Gravitation as deduced from submicroscopic quantum mechanics

Volodymyr Krasnoholovets

Institute of Physics, National Academy of Sciences, Prospect Nauky 46, UA-03028 Kyїv, Ukraine
(web page http://www.inerton.kiev.ua)

16 May 2002, 4 April 2003, 23 April 2004, 25 April 2005

Abstract

Based on the model of a "soft" cellular space and deterministic quantum mechanics developed previously, the scattering of a free moving particle by structural units of the space – superparticles – is studied herein. The process of energy and inert mass transmission from the moving particle to superparticles and hence the creation of elementary excitations of the space – inertons – are analyzed in detail. The space crystallite made up around a particle in the degenerate space is shown to play the key role in those processes. A comprehensive analysis of the nature of the origin of gravitation, the particle’s gravitational potential $1/r$, and the gravitational interaction between material objects is performed. It seems reasonably to say that the main idea of the work may briefly be stated in the words: No motion, no gravity.

Key words: space, gravitation, inertons, mass dynamics, quantum theory

PACS: 03.65.Bz Foundations, theory of measurement, miscellaneous theories; 03.75.-b Matter waves; 04.60.-m Quantum gravity

The development of knowledge brings other results than those ignorance cultivates. This we learned from sages.

Śrī Īṣopanishad, Mantra ten

1 Introduction

Gravitation still remains the most obscure theme in physics. Notwithstanding this, the conceptual foundations of general relativity and quantum mechanics allow concrete modifications in each other in the interface region, Ahluwalia [1]. Describing conceivable new generation quantum-gravity experiments in certain atomic systems, Ahluwalia [2] then notes that they are based on the possibility that quantum gravity
might affect the nature of general symmetry, or that the theory of general relativity itself may not provide a complete description of gravitation.

Many fundamental problems of gravity and the quantum behavior of the matter have been raised also in remarkable review by Sorkin [3]. In review [4], Dowker and Sorkin investigate some quantum properties of spatial topological geons, particles in 3+1 quantum gravity. Geons allow the construction of spin statistics both fermionic and bosonic and this seems to be a way to an understanding the inner construction of Nature. It is interesting that the authors note that geons are found in some common environment. But what kind of environment? May geons interact with the environment? In particular, what is the law of the motion of the geon? Of course, these and other questions need separate studies. However, the fact of the presence of an environment has engaged a special attention. This signifies that the spacetime, or a vacuum is endowed with structural properties that turn us out to a certain aether, but rather quantum one.

In recent years several serious attempts to re-introduce an aether in physics have been made [5-11]. Similarly, in the previous works of the author [12-19] a simulation of a vacuum in the form of a cellular elastic space has been proposed and deterministic quantum mechanics based on the strong interaction of a moving particle with such a space net has been constructed. The real space has been suggested to consist of peculiar super densely packed superparticles, or balls, which are found in the degenerate state over all the multiplets. A particle is created from a superparticle and hence the particle is interpreted as a local deformation of the degenerate space. Such idea agrees very well with the mathematical study of space carried out by Bounias and Bonaly [20,21] (see also Ref. [22]) who have shown basing on topology and set theory that the necessity of the existence of the empty set leads to the topological spaces resulting in a "physical universe". This allows the investigation of links between physical existence, observability and information. Thus the empty hyperset provides for a formal structure that correlates with the degenerate cell of space and supports conditions for the existence of a universe. Magnon [23] following Bounias also pointed out about some "primordial cell" and "existential principle".

Moreover, the surprising thing is that the information on the existence of the supreme substance and the first cause of matter in the form of an indivisible thing is contained in Bhagavad-gītā (see also Bhaktivedanta Swami Prabhupada [24]): "In spite of that material body is subjected to destruction, [the subtle particle] is eternal" [Bg. 2.18]; "It never takes birth and never dies at any time nor does it come into being again when the material body is created. It is birthless, eternal, imperishable and timeless and is inviolable when the body is destroyed" [Bg. 2.20]; "After some time it is disenthralled by entirely annihilation of the material body. Yet it endures the destruction of the material world" [Bg. 2.22]; "It is not fissionable, not burning out, not soluble, and not drying up" [Bg. 2.23]; "Since it is not visible, its entity does not change, its properties remain unchangeable" [Bg. 2.24]; "Yet there is another nature, which is eternal and is transcendental to this manifested and unmanifested matter. It is supreme and is never annihilated. When all in this world is annihilated, that part remains as it is" [Bg. 8.20].

There are also other approaches introducing an aether and those ones that aims to the examination of properties of space-time at very short distances. In particular, Amelino-Camelia notes that the nature of space-time has to show a "fuzziness"
at distances close to the Planck one [25] and at these distances particles may be described as geometry ”defects” [26]. ”Foamy” quantum gravity fluctuations are studied in Refs. [27,28] and such fluctuations seems can be to modify particle propagation in an observable way [29]. Semi-classical space-time is emerged in canonical quantum gravity in the loop presentation as a polymer-like structure at microscales, which allows a possible correction to the Maxwell equations caused by quantum gravity [30]. Matone [31] has studied the quantum Hamilton-Jacobi equation for a system of two particles and concluded that gravitation in such system had a pure quantum-mechanical origin.

Coming back to the author’s concept of the constitution of space and the creation of elementary particles in it [12-19], we should note that the approach supposes the formation of a deformation coat around the created particle [12-16]. The coat differs from the degenerate space in that its superparticles possess mass. Therefore the coat may be called the space crystallite. Its size corresponds to the Compton wavelength $\lambda_0$ of the created particle, $\lambda_0 = h/M_0c$ [14]. According to the definition [12,13], the induction of mass means that the volume of a superparticle changes from its volume in the degenerate space. In other words, if we set that a superparticle constricts with deformation, the mass will be defined as the ratio of superparticle’s initial and final volumes, $m_0 \propto V/V_0^{sup}$ for the massive superparticle and $M_0 \propto V/V_0^{par}$ for the particle, where $V$ is the typical volume of a degenerate superparticle and the volumes of a deformed superparticle and a particle are respectively $V_0^{sup}$ and $V_0^{par}$. When a particle moves, the crystallite travels together with it. However superparticles themselves are motionless: the crystallite state migrates by a relay mechanism. The rearrangement of superparticles due to the particle’s motion takes place with a velocity that equals or exceeds the speed of light $c$, but the motion of the particle itself occurs with the velocity $v_0 < c$ (hereinafter $v_0$ designates the initial particle’s velocity, which the particle acquires at a momentary push). The moving particle emits and absorbs elementary excitations of the space – inertons, which appear as a result of friction that the particle undergoes when moves against superclosely packed superparticles. Why do inertons not leave the particle totally? Why do they come backwards again? This is because they carry not only mass but electromagnetic polarization as well as particles are charged. However, this is the subject of a separate study. Here we only point out that the velocity $c_{free}$ of an absolutely free inerton that might migrate in the degenerate space should exceed the velocity of light $c$, perhaps several times. Accordingly, the initial velocity $\dot{c}$ of electromagnetic polarized inertons at which they are emitted from the moving particle and accompany it, is not able to reach the threshold value $c_{free}$ and probably $c < \dot{c} < c_{free}$. At the same time the mean value of the velocity of the particle’s inertons over a period still remains less then the velocity of light $c$ even when the velocity $v_0$ of the particle approaches to $c$ (in fact $\frac{1}{2}(\dot{c} + 0) < c$ even at $\dot{c} = \sqrt{c^2 + c^2}$, see expression (7) below).

Detailed theoretical consideration of the motion of a canonical particle has shown [12-14] that owing to the interaction with superparticles the particle looses its kinetic energy on the section $\lambda/2$ of the particle path where $\lambda$ is the amplitude of spatial oscillations of the particle (the de Broglie wavelength). The lost energy is spent on the creation of inertons. On the next section $\lambda/2$ the particle absorbing inertons acquires the velocity $v_0$, and so on. Thus inertons form a substructure of the matter
waves. It should be emphasized that the major theoretical prediction, the existence of clouds of inertons surrounding particles such as electrons and atoms, indeed, has recently been substantiated in a number of experiments [17-19].

In addition to friction, or inertia of the space, which results into the generation of inertons, the moving particle undergoes the dynamic pressure on the side of the whole space [13] (the pressure acts on the particle through its coat). The space pressure causes an additional deformation in the particle volume, $V_{\text{part}}^0 \rightarrow V_{\text{part}}^0 \sqrt{1 - v_0^2/c^2}$, that is, the pressure induces the so-called relativistic mass $M = M_0 / \sqrt{1 - v_0^2/c^2}$. Here we write the speed of light $c$, because the electromagnetic polarization that accompanies any particle imposes a limitation on the speed of inertons. Since the value of $v_0$ varies in the spatial interval $\lambda$ of the particle path, and this is one of the peculiarities of the model constructed, the particle energy should periodically change as well. The energy passing from the particle to its inerton cloud is given by the kinetic energy of the particle $\frac{1}{2}Mv^2$. At this moment the particle mass changes from $Mc^2$ to $Mc^2 - \frac{1}{2}Mv^2$ and hence the behavior of the mass obeys the law

$$M \rightarrow M_0 \rightarrow M \rightarrow M_0 \rightarrow \ldots$$

That is, the value of mass oscillates along the particle path within the spatial period, or amplitude $\lambda$.

