Fast Radio Burst Pulse Widths, Scattering, and Distances

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ABSTRACT

By comparing the dispersion measures and pulse widths of two fast radio bursts (FRB) for which pulse widths were measured we show that if the dispersion measures resulted from propagation through the intergalactic medium at cosmological distances and the widths were a consequence of scattering by single thin screens, then the screens’ electron densities were \( \gtrsim 20/\text{cm}^3 \), 10\(^8\) times the intergalactic density. This problem is resolved if the radiation scattered close to its source, where high densities are possible. Observation of dispersion indices close to their low density limit of \(-2\) sets a model-independent upper bound on the electron density and a lower bound on the size of the dispersive plasma cloud, excluding terrestrial or Solar System origin. Much of the dispersion measures may be attributed to scattering regions about 1 AU from the sources, with electron densities \( \sim 3 \times 10^8/\text{cm}^3 \). Transparency to inverse bremsstrahlung requires that scattering occurred in regions with temperature \( \gtrsim 10^7 \text{K} \), consistent with the environment of an energetic outburst. The inferred parameters are only marginally consistent and suggest re-examination of the assumed relation between dispersion measure and distance. Origin in an ionized starburst or protogalaxy is suggested, but statistical arguments exclude compact young SNR in the Galactic neighborhood.

Subject headings: radio continuum: general — intergalactic medium — plasmas — scattering

1. Introduction

Thornton, et al. (2013) discovered four fast radio bursts (FRB) whose large dispersion measures (DM) and high Galactic latitudes indicated that their sources were at cosmological distances. FRB 110220 had an observed dedispersed width \( W = 5.6 \pm 0.1 \text{ ms} \) (at a frequency \( \nu = 1300 \text{ MHz} \)), while only upper limits on \( W \) were found for the remaining three FRB. Burke-Spolaor & Bannister (2014) discovered FRB 011025 for which \( W = 9.4 \pm 0.2 \text{ ms} \). Fitting \( W \propto \nu^\beta \), both these FRB had \( \beta \) in agreement with the predicted \( \beta = -4 \) for multipath propagation spreading in a scattering plasma medium. Two other FRB, 010621 (Keane, et al. 2012) and 121102 (Spitler, et al. 2014), had
measured widths but these were not attributed to scattering; these FRB occurred at low Galactic latitudes, hinting that they may be associated with our Galaxy. We do not discuss them explicitly, but their parameters are similar to those of FRB 110220 and FRB 011025, with similar implications if the same assumptions are made.

This paper explores the implications of the assumptions that the measured dispersion measures of FRB result from passage through intergalactic plasma and indicate cosmological distances and that the dedispersed pulse widths are a consequence of scattering. Section 2 presents our central result, that the scattering responsible for the pulse widths of these FRB did not occur in the general intergalactic medium. Section 3 discusses where the scattering may have occurred. Section 4 sets bounds on the parameters of the scattering region. Section 5 sets lower limits on the number of active FRB sources from the absence of repetitions. Section 6 considers the possible association of FRB with supernovae or their remnants. Section 7 considers constraints that can be placed on the scattering region by the requirement that it be transparent to the radiation. Section 8 obtains limits on the plasma density in the scattering region that can be inferred from the observed dispersion indices. Section 9 discusses the applicability and implications of the requirement that the dispersive medium be stable against gravitational collapse, and suggests that FRB originate in ionized starbursts or protogalaxies whose plasma provides most of their dispersion measures. Section 10 contains a concluding discussion. Most of the results of this paper, with the exception of those of Sections 6 and 9, are based on the interpretation of the dedispersed pulse widths as the result of scattering. Because useful data are available for two FRB, we present numerical results as ordered pairs (110220, 011025).

2. Pulse Widths

We make the approximation that FRB (110220, 011025) were at distances $D = (2.8, 2.2)$ Gpc (Thornton, et al., 2013; Burke-Spolaor & Bannister, 2014) in a flat static universe. For the estimated redshifts $z = (0.81, 0.61)$ this only introduces an error of a factor $O(1)$, less than other uncertainties. We approximate the propagation paths as produced by a single scattering at a distance $aD$ from us and $(1 - a)D$ from the source. If the scattering angle $\Delta \theta \ll 1$ then the angles $\phi \approx (1 - a)\Delta \theta$ and $\chi \approx a\Delta \theta$; the geometry is shown in Fig. 1.

