Population Inversion, Negative Temperature, and Quantum Degeneracies

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Abstract

We revisit the basic principle for lasing: Population inversion which is nevertheless closely linked to the negative temperature state in non-equilibrium thermodynamics. With the introduction of quantum degeneracies, we also illustrate their relationship with the lasing via the tuning of population inversion.

Introduction

Atomic and molecular physics are certainly the oldest subfields of quantum physics and everybody knows the major role that they played in the early 1920s when the first principles of quantum mechanics were elaborated. After this golden era, many physicists considered research in these subfields essentially complete and in fact, for several decades, activity in atomic and molecular physics decreased steadily in comparison with the large effort directed towards nuclear and high-energy physics. The advent of lasers (LASER means: Light Amplification by Stimulated Emission of Radiation), and more precisely of tunable lasers, changed this situation and since the early 1970s, atomic and molecular physics, gradually intermingling with laser physics, has become an area where more and more activity is contributing substantially to the understanding of many phenomena.

There has been a gradual change in the use of lasers in the teaching as well as research laboratory. Increasingly, the laser is not being used for spectroscopy, but as a tool [1]. However, the art of making a laser operate is to work out how to get population inversion for the relevant transition. Once we have population inversion, we have a mechanism for generating gain in the laser medium.

Thermodynamic arguments tell us, in addition to the black-body law of radiation, that the interaction between electromagnetic waves and matter at any temperature cannot produce amplification, for radiation at the temperature of matter cannot be made more intense by interaction of the two without violating the second law [2].

Two-Level System

We now wish to study how we can use stimulated emission to make a light amplifier. In a gas of atoms in thermal equilibrium, the population of the lower level \(N_1\) will always be greater...
than the population of the upper level ($N_2$). (Please see Eq. (2) below). Therefore, if a light beam is incident on the medium (cf. Fig. 1), there will always be more upward transitions due to absorption than downward transitions due to stimulated emission (as the absorption is proportional to $N_1$ and the stimulated emission is proportional to $N_2$). Hence there will be net absorption, and the intensity of the beam will diminish on progressing through the medium. To amplify the beam, we require that the rate of stimulated emission transitions exceeds the rate of absorption. If the light beam is sufficiently intense that we can ignore spontaneous emission and the levels are non-degenerate, this implies that $N_2$ must exceed $N_1$. This is a highly non-equilibrium situation, and is called population inversion.

Inspection of Eq. (2) below implies that population inversion corresponds to negative temperatures! This is not as ridiculous as it sounds, because the atoms are not in thermal equilibrium. Once we have population inversion, we have a mechanism for generating gain in the laser medium. The art of making a laser operate is to work out how to get population inversion for the relevant transition.

The rate of change of electromagnetic energy confined in a region where it interacts with a group of particles must, from Einstein’s work, have the form

$$B_{12}N_1 u(\nu) = A_{21}N_2 + B_{21}N_2 u(\nu) \quad (1)$$

where $N_1$ and $N_2$ are the numbers of molecules in the lower and upper of two quantum states. In thermal equilibrium the ratio of $N_2$ to $N_1$ at temperature $T$ is given by Boltzmann’s law:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\hbar \nu / k_B T} \quad (2)$$

where $g_2$, $g_1$ are the degeneracies of level 2,1 respectively, and $\hbar \nu = E_2 - E_1$ (if the energy of thermal motion is sufficient: $k_B T > E_2 - E_1$, then a part of particles are thrown into the upper level).

In fact, we may have the following case: e.g., a light quantum may be absorbed by the medium: and, in this case an absorption is produced. The difference in energy between the upper and lower levels is equal to the quantum energy. This process is connected with the decrease in energy of the electromagnetic field and is called resonance absorption [3].

On the other hand, we may also have: Under the influence of a quantum, a particle may be transferred from the upper level to the lower level. Such a transfer will be accompanied by the emission of a light quantum identical in frequency, direction of propagation and polarization to the quantum which produced the emission. This process is connected with an increase of the field energy and is called stimulated emission.