De Broglie [32] was the first to indicate that the corpuscle dynamics was the basis for the wave mechanics. With the variational principle, he obtained and studied the equations of motion of a massive point reasoning from the typical Lagrangian

$$\mathcal{L} = -M_0 c^2 \sqrt{1 - v^2/c^2}$$

in which, however, the velocity $v$ of the point was constant along a path. The study showed that the dynamics had the characteristics of the dynamics of the particles with a variable proper mass. Oudet [33] conjectured similar peculiarities for the electron. Papini and Wood [34] studied a geometrical solution to the de Broglie variable mass problem. What is more, the de Broglie view is well substantiated as it immediately follows from the Schrödinger equation: the equation giving quantum solutions contains a pure classical parameter – the unchangeable particle mass.

Based on the theory developed in recent author’s works [12-19], the present paper shows that the creation of inertons from a moving particle is stipulated by the existence of the space crystallite around the particle. The de Broglie’s idea and the author’s previous results are taken as the starting point. The study includes an extensive description of the processes of the energy and mass transmission from the particle to superparticles when the particle and coming superparticles collide. Besides for the first time the theory arises the question, how is the gravitational potential induced by a particle/object in the ambient space? It is read that such induction is caused by inertons enclosing any material object. Thereby it is the dynamic inerton field that is responsible for the generation of the static Newton potential $1/r$. The appearance of the gravitational interaction between both particles and material objects is elucidated in some detail.
### 2 Emission of inertons

For the solution of equations of motion of the particle and inertons the relation

\[ M \dot{X}_i^2 = m_i \hat{c}^2 \]  

has been used in the preceding papers of the author [12-14]. In (1) \( M \) is the particle mass, \( \dot{X}_i \) is the vector of the particle velocity at the moment of the \( i \)th inerton emission (\( X_i \) is the radius vector of the particle and the dot over \( X_i \) means the differentiation in respect to the proper time of the particle), \( m_i \) is the mass of the \( i \)th inerton and \( \hat{c} \) is its initial velocity. Relation (1) is the consequence of the intersection of geodesics of the particle and the \( i \)th inerton. At the moment of the \( i \)th collision of the particle and the superparticle, the former emits the inerton, as follows from (1), whose energy is equal to the double kinetic energy of the particle itself. It turns out that a moving object emits an enormous quantity of inertons \( N \) within a half-period of its spatial oscillation \( N = \lambda / V^{1/3} \) where \( \lambda \) is the spatial period identical to the de Broglie wavelength and \( V^{1/3} \sim 10^{-30} \) m, or \( V^{1/3} \sim l_{\text{Planck}} \approx 10^{-35} \) m, is the suggested size of the superparticle). So the energy of the emitted inerton cloud is of the order of the huge magnitude \( NM \hat{c}^2/2 \).

This situation is possible when the \( i \)th inerton gains the energy because the medium around the particle is in the vibrating state rather than because of loss in kinetic energy of the particle itself (its energy suffices for the creation of one inerton only [12]). The availability of the crystallite in the space around the particle has been proven theoretically in paper [14]. The crystallite’s superparticles are characterized by mass. Hence collective vibrations of massive superparticles, similarly to vibrations of atoms in an ordinary solid crystal, should be inherent to the crystallite. Therefore, vibrating superparticles are able to strike the particle in such a way that the particle will loss energy in collisions and then will generate excitations in the surrounding. Excitations caught by the vibrating superparticles will carry away from the particle.

At the formal consideration of quantum gravity, for instance in the model of D-brane string solutions, researchers also face the problem of scattering. Kabat and Pouliot [35] treated the zero-brane dynamics in which one could probe distances much shorter than the string space. Ellis et al. [36] have studied the gravitational recoil effects induced by energetic particles. In their approach defects in space-time are derived as an energy-dependent refractive index and D-brane foam has corresponded to a minimum-uncertainty wave-packet. The shift of D-brane particle has been induced by the scattering after interaction with a closed-string state.

Let us treat now the process of the particle scattering by superparticles in the model discussed in detail.

Two relationships were previously derived [13,14]:

\[ \Lambda = \lambda \hat{c} / v_0, \]  

\[ \Lambda = \tilde{\lambda} v_0 \hat{c}^2 / v_0^2. \]  

Here expression (2) connects the spatial period \( \lambda \) of the particle oscillations (the de Broglie wavelength) with the specific enveloping amplitude \( \Lambda \) of the inerton ensem-
ble. Expression (3) connects the effective size of the crystallite \( \bar{\lambda}_{v_0} \), defined by the Compton wavelength of the particle, with the specific amplitude \( \Lambda \) of inertons.

However the crystallite is dynamic: along the particle’s velocity vector permanently occurs the relay readjustment of superparticles from the massless to massive state and again to the massless one (Fig. 1). The resultant of the movement of superparticles is directed antiparallel to the vector \( \vec{v}_0 \). Thus along the line of the particle motion the state of superparticles changes dynamically: all the time in the course of the particle motion superparticles change their state from the degenerate one to the massive one. But in transversal directions the state of superparticles remains practically unaltered: superparticles surrounding the particle continuously save the same massive state, i.e., in these directions superparticles in the crystallite might be considered as hard. Since in any crystal atoms vibrate, massive superparticles should vibrate in the said crystallite as well. However from the pattern above it turns out that superparticles in the crystallite suffer rather pure transversal vibrations – their equilibrium positions transversely vibrate in reference to the vector \( \vec{v}_0 \). Such transversal vibrations of the crystallite we may call the transversal vibrating mode.

The elasticity constant \( \gamma \) of the crystallite and the average mass \( m_{cr} \) of a crystallite’s superparticle determine the cyclic frequency of these collective vibrations [14]

\[
\omega_{k_{v_0}} = \sqrt{\gamma/m_{cr}}.
\]

Note that the value of \( \gamma \) is given by the particle mass at rest \( M_0 \), so \( \gamma \) is not universal.

Now one can reproduce in detail the picture of the particle motion with the emission and absorption of inertons. Let us assume that at some initial moment, the particle has the velocity \( v_0 \) and mass \( M = M_0/\sqrt{1-v_0^2/c^2} \), equivalently, the particle is characterized by the additional deformation to its own volume \( V_0^{\text{part}} \) due to the received velocity \( v_0 \): \( V_0 \rightarrow V^{\text{part}} = V_0^{\text{part}} \sqrt{1-v_0^2/c^2} \). The particle in motion runs into superparticles of the crystallite, which vibrate in directions transversal to the vector \( \vec{v}_0 \). The superparticles’ vibratory longwave modes can be considered as a
longwave excitation of the crystallite:

$$k v_0 a \ll 1$$  \hspace{1cm} (5)\

where \( k v_0 = \pi / \lambda v_0 \) is the wave number and \( a \) is the size of a superparticle in the crystallite. The longwave approximation (5) is true not only for \( v_0 \ll c \) but also for the velocity \( v_0 \) close to \( c \). The violation of inequality (5) happens only at the up-relativistic velocity \( v_0 \) of the particle when \( \lambda v_0 \) is brought near to \( a \) (but this value \( v_0 \) is certainly unattainable experimentally). Note that here we have the absolute analogy with the solid because it is known from the theory of solid state (see, e.g. Ref. [37]) that longwave excitations are elastic waves of the medium. The velocity of these waves – the sound velocity \( v_{\text{sound}} \) – is determined from the relation

$$v_{\text{sound}} = \bar{a} \sqrt{\bar{\gamma}/\bar{M}}$$

where \( \bar{a} \) is the lattice constant, \( \bar{M} \) is the atom mass and \( \bar{\gamma} \) is the elastic constant of a crystal. In this approximation, the group and phase velocities coincide.

