Decoupling of Massive Right-handed Neutrinos

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Abstract

We investigate the effect of $B+L$-violating anomalous generation of massive right-handed neutrinos on their decoupling, when the right-handed neutrino mass is considerably greater than the right-handed gauge boson masses. Considering normal annihilation channels, the Lee-Weinberg type of calculation, in this case, gives an upper bound of about 700 Gev, which casts doubt on the existence of such a right-handed neutrino mass greater than right-handed gauge boson masses. We examine the possibility that a consideration of anomalous effects related to the $SU(2)_R$ gauge group may turn this into a lower bound $\sim 10^2$ Tev.

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I. INTRODUCTION

Neutrino oscillation interpretation of recent observations of solar and atmospheric neutrino fluxes, although presenting some inconsistencies, may be taken to strengthen the idea of non-zero neutrino masses. In this situation, in addition to the standard model left-handed neutrinos, the existence and masses of right-handed neutrinos assume topical interest.

The contribution of massive neutrinos to the mass-density of the universe...
allows the setting of a lower bound to such a neutrino mass from the usual cosmological constraints on the age and mass-density of the universe [1,2,3]. The standard calculations consider a neutrino mass less than gauge boson masses. In the present paper, working in a L-R symmetric extension of the standard model [4,5,6], we investigate how the nature of the bound is altered when the right-handed neutrino mass is greater than gauge boson masses.

In these L-R symmetric models, the breaking of $SU(2)_R$ gauge symmetry is associated with a critical temperature. This may, typically, be of the order of 1-10 Tev [7,8,9], and right-handed electron neutrino masses $\approx 10$ Tev have been considered, yielding a left-handed electron neutrino mass $\approx 10^{-10}$ Gev, by a see-saw mechanism [9]. Now, B+L is not conserved in standard electroweak theory due to an anomaly involving the SU(2) gauge group [10], and, at temperatures $\geq 1$ Tev, B+L-violating transitions occur classically, via thermal fluctuations, at rates higher than the expansion rate of the universe [11]. So, we may expect that similar anomalous generation of right-handed neutrinos (in addition to the left-handed ones), via the L-R symmetric gauge group, may become important near the $SU(2)_R$-breaking critical temperature.

Although there is still a lot of fluidity in the matter, particle physics and cosmological bounds usually suggest right-handed $Z_R$ and $W_R$ boson masses with values $\geq 0.5$ TeV and 1.6-3.2 TeV, respectively [12]. If, now, right-handed neutrinos of mass $\geq 10$ TeV come under consideration in the literature, then it becomes necessary to investigate whether anomalous effects can, indeed, modify significantly the decoupling of right-handed neutrinos with mass greater than right-handed gauge boson masses.

The plan of the paper is as follows. Section I is the Introduction. In Section II, the L-R symmetric model is used to evaluate the reduction rate of the right-handed neutrinos, in a standard Lee-Weinberg type of calculation,
and to observe how the cosmological bound on their mass becomes an upper one, when this mass is greater than the right-handed gauge boson masses. In Section III, the anomalous rate of reduction of right-handed neutrinos is related to the general anomalous rate of B+L-violating transitions, and the qualitative effect of the anomalous rate on the previously obtained mass bound is estimated, assuming a generic form for the B+L-violating rate arising from the anomaly involving the $SU(2)_R$ gauge group. In Section IV, the influence of these anomalous effects on the mass bound is studied numerically, using numbers obtained by a simple extrapolation from the $SU(2)_L$ result.

II. DECOUPLING WITHOUT ANOMALOUS EFFECTS

A. Boltzmann equation for processes $N\bar{N} \rightarrow F\bar{F}$.

We wish to set up a Boltzmann equation for the number density of right-handed neutrinos, and, from a calculation of the asymptotic number density, estimate the contribution of these neutrinos to the mass-density of the universe, and, hence, set bounds to the right-handed neutrino mass [1,3].

We will simplify matters by neglecting the decay of right-handed neutrinos. Then, the normal electro-weak process chiefly responsible for the reduction of right-handed neutrinos may be written as $N\bar{N} \rightarrow F\bar{F}$, where F is a quark or a lepton, lighter than N.

We are interested in investigating the situation when the right-handed neutrino mass is considerably greater than the right-handed gauge boson masses.

To calculate the rate of reduction of right-handed neutrinos we consider the L-R symmetric model [5,6]. This model has pairs of fermion doublets $f'$ belonging to different representations of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, like

\[
\begin{pmatrix}
\nu_L \\
e_L
\end{pmatrix}
\begin{pmatrix}
\frac{1}{2}, 0, -1
\end{pmatrix},
\begin{pmatrix}
\nu_R \\
e_R
\end{pmatrix}
\begin{pmatrix}
0, \frac{1}{2}, -1
\end{pmatrix};
\]
\[ \begin{pmatrix} u_L \\ d_L \end{pmatrix} (1/2, 0, 1/3), \begin{pmatrix} u_R \\ d_R \end{pmatrix} (0, 1/2, 1/3). \]

The numbers refer to the quantum numbers \( T_{3L}, T_{3R}, B - L \), respectively. We will also write \( \nu_R \equiv N, \nu_L \equiv \nu \).

The fermion gauge-boson interaction Lagrangian is

\[ L_{\text{int}} = g (f' \gamma_\mu P_L T_L f', \bar{W}_L \gamma_\mu + \bar{f}' \gamma_\mu P_R T_R f', \bar{W}_R \gamma_\mu + \frac{1}{2} g' \bar{f}' \gamma_\mu (B - L) f' B^\mu, \]

where \( P_{L,R} = \frac{1}{2} (1 \pm \gamma_5) \), \( \bar{T} \) is the isospin operator and \( \bar{W}_\mu, B^\mu \) are the gauge bosons. The neutral currents are set out in the basis

\[ A^\mu = \sin \theta (W_{3L}^\mu + W_{3R}^\mu) + \sqrt{\cos 2 \theta} B^\mu, \]

\[ Z^\mu = \cos \theta W_{3L}^\mu - \sin \theta \tan \theta \sin W_{3R}^\mu - \tan \theta \sqrt{\cos 2 \theta} B^\mu, \]

\[ Z'^\mu = \frac{\sqrt{\cos 2 \theta}}{\cos \theta} W_{3R}^\mu - \tan \theta B^\mu, \]

where \( \theta \) is the Weinberg angle.

