Charmed $B$ Decays to $\eta'$ and $\eta$

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Abstract

Exclusive charmless $B$ decays to $\eta'$ and $\eta$ are studied in the standard effective Hamiltonian approach. The effective coefficients that take into account nonfactorizable effects in external and internal $W$-emission amplitudes are fixed by experiment. Factorization is applied to the matrix elements of QCD, electroweak penguin operators and next-to-leading corrections to penguin Wilson coefficients, which are renormalization-scheme dependent, are included, but the resultant amplitude is renormalization-scheme and -scale independent. We show that the unexpectedly large branching ratio of $B^{\pm} \to \eta' K^{\pm}$ measured by CLEO can be explained by the Cabibbo-allowed internal $W$-emission process $b \to c\bar{c}s$ followed by a conversion of the $c\bar{c}$ pair into the $\eta'$ via gluon exchanges characterized by a decay constant of order $-50$ MeV. Implications are discussed. The mechanism of the transition $c\bar{c} \to \eta'$, if correct, will enhance the decay rates of $B \to \eta' K^*$ by more than an order of magnitude. However, it plays only a minor role to the rare decays $B \to \eta' (\eta) \pi (\rho)$. The predicted branching ratio of $B^{\pm} \to \eta \pi^{\pm}$ for positive $\rho$ is marginally ruled out by experiment, indicating that the favored Wolfenstein parameter $\rho$ is negative.
1. The inclusive and exclusive rare $B$ decays to $\eta'$ have recently received a great deal of attention. The CLEO collaboration has reported very large branching ratios for inclusive $\eta'$ production \cite{1}

$$\mathcal{B}(B^\pm \to \eta' X; \ 2.0 \text{ GeV} \leq \rho_{\eta'} \leq 2.7 \text{ GeV}) = (6.2 \pm 1.6 \pm 1.3) \times 10^{-4},$$

and for the exclusive decay $B \to \eta' K$ \cite{2}

$$\mathcal{B}(B^{\pm} \to \eta' K^{\pm}) = \left(7.1^{+2.5}_{-2.1} \pm 0.9\right) \times 10^{-5},$$
$$\mathcal{B}(B^{(0)} \to \eta' (0^0)) = \left(5.3^{+2.8}_{-2.2} \pm 1.2\right) \times 10^{-5}. \tag{2}$$

These results are abnormal large; for example, the branching ratio of $B^{\pm} \to \eta' K^{\pm}$ is theoretically estimated to be of order $(1 - 2) \times 10^{-5}$ in the standard approach, which is too small by $2.5\sigma$ compared to experiment. It is natural to speculate that the unexpected observation has something to do with the unique and special feature of the $\eta'$ meson.

Several mechanisms have been advocated to explain the copious $\eta'$ production in the inclusive process $B \to \eta' + X$: (i) the penguin mechanism $b \to s + g^*$ followed by $g^* \to \eta' + g$ via the QCD anomaly \cite{3, 4}. Whether or not the standard $b \to s + g^*$ penguin alone is adequate to explain the data (1) is still under debate, (ii) the Cabibbo-allowed process $b \to (c\bar{c})_1 + s$ (the subscript 1 denotes color singlet), followed by the transition $(c\bar{c})_1 \to$ two gluons $\to \eta'$. It was argued in \cite{5} that the decay constant $f_{\eta'}^{(c\bar{c})}$, to be defined below, should be as large as 140 MeV in order to explain the inclusive $\eta'$ data solely by this mechanism, and (iii) the process $b \to (c\bar{c})_8 + s$, analogous to (ii) but with the $c\bar{c}$ pair in a color-octet state, followed by $(c\bar{c})_8 \to \eta' + X$ \cite{3}.

In this Letter we will study the exclusive charmless $B$ decays to $\eta'$ and $\eta$ and show that the CLEO measurement of $B^{\pm} \to \eta' K^{\pm}$ is indeed larger than what expected from the standard approach based on the effective Hamiltonian and factorization. We then proceed to suggest that two of the aforementioned mechanisms, though important for inclusive $\eta'$ production, do not play any significant role to the two-body decay $B \to \eta' K$ and that a reasonable amount of the charm content of the $\eta'$ arising from the transition $c\bar{c} \to \eta'$, corresponding to $|f_{\eta'}^{(c\bar{c})}| \sim 50$ MeV, suffices to account for the data. Many interesting implications due to this mechanism are discussed.

