Distinguishable RGE running effects between Dirac neutrinos and Majorana neutrinos with vanishing Majorana CP-violating phases

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Abstract

In a novel parametrization of neutrino mixing and in the approximation of $\tau$-lepton dominance, we show that the one-loop renormalization-group equations (RGEs) of Dirac neutrinos are different from those of Majorana neutrinos even if two Majorana CP-violating phases vanish. As the latter can keep vanishing from the electroweak scale to the typical seesaw scale, it makes sense to distinguish between the RGE running effects of neutrino mixing parameters in Dirac and Majorana cases. The differences are found to be quite large in the minimal supersymmetric standard model with sizable $\tan \beta$, provided the masses of three neutrinos are nearly degenerate or have an inverted hierarchy.

PACS number(s): 14.60.Pq, 13.10.+q, 25.30.Pt

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I. INTRODUCTION

Since 1998, a number of successful neutrino oscillation experiments [1–5] have provided us with very convincing evidence that neutrinos are massive and lepton flavors are mixed. A global fit of current experimental data [6] yields two neutrino mass-squared differences:

$$\Delta m_{\odot}^2 \equiv m_2^2 - m_1^2 = (8.0 \pm 0.3) \times 10^{-5} \text{ eV}^2$$

and

$$\Delta m_{\text{atm}}^2 \equiv m_3^2 - m_2^2 = \pm(2.5 \pm 0.3) \times 10^{-3} \text{ eV}^2.$$ 

The upper bound of every neutrino mass is expected to be $m_i < 0.23 \text{ eV}$ (for $i = 1, 2, 3$) [7].

Three neutrino mixing angles, associated respectively with solar, atmospheric and CHOOZ experiments, are well restricted: $30^\circ \lesssim \theta_{12} \lesssim 38^\circ$, $36^\circ \lesssim \theta_{23} \lesssim 54^\circ$ and $\theta_{13} < 10^\circ$ at the 99% confidence level [6]. But whether neutrinos are Dirac or Majorana particles remains an open question. If the neutrinoless double-beta ($\beta\beta_{0v}$) decay is eventually observed, we shall make sure that neutrinos are Majorana particles. If there is no experimental signal for the $\beta\beta_{0v}$ decay, however, we shall be unable to conclude that neutrinos are just Dirac particles [8]. Although most theorists believe that neutrinos should be Majorana fermions, there do exist some interesting models which treat massive neutrinos as Dirac fermions [9].

Before the nature of neutrinos is ultimately identified by the future neutrino experiments, it is worthwhile to study the phenomenology of both Dirac and Majorana neutrinos.

Since many viable neutrino models are proposed at a superhigh energy scale, one has to take account of radiative corrections to their consequences on lepton flavor mixing and neutrino oscillations at low energy scales. Inversely, one may examine the evolution of those observed neutrino mixing parameters from the electroweak scale $\Lambda_{\text{EW}} \sim 10^2 \text{ GeV}$ up to a superhigh energy scale by using their renormalization-group equations (RGEs). This kind of study is very useful, both for model building itself and for distinguishing different models. It is also useful to reveal the intrinsic differences between Dirac and Majorana neutrinos, as we shall explicitly show in this paper.

A simple extension of the standard model (SM) or the minimal supersymmetric standard model (MSSM) is to introduce very heavy right-handed Majorana neutrinos and keep the Lagrangian of electroweak interactions invariant under $SU(2)_L \times U(1)_Y$ gauge transformation. The smallness of left-handed Majorana neutrino masses can then be explained via the well-known seesaw mechanism [10]. Below the typical seesaw scale $\Lambda_{\text{SS}} \sim 10^{14} \text{ GeV}$, where heavy Majorana neutrinos become decoupled, the effective neutrino coupling matrix $\kappa$ obeys

$$16\pi^2 \frac{d\kappa}{dt} = \alpha_M \kappa + C \left[ \left( Y_i Y_i^\dagger \right) \kappa + \kappa \left( Y_i Y_i^\dagger \right)^T \right]$$

at the one-loop level [11], where $t \equiv \ln(\mu/\Lambda_{\text{SS}})$ with $\mu$ being an arbitrary renormalization scale between $\Lambda_{\text{EW}}$ and $\Lambda_{\text{SS}}$, $Y_i$ is the charged-lepton Yukawa coupling matrix, $C = -1.5$ (SM) or $C = 1$ (MSSM), $\alpha_M \approx -3g_2^2 + 6g_1^2 + \lambda$ (SM) or $\alpha_M \approx -1.2g_1^2 - 6g_2^2 + 6g_1^2$ (MSSM). Here $g_1$ and $g_2$ are the gauge couplings, $y_t$ stands for the top-quark Yukawa coupling, and $\lambda$ denotes the Higgs self-coupling in the SM.

