Worldline Superfield Actions for N=2 Superparticles

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Abstract

We propose doubly supersymmetric actions in terms of $n = 2(D - 2)$ worldline superfields for $N = 2$ superparticles in $D = 3, 4$ and Type $IIA$ $D = 6$ superspaces. These actions are obtained by dimensional reduction of superfield actions for $N = 1$ superparticles in $D = 4, 6$ and 10, respectively. We show that in all these models geometrodynamical constraints on target superspace coordinates do not put the theory on the mass shell, so the actions constructed consistently describe the dynamics of the corresponding $N = 2$ superparticles.

We also find that in contrast to the $IIA$ $D = 6$ superparticle a chiral $IIB$ $D = 6$ superparticle, which is not obtainable by dimensional reduction from $N = 1$, $D = 10$, is described by superfield constraints which produce dynamical equations. This implies that for the $IIB$ $D = 6$ superparticle the doubly supersymmetric action does not exist in the conventional form.

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1 Introduction

An initial motivation for considering doubly supersymmetric models \[1\]–\[14\] was to better understand the relationship between Ramond–Neveu–Schwarz and Green–Schwarz formulation of superstrings \[15\]. The former possesses manifest local supersymmetry on the worldsheet of the superstring, while the latter is manifestly supersymmetric in target superspace and, in addition, has a non–manifest local fermionic (so called kappa) symmetry \[16\] on the worldsheet. The doubly supersymmetric models possess both types of supersymmetry simultaneously. In general they describe wider spectrum of physical states than the single supersymmetric counterparts they stemmed from \[1\]. At the same time in \[2\] it was realized that the \(\kappa\)–symmetry of Casalbuoni–Brink–Schwarz superparticles \[17\] and Green–Schwarz superstrings \[15\] can be a manifestation of a hidden local supersymmetry on the world surface, and worldline superfield actions for superparticles in \(N = 1, D = 3\) and \(N = 1, D = 4\) target superspace were constructed as an implementation of this idea. Since then all presently known super–p–branes have acquired doubly supersymmetric description (see \[14\] for a review and \[18\] for recent progress) thus forming a subclass of more general class of the doubly supersymmetric models.

Having replaced the \(\kappa\)–symmetry with the local supersymmetry one got a covariant algebra of irreducible first–class constraints generating this symmetry, which has influenced the development of new methods of covariant quantization of superparticles, superstrings \[19\]–\[24\] and null–super–p–branes \[25\].

The doubly supersymmetric approach has also proved to be the most appropriate for the application to studying super–p–brane dynamics of geometrical methods of surface theory describing properties of embedding world (super)surfaces into target (super)spacetimes \[12, 13, 18\]. This has helped one to get superfield equations of motion for new important types of super–p–branes, such as Dirichlet branes and a five–brane of M theory \[26\], without knowledge of their actions, which were constructed later on \[27, 28\].

A basic condition which determines the embedding of world supersurface of any super–p–brane into target superspace, is a so–called geometrodynamical condition. It prescribes target–space supervielbein vector components be zero along the Grassmann directions of the world supersurface. Depending on the dimensions of worldvolume and target superspaces this condition can define either non–minimal or minimal embedding. For \(N = 1\) superparticles \[2\]–\[4, 6\], Type I superstrings in \(D = 3, 4, 6\) and \(10\) \[4\], and for an \(N = 2\) \(D = 3\) superparticle and superstring \[8\] the geometrodynamical constraint defines non–minimal embedding. From the dynamical point of view this means that the geometrodynamical constraint does not put these theories on the mass shell, i.e. it produces no dynamical equations of motion. In this case this condition can be incorporated into the action with a superfield Lagrange multiplier, and such an action will consistently describe the dynamics of the supersymmetric object. All known superfield actions of the models mentioned above contain this geometrodynamical term.

However, for a wide class of models, such as Type II \(D = 10\) superstrings and D–branes, the \(N = 1, D = 11\) supermembrane and the five–brane the situation is different. The geometrodynamical condition defines the minimal embedding of the corresponding superworldvolumes into target superspaces, i.e. it puts the theory on the mass shell \[12, 18\]. Now we cannot construct the geometrodynamical action as in the case of non–minimal embedding since the superfield Lagrange multiplier accompanying the geometrodynamical constraint would contain redundant propagating degrees of freedom spoiling the spec-
trum of physical states of the model. Thus, when the geometrodynamical constraint contains superfield equations of motion of a super–p–brane one encounters a problem in constructing a worldvolume superfield action for the super–p–brane. In this case one can use a generalized action principle of the group–manifold approach. It allows one to construct a worldvolume functional which, however, is not a superfield action in the conventional sense.

On the contrary, conventional superfield actions should exist for the models with off–shell superfield constraints and it seems of interest to find wider class of super–p–branes (than that known so far) for which the off–shell geometrodynamical constraint does not produce equations of motion, and to construct superfield actions for them. A possible way of getting, at least, some of these models is to consider the dimensional reduction of super–p–brane models for which superfield actions are already known. For instance, we can take an $N = 1, D = 10$ superparticle and dimensionally reduce it down to $N = 2, D = 6$, or reduce an $N = 1, D = 6$ superparticle to $N = 2, D = 4$ and see what form of doubly supersymmetric actions one gets for these superparticles with extended supersymmetries. This is the main purpose of this paper.

The paper is organized as follows:

In Section 2 we describe the main features of the doubly supersymmetric formulation of an $N = 1$ superparticle which will be then used to obtain superfield actions of $N = 2$ superparticles in $D = 3, 4$ and $D = 6$.

In Section 3 the action of an $N = 2, D = 3$ superparticle is obtained by the dimensional reduction of the superfield action of the $N = 1$ superparticle in $D = 4$. In addition to the geometrodynamical term (which was considered earlier by Galperin and Sokatchev) this action includes a Lagrange multiplier term with a bilinear combination of Grassmann derivatives of the fermionic superfields. The Lagrange multiplier is a purely auxiliary degree of freedom, so its presence does not spoil the physical content of the model. The second term in the action ensures its invariance under a local symmetry with superfield parameters, analogous to that of $N = 1$ superparticles, which allows one to gauge away auxiliary fields from the Lagrange multipliers.

In Sections 4 and 5 we apply the dimensional reduction procedure to get doubly supersymmetric actions for an $N = 2$ superparticle in $D = 4$ and a Type IIA superparticle in $D = 6$. We analyze the geometrodynamical constraints in the both cases and show that they do not produce equations of motion. We also demonstrate that for the Type IIB $D = 6$ superparticle it turns out to be impossible to construct the superfield action in the form mentioned above because in this case the geometrodynamical constraint puts the theory on the mass shell. The dynamical contents of the models constructed coincide with that of the $D = 4$ and $6 N = 2$ Casalbuoni–Brink–Schwarz superparticles.

Conventions and notation

We use the mostly negative signature for the Minkowski metric tensor

$$\eta^{mn} = \text{diag}(+, -, \ldots, -).$$

In what follows the indices corresponding to vector and spinor representations of target space Lorentz groups $SO(1, D - 1)$ are underlined; non–underlined indices with and

1This problem of supersymmetric theories is well known. When the number of supersymmetries is too large and/or the dimension of space–time is too high, superfield constraints which are required for diminishing a number of independent fields put the theory on the mass shell.
without hats are reserved for representations of orthogonal and unitary groups, respectively.

\( SU(2) \)-indices are raised and lowered by the unit antisymmetric tensors \( \epsilon^{AB} \) and \( \epsilon_{AB} \) (\( \epsilon_{21} = \epsilon^{12} = 1 \)) as follows

\[
\xi^A = \epsilon^{AB} \xi_B, \quad \xi_A = \epsilon_{AB} \xi^B.
\]

\{ ... \} and \[ ... \] denote symmetrization and antisymmetrization of indices with the weight 1 respectively. Other notation are introduced below.

## 2 \( N = 1 \) superparticle in terms of unrestricted world line superfields

In this section we briefly consider the doubly supersymmetric formulation of \( N = 1 \) superparticle mechanics in \( D = 3, 4, 6 \) and 10 \([2, 6]\) in terms of unrestricted worldline superfields.

