Cosmological constant, renormalization group and Planck scale physics

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Starting from generic quantum effects at the Planck scale $M_P$, we find that the renormalization group running of the cosmological constant (CC) at low energies is possible if there is a smooth decoupling of all massive particles from $M_P$ to the mass of the lightest neutrino, $m_{\nu}$. We discuss the theoretical implications of this running for the “old” and “new” cosmological constant problems. Interestingly enough, the CC running implies a strong relationship between quantum field theory and cosmology, which should be observable in the near future in experiments such as SNAP through the measurement of a cubic redshift dependence of the CC.

1. Introduction

The concept of vacuum is one of the most intriguing ones in modern science, for it shows how the human views on the nature evolve with time. In classical physics, the vacuum is just empty space or the place prepared for the dynamics of particles. However, in quantum mechanics and QFT the vacuum is full of interesting phenomena such as creation and annihilation of virtual particles. This notion of vacuum led to the exceptional success of QED. Here, and also in the Standard Model (SM) of strong and electroweak interactions, the effects of virtual particles manifest themselves through quantum corrections to ordinary observables (e.g. cross-sections). However, switching on the gravitational field, one gets a chance to observe the energy and pressure of vacuum itself, for it appears as a component of the Einstein equations: that one called the cosmological constant (CC). According to the recent supernova data [1], the energy of vacuum is of the order of the present critical density and hence this “dark energy” dominates over the density of ordinary matter, radiation and even over the density of the dark matter which is requested by the astrophysical observations. Taking into account that all the matter content of the Universe has been, probably, created out of the vacuum during the reheating period after inflation, the non-zero energy of vacuum can not be seen as a total surprise. In actual fact the most shocking issue is why there is nowadays a residual vacuum energy (the observed CC) so close to the matter density. Furthermore, there is another, even greater, mystery in this story. The naive estimate of the tree-level contribution of the energy of vacuum in the SM (induced CC) is some 55 orders of magnitude greater than the cosmological constant which has been detected via the supernovae observations. In order to cure this problem one has to introduce another CC which is a characteristic of the vacuum itself (vacuum CC). Then one is forced to fine-tune this independent parameter with a tremendous precision. To explain this fine-tuning in a natural way is the “old” CC problem [2], while to explain the approximate coincidence of the CC and the matter density is the “new” CC problem [3]. Here we wish to address the “new” CC problem in the light of the Renormalization Group (RG) in QFT in a curved background.

2. Renormalization group and decoupling

In the standard picture the CC does not change when the Universe expands in the FLRW phase,
while the matter-radiation density is rapidly decreasing. The only changes of the CC are associated to phase transitions in the early Universe and should be constant in the later epochs. But there is another possible source of time dependence of the CC. Both induced and vacuum CC are subject to the RG running. However, this running must be suppressed at low energies because it is produced by the Feynman diagrams with loops of massive matter fields with external gravitational tails. If we use Einstein equations to estimate the typical energy of the gravitational quanta, this energy must be associated to the Hubble parameter \( H \). In the present universe this parameter has an approximate value \( H_0 \approx 1.5 \times 10^{-42} \text{GeV} \). If we compare this number with the assumed mass of the presumably lightest neutrino, \( m_\nu \approx 10^{-12} \text{GeV} \), at first sight it is clear that the corresponding loop must be completely decoupled and the running of the CC at low energy has no sense. But this is not the whole story. Consider the contribution of the particle with mass \( m \) to the \( \beta_\Lambda \)-function. The decoupling of massive particles at low energies is not abrupt, and the usual form of the scale dependence of this \( \beta \)-function at \( p^2 \ll m^2 \) (here \( p^2 \) is the square of the Euclidean momenta) is expected to be

\[
\beta^{(IR)}_\Lambda \sim \frac{p^2}{m^2} \times \text{const.} \tag{1}
\]