If neutrinos are Dirac particles, their Yukawa coupling matrix $Y_{\nu}$ must be extremely suppressed in magnitude to reproduce the light neutrino masses of $\mathcal{O}(1) \text{ eV}$ or smaller at low energy scales. The running of $Y_{\nu}$ from a superhigh energy scale (e.g., $\Lambda_{\text{SS}}$) down to $\Lambda_{\text{EW}}$ is governed by the one-loop RGE [12]

$$16\pi^2 \frac{d\omega}{dt} = 2\alpha_D \omega + C \left[ \left( Y_i Y_i^\dagger \right) \omega + \omega \left( Y_i Y_i^\dagger \right) \right],$$

where $\omega$ denotes the Higgs self-coupling in the SM.
where $\omega \equiv Y_\nu Y_\nu^T$, $\alpha_D \approx -0.45 g_1^2 - 2.25 g_2^2 + 3 y_t^2$ (SM) or $\alpha_D \approx -0.6 g_1^2 - 3 g_2^2 + 3 y_t^2$ (MSSM). In writing out Eq. (2), we have safely neglected those tiny terms of $\mathcal{O}(\omega^2)$.

One may use Eq. (1) or (2) to derive the explicit RGEs for neutrino mixing parameters in the flavor basis where $Y_\nu$ is diagonal and real (positive). In this basis, we have $\kappa = Y_M \vec{\kappa} Y_M^T$ with $\vec{\kappa} = \text{Diag}\{\kappa_1, \kappa_2, \kappa_3\}$ for Majorana neutrinos; or $\omega = Y_D \vec{\omega} Y_D^T$ with $\vec{\omega} = \text{Diag}\{y_1^2, y_2^2, y_3^2\}$ for Dirac neutrinos. $Y_M$ or $Y_D$ is just the lepton flavor mixing matrix. The unitarity violation of $Y_M$ is extremely tiny and can safely be neglected in all realistic seesaw models [13]. At $\Lambda_{EW}$, Majorana neutrino masses are $m_i = v^2 \kappa_i$ (SM) or $m_i = v^2 \kappa_i \sin^2 \beta$ (MSSM) and Dirac neutrino masses are $m_i = v y_i$ (SM) or $m_i = v y_i \sin \beta$ (MSSM) with $v \approx 174$ GeV. A general parametrization of $Y_M$ or $Y_D$ is $Y_M = Q_M U P_M$ or $Y_D = Q_D U P_D$, where $P_M$ (or $P_D$) and $Q_M$ (or $Q_D$) are two diagonal phase matrices, and $U$ is a unitary matrix containing three mixing angles and one CP-violating phase. Note that $P_M$ and $Q_D$ have no physical significance, but $P_M = \text{Diag}\{e^{i\phi}, e^{i\sigma}, 1\}$ consists of two non-trivial (physical) phases which are commonly referred to as the Majorana CP-violating phases. A novel parametrization of $U$ is [14]

$$U = \begin{pmatrix} s_t s_c c + c_t c_v e^{-i\phi} & s_t c_v c - c_t s_v e^{-i\phi} & s_t s \cr s_t c_c c - c_t s_v e^{-i\phi} & c_t c_v c + s_t s_v e^{-i\phi} & c_t s \cr -s_v s & -c_v s & c \end{pmatrix},$$

(3)

where $c_t \equiv \cos \theta_t$, $s_v \equiv \sin \theta_v$, $c \equiv \cos \theta$, and so on. This parametrization, together with the approximation of $\tau$-lepton dominance (i.e., $Y_{\nu} Y_{\nu}^T \approx \text{Diag}\{0, 0, y_\tau^2\}$ in view of $y_\tau^2 \ll y_\mu^2 \ll y_\nu^2$), allows us to obtain the following RGEs for two Majorana phases $\rho$ and $\sigma$ [12]:

$$\dot{\rho} = \frac{C y_\tau^2}{16 \pi^2} \left[ \tilde{\zeta}_{12} c_\rho s_\rho c_{(\sigma - \rho)} s_{(\sigma - \rho)} \tilde{\zeta}_{13} \left( c_\rho s_\rho c_{(\sigma - \rho)} s_{(\sigma - \rho)} - 2 c_\rho c_\sigma s_\sigma c_\sigma s_\sigma \right) \right],$$

$$\dot{\sigma} = \frac{C y_\tau^2}{16 \pi^2} \left[ \tilde{\zeta}_{12} c_\rho s_\rho c_{(\sigma - \rho)} s_{(\sigma - \rho)} \tilde{\zeta}_{13} \left( c_\rho s_\rho c_{(\sigma - \rho)} s_{(\sigma - \rho)} - 2 c_\rho c_\sigma s_\sigma c_\sigma s_\sigma \right) \right],$$