We can easily understand that, from above expression, we have, once both of the two levels are non-degenerate,

$$\frac{N_2}{N_1} \leq 1 \quad \text{if} \quad T \geq 0. \quad (3)$$
It is just this property of high ordering of a system with negative temperature which makes it possible to produce high-coherent emission in quantum oscillators, to produce high sensitive quantum amplifiers, and to separate the energy stored in the state with negative temperature in a very short time, of the order of the reciprocal of the emission frequency [2].

Meanwhile from the expression of Eq. (2), if both of the two levels are degenerate and $e^{-\frac{\hbar \nu}{k_B T}}$ is not the dominated term at the right-hand-side of Eq. (2), we can still have the population inversion (i.e., $N_2/N_1 > 1$) via the tuning of ratio of $g_2/g_1$ or degeneracies of level 2,1. The possible illustration of equation (2) is shown in Fig. 2 where some cases of $g_2/g_1$ and $\hbar \nu/k_B T$ were selected. Note that there is no population inversion ($N_2/N_1 \leq 1$) for $g_2/g_1 \sim 1$ and $\hbar \nu/k_B T > 0.1$.

**Example : Argon Ion Laser**

To give an example for the latter situation, we shall consider Argon which has 18 electrons with the configuration $1s^22s^22p^63s^23p^6$. Argon atoms incorporated into a discharge tube can be ionized by collisions with the electrons Since there are six 4p levels as compared to only two 4s levels, the statistics of the collisional process leaves three times as many electrons in the 4p level than in the 4s level ($g_2/g_1 = 3$). Hence we have population inversion. Moreover, cascade transitions from higher excited states also facilitates the population inversion mechanism. The lifetime of the 4p level is 10 ns, which compares to the 1 ns lifetime of the 4s level. The partial application of above reasoning leads to the Argon ion (normally Ar$^+$) Lasers.

**Higher-Level Systems**

Some lasers are classified as being three-level systems. The standard example is ruby, which was the first laser ever produced. The key difference between a three-level laser and a four-level laser is that the lower laser level is the ground state. It is much more difficult to obtain population inversion in three-level lasers because the lower laser level initially has a very large population. Let this population be $N_g$. By turning on the pump, we excite $dN$ atoms to level 1, which then decay to level 2. Thus the population of Level 2 will be $dN$, and the population of the ground state will be ($N_g - dN$). Hence for population inversion we require $dN > (N_g - dN)$, that is $dN > N_g/2$. Therefore, in order to obtain population inversion we have to pump more than half the atoms out of the ground state into the upper laser level. This obviously requires a very large amount of energy. This contrasts with the four-level lasers in which the lower laser level is empty before the pumping process starts, and much less energy is required to reach threshold. Despite the fact that the threshold for population inversion is very high in a three-level system, they can be quite efficient once this threshold is overcome. Ruby lasers pumped by bright flash lamps actually give very high output pulse energies. However, they only work in pulsed mode. Continuous lasers tend to be made using four-level systems.
Conclusion

In this short paper by using the energy-level diagram for atoms with two energy levels we revisit the population inversion that is the fundamental principle for lasing and although it is closely related to the negative temperature state in non-equilibrium thermodynamics. We also illustrate the role of quantum degeneracies or their relationship with the lasing via the tuning of population inversion.

References

[1] Siegman A E 1986 *Lasers* (Oxford: Oxford University Press)
   Kocchner W 1977 *Solid-state Laser Engineering* (2nd. ed., Springer series in optical sciences I) (Berlin: Springer)

[2] Townes CH 1964 *Production of coherent radiation by atoms and molecules* (Nobel Lecture, December 11, 1964)
   http://nobelprize.org/nobel_prizes/physics/laureates/1964/Townes-lecture.html

[3] Basov NG 1964 *Semiconductor lasers* (Nobel Lecture, December 11, 1964)
   http://nobelprize.org/nobel_prizes/physics/laureates/1964/Basov-lecture.html
Fig. 1 Schematic 2-level system for absorption and stimulated emission transitions (atoms with two energy levels).
Fig. 2 Illustration of population inversion ($N_2/N_1 > 1$, cf. Eq. (2)) for some cases of degeneracies ($g_2/g_1$) and $\hbar \nu/k_B T$. There is no population inversion ($N_2/N_1 \leq 1$) for $g_2/g_1 \sim 1$ and $\hbar \nu/k_B T > 0.1$ (the lower region where the (black) arrow points out).