Negative-Parity States and $\beta$-decays in odd Ho and Dy Nuclei with $A=151,153$

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We have investigated the negative-parity states and electromagnetic transitions in $^{151,153}$Ho and $^{151,153}$Dy within the framework of the interacting boson fermion model 2 (IBFM-2). Spin assignments for some states with uncertain spin are made based on this calculation. Calculated excitation energies, electromagnetic transitions and branching ratios are compared with available experimental data and a good agreement is obtained. The model wave functions have been used to study $\beta$-decays from Ho to Dy isotones, and the calculated log $ft$ values are close to the experimental data.

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I. INTRODUCTION

The interacting boson model (IBM) has been remarkably successful in describing the collective phenomena observed in even-even medium to heavy mass nuclei [1, 2, 3]. In the simplest version of this model, the IBM-1, the nuclear properties are described by a system of a fixed number of boson. In this version no distinction is made between proton boson and neutron boson, therefore all states in IBM-1 are $F$-spin symmetric [4, 5, 6]. The building blocks are $(s^1, s)$ for $s$-boson and $(d^1, d^1)$ for $d$-boson. The second version, the IBM-2, does distinguish between proton boson and neutron boson. The states in IBM-2 include all the $F$-spin symmetric states as well as mixed symmetry states belonging to the U(6) representation $[N-1,1]$. An important property of this new version is that the proton-neutron symmetry character of each states is specified in terms of a new quantum number called $F$-spin $[7, 8, 9, 10, 11]$. For lighter nuclei, the IBM has been extended to the interacting boson model with isospin (IBM-3) [12]. Within the IBM-3, the neutron-proton pair must be included in addition to the two other types of bosons in the IBM-2, and they form an isospin triplet $[13, 14, 15, 16, 17]$. In the interacting boson fermion model (IBFM) [18], odd-A nuclei are described by coupling the degrees of freedom of odd particle to a core which is described in the IBM. Calculations of positive and negative-parity states and the electromagnetic transitions of odd mass nuclei have performed within the framework of IBFM, for instance in Refs. [19, 20, 21, 22, 23, 24]. One of most important property in nuclear structure study is the $\beta$ decay rates. The $\beta$ transition for odd-nuclei has received intensive interests in the last few years [25, 26, 27, 28, 29, 30, 31, 32]. Theoretical contribution to the study of nuclear beta decay have been made over the years using the IBFM [33, 34, 35], and good agreement has been found with available experimental data.

The purpose of the present work is to investigate the negative-parity states and electromagnetic transitions in the $^{151,153}$Ho and $^{151,153}$Dy isotopes by using the IBFM-2 model. More importantly, $\beta$ decay between the levels are studied by using the wave functions obtained from the structure calculation of this model. In particular, the influence of different values of hamiltonian parameters on the energies and decay probabilities is investigated.

In order to calculate an odd-nucleus in the IBFM-2 model, we need to choose an even-even core. Here, the even-even $^{150,152}$Dy nuclei have been chosen as the respective core for $^{151,153}$Dy isotopes. They have 66 protons in the 50 – 82 shell and 84 and 86 neutrons in shell 82 – 126, respectively. Both nucleons lie in the first half shell, therefore they should be considered as particle bosons. For $^{151,153}$Ho isotopes, we considered them as resulting from coupling a proton hole to the even-even Er nuclei.

In section II we briefly review the interacting boson fermion model. In section III we present our calculation results for the energy levels of the core nuclei and compared with available data. The negative-parity states of $^{151,153}$Ho and $^{151,153}$Dy nuclei are presented in sections IV and V respectively. A discussion of electromagnetic transitions follows in section VI. In section VII the $\beta$ decay from levels of odd-proton Ho-isotopes to levels in odd-neutron Dy-isotopes levels are studied. Finally, in section VIII we summarize our results.

II. THE IBFM-2 MODEL

The low lying levels in odd nuclei are described as combined system of a group of bosons with one fermion. In general, the Hamiltonian for this coupled system can be written as [34],

$$H = H_B + H_F + V_{BF}. \quad (1)$$
Here $H_B$ is the usual IBM-2 Hamiltonian which describes the system of $(s_\nu, s_\pi)$ and $(d_\nu, d_\pi)$-bosons

$$H = \varepsilon_d(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa_{\pi\nu} \hat{Q}_\pi \cdot \hat{Q}_\nu + \sum_{\rho = \pi, \nu} \hat{V}_\rho + \hat{M}_{\pi\nu},$$

(2)

where $\varepsilon_d$ is the d-boson excitation energy, and $n_{d\pi}$ and $n_{d\nu}$ are the neutron and proton $d$-boson number operator respectively. $\kappa_{\pi\nu} \hat{Q}_\pi \cdot \hat{Q}_\nu$ is the quadruple interaction between proton and neutron boson, and $Q_{\rho}$, the quadruple operator, is given by

$$\hat{Q}_\rho = (s_\rho \hat{d}_\rho^\dagger + s_\rho \hat{d}_\rho)^2 + \chi_\rho (d_\rho^\dagger \hat{d}_\rho)^2.$$

(3)

The Majorana interaction is

$$\hat{M}_{\pi\nu} = \xi_2[\hat{d}_\nu \hat{s}_\pi^\dagger - \hat{d}_\pi \hat{s}_\nu^\dagger, (\hat{d}_\pi \hat{s}_\nu - \hat{d}_\nu \hat{s}_\pi)](2) + \frac{1}{2} \sum_{k=1,3} \xi_k [\hat{d}_\nu \hat{d}_\nu^\dagger](k),$$

(4)

and it only affects the positions of the mixed symmetry states. The $V_{\rho\rho}$ term represents the interaction between like bosons,

$$\hat{V}_{\rho\rho} = 1/2 \sum_{L=0,2,4} [2L + 1]C^{(L)}_{\rho}\left[\hat{d}_{\rho\nu} \hat{d}_{\rho\nu}^\dagger\right](L),$$

(5)

where $\rho = \pi, \nu$.

The term $H_F$ is the Hamiltonian of odd fermion,

$$H_F = \sum_i \epsilon_i n_i,$$

(6)

where $\epsilon_i$ is the quasi-particle energy of the $i$th orbital, and $n_i$ is fermion number operator. The quasi-particle energies and occupation probabilities are usually calculated using the BCS approximation in terms of the Fermi energy $\lambda$, the paring gap $\Delta$ and the single-particle energies $E_i$,

$$\epsilon_i = \sqrt{(E_i - \lambda)^2 + \Delta^2}.$$

(7)

The occupation probabilities are then given by

$$v_i = \left[\frac{1}{2}(1 - \frac{E_i - \lambda}{\epsilon_i})\right]^{1/2}, \quad u_i = (1 - v_i^2)^{1/2}.$$

(8)

The bosons-fermion interaction $V_{BF}$ is, in general, rather complicated but it has been shown to be dominated by three terms,

$$V_{BF} = \sum_i A_i n_i n_{d\pi} + \sum_{i,j} \Gamma_{ij} \{[a_{i\pi}^\dagger a_{j\pi}^\dagger]^2 \cdot Q_{\rho\rho}^B \} + \sum_{i,j} \Lambda_{i,j} \{[d_{i\pi}^\dagger d_{j\pi}^\dagger]^2 (k) a_{i\pi}^\dagger s_{j\pi}^\dagger]^2 + [s_{i\pi}^\dagger d_{j\pi}^\dagger]^2 + H.c.\},$$

(9)

where $d_{\mu} = (-1)^\mu d_{-\mu}$, $a_{j\mu} = (-1)^{-\mu} a_{j-\mu}$ and $Q_{\rho\rho}^B$ is the boson quadrupole operator which is defined in eq. (8). The symbols $\rho$ and $\hat{\rho}$ denote $\pi(\nu)$ and $\nu(\pi)$ if the odd fermion is a proton (neutron). The first term in Eq.(9) is a monopole-monopole interaction, which is proportional to the number of $d$-bosons. Therefore it only gives rise to a renormalization of the boson energy $\varepsilon = \varepsilon_d - \varepsilon_s$ and it does not affect the main structure of energy spectrum. The second term is a quadrupole-quadrupole interaction, and the last term is the exchange interaction. The origin of the exchange force is closely related to the presence of the Pauli principle. Both terms are dominant terms and appear to arise from the strong neutron-proton quadrupole force. The orbital dependence of the coupling coefficients has been microscopically estimated.
III. EVEN-EVEN NUCLEI STRUCTURE

