Charge non-conservation, dequantisation, and induced electric dipole moments in varying-\(\alpha\) theories

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Abstract

We note that in extensions of the Standard Model that allow for a varying fine structure constant, \(\alpha\), all matter species, apart from right-handed neutrinos, will gain an intrinsic electric dipole moment (EDM). In a large subset of varying-\(\alpha\) theories, all such particle species will also gain an effective electric charge. This charge will in general not be quantised and can result in macroscopic non-conservation of electric charge.

*Key words:* varying fine-structure constant, electroweak theory, charge non-conservation

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1 Introduction

Motivated by the studies of fine-structure in the absorption lines of dust-clouds about quasars by Webb *et al.*, [1], recent years have seen a growing interest in the possibility that the fine-structure constant, \(\alpha_{em} = e^2/\hbar c\), can vary in space and time. The observations of Webb *et al.* favour a value of \(\alpha_{em}\) at redshifts 1-3.5 that is lower that it is today: \(\Delta \alpha_{em}/\alpha_{em} \equiv [\alpha_{em}(z) - \alpha_{em}(0)]/\alpha_{em}(0) = -0.57 \pm 0.10 \times 10^{-5}\). A similar observational study using a different data set did not however see a variation in \(\alpha_{em}\), [2]. There is no shortage of other astrophysical, geological, and experimental bounds on the time-variation of \(\alpha_{em}\). An excellent review of these matters has been given by Uzan in ref. [4]. There has also been a great deal of effort concentrated on constructing and

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constraining a self-consistent theoretical framework to explain $\alpha_{em}$'s apparent cosmological change [5,6]. It has been noted by several authors that, if $\alpha_{em}$ can vary, then other Standard Model gauge coupling ‘constants’ should also be able to change, [3]. Indeed, as a result of electroweak unification a change in $\alpha_{em}$ implies that at least one of the weak coupling constants, $g_W$ and $g_Y$, must also change, [7,9]. In this letter we will show that, with the exception of the cases where the ratio $g_Y/g_W := \tan \theta_W$ is a true constant, charge non-conservation, dequantisation and a charged neutrino are generic features of almost all varying-coupling electroweak theories. Such theories will ensure that all matter species gain an electric dipole moment (EDM).

2 General theory

The electroweak couplings, $g_W$ and $g_Y$, are made spacetime dependent by the definitions: $g_W := e^\varphi$ and $g_Y := e^\chi$; $\varphi(x)$ and $\chi(x)$ are scalar fields. We do not exclude the possibility that they may be functions of each other. In general we assume $\nabla_\mu \varphi \neq 0$ and $\nabla_\mu \chi \neq 0$. It is not necessary for what follows to say anything more specific about the dynamics of the dilaton fields, $\chi$ and $\varphi$. Gauge-invariance fixes the gauge-kinetic sector of such a theory to be:

$$\mathcal{L}_g = -\frac{1}{4} \text{tr} \mathcal{F}_{W \mu \nu} \mathcal{F}^{\mu \nu}_W - \frac{1}{4} \mathcal{F}_{Y \mu \nu} \mathcal{F}^{\mu \nu}_Y$$

where the field strengths, $\mathcal{F}^{\mu \nu}_W$ and $\mathcal{F}^{\mu \nu}_Y$ are given by:

$$\mathcal{F}^{\mu \nu}_W = \mathcal{F}^{\mu \nu}_W + \partial^\mu \varphi \mathbf{W}^\nu - \partial^\nu \varphi \mathbf{W}^\mu,$$
$$\mathcal{F}^{\mu \nu}_Y = \mathcal{F}^{\mu \nu}_Y + \partial^\mu \chi \mathbf{Y}^\nu - \partial^\nu \chi \mathbf{Y}^\mu.$$ (2)

The $\mathcal{F}^{\mu \nu}_W$ and $\mathcal{F}^{\mu \nu}_Y$ are respectively given by standard expressions for the $SU(2)$ and $U(1)$ Yang-Mills field strengths; $\mathbf{W}^\mu$ and $\mathbf{Y}^\mu$ are the gauge fields. In all but the special case where $\varphi \equiv \chi + const$, the ratio of $g_W$ and $g_Y$, and hence $\theta_W := \arctan (g_Y/g_W)$ will not be constant. Moreover, as a result of renormalisation, even if $\theta_W$ is spacetime independent at one particular energy scale it will not be at all others. The fine structure constant is given by:

$$\alpha = g_W^2 \sin^2 \theta_W := e^{2\phi}; \quad \phi = \phi(\varphi, \chi).$$ (4)
3 Charge non-conservation and simple varying-\(\alpha\) theories

