Newton Equations May Be Treated as Diffusion Equations in the Real Time and Space Fields of Multifractal Universe (Masses are Diffusion Coefficients of Diffusion-like Equations)

L.Ya.Kobelev
Department of Physics, Urals State University
Lenina Ave., 51, Ekaterinburg, 620083, Russia
E-mail: leonid.kobelev@usu.ru

In thirties years of last century Dirac proposed to treat Schrödinger equation as the equation of diffusion with imaginary diffusion coefficient. In the frame of multifractal theory of time and space (in this model our the multifractal universe is consisting of real time and space fields) in the works [1]-[16] was analyzed how the fractional dimensions of real fields of time and space influence on behavior of different physical phenomena. In this paper the Newton equations of the multifractal universe (considered for the first time in [1]-[3]) are generalized and is treated as the equations of diffusion with mass of bodies (depending of fractional dimension of place, where these bodies located) as a coefficient of diffusion. The realization of this point of view for inhomogeneous time equations (the analogies of Newton equations) is carried out too. The last lead to introducing new sort of masses: the masses that characterize the inertia of inhomogeneous time flows with space coordinates changing.

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I. INTRODUCTION

This paper based on some concepts of physical models of the theory of multifractal time and space which are consequences of this theory (Kobelev [1]-[16]). The multifractal model of space and time treats the time and the space with fractional dimensions as real fields. Universe is formed only by these fields, i.e. our universe is the fractional real material time and fractional real material space. All other fields are born by the fields of time and space by means of their fractional dimensions. As the time and the space are material fields with fractional dimensions and multifractal structure (multifractal sets,) they are defined on the sets of their carriers of measure (physical vacuum for our universe which born her when "big-bang" happened). In each the time (or the space) point ("points" are approach for very small "intervals" of time and space and "intervals" are the multifractal sets with global dimensions for their sets, that play role of local dimensions for universe in whole) the dimensions of time (or space) determine the densities of Lagrangians energy for all physical fields (or new physical fields for space) in these points. Time and space are binding by relation \( dt^2 - c^2 dr^2 = 0 \) (this relation is only a good approach, more precise relations see at [3]). The purpose of this paper is consideration of very interesting problem (in the frame of mathematical formalism of the multifractal model of time and space introduced in the papers of Kobelev and presented in [1]-[16]): may masses of bodies be treated as characteristic of the real space that are analogies of the diffusion coefficient? Such interpretation will be the analogy of Dirac interpretation of Schrödinger equation. In that case main equations of modern physics (Newton and Schrödinger equations) are the analogy of diffusion equation and their nature caused by diffusion processes in the real time and space fields of our multifractal universe.

II. NEWTON EQUATIONS IN THE MULTIFRACTAL UNIVERSE

Newton equations in the multifractal universe considered in [1]-[3]. These equations read

\[
D_{-t}^{d_t(\mathbf{r}, t)} [mD_{+d_t(\mathbf{r}, t)} \mathbf{r}(t)] = D_{+t}^{d_t} [m\Phi_g(\mathbf{r}(t))] \quad (1)
\]

\[
D_{-t}^{d_t} D_{+d_t} \Phi_g(\mathbf{r}(t)) + \frac{b_g^2}{2} \Phi_g(\mathbf{r}(t)) = \gamma \quad (2)
\]

In the Eq.(2) the constant \( b_g^{-1} \) has order of size of universe and introduced with purpose to extend the class of
functions on which the generalized fractional derivatives concept is applicable. These equations do not present a closed system because of the fractionality of spatial dimensions. Therefore we approximate the fractional derivatives with respect to space coordinate as \( D^d_{x,r} \approx \nabla \), i.e. approximate by usual space derivatives. In the Eqs. (1)-(3) is used the integral functional \( D^d_{x,r} \) (both left-sided and right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets (see [1]-[3]). These functionals are simple and natural generalization of the Riemann-Liouville fractional derivatives and integrals and read:

\[
D^d_{+,t} f(t) = \left( \frac{d}{dt} \right)^n \int_0^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}}
\]

(3)

\[
D^d_{-,t} f(t) = (-1)^n \left( \frac{d}{dt} \right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}}
\]

