Chapter
Evolutionary Design of Heat Exchangers in Thermal Energy Storage
Miguel Rosa Oliveira Panão

Abstract

The efficiency and ability to control the energy exchanges in thermal energy storage systems using the sensible and latent heat thermodynamic processes depends on the best configuration in the heat exchanger’s design. In 1996, Adrian Bejan introduced the Constructal Theory, which design tools have since been explored to predict the evolution of the architecture in flow systems. This chapter reviews the fundamental knowledge developed by the application of the constructal principle to the energy flows in the design of heat exchangers of thermal energy storage systems. It introduces the Svelteness and scale analysis, as two constructal tools in the evolutionary design of engineering flow systems. It also includes the analysis on essential scales of several configurations, or energy flow architectures, toward establishing the main guidelines in the design of heat exchangers for storing thermal energy.

Keywords: thermal energy storage, heat exchangers, constructal theory, phase-change materials, flow architecture

1. Introduction

Engineering systems capture a fraction of the total amount of thermal energy available from renewable sources, and to increase the energy system reliability, the research and development of the flow architecture in thermal energy storage systems is of paramount importance.

One of the crucial issues is the characteristic fluctuations in the availability of energy from renewable resources and wasted energy in industrial processes. The design of efficient thermal energy storage systems is an essential step toward meeting the consumption demands of electricity and heat [1]. Therefore, there is growing attention in the development of thermal energy storage systems to produce adequate energy savings and utilization, with a relevant impact on numerous and diverse applications [2, 3].

There are two basic approaches to thermal energy storage. One using the sensible heat without phase-change (SHS - Sensible Heat Storage), and another using the sensible heat and phase-change (LHS - Latent Heat Storage), as depicted in Figure 1. The thermal balance describing each approach is given by

\[ Q_{st}^{SHS} = V_{SHS} \rho_l c_p l (T_f - T_i) \]
where $V$ is the storage material volume, $\rho_l, c_{p,l}$ are the fluid or melted material’s density and specific heat, respectively, $h_{sl}$ is the latent heat of fusion of the Phase-Change Material (PCM), and $\rho_s, c_{p,s}$ are the PCM density and specific heat in its solid state, $T_i$ and $T_f$ are the initial and final temperatures of the energy storage process and $T_m$ corresponds to the melting temperature of the PCM in the LHS case.

In both approaches, the upper limit for the final temperature is the saturation value associated with the vaporization of the liquid. Because of its high heat capacity, water is the most used fluid for SHS. However, theoretically, Huang et al. [4] showed the energy stored in a water-based system is one order of magnitude lower than the energy stored in a PCM. And, experimentally, Kaygusuz [5] showed evidence of a PCM system able to store up to 60% of its theoretical maximum, which represents almost the double value of the theoretical storage capacity in water-based systems. These results were the motivation for further investment in the development of LHS systems. Compared to single-phase heat storage systems, LHS systems store the same amount of energy using more compact systems, reducing production and maintenance costs.

In LHS systems, Figure 1 on the right represents the three theoretical stages of the storage process. Initially, the Thermal Storage Material (TSM) is in its solid-state, and it stores (charges) energy through:

Step 1) the sensible heat until the melting temperature;

Step 2) the latent heat component until a complete phase-change of the TSM from solid to liquid. In this stage, the time of the melting process depends on the advancement of the solid–liquid interface (melting front), according to the configuration of the thermal fluid circuit of the heat exchanger (HE) immersed in the TSM;

Step 3) after the total liquefaction of the TSM, energy storage continues until the liquid reaches the saturation temperature of the next phase-change without compromising the volume of the Thermal Storage facility.

The storage of energy in these steps depends on the thermal properties of TSM, and the thermodynamics of the storage process. Still, the main challenge is the design of heat exchangers, as the engineering system that enables the flow of energy from the sources (renewable and non-renewable) to the TSM, disregarded in recent comprehensive reviews on thermal energy storage [6, 7]. Namely, this design has a significant impact on the charging and discharging times, if using renewable energy sources, given their limited time-window throughout the day.

The standard approach in the design of heat exchangers is to optimize the thermal and hydrodynamic energy flows. It uses an iterative process based on
previous work, and typical working conditions, such as the amount of fouling and pressure drop in the system, testing a significant number of trial-and-error designs until the values for the heat transfer performance, hydrodynamic effects and longevity are within pre-established requirements. A common trait in this standard approach is the lack of evaluation criteria grounded on the underlying physical processes. Constructal design distinguishes from the standard approach in providing the evaluation criteria in such a way. Therefore, the purpose of this chapter is to synthesize and present an evolutionary design approach (not optimization) using tools based on constructal theory. Therefore, the novelty is to include the architecture of thermal energy storage systems at the design stage [8] and investigate the best way to introduce the freedom to morph to overcome the shortcomings on charging and discharging periods due to prescribed, rigid and fixed designs when subjected to daily and seasonal changes.