Despite that a recent attempt to verify the Appelquist-Carazzone theorem in curved space-time failed in the part concerning the CC, we have a very strong argument in favor of the formula (1). In order to see the decoupling one has to apply the physical mass-dependent renormalization scheme, which requires an explicit control of the energy of the particles. For this reason, the calculations can be performed straightforwardly only for the metric perturbations on the flat background, and this restricts too much the form of the covariant non-local terms in the vacuum effective action which may be responsible for the running of the CC. The root of the problem is that we do not possess, at the moment, a completely covariant calculational scheme compatible with the mass-dependent renormalization scheme. However, let us suppose that we have the instruments for this calculation. Then, in the cosmological setting, we are going to meet a general \( H \)-dependent expression for the contribution of the particle with mass \( m \) and spin \( J \):}

\[
\beta_\Lambda(m, H) = F\left(\frac{H^2}{m^2}\right) \times \beta^{(UV)}_\Lambda, \tag{2}
\]

where \( \beta^{(UV)}_\Lambda = N_J m^4/16\pi^2 \) is the non-suppressed UV contribution to the CC \( \beta \)-function. Here \( N_J = (-1)^{2J}(J + 1/2) n_J n_e \) is a multiplicity factor, with \( n_{\{0,1,1/2\}} = (1,1,2) \) and \( n_e = 1,3 \) for uncolored and colored particles respectively. It is clear that the function \( F(x) \) in (2) equals one in the UV limit \( x \to \infty \), because this is required by the correspondence between the mass-dependent renormalization and minimal subtraction schemes at high energies. At the same time, in the opposite limit \( x \to 0 \) this function has to vanish, because the non-decoupling of the \( \beta_\Lambda \)-function would lead to the extremely fast variation of the CC with the obvious untenable consequences in cosmology. This means that in the power series expansion \( F(x) = F_0 + x F_1 + x^2 F_2 + \ldots \), the coefficient \( F_0 \) is zero. However, we do not have any reason to suppose that the other coefficients of this expansion must be zero. It is easy to see that assuming \( F_1 \neq 0 \) leads to (1). In fact, the equality \( F_1 = 0 \) means that we impose one more constraint on the CC, and this is not what we want, from the theoretical point of view. So, let us suppose that \( F_1 \neq 0 \) and hence that the Eq. (1) is true. Then the total \( \beta \)-function will be the sum over all the fields and we reach the following relation for the low-energy running of the cosmological constant:

\[
\frac{d\Lambda}{d\ln H} = \frac{F_1}{(4\pi)^2} \sum_i \left(\frac{H^2}{m_i^2}\right) N_J m_i^4 \equiv \sigma H^2 M^2, \tag{3}
\]

where the parameter \( \sigma M^2 (\sigma = \pm 1) \) is defined by the sum of all existing particles: light and heavy. Let us notice that the leading contribution to (3) are those of the heaviest particles. Hence the details of the low-energy physics has

\[\text{Notice that every phase transition should change the induced CC and leave the vacuum CC intact. That is why the fine-tuning is so weird!}\]
no impact on the possible infrared (IR) running of the CC. This concerns, in particular, the non-perturbative effects of the low-energy QCD and the higher loop contributions to the $\beta_\Lambda$-function. By dimensional reasons and due to covariance, these contributions will always fall into the same form \((3)\). After all, this formula seems to have a universal form and emerges from a vast class of QFT models\(^2\). Equation \((3)\) shows that the low-energy dynamics of the CC is completely defined by the spectrum of the heaviest particles which become active only at extremely high energies. We stress that, despite this looks paradoxical, it is rather robust and the only one phenomenological input which we used was the hypothesis $F_1 \neq 0$. In fact, the nontrivial role of the high energy spectrum for the low-energy dynamics of the CC \((3)\) just means that we do not like to introduce an extra unnecessary fine-tuning $F_1 = 0$ to the CC problem.

3. Impact for the early and late universes

Equation \((3)\) includes a parameter $\sigma M^2$ which we cannot define from the present-day particle physics because it may naturally involve Planckian-size masses of particles which are inaccessible to all future accelerators. For the sake of simplicity, we can just take $M^2 = M_P^2$. Let us clarify that this choice does not necessary mean that the relevant high energy particles have the Planck mass. The mass of each particle may be smaller than $M_P$, and the equality, or even the effective value $M \gtrsim M_P$, can be achieved due to the multiplicities of these particles. Finally, the sign $\sigma = \pm 1$ depends on whether bosons or fermions dominate at the sub-Planck scale.

