Electron correlation effects in a wide channel from the $\nu = 1$ quantum Hall edge states

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The spatial behavior of Landau levels (LLs) for the $\nu = 1$ quantum Hall regime at the edge of a wide channel is studied in a self-consistent way by using a generalized local density approximation proposed here. Both exchange interaction and strong electron correlations, due to edge states, are taken into account. They essentially modify the spatial behavior of the occupied lowest spin-up LL in comparison with that of the lowest spin-down LL, which is totally empty. The contrast in the spatial behavior can be attributed to a different effective one-electron lateral confining potentials for the spin-split LLs. Many-body effects on the spatially inhomogeneous spin-splitting are calculated within the screened Hartree-Fock approximation. It is shown that, far from the edges, the maximum activation energy is dominated by the gap between the Fermi level and the bottom of the spin-down LL, because the gap between the Fermi level and the spin-up LL is much larger. In other words, the maximum activation energy in the bulk of the channel corresponds to a highly asymmetric position of the Fermi level within the gap between spin-down and spin-up LLs in the bulk. We have also studied the renormalization of the edge-state group velocity due to electron correlations. The results of the present theory are in line with those suggested and reported by experiments on high quality samples.

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I. INTRODUCTION

Even though most previous theoretical works have developed a noninteracting picture of the edge states in quantum Hall regime, the influence of electron-electron interactions on the edge-state properties in a channel and on the subband structure of quantum wires has been the subject of intense study in the recent years. In Refs., only the direct Hartree interaction was taken into account. Nevertheless, edge states played no fundamental role in many studies that develop or use theoretical pictures of the spin-splitting Landé $g^*$ factor enhanced by the exchange interaction. On the other hand, in Ref., correlation effects due to edge states on the effective $g^*$ factor for a quantum wire at the $\nu = 1$ quantum Hall regime were considered, while for a wide channel only some qualitative estimation was given. It was shown that correlations, due to the screening at the edges, strongly suppress the exchange splitting and smoothen the energy dispersion at the edges.

This paper provides a step further towards the understanding of the role of electron-electron interactions in the $\nu = 1$ quantum Hall effect regime, in particular, the very important role of edge states, by extending the approach of Ref., which was based on the screened Hartree-Fock approximation (SHFA). By using the SHFA, we develop a generalized local density approximation (GLDA), which is an essential improvement of the previous modified local density approximation (MLDA) of Ref. The main objective is to determine the enhanced spin-splitting for the two-dimensional electron system (2DES) in a wide channel and the position of the Fermi level within the exchange enhanced gap. A realistic model for the edge regions of a wide channel in a strong magnetic field is solved self-consistently when the lowest spin-polarized Landau level (LL) is occupied, i.e., when $\nu = 1$ in the inner part of the channel and the formation of a dipolar strip at the channel edges does not occur. Moreover, we assume that a bare confining potential is rather steep that prevents the flattening of edge states in the vicinity of the Fermi level. The confining potential of the model is obtained in the Hartree approximation as a self-consistent one-electron potential, which in addition to a bare confining potential includes the screening by the 2DES and pertinent electron-electron interactions. As we assume that there is no flattening at the edge regions, the slope of the confining potential, proportional to the group velocity $v^H_g$ in the Hartree approximation, is finite. It is also assumed that the confining potential, without many-body interactions, is smooth on the $\ell_0$ scale, where $\ell_0 = (\hbar/m\omega_c)^{1/2}$ with $\omega_c = |e|B/mc$ the cyclotron frequency, and, hence, leads to a rather small $v^H_g$. 

1
In this work we show that, if we go beyond the exchange interaction, the spatial behavior of the LLs is strongly modified between the middle part of the channel and the region near the edges due to electron correlations. Furthermore, the position of the Fermi level in the gap at the inner part of the channel is highly asymmetric. In the region, where the LLs are flat, the Fermi level is much closer to the upper spin-split LL than to the occupied lower spin-split LL. The most essential role played by correlations is related to the screening by the edge states which in turn depends strongly on their group velocity $v_{g}$. We notice that the exchange interaction leads to an infinite (logarithmically divergent) $v_{g}$. Electron correlations may restore a smooth dispersion of the single-particle energy, on the $l_{0}$ scale, as a function of the oscillator center $y_{0} \approx k_{x} l_{0}^{2}$. Because, in typical experiments, the condition of strong $B$ limit, $r_{0} = \varepsilon_{0}^{2}/(\varepsilon_{0} \hbar \omega_{c}) \ll 1$, is not satisfied, we point out that the proposed GLDA gives and adequate self-consistent treatment to many-body effects in a strong $B$, when $r_{0} \ll 1$. The paper is organized as follows. In Sec. II, we describe a new microscopic nonperturbative approach for the calculation of the screened Coulomb interaction for laterally nonhomogeneous 2DES in a wide channel in the strong magnetic field limit $r_{0} \ll 1$. In Sec. III, we study exchange correlations effects for $\nu = 1$ by using a proposed GLDA to obtain the activation gap, the asymmetry of the Fermi level position within the Fermi gap and investigate the renormalization of the LLs due to exchange and correlations for the case more experimentally feasible strong $B$, i.e. $r_{0} \ll 1$. In addition, we apply our theory to the experiment described in Ref. [1]. We summarize our results with some remarks in Sec. IV.

