Derivation of Dielectric Model of Confimenent in QCD

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Abstract

After the gauge invariant gluon condensation, gluons remain as massless excitations. When an effective theory describing the condensation and the excitation is constructed, a constraint has to be imposed for the vacuum to be stable. The constraint implies the perfect dielectricity and assures the solution of color flux tube when quarks are introduced. Thus the dielectric model of Kogut-Susskind [5] and 't Hooft [6] are derived by the stability of the condensed vacuum.

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Introduction;

Understanding of the confinement mechanism based on QCD Lagrangian remains as a fundamental problem in the strong interaction regime. Besides lattice QCD [11], active researches have been performed along the line of the dual superconductivity with the abelian gauge fixing [2,3], infrared structure of the gluon propagators [4], e.t.c.. In continuum QCD, the key point will be how to extract the effect of the gluon condensation.

In this letter, we propose an approach to construct the effective Lagrangian and derive the dielectric model of Kogut-Susskind [5] and 't Hooft [6]. We assume that gluons condense in gauge invariant form, so the gauge invariance is not broken in the vacuum and gluons which are excitations above the condensed vacuum remain massless. Because of masslessness, they may condense further in color singlet form. Therefore, when we construct the effective Lagrangian describing the interaction between the condensation and excitation field, non-trivial conditions have to be imposed to assure the the stability of the vacuum. Remarkably, these conditions imply the perfect dielectricity of the vacuum and guarantee the existence of the color flux tube of infinite length when quarks are introduced. As for the gluon condensation, the magnetic type condensation $<G^{2}_{\mu \nu}> 0$ has been established [17] but the advantage of the present work is that we do not have to ask the precise mechanism of the condensation, the magnetic monopole[8] or the gluon pairs[9] e.t.c..

Effective Lagrangian $L_{\text{eff}}$;

To measure the singlet condensation, any gauge invariant operator can be used but we choose for simplicity the normal ordered form of $G^{2}_{\mu \nu}$: $\phi(x) = N G^{2}_{\mu \nu}(x) = G^{2}_{\mu \nu}(x) - <0|G^{2}_{\mu \nu}(x)|0>$. ($|0> \text{ is the}$
normal vacuum and the dimensional regularization is adopted.) With $|0>_{c}$ the condensed vacuum, we know that $\epsilon<0|\mathcal{N}G_{\mu\nu}^{2}|0>_{c}=\phi_{c}>0$. We regard gluon fields $A_{\mu}^{a}(x)$ and $\phi(x)$ as independent fields. (The general formalism to achieve this is shown below.) As in the case of the free energy of the phenomenological theory of Ginzburg-Landau, the low energy effective Lagrangian $\mathcal{L}_{eff}$ is obtained by the hydrodynamic expansion. It agrees with the expansion according to the operator dimension in $A_{\mu}^{a}(x)$. By gauge invariance and keeping lowest non-trivial terms, we get

\begin{equation}
\mathcal{L}_{eff} = \partial_{\mu}\phi(x)\partial^{\mu}\phi(x)/2 - V(\phi(x)) - \epsilon(\phi(x))G_{\mu\nu}(x)/4 \equiv \mathcal{L}_{\phi} + \mathcal{L}_{c,A},
\end{equation}

\begin{equation}
G_{\mu\nu}^{a}(x) = \partial_{\mu}A_{\mu}^{a}(x) - \partial_{\nu}A_{\mu}^{a}(x) + gf^{abc}A_{\mu}^{b}(x)A_{\nu}^{c}(x).
\end{equation}

Here, $g$, $f^{abc}$ is the coupling constant and the structure constant of QCD, respectively. The effect of the condensation on the excitation appears as a dielectric factor $\epsilon(\phi)$. Assuming $\phi_{c} > 0$, define $V(0) = 0$, $V(\phi_{c}) < 0$. We require $\phi = 0$ ($\phi = \phi_{c}$) is the maximum (minimum) of $V(\phi)$; $V'(0) = V'(\phi_{c}) = 0$, $V''(0) < 0$, $V''(\phi_{c}) > 0$. When $\phi = 0$, $\mathcal{L}_{QCD}$ has to be recovered, so $\epsilon(0) = 1$. For all $\phi$, $\epsilon(\phi)$ has to be positive in order for $\mathcal{L}_{c,A}$ to be a sensible theory. The gauge fixing term and resulting ghost fields are not written explicitly. Our conclusions will not change if these are introduced since only a gauge invariant degree $\phi(x)$ is added to the ordinary QCD, where we know how to extract physical sectors [10].

