PRODUCTION OF THE $B_c$ MESON

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The production of the $B_c$ and $B_c^*$ mesons was studied in the framework of the factorization formalism and perturbative QCD. Predicted results implied that an observable number of $B_c$ and $B_c^*$ events can be produced at LEP I and at Tevatron. The fragmentation approximation describes the production of the $B_c$ and $B_c^*$ mesons very well in high energy $e^+e^-$ collisions. In hadronic collisions, it is valid when and only when the transverse momentum $P_T$ of the produced $B_c$ and $B_c^*$ is much larger than the mass of the $B_c$ meson.

1. Introduction

The $B_c$ meson is very interesting because it is the only flavored meson containing a heavy quark and a heavy antiquark. However, the study of the $B_c$ meson suffers from the difficulty of producing it in experiments. The conventional production mechanisms for the $B_u$ and $B_d$ mesons and for heavy quarkonia $J/\psi$ and $\Upsilon$ are not available for the $B_c$ and $B_c^*$ mesons. A question is whether a sufficiently large number of $B_c$ events can be produced in experiments. If not, the study of the $B_c$ meson would be only of theoretical interest.

Theoretical studies of the production for the $B_c$ meson over the past few years give a positive answer to this question. The dominant mechanism for the production of the $B_c$ meson in high energy collisions has been found to be the following. First two heavy quark pairs $c\bar{c}$ and $b\bar{b}$ are produced in a high energy process, and then the $c$ quark from the $c\bar{c}$ pair and the $\bar{b}$ quark from the $b\bar{b}$ pair bind to form a $B_c$ meson. At the $e^+e^-$ collider LEP I, the $B_c$ meson can be produced via the $Z^0$ decay $e^+e^- \rightarrow Z^0 \rightarrow B_c + b + \bar{c}$. At the hadron-hadron colliders Tevatron and LHC, it can be produced by gluon-gluon fusion and quark-antiquark annihilation subprocesses such as $g + g \rightarrow B_c + b + \bar{c}$, $q + \bar{q} \rightarrow B_c + b + \bar{c}$. Calculations show that quite a large number of the $B_c$ events can be produced both at LEP I and at the Tevatron and the LHC. The experimental search for the $B_c$ meson is now under way at LEP and the Tevatron. Some preliminary results from LEP I experiments were reported in this workshop. Results from the Tevatron are expected in the near future.

Like the $c\bar{c}$ and $b\bar{b}$ systems, the $b\bar{c}$ is a nonrelativistic bound state system of heavy quark and antiquark.

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A heavy quark and a heavy antiquark. According to the nonrelativistic QCD (NRQCD) factorization formalism, the production cross section of the $B_c$ and the $B^*_c$ mesons can be factorized as the products of short distance coefficients and long distance matrix elements. The production of the $c\bar{c}$ and the $b\bar{b}$ pairs is a short-distance process that occurs at a distance scale of order $1/m_Q$ or smaller, and it can be accurately calculated using perturbative QCD to any order in $\alpha_s(m_Q)$. The formation of the $B_c$ meson is a long-distance process which occurs at a scale of order $1/(m_Qv)$, and it can be described by NRQCD matrix elements which scale in a definite way with $v$, the typical relative velocity of the bound states. In the production of the $B_c$ or $B^*_c$ mesons, the contribution from color-octet processes is smaller than that from color-singlet process simply because the short-distance coefficients are of the same order in $\alpha_s$, but the color-octet matrix elements are higher order of $v$. Thus the production is dominated by color-singlet processes. The relevant color-singlet matrix elements can be estimated well by heavy-quark potential models or by lattice calculations. Therefore, the production of the $B_c$ is predictable without any free parameters.