(4)

where \( \Gamma(x) \) is Euler’s gamma function, and \( a \) and \( b \) are some constants which take values from interval \((0, \infty)\). In these definitions, as usually, \( n = \{d\}+1 \), where \( \{d\} \) is the integer part of \( d \) if \( d \geq 0 \) (i.e. \( n-1 \leq d < n \)) and \( n = 0 \) for \( d < 0 \). If \( d = \text{const} \), the generalized fractional derivatives (GFD) \((1)-(4)\) coincide with the Riemann-Liouville fractional derivatives \((d \geq 0)\) or fractional integrals \((d < 0)\). When \( d = n+\varepsilon(t), \varepsilon(t) \to 0 \), GFD can be represented by means of integer derivatives and integrals. For \( n = 1 \), i.e. \( d = 1+\varepsilon, |\varepsilon| << 1 \) it is possible to obtain:

\[
D^d_{+,t} f(r(t),t) \approx \frac{\partial}{\partial t} f(r(t),t) + \alpha \frac{\partial}{\partial t} \left[ \varepsilon(r(t),t) f(r(t),t) \right] + \frac{\varepsilon(r(t),t) f(r(t),t)}{t}
\]

(5)

where \( \alpha \) is a constant and determined by choice of the rules of regularization of integrals \((1)-(3)\) (for more detailed see [2]) and the last member in the right hand side of (3) is very small. The selection of the rules of regularization that gives a real additives for usual derivative in (3) yields \( a = 0.5 \) or \( a = 1 \) for \( d < 1 \) [4]. The functions under the integral sign in the (3)- (4) we consider as the generalized functions defined on the set of the finite or generalized functions \([5]\). The notions of the GFD, similar to \([1]-[4]\), can also be defined and for the space variables \( r \). The definitions of GFD \([3]-[4]\) need in the connections between the fractal dimensions of time \( d_i(r(t),t) \) and the characteristics of physical fields \( (\Phi_i(r(t),t), i = 1, 2, \ldots) \) or densities of Lagrangians \( L_i \) and such connections were defined in the cited works. Following \([1]-[15]\), we define this connections by the relation:

\[
d_i(r(t),t) = 1 + \sum \beta_i L_i(\Phi_i(r(t),t))
\]

(6)

where \( L_i \) are densities of energy \( (\text{Lagrangian densities}) \) of physical fields, \( \beta_i \) are dimensionless constants with physical dimension of \( [L_i]^{-1} \) (it is worth to choose \( \beta_i \) in the form \( \beta_i = \alpha^{-1} \beta_i \) for the sake of independence from the regularization constant and select the \( \beta = Mc^2 \) where \( M \) is the mass of the body that born considered gravitational field). The definition of the time as the system of subsets and definition of the FD for \( d_i \) (see [4]) connects the value of fractional (fractal) dimension \( d_i(r(t),t) \) with each time instant \( t \). The latter depends both on time \( t \) and coordinates \( r \). If \( d_i = 1 \) (an absence of physical fields) the set of time has topological dimension equal to unity. For large energy densities \( (e.g., \text{for cases when gravitational field is large (the domain of space where } r < r_0) \) Eqs. (1)-(3) contain no divergencies \([1]\) since integral-differential operators of the generalized fractional differentiation are reduced to the generalized fractional integrals \((\text{see } [1])\).

We bound the consideration only by the case when relations \( d_i = 1 - \varepsilon(r(t),t) \), \( |\varepsilon| \ll 1 \) are fulfilled. In that case the GFD (as was shown in [4]) may be represented \((\text{as a good approach})\) by ordinary derivatives and relations \([1]-[4]\) are valid. Now we can determine the \( d_i \) for distances much larger than the gravitational radius \( r_0 \) \((\text{for the problem of a body motion in the field of spherical-symmetric gravitating center})\) as (see [4] for more details)

\[
d_i \approx 1 + \beta_g \Phi_g + \beta_m \Phi_m
\]

(7)

where \( \Phi_g \) is the gravitational potential of mass \( M \) and \( \Phi_m \) is the gravitational potential born by the body with mass \( m \) in its center. So the equations \((1)-(2)\) reed \((\text{we used for GFD approach of } (1)-(3)\))

\[
1 - 2\varepsilon(r(t),t) \frac{d^2}{dt^2} r = F_g
\]

(8)

where

\[
F_g = -\frac{n M}{r}, \varepsilon = \beta_g \Phi_g + \beta_m \Phi_m
\]

(9)

We had neglected by the fractional parts of spatial dimensions and by the contributions from the term with \( b_g^{-1} \). Now we take the \( \beta_m = \beta_g = c^{-2} \) for potentials \((\text{or } \beta = (Mc)^{-2} \text{ for Lagrangian density } L) \). Then equation (8) gives the small corrections to Mercury perihelion rotations of general relativity \((\text{see } [17])\). It is the example of limited character of general relativity principle of equivalence.

