Reheating and leptogenesis in a SUGRA inspired brane inflation

Sayantan Choudhury\textsuperscript{1}\textsuperscript{*} and Supratik Pal\textsuperscript{1,2}\textsuperscript{†}
\textsuperscript{1}Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B.T. Road, Kolkata 700 108, India
\textsuperscript{2}Bethe Center for Theoretical Physics and Physikalisches Institut der Universität Bonn, Nussallee 12, 53115 Bonn, Germany

We have studied extensively phenomenological implications in a specific model of brane inflation driven by background supergravity\textsuperscript{[1]}, via thermal history of the universe and leptogenesis pertaining to the particle physics phenomenology of the early universe. Using the one loop corrected inflationary potential we have investigated for the analytical expression as well as the numerical estimation for brane reheating temperature for standard model particles. This results in some novel features of reheating from this type of inflation which have serious implications in the production of heavy Majorana neutrinos needed for leptogenesis through the reheating temperature. We have also derived the expressions for the gravitino abundance during reheating and radiation dominated era. We have further estimated different parameters at the epoch of phase transition and revealed their salient features. At the end we have explicitly given an estimate of the amount of CP violation through the effective CP phase which is related to baryon asymmetry as well as gravitino dark matter abundance.

I. INTRODUCTION

It is now well accepted that the post big bang universe passed through different phases having two-fold significance – phenomenological and cosmological. One of the significant phases, namely, reheating plays the pivotal role in explaining production of different particles from inflaton/ vacuum energy. As we look back in time reheating was completed within the first second (and probably much earlier) after the big bang. At that time nucleosynthesis, or the formation of light nuclei occurred. Particle physicists as well as cosmologists have a clear picture of this hot big bang phase because ordinary matter and radiation were driving it and also the physical processes that characterize it involve terrestrial physics. On the other hand the mysterious force that drives the inflationary phase is conventionally described by a scalar field, named inflaton which oscillates near the minimum of its effective potential and produces elementary particles. These particles interact with each other and eventually they come to a state of thermal equilibrium at some arbitrary temperature $T$. This process completes when all the energy of the classical scalar field transfer to the thermal energy of elementary particles. Since long theoretical physicists have been investigating reheating as a perturbative phase, or one in which single inflaton quanta decayed individually into ordinary matter and radiation. The recent theoretical studies have shown that in many cases the decay occurs through a non-perturbative process in which the particles behave in an ordered manner. Non-perturbative processes involved at reheating are extremely more efficient than the perturbative ones and often more difficult to investigate in practice. In short there is no existence of a complete theory which explains non-perturbative effects during reheating for the total time scale.

Besides production of gravitinos during perturbative reheating its decay plays a significant role in the context of leptogenesis. More precisely two types of gravitinos are produced in this epoch - stable and unstable. Stable ones and decay products of unstable ones directly or indirectly stimulate the light element abundances during big bang nucleosynthesis. Most importantly the unstable one has important cosmological consequences out of which the major one directly affects the expansion rate of the universe. In order to explain cosmological consequences at a time by a single physical entity, it is customary to explain everything in terms of gravitino energy density which is directly proportional to the gravitino number density or gravitino abundance. This gravitino abundance is obtained by considering gravitino production in the radiation dominated era following reheating. Gravitinos are originated through thermal scattering in the early universe and are usually related to the reheating temperature ($T_{reh}$). Particle physics phenomenology usually requires that under instantaneous decay approximation $T_{reh}$ is maximum during reheating.

In the present article we have studied extensively reheating phenomenology and leptogenesis in a typical brane inflation model which was proposed earlier by us. Precisely, the model includes one loop radiative correction in the framework of local brane version of the supersymmetric theory i.e. $N = 1, D = 4$ SUGRA which is derived from the background $N = 2, D = 5$ SUGRA in the bulk (for details please refer to\textsuperscript{[1]})

\textsuperscript{*}Electronic address: sayanphysicsis@gmail.com
\textsuperscript{†}Electronic address: supratik@isical.ac.in
will be revealed, the scenario is somewhat different in the context of reheating from brane inflation which results in novel features worth studying in details. This has serious implication for the production of the heavy Majorana neutrinos needed for leptogenesis [13]. We further estimate different parameters related to reheating and leptogenesis at the epoch of phase transition [20]. Last but not the least we have given an estimate of CP violation which is the indirect evidence of the baryon asymmetry and connected with gravitino dark matter abundance.

II. REHEATING PHENOMENOLOGY ON THE BRANE FOR SU(2)L \otimes U(1)_Y

A. Model Building from background supergravity

For systematic development of the formalism, let us briefly review from our previous paper [1] how one can construct the effective 4D inflationary potential of our consideration starting from N = 2, D = 5 SUGRA in the bulk which leads to an effective N = 1, D = 4 SUGRA in the brane. Considering the fifth dimension is compactified on the orbifold S^1/Z_2 of comoving radius R, the N = 2, D = 5 bulk SUGRA is described by the following action

\[ S = \frac{1}{2} \int d^4x \int_{-\pi R}^{+\pi R} dy \sqrt{-g} \left[ M_5^2 (R_5 - 2\Lambda_5) + L_{\text{SUGRA}}^{(5)} + \sum_{i=1}^2 \delta(y - y_i) L_{4i} \right]. \]  

(1)

