**Fuzzy Adaptive Sliding Mode Impedance Control of Fracture Reduction Robot**

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This work was supported by the National Key Research and Development Program of China under Grant 2017YFB1304202.

**ABSTRACT** With the development of robotics, it will be possible for fracture reduction robots to replace doctors in performing surgery operations. The surgery accuracy and safety by the reduction robot is important. In order to compensate various uncertainties and nonlinear problems in the reduction robot impedance control system, this paper proposes a fuzzy adaptive sliding mode impedance control of the reduction robot, which includes force residual observer (FRO) and fuzzy adaptive sliding mode controller (FASMC). The FRO is used to estimate the external force between the reduction robot and the fractured musculoskeletal tissue in real-time, no force sensor is installed at the end-effector of reduction robot. The FASMC mainly includes three parts: linearizing the system by inverse dynamics and PD control, improving the system’s robustness by the sliding mode control and fuzzy adaptive switching gain to reduce chattering, eliminating the nonlinear disturbance term of the system, and the normalized factor linear combination fuzzy system is used to compensate. The co-simulation of the fracture reduction robot shows that the FASMC impedance control method has a better steady-state position and force tracking accuracy, the steady-state position accuracy is −0.733 mm, and the steady-state force accuracy is −2.12 N. This paper provides an effective method for impedance control of the fracture reduction robot.

**INDEX TERMS** Impedance control, force residual observer (FRO), fuzzy adaptive sliding mode controller (FASMC), reduction robot, femoral shaft fracture.

**I. INTRODUCTION**

The surgical effect of the traditional fracture reduction surgery mainly depends on the doctor’s experience. Compared with traditional reduction surgery, the robotic surgery has advantages, such as reducing the labor intensity of doctors, reducing radiation and infection, which will have a good application prospect [1]. Several fracture reduction robots have been developed, and reduction path planning, intra-operative tracking, human-computer interaction, automatic reduction has been studied. For robot-assisted fracture reduction surgery, the reduction path and the reduction force are very important. Researching the position/force control of the reduction robot is of great significance to realize the automatic reduction of fracture surgery.

Many scholars have studied robot-assisted fracture reduction, mainly involving navigation technology [2], remote control technology [3], image matching automatic control technology [4], and human–computer interaction [5]. At present, the research on reduction force control of reduction robot mainly focuses on the measurement of reduction force [6] and the mathematical modeling of muscle force [7] to obtain accurate reduction force. Georgilas et al. [8] clinically measured the reduction force of femoral fractures in patients as high as 604.53 N. In order to prevent excessive reduction force in the process of reduction from damaging patients’ bones, muscles, and nerves, it is necessary to control the position/force in the reduction process. Kim et al. [9] used the force sensor for admittance control, and obtained real-time environmental contact force through the force sensor installed at the serial robot’s end-effector, forming a force closed-loop control system. However, there is a lack of research on impedance control of reduction robot to achieve accurate tracking of reduction path and desired force.

In order to improve the impedance control performance of robot, many researchers have combined different advanced control methods with impedance control, including sliding mode control (SMC) [10], fuzzy control (FC) [11], neural
network [12], adaptive control [13], learning algorithm [14], reinforcement learning [15], and compound learning [16]. Compared with the serial manipulator, the Stewart-based parallel manipulator has the characteristics of high stiffness, large load, good absolute positioning accuracy, and repeated positioning accuracy, which is more suitable for fracture reduction surgery with large load and high accuracy requirements. The impedance control of parallel robot can be divided into position modified impedance control [17]–[20] and inverse dynamic impedance control [21] according to different implementation methods. Kizir et al. proposed a position-based impedance control for the Stewart platform (SP), mainly divided into three parts: position controller, impedance filter, and FC. When the interaction occurs, the FC adjusts the parameters of the impedance filter to achieve force control [17]. Kalani et al. [18] proposed a fuzzy impedance control strategy for jaw rehabilitation training. Aminiazar et al. [19] proposed a control strategy combining adaptive control and impedance control and used artificial neural networks and genetic algorithms to estimate and optimize controller parameters. Zhou et al. [20] proposed a SMC for position control in the position-based impedance control strategy to ensure that the system can track the force and position under the influence of external disturbance and modeling error. Because the dynamic equation of parallel robot is nonlinear, it depends on the system state. The position correction impedance control based on PID controller has poor dynamic tracking performance, and the controller gain is difficult to meet all States. Excessive controller gain will cause high energy consumption. The impedance control based on inverse dynamics can transform the nonlinear dynamic problems into linear problems, which is the basis of the design of forward controller [21]. At present, the research on impedance control of parallel robot based on inverse dynamics has not attracted enough attention.

There are various uncertainties and nonlinear disturbances during the reduction process. An adaptive fuzzy sliding mode control (AFSMC) scheme can effectively improve the system’s robustness and has little dependence on the model to solve these problems. SMC compensates for the uncertainty of the system to ensure the robustness of the system, and FC compensates for the nonlinear disturbance of the system [22]–[28]. Some new SMC and FC designs are used to improve the performance of adaptive control systems [29]–[33]. Zhang et al. [29] proposed an SMC based on stochastic sliding mode surface (SSS), which can effectively avoid the state trajectory overflow from the sliding mode surface and solve the stability problem of SMC. Pan et al. proposed a disturbance compensator based on SMC to compensate for the uncertainty or external disturbance in the vehicle active suspension system, which can completely eliminate the matching disturbance term [30]. An adaptive fault-tolerant method is designed based on the adaptive backstepping method, which can realize any small error tracking when the actuator has random faults [31]. Shi et al. [32] designed a dynamic SMC to stabilize TS Fuzzy Singular time-delay systems in the presence of uncertainties and disturbance mismatches. Zheng et al. designed adaptive fuzzy backstepping controllers for master and slave respectively to deal with the nonlinear and uncertain problems in robot teleoperation. By adjusting the model system parameters online, the adaptive law is designed to achieve global stability [33]. AFSMC is used in Stewart position control [34] and position/force hybrid control [35]. The upper bound of system state uncertainty is variable, which reduces the dependence of controller on parameters. However, the steady-state position error of the position/force hybrid control simulation is about 2.1 mm [35], which is difficult to meet higher fracture reduction accuracy requirements.

