The Progress of Three Astrophysics Simulation Methods: Monte-Carlo, PIC and MHD

Jiawei Guo*

Chemical and biomedical engineering, University of South Florida, Tampa, Florida, 33620, United States.
*jiaweiguo@usf.edu

Abstract. Contemporarily, the dramatic progress that proceeded in astrophysical simulation stems from the computational upgrade. Simultaneously, the relative algorithms are from naive to mature. Some imaginary algorithms spring up like bamboo. The typical methods are Monte-Carlo, Particle-in-Cell, and Magneto-hydrodynamic considering the relativistic effect. This paper reviews the typical progress of the above three methods as well as their hybrid method in astrophysics. These results offer a guideline for further research in astrophysics simulation.

1. Introduction

In the past few decades, due to the upgrading of computer hardware, some methods based on high-performance brute force solutions can be applied to astrophysics [1]. The previously tedious calculation process has been simplified through the computer to retouch the repetitive steps, including the classification of data and statistics. It sets up the foundation for some algorithms that were previously idealized. Generally, there are three typical simulation methods for astrophysics: Monte-Carlo, Particle-in-Cell, and Magnetohydrodynamic methods.

The Monte-Carlo Algorithm is based on random number generation. The given random initial quantity is substituted into the given equation to find the corresponding solution, and the quantity of into the equation in different states is depicted. Additionally, the various parameters in the equation delve into the typically conformed solution to the experiment expectations [2].

The Particle-in-Cell method separates the calculation spaces into fine grids in a multi-dimensional structure. To simplify the multi-body problem, the location and intensity distribution of the grid in the radiative pattern fit the given rule. Therefore, the model solves the N-body question under given data in the experiment [3].

The Magnetohydrodynamics method describes the object's motion by solving the mass continuity equation, Cauchy momentum equation, energy equation, Ohm’s law, and Lorentz equation. Due to the complex properties in Galaxy, these equations are the basic conditions that constitute the objective motion and consider the relativistic motion [4].

These algorithms are combined to form a frontier algorithm in astrophysics. By changing the initial value, the solution of simulation for nebular formation was picked and solved. Mark’s review also detailed some general simulations of multi-body problems and tree models to simulate ordinary matter, dark energy, dark matter [5].

This paper describes the mainstream method concerning computational astrophysics. The rest of the paper is organized as follow: primarily, the Monte-Carlo method is introduced in Sec. 2; then, Particle-in-Cell method is demonstrated in Sec. 3 while Magnetohydrodynamics method is discussed in Sec. 4;
Subsequently, hybrid simulations are reviewed in Sec. 5; a brief summary and perspective are given in Sec. 6 eventually.

2. Monte-Carlo method

The Monte-Carlo method, which traces the rule in chaos by a random number, is a useful method to do statistics in Big Data [6]. The Monte-Carlo has applied in economy [7], particle physics and astrophysics [8-10], hydrology [11], material science [12,13], statistical physics [14], biology [15], and computer science [16,17].

2.1 Particle physics and astrophysics

In cosmic, the universe provides some speculatively extreme environment that is impossibly existed on earth. Simultaneously, the intensive properties of tiny particles also illuminate the cosmological phenomena [18,19]. The simulation results of the program SOPHIA are shown in Figure 1, which simulates the interaction between photons and nucleons and illustrates the rate of distribution for three different radiational beams [20]. The energy was selected in a wide range. Twenty-one input parameters are utilized to simulates ten thousand models about the solar. Each parameter is distributed a random probability and simulates the result to fit the experimental data. The 19 parameters are nuclear fusion cross sections, the solar age and luminosity, the diffusion coefficient, and the nine most important heavy element abundances on the Sun's surface. Besides, the other two additional inputs are radiative opacity and the equation of state [21]. The model includes 95% type of stars of the universe by TRILEGAL model of the galaxy and adjusts the properties of the stars and the figures subsequently to fit the observation of Transiting Exoplanet Survey Satellite (TESS) [22].

