Gravitational wave detection by bounded cold electronic plasma in a long pipe

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Abstract

We intend to propose an experimental sketch to detect gravitational waves (GW) directly, using an cold electronic plasma in a long pipe. By considering an cold electronic plasma in a long pipe, the Maxwell equations in 3+1 formalism will be invoked to relate gravitational waves to the perturbations of plasma particles. It will be shown that the impact of GW on cold electronic plasma causes disturbances on the paths of the electrons. Those electrons that absorb energy from GW will pass through the potential barrier at the end of the pipe. Therefore, crossing of some electrons over the barrier will imply the existence of the GW.

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1 Introduction

Stars and spaces between them are composed of matter in plasma phase. Since gravitational field is the most effective force in the macroscopic scale it will determine the dynamics of astrophysical objects. So it is necessary to study the effects of gravity on plasma phase of matter. For instance, the major reason for spiral shape of galactic arms has been known to be the effect of GW at the first stages of forming spiral galaxies [1]. Thus the theory of plasma state of matter in curved spacetime has been carried out [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. An interesting phenomenon that happens is creation of electromagnetic field by GW. Gravitational fields enters into Maxwell Eqs as current terms, and like the electrical current create a electromagnetic field [12]. An interesting point in such studies is the emergence of resonance phenomenon in the interaction of GW with plasma[13]. This fact is used to propose a new method for detecting GW, beside standard methods[14, 15].

As we know gravitational waves have not been detected directly in the laboratory. The very small value of gravitational constant in comparison with Coulomb constant causes such failures in detection of GW in laboratory efforts. There have been many efforts to suggest experiments in order to detect GW. Metal rod resonance and optical interferometry have been the most common methods in efforts to observe GW [14, 15]. There are some other methods too [16]. The most successful method implying the existence of GW has been the energy reduction of binary stars [17, 18]. In fact we have never

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observed the gravitational waves. Taylor has computed the energy of gravitational radiation of binary stars rotating about each other. Observed energy reduction of such stars is in full agreement with Taylor calculation.

The most promising idea is that of Weber. To observe GW effects directly in the laboratory, Weber has computed the effect of GW on the length of an Aluminum bar. We have used the basic Weber’s idea but changed some ingredients. We thought that replacing Weber’s rod with an electronic bar may be useful. Actually electronic bar has a great advantage; the electronic bar does not have the unwanted solid structure and then, atomic structures does not have noisy effects on it. Eliminating noisy effects is very important because they may interfere with GW. Our main purpose is to separate GW from environmental effects as much as possible. In order to do so, we enclose a cold electronic plasma in a potential well First, figure 1 and then two negative rings will be placed at the ends of the pipe to create a potential barrier. The potential barrier will enclose electronic cloud within the cylinder. It can be shown that the slope of this barrier is very sharp, so the trapped electrons will encounter to the potential barrier very close to the two rings [19]. The rings will create radial fields that will be discussed later. We will place a shield on the whole apparatus to prevent it from environmental noisy effect. Thus there is no any external electromagnetic field affecting on the pipe. But GW will pass through the metallic wall and will affect on the dynamics of the internal electrons. Length of the pipe should be suitably chosen [20]. Electrons within the pipe will get energy from GW and will eventually pass through the ring’s potential barrier. Almost all electron are trapped in the pipe but small number of electrons can cross the barrier due to energy absorption from GW. The escaped electrons may be simply detected by an electronic detector.

In this paper we consider first bounded cold electronic plasma in a long pipe. The effect of Perturbation in electronic motion will be considered in section 3. Then we review 3+1 formalism to explain why GW has real existence. We will consider in section 5 the effect of GW on cold electronic plasma that is enclosed in a long pipe. Finally we propose this apparatus for a possible detector of GW.

2 Enclosed cold electronic plasma in a long pipe

We have previously studied the electric field inside a long pipe equipped two negative charge rings and shown that it is damped as cosine hyperbolic function [19]. Actually there can not be any sign of electric field created by the rings within the pipe. Suppose an amount of cold electronic cloud within the pipe is imported. We impose a longitudinal magnetic field \( B_0 \) to radially enclosed electronic cloud, Fig 1. The electrons will rotate around the pipe axis. During rotation, they will move to the ends of the pipe. At the ends, the electrons will confront the barrier and will be reflected. We will use the fluid model to describe the situation, so there will not be pure longitudinal motion and any \( B_\phi \) due to the longitudinal motion of electrons. Rotational motion of the electrons will create a \( B_z \) that will oppose the \( B_0 \) (Lens law). There is some \( E_r \) due to non-neutral nature of the electronic plasma. We will now compute this \( E_r \) field from Poisson equation:

\[
\nabla \cdot \vec{E} = -4\pi n^0 e \Rightarrow \frac{1}{r \frac{\partial}{\partial r}} (r E_r) = -4\pi n^0 e \Rightarrow E_r = -\frac{2\pi n^0}{r}.
\]

