Angular resolved phonon emission from excited quantum dots

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Abstract. We calculate the angular emission characteristics of phonons from parabolic confined quantum dots containing a single or two interacting electrons. The emission spectra are shown to be generally characterized by a narrow polar angle giving a phonon propagation direction explicitly characterized by the energy difference of the transition. In addition, the phonon emission spectra contain a given number of azimuthally oriented lobes which reflect the quantum structure of the initial excited state. This implies that measurements of angular resolved phonon emission spectra can give detailed information on the electronic charge distribution and energy spectra of excited quantum states. When such a structure is known, a large-scale ordering of identical quantum dots with respect to the emission angles may realize phonon amplification by stimulated emission.

Contents

1. Introduction 2
2. Emission from one-electron dots 3
3. Spin-conserving two-electron emission 5
4. Spin–orbit-mediated transitions 9
5. Conclusion and outlook 11
Acknowledgment 13
References 13

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1. Introduction

Decay and breakup mechanisms are among the most central and fundamental research areas in quantum mechanics, from high-energy physics to solid-state systems [1–3]. The angular distribution of the subsequent outgoing energy quanta gives insight into the reaction dynamics and the nature of the interaction. In nuclear, atomic and molecular systems, the excess energy in terms of charged particles, neutrons or photons carries detailed information on the symmetry, localization and entanglement of the initial quantum state [4]. For solid-state systems, however, the decay mechanisms are often more involved, and as such the outcome may stem from competing simultaneous interactions. Thus, the emission characteristics of each process in the outgoing decay product are of fundamental interest.

The coupling between electronic quantum states and lattice vibrations in semiconductor crystals [5, 6] and nanostructures [7–12] has been studied for decades. An example of such a system is the quantum dot that, among many other potential applications in both medicine and technology, has been experimentally realized as a controllable quantum bit [13]. In this context, the study of electronic relaxation processes in semiconductor quantum dots is important [14, 15]. Spin-conserving phonon emission rates have been calculated for one-electron single [16] and double [16–18] quantum dots. In particular, the (polar) directional emission characteristics from quantum dots as a function of dot confinement have been analysed based on numerical calculations in [19].

In this paper, we further develop the understanding of angular phonon emission from one- and two-electron sub-surface gate-controlled quantum dots. Phonon emissions in the acoustic as well as in the optical branch occur in general as competing processes. We will, in the following, limit our study to the emission of acoustic phonons alone, since this process can be assumed to dominate at small confinement strengths (implying low phonon energies). However, the formalism below can be directly extended to the emission of optical phonons as well. In contrast to radiative emission from atoms, the phonon emission characteristics are much less understood, and yet are of importance for future quantum information systems based on communicating quantum dots. The results are based on Fermi’s golden rule for transitions between the lowest states in one- and two-electron quantum dots, which are modelled by harmonic basis states entirely. This is an alternative numerical method as compared with [19] and leads to analytical expressions for the differential emission amplitudes. We find that the polar emission spectrum is generally peaked around a narrow central angle and that the most probable polar emission angle is derived analytically in terms of parameters for the single-electron dot. Furthermore, it is shown that a departure from circular planar symmetry leads to a corresponding azimuthal dependence in the phonon emission spectra, which in turn can be directly related to the structure of the initial excited state. This is also true for spin-mixing transitions mediated by the spin–orbit interaction.

The section that follows describes the numerical model and presents a discussion of the phonon emission characteristics of one electron confined in a quantum dot potential. In the subsequent section, the discussion is extended to include two interacting electrons in the same potential. Finally, in the last section the spin–orbit interaction is taken into account and spin-mediated transitions are studied.
2. Emission from one-electron dots

An electron confined in a two-dimensional (2D) parabolic quantum dot is described by the Hamiltonian
\[ h = h_0 + h_{e-ph}, \]
where the first part describes a confined electron,
\[ h_0 = -\frac{\hbar^2}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \frac{1}{2} m^* \omega_0^2 (x^2 + y^2), \]
and the latter describes the electron–phonon interaction,
\[ h_{e-ph} = \sum_{j=1}^{3} M_j \left( \hat{a}_{q_j} e^{i\mathbf{qr}} + \hat{a}^\dagger_{q_j} e^{-i\mathbf{qr}} \right), \]
where the operator \( \hat{a}_{q_j} \) (\( \hat{a}^\dagger_{q_j} \)) annihilates (creates) a transverse or longitudinal phonon with momentum \( \mathbf{q} \) on the lattice. We will consider phonon emission into the surrounding structure, which we assume to be a 4\( \pi \) spherical slab of GaAs. The material parameters are therefore chosen to be those of GaAs, i.e. the effective mass \( m^* = 0.067 m_e \). Furthermore, \( M_j \) represents the material-sensitive coupling strengths due to the deformation potential or the piezoelectric field interactions (to be defined below).