This paper’s main contributions are as follows:

1) In the paper, an impedance control method is proposed to study the position/force control of reduction robots, including fuzzy adaptive sliding mode controller (FASMC) and force residual observer (FRO). Compared with the existing position/force hybrid control, this impedance control method can obtain better position and force tracking accuracy.

2) The FASMC is designed and applied to impedance control. The inverse dynamic model is used to linearize the system; The SMC and FC are used to reduce the chattering phenomenon and improve the system’s robustness; the linear combination fuzzy system with normalization factor is used to compensate for the non-linear disturbance term of the system. The stability and convergence of the system are proved by Lyapunov stability theory.

3) The FRO is used to estimate the environmental contact force in real-time as impedance control feedback signal. The FRO reduces the reduction robot’s failure rate and production cost, and avoids installing force sensor at the end of the reduction robot.

This paper is structured as follows. Section II introduces the PA-MTM skeletal muscle model and FRO. In section III, a position correction impedance control based on FRO feedback is proposed. In section IV, the FASMC is designed, and its stability is analyzed. In section V, MATLAB/Simulink software is used to conduct impedance control for the fracture reduction robot for 3D visualization simulation verification. This paper is summarized in section VI.

II. DYNAMICS OF THE REDUCTION ROBOT

The fracture reduction surgery robot is shown in Fig. 1, which consists of a host computer, a mobile device, a reduction robot, a patient model, a distal gripper, an optical surgical navigation system, and a proximal gripper.

During the process of femoral fracture reduction, muscle force is mainly generated from the muscle stretching, which is the main affecting factor [36]. Generally, the muscle force model is Hill three-element model [37] or Haeufle four-element model [7]. Lei et al. [38] proposed a kind of four element musculoskeletal model named PA-MTM to study...
the biomechanical properties of the fractured femoral shaft, and the reduction path in three-dimensional space is planned by the A* algorithm. In this paper, the PA-MTM muscle force model was adopted. Considering the influence of lower limb gravity, the force analysis of the fractured femoral shaft during the reduction process can be expressed as:

\[
F_r' + F_r = F_{\text{axial}} + F_m + G_l = 0
\]  

where \(F_r\) is the reduction force provided by the reduction robot, \(F_m\) is the muscle force, \(G_l\) is the lower limb gravity, \(F_r\) is the lower limbs force, \(F_{\text{axial}}\) is composed of muscle force \(F_m\) and lower limb gravity \(G_l\). The main muscles that produce muscle force \(F_m\) are rectus femoris muscle, biceps femoris muscle, sartorius muscle, semitendinosus muscle, and semimembranosus muscle [39].

According to the relevant muscle parameters in the literature [38], \(F_r = [F_{rx} \; F_{ry} \; F_{rz} \; 0 \; 0 \; 0]^T\) change is shown in Fig. 2. At the end position, the resultant force \(F_r\) of the lower limbs reaches the maximum value 497.28 N. At this time, \(F_r = [472.2 \; -69.5 \; -139.6 \; 0 \; 0 \; 0] \text{N} \cdot \text{m}^T\), where the component force along the \(x\)-direction (i.e., the axial force) reaches the largest value 472.2 N. As shown in Fig. 2, the axial force \(F_{rx}\) of the lower limb force \(F_r\) along the \(x\)-direction plays a major role in the reduction robot. To simplify the complexity of the system and avoid singularity in the impedance control process of the reduction robot, only the axial force \(F_{rx}\) is studied for impedance control.

At the beginning of the reduction process, the muscle force changes suddenly, so the system will be impacted, which will affect the safety of fracture reduction. During actual process of fracture reduction, the muscle force gradually increased. Therefore, it is necessary to use a first-order low-pass filter to smoothly transition the muscle force signal, and obtain

\[
Q(s) = \frac{1}{\tau s + 1}
\]  

where \(\tau\) is the time constant, the smaller \(\tau\), the stronger the anti-interference ability of the filter, but the weaker the noise suppression ability, and vice versa. \(\tau\) is determined by the designer experience, \(\tau = 0.09\).

The configuration of the reduction robot is parallel robot. When unknown disturbing force is considered during the reduction process, the dynamics of the reduction robot in the workspace can be derived as:

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = F - F_r - F_f(q, \dot{q}, \ddot{q})
\]  

where \(q, \dot{q}\) and \(\ddot{q}\) respectively represent the end position and orientation, velocity, and acceleration of the parallel robot in the workspace. \(M(q), C(q, \dot{q})\) and \(G(q)\) represent the inertia matrix, Coriolis and centripetal force matrix, and gravity vector in the workspace. \(F_f(q, \dot{q}, \ddot{q})\) is unknown interference force, including modeling error, friction, and other external interference.
When the fracture reduction force is considered, the dynamic equation of the reduction robot can be expressed in the joint space as:

\[
D(q) \ddot{\mathbf{L}} + H(q, \dot{q}) \dot{\mathbf{L}} + \mathbf{P}(q) = \tau - \tau_r - \tau_f \tag{4}
\]

where \(\mathbf{L}, \dot{\mathbf{L}},\) and \(\ddot{\mathbf{L}}\) respectively represent the joint displacement, velocity, and acceleration of the parallel robot. \(D(q)\) is the inertia matrix, \(H(q, \dot{q})\) is Coriolis and centrifugal force matrix, \(\mathbf{P}(q)\) is gravity vector, \(\tau = J^T \mathbf{F}\) is the joint driving force, \(\tau_r\) is the force of the lower limbs on the reduction robot, and \(\tau_f\) is the unknown interference force.

