In the original BCS theory of superconductivity (SC) excited states are separated from the ground state by an energy gap. SC does not necessarily lead to a fully gapped energy spectrum of quasiparticle excitations. It is well known that for unconventional SC, where the mechanism of SC is different from the BCS, non-trivial symmetry of the order parameter allows the existence of point or line nodes on the Fermi surface (FS) in momentum space\[1, 2, 3]. For ordinary s-wave pairing gapless SC appears in the presence of paramagnetic impurities, when the time-reversal $t \to -t$ symmetry is broken\[4]. Neglecting the orbital effects, for a SC placed in magnetic field one expects the realization of inhomogeneous Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state\[5, 6], in which the superconducting gap also passes through zeros for each Fermi surface (FS) as:

$$\Delta_{i}(p) = -T \sum_{n,k,p'} U_{ik}(p,p') F_{k}(i\omega_{n}, p'),$$  \hspace{0.5cm} (1)

where $F_{k}(i\omega_{n}, p)$ is the anomalous Gor’kov function, $k, i$ are band indices, $U_{ik}$ is the interaction between bands $i$ and $k$.

The type of the superconducting state below $T_{c}$ depends on the choice of the pairing ansatz:

$$U_{ik}(p,p') = \chi(\varphi) U_{ik}(\varphi'),$$ \hspace{0.5cm} (2)

where $\chi(\varphi)$ is the appropriate irreducible representation; we take $\chi(\varphi)$ below as a const (1) for the s-wave pairing, or as cos (2\varphi) for the d-wave pairing. Solutions of the multiband Gor’kov equations\[7, 8, 9, 10] for the Green’s functions in magnetic field $I = \mu_{B}B$ can be written as:

$$\hat{F}_{k}^{\dagger}(\omega_{n}, p) = \frac{i\delta^{2} \Delta_{i}^{*}(p)}{(i\omega_{n} - I\delta^{2})^{2} - \xi_{k}(p)^{2} - |\Delta_{k}(p)|^{2}}$$  \hspace{0.5cm} (3)

and

$$\hat{G}_{k}(\omega_{n}, p) = \frac{i\omega_{n} + \xi_{k}(p) - I\delta^{2}}{(i\omega_{n} - I\delta^{2})^{2} - \xi_{k}(p)^{2} - |\Delta_{k}(p)|^{2}}$$ \hspace{0.5cm} (4)

In this letter we show that similar features in the energy spectrum should naturally appear in quasi-2D multiband SC, such as some organic SC or “115” heavy fermion materials, CeMIn$_5$ ($M = Co, Ir$). Most recent experimental activity has been devoted to finding the LOFF state\[11, 12, 13, 14, 15] in CeCoIn$_5$. The latter material is characterized by quasi-2D Fermi surfaces and a multiband energy spectrum. We consider a model of quasi-2D two-band SC of both s-wave and d-wave type in magnetic field applied parallel to the plane. Apart from the LOFF state in higher magnetic fields, for $\mu_{B}B \sim I$ comparable with the smaller gap we observe whole regions of open FS, similar to the situation considered in Ref.\[7]. For an s-wave SC we investigate analytically in detail this low-temperature and low field region of the phase diagram.

We adopt the standard multiband interaction scheme (see, e.g., Ref.\[16]). The matrix elements $U_{ik}(p,p')$ for the interaction enter the definitions of the gaps, $\Delta_{i}(p)$ for each Fermi surface (FS) as:

$$\Delta_{i}(p) = -T \sum_{n,k,p'} U_{ik}(p,p') F_{k}(i\omega_{n}, p'),$$  \hspace{0.5cm} (1)

where $F_{k}(i\omega_{n}, p)$ is the anomalous Gor’kov function, $k, i$ are band indices, $U_{ik}$ is the interaction between bands $i$ and $k$.

