Measurement-induced skin effect and the absence of entanglement phase transition

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Recent developments on the measurement-induced phase transition provide new insight into the dynamical phases of matters. A monitored system undergoes an entanglement transition from (sub)extensive to area law entropy scaling with an increasing measurement rate. Despite the knowledge of this universal behavior observed in both interacting and free systems, we propose that the entanglement transition may break down for (open boundary) systems subjected to some generalized measurement taking the form of projective monitoring with conditional feedback. We show that some generalized measurements, however weak they are, cause the particles to accumulate toward the edge, leading to an anomalous late-time localization phenomenon reminiscent of the “skin effect” in non-Hermitian systems. Such particle localization, named the measurement-induced skin effect, will suppress the entanglement generation and render the system short-range entangled without any entanglement transition. This scheme also provides a route to experimentally implement the skin effect without post-selections.

Introduction.— The competition between the measurement and unitary evolution produces a novel entanglement phase transition [1–9]. In contrast to the generic thermalization dynamics where a closed system eventually evolves to a long-range entangled state with local observables relaxing to their thermal values [10–13], measurements destroy the quantum coherence and render the system short-range entangled. A prototypical model displaying this competition is a random circuits [14–18] interspersed by onsite measurements [1–3]. Numerical simulations reveal an entanglement transition, called the measurement-induced phase transition (MIPT), from a volume-law weakly-measured regime to an area-law strongly-measured regime. At the transition point, the system appears to have conformal invariance, implying an underlying (non-unitary) conformal field theory [19–21]. Besides the random circuit systems, later researches find entanglement transitions in the context of monitored fermions [22–27], monitored open systems [28, 29], circuits with pure measurements [30, 31], random tensor network [32–35], and quantum error correction thresholds [36–39].

Quantum measurements introduce intrinsic randomness to the originally deterministic dynamics, with each set of recorded measurement results corresponding to a specific trajectory [40]. Among various frameworks trying to explain the MIPT [23, 27, 36–39, 41–47], one approach boils down to focusing on a specific trajectory, of which the dynamics is described by a non-Hermitian Hamiltonian [48–52]. On the other hand, it is known that non-Hermitian Hamiltonians may induce the so-called non-Hermitian skin effect [53–57], which cause an extensive number of particles localizing near the open edges. The skin effect significantly impacts the steady-state entanglement since the accumulation of particles near the boundary and the Pauli exclusive principle forbid a large amount of entanglement. Therefore, the appearance of the skin effect suggests the absence of MIPT. This simple argument, however, relies on the assumption that the particular non-Hermitian evolution is enough to represent the whole ensemble of trajectories, which is generally not true. The non-Hermitian dynamics, nevertheless, can be enforced by post-selection where there are exponentially many experiments carried out, from which the desired measurement record is selected. Due to the experimental difficulty in implementing the post-selected many-body dynamics, a natural question is whether the skin effect can appear without resorting to post-selection.

We address this question assertively, provided that the measurement is in its general form. An axiom in quantum mechanics says that measurement is a complete set of projection operators. Such a set of projection operators is described by a non-Hermitian Hamiltonian [48–52]. On the other hand, it is known that non-Hermitian Hamiltonians may induce the so-called non-Hermitian skin effect [53–57], which cause an extensive number of particles localizing near the open edges. The skin effect significantly impacts the steady-state entanglement since the accumulation of particles near the boundary and the Pauli exclusive principle forbid a large amount of entanglement. Therefore, the appearance of the skin effect suggests the absence of MIPT. This simple argument, however, relies on the assumption that the particular non-Hermitian evolution is enough to represent the whole ensemble of trajectories, which is generally not true. The non-Hermitian dynamics, nevertheless, can be enforced by post-selection where there are exponentially many experiments carried out, from which the desired measurement record is selected. Due to the experimental difficulty in implementing the post-selected many-body dynamics, a natural question is whether the skin effect can appear without resorting to post-selection.

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tored dynamics, formulated as the stochastic Schrödinger equation (see Appendix A for details), consisting of an effective Hamiltonian and random quantum jump processes [60–62]. In this study, we consider the generalized monitorings taking the form of projective quantum jumps followed by conditional unitary operators (as shown in Fig. 1). We note that such measurements with conditional feedback have appeared in the literature as a scheme to construct desired quantum states [63] or dynamics [64].