Neglecting \( Z-Z' \) mixing, one gets the \( Z' \) neutral current Lagrangian [12]

\[ L_{\text{Z'^NC}} = g \frac{\sin^2 \theta}{\cos \theta} \sum_{f'} \bar{f}' \gamma_\mu [P_L T_3] - Q \sin^2 \theta] f' + \cos^2 \theta \sum_{f'} \bar{f}' \gamma_\mu [P_R T_3] - Q \sin^2 \theta] f' Z'_\mu. \quad (1) \]

Q is the charge operator.

The charged current Lagrangian consists of terms of the form

\[ L_{\text{CCN}} = \frac{g}{\sqrt{2}} (\bar{\nu} \gamma_\mu e_L W^\mu L + h.c.) + \frac{g}{\sqrt{2}} (\bar{N} \gamma_\mu e_R W^\mu R + h.c.). \quad (2) \]

Assuming CP-symmetry, and equilibrium conditions for all relevant particles except the N neutrinos, the rate of reduction of N neutrinos per unit volume can be obtained from the Boltzmann collision integral for the processes \( N \bar{N} \rightarrow F \bar{F} \) [13,14]:

\[ \Gamma_a = \sum_{F} \int d\pi_N d\pi_{\bar{N}} d\pi_F d\pi_{\bar{F}} (2\pi)^4 \delta^4 (p_N + p_{\bar{N}} - p_F - p_{\bar{F}}) \]

\[ |\mathcal{M}|^2 (f_N f_{\bar{N}} - f_{\bar{N}} f_{\bar{N}}). \quad (3) \]
Here, \( f \) is the phase space distribution function and \( f_{eq} \) is its equilibrium value. \(|\mathcal{M}_F|^2\) is the spin averaged matrix element squared, with proper symmetry factor, for the process \( N\bar{N} \rightarrow F\bar{F} \), assumed, by CP-symmetry, to be the same as that for the process \( F\bar{F} \rightarrow N\bar{N} \). The measure \( d\pi_i = g_i d^3p/(2\pi)^32E \), \( g_i \) being the degeneracy number. We assume that there is no significant Fermi degeneracy, so that \( 1 - f \approx 1 \).

Because CP-symmetry has been assumed, we further assume that there is no \( N \) or \( \bar{N} \) excess, and we can set \( n = \bar{n} \), as well as \( \mu_N = 0 = \mu_{\bar{N}} \). We can, then, take

\[
f_{Neq} = e^{-E_N/T}.
\]

The summation is over quarks and leptons lighter than \( N \). Let us take \( \nu \) and \( N \) to be electron neutrinos. We assume right-handed neutrinos of the other two generations to be much more massive than the \( N \) neutrinos, so that they are not relevant here.

It is usual to introduce the thermal average of the annihilation cross-section times relative velocity

\[
<\sigma|v> = \frac{1}{n_{eq}^2} \sum_F \int d\pi_N d\pi_{\bar{F}} d\pi_F d\pi_{\bar{F}} (2\pi)^4\delta^4(p_N + p_{\bar{F}} - p_F - p_{\bar{F}}) |\mathcal{M}_F|^2 e^{-E_N/T} e^{-E_{\bar{F}}/T},
\]

and write (3) in the form [1]

\[
\Gamma_a = <\sigma|v>(n^2 - n_{eq}^2),
\]

where \( n \) is the number density of the \( N \) neutrinos and \( n_{eq} \) is its equilibrium value.

Then the Boltzmann equation for the reduction of \( N \) neutrinos by these processes, in a universe expanding with \( \dot{R}/R = H \), becomes [14]

\[
\frac{dn}{dt} + 3Hn = - <\sigma|v>(n^2 - n_{eq}^2). \tag{4}
\]
B. Calculation of $< \sigma |v| >$ from L-R symmetric model.

The Feynman diagram for $N \bar{N} \rightarrow Z' F \bar{F}$, from (1), is given in Fig. 1. For the $N \bar{N} \rightarrow e \bar{e}$ amplitude, we get, using (2), an additional contribution from the diagram shown in Fig. 2.

We will work at temperatures $T < T_{CR}$, the critical temperature corresponding to the breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ to $SU(2)_L \times U(1)_Y$.

We are going to consider N-type neutrinos with a mass $M$, which is at least an order or two of magnitude larger than $M_{Z'} (M_{Z'} \geq 0.5$ Tev [12]). At this energy scale, we will approximate all quark and the $e, \mu, \tau, \nu$ masses to zero (mass of top $\approx 175$ Gev).

Next, we assume that $\nu$ and $N$ have Majorana mass eigenstates [15]

$$\chi = \nu + \nu^c, \quad \omega = N + N^c,$$

where the superscript ”$c$” refers to the charge conjugate field.

It is usual to consider a bidoublet and two triplet Higgs particles to generate Majorana states [6]. In this paper, however, we do not go into the details of any specific model of the Higgs sector. While evaluating the matrix element,
we have considered N to be purely Majorana, i.e. we have neglected the contribution of the vector current and doubled that of the axial current by replacing \((1 + \gamma_5)\) with \(2\gamma_5\) [12].

The spin-averaged matrix element squared, with symmetry factor \(\frac{1}{2!}\), for the first diagram gives, from (1),

\[
|M_F|^2 = \frac{1}{2}(g^4/(2\cos^2\theta))(C_{VF}^2 + C_{AF}^2)\left[\frac{1}{(q^2 - M_{Z'}^2)^2}\right],
\]

where,

\[
C_{VF} = T_3 - 2Q \sin^2\theta, \quad C_{AF} = T_3 \cos^2\theta.
\]

Now,

\[
q^2 = (k + \bar{k})^2 = s = 4E_{CM}^2
\]

\[
= 4(M^2 + k_{CM}^2)
\]

\[
>> M_{Z'}^2,
\]

where \((E_{CM}, k_{CM})\) is the 4-vector k in the CM frame. So, we approximate \(1/(q^2 - M_{Z'}^2)^2\) by \(1/q^4\).

