Deterministic Teleportation and Universal Computation Without Particle Exchange

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Abstract

Teleportation is a cornerstone of quantum technologies, and has played a key role in the development of quantum information theory. Pushing the limits of teleportation is therefore of particular importance. Here, we apply a different aspect of quantumness to teleportation—namely exchange-free computation at a distance. The controlled-phase universal gate we propose, where no particles are exchanged between control and target, allows the full repertoire of quantum computation, including complete Bell detection among two remote parties, and is experimentally feasible. Our teleportation-with-a-twist, which we extend to telecloning, then requires no pre-shared entanglement nor classical communication between sender and receiver, with the teleported state gradually appearing at its destination.

I. INTRODUCTION

In the popular imagination, teleportation has come to refer to the process by which a body or an object is transported from one place to another without taking the actual journey. While a staple of science fiction, science by contrast seemed to rule it out based on the uncertainty principle, which placed a fundamental limit on the accuracy of measurement [15]. No wonder when in 1993 Bennett and colleagues proposed the first quantum teleportation protocol [5], it was soon recognised as a seminal moment in physics. Relying on the non-classical resource of pre-shared entanglement between the communicating parties, an unknown quantum state of a physical system is jointly measured by the sender with one part of an entangled pair, in such a way that allows its reconstruction at the remote entangled partner at the receiver, while leaving behind its physical constituents. Classical communication is typically required to complete this disembodied transport.

Not only has quantum teleportation become a backbone of quantum technologies such as quantum communication, quantum computing, and quantum networks, it has also played a crucial role in the development of formal quantum information theory. As such, pushing the limits of quantum teleportation is of significant importance, which is what we intend to do here by invoking yet another aspect of quantumness: exchange-free computation at a distance. While Gedanken, or thought experiments have historically played a crucial role...
conceptual role in physics (with the EPR proposal for instance being famously conceived as such) we try to go beyond theory to proposing feasible demonstration.

In exchange-free communication, also known as counterfactual communication, a classical message is sent by means of quantum processes without the communicating parties exchanging any particles. (We use the term exchange-free in place of counterfactual since the term counterfactual has historically been used differently in the literature. Moreover, the term exchange-free more accurately describes the protocol, namely information being sent without exchange of particles.) With its roots in the phenomena of interaction-free measurement and the quantum Zeno effect \([8, 12, 13, 16, 17, 19, 27]\), the first such deterministic protocol was proposed by Salih et al \([32]\), before being experimentally demonstrated by Pan and colleagues \([7]\). While once controversial, the once-heated debate over whether exchange-free communication was permitted by the laws of physics (for both bit values) seems now to be resolving; Nature does allow exchange-free communication, and consequently computation at a distance \([3, 9, 11, 29, 33, 34, 36]\).

This counterfactual communication was generalised to sending quantum information exchange-free for the first time in the Salih14 protocol \([27, 28]\), also known as counterportation, proposing an exchange-free quantum CNOT gate as a new computing primitive. The exchange-free CNOT was later employed by Zaman et al to propose exchange-free Bell analysis, albeit with a 50% theoretical efficiency limit \([41]\).

By contrast, the controlled \(\hat{R}_z\)-rotation we propose here, based on the above-mentioned CNOT gate, is universal and has no theoretical limit on efficiency. We then combine quantum teleportation with exchange-free computation at a distance to propose deterministic exchange-free teleportation. The core of this gate is set up by the entangling operation enabled by a one-dimensional cavity atom system. The ground state of an atom in a cavity can be put into a superposition of being on-resonant (a zero) and reflecting, or off-resonant (a one) and transmitting, a photon \([14, 26]\). We then construct a counterfactual way of probing whether Bob is blocking/not blocking (transmitting/reflecting) using the standard counterfactual communication-style protocol.
FIG. 1. Our setup for an experimentally feasible, exchange-free controlled-$\hat{R}_z$, universal gate. This is based on Salih’s exchange-free CNOT gate which has Bob enacting a superposition of blocking and not blocking the communication channel by means of a trapped atom [27, 28, 30]. With the addition of a phase-shift plate applying a $(\theta + \pi)/2$ rotation, switchable mirror SM0, a $\pi/2$ half wave plate that flips polarisation, polarising beamsplitter PBS1, and an optical delay loop, the chained quantum Zeno effect unit becomes the basis of our controlled phase-rotation universal gate, entangling the states of Alice’s photonic qubit and Bob’s trapped atom qubit.

