Coding Techniques for Future Wi-Fi

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Abstract

Future wireless applications such as high definition video streaming, wireless cloud radio access networks, and cellular data offload are bandwidth-hungry, which highlights the need for Wi-Fi links exceeding 1 Gb/s. This article discusses coding schemes and, in particular, the use of Low-Density Parity-Check (LDPC) codes as Forward Error Correction (FEC) in future Wi-Fi standards. Moreover, we consider advanced design strategies such as root-check LDPC structures, computer-aided design for short blocks and high-performance decoding algorithms with low latency. We then conclude the article with a discussion of FEC challenges for future Wi-Fi applications.

I. INTRODUCTION

Driven by the ever-increasing user connectivity demands, emerging and future generations of Wi-Fi, such as IEEE 802.11ac and WiGig IEEE 802.11ad, will be capable of achieving multiple gigabits per second speeds. Furthermore, the future Wi-Fi will be used to do everything from simple web browsing and peer-to-peer sharing, to multimedia streaming, real-time teleconferencing, cable replacement, and wireless docking, to name a few. Therefore, one key element will be the reliable delivery of information at the final destination. One strategy to improve the reliability of the received information is the use of Forward Error Correction (FEC). FEC relies on a mathematical mapping of messages to include a special kind of redundancy that enables the receiver to correct the errors caused by the channel [1].

There are two main types of FEC used by Wi-Fi: LDPC codes and convolutional codes. LDPC codes belong to the class of block codes, can perform close to channel capacity and achieve excellent performance. Block codes work on fixed-size blocks (packets) of bits or
symbols of predetermined size. A practical block code can be decoded in polynomial time
to its block length. In contrast, convolutional codes work on bit or symbol streams of arbitrary
length. Convolutional codes are most often decoded with the Viterbi algorithm [1], though other
algorithms are sometimes used. Nevertheless, the Viterbi algorithm can lead to high complexity in
terms of decoding for codes with large constraint length. Recent studies comparing convolutional
and LDPC codes have reported that convolutional codes can be advantageous for low latencies,
whereas LDPC codes are preferred for higher latencies or medium to large blocks [2].

In Fig. 1 an example of a system model for a Wireless Local Area Network (WLAN) is
presented. In Fig 1 a), we have a Wi-Fi communications system where a tablet, a laptop and
a mobile phone exchange information with an Access Point (AP). In Fig. 1 b) a simplified
structure of a Wi-Fi communication system between a transmitter (TX) and a receiver (RX),
for example, a laptop and an AP is shown. In Fig. 1 b) we describe the encoding, transmit and
receive processing, and the decoding of a message \( m = [m_0 \ldots m_k] \). Specifically, the message
\( m \) is encoded by an LDPC encoder to produce the codeword \( c = [c_0 \ldots c_n] \), where the rate is
\( R = k/n \). Then, the codeword \( c \) is modulated to one of the available schemes, namely, BPSK,
QPSK, 16-QAM, 64-QAM and 256-QAM generating the symbol stream \( s \). In the case of IEEE
802.11ac the symbol stream \( s \) is transmitted by an OFDM technique which involves additional
steps: a serial to parallel (1 up to 8 streams) conversion to calculate the Inverse Fast Fourier
Transform (IFFT) with a maximum of 114 sub-carries/pilots, a parallel to serial conversion is
performed, and the resulting stream is sent to the channel. At the receiver side the inverse process
is carried out. For the case of IEEE 802.11ad the transmission is omitted (dashed blocks), so
the resulting stream can be transmitted through either an OFDM scheme or a single-carrier one.
Extensions to OFDM systems [3], [4] and multiple-antenna systems [5] can also be considered.
The focus of this article is on LDPC design and decoding strategies for future Wi-Fi.