We calculate \(<\sigma|v|>_\text{CM}\) in two steps. First, we calculate

\[
I_F = \int d\pi_F d\pi_{\bar{F}}(2\pi)^4 \delta^4(k + \bar{k} - p - \bar{p})|M_F|^2
\]

in the CM frame. The result is

\[
I_F = [g^4/(64\pi\cos^22\theta)](C_{VF}^2 + C_{AF}^2)(2/3)\beta^2,
\]

where \(\beta\) is the relative velocity = \(2|k_{CM}|/E_{CM}\). This "p-wave" term is a signature of Majorana neutrino annihilation.

In the Lee-Weinberg type of decoupling calculation, the N neutrinos may be considered to be non-relativistic, as the relevant temperatures are of the order
of M. Then, in the comoving "lab" frame, where \( \tilde{k} \) makes an angle \( \alpha \) with \( k \),

\[
I_F = \left[ g^4/(64\pi \cos^2 2\theta) \right] (C_{V_F}^2 + C_{AF}^2) (2/3)(k^2 + \tilde{k}^2 - 2|k||\tilde{k}| \cos \alpha)/M^2. \tag{5}
\]

In the second step we do the thermal averaging. Then,

\[
< \sigma|v| >_{F} = \frac{\int d\pi N e^{-E_N/T} \int d\pi \bar{N} e^{-E_{\bar{N}}/T} I_F}{\int g_N d^3 k (2\pi)^3 e^{-E_N/T} \int g_{\bar{N}} d^3 \bar{k} (2\pi)^3 e^{-E_{\bar{N}}/T}}.
\]

Calculation, in the non-relativistic approximation for the \( N \) neutrinos, gives

\[
< \sigma|v| >_{F} = \frac{g^4}{(64\pi \cos^2 2\theta)} (C_{V_F}^2 + C_{AF}^2) \frac{1}{M^2} \frac{T}{M}. \tag{6}
\]

In the case of \( N\bar{N} \to e\bar{e} \), the effect of the extra diagram can be taken into consideration by the usual Fierz rearrangement, which gives, in this case,

\[
C_{V_e}/\cos 2\theta \to (C_{V_e}/\cos 2\theta) + 1,
\]

\[
C_{Ae}/\cos 2\theta \to (C_{Ae}/\cos 2\theta) + 1.
\]

Finally, we get

\[
< \sigma|v| > = \frac{g^4}{(64\pi \cos^2 2\theta)} \frac{1}{M^2} \frac{T}{M} \sum_F (C_{V_F}^2 + C_{AF}^2). \tag{6}
\]

So, effectively, \( < \sigma|v| > \sim \frac{1}{M^2} \), as \( T \sim M \).

**C. Mass Bound for Right-handed Neutrinos.**

Introducing \( x = M/T \) and \( Y = n/s \), where \( s \) is the entropy density, (4) becomes

\[
(1.66g^* \sqrt{x}/x^4)(M^5/M_{Pl}) \frac{2\pi^2}{45} g_* \frac{dy}{dx} = \left[ \frac{2\pi^2}{45} g_*^2 \frac{M^6}{x^6} \right] < \sigma|v| > (Y^2 - Y_{eq}^2). \tag{7}
\]

or,

\[
dY = -0.26g^* \sqrt{x} < \sigma|v| > (M M_{Pl}/x^2)(Y^2 - Y_{eq}^2). \tag{8}
\]

We take \( M_{Pl} = 1.22 \times 10^{19} \) Gev, and \( g^* \approx g_*^* \approx 100 \) just below the critical temperature, considering \( N, W_R, Z' \) (and \( N_\mu, N_\tau \)) to be massive (we have not counted Higgs degrees of freedom).
Summing over all quarks and leptons, except the three right-handed neutrinos, we get, on calculation,

$$\sum_F (C_{V,F}^2 + C_{A,F}^2) = 3.28$$

(taking $\sin^2 \theta = 0.23$).

Taking $g=0.65$, $\langle \sigma |v| \rangle = 0.01/(M^2 x)$.

For massive Majorana neutrinos, we get, in the non-relativistic approximation [14],

$$Y_{eq} = 2.89 \times 10^{-3} x^3 e^{-x}. \quad (9)$$

From (8),

$$\frac{dY}{dx} = -(3.16 \times 10^{17}/M x^3)(Y^2 - Y_{eq}^2). \quad (10)$$

We write $\Delta = Y - Y_{eq}$.

Then, before decoupling, $Y \approx Y_{eq}$, and $\Delta' \sim 0$, giving

$$\Delta \approx -M x^3 Y_{eq}'/[3.16 \times 10^{17}(2Y_{eq} + \Delta)]$$

Now, we put $\Delta = cY_{eq}$ at decoupling, where $c \sim 1$. According to the general numerical analysis of this type of decoupling [14], $c(c + 2) = 2$ when $\langle \sigma |v| \rangle \sim T$.

At decoupling, when $x = x_d > 1$, $Y_{eq}' \approx -Y_{eq}$, and

$$\Delta(x_d) \approx cY_{eq}(x_d) = M x_d^3/[3.16 \times 10^{17}(c + 2)]. \quad (11)$$

This leads to

$$x_d \approx 35.14 - \ln M - 1.5 \ln(35.14 - \ln M). \quad (12)$$

After decoupling, $Y \gg Y_{eq}$ and $\Delta \approx Y$.

From (10), we get

$$\Delta' = -3.16 \times 10^{17} \Delta^2/(M x^3),$$

which gives, on integration, at $t \to \infty$,

$$\Delta_\infty = Y_\infty = 2M x_d^2/(3.16 \times 10^{17}),$$

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assuming, \( Y(x_d) \gg Y_\infty \).

We will take as our cosmological bound [14]:

\[
\Omega_N h^2 < 1, \text{ where } \Omega_N = \rho_N / \rho_c = M s_0 Y_\infty / \rho_c.
\]

Here, it is assumed that \( h > 0.4 \).

