Quark–hadron phase transition in Brans–Dicke brane gravity

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Abstract

A standard picture in cosmology has been emerging over the past decade in which a phase transition, associated with chiral symmetry breaking after the electroweak transition, has occurred at approximately $10^{-6}$ s after the Big Bang to convert a plasma of free quarks and gluons into hadrons. In this paper, we consider the quark–hadron phase transition in a Brans–Dicke brane world scenario within an effective model of QCD. We study the evolution of the physical quantities relevant to a quantitative description of the early universe, namely, the energy density, temperature and the scale factor before, during, and after the phase transition. We show that for different values of the Brans–Dicke coupling, $\omega$, phase transition occurs and results in decreasing the effective temperature of the quark–gluon plasma and of the hadronic fluid. We then move on to consider the quark–hadron transition in the smooth crossover regime at high and low temperatures and show that such a transition occurs and results in decreasing the effective temperature of the quark–gluon plasma during the process of quark–hadron phase transition.

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(Some figures in this article are in color only in the electronic version)

1. Introduction

Over the past decade the possibility that the observable universe is a brane [1] embedded in a higher dimensional spacetime has been the hallmark of many great research works. This scenario has motivated intense efforts to understand the case where the bulk is a five-dimensional anti de-Sitter space. In this setup, gravitons are allowed to penetrate into the bulk but are localized on and around the brane [2]. It was then shown that in a background of a
non-factorizable geometry an exponential warp factor emerges which multiplies the Poincaré invariant 3+1 dimensions in the metric. The existence of branes and the requirement that matter fields should be localized on the brane lead to a non-conventional cosmology which has seeded a large number of studies. Of interest in the present study are the brane-world models in the context of Brans–Dicke (BD) gravity. Interestingly, it has been shown that in such a scenario and in the presence of a BD field in the bulk [3] the conservation equation for the matter field can be satisfied. It would therefore be of interest to study the quark–hadron phase transition in the context of BD brane world theory. The question of quark–hadron phase transition in the context of conventional brane-world models has been addressed previously [4, 5].

Standard cosmology suggests that as the early universe expanded and cooled, it underwent a series of symmetry-breaking phase transitions, causing topological defects to form. It is the study of such phase transitions that would pave the way for a better understanding of the evolution of the early universe, characterized by the existence of a quark–gluon plasma undergoing a phase transition. In what follows we focus attention on possible scenarios which might have occurred to allow for such a phase transition come to the fore. We generally follow the discussion presented in [4] which puts the quark–gluon phase transition in a cosmologically transparent perspective.

The existence of phase transition from the quark–gluon plasma phase to hadron gas phase is a definite prediction of QCD. However, the phase transition in QCD can be characterized by a truly singular behavior of the partition function leading to a first- or second-order phase transition, and also it can be only a crossover with rapid changes in some observables, strongly depending on the values of the quark masses. The possibility of a phase transition in the gas of quark–gluon bags was demonstrated for the first time in [6]. Most studies have shown that one may obtain first, second- and higher-order transitions. In addition, the possibility of no phase transitions was pointed out in [7]. Recently, lattice QCD calculations performed for two quark flavors suggest that QCD makes a smooth crossover transition at a temperature of $T_c \sim 150$ MeV [8]. Such a phase transition could be responsible for the formation of relic quark–gluon objects in the early universe which may have survived. In this paper, our study of phase transition is based on the ideas proposed in the first reference in [6], where it was shown that under certain conditions a gas of extended hadrons could produce phase transitions of the first or second order, and also a smooth crossover transition that might be qualitatively similar to that of lattice QCD.

The cooling down of the color deconfined quark–gluon plasma below the critical temperature believed to be around $T_c \approx 150$ MeV makes it energetically favorable to form color confined hadrons (mainly pions and a tiny number of neutrons and protons, since the net baryon number should be conserved). However, such a new phase does not form promptly. Generally speaking, a first-order phase transition needs some supercooling to achieve the energy used in forming the surface of the bubble and the new hadron phase. A brief account of a first-order quark–hadron phase transition in the expanding universe may be envisioned as follows [9]. When a hadron bubble is nucleated, latent heat is released and a spherical shock wave expands into the surrounding supercooled quark–gluon plasma. The plasma thus formed is reheated and approaches the critical temperature, preventing further nucleation in a region passed by one or more shock fronts. Bubble growth is generally described by deflagrations where a shock front precedes the actual transition front. The stopping of the nucleation occurs when the whole universe has reheated $T_c$. The prompt ending of this phase transition, in about 0.05 μs, renders the cosmic expansion completely negligible over this period. Afterwards, the hadron bubbles grow at the expense of the quark phase and eventually percolate or coalesce. Eventually, when all quark–gluon plasma has been converted into hadrons, neglecting possible
quark nugget production, the transition ends. The physics of the quark–hadron phase transition and its cosmological implications have been extensively discussed in the framework of general relativistic cosmology in [10–23].

As is well known, the Friedmann equations in brane-world scenarios differ from that of the standard 4D cosmology in that they result in an increased expansion rate at early times. We expect this deviation from the standard 4D cosmology to have noticeable effects on the cosmological evolution, especially on cosmological phase transitions. In the context of brane-world models, the first-order phase transitions have been studied in [24] where it has been shown that due to the effects coming from higher dimensions, a phase transition requires a higher nucleation rate to complete and baryogenesis and particle abundances could be suppressed. Recently, the quark–hadron phase transition was studied in a Randall–Sundrum (RS) brane-world scenario [4]. Within the framework of first-order phase transitions, the authors studied the evolution of the relevant cosmological parameters (energy density, temperature, scale factor, etc) of the quark–gluon and hadron phases and the phase transition itself. In another attempt, the phase transition of quarks and gluons was studied in a brane-world model in which the confinement of matter fields on the brane is achieved through a confining potential [5]. As was mentioned above, it would therefore be of interest to study phase transitions of this nature in the context of a BD brane-world scenario and this is what we intend to do in what follows. In addition, as recent calculations in lattice QCD strongly favor a smooth crossover transition [8], the study of the formation of hadrons in such a scenario seems to be of particular importance for a better understanding of the subtleties of the early universe and this is what we shall present in the last section.