If the $B_c$ is produced with sufficiently large transverse momentum, then the fragmentation approximation is expected to be available. In this approximation, the cross section factors into a cross section for producing a $b$ quark and a fragmentation function that gives the probability for the $b$ quark to hadronize into a $B_c$ meson. The fragmentation functions are universal, so they can be used to calculate the production of the $B_c$ both in $e^+e^-$ process and in hadron-hadron collisions. An interesting question is how well the fragmentation approximation to the full calculation works for the production of the $B_c$ mesons. In the following sections, this question will be addressed for the production of $B_c$ and $B^*_c$ at LEP and at hadronic colliders.

2. Production of the $B_c$ Meson in $e^+e^-$ Collision

At LEP I, millions of $Z^0$ events have been accumulated. Such a large number of events provides us with an opportunity to search for the $B_c$ meson through the process $Z^0 \rightarrow B_c(B^*_c) + b + \bar{c}$. Calculations of the production rate show that the branching ratio for $Z^0 \rightarrow B_c + X$ is around $10^{-4}$.

The process can be calculated either using the covariant projection method or using the recently developed threshold expansion method. At the lowest order in $\alpha_s$, there are four Feynman diagrams responsible for the process $Z^0 \rightarrow B_c + b + \bar{c}$. Let $k$, $P$, $q_1$, and $q_2$ be the momenta of the $Z^0$ boson, $B_c$ meson, $\bar{c}$ quark and $b$ quark, respectively. We can define three Lorentz invariant variables

$$x \equiv \frac{2k \cdot q_1}{M_Z^2}, \quad y \equiv \frac{2k \cdot q_2}{M_Z^2}, \quad z \equiv \frac{2k \cdot P}{M_Z^2},$$

(1)

which are twice the energy fractions of the $B_c$, $\bar{c}$ quark and $b$ quark in the $Z^0$ rest frame, respectively. They are constrained by $x + y + z = 2$ and $0 < x, y, z < 1$.

The full calculation for $Z^0 \rightarrow B_c + X$ is straightforward. The decay width
and various distributions in \(x, y,\) and \(z\) can be expressed as products of short-distance coefficients and long-distance color-singlet matrix elements \(\langle 0|O_1^{B_c}(1S_0)|0\rangle\) and \(\langle 0|O_1^{B_s}(3S_1)|0\rangle\). Using the approximate heavy-quark spin symmetry of NRQCD, these two matrix elements are equal to \(f_{B_c}^2 m_c\) and \(3f_{B_s}^2 m_c\), respectively, up to corrections of order \(v^2\).

The fragmentation functions for the production of the \(B_c\) can be extracted by taking the limit \(M_Z/m_{B_c} \to \infty\). In this limit, the square of the matrix element is singular when the produced \(B_c\) is collinear either with the \(b\) quark or with the \(\bar{c}\) quark. In this region, either \(x \to 1\) or \(y \to 1\) but \(z\) varies from 0 to 1. In the limit, the \(z\) distribution comes from those most singular terms and can be factorized as

\[
\frac{d\Gamma_{Z \to B_c^{(*)}}}{dz} = \Gamma(z \to \bar{b}b) \cdot D_{\bar{b} \to B_c^{(*)}}(z) + \Gamma(z \to \bar{c}c) \cdot D_{c \to B_c^{(*)}}(z),
\]

where \(D_{\bar{b} \to B_c^{(*)}}(z)\) and \(D_{c \to B_c^{(*)}}(z)\) are the fragmentation functions for \(\bar{b}\) or \(c\) into a \(B_c^{(*)}\) meson. The \(\bar{b}\) quark fragmentation functions are

\[
D_{\bar{b} \to B_c}(z) = \frac{4\alpha_s(2m_c)^2 \langle 0|O_1^{B_c}(1S_0)|0\rangle}{243m_c^4} \frac{rz(1-z)^2}{(1-(1-r)z)^6} \left(6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 - 2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1 - 2r + 2r^2)z^4\right),
\]