Here the sum includes the walls at the orbifold points y_i = (0, \pi R) and 5-dimensional coordinates x^\mu = (x^\alpha, y), where y parameterizes the extra dimension compactified on the closed interval [-\pi R, +\pi R]. Written explicitly, the contribution from bulk SUGRA in the action

\[ e^{-1} L_{\text{SUGRA}}^{(5)} = -\frac{M_5^2 \mu^{(5)}}{2} + \frac{1}{16\pi^2} \left( \bar{\psi}_I \gamma^{\mu} \nabla_\mu \psi_I - \bar{\psi}_I F_{\alpha \beta}^I \epsilon^{\alpha \beta} \right) + \text{Fermionic} + \text{Chern - Simons}, \]

and including the radion fields (\chi, T, T^\dagger) the effective brane SUGRA counterpart turns out to be

\[ \delta(y) L_4 = -e^{(5)} \Delta(y) \left[ (\partial_\mu \phi) (\partial^\mu \phi) + i \lambda \bar{\phi} \gamma^5 D_\alpha \phi \right]. \]

The Chern-Simons terms can be gauged away assuming cubic constraints and Z_2 symmetry. Further, S^1/Z_2 orbifold setting allows us to express the 4-dimensional part of the action (after dimensional reduction) as

\[ S = \frac{M_{PL}^2}{4L} \int d^4x \sqrt{-g} \left[ R_4^{(4)} + (\partial_\mu \phi^{(4)}) (\partial^\mu \phi^{(4)}) - Q F_{\alpha \beta}^4 \epsilon^{\alpha \beta} \right]. \]

(3)

where \( P = 2 M_{PL}^2 \mu^{(5)} \), \( Q = \frac{C(T, T^\dagger)}{4 \pi^2 R^2} \), and the 4D Planck mass \( M_{PL} = \sqrt{\frac{8}{5} \epsilon^{(5)}} = \sqrt{8 \pi M = \frac{M_5^2}{\lambda^2} \sqrt{\frac{8}{5}} = 1.22 \times 10^{19} \text{GeV}} \). Here we have introduced the reduced 4D Planck mass \( M = 2.43 \times 10^{18} \text{GeV} \), 5D and 4D charge \( e_5 \) and \( e_4 \), 5D Planck mass \( M_5 \) and the brane tension \( \lambda \) and two constants \( \beta \) and \( b_0 \) comes from the metric structure. Here \( C(T, T^\dagger) \) represents an arbitrary function of \( T \) and \( T^\dagger \). This leads to an effective \( N = 1, D = 4 \) SUGRA in the brane with the F-term potential

\[ V = V_F = \exp \left( \frac{K(\phi, \phi^\dagger)}{M^4} \right) \left[ \left( \frac{\partial W}{\partial \phi^\dagger} + \left( \frac{\partial K}{\partial \phi^\dagger} \right) \left( \frac{W}{M^2} \right) \right]^\dagger \right. \]

\[ \left. \left( \frac{\partial^2 K}{\partial \phi \partial \phi^\dagger} \right)^{-1} \left( \frac{\partial W}{\partial \phi} + \left( \frac{\partial K}{\partial \phi} \right) \left( \frac{W}{M^2} \right) - \frac{1}{2} \frac{W^2}{M^4} \right) \right]. \]

(4)

Here \( \Psi^\alpha \) is the chiral superfield and \( \phi^\alpha \) be the 4D complex scalar field. In this context the Kähler potential is dominated by the leading order term i.e. \( K = \sum_n \phi^n \phi^\dagger^n \). The superpotential in eq\( (1) \) is given by \( W = \sum_{n=0}^{\infty} D_n W_n(\phi^n) \) with the constraint \( D_0 = 1 \). Expanding the slowly varying inflaton potential around the value of the inflaton field along with \( Z_2 \) symmetry the required renormalizable one-loop corrected inflaton potential turns out to be

\[ V(\phi) = \Delta^4 \left[ 1 + \left( D_4 + K_4 \ln \left( \frac{\phi}{M} \right) \right) \left( \frac{\phi}{M} \right)^4 \right], \]

(5)

where \( K_4 = \frac{\partial^4}{2\pi^2 M^4} \) and \( D_4 = C_4 - \frac{2\Sigma K_2}{M^2} \) where \( C_4 \) is negative constant appearing at the tree level. Here \( \Delta \) represents the energy scale of brane inflation which can be expressed in terms of the slow roll parameter \( \eta_s \), explicitly derived in [1]. For our model \( \Delta \simeq 0.2 \times 10^{18} \text{GeV} \) for the window \( 0.70 < D_4 < -0.60 \).

With this brief review of the construction of the potential we are now in a position to investigate for its phenomenological significances.

From the knowledge of particle physics it is known that during the epoch of reheating inflatons decay into different particle constituents [3] [21] are directly related to the trilinear coupling of the inflaton field. There might be a possibility of collision originated through quartic coupling and driven by background scalar field. For example here the contribution from the heavy Majorana neutrino comes from the seesaw Lagrangian \( L_{\text{Majo}} = -\bar{l}_L H \psi - \frac{1}{\Lambda} M \bar{\psi} \psi \) and h.c., where \( l_L \) and \( H \) are the lepton and the Higgs doublets, respectively, and \( M \) is the lepton-number-violating mass term of the right-handed neutrino. Now using the assumption \( m_\phi \gg m_\sigma, m_\phi \gg m_e \) the total inflaton decay width for the positively and negatively charged \( (\phi^+, \phi^-) \) scalar fields as well as the fermionic field \( \psi \) (Example: For the heavy Majorana neutrinos the decay process \( \psi \rightarrow l_L H \psi \rightarrow l_L H \text{predominates} \) ) is given by \( \Gamma_{\text{total}} \simeq \frac{1}{4\pi} \left( \frac{\phi_0^2}{M^2} \right) \sim \frac{1}{(2\pi)^2} \left( \frac{\Delta^2}{M^2} \right) \)

where the coupling strength \( C \simeq m_\phi \left( \frac{\phi_0^2}{M^2} \right) \) and h \( \sim \left( \frac{\Delta^2}{M^2} \right) \) and the background scalar field is \( \sigma \).