The vibration energy saved in the crystallite of a moving particle can be found from the equation [14]

$$\hbar \omega_{k v_0} = M c^2$$  \hspace{1cm} (6)\

where \( M = M_0 / \sqrt{1 - v_0^2/c^2} \) is the total mass (we ignore the energy that stemming from the particle spin, see Ref. [14] for details). Essentially, collisions of the particle with the crystallite mode represent an impetuous attack (with the speed of \( c \)) to which the particle is subjected on the part of the front of this mode. Each of the \( i \)th action of collisions results in the emission of the \( i \)th inerton: the mode knocks out the inerton from the particle. The \( i \)th inerton has a mass \( m_i \) and two velocity components: \( \dot{x}_i^\parallel \) directed perpendicular to the vector \( \vec{v}_0 \) and \( \dot{x}_i^\parallel \) along this vector. It is obvious that the component \( \dot{x}_i^\parallel \) passes over from the particle to the \( i \)th inerton; the component \( \dot{x}_i^\perp \) is caused by the momentum of the crystallite mode. Supposing the speed \( c \) is the velocity of the crystallite mode, we get relation

$$\sqrt{(\dot{x}_i^\parallel)^2 + (\dot{x}_i^\perp)^2} = \dot{c}, \quad \text{or} \quad \sqrt{(\dot{x}_i^\parallel)^2 + c^2} = \dot{c}. \hspace{1cm} (7)$$

Let \( m_i \dot{c}^2 \) be the total energy of the \( i \)th inerton. After a series of \( N \) collisions of the particle with the mode, the energy of the latter should gradually decrease:

$$\hbar \omega_{v_0} > \cdots > \hbar \omega_i > \hbar \omega_{i+1} > \cdots > \hbar \omega_N.$$  

Therefore, the mode spends the energy \( \hbar \omega_i \) for splitting of the particle and the creation of the inerton that appears with the same energy \( \hbar \omega_i = m_i \dot{c}^2 \). Immediately after the \( i \)th collision the whole space net readjusts the crystallite to a new quasi-equilibrium state in which the vibration energy \( \hbar \omega_{i+1} \) corresponds to a new total particle energy \( M_{i+1} \dot{c}^2 \). And then again a collision occurs and the energy passes from the crystallite mode to the \((i+1)\)th inerton, etc. until the particle stops after the \( N \)th collision, i.e., when its velocity and inert mass are exhausted, \( \dot{X} \rightarrow 0, M_i \rightarrow M_0 \).

However, how is the inerton mass created? Obviously the process of mass creation takes place at the sacrifice of the particle mass. It should be assumed that at the \( i \)th collision of the particle with the mode, the particle loses a portion \( \delta V_i \) of its initial relativistic deformation \( V_0^{\text{part}} \sqrt{1 - v_0^2/c^2} \), or, in other words, the relativistic mass \( M \) decreases on a value of the inerton mass \( m_i \) created on the \( i \)th coming superparticle. Thus the particle mass also acquires the index \( i \) and the generalization
for expression (1) should be the relationship

\[ M_i \dot{X}_i^2 = m_i \dot{c}^2. \tag{8} \]

Now let us dwell on the inerton motion. The inerton is created in the elastic medium, i.e. crystallite, which is specified by the elasticity constant \( \gamma \). The constant \( \gamma \) determines the interaction of the created inerton with the elastic force of the crystallite. As a result of such an interaction, the elastic force aspires to make the inerton return to the particle. Thereby the force sets the inerton into oscillation along the line which is superimposed with the vector of the initial inerton velocity \( \vec{c} \). The cycle frequency of oscillation of the \( i \)th inerton is

\[ \omega_i = \sqrt{\gamma/m_i}. \tag{9} \]

The maximum distance to which the inerton migrates from the particle is the amplitude \( \Lambda_i \), or \( \Lambda_i = \dot{c}/(\omega_i/2\pi) \). Behind the crystallite boundary (in the case when \( v_0 < c \), i.e., when \( \Lambda_i \) prevails the Compton wavelength \( \lambda_{v_0} \)) the inerton is guided by the degenerate space whose elasticity is adjusted to the mass of the moving inerton in conformity with relation (9), i.e., \( \gamma_{\text{space}} = \gamma = \omega_i^2 m_i \). Note once again that \( \gamma \) is not a universal parameter of the universe. This is a consequence of the axiom [12,13] of adiabatic motion of particles/quasi-particles in space when a moving object does not leave faults into the passed range of the cellular space. In this event spatial oscillations of the particle/quasi-particle are exemplified by the adiabatic invariant of space, Planck's constant \( \hbar \), and in the case of our inerton this adiabatic invariant can be written as \( \hbar = (\frac{1}{2} m_i \dot{x}_i^2)/\omega_i \).

On the other hand, in papers [12-14] we characterized the \( i \)th inerton by the frequency of its collisions with the particle, \( 1/T_i \). Therefore we can relate this frequency to the oscillation frequency (9), i.e. \( \omega_i = 2\pi/2T_i \). The parameter \( 1/T_i \) provided for the energy exchange between the particle and the \( i \)th inerton. It is this parameter that allowed us to obtain the periodicity in solutions for dynamic variables of the particle and inertons. According to these solutions, within even time half-period of collisions \( T_i/2 \) and the spatial one \( \lambda_i/2 \), the particle is scattered by the crystallite vibrations with the subsequent formation of inertons. Within odd half-period the inertons are absorbed by the particle; they transmit the mass and the longitudinal component of the velocity to the particle, i.e., the inertons guide the particle. The guided particle is followed by the restoration of the vibrating crystallite mode, or in other words, the whole degenerate space restores the crystallite state to the initial dynamic state at which the particle possesses the initial velocity vector \( \vec{v}_0 \).

We have stressed that the energy of the emitted inerton is in the direct proportion to the particle energy at the moment of collision with the crystallite mode. Since both the energy of the particle and that of the mode decrease from collision to collision, the same relation should be true for the energy of inertons. It follows that the inequality \( m_{i+1} < m_i \) holds for the mass of emitted inertons.

The value of mass of inertons, which carry out inert and gravitational properties of particles, has been evaluated in paper [38]. It has been shown that masses of inertons emitted and then absorbed by a moving particle are not strongly fixed but distributed in a wide spectral range much as the photon frequency varies from zero to the frequency of high-level \( \gamma \)-photon.
3 Mass dynamics

In our model the canonical particle is considered as a stationary deformed elementary cell, or superparticle, of the real space. Hence the origin of the initial particle velocity $\vec{v}_0$ is not a moving point in the space as in contemporary geometry [39] but a cell whose volume is different from that in the degenerate state. Thereby we can decompose real space in two subspaces: the external and internal subspaces. Namely, in the first space the entire cell is considered; it is characterized by an observable trajectory $l$ and the vector of the particle velocity $\vec{v}_0$ which sticks out of the particled cell belongs to this subspace as well. The second space represents the cell itself: the cell’s size, its inner substructure, degree of its global surface curvature (or deformation), and hidden motion of the cell’s centre-of-mass. For example, the notion of spin-1/2 determined as the proper pulsation of the particle [14] should be related to the internal space. Thus a moving particled cell is scattered by surrounding superparticled cells and produces changes in both the external and internal characteristics.

So, due to the interaction between a moving particle and oncoming superparticles the particle undergoes a peculiar splitting, or fission. The particle mass decays and is spent for the creation of inertons. Apparently this is the process that falls under the study in the internal space.

The particle moves in the external space and its behavior here is characterized by its proper time $t = l/v_0$, where $l$ and $v_0$ is the particle trajectory and the particle velocity respectively. Let us refer all processes which occur in the internal space to the proper time $t$ of the particle. This means that the dynamics of a global cellular deformation of the particled cell, i.e., the dynamics of the inert mass $M$, should be treated as a function of $t$. Thus two masses may be distinguished in our system: the particle mass $M$ and the mass of particle’s inerton cloud $m$. Besides we have to operate with the rate of change of mass per time $t$. For this purpose, let us introduce values $\dot{M}$ and $\dot{m}$, which are the rates of change of the particle inert mass and the inerton cloud mass respectively.

3.1 Longitudinal mass

To describe a dynamics of the particle mass along the particle path $l$, we need a model Lagrangian written in the form equivalent to the classical one. Let us start from the following specific Lagrangian of the internal space, i.e., the original mass function

$$L_\parallel = \dot{M}_\parallel^2/2 + \dot{m}_\parallel^2/2 - \frac{\pi}{T} m_\parallel \dot{M}_\parallel.$$  

(10)

Here, the first and the second terms are peculiar kinds of 'kinetic energies' of the particle inert mass and its inerton cloud mass respectively; the third term describes the mass exchange; $\pi/T$ is the cyclic frequency of collisions between the particle and the inerton cloud.

