - D \cdot \sum_{\langle i,j \rangle} \mathbf{m}_i \times \mathbf{m}_j - B \cdot \sum_i \mathbf{m}_i - A \sum_i m_{i,z}^2.
\]

(1)

Here, \( \mathbf{m}_i \) denotes the normalized magnetic moment at lattice site \( i \), and the summation \( \langle i,j \rangle \) is over the nearest lattice sites \( i \) and \( j \); the first term in (1) describes the ferromagnetic exchange interaction, where \( J > 0 \) is the interaction strength; the second term in (1) describes the DM interaction, where the form of \( D \) can be either \( D \mathbf{r}_i \) or \( D \mathbf{r}_i \times \hat{\mathbf{e}}_z \), with the notations \( \mathbf{r}_i = \mathbf{r} - \mathbf{r}_i \) and \( \hat{\mathbf{e}}_z = (0, 0, 1) \); the third term in (1) describes the Zeeman effect, where \( B \) is the external magnetic field; the last term describes the anisotropy effect at impurity sites \( i \), the strength of which is determined by \( A \) [29]. For the case under consideration here, the magnetic dipole–dipole interaction has negligible effect on the magnetostatic configuration [29, 30] and thus is not included in the energy functional \( E(\{ \mathbf{m}_i \}) \).

For a given parameter set \( \{ J, D, B, A \} \), the stable magnetization configuration \( \{ \mathbf{m}_i^0 \} \) is achieved by minimizing the energy functional \( E \) via the stochastic Landau–Lifshitz–Gilbert (LLG) equation (see appendix for details). Depending on the relative direction of \( D \) and \( \mathbf{r}_i \), the obtained SkXs can be classified with four different helicities [10] (\( \gamma = 0, \pi \) for Néel-type SkXs and \( \gamma = \pm \pi/2 \) for Bloch-type SkXs), where \( \mathbf{D} \cdot \mathbf{r}_i = \sin \gamma \) and \( \mathbf{D} \cdot (\mathbf{r}_i \times \hat{\mathbf{e}}_z) = \cos \gamma \). The resulting dimensionless magnetic field \( \mathbf{B}(\mathbf{r}) \) is the summation over the magnetic dipole field generated by each magnetic moment \( \mathbf{m}_i^0 \), i.e.

\[
\mathbf{B}(\mathbf{r}) = \sum_i \frac{3(\mathbf{m}_i^0 \cdot \mathbf{R}_i) \mathbf{R}_i}{\mathbf{R}_i^3} - \mathbf{m}_i^0.
\]

(2)

Here, \( \mathbf{R}_i = \mathbf{r} - \mathbf{r}_i \) denotes the displacement vector from the \( i \)-th lattice site \( \mathbf{r}_i \) to the spatial point \( \mathbf{r} \).

A unit cell with periodic boundary condition is used to construct the infinite square lattice of the discretized magnetic moments. In order to get a perfect SkX, where the magnetic skyrmions form a triangular array [10], a suitable size should be chosen for the unit cell. In figure 1, the magnetic SkXs with four different helicities \( \gamma \) and the associated magnetic field distributions are shown within a unit cell of \( 80 \times 69 \) lattice sites, which approximately satisfies the ratio \( 2: \sqrt{3} \) for the triangular array. Here, we set the parameters as [29] \( J = 1 \) meV, \( D = \pm 0.18 \) meV, \( B = (0, 0, 0.02) \) meV, \( A = 0.2 \) meV, while the concentration of impurity sites is set as 0%.

