Systematic study of the elliptic flow parameter using a heavy-ion collision model

Md. Nasim and Bedangadas Mohanty
School of Physical Sciences, National Institute of Science Education and Research, Bhubaneswar-751005, India

Elliptic flow parameter, \( v_2 \) is consider as a sensitive probe for early dynamics of the heavy-ion collision. In this work we have discussed the effect of detector efficiency, procedure of centrality determination, effect of resonance decay, procedure to obtain event plane resolution on the measured \( v_2 \) by standard event plane method within the framework of a transport model. The measured value of \( v_2 \) depends on the detector efficiency in particle number counting. The effect of centrality determination is found to be negligible. The method of event-by-event correction of event plane resolution for wide centrality bin yields in results closer to the true value of \( v_2 \). The effect of resonance decay is seen to decrease the \( v_2 \) of \( \pi, K \) and \( p \). We also propose a procedure to correct for an event bias effect on \( v_2 \) while comparing the minimum bias centrality \( v_2 \) values for different multi-strange hadrons. Finally we have presented a model based confirmation of the recently proposed relation between \( v_2 \) obtained using event plane method and scalar product method to the true value of \( v_2 \).

PACS numbers: 25.75.Ld and 25.75.-q and 25.75.Ag

I. INTRODUCTION

The study of the azimuthal angle distribution for hadrons produced in high energy heavy-ion collisions is considered as a very useful observable for understanding the properties of the hot and dense matter formed in the collisions \([1–8]\). The second Fourier coefficient of the azimuthal angle \((\Theta)\) is called as the elliptic flow parameter \((v_2)\) \([2]\). It is defined as

\[
v_2 = \langle \cos(2(\phi - \Psi)) \rangle = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle,
\]

where \(p_x\) and \(p_y\) are the \(x\) and \(y\) components of the particle momenta, respectively. The \(\langle \cdot \rangle\) denotes the average over all particles in all events. The \(\Psi\) is an estimate of the angle subtended by the plane formed by impact parameter and beam \((z)\) axis with respect to the \(x\)-axis. The magnitude of \(v_2\) is found to be sensitive to the equation of state, thermalization, transport coefficients of the medium, and initial conditions in the heavy-ion collisions \([10,12]\).

Since \(v_2\) is an important observable in high energy heavy-ion collisions, it is necessary to study the effect of various experimental conditions towards the measurement of its true value. In this work we study the effect of finite particle number counting (detector efficiency), method of centrality determination, effect of correcting for finite resolution in obtaining \(\Psi\) and effect of resonances. For this study we use transport based models, which provides an ideal set up to study and quantify the above effects. From the models we know the true value of \(v_2\) at the same time it provides information of particles produced in heavy-ion collisions that can be treated in a manner similar to the actual experimental conditions. For minimum bias collisions the event class (in terms of average initial spatial anisotropy) having a rare heavy particle like \(\Omega\) baryon could be different from that having a proton (copious produced). This may necessitate an appropriate correction to the measured \(v_2\) of different hadrons for wide centrality class (0-80%) so that they can be compared among themselves. In addition, we also use this model framework to verify the recently proposed relation that \(v_2\) obtained using event plane method approaches the root-mean square \(v_2\) (as obtained from the scalar product method) in the small event plane resolution limit while it approaches the mean \(v_2\) value in the high event plane resolution limit \([13]\).

The paper is organised as follows. In section \(\text{II}\) we describe the transport based models used in this study. The effect of detector efficiency on measured \(v_2\) is presented in section \(\text{III}\). In section \(\text{IV}\) we investigate the effect of centrality selection procedure on measured \(v_2\). The event plane resolution correction methods, event bias correction method and effect of resonance decay on \(v_2\) has been discussed in sections \(\text{V}, \text{VI}\) and \(\text{VII}\) respectively. In section \(\text{VIII}\) we discuss the recently proposed relation between \(v_2\) obtained from event plane and scalar product method. Finally section \(\text{IX}\) summarises our findings.