Following the system model of Fig. 1 b) a \( G \) generator matrix to encode the message \( m \) is
used and a Parity Check Matrix (PCM) \( H \) for the decoding process. If the estimated codeword
\( \hat{c} \) is correct then \( \hat{c} \cdot H^T = 0 \) must be satisfied, where \((.)^T\) stands for the transpose operation. An
LDPC code is characterized by its sparse PCM \( H \). For \( H \) to be sparse the number of entries
equal to one must be much less than the number of zeros.

In this paper we discuss the perspectives of LDPC coding for the future of Wi-Fi. Furthermore,
the LDPC codes used in the Wi-Fi standard are reviewed. Moreover, we draw attention to
advanced design strategies such as Root-Check LDPC structures, computer-aided methods for
short blocks and how they can be used in future standards. In addition, we also consider advanced LDPC decoding techniques such as reweighting and scheduling methods, which have the potential to reduce the decoding latency in several important applications. An analysis of fast LDPC decoding methods to decrease the computational complexity in hardware is also included. Finally, some research challenges are identified.

The rest of this paper is organized as follows. In Section II a general discussion about LDPC codes for wireless communication standards is developed. In Section III we describe the LDPC codes used in the Wi-Fi standard. In Section IV we review the use of Root-Check LDPC codes and other techniques for the future Wi-Fi. In Section V we discuss advanced decoding techniques such as reweighting, scheduling and reduced complexity decoding methods. Section VI presents the challenges, while Section VII concludes this paper.

II. LDPC CODES IN WIRELESS COMMUNICATION STANDARDS

LDPC codes with randomly generated parity check matrices and large blocks generally have good performance, but they lack enough structure to facilitate efficient encoding methods. In practical applications, structured and short length LDPC codes are highly recommended as modern wireless standards adopt Quasi-Cyclic LDPC (QC-LDPC) codes and Irregular Repeat and Accumulate LDPC (IRA-LDPC) codes.

QC-LDPC codes are designed by tiling circulant matrices. A circulant matrix is a square matrix in which each row is a right cyclic shift of the previous row and the first row is a cyclic shift of the last row. A circulant matrix can be totally characterized by its first row. If a circulant...
matrix has weight \( w = 1 \) per row, it is called a circulant permutation matrix. The all-zero matrix, also called null matrix, is also circulant matrix with all its elements being zero. If a PCM \( H \) consists of \( m \times n \) circulant sub-matrices of dimensions \( s \times s \) with \( M = ms \) and \( N = ns \), the resulting linear block code will be a QC code with a period of \( n \). In such code, the \( n \)-bit shift of any codeword is another codeword. The generator matrices \( G \) of QC-LDPC codes are in systematic-circulant form with the requirement that the PCM are full rank. The memory cost for encoding QC-LDPC codes is greatly reduced and the encoder can be implemented by using simple shift registers \([1]\).

The accumulator-based codes that were invented first are the so-called repeat-accumulate (RA) codes \([1]\). Despite their simple structure, they were shown to provide good performance and, more importantly, they pioneered the design of efficiently encodable LDPC codes. The key points in using IRA-LDPC codes are the simplicity in designing such codes and faster encoding than conventional LDPC methods. The Wi-Fi standard uses a type of QC-LDPC codes, whereas other standards work with a combination of QC and IRA LDPC codes known as QC-IRA LDPC codes. There are two main advantages in QC-IRA-LDPC codes: the memory requirements to store the matrices by QC codes and the simple encoding process provided by IRA codes. For instance, Fig. 2 presents an example of a PCM for a QC-IRA-LDPC code with dimension \( 288 \times 576 \). The right hand side of Fig. 2 is a dual diagonal associated with an RA based code. On the left hand side of Fig. 2 are the circulant matrices.