We assume that the origin of the pulse width $W$ is dispersion in propagation path lengths, not in group velocity along different paths. The total propagation delay corresponding to the reported (after subtracting estimated Galactic contributions) DM $\approx (910, 680)$ pc-cm$^{-3}$ (Thornton, et al., 2013; Burke-Spolaor & Bannister, 2014)

$$\Delta t_{DM} = \frac{2\pi e^2}{m_e c^2} \omega^2 DM \approx (2.2, 1.7) s \approx (400, 180) W$$

at 1300 MHz. This assumption implies an assumption about the homogeneity of the intervening medium, in which most of the dispersion is presumed to originate, on scales $O(\Delta \theta D) \ll D$ of the
Fig. 1.— Path of scattered radiation

separation, perpendicular to the propagation direction, of the weakly scattered paths.

The incremental delay attributable to scattering by an angle $\Delta \theta$ is

$$W \approx \frac{D}{2c}(\Delta \theta)^2 a(1 - a). \quad (2)$$

Then

$$\Delta \theta \gtrsim \sqrt{\frac{8cW}{D}} \approx (4 \times 10^{-10}, 6 \times 10^{-10}), \quad (3)$$

with the minimum value obtained for $a = 1/2$. The results (2), (3) reproduce (52), (53) of Kulkarni, et al. (2014).

Refraction by a surface tilted from the direction of propagation by an angle $\theta$, with a ratio $n$ of refractive indices between the two sides of the surface, leads to a deflection, unless $|\pi/2 - \theta| \lesssim O(\sqrt{|1 - n|}) \ll 1$,

$$\Delta \theta \approx |1 - n| \tan \theta. \quad (4)$$

In general $\tan \theta = O(1)$. Taking this as an approximate equality

$$4 \times 10^{-10} \lesssim \Delta \theta \approx 1 - n \approx \frac{1}{2} \frac{\delta \omega_p^2}{\omega^2}. \quad (5)$$

From the expression for the plasma frequency

$$\delta \omega_p^2 = \frac{4\pi \delta n e^2}{m_e}, \quad (6)$$
where $\delta n_e$ is the magnitude of fluctuations in the electron density, we find

\[ \delta n_e \gtrapprox \frac{m_e \omega^2}{2\pi e^2} \sqrt{\frac{8eW}{D}} \approx (17, 24) \text{ cm}^{-3}. \] (7)

### 3. Where Scattered?

The inferred $\delta n_e$ (7) is more than seven orders of magnitude greater than the maximum cosmologically allowable intergalactic $\langle n_e \rangle \leq 2 \times 10^{-7} (1 + z)^3 \text{ cm}^{-3} = \mathcal{O}(10^{-6} \text{ cm}^{-3})$. A single scattering screen must have $\delta n_e \gtrsim 10^7 \langle n_e \rangle$, a density much too great to be confined in intergalactic space.

The pulse width could be explained as the result of $\mathcal{O}(10^{14})$ independent uncorrelated scatterings, each by a scatterer with $\delta n_e \sim \langle n_e \rangle$. The scattering regions must be $\lesssim 10^{14}$ cm in size. There is no evident source of such fine scale structure in the intergalactic medium, and it would be difficult to maintain because at intergalactic densities the particle mean free paths are $\mathcal{O}(10^{18} T_e^2 \text{ cm})$, much longer than the putative structure size. It would be smoothed rapidly by free particle flow, both of electrons and of ions. Henceforth we assume $\mathcal{O}(1)$ scattering between emitter and detection, rather than a large number of independent scatterings.

An additional argument against the hypothesis of intergalactic scattering is that if scattering were distributed through the intergalactic medium, all FRB should be broadened, with $W \propto D^2$ (2). This is inconsistent with the upper bounds on $W$ found (Thornton, et al. 2013) for three other FRB. The large inferred $\delta n_e$ requires that the pulse was scattered in a dense localized region.

This cannot have been general intergalactic space. It is unlikely to have been Galactic because the giant nanoshots of at least two Galactic pulsars (Hankins, et al. 2003, Hankins & Eilek 2007, Soglasnov, et al. 2004) exclude Galactic broadening of more than $\mathcal{O}(2–10 \text{ ns})$. The proportionality of $W$ to $D^2$ suggests that Galactic scattering might still be consistent with the observed ms widths of two FRB at cosmological distances, but their high Galactic latitudes indicate smaller $W$ than would be obtained by $W \propto D$ scaling from Galactic pulsars, and the upper bounds on the widths of three FRB argue against an origin in the general interstellar medium through which all FRB ray paths travel.