The transition rates for decay due to the electron–phonon interaction can, to a first-order approximation, be described by Fermi’s golden rule
\[ w_{I \rightarrow F} = \frac{V}{\hbar (2\pi)^2} \sum_{j=1}^{3} \int d^3 q |M_j(q)|^2 \left| \langle \Psi_F | h_{e-ph} | \Psi_I \rangle \right|^2 \delta \left( \Delta \epsilon_{F,I} - \hbar c_j q \right), \]
where \( V \) is an auxiliary normalization volume and \( \Delta \epsilon_{F,I} = \epsilon_F - \epsilon_I \) is the energy of the released energy quantum. The linear isotropic dispersion relation, \( \omega_{F,I} = c_l q \), is applied, where \( c_l = 4720 \text{ m s}^{-1}, c_t = 3340 \text{ m s}^{-1} \) denotes the longitudinal (transverse) speed of sound. This commonly used approximation is very good in the case of zone-centred phonons, i.e. for small values of \( q \) \([16, 18]\). Furthermore, \( \Psi_I \) and \( \Psi_F \) refer to the initial and final states of the electrons corresponding to the energies \( \epsilon_I \) and \( \epsilon_F \), respectively. The electron–phonon coupling strengths \( M_j \) are explicitly given by
\[ |M_{dp,1}(q)|^2 = \frac{\hbar \Xi_d^2}{2\rho c_l V} |q|, \]
\[ |M_{pz,1}(q)|^2 = \frac{32\pi^2 \hbar e^2 \hbar^2_4 (3q_x q_y q_z)^2}{\epsilon^2 \rho c_l V} \left| q \right|^7, \]
\[ |M_{pz,1}(q)|^2 = \frac{32\pi^2 \hbar^2 e^2 \hbar^2_4}{\epsilon^2 \rho c_l V} \left[ (q_x^2 q_y^2 + q_y^2 q_z^2 + q_z^2 q_x^2) \left| q \right|^5 - (3q_x q_y q_z)^2 \left| q \right|^7 \right] \]
for coupling via the deformation potential and the longitudinal and transverse piezoelectric potentials, respectively. In these expressions \( \rho = 5300 \text{ kg m}^{-3} \) is the GaAs mass density,
$\epsilon_i = 13.1$ is the relative permittivity, $\Xi_d = 6.7$ eV is the deformation potential constant, $h_{14} = 1.4 \times 10^9$ V m$^{-1}$ is the piezoelectric constant and $e$ is the electronic charge.

We will consider a quasi-2D quantum dot with an order of magnitude stronger confinement in the vertical direction. Within this approximation the vertical degrees of motion become frozen and the total wavefunction is safely augmented by a ground-state component in the vertical direction, $\Psi(r) = \psi_n(r)\psi_0(z)$. In previous works [16, 19], the $z$-direction has been modelled in terms of the ground state of a finite square well. Here we apply the harmonic approximation also in the vertical direction but with a much stronger confinement compared to the parallel one, i.e. $\hbar \omega_z \gg \hbar \omega_0$. The perpendicular confinement strength $\hbar \omega_0$ then gives an effective vertical extension $L_z = \sqrt{\hbar/(m^*\omega_z)}$ defined by the classical turning point of the classical harmonic oscillator. In experiments it is believed that $L_z$ is at maximum a few atomic layers thick [20], defining the orders of magnitude of the confinement strengths $\hbar \omega_z$ applied in the present work. In this range, the emission spectrum is essentially insensitive to the precise value of $\hbar \omega_z$.

We will now consider transitions from the first excited level to the ground state. The initial and final states are harmonic oscillator eigenstates with quantum numbers $n_x$, $n_y$ and $n_z$, and the spatial integral pertaining to the particular transition $n_z = 1 \rightarrow 0$ becomes

$$
\langle \Psi_f | h_{e-ph} | \Psi_i \rangle = -i q_z \sqrt{\frac{\hbar}{2m^*\omega_0}} \exp \left[ -\frac{\hbar}{4m^*} \left( \frac{q_z^2 + q_y^2}{\omega_0} + \frac{q_x^2}{\omega_z} \right) \right].
$$

(6)

