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Spectral statistics of Erdős-Rényi graphs II: eigenvalue spacing and the extreme eigenvalues.
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Summary: We consider the ensemble of adjacency matrices of Erdős-Rényi random graphs, i.e. graphs on $N$ vertices where every edge is chosen independently and with probability $p \equiv p(N)$. We rescale the matrix so that its bulk eigenvalues are of order one. Under the assumption $pN \gg N^{2/3}$, we prove the universality of eigenvalue distributions both in the bulk and at the edge of the spectrum. More precisely, we prove

(1) that the eigenvalue spacing of the Erdős-Rényi graph in the bulk of the spectrum has the same distribution as that of the Gaussian orthogonal ensemble; and

(2) that the second largest eigenvalue of the Erdős-Rényi graph has the same distribution as the largest eigenvalue of the Gaussian orthogonal ensemble.

As an application of our method, we prove the bulk universality of generalized Wigner matrices under the assumption that the matrix entries have at least $4 + \epsilon$ moments.

MSC:
05C80 Random graphs (graph-theoretic aspects)
60B20 Random matrices (probabilistic aspects)

Full Text: DOI arXiv

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