II. RESULTS

We first go through the chained quantum Zeno effect (CQZE) unit, as given in Fig. 1. This is based on Salih’s exchange-free CNOT gate, which has Bob enacting a superposition of blocking and not blocking his side of the communication channel [27, 28, 30]. The switchable mirror, SM1, is first switched off to allow the photon into the outer interferometer, before
being switched on again. The switchable polarisation rotator, SPR1, rotates the photon’s polarisation from $H$ to $V$, by a small angle $\frac{\pi}{2M}$:

$$|H\rangle \rightarrow \cos \frac{\pi}{2M}|H\rangle + \sin \frac{\pi}{2M}|V\rangle$$

$$|V\rangle \rightarrow \cos \frac{\pi}{2M}|V\rangle - \sin \frac{\pi}{2M}|H\rangle$$  \hspace{1cm} (1)

The polarising beam-splitter, PBS2, passes the $H$-polarised component towards the mirror below it, while reflecting the small $V$-polarised component towards the inner interferometer. The switchable mirror, SM2, is then switched off to allow the $V$-polarised component into the inner interferometer, before being switched on again. The switchable polarisation rotator, SPR2, rotates the $V$-polarised component by a small angle $\frac{\pi}{2N}$:

$$|V\rangle \rightarrow \cos \frac{\pi}{2N}|V\rangle - \sin \frac{\pi}{2N}|H\rangle$$  \hspace{1cm} (2)

The polarising beamsplitter, PBS3, then reflects the $V$-polarised component up, towards a mirror while passing the $H$-polarised component towards Bob’s trapped atom, which is in a superposition, $\alpha |0\rangle + \beta |1\rangle$, of reflecting back any photon, and transmitting it through to the loss detector. If the atom reflects (is in state $|0\rangle$), the $H$-polarised component survives—if the atom transmits ($|1\rangle$), the component is lost. This means the overall action of the inner interferometer, assuming the photon is not lost to Bob’s detector $D_B$, can be described as

$$|V\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow$$

$$\alpha(\cos \frac{\pi}{2N}|V\rangle - \sin \frac{\pi}{2N}|H\rangle)|0\rangle + \beta \cos \frac{\pi}{2N}|V\rangle|1\rangle$$  \hspace{1cm} (3)

This represents one inner cycle. The photonic superposition has now been brought back together by PBS3 towards SM2. After $N$ such cycles the state is,

$$|V\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha |H\rangle |0\rangle + \beta \cos^N \frac{\pi}{2N}|V\rangle|1\rangle$$  \hspace{1cm} (4)

The switchable mirror, SM2, is then switched off to let the photonic component inside the inner interferometer out. Since for large $N$, $\cos^N \frac{\pi}{2N} \rightarrow 1$, the state becomes,

$$|V\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha |H\rangle |0\rangle + \beta |V\rangle|1\rangle$$  \hspace{1cm} (5)
Similarly, the first outer cycle, starting with the photon at SM1, assuming the photon is neither lost to Alice’s detector $D_A$, nor to Bob’s $D_B$ inside the inner interferometer, implements

$$|H\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow\alpha \cos\frac{\pi}{2M} |H\rangle |0\rangle + \beta (\cos\frac{\pi}{2M} |H\rangle + \sin\frac{\pi}{2M} |V\rangle ) |1\rangle$$  \hspace{1cm} (6)

This represents one outer cycle, containing $N$ inner cycles. The photonic superposition has now been brought back together by PBS2 towards SM1. After $M$ such cycles, the state is

$$|H\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha \cos M \frac{\pi}{2M} |H\rangle |0\rangle + \beta |V\rangle |1\rangle$$  \hspace{1cm} (7)

Since for large $M$, $\cos M \frac{\pi}{2M}$ approaches 1, this goes to,

$$|H\rangle (\alpha|0\rangle + \beta|1\rangle) \rightarrow \alpha |H\rangle |0\rangle + \beta |V\rangle |1\rangle$$  \hspace{1cm} (8)

The switchable mirror SM1 is now switched off to let the photon out. Crucially, this last equation describes the action of a quantum CNOT gate with Bob’s as the control qubit, acting on Alice’s $H$-polarised photon.

