Present status of experimental determination of $|V_{cb}|$*

Marina Artuso$^1$ and Elisabetta Barberio$^2$

October 28, 2018

$^1$Syracuse University, Syracuse, NY 13244, USA
$^2$CERN, Geneva, Switzerland

Abstract

The present status of our knowledge of the magnitude of the quark mixing parameter $|V_{cb}|$ is reviewed, with particular emphasis on the factors affecting experimental and theoretical errors and on prospects for a more precise determination.

*An abbreviated version will be included in PDG2002 as a minireview on $|V_{cb}|$. 
1 Introduction

In the framework of the Standard Model, the quark sector is characterized by a rich pattern of flavor-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

\[
V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\] (1)

Since the CKM matrix must be unitary, it can be expressed as a function of only four parameters. A commonly used approximate parameterization was originally proposed by Wolfenstein [2]. It reflects the hierarchy between the magnitude of matrix elements belonging to different generations. Very frequently it is quoted in the approximation valid only to \(\lambda^3\). We need to carry out this expansion further in order to incorporate CP violation in neutral \(K\) decays. This expression, accurate to \(\lambda^3\) for the real part and \(\lambda^5\) for the imaginary part, is given by:

\[
\begin{pmatrix}
1 - \lambda^2/2 & \lambda - i\eta A\lambda^3(1 - \rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 - i\eta A\lambda^4 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & A\lambda^2(1 + i\eta\lambda^2)
\end{pmatrix}.
\] (2)

The parameter \(\lambda\) is well measured as 0.2196\(\pm\)0.0023 [1], constraints exist on \(\rho\) and \(\eta\) from measurements of \(V_{ub}\) and \(B^0\overline{B}^0\) mixing. This report focuses on the magnitude of the CKM element \(|V_{cb}|\), related to the Wolfenstein parameter \(A\) [4].

Two different methods have been used to extract this parameter from data: the exclusive measurement, where \(|V_{cb}|\) is extracted by studying exclusive \(B \to D^*\ell\nu\) and \(B \to D\ell\nu\) decay processes; and the inclusive measurement, which uses the semileptonic width of b-hadron decays. Theoretical estimates play a crucial role in extracting \(|V_{cb}|\) and an understanding of their uncertainties is very important.

2 Exclusive \(|V_{cb}|\) determination

The exclusive \(|V_{cb}|\) determination is obtained studying the \(B \to D^*\ell\nu\) and \(B \to D\ell\nu\) decays, using Heavy Quark Effective Theory (HQET), an exact theory in the limit of infinite quark masses. Presently the \(B \to D\ell\nu\) transition provides a less precise value and is used as a check.

2.1 The decay \(B \to D^*\ell\nu\) in HQET

HQET predicts that the differential partial decay width for this process, \(d\Gamma/dw\), is related to \(|V_{cb}|\) through:

\[
\frac{d\Gamma}{dw}(B \to D^*\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3} \mathcal{K}(w)\mathcal{F}(w)^2,
\] (3)
where \( w \) is the inner product of the \( B \) and \( D^* \) meson 4-velocities, \( K(w) \) is a known phase space factor and the form factor \( F(w) \) is generally expressed as the product of a normalization factor \( F(1) \) and a function, \( g(w) \), constrained by dispersion relations [3].

There are several different corrections to the infinite mass value \( F(1) = 1 \) [4]:

\[
F(1) = \eta_{\text{QED}} \eta_A \left[ 1 + \delta_{1/m_Q^2} + \ldots \right] \tag{4}
\]

where \( Q = c \) or \( b \). By virtue of Luke’s theorem [5], the first term in the non-perturbative expansion in powers of \( 1/m_Q \) vanishes. QED corrections up to leading logarithmic order give \( \eta_{\text{QED}} \approx 1.007 \) [1] and QCD radiative corrections to two loops give \( \eta_A = 0.960 \pm 0.007 \) [2]. Different estimates of the \( 1/m_Q^2 \) corrections, involving terms proportional to \( 1/m_Q^2 \), \( 1/m_c^2 \) and \( 1/(m_b m_c) \) have been performed in a quark model [7], [8], with OPE sum rules [1], and, more recently, with an HQET based lattice gauge calculation [10]. The value from this quenched lattice HQET calculation is \( 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014} \approx 0.000 \pm 0.006 \). The errors quoted reflect the statistical accuracy, the matching error, the lattice finite size, the uncertainty in the quark masses and an estimate of the error induced by the quenched approximation, respectively. The central value obtained with OPE sum rules is similar, with an error of \( \pm 0.04 \) [11]. Consequently, we will use \( F(1) = 0.91 \pm 0.04 \) [11].

The analytical expression of \( F(w) \) is not known a-priori, and this introduces an additional uncertainty in the determination of \( F(1)|V_{cb}| \). First measurements of \( |V_{cb}| \) were performed assuming a linear approximation for \( F(w) \). It has been shown [12] that this assumption is not justified and that linear fits systematically underestimate the extrapolation at zero recoil \( (w = 1) \) by about 3%. Most of this effect is related to the curvature of the form factor, and does not depend strongly upon the details of the non-linear shape chosen [12]. All recent published results use a non-linear shape for \( F(w) \), approximated with an expansion around \( w = 1 \) [13]. \( F(w) \) is parameterized in terms of the variable \( \rho^2 \), which is the slope of the form-factor at zero recoil of Ref. [13].

