Auction Mechanism-Based Sectored Fractional Frequency Reuse for Irregular Geometry Multicellular Networks

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Abstract: Modern cellular systems have adopted dense frequency reuse to address the growing amount of mobile data traffic. The system capacity is improved accordingly; however, this is at the cost of augmented Inter-Cell Interference (ICI). Recently, Fractional Frequency Reuse (FFR) has emerged as an efficient ICI management scheme in Orthogonal Frequency-Division Multiple Access (OFDMA)-based cellular systems. However, the FFR scheme that leads to optimized spectrum allocation for individual users in the irregular geometry networks is not considered in the literature. Meanwhile, in the practical wireless scenario, the users are non-cooperative and want to maximize their demands. A game-theoretic Auction Mechanism-based Sectored-FFR (AMS-FFR) scheme is proposed in this paper to optimally distribute the bandwidth resources to the individual users in the realistic multicellular network deployment. In the proposed auction mechanism, the Base Station (BS) acts as an auctioneer and is the owner of sub-carriers. The users are permitted to bid for a bundle of sub-carriers corresponding to their traffic requirements. The Monte Carlo simulation results show that the presented AMS-FFR scheme outperforms the prevailing FFR schemes in terms of achievable throughput by 65% and 46% compared to the basic FFFR and dynamic FFR-3 schemes, respectively. Moreover, the average sum rate along with the user satisfaction is significantly increased while considering a full traffic load.

Keywords: Inter-Cell Interference; irregular geometry; Fractional Frequency Reuse; combinatorial auction

1. Introduction

The exponential growth in mobile data traffic along with the ever-increasing demands for mobile broadband application services have triggered the development of fifth-generation (5G) cellular networks [1]. The scarcity of spectrum resources and increased data rate demands are challenging future cellular operators to improve their spectral efficiency [2,3]. This challenge is addressed in the literature by full frequency reuse, in which all spectrum resources are employed at each network cell [4]. Accordingly, spectrum efficiency is improved; however, this is at the cost of escalated ICI [5]. ICI is among the most significant performance-limiting constraints and is worsened at the cell edge if spectrum management is not done properly [6]. Therefore, interference mitigation in future cellular systems is a major concern in both the academic research and industry communities [7].

In the literature, related work on interference management indicates that FFR has great potential to mitigate ICI and can enhance system performance [8,9]. FFR was developed as an extension of the fundamental frequency reuse concept [10]. It is a frequency reuse-based...
Inter-Cell Interference Coordination Cooperation (ICIC) scheme developed for fourth-generation (4G) cellular systems [11] and can be applied to 5G networks as well [12]. In FFR (Strict-FFR), the cell area is parted into cell-center and cell-edge sub-regions. In this division, the benefits of both low- and high-frequency reuse factors are exploited. Accordingly, the total system spectrum is also fragmented into two sub-bands. One sub-band is assigned to the center users of each cell with a reuse factor of one. Meanwhile, to use the other part of the spectrum with a high reuse factor, it is additionally split into numerous sub-bands; accordingly, one of the subsequent sub-bands is assigned to the users of each cell located at the cell-edge region [13]. Thus, the cell-edge region has a different frequency band and hence ICI is avoided [14]. Since only one sub-band out of the total sub-bands specified for the cell-edge region is utilized at each cell, Strict-FFR results in underutilization of the spectrum resources. To overcome this limitation, Sectored-FFR or FFR-3 has been proposed in the literature, where cell sectoring has been integrated, which results in the further division of the cell-edge region into several sectors [15]. Accordingly, the cell-edge region sub-bands are assigned to each sector and hence full spectrum utilization is realized.

To analyze the performance of ICI mitigation approaches, the consideration of the network geometry or topology and related parameters plays a significant role [16]. The performance analysis of FFR has been carried out in the literature while using a perfect hexagonal geometry model [5,13]. In regular geometry models, BSs are placed ideally at the cell center while considering interference from an equal number of adjacent cells [17]. Furthermore, in the Wyner Model with regular geometry, it is expected that all adjoining cells produce an identical amount of interference [18]. The received Signal to Noise plus Interference Ratio (SINR) of a mobile user is highly dependent on its distance from the serving BS. Therefore, the network geometry and the distribution of the BSs and mobile users have a considerable impact on the SINR statistics [19]. Accordingly, in a cellular network, the spatial configuration of users, BSs, and network topology have a considerable effect on its performance [20].