We now explain the rest of the setup, which uses the CQZE unit to implement a universal, general-input controlled-$\tilde{R}_z$ rotation gate (Fig.1). We begin with a superposition state at Alice, $a|V\rangle + b|H\rangle$. This is split at PBS1, with the $H$-polarised component going into an optical loop, and the $V$-polarised component going through a $\pi/2$ half wave plate flipping its polarisation to $H$, before being admitted into the CQZE unit by turning off the switchable mirror SM0, before turning it on again. Upon exiting the CQZE unit, it is reflected back by SM0, having a phase of $\theta + \pi$ if $V$-polarised ($0$ if $H$) applied to it by the phase shifter, before going through another run of the CQZE unit. This always produces an $H$-polarised state, as noted in [18]. Note that the $\pi$ term in the phase shifter is a correction term. The photonic component now exits through SM0, which is switched off, before being flipping back to $V$-polarisation at the $\pi/2$ half wave plate, having acquired a $\theta$ phase shift. It then recombines with the $H$-polarised component at PBS1.
Given the initial state of the overall system is

\[ (a |V \rangle + b |H \rangle) \otimes (\alpha |0 \rangle + \beta |1 \rangle) \] (9)

and that only the initially V-polarised component “interacts” with the trapped atom, we get the final state

\[ a |V \rangle (\alpha |0 \rangle + \beta e^{i\theta} |1 \rangle) + b |H \rangle (\alpha |0 \rangle + \beta |1 \rangle) \] (10)
This is an entangled state between Alice’s polarisation qubit and Bob’s trapped ion qubit: a controlled-phase rotation, with Alice’s as the control qubit and Bob’s as the target. Due to the symmetry of control and target qubits for this type of rotation, it can also be factorised as

\[
\alpha |0\rangle (a |V\rangle + b |H\rangle) + \beta |1\rangle (ae^{i\theta} |V\rangle + b |H\rangle)
\]  

the same controlled-phase rotation expressed differently, now with Bob’s as the control qubit and Alice’s as the target. Taking the special case when \(\theta = \pi\), we get a controlled-Z gate.

On universality, our exchange-free controlled-\(\hat{R}_z\), as a two-qubit gate, allows efficient implementation of any quantum circuit when combined with local operations. But there’s another sense in which it is universal. As explained later, this gate can be operated differently, allowing one party with classical action to enact any desired operation on a second party’s remote photonic qubit, exchange-free. This classical action can even control a two-qubit gate at the second party, as shown in [31]. Our controlled-\(\hat{R}_z\) gate can therefore be thought of as a universal set in its own right.

Bob needs a way to implement a superposition of reflecting, bit “0”, and blocking, bit “1”, Alice’s photon. There are many ways to go about this; however, recent breakthroughs in trapped atoms inside optical cavities [26], such as the demonstration of light-matter quantum logic gates [25, 35], make trapped atoms an obvious choice.

Bob’s qubit is a single \(^{87}\text{Rb}\) atom trapped inside a high-finesse optical resonator by a three-dimensional optical lattice [24, 25]. Depending on which of its two internal ground states the \(^{87}\text{Rb}\) atom is in, a photon impinging on the cavity in Fig. 1 will either be reflected as a result of strong coupling, or otherwise enter the cavity on its way towards detector \(D_B\). For this, it needs to have mirror reflectivities such that a photon entering the cavity exists towards detector \(D_B\), similar to [20]. By placing the \(^{87}\text{Rb}\) in a superposition of its two ground states, by means of Raman transitions applied through a pair of Raman lasers, Bob implements the desired superposition of reflecting Alice’s photon back and blocking it. Note that coherence time for such a system is of \(O(10^{-4}s)\) [24], with longer times possible. Therefore, if the protocol is completed within \(O(10^{-5}s)\), lower-bounded by the \(O(10^{-9}s)\) switching speed of switchable components, then decoherence effects can be ignored.