It follows from Noether’s principle that any gauge-invariant varying-alpha theory contains a conserved current. The class of theories described above is symmetric under modified \(U(1)_{\text{em}}\) gauge transformations \(A_\mu \rightarrow A_\mu + e^{-\phi} \nabla_\mu \Lambda\), where \(\alpha = e^{2\phi}\). The rest of the gauge symmetry must be broken by a Higgs sector if it is to describe our universe. The conserved current however is not the one conjugate to \(A_\mu\), i.e. \(J^\mu(x) := \frac{\delta S_{\text{matter}}}{\delta A_\mu(x)}\) (with \(S_{\text{matter}}\) being the matter action). Noether’s principle says \(\nabla_\mu \left( e^\phi J^\mu \right) = 0\) so it is \(j^\mu := e^\phi J^\mu\) that is conserved. The question of which of \(J^\mu\) and \(j^\mu\) should be considered physical rests in the form of the dilaton-to-matter coupling. Refs. [8] and [9] respectively take different stances on the issue. In ‘\(j^\mu\)-physical’ theories charge is clearly conserved.

If \(J^\mu\) is naturally interpreted as the physical current then there is a form of non-conservation of charge. The total charge, \(Q\), in a volume \(V\) is:

\[
Q := \int_V j_0 e^{\phi(t,x)} \, dV = \int_V j_0 \left( e^{\phi(t,0)} + x \cdot \nabla e^{\phi(t,0)} + \ldots \right) = e^{\phi(t,0)} q + \nabla e^{\phi(t,0)} \cdot d + \ldots + \nabla^i \nabla^j \ldots \nabla^k e^{\phi(t,0)} d_{ij..k}^{(n)} + \ldots
\]

where \(d_{ij..k}^{(n)}\) is the \(n\)th electric multipole moment w.r.t. to the conserved current \(j^\mu\). A collection of neutral particles cannot develop an electric charge in such theories. Similarly an initially electrically neutral, perfect fluid (containing a mixture of negatively and positively charged components) cannot become charged since all multipole moments will vanish for such a fluid. This implies that cosmologically, at least, charge will be conserved to a very good approximation. The universe cannot develop a non-negligible overall charge in this way. Particle level interactions will also conserved charge at each vertex as a result of the conservation of \(j^\mu\).

We will now show that when \(\theta_W\) and \(\alpha\) vary then a stronger form of non-conservation of charge arises in ‘\(J^\mu\)-physical’ theories, and that even in ‘\(j^\mu\)-physical’ theories the fermions develop an EDM.

4 A new interaction from varying-\(\theta_W\)

A Higgs sector must break the \(SU(2)_L \times U(1)_Y\) symmetry down to the \(U(1)\) of electromagnetism. The physically propagating fields, the photon, \(A^\mu\), and the Z-boson, \(Z^\mu\), are given in terms of \(Y^\mu\) and \(W_3^\mu\) in the usual way. Their field strengths are:
\[ F_{\mu\nu}^A = 2e^{-1}\partial[\mu (eA^\nu)] , \]  
\[ F_{\mu\nu}^Z = 2(g_W \cos \theta_W)^{-1}\partial[\mu (g_W \cos \theta_W Z^\nu)] , \]  
(6)

where \( e = g_W \sin \theta_W := e^\phi \) is the fundamental electric charge. The kinetic terms for \( W^3 \) and \( Y \) now become:

\[ L_{Z,A} := -\frac{1}{4}F_{W3}^2 - \frac{1}{4}F_Y^2 = -\frac{1}{4}F_A^2 - \frac{1}{4}F_Z^2 + 2F_{\mu\nu}^A \partial_\mu \theta_W \partial_\nu \theta_W - 2 \tan \theta_W F_{\mu\nu}^Z \partial_\mu \theta_W \partial_\nu \theta_W Z^\nu - \frac{2}{\sin^2 \theta_W \cos^2 \theta_W} \left[ (\partial \theta_W)^2 Z^2 - \partial_\mu \theta_W \partial_\nu Z^\mu Z^\nu \right] \]

(8)

The first two terms are the standard kinetic terms for the photon and \( Z^- \) boson. The term in square brackets and the one before provide only minor corrections to the \( Z^- \) boson propagator. The third term, \( 2F_{\mu\nu}^A \partial_\mu \theta_W \partial_\nu \theta_W Z^\nu \), is the one that interests us. It produces a coupling between the photon and the \( Z^- \) boson that was not previously present. It means that all particle species with weak neutral charge will induce an electric current density.