II. MODEL DESCRIPTION

The A Multi Phase Transport (AMPT) model \([14]\) (version: 25t7d) uses the same initial conditions as in HIJING \([13]\). However the minijet partons are made to undergo scattering before they are allowed to fragment into hadrons. The string melting (SM) version of the AMPT model is based on the idea that for energy densities beyond a critical value of \(\sim 1\ \text{GeV/fm}^3\), it is difficult to visualize the coexistence of strings (or hadrons) and partons. Hence the need to melt the strings to partons. This is done by converting the mesons to a quark and anti-quark pair, baryons to three quarks etc. The scattering of the quarks are based on parton cascade \([14]\). Once the interactions stop, the partons then hadronizes.
through the mechanism of parton coalescence. The interactions between the minijet partons in AMPT model and those between partons in the AMPT-SM model could give rise to substantial $v_2$. All results presented here using this model is for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at midrapidity (-0.5 < $\eta$ < 0.5) with total event statistics of 1.8 million minimum bias (0-80%) events. The true event plane angle is fixed to zero degree for the simulation results presented.

The Ultra relativistic Quantum Molecular Dynamics (UrQMD) model (version: 3.3) is based on a microscopic transport theory where the phase space description of the reactions are important [10]. It allows for the propagation of all hadrons on classical trajectories in combination with stochastic binary scattering, color string formation and resonance decay. It incorporates baryon-baryon, meson-baryon and meson-meson interactions, the collisional term includes more than 50 baryon species and 45 meson species. This model is used to understand the resonance decay effect on measured $v_2$. The analysis makes use of 1.5 million minimum bias (0-80%) events for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at midrapidity.

III. EFFECT OF DETECTOR EFFICIENCY

In this section we discuss the effect of detector efficiency on measured $v_2$ in a typical heavy-ion experiment. The AMPT model is used to simulate Au+Au collision and introduced the effect of finite detector efficiency on particle number counting. A realistic detector efficiency for reconstruction of charged kaon and $K_S^0$ as a function of the transverse momentum ($p_T$) of the measured hadron from a typical heavy-ion experiment [17, 18] as shown in Fig. 1 is considered. The reconstruction efficiency for neutral kaons, which have to be reconstructed from the decayed charged pion daughters are smaller compared to those from more directly reconstructed charged kaons. Two sets of events were considered, one with 100% reconstruction efficiency versus $p_T$ termed as “default” and the other where the kaon tracks in a given event for a given $p_T$ range were randomly removed as per the efficiency shown in Fig. 1. The resultant yield of the charged kaons from AMPT model with $K_S^0$ reconstruction efficiency effect incorporated has been compared to the default case in Fig. 2. The two distributions have been normalised at their respective yield values at $p_T = 1$ GeV/c. The effect of finite particle reconstruction efficiency can be seen from the shape of yield vs. $p_T$ distribution below $p_T = 1$ GeV/c and as the efficiency values are constant with $p_T$ beyond 1 GeV/c the spectra shape are similar at high $p_T$ (consistent with Fig. 4).

The elliptic flow of charged kaon has been calculated for three different condition: (a) with 100% particle reconstruction efficiency (labeled as default), (b) with charged kaon reconstruction efficiency and (c) with $K_S^0$ reconstruction efficiency. Figure 3 shows the comparison of kaon $v_2$ for the above three different cases. The kaon $v_2$ from AMPT for default case, with charged kaon reconstruction efficiency and with $K_S^0$ reconstruction efficiency are shown as solid black circle, open blue square and open red circle, respectively. The bottom panel of Fig. 3 shows the ratios of default kaon $v_2$ to that obtained with two different particle reconstruction efficiency. There is a change due to the finite particle reconstruction efficiency.

![FIG. 1: Typical particle reconstruction efficiency as function of $p_T$ for charged kaon and $K_S^0$ at midrapidity in Au+Au collisions at high energy.](image1.png)

![FIG. 2: (Color online) Kaon yield as function of $p_T$ from AMPT model for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Red dash line corresponds to the yield of kaon obtained directly from the AMPT model and black solid line corresponds to the kaon yield after incorporating the finite $K_S^0$ reconstruction efficiency values as shown in Fig. 1]
charged kaon reconstruction efficiency and with
condition: 100% kaon reconstruction efficiency (default), with collision for 0-80% from AMPT model in three different con-
FIG. 3: (Color online)
von the measured
struction efficiency between charged kaon and
need for correcting the effect of difference in finite recon-iciency the change is less than 5%. Our study shows a
results using the higher charged kaon reconstruction ef-construction efficiency is about 10% to 30% while for the
measurement (70-80%) and 9 corresponds to most central (0-5%) Au + Au
collisions studied.