2. The hadronic rare $B$ decays and their CP violation have been studied in detail in \cite{7, 8} (see also \cite{3, 11, 12}). The relevant effective $\Delta B = 1$ weak Hamiltonian is

$$\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[V_{ub}V_{uq}^*(c_1O_1^u + c_2O_2^u) + V_{cb}V_{cq}^*(c_1O_1^c + c_2O_2^c) - V_{tb}V_{tq}^* \sum_{i=3}^{10} c_i O_i \right] + \text{h.c.}, \tag{3}$$

where $q = u, d, s$, and

$$O_1^u = (\bar{u}b)_{V-A}(\bar{q}u)_{V-A}, \quad O_1^c = (\bar{c}b)_{V-A}(\bar{q}c)_{V-A},$$
$$O_2^u = (\bar{q}b)_{V-A}(\bar{u}u)_{V-A}, \quad O_2^c = (\bar{q}b)_{V-A}(\bar{c}c)_{V-A},$$
$$O_3 = (\bar{q}b)_{V-A} \sum_{q'}(\bar{q}'q')_{V-A}, \quad O_4 = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A},$$
$$O_5 = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A}, \quad O_6 = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A},$$
$$O_7 = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A}, \quad O_8 = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A},$$
$$O_9 = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A}, \quad O_{10} = (\bar{q}b_{V-A})(\bar{q}b_{V-A}) \sum_{q'}(\bar{q}'q')_{V-A}.$$
The Wilson coefficients $c_i(\mu)$ in Eq. (1) have been evaluated at the renormalization scale $\mu \sim m_b$ to the next-to-leading order. Beyond the leading logarithmic approximation, they depend on the choice of the renormalization scheme (for a review, see [12]). The mesonic matrix elements are customarily evaluated under the factorization hypothesis. In order to ensure the $\mu$ and renormalization scheme independence for the physical amplitude, the matrix elements have to be computed in the same renormalization scheme and renormalized at the same scale as $c_i(\mu)$. In the naive factorization approach, the relevant Wilson coefficients for external $W$-emission (or so-called “class-I”) and internal $W$-emission (or “class-II”) nonpenguin amplitudes are given by $a_1 = c_1 + c_2/N_c$ and $a_2 = c_2 + c_1/N_c$, respectively, with $N_c$ the number of colors. However, it is well known that the factorization approximation does not work for color-suppressed class-II decay modes (for a review, see [13]). This means that it is mandatory to take into account the nonfactorizable effects, especially for class-II modes.

For $B \to PP$ or $VP$ decays ($P$: pseudoscalar meson, $V$: vector meson), nonfactorizable contributions can be lumped into the effective parameters $a_1$ and $a_2$ [14, 15, 16]:

$$a_1^{\text{eff}} = c_1(\mu) + c_2(\mu) \left( \frac{1}{N_c} + \chi_1(\mu) \right), \quad a_2^{\text{eff}} = c_2(\mu) + c_1(\mu) \left( \frac{1}{N_c} + \chi_2(\mu) \right),$$  

(5)

where $\chi_i$ are nonfactorizable terms and receive main contributions from the color-octet current operators. Without loss of generality, the effective parameters $a_i^{\text{eff}}$ include all the contributions to the matrix elements and the nonfactorizable terms $\chi_i(\mu)$ take care of the correct $\mu$ dependence of the matrix elements [17]. Schematically, we have

$$\langle c(\mu)O(\mu) \rangle \to a^{\text{eff}} \langle O^{\text{tree}} \rangle_{\text{fact}},$$

(6)

where $\langle O^{\text{tree}} \rangle_{\text{fact}}$ is the matrix element of the tree operator $O^{\text{tree}}$ evaluated using the factorization method. Therefore, $a_i^{\text{eff}}$ are renormalization-scheme and -scale independent. Although nonfactorizable contributions are nonperturbative in nature and thus difficult to estimate, the effective coefficients $a_i^{\text{eff}}$ can be determined from experiment. From $B \to D^{(*)}\pi(\rho)$ and $B \to J/\psi K^{(*)}$ decays, we found that $a_1 \sim (0.96 - 1.10)$ [18] and $a_2 = 0.26 \sim 0.30$ [15]. In the present analysis we assign the values

$$a_1^{\text{eff}} = 1.02, \quad a_2^{\text{eff}} = 0.28,$$

(7)

for exclusive two-body decays of $B$ mesons. By comparing (5) with (7), we learn that factorization works reasonably well for class-I decay modes, but nonfactorizable effects play an essential role for class-II channels.
Unlike the effective coefficients $a_{1,2}^\text{eff}$, which can be empirically determined from experiment, \textit{a priori} it is not clear how reliable is the factorization approach for the matrix elements of the QCD and electroweak penguin operators. Recently CLEO II has reported preliminary results of charmless hadronic $B$ decays, namely, $B \to K^{\pm} \pi^\mp$, $B \to K^0 \pi^\pm$ \cite{1,13}, dominated by the penguin mechanism. It turns out that the predictions based on factorization are in fair agreement with data \cite{20}. Although at present we are not able to determine the analogous effective penguin coefficients that include nonfactorizable effects from experiment, we can nevertheless construct penguin Wilson coefficients that are independent of the choice of the renormalization scale and the renormalization scheme (for a review, see \cite{21}). The one-loop QCD and electroweak corrections to matrix elements are parametrized by the matrices $\hat{m}_s$ and $\hat{m}_e$, respectively \cite{22,23}:

$$\langle O_i(\mu) \rangle = \left[ I + \frac{\alpha_s(\mu)}{4\pi} \hat{m}_s(\mu) + \frac{\alpha}{4\pi} \hat{m}_e(\mu) \right]_{ij} \langle O^\text{tree}_j \rangle.$$  \hspace{1cm} (8)

