Null-Brane Solutions in Supergravities

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Abstract

We find a new class of time-dependent brane solutions in supergravities in arbitrary dimensions $D$. These are general intersecting light-like branes (null-branes), and their superposition and intersection rules are obtained. This is achieved by directly solving bosonic field equations for supergravity coupled to a dilaton and antisymmetric tensor fields. We discuss their possible significance.

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There has been much interest in time-dependent and spacelike brane solutions (S-branes) of supergravities in eleven and ten dimensions because of its possible connection with tachyon condensations and dS/CFT correspondence [1]-[5] (see also refs. [6, 7] for related solutions). These theories are the low-energy limits of the string theories and supposedly unifying M-theory of strings. Following the usual convention, S\(p\)-branes are used for those with \((p+1)\)-dimensional Euclidean world-volume. The more general solutions can be understood as intersecting ones of these fundamental S\(p\)-branes [8, 9, 10] and the rules how the branes intersect with each other are given in analogy to the usual branes [11]-[15]. Possible physical implications of these solutions are discussed in [16, 17, 18].

The existence of S-brane solutions is inferred from the following argument [1]. Consider initial data at time \(t = 0\) with the tachyon field sitting at the unstable maximum with a small velocity in a double-well potential of unstable D3-brane in type IIA theory. As time evolves into the future, the tachyon rolls off the top, emits closed string radiation and settles down to the minimum. Evolving into the past, one finds the time-reversed process with the tachyon approaching the other minimum. The overall picture is that the finely tuned incoming radiation conspires to excite the tachyon field to the top of the barrier and then down to the other side, dissipating back into the radiation. The obtained result is a timelike kink in the tachyon field. This picture was used for constructing S-brane solutions in field theories as well as superstrings/supergravities, and produces an interesting class of time-dependent solutions. Time-dependent solutions are investigated rather recently. It is thus interesting to try to find other possible time-dependent \(p\)-brane solutions.

The above argument can be immediately used to argue the existence of null-brane solutions (N-branes) with initial data on null hypersurface [1]. This class of solutions are also interesting from the viewpoint of closed/open string correspondence and stringy explanation of entropy of the black holes as discussed in ref. [19], where such solutions were discussed in the string worldsheet picture. However, to the best of our knowledge, no corresponding solutions in supergravities have been constructed.\(^1\) In order to gain insight into the geometric meaning, it is important to have such solutions explicitly. It is the purpose of this note to give this class of solutions in supergravities. In particular, we not only construct such solutions but also give the intersection rules for the way how the

\(^1\)“N-brane” solution is given in ref. [20], but it is called so because the field strength has the component in null direction, and is not the brane solution localized in the light-like direction.
solutions can intersect with each other by extending the method of [10, 15]. We show that the rules are simple consequences of the field equations, which can be easily integrated and the consistency of the solutions reduces the problem of solving the field equations to an algebraic one.

The results of our analysis turn out to be rather similar to the superposition rules for other types of branes [10, 15]. We show that the requirement that the field for each brane be independent is sufficient to give the solutions and intersection rules.

Let us start with the general action for gravity coupled to a dilaton \( \phi \) and \( m \) different \( n_A \)-form field strengths:

\[
I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \phi)^2 - \sum_{A=1}^{m} \frac{1}{2 n_A!} e^{a_A \phi} F_{n_A}^2 \right]. \tag{1}
\]

This action describes the bosonic part of \( D = 11 \) or \( D = 10 \) supergravities; we simply drop \( \phi \) and put \( a_A = 0 \) and \( n_A = 4 \) for \( D = 11 \), whereas we set \( a_A = -1 \) for the NS-NS 3-form and \( a_A = \frac{1}{2} (5 - n_A) \) for forms coming from the R-R sector.\(^2\) To describe more general supergravities in lower dimensions, we should include several scalars, but for simplicity we disregard this complication in this paper.