for the \(B_c\) meson and

\[
D_{\bar{b} \to B_s^*}(z) = \frac{4\alpha_s(2m_c)^2 \langle 0|O_1^{B_s^*}(3S_1)|0\rangle}{243m_c^4} \frac{rz(1-z)^2}{(1-(1-r)z)^6} \left(2 - 2(3 - 2r)z + 3(2 - 2r + 4r^2)z^2 - 2(1-r)(4 - r + 2r^2)z^3 + (1 - r)^2(3 - 2r + 2r^2)z^4\right),
\]

for the \(B_s^*\) meson, where \(r = m_c/(m_b + m_c)\). The \(c\) quark fragmentation functions can be obtained by exchanging \(m_b\) and \(m_c\). These fragmentation functions describe the probability for a jet initiated by a high energy \(\bar{b}\) quark to include the hadron \(B_c\) carrying a fraction \(z\) of the jet momentum. They were first calculated by Chang and Chen and Braaten, Cheung, and Yuan. These are the lowest order fragmentation functions at the energy scale around \(m_{B_c}\). All leading logarithmic terms coming from the collinear emission of gluons can be summed up by solving the Altarelli-Parisi evolution equations from the energy scale at order \(M_Z\) to that at order \(m_{B_c}\),

\[
\frac{\partial}{\partial \mu} D_{i \to B_c}(z, \mu) = \sum_j \int_0^1 \frac{dy}{y} P_{i \to j}(z/y, \mu) D_{j \to B_c}(y, \mu),
\]

where \(P_{i \to j}(x, \mu)\) is the Altarelli-Parisi function for the splitting of the parton of type \(i\) into a parton of type \(j\) with longitudinal momentum fraction \(x\).
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Explicit calculations show that the result predicted by the fragmentation approximation Eq. (2) is very close to that from the full calculation. Integrating over the variable $z$, in eqs. (3), (4), the probabilities for a high energy $\bar{b}$ antiquark or $c$ quark fragmentating into a $B_c$ or $B^*_c$ meson can be obtained. Taking the parameters adopted by Braaten, Cheung, and Yuan, the numerical values for the fragmentation probabilities are $2.2 \times 10^{-4}$ for $\bar{b} \to B_c$ and $3.1 \times 10^{-4}$ for $\bar{b} \to B^*_c$. The fragmentation probabilities for $c$ are smaller than those for $\bar{b}$ by almost two orders of magnitude, because it is much harder to create an extra $b\bar{b}$ pair by emitting a virtual gluon than to create an extra $c\bar{c}$ pair.

3. Production of the $B_c$ Meson in Hadron-Hadron Collision

Hadronic production, as first pointed out by Chang and Chen, is dominated at high energies by the subprocess $g + g \to B_c(B^*_c) + b + \bar{c}$. It can be calculated fully to order $\alpha_s^4$ in PQCD. Since a difficult numerical calculation is involved, confirmation from some other independent calculations are necessary. Since Chang and Chen first presented the numerical results for the hadronic production, calculations have also been done by several other authors, not all of which are in agreement. Slabospitsky claimed an order of magnitude larger result than Chang and Chen. Berezhnoy et al. obtained a result larger than Chang and Chen and smaller than Slabospitsky. Masetti and Sartogo found a result similar to Berezhnoy et al. More recently, Berezhnoy et al. found that a color factor $\frac{1}{3}$ was overlooked in their previous work. After including this factor, their revised result is in agreement with Chang and Chen. Baranov independently also obtained results similar to Chang and Chen. Kolodziej et al. presented results using different parton distribution functions and energy scale from that used by Chang and Chen so it difficult to directly compare their final results. However, their results for the cross sections for the parton subprocess are similar to others. Kolodziej et al. pointed out that the cross section for the subprocess calculated by Berezhnoy et al. for $\sqrt{s} = 1$ TeV was incorrect. The error probably came from the numerical calculation of the subprocess cross section since the the square of the matrix element is highly singular at such a high energy. It would not affect the cross section from hadron collision at Tevatron or LHC energies because the contributions from gluons with such a high energy center-of-mass are negligible. Therefore, we are confident that the results of the original order-$\alpha_s^4$ PQCD calculation by Chang and Chen and the more recent one are now in agreement.

