Concealing Dirac neutrinos from cosmic microwave background

Anirban Biswas, Dilip Kumar Ghosh and Dibyendu Nanda

Abstract. The existence of prolonged radiation domination prior to the Big Bang Nucleosynthesis (BBN), starting just after the inflationary epoch, is not yet established unanimously. If instead, the universe undergoes a non-standard cosmological phase, it will alter the Hubble expansion rate significantly and may also generate substantial entropy through non-adiabatic evolution. This leads to a thumping impact on the properties of relic species decoupled from the thermal bath before the revival of the standard radiation domination in the vicinity of the BBN. In this work, considering the Dirac nature of neutrinos, we have studied decoupling of ultra-relativistic right-handed neutrinos ($\nu_R$s) in presence of two possible non-standard cosmological phases. While in both cases we have modified Hubble parameters causing faster expansions in the early universe, one of the situations predicts a non-adiabatic evolution and thereby a slower redshift of the photon temperature due to the expansion. Considering the most general form of the collision term with Fermi-Dirac distribution and Pauli blocking factors, we have solved the Boltzmann equation numerically to obtain $\Delta N_{\text{eff}}$ for the three right-handed neutrinos. We have found that for a large portion of parameter space, the combined effect of early decoupling of $\nu_R$ as well as the slower redshift of photon bath can easily hide the signature of right-handed neutrinos, in spite of precise measurement of $\Delta N_{\text{eff}}$, at the next generation CMB experiments like CMB-S4, SPT-3G etc. This however will not be applicable for the scenarios with only fast expansion.

Keywords: neutrino properties, neutrino theory, particle physics - cosmology connection, physics of the early universe

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1 Introduction

The anisotropies in leftover radiation from the early universe, known as the cosmic microwave background (CMB), is highly sensitive to the presence of extra radiation energy at the time of recombination [1]. The amount of extra radiation energy density is usually parameterized in terms of the effective numbers of neutrinos as [2]

\[ N_{\text{eff}} = \frac{(\rho_{\text{rad}} - \rho_{r})}{\rho_{\nu_L}} \]  

(1.1)

where \( \rho_{\text{rad}} \) is the total radiation energy density, \( \rho_{r} \) is the energy density of photon and \( \rho_{\nu_L} \) is the energy density of a single active neutrino species. The current data from the measurement of CMB by the Planck satellite [3] suggests \( N_{\text{eff}} = 2.99^{+0.34}_{-0.33} \) at 95% C.L. (including the baryon acoustic oscillation (BAO) data) which perfectly agrees with the Standard Model (SM) prediction \( N_{\text{eff}}^{\text{SM}} = 3.045 \) [2, 4, 5]. The next generation CMB experiments particularly CMB-S4 [6] will be sensitive to a precision of \( \Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}} = 0.06 \) at 95% C.L., which is expected to test all such beyond Standard Model (BSM) scenarios with light degrees of freedom (DOF) that were in equilibrium with the SM at some point of the evolution of our universe or produced non-thermally from the decay or annihilation of other heavy species [7–21]. In many of such BSM scenarios, the primary motivation is to explain the tiny nonzero neutrino masses (see for example [9, 10]) as suggested by neutrino oscillation experiments [22–26]. Besides, the nature of neutrinos, whether they are Dirac or Majorana fermion, is one of the most fundamental open questions in particle physics and there has not been any preference to any of the particular scenario from the existing experimental data till date. However, from the theoretical point of view, the Dirac nature essentially demands at least two extremely light right-chiral components like \( \nu_{L_S} \) compared to the Majorana case where we usually requires heavy fermionic DOF for the seesaw mechanisms (see [27] for a review). Therefore depending on their interactions with the SM particles, these ultra-relativistic DOF in the early universe may have substantial contribution to the radiation energy density and hence to the parameter \( N_{\text{eff}} \) that leads to severe constraints on the interactions of \( \nu_{RS} \) with the bath particles [28–36]. For instance, if there are three \( \nu_{RS} \) and they were in thermal bath at the early universe, the Planck 2018 data suggests that \( \nu_{RS} \) have to be decoupled from
the SM plasma at temperature higher than 600 MeV \cite{37}, otherwise they will contribute more than the current allowed limit of $\Delta N_{\text{eff}} \leq 0.285$ at 95\% C.L. Therefore, the decoupling temperature of $\nu_R$ is very crucial as this will decide $T_{\nu_R}$ at the later epoch which eventually fixes the contribution to $N_{\text{eff}}$. Note that if the neutrino masses are generated only by the standard Higgs mechanism like other SM fermions, the impact of $\nu_R$ into the parameter $N_{\text{eff}}$ would be extremely small (O($10^{-12}$)) \cite{14} due to minuscule Yukawa couplings not allowing $\nu_R$ to attain thermal equilibrium with the SM bath. Therefore, cosmological probe of the Dirac neutrinos in the upcoming CMB experiments will confirm new interactions in the neutrino sector.

In \cite{13}, the authors have introduced new interactions between $\nu_L$ and $\nu_R$ as effective four-fermion interactions and set upper limits on such couplings by considering the impact of new physics in $\Delta N_{\text{eff}}$. In this article, we show that the upper bound discussed in \cite{13}, assuming the standard radiation dominated era prior to the Big Bang Nucleosynthesis (BBN), can be significantly relaxed if one alters the cosmological history of the corresponding epoch \cite{38-56}. Although it has been known with some precision that the universe was radiation dominated at the time of BBN \cite{57, 58}, one cannot exclude the possibility of some component other than radiation dominating the total energy budget of the universe before BBN. Here we explore this possibility and discuss the influence of such non-standard cosmological evolution of the universe on the decoupling of right-handed neutrinos ($\nu_R$) from the thermal bath of the SM particles. We consider that the early universe was dominated by a species whose energy density redshifts with the cosmic scale factor $a$ as $\rho_i \propto a^{-(4+n)}$. At first, we take $n = -1$ which represents an early matter dominated universe \cite{50-55, 59} where the energy density is dominated by a non-relativistic species $M$. In the second case, we consider $n > 0$ and that leads to the scenario discussed in \cite{48} where a species $\Phi$, other than the usual matter and radiation, becomes the dominant component in the energy density. However, one of the fundamental differences between the two above mentioned scenarios is that in the case of early matter domination (i.e. for $n = -1$) the species $M$ should not be absolutely stable like $\Phi$, otherwise it will always remain the dominate source in the energy budget since the energy density of $M$, compared to the radiation, redshifts slowly due to the cosmic expansion. On the other hand, as the energy density of $\Phi$ falls faster than the radiation ($\propto a^{-4}$), it will eventually become sub-dominant at some point of time as the universe expands. Nevertheless, in both the cases, the expansion rate of our universe, denoted by the Hubble parameter $H$, increases due to presence of additional source of energy over the radiation. This gives rise to a faster expanding universe where right-handed neutrinos are decoupled from the thermal bath at some higher temperature for a given interaction strength. The higher decoupling temperature of $\nu_R$ eventually generates a smaller value of $\Delta N_{\text{eff}}$. Moreover, in the first case with $n = -1$, the decay of $M$ into the SM particles leads to a non-adiabatic expansion with entropy production which results in a slowly cooling universe compared to the standard case of adiabatic expansion \cite{60}. This further reduces the ratio $T_{\nu_R}/T$ and hence the contribution of $\nu_R$ in $N_{\text{eff}}$. Therefore, due to the combined effect of both the faster expanding universe and entropy injection in the visible sector, the impact of $\nu_R$ in the parameter $N_{\text{eff}}$ gets heavily suppressed compared to the case with standard $\Lambda$CDM cosmology.

