Mathematical model for reliability indicators calculation of tethered UAV hybrid navigation system

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Abstract. The mathematical model for reliability indicators calculation of the hybrid navigation system containing microwave and technical vision subsystems is proposed in this paper for the first time. The proposed method is based on the translation matrix concept of solutions to the Kolmogorov equation system and it allows us to obtain the mathematical expression of availability factor, downtime ratio, and other reliability indicators. Also the presented approach allows finding the reliability indicators for the cases of jump change of transition intensities caused by external influences. Besides the analytical method can be used for investigation of hybrid navigation system transient mode functioning. The results of the numerical calculations clearly demonstrated correctness of the proposed approach.

1. Introduction
Unmanned aerial vehicles (UAVs) are widely used in various fields of economics, industry and military [1]. The disadvantage of most UAVs using is necessarily of recharging of a rechargeable battery, which is not always which does not always meet the requirements of providing uninterrupted high-quality communication. For example, in the case of placing a mobile base station or an equipment of video surveillance system on UAV it is important problem to provide long working time of it. In such cases, the so-called tethered UAVs are widely used, which receive power from a generator located at the ground station via an electric cable [1-3]. At the present time the papers describing the features of building communication systems on base of tethered UAVs, the features of power supply of such a system, the functioning principles of autonomous navigation system are presented in scientific literature [1-5]. For example, the paper [2] presents the features of building an autonomous navigation system based on radar methods. The proposed navigation method [2] allows us to accurately determine the coordinates of UAV at an altitude of 10 meters and above. But at altitudes up to 10 meters, the accurate of the navigation system is insufficient and vision-based navigation systems are more are more suitable for solving this problem [6]. So, a hybrid navigation system based on radar and vision-based navigation systems is the most effective solution. But there are some cases when such a hybrid navigation system may be in unsteady state. For example, the process of take-off or landing of an UAV leads to switching from one navigation system to another. In such cases the system is in a transient mode. During this period, the reliability indicators of the hybrid navigation system can differ significantly from the parameters in the stationary mode. Therefore, the actual problem is calculating the reliability indicators of a hybrid navigation system in steady and transient mode.
2. The statement of the problem

Let us consider the hybrid navigation system consists of the two subsystems based on different principals. The first one is the radar subsystem of the terahertz domain and the second one is the vision-based navigation subsystem. Such the system has been proposed and described recently in [7]. However, the problem of reliability of its functioning has not been studied in [7]. First of all the probability of operation of each of the subsystems depends on the height. This probability is also determined by the technical characteristics of the noted above subsystems. The main navigation system is the radar subsystem at an altitude above 10m, and the main navigation system is the technical vision subsystem at an altitude of less than 10m. It should be noted that when the UAV is raised above 10 meters, the vision-based navigation subsystem does not turn off, it is switched in a cold standby mode. Analogously, the radar navigation subsystem transits on in the reserve mode after the UAV is descended below the level of 10 meters. It is obvious that the conditions for switching on one or the other subsystem are very important both theoretical and practical problems.

The aim of this treatment is investigation of reliability indicators in transient mode which accrues after switching from one subsystem to another. Such hybrid system can be described by using the theory of Markov processes [8,9]. Figure 1 shows the transition graph of the corresponding Markov chain. In our case the Markov chain consists of the four states. The first state \( S_1(t) \) is the one of functioning of the radar navigation subsystem only [2], the second state \( S_2(t) \) is the one of switching from the radar navigation subsystem to the technical vision navigation subsystem, \( S_3(t) \) is the state of functioning only of the technical vision subsystem [5], and \( S_4(t) \) is the state of switching from the technical vision navigation subsystem to the radar navigation subsystem. The intensities of transitions \( \lambda_{ij}(t) \) are generally functions of altitude, time, and many other parameters characterizing the navigation system. A change of transition intensities is associated with a change of external weather conditions during take-off or landing of the UAV. Moreover an altitude is also function of \( t \). For the possibility of obtaining the analytical expressions in this work, a piecewise-constant approximation of the original dependencies is used. Then we can write

\[
\lambda_{ij}(h,t) = \begin{cases} 
\lambda_{ij}^{(1)}, & 0 < t < t_1 \\
\lambda_{ij}^{(2)}, & t_1 < t < t_2 \\
\ldots \\
\lambda_{ij}^{(N)}, & t_N < t < t_N 
\end{cases}
\]

(1)

Thus, first of all we must find the dependences of the state probabilities on the time \( t \) and obtain the mathematical expression of availability factor, downtime ratio, and other reliability indicators by using the state probabilities.

