Selection of Nearby Microlensing Candidates for Observation by SIM

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ABSTRACT

I investigate the prospects for using the Space Interferometry Mission (SIM) to measure the masses of nearby stars from their astrometric deflection of more distant sources, as originally suggested by Paczyński and by Miralda-Escudé. I derive an analytic expression for the total observing time $T_{\text{tot}}$ required to measure the masses of a fixed number of stars to a given precision. I find that $T_{\text{tot}} \propto r_{\text{max}}^{-2}$, where $r_{\text{max}}$ is the maximum radius to which candidates are searched, or $T_{\text{tot}} \propto \mu_{\text{min}}^2$, where $\mu_{\text{min}}$ is the minimum proper motion to which candidates are searched. I show that $T_{\text{tot}}$ can be reduced by a factor 4 if source availability is extended from $V_s = 17$ to $V_s = 19$. Increasing $r_{\text{max}}$ and $V_s$ and decreasing $\mu_{\text{min}}$ all require a significantly more aggressive approach to finding candidates. A search for candidates can begin by making use of the Luyton proper motion catalog together with the USNO-A2.0 all-sky astrometric catalog. However, a thorough search would require the all-sky USNO-B proper-motion catalog which is not yet available. The follow-up observations necessary to prepare for the mission will become more difficult the longer they are delayed because the candidate pairs are typically already within 1” and are getting closer.

Subject headings: astrometry – Galaxy: stellar content – gravitational lensing
1. Introduction

Refsdal (1964) pointed out that it should be possible to measure the masses of nearby field stars from the astrometric deviation they induce on more distant sources as they pass by the latter. To be practical, the two stars must pass within $\mathcal{O}(1''')$ of each other. Paczyński (1995,1998) and Miralde-Escudé (1996) examined this idea in the context of the current rapid improvements in astrometric capability. They made rough estimates of the number of mass measurements that could be obtained using various ground-based and space-based facilities. Here I re-examine this problem specifically guided by the capabilities and requirements of the Space Interferometry Mission (SIM).

The planned SIM launch date is 2005 and the minimum mission lifetime is 5 years. In order to carry out mass measurements, two steps must be completed prior to launch. First, one must identify candidate pairs of stars from a proper-motion catalog: a nearby “lens” star must be found that is likely to pass sufficiently close to a more distant “source” star to cause a large deflection of light and so permit a precise measurement of this deflection. Second, given the quality of the catalogs that will be available in the near future, it will generally not be possible to predict which of the candidates will be the best to make precise mass measurements with a modest amount of observing time. Rather, it will be necessary to perform follow-up observations of these candidates prior to the event in order to determine the impact parameter (angular separation $\beta$ at the point of closest approach).

Typically, the candidates are already closer than $1''$, often much closer. Moreover, in many cases, one star is substantially brighter than the other. Hence, the follow-up observations will usually require adaptive optics or the Hubble Space Telescope. These requirements will grow more severe as time passes. In brief, preparation for mass measurements using SIM should proceed without delay.

Because SIM observing time comes at a high premium, my approach is to rank candidates by the amount of observing time that is required to make a mass measurement of fixed precision. I then use this framework to characterize and evaluate various selection strategies.

The probability that it is possible to measure the mass of a given foreground star grows monotonically with its proper motion and is linear in the proper motion in most cases. Hence, a survey based on an ideal star catalog (not affected by magnitude limits or crowding) would investigate foreground stars down to some
minimum proper motion $\mu_{\text{min}}$. On the other hand, for stars of sufficiently low luminosity, the magnitude limits of the underlying catalog will impose an effective distance limit, $r_{\text{max}}$. Thus, it is important to consider both forms of selection. In practice, the actual selection process may also be affected by crowding, but I will not consider crowding explicitly in this paper. Rather, one may think of crowding as imposing an indirect constraint on $\mu_{\text{min}}$ or $r_{\text{max}}$.