Scattering must have occurred close to the source. We write $a = 1 - \epsilon$, with $\epsilon \ll 1$. Then (2) becomes

\[ 1 \gtrapprox \Delta \theta \approx \sqrt{\frac{2cW}{\epsilon D}} \approx (2 \times 10^{-10}, 3 \times 10^{-10}) \epsilon^{-1/2}, \] (8)

or

\[ \epsilon D \gtrapprox 2cW \approx (3 \times 10^8, 6 \times 10^8) \text{ cm}. \] (9)

Then (5), (6) and (8) yield

\[ \epsilon \delta n_e^2 \gtrapprox \frac{eW \omega^4 m_e^2}{2\pi^2 D c^4} \approx (70, 150) \text{ cm}^{-6}. \] (10)
This also illustrates the familiar result $W \propto \omega^{-4}$, independent of any specific model of the distribution of $\delta n_e$, provided all the structure occurs on scales $\gg \lambda/2\pi$ so that geometrical optics applies.

4. Fluctuation Density and Location

Make the plausible, but unproven, assumption $\delta n_e \sim n_e$. For scattering occurring within a distance $\epsilon D$ of the source (or the observer)

$$n_e \epsilon D \equiv \text{DM}_{\text{local}} \leq \text{DM} = (910, 680) \text{ pc-cm}^{-3},$$

where $\text{DM}_{\text{local}}$ is the dispersion local to the source. Use (10) to eliminate $n_e$, obtaining

$$\epsilon D \lesssim \frac{2 \pi^2 \text{DM}^2 \epsilon^4}{c W m_e^2 \omega^4} \approx (1.3 \times 10^{13}, 4.4 \times 10^{12}) \text{ cm};$$

the corresponding density of the scattering matter

$$\delta n_e \sim n_e \gtrapprox \frac{c W m_e^2 \omega^4}{2 \pi^2 \text{DM}_{\text{local}} \epsilon^4} \approx (2 \times 10^8, 5 \times 10^8) \text{ cm}^{-3}.$$ (13)

These limits correspond to $\text{DM}_{\text{local}} \approx \text{DM}$, in which case $\text{DM}$ cannot be used to infer the distance because an unknown fraction, perhaps nearly all, of the dispersion is local to the source. For FRB for which only upper bounds on $W$ exist, there is neither a lower bound on $n_e$ nor an upper bound on $\epsilon D$.

If we take the lower bound (9) on $\epsilon D$ rather than the upper bound (12) then, using (10),

$$\delta n_e \lesssim \frac{\omega^2 m_e}{(2\pi^2)} \approx 4 \times 10^{10} \text{ cm}^{-3}.$$ (14)

independent of the parameters of any particular FRB. This amounts, aside from a factor of two, to the condition that the radiation propagate through the scattering plasma. If $n_e$ approaches this bound then $\omega_p \approx \omega$ and $\Delta \theta = O(1)$. Such a dense cloud may also have been the source of the FRB emission (Katz 2014a). The limits (9) and (13) imply

$$\text{DM}_{\text{local}} \gtrapprox \frac{c^2 W^2 \omega^4 m_e^2}{\pi^2 \text{DM} \epsilon^4} \approx (0.02, 0.09) \text{ pc-cm}^{-3}.$$ (15)

The two limits (11) and (15) bound the possible range of $\text{DM}_{\text{local}}$, and correspond to the bounds (13) and (14) on $\delta n_e$. The local contribution to the dispersion measure may, but need not, be very small.

5. Number of FRB Sources

There are two constraints on the number of presently active FRB sources $N_{\text{sources}} \equiv BT$, where $B$ is their birth rate within the volume from which FRB may be detected and $T$ is their
active lifetime (consistent with the known properties of FRB, such as their dispersion measure). If the bursts occur stochastically, without any latency period following a burst, then the absence of coincidences among \( N_{\text{FRB}} \) observed FRB implies

\[
N_{\text{sources}} \gtrsim N_{\text{FRB}}^2. \tag{16}
\]

The absence of repetitions of any individual FRB implies

\[
N_{\text{sources}} \gtrsim \Omega_{\text{FRB}} \tau_{\text{min}}, \tag{17}
\]

where \( \Omega_{\text{FRB}} \) is the all-sky FRB rate and \( \tau_{\text{min}} \) is the empirical lower bound on the repetition time of an individual source. \( \text{Thornton, et al.} \ (2013) \) estimate \( \Omega_{\text{FRB}} \sim 0.3/s \) while \( \text{Kulkarni, et al.} \ (2014) \) estimate \( \Omega_{\text{FRB}} \sim 0.1/s \); the spread between these two values is an indication of their uncertainty.

