Impact of a topological defect and Rashba spin-orbit interaction on the thermo-magnetic and optical properties of a 2D semiconductor quantum dot with Gaussian confinement

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In this paper we examine the effect of introducing a conical disclination on the thermal and optical properties of a two dimensional GaAs quantum dot in the presence of a uniform and constant magnetic field. The model consists of a single-electron subject to a confining Gaussian potential with a spin-orbit interaction in the Rashba approach. We compute the specific heat and the magnetic susceptibility from the exact solution of the Schrödinger equation via the canonical partition function; while the total absorption coefficient and the refractive index changes were calculated in the density matrix formalism iteration approach. We show that the peak structure of the Schottky anomaly is linearly displaced as a function of the topological defect. We found that such defect and the Rashba coupling modify the values of the temperature and magnetic field in which the system behaves as a paramagnetic material. Remarkably, the introduction of a conical disclination in the quantum dot opens a new set of available electronic transitions when an external electromagnetic field is applied. These selection rules are observed in the absorption coefficient and the refraction index changes as different blue-shifted peaks.

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I. INTRODUCTION

The quantum dots (QDs) are considered to be the keystone for building solid-state nanodevices with applications in quantum information technologies, since it is currently possible to control the number of electrons in such mesoscopic systems [14]. In particular, spin-related phenomena in QD’s has been studied in extension in the last decades as they are crucial in the semiconductor technology called spintronics [5]. Among these, spin-orbit (SO) coupling mechanisms in semiconductors provide a basis for device applications and a source of interesting physics, such as the spin transistor [9]. The Rashba effect is of special interest as it provides a SO coupling whose tunability allows SO effects to occur in QD with few electrons [7]. Several studies were carried around the impact of Rashba-SO interaction (SOI): In fact, theoretical studies were carried on the impact of Rashba-SOI, and particularly on the calculation of the optical properties in a disk-like QD in the presence of an external magnetic field within the framework of the density matrix approach [8]. Interestingly, it was found that the resonance peaks on both the absorption coefficients and refraction index shift to the red with increasing strength of the SOI. Also, it has been shown that the SO effects modify the fluctuations of the conductance of a QD consisting of a GaAs heterostructure when a parallel magnetic field is applied [9]. An interesting behavior in the magnetization and the susceptibility in a parabolic QD at low magnetic fields have been observed. This fact has been attributed from a theoretical point of view as a consequence of the presence of the Rashba term [10]. Similar studies have focused on both Rashba and Dresselhaus spin-orbit coupling mechanisms for explaining the level anticrossing in low-dimensional systems [11][12]. In addition, other related works in this field have also been investigated the influence of the SOI on the energy levels of electrons with parabolic confinement [13][14]. More recently, it has been found that, there is a significant dependence of the Rashba contribution on the electronic, thermo-magnetic and transport properties [15].

On the other hand, the shape of the electron confining potential is crucial for the correct description of QD dynamics. It is well established that a harmonic potential is a good approximation which reproduces the main characteristics of such systems. Recently, Castaño et al. [16] studied the thermal and magnetic properties of a parabolic GaAs-QD in the presence of external magnetic and electric fields, resulting in a good description of the thermo-magnetic phase diagram. Kumar [17] included a SOI term and electron-electron interactions to the parabolic potential model (PPM) in order to compute the ground state of the GaAs QD. Further experimental investigations have shown that the confining potential is rather anharmonic and has a finite depth which has been simulated by several authors using a Gaussian potential model (GPM) [18][24]. In fact, this potential model has been widely used in several branches of physics, for example, in the description of the Gaussian core model of interacting particles [25][26], and stability diagrams in double quantum dot systems [27].

The aim of this work is to investigate the thermal, mag-
netic and optical properties of a single-electron system incorporating the Rashba SOI within a two dimensional QD (2D-QD), that displays a topological defect given by a conical disclination \cite{28,30}. Moreover, our model takes into account that particle is trapped in a potential that interpolates between a parabolic and a Gaussian potential, as well as that the whole system is subjected to a uniform external magnetic field.