2. Field equations in the BD brane scenario

We start by writing the action for the BD brane-world [25]

\[ S = -\frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( \phi R - \frac{\omega}{\phi} \partial_A \phi \partial^A \phi \right) + \int d^5x \sqrt{-g} L_m, \]  

where \( R \) is the Ricci scalar associated with the five-dimensional spacetime metric \( g_{AB} \), \( \phi \) is a scalar field which we shall call the BD field, \( \omega \) is a dimensionless coupling constant which determines the coupling between gravity and the BD scalar field and \( L_m \) represents the Lagrangian for the matter fields. Latin indices denote five-dimensional components \( (A, B = 0, \ldots, 5) \) and for convenience we choose \( \kappa^2 = 8\pi G_n = 1 \). The variation of the action with respect to \( g_{AB} \) and \( \phi \) yields the field equations

\[ G_{AB} \equiv R_{AB} - \frac{1}{2} g_{AB} R = \frac{1}{\phi^2} \left[ T^\phi_{AB} + T_{AB} \right], \]  

where

\[ T^\phi_{AB} = \frac{\omega}{\phi} \left[ \phi_A \phi_B - \frac{1}{2} g_{AB} \phi^C \phi^C \right] + \left[ \phi_{AB} - g_{AB} \phi^C \phi_C \right], \]

\[ \Box \phi = \frac{T}{3\omega + 4}, \]

and \( T = T^C_C \) is the trace of the energy–momentum tensor of the matter content of the five-dimensional spacetime. Note the factor \( 3\omega + 4 \) in the denominator on the right-hand side of the BD field equation instead of the familiar \( 2\omega + 3 \) in the four-dimensional case [26]. This is determined by requiring the validity of the equivalence principle in our setup, see [27] for a discussion of this topic in the context of four-dimensional BD theory.
Being interested in cosmological solutions, we take the spatially flat cosmology \((k = 0)\) and consider a five-dimensional flat metric of the following form:

\[
ds^2 = -n^2(t, y) \, dt^2 + a^2(t, y) \delta_{ij} \, dx^i \, dx^j + b^2(t, y) \, dy^2,
\]

where \(i, j = 1, 2, 3\). We also assume an orbifold symmetry along the fifth direction \(y \to -y\).

Next we define the energy–momentum tensor

\[
T^A_B = T^A_{\text{brane}} + T^A_{\text{bulk}},
\]

where the subscripts ‘brane’ and ‘bulk’ refer to the corresponding energy–momentum tensors. For simplicity we assume that the bulk is devoid of matter other than the BD scalar field. The brane matter field is held at \(y = 0\) with the following energy–momentum tensors:

\[
T^A_{\text{brane}} = \frac{\delta(y)}{b} \text{diag}(-\rho, p, p, p, 0) \quad \text{and} \quad T^A_{\text{bulk}} = \text{diag}(0, 0, 0, 0, 0),
\]

where \(\rho = \rho^b + \lambda\) and \(p = \rho^p - \lambda\). The above expressions are written assuming that the brane has ordinary matter with tension \(\lambda\) and that the bulk is empty. There are several constraints suggested for the brane tension \(\lambda\). One is that suggested by the Big Bang nucleosynthesis, \(\lambda \geq \lambda_0\) MeV\(^4\) [28]. A much stronger bound for \(\lambda\) is due to the null results of submillimeter tests of Newton’s law, giving \(\lambda \geq 10^8\) GeV\(^4\) [29]. An astrophysical lower limit on \(\lambda\) which is independent of Newton’s law and cosmological limits has been studied in [28], leading to the value \(\lambda > 5 \times 10^6\) MeV\(^4\) which is the constraint we will be using in our model.

Using metric (5) we are now able to write the equations of motion. The \((0, 0)\) component reads

\[
3 \left[ \frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{n^2}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right) \right] = \phi \left( T_{00}^\phi + T_{00} \right),
\]

where

\[
T_{00}^\phi = -\frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{n^2}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right) \right)\frac{n}{b} \left( \frac{\phi'' + \phi' \left( 3 \frac{a'}{a} - \frac{b'}{b} + \frac{\omega \phi'}{2} \right) }{\phi} \right).
\]

The \((i, j)\) components are given by

\[
\{-2 \frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \left[ \frac{\dot{a}}{a} \left( -\frac{2}{a} + \frac{n'}{n} \right) + \frac{\dot{b}}{b} \left( -2 \frac{\ddot{a}}{a} + \frac{n'}{n} \right) \right] \delta_{ij}
+ \left\{ \left( \frac{n}{b} \right)^2 \left[ \frac{2 a''}{a} + \frac{n''}{n} + \frac{a'}{a} \left( -\frac{2 a'}{a} + \frac{n'}{n} \right) \right] - \frac{b'}{b} \left( \frac{n''}{n} + 2 a'' \right) \right\} \delta_{ij} \}
= \frac{1}{\phi} \left( \frac{n}{a} \right)^2 \left( T_{ij}^\phi + T_{ij} \right),
\]

where

\[
T_{ij}^\phi = \left\{ \frac{\ddot{a}}{a} + \ddot{b} + \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{n}{n} \right) \frac{\omega \phi}{2} \right\} \left( \frac{n}{b} \right)^2 \left[ \frac{\phi'' + \phi' \left( 2 \frac{a'}{a} + \frac{n'}{n} + \omega \phi' \right) }{\phi} \right] \delta_{ij}.
\]

The \((0, 5)\) component takes the form

\[
3 \left( \frac{\dot{a} n' + \dot{b} a' - \dddot{a}}{a} \right) = \frac{1}{\phi} T_{05}^\phi,
\]

where

\[
T_{05}^\phi = \phi' - \phi \left( \frac{n'}{n} - \frac{\omega \phi'}{\phi} \right) - \frac{\dot{b}}{b} \phi'.
\]
Finally, for the \((5,5)\) component one has

\[
3 \left[ -\left( \frac{\dot{a}}{a} + \frac{\dot{a}}{a} \left( \frac{\dot{n}}{n} \right) \right) + \left( \frac{n}{b} \right)^2 \left( \frac{a'}{a} \frac{n'}{n} \right) \right] = \frac{1}{\phi} \left( \frac{n}{b} \right)^2 \left[ T_{55}^\phi + T_{55} \right], \tag{14}
\]

where

\[
T_{55}^\phi = \dot{\phi} + \frac{\phi}{2} \left( 3 \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) - \left( \frac{n}{b} \right)^2 \phi' \left( \frac{3 a'}{a} + \frac{n'}{n} - \frac{\omega \phi}{2} \right). \tag{15}
\]

The equation of motion for the BD field reads

\[
\ddot{\phi} + \frac{\phi}{2} \left( 3 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{\dot{n}}{n} \right) - \left( \frac{n}{b} \right)^2 \left[ \phi'' + \phi' \left( 3 \frac{a'}{a} - \frac{b'}{b} + \frac{n'}{n} \right) \right] = -n^2 \frac{T}{3\omega + 4}, \tag{16}
\]

where a dot represents the time derivative with respect to \(\tau\) and the prime corresponds to derivatives with respect to \(y\). Note that in the above derivation we have assumed \(\phi = \phi(\tau, y)\).

