Persons Camp Using Interpolation Method

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Abstract. The aim of this paper is to estimate the rate of contaminated soils by using suitable interpolation method as an alternative accurate tool to evaluate the concentration of heavy metals in soil then compared with standard universal value to determine the rate of contamination in the soil. In particular, interpolation methods are extensively applied in the models of the different phenomena where experimental data must be used in computer studies where expressions of those data are required.

In this paper the extended divided difference method in two dimensions is used to solve suggested problem. Then, the modification method is applied to estimate the rate of contaminated soils of displaced persons camp in Diyala Governorate, in Iraq.

Keywords: Interpolation method, divided difference method, Soil contamination, Heavy metals.

1. Introduction

The paper emphasizes the importance of the numerical analysis in applications, being provided a systematic presentation of the methods and techniques of numerical analysis and interpolation of the functions. Basically, there are many types of approximating functions. Thus, any analytical expression may be expressed as an approximating function, the most common types being: polynomials, trigonometric and exponential functions. Special attention is dedicated to polynomials which are the oldest and simplest methods of approximation.

A particular and important aspect in the numerical methods subject is the approximation of the different values, operation designated as interpolation, which is employed in most of the branches of the science, such as: engineering (Oanta, 2001, [1]), economics (Oanta, 2007, [2]), etc.

The problems of interpolation and approximate the functions of several independent variables are important but the methods are less well developed than in the case of functions of a single variable. An
immediate indication of the difficulties inherent in the higher dimensional case can be seen in the lack of uniqueness in the general interpolation problem. In many problems in engineering and science, the data consist of sets of discrete points, being required approximating functions which must have the following properties:

- The approximating function should be easy to determine;
- It should be easy to evaluate;
- It should be easy to differentiate;
- It should be easy implemented.

It can be noticed that polynomials satisfy all four these properties, moreover it have many important properties say: continuity and orthogonally.

The study of interpolation method and its applications in contamination of soil by heavy metals are firstly beginning with Tawfiq, et al., in 2015 [3-7]. In this paper the extending of divided difference method in two dimension are proposed then applied to estimate the concentration of heavy metals for displaced persons camp in Diyala Governorate in Iraq, then estimate the rate of contamination in that soil.

2. Polynomial Interpolation

We must find a real interpolation function $F$, which has to satisfy the following conditions:

$$F(x_i) = y_i, i = 0, ..., n.$$ 

The theoretical base of polynomial approximation is the Weierstrass theorem [8]. This theorem shows that any continuous function can be approximated with accuracy on an interval, using a polynomial function. The interpolation polynomial function is unique for a function on any given interval. The most know methods for polynomial interpolation are: Lagrange, Newton, divided difference, Hermite, spline, Birkhoff polynomial interpolation, trigonometric and rational interpolation (for details see [8-9]). In this paper we suggest divided difference method and its extended in two variable to solve suggested problem.

3. Divided Difference Method

Let $f(x)$ be a continuous function given at the distinct point $x_i, i = 0, 1, 2, ..., n$, define the zeros divide difference of the function $f$ with respect to $x_i$ denoted $f[x_i]$ by

$$f[x_i] = f(x_i).$$

The first divided difference of the function $f$ with respect to $x_i$ and $x_{i+1}$ denoted $f[x_0, x_1]$ and is defined as:

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}.$$

The second divided difference denoted $f[x_0, x_{i+1}, x_{i+2}]$ and is defined as:

$$f[x_0, x_{i+1}, x_{i+2}] = \frac{f(x_{i+2}) - f(x_{i+1})}{x_{i+2} - x_{i+1}}.$$

Similarly the nth divided difference relative to $x_0, x_1, ..., x_{n-1}, x_n$ is given by

$$f[x_0, x_1, ..., x_{n-1}, x_n] = \frac{f(x_{n-1}) - f(x_{n-2})}{x_{n-1} - x_{n-2}}. $$

The value of $f[x_0, x_1, ..., x_n]$ is independent of the order number $x_0, ..., x_n$.

i.e., if $i = 0, 1, 2$, then $f[x_0, x_1, x_2] = f[x_1, x_0, x_2] = f[x_2, x_1, x_0]$.

Then a polynomial of degree $n$ in $x$ can be construct to interpolate through $x_i, i = 0, 1, ..., n$, as

$$P_n(x) = \sum_{i=0}^{n} f[x_0, x_1, ..., x_i] (x - x_0) ... (x - x_{i-1}),$$  \hspace{1cm} (1)
Let $w_{i-1}(x) = \prod_{k=0}^{i-2}(x-x_k)$, so rewrite equation (1) as follow:

$$P_n(x) = \sum_{i=0}^{n} f[x_0, x_1, ..., x_i] w_{i-1}(x),$$  \hspace{1cm} (2)

4. Interpolation in Two Dimensions
In this section we define the interpolation in two dimensions and generalized divided difference formula in two dimensions. Given distinct point $(x_0, y_0), (x_1, y_1), ..., (x_m, y_n)$ in the $(x, y)$-plane, we have to find polynomial passing through these points by divided difference formula.