### 2.2 Experimental techniques to study the decay \( B \to D^*\ell\nu \).

The decay \( B \to D^*\ell\nu \) has been studied in experiments performed at the \( \Upsilon(4S) \) center of mass energy and at the \( Z^0 \) center of mass energy at LEP. At the \( \Upsilon(4S) \), experiments have the advantage that the \( w \) resolution is quite good. However, they have more limited statistics near \( w = 1 \) in the decay \( B^0 \to D^{*+}\ell\nu \), because of the lower reconstruction efficiency of the slow pion, from the \( D^{*+} \to \pi^+D^0 \) decay. The decay \( B^- \to D^{0}\ell\nu \) is not affected by this problem and CLEO [14] uses both channels. In addition, kinematic constraints enable \( \Upsilon(4S) \) experiments to identify the final state including the \( D^* \) without a large contamination from the poorly known semileptonic decays including a hadronic system heavier than \( D^* \), commonly identified as \( \text{‘D}^{**} \). At LEP, \( B^0 \)'s are produced with a large momentum (about 30 GeV on average). This makes the determination of \( w \) dependent upon the neutrino four-momentum reconstruction, thus giving a relatively poor resolution and limited physics background rejection capabilities. By contrast, LEP experiments benefit from an efficiency only mildly dependent upon \( w \).
Table 1: Experimental results as published by the collaborations. LEP numbers use theoretical predictions for $R_1$ and $R_2$. The published ALEPH result is obtained using a linear fit and the old Isgur-Wise model \cite{26} for $D^{*\ast}$. The updated ALEPH numbers (used in our average) are obtained using the same fit parameterization and $D^{*\ast}$ models as the other LEP experiments \cite{27}. The Belle result quoted here uses $R_1$ and $R_2$ from CLEO data.

Experiments determine the product $(\mathcal{F}(1) \cdot |V_{cb}|)^2$ by fitting the measured $d\Gamma/dw$ distribution. Measurements at the $\Upsilon(4S)$ have been performed by CLEO \cite{14} and Belle \cite{15}. Figure 1 shows the latest CLEO measurement \cite{14} of $\mathcal{F}(w)|V_{cb}|$ as a function of $w$. At LEP data are available from ALEPH \cite{16}, DELPHI \cite{17} and OPAL \cite{18}. DELPHI fits for $\mathcal{F}(w)|V_{cb}|$, including a free-curvature parameter; the result of this fit, Fig. 2, agrees with the shape parameterization of \cite{13}, used by all experiments. Both the CLEO and DELPHI spectra are corrected for smearing, as well as for efficiency.

At LEP, the dominant source of systematic error is the uncertainty on the contribution to $d\Gamma/dw$ from semileptonic $B$ decays with final states including a hadron system heavier than the $D^*$, either narrow orbitally excited charmed meson or non-resonant or broad species. The existence of narrow resonant states is well established \cite{1} and a signal of a broad resonance has been seen by CLEO \cite{19}, but the decay characteristics of these states in $b$-hadron semileptonic decays have large uncertainties. The average of ALEPH \cite{20}, CLEO \cite{21} and DELPHI \cite{22} narrow state branching fractions show that the ratio $R_{\ast\ast} = \frac{B(B\to D\ell\nu)}{B(B\to D_1\ell\nu)}$ is smaller than one ($< 0.6$ at 95% C.L. \cite{23}), in disagreement with HQET models where an infinite quark mass is assumed \cite{24}, but in agreement with models which take into account finite quark mass corrections \cite{25}. Hence, LEP experiments use the treatment of narrow $D^{*\ast}$ proposed in \cite{25}, which accounts for $O(1/m_c)$ corrections. Ref. \cite{25} provides several possible approximations of the form factors, that depend on five different expansion schemes and on three input parameters. To calculate the systematic errors each proposed scheme is tested, with the relevant input parameters varied over a range consistent with the experimental limit on $R_{\ast\ast}$. The quoted systematic error is the maximal difference from the central value obtained with this method. Broad resonances or other non-resonant terms may not be modelled correctly with this approach.

Table \ref{tab:results} summarizes all published data as quoted in the original papers. To combine the published data, the central values and the errors of $\mathcal{F}(1)|V_{cb}|$ and $\rho^2$ are re-scaled to the same set of input parameters and their quoted uncertainties. These common inputs are listed in

| experiment     | $\mathcal{F}(1)|V_{cb}|\times10^3$ | $\rho^2$       | Corr_{stat} |
|----------------|----------------------------------|----------------|-------------|
| ALEPH published| 31.9±1.8±1.9                     | 0.31±0.17±0.08 | 92%         |
| ALEPH update   | 31.5±2.1±1.3                     | 0.58±0.25±0.11 | 94%         |
| DELPHI         | 35.5±1.4±2.4                     | 1.34±0.14±0.23 | 94%         |
| OPAL           | 37.1±1.0±2.0                     | 1.21±0.12±0.20 | 90%         |
| Belle          | 35.8±1.9±1.8                     | 1.45±0.16±0.20 | 90%         |
| CLEO           | 43.1±1.3±1.8                     | 1.61±0.09±0.21 | 86%         |
Table 2: Values of the most relevant parameters affecting the measurement of $|V_{cb}|$. The three $D^{**}$ production rates are fully correlated.