The rest of the paper is organized as follows: in section 2, we describe the effective four-fermion operators responsible for the thermalisation of $\nu_R$. In section 3, we have discussed the impact of $\nu_R$ to $\Delta N_{\text{eff}}$ in the standard cosmological scenario while the section 4 is devoted to analyse the effect of non-standard cosmological histories. Finally, we present our conclusion in section 5. A procedure for simplifying the general collision term of the Boltzmann equation has been presented in appendix A.
possible process & $S \times |\mathcal{M}|^2$\\
\hline $
u_R(p_1) + 
u_R(p_2) \leftrightarrow \nu_L(p_3) + \nu_L(p_4)$ & $8G_S - 12G_T^2(p_1 \cdot p_2)(p_3 \cdot p_4)$ \\
$
u_R(p_1) + \bar{\nu}_R(p_2) \leftrightarrow \nu_L(p_3) + \bar{\nu}_L(p_4)$ & $4G_P - 2G_V|\nu_L(p_1 \cdot p_2)(p_3 \cdot p_4)$ \\
$
u_R(p_1) + \nu_L(p_2) \leftrightarrow \nu_R(p_3) + \nu_L(p_4)$ & $4G_P - 2G_V^2(p_1 \cdot p_2)(p_3 \cdot p_4)$ \\
$
u_R(p_1) + \bar{\nu}_L(p_2) \leftrightarrow \nu_R(p_3) + \bar{\nu}_L(p_4)$ & $4G_P - 2G_V^2(p_1 \cdot p_2)(p_3 \cdot p_4)$ \\
$
u_R(p_1) + \bar{\nu}_L(p_2) \leftrightarrow \nu_R(p_3) + \nu_L(p_4)$ & $16G_S - 12G_T^2(p_1 \cdot p_2)(p_3 \cdot p_4)$ \\
\hline

Table 1. Different processes involved in the thermalisation of $\nu_R$ and the corresponding amplitude square where $S$ represents the symmetry factor corresponding to the matrix elements.

2 Four-fermion interactions of $\nu_R$

In the aforementioned discussion, we have stated that the main motivation of this work is to study the impact of non-standard cosmology on the decoupling temperature of right-handed neutrinos $\nu_R$ and its consequences in $\Delta N_{\text{eff}}$. The right-handed neutrinos can be thermalised in the early universe through their interactions with the SM bath. One can write down the following effective four-fermion operators [13]:

$$\mathcal{L} = G_S \bar{\nu}_R \nu_R \bar{\nu}_L \nu_L + G_P \bar{\nu}_R \nu_L \bar{\nu}_L \nu_R + G_V \bar{\nu}_L \gamma_{\mu} \nu_L \nu_R \gamma_{\mu} \nu_R$$

where $G_S, G_P, G_V, G_T$ are the effective coupling constants for scalar, pseudo scalar, vector and tensor type interactions respectively and have dimension similar to the Fermi constant $G_F$. Here, we have considered that $\nu_R$ interacts only with the active neutrinos ($\nu_L$). In principle, one should consider interactions of $\nu_R$ with all other SM particles that were in thermal bath during decoupling of $\nu_R$ which typically occurred at $T \sim \mathcal{O}(100)$ MeV. However, for simplicity, in this work we have assumed that $\nu_R$ has interaction with the left-handed neutrinos only and based on this assumption we have performed a model independent analysis in an effective theory framework. In table 1, we present different processes involved in the thermalisation of $\nu_R$ and the corresponding amplitude square. In the next section, we discuss briefly about the contribution of $\nu_R$ in $\Delta N_{\text{eff}}$ within the standard cosmological evolution of the universe.

3 Contribution to $\Delta N_{\text{eff}}$ from $\nu_R$ in the standard cosmology

As discussed earlier, the Dirac nature of neutrinos requires the newly added right-chiral parts are as light as the left-handed SM neutrinos. The presence of additional ultra-relativistic species in the thermal plasma at the early universe can give substantial contribution to the effective relativistic degrees of freedom, $N_{\text{eff}}$ that can be probed by the CMB experiments. From eq. (1.1), the additional contribution coming from $\nu_R$ at the time of CMB can be written as

$$\Delta N_{\text{eff}} = \sum_{\alpha} \frac{\rho_{\nu^\alpha}}{\rho_{\nu_L}},$$

$$= 3 \times \frac{\rho_{\nu_R}}{\rho_{\nu_L}},$$

$$= 3 \times \left(\frac{T_{\nu_R}}{T_{\nu_L}}\right)^4,$$

(3.1)
where \( \alpha = 3 \), represents the number of right-handed neutrinos in the theory. In the above equation we have assumed all the three \( \nu_R \)s behave identically and accordingly \( \sum \rho_{\nu_R} = 3 \times \rho_{\nu_R} \), where \( \rho_{\nu_R} \) is the energy density of a single right-handed neutrino species. To estimate \( \Delta N_{\text{eff}} \) due to \( \nu_R \), we need to know the temperature \( T_{\nu_R} \) at the time of CMB, which evolves independently after the decoupling of \( \nu_R \) from the thermal bath. The decoupling temperature \( T_{\text{dec}} \) is usually defined as the temperature when the expansion rate of the universe dominates over the interaction rate \( \Gamma \) and hence at \( T = T_{\text{dec}} \),

\[
\Gamma (T_{\text{dec}}) = H (T_{\text{dec}}). \tag{3.2}
\]

After decoupling from the thermal bath, the energy density of \( \nu_R \) redshifts as \( a(t)^{-4} \), where \( a(t) \) is the cosmic scale factor at any given time \( t \). As the SM neutrinos \( (\nu_L) \) also show the similar behavior due to relativistic decoupling at \( T_{\nu_L}^{\text{dec}} \sim 1 \) MeV, the ratio \( T_{\nu_R}/T_{\nu_L} \) remains unchanged afterwards. So, practically we do not need to compute the ratio at the time of CMB, rather it is sufficient to evaluate the ratio at a much higher temperature \( T \) \( (T > T_{\nu_L}^{\text{dec}} \gg T_{\text{CMB}}) \) when \( \nu_L \) shares same temperature with the photon bath. Accordingly, the eq. (3.1) can also be written as

\[
\Delta N_{\text{eff}} = 3 \times \left( \frac{T_{\nu_R}}{T} \right)^4 \bigg|_{T>T_{\nu_L}^{\text{dec}}},
\]

\[
= 3 \times \xi^4 \bigg|_{T>T_{\nu_L}^{\text{dec}}}, \tag{3.3}
\]

here \( \xi = T_{\nu_R}/T \). Therefore, to evaluate \( \Delta N_{\text{eff}} \) due to \( \nu_R \), all we need is \( T_{\nu_R} \) at a temperature \( T \) just before the decoupling of \( \nu_L \).

In order to proceed further, we need the Boltzmann equation for the energy density of \( \nu_R \) which can also be expressed in terms of \( \xi \) as [15]

\[
x \frac{d\xi}{dx} + (\beta - 1) \xi = \frac{\beta x^4}{4 \kappa \xi^3 H M_0} C_{2 \rightarrow 2}, \tag{3.4}
\]

where, \( M_0 \) is any arbitrary mass scale and \( x = M_0/T \). The quantity \( \beta \) depends on the variation of DOF \((g_*)\) related to the entropy density with \( T \) and its expression is given in appendix A. The collision term for \( 2 \rightarrow 2 \) scatterings listed in table 1 is denoted by \( C_{2 \rightarrow 2} \). In this work, we have considered the most general collision term with quantum statistics (FD distribution) and Pauli blocking factors. To simplify the collision term we have followed the prescription given in [61, 62] which reduces the initial twelve dimensional integration into four dimension and the detailed procedure has been given in appendix A. We have also checked that our result matches with the one given in [13] where the authors had found out the contribution of \( \nu_R \) to \( \Delta N_{\text{eff}} \) by considering the standard cosmological evolution of our universe and put upper limits on the effective coupling constants for different types of interactions as shown in table 1. In figure 1, we show the contributions of various four-fermion operators to \( \Delta N_{\text{eff}} \) as a function of the corresponding effective coupling constant. The most stringent constraint is coming for the tensor type interactions. In terms of the Fermi constant \( G_F = 1.1664 \times 10^{-5} \) GeV\(^{-2} \) the upper bonds that we obtain can be expressed as,

\[
G_S < 5.52 \times 10^{-4} G_F, \quad G_P < 1.28 \times 10^{-3} G_F, \quad G_V < 6.4 \times 10^{-4} G_F, \quad G_T < 4.56 \times 10^{-5} G_F. \tag{3.5}
\]
Figure 1. The impact of $\nu_R$ to the effective relativistic degrees of freedom or $\Delta N_{\text{eff}}$ in presence of different interactions ($G_S, G_P, G_V, G_T$) as shown in table 1. The present and the future experimental bounds are also indicated in the same figure.