Taking \( s_0 = 2970 \text{ cm}^{-3} \) and \( \rho_c = h^2 1.88 \times 10^{-29} \text{ g cm}^{-3} \), we get

\[
\Omega_N h^2
\]

\[
= 2.80851 \times 10^8 M Y_\infty \quad \text{(13)}
\]

\[
= 3.62 \times 10^{-9} M^2 x_d^2, \quad \text{(13a)}
\]

where \( M \) is to be taken in Gev.

At the bound,

\[
3.62 \times 10^{-9} M^2 x_d^2 = 1.
\]

Solving this equation and (12) simultaneously, using simple numerical methods, we get

\[
x_d = 23.55, \quad M = 706 \text{ Gev}.
\]

Now, if we omit \( \ln M \) in the third term on the R.H.S. of (12), we get, approximately,

\[
x_d = 29.80 - \ln M. \quad \text{(14)}
\]

If we make this approximation, the error in \( x_d \) is less than 5 percent, even if \( M \) is as large as \( 10^6 \) Gev. Using (14), we get

\[
d(\Omega_N h^2)/dM = 3.62 \times 10^{-9} \times 2M(29.80 - \ln M)(28.80 - \ln M),
\]

which is positive for all practical purposes.

This means that \( \Omega_N h^2 < 1 \) fixes an upper bound for \( M \).

This can be seen transparently if we work in the approximation

\[
Y_\infty \approx Y(x_d) \approx 2Y_{eq}(x_d),
\]
Table 1

| M(Gev) | $\Omega_N h^2$ |
|--------|----------------|
| 10,000 | 160            |
| 5,000  | 42.6           |
| 1,000  | 1.95           |
| 750    | 1.12           |
| 706    | 1.00           |
| 500    | 0.516          |
| 250    | 0.136          |

taking $c \approx 1$ in (11).

Then,

$$\Omega_N h^2 \sim M x_d^2 e^{-x_d}.$$  

(14) shows that $\Omega_N h^2 \sim M^2 (29.80 - \ln M)^{\frac{3}{2}}$, and, so, as $M$ increases, $\Omega_N h^2$ increases, for all practical values of $M$.

This conclusion can be verified, numerically, by giving $M$ different values in (12) and substituting the resulting $x_d$ in (13a). The results are shown in Table 1.

In the usual Lee-Weinberg case, with $M << \text{gauge boson masses}$, one gets a lower bound because $<\sigma|v|> \sim M^2$, which leads to $x_d \sim 3 \ln M + \text{constant}$, and $\Omega_N h^2 \sim \frac{1}{M^2}$. With $M >> \text{gauge boson masses}$, $<\sigma|v|> \sim \frac{1}{M^2}$, and this makes the difference.

$M < 706 \text{ Gev}$ is, in effect, incompatible with our assumption of $M > \text{right-handed gauge boson masses}$, because, as we have remarked earlier, particle physics and cosmological bounds suggest right-handed gauge boson masses $\sim 0.5 - 1 \text{ Tev}$ or more. We have to conclude that the assignment of any real-
istic mass, greater than right-handed gauge boson masses, to the right-handed neutrinos will violate the cosmological bound $\Omega_N h^2 < 1$.

However, we have considered only the normal electroweak process $N \bar{N} \rightarrow F \bar{F}$. In the next section, we consider, in addition, anomalous processes.

III. INTRODUCTION OF ANOMALOUS EFFECTS.

A. Anomalous generation of right-handed neutrinos.

For the standard model, a classical, unstable, time-independent solution of the equations of motion has been identified [16,17]. This sphaleron solution corresponds to the barrier between vacua with different topological numbers. A sphaleron-mediated transition over the barrier leads to a fermion-number violating transition with $|\Delta L| = 3, |\Delta B| = 3$, of the type

$$|W^{cl}_\mu \alpha > \rightarrow |W^{cl}_\mu \alpha' >,$$

where $\alpha, \alpha'$ are fermion states, differing by $|\Delta L| = 3, |\Delta B| = 3$, and $W^{cl}_\mu, W^{cl'}_\mu$ are the initial and final SU(2) gauge boson configurations, which are essentially classical. (We are neglecting the small effect of the U(1) part [17].)

All colours and families of quarks and leptons will be generated equally, but, in any one transition, only one member per doublet will be found. For the rest of this paper, we will neglect family mixing and consider anomalous generation for a single (the lightest) family. In this case, $|\Delta L| = 1, |\Delta B| = 1$. $\alpha, \alpha'$ will be restricted by the requirement that the sphaleron must be a colour singlet, SU(2) singlet and neutral mediator. There are then two relevant amplitudes, which we may write formally as

$$< W^{cl}_\mu uud eW^{cl'}_\mu > \text{ and } < W^{cl}_\mu udd \nu W^{cl'}_\mu >.$$

All processes with these amplitudes can occur. For example, $\alpha$ may be the vacuum and $\alpha'$ may represent uude or udd$\nu$. 

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In the L-R symmetric model, we expect, on general grounds [16,11,9], anomalous B+L generation above or just below $T_{CR}$, from both $SU(2)_L$ and $SU(2)_R$ gauge boson field configurations with non-vanishing topological charge.

However, the actual construction of the sphaleron solution depends on the details of the Higgs multiplet. The $SU(2)_L$ sphaleron [17] was worked out with a complex doublet. In the L-R symmetric case, the generation of Majorana masses at the higher energy scale results from spontaneous symmetry-breaking associated with a $SU(2)_R$ triplet scalar field (in addition to a $SU(2)_L$ triplet and a bidoublet which develop v.e.v. at the lower energy scale) [6]. It has been shown [9] that the topological condition necessary for the existence of a sphaleron solution is fulfilled for a simplified model of $SU(2)_R$ symmetry-breaking at the higher energy scale via a triplet complex scalar field. But, the construction of an explicit solution has proved very difficult.

In this situation, one has to assume [9,18] the occurrence of B+L violation via sphalerons for the $SU(2)_R$ gauge group, in addition to B+L violation for $SU(2)_L$ at the higher energy scale. Neglecting mixing parameters between left-handed and right-handed gauge bosons, we work with a highly simplified model in which the $W^\mu_L$ give rise to anomalous generation of leptons and baryons from left-handed doublets, and the $W^\mu_R$ from right-handed doublets. In particular, the $W^\mu_R$ will generate, anomalously, right-handed N neutrinos.

