Decay properties of the heavy-light mesons

Takayuki MATSUKI,¹, *) and Koichi SEO²,**)

¹Tokyo Kasei University, Tokyo 173-8602, Japan
²Gifu City Women’s College, Gifu 501-0192, Japan

Abstract

We study the decay properties of a heavy-light meson. We reformulate the decay amplitudes for the heavy-light systems and find a new way to calculate decay rates. Applying this formulation, we find a new sum rule for the radiative decays of one heavy-light meson into another, $H_1 \rightarrow H_2 + \gamma$ with various combinations of $H_i$.

*) E-mail: matsuki@tokyo-kasei.ac.jp; an invited talk at "New Frontiers in QCD 2010" held at Kyoto.
**) E-mail: seo@gifu-cwc.ac.jp
§1. Introduction

Ever since the discovery of the so-called $D_{sJ}$ in 2003 by BaBar\textsuperscript{1} and CLEO\textsuperscript{2} which are now identified as $D_s(2317)$ and $D_s(2460)$ with $j_q^P = (1/2)^+$ and $(3/2)^+$, respectively, the particles are discovered in the vicinity of this energy or above one after another. Theorists as well as experimentalists are trying to explain these particles as ordinary $Q\bar{q}$, or molecular, or tetra-quark, or a bound state of diquark states, \textit{etc.}\textsuperscript{3} At the present time there are several explanations for the same state and the final decisive answer is not yet given.

As well as explanations of its origin, many people study the properties of these heavy mesons, spin, parity, decay rates, etc. In this paper, we report our study on the formulation to calculate decay rates of a heavy-light meson which are Lorentz-invariant and on the kinematical results derived from it. We assume that the $D_{sJ}$ are heavy-light composite mesons $Q\bar{q}$, whose assumption is supported by experimentally compatible numerical calculations of the lattice gauge theory.\textsuperscript{4,5}

There are several ways to calculate decay rates of the heavy-light mesons proposed so far; one is by Goity and Roberts\textsuperscript{6} which is applied to newly discovered heavy mesons by Di Pierro and Eichten,\textsuperscript{7} and another by Bardeen, Eichten, and Hill.\textsuperscript{8} The former is based on the ordinary dipole expansion of a bound state with the heavy quark at rest at the center and the latter utilizes the effective chiral Lagrangian which describes coupling of heavy-light mesons with chiral particles.

We propose a different method from these to calculate decay rates, which are Lorentz-invariant and are calculated in the Breit frame where initial and final mesons have the same velocity in the opposite directions.

§2. Formulation

We start to construct formulation how to calculate a decay rate of a bound state because the former methods are not Lorentz-covariant\textsuperscript{6,7} or do not use meson wave functions to obtain the numerical results. We would like to solve these problems in this paper. For instance, consider the method proposed in Refs. \textsuperscript{6} and \textsuperscript{7}, in which, \textit{e.g.}, pion decay is described by the following equation.

$$gk_\mu \int d^3x \, \psi_j^\dagger (\vec{x}) \, O^\mu \psi_i (\vec{x}) \exp \left( i\vec{k} \cdot \vec{x} \right), \quad (2.1)$$

where the interaction is assumed to be

$$\mathcal{L}_{\text{int}} = ig\partial^\mu \phi j_{5\mu}, \quad (2.2)$$
with axial vector current $j_{3\mu}$. Here one uses the composite meson wave functions, $\psi_{i,f}$, at the rest frame even for the final state $\psi_f$. Another point is that a plane wave pion wave function $\exp (i\vec{k} \cdot \vec{x})$ is inserted ad hoc. Without phase factors in the initial and final wave functions $\exp (-iP_{i,f} \cdot x)$, the expression of Eq. (2.1) neglects the recoil of heavy-light mesons. We claim the factor in Eq. (2.1) should be replaced with the following expression in the Breit frame.

$$\exp (-ik \cdot x) \to \exp (-2im_Q V z),$$

(2.3)

where $m_Q$ is a heavy quark mass. The factor 2 on the r.h.s. of Eq. (2.3) in the exponent appears because the initial heavy quark has a momentum $m_Q \gamma V$ in the $+z$ direction and the final one has $m_Q \gamma V$ in the $-z$ in the Breit frame, hence the recoil momentum becomes $2m_Q \gamma V$. The factor $\gamma$ is absorbed into integral variable $z' = \gamma z$.