As shown in figure 1, the magnetic moment in the center of each skyrmion will point towards the \( -\hat{\mathbf{e}}_z \) direction, which is opposite to the applied magnetic field. As expected, all the four calculated magnetostatic fields have the same period as the original SkXs, and the field strength will decay at distance away from the crystal plane at \( z = 0 \). Impressively, for the Néel-type SkXs with helicity \( \gamma = 0 \) (\( \gamma = \pi \)), the field strength in the upper half-space \( z > 0 \) is much stronger (weaker) than that in the lower half-space \( z < 0 \), and the field components satisfy the symmetry relations \( \mathbf{B}_{\gamma=0,z}(x, y, z) = -\mathbf{B}_{\gamma=\pi,z}(x, y, -z) \) and \( \mathbf{B}_{\gamma=\pi,z}(x, y, z) = \mathbf{B}_{\gamma=0,z}(x, y, -z) \), as shown in figure 1(a), (b). For the Bloch-type SkXs with helicity \( \gamma = \pm \pi/2 \), the strength of magnetostatic fields show a symmetric distribution over the crystal plane, and they are exactly the same, i.e. \( \mathbf{B}_{\gamma=\pm \pi/2}(\mathbf{r}) = \mathbf{B}_{\gamma=-\pm \pi/2}(\mathbf{r}) \), which implies that the helicity plays no role here. Moreover, the symmetry relations \( \mathbf{B}_{\gamma=\pm \pi/2,y}(x, y, z) = -\mathbf{B}_{\gamma=-\pm \pi/2,y}(x, y, -z) \) and \( \mathbf{B}_{\gamma=\pm \pi/2,x}(x, y, z) = -\mathbf{B}_{\gamma=-\pm \pi/2,x}(x, y, -z) \) also exist for the Bloch-type SkXs, as shown in figure 1(c), (d).

Due to the anisotropy term in equation (1), the magnetic skyrmions will be pinned by the presence of impurities, which then makes the magnetic SkXs imperfect. This phenomena is shown in the magnetization configurations in figure 2, where the impurity concentration is set as 3%. The lattice structures of the magnetic SkXs have been distorted by the impurities, which is accompanied by the deformation of the magnetostatic field distributions. However, we found that the main features of the magnetostatic field described above are still robust against the impurities. Explicitly, the magnetostatic fields of the distorted Néel-type SkXs (\( \gamma = 0 \) or \( \pi \)) remain very weak on one side of the crystal plane, while the symmetry relations \( \mathbf{B}_{\gamma=\pm \pi/2,y}(x, y, z) = -\mathbf{B}_{\gamma=-\pm \pi/2,y}(x, y, -z) \) and \( \mathbf{B}_{\gamma=\pm \pi/2,x}(x, y, z) = -\mathbf{B}_{\gamma=-\pm \pi/2,x}(x, y, -z) \) for the magnetostatic fields also hold for the distorted Bloch-type SkXs (\( \gamma = \pm \pi/2 \)). The same phenomena has also been observed in the simulation results for other impurity concentrations up to 5%.
3. Magnetic potentials

3.1. Magnetic scalar potential

The magnetic Skyrmions (SkXs) can be analytically described as the hybridized triple-$Q$ state, namely, the superposition of three helical states with the same pitch length and chirality on the uniform ferromagnetic magnetization $\mathbf{m}_0$ align along the $\hat{z}$ direction \[6, 10\].

$$\mathbf{m}(\mathbf{r}) = \mathbf{m}_0 \delta(z) + \mathcal{A} \sum_{i=1}^{3} \left( \hat{\mathbf{e}}_i \cos(\mathbf{Q}_i \cdot \mathbf{r}) + \hat{\mathbf{e}}_i \sin(\mathbf{Q}_i \cdot \mathbf{r}) \right) \delta(z).$$

(3)

Here, $\mathcal{A}$ denotes the magnetization of a single helical state; the three wavevectors $\mathbf{Q}_i = 1, 2, 3$ form an angle of $2\pi/3$ with each other in the crystal plane and satisfy the relation $\mathbf{Q}_i \cdot \mathbf{Q}_j = 0$. $\hat{\mathbf{e}}_i$ is the unit vector normal to the crystal plane as defined above; $\hat{\mathbf{e}}_i$ are determined by the helicity $\gamma$, where $\hat{\mathbf{e}}_1 = -\cos \gamma \hat{\mathbf{Q}}_i$ for Néel-type SkXs and $\hat{\mathbf{e}}_2 = \sin \gamma \hat{\mathbf{e}}_1 \times \hat{\mathbf{Q}}_i$ for Bloch-type SkXs.