First, we want to relate the rate of production of the right-handed N neutrinos per unit volume to the total rate $\Gamma$ of $\Delta B = 1, \Delta L = 1$ anomalous transitions per unit volume. We divide $\Delta L = +1$ processes into four types (assuming distinct flavour eigenstates for N and $\bar{N}$):

- $l$: processes with a N in the final state, e.g.,

$$|W^\mu_{\mu R} vac > \rightarrow |W'^{\mu}_{\mu R} uddN >,$$
\( \bar{l} \): processes with a \( \bar{N} \) in the initial state, e.g.

\[ |W_{\mu R}^{\bar{e}d} \bar{N} \bar{u} \bar{d} > \rightarrow |W_{\mu R}^{e'd} \text{vac} >, \]

\( m \): processes with an \( e^- \) in the final state, e.g.

\[ |W_{\mu R}^{\bar{e}d} \text{vac} > \rightarrow |W_{\mu R}^{e'd} u u d^- >, \]

\( \bar{m} \): processes with an \( e^+ \) in the initial state, e.g.

\[ |W_{\mu R}^{\bar{e}d} \bar{u} \bar{d} e^+ > \rightarrow |W_{\mu R}^{e'd} \text{vac} >. \]

Therefore,

\[ \Gamma = \sum_l \Gamma_l + \sum_{\bar{l}} \Gamma_{\bar{l}} + \sum_m \Gamma_m + \sum_{\bar{m}} \Gamma_{\bar{m}}, \]

where \( \sum_i \) is a sum over all processes of type \( i \).

Each process has a rate which is determined in an essentially classical way: if the thermal fluctuation has sufficient energy to cross the barrier, the process will occur. If \( i \omega^- \) is the frequency of the unstable (sphaleron) mode, a classical statistical mechanics calculation gives \([19,20]\)

\[ \Gamma_i = (\omega^-/\pi)(\text{Im}\mathcal{F}/T), \quad (15) \]

where \( \mathcal{F} \) is the free energy. Also,

\[ (\text{Im}\mathcal{F}/T) \sim e^{-V_0/T}, \quad (16) \]

where \( V_0 \) is the barrier height.

Because of this essentially classical nature, the barrier-crossing rate, at a given temperature, under equilibrium conditions, should be of the same order in different channels. In other words, we may expect that the rate of \( \Delta L = 1, \Delta B = 1 \) transitions, featuring one member of a lepton doublet, will be of the same order as the rate of such transitions, featuring the other member of the doublet. As a first approximation, we may take

\[ \sum_l \Gamma_l + \sum_{\bar{l}} \Gamma_{\bar{l}} \approx \sum_m \Gamma_m + \sum_{\bar{m}} \Gamma_{\bar{m}} \approx \frac{1}{2} \Gamma. \quad (17) \]
The approximation will be bad when the N neutrinos are way out of equilibrium. In a decoupling study, however, one is interested in finding out when the species just falls out of equilibrium.

Let us now interpret \( l, \bar{l} \) formally as Boltzmann collisional processes

\[
\begin{align*}
  l : i + j + \ldots & \rightarrow N + a + b + \ldots \\
  \bar{l} : \bar{N} + \bar{a} + \bar{b} + \ldots & \rightarrow \bar{i} + \bar{j} + \ldots
\end{align*}
\]

CPT ensures that for every process of type \( l \), there is a process of type \( \bar{l} \) with the same matrix element \( \mathcal{M}_l \). Then, we can write, formally, [13,14]

\[
\Gamma_l = \int d\pi_N d\pi_a d\pi_b \ldots d\pi_i d\pi_j |\mathcal{M}_l|^2 \\
(2\pi)^4 \delta^4 (p_N + p_a + p_b - p_i - p_j) f_N f_a f_b \ldots \quad (18)
\]

We have again assumed that all relevant species are in equilibrium except the right-handed neutrinos, and that there is no significant Fermi degeneracy or Bose condensation. Also,

\[
\Gamma_{\bar{l}} = \int d\pi_{\bar{N}} d\pi_{\bar{a}} d\pi_{\bar{b}} \ldots d\pi_{\bar{i}} d\pi_{\bar{j}} |\mathcal{M}_{\bar{l}}|^2 \\
(2\pi)^4 \delta^4 (p_{\bar{N}} + p_{\bar{a}} + p_{\bar{b}} - p_{\bar{i}} - p_{\bar{j}}) f_{\bar{N}} f_{\bar{a}} f_{\bar{b}} \ldots \quad (19)
\]

In these formal expressions, \( |\mathcal{M}_l|^2 \) is related to the classical probability and is not to be interpreted perturbatively.

As we are interested here in decoupling and not in baryogenesis, we will neglect small CP-asymmetric effects and assume CP-symmetry. Then, we can assume, as in Sec.II.A.,

\[
n = \bar{n}, \quad n_{eq} = \bar{n}_{eq}; \quad \text{also,} \quad f_a = e^{-\frac{E_a}{T}}, \quad f_{\bar{a}} = e^{-\frac{E_{\bar{a}}}{T}} \quad \text{etc.} \quad (20)
\]

In this case, we can write, from (18) and (19),

\[
\Gamma_l = I_l n_{eq}, \quad \text{and} \quad \Gamma_{\bar{l}} = I_l n, \quad (21)
\]
where \( I_l \) contains the result of the phase space integrations, apart from \( n_{eq} \) and \( n \) \([14,21]\). \( I_l = I_\bar{l} \) due to (20). It can be interpreted as a thermally averaged width in mode \( l \).

