Unusual reflection of electromagnetic radiation from a stack of graphene layers at oblique incidence

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Abstract

We study the interaction of electromagnetic (EM) radiation with single-layer graphene and a stack of parallel graphene sheets at arbitrary angles of incidence. It is found that the behavior is qualitatively different for transverse magnetic (or p-polarized) and transverse electric (or s-polarized) waves. In particular, the absorbance of single-layer graphene attains a minimum (maximum) for the p (s)-polarization at the angle of total internal reflection when the light comes from a medium with a higher dielectric constant. In the case of equal dielectric constants of the media above and beneath graphene, for grazing incidence graphene is almost 100% transparent to p-polarized waves and acts as a tunable mirror for the s-polarization. These effects are enhanced for a stack of graphene sheets, so the system can work as a broad band polarizer. It is shown further that a periodic stack of graphene layers has the properties of a one-dimensional photonic crystal, with gaps (or stop bands) at certain frequencies. When an incident EM wave is reflected from this photonic crystal, the tunability of the graphene conductivity renders the possibility of controlling the gaps, and the structure can operate as a tunable spectral-selective mirror.

Keywords: metamaterial, graphene

(Some figures may appear in colour only in the online journal)

1. Introduction

Electromagnetic (EM) metamaterial engineering yields specific optical properties which do not exist in natural materials [1]. These properties include EM energy concentration in sub-wavelength regions and radiation guiding [2], enhanced absorption [3], reflection [4] and transmission [5], and color filtering [6]. An important and prominent example of metamaterials and their specific properties are photonic crystals (PCs)\(^1\), where the propagation of electromagnetic waves of certain frequencies, belonging to gaps (or stop bands) in the spectrum, can be prohibited or allowed in certain directions only [4]. Thus, the so-called three-cylinder structure [7] was the first experimental realization of a full photonic band-gap, where the propagation of electromagnetic waves is not possible in any direction. The photonic band-gap structure of PC resembles and appears in full analogy to the electronic band-gap structure in solid-state crystals.

In metamaterial engineering it is useful to implement some tools for adjusting the EM properties, thus achieving tunability. Tunable metamaterials allow for continuous variation of their properties through a certain external influence (for a review see, e.g. [8–10]). Among the possible instruments to achieve the dynamical tunability of PCs we can mention the optical beam intensity in a nonlinear material [11], electric field in ferroelectrics [12], applied voltage in liquid crystals [13, 14], magnetic field in

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\(^1\) Although photonic crystals are different from ‘classical’ metamaterials (devices with negative refractive index) since the wavelength of operation is comparable to their structural period, the term ‘metamaterial’ nowadays is applied to any artificial structure with designed material (in our case, optical) properties.
ferromagnets or ferrimagnets [12, 15], and mechanical force as a means for changing the PC period [16]. There are also possible tuning mechanisms in crystalline colloidal arrays of high refractive index particles [17], magnetic fluids [18] or superconductors [19, 20].

The two-dimensional carbon material graphene possesses a number of unique and extraordinary properties, such as high charge carrier mobility, an electronic energy spectrum without a gap between the conduction and valence bands, and frequency-independent absorption of EM radiation. The optical properties of graphene have been extensively studied both theoretically [21–34] and experimentally [35–42]. Since the carrier concentration in graphene (and, hence, its frequency-dependent conductivity) can be effectively tuned with the gate voltage, giving the possibility of creating superconductors [19, 20],

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dependence. The frequency-dependent (optical) conductivity of graphene is a sum of two contributions: (i) a Drude term describing intra-band processes, and (ii) a term taking into account inter-band transitions. At zero temperature the optical conductivity has a simple analytical expression [21, 22, 29, 30, 25]. The inter-band contribution has the form \( \sigma_I = \sigma'_I + i\sigma''_I \), where

\[
\sigma'_I = \sigma_0 \left(1 + \frac{1}{\pi} \arctan \frac{\hbar \omega - 2E_F}{\hbar \Gamma} \right) - \frac{1}{\pi} \arctan \frac{\hbar \omega + 2E_F}{\hbar \Gamma} ,
\]

