Neutrinos in an expanding Universe

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Abstract. The Universe contains several billion neutrinos for each nucleon. In this paper, we follow the history of these relic neutrinos as the Universe expanded. At present, their typical velocity is a few hundred km/s and, therefore, their spectra are affected by gravitational forces. This may have led to a phenomenon that could explain two of today's great mysteries: the large-scale structure of the Universe and the increasing rate at which it expands.

1. Introduction
According to the Big Bang model, large numbers of neutrinos and antineutrinos have been around since the earliest stages of the evolving Universe. During the Leptonic Era, the density was so large that the (anti-)neutrinos were in thermal equilibrium with the other particles that made up the primordial soup: photons, electrons, positrons and nucleons. When the Universe was about 1 second old, the neutrinos decoupled and thus turned into relics of the Big Bang. Since that time, they have behaved like a gas of free particles. They are now the most abundant species, outnumbering baryons by more than 9 orders of magnitude. Yet, despite their enormous abundance (estimated at $O(10^{87})$ for the observable Universe), relic neutrinos have until now escaped direct detection. Their extremely small kinetic energy makes it all but impossible to find a process through which they might reveal themselves as individual particles.

In this paper, we follow the relic neutrinos throughout the history of the Universe. In Section 2, their properties are briefly reviewed. In Section 3, the effects of gravitational acceleration and deceleration on the spectra and local densities of massive neutrinos are studied. It turns out that the density may exceed the maximum density allowed by the Pauli principle in the field-free region around the center-of-mass of galaxy clusters. In Section 4, we discuss the mechanism that prevents this from happening and analyze some of the consequences of this mechanism. Conclusions are presented in Section 5.

2. Properties of relic neutrinos
2.1. Spectra and densities
During the Leptonic Era, neutrinos and antineutrinos were in thermal equilibrium with the other constituents of the Universe. They converted protons into neutrons and vice versa. However, when the Universe was $\approx 1$ second old and the temperature (or rather $kT$) had dropped to $\sim 1$ MeV, this thermal equilibrium ceased to exist, for two reasons. First, the production of neutrons through the reactions $\bar{\nu}_e + p \rightarrow n + e^+$ and $e^- + p \rightarrow \nu_e + n$ became energetically impossible, whereas the reverse reactions still occurred. This led to the proton/neutron asymmetry. Second, the density dropped to the point where the average lifetime of the neutrinos and antineutrinos became longer than the age of the Universe (i.e., 1 second at that time).
The decoupling of the neutrinos$^1$ marked the end of the Leptonic Era. At that time, the neutrino spectrum was described by a relativistic Fermi-Dirac distribution function. The number of particles per unit volume in the momentum bin $(p, p + dp)$ is given by

$$N(p)dp = \frac{\pi p^2 dp}{\hbar^3[\exp(pc/kT_\nu) + 1]^{1/2}}$$

where $kT_\nu \approx 1$ MeV, $p_\nu \approx 1$ MeV/c and $g_\nu$ denotes the number of neutrino helicity states: $g_\nu = 2$ for Majorana neutrinos, $g_\nu = 4$ for Dirac neutrinos $^1$.

The neutrino density, which can be found by integrating Equation 1 over the entire spectrum, was during the Leptonic Era related to the photon density as:

$$\frac{n_\nu}{n_\gamma} = \frac{3}{4} \text{ (Majorana)}, \frac{6}{4} \text{ (Dirac)}$$

About 10 seconds later, when $kT$ had dropped below 0.511 MeV, the photon spectrum was reheated because of the electron-positron annihilation, and the photon density increased by a factor of $11/4$. As a result, the neutrino and photon temperatures were no longer the same either:

$$T_\nu = \sqrt{\frac{4}{11}} T_\gamma$$

After the decoupling, the de Broglie wavelengths ($\lambda = h/p$) of these relic neutrinos have been expanding in proportion to the size of the Universe. Even though the particles may well be non-relativistic at the present time, their momentum spectrum is still described by the same relativistic equation (1), albeit that the momenta have been redshifted by 10 orders of magnitude and the temperature $T_\nu$ has decreased correspondingly. Their present spectrum is thus believed to be a momentum-redshifted relativistic Fermi–Dirac distribution. Since the present value of $T_\gamma$ amounts to $2.725 \pm 0.001$ K $^2$, this neutrino spectrum is characterized by a temperature $T_\nu = 1.95$ K. In the following we will define the Fermi momentum$^3$ $p_F$ as $kT_\nu/c$. For a neutrino temperature of 1.95 K, this momentum is $1.68 \cdot 10^{-4}$ eV/c, or $9.0 \cdot 10^{-32}$ kg.m.s$^{-1}$.

