Magnitude of Solar Radiation Torque in the Transition Region from the Umbra to the Dark Shadow of the Earth

R E S Cabette, M C Zanardi and I Kolesnikov

1São Paulo Salesian University, UNISAL, Dom Bosco Street, 284, Lorena, SP, Brazil
2Federal University of ABC, UFABC, Avenue of the States, 5001, Santo André, SP, Brazil

E-mail: recabette@uol.com.br

Abstract. The analysis of solar radiation pressure force and its influence on the motion of artificial satellites has been developed by researchers. Accurate models to describe the influence of the Earth’s shadow on the torque and force due to solar radiation pressure have been presented. In this work the solar radiation torque (SRT) and its influence on the attitude of an artificial satellite are taken into account by the introduction of the Earth’s shadow function in the equations of motion. This function assumes a unitary value when the satellite is in the fully illuminated region of its orbit, and the value zero for the full shade region. The main objective of this study is to analyze the magnitude of SRT using the equations described by quaternions during a 35 day period and to compare the results with the satellite transition through the shadow region and the time interval in this region. The duration and transition through the shadow region were obtained using the software "Shadow Conditions of Earth Satellites". The formulation is applied to the Brazilian Data Collection Satellites SCD1 and SCD2, and the torque model is presented in terms of the satellite attitude quaternion, distance of the satellite to the Sun, orbital elements, right ascension and declination of the Sun.

1. Introduction

The study of the rotational motion of artificial satellites leads to a more precise results when the torques caused by external forces that influence their motion are included. In this paper, the force caused by the solar radiation pressure is one of the external forces that has been modeled, taking into account the effects of Earth’s shadow, and inserted in the equations in order to propagate the rotational motion of the artificial satellites.

In the literature several papers considered the influence of the solar radiation torque (SRT) on the attitude of the artificial satellite [3,6,7,9,11,12]. Most of them modeled the torque acting on the artificial satellite and determine analytical and numerical solutions for the dynamic system of equations describing the rotational motion of the satellite, including the shadow region of the Earth.

The goal of this paper is the analysis of the magnitude of the SRT during a 35 day period using the Brazilian Satellites SCD1 and SCD2 data. Here it is assumed that the orbital motion is known and given by the problem of two bodies and quaternions are used to represent the space orientation (attitude) of the satellite. The equations of motion are given by Euler’s equation, relating the rate of change of the components of the satellite angular velocity with the external torques, and by the attitude
kinematic equations, relating the rate of change of the components of the quaternion with the components of the angular velocity. To determine the components of the solar radiation torque a cylindrical satellite is admitted, so that the component of the torque on the Z axis of the system fixed in the satellite is zero. Therefore, the magnitude of the angular velocity is not affected by this torque. Also, the applications are made for a spin stabilized satellite.

In spite of the applications were performed for the spin stabilized satellites SCD1 and SCD2, this formulation can be apply for any kind of satellite since it uses the attitude quaternions and the components of the angular velocity.

2. Attitude Motion

The motion of a satellite is specified by its position, velocity, space orientation and angular velocity. The last two quantities describing the rotational motion around its center of mass, and the space orientation is called attitude and they are the focus of this paper.

The attitude expresses the relation between two coordinate systems, one system fixed in the satellite (herein given by principal axis of inertia of the satellite and named principal system) and another system having the same origin, but with axes parallel to a inertial reference system (here called equatorial system, with axes parallel to the Earth's equatorial system, with Z-axis along the north pole, XY plane being the Earth equatorial plane and the X-axis along the spring equinox). The Equator system, the orbital system and the orbital elements are represented in the Figure (1).

![Figure 1: Equatorial and Orbital systems: Ω - longitude of the ascending node, ω - argument of perigee, I - orbital inclination, v - true anomaly.](image)

If the satellite has a rotation around some axis, then its orientation or attitude changes every instant. Thus, the angular velocity and the attitude time derivatives are related through the kinetics equations.

