FACILITATING MATHEMATICAL UNDERSTANDING IN THREE-DIMENSIONAL GEOMETRY USING THE SOLO TAXONOMY

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ABSTRACT
This study aims to examine: (1) an increase in mathematical understanding of students whose learning with SOLO taxonomy of Superitem techniques compared to conventional learning, (2) completeness of mathematical understanding of students using SOLO taxonomy of Superitem techniques, (3) student attitudes toward learning using the SOLO taxonomy of Superitem techniques. The study was a quasi-experimental study in one of the high schools in Bandung with a research design control group design posttest. The population is all class X high school students. The sample was taken by the Purposive Random Sampling technique as much as two classes. Data collection is done by giving tests and non-tests. The written test is mathematical understanding in the concept of three dimensional geometry. Non tests are attitude questionnaires with a Likert scale. The results showed (1) an increase in mathematical of students whose learning using SOLO taxonomy with Superitem techniques was higher than that of students with conventional learning methods, (2) classical mastery learning in experimental classes was achieved, and (3) positive student attitudes in all indicator.

KEYWORDS
Mathematical understanding, SOLO taxonomy, Superitem technique, quasi experiment, three-dimensional geometry

INTRODUCTION
Mathematical understanding and attitudes towards mathematics are important. Mathematical understanding is one of the developments of contemporary mathematics education Sumarmo (2002). Having a mathematical understanding and having an attitude of appreciating the usefulness of mathematics in life are some of the goals of students in mathematics (BSNP (2016); Budiharjo (2006)). Almost all learning theories make understanding the goal of learning (Dahlan, 2004). Understanding is a fundamental aspect of learning (Hiebert & Carpenter in Dahlan, 2004). Having the skills to be confident in their mathematical abilities is the goal of school mathematics (NCTM 2000). Mathematical understanding and student attitudes influence student mastery learning. In accordance with Ruseffendi statement that positive attitudes towards mathematics are positively correlated with mathematics learning achievement (Sudihartinih, 2012).

Mastery learning is often used as an indicator to state a group of people or individuals have succeeded in the learning process. Sabandar (2008) through the study of metaanalysis that he did on many studies at various levels of higher education, it was found that completeness in various aspects of ability in mathematics education still had not been achieved evenly. The ability to understand the concepts of several students is also still low (Wahyudin, 1999). The results of the 2015 PISA tests (OECD, 2018) and 2015 TIMMS (Mullis, 2015) also showed some of the students' low conceptual comprehension abilities. Even though the data is from middle school
students in Indonesia, this situation will be similar to high school students. So that a model / strategy / technique / learning approach is needed to improve student mastery learning.

In order for students to succeed in learning a material, learning readiness must be possessed (Firdaus, 2004). Readiness of student learning can be accelerated like using a spiral approach from Bruner (Ruseffendi, 1980). A spiral approach is a method used to develop concepts, starting from an intuitive way to analysis, from exploration to mastery by giving enough space between the lowest and the highest stages. The spiral approach is relevant to the characteristics of mathematics learning, namely learning from the concrete to the abstract; from simple to complex; and tiered concepts or principles.

Biggs and Collis (Alagmulai, 2006) conducted a study of the structure of learning outcomes with tests compiled in the form of Superitem, in their findings suggesting that at each stage or cognitive level there is the same and increasing response structure from simple to abstract. The structure is called SOLO Taxonomy. The study of the SOLO stage was also carried out by Sumarmo (Firdaus, 2004). His findings increase the belief that in mathematics learning, an explanation of mathematical concepts should not be directly on complex concepts or processes, but must begin with simple concepts and processes. In his research has provided an alternative learning that starts from the simple increase in the complex, the learning uses the task of Superitem. The following are the learning steps. First, illustrating concrete concepts or processes, namely by providing real data (problems in everyday life), then gradually students are guided to compile their analogies in the form of concepts or processes being discussed. Second, problem training from the simple to the complex, among others, with Superitem. The three tests are in the form of Superitem that are in accordance with the taxonomy of SOLO.