Table 2. The $\mathcal{F}(1)|V_{cb}|$ values used for this average are extracted using the parametrization in [14], based on the experimental determinations of the vector and axial form factor ratios $R_1$ and $R_2$ [30]. The LEP data, which originally used theoretical values for these ratios, are re-scaled accordingly [27]. Table 3 summarized the corrected data. The averaging procedure [27] takes into account statistical and systematic correlations between $\mathcal{F}(1)|V_{cb}|$ and $\rho^2$. Averaging the measurements in Table 3, we get:

$$\mathcal{F}(1)|V_{cb}| = (38.3 \pm 1.0) \times 10^{-3}$$

and

$$\rho^2 = 1.5 \pm 0.13$$

with a confidence level [1] of 5.1%. The error ellipses for the corrected measurements and for the world average are shown in Figure 3.

The main contributions to the $\mathcal{F}(1)|V_{cb}|$ systematic error are from the uncertainty on the $B \rightarrow D^{**}\ell \nu$ shape and $\mathcal{B}(b \rightarrow B_d)$, $(0.57 \times 10^{-3})$, fully correlated among the LEP experiments, the branching fraction of $D$ and $D^*$ decays, $(0.4 \times 10^{-3})$, fully correlated among all the experiments, and the slow pion reconstruction from Belle and CLEO which are uncorrelated, $(0.28 \times 10^{-3})$. The main contribution to the $\rho^2$ systematic error is from the uncertainties in the measured values of $R_1$ and $R_2$ (0.12), fully correlated among experiments. Because of the large contribution of this uncertainty to the non-diagonal terms of the covariance matrix, the averaged $\rho^2$ is higher than one would naively expect.

Using $\mathcal{F}(1) = 0.91 \pm 0.04$ [11], we get $|V_{cb}| = (42.1 \pm 1.1_{\text{exp}} \pm 1.9_{\text{theo}}) \times 10^{-3}$. The dominant error is theoretical, but there are good prospects that lattice gauge calculations will improve significantly the accuracy of their estimate.

---

1 The $\chi^2$ per degree of freedom is less than 2, and we do not scale the error.
Table 3: Experimental results after the correction to common inputs and world average. The LEP numbers are corrected to use $R_1$ and $R_2$ from CLEO data. $\rho^2$ is the slope of the form-factor at zero recoil as defined in [13].

2.3 The decay $B \to D\ell\nu$

The study of the decay $B \to D\ell\nu$ poses new challenges both from the theoretical and experimental point of view.

The differential decay rate for $B \to D\ell\nu$ can be expressed as:

$$\frac{d\Gamma_D}{dw}(B \to D\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3}K_D(w)G(w)^2,$$

where $w$ is the inner product of the $B$ and $D$ meson 4-velocities, $K_D(w)$ is the phase space and the form factor $G(w)$ is generally expressed as the product of a normalization factor $G(1)$ and a function, $g_D(w)$, constrained by dispersion relations [3].

The strategy to extract $G(1)|V_{cb}|$ is identical to that used for the $B \to D^*\ell\nu$ decay. However, in this case there is no suppression of $1/m_Q$ (i.e. no Luke theorem) and corrections and QCD effects on $G(1)$ are calculated with less accuracy than $F(1)$ [31] [32]. Moreover, $d\Gamma_D/dw$ is more heavily suppressed near $w = 1$ than $d\Gamma_{D^*}/dw$ due to the helicity mismatch between initial and final states. This channel is much more challenging also from the experimental point of view as it is hard to isolate from the dominant background $B \to D^*\ell\nu$ as well as from fake $D$-$\ell$ combinations. Thus, the extraction of $|V_{cb}|$ from this channel is less precise than the one from the $B \to D^*\ell\nu$ decay. Nevertheless, the $B \to D\ell\nu$ channel provides a consistency check and allows a test of heavy-quark symmetry [32] through the measurement of the form factor $G(w)$, as HQET predicts the ratio $G(w)/F(w)$ to be very close to one.

Belle [33] and ALEPH [16] studied the $B^0 \to D^+\ell^-\bar{\nu}$ channel, while CLEO [34] studied both $B^+ \to D^0\ell^+\bar{\nu}$ and $B^0 \to D^+\ell^-\bar{\nu}$ decays. The published results are shown in Table 4 as well as the results scaled to common inputs. Averaging the latter data, using the procedure of [27], we get $G(1)|V_{cb}| = (41.3 \pm 4.0) \times 10^{-3}$ and $\rho^2_D = 1.19 \pm 0.19$, where $\rho^2_D$ is the slope of the form-factor given in [13] at zero recoil.
Table 4: Experimental results before and after the correction to common inputs and world average. $\rho_D^2$ is the slope of the form-factor given in [13] at zero recoil.

| Experiment | $|V_{cb}|(\times 10^3)$ | $\rho_D^2$ |
|------------|-------------------------|-----------|
| Published values | | |
| ALEPH | 31.1 ± 9.9 ± 8.6 | 0.20 ± 0.98 ± 0.50 |
| Belle | 41.1 ± 4.4 ± 5.2 | 1.12 ± 0.22 ± 0.14 |
| CLEO | 44.4 ± 5.8 ± 3.7 | 1.27 ± 1.25 ± 0.14 |
| Scaled values | | |
| ALEPH | 37.7 ± 9.9 ± 6.5 | 0.90 ± 0.98 ± 0.38 |
| Belle | 41.2 ± 4.4 ± 5.1 | 1.12 ± 0.22 ± 0.14 |
| CLEO | 44.6 ± 5.8 ± 3.5 | 1.27 ± 0.25 ± 0.14 |
| World average | 41.3 ± 2.9 ± 2.7 | 1.19 ± 0.15 ± 0.12 |

Theoretical predictions for $G(1)$ are consistent: 1.03 ± 0.07 [39], and 0.98 ± 0.07 [32]. A quenched lattice calculation gives $G(1) = 1.058^{+0.020}_{-0.017}$ [30], where the errors does not include the uncertainties induced by the quenching approximation and lattice spacing. Using $G(1) = 1.0 ± 0.07$, we get $|V_{cb}| = (41.3 ± 4.0 \text{exp} ± 2.9 \text{theo}) \times 10^{-3}$, consistent with the value extracted from $B \to D^*\ell\nu$ decay, but with a larger uncertainty.

