THE DECIHERTZ LASER INTERFEROMETER CAN DETERMINE THE POSITION OF THE COALESCING BINARY NEUTRON STARS WITHIN AN ARCMINUTE A WEEK BEFORE THE FINAL MERGING EVENT TO THE BLACK HOLE

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Received 2003 July 22; accepted 2003 August 18; published 2003 October 1

ABSTRACT

It may be possible to construct a laser interferometer gravitational wave antenna in space with $h_{\text{rms}} \sim 10^{-23}$ at $f \sim 0.1$ Hz in ~2020. This decihertz antenna may be called the Decihertz Interferometer Gravitational Wave Observatory and Big Bang Observer (DECIGO/BBO). The analysis of 1–10 yr of observational data of the coalescing binary neutron stars or black holes at the distance of $\sim 300$ Mpc will give us the spatial position within approximately an arcminute and the time of the coalescence within $\sim 0.1$ s beforehand. With the knowledge of the accurate position and the time of the final merging event, the follow-up simultaneous observation using high-frequency ($f \sim 100$ Hz) gravitational wave antennae as well as electromagnetic wave antennae from the radio frequency to the ultra–high-energy gamma ray will reveal the physics in the enigmatic event of the coalescence and the formation of the black hole.

Subject headings: binaries: general — gravitation — gravitational waves

1. INTRODUCTION

There are at least four frequency bands in which active or planned gravitational wave antennae exist. The resonant bars as well as ground laser interferometers such as TAMA300, LIGO I, VIRGO, and GEO600 are covering the frequency band of 10 Hz–1 kHz while the timing analysis of pulsars is covering the $\sim 10^{-6}$ Hz band. The Laser Interferometer Space Antenna (LISA) will cover $10^{-4}$ to $10^{-2}$ Hz from ~2010. Very recently the space antenna in the decihertz band (10$^{-2}$ to 10 Hz) came into the group. According to Seto, Kawamura, & Nakamura (2001), it might be possible to construct a laser interferometer gravitational wave antenna in space with $h_{\text{rms}} \sim 10^{-27}$ at $f \sim 0.1$ Hz in this century. Using this antenna, they show that (1) the time variation of the Hubble parameter of our universe may be determined for 10 yr observation of binary neutron stars per year may be detected with a signal-to-noise ratio $(S/N) \sim 10^5$, (2) the stochastic gravitational wave that is predicted by the inflationary universe paradigm may be detected.

Seto et al. (2001) call the decihertz antenna the Decihertz Interferometer Gravitational Wave Observatory (DECIGO), while recently in the NASA SEU 2003 Roadmap “Beyond Einstein,” the decihertz antenna is called the Big Bang Observer (BBO)$^1$ to stress the detection of the stochastic gravitational waves from inflation. Since DECIGO and BBO are similar antennae, in this Letter we call the decihertz antenna DECIGO/BBO. In DECIGO, $h_{\text{rms}} \sim 10^{-27}$ around 0.1 Hz is the ultimate goal in the future, assuming the quantum limit sensitivity for a 100 kg mass.

Let us consider more realistic parameters that might be achieved in ~2020, such as 300 W laser power, 3.5 m mirrors, $5 \times 10^4$ km arm length, and 0.01 LISA acceleration noise, which will give $h_{\text{rms}} \sim 10^{-23}$ around 0.1 Hz (e.g., Larson, Hiscock, & Hellings 2000). In this sensitivity, the S/N of the chirp signal from the coalescing binary neutron stars at $z \sim 1$ will be $\sim 1$, while that of the 10 $M_\odot$ black hole (BH) binary will be $\sim 10$, so that we may directly measure the acceleration of the universe similar to the ultimate DECIGO. After subtracting the signals from neutron star binaries and BH binaries, we may detect the stochastic gravitational waves from inflation if $\Omega_{\text{gw}} \approx 10^{-15}$, which is the upper bound from cosmic microwave background quadrupole anisotropies measured by COBE (Gorski et al. 1996).