Consider the distinct point $(x_i, y_j)$ are given as rectangular grid such: $x_i, i = 0, 1, 2, ..., m$ and $y_j, j = 0, 1, 2, ..., n$, then a polynomial of degree $m$ in $x$ and $n$ in $y$ can be construct to interpolate through $(x_i, y_j)$, by extend one dimension formula equation (2), this is, given by:

$$P_{m,n}(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} w_{i-1}(x) w_{j-1}(y) f[x_0, x_1, ..., x_i, y_0, y_1, ..., y_j]$$  \hspace{1cm} (3)

where; $w_{i-1}(x) = \prod_{k=0}^{i-1}(x-x_k)$, $w_{j-1}(y) = \prod_{k=0}^{j-1}(y-y_k)$, $w_{-1}(x) = 1$ and $w_{-1}(y) = 1$.

5. Suggested Modification
There is an alternative method for generating approximations that has as it basis the divided difference at $x_0, x_1, ..., x_m$, this alternative method uses the connection between the $n$th divided difference and the $n$th derivative of $f$ and given in the following theorem

**Theorem 1** [10]
If $f^{(n)}(x)$ is continuous in $[a,b]$ and $x_0, x_1, ..., x_n$ are in $[a, b]$ than

$$f[x_0, x_1, ..., x_n] = \frac{f^{(n)}(\xi)}{n!}$$

where; $\min (x_0, x_1, ..., x_n) \leq \xi \leq \max (x_0, x_1, ..., x_n)$

A particular case of Theorem 1, is the following corollary.

**Corollary 1**
If $f^{(n)}(x)$ is continuous in a neighborhood of $x$ than

$$f[x, x_1, ..., x_n] = \frac{f^{(n)}(x)}{n!}$$

Since $z_{2i} = z_{2i+1} = x_i$ for each $i$, we cannot define $f[z_{2i}, z_{2i+1}]$ by divided difference formula, however, if we assume based on theorem (1) that the reasonable substitution in this situation is $f[z_{2i}, z_{2i+1}] = f(x_i)$, we can use the entries: $f'(x_0), f'(x_1), ..., f'(x_n)$, in place of undefined first divided differences: $f[z_0, z_1], f[z_2, z_3], ..., f[z_{2n}, z_{2n+1}]$ respectively.

The remaining entries are generated in the same manner as in the Table (1). The modify polynomial is then given by

$$P_{2n+1}(x) = f[z_0] + \sum_{i=1}^{2n+1} f[z_0, z_1, ..., z_i](x-z_0)(x-z_1)...(x-z_{i-1}).$$  \hspace{1cm} (4)

Then equation (4) can be generalized in two variables formula as follows
\[ P_{2m+1,2n+1}(x,y) = f[z_0, v_0] + \sum_{i=1}^{2m+1} \sum_{j=0}^{2n+1} f[z_0, \ldots, z_i, v_0, \ldots, v_j] (x - z_0) \ldots (x - z_{i-1}) (y - v_0) \ldots (y - v_{j-1}) \]  

(5)

For simplification,

\[ P_{2m+1,2n+1}(x,y) = \sum_{i=0}^{2m+1} \sum_{j=0}^{2n+1} f[z_0, \ldots, z_i, v_0, \ldots, v_j] w_{i-1}(z) w_{j-1}(v) \]  

(6)

where; \( z_{2i} = z_{2i+1} = x_i \) and \( v_{2j} = v_{2j+1} = y_j \), for all \( i = 1,2, \ldots, m \) and \( j = 1,2, \ldots, n \)

| \( F(x) \) | \( z \) | First divided difference | Second divided difference |
|---|---|---|---|
| \( f[z_0] = f(x_0) \) | \( Z_0 \) | \( f[z_0, z_1] = f(x_0) \) | \( f[z_1, z_2, z_3, z_4] = f[z_1, z_2] - f[z_0, z_1] \) |
| \( f[z_1] = f(x_0) \) | \( Z_1 \) | \( f[z_1, z_2] = f[z_2] - f[z_1] \) | \( z_2 - z_0 \) |
| \( f[z_2] = f(x_1) \) | \( Z_2 \) | \( f[z_1, z_2, z_3] = f[z_2, z_3] - f[z_1, z_2] \) | \( z_3 - z_1 \) |
| \( f[z_3] = f(x_1) \) | \( Z_3 \) | \( f[z_2, z_3, z_4] = f[z_3, z_4] - f[z_2, z_3] \) | \( z_4 - z_2 \) |
| \( f[z_4] = f(x_2) \) | \( Z_4 \) | \( f[z_3, z_4, z_5] = f[z_4, z_5] - f[z_3, z_4] \) | \( z_5 - z_3 \) |
| \( f[z_5] = f(x_2) \) | \( Z_5 \) | \( f[z_4, z_5] = f(x_2) \) | |