When inserting this expression into Fermi’s golden rule, (4), the final $q$-integral for the deformation coupling takes the form

$$
w_{DP} = \frac{d_1 h}{2m^*\omega_0} \int d^3 q_0 \sin^2 \theta \cos^2 \phi q^3 G_{\omega_0,\omega_z}^{q}(\theta) \delta(q - q_0).
$$

(7)

We have introduced here the auxiliary quantity $G_{\omega_0,\omega_z}^{q}(\theta) = \exp[-\frac{\hbar q^2}{2m^*} (\sin^2 \theta + \cos^2 \phi)]$ and defined $d_1 = \Xi_d^2/(2^3 \pi^2 \hbar c^2)$. Thus, the differential emission probability can be expressed in a simple analytical form

$$
\frac{dw_{DP}}{d\Omega} = \frac{d_1 h}{2m^*\omega_0} q_0^5 \sin^2 \theta \cos^2 \phi G_{\omega_0,\omega_z}^{q}(\theta).
$$

(8)

Similarly, for the piezoelectric parts,

$$
\frac{dw_{PZL}}{d\Omega} = \frac{p_l h}{2m^*\omega_0} q_0^3 \sin^6 \theta \cos^2 \theta \cos^4 \phi \sin^2 \phi G_{\omega_0,\omega_z}^{q}(\theta),
$$

(9a)

$$
\frac{dw_{PZZ}}{d\Omega} = \frac{p_l h}{2m^*\omega_0} q_0^3 \left[ \sin^6 \theta (1 - 9 \cos^2 \theta) \cos^4 \phi \sin^2 \phi + \sin^4 \theta \cos^2 \theta \cos^2 \phi \right] G_{\omega_0,\omega_z}^{q}(\theta),
$$

(9b)

where $p_l = 72 e^2 h_{14}/\epsilon_i^2 \hbar c_i^2$ and $p_i = 8 e^2 h_{14}/\epsilon_i^2 \hbar c_i^2$. With these formulae at hand the angular properties of phonon emission may be analysed directly. Firstly, we note that an initially asymmetric electron density distribution with respect to the azimuthal angle results in a corresponding $\phi$ dependence in the emission spectrum. However, by constructing initial states independent of the azimuthal angle, $|n_x, n_y \rangle \rightarrow (|1, 0 \rangle \mp i|0, 1 \rangle) / \sqrt{2}$, the emission naturally becomes $\phi$ independent [19]. Secondly, we note that the angular dependence for the piezoelectric decay is more complex than the deformation part, with a leading sin-power of 4.
and 6 versus 2. For the deformation potential we then obtain maximum emission probability along the angle

$$\theta_{\text{max}}^{\text{DP}} = \sin^{-1} \left( c_1 \sqrt{\frac{2\hbar}{m^*\omega_0(1 - \omega_0/\omega_z)}} \right).$$  \hspace{1cm} (10)

In figure 1, we plot the angle of the most probable phonon emission as a function of lateral confinement strength. The analytical formula (10), giving the most probable emission angle of the deformation phonons, is seen to be in excellent agreement with the corresponding numerical result when all phonons are considered. In both cases, the dominant emission angle switches from 90° in the limit of zero \(xy\)-confinement (emission in the \(xy\)-plane) to 0° (emission perpendicular to the \(xy\)-plane) for strong confinement. In the inset of the figure, the actual angular distribution is shown for two different cases. The upper left part of the inset shows the electronic density pertaining to an eigenstate with \((n_x, n_y) = (1, 0)\). In this case, the angular distribution becomes most dominant in the \(xz\)-plane with two characteristic spikes in each hemisphere as a clear signature of the symmetry of the initial state, as seen from (8) and (9). Furthermore, the differential emission probability vanishes in the \(y = 0\) plane. In contrast, for the case of azimuthal symmetric initial (and final) states, constructed as \((|1, 0\rangle \mp i|0, 1\rangle)/\sqrt{2}\), the emission dependence of the azimuthal angle disappears and we are left with a cone-like structure. Note that in both cases the angular distribution of the spectrum in the polar direction is very narrow.

