On a new class of analytic functions with respect to symmetric points involving the q-derivative operator

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Abstract. In this paper we introduce a new subclass of analytic functions with respect to symmetric points involving the q-derivative operator. We find coefficient bounds and Fekete-Szego inequalities for functions belonging to these class. Relevant connections with previous results are pointed out.

1. Introduction

The quantum calculus has called the attention of many great researchers from Geometric function theory field due to its numerous applications in mathematics and physics (see the recent papers [1, 2, 3, 6, 9, 12, 14, 15] and the references therein).

Let $A$ be the class of all functions $f$ of the form

$$f(z) = z + \sum_{n \geq 2} a_n z^n$$  \hspace{1cm} (1)

which are analytic in the open unit disk $U = \{ z \in \mathbb{C} : |z| < 1 \}$. If $f$ and $g$ are two analytic functions in $U$, we say that $f$ is subordinate to $g$, written as $f \prec g$, if there exists a Schwarz function $w$, analytic in $U$ with $w(0) = 0$ and $|w(z)| < 1$, such that $f(z) = g(w(z))$, for all $z \in U$ (see [11]).

For $0 < q < 1$, the q-derivative of a function $f \in A$ is defined by (see [5])

$$d_q f(z) = \frac{f(qz) - f(z)}{(q - 1)z}, \hspace{1cm} (z \neq 0),$$  \hspace{1cm} (2)

$$d_q f(0) = f'(0).$$

Therefore, if function $f$ is given by (1), then

$$d_q f(z) = 1 + \sum_{n \geq 2} [n]_q a_n z^{n-1},$$  \hspace{1cm} (3)

where

$$[n]_q = \frac{1 - q^n}{1 - q}.$$  \hspace{1cm} (4)
We note that as \( q \to 1^- \), \( |n| q \to n \) and \( d_q f(z) \to f'(z) \).

Using subordination and the q-derivative we define a new subclass of analytic functions with respect to symmetric points which allows us to study various subclasses of functions in a unified manner.

**Definition 1.1.** Let \( \varphi(z) = 1 + B_1 z + B_2 z^2 + \ldots \) be a univalent starlike function with respect to 1 which maps the unit disk \( U \) onto a region in the right half plane which is symmetric with respect to the real axis and let \( B_1 > 0 \). The function \( f \in \mathcal{A} \) belongs to the class \( M_{q,s,\alpha}(\varphi), 0 \leq \alpha \leq 1 \) if

\[
\frac{2\alpha zd_q(zd_q f(z)) + 2(1 - \alpha)zd_q f(z)}{\alpha zd_q(f(z) - f(-z)) + (1 - \alpha)(f(z) - f(-z))} < \varphi(z), \quad z \in U.
\]

We remark the following:

(i) For \( \alpha = 0 \) we obtain the class \( S_{q,s}^*(\varphi) \) and for \( \alpha = 1 \) we get the class \( C_{q,s}(\varphi) \) (see [8]);

(ii) If we let \( q \to 1^- \) and \( \alpha = 0 \) we obtain the class \( S_{q}^*(\varphi) \) and for \( \alpha = 1 \) we find the class \( C_{q}(\varphi) \) (see [10]);

(iii) When \( q \to 1^- \) and \( \varphi(z) = \frac{1 + A_1 z}{1 + B_1 z}, -1 \leq B < A \leq 1 \) the class \( M_{q,s,\alpha}(\varphi) \) reduces to the class \( M_{s,\alpha}(A, B) \) (see [13]);

(iv) If \( q \to 1^- \) the class \( M_{q,s,0}(\frac{1 + A_1 z}{1 + B_1 z}) \equiv S_{q}^*(A, B) \) and \( M_{q,s,1}(\frac{1 + A_1 z}{1 + B_1 z}) \equiv C_{q}(A, B) \), where \(-1 \leq B < A \leq 1\) (see [4, 13]).

