Analysis of "working together" based on momentum conservation

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Abstract. This paper systematically analyzes and evaluates the team players, concentric drum face, flexible rope and volleyball in the team game system of "working together", and studies a series of problems including the optimal strategy and error adjustment of the team game. By looking for the mechanical connection of the system composed of players, concentric drum surface, flexible rope and volleyball, using the conservation of momentum theorem and differential equation, this paper gives an accurate mathematical description of the movement of each part of the system, and concludes that the necessary condition of the optimal strategy under ideal conditions is to make the forced vibration of the system reach resonance, that is, to keep the period consistent. Considering that the size and time of the players' exertion will have an internal impact on the times of hitting the ball, this paper uses the analytic hierarchy process to objectively evaluate the relative weight of the two effects, and uses the weighted method to establish a comprehensive evaluation function to solve the optimal solution of the binary objective decision-making problem. In order to ensure the accuracy of the model, the influence of air resistance on the system is evaluated by hydrodynamics.

Keywords: momentum conservation, analytic hierarchy process, fluid mechanics.

1. Introduction

Concentric drum is an expansion project to test team cooperation. Its props are a double-sided drum with cowhide. There are several flexible ropes with equal length and evenly distributed along the circumference in the drum body. When the ball falls freely from a certain height towards the center of the drum, the game begins. Players need to work together to operate the drum, which is in the initial state of horizontal static, to bump the ball rhythmically and try their best to maximize the number of strokes. The players can’t touch the rope or other position of the drum in the process of hitting the ball.

2. Problem analysis

This paper attempts to establish a mathematical model to describe the problem, and design algorithm to solve the model.

In the ideal state, everyone can accurately control the direction, timing and strength of the force. This paper discusses the best cooperation strategy of the team under this ideal condition, and gives the height of hitting the ball under this strategy.
In fact, the research of concentric drum cooperation strategy is to make a reasonable stress analysis of the concentric drum system, establish a mechanical model and apply it to the mechanical parameters of each individual, such as the acceleration of volleyball, the displacement of the drum surface and the force application time of the players. The core of the model is how to describe the motion of the "concentric drum" system.

Under the given basic constraints, the additional condition of "ideal team member" is added, that is, each team member can accurately control the direction, timing and strength of the force. It is necessary to analyze the overall and local forces respectively according to the mechanical system, and establish the theoretical kinematic equation description system for solving the system through the reasonable selection of micro elements.

In the whole sports system, drum and players are connected by rigid constraints, while drum and volleyball are connected by collision model. Since the "ideal conditions" are limited to the players, it is necessary to analyze the irresistible factors such as air resistance.

3. Theoretical basis
Theoretical mechanics has the following basic axioms:

3.1. Conservation of momentum:
When the interaction force between objects is much greater than the external force on the system and the interaction time is very short, the momentum of the system is considered to be conserved.

3.2. Micro element thought:
When the effect of an object in an infinitesimal is far less than the total process, the state in a infinitesimal can be considered constant.

3.3. Forced vibration:
The forced vibration of the object is the most intense at resonance, that is, the gain of the system takes the maximum value.

4. Differential equation model of motion

4.1. Condition demonstration
Some of the conditions in the problem have not been proved mathematically.

Conclusion 1: the air resistance of volleyball free fall can be ignored.

The motion of anybody in nonvacuum will be affected by air resistance, and the resistance is usually a complex variable force, so it can be simplified as much as possible in the scope of theoretical mechanics to make it solvable. When the speed of the object is not too high, it can be considered that the resistance of the object in the air is linear with the speed. The drag coefficient is related to the size and shape of the object, where $\rho$ is the density of the medium and $S$ is the middle section of the object. Then the following equations are satisfied when the object falls freely

$$\begin{cases} f = k \ddot{x} \\ k = ps \xi \\ m_1 g - f = m_1 \ddot{x} \end{cases}$$

(1)

The constants meet the requirements of Table 1
Table 1. Constant reference table

| SI | C  | ρ   | s     |
|----|----|-----|-------|
| Constant name | Air resistance coefficient | Dry air density | Center area |
| Constant number | 0.30 | 1.2 | 0.0314 |

In this problem, the maximum velocity of the ball falling from the height of 0.4m does not exceed the maximum velocity without resistance of 0.2856m/s, so the maximum air resistance does not exceed \( f_{max} \approx 0.0032 \text{(N)} \), so \( f_{max} \ll G \). Condition one is tenable.