The Lagrangian (10) of the internal space is similar in its form to the Lagrangians of the external space used for the particle motion in papers [12,13,16]. The Euler-Lagrange equations of motion are as follows

$$\ddot{M}_\parallel - \frac{\pi}{T} \dot{m}_\parallel = 0,$$  

(11)
\[ \ddot{m}_\parallel + \frac{\pi}{T} \dot{M}_\parallel = 0 \]  

(12)

If we pay attention to the fact that at the initial moment \( t = 0 \) the total mass was concentrated in the particle, we come to the solution

\[ M_\parallel = \mu_\parallel \cdot |\cos(\pi t/T)|, \]  

(13)

\[ m_\parallel = \mu_\parallel \cdot |\sin(\pi t/T)| \]  

(14)

where the amplitude, i.e. the value of the exchange mass is

\[ \mu_\parallel = M_0 \cdot \left(1/\sqrt{1-v_0^2/c^2} - 1\right) \]  

(15)

As follows from solutions (13) to (15), the mass of the particle changes periodically along its path from the initial value \( M = M_0/\sqrt{1-v_0^2/c^2} \) in node points, which corresponds to moments of time \( t = nT \) where \( n = 0, 1, 2, \ldots \), to the rest mass \( M_0 \) in antinode points.

### 3.2 Transversal mass

To the careful observer, the solutions (13) - (15) correct only along the particle path \( l \). However, the particle mass obeys changes also in transversal directions and expression (7) indicates conclusively that its change is different from that prescribed by the solution (13) and (15).

Indeed, if in the longitudinal direction the velocity of inertons is \( \dot{x}_\parallel = v_0 \), in transversal directions the velocity is other, \( \dot{x}_\perp = c \) (7), and, therefore, in the latter case the Lagrangian must also be distinguished from (10). Namely, it has to look as follows

\[ L_\perp = \dot{M}_\perp^2/2 + \dot{m}_\perp^2/2 - \frac{\pi}{T} m_\perp \dot{M}_\perp \]  

(16)

where the first and the second terms describe the 'kinetic energies' of the particle proper mass and its inerton cloud mass respectively; the third term describes the mass exchange; \( \pi/T \) is the same cyclic frequency of collisions between the particle and the inerton cloud.

The solutions to the Euler-Lagrange equations derived on the basis of the Lagrangian (16) are

\[ M_\perp = \mu_\perp \cdot |\cos(\pi t/T)|, \]  

(17)

\[ m_\perp = \mu_\perp \cdot |\sin(\pi t/T)| \]  

(18)

where the amplitude, i.e. the value of the exchange mass is

\[ \mu_\perp = M_0. \]  

(19)

Consequently, the particle mass is pumped over from the particle to the inerton cloud, i.e. to the ambient space, and then comes back from the cloud to the particle. Thus, the periodical transfer of the particle mass occurs by a tensorial law: it varies from \( M \equiv M_0\sqrt{1-v_0^2/c^2} \) to \( M_0 \) along the particle pass \( l \) and changes from \( M_0 \) to 0 in transversal directions.
Figure 2: Two limiting cases for the state of an inerton from the particle’s environment in the space net: (a) the deformation, i.e., the volume change is localized in the cell (here, the inerton mass \( m_i \neq 0 \)); (b) there is no deformation in the cell (here, \( m_i = 0 \), or in other words, the volume of the cell does not distinguish from that of nearest cells) while the local deformation is completely transferred into the rugosity of the space net.

4 Mass dynamics of inertons

In the two previous sections inertons have been treated as quasi-particles which migrate hoping from superparticle to superparticle by a relay mechanism. At the same time, it is the vibrating motion of crystallite’s superparticles that knocks inertons out of the particle. Therefore it is reasonable to attempt to consider the splitting of the particle mass and its transformation to the ensemble of inertons from the point of view of wave process.

The \( i \)th inerton created by the particle starts to migrate having a mass \( m_i \). The appearance of the \( i \)th inerton means that the superparticle next to the particle takes on the deformation, on the mass \( m_i \). In other words, the volume of the corresponding superparticle located at this place is compressed. So the inerton, i.e. the mass \( m_i \), starts to barrel in with the initial velocity \( \hat{c} \) from superparticle to superparticle. Space gradually brakes the moving inerton and that is why the value of inerton mass \( m_i \) decreases with \( r \) passing to zero at distance \( \Lambda_i \) from the particle. At the same time the substrate elastically wrinkles. In other words, the local deformation is sensibly transformed into the rugosity [40] of the cellular space net, which reaches the maximum value at \( \Lambda_i \). Then the space net begins to straighten the rugosity and initiates the reverse inerton motion: the rugosity grades again into the local deformation, the inerton takes on the original mass \( m_i \) and after all it arrives to the particle and returns the mass \( m_i \) to it. It stands to reason that for the discrete elastic medium the most straightforward pattern for the motion of the \( i \)th inerton from the particle’s environment can be presented in Fig. 2.

In such a way in the system studied we can distinguish two variables: the inerton mass \( m_i \) that describes a compressed cell of the space net (Fig 2, a) and the space net rugosity \( \vec{u}_i \) that may be regarded as some kind of a cell displacement (Fig. 2, b). The mass \( m_i \) is viewed as a function of the proper time \( t_i \) of the \( i \)th inerton. The rugosity is supposed is a function of the radius vector \( \vec{x}_i \) of the \( i \)th inerton and its proper time \( t_i \), i.e. \( \vec{u}_i = \vec{u}_i(\vec{x}_i, t) \). Further we will deal with dimensionless variables \( \varrho_i = m_i(\vec{x}_i, t)/m_{i0} \) (here \( m_{i0} \) is the initial maximum value of mass of the
ith inerton) and $\vec{\xi}_i = \vec{u}_i(\vec{x}_i, t)/(V^\text{sup}_0)^{1/3}$, which may be considered as two potentials of the inerton.

Let us study a dynamics of the system in the framework of the following specific Lagrangian

$$L = \sum_i \left[ \frac{1}{2} \ddot{\varrho}_i^2 + \frac{1}{2} \dddot{\varrho}_i^2 - \hat{c} \dddot{\varrho}_i \nabla \vec{\xi}_i \right]. \quad (20)$$

Here, the sum spreads to all emitted inertons; the first two terms are the rates of change of the mass $\varrho_i$ and the rugosity $\vec{\xi}_i$ respectively, the third term describes their interaction. In the Lagrangian (16) the parameter $\hat{c}$ specifies the speed of interplay between the two potentials, namely, the mass potential and the rugosity one.

The Lagrangian (20) includes the function $\nabla \vec{\xi}_i$. In this case, the Euler-Lagrange equations take the form (see, e.g. ter Haar [41]):

$$\frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{Q}} - \frac{\delta L}{\delta Q} = 0 \quad (21)$$

where the functional derivative

$$\frac{\delta L}{\delta Q} = \frac{\partial L}{\partial Q} - \frac{\partial}{\partial x} \left( \frac{\partial L}{\partial (\frac{\partial Q}{\partial x})} \right) - \frac{\partial}{\partial y} \left( \frac{\partial L}{\partial (\frac{\partial Q}{\partial y})} \right) - \frac{\partial}{\partial z} \left( \frac{\partial L}{\partial (\frac{\partial Q}{\partial z})} \right). \quad (22)$$

For the Lagrangian (20) Eqs. (21) and (22) yield the following two equations:

$$\frac{\partial^2 \varrho_i}{\partial t^2} - \hat{c} \nabla \dddot{\varrho}_i = 0; \quad (23)$$

$$\frac{\partial^2 \vec{\xi}_i}{\partial t^2} - \hat{c} \nabla \dddot{\varrho}_i = 0. \quad (24)$$

These equations being uncoupled change to the following

$$\frac{\partial^2 \varrho_i}{\partial t^2} - \hat{c}^2 \nabla^2 \varrho_i = 0; \quad (25)$$

$$\frac{\partial^2 \vec{\xi}_i}{\partial t^2} - \hat{c}^2 \nabla \cdot \nabla \vec{\xi}_i = 0 \quad (26)$$

(in general case the right hand sides of Eqs. (25) and (26) are equal to constants $C_1$ and $C_2$, however, the appropriate initial and boundary conditions for $\varrho_i$ and $\vec{\xi}_i$ make it possible to put $C_1, C_2 = 0$).

Eq. (25) is a typical wave equation; in our case it describes the behavior of the mass potential $\varrho_i$ of the $i$th inerton. Thus this characteristic, the mass of the inerton, indeed changes periodically as we initially suggested. Let us treat the solution to equation (25), because it is this equation that is most informative at the study of the mass behavior of the particle’s inertons. The initial conditions are

$$\varrho_i(\vec{x}_i, 0) = \varrho(\vec{x}_i); \quad \frac{\partial \varrho_i(\vec{x}_i, 0)}{\partial t} = 0. \quad (27)$$
The Cauchy problem to Eq. (25) with conditions (27) reduces to the problem on oscillations of a string with the fixed end \( \vec{x}_i = 0 \). In our case the solution – in the form of a standing wave – will show how the value of inerton mass decreases along the inerton path, from \( x_i = 0 \) to \( x_i = \Lambda_i \).

