Robust Optimization Design for Turbine Blade-Tip Radial Running Clearance using Hierarchically Response Surface Method

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Abstract. Considering the robust optimization computational precision and efficiency for complex mechanical assembly relationship like turbine blade-tip radial running clearance, a hierarchically response surface robust optimization algorithm is proposed. The distribute collaborative response surface method is used to generate assembly system level approximation model of overall parameters and blade-tip clearance, and then a set samples of design parameters and objective response mean and/or standard deviation is generated by using system approximation model and design of experiment method. Finally, a new response surface approximation model is constructed by using those samples, and this approximation model is used for robust optimization process. The analyses results demonstrate the proposed method can dramatic reduce the computational cost and ensure the computational precision. The presented research offers an effective way for the robust optimization design of turbine blade-tip radial running clearance.

1. Introduction

Turbine blade-tip clearance continues to be a concern in the design of gas turbines, because of a reasonable implemented active tip clearance control benefits to increase efficiency, reduce specific fuel consumption, and improve the performance and security of aeroengine [1-3]. The experiments and numerical simulation analysis of BTRRC have been studied [4-6]. In those researches, the effects of dynamic characteristics of mechanisms on turbine blade-tip clearance are detailed analyzed and experimented. Kypuros and Melcher [4] developed a simplified model from first principles that grossly captures the dynamic change in turbine tip clearance during take-off. Much of those work identified the influence of dynamic characteristics to BTRRC by experiments and deterministic analysis which predict initially the radial deformations of disk, blade and casing and then to estimate the BTRRC. To some extent, these efforts possess great blindness in BTRRC design due to scarcely consider the randomness of various impact factors [7]. Considering the influence of parameters uncertainty, Fei and Bai [8] presented distributed collaborative probabilistic design method for BTRRC. Compare with the deterministic analysis method, the probabilistic design method is more reasonable for design turbine blade-tip clearance. However, it is difficult for the probabilistic design to decrease the impacts of parameters variables to blade-tip clearance under operating conditions, so it is necessary to implement the BTRRC robust optimization design so that to decrease the degree of influence of parameters variation to BTRRC.

For complex mechanical assembly structure directly FE (Finite Element) method based robust optimization process has to be confronted with unaffordable efficiency resulting from too large computational load, so several surrogate based robust optimization methods such as Kriging method,
polynomial response surface and artificial neural network have been studied [9-11]. However, the robust design of BTRRC is an inherently multi-component multi-discipline (MOMD) nonlinear dynamic design problem, which is involving multiple objects (turbine disk, blade and casing shroud), multiple disciplines (heat, centrifugal force, mechanical vibration, etc.). Therefore, it is difficult to directly construct a system approximation function model of parameters and blade-tip clearance. Bai and Fei [8, 12] presented a distributed collaborative response surface method (DCRSM) based on quadratic polynomial function for complex mechanical dynamic assembly reliability design, and the result reveals DCRSM has high efficiency and high precision in solving the complex structure assembly relationship probability analysis. The robust optimization objective is minimizing mean and standard deviation of objective response value [13], it means that robust optimization iteration process has to implement thousands of sampling so that to obtain mean and/or standard deviation of objective response. So traditional directly surrogate model based robust optimization process is very time-consuming.

In this paper a hierarchically response surface robust optimization algorithm is proposed for BTRRC robust optimization design. The theory idea and mathematic model of hierarchically response surface method is described in section 2. And then the robust optimization design of HPT BTRRC is to be accomplished. The efforts offer an effective way for the robust design of mechanical dynamic assembly and enrich mechanical robust optimization theory and method.

2. The mathematic model of hierarchically response surface method
In this section, the basic theory of hierarchically response surface robust optimization algorithm has been introduced briefly and the mathematic model of presented method is established.