The central result of this work is the discovery of a measurement-induced skin effect (MISE) in the monitored fermion systems, featuring a particle localization near the edge (under open boundary conditions), as displayed in Fig. 2(b). We demonstrate that for some types of monitoring, MISE appears at an arbitrarily small measurement rate, and thus implies an anomalous boundary sensitivity: under the open boundary condition, the accumulation of particles suppresses the formation of entanglement, leading to an area-law entangled state, while under periodic boundary conditions, regular MIPT persists [see Fig. 2(c)(d)]. We further argue that the MISE survives when the interaction is turned on, which predicts the absence of the universal volume-to-area-law transition for monitored interacting systems. Besides, in contrast to the single-body non-Hermitian dynamics from post-selection, MISE provides a many-body version of the skin effect, which appears naturally in the context of the open quantum system.

**Skin effect in monitored dynamics.**—The evolution of the monitored fermion chain can be described by the stochastic Schrödinger equation [60–62]:

\[
\frac{d|\psi\rangle}{dt} = -i H_{\text{eff}}|\psi\rangle \, dt + \sum_m \left[ \frac{L_m}{(L_m L_m^\dagger)^{1/2}} - 1 \right] |\psi\rangle dW_m, \tag{1}
\]

where the effective non-Hermitian Hamiltonian is

\[
H_{\text{eff}} = H - \frac{i \gamma}{2} \sum_m L_m^\dagger L_m, \tag{2}
\]

in which \(H\) is the system Hamiltonian, \(L_m\) the jump operators specifying the observables we monitor, and \(\gamma\) the measurement rate. Each \(dW_m\) is an independent Poisson random variable describing the event of random quantum jumps (see Appendix A for detail). Unlike many previous studies, in this work, we consider \(L_m\) that acts on two adjacent sites and takes the form

\[
L_m = U_m P_m \tag{3}
\]

instead of being a projector. The prototypical model we consider involves the measurements described by the projector

\[
P_i = \frac{1}{2} (c_i^\dagger - ic_{i+1}^\dagger)(c_i + ic_{i+1}) \tag{4}
\]

followed by a conditional unitary operator \(U_i\) not yet specified. The effective non-Hermitian Hamiltonian is (note that the choice of \(U_m\) does not affect \(H_{\text{eff}}\))

\[
H_{\text{eff}} = \sum_i \left[ t_L c_i^\dagger c_{i+1} + t_R c_{i+1}^\dagger c_i - \frac{i \gamma}{4} (n_i + n_{i+1}) \right], \tag{5}
\]

where \(t_{L/R} = 1 \pm \gamma/4\), is the Hatano-Nelson model [65] displaying non-Hermitian skin effect. For the particular trajectory with no jump recorded, no MIPT will be expected as the result of the skin effect. On the other hand, the (averaged) dynamic of particle density is equivalently described in the density matrix formalism by the Lindblad master equation [66–68]:

\[
\frac{d}{dt} \rho = -i[H, \rho] - \frac{\gamma}{2} \sum_m \{L_m^\dagger L_m, \rho\} + \gamma \sum_m L_m \rho L_m^\dagger, \tag{6}
\]

where \(\rho = \langle \psi|\psi\rangle\) is the trajectory-averaged density matrix. For the projective measurements \(L_i = P_i\), the maximally mixed state (within a given particle-number sector) is a nonequilibrium steady state. Moreover, the steady state of Lindblad equation we consider, within a given symmetry sector, is unique [69–72] (see Appendix B for the proof). Therefore, the particle distribution is homogeneous when considering the trajectory averaging, without any particle localization.
Such a no-go principle for skin effect also manifests in the stochastic dynamics picture. The projective measurement $P_i$ measures the occupation of the local quasi-mode $d^i = c_i^\dagger - ic_{i+1}^\dagger$ which is a right-moving wave packet \cite{73}. The effective Hamiltonian $H_{\text{eff}}$ describes the dynamics where no such mode is detected, and therefore the left-moving mode is probabilistically favored. In the projective measurement case, the detected right-moving quasimodes will balance out the momentum distribution, leaving a steady state of homogeneity. The balance will be broken by the measurement feedback $U_i \neq I$, and the steady state can be qualitatively different. Here we consider a simple case where

$$U_i = \exp(i\theta n_{i+1})$$  \hspace{1cm} (7)

is a $\theta$-phase gate acting on site $(i+1)$. Taking the $\theta = \pi$ case for example, the unitary feedback will convert the detected $d^i$ mode to $\tilde{d}^i = c_i^\dagger + ic_{i+1}^\dagger$, a left-moving quasi-mode. The net effect of non-Hermitian and quantum jumps thus leads to a left-moving tendency for all particles, resulting in the MISE.