In the lower part of figure 1, we plot the transition rate from \((1, 0) \to (0, 0)\) as function of lateral confinement and for two vertical confinement strengths. Bearing in mind the auxiliary function \(G^{q}_{\omega_0, \omega_z}(\theta)\) of (8) and (9) the rate is expectedly seen to have a very sharp decay of more than 10 orders of magnitude over a confinement range from 0 to 5 meV. In addition, we note the exponential dependence on the vertical confinement and the opposite lifetime behaviour when increasing the vertical versus the lateral confinement. Furthermore, the piezoelectric part is seen to dominate at the smallest confinement strength, as expected from (5). Finally, we remark that these results are in overall agreement with the results in [19] with one exception: scattering minima, i.e. spike-like structures in the transition rate, are not present with the harmonic confinement. The origin of this discrepancy can be traced back to the treatment of the vertical degree of freedom. By introducing steep anharmonic terms away from the equilibrium, i.e. an anharmonic potential of the form

$$V(z) = \left( \frac{1}{2} m^*\omega_z^2 - 3\hbar\omega_z \frac{b}{z_0^2} \right) z^2 + 2m^*\omega_z^2b \left( \frac{z^4}{4z_0^4} + b \frac{z^6}{z_0^6} \right),$$  \hspace{1cm} (11)

with \(b = 1/4\) and \(z_0 = 10.35\) nm, we obtain a similar inverse lifetime as compared with the harmonic case, see figure 1 (lower panel). The appearance of the spikes at certain values of \(\hbar\omega_0\) can be attributed to the anharmonic terms in the potential. The dominant phonon emission angles, in the case of a harmonic potential, are shown in figure 1 (upper panel). By inspection, the corresponding results in the anharmonic case are seen to deviate very slightly from the harmonic results.

3. Spin-conserving two-electron emission

Motivated by the simple description of the single-electron problem, we now generalize to the two-electron case. The Hamiltonian of two interacting electrons within the same parabolic
Figure 1. Upper panels: most probable polar angle of phonon emission for the $(n_x, n_y) = (1, 0) \rightarrow (0, 0)$ transition in a single-electron harmonically confined quantum dot when varying the confinement strength $\hbar \omega_0$. Full line: only the deformation potential is taken into account, cf (10). Red circles: the same result when all three couplings are accounted for. In the left column of the inset, the one-electron densities of an initial aligned versus a circular state for $\hbar \omega_0 = 0.2 \text{ meV}$ and $\hbar \omega_z = 31 \text{ meV} (L_z = 12.1 \text{ nm})$ are shown. The corresponding angular spectra are displayed in the right column, with the most probable emission angle given by the angle between the peak emission direction and the $z$-axis. Lower panel: transition rates from the first excited level as a function of planar confinement strength $\hbar \omega_0$ for two vertical confinements; $\hbar \omega_z = 20 \text{ meV} (L_z = 15.1 \text{ nm})$ (blue curves) and $\hbar \omega_z = 60 \text{ meV} (L_z = 8.7 \text{ nm})$ (thick red curves). Full, dash-dotted and dashed lines are contributions from the deformation potential and the longitudinal and transverse piezoelectric potentials, respectively. The thin black lines show the transition rate via the deformation potential in the case of anharmonic vertical confinement (see text for details).
as a sum of products of basic integrals, involving only one variable of the phonon interaction,
\[ H = h(r_{i1}) + h(r_{i2}) + \frac{e^2}{4\pi \varepsilon_0 |r_{i1} - r_{i2}|}, \]

where \( h_{i1,2} \) is the single-particle Hamiltonian (cf (1)), \( \varepsilon_0 \) is the permittivity, \( \varepsilon_r = 13.1 \) is the relative permittivity in GaAs and the symbol ‘\( \parallel \)’ is used to denote the in-plane coordinates.

Now, an approximate or numerical representation of the initial and the final state is necessary. In view of the intuitive and analytical results obtained for the single-electron wavefunction, we expand the two-electron wavefunction in a basis of one-electron states, and obtain eigenstates by diagonalizing the two-electron Hamiltonian. The initial (excited) and final (ground) states are then achieved as linear combinations of basis states, \( |\Psi_{1\parallel}\rangle = \sum_i a_{i|\parallel} |i\rangle \) and \( |\Psi_{1\parallel}\rangle = \sum_j a_{j|\parallel} |j\rangle \), where \( |i\rangle, |j\rangle \) denotes pairs of spin symmetrized two-electron wavefunctions. The spatial parts are one-electron eigenstates in both the \( x \)- and \( y \)-direction, symmetric or antisymmetric with respect to particle exchange depending on the total spin \( S \) being 0 or 1. The total wavefunction can then be expressed as
\[
\Psi_{F/1}(r_{i1}^{\parallel}, z_{1}, r_{i2}^{\parallel}, z_{2}) = \Psi_{F/1}(r_{i1}^{\parallel}, r_{i2}^{\parallel}) \left( \frac{m^+\omega_c}{\pi \hbar} \right)^{1/2} \exp \left[ -\frac{m^+\omega_c}{\hbar} (z_1^2 + z_2^2) \right].
\]

