Scalable Uplink Signal Detection in C-RANs via Randomized Gaussian Message Passing

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Abstract

Cloud Radio Access Network (C-RAN) is a promising architecture for unprecedented capacity enhancement in next-generation wireless networks thanks to the centralization and virtualization of base station processing. However, centralized signal processing in C-RANs involves high computational complexity that quickly becomes unaffordable when the network grows to a huge size. This paper endeavours to design a scalable uplink signal detection algorithm, in the sense that the complexity per unit network area remains constant when the network size grows. To this end, we formulate the signal detection in C-RAN as an inference problem over a bipartite random geometric graph. By passing messages among neighboring nodes, message passing (a.k.a. belief propagation) provides an efficient way to solve the inference problem over a sparse graph. However, the traditional message-passing algorithm is not guaranteed to converge, because the corresponding bipartite random geometric graph is locally dense and contains many short loops. As a major contribution of this paper, we propose a randomized Gaussian message passing (RGMP) algorithm to improve the convergence. Instead of exchanging messages simultaneously or in a fixed order, we propose to exchange messages asynchronously in a random order. Numerical results show that the proposed RGMP algorithm has significantly better convergence performance than conventional message passing. The randomness of the message update schedule also simplifies the analysis, and allows the derivation of the convergence conditions for the RGMP algorithm.

Index Terms

Cloud-RAN; signal processing; message passing; belief propagation

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I. Introduction

A. Background and Motivations

Cloud Radio Access Networks (C-RANs) have drawn considerable attention for their potential to sustain the explosive traffic demand in wireless communications. Unlike traditional cellular networks, a C-RAN splits the low-cost and light-weighted remote radio heads (RRHs) from the baseband processing units (BBUs), and merges the latter into a data center. The RRHs and BBUs are connected by a low-latency, high-bandwidth fiber network. The special architecture of C-RAN allows full-scale RRH coordination, which enables flexible interference management, dynamic resource allocation, and collaborative radio technology. This consequently leads to significant capacity enhancement. The full-scale coordination, however, also introduces a severe complexity issue. The state-of-the-art C-RAN technology is able to support thousands of RRHs [1]. Full-scale RRH coordination over such a large network involves prohibitively high computational complexity. For example, the linear minimum mean square error (MMSE) detector requires cubic complexity in the network size (in terms of the number of RRHs), or equivalently a quadratic complexity normalized by the number of RRHs [2]. This implies that the detection complexity quickly becomes unaffordable as the network size grows. As such, a main challenge of C-RAN is to design scalable coordination algorithms, where scalable means: 1) the performance is near the optimum performance of full-scale RRH coordination, 2) the normalized computational complexity per RRH does not grow with the network size, or equivalently, the total computational complexity grows linearly with the network size.

In a C-RAN, users and RRHs are scattered over a large area. Due to the attenuation effect in the propagation of electromagnetic waves, an RRH usually receives strong signals from only a small number of nearby users. Intuitively, ignoring the signals from far-away users in general does not cause much performance loss. As shown in our previous work [4], with a distance-threshold-based channel sparsification approach, a vast majority of signals over the transmission links can be ignored if we can tolerate a small degradation in the signal-to-noise-plus-interference ratio (SINR). As such, each RRH only needs to serve its nearby users whose distances to the
RRH are below a certain threshold. Based on the sparsified channel matrix, [4] proposes an algorithm that greatly reduces the computational complexity of MMSE detection from $O(N^3)$ to $O(N^a)$, where $N$ is the total number of RRHs and $a \in (1, 2]$ is a constant depending on the computing implementations. However, the algorithm is not perfectly scalable yet, in the sense that the complexity grows faster than linear with the network size.

In this paper, we are interested in designing a perfectly scalable algorithm for joint signal detection in the uplink of C-RAN. With channel sparsification [4], a C-RAN system can be represented by a bipartite random geometric graph, as shown in Fig. 1. Here, RRHs and users are treated as vertices/nodes, and an edge connects an RRH and a user if the distance between them does not exceed the threshold. Then, signal detection in a C-RAN is converted to a statistical inference problem over a bipartite random geometric graph, where an inference problem is to estimate the signals from unobserved nodes (i.e., user nodes), conditional on signals from observed nodes (i.e., RRH nodes).

![Graphical representation of a C-RAN](image)

Fig. 1. Graphical representation of a C-RAN

Message passing (a.k.a. belief propagation) is well known as a low-complexity iterative solution to inference over random sparse graphs [3]. In a message-passing algorithm, messages are exchanged between nodes with edge connection. Thus, the complexity of message passing is proportional to the number of edges in the network. In a C-RAN with channel sparsification, the
number of messages per RRH is proportional to the number of nearby users in its neighborhood, which does not scale with the network size. Thus, the total complexity per iteration of message passing in C-RANs is linear in the network size. Unfortunately, the convergence of message passing over a bipartite random geometric graph is not guaranteed. The reason is that the random geometric graph is locally dense and always contains loops. It is well-known that message passing is not guaranteed to converge when the graph is loopy [9]. Indeed, numerical simulations indicate a non-trivial probability that the message-passing algorithm for C-RANs does not converge.

B. Contributions

In this paper, we propose a novel algorithm, referred to as randomized Gaussian message passing (RGMP), to address the convergence issue of message passing over a bipartite random geometric graph. Unlike conventional message passing with synchronous message updating, the RGMP algorithm updates messages asynchronously with a random sequential order. To the best of our knowledge, this is the first work to introduce randomness to the update schedule of message passing algorithms. Intuitively, when messages are exchanged among nodes of a loopy graph, errors may accumulate along loops, which eventually leads to the divergence of the algorithm [5]. Updating messages sequentially in a random order effectively weakens the effect of loops and thus improves the convergence. The randomness of the message update schedule also simplifies the analysis, and allows to derive a necessary and sufficient condition for the expected convergence of the RGMP algorithm. We show by both analysis and numerical results that the RGMP algorithm converges with a much higher probability than conventional message passing. Indeed, we have never observed a single divergence in our simulations when the network size is moderately large (i.e., when the network has more than five RRHs). Moreover, our numerical results indicate that the number of iterations of RGMP does not increase with the network size. This implies that the total computational complexity is linear with the network size, and thus the algorithm is perfectly scalable.
C. Related Work

In the previous work \cite{4}, we proposed a dynamic clustering algorithm to reduce the computational complexity of the MMSE detector. The complexity of the algorithm is reduced from cubic to no more than quadratic in the number of RRHs. In \cite{6}, Shi et al. presented a two-stage approach to solve large-scale convex optimization problems for dense wireless cooperative networks, such as C-RANs. Matrix stuffing and alternating direction method of multipliers (ADMM) were used to speed up the computation. Particularly, it was shown in \cite{7} that the expected output of randomly permuted ADMM converges to the unique solution of the optimal linear detector. However, we will see that the ADMM algorithm converges much more slowly than the proposed RGMP algorithm when applied to large networks like C-RANs. In addition, there are other iterative solvers of the linear MMSE detector, such as the preconditioned conjugate gradient (PCG) method \cite{13} and the generalized approximate message passing (GAMP) algorithm \cite{8}. We show by numerical simulation that the number of iterations needed for convergence in these algorithms are roughly linear in the network size. This translates to quadratic computational complexity in total. Therefore, all these algorithms are not scalable.