2. Design tools in constructal theory

In 1996, Adrian Bejan [9], professor at Duke University, proposed a Constructal Theory to explain the evolution of configurations in nature stating, "for a finite-size flow system (not infinitesimal, one particle, or sub particle) to persist in time (to live) it must evolve with freedom such that it provides easier and greater access to what flows."

In practice, when using the constructal theory in engineering, one finds the best direction for the flow structures emerging from of what facilitates movement, designated as constructal design. And this design establishes a relation between what flows (energy, fluids, people, etc.) and the geometry of the flow architecture, in such a way that it becomes a global property of the engineering system. This property, the Svelteness, is a tool in constructal design.

In thermal energy storage, there are several length and time scales competing in the unfolding heat transfer processes, characterized by mass, momentum and energy balances of the system. However, not all the terms correspond to the dominant scales setting the overall result of charging and discharging of energy. A scale analysis is the second tool in constructal design explored in this section, allowing a proper definition of the relevant scales, and their implication to the heat exchanger design.

2.1 Svelteness of flow configuration

The Svelteness (Sv) is this global geometric property of the flow space, which guides the engineering practice in the assessment of the flow design performance. This property corresponds to the relation between two length scales of the flow system configuration: an external \( L_e \); and an internal length \( L_i \), usually associated with the volume as \( V^{1/3} \).

\[
Sv = \frac{L_e}{L_i} \tag{3}
\]

According to Bejan and Lorente [10], the evolutionary direction is that of vascularization, implying an increase of the Svelteness. Therefore, as an example applied to TES, what is the best constructal design solution for a simple tube inside a tank?
Should the design be a PCM in the inner tube and having the thermal fluid flowing through the tank (see [11]), or the opposite (see [12])?

In this example, the inner tube and tank are cylindrical with a length of \(L\), and diameter of \(d\) and \(D\), respectively. Therefore, the Svelteness in both cases has \(L\) as its external length scale. However, the volume associated to the thermal fluid depends on the situation. When the PCM is inside the inner tube, the volume where the thermal fluid flows is \(V = \frac{\pi}{4}(D^2 - d^2)L\), and in the opposite case, \(V = \frac{\pi}{4}d^2L\). If \(Sv'\) corresponded to the case where the energy flows inward to the PCM on the inner tube, and \(Sv^\circ\) when it flows outward to the PCM in the tank, the relation between Sveltenesses would be

\[
\frac{Sv^\circ}{Sv'} = \left[\left(\frac{D}{d}\right)^2 - 1\right]^{1/3} \tag{4}
\]

If a larger Svelteness points in the evolutionary design of the outward energy flow from the thermal fluid inside the inner tube, \(Sv^\circ/Sv' > 1\), thus \(D^2/d^2 > 2\), and \(D^2/d^2 < 2\) otherwise. Consider the case of choosing the best design to favor the flow of energy inward to a PCM inside a tube, as in the work of Ghoneim [11]. What should be the value of the void fraction \(\phi\), which is the ratio between the thermal fluid volume \((V_{tf})\) and the storage tank volume \((V_{st})\)? First, the volume of thermal fluid is the difference between the storage tank volume and the volume of all tubes containing the PCM energy storage material, \(V_{tf} = V_t - n_t V_t\), with \(n_t\) as the number of tubes and \(V_t = \frac{\pi}{4}d^2L\) as the volume of each tube. Therefore, one defines the void fraction as

\[
\phi = 1 - \frac{n_t V_t}{V_{st}} \tag{5}
\]

Considering \(D\) as the external diameter containing the tube diameter \(d\), and the thermal fluid circulating in the tank, the storage tank volume should equal the total volume of all the tubes with the “necessary” thermal fluid volume, \(V_{st} = n_t(\pi/4)D^2L\), thus, using this reasoning in the void fraction implies that

\[
\frac{D^2}{d^2} = \frac{1}{1 - \phi} \tag{6}
\]

Considering the previous constructal analysis using the Svelteness, storing energy with PCM material inside the tubes is only worthy when \(D^2/d^2 < 2\). Therefore, when applied to Eq. (6), it points to the need of void fraction values of \(\phi < 0.5\). In fact, all the LHS systems investigated using the configuration of Ghoneim [11], choose \(\phi = 0.3 < 0.5\). Constructual theory corroborates this option, but indicates that \(\phi\) could assume higher values, eventually leading to the insertion of more tubes with PCM, allowing the storage of more energy.

On the other hand, recent works as that of Agyenim et al. [12], point toward having the PCM in the tank, instead of inside the inner tubes, and in these cases with a single tube through which circulates the thermal fluid, \(D^2/d^2 = 7.35 > 2\).

2.2 Method of scale analysis

*Scale analysis* or *scaling* is a problem solving method useful to obtain essential and expedite information of several energetic processes [13]. It is not the same as a
the dimensional analysis performed in fluid mechanics, but to assess the importance of the order of magnitude of the parameters involved in heat transfer processes, and extract the relevant scales from their governing equations. For more details on the principles of scale analysis, see Bejan [13] (pp. 17–20). Here, one uses an example to illustrate the method.

