Popularity enhances the interdependent network reciprocity

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Interdependent networks (IN) are collections of non-trivially interrelated graphs that are not physically connected, and provide a more realistic representation of real-world networked systems as compared to traditional isolated networks. In particular, they are an efficient tool to study the evolution of cooperative behavior from the viewpoint of statistical physics. Here, we consider a prisoner dilemma game taking place in IN, and introduce a simple rule for the calculation of fitness that incorporates individual popularity, which in its turn is represented by one parameter \( \alpha \). We show that interdependence between agents in different networks influences the cooperative behavior trait. Namely, intermediate \( \alpha \) values guarantee an optimal environment for the evolution of cooperation, while too high or excessively low \( \alpha \) values impede cooperation. These results originate from an enhanced synchronization of strategies in different networks, which is beneficial for the formation of giant cooperative clusters wherein cooperators are protected from exploitation by defectors.

1. Introduction

How cooperation in social dilemmas survives and is enhanced is still an open question, attracting a lot of interest across a myriad of disciplines, such as physics, mathematics, biology, computer science and ecology. With the aid of evolutionary game theory, many statistical physics methods (like Monte Carlo, MC simulation, mean-field theory and pair approximation) have been successfully applied to cope with this issue [1–6], and several mechanisms have been proposed to be of the basis of cooperation enhancement, such as tit–for–tat [7], win-stay-lose shift [8, 9], voluntary participation [10, 11], spatially structured populations [12–18], heterogeneity or diversity [19, 20], mobility of players [21–23], and co-evolution of dynamical rules [24, 25]. In particular, Nowak [3] categorized all these cases under five possible scenarios: kin selection, direct reciprocity, indirect reciprocity, group selection and network reciprocity, (for a comprehensive review of the subject, we address the reader to [26, 27]).

On the other hand, interdependent networks (IN) are those graphs where one seemingly irrelevant change in a network might induce catastrophic and unexpected consequences in another network [28]. IN have been used to study the evolution of cooperation, and to try resolving social dilemmas [29–34]. For instance, the individual reputation between two IN has been shown to greatly promote the cooperation level [35, 36]. In addition, [37] considered coupled fitness on IN, and found the existence of a coupling threshold \( \gamma_c \) below which cooperation is promoted, and above which spontaneous symmetry breaking of cooperation levels occur on different
networks. The influence of individual particularity has also attracted attention in traditional single networks. Szolnoki et al [38] defined the individual’s learning ability as one function of the collective opinion of groups, and found that wisdom of groups can indeed promote cooperation. Similarly, individual popularity combined with preferential selection can effectively resolve the so-called social dilemma [39, 40]. However, so far a mutual influence between networks was not considered, and it is therefore natural to study how network reciprocity is affected if one incorporates popularity into IN, combining it with the calculation of the fitness.

In this paper, we overcome this limitation, and study how individual popularity (defined as the number of the same strategies adopted in the corresponding group of the other, dependent, network) affects the evolution of cooperation. On its turn, popularity is used as a criterion of interdependence strength, which directly decides individual’s fitness. That is, the higher the individual popularity is, the larger is the additional payoff that agents obtain. Some real-life examples support such a hypothesis: good public management policies in a country, for instance, will become popular within various hierarchies of departments, and may produce collective welfare for citizens; sport or film stars with higher popularity are likely to obtain higher incomes than common individuals in the society.

The remainder of the paper is organized as follows. We first describe the model, and then we present extensive MC simulation results, which allow us to unveil the reason why popularity favors cooperation spreading on IN. Finally, we summarize the main conclusions, and discuss their potential implications.

2. Methods

We consider a prisoner’s dilemma game (PDG) on IN consisting of two \(L \times L\) square lattices with periodic boundary conditions. Each player on both networks is initially designed as either a cooperator (C) or defector (D) with equal probability. PDG is characterized by a temptation to defect \(T\) (the highest payoff obtained by a defector if playing against a cooperator), a reward for mutual cooperation \(R\), a punishment for mutual defection \(P\) and a sucker’s payoff \(S\) (the payoff received by a cooperator against a defector). For simplicity (and yet without loss generality) we use the weak PDG model [3, 41] where the payoff parameters are set as follows: \(T = b, R = 1, P = S = 0.1 \leq b < 2\) ensures a proper payoff ranking \(T > R > P > S\) and captures the essential social dilemma, where private interests are at odds with collective welfare. The accumulated payoff of player \(x\) and player \(x'\) on both networks follows the same procedure: first one obtains payoffs \(P_x\) and \(P_x'\) via playing with their four direct neighbors, and then their fitness is evaluated as follows:

