LGLG-WPCA: An Effective Texture-based Method for Face Recognition

Chaorong Li\(^1,2\), Wei Huang\(^1\), and Huafu Chen\(^1\)

\(^1\)University of Electronic Science and Technology of China, Chengdu 611731, China
\(^2\)Department of Computer Science and Information Engineering, Yibin University, Yibin 644000

This paper proposes an effective texture-based face feature extraction method which is based on Learning Gabor Log-Euclidean Gaussian, called LGLG-WPCA. LGLG-WPCA has the robust performance for adverse conditions such as varying poses, skin aging and uneven illumination. LGLG learns face features from the embedded multivariate Gaussian in Gabor wavelet domain using Whitening Principal Component Analysis (WPCA). In LGLG, we first employ Gabor wavelet to decompose the face, and then use the multivariate Gaussian distribution to fit Gabor subbands. Because the space of Gaussian is a Riemannian manifold and it is difficult to incorporate learning mechanism in the model. To address this issue, we use L\(^2\)-EMG\(\[1\]\) to map the multidimensional Gaussian model to the linear space, and then use WPCA to learn facial features. Experiments show that our proposed method is an effective and promising face texture feature extraction technique.

Index Terms—Gabor wavelet, Multivariate Gaussian, Log-Euclidean Gaussian, Face recognition, WPCA.

I. INTRODUCTION

Because of the potential application value, face recognition has always been a research hotspot in the field of machine vision, such as identity authentication, access control system and online transaction. Unfavorable factors constitute great challenges to the automatic face recognition. These unfavorable factors include: image noise, low resolution image, uneven illuminations and poses, skin aging, facial expressions and facial occlusion. Among these factors, uneven illuminations, facial occlusion and varying poses are most prone to occur in unconstrained environments.

To face recognition, the effects of uneven illumination can be counteracted by conventional image processing techniques like the simple and efficient preprocessing chain \[2\]. However, facial occlusion and varying pose are more difficult than uneven illuminations, because the conventional image processing methods cannot correct or reconstruct a high fidelity face image for the damaged image\[3\].

In general, face recognition methods against adverse factors fall into two categories. (1) Directly calculate the face features from face image using Deep Convolutional Network (DCNN) based methods such as DeepID\[4\], FaceNet\[5\], VGGFace\[6\] and PCANet\[7\]. These methods use a large number of samples to adapt to face recognition in different poses, different expressions, and lighting. However, such methods are less effective for large varying poses. (2) Preprocessing-based feature extraction method (see Figur\[4\], which is the approach adopted by most face recognition methods. First, the preprocessing step is used to remove or counteract the adverse factors including preprocessing the illumination, noise, varying poses and facial expressions, etc. Then texture feature extraction descriptors (like LBP\[8\], LTP\[2\], SIFT\[9\] HOG\[10\] and Gabor\[11\]) or DCNNs are used to produce the face features.

Recently, a few preprocessing based methods have been developed to deal with the most common and difficult large-varying-pose problem: For example, DeepFace\[12\] uses 3D to align faces; TP-GAN\[13\] uses Generative Adversarial Nets(GAN) \[14\] to rectify (frontalize) face image and Light CNN\[15\] for face recognition; HF-PIM\[16\] combines GAN and 3D Morphable Model\[17\] to frontalize the face and then use Light CNN to extract facial features; DR-GAN\[18\] trains the Encoder-Decoder network while using GAN to frontalize the faces. Large varying poses or occlusions may be corrected (frontalized) by using 3D Morphable model or GAN. However, the 3D Morphable or GAN based methods cannot rectified the small varying expressions or poses and furthermore; moreover, the rectified images will produce more or less deviation compared with the ground truth images.

In fact, the small varying expression and poses are inevitable and therefore it is important that the feature extraction method has robustness to small pose or expression variations or difference between the original image and the resurrected image.
In this paper, we propose a preprocessing-texture feature extraction method (called LGLG-WPCA) for face recognition. LGLG extracts facial features using multidimensional Gaussian models on Gabor wavelet which is an excellent texture feature descriptor and is insensitive to noise and it can extract discriminative face features under the condition of small pose or expression variations. Previous methods established a covariance matrix on the Gabor wavelet domain to represent the human face and these methods ignore the mean information of the Gabor wavelet subband. Multidimensional Gaussian model is an extension of the covariance matrix, and Gaussian model is superior to the covariance model for multivariate data. Similar to covariance matrix, multidimensional Gaussian model also belongs to the Riemannian manifold. The computational cost in Riemannian space is much higher than that of in Euclidean space; furthermore it is difficult to incorporate a learning mechanism to improve the performance of the model in Riemannian space. In order to reduce the computational cost, we use $L^2$EMG\(^1\) embed multivariate Gaussian model in Euclidean space and Whitening Principal Component Analysis (WPCA) is used to learning discriminative face features.