The superparticle dynamics is defined by the minimal embedding of the one-dimensional \( n = D - 2 \) worldline supersurface into the \( D \)-dimensional \( N = 1 \) target superspace \([2]\) (we will consider flat target superspaces only). In the case of the doubly supersymmetric particles \([2, 3, 6]\), \( \hat{\kappa} \) the intrinsic geometry of the worldline is superconformally flat and can be described with the use of the flat Cartan forms \([2, 3, 6]\)

\[
e_\tau = d\tau - i\eta_\hat{q} \eta_{\hat{q}}, \quad e_\hat{q} = d\eta_\hat{q}, \quad e_\bar{q} = d\bar{\eta}_\bar{q}, \quad (1)
\]

where \( \hat{q} \) is the index of the representation of the \( SO(D - 2) \) group, which is the automorphism group of the \((1|D - 2)\) worldline supersymmetry algebra. These forms constitute a non-degenerate supervielbein in the cotangent space of the worldline supermanifold. It is used as a basis of the superworldline differential forms. For example, pullbacks of superinvariant one-forms \( \Pi^m = dX^m - i\bar{\Theta} \Gamma^m \Theta \) and \( \Pi^\mu = d\Theta^\mu \) of the flat target superspace onto the superworldline are

\[
\Pi^m = e_\tau \Pi^m_\tau + e_\hat{q} \Pi^m_\hat{q}, \quad (2)
\]

\[
\Pi^\mu = e_\tau \Pi^\mu_\tau + e_\hat{q} \Pi^\mu_\hat{q}. \quad (3)
\]

Embedding equations adjust the target space supervielbein to the worldline one in such a way that the worldline vector component of \( \Pi^m \) is directed along the bosonic component of the superworldline frame, the fermionic components of the target space supervielbein lie along Grassmann directions of the worldline superspace and all other (bosonic) components of the target supervielbein are orthogonal to the worldline. In particular, projections of the form \( \Pi^m_\hat{q} \) onto the intrinsic vielbein forms \( e_\hat{q} \) vanish:

\[
\Pi^m_\hat{q} = D_\hat{q} X^m - iD_\bar{q} \bar{\Theta} \Gamma^m \Theta = 0, \quad (4)
\]

where \( D_\hat{q} = \frac{\partial}{\partial \eta_\hat{q}} + i\eta_\hat{q} \frac{\partial}{\partial \bar{\eta}_\bar{q}} \) are flat fermionic covariant derivatives of the worldline superspace and the superparticle target space coordinates \( X^m \) and \( \Theta^\mu \) are scalar unrestricted worldline superfields.

\(^2\)The number of worldline supersymmetries is chosen to be equal to the number of independent \( \kappa \)-symmetry transformations of the corresponding Brink–Schwarz superparticle action, i.e. half of the number of target space supersymmetries \([2]\).
Equation (4) is called the geometrodynamical constraint. Its left–hand side $\Pi_{q}^{m}$ is incorporated into superfield actions of superparticles with a superfield Lagrange multiplier $P_{m\hat{q}}$:

$$S_{D}^{N=1} = \int d\tau d^{D-2}\eta P_{m\hat{q}}\Pi_{q}^{m}. \quad (5)$$

The integration is performed over the worldline supersurface, which is supposed to be non–degenerate. This means [6] that the vector $\frac{\partial X^{m}}{\partial \tau}$ is non–vanishing and the matrix $D_{q}\Theta^{\mu}$ is of the maximal rank $3$:

$$\frac{\partial X^{m}}{\partial \tau} \neq 0; \quad \text{rank}(D_{q}\Theta^{\mu}) = n. \quad (6)$$

The variation of (5) with respect to $P_{m\hat{q}}$ yields the geometrodynamical constraint (4) as a superfield equation. In the case of $N = 1$ superparticles (4) defines a non–minimal embedding of the superparticle worldline into the target superspace, and (5) consistently describes the dynamics of the $N = 1$ superparticles in $D = 3, 4, 6$ and 10.

The superfield action (5) is invariant with respect to local worldline superdiffeomorphisms

$$\tau \to \tau'(\tau, \eta), \quad \eta_{q} \to \eta'_{q}(\tau, \eta), \quad (7)$$

which are restricted to transform the flat supervielbein form $e_{\tau}(\eta)$ homogeneously:

$$e'_{\tau} = W(\tau, \eta)e_{\tau}. \quad (8)$$

This requirement imposes the constraint on the superreparametrization functions $\tau'(\tau, \eta)$ and $\eta'_{q}(\tau, \eta)$:

$$D_{q}\tau' - i(D_{\hat{q}}\eta'_{\hat{p}})\eta'_{\hat{p}} = 0, \quad (9)$$

and ensures that the covariant derivatives $D_{q}$ transform homogeneously as well

$$D'_{q} = (D_{\hat{q}}\eta_{\hat{p}})D_{\hat{p}}. \quad (10)$$

In the infinitesimal form these transformations are described by a single superfield parameter $\Lambda$ [6, 8]:

$$\delta \tau = \Lambda - \frac{1}{2}\eta_{q}D_{q}\Lambda; \quad \delta \eta_{q} = -\frac{i}{2}D_{q}\Lambda; \quad \delta D_{q} = -\frac{1}{2}\frac{\partial \Lambda}{\partial \tau}D_{q} + \frac{i}{4}[D_{q}, D_{\hat{p}}]\Lambda D_{\hat{p}}; \quad (11)$$

$$\delta(d\tau d^{D-2}\eta) = (1 - \frac{n}{2})\frac{\partial \Lambda}{\partial \tau}(d\tau d^{D-2}\eta).$$

The invariance of the action (5) under (11) requires the following transformation properties of the Lagrange multiplier:

$$\delta P_{m\hat{q}} = \frac{n - 1}{2}\frac{\partial \Lambda}{\partial \tau}P_{m\hat{q}} + \frac{i}{4}[D_{q}, D_{\hat{p}}]\Lambda P_{m\hat{p}}. \quad (12)$$

If the dimension of space–time is more than 3, the action (5) possesses [8] an infinitely reducible symmetry [8]

$$\delta P_{m\hat{q}} = D_{\hat{p}}(\xi_{\hat{p}q}\Gamma^{\mu}D_{z}\Theta) \quad (13)$$

\[3\]The requirement (3) is introduced to exclude a non–physical solution corresponding to a particle “frozen” into a point of the target superspace.
with the spinor parameter $\xi_{\hat{m}\hat{q}\hat{s}}$, which is symmetric and traceless in $\hat{p}, \hat{q}, \hat{s}$. This symmetry allows one to gauge away all $P_{\hat{m}\hat{q}}$ components except

$$p_{\hat{m}} = \frac{1}{(D - 2)!} \hat{\epsilon}_{\hat{q}_1 \ldots \hat{q}_{D-2}} D_{\hat{q}_1} \ldots D_{\hat{q}_{D-3}} P_{\hat{m}\hat{q}_{D-2}} |\eta = 0,$$

which satisfies the equation $\frac{\partial p_{\hat{m}}}{\partial \tau} = 0$ and plays the role of the superparticle momentum. After elimination of all auxiliary fields the dynamical content of the action (5) coincides with that of the Casalbuoni–Brink–Schwarz superparticle in $D = 3, 4, 6$ and 10.

### 3 $N = 2$ superparticle in $D = 3$

Doubly supersymmetric action for the $N = 2$ $D = 3$ superparticle in terms of $n = 2$ worldline superfields, constructed in [8], consists of the geometrodynamical term only:

$$S_{GS} = \int d\tau d^2\eta P_{\hat{m}\hat{q}}(D_{\hat{q}} X_{\hat{m}} - i (D_{\hat{q}} \Theta^{\hat{A}}) \gamma_{\hat{m}\hat{p}} \Theta^{\hat{A}}). \quad (14)$$

Here $\hat{m} = 0, 1, 2$ is the $D = 3$ vector index, $\hat{\mu}, \hat{\nu} = 1, 2$ are indices of three–dimensional Majorana spinors, $\gamma_{\hat{m}\hat{\mu}}$ are $D = 3$ Dirac matrices in the real representation, $\hat{q}$ and $\hat{A}$ are indices, corresponding to the local $n = 2$ worldline supersymmetry of the worldline and the global $N = 2$ supersymmetry of the target space, respectively.

In this section we will show that the action (14) can be obtained from the four–dimensional action (5) by dimensional reduction and upon eliminating some of the auxiliary fields.

We start with studying the structure of the geometrodynamical constraint of the $N = 2$ $D = 3$ superparticle and show that it does not put the theory on the mass shell. In order to do this it is convenient to rewrite this constraint in terms of superinvariant one–forms:

$$\Pi^{\hat{m}} \equiv dX^{\hat{m}} - i d\Theta^{\hat{A}} \gamma_{\hat{m}\hat{p}} \Theta^{\hat{A}} = e_\tau \Pi_{\tau}^{\hat{m}}, \quad (15)$$

$$\Pi^{\hat{\mu}\hat{A}} = d\Theta^{\hat{A}} \equiv e_\tau \Pi_{\tau}^{\hat{\mu}\hat{A}} + e_{\hat{q}} \Pi_{\hat{q}}^{\hat{\mu}\hat{A}}, \quad (16)$$

where $e_\tau$ and $e_{\hat{q}}$ are intrinsic vielbeins defined in (I), and

$$\Pi_{\tau}^{\hat{m}} = \frac{\partial X^{\hat{m}}}{\partial \tau} - i \frac{\partial \Theta^{\hat{A}}}{\partial \tau} \gamma_{\hat{m}\hat{p}} \Theta^{\hat{A}},$$

$$\Pi_{\tau}^{\hat{\mu}\hat{A}} = \frac{\partial \Theta^{\hat{A}}}{\partial \tau}; \quad \Pi_{\hat{q}}^{\hat{\mu}\hat{A}} = D_{\hat{q}} \Theta^{\hat{A}}. \quad (17)$$