To obtain \( B_z \), we use the Steady state Ampere law:

\[
\nabla \times \vec{B} = \frac{4\pi}{c} (-n^0 e \vec{v}).
\]

Since the vector \( \vec{v} \) merely have \( v_\phi \) component, we conclude from above equation that

\[
\frac{\partial B_z^0}{\partial r} = \frac{4\pi}{c} n^0 e v_\phi.
\]
Figure 1: Enclosed cold electronic plasma in a long pipe. Two negative potential rings at the end of the pipe create a potential barrier.

Thus longitudinal magnetic field will be changed with radius by the rotational electron motion and then, longitudinal magnetic field will never be a uniform field. We will begin with momentum equation to find rotational frequency of the electrons around axis. The plasma is so cold that there is no pressure or impact terms in momentum equation. Thus:

\[nm \left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} = -e n^0 \left( \vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right),\]  

(4)

where \( n^0 \) is the electrons density, \( m \) the electron mass, and \( e \) the electron charge. Electric field in the cylinder is

\[\vec{E} = E_r \vec{a}_r + E_z \vec{a}_z.\]  

(5)

\( E_z \) is largely due to lateral potential rings and a little due to finiteness of electronic column. As we have quoted earlier [19], the \( E_z \) of the rings will rapidly be damped, thus \( E_z \) will only be non-vanishing closely near (in some millimeter) the ends. We will ignore the \( E_z \) that belongs to the finiteness of the electronic column. \( E_r \) can be obtained from Eq (2). The \( \vec{B} \) field has two components:

\[\vec{B} = B_\phi \vec{a}_\phi + B_z \vec{a}_z.\]  

(6)

\( B_z \) is due to longitudinal motion that will be ignored in the fluid model, because the longitudinal current average is zero. \( B_z \) has two components too:

\[B_z = B^0 + B^0_z,\]  

(7)

where \( B^0 \) is the confining longitudinal external field and \( B^0_z \) is due to the rotational motion. \( B^0_z \) can be ignored in comparison with \( B^0 \). We then have from Eq(4):

\[\frac{1}{r} (v_\phi) = \frac{e}{m} (E_r + \frac{v_\phi B^0}{c} ) \vec{a}_r.\]  

(8)

And then:

\[\omega_r = \frac{v_\phi}{r} = \frac{\omega_c}{2} \left[ 1 \pm \left( 1 - \frac{2 \omega_p^2}{\omega_c^2} \right)^{\frac{1}{2}} \right],\]  

(9)

where \( \omega_c = \frac{e B_0}{mc} \) is the cyclotron angular frequency of pseudo-neutral plasma and \( \omega_p = \sqrt{\frac{4 \pi e^2}{m}} \) is the angular frequency of the electronic plasma. If the plasma was not pseudo-neutral then there would not be any \( E_r \). So we can conclude from Eq(4): \( \omega_r = \omega_c \). It is obvious from Eq(9) that if we did not ignore \( B^0_z \) in comparison with \( B^0 \) then \( B_z \) would be changed with radius (Eq(3)). If doing so, \( \omega_c \) will vary with \( r \) too, and then \( \omega_r \) vary with \( r \). But the following calculation shows that the variation is so small:

\[\frac{\partial B^0_z}{\partial r} = 4 \pi n^0 e v_\phi = \frac{4 \pi n^0 e \omega_c}{c} \frac{1}{2} \left[ 1 \pm \left( 1 - \frac{2 \omega_p^2}{\omega_c^2} \right)^{\frac{1}{2}} \right] r = \frac{2 \pi n^0 e^2 B^0}{mc^2} \left[ 1 \pm \left( 1 - \frac{2 \omega_p^2}{\omega_c^2} \right)^{\frac{1}{2}} \right] r \ll 1.\]  

