Information sharing and sorting in a community

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We present the results of detailed numerical study of a model for the sharing and sorting of informations in a community consisting of a large number of agents. The information gathering takes place in a sequence of mutual bipartite interactions where randomly selected pairs of agents communicate with each other to enhance their knowledge and sort out the common information. Though our model is less restricted compared to the well established naming game, yet the numerical results strongly indicate that the whole set of exponents characterizing this model are different from those of the naming game and they assume non-trivial values. Finally it appears that in analogy to the emergence of clusters in the phenomenon of percolation, one can define clusters of agents here having the same information. We have studied in detail the growth of the largest cluster in this article and performed its finite-size scaling analysis.

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1. INTRODUCTION

The naming game (NG) ¹,² is a simple multi-agent model that employs mutual bipartite interactions within a population of individuals which leads to the emergence of a shared communication scheme. In each game a randomly selected pair of agents interact to negotiate conventions, i.e., associations between forms (names) and meanings (for example objects in the environment, linguistic categories etc.). The negotiation of conventions takes place as follows: one of the agents (acting as a speaker) attempts to draw attention of the other agent (acting as the hearer) toward the external meaning (i.e., an object or a category) by the production of a conventional form. In case the hearer is able to express the proper meaning of the form uttered by the speaker the pair of agents is assumed to meet a mutual consensus and the interaction is called a “success”. Consequently, both the agents update their form-meaning repertoire by removing all competing forms corresponding to the meaning except the “winning” one currently uttered by the speaker. On the other hand, if the hearer produces a wrong interpretation then she takes lesson from the meeting by learning this new form-meaning association and in this case the interaction is termed as a “failure”. Thus, on the basis of success and failure of the hearer in producing meaning of the name, both the interacting agents reshape their internal form-meaning association. Through successive interactions, the adjustment of such individual associations collectively leads or should lead to the emergence of a global consensus.

The naming game model is one of the simplest examples of a framework progressively leading to the establishment of human-like languages. It was initially formulated to understand the role of self-organization in the evolution and change of human languages ³ ⁴ ⁵. Since then, this model has acquired a paradigmatic role in the novel field of semiotic dynamics (see ⁴ ⁵ ⁶ ⁷ ⁸ ⁹ for a series of references) that primarily investigates how language evolves through the invention and successive adoption of new words and grammatical constructions. NG finds wide applications in various fields ranging from artificial sensor network as a leader election model ⁷ to the social media as an opinion formation model ⁸. More advanced models ⁹ ¹⁰ attempting to explain further complex processes like categorization have also been built on top of the basic naming game framework.

In this paper, we revisit the basic construction of this model and argue that it is too stringent in removing all the entries except the winning one from the agent repertoire after a successful interaction. Further, it has to be noted that learning is seldom unidirectional as in case of a failure in the original naming game; in contrast, we believe that learning activity most of the times is reciprocal ¹¹ ¹². Therefore, here we redefine the interaction rules in order to address the above limitations by having a symmetric model where on a success both the agents sort out all the common information that they have while on a failure enhance each of their knowledge by learning all the form-meaning associations that the other partner only knew so far. One can intuitively posit that this process should lead to the emergence of a faster consensus than the original naming game owing to the fact that (a) the agents learn more and (b) the agreement criteria is relaxed, thereby, increasing manifolds the probability of successful communication. We perform rigorous numerical simulations to obtain the scaling relations for this revised model and explicitly show that for a population of N agents the time to reach global consensus indeed scales as N¹.¹³ as opposed to N¹.⁵ for the original naming game.

2. THE MODEL

There are N agents in a community. Each agent i (i = 1, ..., N) has an inventory of words whose length
and \( \ell_j \) respectively are selected randomly with uniform probability from all the agents. One of them, say the \( i \)-th agent is randomly selected between the two and is termed as the ‘speaker’ where as the \( j \)-th agent is called the ‘hearer’. The time \( t \) is discrete and is measured in terms of the number of interactions. The interaction between them can take place in the following three possible ways:

A. **Invention:** In this case the inventory list of the speaker is empty. The speaker picks up a new word and keeps it at the bottom of his inventory. Since this is a new word, it cannot be present in the inventory of the \( j \)-th agent. Therefore this new word is simply added at the top of the inventory of the \( j \)-th agent.