Table 1: divided difference with suggested modification

Thus, theorem 1, can be generalize in more than one variable by the following corollary

**Corollary 2**

If \( f(x) \) has a continuous derivative of order \( m \) in \([a, b] \); \( x_0, \ldots, x_p, y_0, \ldots, y_q, z_0, \ldots, z_r \) are in \([a, b] \);

\[ x_i \neq y_i, x_i \neq z_k, y_i \neq z_k, \forall i, j, k, 0 \leq p, q, r \leq m; \text{then} \]

\[ f[x_0, \ldots, x_p, y_0, \ldots, y_q, z_0, \ldots, z_r] = \frac{1}{p!q!r!} \frac{\partial^p}{\partial x^p} \frac{\partial^q}{\partial y^q} \frac{\partial^r}{\partial z^r} f(x, y, z)(\xi, \eta, \zeta) \]

where; \( \min(x_0, \ldots, x_p) \leq \xi \leq \max(x_0, \ldots, x_p) \), \( \min(y_0, \ldots, y_q) \leq \eta \leq \max(y_0, \ldots, y_q) \), \( \min(z_0, \ldots, z_r) \leq \zeta \leq \max(z_0, \ldots, z_r) \).

Now, we can generalize the suggested modification in two variables and construct table (2) as the same manner of table (1) but in two variables.
Table 2: divided difference with suggested modification in two variables

| \((x_0, y_0)\) | \(f[x_0, y_0]\) | First divided difference | Second divided difference |
|----------------|----------------|-------------------------|--------------------------|
| \((x_0, y_0)\) | \(f[x_0, y_0]\) | \(\frac{\partial}{\partial x}\frac{\partial}{\partial y}f[x_0, y_0]\) | \(\frac{f[x_1, y_0] - f[x_0, y_0]}{(x_1 - x_0)}\) |
| \((x_0, y_0)\) | \(f[x_0, y_0]\) | \(\frac{\partial}{\partial x}f[x_0, y_0]\) | \(\frac{f[x_1, y_0] - f[x_0, y_0]}{(x_1 - x_0)}\) |
| \((x_1, y_0)\) | \(f[x_1, y_0]\) | \(\frac{\partial}{\partial x}f[x_1, y_0]\) | \(\frac{f[x_1, y_0] - f[x_0, y_0]}{(x_1 - x_0)}\) |
| \((x_0, y_1)\) | \(f[x_0, y_1]\) | \(\frac{\partial}{\partial x}\frac{\partial}{\partial y}f[x_0, y_1]\) | \(\frac{f[x_1, y_0, y_1] - f[x_0, x_1, y_0]}{(y_1 - y_0)}\) |
| \((x_1, y_0)\) | \(f[x_1, y_0]\) | \(\frac{\partial}{\partial x}f[x_1, y_0]\) | \(\frac{f[x_1, y_0, y_1] - f[x_0, x_1, y_0]}{(y_1 - x_0)}\) |
| \((x_0, y_1)\) | \(f[x_0, y_1]\) | \(\frac{\partial}{\partial x}\frac{\partial}{\partial y}f[x_0, y_1]\) | \(\frac{f[x_1, y_0, y_1] - f[x_0, x_1, y_0]}{(y_1 - x_0)}\) |
| \((x_1, y_0)\) | \(f[x_1, y_0]\) | \(\frac{\partial}{\partial x}f[x_1, y_0]\) | \(\frac{f[x_1, y_0, y_1] - f[x_0, x_1, y_0]}{(y_1 - x_0)}\) |

Now, to illustrate the importance of suggested method, we applied it to determine the concentration of heavy metals in soil, for estimate the rate of contamination in its soil.

