Requirement for quantum computation

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Abstract

We identify “proper quantum computation” with computational processes that cannot be efficiently simulated on a classical computer. For optical quantum computation, we establish no-go theorems for classes of quantum optical experiments that cannot yield proper quantum computation, and we identify requirements for optical proper quantum computation that correspond to violations of assumptions underpinning the no-go theorems.

1 Introduction

Quantum computation \cite{1, 2} offers the possibility of (i) efficiently simulating quantum dynamics for which classical simulations are hard and (ii) solving computational problems for which no efficient algorithm is known for classical computation. In the latter case, Shor’s algorithm for efficient factorization \cite{3} is a famous example of the potential of quantum computation, which has motivated a global effort to develop quantum computers. As quantum mechanics underpins all physical theories, it is important to identify the requirements for “proper quantum computation,” where we use this term to refer to quantum computations that outperform those allowed by a strict application of the laws of classical physics.

A key issue in the design of these quantum computers is identifying requirements such as gates with entanglement capability and appropriate sources and detectors that can perform tasks not possible with a classical machine. An important question is: How do we know when a quantum process may yield a computational advantage?

One approach to identifying quantum processes that may potentially lead to quantum computation is to identify what is not proper quantum computation. To address this question in a quantum optics setting, we have identified a large “toolkit” consisting of sources, processors and detectors such that any network consisting of these devices can be efficiently simulated on a classical computer.
With this toolkit, we construct no-go theorems that elucidate requirements for quantum computation. On one hand, a surprisingly large class of quantum networks are shown not to allow for quantum computation in the sense of solving problems that are intractable on a classical machine. On the other hand, the critical resources for performing quantum computation are exposed as violations of assumptions in the theorems. Identifying resources through violations of assumptions is particularly useful in understanding schemes that eliminate the apparent requirement of an optical nonlinearity by utilizing certain measurements. Our no-go theorems also suggest how stringent criteria might be relaxed and still be able to deliver powerful quantum computation.

2 Computing: problems and algorithms

Quantum computation is often regarded as a powerful, “non-classical” computation, i.e., a computation that cannot be performed “easily” on a conventional computer. However, this description requires us to be specific about what we mean by a computation, by “non-classical”, and by computationally “easy”. Because we expect a quantum computer to be able to do anything that a classical computer can do (and possibly much more), we need to rule out classes of computations on a quantum computer that are not proper, in the sense that they could equally well be performed on a classical computer. In this section, we define relevant computing concepts and terms with the aim of identifying what constitutes proper quantum computation.

A computational problem is a mathematical function that maps an instance (i.e., an input) to a solution. The sets of instances and solutions are classical sets of distinguishable elements, e.g., integer numbers. For example, the problem, “Is an integer \( p \) a prime number?” is a computational problem, one for which the set of instances is the set of integer numbers and the set of solutions consists of the logical output “yes (it is prime)” and “no (it is not)”.

For a specific computational problem, an algorithm is a detailed, step-by-step method or recipe for finding the solution corresponding to a given instance [4], using an agreed set of operations. These operations may include boolean operations or unitary quantum transformations or something else. An algorithm is thus a prescription, using specified operations, for solving the problem in general. Finding the solution for a given instance simply becomes a mechanistic process: follow the steps of the algorithm to obtain the solution.

Once an algorithm for a problem is devised, the process of calculating solutions for various instances can be automated. A computer is a physical, mechanistic device used to implement the steps of an algorithm, thus calculating a solution for a given instance of the problem. A computer is mechanistic and employs specific operations, and it is important to identify the physical laws governing the computer’s operations; different physical laws underpinning the computation could affect the computer’s capabilities. We define a classical computer as a computer whose architecture obeys the laws of classical physics. This point is rather subtle: the architecture of existing computers is classical because
it can, in principle, be implemented in systems that obey the laws of classical physics; in reality devices such as transistors, which are described as quantum devices, does not render such architectures quantal because quantum devices are a technical convenience, not an in principle requirement.

The definition of a classical computer is now clear and can be extended to define the quantum computer. A quantum computer is one whose processes obey the laws of quantum physics. Because quantum physics is fundamental (and classical physics emerges from quantum laws in particular circumstances), our definition leads to a classical computer being a special case of a quantum computer, one for which the governing natural laws are restricted to the limit where classical physics applies.