2.1. Hierarchically response surface robust optimization algorithm.
The distributed collaborative response surface method is divided complex assemblage model into several sub-components model, and then the appropriate response surface method is selected to generate the approximation function model of each sub-component. Finally, the system response surface approximation model is constructed according to the relationship of each sub-model. The advantages of distributed collaborative response surface method are relative to the overall system “big” model, the number of parameters and the nonlinearity between input parameters and output response is greatly reduced of sub-model, so the accuracy can dramatic increase of each sub-structure response surface approximation model.

The key ideal of proposed method is using distributed collaborative response surface method to construct the System assemblage response surface model. A set of new samples of design parameters and robust objective response value is generated by using System response surface model and design of experiment (DOE) method, and then a new response surface model is constructed by using those samples. The basic flowchart of proposed method is shown in figure 1.

In figure 1, the $y(z_i, p_i)$ ($i = 1, 2, \cdots, n$) is sub-model response surface approximation function, $z_i$ and $p_i$ are design parameters and random parameters vector, respectively. $\mu(z)$ and $\sigma(z)$ are approximation function of design variables and objective response mean and standard deviation.

The advantages of proposed method are summarized as follows:

1) Compare with finite element method the distributed collaborative response surface method can dramatic decrease the computation cost to generate the samples of design parameters and robust optimization objective value (mean and standard deviation).

2) The design parameters response surface model is applied to robust optimization process can convert indeterminacy optimization to c deterministic optimization that can save the optimization times.
2.2. The mathematic model of presented method.

For many complex mechanical assembly structures like blade tip clearance, they are implicit and highly nonlinear function relations of input parameters and blade tip clearance. Compared with several flexible approximations model, the Kriging method is considered to be a reliable surrogate method, because that provides an accurate approximation model of a highly nonlinear function [14, 15]. In this work, the Kriging model is selected to construct the distributed collaborative response surface model.

Assuming that a mechanical assemblage comprises \( n \) (\( n \in \mathbb{Z} \)) components and the each component constructs a sub-model via Kriging method [16].

\[
\hat{y}_l(z_0, p_l) = f_l^T(z_0, p_l)\hat{\beta}_l + r_l^T(z_0, p_l)R_l^{-1}(Y_l - F_l\hat{\beta}_l) \tag{1}
\]

where, \( \hat{y} \) is output response, \( z_l = [z_{l1}, z_{l2}, \cdots, z_{lj}] \ j = 1,2,\cdots m \) and \( p_l = [p_{l1}, p_{l2}, \cdots, p_{lk}] \) \( k = 1,2,\cdots q \) are design variables and random variables of each component, respectively.

Let a set of \( n \) known input samples \( X = [x_1, x_2, \cdots, x_n] \) with \( x_i \in \Omega \) and output responses \( Y = [y_1, y_2, \cdots, y_n] \). The \( R \) is the correlation matrix of known samples, \( r(x) \) is the vector of correlations between the sample \( x = (i=1, \cdots, m) \) and sample \( x \):

\[
r(x) = [R(\theta, x_1, x), \cdots, R(\theta, x_m, x)]^T \tag{2}
\]

The Gaussian function provides a relatively smooth and infinitely differentiable surface and defined with only one parameter \( \theta \), [17, 18]. So the Gaussian function is selected for this work and the mathematics as follows:

\[
R(\theta, x_i, x) = e^{-\theta|x_i-x|^2} \text{ where } \theta > 0 \tag{3}
\]

The least-squares estimate of \( \hat{\beta} \) is:

\[
\hat{\beta} = (F^T R^{-1} F)^{-1} F^T R^{-1} Y \tag{4}
\]

The vector \( F \) is generated by estimating \( f(x) \) at the n known observations:

\[
F = [f(x_1), f(x_2), \cdots, f(x_n)]^T \tag{5}
\]
where, \( f(x) \) is the known approximation function. In general \( f(x) \) is assume a constant [19, 20], so we employ a constant term for \( f(x) \) in this paper.