I derive simple expressions for the total observing time $T_{\text{tot}}$ needed to make $N$ mass measurements. For fixed $N$, I show that $T_{\text{tot}} \propto r_{\text{max}}^{-2}$ for distance limited surveys, and $T_{\text{tot}} \propto \mu_{\text{min}}^2$ for proper-motion limited surveys. Hence, minimization of the observing time requires pushing $r_{\text{max}}$ out as far as possible or pushing $\mu_{\text{min}}$ as low as possible. The sample of candidates will then have on average smaller proper motions meaning that they are even closer on the sky today, thus making follow-up observations even more difficult. In addition, I show that by extending the available sources from $V_s = 17$ to $V_s = 19$, one can decrease $T_{\text{tot}}$ by a factor of 4 despite the lower flux from these fainter sources. However, to determine which of these fainter sources are really usable requires a much more precise estimate of their expected impact parameters, that is, even more precise measurements of their current positions despite the larger disparity in the source/lens flux ratio. Obviously these measurements will also become more difficult with time.

To carry out a search for candidates it would be best to begin with an all-sky proper motion catalog. Such a catalog is currently being prepared by the US Naval Observatory (D. Monet 1999, private communication) but has not yet been released. In the meantime, one can make a good start using the Luyton (1979) proper motion catalog in combination with the USNO-A2.0 astrometric catalog (Monet 1998). I briefly describe how to carry out such a search.

### 2. Required Observation Time For an Individual Lens

Consider a nearby star ("the lens") of mass $M$ and at distance $r$ that passes within an angle $\beta$ of a more distant star ("the source") at $r_s$. The source light will then be deflected by an angle $\alpha = 4GM/(\beta rc^2)$ at the point of closest approach. Consequently, the source will appear displaced by $\tilde{\alpha} \equiv \alpha(1 - r/r_s) = 4GM\pi_{\text{rel}}/(\text{AU} \beta c^2)$ relative to the position expected in the absence of lensing. One can therefore determine the mass of the lens by measuring this
displacement, provided that $\beta$ and the relative parallax $\pi_{\text{rel}}$ are known. Note that

$$\alpha = 80 \, \mu\text{as} \frac{M}{M_\odot} \left( \frac{\beta r}{100 \, \text{AU}} \right)^{-1}.$$  \hfill (1)

Assuming photon-limited astrometry measurements, the total amount of observing time $\tau$ required to achieve a fixed fractional error in the mass measurement then depends on three factors. First, the measurement is easier the bigger $\tilde{\alpha}$: the fractional error for fixed observing time falls as $\tilde{\alpha}^{-1}$ and so for fixed fractional error, $\tau \propto \tilde{\alpha}^{-2}$. Second, the measurement is easier the brighter the source magnitude $V_s$, $\tau \propto 10^{0.4V_s}$. Third, the time required depends on the geometry of the encounter. The geometry can be described in terms of the angular coordinates $(\beta, \lambda)$ of the source-lens separation vector at the time $(t = 0)$ of the midpoint of the mission and the angular displacement $\mu \Delta t$ of the lens relative to the source during the course of the mission. Here $\beta$ is the source-lens separation at the time of closest approach $(t = t_0)$, $\mu$ is the relative source-lens proper motion, $\Delta t$ is the duration of the mission, and $\lambda = -\mu t_0$. I therefore write,

$$\tau = T_0 \left( \frac{\tilde{\alpha}}{\alpha_0} \right)^{-2} 10^{0.4(V_s-17)} \gamma \left( \frac{\lambda}{\beta}, \frac{\mu \Delta t}{\beta} \right),$$  \hfill (2)

where $\gamma$ is a function to be described below, and where $\alpha_0$ and $T_0$ are convenient normalization factors. For definiteness, I will take the required mass precision to be $\sigma_M/M = 1\%$ and will arbitrarily adopt $\alpha_0 = 100 \, \mu\text{as}$. I will normalize $\gamma$ so that it is effectively the number of equal-duration measurements that must be made. I characterize SIM astrometry as requiring 1 minute to achieve 40 $\mu\text{as}$ precision in 1 dimension at $V_s = 17$. Then

$$T_0 = \left( \frac{\sigma_M}{M} \right)^{-2} \left( \frac{40 \, \mu\text{as}}{\alpha_0} \right)^2 \text{min} = 27 \, \text{hours}.$$  \hfill (3)