II. EXCHANGE-CORRELATION EFFECTS IN A WIDE CHANNEL

We consider a strictly two-dimensional electron system confined in the $(x,y)$ plane to a wide channel of width $W$ and length $L$, in the presence of a strong magnetic field $B$ along the $z$ axis. Taking the vector potential $A = -By\hat{x}$, we write the single-particle Hamiltonian as $\hat{H} = [(\hat{p}_{x} + eBy/c)^{2} + m^{2}]/2m + V_{a} + g_{\mu B}\hat{\sigma}_{z}B/2$, where the confining potential $V_{a} = 0$ at the inner part of the channel and $V_{a} = m\Omega^{2}(y - y_{r})^{2}/2$, $y \geq y_{r}$, at the right edge, with $\Omega \ll \omega_{c}$; $y_{r}$ being the bare Landé $g$-factor, $\mu_{B}$ the Bohr magneton, and $\sigma_{z}$, $z$-component Pauli matrix. The eigenvalues and eigenfunctions near the right edge of the channel are well approximated by $\epsilon_{n,k_{x},\sigma} = (n + 1/2)\hbar \omega_{c} + m^{2}\Omega^{2}(y_{0} - y_{r})^{2}/2 + g_{\mu B}k_{x}B/2$ and $\psi_{nk_{x},\sigma}^{(r)}(r,\sigma) = \langle r | n_{k_{x}} \rangle | \sigma \rangle$, with $\langle r | n_{k_{x}} \rangle = \exp(ik_{x}x)\Psi_{n}(y - y_{0}(k_{x}))/\sqrt{\mathcal{T}}$ and spin function $| \sigma \rangle = | \psi_{\sigma}(\sigma_{1}) = \delta_{\sigma_{1}}, \sigma_{1} = \pm; 1, y_{0} = y_{0}(k_{x}) = \ell_{0}^{2}k_{x}, \Psi_{n}(y) \rangle$ is a harmonic oscillator function, $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ for $x < 0$, $r = \{x, y\}$. The edge of the $(n, \sigma)$ LL is denoted by $y_{n}^{(n,\sigma)} = y_{r} + \ell_{0}^{2}k_{n}^{(n)}\sigma = \ell_{0}^{2}k_{n}^{(n)}$, where $k_{n}^{(n)} = k_{x} + k_{y}^{(n)}$, and $W = 2y_{0}^{(1)}$ and the group velocity of the edge states $v_{g_{n}}^{(n,H)} = \partial_{k_{x}} + v_{g}k_{y}^{(n)}$ with wave vector $k_{y}^{(n)} = (\omega_{c}/\hbar \Omega)\sqrt{2m^{2}\Delta_{F}^{(n)}}$, $\Delta_{F}^{(n)} = B_{F}^{H} - (n + 1/2)\hbar \omega_{c} - g_{\mu B}B/2$, and $E_{F}^{H}$ is the Fermi energy in the Hartree approximation. We can also write $v_{g_{n}}^{(n,H)} = eE_{n}^{(n)}/B$, where $E_{n}^{(n)} = \Omega\sqrt{2m^{2}\Delta_{F}^{(n)}}/|e|$ is the electric field associated with the confining potential $V_{a}$ at $y_{n}^{(n)}$. We also introduced the wave vector $k_{r} = y_{r}/\ell_{0}^{2}$. For definiteness, we take the background dielectric constant $\varepsilon$ to be spatially homogeneous. Because we will apply our theory for the case of GaAs based samples, where $y_{0} = -0.44$, it is assumed in our study that $y_{0} < 0$.

We begin by considering the strong magnetic field limit when $r_{0} \ll 1$ and only the lowest upper-spin level is occupied. It was shown in Ref. [1], that the exchange and correlation contributions to the single-particle energy $E_{0,k_{x},1} = \epsilon_{0,k_{x},1} + \epsilon_{0,k_{x},1}^{c}$ in the SHF can be written as

$$\epsilon_{0,k_{x},1} = \frac{1}{8\pi^{2}} \int_{k_{x}^{(1)}}^{k_{x}^{(1)}} dk_{x}^{(1)} d\epsilon_{y}^{(1)} \int_{-\infty}^{\infty} dq_{x} d\epsilon_{y}^{(1)} V^{*}(k_{x}, q_{y}; q_{y}^{(1)}) (0k_{x}^{(1)}|e^{iq_{x}y^{(1)}}|0k_{x}^{(1)}) (0k_{y}^{(1)}|e^{iq_{y}y^{(1)}}|0k_{y}^{(1)}) ,$$  

where $V^{*}(q_{x}, q_{y}; q_{y}^{(1)})$ is the Fourier transform of the screened Coulomb interaction which can be evaluated within the random phase approximation (RPA), $(k_{x} = k_{x} \pm k_{x}^{(1)},$ and the matrix element $(0k_{x} | \exp[iq_{y}y] | 0k_{x}^{(1)}) = \exp[-|k_{x}^{2} + q_{y}^{2} - 2q_{y}k_{y}^{(1)}|^{2}/4]$. Notice that close to the edges, if neglect by screening in $V^{*}(q_{x}, q_{y}; q_{y}^{(1)})$, Eq. (1) takes into account exchange in the first order of $r_{0}$, while in the inner part of the channel it is taken into account practically exactly.

