Threshold effects in open-string theory

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ABSTRACT

We analyze the one-loop effective gauge-field action in $Z_2$-orbifold compactifications of type-I theory. We show how, for non-abelian group factors, the threshold effects are ultraviolet finite though given entirely by a six-dimensional field theory expression.
1. Introduction

Threshold effects in heterotic string theory [1, 2, 3] have been studied intensively in the past, both in relation to gauge coupling unification [4], and in the context of 4d N=2 heterotic-type II duality [5, 6, 7]. In open string theory, on the other hand, they only now start to receive attention [8]. This is in part due to the fact that exact threshold calculations are usually done in orbifold limits, and that the rules of orbifold compactifications for type-I theory have proven much harder to elucidate [9, 10, 11, 12]. In this paper we will analyze the full one-loop Lagrangian for slowly-varying gauge-field strengths in the Z₂-orbifold models constructed recently by Gimon and Polchinski [11]. These models have N=1 supersymmetry in six dimensions, and a maximal gauge group \( G = U(16) \times \tilde{U}(16) \), which can be broken by both discrete and continuous (antisymmetric) moduli. Upon toroidal compactification to four dimensions, one finds N=2 supersymmetries and extra (adjoint) Wilson-line moduli. In calculating the effective gauge-field action we will follow reference [13], where a similar calculation was carried out for toroidal compactifications of the type-I SO(32) superstring. Our conclusions can be summarized as follows:

(a) the structure of ultraviolet divergences is identical in the toroidal and orbifold models, and can be traced to tadpoles of the dilaton and graviton, provided the background field has no component along one of the simple U(1) factors in six-dimensions;
(b) gauge coupling renormalization is entirely due to six-dimensional field-theory states, but is ultraviolet finite due to the (unconventional) cutoff prescription dictated by the string.

To understand the above results heuristically, note that the divergent parts of one-loop string amplitudes are proportional to the square of closed-string tadpoles, which in a consistent theory are in turn \( o(F_{\mu\nu}^2) \) each. The only possible exception to this rule comes from Green-Schwarz couplings of twisted closed-string states, required to cancel the anomalies of simple U(1) gauge-group factors [15], and giving rise to tadpoles of \( o(F_{\mu\nu}) \).
traceless background fields the quadratic part of the induced Lagrangian is thus ultraviolet finite. As for the fact that only massless six-dimensional states contribute, it follows from the observation that these are the only open-string BPS states: all other states have the spin content of N=4 multiplets and do not therefore renormalize the gauge coupling \[7\]. This last conclusion has been also reached in a somewhat different context by Douglas and Li [8], though in their D-brane realization of N=2 super Yang-Mills ultraviolet finiteness of the thresholds is not explicit.

The structure of this paper is as follows: in section 2 we will review the one-loop calculation of the gauge action for toroidal compactifications of the SO(32) type-I superstring. In section 3 we will analyze ultraviolet divergences, and rederive in particular the relation between the gauge and gravitational couplings and the string scale, obtained previously by Abe and Sakai [16, 17]. The results of these two sections are standard [18, 19], but we include them as a warm up for the orbifold calculation which follows in section 4. In section 5 we will consider in particular the quadratic piece of the induced action, and show that it can be obtained from 6d field theory with a subtle cutoff prescription. We will conclude with some comments and perspective.

2. Annulus, Möbius strip and Klein bottle

We will first consider toroidal compactifications of the SO(32) type-I theory down to four space-time dimensions, that is on \( R^4 \times T^6 \). Many technical steps are best illustrated in this simple context, even though maximal unbroken supersymmetry implies that the \( \beta \)-functions and threshold corrections vanish. Besides the torus \( \mathcal{T} \), one-loop vacuum diagrams include the annulus \( \mathcal{A} \), the Möbius strip \( \mathcal{M} \) and the Klein bottle \( \mathcal{K} \). These are given

\[ \text{We thank E. Kiritsis for a clarification of this point.} \]
respectively by

\[ A(i, j) = -\frac{1}{2} \int_0^\infty \frac{dt}{t} \text{Str}_{\text{open}}(i, j) e^{-\frac{ix}{2}(k^\mu k_\mu + M^2)} \]

\[ M(i) = -\frac{1}{2} \int_0^\infty \frac{dt}{t} \text{Str}_{\text{open}}(i) \Omega e^{-\frac{ix}{2}(k^\mu k_\mu + M^2)} \]

\[ K = -\frac{1}{2} \int_0^\infty \frac{dt}{t} \text{Str}_{\text{closed}} \Omega e^{-\frac{ix}{2}(k^\mu k_\mu + M^2)} \]

where \( \Omega \) is the world-sheet reflection operator, \( M^2 \) the mass-squared operator in four dimensions, \( (i, j) \) are Chan-Paton labels of the open-string endpoints, and the supertrace includes a sum over energy-momentum,

\[ \text{Str} = V(4) \int \frac{d^4 k}{(2\pi)^4} \left( \sum_{\text{bos}} - \sum_{\text{ferm}} \right) \]

with \( V(4) \) the regulated volume of \( R^4 \). Only open strings with identical endpoint charges contribute to the Möbius trace, as follows also from the fact that the corresponding surface has a single boundary. Likewise, only closed strings with identical left- and right-moving excitations contribute to the Klein-bottle trace.