The experiments have also measured the differential decay rate distribution to extract the ratio $G(w)/F(w)$. The data are compatible with a universal form factor as predicted by HQET. From the measured values of $G(1)|V_{cb}|$ and $F(1)|V_{cb}|$, we get $G(1)/F(1) = 1.08 ± 0.09$, consistent with the form-factor values we used.

3 $|V_{cb}|$ determination from inclusive B semileptonic decays

Alternatively, $|V_{cb}|$ can be extracted from the inclusive branching fraction for semileptonic $b$ hadron decays $B(B \to X_c\ell\nu)$ [37], [38]. Several studies have shown that the spectator model decay rate is the leading term in a well defined expansion controlled by the parameter $\Lambda_{QCD}/m_b$. Non-perturbative corrections to this leading approximation arise only to order $1/m_b^2$. The key issue in this approach is the ability to separate non-perturbative corrections, that can be expressed as a series in powers of $1/m_b$, and perturbative corrections, expressed in powers of $\alpha_S$. Quark-hadron duality is an important $ab\ initio$ assumption in these calculations. While several authors [39] argue that this ansatz does not introduce appreciable errors as they expect that duality violations affect the semileptonic width only in high powers of the non-perturbative expansion, other authors recognize that an unknown correction may be associated with this assumption [40]. Arguments supporting a possible sizeable source of errors related to the assumption of quark-hadron duality have been proposed [41]. This issue needs to be resolved.
with further measurements. At present, no explicit additional error has been added to account for a possible quark-hadron duality violations.

The coefficients of the $1/m_b$ power terms that are valid through order $1/m_b^2$ include four parameters: the expectation value of the kinetic operator, corresponding to the average of the square of the heavy quark momentum inside the hadron, the expectation value of the chromomagnetic operator and the heavy quark masses ($m_b$ and $m_c$). The expectation value of the kinetic operator is introduced in the literature as $\lambda_1$ [42], [44] or $\mu_1^2$ [37] [38], whereas the expectation value of the chromomagnetic operator is defined as $\lambda_2$ [42], [44] or $\mu_2^2$ [37] [38]. The two notations reflect a difference in the approach used to handle the energy scale $\mu$ used to separate the long distance from the short distance physics. HQET is most commonly renormalized in a mass-independent scheme, thus making the quark masses the pole masses of the underlying theory (QCD). The second group of authors prefer the definition of the non-perturbative operators using a mass scale $\mu \approx 1$ GeV.

The equation for the semileptonic width according to the first set of authors can be found in Ref. [40], that has been used to extract $|V_{cb}|$ from the semileptonic branching fraction:

$$
\Gamma_{sl} = \frac{G_F^2 |V_{cb}|^2 M_B^5}{192\pi^3} [0.3689[1 - 1.54\frac{\alpha_s}{\pi} - 1.43\beta_0 \frac{\alpha_s^2}{\pi^2} - 1.648 \frac{\lambda_1}{M_B} (1 - 0.87 \frac{\alpha_s}{\pi}) - 0.946 \frac{\lambda_2}{M_B} - 3.185 \frac{\lambda_1}{M_B}] + 0.02 \frac{\lambda_1}{M_B} - 0.298 \frac{\lambda_1}{M_B} - 3.28 \frac{\lambda_1}{M_B} - 10.47 \frac{\lambda_1}{M_B} - 6.153 \frac{\alpha_1}{M_B} + 7.482 \frac{\alpha_1}{M_B} - 7.4 \frac{\alpha_1}{M_B} + 1.491 \frac{\alpha_1}{M_B} - 10.41 \frac{\alpha_1}{M_B} - 7.482 \frac{\alpha_1}{M_B} + O(1/M_B^4)] .
$$

Eq. 6, in the $\overline{MS}$ scheme, is calculated to order $1/M_B^4$ and $\beta_0\alpha_s^2$. In order to derive this equation, the quark masses are related to the corresponding meson masses through [47]:

$$
m_Q = \frac{M_M - \Lambda}{2m_Q} + \frac{\lambda_1}{2m_Q},
$$

where $m_Q$ is the heavy quark mass, $\overline{M}_M$ is the spin averaged heavy meson mass, $(\overline{M}_B = 5.313\, GeV/c^2$ and $\overline{M}_D = 1.975\, GeV/c^2$). The constant coefficients $\rho_1$, $\rho_2$, $\tau_1$, $\tau_2$, $\tau_3$ and $\tau_4$ are form factors of different contributions that arise to order $1/m_b^2$ and are described in more detail in Ref. [42].

