One-Hadron-Exchange K–p interaction model in nonrelativistic energy region

H Fadhlurahman¹, A Salam¹ and I Fachruddin¹

¹Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424, Indonesia

E-mail: agussalam@gmail.com

Abstract. A nonrelativistic K–p interaction model is derived as one-hadron-exchanges. The exchanged particles are the scalar meson σ, the vector mesons ω, ρ, the hyperons Λ, Σ, and the resonances Λ'(1600), Σ'(1385). The models parameters are determined by fitting to experimental data of spin-averaged differential cross section for kaon laboratory energy range of 51.27 MeV up to 376.87 MeV. We use a 3D technique without partial wave expansion to calculate K–p scattering.

1. Introduction
In particle physics, one of the interesting topics is how hadrons interact with each other. This work intends to derive an interaction model for K–p to be used in calculations of nonrelativistic reactions. The formulated interaction model is of the type of one-hadron-exchange potential, which is more practical in applications. The exchanged particles are the scalar meson σ, the vector mesons ω, ρ, the hyperons Λ, Σ, and the resonances Λ'(1600), Σ'(1385). In taking the hadrons being exchanged and the diagrams being considered we are inspired by the work in References [1, 2]. To make it suitable for nonrelativistic calculations, we derive the interaction model within the Blankenbecler-Sugar reduction [3]. The model's parameters are determined by fitting to experimental data of spin-averaged differential cross-section for kaon laboratory energy range of 51.27 MeV up to 376.87 MeV [4, 5].

New technology enables experiments to be performed at higher energies. Thus, a more suitable calculation technique is needed. This technique is usually called 3D technique that does not use the standard partial wave expansion. We use a 3D technique for the KN interaction with hadron exchanges derived in References [6, 7].

Being one-hadron-exchanges, the K–p potential model derived in this research allows to perform scattering calculations in a simpler manner. By using a 3D technique, this assures that in the process to obtained the potential parameters all the partial waves taking part in the scattering are taken into account. Several works prior to our research can be found in References [8, 9, 10], but these works apply to relativistic scattering.

2. Method
We construct the K–p interaction model as a one-hadron-exchange potential described as the Feynman diagram shown in Figure 1. After applying the Blankenbecler-Sugar reduction, we obtain the interaction in operator form as shown in Equations (1)-(5).
Figure 1. Feynman diagram of the mesons $\omega$, $\rho$, $\sigma$ exchange and baryons $\Lambda$, $\Sigma$ exchange.

The second diagram applies also to $\Lambda^*$ and $\Sigma^*$ exchange.

\[
V_\sigma(p', p) = \frac{-g_{KK}\sigma g_{NN}\sigma}{32\pi^3 m_N m_K} \frac{(m_N + m_K)}{(E_N + \omega'_K \sqrt{E_n + \omega_K}) (p' - p)^2 + m^2_\sigma} 
\times F_{NN\sigma}(p' - p)^2 F_{KK\sigma}(p' - p)^2 \frac{\delta_\sigma(p', p)}{(\omega'_K \omega_K W' W)^{1/2}},
\]

\[
V_\omega(p', p) = \frac{g_{KK}\omega g_{NN}\omega}{64\pi^3 m_N m_K} \frac{(m_N + m_K)}{(E_N + \omega'_K \sqrt{E_n + \omega_K}) (p' - p)^2 + m^2_\omega} 
\times \left(1 + \frac{f_N N^\rho_N}{g_N N^\rho_N} \delta_{\nu_1}(p', p) + \frac{f_N N^\rho_N}{g_N N^\rho_N} \delta_{\nu_2}(p', p) \right) \frac{\delta_\omega(p', p)}{(\omega'_K \omega_K W' W)^{1/2}},
\]

\[
V_\Gamma(p', p) = \frac{-g_{KK}\Gamma g_{NN}\Gamma}{128\pi^3 m_N m_K} \frac{(m_N + m_K)}{(E_N + \omega'_K \sqrt{E_n + \omega_K}) (p' + p)^2 + m^2_\Gamma} 
\times F_{KK\Gamma}(p' - p)^2 \frac{\delta_\Gamma(p', p)}{(\omega'_K \omega_K W' W)^{1/2}},
\]

\[
V_{A^*}(p', p) = \frac{g_{NN^*A}}{128\pi^3 m_N m_{A^*}} \frac{(m_N + m_{A^*})}{(E_N + \omega_{A^*} \sqrt{E_n + \omega_K}) (1 - \mathbf{r}_1 \cdot \mathbf{r}_2)} 
\times \frac{F_{NN^*A}(p' + p)^2}{E_N + \omega_{A^*} - m_{A^*}} \frac{\delta_{A^*}(p', p)}{(\omega'_K \omega_K W' W)^{1/2}},
\]