IV. EFFECT OF CENTRALITY
DETERMINATION PROCEDURE

Determination of centrality selection in experiments is
found to play an important role in measurements related

to particle correlations [10]. Specifically when the same
particles that are used for the correlation studies also
forms a subset of the particles used to obtain the cen-
trality selection. Hence it is important to study the effect
of centrality determination procedure on measurement of
v_{2}. Further different experiments use different methods
of centrality selection. Hence such a study is necessary
to see if measured v_{2} values across different experiments

can be compared.

The simulated charged particle average v_{2} (∆v_{2}) values are obtained for three different ways for centrality se-
lection. They are: (a) centrality obtained using charged
particle multiplicity within |η|< 0.5 (labeled as centrality
1), centrality obtained using charged particle multiplicit-
y within |η|> 0.5 and |η|< 1.0 (labeled as centrality 2), and that obtained using spectator neutrons (labeled as
centrality 3). Experiments at the Relativistic Heavy-Ion
Collider facility commonly uses these methods to select
on collision centrality. Figure 4 show ⟨v_{2}⟩ of charged par-
ticles measured at midrapidity (|η| < 0.5) as function of
centrality for the above three different cases using AMPT
model. We observed that the maximum difference in ⟨v_{2}⟩
due to different centrality selection procedure is ∼ 2%.
The agreement between the ⟨v_{2}⟩ result from centrality 1 and centrality 2 shows there is no auto-correlation ef-
due to using common set of particles for both the
centrality determination and ⟨v_{2}⟩ calculation.

V. EVENT PLANE RESOLUTION
CORRECTION

Event plane (Ψ) is an estimation of true reaction plane
Ψ_{r}). As the estimated reaction plane fluctuates owing
to finite number of particles, one has to correct observed
v_{2}^{obs} by the corresponding event plane resolution. The
event plane resolution is defined by the correlation of the
event plane with the reaction plane [21]:

R = ⟨cos(2(Ψ − Ψ_{r})))⟩. \tag{2}

The reaction plane is not measurable in experiments
hence resolution can not be calculated using above rela-
tion. To estimate the event-plane resolution we measure
the correlation between the azimuthal angles of two sub-
set groups of tracks, called sub-events (labeled as A and
B):

R = ⟨cos(2(Ψ − Ψ_{r})))⟩ = C.Cos(2(Ψ^{A} − Ψ^{B})). \tag{3}

where C is factor that depends on the multiplicity of the
event, Ψ^{A,B} are sub event plane angles and ⟨ ⟩ denote
the average over events. The resolution corrected $v_2$ is given as,

$$v_2 = \frac{\langle\cos(2(\phi - \Psi))\rangle}{\langle\cos(2(\Psi - \Psi^B))\rangle} = \frac{v_2^{\text{obs}}}{R}. \quad (4)$$

Most commonly used method for resolution correction for an average $v_2$ over a wider centrality range (like 0-80%) is

$$\langle v_2 \rangle = \frac{\langle v_2^{\text{obs}} \rangle}{\langle R \rangle}. \quad (5)$$

Here $\langle R \rangle$ is the mean resolution in that wide centrality bin and can be calculated as

$$\langle R \rangle = \frac{\sum N_i \langle R \rangle_i}{\sum N_i}, \quad (6)$$

where $N_i$ and $\langle R \rangle_i$ is the multiplicity and resolution of the $i^{th}$ narrow centrality bin (typical centrality bin widths of 5% or 10%). However as shown using AMPT model simulations for charged particles in Fig. 5, such a procedure does not recover back the true $v_2$ denoted as $v_2(\text{RP})$.

Therefore another approach for event plane resolution correction for wide centrality bin has been proposed \cite{21}. In this method resolution correction for wide centrality bin is done by dividing the term $\cos(2(\phi - \Psi))$ by the event plane resolution $\langle R \rangle$ for the corresponding centrality for each event.

$$\langle v_2 \rangle = \langle v_2^{\text{obs}} \rangle \div \langle R \rangle. \quad (7)$$

These two method would not yield the same value of $\langle v_2 \rangle$ because

$$\langle v_2^{\text{obs}} \rangle \div \langle R \rangle \neq \langle v_2^{\text{obs}} \rangle \div R. \quad (8)$$

Figure 5 shows charged particles $v_2$ as function of $p_T$ for 0-80% centrality bin in Au + Au collisions. The red marker corresponds to $v_2$ measured with respect to true reaction plane. Open black and solid blue circle corresponds to $v_2$ measured with respect to event plane and resolution correction done using method described in equation 5 and 7, respectively. The $v_2$ measured with respect to true reaction plane is the actual $v_2$ in the AMPT model. The results in Fig. 5 shows that event-by-event resolution correction method using the equation 7 gives $v_2$ values closer to the true $v_2$.