We now move to an exchange-free implementation of teleportation. This is based on the quantum teleportation first devised by Bennett et al [5], but recast such that there is no need for previously-shared entanglement between Alice and Bob, nor classical communication.
between then. Our teleportation scheme is shown in Fig.2.

In this protocol, we have a photon-polarisation qubit at Alice, and an entangled pair of qubits, one trapped-atom and the other photon-polarisation, at Bob. Alice’s qubit is instantiated in the state to be teleported, e.g. by a third party, while Bob’s two modes are in the maximally entangled state

\[
\frac{|H\rangle |0\rangle + |V\rangle |1\rangle}{\sqrt{2}}
\]  

(12)

(which can be created by Bob sending an \(H\)-polarised photon into the cavity, when it is in the superposition \(|0\rangle + |1\rangle\)/\(\sqrt{2}\), and then recombining the photon’s components from the transmitted and reflected outputs at a polarising beamsplitter, after applying a \(H \rightarrow V\) rotation on the path from the transmitted side).

To enact teleportation, Bob first applies a Hadamard gate to his trapped-atom qubit, before Alice applies an exchange-free controlled-\(Z\) gate, with her photonic qubit as the control and and Bob’s trapped-atom qubit as the target. Bob and Alice then apply Hadamard gates onto their respective qubits, before measuring the states in the computational basis for Bob, and in the \(H/V\) basis for Alice, together performing a complete Bell measurement. Bob then either flips or doesn’t flip the polarisation of his photonic qubit based on the classical measurement outcome of his trapped-atom qubit. Alice then, based on the classical measurement outcome of her qubit, either performs an exchange-free controlled-\(Z\) on Bob’s photonic qubit with her control set to |1\rangle by blocking both runs, or else sets her control to |0\rangle by not blocking both runs. These last two steps by Bob and Alice respectively act as the feed-forward step of teleportation (which next-generation trapped atoms are expected to allow) leaving Bob’s photonic qubit in the state of Alice’s original qubit.

III. DISCUSSION

Our exchange-free protocol bears all the hallmarks of teleportation as given by Pirandola et al [23]. Alice’s input state is unknown to her, and can be provided by a third party who also verifies the teleported state at Bob. The protocol allows complete Bell detection, which in our case is jointly carried out by Alice and Bob exchange-free. The protocol allows the possibility of real-time correction on Bob’s photonic qubit. Lastly, even for the smallest number of cycles, achievable fidelity for our protocol exceeds the 2/3 limit of
FIG. 3. Average fidelity of our exchange-free teleportation protocol, shown as a function of the number of outer and inner cycles, M and N. This is for an imperfect trapped-atom at Bob that fails to reflect an incident photon 34% of the time when it should reflect, and fails to block the photon 8% of the time when it should block. Fidelity is averaged over 100 points evenly distributed on the Bloch sphere of possible states Alice could send.

“classical teleportation”, which comes from the no-cloning theorem [40]. Fig.3 gives the average fidelity $F(\theta, \phi)$, where

$$F(\theta, \phi) = \langle \Psi_{in}\mid \Psi_{out}\rangle\langle \Psi_{out}\mid \Psi_{in}\rangle$$

and $\Psi_{in}$ and $\Psi_{out}$ are the input and output states of the protocol, $\theta$ and $\phi$ parameterise the input state’s Bloch sphere (azimuthal and radial angle respectively), and the average fidelity is $F(\theta, \phi)$ averaged over $\theta$ and $\phi$ [22]. The average efficiency of the protocol is 30% for a number of cycles $M$ equal to 10 and $N$ equal to 100, but improves for larger numbers of cycles.
FIG. 4. An entanglement diagram for exchange-free telecloning. The diagram shows the initial entangled state between port-qubit P, copy-qubits C_q, and ancilla-qubits A_{q-1}, all at Bob(s). Also shown is the Bell Measurement done on Bob’s port qubit P and Alice’s initial state qubit X. The thick black lines mark entanglement, while the red dashed box indicates an exchange-free Bell measurement. This forces the system into one of four states. Alice applies suitable exchange-free controlled-rotations (Pauli operations) to recover the approximate copies at Bob.