Consider the example above of an LHS system with the PCM inside a tube and the thermal fluid circulating around it. If there is a sudden change in the thermal fluid temperature \( T_{tf} \), how long will it take for that perturbation to reach the PCM material at the central axis of the tube? Assuming heat transfer by diffusion in cylindrical coordinates, and that changes in the thermal diffusivity \( \alpha \) are negligible within that time scale, the energy equation for the thermal energy storage process is

\[
\frac{\partial T}{\partial t} = \frac{\alpha \partial^2 T}{r \partial r^2} \tag{7}
\]

Scaling means using the symbol \( \sim \) to establish the order of magnitude of a differential term with the main parameters of the flow configuration. Therefore,

\[
\frac{\partial T}{\partial t} \sim \frac{\Delta T}{\tau} \tag{8}
\]

\[
\frac{\alpha \partial^2 T}{r \partial r^2} \sim \frac{\alpha}{(d/2)^2} \frac{\Delta T}{\tau} \tag{9}
\]

where \( \tau \) corresponds to the time scale under evaluation, \( \Delta T \) to the temperature difference between the tube’s boundary and the center, and \( d \) is the tube’s diameter. In scaling terms, Eq. (7) becomes,

\[
\frac{\Delta T}{\tau} \sim \frac{\Delta T}{(d/2)^2} \tag{10}
\]

which solved for the time scale results in

\[
\tau \sim \frac{(d/2)^2}{\alpha} \tag{11}
\]

Bejan [13] contains the synthesis for all the rules in a scale analysis. However, the example above is enough to explain the procedure applied later in section 3. The following section exemplifies the application of constructal theory as an evolutionary design method to develop heat exchangers in sensible and latent heat storage engineering systems.

3. Constructal theory in thermal energy storage heat exchangers

One of the essential elements in a constructal theory analysis is the freedom to morph of flowing configurations. Therefore, once we identify what is the flow under analysis, one can better understand what its freedom to morph means. On the other hand, the heat exchanger in thermal energy storage corresponds to the structure obtained after morphing through which energy flows from a source, usually the thermal fluid, to the storage material (e.g. a solid or a phase-change material, PCM). Depending on the storage material, the heat transfer mechanisms vary, and, accordingly, the energy storage scales. For example, if the material is solid, the heat
transfer mechanism is diffusion and the mode is the one on the left of Figure 1. But if one uses a PCM, the energy storage story follows the second mode on the right, involving natural convection, a melting process, a solid–liquid interface moving boundary, and all these elements lead to additional complexity of the heat exchanger, affecting the energy storage scales.

3.1 Heat exchangers in sensible heat storage

Consider an underground volume of solid with a network of channels through which a thermal fluid transports energy for storage purposes. The storage mechanism is heat diffusion from the channels outer area to the volume of solid. What should be the structure of the channels network? Combelles et al. [14] explored this TES system with tree-shaped configurations of 2D channels made of parallel plates the length \( L_1 \) and \( D_1 \) of width within an area of \( 2L_1 \times 2L_1 \), and 3D pipes configured within a solid of \( 2L_1 \times 2L_1 \times L_1 \) of volume, as depicted in Figure 2.

The first step is to characterize the architecture of each configuration type (2D or 3D) in terms of their Svelteness, as the global property of the system, which relates the external length scale given by the total length of the flow network, \( L_{\text{total}} = \sum_{i=1}^{n} L_i \), depending on its complexity \((n)\), and the internal flow length scale varying with the 2D or 3D nature of the flow. If the flow network is bi-dimensional, the internal length scale corresponds to \( A_f^{1/2} \) with \( A_f = \sum_{i=1}^{n} 2^{i-1} L_i D_i \), while in the three-dimensional configuration, this scale is \( V_f^{1/3} \) with \( V_f = \sum_{i=1}^{n} 2^{i-1} \frac{4}{\pi} D_i^2 L_i \). When one increases the complexity of the flow network, Lorente et al. [15] show the relation between the length and diameter of one branch \((i)\) and the next ramification \((i+1)\) follows the Hess-Murray rule. Thus,

\[
\frac{L_{i+1}}{L_i} = \frac{D_{i+1}}{D_i} = 2^{-1/2}
\]

---

**Figure 2.**
2D and 3D diffusive TES tree-shape configurations.
which developed to depend on the length and diameter of the first branch \((L_1, D_1)\), simplify to

\[
L_i = L_1 \left(2^{-1/2}\right)^{i-1} \tag{13}
\]

\[
D_i = D_1 \left(2^{-1/2}\right)^{i-1} \tag{14}
\]

The Svelteness for both general configurations, considering the relations in Eqs. (13) and (14), depends on two major features of the configuration: its complexity \((\psi(n))\); and the geometrical relation between the length and diameter of the first channel \((L_1/D_1)\).