According to each component assembly relationship, the assemblage approximation mathematic model is constructed by using Kriging based distributed collaborative response surface model [8, 12]. The mathematic model as shown in equation (6).

\[
\hat{y}(z, p) = f^T(z, p)\hat{\beta} + r^T(z, p)R^{-1}(Y - F\hat{\beta})
\]

where, \( z = [z_1, z_2, \cdots z_l] \) and \( P = [p_1, p_2, \cdots p_l](i = 1, 2, \cdots, n) \) are design variables vector and random variables vector of mechanical assemblage system.

The robust optimization objective is trying to choose properly levels of the design parameters to ensure structural performance insensitivity to change of the random parameters. It means that the robust optimization process just select a set of design parameters to minimize the mean value \( \mu \) and standard deviation \( \sigma \) of objective response.

So the design parameters \( z \) design space is divided into \( k_i \) levels and the design of experiment method is used to generate a set of sample design point of design parameters. According to the number of parameters and the number of levels of each design parameter, full combination, uniform design or orthogonal array can be selected as the design of experiment method. And then, each experiment the design parameter vector \( z_i(i = 1, \cdots, n) \) and random parameters vector \( p \) generate a set of sample points that as shown in table 1. The each experiment sample point substituting into equation (6) and be implemented thousands sampling by using Monte Carlo simulation. Finally, the objective response mean \( \mu_i \) and standard deviation \( \sigma_i \) of each experiment are calculated by using mathematical statistics method.

Table 1. The sample point of design parameters and random parameters.

| Run number | Design parameters | Random parameters | Mean | Standard deviation |
|-----------|-------------------|-------------------|------|-------------------|
| 1         | \( z_1 \)         | \( p \)           | \( \mu_1 \) | \( \sigma_1 \) |
| 2         | \( z_2 \)         | \( p \)           | \( \mu_2 \) | \( \sigma_2 \) |
| \vdots   | \vdots            | \vdots            | \vdots | \vdots            |
| \( n \)  | \( z_n \)         | \( p \)           | \( \mu_n \) | \( \sigma_n \) |

Once the samples of design parameters and objective response mean \( \mu \) and standard deviation \( \sigma \) is generated. The approximation function model \( \mu(z) \) and \( \sigma(z) \) are constructed by select proper response surface approximation model and the robust optimization can be formulated as an optimization problem as follows:

\[
\text{Find } z \\
\text{Minimizing } \alpha \mu(z) + (1-\alpha) \sigma(z) \\
\text{Subject to } z_L \leq z \leq z_U
\]

where, \( z \) is the design parameters vector, The \( \alpha \) is weighting factor. \( z_L \) and \( z_U \) are the lower-limit and upper-limit of the control parameters. \( \mu(z) \) and \( \sigma(z) \) are the mean value and standard deviation of the objective response, respectively.

3. Robust optimization for BTRRC

3.1. The FE model and parameters study.

The robust optimization design of BTRRC involves three basic components – the casing shroud, turbine disk, and turbine blade [4]. The FE models of three components were established for their radial deformations analysis as shown in figure 2. The casing shroud is simplified as the bushing ring, and the axial cross section of bushing ring is only analyzed to build FE model shown in figure 2(a). The turbine blade model is shown in figure 2(b). The turbine disk structure is modeled shown in figure
2(c), disk mortises are assumed to be negligible and the loads and constraint conditions on disk are axisymmetric.

The input variables of blade and disk are consists of rotor speed $\omega$, gas temperature $t$ and corresponding material parameters are selected [8, 12]. These parameters are assumed to be mutually independent and obey a Gaussian normal distribution. These random variables mean and standard deviation are shown in table 2.