It is now possible to express the integrals over initial and final states, including the electron–phonon interaction, as a contribution from the planar diagonal states multiplied by the augmented perpendicular part
\[
\langle \Psi_{1\parallel} | \sum_{j=1,2} e^{-iqr_{j}} |\Psi_{1\parallel}\rangle = \left( \frac{m^+\omega_c}{\pi \hbar} \right) \int d^3r_1 \, d^3r_2 \times \Psi_{F}^{\parallel}(r_{i1}^{\parallel}, r_{i2}^{\parallel}) \left( \sum_{j=1,2} e^{-iqr_{j}} \right) \Psi_{1}(r_{i1}^{\parallel}, r_{i2}^{\parallel}) \exp \left[ -\frac{m^+\omega_c}{\hbar} (z_1^2 + z_2^2) \right]
\]
\[
= \int_{F}^\parallel(q_{x}, q_{y}) \exp \left( -\frac{\hbar q_z^2}{4m^+\omega_c} \right).
\]

The planar 4D integral takes the form
\[
\int_{F}^\parallel(q_{x}, q_{y}) = \int d^2r_1 \, d^2r_2 \Psi_{F}^{\parallel}(r_{i1}^{\parallel}, r_{i2}^{\parallel}) \left( \sum_{j=1,2} e^{-iqr_{j}} \right) \Psi_{1}(r_{i1}^{\parallel}, r_{i2}^{\parallel})
\]
\[
= 2 \int d^2r_1 \, d^2r_2 \Psi_{F}^{\parallel}(r_{i1}^{\parallel}, r_{i2}^{\parallel}) e^{-iqr_{1}} \Psi_{1}(r_{i1}^{\parallel}, r_{i2}^{\parallel}).
\]

As the eigenstates are also a sum of antisymmetric basis states, the integral can be written as a sum of products of basic integrals, involving only one variable of the phonon interaction,
\[
\int_{F}^\parallel(q_{x}, q_{y}) = \sum_{i,j,i',j'} a^*_{i,j} a_{i',j'} \delta_{ij,i'} e^{-iqr_{1}}
\]
\[
= 2 \sum_{i,j,i',j'} a^*_{i,j} a_{i',j'} \delta_{ij,i'}
\]
\[
= 2 \sum_{i,j,i',j'} a^*_{i,j} a_{i',j'} \delta_{ij,i'} \Phi_{m_{i},n_{j}} \Phi_{m_{i},n_{j}}.
\]
Here \( a_{i,j} \) are one-electron expansion coefficients, and in the last line, the \( n, ms \) now refer to quantum numbers of the basis states in the \( x \) and \( y \)-directions. The quantity \( \Phi \) becomes

\[
\Phi_{n,n'} = (\pm i)^n \frac{N!}{\sqrt{n! \cdot n'}} \left( q_y \sqrt{\frac{\hbar}{2m^*\varepsilon_0}} \right)^{2M+n} \exp \left(-\frac{\hbar q_y^2}{4m^*\varepsilon_0}\right) L_N^{2M+n}(\hbar q_y^2/2m^*\varepsilon_0),
\]

(17)

which is a well-known Fourier transform between two harmonic oscillator eigenstates. The factor \( \pm i \) stems from the sign of the exponent in the phonon coupling, \( L'_N \) denote the associated Laguerre polynomials, \( N = \min(n, n') \) and \( M = (|n - n'| - \eta)/2 \), where \( \eta = 1 \) if \( n + n' \) is odd and 0 otherwise. The final integral then appears as a general sum of powers of \( q_x, q_y \),

\[
f_{Fl}(q_x, q_y) = 2 \sum C_i \left( q_x \sqrt{\frac{\hbar}{2m^*\varepsilon_0}} \right)^{k_i} \left( q_y \sqrt{\frac{\hbar}{2m^*\varepsilon_0}} \right)^{l_i} e^{-\hbar q_y^2/4m^*\varepsilon_0},
\]

(18)

where \( k_i + l_i \) can be both even and odd integers, and the \( C_i \)'s are the expansion coefficients of (16) when rewritten as a single sum. As in the one-electron case the expression above is squared and the \( q \)-integration may be performed in spherical coordinates taking advantage of the delta-function. From this, it follows that the differential emission probability due to the deformation interaction is given by

\[
\frac{\text{d}w_{\text{DP}}}{\text{d}\Omega} = 4dG_{q_0,0,0}(\theta) \sum C_i q_0^{3+k_i+l_i} \left( \frac{\hbar}{2m^*\varepsilon_0} \right)^{(k_i+l_i)/2} \sin^{k_i+l_i} \theta \cos^{k_i} \phi \sin^{l_i} \phi,
\]

