Observation of spin-1 tunneling on a quantum computer

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Abstract Spin-1 tunneling and splitting of energy level as result of tunneling are observed explicitly on IBM’s quantum computer, ibmq-bogota. The spin-1 is realized with two spins-1/2. We detect oscillations of spin-1 between opposite directions in the result of tunneling on the basis of studies of time dependence of the mean value of z-component of spin-1 on the quantum device. The energy level splitting is observed quantifying on the IBM’s quantum computer the eigenvalues of Hamiltonian which describes the spin tunneling.

1 Introduction

Quantum computers are natural devises for simulating quantum systems [1–3]. Quantum bit in native way represents spin-1/2. Therefore quantum spin-1/2 systems or in general systems with binary degrees of freedom are the most suitable for simulating them with quantum processor [4–14].

Quantum spin tunneling is phenomena where single spin tunnels between two opposite directions. This leads to degeneracy of energy levels related with opposite states which is called quantum spin tunneling splitting (see, for instance, [15, 16]). Experimental observation of quantum tunneling of the magnetization of cluster with \( S = 10 \) as reported in [17]. Latter in [18, 19] a direct measurement of the quantum tunneling splitting energy of the spin \( S = 1 \) was done. Note that quantum spin tunneling splitting for zero field is only possible for integer spins. The smallest spin for which this phenomena can be observed is \( S = 1 \).

In this paper we simulate phenomena spin-1 quantum tunneling on quantum computer. We observe explicitly spin-1 tunneling and splitting of energy level as result of tunneling. The spin tunneling is detected on the basis of studies of evolution of mean value of z-component of spin-1. The splitting of energy level is observed detecting energy levels of the Hamiltonian that describes single-spin tunneling on a quantum computer. The studies are done using the method of detection of the energy levels of spin system on a quantum computer with probe spin evolution proposed in [9, 10].

The paper is organized as follows. In Sect. 2 the spin-1 tunneling is considered and the way to detect it on a quantum device is presented. In Sect. 3 the spin tunneling and the energy level splitting in the result of tunneling are detected on IBM’s quantum computer. Conclusions are presented in Sect. 4.

2 Spin-1 tunneling and its studies on a quantum computer

The Hamiltonian that describes single-spin tunneling reads

\[
H = DS_z^2 + \gamma(S_z^2 - S_x^2),
\]

here \( S_{\alpha} \) are spin-1 operators, \( D \) is the axial constant which determines the magnetic anisotropy, constant \( \gamma \) is responsible for single-spin tunneling effect (see, for instance, [20]).

Energy levels and corresponding eigenstates of Hamiltonian (1) are well known. They are as follows

\[
E = 0, \quad |\psi\rangle = |0\rangle,
\]

\[
E = E_{\pm} = D \pm |\gamma|, \quad |\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm |−1\rangle).
\]

Tunneling leads to splitting of energy level, it reads \( \Delta = 2|\gamma| \). States \( |−1\rangle, |0\rangle, |1\rangle \) are eigenstates of \( S_z \) for spin-1.
The processes of tunneling can be seen explicitly studying dynamical properties of spin-1. In relation with this it is worth mentioning paper [21] where dynamical problems of spin-1 were considered to study quantum brachistochrone problem.

Let in the initial time $t = 0$ the spin is in the state $|1\rangle$ and is positively directed along z-axis, $\langle S_z \rangle_{t=0} = 1$. One can find that the evolution of the state vector reads

$$|\psi(t)\rangle = e^{-i\frac{Ht}{\hbar}}|1\rangle = \cos \omega t|1\rangle - i \sin \omega t|-1\rangle,$$

(4)

where $\omega = \gamma/\hbar$. From (4) we have, that in result of tunneling the spin oscillates between two opposite directions described by the state vectors $|1\rangle$, $|-1\rangle$. These oscillations are reflected in the time dependence of the mean value of z-component of spin

$$\langle S_z(t)\rangle = \langle \psi(t)|S_z|\psi(t)\rangle = \cos 2\omega t,$$

(5)

and can be detected on a quantum device. Here $2\omega$ is related with the energy splitting, namely $2\omega = \Delta/\hbar$.