From (17) and (21),
\[
\sum_l I_l (n + n_{eq}) = \frac{1}{2} \Gamma, \quad \text{and}
\]
\[
\sum_l \Gamma_l = \frac{n_{eq}}{2(n + n_{eq})} \Gamma. \tag{22}
\]
For \( \Delta L = -1 \) anomalous transitions, we will get, similarly, a process \( l' \) with a \( \bar{N} \) in the final state, and a process \( \bar{l}' \) with a \( N \) in the initial state, and a similar result
\[
\sum_{l'} \Gamma_{\bar{l}'} = \frac{n}{2(n + n_{eq})} \Gamma'. \tag{23}
\]
Now, we are neglecting baryon- and lepton-number excess or deficit. In this approximation, we can set \( \mu_N = 0 \). We can, then, take \([20,22]\) \( \Gamma = \Gamma' \), i.e. the rate of \( \Delta L = 1 \) transitions \( \approx \) the rate of \( \Delta L = -1 \) transitions.

Hence, the net rate of reduction of \( N \) neutrinos by anomalous processes, per unit volume, \( \Gamma_N = \) rate of such processes, per unit volume, with a \( N \) in the initial state \( \) – rate of such processes, per unit volume, with a \( N \) in the final state
\[
= \sum_{l'} \Gamma_{\bar{l}'} - \sum_l \Gamma_l.
\]
We finally get
\[
\Gamma_N = \frac{n - n_{eq}}{2(n + n_{eq})} \Gamma. \tag{24}
\]
As expected, the anomalous rate vanishes if the \( N \) neutrinos are in equilibrium.

Since, we have Majorana mass states, lepton number violating processes with \( \Delta L = 2 \) can arise from \( L \)-violating terms in the Lagrangian. A variety of such processes has been considered in the literature \([9,18,23]\). These theories are, broadly, of two types.

In one type of theory, a massive right-handed Majorana neutrino is allowed to decay. As we are neglecting decay, we have not considered such theories.
In the other type of theory, the L-violating term is, effectively, of the type \((m_\nu/v^2)\nu\nu HH\), where \(H\) is a Higgs scalar, \(m_\nu\) is the \(\nu\) mass, and \(v\) is the Higgs v.e.v. Usually, in these theories, one takes, \(v^2/m_\nu = M\). If we consider a similar term \((1/M)\nu\nu HH\), we will find a cross-section \(\sim (1/M^2)\). So, such terms will not give processes significantly faster than \(N\bar{N} \rightarrow F\bar{F}\), with \(M >> g\)auge boson masses, and may be left out in an exercise where the emphasis is on qualitative features of the decoupling.

Assuming, therefore, CP-symmetry and equilibrium conditions for all relevant particles except right-handed neutrinos, the rate of reduction, per unit volume, of the \(N\) neutrinos can be written in the form of a Boltzmann equation

\[
\frac{dn}{dt} + 3Hn = -\frac{(n - n_{eq})}{2(n + n_{eq})} \Gamma_R - <\sigma|v|>(n^2 - n_{eq}^2) \quad (25)
\]

This is our basic equation.

We have put \(\Gamma = \Gamma_R\) to indicate that we are considering the anomalous rate for right-handed neutrinos.

**B. Anomalous effects in decoupling.**

For \(T < T_{CL}\), the critical temperature for the spontaneous breakdown of \(SU(2)_L \times U(1)_Y\), \(\Gamma = \Gamma_L\) has been calculated [20,24]. For the right-handed case, with \(T < T_{CR}\), the complication of the Higgs sector has obstructed a calculation of \(\Gamma_R\). However, the very general considerations mentioned in Sec.III.A. imply that \(\Gamma \sim e^{-V_0/T}\).

\(V_0\) can be estimated heuristically, as follows [25,26]. If we assume a sphaleron solution with energy \(E_{sp}\), we can put \(V_0 = E_{sp}\). But \(E_{sp}\) arises from a classical solution, i.e. from a limit where many quanta are involved. We can take the energy per quantum \(\sim M_W\), and the average number of quanta \(\sim (1/\alpha_W)\), where \(\alpha_W = g^2/4\pi\). Then, \(E_{sp} \sim \frac{M_W}{\alpha_W}\) is a very general estimate, which should hold for the right-handed case also, with \(M_W = M_WR\). So, we can write, on
general grounds,

\[ \Gamma_R = \tilde{R}(M_{WR}, T)e^{-B_{MW}/(\alpha W T)}, \quad (26) \]

where \( \tilde{R}, B \) depend on the precise form of the symmetry-breaking. However, we can say that \( \tilde{R} \) will have dimension \((\text{mass})^4\), and \( B \) will be dimensionless and of order 1. Also, the whole idea of separating out the exponential is to isolate a prefactor \( \tilde{R} \) which can be assumed to vary more slowly (in the left-handed case, the prefactor varies as powers of the arguments [20]).

Introducing \( x \) and \( Y \), the Boltzmann equation (25) becomes, from (7), (10), and (26),

\[
\frac{dY}{dx} = -\frac{1}{(1.66g^*/x^4)(M^5/M_p^2)2\pi^2} \frac{2}{45} g^*_s \frac{Y - Y_{eq}}{2(Y + Y_{eq})} \tilde{R}'(M, a, x) e^{-Bx/(\alpha W a)}
- \frac{3.16 \times 10^{17}}{Mx^3}(Y^2 - Y_{eq}^2), \quad (27)
\]

writing \( a = (M/M_{WR}) \). We are considering \( a > 1 \).

Compactly, we can write

\[
\frac{dY}{dx} = -R(M, a, x)e^{-Bx/(\alpha W a)} \frac{Y - Y_{eq}}{2(Y + Y_{eq})} - \frac{3.16 \times 10^{17}}{Mx^3}(Y^2 - Y_{eq}^2), \quad (28),
\]

where \( \tilde{R}' \) and \( R \) are obtainable from \( \tilde{R} \).

As "a" (the quotes are to avoid confusion with the singular a) increases, the first term gains importance because of the exponential. Suppose "a" has a value for which the first term is predominant near decoupling. Let us characterise decoupling by the simple criterion

\[
\Delta(x_d) = Y_{eq}(x_d). \quad (29)
\]

This is equivalent to taking \( c=1 \) in (11).