Conclusion 2: if volleyball wants to be stable at a fixed height, the collision between drum and volleyball must be periodic.

The vibration of the system composed of drum and volleyball is forced vibration. It can be seen from the amplitude frequency curve that in the case of small damping, when the excitation frequency is close to the natural frequency, the amplitude increases sharply, as shown in Figure 1. the forced vibration resonance axiom given in the model preparation stage is combined with the frequency amplitude curve. If and only if the volleyball motion frequency is equal to the drum motion frequency, the system gain is the maximum.

![Amplitude curve](image)

**Figure 1. Amplitude curve**

4.2. Model establishment

In order to achieve the maximum gain of the system, the speed of the ball reaching the collision point must be the same each time. Under the condition of considering only the gravity field, the following equations are satisfied by volleyball and drum

\[
\begin{align*}
\dot{v}_1 &= \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \\
\dot{v}_2 &= \frac{m_2 - m_1}{m_1 + m_2} v_1 + \frac{2m_1}{m_1 + m_2} v_2 \\
v_1 &= -v_2
\end{align*}
\]  

(2)

The results are shown in formula (3)

\[
v_2 = -0.075v_1
\]

(3)
In concentric drum competition, players usually pull the rope in a certain horizontal plane. The action process is shown in Fig. 2 and Fig. 3.

**Figure 2.** Tension diagram (front view)

**Figure 3.** Tension diagram (top view)

AB is the horizontal plane where the players pull the rope $p2p1$ and $p2 \ 'p1$ 'drum surface at different times. Note that the starting position of $<BAP$ is $\theta$, $p2p1$ and $p2 \ 'p1$ ' are the origin of x-axis. The following equations can be constructed:

\[
\begin{align*}
\sin (\theta) &= \frac{h - x}{t_0} \\
F_x \sin(\theta) &= m_2 \ddot{x} \\
F_s &= NF_0 \\
\int_0^r \ddot{x} + \frac{v_2 \ddot{e}}{2} &= H \\
\int_0^r \dddot{x} &= v_2 \\
v_1 &= \dot{e} 
\end{align*}
\]

(4)

4.3. **Model solving**

As the ideal condition given in the question, in theory, as long as the equation (4) is satisfied, the players can achieve infinite times of batting, but although they can accurately control the timing and size of the force, their physical strength is not unlimited. Therefore, the linear comprehensive weighting method is used to combine multiple indexes into a comprehensive evaluation index to accurately examine the physical consumption brought by the strategy to the players. The linear comprehensive evaluation
function is as follows, in which the force duration $T$ and $F$ are obtained after dimensionless processing, and the linear weight $\omega_i$ is obtained by analytic hierarchy process

\[
\begin{align*}
\min \left\{ y = \sum_{j=1}^{2} \omega_j x_j \right\} \\
x_2 = \frac{F}{\omega_1} \\
\omega_1 = 0.7345 \\
\omega_2 = 0.2655
\end{align*}
\]

The results are as follows:

\[
\begin{align*}
F &= 79.8371(N), \quad t = 0.2887(s), \quad T = 2nt
\end{align*}
\]

Every $2T$, the drum surface will fall from the impact point to the lowest point, and then rise from the lowest point to the impact point. The results are shown in Figure 4

![Figure 4. Periodicity of displacement with time](image)

**References**

[1] Department of theoretical mechanics, Harbin Institute of technology. Theoretical mechanics [i]. Beijing: Higher Education Press, 2016.

[2] Wang Lihua. The influence of air resistance on free fall [g]. Journal of Cangzhou Teachers College, 2000, (2).

[3] Han Zhonggeng. Mathematical modeling method and its application [M]. Beijing: Higher Education Press, 2016.