Spin-1 can be realized with two spins-1/2 (see [22]). The operator of spin-1 can be represented as a sum of two spin-1/2 operators as follows

$$S_\alpha = \frac{1}{2}(\sigma_1^\alpha + \sigma_2^\alpha),$$

(6)

where $\sigma_\alpha^\alpha$ are Pauli operators, $\alpha = (x, y, z)$. For spin-1 the eigenvalue of $S^2$ is $j(j + 1) = 2$, ($j = 1$). In order to satisfy this relation the action of spin-1/2 operators has to be restricted on subspace spanned by vectors

$$|00\rangle \equiv |1\rangle, \quad |11\rangle \equiv |-1\rangle, \quad \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \equiv |0\rangle,$$

(7)

(see [22]). Note that singlet state of two spins-1/2 is annihilated by operators (6) and does not belong to this subspace. Representation of spin-1 by two spins-1/2 allows us to model and study spin-1 systems on a quantum computer, in particular to examine quantum spin-1 tunneling with quantum calculations.

In the next section we present results of simulation of spin-1 tunneling on the IBM’s quantum computer.

3 Detection of spin-1 tunneling on IBM’s quantum computer

Using representation for spin-1 (6) we rewrite Hamiltonian (1) as follows

$$H = \frac{D}{2}(1 + \sigma_1^x \sigma_2^x) + \frac{\gamma}{2}(\sigma_1^y \sigma_2^y - \sigma_1^z \sigma_2^z).$$

(8)

Expression (8) corresponds to Hamiltonian of two spins-1/2 with anisotropic Heisenberg interaction. Evolution operator of this system can be realized on a quantum computer. Due to commutation relation $[\sigma_1^i, \sigma_2^i] = 0$, the evolution operator can be factorized. It reads

$$U(t) = e^{-iHt} = e^{-i\frac{D}{2}t}e^{-i\frac{\gamma}{2}t\sigma_1^y \sigma_2^y}e^{-i\frac{\gamma}{2}t\sigma_1^z \sigma_2^z}e^{-i\frac{\gamma}{2}t\sigma_1^z \sigma_2^z}.$$

(9)

Here for convenience we put $\hbar = 1$.

In order to show quantum tunneling of spin-1 explicitly, we detect evolution of the mean value

$$\langle S_z(t)\rangle = \frac{1}{2}(\langle \sigma_1^z(t)\rangle + \langle \sigma_2^z(t)\rangle),$$

(10)

governed by (1) on a quantum computer. Two-qubit quantum protocol for this studies is presented on Fig. 1.

In the quantum protocol we consider the initial state of spin-1 to be $|1\rangle$, that corresponds to the state of two spins-1/2 (qubits) as $|00\rangle$. Also, to construct protocol Fig. 1 we take into account that with the exactness to the total phase the operator $\exp(-i\gamma\alpha_0^\alpha_0 \sigma_1^z/2)$ can be represented as $\text{CNOT}_{01} H_0 p_0(\gamma\alpha) H_0 \text{CNOT}_{01}$, where $H_0$ is the Hadamard gate acting on $q[i]$, $\text{CNOT}_{ij}$ is the controlled NOT-gate acting on qubit $q[i]$ as on the controlled and on the qubit $q[j]$ as on the target. Operator $\exp(i\gamma\alpha_0^\alpha_0 \sigma_1^z/2)$ can be rewritten as $\text{CNOT}_{01} C\text{Z}_{01} H_0 p_0(\gamma\alpha) H_0 \text{CNOT}_{01}$, where $\text{CNOT}_{ij}$ is the controlled Z-gate acting on qubits $q[0]$, $q[1]$. For $\exp(-iD\alpha_0^\alpha_0 \sigma_1^z/2)$ we have representation $\text{CNOT}_{01} R_\alpha Z_1(D\alpha) \text{CNOT}_{01}$, where $R_\alpha Z_1(D\alpha)$ is $Z$-rotation gate that acts on $q[1]$. In quantum protocol Fig. 1 we also take into account that $\text{CNOT}_{ij}$

governed by Hamiltonian (1).

![Fig. 1](image)

$q[1] |0\rangle \rightarrow \text{H} \rightarrow \text{H} \rightarrow \text{H} \rightarrow \text{P}_{\text{D}} \rightarrow \text{Z}[\alpha] \rightarrow |\rangle$

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4 Quantifying of the energy spectrum on a quantum computer