![Figure 2. An example of a parity check matrix for a QC-IRA-LDPC code. The black dots represent the ones and the white space represent the zeros.](image)
III. LDPC CODES USED IN THE WI-FI STANDARD

In this section, the LDPC codes used in the Wi-Fi standard are detailed. The Wi-Fi standard adopts Quasi-Cyclic LDPC (QC-LDPC) codes. As discussed previously, FEC introduces some kind of redundancy in the form of $n - k$ extra bits in a codeword of length $n$ to improve the reliability of a delivered message which results in the code rate $R = k/n$. For instance, the PCM presented on Fig. 2 is an LDPC code with code rate $R = \frac{1}{2}$. The rate $\frac{1}{2}$ means that half of the codeword is the message (information) and the other half is the redundancy. The Wi-Fi standard operates with the following code rates, $R = \frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$ and $\frac{13}{16}$ with a fixed block length of $L = 672$ bits. For example, with a code rate $\frac{1}{2}$ we have 336 information bits plus 336 bits of redundancy, for code rate $\frac{13}{16}$ we have 546 information bits plus 126 bits of redundancy.

Fig. 3 presents the PCM $H$ of the Wi-Fi standard with code rate $R = \frac{1}{2}$ and dimensions $336 \times 672$. The other PCMs for the rest of code rates follow a similar structure as for code rate $R = \frac{1}{2}$. Once a standard is defined, a design is adopted and the PCM $H$ must be stored in all devices for compatibility issues. Whenever the standard is updated there is an opportunity to incorporate more sophisticated PCM designs which could lead to improved performance.

IV. DESIGNING HIGH-PERFORMANCE LDPC CODES

In this section we review advanced LDPC code design strategies such as Progressive Edge Growth (PEG) algorithms, Root-Check LDPC structures, QC and IRA structures.
A. PEG-Based Algorithms

Among the algorithms capable of producing LDPC codes of highest performance and efficiency for short to moderate lengths are the PEG-based algorithms [6] and QC and IRA structures. The codes produced by the PEG algorithm exhibit improved performance compared to random construction methods [6] as PEG optimizes the underlying graph structure of the PCM. In particular, PEG strategies attempt to increase the girth of the graph and improve its general connectivity. Improvements and modifications in the original PEG algorithm have provided a better performance than the original PEG algorithm. For instance, Healy and de Lamare in [7] have proposed an algorithm to design LDPC codes which involves the use of decoder-based optimisation with the sum-product algorithm (SPA) and also considered multiple candidates to optimize the graph connections [8]. Therefore, LDPC codes for future Wi-Fi standards can take advantage if they are designed by PEG-based algorithms and several detection and decoding strategies [9], [10], [11], [12], [13], [14].

B. Structured Code Designs

A drawback of standard unstructured LDPC codes designed with conventional and PEG-based algorithms is their high encoding complexity, which impacts the cost and power consumption of devices. Fortunately, the memory and computational cost for encoding LDPC codes can be greatly reduced by adopting QC-LDPC codes because the encoder can be implemented by simple shift registers. Furthermore, IRA LDPC codes are also useful due to their simplicity in designing and faster encoding than conventional LDPC codes. In addition, designers can resort to QC-IRA LDPC codes which brings further cost reduction by combining the features of QC and IRA LDPC codes [15], using PEG-based optimization of the graphs and statistically-driven approaches [16]. These highly structured LDPC codes could play an important role in the design of future Wi-Fi standards.

C. Root-Check Codes

Another family of LDPC codes called Root-Check for block-fading channels were proposed in [17]. Root-Check codes are able to achieve the maximum diversity of a block-fading channel and have a performance near the limit of outage. Several types of Root-Check LDPC codes were developed, e.g., [18], [19], [8]. Among the Root-Check based LDPC codes the ones designed with the PEG algorithm have shown the best performance [19]. In particular, we have designed
a QC-IRA Root-Check LDPC code PEG-based with the same block length as that of the Wi-Fi standard $L = 672$ and with code rate $\frac{1}{2}$. The key point in this example is to demonstrate by simulations that a Root-Check based code can significantly outperform the Wi-Fi LDPC code. For example, consider a simple scenario where the channel is a block-fading with $F = 2$ fadings, BPSK modulation is used and a maximum of 20 decoding iterations are allowed.