**Table 2.** The variables mean and standard deviation of turbine blade and disk.

| Variables       | Mean (rad/s) | Standard deviation |
|-----------------|--------------|--------------------|
| $\omega$        | 1168         | 15                 |
| $t$ (°C)        | 1050         | 12                 |
| $E_p$ (Gpa)     | 179          | 5                  |
| $K_{st}/(10^{-3} / ^{°}C)$ | 1.549      | 0.0871             |
| $\lambda_b/(w/(m^2 ∙ °C))$ | 26.8       | 0.834              |
| $\alpha_b/(w/(m^2 ∙ °C))$ | 11756      | 352.7              |
| $\rho_b/(kg/m^3)$ | 8500        | 125                |

| Variables       | Mean (rad/s) | Standard deviation |
|-----------------|--------------|--------------------|
| $\omega$        | 1168         | 15                 |
| $t$ (°C)        | 1050         | 12                 |
| $E_p$ (Gpa)     | 184          | 6.4                |
| $K_{st}/(10^{-3} / ^{°}C)$ | 1.507      | 0.0871             |
| $\lambda_b/(w/(m^2 ∙ °C))$ | 20.8       | 0.834              |
| $\alpha_b/(w/(m^2 ∙ °C))$ | 1500       | 45                 |
| $\rho_b/(kg/m^3)$ | 8200        | 123                |

In table 2, the $E$ and $\rho$ are Modulus of Elasticity and material density; $\alpha, K$ and $\lambda$ are surface coefficients of heat transfer, Expansion Coefficients and Thermal Conductivity, respectively. The subscripts $b$ and $d$ are describe the turbine blade and turbine disk.

In this example the casing shroud is hoop-like structure, so the shroud radial deformation due to thermal stresses [4] that due to the temperatures differentials of outer and inner surfaces of the casing shroud. The selected random parameters of casing shroud are shown in table 3 [8, 12].

In table 3, the $h$ is shroud thickness, $t_1$ is outside temperature of shroud, the $\alpha_c$ and $\alpha_c^*$ are inside and outside surface coefficients of heat transfer of casing shroud, respectively. The subscript $c$ denoted the casing shroud.

**Table 3.** The variables mean and standard deviation of casing shroud.

| Variables       | Mean | Standard deviation |
|-----------------|------|--------------------|
| $h/(mm)$        | 3    | 0.01               |
| $t_1/(°C)$      | 1050 | 12                 |
| $t_2/(°C)$      | 320  | 6                  |
| $K_s/(10^{-3} / ^{°}C)$ | 1.8   | 0.39               |
| $\alpha_c/(w/(m² ∙ °C))$ | 6000  | 180                |
| $\alpha_c^*/(w/(m² ∙ °C))$ | 2600  | 78                 |
| $\lambda_c/(w/(m°C))$ | 26.4  | 0.0834             |

3.2. Robust optimization model construct of BTRRC.
The mathematic of BTRRC is described:

\[ \delta = \delta^* - Y \]  \hspace{1cm} (8)

where, \( \delta \) is the BTRRC, \( \delta^* \) is initial assembly blade-tip clearance, respectively. In this example the initial assembly blade-tip clearance is 2.5(mm). \( Y \) is the radial deformations of blade-tip clearance, and that can be calculated by [4]:

\[ Y = Y_1' + Y_2' - Y_3' \]  \hspace{1cm} (9)

where, \( Y_i' \) are the radial deformations of disk, blade and casing, respectively, and those can be obtained using equation (1):

\[ Y_i = \hat{y}_i(z_i, p_i) = f_i^T(z_i, p_i)\hat{\beta}_i + r_i^T(z_i, p_i)R_i^{-1}(Y_i - F_i\hat{\beta}_i) \quad i = 1 \cdots 3 \]  \hspace{1cm} (10)

According to proposed method the radial deformations \( Y \) of blade-tip clearance can be obtained by equation (6):

\[ Y = \hat{y}(z, p) = \begin{bmatrix} 1, 1, -1 \end{bmatrix}[\hat{y}_1(z_1, p_1), \hat{y}_2(z_2, p_2), \hat{y}_3(z_3, p_3)]^T \]  \hspace{1cm} (11)