**Free fermion model.** We first consider the free fermion case

$$H = \sum_i (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i)$$  \hspace{1cm} (8)

and $U_i = e^{i\pi n_{i+1}}$. Note that the corresponding Lindblad equation for operator $L_i = U_iP_i$ is generally non-integrable and thus computationally inaccessible. Nonetheless, by taking the trajectory average of observables, the stochastic evolution (1) provides an efficient numerical method (see Appendix C for details) for the dynamics of averaged observable (or entanglement), as long as $U_i$ preserves the Gaussian state \cite{74}.

The steady-state displaying MISE [as shown in Fig. 2(b)] features in two domains where $n_i$ only takes the value of 1 or 0. The “domain wall” in between is the only nontrivial part of the steady state. We characterize the particle localization by the classical entropy of particle distribution:

$$S_{\text{cl}} \equiv - \sum_i [(n_i) \log(n_i) + (1 - \langle n_i \rangle) \log(1 - \langle n_i \rangle)].$$  \hspace{1cm} (9)

This quantity is closely related to MISE since only the nontrivial density (i.e. $\langle n_i \rangle \neq 0,1$) contributes to $S_{\text{cl}}$. Therefore the asymptotic behavior $S_{\text{cl}} \sim O(1)$ for $L \to \infty$ implies MISE. In addition, $S_{\text{cl}}$ also imposes an upper bound for the entanglement entropy. Note that the entanglement entropy satisfies the subadditivity \cite{58, 59}:

$$S_{AB} \leq S_A + S_B.$$  \hspace{1cm} (10)

For any bipartite system with subsystems $A$ and $B$,

$$S_A + S_B \leq \sum_i S_i \leq S_{\text{cl}}.$$  \hspace{1cm} (11)

where the last inequality follows from the fact that the eigenvalues of a positive $2 \times 2$ matrix majorize \cite{75} the diagonal elements, and thus have less entropy. Since $S_A = S_B$, the entanglement entropy of any possible bipartition is bounded by half of the classical entropy.

In Fig. 3(a), we simulate the evolution of the classical entropies, starting from a product state with the left half part filled: $|\psi_0\rangle = |1 \cdots 10 \cdots 0\rangle$. Note that for a fixed measurement rate $\gamma$ and sufficiently large size $L$, the evolution of $S_{\text{cl}}$ becomes independent of $L$: the sites near the boundary are static, and the sharp edge in the middle blur to a domain wall [see Fig. 2(b)]. The system reaches the steady state with characteristic relaxation time $t^* \sim \gamma^{-1}$, the classical entropy for which, as shown in Fig. 2(b), has the asymptotic behavior

$$S_{\text{cl}}(\gamma, L \to \infty) = \frac{3.3 \cdots}{\gamma}$$  \hspace{1cm} (12)

in the thermodynamic limit. This implies that the characteristic length for the domain wall is $L^* \sim \gamma^{-1}$. Eq. (12) therefore predicts the area-law entanglement scaling $S \leq c/\gamma$ for arbitrary small $\gamma$. Moreover, in Fig. 2(c), we observe the density profile has a scale invariance:

$$S_{\text{cl}}(\gamma, L) = L f(\gamma L),$$  \hspace{1cm} (13)

where the scaling function $f(x) \to 0$ in the $x \to \infty$ limit. This provides another evidence for MISE in the $\gamma \to 0$ limit: the scaling behavior (13) implies that even for tiny
\( \gamma \), when \( L \gg \gamma^{-1} \), the density of classical entropy, and thus the entanglement entropy, approach zero.