One of the main targets of the ground laser interferometer gravitational wave antennae is the chirp signal from the coalescing binary neutron stars or BHs at the distance of 200–400 Mpc with the expected event rate of $\sim 1$ yr$^{-1}$ (e.g., Phinney 1991). Since the event rate is $\sim 1$ yr$^{-1}$, the same binary is in the band of DECIGO/BBO, 1–10 yr before the ground laser interferometers and bar detectors detect the gravitational waves from the final merging event. This means that in practice, we may observe the evolution of the coalescing binary from the $10^{-2}$ Hz band to the $\sim 1$ kHz band. The duration in the band of the ground laser interferometer is 3 minutes or so, while the duration in the DECIGO/BBO band is 1–10 yr, so that we may extract the information of the binary from the DECIGO/BBO band observation first. The point here is that we can know the accurate position and the distance to the binary beforehand, so that we can prepare various detectors for the observation of the final merging phase; that is, we can point electromagnetic antennae from radio to ultra–high-energy gamma rays and cosmic rays to the direction of the coalescing binary at the expected final merging time. We can also tune the gravitational wave antennae to various sensitivity bands to detect the characteristics of the merging event, such as the frequency at the innermost stable circular orbit and the quasi-normal mode of the final BH.

In this Letter, using the practical DECIGO/BBO in ~2020, we show how accurately we can determine the spatial position and the distance to the coalescing binary neutron stars or BHs. Since the angular resolution of LISA was calculated (Cutler 1998; Hughes 2002; Seto 2002; Vecchio 2003), we apply these methods to the case of DECIGO/BBO. We adopt the Hubble parameter $h \equiv H_0$ 100 km s$^{-1}$ Mpc$^{-1} = 0.7$ and the units of $c = G = 1$.

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1 See the LISA Web site at http://lisa.jpl.nasa.gov/index.html.
2 See http://universe.gsfc.nasa.gov/be/roadmap.html.
3 There may be other sources that we do not know about at this point.
2. GRAVITATIONAL WAVEFORM AND PARAMETER ESTIMATION

2.1. Sensitivity of DECIGO/BBO

DECIGO/BBO would consist of three spacecraft separated by \(5 \times 10^4\) km, which is \(1/100\) times smaller than the size of LISA (see Bender et al. 2000). They would be spaced in an equilateral triangle and be orbiting around the Sun. The change of the detector’s orientation and the position enables us to obtain the source position in the following two ways: (1) DECIGO/BBO’s orientation rotates with a period of 1 yr, which imposes the modulation on the measured signal, and (2) DECIGO/BBO’s orbital motion around the Sun imposes the periodic Doppler shift on the signal frequency (see also Cutler 1998 in the case of LISA). For the monochromatic signal of the frequency \(f\), (1) the rotational modulation changes the frequency \(f\) to \(f \pm 2/\tau\), where \(\tau = 1\) yr \((2/\tau \sim 10^{-7}\) Hz\), and (2) the Doppler modulation changes \(f\) to \(f(1 \pm v)/v\). \(\pm \sim 10^{-3}\) Hz as in DECIGO/BBO, the effect of the Doppler modulation is more important for the determination of the source position than the rotational modulation, so that we consider only the former effect in the measured signal. The observed waveform \(\tilde{h}(f)\) in the frequency domain \(h_0(f)\) is given by

\[
\tilde{h}_0(f) = \tilde{h}(f)e^{i\phi_0(f)},
\]

where \(\tilde{h}(f)\) is the waveform at the solar system barycenter and \(\phi_0(f)\) is the Doppler phase, which is the difference of the phase between the detector and the Sun; \(\phi_0 = 2\pi R \times \sin \theta_\odot \cos(2\pi fT - \phi_\odot)\), where \(R = 1\) AU, \(T = 1\) yr, and \((\theta_\odot, \phi_\odot)\) is the direction to the source. These angular coordinates are defined in a fixed barycenter frame of the solar system (see Cutler 1998).