Equation (3) implies that the magnetostatic field can be decomposed into two parts $\mathbf{B}_m(\mathbf{r})$ and $\mathbf{B}_p(\mathbf{r})$, which are generated by the perpendicular magnetization component $\mathbf{m}_p(\mathbf{r})$ and the planar magnetization component $\mathbf{m}_l(\mathbf{r})$, respectively. Since only the planar magnetization component is related to the helicity $\gamma$, $\mathbf{B}_l(\mathbf{r})$ of the four types of SkXs in figure 1 should be the same, and $\mathbf{B}_p(\mathbf{r})$ will be the characteristic quality to distinguish their helicities.

It is informative to understand the magnetostatic field generated by $\mathbf{m}(\mathbf{r})$ from the viewpoint of magnetic scalar potential $\Phi(\mathbf{r})$, which is defined as $\mathbf{B}(\mathbf{r}) = -\nabla \Phi(\mathbf{r})$ and is given by Poisson’s equation $\nabla^2 \Phi(\mathbf{r}) = -\rho_m(\mathbf{r})$ \[31\]. Here, $\rho_m(\mathbf{r}) = -\nabla \cdot \mathbf{m}(\mathbf{r})$ is the effective ‘magnetic charge’ of the SkXs, and the vacuum permeability $\mu_0$ is temporarily neglected for simplicity. For the planar magnetization component $\mathbf{m}_l(\mathbf{r})$, one has

$$\rho_{m,l}(\mathbf{r}) = \mathcal{A} \mathbf{Q} \cos \gamma \sum_{i=1}^{3} \cos(\mathbf{Q}_i \cdot \mathbf{r}) \delta(z).$$

(4)

For Bloch-type SkXs with $\gamma = \pm \pi/2$, the magnetic charge $\rho_{m,l}$ will vanish, thus the generated magnetostatic field will be solely determined by $\mathbf{m}_l(\mathbf{r})$ and is independent on the helicity. For Néel-type SkXs, the polarity of

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Figure 1. The magnetization configurations $\mathbf{m}(\mathbf{r})$ and the associated magnetostatic field distributions $\mathbf{B}(\mathbf{r})$ (arbitrary unit) in the $x$-$y$ planes at $z = \pm 5$ and in the $x$-$z$ plane at $y = 35$ for magnetic skyrmion crystals with different helicities $\gamma$: (a) $\gamma = 0$; (b) $\gamma = \pi$; (c) $\gamma = -\pi/2$; (d) $\gamma = \pi/2$. Here, the lattice constant is set as 1; the parameters are set as $J = 1$ meV, $D = \pm 0.18$ meV, $B = (0, 0, 0.02)$ meV, $A = 0.2$ meV, and the impurity concentration is 0%.
magnetic charge $\rho_{m,\perp}(r)$ will be dependent on the helicity. Therefore, the magnetostatic field of SkXs can be classified into three types according to the helicity $\gamma = 0, \pi, \pm \frac{\pi}{2}$, respectively.

By solving the Poisson’s equation, the magnetic scalar potential $\Phi_{\parallel}(r)$ generated by the planar magnetization $\mathbf{m}_{\parallel}(r)$ is obtained as

$$\Phi_{\parallel}(r) = \cos \gamma \frac{A}{2} e^{-Q|z|} \sum_{i=1}^{3} \cos(Q_i \cdot \mathbf{r}), \quad (5)$$

then the corresponding magnetostatic field $\mathbf{B}_{\parallel}(r)$ is

$$\mathbf{B}_{\parallel}(r) = \cos \gamma \frac{A Q}{2} \sum_{i=1}^{3} \hat{n}(Q_i, \mathbf{r}), \quad (6)$$

where the unit vector $\hat{n}(Q_i, \mathbf{r})$ is defined as

$$\hat{n}(Q_i, \mathbf{r}) = \hat{Q}_i \sin(Q_i \cdot \mathbf{r}) + \hat{e}_z \text{sgn}(z) \cos(Q_i \cdot \mathbf{r}). \quad (7)$$

Similarly, the magnetic charge $\rho_{m,\perp}(r)$ for the perpendicular magnetization $\mathbf{m}_{\perp}(r)$ is

$$\rho_{m,\perp}(r) = -(m_0 + A \sum_{i=1}^{3} \cos(Q_i \cdot \mathbf{r})), \quad (8)$$

then the magnetic scalar potential $\Phi_{\perp}(r)$ and the corresponding magnetostatic field $\mathbf{B}_{\perp}(r)$ will be

$$\Phi_{\perp}(r) = \text{sgn}(z) \frac{A}{2} e^{-Q|z|} \sum_{i=1}^{3} \cos(Q_i \cdot \mathbf{r}), \quad (9)$$

$$\mathbf{B}_{\perp}(r) = \text{sgn}(z) \frac{A Q}{2} \sum_{i=1}^{3} \hat{n}(Q_i, \mathbf{r}). \quad (10)$$