With these considerations in mind, our first observation is that the natural value of the $\beta$-function \((3)\) at the present time is

$$|\beta_\Lambda| = \frac{c}{(4\pi)^2} M_P^2 \cdot H_0^2 \sim 10^{-47} \text{ GeV}^4,$$  \hspace{1cm} \(4\)

where $c = \mathcal{O}(1 - 10)$. Hence $\beta_\Lambda$ is very much close to the experimental data on the CC\([1]\). This is highly remarkable, because two vastly different and (in principle) totally unrelated scales are involved to realize this “coincidence”: $H_0$ and $M_P$, being these scales separated by more than 60 orders of magnitude! The resemblance between the renormalization group eq. \((3)\) and the Friedmann equation $H^2 \sim \Lambda/M_P^2$ for the modern, CC-dominated, Universe looks rather intriguing and is worth exploring. The RG equation \((3)\) links the value of the CC with the one of the Hubble parameter. The latter depends on the conformal factor of the metric because of the dynamics of the matter-energy density. Let us emphasize that the RG-based dependence between the CC and the matter density means that we can essentially alleviate the “new” CC problem, because the coincidence is not directly related to a particular cosmological epoch anymore! At the same time, the dynamics of the CC may jeopardize the well-known results in the Standard Cosmological Model, primarily for the nucleosynthesis.

Let us consider the cosmological solution corresponding to \((3)\). For the sake of simplicity we restrict our consideration to the case of the conformally flat $k = 0$ FLRW metric. The more general formulas for an arbitrary $k$ can be found in the parallel paper \([9]\). It proves useful to solve for the CC and matter density in terms of the redshift variable $z$, defined as $1 + z = a_0/a$, where $a_0$ is the present-day scale factor. Along with Eq. \((3)\) we shall use the Friedmann equation

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \rho + \Lambda \right),$$ \hspace{1cm} \(5\)

where $\rho$ is the matter-radiation density, and the energy conservation law

$$\frac{d\Lambda}{dt} + \frac{d\rho}{dt} + 3H(\rho + p) = 0,$$ \hspace{1cm} \(6\)

where $p$ is the pressure. Since we need to deal with both matter and radiation dominated regimes, it is useful to solve the coupled system of differential equations \((3), (5)\) and \((6)\) using an arbitrary equation of state $p = \alpha \rho$. The time derivative in \((6)\) can be easily traded for a derivative in $z$ via $d/dt = -H(1+z) d/dz$. The solution is completely analytical and takes the form

$$\rho(z; \nu) = \rho_0 (1+z)^\nu \text{ and }$$  \hspace{1cm} \(7\)

\(\text{in string theory.}\)
\[ \Lambda(z; \nu) = \Lambda_0 + \frac{\nu}{1-\nu} \left[ \rho(z; \nu) - \rho_0 \right], \quad (8) \]

where \( \rho_0, \Lambda_0 \) are the present day values of the matter density and CC, and we have introduced the following notations:

\[ \nu = \frac{\sigma M^2}{12\pi M_P^2}, \quad r = 3(1-\nu)(\alpha + 1). \quad (9) \]

In order to avoid confusion, we note that the above solution for \( \Lambda(z; \nu) \) has no singularity in the limit \( \nu \to 1 \). Also, as expected, for \( \nu \to 0 \) we recover the standard result for \( \rho(z) \) with constant CC.

Consider the nucleosynthesis epoch when the radiation dominates over the matter, and derive the restriction on the parameter \( \nu \). In the radiation-dominated regime, the solution for the density \( \rho \) can be rewritten in terms of the temperature and the number of effectively massless or relativistic degrees of freedom,

\[ \rho_R(T) = \frac{\pi^2 g^* T^4}{30} \left( \frac{T_0}{T} \right)^{4\nu} \quad (10) \]

with \( T_0 \approx 2.75 K = 2.37 \times 10^{-4} eV \) being the present CMB temperature.

It is easy to see that the size of the parameter \( \nu \) gets restricted, because for \( \nu \geq 1 \) the density of radiation (in the flat case) would be the same or even below the one at the present universe. Hence, in order not to be ruled out by the nucleosynthesis, our model has to satisfy

\[ |\Lambda_R/\rho_R| \simeq |\nu/(1-\nu)| \simeq |\nu| \ll 1. \quad (11) \]

A nontrivial range could e.g. be \( 0 < |\nu| \leq 0.1 \). Both signs of \( \nu \) are in principle allowed provided the absolute value satisfies the previous constraint. Let us notice that, in view of the definition \( \beta \), the condition \( \nu \ll 1 \) also means that \( M \lesssim M_P \). Hence, the nucleosynthesis constraint coincides with our general will to remain in the framework of the effective approach. It is remarkable that the two constraints which come from very different considerations, lead to the very same restriction on the unique free parameter of the model. The canonical choice \( M^2 = M_P^2 \), corresponds to

\[ |\nu| = \nu_0 \equiv \frac{1}{12\pi} \simeq 2.6 \times 10^{-2}. \quad (12) \]