\[
V_{\Sigma^*}(p', p) = \frac{g_{NN^*\Sigma}}{256\pi^3 m_N m_{\Sigma^*}} \frac{(m_N + m_{\Sigma^*})}{(E_N + \omega_{\Sigma^*} \sqrt{E_n + \omega_K}) (p' + p)^2 + m^2_{\Sigma^*}} 
\times \frac{F_{NN^*\Sigma}(p' + p)^2}{(p' + p)^2 + m^2_{\Sigma^*}} \frac{\delta_{\Sigma^*}(p', p)}{(\omega'_K \omega_K W' W)^{1/2}},
\]

with $\delta_\sigma(p', p)$, $\delta_{\nu_1}(p', p)$, $\delta_{\nu_2}(p', p)$, $\delta_\omega(p', p)$, $\delta_{A^*}(p', p)$, and $\delta_{\Sigma^*}(p', p)$ being given as

\[
\delta_\sigma(p', p) = m_K (W' W - \sigma \cdot p' \sigma \cdot p),
\]

\[
\delta_{\nu_1}(p', p) = W p'^2 + W' p^2 + W' W (\omega'_K + \omega_K) + (W' + W + \omega'_K + \omega_K) \sigma \cdot p' \sigma \cdot p.
\]
In Equations (1)-(5) the form factors are given as

\[ \hat{Q}_3(p', p) = W'p^2\omega_K(E'_N - 3\omega_K + m_{\Sigma^+}) + Wp'^2\omega_K(\omega_K - 2\omega_K' - E'_N + m_{\Xi^0}) \]

\[ + \{2\omega_K\omega_K'(E'_N - \omega_K + m_{\Xi^0}) + \omega_Kp'^2 - \omega_Kp^2\}W'W \\
+ (W' - W - E'_N - \omega_K + m_{\Sigma^+})p'p^2 \\
+ \{W'W(3\omega_K - E'_N - m_{\Xi^0}) + 3Wp'^2 + Wp^{(2)}p \cdot p' \\
+ \{W'\omega_K(E'_N - \omega_K - m_{\Sigma^+}) - W'\omega_K(E'_N - \omega_K + m_{\Sigma^+}) \}
+ 2\omega_K\omega_K'(E'_N - 3\omega_K - W' - W - m_{\Xi^0}) \]

\[ + p'^2(W + \omega_K) - p^2(W' + \omega_K' - (E'_N + \omega_K + m_{\Sigma^+})W'W \\
+ (3\omega_K - E'_N + W' + 3W + m_{\Xi^0})p \cdot p')\sigma \cdot p' \sigma \cdot p. \]  

In Equations (1)-(5) the form factors are given as

\[ F_0(q^2) = \left( \frac{A_0^2 - m^2}{A_0^2 - q^2} \right)^n, F_0(q^2) = \left( \frac{A_0^2 - m^2}{A_0^2 - q^2} \right)^n, \]

with \( q^2_r(r = \sigma, \omega, \rho) \) being the momentum transfers or momenta of the particles being exchanged:

\[ q^2_r = p' - p, \]

\[ q^2_\mu = \left( p'_N - p_K \right)^\mu = \left( E'_N - \omega_K, p'_N - p_K \right), \]

\[ = \left( E'_N - \omega_K, p' - p \right). \]  

Additional isospin factors of \( \tau_1 \cdot \tau_2 \) for \( \rho \) exchange, \( \frac{1}{2}(1 - \tau_1 \cdot \tau_2) \) for \( \Lambda \) exchange, and \( \frac{1}{2}(3 + \tau_1 \cdot \tau_2) \) for \( \Sigma \) exchange are added.

3. Results and Discussion

We fit calculations to experiment data of the spin-averaged differential cross section for kaon laboratory energies of 51.27 MeV to 376.87 MeV. We take the initial values of the parameters given in Table 1, which are taken from References [1, 2] and from Reference [11] for the nucleon-meson vertices. The fitting process gives \( \chi^2/N \) value of 31.72 obtained from Equation (14) and new values of the parameters shown in Table 2. Note that some parameters are fixed, not fitted. The fixed parameters are particle’s masses except for the \( \sigma \) meson mass, \( f/g \), and the exponent. The mass of the \( \sigma \) meson is not a fixed parameter. Thus, it has to be fitted. The indices 0 and 1 for the \( \sigma \) meson in Tables 1 and 2 are for the total isospin of the \( K \bar{p} \) system.