![Figure 1. Transition graph of a Markov stochastic process.](image)

We also intend to numerically investigate the behavior of the system with large fluctuations in altitude and the effect of small fluctuations in altitude when landing the drone on the system reliability indicators.
2.1. Translation matrix
At first let us to write the Kolmogorov equation system corresponding to the transition diagram presented in figure 1:

\[
\begin{align*}
\frac{dp_1}{dt} &= -p_1(t)\lambda_{12}(t) + p_2(t)\lambda_{21}(t) + p_4(t)\lambda_{41}(t) \\
\frac{dp_2}{dt} &= p_1(t)\lambda_{12}(t) - p_2(t)\lambda_{21}(t) + p_3(t)\lambda_{32}(t) \\
\frac{dp_3}{dt} &= p_2(t)\lambda_{23}(t) - p_3(t)\lambda_{33}(t) + p_4(t)\lambda_{43}(t) \\
\frac{dp_4}{dt} &= p_3(t)\lambda_{34}(t) - p_4(t)\lambda_{44}(t) + \lambda_{41}(t)
\end{align*}
\]  

(2)

Here \( p_i \) is the probability of the system being in state \( i \).

To find the reliability indicators in this work, it is proposed to use the method of the so-call translation matrix of solutions to the Kolmogorov system (2). This matrix \( L(t) \) relates the probabilities of the states of the system at an arbitrary time \( t \) with these probabilities at the initial time.

\[
P(t) = L(t)P(t_0).
\]  

(3)

The various methods for finding the translation matrix \( L(t) \) are described in mathematics, radio engineering and microwave theory \[7\] in detail, therefore, we do not consider this issue in detail here. It is important that the translation matrix has the same order as the Kolmogorov system. In this case it is a 4x4 matrix.

In the case of the abrupt change transition intensities, the translation matrix after the jump is found as the multiplication of the matrix of the first interval and the translation matrix of the second interval

\[
L(t) = L_2(t)L_1(t_1).
\]  

(4)

Analogously, in the case of \( N \) jumps the behavior of the system on the \( i \)-th time interval is described by the matrix

\[
L(t) = L_{N+1}(t - \sum_{i=1}^{N} t_i)\prod_{i=N}^{1} L_i(t_i).
\]  

(5)

The matrix (5) allows us to determine the state probabilities (figure 1) at any time and it can be used for obtaining the reliability parameters of the hybrid navigation system.

2.2. Reliability parameters
Since the availability factor at an arbitrary time moment \( t \) is equal to the sum of the probabilities that either the radar navigation subsystem or technical vision navigation subsystem is working, then we can write

\[
R(t) = p_1(t) + p_3(t).
\]  

And taking into account the kind of the fundamental matrix, we can write the following expression

\[
R(t) = \sum_{i=1}^{4} \left[ L_{i1}(t) + L_{i3}(t) \right] \lambda_i(t_0),
\]  

(6)

for the availability factor, where \( L_{i1}(t) \) and \( L_{i3}(t) \) are the elements of the fundamental decision matrix.

Analogously the downtime ratio at an arbitrary time moment \( t \) is equal to the sum of probabilities that both subsystems are not functioning and the system is in switching mode. Taking into account this fact we can write the expression for this parameter...
The important problem for investigation of estimation of transient time. The transient time is the time period from the beginning of the system functioning to the transition it in the stationary mode when the probabilities of system states do not depend on time. Now, the general solution of (2) can be written in the form

\[ Q(t) = \sum_{i=1}^{4} [L_{2i}(t) + L_{4i}(t)]p_i(t_0) \]  

(7)

The important problem for investigation of estimation of transient time. The transient time is the time period from the beginning of the system functioning to the transition it in the stationary mode when the probabilities of system states do not depend on time. Now, the general solution of (2) can be written in the form

\[ p_i(t) = \sum_{k=1}^{N} A_{ik} \xi_{il} \exp(\gamma_i t) \]  

(8)

As we can see from (8) the transient time is determined by \( \gamma_k \). Let us define the transient time as \( \tau_r = \frac{5}{\gamma_{k_{\min}}} \). Indeed, the probability \( p_i(t) \) changes \( e \) times for the time \( 1/\gamma_i \), and this probability changes \( 5e \) times for the time \( \tau_r \). Moreover, we have four values \( \gamma_k \), and the maximal transient time is determined by the minimal value of \( \gamma_k \).