To find the energy spectrum of spin-1 Hamiltonian (1) on a quantum device and detect splitting of the energy levels of spin systems proposed in our papers [9, 10]. In [9] we presented a method for detecting the energy levels of a spin system which is based on the studies of evolution of the mean value of operator of a physical quantity anticommuting with the Hamiltonian of the system. Because an operator anticommuting with Hamiltonian does not exist for all spin systems, in [10] we generalized the proposed method of detecting energy levels to the case of arbitrary spin Hamiltonians. In [10] it was proposed to build the total Hamiltonian adding probe (ancilla) spin-1/2 and to detect splitting of the energy level as result of tunneling we use the method of quantifying the energy levels of spin systems proposed in our papers [9, 10]. In [9] we represent a method for detecting the energy levels of a spin system which is based on the studies of evolution of the mean value of operator of a physical quantity anticommuting with the Hamiltonian of the system. Because an operator anticommuting with Hamiltonian does not exist for all spin systems, in [10] we generalized the proposed method of detecting energy levels to the case of arbitrary spin Hamiltonians. In [10] it was proposed to build the total Hamiltonian adding probe (ancilla) spin-1/2 and to detect the energy levels on the basis of studies of the probe spin evolution.

In this section we apply the method of detection of the energy levels of spin systems presented in [10] to Hamiltonian (1) describing spin-1 tunneling problem. So, let us construct the total Hamiltonian as follows

\[ H_T = \sigma_0^z (H + C), \]  

(11)

here \( H \) is given by (1), \( \sigma_0^z \) is \( z \)-component of the Pauli matrix corresponding to the probe spin-1/2. Constant \( C \) has to be chosen to shift the energy levels of the Hamiltonian (1) to the positive or negative ones. The energy levels of \( H \) for \( D < 0 \) and \( |D| > |\gamma| \) are nonpositive. Therefore we can put \( C = 0 \). In this case the energy levels of \( H_T \) are related with the energy levels of the Hamiltonian (1) as \( E_T = \pm E \), where we use notation \( E_T \) for the energy levels of \( H_T \) and notation \( E \) for the energy levels of \( H \).

Operator \( \sigma_0^z \) of the probe spin anticommutes with the total Hamiltonian. Let us study its evolution and detect the energy levels of \( H_T \) and as result the energy levels of \( H \) as was proposed in [9]. We consider the initial state as

\[ |\psi_0\rangle = |+\rangle |\chi, \chi\rangle, \]  

(12)

where \(|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\) is the initial state of the probe spin. State \(|\chi, \chi\rangle\) is the initial state of two spins-1/2 representing spin-1 with Hamiltonian (1), \(|\chi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}|1\rangle)\). The state \(|\chi, \chi\rangle\) belongs on the subspace (7). Note that state \(|\chi, \chi\rangle\) with \( \phi \neq 0 \) includes all eigenstates of \( H \). Calculating the evolution of the mean value of \( \sigma_0^z \), we find

\[ \langle \sigma_0^z (t) \rangle = \frac{1}{2} (\cos^2 \varphi \cos 2\omega_t t + \sin^2 \varphi \cos 2\omega_{-} t + 1). \]  

(13)

Then the Fourier transformation of the time evolution of the mean value \( \langle \sigma_0^z (t) \rangle \) has \( \delta \)-peaks at the frequencies related with the energy levels of \( H_T \) and \( H \). We obtain

\[ \sigma_0^z (\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \langle \sigma_0^z (t) \rangle e^{i\omega t} = \frac{1}{4} \cos^2 \varphi (\delta(\omega - 2\omega_t) + \delta(\omega + 2\omega_{-})), \]
where we use notations \( \omega_+ = E_+/\hbar \) and \( \omega_- = E_-/\hbar \) for \( h = 1 \) \( \omega_\pm = E_\pm \). To find the frequencies on a quantum device the mean values of physical quantity \( \langle \sigma^z_\omega \rangle \) have to be calculated at fixed moments of time \( t = \tau n \), where \( n = N, \, -N + 1, \ldots, N - 1 \). The delta-peaks are more thin and have higher amplitude for larger \( T = N \tau \). Therefore for more accurate results for the frequencies reducing of the time step \( \tau \) and increasing time duration \( T \) are needed [9].

To study \( \langle \sigma^z_\omega \rangle \) on a quantum device we construct quantum protocol presented on Fig. 3.