\[
Sv_k = \psi_k(n) \left(\frac{L_1}{D_1}\right)^{q_k} \quad \text{with} \quad k = \{2D, 3D\} \tag{15}
\]

and

\[
\begin{align*}
\psi_{2D}(n) &= \frac{1 - 2^{-n/2}}{1 - 2^{-1/2} n^{-1/2}} \quad \wedge \quad q_{2D} = 1/2 \\
\psi_{3D}(n) &= 2 \left(\frac{1 + 2\sqrt{2}}{1 - 2^{1/2} n^{1/2}}\right)^{1/3} \left(1 - 2^{-n/2}\right) \quad \wedge \quad q_{3D} = 2/3
\end{align*}
\]

If the Svelteness indicates the evolution of the flow configuration, one should connect the flow architecture complexity, \(\psi_k(n)\), and the geometry of the initial channel, with the scales associated to the storage of energy.

The amount of energy stored depends on the material, but in this TES system, the relevant scale is the storage time and the evolution of the temperature in the conductive solid. Considering the solid is, initially, at \(T_0\), and the inflowing thermal fluid is at \(T_{in}\), in time, the average temperature in the solid \((T_{avg})\) evolves toward \(T_{in}\). Therefore, Combelles et al. [14] analyzes the evolution of this diffusive thermal storage configuration with a dimensionless thermal potential as

\[
\theta_{avg} = \frac{T_{avg} - T_0}{T_{in} - T_0} \tag{16}
\]

This analysis focuses on the timescales of energy storage and the corresponding effect of the configuration complexity (number of bifurcations, \(n\)). Considering the fluid, there are two essential timescales:

- the timescale of fluid traveling the \(n\) channels;

\[
t_f = \sum_{i=1}^{n} \frac{L_i}{V_i} = \psi_f(n) \left(\frac{L_1}{V_1}\right) \sim \frac{L_1}{V_1} \tag{17}
\]

since \(\psi_f(n) = \frac{2^{3/2} - 1}{2^{1/2} - 1}\) varies between 1 and 1.55, which means \(\psi_f(n) \sim 1 \forall n \geq 1\); and

- the timescale of thermal diffusion across the channel;

\[
t_c \sim \frac{D_1^2}{\alpha_f} \tag{18}
\]

Combelles et al. [14] argue that in the case where thermal diffusion in the channel’s boundary layer is a slower process than the fluid traveling through the
channels, $t_f < t_c$, and the temperature of the fluid at exit is practically unchanged. It is a relevant result to establish a stable boundary condition.

The third timescale, and the longer, corresponds to the time it takes to store energy in the solid volume, $t_s$, meaning the timescale to heat the entire volume by thermal diffusion, expressed as

$$t_s \sim \frac{L_1^2}{\alpha_s}$$  \hspace{1cm} (19)

From the numerical simulations, the evolution of the dimensionless thermal potential $\theta_{avg}(t)$ is given by

$$\theta_{avg}(t) = 1 - \exp \left( -C \frac{t}{\tau} \right)$$  \hspace{1cm} (20)

where $C$ is a scale parameter, and $\tau$ is the TES response time given by

$$\tau = \frac{m_s c_{p,s}}{m_f c_{p,f}}$$  \hspace{1cm} (21)

with $m_s = \rho_s 4L_1^3$ as the solid mass (and $\rho_s$ its density), $c_{p,s} = \frac{k_s}{\rho_s \alpha_s}$ is the solid specific heat, and $m_f c_{p,f} = \left( \frac{k_f}{\alpha_f} \right) \frac{\pi}{4} D_1^2 V_1$ is the thermal capacity rate of the fluid.

Considering Eq. (15), and introducing it in the TES response time results in

$$\tau = \frac{16}{\pi} \left( \frac{S v_k}{\psi_k(n)} \right)^{2/q_k} \frac{\tilde{\alpha}}{\tilde{k}} t_f$$  \hspace{1cm} (22)

with $\tilde{\alpha} = \alpha_f/\alpha_s$ and $\tilde{k} = k_f/k_s$. The results from the numerical simulations in both 2D and 3D configurations reported in Combelles et al. [14] evidence the decrease of the diffuse TES system response time with a higher complexity of the flow network, which is consistent with the relation obtained in Eq. (22) for a fixed $S v_k$ as considered in their simulations.

**Figure 3** shows the results for the complexity degree scale ($\psi_k(n)$) normalized by the value obtained for $n = 4$: $\tilde{\psi}_k(n) = \psi_k(n)/\psi_k(4)$. In a 2D configuration, one obtains a maximum of the scale associated to the complexity degree at $n = 4$ (which

![Figure 3](image-url)
is the maximum complexity investigated in Combelles et al. [14]). In the 3D configuration, $\psi_{3D}(n)$ shows a monotonic behavior, although for $n > 10$, the increase of one level of complexity generates a variation of less than 1% in diminishing returns. The maximum complexity explored by Combelles et al. [14] was $n = 4$ and adding one level of complexity would produce an increment of only 6.4% compared to 10.6% between $n = 3$ and 4.

