1. Introduction

The study of quasi-one dimensional strongly correlated electron systems (SCES’s) has received a lot of attention in the past few years. This interest was mainly triggered by the synthesis of materials which in a wide range of temperatures can be well modeled by a 3D system of (almost) decoupled spin chains, spin ladders or more generally, Hubbard chains and ladders [1]. An important reason that has put these 1D systems again on the scene is the appearance of the so-called “stripe” phases which have shown up in very different contexts, such as high-$T_c$ cuprates [2], integer quantum Hall effect systems at high Landau levels [3], materials showing colossal magnetoresistance (manganites) [4], etc.

The present article contains a brief review of theoretical work on magnetization plateaux in SCES’s in 1D. The presentation reflects the authors’ perspective and due to lack of space, not all important contributions to this subject can be mentioned. The first studies on this issue were performed by Hida [5] and Okamoto [6] in an attempt to describe some organic compounds with periodic couplings. After a few other isolated cases were studied [7,8], Oshikawa and collaborators [9] have undertaken the first systematic study of this problem and, by extending the Lieb-Schultz-Mattis theorem to systems in a magnetic field, they provided a necessary condition for the appearance of magnetization plateaux in 1D systems. When the magnetization $\langle M \rangle$ is normalized to saturation values $\pm 1$, this condition for the appearance of a plateau with magnetization $\langle M \rangle$ can be cast in the form

$$SV(1 - \langle M \rangle) \in \mathbb{Z}. \quad (1)$$
Here $S$ is the size of the local spin and $V$ the number of spins in the unit cell for the translation operator acting on the magnetization $\langle M \rangle$ groundstate. It should be noted that translational invariance can be spontaneously broken in the groundstate and then $V$ would be larger than the unit cell of the Hamiltonian.

Spin ladders in a magnetic field constitute a class of systems where the full phase diagram was explored and where it was checked when the necessary condition (1) becomes also sufficient [15-20].

A famous result on pure 1D spin chains is Haldane’s conjecture [21] (see also [22] for a new field theoretical approach). It states that such chains with integer $S$ are gapful while those with half-integer $S$ remain gapless, corresponding to the condition (1) with $\langle M \rangle = 0$ and $V = 1$ since a spin gap is equivalent to an $\langle M \rangle = 0$ plateau. This feature is indeed confirmed in seminal numerical studies of Heisenberg chains [23,24], but no other plateaux are found for these systems. Other non-trivial plateaux, which for $S > 1$ would be permitted by (1), arise only when the model is modified, e.g. by adding a single-ion anisotropy [25,26].

The existence of real materials with trimer constituents [27-31] motivated the investigation of the magnetization process of trimerized [5,6,32], frustrated trimerized [33,34], quadrumerized [35] and more generally periodically modulated $S = 1/2$ Heisenberg spin chains with period $p$ (so-called $p$-merized chains) [36-40]. Ladders consisting of both staggered and non-staggered dimerization along the chains were also studied [41].

From the experimental point of view, one of the most exciting materials is NH$_4$CuCl$_3$ where just two plateaux with $\langle M \rangle = 1/4$ and $3/4$ have been observed [42]. The crystal structure of NH$_4$CuCl$_3$ suggests to model it as a two-leg zig-zag ladder, however with room for some modifications. While one can indeed obtain plateaux with $\langle M \rangle = 1/4$ and $3/4$ in a zig-zag ladder with dimerized legs [43,44], neither this point of view nor other proposals [45] have so far resulted in a really satisfactory theoretical description of the experimental observations. In short, if NH$_4$CuCl$_3$ is a quasi-one-dimensional system, eq. (1) should apply with $S = 1/2$ since the spin is carried by Cu$^{2+}$ ions. Then one needs $V = 8$ in order to permit plateaux with $\langle M \rangle = 1/4$ and $3/4$. However, $V = 8$ would permit also plateaux with $\langle M \rangle = 0$ and $1/2$ which are not observed experimentally [42], but are very pronounced e.g. in the dimerized zig-zag ladder [44].

