The Application of Transfer Matrix Method in Acoustic Performance Analysis of Composite Materials and Acoustic Protection Design of Ship

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Abstract. This research introduced the transfer matrix method to calculate the acoustic performance of composite materials, which solving the problem of traditional methods existing in calculating acoustic performance of complex composite structure. Based on the impedance tube method and the reverberation chamber method, the experiment of measuring sound absorption and insulation parameters of typical composite materials was conducted. And then, the validity of theoretical calculation of the transfer matrix method was verified. In addition, under the application of the statistical energy method, the model simulation was carried out by taking a ship as an example, and the transfer matrix method in the noise protection design of the ship was discussed.

1. Introduction

Compared with traditional metal materials, composite materials have some advantages in acoustic performance, damping performance and designability. They have been so widely used in noise control of aerospace, underwater ships, automobile and other industries that it has great significance to study the acoustic properties of composite materials.

At present, there are quite a lot of theoretical researches and applications on the acoustic performance calculation of single-layer and double-layer composite materials, but the theoretical calculation of acoustic performance of multi-layer complex composite structures as a hot issue that has not been solved well. The acoustic performance of the composite material is affected by the important parameter of porosity, thickness, Young's modulus and the lamination sequence of the composite structure. Due to the above characteristics, the traditional methods of composite performance mainly...
include the equivalent circuit method\textsuperscript{[1]}, the finite element method\textsuperscript{[2]}, etc., which are difficult to deal with the acoustic performance of complex structures.

The transfer matrix method has a superiority in the calculation of chain structure\textsuperscript{[3]}, so it has been introduced into the calculation of acoustic performance of composite materials. The impedance performance and the reverberation chamber method are used to measure the acoustic performance and verify the effectiveness of the transfer matrix method in dealing with such problems. In addition, a statistical energy model simulation is carried out by taking a ship as an example. The application of this method in noise prediction of ship is discussed, which provides the support for the application of noise protection design of ship.

2. Analysis of the acoustic performance of composite based on the transfer matrix method

The principle of acoustic performance analysis of composite materials based on transfer matrix method is as follows. It is assumed that the incident wave is incident at the incident angle \(\theta_0\), and the theoretical model of the composite material is shown in figure 1.

\[
\begin{align*}
Z &= Z_0 = Z_{n=1} = \cdots = Z_N = Z_H \\
\theta &= \theta_0 \\
N &+ 1
\end{align*}
\]

Figure 1. Multilayer composite material schematic

It is also assumed that the composite interface is a plane parallel to each other, the interface is marked as \(z=z_n\), and the outermost layer of the structure is a semi-infinite uniform medium. A plane wave (frequency = \(f\)) is incident at an incident angle \(\theta_0\), defining in the \(x-o-z\) coordinate system. Meanwhile, the sound pressure in each layer of the composite material is \(p_n\), and the sound velocity of normal direction is \(v_{nz}\), wherein the subscript \(n\) represents the number of composite material layer. The sound pressure \(p_n\) can be expressed as:

\[
p_n = a_n e^{ik_n x \sin \theta_0 + ik_n z \cos \theta_0} + b_n e^{ik_n x \sin \theta_0 - ik_n z \cos \theta_0}
\]

The limited condition is that the sound pressure and the sound velocity of normal direction are continuous at the interface:

\[
p_n = p_{n+1}, \quad v_{nz} = v_{(n+1)z}(z = z_n)
\]

The transfer matrix is a relationship matrix between \(p_n\) and \(v_{nz}\) in two adjacent interfaces, and in the subsequent derivation process, the factor \(e^{ik_n x \sin \theta_0}\) is omitted for the operation of the \(x\) coordinate is not involved, then the following matrix is obtained:

\[
q_n = \begin{pmatrix} p_n \\ v_{nz} \end{pmatrix} = \begin{pmatrix} a_n e^{ik_n z \cos \theta_0} + b_n e^{-ik_n z \cos \theta_0} \\ 1/(z_n)(a_n e^{ik_n z \cos \theta_0} - b_n e^{-ik_n z \cos \theta_0}) \end{pmatrix}, \quad z_n = \frac{d_n e_n}{\cos \theta_n} = \frac{E_n}{\cos \theta_n}
\]