If the bursts are stochastic then \( \tau_{\text{min}} \approx \tau_{\text{tot}} \), the total time beams pointed in the known directions to FRB, summed over all FRB, without observing a repetition\(^1\). On the other hand, if there is a latency period between FRB from a single source then \( \tau_{\text{min}} \approx \tau \), the longest duration of continuous observation of an individual FRB location. The conditions (16) and (17) may be used to test models of \( N_{\text{sources}} \) against the empirical parameters \( N_{\text{FRB}}, \tau_{\text{min}} \) and \( \Omega_{\text{FRB}} \), and thereby to constrain models of the sources, of their astronomical environments, and of their distances. If more than one FRB were observed from the same direction then (16), with the right hand side divided by the number of coincidences, would become an approximate equality and consistency with (17) could be tested.

6. Supernovae and Their Remnants

The discovery \( \text{Keane, et al.} \ (2012; \text{Spitler, et al.} \ (2014) \) of two apparent FRB at low Galactic latitude suggests they may be cosmologically local and associated with our Galaxy. \( \text{Kulkarni, et al.} \ (2014) \) suggest an association with the giant flares of SGR, with dispersion originating in the surrounding young SNR, and a lower bound on FRB distances of 300 kpc (for an assumed temperature of the dispersive plasma of 8000°K). The local dispersion measure of a source at the center of a spherical cloud of ionized gas of mass \( M \) and radius \( R \) is

\[
DM_{\text{local}} = 818 \frac{M}{M_\odot} \left( \frac{R}{0.1 \text{ pc}} \right)^{-2} f \text{ pc-cm}^{-3}, \tag{18}
\]

where \( f = 1 \) for a homogeneous sphere and \( f = 1/3 \) for a thin shell, implying \( R \approx 0.1 \) pc for a SNR, lost stellar envelope, etc., that provides much of the dispersion measure of a FRB. Note, however, that by the argument of Section 4 such a cloud cannot explain the observed pulse widths.

\(^1\)It is not necessary that a beam be pointed to a single FRB for this time because, if they all have the same properties, staring at each position at which an FRB has been observed is equivalent.
The age and lifetime $T$ of an expanding cloud

$$T \approx \frac{R}{V} \approx 30 \frac{R}{0.1 \text{ pc}} \frac{3000 \text{ km/s}}{V} y \approx 30 \sqrt{\frac{f(M/M_{\odot})}{\text{DM}_{1000}}} \frac{3000 \text{ km/s}}{V} y,$$

where $V$ is the expansion velocity. At $R = 0.1$ pc only $\sim 10^{-4} n M_{\odot}$ of interstellar material will have been swept up, for an interstellar density of $n$ atoms/cm$^3$, so $V$ is nearly the initial explosion velocity. If $V$ is within the range 3000–30000 km/s of SN ejecta then the age of the dispersing cloud $T \lesssim 30$ years. If FRB are found within such clouds then if repetitive bursts are observed their dispersion measures will decrease monotonically and smoothly according to (18) with $R = V t$. The hypothesis that the dispersion is produced by very young SNR is contradicted by the absence of recent SN at the high Galactic latitudes of most FRB.

The number of SNR with ages $t < T$ (19) associated with our Galaxy (out to distances $\sim 1$ Mpc) is inferred from the SN rate to be $N_{\text{SNR}} t < T \lesssim O(1)$. The hypothesis that the dispersion measures of FRB result from propagation through such young and nearby SNR is also contradicted by the fact that no repeaters are observed among seven FRB when only $\lesssim O(1)$ SNR young enough to meet this requirement likely exists within 1 Mpc. Further, the all-sky FRB rate $\Omega_{\text{FRB}} \sim 0.1$–0.3/s would imply a repetition time of an individual source $\tau \sim N_{\text{sources}}/\Omega_{\text{FRB}} = N_{\text{SNR}} t < T/\Omega_{\text{FRB}} \sim 3$–10 s. If the shells expand more slowly than typical SN then $T$, $N_{\text{sources}}$ and $\tau$ are increase $\propto 1/V$. Such rapid repetitions of FRB may be tested empirically.

