Construction of True Stress-Strain Curve of Metallic Material by Artificial Neural Network and Small Punch Test

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Abstract. Small punch test (SPT) is used to evaluate mechanical properties of metallic materials by a miniature specimen. A method combining SPT and artificial backpropagation neural network for determining the true stress-strain curve of metallic materials is proposed. The load-displacement curves of different hypothetical materials were obtained by the finite element model of SPT with considering Gurson-Tvergaard-Needleman (GTN) damage parameters and used to train a backpropagation neural network. The relationship between the load-displacement curve of SPT and the true stress-strain curve of the conventional uniaxial tensile test was established based on the trained neural network, which is validated by the experimental results of X80 pipeline steel. The results demonstrate that the established relationship can be used to predict the true stress-strain curve of the metallic materials and then to determine their elastoplastic properties by SPT.

1. Introduction
Small punch test (SPT) [1-3] is one of nearly non-destructive methods for miniaturized specimen techniques. It has been applied for the evaluation of a wide range of mechanical properties, including strength, ductility, fracture toughness, ductile-brittle transition temperature (DBTT) and high temperature mechanical behavior, i.e. creep. Mao correlated the yield load and the maximum load of the SPT with the yield strengths and the ultimate tensile strengths of various kinds of materials, and gave the corresponding empirical formula [4]. Chica analyzed the initial stage of the load-displacement curve of SPT, correlated the slope of the initial stage with the elastic modulus of alloys, and obtained the empirical formula between them [5]. Based on the principle of equivalent energy, Peng [6] analyzed the third stage of load-displacement curve of SPT, obtained the true stress-strain curve and ultimate tensile strength of the materials. In general, the empirical correlation method was used by the most of researchers to obtain the correlation formula of material strength and fracture toughness based on their own test results. However, due to a wide variety of differences in the nature of material used in the test, the configuration of the testing machine, and the definition methods of yield load on the load-displacement curve of SPT, the empirical formulas proposed by different researchers are also different [7-9]. These empirical formulas are not comparable to each other, resulting in the consumption of research efforts and research resources. Therefore, it is urgent to find a more reasonable and effective method to correlate the small punch test with the conventional standard tests, and then the accurate elastoplastic mechanical properties of materials would be extracted.
In this study, a method was proposed to determine the true stress-strain curve of metallic materials through SPT and backpropagation (BP) neural network, which is an algorithm of artificial neural network and is widely used in the training of feedforward neural networks for supervised learning. The complete elastoplastic mechanical properties of materials could be obtained by this method. The load-displacement curves of SPTs were obtained by the finite element model within Gurson-Tvergaard-Needleman (GTN) damage parameters. The BP neural network was trained with the load-displacement curves and true stress-strain curves, and the relationship between them was established. Compared with the conventional empirical correlation method, this method has the advantages of lower cost, higher accuracy and higher efficiency.

2. Experiment and Finite Element Modeling

The chemical composition of X80 pipeline steel used in the test was shown in Table 1. The engineering stress-strain curve of X80 pipeline steel was measured by conventional uniaxial tensile test, and showed in Fig. 1. Three groups of tests were carried out and average values of the load and displacement were used to plot the curve. The elastic modulus $E$, yield strength $\sigma_y$ and ultimate tensile strength of X80 pipeline steel were 206 GPa, 594 MPa and 713 MPa, respectively.

![Fig. 1 Engineering stress-strain curve of X80 pipeline steel obtained by uniaxial tensile test](image1)

![Fig. 2 True stress-strain curve of X80 pipeline steel](image2)

| Table 1 Chemical composition of X80 pipeline steel (wt. %) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| C               | Si              | Mn              | P               | S               | Cr              | Mo              | Ni              | Nb              | V               | Ti              | Cu              | Fe              |
| 0.043           | 0.23            | 1.87            | 0.01            | 0.0028          | 0.025           | 0.27            | 0.23            | 0.06            | 0.006           | 0.017           | 0.13            | Bal.            |