In the calculation the proton and neutron shells are assumed to closed at \( Z = 50 \) and \( N = 82 \) magic shells. For \(^{150,152}\)Dy nuclei, there are eight proton-bosons and one and two neutron-bosons, respectively. On the other hand, there are seven hole-like proton bosons and one and two particle-like neutron bosons for \(^{152,154}\)Er nuclei, respectively. The calculated excitation energies are obtained by diagonalizing the Hamiltonian in Eq. (2), usually using the NPBOS code \(^{[38]}\). The parameters \( \epsilon_d \) and \( \kappa_{\pi \nu} \) have been determined so as to reproduce as closely as possible the energy of the low lying positive parity states. The energy of the ground state band levels were optimized by varying the proton anharmonicity parameters \( C_{2l}^p (L = 0, 2, 4) \). The parameters \( \chi_\pi \) and \( \chi_\nu \) have been kept constant in the two isotopes, and are taken as the same as for the SU(3) limit: \( \chi_\pi = \chi_\nu = -\sqrt{7}/2 \). In order to identify mixed symmetry states, we fitted the energy of all the \( J = 2^+ \) states below 3 MeV by smoothly changing \( \xi_2 \). The best fit values for the Hamiltonian parameters are given in Table II and the calculated energy levels are compared with available experimental data as shown in Figs. I-IV. Good agreement between the calculated and observed spectrum is obtained for the ground state band. From Fig. II one can see that the sequence of levels is well reproduced, though the calculated excitation energies of \( 6_1^+ \) and \( 8_1^+ \) are somewhat higher than the experimental ones. The splitting of \( 2_2^+ \) and \( 4_1^+ \) in the "two phonon states" is well reproduced, and it justifies the value of \( C_{2l}^p \) that are used. The energy of \( 0_2^- \) of the two phonon states equals to 1.757 MeV in the IBM-2, and this remains to be seen in future experiment.

In \(^{152}\)Dy nucleus, the level at 1.452 MeV have possible \( 1^+, 2^+ \) assignments in experiment. It is close to a level at 1.452 MeV with \( J = 1^+ \) in our IBM-2 result. The \( 4_1^+ \) state is higher than the data. There is no suitable solution in the present scheme for this problem. One possible explanation is the effect of the g-boson. The second \( J = 2^+ \) state at energy 1.379 MeV is close to the experimental level at 1.313 MeV. The calculated \( J = 3_2^+ \) state at 1.470 MeV is in good agreement with the experimental level at 1.448 MeV, and this state has mainly the \( F = F_{\text{max}} \) component. For \(^{152,154}\)Er nuclei, the ground state band fits well with data, although we used the same parameters that adjusted for \(^{150,152}\)Dy nuclei as shown in Figs. III and IV. In \(^{152}\)Er, the calculated \( J = 2_2^+ \) state at 1.758 MeV is close to the experimental one at 1.715 MeV, and this state has a very pure \( F_{\text{max}} \) character. The calculation result shows that the wave function for \( J = 2_2^+ \) is composed of 100% of the \( sN^-2d^2 \) configuration.

IV. ODD-PROTON \(^{151,153}\)HO STRUCTURE

In order to study especially the influence of fermion degrees of freedom, we have investigated odd-neutron Dy nuclei and odd-proton Ho nuclei with the same boson parameters for each odd-A number. Thus, the differences in the nuclear structures of these two nuclei can arise only from the boson-fermion interaction and the odd-particle Hamiltonian. The quasi-particle energies \( \epsilon_i \) and occupation probabilities \( \nu_i^2 \) are determined from a simple BCS calculation using Reeehal and Sorensen \(^{[39]}\) single-particle energies. The single-particle energies taken are given in Table II. The BCS equations are resolved with the single-particle orbitals \( g_{9/2}, g_{7/2}, d_{5/2}, h_{11/2}, d_{3/2} \) and \( s_{1/2} \), and with \( \Delta = 12/\sqrt{A} \). In the description of negative-parity states in Ho-isotopes, the odd proton is taken to be in the \( h_{11/2} \) orbital. We searched the optimal values of the interaction parameters that describe well the experimental levels and electromagnetic transitions. The found values of the parameters are shown in Table II. In Ho-isotopes which have 67 protons, the occupation probabilities of \( \pi h_{11/2} \) is \( \nu^2 \simeq 0.32 \). As a result, the parameter of exchange force in \( V_{B,F} \) plays a crucial role in fitting the experimental data. The strength of the exchange force in the \( V_{B,F} \) has to be increased in order to lower the energy of the first \( 2^- \) state in \(^{153}\)Ho. The \( \Gamma \) was kept constant for both isotopes. The calculated and observed \(^{[40]}\) energy spectra of \(^{151,153}\)Ho are shown in Figs. V and VI respectively. The calculation gives a number of predictions, and they are presented in the figures which are helpful to future experiments. From the figures, we see that the calculated energy levels agree with the experimental data quite well in general. Reproduction of the trend in the experimental data is clear, especially those of the first and second appearance of negative-parity states.

Experimentally, the first and second negative parity excited states have possible \( J = (0_2^- 2_2^-, 2_2^- 4_1^-) \) assignments, and they are close to states with \( J = 3_2^- \) and \( 7_2^- \), respectively in the IBFM-2 calculation for both isotopes. In \(^{151}\)Ho the states at 0.869 and 0.910 MeV in the experimental data are close to the states with \( J = (3_2^- 7_2^-) \) and \( J = (3_2^- 7_2^-) \) in the IBFM-2 results at 0.826 and 0.878 MeV, respectively. The observed order inversion in Ho-isotopes, namely \( 21^- 23^- \) has also been reproduced. However, the calculated energy of the \( (21^- 23^-)1 \) and \( (23^- 21^-)1 \) state in \(^{151}\)Ho are larger than the experimental value. This is probably due to the restriction of the limited single-particle space in the \( h_{11/2} \) orbital \(^{[19]}\).

In \(^{153}\)Ho the calculated \((23^- 21^-)1 \) state at 0.534 MeV is close to the experimental level at 0.576 MeV, while the IBFM-2 calculation gives the \((45^- 23^-)2 \) at 1.046 MeV and it is far from the experimental one at 0.727 MeV. On other hand, the
energy of the \((\frac{7}{2}^-)_1\) state in the model calculations equals to 0.725 MeV. We have considered two possible theoretical assignments for the experimental state at 0.926 MeV: \(\frac{9}{2}^-\) and \(\frac{9}{2}^+\). The calculations indicate that the \((\frac{9}{2}^-)_1\) is more probable. The characteristics of the experimental level at 1.700 MeV with possible \((\frac{5}{2}^-;\frac{7}{2}^-;\frac{9}{2}^-)\) assignment is in good agreement with our calculated \(\frac{7}{2}^-\) state at 1.624 MeV.

The higher spin states, such as \(\frac{25}{2}^-\) and \(\frac{27}{2}^-\) of \(^{153}\)Ho, have also been reproduced. The calculated energies for the \(\frac{25}{2}^-\) and \(\frac{27}{2}^-\) states in \(^{153}\)Ho are 2.490 and 2.201 MeV, respectively, which are close to experimental levels at 2.358 and 2.297 MeV respectively.