An alternative way to calculate the hadronic production is to use the fragmentation approximation. From general factorization theorems it is clear that, for sufficiently large transverse momentum $P_T$ of the $B_c$ or $B^*_c$, hadronic production is dominated by fragmentation. The calculation can then be considerably simplified using this approximation, as was first done by Cheung. Subsequently, the comparison between the full $\alpha_s^4$ calculation and the fragmentation approximation has been discussed by several authors. However, very different conclusions have
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been drawn, although their numerical results are similar. Chang et al.\textsuperscript{13} found that when $\sqrt{s}$ and $p_T$ are small the difference between the fragmentation approximation and the $\alpha_s^4$ calculation is large. Berezhony et al.\textsuperscript{14} claimed that the fragmentation approximation breaks down even for very large $P_T$ by examining the ratio of $B_c^*$ to $B_c$ production. Kołodziej et al.\textsuperscript{16} claimed that the fragmentation approximation works well if $P_T$ exceeds about $5 - 10$ GeV, which is comparable to the $B_c$ mass, by investigating the $P_T$ distribution of the $B_c$ meson.

It is nontrivial to clarify these points since the full calculation is quite complicated, with the dominant subprocess involving 36 Feynman diagrams, but the importance of investigating this issue is twofold. From the theoretical perspective, it provides an ideal example to quantitatively examine how well the fragmentation approximation works for calculating the hadronic production of heavy flavored mesons. In this process both the full $\alpha_s^4$ contributions and the fragmentation approximation can each be calculated reliably. Experimentally, it is important to have a better understanding of the production of the $B_c$ mesons in the small $P_T$ region where the $B_c$ production cross section is the largest. The $P_T$ distribution decreases very rapidly as $P_T$ increases.

Recently, Chang, Chen and Oakes\textsuperscript{8} carried out a detailed comparative study of the fragmentation approximation and the full $\alpha_s^4$ QCD perturbative calculation. We first studied the subprocesses carefully. By analyzing the singularities appearing in the amplitudes and in the $P_T$ distributions of the subprocesses, we gained some important insight into the processes. We then calculated the entire hadronic production cross section for $p\bar{p}$ (or $pp$) collisions at the Tevatron energy. To facilitate a quantitative comparison we found it very instructive to examine the distribution in variable $z$, defined to be twice the energy fraction carried by the $B_c$ meson in the center of mass of the subprocess, which is in principle observable. Investigating the $z$ distribution we found that the fragmentation contribution dominates when and only when $P_T \gg M_{B_c}$. It is insufficient to evaluate the validity of the fragmentation approximation by only examining the $P_T$ distribution.

In order to obtain some insight into the process, we first focus the discussion on the subprocess. At the lowest order, $\alpha_s^4$, there are 36 Feynman diagrams responsible for the dominant gluon fusion subprocess $g + g \rightarrow B_c + b + \bar{c}$. When the energy in the center of mass system, $\sqrt{s}$, is much larger than the heavy quark mass, the main contributions to the cross section come from the kinematic region where certain of the amplitudes in the matrix element are nearly singular; i.e., some of the quark lines or gluon lines are nearly on-shell. In the large $P_T$ region, the only singularity is the collinear one for which the $B_c$ is collinear with fragmenting $b$ antiquark or collinear with the fragmenting $c$ quark. Therefore, at sufficiently large $P_T$, the subprocess is dominated by fragmentation. However, when the $P_T$ of the $B_c$ meson is small the produced $B_c$, as well as the $b$ and the $\bar{c}$ quarks, can be soft or collinear with the beam. In this region the square of the matrix element is highly singular because two or more of the internal quarks or gluons in certain Feynman diagrams can simultaneously be nearly on-mass-shell. Although this region is a smaller part of the phase space, these
nearly singular Feynman diagrams, in fact, make a large contribution to the cross section and dominate the small $P_T$ region. The contributions to the cross sections can therefore be decomposed into two components: the fragmentation contribution, which dominates in the large $P_T$ region and the non-fragmentation component, which dominates at smaller $P_T$. The contributions of these two components are quite clearly distinguishable in the $P_T$ distribution of the subprocess, particularly at large $\sqrt{s}$. In Fig. 1 we show the $P_T$ distribution of the subprocess when $\sqrt{s} = 200$ GeV for both the full $\alpha_s^4$ calculation and the fragmentation approximation with $\alpha_s = 0.2$, $m_b = 4.9$ GeV, $m_c = 1.5$ GeV, and $f_{B_c} = 480$ MeV. It is easy to see in Fig. 1 that when $P_T$ is larger than about 30 GeV for the $B_c$ and about 40 GeV for the $B_c^*$, the fragmentation approximation is close to the full $\alpha_s^4$ calculation. However, when the $P_T$ is smaller than these values, the deviation between the