VI. EVENT BIAS CORRECTION

In the particular case of comparing the $v_2$ values for heavier hadrons such as $\Omega$ to those copiously produced such as pions or protons in minimum bias collisions there is an inherent bias towards the event class. This is illustrated in the top panel of Fig. 6. It shows the number of events as function of particle multiplicity in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from AMPT Model. The black, red, blue and green histogram corresponds to the events which contain at least one proton, $\phi$, $\Xi$ and $\Omega$, respectively. The minimum bias multiplicity distribution with heavier hadrons like $\Xi$ and $\Omega$ are very different from those for protons. The participant eccentricity ($\varepsilon_{\text{part}}$) obtained using the initial position of the participating nucleons \cite{22, 23} is correlated with the particle multiplicity as seen from Fig. 6 bottom panel. Hence the average $\varepsilon_{\text{part}}$ of events containing multi-strange hadron like $\Omega$ in 0-80% wide centrality would be smaller than the average $\varepsilon_{\text{part}}$ determined for events containing protons. Since $v_2$ is driven by the anisotropy of the initial spatial geometry, therefore the event bias is naturally introduced when comparison is made between $v_2$ measured in a wide centrality bin especially for the rarely produced particles like $\Omega$ to that for protons. This event bias effect needs to be corrected before comparisons of minimum bias $v_2$ values for different types of hadrons. This bias could be corrected by normalising the measured $v_2$ by the ratio of standard average $\varepsilon_{\text{part}}$ (for charged particle) to the average $\varepsilon_{\text{part}}$ of the events which contains the particle of interest weighted by the corresponding yield. The correction factor in the present calculations using AMPT data for $\Omega$ is 1.15 where as for $\Xi$ and $\phi$ it is 1.023 and 1.010, respectively. The lighter hadrons do not have a large event bias correction, due to their copious production in nuclear collisions at RHIC energy.

The physical consequence of such an event bias is shown in Fig. 7. The number of constituent quark scaling between $\Omega$ and proton which is naturally expected in AMPT model holds better at the intermediate $p_T$ only
VII. RESONANCE DECAY EFFECT

In the heavy-ion collision a large fraction of stable hadrons are from resonance decays. To study the effect of resonance decays on the elliptic flow of stable hadrons, we have used the UrQMD model.

![Graph showing resonance decay effect](image)

FIG. 8: (Color online) The $v_2$ of $\pi$, $K$ and $p$ as function of $p_T$ at 0-80% centrality with decay off and decay on condition from minimum bias Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV from UrQMD model.

Specifically we study the effect of resonances, such as $\rho$, $\Lambda$, $\phi$, $\eta$, $\Omega$, $\Sigma$ and $\Delta$ on measured $v_2$ of inclusive pion, kaon and proton. In the UrQMD model there is a option for switching off and on the decay of each resonance. Figure 8 shows $v_2$ of $\pi$, $K$ and $p$ as function of $p_T$ with decay off and decay on condition in Au+Au collision from UrQMD model. The ratios shown in the lower panels of Fig 8 shows that there is change of 10% to 15% in the $v_2$ values of pion, less than 5% for the kaon and the $v_2$ values for the proton is almost unaffected. There is a decrease in $v_2$ values due to the decay of resonances. However, one could expect a higher value of $v_2$ from decay of resonances. The decay particle at given transverse momentum arises mostly from a resonance at higher momentum. The $v_2$ value in general increases with $p_T$. However, it seems the decay process being isotropic in the rest frame of the resonance, reduces the $v_2$ [24]. To understand the results better, we have further studied the effect from the decay of $\rho \rightarrow \pi^+ + \pi^-$ on $v_2$ of pions. This decay process is isotropic in the rest frame of the resonance and hence one can expect reduction in the momentum anisotropy of the daughter pions.