The concept in this research is geometry, because it is important. Geometry plays a role in the concepts of astronomy, chemistry, biology, algebra, statistics, calculus (Luneta, 2015). Geometry plays a role in logical thinking (Fichte in Wood 2012); it becomes a problem solver (Bobango in Ramlan (2016)). So the purpose of this study is to examine the mathematical understanding and attitudes of high school students with SOLO taxonomy with Superitem techniques.

LITERATURE REVIEW

Mathematical Understanding
Skemp stated that understanding the concept is divided into two, namely instrumental understanding and relational understanding. Instrumental understanding is the understanding of concepts that are mutually separate and only memorize formulas in simple calculations, doing things according to the algorithm. While relational understanding means understanding that can associate something with other things correctly and realize the process carried out (Pollatsek in Sumarmo, 1987). Indicators of mathematical understanding are: (1) classifying objects according to certain characteristics (according to the concept), (2) giving examples and non-examples of concepts, (3) using, utilizing, and choosing certain procedures or operations, (4) applying problem solving algorithms or algorithms.

SOLO Taxonomy
According to Biggs and Collis (Alagmulai, 2006) based on the quality of the child's response model, the SOLO stage of children is classified into five levels, namely:
(1) Pre structural, rejects giving responses or answers without a logical basis.
(2) Structural Union, can draw conclusions based on one relationship, data or information concretely.
(3) Multi structural, can draw conclusions based on two or more relationships, data or information, but still separately.
(4) Relational, can think deductively and draw conclusions based on two or more relationships, data or information in an integrated manner.
(5) Abstract, can think inductively or deductively and can form general principles or hypotheses based on information provided.

The study conducted by Sumarmo (Firdaus, 2004) provides an alternative mathematics learning that links SOLO taxonomy, starting from simple to complex. The learning uses Superitem assignments, namely:
(1) Illustration of concrete concepts or processes, providing real data (problems in daily life), then gradually students are guided to form analogies in the form of concepts or processes being discussed.
(2) Problem training from the simple to the complex, among others, with Superitem.
(3) In addition to the description or objective questions, also give a test in the form of a Superitem that is in accordance with the taxonomy of SOLO.

Questions can be made from the structural stage (small percent) to relational, then for some essential concepts students are gradually guided to the relational-abstract SOLO stage.

**RESEARCH METHOD**

The method of this research is quasi-experimental design with pre-post test control group design. This study follows the research of Sudihartinih (2012). The experimental class uses learning with using SOLO taxonomy of Superitem techniques, while the control class uses conventional learning (expository). The population is all students of class X in one of the high schools in Bandung. Samples were taken using the Purposive Random Sampling technique, as many as two classes. The experimental class participants were 31 students. The control class participants are 30 students. The instruments of data collection are non-tests and tests. Non-test instruments in the form of Likert scale which contains a statement about student attitudes. The test instrument is in the form of a written type of description to measure the mathematical understanding. Before being used for the pretest and posttest the questions were tested first to be analyzed. Following are the results of the analysis.

| Table 1. Test results problem |
|-----------------------------|
| Indicator of mathematical understanding | Number of Problem | Item Validity | Item Difficulty | Information |
| Classifying objects according to certain characteristics (according to the concept) | 1 | Low | Easy | Revision |
| Give examples and non-examples of concepts | 2 | Enough | Moderate | Used |
| Use, utilize, and choose certain procedures or operations, | 3 | Enough | Moderate | Used |
| Apply problem solving algorithms or algorithms | 4 | Enough | Moderate | Used |
The results of the analysis show that the reliability coefficient of the ability to understand concepts and mathematical reasoning on the three dimensional geometry material is 0.62 with a moderate reliability category.

To support learning using SOLO / Superitem techniques prepared teaching materials. Teaching materials consist of material and a set of tasks, either ordinary or problem description questions according to the taxonomy of SOLO / Superitem. Covers the subject: three dimensional geometry with the following sub-topics.
1. Point, line and plane in three dimensional space.
2. Determine the distance from point to line and from point to plane in three dimensional space.
3. Specifies the angle between lines and plane and between two plane in a three dimensional space.