In order to evaluate the screened interaction, we use the self-consistent field form of the random phase approximation (RPA). [2] Let us consider the static screened potential $\varphi(x - x_{0}, y; y_{0})$, where the argument $(x - x_{0})$ takes into account translational invariance in the $x$ direction, of an electron charge $e\delta(r - r_{0})$ located at $(x_{0}, y_{0})$. The one-electron Hamiltonian, in the presence of a self-consistent potential $V^{*}(x - x_{0}, y; y_{0}) = e\varphi(x - x_{0}, y; y_{0})$, is $\hat{H} = \hat{H}^{0} + V^{*}(x - x_{0}, y; y_{0})$. The equation of motion for the one-electron solution $\hat{\phi}$ is solved together with Poisson’s equation for the self-consistent potential. Following closely the approach of Ref. [2], we obtain the integral equation for the Fourier components of the induced charge density as
\[ \rho(q_x, y; y_0) = \frac{2e^2}{\varepsilon L} \sum_{n=0}^{\tilde{n}} \sum_{k_x, \alpha} \sum_{\sigma} F_{n, \sigma}(k_x, q_x) \Pi_{n, \sigma}(y, k_x, k_x - q_x) \times \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \Pi_{n, \sigma}(\hat{y}, k_x, k_x - q_x) K_0(|q_x||\hat{y} - y'|) [\rho(q_x, y'; y_0) + e\delta(y' - y_0)], \] (2)

where

\[ F_{n, \sigma}(k_x, q_x) = \frac{f_{n, k_x, q_x, \sigma} - f_{n, k_x, q_x, \sigma}}{\epsilon_{n, k_x, q_x, \sigma} - \epsilon_{n, k_x, q_x, \sigma} + i\hbar / \tau}, \]

and \( \Pi_{n, \sigma}(y, k_x, q_x) = \Psi_{n, \sigma}(y - y_0(k_x)) \Psi_{n, \sigma}(y - y_0(k_x)). \) Here \( f_{n, k_x, q_x, \sigma} \) is the Fermi-Dirac function, \( \tilde{n} \) denotes the highest occupied LL and \( K_0(x) \) is the modified Bessel function. To obtain Eq. (2), we have used the condition \( q_x \leq 1, q_y = \omega_c, \) which should be well satisfied for actual \( q_x \approx q_y^{-1} \) due to the smoothness of the confining potential.

For \( \nu = 1 \) we have \( \tilde{n} = 0, \sigma = 1 \) and Eq. (2) takes the form

\[ \rho(q_x, y; y_0) = \frac{e^2}{\pi \varepsilon} \int_{-\infty}^{\infty} dk_x F_{0, 1}(k_x, q_x) \Pi_0(y, k_x, k_x - q_x) \times \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \Pi_0(\hat{y}, k_x, k_x - q_x) K_0(|q_x||\hat{y} - y'|) [\rho(q_x, y'; y_0) + e\delta(y' - y_0)], \] (3)

Assuming that actual \( |q_x| \ll q_y^{-1} \) and considering only the right edge region or the inner part of the channel, we can approximate the numerator of the expression for \( F_{0, 1}(k_x, q_x) \) as \( (f_{0, k_x, q_x, 1} - f_{0, k_x, q_x, 1}) \approx -q_x(\partial f_{0, k_x, q_x, 1}/\partial k_x) = q_y \delta(k_x - k_y^{-1}). \) In addition, taking into account the smoothness of \( \epsilon_{0, k_x, q_x, 1} \) at the edge (i.e., \( \epsilon_{0, k_x, q_x, 1} \approx 1 \) for \( |q_x| \ll q_y^{-1} \)), it follows that \( F_{0, 1}(k_x, q_x) \approx -\delta(k_x - k_y^{-1})/h \omega_c^3. \) Substituting the latter in Eq. (3) and then integrating over \( k_x, \) we obtain

\[ \rho(q_x, y; y_0) = -r_H^1 \Pi_0(y, k_y^{-1}, k_y^{-1} - q_x) \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \Pi_0(\hat{y}, k_y^{-1}, k_y^{-1} - q_x) K_0(|q_x||\hat{y} - y'|) [\rho(q_x, y'; y_0) + e\delta(y' - y_0)], \] (4)

where \( r_H^1 = e^2/(\pi h \omega_c^3 q_y^{-1}) \) is a characteristic dimensionless parameter for the system. Notice that for the assumed symmetric wide channel, one can neglect the effect of the left edge states on the right edge region or the inner part of the channel. The solution of Eq. (4) can be written as

\[ \rho(q_x, y; y_0) = \rho(q_x, y_0) \Psi_0(y - y_0^{-1}) \Psi_0(y - y_0^{-1} + q_x \ell_0^2), \] (5)

where

\[ \rho(q_x, y_0) = -e r_H^1 [1 + r_H^1 M(0, q_x)]^{-1} \times \int_{-\infty}^{\infty} dx \Psi_0(x) \Psi_0(x + q_x \ell_0^2) K_0(|q_x||x - (y_0 - y_0^{-1})|). \] (6)

Here \( M(0, q_x) = \exp(-q_x^2 \ell_0^2/4) K_0(q_x^2 \ell_0^2/4)/2 \) is a special case of the matrix element

\[ M(k_x - k_y^{-1}, q_x) = e^{-q_x^2 \ell_0^2/2} \int_0^{\infty} dy e^{-q_x^2 \ell_0^2/2} \frac{\cos(q_y (k_x - k_y^{-1}) \ell_0^2)}{\sqrt{q_x^2 + q_y^2}}, \]

where \( M(x, y) \equiv M(x, -y) \equiv M(-x, y). \)

Substituting Eq. (6) into Eq. (3), using the latter in the Poisson’s equation for the total electric potential \( \varphi(q_x, y; y_0) \) induced by the total charge density \( [\rho(q_x, y; y_0) + e\delta(y - y_0)] \), and Fourier transforming the result, we obtain the screened Coulomb potential \( V^*(q_x, q_y; q_y') = e \varphi(q_x, q_y; q_y') \) as

\[ V^*(q_x, q_y; q_y') = \frac{2\pi e^2}{\varepsilon} \int_0^{\infty} dy e^{-q_x^2 \ell_0^2/2} \frac{\cos(q_y (2k_y^{-1} - q_x) \ell_0^2/2)}{1 + r_H^1 M(0, q_x)} \}