The force of the fracture reduction robot shown in Fig. 3. The proximal gripper is used to fix the proximal femur fracture. The distal gripper is installed on the reduction robot to clamp the femur fracture. The driving force of the reduction robot \(\tau_i (i = 1, \ldots, 6)\) is provided by six electric cylinders during the reduction process, as \(\mathbf{F}\) in the workspace. The reduction force \(\mathbf{F}_r\) is provided by driving force \(\mathbf{F}\), which is used to overcome muscle force \(\mathbf{F}_m\), lower limb gravity \(\mathbf{G}_l\), and unknown interference force \(\mathbf{F}_f\).

The construction FRO is expressed as [40]:

\[
r = K_1 \int_0^t [-K_2 r + p - \int_0^t (\tau + H^T(q, \dot{q}) \dot{\mathbf{L}} - \mathbf{P}(q) - r) dt] dt + K_3 f_s \tag{5}
\]

where \(p\) is the generalization momentum, \(K_1 > 0, K_2 > 0, K_3 > 0\) is the gain matrix, \(r\) is the force residual, \(f_s\) is the adjustment function, \(f_c = p - \int_0^t (\tau + H^T(q, \dot{q}) \dot{\mathbf{L}} - \mathbf{P}(q) - r) dt\).

The FRO only needs to obtain the displacement and velocity information \((\mathbf{L}, \dot{\mathbf{L}})\) feedback by the electric cylinder encoder and detect the output torque \(\tau\) of the electric cylinder, without measuring the acceleration \(\ddot{\mathbf{L}}\). The FRO is equivalent to building a virtual force sensor in the broken bone area of the reduction robot, which can detect the reduction force in the fracture reduction process in real-time. Therefore, there is no need to install an expensive force sensor at the end-effector.

III. POSITION IMPEDANCE CONTROL BASED ON FRO FEEDBACK

The dynamic characteristics of the impedance control are determined by the mechanical impedance. The impedance control performance can be adjusted by selecting the impedance matrix parameters to realize the position/force control of the reduction robot [41]. The expected impedance model can be expressed as:

\[
M_d \ddot{\mathbf{q}} + B_d \dot{\mathbf{q}} + K_d \mathbf{q} = -\mathbf{F} = -\mathbf{F}_d + \mathbf{F}_r \tag{6}
\]

where \(M_d, B_d, K_d\) represent the inertia, damping, and stiffness matrix of the expected impedance model, respectively. \(\ddot{\mathbf{q}}\) represents the actual position, velocity, and acceleration of the end-effector, \(\dot{q}_d, \ddot{q}_d\) represent the expected position, velocity, and acceleration of the end-effector, respectively. \(\mathbf{F} = \mathbf{F}_d - \mathbf{F}_r\) represents the force error, \(\mathbf{F}_d\) is the expected contact force between the end-effector and the environment.

According to the standard impedance model (6), the position correction \(\ddot{\mathbf{q}}\) is expressed in the frequency domain as:

\[
\ddot{\mathbf{q}}(s) = \frac{\tilde{\mathbf{F}}(s)}{M_d s^2 + B_d s + K_d} \tag{7}
\]

where \(\ddot{\mathbf{q}}(s) = \mathbf{q}(s) - q_d(s)\). Equation (7) can be regarded as a second-order low-pass filter, which filters the difference \(\tilde{\mathbf{F}}\) between the expected force of the reduction robot and the force of the lower limbs, and obtains the position correction \(\ddot{\mathbf{q}}\).

When the reduction robot is not in contact with the patient’s lower limbs, \(\tilde{\mathbf{F}} = \mathbf{F}_d = \mathbf{F}_r = 0\), the position correction \(\ddot{\mathbf{q}} = 0, \dot{q}_d = q_d\), the reduction robot only controls the motion.

The position controller is used to track the moving platform’s expected position, and the equivalent expected impedance model of the interaction between the robot and the environment is realized. As shown in Fig. 4, it is the position correction impedance control schematic diagram based on the external force estimation feedback. Through the information acquisition device of the electric cylinder of the reduction robot, the driving force \(\tau\), the displacement \(\mathbf{L}\) and the velocity \(\dot{\mathbf{L}}\) of the electric cylinder movement are obtained in real-time, and the contact force between the reduction robot and the environment is estimated through the FRO. The difference \(\tilde{\mathbf{F}}\) between the expected force \(\mathbf{F}_d\) of the reduction robot and the actual lower limb force \(\mathbf{F}_r\) of the patient is filtered by impedance filter, and the position correction \(\ddot{\mathbf{q}}\) is obtained. \(\ddot{\mathbf{q}}\) corrects the expected position \(q_d\) to obtain the reference position \(q_r, \mathbf{q} = q_d - \ddot{\mathbf{q}}\), which is used as the input of the position controller.