Before decoupling, \( Y \approx Y_{eq}, \Delta' \sim 0, \) and

\[
Y_{eq}' = -Re^{-Bx/(\alpha W a)} \frac{\Delta}{2(Y_{eq} + \Delta)}, \quad (30)
\]
At decoupling, when $Y'_{eq} = -Y_{eq}$, (30) gives

$$[6Y_{eq}(x_D)/R]e^{Bx_d/(\alpha_W a)} = 1,$$

and, using the value of $Y_{eq}$ from (9),

$$e^{[\frac{B}{\alpha_W a} - 1]x_d} G(M, a, x_d) = 1,$$

where the prefactor G can be assumed to have a slower variation with $x_d$ and "$a" than the exponential, because R is a prefactor for which this has been assumed. Then, assuming the exponential to dominate, we expect, approximately,

$$\left(\frac{B}{\alpha_W a} - 1\right)x_d = \tilde{B}x_d \approx \text{constant}. \quad (31)$$

Now, the sphaleron decay will produce a N neutrino only if the kinematic constraint $E_{sp} > M$ is satisfied. As $E_{sp} = BM_{WR}/\alpha_W$, so, $B/\alpha_W > a$ gives an upper limit $a'$ on "$a" for anomalous effects to occur. For $a < a'$, $\tilde{B} > 0$, and, if "$a" is increased, $\tilde{B}$ decreases, so that, $x_d$ increases.

We approximate $Y_\infty$ by $Y(x_d)$, so that, from (29),

$$Y_\infty \approx 2Y_{eq}(x_d).$$

Then, we get, from (13) and (9),

$$\Omega_N h^2 = 1.62332 \times 10^6 x_d^3 e^{-x_d} aM_{WR}. \quad (32)$$

Since, from (31), $x_d \sim \frac{B}{a} / (1 - \frac{a}{a'})$, the exponential will dominate, and we can expect that, as "$a" increases, $\Omega_N h^2$ will decrease. This means that $\Omega_N h^2 < 1$ will give a lower bound on "$a", and, hence, on M, for a given $M_{WR}$, for $a < a'$.

If we can actually find values of the parameter "$a" = M/M_{WR}$, within the range $1 < a < a'$, for which the anomalous term in (28) predominates, there will not be any hindrance from the Lee-Weinberg type of cosmological bound to right-handed neutrinos having masses greater than right-handed gauge boson masses.

So, we find that anomalous reduction of right-handed neutrino number may have important effects on the decoupling of such neutrinos.
Whether these formal expectations will be borne out depends on the actual numbers in $\Gamma_R$. Extrapolating the known result for $\Gamma_L$ to the right-handed case, keeping wide leeway, we will find that numerical results give cause for optimism.

IV. NUMERICAL RESULTS

We will take the $\Gamma_L$ given in Reference [20].

\[
\Gamma_L = \frac{(1.4 \times 10^6) M_W^7}{g^6 T^3} \exp\left[-\frac{16\pi M_W}{g^2 T}\right].
\] (33)

Here, the unstable mode $\omega^-$ is taken $\approx M_W$, and $E_{sp} = 2 (M_W/\alpha_W) \bar{E}$. $\bar{E}$ is a number which depends on $(\lambda/g^2)$: $1.56 < \bar{E} < 2.72$ for $0 < \lambda < \infty (\lambda$ is the 4-Higgs self-coupling constant). We take $\bar{E} = 2$. $M_W$ is temperature dependent.

\[
M_W = M_W(0)\left[1 - (T/T_C)^2\right]^{\frac{1}{2}},
\] (34)

and $T_C = 3.8 M_W(0)$ [20]. There is an overall constant $\kappa \sim 1$ [20,24]. We take $\kappa = 1$.

This expression is valid for $2M_W \ll T \ll 2M_W/\alpha_W$. However, the range of $T$ may be taken to be $M_W \ll T \ll M_W/\alpha_W$. [25]

We extrapolate this rate to get $\Gamma_R$, in a simple way, using the following prescription:

(i) replace $M_W$ by $M_{WR}$,

(ii) write $T_{CR} = z M_{WR}(0),$

and (iii) include an overall factor $b$.

$z$ is not known reliably, because, as yet, there isn’t sufficient experimental data to evaluate the full L-R Lagrangian, including the Higg sector [27]. For large $z$, (34) shows that $M_{WR} \approx M_{WR}(0)$. If $z$ is too small, $M_{WR}$ will become imaginary. We will vary $z$ between 2 and 10. The numerical work will show that below $z=2$, the mass is not real, while there is little change above $z=10$.

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Whereas the exponential part in (33) will almost certainly be right for \( \Gamma_R \) (apart from the order one quantity \( \bar{E} \)), the prefactor is bound to require considerable modification. Considering the prefactor to be a slowly varying quantity, whose main function is to set the numerical scale of the essentially exponential variation of \( \Gamma_R \) with \( (1/T) \), we will allow \( b \) to vary from \( 10^{-3} - 10^3 \), i.e. the decoupling will be investigated with anomalous rates for right-handed neutrino reduction varying over 6 orders of magnitude around the rate obtained by simple substitution of the right-handed W boson mass in the formula for the left-handed case.

We have, then,

\[
\Gamma_R = \frac{(b \times 10^6) M_{WR}(0)^7}{g^6 T^3 [1 - \left( \frac{T}{z_{MW}(0)} \right)^2]^{\frac{7}{2}}} \exp\left[-\frac{16\pi M_{WR}(0)}{g^2 T} \left(1 - \left( \frac{T}{z_{MW}(0)} \right)^2 \right)^\frac{7}{2}\right]
\]

Introducing \( x \) and \( Y \) in the above expression, the Boltzmann equation (25) becomes

\[
\frac{dY}{dx} = -f(x)(Y^2 - Y_{eq}^2) - g(x)\frac{Y - Y_{eq}}{Y + Y_{eq}}.
\]

From (10), \( f(x) = \frac{3.16 \times 10^{17}}{a M_{WR}(0)x^3} \).

\( \Gamma_R \) gives \( g(x) = \frac{b 1.53 \times 10^{23} x^7}{a^8 M_{WR}(0)} \left(1 - \left( \frac{a}{x} \right)^2 \right)^\frac{7}{2} \exp\left[-\frac{118.98 x}{a} \left(1 - \left( \frac{a}{x} \right)^2 \right)\right], \)

where \( a = M/M_{WR}(0) \).