As a consequence,

$$c_i(\mu)\langle O_i(\mu) \rangle = c_i(\mu) \left[ I + \frac{\alpha_s(\mu)}{4\pi} \hat{m}_s(\mu) + \frac{\alpha}{4\pi} \hat{m}_e(\mu) \right]_{ij} \langle O^\text{tree}_j \rangle \equiv \tilde{c}_j \langle O^\text{tree}_j \rangle.$$  \hspace{1cm} (9)

One-loop penguin matrix elements of the operators $O_{1,2}$ have been calculated with the results \cite{22,23} (penguin diagrams for the matrix elements of penguin operators are calculated in \cite{10}):

$$\hat{m}_s(\mu)_{13} = \hat{m}_s(\mu)_{15} = -3\hat{m}_s(\mu)_{14} = -3\hat{m}_s(\mu)_{16} = \frac{1}{6} \left[ \frac{2}{3} \kappa + G(m_c, k, \mu) \right],$$

$$\hat{m}_e(\mu)_{17} = \hat{m}_e(\mu)_{19} = 3\hat{m}_e(\mu)_{27} = 3\hat{m}_e(\mu)_{29} = -\frac{4}{9} \left[ \frac{2}{3} \kappa + G(m_c, k, \mu) \right],$$  \hspace{1cm} (10)

where $\kappa$ parametrizes the renormalization scheme dependence. For example, for the naive dimension regularization (NDR) scheme and the ‘t Hooft-Veltman (HV) scheme, $\kappa$ is given by \cite{23}:

$$\kappa = \begin{cases} -1 & \text{(NDR)}, \\ 0 & \text{(HV)}. \end{cases}$$  \hspace{1cm} (11)

The function $G(m_c, k, \mu)$ in Eq. (10) defined by

$$G(m_c, k, \mu) = -4 \int_0^1 dx x(1-x) \ln \left[ \frac{m_c^2 - k^2 x(1-x)}{\mu^2} \right],$$  \hspace{1cm} (12)

has the analytic expression:

$$\text{Re} \ G = \frac{2}{3} \left( -\ln \frac{m_c^2}{\mu^2} + \frac{5}{3} + \frac{4}{3} \frac{m_c^2}{k^2} - (1 + 2 \frac{m_c^2}{k^2}) \sqrt{1 - 4 \frac{m_c^2}{k^2}} \ln \frac{1 + \sqrt{1 - 4 \frac{m_c^2}{k^2}}}{1 - \sqrt{1 - 4 \frac{m_c^2}{k^2}}} \right),$$

$$\text{Im} \ G = \frac{2}{3} \pi \left( 1 + 2 \frac{m_c^2}{k^2} \right) \sqrt{1 - 4 \frac{m_c^2}{k^2}},$$  \hspace{1cm} (13)
for \(k^2 > 4m_c^2\), where \(k^2\) is the momentum squared of the virtual gluon. Note that the imaginary part of \(G(m, k, \mu)\), which furnishes the desired dynamic strong phase necessary for generating CP asymmetries in charged \(B\) decays, arises only in the time-like penguin diagram with \(k^2 > 4m^2\). Eqs. (11-15) we find that \(\tilde{c}_i\), \(c_i\) dependent Wilson coefficients \(\bar{c}_i\), renormalization-scheme independent \(\bar{c}_i\), diagram with \(k^2 > 4m^2\). Eqs. (9) and (10) lead to the renormalization scale and scheme independent penguin Wilson coefficients:

\[
\begin{align*}
\tilde{c}_3 &= c_3(\mu) + \frac{1}{3} \bar{P}_s(\mu), & \tilde{c}_4 &= c_4(\mu) - \bar{P}_s(\mu), & \tilde{c}_5 &= c_5(\mu) + \frac{1}{3} \bar{P}_s(\mu), \\
\tilde{c}_6 &= c_6(\mu) - \bar{P}_s(\mu), & \tilde{c}_7(9) &= c_7(9)(\mu) - \bar{P}_e(\mu), & \tilde{c}_8(10) &= c_8(10),
\end{align*}
\]

where

\[
\begin{align*}
\bar{P}_s(\mu) &= \frac{\alpha_s(\mu)}{8\pi} \left[\frac{2}{3} \kappa + G(m_c, k, \mu)\right] c_1(\mu), \\
\bar{P}_e(\mu) &= \frac{\alpha}{9\pi} \left[\frac{2}{3} \kappa + G(m_c, k, \mu)\right] [c_1(\mu) + 3c_2(\mu)].
\end{align*}
\]

