The valley splitting in Si two-dimensional electron systems is studied using Si/SiGe single quantum wells (QWs) with different well widths. The energy gaps for 4 and 5.3 nm QWs, obtained from the temperature dependence of the longitudinal resistivity at the Landau level filling factor \( \nu = 1 \), are much larger than those for 10 and 20 nm QWs. This is consistent with the well-width dependence of the bare valley splitting estimated from the comparison with the Zeeman splitting in the Shubnikov-de Haas oscillations.

Low disorder silicon 2DESs are realized in high-quality strained silicon quantum wells (QWs) of Si/Si\(_{1-x}\)Ge\(_x\) heterostructures. Recent calculations indicate that the valley splitting in Si/SiGe heterostructures strongly depends on the well width. However, the well-width dependence has not been studied experimentally, while the effect of lateral confinement was investigated using a quantum point contact. In this letter, we report magnetotransport measurements on silicon QWs with different well widths. The results indicate that the valley splitting is much larger in narrower QWs than in wide QWs.

Four samples of well width \( w = 4, 5.3, 10 \), and 20 nm, were grown by the molecular beam epitaxy technique. A strained Si QW is sandwiched between relaxed Si\(_{0.8}\)Ge\(_{0.2}\) layers. The electrons are provided by a Sb-\( \delta \)-doped layer 20 nm above the channel. The electron density \( N_s \) can be controlled by varying back-gate voltage \( V_{BG} \) of a \( p \)-type Si(001) substrate 2.1 \( \mu \)m below the channel at 20 K after brief illumination of red light emitting diode.

Sample preparation and characterization were described in detail elsewhere. Standard four-probe resistivity measurements were performed for 1.8 \( \times \) 0.2 mm\(^2\) Hall bars in a pumped \(^3\)He refrigerator.

Figure 1(a) shows the longitudinal resistivity \( \rho_{xx} \) versus perpendicular magnetic field \( B \) of the 20 nm sample at 0.38 K. The integer quantum Hall (QH) states are clearly observed for the Landau level (LL) filling factors \( \nu = 1, 2 \), and 4. As shown in Fig. 1(b), they are related to the valley splitting \( E_v \), the Zeeman splitting \( E_z = g\mu_B B \) and the cyclotron gap \( h\omega_c \), respectively. Here \( g \) is the g-factor and \( \omega_c \) is the cyclotron frequency. The \( \nu = 3 \) QH state is not observed at this temperature. On the other hand, the valley splitting at \( \nu = 3 \) appears in narrower QWs as shown in Figs. 1(c) and 1(d). In these samples, \( \rho_{xx} \) reaches zero in the vicinity of \( \nu = 1 \).

In order to study the valley splitting quantitatively, we investigate the temperature dependence of the \( \rho_{xx} \) minimum. Typical data are shown in Fig. 2(a). The results are described well by the thermally activated form \( \rho_{xx} = \rho_0 \exp (-\Delta / k_B T) \), where \( \Delta \) is the energy gap. Note that the slope in the Arrhenius plots corresponds to \( -\Delta / 2 \) since the chemical potential lies at the middle

![FIG. 1: (a) Low temperature longitudinal resistivity \( \rho_{xx} \) of the 20 nm sample. The electron density is in units of \( 10^{15} \) m\(^{-2}\). Arrows indicate magnetic fields for integer LL filling factors. (b) Single particle LL energy diagram. LLs spaced by the cyclotron energy \( h\omega_c \) are split by \( g\mu_B B \) and \( E_v \). The Fermi energy is located in the valley splitting for odd integer values of \( \nu = N_s h/eB \). For \( \nu = 4n \) and \( \nu = 4n - 2 \) \((n = 1, 2, 3, ...)\), it is in the cyclotron gap and the Zeeman splitting, respectively. (c) \( \rho_{xx} \) of the 5.3 nm sample. (d) \( \rho_{xx} \) of the 4 nm sample.](image)
that the effective thickness of the electron layer does not

do not contradict them. Due to modulation doping and

as a function of \(N_s\). The energy gaps of the \(\nu = 1\) QH

of \(\nu = 1\) and low temperature.

In Fig. 2(b), \(\Delta\) obtained for different samples is shown

of the gap for integer \(\nu\) and low temperature.

\(\nu = 1\) QH state for the 4 and 5.3 nm samples are much larger than

the theories. However, we should consider the effects of
electron-electron Coulomb interactions since they play

the crossover region is estimated to be about 2 MV/m in our samples. If

used the simple approximation of Ref. 9, the crossover length. This is

an order magnitude larger than the single

For small \(g\), the lowest-energy charged excitations at \(\nu = 1\)

in the Hartree-Fock (HF) approximation for ideal (zero-

and LL mixing. Disorder broadening of LLs is expected to be strong in low

state for the 4 and 5.3 nm samples are much larger than

is calculated to be \(\Delta_{SK} = 88\) K for \(\nu = 1\) and \(\Delta_{SK} = 51\) K for

of disorder, finite thickness, and LL mixing. Disorder broadening of LLs is

is the magnetic length. This is exactly half the exchange energy cost of a

the disorder broadening effect. However, it does not account for the

theories. However, we should consider the effects of
electron-electron Coulomb interactions since they play

in usual 2DESs. In the case of GaAs QH systems, the Zeeman splitting observed at \(\nu = 1\) or

3 is an order magnitude larger than the single

particle Zeeman splitting \(|g\mu_B B|\) with the bare \(g\)-factor

\(g = -0.44\). Furthermore, it is widely accepted that

exchange interactions stabilize the ferromagnetic state at \(\nu = odd\) even in the limit of \(g = 0\). For

are spin-texture excitations known as Skyrmions. In the

the Hartree-Fock (HF) approximation for ideal (zero-

2DESs with wave functions projected onto a single LL, the gap for \(g = 0\) is given by

\[
\Delta_{SK} = \frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{e^2}{4\pi\epsilon_0\kappa_{sc}\ell_B},
\]

where \(\kappa_{sc}\) is the dielectric constant in the semiconduc-
tor and \(\ell_B \equiv (\hbar/eB)^{1/2} = (\nu/2\pi N_s)^{1/2}\) is the magnetic

In 2DESs, it is calculated to be \(\Delta_{SK} = 88\) K for \(\nu = 1\) and \(\Delta_{SK} = 51\) K for

Note that we consider the lowest Landau orbital even for \(\nu = 3\) in Si 2DESs.

