Flavor Symmetry Breaking and Strangeness in the Nucleon

H. Weigel

Institute for Theoretical Physics, Tübingen University, D-72076 Tübingen, Germany

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Abstract. We suggest that breaking of SU(3) flavor symmetry mainly resides in the baryon wave–functions while the charge operators have no (or only small) explicit symmetry breaking components. We utilize the collective coordinate approach to support this picture. In particular we compute the $g_A/g_V$ ratios for hyperon beta–decay and the strangeness contribution to the nucleon axial current matrix elements and analyze their variation with increasing flavor symmetry breaking.

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1 Introduction and Motivation

There has been much interest in the strangeness content of the nucleon ever since the analysis of the DIS data [1] suggested a large (negative) polarization of strange quarks of the nucleon [2, 3], $\Delta S_N \approx -0.15$. This surprising result particularly relies on the assumption of flavor covariance for the axial current matrix elements of the $\frac{1}{2}^+$ baryons. This assumption originates from the feature that the Cabibbo scheme [4], that utilizes the flavor symmetric predictions are presented using $F = 0.459$ and $D = 0.799$. Analytic expressions which relate these parameters to the $g_A/g_V$ ratios may e.g. be found in table I of [5].

Table 1. The empirical values for the $g_A/g_V$ ratios of hyperon beta–decays [6], see also [5]. For $\Sigma \to \Lambda$ only $g_A$ is given. Also the flavor symmetric predictions are presented using $F = 0.459$ and $D = 0.799$. Analytic expressions which relate these parameters to the $g_A/g_V$ ratios may e.g. be found in table I of [5].

|         | $A \to p$       | $\Sigma \to n$ | $\Xi \to \Lambda$ |
|---------|-----------------|-----------------|-------------------|
| emp.    | 0.718 ± 0.015   | 0.340 ± 0.017   | 0.25 ± 0.05       |
| $F&D$   | 0.725 ± 0.009   | 0.339 ± 0.026   | 0.19 ± 0.02       |
| emp.    | 1.258 ± 0.158   | 0.61 ± 0.02     | 0.65 ± 0.01       |
| $F&D$   | 1.258 = $g_A$   |                 |                   |

Here we will investigate in how far this agreement justifies to carry over flavor covariance to strangeness conserving axial current matrix elements in order to disentangle the various quark flavor components of the nucleon axial current matrix element. This investigation requires baryon axial current matrix elements as functions of the (effective) strength of flavor symmetry breaking. This can be achieved within the three flavor version of the Skyrme model (and generalizations thereof) in which baryons emerge as solitons. In such models baryon states are constructed by quantizing the large amplitude fluctuations about the soliton and constructing exact eigenstates in the presence of symmetry breaking. We focus on a picture where symmetry breaking mainly resides in the baryon wave–functions, including important contributions which would be missed in a first order treatment. In contrast, we assume that the current operators, from which the charges are computed, are dominated by flavor covariant components. In a first step we do not specify the model Lagrangian but adjust the prefactors of the few possible flavor covariant components of the axial current operator to observables in hyperon beta–decay and analyze their matrix elements as functions of flavor symmetry breaking. We also present results obtained from a realistic vector meson soliton model that supports the suggested picture. Details omitted here may be traced from ref [6].

2 Symmetry Breaking in the Baryon Wave–Functions

The collective coordinates $A$ that parameterize the large amplitude fluctuations off the soliton are introduced via

$$U(r, t) = A(t)U_0(r)A^\dagger(t), \quad A(t) \in SU(3). \quad (1)$$

$U_0(r)$ describes the soliton embedded in the isospin subgroup. A prototype model Lagrangian for $U(r, t)$ consists of the Skyrme model supplemented by the Wess–Zumino–Witten term and suitable symmetry breaking pieces. We parameterize the collective coordinates by “Euler–angles”

$$A = D_2(\hat{I}) e^{-i\nu \lambda_4} D_2(\hat{R}) e^{-i(\nu/\sqrt{3})\lambda_8}. \quad (2)$$