For a specified computation problem, the chosen algorithm determines if a quantum computer is required or if a classical computer will suffice. If the algorithm employs classical operations, a classical computer suffices; if quantum operations are employed, then a quantum computer is needed (although of course a classical computer may be able to simulate the quantum algorithm). Thus, we define a quantum algorithm as one that requires quantum operations to implement. In contrast, a classical computer program is a classical algorithm that requires only classical operations for its execution.

With these definitions, we can address the question, “What is a quantum computation?” It seems natural to define quantum computation as a physical process, obeying the laws of quantum physics, that provides a solution for a given instance of a computational problem. This definition is unsatisfactory for our purposes: any computation on a conventional computer (which fundamentally obeys the laws of quantum physics) would thus be considered a quantum computation.

A better definition might be obtained by demanding that demonstratable quantum dynamics (such as quantum tunnelling, quantum superpositions or entanglement) occur during the computation. This approach is appealing because one could experimentally test for quantum effects and claim that a quantum computation is occurring. However, the problem with this extended definition is that the presence of quantum effects may not yield a quantum computation that performs better than a classical computation. Quantum effects could enhance the performance of computation, but incorporating entanglement does not guarantee this performance enhancement. The definition needs to consider the performance of the computation.

Proper quantum computation may be described as computation that outperforms comparable (or possibly any) classical computation. In order to quantify this concept of outperformance, we appeal to the subject of computational complexity.

### 2.1 Computational complexity

Quantum computation, per se, had its origins in Feynman’s musings [1], motivated by using quantum processes to efficiently simulate quantum dynamics for cases where classical computers appear to be grossly inadequate. The issues of
efficiency and efficient simulation are an active area of research in computer science known as computational complexity \cite{4}, and will be useful in defining a concept of proper quantum computation.

The motivation of computational complexity is to quantify the difficulty of an algorithm for a computational problem. Note that “difficulty” does not quantify how challenging it is for a researcher to devise an algorithm for a problem, but instead quantifies the physical resources that are required by a computer to implement an algorithm for a given instance size. More precisely, it is a comparison of the amount of physical resources $R$ (e.g., the number of computational steps, or the amount of physical memory) required in order to obtain a solution for an instance with size $S$ (e.g., the number of bits needed to write the instance into memory).

The computational complexity class $P$ is, loosely, the set of algorithms for which $R$ is bounded by a polynomial function of $S$. For such an algorithm, it is often said that the amount of physical resources required to solve the problem “scales polynomially” in the size of the instance. Classical algorithms that are in $P$ are generally considered to be easy, or efficient, whereas an algorithm that is not in $P$ is considered hard and inefficient. Note that this classification is a loose guide (an algorithm where $R$ behaves as $S^{100}$ is in $P$ but may be practically intractable), but is one that in practice serves as an excellent classification of easy and hard algorithms.

It could be argued that the goal of quantum computer science is to devise quantum algorithms for which the number of quantum operations required scales polynomially in the size of the instance, but where no classical algorithm in $P$ is known or even possible. In such cases, allowing for quantum operations clearly results in an advantage from a computational perspective. Shor’s quantum algorithm for efficient factorization is one such example; finding more examples may lead to an understanding of how quantum computation may be more powerful than classical.

2.2 Classical simulation of quantum processes

As discussed in the previous section, allowing quantum operations in a computation does not guarantee an advantage from a computational complexity perspective. In order to identify requirements for proper quantum computation as distinct from classical computing, we require that proper quantum computation offers an advantage over classical computing. Consider a quantum algorithm, or even more generally, a quantum process. We say that this process can be efficiently simulated on a classical computer if the classical computing resources required to simulate it scale polynomially in the size of the quantum process; i.e., if the classical algorithm simulating the quantum one is in $P$. The concept of efficient classical simulation leads us to the following definition.

**Definition (proper quantum computation):** A quantum computation that cannot be efficiently simulated on a classical computer.

We immediately see that this definition is unsatisfactory in some respects: some quantum computations offer a polynomial speedup over classical algo-
rithms (such as Grover’s search algorithm, which offers a quadratic speedup over classical searches). These computations can be efficiently classically simulated and are thus not proper quantum computations by our definition; nevertheless they still qualify as quantum algorithms. However, quantum computations that are proper (and thus not efficiently classically simulatable) are of paramount importance, and thus we focus our attention on identifying requirements for them.