The expressions of these amplitudes for zero gauge-field strength and arbitrary Wilson-line backgrounds are

\[ A(i, j) = -\frac{i}{2} V(4) \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{\alpha_i + \alpha_j + * \Gamma_6} e^{-\pi t \alpha^2 / 2} \right) Z(it/2) \]

\[ M(i) = \frac{i}{2} V(4) \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{\alpha_i + * \Gamma_6} e^{-\pi t \alpha^2 / 2} \right) Z(it/2 + 1/2) \]

\[ K = -\frac{i}{2} V(4) \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{* \Gamma_6} e^{-\pi t \alpha^2 / 2} \right) Z(2it) \]

where

\[ Z(\tau) = \frac{1}{\eta^{12}(\tau)} \times \sum_{\alpha=2,3,4} \frac{1}{2} s_\alpha \theta_\alpha^4(0|\tau) \]

is the usual open-string oscillator sum, \( s_3 = -s_2 = -s_4 = 1 \) are the GSO projection signs, and we have set

\[ 2\alpha' = 1. \]
The sums inside the large parentheses run over the internal momentum lattice $^\ast \Gamma_6$, shifted from the origin by the Wilson-line backgrounds.Explicitly, $a_i^I$ is the eigenvalue on Chan-Patton state $|i>$ of the constant gauge-field background $A^I$, pointing in the $I$th direction on the torus, and lying in the Cartan subalgebra of SO(32). Besides these shifts, the three expressions in (2.3) differ only in the argument of the modular functions [10]: the argument in the Klein bottle is four times that in the annulus, because closed-string Regge trajectories have a mass spacing twice as large as the corresponding open-string ones. The extra $\pm \frac{1}{2}$ in the argument of the Möbius amplitude takes into account properly the eigenvalues of oscillator excitations under the reflection operator $\Omega$. The sign of $\mathcal{M}$ has been chosen so that the eigenvalue of massless open-string states under $\Omega$ be minus one. Finally notice that the Klein bottle symmetrizes the Neveu-Schwarz Neveu-Schwarz states, and antisymmetrizes the Ramond-Ramond states, as required by space-time supersymmetry.

The presence of a constant electromagnetic background modifies these amplitudes in a simple way [13,20]. We will choose for definiteness a (positive) magnetic field in the $X^1$ direction

$$F_{23} = BQ \ ,$$

(2.5)

with $Q$ a Cartan-subalgebra generator normalized so that $tr_{\text{fund}}Q^2 = \frac{1}{2}$. The net effect [20] of the field on the open-string spectrum is a shift of the oscillator frequencies of the complex coordinate $X_2 + iX_3$ by an amount $\epsilon$, where

$$\pi \epsilon = \arctan(\pi q_i B) + \arctan(\pi q_j B)$$

(2.6)

and $q_i$, $q_j$ are defined as above. For simplicity of notation, the dependence of $\epsilon$ on the endpoint states $i, j$ will be implicit in the sequel. The annulus and Möbius-strip amplitudes are now given by eqs. (2.1) with the replacements

$$k_{\mu}k^\mu \rightarrow -(k_0)^2 + (k_1)^2 + (2n + 1)\epsilon + 2\epsilon \Sigma_{23}$$

$$\text{Str} \rightarrow V^{(4)}(q_i + q_j)B \int \frac{d^2 k}{(2\pi)^2} \sum_n \left( \sum_{\text{bos}} - \sum_{\text{ferm}} \right)$$

(2.7)
Here $\Sigma_{23}$ is the spin operator in the (23) direction, and $n = 0, 1, \ldots$ labels the Landau levels whose degeneracy per unit area is $\frac{(q_i + q_j)B}{2\pi}$. The above formula encodes in particular the fact that all open-string states have the same gyromagnetic ratio $g = 2$. The reflection operator $\Omega$ acts as in the zero-field limit. Performing the supertraces explicitly leads to the following expressions for the amplitudes

$$A(i, j|B) = -\frac{i}{2} V^{(i)} \int_0^{\infty} \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{a_i + a_j + \gamma_6} e^{-\pi t^2/2} \right) \frac{1}{\eta^{12}(\frac{it}{2})} \times$$

$$\times \frac{i}{2} (q_i + q_j) B t \frac{\theta_1'(0|\frac{it}{2})}{\theta_1(\frac{it}{2}|\frac{it}{2})} \sum_{\alpha=2,3,4} \frac{1}{2} s_\alpha \theta_\alpha(i\epsilon t|\frac{it}{2}) \theta_\alpha^3(0|\frac{it}{2})$$

and

$$M(i|B) = \frac{i}{2} V^{(i)} \int_0^{\infty} \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{2a_i + \gamma_6} e^{-\pi t^2/2} \right) \frac{1}{\eta^{12}(\frac{it}{2} + \frac{1}{2})} \times$$

$$\times iB q_i t \frac{\theta_1'(0|\frac{it}{2} + \frac{1}{2})}{\theta_1(\frac{it}{2}|\frac{it}{2} + \frac{1}{2})} \sum_{\alpha=2,3,4} \frac{1}{2} s_\alpha \theta_\alpha(i\epsilon t|\frac{it}{2} + \frac{1}{2}) \theta_\alpha^3(0|\frac{it}{2} + \frac{1}{2})$$

The Klein bottle is of course unmodified. The reader can verify as a check that these amplitudes reduce to (2.4) in the $B = 0$ limit.

The full one-loop free energy reads

$$i\mathcal{F} = \frac{1}{2} (\mathcal{T} + \mathcal{K}) + \frac{1}{2} \sum_{i, j} A(i, j|B) + \frac{1}{2} \sum_i M(i|B) .$$

with $\mathcal{T}$ the contribution of the torus. The $o(B^2)$ Maxwell term vanishes for toroidal compactifications as a result of $N = 4$ supersymmetry. In the form (2.8)-(2.9) of the amplitudes, this follows from the well known identity

$$\sum_\alpha s_\alpha \theta_\alpha^3(\nu|\tau) \theta_\alpha^3(0|\tau) = 0$$

and the fact that $\theta_2$, $\theta_3$ and $\theta_4$ are even functions of their (first) argument. Alternatively, from eqs. (2.1)-(2.7) one obtains the following contribution of
a supermultiplet to $F$:

$$V^{(4)} \int_0^\infty \frac{dt}{t} e^{-\frac{4t}{\pi^2}M^2} \left\{ -\frac{1}{8\pi^4t^2} \text{str}(1) + \frac{\mathcal{B}^2(q_i + q_j)^2}{16\pi^2} \text{str}(\frac{1}{12} - \Sigma^2_{23}) \right.$$ 

$$- \frac{\mathcal{B}^2(q_i^2 + q_j^2 - q_i q_j)}{24\pi^2t^2} \text{str}(1) + o(B^4) \right\}$$

(2.12)

where $\text{str}$ sums over spin states of the supermultiplet. The first two terms in the above expansion are the one-loop field-theory corrections to the vacuum energy and Maxwell action, while the third term is due to the stringy nature of the particles. All of these terms vanish identically for complete $N = 4$ supermultiplets, consistently with the fact that such multiplets do not renormalize the gauge coupling.