To further verify the insight from the analytical results above, the magnetization configurations $\mathbf{m}(r)$ in figure 1 are decomposed into the planar and perpendicular components $\mathbf{m}_{\parallel}(r)$ and $\mathbf{m}_{\perp}(r)$, respectively, and the

![Figure 2](image-url)
effective magnetic charge densities $\rho_{ij}(r)$ are also obtained from $m_i(r)$. As shown in figure 3, their associated magnetostatic fields $B_i(r)$ and $B_0(r)$ calculated with equation (2) indeed agree well with equation (6) and (10).

The features of the magnetostatic fields shown in figure 1 can now be well understood with equation (6) and (10). First, each component of the fields has the same modulation period in the $x$-$y$ plane as the underlying SkXSs, and will decay exponentially with characteristic length $1/\lambda$ away from the crystal plane. Second, $\Phi_{ij}(r)$ and $\Phi_{0}(r)$ can be regarded as the contributions from ‘inner’ and ‘outer’ magnetic charge density, which are even and odd function of $z$, respectively, and their summation will vanish at the down (upper) half-plane for Néel-type SkXSs with helicity $\gamma = 0$ ($\gamma = \pi$); for Bloch-type SkXSs $\gamma = \pm \frac{\pi}{2}$, $\Phi_{ij}(r)$ and $B_i(r)$ will vanish, and the magnetostatic fields will be the same no matter what the helicities are. In fact, the magnetization configurations of Néel-type SkXSs form the so-called ‘Halbach arrays’ at nanoscale [32–34], which have the feature of ‘one-sided flux’ [32]. Finally, the symmetry relations of $B_i(r)$ revealed in figure 1 can be easily verified with the expressions of equation (6) and (10). It is worth noting that the analytical results here give the absolutely zero field on one side of the Néel-type SkXSs, which is in contrast to the weak residual field shown in the numerical results in figure 1. The reason is that the triple-$Q$ state can only approximately describe the magnetization configuration of the magnetic SkXSs, which causes the slight discrepancy between the two approaches.

3.2. Magnetic vector potential

An alternative viewpoint to understand the magnetostatic field is based on the magnetic vector potential $A(r)$ generated by the ‘magnetic current density’ $J_m(r) = \nabla \times m(r)$ [31], which is calculated to be

$$J_m(r) = AQ \sum_{i=1}^{3} (w(Q_i, r) \gamma \delta(z) + (\hat{e}_z \times \hat{e}_i) \sin(Q_i \cdot r) \delta'(z)),$$

(11)

where the vector $w(Q_i, r)$ is defined as

$$w(Q_i, r) = (\hat{e}_z \times \hat{Q}_i) \sin(Q_i \cdot r) + \hat{e}_z \sin \gamma \cos(Q_i \cdot r).$$

(12)

In the Coulomb gauge ($\nabla \cdot A = 0$), the magnetic vector potential $A(r)$ satisfies Poisson’s equation

$$\nabla^2 A(r) = -J_m(r),$$

which results in

$$A(r) = \frac{A}{2} e^{-Q|z|} \sum_{i=1}^{3} (w(Q_i, r) - \text{sgn}(z) (\hat{e}_z \times \hat{e}_i) \sin(Q_i \cdot r)).$$

(13)

Therefore, the current density $J_m(r)$ and magnetic vector potential $A(r)$ can also be decomposed into ‘inner’ and ‘outer’ contributions, which are even and odd functions of $z$, respectively. For Néel-type SkXSs ($\gamma = 0, \pi$), there is no $z$-component in $J_m(r)$ and $A(r)$, and $A(r)$ will vanish at the down half-plane when $\gamma = 0$ or upper half-plane when $\gamma = \pi$, considering that $\hat{e}_i = -\cos \frac{\gamma}{2} \hat{Q}_i$. For Bloch-type SkXSs ($\gamma = \pm \frac{\pi}{2}$), the second term in (13) is an irrotational vector field, which suggests that the ‘outer’ current density has no contribution to the magnetic field in this case.