The corresponding equation for the semileptonic width in the second approach is [43]:

$$
\Gamma_{sl} = \frac{G_F^2 m_b^5}{192\pi^3} \cdot [z \left( \frac{m_c^2}{m_b^2} \right)^2 \left[ 1 - a_1 \frac{\alpha_s}{\pi} - a_2 \frac{\alpha_s}{\pi} \right] \cdot \left( 1 - \frac{\mu_1^2 - \mu_2^2}{2m_b^2} \right) - (1 + b_1 \frac{\alpha_s}{\pi} + ...) \cdot (1 - \frac{m_c^2}{m_b^2})^4 \frac{2\mu_2^2}{m_b^2} - d \frac{\rho_D^2}{m_b^2}],
$$

where $m_b(\mu)$ and $m_c(\mu)$ are short scale quark masses, $z$ is a known parton phase space factor dependent upon $m_c^2/m_b^2$, $a_i$ and $b_i$ parameters are coefficient of the perturbative expansion [43]: they, as well as the parameter $d$, are function of $(m_c^2/m_b^2)$. The parameter $\rho_D$ gives the strength of the so called ‘Darwin term’, the dominant $1/m_b^3$ correction in this approach. The short distance mass $m_b(\mu \approx 1 GeV)$ has been evaluated with different techniques. The central value is rather consistent and the errors vary from 30 to 110 MeV [45]. Thus we consider $m_b(\mu \approx 1 GeV) = 4.58 \pm 0.09$ GeV the best representation of our present knowledge.
Experimental determinations of the HQE parameters are important in several respects. In particular, redundant determinations of these parameters may uncover inconsistencies, or point to violation of some important assumptions inherent in these calculations. The parameter $\lambda_2$ can be extracted from the $B^* - B$ mass splitting and found to be $\lambda_2 = 0.128 \pm 0.010$ GeV$^2$ [12], whereas the other parameters need more elaborate measurements.

The first stage of this experimental program has been completed recently. The CLEO collaboration has measured the shape of the photon spectrum in $b \rightarrow s\gamma$ inclusive decays. Its first moment, giving the average energy of the $\gamma$ emitted in this transition, is related to the $b$ quark mass. In the formalism of Ref. [46] this corresponds to the measurement of the parameter $\Lambda = 0.35 \pm 0.07 \pm 0.10$ GeV.

The parameter $\lambda_1$ is determined experimentally through a measurement of the first moment of the mass $M_X$ of the hadronic system recoiling against the $\ell\nu$ pair. This CLEO measurement takes advantage of the ability of reconstructing the $\nu$ 4-momentum with high efficiency and resolution, by virtue of the hermeticity of the detector and the simplicity of the initial state. The relationship between the first moment of $M_X$, defined as $M_1 \equiv \langle M_X^2 - M_D^2 \rangle / M_B^2$, and the parameters $\Lambda$ and $\lambda_1$ is given in [47].

$$M_1 = \frac{\langle M_X^2 - M_D^2 \rangle}{M_B^2} = [0.0272 \alpha_s + 0.058 \beta_0 \alpha_s^2 + 0.207 \frac{\Lambda}{M_B} (1 + 0.43 \alpha_s) + 0.193 \frac{T_3}{M_B} + 1.38 \frac{\lambda_1}{M_B} + 0.203 \frac{T_3}{M_B} + 0.19 \frac{T_3}{M_B} + 3.2 \frac{\lambda_2}{M_B} + 1.4 \frac{T_3}{M_B} + 4.3 \frac{\rho_1}{M_B} - 0.56 \frac{\rho_2}{M_B} + 2.0 \frac{T_3}{M_B} + 1.8 \frac{T_3}{M_B} + 1.7 \frac{T_3}{M_B} + 0.91 \frac{T_3}{M_B} + O(1/M_B^4)].$$

(9)

The measured value for $\langle M_X^2 - M_D^2 \rangle$ is $0.251 \pm 0.066$ GeV$^2$. This constraint, combined with the measurement of the mean photon energy in $b \rightarrow s\gamma$, implies a value of $\lambda_1 = -0.24 \pm 0.11$ GeV$^2$, to order $1/M_B^2$ and $\beta_0 \alpha_s^2$ in $(M_S)$. The quoted theoretical uncertainty of 2% accounts for the $1/M_B^2$ and $\alpha_s$ uncertainties, but not for possible violations of quark-hadron duality. Thus more conservative uncertainties have been used in the literature for our present knowledge of this parameter [48]. The next step involves a determination of the HQE parameters with independent measurements, for example, the moments of the lepton energy spectrum [49], [50]. Preliminary data show contradictory implications [50], [51]; this issue should be settled soon with definitive results from several different experiments.

### 3.2 Experimental determination of the semileptonic branching fraction

The value of $\mathcal{B}(B \rightarrow X_c \ell\nu)$ has been measured both at the $\Upsilon(4S)$ and LEP.

The most recent CLEO data, published in 1996 and based on a subset of the data sample
accumulated now, obtains this branching fraction using a lepton tagged sample \[52\]. In this approach, a di-lepton sample is studied, and the charge correlation between the two leptons allows to disentangle leptons coming from the direct decay \( B \to X_c \ell \nu \) and the dominant background at low lepton momenta, the cascade decay \( B \to X_c X_s \ell \nu \). This method, pioneered by the ARGUS collaboration \[53\] allows the electron spectrum from \( B \to X_c \ell \nu \) to be measured down to 0.6 GeV/c. Thus, it allows to reduce the model dependence of the extracted semileptonic branching fraction very substantially. They obtain \( \mathcal{B}(B \to X_c \ell \nu) = (10.49 \pm 0.17 \pm 0.43)\% \).