Now let us turn to the generalized problem, namely, let us treat the behavior of the mass of particle’s inerton cloud considered as a single object. The single cloud of inertons is periodically emitted by the particle and then absorbed again. Besides one should go on to the proper time \( t \) of the inerton cloud and to the cloud amplitude \( \Lambda \) that is found from relationship (2) (note that such a procedure was already made in papers [13,14] for other problems). We may describe such a system by the equation

\[
\varrho_{tt} - \hat{\gamma}^2 \Delta \varrho = 0,  \tag{28}
\]

which is the generalized one to Eq. (25). Here \( \varrho \) comprises the total mass potential of the inerton cloud (and hence the total inerton mass (11), i.e. \( \varrho = m/\mu \)). Apparently, the system studied features the central symmetry in respect to the particle since inertons issue from the particle throughout the azimuth angle \( 2\pi \) around its path (in a truncated view, it may be called the radial symmetry of the cloud around the supposedly motionless particle). Nevertheless the distribution of inerton trajectories, peculiar rays-strings, and the cause of this distribution is not discussed herein.

In such a manner we have reduced the problem to the treatment of wave equation (28) that possesses the central symmetry with the initial conditions

\[
\begin{align*}
\varrho(r, 0) &= \varrho(r) = f(r), \\
\frac{\partial \varrho(r, 0)}{\partial t} &= 0;  \tag{29}
\end{align*}
\]

the second condition here means that the initial mass of the inerton cloud is zero (i.e. no any inert mass around a particle in the beginning). The boundary condition is

\[
\frac{\partial \varrho}{\partial r} \bigg|_{r=\Lambda} = F(\Lambda, t).  \tag{30}
\]

Because, \( \varrho = \varrho(r, t) \), then the Laplace operator in the spherical coordinates with the center located in the particle changes to the following

\[
\Delta \varrho \rightarrow \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \varrho).  \tag{31}
\]

Therefore equation (28) takes the form of the equation of radial oscillations

\[
\frac{1}{r} (r \varrho)_{rr} = \frac{1}{\hat{\gamma}^2} \varrho_{tt},  \tag{32}
\]

whose solutions (with conditions (29) and (30)) are well-known in classical mathematical physics (see, e.g. Refs. [42,43]). In our case the solution has the form

\[
\varrho(r, t) = \frac{C}{r} \left| \cos \left( \frac{\pi r}{2\Lambda} \right) \right| \left| \cos \left( \frac{\pi t}{2T} \right) \right|  \tag{33}
\]

and then functions \( f(r) \) and \( F(\Lambda, t) \) in the conditions (29) and (30) are equal to
\[ f(r) = C \frac{1}{r} \cos \left( \frac{\pi r}{2 \Lambda} \right), \]  
\[ F(\Lambda, t) = C \frac{\pi}{2\Lambda} \left| \cos \left( \frac{\pi t}{2T} \right) \right|, \]  
then \( g_r(r, t) \bigg|_{r=\Lambda, t=T} \equiv F(\Lambda, T) = 0. \)

The solution (33) can be rewritten explicitly, namely, for two components of the inerton cloud mass, respective longitudinal (∥) and transversal (⊥):

\[ m_{∥,⊥}(r, t) = C \frac{\mu_{∥,⊥} r}{r} \left| \cos \left( \frac{\pi r}{2\Lambda} \right) \cos \left( \frac{\pi t}{2T} \right) \right| \]  
where amplitudes are given in expressions (15) and (19), i.e., respectively

\[ \mu_∥ = M - M_0 \equiv M_0 \cdot \left( 1/\sqrt{1 - v_0^2/c^2} - 1 \right), \quad \mu_⊥ = M_0. \]

In equations from (31) to (35) the variable \( r \) changes in the range from the size of the particle, \( r = \left( V_{0}^{sup} \right)^{1/3} = 10^{-30} \) m (or maybe rather \( l_{Planck} \sim 10^{-35} \) m), to the inerton cloud amplitude \( r = \Lambda = \lambda \hat{c}/v_0 \) where \( \lambda \) is the de Broglie wavelength.

The solution obtained, (33) to (35), directly demonstrates that the particle periodically throws about its mass and then takes it back. The distribution of mass around the particle follows the amplitude of mass oscillation of the inerton cloud

\[ \mu_{∥,⊥} \cos \left( \pi r / 2\Lambda \right). \]

where \( (V_{0}^{sup})^{1/3} < r \leq \Lambda \). But this means that the inerton cloud forms the gravitational potential of the particle! Indeed, the two orthogonal components of mass (36) being distributed around the particle signifies the formation of a mass field around the particle. In other words, the mass field is the result of the defractalization of parts of the particle when its mass gets asymmetrically smeared around the “core” superparticle in the range covered by the inerton cloud amplitude \( \Lambda \) (2).

Thus in this range the cloud’s superparticles acquire additional deformations, i.e. become massive and their mass decreases with \( r \) in compliance with expression (35). Inertons migrating from the particle and then turning backward to it densely fill the environment. Superparticles by which inertons migrate contract and this means that the entire space net around the particle contracts as well. In such a manner any test particle having occurred under the mass field (35) will follow its gradient. That is, it is the contraction of the space net between two massive particles that realizes the attraction between them. This is the inner reason of the phenomenon of gravity, or attraction.

In a region confined by the size of a superparticle and the inerton cloud amplitude, \( (V_{0}^{sup})^{1/3} \ll r \ll \Lambda \), and at \( v_0 \ll c \) the time-averaged mass field (35) is reduced to a good approximation

\[ m_{∥}(r) = (V_{0}^{sup})^{1/3} \left( v_0^2 / c^2 \right) \frac{M_0}{r}; \]

\[ m_{⊥}(r) = (V_{0}^{sup})^{1/3} \frac{M_0}{r}. \]
Going over to conventional physical units, we have to multiply the both sides of expressions (37), (38) by a factor \(-G/(V_0^\text{sup})^{1/3}\) where \(G\) is the Newton constant of gravitation; as a results we obtain

\[ U_\parallel(r) = -(v_0^2/c^2) G \frac{M_0}{r}. \]  

\[ U_\perp(r) = -G \frac{M_0}{r}. \]

Thus the longitudinal component (39) of the gravitational potential of a particle depends on its velocity \(v_0\), though the transversal component (40) exactly represents Newton’s gravitational law.

Consequently, the introduction of inertons means that a new kind of a mechanics makes its appearance. Actually, in addition to contact and elastic interactions, which are characteristics of classical mechanics, and in addition to the specificity of orthodox quantum mechanics, which repeats the scheme of classical mechanics by using statistical tools, we gain a field mechanics. This one is based on the concept of the elastic tessellation space and, moreover, the concept treats an object as an element of the tessellattice [22]. And just that aspect is responsible for the field mechanics: inertons carry not only the momentum and kinetics energy from one particle to another, but in addition they transfer the local deformation into the surrounding space. When a test particle is thrown into the massive field induced by the other particle, it is contracted and, therefore, the gravitational law (39), (40) prescribes alterations in energetic and force characteristics of the test particle caused by the particle contraction due to its embedding into the contracted tessellation space.

5 Discussion

In the quantum substrate studied both the substrate itself and its components – superparticles-cells are elastic. They are those peculiarities that enable one to interpret mass as a deformation of a superparticle. Such a deformation looks like an uniform reduction of the superparticle’s volume (though the deformation is fractal [22]). Peculiarities of the particle motion in the space involve the particle motion itself including changes in behavior of center-of-mass, i.e. spin components (↑, ↓), the motion of the particle’s deformation coat (equivalently a crystallite) along with the particle, and the creation and migration of inertons. The dynamics of the system under consideration exhibits the periodical decay of the particle mass into mass of inertons. At a distance of \(\Lambda\) from the particle the inert mass carried by the inerton cloud completely turns into the rugosity of the space. Such periodical transformation of the deformation from the particle to the spatial rugosity permits the description of the inerton motion in terms of standing elastic spherical waves. They have been these waves that induce the deformation, i.e. gravitational potential \(U \propto M/r\) (39), (40) surrounding the particle (but the distance \(r\) is limited to the enveloping amplitude \(\Lambda\) of the particle’s inerton cloud). Therefore, standing spherical inerton waves are carriers of the real gravitational interaction between canonical elementary particles. It is significant that the mutual interaction appears only as a result of the
motion of particles because there is no information about an absolutely motionless particle relative to the space beyond the border of the crystallite (see Ref. [14] for details). In such a manner the crystallite is a peculiar kind of the screen between the particle and the degenerate space tessellattice.