(10)
3 The effect of perturbation on the dynamic of electrons

Maxwell’s Eqs and Newton’s Eqs dominate on perturbations namely perturbation can not get arbitrary values. Suppose an external agitations cause a first order perturbation:

\[ n = n_0 + n_1, \quad \vec{B} = \vec{B}_0 + \vec{B}_1, \quad \vec{v} = \vec{v}_0 + \vec{v}_1, \quad \vec{E} = \vec{E}_1. \]  
(11)

We ignored the constant part of \( E \). Starting by the first order momentum equation, one finds:

\[ \frac{\partial \vec{v}_1}{\partial t} + \vec{v}_0 \cdot \vec{\nabla} \vec{v}_1 + \vec{v}_1 \cdot \vec{\nabla} \vec{v}_0 = \frac{-e}{m} (\vec{E}_1 + \frac{\vec{v}_0 \times \vec{B}_1 + \vec{v}_1 \times \vec{B}_0}{c}). \]  
(12)

Suppose the perturbation merely has radial and polar components:

\[ \vec{v}^1 = v_\phi \vec{\alpha}_\phi + v_r \vec{\alpha}_r, \]  
(13)

where it is a mode. There are many electromagnetic modes that correspond to excitant fields. We will see that in the first order, the shear tensor can only exist. Then only deformation in \( x \) and \( y \) components will exist. Expressing Eq(2) in cylindrical coordinates:

\[ \frac{\partial v_r}{\partial t} + \omega_r \left( \frac{\partial v_r}{\partial \phi} - v_\phi \right) - \omega_r v_\phi = \frac{-e}{m} E_r - \frac{e}{mc} (v_0 B_z + v_\phi B_0), \]  
(14)

\[ \frac{\partial v_\phi}{\partial t} + \omega_r \left( \frac{\partial v_\phi}{\partial \phi} + v_r \right) + v_r \frac{\partial v_0}{\partial r} = \frac{-e}{m} E_\phi + \omega_e v_r, \]  
(15)

We can express the above equations in Fourier domain \( \frac{\partial}{\partial t} = -i\omega \) and \( \frac{\partial}{\partial \phi} = ik \), and using \( \frac{\partial v_0}{\partial r} = \omega_r \) approximation to obtain,

\[ i(k \omega_r - \omega) v_r - \omega_r v_\phi = \frac{-e}{m} E_r - \frac{e v_0}{mc} B_z, \]  
(16)

\[ \omega_r v_r + i(k \omega_r - \omega) v_\phi = \frac{-e}{m} E_\phi. \]  
(17)

If we solve this algebraic equations, we can express velocity in term of electric and magnetic field components

\[ v_\phi = a E_r + br B_z + f E_\phi, \quad v_r = AE_\phi + DE_r + Sr B_z, \]  
(18)

where \( a, b, f, A, D, S \) are defined as follows

\[ a = \frac{-e \omega_r}{m (\omega_r - \omega)^2 - \omega_r^2}, \quad b = \frac{-e \omega_r^2}{mc (\omega_r - \omega)^2 - \omega_r^2}, \quad f = \frac{i(k \omega_r - \omega)}{m (\omega_r - \omega)^2 - \omega_r^2}. \]  

\[ A = \frac{-e}{m \omega_r} + \frac{(k \omega_r - \omega)^2}{\omega_r m[(k \omega_r - \omega)^2 - \omega_r^2]}, \quad D = \frac{i(k \omega_r - \omega)e/m}{(k \omega_r - \omega)^2 - \omega_r^2} = ef, \quad S = \frac{i e \omega_r (k \omega_r - \omega)/mc}{(k \omega_r - \omega)^2 - \omega_r^2}. \]  
(19)

We need electron density and velocity to compute electromagnetic’s field in the cylinder. Eqs(18) shows that velocity can be expressed in terms of fields. Now we write density in terms of velocity. Since unperturbed velocity has only \( \phi \) component, we have from linearized momentum equations;

\[ \frac{\partial n_1}{\partial t} + n_0 \vec{\nabla} \cdot \vec{v}_1 + \omega_r \frac{\partial n_1}{\partial \phi} = 0. \]  
(20)

Converting into Fourier domain gives:

\[ -i \omega n_1 + n_0 \vec{\nabla} \cdot \vec{v}_1 + ik \omega_r n_1 = 0 \implies n_1 = \frac{n_0 \vec{\nabla} \cdot \vec{v}_1}{i(\omega - k \omega_r)}. \]  
(21)
Now, we begin with linearized Ampere Eqs and replace electron’s velocity and density from Eq(18) and Eq(21). First order Ampere Eqs are

\[ \nabla \times \vec{B}_1 = \frac{1}{c} \frac{\partial \vec{E}_1}{\partial t} - \frac{4\pi e}{c} (n_0 \vec{v}_1 + n_1 \vec{v}_0). \]  