As noted before, the Wilson coefficients \(c_i(\mu)\) have been evaluated to the next-to-leading order in the NDR, HV and RI (regularization independent) renormalization schemes [12, 23]. For the purpose of illustration, we use the \(\Delta B = 1\) Wilson coefficients obtained in HV and NDR schemes at \(\mu = 4.4\) GeV, \(\Lambda_{\overline{MS}}^{(5)} = 225\) MeV and \(m_t = 170\) GeV in Table 22 of [12]. From Eqs. (11-15) we find that \(\tilde{c}_3 - \tilde{c}_{10}\), obtained from \(c_i(\mu)\) in the HV and NDR schemes are numerically very similar, implying the renormalization scheme independence of \(\tilde{c}_i\). The values of \(\tilde{c}_i\) at \(k^2 = m_b^2/2\) are

\[
\begin{align*}
\tilde{c}_3 &= 0.0200 + i 0.0048, & \tilde{c}_4 &= -0.0515 - i 0.0143, \\
\tilde{c}_5 &= 0.0155 + i 0.0048, & \tilde{c}_6 &= -0.0565 - i 0.0143, \\
\tilde{c}_7 &= -(0.0757 + i 0.0558)\alpha, & \tilde{c}_8 &= 0.057\alpha, \\
\tilde{c}_9 &= -(1.3648 + i 0.0558)\alpha, & \tilde{c}_{10} &= 0.264\alpha.
\end{align*}
\]

One can also obtain \(\tilde{c}_i\) by first evaluating the renormalization scheme independent but \(\mu\) dependent Wilson coefficients \(\bar{c}_i(\mu)\), and then connecting them to \(\tilde{c}_i\) via the relation

\[
\tilde{c}_i = \left[1 + \frac{\alpha_s(\mu)}{4\pi}(\tilde{m}_s(\mu) - \tilde{r}_s(\mu)) + \frac{\alpha}{4\pi}(\tilde{m}_c(\mu) - \tilde{r}_c(\mu))\right]_{ji} c_j(\mu),
\]

where the expressions of the matrices \(\tilde{r}_{s,c}(\mu)\) are given in [22, 23]. Using the values of \(\bar{c}_i(\mu)\) obtained at \(\mu = m_b = 5\) GeV in [23], the reader can check that \(\tilde{c}_3 - \tilde{c}_{10}\), obtained in this way are numerically in accordance with (16) except for a small discrepancy with \(\tilde{c}_7\). This shows the renormalization scale \(\mu\) independence of the Wilson coefficients \(\tilde{c}_i\).

Two remarks are in order. (i) One can equally use the Wilson coefficients \(c_i(\mu)\) or the renormalization-scheme independent \(\bar{c}_i(\mu)\) to calculate the physical amplitude. However, the matrix elements of four-quark operators must be renormalized at the same scale \(\mu\) and evaluated in the same renormalization scheme for the latter. The great advantage of working with the renormalization-scale and -scheme independent \(a_{1,2}^{\text{eff}}\) and \(\tilde{c}_3 - \tilde{c}_{10}\) is two-fold: First, the
corresponding tree-level operators are scale and scheme independent and hence they can be evaluated using factorization. Second, long-distance nonfactorizable effects for external and internal $W$-emissions are already included in the effective coefficients $a_{1,2}^{\text{eff}}$. (ii) It is instructive to compare $\tilde{c}_i$ with the leading-order Wilson coefficients $c_i(\mu)$ evaluated at $\mu = 4.4$ GeV and $\Lambda_{\overline{\text{MS}}}^{(5)} = 225$ MeV (see Table 22 of [12]):

$$
c_3 = 0.014, \quad c_4 = -0.030, \quad c_5 = 0.009, \quad c_6 = -0.038, \quad c_7 = 0.045 \alpha, \quad c_8 = 0.048 \alpha, \quad c_9 = -1.280 \alpha, \quad c_{10} = 0.328 \alpha. \quad (18)
$$

It is evident that next-to-leading order corrections to the Wilson coefficients of QCD penguin operators are quite sizable.

3. We are ready to employ the weak Hamiltonian (1) and the factorization approximation to calculate exclusive charmless rare $B$ decays. We take the decay $B^- \to \eta'K^-$ as an example. Factorization leads to

$$
A(B^- \to \eta'K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* \left( a_{11}^\text{eff} X_1 + a_{12}^\text{eff} X_{2u} + a_{13}^\text{eff} X_3 \right) + V_{cb}V_{cs}^* a_2^\text{eff} X_{2u} (f_{\eta'}^{(\text{eff})}/f_{\eta'}^{(\text{eff})}) \right\},
$$

where $a_{2i} \equiv \tilde{c}_{2i} + \frac{1}{N_c} \tilde{c}_{2i-1}$, $a_{2i-1} \equiv \tilde{c}_{2i-1} + \frac{1}{N_c} \tilde{c}_{2i}$ ($N_c = 3$) for $i \geq 2$, $X_i$ are factorizable terms:

$$
X_1 \equiv \langle \tilde{K}^- | (\bar{s}u)_{\nu-A} | 0 \rangle \langle \eta' | (\bar{u}b)_{\nu-A} | B^- \rangle = i f_{\tilde{K}} (m_{\tilde{K}}^2 - m_{\eta'}^2) F_0^{\tilde{B}_0}(m_{\tilde{K}}^2),
$$