The BTRRC is given by substituting equation (11) into equation (8):

\[ \delta = \delta^* - \hat{y}(z, p) = \begin{bmatrix} 1, -1, -1 \end{bmatrix}[\delta^*, \hat{y}_1(z_1, p_1), \hat{y}_2(z_2, p_2), \hat{y}_3(z_3, p_3)]^T \]  \hspace{1cm} (12)

According to the statistical characteristics of random variables in table 2 and table 3, the Latin Hypercube sampling (LHS) method is used to generate 150 input samples and corresponding output response is calculated using FE method. The Kriging approximation models are constructed by three components using 130 samples, and the others 20 samples are used to validate the Kriging model estimate accuracy. The result of Kriging model predict is shown in figure 3, and the predict value root mean square deviation (RMSE) of Kriging model can be seen in table 4.

**Figure 3.** The validate samples and Kriging based DCRSM predict value of blade tip clearance.

**Table 4.** The RMSE of Kriging based DCRSM estimate value of blade tip clearance.

| Method              | Validate samples number | RSME       |
|---------------------|-------------------------|------------|
| Kriging based DCRSM | 20                      | 6.5206×10^{-4} |

According to the characteristic of all parameters and the result of sensitivity analysis [8], the rotor speed \( \omega \), gas temperature \( t \), shroud thickness \( h \) and outer temperature \( t_1 \) of shroud are selected as the design parameters, and those design parameters work point design interval is presented in table 5. The low-limit and upper-limit of table 5 are design parameters work point optimization interval.
Table 5. Design parameters optimization interval.

| Design parameter | Low-limit | Upper limit |
|------------------|-----------|-------------|
| $\omega$ (rad/s) | 1100      | 1200        |
| $t_1$ (°C)      | 1000      | 1100        |
| $h$ (mm)        | 2.6       | 3.4         |
| $t_2$ (°C)      | 300       | 350         |

The LHS method is selected to generate 120 design parameters samples in optimization space, and the equation (12) to implement $10^5$ times sampling by using Monte Carlo method in each experiment run so that to calculate the corresponding mean value $\mu$ and standard deviation $\sigma$. Considering the parameters number and robust optimization iterative computing time, the polynomial response surface model is selected to fitting the approximation function model. The fitting result as shown in equation (13) and equation (14).

\begin{align*}
\mu &= 3.6654 + 6.5478 \times 10^{-2} \omega - 0.0029 t_1 - 0.0105 h - 5.3401 \times 10^{-4} t_2 \\
&- 2.1536 \times 10^{-4} \omega t_1 + 1.2911 \times 10^{5} \omega h + 5.6580 \times 10^{-2} \omega t_1 + 1.3244 \times 10^{-2} t_1 h \\
&+ 9.9553 \times 10^{5} t_1 + 1.1673 \times 10^{6} h t_1 - 5.7033 \times 10^{-2} \omega t_1 + 7.3360 \times 10^{-2} t_1^2 \\
&+ 0.0022 h^2 + 2.9914 \times 10^{-2} t_1^2 \\
\sigma &= 0.1352 - 2.4763 \times 10^{-5} \omega - 4.9894 \times 10^{-6} t_1 - 0.0149 h - 3.6864 \times 10^{-4} t_1 \\
&- 1.3180 \times 10^{-4} \omega t_1 + 5.5909 \times 10^{-6} \omega h + 2.3628 \times 10^{-2} \omega t_1 + 7.4843 \times 10^{-4} t_1 h \\
&+ 5.6335 \times 10^{-5} t_1 + 4.8232 \times 10^{-6} h t_1 - 2.1406 \times 10^{-6} \omega h + 2.8055 \times 10^{-6} t_1^2 \\
&- 1.4935 \times 10^{-5} h t_1 + 4.7324 \times 10^{-4} t_1^2
\end{align*}

The 10 samples are selected to validate estimate precision of polynomial response surface approximation model as shown in figure 4. The figure 4 (a) is mean $\mu$ predict value and the figure 4 (b) is standard deviation $\sigma$ predict value. The RSME of estimated mean value $\mu$ and standard deviation $\sigma$ are shown in table 6.