When charge degrees of freedom are dynamical, as in Hubbard chains, the situation turns out to be quite different and very interesting, although the effect of a magnetic field has been studied so far essentially only in two cases: An integrable spin-$S$ generalization of the $t-J$ model [46] and doped Hubbard chains with periodically modulated hopping matrix elements or on-site energies [47]. In addition to fully gapped situations (both a charge gap and a spin gap), situations arise in which magnetization plateaux appear at *irrational* doping-dependent values of the magnetization, in contrast to what we have just reviewed for purely magnetic systems. Interestingly, under such circumstances, gapless degrees of freedom are present and consequently certain superconducting correlations have power law behavior [47].

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1 In cases where the sites carry different spins [10-14], the combination $SV$ should be replaced by the maximal spin in the unit cell.
Recently, the effect of disorder on plateau systems, such as the \( p \)-merized XXZ chains, has also been studied. It was found that, for binary disorder, plateaux do not disappear but are rather shifted in a precise amount which can be predicted by means of a simple argument to be \( \langle M \rangle = 1 + \frac{2}{p} (p - 1) \), where \( p \) is the strength of disorder. Conversely, continuous distributions of disorder erase completely the plateau structure. Interestingly, the effect of disorder on the susceptibility leads to an even-odd effect similar to what is found in \( N \)-leg ladders \([49]\).

2. Field theory approach

The field theory description of spin chains is a useful technique for treating problems of weakly coupled spin ladders, weak dimerization in chains, etc. as we will illustrate below for the case of \( S = 1/2 \). For higher spin chains, non-Abelian bosonization has proven to be better suited. For spin 1/2 chains, the Abelian bosonization technique has proven to be very efficient. It describes the low energy, large scale behavior of the system, and can be extended to the case of an easy axis anisotropy. More specifically, the continuum limit of the Hamiltonian

\[
H_{XXZ} = J \sum_{x=1}^{L} \left\{ \Delta S_x^z S_{x+1}^z + \frac{1}{2} \left( S_x^+ S_{x+1}^- + S_x^- S_{x+1}^+ \right) \right\} - h \sum_{x=1}^{L} S_x^z
\]

is given by the Tomonaga-Luttinger Hamiltonian

\[
H = \frac{1}{2} \int dx \left( v K (\partial_x \phi)^2 + \frac{v}{K} (\partial_x \phi)^2 \right).
\]

The bosonic field \( \phi^i \) and its dual \( \tilde{\phi}^i \) are given by the sum and difference of the lightcone components, respectively. The constant \( K = K(\langle M \rangle, \Delta) \) governs the conformal dimensions of the bosonic vertex operators and can be obtained exactly from the Bethe Ansatz solution of the XXZ chain (see e.g. [16] for a detailed summary). One has \( K = 1 \) for the \( SU(2) \) symmetric case \( (\Delta = 1) \) and it is related to the radius \( R \) of [16] by \( K^{-1} = 2\pi R^2 \).

In terms of these fields, the spin operators read

\[
S_x^z = \frac{1}{\sqrt{2\pi}} \partial_x \phi + a : \cos(2k_F x + \sqrt{2\pi} \phi) : + \frac{\langle M \rangle}{2},
\]

\[
S_x^\pm = (-1)^x : e^{\pm i\sqrt{2\pi} \phi} \left( b \cos(2k_F x + \sqrt{2\pi} \phi) + c \right) : ,
\]

where the colons denote normal ordering with respect to the groundstate with magnetization \( \langle M \rangle \). The Fermi momentum \( k_F \) is related to the magnetization of the chain as \( k_F = (1 - \langle M \rangle) \pi / 2 \). An XXZ anisotropy and/or the external magnetic field modify the scaling dimensions of the physical fields through \( K \) and the commensurability properties of the spin operators, as can be seen from (4), (5).

If one introduces periodically modulated couplings between the spins \( J_x = J \) if \( x \neq np \) and \( J_x = (1 - \delta)J \equiv J' \) if \( x = np \), this amounts to the following perturbation for the original Hamiltonian in the continuum limit [36]

\[
H_{pert} = \lambda \int dx \cos(2pk_F x + \sqrt{2\pi} \phi),
\]
where $\lambda$ is proportional to $\delta J$.

