Small-scale structure of cold dark matter
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We investigate the clumping of cold dark matter (CDM) at small scales. If the CDM particle is the neutralino, we find that collisional damping during its kinetic decoupling from the radiation fluid and free streaming introduce a small-scale cut-off in the primordial power spectrum of CDM. This cut-off sets the scale for the very first CDM objects in the Universe, which we expect to have a mass of $\sim 10^{-12} M_{\odot}$. For non-thermal CDM candidates, such as axions, wimpzillas, or primordial black holes, the cosmological QCD transition might induce features in the primordial spectrum at similar mass scales.

1. Introduction

Dark matter with an equation of state that has been non-relativistic ($p \ll \rho$) since structure formation started (around matter-radiation equality) is called cold dark matter \cite{1}. CDM gives rise to the hierarchical formation of large structures ($> 1$ Mpc) in the Universe, i.e. small structures form first and grow to larger structures later. The growth of CDM density fluctuations is suppressed during the radiation dominated epoch. Thus the rms CDM density fluctuations go like $k^3$ at large scales, but increase only logarithmically with wavenumber at scales well below the Hubble scale at matter-radiation equality.

At very small scales ($\ll 1$ Mpc), the power spectrum and the evolution of CDM density fluctuations has not been discussed in detail so far, although the understanding of the small-scale behavior of CDM density fluctuations is essential for a realistic estimate of expected rates in CDM searches. The reason is that analytic calculations in this deeply non-linear regime are very challenging and numerical simulations do not have the dynamical range to resolve scales as small as the solar system. A priori one can say that there has to be a cut-off in the CDM power spectrum at some very small scale, otherwise the energy density in the fluctuations itself would be infinity.

The nature of CDM is unknown. Here we mention three popular candidates with very different properties: The first one is the lightest neutralino, which probably is the lightest supersymmetric particle \cite{2}. Its mass is expected to be in the range $10^{-6}$ eV – $10^{-2}$ eV \cite{3} and contributes to the dark matter if the mass is small. Axions interact much weaker than weakly interacting particles, the interactions are suppressed by the Peccei-Quinn scale, which is $\sim 10^{12}$ GeV. Thus axions are never in thermal equilibrium with the radiation fluid. An example of small-scale structure in CDM has been found from the initial misalignment mechanism of axions by Hogan and Rees \cite{4}. It turns out that, if the Peccei-Quinn scale is below the reheating temperature after inflation, large isocurvature
perturbations in the axion density are created once the axion mass is switched on during the QCD transition. It has been shown that axion mini-clusters with $\sim 10^{-12} M_\odot$ and radii of $\sim 0.1 R_\odot$ might emerge, which may be observed by means of pico- and femtolensing.

Let us mention a third CDM candidate: primordial black holes. Their mass should be $> 10^{-6} M_\odot$ in order to survive until today. Primordial black holes interact with the rest of the Universe via gravity only. They may be found or excluded by gravitational lensing.

2. Damping scales for CDM

For any thermal CDM species there are two mechanisms that contribute to the damping of small-scale fluctuations: During the kinetic decoupling of CDM from the radiation fluid the mean free path is finite and thus collisional damping occurs. After the kinetic decoupling has been completed free streaming can further wash out the remaining fluctuations.

For neutralinos, chemical freeze out happens at $\sim m/20 > 2$ GeV. In contrast kinetic decoupling happens at much smaller temperatures, because elastic interactions with the radiation fluid are possible at temperatures as low as 1 MeV. However, due to the large momentum of the neutralinos, many collisions are needed for a significant change of the momentum. It turns out that $N \sim m/T$ collisions can keep the neutralinos in kinetic equilibrium and thus the relaxation time can be estimated as $\tau \sim N \tau_{\text{coll}}$, where the collision time for a bino is given by

$$\tau_{\text{coll}} \approx \left[ 5.5 \left( \frac{G_F m_W^2}{M^2 - m^2} \right)^2 T^5 \right]^{-1},$$

with $M$ being the slepton mass and $m$ the mass of the bino. With a slepton mass $M = 200$ GeV and a bino mass of $m = 100$ GeV, the relaxation time is given by $\tau \sim (10$ MeV$/T)^4 t_H$, where $t_H$ is the Hubble time. Kinetic decoupling of neutralinos happens at $T \sim 10$ MeV.