![Figure 1. The rate of distribution under the different radiational beams of different energy](image-url)
2.2 Cosmological physics

In cosmological physics, the Monte-Carlo method can apply to the Reduced Basis (RB) and the Empirical Interpolation Method (EIM) [23]. The surrogate model is related to the Reduced Order Modeling due to the correlated parameters in the equation. The Symbolic regression can be used to modify the pattern in the existing database.

2.2.1 Surrogate model

In some cases, there are several parameters to drive an equation. After some of the parameters are given, the brief pseudo model can substitute the initial function, modify the model and test the error until the error is satisfying. This method effectively reduces the basis and simplifies the process. In the large-scale calculation, the subtle structure and function can be simplified to the pseudo model and apparently reduce the calculation order. The subsequent calculation can follow the previous experience and accelerate the calculation but retain the accuracy. For example, in the calculation of Gravitational Wave, the no hair effect allows us to use eight parameters to describe the state function in the collision of two black holes. However, solving the equation with the higher order factor consumes lots of computational effort. Therefore, it is feasible to use a random number to test the possible solution with given data and find the main effect on the various scale. For the cluster, it is nearly impossible to describe the motion of the N-body model. The necessary approximation is to regard the cluster as a whole object, i.e., record the trace, draw the equation, and test error for the object. Repeat the process if the error is comparable to the data [23].

2.2.2 Symbolic regression

The symbolic regression is a regression analysis that applies to the pattern of the function. The logic signs are the knots, and the bottom branch is the data. Changing the signal delves the inertial correlation among this given data. In this way, the candidate can substitute the existing frame if the accuracy and complexity are both satisfying. The Figure 2 depicts the process of using the statistical method to find the functional pattern after mutation and the inherited gene crossover from the parents to their children [24]. Besides, this functional pattern may apply to astrophysics to find the inertial connection between different particles or galaxies.

![Figure 2. Influence of genetic programming tree with different signs [24].](image-url)
3. Particle in Cell method
The particle in cell (PIC) method, which throws particles into the finite mesh, is a method to simulate the specific regional kinetic and potential. The PIC method may simultaneously illuminate the continuous and discrete method and apply to astrophysics, material science, biology.

3.1. N-body simulation for superparticles
N discrete simulation particles are randomly drop in large scale space, which can simulate the diffusion of the particles. Solving the Poisson-Vlasov Equation,

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} v + \frac{\partial f}{\partial v} \left( -\frac{\partial \phi}{\partial x} \right) = 0
\]

one acquires the dynamical information of each particle. For the N-body problem, following the Gravitational equation and some correction due to the collision among particles. The flow cha of the algorithm follows:

1. Randomly select a particle in the space.
2. Regarding the force from the other particles as potential force, calculate the gravitational interaction from other N-1 particles, and let the direct sum of the interaction as a function.
3. Move the particle by the total force and change the velocity and coordinates until the total force \(f=0\).
4. Repeat from step1.

Therefore, the final equilibrium is shown. The Figure 3 describes the N-body simulation by inserted the particle and keep a balance between different particles [25].

Figure 3. The N-body simulation from \(z=8\) to \(z=3\), the left picture depicts the distribution of halos and the right picture draws the related particle distribution [25].
3.2 Particle-Particle Particle Mesh
The Particle-Particle Particle Mesh (P3M) method, based on N-body simulation, computes the potential for selected particles among the rest particles by mesh. In this method, the potential is classical for the particle that is too far away from each other. I.e., the Poisson equation can be applied to solve the equation. The compensation for the particle that is closed together is combining the two particles into one particle (particle merge). Based on the given dynamical model and potential, the trace is predictable. Nevertheless, the accuracy is based on the density of the mesh. Recently, an adaptive mesh method contains the equilibrium between accuracy and the computational burden [26].