(7)

\[ \times e^{-[q_y^2 + (q_y')^2] \ell_0^2/4} \exp(-i(q_y + q_y')(2k_y^{-1} - q_x) \ell_0^2/2} \]

(8)
The first term in the curly brackets of Eq. (8) is the bare Coulomb interaction which leads to the exchange contribution. Substituting it in Eq. (1), leads to

\[ \epsilon_{0,k_x,1}^x = -\frac{e^2}{2\varepsilon \ell_0} \int_{\tilde{k}_x-k_0^{(1)}}^{\tilde{k}_x+k_0^{(1)}} dt e^{-t^2/4} K_0(t^2/4), \]  

(9)

where \( \tilde{k}_{x,r_0} = k_{x,r_0} \kappa_0 \). For \( \tilde{k}_0^{(1)} \) \( - \tilde{k}_x \gg 1 \), \( \epsilon_{0,k_x,1}^x \approx \epsilon_0^x = -(\pi/2)/2(\varepsilon^3/\varepsilon \ell_0) \) and \( \epsilon_{0,\pm k_0^{(1)},1}^x = \epsilon_0^x/2 \), for \( \tilde{k}_x = \pm \tilde{k}_0^{(1)} \).

Now, substituting Eq. (8) into Eq. (1), we obtain, for the right edge region or the inner part of channel, that

\[ \epsilon_{0,k_x,1}^{xc} = -\frac{e^2}{\pi \varepsilon} \int_{-\infty}^{k_0^{(1)}} \frac{1}{1 + r_1^H M(0,k_x-k_x') \kappa_0^{(1)}} \times \left\{ M(0,k_x-k_x') + r_1^H [M^2(0,k_x-k_x') - M^2(k_x-k_0^{(1)},k_x-k_x') \kappa_0^{(1)}] \right\}. \]  

(10)

The first two positive terms in the curly brackets of Eq. (10) lead to the exchange contribution given by Eq. (1). The third negative term is the important contribution coming from electron correlations.

In order to estimate the correlations role, let us take \( \epsilon_{0,k_x,1}^{xc} \) at the Fermi level, i.e., for \( k_x = k_0^{(1)} \). Then, from Eq. (10), it follows that

\[ \epsilon_{0,k_x,1}^{xc} = -\frac{e^2}{\pi \varepsilon} \int_{0}^{\infty} dx [\exp(-x^2/4)K_0(x^2/4)]/2 \times \{1 + r_1^H [\exp(-x^2/4)K_0(x^2/4)]/2\}^{-1}. \]  

(11)

Notice that \( r_1^H \) is typically a large parameter in GaAs (\( \varepsilon \approx 12.5 \)) based samples. Indeed, the characteristic velocity \( e^2/\pi \varepsilon \approx 5.6 \times 10^6 \) cm/s and due to its big value usually can be considered much larger than \( \epsilon_0^H \). Then assuming that \( r_1^H \gg 1 \) we obtain, from Eq. (11), that \( \epsilon_{0,k_x,1}^{xc} \approx -\hbar^2 r_1^H \kappa_0^{(1)} \varepsilon \), which estimates the many-body lowering of the Fermi level. Now, if we compare with the Hartree-Fock approximation (HFA) result, we see that \( \epsilon_0^x/2\epsilon_{0,k_x,1}^{xc} \approx (\pi/2)^{3/2} r_1^H \) \( \gg 1 \), which implies that correlations essentially contributes to diminish the Fermi level lowering with respect to the bottom of the upper spin-split LL (\( n = 0, \sigma = -1 \)).

### III. CORRELATION EFFECTS ON THE ENHANCED ACTIVATION GAP

In the strong magnetic field limit, \( r_0 \ll 1 \), the total single-particle energy of the \( (n = 0, \sigma = 1) \) LL \( E_{0,k_x,1} = \epsilon_{0,k_x,1}^{xc} + \epsilon_{0,k_x,1}^{xc} \), where \( \epsilon_{0,k_x,1}^{xc} \) is given by Eq. (10), can be written, in the right edge region or the inner part of the channel, as

\[ E_{0,k_x,1} = \frac{\hbar \omega_e}{2} - |g_0| \mu_B B/2 + \frac{m^* \Omega^2 \varepsilon^x}{2} (k_x-k_{x'})^2 \Theta(k_x-k_{x'}) - \frac{e^2}{\pi \varepsilon} \int_{-\infty}^{k_0^{(1)}} \frac{dk_x'}{1 + r_1^H M(0,k_x-k_x')} \times \left\{ M(0,k_x-k_x') + r_1^H [M^2(0,k_x-k_x') - M^2(k_x-k_{0^{(1)},k_x-k_x'}) \]  

(12)

Notice that \( \epsilon_{0,k_0^{(1)},1} = E_{1}^H \) and only the \( (n = 0, \sigma = 1) \) LL is assumed to be occupied. Moreover, \( E_{1}^H \) is the quasi-Fermi level for this LL, when it appears above the bottom of the \( (n = 0, \sigma = -1) \) LL. However, we demand that the quasi-Fermi level \( E_F \) of the \( (n = 0, \sigma = 1) \) LL, renormalized by exchange, or by both exchange and correlations, is the actual Fermi level, i.e., it is below the bottom of \( (n = 0, \sigma = -1) \) LL. In addition, we assume that exchange correlation effects do not change the Fermi wave vectors \( k_0^{(1)} \) and \( k_{e,1}^{0,1} \). We point out that by taking the exchange interaction into account, \( E_F \) is already different from the \( E_{1}^H \), since \( |\epsilon_0|/\hbar \omega_e = \sqrt{\pi}/2 r_0 \), even for \( r_0 \ll 1 \).