## II. RELATED WORKS

Gabor wavelet (Gabor filters) is an important texture extraction tool in the field of computer vision. In a number of works, Gabor wavelet is used to face recognition. Because Gabor wavelet is a very redundant transform, researchers used PCA or LDA to compress the Gabor subbands (responses) and the discriminative face information are yielded. For example, Gabor-Fisher classifier\(^{19}, 20\) applies the Fisher linear discriminant model to Gabor subbands derived from the Gabor wavelet representation of face images; Gabor wavelet based kernel PCA method\(^21\) used kernel PCA to compress the Gabor features. Some researchers\(^22, 23\) encoded the Gabor subbands by using LBP, and then obtained the face features.

Mean and standard-deviation of the Gabor wavelet subband are the discriminative information of images, and they can be used as the image features; however, the low-dimensional features composed of mean and standard-deviation are difficult to distinguish between different faces which are highly similar between-classes and large varying within-classes. Covariance matrix is an effective measure for augmenting standard deviation features. Given the number of Gabor subbands is $d$, the dimensionality of mean and standard-deviation features is $2d$, whereas the size of covariance matrix features is $d \times d$ and the different value of covariance matrix is $(d^2 + d)/2$. Therefore, covariance matrix has stronger discriminative capability than that of the features composed of mean and standard deviation.

Covariance/covariance matrix has been widely used to represent images\(^24, 25, 26, 27, 28, 29\). In \(^24\), Tuzel et al. map each pixel to a 5-dimensional feature space (intensities, norm of first and second order derivatives of intensities), and use a covariance matrix to model these features. Yanwei et al.\(^25\) proposed a covariance matrix based method to model Gabor subbands; in their work, both pixel locations and Gabor features are employed to construct the covariance matrices. Wang et al.\(^26\) use covariance matrix for image set based face recognition. Recent years, covariance or covariance matrix are introduced into deep learning based networks\(^30, 31\). Because covariance matrix is Riemannian manifold, Euclidean distance cannot be directly used as the measure of covariance matrix. Therefore, Tuzel et al.\(^24\) proposed Riemannian distance as the measure of covariance matrix. Given two covariance matrices $C_1$ and $C_2$, Riemannian distance is defined as:

$$RD(C_1, C_2) = \sqrt{\sum_{i=1}^{d} ln\lambda_i^2(C_1, C_2)},$$  \hspace{1cm} (1)

where $\lambda_i(C_1, C_2)_{i=1,...,d}$ are the generalized eigenvalues of $C_1$ and $C_2$. The computation cost of Riemannian distance in the feature matching step is expensive because we should calculate the generalized eigenvalues of $C_1$ and $C_2$ in \(^1\). Researchers have developed a embedding approach which transforms the covariance matrix into a linear space, called Log-Euclidean distance (LED)\(^32, 33\). LED is expressed as:

$$LED(C_1, C_2) = ||log(C_1) - log(C_2)||_F,$$  \hspace{1cm} (2)

where log is matrix logarithm operator and $\cdot||_F$ denotes the Matrix Frobenius Norm (MFN). Besides, Minh et al.\(^27\) provided a finite-dimensional approximation of the Log-Hilbert-Schmidt (Log-HS) distance between covariance operators to image classification.