The systematic error (4\%) is dominated by experimental uncertainties. Lepton identification efficiency, fake rate determination and tracking efficiencies contribute to 3\% of this overall error. The remaining error is a sum of several small corrections associated to the uncertainty in the mixing parameter, and additional background estimates \[52\].

Combining \( \Upsilon(4S) \) results \[1\], we obtain: \( \mathcal{B}(b \to X \ell \nu) = (10.38 \pm 0.32)\% \). Using \( \tau_B^+, \tau_{B^0} \) \[54\], \( f_+/f_0 = 1.04 \pm 0.08 \) \[54\] and subtracting \( \mathcal{B}(b \to u \ell \nu) = (0.17 \pm 0.05)\% \), we get: \( \mathcal{B}(b \to X_c \ell \nu) = (10.21 \pm 0.32)\% \) and \( \Gamma(b \to X_c \ell \nu) = (0.419 \pm 0.013 \pm 0.003) \times 10^{-10} \) MeV, where 0.003 \( \times \) 10\(^{-10} \) MeV includes the uncertainties from \( \mathcal{B}(b \to u \ell \nu) \) and the model dependence, correlated with LEP.

At LEP, \( B_d^0, B^-, B_s \) and b-baryon are produced, so the measured inclusive semileptonic branching ratio is an average over the different hadron species. Assuming that the semileptonic widths of all b-hadrons are equal, the following relation holds:

\[
\mathcal{B}(b \to X_c \ell \nu)_{LEP} = \frac{f_{B^0} \Gamma(B^0 \to X_c \ell \nu)}{\Gamma(B^0)} + f_{B^-} \frac{\Gamma(B^- \to X_c \ell \nu)}{\Gamma(B^-)} + f_{B_s} \frac{\Gamma(B_s \to X_c \ell \nu)}{\Gamma(B_s)} + f_{\Lambda_b} \frac{\Gamma(\Lambda_b \to X_c \ell \nu)}{\Gamma(\Lambda_b)} =
\]

\[
\Gamma(B \to X_c \ell \nu) \tau_b
\]

where \( \tau_b \) is the average b-hadron lifetime. Taking into account the present precision of LEP measurements of b-baryon semileptonic branching ratios and lifetimes, the estimate uncertainty for a possible difference for the width of b-baryons is 0.13%.

At LEP, \( \mathcal{B}(b \to X \ell \nu) \) is measured with dedicated analyses \[55, 57, 58, 59\], summarized in (Table 3). The average LEP value for \( \mathcal{B}(b \to X \ell \nu) = (10.59 \pm 0.09 \pm 0.30)\% \) is taken from a fit \[55\], which combines the semileptonic branching ratios, the \( B^0 - \overline{B^0} \) mixing parameter \( \chi \), and \( R_b = \Gamma(Z \to b \overline{b})/\Gamma(Z \to \text{had}) \).

Ref. \[56\] shows that the main contribution to the modelling error is the uncertainty in the composition of the semileptonic width including the narrow, wide and non-resonant D** states. \( B_s \) and b-baryons are about 20\% of the total signal and their contribution to the uncertainty of the spectrum is small. In this average, we use the modelling error quoted by \[56\] rather than the error from the combined fit, as the ALEPH procedure is based on more recent information. The dominant errors in the combined branching fraction are the modelling of semileptonic decays (2.6\%) and the detector related items (1.3\%).

Subtracting \( \mathcal{B}(b \to u \ell \nu) \) from the LEP semileptonic branching fraction, we get: \( \mathcal{B}(b \to X_c \ell \nu) = (10.42 \pm 0.34)\% \), and using \( \tau_b \) \[55\]: \( \Gamma(b \to X_c \ell \nu) = (0.439 \pm 0.010 \pm 0.011) \times 10^{-10} \) MeV, where the systematic error 0.011 \( \times \) 10\(^{-10}\) MeV reflects the \( \mathcal{B}(b \to u \ell \nu) \) uncertainty and the model dependence, correlated with the \( \Upsilon(4S) \) result.
Table 5: $\mathcal{B}(b \to \ell \nu)$ measurement from LEP and their average. The errors quoted reflect statistical, systematic and modelling uncertainties respectively.

| Experiment         | $\mathcal{B}(b \to \ell \nu)\%$            |
|--------------------|-------------------------------------------|
| ALEPH              | $10.70 \pm 0.10 \pm 0.23 \pm 0.26$        |
| DELPHI             | $10.70 \pm 0.08 \pm 0.21 \pm 0.30$        |
| L3                 | $10.85 \pm 0.12 \pm 0.38 \pm 0.26$        |
| L3 (double-tag)    | $10.16 \pm 0.13 \pm 0.20 \pm 0.22$        |
| OPAL               | $10.83 \pm 0.10 \pm 0.20 \pm 0.26$        |
| LEP Average        | $10.59 \pm 0.09 \pm 0.15 \pm 0.26$        |

Combining the LEP and the $\Upsilon(4S)$ semileptonic widths, we get: $\Gamma(b \to X_c \ell \nu) = (0.43 \pm 0.01) \times 10^{-10}$ MeV, which is used in the formula of Ref. [42] to get:

$$|V_{cb}|_{\text{incl}} = (40.4 \pm 0.5_{\text{exp}} \pm 0.5_{\lambda_1, \Lambda} \pm 0.8_{\text{theo}}) \times 10^{-3},$$

where the first error is experimental, the second from the measured value of $\lambda_1$ and $\Lambda$, assumed to be universal up to higher orders. The third error is from $1/m_b^3$ corrections and from the ambiguity in the $\alpha_s$ scale definition. The error on the average b-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching ratio.