(19)

and the piezoelectric contributions correspondingly become

\[
\frac{\text{d}w_{\text{PZL}}}{\text{d}\Omega} = 4p_i G_{q_0,0,0}(\theta) \sum C_i \left( \frac{\hbar}{2m^*\varepsilon_0} \right)^{(k_i+l_i)/2} q_0^{1+k_i+l_i} \sin^{k_i+l_i+2} \theta \cos^{k_i+2} \phi \sin^{l_i+2} \phi
\]

\[
+ \sin^{2+k_i+l_i} \theta \cos^{2} \theta \cos^{k_i} \phi \sin^{l_i} \phi.
\]

(20)

The expansion coefficients of the squared sum, when written as a single sum, are then denoted by \( C_i \).

In essence, the two-electron expressions for the angular profiles become generalizations of the one-electron expressions (8) and (9), involving essentially sums over analytic components from each basis state. The rates and the characteristic emission angles will, however, be affected by the electron–electron interaction since the one-electron relation \( \Delta_E_{\text{PE}} = \hbar\omega_0 \) is no longer valid. Due to the electron–electron interaction the spectrum becomes more complex compared to what is the case for the single-electron dot. In the absence of the spin–orbit interaction the energy spectrum splits into singlet and triplet states, each with variable degeneracy.

In figure 2, we display the most probable angle of emission for parabolic quantum dots as a function of planar confinement \( \hbar\omega_0 \). The considered transition is from the first excited singlet state to the ground state with zero spin–orbit interaction. The states are obtained from diagonalization as described above. We note that the most probable angle of emission follows the same general trend as in the one-electron case. We also observe that for a given energy separation the maximum polar emission angles are slightly larger than the corresponding angles in the one-electron case and that they increase with decreasing vertical confinement.

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Figure 2. The most probable polar angle as a function of energy separation between the two lowest singlet states, for three perpendicular confinements, in the case of a two-electron quantum dot. Black asterisks: $\hbar \omega_z = 10 \text{ meV}$ ($L_z = 21.3 \text{ nm}$). Red circles: $\hbar \omega_z = 20 \text{ meV}$ ($L_z = 15.1 \text{ nm}$). Blue crosses: $\hbar \omega_z = 60 \text{ meV}$ ($L_z = 8.7 \text{ nm}$).

In figure 3, we display the lowest singlet and triplet states of a quantum dot with confinement strengths $\hbar \omega_y = 2.7 \text{ meV}$ and $\hbar \omega_x = 0.9 \hbar \omega_y$. In the lower left part of the figure, the single-electron densities of the three first excited states are shown and the corresponding phonon emission spectra are presented in the right column. As seen from the electron densities the first two excited states have a p-state character oriented along the $x$- and $y$-axis, respectively. The corresponding phonon spectra are seen to reflect the character of the initial (excited) states. The first excited state emits phonons predominantly in the $xz$-plane, while the second one does this in the $yz$-plane. The third excited state has a d-state character, though with a dominant electron density in the $x = 0$ plane. The corresponding emission spectrum gives a similar variation in azimuthal angle. When considering transitions between excited states the emitted spectrum becomes more complex, since both the initial and the final state may have a significant degree of azimuthal structure.

We note from numerical values, and in agreement with (8) and (9), that the most probable decay scenario from a highly excited two-electron dot is dominated by a series of (rapid) transitions down to the first excited state, rather than a direct transition down to the ground state. With targeted or random excitation of an ensemble of identical dots to higher excited states, which can be realized on a picosecond timescale [21], population inversion with a dominant number of dots in the first excited state is thus possible. From this state, stimulated emission of coherent phonons could take place.

4. Spin–orbit-mediated transitions

So far the electronic spin has been preserved in all transitions. In the following, we also allow for the states to change their spin during the interaction. To this end, we append two new parts.
Figure 3. Upper panels: the lowest singlet energy levels of a two-electron parabolic quantum dot with $\hbar\omega_y = 2.7$ meV and $\hbar\omega_x = 0.9\hbar\omega_y$ ($\hbar\omega_z = 40$ meV, i.e. $L_z = 10.7$ nm). The considered transitions are indicated by arrows. Lower panels: the left column shows the single-electron densities for each of the initial states (1), (2) and (3), and the corresponding phonon emission spectra are displayed in the right column. The axes in these panels, for (1), (2) and (3), extend to $(x, y, z) = 1.5 \times [(0.58, 0.18, 4.13) \times 10^4, (0.19, 0.41, 3.51) \times 10^3, (28.49, 13.76, 247.77)]$, respectively.