Covariance matrix is a special case of multivariate Gaussian distribution which parameters consist of mean vector and covariance matrix. Similar to covariance matrix, the space of Gaussian is not a linear space but a Riemannian manifold. Peihua et al.\(^1\) used Lie group to embed multivariate Gaussian into Euclidean space, called Local Log-Euclidean Multivariate Gaussian ($L^2$EMG). Embedding form of Gaussian Euclidean space is denoted by

$$B = log\left[\begin{bmatrix} C + \mu\mu^T \mu \\ \mu \end{bmatrix} \mu \right]^{\frac{1}{2}}$$  \hspace{1cm} (3)

where $C$ and $\mu$ are the covariance matrix and mean parameters of Gaussian. The similarity of two Gaussian models is denoted by

$$L^2EMG(C_1, C_2) = ||B_1 - B_2||_F$$  \hspace{1cm} (4)

Multivariate statistical models including covariance based methods have two shortcomings. First, because of the complicated matrix operation, the computation of the measure in a Riemannian space is time-consuming compared to the computation in a Euclidean space. Second, it is difficult to resort to an effective learning approach to improve the performance. Recently a few methods have been proposed to address these shortcomings. For example, Harandi et al.\(^84\) use an orthonormal projection model to project the high-dimensional manifold to low-dimensional. Wang et al.\(^28\) developed a discriminative covariance oriented representation learning framework to deal with face recognition. Different from the methods mentioned above, we proposed an efficient method by WPCA to learn the face features from the multivariate Gaussian in Gabor domain. Our methods can efficiently describe the texture feature of face and it is robust to small variance of face such as facial expressions.
III. Gabor Wavelet

Gabor Wavelet is an efficient tool for image analysis. It is defined as the convolution on the image with a set of Gabor filters. The 2D Gabor filter is the product of a Gaussian function and the complex exponential function, denoted by \([19]\),

\[
\psi_{u,v}(z) = \frac{\|k_{u,v}\|^2}{\sigma^2} e^{-\left|\frac{k_{u,v}^2}{\sigma^2} - e^{-\frac{z^2}{\sigma^2}} \right|} (5)
\]

where \(z = (x,y)\); \(u\) and \(v\) define the direction and scale of the Gabor filters (Therefor, \(\psi_{u,v}\) is called Gabor wavelet), \(\|\cdot\|\) denotes the norm operator; \(k_{u,v}\) is the wave vector which has the following express

\[
k_{u,v} = k_{\text{max}}/f^v, \quad k_{\text{max}} \text{ is the maximum frequency; }
\]

\[
\psi_v = \frac{2\pi}{f}; \quad f \text{ is the spacing between kernels in the frequency domain. In this work, } f = \sqrt{2} \text{ and } u \text{ takes the values from set } \{1, 2, \cdots, U\} \text{ and } v \text{ takes the values from set } \{1, 2, \cdots, V\}, \text{ where } U \text{ and } V \text{ are the maximum number of directions and scales. Given } U \text{-direction and } V \text{-scale of Gabor wavelet is performed on an image, there will be produced } U \times V \text{ responses (subbands). The subbands of the Gabor wavelet are complex and the amplitudes of subbands are used in this paper. Fig.2 shows an 8-direction and 4-scale decomposition of Gabor wavelet.}

IV. Learning Gabor Log-Euclidean Gaussian (LGLG) for Face Recognition

In the proposed method, Gabor wavelet is used to extract the texture feature of face. However there are very a few number subbands produced by Gabor wavelet. Previous work use covariance matrix to capture the subbands\([25]\). However, covariance matrix ignores the means information of subbands. LGLG uses multivariate Gaussian which contains both the covariance and means to model the subbands. There are two steps in LGLG. The first step is to extract the local feature using Gabor Log-Euclidean Gaussian (GLG); the second step is to Learn GLG (LGLG) for face recognition.

A. Extract local feature using Gabor Log-Euclidean Gaussian

The detailed scheme of constructing a GLG on a block of face is shown in Fig.3. Gabor wavelet is first used decompose the local block of face image and each of the decomposed magnitude subband of Gabor wavelet is vectorized into 1-dimension vector, denoted by \(x = \{x_1, x_2, \cdots, x_L\}\), where \(L\) is the length of \(x\). To perform a \(U\)-direction and \(V\)-scale decomposition, \(P (P = U \times V)\) 1-dimension vectors are to be yielded. Then all of vectors produced from the subbands are formatted as following matrix:

\[
X = [x_1, x_2, \cdots, x_P]
\]

(7)

If \(X\) is regarded as observation matrix of a random vector and each column of the matrix corresponds to the observations of a random variable, then \(X\) is approximately Gaussian distribution \([35]\). We can calculate the parameter covariance matrix \(C\) and mean \(m\) of Gaussian on \(X\) by using maximum Likelihood Estimation (MLE).