To estimate $\gamma$, I consider sets of observations over the angular interval $[\lambda_-, \lambda_+]$ where $\lambda_{\pm} = \lambda \pm \mu \Delta t/2$, and solve simultaneously for six source parameters: the two-dimensional angular position at the midpoint, the two-dimensional proper motion, the parallax, and $\tilde{\alpha}$. Even though very little information can be obtained about $\tilde{\alpha}$ from astrometry measurements parallel to its direction of motion, I include such measurements in order to be sensitive to other kinds of apparent source acceleration (e.g. gravitational). Without such a check, the mass measurement could not be considered reliable. I then optimize these observations for the measurement.
of $\tilde{\alpha}$. Generally, the optimum configuration has roughly equal total exposure times at the point of closest approach and near the beginning and end of the experiment, and has no observations at other times. I take $\Delta t$ to be 5 years, but the results would be the same for any value provided that $\Delta t \gg 1$ yr. I find that $\gamma$ achieves a minimum, $\gamma \sim 10$ when $\lambda^- < -2\beta$ and $\lambda^+ > +2\beta$. For example, $\gamma(0, x) = 10$ for $x \geq 4$. Some other indicative values are $\gamma(0, 2) = 21$, $\gamma(0, 1) = 99$, $\gamma(0.75, 2.5) = 24$, $\gamma(0.25, 1.5) = 39$. Thus, if the source does not move by a distance at least equal to the impact parameter on either side of the lens during the course of the observations, $\gamma$ becomes very high. In my analysis below, I will incorporate the exact values of $\gamma$ for each configuration. But qualitatively one can think of $\gamma$ as being

$$\left[\gamma\left(\frac{\lambda}{\beta}, \frac{\mu \Delta t}{\beta}\right)\right]^{-1} \sim \gamma^* \Theta\left(\lambda - \beta + \frac{\mu \Delta t}{2}\right)\Theta\left(-\lambda - \beta + \frac{\mu \Delta t}{2}\right),$$

(4)

where $\Theta$ is a step function, and $\gamma^* = 10$.

3. Observing-time Distribution

From the previous section, a star with $M = M_\odot$, $r = 100$ pc, $\beta = 1''$, and a minimal $\gamma$ would require about 420 hours of observation time for a 1% mass measurement. Hence, it is prudent to consider how one might find pairs of stars with the most favorable characteristics. I begin by writing down the observing-time distribution for an arbitrarily selected sample but with lenses of fixed mass $M$,

$$\frac{dN}{dT} = \int d^3r d^3r_s dV_s dv_\perp n(r)n_s(V_s, r_s)f(r, v_\perp)S(r, r_s, v_\perp, V_s, ...)\delta[T - \tau(V_s, M, b, \ell, v_\perp \Delta t, r, r_s)].$$

(5)

Here $n(r)$ is the number density of lenses as a function of their position, $n_s(V_s, r_s)$ is the number density of sources as a function of their magnitude and position, $v_\perp = r\mu$ is the transverse speed of the lens relative to the observer-source line of sight, $f(r, v_\perp)$ is the transverse speed distribution as a function of position, $S$ is the selection function (with possibly many variables in addition to those explicitly shown), $b = r\beta$, $\ell = r\lambda$, and $\delta$ is a Dirac delta function. Equation (5) cannot be simplified without additional assumptions. As I introduce these assumptions, I will briefly outline their impact. Some of the simplifications will then be discussed in greater detail below.

I first assume that $r \ll r_s$. This is an excellent approximation for disk lenses although it is not as good for halo lenses. It has two simplifying effects: $\tilde{\alpha} \to \alpha,$
so \( \tau = \tau(V_s, M, b, \ell, v_{\perp} \Delta t) \), and the 3-space density of sources \( n_s \) can be replaced with the projected surface density \( \phi(V_s, \Omega) \), where \( \Omega \) is position on the sky. (To be more precise \( \phi \) is the density of sources in the neighborhood of the lens position \( \Omega \).) Second, I assume that the product of the selection function and number density can be written,

\[
n(r)S(r, r_s, v_{\perp}, V_s, \ldots) \rightarrow n\Theta(r_{\text{max}} - r),
\]

where \( n \) is now assumed to be uniform, and \( r_{\text{max}} \) is a maximum search radius. In fact, this is an oversimplification. A major focus of the present study is to determine what effect the selection function has on the observing-time distribution. The best way to do this is to begin with this simplified picture. With these assumptions, equation (3) can be written,

\[
\frac{dN}{dT} = \int dv_{\perp} dV_s \int d\beta d\lambda dr r^2 n\Theta(r_{\text{max}} - r) \delta[T - \tau(V_s, \Omega)] f(v_{\perp}, r).
\]