The skin effect also manifests itself in the periodic boundary condition. By simulating the system with identical parameters but under periodic boundary conditions, we show in Fig. 2(d) that there is an imbalance in the momentum distribution even for the \( \gamma \ll 1 \) case. We can characterize such imbalance by the current:

\[
J[n_k] = \int_{-\pi}^{\pi} dk \, v_k n_k,
\]

where in the \( \gamma \ll 1 \) limit, we can approximate the velocity by the dispersion of free Hamiltonian:

\[
v_k \approx \partial_k E(k) = \sin(k).
\]

For \( \gamma = 0.01 \) case, \( J \approx 0.94 \). The nonzero current suggest the skin effect under open boundary condition.

In the above analysis, we consider only the \( \theta = \pi \) case in Eq. (7), while similar MISE appears for arbitrary \( \theta \). In Appendix D, we show the numerical results for \( \theta \neq \pi \). The length of the domain wall, however, is minimized at \( \theta = \pi \). When \( \theta \) approaches zero, we expect that the MISE still appears, although with a large domain wall that may exceed the numerical simulation capability. Therefore, for the projector \( P^1 \) in Eq. (4), arbitrary measurement feedback can result in an extreme boundary sensitivity due to MISE.

**Interacting system.**— In the following, we demonstrate that MISE persists in the interacting system. We will focus on the Hamiltonian with nearest neighbor density-density interaction:

\[
H = \sum_i (c^\dagger_i c_{i+1} + c^\dagger_{i+1} c_i + g n_i n_{i+1}).
\]

The continuous monitoring is the same as the free fermion case. We note that previously, the MIPT for the interacting system is usually on the quantum circuit system with projective measurement. Meanwhile, we show in Fig. 4(a) that systems under continuous monitoring also support a similar entanglement transition (under periodic boundary conditions). Under open boundary conditions, we show in Fig. 4(b) a typical dynamical trajectory, which is qualitatively the same as MISE appearing in monitored free fermion systems.

In the presence of MISE, the only active part of the dynamics is the formation and fluctuation of the domain wall. This enables us to simulate the time evolution for large system sizes using the TEBD algorithm [76, 77] based on the matrix-product states [78, 79]. We calculate the steady-state classical entropy for different \( \gamma \), with system size \( L \) up to 50 [see Fig. 4(c)]. The numerics shows that at least for \( \gamma > 0.4 \), the classical entropy satisfies the similar scaling law:

\[
S_{cl}(\gamma, L \to \infty) \propto \gamma^{-1}.
\]

This scaling implies the numerical hardness in the small \( \gamma \) regime. Although we are not able to numerically check the scaling behavior in the small \( \gamma \) limit, from the finite size simulation, we obtain the same scaling law as in Eq. (13) [as displayed in Fig. 4(d)]. Therefore, we expect that in the thermodynamics limit, when \( L \gg \gamma^{-1} \), the density of the entanglement entropy approaches zero, indicating an area law.

**Discussion.**— In this work, we consider the effect of imperfect monitoring, formulated as a projective measurement followed by unitary feedback, on the measurement-induced phase transition. The discovery of the measurement-induced skin effect suggests that conditional feedback can qualitatively change the nonequilibrium steady state. Therefore, the entanglement phase structure may be enriched when considering dynamics under generalized measurements. Also, this study shows that MISE may appear in open or (generalized) monitored systems. This provides a route to experimentally implement a many-body skin effect without post-selection.

**Note added.**— In the middle of this work, we became aware of a recent work [80], which also considers the effect of conditional feedback in the context of MIPT. In Ref. [80], the authors utilize the feedback (called the pre-selection) to map the MIPT to a quantum absorbing state transition, which overcomes the exponential overhead of detecting MIPT experimentally. On the other hand, our
work can be regarded as complementary to the problem: instead of revealing, we show that conditional feedback can also destroy the MIPT.