The plastic parameters of X80 pipeline steel required for numerical simulation were obtained from the true stress-strain curve. Hollomon formula were used to describe the elastoplastic constitutive relations of most ductile materials as follows,

$$\sigma = \begin{cases} E\varepsilon & \sigma > \sigma_y \\ K\varepsilon^n & \sigma < \sigma_y \end{cases}$$  \hspace{1cm} (1)

where $\sigma$ is true stress, $\sigma_y$ is yield strength, $\varepsilon$ is true strain, $E$ is elastic modulus, $K$ is strength coefficient and $n$ is strain hardening index. The true stress and true strain of materials can be calculated by Eq. 2 and Eq. 3.

$$\sigma = \sigma_E \left(1 + \varepsilon_E \right)$$ \hspace{1cm} (2)

$$\varepsilon = \ln \left(1 + \varepsilon_E \right)$$ \hspace{1cm} (3)

where $\sigma_E$ is engineering stress and $\varepsilon_E$ is engineering strain. The true stress-strain curve of material is shown in Fig. 2, the part of the curve between yield strength and ultimate tensile strength was fitted.
by power function, and the parameters K and n in Hollomon formula were obtained.

In addition, the relationship between engineering stress and real stress-strain can be obtained by deducing the above formulas, which can be expressed as follows,

\[
\sigma_E = \sigma e^{-n}
\]

where the \(e=2.718\) is a natural constant. When the engineering stress reaches the maximum value, i.e. \(\sigma_b\), the ultimate strain \(\varepsilon_E\) equals the strain hardening index \(n\) \[10\], so the ultimate tensile strength can be given as follows \[11\],

\[
\sigma_b = K\left(\frac{n}{e}\right)^n
\]

from the above relationship, it can be seen that the ultimate tensile strength of different materials can be deduced from the corresponding Hollomon formula. The SPT device was shown in Fig. 3. The punch head's radius \(r\) was 1.25 mm and the lower die diameter \(D\) was 4 mm, with 0.5 mm chamfer edge. The thickness \(t\) and diameter \(d\) of SPT specimen were 0.5 mm and 10 mm, respectively. The specimen was placed on the lower die and pressed by the upper die to prevent sliding. During the test, the punch was loaded downward at a constant rate of 0.2 mm/min. The center of the specimen was pressed down until the specimen breaks. The load and the displacement of the punch were recorded during the whole test process, and the load-displacement curve of the SPT was obtained, as shown in Fig. 4.

These parameters were closely related to the elastic modulus \(E\), yield strength \(\sigma_y\) and ultimate tensile strength \(\sigma_b\) obtained by uniaxial tensile test. Mao \[4\] conducted SPT and uniaxial tensile tests on various materials, and correlated the test results. With Mao’s equation, the elastic modulus \(E = 225\text{GPa}\), yield strength \(\sigma_y = 473\text{MPa}\) and ultimate tensile strength \(\sigma_b = 982\text{MPa}\) were obtained by substituting the parameters obtained from the test results of SPT of X80 pipeline steel. Compared with the uniaxial tensile test results, the errors were 9.2%, 20.3% and 37.8% respectively. It can be found that the mechanical properties of X80 pipeline steel can be obtained directly from empirical formulas given by other researcher, but the errors were larger than the standard conventional test. In order to obtain more accurate mechanical properties such as elastic modulus, yield strength and ultimate tensile strength of pipeline steel materials by SPT, it was necessary to conduct SPT and uniaxial tension test for a variety of pipeline steel materials to fit new empirical formulas. However, if the in-service pipeline is inspected without shutting down transportation, the samples required for standard test cannot be taken out. Therefore, it is necessary to develop a novel method to obtain the mechanical properties of in-service pipeline materials under the condition of ensuring the integrity of pipeline. Therefore, in this study, BP neural network combined with SPT was proposed and used to obtain the elastoplastic mechanical properties parameters of in-service pipeline materials.

The finite element software ABAQUS was used to simulate the SPT process. Fig 4 shows the 2D
axisymmetric finite element model of SPT specimen. The punch, upper die and lower die were considered as rigid body without considering the deformation of them. The upper die and lower die were fixed in all degrees of freedom, whereas the punch can be moved vertically by a displacement boundary condition. The contact between the specimen and punch, upper die and lower die was modeled, and the friction coefficient $\mu$ was 0.15. Element CAX4R was selected for the element. Ten layers of meshes are divided in thickness direction.