Another interesting situation is the one of spin ladders [15,16], where $N$ identical chains are coupled with a transversal coupling $J'$

$$H = \sum_{a=1}^{N} H_{XZ}^a + J' \sum_{x,a=1}^{a=N} \vec{S}_x^a \cdot \vec{S}_{x+1}^a.$$

For simplicity we used here periodic boundary conditions (PBC’s) along the transverse direction. One obtains in the continuum a collection of identical Hamiltonians like (3), with perturbation terms which couple the fields of the different chains. After a careful renormalization group (RG) analysis, one can show that at most one degree of freedom, given by the combination of fields $\phi_D = \sum_{a} \phi_a$, remains massless. The large scale effective action for the ladder systems is then given again by a Hamiltonian (3) for $\phi_D$ and the perturbation term

$$H_{pert} = \lambda \int dx \cos(2Nk_F x + \sqrt{2\pi\phi_D}),$$

where $k_F = (1 - \langle M \rangle)\pi/2$ is related to the total magnetization $\langle M \rangle$.

In both cases, the key point is to identify the values of the magnetization for which the perturbation operators (6) or (7) can play an important rôle. In fact, this operator is commensurate at values of the magnetization given by (1) with $S = 1/2$ and $V = p$ or $V = N$, respectively. If this operator turns out to be also relevant in the RG sense (this depends on the parameters of the effective Hamiltonian (3), the model will have a finite gap, implying a plateau in the magnetization curve. The generalization to $p$-merization in $N$-leg ladders results in $V = pN$, though in some cases the topology of couplings has to be carefully analyzed [41].

The case of a single $p$-merized chain can be generalized to the presence of charge carriers [47]. Let us consider the Hubbard model, representing interacting electrons with spin 1/2. The Hamiltonian is given by

$$H = -\sum_{x,\alpha} t(x)(c_{x+1,\alpha}^{\dagger}c_{x,\alpha} + H.c.) + U \sum_{x} c_{x,\uparrow}^{\dagger}c_{x,\uparrow}c_{x,\downarrow}^{\dagger}c_{x,\downarrow} + \frac{\mu(x)}{2} \sum_{x} (c_{x,\uparrow}^{\dagger}c_{x,\uparrow} - c_{x,\downarrow}^{\dagger}c_{x,\downarrow}),$$

where $t(x)$ and $\mu(x)$ are taken as periodic in the variable $x$ with period $p$. The bosonized version of this Hamiltonian can be written as a Gaussian part, given by

$$\sum_{i=c,s} \frac{u_i}{2} \int dx \left[ (\partial_x \phi_i)^2 + (\partial_x \theta_i)^2 \right],$$

where the fields labeled by $c$ and $s$ are particular combinations of the bosonic fields dictated by the Bethe Ansatz solution of the homogeneous model. The most relevant of several perturbation terms is given by

$$O_{pert} = \lambda_1 \sin[\frac{n}{2} + pn\pi x - \sqrt{\pi\phi_c}] \cos[\sqrt{2\pi\phi_s}] + \lambda_2 \sin[\pi n + 2pn\pi x - \sqrt{4\pi\phi_c}].$$
We have written the perturbation term for the case of zero magnetic field for simplicity. Using the same arguments of commensurability and relevance as above, we can show that for \( p n \in \mathbb{Z} \), we have a charge gap. If the condition is further constrained to \( p n/2 \in \mathbb{Z} \), for zero magnetic field we have also a spin gap, implying a plateau with \( \langle M \rangle = 0 \). In the case of non-zero magnetic field, if one of the conditions
\[
\frac{p}{2} (n \pm \langle M \rangle) \in \mathbb{Z}
\]
is satisfied, and the doping is kept fixed (as is natural from the point of view of experimental realizations of doped systems), the system has a magnetization plateau, but still exhibits massless behavior as well, e.g. in the specific heat which vanishes linearly as the temperature goes to zero. If both conditions are simultaneously satisfied, the system is gapped in the charge and spin sectors; this situation is in fact the generalization to arbitrary doping of the results for \( p \)-merized Heisenberg chains discussed above.