(22)

Expressing Eqs(22) in cylindrical coordinates and then in Fourier domain:

\[ \frac{ik}{r} B_z - ik' B_\phi = -i\omega c E_r - \frac{4\pi e n_0}{c} v_r, \]
\[ -\partial B_z \partial r = -i\omega c E_\phi - \frac{4\pi e n_0}{c} v_\phi - \frac{4\pi e n_0}{c} n_1, \]
\[ \frac{\partial}{\partial r} (r B_\phi) = 0. \]  

(23)

Now, we put \( n \) and \( v \) from Eq(18) and Eq(21) in Eqs(23):

\[ \frac{ik}{r} B_z - ik' B_\phi = -i\omega c E_r + (A' E_\phi + D' E_r + S' r B_z), \]
\[ -\partial B_z \partial r = -i\omega c E_\phi + (a' E_r + b' r B_z + f' E_\phi) + [A'' \partial r (r E_\phi) + D'' \partial r (r E_r) + S'' \partial r (r^2 B_z)], \]  

(24)

where

\[ A' = -\frac{4\pi e n_0}{c} A, \quad D' = -\frac{4\pi e n_0}{c} D, \quad S' = -\frac{4\pi e n_0}{c} S, \]
\[ a' = -\frac{4\pi e n_0}{c} (1 + \frac{k\omega_r}{\omega - k\omega_r}) a, \quad A'' = \frac{4\pi e \omega r n_0}{ic(\omega - k\omega_r)} A. \]  

(25)

We must eliminate one of \( E \) or \( B \) to solve Eq(24). Using Faraday law in cylindrical coordinates, we can write \( B \) in term of \( E \)

\[ B_\phi = \frac{ck'}{\omega} E_r, \quad B_z = \frac{c}{ir\omega} (\frac{\partial}{\partial r} (r E_\phi) - ik E_r). \]  

(26)

Putting Eq(26) in Eq(24) and solving \( E_\phi \), gives:

\[ \left( -\frac{i\omega}{c} + f' \right) E_\phi + A'' \frac{\partial}{\partial r} (r E_\phi) + a' M(r) \frac{\partial}{\partial r} (r E_\phi) - a' N(r) E_\phi = \frac{-c}{i\omega} \frac{\partial}{\partial r} (\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi)) + \frac{kc}{\omega} \frac{\partial}{\partial r} \left( \frac{M(r)}{r} \frac{\partial}{\partial r} (r E_\phi) \right) \]
\[ -\frac{kc}{\omega} \frac{\partial}{\partial r} \left( \frac{N(r)}{r} E_\phi \right) - \frac{b' c}{i\omega} \frac{\partial}{\partial r} (r E_\phi) + \frac{b' c M(r)}{\omega} \frac{\partial}{\partial r} (r E_\phi) - \frac{b' c N(r)}{\omega} E_\phi - \frac{S'' c}{i\omega} \frac{\partial}{\partial r} (r E_\phi), \]  

(27)

where

\[ M(r) = \frac{(ik \frac{r}{r} - S' r)}{(\frac{i\omega}{c} + D')} + i k \left( \frac{1}{i\omega} - S' r \right) \left( \frac{c}{i\omega} + i \frac{k r^2 c}{\omega} \right), \]
\[ N(r) = \frac{A'}{\left( \frac{i\omega}{c} + D' \right) + i k \left( \frac{1}{i\omega} - S' r \right) \left( \frac{c}{i\omega} + i \frac{k r^2 c}{\omega} \right).} \]  

(28)

Solving Eq(27) and putting the field equal to zero at \( r = R \), we can obtain dispersion relation. Homogenous second order differential Eq(27) has two solutions:

\[ E_\phi(r) = c_1 E_{1\phi} + c_2 E_{2\phi}. \]  

(29)

One of this independent solution, for example \( E_{2\phi} \), is singular at origin and then is not physical. Reminder field must satisfy the following boundary condition:

\[ E_{1\phi}(r = a) = 0. \]  

(30)
By solving the obtained algebraic equation, one should obtain desired dispersion relation:

\[ k = k(\omega). \]  \hspace{1cm} (31)