V. ODD-NEUTRON \(^{151,153}\)DY STRUCTURE

For Dy-isotopes, the BCS equations are solved with the single-particle orbitals \(f_{7/2}, h_{9/2}, p_{3/2}, f_{5/2}, i_{13/2}, h_{11/2}\) and \(p_{1/2}\) with \(\Delta = 12/\sqrt{\mathcal{A}}\). The values of single-particle energies are extracted from Ref. \(^{41}\), and they are very similar to the ones used in Ref. \(^{12}\), except for the \(i_{13/2}\) orbital. In our calculations the \(i_{13/2}\) is still present between the \(h_{9/2}\) and \(p_{3/2}\) orbitals but close to \(p_{3/2}\) orbital in both isotopes. The adopted single-particle energies are listed in Table \(^{13}\). In \(^{151,153}\)Dy-isotopes, the \(\nu p_{1/2}\) level is almost completely empty \((\nu^2 \approx 0)\) while the \(\nu h_{11/2}\) level is almost completely occupied \((\nu^2 \approx 1)\), thus the two orbitals were omitted from the calculation. According to this result, we include only the first four orbitals in calculating the negative-parity states.

A comparison of the results of our IBFM-2 calculations with the experimental results \(^{10}\) on the low-lying negative-parity states of \(^{151,153}\)Dy isotopes is shown in Figs. \(^{4}\) and \(^{5}\) respectively. All levels below 2.5 MeV known from experiment are included. From Fig. \(^{7}\) one can see that the \((\frac{9}{2}^-)_1\) and \((\frac{11}{2}^-)_1\) states agree very well between calculation and experiment. For \(^{151}\)Dy the experimental data up to 2.5 MeV is scarce. The low-lying state with 0.984 MeV has not been assigned any other quantum number. It has a transition \(E_\gamma = 0.209\) and \(log ft = 5.8\) from \(^{151}\)Ho ground state \(\frac{9}{2}^-\). From our calculation, it is close to the calculated state at 0.886 MeV with \(J^- = (\frac{4}{2}^-)_2\). This assignment is further enforced by our \(log ft\) studies to be presented shortly.

Another state with uncertain spin assignment \((\frac{9}{2}^-;\frac{11}{2}^-)\) is at an excitation energy of 1.549 MeV with \(\gamma\)-transition to \((\frac{7}{2}^-)_1\) state, and it has a \(log ft = 5.1\) from \(^{151}\)Ho ground state \(\frac{9}{2}^-\). It is reproduced very well in our IBFM-2 calculation with an excitation energy of 1.523 MeV with \(J^- = (\frac{9}{2}^-)_3\). There is not \(J^- = \frac{11}{2}^-\) state in energy range between 1.40 MeV and 1.68 MeV in our calculation. Our calculation predicts \((\frac{3}{2}^-)_1\) and \((\frac{5}{2}^-)_2\) at 0.657 MeV and 1.369 MeV in \(^{153}\)Dy, and they have not been observed experimentally, while in \(^{153}\)Dy the \((\frac{3}{2}^-)_1\) state is at 0.104 MeV in our IBFM-2 calculation, and this very well reproduced the experimental level at 0.108 MeV.

In Fig. \(^{5}\) we present a more detailed comparison between experimental and calculated energy states in \(^{153}\)Dy. Because many states have no clear assignment, so special attention is given to these levels with the hope to give assignment to them. Indeed, it is found that many calculated states are quite close to the experimental ones, and it will be highly desirable to substantiate this model prediction in future experiment. A detailed presentation is given in Table \(^{15}\) where we list the calculated energy levels, available experimental assignments (certain and uncertain spin assignment).

We also analyzed the single particle occupation probability for interested states in Dy-isotopes, and they are summarized in Table \(^{15}\). It is apparent that for the first few states, they are mainly single quasi-particle excited states where one of the occupation probability is dominant. As we go to higher excited states, we see a spreading of the occupation into more single particle states. The most important single-particle orbitals are those closest to the Fermi level, and they are \(f_{7/2}\) and \(h_{9/2}\). Clearly, for \(^{153}\)Dy, the \(\frac{4}{2}^-\) state is lower than that of the \(\frac{4}{2}^-\) in energy, and there is no order inversion here.

VI. ELECTROMAGNETIC TRANSITIONS

In the IBFM-2 model the electromagnetic transition operator is described by the following operator

\[
T^{(\lambda)} = T_B^{(\lambda)} + T_F^{(\lambda)},
\]

which contains a boson part and fermion part. The \(E2\) transition operator is expressed \(^{34}\)

\[
T^{E2} = e^B e^B Q^B + e^B Q^B + \sum_{i,j} e_i^{(2)} e_j^{(2)} a_i^T a_j(2),
\]

where

\[
e_i^{(2)} = \langle i|\gamma|\gamma = 2\rangle.
\]
where the quadrupole operators \( Q_x \) and \( Q_y \) are defined in Eq. (3), \( e_x \) and \( e_y \) the proton and neutron boson effective charges and

\[
e^{(2)}_{i,j} = -\frac{1}{\sqrt{3}} (u_i u_j - v_i v_j) \langle l_i, \frac{1}{2}, j_i \| r^2 Y(2) \| l_j, \frac{1}{2}, j_j \rangle.
\]

The M1 transition operator in IBFM-2 is

\[
T^{M1} = \sqrt{\frac{3}{4\pi}} (g_x B L_x^B + g_y B L_y^B + \sum_{i,j} e^{(1)}_{i,j} (a_i^\dagger a_j)^{(1)})
\]

where \( g_x \) and \( g_y \) are g factors for proton and neutron boson, \( \hat{L} \) is the angular momentum operator

\[
L_\rho = \sqrt{10} [d^\dagger_\rho d_\rho]^{(1)},
\]

and the coefficient

\[
e^{(1)}_{i,j} = \frac{1}{\sqrt{3}} (u_i u_j + v_i v_j) \langle l_i, \frac{1}{2}, j_i \| (g_1 l + g_s s) \| l_j, \frac{1}{2}, j_j \rangle,
\]

where \( g_1 \) and \( g_s \) are the single particle g-factors of the odd nucleon. For boson part, the E2 matrix elements are very sensitive to the difference between neutron boson and proton effective charge, and they are kept constant at \( e_x = 2 e_y = 0.1 \) e.b for all isotopes. The values of \( e_x^B \) were determined from the experimental \( B(E2; 2^+ \rightarrow 0^+) \) of even-even \(^{152}\)Dy nucleus. For the odd nucleon, the effective charge 1.5 e and 0.5 e are taken for the proton and the neutron, respectively. The parameter \( \chi \) in the E2 transition operator has the same value as in the Hamiltonian, though they are not necessary \(^{43,44}\). The standard boson g factor values \( g_x = 1 \mu_N \) and \( g_y = 0 \mu_N \) are used for all isotopes. We have estimated the single particle \( g_1 \) and \( g_s \), and taken them as \( g_1^\rho = 0 \mu_N \) and \( g_1^\nu = 1 \mu_N \), while the spin \( g \)-factors are taken as the free values quenched by a factor of 0.7 and 0.5 for proton and neutron, respectively, which is the common practice as in Refs. \(^{20,43,44}\). Using this procedure we have calculated the electromagnetic transitions, and a very good agreement between calculated and experimental magnetic moment of the ground states is obtained in both magnitude and sign as shown in Tables \[X\] and \[XI\].

The resulting branching ratios for the \(^{151,153}\)Ho and \(^{151,153}\)Dy nuclei are listed in Tables \[VII\] and \[IX\] respectively, in comparison with the experimental data \(^{40}\). In these tables, the strongest branch is correctly predicted. The other transitions are in qualitative agreement with experiment. The deviations can be reduced by changing \( g_s \), as well as by changing the effective charges. From Tables \[VII\] and \[IX\] one can see that the strongest transitions are between those states having the same dominated single-particle orbital. This means that a strong transition should occur between levels in the same band. Our results for electromagnetic transition probabilities are summarized in Tables \[X\] and \[XI\] respectively. In \(^{151,153}\)Dy the \( \frac{5}{2}^- \) states decay predominantly to the \( \frac{5}{2}^- \) states via a pure M1 transition. In \(^{153}\)Dy isotope, the \( \frac{5}{2}^- \) is associated with \( \nu f_7/2 \) and \( \nu h_9/2 \), the exhibit sizable mixing of these two quasi-particle orbitals. Due to the mixing of the two components in the wave function in this state, the \( \frac{5}{2}^- \) and \( \frac{5}{2}^+ \) states are connected by strong E2 transition and weak M1 transition in \(^{153}\)Dy isotope. In contrast, we see that the \( \frac{7}{2}^- \) state decays to \( \frac{5}{2}^- \) states by very strong E2 and M1 transitions in \(^{151}\)Dy isotope.