![Graph showing $P_T$ distributions](image)

Fig. 1. The $P_T$ distributions of the $B_c$ and $B_c^*$ meson for the subprocess with $\sqrt{s} = 200$ GeV. The solid and the dotted lines correspond to the full $\alpha_s^4$ calculation and the fragmentation approximation, respectively.

fragmentation approximation and the full calculation becomes large and the non-fragmentation component clearly dominates the production. This critical value of $P_T$ is certainly much larger than the heavy quark masses, or the $B_c$ meson mass; it slowly increases with increasing $\hat{s}$, which may indicate that there is an additional enhancement due to logarithmic terms such as $\ln \hat{s}/m^2$ in the non-fragmentation component compared to the fragmentation component. When $\sqrt{s}$ is not very large this two component decomposition is less distinct, since the higher twist terms can not be ignored. In this case, the fragmentation approximation is not a very good approximation.

Let us now turn to the calculation of $B_c$ and $B_c^*$ production at $p\bar{p}$ colliders, particularly the Fermilab Tevatron. In Fig. 2, the $P_T$ distributions of the $B_c$ and the
$B_c^*$ mesons coming from the full $\alpha_s^4$ calculation is compared with the fragmentation approximation. In these calculations we used the CTEQ3M parton distribution functions. For simplicity, we use a uniform choice of energy scales, choosing the same scale as set in the $\bar{b}$ fragmentation component; i.e. $\alpha_s^2(P_T) \times \alpha_s^2(2m_c)$. From

Fig. 2, we see that for the $B_c$ meson the $P_T$ distributions for $P_T > 5$ GeV are very similar for the full $\alpha_s^4$ calculations and for the fragmentation approximation. This critical value of $P_T$ is much smaller than that found above in the study of the subprocess. However, for the $B_c^*$ meson, the result predicted by the full $\alpha_s^4$ calculation in Fig. 2 is a factor 1.5 to 2.0 greater than the fragmentation calculation over a much larger range of $P_T$, more consistent with what was found in the study of the subprocess. Therefore, it is difficult to decide from only the $P_T$ distributions where the fragmentation approximation is reliable for the hadronic production of the $B_c$ and $B_c^*$ mesons. The use of $P_T$ distribution alone can be misleading.

To clarify this issue, we introduce the distribution

$$C(z) = \frac{d\sigma}{dz} = \int dx_1 dx_2 g(x_1, \mu) g(x_2, \mu) \frac{d\hat{\sigma}(\sqrt{\hat{s}}, \mu)}{dz},$$

where $z \equiv 2(k_1 + k_2) \cdot p/\hat{s}$ and $g(x_i, \mu)$ is the gluon distribution function. The distribution $C(z)$ provides a sensitive means to investigate the dynamics of the production process and the fragmentation approximation. Clearly, if the fragmentation approximation is valid, $d\hat{\sigma}(\sqrt{\hat{s}}, \mu)/dz$ can be factorized as