(7)

In this form, the integration still cannot be factored because of the correlation between the speed distribution of the lenses and the distribution of sources. I therefore assume that \( f(v_{\perp}, r) \rightarrow f(v_{\perp}) \), i.e., that the speed distribution does not depend on position. This is actually a very minor assumption, provided that the speed distribution is taken to be the average over the Galactic plane where the majority of the source stars are. Two of the integrals can then be evaluated directly, and equation (7) becomes,

\[
\frac{dN}{dT} = nr_{\text{max}} \int db d\ell dV_s \int dv_{\perp} f(v_{\perp}) \phi(V_s) \delta[T - \tau(V_s, \Gamma)] H(\Gamma),
\]

(8)

where \( \phi(V_s) \equiv \int d\Omega \phi(V_s, \Omega) \) is the luminosity function integrated over the entire sky. Equation (8) already contains an important result: the number of lenses available for measurement at fixed observing time is directly proportional to \( r_{\text{max}} \), the physical depth to which they are searched.

To further evaluate the integral, first define,

\[
G(\gamma'; x) = \int dy \delta[\gamma' - \gamma(y, x)].
\]

(9)

Then the integral can be written

\[
\frac{dN}{dT} = nr_{\text{max}} \int db d\ell dV_s \int dv_{\perp} f(v_{\perp}) \phi(V_s) \delta[T - \tau(V_s, \Gamma)] H(\Gamma),
\]

(10)

where

\[
H(\Gamma) = \int d\gamma db dv_{\perp} bf(v_{\perp})G\left(\gamma; \frac{v_{\perp} \Delta t}{b}\right) \delta\left[\Gamma - \left(\frac{4GM}{\alpha_0 bc^2}\right)^{-2} \gamma\right].
\]

(11)
3.1. Analytic Estimate

I will evaluate equation (10) explicitly in § 3.2 below. However, it is also instructive to make an analytic estimate of this equation with the help of a few approximations. First, I assume that all the sources have the same magnitude, \( \phi(V_s) = N\delta(V_s - 17) \), where \( N \) is the total number of source stars. Hence,

\[
\frac{dN}{dT} = \frac{nNr_{\text{max}}}{T_0} H\left(\frac{T}{T_0}\right),
\]

(12)

Second, I use the approximation (4) to estimate \( G \),

\[
G(\gamma; x) = (x - 2)\Theta(x - 2)\delta(\gamma_s - \gamma), \quad (\gamma_s = 10).
\]

(13)

Third, I take \( f(v_\perp) = \delta(v_\perp - v_*) \), where \( v_* \) is a typical transverse speed for the lens population. Then

\[
H(\Gamma) = \Gamma^{-1/2}b_0\gamma_s^{-1/2}\left[v_*\Delta t - 4b_0\left(\frac{\Gamma}{\gamma_s}\right)^{1/2}\right], \quad b_0 \equiv \frac{2GM}{\alpha_0c^2},
\]

(14)

where I have suppressed the \( \Theta \) function that limits the range of validity to \( \Gamma < \gamma_s(\alpha_0c^2v_*\Delta t/8GM)^2 \). Combining equations (12) and (14), I obtain,

\[
\frac{dN}{dT} = \frac{2G\rho Nr_{\text{max}}v_*\Delta t}{(\gamma_sTT_0)^{1/2}\alpha_0c^2}, \quad \left[T \ll T_0\gamma_s\left(\frac{\alpha_0c^2v_*\Delta t}{8GM}\right)^2\right],
\]

(15)

where \( \rho \equiv nM \). The limiting condition in equation (13) comes from assuming that the first term in brackets in equation (14) is much greater than the second. Equation (15) tells us that the observing-time distribution depends on the type of lens only through its mass density \( \rho \), its typical velocity \( v_* \), and the cutoff which scales as \( (v_*/M)^2 \).