For instance, the electron’s spatial crystallite of the electron (the crystallite shields the electron from the degenerate space) in the state of rest has the size of the Compton wavelength \( \lambda_0 = \frac{h}{M_0 c} \simeq 2.42 \times 10^{-12} \) m. The electron of a hydrogen atom is characterized for the lowest level by the following parameters: 
\[
\nu_0 \simeq 2.1 \times 10^6 \text{ m/s}, \quad \lambda = \frac{h}{M_0 \nu_0} \simeq 0.35 \text{ nm}, \text{ and the amplitude of the inerton cloud (2) } \Lambda = \frac{\lambda c}{\nu_0} \approx 20 \text{ nm.}
\]
Thus the amplitude of the cloud far exceeds the atom size \( a_H \simeq 0.1 \text{ nm}, \text{ i.e., } \Lambda/a_H \sim 200. \) The crystallite size of the nucleus of a hydrogen atom is determined by the Compton wavelength of the proton which equals \( 1.32 \times 10^{-15} \) m. This value is much smaller than all the above-mentioned parameters. Hence it is reasonable to conclude that it is not an atomic nucleus that influences the electron, but on the contrary, the field of transversal potential \( U_{\perp} \) (40) of the electron’s spherical inerton wave embraces the nucleus. This is evidently true for the electromagnetic interaction as well, because the electromagnetic field may be regarded as some kind of a polarization, which is superimposed on inertons.

What is a radius of the gravitational potential created by a heavy material object placed in the degenerate space? It is apparent that the components of the compound object are in ceaseless motion. For example, in a solid atoms vibrate in the neighborhood of their equilibrium positions. These vibrations result from the atom-atom interaction, which consists of both the elastic component of electromagnetic nature and the inertonic one [18]. Amplitudes of vibrating entities in a solid play a role of the de Broglie wavelengths of these entities [19]. If, say, a solid sphere with a radius \( R_{\text{sph}} \) consists of \( N_{\text{sph}} \) atoms and the interatomic distance is \( \bar{a} \), the spectrum of wavelengths of acoustic waves of the sphere is defined as \( 2\bar{a}n \) where \( n = 1, 2, \ldots, N_{\text{sph}}/2 \). The vibrating motion of atoms generates respective inerton clouds in the degenerate space, which move in synchronism with atoms in the acoustic waves. The value of the corresponding amplitude of the \( n \)th inerton cloud can be obtained from the relationship for the frequency of collisions of the corresponding acoustic wave with the inerton cloud, i.e. amplitude of inerton cloud \( \Lambda_n \) that accompanies the \( n \)th acoustic wave is (compare with expression (2))
\[
\Lambda_n = 2\bar{a}n \frac{c}{v_{\text{sound}}},
\]
where \( v_{\text{sound}} \) is the sound velocity of the sphere. In the classical limit the object size far exceeds its de Broglie wavelength. In that case long wavelength harmonics should determine the inerton field structure around the object and this means that any reasonable laboratory time interval \( t \) is still very small in comparison with \( T \), so in expression (35) we may set \(|\cos(\pi t/2T)| \simeq 1. \) For instance, if the sphere has the volume \( 1 \text{ cm}^3 \), \( \bar{a} = 0.5 \text{ nm, } N_{\text{sph}} = 10^{22} \text{ atoms, and } v_{\text{sound}} = 10^3 \text{ m/s expression (41) gives for the biggest amplitude: } \Lambda_{N_{\text{sph}}} \sim 10^{18} \text{ m. Thus the amplitude of the longest inerton wave } \Lambda_{N_{\text{sph}}}, \text{ which the material sphere generates as a whole is the maximum distance to which the inerton field of the sphere propagates in the form of the standing spherical inerton wave.}

Thereby, we can write the gravitational potential of a compound spherical object,
which is similar to the particle’s (40),

\[ U = -\frac{GM}{r} \]  

where \( r \) is limited by inequalities \( R_{sph} \leq r \ll \Lambda_{N_{sph}} \). Let us roughly estimate the time necessary for the emitted inertons to return. For simplicity we assumed that the velocity of inertons \( \hat{c} = c \). Then expressions above yield for the corresponding time, namely, for time wave period:

\[ T_{N_{sph}} = \frac{\Lambda_{N_{sph}}}{c} \sim 300 \text{ years!} \]

The longitudinal gravitational component proportional to \( v_0^2/c^2 \) is also available around macroscopic objects, though its manifestation will be treated in a separate work.

On the other hand, the description of the quasi-stationary potential (34a) around the object can mathematically be presented in the form of metric tensor components \( g_{ij} \), which are fundamental in Einstein’s general relativity, because of the alteration of the size of superparticles surrounding the central object. Such size alteration we associate with the induction of the gradient of the deformation field (i.e. inerton field). Consequently at the macroscopic approximation the metric tensor of the deformation potential of the degenerate space may really be regarded as an effective gravitational one.

We also enlarge on special relativity. The microscopic mechanism forming the basis for Lorentz transformations in the space substrate has been analyzed by the author in paper [13]. In essence, the microscopic theory takes into account that any material object consists of elementary particles and each particle is surrounded by its own deformation coat, equivalently, the crystallite. The behavior of the crystallite has been described in terms of the discrete hydrodynamics where the point is regarded to be equal to the size of the crystallite, i.e., the Compton wavelength \( \tilde{\lambda}_{v_0} \). With the whole object, one may introduce some effective crystallite as well. Then the size of the object crystallite will be defined by the object’s Compton wavelength \( \lambda_{\text{Comp}} \). However in the limit \( l_{\text{obj}} \gg \lambda_{\text{Comp}} \) (for space) where \( l_{\text{obj}} \) is the object typical size and \( t \gg \lambda_{\text{Comp}}/v_0 \) (for time) where \( v_0 \) is the object velocity, the discrete hydrodynamics can be easily replaced by the kinematics of special relativity which correctly depicts all the parameters of the object but does not reflect the actual (microscopic) origin that is covert by the relativity formalism.

6 Concluding remarks

Several important aspects of motion of a canonical particle have been studied in the present paper. The study based on the submicroscopic approach has allowed the consideration of behavior of a particle and the surrounding space along the whole particle path. It has been shown that the availability of the deformation coat, i.e. crystallite, whose massive superparticles are found in the ceaseless vibrating motion plays the role of an original generator that knocks inertons out of the particle, as a result of which its mass decays from \( M_0/\sqrt{1-v_0^2/c^2} \) to \( M_0 \) along the particle path and from \( M_0 \) to zero in transversal directions. Then the elastic space gives back inertons to the particle restoring its total mass. All these changes occur within the spatial period of particle oscillation, i.e., the de Broglie wavelength \( \lambda \). Thereby we can say such mechanism represents a real perpetual motion machine, which is launched by an initial push transmitting the velocity \( \vec{v}_0 \) to the particle.
It is apparent the particle's inerton migrating in the space is specified by coordinates and the velocity and the equation of motion

\[ \ddot{r} + \frac{\gamma}{m} r = 0 \]  

(43)
gives the information about these parameters [12-14]. Yet the inerton as a quasi-particle carries the local deformation \( \phi \) whose migration in the degenerate space is subjected to wave equation (25)

\[ \phi_{tt} - \hat{c}^2 \Delta \phi = 0. \]  

(44)

A couple of equations (43) and (44) completely describes the behavior of the inerton. The first equation, (43), describes the motion of the "core" of the inerton; this equation enables the submicroscopic interpretation of quantum mechanics. The second one, (44), describes the value of deformation that the inerton transfers from the particle into the tessellattice: the value \( \phi \) decreases by the law of inverse distance \( 1/r \) where \( r \) is the space between the particle and the inerton (note that \( r \) obtained from Eq. (43) is entered into Eq. (44)). Practically speaking, this allows the interpretation of phenomenon of attraction as a contraction of the space tessellattice between material objects.

This can readily be visualized by the following pattern. A physical point being embedded into a medium radiates standing acoustic (longitudinal type) radial waves which spread down to the distant spherical surface; hence, the medium vibrates in the range covered by the sphere's radius. A standing wave, as is well-known, is characterized by ranges of stretching and contraction of the medium. Thus our central point should be treated as a node point and the distant spherical surface's points to be antinode points, that is, the medium is contracted in the vicinity of the central point and stretched at the distant surface (see Fig. 2,b). As a result, the gradient of tension will exactly be directed to the central point simulating the phenomenon of classical "static" gravity.

Now let \( r \) be a space between two particles and let it obeys the inequality \( r < (\Lambda_1 + \Lambda_2) \) where \( \Lambda_{1(2)} \) are envelope amplitudes of inerton clouds of the two particles. In this situation the particles will fall under the behavior predicted by Newton/Coulomb law. This has been shown in the previous Section. However, such kind of the interaction between canonical particles is correct only when velocities of interacting particles are widely different.