In protocol Fig. 3 the Hadamard gates and the phase shift gates are applied to prepare the initial state (12). We have \( |\psi_0\rangle = \tilde{P}_1(\varphi)|\tilde{P}_2(\varphi)\rangle|H_0\rangle|H_1\rangle|H_2\rangle|000\rangle \). To realize the operator of evolution with Hamiltonian (11) we take into account that with the exactness to the total phase can be represented as

\[
\exp(-i \gamma_\varphi \sigma^y_0 \sigma^z_\omega /2) = \exp\left(\frac{\gamma_\varphi}{2} \sigma^z_0 \sigma^y \right) = \exp\left(\frac{i \gamma_\varphi}{2} \sigma^y_0 \sigma^z \right),
\]

therefore operators \( \exp(-i \gamma_\varphi \sigma^z_0 \sigma^z_\omega /2) / \exp(i \gamma_\varphi \sigma^y_0 \sigma^z_\omega /2) \) with the exactness to the total phase can be represented as

\[
R_Y\left(\frac{\pi}{2}\right) R_Y\left(\frac{\pi}{2}\right) CNOT_{01} CNOT_{12} R_Z(\gamma_\varphi) CNOT_{12} CNOT_{01} R_Y\left(-\frac{\pi}{2}\right) R_Y\left(-\frac{\pi}{2}\right),
\]

\[
R_X\left(-\frac{\pi}{2}\right) R_X\left(-\frac{\pi}{2}\right) CNOT_{01} CNOT_{12} R_Z(-\gamma_\varphi) CNOT_{12} CNOT_{01} R_X\left(\frac{\pi}{2}\right) R_X\left(\frac{\pi}{2}\right),
\]

respectively. Here \( R_X(\pi/2), \, R_Y(\pi/2) \) are \( X \) - and \( Y \)-rotation gates acting on \( q[i] \). Finally, to find the mean value of \( \sigma^z_\omega \) operator we apply \( R_Y(\pi/2) \) because operator \( \sigma^z_\omega \) can be represented as \( \sigma^z_0 = e^{i \frac{x}{2} \sigma^z_0} e^{-i \frac{x}{2} \sigma^z_0} \). So, to calculate the mean value of \( \sigma^z_0 \) on the basis of the results of measurement in the standard basis the state of qubit \( q[0] \) has to be rotated around the \( Y \) axis.

The protocol Fig. 3 was realized on IBM’s quantum computer ibmq-bogota for parameter \( \alpha \) changing from \(-8\pi \) to \( 8\pi \) with the step \( \pi/24 \) specifying the number of shots to be 1024. We choused \( \varphi = \pi/4 \). In this case terms with \( \omega_+ \) and \( \omega_- \) in (14) have equal contribution. To detect the energy level splitting we performed quantum calculations for \( D = -2 \) and different parameters \( \gamma \) possessing the following values 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75. The results of quantum calculations are shown on Figs. 4, 5. The results of calculations of \( \sigma^z_\omega \) on qasm-simulator with depolarizing noise model with error probabilities for 1-qubit gates 0.001 and 2-qubit gates 0.01 (marked by crosses), and on qasm-simulator with readout errors 0.05 (marked by circles) are presented on Fig. 4b. Note that the gate errors have larger effect on the results of calculations of \( \sigma^z_\omega \) than the readout errors. This is because the quantum protocol Fig. 3 is gate expensive and requires the measurement of the state of only one qubit. The shift down of the results of quantum calculations on Fig. 4b can be related with some systematic errors. It is worth noting that even in this case we obtain correct result for the energy levels. This is because this shift effects only on the magnitude of the peak at the zero point.

Note that because of errors of calculations on the quantum device we have also obtained the imaginary part of \( \langle \sigma^z_\omega \rangle \) that looks as a noise. Therefore, on Figs. 4, 5 we plot the real part of the mean value \( \langle \sigma^z_\omega \rangle \).

Delta peaks of \( \text{Re} \langle \sigma^z_\omega \rangle \) at \( \omega = 0 \), \( \omega_- = -4.5 \), \( \omega_+ = -3.5 \) correspond to the energies \( E = 0 \), \( E_- = -2.25 \), \( E_+ = -1.75 \) of (1) (see Fig. 4b). Similarly detecting delta peaks at \( \omega = 0 \), \( \omega_- = -5 \), \( \omega_+ = -3 \) we obtain \( E = 0 \), \( E_- = -2.5 \), \( E_+ = -1.5 \) (see Fig. 5a), delta-peaks at \( \omega = 0 \), \( \omega_- = -5.5 \), \( \omega_+ = -2.5 \) correspond to \( E = 0 \), \( E_- = -2.75 \), \( E_+ = -1.75 \) (see Fig. 5b). Obtaining delta peaks at \( \omega = 0 \), \( \omega_- = -6 \), \( \omega_+ = -2 \) we quantify \( E = 0 \), \( E_- = -3 \), \( E_+ = -1 \) Fig. 5c, and delta peaks at \( \omega = 0 \), \( \omega_- = -6.5 \), \( \omega_+ = -1.5 \) give us energy levels \( E = 0 \), \( E_- = -3.25 \), \( E_+ = -0.75 \) (see Fig. 5d). So, we detect the quantum tunneling splitting energy of the spin \( S = 1 \) on IBM’s quantum computer ibmq-bogota. The results of quantum calculations correspond to the analytical ones.