A sensible observing strategy will naturally focus on the lenses that require the least observing time. I therefore consider a program that measures the masses of all lenses requiring observing times less than some maximum, \( T_{\text{max}} \). The total observing time \( T_{\text{tot}} \) can then be expressed as a function of the total number of stars observed, \( N \), and of the other parameters:

\[
T_{\text{tot}} = \int_0^{T_{\text{max}}} dT T \frac{dN}{dT} \quad \text{and} \quad N = \int_0^{T_{\text{max}}} dT \frac{dN}{dT},
\]

(16)

\[
T_{\text{tot}} = \frac{1}{3}N^3\left(\frac{4G\rho Nr_{\text{max}}v_*\Delta t}{\alpha_0c^2}\right)^{-2}\gamma_sT_0.
\]

(17)
Equation (17) is one of the major results of this paper. It states that the total observing time required to measure the masses of a fixed number of lenses scales inversely as the square of the search radius $r_{\text{max}}$ of the sample. Given the premium on SIM time, this result implies that the search should be pushed to as large a radius as possible. I discuss the prospects for doing this in § 4.

The total time can be written out explicitly

$$T_{\text{tot}} = 230 \text{ hours} \left( \frac{N}{5} \right)^3 \left( \frac{\rho}{0.01 M_{\odot} \text{ pc}^{-3}} \right)^{-2} \left( \frac{v_\star \Delta t}{35 \text{ AU}} \right)^{-2} \left( \frac{r_{\text{max}}}{100 \text{ pc}} \right)^{-2} \left( \frac{N}{10^8} \right)^{-2},$$

where I have assumed a mission lifetime of $\Delta t = 5$ yrs and normalized the transverse speed to a typical disk value $v_\star = 33 \text{ km s}^{-1}$ and the density to approximately 1/3 of the local stellar disk density (Gould, Bahcall, & Flynn 1997). That is I consider that one is interested in one (or perhaps several) subsets of the whole disk population. I have also assumed a total of $N = 10^8$ stars at $V_s = 17$ over the whole sky (Mihalas & Binney 1981). This estimate incorporates a maximum observing time per object,

$$T_{\text{max}} = \frac{3}{N} T_{\text{tot}},$$

which must be well under the cutoff in equation (15) given by

$$T_{\text{cut}} = 13 \text{ hours} \left( \frac{v_\star \Delta t}{35 \text{ AU}} \right)^2 \left( \frac{M}{M_{\odot}} \right)^{-2}.$$

If $T_{\text{cut}} \gg T_{\text{max}}$, then the scaling relation (17) ($N \propto T_{\text{tot}}^{1/3}$) is no longer satisfied. See Figure 1 below. The cutoff is satisfied for low mass disk stars (assuming only 5 mass measurements are desired) but becomes more difficult for higher masses.

Another important feature of equation (17) is that $T_{\text{tot}} \propto 10^{0.4(V_s-17)} T_0 / N^2$. Thus, if we compare $V_s = 17$ and $V_s = 18$, the latter are 2.5 times fainter and so require 2.5 times greater $10^{0.4(V_s-17)}T_0$, the observing time for a single astrometric measurement of precision $\alpha_0$. On the other hand, there are approximately 1.9 times as many stars (Mihalas & Binney 1981) and so $T_{\text{tot}}$ is actually smaller by a factor $\sim 0.7$. It should be noted, however, that the shorter observing time comes about because the impact parameter $b$ is typically 1.9 times smaller. In § 4, I will discuss the prospects for recognizing when such close encounters will occur.
3.2. Numerical Estimates

To test the estimates derived in § 3.1, I continue to approximate $\phi$ as a $\delta$ function, but otherwise carry out the full integration indicated by equations (14) and (15). I take the velocity distribution to be a two-dimensional Gaussian with (one-dimensional) dispersion typical of foreground objects in the Galactic plane: $\sigma^2 = \sigma_U^2/4 + \sigma_V^2/4 + \sigma_W^2/2$, where $\sigma_U = 34 \text{ km s}^{-1}$, $\sigma_V = 28 \text{ km s}^{-1}$, and $\sigma_W = 20 \text{ km s}^{-1}$. This yields $\sigma = 26 \text{ km s}^{-1}$ for which the mean speed is $v_* = (\pi/2)^{1/2} \sigma = 33 \text{ km s}^{-1}$ (as used in § 3.1). Figure 1 shows the results for $M = M_\odot$ (bold curve) and $M = 0.1 M_\odot$ (solid curve). The agreement with the analytic prediction from equation (18) (dashed line) is excellent. Equations (19) and (20) predict that the cutoff should be at $N \sim 1.5(M/M_\odot)^{-1}$, or at $N \sim 15$ for the two cases shown. In fact the actual values are about 2.5 times higher. Most of this difference (a factor of 2) is due to the fact that the velocity distribution is not a $\delta$ function, and the higher-speed stars are more likely to be candidates and are less affected by the threshold.