Now to construct the thermodynamical observable the effective number of particles incorporating relativistic degrees of freedom is defined [22] as \( N^* = N_B^* + \frac{7}{8} N_F^* \),
where $N_B = \sum_i N_{B_i}$ and $N_F = \sum_j N_{F_j}$. Here $N_B$ represents the number of bosonic degrees of freedom with mass $m_\phi \ll T$ and $N_F$ represents number of fermionic degrees of freedom with mass $m_\psi \ll T$. Here 'i' and 'j' stand for different bosonic and fermionic species respectively. For the phenomenological estimation $N^* \sim 10^2 - 10^4$ and for realistic models $N^* \sim 10^2 - 10^3$. For convenience let us express reheating temperature on the brane as

$$\Gamma_{total} = 3H(T^{br}) = \sqrt{\frac{3\rho(t_{reh})}{M^2} \left[ 1 + \frac{\rho(t_{reh})}{2\lambda} \right]}, \quad (6)$$

where $H(T^{br})$ and $\rho(t_{reh})$ be the Hubble parameter and energy density during reheating respectively. It is worth mentioning that the brane reheating temperature does not depend on the initial value of the inflaton field and is solely determined by the elementary particle theory of the early universe.

B. Phase transition in brane inflation

Phase transition in braneworld scenario is weakly first order in nature [23]. So it is convenient to write the brane reheating temperature in terms of the critical parameters. To serve this purpose the critical density and the critical temperature or transition temperature can be written as:

$$\rho(t_c) = 2\lambda = \frac{3}{16\pi^2} \frac{M_5^2}{\pi^2} \sqrt{\frac{5}{\pi} N^* M_{PL}^2}, \quad T_c = \sqrt{\frac{3}{\pi} \frac{5}{\pi} N^* M_{PL}^2} \quad (7)$$

which makes a bridge between the phenomenology and observation. In the high energy limit 5D Planck mass ($M_5$) can be expressed in terms of our model parameters as $M_5 = \sqrt{\frac{5(1+4D_4)}{\alpha^2}} \phi_*$. Here $\Delta^2$ represents the amplitude of the scalar perturbation defined as $\Delta^2 = \frac{512\pi^4}{16M_{PL}^2} \left[ \frac{\nu^2}{\nu^3} \left[ 1 + \frac{\nu^2}{2\alpha} \right]^3 \right]$. Most importantly here the subscript * represents here the epoch of horizon crossing ($k = aH$) and $\alpha$ represents a dimensionless model parameter defined as $\alpha = \frac{\Delta^2}{\nu}$. The major thermodynamic quantities – critical density ($\rho_c$), critical pressure ($P_c$), critical entropy ($S_c$) – and the Hubble parameter at the critical temperature ($H_c$) related to the phase transition designated by a four tuple critical set $U(c_\gamma)$ by the following fashion for our model:

$$U(c_\gamma) : \{ \rho_c, P_c, S_c, H_c \}$$

$$\equiv \left\{ \phi^4 A(\phi), \left\{ 1200, 400, 1600, \frac{20}{\sqrt{A(\phi) M^2}} \right\} \right\} \forall \gamma \in J$$

where we have defined a dimensionless characteristic quantity $A(\phi) = \frac{\pi^2 (K_4 + 4D_4) \Delta^2}{\alpha M^2}$ at the horizon crossing in this context. The above mentioned physical quantities are function of the critical or transition temperature which is defined as

$$T_c := \left[ T_{c\gamma} = \sqrt{\frac{C\gamma A(\phi) \phi^4}{\pi^2 N^*}} \frac{\Gamma_{total} M_{PL}^2}{T_c^2} \right]$$

with $C\gamma = \{ 36000, 288000, 19200 \} \forall \gamma \in J$ (9)

with gauge group $J := SU(2)_L \otimes U(1)_Y$ and the species index $\gamma = 1(B \Rightarrow Boson), 2(F \Rightarrow Fermion), 3(M \Rightarrow Mixture)$.

C. Brane reheating temperature

In this context the reheating temperature can be written [25] as a one to one mapping $(\Omega)$ in parameter space as

$$\Omega : \left\{ T^{br} := \frac{T_c}{\sqrt{2}} \left[ 1 + \frac{5}{\pi^2 N^*} \left( \frac{\Gamma_{total} M_{PL}^2}{T_c^2} \right)^2 \right] - 1 \right\}$$

$$\Rightarrow T^{br} = \left\{ \left\{ \frac{10}{3N^*} \frac{\Gamma_{total} M_{PL}^2}{T_c^2} \right\} \right\} \in \mathbb{C}$$

where $\mathbb{C}$ represents collection of all gauge group which supports particle theory. But in this context we are confining ourselves into the Standard Model regime. So to construct a fruitful model of reheating in the context of Standard Model gauge group, we rewrite all general principal components in terms of physical degrees of freedom.
in a compact fashion. We consider a one to one high energy mapping $Q[\gamma]$ in a physical space such that