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**Appendix A: Stochastic Schrödinger Equation**

Microscopically, a measurement process involves a short-time interaction between the system and the probe, which are initially separable:

$$|\psi_{AB}\rangle = e^{-iH_{\text{int}} \Delta t} |\psi_A\rangle \otimes |\psi_B\rangle,$$  \hspace{1cm} (A1)

where the wave function of the measured system is denoted as $|\psi_A\rangle$ and the probe $|\psi_B\rangle$. When $\Delta t$ is much smaller than the time scale of the system, the system can be regarded as static during the measurement. Such measurement is called the strong measurement. The probe is thought to be a device that can convert quantum information to the classical one, which takes the form of standard projective measurement. That is, suppose the eigenbasis of the probe is $\{|\phi_n\rangle\}$, the probability of getting a record $n$ is

$$p_n = \langle \phi_n | \rho_B | \phi_n \rangle, \quad \rho_B \equiv \text{Tr}_{A} |\psi_{AB}\rangle \langle \psi_{AB}|,$$  \hspace{1cm} (A2)

and the feedback of the measurement to the system is

$$|\tilde{\psi}_A^{(n)}\rangle = \langle \phi_n | e^{-iH_{\text{int}} \Delta t} |\psi_A\rangle \otimes |\psi_B\rangle \equiv M_n |\psi_A\rangle.$$  \hspace{1cm} (A3)

The completeness condition requires

$$\sum_n \langle \tilde{\psi}_A^{(n)} | \tilde{\psi}_A^{(n)} \rangle = 1 \implies \sum_n M_n^\dagger M_n = 1.$$  \hspace{1cm} (A4)

This is the general form of the measure. In the language of density operator, a measurement is described by a set of operators $\{M_n\}$. A measurement process may record a result $n$ with probability $p_n$ and change the state to:

$$\rho \rightarrow \frac{M_n \rho M_n^\dagger}{\|M_n \rho M_n^\dagger\|}.$$  \hspace{1cm} (A5)

If the measurement result is not known, the averaged density matrix after the measurement is

$$\rho \rightarrow \sum_n M_n \rho M_n^\dagger.$$  \hspace{1cm} (A6)

Such a map is called the quantum channel [58].

On the other hand, if the strength of system-probe coupling is comparable with the energy scale of the system, which is the case for an open quantum system, the quantum channel expression should depend on time $\Delta t$. Such measurement process is called weak measurement. When the system is Markovian (the equation of motion depends only on the near past), the course-grained dissipation process can be described by the channel:

$$M_n = L_n \sqrt{\gamma \Delta t},$$

$$M_0 = \left( 1 - \sum_{n>0} M_n^\dagger M_n \right)^{1/2},$$  \hspace{1cm} (A7)

$$= 1 - \frac{\gamma}{2} \sum_{n>0} L_n^\dagger L_n \Delta t + O(\Delta t^2).$$

For density matrix, the coarse-grained differential equation is the Lindblad equation:

$$\frac{d\rho}{dt} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \sum_n M_n e^{-iH_{\text{int}} \Delta t} \rho e^{iH_{\text{int}} \Delta t} M_n^\dagger$$

$$= -i[H,\rho] - \frac{\gamma}{2} \sum_i [L_i^\dagger L_i, \rho] + \gamma \sum_i L_i \rho L_i^\dagger.$$  \hspace{1cm} (A8)

The joint dynamics of Hamiltonian evolution and measurement can be equivalently described by the stochastic process, as shown in Fig. 5, where for each time step $\Delta t$, the system first undergoes a coherent evolution $|\psi\rangle \rightarrow e^{-iH \Delta t} |\psi\rangle$, then the application of measurement produces a random process:

$$|\psi\rangle \rightarrow \begin{cases} M_n(\Delta t) |\psi\rangle & P_n = \langle \psi | L_n^\dagger L_n |\psi\rangle \gamma \Delta t \\ M_0(\Delta t) |\psi\rangle & P_0 = 1 - \sum_{n>0} P_n \end{cases}.$$  \hspace{1cm} (A9)