3. Strong coupling arguments

The magnetization process is easy to understand if the system decouples into clusters of \( V \) sites \([15,16,37,47]\). These ‘strongly coupled’ clusters magnetize independently such that at zero temperature the magnetization \( \langle M \rangle \) can only take finitely many values. For a spin-system with spin \( S \) they are subject to the quantization condition \( (1) \) where \( V \) is directly the number of spins in one of the decoupled clusters. Using (higher order) series expansions around this decoupling point \([15,16,34,37]\), the quantization condition obtained in this simple manner can be argued to be valid also for more general parameters.

In its simplest form, this argument explains magnetization plateaux which arise because the unit cell of the Hamiltonian contains several spins. However, the argument can be refined to account also for spontaneous breaking of translational symmetry by a period of two. The transitions between plateau states can be treated by degenerate perturbation theory, leading to effective Hamiltonians which in many cases turn out to be effectively an \( XXZ \) chain \( (2) \) if only the lowest order(s) are kept \([50-54,17,43,34]\). If the effective \( XXZ \) anisotropy turns out to be sufficiently large \( (\Delta > 1) \), a gap opens and translational symmetry is spontaneously broken. In this manner one finds a further plateau precisely in the middle between the two values of \( \langle M \rangle \) predicted by considering just the decoupling limit itself. This illustrates that \( V \) in \( (1) \) should be taken from the unit cell of the groundstate whose size is an integer multiple (in general larger than 1) of the unit cell of the Hamiltonian.

Amusingly, first-order strong-coupling results for the magnetization process remain exact over a finite range of parameters in certain systems with local conservation laws \([12,55]\).

4. Numerical approaches

In many cases coupling constants do not lie in any of the weak- or strong-coupling regimes discussed before. In such cases, accurate results can be obtained
Figure 1. Magnetization curves of (a) the planar 3-leg ladder at \( J' = 3J \) [15] and (b) the quadrumerized chain with \( J' = J/2 \) [36]. The diamonds show results of (a) fourth-order [16] and (b) second-order [37] strong-coupling expansions for the plateau boundaries. The bold line is an extrapolation to the thermodynamic limit.

only numerically. Since on the one hand, vector spaces are large already for moderate system sizes and on the other hand, only a few extreme eigenvalues are needed to compute the zero-temperature magnetization curves, one frequently resorts to the Lanczos method as was already done in pioneering works [23,24]. In the present situation the magnetic field couples to a conserved quantity \( S^z_{\text{tot}} \). One therefore only needs the groundstate energy \( E(S^z_{\text{tot}}, h = 0) \) at \( h = 0 \) in each of the magnetization subspaces with \( S^z_{\text{tot}} \in \{0, 1, ..., SL\} \) (here \( L \) is the total number of spins). Then one can readily obtain the energy in a finite field \( h \) through the relation \( E(S^z_{\text{tot}}, h) = E(S^z_{\text{tot}}, 0) - hS^z_{\text{tot}} \) and then construct the groundstate magnetization curve. Note that the magnetization has steps on a finite system since the possible values of the magnetization are quantized.

Among the systems whose magnetization process has been studied by the Lanczos method, the two-leg zig-zag ladder (which is equivalent to the \( J_1-J_2 \) chain) has a particularly long tradition [56-61]. However, plateaux with \( \langle M \rangle \neq 0 \) are observed only for more than two legs [43,38] or if the coupling constants are modulated [41,44,50,61,62].

Fig. 1a) shows the magnetization curve of an \( S = 1/2 \) 3-leg ladder with open boundary conditions along the rungs [15]. As expected, there is a clear plateau with \( \langle M \rangle = 1/3 \) and the fourth-order strong coupling series for the boundaries of the plateau [16] are in good agreement with the Lanczos results. The effect of \( p \)-merization is illustrated in Fig. 1b) which shows the magnetization curve of a quadrumerized \( S = 1/2 \) Heisenberg chain [36]. Here one observes two clear plateaux at \( \langle M \rangle = 0 \) and 1/2, as expected. The series for their boundaries [37] are in good agreement with the numerical data even if they are only of second order. Moreover,
in the latter case bosonization predicts the plateaux to open with power laws in $\delta$ which can indeed be verified numerically [36].