With series method we have solved Eq(27) by MAPLE software. Based on what has been said before, we have obtained dispersion relation. The result is sketched in Fig 2. One can clearly see the Langmuir and electromagnetic parts in Fig 2. For better viewing we have changed the limiting value in the MAPLE program to obtain Fig 3. As we can see in Fig 3, the cutoff frequency of electromagnetic wave is:

\[ \omega_0 = 7.053\omega_r. \]  \hspace{1cm} (32)

In this study we will suppose only transverse wave exist. This assumption is based on the fact that the shear tensor create transversal motion. Due to viscosity of the electronic fluid, transversal motion will create longitudinal motion. We will also suppose that the electronic fluid is very dilute and hence there is not any longitudinal motion. Thus to first order, gravitational wave will create a consistent transversal mode. Note that we could use proper dielectric tensor:

\[ \epsilon = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \omega_p^2/\omega^2 \end{pmatrix}. \]  \hspace{1cm} (33)

In the magnetized non neutral electronic plasma confined in a metallic cylinder, the dielectric tensor is given by this matrix. With this dielectric tensor we arrive at dispersion relation [21]:

\[ (ka)^2 = \frac{\omega^2 a^2}{c^2} - \frac{\nu^2}{1 - \omega_p^2/\omega^2}, \]  \hspace{1cm} (34)
where \( a \) is cylinder radius and \( p_{\nu n} \) is \( \nu \)th root of first type Bessel function of order \( n \). This dispersion relation has a cutoff frequency that is in full agreement with our previous result Eq(32). Since we want to extend the problem to the relativistic case, we have not used dielectric tensor. In our consideration both Maxwell and momentum Equations have been changed. To compute electric field \( E_\phi \), we first obtain \( k \) from 
\[ k = \frac{2\pi n}{L}, \]
where \( L \) is the length of the pipe and \( n \) is an integer number. With a specified \( k \), one obtains \( \omega \) from dispersion relation and then electric field can be obtained. The result is sketched in Fig 4.

We must note that \( E_\phi \) is the solution of a homogenous differential equation and hence it can be specified up to a multiplication factor.

### 4 3+1 formalism

Although relativity joins space and time but sometimes splitting of space and time is useful. We usually like to think physical events as spacial events in different times. This is the 3+1 formalism; 3 for spatial dimensions and 1 for time dimension [22, 23, 24, 25, 26, 27, 28]. In 3+1 formalism, every physical quantity splits to two components, one in the space like hypersurface and the other in the orthogonal direction to it. Energy momentum tensor in this formalism is [25, 29]:

\[
T_{ab}^{dust} = \mu u_a u_b + q_a u_b + u_a q_b + p\gamma_{ab} + \pi_{ab},
\]

where \( u_a \) is the dust velocity component that have filled the whole spacetime manifold, \( \mu \) energy density, \( q \) momentum density, \( p \) isotropic pressure, \( \pi \) traceless anisotropic pressure, and \( \gamma \) is projection operator. In this formalism, energy momentum tensor of Maxwell’s field is [25, 30, 31]

\[
T_{ab}^{(em)} = \frac{1}{2}(E^2 + H^2)U_a U_b + \frac{1}{6}(E^2 + H^2)\gamma_{ab} + 2Q_{(a} U_{b)} + P_{ab},
\]
where \( E^2 = E_a E^a, \quad H^2 = H_a H^a, \quad Q_a = \eta_{abc} E^b H^C \) Poynting vector, and \( P_{ab} \) traceless tensor:

\[
P_{ab} = P_{\langle ab \rangle} = \frac{1}{2} (E^2 + H^2) \gamma_{ab} - E_a E_b - H_a H_b.
\]

\( E_a \) and \( H_a \) are the electromagnetic field components that can be obtained from Faraday tensor, \( F_{ab} \), by

\[
E_a = F_{ab} u^b, \quad H_a = \frac{1}{2} \eta_{abc} F^{bc}.
\]

(37)

Where \( \eta_{abc} \) is Levi-Civita tensor. In 3+1 formalism, matter and energy will be specified by \( \mu, q^a, P, \pi^{ab}, E^a, H^a, Q_2, \) and \( P_{ab} \). Spacetime structure will be specified by, \( \dot{u}_a \) fluid acceleration, \( \theta \) expansion coefficient, \( E^{ab} \) electric Weyl tensor, and \( H^{ab} \) magnetic Weyl tensor. These quantities are defined as follows:

\[
\theta = \tilde{\nabla}_a U^a, \quad (\theta = 3H), \quad \sigma_{ab} = \tilde{\nabla}_{\langle a} U_{b \rangle}, \quad \omega_{ab} = \tilde{\nabla}_{[a} U_{b]}\\
E_{ab} = c_{acbd} U^c U^d, \quad H_{ab} = \frac{1}{2} \eta_{ade} c_{bc} U^c,
\]