Both sides of Eq. (2.3) are equal to each other when one replaces $m_Q$ on the r.h.s. with an average of hadron masses as $(m_1 + m_2)/2$. When we include phase factors of initial and final wave functions $\psi_{i,f}$, together with the plane wave pion wave function, we obtain four-delta function meaning four-momentum conservation. That is, the plane wave pion wave function disappears from the final expression but there remains the recoil phase factor.

Our derivation of Eq. (2.3) is based on the field theory and is given as follows.

$$\langle 0 | q^c (t, \vec{x}) Q (t, \vec{y}) | P \rangle = \int d^4 x \langle P', k | \mathcal{L}_{\text{int}} | P \rangle$$

$$= (2\pi)^4 \delta^4 (P - P' - k) \int d^3 z \begin{bmatrix} \psi_{\gamma}^0(\vec{z}; P') O \psi_\gamma^0(\vec{z}; P) \end{bmatrix} e^{-ik \cdot \vec{z}}$$

(2.5)

$$= (2\pi)^4 \delta^4 (P - P' - k) \int d^3 z \begin{bmatrix} \psi_{\gamma}^1(\vec{z}; P') O \psi_\gamma^1(\vec{z}; P) \end{bmatrix}.$$  

(2.6)

These expressions seem to be different and contradict from each other. However, when one rewrites Eqs. (2.5, 2.6) in terms of the rest frame wave functions, these become the same expression given by

$$(G^{-1} O G)^{\alpha \beta} \int d^3 z' \psi_{\alpha \gamma}^+ (\vec{z}' ; M) \psi_{\beta \gamma} (\vec{z}' ; M) e^{-2im_Q V z'_z}.$$  

(2.7)
where $G$ is a Lorentz transformation matrix.

The rest of our formulation is based on our former paper\textsuperscript{19} in which it is shown how to construct the Lorentz-boost wave function from the rest frame wave function. In that paper, it is also shown that the Breit frame only with $t' = x' y', i.e., times of the final heavy quark $Q(0' \to 0')$ and light anti-quark $q(x' \to x')$ are equal to each other, gives the Lorentz-invariant results, and hence in this paper we also adopt this frame. We study main decay modes of the heavy-light mesons, that is, one heavy-light meson decays into another with one chiral particle or one photon (radiative decay), e.g., $D_s(0^+) \to D_s(0^-) + \pi^0$ or $D_s(0^+ \to D_s(1^-) + \gamma$. The amplitudes for each process can be written in terms of form factors which are given in the next section.

\section*{§3. Form Factors}

In this section, we study the form factors with an initial state of spin up to $2^+$. In the case of radiative decays, amplitudes are given in terms of thirty electromagnetic form factors as follows (if one takes into account the gauge invariance, the number of independent form factors is reduced to twenty-three.)

\begin{equation}
\langle 0^- | j_\mu | 1^- \rangle = \epsilon_{\mu\nu\rho\sigma} v_1^\nu v_2^\rho \epsilon^\sigma \xi_V^{(1)},
\end{equation}

\begin{equation}
\langle 0^- | j_\mu | 0^+ \rangle = 0,
\end{equation}

\begin{equation}
\langle 1^- | j_\mu | 0^+ \rangle = (\omega + 1) \epsilon^\mu \xi_V^{(2)} + (\epsilon^* \cdot v_1) \left( (v_1 + v_2)_\mu \xi_V^{(2)} + (v_1 - v_2)_\mu \xi_V^{(2)} \right),
\end{equation}

\begin{equation}
\langle 0^- | j_\mu | 1^+ \rangle = (\omega + 1) \epsilon^\mu \xi_V^{(i)} + (\epsilon \cdot v_2) \left( (v_1 + v_2)_\mu \xi_V^{(i)} + (v_1 - v_2)_\mu \xi_V^{(i)} \right),
\end{equation}