The type of the superconducting state below $T_{c}$ depends on the choice of the pairing ansatz:

$$U_{ik}(p,p') = \chi(\varphi) U_{ik}(\varphi'),$$ \hspace{0.5cm} (2)

where $\chi(\varphi)$ is the appropriate irreducible representation; we take $\chi(\varphi)$ below as a const (1) for the s-wave pairing, or as cos (2\varphi) for the d-wave pairing. Solutions of the multiband Gor’kov equations\[7, 8, 9, 10] for the Green’s functions in magnetic field $I = \mu_{B}B$ can be written as:

$$\hat{F}_{k}^{\dagger}(\omega_{n}, p) = \frac{i\delta^{2} \Delta_{i}^{*}(p)}{(i\omega_{n} - I\delta^{2})^{2} - \xi_{k}(p)^{2} - |\Delta_{k}(p)|^{2}}$$  \hspace{0.5cm} (3)

and

$$\hat{G}_{k}(\omega_{n}, p) = \frac{i\omega_{n} + \xi_{k}(p) - I\delta^{2}}{(i\omega_{n} - I\delta^{2})^{2} - \xi_{k}(p)^{2} - |\Delta_{k}(p)|^{2}}$$ \hspace{0.5cm} (4)
The energy spectrum of the system for excitations near each FS is given by the poles of the $G_k(\omega_n, \mathbf{p})$:

$$
\tilde{E}_k(p) = \sqrt{\xi_k(p)^2 + |\Delta_k(p)|^2} + I\sigma^2
$$

(5)

The bands with different $k$ are coupled by the gap equation, Eq. (1). We consider below a model with 2 FS.

While the search for the LOFF-state commonly starts from the side of higher fields, we study the effects in small magnetic fields of the order of the smaller gap, $\Delta_2$. Our main interest lies in the field range where $\Delta_2 < I \ll \Delta_1$. Once the magnetic field $I = \mu_B B$ exceeds the smaller gap, then, according to Eq. (5), electron- and hole-pockets will open, forming an ungapped area near the second Fermi surface (FS2). In the s-wave case this process is accompanied by a weak 1st order phase transition. For a d-wave SC the energy gaps $\Delta_\lambda(p)$ have line nodes and associated gapless states from the start. Nevertheless, depending on the strength of interactions, an irregular behavior of the gap amplitude as a function of magnetic field also occurs in some region of model parameters (see Fig. (2)), which indicates a 1-st order transition.

It is convenient to express the solution of Eq. (1) in terms of dimensionless coupling constants, $\lambda_{ik} = U_{ik} v_k$, where $v_k$ is the density of states on the $k$-th Fermi surface (FSk). The linearized gap equation Eq. (1) leads to the familiar instability curve for $T_c$, which we find, is independent of the number of FS involved:

$$
\ln \frac{T_c}{T_{c0}} = \Psi \left( \frac{1}{2} \right) - Re \left[ \Psi \left( \frac{1}{2} + i \frac{I}{2\pi T_c} \right) \right].
$$

(6)

Here $T_{c0}$ is the superconducting transition temperature without the magnetic field,

$$
T_{c0} = \frac{2\Delta_1}{\pi} e^{1/g} \quad (g < 0)
$$

$$
g = \frac{\lambda_{11} \lambda_{22} - \lambda_{12} \lambda_{21}}{\lambda_{11} + \lambda_{22} + (\lambda_{11} - \lambda_{22})^2 + 4\lambda_{12} \lambda_{21}},
$$

(8)

$\gamma \simeq 1.781$, and $\Lambda$ is the upper cut-off for the interactions in Eq. (2). However, the total $(T, B)$ phase diagram for two bands changes significantly, especially at lower temperatures and fields. Some main qualitative changes in the physics of multiband SC in this area can already be seen in a simplified model with $\Delta_1$ as the primary gap, and $\Delta_2 \ll \Delta_1$ induced by the SC order on FS1 [21]. When $I$ is close to the primary energy gap, $\Delta_1$, an inhomogeneous LOFF state will appear [22, 23]. For $\Delta_1 \simeq \Delta_2$ there could be significant modifications for the LOFF state, and the boundaries of the LOFF state on the $(T, B)$ phase diagram. We assume that two gaps differ enough for the LOFF state not to change significantly from the single-band model [22, 23]. Below we consider in more detail the low-field region of the $(T, B)$-plane.