For this control scheme, the control law design of the position controller need not to establish the dynamics model of the reduction robot, so as to avoid the calculation of the forward solution of the reduction kinematics. The position controller adopts PID control, so the control system is simple, and the algorithm’s calculation is less. The contact force of the environment can be obtained through the FRO, which is easy to implement in practical engineering applications.

IV. FUZZY ADAPTIVE SLIDING MODE CONTROLLER (FASM C)

According to (6), we can get:

\[
\ddot{\mathbf{q}} = M_d^{-1} \left[ \mathbf{F}_d - \mathbf{F}_r + B_d \dot{\mathbf{q}} + K_d \mathbf{q} \right] + \ddot{q}_d \tag{8}
\]

Incorporating (8) into (3), the output torque of the controller can be expressed as:

\[
\mathbf{F} = M(q) \left[ M_d^{-1} \left[ \mathbf{F}_d - \mathbf{F}_r + B_d \dot{\mathbf{q}} + K_d \mathbf{q} \right] + \ddot{q}_d \right] + C(q, \dot{q}) \dot{\mathbf{q}} + G(q) + F_r + F_f (q, \dot{q}, \ddot{q}) \tag{9}
\]
The control law $u$ is designed, assuming that the reduction robot has two parts of certainty and uncertainty, then:

$$u = M\ddot{q} + C\dot{q} + G + F_f + F_j = u_{IDC} + \tilde{u}$$

(10)

where $\ddot{q}_r, \dot{q}_r$ are the reference acceleration and velocity, respectively. $u$ is divided into the known part $u_{IDC} = M\ddot{q} + C\dot{q} + G + F_f$ [21] and the unknown part $\tilde{u}$, where:

$$\tilde{u} = F_f (q, \dot{q}, \ddot{q}) = M\ddot{q} + C\dot{q} + G + F_f + d$$

(11)

where $\tilde{M}$, $\tilde{C}$, and $\tilde{G}$ are the uncertain terms of the dynamic equation, $\ddot{F}_f$ is the uncertainty of the reduction force, $r$ is the reference acceleration and velocity, $\dot{r}$, $\ddot{r}$ are the uncertain terms of the dynamic equation, $d$ is external interference such as friction.

According to (9) and (10), when the system has no uncertain terms, the known part of the system $u_{IDC} = F_f$. $F_f$ contains the impedance model and reduction robot dynamics information.

**A. SLIDING MODE CONTROLLER (SMC)**

The SMC is designed to compensate for the nonlinear disturbance of the system $F_f (q, \dot{q}, \ddot{q})$, and the sliding surface (switching function) is designed as:

$$s = \ddot{q} + \lambda \ddot{q} = \ddot{q} - \ddot{q}_r$$

(12)

where $\lambda = \text{diag} \{\lambda_1, \ldots, \lambda_6\}$, $\lambda_i > 0$ ($i = 1, \ldots, 6$) is adjustable and satisfies the Hurwitzian stability condition. $\ddot{q}_r$ is obtained by correcting the position error $\ddot{q}$ from the desired velocity $\dddot{q}_d, \ddot{q}_r = \dddot{q}_d - \dddot{q}_r$. The time derivative of (11) is obtained as follows:

$$\dot{s} = \ddot{q} - \ddot{q}_r = M^{-1} (F - C\dot{q} - G - F_f - F_f) - \ddot{q}_r$$

(13)

Take the law of equal velocity approach:

$$\dot{s} = -\sigma \text{sgn}(s), \quad \text{sgn}(s) = \begin{cases} +1, & s > 0 \\ -1, & s < 0 \end{cases}$$

(14)

where $\sigma > 0$, $\sigma$ is the approaching velocity, the larger the $\sigma$, the more serious the system chattering. On the contrary, when $\sigma$ decreases, the system chattering decreases. Since $\text{sgn}(s)$ indicates that the system has an infinite switching frequency, the actual system cannot be realized. At present, the saturation function $\text{sat}(s)$ is used to modify the switching sign function $\text{sgn}(s)$ to eliminate chattering, which is expressed as [42]:

$$\text{sat}(s) = \begin{cases} \text{sgn}(s), & |s| > \epsilon \\ s/|s|, & |s| \leq \epsilon \end{cases}$$

(15)

where $\epsilon > 0$, $\epsilon$ is the thickness of the sliding mode area.

Then the (13) of the constant velocity approaching law can be rewritten as:

$$\dot{s} = -\sigma \text{sat}(s)$$

(16)

where the switching term $\sigma \text{sat}(s)$ is the cause of the control system’s chattering. The $\sigma \text{sat}(s)$ compensates for the uncertain terms $F_f$ such as model errors and environmental interference, and ensures the sliding mode area on the switching surface. To reduce chattering caused by excessive switching gain, fuzzy rules, and expert experience are used to estimate the switching gain, so that the switching gain $\sigma$ changes with the change of $F_f$.

The fuzzy rules are as follows: when the system state is far from the sliding mode surface, $|s|$ increases, $\sigma$ increases; when the system state is close to the sliding mode surface, $|s|$ decreases, $\sigma$ decreases. Design the fuzzy system, $|s|$ is the input of the fuzzy system, and $\sigma$ is the output of the fuzzy system. Define the fuzzy set of input $s$ and output $\Delta\sigma$ of the fuzzy system are: [NB, NM, NS, ZO, PS, PM, PB]. The membership function of the input variable $s$ is as shown in Fig. 5, and the membership function of the output variable $\Delta\sigma$ is as shown in Fig. 6.