Speaking of cloning, since quantum telecloning combines approximate cloning with teleportation to transport multiple approximate copies of a state, one would think that our exchange-free teleportation protocol might allow telecloning to be carried out exchange-free. In fact the telecloning scheme of Murao et al [21], which employs a Bell measurement, along with local operations at the receiver based on the Bell detections, can be made exchange-free in a similar manner to how we made teleportation exchange-free. Their scheme starts with an already prepared multipartite entangled state [21], which for our purposes we take to be located at Bob, with one of the entangled qubits in the form of say a trapped-atom, and the output qubits where the approximate copies appear, all photonic. Alice and Bob jointly perform an exchange-free Bell measurement between Alice’s photonic input qubit, and Bob’s trapped-atom ‘port’ qubit, as we show in Fig.4. Based on the classical outcomes of the Bell measurement, Alice applies suitable exchange-free controlled-rotations (Pauli op-
erations) to recover the approximate copies at Bob. The fidelity of these copies is limited by the no-cloning theorem to
\[ \gamma = \frac{2q + 1}{3q} \] (14)
where \( q \) is the number of approximate copies of our state that we want to send. For the protocol, when the trapped-atom interaction is ideal, we always reach this limit of fidelity (5/6 for two copies) even for the smallest number of cycles. In Fig.5, we give the fidelity for an imperfect trapped-atom at Bob that fails to reflect an incident photon 34% of the time when it should reflect, and fails to block the photon 8% of the time when it should block, for different values of \( M \) and \( N \). Average efficiency is 14% for \( M \) equal to 10 and \( N \) equal to 100, but improves for larger numbers of cycles.

An interesting modification of Salih et al.’s 2013 protocol was recently proposed by Aharonov and Vaidman, satisfying their criterion, based on weak trace, for exchange-free communication [3]. Following Salih’s 2018 paper on counterportation [30], we now show how to implement it in our protocol. In the CQZE module, after applying SPR2 inside the inner interferometer for the \( N \)th cycle, Alice now makes a measurement by blocking the entrance to channel leading to Bob. (She may alternatively flip the polarisation and use a PBS to direct the photonic component away from Bob.) Instead of switching SM2 off, it is kept turned on for a duration corresponding to \( N \) more inner cycles, after which SM2 is switched off as before. One has to compensate for the added time by means of optical delays. The idea here is that, for the case of Bob not blocking, any lingering \( V \) component inside the inner interferometer after \( N \) inner cycles (because of weak measurement or otherwise) will be rotated towards \( H \) over the extra \( N \) inner cycles. This has the effect that, at least as a first order approximation, any weak trace in the channel leading to Bob is made negligibly small.

While alternative proposals have been given for counterfactual communication by Arvidsson-Shukur et al. [4-6], the protocols’ definition of counterfactuality have been the subject of debate [11, 38, 39]. Their adoption of Fisher Information as a tool for analysing counterfactuality is interesting nonetheless.

As Vaidman points out in [37], Salih’s 2014 protocol—also known as counterportation [30]—achieves the same end goal of Bennett et al.’s (1993) teleportation [5], namely the disembodied transport of an unknown quantum state, albeit over a large number of protocol cycles. It is an entirely different protocol though, as can be seen from their respective
FIG. 5. Average fidelity for exchange-free telecloning using the exchange-free controlled-Z gate described above, plotted for different numbers of outer \((M)\) and inner \((N)\) cycles. Fidelity is calculated for an imperfect trapped-atom at Bob that fails to reflect an incident photon 34\% of the time when it should reflect, and fails to block the photon 8\% of the time when it should block. Fidelity is averaged over 100 points evenly distributed on the Bloch sphere of possible states Alice could send.

quantum-circuit diagrams. Our current protocol, by contrast to counterportation, is directly based on the 1993 teleportation protocol, but implemented using our universal, newly proposed counterfactual Z-gate, with the aim of exploring the foundations of this most central of quantum information protocols.