Selfconsistency conditions for the Eqs. (15) and (16) are:

$$- i d\Theta^{\hat{A}} \gamma_{\hat{m}\hat{\mu}} d\Theta^{\hat{A}} = d(e_\tau \Pi_{\tau}^{\hat{m}}), \quad (18)$$

$$d(e_\tau \Pi_{\tau}^{\hat{\mu}\hat{A}}) + d(e_{\hat{q}} \Pi_{\hat{q}}^{\hat{\mu}\hat{A}}) = 0. \quad (19)$$

Expanding the l. h. s. of (18) in components (I) and using the “constraints” on $e_\tau$ and $e_{\hat{q}}$

$$de_\tau = - i e_{\hat{q}} e_{\hat{q}}, \quad dc_{\hat{q}} = 0,$$
which follow from (1), one can rewrite (18) and (19) as follows

$$\Pi^{\hat{\mu}\hat{A}}_{\{q} \gamma_{\mu}^{\hat{\nu}} \Pi^{\hat{A}}_{\nu\}} = \frac{1}{2} \delta_{\hat{q}\hat{p}} \Pi^{\hat{\mu}\hat{A}}_{\hat{p}} \gamma_{\mu\nu}^{\hat{\nu}} \Pi^{\hat{A}}_{\nu\},$$

(20)

$$\Pi^{\hat{m}}_{\hat{r} \hat{r}} = \Pi^{\hat{\mu}\hat{A}}_{\hat{q}} \gamma_{\mu}^{\hat{\nu}} \Pi^{\hat{A}}_{\nu\hat{q}}.$$

(21)

Eq. (21) is called the twistor constraint since it expresses the vector $\Pi^{\hat{m}}_{\hat{r}}$ as a bilinear combination of commuting spinors $\Pi^{\hat{\mu}\hat{A}}_{\hat{r}}$. Eqs. (20) are algebraic constraints, relating the bosonic spinor superfields $\Pi^{\hat{\mu}\hat{A}}_{\hat{s}}$ to each other.

Assume that spinors $\Pi^{\hat{\mu}1}_{\hat{s}}$ form a complete non–degenerate spinor basis in $D = 3$ (i.e. $\text{det} \, \Pi^{\hat{\mu}1}_{\hat{s}} \neq 0$). Then $\Pi^{\hat{\nu}2}_{\hat{p}}$ can be represented as a linear combination of basic spinors:

$$\Pi^{\hat{\mu}\hat{A}}_{\hat{p}} = a_{\hat{p}\hat{q}} \Pi^{\hat{\mu}1}_{\hat{q}}.$$

(22)

Substituting this expression into (20) and taking into account the linear independence of three non–vanishing vectors $\Pi^{\hat{\mu}\hat{A}}_{\hat{p}} \gamma_{\mu}^{\hat{\nu}} \Pi^{\hat{A}}_{\nu\hat{q}}$, one obtains that the constraint (20) reduces to conditions on the coefficients $a_{\hat{p}\hat{q}}$:

$$a_{\hat{p}\hat{q}} \delta_{\hat{q}\hat{i}} + \delta_{\{\hat{p}\hat{q}\}}^{\hat{i}} a_{\hat{i}\hat{a}} a_{\hat{a}\hat{a}} + \delta_{\hat{i}\hat{a}} = \frac{1}{2} \delta_{\hat{p}\hat{q}} (a_{\hat{i}\hat{a}} a_{\hat{a}\hat{a}} + \delta_{\hat{i}\hat{a}})$$

(23)

(on the l. h. s. of this expression only the indices $\hat{p}, \hat{q}$ are symmetrized). The general solution to the system (23) has the following form:

$$a_{\hat{p}\hat{q}} = \pm \epsilon_{\hat{p}\hat{q}}.$$

(24)

One can select the sign (say, +) without the loss of generality, the system (20) reducing to

$$\Pi^{\hat{\mu}\hat{2}}_{\hat{p}} = \epsilon_{\hat{p}\hat{q}} \Pi^{\hat{\mu}\hat{1}}_{\hat{q}}.$$

(25)

Hitting (25) by the Grassmann covariant derivatives $D_{\hat{q}}$, one obtains the constraint which expresses the field $\epsilon_{\hat{q}\hat{p}} D_{\hat{q}} D_{\hat{p}} \Theta^{\hat{A}A}$ in terms of $\Pi^{\hat{\mu}\hat{A}}_{\hat{r}}$:

$$\epsilon_{\hat{q}\hat{p}} D_{\hat{q}} D_{\hat{p}} \Theta^{\hat{A}A} = 2i \epsilon^{\hat{A}B} \Pi^{\hat{\mu}\hat{B}}_{\hat{r}}$$

(26)

Taking into account (27) one gets the twistor constraint in the following form:

$$\Pi^{\hat{m}}_{\hat{r} \hat{r}} = 2 \Pi^{\hat{\mu}\hat{1}}_{\hat{s}} \gamma_{\mu}^{\hat{\nu}} \Pi^{\hat{A}}_{\nu\hat{q}}.$$

(27)

One can see that the geometrodynamical constraint does not contain dynamical equations. It includes only constraints which express higher components of the superfields $X_{\hat{m}A}$ and $\Theta^{\hat{A}A}$ in terms of their leading components. This means that the superfield $P^{\hat{m}A}_{\hat{p}}$ does not contain any dynamical degrees of freedom except the superparticle momentum

$$P^{\hat{m}}_{\hat{p}} = \frac{1}{2} \epsilon_{\hat{q}\hat{p}} D_{\hat{q}} ( \Pi^{\hat{m}A}_{\hat{p}} |_{\eta=0} ).$$

(28)
All other $P_{\hat{\mu}\hat{\nu}}$ components are auxiliary fields which can be eliminated either by explicit solution of constraints or by fixing gauges of available local symmetries [5].

In order to clarify the structure of the local symmetries and with the purpose of the generalization to higher space–time dimensions we shall obtain a superfield action for the $N = 2 \ D = 3$ superparticle once again by the dimensional reduction of the doubly supersymmetric formulation of an $N = 1 \ D = 4$ superparticle considered in the previous Section. A reason for this is that the symmetry structure of the doubly supersymmetric formulation of $N = 1 \ D = 4$ is known and it is retained after dimensional reduction.

The $D = 4$ action [5], written in terms of two–component Weyl spinors, is

$$S^{N=4}_{D=4} = \int d\tau d^{3}\eta P_{\hat{\mu}\hat{\nu}}(D_{\hat{\mu}}X^{\hat{\nu}} - i(D_{\hat{\nu}}\Theta^{\hat{\nu}})\sigma^{\mu\alpha}_{\hat{\nu}}\tilde{\Theta}^{\bar{\alpha}} \sigma_{\bar{\alpha}\hat{\mu}} - i(D_{\hat{\mu}}\tilde{\Theta}^{\hat{\alpha}})\sigma^{\mu\alpha}_{\hat{\nu}}\Theta^{\bar{\alpha}}) = i(\bar{\Theta}^{\hat{\alpha}}\sigma^{\mu\alpha}_{\hat{\nu}}\Theta^{\bar{\alpha}} - i(\bar{\Theta}^{\hat{\alpha}}\sigma^{\nu}_{\hat{\mu}}\Theta^{\bar{\alpha}})) = i(\bar{\Theta}^{\hat{\alpha}}\sigma^{\mu\alpha}_{\hat{\nu}}\Theta^{\bar{\alpha}} - i(\bar{\Theta}^{\hat{\alpha}}\sigma^{\nu}_{\hat{\mu}}\Theta^{\bar{\alpha}})).$$  \hspace{1cm} (29)

Here $m = 0, 1, 2, 3$ is the $D = 4$ vector index and $\alpha, \hat{\alpha}$ are indices of the fundamental representations of $SL(2, \mathbb{C})$.