The SMC is designed as:

$$u_{SMC} = \sigma \text{sat}(s)$$

(17)

The upper limit of $\sigma$ is estimated adaptively by the integral method:

$$\dot{\sigma} = \alpha \int_0^t \Delta\sigma dt$$

(18)

where $\alpha$ is the adjustment coefficient, which is determined by experience, and the $\sigma$ estimated error of $\tilde{\sigma} = \sigma - \sigma$.

**B. FUZZY CONTROL (FC)**

The FC is used to compensate for the nonlinear disturbance term $F_f (q, \dot{q}, \ddot{q})$ of the system. When the sliding mode surface $s$ and its change rate $\dot{s}$ are changed, the FC $u_{FC}$ changes accordingly. For example, when $s$ and $\dot{s}$ are NB, the system state is far away from the sliding mode surface, then the input $u_{FC}$ is PB to ensure that $\dot{s}$ decreases rapidly, so that the system state quickly approaches the sliding mode surface $s = 0$. The fuzzy rules are: IF (s is NB) and (\dot{s} is NB), THEN ($u_{FC}$ is PB). According to the input $s$ and $\dot{s}$ of fuzzy system, 7 fuzzy sets are designed respectively, and there are 49 fuzzy rules in total. The following two steps are used to construct the fuzzy compensation system $u_{FC}$ [43]:

1) For the variable $x_i$ ($i = 1, 2$), define $p_l$ fuzzy sets $A^l_i$ ($l = 1, \ldots, 7$);

2) Use $\prod p_i = p_1p_2 = 49$ fuzzy rules to construct $u'_{FC}(x | \theta) = u'_{FC}(s, \dot{s} | \theta)$, $x_1 = s$, $x_2 = \dot{s}$, then the $j$th fuzzy rule is expressed as:

$$R^j : \text{IF } x_1 \text{ is } A^l_1, \text{ and } x_2 \text{ is } A^l_2, \text{ then } u'_{FC} \text{ is } B^{l_1l_2}$$

(19)

where $B^{l_1l_2}$ is the output fuzzy set of the fuzzy system.

Using product inference engine, singleton fuzzifier, and center average defuzzifier, the fuzzy system can be written as:

$$u'_{FC}(x | \theta) = \frac{\sum_{i=1}^{7} \sum_{j=1}^{7} y^{l_1l_2} \left( \sum_{i=1}^{7} u_{A^l_i}(x_i) \right)}{\sum_{i=1}^{7} \sum_{j=1}^{7} \left( \sum_{i=1}^{7} u_{A^l_i}(x_i) \right)}$$

(20)
TABLE 1. Fuzzy rules.

| u'_{FC} | s | NB | NM | NS | ZO | PS | PM | PB |
|---------|---|----|----|----|----|----|----|----|
| NB      | PB | PB | PB | PB | PB | PS | PM | ZO |
| NM      | PB | PB | PB | PS | PM | ZO | NS | NM |
| NS      | PB | PB | PM | PS | ZO | NS | NM | PB |
| ZO      | PB | PM | PS | ZO | NS | NM | NB | NB |
| PS      | PM | PS | ZO | NS | NM | NB | NB | NB |
| PM      | PS | ZO | NS | NM | NB | NB | NB | NB |
| PB      | ZO | NS | NM | NB | NB | NB | NB | NB |

FIGURE 5. Fuzzy system input membership function.

FIGURE 6. Fuzzy system output membership function.

where \( u_{A_i} (x_i) \) is in the \( l \) fuzzy rules, the \( i \)th input \( x_i \) belong to fuzzy set \( A_i \). \( \Pi \) is a fuzzy operator, which is usually minimized. Let \( \lambda_i \) be a free parameter, which is put in the set \( \theta \in R^{(49)} \), and introduce the vector \( \xi (x) \), the fuzzy compensation is:

\[
u'_{FC} (x | \theta) = \theta \xi (x) \tag{21}\]

where \( \theta \in R^{(49)} \) is the unknown parameter vector set, \( \xi (x) \) is the 49-dimensional fuzzy base vector, and the \( l \)th element is \( \xi_1 = \frac{2 \sum_{i=1}^{l} u_{A_i} (x_i) / \sqrt{\sum_{i=1}^{l} u_{A_i} (x_i)}}{\sqrt{\sum_{i=1}^{l} u_{A_i} (x_i)}} \).

The membership function is Gaussian:

\[
u_{A_i} (x_i) = \exp \left( -\frac{(x_i - \alpha)^2}{\beta} \right) \tag{22}\]

where \( \alpha \) and \( \beta \) are the center and width of the membership function, respectively, \( \beta > 0 \).

Setting the fuzzy variable set \{NB, NM, NS, ZO, PS, PM, PB\}, and get fuzzy rules are shown in Table 1.

An adaptive fuzzy compensation system for the reduction robot is established, and the normalized factor linear combination fuzzy system is used to output [35]. Then the reduction robot fuzzy compensation is:

\[
u_{FC} (s, \dot{s} | \phi) = \phi \theta \xi (s, \dot{s}) = \phi u'_{FC} (s, \dot{s} | \theta) = \phi u'_{FC} \tag{23}\]

where \( \phi \) is the output linearization normalization factor of the fuzzy system, \( \xi (s, \dot{s}) \) is the fuzzy basis vector of the input variable \( (s, \dot{s}) \). The normalization factor \( \phi \) is estimated, and the estimated value is \( \hat{\phi} \), then the estimation error is:

\[
\tilde{\phi} = \phi - \hat{\phi} \tag{24}
\]