Interestingly, Aharonov et al. have recently shown that counterfactual processes such as the ones presented here conserve modular angular momentum, much like the quantum Cheshire Cat effect [1, 2].
In our recent paper, *Exchange-Free Computation on an Unknown Qubit at a Distance*, we give a protocol that allows Bob to implement any phase on Alice’s qubit, exchange-free [31]. This then forms the basis of a device that we called a phase unit, allowing Bob to apply any arbitrary single-qubit unitary to the qubit, exchange-free. However, an issue that phase unit displayed was that the time the photon exited the device was correlated with the phase applied by Bob. While we provided a way for Bob to undo this time-binning after the fact, it is generally desirable to remove it altogether.

By adapting the controlled phase-rotation above, a phase unit can be constructed that doesn’t exhibit this time-binning. We use the set-up in Fig.1, but instead with a classical Bob either blocking or not-blocking, and with SM0 keeping Alice’s photon in the device for 2L runs (rather than 2). Bob sets $\theta$ to $\pi/L$, blocking for $2k$ of the runs and not blocking for $2(L - k)$, in units of 2 runs where he either blocks for both or does not block for both. This allows Bob to set a phase on Alice’s photon of $2\pi k/L$. We place three of these devices in series, interspersed with a $-\pi/4$-aligned Quarter Wave Plate, $\hat{U}_{QWP}\hat{R}_x(-\pi/2)$, and its adjoint, $\hat{U}_{QWP}^\dagger\hat{R}_x(\pi/2)$, to create a chained-\(\hat{R}_z\hat{R}_x\hat{R}_z\) a set of rotations, into which any single qubit unitary can be decomposed. Bob can thus apply any arbitrary single-qubit unitary to Alice’s qubit—both exchange-free, and without time-binning. This also, as we show in [31], allows us to classically control of a universal two-qubit gate, which enables Bob to directly enact in principle any desired algorithm on a remote Alice’s programmable quantum circuit.

In conclusion, we have shown how the chained quantum Zeno effect can be employed to construct an experimentally feasible, exchange-free controlled-$\hat{R}_z$ operation, which is not only a universal gate, but can be considered a universal set. This allowed us to propose a protocol for deterministic teleportation of an unknown quantum state between Alice and Bob, without exchanging particles. The fact that the multiple cycles cause teleportation to happen gradually, in slow-motion so to speak, as opposed to standard quantum teleportation where the teleported state appears at once, is as interesting as it is surprising.

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[1] Yakir Aharonov, Eliahu Cohen, and Sandu Popescu. A dynamical quantum cheshire cat effect and implications for counterfactual communication. Nature Communications, 12(1):1–8, 2021. doi:10.1038/s41467-021-24933-9

[2] Yakir Aharonov and Daniel Rohrlich. What is nonlocal in counterfactual quantum communication? Physical Review Letters, 125(26):260401, 2020. doi:10.1103/PhysRevLett.125.260401

[3] Yakir Aharonov and Lev Vaidman. Modification of counterfactual communication protocols that eliminates weak particle traces. Phys. Rev. A, 99:010103, Jan 2019. doi:10.1103/PhysRevA.99.010103

[4] D. R. M. Arvidsson-Shukur and C. H. W. Barnes. Quantum counterfactual communication without a weak trace. Phys. Rev. A, 94:062303, Dec 2016. doi:10.1103/PhysRevA.94.062303

[5] Charles H Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K Wootters. Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels. Physical review letters, 70(13):1895, 1993. doi:10.1103/PhysRevLett.70.1895

[6] I Alonso Calafell, T Strömberg, DRM Arvidsson-Shukur, LA Rozema, V Saggio, C Greganti, NC Harris, M Prabhu, J Carolan, M Hochberg, T. Baehr-Jones, D. Englund, C.H.W. Barnes,
and P. Walther. Trace-free counterfactual communication with a nanophotonic processor. *npj Quantum Information*, 5(1):61, 2019. doi:10.1038/s41534-019-0179-2.