The performance of conventional FFR algorithms applied to regular geometry-based hexagon models is insufficient in the realistic network scenario [21]. In realistic deployment, network cells are of irregular geometry, and the propagation conditions significantly vary for each cell. Furthermore, because the BSs are not properly positioned at the cell center, each cell experiences a varying proportion of ICI [16]. These factors have sparked investigation, and FFR has been combined with cellular networks of irregular geometry [4,22]. However, the simplest form of FFR (Strict-FFR) is adopted with irregular geometry with only two sub-regions, with no further sectoring of the cell-edge region. Therefore, such deployment results in the underutilization of the spectrum. Furthermore, the sub-regions’ coverage areas and user counts differ as a result of the integration of FFR into the network with irregular geometry. Therefore, spectrum partition is also required to be dynamic. The dynamic spectrum allocation enables the division of system bandwidth under the network variations and diverse traffic demands [23]. Moreover, the dynamic spectrum partitioning configuration needs to be optimized in the FFR when executed for the irregular geometry-based cellular network.

A Sectored-FFR ICI mitigation algorithm has been proposed in our previous work [24] for an irregular geometry-based network. The available bandwidth was utilized with a full frequency reuse factor at each cell of the network. The cell edge was partitioned into three sub-regions. Accordingly, the available bandwidth was partitioned and dynamically assigned. However, the spectrum allocation for the individual user was not optimized in this scheme. Meanwhile, in the practical wireless scenario, the users are non-cooperative, and each user wants to maximize their demands. Thus, the users may behave selfishly and may try to compete with each other to gain more radio resources. Since the users are competing for radio resources, there will be situations where some users will be deprived of bandwidth.

In this paper, a game-theoretic Auction Mechanism-based Sectored-FFR (AMS-FFR) scheme is proposed for an irregular-geometry multicellular network. The proposed AMS-
FFR scheme is designed to optimally assign the spectrum resources to the individual users, leading to the optimal spectrum allocation for every sub-region of the cell. The BS serves as the auctioneer and is the owner of the sub-carriers in the proposed AMS-FFR scheme, and non-cooperative competing users are permitted to bid for different numbers of sub-carriers. The choice of the combinatorial auction is based on the fact that each user requires more than one sub-carrier. To meet the diverse traffic demand, the users are allowed to bid for any number of sub-carriers. Moreover, as the sub-carriers fade independently, the users are allowed to bid for any combination of sub-carriers in a distributive fashion. However, the users are restricted to a single bid in one allocation time to avoid complexity. In the proposed scheme, sub-bands are specified for each sub-region to mitigate ICI, and the users are allowed to bid for any combination of sub-carriers from the specified sub-band. The proposed AMS-FFR scheme significantly enhances the users’ throughput, average sum rate, and user satisfaction compared to the existing FFR-3 and Dynamic-FFR3 techniques.

The paper is organized as follows. Section 2 presents the system model along with the optimization problem formulation, followed by the development of the proposed AMS-FFR scheme in Section 3; the performance evaluation is described in Section 4, and the conclusions in Section 5.

2. System Model

The system model considers an OFDMA-based Irregular Geometry Multicellular downlink network, where the fixed-powered BSs are supposed to be located according to the Poisson Hard Core Process (PHCP) [25]. The randomly distributed mobile users are linked to their closest BSs. The coverage area of a BS is depicted by the Voronoi tessellations [26]. The full buffer model is considered for the user traffic demand. Specifically, each user’s minimum data traffic demand is generated as a random function within a particular range of data rate values. As a result, the simulation anticipates that users in the network will always have data to send. For an operating BS $i$, the downlink received SINR of an individual user $n$ on sub-carrier $k$ can be determined using the equation presented in [24].

$$SINR_{n,k} = \frac{P_{n,k}^i G_{n,k}^i}{N_0 \Delta f + \sum_{j \in \phi \setminus i} P_{n,k}^j G_{n,k}^j}$$

(1)

where the received power of a user $n$ is $P_{n,k}^i$ on the sub-carrier $k$. The channel gain on $k$ for $n$ is represented by $G_{n,k}^i$ for the BS $i$. $N_0$ represents the power spectral density of the noise, and $\phi$ stands for BSs in the network. The transmit power and channel gain from the interferers are $P_{n,k}^j$ and $G_{n,k}^j$, respectively.