We calculate the differential cross section using the new values of the parameters. We choose two energies as examples. To see the significance of the exchanged particles, we eliminate the contribution of the corresponding exchanged particles and see the differences from the calculations with complete exchanged hadrons. If the differences are large, the contribution of the corresponding exchanged hadron is large.
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\[
\frac{\chi^2}{N} = \frac{1}{N_{\text{data}} - N_{\text{par}}} \sum_{i=1}^{N_{\text{data}}} \left( \frac{z_{\text{th}} - z_{\text{exp}}}{\Delta z} \right)_i^2
\]  
(14)

**Table 1. Initial value of the parameters.**

| Particle | Mass (MeV) | $g_N/\sqrt{4\pi}$ | $g_K/\sqrt{4\pi}$ | $\Lambda_N$ (MeV) | $\Lambda_K$ (MeV) | $\nu/g$ | Exponent |
|----------|------------|--------------------|--------------------|-------------------|-------------------|--------|----------|
| $\sigma_0$ | 720 | 4.2869 | 0.3009 | 2000 | 1400 | 0 | 1 |
| $\sigma_1$ | 550 | 2.9906 | 0.3009 | 1900 | 1400 | 0 | 1 |
| $\rho$ | 769 | 0.9487 | 0.8285 | 1850 | 1550 | 6.1 | 2 |
| $\omega$ | 782.6 | 4.9497 | -0.4762 | 1850 | 1600 | 0 | 2 |
| $\Lambda$ | 1116 | -3.944 | -3.944 | 1400 | 1400 | 0 | 1 |
| $\Sigma$ | 1193 | 0.759 | 0.759 | 1400 | 1400 | 0 | 1 |
| $\Lambda^*$ | 1600 | 0.2 | 0.2 | 2000 | 2000 | 0 | 1 |
| $\Sigma^*$ | 1385 | -0.193 | -0.193 | 2000 | 2000 | 0 | 1 |

**Table 2. New parameters value with $\chi^2/N$ value of 31.72.**

| Particle | Mass (MeV) | $g_N/\sqrt{4\pi}$ | $g_K/\sqrt{4\pi}$ | $\Lambda_N$ (MeV) | $\Lambda_K$ (MeV) | $\nu/g$ | Exponent |
|----------|------------|--------------------|--------------------|-------------------|-------------------|--------|----------|
| $\sigma_0$ | 671.1 | 3.384 | 1.971 | 1700 | 1200 | 0 | 1 |
| $\sigma_1$ | 550.8 | 3.504 | 0.8499 | 2000 | 2000 | 0 | 1 |
| $\rho$ | 769 | 1.668 | 0.9783 | 1600 | 1880 | 6.1 | 2 |
| $\omega$ | 782.6 | 4.242 | -0.3193 | 1812 | 1997 | 0 | 2 |
| $\Lambda$ | 1116 | -3.466 | -3.466 | 1230 | 1230 | 0 | 1 |
| $\Sigma$ | 1193 | 0.8816 | 0.8816 | 1287 | 1287 | 0 | 1 |
| $\Lambda^*$ | 1600 | 1.427 | 1.427 | 1726 | 1726 | 0 | 1 |
| $\Sigma^*$ | 1385 | -0.1921 | -0.1921 | 1969 | 1969 | 0 | 1 |
Figure 2. Differential cross sections for energy (a) 82.27 MeV and (b) 174.43 MeV without mesons $\sigma$, $\rho$, and $\omega$ exchange.

Figure 3. Differential cross sections for energy range (a) 82.27 MeV and (b) 174.43 MeV without baryons $\Lambda$ and $\Sigma$ exchange.

Figure 4. Differential cross sections for energy range (a) 82.27 MeV and (b) 174.43 MeV without $\Lambda^*$ and $\Sigma^*$ exchange.

Figures 2–4 show comparisons between experiment data, the calculations with complete exchanged hadrons named “Scattering Calculation Result”, and the calculations without one exchanged particle's contributions. The calculations with complete exchanged hadrons can relatively follow the experiment data trends but still do not hit the data. It is clearly shown that all the exchanged particles have a large
contribution. The \( \sigma, \rho, \) and \( \omega \) mesons have significant contributions at backward angles but the contributions become smaller in higher energies. \( \Lambda \) and \( \Sigma \) baryons contribute largely at all angles, and the contributions get larger as energy raises. \( \Sigma^* \) and \( \Lambda^* \) particles show significant contributions at all angles, and the contributions get smaller in lower energies.

4. Conclusion
We have derived a \( Kp \) interaction model as one-hadron-exchanges for nonrelativistic calculations. The potential parameter values are determined by fitting to the differential cross-section experimental data for kaon laboratory energies up to around 400 MeV. The \( \chi^2/N \) value obtained is 31.71, which is a large number, meaning that the model needs to be improved. We found that the \( \sigma, \rho, \) and \( \omega \) mesons contribution gets smaller at the higher energy. On the contrary, baryons \( \Lambda, \Sigma, \Lambda^* \) and \( \Sigma^* \) contribute more significant at the higher energy.

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