However, as we have mentioned earlier, these upper bounds are true provided the universe was radiation dominated throughout its evolution starting from inflation to the matter radiation equality (redshift $z \sim 3400$). This is not a necessary condition from any cosmological observations so far and one can always consider some alternative cosmological histories. In the next section, we will show that these upper limits can be significantly relaxed if we consider non-standard cosmological history instead of the standard $\Lambda$CDM cosmology. Moreover, in figure 1 it appears that $\Delta N_{\text{eff}}$ saturates to a particular value as we lower the coupling $G_{\text{eff}}$ below $10^{-12}$ GeV$^{-2}$. This is mainly due to the reason that a relativistic species which was in thermal equilibrium in the early universe, always has a minimum contribution to $\Delta N_{\text{eff}}$ and it does not depend on the decoupling temperature ($T_{\text{dec}}$). For a single species of $\nu_R$, the minimum value is $\Delta N_{\text{eff}} \simeq 0.027 \times 2 \times \frac{7}{8} \left( \frac{106.75}{g_*(T_{\text{dec}})} \right)^{4/3} = 0.04725$ [7]. In this work, we have considered $G_{\text{eff}} \geq 10^{-14}$ GeV$^{-2}$ such that the thermalisation condition is always maintained.

4 Non-standard cosmological scenarios

4.1 Matter dominated universe

Let us consider that at the early universe just after the inflation, the radiation wasn’t the only component that had significantly contributed to the total energy density. Rather, for some epoch, the total energy budget was dominated by some pressureless fluid, denoted as $M$, and the energy density of such species depends on the cosmic scale factor $a$ like the usual non-relativistic species (referred as matter) as

$$\rho_M \propto a^{-3}. \quad (4.1)$$

In such a scenario, the universe went through different cosmological epochs as shown in figure 2. After starting with the inflationary epoch of rapid expansion, the universe enters
Figure 2. The evolution history of the universe in a early matter dominated universe.

into an early radiation dominated epoch (ERD) at the reheating temperature $T_{RH}$ which is usually defined as the maximum temperature in the radiation dominated universe. The early matter domination (EMD) begins when energy density associated with the species $M$ starts to dominate over that of the radiation at some temperature ($T_i$). As the rate at which the energy density of matter redshifts is slower than the radiation, therefore unless $M$ decays into the radiation at some later epoch, the matter would always dominate the total energy budget. However, as we know that the universe was radiation dominated (RD) at the time of BBN, the species $M$ must decay to the radiation prior to the formation of light elements. Let us consider that at temperature $T_e$ the decay becomes effective and this results in an enhancement in $\rho_R$ and the radiation again starts to dominate the universe’s energy budget at $T_r$ after $M$ decays completely.\footnote{Here, we will study the effect of EMD and EP eras on the decoupling temperature of $\nu_R$, which also depends on the coupling $G_{\nu R}$. To understand the maximum possible impact of non-standard cosmology, we have set $G_{\nu R}$ in such a way that $\nu_R$ always decouples between $T_i$ and $T_r$ when, the total energy density is dominated by the species $M$.}

The cosmological era between $T_e$ and $T_r$ when the species $M$ decays completely into the radiation and creates entropy in the SM bath is known as the entropy production era (EP). The phenomenological consequences of such non-standard cosmological history have been studied in many different contexts [50–55, 63–65].

Now, we shall describe briefly the three non-standard epochs occurred before the usual radiation domination in the vicinity of BBN. The basic difference of the EP era with others is that during this particular epoch the universe undergoes a non-adiabatic expansion and as a result the entropy per co-moving volume $S = sa^3$ is not conserved. When there is both matter as well as radiation and both have non-negligible contributions in the energy density ($\rho$), the Hubble parameter is given by $H = \frac{1}{M_{Pl}}\sqrt{\frac{8\pi}{3}(\rho_R + \rho_M)}$ with $M_{Pl} = 1.22 \times 10^{19}$ GeV, the value of the Planck mass. In the ERD era (region I), $\rho_R \gg \rho_M$ and $\rho \simeq \rho_R = \frac{\pi^2}{30}g_\rho T_i^4$, where $g_\rho$ is known as the number of effective relativistic degrees freedom associated with the radiation energy density. Therefore the expansion rate is same as the usual radiation dominated era i.e.

$$H_{ERD}(T) = \sqrt{\frac{4\pi^3}{45}g_\rho(T)\frac{T^2}{M_{Pl}}}, \text{ for } T \geq T_i \tag{4.2}$$

At $T = T_i$, energy density of the species $M$ becomes equal to the radiation density i.e. $\rho_M = \rho_R(T_i)$ and thereafter the EMD era (region II) starts. As the entropy per comoving volume is conserved before $T \geq T_e$, we can write the energy density of $M$ in any arbitrary temperature ($T \geq T_e$) as

$$\rho_M(T) = \rho_M(T_i)\frac{g_s(T)}{g_s(T_i)}\frac{T^3}{T_i^3},$$

$$= \frac{\pi^2}{30}g_\rho(T_i)\frac{T_i^4}{g_s(T_i)}\frac{g_s(T)}{g_s(T_i)}\frac{T^3}{T_i^3}, \tag{4.3}$$
where \( g_s \) is known as the number of effective relativistic degrees freedom associated with the entropy density of the universe. Using \( \rho_M(T) \) one can easily write the Hubble parameter in the EMD era between \( T_i \) and \( T_e \) as

\[
H_{\text{EDM}}(T) = \frac{1}{M_{\text{Pl}}} \sqrt{\frac{4\pi^3}{45} \frac{g_s(T_i)}{g_s(T_e)} \frac{T^4}{T_i} \frac{\rho_s(T)}{\rho_s(T_i)}}.
\]

\[
= H_{\text{ERD}}(T_i) \sqrt{\frac{g_s(T)}{g_s(T_i)}} \left( \frac{T}{T_i} \right)^{3/2}, \quad \text{for } T_i \geq T \geq T_e. \tag{4.4}
\]

Comparing eqs. (4.2) and (4.4) we can easily notice the different temperature dependence of \( H \) in radiation and matter dominated eras. In the ERD epoch, \( H \propto T^2 \) while matter domination with adiabatic expansion leads to \( H \propto T^{3/2} \).

Region III in figure 2 between \( T_e \) and \( T_r \) is the epoch where decay of the species \( M \) into radiation happens thereby getting back the usual radiation domination at the end. Since the energy is injected from the species \( M \) to the SM by the decay, the universe undergoes a non-adiabatic expansion with entropy production. The evolution of matter density \( \rho_M \), radiation density \( \rho_R \) and entropy per comoving volume \( S \) in this era are given by,

\[
\frac{d\rho_M}{dt} + 3 \rho_M H = -\Gamma \rho_M, \tag{4.5}
\]

\[
\frac{d\rho_R}{dt} + 4 \rho_R H = \Gamma \rho_M, \tag{4.6}
\]

and

\[
\frac{dS}{dt} = \Gamma \rho_M \frac{a^3}{T}. \tag{4.7}
\]

Where \( \Gamma \) is the total decay width of \( M \). One can easily notice that when \( \Gamma \ll H \), we recover the usual properties of adiabatic expansion. The solutions of eqs. (4.5) and (4.6) are

\[
\rho_M = \rho_M(T_e) \left( \frac{a_e}{a} \right)^3 e^{-\Gamma(t-t_e)}, \tag{4.8}
\]

\[
\rho_R = \rho_R(T_e) \left( \frac{a_e}{a} \right)^4 \left( 1 - \left( \frac{a_e}{a} \right)^{5/2} \right) + \frac{\Gamma M}{5 H(T_e)} \rho_M(T_e) \left( \frac{a_e}{a} \right)^{3/2} \left( 1 - \left( \frac{a_e}{a} \right)^{5/2} \right) . \tag{4.9}
\]