While Eq. (1) gives only the lowest exchange energy cost, these values are much larger than the experimental ones. The discrepancy between Eq. (1) and experiment is well known for the Zeeman splitting in GaAs QH systems, and discussed in relation to the effects of disorder, finite thickness, and LL mixing. Disorder broadening of LLs is expected to be strong in low mobility samples. In our case, the zero-field mobility \(\mu\) decreases rapidly with decreasing \(w\). The 4 nm sample has \(\mu = 0.62\) m²/Vs at \(N_s = 1.33 \times 10^{15}\) m⁻². The half width of the level broadening \(\Gamma = 1/2\tau\) is calculated to be 5.7 K, where \(\tau\) is the scattering time. As shown in Fig. 2(b), the measured gap for \(w = 4\) nm is slightly smaller than that for \(w = 5.3\) nm. This may be caused by the disorder broadening effect. However, it does not account for the observed small gaps of the 10 nm and 20 nm samples with much higher mobility. The finite thickness of the electron layer leads to the softening of the Coulomb interaction and reduces \(\Delta_{SK}\). However, the correction for Eq. (1) is calculated to be small in our QWs. LL mixing may be the most important effect for our samples. In Ref. 22, it is shown that the Skyrmion gap is substantially reduced in comparison with the HF calculations when the ratio of the typical Coulomb energy \(E_C = e^2/(4\pi\epsilon_0\kappa_{sc}\ell_B)\) to \(\hbar\omega_c\) becomes comparable with or larger than unity. In Si 2DESs, the ratio is given by \(r_C = 3.9(\nu/N_s[10^{15} m^{-2}])^{1/2}\). Thus the effect of LL mixing is expected to be strong in our samples. As \(B\) decreases, the Coulomb energy decreases and the effect of LL mixing increases. This explains the observed \(N_s\) dependence of \(\Delta\) and the large difference between \(\nu = 1\) and \(\nu = 3\).

To obtain the bare valley splitting \(E_v\) from the mea-
TABLE I: Bare valley splitting estimated from the Shubnikov-de Haas oscillations. The average electric field $\bar{F}$ in the QW region at the corresponding back-gate voltage is also presented for future calculations.

| $w$ (nm) | $E_\nu$ (K) | $\bar{F}$ (MV/m) |
|----------|-------------|------------------|
| 4        | $1.8 - 3.6$ | 1.0              |
| 5.3      | $1.4 \pm 0.4$ | 1.3          |
| 10       | $< 0.7$     | 2.0              |
| 20       | $< 0.2$     | 2.3              |

sured gap $\Delta$, we need to evaluate the exchange enhancement. To our knowledge, however, there is no available theory for large $r_c$ at present. Instead, here we estimate the bare valley splitting from the comparison with the Zeeman splitting in the Shubnikov-de Haas oscillations. We assume that $E_\nu$ does not depend on $B$ and $N_s$, and the exchange enhancement and the disorder smearing occur equally for the valley and Zeeman splittings. For the 4 nm sample, the $\nu = 3$ QH state is stronger than the $\nu = 2$ QH state, as shown in Fig. 1(d), although the enhancement is expected to be smaller for larger $\nu$. Thus the bare valley splitting $E_\nu$ should be larger than $g\mu_B B - E_\nu$. This leads to $E_\nu > g\mu_B B/2 = 1.8$ K with $g = +2.0$ and $B = 2.7$ T. On the other hand, we do not expect a crossing between the second and third LLs at $\nu = 2$ since the QH state becomes stronger as the magnetic field is tilted away from normal to the 2D plane (not shown here). This indicates $E_\nu < g\mu_B B = 3.6$ K. For the 5.3 nm sample, we were able to compare the $\nu = 2$ and $\nu = 3$ QH states at the same $B$ by adjusting $N_s$. From the measured gaps, we obtain $E_\nu = 1.4 \pm 0.4$ K taking into account small $N_s$-dependence of the mobility. For the 10 nm sample, some of the present authors have studied the Shubnikov-de Haas oscillations in low magnetic fields below 0.1 K. The Zeeman splitting can be observed even at $B = 0.29$ T ($\nu = 34$) while the valley splitting is smeared out. Thus the bare valley splitting in this sample is estimated to be less than 0.2 K. Using similar procedures, we obtain $E_\nu < 0.7$ K for the 10 nm sample. The results are summarized in Table I. Although there is some uncertainty, it is confirmed that the bare valley splitting increases rapidly with decreasing well width.

In summary, we have studied the valley splitting in Si/SiGe heterostructure samples with different well widths. The energy gap obtained from the $T$-dependence of $\rho_{xx}$ is much larger in narrow QWs than in wide QWs, while the degree of the exchange enhancement is unknown at present. The bare valley splitting estimated from the comparison with the Zeeman splitting also exhibits a rapid increase with decreasing well width.

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