4 Conclusions

The values of $|V_{cb}|$ obtained both from the inclusive and exclusive method agree within errors. The value of $|V_{cb}|$ obtained from the analysis of the $B \to D^* \ell \nu$ decay is:

$$|V_{cb}|_{\text{exclusive}} = (42.1 \pm 1.1_{\text{exp}} \pm 1.9_{\text{theo}}) \times 10^{-3}$$  \hspace{1cm} (11)

where the first error is experimental and the second error is from the $1/m_b^3$ corrections to $F(1)$. The value of $|V_{cb}|$, obtained from inclusive semileptonic branching fractions is:

$$|V_{cb}|_{\text{incl}} = (40.4 \pm 0.5_{\text{exp}} \pm 0.5_{\lambda_1, \Lambda} \pm 0.8_{\text{theo}}) \times 10^{-3},$$  \hspace{1cm} (12)

where the first error is experimental, the second error is from the measured values of $\lambda_1$, and $\Lambda$, assumed to be universal up to higher orders, and the last from $1/m_b^3$ corrections and $\alpha_s$. Non-quantified uncertainties are associated with a possible quark-hadron duality violation. For this reason, we chose not to average the two numbers.

While experimental errors have reached 2.7% and 1.2% levels respectively, the dominant uncertainties remain of theoretical origin. Thus an unambiguous statistical treatment of these uncertainties is very difficult. High precision tests of HQET, checks on possible violations of quark-hadron duality in semileptonic decays, experimental determination of $m_b$, $m_b - m_c$ and $\mu^2_{\pi}$ are needed to complete this challenging experimental program.
5 Acknowledgements

We would like to acknowledge useful contributions from K. Ecklund, J.H. Ki, K. Moenig and P. Roudeau. Moreover we would like to thank C. Bauer, I.I. Bigi, A. Falk, Z. Ligeti, A. Kronfeld, T. Mannel, S. Stone and N.G. Uraltsev for interesting discussions. This work was supported in part by National Science Foundation.

References

[1] Particle Data Group, C. Caso et al., this edition.
[2] L. Wolfenstein Phys. Rev. Lett. 51 (1983), 1945.
[3] C. Glenn Boyd, B. Grinstein and R.F. Lebed, Phys. Lett. B 353 (1995), 306.
[4] P.F. Harrison and H.R. Quinn, editors The BaBar Physics Book (1998) and references therein.
[5] M. Luke, Phys. Lett. B 252 (1990) 447.
[6] A. Czarnecki, Phys. Rev. Lett. 76 (1993), 4124
[7] A.F. Falk and M. Neubert, Phys. Rev. D 47 (1993), 2965 and 2982.
[8] T. Mannel, Phys. Rev. D 50 (1994), 428.
[9] M. Shifman, N.G. Uraltsev and A. Vainshtein, Phys. Rev. D 51 (1995), 2217.
[10] J.N. Simone et al., Nucl. Phys. Proc. Suppl. 83(2000), 334; hep-ph/0110253
[11] Working Group 1 Summary, CKM Workshop, CERN, CH (2002); http://ckm-workshop.web.cern.ch/ckm-workshop/
[12] S. Stone, in B Decays, 2nd Edition, S. Stone editor (1994), 283.
[13] I. Caprini, L. Lellouch and M. Neubert, Nucl. Phys. B 530 (1998) 153; C.G. Boyd, B. Grinstein, R.F. Lebed, Phys. Rev. D56 (1997) 6895.
[14] The CLEO Collaboration, LNS Preprint CLNS 01-1776 (2002); hep-ex/0203032
[15] K. Abe et al. (BELLE), Phys. Lett. B 526 (2002), 247; hep-ex/0111060
[16] D. Buskulic et al. (ALEPH), Phys. Lett. B 935 (1997), 373.
[17] P. Abreu et al. (DELPHI), Phys. Lett. B 510 (2001), 55.
[18] G. Abbiendi et al. (OPAL), Phys. Lett. B 482 (2000), 15.
[19] CLEO coll, CLEO CONF 99-6, hep-ex/9908009 (1999).
[20] D. Buskulic et al. (ALEPH), *Z. Phys. C* **73** (1997), 601

[21] A. Anastassov et al. (CLEO), *Phys. Rev. Lett.* **80** (1998), 4127

[22] D. Block et al. (DELPHI), *Contributed Paper to ICHEP 2000*, DELPHI 2000-106 Conf. 45 (2000).

[23] ALEPH, CDF, DELPHI, L3, OPAL, SLD, CERN-EP/2001-050.

[24] V. Morenas et al., *Phys. Rev. D* **56** (1997), 5668;
M.Q. Huang and Y.B. Dai, *Phys. Rev. D* **59** (1999), 34018;
M.Oda et al., [hep-ph/0005102](https://arxiv.org/abs/hep-ph/0005102) (2000).

[25] A.K. Leibovich, Z. Ligeti, I.W. Stewart, M.B. Wise *Phys. Rev. D* **57** (1998), 308;
*Phys. Rev. Lett.* **78** (1997), 3995.

[26] N. Isgur et al., *Phys. Rev. D* **39** (1989), 799.