(4)

where $\dot{\rho} \equiv \lim_{\Delta \to 0} \frac{\rho(\Delta \tau) - \rho(\tau)}{\Delta \tau}$, $\dot{\sigma} \equiv \lim_{\Delta \to 0} \frac{\sigma(\Delta \tau) - \sigma(\tau)}{\Delta \tau}$, $\zeta_{ij} \equiv 4 \kappa_i \kappa_j / (\kappa_i^2 - \kappa_j^2)$, $c_a \equiv \cos a$ and $s_a \equiv \sin a$ (for $a = \rho, \sigma$ or $\sigma - \rho$). A particularly interesting feature of Eq. (4) is that $\rho = \sigma = 0$ at a specific energy scale leads to $\dot{\rho} = \dot{\sigma} = 0$, implying that $\rho$ and $\sigma$ can keep vanishing at any energy scales between $\Lambda_{EW}$ and $\Lambda_{SS}$. In this case, only three mixing angles $(\theta_1, \theta_\mu, \theta)$ and the so-called Dirac CP-violating phase $\phi$ undergo the RGE evolution. Note that a kind of underlying flavor symmetry may actually forbid two Majorana phases to take non-zero values in a concrete neutrino model. It is therefore meaningful to ask whether the RGE running behaviors of Majorana neutrinos with $\rho = \sigma = 0$ are identical to those of Dirac neutrinos. The purpose of this paper is just to answer such a question.

In section II, we show that the one-loop RGEs of Majorana neutrinos with $\rho = \sigma = 0$ are analytically similar to those of Dirac neutrinos, but their expressions are not exactly identical. Section III is devoted to a numerical analysis of the RGE running behaviors of three mixing angles and the Dirac CP-violating phase, and to a careful comparison between the cases of Dirac and Majorana neutrinos. Four different neutrino mass spectra are taken into account in our calculations. We find that the differences between Dirac and Majorana neutrinos in their RGE running effects can be quite large in the MSSM with sizable $\tan \beta$, provided the masses of three neutrinos are nearly degenerate or have an inverted hierarchy. A brief summary of the main results is given in section IV.
II. RGES OF DIRAC AND MAJORANA NEUTRINOS

The one-loop RGEs for three Yukawa coupling eigenvalues of Dirac neutrinos \( y_i \) with \( i = 1, 2, 3 \) and their four flavor mixing parameters \( (\theta_i, \theta_{i'} , \theta, \rho \) and \( \phi \) have been derived in Ref. [12]. Here we replace \( y_i \) by \( m_i \). The RGEs of three neutrino masses, three mixing angles and one CP-violating phase can then be written as

\[
\begin{align*}
\dot{m}_1 &= \frac{m_1}{16\pi^2} \left( \alpha_D + Cy_r^2 s^2 s^2 \right), \\
\dot{m}_2 &= \frac{m_2}{16\pi^2} \left( \alpha_D + Cy_r^2 c^2 s^2 \right), \\
\dot{m}_3 &= \frac{m_3}{16\pi^2} \left( \alpha_D + Cy_r^2 c^2 \right); \\
\end{align*}
\]

and

\[
\begin{align*}
\dot{\theta}_l &= \frac{Cy_r^2}{8\pi^2} c_{\nu} s_{\nu} c_{\phi} m_3 (m_2^2 - m_1^2) \left( \frac{m_3^2 - m_1^2}{(m_2^2 - m_1^2)} \right), \\
\dot{\theta}_\nu &= \frac{Cy_r^2}{16\pi^2} c_{\nu} s_{\nu} \left[ s_{\tau}^2 m_2^2 + m_1^2 - c_{\tau}^2 2m_3^2 (m_2^2 - m_1^2) \right], \\
\dot{\theta} &= \frac{Cy_r^2}{16\pi^2} c_{\nu} s_{\nu} \left[ s_{\tau}^2 m_3^2 + m_1^2 + c_{\tau}^2 2m_3^2 (m_2^2 - m_1^2) \right], \\
\dot{\phi} &= \frac{Cy_r^2}{8\pi^2} \left( c_{\tau}^2 - s_{\tau}^2 \right) c_i^{-1} s_i^{-1} c_{\nu} s_{\nu} \cos \phi \left( m_3^2 (m_2^2 - m_1^2) \right) \left( \frac{m_3^2 - m_1^2}{(m_3^2 - m_2^2)} \right),
\end{align*}
\]