$$Q[\gamma]: \left\{ \begin{array}{l}
T^{{br}} = T^{{c}} \sqrt{2} \left[ \frac{1}{\pi^2 N^*_\gamma} \left( \frac{\Gamma_{total} M_{PL}}{T^2_{c}} \right)^2 - 1 \right] ^{1/2} \\
\Rightarrow T^{{brh}} = \sqrt{2} \left[ \frac{W_\gamma (K_1 + 4 D_4) \Delta_\delta \phi_0^4 \Gamma_{total}}{\pi^2 N^*_\gamma \alpha^2} \right] \forall \gamma \in J
\end{array} \right.$$ (11)

it maps the actual brane reheating temperature ($T^{{br}}$) to its high energy value ($T^{{brh}}$) in the Standard Model gauge group $J := SU(2)_L \otimes U(1)_Y$ with $Z_\gamma = (5, 40/3, W_\gamma = (600, 320)$ and $\gamma = 1(B), 2(F), 3(M)$. Here $\bigcup \gamma \subset \mathcal{Q}[\gamma] \subseteq \mathcal{O}$ for which $J \in \mathcal{G}$. Most importantly the superscript ‘br’ and ‘brh’ stands for parameters before and after high energy mapping respectively. Here it should be mentioned that the brane reheating temperature incorporates all the effects of heavy Majorana neutrinos as well as the other fermions and bosons through the total decay width $\Gamma_{total}$.

The reheating temperature for different species can readily be calculated from our model. For a typical value of $C_4 \simeq D_4 = -0.7$ (consistent with [1]), we have: for boson $T^{{brh}}_B \simeq 7.6 \times 10^{10} \text{GeV}$, for fermion $T^{{brh}}_F \simeq 7.8 \times 10^{10} \text{GeV}$ and for mixture of species $T^{{brh}}_M \simeq 6.5 \times 10^{10} \text{GeV}$. This is significantly different from GR where $T^{{reh}} \leq 10^6 - 10^7 \text{GeV}$ and is a characteristic feature of brane inflation.

## III. GRAVITINO PHENOMENOLOGY ON THE BRANE FOR SU(3)_C \otimes SU(2)_L \otimes U(1)_Y

Let us now move on to studying how the self interacting term of our model is directly related to the leptogenesis through the production of thermal gravitinos which is a special ingredient for the heavy Majorana neutrinos in the leptogenesis. Let us start with a physical situation where the inflaton field starts oscillating when the inflationary epoch ends at a cosmic time $t = t_{osc} \simeq t_f$. Throughout the analysis we have assumed that the universe is reheated through the perturbative decay of the inflaton field for which the reheating phenomenology in brane is described by the Boltzmann equation

$$\dot{\rho}_r + 4H \rho_r = \Gamma_\phi \rho_\phi,$$ (12)

where in braneworld

$$H^2 = \frac{8\pi}{3 M_{PL}^2} (\rho_r + \rho_\phi) \left[ 1 + \frac{(\rho_r + \rho_\phi)}{2\lambda} \right]$$

$$= H_{osc}^2 \left( \frac{a_{osc}}{a} \right)^4 \left[ 1 + \frac{\alpha}{2} \left( \frac{a_{osc}}{a} \right)^4 \right] \cdot$$ (13)

Here $\rho_r$ and $\rho_\phi$ represent the energy density of radiation and inflaton respectively and $\Gamma_\phi$ is the rate of dissipation of the inflaton field energy density. At $t = t_{osc}$ epoch the Hubble parameter is designated by

$$H_{osc} = \sqrt{\frac{8\pi}{3}} \frac{\Delta^2}{M_{PL}} = \frac{\Delta^2}{\sqrt{3}M}.$$ (14)

Assuming $\Gamma_\phi \gg H$ from we get

$$\rho_\phi = \Delta^4 \left( \frac{a_{osc}}{a} \right)^4 \exp \left[ -\Gamma_\phi(t - t_{osc}) \right].$$ (15)

It is worthwhile to mention here that the inflaton field $\phi$ follows an equation of state similar to radiation rather than matter i.e. $\omega_\phi = \frac{P_\phi}{\rho_\phi}$, where $P_\phi = \rho_\phi - \Delta^4 \left[ 1 + \left( D_4 + K_4 \ln \left( \frac{\phi}{M} \right) \right) \left( \frac{\phi}{M} \right)^4 \right]$. Now solving Friedmann equation the dynamical character of the scale factor can be expressed as

$$a(t) = a_{osc} \left[ \frac{\sqrt{1 + \frac{\alpha}{2} + 2H_{osc}(t - t_{osc})^2} - \frac{\alpha}{2}}{2} \right],$$ (16)

where we use a specific notation $a(t_{osc}) = a_{osc}$.