$$
X_{2d} \equiv \langle \eta' | (\bar{q}q)_{\nu-A} | 0 \rangle \langle \tilde{K}^- | (\bar{s}b)_{\nu-A} | B^- \rangle = i f_{\eta'}^{(\text{eff})} (m_B^2 - m_{\eta'}^2) F_0^{\tilde{B}_0}(m_{\eta'}^2),
$$

$$
X_3 \equiv \langle \eta' \tilde{K}^- | (\bar{s}u)_{\nu-A} | 0 \rangle \langle 0 | (\bar{u}b)_{\nu-A} | B^- \rangle,
$$

and $F_0$ is the form factor defined in [26]. In the derivation of Eq. (19) we have applied the isospin relation $X_{2d} = X_{2u}$ and equations of motion for $(S - P)(S + P)$ penguin matrix elements. The wave functions of the physical $\eta'$ and $\eta$ states are related to that of the SU(3) singlet state $\eta_0$ and octet state $\eta_8$ by

$$
\eta' = \eta_8 \sin \theta + \eta_0 \cos \theta, \quad \eta = \eta_8 \cos \theta - \eta_0 \sin \theta \quad (21)
$$

with $\theta \approx -20^\circ$. When the $\eta - \eta'$ mixing angle is $-19.5^\circ$, the $\eta'$ and $\eta$ wave functions have simple expressions [8]

$$
|\eta'\rangle = \frac{1}{\sqrt{6}} |\bar{u}u + \bar{d}d + 2\bar{s}s\rangle, \quad |\eta\rangle = \frac{1}{\sqrt{3}} |\bar{u}u + \bar{d}d - \bar{s}s\rangle. \quad (22)
$$
For simplicity we will employ $\theta = -19.5^\circ$ in ensuing discussion. In SU(3) limit, the $\eta'$ and $\eta$ decay constants become

\[
\begin{align*}
F^{(uu)}_{\eta'} &= F_{\eta'}(dd) = F_{\eta'}(ss)/2 = f_{\pi}/\sqrt{6} = 54 \text{ MeV}, \\
F_{\eta'}(uu) &= -F_{\eta'}(dd) = -f_{\eta}/\sqrt{3} = 77 \text{ MeV}.
\end{align*}
\]

(23)

The measured physical decay constants $f_\eta$ and $f_{\eta'}$ are fairly close to $f_{\pi}$ [27]. As a result, in SU(3) limit, $X_{2s} = 2X_{2u}$ for $\eta'$ production and $X_{2s} = -X_{2u}$ for the $\eta$. In the limit of SU(3)-flavor symmetry, we also have

\[
F_0^{B_K}(0) = \sqrt{6}F_0^{B_{\eta'}}(0) = F_0^{B_{\pi \pm}}(0),
\]

(24)

where the factor of $\sqrt{6}$ comes from the normalization constant of the $\eta'$ wave function [see Eq. (22)].

For non-penguin external and internal $W$-emission contributions to $B^{\pm} \rightarrow \eta'K^{\pm}$, we employ the effective coefficients $a_{1,2}^{qq}$ [Eq. (7)] since they take into account nonfactorizable effects. In Eq. (19) $X_3$ stands for the $W$-annihilation contribution. In the penguin mechanism, $X_3$ arises from the space-like penguin diagram. It is common to argue that $W$-annihilation is negligible due to helicity suppression, corresponding to form factor suppression at large momentum transfer, $q^2 = m_B^2$ (for a recent study, see [29]). However, we see from Eq. (19) that it is largely enhanced in the space-like penguin diagram by a factor of $m_B^2/(m_u m_s)$. Unfortunately, we do not have a reliable method for estimating $W$-annihilation, though recent PQCD calculations suggest that space-like penguins are small compared to the time-like ones [30].

In terms of the Wolfenstein parametrization [31], the relevant quark mixing matrix elements for $B \rightarrow \eta'K$ are

\[
V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta), \quad V_{cb}V_{cs}^* = A\lambda^2(1 - \frac{\lambda^2}{2}), \quad V_{tb}V_{ts}^* = -A\lambda^2,
\]

(25)

where $\lambda = 0.22$ and $A = 0.804$ for $|V_{cb}| = 0.039$. Since the parameters $\rho$ and $\eta$ have not been measured separately, it is desirable to have the results presented for allowed regions of $\rho$ and $\eta$. As $\rho$ and $\eta$ are approximately constrained by $0.2 < \eta < 0.4$ and $-0.3 < \rho < 0.3$ (see, e.g., [14]), we will fix $\eta \sim 0.30$ and take $\rho = 0.30$ as well as $\rho = -0.30$. We also need input of other parameters. For quark masses we use $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 175$ MeV, $m_c = 1.5$ GeV and $m_b = 5.0$ GeV. Heavy-to-light mesonic form factors have been evaluated in the relativistic quark model [26, 32]. To be specific, we use $F_0^{B\eta'}(0) = 0.254/\sqrt{6}$ [24] and $F_0^{B_K}(0) = 0.34$ [32]. For the lifetime of $B$ mesons, the world average is $\tau(B_d) = (1.55 \pm 0.04)$ ps and $\tau(B^{\pm}) = (1.66 \pm 0.04)$ ps [33]. Since what CLEO has measured is the combined branching ratio

\[
\mathcal{B}(B^{\pm} \rightarrow \eta'K^{\pm}) \equiv \frac{1}{2} \left[ \mathcal{B}(B^+ \rightarrow \eta'K^+) + \mathcal{B}(B^- \rightarrow \eta'K^-) \right],
\]