With the magnetic vector potential $A(r)$ in equation (13), the magnetic field $B(r)$ is straightforwardly obtained as

![Figure 3](image-url)
\[ \mathbf{B}(\mathbf{r}) = \frac{A Q}{2} e^{-Q|z|} \sum_{i=1}^{3} (\text{sgn}(z) \hat{Q}_i - \hat{e}_z) \sin(\mathbf{Q}_i \cdot \mathbf{r}) + \frac{A Q}{2} e^{-Q|z|} \sum_{i=1}^{3} \hat{e}_z \cos(\mathbf{Q}_i \cdot \mathbf{r}) + \sin \gamma \frac{A Q}{2} e^{-Q|z|} \sum_{i=1}^{3} (\hat{e}_z \times \hat{Q}_i) \sin(\mathbf{Q}_i \cdot \mathbf{r}). \]

When \( \gamma = 0, \pi \), equation (14) will reduce to the magnetic field of Néel-type SkyXs, i.e. the summation of \( \mathbf{B}(\mathbf{r}) \) and \( \mathbf{B}(\mathbf{r}) \); when \( \gamma = \pm \frac{\pi}{2} \), equation (14) will reduce to the magnetic field of Bloch-type SkyXs, i.e. \( \mathbf{B}(\mathbf{r}) \).

Therefore, the results obtained from the 'magnetization current' picture are consistent with the 'magnetic charge' picture.

4. Magnetostatic field between two magnetic skyrmion crystals

We now discuss the magnetostatic fields generated by stacking two chiral ferromagnetic layers, which provide us more flexibility to construct magnetic field at nanoscale. Considering that two layers with Bloch-type SkyXs are located at the planes \( z = \pm d/2 \), and their magnetization configurations are \( \mathbf{m}_i(\mathbf{r}) \), there can be two types of magnetostatic fields between the two layers depending on the relative direction of \( \hat{e}_{z, \pm} \). For the parallel case with \( \hat{e}_{z, \pm} = \hat{e}_z \), the magnetostatic field \( \mathbf{B}^P(\mathbf{r}) \) is

\[ \mathbf{B}^P(\mathbf{r}) = -A Q e^{-Qd} \sum_{i=1}^{3} [\sin(\mathbf{Q}_i \cdot \mathbf{r}) \sinh(Qz) \hat{Q}_i + \cos(\mathbf{Q}_i \cdot \mathbf{r}) \sinh(Qz) \hat{e}_z]. \]  \( \text{(15)} \)

While for the antiparallel case with \( \hat{e}_{z, \pm} = -\hat{e}_{z, \mp} = \hat{e}_z \), the magnetostatic field \( \mathbf{B}^{AP}(\mathbf{r}) \) will be

\[ \mathbf{B}^{AP}(\mathbf{r}) = -A Q e^{-Qd} \sum_{i=1}^{3} [\sin(\mathbf{Q}_i \cdot \mathbf{r}) \sinh(Qz) \hat{Q}_i + \cos(\mathbf{Q}_i \cdot \mathbf{r}) \cosh(Qz) \hat{e}_z]. \]  \( \text{(16)} \)

Equation (15) and (16) suggest that two layers of SkyXs can generate magnetostatic fields periodically modulated in the \( x-y \) plane, and the field magnitudes depend exponentially on the layer distance \( d \). With the approximate relations \( e^{-Qd} \approx 1, \sinh(Qz) \approx Qz \) and \( \cosh(Qz) \approx 1 \) when \( d \ll 1/Q \), \( \mathbf{B}^P(\mathbf{r}) \) will be proportional to \( z \) and thus has a constant gradient along the \( \hat{e}_z \) direction, while the \( z \)-component of \( \mathbf{B}^{AP}(\mathbf{r}) \) will be much stronger than the planar component and is near constant along the \( \hat{e}_z \) direction. Besides, the magnetostatic fields can be further manipulated by translating or rotating the SkyXs, which then give more types of field distributions. Considering that the magnetostatic fields of other magnetised microstructures have been successfully applied to trap and manipulate ultracold atoms in the past [27, 28], we expect that SkyXs would also play an unique role in atom optics.