and

\[
\sigma''_I = -\sigma_0 \frac{1}{2\pi} \ln \left( \frac{(2E_F + \hbar \omega)^2 + \hbar^2 \Gamma^2}{(2E_F - \hbar \omega)^2 + \hbar^2 \Gamma^2} \right),
\]

where \( \sigma_0 = \pi e^2/(2\hbar) \) is the so-called universal conductivity of graphene. The Drude conductivity term is

\[
\sigma_D = \sigma_0 \frac{4E_F}{\pi} \frac{1}{\hbar \Gamma - i\hbar \omega}.
\]

We can write \( \sigma_g = \sigma_0 f(\omega) \), where \( f(\omega) \) is a dimensionless function. In figure 2 we depict the Drude and inter-band contributions to the total optical conductivity of graphene.

It is evident that at low frequencies (left panel in figure 2) the Drude term significantly exceeds the inter-band term (both real and imaginary parts), while in the high-frequency range (right panel in figure 2) the inter-band term dominates. Moreover, in the vicinity of the threshold frequency, \( \omega = 2E_F/h \), the real part of the conductivity increases drastically and achieves the universal value, \( \sigma_0 \) (onset of inter-band transitions), while the imaginary part is minimal, negative and of the order of several universal conductivities in modulus. As a result, at low frequencies the presence of graphene at the interface between two dielectrics influences significantly the reflectance and the transmittance of the structure. This effect, due to the high value of the Drude conductivity, is clearly seen in figures 3 and 4(a)–(c), where the low-frequency region is characterized by the lower transmittance (see figures 3(a), (c), (e) and the respective insets for \( \omega = 0.01E_F \)), higher reflectance (figures 3(b), (d), (f) and the respective insets for \( \omega = 0.01E_F \)) and enhanced absorbance (figures 4(a)–(c)) for all parameter values. In fact, the transmittance and the reflectance are mainly determined by the real part of the conductivity, except when the imaginary part is large in modulus (at low frequencies and near \( \hbar \omega = 2E_F \)).

At normal incidence (\( \theta = 0 \)), we have (for any polarization):

\[
T_p = T_s = \sqrt{E_1 E_3} \left| \frac{2}{\sqrt{E_1} + \sqrt{E_3} + \pi \alpha f(\omega)} \right|^2 ,
\]

\[
R_p = R_s = \frac{E_2^2}{E_3^2} \left| \frac{2}{\sqrt{E_1} + \sqrt{E_3} + \pi \alpha f(\omega)} \right|^2 ,
\]
where $\alpha$ is the fine structure constant. For oblique incidence ($\theta \neq 0$), the dependences of the transmittance and the reflectance on $\omega$ are strongly affected by the relation between the dielectric permittivities of the substrate and the capping dielectric. Therefore, we considered all three possible situations: (i) $\epsilon_1 = \epsilon_2$ (figures 3(a) and (b)), (ii) $\epsilon_1 > \epsilon_2$ (figures 3(c) and (d)), and (iii) $\epsilon_1 < \epsilon_2$ (figures 3(e) and (f)). In the ‘symmetric’ case of $\epsilon_1 = \epsilon_2 = \epsilon$, the transmittance and the reflectance can be expressed by simple formulas,

$$
R_p = \frac{\pi \alpha f(\omega) \cos \theta / \sqrt{\epsilon}}{2 + \pi \alpha f(\omega) \cos \theta / \sqrt{\epsilon}}^2, \quad (20)
$$

$$
T_p = \frac{2}{2 + \pi \alpha f(\omega) \cos \theta / \sqrt{\epsilon}}^2, \quad (21)
$$

$$
R_s = \frac{\pi \alpha f(\omega) / \sqrt{\epsilon}}{2 \cos \theta + \pi \alpha f(\omega) / \sqrt{\epsilon}}^2, \quad (22)
$$

$$
T_s = \frac{2 \cos \theta}{2 \cos \theta + \pi \alpha f(\omega) / \sqrt{\epsilon}}^2. \quad (23)
$$