(26)

\footnote{It should be stressed that the form factors given in [20] for $B(D) \rightarrow \pi^0$, $\eta$, $\eta'$ transitions do not take into account the normalization constant of the neutral meson wave functions.}
we shall present the results for the combined one. Neglecting the $W$-annihilation contribution characterized by the parameter $X_3$, we find that in the absence of the transition $c\bar{c} \to \eta'$ (see Table I),

$$B(B^\pm \to \eta' K^\pm) = (1.4 - 1.8) \times 10^{-5} \quad \text{for } f_{\eta'}^{(c\bar{c})} = 0,$$

which is substantially smaller than the CLEO measurement (2). It is worth emphasizing that if the Wilson coefficients are evaluated only to the leading order [cf. Eq. (18)], then the resultant predictions become even worse: $B(B^\pm \to \eta' K^\pm)_{\text{L.O.}} = (0.5 - 0.8) \times 10^{-5}$. In fact, the leading order calculation is already ruled out by the recent CLEO measurement of $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\pm K^\pm$.

We briefly discuss the uncertainties associated with the prediction of $B(B^\pm \to \eta' K^\pm)$ and the numerical results given in Table I. We have neglected $W$-annihilation, space-like penguin diagrams and nonfactorizable contributions to the matrix elements of penguin operators; all of them are difficult to estimate. Other major sources of uncertainties come from the choice of form factors and light quark masses. It is natural to ask if the CLEO result for $B^\pm \to \eta' K^\pm$ can be explained by different choices of the strange quark mass $m_s$ and form factors. For a given set of form factors, it is easily seen that the branching ratio increases with decreasing $m_s$. For example, we find a large branching ratio $B(B^\pm \to \eta' K^\pm) = 6.5 \times 10^{-5}$ for $m_s = 60$ MeV and $\rho < 0$. However, the use of a small strange quark mass is in conflict with a recent measurement in $\tau$ decay, $m_s(m_\tau) = 212^{+30}_{-35}$ MeV [34]. It is also true that the measured branching ratio of the $\eta'K$ mode can be accommodated by choosing large form factors, e.g., $F_0^{B^\pm K}(0) \approx \sqrt{6} F_0^{B\eta'}(0) \approx 0.61$. However, this will break the SU(3)-symmetry relation (24) very badly. All the existing QCD sum rule, lattice and quark model calculations indicate that $F_0^{B\pm}(0) \approx 0.33$ or less (for a review, see [33]). The recent observed charmless decays $B \to K^\pm \pi^\mp, B \to K^0 \pi^\pm$ dominated by the penguin diagram are governed by the form factor $F_0^{B^\pm}$. We find that the predictions for $B \to K\pi$ based on the standard approach with $F_0^{B^\pm}(0) = 0.33$ are in agreement with data. Therefore, in view of the SU(3) relation (24) for form factors, it is very unlikely that $F_0^{B\eta'}(0)$ can deviate much from 0.33. We should also stress that it is dangerous to adjust the form factors in order to fit a few particular modes; the comparison between theory and experiment should be done using the same set of form factors for all channels. We thus conclude that the large branching ratio of $B^\pm \to \eta' K^\pm$ observed by CLEO cannot be explained by the conventional approach.

The sizable discrepancy between theory (27) and experiment (2) strongly suggests a new mechanism unique to the $\eta'$ meson. It appears that the most natural one is the Cabibbo-allowed process $b \to c\bar{c}s$ followed by a conversion of the $c\bar{c}$ pair into the $\eta'$ via two gluon exchanges [see Eq. (19)]. This new internal $W$-emission contribution is important since its mixing angle $V_{cb}V_{cs}^*$ is as large as that of the penguin amplitude and yet its Wilson coefficient $a_2^{\text{eff}}$ is larger than that of penguin operators. The decay constant $f_{\eta'}^{(c\bar{c})}$, defined as $(0|\bar{c}c_{\mu}I_{\gamma}c|\eta') = f_{\eta'}^{(c\bar{c})} q_\mu$, can be reduced via the OPE to a matrix element of a particular