[27] LEP $V_{cb}$ Working Group, Internal Note,
[http://lepvcb.web.cern.ch/LEPVCB/](http://lepvcb.web.cern.ch/LEPVCB/)

[28] CDF, LEP, SLD B Oscillations Working Group, Internal Note,
[http://lepbososc.web.cern.ch/LEPBOSC/](http://lepbososc.web.cern.ch/LEPBOSC/)

[29] CDF, LEP, SLD B-hadron Lifetime Working Group, Internal Note,
[http://claires.home.cern.ch/claires/lepblife.html](http://claires.home.cern.ch/claires/lepblife.html)

[30] J.E. Duboscq et al. (CLEO), *Phys. Rev. Lett.* **76** (1996), 3898.

[31] M. Neubert, *Phys. Lett.* B **264** (1991), 455.

[32] Z. Ligeti, Y. Nir and M. Neubert, *Phys. Rev. D* **49** (1994), 1302; [hep-ph/9908432](https://arxiv.org/abs/hep-ph/9908432).

[33] K. Abe et al. (Belle), *Phys. Lett. B* **526** (2002), 258; [lep-ex/0111082](https://arxiv.org/abs/lep-ex/0111082).

[34] J. Bartelt et al. (CLEO), *Phys. Rev. Lett.* **82** (1999), 3746.

[35] D. Scora and N. Isgur, *Phys. Rev. D* **52** (1995), 2783.

[36] S. Hashimoto et al., *Phys. Rev. D* **61** (1999), 014502.

[37] I. Bigi, M. Shifman and N.G. Uraltsev, *Annu. Rev. Nucl. Part. Sci.* **47** (1997), 591.

[38] A.V. Manohar and M.B. Wise, *Phys. Rev. D* **49** (1994), 110;
A.F. Falk, Lectures presented at TASI-2000, [hep-ph/0007339](https://arxiv.org/abs/hep-ph/0007339).

[39] I.I. Bigi and N.G. Uraltsev, *Int.J.Mod.Phys. A* **16** (2001) 5201;
A. Le Yaouanc et al., *Phys. Lett. B* **488** (2000), 153;
A. Le Yaouanc et al., *Phys. Rev. D* **62** (2000), 74007;
A. Le Yaouanc et al., *Phys. Lett. B* **517** (2001), 135.

[40] G. Buchalla, [hep-ph/0202092](https://arxiv.org/abs/hep-ph/0202092) (2002) and references therein.
[41] N. Isgur, *Phys. Lett.* B **448** (1999), 111.

[42] M. Gremm and N. Kapustin, *Phys. Rev.* D **55** (1997), 6924.

[43] N.G. Uraltsev, *Int. J. Mod. Phys.* A **11** (1996) 515; N.G. Uraltsev, *talk given at CKM Workshop*, CERN, Geneva, Switzerland, (2002).

[44] A. Falk, M. Luke and M.J. Savage, *Phys. Rev.* D **33** (1996), 2491.

[45] A. Hoang, *talk given at CKM Workshop*, CERN, Geneva, Switzerland, (2002).

[46] A. Falk and M. Luke, *Phys. Rev.* D **57** (1998), 424

[47] D. Cronin-Hennessy *et al.* (CLEO), *Phys. Rev. Lett.* **87** (2001), 251808.

[48] A. Falk, *Presentation at the Fifth KEK Topical Conference, ”Frontiers in Flavor Physics”, November 20-22, 2001*; [hep-ph/0201094].

[49] M. Voloshin, *Phys. Rev.* D **51** (1995), 4934.

[50] M. Gremm *et al.*, *Phys. Rev. Lett.* **77** (1996), 20.

[51] CLEO coll., CLEO CONF 98-21.

[52] B.C. Barish *et al.* (CLEO), *Phys. Rev. Lett.* **76** (1996), 1570; S. Henderson *et al.* (CLEO), *Phys. Rev.* D **45** (1992), 2212.

[53] H. Albrecht *et al.* (ARGUS), *Phys. Lett.* B **318** (1993), 397.

[54] J.P. Alexander *et al.* (CLEO), *Phys. Rev. Lett.* **86** (2001), 2737.

[55] LEP/SLD Electroweak Heavy Flavor Results Winter 2001 Conferences, [http://lepewwg.web.cern.ch/LEPEWWG/heavy/](http://lepewwg.web.cern.ch/LEPEWWG/heavy/)

[56] A. Heister *et al.* (ALEPH), *Eur. Phys. J.* C **22** (2002), 613.

[57] P. Abreu *et al.* (DELPHI), *Eur. Phys. J.* C **20** (2001), 455.

[58] M. Acciari *et al.* (L3), *Z. fur Phys.* C **71** (1996), 379; M. Acciari *et al.* (L3), *Eur. Phys. J.* C **13** (2000), 47.

[59] G. Abbiendi *et al.* (OPAL), *Eur. Phys. J.* C **13** (2000), 225.
Figure 1: $|V_{cb}| F(w)$ CLEO unfolded spectra, where the solid circles (squares) are derived from the $D^{*+}\ell\nu$ ($D^{*0}\ell\nu$) data samples respectively. The curve shows the result of the fit described in the text.

Figure 2: a) Decay form factor from the unfolded DELPHI data. The dotted line shows the fit result neglecting the bin-to-bin correlation. The continuous line shows the result when the correlation are taken into account.
Figure 3: The error ellipses for the corrected measurements and world average for $|F(1)V_{cb}|$ versus $\rho^2$. The ellipses are obtained from product between the 1 $\sigma$ error of $|F(1)V_{cb}|$, $\rho^2$ and the correlation between the two. Consequently, the ellipses correspond to about 37% C.L.