In the weak coupling approach, $g \ll 1$, the ratio of $\Delta_2(T, B)$, the driven gap, and $\Delta_1(T, B)$, the primary gap, which we define as model parameter $t$, sets in at

$T_c$ and is temperature- and magnetic field-independent:

$$
\frac{\Delta_2}{\Delta_1} = \frac{t}{2\lambda_{12}} = \frac{2\lambda_{12}}{\lambda_{22} - \lambda_{11} + \sqrt{(\lambda_{11} - \lambda_{22})^2 + 4\lambda_{12} \lambda_{21}}},
$$

(9)

Eq. (10) for the s-wave case can be easily solved analytically at $T = 0$. Introducing new parameters,

$$
\alpha = t^2 v_2 (v_1 + v_2 t^2)^{-1}, \quad \Delta_0 \equiv (\pi/\gamma) T_{c0},
$$

we find two different solutions for Eq. (1) for $I \leq I_{cr} = \Delta_0 (1 + \sqrt{1 - t^2})^{-\alpha}$, and no solutions for $I > I_{cr}$. The first solution is

$$
\Delta_1 = \Delta_{10} = t^{-\alpha} \Delta_0, \quad I < \Delta_{20}
$$

$$
\frac{\Delta_1}{\Delta_0} = \left( \frac{1}{I + \sqrt{I^2 - I^2 \Delta_1^2}} \right)^\alpha, \quad \Delta_{20} < I < I_{cr}. \quad (12)
$$

Here $\Delta_{10} = \Delta_0 (T = 0, B = 0)$, $i = 1, 2$. The second solution exists for magnetic fields $\Delta_0 / 2 < I < I_{cr}$,

$$
(I + \sqrt{I^2 - \Delta_1^2})^{1-\alpha} (I + \sqrt{I^2 - I^2 \Delta_1^2})^\alpha = \Delta_0,
$$

(13)

and is the familiar [3, 13] unstable solution for the energy gap in high magnetic fields. The two solutions are plotted in Fig. 1. The re-entrant behavior in magnetic fields $I \simeq \Delta_{20} = t \Delta_{10}$ clearly indicates the 1-st order character of transition into the gapless state. At this transition an open Fermi surface is formed, according to the energy spectrum given by Eq. (6). The position of the 1-st order phase transition is found from the energy at $T = 0$, which for $I < \Delta_{20}$ has the usual form,

$$
\Delta E \equiv E_S - E_{N0} = - (v_1 \Delta_1^2 + v_2 \Delta_2^2)/4.
$$

(14)
In the vicinity of this transition Eq.(12) is considerably simplified,

$$\tau_{I} = (1/2)\tau_{I}^2\alpha^{-2} + \tau_{\Delta}. \quad (16)$$

Expanding the energy Eq.(15) in the vicinity of this transition, we find:

$$E_{S} - E_{NO} = E_{0}(1 - 2\tau_{I}^2 - (4/3)\tau_{I}^3\alpha^{-2}), \quad (17)$$

where $E_{0}$ is the energy of the superconducting state at $I = 0$. The cubic terms in the energy, and the form of Eq.(16) clearly indicate a first order transition. After a simple calculation, we find that the 1-st order transition occurs at $\tau_{Icr} = -3\alpha^2/8$. The energy gap $\tau_{\Delta}$ changes abruptly from $\tau_{\Delta} = 0$ to $\tau_{\Delta} = -3\alpha^2/2$, which corresponds to a metamagnetic transition, with a paramagnetic jump in the magnetic moment $M = (3\alpha/2)\mu_{B}\nu_{2}\Delta_{20}$ (or to a sudden appearance of the finite DOS). We find a simple expression for the magnetic moment for $I > I_{cr}$ in the vicinity of this transition:

$$M = \mu_{B}\nu_{2}\Delta_{20}(\alpha + \gamma\alpha^2 + 2\tau_{I}). \quad (18)$$