to the Hamiltonian: firstly, one part modelling the interaction with an external magnetic field $B$,

$$h_B = \frac{e^2}{8m^*} B^2 (x_j^2 + y_j^2) + \frac{e}{2m^*} B L_{zj} + g^* \frac{e}{2m_e} BS_{zj},$$  \hspace{1cm} (21)

and secondly, one part that enables spin–orbit interactions,

$$h_{SO} = \frac{\beta}{\hbar} (p_j^x \sigma_j^y - p_j^y \sigma_j^x).$$  \hspace{1cm} (22)
Here, $m_e$ is the (non-scaled) electronic mass, $g^* = -0.44$ is the effective $g$-factor, $\beta$ is the coupling strength of the linear Dresselhaus term, and $p_j = -i\hbar \partial/\partial r_j + A(r_j)$, where $A(x_j, y_j) = (-y_j, x_j)B/2$.

In the case of spin–orbit-mediated phonon emission, we consider the transition from the lowest triplet states, $|\tilde{T}_\pm\rangle$, to the ground (singlet) state. Note that the third state in the triplet, $|\tilde{T}_0\rangle$, does not couple directly to the ground state for symmetry reasons and will therefore not be considered. Transition rates have recently been measured [20] and shown to be in qualitative agreement with theory [22]. Here we point out that the corresponding phonon spectrum will display the details of the spin–orbit interaction at the most fundamental level as each of the triplet states will give signals in separated angular regions.

In figure 4, the upper left panel shows the well-known spectrum of the lowest singlet state and the triplet states as a function of magnetic field strength. The degeneracy in the triplet states is gradually lifted when approaching the anti-crossing at $B \sim 1.3$ T due to the Zeeman coupling and the dependence of the spin–orbit coupling on magnetic field strength. The average relaxation rate from this band of triplet states to the ground state is displayed in the panel to the right. Also shown are the contributions from the different electron–phonon couplings, as well as the contributions from $|\tilde{T}_+\rangle$ and $|\tilde{T}_-\rangle$. The contribution to the rate from the latter is seen to be stronger in the presence of the Dresselhaus coupling as compared to $|\tilde{T}_+\rangle$. At small magnetic field strengths the emission is seen to be dominated by the deformation potential and occurs, as shown in the lower left panel, with a large characteristic polar emission angle of about $30^\circ$. The situation reverses near the anti-crossing where the emission is close to or in the $xy$-plane. We note that the phonon spectra in both cases depend strongly on the initial state. For example, the emission from the $|\tilde{T}_-\rangle$ state has maximum azimuthal angle at $\phi \sim 45^\circ$, while the $|\tilde{T}_+\rangle$ state sends out the phonons essentially at $\phi \sim 90^\circ$. When varying the Dresselhaus parameter the azimuthal character of the phonon spectrum changes accordingly.

5. Conclusion and outlook

The study of angle-resolved phonon emission from quantum dots will require a new type of detectors that we, in the present work, term a ‘single-phonon counter’. The possibility of detecting such pulses seems not unrealistic since the mean free path of phonons in semiconductor systems can become as large as $10^{-2}$ m at low temperatures. The critical aspect of a working device in order to detect the emission will be the detector efficiency of phonons as they arrive at the surface of the material embedding the quantum dot. However, methods for detecting such waves in GaAs have been reported [23, 24]. By manufacturing a grid of two-electron dots the signal may also be magnified by orders of magnitude in intensity due to stimulated emission. Sub-surface gate-controlled quantum dots with a controllable number of electrons are at present produced by lithographic techniques, which imply that grids with a large number of dots can be produced. Nevertheless, the efficiency of the phonon detection will depend on the ability to fabricate nearly identical quantum dots. We also remark that grids of other quantum dot structures can be produced by a range of other methods such as epitaxial growth, nucleation and self-organization [25].