The strain sensitivity of DECIGO/BBO is about 1000 times better than that of LISA (i.e., \(10^{-23}\) Hz\(^{-1/2}\) at \(f = 0.1\)–1 Hz), and the acceleration noise is 100 times lower than that of LISA (i.e., \(3 \times 10^{-17}\) m s\(^{-2/2}\)). Adopting these parameters, we show the sensitivity of DECIGO/BBO as compared with LISA and LIGO II in Figure 1 (Larson et al. 2000). Note that we are using a smooth curve and neglecting the wavy behavior of the transfer function for simplicity.

2.2. Gravitational Waveform

We consider the equal mass neutron star and BH binaries as sources of DECIGO/BBO. We consider only a circular orbit, since the expected eccentricity is \( \sim 10^{-3} \) for the neutron star binary.

We use the restricted 1.5 post-Newtonian approximation as the in-spiral waveform (Cutler & Flanagan 1994) for simplicity. The waveform in the frequency domain is given by

\[
\tilde{h}(f) = Af^{-7/6}e^{i\Psi(f)},
\]

where \(A\) is the amplitude and \(\Psi(f)\) is the phase. They depend on six parameters: the redshifted chirp mass \(M_* = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}(1 + z_\odot)\), the reduced mass \(\mu_* = M_1 M_2(1 + z_\odot)/(M_1 + M_2)\), the spin-orbit coupling constant \(\beta\), the coalescence time \(t_c\), the phase \(\phi_\odot\), and the luminosity distance to the source \(D_\odot\). The amplitude is given by

\[
A = K(5/96)^{1/2}M_*^{25/12}(\pi/2D_\odot),
\]

where \(K\) is the constant determined by the inclination of the source, the relative orientation of the source, and the detector. Since the average value of \(K\) is about unity (Finn & Chernoff 1993), we assume \(K = 1\) for the following calculation. The phase \(\Psi(f)\) in equation (2) is a rather complicated function of \(M_*\), \(\mu_*\), \(\beta\), \(\phi_\odot\), and \(t_c\), which is given in equation (3.24) of Cutler & Flanagan (1994).

2.3. Parameter Estimation

The signal observed by DECIGO/BBO, \(\tilde{h}_0(f)\), can be obtained inserting equation (2) in equation (1). The signal \(\tilde{h}_0(f)\) is characterized by eight parameters \((M_*, \mu_*, \beta, \phi_\odot, t_c, D_\odot, \theta_\odot, \phi_\odot)\). In the matched filter analysis with the template, these parameters can be determined. We compute the errors in the estimation of these parameters using the Fisher matrix formalism (Finn 1992; Cutler & Flanagan 1994). The variance-covariance matrix of the parameter estimation error \(\Delta \gamma\) is given by the inverse of the Fisher information matrix \(\Gamma_{ij}\) as

\[
\Delta \gamma_{ji} = (\Gamma^{-1})_{ji},
\]

where \(Sn(f)\) is the noise spectrum. We regard \(Sn(f)\) as the instrumental noise in Figure 1, neglecting the binary confusion

| Binary Masses \((M_*)\) | S/N | \(\Delta M/M_*\) | \(\Delta t_c\) | \(\Delta D_\odot/D_\odot\) |
|--------------------------|-----|-----------------|--------------|-----------------|
| 1.4 + 1.4 \ldots | 62 | 6.0 \times 10^{-7} | 7.4 \times 10^{-4} | 6.6 \times 10^{-2} | 1.6 \times 10^{-2} |
| 10 + 10 \ldots | 702 | 3.4 \times 10^{-4} | 8.4 \times 10^{-3} | 1.3 \times 10^{-2} | 1.4 \times 10^{-2} |
| 10^{-2} + 10^{-2} \ldots | 374 | 2.7 \times 10^{-2} | 2.8 \times 10^{-1} | 0.18 | 2.7 \times 10^{-1} |
| 10^{-4} + 10^{-4} \ldots | 375 | 5.9 \times 10^{-6} | 9.5 \times 10^{-3} | 0.15 | 2.7 \times 10^{-3} |
| 10^{-5} \ldots | 2557 | 3.3 \times 10^{-7} | 1.4 \times 10^{-4} | 0.71 | 3.9 \times 10^{-4} |
| 2557 \ldots | 1.2 \times 10^{-2} | 7.5 \times 10^{-3} | 0.47 | 3.9 \times 10^{-4} |