We make the assumption that the metric and the BD field are continuous across the brane localized at \(y = 0\). However, their derivatives can be discontinuous at the brane position in the \(y\) direction. This suggests the second derivatives of the scale factor and the BD field will have a Dirac delta function associated with the positions of the brane. Since the matter is localized on the brane it will introduce a delta function in the Einstein equations which will be matched by the distributional part of the second derivatives of the scale factor and the BD field. For instance at \(y = 0\), we have \[25\]

\[
f'' = \hat{f}'' + [f']\delta(y), \tag{17}
\]

where the hat marks the non-distributional part of the second derivative of the quantity. The part associated with a delta function, \([f']\), is a jump in the derivative of \(f\). Here \(f\) could be any of the three quantities \(a\), \(n\) or \(\phi\). The jump in \(f\) at \(y = 0\) can be written as

\[
[f'] = f'(0^+) - f'(0^-), \tag{18}
\]

and the mean value of the function \(f\) at \(y = 0\) is defined by

\[
\mu f = \frac{f'(0^+) - f'(0^-)}{2}. \tag{19}
\]

After substituting equation (17) in the Einstein field equations it is possible to find the jump conditions for \(a\) and \(n\) by matching the Dirac delta functions appearing on the left-hand side of the Einstein equations to the ones coming from the energy–momentum tensor, equation (6). For the BD field one has to use equation (16) to evaluate the jump conditions. We therefore find

\[
[a']_b = -\frac{1}{(3\omega + 4)\phi} \left[ p + (\omega + 1) \rho \right], \tag{20}
\]

\[
[n']_b = \frac{1}{(3\omega + 4)\phi} \left[ 3(\omega + 1) p + (2\omega + 3) \rho \right], \tag{21}
\]

\[
[\phi']_b = \frac{2}{(3\omega + 4)\phi} \gamma \rho, \tag{22}
\]

where

\[
\gamma = \frac{1}{2} (3 w_m - 1), \tag{23}
\]

with \(w_m = \frac{\xi}{\rho}\) and the subscript 0 stands for the brane at \(y = 0\). The first two conditions, equations (20) and (21), are equivalent toIsrael’s junction conditions in general relativity (see...
[30] for a discussion of its application in the context of brane-worlds). It is important to note
that the above jump conditions at $y = 0$ depend on the energy density and pressure component
of the brane and the induced curvature on the brane. Interestingly, for the radiation dominated
phase on the brane, $\rho = 3p$, the jump condition for $\phi$ does not vanish and is proportional to
the energy density and pressure component of the induced curvature on the brane.

Using the $(0, 0)$ component of the Einstein field equations for the brane located at $y = 0$
and the equations representing the jump conditions (20), (21) and (22) one gets the Friedmann
equation as follows:

$$H^2 + \Upsilon \left( H - \frac{\omega}{6} \Upsilon \right) = \frac{1}{4(3\omega + 4)^2 \phi_0^2} \left[ \frac{\omega}{6} (3p - \rho)^2 + (2 + 3\omega + \omega^2) \rho^2 - \omega p \rho - 2p^2 \right].$$

(24)

where $H = \frac{\dot{a}_0}{a_0}$ and $\Upsilon = \frac{\dot{\phi}_0}{\phi_0}$. Note that $\rho$ and $p$ consist of two parts, that is $\rho = \rho^b + \lambda$ and
$p = p^b - \lambda$, where $\lambda$ is tension on the brane. Using the $(0, 5)$ component of the Einstein
equation and substituting equations (20) and (21) we get the continuity equation for the matter
on the brane

$$\dot{\rho} + 3(\rho + p)H = 0.$$

(25)

While deriving the above equations we have assumed that, from the point of view of the
brane observer, the extra dimension is static, that is $b = b_0$. We have also fixed the time in
such a way that $n_0 = 1$, corresponding to the usual choice of time in conventional cosmology.
Taking the mean value of the BD field equation we obtain an equation of motion for $\phi$ on the
brane

$$\frac{\ddot{\phi}_0}{\phi_0} + 3H \Upsilon = \frac{\omega(\rho - 3p)^2}{(3\omega + 4)^2 \phi_0^2}.$$

(26)

Note that in order to obtain equation (26) we also have to assume that the non-distributional
part of $\phi''$ vanishes, otherwise, a term involving $\dot{\phi}''$ will appear in the BD field equation. As
we shall see in the next section it is possible to obtain cosmologically interesting solutions
which verify this condition. Equations (24) and (25) are dynamical equations in our BD brane
scenario describing the evolution of the universe. In the next section we shall examine these
equations for the quark–hadron phase transition in the early universe.

3. Quark–hadron phase transition

The quark–hadron phase transition is a notion fundamental to the study of particle physics,
particularly in the context of lattice gauge theories. However, it is an integral part of any
study dealing with the underlying mechanisms responsible for the evolving universe at its
early stages of formation in which a soup of quarks and gluons interact and undergo a phase
transition to form hadrons. It is therefore essential to have an overview of the basic ideas
before attempting to use the results obtained from such a phase transition and apply them to the
study of the evolution of the early universe within the context of the BD brane-world scenario.
In this regard, a well written and concise review can be found in [4] and the interested reader
should consult it. Here, it would suffice to mention the results relevant to our study and leave
the details of the discussion to the said reference.