D. Organization

The rest of the paper is organized as follows. In Section II, we describe the system model. In Section III, we introduce a Gaussian message-passing algorithm with channel sparsification for signal detection in C-RANs with linear complexity per iteration, and then discuss the convergence issue. In Section IV, we propose the RGMP algorithm to address the convergence issue of Gaussian message passing. In Section V, the convergence condition of the RGMP algorithm are analysed. In Section VI, simulation results are demonstrated to compare the RGMP algorithm with other existing algorithms. Conclusions and future works are discussed in Section VII.

II. SYSTEM MODEL

In this paper we consider the uplink transmission of a C-RAN with $N$ single-antenna RRHs and $K$ single-antenna users. Suppose that both the RRHs and the users are randomly located
over an area. Let $x_k$ be the signal transmitted by user $k$, and $y_n$ be the received signal at RRH $n$. Denote $x = [x_1, \ldots, x_K]^T$ and $y = [y_1, \ldots, y_N]^T$. Then, the received signal vector $y \in \mathbb{C}^{N \times 1}$ at the RRHs is

$$y = P^{1/2}Hx + n, \quad (1)$$

where $H \in \mathbb{C}^{N \times K}$ denotes the channel matrix, with the $(n, k)$-th entry $H_{n,k}$ being the channel coefficient between the $k$-th user and the $n$-th RRH; $P$ is the transmission power allocated to each user; and $n \sim \mathcal{CN}(0, N_0 I)$ is a noise vector received by the RRHs. The transmitted signals are assumed to have zero mean and unit variance, i.e., $E[x] = 0$ and $E[xx^H] = I$. We further assume $H_{n,k} = \gamma_{n,k}d_{n,k}^{-\alpha}$, where $\gamma_{n,k}$ is the i.i.d. Rayleigh fading coefficient with zero mean and unit variance, $d_{n,k}$ is the distance between the $n$-th RRH and the $k$-th user, and $\alpha$ is the path loss exponent. Here, $d_{n,k}^{-\alpha}$ is the path loss from the $k$-th user to the $n$-th RRH.

In this paper, we employ linear MMSE detection to estimate the transmit signal vector $x$, with the decision statistics given by

$$\widehat{x} = P^{1/2}H^H(PHH^H + N_0 I)^{-1}y. \quad (2)$$

In the above, the inversion of the $N \times N$ matrix $PHH^H + N_0 I$ requires computational complexity of $O(N^3)$. This complexity is prohibitively high for a large-scale C-RAN with hundreds and thousands of RRHs, thus posing a serious scalability problem. In what follows, we endeavour to develop a scalable algorithm to detect $x$ with complexity $O(N)$ under the assumption that $K$ grows at the same rate as $N$. In other words, the average computational complexity per RRH (or per unit network size) does not scale with $N$.

### III. Gaussian Message Passing with Channel Sparsification

In this section, we first describe the channel sparsification approach introduced by the authors in [4] to model a C-RAN as a bipartite random geometric graph. Then, we apply the Gaussian message-passing algorithm over bipartite random geometric graphs for signal detection.
A. Channel Sparsification

We borrow the channel sparsification approach in our recent work [4] to sparsify the channel matrix, as described below. The entries of $\mathbf{H}$ are discarded based on the distances between RRHs and users. Specifically, the $(n,k)$-th entry in the resulting sparsified channel matrix $\hat{\mathbf{H}}$ is given by

$$
\hat{H}_{n,k} = \begin{cases} 
H_{n,k}, & d_{n,k} < d_0 \\
0, & \text{otherwise,}
\end{cases}
$$

where $d_0$ is a distance threshold. Given the sparsified channel matrix $\hat{\mathbf{H}}$, the received signal can be represented as

$$
y = PH\hat{\mathbf{H}}x + P\tilde{\mathbf{H}}x + n,
$$

where $\tilde{\mathbf{H}} = \mathbf{H} - \hat{\mathbf{H}}$. The MMSE estimator of $\mathbf{x}$ is approximated by

$$
\hat{\mathbf{x}} \approx P\frac{1}{2}\hat{\mathbf{H}}^H(P\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \hat{\mathbf{N}}_0)\mathbf{I}^{-1}y,
$$

with $\hat{\mathbf{N}}_0 = PE[\sum_{j \neq k} |\tilde{H}_{n,j}|^2] + N_0$ for arbitrary RRH $n$.

As proven in [4], the channel matrix can be sparsified without considerably compromising the SINR. The reason is that as the RRHs and users are uniformly distributed over a large area, an RRH can only receive reasonably strong signals from a small number of nearby users, and vice versa. Therefore, the majority of the elements of $\mathbf{H}$ are relatively small in magnitude, and ignoring them in signal detection leads to marginal loss in the overall system performance. Indeed, according to [4], when $N$ scales in the same order as $K$, the distance threshold $d_0$ does not increase with the network size to achieve a certain SINR performance. Thus, in this paper, we assume that $d_0$ is a predetermined constant regardless of the network size. This implies that the average number of users connecting to an RRH does not scale with the network size.

B. Bipartite Random Geometric Graph

Channel sparsification simplifies the signal detection in a C-RAN to an inference problem over a bipartite random geometric graph (see Fig. 1). In the bipartite random geometric graph,
RRHs and users in a C-RAN are referred to RRH nodes and user nodes respectively, and edge connections exist only between RRH nodes and user nodes. More specifically, an RRH node is connected to a user node only if the distance between them falls below the threshold $d_0$, and the weight over such an edge is the channel coefficient from the corresponding user to the corresponding RRH.

Suppose that the entries in $\mathbf{x}$ follow an independent complex Gaussian distribution. Then, $\mathbf{y}$ and $\mathbf{x}$ are jointly Gaussian, and therefore the MMSE detector in (2) is also the maximum a posteriori probability (MAP) detector that maximizes the a posteriori probability $p(\mathbf{x}|\mathbf{y})$. That is,

$$\hat{\mathbf{x}} = \arg \max p(\mathbf{x}|\mathbf{y}). \tag{6}$$

The probability density function $p(\mathbf{x}|\mathbf{y})$ can be factorized as

$$p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$$

$$= p(y_1|x) \cdots p(y_n|x) \cdots p(y_N|x)$$

$$\times p(x_1) \cdots p(x_k) \cdots p(x_K). \tag{7}$$

Recall that we sparsify the channel matrix by using the channel sparsification approach given in [4]. Based on (5), the factorization of $p(\mathbf{x}|\mathbf{y})$ is approximated as

$$p(\mathbf{x}|\mathbf{y}) \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x})$$

$$\approx p(y_1|x_{I_1}) \cdots p(y_n|x_{I_n}) \cdots p(y_N|x_{I_N})$$

$$\times p(x_1) \cdots p(x_k) \cdots p(x_K), \tag{8}$$

where $x_{I_n}$ contains all $x_i$ with $i \in I_n$ and $I_n$ is the set of user indices with $d_{n,k} < d_0$.