There are many ways to represent the attitude, but in this study, the quaternions are used. The attitude quaternion \( q \) is a 4x1 vector given by [8,12]:

\[
q = [q_1 \quad q_2 \quad q_3 \quad q_4]^t = [\tilde{q} \quad q_4]^t,
\]  

(1)

where \( t \) represents the transposed of the matrix and it can be expressed in function of the rotation angle \( \phi \) and of the axis of rotation \( \vec{w} \):

\[
\tilde{q} = [q_1 \quad q_2 \quad q_3]^t = \sin(\phi/2) \vec{w} ; \quad q_4 = \cos(\phi/2).
\]  

(2)
There are two reasons to use the quaternion formulation: first one is to save computational time by avoiding integrating trigonometric functions and the second is to avoid the singularity problem that is characteristic of the Euler’s angles formulations [8,12].

2.1. Equation of the rotational motion
2.1.1. Dynamic Equations
The dynamics equations of the artificial satellite rotational motion express in the principal system $(Oxyz)$, are given by [8,12]:

\[
\begin{align*}
p &= \frac{N_x}{I_x} + \left( \frac{I_y - I_z}{I_x} \right) q r \\
q &= \frac{N_y}{I_y} + \left( \frac{I_z - I_x}{I_y} \right) p r \\
r &= \frac{N_z}{I_z} + \left( \frac{I_x - I_y}{I_z} \right) p q
\end{align*}
\]

(3)

where: $I_x$, $I_y$, and $I_z$ are the Principal Moments of Inertia of the satellite; $p$, $q$, $r$ and $N_x$, $N_y$, $N_z$ are the components of the angular velocity and the external torques in the principal system, respectively. These equations will be used in the attitude propagation with the inclusion of the components SRT in these equations. The mathematic model for SRT is discussed in the topic 2.2.

2.1.2. Kinematic Equation for the quaternion
The kinematic equations, describing the time derivatives of the attitude quaternion components, due to the rotation of the satellite, are given by [8,10]:

\[
\begin{align*}
q_1 &= \frac{1}{2} \left[ pq_4 - q q_3 + rq_2 \right] \\
q_2 &= \frac{1}{2} \left[ q q_4 - rq_1 + pq_3 \right] \\
q_3 &= \frac{1}{2} \left[ rq_4 - p q_2 + q q_1 \right] \\
q_4 &= -\frac{1}{2} \left[ pq_1 + q q_2 + rq_3 \right]
\end{align*}
\]

(4)

(5)

The great advantage of the use of quaternions in kinematic equations is the no existence of a zero denominator, which means there isn’t singularity, such as those occur in Euler angles [4].

2.2. Direct Solar Radiation Torque
Solar radiation pressure is generated by the continuous stream of photons that collide with the surface of the satellite. The rate of momentum of all incident photons in the satellite's surface originates the Solar Radiation Force, which can cause disturbances in the satellite orbit and may result in a torque depending on the physical and geometric characteristics of the satellite. However, the solar radiation torque exists only when the satellite is partially or fully illuminated [11,12]. Thus, there are three transition phases, represented in Figure 2:

- Transition through dark region (shadow);
- Transition through partially illuminated region (penumbra);
- Transition through fully illuminated region.

![Figure 2: Transition Phases: umbra, penumbra and illuminated regions.](image)
Solar radiation pressure force on the satellite is given by the surface integral over the entire surface of the satellite in which there is the incidence of sunlight [11]:

\[
\vec{F} = -\int_{S} \frac{K}{R^2} \left[ \frac{2\gamma}{3} (1 - \beta) \cos \theta + 4\beta \gamma \cos^2 \theta \right] \hat{n} + \left[ (1 - \beta \gamma) \cos \theta \right] \hat{u} \cdot dS
\]  

where \( K \) is the solar parameter and assume the value \( 1.01 \times 10^{17} \) kgm/s [6], \( \gamma \) is the total reflection coefficient, \( \beta \) is the specular reflection coefficient, \( \hat{n} \) is the normal unit vector of the satellite surface, \( \hat{u} \) is the direction of the incident solar flux over the element of surface \( dS \), \( \theta \) is the angle between \( \hat{n} \) and \( \hat{u} \) (see Figure 3), \( R \) is the distance of de element \( dS \) and the Sun (here is adopted equal to the distance of the satellite center to the Sun).