From the action (1), one derives the field equations

\[
R_{\mu \nu} = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \sum_{A} \frac{1}{2 n_A!} e^{a_A \phi} \left[ n_A (F_{n_A}^2)_{\mu \nu} \right] - \frac{n_A - 1}{D - 1} F_{n_A}^2 g_{\mu \nu},
\]

\[
\Box \phi = \sum_{A} \frac{a_A}{2 n_A!} e^{a_A \phi} F_{n_A}^2,
\]

\[
\partial_{\mu_1} \left( \sqrt{-g} e^{a_A \phi} F^{\mu_1 \cdots \mu_{n_A}} \right) = 0,
\]

\[
\partial_{[\mu} F_{\mu_1 \cdots \mu_{n_A}]} = 0. \tag{2}
\]

The last equations are the Bianchi identities.

We take the following metric for our system:

\[
ds_D^2 = -2e^{2u_0} dv du - 2e^{2u_1} dv^2 + \sum_{\alpha=2}^{p} e^{2u_\alpha} dy_\alpha^2 + e^{2B} d\Sigma_k^2_{\kappa,\sigma}, \tag{3}
\]

where \( D = p+k+1 \), the coordinates \( v = (t+x)/\sqrt{2} \) and \( y_\alpha, (\alpha = 2, \ldots, p) \) parametrize the \( p \)-dimensional world-volume directions and the remaining coordinates of the \( D \)-dimensional

\(^2\)There may be Chern-Simons terms in the action, but they are irrelevant in our following solutions.
spacetime are the lightcone coordinate $u = (t - x)/\sqrt{2}$ and those for $k$-dimensional spherical ($\sigma = +1$), flat ($\sigma = 0$) or hyperbolic ($\sigma = -1$) spaces, whose line elements are $d\Sigma_{k,\sigma}^2$. Since we are interested in solutions localized in the lightcone directions, all the functions appearing in the metrics as well as dilaton $\phi$ are assumed to depend only on $u$. The Ricci tensors for the metric (3) are

$$R_{uu} = -\sum_{a=2}^{p} [u''_a + (u'_a)^2 - 2u'_a u'_0] - k[B'' + (B')^2 - 2B'u'_0],$$

$$R_{vv} = 4e^{-4u_0+4u_1} \left[ u''_1 + u'_1 \left( -2u'_0 + 2u'_1 + \sum_{a=2}^{p} u'_a + kB' \right) \right],$$

$$R_{uv} = 2e^{-2u_0+2u_1} \left[ u''_1 + u'_1 \left( -2u'_0 + 2u'_1 + \sum_{a=2}^{p} u'_a + kB' \right) \right],$$

$$R_{\alpha\beta} = -2e^{2(-2u_0+u_1+B)} \left[ u''_\alpha + u'_\alpha \left( -2u'_0 + 2u'_1 + \sum_{a=2}^{p} u'_a + kB' \right) \right] \delta_{\alpha\beta},$$

$$R_{ab} = -2e^{2(-2u_0+u_1+B)} \left[ B'' + B' \left( -2u'_0 + 2u'_1 + \sum_{a=2}^{p} u'_a + kB' \right) \right] g_{ab} + \sigma(k-1)\bar{g}_{ab},$$

where $\bar{g}_{ab}$ is the metric for the hypersurface $\Sigma_{k,\sigma}$. Here and in what follows, a prime denotes a derivative with respect to $u$. We note that the above metric (3) is similar to but not quite the same as what is considered in the pp-wave solutions [21] with the metric $ds_D^2 = -2e^{2u_0}du^2 - 2e^{2u_1}du^2 + \cdots$. One may wonder if any interesting solutions exist for this case or for the metric (3) without $du^2$ term. We find that neither of these metrics give solutions with nontrivial field strengths corresponding to $N$-branes of our interest.