We now transfer the bipartite random geometric graph to a factor graph with the factorization in (8). As illustrated in Fig. 2, a factor graph is also a bipartite graph comprising two types of nodes, namely, variable nodes (denoted by circles) and check nodes (denoted by squares), together with edges connecting these two types of nodes. The relation between the factorization

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1If $\mathbf{x}$ does not follow a Gaussian distribution, the message-passing algorithm presented in this paper gives an approximation of the linear MMSE estimation [9].
and its associated factor graph is as follows. A check node $p(y_n | x_{I,n})$ is connected to a variable node $x_k$ by an edge when there is an edge connecting the $n$-th RRH node and the $k$-th user node in the corresponding random geometric graph (i.e., $d_{n,k} < d_0$), or equivalently, when the function $p(y_n | x_{I,n})$ takes $x_k$ as input.

C. Gaussian Message Passing

We are now ready to introduce the Gaussian message-passing algorithm for signal detection. The algorithm will be implemented in the centralized data center. The messages, namely, the marginals of $\{x_k\}$ and $\{y_n\}$, are exchanged along the edges. In this paper, both $\{x_k\}$ and $\{y_n\}$ are Gaussian distributed, and therefore the messages are Gaussian probability density functions and can be completely characterised by mean and variance. Denote by $m_{y_n \to x_k}^{(t)}$ and $v_{y_n \to x_k}^{(t)}$ the mean and variance sent from check node $p(y_n | x_{I,n})$ to variable node $x_k$ at iteration $t$, respectively, and denote by $m_{x_k \to y_n}^{(t)}$ and $v_{x_k \to y_n}^{(t)}$ the mean and variance sent from variable node $x_k$ to check node $p(y_n | x_{I,n})$ at iteration $t$, respectively. The detailed steps of message passing are presented in Algorithm 1. We refer to this algorithm as Gaussian message passing (GMP), as all the messages involved are Gaussian marginals. Note that each RRH only serves users located in a circle with
Algorithm 1 Gaussian Message-Passing (GMP) Algorithm

**Input:** \( \hat{H}, y \)

**Output:** \( \hat{x}_k \) for all \( k \)

1. initial \( t = 0, m^{(0)}_{x_k \rightarrow y_n} = 0, v^{(0)}_{y_n \rightarrow x_k} = 1 \), for all \( k, n \)
2. repeat
   3. Set \( t \leftarrow t + 1 \)
   4. For all \( n, k \) such that \( \hat{H}_{n,k} \neq 0 \), compute
      \[
      v^{(t)}_{y_n \rightarrow x_k} = \frac{1}{P|\hat{H}_{n,k}|^2} \left( \hat{N}_0 + P \sum_{j \neq k} |\hat{H}_{n,j}|^2 v^{(t-1)}_{x_j \rightarrow y_n} \right) \tag{9}
      \]
      \[
      m^{(t)}_{y_n \rightarrow x_k} = \frac{1}{P^2 \hat{H}_{n,k}} \left( y_n - P^{\frac{1}{2}} \sum_{j \neq k} \hat{H}_{n,j} m^{(t-1)}_{x_j \rightarrow y_n} \right) \tag{10}
      \]
      \[
      v^{(t)}_{x_k \rightarrow y_n} = \left( \sum_{\hat{H}_{j,k} \neq 0, j \neq n} \frac{1}{v^{(t)}_{y_j \rightarrow x_k}} + 1 \right)^{-1} \tag{11}
      \]
      \[
      m^{(t)}_{x_k \rightarrow y_n} = v^{(t)}_{x_k \rightarrow y_n} \left( \sum_{\hat{H}_{j,k} \neq 0, j \neq n} \frac{m^{(t)}_{y_j \rightarrow x_k}}{v^{(t)}_{y_j \rightarrow x_k}} \right) \tag{12}
      \]
5. until the stopping criterion is satisfied
6. Compute
   \[
   v_k = \left( \sum_{\hat{H}_{n,k} \neq 0} \frac{1}{v^{(t)}_{y_n \rightarrow x_k}} + 1 \right)^{-1} \tag{13}
   \]
   \[
   \hat{x}_k = v_k \left( \sum_{\hat{H}_{n,k} \neq 0} \frac{m^{(t)}_{y_n \rightarrow x_k}}{v^{(t)}_{y_n \rightarrow x_k}} \right) \tag{14}
   \]

a constant radius \( d_0 \). Thus, the average number of messages to be exchanged and computed at each node does not scale with the network size. Therefore, the complexity per iteration of the GMP algorithm is linear in the number of RRHs and users.

In spite of its linear complexity per iteration, the GMP algorithm is not guaranteed to converge on the factor graphs induced by C-RANs. It is known that the Gaussian message-passing
algorithm always converges to the optimal solution on a tree-type factor graph\footnote{2}. It is also known that, if a factor graph is random and sparse enough, the corresponding message-passing algorithm converges asymptotically as the network size grows to infinity \footnote{16}. However, the factor graph for a bipartite random geometric graph induced from a C-RAN is locally dense and far from being a tree. This is due to the fact that every RRH needs to simultaneously serve multiple nearby users. For example, \{R2, U1, R3, U2\} in Fig. 1 form a loop\footnote{3} of length 4. Indeed, we observe in simulations that the GMP algorithm diverges in C-RAN with a non-trivial probability. Even worse, the probability of divergence grows with the network size, as illustrated later in Fig. 4. We focus on improving the convergence performance of GMP in the rest of the paper.

\textbf{Remark 1.} The GMP algorithm for a C-RAN with channel sparsification can be simply extended to the case without channel sparsification by setting the distance threshold to infinity. However, this leads to an increase of the computational complexity per iteration. We see that in each iteration of Algorithm 1, messages need to be updated on every edge of the factor graph. From channel randomness, the entries of \(H\) are non-zero with probability one. Thus, in the factor graph without channel sparsification, every RRH check node \(p(y_n|x)\) is connected to all user variable nodes \(\{x_k\}_{k=1}^K\). This implies that the total number of edges in the factor graph is \(NK\), implying that the complexity of the GMP algorithm is \(O(NK)\) per iteration, which is unaffordable for a large-scale C-RAN.