Plugging eqn (10) and eqn (15) in eqn (12) we get

$$\dot{\rho}_r + \frac{2H_{osc}}{\Gamma_\phi^4 \exp \left[ -\Gamma_\phi(t - t_{osc}) \right]} \rho_r$$

$$= \left[ \sqrt{1 + \frac{\alpha}{2} + 2H_{osc}(t - t_{osc})^2} - \frac{\alpha}{2} \right],$$ (17)

As a whole phenomenological construction of gravitino abundance is governed by the above equation. But eqn (17) is not exactly analytically solvable. So we are confining our attention to the high energy limit where the Friedmann equation (13) can be approximated as

$$H^2 = \frac{8\pi}{6 \lambda M_{PL}^2} (\rho_r + \rho_\phi)^2 = \frac{\alpha}{2} H_{osc}^2 \left( \frac{a_{osc}}{a} \right)^8 ,$$ (18)

whose solution is given by

$$a(t) = a_{osc} \sqrt{1 + 2\sqrt{2\alpha} H_{osc}(t - t_{osc})}.$$ (19)

Now using an physically viable assumption $t \leq \Gamma^{-1}_\phi$ the exact solution of the eqn (17) in the high energy limit can be written as

$$\rho_r \simeq \frac{3M^2 H_{osc}^2 \Gamma_\phi(t - t_{osc})}{\left[ 1 + 2\sqrt{2\alpha} H_{osc}(t - t_{osc}) \right]}$$

$$= \frac{3M^2 H_{osc} \Gamma_\phi}{2\sqrt{2\alpha}} \left( \frac{a_{osc}}{a} \right)^4 \left[ \left( \frac{a}{a_{osc}} \right)^4 - 1 \right].$$ (20)

Our intention is to find out the extremum temperature during reheating epoch which is one of the prime components for the determination of gravitino abundance. In
the braneworld scenario this extremum temperature is given by
\[ T_{ex}^{bh} = \frac{\sqrt{13\sqrt{3}^2 MT^2}}{N\pi^2} \sqrt{\frac{1}{2\alpha}} = \sqrt{\frac{45\Gamma_\phi M^2}{8N\pi^2}} \] (21)
and it is less than the reheating temperature in brane (Tbrh). This phenomenon is different from standard GR results [10] where we see that the reheating temperature shoots up to a maximum value and it gives the upper bound of the reheating temperature. But in the present context of brane inflation this situation is completely different i.e., at first temperature falls down to a minimum which fixes the lower bound of the reheating temperature and rises to a maximum at the end of reheating epoch. Using eqn (17), eqn (21) and the thermodynamic background of energy density of radiation we can express the scale factor in terms of temperature as
\[ a(T) = \begin{cases} \sqrt{1 - 32T_0^2} & \text{if } t_{osc} \simeq t_f \\ \sqrt{32 - (\frac{T}{T_{osc}})^4 - 1} & \text{if } t_{osc}(\simeq t_f) < t \leq t_{reh} \end{cases} \] (22)
It is worth mentioning that if we break the time scale into two parts t_{osc} < t < t_{ex} and t_{ex} < t < t_{reh} as done in GR the scale factor and hence the remaining results have same expressions in these two different zones. This is in sharp contrast with standard GR results except at t = t_f, where they have different values in the two different regimes.

Let us now use this phenomenological background to derive the expression of the gravitino production during two thermal epochs - reheating and radiation dominated era. It is well known that gravitinos are produced by the scattering of the inflaton decay products [26]. The master equation of gravitino phenomenology as obtained from ‘Boltzmann equation,’ is given by [18, 27]
\[ \frac{dn_G}{dt} + 3Hn_G = \langle \Sigma_{total} | v \rangle n^2 - \frac{m^4 T^4}{E_\gamma^2 \tau_\gamma}, \] (23)
where n = \frac{\zeta(3)Y^3}{\pi^2} is the number density of scatterers (bosons in thermal bath) with \zeta(3)=1.20206.... Here \Sigma_{total} is the total scattering cross section for thermal gravitino production, v is the relative velocity of the incoming particles with \langle v \rangle = 1 where (...) represents the thermal average. The factor \frac{m^4}{E_\gamma \tau_\gamma} represents the averaged Lorentz factor which comes from the decay of gravitinos can be neglected due to weak interaction. For the gauge group E := SU(3)_C \otimes SU(2)_L \otimes U(1)_Y the thermal gravitino production rate is given by,
\[ \langle \Sigma_{total} | v \rangle = \frac{\tilde{a}}{M^2} = \frac{3\pi}{16\zeta(3)M^2} \sum_{i=1}^{3} \left[ 1 + \frac{M_i^2}{3m_i^2} \right] C_i g_i^2 \ln \left( \frac{K_i}{g_i} \right), \] (24)
\[ \frac{dn_G}{dT} + 3Hn_G = \langle \Sigma_{total} | v \rangle n^2, \] (26)
where a boundary condition T = T_{ex}^{bh}, T = 0 is introduced. In terms of a dimensionless variable
\[ x = 32 \left( \frac{T}{T_{ex}^{bh}} \right)^4 - 1 \] (27)
eqn (26) can be expressed as
\[ \frac{dn_G}{dx} + \frac{d}{x}n_G = -\frac{d_3(x+1)^2}{x^2} \] (28)
where \( d_1 = -\frac{3}{4}, \quad d_3 = \frac{(T_{bh})^6}{32} \frac{\bar{\alpha}}{M^2} \left( \frac{\zeta(3)}{\pi^2} \right)^2 \frac{\sqrt{x}}{4 \sqrt{M^2_{\text{br}}}}. \) The exact solution of the eqn(28) is given by

\[
n_G(x) = \frac{2d_3}{\sqrt{x}} \left( x + 1 \right) \left( \frac{\Gamma(\frac{3}{4})^2}{\Gamma(\frac{1}{4})^2} \right) \left( \frac{1}{\Gamma(\frac{3}{4})} + \frac{1}{\Gamma(\frac{1}{4})} - \frac{2}{\Gamma(\frac{1}{2})} \right) \cdot (29)
\]