The approximation disturbance term of the fuzzy system can be expressed as:

\[
F_f = \phi u'_{FC} + \varepsilon \tag{25}
\]

where \( \varepsilon \) is the approximation disturbance term error of the fuzzy system. After the online normalization factor is estimated, the approximation error can be expressed as:

\[
\tilde{u}_{FC} = F_f - u_{FC} = \phi u'_{FC} + \varepsilon - \phi u'_{FC} = \tilde{\phi} u'_{FC} + \varepsilon \tag{26}
\]

where \( \varepsilon \) is the approximation disturbance term error of the fuzzy system. After the online normalization factor is estimated, the approximation error can be expressed as:

\[
\tilde{u}_{FC} = F_f - u_{FC} = \phi u'_{FC} + \varepsilon - \phi u'_{FC} = \tilde{\phi} u'_{FC} + \varepsilon \tag{26}
\]

The design of the normalized factor estimates \( \hat{\phi} \) can be adjusted adaptively, and the adaptive law is as follows:

\[
\tilde{\phi} = -\gamma su_{FC} \tag{27}
\]

where \( \gamma \) is the adaptive law coefficient, which is adjusted according to the designer’s experience.

C. SUMMARY OF FASMC

The FASMC control law is designed as follows:

\[
u = M \ddot{q} + C \dot{q} + G + F_r + \hat{\phi} \theta \xi (s, \dot{s}) - K_D s - \hat{\sigma} \text{sat}(s) = u_{IDC} + u_{FC} - u_{SMC} - u_{PD} \tag{28}
\]

where \( \varepsilon_{max} \leq \sigma, \sigma \) is the upper limit of \( \varepsilon, K_D > 0, u_{IDC} = M \ddot{q}_r + C \dot{q}_r + G + F_r, \) is the inverse dynamics controller (IDC), \( u_{FC} = \hat{\phi} \theta \xi (s, \dot{s}) \) is the FC, \( u_{SMC} = \hat{\sigma} \text{sat}(s) \) is the SMC, \( u_{PD} = K_D s \) is the Proportion Differentiation (PD) controller. Its functions are as follows: The \( u_{IDC} \) realizes the linearization of the nonlinear dynamic equation; The \( u_{FC} \) can estimate and update the system’s nonlinear disturbance term online; The \( u_{SMC} \) is used to compensate the system uncertainty improves the system’s robustness. The \( u_{PD} \) realizes the system’s rapid response and reduces the system’s steady-state error [21].

The FASMC can effectively suppress the influence of disturbance, friction, and other factors, which has the advantages of fast response and insensitive disturbance. The reduction robot control strategy is shown in Fig. 7. There is no need...
to install a force sensor at the end-effector of reduction robot, and the contact force can be obtained from the FRO. The IDC of impedance control mainly consists of two parts: the inner loop is the inverse dynamics part, and the outer loop is the motion controller based on the impedance control model. The main controller of the controller is the position controller to ensure the accuracy of reduction, and the secondary controller is the force controller to achieve desired force tracking and ensure reduction safety [21].

**D. STABILITY ANALYSIS**

Defining the Lyapunov function:

\[
V = \frac{1}{2} s^T M s + \frac{1}{2\gamma} \tilde{\sigma}^2 + \frac{1}{2\alpha} \tilde{\sigma}^2
\]  

(29)

where \( \gamma > 0, \alpha > 0 \). The time derivative of \( V \) is as follows:

\[
\dot{V} = s^T M \left[ \dot{u} - Cq - G - F_r - F_f - M\dot{q}_r + C(\dot{q} - \ddot{q}_r) \right] - \frac{1}{\gamma} \tilde{\sigma} \dot{\sigma} - \frac{1}{\alpha} \tilde{\sigma} \dot{\sigma}
\]

(30)

Since \( \dot{M} = 2C \) is an antisymmetric matrix, so \( s^T (\dot{M} - 2C)s = 0 \), then:

\[
\dot{V} = s^T (u - Cq - G - F_r - F_f - M\dot{q}_r + Cs) - \frac{1}{\gamma} \tilde{\sigma} \dot{\sigma} - \frac{1}{\alpha} \tilde{\sigma} \dot{\sigma}
\]

(31)

Combining with (25) and (27), the results are as follows:

\[
\dot{V} = s^T \left( \dot{\phi} \theta \xi (s, \dot{s}) - u_{FC} - \ddot{\phi} u_{FC} - \varepsilon \right) - \frac{1}{\gamma} \tilde{\sigma} \dot{\sigma} - \frac{1}{\alpha} \tilde{\sigma} \dot{\sigma}
\]

(32)

Since \( \sigma > 0, K_D > 0, \varepsilon_{\text{max}} \leq \sigma \), then:

\[
\dot{V} = -s^T e - ||s|| \sigma - ||s||^2 K_D \leq 0
\]

(33)

Since \( V \geq 0 \) and \( \dot{V} \leq 0 \), the system is gradually stable. When \( s \equiv 0, \dot{V} \equiv 0 \), according to the LaSalle theorem,
when $t \to \infty$, $s \to 0$, that is, the tracking error $\tilde{q}$ converges to the sliding surface and is limited to the surface [44]. The convergence rate of the system depends on $\sigma$ and $K_D$.