![Figure 4](image_url)

**Figure 4.** The Kriging based DCRSM and polynomial predict result: (a) Mean $\mu$ (b) Standard deviation $\sigma$.

Table 6. The RMSE of polynomial estimate value.

| Objective | Validate samples number | RSME |
|-----------|-------------------------|------|
| $\mu$ (mm) | 10                      | $7.4852 \times 10^{-4}$ |
| $\sigma$ (mm) | 10                      | $4.0975 \times 10^{-4}$ |

The robust optimization of BTRRC can be formulated by substituting equation (13) and equation (14) into equation (7):

\begin{align*}
\text{Find} & \quad z = [\omega, t_1, h, t_2] \\
\text{Minimizing} & \quad q\mu(z) + (1-q) \sigma(z) \\
\text{Subject to} & \quad z_L \leq z \leq z_U
\end{align*}

where, $z$ is the design parameters vector that shown in table 5. The weighting factor $q$ is 0.2. $z_L$ and $z_U$
are the lower-limit and upper-limit of the design parameters that describes in Table 5. $\mu$ and $\alpha$ are the mean value and standard deviation of the objective response, respectively.

3.3. Result and discussion.

In order to discuss the effectiveness and precision of the presented method, the Kriging based DCRSM robust optimization algorithm is applied, and the numerical results are compared with the presented method. The Fruit Fly Optimization Algorithm is selected to simulation the robust optimization process in this article.

The optimization results of is shown in table 7. The “Error” means relative error of the proposed method estimated solution for the solution by directly using Kriging based DCRSM robust optimization method.

Table 7. Robust optimization result of Kriging based DCRSM and proposed method.

| Methods            | $\omega$(rad/s) | $t$(°C) | $h$(mm) | $t_i$(°C) | $\mu$(mm) | $\sigma$(mm) | Simulation times(s) |
|---------------------|-----------------|---------|---------|-----------|------------|--------------|--------------------|
| Kriging based DCRSM | 1190.5          | 1043    | 3.1081  | 308       | 0.5211     | 0.0814       | 1785               |
| Proposed method     | 1199.5          | 1035    | 3.105   | 305       | 0.5236     | 0.0813       | 2.89               |
| Error               | 0.76%           | 0.77%   | 0.1%    | 0.97%     | 0.48%      | 0.12%        |                    |

As demonstrated by table 7, the result of optimized reveals that maximum relative error is 0.97%. This result illustrates the proposed method has higher calculate accuracy.

From table 7, the simulation time (2.89 s) of proposed method is far less than that (1785 s) of Kriging based DCRSM because of the polynomial can directly calculate the mean value $\mu$ and standard deviation $\sigma$ not need implement 5000 times sampling to calculate the mean and standard deviation of objective response in each optimum iterative procedure. The results demonstrate that the proposed hierarchically response surface method is able to further improve computational efficiency for complex mechanical dynamic assembly relation.

4. Conclusion

This article presented a hierarchically response surface method based robust optimization algorithm for BTRRC robust optimization design. The Kriging based DCRSM is used to construct the system approximation model of overall parameters and blade tip clearance, and then a set of samples of design parameters and robust optimization objective (mean and /or standard deviation) is generated by using this system surrogate model and design of experiment method. Finally, the polynomial response surface is selected to construct the approximation model of design parameters and robust optimization objective (mean and /or standard deviation) by using those samples, and polynomial approximation model is used for robust optimization process.

By comparing the results with Kriging based DCRSM, the precision and calculation cost were discussed. Through the presentation of the numerical result, we can conclude that the presented method has strong advantages with high calculating efficiency and accuracy in complex mechanical assembly relation robust optimization. It means that the proposed method is suitable for industrial applications.

This paper provides a novel robust optimization idea and approach for the design of reasonable mechanical assembly relationship, as well as develops the theory of mechanical assembly relation robust optimization design.

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