In this paper, we have investigated the decay characteristics of excited states of quantum dots containing one or two electrons. Our results regarding lifetimes and dominant mechanisms are based on an original numerical approach and found to be in overall agreement with earlier
Figure 4. Upper left panel: energy as a function of magnetic field strength for the four lowest states of a two-electron quantum dot, i.e. $|\tilde{S}\rangle$, $|\tilde{T}^+\rangle$, $|\tilde{T}_0\rangle$ and $|\tilde{T}^-\rangle$ from the bottom to the top. The arrows mark the spin-mixing phonon emission spectra calculated in the lower panels. Upper right panel: total relaxation rate (solid thick line) between the triplet states and the (singlet) ground state. Also shown are the contributions to the relaxation rate from the deformation potential (dash-dotted thin line) and the piezoelectric potential (dashed thin line), as well as the contributions from $|\tilde{T}^+\rangle$ (dashed thick line) and $|\tilde{T}^-\rangle$ (dash-dotted thick line). Lower panel: the left column shows phonon emission spectra from the uppermost triplet state, while the lower shows the spectra from the lowermost triplet state at $B = 0.28$ T (arrows to the left). Corresponding spectra at $B = 1.3$ T (arrows to the right) are shown in the right column. The Dresselhaus parameter is $\beta = 1$ meVÅ, and the confinements are $\hbar\omega_0 = 2.7$ meV and $\hbar\omega_z = 40$ meV ($L_z = 10.7$ nm). The axes shown in the lower panels for (1)–(4) extend to $(x, y, z) = 1.5 \times [(0.19, 0.19, 0.38), (0.27, 0.27, 0.62), (0.04, 0.04, 0.02), (0.40, 0.39, 0.14)]$, respectively.
works. We have shown here that the emitted phonons have very characteristic spatial profiles: acoustic phonons are emitted in a narrow range of polar angles, related intimately to the energy splitting of the initial and final states. Moreover, the angular propagation pattern of the outgoing phonons reflects the character of the initial excited quantum state. In this respect, the development of a ‘single-phonon counter’ would be important, as it could be an accurate device for classifying energy spectra and quantum structures in semiconductors.

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References

[1] Rose H J and Brink D M 1967 Rev. Mod. Phys. 39 306
[2] Buchalla G, Buras A J and Lautenbacher M E 1996 Rev. Mod. Phys. 68 1125
[3] Tanner G, Richter K and Rost J M 2000 Rev. Mod. Phys. 72 497
[4] Schöffer M S et al 2008 Science 320 920
[5] Meijer H and Polder D 1953 Physica 19 255
[6] Zook J D 1964 Phys. Rev. 136 A869
[7] Leburton J P, Pascual J and Sotomayor-Torres C M 1993 Phonons in Semiconductor Nanostructures (Dordrecht: Kluwer)
[8] Calvani P 2001 Optical Properties of Polarons (Bologna: Editrice Compositori)
[9] Stroscio M A and Dutta M 2001 Phonons in Nanostructures (Cambridge: Cambridge University Press)
[10] Devreese J T 2007 J. Phys.: Condens. Matter 19 255201
[11] Devreese J T and Alexandrov A S 2009 Rep. Prog. Phys. 72 066501
[12] Devreese J T, Fomin V M and Gladilin V N 2011 Exciton–Phonon Interaction in Quantum Dots (Encyclopedia of Nanoscience and Nanotechnology) vol 14 ed H S Nalwa (Valencia, CA: American Scientific Publishers) pp 1–18
[13] Petta J R, Johnson A C, Taylor J M, Laird E A, Yacoby A, Lukin M D, Marcus C M, Hanson M P and Gossard A C 2005 Science 309 2180
[14] Hanson R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2007 Rev. Mod. Phys. 79 1217
[15] Fujisawa T, Oosterkamp T H, van der Wiel W G, Broer B W, Aguado R, Tarucha S and Kouwenhoven L P 1998 Science 282 932
[16] Bockelmann U 1994 Phys. Rev. B 50 17271
[17] Climente J I, Bertoni A, Goldoni G and Molinari E 2006 Phys. Rev. B 74 035313
[18] Stavrou V N and Hu X 2005 Phys. Rev. B 72 075362
[19] Climente J I, Bertoni A, Goldoni G and Molinari E 2007 Phys. Rev. B 75 245330
[20] Meunier T, Vink I T, van Beveren L H W, Tielrooij K J, Hanson R, Koppens F H L, Tranitz H P, Wegscheider W, Kouwenhoven L P and Vandersypen L M K 2007 Phys. Rev. Lett. 98 126601
[21] Sælen L, Nepstad R, Degani I and Hansen J P 2008 Phys. Rev. Lett. 100 046805
[22] Climente J I, Bertoni A, Goldoni G, Rontani M and Molinari E 2007 Phys. Rev. B 76 085305
[23] Ramsbey M T, Szafranek I S G and Wolfe J P 1994 Phys. Rev. B 49 16427
[24] Pfeifer T, Dekorsy T, Kutt W A and Kurz H 1992 Appl. Phys. A 55 482
[25] Weaver J H and Antonov V N 2004 Surf. Sci. 557 1