For the field strengths, we take the most general ones consistent with the field equations and Bianchi identities. Those for an electrically charged $Nq$-brane (whose world-volume is $(q + 1)$-dimensional) is given by

$$F_{uv\alpha_2\ldots\alpha_{q+1}} = \epsilon_{v\alpha_2\ldots\alpha_{q+1}} E', \quad (n_A = q + 2),$$

where $v, \alpha_2, \ldots, \alpha_{q+1}$ stand for the tangential directions to the $Nq$-brane. The magnetic case is given by

$$F^{\alpha_{q+2}\ldots\alpha_p a_1\ldots a_k} = \frac{1}{\sqrt{-g}} e^{-\alpha q + 2\ldots\alpha_p a_1\ldots a_k} \tilde{E}', \quad (n_A = D - q - 2)$$

where $a_1, \ldots, a_k$ denote the coordinates of the $k$-dimensional hypersurface $\Sigma_{k,\sigma}$. The functions $E$ and $\tilde{E}$ are again assumed to depend only on $u$. 

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The electric field (5) trivially satisfies the Bianchi identities but the field equations are nontrivial. On the other hand, the field equations are trivial but the Bianchi identities are nontrivial for the magnetic field (6).

We will solve the field eqs. (2) with the ansatz

\[ 2u_0 = 2u_1 + \sum_{\alpha=2}^{p} u_\alpha + kB, \] (7)

which simplifies the field equations (2) considerably. For both cases of electric (5) and magnetic (6) fields, we find that the field eqs. (2) are cast into

\[ -2u''_0 + 2u''_1 + 4u'_0(u'_0 - u'_1) - \sum_{\alpha=2}^{p} (u'_\alpha)^2 - k(B')^2 = \frac{1}{2} \phi'^2; \] (8)

\[ u''_1 = \sum_{A} \frac{D - q_A - 3}{4(D - 2)} S_A(E'_A)^2, \] (9)

\[ u''_\alpha = \sum_{A} \frac{\delta^{(\alpha)}_A}{4(D - 2)} S_A(E'_A)^2, \quad (\alpha = 2, \ldots, p), \] (10)

\[ B'' - \frac{\sigma(k - 1)}{2} e^{4u_0 - 2u_1 - 2B} = -\sum_{A} \frac{q_A + 1}{4(D - 2)} S_A(E'_A)^2, \] (11)

\[ \phi'' = -\sum_{A} \frac{\epsilon_A q_A}{4} S_A(E'_A)^2, \] (12)

\[ (S_A E'_A)' = 0, \] (13)

where \( A \) denotes the kinds of \( q_A \)-branes and we have defined

\[ S_A \equiv \exp \left( \epsilon_A q_A \phi - 2 \sum_{\alpha \in q_A} u_\alpha \right), \] (14)

(here and in what follows \( \alpha = 1 \) is included in the sum when written as \( \alpha \in q_A \)) and

\[ \delta^{(\alpha)}_A = \begin{cases} D - q_A - 3 & \text{for } y_\alpha \text{ belonging to } q_A \text{-brane} \\ -(q_A + 1) & \text{otherwise} \end{cases}, \] (15)

and \( \epsilon_A = +1(-1) \) corresponds to electric (magnetic) fields. For magnetic case we have dropped the tilde from \( E_A \). Equations (8), (9), (10) and (11) are the \( uu, vv, \alpha\alpha \) and \( ab \) components of the Einstein equation in (2), respectively. (\( uv \) and \( vu \) components give the same eq. (9)). We also define \( \delta^{(1)}_A = D - q_A - 3 \), so that eq. (9) can be written as (10). The last one is the field equation for the field strengths of the electric fields and/or Bianchi
identity for the magnetic ones. It is remarkable that both the electric and magnetic cases can be treated simultaneously just by using the sign $\epsilon_A$. This is because the original system (1) has the S-duality symmetry under

$$g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad F_{nA} \rightarrow e^{-aA} * F_{nA}, \quad \phi \rightarrow -\phi.$$  \hspace{1cm} (16)