5 Induced currents

At energies well below the \( Z^- \) boson mass, \( M_Z \sim 91 \text{ GeV} \), \( Z^\mu \approx J^\mu_N/M_Z^2 ; J_N^\mu \) the weak neutral current density. The interaction term of section 4 therefore produces an effective electromagnetic current density, \( \hat{J}^\mu \), given by:

\[ \hat{J}^\mu \equiv e^\phi \hat{j}^\mu := 2e^\phi \nabla_\nu \left( \frac{\nabla[\mu \theta_W J_N^\nu - \nabla[\mu \theta_W J_N^\nu]}{e^\phi M_Z^2} \right) . \]

(7)

The nature of the physical electric potential depends on whether \( J^\mu \) or \( j^\mu \) is the physical current density. When the magnetic field vanishes, \( B = 0 \), the physical potential is defined by the condition that the electric field \( E \) should vanish if and only if the potential is constant. When \( B = 0 \), the modified Maxwell equations are:

\[ e^\phi \nabla \cdot (e^{-\phi}E) = \rho := J^0 \]
\[ \nabla \times (e^\phi E) = 0 \]

(8)

So long as the gradients in \( \phi \) varying only very slightly within the region of space where \( \rho(x) \) has support, then \( \mu = \left( \nabla \phi \right)^2 - \nabla^2 \phi \approx \text{const} \). The electric field is given by \( E := e^{-\phi} \nabla \left( e^\phi \Psi \right) \) where:
\[ \Psi(x) = -\frac{1}{4\pi} \int d^3x' \text{Re} \left( \frac{e^{-\sqrt{M|x-x'|}}}{|x-x'|} \right) \rho(x') \] (10)

This case is the one that corresponds to \( J^\mu \) being the physical current density. Here, \( \Phi := e^\phi \Psi(x) \) is deemed to be the physical potential.

This is not the only possibility. If the \( \phi \) equation of motion is such that \( \vec{\nabla} \phi \times \textbf{E} = 0 \) whenever \( \textbf{B} = 0 \), then \( \textbf{E} = e^\phi \vec{\nabla} \Upsilon \) with:

\[ \Upsilon(x) = -\frac{1}{4\pi} \int d^3x' \frac{e^{-\phi} \rho(x')}{|x-x'|} . \] (11)

It is clear that here \( e^{-\phi} \rho(x) = j^0 \) is the physical charge density; \( \Upsilon(x) \) is identified as the physical potential. The requirement that \( \vec{\nabla} \phi \times \textbf{E} = 0 \) might seem quite contrived. It might, however, arise as an integrability condition for the \( \phi \)-equation of motion; for example as in ref. [8]. This condition defines how the mass of any charged particle should depend on \( \alpha \). All charged particles must develop this \( \alpha \)-dependent mass through photon and dilaton loop corrections. Chiral fermions are protected against becoming massive in this way, therefore all viable ‘\( j^\mu \)-physical’ theories cannot contain charged chiral fermions. This statement applies equally to all charges associated with varying-gauge couplings. Weakly charged neutrinos must therefore be massive in ‘\( j^\mu \)-physical’ varying-\( \alpha \) theories.

Consider a point particle, weak neutral charge \( Q_N \), at \( x = 0 \). In a ‘\( J^\mu \)-physical’ theory the new interaction term described above makes the following contribution to the physical electric potential \( \Phi(x) \):

\[ \Phi(x) \approx \left( -\frac{Q_N \vec{\nabla} \theta_W \cdot \vec{\nabla} \phi}{M_Z^2} \right) \frac{e^{\phi(x)}}{2\pi r} + \frac{Q_N \vec{\nabla} \theta_W}{M_Z^2} \cdot \frac{x e^{\phi(x)}}{2\pi r^3} , \] (12)

where \( r = |x| \). The first term in \( \Phi \) represents a point electric charge \( q_{eff} = \frac{2Q_N \vec{\nabla} \theta_W \cdot \vec{\nabla} \phi}{M_Z^2} \). The second term is the potential of an electric-dipole moment \( d_{eff} = \frac{-2Q_N \vec{\nabla} \theta_W}{M_Z^2} \). In ‘\( J^\mu \)-physical’ theories all weak neutrally-charged particles will become effectively electrically charged when \( \theta_W \) varies. Such particles will also develop an effective EDM. The form of \( q_{eff} \) means that it will not be quantised in units of \( e \). There is effective dequantisation of electric charge in these theories. In ‘\( j^\mu \)-physical’ theories we do not see an induced charge effect. Weak neutrally-charged particles will still develop an EDM. The electric potential, \( \Upsilon(x) \), is:
\[ \Upsilon(x) = \frac{Q_N \vec{\nabla} \theta_W}{e^{\phi(0)} M_Z^2} \cdot \frac{x}{2\pi r^3}. \]  