$$d\hat{\sigma}(\sqrt{\hat{s}}, \mu)/dz = \sum_i \hat{\sigma}_{gg \to Q_i\bar{Q}_i} \otimes D_{Q_i\to B_c}(z, \mu).$$
where \( D_{Q_i \to B_c}(z, \mu) \) are the usual fragmentation functions and \( \sigma_{gg \to Q_i \bar{Q}_i} \) is the gluon fusion subprocess cross section for production of the heavy quark pair \( Q_i \bar{Q}_i \). In this approximation, the integrals over \( x_1 \) and \( x_2 \) can be performed, the fragmentation function can be factored out, and \( C(z) \) is simply proportional to a sum of the usual fragmentation functions which is insensitive to the parton distribution functions and to the kinematic cuts. However, if the distribution \( C(z) \) is quite different from the fragmentation functions, it is an indication that the fragmentation approximation is not valid. Therefore, comparing \( C(z) \) calculated in the fragmentation approximation with the full order \( \alpha_s^4 \) calculation, provides a quantitative criterion to judge the validity of the fragmentation approximation.

In Fig 3, the distribution \( C(z) \) calculated in the fragmentation approximation is compared with the full order \( \alpha_s^4 \) calculation for \( B_c \) and \( B_c^* \) meson production with a cut of \( P_T > 10 \) GeV (Fig. 3a) and also with cuts of \( P_T > 20 \) GeV and \( P_T > 30 \) GeV (Fig. 3b). The \( z \) distribution \( C(z) \) is sensitive to the smallest \( P_T \) region for a given \( P_T \) cut because the \( P_T \) distributions of the \( B_c \) and \( B_c^* \) mesons decrease very rapidly with increasing \( P_T \). From Fig. 3, some general features are evident. For the \( B_c \) meson, with smaller \( P_T \) cuts, the distribution \( C(z) \) is overestimated in the higher \( z \) region by the fragmentation approximation, while it is underestimated in the lower \( z \) region, however, after integration over \( z \), the result is similar to the full \( \alpha_s^4 \) calculation. For the \( B_c^* \) meson, even for the largest \( P_T \) cut, the distribution \( C(z) \) is underestimated the fragmentation approximation at all values of \( z \) and, after integration over \( z \), the result is definitely smaller than the full \( \alpha_s^4 \) calculation. This feature explains why the \( P_T \) distributions shown in Fig. 2 are similar for the \( B_c \) meson even down to \( P_T \sim M_{B_c} \) but are different for the \( B_c^* \) meson. This also shows

![Fig. 3. The \( z \) distributions \( C(z) \) of the \( B_c \) and \( B_c^* \) at the Tevatron energy \( \sqrt{s} = 1.8 \) TeV. The solid lines are the full \( \alpha_s^4 \) calculation and the dotted lines are the fragmentation approximation (a) with the cut \( P_T > 10 \) GeV and (b) with the cuts \( P_T > 20 \) GeV and \( P_T > 30 \) GeV.](image-url)
that it is simply fortuitous that the $P_T$ distribution of the $B_c$ calculated in the fragmentation approximation is similar to that from the full $\alpha_s^4$ calculation for $P_T$ below this value, particularly down to $P_T \sim M_{B_c}$. It is also clear that when $P_T$ is increased the distributions become closer. As shown in Fig. 3b, when $P_T$ is as large as 30 GeV the curves calculated in the fragmentation approximation are quite close to the full $\alpha_s^4$ calculation. This indicates that the fragmentation approximation is valid in the large $P_T$ region, as expected. We emphasize here that the difference between the fragmentation approximation and full calculation is not universal, but is process–dependent.

