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Advanced coding techniques for few mode transmission systems

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Abstract: We experimentally verify the advantage of employing advanced coding schemes such as space-time coding and 4 dimensional modulation formats to enhance the transmission performance of a 3-mode transmission system. The performance gain of space-time block codes for extending the optical signal-to-noise ratio tolerance in multiple-input multiple-output optical coherent spatial division multiplexing transmission systems with respect to single-mode transmission performance are evaluated. By exploiting the spatial diversity that few-mode-fibers offer, with respect to single mode fiber back-to-back performance, significant OSNR gains of 3.2, 4.1, 4.9, and 6.8 dB at the hard-decision forward error correcting limit are demonstrated for DP-QPSK 8, 16 and 32 QAM, respectively. Furthermore, by employing 4D constellations, $6 \times 28$Gbaud 128 set partitioned quadrature amplitude modulation is shown to outperform conventional 8 QAM transmission performance, whilst carrying an additional 0.5 bit/symbol.

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References and links

1. R. W. Tkach, “Scaling optical communications for the next decade and beyond,” Bell Labs Tech. J.,4 (14), pp. 3–9, Feb. (2010).
2. A. D. Ellis, “The Nonlinear Shannon limit and The Need for New Fibres,” Proc. SPIE 8434, 84340H (2012).
3. R.-J. Essiambre and R. W. Tkach, “Capacity Trends and Limits of Optical Communication Networks,” Proc. IEEE 100(5), 1035–1055 (2012).
4. J. Sakaguchi, B. J. Puttnam, W. Klaus, Y. Awaji, N. Wada, A. Kanno, T. Kawanishi, K. Imamura, H. Inaba, K. Mukasa, R. Sugizaki, T. Kobayashi, and M. Watanabe, “305Tb/s Space Division Multiplexed Transmission Using Homogeneous 19-Core Fiber,” J. Lightwave Technol. 31(4), 554–562 (2013).
5. K. Igarashi, T. Tsuritani, I. Morita, Y. Tsuchida, K. Maeda, M. Tadakuma, T. Saito, K. Watanabe, K. Imamura, R. Sugizaki, and M. Suzuki, “1.03-Exabit/s.km Super-Nyquist-WDM transmission over 7,326-km seven-core fiber” Proc. European Conference on Optical Communications (ECOC’13), paper PD3.E.3 (2013).
6. V. A. J. M. Sleiffer, Y. Jung, B. Inan, H. Chen, R. G. H. van Uden, M. Kuschn erov, D. van den Borne, S. L. Jansen, V. Veljanovski, A. M. J. Koonen, D. J. Richardson, S. Alam, F. Poletti, J. K. Sahu, A. Dhar, B. Corbett, R. Winfield, A. D. Ellis, and H. de Waardt, “Mode-division-multiplexed 3x112-Gb/s DP-QPSK transmission over 80-km few-mode fiber with inline MM-EDFA and Blind DSP,” Proc. European Conference on Optical Communication (ECOC’12), paper Tu.1.C.2 (2012).
7. P. Sillard, D. Molin, M. Bigot-Astrue, H. Maerten, D. van Ras and F. Achten, “low-DMGD 6-LP-Mode Fiber” Optical Fiber Communication Conference (OFCC’14) paper M3F.2 (2014).
8. R. G. H. van Uden, R. Amezcua Correa, E. Antonio Lopez, F. M. Huiskens, C. Xia, G. Li, A. Schulgen, H. de Waardt, A. M. J. Koonen, C. M. Okonkwo, “Ultra-high density spatial division multiplexing with a few-mode multi-core fiber,” Nat. Photonics 8(11), 865–870 (2014).
9. K. Saitoh, T. Matsui, T. Sakamoto, M. Koshiba, and S. Tomita, “Multicore hole assisted fibres for high core density space division multiplexing,” 15th Optoelectronics and Communications Conference (OECC), 164–165 (2010).
10. H. Chen, V. Sleiffer, B. Snyder, M. Kuschn erov, R. van Uden, J. Yongmin, C. M. Okonkwo, O. Raz, P. O’Brien, H. de Waardt, and T. Koonen, “Employing prism based three-spot mode couplers for high capacity MDM/WDM transmission,” IEEE Photon. Technol. Lett. 25(24), 2474–2477 (2013).
11. H. Chen, R. van Uden, C. Okonkwo, B. Snyder, O. Raz, P. O’Brien, H. van den Boom, H. de Waardt, and T. Koonen, “Employing an integrated mode multiplexer on silicon-on-insulator for few mode fiber transmission” Proceedings of the European Conference on Optical Communications (ECOC’13) paper Tu.1.B.4 (2013).