**Stability of the condensed vacuum:**

Below, the fluctuation of the condensation is neglected, so $\phi(x)$ is treated as a c-number field. In order for $\mathcal{L}_{eff}$ to describe a consistent theory, i.e. $\phi = \phi_{c}$, actually realizes the lowest energy state of $\mathcal{L}_{eff}$, we have to require first of all that excitation fields $A_{\mu}^{a}(x)$ do not condense any more when $\phi = \phi_{c}$. If $\epsilon(\phi_{c}) \neq 0$, $\mathcal{L}_{c,A}$ is proportional to $\mathcal{L}_{QCD}$, so $A_{\mu}^{a}(x)$ condenses in the color singlet channel. In that case $\phi$ has to be redefined, and after that $\epsilon(\phi_{c}) = 0$ is satisfied. Such a perfect dia-electricity of the stable vacuum is specific to QCD; the gauge invariance is preserved after the condensation. Note that if the excitation acquires a mass gap, as in the case of the superconductor, such an instability never occurs. Assuming the homogeneous case $\phi = \text{constant}$, we require that the energy is indeed increased for $\phi \neq \phi_{c}$. Consider the trace of the energy momentum tensor in n-dimension by using $\mathcal{L}_{eff}$: $\Theta_{\mu}^{\mu} = 4V(\phi) + \epsilon(\phi)(n - 4)G_{\mu\nu}^{2}$. In calculating the anomalous term for $n \rightarrow 4$, we have to define $\mathcal{N}G_{\mu\nu}^{2}$, by subtracting the expectation value taken by the perturbative vacuum of the ordinary QCD. In this connection, notice that our origin of the energy is that of the perturbative vacuum of QCD, for example we have defined $V(\phi = 0) = 0$. Thus the situation is that the whole system evolves with $\mathcal{L}_{eff}$ while the normal order has to be defined by the perturbative vacuum of $\mathcal{L}_{QCD}$. In such a case, it is convenient to adopt the interaction picture with $\mathcal{L}_{\epsilon=1,A} = \mathcal{L}_{QCD}$ as the “free Lagrangian”, and the rest as the “interaction part”, the latter being converted into the state vector. We can minimize the expectation value of $\Theta_{\mu}^{\mu}$ in the interaction picture thus defined. (See below for details.) Now the operator evolves by $\mathcal{L}_{QCD}$, so the trace anomaly of QCD holds:

\begin{equation}
\Theta_{\mu}^{\mu}(x) = 4V(\phi) + \epsilon(\phi)\beta(g)/2g)\mathcal{N}G_{\mu\nu}^{2}(x),
\end{equation}

Here $\beta(g)$ is the usual $\beta$ function with $g$ the renormalized coupling constant. We know that the excitation
field $A^a_\mu$ condenses with the amount

$$B = (\beta(g)/8g)_c < 0 |N\mathcal{G}_{\mu\nu}^a(x)|0>_c < 0. \quad (3)$$

With fixed $\phi$, the energy density (measured from the normal vacuum) of such a state is given by $e = V(\phi) + \epsilon(\phi)B$. Here we have used the relation valid for any homogeneous vacuum: $<\Theta_{\mu\nu}(x)> = g_{\mu\nu}e$, so $<\Theta^\mu_\mu> = 4e$. We require the above condensation does not lower the energy of the state $\phi = \phi_c$, otherwise $|0>_c$ is unstable. Here we note that $B$ introduced in (3) is the energy of condensed vacuum $|0>_c$, since it is calculated by $\mathcal{L}_{QCD}$, therefore it is nothing but $V(\phi_c)$; $V(\phi_c) = B$. In this way, the stability of $|0>_c$ requires

$$B < V(\phi) + \epsilon(\phi)B, \quad (4)$$

A remarkable fact is that the stability condition (4) assures the existence of the solution of color flux tube of infinite length when quarks are introduced. Near $\phi = \phi_c$, let us put $\phi = \phi_c + \Delta \phi$ in (4) and expanding $V(\phi)$ and $\epsilon(\phi)$ up to $(\Delta \phi)^2$. One arrives at

$$\epsilon(\phi_c) = \epsilon'(\phi_c) = 0, \quad V''(\phi_c) + \epsilon''(\phi_c)B > 0 \quad (5)$$

In case $\epsilon(\phi)$ is parametrized as $\epsilon(\phi) = C(\phi - \phi_c)^{2\alpha}$ with $C > 0$ near $\phi = \phi_c$, then $\alpha \geq 1$. Expanding in $\phi$ for small $\phi$, one obtains from (4)

$$\epsilon'(0) = 0, \quad V''(0) + \epsilon''(0)B > 0. \quad (6)$$

By $V''(0) < 0$ and $B < 0$, we get $\epsilon''(0) < 0$. Thus near $\phi = 0$, $\epsilon(\phi)$ behaves as $1 + a \phi^2$ with $a < 0$.