### 3. Ultraviolet divergences

In heterotic string theory ultraviolet finiteness at one loop follows from the restriction of the integration over all world-sheet tori to a single fundamental domain. This presupposes conformal invariance, or equivalently the absence of classical tadpoles. Since a background $F_{\mu\nu}$ does give rise to tadpoles for the graviton and dilaton, a complete background-field calculation requires an appropriately curved space-time [3]. This is also true in type-I theory, but massless closed-string tadpoles may now manifest themselves as ultraviolet divergences from small holes, rather than as a violation of conformal invariance in the world-sheet interior. Such divergences should be cancelled by a generalized Fishler-Susskind mechanism [22, 18, 19]. As we will see this only affects the results at $o(B^4)$, so that the threshold calculation can be consistently performed in flat space-time.

In order to study the ultraviolet behaviour of the amplitudes we must rewrite them in terms of the proper time in the (transverse) closed string channel [23]. We normalize this proper time through the closed-string propagator,

$$\Delta_{\text{closed}} = \frac{\pi}{2} \int_0^\infty dl e^{-\frac{\pi}{2} (k^\mu k_\mu + M_{\text{closed}}^2)}$$

(3.1)
for scalar states. The relation between $l$ and the proper time in the direct channel is different for each surface

$$l = \begin{cases} 
1/t & \text{annulus} ; \\
1/4t & \text{M"obius} ; \\
1/4t & \text{Klein bottle} .
\end{cases} \quad (3.2)$$

To study the $l \to \infty$ limit we must use the modular properties of the elliptic functions. For the annulus we use their transformation under $\tau = it/2 \to -1/\tau = 2il$, together with the Poisson resummation formula to find

$$A(i, j|B) = -\frac{i}{32} v^{(4)} v^{(6)}(q_i + q_j) B \int_0^\infty dt \left( \sum_{w \in \Gamma_6} e^{-w^2 i/2 \pi - i(a_i + a_j) \cdot w} \right) \times \eta^{-12} (2il) \frac{\theta_1'(0|2il)}{\theta_1(0|2il)} \sum_{\alpha = 2, 3, 4} \frac{1}{2} s_\alpha \theta_\alpha (\epsilon|2il) \theta_\alpha^3 (0|2il) \quad (3.3)$$

where $V^{(6)}$ is the volume of the compact six-torus, $\Gamma_6$ the (winding) lattice of points identified with the origin, and

$$v^{(d)} = V^{(d)}/(2\pi \sqrt{\alpha'})^d .$$

Likewise for the M"obius strip we use the sequence of modular transformations

$$\tau = \frac{it}{2} + \frac{1}{2} \to -1/\tau \to -1/\tau + 2 \to (1/\tau - 2)^{-1} = 2il - \frac{1}{2}$$

with the result

$$M(i|B) = 2i v^{(4)} v^{(6)} q_i B \int_0^\infty dl \left( \sum_{w \in \Gamma_6} e^{-2w^2 i/\pi - 2ia \cdot w} \right) \eta^{-12} (2il - \frac{1}{2}) \times \frac{\theta_1'(0|2il - \frac{1}{2})}{\theta_1(\frac{1}{2}|2il - \frac{1}{2})} \sum_{\alpha = 2, 3, 4} \frac{1}{2} s_\alpha \theta_\alpha (\frac{\epsilon}{2}|2il - \frac{1}{2}) \theta_\alpha^3 (0|2il - \frac{1}{2}) \quad (3.4)$$

\footnote{Our conventions differ from those of Gimon and Polchinski \cite{GimonPolchinski}. With our normalization of the direct-channel proper time, a cutoff $t > \Lambda^{-2}$ is equivalent to a universal momentum cutoff on all open and closed-string states.}
Finally, after a change of variables $\tau = 2it \rightarrow -1/\tau = 2il$, the Klein bottle amplitude takes the form

$$K = -32i v^{(4)}v^{(6)} \int_0^\infty dl \left( \sum_{w \in \Gamma_6} e^{-2w^2 l/\pi} \right) \times \eta^{-12}(2il) \sum_{\alpha=2,3,4} \frac{1}{2} s_\alpha \theta^4(0|2il).$$

(3.5)

Using the appropriate expansions of the elliptic functions and some simple trigonometry, we can extract the infrared divergences of these expressions with the result

$$\mathcal{A}(i, j|\mathcal{B}) \sim \frac{i}{4} v^{(4)}v^{(6)} \int_0^\infty dl \left[ \frac{1 - q_i q_j \pi^2 B^2}{\sqrt{1 + q_i^2 \pi^2 B^2}} \right] + \left[ \frac{1}{2} (q_i + q_j)^2 \pi^2 B^2 \div \sqrt{1 + q_i^2 \pi^2 B^2} \sqrt{1 + q_j^2 \pi^2 B^2} \right]$$

$$\mathcal{M}(i|\mathcal{B}) \sim -16i v^{(4)}v^{(6)} \int_0^\infty dl \left[ \frac{1}{RR} \div \frac{1}{2} \sqrt{1 + q_i^2 \pi^2 B^2} \right]$$

$$\mathcal{K}(\mathcal{B}) \sim 256i v^{(4)}v^{(6)} \int_0^\infty dl \left[ \frac{1}{R-R} - \frac{1}{NS-NS} \right]$$

(3.6)

We have identified in the above expressions the contributions from the Ramond-Ramond or Neveu-Schwarz intermediate closed-string states. These correspond to the $\alpha = 2$ and $\alpha = 3, 4$ terms, respectively, of the spin structure sums in the $l$-channel.