V. VIRTUAL PROTOTYPE SIMULATION

In order to verify the performance of the proposed impedance control algorithm, the reduction process of the robot is simulated in Matlab/Simulink software. 3D model of the reduction robot is established in SolidWorks, which is imported into Simulink. The software automatically generates the Mechanics Explorer visualization interface. 3D virtual prototype contains the attributes of the physical reduction robot (including unknown interference force, gravity and inertial force, etc.). As shown in Fig. 8, visual simulation system of the fracture reduction robot is mainly composed of the user interface, the fracture reduction robot, the Mechanical Explorer visual interface, and the oscilloscope module. The user interface is used for simulation operation, which is convenient for users to choose an algorithm, set parameters, and debug algorithms. The fracture reduction robot includes kinematics, dynamics, and various control algorithms. The Mechanics Explorer visualization interface is used to observe the motion state of reduction robot 3D virtual prototype in real-time. The oscilloscope module is used to display the change of control system parameters of the reduction robot in real-time, and output the results of each data.

The parameters of the FASMC are shown in Table 2.

| Parameter | Name                          | Value  |
|-----------|-------------------------------|--------|
| $K_s$     | Impedance model stiffness matrix | 500000 |
| $B_s$     | Impedance model damping matrix | 4000   |
| $M_s$     | Impedance model inertia matrix | $M_s = M$ |
| $\lambda$ | Error S function coefficient | 2      |
| $K_p$     | PD control part coefficient   | 50000  |
| $\alpha$  | Sliding mode gain integral adjustment coefficient | 0.5 |
| $\gamma$  | Fuzzy adaptive law coefficient | 20000  |

A. FASMC IMPEDANCE CONTROL

As shown in Fig. 3, a characteristic point P on the distal segment of the fractured bone is selected as the marked point. Firstly, according to the relative position and orientation deviation between the distal and proximal segments, the reduction ending point is marked as point B ($-30.02$ mm, 8.8 mm, 41.69 mm). Secondly, the reduction process’s obstacle points are determined, and according to the fracture contour information, the 3D A* algorithm is adopted to plan the reduction path [45]. Finally, the desired 3D reduction path is shown as “—” in Fig. 9, where A and B are the starting point and ending point, respectively.

During the reduction process, only the axial force $F_{rx}$ is studied for impedance control, and the force acting position is point P, then the force $F_r$ can be expressed as $F_r = [F_{rx}, 0, 0, 0, 0]^T$. When the $x$-direction displacement reaches the maximum, $F_{rx}$ reaches the maximum 472.2 N.
To verify the validity of FASMC, the algorithm is compared with four other similar algorithms based on IDC. The IDC needs to establish an accurate reduction robot, which can effectively weaken the system’s nonlinearity and coupling. The FC adds an adaptive FC part to the IDC to estimate and update the unknown nonlinear disturbance term in the system online to approximate the unknown nonlinear disturbance term. The SMC adds an SMC part to the IDC to compensate for the system uncertainty and improve its robustness. The PD controller adds the PD part to the IDC to realize the system’s rapid response and reduce the system’s steady-state error [21].

The FRO is used to estimate the reduction force in real-time, which replaces the force sensor to achieve impedance control, and complete the tracking of the desired trajectory and reduction force. In the impedance control closed loop, the time from the FRO feedback signal to the controller is about 4ms.

The marked point P’s position and force tracking error along the x-direction are shown in Fig.10 and Fig.11, respectively. In combination with Fig. 2, during the reduction process, the position and force tracking error increase along the x-direction with the increase of the reduction force in the AB phase. In the BC phase, the reduction force increases slowly, and the rate of change of the reduction force decreases. Under the controller’s action, the position tracking error decreases rapidly and then increases slowly, and the force tracking error decreases rapidly and then increases slowly. In the CD phase, the reduction force remains unchanged, the position error reaches a stable value, and the force tracking error increases slowly.

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**FIGURE 10.** Position tracking error along the x-direction.

**FIGURE 11.** Force tracking error along the x-direction.

**TABLE 3.** x-direction position simulation results.

| Algorithm | MAE   | SD    | Max   | Final  |
|-----------|-------|-------|-------|--------|
| IDC       | -0.602| 0.310 | -0.944| -0.944 |
| PD        | -0.933| 0.227 | -1.413| -0.787 |
| FC        | -0.736| 0.219 | -1.129| -0.876 |
| SMC       | -0.588| 0.304 | -0.931| -0.930 |
| FASMC     | -1.043| 0.287 | -1.561| -0.738 |

**TABLE 4.** x-direction force simulation results.

| Algorithm | MAE   | SD    | Max   | Final  |
|-----------|-------|-------|-------|--------|
| IDC       | 2.049 | 2.213 | 4.423 | -2.128 |
| PD        | 3.103 | 2.494 | 5.328 | -2.121 |
| FC        | 1.898 | 2.303 | 6.855 | -2.126 |
| SMC       | 2.026 | 2.124 | 4.385 | -2.128 |
| FASMC     | 3.179 | 2.418 | 5.328 | -2.120 |
In the FASMC, the parameters change with the error $s$. Under the action of the reduction force, the position and force tracking errors increase with the increase of the reduction force. Since the other four algorithms have significant fixed controller parameters, when the error of the 0-4s stage gradually increases with the reduction force, the other four algorithms can track the desired trajectory and force more effectively. However, when the reduction force reaches the maximum and remains unchanged, the FASMC can adjust the control parameters according to the error, so that FASMC has minor stability error and better reduction accuracy than other algorithms.