We have also calculated the quadrupole moments of the ground states and some low-lying states in \(^{151,153}\)Ho and \(^{151,153}\)Dy. The results are given in Tables \[XI\] and \[XII\] respectively. The calculated results are in the same order of magnitude as the available experimental data.

### VII. \( \beta \)-Decay

In IBFM-2, a relation among the IBM and the underlying shell model has been established by including the proton and neutron degree of freedom \(^{7}\). This offers one the capability to compute the probabilities of \( \beta \)-decay. The decay of odd-nuclei proceeds predominantly through the conversion of the odd particle from neutron to proton \((\beta^-\text{-decay})\) or from proton to neutron \((\beta^+\text{-decay})\). There are two types of beta decay, the Fermi decay and Gamow-Teller decay. In the framework of IBFM both transitions can be calculated \(^{17}\). First define the following operators,

\[
\Lambda_m^{(j)} = \xi_j a^{\dagger}_j m + \sum_j \xi_{j^*} s [d_j^{\dagger} a^{\dagger}_j m] \quad (\Delta n_j = 1, \Delta N = 0),
\]

(20)
IBFM-2 is well suited for understanding 

Examining the wave function of daughter nuclei 

The adjustable parameters in the beta decay calculation, consequently the log of the calculation is that it indicates the observed state at 1.549 MeV in states have the approximately the same values and in good agreement with experimental ones. An interesting result in units of second where 

\[ f_t = \frac{6163}{\langle M_F \rangle^2 + (G_A/G_V)^2 \langle M_{GT} \rangle^2} \] 

in units of second where \( G_A/G_V \)^2=1.59, and 

\[ \langle M_F \rangle^2 = \frac{1}{2I_1+1} |\langle I_f || Q^F || I_i \rangle|^2, \] 

\[ \langle M_{GT} \rangle^2 = \frac{1}{2I_1+1} |\langle I_f || Q_{GT} || I_i \rangle|^2. \] 

Having obtained the wave functions, we can calculate the \( \beta \)-decay rates. It should be stressed that there is no adjustable parameters in the beta decay calculation, consequently the \( \log f_t \) is obtained in a parameter free manner. Examining the wave function of daughter nuclei \(^{151,153}\)Dy, the first and second excited \( \frac{9}{2}^- \) states are dominated by \( h_{9/2} \) and \( f_{7/2} \) orbitals in both isotopes, respectively. According to these components the \( \log f_t \) values of the two states have the approximately the same values and in good agreement with experimental ones. An interesting result of the calculation is that it indicates the observed state at 1.549 MeV in \(^{151}\)Dy, corresponds to the state \( \frac{9}{2}^- \rangle \) at 1.523 MeV in the IBFM-2 result, and the main single-particle component of this state is \( h_{9/2} \). Because the \( \frac{9}{2}^- \rangle \) state in \(^{153}\)Dy at 0.862 MeV in IBFM-2 results, has a dominant component of \( f_{7/2} \), the beta decay rate is small, hence the \( \log f_t \) value of the \( (\frac{9}{2}^-) \) in \(^{153}\)Dy is larger than for the one in \(^{151}\)Dy. This enforces our spin assignment in the structure calculation.

The \( \beta \)-decays of ground state \( J^- = \frac{11}{2}^- \) of \(^{151,153}\)Ho to \(^{151,153}\)Dy have many branches as shown in Table XI, but the strong ones are to the level \( \frac{9}{2}^- \), which are 60 and 70 percent \[48]. The \( \log f_t \) are equal to \( (4.6 \) and 3.5111) and \( (4.7 \) and 3.612) in the experiments and the model results for two isotopes, respectively. In \(^{153}\)Dy, with the predicted assignments \( J^- (1.092) \) and \( J^- (1.189) = \frac{11}{2}^- \), the calculated \( \log f_t \) of the two branches are 6.707 and 7.358 respectively, which are close to the experimental value of 6.3 for both branches, respectively.

From Table XI it can be concluded that IBFM-2 provides a meaningful framework for describing \( \beta \)-decay transitions between \(^{151,153}\)Dy and \(^{151,153}\)Ho. Together with other recent \( \beta \)-decay calculations \[33 \ 34 \ 55], it shows that the IBFM-2 is well suited for understanding \( \beta \)-decay properties of rare-earth nuclei.

**VIII. SUMMARY**

In this work IBFM-2 calculations for the odd-mass Ho and Dy (A=151,153) have been presented. For Dy-isotopes four fermion single-particle \( f_{7/2}, h_{9/2}, p_{3/2} \) and \( f_{5/2} \) orbitals were used to study the negative-parity states. For
study the negative-parity states in Ho-isotopes only $h_{11/2}$ has been taken into account. The boson-boson interaction parameters were fixed by the calculation on the boson core nuclei. The boson-fermion interaction parameter is kept constant of each element, and there are only two free varying boson-fermion quadrupole interaction parameters for each even-odd nucleus. The analysis of the wave functions indicates that the $f_7/2$ and $h_9/2$ orbitals are dominate in the wave functions in Dy-isotopes. The known quadrupole and magnetic moments in these nuclei are reasonably well described by the model. The present IBFM-2 calculations provide a satisfactory framework for describing the $\beta$-decay rates in the odd-mass nuclei with $A \sim 150$. The predictions of this work can serve as a good reference to experimentalists. Further experimental study will be very helpful to further test the present IBFM-2 calculation.

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FIG. 1: The calculated and observed energy spectra for the $^{150}$Dy isotope.
FIG. 2: The calculated and observed energy spectra for the $^{152}$Dy isotope.

TABLE I: The parameters of the IBM-2 Hamiltonian. $\chi_\pi = \chi_\nu = -1.323$, have been chosen for $^{150,152}$Dy and $^{152,154}$Er. All the parameters are in MeV unit.

| nucleus | $k_{\pi\nu}$ | $\varepsilon_d$ | $\xi_1$ | $\xi_2$ | $\xi_3$ | $C^{L}_{\pi}(L = 0, 2, 4)$ |
|---------|---------------|-----------------|---------|---------|---------|-----------------|
| $^{150}$Dy, $^{152}$Er | $-0.010$ | $0.820$ | $0.300$ | $0.300$ | $0.300$ | $0.210, 0.210, -0.130$ |
| $^{152}$Dy, $^{154}$Er | $-0.010$ | $0.650$ | $-0.400$ | $0.140$ | $0.400$ | $0.500, 0.500, 0.000$ |

TABLE II: Single-particle energies (MeV) of proton orbitals in Ho-isotopes and parameters in the boson-fermion interaction (MeV).

| nucleus | $g_{9/2}$ | $g_{7/2}$ | $d_{5/2}$ | $h_{11/2}$ | $d_{3/2}$ | $s_{1/2}$ | $\Gamma$ | $A$ | $\Lambda$ |
|---------|-----------|-----------|-----------|------------|-----------|-----------|---------|-----|--------|
| $^{151}$Ho | $-5.000$ | $0.123$ | $0.800$ | $2.202$ | $3.021$ | $3.277$ | $1.200$ | $-0.500$ | $0.250$ |
| $^{153}$Ho | $-5.000$ | $0.148$ | $0.798$ | $2.252$ | $3.101$ | $3.337$ | $1.200$ | $-0.240$ | $0.520$ |
FIG. 3: The calculated and observed energy spectra for the $^{152}$Er isotope.