Proving that a quantum algorithm is proper and will outperform any classical algorithm is difficult. One fruitful direction towards this goal is to identify large classes of quantum processes that do not offer an advantage over a classical system: this exclusive direction serves to focus the search for proper quantum algorithms. Such efficient simulation does not imply that quantum effects are not present in the quantum system, but simply notes that the quantum system does not provide a computational advantage. Appealing to Feynman’s original concept, the interesting applications for a quantum computer are for quantum processes that cannot be efficiently simulated on a classical computer.

3 Optical realizations of quantum computation

We now turn our attention to performing quantum computation with quantum optics. Optical realizations of quantum information processing benefit from advanced techniques in quantum optics for state preparation, unitary evolution with low decoherence and high-efficiency measurement. Both qubit and continuous-variable schemes allow optical quantum information processing; experiments demonstrating optical quantum teleportation and proposals for schemes such as optical quantum secret sharing are testimony to advances in quantum optical quantum information tasks and processes.

It is important to determine the useful and necessary optical processes to perform proper quantum computation, and we specifically identify classes of processes that can be efficiently simulated on a classical computer. The Gottesman-Knill (GK) theorem for qubits shows that it is sometimes possible to efficiently simulate a restricted set of quantum operations on a classical computer via a clever representation. In the following, we construct a large toolkit of quantum optics sources, processors and detectors that can be efficiently simulated on a classical computer. This construction allows us to identify key resources outside of this toolkit that may allow for proper quantum computation.

3.1 Simulating optical quantum processes

A full quantal treatment of a mode of the electromagnetic field requires the infinite-dimensional Hilbert space of a harmonic oscillator. Clearly, attempting to represent the state of many coupled optical modes on a classical computer is a daunting task due to the shear size of the quantum Hilbert space. Only more compact representations of a restricted set of such quantum states and corresponding transformations could ever be made tractable on a classical computer.
We demonstrate in the following that such compact representations exist for a wide range of optical quantum networks.

Linear optics, consisting of beam splitters, phase-shifters and other linear couplers together with semiclassical sources and coherence measurements can be described purely through a semiclassical description, and thus cannot yield proper quantum computation because the linear semiclassical evolution can be efficiently simulated. Also, single-photon schemes that employ only linear optics [17] are not scalable, in that they require resources that grow exponentially in the number of qubits [18]. Squeezing processes are realised by a $\chi^{(2)}$ non-linearity; if the pump is treated classically using a mean-field approximation, the resulting operation can be viewed as linear on a single mode (one-mode squeezing) or two modes (yielding an entangling transformation). The addition of squeezing and entangling transformations to this scheme is also insufficient, as proven by the continuous-variable classical simulatability theorem of Bartlett et al. [19]:

**Theorem 1 (Efficient Classical Simulation of Continuous Variable Quantum Information):** Any continuous variable quantum information process that initiates with Gaussian product states (products of squeezed displaced vacuum states) and performs only (i) linear phase-space displacements, (ii) squeezing transformations on a single oscillator mode, (iii) two-mode squeezing transformations, (iv) measurements of quadrature phase (i.e., homodyne detection) with finite losses, and (v) any such operations conditioned on classical numbers or homodyne detection (classical feed-forward), can be efficiently simulated using a classical computer.

An outline of the proof of this theorem is as follows. Gaussian states of an $N$-mode optical system are completely characterised by the vector consisting of the mean values of the canonical variables and by the covariance matrix. This representation of the states can be stored efficiently on a classical computer. Linear optics and one- and two-mode squeezing transformations possess a straightforward group action on this representation: these operations displace the means and transform the covariance matrix but maintain the Gaussian property of the multi-mode state. Because of the ease of this representation, these calculations can be simulated efficiently on a classical computer.

We note that the inclusion of squeezing into this list allows for non-Poissonian photon statistics. For example, the output of parametric downconversion is described by a state with only even photon number contributions. Such squeezed states, however, are still Gaussian and fall within the constraints of our theorem. Thus, although techniques of linear optics and squeezing with semiclassical sources and homodyne detection are highly advanced and can demonstrate non-classical properties such as quantum teleportation [11, 12, 13] and quantum secret sharing [14, 15], they are insufficient to perform proper quantum computation.