The RR piece of the ultraviolet divergence is polynomial in the background field. This is because it comes from tadpoles in the Wess-Zumino part of the effective action [14, 15] which is the integral of 10-forms made out of $F_{\mu\nu}$ and the antisymmetric massless RR tensors. The constant part is
a tadpole for the unphysical RR 10-form [18], which cancels in the full free energy, eq. (2.10), after summing over the 32 Chan-Patton charges. The quadratic term, proportional to \( q_i q_j \), also cancels after summing over opposite charges or, in modern language, over conjugate pairs of 9-branes. This is consistent with the fact that the potential coupling \( f(tr F) \wedge A^{(8)} \) is here absent, both because the (high-energy) gauge group contains no simple \( U(1) \) factors, and also because the RR 8-form is projected out of the spectrum by the orientation reversal.

Consider next the Neveu-Schwarz piece of the ultraviolet divergence. This comes from tadpoles of the graviton, dilaton and 2-index antisymmetric tensor, which couple to the background field through a Dirac-Born-Infeld action [24, 18]. The constant term vanishes for SO(32) gauge group, consistently with the fact that the vacuum energy in the absence of a magnetic field should be zero. The 2-index tensor \( B^{\mu\nu} \) is projected out of the spectrum by orientation reversal, consistently again with the fact that the piece proportional to \( q_i q_j \) cancels after summing over opposite endpoint charges. The first physical divergence appears thus at order \( o(B^4) \), and comes from an on-shell graviton or a dilaton. As a check of our formulae let us compute this directly from the effective open-string action

\[
S = \int d^{10}x \sqrt{-g} \left[ -\frac{1}{2\kappa^{(10)}_2} R + \frac{1}{16\kappa^{(10)}_2} (\partial_\mu \phi)^2 + \frac{1}{2g^{(10)}_2} e^{\phi/4} tr F_{\mu\nu} F^{\mu\nu} \right] \tag{3.7}
\]

where \( \kappa^{(10)} \) and \( g^{(10)} \) are the gravitational and gauge couplings, and the \( SO(32) \) generators are normalized to \( tr(t^a t^b) = \frac{1}{2} \delta^{ab} \). A constant magnetic field in flat space with \( <\phi> = 0 \), gives rise to an energy-momentum tensor

\[
j_{\mu\nu} = -\frac{1}{g^{(10)}_2} tr \left( F_{\mu\rho} F^{\nu\rho} - \frac{1}{4} \eta_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \right), \tag{3.8}
\]

as well as to a source for the dilaton

\[
j_{\phi} = \frac{1}{8g^{(10)}_2} tr F_{\mu\nu} F^{\mu\nu}. \tag{3.9}
\]

Using the graviton propagator in the De Donder gauge [23],

\[
\frac{1}{2\kappa^{(10)}_2} \Delta^{\mu\nu,\rho\sigma} = (\eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \frac{1}{4} \eta^{\mu\nu} \eta^{\rho\sigma}) \frac{i}{\kappa^2}, \tag{3.10}
\]
one derives easily the following infrared contribution to vacuum energy due
to the above tadpoles,

\[ \mathcal{F} \sim -\kappa_{(10)}^2 V^{(10)} \left[ 4 j_\phi^2 + 2 j_{\mu \nu} j^{\mu \nu} - \frac{1}{4} (j_\mu^{\mu})^2 \right] \times \frac{\pi}{2} \int_0^\infty dl \]

\[ \sim -\frac{3}{4} \kappa_{(10)}^2 V^{(10)} B^4 \times \frac{\pi}{2} \int_0^\infty dl \]  

(3.11)

To compare with the result of the string calculation, we must expand the
annulhus and Möbius to order \( B^4 \), and perform the summation over endpoint
charges. For a generic normalized generator there are two charged endpoint
states, \( q_1 = -q_2 = \frac{1}{2} \), while \( q_3 = \ldots = q_{32} = 0 \). After some straightforward
algebra one finds agreement with eq. (3.11) provided

\[ g_{(10)}^4 = 2^9 \pi^7 \kappa_{(10)}^2 (2\alpha')^2 \]  

(3.12)

where we have here restored correct mass units \( [1] \). This relation between
the gravitational and gauge coupling constants has been derived previously
by Abe and Sakai \([10]\), and in the case of the bosonic string by Shapiro and
Thorn, and Dai and Polchinski \([17]\). In terms of four-dimensional couplings
it reads

\[ g_{(4)}^4 \alpha' = 16 \pi^2 \kappa_{(4)}^2 \frac{(4\pi^2\alpha')^3}{V^{(6)}} \]  

(3.13)

Contrary to what happens for the heterotic string \([20]\), the compactification
volume enters non-trivially here. Thus in open string theory the string scale
is not irrevocably tied to the Planck scale at tree level, a fact that has received
some attention recently in refs. \([27]\).

The basic lesson to retain here is that the quadratic part of the effective
action is ultraviolet convergent in flat space-time, without evoking space-
time supersymmetry. This will continue to hold, with a slight caveat, in the
orbifold compactification to which we turn now.

\[ ^{\dagger}\text{More appropriately we should have replaced }1/\alpha'\text{ by the mass squared of the first open-string excitation. Equation (3.12) would then be manifestly invariant under Weyl rescalings of the metric.} \]
4. $Z_2$ orbifold

Let us first briefly recall the upshot of the analysis by Gimon and Polchinski \cite{11}. The type-I theory on $R^6 \times T^4/Z^2$ contains untwisted and twisted closed strings, as well as open strings of three different types: those with freely moving endpoints (NN or 99), those whose endpoints are stuck on some 5-branes transverse to the orbifold (DD or 55), and those with one endpoint stuck and one moving freely (DN or 59). Consistency fixes both the number of 9-branes and the number of 5-branes to be 32. It also fixes the action of the orientation reversal ($\Omega$) and orbifold-twist ($R$) on the open-string end-point states. This action does not mix Neumann with Dirichlet endpoints, and can be described in appropriate bases by the four $32 \times 32$ matrices

$$\gamma_{\Omega,9} = 1, \quad \gamma_{R,9} = \gamma_{R,5} = \gamma_{\Omega,5} = \begin{pmatrix} 0 & i1 \\ -i1 & 0 \end{pmatrix}$$

(4.1)

The massless spectrum of this theory in six dimensions has

(i) the N=1 supergravity, one tensor and four gauge-singlet hypermultiplets from the untwisted closed-string sector,

(ii) sixteen gauge-singlet hypermultiplets, one from each fixed-point of the orbifold, in the twisted closed-string sector

(iii) $U(16)$ vector multiplets and two hypermultiplets in the antisymmetric 120 representation from the NN sector,

(iv) identical content, i.e. an extra $\tilde{U}(16)$ gauge group and two antisymmetric hypermultiplets, from the DD sector, and

(v) one hypermultiplet transforming in the representation $(16, 16)$ of the full gauge group and coming from the DN sector.