\[
\begin{align*}
F_x &= P_x + \beta \cdot P_x', \\
F_x' &= P_x' + \beta \cdot P_x,
\end{align*}
\]

where \(\beta\) denotes the interdependence strength. In [42] it has been proved that an optimal value of \(\beta\) exists for the promotion of cooperation, but we consider here the interdependence strength as closely related to player’s popularity. Figure 1 shows a schematic example of our definition of popularity. The red player in the upper network can get additional payoff since it has a large popularity, while the blue player in the upper network cannot, as its popularity is low. In particular, popularity can be quantified via a tunable parameter \(\alpha\) as follows:

\[
\beta = \left(\frac{N_x}{N}\right)\alpha,
\]

where \(N = k + 1\) is the size of the group in the other network which corresponds to the group of neighbors of the local player in the actual network, and \(N_x\) denotes the number of players in that group which are adopting the same strategy of the local player of the actual network. Obviously, \(\alpha = 0\) means that each agent’s fitness is equal to sum of its own payoff and its partner’s payoff on the other network, which eliminates the potential impact of heterogeneous fitness distribution [43]. Positive and large \(\alpha\) values enable \(\beta\) to become smaller and smaller, since \(N_x / N \leq 1\).

With the above definition of fitness, a strategy transition can take place between the direct four neighbors on any given network, but can never happen between players residing on different networks. Here, we adopt the well-known Fermi function as probabilistic updating dynamics. In details, player \(x\) (\(x'\)) adopts the strategy of one of its randomly selected neighbors \(y\) (\(y'\)) with a probability:

\[
W_x(y) = \frac{1}{1 + \exp((F_y - F_x)/K)},
\]

or

\[
W_x'(y') = \frac{1}{1 + \exp((F_{y'} - F_x')/K)},
\]
Figure 1. Schematic illustration of how popularity affects the individual’s fitness. We denote two (randomly selected) players in the upper network and their partners (having four neighbors) in the bottom network via dash lines. The red player in the upper network can get additional payoff since it has a large popularity (which is defined as the number of nodes adopting the same strategy in the bottom network), while the blue player in the upper network has a low popularity due to having more opponents in the bottom network.

where $K$ quantifies the uncertainty related to the strategy adoption process. The Fermi function is originally introduced in statistical physics [44, 45], and became an effective method to quantitatively measure the change of nodes’ states under the impact of neighboring agents in evolutionary game theory. As for uncertainty or noise (which would correspond to temperature in traditional statistical physics), we fix $K$ equal to 0.1 (for simplicity) in this work, and refer the reader to [10, 46] for the role of $K$ in evolutionary games.

As for our simulations, during one full MC step each player has a chance to adopt the neighbor’s strategy once on average. The results of MC simulations presented below were obtained on square lattices with average degree $k = 4$. The network size was varied from $L = 400$ to $L = 1000$ (in order to avoid finite size effects) and the equilibration required up to 50 000 time steps. We also conducted asynchronous updating methods, and found qualitatively similar results (not shown here).

3. Results

Figure 2 shows the fraction of cooperation $\rho_C$, as a function of the temptation to defect $b$, for different values of $\alpha$ ($\alpha = 0, \alpha = 0.5$ (low values scenario), $\alpha = 1.0$ (intermediate values scenario), and $\alpha = 5.0$ (high values scenario)). The first interesting conclusion is that only at intermediate values of $\alpha$ cooperation is optimally guaranteed, whereas for too low or too high $\alpha$ values cooperation is impeded. In fact $\alpha = 0$ (at which popularity is not taken into account) corresponds to the case where all players have the highest reward (interdependence strength equal to 1.0), and thus it implies full interdependence between the two populations. In this case, cooperation is supported by classical IN reciprocity [43]. $\alpha > 0$ incorporates instead individual’s popularity into the model, and yet it weakens the interdependence strength. When $\alpha$ is too high, the model returns to the traditional single network reciprocity case [3].

For intermediate values of $\alpha$ (i.e. $\alpha = 1.0$) cooperation is higher, as compared with the results of the other three cases. This fact goes beyond the boundaries supported by traditional network reciprocity or IN reciprocity. That is to say, when $\alpha$ is intermediate, additional reciprocity mechanisms are responsible for the emergence of cooperation behavior. In what follows, we will give further details about this latter finding.