Note that the factor $4\pi \alpha \sigma_0 / c = \pi \alpha$ multiplying the dimensionless function $f(\omega)$, which represents the frequency dependence of the graphene conductivity, is a small number ($\approx 0.023$). Thus, unless the absolute value of $f(\omega)$ is large, the term related to graphene in equations (10)–(13) and, accordingly, in the above expressions for $T$ and $R$ is small. Therefore, the reflectance and the transmittance of the structure are close to the values defined by the usual Fresnel expressions, except for $\omega \to 0$ and $\omega \approx 2E_F / h$. In particular, the reflectance is proportional to $(\pi \alpha)^2$.

It should also be noticed that in equations (18)–(23) the effect of graphene is stronger for lower dielectric constants and is maximal for free standing graphene ($\epsilon_{1,2} = 1$). As $\theta$ increases (see figures 3(a) and (b) and the insets), the transmittance $T_s$ decreases and attains zero for $\theta = \pi / 2$, while the reflectance $R_s$ increases and tends to unity as $\theta \to \pi / 2$. In contrast, a p-polarized wave is ‘totally transmitted’ as $\theta \to \pi / 2$ ($R_p = 0, T_p \to 1$). With the electric field perpendicular to the graphene sheet (TM wave), no charge oscillations are induced at the interface and the EM field is not perturbed. Also, the low-frequency absorbance at grazing incidence is a decreasing function of the angle for both p- and s-polarized waves and the limit $\theta = \pi / 2$ corresponds to zero absorbance (see figure 4(a)). Note that the absorption is entirely related to graphene because the dielectrics are assumed dispersionless.

The situation is different when the dielectric constants of the substrate and capping dielectric are not equal, because in this case there would be reflection even in the absence of graphene. Here, at grazing incidence the reflectance (transmittance) is close to unity (zero) for both polarizations (see figures 3(c)–(f)), just like for a ‘normal’ interface between two dielectrics (without graphene). In case (ii), although the angular dependence of the low-frequency absorbance (figure 4(b)) is qualitatively similar to the ‘symmetric’ case (i), it is higher for TM waves (cf figures 4(b) and (a)), in contrast to case (i). The particularity of case (ii) ($\epsilon_1 > \epsilon_2$, e.g. uncovered graphene on a substrate) is that the angular dependence of the TM wave reflectance (see figure 3(d)) possesses a minimum at a certain $\theta$ close to the Brewster angle for the two dielectrics considered, $\theta_B = \arctg[(\epsilon_1/\epsilon_2)^{1/2}] [64]$. Owing to the non-zero imaginary part of the graphene conductivity, the TM wave reflectance at this angle is finite, while it would reach zero in the case without graphene. Therefore we call this the ‘quasi-Brewster’ angle, $\theta_{qB}$. It depends upon the frequency and the Fermi energy of graphene; this dependence is depicted in figure 4(d). At low frequencies, where the conductivity $\sigma_F$ is high owing to the Drude term, the quasi-Brewster angle of the structure exceeds the conventional Brewster angle $\theta_{B0}$ of the interface without graphene. This effect is more pronounced for higher values of the Fermi energy—compare the three curves in the plot.

Figure 2. Optical conductivity of uniform graphene: Drude (left) and inter-band (right) contributions. We assumed $E_F = 0.23$ eV and $\Gamma = 2.6$ meV. The solid (dashed) line stands for the real (imaginary) part of the conductivity.
Figure 3. Transmittance $T_{p,s}$ (left column) and reflectance $R_{p,s}$ (right column) of graphene clad by two semi-infinite dielectrics versus angle of incidence $\theta$ and frequency $\omega$. In all cases $\varepsilon_1 = 3.9$ (SiO$_2$) and $\Gamma = 2.6$ meV. Other parameters are: $\varepsilon_2 = 3.9$, $E_F = 0.157$ eV (upper row), $\varepsilon_2 = 1.0$, $E_F = 0.1$ eV (middle row), or $\varepsilon_2 = 11.9$ (Si), $E_F = 0.25$ eV (lower row). In each panel, the angular dependences for two fixed frequencies ($\hbar\omega = 0.01E_F$ and $0.1E_F$) are depicted in the insets.