In order to simulate the elasto-plastic deformation, damage and fracture behavior of SPT specimens, the definition of material parameters was divided into two parts: (1) the definition of material elastoplastic parameters; (2) the definition of material damage parameters. The elastoplastic parameters required in the finite element model include elastic modulus, Poisson's ratio and stress-strain relationship, which can be obtained by uniaxial tensile test. For material damage parameters, the GTN damage model is adopted [12] and corresponding parameters was introduced into the finite element model. $q_1$, $q_2$ and $q_3$ are the material damage parameters. $f_0$ denotes the initial void volume fraction. $f_N$ is volume fraction of particles that can be nucleated. $f_F$ and $f_C$ define the void volume fraction at failure and at a critical situation, respectively.

### 3. Results and Discussions

Experimental Results and Continuous Damage Model. In the finite element model of SPT, the void parameters $f_0$, $f_N$, $f_C$ and $f_F$ of X80 pipeline steel are unknown, so the values of these parameters should be determined. The specimens with diameter of 10.0 mm and thickness of 0.600 mm were cut from the pipeline by wire cutting. Sandpaper was used to grind the specimens to remove the influence areas of overheating and work hardening, so that there were no cracks and other macro defects in the specimens. In order to ensure the accuracy of the test, three groups of tests were prepared. The final specimen thickness $t$ was 0.495 mm, 0.498 mm and 0.499 mm, respectively. The load-displacement data of each specimen were recorded and three groups of load-displacement curves of SPT were obtained. The finite element model of SPT with different mechanical properties were simulated to ensure that the plastic parameters of the material were unchanged, and only the GTN damage parameters were changed. Finally, the specific material damage parameters could be determined through comparing the results obtained from the experiment and finite element analysis, respectively. Fig. 5 shows the curve with the highest matching degree with test. In this paper, the GTN damage parameters are finally determined as shown in Table 2.

| $q_1$ | $q_2$ | $q_3$ | $\varepsilon_N$ | $\delta_N$ | $f_0$ | $f_N$ | $f_C$ | $f_F$ |
|------|------|------|-------------|---------|------|------|------|------|
| 1.5  | 1    | 2.25 | 0.3         | 0.1     | 0.0025 | 0.0008 | 0.03  | 0.15 |

BP Neural Network Modeling. BP neural network can train a lot of data and establish corresponding relationship between two different factors. In this paper, the load-displacement curve of SPT was taken as input set and the true stress-strain curve as output set. The main training scheme of BP neural network is shown in Fig. 6.

In the training process of BP neural network, a large number of data need to be calculated to reduce the error continuously. The calculation process, it is divided into two steps: (1) Firstly, the target error value was set, and a group of load-displacement data and real stress-strain data of SPT of the same material were input. After receiving a set of load-displacement data of the SPT, the activation function $f_1(e)$ calculated these data in this layer, and the calculated result $y_1$ was output to the second layer neurons. The output set $y$ was calculated by the second layer neurons, and the error $\delta$ between the result and the input true stress-strain data was obtained; (2) The calculated error $\delta$ was fed back to the layers of neurons, and the weight value of each layer of neurons were corrected, respectively. The corrected weight value was used to repeat step (1) until the error was less than the preset target error.
\[
\begin{align*}
    w'_1 &= w_1 + \eta \delta_1 \frac{df_1(e)}{de} x_1 \\
    w'_2 &= w_2 + \eta \delta_1 \frac{df_2(e)}{de} x_2 \\
    w'_3 &= w_3 + \eta \delta_1 \frac{df_3(e)}{de} y_1
\end{align*}
\]

where \(w\) is weight value, \(\eta\) is the learning rate, \(f(e)\) is the activation function and \(\delta\) is the calculation error.