Figure 8 shows the $v_2$ of pion as a function of $p_T$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV from UrQMD model for three different cases: (a) All resonances are decayed (shown by red inverted triangles), (b) $\rho$, $\Lambda$, $\eta$, $\Sigma$ and $\Delta$ are not decayed (shown by solid blue circle) and, (c) $\Lambda$, $\eta$, $\Sigma$ and $\Delta$ are not decayed (shown by black triangle). There is decrease in the $v_2$ values of pion due to the decay of $\rho \rightarrow \pi^+ + \pi^-$ as expected from the decay kinematics (comparison between cases (b) and (c)). However, the pion $v_2$ values increases due to decay of $\Lambda$, $\eta$, $\Sigma$ and $\Delta$ (comparing between cases (a) and (c)).

after the event bias correction as described above.
Similarly we have observed that the decreases in kaons \(v_2\) is due to \(\phi \rightarrow K^+ + K^-\) decay (not shown here). There is an overall decrease in the measured value of pion and kaon \(v_2\) due to the decay of the resonances.

\[
\langle v_2^{n=2}\rangle = \langle v_2^{\text{SP}} \rangle = \sqrt{\langle v_2^2 \rangle}.
\]

VIII. ELLIPTIC FLOW FROM SCALAR PRODUCT METHOD

In a scalar product method \([25]\) of determining \(v_2\) each event is partitioned into two sub events, labeled here as A and B. If \(Q_n^A\) and \(Q_n^B\) are the flow vectors of sub events A and B for the \(n^{th}\) harmonic then the correlation between the two sub events is given as

\[
\langle Q_n^A Q_n^{B*}\rangle = \langle v_2^n \rangle M^A M^B,
\]

where \(M^A\) and \(M^B\) are the multiplicities for sub events A and B, respectively. Elliptic flow parameter in this method is given as

\[
v_2(\text{SP}) = \frac{\langle Q_2 u_2^2 \rangle}{\sqrt{\langle Q_2^2 Q_2^{B*} \rangle}}.
\]

Here \(Q_2 = \sum u_i^j\) and \(u_i^j\) is a unit vector associated with the \(i^{th}\) particle. The scalar-product method always yields the root-mean-square \(v_2\), regardless of the details of the analysis \([13]\).

\[
v_2(\text{SP}) = \frac{\langle Q_2 u_2^2 \rangle}{\sqrt{\langle Q_2^2 Q_2^{B*} \rangle}} = \sqrt{\langle v_2^2 \rangle}.
\]

But this is not true for \(v_2(\text{EP})\) measured by conventional event plane method. Recently it has been argued \([13]\) that in the limit of perfect resolution (i.e. \(R \rightarrow 1\))

\[
v_2(\text{EP}) \rightarrow \langle v_2 \rangle.
\]

and in the limit of low resolution

\[
v_2(\text{EP}) \rightarrow \sqrt{\langle v_2^2 \rangle}.
\]

We have investigate this using AMPT model where the actual \(\langle v_2 \rangle\) in known \(\langle v_2(\text{RP}) \rangle\). The event plane resolution from AMPT model is shown in Fig. 10 for nine centrality bins, corresponding to 0-5%, 5-10%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%, 60-70% and 70-80% for the cross section. Resolution is poor for the peripheral centrality and maximum at mid-central and then slightly decreases for the most central collisions. This is due to the interplay of multiplicity and \(v_2\) for different centrality bins. Figure 11 shows charged particle \(v_2\) as a function of centrality in Au+Au collisions at 200 GeV from AMPT model using scalar product, event plane and reaction plane methods. The most central collisions studied corresponds to a value of 9 in the \(x\)-axis and most peripheral collisions corresponds to a value of 0 in the \(x\)-axis. For the peripheral collision where resolution is
poor, $v_2(EP)$ and $v_2(SP)$ are very close to each other that means $v_2(EP)$ is equivalent to $\sqrt{\langle v_2^2 \rangle}$. However for central to mid-central collisions where resolution is high, $v_2(EP)$ is closer to $v_2(RP)$ or $\langle v_2 \rangle$. The results are consistent with with equations 12-13 and as proposed in Ref. [13].