Where \(Z_n\) is the acoustic impedance of normal direction, \(d_n\) is the density of the \(n\)-th layer composite and \(E_n\) is the elastic modulus of the \(n\)-th layer composite. The matrixes of the upper and lower surfaces of the \(n\)-th composite material are respectively denoted as \(q_n^{up}\) and \(q_n^{down}\), and the
equation 4 is applied to the interface of the incident layer \((z=0)\) and the interface of the transmission layer \((z=H)\):

\[
q_0^{up} = \left( \frac{1 + R_p}{z_0} \right) q_0^{down} = \left( \frac{T_p}{z_0} \right) T_p \frac{T_p}{z_{N+1}}, \text{ and } q_n^{down} = T \cdot q_n^{up}
\]  

(4)

Where \(T\) is the transfer matrix between \(q_n^{up}\) and \(q_n^{down}\), which is a second-order square matrix. It is proved below that \(T\) is the transfer matrix between the incident surface and the exit surface of the composite. The sound pressure reflection coefficient can be obtained as follow:

\[
R_p = \frac{(T_{11} + \frac{1}{z_0} T_{12}) - z_{N+1}(T_{21} + \frac{1}{z_0} T_{22})}{z_{N+1}[(T_{21} + \frac{1}{z_0} T_{22}) - \frac{1}{z_0} (T_{11} + \frac{1}{z_0} T_{12})]}
\]  

(5)

Then the acoustic energy reflection coefficient \(R_e\) and sound absorption coefficient \(\alpha\) can be obtained as follow:

\[
R_e = \left| R_p \right|^2 \cdot \frac{z_0}{z_{N+1}}, \alpha = 1 - R_e \cdot R_e^*
\]  

(6)

According to the equation 3, the matrix of the \(n\)-th layer can be expressed as:

\[
q_n^{down} = \begin{pmatrix} e^{ik_z z_n \cos \theta_n} & e^{-ik_z z_n \cos \theta_n} \\ e^{ik_z z_n \cos \theta_n} & -e^{-ik_z z_n \cos \theta_n} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}, \quad q_n^{up} = \begin{pmatrix} e^{ik_z z_n \cos \theta_n} & e^{-ik_z z_n \cos \theta_n} \\ e^{ik_z z_n \cos \theta_n} & -e^{-ik_z z_n \cos \theta_n} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}
\]  

(7)

According to boundary conditions of the composite sound absorbing system, the matrixes \(q_n\) above the \(n\)-th layer material and under the \((n+1)\)-th layer material has the following relationship: \(q_n^{up} = q_{n+1}^{down} = E_n\), it can be obtained: \(E_n = t_n \cdot E_{n-1}\). \(T_n\) is the transfer matrix of the \(n\)-th layer material, and the specific form is:

\[
t_n = \begin{pmatrix} \cos O_n & i z_n \sin O_n \\ i \sin O_n & \cos O_n \end{pmatrix}
\]  

(8)

Where \(O_n = k_n h_n \cos \theta_n\). From the equation 4 to equation 8, the relationship between the total transfer matrix \(T\) and the transfer matrix \(t_n\) of each layer is obtained:

\[
T = t_n \cdot t_{n-1} \cdot \cdots \cdot t_2 \cdot t_1
\]  

(9)

3. Comparative analysis of test results and theoretical calculations

In this section, we apply the impedance tube method and the reverberation chamber method to respectively test the sound absorption coefficient and sound insulation of composite structures.

3.1. The principle of the impedance tube method and the reverberation chamber method

3.1.1. The impedance tube method for measuring sound absorption coefficient. The basic principle of the impedance tube method for measuring the sound absorption coefficient is that the normal-incident reflection factor \(r\) can be determined by the transfer function \(H_{12}\) measured by the two microphones in front of the sample. The schematic of the measurement principle is shown in the figure 2.
The sound pressure at the position of the two microphones is expressed as:

\[ P(x_i) = P_i \cdot e^{ikx_i} + P_r \cdot e^{-ikx_i}, \]

where \( P_i \) and \( P_r \) are the amplitudes of the incident sound pressure and the reflected sound pressure, \( k \) is the complex wave number. The transfer function \( H_{12} \) of the total sound field is:

\[ H_{12} = \frac{p(x_2)}{p(x_1)} = \frac{e^{ikx_2} + r \cdot e^{-ikx_2}}{e^{ikx_1} + r \cdot e^{-ikx_1}} \]

This gives the normal-incident reflection factor \( r \) and the sound absorption coefficient \( \alpha \):

\[ r \cdot e^{i\theta} = \frac{H_{12} \cdot e^{i\theta} - e^{ikx_1}}{e^{-ikx_1} - H_{12} \cdot e^{i\theta} \cdot e^{ikx_1}}, \]

and \( \alpha = 1 - |r|^2 \)

It can be seen that the sound absorption coefficient can be obtained from the inter-microphone transfer function \( H_{12} \) and the mounting position parameters \( x_1, x_2 \).