Finally, we estimate the amplitudes of the magnetostatic field and field gradient, which are \( \mathbf{B} \sim \mu_0 A Q e^{-Qd} \) and \( \nabla \mathbf{B} \sim \mu_0 A Q e^{-Qd} \), respectively. Here, we retrieve the vacuum permeability \( \mu_0 = 4\pi \times 10^{-7} \text{T m A}^{-1} \). Assuming a ferromagnetic film with the magnetization \( M_s \sim 1000 \text{kA m}^{-1} \), the thickness \( t \sim 5 \text{ nm} \), the period of SkyXs \( \lambda \sim 50 \text{ nm} \), the layer distance \( d = 50 \text{ nm} \) and utilizing the relations \( Q = 2\pi/\lambda, A = M_s t \), one gets \( B \sim 1.5 \text{ mT} \) and \( \nabla \mathbf{B} \sim 1.8 \times 10^4 \text{ T cm}^{-1} \). By decreasing the layer distance \( d \), the amplitudes \( \mathbf{B} \) and \( \nabla \mathbf{B} \) can be further increased exponentially. Therefore, the magnetostatic fields generated by the SkyXs are strong enough to trap and manipulate neutral atoms with magnetic moments at nanoscale by controlling these topological magnetic textures.

5. Conclusion

In conclusion, we have revealed the fundamental features of magnetostatic fields generated by magnetic skyrmion crystals based on micromagnetic simulations, which can further be verified from the analytical solutions within the triple-Q state description. The field generated by Néel-type SkyX distributes only on one side of the crystal plane depending on its helicity, while the field of Bloch-type SkyX distributes on both sides of the crystal plane and is irrelevant to the helicity. The symmetry relations between the field components are also discussed, and the phenomena revealed here is robust in the presence of impurities. Finally, we have investigated the magnetostatic field distributions by stacking two magnetic SkyXs together. The results here will not only deepen our understanding of the magnetostatic characteristics of SkyXs, which are important to observe SkyXs with field sensing techniques and design skyrmion-based electronics devices, but also provide the possibility to trap and manipulate neutral atoms with magnetic moments at nanoscale by controlling these topological magnetic textures.

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Appendix. Micromagnetic Simulation with stochastic LLG equation

The micromagnetic simulation is performed with the stochastic LLG equation \[ \frac{d\mathbf{m}_i}{dt} = -\gamma \mathbf{m}_i \times \mathbf{B}^\text{eff}_i + \frac{\alpha}{M_i} \mathbf{m}_i \times \frac{d\mathbf{m}_i}{dt}, \] \hspace{1cm} (A.1)

where the effective field \( \mathbf{B}^\text{eff}_i = -\frac{\partial \mathcal{H}[\mathbf{m}_i]}{\partial \mathbf{m}_i} + \sqrt{2} \mathbf{W}(t) \) consists of the deterministic part from the energy functional and the random part due to thermal fluctuation. Especially, we have \( \epsilon = \frac{2k_B T}{M_i V} \), where \( \gamma \) is the gyromagnetic ratio, \( \alpha \) is the Gilbert damping coefficient, \( M_i \) is the saturation magnetization, \( V \) is the volume of magnetic moment, \( k_B \) is the Boltzmann constant, \( T \) is the temperature, and \( \mathbf{W}(t) \) is the normalized Gaussian white-noise process \[35\]. The thermal fluctuation is included for the magnetization configuration to overcome possible energy barriers.

For a given parameter set \( \{ J, D, B, A \} \), the stable magnetization configuration \( \{ \mathbf{m}_i \} \) is achieved by simulating the stochastic LLG equation \( (A.1) \) numerically. The initial magnetization configuration is set in an arbitrary distribution, and the temperature is decreased slowly from a high initial value to zero. Since only the stationary solution of the LLG equation is needed, we can set arbitrary values for \( \gamma, \alpha \) during the practical simulations without hindering the final results. Here, we have set \( \gamma = \alpha = 1 \), and the dimensionless amplitude of the random magnetic field \( \epsilon \) is slowly decreased from 1 to 0.

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