Here $D_2$ denote the rotation matrices for rotations in isospin ($\hat{I}$) and coordinate–space ($\hat{R}$). Substituting (2) into the model Lagrangian yields upon canonical quantization the Hamiltonian for the collective coordinates $A$:

$$H = H_n + \frac{3}{4} \gamma \sin^2 \nu. \quad (3)$$
The symmetric piece of this collective Hamiltonian only contains Casimir operators and may be expressed in terms of the $SU(3)$–right generators $R_a (a = 1, \ldots, 8)$:

$$H_s = M c_1 + \frac{1}{2a^2} \sum_{i=1}^{3} R_i^2 + \frac{1}{2b^2} \sum_{\alpha=4}^{7} R_\alpha^2. \quad (4)$$

$M, a^2, b^2$ and $\gamma$ are functionals of the soliton, $U_0(r)$. The generators $R_a$ can be expressed in terms of derivatives with respect to the ‘Euler angles’. The essential feature of the parameterization (3) is that the flavor symmetry breaking part of the full Hamiltonian (2) only depends on the flavor changing angle $\nu$. Therefore the eigenvalue problem $H \Psi = \nu \Psi$ reduces to ordinary second order differential equations for isoscalar functions which only depend on $\nu$ (4). Solely the product $\omega^2 = \frac{4}{3} \gamma b^2$ appears in these differential equations as the effective strength of symmetry breaking on which the eigenfunctions of $H$ depend parameterically. A value in the range $5 \lesssim \omega^2 \lesssim 8$ is required to obtain reasonable agreement with the empirical mass differences for the $^{1+}_2$ and $^{3+}_2$ baryons (2). Such large a value for $\omega^2$ is without reach of a perturbation expansion as the resulting baryon wave–functions exhibit strong distortion from flavor covariance.

3 Charge Operators

In the soliton description the effect of the derivative type symmetry breaking terms is mainly indirect. They provide the splitting between the various decay constants and thus increase $\gamma$ since it is proportional to $f_K^2 m_K^2 - f^2 m_N^2 \approx 1.5 f^2 (m_K^2 - m_N^2)$. Otherwise the derivative type symmetry breaking terms are negligible. Whence symmetry breaking terms can be omitted in the current operators and the non–singlet axial charge operator is parameterized as $(a = 1, \ldots, 8, i = 1, 2, 3)$

$$\int d^3 r A_{i}^{(a)} = c_1 D_{ai} - c_2 D_{a8} R_i + c_3 \sum_{\alpha,\beta=4}^{7} d_{a\alpha\beta} D_{a\alpha} R_\beta,$$  

where $D_{ab} = \frac{1}{2} \text{tr} (\lambda_a \lambda_b A^\dagger)$. For $\omega^2 \to \infty$ (infinitely heavy strange degrees of freedom) the strangeness contribution to the nucleon axial charge should vanish. Noting that $\langle N \mid D_{83} \mid N \rangle \to 0$ and $\langle N \mid \sum_{\alpha,\beta=4}^{7} d_{a\alpha\beta} D_{a\alpha} R_\beta \mid N \rangle \to 0$ while $\langle N \mid D_{88} \mid N \rangle \to 1$ for $\omega^2 \to \infty$, we demand

$$\int d^3 r A_i^{(0)} = -2 \sqrt{3} c_2 R_i \quad i = 1, 2, 3 \quad (6)$$

for the axial singlet current because it leads to the strangeness projection, $A_i^{(s)} = (A_i^{(0)} - 2 \sqrt{3} A_i^{(8)})/3$ that vanishes for $\omega^2 \to \infty$. Actually all model calculations in the literature (10,12) are consistent with this relation between singlet and octet currents. The singlet current matrix element, $\Delta \Sigma_B = \sqrt{3} c_2$, is the quark spin contribution to the spin of the considered baryon, $B$. It is well known that the empirical value for the nucleon matrix element, $\Delta \Sigma_N \approx 0.20 \pm 0.10$ (3) is insensitive to the strength of flavor symmetry breaking (2). This suggests to adjust $c_2$ accordingly. In order to completely describe the hyperon beta–decays we also demand matrix elements of the vector charges. These are obtained from the operator

$$\int d^3 r V_0^{(a)} = \sum_{b=1}^{8} D_{ab} R_b = L_a,$$  

which introduces the $SU(3)$–left generators $L_a$.