Figure 2 shows the same quantities for six different magnitude bins of the luminosity function (Mihalas & Binney 1981), and $M = M_\odot$. The $V_s = 17$ curve (same as in Fig. 1) is shown as bold dashed line, and the others $V_s = 14, 15, 16, 18, 19$ are shown as solid lines. The curves can be separately identified by noting that the cutoff increases with magnitude. The upper bold line shows the result of combining all of these while the lower bold line shows the result of combining the four bins with $V_s \leq 17$ together. Note that each of three bins $V_s = 17, 18, 19$ contribute about equally (for $T_{\text{tot}} \lesssim 100$ hours). This is because the longer integration times required for the fainter sources are compensated by the fact that they are more numerous and hence closer on average to the lenses.

3.3. Proper-motion selection

As I discussed in the introduction, in some regimes the selection function is best described as a cut on distance and in other it is best described as a cut on proper motion. So far, I have focused on selection by distance. See equation (13). Had I instead selected on proper motion,

$$n(r)S(r, r_s, v_\perp, V_s, ...) \to n \Theta \left( \frac{v_\perp}{r} - \mu_{\text{min}} \right),$$

(21)
then equations (10) and (11) would be replaced by
\[ \frac{dN}{dT} = n \int drd\Gamma dV_s \phi(V_s) \delta[T - \tau(V_s, \Gamma)] H(\Gamma; r \mu_{\text{min}}), \]

where
\[ H(\Gamma; u) = \int_{\infty}^{\infty} dv_{\perp} f(v_{\perp}) \int d\gamma db b G(\gamma; \frac{v_{\perp} \Delta t}{b}) \delta \left[ \Gamma - \left( \frac{4GM}{\alpha_0 c^2} \right)^{-2} \gamma \right]. \]

Carrying through the derivation, one obtains the analog of equation (17),
\[ T_{\text{tot}} = \frac{1}{3} N^3 \left( \frac{4G \rho N \langle v_{\perp}^2 \rangle \Delta t}{\mu_{\text{min}} \alpha_0 c^2} \right)^{-2} \gamma^* T_0, \]

where \( \langle v_{\perp}^2 \rangle \) is the mean square transverse speed. That is, equations (17) and (24) are identical except \( r_{\text{max}} v^* \rightarrow \langle v_{\perp}^2 \rangle / \mu_{\text{min}} \). For a Gaussian, \( \langle v_{\perp}^2 \rangle = 2\sigma^2 \), so this relation can be written
\[ r_{\text{max}} \rightarrow \frac{4}{\pi} \frac{v^*}{\mu_{\text{min}}} = 90 \text{ pc} \frac{v^*}{33 \text{ km s}^{-1}} \left( \frac{\mu_{\text{min}}}{100 \text{ mas yr}^{-1}} \right)^{-1}. \]

I find that with this substitution, the curves produced by equations (22) and (23) are almost identical to those produced by equations (10) and (11) except that the cutoffs are increased by a factor 1.2. Thus, proper-motion selection and distance selection produce essentially the same results, provided they are converted using equation (25).

4. Identification of Pairs

From equation (18), the total observing time required to measure the mass of a fixed number of stars declines as \( r_{\text{max}}^{-2} \). From Figure 2, one sees that including the magnitude bins \( V_s = 18, 19 \) is roughly equivalent to increasing the total number of sources \( N \) by a factor 2, and from equation (13), the observing time is reduced as \( N^{-2} \sim 0.25 \). This estimate is confirmed by the offset between the two bold curves in Figure 2.

Hence, if it were possible to push out to fainter sources and more distant lenses, it would certainly be profitable to do so. I therefore now investigate the constraints governing the identification of lens-source pairs. The basic problem is that if these
pairs are to be close enough for astrometric microlensing in, say, 2008, then they are already very close. That is, their separation $\Delta \theta_{\text{now}}$ is

$$\Delta \theta_{\text{now}} = 0''5 \frac{v_\perp}{30 \text{ km s}^{-1}} \frac{T_{\text{event}} - T_{\text{now}}}{8 \text{ yrs}} \left( \frac{r}{100 \text{ pc}} \right)^{-1}. \quad (26)$$

Thus, it would be difficult to conduct a large-scale survey for such pairs.