**III. GENERALIZED NEWTON EQUATIONS AND THEIR DIFFUSION INTERPRETATION**

In this paragraph we write down the generalization of modified Newton equations \((1)-(2)\) in the multifractal time universe. Let the gravitational forces only is presence . This generalization based on a quite natural assumption: if the every point of the real spatial field determines their fractional \( (\text{temporal and spatial}) \) dimensions, the masses in this point \((\text{as the one of characteristics of the real spatial field})\) must have dependence
of the fractional dimensions in this point (i.e. at \( d_t \) and \( d_r \)). So, the Eqs. (1)-(2) will have the form

\[
D_{-t}^{d_t}(r,t) [m_{d_t,d_r} D_{+r}^{d_r}(r,t)] = D_{-t}^{d_t}[m_{d_t,d_r} \frac{\gamma M}{r}]
\quad \text{(10)}
\]

If the mass of accelerated body \( m_d \) has no dependencies of fractional dimensions the Eq. (10) and Eqs. (1)-(2) are coinciding. We consider now the simple method of receiving of the equation (11) from qualitative reasoning. Let \( E_r \) is the density of energy at the point \( r \) of real space field. On the one hand the gradient of it gives for current \( j \) equation

\[
j_E \sim \nabla E(r)
\quad \text{(11)}
\]

On another hand this current is proportional to changing (with changing of time) of gradient \( E(r) \) in the temporal space

\[
j_E \sim \frac{\partial^2}{\partial t^2} E(r)
\quad \text{(12)}
\]

The latter equation describes the diffusion of the energy at space point \( r \) in the temporal space under the influence of gradient of it. So designate a diffusion coefficient in temporal space as \( m_{d_t,d_r} \). We may compare now the (12) to the (11) and separate \( E \) in the (13) into the coefficient of diffusion \( m \) and the space field transferring the energy \( r \) and replace \( E \) by gravitational energy in the (11). We receive the Newton equations in the form (10). Thus we have the interpretation of Newton equations as the equations are describing the masses of bodies as the diffusion coefficients of transferring of energy density of the real space field of the point \( r \) at the temporal space (at the spatial point \( r \)). We consider the case when it is possible to neglect by corrections of order \( r_0 r^{-1} \) where \( r_0 = \frac{2\sqrt{\gamma M}}{c^2} \) is the gravitational radius, in the generalized fractional derivatives and replaced them by usual derivatives, then the Eq. (11) reads

\[
m_{d_t} \frac{\partial^2}{\partial t^2} r + \frac{\partial m_{d_t}}{\partial t} \frac{\partial}{\partial t} r = \nabla \frac{m M \gamma}{r}
\quad \text{(13)}
\]

In the Eq. (13) there is the additional member with derivatives \( m_t \) with respect to \( t \). This member describes the change of mass \( m \) with time. As in the multifractal theory of time and space (see [1]-[4]) there are no the constant values, the appearing of this member is very natural because all physical values in multifractal universe are changing in time. For \( d_t = 1 + \varepsilon, \varepsilon << 1 \) this member is very small and not essential.

IV. GENERALIZED INHOMOGENEOUS TIME EQUATIONS AND THEIR DIFFUSION INTERPRETATION

In the multifractal universe the time and the space are real and inhomogeneous fields. So for the time \( t(r) \) that depends of the spatial points of space field \( r \) there is the equation that was found for the first time in the paper [3]. If we are to take into account the fractionality of spatial dimensions \( (d_x \neq 1, d_y \neq 1, d_z \neq 1) \) (see [3], [4]), we arrive to a new class of equations that describe "temporal" physical fields (we shall call them the "temporal" fields) generated by the space with fractional dimensions. These equations are quite similar to the corresponding equations that appear due to fractionality of time dimensions (the latter were given earlier). In Eqs. (10), we must take \( x = r, \alpha = r \) and fractal dimensions \( d_t(t(r), r) \) will be described by the Eq. (12) of this paper with \( t \) being replaced by \( r \). Thus for time \( t(r(t)) \) and potentials \( \Phi_g(t(r), r) \) (analogous of gravitational field) the equations analogous to Newton’s will read (here the spatial coordinates play the role of time)