When the frequency grows (and the Drude term in $\sigma_g$ becomes smaller) $\theta_{br}$ approaches the value of the conventional Brewster angle $\theta_{0br}$. However, at $\omega \sim 2E_F$ the quasi-Brewster angle jumps up because of the Fermi step in the real part of $\sigma_g$ (onset of inter-band transitions). The difference between $\theta_{br}$ and the conventional Brewster angle, $\theta_{0br}$, characteristic of a graphene-free interface between the same two dielectrics, can be used for visualization of graphene, which constitutes a considerable problem [65]. At low frequencies, $\theta_{br} - \theta_{0br}$ can easily reach detectable values of $\sim 5^\circ$.

In the last possible situation (iii), there is a critical angle of total internal reflection ($\approx 35^\circ$ for the parameters of figures 3(e) and (f)). Above this critical angle, $\theta_c$, the transmittance vanishes and the reflectance is close to unity (see figures 3(e) and (f)). Notice that the value of $\theta_c$ does not depend on the graphene parameters. In the case under
consideration, the low-frequency absorbance (figure 4(c)) exhibits the most interesting behavior: in the vicinity of the total internal reflection angle the absorbance of the p-polarized wave is almost zero, while that of the s-polarized wave reaches its maximum of $\approx 75\%$.

**3. Graphene multilayer photonic crystal**

Now we shall consider an external EM wave falling on a periodic multilayer structure (see figure 1(b)) consisting of an infinite number of parallel monolayer graphene sheets separated by dielectric slabs of thickness $d$; in practical terms a few graphene layers play the same role as an infinite number of them [66]. The geometry of the problem is similar to that considered in section 2, but with graphene layers (for which we shall assume the same Fermi energy) located at positions $z = md$, $m \in [0, \infty)$. Thus, the considered structure is a semi-infinite 1D PC, terminated by the graphene layer at $z = 0$. We shall still consider a capping dielectric (with arbitrary real $\varepsilon_2$) on top of it.

In order to find the reflectance of this PC, we notice that the structure of the electromagnetic field in the capping dielectric is the same as represented by equations (1) and (3) and (6) and (8) for p- and s-polarized waves, respectively. At the same time, the fields in the substrate should be considered separately in each layer between adjacent graphene sheets at planes $z = md$ and $(m + 1)d$. Namely, solutions of the Maxwell equations at spatial domain $md \leq z \leq (m + 1)d$ can be represented as

$$H_y^{(1)}(x, z) = \left\{ H_y^{(m)} \exp[i k_1 z (z - md)] + H_y^{(0)} \exp[-i k_1 z (z - md)] \right\} \exp(i k_s x),$$

$$E_x^{(1)}(x, z) = \left\{ \frac{k_1}{\varepsilon_1} H_y^{(m)} \exp[i k_1 z (z - md)] - H_y^{(0)} \exp[-i k_1 z (z - md)] \right\} \exp(i k_s x)$$

and

$$E_y^{(1)}(x, z) = \left\{ E_y^{(m)} \exp[i k_1 z (z - md)] + E_y^{(0)} \exp[-i k_1 z (z - md)] \right\} \exp(i k_s x).$$

Here $H_y^{(m)}$ are the amplitudes for forward (sign ‘+’) or backward (sign ‘−’) propagating TM waves. Correspondingly, $E_y^{(m)}$ represent the amplitudes of the TE waves. The amplitudes $H_y^{(m+1)}$ can be related to $H_y^{(m)}$ by matching boundary conditions at $z = (m + 1)d$ on graphene (similar to that used in section 2), namely:

$$\begin{pmatrix} H_y^{(m+1)} \\ H_x^{(m+1)} \end{pmatrix} = \tilde{M}_p \begin{pmatrix} H_y^{(m)} \\ H_x^{(m)} \end{pmatrix},$$

$$\tilde{M}_p = \begin{pmatrix} e^{ik_s d} \left[ 1 - \frac{2 \pi k_1 z}{\omega \varepsilon_1} \sigma_g \right] & e^{-ik_s d} \frac{2 \pi k_1 z}{\omega \varepsilon_1} \sigma_g \\ -e^{ik_s d} \frac{2 \pi k_1 z}{\omega \varepsilon_1} \sigma_g & e^{-ik_s d} \left[ 1 + \frac{2 \pi k_1 z}{\omega \varepsilon_1} \sigma_g \right] \end{pmatrix}.$$
Similarly, for the s-polarization, allowed bands, 1 for the surface mode, 2, 3, etc for bulk modes.