This paper is organized as follows. In Sec. II we present the theoretical model for describing a single-electron subject to a confining Gaussian potential with a spin-orbit interaction in the Rashba approach. Additionally, a topological defect is included in the model by considering a conical disclination. In the same section, we compute the conical disclination \cite{28–30}. Moreover, our model takes into account that particle is trapped in a potential that interpolates between a parabolic and a Gaussian potential.

In the present work \cite{31–33}. The term $\hat{H}_{\text{Gauss}}$ corresponds to the confining potential, $V_{SO}(\rho, \phi)$ is the SOI and the last term is the Zeeman coupling of the external magnetic field with the electron spin. The vector potential $A$ expressed in the symmetric gauge $A = \frac{B}{2}(-y, x, 0)$ which in the $(r, \theta)$-coordinate system has the form

$$A(r) = \frac{B r}{2\alpha} \hat{e}_\theta. \tag{5}$$

The confining potential $\hat{H}_{\text{Gauss}}$ consists of a parametrization that interpolates between a parabolic and a Gaussian potential as shown in \cite{19,20}. In such manner, the Gaussian model can be approximated as a parabolic potential plus a perturbation:

\begin{align*}
\hat{H}_{\text{Gauss}} &= -V_0 e^{-r^2/2R^2}, \\
&\approx \frac{m^*}{2} \left[ (1 - \kappa) \omega^2_c + 2V_0 \kappa \left( \frac{\tilde{\omega}}{\hbar + 2m^* \tilde{\omega} R} \right) \right] r^2 \\
&- V_0, \\
&\equiv \frac{m^*}{2\alpha^2} \omega^2(\kappa)r^2 - V_0. \tag{6}
\end{align*}

Where $\tilde{\omega} = \sqrt{\omega_c^2 + \omega_R^2}$, $\omega_c = qB/m^*$, and $\omega_R^2 = V_0/m^* R^2$. Here the parameter $\kappa$ controls the form of the confinement: $\kappa = 0$ for the PPM and $\kappa = 1$ for the GPM. The SOI term is given by the general expression \cite{34,35}

$$\hat{V}_{SOI} = \frac{\gamma_s}{\hbar} \sigma \cdot \left[ \nabla V \times \left( \frac{p - q}{c} A \right) \right], \tag{7}$$

where the normal to the surface is chosen along the $z$-axis. The Rashba spin-orbit coupling is denoted by $\gamma_s$ and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli matrices vector. Thus, $\hat{V}_{SOI}$ in the coordinate representation has the form

$$\hat{V}_{SOI}(\rho, \phi) = \gamma_s \sigma_z \frac{dV_c}{d\rho} \left[ -i \left( \frac{1}{\rho} \right) \frac{\partial}{\partial \phi} + \frac{q}{2\hbar} \hat{B} \right]. \tag{8}$$
The confining potential $V_c$ corresponds to the Gaussian potential $H_{\text{Gauss}}$. Therefore, we have that

$$
\hat{V}_{SOI}(r, \theta) = \frac{m^*}{2} s \gamma m^* \omega_c \omega^2(\kappa) \frac{\alpha^2 \hbar}{2} r^2 \\
- i \sigma_z \frac{\gamma m^* \omega^2(\kappa)}{\alpha} \frac{\partial}{\partial \theta},
$$

with $s = \pm 1$ referring to the spin projection. Finally, the Hamiltonian of the system becomes

$$
\hat{H} = -\frac{\hbar^2}{2m^*} \left[ \frac{\alpha^2}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] + \omega_c \frac{\alpha^2}{2} \hat{L}_z \\
+ \frac{1}{2} \alpha^2 \Omega^2_s(\alpha, \kappa) r^2 - V_0 \\
+ \mu_B g_s \hat{S} \cdot \mathbf{B} - i \sigma_z \frac{\gamma m^* \omega^2(\kappa)}{\alpha} \frac{\partial}{\partial \theta}.
$$

The effective harmonic frequency in Eq. (10) is given by

$$
\Omega_s^2 = \Omega_s^2(\alpha, \kappa) = \left( 1 + s \gamma m^* \omega_c \right) \omega^2(\kappa) + \left( \frac{\omega_c}{2\alpha} \right)^2.
$$