**Figure 3:** Geometry of the solar incident light over satellite surface \( dS \).

The element of Radiation Solar Torque (\( d\vec{N} \)) around the satellite center of mass (CM) due to the solar radiation force (\( \vec{F} \)) is given by:

\[
d\vec{N} = \vec{r} \times d\vec{F}
\]  

where \( \vec{r} \) is the position vector of the surface element \( dS \) of the satellite in relation to the CM. After performing the cross product, the solar radiation torque (SRT) over all the satellite surface can be given by [11]:

\[
\vec{N} = -\int_{S} \frac{K}{R^2} \left[ \frac{2\gamma}{3} (1 - \beta) \cos \theta + 4\beta \gamma \cos^2 \theta \right] \vec{r} \times \hat{n} + \left[ (1 - \beta \gamma) \cos \theta \right] \vec{r} \times \hat{u} \cdot dS
\]  

In the development of the integrals of the Eq. (8) it is necessary to consider the shape of the satellite. Depending on the satellite shape, the SRT can be zero if there is symmetry in the satellite or if the center of mass is coincident with the application point of the radiation pressure force. In this paper the considered satellite is cylindrical, with symmetry along the z-axis.

In order to get the components of the SRT in the body system it is assumed that the position vector \( \vec{R} \) is equal to the position vector \( \vec{R}^* \) of the satellite CM in relation to the Sun (see Figure 4) since the satellite dimensions are negligible in relation to the satellite distance to the Sun.
After some algebraic manipulations, which are associated with the developments of the integral in Eq. (8) for a cylindrical satellite, the $z$-axis component of the SRT is zero (because of the satellite symmetry) and the others SRT components in the principal system [3,5,6] are given by:

\[ N_z = \frac{K}{R^2} \left( \beta_1 \gamma_1 - \beta_2 \gamma_2 \right) \frac{h}{2} \pi \sigma^2 \{ a_s^2 R_x R_z + a_s r' (R_x a_{31} + R_z a_{21}) + r^2 a_{51} a_{31} \} \]  

\[ N_y = \frac{K}{R^4} \left( \beta_1 \gamma_1 - \beta_2 \gamma_2 \right) \frac{h}{2} \pi \sigma^2 \{ a_s^2 R_x R_z + a_s r' (R_x a_{31} + R_z a_{11}) + r^2 a_{41} a_{31} \} \]  

where $h$ and $\sigma$ are the height and radius of the circular cylindrical satellite; $\beta_i, \gamma_i, i = 1,2$ are specular and total reflection coefficients respectively, for each satellite surface (which assume constant values); $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}$ and $a_{33}$ are the elements of the rotation matrix between the principal system and orbital system [3,5,8] and depend on the attitude quaternion, $(q_1, q_2, q_3, q_4)$ and the orbital elements (orbital inclination, perigee argument, longitude of the ascendant node and true anomaly). $R_x, R_y, R_z$ give the Sun direction in the principal system and are expressed by:

\[ R_x = q_A \cos \delta_s \cos \alpha_s + q_D \cos \delta_s \sin \alpha_s + q_G \sin \delta_s \]  

\[ R_y = q_B \cos \delta_s \cos \alpha_s + q_E \cos \delta_s \sin \alpha_s + q_H \sin \delta_s \]  

\[ R_z = q_C \cos \delta_s \cos \alpha_s + q_F \cos \delta_s \sin \alpha_s + q_I \sin \delta_s \]  

where $\alpha_s, \delta_s$ are the Sun's right ascension and declination, respectively; $q_A, q_B, q_C, q_D, q_E, q_F, q_G, q_H, q_I$ are the elements of the rotation matrix between the principal system and Equatorial system, and depends on the attitude quaternion [8,10].