We start from the equation of state of matter in the quark phase which can generally be
given in the form

$$\rho^b_v = 3a_v T^4 + V(T), \quad \rho^b_p = a_p T^4 - V(T).$$

(27)
where \( a_q = (\pi^2 / 90) g_q \), with \( g_q = 16 + (21/2) N_F + 14.25 = 51.25 \) and \( N_F = 2 \) with \( V(T) \) being the self-interaction potential. For \( V(T) \) we adopt the expression \[ V(T) = B + \gamma_T T^2 - \alpha_T T^4, \] (28)

where \( B \) is the bag pressure constant, \( \alpha_T = 7\pi^2 / 20 \) and \( \gamma_T = m_s^2 / 4 \). The critical temperature \( T_c \) is defined by the condition \( \rho_q(T_c) = p_h(T_c) \) [9], and is given by

\[
T_c = \left[ \frac{\gamma_T + \sqrt{\gamma_T^2 + 4B(a_q + \alpha_T - a_T)}}{2(a_q + \alpha_T - a_T)} \right]^{1/2}.
\] (30)

If we take \( m_s = 200 \) MeV and \( B^{1/4} = 200 \) MeV, the transition temperature is of the order \( T_c \approx 125 \) MeV. It is worth mentioning that since the phase transition is assumed to be of first order, all the physical quantities exhibit discontinuities across the critical curve.

4. Behavior of BD brane universe during quark–hadron phase transition

We are now in a position to study the phase transition described above. The framework we are working in is defined by the BD brane-world scenario for which the basic equations were derived in section 2. The physical quantities of interest through the quark–hadron phase transition are the energy density \( \rho \), temperature \( T \) and scale factor \( a_0 \). These parameters are determined by the Friedmann equation (24), conservation equation (25) and the equations of state, namely (27), (28) and (29). To start, we consider the evolution of the BD brane-world before, during and after the phase transition era.

4.1. Behavior of temperature

Let us consider the era preceding the phase transition for which \( T > T_c \) and the universe is in the quark phase. Use of equations of state of the quark matter and the conservation of matter on the brane, equation (25), leads to

\[
H = \frac{\dot{a}_0}{a_0} = -\frac{3a_q - \alpha_T T}{3a_q} - \frac{1}{6} \frac{\gamma_T T}{a_q T^3}.
\] (31)

Integrating the above equation immediately gives

\[
a_q(T) = c T^{a_T - \frac{3a_q}{a_T}} \exp \left( \frac{1}{12} \frac{\gamma_T}{a_q} \frac{1}{T^2} \right),
\] (32)

where \( c \) is a constant of integration.
To consider the phase transition in the BD brane model, for simplicity, we take an ansatz in the form of a relationship between scale factor on the brane \(a_0\) and the BD scalar field \(\phi_0\) as follows:

\[
\phi_0(\tau) = \mu a_0^n(\tau),
\]

(33)

where \(\mu\) and \(n\) are constant. Thus, using equation (33), the Friedmann equation (24) can be written as

\[
H^2 = \frac{1}{2\left[(3\omega + 4)^2(2 + 2n - \frac{\omega}{3}n^2)\right]}\phi_0^2\left[\frac{2\omega}{3}(3\rho - \rho^\ast) + (2 + 3\omega + \omega^2)\rho^2 - pp\omega - 2p^2\right].
\]

(34)

We may now proceed to obtain an expression describing the evolution of temperature of the BD brane universe in the quark phase by combining equations (27), (28), (31), (32), (33) and (34), leading to

\[
d\frac{T}{dT} = \frac{-T^{(nA_0+3)}}{\mu(3\omega+4)(A_0T^2 + A_1)}\sqrt{2(2 + 2n - \frac{\omega}{3}n^2)}\exp\left(-\frac{nA_1}{2T^2}\right)
\]

\[
\times \left[\frac{2\omega}{3}(V(T) - \lambda)^2 + (2 + 3\omega + \omega^2)(\rho^b(T) + \lambda)^2 - \omega(\rho^b(T) - \lambda)(\rho^b(T) + \lambda) - 2(\rho^b(T) - \lambda)^2\right]^{1/2},
\]

(35)

where we have denoted

\[
A_0 = 1 - \frac{\alpha\tau}{3a_y},
\]

\[
A_1 = \frac{\gamma\tau}{6a_y}.
\]

(36)

Equation (35) may be solved numerically and the result is presented in figure 1 which shows the behavior of temperature as a function of the cosmic time \(\tau\) in a BD brane-world filled with quark matter for different values of \(\omega\) with \(n = 0.05, \mu = 2 \times 10^5\) and \(\lambda = 10 \times 10^8\) MeV\(^4\).

### 4.2. Temperature behavior with \(V(T) = B\)

One may gain considerable insight in the evolution of cosmological quark matter in our BD brane-world by taking the simple case in which temperature corrections can be neglected in the self interacting potential \(V\). Then \(V = B = \text{const.}\) and equation of state of the quark matter is given by that of the bag model, namely \(p^b = (\rho^b - 4B)/3\). Equation (25) may then be integrated to give the scale factor on the brane as a function of temperature

\[
a_0(T) = \frac{c}{T}, \quad \phi_0 = \mu e^\omega T^\omega.
\]

(37)

where \(c\) is a constant of integration.

Using equations (34) and (37), the time dependence of temperature can be obtained from equation...
The behavior of $T(\tau)$ as a function of time ($\tau$) for $\mu = 2 \times 10^5$, $\lambda = 10 \times 10^8$ MeV$^4$, $n = 0.05$ and different values of $\omega$: $\omega = 1 \times 10^3$ (solid curve), $\omega = 2.3 \times 10^3$ (dashed curve), $\omega = 2.4 \times 10^3$ (dotted curve) and $\omega = 2.5 \times 10^3$ (dotted-dashed curve). We have taken $B^{1/4} = 200$ MeV.

\[
\frac{dT}{d\tau} = \frac{-T^{\ast\ast}}{\mu(3\omega + 4)\sqrt{2(2 + 2n - \frac{\omega}{2}n^2)}} \times \left[ \frac{2\omega}{3}(B - \lambda)^2 + (2 + 3\omega + \omega^2)(\rho^b(T) + \lambda)^2 - \omega \left( \frac{\rho^b - 4B}{3} - \lambda \right) \left( \rho^b(T) + \lambda \right) - 2 \left( \frac{\rho^b - 4B}{3} - \lambda \right)^2 \right]^{1/2},
\]

where we have taken $c = 1$ and $\rho^b(T)$ is given by

\[
\rho^b(T) = 3a_s T^4 + B.
\]

The above equation can be solved numerically and the result is given in figure 2 which shows the behavior of temperature as a function of cosmic time $\tau$ in a BD brane-world filled with quark matter for different values of $\omega$ with $\mu = 2 \times 10^5$, $n = 0.05$ and $\lambda = 10 \times 10^8$ MeV$^4$ as the self-interacting potential, $V(T)$, is considered to be a constant.