6. Sampling

The area of study is muasker Saad displaced persons camp in Diyala Governorate, Iraq. The 8 soil samples were collected with depth (0- 10 cm) using iron shovel (the quantity of each sample was 1 kg), isolation of foreign materials such as plant leaves, debris etc. were removed from the collected soil samples then all the samples were put in plastic bags to measure the concentration of heavy metals (Cd, Cu, Cr, Fe, Ni, Pb and Zn). The samples are carefully collected from each source area in different land using types with a stainless steel spatula. They were air–dried in the laboratory, homogenized and sieved through a 2mm polyethylene sieve to remove large debris, stones and pebbles, after they were disaggregated with a porcelain pestle and mortar. Then these samples were stored in clean self–sealing plastic bags for further analysis. Metal determinations were done by X– ray fluorescence analysis (XRF). pH for all samples was measured in a study soil. Table 3 gave the laboratory results, which represent the initial data which used to interpolate and get approximate function depending on suggested method.
Table 3. Concentration of heavy metals calculated in laboratory.

| Samples  | PH (S.M.C.>50) conc. | Poll. Am. | Pb (S.M.C.>50) conc. | Poll. Am. | Ni (S.M.C.>50) conc. | Poll. Am. | Zn (S.M.C.>70) conc. | Poll. Am. | Fe (S.M.C.>38000) conc. | Poll. Am. | Cr (S.M.C.=100) conc. | Poll. Am. | Cu (S.M.C.=20) conc. | Poll. Am. | Cd (S.M.C.=1) conc. |
|----------|----------------------|----------|----------------------|----------|----------------------|----------|----------------------|----------|----------------------|----------|----------------------|----------|----------------------|----------|
| Sample 1 | 6                    | 20       | -30                  | 64       | 14                   | 124      | 54                   | 12500    | -25500               | 59.3     | -40.7                | 14.6     | -5.4                 | ND       |
| Sample 2 | 6.2                  | 20.6     | -29.4                | 64.2     | 14.2                 | 126.5    | 56.5                 | 13320    | -24680               | 64.6     | -35.4                | 16.6     | -3.4                 | ND       |
| Sample 3 | 6.2                  | 20.6     | -29.4                | 64.2     | 14.2                 | 126.5    | 56.5                 | 13320    | -24680               | 64.6     | -35.4                | 16.6     | -3.4                 | ND       |
| Sample 4 | 6.4                  | 26.9     | -23.1                | 96.6     | 46.6                 | 113.2    | 43.2                 | 24975    | -13025               | 38.2     | -61.8                | 23.6     | 3.6                  | ND       |
| Sample 5 | 6.4                  | 26.9     | -23.1                | 96.6     | 46.6                 | 113.2    | 43.2                 | 24975    | -13025               | 38.2     | -61.8                | 23.6     | 3.6                  | ND       |
| Sample 6 | 6.6                  | 28.6     | -21.4                | 104.9    | 54.9                 | 126.5    | 56.5                 | 21978    | -16022               | 40.9     | -59.1                | 26.9     | 6.9                  | ND       |
| Sample 7 | 6.6                  | 28.6     | -21.4                | 104.9    | 54.9                 | 126.5    | 56.5                 | 21978    | -16022               | 40.9     | -59.1                | 26.9     | 6.9                  | ND       |
| Sample 8 | 6.6                  | 30.5     | -19.5                | 128      | 78                   | 128      | 58                   | 25650    | -12350               | 45       | 55                   | 30.3     | 10.3                 | ND       |

7. Results and Discussion
The data obtained from laboratory dissecting represent the concentration of heavy metals for selected soils are interpolated by suggested method and get the concentration of those heavy metals but in any desire time and any neighboring area, then compared with standard universal for concentration of heavy metals in soil depending on [11] to determine the rate of contamination in soil in that time. We applied suggested method and the results illustrated in figure (1) for each heavy metals. Also, figure (2) illustrate PH of study area.
Figure 1: concentration of heavy metals by suggested method

Figure 2: PH of study area by suggested method

'figure(3)', illustrate the accuracy of suggested method by using a comparison between the results of suggested method and laboratory dissecting. 'figure( 4)', illustrate the concentration of heavy metals by suggested method for different years.
Figure 3: Comparison between suggested method and laboratory dissecting

Figure 4: concentration of heavy metals by suggested method for different years

8. Conclusions
New approaches to interpolation of two variables is proposed by generalized one variable divided difference method then developed to increase the accuracy of results based on increase the data. Application in contamination soil is establish to illustrate the importance and efficiency of suggested method.

The results which obtained from the present work show that soil of muasker Saad displaced persons camp in Diyala Governorate, in Iraq were found to be significantly contaminated with metals like Cr, Ni, Pb and Zn at levels above the background concentration in the international soils, which may give rise to various health hazards, while the concentrations of Cd, Co and Fe were under the background concentration in soil. We noted that the comparison between concentrations of four years is introduced to determine the effect of displaced persons for increasing the contamination and figure (5) illustrate the rate of concentration for each heavy metals and sample. In addition the practical results showed the suggested method is easy implemented, high accuracy, efficient and rapid compared to other method.
Figure 5: Rate of concentration for heavy metals in each sample

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