If FRB are associated with SN, at a rate of order one-to-one (the FRB do not repeat), comparison of the rates of the two classes of events shows that their distances must be cosmological: The SN rate is estimated (Sharon, et al. 2007) to be $\Omega_{\text{SN}} \approx 0.098 \times 10^{-12}/M_{\odot}$-y. Standard cosmological parameters indicate a local baryon density $\rho_{\text{baryon}} = 1.9 \times 10^{-27} M_{\odot}/\text{cm}^3$ and then a SN rate of $1.9 \times 10^{-77}/\text{cm}^3$-y. Comparison to the all-sky FRB rate $\Omega_{\text{FRB}} \approx 0.1$–0.3/s indicates that SN out to a distance of $\sim 1$ Gpc must contribute. Unless the volumetric FRB rate is much higher than the SN rate, as might be the case if FRB are giant pulsar pulses, SGR outbursts (Kulkarni, et al. 2014), or other phenomena that repeat many times in their sources’ lifetimes, FRB originate at cosmological distances, even if much of their dispersion measures is local to their sources.

The absence of obvious correlation with cosmologically local structure such as the Coma cluster indicates a lower bound on their distance $O(100 \text{ Mpc})$, but does not directly indicate either the FRB mechanism or their astronomical environment.

7. Inverse Bremsstrahlung

If $\delta n_e \approx n_e$ then (10) implies an inverse bremsstrahlung optical depth ($\tau_{ff} \propto \epsilon D n_e^2$) in the scattering medium, independent of the particular values of $\epsilon$ and $\delta n_e$. Aside from the medium

$$\text{Inverse Bremsstrahlung}$$
temperature, it depends only on observed quantities:

\[
\tau_{ff} \approx \frac{4}{3} \sqrt{\frac{\pi n_e^2 D e_6}{3 k_B T k_B T c m_e^3 / 2}} \sim 8 \sqrt{\frac{\pi m_e W \omega^2 e_2}{3 k_B T}} g_{ff} \approx (2.3, 3.9) \left(\frac{10^7 \circ \text{K}}{T}\right)^{3/2},
\]

where the Gaunt factor \( g_{ff} \approx 11.5 \) (Spitzer 1962). In order that \( \tau_{ff} \lesssim 1 \) it is necessary that either \( T \gtrsim 10^7 \circ \text{K} \) or \( \langle \delta n_e^2 \rangle \gg \langle n_e \rangle^2 \) (the scattering matter be a thin dense screen) in a much more dilute medium. The first possibility is consistent with a region of high energy density; the second is also possible but would vitiate the assumption \( \delta n_e \sim n_e \). If \( W \) is not measured then \( \tau_{ff} \lesssim 1 \) still imposes a temperature-dependent constraint on the emission measure \( \int n_e^2 d\ell = \int n_e^2 D d\epsilon \) along the path between the source and the observer.

8. Dispersion Index

The dispersion index \( \alpha \) is defined by the dispersion delay \( \Delta t \propto \nu^\alpha \). For FRB110220 \( \alpha = -2.003 \pm 0.006 \) (Thornton, et al. 2013) while for FRB 011025 \( \alpha = -2.00 \pm 0.01 \) (Burke-Spolaor & Bannister 2014). The value of \( \alpha \) provides an additional constraint on the density \( n_e \) in the scattering region.

Expansion of the dispersion relation for electromagnetic waves in a cold (nonrelativistic) plasma in powers of \( \omega_p^2 / \omega^2 \ll 1 \) yields (Katz 2014b)

\[
\Delta t = \int \frac{d\ell}{c} \frac{\omega_p^2}{2 \omega^2} \left( 1 + \frac{3 \omega_p^2}{4 \omega^2} + \cdots \right).
\]

Then

\[
\alpha \equiv \frac{d \ln \Delta t}{d \ln \omega} = -2 - \frac{3 \omega_p^2}{2 \omega^2} + \cdots .
\]

In order to constrain \( n_e \) in the scattering region we must allow for the fact that it contributes only a fraction of the (extra-Galactic) dispersion of the pulse. The remainder, perhaps nearly all, is attributed to intergalactic propagation, for which the higher terms in (22) are negligible. From the observed bounds on \( \alpha \), (22) yields

\[
\frac{\omega_p^2}{\omega^2} \frac{\text{DM}_{\text{local}}}{\text{DM}} \leq \frac{2}{3} \max (-\alpha - 2) = (0.006, 0.007),
\]

where \( \max (-\alpha - 2) \approx 0.01 \) is the observed upper bound on \( -\alpha - 2 \) for the FRB for which values are reported. These upper bounds on \( \text{DM}_{\text{local}} \) are consistent with the lower bounds (15).