From eq. (13) one finds

$$S_A E_A' = c_A,$$  \hspace{1cm} (17)

where $c_A$ is a constant. With the help of eq. (17), we find that eqs. (10) and (12) give

$$u'_\alpha = \sum_A \frac{\delta^{(\alpha)}_A}{4(D-2)} c_A E_A + c_\alpha, \quad (\alpha = 1, \ldots, p),$$

$$\phi' = -\sum_A \frac{\epsilon_{A\alpha A}}{4} c_A E_A + c_\phi,$$  \hspace{1cm} (18)

where $c_\alpha$ and $c_\phi$ are integration constants. Let us next define

$$g(u) = (2u_0 - u_1 - B)/(k - 1).$$  \hspace{1cm} (19)

We find from (7)

$$B = g - \frac{1}{k-1} \sum_{\alpha=1}^p u_\alpha, \quad 2u_0 = kg + u_1 - \frac{1}{k-1} \sum_{\alpha=1}^p u_\alpha,$$  \hspace{1cm} (20)

Using (18), we get

$$B' = g' - \sum_A \frac{q_A + 1}{4(D-2)} c_A E_A - \frac{1}{k-1} \sum_{\alpha=1}^p c_\alpha,$$  \hspace{1cm} (21)

$$2u'_0 = kg' + \sum_A \frac{D - 2q_A - 4}{4(D-2)} c_A E_A + c_1 - \frac{1}{k-1} \sum_{\alpha=1}^p c_\alpha,$$  \hspace{1cm} (22)

Substituting (17), (19) and (21) into (11), we obtain

$$g'' - \frac{\sigma(k-1)}{2} e^{2(k-1)g} = 0,$$  \hspace{1cm} (23)

which yields

$$g^2 - \frac{\sigma}{2} e^{2(k-1)g} = \beta^2,$$  \hspace{1cm} (24)
where $\beta$ is an integration constant. The solution to eq. (24) is given by

$$g(u) = \begin{cases} 
\frac{1}{k-1} \ln \frac{\sqrt{2} \beta}{\cosh[(k-1)\beta(u-u_1)]} : \sigma = +1, \\
\pm \beta (u - u_1) : \sigma = 0, \\
\frac{1}{k-1} \ln \frac{\sqrt{2} \beta}{\sinh[(k-1)\beta(u-u_1)]} : \sigma = -1,
\end{cases}$$

where $u_1$ is another integration constant.

Substituting eqs. (18) and (21)-(24) into (8) yields

$$\left( \sum_A D - 2q_A - 4 \right) c_A E_A + c_1 - \frac{1}{k-1} \sum_{\alpha=1}^{p} c_\alpha \left( \sum_A \frac{c_A E_A}{4} + c_1 + \frac{1}{k-1} \sum_{\alpha=1}^{p} c_\alpha \right) + \frac{1}{k-1} \sum_{\alpha=2}^{p} \left( \sum_A \frac{\delta_A^{(\alpha)}}{4(D-2)} c_A E_A + c_\alpha \right)^2 k \left( \sum_A \frac{q_A + 1}{4(D-2)} c_A E_A + \frac{1}{k-1} \sum_{\alpha=1}^{p} c_\alpha \right)^2 + \frac{1}{2} \left( \sum_A \frac{\epsilon_A a_A}{4} c_A E_A - c_\phi \right)^2 - \sum_A \frac{c_A}{4} E_A' - k(k-1)\beta^2 = 0. \tag{26}$$

This equation must be valid for arbitrary functions $E_A$ of $u$. From the $E_A$-independent part of eq. (26), one finds

$$\frac{1}{k-1} \left( \sum_{\alpha=1}^{p} c_\alpha \right)^2 + \sum_{\alpha=1}^{p} c_\alpha^2 + \frac{1}{2} c_\phi^2 = k(k-1)\beta^2. \tag{27}$$