4.3. Formation of hadrons

During the phase transition, the temperature and pressure are constant and quantities like the entropy $S = sa^3$ and enthalpy $W = (\rho + p)a^3$ are conserved. Also, during the phase transition $\rho^b(\tau)$ decreases from $\rho^b(T_c) \equiv \rho_Q$ to $\rho^b(\tau_b) \equiv \rho_w$. For phase transition temperature of $T_c = 125$ MeV we have $\rho_Q \approx 5 \times 10^8$ MeV$^4$ and $\rho_H \approx 1.38 \times 10^9$ MeV$^4$, respectively. For the same value of the temperature the value of the pressure of the cosmological fluid during the phase transition is $p_b^{\ast} \approx 4.6 \times 10^8$ MeV$^4$. Following [4, 9], we replace $\rho^b(\tau)$ by $h(\tau)$, the volume fraction of matter in the hadron phase, by defining

\[
\rho^b(\tau) = \rho_w h(\tau) + \rho_Q [1 - h(\tau)] = \rho_Q [1 + mh(\tau)],
\]

where $m = (\rho_w - \rho_Q)/\rho_Q$. The beginning of the phase transition is characterized by $h(\tau_c) = 0$ where $\tau_c$ is the time representing it and $\rho^b(\tau_c) \equiv \rho_Q$, while the end of the transition is
characterized by $h(t_h) = 1$ with $t_h$ being the time signaling the end and corresponding to $ρ_b^h(t_h) ≡ ρ_μ$. For $τ > t_h$ the universe enters into the hadronic phase.

Equation (25) now gives

$$\frac{d_h}{a_0} = -\frac{1}{3} \frac{(ρ_μ - ρ_Q)h}{ρ_μ + p_e + (ρ_μ - ρ_Q)h} = -\frac{1}{3} \frac{r h}{1 + r h},$$

(41)

where we have denoted $r = (ρ_μ - ρ_Q)/(ρ_μ + p_e)$. The relationship between the scale factor on the brane and the hadronic fraction $h(τ)$ may now be obtained from the above equation

$$a_0(τ) = a_0(t_c)(1 + r h(τ))^{-1/3}, \quad φ_0 = μ a_0^n t_c(1 + r h(τ))^{-n/3},$$

(42)

where use has been made of the initial condition $h(t_c) = 0$. Now, using equations (34) and (42) we obtain the time evolution of the matter fraction in the hadronic phase

$$\frac{dh}{dτ} = -\frac{3(1 + r h(t))^{1/3}1}{μr(3ω + 4)a_0^n(t_c)} \sqrt{2(2 + 3ω + ω^2)(χ(t)ρ_μ + λ)^2 - (p_μ - λ)(χ(t)ρ_μ + λ) - 2(p_μ - λ)^2}.$$

(43)

Here we take $a_0^n(t_c) = 1$ and $χ(t) = 1 + m h(t)$. Figure 3 shows variation of the hadron fraction $h(τ)$ as a function of $τ$ for different values of $ω$ with $μ = 20000$, $n = 0.05$ and $λ = 8 \times 10^8$ MeV$^4$.

4.4. Pure hadronic era

After the phase transition, the energy density of the pure hadronic matter is given by $ρ_b^h = 3 p_b^h = 3 a_0 T^4$. The conservation equation on the brane (25) leads to

$$a_0(T) = a_0(t_c)T_c/T, \quad φ_0 = μ a_0^n(t_c)(T_c/T)^n.$$

(44)
The behavior of \( h(\tau) \) as a function of time (\( \tau \)) for \( \mu = 20000, \lambda = 8 \times 10^8 \text{MeV}^4 \), \( n = 0.05 \), \( V(T) = B \) and different values of \( \omega \): \( \omega = 1 \times 10^3 \) (solid curve), \( \omega = 2.1 \times 10^3 \) (dashed curve), \( \omega = 2.3 \times 10^3 \) (dotted curve) and \( \omega = 2.47 \times 10^3 \) (dotted–dashed curve). We have taken \( B^{1/4} = 200 \text{MeV} \).

The temperature dependence of the BD brane universe in the hadronic phase is governed by the equation

\[
\frac{dT}{d\tau} = -\frac{T^{n+1} \left[ \frac{5}{2} \omega \lambda^2 + \left( \omega^2 + \frac{5}{2} \omega + \frac{16}{7} \right) (a_\sigma T^4 + \lambda)^2 \right]^{\frac{1}{2}}}{\mu [T_c a_\sigma (T_h)]^n \left( 3 \omega + 4 \right) \sqrt{2 \left( 2 + 2n - \frac{\omega}{3} n^2 \right)}}. \tag{45}
\]

Variation of temperature of the hadronic fluid filled BD brane universe as a function of \( \tau \) for different values of \( \omega \) with \( \mu = 20000, n = 0.02 \) and \( \lambda = 5 \times 10^8 \text{MeV}^4 \) is represented in figure 4.

4.5. Effects of the BD coupling on phase transition

It is important to emphasize that the BD coupling, \( \omega \), plays an essential role in the model at hand and it is thus appropriate at this point to consider the effects of \( \omega \) during the phase transition.

We know that in the early universe the energy density is extremely high and thus in the high-density regime the Hubble function in the DB brane world is proportional to both the energy density of the cosmological matter and \( \omega \), as can be seen from terms in the square bracket in equation (24). Moreover, in the early universe the BD coupling, which is an indication of the strength of the DB scalar field, is expected to play an important role and therefore its numerical value would characterize its influence. In general, in the limit \( \omega \rightarrow \infty \), we recover the FRW equation in the Randall–Sundrum (RS) model [30] from equation (24).

In the context of the BD brane world model, we have found that the temperature evolution of the universe is different from that of the RS brane world model. The temperature of the early universe in the quark phase is higher in the BD brane world scenario, as can be seen from the left graph in figure 5, where the dotted–dashed curve, to a high degree, corresponds to the RS brane world limit of large \( \omega \). Hence, a large value of the BD coupling, \( \omega \), would significantly reduce the temperature of the quark–gluon plasma, and accelerates the phase transition to the hadronic era. Once the quark–hadron phase transition starts, the hadron fraction \( h \) is again
Figure 4. The behavior of $T(\tau)$ as a function of time ($\tau$) for $\mu = 20000$, $\lambda = 5 \times 10^8$ MeV$^4$, $n = 0.02$ and different values of $\omega$: $\omega = 15 \times 10^2$ (solid curve), $\omega = 13 \times 10^3$ (dashed curve), $\omega = 9 \times 10^3$ (dotted curve) and $\omega = 1 \times 10^3$ (dotted-dashed curve). We have taken $B^{1/4} = 200$ MeV and $T_c = 125$ MeV.