**Figure 2.** Schematic diagram of the impedance tube method

### 3.1. The reverberation chamber method for measuring sound insulation.

The sound insulation measurement of large sample is carried out in two connected chambers. The schematic diagram is shown in figure 3. The sound pressure level measured by the reverberation chamber is SPL1 while the sound pressure level measured by the receiving chamber is SPL2. In addition, it is necessary to correct the sound absorption effect of the receiving room. Therefore, the measurement formula of the sound insulation amount is:

\[ SPL = SPL_1 - SPL_2 + 10 \log \left( \frac{1}{4} + \frac{A}{\alpha S} \right) \]

Where \( A \) is the surface area of the sample, and \( S \) is the total area of the receiving chamber.

**Figure 3.** The sound insulation measured by the reverberation chamber method

The layout of sample is shown in figure 4.

**Figure 4.** The layout of the specimen

### 3.2. The result

Figure 5-figure 6 shows the comparison between measured values and the theoretical value calculated by the transfer matrix method. The error of the sound absorption coefficient is within 0.1, but it gets...
bigger and bigger with the increase of the frequency, as seen in figure 5. In the other hand, it can be seen from the figure 6 that the error of the calculated value of the sound insulation is within 5dB, which meets the requirements of engineering application. The theoretical value has less error in the middle frequency band, and the error increases with the increase of frequency. Altogether, the effectiveness of the composite matrix based on the transfer matrix method is verified.

Figure 5. Comparison of experimental and theoretical values of Sound absorption coefficient

Figure 6. Comparison of experimental and theoretical values of Sound transmission

4. Application of transfer matrix method in noise protection design of ship

This section is based on the statistical energy method to simulate the noise protection design of a ship as an example. It is used to investigate the application of the calculation of acoustic parameters by transfer matrix method in the acoustic protection design of ships. The geometric model of ship is introduced into VAOne and the statistical energy model is established. Then, the acoustic loads are applied in the ship's statistical energy model. Figure 7 shows the layout of the load application position in 1800mm deck while figure 8 shows the nephogram of cabin noise.

Figure 7. The load layout diagram

Figure 8. The nephogram of cabin noise

Take the running of all equipment as the calculation conditions, and the conference room in the living compartment is selected as the assessment cabin. There are two types of the outfitting materials including sound insulation materials and sound absorption materials. The former laid in the assessment cabin, and the later laid in the cabin near the noise source. Through the transfer matrix method, the acoustic parameters of two materials can be calculated. Then the results are input into the statistical energy model, and then the noise values of the assessment cabin, whether laying the sound insulation composite material or not, are calculated and compared. Finally, the effect of cabin noise reduction by laying composite material can be obtained and shown in figure 9(a). It can be seen that the noise reduction of the cabin is more obvious, and the total noise reduction reaches 11.2dB.
The noise values of the cabin near the noise source and the assessment cabin after laying the sound absorbing composite material are shown in figure 9(b) and figure 9(c). The noise reduction effect of the evaluation cabin is obvious by using the sound absorbing composite materials and the noise reduction amount is 16.1dB, as seen in figure 9(b). But the noise reduction on the cabin near the sound source is only 3.1dB, as seen in figure 9(c), which indicates that the sound absorption composite materials has a good effect on blocking noise transmission, but the noise caused by the noise source inside the cabin cannot be effectively treated.

Figure 9. The noise reduction effect of outfitting materials
(a) sound insulation; the evaluation cabin, (b) sound absorbing composite materials; the cabin near the sound source, (c) sound absorbing composite materials; the evaluation cabin.

5. Conclusion
In this paper, the transfer matrix method is used to theoretically deduct the acoustic performance of composite materials. The excellent accuracy and reliability of this method are confirmed by comparing the calculated results with experimental values. Using the transfer matrix method to calculate the acoustic performance of composite materials has the characteristics of high calculation efficiency, great reliability and wide applicability. Then the model simulation was carried out by taking a ship as an example, and the transfer matrix method in the noise protection design of the ship was discussed.

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