In absolute terms, the constructal design of heat exchangers in diffusive TES systems suggests the choice of a 3D configuration, rather than a 2D, for a faster energy storage, since it leads to $\psi_{3D}$ complexity scale factors of 1.6 to 2.3 times higher than the 2D scale, allowing shorter charging times. However, in applications where a 2D configuration is more appropriate, the level of complexity should not go beyond 4 dendritic bifurcations.

3.2 Heat exchangers in latent heat storage

Energy storage systems using the latent heat of a certain phase-change material (PCM) rely on the heat transfer mechanisms of diffusion and natural convection. Initially, the PCM is in its solid state and it stores heat by diffusion close to the channel containing the thermal fluid, or fin, until it reaches the fusion (or melting) temperature ($T_m$), creating a melting frontline. Thereafter, a solid–liquid interfacial boundary develops and the melting history consists in two distinct periods: invasion; and consolidation.

The invasion period corresponds to the time interval until the solid–liquid interface reaches a distance equivalent to the process characteristic length. The consolidation period corresponds to the remaining time until all the PCM in the LHS system is in its liquid state. These periods do not, necessarily, correspond to the timescales associated to the diffusive and convective heat transfer processes. The charging and discharging times of LHS depend on the heat exchanger design and the dominant heat transfer mechanisms through which energy flows from its source to the PCM.

There are several design configurations investigated with a constructal approach for the heat exchangers using phase-change to store energy in PCM. Figure 4 presents three configurations reported in the literature. The configurations with a vertical pipe [15] is the less prone to morphing. The helical pipe [16] in a cylindrical PCM enclosure is fixed, but the ability to vary the number of turns and the diameter of each turn increases the system’s freedom to morph. Finally, an advancing heat source line invading the PCM material aims at the theoretical design with the greatest freedom to morph [17]. One of the novelties in constructal design of engineering systems is determining the Svelteness as expression of its architecture, and the system’s freedom to morph, considering an evolutionary path toward

![Figure 4. Heat exchanger configurations investigated with a constructal theory approach.](image)
vascularization, i.e. an increase of its Svelteness. The constructal analysis of all designs also implies the investigation of length and timescales associated with heat transfer mechanisms, and the possible effect of the Svelteness in these scales.

Lorente et al. [15] performed a scale analysis to analyze the latent thermal energy storage where energy flows from the thermal fluid circulating inside a central vertical pipe and the surrounding PCM. The dominant heat transfer mechanism is natural convection. The Svelteness in this case would have the height of the enclosure ($L_e = H$) as external length scale, and an internal length scale based on the volume occupied by the thermal fluid as $L_i = \left(\frac{\pi d^2 H}{4}\right)^{1/3}$, with $d$ as the diameter of the vertical pipe. The final outcome relating the Svelteness with the heat exchanger geometry leads to

$$H/d = \left(\frac{\pi}{4}\right)^{1/3}Sv^{3/2} \tag{23}$$

The maximum energy one can store in this first TES configuration is $Q_{max} = \frac{\rho}{\Delta h_s} \left(\frac{D^2 - d^2}{4}\right)H\rho h_{sl}$, with $\rho$ as the PCM density and $h_{sl}$ as its latent heat of fusion. According to Lorente et al. [15], the time-scale of the melting history ($\tau_m$) considers natural convection as the dominant heat transfer mechanism where friction dominates buoyancy forces, thus, resulting in

$$\tau_m = \frac{Q_{max}}{\pi d k \Delta h_s \text{Ra}^{1/4}} \tag{24}$$

The relation between the maximum amount of energy stored and this time-scale, including Eq. (23) in (24) leads to $Q_{max} / \tau_m \sim Sv^{-3/2}$. Considering that evolution in constructal theory occurs toward the vascularization of flow architectures, implying the increase of their Svelteness, in this case, it leads to decreasing $Q_{max} / \tau_m$, instead of increasing as desired. This is an interesting result from the constructal design point of view because it indicates that natural convection generated by the vertical pipe alone is not the best heat transfer mechanism for a faster energy storage in LHS. However, one should point that their simplified scale analysis, and numerical simulations, which considers an annular moving melting front, seems unrealistic when confronted with later experimental works such as Zhang et al. [18], with time-scales of one order of magnitude lower than those predicted in Lorente et al. [15] – $O(\tau_m) \sim 10^3$ h. Nonetheless, this result points to the need of a better solution to facilitate the flow access of energy between a surface heated by a thermal fluid and the PCM. The works of Ogoh and Groulx [19], and Kamkari and Shokouhmand [20], are examples where fins around the main energy source facilitate its flow to the PCM for storage during charging, promoting heat transfer by diffusion and mitigating natural convection.