IV. RANDOMIZED GAUSSIAN MESSAGE PASSING WITH CHANNEL SPARSIFICATION

A. Randomized Gaussian Message Passing

In this section, we propose the RGMP algorithm to address the convergence issue of GMP. The main novelty of the RGMP algorithm is on the scheduling strategy for message updating. The

\footnote{2}{A tree-type graph is an undirected graph in which any two nodes are connected by exactly one path, where a path is a sequence of edges which connect a sequence of vertices without repetition.}

\footnote{3}{A loop in a graph is a path that starts and ends at the same node.}
Algorithm 2 Randomized Gaussian Message-Passing (RGMP) Algorithm

**Input:** $\widehat{H}$, $y$

**Output:** $\widehat{x}_k$ for all $k$

1: initialize $t = 0$, $m_{x_k \rightarrow y_n}^{(0)} = 0$, $v_{x_k \rightarrow y_n}^{(0)} = 1$, for all $k, n$.  
2: repeat
3:   Set $t \leftarrow t + 1$.
4:   Pick a permutation $\sigma$ of $\{1, \cdots, K\}$ uniformly at random.
5:   For $i = 1, \cdots, K$, and $\widehat{H}_{n,\sigma(i)} \neq 0$, compute
6:     \[
v_{x_n \rightarrow x_{\sigma(i)}}^{(t)} = \frac{1}{P[H_{n,\sigma(i)}]} \left( \sum_{j < i} \left| \widehat{H}_{n,\sigma(j)} \right|^2 v_{x_{\sigma(j)} \rightarrow y_n}^{(t)} + \sum_{j > i} \left| \widehat{H}_{n,\sigma(j)} \right|^2 v_{x_{\sigma(j)} \rightarrow y_n}^{(t-1)} \right) \]
7:   \[
m_{x_n \rightarrow x_{\sigma(i)}}^{(t)} = \frac{1}{P[H_{n,\sigma(i)}]} \left( y_n - P^2 \sum_{j < i} \left| \widehat{H}_{n,\sigma(j)} \right|^2 m_{x_{\sigma(j)} \rightarrow y_n}^{(t)} - P^2 \sum_{j > i} \left| \widehat{H}_{n,\sigma(j)} \right|^2 m_{x_{\sigma(j)} \rightarrow y_n}^{(t-1)} \right) \]
8:     \[
v_{x_{n} \rightarrow y_n}^{(t)} = \left( \sum_{\widehat{H}_{j,\sigma(i)} \neq 0, j \neq n} \frac{1}{v_{y_j \rightarrow x_{\sigma(i)}}^{(t)}} + 1 \right)^{-1} \]
9:     \[
m_{x_n \rightarrow y_n}^{(t)} = v_{x_n \rightarrow y_n}^{(t)} \left( \sum_{\widehat{H}_{j,\sigma(i)} \neq 0, j \neq n} m_{y_j \rightarrow x_{\sigma(i)}}^{(t)} \right) \]
10: until stopping criteria is satisfied
11: Compute
12:     \[
v_k = \left( \sum_{\widehat{H}_{n,k} \neq 0} \frac{1}{v_{y_n \rightarrow x_k}^{(t)}} + 1 \right)^{-1} \]
13:     \[
\widehat{x}_k = v_k \left( \sum_{\widehat{H}_{n,k} \neq 0} \frac{m_{y_n \rightarrow x_k}^{(t)}}{v_{y_n \rightarrow x_k}^{(t)}} \right) . \]

conventional GMP algorithm employs synchronous message passing, i.e., messages are updated in parallel. As aforementioned, synchronous message passing does not work well in C-RANs due to local loops in the factor graph. To address this issue, we propose the RGMP algorithm with random asynchronous scheduling, i.e., messages are updated sequentially in a randomly permuted order.

The RGMP algorithm is described as follows. Define $\Sigma$ as

\[
\Sigma \triangleq \{ \sigma | \sigma \text{ is a permutation of } \{1, \cdots, K\} \}. \]
At each iteration, we draw a permutation $\sigma$ of $\{1, \cdots, K\}$ uniformly from $\Sigma$, and update the messages at the user variable node side in the order of $\sigma$. For example, at the $t$-th iteration, when $K = 3$ and $\sigma = (2, 1, 3)$, we first update all the messages on the edges connecting the user variable node $x_2$. Then, the messages on the edges connecting the user variable node $x_1$ are updated. Finally, messages related to variable node $x_3$ are updated. The RGMP algorithm is given in Algorithm 2.

B. Numerical Examples

![Graph](a) RGMP

![Graph](b) Synchronous Gaussian message passing

![Graph](c) Asynchronous Gaussian message passing with schedule $(1, 2, 4, 3)$

![Graph](d) Asynchronous Gaussian message passing with schedule $(1, 2, 3, 4)$

Fig. 3. Relative error vs number of iterations.
In this subsection, we use a toy example to illustrate the difference between our proposed RGMP algorithm and synchronous/asynchronous GMP. Consider the following randomly generated channel matrix

$$\mathbf{H} = 10^{-5} \begin{bmatrix} -0.1458 + 0.2401i & -2.0998 - 0.7353i & -2.1459 - 2.0284i & 0.6130 + 2.0420i \\ 17.7199 + 18.8315i & 1.8431 - 2.4183i & 5.7441 + 2.0536i & 0.4837 - 3.0383i \\ 5.1714 - 14.5292i & 0.1184 - 1.5314i & -10.3012 + 0.1049i & 2.4388 - 0.8546i \\ -25.2041 - 16.2758i & 1.1697 - 0.3792i & 2.2858 - 0.2858i & 6.0425 - 2.6317i \end{bmatrix}, \quad (22)$$

and let the transmit SNR (i.e., $\frac{P}{N_0}$) be 100dB. The corresponding received signal $\mathbf{y}$ is

$$\mathbf{y} = [1.6847 - 7.1280i, -20.9794 + 3.6052i, -3.0214 + 3.8041i, 21.5306 + 6.5308i]^T. \quad (23)$$

For fairness of comparison, we do not conduct channel sparsification in this example. That is, the distance threshold is set to infinity. Fig. 3 plots the relative error versus the number of iterations for the RGMP algorithm and the GMP algorithm with different message update strategies, i.e., synchronous update and asynchronous update with different fixed orders, $(1, 2, 4, 3)$ and $(1, 2, 3, 4)$. The relative error is defined as $\frac{\|\mathbf{P}^\frac{1}{2} \mathbf{H}^H \mathbf{x}^{(t)} - \mathbf{P}^\frac{1}{2} \mathbf{H}^H \mathbf{y}\|}{\|\mathbf{P}^\frac{1}{2} \mathbf{H}^H \mathbf{y}\|}$, where $\mathbf{x}^{(t)}$ is the estimation of the transmitted signal after the $t$-th iteration. We see that the synchronous GMP algorithm and the asynchronous one with order $(1, 2, 3, 4)$ diverge, but the asynchronous GMP with order $(1, 2, 4, 3)$ and the proposed RGMP algorithm converge.