4. The running and the “old” CC problem

One can observe, at this point, some relation between our RG approach and the “old” CC problem. There are two leading ideas concerning the solution of this problem: The first one supposes the existence of an unknown symmetry characterizing the fundamental theory such as (super)string or M-theory. It is supposed that this symmetry (an unbroken supersymmetry is the simplest example) must preclude the contributions of the virtual particles to the CC, and thus reduce the order of the fine-tuning or even make it unnecessary (see, e.g. [10]). An obvious difficulty is that the corresponding symmetry must take place at the very low energy scale, most naturally in the remote future when the matter density will become zero. Also, this means that the information about this symmetry was somehow encoded into the early Universe, before all the supposed phase transitions took place. We remark that our formula (3) is the first explicit example of the possible relation between the Planck scale and the cosmic scale physics.

The second way of thinking about the CC problem is to assume that its solution cannot be attained from first principles, and that one must resort to some sort of anthropic hypothesis [2,3]. Both points of view have some peculiarities. The difficulty of the “symmetry approach” is that one needs this symmetry not at high, but at very low energy. Hence, many candidate symmetries are useless or at least looks to be so. In particular, this concerns supersymmetry which (if exists at all!) should be broken at low energies. In turn, the anthropic hypothesis may be interpreted as an indication to the existence, at some instant in the past, of many universes with some random distribution of the values of the CC. One can, e.g., associate the existence of the numerous choices of the universes with the indefiniteness of vacuum in string theory which is indeed the main candidate theory and is supposed to solve all problems of physics including, of course, the CC one.

It is obvious that the first (symmetry-based) sort of solution for the CC problem corresponds to the zero value of the CC in a remote future, when the density of matter \( \rho_M \) will become negli-
gible due to the further expansion of the Universe. At the same time, the anthropic solution does not imply this requirement, because the choice of the vacuum is performed in the chaotic way and we are presumably living in just one of those universes which permit the comfortable discussion of the CC problem. Actually, one can get some hint about which way is correct by just applying the solution \[5\] with the purpose to see whether the CC can tend to zero in the remote future. It is easy to solve the corresponding equation in the flat case:

\[
\Lambda(z \to -1) = \Lambda_0 - \frac{\nu}{1-\nu} \rho_0 = 0. \tag{13}
\]

We arrive at the suggestive value \(\nu = \Omega^0_{\Lambda}\) where the present day estimate is \(\Omega^0_{\Lambda} \simeq 0.7\) \[1\]. This value of \(\nu\) is smaller than one, but it implies a fairly large correction to some standard laws of conventional FLRW cosmology. Whether we can accept it or not is not obvious at present. However, if accepted, then it would hint at the “symmetry” approach to the old CC problem, in the sense that string theory itself could perhaps provide that value of \(\nu\) as a built-in symmetry requirement. In Ref. \[11\] we test explicitly the cosmological laws using experimental data and simulations. We find that even thinking of \(\nu\) in Eqs.\[11,5\] as a mere phenomenological parameter that gauges the departure of these laws from the conventional FLRW solution, the tolerance in \(\nu \neq 0\) is still remarkably high.

5. CC running and SNAP/HST testing

We now ask whether even the most obviously permitted values of the parameter \(\nu \ll 1\) may lead to observable consequences. The remarkable answer is: yes. In order to see this, we consider the “recent” Universe characterized by the redshift interval \(0 < z \lesssim 2\), and evaluate some cosmological parameters which can be, in principle, improved by the future observations, say by the SNAP project and beyond HST \[12\]. The first relevant exponent is the relative deviation \(\delta_{\Lambda}(z;\nu) \equiv (\Lambda(z;\nu) - \Lambda_0)/\Lambda_0\) of the CC from the constant value \(\Lambda_0\). One has to remember that the existing estimates for the CC from the supernovae data \[1\] correspond to the supernovae measurements at some \(z = z_0\). Then, using our solution \[5\] we obtain, in first order of \(\nu\),

\[
\delta_{\Lambda}(z;\nu) = \frac{\nu \omega^0_{\Lambda}}{\omega^0_{\Lambda}} (1 + z)^3 - (1 + z_0)^3. \tag{14}
\]

Taking \(z_0 \simeq 0.5\) (the approximate central value of the sample of high redshift supernovae from \[1\]), with \(\omega^0_{\Lambda} = 0.3\) and \(\omega^0_\Lambda = 0.7\), and \(\nu = \nu_0\), we find e.g. \(\delta_{\Lambda}(z = 1.5;\nu_0) \simeq 14\%\). In general, the strong cubic \(z\)-dependence in \(\delta_{\Lambda}(z;\nu)\) should manifest itself in the future CC observational experiments where the range \(z \gtrsim 1\) will be tested. It is important to emphasize that \(\nu\) is the unique arbitrary parameter of this model for a variable CC. Therefore, the experimental verification of the above formula must consist in: i) pinning down the sign and value of the parameter \(\nu\); and ii) fitting that formula to the experimental data \[3\].