Considering the case of Ogoh and Groulx [19], Figure 5 depicts the adding of disc-shape fins to the original vertical pipe in cylindrical PCM enclosure configuration, where each disc-shape fin corresponds to a construct (represented on the right).

The total height of the cylinder ($H$), in terms of constructs corresponds to $H = nh + h_f$, with $n$ as the number of constructs, $h$ the total height of the PCM inside a construct, and $h_f$ as the fin thickness. Therefore, the interval between annular fins becomes

$$h = \frac{H}{n} - h_f \tag{25}$$

Considering the Svelteness ($Sv$) for this construct as the ratio between the external characteristic length based on the upper and bottom areas of the disc-shape fin, $L_e = \left[\frac{\pi}{2}(D^2 - d^2)\right]^{1/2}$, and the internal characteristic length given by the annular
disc volume, \( L_i = \left[ \frac{\pi}{4} (D^2 - d^2) h_f \right]^{1/3} \), considering Eq. (25), one can express the ratio between the space between fins \( (h) \) and the external diameter of the enclosure \( (D) \) as

\[
\frac{h}{D} = n^D 
\sqrt{\frac{2\pi}{Sv^3}} 
\]  

(26)

Ogoh and Groulx [19] argue for a neglecting effect of convection between annular fins, thus, the energy stored by phase-changing the PCM from solid to its liquid state occur by conduction. In this sense, the most important scale characterizing the melting front \( (\delta) \) departs from the annular disc fin. The balance between the conduction heat flux \( (q'') \) supplied to the melting front and the rate of melting can be described as

\[
q'' dt = \rho h s l d\delta 
\]  

(27)

Assuming a linear temperature distribution across this layer where the heat transfer occurs by diffusion, one could quantify the heat flux as \( q'' = k \Delta T \), which applied in Eq. (27) results in

\[
\delta = \sqrt{a \tau} 
\]  

(28)

with \( a = \frac{2k \Delta T}{\rho h_s} \), where \( \Delta T = T_w - T_m \), with \( T_w \) as the temperature of the fin wall, \( T_m \) the fusion temperature of the PCM, and \( \tau \) corresponds to the timescale of energy storage. To understand the evolution of the configuration based on this scale, one could argue that the charging finishes when \( h \sim 2\delta \), thus, replacing this scale in Eq. (26), and solving it as a function of the Svelteness, results in

\[
Sv \sim \sqrt{2\pi} \left( \frac{nD}{H - 2n\sqrt{a \tau}} \right) 
\]  

(29)

Therefore, since \( Sv > 0 \), it implies \( H - 2n\sqrt{a \tau} > 0 \), resulting in an upper theoretical limit for this timescale as

\[
\tau < \left( \frac{H}{2n} \right)^2 \frac{1}{a} = \tau_{max} 
\]  

(30)
Figure 6 shows the results for this limit considering the properties reported in [19].

The constructive design analysis of this heat exchanger indicates diminishing returns of less than 10% for a number of fins above \( n > 18 \), which is coherent with the numerical results presented by Oghoh and Groulx [19]. Applying the timescale defined by Eq. (30) to the experimental conditions in the work of Kamkari and Shokouhmand [20], which explore the effect of no fins with the cases of 1 and 3 fins, for the later case, \( \tau_{\text{max}} \) is roughly 1.3 \( \times \) the value measured for the total melting process. In the case of the experiments with 1 fin, the results for \( \tau_{\text{max}} \) are significantly larger, evidencing the role of natural convection in delaying the heat transfer to the PCM.

The helical coil illustrated in Figure 4 is an alternative to the vertical pipe, and a geometry where the freedom to morph is larger because of the ability to change the helix diameter, number of turns and pitch angle. Alailami et al. [16] explored the morphing ability of the system in its design stage to optimize the storage of energy analyzing two scales. The timescale of heat penetrating the storage material from the boundaries of the helical coil to the cylinder diameter, \( \tau_c = D^2/\alpha \); and the temperature difference, \( T(t) - T_{gf} \), scaled by the initial condition, \( T(0) - T_{gf} \), with \( T_{gf} \) as the temperature of the thermal fluid. When Alailami et al. [16] simulated the evolution of the scaled average temperature \( -\bar{T}_{\text{avg}} = \frac{T_{avg}(t^*) - T_{gf}}{T(0) - T_{gf}} \) its value decreased with the scaled time \( -t^* = t/\tau_c \) meaning the average temperature of the energy storage material approaches the temperature of the thermal fluid.

Afterward, focusing the analysis on \( t^* = 0.3 \), and varying the helical coil diameter \( -D_h = \zeta D \) and pitch height \( -H_h = \epsilon H \) obtained as function of the cylinder diameter (\( D \)) and height (\( H \)), the authors reached an optimum diameter and pitch length of the helical coil, corresponding to \( \zeta = 0.6 \), and \( \epsilon = 0.3 \), respectively.