\[
\mu = \frac{1}{P} \sum_{k=1}^{P} x_k
\]

(8)

\[
C = \frac{1}{P} \sum_{k=1}^{P} (x_k - \mu)(x_k - \mu)^T
\]

(9)

According to the estimated \(C\) and \(\mu\), we use EQ\([4]\)to embed the Gaussian model constructed from subbands of Gabor wavelet in Euclidean space and vectorize it into a local feature vector, denoted by

\[
F_i = \text{vec} \left[ \log \left( \frac{C + \mu \mu^T}{\mu^T \mu} \right)^{\frac{1}{2}} \right]
\]

(10)

where \(i = 1, 2, \cdots, P\). Sign \(\text{vec}()\) denotes the vectorization of matrix.

B. Learning GLG using WPCA (LGLG-WPCA) for face recognition

In LGLG, we use three preprocessing approaches (Gamma correction, Difference of Gaussian filter and contrast normalization) described in \([2]\) to counter the effects of illumination variations. Before extracting face features, the image will be divided into \(m \times n\) local blocks and GLG is applied to all the blocks producing \(L (L = m \times n)\) vectors denoted by \(F_1, F_2, \cdots, F_L\). The feature vectors \(F_i\) of the blocks in the image are concatenated into a high-dimensional vector \(F\), and WPCA is used to project the high-dimensional vector into a low-dimensional feature vector. Whitening Principal component analysis (WPCA) is more efficient than Principal component analysis (PCA) for face recognition under the condition of the training set has single sample per person. Compared to PCA, the extra benefit of WPCA is that it normalizes the contribution of each principal component by whitening transformation which divides the principal components by standard deviations.

The columns of \(U\) are composed of the eigenvectors of the covariance matrix. A high-dimensional vector can be compressed into a low-dimensional vector \(y\) by projecting it on \(U\), that is:

\[
y = U^T x.
\]

(11)

The second step of WPCA is to transform the \(U\) into \(W\) by whitening approach:

\[
W = U (D)^{-\frac{1}{2}}
\]

(12)

where \(D = \text{diag}(\lambda_1, \lambda_2, \cdots)\). Then the projected WPCA features \(y\) are

\[
y = W^T x = \left( U (D)^{-\frac{1}{2}} \right)^T x.
\]

(13)

However, WPCA may suffer performance degradation problem when the eigenvalues of the covariance matrices are very small or close to zero. If the eigenvalues of covariance matrix are too small and we use WPCA to whitening the features, it will over-amplify the influence of the small eigenvalues.
in feature matching. To address the issue, we standardize the WPCA features with z-score standardization (ZSCORE), which is denoted as:

$$z = \frac{y - \text{MEAN}(y)}{\text{STD}(y)}$$  \hspace{1cm} (14)

where \(\text{MEAN}(y)\) and \(\text{STD}(y)\) denote the mean and standard deviation of \(y\), respectively. In face recognition, standard Euclidean distance is used as the similarity between two face image.

V. EXPERIMENTS

Our evaluation is based on standard FERET database. Standard FERET contains four subsets: a gallery, a facial expression subset (fb), an illumination subset (fc), and two duplicate subsets (dup I and dup II). Dup I probe images were obtained at different times. The harder dup II probe subset is a subset of dup I; the images are in dup II are taken only at least 18 months. In this dataset, the images were cropped to \(150 \times 90\) and each image was divided into \(10 \times 6\) blocks (the block size is \(15 \times 15\)). In LGLG, 4-scale and 8-direction Gabor decomposition is performed and parameter \(\sigma\) of Gabor is set to \(1.7\pi\). Because there are 1196 person in the gallery, we use WPCA to compress the GLG features in all the block into an 1196-dimension feature and Euclidean norm 2 is used as the similarity.