where \( c_{\phi} \equiv \cos \phi \) and \( s_{\phi} \equiv \sin \phi \). Note that the neutrino mass-squared differences \( m_3^2 - m_1^2 \equiv \Delta m_{31}^2 \) and \( m_2^2 - m_1^2 \equiv \Delta m_{21}^2 \) are much larger in magnitude than \( m_2^2 - m_1^2 \equiv \Delta m_{32}^2 \), as indicated by current experimental data. Typically, \( \Delta m_{21}^2 \approx 8.0 \times 10^{-5} \text{ eV}^2 \) and \( \left| \Delta m_{31}^2 \right| \approx \left| \Delta m_{32}^2 \right| \approx 2.5 \times 10^{-3} \text{ eV}^2 \) [6]. Among three neutrino mixing angles, the RGE running of \( \theta_\nu \) is expected to be most significant. The CP-violating phase \( \phi \) may significantly evolve from one energy scale to another, if \( \theta_l \) takes sufficiently small values. These qualitative features will become clearer in our subsequent numerical calculations.

The one-loop RGEs for three effective coupling eigenvalues of Majorana neutrinos \( \kappa_i \) with \( i = 1, 2, 3 \) and their six flavor mixing parameters \( (\theta_i, \theta_{i'}, \theta, \rho, \sigma) \) can be found in Ref. [12]. Here we replace \( \kappa_i \) by \( m_i \) and take \( \rho = \sigma = 0 \) at either \( \Lambda_{\text{EW}} \) or \( \Lambda_{\text{SS}} \). As pointed out in section I, Eq. (4) assures two Majorana phase \( \rho \) and \( \sigma \) to keep vanishing at any energy scales between \( \Lambda_{\text{EW}} \) and \( \Lambda_{\text{SS}} \). One may safely simplify the RGEs of \( \theta_i, \theta_{i'}, \theta, \rho \) and \( \phi \) obtained in Ref. [12] for Majorana neutrinos by setting \( \rho = \sigma = 0 \), and then compare them with their Dirac counterparts on the same footing. In this case, we arrive at

\[
\begin{align*}
\dot{m}_1 &= \frac{m_1}{16\pi^2} \left( \alpha_M + 2Cy_r^2 s^2 s^2 \right),
\dot{m}_2 &= \frac{m_2}{16\pi^2} \left( \alpha_M + 2Cy_r^2 c^2 s^2 \right),
\dot{m}_3 &= \frac{m_3}{16\pi^2} \left( \alpha_M + 2Cy_r^2 c^2 \right);
\end{align*}
\]

and

\[
\begin{align*}
\dot{\theta}_l &= \frac{Cy_r^2}{8\pi^2} c_{\nu} s_{\nu} c_{\phi} m_3 (m_2^2 - m_1^2) \left( \frac{m_3^2 - m_1^2}{(m_2^2 - m_1^2)} \right),
\dot{\theta}_\nu &= \frac{Cy_r^2}{16\pi^2} c_{\nu} s_{\nu} \left[ s_{\tau}^2 m_2^2 + m_1^2 - c_{\tau}^2 2m_3^2 (m_2^2 - m_1^2) \right],
\dot{\theta} &= \frac{Cy_r^2}{16\pi^2} c_{\nu} s_{\nu} \left[ s_{\tau}^2 m_3^2 + m_1^2 + c_{\tau}^2 2m_3^2 (m_2^2 - m_1^2) \right],
\dot{\phi} &= \frac{Cy_r^2}{8\pi^2} \left( c_{\tau}^2 - s_{\tau}^2 \right) c_i^{-1} s_i^{-1} c_{\nu} s_{\nu} \cos \phi \left( m_3^2 (m_2^2 - m_1^2) \right) \left( \frac{m_3^2 - m_1^2}{(m_3^2 - m_2^2)} \right),
\end{align*}
\]
\[ \dot{\theta}_l = \frac{C y^2_{\tau}}{8 \pi^2} c_\nu s_\nu C c_\phi \frac{m_3 (m_2 - m_1)}{(m_3 - m_1) (m_3 - m_2)} , \]
\[ \dot{\theta}_\nu = - \frac{C y^2_{\tau}}{16 \pi^2} c_\nu s_\nu \left[ s^2 m_2 + m_1 \right] \left( s^2 m_3 + m_2 \right) \left( c^2 \frac{2 m_3 (m_2 - m_1)}{(m_3 - m_1) (m_3 - m_2)} \right) , \]
\[ \dot{\phi} = - \frac{C y^2_{\tau}}{8 \pi^2} \left( c^2 - s^2 \right) \left( c^2 - s^2 \right) \frac{m_3 (m_2 - m_1)}{(m_3 - m_1) (m_3 - m_2)} . \] (8)

As a consequence of \( \Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \), the mixing angle \( \theta_\nu \) is most sensitive to radiative corrections. The RGE evolution of the CP-violating phase \( \phi \) depends strongly on the smallness of \( \theta_l \), on the other hand. These qualitative features are essentially analogous to what we have pointed out for Dirac neutrinos.