In terms of subordination, function \( f \in M_{q,s,\alpha}(\frac{1 + A_1 z}{1 + B_1 z}), 0 \leq \alpha \leq 1 \) if and only if

\[
\frac{2\alpha zd_q(zd_q f(z)) + 2(1 - \alpha)zd_q f(z)}{\alpha zd_q(f(z) - f(-z)) + (1 - \alpha)(f(z) - f(-z))} = p(z), \quad z \in U,
\]

where \( p \) is given by

\[
p(z) = \frac{1 + A w(z)}{1 + B w(z)} = 1 + b_1 z + b_2 z^2 + \ldots.
\]

Goel and Mehrok[4] proved the following result:

**Lemma 1.1.** [4] If function \( p \) is given as in (7), then

\[
|b_n| \leq A - B, \quad n \geq 1.
\]

**Lemma 1.2.** [7] If \( p(z) = 1 + c_1 z + c_2 z^2 + \ldots, z \in U \) is a function with positive real part in \( U \) and \( \zeta \) is a complex number, then

\[
|c_2 - \zeta c_1^2| \leq 2\max\{1; |2\zeta - 1|\}.
\]

The result is sharp for the function given by

\[
p(z) = \frac{1 + z^2}{1 - z^2} \text{ and } p(z) = \frac{1 + z}{1 - z}, \quad z \in U.
\]

**Lemma 1.3.** [7] If \( p(z) = 1 + c_1 z + c_2 z^2 + \ldots, z \in U \) is a function with positive real part in \( U \), then

\[
|c_2 - \nu c_1^2| \leq \begin{cases} 
-4\nu + 2 & \text{if } \nu \leq 0 \\
2 & \text{if } 0 \leq \nu \leq 1 \\
4\nu - 2 & \text{if } \nu \geq 1
\end{cases}
\]

where \( \nu < 0 \) or \( \nu > 1 \), the equality holds if and only if \( p(z) \) is \( \frac{1 + z}{1 + \zeta z} \) or one of its rotations. If \( 0 < \nu < 1 \), then the equality holds if and only if \( p(z) \) is \( \frac{1 + z^2}{1 + z^2} \) or one of its rotations. If \( \nu = 0 \), the
equality holds if and only if \( p(z) = \left( \frac{1+\gamma}{2} \right) \frac{1+\gamma}{1+\gamma}, 0 \leq \gamma \leq 1 \) or one of its rotations. If \( \nu = 1 \), the equality holds if and only if \( p \) is the reciprocal of one of the functions such that the equality holds in the case of \( \nu = 0 \). Also, the above upper bound is sharp and it can be improved as follows, when \( 0 < \nu < 1 \):

\[
|c_2 - \nu c_2^2| + \nu |c_1| \leq 2, \quad 0 < \nu \leq \frac{1}{2},
\]

and

\[
|c_2 - \nu c_2^2| + (1 - \nu) |c_1| \leq 2, \quad \frac{1}{2} \leq \nu < 1.
\]

Unless otherwise mentioned, we assume throughout this paper that \(-1 \leq B < A \leq 1\), \(0 \leq \alpha \leq 1\), \(0 < q < 1\), \(n \in \mathbb{N}\) and \(z \in U\).

2. Coefficient Inequalities

**Theorem 2.1.** Let function \( f \) as defined in (1) be in the class \( M_{q,s}(1+Bz) \). Then for \( n \geq 1 \)

\[
|a_{2n}| \leq \frac{A-B}{[2n]_q \{\alpha[2n]_q+1-\alpha\}} \prod_{j=1}^{n-1} \frac{A-B + [2j+1]_q - 1}{[2j+1]_q - 1} \tag{9}
\]

and

\[
|a_{2n+1}| \leq \frac{A-B}{([2n+1]_q - 1) \{\alpha[2n+1]_q+1-\alpha\}} \prod_{j=1}^{n-1} \frac{A-B + [2j+1]_q - 1}{[2j+1]_q - 1}. \tag{10}
\]

**Proof.** Considering (6) and (7) and equating the coefficients of like powers of \( z \), it follows that

\[
[2]_q \{\alpha[2]_q + 1 - \alpha\} a_2 = b_1, \quad ([3]_q - 1) \{\alpha[3]_q + 1 - \alpha\} a_3 = b_2, \tag{11}
\]

\[
[4]_q \{\alpha[4]_q + 1 - \alpha\} a_4 = b_3 + \{\alpha[3]_q + 1 - \alpha\} a_3 b_1, \tag{12}
\]

\[
([5]_q - 1) \{\alpha[5]_q + 1 - \alpha\} a_5 = b_4 + \{\alpha[3]_q + 1 - \alpha\} a_3 b_2, \tag{13}
\]

and so on

\[
[2n]_q \{\alpha[2n]_q + 1 - \alpha\} a_{2n} = b_{2n-1} + \{\alpha[3]_q + 1 - \alpha\} a_3 b_{2n-3} + \cdots + \{\alpha[2n-1]_q + 1 - \alpha\} a_{2n-1} b_1, \tag{14}
\]

\[
([2n+1]_q - 1) \{\alpha[2n+1]_q + 1 - \alpha\} a_{2n+1} = b_{2n} + \{\alpha[3]_q + 1 - \alpha\} a_3 b_{2n-2} + \cdots + \{\alpha[2n-1]_q + 1 - \alpha\} a_{2n-1} b_2. \tag{15}
\]