In Fig. 4 we depict the Frame Error Rate (FER) performance of a Root-Check LDPC code versus the LDPC code from Wi-Fi standard both are with the same code rate and with the same block length. The outage curve is also plotted as reference which represents the channel capacity. From Fig. 4 we can see that the QC-IRA Root-Check LDPC code outperforms the Wi-Fi LDPC code by $6.5dB$ in terms of SNR.

![Figure 4. FER performance comparison for QC-IRA Root-Check LDPC code and Wi-Fi LDPC code over a block-fading channel with $F = 2$ and $L = 672$. The maximum number of iterations is 20. Code rate $R = \frac{1}{2}$.](image)

Fig. 5 shows the average number of iterations required by the QC-IRA Root-Check LDPC code versus the Wi-Fi LDPC code. For the entire SNR region, in average, we observe that the QC-IRA Root-Check LDPC code requires less decoding iterations than other LDPC code designs. It must be mentioned that for medium to high SNR the average required number of iterations is less than 2 iterations. The average number of iterations, less than 2 at medium to high
V. ADVANCED DECODING TECHNIQUES

In this section, we discuss advanced LDPC decoding techniques which include reweighting and scheduling approaches, and low-complexity decoding algorithms. These algorithms offer considerable advantages over standard belief propagation (BP) algorithms and can address some of its limitations with regards to performance, delay issues and computational cost.

A. Reweighting Methods

The BP algorithm, sometimes also called sum-product algorithm (SPA), is a powerful algorithm to approximately solve several inference problems which can be used for error control coding.
Once the BP algorithm was applied as a decoding algorithm for LDPC codes, various versions of BP graph-based decoding algorithms have been reported in the area. However, the lack of a convergence guarantee and the high-latency due to many decoding iterations are still open issues for researchers when it comes to effectively decoding LDPC codes in Wi-Fi applications, where such applications are bandwidth-hungry.

Recently, Wyneersch et al. [20] introduced the uniformly reweighted BP (URW-BP) algorithm which exploits BP’s distributed nature and reduces the factor appearance probability (FAP) to a constant value. In [20], the URW-BP has been shown to outperform the standard BP in terms of LDPC decoding among other applications. In terms of BP algorithm the URW-BP makes its main modification on the beliefs sent by check nodes to variable nodes. Therefore, the beliefs of the j-th estimated received vector \( b(x_j) \) is:

\[
b(x_j) = \mathcal{L}(x_j) + \sum_{i \in \mathcal{N}(j)} \rho \Lambda_{i,j},
\]

where \( \mathcal{L}(x_j) \) is the a priori Log Likelihood Ratio (LLR) from the channel, \( \rho \) is the constant FAP value, \( \Lambda_{i,j} \) are the messages sent from check node \( c_i \) to variable node \( v_j \) and \( i \in \mathcal{N}(j) \) is the neighbouring set of check nodes of \( v_j \).

Liu and de Lamare in [21] have investigated the idea of reweighting a suitable part of the factorized graph while also statistically taking the effect of short cycles into account. By combining the reweighting strategy with the knowledge of the short cycles obtained by the cycle counting algorithms they have presented the variable FAP BP (VFAP-BP) algorithm. The VFAP-BP algorithm assigns distinct FAP values to each check node on the basis of the structure of short cycles rather than a complex global graphical optimization. VFAP-BP outperforms standard algorithms. Furthermore, the VFAP-BP algorithm can be applied with both regular and irregular LDPC codes while URW-BP is only advantageous for regular LDPC codes.