(38)

where \( \tilde{\nabla} \) is projection of covariant derivative in hypersurface, \( c_{abcd} \) the Weyl conformal curvature tensor, and \( H \) the Hubble constant. If \( \omega^{ab} = 0 \), \( \tilde{\nabla} \) will then equal to 3 dimensional covariant derivative in the hypersurface, in such case we denote it with \( \nabla \). Dynamic of energy-matter quantities and geometrical quantities can be obtained by projection of Einstein equation along normal vector (orthogonal to hypersurface). We list some of these equations that will be needed later [25]:

\[
\dot{\mu} + \tilde{\nabla}_a q^a = -\theta(\mu + p) - 2(\dot{U}_a q^a) - (\sigma_0^a \pi_a^b), \quad (\mu \text{ propagation Eq.})
\]

\[
\dot{q}^{(a)} + \tilde{\nabla}_a p + \tilde{\nabla}_b \pi^{ab} = -\frac{4}{3} \theta q^a - \sigma_0^a q^b - (\mu + p) \dot{U}^a - \dot{U}_b \pi^{ab} - \eta^{abc} \omega_b q_c, \quad (q \text{ propagation Eq.})
\]
\[ \dot{E}_{(a)} = (\sigma_{ab} + \eta_{abc}\omega^c - \frac{2}{3}\theta_{cab})E^b + \eta_{abc}U^bH^c + \text{curl}H_a - j_a, \quad (E_a \text{ propagation Eq.}) \]

\[ \dot{H}_{(a)} = (\sigma_{ab} + \eta_{abc}\omega^c - \frac{2}{3}\theta_{cab})H^b + \eta_{abc}U^bE^c - \text{curl}E_a, \quad (H_a \text{ propagation Eq.}) \]

\[ \sigma^{(ab)} - \nabla^a U^b = -\frac{2}{3}\sigma^{ab} + \dot{\pi}^{(a)} \dot{U}^{b} - \sigma^{(a}_{c}\sigma^{b)\epsilon - \omega^{(a}_{,b)} - (E^{ab} - \frac{1}{2}\pi^{ab}), \quad (\sigma_{ab} \text{ propagation Eq.}) \]

\[ \dot{E}^{(ab)} + \frac{1}{2}\pi^{(ab)} - (\text{curl}H)^{ab} + \frac{1}{2}\nabla^a q^b = -\frac{1}{2}(\mu + p)\sigma^{ab} - \theta(E^{ab} + \frac{1}{6}\pi^{ab}) + 3\sigma^{(a}_{c}(E^{b)\epsilon - \frac{1}{6}\pi^{b)\epsilon} \]

\[ - \dot{U}^{(a)} q^b + \eta^{cd(a}[2\dot{U}^{b}_d + \omega_c(E^b_d + \frac{1}{2}\pi^b_d)] \], \quad (E_{ab} \text{ propagation Eq.}) \]

\[ \dot{H}^{(ab)} + (\text{curl}E)^{ab} - \frac{1}{2}(\text{curl}\pi)^{ab} = -\theta H^{ab} + 3\sigma^{(a}_{c}(H^{b)\epsilon + \frac{3}{2}\omega^{(a}_{,b)} - \eta^{cd(a}[2\dot{U}^{b}_d}

\[ - \frac{1}{2}\sigma^{b}_{c} q^d - \omega_c H^b_d], \quad (H_{ab} \text{ propagation Eq.}) \quad (39) \]

where bracket in tensor or vector means as:

\[ v^{(a)} = \gamma^a b^{(b} \quad \& \quad T^{(ab)} = [\gamma^{(a}_{c}\gamma^{b)d} - \frac{1}{3}\gamma^{a}_{c}\gamma^{b}d]T^{cd}. \]