5 Errors in the detection of the energy levels related with the finite step and duration of time

Let us estimate errors in the detection of the energy levels related with the finite step \( \tau \) and duration of time \( T = N \tau \) in calculation of the mean value \( \langle \sigma^z_\omega \rangle \). In a general case of spin system we have

\[
\sigma(\omega) = \sum_j \frac{1}{2\pi} \sum_{n=-N}^N \tau e^{-i(\omega - 2\omega_\tau)n} r_n
\]

\[\odot\text{Springer}\]
Evolution of the mean value of $\sigma^x_0$ in the case of $D = -2$ and $\gamma = 0.25$ detected on ibmq-bogota (marked by blue crosses on plot a) on qasm-simulator with depolarizing noise model (marked by green crosses on plot b), on qasm-simulator with redout error (marked by circles on plot b) and analytical results (marked by line on plots a and b). The real part of $\sigma^x_0(\omega)$ found on the basis of results of quantum calculations.

\[ \sigma^x_0(\omega) = \sum_j g_j \frac{1}{2\pi} \tau \left( 1 + 2 \cos((\omega - 2\omega_j)(T + \tau)/2) \right) \frac{\sin((\omega - 2\omega_j)T/2)}{\sin((\omega - 2\omega_j)\tau/2)}, \]

(19)

here $g_j$ are amplitudes corresponding to frequencies $\omega_j$, see Eq. (7) in [10]. For small errors we can consider $\delta\omega\tau$ related with finite step $\tau$ independently of $\delta\omega N$ related with the duration of time $T = N\tau$.

For calculation $\delta\omega\tau$ we consider fixed small $\tau$ and $N \to \infty$. In this limit (19) reads

\[ \sigma^x_0(\omega) = \frac{\tau}{2\pi} \sum_j g_j \sum_j g_j \delta(\omega - 2\omega_j^T). \]

(20)

We see that for the finite step $\tau$ in the limit $N \to \infty$ (in this case duration of time tends to infinity) $\sigma^x_0(\omega)$ has $\delta$-peaks and $\tau$ leads only to a constant shift that does not effect the peaks. Thus in this limit $\delta\omega\tau = 0$.

Now let us consider $\delta\omega N$. In this case the duration of time $T$ is large but fixed and $\tau \to 0$. In this limit equation (19) is reduced to

\[ \sigma^x_T(\omega) = \sum_j f_j(\omega), \quad f_j(\omega) = \frac{g_j}{\pi} \frac{\sin((\omega - 2\omega_j^T)T)}{(\omega - 2\omega_j^T)}, \]

(21)

see equation (8) in [10]. Each of these functions $f_j(\omega)$ posses maximum value at the point $\omega = 2\omega_j^T$. But in result of overlaping of different functions $f_j(\omega)$ the sum of them $f(\omega)$ possesses maximal value at shifted points. Let us consider the shift of point $\omega = 2\omega_k^T$. For this we develop $f(\omega)$ in series at point $\omega_k^T$:

\[ f(\omega) = \sum_j f(2\omega_k^T) + \frac{1}{2} f''(2\omega_k^T)(\omega - 2\omega_k^T)^2 + \sum_{j \neq k} f_j(2\omega_k^T)(\omega - 2\omega_k^T), \]

(22)

here we take into account that $f''(2\omega_k^T) = 0$ and for $f_j(\omega)$ ($j \neq k$) we can take into account only the first linear term.