Next we present the relative deviation of the square of the Hubble parameter \(H^2(z;\nu)\) with respect to the conventional one \(H^2(z;\nu = 0)\). Again we just quote the flat case. The resulting deviation \(\delta H^2(z;\nu) = (H^2(z;\nu) - H^2(z;0))/H^2(z;0)\) is

\[
\delta H^2 = -\nu \omega^0_\Lambda \frac{1 + (1 + z)^3 [3 \ln(1 + z) - 1]}{1 + \omega^0_\Lambda (1 + z)^3 - 1}. \tag{15}
\]

Equation \[15\] gives the leading quantum correction to the Hubble parameter \(\delta H^2(0;\nu) = 0\), because for all \(\nu\) we have the same initial conditions. Then for \(z \neq 0\) we have e.g. \(\delta H^2(1.5;\nu_0) \simeq -4.2\%\) and \(\delta H^2(2;\nu_0) \simeq -5.7\%\). For larger \(\nu\), we get quite sizeable effects like \(\delta H^2(z;0.1) \simeq -16\%\) and \(-21\%\) for \(z = 1.5\) and \(z = 2\) respectively.

The last exponent of interest that we wish to remark here makes use of our previous results for \(\Lambda(z;\nu)\) and \(H(z;\nu)\). Then we can compute the relative deviation of the renormalized cosmological constant parameter \(\Omega_{\Lambda}(z;\nu) = 8\pi GA(z;\nu)/3H^2(z;\nu)\) at redshift \(z\) with respect to the standard one, \(\Omega_{\Lambda}(z;0)\). At leading order,

\[
\delta \Omega_{\Lambda}(z;\nu) = \frac{\Omega_{\Lambda}(z;\nu) - \Omega_{\Lambda}(z;0)}{\Omega_{\Lambda}(z;0)} = \]

\[\text{For alternative RG scenarios, in the context of quantum gravity, see \[13\] and references therein.}\]
As we could avoid the embarrassing event horizon problem, a symmetry requirement, e.g., within M-theory, has just one arbitrary parameter $\nu < 0$. For $\nu < 0$ the effects go in the opposite direction. If some future experiments can reach the far $z = 2$ region with enough statistics, the effects on $\Omega_{\Lambda}(z)$ are even more dramatic. At present $\Omega_{\Lambda}^0$ has been determined at roughly 10% from both supernovae and CMB measurements, and in the future SNAP will pin $\Omega_{\Lambda}$ down to within $\pm 0.05$ [12]. The previous numbers show that for $z \gtrsim 1$, the cosmological quantum corrections can be measured already for a modest $\nu \gtrsim 10^{-2}$. A complete numerical analysis of this kind of FLRW models, including both the flat and curved space cases, together with a detailed comparison with the present and future Type Ia supernovae data, will be presented elsewhere [11].

6. Conclusions

We have exemplified the possible running of the CC at the present cosmic scale due to the renormalization group and the smooth decoupling of the massive fields at low energies, assuming the Hubble parameter $H$ as the RG scale. A time dependence of the CC may therefore be achieved without resorting to scalar fields mimicking the cosmological term. It turns out that the $\beta_{\Lambda}$ function has just one arbitrary parameter $\nu < 1$. For $\nu \ll 1$, we insure the absence of the trans-planckian energies and also consistency of the CC with the nucleosynthesis calculations. However, larger values of $\nu$ cannot be completely excluded at present [11]. For example, if $\nu = 2\nu_0$, a flat universe would have exactly zero CC in the infinite future. This would open the possibility that a symmetry requirement, e.g., within M-theory, could avoid the embarrassing event horizon problem in this string framework where an asymptotic positive CC is not welcome. In fact, our model with running CC could represent the effective behavior of many high energy theories. Last, but not least, there are excellent prospects for testing this RG cosmological model in the future by SNAP and the upgraded HST experiments.

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