Using (6), (10), (23) and (22),

\[
n_e \approx \sqrt{-\frac{(\alpha - 2) c W e_6 m_e^2}{12 \pi^3 e_6 \text{DM}}} \lesssim (1.6 \times 10^8, 2.6 \times 10^8) \text{ cm}^{-3},
\]

where the inequality results from the most negative values of \( \alpha (-2.009,-2.01) \) permitted by the data. The two bounds (13) and (24) are slightly inconsistent for both FRB, but because of the
necessarily rough approximations made, this discrepancy is not significant. Their nearness does indicate that \( n_e \) is near the upper limit of the range allowed by (13) and that a significant fraction of the dispersion measure may be local to the source. Unlike the temperature-dependent bound (20), the bound (24) is independent of temperature (provided it is nonrelativistic).

The lower bound (17) on \( \delta n_e \) may be combined with the upper bound on \( n_e \) implied by the maximum value of \( (\alpha - 2) \) in (24), assuming \( \delta n_e \sim n_e \), to yield a lower bound

\[
D \gtrsim \frac{24\pi^2 e^2 DM}{\max (-\alpha - 2) \omega^2 m_e} \approx 10^{14} \text{ cm.} \tag{25}
\]

This bound is more than the statement that the FRB occur outside the inner Solar System. Combined with (12) it also indicates that, whatever the distance to the FRB, scattering occurs over a small fraction of the distance to them.

### 9. Jeans Limit

If the dispersion occurs in a stable static plasma cloud, then the Jeans condition that the cloud be stable against gravitational collapse imposes further constraints on its parameters:

\[
\sqrt{\frac{GM}{R}} \lesssim c_s = \sqrt{\frac{5k_B T (1 + \mu)}{3m_p}}, \tag{26}
\]

where \( c_s \) is the sound speed and \( \mu \approx 0.85 \) is the number of electrons per baryon. Substituting \( M \approx R^3 m_p n_e / \mu \) and \( DM \approx n_e R \), we find

\[
R \lesssim \frac{5(1 + \mu) \mu k_B T}{3GDM m_p^2} \approx 5 \times 10^{21} \frac{T_{8000}}{DM_{1000}} \text{ cm} \tag{27}
\]

and

\[
n_e \sim \frac{DM}{R} \gtrsim 0.6 \frac{DM_{1000}^2}{T_{8000}^{-1}} \text{ cm}^{-3}, \tag{28}
\]

where we normalize the temperature \( T_{8000} \equiv T/8000 \text{ K} \) (following Kulkarni, et al. (2014)) and the dispersion measure \( DM_{1000} \equiv DM/1000 \text{ pc-cm}^{-3} \), and assume complete ionization and cosmic abundances. The corresponding mass

\[
M \approx \frac{25k_B T (1 + \mu)^2}{9G^2 m_p^3 DM} \approx 8 \times 10^7 \frac{T_{8000}^2}{DM_{1000}} M_\odot. \tag{29}
\]

The hydrodynamic time

\[
T_J \sim \frac{R}{c_s} \gtrsim \sqrt{\frac{5k_B T (1 + \mu)}{3m_p} \frac{\mu}{Gm_p DM}} \approx 10^6 \frac{T_{8000}^{1/2}}{DM_{1000}} \text{ yr} \tag{30}
\]

has no explicit dependence on the unknown parameters \( n_e, R \) and \( M \). \( T_J \) is long enough to avoid the statistical problems (Section 6) posed by attributing the dispersion measures to young Galactic
SNR, whose youth implies that only a very few are active with the observed dispersion measures at any time. The dispersive cloud could be more compact and dense than the bounds (27) and (28), perhaps by a large factor.