To find the maximum points we consider $f'(\omega) = 0$ that explicitly reads

\[ f''(2\omega_k^T)(\omega - 2\omega_k^T) + \sum_{j \neq k} f_j'(2\omega_k^T) = 0. \]

(23)
Fig. 5 The real part of $\sigma^0_0(\omega)$ obtained on the basis of results of quantum calculations for $\langle \sigma^0_0(t) \rangle$ obtained on ibmq-bogota for $D = -2$ and $\gamma = 0.5$, $b \gamma = 0.75$, $c \gamma = 1$, $d \gamma = 1.25$

Solution of this equation is the following

$$\omega = 2\omega_k^j - \frac{1}{f''_k(2\omega_T^j)} \sum_{j \neq k} f'_j(2\omega_T^j) = 2(\omega_k^j + \delta \omega_k^j),$$

(24)

where

$$\delta \omega_k = \frac{1}{2f''_k(2\omega_T^j)} \sum_{j \neq k} f'_j(2\omega_T^j)$$

(25)

is the correction to the energy level $\omega_k$. For our case we find

$$f'_j(\omega) = \frac{g_j}{\pi} \left( T \frac{\cos((\omega - 2\omega_T^j)T)}{(\omega - 2\omega_T^j)} - \sin((\omega - 2\omega_T^j)T) \right),$$

(26)

$$f''_j(\omega) = \frac{g_j}{\pi} \left( -T^2 \frac{\sin((\omega - 2\omega_T^j)T)}{(\omega - 2\omega_T^j)} - 2T \frac{\cos((\omega - 2\omega_T^j)T)}{(\omega - 2\omega_T^j)^2} + 2 \frac{\sin((\omega - 2\omega_T^j)T)}{(\omega - 2\omega_T^j)^3} \right).$$

(27)

For large $T$, considering the leading term over $T$, we can write

$$f'_j(2\omega_T^j) = \frac{g_j}{\pi} T \frac{\cos(2(\omega_T^j - \omega_T^j)T)}{2(\omega_T^j - \omega_T^j)},$$

(28)

$$f''_j(2\omega_T^j) = -\frac{g_j}{\pi} T^3.$$

(29)
Then according to (25) the frequency $\omega_k$ is shifted by the value

$$\delta \omega_T^k = \frac{1}{2g_k T^2} \sum_{j \neq k} \frac{g_j \cos(2(\omega_k^T - \omega_j^T)T)}{2(\omega_k^T - \omega_j^T)}.$$  

(30)

Thus $\delta \omega_T^k$ is changed in the region

$$-|\delta \omega_T^k|_{\text{max}} \leq \delta \omega_T^k \leq |\delta \omega_T^k|_{\text{max}},$$

(31)

where

$$|\delta \omega_T^k|_{\text{max}} = \frac{1}{2|g_k|T^2} \sum_{j \neq k} \frac{|g_j|}{2|\omega_k^T - \omega_j^T|}.$$  

(32)

Therefore, the errors in estimation of the energy levels are in the region given by (31). The maximal possible error is given by (32) and is proportional to $1/T^2$. Thus the error is less when the time of duration is larger. Note, that in fact errors can be less because of possible canceling of the terms with different signs in (30).

Using this result we calculate maximal errors in estimation of the energy levels for the spin-1 system. For energy levels presented on Fig. 4b (where $h = 1$, $T = 8\pi$) we find that for $E = 0$ (with $g=1/2$) the maximal error is $0.0007$, for $E = -1.75$ (with $g=1/2$) the maximal error is $0.0076$, for $E = -3.5$ (with $g=1/8$) the maximal error is $0.0067$. Therefore the errors of the detection of the energy levels related with the finite step and duration of time are very small.

6 Conclusions

Quantum spin-1 tunneling has been observed on IBM’s quantum computer, ibmq-bogota. To model spin-1 on the quantum devise we have used its representation by two spins-1/2.

We have proposed to detect spin-1 tunneling studying time evolution of mean value of $z$-component of spin-1 operator $\langle S_z^i(t) \rangle$. It has been shown that the time dependence of $\langle S_z^i(t) \rangle$ reflects oscillations of spin-1 between two opposite directions in the result of tunneling.

The evolution of mean value of $z$-component of spin-1 operator governed by Hamiltonian describing spin-1 tunneling has been found on IBM’s quantum computer (see Fig. 2). For this purpose quantum protocol Fig. 1 has been realized on the ibmq-bogota. As a result, the spin-1 tunneling between two opposite directions described by the state vectors $|1\rangle$, $|-1\rangle$ has been observed on a quantum device.

The energy levels of Hamiltonian describing spin-1 tunneling have been quantified on the quantum computer on the basis of studies of a probe spin evolution Figs. 4, 5. The splitting of the energy level of spin-1 has been detected (see Fig. 5). The results of quantum calculations are in agreement with the theoretical ones.

We would like to stress that the method for detecting of the energy levels of a spin system proposed in [9] gives correct results even in the case of great quantum errors (see, for instance, Fig. 4 in the present paper and Fig. 10 in the paper [9]). This is an important advantage of the method.

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