Where \( T_e \) is the initial temperature and the corresponding time and scale factor are \( t_e \) and \( a_e \) respectively. The first term of eq. (4.9) is the usual evolution of \( \rho_R \) due to expansion while the second term has the origin from the decay of \( M \). Although, the species \( M \) decays into the radiation during this epoch, we still have the matter dominance in total energy density and using the relation \( a \propto t^{2/3} \) one can easily solve \( \rho_R \) in terms of time \( t \) as

\[
\rho_R = \rho_R(t_e) \left( \frac{t_e}{t} \right)^{8/3} + \frac{3}{5} \Gamma \rho_M(t_e) \left( \frac{t_e}{t} \right)^{2/5} \left( 1 - \left( \frac{t_e}{t} \right)^{5/3} \right) . \tag{4.10}
\]

From the above expression of \( \rho_R \), it is clearly seen that, the first term dies out more quickly compared to the term coming from the decay of \( M \) with respect to the scale factor \( a \). Therefore, within the EP era for \( t \gtrsim t_e \left( \frac{5}{3} \frac{t_e}{3} \right)^{3/5} \), we will eventually have a situation when the second term takes over the first term. As this term has different scale factor dependence
compared to the first one, we have different temperature-scale factor relationship in the EP era between $t \gtrsim t_e \left( \frac{3}{3 \tau_M} \right)^{3/5}$ to $t \simeq \tau_M$, which is given by (in the limit $a \gg a_e$)

$$T \simeq \left( \frac{12}{\pi^2} \frac{\Gamma M}{H(T_e)} \rho_M(T_e) a_e^{3/2} \right)^{1/4} g_r^{-1/4} a^{-3/8}. \quad (4.11)$$

The physical meaning of $T \propto a^{-3/8}$ rather than $T \propto a^{-1}$ is that during ER era, due to energy injection, the temperature of the universe redshifts slowly with the expansion. As a result, the entropy per comoving volume, $S \propto g_s a^{3/8} \sim g_s g_r^{-3/4} a^{15/8}$, increases\(^2\) with the cosmic scale factor. The corresponding temperature-time relationship can easily be obtained from eq. (4.10) as

$$T \simeq \left( \frac{18}{\pi^2} \Gamma M \rho_M(T_e) a_e^2 \right)^{1/4} g_r^{-1/4} t^{-1/4}. \quad (4.12)$$

Therefore, the differential form of temperature-time relationship during the phase of non-adiabatic expansion is

$$\frac{dT}{dt} = -\frac{HT}{\gamma}, \quad (4.13)$$

where

$$\gamma = \frac{8}{3} \left( 1 + \frac{1}{4} T \frac{d g_r}{d T} \right). \quad (4.14)$$

The Hubble parameter in the EP era (region III) is given by

$$H_{EP} = \frac{1}{M_{Pl}} \sqrt{\frac{8\pi^3}{3}} \rho_M(T_e) \left( \frac{a_e}{a} \right)^{3/2}. \quad (4.15)$$

Now using $(\frac{a_e}{a})^{3/2} \simeq \frac{\pi^2}{12} \frac{H(T_e)}{\Gamma M \rho_M(T_e)} g_r T^4$ from eq. (4.11), we get

$$H_{EP} = \frac{H(T_e)^2}{\Gamma M \rho_M(T_e)} \frac{\pi^2}{12} g_r(T) T^4, \quad (4.16)$$

The decay width of $M$ should be of the order of the Hubble parameter when maximum decay occurs i.e. $\Gamma_M = \zeta H(T_r)$ and this leads to the end of the EP era at $T_r$. Here $\zeta = \frac{5}{2}$, a constant which we fix from continuity of the Hubble parameter across the boundary between EP and RD era (at $T = T_r$). After a few mathematical simplifications the Hubble parameter during the EP era is given by

$$H_{EP}(T) = \frac{1}{H_{RD}(T_r)} \frac{4\pi^3}{45} g_r(T) T^4 M_{Pl}^{-1}, \quad (4.17)$$

where $H_{RD}$ is the Hubble parameter in the radiation dominated era (region IV) which has the following well known form

$$H_{RD}(T) = \sqrt{\frac{4\pi^3}{45} g_r(T)} \frac{T^2}{M_{Pl}}. \quad (4.18)$$

\(^2\)During the adiabatic expansion, $T \propto g_s^{1/3} a^{-1}$ and hence $S$ does not have any $a$ (or $T$) dependence.
Moreover, continuity of the Hubble parameter across the boundary between EDM era and EP era i.e. $H_{\text{EDM}}(T_e) = H_{\text{EP}}(T_e)$ correlates the three temperatures $T_i$, $T_e$ and $T_r$ in the following way

$$\frac{T_e}{T_r} = \left( \frac{g_\rho(T_r) g_\rho(T_i) g_s(T_e)}{g_s(T_i) g_s(T_e)} \right)^{1/5}.$$ (4.19)

Therefore, only two among the three temperatures are independent, the rest can be determined by solving the relation iteratively.

**Entropy production.** Here we would like to discuss in more detail about the entropy production in the region III with necessary expressions. From eq. (4.7), it is evident that $S$ is not conserved in the EP era particularly during the decay of $M$ into radiation. The actual amount of entropy increment can be found after solving eq. (4.7). In order to solve the entropy equation let us write it in a more convenient form by replacing $T$ by $S$. After the replacement eq. (4.7) now takes the following form

$$S^{1/3} \frac{dS}{dt} = \left( \frac{2\pi^2}{45} g_s \right)^{1/3} \frac{\Gamma M \rho_M a^4}{S_e}.$$ (4.20)

Substituting $\rho_M$ from eq. (4.8), we can get the fractional change in $S$ after solving the above equation between $t_e$ and $t_r$ (i.e. from temperature $T_e$ up to $T_r$) as [59, 60]

$$\frac{S(t_r)}{S(t_e)} = \frac{S_r}{S_e} = \left( 1 + \frac{4}{3} \left( \frac{2\pi^2}{45} \frac{\rho_M(T_e) a_e^4}{S_e^{4/3}} \right) \frac{\Gamma M}{S_e^{4/3}} I \right)^{3/4},$$ (4.21)

where, the enhancement factor $I$ is given by

$$I = \Gamma M \int_{t_e}^{t_r} a_e^{1/3} g_s \frac{a}{a_e} e^{-\Gamma M (t-t_e)} dt.$$ (4.22)

One can simplify $I$ under certain assumptions that $t_r \gg t_e$, $\Gamma M t_e \ll 1$ and $g_s$ is not changing significantly between $t_e$ and $t_r$ then the enhancement factor has the following simplified expression

$$I \simeq \Gamma \left[ \frac{5}{3} \right] g_s^{1/3} \left( \frac{\Gamma M}{t_e} \right)^{2/3}.$$ (4.23)

Therefore, it is evident that the entropy generation will be maximum when $M$ has a longer lifetime ($\tau_M$) compared to $t_e$, the starting point of the region III in figure 2, or in other word the universe undergoes a prolonged EP era. Note that the matter domination epoch can be controlled by only two parameters $T_i$ and $T_r$ and the other one can be expressed in terms of these two as shown in eq. (4.19).

In figure 3, we have shown the variation of the energy densities of matter ($\rho_M$) and radiation ($\rho_R$) as a function of temperature ($T$). Both the energy densities decreases as the temperature goes down with the expansion of our universe. The epoch of matter domination solely depends on decay width $\Gamma_M$ as shown in eqs. (4.5) and (4.6). It can be seen that the epoch of matter domination which ends through the entropy injection into the radiation bath becomes longer with the decrease of $\Gamma_M$. That means, $T_r$, the temperature when $\rho_M$ becomes subdominant, decreases with the decrease in $\Gamma_M$. In figure 4, we have shown the impact of the two parameters ($T_i$ and $T_r$) on the expansion rate of the universe as a function
\( M_\oplus \Gamma_M = 0.1 \)
\( M_\oplus \Gamma_M = 0.01 \)
\( M_\oplus \Gamma_M = 0.001 \)

\[ \rho \geq 10^{-6} \]

\[ \rho^{1/4} T \geq 10^{-3} \]

\[ H \geq 10^{-25} \]

**Figure 3.** Evolution of energy densities of matter and radiation as a function of temperature has shown for three benchmark values of \( \Gamma_M \) represented in different color and \( M_\oplus \Gamma_M \) is defined as \( M_\oplus = \sqrt{\frac{3}{8\pi}} M_{\text{Pl}} \).