The Bloch wavevector \( \mathbf{q} \) use the Bloch theorem, which determines the proportionality since the considered structure is periodic.

\[
E^{(m+1)}_+ = \hat{M}_s \left( e^{i k_{1,z} d} \left[ 1 - \frac{2 \pi \omega}{c^2 k_{1,z} \sigma_g} \right] - e^{-i k_{1,z} d} \frac{2 \pi \omega}{c^2 k_{1,z} \sigma_g} \right),
\]

\[
\hat{M}_s = \begin{pmatrix} e^{i k_{1,z} d} & 0 \\ e^{i k_{1,z} d} & e^{-i k_{1,z} d} \end{pmatrix}.
\]  \( (29) \)

Since the considered structure is periodic, it is possible to use the Bloch theorem, which determines the proportionality between the field amplitudes in the adjacent periods through the Bloch wavevector \( \mathbf{q} \):

\[
H_{x}^{(m+1)} = \exp(i q d) H_x^{(m)},
\]

\[
E_{x}^{(m+1)} = \exp(i q d) E_{x}^{(m)}.
\]

After substitution of these relations into equations \( (28) \) and \( (29) \), the compatibility condition of the resulting linear equations requires:

\[
\text{Det} \left[ \hat{M}_p - \exp(i q d) \hat{I} \right] = 0,  \quad (30)
\]

\[
\text{Det} \left[ \hat{M}_s - \exp(i q d) \hat{I} \right] = 0,  \quad (31)
\]

2 The surface at \( z = 0 \), of course, breaks the translational symmetry. This symmetry breaking can give rise to local (surface) modes, known as Tamm states in the case of electronic structure of crystals (not to be confused with the Bloch-type TM surface wave discussed below!). However, the band structure of the spectrum is preserved and the surface EM mode can be excited only under special conditions, enhancing the in-plane wavevector \( k_{1,z} \).
The former is a ‘Bloch surface plasmon-polariton’, with the electric and magnetic fields strongly localized at the graphene sheets [49] but with a real Bloch wavevector. In the case of s-polarization such a surface mode does not exist and only bulk modes are present in the spectrum (see figures 6(b) and (c)).

Particular solutions of the dispersion relation for the p-polarization, equation (32), for \( q = \pi/d \) (so called ‘Bragg modes’) can be represented as

\[
k_{1,z} = \left( \frac{\omega^2}{c^2} \varepsilon_1 - k_x^2 \right)^{1/2} = (2n + 1) \pi/d, \quad n \in [0, \infty).
\]

For \( q = 0 \) we have:

\[
k_{1,z} = \left( \frac{\omega^2}{c^2} \varepsilon_1 - k_x^2 \right)^{1/2} = 2n' \pi/d, \quad n' \in [0, \infty).
\]

s-polarized Bragg modes (solutions of (33)) for \( q = \pi/d \) are exactly the same as (34), but for \( q = 0 \) they are similar to (35) (except that \( n' \neq 0 \)). The modes with Bragg wavevectors, (34) and (35), have nodes at graphene layers and, therefore, these solutions do not involve the graphene conductivity, \( \sigma_g \). As a matter of fact, these solutions correspond to \( H_{1+}^{(m)} = H_{1-}^{(m)} \) for the p-polarization and \( E_{1+}^{(m)} = -E_{1-}^{(m)} \) for the s-polarization. It implies zero in-plane components of the electric field in both cases, consequently, no electric current is induced in graphene sheets located at \( z = md \) (see equations (25) and (26)).

