Predesign of Nodes of Cable Net for Umbrella Reflectors Based on Bezier Surface

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Abstract. The umbrella antenna cable net is different from the ring truss antenna. Most umbrella antennas do not use triangular cable nets. In this case, a predesign method based on Bezier surface is proposed for a common umbrella antenna cable net, and the corresponding formula is derived. The relationship between the shape accuracy and the cable net shape is obtained. From the extremum condition of the function, the distribution form of the node with the highest accuracy is obtained. The new cable net structure is parallelogram and triangle alternation. The cable net can meet the requirements of umbrella antennas with different number of antenna ribs. Finally, the accuracy of the surface of the reflector obtained by this method is verified, and the design can meet the requirements of engineering accuracy. This method can quickly calculate the distribution of the reflector nodes meeting the engineering accuracy. This method is a useful attempt to design the reflector of the umbrella cable-net antenna, and it has some inspiration to the design of this kind of antenna.

1. Introduction
The large aperture umbrella satellite antenna consists mainly of the reflector and the expansion mechanism, in which the reflector is composed of several antenna ribs and the cable net attached to them, as shown in Figure 1 [1]. The cable net structure uses tension cable to divide the metal mesh into several units, and connects with the antenna ribs through the nodes. In the prestress state, the ideal paraboloid is fitted by some small planes or surfaces. In the antenna design stage, the fitting error of the cable net structure to the ideal paraboloid is the main factor affecting the accuracy of the antenna. For the umbrella antenna, the triangular cable net can not be used in many cases because of the characteristics of the structure, but the cable net structure shown in Figure 2 is used, which makes the unit surface not be a plane, so the empirical formula [2] that uses the maximum length of the triangular element surface will cause greater error.

![Figure 1. Umbrella reflector](image1.jpg)

![Figure 2. Cable net of an umbrella reflector](image2.jpg)
At present, the cable network structure predesign shown in Figure 2 is often reversed through experience or finite element method, and there is no convenient method to obtain the length of cable segment which is satisfactory for requirement. In this paper, a method based on the double Bezier surface is proposed for this cable net structure, and an empirical formula is obtained, and a cable net form suitable for the umbrella antenna is obtained, which makes the pre design of the cable net possible.

2. Pre design method of cable net of umbrella antennas
The basic principle is to use double one Bezier surface to fit the unit surface, and get the relationship between the root mean square error and the cell size by integration.

2.1 Double one power Bezier surface
In three-dimensional space, there are \((n + 1) \times (m + 1)\) points \(P_{ij}(i = 0, 1, ..., n; j = 0, 1, ..., m)\), we call the \(n \times m\) power parametric surface

\[
P(u, v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{ij}B_{i,n}(u)B_{j,m}(v)
\]

(1)

the \(n \times m\) power Bezier surface.

in which:

\[
B_{i,n}(u) = C_i^n u^i (1 - u)^{n-i}
\]

(2)

\[
B_{j,m}(v) = C_j^m v^j (1 - v)^{m-j}
\]

(3)

The matrix form of Bezier surface is

\[
P(u, v) = \begin{bmatrix} P_{00} & P_{01} & \cdots & P_{0m} \\ P_{10} & P_{11} & \cdots & P_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n0} & P_{n1} & \cdots & P_{nm} \end{bmatrix} \begin{bmatrix} B_{0,n}(v) \\ B_{1,n}(v) \\ \vdots \\ B_{n,n}(v) \end{bmatrix}
\]

(4)

When \(n = m = 1\), it is a double one power Bezier surface. If there are \((n + 1) \times (m + 1) = 2 \times 2 = 4\) control points \(P_{00}, P_{01}, P_{10}, P_{11}\), that

\[
P(u, v) = [1 - u] \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} [1 - v] = (1 - u)(1 - v)P_{00} + u(1 - v)P_{10} + (1 - u)vP_{01} + uvP_{11}
\]

(5)

Its graphical representation is shown in Figure 3. We can prove that it is a piece of curved surface on a hyperbolic paraboloid. When \(u=0\) and \(u=1\), for them in (5), the two boundaries we get are straight lines; when \(v=0\) and \(v=1\), for them in (5), the two boundaries are straight lines too. The double one power Bezier surface is surrounded by four straight lines.

![Figure 3. Double one power Bezier surface](image)

The coordinate values of four vertices are substituted into the equation, and the parametric equation of the surface is obtained.
\[ \begin{align*}
  x &= w - uw \\
  y &= u - uw \\
  z &= uw
\end{align*} \tag{6} \]

The surface equation can be obtained by eliminating the parameters.