3.3 Tree-Algorithm
The Tree-Algorithm is a bisection method applied in more than one dimension. On a given two-dimensional picture, several particles are thrown into this plane to analyze the properties of those particles. Firstly, the picture is equally divided into four parts by two lines and now, the picture has $2 \times 2$ mesh, the particles are counted in each part. When precise data is needed, the $2 \times 2$ mesh is applied to the divided part based on the previous $2 \times 2$ mesh. The outcome roughly appears. Compared to the P3M method, the algorithm's complexity is lower in large amounts of particles [26]. Figure 4 illustrates how the Tree-algorithm processing [27].

![Figure 4. The tree model recognizing process [27].](image)

4. The Magnetic hydrodynamics method
The Magnetic Hydrodynamics method (MHD) explores the behavior of electrically conducting fluid in a magnetic field. MHD is based on several equations: mass continuity equation, Cauchy momentum equation, energy equation, Maxwell equation, Ohm’s law, Lorentz equation. Recently, the properties are improved with high performance based on computational methods [28]. The MHD applies to astrophysics, material science and engineering, and Targeted therapy in medical science [29-31]. In terms of astrophysics, there are several parts listed.

4.1 The magnetic hydrodynamics method in Astrophysics
In the past several decades, the MHD is a powerful instrument for interpreting the movement and trace of the electromagnetic objection and precisely predicting the tendency by high order parameters. The MHD was also designed to determine the magnetic effect on the planet by Adaptive Mesh Refinement (AMR). The Figure 5 exhibits the practical MHD simulation with the adaptive size [32].
Figure 5. The empty space occupies the large-scale grids and the space which existed the fluid is meticulously refined [32].

4.2 The relativistic magnetic hydrodynamics method
In recent years, the general relativistic mixed with the MHD method attracts scientist’s attention. Some developments concentrate on the general relativistic correct MHD due to the universe scale [33,34]. The large mass planets distort the kinetic trace of light by the general relativity. The gravitational wave verified the effects [35].

5. Hybrid Method
Prior studies have noted the importance of the Monte-Carlo method, PIC method, and MHD method. Some simulations that concatenate two or three of the above method are shown in this section.

5.1 Monte-Carlo and MHD method
Ref. [36] designed a scheme to distinguish unknown parameters in the astrophysics model, which combines the simulation result with the experimental result (shown in Figure 6). Fortunately, the elliptical and spiral galaxies are formed with the Illustris simulation, based on the adjustment of the model. This method applies the Monte-Carlo method and MHD method.
5.2 Monte-Carlo, PIC, and MHD method

Some developments concentrate on magnetized fusion Plasmas [37]. The Completely ionized particle applies the PIC method and the weakly ionized particles. The inner properties of the plasma relevant to the magnetic field due to the electricity. The hybrid method initially employs the ions as the large particles while electrons as massless granular. Furthermore, magnetic perturbation is considered in this model. Eventually, the Monte-Carlo method supports the kinetic and potential relationships among those

Figure 6. (a) The simulation of the universe generates the classification of spiral galaxies with ellipticals, irregular, and Disk galaxies. (b) Hubble Space Telescope image in a certain range. (c) simulation observation from Illustris[36].
particles. When the magnetic property appears, the fluid constricts to be a closed ring [38]. A typical ion density distribution based on such a method is given in Figure 7.

![Ion density distribution constricts to a closed loop][38].

6. Conclusions and Perspectives
In summary, this paper roughly introduces the application of Monte-Carlo, PIC, MHD, and hybrid methods in astrophysics. Specifically, the Monte-Carlo methods are reviewed in applying the theory of statistical and linear or nonlinear regression. The PIC simulations are discussed for the N-body simulation and the tree algorithm employed in astrophysics. The MHD is presented in consideration of the several equations and the relativistic to consider the time and space. Moreover, the hybrid methods combine with two or three methods are demonstrated, which can delve into the discrete and continuous question to weigh the distribution. In the future, the hybrid method is more favorable to solve the unknown phenomenon because of the complete self-consistent field. The boost of computational hardware simultaneously resolves the brute force to solve previously inaccessible problems.

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