Recently, non-unitary processes such as measurement have been identified as a means to extend the power of optical quantum information processing [16, 17]. The essence of such schemes is that two optical systems (e.g., modes) are entan-
gled, followed by a measurement on one system. The other system “collapses” into a state that depends on the measurement outcome; for certain outcomes this collapse can be seen as equivalent to a unitary transformation. Proposals by Knill, Laflamme and Milburn [7] (KLM) and Gottesman, Kitaev and Preskill [8] (GKP) employ photon counting to induce unitary transformations in optical systems non-deterministically (i.e., they occur when certain measurement outcomes are observed), which leads to potential experimental schemes for optical quantum computation.

The above classical simulatability theorem can be extended to include non-unitary processes such as measurement. The classical simulatability theorem of Bartlett and Sanders [20] employs the powerful formalism of Gaussian completely positive (CP) maps [21] to describe efficiently simulatable operations (including some non-unitary processes such as measurement) on Gaussian states. 

**Theorem 2 (Efficient Classical Simulation of Optical Processes):** Any quantum information process that initiates in a Gaussian state and that performs only Gaussian CP maps can be efficiently simulated using a classical computer. These maps include (i) the unitary transformations corresponding to linear optics and squeezing, (ii) linear amplification (including phase-insensitive and phase-sensitive amplification and optimal cloning), linear loss mechanisms or additive noise, (iii) measurements that are Gaussian CP maps including, but not limited to, projective measurements in the position/momentum eigenstate basis or coherent/squeezed state basis, with finite losses, and (iv) any of the above Gaussian CP maps conditioned on classical numbers or the outcomes of prior Gaussian CP measurements (classical feedforward).

Again, the proof of this theorem lies in the simple representation for Gaussian states given by the means and covariance matrix. The non-unitary operations covered by this theorem form a semigroup (similar to a group but without the guarantee that every element is invertible) that again preserve the Gaussian nature of the states. This theorem for efficient classical simulation provides a powerful tool in assessing whether a given optical process can enhance linear optics to allow for proper quantum computation. Algorithms or circuits employing Gaussian-preserving maps can be efficiently simulated on a classical computer, and thus cannot lead to proper quantum computation.

The results of this section reveal the limitation of using Gaussian states for quantum information processing: the existence of a compact representation for these states (and Gaussian-preserving transformations and measurements on them) leads to no-go theorems for proper quantum computation. Clearly, “going beyond” Gaussian states in optical quantum computation is necessary (although possibly not sufficient).

### 3.2 Requirement of optical nonlinearity

In particular, higher-order optical nonlinear processes (such as a Kerr nonlinearity [22]), which can yield non-Gaussian states, have been identified as a
necessary requirement [9] for proper quantum computation with optics. Unfortunately, Kerr nonlinearities suffer either from weak strengths or high losses, and the lack of appropriate nonlinear materials greatly restricts the type of processes that can be performed in practice. Optical quantum computation schemes such as [7, 8] use measurements to induce a nonlinear transformation; however, as shown in the previous theorems, not all forms of measurement can yield proper quantum computation. In this section, we discuss measurements that may be used to induce an optical nonlinearity and those which cannot by employing the results of our no-go theorems.

First, our theorem provides a strong no-go result for the use of homodyne measurement:

**Corollary 1:** Linear optics or squeezing transformations conditioned on the measurement outcome of homodyne detection with finite losses using Gaussian states cannot induce a nonlinearity.

Thus, initiating with Gaussian states, it is not possible to use homodyne measurements and feedforward of measurement results to induce a (possibly nondeterministic) optical nonlinearity in the way that photon counting allows in the KLM scheme. In terms of optical implementations of quantum computing, this theorem reveals why all previous schemes either propose some form of optical nonlinearity [6, 9], use other forms of measurement such as photon counting [7, 8] or are not efficiently scalable [17].

This theorem also places severe constraints on the use of photodetection to perform nonlinear transformations in realizations of optical quantum computing. For a threshold photodetector [7, 23, 24] with perfect efficiency, the POVM is given by two elements, corresponding to “absorption” and “no-absorption” of light. Photon counters are effectively constructed as arrays of such detectors [24]. The vacuum projection describes the non-absorption measurement, and the corresponding map describing this measurement result is Gaussian CP. However, the absorption outcome is not.

**Corollary 2:** Gaussian-preserving maps conditioned on the no-absorption outcome of a photodetection measurement can be efficiently simulated on a classical computer; transformations conditioned on the absorption outcome cannot be efficiently simulated in this manner.