For \( x < x_d \), this equation simplifies, as in Section II.C., to

\[
\Delta = -\frac{Y_{eq}'}{f(x)(2Y_{eq} + \Delta) + \frac{g(x)}{2Y_{eq} + \Delta}}.
\]

We choose, again, as an approximate criterion for decoupling:

\[
\Delta(x_d) \approx Y_{eq}(x_d) \quad \Rightarrow \quad Y(x_d) \approx 2Y_{eq}(x_d).
\]

At decoupling, \( Y_{eq}' = -Y_{eq} \). (36), then, leads to the decoupling condition

\[
3f(x_d)Y_{eq}(x_d) + \frac{g(x_d)}{3Y_{eq}(x_d)} = 1.
\]
Table II

Effect of uncertainty in $T_{CR}$

| z  | X  | A  |
|----|----|----|
| 2  | 31.5 | 41 |
| 3  | 31.6 | 48 |
| 4  | 31.7 | 50 |
| 5  | 31.7 | 52 |
| 6  | 31.7 | 53 |
| 8  | 31.8 | 54 |
| 10 | 31.8 | 54 |
| 100 | 31.8 | 55 |

Again, using the approximation

$Y_\infty \approx Y(x_d) \approx 2Y_{eq}(x_d)$,

the cosmological bound becomes, from (32),

$$1.62332 \times 10^6 x_d^2 e^{-x_d} a M_{WR}(0) < 1.$$  

At the bound,

$$1.62332 \times 10^6 x_d^2 e^{-x_d} a M_{WR}(0) = 1. \quad (38)$$

First, we check the effect of $z$. Taking $M_{WR}(0) = 4000$ Gev and $b=1$, we solve (37) and (38), numerically, to obtain values of $x_d = X$ and "a"=A, for which $\Omega_N h^2$ is just equal to 1. The results, displayed in Table II, show that, as $z$ varies in the range $2 \leq z \leq 10$, $X$ varies from 31.5 to 31.8, and $A$ from 41 to 54. For $z=1$, $M_{WR}$ is no longer real. Also, as expected, $z=100$ gives for $X$ and $A$ practically the same values as given by $z=10$.

Having seen that the effect of varying $z$ is small, we set $z=4$ for subsequent numerical work.
We next check that the bound obtained is actually a lower bound. We do this by varying "a" around the value $A$. For each assigned value of "a", we solve (37) for $x_d$, and evaluate $\Omega_N h^2$ for this $x_d$ from the LHS of (38). The results, displayed in Table III, show that as "a" increases through the value $A$, $\Omega_N h^2$ falls through 1, from higher to lower values.

Finally, we vary $b$ from $10^{-3}$ to $10^3$. The results are shown in Table IV. We find that $X$ changes from 31.8 to 31.6, and $A$ changes from 56 to 46. In every case, we have verified the nature of the bound, numerically. The results (not exhibited) parallel Table III. The bound remains a lower one. If, of course, smaller and smaller values of $b$ are considered, eventually the anomalous effects term will be swamped by the second term in (25) or (28), and the bound will revert to an upper one. However, the numerical work shows that this does not happen even for $b=10^{-3}$.

It is necessary to verify that the restriction $M_{WR} < T < M_{WR}/\alpha_W$ is satisfied. For the lower limit, the worst case occurs when $M_{WR} \approx M_{WR}(0) = 4000$ Gev. Now, $T = 4000A/X$, and the restriction is satisfied if $A > X$. A
Table IV

Effect of overall uncertainty factor

| b    | X  | A |
|------|----|---|
| 0.001| 31.8| 56 |
| 0.01 | 31.8| 54 |
| 0.1  | 31.7| 52 |
| 1    | 31.7| 50 |
| 10   | 31.7| 49 |
| 100  | 31.6| 47 |
| 1000 | 31.6| 46 |

Perusal of Tables III and IV will show that this is, indeed, so, for the parameter ranges considered by us. The stronger restriction, with $M_{WR}$ replaced by $2M_{WR}$, is, however, not obeyed.

For the upper limit, the worst case occurs when $z$, and, hence, $M_{WR}$ is the least. Taking $z=2$, $X=31.5$, and $A=41$, from Table II, we find that $M_{WR}/\alpha_W \approx 90,000$ Gev, while $T \approx 5200$ Gev. The restriction is obeyed.

We check the kinematical constraint $E_{sp} > M$. As $E_{sp} = 2(M_{WR}/\alpha_W)\bar{E}$, we look only at the case when $M_{WR}$ is the least, viz. $z=2$. $E_{sp}$ comes out to be $> 360,000$ Gev, in this case, while, even for $A=55$, $M=220,000$ Gev, less than $E_{sp}$, as required.

V. CONCLUSIONS

Analysing the decoupling of right-handed neutrinos with mass greater than right-handed gauge boson masses, using normal electroweak annihilation channels, we find that the cosmological bound $\Omega_N h^2 < 1$ leads to an upper bound on the right-handed neutrino mass $M$, of about 700 Gev. What this really
means is that a right-handed neutrino mass greater than right-handed gauge 
bozon masses is unlikely to be allowed cosmologically.

If we now assume that anomalous $B+L$–violating processes work at the 
right-handed symmetry-breaking scale by thermal diffusion over a barrier, sepa-
rating states of different $B+L$, in the same way as this happens at the left-
handed symmetry-breaking scale, then, we find that it is possible to have a 
lower bound for a right-handed neutrino mass greater than right-handed gauge 
bozon masses.

A numerical extrapolation of the anomalous rate from the lower to the 
higher energy scale, allowing a leeway of six orders of magnitude, confirms 
this possibility. Taking $M_{WR} = 4$ Tev, a lower bound appears for the right-
handed neutrino mass at about 50 times the $WR$ boson mass. However, in the 
absence of an explicit calculation of the anomalous rate for the right-handed 
case, the numbers must only be considered as giving qualitative support to 
the idea that, at Tev energy scales, anomalous generation plays an important 
part in decoupling, and may take away cosmological obstacles to the existence 
of right-handed neutrinos with mass greater than right-handed gauge bozon 
masses. To obtain reliable bounds, it is necessary to solve the problem of 
constructing explicitly the sphaleron solution for the right-handed case.

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