The values for $g_A$ and $g_V$ (only $g_A$ for $\Sigma^+ \to \Lambda e^+ \nu_e$) are obtained from the matrix elements of the operators in eqs (3) and (6), respectively, sandwiched between the eigenstates of the full Hamiltonian (2). We still have to specify $c_1$ and $c_3$. We determine these two parameters such that nucleon axial charge, $g_A$ and the $g_A/g_V$ ratio for $\Lambda \to pe^-\bar{\nu}_e$ are reproduced at a prescribed strength of flavor symmetry breaking, $\omega^2_{\text{fix}} = 6.0$. Then we are not only left with predictions for the other decay parameters but we can in particular study the variation with symmetry breaking. This is shown in figure 4. The dependence on flavor symmetry breaking is very moderate, and the results can be viewed as reasonably agreeing with the empirical data, cf. table 4. The observed independence of $\omega^2$ shows that these predictions are not sensitive to the choice of $\omega^2_{\text{fix}}$. The two transitions, $n \to p$ and $\Lambda \to p$, which are not shown in figure 4, exhibit a similar negligible dependence on $\omega^2$. We therefore have a two parameter ($c_1$ and $c_3$, $c_2$ is fixed from $\Delta \Sigma_N$) fit of the hyperon beta–decays. Comparing the results in figure 4 with the data in table 4.

1. In this section we will not address the problem of the too small model prediction for $g_A$.

2. However, the individual matrix elements entering the ratios $g_A/g_V$ vary strongly with $\omega^2$. (8).
we see that the present calculation using the strongly distorted wave\textendash functions agrees equally well with the empirical data as the flavor symmetric F&D fit. On the other hand, the strangeness contribution to the nucleon axial current matrix element reduces from $\Delta S_N \approx -0.13$ in the symmetric treatment to $\Delta S_N \approx -0.07$ in the realistic case.

### 4 Model Calculation

We consider a realistic soliton model containing pseudoscalar and vector meson fields. It has been established for two flavors in ref [4] and been extended to three flavors in ref [1] where it has been shown to fairly well describe the parameters of hyperon beta\textendash decay (cf. table 4 in ref [1]). The model Lagrangian contains terms which involve the Levi\textendash Civita tensor $\epsilon_{\mu\nu\rho\sigma}$, to accommodate processes like $\omega \to 3\pi$. Such terms contribute to $c_2$ and $c_3$. A minimal set of symmetry breaking terms is included [6] to account for different masses and decay constants. They add symmetry breaking pieces to the axial charge operator,

$$\delta A_i^{(a)} = c_4 D_{a8} D_{8i} + c_5 \sum_{\alpha, \beta = 4}^7 d_{\alpha\beta} D_{a\alpha} D_{8\beta} + c_6 D_{a4}(D_{88} - 1),$$

$$\delta A_i^{(0)} = 2\sqrt{3} c_4 D_{8i}.$$  

Unfortunately the model parameters cannot be completely determined in the meson sector [4]. We use the remaining freedom to accommodate baryon properties in three different ways as shown in table 4. The set denoted by ‘b.f.’ refers to a best fit to the baryon spectrum. It predicts the axial charge somewhat on the low side, $g_A = 0.88$. The entry ‘mag. mom.’ labels parameters that yield magnetic moments close to the empirical data (with $g_A = 0.98$) and finally the set labeled ‘$g_A$’ reproduces the axial charge of the nucleon [4]. We observe that in particular the strangeness projection of the nucleon axial current is very small and depends only mildly on the model parameters. This confirms the above conclusion from the general structure of the axial current matrix elements that the strangeness admixture in the nucleon is significantly smaller than an analysis based on flavor covariance suggests. Also the predictions for the axial properties of the $\Lambda$ hyperon are quite insensitive to the model parameters. Sizable polarizations of the up and down quarks in the $\Lambda$ are predicted; comparable to those obtained from the SU(3) analysis [17] of the available data.

### 5 Conclusions

We have suggested a picture for the axial charges of the low\textendash lying $\frac{3}{2}^+$ baryons which manages to reasonably reproduce the empirical data without introducing (significant) flavor symmetry breaking components in the corresponding operators. Rather, a sizable symmetry breaking, as demanded by the baryon spectrum, resides almost completely in the baryon wave\textendash functions. In this picture the empirical data for hyperon beta\textendash decay are as reasonably reproduced as in the Cabibbo scheme. We emphasize that the present picture is not a re\textendash application of the Cabibbo scheme since here the ‘octet’ baryon wave\textendash functions have significant admixture of higher dimensional representations. Especially, when compared with the flavor covariant treatment, the present approach predicts a sizable suppression of strangeness in the nucleon.

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