**Figure 4.** The expansion rate for different combinations of \( T_i \) and \( T_r \) as a function of the bath temperature in the matter dominated universe.

of the temperature. The left panel shows the dependence on \( T_i \) (blue for \( T_i = 10^{10} \) GeV, red for \( T_i = 10^9 \) GeV, and brown for \( T_i = 10^4 \) GeV) where we have kept \( T_r \) to be fixed at 10 MeV whereas the right panel represents the impact of \( T_r \) (blue for \( T_r = 10 \) MeV, red for \( T_r = 50 \) MeV, and brown for \( T_i = 100 \) MeV) where \( T_i \) remains fixed at \( 10^4 \) GeV. The green line in both the figure shows the expansion rate in usual standard radiation dominated universe (\( \rho_M = 0 \)). It is clearly seen that in the matter dominated era, the universe expands much
Figure 5. Evolution of $T_{\nu_R}$ for different values of the parameters controlling the matter domination for a given $G_V = 10^{-7}$ GeV$^{-2}$. In the left panel we have fixed $T_r$ at 10 MeV whereas in the right panel $T_i$ is fixed at $10^4$ GeV.

faster than the usual radiation dominated universe. One can also note that, in all these cases, at $T < T_r$ the expansion rate exactly coincide with the standard radiation dominated universe which means that once the matter domination ends we can longer see the impact of the parameters responsible for the non-standard evolution of history. However, as stated before, the non-standard evolution history can affect different cosmological phenomenon and we can always look for their imprints on different cosmological observable and one such observable is $\Delta N_{\text{eff}}$ which can carry the information of such non-standard history. Lets us now discuss consequence of early matter domination era on the $\Delta N_{\text{eff}}$ coming from the thermalised $\nu_R$ in the early universe. Due to the faster expansion, $\nu_{R\bar{R}}$ can be decoupled from the thermal bath at some earlier temperature and reduce their final temperature at some later time. However, one important point to note here is that $\Delta N_{\text{eff}}$ depends on the ratio $T_{\nu_R}/T$ which will not only be affected by the faster expansion of the universe but also the entropy injection in the SM sector from the decay of $M$. The energy density frozen in the non-relativistic matter $M$ had to be transferred to the radiation sector to end the matter domination in order to begin the radiation domination on the on set of BBN. Due to this entropy injection the total entropy in the co-moving volume would no longer be constant and the temperature of the thermal bath will start falling slower than the usual as $a^{-3/8}$ [41, 66]. The impact of the faster expansion and entropy injection to the SM bath on the evolution of $T_{\nu_R}/T$ is shown in figure 5. The left panel shows the evolution for three benchmark values of $T_i$ by keeping the temperature $T_r$ fixed at 10 MeV whereas in the right panel we have varied $T_r$ by keeping $T_i$ fixed at $10^4$ GeV. One can clearly notice that due to the faster expansion the ratio of the temperature drops below 1 at some higher temperature in comparison to the standard radiation domination (the blue dot-dashed line) and then sharply falls due to the entropy injection in the thermal plasma and becomes constant when the entropy injection ends. As a result of this significant drop of $T_{\nu_R}/T$, the value of $\Delta N_{\text{eff}}$ becomes much smaller in comparison to the standard radiation dominated universe. While showing the evolution of the temperature ratio we have considered the vector type interactions only and set $G_V = 10^{-7}$ GeV$^{-2}$ whereas all the other kind of interactions has been set to zero. As the early matter domination decreases the final value of $T_{\nu_R}/T$, stronger interactions between $\nu_R$ and SM plasma can be allowed which are excluded in the radiation dominated universe.
Finally, in figure 6, we present the main result of this section. We have shown the variation of $\Delta N_{\text{eff}}$ as a function of four-fermion interaction strength and here we have considered only the vector type coupling $G_V$. The other kind of interactions will also show the similar behaviour. As discussed earlier, the faster expansion and entropy injection push $\Delta N_{\text{eff}}$ to much smaller value than the standard cosmological scenario for a given interaction strength. In the left panel of figure 6, we show that for a fixed $T_r = 100 \text{MeV}$, $\Delta N_{\text{eff}}$ decreases with increasing $T_i$. This is because lower value $T_i$ means the lowering the entropy injection to the SM bath. The right panel shows the impact of $T_r$ for three benchmark values where we have kept $T_i$ fixed at $10^4 \text{GeV}$. In both the cases, for any given interaction strength, $\Delta N_{\text{eff}}$ becomes much less than the standard radiation dominated universe (the blue dashed line corresponds to $\rho_M = 0$). In the next section, we have discussion another alternative non-standard cosmological history where the early epoch of the universe was dominated by some species $\Phi$ whose energy density falls even faster than the radiation which can also enhance the expansion rate of the universe and can significantly affect the decoupling temperature of $\nu_R$ and the $\Delta N_{\text{eff}}$.

4.2 Fast expanding universe

Let us consider another non-standard cosmological scenario where the early universe was dominated by a species ($\Phi$) which redshifts faster than radiation,

$$\rho_{\Phi} \propto a^{-(4+n)},$$

where $n > 0$ for $\Phi$ and $n = 0$ for radiation. One can express $\rho_{\Phi}$ as a function of the bath temperature ($T$) which can be achieved by incorporating the conservation of total entropy in a comoving volume $S = sa^3 = \text{constant}$, where the entropy density(s) can be expressed as,

$$s(T) = \frac{2\pi^2}{45} g_s(T) T^3.$$  

As we already know that the universe was radiation dominated at the time of the BBN, let us now define a temperature $T_r$ at which the $\rho_{\Phi}$ becomes equal to $\rho_R$ below which the $\rho_R$
dominates the total energy budget. From eq. (4.24), one can write

\[ \rho_\Phi(T) = \rho_\Phi(T_r) \left( \frac{a(T_r)}{a(T)} \right)^{4+n} \] (4.26)

\[ = \rho_\Phi(T_r) \left( \frac{g_s(T)}{g_s(T_r)} \right)^{4+n} \left( \frac{T}{T_r} \right)^{4+n} \] (4.27)

where in the last line we use the entropy conservation condition at temperature $T$ and $T_r$. Finally, the total energy can be expressed as

\[ \rho(T) = \rho_R(T) + \rho_\Phi(T) = \rho_R(T) \left[ 1 + \frac{\rho_\Phi(T_r)}{\rho_R(T)} \left( \frac{g_s(T)}{g_s(T_r)} \right)^{4+n} \left( \frac{T}{T_r} \right)^n \right] \] (4.28)

by setting $\rho_\Phi(T_r) = \rho_R(T_r)$ as mentioned above. The expansion rate of the universe at the $\Phi$ dominated era (at $T > T_r$) can be controlled by two different parameters $T_r$ and $n$. We use this expression to evaluate the Hubble parameter as,

\[ H(T) = \frac{1}{M_{Pl}} \sqrt{\frac{8\pi \rho(T)}{3}}. \] (4.29)

In figure 7, we show the expansion rate for different combinations of $T_r$ and $n$ as a function of the bath temperature and compared it with the standard cosmology. In the left panel, we show the expansion rate for three different $n$ ($n = 1$ for blue, $n = 2$ for red, and $n = 3$ for brown) by keeping $T_r$ fixed at 5 MeV. The green line corresponds to standard radiation dominated universe where $\rho_\Phi = 0$. We see that the expansion rate increases with increasing $n$ as it increases the total energy content of the universe. Similarly, the right panel shows the dependence on $T_r$, the temperature below which $\rho_\Phi$ becomes subdominant. One can notice that smaller the $T_r$ longer the $\Phi$ domination and faster the expansion as shown in the right panel of the figure 7. However, $T_r$ and $n$ are not completely independent parameters as the lower value of $T_r$ must be larger than $T_{BBN} \approx 1$ MeV. For $T_r$ very close to the BBN temperature, the faster expansion of the universe can modify the prediction of the...
abundance of the light elements prior to BBN. As discussed in [13], to avoid such effects we must satisfy the following condition,

\[ T_r \geq (15.4)^{\frac{1}{5}} \text{ MeV}. \]  