The inequality (6) becomes an equality for $\phi = \phi_c$ and for $\phi = 0$. This should be the case since both represent $|0>_c$: for $\phi = 0$ the excitation $A^a_\mu$ condenses producing $B$. However we have to select $\phi = \phi_c$ as $|0>_c$, because our starting definition of $A^a_\mu$ is that it represents the exitation field without the condensed part. If $\phi = 0$ is selected, the role of $\phi$ and $\mathcal{N}\mathcal{G}_{\mu\nu}^a$ is interchanged, so we have to restart our discussions.

We conclude that $\phi = \phi_c$ is the lowest state of energy for $0 < \phi < \phi_c$, without $\mathcal{N}\mathcal{G}_{\mu\nu}^a$ condensing anymore, since the condensation does not lower the energy. When $\phi = \phi_c$, we have no observable effect of $\mathcal{N}\mathcal{G}_{\mu\nu}^a$ due to $\epsilon = 0$. (See below for more details.) Thus our effective Lagrangian is (1), with $V(\phi)$ and $\epsilon(\phi)$ satisfying (4). Before showing the tube-like solution, we discuss two subjects whose results have been utilized above.

**Constructing $\mathcal{L}_{eff}$:**

We present first the method of obtaining $\mathcal{L}_{eff}$ by the Fourier tranform and its inverse, with $\phi$ and $A^a_\mu$ being treated as independent fields. Let $\Psi[A^a_\mu]$ be an arbitrary functional of $A^a_\mu(x)$ and start from

$$Z = \int [dA^a_\mu] \Psi[A^a_\mu] \exp i \int \mathcal{L}_{QCD} dx \quad (7)$$

Let $O(A^a_\mu(x))$ be a gauge invariant operator and insert a functional identity $\int [d\phi] \int [dJ] \exp i \int J(x)(O(A^a_\mu(x))-\phi(x))dx$ and write $\Psi[A^a_\mu]$ by a functional Fourier transform $\Psi[j^a_\mu]$. Then the integration over $A^a_\mu$ is done with $\mathcal{L}_{QCD} + j^a_\mu A^{a\mu} + JO(A^a_\mu)$ in the exponential. Writing the result as $\exp i W[j^a, J]$, we have

$$Z = \int [dJ] \int [dj^a_\mu] \int [d\phi] \Psi[j^a_\mu] \exp iW[J, \phi, j]$$

$$= \int [dA^a_\mu] \int [d\phi] \Psi[A^a_\mu] \exp i\Gamma[\phi, A^a_\mu]. \quad (8)$$
is obtained. Here \( W[J, \phi, j] = \int W[J, j_\mu]\) has been transformed back to \( \Psi[A_\mu^a] \), and we have done integrations over \( j_\mu \) and \( J \). Now after \( \phi \) integration, we get back to \( \mathcal{L}_{QCD} \) since only identical transformations are done. However, when the stationary phase of \( \phi \) integration is present, it corresponds to a nontrivial phase. Note that if the integral defining \( Z \) is dominated by a special value of \( \phi \), \( Z \) cannot be transformed back to \( \mathcal{L}_{QCD} \), inequivalent vacuum being selected. Since \( \Psi[A_\mu^a] \) is arbitrary, \( \Gamma[\phi, A_\mu^a] \) can be regarded as the Lagrangian of the condensed phase. \( (\Psi[A_\mu^a]) \) can be generalized to \( \Psi[A_\mu^a, \phi] \) in (3). We have checked our useful formula (3) for several model Lagrangians. In the case of QCD, we know that \( \phi \) condenses, so \( \Gamma[\phi, A_\mu^a] \) has a non-trivial stationary solution corresponding to this condensate. Eq. (1) is the hydrodynamic expansion of \( \Gamma[\phi, A_\mu^a] \) and we have regarded \( \phi \) as a c-number field since \( \phi \) loses fluctuations by the stationary phase mechanism. The stationary condition is represented by the stationary equation of \( \mathcal{L}_\phi \) and we require \( \mathcal{L}_{\epsilon, A} \) not to cause the instability of the of \( V'(\phi) = 0 \).