12. L. Grüner-Nielsen, Y. Sun, J. W. Nicholson, D. Jakobsen, K. G. Jespersen, R. Lingle, Jr., and B. Palsdottir, “Few Mode Transmission Fiber with Low DGD, Low Mode Coupling, and Low Loss,” J. Lightwave Technol. 30(23), 3693–3698 (2012).
13. R. Randel, D. Pilori, S. Corteselli, G. Raybon, A. Adamiecki, A. Gnauck, S. Chandrasekhar, P. J. Winzer, L. Altenhain, A. Bielik, and R. Schmid, “All Electronic Flexibly Programmable 864-Gb/s Single Carrier PDM-64-QAM” Optical Fiber Communication Conference (OFC’14) paper Th5C.8 (2014).
14. S. Savory, “Digital Coherent Optical Receivers: Algorithms and Subsystems,” IEEE J. Sel. Top. Quantum Electron. 16(5), 1164–1179 (2010).
15. R. G. H. van Uden, C. M. Okonkwo, V. A. Sleiffer, H. de Waardt, and A. M. J. Koonen, “MIMO equalization with adaptive step size for few-mode fiber transmission systems,” Opt. Express 22(1), 119–126 (2014).
16. R. G. H. van Uden, C. M. Okonkwo, V. A. J. M. Sleiffer, M. Kuschnerov, H. de Waardt, and A. M. J. Koonen, “Single DPLL Joint Carrier Phase Compensation for Few-Mode Fiber Transmission,” IEEE Photon. Technol. Lett. 25(14), 1381–1384 (2013).
17. V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-Time Block Codes from Orthogonal Designs,” IEEE Trans. Inf. Theory 45(5), 1456–1467 (1999).
18. S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” IEEE J. Sel. Areas Commun. 16(8), 1451–1458 (1998).
19. G. Ungerboeck, “Channel coding with multilevel/phase signals,” IEEE Trans. Inf. Theory 28(1), 55–67 (1982).
20. J. H. Conway and N. J. A. Sloane, “Sphere packings, lattices and groups,” Bull. Am. Math. Soc. 21(1), 1 (1989).
21. E. Agrell and M. Karlsson, “Power Efficient Modulation Formats in Coherent Transmission Systems,” J. Lightwave Technol. 27(22), 5115–5126 (2009).
22. V. A. J. M. Sleiffer, Y. Jung, V. Veljanovski, R. G. H. van Uden, M. Kuschnerov, Q. Kang, L. Grüner-Nielsen, Y. Sun, D. J. Richardson, S. U. Alam, F. Poletti, J. K. Sahu, A. Dhar, H. Chen, B. Inan, A. M. J. Koonen, B. Corbett, R. Winfield, A. D. Ellis, and H. de Waardt, “73.7 Tbps (96x3x256-Gb/s) mode-division-multiplexed DP-16QAM transmission with inline MM-EDFA,” Proc. European Conference on Optical Communication (ECOC’12), paper Th.3.C.4–1 (2012).
23. R. G. H. van Uden, C. M. Okonkwo, H. Chen, H. de Waardt, and A. M. J. Koonen, “Time domain multiplexed spatial division multiplexing receiver,” Opt. Express 22(10), 12668–12677 (2014).
24. L. D. Coelho and N. Hanik, “Global optimization of fiber-optic communication systems using four-dimensional modulation formats,” in Optical Communication (ECOC’11) paper Mo.2.B (2011).
25. G. J. Foschini, “Layered Space-Time Architecture for Wireless Communications in a Fading Environment When Using Multi-Element Antennas,” Bell Labs Tech. J. 1(2), 41–59 (1996).

1. Introduction

According to the forecasted impending “capacity crunch”, we are on the verge of approaching the fundamental limit of existing standard single-mode fiber (SSMF), which is mainly attributed to fiber non-linearity [1–3]. Hence, the ongoing rapid development of space-division multiplexing (SDM) transmission systems is expected to provide a viable strategy to increasing capacity of optical links. To support SDM, emerging transmission systems primarily employ multiple single-mode cores [4,5], multi-mode transmission [6,7], or a combination of both spatial aspects [8,9]. In this work, note that we employ a specific case of multi-mode transmission by exploiting 3 linearly polarized (LP) modes; LP_{01}, LP_{1a} and LP_{1b} which are excited and co-propagated in a few-mode fiber (FMF) transmission system. In this case, the throughput of the fiber is increased by 3 fold.