IX. SUMMARY

We have presented a transport model based study of various effects on experimentally measured $v_2$. The results are presented using Au+Au collisions at midrapidity at $N_{NN} = 200$ GeV from AMPT and UrQMD model. We find that finite particle counting efficiency of the detectors used in real experiments affects the measured $v_2$ values. Specifically due to the difference in the reconstruction efficiencies of charged kaon and neutral kaons, the measured $v_2$ values could differ by 10-30\% as a function of $p_T$. Experiments needs to account for this inefficiencies while obtaining $v_2$ using event plane method, before comparing the results for measured hadrons with very different reconstruction efficiencies. We observe that the measured $v_2$ values remain insensitive to the method of centrality determination used in the experiments. However the procedure to correct for event plane resolution in wide centrality bin results affects the extracted $v_2$ values. Event-by-event resolution correction seems to give $v_2$ values that are closer to the true $v_2$ values. The over all effect of resonance decay is to reduce the $v_2$ values relative to the true $v_2$. This is dominated due to kinematic effect of the decay process being isotropic in the rest frame of the resonance and such resonances contributing more in terms of the yields of the measured hadrons. The minimum bias event class in terms of average initial $p_{part}$ value for events having rare heavier particle like $Ω$ is different from those having protons. In order to appropriately compare the minimum bias $v_2$ values of various hadrons, we propose an event bias correction procedure. We also demonstrate that this procedure seems to work by showing that number of constituent quark scaling for $Ω$ and protons $v_2$. Finally we have demonstrated through the model study that in the limit of high resolution for event plane determination the $v_2(EP) \rightarrow \langle v_2 \rangle$ and in the limit of small event plane resolution the $v_2(EP) \rightarrow \sqrt{\langle v_2^2 \rangle}$.

Acknowledgments

Financial assistance from the SwarnaJayanti Fellowship of the Department of Science and Technology, Government of India is gratefully acknowledged. We thank Dr. H. Masui, Dr. S. Shi and Dr. N. Xu for useful discussions on event bias correction method.

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 707, 330 (2012).
[2] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 109, 022301 (2012).
[3] K. Aamodt et al. [ALICE Collaboration], Phys. Rev. Lett. 105, 252302 (2010).
[4] K. Aamodt et al. [ALICE Collaboration], Phys. Rev. Lett. 107, 032301 (2011).
[5] K. H. Ackermann et al. [STAR Collaboration], Phys. Rev. Lett. 86, 402 (2001).
[6] C. Adler et al. [STAR Collaboration], Phys. Rev. Lett. 87, 182301 (2001).
[7] S. S. Adler et al. [PHENIX Collaboration], Phys. Rev. Lett. 91, 182301 (2003).
[8] A. Adare et al. [PHENIX Collaboration], Phys. Rev. Lett. 98, 172301 (2007).
[9] S. A. Voloshin, A. M. Poskanzer and R. Snellings, arXiv:0809.2949
[10] J. Y. Ollitrault, Phys. Rev. D 46, 229 (1992); H. Sorge, Phys. Rev. Lett. 78, 2309 (1997).
[11] P. Huovinen, P. F. Kolb, U. Heinz, P. V. Ruuskanen, and S.A. Voloshin, Phys. Lett. B503, 58 (2001).
[12] P. Romatschke and U. Romatschke, Phys. Rev. Lett. 99, 172301 (2007).
[13] M. Luzum and J. -Y. Ollitrault, Phys. Rev. C 87, 044907 (2013).
[14] Zi-Wei Lin, C. M. Ko, Phys. Rev. C 65, 034904 (2002); Zi-Wei Lin et al., Phys. Rev. C 72, 064901 (2005); Lie-Wen Chen et al., Phys. Lett. B 605, 95 (2005).
[15] X. N. Wang and M. Gyulassy, Phys. Rev. D 44, 3501 (1991).
[16] S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998); M. Bleicher et al., J. Phys. G 25, 1859 (1999).
[17] M. M. Aggarwal et al. [STAR Collaboration], Phys. Rev. C 83, 024901 (2011).
[18] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. C 78, 034909 (2009).
[19] L. Adamczyk et al. [STAR Collaboration], arXiv:1309.5681 [nucl-ex].
[20] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C 58, 1671 (1998) [nucl-ex/9805001].
[21] H. Masui and A. Schmah, arXiv: 1212.3650v1 [nucl-ex].
[22] B. Alver and G. Roland, Phys. Rev. C 81, 054905 (2010) [Erratum-ibid. C 82, 039903 (2010)].
[23] M. .R. Haque, M. .Nasim and B. Mohanty, Phys. Rev. C 84, 067901 (2011) [arXiv:1111.5095 [nucl-ex]].
[24] T. Hirano, Phys. Rev. Lett. 86, 2754 (2001).
[25] STAR Collaboration, C. Adler et al., Phys. Rev. C 66, 034904 (2002).