Different records of the measurement result correspond to different trajectories, and the Lindblad equation is equivalent to the trajectory averaged of such stochastic processes. In the continuum limit, the stochastic differential equation can be formulated by introducing a Poisson random variable $dW_n$, satisfying the orthogonal condition

$$dW_m dW_n = \delta_{mn} dW_m,$$  \hspace{1cm} (A10)

and has the expectation value:

$$\langle dW_n \rangle = \langle \psi | L_n^\dagger L_n |\psi\rangle \gamma dt.$$  \hspace{1cm} (A11)
For the particle number conserving case, as we considered above, one only assumes no conserved quantity for the Lindblad equation. This is equivalent to the algebra within a fixed particle-number sector since any two product states in the sector can be related by applying several fermion hopping terms. To proceed, we subtract the hopping terms between sites 1 and 2 by the Gaussian state. For a particle number conserving calculation above to obtain all \( c_i \) \( c_j \) terms on sites \( (1,2,3) \). We first prove that the Hamiltonian (under open boundary conditions)
\[
H = \sum_i \left( c_i^+ c_{i+1} + c_{i+1}^+ c_i \right)
\]
and the projectors
\[
P_i = \frac{1}{2} (c_i - ic_{i+1}^+)(c_i + ic_{i+1}^+)
\]
generate the whole algebra. Note \( H \) and \( P_1, \ P_2 \) together generate the following particle number operators:
\[
\begin{align*}
&n_1 - n_3 = P_2 - P_1 + i[H, \ P_1 + \ P_2], \\
n_1 + n_2 &= \frac{1}{2} \left( P_1 + P_2 + i[H, n_1 - n_3] + n_1 - n_3 \right), \\
n_2 + n_3 &= (n_1 + n_2) - (n_1 - n_3).
\end{align*}
\]
Then, some straightforward algebra lead to
\[
\begin{align*}
&c_1^+ c_2 - c_2^+ c_1 = i(n_1 + n_2) - iP_1, \\
&c_1^+ c_2 + c_2^+ c_1 = [c_1^+ c_2 - c_2^+ c_1, n_2 + n_3], \\
&c_2^+ c_3 - c_3^+ c_2 = i(n_2 + n_3) - iP_2, \\
&c_2^+ c_3 + c_3^+ c_2 = [n_1 + n_2, c_2^+ c_3 - c_3^+ c_2].
\end{align*}
\]
Upon some addition among Eqs. (B5), we obtain the operator \( c_1^+ c_2 \), \( c_2^+ c_3 \) and their Hermitian conjugates. The commutations of them further produce \( c_1^+ c_3 \) and its conjugate. Also, note that
\[
[c_1^+ c_2, c_2^+ c_1] = n_1 - n_2.
\]
Together with Eqs. (B4), we generate all fermion bilinear terms \( c_i^+ c_j \) (including \( i = j \) case) on sites \( (1,2,3) \). To proceed, we subtract the hopping terms between sites \( (1,2) \) from \( H \). The resulting operator is equivalent to a shorter chain starting from site \( 2 \). We can then utilize the calculation above to obtain all \( c_i^+ c_j \) terms on sites \( (2,3,4) \). We eventually obtain all fermion bilinear terms on the chain by applying the strategy iteratively. Note that fermion bilinear terms \( c_i^+ c_j \) generate the complete algebra within a fixed particle-number sector since any two product states in the sector can be related by applying several fermion hopping terms.

The proof of the uniqueness of steady state for \( \{ L_n = \mathcal{U}_n P_n \} \) is essentially the same. Note that we can generate all \( P_j \) terms by \( P_j = L_j^+ L_j \). The rest of the proof is the same as above.

**Appendix B: Uniqueness of nonequilibrium steady state**

In Refs. [69, 70] (see review in Ref. [71] and application in Ref. [72]), it was shown that a Lindblad equation has unique nonequilibrium steady state if and only if the set
\[
\{ H, L_1, L_1^+, L_2, L_2^+, \ldots \}
\]
generates (under multiplication and addition) the complete algebra on the Hilbert space. The general proof assumes no conserved quantity for the Lindblad equation. For the particle number conserving case, as we considered in the main text, we can focus on the Hilbert subspace \( \mathcal{H}_N \) spanned by \( N \)-particle states. The uniqueness condition then says if \( \{ H, L_1, L_1^+, L_2, L_2^+, \ldots \} \) generates the complete algebra on \( \mathcal{H}_N \), the steady state in \( \mathcal{H}_N \) will be unique.