The position and force error of are shown in Table 3 and Table 4, respectively. The maximum position error are $-0.944$ mm (IDC), $-1.413$ mm (PD), $-1.129$ mm (FC), $-0.930$ mm (SMC), $-1.561$ mm (FACMC), and the maximum force error are $4.423$ N (IDC), $5.328$ N (PD), $6.855$ N (FC), $4.385$ N (SMC), $-5.328$ N (FACMC). The steady-state positional errors are $-0.944$ mm (IDC), $-0.787$ mm (PD), $-0.876$ mm (FC), $-0.930$ mm (SMC), $-0.733$ mm (FACMC), respectively. At the end of the simulation, the force errors obtained by the five methods are about $-2.12$ N. Since the reduction accuracy is the priority
factor in fracture surgery, the priority is given to ensuring position control accuracy. The proposed FACMC has the smallest steady-state error and the highest reduction accuracy. When the reduction robot is at the reduction end position, it is difficult for its controller to maintain the desired position and force simultaneously. The impedance control based on the FACMC can ensure that the reduction end position remains unchanged and achieve the requirements of safe reduction.

According to the impedance control of the FASMC, the actual end-effector position and orientation are shown in Fig. 12. The top position and orientation error during the reduction process are $[−1.561 \text{ mm}, 1.087 \text{ mm}, 0.766 \text{ mm}, 0^\circ, 0^\circ, 0^\circ]^T$. At the end of the reduction, the position and orientation errors are $[−0.733 \text{ mm}, 0.166 \text{ mm}, −0.146 \text{ mm}, 0^\circ, 0^\circ, 0^\circ]^T$.

The maximum contact force error is shown in Fig. 13. The maximum end contact force error during reduction are $[5.328 \text{ N}, −3.088 \text{ N}, −2.921 \text{ N}, −0.038 \text{ N-m}, −0.049 \text{ N-m}, 0.025 \text{ N-m}]^T$. When the simulation is terminated, the reduction force errors are $[−2.12 \text{ N}, −0.003 \text{ N}, −0.025 \text{ N-m}, 0 \text{ N-m}, 0 \text{ N-m}, 0 \text{ N-m}]^T$.

The FRO obtains the actual reduction force change, as shown in Fig. 14.

The uncertainties estimated by the fuzzy system in FASMC is shown in Fig. 15. At the beginning of the reduction process, the uncertainties of each link of the reduction robot are significant. With the compensation of the fuzzy system, the uncertainties of the system decrease. At the reduction end position, the uncertainties of each link remain stable.

**B. POSITION IMPEDANCE CONTROL**

The impedance control based on position correction is adopted, and the actual terminal position and orientation error

**FIGURE 17.** End-effector reduction force error.

**FIGURE 18.** Under the collision force of 100N in the $y$-direction.

**FIGURE 19.** The trajectory of the reduction force.

**FIGURE 20.** Force residual changes in joint space.
of the marked point P is shown in Fig. 16. The maximum position and orientation error during the reduction process are \([-0.109 \text{ mm}, 1.159 \text{ mm}, -0.078 \text{ mm}, 0.038^\circ, 0^\circ, 0.023^\circ]\)^T. The position and orientation errors at the reduction endpoint are \([0.005 \text{ mm}, 0.001 \text{ mm}, -0.006 \text{ mm}, 0^\circ, 0^\circ, 0^\circ]\)^T.

The actual reduction force error is shown in Fig. 17. The maximum reduction force error during reduction are \([3.4 \text{ N}, 4.97 \text{ N}, -6.233 \text{ N}, -0.033 \text{ N-m}, -0.064 \text{ N-m}, 0.027 \text{ N-m}]^T\). When the simulation is terminated, the reduction force error are \([-2.13 \text{ N}, -0.001 \text{ N}, -0.04 \text{ N}, 0 \text{ N-m}, 0 \text{ N-m}, 0 \text{ N-m}]^T\).

It is known that the FRO is used to estimate the reduction force online, which can replace the force sensor to realize the impedance control of the reduction robot based on position correction, and complete the tracking of the desired trajectory and reduction force.

C. ACCIDENTAL COLLISION

The effect of the FASMC impedance control system based on FRO feedback under external disturbance is verified. During the reduction process, when the time is 4 s, the end-effector is subjected to an accidental collision force \(F_{y4} = 100 \text{ N} \) along the y-direction, as shown in Fig. 18. Set the safety threshold \(Thr = \pm 20 \text{ N} \) in the y-direction of the system. When the force \(F_{xy} \) in the y-direction is estimated by the FRO to exceed the range of \(\pm 20 \text{ N} \), the system will issue a warning. During the reduction process, the variation of external force estimated by the FRO is shown in Fig. 19, and the external force changes in the joint space are shown in Fig. 20.

VI. CONCLUSION

To improve the performance of fracture reduction robot during surgery, this paper proposes an impedance control method, including a FRO and a FASMC. The FRO is used to estimate the contact force between the reduction robot and the musculoskeletal tissue environment in real-time, which is used as the feedback signal of impedance control. The FASMC is designed based on the inverse dynamics of the robot, which can effectively suppress the influence of significant disturbances, noise, and other factors. The system has the advantages of small steady-state error and insensitive disturbance. Lyapunov stability theory is used to prove the stability of the system. The visualization simulation of the fracture reduction process is performed in Matlab/Simulink software. The simulation results show that at the end of the reduction, the position and orientation errors are \([-0.733 \text{ mm}, 0.166 \text{ mm}, -0.146 \text{ mm}, 0^\circ, 0^\circ, 0^\circ]\)^T, the reduction force errors are \([-2.12 \text{ N}, -0.003 \text{ N}, -0.025 \text{ N}, 0 \text{ N-m}, 0 \text{ N-m}, 0 \text{ N-m}]^T\). The reduction robot can estimate the collision force when the external collision occurs.

In the future, the experimental study of the fracture reduction will be done to verify the control effect of the proposed control method. The error compensation techniques based on neural networks, machine learning or deep learning will improve the accuracy of the reduction robot and further improve the robot’s performance.

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