Figure 5. Left, the behavior of $T(\tau)$ as a function of time ($\tau$) for $\mu = 2 \times 10^5$, $\lambda = 10 \times 10^8$ MeV$^4$, $n = 0.05$ and small and large values of $\omega$, respectively: $\omega = 1 \times 10^0$ (solid curve) and $\omega = 2.518 \times 10^3$ (dotted–dashed curve). Right, the behavior of $h(\tau)$ as a function of time ($\tau$) for $\mu = 20000$, $\lambda = 8 \times 10^8$ MeV$^4$, $n = 0.05$, $V(T) = B$ and small and large values of $\omega$, respectively: $\omega = 1 \times 10^1$ (solid curve) and $\omega = 2.5 \times 10^3$ (dotted–dashed curve). We have taken $B^{1/4} = 200$ MeV.

strongly dependent on the BD coupling $\omega$. From the right-hand graph in figure 5 it is seen that for large values of $\omega$, $h(\tau)$ is much higher (dotted–dashed curve) than in the RS model and standard general relativity. The effect of an increase in $\omega$ on the brane is to strongly accelerate the formation of the hadronic phase and shorten the time interval necessary for the transition. A large $\omega$ tends to reduce the temperature of the hadronic fluid. From these figures it can be seen that for small values of $\omega$, the rate of the phase transition is slower than the large values of $\omega$. 


5. Lattice QCD phase transition

As was mentioned in the introduction, lattice QCD calculations for two quark flavors suggest that QCD makes a smooth crossover transition at a temperature of $T_c \sim 200$ MeV [8]. It is therefore necessary to have a brief review of the basic notions of this subject before using the results to study the universe at early times within the context of a BD brane scenario without the cosmological constant, $\lambda$, on the brane.

Lattice QCD is an approach which allows one to systematically study the non-perturbative regime of the QCD equation of state. This approach has enabled the calculation of the QCD equation of state using supercomputers [32] with two light quarks and a heavier strange quark on a $(N_t = 6) \times 32^3$ size lattice. The quark masses have been chosen to be close to their physical value, i.e. the pion mass is about 220 MeV. The equation of state was calculated at a temporal extent of the lattice $N_t = 6$ for which sizable lattice cut-off effects are still present [33]. The data for energy density $\rho(T)$, pressure $p(T)$ and trace anomaly $\rho - 3p$ and entropy $s$, used in what follows are taken from [32]. We also note that besides the strange quark, one can also include the effect of the charm quark as well as photons and leptons on the equation of state. These have important cosmological contributions as was shown in [34]. Recent references on lattice QCD at high temperature can be found in [35].

Over the high temperature regime, radiation-like behavior is seen as expected. However, in the region at and below the critical temperature $T_c (\approx 200$ MeV) of the deconfinement transition, the behavior changes drastically. This behavior change is also relevant for cosmological observables as we will see in the following. For high temperature, that is between 2.82 (100 MeV) and 7.19 (100 MeV), one can fit the data to a simple equation of state of the form

$$\rho(T) \approx \alpha T^4, \quad (46)$$
$$p(T) \approx \sigma T^4.$$

The values of $\alpha = 14.9702 \pm 0.09997$ and $\sigma = 4.99115 \pm 0.04474$ are found using a least squares fit [32].

While for times before the phase transition the lattice data matches the radiation behavior very well, for times corresponding to temperatures above $T_c$ the behavior of the lattice data changes toward matter dominated behavior. We remark that lattice studies show that the QCD phase transition at its physical values is actually a crossover transition.

5.1. High temperature regime

Let us first consider the era before phase transition at high temperature where the universe is in the quark phase. Using the conservation equation of matter together with the equation of state of quark matter (46), one finds the following relationship for the Hubble parameter

$$H = \frac{\dot{a}}{a} = -\frac{4\alpha}{3(\alpha + \sigma)} \frac{T}{T}, \quad (47)$$

whose solution is given by

$$a(T) = cT^{3/4(3\alpha + \sigma)}, \quad (48)$$

where $c$ is a constant of integration.

One may now proceed to obtain an expression describing the behavior of temperature of the BD brane universe with respect to time in the quark phase. Using equations (34), (46), (47) and (48) one finds a differential equation for the temperature

$$\frac{dT}{d\tau} = -\frac{3(\alpha + \sigma)T}{4\mu(3\omega + 4)T^{3/4(3\alpha + \sigma)}} \left( \frac{\mu(3p - \rho) + (2 + 3\omega + 6\omega^2)\rho^2 - \omega p\rho - 2p^2}{2(2(n + 1) - n\frac{\omega}{\omega} - \frac{\omega}{\omega}^2)} \right)^{1/2}, \quad (49)$$
where we have set the constant $c$ to unity. The transition region in the crossover regime can be defined as the temperature interval $282 \text{ MeV} < T < 719 \text{ MeV}$. In view of the crossover nature of the finite temperature QCD transition, such definition is equivocal [36].

Equation (49) can be solved numerically and the result is plotted in figure 5 which shows the behavior of temperature of the universe in the quark phase as a function of the cosmic time in BD brane cosmology for $\omega = 3 \times 10^5$, in the interval $282 \text{ MeV} < T < 719 \text{ MeV}$ in the high temperature regime. We see that as the time evolves the universe becomes cooler.

5.2. Low temperature regime

Besides lattice QCD there are other approaches to the low temperature equation of state. In the framework of the hadronic resonance gas model (HRG), QCD in the confinement phase is treated as a non-interacting gas of fermions and bosons [37]. The fermions and bosons in this model are the hadronic resonances of QCD, namely mesons and baryons. The idea of the HRG model is to implicitly account for the strong interaction in the confinement phase by looking at the hadronic resonances only since these are basically the relevant degrees of freedom in that phase. The HRG model is expected to give a good description of thermodynamic quantities in the transition region from high to low temperature [38]. The HRG result for the trace anomaly can also be parameterized by a simple form [36]

$$\frac{\Theta(T)}{T^4} = \frac{\rho - 3p}{T^4} = a_1T + a_2T^3 + a_3T^4 + a_4T^{10},$$

with $a_1 = 4.654 \text{ GeV}^{-1}$, $a_2 = -879 \text{ GeV}^{-3}$, $a_3 = 8081 \text{ GeV}^{-4}$, $a_4 = -7039 \text{ GeV}^{-10}$.