It is interesting to note that Eq. (7) can actually be obtained from Eq. (5) with the replacements \( \alpha_D \rightarrow \alpha_M \) and \( C \rightarrow 2C \), while Eq. (8) can be achieved from Eq. (6) with the replacements \( m_i^2 \rightarrow m_i \) (for \( i = 1, 2, 3 \)). These similarities and differences imply that it is very non-trivial to distinguish between the RGE running behaviors of Dirac neutrinos and Majorana neutrinos with vanishing Majorana CP-violating phases.

A rephasing-invariant description of leptonic CP violation in neutrino oscillations is to make use of the Jarlskog parameter \( J \) [15]. It explicitly reads as \( J = c_\nu s_\nu c_\phi s_\phi \) in the parametrization taken in Eq. (3) for either Dirac or Majorana neutrinos. The one-loop RGE of \( J \) has been derived in Ref. [12]. Taking \( \rho = \sigma = 0 \) in the Majorana case, we obtain a simplified expression of \( J^M \),
\[ J^M = \frac{C y^2_{\tau}}{16 \pi^2} J^M \left[ \frac{c^2 - s^2}{s^2} \frac{2 m_2 + m_1}{m_2 - m_1} + \left( c^2 - s^2 \right) \frac{m_3 + m_1}{m_3 - m_1} + \left( c^2 - s^2 \right) \frac{m_3 + m_2}{m_3 - m_2} \right] , \] (9)
which is very analogous to \( J^D \) of Dirac neutrinos,
\[ J^D = \frac{C y^2_{\tau}}{16 \pi^2} J^D \left[ \frac{c^2 - s^2}{s^2} \frac{2 m_2 + m_1}{m_2 - m_1} + \left( c^2 - s^2 \right) \frac{m_3 + m_1}{m_3 - m_1} + \left( c^2 - s^2 \right) \frac{m_3 + m_2}{m_3 - m_2} \right] . \] (10)

It is obvious that Eq. (10) can be obtained from Eq. (9) with the replacements \( m_i \rightarrow m_i^2 \) (for \( i = 1, 2, 3 \)). Note that \( J^D \propto J^D \) (or \( J^M \propto J^M \)) holds. This result implies that the Jarlskog parameter will keep vanishing at any energy scales between \( \Lambda_{EW} \) and \( \Lambda_{SS} \), if it initially vanishes at either \( \Lambda_{EW} \) or \( \Lambda_{SS} \).

Two comments are in order. First, three mixing angles \( \theta_l, \theta_\nu \) and \( \theta \) in our parametrization can simply be related to \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) in the standard parametrization advocated by the Particle Data Group [16]. The relations are [17]
\[ \theta_{12} \approx \theta_\nu , \quad \theta_{23} \approx \theta , \quad \theta_{13} \approx \theta \sin \theta \] (11)
in the leading-order approximation. Thus \( 30^\circ \lesssim \theta_\nu \lesssim 38^\circ , \ 36^\circ \lesssim \theta \lesssim 54^\circ \) and \( \theta_l < 17^\circ \) are expected to hold, as indicated by the global fit done in Ref. [6]. Second, the approximation of \( \tau \)-lepton dominance used in our analytical calculations is very reliable. In other words, the contributions of \( y_e^2 \) and \( y_\mu^2 \) to all of the RGEs listed above are negligibly small. This observation has been verified by our numerical calculations, in which there is no special assumption or approximation.
III. NUMERICAL ILLUSTRATION AND COMPARISON

In view of the fact that the absolute mass scale of three light neutrinos and the sign of ∆m_{32} remain unknown at present, let us consider four typical patterns of the neutrino mass spectrum:

- Normal hierarchy (NH): m_1 < m_2 < m_3. For simplicity, we typically take m_1 = 0 at Λ_{EW} in our numerical calculations. Then m_2 = √(Δm_{21}^2) and m_3 = √(|Δm_{32}^2| + Δm_{21}^2) can be determined from current experimental data.

- Inverted hierarchy (IH): m_3 < m_1 < m_2. For simplicity, we typically take m_3 = 0 at Λ_{EW} in our numerical calculations. Then m_2 = √|Δm_{32}^2| and m_1 = √(|Δm_{32}^2| - Δm_{21}^2) can be determined from current experimental data.

- Near degeneracy (ND) with Δm_{32} > 0: m_1 ≲ m_2 ≲ m_3. For simplicity, we typically take m_1 = 0.2 eV at Λ_{EW} in our numerical calculations.

- Near degeneracy (ND) with Δm_{32} < 0: m_3 ≲ m_1 ≲ m_2. For simplicity, we typically take m_1 = 0.2 eV at Λ_{EW} in our numerical calculations.