**Appendix C: Free fermion simulation**

The free fermion system can be efficiently represented by the Gaussian state [74]. For a particle number simulation of Eq. (A14), we can first discretize the time into small interval \( \Delta t \). The discrete evolution is then
\[
|\psi(t + \Delta t)\rangle = \mathcal{M}_{\Delta t}[e^{-iH_{\text{eff}} \Delta t} |\psi(t)\rangle],
\]
where \( \mathcal{M}_{\Delta t} \) represents the quantum jump that randomly happened in time interval \( \Delta t \):
\[
\mathcal{M}_{\Delta t}[|\psi\rangle] \propto \prod_{m \in I} L_m |\psi\rangle.
\]
The set
\[
I = \{ n | r_n < \gamma \langle L_n^+ L_n \rangle \Delta t \}
\]
denotes the jump processes, where \( \{ r_n \in (0,1) \} \) is a set of independent random variables (with evenly distributed probability).

We first prove that the Hamiltonian (under open boundary conditions)
\[
H = \sum_i (c_i^+ c_{i+1} + c_{i+1}^+ c_i)
\]
and the projectors
\[
P_i = \frac{1}{2} (c_i - ic_{i+1}^+)(c_i + ic_{i+1}^+)
\]
generate the whole algebra. Note \( H \) and \( P_1, P_2 \) together generate the following particle number operators:
\[
\begin{align*}
n_1 - n_3 &= P_2 - P_1 + i[H, P_1 + P_2], \\
n_1 + n_2 &= \frac{1}{2} (P_1 + P_2 + i[H, n_1 - n_3] + n_1 - n_3), \\
n_2 + n_3 &= (n_1 + n_2) - (n_1 - n_3).
\end{align*}
\]
Then, some straightforward algebra lead to
\[
\begin{align*}
c_1^+ c_2 - c_2^+ c_1 &= i(n_1 + n_2) - iP_1, \\
c_1^+ c_2 + c_2^+ c_1 &= [c_1^+ c_2 - c_2^+ c_1, n_2 + n_3], \\
c_2^+ c_3 - c_3^+ c_2 &= i(n_2 + n_3) - iP_2, \\
c_2^+ c_3 + c_3^+ c_2 &= [n_1 + n_2, c_2^+ c_3 - c_3^+ c_2].
\end{align*}
\]
Upon some addition among Eqs. (B5), we obtain the operator \( c_1^+ c_2 \), \( c_2^+ c_3 \) and their Hermitian conjugates. The commutations of them further produce \( c_1^+ c_3 \) and its conjugate. Also, note that
\[
[c_1^+ c_2, c_2^+ c_1] = n_1 - n_2.
\]
conserving system, the Gaussian state is a quasimode-occupied state, represented by a matrix $B$:

$$
|B\rangle = \prod_{j=1}^{N} \sum_{i} B_{ij} c_{i}^{\dagger} |0\rangle = \bigotimes_{j=1}^{N} (B_j),
$$

(C1)

where each column $B_j$ is an occupied quasimode. Note that there is an SU($N$) gauge freedom for the matrix $B$, i.e.,

$$
|B'\rangle = |BU\rangle = |B\rangle,
$$

(C2)

where $U$ is an arbitrary SU($N$) matrix. Such gauge freedom implies that a Gaussian state is entirely specified by the linear subspace spanned by the quasimodes $B_i$’s.

The random Schrödinger equation can be Trotterized as Eq. (A15). Using the Baker-Campbell-Hausdorff formula $e^A e^{-A} = e^{[A, A]}$, the nonunitary evolution is

$$
e^{-iH_{eq} \Delta t} |B_t\rangle = \prod_{j=1}^{N} \sum_{i} B_{ij} e^{-iH_{eq} \Delta t} c_{i}^{\dagger} e^{iH_{eq} \Delta t} |0\rangle
$$

$$= \prod_{j=1}^{N} \sum_{i} B_{ij} e^{-i\Delta t} [H_{eq}, c_{i}^{\dagger}] |0\rangle
$$

$$= \prod_{j=1}^{N} \sum_{i} \sum_{k} B_{ij} c_{k}^{\dagger} e^{-i\Delta t} [H_{eq}, c_{k}] |0\rangle
$$

$$= \prod_{j=1}^{N} \sum_{i} \sum_{k} [e^{-i\Delta t} B_{ij} c_{k}^{\dagger} |0\rangle
$$

$$= [e^{-iH_{eq} \Delta t} B_t].
$$

(C3)