Since the current photon density has been measured to be 411 cm$^{-3}$, the neutrino density may be estimated to be $\sim 220$ cm$^{-3}$, for each flavor, if neutrinos and antineutrinos are fundamentally different (Dirac) particles, as in the case of neutral kaons. If they are indistinguishable (Majorana particles), such as $\pi^0$s, then the densities are a factor of two smaller.

2.2. Neutrino masses and velocities

At these densities, there are thus $\mathcal{O}(10^{87})$ relic neutrinos in the observable Universe. Given the measured baryon/photon ratio of $(4.5 \pm 1.9) \cdot 10^{-10}$ $^3$, and the fact that there are three generations of neutrinos, there are thus $\approx 5$ billion neutrinos for every baryon. Even a very small neutrino rest mass may thus have very significant consequences for the total mass of the Universe.

The experimental upper limit on the mass of the electron antineutrino$^3$ was determined at 2.2 eV (95% C.L.), from a study of the electron spectrum of $^3$H decay $^4$. The experimental results on atmospheric neutrinos obtained by the Superkamiokande $^5$ and SNO $^6$ Collaborations suggest that at least one of the neutrino mass eigenstates has a value in excess of 0.04 eV. Interpretations of other experimental results suggest mass values of the electron neutrino in the 0.2 - 0.6 eV range $^7, 8$. There is no experimental information that rules out a neutrino rest mass of the order of 0.1 - 1 eV.

It is important to realize that, depending on their mass, the relic neutrinos might be very non-relativistic at the current temperature. Since they decoupled, their momenta have been stretched by a factor $10^{10}$, from $\sim 1$ MeV/c to $\sim 10^{-4}$ eV/c. If their rest mass were 1 eV/c$^2$, their maximum velocity would thus be $\sim 10^{-4}c$.

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1 In the following, statements about “neutrinos” are intended to mean “neutrinos and antineutrinos”, unless stated otherwise.

2 This definition deviates from the one used in solid state physics, in which a non-zero chemical potential is used.

3 In the following, we express masses in energy units, omitting the $c^{-2}$ factor.
The relationship between the neutrino mass and the Fermi velocity \( v_F = kT_\nu/cm_\nu \) at the current neutrino temperature \( T_\nu = 1.95 \) K is shown in Figure 1, together with the limits on the neutrino mass derived from the aforementioned direct measurements and interpretations. The latter suggest \( v_F \) values between 70 and 200 km/s, while direct measurements put the velocity interval between 25 and 1200 km/s.

2.3. Degeneracy and expansion

In a Fermi-Dirac gas, the fermions are stacked into progressively higher energy states, beginning with the ground state. When all available states are filled, i.e., when the gas is said to be degenerate, one cannot squeeze any more particles with momenta in the range \( 0 < p < p_{\text{max}} \) into the volume occupied by the gas without violating the Pauli principle. We may estimate the density at which violation of the Pauli principle becomes an issue with the aid of the Heisenberg uncertainty relation:

\[
\rho_f(\text{max}) = \frac{8\pi}{3} \left( \frac{p_{\text{max}}}{\hbar} \right)^3
\]

In non-relativistic Fermi-Dirac gases, the maximum fermion density at a given temperature is thus proportional to the fermion mass cubed. This density cannot be exceeded. Attempts to squeeze more fermions into the available volume than allowed by this maximum will trigger the fermion degeneracy pressure, which prevents this from happening. The outside pressure forces the fermions into higher and higher quantum states, up to the point where the pressure exerted as a result of their thermal motion balances the external force. This is the mechanism that prevents the gravitational collapse of White Dwarfs and neutron stars. In the case of White Dwarfs, the fermions are electrons, and the momenta are typically of the order of MeV/c. In neutron stars, the fermions are neutrons and the momenta are measured in units of GeV/c.