\[ \sqrt{6} F_0^{B\eta'}(0) \approx 0.61. \]
dimension-6 pseudoscalar gluonic operator [28]. It has been estimated in [28], based on the OPE, large-\(N_c\) approach and QCD low energy theorems, that \(|f^{(sc)}_{\eta'}| = (50 - 180)\) MeV. (An independent estimate of \(f^{(sc)}_{\eta'}\) based on the instanton-liquid model is given in [37].) It was claimed in [28] that \(|f^{(sc)}_{\eta'}| \sim 140\) MeV is needed in order to exhaust the CLEO observation of \(B^\pm \to \eta'K^\pm\) and \(B \to \eta' + X\) by the mechanism \(b \to c\bar{c} + s \to \eta' + s\) via gluon exchanges. In view of the fact that \(f^{(ua)}_{\eta'}\) is only of order 50 MeV, a large value of \(f^{(sc)}_{\eta'}\) seems to be very unlikely. We believe that it is more plausible to take the lower side of the theoretical estimate, namely \(|f^{(sc)}_{\eta'}| \sim 50\) MeV. \(\Box\) On the other hand, it is claimed in [28] that \(|f^{(cc)}_{\eta'}| \leq 40\) MeV. From Eq. (19) it is clear that this additional contribution will enhance the decay rate of \(B^\pm \to \eta'K^\pm\) provided that the sign of \(f^{(cc)}_{\eta'}\) is opposite to that of \(f^{(ua)}_{\eta'}\). (Note that the estimate of \(f^{(cc)}_{\eta'}\) in [28] does not fix its sign.) For \(f^{(cc)}_{\eta'} = -50\) MeV, we obtain

\[
\begin{align*}
B(\eta^\pm \to \eta'K^\pm) & = (5.8 - 6.7) \times 10^{-5}, \\
B(\eta^0 \to \eta'K^0) & = (5.7 - 5.9) \times 10^{-5},
\end{align*}
\]  

(28)

which are in agreement with the CLEO measurement (2), especially when \(\rho\) is negative. Contrary to what has been claimed in [28], it is not necessary to introduce a large value of \(f^{(cc)}_{\eta'}\) to account for the experimental observation of \(B^\pm \to \eta'K^\pm\).

Suppose the dominant mechanism for the observed large inclusive \(B^\pm \to \eta' + X\) signal comes either from the \(b \to s g^*\) penguin followed by the transition \(g^* \to g\eta'\) via the QCD anomaly, or from the process \(b \to (c\bar{c})_8 + s\), followed by the conversion \((c\bar{c})_8 \to \eta' + X\). Will these mechanisms still play an important role in exclusive \(B^\pm \to \eta'K^\pm\) decays? It is easily seen that since the above two mechanisms involve a production of a gluon before hadronization, they will not make contributions to two-body exclusive decays unless the gluon is soft and absorbed in the wave function of the \(\eta'\). Another possibility is the penguin-like process \(b \to s + g^*g^* \to s + \eta'\), but it is a higher order penguin mechanism.

The factorizable contribution to \(B^- \to \eta K^-\) has a similar expression as (19) for \(B^- \to \eta'K^-\) with the replacement

\[
f^{(cc)}_{\eta'} \to f^{(cc)}_{\eta'} \approx -\sin \theta f^{(cc)}_{\eta'},
\]  

(29)

where \(\theta \approx -20^\circ\) is the \(\eta - \eta'\) mixing angle. Because of the destructive interference in the penguin diagrams due to the relation \(X_{2s} = -X_{2u}\) in SU(3) limit and because of the smallness of the \(c\bar{c} \to \eta_0\) contribution to \(\eta\) production, the decay \(B \to \eta K\) is suppressed. We see from Table I that the branching ratio of \(B \to \eta K\) is smaller than that of \(B \to \eta'K\) by an order of magnitude. Nevertheless, a measurement of \(B(\eta \to \eta K)\) of order \((2 - 5) \times 10^{-6}\) will confirm the importance of the \(c\bar{c} \to \eta'\) mechanism. From Table I we see that the electroweak penguin effects are in general very small, but they become important for \(B \to \eta K\) and \(B \to \eta K^*\) decays due to a large cancellation of QCD penguin contributions in these decay modes. It is worth stressing that CP asymmetries in \(B^\pm \to \eta(\eta')K^\pm(K^{*\pm})\) decays will be largely diluted in the presence of the \(c\bar{c}\) conversion into the \(\eta'\). \(\Box\)

\(^3\)The mixing angle of the \(\eta'\) with the \(c\bar{c}\) state is of order 7° for \(|f^{(cc)}_{\eta'}| \sim 50\) MeV.
The mechanism of $c\bar{c}$ conversion into $\eta'$, if correct, is probably most dramatic in the decay $B \to \eta'K^*$. The factorization amplitude for $B^- \to \eta'K^{*-}$ is given by

$$A(B^- \to \eta'K^{*-}) = \frac{G_F}{\sqrt{2}} \left[ V_{tb}V_{ts}^* \left( e_X X'_1 + a_X X'_2 + a_1 X'_3 \right) + V_{cb}V_{cs}^* f^{(cc)}(f_{\eta'}/f_{\eta'}) \right]$$

where

$$X'_1 \equiv \langle K^{*-} | (\bar{s}u)_{v-A} | 0 \rangle \langle \eta' | (\bar{u}b)_{v-A} | B^- \rangle = -2 f_{K^*} m_{K^*} F_1^{B\eta'}(m_{K^*}) (\varepsilon \cdot p_B),$$

$$X'_2 \equiv \langle \eta' | (\bar{q}q)_{v-A} | K^{*-} \rangle \langle (s \bar{b})_{v-A} | B^- \rangle = -2 f_{\eta'}(m_{K^*}) A_0^{BK^*}(m_{\eta'}^2) (\varepsilon \cdot p_B),$$

$$X'_3 \equiv \langle \eta' | K^{*-} | (\bar{s}u)_{v-A} | 0 \rangle \langle 0 | (\bar{u}b)_{v-A} | B^- \rangle.$$ 