In lattice QCD the calculation of the pressure, energy density and entropy density usually proceeds through the calculation of the trace anomaly. Using the thermodynamic identities, the pressure difference at temperatures $T$ and $T_{\text{low}}$ can be expressed as the integral of the trace anomaly

$$\frac{p(T)}{T^4} - \frac{p(T_{\text{low}})}{T_{\text{low}}^4} = \int_{T_{\text{low}}}^{T} \frac{dT'}{T'^5} \Theta(T').$$

By choosing a sufficiently small lower integration limit, $p(T_{\text{low}})$ can be neglected due to the exponential suppression. Then the energy density $\rho(T) = \Theta(T) + 3p(T)$ and the entropy density $s(T) = (\rho + p)/T$ can be calculated. This procedure is known as the integral method [39]. Using equations (50) and (51) we obtain

$$\rho(T) = 3\eta T^4 + 4a_1T^5 + 2a_2T^7 + \frac{7a_3}{4}T^8 + \frac{13a_4}{10}T^{14},$$

$$p(T) = \eta T^4 + a_1T^5 + \frac{a_2}{3}T^7 + \frac{a_3}{4}T^8 + \frac{a_4}{10}T^{14},$$

where $\eta = -0.112$. The trace anomaly plays a central role in lattice determination of the equation of state. The equation of state is obtained by integrating the parameterizations given in equations (50) over the temperature as shown in equation (51).

Let us now consider the era before phase transition at low temperature where the universe is in the confinement phase and is treated as a non-interacting gas of fermions and bosons [37]. Using the conservation equation of matter together with equation of state (52), one gets the following relationship for the Hubble parameter

$$H \equiv \frac{\dot{a}}{a} = -\frac{12\eta T^3 + 20a_1T^4 + A(T)}{3[4\eta T^4 + 5a_1T^5 + B(T)]} T,$$

where

$$A(T) = 14a_2T^6 + 14a_3T^7 + \frac{a_4}{4}T^{13},$$

$$B(T) = \frac{5}{2}a_2T^7 + 2a_3T^8 + \frac{5}{2}a_4T^{14}.$$
Figure 6. The behavior of $T(\tau)$ in the interval $282\text{ MeV} < T < 719\text{ MeV}$ as a function of $\tau$ for $\mu = 10^4$, $n = 0.000 04$ and $\omega = 3 \times 10^5$.

One can solve for the scale factor as

$$a(T) = \frac{c}{T(75a_1 T + 35a_2 T^3 + 30a_3 T^4 + 21T^{10} + 60\eta)^{1/3}},$$

where $c$ is a constant of integration.

We can obtain an expression describing the behavior of temperature of the BD brane universe with respect to time in the quark phase. Upon using equations (34), (52), (53) and (55) one finds a differential equation for the temperature as follows:

$$\frac{dT}{d\tau} = -\frac{3(4\eta T^4 + 5a_1 T^5 + B)}{\mu(3\omega + 4)a^n(T)(12\eta T^3 + 20a_1 T^4 + A)} \times \left( \frac{\omega \rho (3\rho - \rho) + (2 + 3\omega + \omega^2)\rho^2 - \omega \rho - 2\rho^2}{2[2(n + 1) - n\eta \frac{\rho}{T}]} \right)^{1/2},$$

where we have set the constant $c$ to unity.

We can solve equation (56) numerically and the results are plotted in figures 6 and 7 where the latter shows the behavior of temperature of the universe in the quark phase as a function of cosmic time in BD brane cosmology for $\omega = 2 \times 10^4$, in the interval $70\text{ MeV} < T < 180\text{ MeV}$ in the low temperature regime.

The behavior of temperature with time can now be compared in the two regimes considered above, namely the first-order phase transition and the smooth crossover transition. Figures 2 and 4 show the variation of temperature in the former while figures 5 and 6 show such a variation in the latter. As can be seen, the general behavior is similar, that is, in both regimes the temperature drops as the time passes. We also note that the variation of $T$ with cosmic time is somewhat different in the two approaches when considered in detail. In the smooth crossover regime where lattice QCD is used to investigate the high temperature behavior, we see that the slope is smooth relative to the first-order phase transition while at lower temperatures where HRG is used, the slope is steep compared to first-order phase transition. Taking into account the energy range in which the calculations are done, one might
conclude that these two approaches to the quark–hadron transition in the early universe do not predict fundamentally different ways of the evolution of the early universe.

6. Conclusions

In this paper, we have discussed the quark–hadron phase transition in a BD brane cosmological setting in which our universe is a three-brane embedded in a five-dimensional bulk spacetime within an effective model of QCD. We studied the evolution of the physical quantities relevant to the physical description of the early universe; the energy density, temperature and scale factor, before, during, and after the phase transition. We found that for different values of $\omega$ phase transition occurs and results in decreasing the effective temperature of the quark–gluon plasma and of the hadronic fluid. We then compared our results with the results presented in [4] and [5]. In [4] the authors studied quark–hadron phase transition in an RS brane model and showed that for different values of the brane tension $\lambda$, phase transition occurs. Also in [5], the authors investigated the quark–hadron phase transition in a brane-world scenario where the localization of matter on the brane is achieved through the action of a confining potential and showed that for different values of the parameters in their model, phase transition takes place. What has been lacking is a study of such phase transitions in the context of a BD brane world model, presented in this work, emphasizing the evolution of the physical quantities relevant to the physical description of the early universe.

Finally, in section 5 we have considered the quark–hadron transition in the context of a smooth crossover regime at high and low temperatures. Such a study is of particular interest since extensive studies in lattice QCD over the past few years have led to a consensus that leans firmly in favor of a smooth crossover quark–hadron transition in the early universe. We showed that such a quark–hadron transition occurs and results in decreasing the effective temperature of the quark–gluon plasma during the process of the quark–hadron transition. The generic behavior of the temperature of the early universe in such a scenario is similar to that of a first-order phase transition, although the differences in the energy should be taken into account.
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References

[1] Rubakov V A and Shaposhnikov M E 1983 *Phys. Lett.* B 125 136
Akama K 1982 Pregeometry *Gauge Theory and Gravitation, Proceedings, Nara (Lecture Notes in Physics* vol 176) ed K Kikkawa, N Nakanishi and H Nariai (Berlin: Springer) p 267
Akama K 1982 *Lect. Notes Phys.* 176 267 (arXiv:hep-th/0001115)