Using the properties of Gaussian hypergeometric function for \( x \gg 1 \) eqn(29) reduces to the following simpler form:

\[
n_G(x) \approx 2d_3 x^{\frac{3}{2}} \sqrt{1 + \frac{x}{\Gamma(\frac{3}{4})^2/\Gamma(\frac{1}{4})^2}} \cdot (30)
\]

Using the boundary condition \( T = T_{br} \) in eqn(29) the numerical value of gravitino abundance turns out to be \( n_G(x_{br}) = 62.023d_3 \),

![FIG. 3: In the above diagram we have plotted variation of gravitino number density in a physical volume vs scaled temperature in braneworlds. Here we have used the fundamental scale \( d_3 = 4.596 \times 10^{-44} \bar{a}M^2 \), where \( \bar{a} \) is a dimensionless number depends on the species of the MSSM gauge group \( E \). For an example \( n = 4 \) level flat direction content QQQL, QuQd, QuLe, uude of MSSM gives \( \bar{a} \approx 15.694 \) in the absence of top Yukawa coupling. Most importantly 4D effective Planck mass \( M = 2.43 \times 10^{18} \text{GeV} \). From the plot it is obvious that the gravitino number density is monotonically increasing function of the dimensionless variable \( \frac{T}{T_{br}} \) except at \( x \leq 0 \) which implies \( \frac{T}{T_{br}} < \frac{1}{\sqrt{32}} \).

Let us now find out the exact analytical expression for the gravitino abundance at reheating temperature \( T_{br} \) in the high energy limit. To serve this purpose substituting \( T = T_{br} \) in eqn(29) we get

\[
n_G(T_{br}) = 8\sqrt{2}d_3 \left( \frac{2d_3}{T_{br}} \right)^4 \left( \frac{T_{br}}{T_{ex}} \right) \left( T_{br} \right)^2 \left( \frac{T_{br}}{T_{ex}} \right)^2 \left( G \right) \left( \frac{T_{br}}{T_{ex}} \right) \cdot (31)
\]

where

\[
G \left( \frac{T_{br}}{T_{ex}} \right) = \left( -2F_1 \left[ \frac{1}{4}; 1; 1; 1; 1; 1; x + 1 \right] + 2F_1 \left[ \frac{1}{4}; 2 - d_1; 1; 1; 1; x + 1 \right] + 2F_1 \left[ \frac{1}{4}; 1; 1; 1; 1; x + 1 \right] \right) \cdot (32)
\]

along with an extra constraint \( G \left( \frac{T_{br}}{T_{ex}} \right) > > \frac{1}{\sqrt{32}} \).

It is convenient to express the abundance of any species ‘\( \sigma \)’ as

\[
\bar{Y}_{b} = \frac{n_{b}}{s} \quad \text{where} \quad n_{b} \quad \text{is the number density of the species ‘} b \text{‘ in a physical volume and ‘} s \text{‘ is the entropy density given by} \quad s = 2\pi^2 N^*T^3. \quad \text{Here the master equation. for gravitino can be expressed as}
\]

\[
\frac{\dot{Y}_G}{G} = \langle \Sigma_{\text{total}}|v| \rangle Y_{G}^{-1} \cdot (33)
\]

Using eqn(10) and eqn(22) the time-temperature relation can be found as:

\[
\frac{T}{T_{br}} = \frac{\Gamma \left[ \left( 1 + \frac{\pi^2}{2} + 2H_{r_{br}}(t - t_{r_{br}}) \right)^2 - \frac{\alpha^2}{4} \right]}{\left[ \left( 1 + \frac{\pi^2}{2} + 2H_{r_{br}}(t - t_{r_{br}}) \right)^2 - \frac{\alpha^2}{4} \right]}. \quad (34)
\]

Eliminating \( \dot{T} \) we get the solution of the master equation(33) in the radiation dominated era as

\[
Y_{G}^{br}(T_f) = Y_{G}^{br}(T_{br}) + Y_{G}^{br-\text{rad}}(T_f) \quad (35)
\]

where

\[
Y_{G}^{br-\text{rad}}(T_f) = \sqrt{\frac{90}{\pi^* N^*}} \left( \frac{45\sqrt{2}}{2\pi^4 N^* \sqrt{\alpha}} \right) \left( \frac{\alpha}{M} \right)^2 \left( \frac{\zeta(3)}{\pi^2} \right)^2 \times \left[ \frac{T_{br}}{T_f} \left( F_1 \left[ \frac{1}{4}; \frac{1}{4}; \frac{1}{4}; \frac{1}{4}; 1; 1; 1; 1; 1; x + 1 \right] + 2F_1 \left[ \frac{1}{4}; \frac{1}{4}; \frac{1}{4}; \frac{1}{4}; 1; 1; 1; 1; 1; x + 1 \right] \right) \right] \cdot (36)
\]

But in eqn(36) the first term on the right-hand side is not exactly computable. As mentioned earlier to find out exact expression we have used here the high energy mapping.