Normal ordering and interaction picture;

Consider the expectation value of an arbitrary local operator \( Q(A_\mu^a(x)) \equiv Q(x) \) in the theory \( \mathcal{L}_{\text{eff}} \) for the homogeneous case of \( \phi(x) = \phi \). Omitting the c-number \( \mathcal{L}_\phi \), here we introduce the \( U \) operator in the path integral form by

\[
U(t_2, t_1 : \epsilon) = \int [dA_\mu^a] \exp \left( \int_{t_1}^{t_2} dt d^3x \mathcal{L}_{\epsilon, A} \right)
\]

([\( dA_\mu^a \)] contains the factor \( \sqrt{\epsilon(\phi)} \).) The matrix element of \( U \) between the states \( |A_\mu^a(t_1, 2, x) \rangle \) (in coordinate representation) is equal to the right-hand side with \( A_\mu^a(x) \) fixed to the value \( A_\mu^a(t_1, 2, x) \) at \( t_{1,2} \). Now we want to rewrite \( U(t_2, t_1 : \epsilon) \) by the interaction representation where “free part” is defined to be \( \epsilon = 1 \). (This method of introducing the interaction representation in path-integral form is quite convenient since it keeps the Lorentz invariance explicitly.) By the homogeneity of the system, we take \( x = 0 \). With \( |\Psi\rangle \) an arbitrary state and taking the coordinate representation in mind, let us consider

\[
< Q(0) > = \langle \Psi | U(0, -\infty : \epsilon) Q(0) U(0, -\infty : \epsilon) | \Psi \rangle
= \langle \Psi, \phi | U(-\infty, 0 : 1) Q(0) U(0, -\infty : 1) | \Psi, \phi \rangle,
\]

\[
|\Psi, \phi \rangle \equiv U(-\infty, 0 : 1) U(0, -\infty : \epsilon) | \Psi \rangle.
\]

In this representation, \( Q \) evolves by \( \mathcal{L}_{QCD} \) so for \( Q(0) = (n - 4) G_{\mu\nu}^2(0) \), the ordinary trace anomaly (2) holds. Here the normal order is defined with respect to the perturbative vacuum of QCD. As for the state vector, if we choose \( |\Psi, \phi \rangle = |0\rangle_c \), then it is the lowest energy state in the theory \( \mathcal{L}_{QCD} \) obtaining (3).

A comment here; if we insert the adiabatic factor connecting \( \mathcal{L}_{QCD} \) and \( \mathcal{L}_{\text{eff}} \), the difference of the energy of the ground states of two theories (difference of both-hand sides of (1)) is given by the adiabatic formula obtained in the interaction representation (11).

Color flux tube;

When color sources are present, they couple to the excitation field \( A_\mu^a(x) \). We show here that the c-number solution of the infinite tube exists, where abelian components \( A_0^a \) and \( A_0^b \) are non-zero. This
is sufficient for the confinement of color non-singlet state. Such a solution, if it exists, has a large c-number value, so it is stable against quantum fluctuations of other gluonic fields. (Quark pair creation is not discussed.) Below, the color index \( a \) takes 3 or 8. Now consider the case of static quark-antiquark point-like source with the strength \( \pm g(\lambda^a/2) \) at \( z = \pm \infty \). (\( \lambda^a \) is the representation matrix of the quark.) Define \( G_{k0}^a \equiv \epsilon(\phi)E_k^a \) \( (k = x, y, z) \), then \( \text{div}(\epsilon(\phi)E^a) \) is the color density. For an infinite tube, \( \phi \) depends only on \( \rho \) in the cylindrical coordinate \( (z, \rho, \phi) \) and \( E^a \) is directed along \( z \) axis, which is constant over all space because of \( \text{rot}E^a = 0 \). The equation of \( \phi \) is

\[
\frac{d^2\phi}{d\rho^2} + \frac{1}{\rho} \frac{d\phi}{d\rho} = V'(\phi) - \epsilon'(\phi)E^2/2 \equiv |B|K'(\phi, \theta),
\]

where we have written \( E_z^a = E_z \lambda^a/2, E^2 = \sum_a E_z^a \) for any color of quark. We have also defined