In addition, we take Δm_{32} spectrum:

Note that (7) becomes important only when tan β R behaves of M the neutrino mass spectrum. We observe that α governed by neutrinos are twice as large as those for Dirac neutrinos.

Energy scale µ = 0 particular when the energy scale µ that R tan β µ distinguishable at the scales R

Near degeneracy (ND) with ∆m_{32} > 0: m_1 ≲ m_2 ≲ m_3. For simplicity, we typically take m_1 = 0.2 eV at Λ_{EW} in our numerical calculations.

In the Dirac case, and their discrepancy can be as large as 0.7 at i = 180 GeV (the Higgs mass) have typically been input. Since the running of m_i is governed by α_D or α_M, R_1 ≈ R_2 ≈ R_3 holds to a high degree of accuracy. Furthermore, the behaviors of R_i are actually independent of the initial value of m_1 and possible patterns of the neutrino mass spectrum. We observe that R_i in the Majorana case is always larger than R_i in the Dirac case, and their discrepancy can be as large as 0.7 at µ = Λ_{SS} ≈ 10^{14} GeV.

The first plot in Fig. 1 illustrates the ratios R_i ≡ m_i(µ)/m_i(M_Z) changing with the energy scale µ in the SM for Dirac and Majorana neutrinos, where m_i(M_Z) = 0.2 eV and M_H = 180 GeV (the Higgs mass) have typically been input. Since the running of m_i is governed by α_D or α_M, R_1 ≈ R_2 ≈ R_3 holds to a high degree of accuracy. Furthermore, the behaviors of R_i are actually independent of the initial value of m_1 and possible patterns of the neutrino mass spectrum. We observe that R_i in the Majorana case is always larger than R_i in the Dirac case, and their discrepancy can be as large as 0.7 at µ = Λ_{SS} ≈ 10^{14} GeV.

The relation R_1 ≈ R_2 ≈ R_3 is also a very good approximation in the MSSM with small tan β, as shown by the second plot in Fig. 1, where tan β = 10 has been input. It is clear that R_i in the Dirac case is numerically distinguishable from R_i in the Majorana case, in particular when the energy scale µ far exceeds M_Z.

If tan β is sufficiently large, the common scaling of three neutrino masses in the RGE evolution will fail [18]. The splitting of R_1, R_2 and R_3, which increases with the energy scale µ, is illustrated by the third plot in Fig. 1 with the input tan β = 50. One can see that R_i in the Dirac case is always smaller than R_i in the Majorana case, and their discrepancy is distinguishable at the scales µ ≫ M_Z.

A. Neutrino masses

In either the SM or the MSSM with small tan β, the RGE running behaviors of three neutrino masses are dominated by α_D or α_M. The y^2-associated term of m_i^2 in Eq. (5) or (7) becomes important only when tan β takes sufficiently large values in the MSSM [18]. Note that α_M = 2α_D holds in the MSSM, in which the running effects of m_i for Majorana neutrinos are twice as large as those for Dirac neutrinos.

The first plot in Fig. 1 illustrates the ratios R_i ≡ m_i(µ)/m_i(M_Z) changing with the energy scale µ in the SM for Dirac and Majorana neutrinos, where m_i(M_Z) = 0.2 eV and M_H = 180 GeV (the Higgs mass) have typically been input. Since the running of m_i is governed by α_D or α_M, R_1 ≈ R_2 ≈ R_3 holds to a high degree of accuracy. Furthermore, the behaviors of R_i are actually independent of the initial value of m_1 and possible patterns of the neutrino mass spectrum. We observe that R_i in the Majorana case is always larger than R_i in the Dirac case, and their discrepancy can be as large as 0.7 at µ ≈ Λ_{SS} ≈ 10^{14} GeV.

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B. Neutrino mixing parameters

Radiative corrections to three neutrino mixing angles, the Dirac CP-violating phase and the Jarlskog parameter are all controlled by the \( \tau \)-lepton Yukawa coupling eigenvalue \( y_\tau \). Because of \( y^2_\tau/(8\pi^2) \approx 1.3 \times 10^{-6} \) (SM) or \( y^2_\tau/8(\pi^2) \approx 1.3 \times 10^{-6} (1 + \tan^2 \beta) \) (MSSM) at \( M_Z \), significant RGE running effects are expected to appear in the MSSM case when \( \tan \beta \) is sufficiently large [19]. To illustrate, here we simply concentrate on the MSSM with \( \tan \beta = 50 \) and consider four typical patterns of the neutrino mass spectrum in our subsequent discussions and calculations.