In tandem with technological improvements in mode multiplexers (fiber-based and integrated [10,11]) and recent research in fibers for SDM [7,12], there are ongoing developments in digital-to-analogue converters (DACs) in the transmitter [13], and coherent reception techniques at the receiver [14], where the development of digital signal processing (DSP) algorithms is key [15,16]. Further developments in transmitter DSP presents the opportunity to investigate novel coding at the symbol level to generate advanced modulation formats. Additionally, as in wireless communication, the advantages of diversity in multiple-input multiple-output (MIMO) transmission in few-mode propagation can be exploited to improve transmission performance. In principle, by exploiting the diversity offered by MIMO optical systems, added flexibility in terms of a trade-off between throughput and optical transmission performance can be obtained.
Similar to a wireless system with multiple antennas, space-time codes (STC) can exploit the diversity in few-mode transmission systems which offer more spatial channels with respect conventional SSMFs. The main contenders are space-time trellis codes (STTC) [17], orthogonal space time codes [18] and delay diversity. The key flexibility added by STC is that space-time blocks can be designed to achieve \( 1/2 \), \( 3/4 \) or the full maximum possible transmission rate thereby gaining significant optical signal to noise ratio (OSNR) margins.

On the other hand, conventional two dimensional modulation formats such as quadrature phase shift keying (QPSK) and 16-ary quadrature amplitude modulation (16-QAM), do not exploit the full four-dimensional signal space available with the optical carrier. These are typically achieved from transmitting in each of the two orthogonal states and independently on each of the fiber polarization states for polarization multiplexed transmission. However, to improve the robustness of digital transmission against additive noise, it is recognized that maximizing the Euclidean distance of the constellation points can be carried out by exploiting higher order dimensions. In SSMF transmission systems, 4 dimensional (4D) modulation formats have been demonstrated to unlock capacity and improve transmission reach [19]. Typically, square lattice constellations also known as D4 lattices are employed. Although, sphere-packed constellations are more power efficient, and hence, provide better OSNR tolerances than square lattices [20]. However, due to the complexity in generating spherical 4D constellations, in this work we focus on the performance of D4 lattice constellations. Typically, to generate a 4D symbol, two 2D symbols arbitrarily chosen are concatenated where the inphase and quadrature components of two polarizations are employed as one 4D symbol [21]. However this increases timing constraints in MIMO equalization, as synchronization between multiple output channels is required. To minimize timing complexity, two consecutive 2D symbols in time are employed as one 4D symbol.

In this work, we experimentally verify the transmission performance of 2D modulation formats over a 3-mode FMF transmission system. The OSNR tolerance gain is achieved through exploiting the spatial diversity few-mode-fibers offer. With respect to a conventional single-mode back-to-back transmission, an OSNR gain of upto 3.2, 4.1, 4.9, and 6.8 dB at the hard-decision forward error correcting (HD-FEC) limit is shown for DP-QPSK, 8, 16, and 32 DP-QAM, respectively. Further in this work, the progress made in FMF transmission systems is combined with the progress in 4D digital modulation formats. The transmission performance of the 28 GBaud 4D formats time switched (TS) QPSK, 32 set partitioned (SP) QAM, and 128-SP-QAM are compared with the regular 2D formats QPSK, 8, and 16QAM. Using a 6 × 6 MIMO frequency domain equalizer (FDE), we show that 128-SP-QAM outperforms 8 QAM after 41.7 km few-mode fiber transmission, whilst carrying 0.5 bit/symbol more information. This demonstrates the robustness of 4D formats with respect to 2D formats and adds flexibility in optimizing system performance by trading throughput for transmission performance and vice versa.

This paper is organized as follows: In section 2, we describe the generation of conventional 2D modulation formats and the experimental few-mode transmission system in section 3. The concept of space-time coding with experimental validation is outlined in Section 4. In Section 5, the 4D modulation formats are discussed including experimental results. Finally, the paper is concluded in section 6.