TABLE III: Single-particle energies (MeV) of neutron orbitals in Dy-isotopes and parameters in the boson-fermion interaction (MeV).

| nucleus | $h_{11/2}$ | $h_{9/2}$ | $f_{7/2}$ | $f_{5/2}$ | $i_{13/2}$ | $p_{3/2}$ | $p_{1/2}$ | $\Gamma$ | $A$ | $\Lambda$ |
|---------|------------|------------|------------|------------|------------|------------|------------|--------|-----|-------|
| $^{151}$Dy | $-5.800$ | $-0.4300$ | $-1.450$ | $1.760$ | $1.350$ | $1.450$ | $2.740$ | $0.500$ | $-0.300$ | $0.350$ |
| $^{153}$Dy | $-5.800$ | $-0.450$ | $-1.500$ | $1.800$ | $1.350$ | $1.500$ | $2.800$ | $0.500$ | $-0.440$ | $0.550$ |
FIG. 4: The calculated and observed energy spectra for the $^{154}$Er isotope.
FIG. 5: The calculated and observed energy spectra for the $^{151}$Ho isotope.
FIG. 6: The calculated and observed energy spectra for the $^{153}$Ho isotope.
FIG. 7: The calculated and observed energy spectra for the $^{151}$Dy isotope.
FIG. 8: The calculated and observed energy spectra for the $^{153}$Dy isotope.
TABLE IV: Available experimental and calculated energy levels for $^{153}$Dy isotope. For details calculation see figure 8.

| $IBFM - 2$ | $EXP.$ |
|------------|--------|
| $J^-/2$ | Energy | $J^-/2$ | Energy |
| 7$^-$/2 | 0.000 | 7$^-$/2 | 0.000 |
| 3$^-$/2 | 0.104 | 3$^-$/2 | 0.108 |
| 5$^-$/2 | 0.327 | 3$^-$/2, 5$^-$/2 | 0.270 |
| 9$^-$/2 | 0.331 | 9$^-$/2 | 0.295 |
| 1$^-$/2 | 0.438 | (-) | 0.500 |
| 9$^-$/2 | 0.600 | 5$^-$/2, 7$^-$/2, 9$^-$/2 | 0.365 |
| 7$^-$/2 | 0.606 | 3$^-$/2, 5$^-$/2, 7$^-$/2 | 0.576 |
| 5$^-$/2 | 0.646 | 3$^-$/2, 5$^-$/2 | 0.565 |
| 11$^-$/2 | 0.657 | 11$^-$/2 | 0.637 |
| 7$^-$/2 | 0.695 | 5$^-$/2, 7$^-$/2 | 0.688 |
| 7$^-$/2 | 0.879 | 7$^-$/2 | 0.830 |
| 13$^-$/2 | 0.877 | 13$^-$/2 | 0.837 |
| 13$^-$/2 | 1.195 | 13$^-$/2 | 1.041 |
| 11$^-$/2 | 1.151 | 11$^-$/2 | 1.068 |
| 11$^-$/2 | 1.236 | 9$^-$/2, 11$^-$/2 | 1.092 |
| 11$^-$/2 | 1.252 | 7$^-$/2, 9$^-$/2, 11$^-$/2 | 1.189 |
| 15$^-$/2 | 1.254 | 15$^-$/2 | 1.273 |
| 13$^-$/2 | 1.277 | 13$^-$/2 | 1.321 |
| 17$^-$/2 | 1.394 | 17$^-$/2 | 1.455 |
| 15$^-$/2 | 1.543 | 15$^-$/2 | 1.584 |
| 17$^-$/2 | 1.752 | 17$^-$/2 | 1.602 |
| 19$^-$/2 | 1.796 | 19$^-$/2, 17$^-$/2 | 1.753 |
| 17$^-$/2 | 1.846 | 17$^-$/2 | 1.862 |
| 21$^-$/2 | 1.892 | 21$^-$/2 | 2.043 |
| 19$^-$/2 | 1.922 | 19$^-$/2 | 2.150 |
| 21$^-$/2 | 2.288 | 21$^-$/2 | 2.454 |
| 25$^-$/2 | 2.621 | 25$^-$/2 | 2.523 |
TABLE V: The percentage components of wave functions for $^{151,153}$Dy-isotopes.

| $J^-$ | $p_{3/2}$ | $f_{7/2}$ | $h_{9/2}$ | $f_{5/2}$ | $p_{3/2}$ | $f_{7/2}$ | $h_{9/2}$ | $f_{5/2}$ |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 7/2$^-$ | 2 | 97 | 0 | 0 | 2 | 97 | 1 | 0 |
| 3/2$^-$ | 13 | 86 | 1 | 1 | 9 | 90 | 0 | 1 |
| 9/2$^-$ | 2 | 95 | 1 | 1 | 2 | 80 | 17 | 1 |
| 1/2$^-$ | 18 | 78 | 2 | 1 | 15 | 82 | 2 | 1 |
| 9/2$^-$ | 1 | 91 | 7 | 0 | 1 | 93 | 5 | 0 |
| 7/2$^-$ | 4 | 95 | 1 | 0 | 8 | 90 | 1 | 1 |
| 5/2$^-$ | 1 | 4 | 81 | 14 | 4 | 53 | 38 | 5 |
| 11/2$^-$ | 5 | 93 | 1 | 0 | 3 | 95 | 1 | 0 |
| 7/2$^-$ | 13 | 83 | 4 | 1 | 8 | 64 | 22 | 6 |
| 3/2$^-$ | 12 | 85 | 1 | 1 | 8 | 88 | 3 | 1 |
| 9/2$^-$ | 1 | 14 | 81 | 4 | 4 | 92 | 4 | 0 |
| 3/2$^-$ | 1 | 10 | 75 | 14 | 13 | 77 | 9 | 2 |
| 11/2$^-$ | 0 | 8 | 89 | 3 | 5 | 91 | 3 | 1 |
| 13/2$^-$ | 0 | 6 | 87 | 7 | 0 | 1 | 93 | 6 |
| 1/2$^-$ | 1 | 5 | 78 | 17 | 10 | 50 | 34 | 7 |
| 11/2$^-$ | 5 | 88 | 6 | 1 | 0 | 3 | 93 | 3 |
| 13/2$^-$ | 3 | 87 | 9 | 1 | 0 | 6 | 89 | 5 |
| 15/2$^-$ | 10 | 88 | 1 | 1 | 5 | 94 | 1 | 0 |
| 17/2$^-$ | 1 | 8 | 82 | 9 | 0 | 0 | 92 | 8 |
| 19/2$^-$ | 16 | 81 | 2 | 1 | 60 | 39 | 0 | 1 |
| 21/2$^-$ | 3 | 13 | 74 | 10 | 0 | 0 | 91 | 9 |
| 23/2$^-$ | 24 | 73 | 2 | 1 | 57 | 41 | 1 | 1 |
| 25/2$^-$ | 15 | 32 | 45 | 8 | 1 | 1 | 88 | 10 |
### TABLE VI: Branching ratios in $^{151}$Ho isotope, the order of states corresponding to IBFM-2 calculation.