Notice that each twisted-sector hypermultiplet contains a RR 4-form field $C^{(I)}$ localized at the $I$th fixed point of the orbifold, which plays a special role in what follows.

This model has a T-duality, which interchanges NN and DD sectors and hence also the two $U(16)$ gauge groups. Without losing generality, we may thus restrict ourselves to background fields $BQ$ arising from the NN sector. The antisymmetric hypermultiplets are six-dimensional moduli. They have a simple geometric interpretation in the DD sector \cite{11}, where they correspond
to motion of a pair of 5-branes with their mirror image away from a fixed-point of the orbifold. Together with the discrete moduli, that correspond to jumps of a 5-brane pair between fixed points, these can be used to break the gauge symmetry to various unitary and symplectic components. To simplify the calculation we will turn off all six-dimensional moduli in what follows, i.e. we will assume maximal unbroken gauge symmetry in six dimensions. The more general calculation presents no real technical difficulty. We will furthermore choose to work on

\[ R^4 \times T^2 \times T^4 / Z_2 \]

so that we may break the gauge group by Wilson-line moduli in four dimensions. We denote volumes as \( V^{(4)} \), \( V^{(2)} \) and \( V^{\text{orbifold}} \), the latter being half the volume of the corresponding torus. There is one final detail to settle: in the basis of Chan-Patton charges of ref. [11], the wavefunctions of NN gauge bosons take the form

\[ \lambda = \begin{pmatrix} A & S \\ -S & A \end{pmatrix} \]  

(4.1)

where \( A \) and \( S \) are arbitrary 16 \( \times \) 16 antisymmetric and symmetric matrices. Since in order to perform our calculation we need to diagonalize the background gauge field, we will make the unitary change of basis of Neumann endpoint states

\[ U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i1 \\ 1 & -i1 \end{pmatrix} \]  

(4.2)

In this new basis the 16 and \( \overline{16} \) representations of the gauge group are disentangled, and the orbifold-twist operator acts as a simple sign,

\[ s_{ij} = -1 \quad \text{or} \quad +1 \]

according to whether the end-point states \( |i> \) and \( |j> \) belong to the same or to conjugates representations.

We are now ready to proceed with the calculation of the one-loop free energy, which is the sum of contributions from the various sectors,

\[ \mathcal{F}^{\text{orbifold}} = \mathcal{F}_{\text{closed}} + \mathcal{F}_{NN} + \mathcal{F}_{ND} + \mathcal{F}_{DD} \]  

(4.3)
Since only Neumann endpoints couple to the background field, we can ignore $F_{\text{closed}}$ and $F_{DD}$ which vanish by space-time supersymmetry. The remaining two contributions read

$$i\mathcal{F}_{NN} = \frac{1}{4} \left\{ \sum_{ij} \mathcal{A}(i, j|B) + \sum_{ij} \mathcal{A}^{(R)}(i, j|B) + \sum_i \mathcal{M}(i|B) + \sum_i \mathcal{M}^{(R)}(i|B) \right\}$$

and

$$i\mathcal{F}_{ND} = \frac{1}{2} \times 32 \sum_i \mathcal{A}_{ND}(i|B)$$

where the superscript $\mathcal{R}$ here indicates the insertion of the orbifold-twist operator inside a trace. Only a single amplitude contributes in the ND sector: indeed, there are no Möbius diagrams, since the action of $\Omega$ does not mix Neumann and Dirichlet states, and $\mathcal{A}_{ND}^{(R)} \propto tr(\gamma_5, \mathcal{R}) = 0$, by eq. (4.1). Notice also that the overall factor in front of $\mathcal{A}_{ND}$ takes into account the multiplicity of 5-branes, as well as the two possible orientations of a ND string.

The NN annulus and Möbius, without insertion of the $\mathcal{R}$ operator, are given by eqs. (2.8-9), with $\star \Gamma_6 = \star \Gamma_2 \oplus \star \Gamma_4$ being the direct sum of the lattices of momenta on the two-torus and the orbifold, and with the Wilson-lines shifting only the former momenta. The other two NN amplitudes read

$$\mathcal{A}^{(R)}(i, j|B) = -\frac{i}{2} V^{(4)} s_{ij} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{a_i + a_j + \star \Gamma_2} e^{-\eta t p^2/2} \right) \frac{1}{\eta^8/\theta_2^2} \times$$

$$\times \frac{i}{2} (q_i + q_j) \mathcal{B} t \frac{\theta_1'(0)}{\theta_1(\frac{\mu t}{2})} \frac{1}{2} \left( \theta_3(\frac{i e t}{2}) \theta_3^2 - \theta_4(\frac{i e t}{2}) \theta_4^2 \right)$$

and

$$\mathcal{M}^{(R)}(i|B) = -\frac{i}{2} V^{(4)} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-2} \left( \sum_{2a_i + \star \Gamma_2} e^{-\eta t p^2/2} \right) \frac{1}{\eta^8/\theta_2^2} \times$$

$$\times i q_i \mathcal{B} t \frac{\theta_1'(0)}{\theta_1(\frac{\mu t}{2})} \frac{1}{2} \left( \theta_3(\frac{i e t}{2}) \theta_3^2 - \theta_4(\frac{i e t}{2}) \theta_4^2 \right)$$