Fortunately, the US Naval Observatory is soon planning to release an all-sky proper motion survey, USNO-B (D. Monet 1999, private communication). Even so, the identification of pairs is not trivial, and becomes more difficult both for fainter sources and larger lens distances (or lower proper motions).

### 4.1. Unblemished Survey Data

I begin by considering the problem of candidate pair identification when the proper-motion survey data conform to “typical” specifications. In fact, the underlying data sets are heterogeneous, with substantially longer baselines in the north than the south. For definiteness, I will consider proper motions derived from 2 epochs, one in 1955 and the other in 1990. This baseline is appropriate for the northern hemisphere. The anticipated proper-motion error is $4 \text{ mas yr}^{-1}$, corresponding to 100 mas errors in each position measurement. This implies an error of about 120 mas in the predicted positions of the source and the lens in 2008, or 170 mas error in their relative position. (Generally, only the error in one direction – that of the impact parameter – comes into play.) Is this good enough? Let us suppose that all pairs requiring $T < T_{\text{max}} = 200 \text{ hours}$ are to be observed. From equation (19) this corresponds to $N \sim 6$. For $\gamma = 10$, equation (2) implies $\alpha \leq 115 \mu\text{as} \times 10^{0.2(V_s - 17)}$, and so from equation (1)

$$\beta_{\text{max}} = 700 \text{ mas} \frac{M}{M_\odot} \left( \frac{r}{100 \text{ pc}} \right)^{-1} 10^{0.2(V_s - 17)}. \quad (27)$$

Hence, for $M \sim M_\odot$, $\beta_{\text{max}}$ is greater than 170 mas even for $r = 200 \text{ pc}$ or $V_s = 18$. Thus while it would still be necessary to do additional astrometry in order to prepare for the observations, relatively few candidates would be rejected by this astrometry. By contrast, for $M = 0.2 \times M_\odot$, $r = 200 \text{ pc}$, and $V_s = 18$, $\beta_{\text{max}} \sim 30 \text{ mas}$. In this case, it would be necessary to examine about 6 candidates drawn from the proper-motion survey to find one suitable for a mass measurement.
In the southern hemisphere, the baselines are shorter, so the proper-motion errors are about twice as big. Hence, about twice as many candidates need to be examined.

4.2. Compromised second epoch

If the time of closest approach is 2008, then at the time of the second epoch of the proper motion survey, say 1990, a typical foreground star will be separated from the line of sight to the source by of order 100 AU. This corresponds to $1''$ at $r = 100$ pc. As discussed in § 3.1 in the analysis of Figure [1], lens candidates tend to be moving faster than the population as a whole, so the actual typical separation will be somewhat larger. Nevertheless, these values are close to the resolution limit of the surveys and become even less favorable at greater distances. In addition, bright lens stars will entirely blot out a substantial region around them on the survey plates, preventing the detection of candidate source stars at all. For example, I find that on the Palomer Observatory Sky Survey (POSS), $V = 8$ stars (the approximate completeness limit of the Hipparcos catalog) tend to blot out a region with a $10''$ radius.

However, even the complete loss of the second-epoch positions of candidate sources is not crippling. The proper motion of the bright lens candidate can still be measured, and its position in 2008 predicted. This region can then be examined on the first epoch plates for potential candidates (assuming that the lens is not bright enough to have blotted out this region even at this earlier epoch). Of course, in the intervening $\sim 50$ years, these candidates will have moved, but probably not by very much. For example, at high latitudes ($|b| \gtrsim 20^\circ$), disk sources typically lie at 3 disk scale heights or about $|\csc b| \text{ kpc}$. Hence, in 50 years, they will typically move $300|\sin b|$ mas which even at $b = 90^\circ$ is not much more than the error in the expected position (see § 4.1). Closer to the plane, the motion will be even less. Thus, without a second epoch, more candidates will have to be examined at high latitudes (but these contain a minority of the candidates anyway) and there will be hardly any effect at low latitudes.
4.3. What can be done now?

The USNO-B catalog has not yet been released. However, it is still possible to begin the search for candidates using the Luyton (1979) proper-motion catalog in combination with the USNO-A2.0 all sky astrometric catalog (Monet 1998). The conditions of such a search are fairly well described by § 4.2.

