Numerical analysis of Journal Bearing based on Finite Difference Method with Inhomogeneous Grid

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Abstract. Finite Difference Method (FDM) is one of the most commonly used methods for calculating the pressure distribution of journal bearings, and the division of grid is the key node in computing. For the bearings with surface texture, considering the great difference between the size of texture and bearing, sparser grid may cause the larger calculation errors, but denser grid cause the increasing of computing time and great inconvenient for multi data contrast. In this paper, time consuming in calculating oil-film-pressure distribution has been compared, between using inhomogeneous or homogeneous grid in FDM, with the condition of surface texture on radial journal bearing. Calculation accuracy has also been compared. The result shows the grate shorten by using the inhomogeneous grid in calculation time, with the same accuracy of results in case of small rate of texture area.

1. Introduction

FDM is a commonly used method to calculate the characteristics of journal bearing. In recent years, there are many researches on the optimization of FDM. E. Ko C. [4] introduced a finite difference scheme for Thrust bearing and verified its validity. Zheng S. [5] presented an iterative algorithm of FDM with variable step size successive approximation, and showed the efficient, reliable, stable and convergent of improved method. Han Y [2] provided a virtual grid based method for the Herringbone grid, showed that new method is helpful to improve the accuracy of calculation. Vijayaraghavan D [3] and so on used adaptive algorithm to carry out grid transformation, and proved the new method can obtains the same or better precision with coarser grid which using less calculating time. Li Qiang [7] et al. put forward a new calculation method using watershed dynamic grid algorithm, and showed the consistent with classical calculation results.

However, researches are all about Journal bearings with the condition of smooth surfaces but not contained of textured surfaces condition. In this paper, a multi-scale Inhomogeneous grid is proposed for calculating of journal bearings with textured surface.

When calculating the oil-film-pressure distribution of the journal bearing with smooth surface by FDM, the calculation accuracy can meet the general requirements with the range of 4–6 degrees (same as axial grid spacing) per unit [1]. But in the condition of textured surface, 4–6 degrees may be too sparse to covering texture unit and may affect the accuracy of calculation. If the grid density is increased, the calculation time will increase too. However, it has been proved that texture only in loading zone showed the better lubrication characteristics. Therefore, in order to save calculating time and ensure
accuracy, the multi-scale division method using small scale grid in texture area, and large scale grid in smooth area has been promoted in this paper. And the advantage of this method will be obvious when area of texture is relatively small.

2. Geometric model

![Geometrical model of Bearing and Texture](image)

Figure 1. Geometrical model of Bearing and Texture

For radial journal bearing with surface texture (Figure 1.), $\phi$ denotes the circumferential coordinates if take the place of top of the bearing as the origin of circumference, $\varphi = \phi - \alpha$ denotes the relatively circumferential coordinates if take the place of maximum oil film thickness $h_{\text{max}}$ as the origin of circumference, and select the axial axis position as the origin of axial coordinates, $\lambda$ represents dimensionless axial coordinates, The dimensionless Reynolds equation can be:

$$\frac{\partial}{\partial \varphi} \left( H^3 \frac{\partial p}{\partial \varphi} \right) + \frac{\partial}{\partial \lambda} \left( H^3 \frac{\partial p}{\partial \lambda} \right) = \frac{3}{2} \frac{\partial H}{\partial \varphi}$$

(1)

Dimensionless parameters:

Choose half of bearing width $L/2$ as denominator of axial coordinates. Choose radius clearance $c_0$ as denominator of all others parameters.

Now relatively circumferential coordinates $\varphi$, deviation angle $\alpha$, axial coordinates $\lambda = \frac{z}{L/2}$, film thickness $H = \frac{h}{c_0}$, diameter of the rotor $D = \frac{d}{c_0}$, bearing width $L = \frac{l}{c_0}$, Texture depth $H_p = \frac{h_p}{c_0}$, Texture radius $R_p = \frac{r_p}{c_0}$, static oil-film-pressure $P = \frac{p}{P_0}$, $\frac{2\mu u}{\psi^2}$ is the reference pressure, $\Omega$ is the rotating speed of rotor (rad/s), $\psi = \frac{c_0}{r}$ is relative radius clearance, $r$ is the radius of rotor. In order to facilitate the comparison results, the following physical quantities in this paper, if not stated, are all assumed as dimensionless forms by default.