However, in actual experiments typically \( r_0 \sim 1 \). Now, we go beyond the strong magnetic field limit for \( E_{0,k_x,1} \), given by Eq. (12), by using \( r_1 = e^2/(\pi \varepsilon \varepsilon_{0,k_x,1}^{xc}) \) instead of \( r_1^H \), and assuming that the approximation is still valid for \( r_0 \approx 1 \). Then Eq. (12) converts into
\[ E_{0,k_x,1} = \frac{\hbar \omega_c}{2} - |g_0|\mu_B B/2 + \frac{m^* \omega_c^2 \ell^4_0}{2} (k_x - k_x)^2 \Theta(k_x - k_x) - \frac{e^2}{\pi \hbar} \int_{-\infty}^{k^{(1)}_{x_0} - k_x} \frac{dx}{1 + r_1 M(0, \sqrt{x^2 + \delta^2/\ell^2_0})} \]

where the group velocity of edge states \( v_{g0}^{(1)} = \partial E_{0,k_x,1}/\hbar \partial k_x \), renormalized by exchange and correlations, which follows from the quadratic equation after using Eq. (13), is given by

\[ v_{g0}^{(1)} = v_{g0}^{1,H} + \frac{e^2}{\pi \hbar \varepsilon} \frac{M(0, \delta/\ell_0)}{1 + r_1 (v_{g0}^{(1)})(0, \delta/\ell_0)}, \]

which was calculated by considering that \( \partial M(k_x - k_{x_0}^{(1)}, \sqrt{x^2 + \delta^2/\ell^2_0})/\partial k_x \) is zero. Equations (13) and (14) provide the self-consistent scheme and are the basic equations of our GLDA. We have introduced \( \delta \ll 1 \) in the second argument of \( M(a, x) \), by changing \( x = k_x' - k_x \) to \( \sqrt{x^2 + \delta^2/\ell^2_0} \), in order to avoid very weak logarithmic divergence for \( x \to 0 \). Indeed, \( M(0, x) \approx \ln(2\sqrt{2}/\ell_0|x|) - \gamma/2 \), for \( x \to 0 \), where \( \gamma \) is the Euler constant. As one will see, the results are very weakly dependent on \( \delta \). This small parameter can be estimated as \( \delta \sim \max[\epsilon_0/d, \ell_0/\epsilon_{g0}^{(1)}] \), where \( d \) is a typical distance of a remote screening region (or a gate) and \( \tau \) is a typical lifetime at the edge states.

From Eq. (14), we obtain that exists only one root

\[ v_{g0}^{(1)} = v_{g0}^{1,H} + \frac{e^2}{\pi \hbar \varepsilon} \frac{M(0, \delta/\ell_0)}{1 + r_1 (v_{g0}^{(1)})(0, \delta/\ell_0)}, \]

satisfying the physical requirement \( v_{g0}^{(1)} > 0 \), which means that the LL is below \( E_F \) in the region between the middle and the right edge of the channel \( y_{g0}^{(1)} \). From Eq. (15), we obtain that \( v_{g0}^{(1)}/v_{g0}^{1,H} \approx \ln(2\sqrt{2}/\delta) \sqrt{M(0, \delta/\ell_0)} \gg 1 \). In addition, one see that \( v_{g0}^{(1)}/v_{g0}^{1,H} \propto 1/\sqrt{v_{g0}^{1,H}} \).

The positive gap between the bottom of the upper spin-split LL and the Fermi level of the interacting 2DES, \( G(v_{g0}^{1,H}) = E_{0,0,-1} - E_{0,k_x^{(1)},1} \), is then given by

\[ G = |g_0|\mu_B B - \frac{m^* \omega_c^2}{2 \Omega^2} (v_{g0}^{1,H})^2 + \frac{e^2}{\pi \hbar \varepsilon} \int_0^\infty dt \frac{M(0, \sqrt{t^2 + \delta^2/\ell^2_0})}{1 + R(v_{g0}^{1,H})M(0, \sqrt{t^2 + \delta^2/\ell^2_0})}, \]

where \( R(v_{g0}^{1,H}) \) is the function which appears, from \( r_1 (v_{g0}^{(1)}) \), after taking \( v_{g0}^{(1)} \) in terms of \( v_{g0}^{1,H} \) according to Eq. (15) and \( M(0, t/\ell_0) = \exp(-t^2/4)K_0(t^2/4)/2 \). To obtain Eq. (16), we put Eq. (13) and the expression \( k_{x_0}^{(1)} = (m^* \omega_c^2/\hbar^2 \Omega^2) v_{g0}^{1,H} \) in the term \( k_{x_0}^{(1)} - k_x^2 \). In the inner part of the channel, the total gap between the lowest spin-split LLs is \( G_{-1,1} = (E_{0,0,-1} - E_{0,0,1}) \approx |g_0|\mu_B B + \sqrt{\pi/2} (e^2/\epsilon_0) \), where we have neglected many-body contributions due to the weak “bulk” screening of the fully occupied LL as previously discussed. However, while this “bulk” screening effect is rather weak at the edge region in comparison with the edge-state screening effect, it should be more significant in the inner part of the channel far from the edge. Taking into account this effect, we obtain \( G_{-1,1} = |g_0|\mu_B B + \sqrt{\pi/2} k_{ba}(r_0)(e^2/\epsilon_0) \), where the numerical values of \( k_{ba}(r_0) \) smoothly vary from 0.79 to 0.63 as \( r_0 \) goes from 0.6 to 1.4. Then \( G_{-1,1} = |g_0|\mu_B B/\hbar \omega_c = \sqrt{\pi/2} r_0 k_{ba}(r_0) \) and values of \( k_{ba}(r_0) \) tend rather fast to 1 as \( r_0 \) goes below 1.

