Optimal control methods for vertical and horizontal beam dynamics

A Mitura¹, J Warminski¹ and M Bochenski¹
¹Department of Applied Mechanics, Lublin University of Technology, Nadbystrzycka 36, 20-618 Lublin, Poland
a.mitura@pollub.pl, j.warminski@pollub.pl, m.bochenski@pollub.pl

Abstract. An application of the Macro Fiber Composite (MFC) actuators for damping of a composite beam is presented in this paper. The effectiveness of vibration reduction by a selected control method is tested for vertical and horizontal position of the beam. The original model has been studied numerically by using Galerkin’s discretisation method. The numerical results for the vertical and horizontal beams are compared.

1. Introduction
Structures composed of beams are important in civil and mechanical engineering systems. Examples of such structures are L-shaped and T-shaped coupled beams. In these coupled beams the resonance behaviour depends on internal autoparametric coupling and nonlinearities. Theoretical and experimental investigations of vibrations of an autoparametric L-shaped system are presented in paper [4]. The authors proposed an analytical model and verified it by experiments and the finite element method. They show that the natural frequencies and vibration modes obtained by both methods are consistent. The present work is a continuation of those previous investigations and it is the first step to designing the optimal control of an L-shaped beam structure.

The L-shaped beam structure consists of two lightweight, inextensible beams (horizontal and vertical) connected in a right angled shape. To reduce vibration two active elements, fixed to each beam are used. Macro Fiber Composite (MFC) has been selected as the actuator technology. This kind of piezoactuator allows a proper interaction with flexible structures [3]. Different beam positions (vertical or horizontal) determine different dynamic properties. The vertical and horizontal beams are controlled independently but take into account the interactions between them. The complete L-shaped structure is not tested as a whole in this work but its subsystems. Separate research into the horizontal and vertical beam dynamics allows the determination of the initial control parameters. In the next step these parameters will be used as the initial point in the analysis of the optimal control of the full structure.

In the literature several strategies to control beam dynamics are proposed [1,2,5]. They are for example: proportional and cubic displacement feedback, cubic velocity feedback, positive position feedback (PPF), nonlinear saturation control (NSC) and others. In this paper we have selected the PPF algorithm for vibration suppression of the (a) horizontal beam under vertical excitation near the first natural frequency and (b) vertical beam near the principal parametric resonance.
2. Model of the structure

Mathematical models of (a) horizontal and (b) vertical beams are presented in this section. The subsystems are shown in figure 1. Both structures consist of a composite flexible beam with a rectangular cross-section which can perform large flexural oscillations. The tested composite beams are made of glass-epoxy material with fibre orientation 0/90/45/-45/45/90/0.

(a)                                                                                              (b)

Figure 1. A model of the horizontal (a) and vertical (b) beams.

The horizontal and vertical beams are mounted on an electromagnetic shaker which is a source of excitation along the $X$ and $Y$ axes, respectively. The kinematic excitation can be written as

for the horizontal beam

$$y = y_0 \sin(\Omega t)$$  \hspace{1cm}  (1)

and for the vertical beam

$$x = x_0 \sin(\Omega t)$$  \hspace{1cm}  (2)

The beam is considered as an Euler-Bernoulli model with an added nonlinear curvature component

$$\rho_x = v^\prime + \frac{1}{2} v^{\prime \prime \prime}$$  \hspace{1cm}  (3)

Kinetic and potential energies of the beam and the added mass $M$ take form:

for the horizontal beam

$$T = \frac{1}{2} \int_0^L \rho A \left( \dot{u}^2 + (\dot{v} + \dot{y})^2 \right) ds$$  \hspace{1cm}  $$V = \frac{1}{2} \int_0^L D_z \rho_x^2 ds$$  \hspace{1cm}  (4)

$$T_M = \frac{1}{2} M \left( \dot{u}_M^2 + (\dot{v}_M + \dot{y})^2 \right)$$  \hspace{1cm}  $$V_M = -M g (v_M + y)$$  \hspace{1cm}  (5)

for the vertical beam

$$T = \frac{1}{2} \int_0^L \rho A \left( (\dot{u} + \dot{x})^2 + v^{\prime \prime \prime} \right) ds$$  \hspace{1cm}  $$V = \frac{1}{2} \int_0^L D_z \rho_x^2 ds$$  \hspace{1cm}  (6)

$$T_M = \frac{1}{2} M \left( \dot{u}_M^2 + \dot{x}^2 + v_M^{\prime \prime \prime} \right)$$  \hspace{1cm}  $$V_M = M g (u_M + x)$$  \hspace{1cm}  (7)

where $u = u(s,t)$ and $v = v(s,t)$ describes the position of the arbitrary beam’s cross section.
Following the procedure presented in paper [4] the dimensionless equations of motion take the form:

for the horizontal beam

$$\ddot{v} - (\lambda v')' + v''\left(1 + v'^2\right) + 4v''v' + v'^3 = y_0\Omega^2 \sin(\Omega t)$$

(8)

for the vertical beam

$$\ddot{v} - (\lambda v')' + v''\left(1 + v'^2\right) + 4v''v' + v'^3 = 0$$

(9)

Considering the lumped mass, we get dynamical boundary conditions in the form:

for the horizontal beam

$$v(0,t) = 0, v'(0,t) = 0, v''(L,t) = 0, v'''(L,t) = \lambda v'_M + M\left(\ddot{v}_M + g^*\right)$$