3. Form of Inhomogeneous grid

![Inhomogeneous grid format](image)

(a) Inhomogeneous grid format

![Five-nodes Difference Format diagram](image)

(b) Five-nodes Difference Format diagram

Figure 2. Inhomogeneous grid format and Five-nodes Difference Format diagram
Inhomogeneous grid format shows in Figure 2, grid spacing in area without surface texture (Area 1) is $\Delta \phi_1$ in circumferential, $\Delta \lambda_1$ in axial. And grid spacing in area with surface texture (Area 2) is $\Delta \phi_2$ in circumferential, $\Delta \lambda_2$ in axial. And same area has same grid spacing. For each node in the grid, a five-node difference format has been went up as Figure 2(b), $(i, j)$ is the coordinate of node which to be solved. Differential quotient expression of each derivative term of dimensionless Reynolds Equation is

\[
\left[ \frac{\partial}{\partial \phi} \left( H^3 \frac{\partial P}{\partial \phi} \right) \right]_{i,j} \approx \alpha_1 P_{i+1,j} + \alpha_2 P_{i-1,j} - (\alpha_1 + \alpha_2)P_{i,j}
\]

\[
\alpha_2 = 2 \frac{a_2 H_i \frac{1}{2}^{3+(a_1-a_2)} H_j^{3}}{a_2^2 (a_1+a_2)} \quad \alpha_2 = 2 \frac{a_2 H_i \frac{1}{2}^{3+(a_1-a_2)} H_j^{3}}{a_2^2 (a_1+a_2)}
\]

\[
\beta_1 = 2 \frac{b_1 H_i \frac{1}{2}^{3+(b_1-b_2)} H_j^{3}}{b_2^2 (b_1+b_2)} \quad \beta_2 = 2 \frac{b_1 H_i \frac{1}{2}^{3+(b_1-b_2)} H_j^{3}}{b_1^2 (b_1+b_2)}
\]

\[
\frac{\partial H}{\partial \phi} \approx \frac{2a_1}{a_2(1+a_2)} H_i^{1/2} - \frac{2a_2}{a_1(1+a_2)} H_j^{1/2} - 2 \frac{a_1-a_2}{a_1 a_2} H_{i,j}
\]

$a_1$, $a_2$, $b_1$, $b_2$ is the dimensionless distance between the center node of the five-node difference format and the adjacent four nodes(as Figure 2(b). showed).

Then establish the difference equation of pressure, the overall numerical calculation process is as Figure 3:

![Figure 3. Numerical calculation process](image)

\[ F_n = \frac{f_{\phi}^2}{\mu r l} = - \int_{\phi_1}^{\phi_2} \left( \int_1^1 P d\lambda \right) \cos \phi d\phi \]  \hspace{1cm} (3)

Among them, the values of $d\lambda$ and $d\phi$ are determined according to the area where the node is located (like $d\lambda_1$ in Area 1, $d\lambda_2$ in Area 2).

4. Comparison of examples

4.1. Condition parameters selection

Radial sliding bearings with oil inlets on both sides, with two bearing bushes $2 \times 150^\circ$ has been chosen in the example (as Figure 1. a), the aspect ratio of the bearing $\frac{L}{d} = 0.6$. And the surface texture distribution is showed in Figure 1.a and Figure 2. (Texture has been arranged at Main bearing area of the lower bearing bush [6]).

Basic parameters of the bearing are shown in Table 1:
Table 1. Basic parameters of the Bearing

| Parameter                  | Symbol | Unit | Value |
|----------------------------|--------|------|-------|
| Bearing diameter           | $d$    | mm   | 20    |
| Bearing width              | $l$    | mm   | 12    |
| Radius difference($R-r$)   | $c_0$  | mm   | 0.1   |
| Eccentricity               | $e$    | /    | 0.6   |
| Texture radius             | $r_p$  | mm   | 0.125 |
| Texture depth              | $h_p$  | mm   | 0.125 |
| Lubricating oil viscosity  | $\mu$  | Pa·s | 0.043 |

(a). Oil-film-thickness distribution of lower bush

(b). Spherical texture

(c) Cylindrical texture

Figure 4. Oil-film-thickness distribution of Texture Area

4.2. Determine the mesh density

First, the minimum numbers of grids that can ensure stable calculation results for smooth surfaces and textured surfaces have been determined before comparing the calculation time. Statistics are made on the calculation time, load capacity $F_h$, and deviation angle $\alpha$ calculation results here, as shown in Figure 5.

(a). Dimensionless calculation time

(b). Calculation result of deviation angle

(c). Calculation result of load capacity

Figure 5. Effect of grid density on the calculation
Figure 5. (a) shows the relationship between calculation time $t(s)$ and grid density. As Figure 5. (a) shows, calculating time can increase thousands of times with the increase of grid density from $100 \times 20$ to $1200 \times 240$.