In the spirit of the MLDA, which has some similarity with the local-density approximation (LDA), we claim that the energy dispersion relation, given by Eqs. (13) - (15), comes from the solution of the single-particle Schrödinger equation (for \( \sigma = 1 \)) with the Hamiltonian \( \hat{h} = \hbar \hat{v} + V_{xc}(y) \), where the self-consistent exchange-correlation potential is given as

\[ V_{xc}(y) = E_{0,y/\ell_0^2} - \frac{\hbar \omega_c}{2} - |g_0|\mu_B B/2 + V_y, \]

where \( E_{0,x,1} \) being determined by Eqs. (13) and (14). The validity of GLDA for \( r_0 \ll 1 \) is well justified if \( v_{g0}^{(1)}/\ell_0 \ll \omega_c \). As a consequence, the eigenenergy \( E_{0,x,1} \) as a function of \( x \) is smooth on the scale of \( \ell_0 \) for any occupied state and the eigenfunction of any actual state can be again approximated by the unperturbed \( \psi_{nk_x,\sigma}(r; \sigma_1) \). Using this, we arrive
to Eqs. (13) and (15), which reduces to Eq. (12), for $r_0 \ll 1$ and $4r_0^H \ln(2\sqrt{2}/\delta) \ll 1$. However, as stated before, the latter condition cannot be satisfied in GaAs based samples. Now, assuming that $V_{xc}(y)$ is smooth on the scale of $\ell_0$ we find, neglecting small corrections, that the corresponding energy dispersion is given back by Eqs. (13) and (15) for $(n = 0, \sigma = 1)$ LL, which confirms the successful self-consistent scheme of GLDA. However, in contrast with the LDA, our $V_{xc}(y)$ depends essentially on the slope at the edge $v_{y0}(1)$ of the channel $\{d[V_{0} + V_{xc}(y)\}/dy\}_{y=v_{y0}(1)} \propto v_{y0}(1)$, which can be quite different for almost the same density profile of the 2DES. Moreover, this effect is essential for some regions where $|y - y_{y0}^{(1)}|/\ell_0 \gg 1$. Hence, the GLDA incorporates nonlocal features, as the MLDA, but in contrast with the LDA. The effective one-electron confining potential $V_{y}(y) = V_{c} + V_{xc}(y)$, for $(n = 0, \sigma = 1)$ LL, within GLDA is determined, for $r_0 \lesssim 1$, by Eqs. (13), (14), and (15) and leads to strong modifications in the energy spectrum and activation gap, while keeps the 2DES density profile practically constant, in comparison, e.g., with the Hartree approximation result. We point out that the assumed smoothness of the total confining potential for $(n = 0, \sigma = 1)$ LL implies that $v_{y0}(1)$, given by Eq. (14), should satisfy the condition $v_{y0}(1)/\ell_0 \ll \omega_c$. This condition can be rewritten as $[(\ell_0/\pi)(v_{y0}^{1H}/\omega_c\ell_0)\ln(2\sqrt{2}/\delta)]^{1/2} \ll 1$. Then it is satisfied for typical $r_0 \lesssim 1$, $\delta$ and $v_{y0}^{1H}/\ell_0 \ll 1$.

Now, we define the dimensionless activation gap $G_a(v_{y0}^{1H}) = G/|(g_0)|\mu B/2|$ due to exchange correlation effects. In the absence of many-body interactions, the maximum value of $G_a = 1$. So the activation gap is enhanced when $G_a > 1$. The asymmetry of the Fermi level position within the Fermi gap in the interior part of the channel can be characterized by another dimensionless function $\delta G(v_{y0}^{1H}) = (G_{-1,1} - G_a)/G_a$, where $G_{-1,1} = G_{-1,1}(|g_0|\mu B/2)$. Notice that, when $v_{y0}^{1H} \rightarrow 0$, $E_F^{H}$ tends, from the above side, to the bottom of the $(n = 0, \sigma = 1)$ LL, in the absence of many-body interactions.