Note.—The results are presented for the various binary masses \((1.4, 10^{-2}–10^{-4} M_\odot)\) for 1 yr (top line) and 10 yr (bottom line) observation.
Fig. 2.—Relative probability distribution of the angular resolution of DECIGO/BBO in the case of 1 yr observation. For each mass case we randomly distribute $10^4$ binaries on the celestial sphere at $(hD_h)$, and we show the probability distribution of the angular resolution for these sources. The solid lines and the dashed lines show $\Delta \theta_s$ and $\Delta \phi_s$, respectively. The S/N and the estimation error in the coalescence time are also shown. The S/N is independent of $\theta_r$ and $\phi_r$ since the phase factor $e^{i\omega t}$ becomes unity in eq. (4), while $\Delta t_c$ is found to depend on $\theta_r$ and $\phi_r$ very weakly. (S/N) 

\[ (S/N)^2 = 4 \int \frac{df}{Sn(f)} |F_0(f)|^2 . \]  

We integrate the gravitational waveform in equations (3) and (4) from 1 yr or 10 yr before the final merging to the cutoff frequency $f_{cut}$ when the binary separation becomes $r = 6(M_1 + M_2)$.

The initial frequency is given by $f_{init} = 0.23(M_1/M_2)^{3/8} (T_{obs}/1 \text{ yr})^{-3/8} \text{ Hz}$, where $T_{obs} = 1$ or 10 yr and the cutoff frequency is $f_{cut} = 4.4 \times 10^{-4} [(M_1 + M_2)M_\odot]^{-1/8} \text{ Hz}$. The result does not depend on the value of $f_{cut}$ so much, since $Sn(f)$ is large at $f_{cut}$.

3. RESULTS

We consider the neutron star binaries ($1.4 + 1.4 \, M_\odot$) and the stellar mass BH binaries ($10 + 10 \, M_\odot$) at $D_s = 200 \, h^{-1} \, \text{Mpc}$ and the intermediate mass BH binaries of mass ($10^2 + 10^3 \, M_\odot$) as well as ($10^3 + 10^3 \, M_\odot$) at $D_s = 3000 \, h^{-1} \, \text{Mpc}$ (Hubble distance) as the sources. Note here that we adopt $h = 0.7$. For each mass case, we randomly distribute $10^4$ binaries on the celestial sphere at $D_s$, and we show the probability distribution of the angular resolution for these sources.