Using Lemma 1.1 we obtain the following upper bounds for the coefficients \( a_2, a_3, a_4 \) and \( a_5 \):

\[
|a_2| \leq \frac{A-B}{[2]_q \{\alpha[2]_q + 1 - \alpha\}}, \quad |a_3| \leq \frac{A-B}{([3]_q - 1) \{\alpha[3]_q + 1 - \alpha\}}, \tag{16}
\]

\[
|a_4| \leq \frac{(A-B)(A-B + [3]_q - 1)}{[4]_q \{\alpha[4]_q + 1 - \alpha\} \{[3]_q - 1\}}, \quad |a_5| \leq \frac{(A-B)(A-B + [3]_q - 1)}{\{\alpha[5]_q + 1 - \alpha\} \{[3]_q - 1\} \{[5]_q - 1\}}. \tag{17}
\]

Therefore, (9) and (10) are valid for \( n = 1 \) and \( n = 2 \).

Again, by using Lemma 1.1 in (14) and (15), we find that

\[
|a_{2n}| \leq \frac{A-B}{[2n]_q \{\alpha[2n]_q + 1 - \alpha\}} \left\{ 1 + \sum_{k=1}^{n-1} \{\alpha[2k+1]_q + 1 - \alpha\} |a_{2k+1}| \right\}, \tag{18}
\]

\[
|a_{2n+1}| \leq \frac{A-B}{([2n+1]_q - 1) \{\alpha[2n+1]_q + 1 - \alpha\}} \left\{ 1 + \sum_{k=1}^{n-1} \{\alpha[2k+1]_q + 1 - \alpha\} |a_{2k+1}| \right\}. \tag{19}
\]
Next, we assume that (9) and (10) holds for \( k \in \{3, 4, \ldots, n - 1\} \). Then from (18), we obtain
\[
|a_{2n}| \leq \frac{A - B}{[2n]_q\{\alpha[2n]_q + 1 - \alpha\}} \left\{ 1 + \sum_{k=1}^{n-1} \frac{A - B}{[2k + 1]_q - 1} \prod_{j=1}^{k-1} \frac{A - B + [2j + 1]_q - 1}{[2j + 1]_q - 1} \right\}. \tag{20}
\]

In order to prove (9) it is sufficient to show that
\[
\frac{A - B}{[2m]_q\{\alpha[2m]_q + 1 - \alpha\}} \prod_{j=1}^{m-1} \frac{A - B + [2j + 1]_q - 1}{[2j + 1]_q - 1} = \frac{A - B}{[2m]_q\{\alpha[2m]_q + 1 - \alpha\}} \left\{ 1 + \sum_{k=1}^{m-1} \frac{A - B}{[2k + 1]_q - 1} \prod_{j=1}^{k-1} \frac{A - B + [2j + 1]_q - 1}{[2j + 1]_q - 1} \right\}, \tag{21}
\]
for \( m \in \{3, 4, \ldots, n\} \).

We remark that (21) is true for \( m = 3 \). Let us suppose that (21) holds for \( 4 \leq m \leq n - 1 \). Hence, we find that
\[
\frac{A - B}{[2n]_q\{\alpha[2n]_q + 1 - \alpha\}} \left\{ 1 + \sum_{k=1}^{n-1} \frac{A - B}{[2k + 1]_q - 1} \prod_{j=1}^{k-1} \frac{A - B + [2j + 1]_q - 1}{[2j + 1]_q - 1} \right\} = \frac{A - B}{[2n-1]_q\{\alpha[2(n-1)]_q + 1 - \alpha\}} \prod_{j=1}^{n-2} \frac{A - B + [2j + 1]_q - 1}{[2j + 1]_q - 1} \cdot A - B \cdot \frac{[2(n-1)]_q\{\alpha[2(n-1)]_q + 1 - \alpha\}}{[2n]_q\{\alpha[2n]_q + 1 - \alpha\}}.
\]

Thus (21) is valid for \( m = n \) and hence (9) follows. Similarly, we can prove (10).
\[\square\]

**Remark 2.1.** If we let \( q \to 1^- \) we obtain the same coefficient inequalities for \( f \in M_s(\alpha, A, B) \) as Selvaraj and Vasanthi (see [13], Theorem 3.1).