(4.30)

Let us now understand the impact of \( T_r \) and \( n \) on the decoupling temperature of the right-handed neutrinos (\( \nu_R \)) which is shown in figure 8. From the above discussion, it is clear that both \( T_r \) and \( n \) can significantly increase the expansion rate of our universe and as a result affects the decoupling temperature of \( \nu_R \). For a given interaction rate (\( \Gamma \)), \( \nu_R \) would decouple from the thermal bath at some higher temperature than the standard scenario (as discussed in [13]) because of the faster expansion. Due to their early decoupling, their final temperature also become smaller than the usual scenario and the contribution to \( \Delta N_{\text{eff}} \) would also decrease. In figure 8, we have shown the footprint of \( T_r \) and \( n \) in the final temperature of \( \nu_R \). The left panel shows the dependence on \( n \) while we keep \( T_r \) fixed at 5 MeV and the right panel manifests the effect of \( T_r \) by considering a fixed \( n = 3 \). We have shown the evolution of \( T_{\nu_R} \) in the standard radiation dominated universe by blue dot-dashed line. In both the cases, we set the effective coupling constant \( G_V = 10^{-7} \text{ GeV}^{-2} \) (8.57 \( \times 10^{-3} G_F \)). One very crucial point to note here is that \( G_V = 10^{-7} \text{ GeV}^{-2} \) lies well above the upper limit given in eq. (3.5). This can also be understood by looking at the final value of \( (T_{\nu_R}/T)^4 = 0.146 \) which gives \( \Delta N_{\text{eff}} = 0.438 \) (by using eq. (3.3)) where we consider the standard expansion history. However, for sufficiently fast expansion \( \nu_R \) can decouple at much higher temperature and resulting to smaller \( \Delta N_{\text{eff}} \). For an example, if we set \( T_r = 5 \text{ MeV} \) and \( n = 2 \) the final value of \( (T_{\nu_R}/T)^4 \) would be 0.0676 as shown in the brown line in the left panel of figure 8, corresponding to \( \Delta N_{\text{eff}} = 0.203 \) which becomes allowed from PLANCK data [3] at 2\( \sigma \) C.L. So, we can reclaim the parameter space disfavored from the standard cosmology with the help of faster expansion of our universe.

Finally in figure 9, we present the main result of this work. We have shown that the upper limits given in figure 1 or in eq. (3.5) will drastically change if we consider a non-standard cosmological evolution of the universe. The left panel is showing the impact of \( n \) (\( n = 1 \) (red), \( n = 2 \) (green) and \( n = 3 \) (brown)) while we have kept \( T_r \) fixed at 5 MeV and the right panel manifests the effect of \( T_r \) (\( T_r = 100 \text{ MeV} \) (red), \( T_r = 50 \text{ MeV} \) (green) and \( T_r = 5 \text{ MeV} \) (brown)) by considering a fixed \( n = 3 \). The blue dashed line represents the
\[ \rho_\Phi = 0 \]

\[ T_r = 5 \text{ MeV}, \ n = 1 \]

\[ T_r = 5 \text{ MeV}, \ n = 2 \]

\[ T_r = 5 \text{ MeV}, \ n = 3 \]

\[ \text{CMB-S4} (2\sigma) \]

\[ \text{SPT} - 3G (1\sigma) \]

\[ \text{PLANCK} 2018 (2\sigma) \]

\[ \Delta N_{\text{eff}} \]

\[ 0.1 \]

\[ 0.05 \]

\[ 0.2 \]

\[ 10^{-14} \]

\[ 10^{-13} \]

\[ 10^{-12} \]

\[ 10^{-11} \]

\[ 10^{-10} \]

\[ 10^{-9} \]

\[ 10^{-8} \]

\[ 10^{-7} \]

\[ 10^{-6} \]

Figure 9. The impact of \( \nu_R \) to the effective relativistic degrees of freedom or \( \Delta N_{\text{eff}} \) in an early \( \Phi \) dominated universe. Here, we have considered interaction for \( G_V \).

| \( T_r, n \) | Early \( \Phi \) dominated universe |
|-------------|----------------------------------|
|             | \( G_S^\Phi / G_S \) | \( G_P^\Phi / G_P \) | \( G_V^\Phi / G_V \) | \( G_T^\Phi / G_T \) |
| 5 MeV, 1    | 2.98 | 3.01 | 3.02 | 2.98 |
| 5 MeV, 2    | 7.67 | 7.78 | 7.78 | 7.67 |
| 5 MeV, 3    | 18.45 | 18.79 | 18.79 | 18.45 |
| 50 MeV, 3   | 3.88 | 3.36 | 3.36 | 3.88 |
| 100 MeV, 3  | 2.5  | 2.16 | 2.16 | 2.5  |

Table 2. Ratio of the upper bound on different types of interactions.

\( \Delta N_{\text{eff}} \) in the standard radiation dominated universe where \( \rho_\phi = 0 \). We have presented the results for \( G_V \) ranging from \( 10^{-14} \text{ GeV}^{-2} \) to \( 10^{-6} \text{ GeV}^{-2} \) and set the other couplings to be zero. The behavior will be same for the other kind of interactions as well. In the table 2, we have shown the changes of the upper bound of different kind of interactions as the ratio between the upper limit in the early \( \Phi \) dominated universe and upper limit in the standard cosmology. Notice that the more significant change is happening for large \( n \) and small \( T_r \) as this governs the faster expansion for a longer period of time.

5 Conclusion

The origin of small neutrino masses and the nature of neutrinos are two unsolved puzzles of fundamental particle physics so far and new particles and interactions are necessary to address these issues. The Dirac neutrino framework is particularly interesting to study as it contains at least two right-chiral components as light as the left-handed ones, which can leave their signature in \( N_{\text{eff}} \) and can be probed in the current as well as future experiments measuring cosmic microwave background photons. However, this requires new non-standard interactions to thermalise \( \nu_R \) with the SM bath at the early universe as the contribution to \( N_{\text{eff}} \) by Dirac neutrinos acquiring masses only via the Standard Higgs mechanism (like other SM fermions) is too small to be detected even in the next generation experiments also.
Therefore, we have considered all possible types (e.g. scalar, pseudo scalar, vector, tensor) of interactions between $\nu_L$ and $\nu_R$ in terms of effective four-fermion operators. In order to calculate $\Delta N_{\text{eff}}$ due to $\nu_R$, we need the temperature ratio $T_{\nu_R}/T$ just before the decoupling of left-handed neutrinos. This we have obtained after solving the Boltzmann equation for the most general collision term with FD statistics and Pauli blocking factors. First, we have considered the standard cosmological history and put upper bounds on the effective couplings of four-fermion operators using the latest $2\sigma$ bound on $\Delta N_{\text{eff}}$ by the Planck collaboration. The most stringent constraint is obtained for the tensorial interaction between $\nu_L$ and $\nu_R$ where the upper limit on the coupling is $G_T < 4.56 \times 10^{-5} G_F$. Our results have matched pretty well with earlier study in the literature.

Next, we have considered non-standard cosmological evolution of the universe prior to the BBN, where energy density of the universe is dominated by some species with energy density $\rho_i \propto a^{-(4+n)}$ and $n \neq 0$. We have explored two different cases i) $n = -1$ (early matter ($M$) domination) and ii) $n > 0$ (early $\Phi$ domination, which is neither matter nor radiation). In the case of early $\Phi$ domination, we have only the effect of fast expansion before the BBN as the Hubble parameter in this era is $H \propto T^{2+n/2}$ ($n > 0$). This leads to an early decoupling of $\nu_R$ from the thermal bath which results in a reduced $\Delta N_{\text{eff}}$. However, in this case, the magnitude of $\Delta N_{\text{eff}}$ due to $\nu_R$ is bounded from below. That means, no matter how early the decoupling occurs, there is always a minimum value of $\Delta N_{\text{eff}} \sim 0.14$ for three right-handed neutrinos if they are in thermal bath at some epoch and this minimum value is independent of the type of interaction that thermalises $\nu_R$. The next generation CMB experiments like CMB-S4, SPT-3G etc. with much improved sensitivity can easily validate the idea of thermalised $\nu_R$ in the context of standard cosmology and also for early $\Phi$ domination.