Without using the work of Alailami et al. [16], Joseph et al. [21] performed an experiment of this configuration to store energy in a PCM. The authors used unoptimized values for the helical coil \( (\zeta \approx 0.7, \epsilon \approx 0.2) \), from the Alailami et al. [16] point of view. However, the PCM configuration showed promising results storing 20% more energy than its equivalent mass in water. Also, the charging process in the PCM facility was slower and the authors attribute this result to the unoptimized heat exchanger design, justifying the need of more research on this topic.

![Figure 6](image_url)

**Figure 6.**
Variation of limit timescale for the energy stored in a PCM through annular fins distributed around a vertical pipe inside a cylindrical enclosure.
The last geometry in Figure 4 is theoretical and corresponds to the greatest freedom to morph through an advancing heated line that can bifurcate at some point. The analysis performed by Bejan et al. [17] focus on the invasion (line advances until the storage boundaries) and consolidation (all PCM melts) stages, and introduces a tree invasion pattern with a complexity level up to \( n = 2 \) branching events. The results of this theoretical analysis point to the acceleration of the charging times. However, the conversion of this approach into a practical application is still a challenge, since there is no technology capable of this kind of morphing inside a PCM solid environment.

A final comment concerns the possible contribution of constructal design to optimize the total cost of heat exchanger design methods, which is not a direct correlation. As shown by Azad and Amidpour [22], since the constructal design provides an evolutionary perspective on the optimum geometric features of heat exchangers, it can lead to a substantial reduction of the total cost, compared to more standard design approaches. Namely, in the aforementioned work, the authors used constructal theory to optimize shell and tube heat exchangers, and the new approach allowed to reduce this cost by 50%. However, the application of a similar reasoning in the development of heat exchangers for thermal energy storage is in need of more research.

4. Conclusions

Thermal energy storage is one of the preeminent options to face the energy challenges of this century, providing a high energy saving potential and effective utilization. However, in these systems, the architecture of the heat exchangers through which energy flows, during charge and discharge, is of paramount importance. While most approaches optimize heat exchanger designs, the one presented in this chapter, based on constructal theory, follows an evolutionary design, meaning that the configuration explored at the design stage is dynamic and free to morph. It is not pre-defined, rigid, or still, but considers how it should evolve toward the greater access of the energy currents that flow through it.

Thermal energy storage systems follow two thermodynamic processes using the sensible heat of the energy storage material, or, besides the sensible heat, also the latent heat, as in Phase-Change Material (PCM). After introducing the general considerations on these systems, this chapter presents two design tools in constructal theory: the Svelteness, as a global property of any flow system, which tends to increase and evolve toward vascularization; and the scale analysis, as an expedite problem solving tool that allows obtaining relevant information of the several energetic processes involved.

Using the design tools presented, this chapter reviews and further explores the constructal theory approach in the development of heat exchangers for sensible and latent thermal energy storage configurations. The analysis evidences the explanatory potential of the constructal approach, increasing the sensibility of the engineer to the advantages of including the freedom to morph at the design stage of heat exchangers in thermal energy storage.

Acknowledgements

The author would like to acknowledge project UIDB/50022/2020 and UIDP/50022/2020 of ADAI for the financial support for this publication.
Nomenclatures and Abbreviations

\( c_p \) Specific heat \([\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}]\)
\( d, D \) Diameter \([\text{m}]\)
\( H, h \) Height \([\text{m}]\)
\( h_d \) Latent heat of fusion \([\text{J/kg}]\)
\( k \) Thermal conductivity \([\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}]\)
\( L \) Length \([\text{m}]\)
\( m \) Mass \([\text{kg}]\)
\( \dot{m} \) Mass flow rate \([\text{kg/s}]\)
\( n \) Number of tubes or Complexity degree \([-]\)
\( Q_{st} \) Energy stored \([\text{J}]\)
\( r \) radial coordinate \([\text{m}]\)
\( \text{Ra} \) Rayleigh number \([-]\)
\( S_v \) Svelteness \([-]\)
\( T \) Temperature \([\text{K}]\)
\( t \) time \([\text{s}]\)
\( T_f \) Final temperature \([\text{K}]\)
\( T_i \) Initial temperature \([\text{K}]\)
\( T_m \) Melting temperature \([\text{K}]\)
\( V \) Volume \([\text{m}^3]\)

Greek Symbols

\( \alpha \) Thermal diffusivity \([\text{m}^2/\text{s}]\)
\( \delta \) Length \([\text{m}]\)
\( \Delta T \) Temperature difference \([\text{K}]\)
\( \varepsilon \) Scale factor \([-]\)
\( \zeta \) Scale factor \([-]\)
\( \theta \) Normalized temperature difference \([-]\)
\( \rho \) Density \([\text{kg/m}^3]\)
\( \tau \) Timescale \([\text{s}]\)
\( \phi \) Void fraction