**Remark 2.** The examples in Fig. 3(c) and 3(d) show that convergence of asynchronous GMP heavily depends on the update order. Unfortunately, there is no systematic way to derive a fixed update order that guarantees convergence. In general, finding such an update order is difficult, especially in large networks. This issue is avoided in the proposed RGMP algorithm by randomizing the update schedule instead of fixing one. Indeed, the randomization significantly weakens the loopy effect of the graph, and thus convergence is almost ensured in RGMP.

In Fig. 4, we plot the empirical probability of convergence against the network size, where users and RRHs are uniformly located in a circular network area with user density $8$/km$^2$ and RRH density $10$/km$^2$. The distance threshold $d_0$ is 1000m. For each simulated point in Fig. 4,
both GMP and RGMP are run for over 6000 times that are randomized over both RRH/user location and channel fading. For GMP, the convergence probability decreases when the network size becomes large. In contrast, no divergence has been observed for the RGMP algorithm throughout our simulations.

V. CONVERGENCE ANALYSIS

In this section, we establish a necessary and sufficient condition for the expected convergence of the proposed RGMP algorithm. For self-containedness, we start with existing results on the analysis of the convergence condition for conventional GMP.

A. Convergence of GMP

The factor graph of a C-RAN contains loops with high probability. The convergence of GMP on a loopy factor graph has been previously studied in [10], with the main result summarized below.
From Algorithm 1, we see that the evolution of the variances $v_{y_{n \rightarrow x_k}}$ is independent of the means $m_{y_{n \rightarrow x_k}}$, $m_{x_k \rightarrow y_n}$ and the received signal $y$. Substituting (11) into (9), we obtain

$$v_{y_{n \rightarrow x_k}}^{(t)} = \frac{1}{P|\hat{H}_{n,k}|^2} \left( \frac{N_0 + P \sum_{j \neq k} |\hat{H}_{n,j}|^2}{\sum_{i,j \neq 0,i \neq n} \frac{1}{v_{y_{i \rightarrow x_j}}^{(t-1)}} + 1} \right)^{-1}.$$  \hspace{1cm} (24)

Denote (24) in a vector form as

$$v^{(t)} = f(v^{(t-1)}),$$  \hspace{1cm} (25)

where $f(\cdot)$ is the evolution function determined by (24), and $v^{(t)}$ is a vector consisting of $v_{y_{n \rightarrow x_k}}^{(t)}$ for all $n$ and $k$ with $\hat{H}_{n,k} \neq 0$. Note that $f(\cdot)$ is a standard function, the definition of which is given below.

**Definition 1.** A function $f(v)$ is standard if for all $v \geq 0$ the following properties are satisfied.

- **Positivity:** $f(v) > 0$.
- **Monotonicity:** If $v \geq v'$, then $f(v) \geq f(v')$.
- **Scalability:** For all $\alpha > 1$, $\alpha f(v) > f(\alpha v)$.

From Theorem 5.1 of [10], the sequence of $v^{(t)}$ always converges to a unique fixed point if the initial point $v > 0$ and the evolution function is standard.

We now consider the convergence of means. A vector of means, $m^{(t)}$, is constructed with its $((k-1)N + n)$-th entry being

$$m_{(k-1)N+n}^{(t)} = \begin{cases} m_{y_{n \rightarrow x_k}}^{(t)}, & \text{if } \hat{H}_{n,k} \neq 0, \\ 0, & \text{otherwise}. \end{cases}$$  \hspace{1cm} (26)

The recursion of the means is given by (10) and (12). As the variances always converge, the evolution of the means can be written as follows:

$$m^{(t)} = \Omega m^{(t-1)} + z,$$  \hspace{1cm} (27)
where \( z^{(t)} \) is an \( NK \times 1 \) vector with its \(((k-1)N+n)\)-th entry being
\[
z_{(k-1)N+n} = \begin{cases} 
\frac{y_n}{p^2 \hat{H}_{n,k}}, & \hat{H}_{n,k} \neq 0, \\
0, & \text{otherwise,}
\end{cases}
\] (28)
and \( \Omega \) is an \( NK \times NK \) matrix with the \(((k-1)N+n, (j-1)N+i)\)-th entry being
\[
\Omega_{(k-1)N+n, (j-1)N+i} = \begin{cases} 
-\hat{H}_{n,k}v^*_{y_n \rightarrow x_j}, & \hat{H}_{n,k} \neq 0, \text{ and } \hat{H}_{i,j} \neq 0, \text{ and } n \neq i, \text{ and } j \neq k, \\
0, & \text{otherwise,}
\end{cases}
\] (29)
with \( v^*_{y_n \rightarrow x_j} = \lim_{t \rightarrow \infty} v^{(t)}_{y_n \rightarrow x_j} \) and \( v^*_{x_j \rightarrow y_n} = \left( \sum_{i,j} \hat{H}_{i,j} \neq 0, i \neq n} \frac{1}{v^*_{y_n \rightarrow x_j}} + 1 \right)^{-1} \). Then, a necessary and sufficient condition for the convergence of (27) is given in Theorem 5.2, [10]. That is, in Algorithm 1, the sequence of \( m^{(t)} \) converges to a unique fixed point if and only if the spectral radius \( \rho(\Omega) < 1 \).

**B. Convergence of RGMP**

In this subsection, we first show that the message variances always converge in the RGMP algorithm. Then, we focus on the convergence condition of the means in RGMP.

Recall that the evolution function of the variances in (24) is a standard function. As proven in [15], if the evolution function of a synchronous algorithm is standard, then the corresponding asynchronous algorithm converges. Based on that, we obtain the following theorem.

**Theorem 1.** In the RGMP algorithm, the sequence of \( v^{(t)}_{y_n \rightarrow x_k} \) always converges to the same unique fixed point as in Algorithm 1 if the initial point \( v > 0 \).

With Theorem 1, it suffices to focus on the convergence condition of the means in the RGMP algorithm. We first derive the evolution function of the means. Denote the permutation at the \( t \)-th iteration as \( \sigma_t \). Combining (16) and (18), we obtain the evolution of means \( m^{(t)} \) as
\[
m_{\sigma_t(i)}^{(t+1)} = \sum_{j<i} \Omega_{\sigma_t(i),\sigma_t(j)} m_{\sigma_t(j)}^{(t+1)} + \sum_{j>i} \Omega_{\sigma_t(i),\sigma_t(j)} m_{\sigma_t(j)}^{(t)} + z_{\sigma_t(i)},
\] (30)
where $m^{(t)}_{\sigma_t(i)}$ is an $N \times 1$ subvector of $m^{(t)}$ with the $n$-th entry being

$$m^{(t)}_{\sigma_t(i)}(n) = \begin{cases} m^{(t)}_{yn \rightarrow x_{\sigma_t(i)}}, & \hat{H}_{n,\sigma_t(i)} \neq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (31)$$

and $z_{\sigma_t(i)}$ is an $N \times 1$ subvector of $z$ with the $n$-th entry being

$$z_{\sigma_t(i)}(n) = \begin{cases} \frac{y_n}{\sqrt{2} \hat{H}_{n,\sigma_t(i)}}, & \hat{H}_{n,\sigma_t(i)} \neq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