B. **Success:** In this case the inventory length of the speaker is non-zero. The speaker and the hearer share information about their contents, sort out the common contents and only the common words are retained. That means the inventories of lengths \( \ell_i \) and \( \ell_j \) of the speaker and the hearer respectively are compared and the number \( n \) of common words are sorted out. In case \( n > 0 \) then this possibility is called a success. The inventories of both the speaker and the hearer are then shrunk to \( n \) entries where only the common words are kept.

C. **Failure:** If the inventories have non-zero lengths yet there is no common word between them, then the lists are merged together and both the agents have the same combined list.

It is to be noted that in this model the success and failure rules are symmetric with respect to the speaker and the hearer.

At any arbitrary intermediate time \( t \) the total number of words in the community is denoted by \( N_w(t) \) and the total number of distinct words is denoted by \( N_d(t) \). The dynamics starts with the inventory lengths \( \ell_i \) and \( \ell_j \) of the speaker and the hearer respectively are selected randomly with uniform probability from all the agents. One of them, say the \( i \)-th agent is randomly selected between the two and is termed as the ‘speaker’ where as the \( j \)-th agent is called the ‘hearer’. The time \( t \) is discrete and is measured in terms of the number of interactions. The interaction between them can take place in the following three possible ways:

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fast, i.e., linearly with time. As time proceeds more and more agents have non-zero inventory lengths and therefore the chances of interactions of types B and C become increasingly likely. Consequently $N_w(t)$ reaches a maximum at a specific time $t_m$ and then it decreases with time (Fig. 1(a)). On the other hand the number of distinct words $N_d(t)$ nearly saturates around a fixed mean value. Eventually $N_d(t)$ also decreases gradually and the community finally converges at the time $t_f$ to a stable state which is a fixed point (Fig. 1(b)). In this stable state $N_w(t_f)$ takes a value $gN$ when every inventory has the same set of $g$ common words, $g$ being a small positive integer. Therefore in contrast to the naming game model where $g=1$, there could be multiple globally common words in our model i.e., $g > 1$. Consequently $N_d(t)$ finally reaches the value $g$. In addition a third quantity $S(t)$ is also calculated which measures the success rate of an interaction at time $t$. In other words $S(t)$ is the fraction of a large number of independent runs which have successful moves at time $t$ and its variation with time is shown in Fig. 1(c).

3. THE ALGORITHM

The simulation algorithm can be described as follows. Positive integer numbers starting from unity are used for representing different words. Therefore at any arbitrary intermediate stage if $N_d$ distinct words have already been used, to choose a new word one simply selects the number $N_d + 1$. It turned out that defining an array $b(k)$ is very useful, $b(k)$ keeps track of the number of times the word $k$ has occurred with all agents. In case $A$, $b(k)$ is increased by 2: $b(k) \rightarrow b(k) + 2$. However to check if an interaction is a case of success or failure, one first compares the inventories of the $i$-th and the $j$-th agents. Therefore every word of the list $\ell_i$ has to be checked in the list $\ell_j$ and vice versa. This is easily done by using another array $a(k)$ and for every word $k$ in $\ell_i$ and $\ell_j$ one makes $a(k) \rightarrow a(k) + 1$. After that the number of locations with $a(k) = 2$ are the number of common words between $\ell_i$ and $\ell_j$. Let this number be $n$ and only these common words are kept in another array $a_1$. At the same time we also count that out of $n$ such common words how many have $b$ values greater than 2, i.e., these words have not only occurred in $\ell_i$ and $\ell_j$ but also in the inventories of other agents. Let this number be $n'$. If $n > 0$ it is a case of success and if $n = 0$ its is a case of failure.