3. Numerical results

In this section the numerical results of the hybrid navigation system reliability investigation are presented. Here we consider two cases of switching from one navigation system to another. The first one occurs during take-off or landing of UAV and the second one is the result of its height oscillation process.

3.1. The periodically jumps of transient intensities

The periodically jumps of transient intensities occurs when the UAV is on the determinate altitude but its position is changed in the result of wind or other external factor influences. For example, the case of large height oscillations is considered in figure 2. Here the altitude of UAV position can change within one meter. In other words, we are studying the fluctuations in the height of the drone.

![Figure 2. Dependencies of hybrid navigation system state probabilities.](image)

The values of transient intensities before the jump are taken as \( \lambda_{12} = 0.05, \lambda_{13} = 0, \lambda_{14} = 0, \lambda_{21} = 0.1, \lambda_{23} = 0.9, \lambda_{24} = 0, \lambda_{31} = 0, \lambda_{32} = 0, \lambda_{34} = 0.8, \lambda_{41} = 0.9, \lambda_{42} = 0, \lambda_{43} = 0.1 \). The values of transient
Intensities after the jump are $\lambda_{12} = 1$, $\lambda_{13} = 0$, $\lambda_{14} = 0$, $\lambda_{21} = 0$, $\lambda_{23} = 0.9$, $\lambda_{24} = 0$, $\lambda_{31} = 0$, $\lambda_{32} = 0$, $\lambda_{34} = 0.00002$, $\lambda_{41} = 0.9$, $\lambda_{42} = 0$, $\lambda_{43} = 0.1$. Here we consider the particular case of the eight jumps. The numerical results show that the probability of failure or another words downtime ratio of the hybrid navigation system is changed from 0.25 to 0.35, the availability factor of radar navigation subsystem is changed from 0.08 to 0.35 and the availability factor of technical vision-based navigation subsystem is changed from 0.05 to 0.65. Thus the fluctuations in the height of the drone give the significant decrease in the reliability of the system under consideration. At the same time, the probability of the functioning of the technical vision subsystem increases. The transient time in accordance with the figure 1 is 19 second.

3.2. Investigation of hybrid navigation system reliability in transient mode

The transient mode of UAV hybrid navigation system functioning occurs during its take-off or landing. During landing of UAV the radar navigation subsystem is switching on the vision-based navigation one. And vice versa during take-off the vision-based navigation subsystem of UAV is switching on the radar navigation subsystem. The dependencies of hybrid navigation system state probabilities in transient mode are presented in figure 3. Here the case of transient mode setting in the hybrid navigation system is considered.

![Figure 3. Dependencies of the hybrid navigation system state probabilities during the UAV landing.](image)

In this mode small jumps of transition intensities occur until the system is in stationary mode. For example we consider the transient mode for following transition intensities before the jump $\lambda_{12} = 0.05$, $\lambda_{13} = 0$, $\lambda_{14} = 0$, $\lambda_{21} = 0.01$, $\lambda_{23} = 0.9$, $\lambda_{24} = 0$, $\lambda_{31} = 0$, $\lambda_{32} = 0.08$, $\lambda_{34} = 0.09$, $\lambda_{42} = 0$, $\lambda_{43} = 0.1$ and after the jump $\lambda_{12} = 0.0001$, $\lambda_{13} = 0$, $\lambda_{14} = 0$, $\lambda_{21} = 0$, $\lambda_{23} = 1$, $\lambda_{24} = 0$, $\lambda_{31} = 0$, $\lambda_{32} = 0$, $\lambda_{34} = 0.00002$, $\lambda_{41} = 0.00001$, $\lambda_{42} = 0$, $\lambda_{43} = 0.99999$. The results show that the probability of radar subsystem functioning is gradually decreasing from 1 to 0.5 and the probability of vision-based navigation subsystem functioning is gradually increasing from 0 to 0.23. The transient time is approximately 20 second.

4. Conclusion

Accurate analytical investigation of tethered UAV hybrid navigation system reliability is presented in this paper for the first time. The consider navigation system consists of radar subsystem and vision-based navigation subsystem. The mathematical method used for investigation is based on the
translation matrix of the Kolmogorov equation system and it allows us to obtain the mathematical expression for the availability factor and for the downtime ratio for cases of abruptly changing of transition intensities. Besides the method can be used for investigation of transient mode of the UAV hybrid navigation system reliability. The numerical calculations clearly demonstrate the proposed approach.

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