\[
D_{d_x}^{d_x}(r,t) m_r D_{d_y}^{d_y}(r,t) t(r) = D_{d_z}^{d_z}[m_r \Phi_g(t(r))]
\quad \text{(14)}
\]

\[
D_{d_t}^{d_t} D_{d_t}^{d_t} \Phi_g(t(r)) + \frac{b_{d_t}^2}{2} \Phi_g(t(r)) = \gamma r
\]

The equation (14) describes the changes of time born by existence of fractionality of spatial dimensions. The \( m_r \) is the analogy of mass the \( m_t \) in the ordinary Newton equations and the last usually is treated as measure of inertia of bodies. The fundamental constant \( \gamma_r \) is the analogues of gravitational constant \( \gamma \) in field of time. So the \( m_r \) may be treated as ”measure of temporal inertia” when the inhomogeneous time flows propagate through different spatial domains.

Thus there are two sorts of ”masses” in the multifractal universe: the masses \( m_t \) and the masses \( m_r \). The first describe the ”bodies inertia” or diffusion of energy of the space with current of time. The second masses \( m_r \) describe the inhomogeneous of time flows with changes of spatial places and may be treated as diffusion coefficient of time energy propagation in the space of real field \( r \). So in the multifractal universe the known conception of masses is doubled and it is necessary to introduce the new sort of masses because there are exist not only the space energy \( 2E_t = m_t \frac{\partial m_t}{\partial t} \) (non relativistic case), but the ”time energy”

\[
2E_r = m_r \frac{\partial m_r}{\partial t}^2
\quad \text{(16)}
\]

This relation characterize the time energy of real time field at point \( t(r) \). We may construct for this field the special relativity of almost inertial systems in full analogy with papers [4], [6]. For Lorentz transformation in the real time field than we have

\[
t' = \frac{t + v x}{\sqrt{(1 - \frac{v^2}{c^2})^2 + 4a^2_0 r}}
\quad \text{(17)}
\]

\[
x' = \frac{x + \frac{a t}{c}}{\sqrt{(1 - \frac{v^2}{c^2})^2 + 4a_0^2 t}}
\quad \text{(18)}
\]
In the Eqs. (17)-(18) we use designation: $c_r = c^{-1}$, $v_r = \frac{\partial r}{\partial t}$. The value $a_0$ may be received from correspondent the $a_0$ of paper [3] by replacing the fractional dimensions $d_r$ for $d_r$. For agreement with Lorentz transformations with respect to any moving in the real space it is necessary to use relation: $v_r = \frac{\partial r}{\partial t}c^{-2}$. So there is only two unknown values: the $m_r$, which is determine the time diffusion (or "time inertia") of real time field in the multifractal universe and temporal force $F(t)$ with unknown $\gamma_r$.

V. CONCLUSIONS

The next main results is necessary to note:
1. In this paper Dirac idea about treating quantum mechanics as diffusion process (with imaginary diffusion coefficient) is propagated for the domain of classical physics. The last is possible only because of real nature of time and space fields in the multifractal universe; 2. the consideration of inhomogeneous time equations (these equations are analogies of Newton equations for real time field) is based on consideration of the process of diffusion of "temporal energy" in space when there are space coordinates changing. This energy depends of fractional dimensions $d_r$ and when the last are changing (that depends of changing of $r$) with flow of time, the $d_r$ and the temporal energy change too.
3. Generalized Newton equations for the multifractal universe is obtained;
4. Generalized temporal equation (analogy of Newton equations for $t(r)$) for multifractal universe is obtained;
5. The generalized temporal masses are presented (see also [3])
5. We pay attention once yet that if time and space treat as convenient only marks for system of references (this point of view contradicts the base assumption of the multifractal theory of universe about the nature of time and space fields as the real fields) diffusion interpretation of classic mechanics equations and nature of usual and the temporal masses is impossible.

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