\begin{equation}
\langle 1^- | j_\mu | 1^+ \rangle = \epsilon_{\mu\nu\rho\sigma} \left[ \epsilon^\nu v_1^\rho v_2^\sigma \xi_V^{(i)} + (v_1 - v_2)_\mu \xi_V^{(i)} \right],
\end{equation}

\begin{equation}
\langle 0^- | j_\mu | 2^+ \rangle = \epsilon_{\mu\nu\rho\sigma} \left[ \epsilon^\nu v_1^\rho v_2^\sigma \xi_V^{(i)} \right],
\end{equation}

\begin{equation}
\langle 1^- | j_\mu | 2^+ \rangle = (\omega + 1) \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu v_1^\rho v_2^\sigma \xi_V^{(8)} + \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu v_1^\rho v_2^\sigma \xi_V^{(8)} + \epsilon_{\mu\nu\rho\sigma} \epsilon^\nu v_1^\rho v_2^\sigma \xi_V^{(8)}
\end{equation}
where \( j_\mu = (-e_q)\bar{q} \gamma_\mu q^c + e_Q \bar{Q} \gamma_\mu Q \) with \( e_q \) and \( e_Q \) being electric charges of \( q \) and \( Q \), respectively. In the case of decays with one chiral particle, amplitudes are given in terms of twenty-six chiral particle form factors as follows:

\[
\frac{\langle 0^- | j_{5\mu} | 1^- \rangle}{\sqrt{M_2 M_1}} = (\omega + 1) \epsilon_{1\mu} \xi_{A1}^{(1)} + (\epsilon_1 \cdot v_2) \left[ (v_1 + v_2) \mu \xi_{A2}^{(1)} + (v_1 - v_2) \mu \xi_{A3}^{(1)} \right],
\]

(3.8)

\[
\frac{\langle 0^- | j_{5\mu} | 0^+ \rangle}{i \sqrt{M_2 M_1}} = (v_1 + v_2) \mu \xi_{A1}^{(2)} + (v_1 - v_2) \mu \xi_{A2}^{(2)},
\]

(3.9)

\[
\frac{\langle 1^- | j_{5\mu} | 0^+ \rangle}{\sqrt{M_2 M_1}} = \epsilon_{\mu \nu \rho \sigma} v_1^\nu v_2^\rho \xi_{A}^{(3)},
\]

(3.10)

\[
\frac{\langle 0^- | j_{5\mu} | 1^+ \rangle}{\sqrt{M_2 M_1}} = \epsilon_{\mu \nu \rho \sigma} v_1^\nu v_2^\rho \xi_{A}^{(4)}, \quad (i = 4, 5)
\]

(3.11)

\[
\frac{\langle 1^- | j_{5\mu} | 1^+ \rangle}{i \sqrt{M_2 M_1}} = (\epsilon_2^\mu \cdot \epsilon_1) (v_1 + v_2) \mu \xi_{A1}^{(4)} + (\epsilon_2^\mu \cdot \epsilon_1) (v_1 - v_2) \mu \xi_{A2}^{(4)}
\]

+ (\epsilon_2^\mu \cdot v_1) \epsilon_{1\mu} \xi_{A3}^{(4)} + (\epsilon_1 \cdot v_2) \epsilon_{2\mu} \xi_{A4}^{(4)}, \quad (i = 6, 7)
\]

(3.12)

\[
\frac{\langle 0^- | j_{5\mu} | 2^+ \rangle}{i \sqrt{M_2 M_1}} = \epsilon_{1\mu} \nu v_2^\nu \xi_{A1}^{(8)} + \epsilon_{1\alpha \beta \gamma \delta} v_2^\nu v_2^\alpha v_2^\beta \left( v_{1\mu} \xi_{A2}^{(8)} + v_{2\mu} \xi_{A3}^{(8)} \right),
\]

(3.13)

\[
\frac{\langle 1^- | j_{5\mu} | 2^+ \rangle}{\sqrt{M_2 M_1}} = \epsilon_{\mu \nu \rho \sigma} \left[ \epsilon_{1\nu} \epsilon_{2\rho} v_2^\alpha \left( (v_1 + v_2)^\sigma \xi_{A1}^{(9)} + (v_1 - v_2)^\sigma \xi_{A2}^{(9)} \right) \right.
\]