The cell partition process is crucial in FFR for classifying users as either cell-center or cell-edge users. As a result, the average SINR $T_{\delta}$ is used as a criterion for completing this operation. Users with SINR values larger than $T_{\delta}$ are designated as cell-center users $N^C$, whereas users with SINR values less than $T_{\delta}$ are designated as cell-edge region users $N^E$ by the BS. The system model is illustrated in Figure 1. In addition to cell partition, the cell-edge area is further divided into three sectors by using $120^\circ$ sectoring. The resulting four sub-regions (cell-center, sector-1, sector-2, and sector-3) are also irregular in geometry and hence show a diverse traffic load. The cell partition and sectoring is elaborated in the flow chart shown in Figure 2. As a result, spectrum resources must be optimally assigned based on demand in each sub-region.
Figure 1. System model.

Figure 2. Flow-chart for cell partition and sectoring.
The users of the individual sub-region of a cell are authorized to contest for the sub-carriers of the contest-designated sub-band of that sub-region. For any cell, the total users are \( N = \{N^c, N^1, N^2, N^3\} \), where \( N^c \) corresponds to the number of users in the cell center. Similarly, \( N^1 \) corresponds to sector-1, \( N^2 \) corresponds to sector-2, and \( N^3 \) corresponds to sector-3. The whole bandwidth has \( K = \{K^c, K^1, K^2, K^3\} \) number of sub-carriers, where \( K^c \) represents the cell-center sub-band, and \( K^1, K^2, \) and \( K^3 \) represent the sub-bands designated for the individual sectors of the cell-edge region. Each sub-carrier fades independently since the OFDMA-based downlink network is studied in this work. As a result, different sub-carriers correspond to varying channel statistics (SINR values) for each user. Moreover, each user requires more than one sub-carrier to accomplish the targeted rate \( R^t_n \). The \( R^t_n \) is also diverse for the users while studying heterogeneous traffic demand. The achievable throughput \( R_{n,k} \) of a user \( n \) on the sub-carrier \( k \) can be expressed as follows.

\[
R_{n,k} = \Delta f (\log_2(1 + SINR_{n,k}))
\]  

(2)

The achievable throughput of a user \( n \) can be computed as \( R_n = \sum_{k=1}^{K} (R_{n,k}) \), when allocated with sub-carriers \( K_n = \{1, 2, \ldots K\} \). The assigned sub-carriers and the respective SINR values affect \( R_n \). Consequently, the goal of sub-carrier distribution is to optimize the possible throughput of the users. Since the users are competing for radio resources, the sub-carrier allocation problem can be formulated as a combinatorial auction, where \( N \) users (bidders) are competing for \( K \) sub-carriers (commodities). The BS is the owner of the sub-carriers and wants to auction them to the \( N \) users.

The BS also acts as an auctioneer (intermediate agent), who hosts and directs the auction process. Each user \( n \) requires a set or bundle of sub-carriers \( C_n = \{k_1, k_2, \ldots k_n\} \) to fulfill its traffic demand or target data rate \( R^t_n \). The amount of the sub-carriers and the choice of the sub-carriers in the set of requested sub-carriers \( C_n \) are different for each user, for two reasons. First, the heterogeneous traffic demand of the users is assumed; therefore, each user requires a different number of sub-carriers to fulfill its traffic demand or target data rate \( R^t_n \). Secondly, in the OFDMA system, each sub-carrier has different SINR values for different users because of the different channel conditions for each user. From the game-theoretic point of view, the auction problem can be modeled as a game:

\[
\Gamma = (N, \{B_n\}, \{U_n\})
\]

(3)

where the users of the cells are the ‘players’ competing for the resources; therefore, the set of players is the set of users. Bid \( \{B_n\} \) submitted by user \( n \) is the ‘strategy’ of the user. Each user \( n \) submits his request to the BS in the form of a bid by expressing a subset of sub-carriers \( (C_n \subseteq K) \) and his valuation \( V_n \in \mathbb{R}^+ \) on the sub-carrier set. The valuation \( V_n \) is the measure of how interested the user \( n \) is in the given subset of sub-carriers. In the OFDMA system, the user’s valuation for a sub-carrier is related to the throughput that it can achieve by obtaining the sub-carrier. Therefore, the strategy of a user \( n \) would be a set of \( (C_n, V_n) \). The payoff or the utility function \( U_n \) of user \( n \) is defined as

\[
U_n(C_n) = V(C_n) - P(C_n)
\]