In Fig. 1, using Eq. (14), $G_a$ is depicted as a function of $v_{y0}^{1H}$, $v_{y0}^{1H}/(\hbar\omega_c/m^*)^{1/2}$ for $\delta = 10^{-3}$, such that the condition of smoothness of the confining potential is $v_{y0}^{1H} \ll \ell_0$. The solid and dashed curves correspond to $\omega_c/\Omega = 20$ and 10, respectively. Furthermore, the solid (dashed) curves correspond, from top to bottom, to $r_0 = 1.4, 1.0$ and 0.6, respectively. The maxima of $G_a$ for the solid curves, from top to bottom, are 30.42 (for $v_{y0}^{1H} = v_{y0}^{1H,max} = 0.0138$ and $k_{c}^{0,1}\ell_0 = 5.52$); 23.57 (for $v_{y0}^{H,max} = 0.0119$ and $k_{c}^{0,1}\ell_0 = 4.7$); and 16.16 (for $v_{y0}^{H,max} = 0.0095$ and $k_{c}^{0,1}\ell_0 = 3.8$). For the dashed curves, the maxima are, from top to bottom, 38.18 (for $v_{y0}^{H,max} = 0.0297$ and $k_{c}^{0,1}\ell_0 = 2.97$); 29.30 (for $v_{y0}^{H,max} = 0.0254$ and $k_{c}^{0,1}\ell_0 = 2.54$); and 19.75 (for $v_{y0}^{H,max} = 0.0201$ and $k_{c}^{0,1}\ell_0 = 2.01$). Notice that the maxima occur at values $v_{y0}^{H,max}$ in which all assumed conditions are well satisfied. When exchange and correlations are neglected, $E_F^{H}$ is a quasi-Fermi level at the maxima, because it is located above the bottom of the $(n = 0, \sigma = -1)$ LL. Indeed, $E_F^{H}$ touches the bottom of $(n = 0, \sigma = -1)$ LL at $v_{y0}^{1H} = 0.0086$ and $k_{c}^{0,1}\ell_0 = 3.43$ for solid curve parameters and at $v_{y0}^{1H} = 0.0172$ and $k_{c}^{0,1}\ell_0 = 1.72$ for the dashed curves, if many-body contributions are neglected. In the present study, parameters of GaAs-based sample are used, in particular, $\varepsilon \approx 12.5$ and $m^* = 0.067m_0$.

In Fig. 2, we present $\delta G$ as a function of $v_{y0}^{1H}$ for the same parameters used in Fig. 1. In the same way, the solid and dashed curves correspond to $\omega_c/\Omega = 20$ and 10, respectively. At the right side of the figure, the solid (dashed) curves correspond, from top to bottom, to $r_0 = 0.6, 1.0$ and 1.4, respectively. Notice that a maximum value of $G_a$ and a minimum of $\delta G$ are at the same $v_{y0}^{H,max}$. The minima $\delta G = 4.11, 4.13$ and 4.00 are for solid curves while $\delta G = 3.18, 3.13$ and 2.98 for dashed lines (from top to bottom at the right side). The results, shown in Figs. 1 and 2, demonstrate that the position of the Fermi level, within the gap in the interior part of the channel, is highly asymmetric and this will be even stronger, if the effect of the “bulk” screening in $G_{-1,1}$ should be neglected. So strong electron correlations of the edge states lead to a highly asymmetric position of the Fermi level within the gap defined by the two lowest spin-split LLs.

The results given in the solid curves of Figs. 1 and 2, for $r_0 = 1.4$, correspond to the experiments of Ref. [10], with electron density $n_s \approx 8.1 \times 10^{10}$ cm$^{-2}$, in which a factor of enhancement of the activation gap of the order of 15 was observed. Indeed, for the $\nu = 1$ QHE regime, $B = 2\pi h c n_s/|e| \approx 3.34$ T gives $r_0 \approx 1.4$ and $\omega_c \approx 8.76 \times 10^{12}$ s$^{-1}$. In our theory, $\Omega$ or $\omega_c/\Omega$ is an undetermined parameter related to the parabolic confinement at the edges. It was estimated in Ref. [3] for $\nu = 1$ and a sample with $n_s \approx 1.9 \times 10^{11}$ cm$^{-2}$, as $\Omega \approx 7.8 \times 10^{13}$ s$^{-1}$. However, Ref. [8] gives a smaller value $\Omega \approx 4.16 \times 10^{11}$ s$^{-1}$, and here we implicitly assume that $\omega_c/\Omega = 20$ should be a realistic estimate for the sample of Ref. [10].

Now we present, in Fig. 3, $G_a$ for $r_0 = 1.4$, using Eq. (14). Solid and dashed curves correspond to $\delta = 10^{-3}$ and $\delta = 10^{-2}$, respectively. The solid (dashed) curves correspond, from top to bottom, to $\omega_c/\Omega = 10, 20, 40, 60$ and 80, respectively. One can see that the maximum of $G_a$ for $\omega_c/\Omega = 20$ is in reasonable agreement with experiment. [10] Observe that for $\omega_c/\Omega = 60$ and 80, our result is very close to the observed activation gap. We believe that the measured smaller value of the gap should be related with effects of weak long-range inhomogeneities on the confining potential $V_y$ at the inner part of the channel, which are absent in our model. It is also seen that $G_a$ is weakly
dependent on the small cutting parameter $\delta$.

We show, in Fig. 4, numerical results, within GLDA, for LLs spectra $E_{0,k_x}\pm 1$ as function of $k_x$, where $k_x$ is measured as $(k_x - k_r)\ell_0$, and $\hat{k}_x = k_x\ell_0$ gives the dimensionless oscillator center. We take $r_0 = 1.0$, $\omega_c/\Omega = 20$, $\delta = 10^{-3}$, $k_c^{0,1}\ell_0 = (k^{(1)}_c - k_r)\ell_0 = 4.76$, and $\tilde{v}_g^{1,H} = 0.0119$, which are the parameters corresponding to the middle solid curve of Fig. 1 at the maximum of $G_x$. There, $\tilde{v}_g^{1,H} / \sqrt{\hbar \omega_c / m^*} \approx 0.176 \ll 1$ satisfies the “smoothness” requirement to applicability of the GLDA. The bottom solid curve represents $E_{0,k_x\pm 1}$, from Eqs. (13), (11), and the dotted line gives the exact position of $E_F$, when both exchange and correlations effects are taken into account. The top solid curve is the spectrum of the upper spin-split LL $E_{0,k_x\pm 1}$, and, for a sake of comparison, the close dashed-dotted curve is $\epsilon_{0,k_x\pm 1} = [\hbar \omega_c + m^* \Omega^2 (y_0 - y_r)^2 \Theta(y_0 - y_r) - |g_0| \mu_B B] / 2$, i.e., the spectrum of the lower spin-split LL without many-body interactions. Finally, the dashed curve is $E_{0,k_x\pm 1}$ obtained within the HFA where correlations effects are totally neglected. The horizontal dashed line shows the position of $E_F$ within the HFA.