To find the eigenvalues of the Hamiltonian in Eq. (10), we admit a periodic solution in the $\theta$-space as follows:

$$
\psi(r, \theta) = \frac{1}{\sqrt{2\pi}} e^{i\theta} R_{nls}(r) \chi(\sigma), \text{ such that } l \in \mathbb{Z}.
$$

With the above, the eigenvalues equation is

$$
-\frac{\hbar^2}{2m^*} \left[ \frac{\alpha^2}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) - \frac{l^2}{r^2} \right] R_{nls}(r) + \left[ \frac{m^* \Omega^2_s}{2\alpha^2} r^2 - V_0 + \frac{1}{2} \frac{\hbar \omega_c}{\alpha^2} l + \left( \gamma m^* \omega^2 l + \frac{1}{4} g^* \hbar \omega_c \right) s \right] R_{nls}(r) = E_{nls} R_{nls}(r).
$$

The exact solutions for the eigenfunctions and eigenvalues are:

$$
R_{nls}(r) = \left( \frac{m^* \Omega_s}{\hbar} \right)^{1/2} \sqrt{\frac{2n!}{(n + p|l|)!}} e^{-\frac{r^2}{2}} r^{|l|} L_{n+|l|}^{|l|} (r^2),
$$

(14a)

$$
E_{nls} = \hbar \Omega_s (2n + p|l| + 1) + \frac{\hbar^2}{2} \hbar \omega_c l \\
+ \left( \gamma m^* \omega^2 l + \frac{1}{4} g^* \hbar \omega_c \right) s - V_0,
$$

(14b)

where $\hat{r} = \sqrt{m^* \Omega_s / \hbar} r$ and $p = \alpha^{-1}$ is the inverse kink parameter.

### B. Thermal and magnetic properties

The canonical partition function is calculated from the energy spectrum in Eq. (14b):

$$
Z = \sum_{n=0}^{+\infty} \sum_{l=-\infty}^{+\infty} \sum_{s=-1}^{1} e^{-\beta E_{nls}},
$$

$$
= Z^{(+)}(\pm) + Z^{(-)},
$$

(15)

The Schottky temperature $T_s$ is defined as

$$
\frac{\partial C_v}{\partial T} \bigg|_{T_s} = 0, \text{ and } \frac{\partial^2 C_v}{\partial T^2} \bigg|_{T_s} < 0.
$$

(18)

Such temperature corresponds to the low-energy peak present in the Schottky anomaly. It is closely related to the energy required for a thermal transition between the ground and the first excited state as it can be interpreted as a resonance in $k_B T_s \sim \Delta E_{01}$. 

The canocial partition function is calculated from the energy spectrum in Eq. (14b):

$$
Z = \sum_{n=0}^{+\infty} \sum_{l=-\infty}^{+\infty} \sum_{s=-1}^{1} e^{-\beta E_{nls}},
$$

$$
= Z^{(+)} + Z^{(-)},
$$

(15)
C. Optical Properties

\[ \Delta l \]

FIG. 2: Angular integral \( I_\phi(p, \Delta l) \) for different values of \( \Delta l \) and \( p \). One can notice that the transition is in general allowed for different values of \( \Delta l \). The graphic is symmetric for \( \Delta l < 0 \).

The refractive index changes and optical absorption coefficient of the 2D-QD are calculated by using the density matrix formalism iteration approach [36,38]. We assume that the system is excited by an external electromagnetic plane wave of frequency \( \omega_0 \) and polarized in the \( \hat{x} \) direction

\[ E(t) = E_0 \cos(\omega_0 t) = \tilde{E}_e^{+}e^{-i\omega_0 t} + \tilde{E}_e^{-}e^{i\omega_0 t}, \] (19)

the linear and the third-order nonlinear refractive index changes can be expressed as