The paramagnetic moment in the s-wave case always appears at $I > \Delta_{20}$ in a 1-st order phase transition. Nevertheless, in case of the driven second gap, $\Delta_{20} \ll \Delta_{10}$, the 1-st order transition is weak, and so is the corresponding change in the SC order parameter, $\Delta_{2} = t\Delta_{1}$. In the first approximation this change can be neglected[21]. Then the temperature- and field-dependence of the magnetic moment is completely described by the standard[24] formulas that follow from the energy spectrum Eq.(10), where $\Delta_{2} = \Delta_{20}$ is regarded as a constant. For example, a simple analytic expression for the magnetic moment in the s-wave case at $T = 0$ is:

$$M = \mu_{B}\nu_{2}\Delta_{20}(\alpha + \gamma\alpha^2 + 2\tau_{I}). \quad (19)$$

Note that $M = 0$ for $I < \Delta_{2}$. For a d-wave case, FS2 gives a contribution for $I$ above and below $\Delta_{2}$:

$$M_{2} = \frac{2}{\pi} \mu_{B}\nu_{2} \int_{0}^{\pi} d\varphi \sqrt{T^{2} - \Delta_{2}^{2}\sin^{2}\varphi}, \quad (20)$$

where the upper limit is $A = \pi/2$ for $I > \Delta_{2}$, or $A = \arcsin(I/\Delta_{2})$ for $I < \Delta_{2}$. This is an elliptic integral of second kind. Note that in the d-wave case FS1 gives the usual nodal contribution, $M_{1} = 0.5\mu_{B}\nu_{1}I^{2}/\Delta_{1}$. The density of states for FS2 in the s-wave case is also given by a simple formula:

$$\nu_{2}(I) = \nu_{2}I/\sqrt{T^{2} - \Delta_{2}^{2}}. \quad (21)$$

In the d-wave case one has to introduce the appropriate angular averages of this result, and add the familiar nodal contribution from the first Fermi surface[24].
In summary, we have shown that a gapless Fermi spectrum characterized by open Fermi surfaces is an inevitable feature for a quasi-2D multiband superconductor placed into a large enough field parallel to the plane. The new state is fully analogous to the one studied in Ref. [16] for the unbalanced pairing problem. Unlike Ref. [16], however, such a gapless state sets in as the 1-st order transition in increased magnetic field. Measurements of the specific heat in applied field are the most direct way to observe the effect in s-wave superconductors, such as Hg-NbSe$_2$. The transition also leads to a metamagnetic jump in the magnetization. For a d-wave pairing, because of the nodes, gapless excitations are present even without external field. As the field is increased, the open Fermi surfaces develop gradually, although character of the process may depend on the interaction parameters. Applied fields should be low enough for these phenomena not to interact with the LOFF state. It is broadly believed that the properties of CeCoIn$_5$ may be close enough to a two-dimensional model to display the inhomogeneous LOFF state. If CeCoIn$_5$, indeed, belongs to the strongly quasi-2D class, the low field properties studied above should manifest themselves as well. Then, if interactions in CeCoIn$_5$ were strong enough to result in a 1-st order transition of Fig. 2, the latter could be observed best by calorimetric measurements, as for the s-wave pairing. If not, then one may rely on the NMR methods for the observation of a rather non-monotonic field behavior for non-linear susceptibility shown in the insert of Fig. 2 (we have not considered possible implications of the effect for thermal conductivity in the presence of a magnetic field). The above effects should be expected for other 2D organic compounds. An ideal realization of the scheme would be superconductivity localized at the surface.

The authors are thankful to L. Balicas and Z. Fisk for helpful discussions. This work was supported (VB) by TAML at the University of Tennessee and (LPG) by NHFML through the NSF Cooperative agreement No. DMR-008473 and the State of Florida.