Subscripts

\( \text{avg} \) average
\( c \) cylinder
\( e \) external
\( f \) fluid, fin
\( h \) helical
\( i \) internal, inner
\( m \) melting
\( \text{max} \) maximum
\( o \) outward
\( s \) solid
\( st \) stored
\( t \) tube
\( tf \) thermal fluid
\( 2D \) Two-dimensional
\( 3D \) Three-dimensional
\( \text{HE} \) Heat Exchanger
\( \text{LHS} \) Latent Heat Storage
PCM  Phase-Change Material
SHS  Sensible Heat Storage
TES  Thermal Energy Storage
TSM  Thermal Storage Material

Author details

Miguel Rosa Oliveira Panão
ADAI, LAETA, Department of Mechanical Engineering, University of Coimbra, Coimbra, Portugal

*Address all correspondence to: miguel.panao@dem.uc.pt

IntechOpen

© 2021 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.
References

[1] Zsiborács H, Baranyai NH, Vincze A, Zentkó L, Birker Z, Máté K, Pintér G. Intermittent renewable energy sources: The role of energy storage in the european power system of 2040. Electronics. 2019 Jul;8(7):729.

[2] Wu S, Yan T, Kuai Z, Pan W. Thermal conductivity enhancement on phase change materials for thermal energy storage: A review. Energy Storage Materials. 2020 Mar 1;25:251–95.

[3] Soares N, Costa JJ, Gaspar AR, Santos P. Review of passive PCM latent heat thermal energy storage systems towards buildings energy efficiency. Energy and buildings. 2013 Apr 1;59:82–103.

[4] Huang BK, Toksoy M, Cengel YA. Transient response of latent heat storage in greenhouse solar system. Solar energy. 1986 Jan 1;37(4):279–92.

[5] Kaygusuz K. Experimental and theoretical investigation of latent heat storage for water based solar heating systems. Energy conversion and management. 1995 May 1;36(5):315–23.

[6] Sarbu I, Sebarchievici C. A comprehensive review of thermal energy storage. Sustainability. 2018 Jan;10(1):191.

[7] Alva G, Lin Y, Fang G. An overview of thermal energy storage systems. Energy. 2018 Feb 1;144:341–78.

[8] Clemente MR, Panáèo MR. Introducing flow architecture in the design and optimization of mold inserts cooling systems. International Journal of Thermal Sciences. 2018 May 1;127:288–93.

[9] Bejan A. Freedom and Evolution: Hierarchy in Nature, Society and Science. Springer Nature; 2019 Dec 6.

[10] Bejan A, Lorente S. Design with constructal theory. Wiley; 2008.

[11] Ghoneim AA. Comparison of theoretical models of phase-change and sensible heat storage for air and water-based solar heating systems. Solar Energy. 1989 Jan 1;42(3):209–20.

[12] Agyenim F, Eames P, Smyth M. A comparison of heat transfer enhancement in a medium temperature thermal energy storage heat exchanger using fins. Solar Energy. 2009 Sep 1;83(9):1509–20.

[13] Bejan A, Convection heat transfer. 4th Ed., John Wiley & Sons; 2013.

[14] Combelles L, Lorente S, Anderson R, Bejan A. Tree-shaped fluid flow and heat storage in a conducting solid. Journal of Applied Physics. 2012 Jan 1;111(1):014902.

[15] Lorente S, Bejan A, Niu JL. Phase change heat storage in an enclosure with vertical pipe in the center. International Journal of Heat and Mass Transfer. 2014 May 1;72:329–35.

[16] Alalaimi M, Lorente S, Bejan A. Thermal coupling between a helical pipe and a conducting volume. International Journal of Heat and Mass Transfer. 2015 Apr 1;83:762–7.

[17] Bejan A, Ziaei S, Lorente S. The S curve of energy storage by melting. Journal of Applied Physics. 2014 Sep 21;116(11):114902.

[18] Zhang P, Meng ZN, Zhu H, Wang YL, Peng SP. Melting heat transfer characteristics of a composite phase change material fabricated by paraffin and metal foam. Applied Energy. 2017 Jan 1;185:1971–83.

[19] Ogoh W, Groulx D. Effects of the number and distribution of fins on the storage characteristics of a cylindrical latent heat energy storage system: a
numerical study. Heat and Mass Transfer. 2012 Oct 1;48(10):1825–35.

[20] Kamkari B, Shokouhmand H. Experimental investigation of phase change material melting in rectangular enclosures with horizontal partial fins. International Journal of Heat and Mass Transfer. 2014 Nov 1;78:839–51.

[21] Joseph A, Kabbara M, Groulx D, Allred P, White MA. Characterization and realtime testing of phasechange materials for solar thermal energy storage. International Journal of Energy Research. 2016 Jan;40(1):61–70.

[22] Azad AV, Amidpour M. Economic optimization of shell and tube heat exchanger based on constructal theory. Energy. 2011 Feb 1;36(2):1087–96.