Secondly, in the case of p-polarization \( k_{1,z} = 0 \) is the solution that implies arbitrary \( H_{1+}^{(m)} \) and \( H_{1-}^{(m)} \), and, as a result, \( H_{1}^{(1)} \) independent of \( z \) as well as \( E_{1}^{(1)} \equiv 0 \). At the same time, for the s-polarization the solution \( k_{1,z} = 0 \) corresponds to a trivial solution of the Maxwell equations with zero electric and magnetic fields. For the p-polarization, the line \( k_{1,z} = 0 \) is crossed by another dispersion curve at the point \( k_z = \sqrt{4\alpha E_F/(\hbar c d)} \) (see figure 5(c)), where there is no gap between the surface and bulk mode bands. Below this point, the solution \( k_{1,z} = 0 \) corresponds to the top of the surface mode band, while above this \( k_z \) it corresponds to the bottom of the bulk mode band. Similarly, the upper bands depicted in figures 5(c) and 6(c) are delimited by the Bragg modes, (34) and (35).

At the same time, changing the graphene Fermi energy, \( E_F \), for example, by varying an external gate voltage, it is possible to tune the width of the gaps, as can be seen from figures 7(a) and (b) (for p- and s-polarizations, respectively). In particular, the gaps vanish when the Fermi level coincides with the Dirac point. At the same time, the waveguide modes defined by equations (34) and (35) remain unchanged because of their above-mentioned independence of the graphene conductivity.

In order to obtain the expression for the reflectance of an EM wave from the graphene multilayer stack, we notice that, by virtue of equations (30) and (33), the amplitudes \( H_{1+}^{(m)} \) and \( E_{1+}^{(m)} \) are related by:

\[
H_{1+}^{(m)} = \rho_p H_{1+}^{(m)}, \quad E_{1+}^{(m)} = -\rho_s E_{1+}^{(m)},
\]

where

\[
\rho_p = \frac{\exp(ik_{1,z}d) \left( e^{2\pi k_1 z} \sigma_g \right)^{2\pi k_1 z} \sigma_g}{\exp(-ik_{1,z}d) \left[ 1 + e^{2\pi k_1 z} \sigma_g \right] - \exp(iq d)},
\]

and

\[
\rho_s = \frac{\exp(-ik_{1,z}d) \left( e^{2\pi k_1 z} \sigma_g \right)^{2\pi k_1 z} \sigma_g}{\exp(ik_{1,z}d) \left[ 1 + e^{2\pi k_1 z} \sigma_g \right] - \exp(iq d)}.
\]
Notice that, when $\rho_p$ and $\rho_s$ is obtained from equations (32) and (33), respectively. It should be pointed out that, if the graphene conductivity is complex, so is the Bloch wavevector $q$ for $\rho_p$ and $\rho_s$, respectively. Thus, when $\omega$ and $k_x$ of the incident wave match one of the bands, the reflectance of the graphene multilayer photonic crystal resembles that of single-layer graphene (compare figures 8(a) and 9(a) with 3(b), as well as figures 8(b) and 9(b) with 3(d) and figures 8(c) and 9(c) with 3(f)).

An incident wave with $\omega$ and $k_x$ inside one of the allowed bands of the photonic crystal is (partially) transmitted into the structure. This effect is clearly seen in figures 8 and 9 for $p$- and $s$-polarized waves, respectively. Thus, when $\omega$ and $k_x$ of the incident wave match one of the bands, the reflectance of the graphene multilayer photonic crystal resembles that of single-layer graphene (compare figures 8(a) and 9(a) with 3(b), as well as figures 8(b) and 9(b) with 3(d) and figures 8(c) and 9(c) with 3(f)).