+ \epsilon_{1\nu} \epsilon_{2\sigma} \left( \epsilon_{1\alpha} \epsilon_{2\beta} \xi_{A3}^{(9)} + \epsilon_{1\alpha} v_2^\beta v_2^\alpha \xi_{A4}^{(9)} \right) \left. \right] + \epsilon_{\alpha \beta \gamma \delta} \epsilon_{1\lambda} v_2^\alpha v_2^\beta v_2^\gamma v_2^\delta \left[ (v_1 + v_2) \mu \xi_{A6}^{(9)} + (v_1 - v_2) \mu \xi_{A7}^{(9)} \right].
\]

(3.14)

Here \( \epsilon_{\mu \nu} \) and \( \epsilon_\mu \) are polarization vectors, which could have subindex to distinguish whether it is for initial \( (i = 1) \) or final \( (i = 2) \). All form factors depend only on \( \omega = (v_1 \cdot v_2) \), where \( v_i \) are initial and final velocities of heavy-light mesons, \( H_i \) \((H_1 \rightarrow H_2 + \pi/\gamma)\). These amplitudes can be written in terms of form factors a la semi-leptonic decay of a heavy-light meson. These are, however, not described in terms of just one form factor like the Isgur-Wise function because the interaction can not be regarded as a point contrary to the semi-leptonic decay.

All amplitudes given above Eqs. (3.1)-(3.14) are in principle calculable at the lowest order in \( 1/m_Q \) because we know the explicit forms of wave functions of any spin in the heavy quark symmetric limit\(^{[10]}\) which are described in the next section.

### §4. Sum Rule

In this section, we show one of our results for the radiative decay of a heavy-light meson. By analytically calculating amplitudes given by Eqs. (3.1)-(3.7), we have the following sum
of the wave functions with a quantum number \( k \) which has two spinor indices and is expressed as \( 4 \times 2 \) matrix form, is explicitly given by

\[
\Psi_{jm}(\vec{r}) = \sqrt{\frac{2M}{4\pi r}} \begin{pmatrix} u^k(r) \\ -iv^k(r)(\vec{\sigma} \cdot \vec{n}) \end{pmatrix} y^k_{jm},
\]

where \( \vec{n} = \vec{r}/r, y^k_{jm} \) being of \( 2 \times 2 \) form are the angular part of the wave function, and \( u^k(r) \) and \( v^k(r) \) are the radial parts. The total angular momentum of a heavy meson \( \vec{J} \) is the sum of the total angular momentum of the light quark degrees of freedom \( \vec{j}_q \) and the heavy quark spin \( \frac{1}{2}\vec{\Sigma}_Q \):

\[
\vec{J} = \vec{j}_q + \frac{1}{2}\vec{\Sigma}_Q \quad \text{with} \quad \vec{j}_q = \vec{L} + \frac{1}{2}\vec{\Sigma}_q,
\]

where \( \frac{1}{2}\vec{\Sigma}_q \) (\( = \frac{1}{2}\vec{\sigma}_q \) \( 1_{2\times2} \)) and \( \vec{L} \) are the \( 4 \times 4 \) spin and the orbital angular momentum of a light antiquark, respectively. Furthermore, \( k \) is the quantum number of the following spinor operator \( K \)

\[
K = -\beta_q \left( \vec{\Sigma}_q \cdot \vec{L} + 1 \right), \quad K \Psi^k_{jm} = k \Psi^k_{jm}.
\]

\( j_q \) and \( k \) are good quantum numbers in the heavy quark limit, hence the wave function can be described in these two.