In Figure 2, we show the relative probability distribution of the angular resolution for the various binary masses $M_1 + M_2 = 1.4 \, M_\odot, 10^{-1} - 10^3 \, M_\odot$ in the case of 1 yr observation. The solid lines and the dashed lines show $\Delta \theta_s$ and $\Delta \phi_s$, respectively. We also show the S/N and the estimation error in the coalescence time $\Delta t_c$, assuming that the template is accurate enough. The S/N is independent of $\theta_r$ and $\phi_r$ since the phase factor $e^{i\Omega t}$ becomes unity in equation (4), while $\Delta t_c$ is found to depend on $\theta_r$ and $\phi_r$ very weakly. In general, the errors ($\Delta \theta_s, \Delta \phi_s, \Delta t_c$) simply scale as $(S/N)^{-1}$. As shown in Figure 2, the angular resolutions are typically $\sim 0.1 - 10$’ for the neutron star binaries and the BH binaries. This is about 10–1000 times better than that of $LISA$ ($\sim 1''$). The Doppler modulation ($\phi_0 \propto f$ in eq. [1]) effect is larger than the rotational modulation for the high-frequency band such as DECIGO/BBO. It is shown (Cutler & Vecchio 1998; Moore & Hellings 2002; Takahashi & Seto 2002) that the angular resolution scales as proportional to $f^{-1}(S/N)^{-1}$ for the monochromatic sources. Thus the angular resolution of DECIGO/BBO should be about 100 times better.
The distance to gamma-ray bursts (GRBs) was not determined before 1997. The minimum possible distance was ~100 AU and the maximum was the Hubble distance (Fishman & Meegan 1995), so that the accuracy of the spatial position of GRBs was obtained a few hours after the GRB event. In our case, the observation period is 1–10 yr before the final merging to BHs so that we would distribute the alert of the event beforehand.

Let us consider the observation from 1 yr before the merging up to a day before the merging. In Figure 3, we show the angular resolution $\Delta \phi_s$ for the neutron star binary $(1.4 + 1.4 M_\odot)$ at 200 $h^{-1}$ Mpc ($h = 0.7$). The observational period is from 1 yr before the final merging to 3 months (dotted line), 1 month (short dashed line), 1 week (long dashed line), and 1 day (dot-dashed line) before the merging. The solid line shows the case of full 1 yr observation up to the cutoff frequency.

In Table 1, we show the S/N and the estimation errors of the chirp mass, the reduced mass, the coalescence time, the distance to the source $D_c$. The results are presented for the various binary masses $(1.4, 10^{-1} - 10^2 M_\odot)$ for 1 yr (top line) and 10 yr (bottom line) observation. The errors simply scale as $(S/N)^{-1}$ for the change of the distance to the source. For the accuracy of the chirp mass $\Delta M_c \sim 10^{-7}$ and the reduced mass $\Delta \mu \sim 10^{-4}$, each mass of the binary is determined within $\sim 10^{-4}$ at the distance to the binary is determined by $(S/N)^{-1}$.

4. DISCUSSION

The distance to gamma-ray bursts (GRBs) was not determined before 1997. The minimum possible distance was ~100 AU and the maximum was the Hubble distance (Fishman & Meegan 1995), so that many theoretical models were equally possible. The main reason that the distance to GRBs was not determined is that the accuracy of the spatial position of GRBs using the gamma-ray observation was at most ~1°, so that many possible host galaxies exist in an error box of GRBs. Finding the host galaxy of GRB was akin to searching for a needle in a haystack. In 1997, the Italian-Dutch satellite BeppoSAX succeeded in obtaining the high-resolution (~2°) X-ray images of GRBs, which led to the determination of redshifts and host galaxies of GRBs (Costa et al. 1997). This clearly demonstrates that the accuracy of an arcminute or so is indispensable to study the objects in full details.

In this Letter, we showed that spatial position of coalescing binary neutron stars or BHs of mass $\sim 10 M_\odot$ at the distance of $\sim 300$ Mpc can be determined within an arcminute from 1 yr observation. This accuracy is comparable or better than that of LISA if the S/N is equal. This explains the accuracy of $\sim 0^\prime 1–10^\prime$.

In conclusion, DECIGO/BBO in ~2020 can determine the angular position of the neutron star binary and the BH binary at $\sim 300$ Mpc within an arcminute before the final merging to the BH. In a sense, DECIGO/BBO will truly open gravitational wave astronomy, since the positional accuracy is comparable to the X-ray telescope on BeppoSAX.

We would like to thank T. Tanaka and N. Seto for useful comments and discussions. This work was supported in part by Grants-in-Aid for Scientific Research of the Japanese Ministry of Education, Culture, Sports, Science and Technology, 14047212 and 14204024.

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