These bounds are consistent with dense compact clouds in the Galactic neighborhood while avoiding the rapid expansion and short lifetime implied by attributing them to rapidly expanding young SNR. Much smaller $R$ and $M$ and larger $n_e$ than the bounds are possible. The bounds also admit a protogalaxy or starburst ionized by an initial generation of hot luminous stars, providing the observed dispersion measures. Such sites are plausible locales for FRB, but give no clues to the origin of the FRB themselves beyond indicating a relation with massive stars and high rates of star formation and death. As argued in Sections 3 and 4, these clouds cannot be the origin of the observed pulse widths, but may contribute a major part of the total dispersion measures.

10. Discussion

The central results (7), (10) and (12) of this paper are that the pulses of FRB 110220 and 011025, the two FRB with pulse widths attributed to scattering, scattered in high density regions close to their sources. We also infer, from the closeness of the dispersion indices to their low density value of $-2$, that dispersion occurred in a region where the electron density was close to the bound (13) and that a significant part of the dispersion occurred close to the source. The distances inferred from the dispersion measures are then only upper bounds, although the fact that most FRB occurred at high Galactic latitudes implies that they are either extra-Galactic or very close ($\lesssim 100$ pc).

These results depend on the assumption $\delta n_e \sim n_e$. This assumption could be violated in many ways. For example, the pulse width might have been produced by the reverberation of radio emission in a cavity of size $< cW$ if the walls of the cavity had a plasma density above the critical density $n_e \approx 1.2 \times 10^{10} \text{ cm}^{-3}$ for 1300 MHz radiation and the interior had a lower, perhaps much lower, density.

This paper began by assuming that the FRB are at the cosmological distances inferred from their dispersion measures, allowing only for the estimated Galactic dispersions (Thornton, et al. 2013). As shown in Section 8, this is only marginally consistent with the dispersion indices. The fact that for both FRB (110220 and 011025) whose pulse widths are attributed to scattering the consistency between (13) and (24) is only marginal should be of concern. It is a priori surprising that both objects should be found in the same corner of the allowable parameter space, the range of plasma densities allowed by the pulse widths, which hints at a fundamental problem with the model.

This suggests that for some, as yet undiscovered, FRB, either a significant deviation from the low density plasma dispersion index $\alpha = -2$ will be found, or there will be a frank inconsistency between the observed $\alpha$ and that inferred from (22) and (24). Such an inconsistency may require
reconsideration of the interpretation of the pulse widths as the effects of scattering or of the dispersion measures as indicating cosmological distances, as Burke-Spolaor & Bannister (2014) and Karbelkar (2014) have done on other grounds. If so, the distances are smaller than inferred from the dispersion measures, perhaps by large factors.

If we reject the inference of cosmological distances then various bounds change. The lower bound on $\delta n_e$ scales $\propto D^{-1/2}$, the estimate of $\epsilon\delta n_e^2$ scales $\propto D^{-1}$, but the bounds on density (13), (23) and (24) are independent of $D$. At $D \sim 30$ kpc (7) becomes $\delta n_e \gtrsim 6 \times 10^3$ cm$^{-3}$, consistent with a young SNR (Kulkarni, et al. 2014). The bounds (20), (24) and (25) exclude origin in local plasmas, such as meteor trails, lightning and electric discharges.

Finally, we note that radar systems use chirped emission, compressed upon reception into narrow pulses, in order to obtain accurate range measurements without requiring excessive peak transmitted powers. The observation of FRB in a single beam at Parkes, in contrast to perytons (Burke-Spolaor, et al. 2011), indicates a distance $\gtrsim 20$ km, outside the first Fresnel zone, consistent with a radar satellite. There is no obvious reason for a radar to have a chirp $\omega \propto t^{-1/2}$ as observed, nor is there obvious reason not. However, the observed dispersed pulse durations of several tenths of a second would imply, for monostatic radar, target distances of at least half that many light seconds to avoid interference of the transmission with the received scattered radiation. At such distances $\sim 10^{10}$ cm the return would be undetectably weak. In contrast, bistatic radar can use arbitrarily long pulses. The pulse repetition intervals would have to have been longer than the lengths of time the radars were anywhere in the 13 beams of the Parkes Multibeam Pulsar Survey (about 0.3 s for a radar in low Earth orbit moving perpendicularly to a beam), yet the pulse durations must have been shorter than the time required to cross a single beam. This explanation would also require at least as many radar satellites, each with a different chirp rate, as FRB because each FRB had a different dispersion measure, or satellites whose chirp rates were variable in some non-obvious manner. This combination of requirements makes the hypothesis of interference by an orbital chirped source implausible.

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