B. Scheduling Methods

The studies in [22], [23] have suggested that the use of appropriate scheduling mechanisms for LDPC decoding can significantly improve the convergence speed in terms of number of iterations. In general, BP or SPA consist of the exchange of messages between the nodes of a graph. Each node generates and propagates messages to its neighbours based on its current incoming messages.
The LDPC code graph is a bi-partite graph composed by $N_{VN}$ variable nodes $v_j$ for $j \in \{1, \cdots, N_{VN}\}$ that represent the codeword bits and $M_{CH}$ check nodes $c_i$ for $i \in \{1, \cdots, M_{CH}\}$ that represent the parity-check equations [22]. In the log-domain implementations of the BP algorithm, the exchanged messages correspond to the LLR of the probabilities of the bits. The sign of the LLR indicates the most likely value of the bit and the absolute value of the LLR gives the reliability of the message.

BP decoding consists of the iterative update of the messages until a stopping rule is satisfied. In flooding scheduling, an iteration consists of the simultaneous update of all the messages from variable to check nodes followed by the simultaneous update of all the messages from check to variable nodes. In sequential scheduling, an iteration consists of the sequential update of all the messages from variable to check nodes as well as all the messages from check to variable nodes in a specific pre-defined order. This pre-defined order is usually designed to allow the parallel processing of the messages. For instance, the Layered Belief Propagation (LBP) will do for each check node the update of check to variable nodes followed by the update of the associated variable nodes. This procedure is done in an iterative way. The algorithm stops if the decoded bits satisfy all the parity-check equations or a maximum number of iterations is reached.

RBP was first introduced in [24] and consists of an informed dynamic scheduling strategy that updates messages according to an ordering metric called the residual. The message with the largest residual is updated first. A residual is the norm (defined over the message space) of the difference between the values of a message before and after an update. The intuitive justification of this approach is that as iterative BP converges, the differences between the messages before and after an update diminish. Then, if a message has a large residual, it means that it is located in a part of the graph that has not converged yet. Accordingly, propagating that message first should speed up convergence.

In order to obtain a better performance, a less greedy scheduling strategy can be used. The greediness of RBP comes from the fact that it tends to propagate first the message to the least reliable node. Vila Casado and et.al. in [22] proposed to update and propagate simultaneously all the check-to-variable messages that correspond to the same check node, instead of only updating and propagating the message with the largest residual. This algorithm is less likely to propagate the information from new errors in the next update. This is because there are many variable nodes that change as opposed to RBP where only one variable node changes. This strategy is called
Node Wise Scheduling BP (NWBP). Fig. 6 depicts an example decoding techniques, which illustrates how they can reduce significantly the number of iterations as compared to BP. With the advent of parallel computing in the latest powerful processors, the use of scheduling methods can reduce significantly the overall number of iterations required in the decoding procedure.

Figure 6. FER performance versus iterations with the Wi-Fi LDPC code rate $R = \frac{1}{2}$ for BP, URW, LBP and VFAP decoding algorithms. SNR = 3dB in the AWGN channel.

With the advent of parallel computing in the latest powerful processors, the use of scheduling methods can reduce significantly the overall number of iterations required in the decoding procedure.

C. Reduced Complexity Decoding Methods

Work on LDPC decoding has mainly focused on floating point arithmetic or infinite precision (BP algorithm). However, hardware implementations of decoding algorithms for LDPC codes must address quantization effects in a fixed-point realization [25]. The first approach was to adopt a logarithmic version of the BP algorithm called Log-BP. Nevertheless, Log-BP sacrifices hardware implementation due to the fact it requires many hyperbolic tangent operations.
The Log-BP algorithm can be simplified using the so-called BP-based approximation (also known as the "min-sum" approximation), which greatly reduces the implementation complexity, but incurs a degradation in decoding performance. This has led to the development of many reduced complexity variants of the BP algorithm that deliver near-optimum decoding performance. There are three main reduced complexity methods of decoding LDPC codes: BP-based, Normalized BP-based and Offset BP-based [25].