We can then rewrite eq. (26) as

$$\sum_{A,B} \left[ M_{AB} \frac{c_A}{4} + \delta_{AB} \left\{ \left( \frac{1}{E_A} \right)' + 2\tilde{c}_A \right\} \right] c_B E_A E_B = 0, \tag{28}$$

where

$$M_{AB} = \frac{D - q_A - q_B - 4}{D - 2} + \sum_{\alpha=2}^{p} \frac{\delta_A^{(\alpha)} \delta_B^{(\alpha)}}{(D-2)^2} + k\frac{(q_A + 1)(q_B + 1)}{(D-2)^2} + \frac{1}{2} \epsilon_A a_A \epsilon_B a_B, \tag{29}$$

$$\tilde{c}_A = \sum_{\alpha \in q_A} c_\alpha - \frac{1}{2} c_\phi \epsilon_A a_A. \tag{30}$$

Since $M_{AB}$ is constant, eq. (28) cannot be satisfied for arbitrary functions $E_A$ of $u$ unless the second term inside the square bracket is a constant. Requiring this to be a constant tells us that the function $E_A$ must satisfy

$$\left( \frac{1}{E_A} \right)' + 2\tilde{c}_A \frac{E_A'}{E_A} + \tilde{c}_A N_A = 0, \tag{31}$$
or

\[
E_A = -\frac{e^{\tilde{c}_A(u-u_A)}}{N_A \cosh \tilde{c}_A(u-u_A)};
\]

(32)

where \(N_A\) is a normalization factor and \(u_A\) is an integration constant. In this way, the problem reduces to the algebraic equation (28) supplemented by (31) without making any assumption other than (7).

Equation (28) has two implications if we take independent functions for the fields \(E_A\). In this case, first putting \(A = B\) in eq. (28), we learn that

\[
c_A = \frac{4(D-2)\tilde{c}_A N_A}{\Delta_A},
\]

(33)

where

\[
\Delta_A = (q_A + 1)(D - q_A - 3) + \frac{1}{2} a_0^2 (D - 2).
\]

(34)

By use of eqs. (32) and (33), eqs. (18), (21) and (22) can be integrated with the results

\[
2u_0 = kg(u) - \sum_A \frac{D - 2q_A - 4}{\Delta_A} \ln \cosh \tilde{c}_A(u-u_A) - c_0 u + c'_0,
\]

\[
u_\alpha = - \sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \ln \cosh \tilde{c}_A(u-u_A) - \tilde{c}_\alpha u + c'_\alpha, \quad (\alpha = 1, \cdots, p),
\]

\[
B = g(u) + \sum_A \frac{q_A + 1}{\Delta_A} \ln \cosh \tilde{c}_A(u-u_A) + c_b u + c'_b,
\]

\[
\phi = \sum_A \frac{(D-2)\epsilon_A a_A}{\Delta_A} \ln \cosh \tilde{c}_A(u-u_A) + \tilde{c}_\phi u + c'_\phi,
\]

(35)

where \(c'\)'s are new integration constants and

\[
c_0 = \sum_A \frac{D - 2q_A - 4}{\Delta_A} \tilde{c}_A - c_1 + \frac{\sum_{\alpha=1}^p c_{\alpha}}{k-1}, \quad c'_0 = c'_1 - \sum_{\alpha=1}^p c'_{\alpha}, \quad \tilde{c}_\alpha = \sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \tilde{c}_A - c_\alpha,
\]

\[
c_b = \sum_A \frac{q_A + 1}{\Delta_A} \tilde{c}_A - \frac{\sum_{\alpha=1}^p c_{\alpha}}{k-1}, \quad c'_b = - \frac{\sum_{\alpha=1}^p c'_{\alpha}}{k-1}, \quad \tilde{c}_\phi = \sum_A \frac{(D-2)\epsilon_A a_A}{\Delta_A} \tilde{c}_A + c_\phi.
\]

(36)