| Level (MeV) | Transition       | $J_z$(IBFM-2) | $J_z$(Exp.) |
|------------|------------------|---------------|-------------|
| 0.638      | $9^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100         |
| 0.667      | $7^-/2_1 \rightarrow 9^-/2_1$  | 0.1           |             |
|            | $7^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100         |
| 0.789      | $15^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100         |
| 0.869      | $13^-/2_1 \rightarrow 15^-/2_1$ | 0.2           |             |
|            | $13^-/2_1 \rightarrow 9^-/2_1$  | 0.0           | 9.7(12)     |
|            | $13^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100(9)      |
| 0.910      | $11^-/2_2 \rightarrow 13^-/2_1$ | 0.0           |             |
|            | $11^-/2_2 \rightarrow 15^-/2_1$ | 0.0           |             |
|            | $11^-/2_2 \rightarrow 7^-/2_1$  | 0.0           |             |
|            | $11^-/2_2 \rightarrow 9^-/2_1$  | 7.3           |             |
|            | $11^-/2_2 \rightarrow 11^-/2_1$ | 100           |             |
| 1.129      | $5^-/2_1 \rightarrow 7^-/2_1$  | 100           | 38(3)       |
|            | $7^-/2_2 \rightarrow 9^-/2_1$  | 43.7          |             |
| 1.279      | $7^-/2_2 \rightarrow 5^-/2_1$  | 25.4          |             |
|            | $7^-/2_2 \rightarrow 11^-/2_1$ | 0.6           |             |
|            | $7^-/2_2 \rightarrow 7^-/2_1$  | 69.4          |             |
|            | $7^-/2_2 \rightarrow 9^-/2_1$  | 100           | 100         |
|            | $7^-/2_2 \rightarrow 11^-/2_1$ | 10.3          |             |
| 1.387      | $19^-/2_1 \rightarrow 15^-/2_1$ | 100           | 100         |
| 1.541      | $7^-/2_3 \rightarrow 5^-/2_1$  | 0.0           |             |
|            | $7^-/2_3 \rightarrow 7^-/2_2$  | 0.0           |             |
|            | $7^-/2_3 \rightarrow 11^-/2_2$ | 0.5           |             |
|            | $7^-/2_3 \rightarrow 7^-/2_2$  | 100           | 100         |
|            | $7^-/2_3 \rightarrow 9^-/2_1$  | 85.5          |             |
|            | $7^-/2_3 \rightarrow 11^-/2_1$ | 70.6          |             |

### TABLE VII: Branching ratios in $^{153}$Ho isotope, the order of states corresponding to IBFM-2 calculation.

| Level (MeV) | Transition       | $J_z$(IBFM-2) | $J_z$(Exp.) |
|------------|------------------|---------------|-------------|
| 0.351      | $9^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100         |
| 0.398      | $7^-/2_1 \rightarrow 9^-/2_1$  | 7.9           |             |
|            | $7^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100         |
| 0.576      | $15^-/2_1 \rightarrow 11^-/2_1$ | 100           | 100         |
| 0.706      | $13^-/2_1 \rightarrow 15^-/2_1$ | 0.7           |             |
|            | $13^-/2_1 \rightarrow 9^-/2_1$  | 0.3           |             |
|            | $13^-/2_1 \rightarrow 11^-/2_1$ | 100           |             |
| 0.814      | $7^-/2_2 \rightarrow 7^-/2_1$  | 0.5           |             |
|            | $7^-/2_2 \rightarrow 9^-/2_1$  | 100           | 100         |
|            | $7^-/2_2 \rightarrow 11^-/2_1$ | 0.1           |             |
| 0.926      | $9^-/2_2 \rightarrow 7^-/2_2$  | 4.8           |             |
|            | $9^-/2_2 \rightarrow 13^-/2_1$ | 0.0           |             |
|            | $9^-/2_2 \rightarrow 7^-/2_1$  | 62.5          |             |
|            | $9^-/2_2 \rightarrow 9^-/2_1$  | 54.7          | 100         |
| 1.207      | $19^-/2_1 \rightarrow 15^-/2_1$ | 100           | 100         |
| Level (MeV) | Transition | $I_\gamma$ (IBFM-2) | $I_\gamma$ (Exp.) |
|------------|------------|---------------------|-------------------|
| 0.527      | $9^-/2_1 \rightarrow 7^-/2_1$ | 100                | 100               |
| 0.775      | $11^-/2_1 \rightarrow 9^-/2_1$ | 5.1                |                   |
|            | $11^-/2_1 \rightarrow 7^-/2_1$ | 100                | 100               |
| 0.984      | $9^-/2_2 \rightarrow 11^-/2_1$ | 5.0                | 100               |
|            | $9^-/2_2 \rightarrow 9^-/2_1$  | 0.1                |                   |
|            | $9^-/2_2 \rightarrow 7^-/2_1$  | 100                |                   |
| 1.334      | $11^-/2_2 \rightarrow 9^-/2_2$ | 0.1                | 22(13)            |
|            | $11^-/2_2 \rightarrow 11^-/2_1$| 0.0                | 50(13)            |
|            | $11^-/2_2 \rightarrow 9^-/2_1$ | 100                |                   |
|            | $11^-/2_2 \rightarrow 7^-/2_1$ | 0.2                |                   |
| 1.348      | $13^-/2_1 \rightarrow 11^-/2_2$| 0.0                |                   |
|            | $13^-/2_1 \rightarrow 9^-/2_2$ | 0.0                |                   |
|            | $13^-/2_1 \rightarrow 11^-/2_1$| 0.8                | 7.5(15)           |
|            | $13^-/2_1 \rightarrow 9^-/2_1$ | 100                | 100(2)            |
| 1.511      | $15^-/2_1 \rightarrow 13^-/2_1$| 1.7                |                   |
|            | $15^-/2_1 \rightarrow 11^-/2_2$| 0.0                |                   |
|            | $15^-/2_1 \rightarrow 11^-/2_1$| 100                | 100(3)            |
| 1.549      | $9^-/2_3 \rightarrow 13^-/2_1$ | 0.0                |                   |
|            | $9^-/2_3 \rightarrow 11^-/2_2$ | 5.6                |                   |
|            | $9^-/2_3 \rightarrow 9^-/2_2$  | 0.3                |                   |
|            | $9^-/2_3 \rightarrow 11^-/2_1$ | 23.9               |                   |
|            | $9^-/2_3 \rightarrow 9^-/2_1$  | 100                | 36(8)             |
|            | $9^-/2_3 \rightarrow 7^-/2_1$  | 3.5                | 100(7)            |
| 1.918      | $17^-/2_1 \rightarrow 15^-/2_1$| 0.6                | 43.2(13)          |
|            | $17^-/2_1 \rightarrow 13^-/2_1$| 100                | 100(2)            |
| 2.263      | $21^-/2_1 \rightarrow 17^-/2_1$| 100                | 100               |
TABLE IX: Branching ratios in $^{153}$Dy isotope, the order of states corresponding to IBFM-2 calculation.