\[
\begin{align*}
\Delta n^{(1)}(\omega_0) &= \frac{\sigma_v |M_{21}|^2}{2n_r^2\epsilon_0} \left[ \frac{E_{21} - i\omega_0}{(E_{21} - i\omega_0)^2 + (\Gamma_{12})^2} \right]^\dagger, \\
\Delta n^{(3)}(\omega_0) &= \frac{\Delta n^{(1)}(\omega_0) - \sigma_v |M_{21}|^2 \mu I}{4n_r^2\epsilon_0} \left[ \frac{(E_{21} - i\omega_0)^2 + (\Gamma_{12})^2}{(E_{21} - i\omega_0)^2 + (\Gamma_{12})^2} \right]^\dagger \\
&\times \left\{ 4(E_{21} - i\omega_0) |M_{21}|^2 - \frac{(M_{22} - M_{11})^2}{(E_{21})^2 + (\Gamma_{12})^2} \\
&\times \left[ (E_{21} - i\omega_0) (E_{21} - i\omega_0) - (\Gamma_{21})^2 \right] \\
&\times - (\Gamma_{12})^2 (2E_{12} - i\omega_0) \right\},
\end{align*}
\]

(20)

where \( \mu \) is the permeability, \( \sigma_v \) is the carrier density, \( n_r \) the refractive index, \( M_{ij} = \langle \psi_i | e^x | \psi_j \rangle \) is the electric dipole matrix element, \( E_{ij} = E_i - E_j \) and \( I \) is the optical intensity of the incident wave. The total refractive index change can be written as

\[
\frac{\Delta n(\omega_0)}{n_r} = \frac{\Delta n^{(1)}(\omega_0)}{n_r} + \frac{\Delta n^{(3)}(\omega_0)}{n_r},
\] (21)

In the same order of approximation, the linear and third-order nonlinear absorption coefficient are

\[
\begin{align*}
\alpha^{(1)}(\omega_0) &= \omega_0 \sqrt{\frac{\mu}{\varepsilon_0}} \left[ \frac{\sigma_v \hbar \Gamma_{12} |M_{21}|^2}{(E_{21} - i\omega_0)^2 + (\Gamma_{12})^2} \right], \\
\alpha^{(3)}(\omega_0, I) &= \omega_0^2 \sqrt{\frac{\mu}{2\varepsilon_0 n_r \epsilon}} \frac{\sigma_v \hbar \Gamma_{12} |M_{21}|^2}{(E_{21} - i\omega_0)^2 + (\Gamma_{12})^2} \\
&\times \left\{ |M_{22} - M_{11}|^2 \left[ 4E_{21} \hbar \omega_0 - h^2 \left( \omega_0 - \Gamma_{12}^2 \right) \right] \\
&\times \left[ E_{21} + (\Gamma_{12})^2 \right] \\
&\times \left[ 3E_{21}^2 |M_{22} - M_{11}|^2 + 4 |M_{21}|^2 \right],
\end{align*}
\]

(22)

where \( \Gamma_{12} = 1/\tau \) is the relaxation rate and \( \tau \) is the relaxation time. On account of the above, the total absorption coefficient is given in this order of approximation by

\[
\alpha(\omega_0, I) = \alpha^{(1)}(\omega_0) + \alpha^{(3)}(\omega_0, I).
\]

The second-order nonlinear absorption coefficient vanishes whenever the medium possesses inversion symmetry, which is the case of both the parabolic and the Gaussian potentials. For the sake of further discussion, we define the critical intensity \( I_c \) as the maximal value of the intensity which allows a positive defined total absorption coefficient, i.e., it is defined by the conditions

\[
\frac{\partial \alpha(\omega_0, I)}{\partial \omega_0} \bigg|_{\omega_0, I_c} = 0, \quad \text{and} \quad \alpha(\omega_0, I_c) = 0.
\]

(24)

The critical intensity can be interpreted as a measure of the non-linearity of the system as it represents the moment when the one-electron density matrix can no longer be expanded in powers of the electric field strength of the optical field [39]. A lower value for \( I_c \) represents an easier to obtain non-linear behavior. A large nonlinear susceptibility is indispensable in the creation of all-optical switching, modulating and computing devices [40,41]. The behavior of \( I_c \) as a function of the inverse kink parameter \( p \) will be addressed in Section [III]