Consistency of the theory requires of course ultraviolet finiteness for every individual cross channel. To check it one needs the explicit forms for these amplitudes, before enforcing supersymmetry identities. Since our conventions differ somewhat from those of Gimon and Polchinski, we give these expressions in the appendix for completeness.
The modular parameter of the elliptic functions inside the integrals is \( \tau = \frac{it}{2} \) for the annulus and \( \tau = \frac{it}{2} + \frac{1}{2} \) for the Möbius strip, and the first argument of the theta functions is by default zero. In deriving the above expressions we made use of the fact that \( 4\eta^2/\theta_2^2 \) is the correctly normalized contribution of the four bosonic orbifold coordinates twisted in the time direction, that the (open-string) Ramond sector does not contribute because of fermionic zero modes, and finally that \( s_{ii} = -1 \). Likewise the ND amplitude reads

\[
A_{ND}(i|B) = -\frac{i}{2} V^{(4)} \int_0^\infty dt \left( \frac{2\pi^2}{2t} \right)^{-2} \left( \sum_{a_i+\gamma_2} e^{-\pi t\gamma^2/2} \right) \frac{1}{\eta^8 \theta_4^2} \times \]

\[
\times \frac{i}{2} q_i B t \frac{\theta_1^0}{\theta_1^{(t/2)}} \sum_{w \in \Gamma_2} e^{-2w^2 t / \pi - 2ia_i \cdot w} \times \]

\[
\times \frac{1}{\eta^2 \theta_4^2 \theta_1^{(t/2)}} \left( \theta_3(e) \theta_2^2 - \theta_2(e) \theta_3^2 \right)
\]

(4.7)

where we have taken here into account that ND bosonic coordinates have half-integer frequencies, while their fermionic partners have integer or half-integer frequencies in the Neveu-Schwarz or Ramond sectors.

Let us summarize the calculation as

\[
\mathcal{F}^{orbifold}(B) = \frac{1}{2} \mathcal{F}^{toroidal}(B) + \delta \mathcal{F}(B),
\]

(4.8)

where \( \mathcal{F}^{toroidal} \) is the induced action of the theory before the orbifold projection, and \( \delta \mathcal{F} \) is given by sums of the amplitudes (4.5-7) over endpoint states. What we will now show is that \( \delta \mathcal{F} \) has no ultraviolet divergences, provided the background field has no component along the simple U(1) gauge group factor. To this end let us use the series of transformations of section (3) to put the novel contributions in the form

\[
A^{(R)}(i, j|B) = -\frac{i}{4} V^{(4)} V^{(2)} s_{ij} (q_i + q_j) B \int_0^\infty dl \left( \sum_{w \in \Gamma_2} e^{-\pi t^2 l/2\pi - i(a_i+a_j) \cdot w} \right) \times \]

\[
\times \frac{1}{\eta^2 \theta_4^2 \theta_1^{(t/2)}} \left( \theta_3(e) \theta_2^2 - \theta_2(e) \theta_3^2 \right)
\]

(4.9)

\[
M^{(R)}(i|B) = 4i V^{(4)} V^{(2)} q_i B \int_0^\infty dl \left( \sum_{w \in \Gamma_2} e^{-\pi t^2 l/2\pi - 2ia_i \cdot w} \right) \times \]

\[
\times \frac{1}{\eta^2 \theta_4^2 \theta_1^{(t/2)}} \left( \theta_3(e) \theta_2^2 - \theta_2(e) \theta_3^2 \right)
\]

(4.10)
and

\[
\mathcal{A}_{ND}(i|B) = -\frac{i}{16} v^{(4)} v^{(2)} q_i B \int_0^\infty dl \left( \sum_{w \in \Gamma_2} e^{-w^2 l/2 - ia \cdot w} \right) \times \\
\times \frac{1}{\eta^8 \theta_2^4 \theta_1^4} \left( \theta_3(\epsilon) \theta_4^2 - \theta_4(\epsilon) \theta_3^2 \right),
\]

(4.11)

The elliptic functions in the above expressions are at modulus \( \tau = 2il \) for the annuli and \( \tau = 2il - \frac{1}{2} \) for the Möbius strip, and it is useful to recall that \( \epsilon \) is defined by eq. (2.6) with \( q_i = q_j \) for the Möbius, and \( q_j = 0 \) for the ND contribution. Using asymptotic expansions of the integrands in the large-\( l \) region, we find the following structure of divergences

\[
\mathcal{A}^{(R)}(i, j|B) \sim -i v^{(4)} v^{(2)} s_{ij} \int_0^\infty dl \times \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \sqrt{1 + q_j^2 \pi^2 B^2} \right]_{NS-NS} - \left[ \sqrt{1 + q_j^2 \pi^2 B^2} \right]_{RR} - \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \right]_{RR} - \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \right]_{RR},
\]

\[
\mathcal{M}^{(R)}(i|B) \sim -8i v^{(4)} v^{(2)} \int_0^\infty dl \times \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \right]_{NS-NS} - \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \right]_{RR},
\]

\[
\mathcal{A}_{ND}(i|B) \sim \frac{i}{8} v^{(4)} v^{(2)} \int_0^\infty dl \times \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \right]_{NS-NS} - \left[ \sqrt{1 + q_i^2 \pi^2 B^2} \right]_{RR},
\]

(4.12)

The Möbius and ND annulus divergences cancel exactly each other, after summing the latter over two orientations and 32 possible D-endpoints. Summing over opposite endpoint charges in the remaining annulus diagram, one finds that all but a quadratic RR contribution vanish. The final result therefore reads

\[
\delta F \sim -i v^{(4)} v^{(2)} (tr Q)^2 \pi^2 B^2 \int_0^\infty dl.
\]

(4.13)

with the trace in the fundamental representation of the gauge group.

This ultraviolet divergence comes from tadpoles of the twisted RR 4-forms, which couple to the background field through the generalized Green-
for canonically-normalized 4-forms. The coupling gives mass to the $U(1)$ (abelian) gauge field, rendering a background inconsistent. For non-abelian background fields, on the other hand, the structure of ultraviolet divergences is identical to that of the toroidal model:

$$F_{\text{orbifold}} \sim \frac{1}{2} F_{\text{toroidal}}.$$ 

Taking into account the halving of the volume, we may conclude in particular that, with SO(32) normalizations for the generators of U(16), the relation between gauge and gravitational couplings stays the same. Furthermore since in the toroidal theory the gauge coupling is not renormalized, we may conclude that in the orbifold the renormalization is ultraviolet finite. We will now see explicitly how this comes about.