| Level (MeV) | Transition     | $I_\gamma$(IBFM-2) | $I_\gamma$(Exp.) |
|------------|----------------|---------------------|------------------|
| 0.108      | $3^-/2_1 \rightarrow 7^-/2_1$ | 100                | 100              |
| 0.270      | $5^-/2_1 \rightarrow 3^-/2_1$ | 100(5)             | 100              |
|            | $5^-/2_1 \rightarrow 7^-/2_3$ | 20.2               | 86(4)            |
| 0.295      | $9^-/2_1 \rightarrow 5^-/2_3$ | 0.0                 |                  |
|            | $9^-/2_1 \rightarrow 7^-/2_3$ | 100                 | 100              |
| 0.365      | $9^-/2_2 \rightarrow 9^-/2_1$ | 0.2                 |                  |
|            | $9^-/2_2 \rightarrow 5^-/2_3$ | 0.0                 | $\approx 3.3$   |
|            | $9^-/2_2 \rightarrow 7^-/2_3$ | 100                 | 100(27)          |
| 0.565      | $5^-/2_2 \rightarrow 9^-/2_2$ | 0.0                 |                  |
|            | $5^-/2_2 \rightarrow 9^-/2_3$ | 2.0                 |                  |
|            | $5^-/2_2 \rightarrow 5^-/2_3$ | 11.3                |                  |
|            | $5^-/2_2 \rightarrow 3^-/2_3$ | 50.9                | 100(4)           |
|            | $5^-/2_2 \rightarrow 7^-/2_3$ | 100                 | 48(15)           |
| 0.576      | $7^-/2_2 \rightarrow 5^-/2_2$ | 0.0                 |                  |
|            | $7^-/2_2 \rightarrow 9^-/2_2$ | 38.5                |                  |
|            | $7^-/2_2 \rightarrow 9^-/2_3$ | 1.4                 |                  |
|            | $7^-/2_2 \rightarrow 5^-/2_3$ | 100                 |                  |
|            | $7^-/2_2 \rightarrow 3^-/2_3$ | 30.7                | 88(15)           |
|            | $7^-/2_2 \rightarrow 7^-/2_3$ | 72.9                | 100(19)          |
| 0.637      | $11^-/2_2 \rightarrow 7^-/2_3$ | 0.0                 |                  |
|            | $11^-/2_2 \rightarrow 9^-/2_3$ | 71.6                |                  |
|            | $11^-/2_2 \rightarrow 9^-/2_3$ | 19.3                | 1.2              |
|            | $11^-/2_2 \rightarrow 7^-/2_3$ | 100                 | 100(4)           |
| 0.688      | $7^-/2_4 \rightarrow 7^-/2_2$ | 7.1                 |                  |
|            | $7^-/2_4 \rightarrow 5^-/2_2$ | 38.8                |                  |
|            | $7^-/2_4 \rightarrow 9^-/2_2$ | 75.1                |                  |
|            | $7^-/2_4 \rightarrow 9^-/2_3$ | 100.0               | 9(4)             |
|            | $7^-/2_4 \rightarrow 5^-/2_3$ | 41.7                |                  |
|            | $7^-/2_4 \rightarrow 3^-/2_3$ | 40.3                | 22(9)            |
|            | $7^-/2_4 \rightarrow 7^-/2_3$ | 38.3                | 100(7)           |
| 0.837      | $13^-/2_1 \rightarrow 11^-/2_1$ | 0.1                 |                  |
|            | $13^-/2_1 \rightarrow 9^-/2_2$ | 0.1                 |                  |
|            | $13^-/2_1 \rightarrow 9^-/2_3$ | 100                 | 100(14)          |
| 1.041      | $13^-/2_2 \rightarrow 13^-/2_1$ | 63.2                | $\approx 50$    |
|            | $13^-/2_2 \rightarrow 11^-/2_1$ | 1.3                 | 100(50)          |
|            | $13^-/2_2 \rightarrow 9^-/2_2$ | 0.0                 |                  |
|            | $13^-/2_2 \rightarrow 9^-/2_3$ | 100                 | 100(50)          |
| 1.273      | $15^-/2_1 \rightarrow 13^-/2_2$ | 17.2                |                  |
|            | $15^-/2_1 \rightarrow 13^-/2_1$ | 0.3                 |                  |
|            | $15^-/2_1 \rightarrow 11^-/2_1$ | 100                 | 100(24)          |
| $J_i^{-} \rightarrow J_f^{-}$ | \(^{151}\text{Ho}\) B(E2) | \(^{151}\text{Ho}\) B(M1) | \(^{153}\text{Ho}\) B(E2) | \(^{153}\text{Ho}\) B(M1) |
|---|---|---|---|---|
| $9^{-}/2_{1} \rightarrow 11^{-}/2_{1}$ | 0.0612 IBFM-2 | 0.0216 EXP. | 0.0628 IBFM-2 | 0.0684 EXP. |
| $7^{-}/2_{1} \rightarrow 9^{-}/2_{1}$ | 0.0645 IBFM-2 | 0.0596 EXP. | 0.0628 IBFM-2 | 0.0684 EXP. |
| $7^{-}/2_{1} \rightarrow 11^{-}/2_{1}$ | 0.0150 IBFM-2 | 0.2398 EXP. | 0.0204 IBFM-2 | 0.2956 EXP. |
| $13^{-}/2_{1} \rightarrow 11^{-}/2_{1}$ | 0.0697 IBFM-2 | 0.0083 EXP. | 0.0717 IBFM-2 | 0.0013 EXP. |
| $13^{-}/2_{1} \rightarrow 9^{-}/2_{1}$ | 0.0082 IBFM-2 | 0.0076 EXP. | 0.0082 IBFM-2 | 0.0076 EXP. |
| $11^{-}/2_{2} \rightarrow 13^{-}/2_{1}$ | 0.0004 IBFM-2 | 0.0942 EXP. | 0.0015 IBFM-2 | 0.153 EXP. |
| $11^{-}/2_{2} \rightarrow 15^{-}/2_{1}$ | 0.0039 IBFM-2 | 0.0036 EXP. | 0.0039 IBFM-2 | 0.0036 EXP. |
| $5^{-}/2_{1} \rightarrow 7^{-}/2_{1}$ | 0.0865 IBFM-2 | 0.0041 EXP. | 0.0850 IBFM-2 | 0.0133 EXP. |
| $5^{-}/2_{1} \rightarrow 9^{-}/2_{1}$ | 0.0363 IBFM-2 | 0.0345 EXP. | 0.0363 IBFM-2 | 0.0345 EXP. |
| $7^{-}/2_{2} \rightarrow 7^{-}/2_{1}$ | 0.0449 IBFM-2 | 0.0007 EXP. | 0.0290 IBFM-2 | 0.0002 EXP. |
| $7^{-}/2_{2} \rightarrow 5^{-}/2_{1}$ | 0.0080 IBFM-2 | 0.4656 EXP. | 0.0093 IBFM-2 | 0.7639 EXP. |
| $3^{-}/2_{1} \rightarrow 7^{-}/2_{1}$ | 0.1238 IBFM-2 | 0.1093 EXP. | 0.1238 IBFM-2 | 0.1093 EXP. |
| $3^{-}/2_{1} \rightarrow 7^{-}/2_{2}$ | 0.0249 IBFM-2 | 0.0182 EXP. | 0.0249 IBFM-2 | 0.0182 EXP. |
| $13^{-}/2_{2} \rightarrow 11^{-}/2_{2}$ | 0.0071 IBFM-2 | 0.0019 EXP. | 0.0005 IBFM-2 | 0.0022 EXP. |
| $13^{-}/2_{2} \rightarrow 13^{-}/2_{1}$ | 0.0131 IBFM-2 | 0.0018 EXP. | 0.0046 IBFM-2 | 0.0306 EXP. |
| $9^{-}/2_{2} \rightarrow 7^{-}/2_{2}$ | 0.0015 IBFM-2 | 0.4275 EXP. | 0.0010 IBFM-2 | 0.1388 EXP. |
| $9^{-}/2_{2} \rightarrow 9^{-}/2_{1}$ | 0.0625 IBFM-2 | 0.0035 EXP. | 0.0713 IBFM-2 | 0.0077 EXP. |
| $11^{-}/2_{1}$ | -0.2858 IBFM-2 | 6.9255 EXP. | -0.2303 IBFM-2 | -1.1(5) EXP. |
| $9^{-}/2_{1}$ | -0.0546 IBFM-2 | 5.9162 EXP. | -0.0359 IBFM-2 | 5.9166 EXP. |
| $7^{-}/2_{1}$ | -0.2761 IBFM-2 | 5.4478 EXP. | -0.2538 IBFM-2 | 5.7557 EXP. |
| $15^{-}/2_{1}$ | -0.4559 IBFM-2 | 8.2781 EXP. | -0.4735 IBFM-2 | 7.9279 EXP. |
| $13^{-}/2_{1}$ | -0.2215 IBFM-2 | 7.6754 EXP. | -0.1816 IBFM-2 | 7.7220 EXP. |
| $11^{-}/2_{2}$ | -0.0582 IBFM-2 | 6.7618 EXP. | -0.0271 IBFM-2 | 6.7609 EXP. |
| $7^{-}/2_{2}$ | 0.1773 IBFM-2 | 4.5124 EXP. | 0.2187 IBFM-2 | 4.3634 EXP. |
| $5^{-}/2_{1}$ | 0.1045 IBFM-2 | 3.7040 EXP. | 0.1267 IBFM-2 | 4.0168 EXP. |
| $3^{-}/2_{1}$ | -0.1572 IBFM-2 | 3.1244 EXP. | -0.1507 IBFM-2 | 3.8207 EXP. |
TABLE XI: Experimental and calculated B(E2) (in unit $e^2b^2$) and B(M1) (in unit $\mu_2^2N$), the Quadrupole moment and Magnetic moment of ground state and low-lying states listed in last lines for $^{151,153}$Dy isotopes.