Characteristic snapshots of the evolution are presented in figure 3. Cooperators and defectors are denoted by red and blue, respectively. Furthermore, warm colors mark smaller $\beta$ values, and cold colors mark larger $\beta$ values. One clearly sees that additional reciprocity mechanisms warrant support for cooperators to survive even for larger temptation to defect (see the second column). Figure 4 shows a diagram of typical configuration patterns, where we calculate the fitness of cooperation and defection still with $\alpha = 1.0$. We first consider the case of only one cooperator (red circle) in a sea of defectors (figure 4(a)). The cooperator’s fitness is 0, and the neighboring defector’s (blue circle) fitness is $0.8 \times 1.56$, and thus cooperator is replaced by defectors. As for the case of two cooperators in a sea of defectors (figure 4(b)), the cooperators’ fitness is $1 \times 0.4$, and the defectors fitness is $0.8 \times 1.56$, and once again cooperators will disappear because of inferiority in their fitness. When
Figure 2. Fraction of cooperation $\rho_c$ versus $b$, for different values of $\alpha$. Intermediate $\alpha$ values enable cooperators to survive at large $b$. The critical $b_C$ (see the arrow) at which cooperators dominate first increases and then decreases.

Figure 3. Characteristic snapshots revealing the spontaneous emergence of cooperative clusters due to individual’s popularity. From left to right: snapshots of the upper and lower networks for $\alpha = 1$ and $b = 1.56$ at 0, 1, 100, 200, and 30,000 time steps. Defectors are blue and cooperators are red. Warm colors mark smaller $\beta$ values and cold colors mark larger $\beta$ values.
instead there are three cooperators in a sea of defectors (figure 4(c)), cooperation can spread since the cooperators’ fitness ($2 \times 0.6$) is larger than the defectors’ fitness ($1.56 \times 0.8$). Therefore, the survival of cooperation can be sustained only if at least three adjacent cooperators are formed during the evolution.

From the last two rows in figure 3, one can also notice that boundary cooperators and defectors capture less benefit from the other network, while cooperators and defectors positioned within inner clusters can have more benefits. In other words, our setup helps cooperation to form more compact clusters, which not only gives powerful protection against exploitation of defectors, but also accelerates the spreading of cooperation till a full C phase is attained. In order to verify this assumption, figure 5 shows some characteristic evolution snapshot, obtained starting from a special strategy distribution: two adjacent cooperators located in top left corner on both networks, and a $4 \times 4$ cooperators located in lower right corner on both networks. Such a 16-cooperators configuration can effectively avoid the accidental disappearance of cooperation due to noise. The color code used is the same as that of figure 4. It is clear that cooperators on the left corner disappear soon, while
cooperators on the lower right corner can spread to the whole network (because the minimum requirement for the survival of cooperation configuration is satisfied).

Finally, it is instructive to compare the time courses of different values of $\alpha$ when starting from a random initial state. In figure 6(a), one can observe that cooperation becomes extinct when $\alpha$ is high or very low, since cooperators are forced to rely on IN reciprocity or single network reciprocity, where cooperation cannot be supported at large temptation to defection. However, when $\alpha$ is low or intermediate (i.e. $\alpha = 0.5$ or 1.0), there is a typical negative feedback trend of cooperation, which is the trademark of network reciprocity. It is worth noting that the delay is due to the fact that cooperators need long time to organize themselves forming the required configuration. Figures 6(b) and (c) show how the fraction of CC pairs within a network and between the two networks varies during the evolution. Clearly, there exists a synchronization effect for all the cases, but only low or intermediate $\alpha$ values can support cooperation (whereas high or low $\alpha$ values fail). Eventually, at intermediate $\alpha$ values one can observe a full C phase even when the temptation to defection is relatively large (i.e. $b = 1.56$).

4. Conclusions

In conclusion, we have studied the effect of individual’s popularity (defined as the number of the same strategies adopted in the corresponding group of the other network) on the evolution of cooperation via a single parameter $\alpha$. Through numerical simulations, we have found that intermediate $\alpha$ values can support cooperation, whereas very low or too high $\alpha$ values hinders its evolution. At intermediate $\alpha$ values, an enhanced synchronization effect occurs in the system, visible from typical configuration patterns.

It is shown that cooperation can be promoted by means of self-organization towards optimally IN when co-evolution of teaching ability is considered [34, 47, 48]. In our work, $\alpha = 0$ corresponds to the case where cooperation proceeds with only the support of IN reciprocity. High $\alpha$ values, on the other hand, weaken the interdependence strength nearly to 0, and leave the two populations almost completely independent, i.e. it determines a situation where traditional network reciprocity is the sole supporting mechanism promoting cooperation. Low or intermediate $\alpha$ values introduce instead a time-varying interdependence between the two populations, which generates heterogeneity. In this sense, intermediate $\alpha$ values correspond to optimally IN, in which cooperation can be promoted by the leader role of distinguished players.

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