2.2 RMS error analysis

Figure 2 shows that the four nodes of the cable net structure are taken as one unit, and the projection in the caliber plane is a trapezium. The surface on ideal paraboloid controlled by four node is a space quadrilateral, which satisfies the double Bezier surface. Establish a coordinate system on the projection plane of paraboloid, as shown in Figure 4.

The projective quadrilateral is trapezoid, and the vertex coordinates are as follows:

- \( P'_0(x_0, y_0) \)
- \( P'_0(x_0 + e, y_0 + h) \)
- \( P'_1(x_0 + l, y_0) \)
- \( P'_1(x_0 + e + m, y_0 + h) \)

The linear equation of the line \( P'_0 P'_0 \) is \( x = k_1 y + b_1 \), and we substitute \( P'_0, P'_1 \) into the straight line equation, get the following expression.

\[
  k_1 = \frac{e}{h} \\
  b_1 = -\frac{ey_0 - hx_0}{h}
\] \tag{8}

The linear equation of the line \( P'_1 P'_1 \) is \( x = k_2 y + b_2 \), and we substitute \( P'_1, P'_1 \) into the straight line equation, get the following expression.

\[
  k_2 = \frac{e - l + m}{h} \\
  b_1 = -\frac{ey_0 - hl - hx_0 - ly_0 + my_0}{h}
\] \tag{9}

Since the four vertices of the surface are on the paraboloid, the surface is on one side of the paraboloid. In order to further reduce the error, refer to the present method of triangle plane fitting[3], the surface is moved along the \( z \) axis as a whole, such as Figure 5, the displacement is \( \delta \). Therefore, the coordinates of \( P_0, P_1, P_1, P_1 \) are:
\[ P_0(x_0, y_0, \frac{x_0^2 + y_0^2}{4f} + \delta) \]
\[ P_0(x_0 + e, y_0 + h, \frac{(x_0 + e)^2 + (y_0 + h)^2}{4f} + \delta) \]
\[ P_1(x_0 + l, y_0, \frac{(x_0 + l)^2 + y_0^2}{4f} + \delta) \]
\[ P_1(x_0 + e + m, y_0 + h, \frac{(x_0 + e + m)^2 + (y_0 + h)^2}{4f} + \delta) \]

We replace the four points above (5) and get the following:

\[ (1-u)(1-v) \begin{bmatrix} x_0 \\ y_0 \\ \frac{x_0^2 + y_0^2}{4f} + \delta \end{bmatrix} + u(1-v) \begin{bmatrix} x_0 + l \\ y_0 \\ \frac{(x_0 + l)^2 + y_0^2}{4f} + \delta \end{bmatrix} \]
\[ +(1-u)v \begin{bmatrix} x_0 + e \\ y_0 + h \\ \frac{(x_0 + e)^2 + (y_0 + h)^2}{4f} + \delta \end{bmatrix} + uv \begin{bmatrix} x_0 + e + m \\ y_0 + h \\ \frac{(x_0 + e + m)^2 + (y_0 + h)^2}{4f} + \delta \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \]  

The parametric equation of the surface is obtained as following:

\[ \begin{cases} x = x(u, v) \\ y = y(u, v) \\ z = z(u, v) \end{cases} \]  

We eliminate the parameter to get the surface equation \( z = z(x, y) \).

The root mean square error of the surface[4] relative to the ideal paraboloid is:

\[ F(\delta, m, l, h, e) = \delta_{rms}^2 = \frac{2}{(m+l)h} \int_{y_0}^{y_0+h} \int_{k_1y+b_1}^{k_2y+b_2} \left( z - \frac{x^2+y^2}{4f} \right)^2 \]

We let \( \frac{\partial F}{\partial \delta} = 0 \), and get the following expression:

\[ \delta = -\frac{e^2 - em + h^2 + l^2 - lm + m^2}{24f} \]  

Replace (13) into (12), and we get \( F_{min1}(m, l, h, e) \). We let \( \frac{\partial F_{min1}}{\partial e} = 0 \), and get

\[ \begin{cases} e_1 = \frac{l-m}{2} \\ e_2 = \frac{l-m}{2} + \sqrt{\frac{4h^2 - 3l^2 + 6lm - 3m^2}{2}} \\ e_3 = \frac{l-m}{2} - \sqrt{\frac{4h^2 - 3l^2 + 6lm - 3m^2}{2}} \end{cases} \]