5. Gauge-coupling renormalization

In order to extract the quadratic piece in the weak-field expansion of $\delta F$, we will make use of the identities

$$\theta''_4 \theta_4 \theta_3^2 = 4\pi^2 \eta^6 \theta_2^2,$$  

(5.1)

and

$$\theta''_3 \theta_3 \theta_2^2 = 4\pi^2 \eta^6 \theta_4^2.$$  

(5.2)

The first of these identities follows from eq. (2.11), if one expands to quadratic order around $\nu = \frac{1}{2}$. Note that a shift of the argument of a $\theta$-function by $\frac{1}{2}$ can be absorbed into a change of spin structure, and that $\theta'_1(0) = 2\pi\eta^3$. The second identity is just a modular transformation of the first. Using these two identities, one finds that the amplitudes (4.5-7) expanded to quadratic order in $\epsilon \simeq (q_i + q_j) B \ll 1$, collapse to contributions of six-dimensional massless
states:

$$A^{(R)}(i,j|B) = -iV^{(4)} \frac{B^2}{8\pi^2} s_{ij} (q_i + q_j)^2 \int_0^\infty \frac{dt}{t} \left( \sum_{a_i + a_j + \Gamma_2} e^{-\pi t p^2/2} \right) + o(B^4)$$

$$M^{(R)}(i|B) = -iV^{(4)} \frac{B^2}{2\pi^2} q_i^2 \int_0^\infty \frac{dt}{t} \left( \sum_{2a_i + \Gamma_2} e^{-\pi t p^2/2} \right) + o(B^4)$$

$$A_{ND}(i|B) = +iV^{(4)} \frac{B^2}{32\pi^2} q_i^2 \int_0^\infty \frac{dt}{t} \left( \sum_{a_i + \Gamma_2} e^{-\pi t p^2/2} \right) + o(B^4)$$

(5.3)

This collapse to the zero-mode space, noted previously by Douglas and Li [8], is to be expected: indeed, as argued in the introduction, only BPS states can contribute to threshold effects, and the only BPS states of the open string are the massless (before Higgsing) modes of the six-dimensional model. A similar result is well known in the heterotic string [7] where, however, the spectrum of BPS states includes infinite string excitations with no simple field-theoretic description.

Putting together eqs. (4.4) and (5.3) we arrive at the following expression for the full free energy, including classical and one-loop contributions

$$F(B)/V^{(4)} = \frac{B^2}{2g^{(4)}} + \frac{B^2}{8\pi^2} \int_0^\infty \frac{dt}{t} \left[ \sum_i q_i^2 \left( \sum_{a_i + \Gamma_2} 4 e^{-\pi t p^2/2} - \sum_{2a_i + \Gamma_2} e^{-\pi t p^2/2} \right) - \sum_{ij} s_{ij} (q_i + q_j)^2 \sum_{a_i + a_j + \Gamma_2} \frac{1}{4} e^{-\pi t p^2/2} \right] + o(B^4)$$

(5.4)

where the SU(16) generators are normalized to $\text{tr}_{16} Q^2 = \frac{1}{2}$, and we recall that the Chan-Patton charges run over both the 16 and the $\overline{16}$ representations separately. As a check let us extract the leading infrared divergence of the coupling-constant renormalization in the limit of vanishing Wilson lines. Cutting off $t < 1/\mu^2$ one finds after some straightforward algebra

$$\left| \frac{4\pi^2}{g^{(4)}_{1-loop}} \right| = \frac{4\pi^2}{g^{(4)}_{tree}} - 6 \log \mu + \text{IR finite} \ ,$$

(5.5)

in agreement with the correct $\beta$-function coefficient of the N=2 theory in
four dimensions,
\[ C_{\text{adj}} - 2C_{120} - 16C_{\text{fund}} = -6. \]
Put differently, expression (5.4) correctly reproduces the logarithmic part of the one-loop prepotential for this model.

This expression is formally identical to that of Kaluza-Klein theory compactified from six to four dimensions. If we were to impose a uniform ultraviolet momentum cutoff, \( t > \frac{1}{\Lambda^2} \), the result would therefore be quadratically divergent. The cutoff dictated by string theory is, however, much smarter! It is uniform in transverse time \( l \), which means that if we cutoff the annulus at \( t = \frac{1}{\Lambda^2} \), we must cutoff the Möbius strip at \( t = \frac{1}{4\Lambda^2} \). To render finiteness more explicit, let us perform the Poisson resummations to the transverse channel, and put eq. (5.4) in the form

\[
\delta F(B) = 2\pi^2 B^2 v^{(4)} v^{(2)} \int_0^\infty dl \sum_{w \in \Gamma_2} \left[ \sum_i q_i^2 \left( e^{-w^2l/2\pi + iw \cdot a_i} - e^{-w^2l/\pi + 2iw \cdot a_i} \right) \right] - \frac{1}{16} \sum_{ij} S_{ij} (q_i + q_j)^2 e^{-w^2l/2\pi + iw \cdot (a_i + a_j)} + o(B^4)
\]

(5.6)

We may now perform the \( l \)-integrations, and sum over the two complex conjugate representations \( 16 \) and \( 16^* \) over which \( i, j \) run. Since generic Wilson-line backgrounds break the gauge symmetry to the Cartan subgroup of \( SU(16) \), the answer is better expressed as the one-loop correction, \( \Delta_{ij} \), to the gauge kinetic function defined through

\[
\mathcal{L}_{\text{eff}} = \left( \frac{1}{2g^{(4)}} \delta_{ij} + \Delta_{ij} \right) F_{i\mu} F_{j\mu}
\]

(5.7)

where \( F_{i\mu} \) is a traceless diagonal 16-dimensional matrix. The final result reads

\[
\Delta_{ij}(a, \Gamma_2) = \sum_{w \in \Gamma_2 - \{0\}} v^{(2)} \frac{1}{2\pi w^2} \left[ \cos(w \cdot a_i) \cos(w \cdot a_j) + \delta_{ij} \left\{ 4 \cos(w \cdot a_i) - \cos(2w \cdot a_i) - \sin(w \cdot a_i) \sum_k \sin(w \cdot a_k) \right\} \right]
\]

(5.8)
where in a generic point of moduli space the winding sums are manifestly convergent. Notice that in addition to the Wilson lines, the above result gives the dependence on the moduli \( \text{Im}T \) and \( U \) of the 2-torus. It should be furthermore straightforward to extend the analysis so as to account for the six-dimensional gauge moduli. The dependence on \( \text{Re}T \), which in open-string theory is a RR 2-form background, may however be harder to extract.