We compared the proposed method LGLG with the state-
of-the-art methods including texture descriptors (such as LBP based method MDML-DCPs+WPCA[36] and Gabor based method LGBP+LGXP+LDA[37]) and DCNN methods (such as VGGFace[6] and PCANet [7]), and the recognition results are shown in Table I. The recognition accuracies of LGLG are 99.75%, 100%, 97.23% and 97.44% on fb, fc, dupI and dupII, respectively; and its average recognition accuracy reaches 98.60% which are the best among all the methods. Three multivariate models in Gabor wavelet domain are implemented for the comparison: COV-GW + RD (Covariance matrix in Gabor Wavelet (GW) domain, Riemannian distance[25] is used), COV-GW + LEG(Covariance matrix in Gabor Wavelet domain, Riemannian, Log-Euclidean embedding, Matrix Frobenius Norm (MFN) is used[32]) and GW + L²EMG (L²EMG [1]in Gabor wavelet domain, MFN is used). It can be observed that LGLG obviously outperform the three multivariate model improving by about 9 percentage points and has significantly reduction of computational cost compared with Riemannian distance because WPCA is used to reduce the dimensions of features. We also implemented the WPCA learning on the covariance model in Gabor wavelet domain (called LGLC-WPCA). The recognition accuracies of LGLC-WPCA and LGLG-WPCA are shown in Fig.5 and we observe that LGLG-WPCA always outperforms the LGLC-WPCA on the three subsets except fb.

VI. CONCLUSIONS

We implemented L²EMG in Gabor wavelet domain for face recognition. Our method, called LGLG-WPCA, is an efficient texture feature extract method which has robust performance under the condition of illumination and small pose and expression variations. LGLG-WPCA is superior to the Gabor-based and LBP-based methods and it is also computational efficient due to Log-Euclidean embedding and WPCA are used for producing the features. It should be pointed out that our method LGLG-WPCA may be not efficient under the large varying pose databases like CFP[43] IJB-A[44] and LFW[45] and it cannot compare our LGLG-WPCA with the frontalized based methods such as DR-GAN[18] and HF-PIM[16] on CPF and LFW because the frontalizing preprocessing are not used in LGLG-WPCA. In the future work, we will use the frontalizing approach as the preprocessing step of LGLG-WPCA for face recognition.

REFERENCES

[1] P. Li, Q. Wang, Z. Hui, and Z. Lei, “Local log-euclidean multivariate gaussian descriptor and its application to image classification,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 39, no. 4, pp. 803–817, 2017.
[2] X. Tan and B. Triggs, “Enhanced local texture feature sets for face recognition under difficult lighting conditions,” Image Processing, IEEE Transactions on, vol. 19, no. 6, pp. 1635–1650, 2010.
[3] S. Sengupta, J.C. Cheng, C.D. Castillo, V.M. Patel, R.Chellappa, and D.W.Jacobs, “Frontal to profile face verification in the wild,” February 2016.
[4] Y. Sun, X. Wang, and X. Tang, “Deep learning face representation from predicting 10,000 classes,” in CVPR, 2014, pp. 1891–1898.
[5] F. Schroff, D. Kalenichenko, and J. Philbin, “Facenet: A unified embedding for face recognition and clustering,” pp. 815–823, 2015.
[6] O. M. Parkhi, A. Vedaldi, and A. Zisserman, “Deep face recognition,” in British Machine Vision Conference, 2015, pp. 41.1–41.12.
[7] T. H. Chan, K. Jia, S. Gao, J. Lu, Z. Zeng, and Y. Ma, “Pcanet: A simple deep learning baseline for image classification?” IEEE Transactions on Image Processing, vol. 24, no. 12, pp. 5017–5032, 2015.
[8] T. Ahonen, A. Hadid, and M. Pietikainen, “Face description with local binary patterns: Application to face recognition.” Pattern Analysis and Machine Intelligence, IEEE Transactions on, vol. 28, no. 12, pp. 2037–2041, 2006.
[9] C. Geng and X. Jiang, “Face recognition using sift features,” in IEEE International Conference on Image Processing, 2010, pp. 3313–3316.
[10] O. Dniz, G. Bueno, J. Salido, and F. D. L. Torre, “Face recognition using histograms of oriented gradients,” Pattern Recognition Letters, vol. 32, no. 12, pp. 1598–1603, 2011.
[11] L. Yu, Z. He, and Q. Cao, “Gabor texture representation method for face recognition using the gamma and generalized gaussian models,” Image and Vision Computing, vol. 28, no. 1, pp. 177–187, 2010.
TABLE I
THE RECOGNITION ACCURACY (PERCENT) ON STANDARD FERET.