In the radiation dominated era the dynamical behavior of temperature can be mapped as

\[
\Gamma : \left\{ T = \left( \sqrt{\left[ \left( 1 + \frac{\pi^2}{2} + 2H_{r_{br}}(t - t_{r_{br}}) \right)^2 - \frac{\alpha^2}{4} \right] \left[ \sqrt{1 + \frac{\pi^2}{2} + 2H_{r_{br}}(t - t_{r_{br}})} \right] \right] \right) \in E \right\} \cdot (37)
\]
Using this map we finally have

\[ Y_{G,brh}^{br}(T_f) = \left( \frac{2}{M} \right) \left( \frac{c(T_{brh})^2}{\pi^2} \right) \left[ \left( \frac{60 \sqrt{\pi T_{brh}}}{T_{brh}^2} \right)^2 \left( 1 - \frac{T_f}{T_{brh}} \right) \right] \]

where

\[ Y_{G,brh}^b - rad(T_f) = \left( \frac{65}{M} \right) \left( \frac{c(T_{brh})^2}{\pi^2} \right) \sqrt{\frac{3}{\alpha}} \left( \frac{15}{\pi N} \right)^2 \left( 1 - \frac{1}{T_f} \right) \]

and

\[ Y_{G,brh}^b \approx Y_{G,brh}^f = \frac{n_{\tilde{g}}}{8} \left( \frac{360 \sqrt{3} \Gamma_{\phi}}{2 \pi^2 N (T_{brh})^2} \right) \left( 32 \left( \frac{T_{brh}}{T_{brh}^2} \right)^4 - 1 \right) \times \left( \frac{T_{brh}}{T_{brh}^2} \right)^2 G \left( \frac{T_{brh}}{T_{brh}^2} \right) \]

The gravitino dark matter abundance and the baryon asymmetry is connected through \( Y_{G,brh}^b \sim \frac{\Theta_{CP}}{D} \) where \( D(\leq 1) \) is the dilution factor and the leading contribution is given by the interference between the tree level and the one-loop level decay amplitudes. Here the CP-violating parameter is described as \( \Theta_{CP} = \frac{\Gamma(\nu_{L} \rightarrow \nu_{L} l_{r}) - \Gamma(\nu_{L} \rightarrow l_{r} l_{L})}{\Gamma(\nu_{L} \rightarrow l_{r} l_{L})} = 3 M_{m_{\nu}} \sin \delta_{CP} \), where \( m_{\nu} \) is the heaviest light neutrino mass, \( v = 174 \mathrm{GeV} \) is the VEV of Higgs and \( \delta_{CP} \) is an effective CP phase which parameterize each entries of the CKM matrix. Particularly \( \delta_{CP} \) acts as a probe of flavor structure in supergravity theories. The complete wash out situation corresponds to \( D = 1 \).

![FIG. 4: Here we have plotted the variation of total gravitino abundance vs temperature in the domain \(-0.70 < D_4 < 0.60\), which clearly shows that gravitino abundance at zero temperature shoots up initially to maximum and then becomes constant with respect to temperature during radiation dominated era in braneworld scenario. As mentioned earlier we have used the fundamental scale \( \frac{4 \pi T_{brh}}{\alpha} = 4.596 \times 10^{-44} \bar{\alpha} \), where \( \bar{\alpha} \) is a dimensionless number which depends on the species of the MSSM and \( M = 2.43 \times 10^{18} \mathrm{GeV} \).](image)