3. Numerical Simulations

In this paper the magnitude of the SRT is analyzed, with the numerical simulation developed in a FORTRAN code for a 35 day period, using the data of Brazilian Collecting Data Satellites (SCD1 and SCD2), supplied by INPE’s Satellite Control Center [1,4]. In order to get the satellite transition through umbra region and the permanence time in this region, it was used a MATLAB program available at the Orbital Mechanics with MATLAB website. This site can be used for many practical problems associated with orbital mechanics, applied astrodynamics and astrometry [2].

3.1. Data Satellites

a) SCD 1 – Initial Date: 1993 – July - 24  00:00:00 GMT
Semi-major axis= 7139615.83m  Eccentricity = 0.00453  Inclination = 25.00°
Right ascension of the ascending node= 260.43°  Argument of perigee= 260.23°
Mean anomaly = 102.89°  Moment of inertia in the x-axis=11.06kg.m²
Moment of inertia in the y axis=10.67kg.m²  Moment of inertia in the z axis=13.00kg.m²
b) SCD 2 – Initial Date: 2002 – February -01 00:00:00 GMT
Semi-major axis = 7133679.70m                                    Eccentricity = 0.00175    Inclination = 25.006°
Right ascension of the ascending node = 88.303°                     Argument of perigee = 288.207°
Mean anomaly = 300.030°                                    Moment of inertia in the x-axis =12.33kg.m^2
Moment of inertia in the y axis =12.35kg.m^2                        Moment of inertia in the z axis =14.50kg.m^2

Structural data (similar to SCD1 and SCD2):
Total reflection coefficient of the octagonal base = 0.7
Specular reflection coefficient of the octagonal base = 0.1
Total reflection coefficient of the shell of the prism = 0.5
Specular reflectance of the dark of the prism = 0.1

First we use the script available at the Orbital Mechanics with MATLAB website called "Shadow Conditions of Earth Satellites" that generates the results for the satellite passes through the shadow region and the permanence time in this region. This information is included in the FORTRAN code and the simulations are realized for each day.

3.2. Results for SCD1 and SCD2 Satellites
This section presents the results obtained in the simulations by MATLAB programs (satellite passage through the shadow region) and FORTRAN (calculus of the magnitude of solar radiation torque). These results are compared to verify the variation of the magnitude of the torque during the permanence in the shadow region.

The data for SCD1 satellite passage through the shadow region covers the period from 07/25/1993 to 26/08/1993 (35 days) and for SCD2 satellite from 01/02/2002 to 08/03/2002 (35 days). The complete data can be accessed in the references [4, 5, 6].

Simulations in MATLAB show the passage of SCD1 and SCD2 satellites by the umbra region and its permanence in this region. The results of the simulations for the considered period are presented in the Figure 5 for each satellite.

Using the Software FORTRAN, the SRT magnitude was computed, including the computed values of the Earth umbra. Figure 6 shows the SRT magnitude for the SCD1 and SCD2.

By the comparison of the umbra time of duration (Figure 5) with the SRT magnitude it is possible to observe that this magnitude increase (decrease) when the time of duration of the satellite transition
decrease (increase). It is important to observe that the simulation time interval is different for each satellite, and as the position of the Sun is different for each interval, then the SRT magnitude has a little different value for each satellite.

Figure 6: Magnitude of Solar Radiation Torque

The SCD1 and SCD2 are LEO (Low Earth Satellite) and small in size. Because of that the SRT magnitude is also small and negligible. It is well known that the magnetic torques are more important for this kind of satellite [1,4]. Despite this fact, as the attitude data for real satellites are hard to be obtained, and as the SCD1 and SCD2 data sets were kindly provided by the CRC/INPE, they were used here to validate the developed model. It is important to be pointed out that torque can be significant for other satellite (larger and in high altitude orbit) and this model can be applied for them.

4. Conclusion
The objective of this work was to verify the magnitude variation of the SRT considering the period that the satellite stays inside the umbra. In order to determine the satellite entrance and exit times in this region, a MATLAB program [2] was used. The simulations data were compared and the results showed that the magnitude of the SRT decreases when umbra duration increase while the magnitude increase with the decreasing of umbra duration.