| Method       | tb  | fc  | dup I | dup II | Avg  |
|--------------|-----|-----|-------|--------|------|
| LBP          | 96.90          | 98.45 | 83.93 | 82.48 | 90.44 |
| LTP          | 96.90          | 98.97 | 83.93 | 83.76 | 90.89 |
| IGBP-GLGP+LDA [37] | 99.00 | 99.00 | 94.00 | 93.00 | 96.25 |
| DFD+WPCA [38] | 99.40 | 100.00 | 91.80 | 92.30 | 95.88 |
| MDML+DCC+LDA [36] | 99.75 | 100.00 | 96.12 | 95.73 | 97.90 |
| SCBP [39]    | 98.95          | 99.00 | 85.2  | 85.0  | 92.03 |
| FCC [40]     | 99.50          | 100   | 96.12 | 94.87 | 97.62 |
| PCANet [41]  | 99.58          | 100   | 95.43 | 94.02 | 97.26 |
| VGGface [5]  | 98.74          | 96.39 | 86.28 | 87.61 | 92.26 |
| 2-FCC [41]   | 99.50          | 100   | 96.12 | 94.87 | 97.62 |
| COV-GW + RD [25] | 97.99 | 99.48 | 80.74 | 78.21 | 89.11 |
| COV-GW + LEG [32] | 98.07 | 99.48 | 81.44 | 80.34 | 89.83 |
| GW + L2-EMG [1] | 98.07 | 99.48 | 82.13 | 81.19 | 90.22 |

| Method       | tb  | fc  | dup I | dup II | Avg  |
|--------------|-----|-----|-------|--------|------|
| LGLG-WPCA    | 99.75 | 100   | 97.23 | 97.44 | 98.60 |

[12] Y. Taigman, M. Yang, Marc, and L. Wolf, “Deepface: Closing the gap to human-level performance in face verification,” in CVPR, 2014, pp. 1701–1708.

[13] R. Huang, S. Zhang, T. Li, and R. He, “Beyond face rotation: Global and local perception gan for photorealistic and identity preserving frontal view synthesis,” arXiv, pp. 2458–2467, 2017.

[14] I. J. Goodfellow, J. Pouget-Abadie, M. Mirza, B. Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio, “Generative adversarial nets,” in International Conference on Neural Information Processing Systems, 2014, pp. 2672–2680.

[15] R. He, X. Wu, Z. Sun, and T. Tan, “Learning invariant deep representation for nir-vis face recognition,” in AAAI, 2017.

[16] J. Cao, Y. Hu, H. Zhang, R. He, and Z. Sun, “Learning a high fidelity pose invariant model for high-resolution face frontalization,” arXiv, 2018.

[17] V. Blanz, “A morphable model for the synthesis of 3d faces,” in Conference on Computer Graphics and Interactive Techniques, 1999, pp. 187–194.

[18] T. Luan, X. Yin, and X. Liu, “Disentangled representation learning gan for pose-invariant face recognition,” in Computer Vision and Pattern Recognition, 2017, pp. 1283–1292.

[19] C. Liu and H. Wechsler, Gabor feature based classification using the enhanced fisher linear discriminant model for face recognition. IEEE Press, 2002.

[20] L. L. Shen, L. Bai, and M. Fairhurst, “Gabor wavelets and general discriminant analysis for face identification and verification,” Image and Vision Computing, vol. 25, no. 5, pp. 553–563, 2007.

[21] C. Liu, “Gabor-based kernel pca with fractional power polynomial models for face recognition.” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 26, no. 5, p. 572, 2004.

[22] W. Zhang, S. Shan, W. Gao, and X. Chen, “Local gabor binary pattern histogram sequence (lgbphs): a novel non-statistical model for face representation and recognition,” in Tenth IEEE International Conference on Computer Vision, 2005, pp. 786–791.

[23] Z. Chai, Z. Sun, H. Mendez-Vazquez, R. He, and T. Tan, “Gabor ordinal measures for face recognition,” IEEE Transactions on Information Forensics and Security, vol. 9, no. 1, pp. 14–26, 2014.

[24] O. Tuzel, F. Porikli, and P. Meer, “Region covariance: A fast descriptor for detection and classification,” ECCV 2006, 2006.

[25] Y. Pang, Y. Yuan, and X. Li, “Gabor-based region covariance matrices for face recognition,” IEEE Transactions on Circuits and Systems for Video Technology, vol. 18, no. 7, pp. 989–993, 2008.

[26] R. Wang, H. Guo, L. S. Davis, and Q. Dai, “Covariance discriminating learning: A natural and efficient approach to image set classification,” in CVPR, 2012.