**Corollary 2.1.** Let function \( f \) as defined in (1) be in the class \( M_{s,\alpha}(\varphi) \). Then for \( n \geq 1 \)
\[
|a_{2n}| \leq \frac{2}{[2n]_q\{\alpha[2n]_q + 1 - \alpha\}} \prod_{j=1}^{n-1} \frac{[2j + 1]_q + 1}{[2j + 1]_q - 1} \tag{23}
\]
and
\[
|a_{2n+1}| \leq \frac{2}{([2n + 1]_q - 1)\{\alpha[2n + 1]_q + 1 - \alpha\}} \prod_{j=1}^{n-1} \frac{[2j + 1]_q + 1}{[2j + 1]_q - 1}. \tag{24}
\]

**Proof.** We use Theorem 2.1 and the fact that \( |b_n| \leq 2 \), for all \( n \in \{1, 2, \ldots\} \). \[\square\]
3. Fekete–Szegő Problem

Now, we derive Fekete–Szegő inequalities for functions \( f \in M_{q,s,\alpha}(\varphi) \).

**Theorem 3.1.** Let \( \varphi(z) = 1 + B_1z + B_2z^2 + \ldots \) and \( \varphi'(0) > 0 \). If function \( f \) given by (1) belongs to the class \( M_{q,s,\alpha}(\varphi) \) and \( \mu \) is a complex number, then

\[
|a_3 - \mu a_2^2| \leq \frac{B_1}{(\beta q - 1) \cdot \{\alpha[3]q + 1 - \alpha\}} \cdot \max \left\{ 1, \left| \frac{\mu B_1(\beta q - 1) \cdot \{\alpha[3]q + 1 - \alpha\}}{[2]q \cdot \{\alpha[2]q + 1 - \alpha\}^2} - \frac{B_2}{B_1} \right| \right\}.
\]

(25)

The result is sharp.

**Proof.** If \( f \in M_{q,s,\alpha}(\varphi) \), then there exists a Schwarz function \( w \) which is analytic in \( U \) with \( w(0) = 0, |w(z)| < 1 \), and such that

\[
2\alpha zdq(zd_qf(z)) + 2(1 - \alpha)zd_qf(z)
\]

\[
\alpha zd_q(f(z) - f(-z)) + (1 - \alpha)(f(z) - f(-z)) = \varphi(w(z)).
\]

(26)

Next, we define the function \( p_1 \) by

\[
p_1(z) = \frac{1 + w(z)}{1 - w(z)} = 1 + c_1z + c_2z^2 + \ldots.
\]

(27)

Since \( w \) is a Schwarz function, we see that \( \text{Re}\{p_1(z)\} > 0 \) and \( p_1(0) = 1 \). Therefore,

\[
\varphi(w(z)) = 1 + \frac{1}{2}B_1c_1z + \left[ \frac{1}{2}B_1(c_2 - \frac{c_1^2}{2}) + \frac{1}{4}B_2c_1^2 \right] z^2 + \ldots.
\]

(28)

Further, we consider

\[
p(z) = \frac{2\alpha zdq(zd_qf(z)) + 2(1 - \alpha)zd_qf(z)}{\alpha zd_q(f(z) - f(-z)) + (1 - \alpha)(f(z) - f(-z))} = 1 + b_1z + b_2z^2 + \ldots.
\]

(29)

and we find that

\[
b_1 = \frac{1}{2}B_1c_1 \text{ and } b_2 = \frac{1}{2}B_1(c_2 - \frac{c_1^2}{2}) + \frac{1}{4}B_2c_1^2.
\]

(30)

From (26), (27) and (30) we obtain

\[
a_2 = \frac{B_1c_1}{2[2]q \cdot \{\alpha[2]q + 1 - \alpha\}} \quad \text{and} \quad a_3 = \frac{B_1(c_2 - \frac{c_1^2}{2}) + B_2c_1^2/2}{2[\alpha[3]q + 1 - \alpha]}.
\]

(31)

Therefore,

\[
a_3 - \mu a_2^2 = \frac{B_1}{2[3]q \cdot \{\alpha[3]q + 1 - \alpha\}} \cdot (c_2 - \nu c_1^2),
\]