The procedure for collecting data is as follows. Preparation phase: Making mathematics teaching materials on three dimensional geometry material according to SOLO's taxonomy of Superitem techniques; prepare written test instruments and student attitude scale questionnaire; validating research instruments; conduct trial and analysis of research instruments; prepare and administer the research permit and contact the mathematics teacher concerned to determine the date and research class. Implementation phase: pretest was conducted in the experimental class and the control class; application of learning; posttest of experimental class and control class; analyzing students' written tests (pretest and posttest) and student attitude questionnaires. The final stage: processing research data; analyze, discuss findings, draw conclusions, and report. The way to construct Superitems has been reported by Romberg, et al (1982).

RESEARCH RESULT AND DISCUSSION

Table 2 shows the results of the research.

|                          | Experiment Class | Control Class |
|--------------------------|------------------|---------------|
| N                        | 31               | 30            |
| Normal Parameters        |                  |               |
| Mean                     | 1.677            | 2.333         |
| Std. Deviation           | 0.599            | 0.606         |
| Kolmogorov-Smirnov Z     | 2.128            | 1.691         |
| Asymp. Sig. (2-tailed)   | 0.000            | 0.007         |

Based on Table 2, it is known that the average score of the pretest mathematical understanding of the experimental group is 1.68 and the control group is 2.33 (ideal score 16). The Kolmogorov-Smirnov value of the experimental group and control pretest scores was 2.128; 1.691 with asymptotic values of significance of each is less than 0.05. This means that at the significance level of 5%, the null hypothesis which states the distribution of the above data comes from a population that has a normal distribution rejected. So that the initial similarity test capability uses non parametric tests. The following is a test of two non-parametric similarities with the Mann Whitney test.
Table 3. Mann-Whitney test score pretest

|             | Mathematical Understanding Pretest |
|-------------|-----------------------------------|
| Mann-Whitney U | 233.000                           |
| Wilcoxon W   | 729.000                           |
| Z            | -3.792                            |
| Asymp. Sig. (2-tailed) | .000                              |

Based on Table 3, it is known that the results of the Asymp. Sig (2-tailed) from the Mann-Whitney test the pretest score is less than 0.05, meaning that at the 5% significance level the null hypothesis states that there is no difference in the average pretest score between the experimental groups, whose learning uses SOLO / Superitem techniques and control group students whose learning is conventional, is rejected. Thus the average initial ability to understand the concept between the experimental group and the control group students is different. So to find out an increase in mathematical understanding between the experimental and control groups, an analysis of the normalized gain with the formula was carried out:

\[
\text{Gain}(g) = \frac{\text{Posttestscore} - \text{pretesscore}}{\text{idealscore} - \text{pretesscore}}
\]

(Meltzer, 2002)

Table 4. Descriptive data and normalized gain data

|                     | Gain Normalized Experimental Class | Gain Normalized Control Class |
|---------------------|-----------------------------------|-------------------------------|
| N                   | 31                                | 30                            |
| Normal Parameters   | Mean                              | 0.654                         |
|                     | Std. Deviation                    | 0.154                         |
|                     | Kolmogorov-Smirnov Z             | 0.814                         |
|                     | Asymp. Sig. (2-tailed)            | 0.522                         |

Based on Table 4 descriptively, it can be seen that the normalized gain values of the two different groups are 0.654 and 0.421. So that statistical tests are carried out. The asymptotic value of the significance of the Kolmogorov-Smirnov test from the normalized gain in the two classes was more than 0.05. This means that at the 5% significance level the null hypothesis states that the distribution of the data above comes from populations that are normally distributed. The homogeneity test was then carried out.