Based on a comparison to Hipparcos data, I.N. Reid (1999 private communication) estimates that the Luyton (1979) proper motions are typically accurate to about 10 mas yr\(^{-1}\), and that the positions are accurate to a few arcseconds. By identifying the Luyton (1979) stars in the USNO-A2.0 survey, one could fix the \(\sim 1955\) positions to \(\sim 100\) mas, and hence the 2008 positions to \(\sim 500\) mas. One could then search the USNO-A2.0 catalog for background stars whose \(\sim 1955\) positions lay along the 2008 path of the Luyton star. As discussed in § 4.2, these stars could be expected to move about \(300|\sin b|\) mas which is generally small compared to the uncertainty in the position of the foreground star. Hence, 2008 source-lens separations could be predicted to \(\sim 0''5\). For pairs that were sufficiently close, the separation could be measured on the POSS II plates to refine the prediction. Follow-up observation could then be made of those pairs surviving this test.

5. Stellar Halo Lenses

Halo stars are about 500 times less common than disk stars (Gould, Flynn, & Bahcall 1998), i.e., \(\rho \sim 6 \times 10^{-5} M_\odot \text{pc}^{-3}\), and they are typically moving about 5 times faster. Let us suppose that they could be spotted to \(r_{\text{max}} = 1\) kpc (see below), and let us take \(N = 4 \times 10^8\) in accord with the discussion of Figure 2. Then, from equation (18), it would be possible to obtain the masses of 5 halo stars with about 150 hours of observation.

At \(r_{\text{max}} = 1\) kpc, it is still appropriate to approximate the density of the stellar halo as uniform. However, it is no longer appropriate to treat the sources as being infinitely far away. As mentioned in § 4.2, at \(b = 90^\circ\), typical disk sources are at 1 kpc. However, the disk sources are farther away at lower latitudes where there are the greatest fraction of sources in any case. Hence, given the level of approximation of the present study, I will ignore this modest correction.
Of course, the radius within 1 kpc contains of order 300 times more stars than the radius within 100 pc, so identifying a relatively complete sample of halo stars seems like a formidable task at first sight. In fact, for stars of fixed color (and approximated as black bodies – or at least as deviating from black bodies in similar ways), we have

\[ t_{\text{cross}} = k_{(B-R)_0} 10^{-0.2R_0} \mu^{-1}, \]  

(28)

where \( k_{(B-R)_0} \) is a constant that depends on the dereddened color, \( R_0 \) is the dereddened magnitude, and \( t_{\text{cross}} \) is the time it takes the star to cross its own radius. For halo stars, \( t_{\text{cross}} \sim 10^3 \) s, which is significantly different from the value for other common classes, \( 2 \times 10^2 \) s for disk white dwarfs, \( 5 \times 10^3 \) s for thick-disk stars, and \( 2 \times 10^4 \) s for main sequence stars. Thus it should not be difficult to find halo star candidates in a proper-motion catalog with colors. The sample will be somewhat contaminated with fast-moving thick-disk stars, but these are also of considerable interest because of their low metallicity.

A more significant problem is that if the survey is limited to \( V \sim 20 \), then at 1 kpc, the bottom of the luminosity function \( M_V > 10 \) is not detectable. These fainter stars contain about half the spheroid mass, implying that the above estimate of the observation time required to measure 5 masses should be multiplied by 4 to about 600 hours. Of course, if one were willing to settle for 3% measurements in place of 1%, this estimate would come down by an order of magnitude.

(29)

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Fig. 1.— Total number of mass measurements $N$ as a function of total required observing time $T_{\text{tot}}$. The solid curve is for $M = 0.1 M_\odot$ and the bold curve is for $M = M_\odot$. The dashed line is the analytic approximation (but without the cutoff) given by eq. (18).

Fig. 2.— Total number of mass measurements $N$ as a function of total required observing time $T_{\text{tot}}$, for mass $M = M_\odot$. The $V_s = 17$ curve (same as in Fig. 1) is shown as a bold dashed line, and the others $V_s = 14, 15, 16, 18, 19$ are shown as solid lines. The curves can be separately identified by noting that the cutoff increases with magnitude. The upper bold line shows the result of combining all of these bins ($14 \leq V_s \leq 19$) while the lower bold line shows the result of combining the four brightest bins ($14 \leq V_s \leq 17$).