Performing the dimensional reduction of (29) to $D = 3$, we require the coordinate $X_2$ to be constant and the corresponding component of the particle momentum to be zero, i.e. (see (28)) $\epsilon_{\hat{\rho}\hat{\nu}}D_{\hat{\rho}}P_{2\hat{\nu}}|_{\eta=0} = 0$. Then, redefining the spinors and the $\sigma$–matrices:

$$0, 1, 3 \rightarrow \hat{0}, \hat{1}, \hat{2}$$

$$\sigma^{0}_{\hat{\mu}\hat{\nu}} \rightarrow \gamma^{0}_{\hat{\mu}\hat{\nu}}, \quad \sigma^{1}_{\hat{\mu}\hat{\nu}} \rightarrow \gamma^{1}_{\hat{\mu}\hat{\nu}}, \quad \sigma^{3}_{\hat{\mu}\hat{\nu}} \rightarrow \gamma^{3}_{\hat{\mu}\hat{\nu}}, \quad \sigma^{2}_{\hat{\mu}\hat{\nu}} \rightarrow \epsilon_{\hat{\mu}\hat{\nu}},$$

$$\Theta^{\alpha} \rightarrow \frac{1}{\sqrt{2}}(\Theta^{\hat{\alpha}} + i\Theta^{\hat{\alpha}}), \quad \bar{\Theta}^{\alpha} \rightarrow \frac{1}{\sqrt{2}}(\Theta^{\hat{\alpha}} - i\Theta^{\hat{\alpha}}),$$

we obtain a $D = 3$ functional

$$S^{N=2}_{D=3} = \int d\tau d^{2}\eta[P_{\hat{\rho}\hat{\eta}}(D_{\hat{\rho}}X^{\hat{\eta}} - i(D_{\hat{\eta}}\Theta^{\hat{\eta}})\gamma^{\hat{\rho}\hat{\eta}}\Theta^{\hat{\eta}}) - iP_{\hat{\rho}\hat{\eta}}(D_{\hat{\rho}}\Theta^{\hat{\eta}})\epsilon^{\hat{\rho}\hat{\eta}\hat{\mu}\hat{\nu}}\Theta^{\hat{\mu}\hat{\nu}}],$$  \hspace{1cm} (30)

where one should take into account that the Lagrange multiplier $P_{2\hat{\eta}}$ is now a restricted superfield. It must satisfy the equation $D_{\hat{\eta}}P_{2\hat{\eta}} = 0$, which follows from the action (29) as a result of its variation with respect to $X_2$. The general solution to this equation is:

$$P_{2\hat{\eta}} = iD_{\hat{\eta}}Q_{\hat{\rho}\hat{\eta}} + \frac{1}{2}\epsilon_{\hat{\rho}\hat{\eta}\hat{\mu}}\epsilon_{\hat{\mu}\hat{\nu}}D_{\hat{\nu}}P_{2\hat{\eta}}|_{\eta=0},$$  \hspace{1cm} (31)

where $Q_{\hat{\rho}\hat{\eta}}$ is symmetric and traceless in $\hat{\rho}, \hat{\eta}$. The expression in brackets is nothing but the second component of the $D = 4$ particle momentum $p_2$ which we put to zero in the course of dimensional reduction. Thus one can rewrite $P_{2\hat{\eta}}$ in terms of unrestricted bosonic superfield $Q_{\hat{\rho}\hat{\eta}}$ only

$$P_{2\hat{\eta}} = iD_{\hat{\eta}}Q_{\hat{\rho}\hat{\eta}}.$$  \hspace{1cm} (32)

Substituting this expression back into (30) and performing integration by parts of the second term, one gets

$$S^{N=2}_{D=3} = \int d\tau d^{2}\eta[P_{\hat{\rho}\hat{\eta}}(D_{\hat{\rho}}X^{\hat{\eta}} - i(D_{\hat{\eta}}\Theta^{\hat{\eta}})\gamma^{\hat{\rho}\hat{\eta}}\Theta^{\hat{\eta}}) + Q_{\hat{\rho}\hat{\eta}}(D_{\hat{\rho}}\Theta^{\hat{\eta}})\epsilon^{\hat{\rho}\hat{\eta}\hat{\mu}\hat{\nu}}\Theta^{\hat{\mu}\hat{\nu}}].$$  \hspace{1cm} (33)
One can see that (30) differs from the action (14) by the second term. The action (33) possesses the \( n = 2 \) local worldline supersymmetry and is also invariant under local transformations of the Lagrange multipliers \( \hat{P}_{\hat{m} \hat{n}} \) and \( \hat{Q}_{\hat{p} \hat{q}} \):

\[
\delta P_{\hat{m} \hat{n}} = D_{\hat{p}} (\hat{\xi}^{\hat{A}}_{\hat{p} \hat{q} \hat{s}} \eta D_{\hat{s}} \hat{Q}^{\hat{A}}_{\hat{p}}), \quad \delta Q_{\hat{p} \hat{q}} = -\frac{i}{2} (\hat{\xi}^{\hat{A}}_{\hat{p} \hat{q} \hat{s}} \epsilon^{\hat{A} \hat{B}}_{\hat{p} \hat{q}} \hat{D}_{\hat{s}} \hat{\Theta}^{\hat{B}}),
\]

\[
(34)
\]

\[
\delta Q_{\hat{p} \hat{q}} = D_{\hat{s}} (\Xi_{\hat{p} \hat{q} \hat{s}}), \quad (35)
\]

where the worldline superfield parameters \( \hat{\xi}^{\hat{A}}_{\hat{p} \hat{q} \hat{s}} \) and \( \Xi_{\hat{p} \hat{q} \hat{s}} \) are completely symmetric and traceless in \( \hat{p}, \hat{q}, \hat{s} \).

These symmetries allow one to eliminate all the components of \( Q_{\hat{p} \hat{q}} \) and \( P_{\hat{m} \hat{n}} \) except (28). The remaining action is equivalent to the Casalbuoni–Brink–Schwarz superparticle action. Additional spinor constraints obtained by varying (33) with respect to \( Q_{\hat{p} \hat{q}} \) are consequences of the geometrodynamical condition. Thus the second term in (33) can be dropped away by use of the symmetry (35) and the part of (34) without the loss of any dynamical information about the superparticle motion and one gets the action (14).

Note, however, that in contrast to (14) where a local symmetry structure is hidden at the component level, the action (33) possesses all gauge symmetries in an explicit superfield form, which will be helpful for the construction and the analysis of \( N = 2 \) \( D = 4 \) and \( D = 6 \) superparticle actions.

4  \( N = 2 \) \( D = 4 \) superparticle action

In this Section the \( N = 2 \), \( D = 4 \) superparticle action will be obtained by the dimensional reduction of the action (3) in \( D = 6 \), which now takes the form

\[
S_{D=6}^{N=1} = \int d\tau d^4\eta P_{\hat{M}\hat{n}} (D_{\hat{q}}X^{\hat{m}}_{\hat{n}} - i (D_{\hat{q}}\Theta^{\hat{I}})_{\hat{m} \hat{n}} \hat{\Theta}^{\hat{I}}). \quad (36)
\]

Here \( \hat{M} \) is the \( SO(1,5) \) vector index, \( \hat{q} \) is the index of the \( n = 4 \) worldline supersymmetry algebra. The \( SU(2) \)–Majorana spinor Grassmann coordinates \( \Theta^{\hat{I}} \) carry the \( SU^*(4) \) index \( \hat{\mu} \) as well as the \( SU(2) \)–index \( \hat{I} \) and satisfy the condition (30):

\[
\bar{\Theta}^{\hat{\mu}} \equiv \bar{\Theta}^{\hat{I}} = C^{\hat{\mu} \hat{\nu}}\bar{\epsilon}^{\hat{I} \hat{J}} \Theta^{\hat{\nu}}; \quad (37)
\]

where \( C^{\hat{\mu} \hat{\nu}} \) is a \( D = 6 \) charge conjugation matrix

\[
C^{\hat{\mu} \hat{\nu}} = \begin{pmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{pmatrix}
\]

(index \( \hat{\mu} \) corresponds to a conjugate \( SU^*(4) \) representation). \( \gamma^{\hat{m}}_{\hat{\mu}} \) are the \( D = 6 \) “Pauli” matrices, for which we choose the following realization

\[
\gamma^{\hat{m}} = \begin{pmatrix}
0 & \sigma^{\hat{m}}_{\hat{\mu} \hat{\nu}} \\
\sigma^{\hat{m}}_{\hat{\mu} \hat{\nu}} & 0
\end{pmatrix}, \quad \gamma^{\hat{4}} = \begin{pmatrix}
-\delta^{\hat{\alpha}_{\hat{\beta}}}_{\hat{\mu}} & 0 \\
0 & -\delta^{\hat{\mu}_{\hat{\beta}}}_{\hat{\alpha}}
\end{pmatrix}, \quad \gamma^{\hat{5}} = \begin{pmatrix}
-\delta^{\hat{\beta}_{\hat{\mu}}}_{\hat{\alpha}} & 0 \\
0 & \delta^{\hat{\mu}_{\hat{\alpha}}}_{\hat{\beta}}
\end{pmatrix},
\]

(37)
where $m = 0, 1, 2, 3$ is the $SO(1, 3)$ vector index; $\alpha, \dot{\alpha}$ are $SL(2, C)$ indices and $\sigma^m_{\alpha\dot{\alpha}}$ are the Pauli matrices. Decomposing the $SU(2)$–Majorana spinors in the action (36) into a pair of two–component $D = 4$ Weyl spinors

$$\Theta^{\mu\dot{i}} = (\Theta^{\alpha\dot{i}}, \Theta_{\dot{\alpha}}),$$

substituting the realization (37) and putting $X^{\pm\dot{\alpha}} = \text{const}$, $\dot{p}^{\pm\dot{\alpha}} = 0$, one obtains the following action reduced to $D = 4$

$$S_{D=4}^{N=2} = \int d\tau d^4\eta [P_{\dot{m}\dot{\alpha}}(D_{\dot{q}}X^m - i(D_{\dot{q}}\Theta^{\alpha\dot{i}})\sigma^m_{\alpha\dot{\alpha}}\Theta^\beta_\dot{\beta} - i(D_{\dot{q}}\Theta_{\dot{\alpha}})\sigma^{\dot{\alpha}\dot{\alpha}}\Theta^\beta_{\beta}) - i\pi_{\dot{q}}(D_{\dot{q}}\Theta^{\alpha\dot{i}})\Theta_{\dot{\alpha}i} - i\pi_{\dot{q}}(D_{\dot{q}}\Theta_{\dot{\alpha}})\Theta^\beta_{\dot{\beta}}].$$