From Table 5, it is known that the normalized gain in the ability to understand the concept has a significance value of Levene test of more than 0.05. This means that at the significance level of 5% the gain normalizes the concept comprehension score derived from a homogeneous population. Thus the test of the difference in mean gain is normalized using the t-test. Testing is done by one-way test at a significance value of 0.05 to test \( H_0 \) and the \( H_1 \) match with the decision-making criteria according to Widhiarso (nd) is to reject \( H_0 \) if \( \text{Sig. (1-tailed)} < 0.05 \). The relationship of significance value according to Widhiarso (nd) is \( \text{Sig.(1-tailed)} = \frac{1}{2} \text{Sig.(2-tailed)} \).
T-test calculations are presented in Table 5. The hypothesis proposed in this study is to increase of mathematical understanding of students whose learning using SOLO / Superitem techniques is better than students who are conventional learning. Based on this hypothesis the following hypothesis is formulated.

\( H_0 : \mu_{1e} = \mu_{1k} \)

Increased mathematical understanding of students whose learning using SOLO / Superitem techniques is the same as students whose learning is conventional.

\( H_1 : \mu_{1e} > \mu_{1k} \)

Increased mathematical understanding of students whose learning using SOLO / Superitem techniques is higher than students who are conventional learning. Information:

\( \mu_{1e} = \text{average gain score ability to understand the concept of the experimental group.} \)

\( \mu_{1k} = \text{average gain score ability comprehension concept control group.} \)

**Table 5. Homogeneity test and t test gain score data normalized**

|         | Levene’s Test for Equality of Variances | t-test for Equality of Means | 95% Confidence Interval of the Diff. |
|---------|----------------------------------------|------------------------------|-------------------------------------|
|         | F           | Sig. | T      | Df | Sig. (2-tailed) | Mean Diff. | Std. Error Diff. | Lower | Upper |
| GainT   | Equal variances assumed | 2.45 | 0.12   | 6.67 | 59 | 0.00 | 0.23 | 0.03 | 0.16 | 0.30 |

From the calculations in Table 5, it is obtained \( \text{Sig. (2-tailed)} = 0.00 \) so that \( \text{Sig. (1-tailed)} = 0.00 \) less than 0.05. This means that at the 5% significance level \( H_0 \) is rejected. Thus an increase in mathematical understanding of students whose learning by using Superitem / SOLO techniques is higher than students who have conventional learning.

This success can also be seen from student learning completeness. It is known that the completeness of individual learning by the school is 55%. So that the number of students in the experimental class who achieved mastery learning was as many as 29 students (94%). While the control class is 12 people (40%). Thus classical learning completeness is only achieved in the experimental class.

These successes occur because Superitem are questions that are designed according to the SOLO stage of students, where the characteristics of the questions contain concepts and processes that are increasingly cognitively given the opportunity for students to develop their knowledge and understand the relationship between concepts. Superitem is able to bring mathematical reasoning to mathematical concepts (Collis & Romberg in Romberg, 1995). Lajoie in Romberg (1995) also states that Superitem are designed to bring mathematical reasoning to mathematical concepts. Besides that it is also supported by positive student attitudes. The following are the results of analysis of student attitudes data.

**Table 6. Student attitude data**
Based on the Table 6, it is known that the average student attitude score is more than three for all student attitude indicators. This shows that students have a positive attitude, so that it affects the increase in mathematical understanding of students. In accordance with Ruseffendi statement that positive attitudes towards mathematics are positively correlated with mathematics learning achievement (Sudihartinih, 2012).

CONCLUSION

The conclusion is as follows.
(1) Increased of mathematical understanding that students use SOLO taxonomy with Superitem techniques is higher than students whose learning is conventional.
(2) Classical learning completeness in the experimental class is achieved.
(3) The attitude of students is positive.