$\Omega_{\sigma_t(i),\sigma_t(j)}$ is the $N \times N$ evolution matrix from user $\sigma_t(j)$ to user $\sigma_t(i)$ with the $(n,m)$-th entry being

$$\Omega_{\sigma_t(i),\sigma_t(j)}(n,m) = \begin{cases} -\frac{\hat{H}_{n,\sigma_t(j)} v^*_n x_{\sigma_t(j)}}{\hat{H}_{n,\sigma_t(i)} v^*_n x_{\sigma_t(j)}}, & \hat{H}_{n,\sigma_t(i)} \neq 0 \text{ and } \hat{H}_{n,\sigma_t(j)} \neq 0, \text{ and } n \neq m, \\ 0, & \text{otherwise.} \end{cases} \quad (33)$$

$\Omega_{\sigma_t(i),\sigma_t(j)}$ is the $(\sigma_t(i), \sigma_t(j))$-th submatrix of $\Omega$. More specifically,

$$\Omega = \begin{bmatrix} 0 & \Omega_{1,2} & \cdots & \Omega_{1,K-1} & \Omega_{1,K} \\ \Omega_{2,1} & 0 & \Omega_{2,3} & \cdots & \Omega_{2,K} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \Omega_{K-1,1} & \cdots & \Omega_{K-1,K-2} & 0 & \Omega_{K-1,K} \\ \Omega_{K,1} & \Omega_{K,2} & \cdots & \Omega_{K-1,K} & 0 \end{bmatrix}. \quad (34)$$

We can further rewrite the equation (30) as

$$L_{\sigma_t} m^{(t+1)} = R_{\sigma_t} m^{(t)} + z, \quad (35)$$

where $L_{\sigma_t} = [L_{\sigma_t(i,j)}]_{i,j} \in \mathbb{C}^{NK \times NK}$ with its $(\sigma_t(i), \sigma_t(j))$-th submatrix being

$$L_{\sigma_t}(\sigma_t(i), \sigma_t(j)) = \begin{cases} -\Omega_{\sigma_t(i),\sigma_t(j)}, & i > j \\ I, & i = j \\ 0, & \text{otherwise,} \end{cases} \quad (36)$$
and $R_\sigma = [R_{\sigma(i),j}]_{i,j} \in \mathbb{C}^{NK \times NK}$ with its $(\sigma_t(i), \sigma_t(j))$-th submatrix being

$$R_{\sigma_t}(\sigma_t(i), \sigma_t(j)) = \begin{cases} \Omega_{\sigma_t(i),\sigma_t(j)}, & i < j \\ 0, & \text{otherwise} \end{cases}$$

(37)

Based on the definition of $L_{\sigma_t}$, the determinant of $L_{\sigma_t}$ is always 1 or $-1$. It implies that $L_{\sigma_t}$ is nonsingular. Then, multiplying both sides of (35) by $L_{\sigma_t}^{-1}$, we obtain

$$m^{(t+1)} = L_{\sigma_t}^{-1} R_{\sigma_t} m^{(t)} + L_{\sigma_t}^{-1} z.$$  

(38)

Consequently, we obtain the following condition for the convergence of the RGMP algorithm.

**Proposition 1.** For a given sequence of permutations $(\sigma_1, \sigma_2, \cdots, \sigma_t, \cdots)$, the RGMP algorithm converges to the fixed point $(I - \Omega)^{-1} z$ if and only if $\lim_{t \to \infty} L_{\sigma_t}^{-1} R_{\sigma_t} \cdots L_{\sigma_1}^{-1} R_{\sigma_1} = 0$, where $\sigma_t$ is the permutation at the $t$-th iteration.

**Proof:** For an arbitrary permutation $\sigma$, the fixed point of

$$m = L_{\sigma}^{-1} R_{\sigma} m + L_{\sigma}^{-1} z$$

is given by

$$m^* = (I - L_{\sigma}^{-1} R_{\sigma})^{-1} L_{\sigma}^{-1} z.$$  

(40)

Substituting $R_{\sigma} = \Omega - I + L_{\sigma}$ into (40), we obtain $m^* = (I - \Omega)^{-1} z$. Clearly, $m^*$ is independent of the choice of permutation $\sigma$. Define $e^{(t)} = m^{(t)} - (I - \Omega)^{-1} z$. Then, $e^{(t)} = L_{\sigma_t}^{-1} R_{\sigma_t} e^{(t-1)}$, for any iteration number $t$. By recursion, we obtain $e^{(t+1)} = L_{\sigma_t}^{-1} R_{\sigma_t} \cdots L_{\sigma_1}^{-1} R_{\sigma_1} e^{(0)}$. Therefore, $m^{(t)} \to m^*$ provided $L_{\sigma_t}^{-1} R_{\sigma_t} \cdots L_{\sigma_1}^{-1} R_{\sigma_1} \to 0$ as $t \to \infty$. This concludes the proof. $lacksquare$

Proposition 2 discusses the convergence for a given sequence of permutations $(\sigma_1, \sigma_2, \cdots, \sigma_t, \cdots)$. To quantify the average performance of RGMP over random permutations, we consider expected convergence in the following, where the expectation is taken over all possible permutations. Let the expected output be

$$\phi^{(t)} = E_{\xi_{t-1}}[m^{(t)}],$$  

(41)
where \( \xi_t = (\sigma_1, \ldots, \sigma_t) \) is the set of the permutations after iteration \( t \). We are now ready to present a necessary and sufficient condition for the convergence of \( \phi^{(t)} \).

**Theorem 2.** The expected output \( \phi^{(t)} = \mathbb{E}_{\xi_{t-1}}[m^{(t)}] \) converges to the unique point \((I - \Omega)^{-1}z\) if and only if the spectral radius \( \rho(\Lambda) < 1 \), where

\[
\Lambda \triangleq \mathbb{E}_\sigma[L_\sigma^{-1}R_\sigma] = \frac{1}{K!} \sum_{\sigma \in \Sigma} (L_\sigma^{-1}R_\sigma). \tag{42}
\]

**Proof:** Denote \( A \triangleq \mathbb{E}_\sigma[L_\sigma^{-1}] \). Based on the definition of \( \phi^t \), we obtain

\[
\phi^{(t+1)} = \mathbb{E}_{\xi_t}[L_\sigma^{-1}R_\sigma, m^{(t)} + L_\sigma^{-1}z] \\
= \mathbb{E}_{\sigma_t} \left[ \mathbb{E}_{\xi_{t-1}} [L_\sigma^{-1}R_\sigma, m^{(t)} + L_\sigma^{-1}z] \right] \\
= \Lambda \phi^{(t)} + Az. \tag{43}
\]

From Theorem 5.3 in [12], the sequence of \( \phi^{(t)} \) converges to the fixed point \((I - \Lambda)^{-1}Az\) if and only if the spectral radius \( \rho(\Lambda) < 1 \). Then, it suffices to show that \((I - \Lambda)^{-1}Az = (I - \Omega)^{-1}z\).