In deriving Eqs(39), we suppose that dust tensor prevail to electromagnetic tensor, also we have ignored gravitational self interaction. In 3+1 formalism, \( E^{ab} \) and \( H^{ab} \) represent gravitational wave and \( \sigma^{ab} \) entered as an intermediate field [32, 33, 34, 35]. In Minkowskian space \( \omega = \theta = \dot{u} = \sigma = 0 \) and Eqs(39) reduce to

\[ \frac{\partial n}{\partial t} + \nabla.(n\vec{v}) = 0, \]

\[ (\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v})^a = -\sigma^a_b - \frac{e}{m}(\dot{E} + \frac{v \times \vec{B}}{c})^a, \]

\[ (\nabla \times \vec{B})_a = -\frac{i \omega}{c} E_a + \frac{4 \pi}{c} (-nq\vec{v})_a - \sigma_{ab}E^b, \]

\[ (\nabla \times \vec{E})_a = \frac{i \omega}{c} B_a + \sigma_{ab}H^b. \quad (40) \]

Note that we have added Lorentz force to second Eq of Eqs(39) to obtain the second Eq of Eqs(40) [12]. As we can see from Eqs(40), the gravitational waves itself have not been entered in these Eqs, but correlated field \( \sigma \) have been appeared in these Eqs.

5 Gravitational waves effect on bounded cold electronic plasma

We have seen in section 3 that, if for any reason the fields in the cylinder is perturbed then the resulting fields will satisfy certain homogenous equations. Now, if we expose gravitational waves on the plasma, we will get a non-homogenous deferential equations with gravitational wave term at the right hand side. We observe from dispersion relation that at a certain frequency, the motion of the electrons will resonate and can pass through the potential barrier. We follow section 3 to obtain non-homogenous equations. To do that we replace the leading equations of section 3 with Eqs(40). To solve the resulting equations, we focus on a spacial case that \( \sigma^{ab} \) is diagonal. With this assumption, we write velocity field in term of electromagnetic field and gravitational field, \( \sigma \):

\[ v_\phi = aE_r + brB_z + fE_\phi + gr\sigma, \quad v_r = AE_\phi + DE_r + SrB_z + Gr\sigma, \quad (41) \]
where

\[
\begin{align*}
    a &= \frac{-e\omega_r/m}{(k\omega_r - \omega)^2 - \omega_r^2}, \\
    b &= \frac{-e\omega_r^2/mc}{(k\omega_r - \omega)^2 - \omega_r^2}, \\
    \omega_r &= \frac{\omega}{c}, \\
    f &= \frac{i(k\omega_r - \omega)/m}{(k\omega_r - \omega)^2 - \omega_r^2}, \\
    g &= \frac{i(k\omega_r - \omega)/e}{(k\omega_r - \omega)^2 - \omega_r^2}, \\
    A &= \frac{-e}{m}\frac{(k\omega_r - \omega)^2}{\omega_r m[(k\omega_r - \omega)^2 - \omega_r^2]} + \frac{(k\omega_r - \omega)^2}{\omega_r m[(k\omega_r - \omega)^2 - \omega_r^2]}, \\
    D &= \frac{i(k\omega_r - \omega)e/m}{(k\omega_r - \omega)^2 - \omega_r^2}, \\
    S &= \frac{i\omega_r(k\omega_r - \omega)/mc}{(k\omega_r - \omega)^2 - \omega_r^2}, \\
    G &= \frac{-e}{m}\frac{(k\omega_r - \omega)^2}{\omega_r m[(k\omega_r - \omega)^2 - \omega_r^2]} \frac{m\omega_r}{e} = \frac{A m\omega_r}{e},
\end{align*}
\]  

(42)

Now if we use Eqs(41) for the velocity field we can express magnetic field in term of electric field by the Ampere Eqs(40):

\[
\left(\frac{ik}{r} - S'\right)B_z - ik'B_\phi = (\frac{-i\omega}{c} + D)E_r + A'\phi + G'^r\sigma,
\]

\[-\frac{\partial B_z}{\partial r} - b' r B_z - S'' \frac{\partial}{\partial r} (r^2 B_z) = (\frac{-i\omega}{c} + f')E_\phi + A'' \frac{\partial}{\partial r} (r E_\phi) + a' E_r + D'' \frac{\partial}{\partial r} (r E_r) + g' r \sigma + G'' \frac{\partial}{\partial r} (r^2 \sigma),
\]

where the primed and double primed quantity is defined as follow

\[
A' = \frac{-4\pi\varepsilon_0}{c} A, \quad A' = \left(\frac{-4\pi\varepsilon_0}{c}\right)(1 + \frac{k\omega_r}{\omega - k\omega_r}) A,
\]

(44)