2. Two-dimensional coding

Recently, there has been resurgence in the use of higher order modulation formats mainly attributed to the improvements in the electronic capabilities of digital-to-analog converters and signal processing techniques [4–6]. Well known formats such as QPSK, 8, 16 and 32 QAM have become straightforward to generate. Each constellation carries \( \log_{2}(N_{\text{const}}) \) bits, where \( N_{\text{const}} \) represents the number of constellation points. Note that QPSK and 16 QAM are rectangular QAM constellations, and 8 and 32 QAM are non-rectangular constellations. Rectangular constellations have the advantage of bits being modulated in either \( I_{\text{QAM}}(t) \) or
QAM(t), whereas non-rectangular QAM constellations have a dependency on both dimensions. Figure 1 shows the 2D constellations already well-established for FMF transmission [22]. In order to optimize their performance, Gray coding is employed on the square constellations, the associated bit mapping for QPSK, 8, 16 and 32 QAM is indicated by numerical labels on the constellation points in Fig. 1.

Fig. 1. Transmitted QAM constellations (a) QPSK, (b) 8, (c) 16 and (d) 32QAM.

3. Experimental setup

In this section, the experimental setup employed to generate 2D symbols as depicted in Fig. 2 is described.

3.1 Transmitter

At the transmitter, a 1555.75 nm external cavity laser (ECL) is used. The output is guided through an IQ-modulator, where the laser output is modulated by a 28GBaud signal. The IQ-modulator is driven by two digital-to-analog converters (DACs), which represent the in-phase (real) and quadrature (imaginary) components of the transmitted 2D constellation. The transmitted 2D constellations under investigation are QPSK (2 bits/symbol), 8QAM (3 bits/symbol), and 16QAM (4 bits/symbol). The transmitted sequences are formed in the digital domain by a number of fully uncorrelated pseudo random bit sequences (PRBSs), each of length $2^{15}$, which avoids any correlation within the $2^{15}$ symbol sequence. To emulate the transmission of independent spatial channels, one of the two split output of the IQ modulator is delayed by 1233 symbols to decorrelate one polarization from the other. After recombination, a dual polarization (DP) signal is noise loaded for optical OSNR characterization. To achieve 3 DP multiplexed mode channels, the noise loaded signal is split into three equal tributaries. Two arms are delayed for mode decorrelation by 3714 and 8233
symbols, respectively. Note that these arbitrary values are due to the discrepancies in fiber patch cord lengths employed as optical delay lines. Each arm is separately amplified before going into the mode multiplexer (MMUX). As a MMUX, a single prism spot launcher is used [10], resulting in the equal excitation of the three LP modes, guaranteeing full mixing. For all inputs, the losses are approximately 4.5 dB. The transmission link consists of two spans of few-mode fiber with lengths of 29.9 km and 11.8 km, respectively [12]. The properties of the employed few mode fibers are detailed in Table 1.

3.2 Receiver

At the receiver side, a reciprocal setup of the MMUX comprising a single prism splits the FMF output into 3 separate outputs. By launching at the MUX side, mixing in the FMF, and demultiplexing at the DMUX, each output contains a mix of the 3 transmitted modes. The loss in the mode demultiplexer is approximately 4.5 dB for each output. After transmission, the three outputs from the mode demultiplexer are received using an elegant mode measurement technique was devised and detailed in [23]. In essence, two dual polarization coherent receivers are employed for the simultaneous reception of 3 transmitted modes. The first coherent receiver receives the first mode; the second receives the remaining two modes in a time domain multiplexed fashion. This setup alleviates the requirement for a high number of receivers and oscilloscopes for multi-mode measurements. Note that as different laser acts as the local oscillator (LO) and mixes with the signal. Real-time oscilloscopes act as 12 (3 modes × 2 polarizations × 2 real-valued axes of a complex symbol) analog to digital converters (ADCs). Typically, a capture is >300000 symbols, and during this period, the system was stable. The captured data is post-processed offline.