In case of success we first update the $b$ array. For each entry $k$ in $\ell_i$ and $\ell_j$ we first make $b(k) \rightarrow b(k) - 1$. Therefore during this updating procedure whenever $b(k)$ becomes zero we reduce $N_d$ by one: $N_d \rightarrow N_d - 1$. Let there be $m$ distinct entries in the inventories of $i$ and $j$ where $b(k)$ becomes zero. Then the $n$ words in $a_1$ array are copied to $\ell_i$ and $\ell_j$. $N_w$ is updated like: $N_w \rightarrow N_w - \ell_i - \ell_j + 2n$ and $N_d$ is updated like: $N_d \rightarrow N_d - m + n - n'$.
This completes a successful interaction.

In the case of failure the combined list of \( \ell_i \) and \( \ell_j \) are copied to the inventory lists of \( i \) and \( j \). For each such word the \( b \) value is increased by unity. The total number of words \( N_w \) increased as \( N_w \rightarrow N_w + \ell_i + \ell_j \), the number of distinct words \( N_d \) remains same. This completes an unsuccessful interaction (see Table I).

4. THE RESULTS

It is noticed that on increasing the community size \( N \) the probability that an arbitrary configuration has the same set of \( g \) distinct words per agent in the final stable state decreases for \( g > 1 \) and it increases to unity for \( g = 1 \). We have measured the fraction \( f_N(g) \) of a large sample of uncorrelated configurations that have \( g \) words in the final stable configurations. The variation of \( f_N(1) \) has been shown in Fig. 2. A plot of \( 1 - f_N(1) \) vs. \( N \) on a log-log scale gives a nice straight line for the intermediate range of \( N \). This indicates that the growth of \( f_N(1) \) to unity as \( N \) increases follows a power law like \( 1 - f_N(1) = AN^{-\tau} \) and our measured value of \( \tau \) is 1.13(2).

The mean maximal time \( \langle t_m(N) \rangle \) and the mean convergence time \( \langle t_f(N) \rangle \) have been measured for different values of \( N \) and are plotted using a log-log scale in Fig. 3. The community sizes which have been simulated varied from \( N = 2^4, 2^5, \ldots, 2^{16} \), increased by a factor of 2 in successive steps. These data fit very well to straight lines. Therefore assuming power law variations like

\[
\langle t_m(N) \rangle \sim N^\alpha \quad \text{and} \quad \langle t_f(N) \rangle \sim N^\beta \quad (1)
\]

we obtained \( \alpha = 1.12 \) and \( \beta = 1.14 \).

This observation leads us to conclude that both \( \alpha \) and \( \beta \) are approximately the same and has a value 1.13(1). It may be noted that these exponents are much smaller than the original naming game (both \( \alpha \) and \( \beta \) equal to 1.5) [1]. This faster consensus is possibly a consequence of the fact that the interaction rule here is symmetric thus increasing the possibility of alignment between the agents through fewer interactions as compared to the original naming game. Further, here the stable state criteria is also relaxed, so the agents are assumed to reach consensus even if they do not agree on only a single word.

Next in Fig. 4, we plotted the average maximal number of words \( \langle N_{wm}(N) \rangle \) against \( N \) on a log-log scale for the same community sizes. Here again we assumed a power law variation like

\[
\langle N_{wm}(N) \rangle \sim N^\gamma \quad (2)
\]

and the average slope is measured using a least square fit method. We obtained an average value of \( \gamma = 1.49 \). Further, this analysis has been done in more detail. The intermediate slopes \( \gamma(N) \) between successive pairs of points have been measured and extrapolated against \( N^{-0.44} \). The extrapolation fits very well to a straight line and in the limit \( N \rightarrow \infty \) the value of \( \gamma = \gamma(\infty) = 1.539 \) has been obtained. This value of \( \gamma \) is comparable with 1.5 in the original naming game model [1].