Note that the same result holds for finite-efficiency photodetectors: such detectors can be modelled as unit efficiency photodetectors with a linear loss mechanism describable using Gaussian CP maps. Thus, the absorption outcome of photodetection and the feedforward of this measurement result is a key resource for optical quantum information processing. This corollary also proves that any nonlinear gate employing linear optics and photon counting must be nondeterministic; a photon counting measurement of a Gaussian state could possibly result in an outcome of zero photons, and such a result corresponds to an efficient, classically simulatable process. (Note that nonlinear optics, in contrast, allows deterministic processing.)
Table 1: Efficient classical simulatability for schemes employing various initial states, unitary gates, and measurements.

| Initial States | Unitary Gates                  | Measurements               | Efficiently simulatable |
|----------------|--------------------------------|-----------------------------|------------------------|
| Vacua          | Linear optics, squeezing       | Gaussian CP (i.e., homodyne) | ✓ [19, 20]             |
| Vacua          | Linear optics, squeezing,      | Homodyne                    | × [9]                  |
|               | Kerr nonlinearity              |                             |                        |
| Single photons | Linear optics only             | Photon counting             | × [7]                  |
| Vacua          | Linear optics, squeezing       | Photon counting & homodyne  | × [8]                  |
| Single photons | Linear optics, squeezing       | Homodyne                    | ?                      |

Our classical simulatability theorem may be useful in assessing the minimum requirements for proper quantum computation with optics. Table 1 presents various classes of initial states, unitary gates, and measurements (that can be used for classical feedforward) and their classical simulatability according to our theorem. Employing only Gaussian states and Gaussian CP maps results in an efficiently simulatable circuit; one can now consider supplementing this set with various “resources” that may allow for proper quantum computation. As shown by Lloyd and Braunstein [9], the addition of a Kerr nonlinearity or any higher-order transformation on a single mode results in the ability to efficiently simulate the evolution of any polynomial Hamiltonian. The schemes of KLM and GKP reveal that photon counting is also a resource that allows for universal quantum computation. The KLM scheme also requires single photon Fock states “on demand” as ancilla inputs to their nondeterministic nonlinear gates; such states lie outside the domain of our theorem (they are not Gaussian) and may serve as a resource for performing nonlinear operations.

It is interesting to consider, then, if single photons on demand are by themselves sufficient to bestow Gaussian CP maps with the power to perform nonlinear operations and thus possibly proper quantum computation. Considering the recent progress in creating single photon turnstile devices [25] (with low probability of producing zero or two photons by accident), a scheme that requires single photons but otherwise employs only linear optics, squeezing, and high-efficiency homodyne detection would obviate the need for ultra-high efficiency photon counters [26].

4 Discussion

Quantum optics is challenged by the advent of quantum computation, not only by the technical hurdles that must be overcome to achieve scalable quantum computers, but also by the fundamental question of creating and verifying
proper quantum computation in the laboratory. Here we have elucidated the nature of quantum computation, pointed out that a large quantum optics toolkit is insufficient to realise proper quantum computation and illustrated how violating assumptions in our no-go theorems may correspond to requirements for quantum computation.

We have established that optical transformations that map Gaussian wavefunctions into Gaussian wavefunctions, whether unitary or not, are insufficient to perform proper quantum computation. If the Gaussian nature of wavefunctions is preserved by the operations, then we can exploit the mean-and-covariance representation of states to implement an efficient classical simulation of the quantum system; hence this quantum computation does not sufficiently outperform a classical computation to qualify as proper quantum computation according to our criteria.

We highlight the importance of non-Gaussian transformations, whether they correspond to nonlinear unitary evolutions or to the nonlinear CP-map of conditioning a unitary transformation on detecting photons (the absorption outcome), and also the initiation with non-Gaussian states, to violate our theorems. If the initial states are non-Gaussian, or if the transformations do not preserve the Gaussian representation of wavefunctions, then the “clever” representation of Gaussian states in terms of means and variances is no longer adequate to provide algorithms for efficient classical simulation. Although we cannot guarantee that violating our theorem is enough for proper quantum computation, we can – and do – rule out classes of experiments (as determined by the “toolkit” in the laboratory) as being sufficient for proper quantum computation.

In designing experiments for quantum computation, one must always consider the most efficient means to simulate the processes and outcomes on a classical computer. If a clever means exists to simulate the experimental quantum computation on a classical computer such that this simulation is in \( P \), then the quantum computation is not proper: our requirement is that the toolkit must be sufficient to produce quantum computations that defy efficient classical simulation.

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