Subsequently after reception, first the optical front-end impairments are compensated in the digital domain. Then, the chromatic dispersion (CD) is removed. To invert the channel mixing effects, a 6 × 6 MIMO FDE is used. The weight matrix of the FDE is heuristically updated using the least mean squares (LMS) algorithm during convergence and decision-directed least mean squares (DD-LMS) during data transmission. To compensate the frequency offset between the transmitter laser and LO, one carrier phase estimation (CPE) block per polarization output channel in the form of a digital phase locked loop is used [16]. After this stage, the constellations are demapped, and the bit error rate (BER) is measured.

| Table 1. FMF characteristics. |
|--------------------------------|
| | FMF 1 | FMF 2 | Unit |
|------------------|-------|-------|------|
| Length           | 29.9  | 11.8  | km   |
| Fiber Core Diameter | 125   | 124.9 | μm   |
| Differential Mode Delay | + 2   | + 1.82 | ns   |
| LP01 Chromatic Dispersion | 19.8  | 19.9  | ps/nm/km |
| LP11 Chromatic Dispersion | 20    | 20.1  | ps/nm/km |
| Mode Field Diameter LP01 | 11    | 11    | μm   |
| Mode Field Diameter LP11 | 11    | 11    | μm   |
| Effective area LP01 | 95    | 96    | μm²  |
| Effective area LP11 | 95    | 95    | μm²  |
4. Space-time diversity coding

Orthogonal STCs and delay diversity are linear space-time block codes (STBCs) which have a lower implementation complexity than trellis codes. Although the STTCs offer better performance than the linear variants, for high throughput optical transmission systems complexity is a key factor [17]. Therefore, we focus on the two linear STBCs, where the orthogonal STBCs are linear codes from which the data is formed as unitary transmission matrices. The most well-known orthogonal STBCs are Alamouti and Tarokh codes [17,18]. However, these only exist for certain numbers of transmitters. Note that, the simplest STBCs, purely exploiting the space and delay diversity, where delayed signal copies are transmitted over a number of transmitters in certain time slots depicted in Fig. 3. The Figure depicts 4 possible cases of the simplest STC scheme, delay-diversity STBCs, where (a) 6 channels is considered full capacity [25], (b) 3 channels, (c) 2 channels, or (d) 1 polarization channel is transmitted. Hence, as SMFs allows 2 polarization channels to co-propagate, case (c) is the most interesting for comparison, where maximum blocks are assigned to 2 channels from other spatial channels, thereby increasing its OSNR margin. The block lengths in Fig. 3 must be larger than the MIMO equalizer length to fully exploit the space and delay diversity without data correlation. To investigate the performance improvement 3MFs can offer for conventional 2D constellation transmission, the experimental setup detailed in Section 3 is employed.

4.1 Space-time diversity results and discussion

Figure 4 shows the performance results of the space-time coded FMF transmission experiment for (a) QPSK, (b) 8, (c) 16, and (d) 32 QAM. As a primary benchmark indicator, the 7% overhead HD-FEC limit is used. If the BER outperforms this given threshold, after error correction, theoretically the data transmission can be seen as error free (<10^-9). As there are 3 spatial LP modes transmitted over the 41.7 km fiber, and accordingly 6 polarization channels, the system throughput versus performance can be subdivided into 3 categories; 3, 1.5, 1, and 0.5 × SMF capacity using no STBC, STBC over 2, 3, and all 6 transmitted channels, respectively.
For QPSK transmission, as shown in Fig. 4(a), for threefold throughput increase there is a 0.6 dB OSNR penalty at the HD-FEC limit with respect to SMF back-to-back (BTB) performance. When applying STBC on 2 polarizations for a single channel, the performance is noticeably superior to the theoretical SMF single polarization performance. However, the FMF transmission capacity is reduced to 1.5 × the SMF capacity. When using a 3 channel STBC, the FMF capacity equals the SMF capacity.

Table 2. SMF Back-to-back a transmission OSNR penalties with respect to the theoretical limit. In addition, OSNR values is summarized for 6, 3 and 2 channels at the HD-FEC limit (the OSNR gain with respect to SMF BTB is in brackets)

| Constellation | SMF Theory OSNR @ HD-FEC-limit (dB) | SMF OSNR penalty w.r.t theory | 3 Mode (6 channel STC) Gain w.r.t SMF BTB | 1.5 Mode (3 channel STC) Gain w.r.t SMF BTB | 1 Mode (2 channel STC) Gain w.r.t SMF BTB |
|---------------|-------------------------------------|-------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| QPSK          | 12.1                                | 14.0 (1.9)                    | 14.6 (-0.6)                              | 12.3 (+2.3)                              | 11.8 (+3.2)                              |
| 8QAM          | 16.2                                | 19.3 (3.1)                    | 19.8 (-0.5)                              | 16.6 (+2.7)                              | 15.2 (+4.1)                              |
| 16QAM         | 19                                  | 22.5 (3.5)                    | 24 (-1.5)                                | 20 (+2.9)                                | 17.6 (+4.9)                              |
| 32QAM         | 21.5                                | 32 (11)                       | >34 (n/a)                                | 34 (+2)                                  | 28 (+6.8)                                |