Similar curves to those in Fig. 4 are depicted in Fig. 5, but with the parameters pertinent to experiments of Ref. [10]. In Fig. 5, the following parameters are used: $r_0 = 1.4$, $\omega_c/\Omega = 20$, $\delta = 10^{-3}$, $k_c^{0,1}\ell_0 = 5.52$, $\tilde{v}_g^{1,H} = 0.0138$, which are used to plot the top solid curve in Fig. 1 at its maximum. Notice here $\tilde{v}_g^{1,y} \approx 0.224$ is small, which satisfies the condition of “smoothness” for applicability of the GLDA. The small effect coming from virtual interlevel transitions was neglected in Figs. 4 and 5. However, the most important contribution of such a small effect, related with the 2DES screening in the middle of the channel for $\nu = 1$ is taken into account in Figs. 1-3.

### IV. CONCLUDING REMARKS

The self-consistent treatment developed here, for 2DES in the $\nu = 1$ QHE regime, shows that edge-state correlations drastically modify the ($n = 0, \sigma = 1$) LL spectrum in a wide region (with width $> \ell_0$) nearby the channel edge, i.e. the effect of correlations induced by edge-states is essentially nonlocal. Moreover, we have shown that the position of the Fermi level $E_F$ is highly asymmetric within the gap defined by the ($n = 0, \sigma = 1$) and ($n = 0, \sigma = -1$) LLs in the interior, or “bulk” part of the channel due to such correlations. The activation gap is much smaller than the Fermi gap. Our results were obtained by developing a generalized local density approximation (GLDA) in which the single-particle confining potential $V_{xc}(y)$ incorporates exchange and nonlocal correlation effects.
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Fig. 1 Many-body enhancement of the activation gap \( G_a = G/\langle\mu_B B/2 \rangle \) as a function of the group velocity \( v_{g0}^1 \), in the Hartree approximation, calculated from Eqs. (15) and (14). Solid (for \( \omega_c/\Omega = 20 \)) and dashed (for \( \omega_c/\Omega = 10 \)) curves correspond to \( r_0 = 1.4, 1.0, \) and 0.6, from top to bottom; with \( \delta = 10^{-3} \). In the absence of many-body interactions, the maximum of \( G_a = 1 \).

Fig. 2 Fractional difference \( \delta G = (G_{-1.1} - G)/G \), where \( G_{-1.1} = E_{0,0,-1} - E_{0,0,1} \), as a function of \( v_{g0}^H \), showing the asymmetry of the Fermi level position within the Fermi gap, in the inner part of the channel. Bulk screening effect is included in the calculation of the total value of the Fermi gap \( G_{-1,1} \). The parameters and notations of the curves are the same as in Fig. 1, but the solid and dashed curves here correspond, counting at their right side from top to bottom, to \( r_0 = 0.6, 1.0, 1.4 \). Notice that the minimum of \( \delta G = 1 \), when many-body interactions are neglected.

Fig. 3 Activation gap \( G_a \) as a function of \( v_{g0}^H \) for \( r_0 = 1.4 \), corresponding to the experiment of Ref. [10]. Curves are shown, from top to bottom, for \( \omega_c/\Omega = 10, 20, 40, 60, \) and 80. The solid and dashed lines are given for \( \delta = 10^{-3} \) and \( 10^{-2} \), respectively. It is seen that \( G_a \) is weakly dependent on the small parameter \( \delta \).

Fig. 4 Energy spectra as a function of \( k_x \) for parameters \( r_0 = 1.0, \omega_c/\Omega = 20, \delta = 10^{-3}, k_x^{0.1} \ell_0 = (k_x^{r0} - k_x) \ell_0 = 4.76, \) and \( v_{g0}^H = 0.0119 \). These values correspond to the maximum of \( G_a \) shown by the middle solid curve in Fig. 1. The solid curve at the bottom is \( E_{0,k_x,1} \), evaluated using Eqs. (15) and (14), which can be compared with \( E_{0,k_x,1} \) in the Hartree-Fock approximation (no correlations involved) represented by the dashed curve, and \( E_{0,k_x,1} \) the spectrum of the lower spin-split LL without many-body interactions (dash-dotted curve). The dotted line is the exact position of...
the Fermi level $E_F$, when exchange-correlation effects are taken into account while the dashed line is the position of the Fermi level in the Hartree-Fock approximation. The top solid curve represents the spectrum of the upper-split LL, $E_{0,k_-,1}$.

Fig. 5 Same as in Fig. 4 with the assumed experimental parameters for the sample of Ref. [10]. Here $r_0 = 1.4$, $\omega_c/\Omega = 20$, $\delta = 10^{-3}$, $k_{c_1,1} = 5.52$, $v_{g0}^{1,1} = 0.0138$. These values correspond to those of the top solid curve in Fig. 1 at its maximum.
Fig. 1
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"Electron correlation effects in a wide channel ..."
Fig. 2
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"Electron correlation effects in a wide channel ... "

\[ \delta G \]

\[ v_{g0}^{1, H}/(\hbar \omega_c/m^*)^{1/2} \]
Fig. 3
O. G. Balev and Nelson Studart
"Electron correlation effects in a wide channel ..."
Fig. 4
O. G. Balev and Nelson Studart
"Electron correlation effects in a wide channel ..."
Fig. 5
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