If we put Eq(43) in Faraday Eq(40) and obtain \(E_\phi\) from it, we will obtain the counterpart of Eq(27):

\[
\left(\frac{i\omega}{c} + f'\right)E_\phi + A'' \frac{\partial}{\partial r} (r E_\phi) + a' M(r) \frac{\partial}{\partial r} (r E_\phi) - a' N(r) E_\phi
\]

\[= \frac{-c}{i\omega + \sigma c} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi)\right) + \frac{ikc}{i\omega + \sigma c} \frac{\partial}{\partial r} \left(\frac{M(r)}{r} \frac{\partial}{\partial r} (r E_\phi)\right) - \frac{ikc}{i\omega + \sigma c} \frac{\partial}{\partial r} \left(\frac{N(r)}{r} E_\phi\right) - \frac{b'r}{i\omega + \sigma c} \frac{\partial}{\partial r} (r E_\phi)
\]

\[+ \frac{ikb'c}{i\omega + \sigma c} M(r) \frac{\partial}{\partial r} (r E_\phi) - \frac{ikb'c}{i\omega + \sigma c} N(r) E_\phi - \frac{S''c}{i\omega + \sigma c} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r} (r E_\phi))
\]

\[+ (g' + 2G') r \sigma - \frac{ikc\sigma}{i\omega + \sigma c} \frac{\partial}{\partial r} \left(\frac{L(r)}{r}\right) - \frac{ikb'\sigma}{i\omega + \sigma c} \frac{L(r)}{r},
\]

(45)

where \(M(r), N(r)\) and \(L(r)\) are defined as follows:

\[
M(r) = (\frac{ik}{r} - S') \frac{c}{r}(\frac{1}{i\omega + \sigma c})/R(r), \quad N(r) = A'/R(r), \quad L(r) = G'/R(r),
\]

\[
R(r) \equiv \left(\frac{-i\omega}{c} + D'\right) + (\frac{ik}{r} - S') \frac{c}{r}(\frac{1}{i\omega + \sigma c}) + \frac{ik^2 c}{\omega}
\]

(46)

As what has been said in section 3, \(E_\phi(r)\) can be obtained from Eq(45) and the \(E_r(r)\) can be obtained from following equation:

\[
E_r(r) = M(r) \frac{\partial}{\partial r} (r E_\phi(r)) - N(r) E_\phi(r),
\]

(47)

and magnetic field is:

\[
B_\phi = \frac{ck'}{\omega} E_r,
\]

(48)
Figure 5: Kinetic energy of electronic plasma particle will be increased by GW, for; \( n = 10^{20} \text{cm}^{-3} \), \( l=200 \text{ cm} \), \( B_0 = 10^8 \text{ G} \).

\[
B_z = \frac{c}{r(i\omega + \sigma c)} \left( \frac{\partial}{\partial r}(rE_\phi) - ikE_r \right).
\]  

(49)

Velocity field are obtained from Eqs(41). Kinetic energy of an electron near the far end of the cylinder for \( r = 4 \) is sketched in Fig 5. As we can see the kinetic energy will increase with external gravitational field.

According to general relativity the geometry of spacetime is determined by matter and energy. The motion of heavy mass in the spacetime causes time dependent variation in its curvature that is called GW. The effect of gravitation that entered in this study appears as \( \sigma \) field that are originated from the motion of far heavy (binary) stars. Having the order of magnitude of \( \sigma \) that belongs to the possible sources of GW, one can evaluate the kinetic energy of the electrons. If one knows the system parameters like electron density, the size of the pipe, and potential of the rings, he/she can in principle create such appropriate conditions that the electrons inside the pipe can pass through the potential barrier, that is a direct sign of existence of GW. This is main idea of the present paper. With a proper \( \sigma \) field strength, a typical electron can get enough energy to cross the potential barrier. This is a remarkable result because based on this calculation one can hope to detect the GW directly by the method described here.

6 Conclusions

We have proposed an experimental sketch to detect gravitational waves(GW) directly, using an cold electronic plasma in a long pipe. By considering an cold electronic plasma in a long pipe, the Maxwell equations in 3+1 formalism have been invoked to relate gravitational waves to the perturbations of plasma particles. It has been shown that the impact of GW on cold electronic plasma causes disturbances on the pathes of the electrons. Then we have shown that those electrons that absorb
energy from GW will pass through the potential barrier at the end of the pipe. Therefore, crossing of some electrons over the barrier will imply the existence of the GW.

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