In the min-sum algorithm the key modification is on the horizontal step or the check to variable node update equation. The check to variable node update is given by

$$
L_{i,j} \approx \left( \prod_{j' \in N(i) \setminus j} \text{sign}(\Lambda_{j',i}) \right) \times \left( \min_{j' \in N \setminus j} |\Lambda_{j',i}| \right)
$$

(2)

where, \( \Lambda_{j',i} \) are the messages sent from variable node \( v_j \) to check node \( c_i \) and \( j' \in N(i) \setminus j \) is the neighbouring set of variable nodes of \( c_i \) except \( v_j \). As it can be seen in (2) there is no hyperbolic tangent operation which decreases significantly the overall computational complexity, although it causes a degradation in decoding performance.

The min-sum algorithm can be improved by employing a check node update that uses a normalization constant greater than \( \alpha \). The change made in (2) is on the right hand side and inside the \( \min \) operation that is divided by \( \alpha \). This method is called Normalized BP-based. The \( \alpha \) parameter should be adjusted for different SNRs and iterations to achieve its optimum performance. An effective approach to determine the optimum value of \( \alpha \) is by Density Evolution [25].

A computationally more efficient approach that captures the net effect of the additive correction term applied in each check node update operation is obtained from the BP-based decoding by subtracting a positive constant \( \beta \). This is called Offset BP-based. Eq. (2) is modified such that \( \min_{j' \in N \setminus j} |\Lambda_{j',i}| \) is replaced by \( \max\{\min_{j' \in N \setminus j} |\Lambda_{j',i}| - \beta, 0\} \). This method differs from the normalization scheme in that LLR messages smaller in magnitude than \( \beta \) are set to zero, therefore removing their contributions in the next variable node update step. The min-sum, normalized BP-based, and Offset BP-based decoding methods do not need any channel information, e.g., the SNR, and work with just the received values as inputs. Both offset and normalized BP-based decoding algorithms can achieve performance very close to that of BP decoding while offering significant advantages for hardware implementation.
VI. CHALLENGES

The LDPC codes used in the Wi-Fi standard are not optimally designed. Therefore, it is important to employ design algorithms capable of producing LDPC codes of high performance and efficiency for short to moderate lengths such as PEG-based algorithms. The example shown in Fig. 4 poses the following question: Why are the Wi-Fi LDPC codes not optimally or near optimally designed? We consider this as an open problem for future Wi-Fi standards.

The PEG-based QC-IRA Root-Check LDPC code designed was compared to the Wi-Fi LDPC code to analyse the overall performance in terms of FER. As seen in Fig. 4 the PEG-based designed code has outperformed the Wi-Fi LDPC code by about $6.5dB$. Therefore, future Wi-Fi standards would have benefits if they considered the use of Root-Check based LDPC codes for rate $R = \frac{1}{2}$ scenarios.

The use of scheduling methods to decode LDPC codes can reduce significantly the number of iterations required in the decoding procedure. Accordingly, the resulting number of iterations can increase the advantages LDPC codes over convolutional codes, which are widely used in portable devices due to their simplicity. An open problem in LDPC decoding is: Which is the most cost-effective method for the Wi-Fi standard? This is an investigation that will keep designers of Wi-Fi systems busy in the next few years.

In terms of adoption of LDPC codes as FEC in Wi-Fi standards, key issues are the encoding and decoding operations. The cost of encoding can be decreased significantly by using RA based LDPC codes which are able to provide simple and low complexity encoding methods. The decoding cost is another key issue that must be addressed. One way to addressing such problem is by introducing reduced complexity decoding (RCD) algorithms, e.g., min-sum, normalized BP-based and Offset BP-based. The choice of decoding algorithm is another important design challenge, which must take into account aspects such low complexity operations, stability and performance.

VII. CONCLUSION

This article has discussed coding techniques for Wi-Fi systems. In particular, we have discussed approaches to reducing the complexity of encoding and decoding and improving the design and the decoding of LDPC codes. The main challenge is in decoding which is how to obtain an attractive trade-off between performance and computational complexity. To conclude, we
advocate that future Wi-Fi standards should use LDPC codes as the main FEC to fulfil the needs of bandwidth-hungry and high-performance applications.

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