If velocities of particles have the same order, the pattern of particle-particle interaction radically changes. In this case the elasticity \( \gamma \) of each of the inerton clouds is approximately identical in value and hence the particles will contact each other through the attractive inerton potential that obeys the harmonic law, \( \frac{1}{2} \gamma r^2 \).

Thus if elasticity constants \( \gamma_1 \) and \( \gamma_2 \) of the two inerton clouds have the same order, one can regard that the interaction between the particles is harmonic. (Indeed, let in a many particle system one particle moves towards the other one and let absolute value of their velocities be the same. Denote the mean mass and the mean velocity of particles' inertons as \( \langle m \rangle \) and \( \langle v \rangle \) respectively. Then at the distance between the particles lesser \((\Lambda_1 + \Lambda_2)\) inertons from one cloud begin to contact inertons from the other one. This means that inertons coming from the opposite directions should elastically collide as the absolute value of their momenta is the
same, $\langle m \rangle \langle v \rangle$. Therefore the two particles owing to their inerton clouds will contact each other much like two elastic balls. The particles repulsing move in opposite directions and again interact with similar particles which return these two to their initial positions).

The existence of the harmonic inerton potential in an ensemble of particles has experimentally been substantiated for atoms in the crystal lattice of a metal [18] and quite recently for the KIO$_3$–HIO$_3$ crystal, in which just the harmonic inerton potential $\frac{1}{2} \gamma r^2$ of hydrogen atoms provides for their clustering [19]. Besides it has theoretically been shown in paper [44] that it is the inerton potential $\frac{1}{2} \gamma r^2$ of nucleons that holds them in a nucleus; in other words, the inerton field of nucleons is a real confining field that ensures the nuclei stability.

Similarly, one can set the criterion regarding the interaction of classical objects. Let there be two the same solid spheres, which are characterized by a temperature $\Theta$. This temperature is directly defined by the mean kinetic energy of the spheres’ atoms. From the relationship $\frac{1}{2} M v^2 = \frac{3}{2} k_B \Theta$ one gets the thermal velocity of vibrating atoms $v_{\Theta} = \sqrt{\frac{3 k_B \Theta}{M}}$. Then one can gain the corresponding de Broglie mean thermal wavelength $\lambda_{\Theta} = \frac{h}{M v_{\Theta}} = \frac{h}{\sqrt{3 k_B \Theta M}}$ and further, in agreement with relationship (2), obtain the mean thermal amplitude of the inerton cloud $\Lambda_{\Theta} = \frac{h c}{3 k_B \Theta}$ of the atom. If the distance $r$ between surfaces of the spheres meets the inequality $r < 2 \Lambda_{\Theta}$, a noticeable elastic inerton correlation will be induced between the spheres, i.e. the spheres will suffer the inerton attraction with the potential $\frac{1}{2} \gamma r^2$. For example, $\Lambda_{\Theta} \sim 10 \mu m$ at the room temperature and $\Lambda_{\Theta} \sim 1 mm$ at 4 K. As was mentioned above, such inerton correlations in a solid were studied in paper [18].

It is important to note that in the experimental study of short-range gravitational effects one should leave the induced electromagnetic influence out of the gravitational action. In other words, the van der Waals interaction between macroscopic objects (see, e.g. reviews [45-47]) should be reduced. (The effect of Casimir force at a distance about several $\mu m$ has recently been studied theoretically by Lambrecht et al. [48,49]. Then Long et al. [50] based on the method developed in Refs. [48,49] have calculated the Casimir background and concluded that a gravitational-strength Yukawa force should be distinguishable from the Casimir one at the scale of about 3 $\mu m$; however, any new gravitational force has not been revealed experimentally in the range between 75 $\mu m$ and 1 mm.)

At $r > 2 \Lambda_{\Theta}$, thermal inerton correlations between objects are absent and this is why one can simulate the interaction based on the notion of the whole inerton cloud that oscillates in the space around each of the objects. In this case, as it has been shown above, the Newton law is realized. A deviation from the Newton law uncovered by general relativity in a macroscopic range needs a separate analysis of the paired interaction of two attracting objects. Nevertheless, the problem does not occur challenging from the point of view of the developing submicroscopic concept.

The manifestation of the inerton field is observed not only in gravitation induced by this field. Inerton waves are directly responsible for the inexplicable force that since the old times has been called the force of inertia. Many times each of us has undergone the influence of this force, which continue to move everybody when he/she tries to stop or turn abruptly. In such cases our own clouds of inertons continue
pushing us slightly when we stop after a fast movement. The inerton field solves also the problem of centrifugal force, which so far still remained an understandable phenomenon of classical mechanics! It is obvious from the stated above that the centrifugal force, which acts on a body moving along a curve line, should appear as a response of the space on a centripetal acceleration applied to the body.

Acknowledgement

I am deeply indebted to late Professor Michel Bounias for the critical reading of the manuscript and the valuable remarks.

References

[1] D. V. Ahluwalia, Principle of equivalence and wave-particle duality in quantum gravity. Memorias del III Taller de la DGF-SMF, 2000. "Aspectos de Gravitación y Física-Matemática", Eds. por N. Bretón, O. Pimentel y J. Socorro (also arxiv.org e-print archive [http://arxiv.org/abs/gr-qc/0009033]; Quantum measurement, gravitation, and locality Phys. Lett. B339, 301-303 (1994).

[2] D. A. Ahluwalia, Quantum gravity: Testing for theories, Nature 398, 199-200 (1999) (also gr-qc/9903074); Can general relativistic description of gravitation be considered complete? Modern Phys.Lett. A13, 1393-1400 (1998).

[3] R. D. Sorkin, Forks in the road, on the way to quantum gravity. Int. J. Theor. Phys. 36, 2759-2781 (1997) (also arXiv.org e-print archive gr-qc/9706002).

[4] H. F. Dowker, and R. D. Sorkin, Spin and statistics in quantum gravity. Proceedings of international meeting on "Spin-statistics connections and communication relations, experimental tests and theoretical implications", Capri, Italy, May 31-June 4, 2000; eds. R. C. Hilborn, and G. M. Tino, pp. 205-218 (2001) (also arXiv.org e-print archive gr-qc/0101042).

[5] S. Rado, Aethero-kinematics, CD-ROM (1994), Library of Congress Catalog Card, # TXu 628-060 (also www.aethero-kinematics.com).

[6] H. Aspden, The theory of the gravitational constant, Phys. Essays 2, 173 - 179 (1989); The theory of antigravity, Phys. Essays 4, 13-19 (1991); Aetherth Science Papers (Subberton Publications, P. O. Box 35, Southampton SO16 7RB, England, 1996).

[7] J. W. Vegt, A particle-free model of matter based on electromagnetic self-confinement (III), Ann. de la Fond. L. de Broglie 21, 481-506 (1996).

[8] F. Winterberg, Physical continuum and the problem of a finistic quantum field theory, Int. J. Theor. Phys. 32, 261-277 (1993); Hierarchical order of Galilei and Lorentz invariance in the structure of matter, ibid. 32, 1549-1561 (1993); Equivalence and gauge in the Planck-scale aether model, ibid. 34, 265-285 (1995); Planck-mass-rotons cold matter hypothesis, ibid. 34, 399-409 (1995);
Derivation of quantum mechanics from the Boltzmann equation for the Planck aether, *ibid.* 34, 2145-2164 (1995); Statistical mechanical interpretation of hole entropy, *Z. Naturforsch.* 49a, 1023-1030 (1994); Quantum mechanics derived from Boltzmann’s equation for the Planck aether, *ibid.* 50a, 601-605 (1995); The Planck aether hypothesis. An attempt for a finistic theory of elementary particles, Verlag relativistischer Interpretationen – VRI, Karlsbad (2000).

[9] F. M. Meno, A smaller bang? *Phys. Essays* 11, no. 2, 307-310 (1998).

[10] H. Kubel, The Lorentz transformations derived from an absolute ether, *Phys. Essays* 10, no. 3, 510-523 (1997).

[11] A. Rothwarf, An aether model of the universe, *Phys. Essays* 11, 444-466 (1998).

[12] V. Krasnoholovets, and D. Ivanovsky, Motion of a particle and the vacuum, *Phys. Essays* 6, no. 4, 554-563 (1993) (also arXiv.org e-print archive quant-ph/9910023).

[13] V. Krasnoholovets, Motion of a relativistic particle and the vacuum, *Phys. Essays* 10, no. 3, 407-416 (1997) (also arXiv.org e-print archive quant-ph/9903077).

[14] V. Krasnoholovets, On the nature of spin, inertia and gravity of a moving canonical particle, *Ind. J. Theor. Phys.* 48, no. 2, 97-132 (2000) (also arXiv.org e-print archive quant-ph/0103110).