That is, the matrix is multiplied by the exponential of the effective non-Hermitian (single-body) Hamiltonian matrix. Note that the resulting matrix is not orthogonal anymore, while the state is still well defined by the linear space spanned by those unorthogonal vectors. In general, for a Gaussian state represented by matrix $B$, we can obtain a canonical form for the representing matrix using the QR decomposition

$$
B = Q \cdot R,
$$

(C4)

where $Q$ is a unitary matrix and $R$ is upper triangular. Note that $Q$ and $B$ span the same linear space, so the Gaussian state can be expressed as $(Q)$. The supper operator $M_{\Delta t}$ in Eq. (A15) corresponds to the Poisson jump process, where for each index $i$, we randomly decide whether a quantum jump process

$$
|B\rangle \rightarrow L_i |B\rangle
$$

(C5)

happens, with probability

$$
p_i = \langle B | L_i^\dagger L_i | B \rangle \gamma \Delta t
$$

(C6)

The $L_m$’s we choose in the main text have the form

$$
L_m = e^{ih} d^\dagger d,
$$

(C7)

where

$$
d^\dagger = \sum_i a_i c_i^{\dagger}
$$

is a quasimode, and

$$
h = \sum_{ij} h_{ij} c_i^{\dagger} c_j
$$

is a fermion bilinear. The following shows that the Gaussian form is preserved by such jump operator $L_m$. First, the probability of the jump process is

$$
\langle B | L_{m}^\dagger L_m | B \rangle = \langle B | d^\dagger d | B \rangle = \|d | B \rangle\|^2.
$$

(C10)

The action of annihilation operator $d$ on $|B\rangle$ is

$$
d |B\rangle = \sum_k a_k^* c_k \prod_i c_i^{\dagger} B_{i0} |0\rangle
$$

(C11)

so we can obtain the probability

$$
p_m = \sum_j \langle a | B_j \rangle \prod_{i \neq j} c_i \langle 0 | B_j \rangle.
$$

(C12)

Besides, we can utilize the gauge freedom to choose the basis such that $\langle a | B_j \rangle = 0$ for $j > 1$. Such matrix $B'$ always exists since we can always find a column $j$ that $\langle a | B_j \rangle \neq 0$ (otherwise, the probability of the jump is zero). We then move the column to the first and define the column as

$$
|B'_j\rangle = |B_j\rangle - \langle a | B_j \rangle / \langle a | B_1 \rangle |B_1\rangle, \quad j > 1.
$$

(C13)

Note that such column transformation does not alter the linear space $B$ spans, while the orthogonality and the normalization might be affected and should be renormalized afterward. Eq. (C11) then simplified to:

$$
d |B\rangle = \bigotimes_{j>1} |B'_j\rangle.
$$

(C14)

The result of the quantum jump is

$$
L_m |B\rangle = |e^{ih} a \rangle \bigotimes_{j>1} |e^{ih} B'_j\rangle.
$$

(C15)

Appendix D: Monitored free fermion with different conditional feedback

In this appendix, we provide more numerical results on the monitored free fermion system with Hamiltonian

$$
H = \sum_i (c_i^{\dagger} c_{i+1} + c_{i+1}^{\dagger} c_i)
$$

(D1)
and the generalized monitoring described by the operator

\[ L_\theta = \frac{1}{2} \epsilon^{3n_{i+1}} (c_i^\dagger - ic_{i+1}^\dagger)(c_i + ic_{i+1}) . \]  

(D2)

In Fig. 6, we present the numerical simulation of the monitored system with different \( \theta \)'s. The steady-state classical entropy all agrees with the general scaling behavior: \( S_{cl}(\gamma, L \to \infty) = c(\theta)/\gamma \). The constants \( c(\theta) \) increases as \( \theta \) decreases:

\[ c(\pi) \approx 3.3, \quad c(0.7\pi) \approx 4.5, \quad c(0.5\pi) \approx 9.0, \quad c(0.4\pi) \approx 16, \quad c(0.3\pi) \approx 55. \]  

(D3)

Also, we see that for \( \gamma < 1 \), the data satisfies the scaling law \( S_{cl} = L f_b(\gamma L) \). We thus expect the skin effect to appear for arbitrary \( \theta \).

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