(38)

In (38) we introduced the complex superfield $\pi_{\dot{q}} \equiv P_{\dot{m}\dot{\alpha}} + iP_{\dot{m}\dot{\alpha}}$, which satisfies the equation $D_{\dot{q}}\pi_{\dot{q}} = 0$ following from the variation of the (36) with respect to $X^{\dot{\alpha}}$ and $X^{\dot{\alpha}}$. The general solution to this equation has the form

$$\pi_{\dot{q}} = iP_{\dot{m}q}Q_{\dot{m}q} + \epsilon_{\dot{m}\dot{\rho}\dot{\sigma}\dot{q}}\bar{\eta}\bar{\eta}\bar{\epsilon}(p_{\dot{m}} + ip_{\dot{m}}),$$

where $Q_{\dot{m}q}$ is an unrestricted superfield, which is symmetric and traceless in $\dot{p}$ and $\dot{q}$; $p_{\dot{m}}$ and $p_{\dot{m}}$ are components of the $D = 6$ superparticle considered in the previous Section:

$$p_{\dot{m}} = \frac{1}{24}\epsilon_{\dot{m}\dot{\rho}\dot{\sigma}\dot{q}}D_{\dot{p}}D_{\dot{q}}D_{\dot{s}}P_{\dot{m}}|_{\eta = 0},$$

which we put to zero. So, $\pi_{\dot{q}} = iP_{\dot{m}q}Q_{\dot{m}q}$.

Inserting this expression into the action (38) and performing integration by parts of the last two terms, one gets the action in the form similar to the action of the $N = 2 D = 3$ superparticle considered in the previous Section:

$$S_{D=4}^{N=2} = \int d\tau d^4\eta [P_{\dot{m}\dot{\alpha}}(D_{\dot{q}}X^m - i(D_{\dot{q}}\Theta^{\alpha\dot{i}})\sigma^m_{\alpha\dot{\alpha}}\Theta^\beta_\dot{\beta} - i(D_{\dot{q}}\Theta_{\dot{\alpha}})\sigma^{\dot{\alpha}\dot{\alpha}}\Theta^\beta_{\beta}) + Q_{\dot{m}\dot{q}}(D_{\dot{q}}\Theta^{\alpha\dot{i}})(D_{\dot{q}}\Theta_{\dot{\alpha}}) + c.c.]$$

(39)

The action obtained possesses manifest $n = 4$ local worldline supersymmetry (11) and is invariant under the following transformations of the Lagrange multipliers:

$$\delta P_{\dot{m}\dot{\alpha}} = \frac{1}{2}D_{\dot{p}}(\xi^{\alpha\dot{i}}_{\dot{m}\dot{\alpha}}\sigma^m_{\alpha\dot{\alpha}}D_{\dot{s}}\Theta^\beta_\dot{\beta}) + \frac{1}{2}D_{\dot{p}}(\dot{\xi}_{\dot{m}\dot{\alpha}}\dot{\epsilon}^{\dot{\alpha}\dot{\beta}\dot{q}}\sigma^{\alpha\dot{\alpha}}D_{\dot{s}}\Theta^\beta_\dot{\beta}),$$

$$\delta Q_{\dot{m}\dot{q}} = -i(\xi_{\dot{m}\dot{\rho}\dot{\sigma}\dot{q}}D_{\dot{q}}\Theta^{\alpha\dot{i}}),$$

$$\delta Q_{\dot{m}\dot{q}} = D_{\dot{s}}(\xi_{\dot{m}\dot{\alpha}}),$$

(40)

(41)

with the parameters $\xi_{\dot{m}\dot{\rho}\dot{\sigma}\dot{q}}$ and $\dot{e}^{\dot{\alpha}\dot{\beta}\dot{q}}$ completely symmetric and traceless in $\dot{p}, \dot{q}$ and $\dot{s}$. The variation of (39) with respect to the Lagrange multiplier $P_{\dot{m}\dot{\alpha}}$ gives the geometrodynamical constraint of the $N = 2, D = 4$ superparticle, which can be written in the form

$$\Pi^m = dX^m - id\Theta^{\alpha\dot{i}}\sigma^m_{\alpha\dot{\alpha}}\Theta^\beta_\dot{\beta} - id\Theta_{\dot{\alpha}}\dot{\sigma}^{\alpha\dot{\alpha}}\Theta^\beta_{\dot{\beta}} = e_r\Pi^m,$$

(42)
\[ \Pi^{ul} \equiv d\Theta^{ul} = e\gamma^{u} \Pi_{\gamma}^{ul} + e_{q} \Pi_{\bar{q}}^{ul}. \] (43)

For the analysis of this constraint it is convenient to use the complex \( SU(2) \) vector parametrization for \( n = 4 \) worldline Grassmann coordinates:

\[ \eta_{\bar{q}} \rightarrow (\eta; (\bar{\eta}) = \bar{\eta}). \]

The flat worldline superinvariant one–forms \( \Pi^{\alpha} \) in this notation are

\[ e_{\tau} = d\tau = \frac{i}{2}d\eta^{i}\bar{\eta}_{i} - \frac{i}{2}d\bar{\eta}_{i}\eta^{i}, \quad e^{i} = d\eta^{i}, \quad \bar{e}_{i} = d\bar{\eta}_{i}, \] (44)

so one can rewrite (43) as:

\[ \Pi^{\alpha} = e_{\tau} \Pi_{\tau}^{\alpha} + e^{i} \Pi_{\tau}^{\alpha} + \bar{e}_{i} \bar{\Pi}_{\tau}^{\alpha}; \] (45)

\[ \Pi_{\tau}^{\alpha} = D_{i} \Theta^{\alpha}, \quad \bar{\Pi}_{\tau}^{\alpha} = D^{\prime} \Theta^{\alpha}, \]

where \( D_{i} = \frac{\partial}{\partial \tau} + i\bar{\eta}_{i}\frac{\partial}{\partial \tau} \) and \( D^{\prime} = -\frac{\partial}{\partial \eta^{i}} - i\eta^{i}\frac{\partial}{\partial \tau} \) are complex conjugate Grassmann covariant derivatives, which satisfy the following (anti)commutation relations

\[ \{D_{i}, D_{j}\} = -2i\delta_{ij} \frac{\partial}{\partial \tau}, \quad \{D_{i}, D_{j}\} = 0, \quad [D_{i}, \frac{\partial}{\partial \tau}] = 0. \] (46)

Selfconsistency conditions for (42) and (47)

\[ -2id\Theta^{\alpha} \sigma^{m}_{\alpha \alpha} d\bar{\Theta}^{\alpha} = e_{\tau} d\Pi^{m}_{\alpha} - ic^{i} \bar{e}_{j} \delta_{ij} \Pi^{m}_{\alpha}, \]

\[ e_{\tau} d\Pi^{m}_{\alpha} - ic^{i} \bar{e}_{j} \delta_{ij} \Pi^{m}_{\alpha} + c^{j} d\Pi^{m}_{\alpha} + \bar{e}_{i} d\Pi^{m}_{\alpha} = 0; \]

give algebraic constraints on the fields \( \Pi^{m}_{\alpha} \) and \( \bar{\Pi}^{m}_{\alpha} \):

\[ \Pi^{m}_{\alpha} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{\beta} = 0; \] (47)

\[ \Pi^{m}_{\alpha} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{ij} + \bar{\Pi}^{m}_{ij} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{\alpha} = \]

\[ \frac{1}{2} \delta^{i}_{j} (\Pi^{m}_{k} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{ik} + \bar{\Pi}^{m}_{ik} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{ji}) \] (48)

as well as the twistor constraint

\[ \Pi^{m} = \Pi^{m}_{k} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{ik} + \bar{\Pi}^{m}_{ik} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{ji}. \] (49)

To solve the system of the algebraic constraints (47) and (48) we consider the spinors \( \Pi^{m}_{\alpha} \) as a \( D = 4 \) spinor basis and expand all other spinor superfields in it:

\[ \Pi^{m}_{\alpha} = a_{i}^{j} \Pi^{\alpha}_{j}, \quad \bar{\Pi}^{m}_{\alpha} = b^{ij} \Pi^{\alpha}_{j}, \] (50)