1. Selection Rules

In order to find the optical properties of the system, it is necessary to compute the allowed transitions of the system. In general, one has to obtain the dipole matrix element

\[
\begin{align*}
\langle nls | x | ml's' \rangle &= r_0 I_\phi(p, \Delta l) I_s(n, m, l, \Delta l, p) \delta_{s,s'} \\
&= r_0 \Lambda_{nm}(l, \Delta l, p) \delta_{s,s'},
\end{align*}
\]

(25)
while the radial integral is

\[ I_\rho \phi = \int_0^{2\pi} d\theta e^{i\Delta l \theta} \cos \alpha \theta, \quad (26a) \]

while the radial integral is

\[
I_x(n, m, l, \Delta l, p) = \frac{p}{2\pi} \int_0^{2\pi} d\theta e^{i\Delta l \theta} \cos \alpha \theta, \\
\times \int dx e^{-x^2} e^{\pm x^2(1 + |l| + |\Delta l|)} L_n^{|l|}(x) L_m^{|\pm \Delta l|}(x). \quad (26b)
\]

In Fig. 2 we show the result of the integral in Eq. (26a) as a function of \( \Delta l \) for different values of the inverse kink parameter \( p \). We emphasize that the disclination introduces a nonzero probability for values of the angular momentum difference not necessarily equal to \( \pm 1 \), though the standard selection rule \( \Delta l = \pm 1 \) is recovered when no defect is introduced \((p = 1)\) as the parity is well defined in such scenario. For solving the integral \( I_x(n, m, l, \Delta l, p) \) from Eq. (26b), we make use of the following identity [12, 13]:

\[
\int_0^{\infty} dx e^{-x^2} x^\mu L_n^\alpha(x) L_m^\beta(x) = (-1)^{m+n} \Gamma(\mu + 1) \\
\times \sum_{k=0}^{\min(m, n)} \binom{\mu - \alpha}{m - k} \binom{\mu - \beta}{n - k} \binom{\mu + k}{k}. \quad (27)
\]

As shown in Fig. 3 the topological disclination introduces the possibility of transitions with \( |n - m| > 1 \) which were not allowed in the case \( p = 1 \) and that are suppressed as the difference of the principal quan-
FIG. 5: Magnetic phase diagram of GaAs as function of $B$ and $T$ for (a) $\kappa = 0$, $\gamma = 0$ and $p = 1$, (b)$\kappa = 1$, $\gamma = 20$nm $^2$ and $p = 1$, (c) $\kappa = 1$, $\gamma = 20$nm $^2$ and $p = 2$ and (d)$\kappa = 1$, $\gamma = 20$nm $^2$ and $p = 4$. The gray region correspond to the diamagnetic phase ($\chi < 0$), and the white region to the paramagnetic phase ($\chi > 0$).

III. RESULTS AND DISCUSSION

In order to compute the thermal and magnetic properties of the 2D GaAs QD we use the following material constants: $m^* = 0.067 m_0$ is the effective electron mass for a GaAs QD; $V_0 = 36.7$ meV is the reference potential; $g^* = -0.44$ is the effective Landé constant; $R = 10$ nm is the radius of the QD; $\Gamma_{12} = (0.2 \text{ ps})^{-1}$ is the relaxation rate, which is taken as equal for all the transitions; $\sigma_v = 3 \times 10^{-22}$ m$^{-3}$ is the carriers density taken as a constant even for a varying area of the QD; $\epsilon_r = 13.18$ is the permittivity of the GaAs QD; and the real part of refraction index $n_r = \sqrt{\epsilon_r}$. The Rashba coupling has been taken as $\gamma_s = 20 \times 10^{-18}$ m$^2$ as a reasonable value in which the SOI can be appreciable [19]. The specific heat $C_v$ as a function of temperature is shown in Fig. 4-(a) which is computed by using Eq. (17) for different values of magnetic field $B$. To appreciate the effect of both the topological disclination and the SOI interaction we compare the results of the PPM without Rashba interaction for $p = 1$ (dashed lines) with the behavior of the GPM in the presence of Rashba coupling for $p = 2$ (dot-dashed lines) and $p = 4$ (solid lines). In Fig. 4-(b) we show the behavior of $C_v$ at low temperatures, where the peak structure observed is the well-known Schottky anomaly, which occurs whenever the thermal energy gained by the electrons in the system is enough for only the lowest two levels [21, 45, 46]. From Fig. 4-(a) it can be noticed that the specific heat converges to some finite value ($\leq 2k_B$) at high temperatures. Such limit value decreases as the magnetic field is increased and it remains even when the kink parameter is modified. Additionally, from Fig. 4-(b) one can notice that introducing a SOI sharpens the peaks and shifts them to lower temperature values. The reason is that the SOI lifts the spin degeneracy and more energy levels are available in the unit range...
FIG. 6: Total absorption coefficient $\alpha$ as a function of the incident photon energy for the transition $|0, 0, \pm 1\rangle \rightarrow |1, 1, \pm 1\rangle$ for different values of the Intensity of the external electric field and for (a) $p = 1$ (b) $p = 3$. In both graphics, $\Delta l = 1$, $l = 1$, $B = 15T$, $\kappa = 1$ and $\gamma_s = 20 \text{nm}^2$.