(4)

where \( P_n \) is the price paid by the user \( n \) for the bundle of sub-carriers \( C_n \). The utility of a user is equal to the achievable throughput of the respective user; therefore, in the rest of the paper, these terms may be used interchangeably. Let \( B \) represent the set of bids received at the BS, submitted by \( N \) single-minded users for \( K \) sub-carriers.

\[
B = (B_1, B_2, \ldots B_N)
\]

(5)

where each bid is the tuple of the bundle of sub-carriers \( C_n \) in which bidder \( n \) is interested and the valuation \( V_n \). The valuation of a user \( n \) for a bundle of sub-carriers \( C_n \) depends on
the received SINR and hence the achievable data rate. The request is submitted by user \( n \) to the BS in the form of a bid \( B_n \), and is given by

\[
B_n = \langle C_n, V_n \rangle
\]  

Let bidder \( n \) be interested in obtaining \( m \) number of sub-carriers; then, (8) can be written as

\[
B_n = (\{k_{n,1}, k_{n,2}, k_{n,3}, \ldots, k_{n,m}\}, V_n)
\]  

where \( m \) represents the number of requested sub-carriers in the bid submitted by the user \( n \). To maximize the throughput, the winner determination problem for the BS is to label the bids as winners or losers. The BS sees the winner determination problem as follows:

\[
\max_{K \in (K^c, K^1, K^2, K^3)} \left( \sum_{n=1}^{N} x_n V_n \right)
\]

subject to

\[
\begin{align*}
\sum_{m=1}^{m} x_{n,k} k_{n} & \leq K \\
x_{n,k} & = \begin{cases} 
1 & k \text{ is allocated} \\
0 & \text{otherwise}
\end{cases} \\
\sum_{n=1}^{N} K_n & \subseteq K^c \\
\sum_{n=1}^{N} K_n & \subseteq K^{1,2,3}
\end{align*}
\]  

Constraint (a) shows that the maximum sub-carriers in the \( N \) bids must not exceed the total of \( K \) sub-carriers, whereas \( x_{n,k} \) is the binary indicator, which shows whether the sub-carrier \( k \) is allocated to user \( n \) or not, as given by (b). To avoid ICI, (c) and (d) confirm that users who belong to a sub-region may only compete for sub-carriers \( C_n \) from the sub-band designated for that sub-region. The combinatorial auction winner determination problem for single-minded users is NP-complete to solve. NP-complete means that a polynomial-time (fast) algorithm to guarantee the optimal allocation to the problem is unlikely to exist [27]. To solve the aforementioned winner determination problem, a computationally efficient yet sub-optimal method is applied here for the sub-carrier allocation. The proposed scheme adopts an approximation mechanism with greedy allocation.

3. Proposed AMS-FFR Scheme

In the development of the AMS-FFR scheme, there are multiple challenges to be tackled. First, the heterogeneous traffic demand of users results in a different number of sub-carrier requirements for each user, which greatly increases the complexity of the allocation process. Secondly, each sub-carrier fades independently and hence the quality of each sub-carrier may be different for different users in the cell. Thirdly, users may be selfish and submit false channel conditions to gain more radio resources to maximize their throughput. To address these challenges, the AMS-FFR scheme is designed to resolve the multiple-object competition among selfish users. The AMS-FFR scheme is based on a combinatorial auction mechanism for the sub-carrier allocation problem given by Equation (8) and to optimally assign the sub-carriers between the competing users. The choice of the combinatorial auction, where the users can bid for multiple sub-carriers, instead of a single-item auction, is based on the fact that each user requires more than one sub-carrier. The users are allowed to bid for any number of sub-carriers, according to their traffic demand \( R_{t,n} \). Moreover, as the sub-carriers fade independently, the AMS-FFR scheme allows each user to independently bid for any combination of sub-carriers in a distributive fashion. However, to avoid complexity, the users are restricted to submitting a single bid in one allocation time.
To cope with the selfish behavior of the users, the AMS-FFR scheme is designed to be incentive-compatible, where truth-telling is the dominant strategy of the users competing for resources [28]. The dominant strategy is the one that maximizes the bidder’s utility regardless of the other bidders’ strategies [29]. The mechanism is truthful if it is in the best interest of the users to submit bids that truthfully reveal their channel conditions. The payment mechanism of the proposed scheme guarantees truthfulness, where the users cannot achieve higher throughput by submitting a bid price not equal to their true channel conditions.