6. Concluding Remarks

Our calculation illustrates in a very simple context the way in which string theory produces finite answers: in the case at hand it is simply field theory but with a very smart cutoff on the momenta. These results have non-trivial implications, both in the context of heterotic-type-I duality [28], and for the study of moduli spaces of D-branes [29, 8]. In this latter context in particular it implies that the metric in the moduli space of \( N=2 \) configurations of D-branes is given entirely by simple and massless closed-string exchange. We plan to pursue these issues further in the near future.

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Appendix

The field-independent contributions of closed and DD strings to the one-loop free energy are given by the following amplitudes

$$iF_{\text{closed}} = \frac{1}{4}(\mathcal{T} + \mathcal{T}^{(\mathcal{R})} + \mathcal{K} + \mathcal{K}^{(\mathcal{R})}) + \frac{1}{4}(\mathcal{T}_{\text{twist}} + \mathcal{T}^{(\mathcal{R})}_{\text{twist}} + \mathcal{K}_{\text{twist}} + \mathcal{K}^{(\mathcal{R})}_{\text{twist}})$$ (A.1)

and

$$iF_{\text{DD}} = \frac{1}{4}(A_{DD} + A^{(\mathcal{R})}_{DD} + M_{D} + M^{(\mathcal{R})}_{D})$$ , (A.2)

where summation over Dirichlet endpoint states is implicitly performed in the second line. The torus amplitudes are well known from the type-II string, and are by themselves ultraviolet finite. $\mathcal{K}$ was computed in section 2, while the remaining Klein bottle amplitudes read in six uncompactified dimensions

$$\mathcal{K}^{(\mathcal{R})} = -\frac{i}{2} V^{(6)} \int_{0}^{\infty} \frac{dt}{t} (2\pi^2 t)^{-3} \left( \sum_{w \in \Gamma_4} e^{-tw^2/2\pi} \right) \frac{1}{\eta^{12}} \times \sum_{\alpha=2,3,4} \frac{1}{2} s_{\alpha} \theta_{\alpha}^4$$ (A.3)

and

$$\mathcal{K}_{\text{twist}} + \mathcal{K}^{(\mathcal{R})}_{\text{twist}} = -\frac{i}{2} V^{(6)} \int_{0}^{\infty} \frac{dt}{t} (2\pi^2 t)^{-3} \frac{1}{\eta^{12}} \frac{1}{\theta_{4}^2} \left( \theta_3^2 \theta_2^2 - \theta_2^2 \theta_3^2 \right)$$ (A.4)

The argument of all modular functions is $\tau = 2it$, while the factor 16 appearing in the second line counts the number of fixed-points of the orbifold. Notice that the role of $\mathcal{K}^{(\mathcal{R})}$ is to symmetrize winding Neveu-Schwarz, and antisymmetrize winding RR states, since pure winding is left unchanged by the combined action of the operator $\Omega \mathcal{R}$. The role of $\mathcal{K}_{\text{twist}} + \mathcal{K}^{(\mathcal{R})}_{\text{twist}}$ on the other hand is to symmetrize twisted NS-NS states, and antisymmetrize twisted RR states, giving a net number of respectively 48 and 16 massless space-time singlets $[\square]$. The DD amplitudes, assuming that all 32 5-branes are at the same fixed point, read

$$A_{DD} = -2^{10} \times \frac{i}{2} V^{(6)} \int_{0}^{\infty} \frac{dt}{t} (2\pi^2 t)^{-3} \left( \sum_{w \in \Gamma_4} e^{-tw^2/2\pi} \right) \frac{1}{\eta^{12}} \times \sum_{\alpha=2,3,4} \frac{1}{2} s_{\alpha} \theta_{\alpha}^4$$ (A.5)
and
\[ M_D^{(R)} = 2^5 \times \frac{i}{2} V^{(6)} \int_0^\infty \frac{dt}{t} (2\pi^2 t)^{-3} \left( \sum_{w \in \Gamma_4} e^{-tw^2/2\pi} \right) \frac{1}{\eta^{12}} \times \sum_{\alpha=2,3,4} \frac{1}{2} \widehat{s}_\alpha \theta_\alpha^4 \quad (A.6) \]

where \( \tau = \frac{\eta}{2} \) for the annulus and \( \tau = \frac{\eta}{2} + \frac{1}{2} \) for the Möbius strip, as usual. The other two contributions are zero in the Neveu-Schwarz and Ramond sectors separately. For \( A^{(R)}_{DD} \) this is obvious since it is proportional to \( (tr \gamma_{5,R})^2 \). As for \( M_D \), the reason is more subtle: it can be traced to the fact that the action of \( \Omega \) on a DD (super)coordinate has an extra minus sign compared to its action on a NN (super)coordinate [11]. As a result \( \Omega \) anticommutes with the fermionic DD zero modes in the Ramond sector, so that the corresponding contribution in the Möbius amplitude vanishes. Space-time supersymmetry then ensures that the contribution of Neveu-Schwarz states is also zero.

When transforming to the cross \( l \)-channel, \( K^{(R)} \), \( A_{DD} \) and \( M_D^{(R)} \) give unphysical tadpoles proportional to the inverse volume of the orbifold. These cancel between the three diagrams [11], a phenomenon that is T-dual to the usual cancellation between the NN annulus and Möbius strip, and the (untwisted) Klein bottle. Notice that this T-duality exchanges the orientation-reversing operator \( \Omega \) with \( \Omega R \). Finally the twisted Klein bottles vanish in Neveu-Schwarz or RR \( l \)-channels separately, and hence do not create any anomalies. Conversely, this shows that in the direct channel, we are forced to antisymmetrize RR twisted states, if we have symmetrized the NS-NS ones.
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