\[
K(\phi, \theta) = v(\phi) - \epsilon(\phi)\theta, \quad \theta = E^2/2|B|
\]

with \( v(\phi) = V(\phi)/|B| \). Note that \( K(\phi, 1) > -1 \) by (9). The behavior of \( K(\phi, \theta) \) is shown in the Figure. The solution of (9) with \( \phi(\rho) \) approaching \( \phi_c \) at \( \rho = \infty \) is given by \( \phi(\rho) \sim \phi_c + D_1 \exp(-\sqrt{W}\rho) + D_2 \exp(\sqrt{W}\rho) \), with \( W = |B|K''(\phi_c) \). We see \( W > 0 \) by (11) if \( \alpha > 1 \). (When \( \alpha = 1 \), \( E < E_{\text{max}} \) with \( E_{\text{max}}^2 = 2|B|v''(\phi_c)/\epsilon''(\phi_c) \) has to be satisfied.) Near \( \rho = 0 \), \( \phi(\rho) \sim \phi(0) + Cp^2 + F_1 \ln \rho \), with \( C = |B|K'(\phi(0), \theta)/4 \). The requirement \( F_1 = 0 \) and \( D_2 = 0 \) determines \( \phi(0) \) and \( D_1 \). \( E \) is fixed by the total flux \( 2\pi \int \rho d\rho \epsilon(\phi(\rho))E_z = g \). Now we look for the solution which increases monotonically from \( \phi(0) \) to \( \phi(\infty) = \phi_c \). This requires \( C > 0 \), which is shown to be consistent afterwards. In order for such a solution to exist, \( K(\phi, \theta) \) should have a bump in the region \( 0 < \phi < \phi_c \). To see this, we note that \( \phi''(\rho) \) changes sign at some \( \rho_d \), where \( \phi''(\rho_d) = 0 \). From (9), and \( \phi'(\rho_d) > 0 \), \( K'(\phi(\rho_d), \theta) \) has to be positive, which connects smoothly to the behavior near \( \phi = \phi_c \), where \( K' < 0 \). So \( K' = 0 \) at some \( \phi = \phi_s \), the bump position, in the range \( \phi_d < \phi_s < \phi_c \). If we define \( \rho_s \) by \( \phi(\rho_s) = \phi_s \), it is the measure of the radius of the tube. Below, \( \theta > 1 \) is proved for any solution, so we need the bump structure for \( \theta > 1 \). This condition is met by the existence of the bump for \( \theta = 1 \), as is clear from \( K(0, \theta) = -\theta \). (see the Figure.) Thus we can say that the stability of the condensed vacuum assures the flux tube of infinite length.

To show \( \theta > 1 \), a sum rule is derived by multiplying \( d\phi/d\rho \) on both sides of (9) and integrating from \( \rho = 0 \) to \( \infty \). Using boundary conditions on both ends, we get

\[
\int_0^\infty \frac{1}{\rho} \left( \frac{d\phi(\rho)}{d\rho} \right)^2 d\rho = |B| \left( K(\phi_c, \theta) - K(\phi(0), \theta) \right) > 0.
\]
Since $K(\phi_c, \theta) = -1$, $K(\phi(0), \theta) < -1$ holds, implying $\theta > 1$; the gain of the electric energy inside the tube is larger than the loss in breaking the condensation energy. If the left-hand side of (9) is regarded as the surface energy, the difference of these two energies is sustained by the surface energy. If we neglect completely the surface energy, the solution satisfies $K'(\phi, \theta) = 0$, thus $\phi = \phi_c$, or $\phi = 0$. Setting $\phi = 0$ inside the tube and $\phi = \phi_c$ outside, and denoting by $S$ the cross section of the tube, the energy per unit length $S(|B| + E^2/2)$ is minimized under the constraint $SE = g$. Then $S = g/\sqrt{2|B|}$, $E = \sqrt{2|B|}$ is obtained, leading to $\theta = 1$. The same result of course can be derived by the sum rule since it gives $K(\phi(0), \theta) = -1$ and $\phi(0) = 0$ if the surface energy is neglected. These hold for all $\alpha \geq 1$. If the surface energy is included and becomes large, $\theta$ and hence $E$ increases ($E < E_{\text{max}}$, when $\alpha = 1$) and both the bump position and $\phi(0)$ approach $\phi_c$. Thus a solution to (9) is assured again for any $\alpha \geq 1$.