To fix the normalization \(N_A\), we go back to eq. (14). Using (35), we find

\[
S_A = [\cosh \tilde{c}_A(u-u_A)]^2 e^{\epsilon_A a_A c'_\phi - 2 \sum_{\alpha \notin qA} c'_{\alpha}},
\]

(37)

which, together with (17) and (33), leads to

\[
N_A = \sqrt{\frac{\Delta_A}{4(D-2)}} e^{\epsilon_A a_A c'_\phi/2 - \sum_{\alpha \notin qA} c'_{\alpha}}.
\]

(38)
Our metric and other fields are thus finally given by

\[ ds_D^2 = \prod_A \left[ \cosh \tilde{c}_A (u - u_A) \right]^{2q_A+1} \left[ -2 e^{k g(u)} - 2 e^{u A} \prod_A \left[ \cosh \tilde{c}_A (u - u_A) \right]^{-\frac{D-2}{2}} du dv \\
- 2 \prod_A \left[ \cosh \tilde{c}_A (u - u_A) \right]^{-\frac{D-2}{2}} e^{-2\tilde{c} u + 2\tilde{c}'_d} d\Sigma_{k,\sigma}^2 \\
+ \sum_{\alpha=2}^p \prod_A \left[ \cosh \tilde{c}_A (u - u_A) \right]^{-\frac{1}{2} \gamma_{(\alpha)}} e^{-2\tilde{c}_\alpha u + 2\tilde{c}'_\alpha d\gamma_{(\alpha)}^2} \right], \]

\[ E_A = -\frac{e^{\tilde{c}_A (u - u_A)}}{N_A \cosh \tilde{c}_A (u - u_A)}, \quad \tilde{c}_A = \sum_{\alpha \in q_A} \frac{c_\alpha}{2} \epsilon_A \epsilon a_A. \]  

(39)

where we have defined

\[ \gamma_{(\alpha)} = \begin{cases} D - 2 & \text{for } y_\alpha \text{ belonging to } q_A-\text{brane} \\ 0 & \text{otherwise} \end{cases}. \]  

(40)

These solutions contain 2\(p\) + 2 integration constants \(c_\alpha, c'_\alpha (\alpha = 1, \ldots, p), c_\phi, c'_\phi\), together with \(u_1\) and \(u_A\) with \(\beta\) determined by eq. (27). Among these, \(c'_\alpha\) can be removed by rescaling the coordinates, and \(u_1\) by a shift of the coordinate \(u\). Without any preference of the choice of other parameters, we leave these as free parameters. Thus the general solutions can be constructed by the following rules:

1. All the directions are multiplied by \(\left[ \cosh \tilde{c}_A (u - u_A) \right]^{2q_A+1} \), and in addition,
2. the overall transverse directions \((u\) and \(k\)-dimensional space\)) are multiplied by \(e^{k g(u)}\) and \(e^{2g(u)}\), respectively,
3. the coordinates belonging to the brane are multiplied by \(\left[ \cosh \tilde{c}_A (u - u_A) \right]^{-\frac{D-2}{2}} \).

The second condition following from eqs. (28) is \(M_{AB} = 0\) for \(A \neq B\). This leads to the intersection rules for two branes: If \(q_A\)-brane and \(q_B\)-brane intersect over \(\bar{q}(\leq q_A, q_B)\) dimensions, this gives

\[ \bar{q} = \frac{(q_A + 1)(q_B + 1)}{D - 2} - 1 - \frac{1}{2} \epsilon_A \epsilon a_A \epsilon_B a_B. \]  

(41)

Remember that the world-volume of \(q\)-branes lies in \((q + 1)\)-dimensional space including \(v\). For eleven-dimensional supergravity, we have electric N2-branes, magnetic N5-branes and no dilaton \(a_A = 0\). The rule (41) tells us that N2-brane can intersect with N2-brane over a ‘0-brane’ \((\bar{q} = 0)\) (which actually lives in 1-dimensional space \(v\)) and with N5-brane over a ‘string’ \((\bar{q} = 1)\) (2-dimensional space including \(v\)), and N5-brane can intersect with N5-brane over ‘3-brane’ \((\bar{q} = 3)\) (4-dimensional space). In particular, our results show that
there is no other intersecting solution as long as we treat the functions $E_A$ with different index $A$ as independent. If this condition is relaxed, there may be other solutions. This is again quite similar to the intersection rules for usual branes [11]-[15] and S-branes [10].