Through out all the numerical estimation we have taken decay width \( \Gamma_{\phi} \approx 2.9 \times 10^{-3} \mathrm{GeV} \), mass of the inflaton \( m_{\phi} \approx 10^{13} \mathrm{GeV} \), final temperature and time at the end of reheating \( T_f \approx 10^{16} \mathrm{GeV} \) and \( t_{f} \approx 1.4 \times 10^{31} \mathrm{GeV} \) respectively. For a typical value of \( C_4 \approx D_4 = -0.7 \) extremum (minimum) temperature during reheating can be estimated as \( T_{ex}^{brh} \approx 7.0 \times 10^{10} \mathrm{GeV} \). This clearly shows deviation from standard GR phenomenology where the extremum (maximum) temperature during reheating can be calculated from MSSM RGE flow at the one-loop level for that flat direction. To obtain a conservative estimate of gravitino abundance we have taken here gaugino masses \( M_i \rightarrow 0 \) for all gauge subgroups within MSSM. For example the fourth level flat directions QQQL QuQd, QuLe, ude give \( \bar{\alpha} = 15.694 \) for a specific choice of the \( U_Y(1), SU_L(2) \) and \( SU_C(3) \) gauge couplings \( g_1 = 0.56, g_2 = 0.72 \) and \( g_3 = 0.85 \) respectively obtained from the universal mSUGRA boundary condition and consistent with electroweak extrapolation of the solution of MSSM RGE flow from the energy scale of brane inflation \( \Delta = 0.2 \times 10^{16} \mathrm{GeV} \) for our model. The linear dependence on \( T_{brh} \) makes simple to revise the constraints on \( T_{brh} \) based on the lower limit on the gravitino abundance - the lower bound on \( T_{brh} \) is increased by a factor of 1.074. Since \( T_{brh} \propto T_{brh} \), \( T_{ex}^{brh} \) is not affected much. Therefore models of leptogenesis that invoke a small \( T_{ex}^{brh} \) to create heavy Majorana neutrinos are not significantly affected. Within \( 55 < N < 70 \) and \( T_{brh} \approx 6.5 \times 10^{10} \mathrm{GeV} \) the entropy density changes. As a consequence the total gravitino abundance changes according to FIG. 4. It is easily seen that \( P = \frac{\tilde{\rho}_{\phi}}{\tilde{S}}, S = \frac{\tilde{\rho}_{\phi}}{\tilde{P}} \) consistency relations are valid in this context. It is worthwhile to mention here that in brane pressure and entropy density of the universe falls down to a minimum due to the minimum temperature during reheating epoch. However during radiation dominated era total entropy density is almost constant for both the cases. This clearly shows the deviation from standard GR phenomenology. Throughout the analysis we have not included the effect of \( \exp[-(\Gamma_{\phi}(t - t_{osc})]\) in the energy density of inflaton \( \rho_{\phi} \). One might be concerned that this will lead to inaccuracies close to \( t_{brh} \) when most of the gravitinos are produced. However if one writes \( \rho_{\phi} \approx a^{-4} \exp[-(\Gamma_{\phi} t)] \approx t^{-2} \exp[-(\Gamma_{\phi} t)] \) for \( t >> t_{brh} \) then \( \rho_{\phi}/\rho_{\phi} \approx -2/t - \Gamma_{\phi} \). Therefore even till
close to \( t_{\text{reh}} = \Gamma^{-1} \rho_\phi \) decreases primarily due to the expansion of the universe. Furthermore, near \( t_{\text{reh}} \) it increases as \( T^{-1/2} \simeq \sqrt{a} \simeq t^{4} \) in brane which is again different from GR phenomenology where \( T^{-1/2} \simeq \sqrt{a} \simeq t^{4} \). The thermal leptogenesis in the braneworld can take place if the lightest heavy neutrino mass lying in the range \( T_{\text{reh}} < \mathcal{M} < T_f \). This confirms that the upper bound of 5D Planck mass \( M_5 < 10^{16}\text{GeV} \) (for our model \( M_5 \simeq 7.8 \times 10^{13}\text{GeV} \) for \( C_4 \simeq D_4 = -0.7 \)), which coincides with the leptogenesis bound implied by the observed baryon asymmetry. It is important to mention here that in the standard cosmology, the thermal leptogenesis in supergravity models is hard to be successful, since the reheating temperature after inflation is severely constrained to be \( T_{\text{reh}} \leq 10^6 - 10^7\text{GeV} \) due to the gravitino problem. However, as pointed out in \( \text{25} \), the constraint on the reheating temperature is replaced by the transition temperature in the braneworld cosmology. As a result the gravitino problem can be solved even if the reheating temperature is much higher. In fact, such inflation models are possible but limited and our model is also in that category. Here we are using a preferable value of the heaviest light neutrino mass from atmospheric neutrino oscillation data \( m_{\nu} \simeq 0.05\text{eV} \) and for sufficient baryon asymmetry the lightest neutrino mass \( \mathcal{M} \simeq 10^{10}\text{GeV} \). For complete washout situation \((D = 1)\) in our model the effective CP phase lying within the window \( 2.704 \times 10^{-9} < \delta_{CP} < 2.784 \times 10^{-9} \), where \( \delta_{CP} \) is measured in degree. Most significantly it indicates that the amount of CP violation in braneworld scenario is very small and identified with the soft CP phase. Consequently it has negligibly small contribution to \( K \) and \( B \) physics phenomenology.

IV. SUMMARY AND OUTLOOK

In the present article we have studied reheating in braneworld cosmology on the background of supergravity. We have exhibited the process of construction of a fruitful theory of reheating for an effective 4D inflationary potential in \( N = 1, D = 4 \) supergravity in the brane derived from \( N = 2, D = 5 \) supergravity in the bulk \( \text{3} \). We have employed this potential in reheating model building by analyzing the reheating temperature in the context of brane inflation, followed by analytical and numerical estimation of different phenomenological parameters. It is worthwhile to mention here that we get a lot of new results in the context of braneworld phenomenology compared to standard GR case. Most importantly we get a different numerical value of reheating temperature as well the extremum temperature compared to the standard GR results. Next using the extremization principle we justify that the extremum temperature is the minimum temperature during reheating which again shows deviation from standard GR inspired phenomenology. All these facts are reflected in the numerical results of the gravitino abundance in reheating and radiation dominated era. In the context of phase transition we also get different numerical results for different parameters for standard model particle constituents.

We have further engaged ourselves in investigating for the effect of perturbative reheating. To this end we propose a theory which reflects the effect of particle production through collision and decay thereby showing a direct connection with the thermalization phenomena. To show this link more explicitly we put forward both analytical and numerical expressions for the gravitino abundance in a physical volume in the reheating epoch. Next we have found out the gravitino abundance in the radiation dominated era. Last but not the least we have expressed the total gravitino abundance in a final temperature \( T_f \). Most significantly the precision level of all estimated numerical results is the outcome of the 4D effective field theory which is analyzes with the arrival of lots of sophisticated techniques.

Apart from the aforesaid success in estimating phenomenological parameters there are some added advantages of our model with reheating in brane which are worth mentioning. One of the most significant features in the context of braneworld phenomenology is the validity of leptogenesis for our model which consequently shows the production of heavy Majorana neutrinos in the brane.

In future our aim is to search for the signatures of our model for domain wall formation \( \text{29} \) linked to the topological defects, ‘Q-ball’ formation \( \text{30} \) connected with the non-topological solitons in braneworld, the role of Lee-Wick particles in brane reheating and leptogenesis, primordial non-Gaussianity, baryogenesis etc. Last but not the least the detailed study of quantum phase transition using Monte Carlo simulation technique \( \text{31} \) to minimize rapid fluctuation \( \text{32} \) or oscillation during measurement is also an open issue. We expect to address some of these issues in near future.

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