As summarized in Table 2 (OSNR penalties are indicated with a minus sign and OSNR gains with positive sign), the OSNR gain between STBC transmission and SMF BTB is approximately 3.2 dB for QPSK. The same STBCs can be applied to 8QAM transmission and the performance results are depicted in Fig. 4(b). As shown in Table 2, for threefold capacity increase, there is a 0.5 dB OSNR penalty with respect to SMF BTB performance. A performance increase of 2.7 dB and 4.1 dB is seen when comparing the FMF capacity of 1.5 × and 1 × the SMF capacity to SMF BTB, respectively. The transmission of the 16QAM constellation is shown in Fig. 4(c). The 3 × SMF capacity OSNR penalty with respect to SMF BTB is 1.5 dB. However, when applying STBCs and reducing the FMF transmission system capacity to SMF capacity, an OSNR gain of 4.9 dB OSNR is observed. For 32QAM transmission, as depicted in Fig. 4(d), the 20% overhead soft-decision FEC (SD-FEC) is required for successful transmission for 3-fold capacity increase with respect to SMF.
However, after applying STBCs, a 6.8 dB OSNR performance gain is observed at the HD-FEC limit. Such OSNR gains come at the cost of DSP complexity and coherent receivers with corresponding ADC. In single mode coherent transmission systems, a single DP coherent receiver, whereas for a 3 mode FMF transmission system three DP coherent receivers are required.

5. Generating 4D constellations

In this section, we describe the generation of 4D modulation using 2D symbols. Note that typically in SMF transmission systems, 2D constellations with redundant signal sets have been shown to improve performance mainly based on the square lattices, also known as the D4-lattice [21]. Although sphere packing formats provide a better OSNR tolerance than square lattices, generating these spherical 4D-constellations is more difficult than generating the D4 lattices [24]. Therefore, we focus on the transmission performance of the D4 lattice constellations.

The following 4D constellations: TS-QPSK (1.5 bits/symbol), 32-SP-QAM (2.5 bits/symbol), and 128-SP-QAM (3.5 bits/symbol). Note that by concatenating two consecutive 2D constellations in time, the 4D constellations are formed. TS-QPSK is based on two QPSK symbols, and the latter two are based on two 16 QAM symbols. In two 2D constellations, 2.log2 (Nconst) bits can be transmitted, which is 4 and 8 bits for the QPSK and 16 QAM bases, respectively. TS-QPSK can easily be generated by adding one parity bit to 3 data bits which are being transmitted. The same methodology is applied to 128-SP-QAM, where 7 data bits and one parity bit are transmitted. A slightly more difficult constellation to generate is 32-SP-QAM. First, both the first and second 16 QAM constellations are SP once, as depicted in Fig. 5. This reduces the throughput to 6 bits per 4D symbol. Hence, when converting the bits/symbol to 2D constellation time-slots, the bits/symbol rate of the 4D formats is reduced by 50%. The transmitted constellations are shown in Fig. 6. The second time slot of the 4D constellations contains a parity bit, which is allocated according to Ungerboeck’s set partitioning method [19].

![Fig. 5. Two-step set partitioning of the 16QAM constellation on even or odd parity.](image-url)
Typically for 2D formats, the presented BER is averaged over 2 captures, where each capture represents 310,000 2D symbols. The first 25k symbols are used for convergence, resulting in system BER averaging over 3.42 million 2D symbols. For 4D, 1.71 (3.41/2) million 4D symbols are calculated. In addition, as a 4D constellation consists of the concatenation of two 2D constellations, the updating algorithms are the same for all formats. In [15], we have shown that the 6 × 6 MIMO equalizer can be rewritten as six 1 × 6 MIMO equalizers, without affecting the equalizer’s complexity. The benefit of coding 4D symbols in time is that only one output is used for a 4D symbol. This reduces the timing alignment complexity between MIMO outputs with respect to coding a 4D symbol onto two separate MIMO outputs.

The measurement results for both single mode back-to-back (BTB) and 41.7 km few-mode fiber transmission are shown in Fig. 7. The performance of the 2D and 4D constellations is shown in Fig. 7(a) and Fig. 7(b), respectively. For clarity, the 6 × 6 few-mode back-to-back results have been omitted in Fig. 7, as the OSNR penalty between 6 × 6 few mode BTB and 41.7 km few-mode fiber transmission was measured to be < 0.2 dB for 16 QAM.