The result analysis confirms the importance of the introduction of the Earth shadow influence in the equation of the rotational motion. It can be done by using the Euler equation and the kinetic equation in terms of quaternions, in order to get a propagated attitude closer to the real attitude of the satellite.

The mathematic expression of the components of the SRT is useful for the numerical and analytical propagation of the satellite attitude. However, depending on the size and altitude of the considered satellite, better results for the attitude propagation must include the others external torques.

Also, the obtained results permitted to verify that the analytical equations used to direct solar radiation torque are valid and the simulations demonstrated the expected behavior. As can be observed in the figures, the torque magnitude calculated using the analytical equations (presented in details in [3]) and the simulations during its passage through the Earth's shadow (period in which the satellite does not experience the strength of solar radiation) are consistent.
This fact confirms the validity of the numerical integration of the equations of rotational motion carried out using the FORTRAN code.

It is also important to observe that, in spite of the applications were performed for the spin stabilized satellites SCD1 and SCD2, this theory can also be applied for any kind of satellite since it uses the attitude quaternions and the components of the angular velocity.

ACKNOWLEDGMENTS
This present work was supported by FAPESP (Process No. 2012/21023-6) and by CNPq/PIBITI.

5. Referências

[1] KUGA, H. K.; ORLANDO, V.; LOPES, R. V. F. Flight dynamics operations during leop for the inpe'ssecundenviromental data collecting satellite SCD2. RBCM _ Journal of the Brazilian Society of Mechanical Sciences, v.21-SP ISS, p.339-344, 1999

[2] Orbital Mechanics with MATLAB. Disponível em: https://cdeaglejr.wordpress.com/?ref=spelling. Acesso em: 07/07/2015

[3] MOTTA, G B ; CARVALHO, M V ; ZANARDI, M C . Analytical Prediction of the Spin Stabilized Satellite's Attitude Using The Solar Radiation Torque. Journal of Physics. Conference Series (Print), v. 465, p. 012009, 2013

[4] ORLANDO, V.; LOPES, R. V. F.; KUGA, H. K. INPE’S flight dynamics team experience thought four years of scd1 in orbit operations: main issues, improvements and tends, ESA International Symposium on Spaceflight Dynamics. Darmstadt, Alemanha, p.433-437, 1997

[5] SANTOS, J. C. (2011) Análise da estabilidade do movimento rotacional com quaternions e sob a influência de torques externos. Trabalho de Graduação.UNESP _ Campus de Guaratinguetá

[6] SANTOS, J. C .;ZANARDI, M. C. F. P. S.; MATOS, N. F. O. Semi-analytical study of the rotational motion stability of artificial satellites using quaternions. Journal of Physics. Conference Series (Online), v. 465, p. 012012, 2013

[7] SHIVASTAVA, S. K.; MODI, V. J. Satellite Attitude Dynamics and Control in the Presence Of Enviromental Torques - A Brief Survey _ AIAA Journal, v. 6, no6,1993.

[8] SHUSTER, M. D. A survey of attitude representation, Journal of Astron. Sciences, v. 41. 4, 1993

[9] TAKEICHI, N. Parametric Excitation Induced by Solar Pressure torque on the Roll-Yaw attitude Motion of a Gravity-Gradient Stabilized Spacecraft Celestial Mechanics and DynamicalAstronomy, 2010

[10] WERTZ, J.R.; Spacecraft attitude determination and control. London: Reidel, 1978.v. 73

[11] ZANARDI, M. C. F. P. S .; VILHENA de MORAES, R. Analytical and Semi-Analytical Analysis an Artificial Satellite's Rotational Motion. Celestial Mechanics & Dynamical Astronomy, Kluwer Academic Publishers, v. 75, n.4, p. 227-250, 1999

[12] ZANARDI, M. C. F. P. S .; VILHENA de MORAES, R.; CABETTE, R. E. S.;GARCIA, R. V. Spacecraft's Attitude Prediction: Solar Radiation Torque and the Earth's Shadow. Advances in Space Research, Elsevier Science, v. 36, p. 466-4471, 2005