The Pauli principle applies also to relic neutrinos. Since the momenta of these particles are of the order of \( 10^{-4} \) eV/c, Eq. 3 implies that their current maximum density is 30 orders of magnitude smaller than that of electrons in a White Dwarf. Using Equations 1 and 3, this maximum density is calculated at a temperature \( T_\nu = 1.95 \) K to be \( \sim 225 \) cm\(^{-3}\), for each (Dirac) flavor. This result is in excellent agreement with the estimated present density of relic neutrinos, \( n_\gamma \times 6/11 \sim 220 \) cm\(^{-3}\) (Section 2.1).

The relic neutrinos thus form a degenerate fermion gas, at \( T_\nu = 1.95 \) K. It is important to note that the expansion of the Universe has not changed the degeneracy. Since the neutrino momenta, \( p_\nu(t) \), are inversely proportional to the scale, \( R(t) \), of the Universe, and since the maximum allowed density evolves in proportion to \( p^3 \) (Equation 3), the ratio of the actual neutrino density and the maximum allowed density is independent of \( R! \) The present situation is thus not new. The Universe has always been filled with a degenerate neutrino gas. All available quantum states with \( p/p_F \ll 1 \) are, and always have been, almost completely occupied, regardless of the neutrino temperature. Since the neutrino momenta also scale with the neutrino temperature of the expanding Universe (e.g., \( cp \propto kT_\nu \), see Equation 1), we conclude that this temperature must have evolved inversely proportional to the distance scale of the Universe:

\[
R(t)T_\nu(t) = \text{constant}
\]  

As we will see in the next section, this has important consequences.
3. Massive neutrinos in the Matter Era

3.1. Spectral effects

During the radiation era, the neutrino spectrum was only affected by the gradual expansion, with $p_\nu(t)$ and $T_\nu(t)$ inversely proportional to the evolving distance scale. However, when gravity became the dominant force in the Universe, important changes were about to take place. Initially, during the period following the photon decoupling (at a redshift $z \sim 1100$), matter was still very uniformly distributed. The main effect of the newly dominant gravitational force was a general slowdown of the expansion rate. However, when galaxy formation started, around $z \sim 10$, matter started to cluster in certain areas, leaving other areas devoid. As a result, gravitation was no longer a uniform force. Figure 2 gives a conceptual impression of this phenomenon. It shows the gravitational field lines in a region of space where this process is taking place. The different galaxies manifest themselves as gravitational wells.

![Figure 2. Gravitational field lines in a region of the Universe where several galaxies are forming.](Image)

Since neutrinos are subject to gravitational forces, their spectra were affected by this developing non-uniformity. The neutrino velocities either increased or decreased as a result of gravitational acceleration or deceleration, depending on the direction of motion of the particles with respect to the (dominant) source of gravitational fields. The effects of this extended over intergalactic distances, to regions far away from these sources. The velocity distribution of the relic neutrinos has thus gradually changed in the non-uniform gravitational fields that resulted from baryon clustering, i.e., galaxy formation.

A well known consequence of this phenomenon is the gravitational clustering of relic neutrinos inside and around galaxies. The properties of such neutrino halos were extensively studied by Ringwald and Wong [9]. Their calculations, which concern neutrinos with masses ranging from 0.15 - 0.6 eV, show that the neutrino density in these halos may increase by as much as a factor of 20 (for $m_\nu = 0.6$ eV) with respect to the average universal density.

This is precisely what should be expected as a result of the neutrino “heating” that has occurred as a result of the process described above. Neutrinos roll into the gravitational wells (depicted in Figure 2) that are forming. They are trapped inside these wells if the kinetic energy they acquire in this process is insufficient to escape from the (deepened) well by the time they reach the other side. Because the neutrinos gain momentum in this process, their effective temperature increases, and the number of quantum states available to the higher-temperature Fermi-Dirac gas becomes larger as well.