For all the light-like $Dq$-brane solutions in type II superstrings, we find

$$\epsilon a = \frac{3 - q}{2},$$

which tells us that the intersection rule is

$$\tilde{q} = \frac{q_A + q_B}{2} - 2.$$  

It may be instructive to see how a single $N$-brane solution looks like. The metrics for the $N2$- and $N5$-branes in $D = 11$ supergravity take the form

$$ds^2_{N2} = [\cosh \tilde{c}(u - u_2)]^{1/3} \left[ -2e^{7g(u) + 2cu} [\cosh \tilde{c}(u - u_2)]^{-1/2} dudv 
+ [\cosh \tilde{c}(u - u_2)]^{-1} \{-2e^{4cu} dv^2 + e^{-2cu} (dy_2^2 + dy_3^2) \} + e^{2g(u) + 2c'} d\Sigma^2_{7,\sigma} \right],$$

$$\tilde{c}^2 + 12c^2 = 84\beta^2,$$

$$ds^2_{N5} = [\cosh \tilde{c}(u - u_2)]^{2/3} \left[ -2e^{4g(u) + 5cu} [\cosh \tilde{c}(u - u_2)]^{-1/2} dudv 
+ [\cosh \tilde{c}(u - u_2)]^{-1} \{-2e^{10cu} dv^2 + e^{-2cu} (dy_2^2 + \cdots + dy_6^2) \} + e^{2g(u) + 2c'} d\Sigma^2_{4,\sigma} \right],$$

$$\tilde{c}^2 + 60c^2 = 24\beta^2,$$

where we have imposed the rotational symmetry on the brane world-volume $y_\alpha$, absorbed some constants in the metric by the rescaling of the coordinates and redefined parameters. With eq. (25), these solutions have 5 independent parameters $\tilde{c}, c, c', u_1$ and $u_2$, but $u_1$ may be eliminated by the shift of the coordinate $u$, resulting in 4 parameters. A more interesting solution is the $ND3$-brane in type IIB:

$$ds^2_{ND3} = -2e^{5g(u) + 3cu} dudv + [\cosh \tilde{c}(u - u_2)]^{-1/2} \{-2e^{6cu} dv^2 + e^{-2cu} (dy_2^2 + \cdots + dy_4^2) \} + e^{2g(u) + 2c'} [\cosh \tilde{c}(u - u_2)]^{1/2} d\Sigma^2_{5,\sigma},$$

$$\tilde{c}^2 + 24c^2 + c^2 = 40\beta^2,$$

where again rotational symmetry on the world-volume is imposed.

We have examined if these solutions preserve any supersymmetry. It turns out that there is no remaining supersymmetry in these solutions, similarly to S-branes. In fact, they are supposed to correspond to branes with Dirichlet boundary conditions in the
lightcone direction, and hence describe configurations which exist only for a fixed lightcone coordinate. It would be interesting to examine stability and particle creations in these geometry [17].

To summarize, we have given quite a general model-independent derivation of the N-brane solutions in supergravities in arbitrary dimensions. The intersection rules simply follow from the field equations if we require that the functions $E_A$ with different index $A$ be independent. In all cases, the algebraic eq. (28) (together with (31)) must be satisfied, and this equation should be most useful to examine possible solutions. Our derivation is a simple generalization of the general method developed in refs. [15, 10]. It is quite satisfying to see that this is such a useful method. We hope to discuss various properties of these solutions using the hints from dualities implied by underlying string dynamics elsewhere.

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