Consider that \( \Omega = R_\sigma - L_\sigma + I \). Substituting \( R_\sigma = \Omega + L_\sigma - I \) into (42), we obtain

\[
\Lambda = \mathbb{E}_\sigma[L_\sigma^{-1}(\Omega + L_\sigma - I)] \\
= \mathbb{E}_\sigma[L_\sigma^{-1}(\Omega - I) + I] \tag{44}
\]

Recall that \( \rho(\Lambda) < 1 \), and thus \( I - \Lambda \) is nonsingular. Together with \( I - \Lambda = A(I - \Omega) \) from (44), we see that both \( A \) and \( I - \Omega \) are nonsingular. Hence,

\[
(I - \Lambda)^{-1}Az = (A(I - \Omega))^{-1}Az = (I - \Omega)^{-1}z, \tag{45}
\]

which concludes the proof.

From Theorem 1, both GMP and RGMP have guaranteed convergence for variances. However, they have different conditions to ensure the convergence of means: GMP requires \( \rho(\Omega) < 1 \) while RGMP requires \( \rho(\Lambda) < 1 \). Since both \( \Omega \) and \( \Lambda \) highly depend on the network geometry, it is difficult to theoretically compare these two convergence conditions. To shed light on the difference, we plot the cumulative distribution function (CDF) of the spectral radius of \( \Omega \) and
**C. Special Cases**

In this subsection, we present some special cases to show that the convergence condition in Theorem 3 is less stringent than that of the synchronous GMP.

1) **Ω is Hermitian matrix**: When Ω is a Hermitian matrix, the condition of expected convergence in Theorem 2 can be further simplified as given in the following corollary.

**Corollary 1.** The expected output \( \phi^{(t)} = E_{\xi_{t-1}}[\mathbf{m}^{(t)}] \) converges to the unique point \((I - \Omega)^{-1}z\).
if \( \Omega \) is a Hermitian matrix, and all the eigenvalues of \( \Omega \) is no greater than 1.

Proof: If \( \Omega \) is Hermitian and all the eigenvalues of \( \Omega \) is no greater than 1, the matrix \( I - \Omega \) is a positive-semidefinite matrix and can be rewritten as \( I - \Omega = BB^H \). As proven by Lemma 2 in [7], all eigenvalues of \( ABB^H \), or equivalently \( A(I - \Omega) \), lie in \((0, 4/3)\).

Substituting \( R_\sigma = \Omega + L_\sigma - I \) into (42), we obtain

\[
\Lambda = I - A(I - \Omega). \tag{46}
\]

As all the eigenvalues of \( A(I - \Omega) \) lie in \((0, 4/3)\), \( \rho(\Lambda) < 1 \) holds all the time.

Obviously, the condition that all the eigenvalues of \( \Omega \) is no greater than one is much weaker than \( \rho(\Omega) < 1 \). It implies that in special cases where \( \Omega \) is Hermitian, \( \rho(\Lambda) < 1 \) is a much less stringent condition than \( \rho(\Omega) < 1 \), and thus our RGMP algorithm converges with a higher probability than Gaussian message passing.

2) Two-user C-RAN: We now consider the C-RAN with two users. We show that if the GMP algorithm converges, the expected output \( \phi^{(t)} \) of RGMP always converges, as proven in the following corollary.

Corollary 2. When the number of users \( K = 2 \), the expected output \( \phi^{(t)} \) converges to the unique point if and only if the spectral radius \( \rho(\frac{1}{2}\Omega + \frac{1}{2}\Omega^2) < 1 \).

Proof: From Theorem 2, it suffices to show that \( \Lambda = \frac{1}{2}\Omega + \frac{1}{2}\Omega^2 \). Note that there are only two different permutations when \( K = 2 \). That is, \( \Sigma = \{(1, 2), (2, 1)\} \). For simplicity of notation, let \( \sigma = (1, 2) \) and \( \sigma' = (2, 1) \). Then, \( L_\sigma = I - R_\sigma \). Noting \( R_\sigma^k = 0 \) for all \( k \geq K \), we obtain

\[
L_\sigma^{-1} = \sum_{k=0}^{\infty} R_\sigma^k = I + R_\sigma \]

by using Taylor series expansion. Similarly, \( L_{\sigma'}^{-1} = I + R_{\sigma'} \). Then

\[
\Lambda = \frac{1}{2} ((I + R_{\sigma'})R_\sigma + (I + R_\sigma)R_{\sigma'})
\]

\[
= \frac{1}{2}\Omega + \frac{1}{2}\Omega^2. \tag{47}
\]

Denote by \( \lambda_1, \ldots, \lambda_{NK} \) the eigenvalues of \( \Omega \). Then, the corresponding eigenvalues of \( \frac{1}{2}\Omega + \frac{1}{2}\Omega^2 \) are \( \frac{1}{2}\lambda_1 + \frac{1}{2}\lambda_2, \ldots, \frac{1}{2}\lambda_{NK} + \frac{1}{2}\lambda_{NK} \). Then, the spectral radius of \( \Omega \) and \( \frac{1}{2}\Omega + \frac{1}{2}\Omega^2 \) are
\[ \max_i |\lambda_i| \text{ and } \max_i |\frac{1}{2}\lambda_i + \frac{1}{2}\lambda_i^2| \] respectively. Thus, \( \rho(\frac{1}{2}\Omega + \frac{1}{2}\Omega^2) < 1 \) always holds if \( \rho(\Omega) < 1 \). Therefore, the condition for expected convergence of the RGMP algorithm is less stringent than that of the synchronous message passing.

VI. NUMERICAL COMPARISONS

In this section, we compare the performance of RGMP with other existing algorithms. Unless specified otherwise, we assume that both users and RRHs are uniformly at random located in a circular network area with user density \( \beta_K = 8/\text{km}^2 \) and RRH density \( \beta_N = 10/\text{km}^2 \). The path loss exponent is 3.7, and the average transmit SNR at the user side equals to 95dB. That is \( \frac{P}{N_0} = 95\text{dB} \). Moreover, the stopping criteria is \( \delta(t) < \delta \), where \( \delta(t) \) is the relative error after the \( t \)-th iteration. In particular, \( \delta(t) = \frac{\|PH^Hx(t) - P\frac{1}{2}H^Hy\|}{\|P\frac{1}{2}H^Hy\|} \), with \( x(t) \) being the estimated transmitted signal after \( t \) iteration.

A. Comparisons of Convergence Speed

In this subsection, we compare the converge speed of RGMP with other algorithms including ADMM [7], GAMP [8], and PCG [13]. For fairness, the channel sparsification approach with distance threshold \( d_0 = 1000\text{m} \) is adopted in all algorithms. In this way, all the algorithms have a linear per-iteration computational complexity with the network size. Thus, we only focus on the convergence speed of these algorithms.