In this paper, we concentrate on the neutrinos that experience the opposite effect. Neutrinos climbing out of the gravitational wells are decelerated in that process. As a result, the neutrino gas in the shaded area surrounding the center-of-gravity ($C$) of the galaxy cluster in Figure 2 will gradually be “cooled”, i.e., the effective temperature of the neutrinos in this area decreases. Just as for the formation of neutrino halos around galaxies, the magnitude of the gravitational cooling effect is very sensitive to the velocity distribution, and thus to the rest mass, of the neutrinos.
Figure 3. Gravitational field lines in a region of the Universe where several galaxies are forming (left). The neutrino spectrum inside the box surrounding the center-of-gravity of this galaxy cluster is affected by this process (right), since neutrinos leaving this box are replaced by neutrinos that are, on average, slower. See text for details.

We have simulated the effects of galaxy formation on the neutrino spectra and densities with a computer program that was especially developed for this purpose. Our simulations show that the low-field region around the center-of-mass of this galaxy cluster is gradually populated by neutrinos with a decreasing average velocity. This is schematically illustrated in Figure 3. The neutrinos leaving the bounded region around the center-of-mass are replaced by neutrinos which have, on average, a lower velocity, as a result of the gravitational deceleration they undergo when climbing out of the gravitational wells. In other words, the neutrino gas in this region is “gravitationally cooled” in this process, in the same way as the neutrino gas trapped inside the gravitational wells is gradually heated.

However, if sufficiently large numbers of neutrinos are sufficiently decelerated in the described process, then there may not be enough quantum states in this region to accommodate them. In that case, the Pauli principle is in jeopardy of being violated. Our calculations indicate that violation of the Pauli principle starts to play a role 600 - 800 million years after the galaxies start to form, depending on the time scale of the galaxy formation process and the neutrino mass.

3.2. The Pauli principle
The Pauli Exclusion Principle is one of the cornerstones of Quantum Mechanics. It is responsible for the atomic shell structure and its many implications. It also determines the properties of degenerate matter, and is thus responsible for astrophysical phenomena such as the helium flash in stellar evolution and the mass-volume relationship for White Dwarfs. Combined with Heisenberg’s Uncertainty Principle, it implies that there is a maximum to the number of fermions that can be squeezed into a given volume. The impossibility to exceed this maximum is a source of pressure, which may counteract efforts to achieve otherwise, and thus prevents the gravitational collapse of objects such as White Dwarfs. Despite many attempts, violation of the Pauli principle has never been experimentally observed.

How does nature prevent violation of the Pauli principle in degenerate matter? In shrinking White Dwarfs, the electrons are forced into increasingly higher quantum states. The higher the quantum state, the higher the velocity, and thus the pressure that can be exerted by these electrons on the outside world, in this case the outside layers of the collapsing star. When the mass of the star is less than the Chandrasekhar limit, an equilibrium is found in which the inward gravitational pressure is balanced by the described outward “fermion degeneracy pressure”. Prevention of violation of the Pauli principle is thus achieved through the action of the forces to which the fermions in question are subjected, in this case the electromagnetic force.

However, the relic neutrinos are not affected by the electromagnetic force and are thus incapable of exerting pressure on their environment. Therefore, the question arises what happens when gravitationally
decelerated neutrinos cannot find a quantum state to fit into? How is violation of the Pauli principle prevented in this case? Whatever the answer to these questions, the Pauli principle must be safeguarded through the action of a fundamental force to which neutrinos are sensitive. Neutrinos are only subject to the weak force and to gravity. And since the range of the weak force is many orders of magnitude smaller than the typical distance between the neutrinos in the degenerate conditions considered here, the weak force can be ruled out as a candidate. Violation of the Pauli principle in intergalactic space can thus only be prevented through the action of gravity, the same force that was responsible for the problem in the first place.

In the general-relativistic description, gravity curves space-time in the vicinity of its source. When $10^{57}$ protons cluster together and form a star, space-time around the newly formed massive object is modified as if it were a converging gravitational lens. Massive particles, and even photons traveling in the vicinity follow trajectories that correspond to the geodesics of this curved space.