Finally, we examine the ratio of cross sections for $B_c^*$ production calculated in the fragmentation approximation with the results of the full $\alpha_s^4$ calculation. As discussed above, for $B_c^*$ meson production the fragmentation approximation always underestimates the full $\alpha_s^4$ result. The deviation from the full calculation for the $B_c^*$ meson can be used as a criterion to test the validity of the fragmentation approximation. The results for the total cross section $\sigma(P_T > P_{T\text{min}})$ for various $P_T$ cuts are listed in Table I. Taking agreement within 30% as the criterion for the validity of the fragmentation approximation we also see that $P_T$ should exceed about 30 GeV, a value considerably larger than the heavy quark masses. We note that this conclusion is rather insensitive to the choice of the energy scale $\mu$ and the parton distribution functions.

Table 1. Total cross sections $\sigma(P_T > P_{T\text{min}})$ in nb for hadronic production of the $B_c$ and the $B_c^*$ mesons predicted by the $\alpha_s^4$ calculation and the fragmentation approximation assuming various $P_T$ cuts and $|Y| < 1.5$. The CTEQ3M parton distribution functions were used and the values $f_{B_c} = 480$ MeV, $m_c = 1.5$ GeV, $m_b = 4.9$ GeV, and $M_{B_c} = 6.4$ GeV.

| $P_{T\text{min}}$ (GeV) | 0   | 5   | 10  | 15  | 20  | 30  |
|-------------------------|-----|-----|-----|-----|-----|-----|
| $\sigma_{B_c^*}(\alpha_s^4)$ | 1.8 | 0.57 | 0.087 | 0.018 | $4.8 \times 10^{-3}$ | $6.3 \times 10^{-3}$ |
| $\sigma_{B_c^*}(\text{frag.})$ | 1.4 | 0.47 | 0.071 | 0.014 | $4.0 \times 10^{-3}$ | $5.3 \times 10^{-4}$ |
| $\sigma_{B_c^*}(\alpha_s^4)$ | 4.4 | 1.4 | 0.22 | 0.041 | $1.1 \times 10^{-2}$ | $1.3 \times 10^{-3}$ |
| $\sigma_{B_c^*}(\text{frag.})$ | 2.3 | 0.78 | 0.12 | 0.025 | $6.8 \times 10^{-3}$ | $9.2 \times 10^{-4}$ |
| $\sigma_{B_c^*}(\alpha_s^4)$ | 0.52 | 0.55 | 0.56 | 0.61 | 0.63 | 0.70 |

In summary, from the study of both the parton subprocess and the hadronic process, we can conclude that the fragmentation mechanism dominates when and only when $P_T \gg M_{B_c}$. This conclusion is independent of the choice of the energy scales and the parton distribution functions. It is only fortuitous that the $P_T$ distribution of the $B_c$ in the fragmentation approximation is similar to that of the full calculation for $P_T$ as low as $M_{B_c}$.

4. Conclusion

Both the spectroscopy of $\bar{b}c$ system and the decays of the $B_c$ mesons have been widely studied. The excited states below the threshold will decay to the ground state $B_c$ by emitting the photon(s) or $\pi$ mesons. The golden channel to
detect the $B_c$ meson is $B_c \rightarrow J/\psi + \pi(\rho)$. However, the branching ratio is quite small, around 0.2. The exclusive semileptonic decay mode $B_c \rightarrow J/\psi + l + \nu_l$ has a relatively larger branch ratio, but there is “missing energy”. Comparative studies indicate that while fragmentation approximation works very well for the production of the $B_c$ and $B^*_c$ in high energy $e^+e^-$ collisions, the condition for the applicability of the fragmentation approximation in hadronic collisions is that the transverse momentum $P_T$ of the produced $B_c$ and $B^*_c$ be much larger than the mass of the $B_c$ meson; i.e., $P_T \gg M_{B_c}$. Theoretical studies for the production of the $B_c$ and $B^*_c$ mesons indicate that an observable number of $B_c$ and $B^*_c$ events can be produced at LEP I and at the Tevatron. With the discovery of the $B_c$ in the near future, a new page in heavy flavor physics will be opened and the study of the $B_c$ meson will no longer be academic.

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