On the other hand, the situation is different for the case with early matter domination. Like the universe with $\Phi$ domination, the matter dominated universe ($n = -1$) also expands at a faster rate due to the presence of additional energy in the form of non-relativistic matter over the standard radiation. This also causes an early decoupling of $\nu_R$. However, unlike the previous case, this is not the only thing that affects $\Delta N_{\text{eff}}$. In this case, the universe also undergoes a non-adiabatic expansion when the matter field decays into the radiation composed of the SM particles. This results in a slowly cooling universe since the entropy injection in the SM bath alters the standard temperature-scale factor relation to $T \propto a^{-3/8}$. Accordingly, the early decoupling of $\nu_R$ followed by slower redshift of the SM bath temperature in contrast to the $\Lambda$CDM cosmology, reduces the ratio $T_{\nu_R}/T$ at $T \sim \mathcal{O}(\text{MeV})$ substantially. Therefore, for an early matter dominated universe, the upcoming CMB experiments may not be able to trace $\nu_R$ in $N_{\text{eff}}$ if there exists a prolonged non-adiabatic phase ($\tau_M \gg t_e$).

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A The Boltzmann equations

In this appendix, we have described the Boltzmann equation expressing the evolution of energy density \( \rho_{\nu R} \) of \( \nu_R \) with respect to the photon temperature \( T \). In particular, we have focused on the collision term for a general \( \nu_L - \nu_R \) scattering taking into account the Fermi-Dirac (FD) distribution functions for neutrinos and appropriate Pauli blocking factors. Following our earlier works [15] on the Dirac nature of neutrinos and its impact on the collision term for a general \( 2 \to 2 \) scattering like \( \nu_i(P_1) + \nu_j(P_2) \leftrightarrow \nu_k(P_3) + \nu_l(P_4) \) we can write the Boltzmann equation of \( \rho_{\nu R} \) for a general \( 2 \to 2 \) scattering as follows.

\[
\frac{d}{dt} \rho_{\nu R} + 3H (\rho_{\nu R} + P_{\nu R}) = C_{2\to 2}, \tag{A.1}
\]

where \( P_{\nu R} \) is the pressure of \( \nu_R \) which we have considered equal to \( \rho_{\nu R}/3 \) as in the case of radiation. The collision term for a general \( 2 \to 2 \) scattering like \( \nu_i(P_1) + \nu_j(P_2) \leftrightarrow \nu_k(P_3) + \nu_l(P_4) \) is given by,

\[
C_{2\to 2} = \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 (2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4) S |M|^2 (\Delta E_1 - \Delta' E_3) \Lambda(f_1, f_2, f_3, f_4), \tag{A.2}
\]

As we are interested in the species \( \nu_R \), we are considering only those \( 2 \to 2 \) scatterings where in the initial state we have at least one \( \nu_R \) while the final state depending on a particular process may or may not have \( \nu_R \). The four momentum of a species is denoted by \( P_i \) and the corresponding three momentum and energy are denoted by \( \vec{p}_i \) and \( E_i \) respectively. The phase space element \( d\Pi_i = \frac{d^4 \vec{p}_i}{(2\pi)^4 2 E_i} \), and \( S \) is the symmetry factor which is equal to \( \frac{1}{3!} \) for each pair of identical species in the initial and final states respectively. Moreover, the factors \( \Delta \) and \( \Delta' \) represent number of \( \nu_R \) in the initial and the final state. The distribution functions including the Pauli blocking factors are within the quantity \( \Lambda \) which has the following expression

\[
\Lambda(f_1, f_2, f_3, f_4) = f_3 f_4 (1 - f_1) (1 - f_2) - f_1 f_2 (1 - f_3) (1 - f_4), \tag{A.3}
\]

with \( f_i = \frac{1}{\exp(E_i/T_i) + 1} \) where \( T_i = T \), the photon temperature, for the \( \nu_L \) while the temperature of \( \nu_R \) is denoted by \( T_{\nu_R} \). In eq. \( (A.2) \), \( |M|^2 \) is the Lorentz invariant matrix amplitude square. We have listed the expressions of \( |M|^2 \) in table 1 for all the relevant scattering processes involving \( \nu_R \).

In order to proceed further, we need to simplify the collision term with twelve dimensional integration over the three momenta of initial and final state particles. For that we have followed the prescription given in [61, 62]. Here we have described the procedure mentioning important expressions which we require to simplify the collision term. We first perform the integration over \( \vec{p}_4 \) and for that we use the identity

\[
\frac{d^3 \vec{p}_4}{2 E_4} = d^4 P_4 \delta(P_4^2) \Theta(P_4^0), \tag{A.4}
\]

where, for simplicity, we have neglected the tiny neutrino masses as those are many orders of magnitude smaller than the typical energy of the neutrinos. The time component of the four momentum \( P_4 \) needs to be greater than zero and this has been ensured by the Heaviside step function \( \Theta(P_4^0) \). The integration over \( d^4 P_4 \) in eq. \( (A.2) \) is now done using...
four dimensional Dirac delta function. As a result, we replace $P_4$ by $P_1 + P_2 - P_3$ and $P_4^2 = P_1^2 + P_2^2 + P_3^2 + 2(P_1 P_2 - P_1 P_3 - P_2 P_3)$. Therefore, the collision now reduces to

$$
C_{2\to2} = 2\pi \int d\Pi_1 d\Pi_2 d\Pi_3 S |M|^2 (\Delta E_1 - \Delta'E_3) \delta (2(P_1 P_2 - P_1 P_3 - P_2 P_3))
\times \Theta (|\vec{p}_1 + \vec{p}_2 - \vec{p}_3|) \Lambda(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3)) .
$$

(A.5)

Now, we need to choose a specific reference frame. We consider the vector $\vec{p}_1$ along the Z axis. The vector $\vec{p}_2$ has polar angle $\alpha$ and azimuthal angle $\beta$ while the vector $\vec{p}_3$ has polar angle $\theta$ and azimuthal angle $\phi$ respectively. However, if we express all the dot products in eq. (A.5) in terms of polar and azimuthal angles, we will find that only the difference between the azimuthal angles of $\vec{p}_2$ and $\vec{p}_3$ is important. Hence, in the subsequent calculations we consider $\beta$ as the difference between two azimuthal angles. Therefore all the necessary scalar products of three momenta are given by

$$
\begin{align*}
\vec{p}_1.\vec{p}_2 &= p_1 p_2 \cos \alpha , \\
\vec{p}_1.\vec{p}_3 &= p_1 p_3 \cos \theta , \\
\vec{p}_2.\vec{p}_3 &= p_2 p_3 \left( \sin \alpha \sin \theta \cos \beta + \cos \alpha \cos \theta \right).
\end{align*}
$$

(A.6)

The next step is to do integration over the azimuthal angle $\beta$ and the argument of the Dirac delta function is

$$
g(\beta) = 2(p_1 p_2 (1 - \cos \alpha) - p_1 p_3 (1 - \cos \theta) - p_2 p_3 (1 - \cos \alpha \cos \theta)) + p_2 p_3 \sin \alpha \sin \theta \cos \beta ,
$$

and

$$
\frac{dg(\beta)}{d\beta} = -2p_2 p_3 \sin \alpha \sin \theta \sin \beta .
$$

(A.7)

Using the well known property of the Dirac delta function,

$$
\int d\beta \delta(g(\beta)) = \sum_i \int d\beta \frac{\delta(\beta - \beta_i)}{|\frac{dg(\beta)}{d\beta}|_{\beta = \beta_i}},
$$

(A.9)

the integration over $\beta$ can be done easily where the roots $\beta_i$s can be obtained by setting $g(\beta) = 0$ which gives

$$
\cos \beta_0 = -\frac{p_1 p_2 (1 - \cos \alpha) - p_1 p_3 (1 - \cos \theta) - p_2 p_3 (1 - \cos \alpha \cos \theta)}{p_2 p_3 \sin \alpha \sin \theta} ,
$$