(1) In the NH case with \( m_1 = 0 \), the RGEs of \( \theta_1, \theta_\nu, \theta, \phi \) and \( J \) can be simplified as

\[
\begin{align*}
\dot{\theta}_1 &= + \frac{Cy^2_\tau}{16\pi^2} c_\nu s_\nu c_\phi r, \\
\dot{\theta}_\nu &= - \frac{Cy^2_\tau}{16\pi^2} c_\nu s_\nu \left( s^2 - c^2 r \right), \\
\dot{\theta} &= - \frac{Cy^2_\tau}{16\pi^2} c \left( 1 + r \right), \\
\dot{\phi} &= - \frac{Cy^2_\tau}{16\pi^2} \left( c_f^2 - s_f^2 \right) c r \left( s_\phi - c_\phi c_\theta \right), \\
\dot{J} &= \frac{Cy^2_\tau}{16\pi^2} J \left[ 2 \left( s^2 c^2 - c^2 \right) - \left( c^2 - c_f^2 s^2 \right) r \right],
\end{align*}
\]

(12)

in which \( r = 2m_2/(m_3 - m_2) \) for Dirac neutrinos or \( r = 2m_2/(m_3 - m_2) \) for Majorana neutrinos. Current experimental data yield \( r_D \approx 0.06 \) and \( r_M \approx 0.4 \). Both of them are too small to compensate for the strong suppression induced by \( y^2_\tau \) in Eq. (12). Thus the RGE corrections to those flavor mixing and CP-violating parameters are not important in the NH case. Note, however, that the radiative correction to \( \phi \) can be very significant when \( \theta_1 \) is extremely small or becomes vanishing. We find that \( \phi \) quickly approaches its quasi-fixed point \( \phi_{QF} = 0 \) or \( \pi \) in the \( \theta_1 \to 0 \) limit, an interesting phenomenon which is remarkably different from the non-trivial quasi-fixed point of \( \phi \) discovered in the general \( (\rho \neq \sigma \neq 0) \) case for Majorana neutrinos [20]. One can also see that both \( J = 0 \) and \( \dot{J} = 0 \) hold when \( \theta_1 \) vanishes; i.e., CP is a good symmetry in this limit.

(2) In the IH case with \( m_3 = 0 \), we arrive at

\[
\begin{align*}
\dot{\theta}_1 &= \dot{\phi} = 0, \\
\dot{\theta}_\nu &= - \frac{Cy^2_\tau}{16\pi^2} c_\nu s_\nu s^2 r', \\
\dot{\theta} &= + \frac{Cy^2_\tau}{16\pi^2} c s, \\
\dot{J} &= \frac{Cy^2_\tau}{16\pi^2} J \left[ 3c^2 - 1 + \left( s^2 - c^2 \right) s^2 r' \right],
\end{align*}
\]

(13)

where \( r' = (m^2_2 + m^2_1)/(m^2_2 - m^2_1) \) for Dirac neutrinos or \( r' = (m_2 + m_1)/(m_2 - m_1) \) for Majorana neutrinos. We observe that radiative corrections to \( \theta_1 \) and \( \phi \) are vanishingly small, and the correction to \( \theta \) is also insignificant. Nevertheless, the RGE running effects of \( \theta_\nu \) and \( J \) may get enhanced by \( r' \), whose typical value reads \( r'_D \approx 60 \) or \( r'_M \approx 120 \) at \( M_Z \). Fig. 2 illustrates the evolution of \( \theta_\nu \) and \( J \) in the IH case. The discrepancy between Dirac and Majorana cases is obviously distinguishable for both parameters, when the energy scale is much higher than \( M_Z \). In particular, \( J^D \sim 2J^M \) holds at \( \mu \sim \Lambda_{SSS} \), because the corresponding value of \( \theta_\nu \) for Majorana neutrinos is roughly half of that for Dirac neutrinos.

(3) In the ND case with \( \Delta m^2_{32} > 0 \) and \( m_1 = 0.2 \) eV, the RGE corrections to those neutrino mixing parameters can significantly be enhanced by the ratios \( (m^2_2 + m^2_1)/(m^2_2 - m^2_1) \) in Eqs. (6) and (10) for Dirac neutrinos, or by the ratios \( (m_2 + m_3)/(m_2 - m_3) \) in Eqs. (8)
and (9) for Majorana neutrinos. We illustrate the typical evolution behaviors of $\theta_l$, $\theta_\nu$, $\theta$ and $\phi$ in Fig. 3. One can see that Majorana neutrinos undergo the RGE corrections more significantly than Dirac neutrinos. The discrepancy between these two cases is about $10^\circ$ for either $\theta$ or $\phi$ at $\mu \gg M_Z$. It is therefore possible to distinguish the running of Majorana neutrinos from that of Dirac neutrinos. The difference between $J^D$ and $J^M$ is insignificant even at $\mu \sim \Lambda_{SS}$, as shown in Fig. 4, partly because the increase (or decrease) of $\theta$ can somehow compensate for the decrease (or increase) of $\theta$ and $\phi$ in the Majorana (or Dirac) case.