(Note that, contrary to $B \to \eta'K$ decay, there is no contribution proportional to $X'_1$ arising from the $(S-P)(S+P)$ part of the penguin operators $O_6$ and $O_8$. The $q^2$ dependence of the form factors $F_0$, $F_1$, $A_0$, defined in [20], can be calculated in the framework of the relativistic quark model. A direct calculation of $B \to P$ and $B \to V$ form factors at time-like momentum transfer in the relativistic light-front quark model just became available recently [32]. It is found that $A_0$, $F_1$ exhibit a dipole behavior, while $F_0$ shows a monopole dependence. At $q^2 = 0$, the relevant form factors are [32]

$$F_1^{BK}(0) = 0.34, \quad F_1^{BP}(0) = 0.26, \quad A_0^{BK^*}(0) = 0.32, \quad A_0^{B\rho}(0) = 0.28.$$ 

(32)

We then utilize the above-mentioned $q^2$ behavior to compute the form factors at the desired momentum transfer squared. Owing to the small penguin contributions, the standard approach’s estimate of $B(B \to \eta'K^*)$ is small, of order $(2 - 5) \times 10^{-7}$ for $B^\pm \to \eta'K^{*\pm}$ and $1 \times 10^{-7}$ for $B^0 \to \eta'K^{*0}$. Since the branching ratio due to the process $b \to c\bar{c} + s \to \eta' + s$ is of order $10^{-5}$, $B(B \to \eta'K^*)$ is greatly enhanced to the order of $(1 - 2) \times 10^{-5}$. It has been argued in [5] that $B(B \to \eta'K^*)$ is about twice larger than that of $B \to \eta'K$, which is certainly not the case in our calculation. It is interesting to note that the ratio $B(B \to \eta'K^*)/B(B \to \eta'\eta)$ is much less than unity in the naive estimate [38], but it becomes greater than unity in the presence of $c\bar{c}$ conversion into the $\eta_0$ (see Table 1).

For $B \to \eta'(\eta)\pi(\rho)$ decays, the mechanism of $c\bar{c} \to \eta_0$ is less dramatic since it does not gain mixing-angle enhancement as in the case of $B \to \eta'(\eta)K(K^*)$. We see from Table I that the full branching ratios in general are close to the predictions in the conventional approach, indicating the minor role played by the charm content of the $\eta'$. Three remarks are in order. First, there is a large enhancement of order $m_{\eta'}^2(m_b)/(m_q m_b)$ ($q = u, d$) occurred in the matrix elements of $(S-P)(S+P)$ penguin operators. The calculation in this case is thus sensitive
to the light quark masses $m_u$ and $m_d$. Second, the branching ratio of $B^\pm \to \eta \pi^\pm$ is predicted to be $1 \times 10^{-5}$ for positive $\rho$, while experimentally $\mathcal{B}(B^\pm \to \eta h^\pm) < 8.0 \times 10^{-5}$ for $h = \pi, K$ [1]. This again indicates that a negative $\rho$ is preferable. Third, the decays $B^\pm \to \eta \rho^\pm$ are overwhelmingly dominated by tree diagrams.

4. To summarize, in view of the recent unexpectedly large branching ratios for inclusive and exclusive charmless $B$ decays to $\eta'$, we have analyzed the exclusive $B$ decays to $\eta'$ and $\eta$ in detail. We showed that the process $b \to c\bar{c} + s \to \eta' + s$ via gluon exchanges with $f_{\eta'}^{(cc)} \sim -50$ MeV is adequate to explain the discrepancy between theory and experiment. A measurement of $\mathcal{B}(B \to \eta' K^*)$ of order $10^{-5}$ will give a strong support of this gluon mechanism of $\eta'$ production. Moreover, we found that the decay rate of $B \to \eta' K^*$ is larger than $B \to \eta K^*$, contrary to what expected from the standard approach. The predicted branching ratio of $B^\pm \to \eta \pi^\pm$ for positive $\rho$ is marginally ruled out by experiment, implying that a negative $\rho$ is preferable.

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Note added: The exclusive decays of $B$ to $\eta'$ and $\eta$ were also discussed recently by A. Ali and C. Greub [hep-ph/9707251], A. Datta, X.G. He and S. Pakvasa [hep-ph/9707259] with conclusions different from ours.
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