(A.10)

and this results in two different values of $\beta_0$ lying between $[0, \pi]$ and $[\pi, 2\pi]$ for one particular value of $\cos \beta_0$. The natural condition $\cos^2 \beta_0 \leq 1$ automatically sets $\sin^2 \beta_0 \geq 0$ and it further implies from eq. (A.8) that $|\frac{dg(\beta)}{d\beta}|_{\beta = \beta_0} \geq 0$. As the integrand is symmetric in $\beta$, we can write

$$
\int_0^{2\pi} d\beta \delta(g(\beta)) = 2 \int_0^\pi d\beta \frac{\delta(\beta - \beta_0)}{|\frac{dg(\beta)}{d\beta}|_{\beta = \beta_0}} \Theta \left(\left|\frac{dg(\beta)}{d\beta}\right|_{\beta = \beta_0}^2\right) .
$$

(A.11)

Substituting $\sin \beta_0$ using the expression of $\cos \beta_0$ (eq. (A.10)) in eq. (A.8), we can express $|\frac{dg(\beta)}{d\beta}|_{\beta = \beta_0}$ in the form of a quadratic equation in $\cos \alpha$ as

$$
|\frac{dg(\beta)}{d\beta}|_{\beta = \beta_0} = 2\sqrt{a_0 \cos^2 \alpha + b_0 \cos \alpha + c_0} ,
$$

(A.12)
Finally, the collision term with integrations over \( \theta \) when the discriminant is between the two roots where, to integrate over \( \theta \), will enter in

\[
C_{2\rightarrow 2} = \frac{(2\pi)^2 S}{2^3 (2\pi)^5} \int \frac{p_{21}^2 dp_{11} p_{22}^2 dp_{22} p_{32}^2 dp_{32}}{E_1 E_2 E_3} \frac{d(cos \alpha) d(cos \theta)}{a_\theta \cos^2 \alpha + b_\theta \cos \alpha + c_\theta} |M|^2 (\Delta E_1 - \Delta' E_3)
\]

\[
\times \Theta \left( |\vec{p}_1 + \vec{p}_2 - \vec{p}_3| \right) \Theta \left( a_\theta \cos^2 \alpha + b_\theta \cos \alpha + c_\theta \right) \Lambda(f_1, f_2, f_3, f_4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3)),
\]

where we have trivially integrated over the other azimuthal angle \( \phi \) also. The next step is to integrate over \( \cos \alpha \) and for that one needs the actual expression of \( |M|^2 \). However, since \( |M|^2 \) is Lorentz invariant, we should only have scalar products of four momenta and the polar angles will enter in \( |M|^2 \) through \( P_1, P_2 \) and \( P_1, P_3 \) only.\(^3\) Therefore, in general, the matrix amplitude square can be expressed a quadratic equation of \( \cos \alpha \) whose coefficients depend on three momenta \( p_1, p_2, p_3 \) and also on the other polar angle \( \theta \) similar to eqs. (A.13)–(A.15).

The integration over \( \cos \alpha \) can be done using the following results \([62]\),

\[
\int_{-\infty}^{\infty} \frac{dx}{\sqrt{a_\theta x^2 + b_\theta x + c_\theta}} \Theta \left( a_\theta x^2 + b_\theta x + c_\theta \right) = \frac{\pi}{\sqrt{-a_\theta}} \Theta \left( b_\theta^2 - 4a_\theta c_\theta \right),
\]

\[
\int_{-\infty}^{\infty} \frac{x dx}{\sqrt{a_\theta x^2 + b_\theta x + c_\theta}} \Theta \left( a_\theta x^2 + b_\theta x + c_\theta \right) = \frac{b_\theta}{2a_\theta \sqrt{-a_\theta}} \Theta \left( b_\theta^2 - 4a_\theta c_\theta \right),
\]

\[
\int_{-\infty}^{\infty} \frac{x^2 dx}{\sqrt{a_\theta x^2 + b_\theta x + c_\theta}} \Theta \left( a_\theta x^2 + b_\theta x + c_\theta \right) = \frac{\pi (3b_\theta^2 - 4a_\theta c_\theta)}{8a_\theta^2 \sqrt{-a_\theta}} \Theta \left( b_\theta^2 - 4a_\theta c_\theta \right),
\]

where, \( x = \cos \alpha \). Although, the limit of the integrals is between \([-\infty, +\infty]\), the actual limit is between the two roots \( \cos \alpha_{\pm} = -\frac{b_\theta \pm \sqrt{b_\theta^2 - 4a_\theta c_\theta}}{2a_\theta} \) of the quadratic equation \( a_\theta \cos^2 \alpha + b_\theta \cos \alpha + c_\theta \) due to the Heaviside step function. The roots will be bounded between \([-1, 1]\) when the discriminant \( b_\theta^2 - 4a_\theta c_\theta > 0 \). This will also determine the actual limit of the integral over \( \cos \theta \) between \( \cos \theta_{-\pm} \) where

\[
\cos \theta_{-} = \max \left[ -1, \frac{p_3(p_1 + 2p_2 - 2p_2(p_1 + p_2))}{p_1 p_3} \right] \text{ for } p_1 + p_2 \geq p_3.
\]

Finally, the collision term with integrations\(^4\) over \( p_1, p_2, p_3 \) and two polar angles is given by

\[
C_{2\rightarrow 2} = \frac{S}{2^3 (2\pi)^5} \int_{0}^{\infty} \frac{p_{21}^2 dp_{11}}{E_1} \int_{0}^{\infty} \frac{p_{22}^2 dp_{22}}{E_2} \int_{0}^{\infty} \frac{p_{32}^2 dp_{32}}{E_3} \int_{\cos \theta_{-}}^{\cos \alpha_{+}} d(cos \theta) \int_{\cos \alpha_{-}}^{\cos \alpha_{+}} d(cos \alpha) \frac{|M|^2}{\sqrt{a_\theta \cos^2 \alpha + b_\theta \cos \alpha + c_\theta}}
\]

\[
\times (\Delta E_1 - \Delta' E_3) \Theta \left( |\vec{p}_1 + \vec{p}_2 - \vec{p}_3| \right) \Theta \left( a_\theta \cos^2 \alpha + b_\theta \cos \alpha + c_\theta \right) \Lambda(f_1, f_2, f_3, f_4(\vec{p}_1 + \vec{p}_2 - \vec{p}_3)),
\]

\(^3\)We can replace the scalar product \( P_3.P_3 \) using the relation \( P_2.P_3 = P_1.P_2 - P_1.P_3 \) for \( m_\nu = 0 \).

\(^4\)We have numerically computed the multi-dimensional integration using Cuba library\(^5\) \([67]\).
with $E_i = p_i$ in the present case with $m_\nu = 0$. For general case when the initial and final state particles cannot be neglected, the expressions given in eqs. (A.10), (A.13)–(A.15) and (A.20) will be different and can be found in [61].

Now, substituting $\rho_\nu = \kappa T^{4}_{\nu R}$ with $\kappa = g_{\nu R} \frac{7}{30} \pi^2$ and $g_{\nu R} = 1$, and transforming time $t$ by the photon temperature $T$ in eq. (A.1), we get the evolution equation for $\xi = T_\nu R / T$ with $x = M_0 / T$ as given below

$$x \frac{d \xi}{dx} + (\beta - 1) \xi = \frac{\beta x^4}{4 \kappa \xi^3 H M_0^4} C_{2 \rightarrow 2},$$

where $M_0$ is any arbitrary mass scale, and $\beta = \left(1 - \frac{1}{3} \frac{g_\nu}{g_s} \frac{d g_s}{d x}\right)$ with $g_s$ is the number of degrees of freedom associated with the entropy density.

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\(^6\)Since we have considered the contribution of $\nu_R$ only in $\rho_{\nu_R}$. If we want to include the effect of $\nu_R$ in the energy density (i.e. $g_{\nu R} = 2$), we have to add a collision term for $\nu_R$ also in the right hand side of eq. (A.1). As the collision terms for $\nu_R$ and $\nu_R$ are identical, this results in an extra factor of 2 in the right hand side of eq. (A.1) which eventually is canceled out by $g_{\nu R}$ within the quantity $\kappa$ in the denominator.
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