We incorporate dissipative phenomena by describing the CDM as an imperfect fluid. The coefficients of heat conduction, shear and bulk viscosity are estimated to be $m \chi \sim \eta \sim \zeta \sim n T \tau$, where $n$ is the number density of CDM particles. We find that the damping of density perturbations goes as

$$\delta \propto \exp \left[ - \left( \frac{M_D}{M} \right)^{0.3} \right],$$

where the damping scale depends on the mass of the neutralino and the slepton masses and is typically $M_D = 10^{-13} M_\odot - 10^{-10} M_\odot$. For comparison, the CDM mass within a Hubble patch is $\sim 10^{-4} M_\odot$ at $T \sim 10$ MeV.

Free streaming leads to additional damping. The velocity of neutralinos right after kinetic decoupling is $v \sim (T/m)^{1/2} \sim 10^{-2}$. Free streaming also gives rise to exponential damping, due to the velocity dispersion. The typical free streaming scale is $10^{-12} M_\odot - 10^{-10} M_\odot$. Thus both damping mechanisms operate approximately at the same scale. The power spectrum of neutralino CDM is cut off at $M < 10^{-12} M_\odot$.

The mechanism of collisional damping also works for CDM in the form of a heavy neutrino ($\sim 1$ TeV), but it does not work for wimpzillas, because these are too heavy to ever be in thermal equilibrium. Free streaming induces a cut-off in the power spectrum for all mentioned CDM candidates. The scale and the strength of the damping depend on the masses and the primordial velocity distributions.

3. QCD induced CDM clumps

Besides damping mechanisms there are also processes that might enhance the primordial CDM spectrum at small scales. One is the formation of axion mini-clusters that we mentioned already in the introduction.

Together with Schmid and Widerin one of the present authors has found that large amplifications of density fluctuations might be induced by the QCD transition at scales $10^{-20} M_\odot < M < 10^{-10} M_\odot$. The mechanism is the following: During a first-order QCD transition the sound speed vanishes. Thus the density perturbations of the dominant radiation fluid go into free fall and create large peaks and dips in the spectrum, which grow at most linearly with the wavenumber. These peaks and dips produce huge
gravitational potentials. CDM falls into these gravitational wells. It is important to note that this amplification mechanism works for matter that is kinetically decoupled at the QCD transition around 150 MeV. For neutralinos this is not the case. Large inhomogeneities in the neutralinos would be washed out by collisional damping later on. A structure similar to the acoustic peaks in the photon-baryon fluid might survive. The large inhomogeneities in the radiation fluid are completely washed out during neutrino decoupling at \( \sim 1 \) MeV.

4. The first CDM objects

The smallest scales that survive damping are the first scales that go non-linear, thus these scales form the first gravitationally bound objects (apart from primordial black holes, if they exist) in the Universe. Let us estimate their size if the CDM is the neutralino. The scale is given by the cut-off at \( M \sim 10^{-12} M_\odot \). With a COBE normalized CDM spectrum we find the rms density fluctuations at equality to be

\[
\frac{\delta \rho}{\rho} \sim 2 \times 10^{-4} \left[ \ln \left( \frac{k_D}{k_{eq}} \right) \right]^{3/2} \left( \frac{k_D}{k_{eq}} \right)^{(n-1)/2}, \tag{3}
\]

which is \( 2 \times 10^{-2}(10^{-1}) \) for the spectral index \( n = 1(1.2) \). Thus these objects go nonlinear at or shortly after equality, at a redshift of \( \sim 10^2(10^3) \).

If we assume that these clouds are spherical they would collapse to a radius of \( 10^4(10^2) R_\odot \) today, which is by chance a very interesting scale for observations. Although \( 10^{-12} M_\odot \) in a volume of \((10^2 R_\odot)^3\) seems to be an extremely diluted cloud, the overdensity of such a cloud today would be \( \sim 10^{11} \). In more optimistic scenarios (larger tilt and/or additional peaks in the spectrum from the QCD transition) it is even possible that these clouds are so compact that pico- and femtolensing can be used to search for them.

It is unclear whether some of these first objects can survive up to today, this will be the subject of further studies. We think that understanding and revealing the small-scale structure of CDM might help us to learn more about the nature of CDM.

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