Similarly, we envision that when $O(10^{77})$ relic neutrinos congregate in a region of space where there are insufficient quantum states to accommodate all of them, space-time is modified such as to prevent violation of the Pauli principle. In the example described in Figure 3, this means that the center-of-gravity $C$ starts to act as a diverging gravitational lens. This is illustrated in Figure 4. Particles approaching point $C$ will follow a trajectory that corresponds to a geodesic on the surface of a hyperboloid. The net effect is the same as if a negative mass were placed in point $C$: All neutrinos approaching point $C$ experience a repulsive force that prevents them from reaching an area that cannot accommodate them. In the following section, we examine some of the consequences of what we will call the Pauli acceleration model.

**4. The Pauli acceleration model**

**4.1. Qualitative considerations**

In the previous section, we saw that violation of the Pauli principle may occur (for proper values of the neutrino rest mass) in the low-field region near the center-of-mass of a newly forming galaxy cluster. Preventing such a violation could be achieved if space-time around this center-of-mass were negatively curved. Such a curvature would not only prevent relic neutrinos from reaching areas where quantum mechanics does not allow them to be, but would have other consequences as well.

The Equivalence Principle implies that any massive object in the gravitational field created this way would experience the same consequences as if a negative mass were placed in the center-of-mass of the galaxy cluster. The galaxies constituting the cluster would thus experience a repulsive force driving them away from their common center-of-mass. The void around the center-of-mass of the cluster would thus tend to grow over time. This local expansion of space is superimposed on the overall Hubble expansion.

If the galaxies are also part of another cluster, they may feel a repulsive force from the center-of-mass of that cluster as well. Both forces work in opposite directions and thus (partially) cancel each other. This may explain the Large Scale structure of the Universe, as observed in the Sloan Digital Sky Survey (Figure 5). Strings of galaxies are separated by large voids. This situation can be compared with that in soap foam above hot water. Temperature differences create pressure differences, which causes the simultaneous growth of all soap bubbles in the relatively cold surrounding air, and the molecules arrange themselves in patterns very similar to the one shown in the figure.
It is important to note that the negative curvature around the center-of-mass is not a temporary feature, which disappears as soon as the local expansion of space caused by it has created the necessary quantum states for the relic neutrinos. Since the de Broglie wavelengths of the neutrinos expand in proportion, the degree of neutrino degeneracy is not affected by this local expansion (Section 2.3). Therefore, the phenomenon that necessitated the negative curvature is permanent and the void around the center-of-mass of the galaxy cluster will thus continue to expand over time. The same will be true for any cluster of galaxies.

A second consequence of this model is that to an observer located at any point in space it will seem as if all distant galaxies move away, in an accelerated fashion. This is illustrated in Figure 5, which shows an enlarged detail of the sky mapped by the Sloan Digital Sky Survey. The arrows represent the velocity vectors measured by the observer located at the indicated position. All galaxies in the ring surrounding the void in which the observer is located move away, with velocities proportional to the distance from the observer. Since this motion is governed by a repulsive force, it is accelerated. Had the observer measured these velocities in the (distant) past, smaller values would have been found. The motion of galaxies located outside the inner ring is determined by a superposition of their motion with respect to galaxies closer to the observer. For example, galaxies 1 and 2 both experience a repulsive force originating from point P. Therefore, galaxy 2 undergoes an accelerated motion with respect to galaxy 1. Its motion with respect to the position of the observer is governed by the vector sum of the acceleration vectors of galaxy 1 with respect to the observer and galaxy 2 with respect to galaxy 1. The result is also here an accelerated motion with a component along the direction away from the observer, and velocities measured today are larger than they were in the distant past.

The apparent accelerated motion of distant galaxies, i.e., the fact that the current value of the Hubble constant is larger than it was in the past, has been concluded from careful analyses of the properties of Supernova Type Ia explosions in distant galaxies [10, 11]. This phenomenon has given rise to many speculations as to its origin. If our model were correct, it is in essence a quantum mechanical effect.