In contrast, incident EM waves with $\omega$ and $k_x$ belonging to the gaps of the PC band structure induce evanescent waves (characterized by imaginary Bloch wavevector $q$, in contrast with the PC surface mode with real $q$ and imaginary $k_{x,z}$), and are almost totally reflected from it. The graphene multilayer PC reflectance is considerably higher than that of the single-layer graphene heterostructure, and at certain frequencies can achieve unity (see panels (d)–(f) in figures 8 and 9).
Perhaps the most interesting effects take place when $\varepsilon_2 < \varepsilon_1$ (panels (b) and (e) in figures 8 and 9). As expected, in the vicinity of the Brewster angle of the interface without graphene ($\theta_{br}^0 \approx 63.124^\circ$), the s-polarization reflectance exceeds significantly that of p-polarized waves for all frequencies inside the band (compare dash-dotted lines in figures 8(e) and 9(e)), similar to the case of single graphene layer. However, it is not so for $\omega$ and $k_x$ inside the gaps. Here both polarizations exhibit an enhanced reflectance. Furthermore, we find some features specific to TM waves. As has been shown in section 2, the presence of graphene at the interface modifies the angle at which the reflectivity minimum in the p-polarization occurs and this quasi-Brewster angle ($\theta_{br}$) is frequency dependent (see figure 4(d)). What happens to the minimum reflectivity angle, $\theta_{min}$, when the wave is reflected from the graphene multilayer PC instead of the single interface? The answer follows from figure 10. When $\omega$ and $k_x$ belong to a band of allowed modes, $\theta_{min}$ oscillates around the conventional Brewster angle ($\theta_{br}$) is frequency dependent (see figure 4(d)). What happens to the minimum reflectivity angle, $\theta_{min}$, when the wave is reflected from the graphene multilayer PC instead of the single interface? The answer follows from figure 10. When $\omega$ and $k_x$ belong to a band of allowed modes, $\theta_{min}$ oscillates around the conventional Brewster angle ($\theta_{br}$), dashed horizontal line in the plot), except for very low frequencies ($\hbar \omega < 3$ meV), where the frequency dependence of the difference, $\theta_{min} - \theta_{br}$, resembles that for the single graphene layer structure (compare to figure 4(d)). The most striking feature in figure 10 is the divergence of $\theta_{min}$ for frequencies corresponding to the stop bands of the photonic crystal (compare to the middle column of figure 8).

The particularity of the situation $\varepsilon_2 > \varepsilon_1$ (see panels (c) and (f) in figures 8 and 9) is the possibility to excite the p-polarized surface mode. If the angle of incidence is below the critical one ($\theta_c \approx 35^\circ$), the excitation of bulk PC modes takes place, while for $\theta > \theta_c$, only the surface mode can be excited, as can be seen by the low-frequency minima in the reflectivity spectra (see figures 8(c) and (f)).

A similar spectral shape has been observed experimentally in [54]. One can say that the interface between the PC and the capping dielectric acts as an attenuated total internal reflection structure for single graphene layer, as described in [43]. It should be emphasized that the origin of the low-frequency minimum observed for s-polarized waves (figures 9(c) and (f)) is completely different. The former is a photonic crystal effect, while the latter exists also in the single-layer case (see figure 3(f)) and is unrelated to any PC surface mode.

The possibility of changing gap widths in the graphene multilayer PC spectrum by changing the Fermi level of graphene layers (see figure 7) has an important consequence, the reflectance of the PC can be dynamically varied through electrostatic gating, by changing the voltage applied to the graphene layers. This effect is depicted in figures 11(a) and (b) for p- and s-polarized waves, respectively. It can be used to design a tunable mirror. One has to choose the frequency of the incident wave inside one of the allowed bands for a
Figure 11. (a), (b) Reflectance versus $E_F$ and frequency for p- (a) and s-polarized (b) waves, for the angle of incidence $\theta = 30^\circ$.
(c) Reflectance versus Fermi level for p- and s-polarized waves, for $\hbar\omega = 9.5$ meV (subtracted from panels (a) and (b) along respective horizontal lines). Other parameters are the same as for the left column of figure 8.