Figure 5. (b) (c) shows the calculation result of load capacity $F_{H}$ and deviation angle $\alpha$ with the grid density increase from $100 \times 20$ to $1200 \times 240$. As Figure 5. (b)(c) shows, when grid density is larger than $200 \times 40$, calculation result can be stable in the condition of smooth surface, but should larger than $1000 \times 200$ in the condition of textured surface (According to the model size in this paper, 4 more nodes should included in both circumference length and axial length of each texture unit). So in this paper, Equivalent Grid Number (Indicates the density of the grid in a certain area. For example, equivalent grid number is $1000 \times 200$ in Area 1: means that if the whole area has been covered by grids as such density, the number of grids will be $1000 \times 200$) in Area 1 has chosen to be $1000 \times 200$ and Area 2 with $200 \times 40$. The grid density ratio of Area 1/Area 2 is 5/1. Such Inhomogeneous grid decrease the total number of grids from $1000 \times 200$ to nearly $288 \times 121$, Theoretically, time consuming of single-step iteration shall be decrease by 83%.

4.3. Comparison of actual results

4.3.1. Oil film pressure distribution

(a). Pressure distribution solved by Homogeneous grid (with spherical texture)

(b). Pressure distribution solved by Inhomogeneous grid (with spherical texture)

Figure 6. Oil film pressure distribution of lower bush

Figure 6. Shows the pressure distribution solved by homogeneous and inhomogeneous grid (with spherical texture)

4.3.2. Comparison of calculating results. Here the shortest calculation time (at grid number $100 \times 20$) has been chosen as the denominator $t_0$, so dimensionless calculation time $T = \frac{t}{t_0}$.

Statistical the calculation time $t$ and the calculation results of load capacity $F_{H}$ and deviation angle $\alpha$, with the eccentricity $\varepsilon$ range from $0.1 \sim 0.9$. The homogeneous grid is $1000 \times 200$ and the Inhomogeneous grid is same as 4.2($1000 \times 200$ in Area 1 and $200 \times 40$ in Area 2 for Equivalent Grid Number). Detail result is shown in Figure 7.
Figure 7. Comparison of calculating result

As Figure 7. (a) (b) shows, no significant difference has been found in the calculating result of both load capacity $F_n$ and deviation angle $\alpha$ between homogeneous and Inhomogeneous grid, which proof the reliability of homogeneous grid method.

Figure 7. (c) (d) proof that homogeneous grid method can save lot of time. As Figure 7. (c) shows, with the low eccentricity $\varepsilon$ from 0.1~0.6 , no significant increase has been found in calculation time $t$ with the increase of eccentricity $\varepsilon$, but time with Inhomogeneous grid is significant lower than with homogeneous grid(around 85% lower). And with the high eccentricity $\varepsilon$ from 0.6~0.9, significant increase of $t$ has been found with the increase of eccentricity $\varepsilon$. When eccentricity $\varepsilon = 0.9$, the longest $t = 116s$ has been reached by cylinder texture used homogeneous grid method for calculating, even the spherical texture used inhomogeneous grid method for calculating has reached to $t = 46s$. But regardless of the eccentricity changing, $t$ in Inhomogeneous grid method is always significant lower than inhomogeneous grid method(at least 60% lower), and $t$ in cylinder textured is always slightly larger than in spherical textured.

Figure 7. (d) Shows the investigation of the relationship between time saving rate $s = \frac{t_H-t_I}{t_H}$ ($t_H,t_I$ represent $t$ using homogeneous grid method or inhomogeneous grid method) and eccentricity $\varepsilon$. It can be found that $s$ has decreased when eccentricity $\varepsilon$ increase. When eccentricity $\varepsilon = 0.1$, $s$ can reach 85% when spherical textured, when eccentricity $\varepsilon = 0.9$, $s$ can reduced to 61% when cylinder textured. And $s$ is always slight smaller when cylinder textured than spherical textured, which shows the opposite situation as Figure 7. (c).

As for cylinder textured, the shape mutation at the edge of the texture is more pronounced than spherical textured, which caused the increase of iteration steps and result to the slightly longer of time.

According to result, time saving rate $s$ has not always reach 83% (Theoretically value in 4.2), even only 61% when eccentricity $\varepsilon = 0.9$, spherical textured, blamed on the increase of iteration steps when using Inhomogeneous grid.

In summary, using inhomogeneous grid in FDM for the numerical calculation of textured journal bearing, can significant saving the calculating time when area of textured is small.

5. Conclusion
Under the conditions in this article:
(1) Using inhomogeneous grid in FDM for numerical calculation of journal bearing can significantly shorten the calculation time.
(2) The effect of time saving is positively related to eccentricity.
(3) With the same eccentricity, the effect of time saving is more significant in spherical textured than cylinder textured.

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