| $J_i^- \rightarrow J_f^-$       | $^{151}$Dy B(E2) | $^{151}$Dy B(M1) | $^{153}$Dy B(E2) | $^{153}$Dy B(M1) |
|--------------------------------|------------------|------------------|------------------|------------------|
|                                | IBFM-2 | EXP.  | IBFM-2 | EXP.  | IBFM-2 | EXP.  | IBFM-2 | EXP.  |
| $3^-/2_1 \rightarrow 7^-/2_1$  | 0.1173  | 0.0389| 0.9135(728) |
| $9^-/2_1 \rightarrow 7^-/2_1$  | 0.0079  | 0.0019|                   |
| $11^-/2_1 \rightarrow 7^-/2_1$ | 0.1022  |       | 0.0016 |
| $5^-/2_1 \rightarrow 7^-/2_1$  | 0.0738  | 0.0470| >0.0534 |
| $11^-/2_1 \rightarrow 9^-/2_1$ | 0.0009  | 0.0008| 0.0128 |
| $5^-/2_1 \rightarrow 3^-/2_1$  | 0.0140  | 0.0001| 0.0910 |
| $5^-/2_1 \rightarrow 9^-/2_1$  | 0.0050  | 0.0221|                   |
| $7^-/2_1 \rightarrow 7^-/2_1$  | 0.0640  | 0.0533| 0.0019 |
| $7^-/2_1 \rightarrow 9^-/2_1$  | 0.0671  | 0.0147| 0.0051 |
| $7^-/2_1 \rightarrow 5^-/2_1$  | 0.0094  | 0.0036| 0.0778 |
| $7^-/2_1 \rightarrow 9^-/2_1$  | 0.0001  | 0.0002| 0.0014 |
| $7^-/2_1 \rightarrow 7^-/2_1$  | 0.0002  | 0.0012| 0.0915 |
| $9^-/2_1 \rightarrow 9^-/2_1$  | 0.0028  | 0.0022| 0.0016 |
| $5^-/2_1 \rightarrow 7^-/2_1$  | 0.0004  | 0.0149| 0.0153 |
| $5^-/2_1 \rightarrow 7^-/2_1$  | 0.0010  | 0.0059| 0.0065 |
| $5^-/2_1 \rightarrow 3^-/2_1$  | 0.0048  | 0.0063| 0.0170 |
| $11^-/2_2 \rightarrow 7^-/2_1$ | 0.0028  |       | 0.0015 |
| $11^-/2_2 \rightarrow 9^-/2_1$ | 0.0731  | 0.0014| 0.0043 |
| $11^-/2_2 \rightarrow 9^-/2_1$ | 0.0016  | 0.0018| 0.1213 |
| $13^-/2_1 \rightarrow 9^-/2_1$ | 0.1174  |       | 0.1030 |
| $13^-/2_1 \rightarrow 11^-/2_1$| 0.0047  | 0.0000| 0.0006 |
| $15^-/2_1 \rightarrow 13^-/2_1$| 0.0012  | 0.0001| 0.0001 |

| $J_i^\rightarrow J_f^\prime$ | $^{151}$Dy B(E2) | $^{151}$Dy B(M1) | $^{153}$Dy B(E2) | $^{153}$Dy B(M1) |
|--------------------------------|------------------|------------------|------------------|------------------|
|                                | IBFM-2 | EXP.  | IBFM-2 | EXP.  | IBFM-2 | EXP.  | IBFM-2 | EXP.  |
| $7^-/2_1$                      | -0.3868 | -0.30(5)| -0.8439 | -0.945(7)| -0.1410 | -0.02(5)| -0.8907 | -0.782(6) |
| $9^-/2_1$                      | -0.6378 | 0.9640  | -0.6726 |                   |
| $3^-/2_1$                      | -0.2672 | -1.7209 | -0.2167 | -1.2199 |
| $11^-/2_1$                     | -0.6098 | 1.0877  | -0.3413 | 0.4119 |
| $5^-/2_1$                      | -0.0568 | -0.6930 | -0.0712 | -0.6369 |
| $9^-/2_2$                      | -0.3249 | 0.4675  | -0.1749 | -0.1302 |
| $7^-/2_2$                      | -0.0702 | -0.0038| -0.3029 | 0.1622 |
| $5^-/2_2$                      | -0.5281 | -0.2779| -0.1388 | -0.2006 |
TABLE XII: Comparison of $\log_{10} ft$ values in the decays $^{151}$Ho $\rightarrow$ $^{151}$Dy and $^{153}$Ho $\rightarrow$ $^{153}$Dy, the order of states corresponding to IBFM-2 calculation.

| $^{151}$Ho $\rightarrow$ $^{151}$Dy | $^{153}$Ho $\rightarrow$ $^{153}$Dy |
|----------------|----------------|
| odd-p | odd-n | log ft | Exp. | odd-p | odd-n | log ft | Exp. |
| $11^{-}/2_{1} \rightarrow 13^{-}/2_{1}$ | 7.260 | 6.200 | $11^{-}/2_{1} \rightarrow 13^{-}/2_{1}$ | 7.231 | 6.300 |
| $11^{-}/2_{1} \rightarrow 13^{-}/2_{2}$ | 8.440 | | $11^{-}/2_{1} \rightarrow 13^{-}/2_{2}$ | 7.793 |
| $11^{-}/2_{1} \rightarrow 13^{-}/2_{3}$ | 8.907 | | $11^{-}/2_{1} \rightarrow 13^{-}/2_{3}$ | 9.456 | 6.200 |
| $11^{-}/2_{1} \rightarrow 13^{-}/2_{4}$ | 9.174 | | $11^{-}/2_{1} \rightarrow 13^{-}/2_{4}$ | 10.228 |
| $11^{-}/2_{1} \rightarrow 13^{-}/2_{5}$ | 8.807 | | $11^{-}/2_{1} \rightarrow 13^{-}/2_{5}$ | 8.515 |
| $11^{-}/2_{1} \rightarrow 11^{-}/2_{1}$ | 8.019 | | $11^{-}/2_{1} \rightarrow 11^{-}/2_{1}$ | 7.961 | 6.000 |
| $11^{-}/2_{1} \rightarrow 11^{-}/2_{2}$ | 5.429 | 5.500 | $11^{-}/2_{1} \rightarrow 11^{-}/2_{2}$ | 7.210 |
| $11^{-}/2_{1} \rightarrow 11^{-}/2_{3}$ | 6.877 | | $11^{-}/2_{1} \rightarrow 11^{-}/2_{3}$ | 5.366 |
| $11^{-}/2_{1} \rightarrow 11^{-}/2_{4}$ | 7.742 | | $11^{-}/2_{1} \rightarrow 11^{-}/2_{4}$ | 9.596 |
| $11^{-}/2_{1} \rightarrow 11^{-}/2_{5}$ | 7.195 | | $11^{-}/2_{1} \rightarrow 11^{-}/2_{5}$ | 6.707 | 6.300 |
| $11^{-}/2_{1} \rightarrow 11^{-}/2_{6}$ | 6.753 | | $11^{-}/2_{1} \rightarrow 11^{-}/2_{6}$ | 7.358 | 6.300 |
| $11^{-}/2_{1} \rightarrow 9^{-}/2_{1}$ | 3.511 | 4.600 | $11^{-}/2_{1} \rightarrow 9^{-}/2_{1}$ | 3.612 | 4.700 |
| $11^{-}/2_{1} \rightarrow 9^{-}/2_{2}$ | 4.759 | 5.800 | $11^{-}/2_{1} \rightarrow 9^{-}/2_{2}$ | 7.175 | 6.500 |
| $11^{-}/2_{1} \rightarrow 9^{-}/2_{3}$ | 5.208 | 5.100 | $11^{-}/2_{1} \rightarrow 9^{-}/2_{3}$ | 6.443 |
| $11^{-}/2_{1} \rightarrow 9^{-}/2_{4}$ | 7.320 | | $11^{-}/2_{1} \rightarrow 9^{-}/2_{4}$ | 4.889 |
| $11^{-}/2_{1} \rightarrow 9^{-}/2_{5}$ | 5.506 | | $11^{-}/2_{1} \rightarrow 9^{-}/2_{5}$ | 6.570 |