Figure 12. Reflectance versus frequency for p- (a) and s-polarized (b) waves falling on a finite PC containing 5 (thick blue lines) or 20 (thin red lines) graphene layers. Other parameters are: $\varepsilon_1 = 3.9$, $\Gamma = 2.6$ meV, $\varepsilon_2 = 3.9$, $E_F = 0.157$ eV, $d = 40 \mu$m, $\theta = 30^\circ$ (panel (a)), or $\varepsilon_2 = 11.9$, $E_F = 0.25$ eV, $d = 4 \mu$m, $\theta = 40^\circ$ (panel (b)). In both panels dashed lines correspond to the case of an infinite number of graphene layers in a PC for the same parameters.

low Fermi energy, and inside the gap for a large Fermi energy. Then the reflectance of the structure can be varied in a broad range, as shown in figure 11(c). The dependence $R(E_F)$ can be made even more abrupt by using graphene layers with a smaller damping parameter $\Gamma$.

All the above results have been obtained for an infinite periodic stack of graphene layers. In reality, of course, PCs consist of a finite number ($N$) of layers. How does the value of $N$ affect the mode eigenfrequencies and the frequency dependence of the reflectance? As known from the band theory of crystalline solids, the eigenmode spectrum is quantized and corresponds to a discrete set of 'allowed' Bloch wavevectors, $q_m = (\pi/d)(m/(N + 1)); m = [1, N]$ obtained from the usual Born–von Karman boundary conditions. For $N \to \infty$, the Bloch wavevector varies in a quasicontinuous way within the interval $q \in [0, \pi/d]$ and equations (32) and (33) hold with a very high precision. For relatively small values of $N$, say, $N \sim 10$, the eigenmode band structure is washed away although the density of states retains a qualitative similarity to the case of $N \to \infty$. The ‘stop bands’ are broadened and correspond to a maximum reflectivity well below unity (see figure 12(b)), however, the latter increases rapidly with the number of layers, as known for periodically stratified media [64]. Already for $N = 20$, the reflectance for s-polarized waves is very similar to that for an infinite PC (compare dashed and thin solid lines in figure 12(b)).

Figure 12(a) shows the finite size effect on the frequency dependence of reflectance related to the surface mode for the p-polarization. No qualitative difference between the cases of $N = 5$ and $N \to \infty$ is seen (compare thick solid and dashed lines in figure 12(a)), which can be understood by the low dispersion of the surface-type PC mode with respect to the Bloch wavevector (see figure 5(b)). This mode is, in fact, a Bloch-type surface plasmon-polariton (SPP) excitation induced by the incident wave when the attenuated total reflection conditions are met [49]. The flatness of the $\omega(q)$ dependence for this PC surface mode originates from the
small overlap of the amplitudes of the SPP excitations in the different graphene layers.

4. Conclusions

In conclusion, there are several interesting effects related to the optical properties of graphene, which are revealed at oblique incidence. Some of them are expected already for a single graphene layer or just a few graphene layers. Under total internal reflection conditions at an interface between two dielectrics, the presence of graphene leads to EM energy absorption only for s-polarized waves. The absorbance attains its maximum exactly at the critical angle of incidence for the s-polarization (and the maximum value is higher when the graphene conductivity is large), while it vanishes for the p-polarization (figure 4(c)). The minimum reflectance of p-polarized waves occurs at a (frequency-dependent) quasi-Brewster angle that can differ by several degrees from the conventional Brewster angle for the same pair of dielectrics. Close to grazing incidence, graphene (when dielectric constants of substrate and capping layer are equal) is fully transparent to p-polarized waves and behaves like a mirror for the s-polarization. This effect can be used for polarization-selective guidance of EM radiation. We have shown that a periodic stack of equally spaced parallel layers of graphene has the properties of a 1D photonic crystal, with narrow stop bands that are nearly periodic in frequency. The PC properties are revealed also at oblique incidence. In particular, the stop bands correspond to singularities of the minimum p-polarized reflection angle calculated as a function of frequency, which is an effect of potential interest for optical switching. We investigated the finite PC size effect and found that about 20 periods are sufficient to get properties very close to those of an infinite PC. Finally, we should stress that the possibility of tuning of the gap (stop band) width by changing the graphene conductivity via electrostatic gating would allow the dynamical variation of the reflectance at specific selected frequencies.

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