Near the source:

In the neighbourhood of the point-like colored source of a quark, the condition (1) near $\phi = 0$ becomes essential. The equation to be solved is $\nabla^2 \phi = V'(\phi) - \epsilon'(\phi)E^a \xi^2/2$. We assume $\phi$ approaches 0 near the source and search for the spherical solution with the electric field at the distant $r$ from the quark given by $E^a = g\lambda^a/2r^2$. Neglecting $V'(\phi)$ and by $\epsilon(\phi) \approx 1 + a\phi^2$ ($a < 0$) near $\phi = 0$, we get $\phi(r) = (1/r)\exp(-\sqrt{A}/r)$, $A = -ag^2\sum_{a}(\lambda^a/2)^2 = -ag^2/3 > 0$. The solution is consistent with the starting assumption that $\phi \sim 0$ near the source. The effect of the condensation near the quark is smaller than any power of $r$.

Strong coupling for the canonical field:

Our full quantum theory is defined by regarding (11) as the action which is functionally integrated over $\phi$ and $A^a_\mu(x)$. Let us neglect the fluctuation of $\phi$ first. In the condensed vacuum, since $\epsilon(\phi_c) = 0$, $A^a_\mu(x)$ fluctuate indefinitely but it has no observable effect by the same fact $\epsilon = 0$. In the homogeneous case, $\mathcal{L}_{\epsilon, A}$ can be written by the canonical field $\tilde{A}^a_\mu(x) = \sqrt{\epsilon(\phi)}A^a_\mu(x)$ (unity for the coefficient of the kinetic term). Then we get the same Lagrangian as QCD with $g$ replaced by $g_c = g/\sqrt{\epsilon(\phi)}$. Thus QCD becomes a strong coupling theory near the vacuum $\phi = \phi_c$, where $g_c \to \infty$. Then, the dominant interaction term becomes 4-point vertex and by the stationary phase mechanism, all $\tilde{A}^a_\mu(x)$’s are freezed to zero locally; gluons are confined in the operator level. The mechanisms of confinement of gluons are different for $A^a_\mu$ and $\tilde{A}^a_\mu$ but they are physically equivalent. When the quark field $q(x)$ are introduced, we add the usual term $\bar{q}(x)(\gamma^\mu(i\partial_\mu + gA_\mu(x)) + m)q(x)$ to $\mathcal{L}_{\text{eff}}$ with $A_\mu = \sum_{a=1}^8 A^a_\mu \lambda^a/2$. Note that the gluonic condensation is unrelated to the quark sector. When $\epsilon = 0$, the integration by $A^a_\mu(x)$ leads to the vanishing of the operator local color current of quarks; $j^a_\mu(x) = \bar{q}(x)\gamma_\mu(\lambda^a/2)q(x) = 0$ for any $a$ and $\mu$. To study excitations, we note that fluctuations of $\tilde{A}_\mu$ and $j^a_\mu$ come about only if $\phi$ fluctuates from the value $\phi_c$, which is governed by the glue ball mass. We expect $\phi \neq \phi_c$ in a localized region with size $1/m_g$, where $\tilde{A}^a_\mu$’s and $j^a_\mu$ have non-zero values and the ordinary perturbative picture works. Above picture of strong coupling expansion looks very similar to the same expansion in lattice QCD.

Discussions:
We try to interpret our results by the dual terminology. The permeability $\mu = 1/\epsilon$ is infinite in the vacuum and for the above tube-like solution, the magnetic supercurrent defined by $\text{rot} D^a/\epsilon = (1/\epsilon) \nabla \epsilon \times D^a$ is circulating around the surface of the tube which sustains the electric flux. Let us introduce the dual potential $C^a_\mu$ as a Lagrange multiplier of the Bianchi identity \cite{12, 13} by taking the axial gauge $n^\mu A^a_\mu = 0$ with some constant vector $n^\mu$. By a simple manipulation, we get the relation that $\text{rot} C^a$ is equal to $D^a$ plus the string term which depends on $n^\mu$. Thus the dual potential $C^a$, with the string part subtracted, constitutes the tube decaying to zero for $\rho = \infty$. Although we have not explicitly the mass term $m^2 C^a_\mu$ which breaks the magnetic gauge invariance, the model discussed here behaves as dual to the superconductor in the above sense.

Our starting formula including full quantum effects is \cite{8}, which can be used in principle to determine consistently phenomenological parameters introduced above, or to study the shielding charge due to gluons, or by including quark degrees, the quark pair production which breaks the tube, e.t.c..

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