of energy, thereby reducing the level spacing and shifting the peak to lower thermal energy [45, 46].

FIG. 7: Total absorption coefficient $\alpha$ as a function of the incident photon energy for all of the allowed transitions from the ground state to excited states with $p = 3$. The intensity of the incident plane wave is fixed to $I = 0.3 \text{MW/cm}^2$; while $B = 15T$, $\kappa = 1$ and $\gamma_s = 20 \text{nm}^2$.

In Fig. 3 we show the thermomagnetic phase diagram obtained by evaluating Eq. (17) for different values of the parameters involved. In Fig. 3(a) we use the PPM approximation without including the SOI and for $p = 1$. Such diagram was included for comparative purposes as it represents the case without a topological disclination which has been analyzed before [19]. In Fig. 3(b)-(d) the graphics display the phase diagram in the GPM formalism with $\gamma_s = 20 \text{nm}^2$ and $p = 1, 2$ and $4$, respectively. The gray region corresponds to the diamagnetic phase ($\chi < 0$) and the white area to the paramagnetic phase ($\chi > 0$).

The effect of introducing a SOI is the decrease of the magnetic field intensity and temperature required for achieving the paramagnetic phase. Furthermore, the topological defect displaces the values of the magnetic field from the origin and gives a paramagnetic region which is not symmetric with respect to $B$. Such behavior can be understood from the energy spectrum in Eq. (14b): as the value of $p$ increases, the effects of the spin quantum number become less relevant and therefore, to appreciate its properties it is necessary to increase the external magnetic field or decrease the temperature of the system.

On the other hand, Fig. 6 shows the total absorption coefficient $\alpha$ as a function of the photon energy $h\omega_0$ for the transition $|0, 0, \pm 1\rangle \rightarrow |1, 1, \pm 1\rangle$ for different values of the Intensity $I$ of the external electric field. Fig. 6 (a) represents the case where no topological defect is included ($p = 1$) and 6 (b) is for $p = 3$. A spin dependence of the resonance can be observed, where the transitions for $s = -1$ show a peak in lower energy than the $s = 1$ transitions. This effect is a consequence of the spatial wave functions depending on the spin via the definition of $\Omega_s$, as can be noticed when taking the energy difference from the spectrum in Eq. (14b): and is typical in systems involving a Rashba QD in the presence of an external magnetic field [45]. As described by Eq. (23), the non-linear effects are enhanced by increasing the intensity of the external electric field. It is noteworthy that the spin-down resonances show a lower value for the critical intensity $I_c$ defined in Eq. (24). This behavior is due to $s = -1$ transitions being less energetic than those for $s = 1$. Therefore, the next optical harmonic can be reached with lower external energy.

In Fig. 7 the total absorption coefficient is plotted for all the possible transitions in the case $p = 3$. The same spin and intensity dependence as in Fig. 6 can be observed. In addition, one can notice that the total absorption coefficient corresponding to a transition with a change in the principal quantum number $|n - m| = 2$ is
FIG. 8: Refraction index changes $\Delta n/n_r$ as a function of the energy of the incident photon for the transition $|0,0,\pm 1\rangle \rightarrow |1,1,\pm 1\rangle$, using (a) $p = 1$ and (b) $p = 3$. In both cases, $\Delta l = 1$, $l = 1$, $B = 15T$, $\kappa = 1$ and $\gamma_s = 20nm^2$.