(13)

The induced EDM is

\[ d'_{eff} := -\frac{2Q_N \vec{\nabla} \theta_W}{e^{\phi(0)} M_Z^2}. \]  

(14)

Order of magnitude estimates for \( q_{eff}, d_{eff} \) and \( d'_{eff} \) are given in the section 6 below.

6 Discussion

The weak neutral current, \( J_{\mu}^N \), is not conserved. In general, massive bodies such as our Sun, and the universe as a whole, have a large net weak-neutral charge density compared to their net electric charge density. In ‘\( J^\mu \)-physical’ electroweak theories of varying-\( \alpha \), particles develop an electric charge proportional to their weak-neutral charge. It is possible then to have non-conservation of electric charge. The universe develops a non-negligible overall charge in this way. The charges that are induced are in general not quantised in units of the fundamental charge \( e \). Spatial variations in \( \theta_W \) also induce EDMs on the fundamental fermion species. The EDMs all point in the direction of \( \vec{\nabla} \theta_W \). In a region where \( \vec{\nabla} \theta_W \approx const \), therefore, these EDMs will line up and produce an overall macroscopic EDM. Numerically the sizes of the effective charges and EDMs are:

\[ q_{eff} \approx 10^{-31} e \left( \| \nabla \theta_W \| \cdot \| \nabla \ln \alpha \| \text{ cm}^2 \right) \]  

(15)

\[ d_{eff} \approx d'_{eff} \sim 10^{-31} e - \text{cm} \left( \| \nabla \theta_W \| \text{ cm} \right) \]  

(16)

In many varying-\( \alpha \) theories one finds \( \vec{\nabla} \ln \alpha \approx \zeta_\alpha \vec{\nabla} \phi_N, \vec{\nabla} \theta_W \approx \zeta_\theta \vec{\nabla} \phi_N \), where \( \phi_N = GM/r^2 \) is the Newtonian gravitational potential. We expect \( \zeta_\alpha, \zeta_\theta \ll 1 \). Near the surface of Earth such theories would induce \( q_{eff} \sim \zeta_\alpha \zeta_\theta 10^{-66} e \), \( d_{eff} \approx d'_{eff} \sim \zeta_\theta 10^{-48} e \text{-cm} \).

Any physically viable, varying-\( \alpha \) and varying-\( \theta_W \) theory must satisfy all relevant bounds on the neutrino and neutron charges and on the EDMs of the fundamental particles. The most restrictive upper bound on the electron-neutrino charge, \( q_\nu \), has been given by Caprini and Ferreira in ref. [12]. They considered the isotropy of the Cosmic Microwave Background (CMB) and found: \( q_\nu < 4 \times 10^{-35} e \). In the same way they also bounded the charge difference between a proton and an electron: \( q_{e-p} < 10^{-26} e \). An upper bound on the neutron charge, \( q_n \), is given by Baumann et al. in [10].
The Particle Data Group, see ref. [11], gives the upper bound on the electron EDM as $d_e < 6.9 \pm 7.4 \times 10^{-28} \text{ e-cm}$. Experiments are planned that would be able to detect any electron EDM at the $10^{-31} \text{ e-cm}$ level, [13]. Ref. [11] also gives upper bounds on the EDMs of the proton, $d_p < 0.54 \times 10^{-23} \text{ e-cm}$, and the neutron, $d_n < 0.63 \times 10^{-25} \text{ e-cm}$. It is clear that all current bounds will be easily satisfied by most varying-$\alpha$ theories.

It is normally the case that intrinsic EDMs on Dirac fermions are indicators of CP-violation. In varying-$\alpha$ theories we have seen that it possible to induce such EDMs without adding any explicit CP-violating term to the Lagrangian and that varying-$\alpha$ theories generically result in some manner of charge non-conservation and effective dequantisation of charge without breaking the $U(1)_{em}$ symmetry. These effects, if detectable in the context of a given theory, could provide us with a new way of probing the rate of spatial variation in $\theta_W$ and $\alpha$.

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