Assembling the Data Base. The training set of BP neural network includes two parts: the load-displacement curve of SPT as input set and the true stress-strain curve of the same material as output set. The true stress-strain curve can be obtained by systematically changing the parameters \(K\) and \(n\) in Hollomon formula. The strain hardening index \(n\) of most materials ranges from 0.005 to 0.6, and the strength coefficient \(K\) ranges from 200 MPa to 2000 MPa and incensement gradient of \(n\) is 0.02.

| Sample number | Input set | Output set |
|---------------|-----------|------------|
| 1             | X1₁, …, X₁₂₀₁, Y₁₁, …, Y₁₂₀₁ | A₁₁, …, A₁₅₀₁, B₁₁, …, B₁₅₀₁ |
| 2             | X₁₂, …, X₁₂₀₂, Y₁₂, …, Y₁₂₀₂ | A₁₂, …, A₁₅₀₂, B₁₂, …, B₁₅₀₂ |
| …            | …         | …         |
| n             | X₁ᵣ, …, X₁ᵣ₂₀, Y₁ᵣ₁, …, Y₁ᵣ₂₀ | A₁ᵣ₁, …, A₁ᵣ₁₅₀, B₁ᵣ₁, …, B₁ᵣ₁₅₀ |

A new true stress-strain curve can be obtained by changing the parameters \(K\) and \(n\). Each true stress-strain curve was substituted into the finite element model of the SPT to obtain the complete load-displacement curve. When BP neural network determines input data, each column matrix is identified as a set of input data, so each curve corresponds to a column of data. The load-displacement curve of SPT and the true stress-strain curve of material are composed of data points, and each data point is confirmed by abscissa and ordinate coordinates. Therefore, each data point on the curve needs to be transformed in the way shown in Table 3. Load-displacement curve of SPT is expressed by \((X, Y)\), true stress-strain curve is expressed by \((A, B)\). Each load-displacement curve takes 120 sets of coordinates, and the true stress-strain curve takes 50 set of coordinates. \((X₁, Y₁), (X₂, Y₂), …, (X₁₂₀, Y₁₂₀)\) is transformed into \([X₁, X₂, …, X₁₂₀, Y₁, Y₂, …, Y₁₂₀]\), the transformation mode of material true stress-strain curve is the same as that of load-displacement curve.

Results of BP Neural Network Calculation. The load-displacement curve of X80 pipeline steel obtained from SPT was input into the trained BP neural network to obtain its true stress-strain curve. Compared with the uniaxial tensile test results, as shown in Fig. 7, the matching degree between the calculated results of BP neural network and uniaxial tensile test was higher.
Fig. 7 Comparison of true stress–strain curves obtained by BP neural network and uniaxial tensile test.

The true stress-strain curves obtained by experiment and BP neural network were processed. The parameters K and n in Hollomon formula were obtained by fitting curve. The ultimate tensile strength was obtained by Eq. 5. When the true strain is 0.002, the stress is taken as the yield strength of the material. The uniaxial tensile test results and the results calculated by BP neural network were listed in Table 4.

Table 4 The mechanical properties of materials obtained by BP neural network and uniaxial tensile test

| Material | Yield strength $\sigma_y$ | Yield strength $\sigma_y_{BPNN}$ | Relative errors $(\sigma_y_{BPNN}-\sigma_y)/\sigma_y$ | Ultimate tensile strength $\sigma_b$ | Ultimate tensile strength $\sigma_b_{BPNN}$ | Relative errors $(\sigma_b_{BPNN}-\sigma_b)/\sigma_b$ |
|----------|--------------------------|-------------------------------|-----------------------------------------|-------------------------------|-------------------------------|-----------------------------------------|
| X80      | 607                      | 567                           | -6.6                                    | 713                           | 746                           | 4.6                                     |

From the Table 5, it can be seen that the maximum error of elastoplastic mechanical properties of X80 pipeline steel calculated by BP neural network is 9.3%, compared with uniaxial tensile test results, which is much smaller than the error of 37.8% calculated by conventional empirical correlation method.

4. Conclusions
In this study, a method was proposed to obtain the true stress-strain curve of metallic materials by using SPT, finite element method and BP neural network. With the developed finite element model of SPT within GTN damage parameters, which was verified by the experimental results. A large number of load-displacement curves of different hypothetical materials were obtained, combined with the corresponding true stress-strain curves, BP neural network was trained to establish the relationship between SPT and conventional uniaxial tensile test. The results demonstrate that the true stress-strain curves and elastoplastic properties of metallic materials could be accurately obtained by the method combining SPT and artificial BP neural network without performing conventional tensile test.

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