[15] V. Krasnoholovets, On the way to submicroscopic description of nature, *Ind. J. Theor. Phys.* 49, 81-95 (2001) (also arXiv.org e-print archive quant-ph/9908042).

[16] V. Krasnoholovets, Space structure and quantum mechanics, *Spacetime & Substance* 1, no. 4, 172-175 (2000) (also arXiv.org e-print archive quant-ph/0106106); Submicroscopic deterministic quantum mechanics, *Int. J. Comput. Anticip. Systems*, to appear around May 2002 (also arXiv.org e-print archive quant-ph/0109012).

[17] V. Krasnoholovets, On the theory of the anomalous photoelectric effect stemming from a substructure of matter waves, *Ind. J. Theor. Phys.*, 49, 1-32 (2001) (also arXiv.org e-print archive quant-ph/9906091).

[18] V. Krasnoholovets, and V. Byckov, Real inertons against hypothetical gravitons. Experimental proof of the existence of inertons, *Ind. J. Theor. Phys.* 48, no. 1, 1-23 (2000) (also arXiv.org e-print archive quant-ph/0007027).

[19] V. Krasnoholovets, Collective dynamics of hydrogen atoms in the KIO$_3$·HIO$_3$ crystal dictated by a substructure of the hydrogen atoms’ matter waves, arXiv.org e-print archive cond-mat/0108417.

[20] M. Bouinas, *La creation de la vie: de la matiere a l’esprit*, L’esprit et la matiere, Editions du Rocher, Jean-Paul Bertrand Editeur, 1990, p. 38-123; A theorem proving the irreversibility of the biological arrow of time, based on fixed points
in the brain as a compact or $\Delta$-complete topological space, in: *American Inst. Phys. Conf. Proc.* (2000); The theory of Something: a theorem supporting the conditions for existence of a physical universe, from the empty set to the biological self, *Int. J. Comp. Anticip. Syst.* 5-6, 1-14 (2000).

[21] M. Bounias, and A. Bonaly, On mathematical links between physical existence, observability and information: towards a "theorem of something", *Ultra Scientist of Phys. Sci.* 6, 251-259 (1994); Timeless space is provided by empty set, *ibid.* 8, 66-71 (1996); On metric and scaling: physical co-ordinates in topological spaces, *Ind. J. Theor. Phys.* 44, 303-321 (1996); Some theorems on the empty set as necessary and sufficient for the primary topological axioms of physical existence, *Phys. Essays* 10, 633-643 (1997).

[22] M. Bounias, and V. Krasnoholovets, Scanning the structure of ill-known spaces: I. Founding principles about mathematical constitution of space; II. Principles of construction of physical space; III. Distribution of topological structures at elementary and cosmic scales *Kybernetes: The International Journal of Systems and Cybernetics*, 32, no. 7/8, 976-1020 (2003) (also physics/0211096 physics/0212004 physics/0301049).

[23] A. Magnon, *Arrow of time and reality: a conciliation*, World Scientific, London (1997).

[24] Śrī Śrīmad A. C. Bhaktivedanta Swami Prabhupada, *Beyond time and space* (Russian version of *Easy journey to other planets*), Almqist & Wiksell, Uppsala (1976), pp. 13-19; *Bhagavad-gītā as it is*, Bhaktivedanta Book Trust, Moscow, Leningrad, Calcutta, Bombey, New Dehly (1984), Ch. 2, Verses 17-30; Ch. 8, Verse 20.

[25] G. Amelino-Camelia, Gravity-wave interferometers as quantum-gravity detectors, *Nature* no. 398, 216-219 (1999) (also arXiv.org e-print archive gr-qc/9808029).

[26] G. Amelino-Camelia, Relativity in space-times with short-distance structure governed by an observer-independent (Planckian) length scale, arXiv.org e-print archive gr-qc/0012051.

[27] J. Ellis, N. Mavromatos and D. V. Nanopoulos, String theory modifies quantum mechanics, *Phys. Lett. B* 293, 37-48 (1992).

[28] G. Amelino-Camelia, J. Ellis, N. Mavromatos and D. V. Nanopoulos, Distance measurement and wave dispersion in a Louville string approach to quantum gravity, *Int. J. Mod. Phys.* A 12, 607-623 (1997)

[29] G. Amelino-Camelia, J. Ellis, N. Mavromatos, D. V. Nanopoulos and S. Sarkar, Potential sensitivity of gamma-ray burster observations to wave dispersion in vacuo, arXiv.org e-print archive astro-ph/9712103

[30] R. Gambini and J. Pullin, Nonstandard optics from quantum spacetime, arXiv.org e-print archive gr-qc/9809038
[31] M. Matone, Equivalence postulate and quantum origin of gravity, arXiv.org e-print archive [hep-th/0005274].

[32] L. de Broglie, Sur la Dynamique du corps à masse propre variable et la formule de transformation relativiste de la chaleur, Comptes Rendus 264 B (16), 1173-1175 (1967); On the basis of wave mechanics 277 B, no. 3, 71-73 (1973).

[33] X. Oudet, L’aspect corpusculaire des électron et la notion de valence dans les oxydes métalliques, Ann. de la Fond. L. de Broglie 17, 315-345 (1992); L’état quantique et les notions de spin, de fonction d’onde et d’action, ibid. 20, 473-490 (1995); Atomic magnetic moments and spin notion, J. App. Phys. 79, 5416-5418 (1996); The quantum state and the doublets Ann. de la Fond. L. de Broglie 25, 1-25 (2000).

[34] G. Papini, and W. R. Wood, A geometric solution to the de Broglie variable mass problem, Phys. Lett. A 202, 46-49 (1995).

[35] D. Kabat and O. Pouliot, A comment on zero-brane quantum mechanics, Phys. Rev. Lett. 77, 1004 - 1006 (1996) (also arXiv.org e-print archive hep-th/9603127).

[36] J. Ellis, N. E. Vavromatos and D. V. Nanopoulos, A microscopic recoil model for light-cone fluctuations in quantum gravity, Phys. Rev. D 61, 027503 (2000) (also arXiv.org e-print archive gr-qc/9906029).

[37] A. S. Davydov, The theory of solids, Nauka, Moscow (1976), p. 38 (in Russian).

[38] V. Krasnoholovets, On the mass of elementary carriers of gravitational interaction, Spacetime & Substance 2, 169-170 (2001) (also quant-ph/0201131).

[39] B. A. Dubrovin, S. P. Novikov, and A. T. Fomenko, Modern geometry. Methods and applications, Nauka, Moscow (1986) p. 18 (in Russian).

[40] The rugosity of the space net implies some kind of a deformation as well, but this is the deformation of the entire cellular ordering. To avoid confuse, we will use the term ”rugosity” in an attempt to distinguish it from the local deformation of the net, i.e. the deformation of superparticle. Perhaps in pure physics terms this notion one can indentify with the tension, or stress, of the lattice.

[41] D. ter Haar, Elements of Hamiltonian mechanics, Nauka, Moscow, 1974, p. 211 (Russian translation).

[42] N. S. Koshlyakov, E. B. Gliner, and M. M. Smirnov, Equations in partial derivatives of mathematical physics, Vysshaya Shkola, Moscow, 1970, p. 185.

[43] A. N. Tikhonov, and A. A. Samarski, Equations of mathematical physics, Nauka, Moscow (1972), p. 404 (in Russian).

[44] V. Krasnoholovets, and B. Lev, Systems of particles with interaction and the cluster formation in condensed matter, Condensed Matter Physics 6, 67-83 (2003) (also arXiv.org e-print archive cond-mat/0210131).
[45] T. H. Boyer, Quantum zero-point energy and long-range forces *Ann. Phys.* **56**, no. 2, 474-503 (1970).

[46] A. A. Grib, S. G. Mamaev, and V. M. Mostepanenko, *Quantum effects in intensive applied fields (methods and results are not connected with the perturbation theory)*, Atomizdat, Moscow (1980), p. 40 (in Russian).

[47] Yu. S. Barash, and V. L. Ginzburg, *Uspekhi Fiz. Nauk* **143**, 345-389 (1984) (in Russian).

[48] A. Lambrecht, and S. Reinaud, Casimir force between metallic mirrors. *Eur. Phys. J. D* **8**, 309-318 (2000) (also arXiv.org e-print archive quant-ph/9907105).

[49] C. Genet, A. Lambrecht, and S. Reinaud, Temperature dependence of the Casimir effect between metallic mirrors, *Phys. Rev. A* **62**, 012110 (2000) (also arXiv.org e-print archive quant-ph/9908042).

[50] J. C. Long, A. B. Churnside, and J. C. Price, Gravitational experiment below 1 millimeter and comment on shielded Casimir backgrounds for experiments in the micron regime, arXiv.org e-print archive quant-ph/9908042.