(10)

for the vertical beam

$$v(0,t) = 0, v'(0,t) = 0, v''(L,t) = 0, v'''(L,t) = \lambda v'_M + M\ddot{v}_M$$

(11)

where $g^* = g\rho AL^3/D$ is the dimensionless gravitational acceleration and $\lambda$ is the Lagrange multiplier defined for the horizontal beam

$$\lambda = \int L \rho A\ddot{u} ds - M\ddot{u}_M$$

(12)

for the vertical beam

$$\lambda = \int L \rho A(\ddot{u} + \ddot{x}) ds - M\left(\ddot{u}_M + \ddot{x} - g^*\right)$$

(13)

In order to transfer the partial differential equations (8), (9) into ordinary differential equations the classical Bubnov–Galerkin procedure is applied. Only the first bending mode is taken into account. Then the differential equations can be written:

for the horizontal beam

$$\ddot{v} + 2\mu \omega_x^2 v + \omega_x^2 v^2 + \beta v^3 + \delta \left(v'v^2 + v'^2\right) = \xi y_0 \Omega^2 \sin(\Omega t)$$

(14)

for the vertical beam

$$\ddot{v} + 2\mu \omega_x^2 v + \omega_x^2 v^2 + \beta v^3 + \delta \left(v'v^2 + v'^2\right) = \xi x_0 \Omega^2 \sin(\Omega t)$$

(15)

The dimensionless parameters of the composite flexible beam are taken from paper [5] and they have the following values: for the vertical beam $\xi = 1.49929$, $\omega_x = 9.3728$, $\mu = 0.05$, $\beta = 14.4108$, $\delta = 3.27463$ and for the horizontal beam $\xi = 0.89663$, $\omega_x = 9.3728$, $\mu = 0.05$, $\beta = 14.4108$, $\delta = 3.27463$.

The MFC actuator is bounded on one side of the composite beam as shown in figure 2 while on the opposite side a sensor - strain gauge is glued. The PPF algorithm has been programmed in a DSP system [5]. Differential equations of motion of the considered composite beam with PPF controller take the form:

for the horizontal beam

$$\begin{cases}
\ddot{v} + 2\mu \omega_x v + \omega_x^2 v + \beta v^3 + \delta \left(v'v^2 + v'^2\right) = \xi y_0 \Omega^2 \sin(\Omega t) + \gamma u_c \\
\ddot{u}_c + 2\zeta \omega_x \dot{u}_c + \omega_x^2 u_c = \alpha v
\end{cases}$$

(16)

for the vertical beam

$$\begin{cases}
\ddot{v} + 2\mu \omega_x v + \omega_x^2 v + \beta v^3 + \delta \left(v'v^2 + v'^2\right) = \xi x_0 \Omega^2 \sin(\Omega t) + \gamma u_c \\
\ddot{u}_c + 2\zeta \omega_x \dot{u}_c + \omega_x^2 u_c = \alpha v
\end{cases}$$

(17)
The difference between the horizontal and vertical beam dynamics consists of a different excitation mechanism, in equation (16) the system is forced by an external harmonic force while in equation (17) it is by a parametric excitation.

3. Numerical results

Numerical simulations of the horizontal and vertical beam without (14), (15) and with control (16), (17) are performed in the MATLAB package. Selected results are presented in figure 3 and figure 4. All simulations have been started from the same initial conditions $\nu(t=0) = 0.1$ and $\nu'(t=0) = 0$.

![Figure 3. Numerical calculations for the uncontrolled system: horizontal beam (a), vertical beam (b).](image)

3D characteristics, beam deflection versus amplitude and frequency of harmonic excitation for the uncontrolled system are shown figure 3. We can see a fundamental difference in the position of the resonance region for both beams. The horizontal beam has a resonance region around the natural frequency of the linear system $\omega_s$. In the second case, for the vertical beam, the region is about the principal parametric resonance frequency of the linear system $2\omega_s$.

The PPF method is employed to damp the high amplitude vibration of the systems. The other parameters of the controller are $\omega_c = \omega_s, \gamma = 0.05, \alpha = 0.05$. These parameters give a smaller amplitude in the resonance region than for the uncontrolled system. The disadvantage of the PPF controller is that a single degree of freedom beam model is extended to a two degree of freedom system. Thus, the
system with PPF has two resonance regions (figure 4). In the case of the vertical beam, two resonance regions are observed for large excitation amplitudes $x_0 > 0.05$ (5% of the beam length). The areas below this threshold are characterized by the absence of vibration $v=0$ (figure 4b), therefore the PPF controller works very effectively. For the horizontal beam the PPF controller can only be used to minimize the vibration in the resonance region. Outside of this area, the dynamics of the beam are worse than for the uncontrolled system.

Figure 4. Numerical calculations for the controlled system: horizontal beam (a), vertical beam (b).

4. Conclusions
Numerical results for the positive position control (PPF) method applied to strongly nonlinear horizontal and vertical beam models are presented in this paper. The results show a possibly very good vibration reduction of the vertical beam for low level of excitation amplitudes. In the case of the horizontal beam this control method has to be modified. In future research we will propose more effective model based algorithms for control with a study of the parameters’ influence on the system’s response.

Acknowledgments
The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013), FP7-REGPOT-2009-1, under grant agreement No: 245479. The support by Polish Ministry of Science and Higher Education – Grant No 1471-1/7 PR UE/2010/7 – is also acknowledged.

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