In Fig. 6, the relative error \( \delta(t) \) is plotted against the number of iterations for \( N = 40 \) and \( K = 32 \). We see that both RGMP and PCG converge very fast. For example, the relative error of RGMP reduces to 0.001 within 10 iterations. However, the performance of the ADMM algorithm is unsatisfactory. Around 500 iterations are needed for the ADMM algorithm to reduce the relative error to 0.02. In fact, from simulation results not presented here, ADMM requires over 5000 iterations on average to reduce the relative error to 0.001 for the network configuration in Fig. 6. Therefore, even though the computational complexity per iteration of ADMM is linear in the number of RRHs and the expected convergence is guaranteed [7], it is still impractical to adopt the ADMM algorithm in C-RAN due to the extremely slow convergence.
Fig. 6. Relative error vs number of iterations when the number of RRHs $N = 40$.

In Fig. 7 we plot the convergence speed of the RGMP algorithm against the network size, where the convergence speed is measured by the critical number of iterations to achieve $\delta^{(t)} <$
10^{-5}. Due to the extremely slow convergence speed of ADMM as shown in Fig. 6, we ignore ADMM and only plot the convergence speed of PCG and GAMP for comparison. We observe that the number of iterations needed by both PCG and GAMP grow roughly linearly with the network size. In contrast, the convergence speed of RGMP is constant with the network size. Note that the computational complexity per iteration of PCG/GAMP/RGMP is linear in the network size. Thus, the total computational complexity of the proposed RGMP algorithm is linear in the network size, while that of PCG and GAMP grows quadratically with the network size.

**B. Comparison of Performance**

![Figure 8](image-url)  
Fig. 8. SINR ratio vs the distance threshold $d_0$ when the network area is 200km$^2$.

In this subsection, we compare the performance of the RGMP algorithm with a disjoint clustering algorithm. The disjoint clustering algorithm divides the whole network into disjoint square clusters with area $A_c$, and MMSE detection is done independently in each disjoint cluster. Channel sparsification is also applied in the disjoint clustering algorithm. In Fig. 8 we plot the mean squared error (MSE) against the distance threshold, where MSE refers to $E[|x_k - \hat{x}_k|^2]$. The
network area is 200km². Thus, the numbers of RRHs and users are 2000 and 1600, respectively. We see that the gap between the RGMP algorithm and the disjoint clustering algorithm is very large. For example, when the distance threshold $d_0$ is 4000m, the MSE of the RGMP algorithm is less than 0.13, which is only half of the MSE of the disjoint clustering algorithm with cluster area 49km².

VII. CONCLUSIONS

In this paper, we proposed the RGMP algorithm for scalable uplink signal detection in C-RANs. With channel sparsification, signal detection in a C-RAN was converted to an inference problem over a bipartite random geometric graph. A random message-update schedule was employed to address the convergence issue of GMP over a bipartite random geometric graph. We analysed the convergence condition of the proposed RGMP algorithm and showed that the convergence condition of RGMP is much less stringent than that of GMP. Moreover, numerical results demonstrated that RGMP exhibits much faster convergence than existing algorithms, such as PCG, GAMP, and ADMM. The work in this paper sheds light on the design of message-passing algorithms on general loopy graphs, which has been a challenging topic in the field for years. Future work can be done in a number of interesting directions, including the extension to the downlink processing of C-RANs and the design of signal detectors with limited fronthaul capacity.

REFERENCES

[1] “C-RAN: The road towards green RAN,” China Mobile Res. Inst., Beijing, China, Oct. 2011, White Paper, ver. 2.5.
[2] M. Tuchler, A. Singer, and K. Ralf, “Minimum mean squared error equalization using a prior information,” IEEE Trans. Signal Process., vol. 50, no. 3, pp. 673-683, 2002.
[3] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, “Factor graphs and the sum-product algorithm,” IEEE Trans. Inf. Theory, vol. 47, no. 2, pp. 498519, Feb. 2001.
[4] C. Fan, Y. J. Zhang, and X. Yuan, “Dynamic nested clustering for parallel PHY-layer processing in cloud-RANs,” IEEE Trans. Wireless Commun., vol. 15, no. 3, pp. 1881-1894, Mar. 2016.
[5] A. T. Ihler, J. W. Fisher, and A. S. Willsky, “Loopy belief propagation: Convergence and effects of message errors,” J. Machine Learning Research, vol. 6, pp. 905-936, 2005.
[6] Y. Shi, J. Zhang, B. O’Donoghue, and K. B. Letaief, “Large-scale convex optimization for dense wireless cooperative networks,” IEEE Trans. Signal Process., vol. 63, no. 18, pp. 4729-4743, Sept. 2015.

[7] R. Sun, Z. Q. Luo, and Y. Ye, “On the expected convergence of randomly permuted ADMM,” 2015 arXiv preprint: http://arxiv.org/abs/1503.06387

[8] S. Rangan, “Generalized approximate message passing for estimation with random linear mixing,” in Proc. IEEE Int. Symp. Inf. Theory, St. Petersburg, Russia, Aug. 2011, pp. 2168-2172.

[9] Y. Weiss and W. T. Freeman, “Correctness of belief propagation in Gaussian graphical models of arbitrary topology,” Neural Computation, vol. 13, pp. 2173-2200, 2001.

[10] B. L. Ng, J. S. Evans, and S. V. Hanly, “Distributed downlink beamforming in cellular networks,” in Proc. IEEE Int. Symp. Inf. Theory, Jun. 2007, pp. 610.

[11] D. P. Bertsekas and J. N. Tsitsiklis, Parallel and Distributed Computation. Englewood Cliffs, NJ: Prentice Hall, 1989.

[12] O. Axelssson, Iterative Solution Methods. Cambridge University Press, 1994.

[13] R. Barrett, M. Berry, T. Chan, J. Demmel, J. Donato, J. Dongarra, V. Eijkhout, R. Pozo, C. Romine, and H. van der Vorst. Templates for the solution of linear systems: Building blocks for iterative methods. Philadelphia, PA: SIAM, 1994

[14] H. -A. Loeliger, J. Dauwels, J. Hu, S. Korl, L. Ping, and F. R. Kschischang, “The factor graph approach to model-based signal processing,” Proc. IEEE, vol. 95, no. 6, pp. 1295-1322, June 2007.

[15] R. D. Yates, “A framework for uplink power control in cellular radio systems,” IEEE Journal on Selected Areas in Communications, Vol. 13, no. 7, pp. 1341-1347, Sept. 1995.

[16] T. J. Richardson, M. A. Shokrollahi, and R. L. Urbanke, “The capacity of low-density parity-check codes under message-passing decoding,” IEEE Trans. Inf. Theory, vol. 47, no. 2, pp. 599-618, February 2001.