FIG. 9: Refraction index changes $\Delta n/n_r$ as a function of the pump photon energy $\hbar \omega_0$ for all allowed transitions from the ground state to other excited states for $p = 3$. The parameters $I = 0.3MW/cm^2$, $B = 15T$, $\kappa = 1$, and $\gamma_s = 20nm^2$ were fixed.

FIG. 10-(a) shows the variation of the critical intensity $I_c$, defined in Eq. (24), while Fig. 10-(b) displays the Schottky temperature $T_s$ that was defined in Eq. (18). Both critical parameters are presented as a function of the inverse kink parameter $p$ and expose a change in their behavior near $p \sim 2$. We interpret this property based on the fact that $p = 2$ is a special value where the wave functions have the highest symmetry. It is noticeable from Fig. 10-(a) that the non-linear effects are a non-trivial function of the inverse kink parameter $p$. Other authors have reported that such effects are caused by confinement, as this means a bigger overlap of the wave functions and therefore, bigger dipole matrix elements. Nevertheless, in the present case, the energy difference increases linearly with $p$ and thus the non-linear effects are maximal near $p = 2$ before showing a decrease due to confinement [41, 44].

The functionality shown in Fig. 10-(b) can be interpreted in terms of the lowest lying 2-level system typical for the Schottky anomaly. For a transition from the ground to the first excited state, any external source of energy needs to be comparable with the separation of the energy levels. In the absence of external fields, the very first transition is given by a change of spin orientation. Because of this, the inset in Fig. 10-(b) shows the energy difference between the two lowest energy levels $E(n = 0, l = 0, s = \pm 1)$ obtained by evaluating Eq. (14b) as a function of the inverse kink parameter $p$. For $p \sim 1.75$ the energy levels are close to each other and less external energy is necessary to excite the system. It is notable that the energy difference is linear in $p$, though the aspect as an absolute value function is caused by the transitions in the case $p = 3$. The same spin and intensity dependence as in Fig. 8 can be observed. In addition, one can notice that the $\Delta n/n_r$ corresponding to a transition with a change in the principal quantum number $|n - m| = 2$ is suppressed in magnitude and non-linear effects respect to the other transitions, in concordance with the behavior shown in Fig. 7.
FIG. 10: (a) Critical Intensity of Eqs. (24) of the incident electromagnetic wave as a function of the inverse kink parameter $p$ for different transitions. (b) Shottky temperature of Eqs. (18) as a function of $p$. The inset shows the variation of the GAP energy between the ground and the first excited state. In both figures we use the parameters $B = 15T$, $\kappa = 1$ y $\gamma_s = 20$\,nm$^2$.

The dependence on the inverse kink parameter $p$ shown in the inset of Fig. 10(b) does not exactly correspond to such of the main graph. The reason is that not only the thermal transition between the ground and the first excited state contribute to the Schottky anomaly, but higher energy states contribution cannot be neglected.

IV. SUMMARY

In this paper we explored the effect of introducing a conical disclination on the thermal and optical properties of a two dimensional Rashba quantum dot in the presence of a uniform and constant magnetic field. The obtained specific heat presents the Schottky anomaly in the low temperature regimen while the Schottky peaks are displaced linearly respect to the inverse kink parameter, which controls the topological defect. Such defect and the Rashba coupling modify the values of temperature and magnetic field in which the system behaves as a paramagnetic material. Furthermore, the introduction of a deficit angle relaxes the selection rules for the dipolar transitions, as the parity of the eigenfunctions is mixed. This unusual behavior can be observed in the absorption coefficient and the refraction index changes as semi-suppressed blue-shifted resonances. The spin dependence of the optical-properties caused by the Rashba SOI and the enrichment of the allowed transitions may open a set of technological applications. Finally, we introduced the concept of critical intensity as a measure of non-linear effects and showed that the topological defect enhances them approximately when half of the QD is sliced off.

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