SUPERNOVAE DATA: COSMOLOGICAL CONSTANT OR RULING OUT THE COSMOLOGICAL PRINCIPLE?

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Analysed in the framework of homogeneous FLRW models, the magnitude-redshift data from high redshift supernovae yield, as a primary result, a strictly positive cosmological constant. Another reading of the currently published measurements does not exclude a possible ruling out of the Cosmological Principle and, thus, also, of the cosmological constant hypothesis. It is shown how shortly coming data can be used to settle this fundamental issue, pertaining to both cosmology and particle physics.

1 Introduction

The discovery of high-redshift type Ia supernovae (SNIa) and their use as standard candles have resurrected interest in the magnitude-redshift relation as a tool to measure the cosmological parameters of the universe.

Data recently collected by two survey teams (the Supernova Cosmology Project and the High-z Supernova Search Team), and analysed in the framework of homogeneous FLRW cosmological models, have yielded, as a primary result, a strictly positive cosmological constant, of order unity if these results were to be confirmed, it would be necessary to explain how \( \Lambda \) is so small, yet non zero. Hence a revolutionary impact.

The purpose is here:

1. Assuming every source of potential bias or systematic uncertainties have been correctly taken into account in the data collecting,

2. Probe the large scale homogeneity of the region of the universe available with the SNIa measurements, thus testing the Cosmological Principle and cosmological constant hypotheses.
2 Magnitude-redshift relation to probe large scale (in)homogeneity

Consider any cosmological model for which the luminosity distance $D_L$ is a function of the redshift $z$ and of the parameters $cp$ of the model. Assume that $D_L$ is Taylor expandable near the observer, i.e. around $z = 0$,

$$
D_L(z; cp) = \left( \frac{dD_L}{dz} \right)_{z=0} z + \frac{1}{2} \left( \frac{d^2D_L}{dz^2} \right)_{z=0} z^2 + \frac{1}{6} \left( \frac{d^3D_L}{dz^3} \right)_{z=0} z^3 + \frac{1}{24} \left( \frac{d^4D_L}{dz^4} \right)_{z=0} z^4 + O(z^5).
$$ (1)

The apparent bolometric magnitude $m$ of a standard candle of absolute bolometric magnitude $M$ is also a function of $z$ and $cp$. In megaparsecs,

$$
m = M + 5 \log D_L(z; cp) + 25. \tag{2}
$$

Luminosity-distance measurements of such sources at increasing redshifts $z < 1$ thus yield values for the coefficients at increasing order in the above expansion. For cosmological models with high, or infinite, number of free parameters, the observations only produce constraints upon the parameter values near the observer. For cosmological models with few constant parameters, giving independent contributions to each coefficient in the expansion, the observed magnitude-redshift relation provides a way:

1. To test the validity of the model.
2. If valid, to evaluate its parameters.

For Friedmann models precisely, the expansion coefficients $D_L^{(i)}$ are independent functions of the three parameters $H_0$, $\Omega_M$ and $\Omega_\Lambda$, and can be derived from the well-known expression of $D_L$. Therefore, accurate luminosity-distance measurements of three samples of same order redshift SNIa - one at $z \sim 0.1$, one at $z \sim 0.5$ and one at $z \sim 0.7$, for instance - would yield values for $D_L^{(1)}$, $D_L^{(2)}$ and $D_L^{(3)}$ and thus select a triplet of numbers for the model parameters $H_0$, $\Omega_M$ and $\Omega_\Lambda$.

Would the value of $\Omega_M$, in this triplet, be negative, and thus physically inconsistent - which cannot be excluded from the current data - the Friedmann cosmology would have to be ruled out at this stage. Would this
value be positive, the triplet could be used to provide a prediction for the value of the forth order coefficient $D_L^{(4)}$. Now, if further observations at redshifts approaching unity could be made - $z \sim 0.8 - 0.9$ would suffice for a measurement accuracy of order 5-10% - $D_L^{(4)}$ could be determined and compared to its predicted value, thus providing a test of the FLRW model.

If the ongoing surveys were to discover more distant sources, at redshifts higher than unity, the Taylor expansion would no longer be valid. One would have to consider numerical methods to select the theoretical model best fitting the data and complete the test of the homogeneity assumption.

3 Example of alternative inhomogeneous model of universe

The ruling out of the FLRW paradigm and of the related Cosmological Principle is not a purely academic possibility. Physically robust inhomogeneous models exist, which can verify any observed magnitude-redshift relation. Furthermore, a non-zero cosmological constant is not mandatory, as $\Lambda = 0$ inhomogeneous models can mimic $\Lambda \neq 0$ Friedmann ones.

Lemaître-Tolman-Bondi (LTB) models are spatially spherically symmetrical solutions of Einstein’s equations with dust as source of gravitational energy. They can thus be retained to roughly represent a quasi-isotropic universe in the matter dominated area.

Einstein’s equations with $\Lambda = 0$ imply that the metric coefficients, in proper time and comoving coordinates, are functions of the time-like $t$ and radial $r$ coordinates, and of two independent functions of $r$, which play the role of model parameters. The radial luminosity distance $D_L$ can be expressed as a function of $t$, $r$, the redshift $z$ and the two above cited independent functions of $r$. In the approximation of a centered observer, the $D_L$ expansion coefficients follow, as independent functions of the derivatives of the model parameters, evaluated at the observer ($z = 0$). These parameters, which are implicit functions of $z$ through the null geodesic equations, are present in each coefficient $D_L^{(i)}$ with derivatives of order $i$. LTB models are thus completely degenerate with respect to any magnitude-redshift relation.

One can therefore fit any observed relation with a class of $\Lambda = 0$ LTB models fulfilling the constraints on its parameters proceeding from the data. In fact, a non-zero $\Lambda$ can also be retained in these models. This only adds a
new free parameter in the equations, increasing the degeneracy of the models with respect to magnitude-redshift relations. It is in particular the case for the class of relations selected by the current SNIa measurements, which can be interpreted as implying either a non-zero cosmological constant in a FLRW universe, or large scale inhomogeneity with no constraint on $\Lambda$.

4 Conclusions

Provided SNIa would be confirmed as good standard candles, data from this kind of sources at redshifts approaching unity could, in a near future, be used to test the homogeneity assumption on our past light cone.

Using, as an example, the LTB solutions, it has here been shown that:

- would this assumption be discarded by the shape of the measured magnitude-redshift relation, inhomogeneous solutions could provide good alternative models, as they are completely degenerate with respect to any of these relations, even with a vanishing cosmological constant.

- would a FLRW type distance-redshift relation be observed, it would not be enough to strongly support the Cosmological Principle. Even if this would imply a fine tuning for its parameters, the possibility for an inhomogeneous model to mimic such a relation could not be excluded.

Therefore, at the current stage reached by the observations, a non-zero $\Lambda$ is not mandatory, as, for example, a class of $\Lambda = 0$ LTB models can mimic a $\Lambda \neq 0$ FLRW M-R relation.

In any case, to consolidate the robustness of future magnitude-redshift tests, it would be worth confronting their results with the full range of available cosmological data, analysed in a model independent way.

References

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