4.2. Quantitative considerations

We have tried to estimate the effects of the described Pauli acceleration quantitatively. Parameters that are of crucial importance in this respect are the galaxy masses and the distance separating the galaxies at the time when they started to form, as well as of course the value of the neutrino mass. Therefore, we have also investigated the sensitivity of our results to the choice of the values of these parameters.

Since it was necessitated by gravitationally decelerated relic neutrinos, the Pauli acceleration is a relatively recent phenomenon. It only started to play a role after the first galaxies had formed and the resulting non-uniform gravitational fields had sufficiently reduced the speed of a significant fraction of the relic neutrinos. Based on the current distance typically separating galaxies (≈ 1Mpc), typical galaxy masses \((10^{10} - 10^{11} M_{\odot})\) and the measured value of the Hubble constant \(73 \text{ km.s}^{-1} \text{Mpc}^{-1}\), we find that the turning point, at which the accelerating effects became larger than the deceleration resulting from the gravitational attraction between the galaxies occurred at a redshift \(z \sim 5\). Assuming a flat Universe, this corresponds to a time when \((1 + z)^{-3/2} = 7\%\) of the total history of the Universe had unfolded, i.e., when the Universe was \(\sim 1\) billion years old.

Our model implies that the expansion of the Universe will continue forever, albeit with a gradually decreasing acceleration. In Section 2.4, we saw that the local expansion of the intergalactic voids does not change the degree of degeneracy of the relic neutrinos, since the wavelengths of these neutrinos increase in proportion to the distance scale. Therefore, the mechanism that drives the expansion does...
not go away. Interestingly, this statement is independent of the mass content of the Universe. Our model implies that there is no relationship between the geometry ($k$) and the energy density ($\Omega$) of the Universe. The scenario that leads to the local expansion of intergalactic voids hinges of course critically on the value of the neutrino mass. If this mass were too small, then the neutrino velocities would be so large that gravitational effects on these velocities would be insignificant. Based on the assumptions listed above, we conclude that the rest masses of neutrinos should not be smaller than $\sim 0.5 \text{ eV}/c^2$. This limit implies that direct measurements of the neutrino mass are possibly within reach. Planned efforts to improve the neutrino mass sensitivity in studies of $^3\text{H}$ decay to the $0.2 \text{ eV}/c^2$ level [4] may well pay off in the form of a precise measurement of this important parameter.

5. Conclusions

The relic neutrinos that fill the Universe form a degenerate Fermi-Dirac gas, with typical velocities of the order of 100 km/s. Some fraction of these neutrinos were significantly affected by the non-uniform gravitational fields that developed when galaxy formation started. We have shown that gravitationally decelerated neutrinos accumulating in the low-field region around the center of a developing galaxy cluster may have approached the density limits set by the Pauli Exclusion Principle. The only way to avoid a violation of the Pauli principle was a local expansion of space in that region. Such an expansion could lead to and would provide a natural explanation for the Large-Scale Structure observed in the Sloan Digital Sky Survey and other studies. It could also explain the apparent accelerated expansion suggested by the high-$z$ Supernova data, without the need for “dark energy” or other unknown physics.

Fermion degeneracy pressure is a phenomenon that, until now, has been exclusively associated with the extremely high temperatures and densities that characterize the interior of degenerate stellar objects such as White Dwarfs and neutron stars. It is remarkable that the same phenomenon may also play a crucial role in the extremely cold and empty conditions of intergalactic space.

Of course, the model proposed in this paper has speculative elements. If the neutrino mass is indeed of the order of $0.5 \text{ eV}/c^2$, then violation of the Pauli principle must have been a serious issue when galaxies started to form. Nature might of course have used other ways to prevent such a violation, instead of the one proposed in this paper, although it is unclear what these other ways could be. However, if the model proposed in this paper turned out to be correct, it means that the expansion of the Universe and the observed peculiarities in this process are essentially a consequence of Quantum Mechanics. It would also be the first observation of the effects of gravity at the quantum level. The intimate link between the structure of matter in its smallest details and on its largest imaginable scales is further illustrated by the crucial role played by the value of the neutrino mass in the expansion of the Universe. If our model is correct, this value must be about $0.5 \text{ eV}/c^2$, which might make it accessible to direct measurements planned for the near future.

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