\( a_{i}^{j} \) and \( b^{ij} \) being superfield coefficients. Substituting (50) into (47) and (48) and assuming that the vectors \( \Pi^{\alpha}_{j} \sigma^{m}_{\alpha \alpha} \bar{\Pi}^{\alpha}_{\alpha} \) are non–vanishing and linearly independent, one obtains that the constraints reduce to the following restrictions on the coefficients

\[ a_{i}^{j} = a\delta_{i}^{j}, \quad b^{ij} = -\bar{a} \bar{e}^{i} \bar{e}^{j}, \quad b^{2ij} = e^{i} \bar{e}^{j}, \]

11
where $a \in \mathbb{C}$ and $\varphi \in \mathbb{R}$ are arbitrary superfields which can be gauged away by the use of local $SU(2)$ transformations generated by the fields $D_i D^i \Lambda|_{\eta=0}$, $\frac{1}{2} [D_i, D^j] \Lambda|_{\eta=0}$, being part of the local worldline superdiffeomorphisms. So, one can rewrite the constraints for bosonic superfields \((50)\) as follows

$$\Pi^{\hat{a} \hat{b}} = a \Pi^{\alpha \beta}, \quad \Pi^\alpha = a e^{i\varphi} \Pi^\alpha, \quad \Pi^{\hat{a} \hat{b}} = e^{i\varphi} \Pi^{\alpha \beta}. \quad (51)$$

Hitting these constraints by fermionic covariant derivatives and using the algebra \((46)\), we obtain constraints on higher components of the superfields $a$ and $\varphi$ and expressions for the second order components of the superfield $\Theta$ in terms of the leading ones:

\[
D_i a = 0; \quad D_i \varphi = \frac{i}{1 + a\bar{a}} (a D_i \bar{a} + e^{-i\varphi} \bar{D}_i a),
\]
\[
D_i D^j \Theta^{\alpha \beta} = \frac{4ie^{-i\varphi}}{1 + a\bar{a}} (\frac{\partial \Theta^{\alpha \beta}}{\partial \tau} - \frac{\partial \Theta^{\alpha \beta}}{\partial \tau} + i(D^j \varphi) D_i \Theta^{\alpha \beta} + \frac{i}{1 + a\bar{a}} (a D^j \bar{a} + e^{-i\varphi} \bar{D}^j a) D_i \Theta^{\alpha \beta},
\]
\[
\bar{D}^j D^j \Theta^{\alpha \beta} = \frac{1}{2} e^{i\varphi} \frac{4ie^{-i\varphi}}{1 + a\bar{a}} \left( \frac{\partial \Theta^{\alpha \beta}}{\partial \tau} + \frac{\partial \Theta^{\alpha \beta}}{\partial \tau} \right) + \frac{i}{1 + a\bar{a}} (a D^j \bar{a} + e^{i\varphi} D^j \bar{a}) D_i \Theta^{\alpha \beta} - \sigma^{\hat{a} \hat{b} \hat{c} \hat{d}} (\bar{D}^k \varphi) \sigma_{\hat{a} \hat{b} \hat{c} \hat{d}} D_i \Theta^{\alpha \beta},
\]

($\hat{a}$ is the $SO(3)$ index). We observe that no dynamical equations appear in \((51)\) and \((52)\). The twistor constraint \((49)\) acquires the form

$$\Pi^m = -(1 + a\bar{a}) \Pi^{\hat{a} \hat{b}} \sigma_{\hat{a} \hat{b} \hat{c} \hat{d}} \Pi^{\hat{c} \hat{d}}.$$ 

One can show that the additional constraints on the spinors $\Pi^{\hat{a} \hat{b}}$ and $\Pi^{\alpha \beta}$ obtained by the variation of \((39)\) with respect to $Q_{\hat{m} \hat{n}}$ are identically satisfied if one takes into account \((51)\). The Lagrange multiplier $Q_{\hat{m} \hat{n}}$ is a purely auxiliary degree of freedom which can be completely gauged away by use of the transformations \((40)\) and \((41)\). Their role is to ensure the invariance of the action under the superfield transformations \((40)\) and \((41)\) which simplifies the analysis of the physical degrees of freedom of the model. Using the symmetry \((40)\) and the constraints contained in the superfield equations for $X^m$ and $\Theta^{\alpha \beta}$, one can reduce the superfield $P_{\hat{m} \hat{n}}$ to the form

$$P_{\hat{m} \hat{n}} = \epsilon_{\hat{m} \hat{n} \hat{p} \hat{q}} \eta^{\hat{q}} \eta^{\hat{p}} p^m,$$

where $p^m$ is the superparticle momentum, satisfying the equation $\frac{\partial p^m}{\partial \tau} = 0$. The set of equations, remaining after elimination of all auxiliary fields and an explicit solution of constraints describe the dynamics of the $N = 2 \ D = 4$ Brink–Schwarz superparticle.

### 5 Doubly supersymmetric action of the $D = 6$ Type IIA superparticle.

When considering the $N = 2, \ D = 6$ superparticle, one should distinguish two different kinds of the target superspace, corresponding to so–called IIA and IIB theories. In
the former two fermionic spinor coordinates have opposite chiralities, while in the latter they have the same chirality. The superfield action of the type discussed herein can be constructed for the Type IIA superparticle only. The reason is that the geometrodynamical constraint does not put the IIA theory on the mass shell, while, as we will show, the geometrodynamical constraint of the IIB $D = 6$ superparticle produces dynamical equations.

We start with demonstration that the dimensional reduction of the doubly supersymmetric action of $N = 1$ $D = 10$ superparticle to $D = 6$ yields the action of the Type IIA theory.

The superfield action (3) for $N = 1$ superparticle in $D = 10$ has the following form [3]

$$S_{D=10}^{N=1} = \int d\tau d^8\eta P_{\hat{M}\hat{\rho}}(D_q X_{\hat{M}} - i(D_q \Theta^{\hat{\alpha}})\gamma_{\hat{\alpha}\hat{\beta}} \Theta^{\hat{\beta}}),$$

where $\hat{M} = 0, \ldots, 9$ is a vector $SO(1, 9)$ index, $\hat{\alpha}, \hat{\beta} = 1, \ldots, 16$ are indices of 16-component Majorana–Weyl spinors, $\gamma_{\hat{\alpha}\hat{\beta}}$ are $16 \times 16$ ten–dimensional $\gamma$–matrices, satisfying

$$\gamma_{\hat{\alpha}\hat{\beta}} (\hat{M} \cdot \hat{N})^{\beta\gamma} = \eta^{\hat{M}\hat{N}} \gamma_{\hat{\alpha}\hat{\beta}},$$

$\hat{q}$ is the index, corresponding to the $n = 8$ worldline supersymmetry. To reduce (53) to $D = 6$ we decompose the Majorana–Weyl spinor $\Theta^{\hat{\alpha}}$ into two $D = 6$ $SU(2)$–Majorana spinors of opposite chiralities:

$$\Theta^{\hat{\alpha}} = (\Theta^{(1)\hat{\mu}} I , \Theta^{(2)\hat{I}} J),$$

($D = 6$ notation has been presented in the previous Section) and use the following realization for $D = 10$ matrices $\gamma_{\hat{\alpha}\hat{\beta}}$: $\hat{M} = (\hat{M}, \hat{i})$, $\hat{M} = 0, \ldots, 5$ is the $SO(1, 5)$ index; $\hat{i} = 6, 7, 8, 9$ is an index of the vector representation of $SO(4)$, corresponding to four compactified space–time dimensions:

$$\gamma_{\hat{\alpha}\hat{\beta}} \hat{M} \hat{i} \hat{j} = \begin{pmatrix} \epsilon \alpha \beta \hat{M} \hat{i} \hat{j} & 0 \\ 0 & \epsilon \alpha \beta \hat{M} \hat{i} \hat{j} \end{pmatrix}, \quad \gamma_{\hat{\alpha}\hat{\beta}} \hat{M} \hat{i} = \begin{pmatrix} -\epsilon \alpha \beta \hat{M} \hat{i} \hat{j} & 0 \\ 0 & \epsilon \alpha \beta \hat{M} \hat{i} \hat{j} \end{pmatrix};$$

$$\gamma^i = \begin{pmatrix} 0 & \delta^i_{\hat{I}} \\ \sigma^i_{\hat{I}} \delta^\mu_{\hat{I}} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\bar{\sigma}^i_{\hat{I}} \delta^\mu_{\hat{I}} \\ -\sigma^i_{\hat{I}} \delta^\mu_{\hat{I}} & 0 \end{pmatrix}$$

(55)

where the matrices $\sigma^i_{\hat{I}}$ constitute a non–degenerate basis in the space of $2 \times 2$ matrices, transformed under the representation of $SO(4) \sim SU(2) \times SU(2)$ ($I, J$ are indices of $SU(2)$, $\hat{I}, \hat{J}$ are indices of another $SU(2)$ and $i$ is an index of the vector $SO(4)$ representation). They satisfy the following relations:

$$\sigma^i_{\hat{I}} \sigma^j_{\hat{J}} = \delta^i_{\hat{J}} \delta^j_{\hat{I}}, \quad \bar{\sigma}^j_{\hat{I}} \sigma^i_{\hat{J}} = \delta^i_{\hat{J}} \delta^j_{\hat{I}}, \quad \sigma^i_{\hat{I}} = \sigma^i_{\hat{I}}.$$
\[ Q^i_\hat{q} (D_q \Theta^{(2)i}_\mu) \sigma^i_I (D_p \Theta^{(1)\mu I}) ], \]

where \( Q^i_\hat{q} \) is a superfield Lagrange multiplier, which is symmetric and traceless in \( \hat{p} \) and \( \hat{q} \).