REFERENCES

1. Alagmulai, S. (2006). SOLO, RASCH, QUEST, and Curriculum Evaluation. Paper presented at The Join ERA/AARE Conference, Singapore.
2. BSNP. (2016). Standar Isi Pendidikan Dasar dan Menengah. Jakarta: PERMENDIKBUD No 21.
3. Budiharjo. (2006). Penerapan Aspek Penilaian pada Penulisan Soal dan Pengolahan Nilai Rapor. Unpublished Handout. Bintek Matematika, Semarang.
4. Dahlan, J.A. (2004). Meningkatkan Kemampuan Penalaran dan Pemahaman Matematika Siswa Sekolah Lanjutan Tingkat Pertama Melalui Pendekatan Pembelajaran Open-Ended, Studi Eksperimen Pada Siswa Sekolah Lanjutan Pertama Negeri di Kota Bandung. Unpublished Dissertation. Universitas Pendidikan Indonesia, Bandung.
5. Firdaus, A. (2004). Meningkatkan Kemampuan Pemecahan Masalah Siswa SLTP Melalui Pembelajaran Menggunakan Tugas Bentuk Superitem. Unpublished Thesis. Universitas Pendidikan Indonesia, Bandung.
6. Luneta, K. (2015). Understanding students' misconceptions: An analysis of final Grade 12 examination questions in geometry. Pythagoras, 36(1), 1-11.
7. Meltzer, D.E. (2002). Addendum to: The relationship between mathematics preparation and conceptual learning gains in physics: a possible "hidden variable" in diagnostic pretest score. Am. J. Phys., 70(12), 1259-1268.
8. Mullis, I.V.S., Martin, M.O., Foy, P., & Hooper, M. (2015). TIMMS 2015 International Result in Mathematics. IEA: TIMSS & PIRLS, Boston College.
9. NCTM. (2000). *Principles and Standard for School Mathematics*. Resto, Virginia: The National Council of Teachers of Mathematics, Inc.

10. OECD. (2018). *PISA 2015 Result in Focus*. Paris: OECD.

11. Ramlan, A.M. (2016). The Effect of Van Hiele Learning Model Toward Geometric Reasoning Ability Based on Self-Efficacy of Senior High School Students. *Journal of Mathematics Education, 1*(2), 63-72.

12. Romberg, T.A., Jurdak, M.E., Collis, K.F., & Buchanan, A.E. (1982). *Construct Validity of Set of Mathematical Superitem*. Madison: Wisconsin Center for Education Research.

13. Romberg, T.A. (1995). *Reform in School Mathematics and Authentic Assessment*. New York: SUNY Press.

14. Rusefendi, E.T. (1980). *Pengantar kepada Membantu Guru Mengembangkan Kompetensinya dalam Pengajaran Matematika untuk Meningkatkan CBSA*. Bandung: Tarsito.

15. Sabandar, J. (2008). Pembelajaran Matematika Sekolah dan Permasalahan Ketuntasan Belajar Matematika. *Professorship Inaugural Speech*, Universitas Pendidikan Indonesia, Bandung.

16. Sudihartinih, E. (2012). Meningkatkan Kemampuan Penalaran Matematik Siswa SMA Melalui Pembelajaran Menggunakan Tugas Bentuk Superitem. *Proceeding*. Seminar Nasional Matematika VI at UNNES.

17. Sumarmo, U. (1987). Kemampuan Pemahaman dan Penalaran Matematika Siswa SMA Dikaitkan dengan Kemampuan Penalaran Logik Siswa dan Beberapa Unsur Pembelajaran. *Unpublished Dissertation*. Universitas Pendidikan Indonesia, Bandung.

18. Sumarmo, U. (2002). Alternatif Pembelajaran Matematika dalam Menerapkan KBK. *Proceeding*, The FPMIPA UPI Seminar.

19. Wahyudin. (1999). Kemampuan Guru Matematika, Calon Guru Matematika dan Siswa dalam Mata Pelajaran Bahasa Indonesia. *Unpublished Dissertation*. Universitas Pendidikan Indonesia, Bandung.

20. Widhiarso, W. (n.d.). *Uji Hipotesis Konparatif (Uji t)*. Retrieved from: https://hasanbio.files.wordpress.com/2012/08/lebih-mesra-dengan-ujit.pdf, on June 18th, 2019.

21. Wood, D.W. (2012). Chapter Five: The Relationship between Geometry and the Wissenschaftslehre. *Academic Journal Fichte-Studien-Supplementa*, 29, 243.