[7] Yuan Cao, Yu-Huai Li, Zhu Cao, Juan Yin, Yu-Ao Chen, Hua-Lei Yin, Teng-Yun Chen, Xiongfeng Ma, Cheng-Zhi Peng, and Jian-Wei Pan. Direct counterfactual communication via quantum zeno effect. *Proceedings of the National Academy of Sciences*, 114(19):4920–4924, 2017. doi:10.1073/pnas.1614560114.

[8] Avshalom C. Elitzur and Lev Vaidman. Quantum mechanical interaction-free measurements. *Foundations of Physics*, 23(7):987–997, Jul 1993. doi:10.1007/BF00736012.

[9] Robert B. Griffiths. Particle path through a nested mach-zehnder interferometer. *Phys. Rev. A*, 94:032115, Sep 2016. doi:10.1103/PhysRevA.94.032115.

[10] Robert B. Griffiths. Reply to “comment on ‘particle path through a nested mach-zehnder interferometer’ “. *Phys. Rev. A*, 97:026102, Feb 2018. doi:10.1103/PhysRevA.97.026102.

[11] Jonte R Hance, James Ladyman, and John Rarity. How quantum is quantum counterfactual communication? *Foundations of Physics*, 51(1):12, Feb 2021. doi:10.1007/s10701-021-00412-5.

[12] Jonte R Hance and John Rarity. Counterfactual ghost imaging. *npj Quantum Information*, 7(1):1–7, 2021. doi:10.1038/s41534-021-00411-4.

[13] Onur Hosten, Matthew T Rakher, Julio T Barreiro, Nicholas A Peters, and Paul G Kwiat. Counterfactual quantum computation through quantum interrogation. *Nature*, 439(7079):949, 2006. doi:10.1038/nature04523.

[14] CY Hu, WJ Munro, JL O’Brien, and JG Rarity. Proposed entanglement beam splitter using a quantum-dot spin in a double-sided optical microcavity. *Physical Review B*, 80(20):205326, 2009. doi:10.1103/PhysRevB.80.205326.

[15] Earle H Kennard. Zur quantenmechanik einfacher bewegungstypen. *Zeitschrift für Physik*, 44(4-5):326–352, 1927. doi:10.1007/BF01391200.

[16] Paul Kwiat, Harald Weinfurter, Thomas Herzog, Anton Zeilinger, and Mark A. Kasevich. Interaction-free measurement. *Phys. Rev. Lett.*, 74:4763–4766, Jun 1995. doi:10.1103/PhysRevLett.74.4763.

[17] Paul G Kwiat, AG White, JR Mitchell, O Nairz, G Weihs, H Weinfurter, and A Zeilinger. High-efficiency quantum interrogation measurements via the quantum zeno effect. *Physical Review Letters*, 83(23):4725, 1999. doi:10.1103/PhysRevLett.83.4725.
[18] Zheng-Hong Li, M Al-Amri, Xi-Hua Yang, and M Suhail Zubairy. Counterfactual exchange of unknown quantum states. Physical Review A, 100(2):022110, 2019. doi:10.1103/PhysRevA.100.022110.

[19] Graeme Mitchison and Richard Jozsa. Counterfactual computation. Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 457(2009):1175–1193, 2001. doi:10.1098/rspa.2000.0714.

[20] Martin Mücke, Eden Figueroa, Joerg Bochmann, Carolin Hahn, Karim Murr, Stephan Ritter, Celso J. Villas-Boas, and Gerhard Rempe. Electromagnetically induced transparency with single atoms in a cavity. Nature, 465:755–758, 2010. doi:10.1038/nature09093.

[21] M Murao, D Jonathan, MB Plenio, and V Vedral. Quantum telecloning and multiparticle entanglement. Physical Review A, 59(1):156, 1999. doi:10.1103/PhysRevA.59.156.

[22] Sangchul Oh, Soonchil Lee, and Hai-woong Lee. Fidelity of quantum teleportation through noisy channels. Physical Review A, 66(2), Aug 2002. doi:10.1103/physreva.66.022316.

[23] Stefano Pirandola, Jens Eisert, Christian Weedbrook, Akira Furusawa, and Samuel L Braunstein. Advances in quantum teleportation. Nature photonics, 9(10):641–652, 2015. doi:10.1038/nphoton.2015.154.