It may be noticed that the main difference between the present model and the original naming game arises from the fact that here a failure is caused when none of the words known by the speaker is also known by the hearer as opposed to the mean field case where the sufficient condition for failure is that the one random word selected by the speaker is unknown to the hearer. A similar argument also holds for the success case where the success probability in this case is determined by whether the hearer knows one or two or three or up to any number of words known by the speaker unlike the mean field case where the success is determined by the match of the
one random word selected by the speaker. Therefore, intuitively a single success or failure event in our model corresponds to an accumulation of a set of a number of independent success and failure events in the mean field case thus making the current dynamics faster and the overall exponents different.

5. THE LARGEST CLUSTER

At an intermediate time \( t \) there are \( N_d(t) \) distinct words and in general each word is shared by a number of agents. Similar to the percolation phenomena we define the cluster size \( s_i \) associated with the \( i \)-th word as the number of distinct agents which have the word \( i \) in their inventories. In the algorithm described in section 3 we have stored the cluster sizes in the array \( b(i) \). As time evolves cluster sizes of some words gradually vanish but at the same time the cluster sizes of the other words grow. Finally only \( g \) distinct words survive whose cluster sizes are exactly \( N \) and at this point of time the dynamics reaches the fixed point. It may be noted that the size of a particular cluster increases in the failure rule and decreases in the success rule only by one agent at a time. We keep track of the variation of the size of the largest cluster \( s_m(t,N) \) and observe how it almost monotonically increases and assumes the size \( N \) at the fixed point (Fig. 5). At an intermediate stage there may be a number of distinct clusters whose sizes are equal to the largest cluster size \( s_m(t,N) \). We define the fractional size of the largest cluster at time \( t \) averaged over many independent runs as:

\[
C(t,N) = \frac{s_m(t,N)}{N}.
\]

In addition we define the size \( s_{2m} \) of the second largest cluster as well. In contrast to \( s_m \), the value of \( s_{2m} \) gradually increases to a maximum value and then systematically decreases to zero at the fixed point (Fig. 5). We define another characteristic time \( t_c \) at which the second largest cluster assumes its maximum value. This is the transition time when the second largest cluster starts dismantling and the largest cluster grows at its fastest rate which signifies the onset of correlation in the community.

In Fig. 6(a) the characteristic time \( \langle t_c(N) \rangle \) averaged over many independent runs has been plotted on a log–log scale against the community sizes \( N = 2^6, 2^7, \ldots, 2^{15} \). While the points seem to fit a nice straight line on the average, a closer look reveals that here again the local slopes between successive pairs of points have a systematic variation. Assuming that the functional form would indeed be a power law in the limit of \( N \rightarrow \infty \) as

\[
\langle t_c(N) \rangle \sim N^\delta
\]

we have extrapolated the local slopes \( \delta(N) \) with a negative power of \( N \). The best value of this correction exponent is 0.42 and in Fig. 6(b) a plot of \( \delta(N) \) against \( N^{-0.42} \) gives a nice straight line for large \( N \) values. Extrapolating to \( N \rightarrow \infty \) we obtained \( \delta = 1.12 \).

Finally in Fig. 7 the average value of the largest cluster size \( C(t,N) \) has been plotted for three different community sizes \( N = 2^{12}, 2^{14} \) and \( 2^{16} \). We first scale the time axis \( t/N^{1.13} \) so that the scaled time could be treated similar to the site / bond occupation probability in percolation theory. The scaled axis is then shifted by 2.98 and then again scaled by \( N^{0.13} \) to obtain a data collapse.

To summarize we devised a new model for information sharing and sorting in a community of agents. Three types of mutual bipartite interactions take place among the randomly selected pairs of agents. Here the interactions are more symmetric and less restricted compared to the ordinary naming game. By Invention new words are created, by Failure inventories are shared and by Success only the common words are sorted out. The dynamics of the system is dominated initially by Invention, followed by rapid growth of different words dominated by Failure and finally the system gradually gets rid of uncommon words dominated by Success moves. The system finally reaches the stable state where each agent has the same set of \( g \) words in his inventory. Using extensive numerical studies we find that the exponents describing the characteristic time scales and the maximum number of words of this model assume a completely distinct set of values compared to the ordinary naming game.
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