When \( e = \frac{l-m}{2} \), \( F_{min1} \) gets a minimum of \( F_{min2} \). Through analysis, we can see that the quadrilateral \( P_0'P_{01}'P_{11}'P_{10}' \) is an isosceles trapezium. In particular, when \( m=0 \), the isosceles trapezoidal degenerate into an isosceles triangle.
Considering the geometric form of the umbrella antenna, when \( m = \frac{l}{2} \), the distribution of nodes is regular, it is easy to manufacture and install cable nets. At this time, the projection shape of \( e = \frac{l}{4} \) cable net at the mouth is shown in Figure 6.

\[ e \]

\[ \theta \]

\[ h \]

\[ \frac{e}{h} = \tan\left(\frac{\theta}{2}\right) \] (16)

That is

\[ h = \frac{l}{4\tan\left(\frac{\theta}{2}\right)} \] (17)

The number of equable portions of the antenna ribs is

\[ N = \left[ \frac{L}{h/\cos^2\theta} \right] = \left[ \frac{4L\sin\left(\frac{\theta}{2}\right)}{l} \right] \] (18)

\( L \) is the length of the antenna ribs, and \([*]\) is an integral function.

Substituting \( m, e, h \) into \( F_{min2} \), we get the minimum mean square error at this time.

\[ \delta_{rms}^2 = \frac{l^4(43\cos^2\theta - 84\cos\theta + 42)}{184320f^2(\cos\theta - 1)^2} \] (19)

That

\[ \delta_{rms} = \frac{l^2}{192\sqrt{5}f} \sqrt{42 + \frac{\cos^2\theta}{(\cos\theta - 1)^2}} = \frac{l^2}{192\sqrt{5}f} \sqrt{42 + \cot^2\theta} \] (20)

At this time, the displacement is \( \delta = -\frac{l^2}{384f}(1 + \cot^2\theta) \).

According to this method, the cable net is predesigned. The cable net form is trapezoid and triangle alternating structure, and the distribution of nodes is regular.

3. An example of 2 meter aperture antenna

Suppose the reflector of an umbrella antenna is \( D = 2m \), the focal length is \( f = 4m \), the number of antenna ribs is 8, and the design error is \( \delta_{rms} = 0.3mm \). This method is applied to pre design the cable net form and verify its accuracy.

After calculation, the cable net structure parameters are as follows: \( N = 6 \); \( \delta = -0.61379mm \). The cable network structure is shown in figure 7 and figure 8.
Due to the symmetry of the reflector, one of the sectors is selected as the research object to explore the accuracy of the reflector surface. First, according to the distribution of nodes, the reflector is divided into triangular and trapezoid shaped surface units. By interpolating, several sampling points are generated in each unit, and the shape surface precision of each unit is calculated approximately, and the shape surface precision of the whole sector is [5]. The concrete formula is as follows:

\[
\delta_i = \frac{\sum_{j=1}^{m} (z_{ij}^* - z_{ij})^2}{m} \\
\delta_{rms} = \sqrt{\frac{\sum_{i=1}^{n} A_i \delta_i^2}{A}}
\]

In which \( \delta_i \) is the root mean square error of the \( i \) cable net unit, \( z_{ij}^* \) is the ordinate of the \( j \) interpolation sampling point in the \( i \) cable net unit. \( z_{ij} \) is the ordinate of the corresponding points of \( z_{ij}^* \) on the ideal paraboloid. \( m \) is the number of interpolated post sampling points for the \( i \) cable net unit. \( \delta_{rms} \) is the root mean square error of the sector. \( A_i \) is the area of the projection plane of the \( i \) unit in the \( xoy \) plane. \( A \) is the area of the sector projection plane, and \( n \) is the number of cable net elements in the sector.

The calculated root mean square error of the reflector is \( \delta_{rms} \approx 0.27132 \text{mm} \), which meets the design requirement of 0.3mm. The error distribution is shown in Figure 9.

As we can see, the error is distributed on the upper and lower sides of the ideal parabola. Although
the unilateral error is large, the error of the two sides counteract each other, and the overall error satisfies the requirement of the surface precision.

4. Conclusion
The effectiveness of the proposed method is verified by an example. Using this method, the design of cable net node can be completed quickly, and the precision requirement is satisfied. As long as the number of antenna ribs is adjusted, the node distribution that meets the requirement of precision can be distributed. Compared with the traditional triangular cable net structure, it can adapt to the reflector design of different number of antenna ribs, so that the number of nodes on each antenna rib is equal.

At the same time, because the node has a certain displacement along the Z axis relative to the ideal paraboloid, it can increase the precision of the surface while the number of nodes is certain. When the precision of the surface is fixed, only less nodes are needed.

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