[2] Randall L and Sundrum R 1999 *Phys. Rev. Lett.* 83 3370

[3] Mennim A and Battye R 2001 *Class. Quantum Grav.* 18 2171 (arXiv:hep-th/0008192)

[4] De Risi G, Harko T, Lobo F S N and Pun C S J 2008 *Nucl. Phys.* B 805 190 (arXiv:0807.3066 [gr-qc])

[5] Heydari-Fard M and Sepangi H R 2009 *Class. Quantum Grav.* 26 235021

[6] Gorenstein M I, Petrov V K and Zinovjev G M 1981 *Phys. Lett.* B 106 327
Gorenstein M I, Petrov V K, Shelest V P and Zinovjev G M 1982 *Theor. Math. Phys.* 52 843

[7] Gorenstein M I, Greiner W and Nan Y S 1998 *J. Phys. G: Nucl. Part. Phys.* 24 725
Gorenstein M I, Gazdzicki M and Greiner W 1998 *Phys. Rev.* C 72 w024909
Zakout I, Greiner C and Schaffner-Bielich J 2007 *Nucl. Phys.* A 781 150
Zakout I and Greiner C 2008 *Phys. Rev.* C 78 034916
Bugaev K A 2007 *Phys. Rev.* C 76 014903
Bugaev K A, Petrov V K and Zinovjev G M 2009 arXiv:0904.4420
Bessa A, Fraga E S and Mintz B W 2009 *Phys. Rev.* D 79 034012

[8] Aoki Y, Borsanyi Sz, Durr S, Fodor Z, Katz S D, Krieg S and Szabo K K 2009 *J. High Energy Phys.* JHEP06(2009)088 (arXiv:0903.4155)
Bazavov A et al 2009 *Phys. Rev.* D 80 014504 (arXiv:0903.4379)
Ferroni L and Koch V 2009 *Phys. Rev.* C 79 034905
De Tar C 2008 *PoS LATTICE 2008* 001 (arXiv:0811.2429)
Aoki Y, Endrodi G, Fodor Z, Katz S D and Szabo K K 2006 *Nature* 443 675
Tan Z G and Bonasera A 2007 *Nucl. Phys.* A 784 368

[9] Kajantie K and Kurki-Suonio H 1986 *Phys. Rev.* D 34 1719

[10] Ignatius J, Kajantie K, Kurki-Suonio H and Laine M 1994 *Phys. Rev.* D 49 3854
Ignatius J, Kajantie K, Kurki-Suonio H and Laine M 1994 *Phys. Rev.* D 50 3738

[11] Kurki-Suonio H and Laine M 1995 *Phys. Rev.* D 51 5431

[12] Kurki-Suonio H and Laine M 1996 *Phys. Rev.* D 54 7163

[13] Christiansen M B and Madsen J 1996 *Phys. Rev.* D 53 5446

[14] Rezzolla L, Miller J C and Pantano O 1995 *Phys. Rev.* D 52 3202

[15] Rezzolla L and Miller J C 1996 *Phys. Rev.* D 53 5411

[16] Rezzolla L 1996 *Phys. Rev.* D 54 1345

[17] Rezzolla L 1996 *Phys. Rev.* D 54 6072

[18] Bhattacharyya A, Alam J-E, Roy S S P, Sinha B, Raha S and Bhattacharjee P 2000 *Phys. Rev.* D 61 083509

[19] Davis A C and Lilley M 2000 *Phys. Rev.* D 61 043502

[20] Borghini N, Cottingham W N and Vnih Mau R 2000 *J. Phys. G: Nucl. Part. Phys.* 26 771

[21] Kim H I, Lee B-H and Lee C H 2001 *Phys. Rev.* D 64 067301

[22] Ignatius J and Schwarz D J 2001 *Phys. Rev. Lett.* 86 2216

[23] Davis S C, Perkins W B, Davis A C and Vernon I R 2001 *Phys. Rev.* D 63 083518

[24] Mendes L E and Mazumdar A 2001 *Phys. Lett.* B 501 249

[25] Barrow J D and Mimoso J P 1994 *Phys. Rev.* D 50 3746

[26] Weinberg S 1972 *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (New York: Wiley)

[27] Germani C and Maartens R 2001 *Phys. Rev.* D 54 124010

[28] Maartens R, Wands D, Basset B A and Heard J P C 2000 *Phys. Rev.* D 62 041301(R)

[29] Binetruy P, Deffayet C and Langlois D 2000 *Nucl. Phys.* B 565 269
Binetruy P, Deffayet C, Ellwanger U and Langlois D 2000 *Phys. Lett.* B 477 285
[31] Lee T D and Pang Y 1992 Phys. Rep. 221 251
[32] Cheng M et al 2008 Phys. Rev. D 77 014511 (arXiv:0710.0354)
    McGuigan M and Solner W 2008 arXiv:0810.0265
[33] Gupta R 2008 PoS LAT 2008 170
[34] Laine M and Schroder Y 2006 Phys. Rev. D 73 085009 (arXiv:hep-ph/0603048)
[35] Cheng M (RBC-Bielefeld Collaboration) 2007 PoS LAT 2007 173 (arXiv:0710.4357)
    Endrodi G, Fodor Z, Katz S D and Szabo K K 2007 PoS LAT 2007 228 (arXiv:0710.4197)
    Miller D E 2007 Phys. Rep. 443 55 (arXiv:hep-ph/0608234)
[36] Huovinen P and Petreczky P 2010 Nucl. Phys. A 837 26 (arXiv:0912.2541)
[37] Karsch F, Redlich K and Tawfik A 2003 Eur. Phys. J. C 29 549 (arXiv:hep-ph/0303108)
    Karsch F, Redlich K and Tawfik A 2003 Phys. Lett. B 571 67 (arXiv:hep-ph/0306208)
    Sakthi Murugesan K, Janhavi G and Subramanian P R 1990 Phys. Rev. D 41 2384
    Tawfik A 2005 Phys. Rev. D 71 054502 (arXiv:hep-ph/0412336)
[38] Braun-Munzinger P, Redlich K and Stachel J 2003 arXiv:nucl-th/0304013
    Andronic A, Braun-Munzinger P and Stachel J 2006 Nucl. Phys. A 772 167 (arXiv:nucl-th/0511071)
[39] Boyd G, Engels J, Karsch F, Laermann E, Legeland C, Lutgemeier M and Petersson B 1996 Nucl. Phys. B 469 419 (arXiv:hep-lat/9602007)