In addition to the local \( n = 8 \) worldline supersymmetry the action (56) is invariant under the following transformations of the Lagrange multipliers \( P_{M\hat{q}} \) and \( Q^i_{\hat{p}\hat{q}} \):

\[ \delta P_{M\hat{q}} = D_p (\xi^{(1)\mu i}_\mu \gamma^{M\hat{q}}_\mu D_s \Theta^{(1)\mu}_I) + D_p (\xi^{(2)i}_\mu \gamma^{M\hat{q}}_\mu D_s \Theta^{(2)i}_\mu); \]

\[ \delta Q^i_{\hat{p}\hat{q}} = -i \left[ (\xi^{(1)\mu i}_\mu \gamma^{M\hat{q}}_\mu \Theta^{(1)\mu i}_I) + (\xi^{(2)i}_\mu \gamma^{M\hat{q}}_\mu \Theta^{(2)i}_\mu) \right]; \]

\[ \delta Q^i_{\hat{p}\hat{q}} = D_s (\xi^{(1)\mu i}_\mu); \]

with the parameters \( \xi^{(1)\mu i}_\mu, \xi^{(2)i}_\mu \), which are completely symmetric and traceless with respect to the \( SO(8) \) indices.

Variation of the action (56) with respect to the Lagrange multiplier \( P_{M\hat{q}} \) gives the geometrodynamical constraint of the \( IIA D = 6 \) superparticle, which we rewrite in terms of superinvariant one–forms

\[ \Pi_{IIA}^M \equiv dX^M = \Theta^{(1)\mu i}_\mu \gamma^{M\hat{q}}_\mu \Theta^{(1)\mu i}_I - i d\Theta^{(2)i}_\mu \gamma^{M\hat{q}}_\mu \Theta^{(2)i}_\mu = e_\tau \Pi_{\tau IIA}; \]

Now we demonstrate that this constraint does not produce dynamical equations. In order to analyze its structure we choose the light–cone basis:

\[ X^M = (X^{++} = X^0 + X^5; X^{--} = X^0 - X^5, X^i), \]

and decompose the \( SU^*(4) \) spinor index \( \mu \) of the Grassmann target space coordinates into a pair of \( SU(2) \times SU(2) \) indices \( A \) and \( \hat{A} \), introducing the sign “indices” +, − (weights) of the \( SO(1, 1) \) subgroup of \( SU^*(4) \)

\[ \Theta^{(1)\mu i}_\mu = (\Theta^{(1)+\hat{A}i}_\hat{A}, \Theta^{(1)\hat{A}i}_A), \quad \Theta^{(2)i}_\mu = (\Theta^{(2)\hat{A}i}_\hat{A}, \Theta^{(2)+\hat{A}i}_A); \]

The geometrodynamical constraint of \( IIA \) superparticle in the light–cone notation acquires the form

\[ \Pi_{IIA}^{++} \equiv dX^{++} + 2i d\Theta^{(1)+\hat{A}i}_\hat{A} \Theta^{(1)+}_\hat{A} + 2i d\Theta^{(2)+\hat{A}i}_\hat{A} \Theta^{(2)+}_\hat{A} = e_\tau \Pi_{\tau IIA}; \]

4 In what follows we use the following realization for \( D = 6 \) “Pauli” matrices \( \gamma^{M\hat{q}}_\mu \) and \( \tilde{\gamma}^{\hat{M}\mu\nu} \):

\[ \gamma^0_{\mu\nu} = \begin{pmatrix} -\epsilon_{AB} & 0 \\ 0 & -\epsilon_{AB} \end{pmatrix}, \quad \tilde{\gamma}^{0\mu\nu} = \begin{pmatrix} \epsilon^{AB} & 0 \\ 0 & \epsilon^{AB} \end{pmatrix}, \]

\[ \gamma^5_{\mu\nu} = \begin{pmatrix} -\epsilon_{AB} & 0 \\ 0 & \epsilon_{AB} \end{pmatrix}, \quad \tilde{\gamma}^{5\mu\nu} = \begin{pmatrix} -\epsilon^{AB} & 0 \\ 0 & \epsilon^{AB} \end{pmatrix}, \]

\[ \gamma^i_{\mu\nu} = \begin{pmatrix} 0 & \sigma_i^{\hat{A}A} \\ \hat{\sigma}_i^\mu & 0 \end{pmatrix}, \quad \tilde{\gamma}^{i\mu\nu} = \begin{pmatrix} 0 & -\sigma^i\hat{A}A \\ -\hat{\sigma}^i\hat{A}A & 0 \end{pmatrix}, \]

and \( 2 \times 2 \) matrices \( \sigma_i^{\hat{A}A} \) and \( \hat{\sigma}_i^{i\hat{A}A} \) are defined above.
\[
\Pi_{IIA} = dX^{--} + 2id\Theta^{(1)}(--)\bar{\Theta}^{(1)} + 2id\Theta^{(2)}(--)\bar{\Theta}^{(2)} = e_\tau \Pi^{--}_{IIA} \tag{61}
\]

\[
\Pi^{i}_{IIA} = dX^{i} - id\Theta^{(1)}+A_{i}\sigma^{+}_{A}\bar{\Theta}^{(1)} - i\delta^{(1)} - A_{i}\sigma^{+}_{A}\bar{\Theta}^{(1)} + A_{i} + id\Theta^{(2)}+A_{i}^{+}\bar{\Theta}^{(2)} - A_{i}^{+} = e_\tau \Pi^{i}_{IIA}.
\]

In particular, \(D_q\Theta^+\) satisfy the constraints
\[
(D_q\Theta^{(1)+i}_{A})(D_q\Theta^{(1)+i}_{A}) + (D_q\Theta^{(2)+i}_{A})(D_q\Theta^{(2)+i}_{A}) = -\delta_{pq}\Pi^{++}_{IIA},
\]
which follow from (61) and imply that the number of independent fields in the 8 \(\times\) 8 matrices
\[
(D_q\Theta^{(1)+I}_{A} \ \ D_q\Theta^{(2)+I}_{A})
\]
is equal to the number of independent parameters of the local worldline \(SO(1,1) \times SO(8)\) symmetry, transforming these matrices. Really, the action (56) was obtained by dimensional reduction of the \(D=10\) \(N=1\) superparticle action (53), in which the components of \(D_q\Theta^+\) form matrices of \(SO(1,1) \times SO(8)\) when \(\Theta^+\) is taken in the light–cone basis: \(\Theta^+ = (\Theta^+\bar{\Theta}, \Theta^+\tilde{\bar{\Theta}})\). The reduction prescription applied in this paper uses these matrices with indices \(\bar{\alpha}\) and \(\tilde{\bar{\alpha}}\), \(\tilde{\bar{\alpha}}\) decomposed in accordance with the \(D=6\) spinor structure.

The fields \(\Theta\) being the scalar worldline superfields, the transformation law for (62) is determined by the transformation properties of the fermionic covariant derivatives (10). The transformation matrices \(D_q\eta_q\) are restricted by (4), and, as a consequence, by
\[
(D_q\eta_q)(D_q\eta_q) = \frac{1}{8}\delta_{pq}(D_q\eta_q)(D_q\eta_q)
\]

Hence, they take their values in the \(SO(1,1) \times SO(8)\) subgroup of the \(n=8\) local worldline supersymmetry group. One can use this symmetry to gauge away all the fields in (62). Fixing the gauge, one should take into account that either \((D_q\Theta^{(1)+I}_{A}, D_q\Theta^{(2)+I}_{A})\) or \((D_q\Theta^{(1)+I}_{A}, D_q\Theta^{(2)+I}_{A})\) should have a non–vanishing determinant. In what follows we will consider the first matrix to be non–degenerate, so only the gauges compatible with this requirement are admissible. For example, we can fix the matrix in question to be the unit one. Decomposing the \(SO(8)\)–index \(q\) into three indices \(SU(2) \times SU(2) \times SO(2): \ q \rightarrow (\tilde{\bar{\alpha}}, \tilde{\bar{\alpha}}, \tilde{\bar{\alpha}})\), we write the gauge fixing conditions as
\[
D^\tilde{\bar{\alpha}}_{\tilde{\bar{\alpha}}}\Theta^{(1)+i}_{A} = \delta^{(1)}\tilde{\bar{\alpha}}\delta^i\tilde{\bar{\alpha}}; \quad D^\tilde{\bar{\alpha}}_{\tilde{\bar{\alpha}}}\Theta^{(2)+i}_{A} = \delta^{(2)}\tilde{\bar{\alpha}}\delta^i\tilde{\bar{\alpha}}; \tag{64}
\]