[24] Andreas Reiserer. A controlled phase gate between a single atom and an optical photon. PhD thesis, Technische Universität München, 2014.

[25] Andreas Reiserer, Norbert Kalb, Gerhard Rempe, and Stephan Ritter. A quantum gate between a flying optical photon and a single trapped atom. Nature, 508:237–240, 2014. doi:10.1038/nature13177.

[26] Andreas Reiserer and Gerhard Rempe. Cavity-based quantum networks with single atoms and optical photons. Reviews of Modern Physics, 87(4):1379–1418, 12 2015. doi:10.1103/RevModPhys.87.1379.

[27] T. Rudolph. Better schemes for quantum interrogation in lossy experiments. Phys. Rev. Lett., 85:2925–2928, Oct 2000. doi:10.1103/PhysRevLett.85.2925.

Hatim Salih. Protocol for counterfactually transporting an unknown qubit. arXiv preprint arXiv:1404.2200, 2014. URL: https://arxiv.org/abs/1404.2200.

[28] Hatim Salih. Protocol for counterfactually transporting an unknown qubit. Frontiers in Physics, 3:94, 2016. doi:10.3389/FPHY.2015.00094.
[29] Hatim Salih. Comment on “particle path through a nested mach-zehnder interferometer”. *Phys. Rev. A*, 97:026101, Feb 2018. doi:10.1103/PhysRevA.97.026101

[30] Hatim Salih. From a quantum paradox to counterportation. *arXiv:1807.06586*, 2018. URL: https://arxiv.org/abs/1807.06586

[31] Hatim Salih, Jonte R Hance, Will McCutcheon, Terry Rudolph, and John Rarity. Exchange-free computation on an unknown qubit at a distance. *New Journal of Physics*, 23(1):013004, Jan 2021. doi:10.1088/1367-2630/abd3c4

[32] Hatim Salih, Zheng-Hong Li, Mohammad Al-Amri, and Muhammad Suhail Zubairy. Protocol for direct counterfactual quantum communication. *Phys. Rev. Lett.*, 110:170502, Apr 2013. doi:10.1103/PhysRevLett.110.170502

[33] Hatim Salih, Zheng-Hong Li, Mohammad Al-Amri, and Muhammad Suhail Zubairy. Salih et al. reply. *Phys. Rev. Lett.*, 112:208902, May 2014. doi:10.1103/PhysRevLett.112.208902

[34] Hatim Salih, Will McCutcheon, Jonte R. Hance, and John Rarity. Do the laws of physics prohibit counterfactual communication? *arXiv:1806.01257*, 2018. URL: https://arxiv.org/abs/1806.01257

[35] T. G. Tiecke, J. D. Thompson, N. P. de Leon, L. R. Liu, V. Vuletić, and M. D. Lukin. Nanophotonic quantum phase switch with a single atom. *Nature*, 508(7495):241–244, 4 2014. doi:10.1038/nature13188

[36] Lev Vaidman. Comment on “protocol for direct counterfactual quantum communication”. *Phys. Rev. Lett.*, 112:208901, May 2014. doi:10.1103/PhysRevLett.112.208901

[37] Lev Vaidman. “counterfactual” quantum protocols. *International Journal of Quantum Information*, 14(04):1640012, 2016. doi:10.1142/S0219749916400128

[38] Lev Vaidman. Analysis of counterfactuality of counterfactual communication protocols. *Phys. Rev. A*, 99:052127, May 2019. doi:10.1103/PhysRevA.99.052127

[39] Alon Wander, Eliahu Cohen, and Lev Vaidman. Three approaches for analyzing the counterfactuality of counterfactual protocols. *Physical Review A*, 104(1):012610, 2021. doi:10.1103/PhysRevA.104.012610

[40] William K Wootters and Wojciech H Zurek. A single quantum cannot be cloned. *Nature*, 299(5886):802–803, 1982. doi:10.1038/299802a0

[41] Fakhar Zaman, Youngmin Jeong, and Hyundong Shin. Counterfactual bell-state analysis. *Scientific reports*, 8(1):14641, 2018. doi:10.1038/s41598-018-32928-8