Applying Eqs. (4.1) and (4.2) to actual heavy-light mesons, e.g., to \( D_s \), we have

\[
\Gamma(D_{s1}(2460) \to D_s(1968) + \gamma) + \Gamma(D_{s1}(2460) \to D^*_s(2112) + \gamma) = \Gamma(D_{s0}(2317) \to D^*_s(2112) + \gamma), \tag{4.7}
\]

\[
\Gamma(D_{s1}(2536) \to D_s(1968) + \gamma) + \Gamma(D_{s1}(2536) \to D^*_s(2112) + \gamma) = \Gamma(D_{s2}(2573) \to D^*_s(2112) + \gamma), \tag{4.8}
\]
where experiments give

\[
\Gamma \left( D_{s1}(2460) \rightarrow D_s(1968) + \gamma \right) < 630 \text{ keV},
\]

\[
\Gamma \left( D_{s1}(2460) \rightarrow D_{s1}^*(2112) + \gamma \right) < 280 \text{ keV},
\]

\[
\Gamma \left( D_{s0}(2317) \rightarrow D_{s1}^*(2112) + \gamma \right) \text{ not yet seen},
\]

\[
\Gamma \left( D_{s1}(2536) \rightarrow D_s(1968) + \gamma \right) \text{ not yet seen},
\]

\[
\Gamma \left( D_{s1}(2536) \rightarrow D_{s1}^*(2112) + \gamma \right) \text{ possibly seen},
\]

\[
\Gamma \left( D_{s0}(2573) \rightarrow D_{s1}^*(2112) + \gamma \right) \text{ not yet seen},
\]

We have to wait and see to what extent the sum rules Eqs. (4.1, 4.2) are satisfied until experiments observe the above and other radiative decay modes.

§5. Summary

We have proposed a new method given by Eq. (2.3) how to calculate decay rates for a heavy-light mesons and have applied it to the processes \( H_1 \rightarrow H_2 + \phi^a/\gamma \) with one chiral particle \( \phi^a \) or one photon in the final state. We have derived form factors of vector as well as axial vector currents, which can be used to calculate the above processes.

We have derived the sum rules given by Eqs. (4.1) and (4.2) for the radiative decay rates in the heavy quark symmetric limit,

\[
\sum_{S=0^-,1^-} \Gamma \left( ^3P_1^- \rightarrow S + \gamma \right) = \Gamma \left( 0^+ \rightarrow 1^- + \gamma \right),
\]

\[
\sum_{S=0^-,1^-} \Gamma \left( ^1P_1^- \rightarrow S + \gamma \right) = \Gamma \left( 2^+ \rightarrow 1^- + \gamma \right),
\]

which needs to be checked by future experiments. We still need to give numerical results for all the processes in concern, \( H_1 \rightarrow H_2 + \phi^a/\gamma \), which will be published in near future.\(^{13}\)

References

1) BaBar Collaboration, B. Aubert et al., Phys. Rev. Lett. 90 (2003), 242001.
2) CLEO Collaboration, D. Besson et al., Phys. Rev. D 68 (2003), 032002.
3) See for review: E. S. Swanson, Phys. Rev. 429 (2006), 243; L.-L. Shen et al., arXiv:1005.0994.
4) PACS-CS collaboration, Y. Namekawa et al., \texttt{arXiv:0810.2364}, a talk given at the kick-off meeting of “Elucidation of new hadrons with a variety of flavors” held at Nagoya Univ., Nov. 27-28, 2009.
5) K.F. Liu, a talk given on Feb. 5, 2010 at “New Frontiers of in QCD 2010” held at Yukawa Inst. for Theor. Phys. of Kyoto Univ., Jan. 18 through Mar. 19, 2010.

6) J.L. Goity and W. Roberts, Phys. Rev. D 60 (1999), 034001.

7) M. Di Perro and E.J. Eichten, Phys. Rev. D 64 (2001), 114004.

8) W.A. Bardeen, E.J. Eichten, and C.T. Hill Phys. Rev. D 68 (2003), 054024.

9) T. Matsuki and K. Seo, Prog. Theor. Phys. 118 (2007), 1.

10) T. Matsuki and T. Morii, Phys. Rev. D 56 (1997), 5646

11) T. Matsuki, T. Morii, and K. Sudoh, Prog. Theor. Phys. 117 (2007), 1077.

12) T. Matsuki, T. Morii and K. Seo, Prog. Theor. Phys. 124 (2010), 285, arXiv:1001.4248.

13) T. Matsuki, T. Mawatari, T. Morii, K. Sudoh, hep-ph/0408326

14) T. Matsuki and K. Seo, work in progress.