The general solution to the geometrodynamical constraint (64) can be written in the gauge (64) in the following form:
\[
D^\tilde{\bar{\alpha}}_{\tilde{\bar{\alpha}}}\Theta^{(1)+i}_{A} = -\sigma^i\tilde{\bar{\alpha}}F^{i\tilde{\bar{\alpha}}}; \quad D^\tilde{\bar{\alpha}}_{\tilde{\bar{\alpha}}}\Theta^{(2)+i}_{A} = -\sigma^i\tilde{\bar{\alpha}}F^{i\tilde{\bar{\alpha}}}; \tag{65}
\]

Moreover, the complete worldline supersymmetry can be used to fix a gauge
\[
(\Theta^{(1)+i}_A, \Theta^{(2)+i}_A) = \eta^i,
\]
where all the world–line supersymmetries are broken. This guarantees the possibility of gauge fixing (64).
\[ \Pi^{++}_{\tau} = 2; \quad \Pi^{-\tau} = E^{i}E^{i} + F^{i}F^{i}; \quad \Pi^{i}_{\tau} = F^{i}; \]

Higher components of the superfields \( E^{i} \) and \( F^{i} \) are expressed in terms of the leading components of \( \Theta^{(1)} \) and \( \Theta^{(2)} \):

\[
D^{1}_{\hat{A}\hat{A}}E^{i} = -i\bar{\sigma}^{i}_{\hat{A}\hat{A}} \frac{\partial \Theta^{(2)-\hat{A}}}{\partial \tau}; \quad D^{2}_{\hat{A}\hat{A}}E^{i} = -i\bar{\sigma}^{i}_{\hat{A}\hat{A}} \frac{\partial \Theta^{(1)-\hat{A}}}{\partial \tau}; \\
D^{1}_{\hat{A}\hat{A}}F^{i} = i\bar{\sigma}^{i}_{\hat{A}\hat{A}} \frac{\partial \Theta^{(1)-\hat{A}}}{\partial \tau}; \quad D^{2}_{\hat{A}\hat{A}}F^{i} = i\bar{\sigma}^{i}_{\hat{A}\hat{A}} \frac{\partial \Theta^{(2)-\hat{A}}}{\partial \tau};
\]

and no dynamical equations appear.

One can see that the Type IIA geometrodynamical constraint does not put the theory on the mass shell and thus the Lagrange multiplier \( P^{\hat{M}}_{\hat{q}} \) does not contain redundant propagating degrees of freedom. The superfield equations, which one can obtain by varying the action (66) with respect to the Lagrange multiplier \( Q_{\hat{p}\hat{q}} \), are consequences of the geometrodynamical constraint (50) and produce no additional restrictions on the fields. So, \( Q_{\hat{p}\hat{q}} \) is a purely auxiliary degree of freedom, which can be eliminated (as well as auxiliary fields in \( P^{\hat{M}}_{\hat{q}} \)) with the help of the gauge symmetries (57) and (58) of the action (56).

Hence, this action consistently describes \( D = 6 \) Type IIA superparticle dynamics.

The different situation is in the case of a IIB \( D = 6 \) superparticle. Now the geometrodynamical constraint takes the form

\[
\Pi^{M}_{IIB} \equiv dX^{\hat{M}} - id\Theta^{(\hat{A})}_{\hat{A}}\theta^{\hat{M}}_{\hat{A}}\Theta^{(\hat{A})\hat{M}} = e_{\tau}\Pi^{M}_{\tau IIB},
\]

where (\( \hat{A} \)) is the SO(2) index of \( N = 2 \) target space supersymmetry. Its light–cone decomposition can be written as follows

\[
\Pi^{++}_{IIB} \equiv dX^{++} + 2id\Theta^{(\hat{A})+\hat{A}I_{\hat{A}}}\theta^{(\hat{A})+\hat{A}I_{\hat{A}}} = e_{\tau}\Pi^{++}_{\tau IIB}, \\
\Pi^{-\tau}_{IIB} \equiv dX^{-\tau} + 2id\Theta^{(\hat{A})-\hat{A}I_{\hat{A}}}\theta^{(\hat{A})-\hat{A}I_{\hat{A}}} = e_{\tau}\Pi^{-\tau}_{\tau IIB}, \\
\Pi^{i}_{IIB} \equiv dX^{i} - id\Theta^{(\hat{A})+\hat{A}I_{\hat{A}}}\theta^{(\hat{A})+\hat{A}I_{\hat{A}}} - id\Theta^{(\hat{A})-\hat{A}I_{\hat{A}}}\theta^{(\hat{A})-\hat{A}I_{\hat{A}}} = e_{\tau}\Pi^{i}_{\tau IIB}.
\]

In (66) we decomposed the \( SU^{*}(4) \) spinor index into a pair of \( SU(2) \times SU(2) \) indices and introduced \( SO(1, 1) \) indices + and - as in the IIA case:

\[
\Theta^{(\hat{A})\hat{M}} = (\Theta^{(\hat{A})+\hat{A}I_{\hat{A}}}, \Theta^{(\hat{A})-\hat{A}I_{\hat{A}}});
\]

Following the same arguments that have been used in the analysis of the IIA geometrodynamical constraint, we can gauge fix the matrix \( (D_{\hat{q}}\Theta^{(\hat{A})+\hat{A}I_{\hat{A}}}) \) to be the unit matrix

\[
D^{\hat{A}\hat{A}}_{\hat{A}\hat{A}}\Theta^{(\hat{A})+\hat{A}I_{\hat{A}}} = \delta^{(\hat{A})\hat{A}}\delta_{\hat{A}\hat{A}}\delta^{\hat{A}\hat{A}},
\]

In this gauge the general solution to the IIB geometrodynamical constraint is

\[
D_{\hat{A}\hat{A}}^{\hat{A}\hat{A}}\Theta^{(\hat{A})-\hat{A}I_{\hat{A}}} = \delta^{\hat{A}\hat{A}}\delta_{\hat{A}\hat{A}}\delta^{\hat{A}\hat{A}}; \\
\Pi^{++}_{\tau} = 2; \quad \Pi^{-\tau} = G^{i}G^{i}; \quad \Pi^{i}_{\tau} = 4G^{i};
\]

(67)
One can see that this solution contains twice less independent fields in comparison with the IIA case. So the equations (66) are more restrictive than the constraints (61). This results in the mass shell equations arising from the higher order selfconsistency conditions of (66). Hitting (67) by Grassmann covariant derivatives and using their algebra

\[ \{D^A_{\dot{A}}, D^B_{\dot{B}}\} = 2 \epsilon_{\dot{A}\dot{A}'} \epsilon_{\dot{B}\dot{B}'} \delta_{\dot{A}\dot{B}} \frac{\partial}{\partial \tau}, \]

we obtain the equations of motion of the \( D = 6 \) IIB superparticle in the light–cone gauge:

\[ \frac{\partial \Theta^{(1)-}_{\dot{A}}}{\partial \tau} = 0. \]

This means that if we wrote down a worldline action for the Type IIB superparticle in the form similar to (56), the Lagrange multiplier \( P^M_{\dot{q}} \) would contain redundant propagating degrees of freedom and the classical dynamics of such a model would not correspond to the dynamics of the \( D = 6 \) IIB Casalbuoni–Brink–Schwarz superparticle.

## 6 Conclusion

In this paper the doubly supersymmetric actions have been constructed for \( N = 2 \) superparticles in \( D = 3, 4 \) and for the Type IIA superparticle in \( D = 6 \). These actions have been obtained by the dimensional reduction of the superfield actions for \( N = 1 \) superparticles in \( D > 3 \). They possess an explicit \( n = 2(D - 2) \) local worldline supersymmetry, replacing \( \kappa \)–symmetries of the corresponding Casalbuoni–Brink–Schwarz superparticles which they classically equivalent to. By comparison with the earlier constructed superfield actions for \( N = 1 \) [2]–[6] and \( N = 2 D = 3 \) [8] superparticles the proposed actions contain additional Lagrange multiplier terms, ensuring their invariance under the local superfield transformations of the Lagrange multipliers in the actions. They are crucial for elimination of auxiliary fields from the Lagrange multipliers.

One may also apply the dimensional reduction procedure to the worldsheet superfield actions of \( N = 1 \) superstrings [7] to get superfield actions for superstrings with extended supersymmetries in \( D = 4 \) and 6. The doubly supersymmetric description of such models is of interest, in particular, because of a connection, recently found between the equations of the doubly supersymmetric formulation of \( D = 3 N = 2 \) Green–Schwarz superstring and some exactly solvable models [21]. For instance, it was demonstrated that these equations are related to the equations of a properly constrained supersymmetric WZNW model based on the \( sl(2, \mathbb{R}) \) algebra. The study of the dynamics of \( N = 2 \) superstrings in \( D > 3 \) in the doubly supersymmetric approach may reveal their relation with more complicated WZNW models with extended two–dimensional supersymmetry, based on the algebras \( sl(2, \mathbb{C}), spin(1,5) \) and \( spin(1,9) \).

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