(4) In the ND case with $\Delta m^2_{32} < 0$ and $m_1 = 0.2$ eV, we get similar enhancements in the RGEs of those neutrino mixing parameters induced by the ratios $(m_i^2 + m_j^2)/(m_i^2 - m_j^2)$ for Dirac neutrinos, or by $(m_i + m_j)/(m_i - m_j)$ for Majorana neutrinos. However, only the running of $\theta$ is sensitive to the sign flip of $\Delta m^2_{32}$, as one can see from Eqs. (6) and (8)–(10), in which $\dot{\theta}_l$ and $\dot{\mathcal{J}}$ are dominated by the term proportional to $(m_i^2 + m_j^2)/(m_i^2 - m_j^2)$ (Dirac) or $(m_2 + m_1)/(m_2 - m_1)$ (Majorana). Then the numerical results for $\theta_l$, $\theta_\nu$, $\phi$ and $\mathcal{J}$ in the present case are very similar to those in the ND case with $\Delta m^2_{32} > 0$. For simplicity, we only illustrate the evolution of $\theta$ in Fig. 5 by taking $\Delta m^2_{32} < 0$. It is obvious that the running behavior of $\theta$ for either Dirac or Majorana neutrinos in Fig. 5 is essentially opposite (or complementary) to that in Fig. 3, just due to the sign flip of $\Delta m^2_{32}$.

IV. SUMMARY

The main goal of this paper is to examine whether the RGE running behaviors of Majorana neutrinos are still different from those of Dirac neutrinos, if two Majorana CP-violating phases vanish at a given energy scale. For this purpose, it is essential to choose a suitable parametrization of the $3 \times 3$ lepton flavor mixing matrix, such that its two Majorana phases keep vanishing in the RGE evolution from one scale to another. We have pointed out that the novel parametrization used in Ref. [12], which consists of the mixing angles $(\theta_l, \theta_\nu, \theta)$ and the CP-violating phases $(\phi, \rho, \sigma)$, does fulfill this requirement. Taking $\rho = \sigma = 0$ at the electroweak scale, we have carefully compared the similarities and differences between the RGEs of $\theta_l$, $\theta_\nu$, $\theta$ and $\phi$ for Majorana neutrinos and those for Dirac neutrinos. Our numerical calculations show that it is possible to distinguish between these two cases in the MSSM with sizable $\tan \beta$, in particular when the masses of three neutrinos are nearly degenerate or have an inverted hierarchy.

Of course, the numerical examples presented in this work are mainly for the purpose of illustration. The point is that the nature of neutrinos determines their RGE running behaviors, and the latter may be crucial for building a realistic neutrino model. We expect that our analysis can not only complement those previous studies of radiative corrections to the physical parameters of Dirac and Majorana neutrinos, but also help us understand the dynamical role of Majorana phases in a more general picture of flavor physics.

ACKNOWLEDGMENTS

This work is supported in part by the National Nature Science Foundation of China.
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FIG. 1. The running neutrino mass ratios $R_i = m_i(\mu)/m_i(M_Z)$ (for $i = 1, 2, 3$), where the dashed and solid curves stand respectively for the Dirac and Majorana cases.
FIG. 2. The running behaviors of $\theta_{\nu}$ and $J$ in the IH case with $\tan \beta = 50$ and $m_3 = 0$ at $M_Z$ within the MSSM, where the dashed and solid curves stand respectively for the Dirac and Majorana cases, and $J^D(M_Z) = J^M(M_Z) \approx 0.0014$.

FIG. 3. The running behaviors of $\theta_{t}$, $\theta_{\nu}$, $\theta$ and $\phi$ in the ND case with $\Delta m_{32}^2 > 0$, $\tan \beta = 50$ and $m_1(M_Z) = 0.2$ eV within the MSSM, where the dashed and solid curves stand respectively for the Dirac and Majorana cases.
FIG. 4. The running behavior of \( J \) in the ND case with \( \Delta m_{32}^2 > 0 \), \( \tan \beta = 50 \) and \( m_1(M_Z) = 0.2 \text{ eV} \) within the MSSM, where the dashed and solid curves stand respectively for the Dirac and Majorana cases, and \( J^D(M_Z) = J^M(M_Z) \approx 0.0014 \).

FIG. 5. The running behavior of \( \theta \) in the ND case with \( \Delta m_{32}^2 < 0 \), \( \tan \beta = 50 \) and \( m_1(M_Z) = 0.2 \text{ eV} \) within the MSSM, where the dashed and solid curves stand respectively for the Dirac and Majorana cases.