[1] G. E. Volovik, L.P. Gor’kov, JETP lett. 39, 674 (1984) [Pis’ma Zh. Eksp. Teor. Fiz. 39, 550(1984)]; Zh. Eksp. Teor. Fiz. 88, 1412 (1985) [Sov. Phys. - JETP 61, 843 (1985)].
[2] L.P. Gor’kov, Sov. Sci. Reviews A 9, 1 (1987).
[3] F. W. Anderson, Phys. Rev. B 30, 4000 (1984).
[4] A. A. Abrikosov, L.P. Gor’kov, Sov. Phys. JETP 12, 1243 (1960) [Zh. Eksper. Teor. Fiz. 39, 1781 (1960)].
[5] A.I. Larkin and Yu.N. Ovchinnikov, Sov. Phys. JETP 20, 762 (1965) [Zh. Eksper. Teor. Fiz. 47, 1136 (1964)].
[6] P. Fulde, R.A. Ferrell, Phys. Rev. 135, A550 (1964).
[7] W. V. Liu and F. Wilczek, Phys. Rev. Lett. 90, 047002 (2003).
[8] E. Gubankova, E.G. Mishchenko, and F. Wilczek, Phys. Rev. Lett. 94, 110402 (2005); E. Gubankova, W.V. Liu, and F. Wilczek, Phys. Rev. Lett. 91, 032001 (2003).
[9] G. Sarma, J. Phys. Chem. Solids 24, 1029 (1963).
[10] S. Takada and T.Izuyama, Progr. Theor. Phys. 41, 635 (1969).
[11] Kun Yang, cond-mat/0603190 (2006).
[12] T. Mizushima, K. Machida, M. Ichioka, Phys. Rev. Lett. 95, 117003 (2005).
[13] K. Kakuyanagi et al., Phys. Rev. Lett. 94, 047602 (2005).
[14] V. F. Mitrovic et al., cond-mat/0605305 (2006).
[15] C. Capan et al., Phys. Rev. B 70, 134513 (2004).
[16] D.F. Agterberg, V. Barzykin, and L.P. Gor’kov, Phys. Rev. B 60, 14868 (1999).
[17] A.A. Abrikosov, L.P. Gor’kov, I.E. Dzyaloshinskii, “Methods of Quantum Field Theory in Statistical Physics”, Dover, New York (1963).
[18] L.P. Gor’kov and A.I. Rusinov, Sov. Phys. JETP 19, 922 (1964) [Zh. Eksper. Teor. Fiz. 46, 1363 (1964)].
[19] M. E. Zhitomirsky and V.-H. Dao, Phys. Rev. B 69, 054508 (2003).
[20] L.P. Gor’kov and T.T. Mnatsakanov, Sov. Phys. JETP 36, 361 (1973) [Zh. Eksper. Teor. Fiz. 63, 684 (1972)].
[21] V. Barzykin and L.P. Gor’kov, cond-mat/0606191 (2006).
[22] A.B. Vorontsov, J.A. Sauls, and M.J. Graf, Phys. Rev. B 72, 184501 (2005).
[23] H. Burkhart and D. Rainer, Ann. Physik 3, 181 (1994).
[24] V. P. Mineev and K. V. Samokhin “Introduction to Un-conventional Superconductivity”, Gordon and Breach, Amsterdam, The Netherlands (1999).
[25] T. Yokoya et al., Science 294, 2518 (2001).
[26] A.V. Sologubenko, I.L. Landau, H.R. Ott, A. Bilusic, A. Smontara, H. Berger, Phys. Rev. Lett. 91, 197005 (2003).
[27] In the real situation of a Q2D material the carriers “inside the plane”, in addition to the external field, would, of course, experience a contribution from the inhomogeneous field of penetrating vortices.
[28] V. Barzykin and L.P. Gor’kov, Phys. Rev. Lett. 89, 227002 (2002).