(32)

where

\[
\nu = \frac{1}{2} \left( 1 - \frac{B_2}{B_1} + \frac{\mu B_1(\beta q - 1) \cdot \{\alpha[3]q + 1 - \alpha\}}{[2]q \cdot \{\alpha[2]q + 1 - \alpha\}^2} \right).
\]

(33)

The result follows by an application of Lemma 1.2 and is sharp for \( f \in M_{q,s,\alpha}(\varphi) \) which satisfies

\[
2\alpha zdq(zd_qf(z)) + 2(1 - \alpha)zd_qf(z)
\]

\[
\alpha zd_q(f(z) - f(-z)) + (1 - \alpha)(f(z) - f(-z)) = \varphi(z)
\]

(34)

or

\[
2\alpha zdq(zd_qf(z)) + 2(1 - \alpha)zd_qf(z)
\]

\[
\alpha zd_q(f(z) - f(-z)) + (1 - \alpha)(f(z) - f(-z)) = \varphi(z^2).
\]

(35)
Theorem 3.2. Let \( \varphi(z) = 1 + B_1 z + B_2 z^2 + \ldots \), with \( B_1 > 0 \) and \( B_2 \geq 0 \). Also let

\[
\sigma_1 = \frac{B_2 - B_1}{B_1^2} \frac{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}}, \tag{36}
\]

\[
\sigma_2 = \frac{B_2 + B_1}{B_1^2} \frac{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}} \tag{37}
\]

and

\[
\sigma_3 = \frac{B_2}{B_1^2} \frac{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}} \tag{38}
\]

If \( f \) given by (1) belongs to the class \( M_{q,s,a}(\varphi) \), then we have the following sharp results:

i) If \( \mu \leq \sigma_1 \), then

\[
|a_3 - \mu a_2^2| \leq \frac{B_2}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}} \cdot \frac{\mu B_1^2}{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2} - \frac{B_2}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}}. \tag{39}
\]

ii) If \( \sigma_1 \leq \mu \leq \sigma_2 \), then

\[
|a_3 - \mu a_2^2| \leq \frac{B_1}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}}. \tag{40}
\]

iii) If \( \mu \geq \sigma_2 \), then

\[
|a_3 - \mu a_2^2| \leq \frac{\mu B_1^2}{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2} - \frac{B_2}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}}. \tag{41}
\]

Further, if \( \sigma_1 \leq \mu \leq \sigma_3 \), then

\[
|a_3 - \mu a_2^2| + \left( \frac{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2 (B_1 - B_2)}{B_1^2 ([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}} + \mu \right) |a_2|^2 \leq \frac{B_1}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}}. \tag{42}
\]

If \( \sigma_3 \leq \mu \leq \sigma_2 \), then

\[
|a_3 - \mu a_2^2| + \left( \frac{[2]_q^2 \{ \alpha \| q + 1 - \alpha \}^2 (B_1 + B_2)}{B_1^2 ([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}} - \mu \right) |a_2|^2 \leq \frac{B_1}{([3]_q - 1) \cdot \{ \alpha [3]_q + 1 - \alpha \}}. \tag{43}
\]

The result is sharp.

Proof. The result follows by applying Lemma 1.3 to (32), where \( \nu \) is given in (33). To prove that the result is sharp we consider the function \( p \) given by (29). If \( \mu < \sigma_1 \) or \( \mu > \sigma_2 \), the equality holds if and only if \( p(z) = (1 + z)/(1 - z) \) or one of its rotations. If \( \sigma_1 < \mu < \sigma_2 \) the equality holds if and only if \( p(z) = (1 + z^2)/(1 - z^2) \) or one of its rotations. If \( \mu = \sigma_1 \), the equality holds if and only if \( p(z) = (1 + z^2) \cdot (1 + z^2) + (1 + z^2) \cdot (1 + z^2), 0 \leq \gamma \leq 1 \) or one of its rotations. If \( \mu = \sigma_2 \), the equality holds if and only if \( p \) is the reciprocal of one of the functions such that the equality holds in the case of \( \mu = \sigma_1 \).

\[\square\]

Remark 3.1. If we apply Theorem 3.2 to functions \( f \in S_{q,a}^*(\varphi) \) and \( f \in C_{q,a}(\varphi) \) we improve the results given in [8] for these classes of functions.
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