An alternative method is DMRG [63] which allows to study larger systems. The number of applications of this method or variants thereof to the magnetization process is increasing steadily [11,12,17-19,33,38,62,64-67]. The only technical complication is that the infinite system algorithm does not give good approximations to incommensurate structures and therefore the finite system algorithm should be used if one wants to compute the complete magnetization curve. Fig. 2 illustrates this method with two examples [12]: (a) The magnetization curve of the $S = 1$ Heisenberg chain exhibits only an $\langle M \rangle = 0$ plateau, corresponding to the Haldane gap [21], and (b) the magnetization curve of the $S = 3/2$-$1/2$ ferrimagnetic Heisenberg chain which exhibits a spontaneous magnetization and thus a plateau with $\langle M \rangle = 1/2$.

Long $XX$ chains in a transversal field can be treated by the Jordan-Wigner transformation (see e.g. [40]). This is particularly useful for systems with quenched disorder [48]. Fig. 3 illustrates the effect of a disordered distribution of couplings with a $p$-periodically modulated background on the magnetic behavior. Here, we show the magnetization curves of a quadrumerized $XX$ chain under different disorder strengths $p$ of a binary distribution $P(J_i) = p\delta(J_i - J') + (1-p)\delta(J_i - J - \gamma_i J)$, where $\gamma_i = \gamma$, ($-\gamma$) if $i = pn$, ($i \neq pn$). One observes the appearance of a rather nontrivial phenomenon, namely, the shift of the magnetization values (not necessarily rational), for which certain plateaux emerge, as compared to the pure system. One can show by means of a simple real space decimation procedure [48] that these new plateaux appear at $\langle M \rangle = 1 + 2(p - 1)/p$ for $XXZ$ chains which in the special case of $XX$ chains is consistent with the numerical results. Continuous distributions for the disorder wipe out completely the plateau structure.
Figure 3. Magnetization curves of quadrumerized $XX$ chains with $5 \times 10^4$ sites in disordered binary backgrounds of strength $p = 0.2, 0.4, 0.6, 0.8$ (solid lines in ascending order) [48]. The left and rightmost dotted lines denote respectively the pure uniform and pure modulated cases $p = 1$ and $p = 0$.

5. Experimental realizations

Two concrete plateau substances have already been mentioned earlier: Clear plateaux with $\langle M \rangle = 1/4$ and $3/4$ have been observed in NH$_4$CuCl$_3$ [42], but in this case a thorough theoretical understanding is still lacking. Conversely, a plateau with $\langle M \rangle = 1/3$ is theoretically expected [33,34] in Cu$_3$Cl$_6$(H$_2$O)$_2$·2H$_8$C$_4$SO$_2$, but magnetization experiments [31] would have to be extended to slightly higher fields to confirm its presence. One example where experimental observations do actually confirm theoretical predictions [7,8] are the plateaux with $\langle M \rangle = 1/2$ and 0 which have been observed in the bond-alternating $S = 1$ chain compound [Ni$_2$(Medpt)$_2$(µ-ox)(µ-N$_3$)]ClO$_4$·0.5H$_2$O [65].

$N$-leg spin ladders are realized in modifications of high-$T_c$ materials [1]. However, magnetization plateaux are predicted for these systems at fields of (several) thousand Tesla which is clearly outside the presently accessible experimental range. One alternative is provided by organic compounds and indeed magnetization experiments have been successfully performed on the two-leg ladder material Cu$_2$(C$_5$H$_{12}$N$_2$)$_2$Cl$_4$ (see e.g. [52]). Vanadium oxides may in general provide another alternative and e.g. some PBC 3-leg ladder is suspected in the recently synthesized compound Na$_2$V$_3$O$_7$ [68]. However, there is already one experimental observation which can be interpreted in terms of a 3-leg ladder: A plateau-like feature at $\langle M \rangle = 1/3$ has been observed in CsCuCl$_3$ for a magnetic field perpendicular to the crystal axis [69]. Although CsCuCl$_3$ is strictly speaking three-dimensional, this plateau can be understood in the approximation of a weakly anisotropic PBC 3-leg ladder with ferromagnetic rungs [67].

In general, it is intriguing that doping can push magnetization plateaux into the low-field region at least for $p$-merized Hubbard chains [47]. This points towards
the possibility of observing magnetization plateaux in doped systems whose parent compounds would exhibit plateaux only at inaccessibly large fields.

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