[27] H. Q. Minh, M. S. Biagio, L. Bazzani, and V. Murino, “Kernel methods on approximate infinite-dimensional covariance operators for image classification,” arXiv, 2016.

[28] W. Wang, R. Wang, S. Shan, and X. Chen, “Discriminative covariance oriented representation learning for face recognition with image sets,” in CVPR, 2017.

[29] Z. Zhang, M. Wang, Y. Huang, and A. Nehorai, “Aligning infinite-dimensional covariance matrices in reproducing kernel hilbert spaces for domain adaptation,” in CVPR, 2018.

[30] D. Acharya, Z. Huang, D. P. Paudel, and L. V. Gool, “Covariance pooling for facial expression recognition,” arXiv, 2018.

[31] P. Li, J. Xie, Q. Wang, and Z. Gao, “Towards faster training of global covariance pooling networks by iterative matrix square root normalization,” in CVPR, 2018.

[32] V. Arsigny, P. Fillard, X. Pennec, and N. Ayache, “Geometric means in a novel vector space structure on symmetric positive definite matrices,” Siam Journal on Matrix Analysis and Applications, vol. 29, no. 1, pp. 328–347, 2011.

[33] ———, “Log-euclidean metrics for fast and simple calculations on diffusion tensors,” Magnetic Resonance in Medicine, vol. 56, no. 2, pp. 411–421, 2006.

[34] M. Harandi, M. Salzmann, and R. Hartley, “Dimensionality reduction on spd manifolds: The emergence of geometry-aware methods,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 40, no. 1, pp. 48–62, 2018.

[35] C. Li, Y. Huang, and L. Zhu, “Color texture image retrieval based on gaussian copula models of gabor wavelets,” Pattern Recognition, vol. 64, pp. 118–129, 2017.

[36] C. Ding, J. Choi, D. Tao, and L. S. Davis, “Multi-directional multi-level dual-cross patterns for robust face recognition,” IEEE transactions on pattern analysis and machine intelligence, vol. 38, no. 3, pp. 518–531, 2016.

[37] S. Xie, S. Shan, X. Chen, and J. Chen, “Fusing local patterns of gabor magnitude and phase for face recognition,” IEEE Transactions on Image Processing, vol. 19, no. 5, pp. 1349–1361, 2010.

[38] L. Z. P. M. and L. SZ, “Learning discriminant face descriptor,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 36, no. 2, pp. 289–302, 2014.

[39] W. Deng, J. Hu, and J. Guo, “Compressive binary patterns: Designing a robust binary face descriptor with random-field eigenfilters,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 1, no. 99, pp. 1–1, 2018.

[40] C. Y. Low, B. J. Teoh, and C. J. Ng, “Multi-fold gabor, pca and ica filter convolution descriptor for face recognition,” IEEE Transactions on Circuits and Systems for Video Technology, vol. 1, no. 99, pp. 1–1, 2018.

[41] ———, “Multi-fold gabor, pca and ica filter convolution descriptor for face recognition,” IEEE Transactions on Circuits and Systems for Video Technology, vol. 3, no. 99, pp. 1–1, 2016.

[42] H.-T. Nguyen and A. Caplier, “Local patterns of gradients for face recognition,” IEEE Transactions on Information Forensics and Security, vol. 10, no. 8, pp. 1739–1751, 2015.

[43] S. Sengupta, J.-C. Chen, C. Castillo, V. M. Patel, R. Chellappa, and D. W. Jacobs, “Frontal to profile face verification in the wild,” in Applications of Computer Vision (WACV), 2016 IEEE Winter Conference on, IEEE, 2016, pp. 1–9.

[44] B. F. Klare, B. Klein, E. Taborsky, A. Blanton, J. Cheney, K. Allen, P. Grother, A. Mah, and A. K. Jain, “Pushing the frontiers of unconstrained face detection and recognition: Iarpa janus benchmark a,” in Proceedings of the IEEE conference on computer vision and pattern recognition, 2015, pp. 1931–1939.

[45] G. B. Huang, M. Mattar, T. Berg, and E. Learned-Miller, “Labelled faces in the wild: A database for studying face recognition in unconstrained environments,” in Workshop on faces in Real-Life Images: detection, alignment, and recognition, 2008.