The crossing point at hard-decision forward error correcting (HD-FEC) limit is the primary performance indicator. Hence, Table 3 denotes the HD-FEC limit crossing points of all the transmitted constellations for the theoretical limit in the second column of Table 3. As shown in the third column, the implementation penalty of the various modulation formats is indicated by the measured BTB OSNR penalty with respect to the theoretical crossing point. The measured BTB OSNR penalty with respect to the theoretical crossing point is shown in the third column of Table 2, and the crossing point is denoted in brackets. To fully complete
the table, in the fourth column, the OSNR penalty after 41.7 km few-mode fiber transmission is denoted. Again, in brackets the measured crossing point is given. From the trends shown in Fig. 7(a) and Fig. 7(b) and outlined in Table 3, it is clear that the OSNR penalty substantially increases for the 2D formats when scaling to a higher number of constellation points. However, for the 4D formats, the OSNR penalty increases slower. Note that the performance with respect to theory for 32-SP-QAM is not as good as the performance of 128-SP-QAM. This issue may be caused by the carrier recovery algorithm, which requires more constellation points to perform optimally [16,24]. The difference in performance is also the case in single mode fiber transmission, as shown in [24]. Therefore, the optimum 4D constellations are TS-QPSK and 128-SP-QAM. From Fig. 5 and Table 3, another interesting observation that can be made, is that even though the theoretical limit for 128-SP-QAM is higher than 8QAM, 128-SP-QAM (28 × 6 × 3.5 = 588Gb/s) outperforms 8QAM (28 × 6 × 3 = 504Gb/s) transmission, whilst carrying an additional 0.5 bit/symbol. Finally, from Fig. 7 observe that the 4D constellation BER curves (Fig. 5(b)) after transmission follow the BTB curves better than the 2D constellations (Fig. 5(a)). From the 41.7 km transmission case, we have established that the better OSNR margin provides a case for 4D constellations to be employed in few-mode fiber transmission systems. The OSNR margin improvement was also noticed in SMF transmission systems [24], but is more pronounced in FMF transmission systems. By providing an additional dimension to the transmitted constellations, 4D constellations allow more robust data transmission than their respective 2D constellations.

| Constellation | Theory OSNR @ HD-FEC-limit (dB) | BTB penalty w.r.t. theory | 6 × 6 Transmission penalty w.r.t. theory |
|---------------|---------------------------------|--------------------------|----------------------------------------|
| TS-QPSK       | 9.9                             | 1.9 (11.8)               | 2.1 (12)                                |
| QPSK          | 12.1                            | 1.9 (14)                 | 2.5 (14.6)                              |
| 32-SP-QAM     | 14                              | 2.7 (16.7)               | 3.3 (17.3)                              |
| 8 QAM         | 16.2                            | 2.9 (19.1)               | 3.5 (19.7)                              |
| 128-SP-QAM    | 17                              | 1.8 (18.8)               | 2.5 (19.5)                              |
| 16 QAM        | 18.7                            | 3.9 (22.6)               | 5.3 (24)                                |

6. Conclusion

The successful transmission of 6 × 28 GBaud QPSK, 8, 16, and 32 QAM over 41.7 km FMF have been demonstrated. Space-time coding was applied to improve the FMF transmission quality indicating better performance than single mode fiber at back-to-back. An OSNR gain of 3.2, 4.1, 4.9, and 6.8 dB at the hard-decision forward error correcting limit with respect to single mode fiber back-to-back performance is shown for QPSK, 8, 16, and 32 QAM, respectively. In addition, the transmission of 4D symbols over few-mode fiber (of 41.7 km) has been further detailed in this paper. To minimize the MIMO output timing alignment complexity, the 4D symbols were formed by two concatenating two 2D symbols in time. Therefore, a standard 6 × 6 MIMO frequency domain equalizer could be employed to unravel the fully mixed mode channels. In the demapping stage, the 4D symbols are converted to their respective bit streams. We have hence shown that 128-SP-QAM outperforms 8 QAM, whilst carrying 0.5 bit/symbol more information for FMF transmission. From these results, it is envisaged that a combination of STC and enhanced multi-dimensional symbol coding provides additional dimensionalities that can be exploited for potential FMF based flex (flexible)-grid/flex-rate optical transmission systems.

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