The comprehensive flow chart of the proposed AMS-FFR is depicted in Figure 3. First, the number of users in individual sub-regions and the number of sub-carriers in sub-bands dedicated to the same region are determined. In the proposed auction mechanism, the BS acts as an auctioneer and the owner of $K$ sub-carriers. The sub-carriers are auctioned to $N$ users, competing for sub-carriers. The users are allowed to bid for a bundle of sub-carriers $C_n$ according to their traffic demand $R_{k,n}$. The bidding process of the users is given in the next section. The BS receives sealed bids $B_N$ from all users and optimally decides the sub-carrier allocation. Sealed bids mean that the bidding information of one user is not shared with other users.

3.1. Bidding for Bandwidth

Let $S_{(N,K)}$ be the channel matrix for $N$ user and $K$ sub-carriers, where each row represents one user and the columns represent one sub-carrier, and each element of the matrix represents the SINR value. In the cell-center region, the channel matrix is given by $S_{(N,K)}^C$ accordingly, for sector-1, sector-2, and sector-3, the subsequent channel matrices are represented by $S_{(N,K)}^m$, where $m$ represents the sector. Moreover, each user wants to realize the target data rate $R_{t,c}$. The target data rate of all users in the cell-center region is given by $R_{T,N,c}$. Similarly, the target data rates of all the users in sector-1, sector-2, and sector-3 are given by $R_{T,N,m}^c$ ($m$ represents the sector). For a user $n$ in a cell-center region with the target/demand rate $R_{c,n}^t$, users $n$ have to bid for a bundle of sub-carriers $C_n$ with high SINR values. To achieve $R_{c,n}^t$, users $n$ have to bid for a bundle of sub-carriers $C_n$ with high SINR values, out of the available sub-carriers.

There can be possible $(2^K)$ combinations of the sub-carriers for a user $n$ to achieve its target data rate. However, in AMS-FFR, the users are restricted to submitting only one combination in one allocation time to avoid complexity. In auction theory, this notion of restricted bidders is called single-minded bidders. The request is submitted by user $n$ to the BS in the form of a bid $B_n$ and is given by (7), where the $C_n \subseteq K$ is the component of the bid that represents the set or bundle of sub-carriers that user $n$ is interested in. The $V_n$ in the equation represents the valuation ($V_n \in R^+$) of user $n$ for the bundle of sub-carriers $C_n$. Moreover, $V_n$ depends upon the achievable throughput on the requested bundle of sub-carriers $C_n$. Therefore, for $N$ users, the total bid received at the BS is given by

$$B_N = \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} \end{pmatrix}$$
Initialize
The number of users \((N_c, N_1, N_2, N_3)\) in each sub-region
The number of sub-carriers \((K_c, K_1, K_2, K_3)\) in each sub-band

BS announce the Auction

Collect the bids from all the users for the available sub-carriers

Determine the approximate value for each bid

Sort the approximate values list \(A(N)\) in descending order

Determine the winners

Losers
Update the bidders list
Update the sub-carriers list

All sub-carriers allocate?

Yes

Determines the price for each winner bid
Determine to bandwidth to be allocated to each user

No

No

Yes

Yes

No

END

Figure 3. Detailed flow-chart of AMS-FFR Algorithm.
The bids submitted by all users in the cell-center region are given by $B_{CN}$. Similarly, the bids submitted by all the users in all three sectors are given by $B_{Nm}$ ($m$ represents the sector). The target data rate ($R_{TN}$), SINR matrix ($S_{TN}$), and the corresponding bids ($B_{TN}$ for each sub-region of the cell) are given as follows.

\begin{align}
\text{Cell Center} & \Rightarrow \\
R_{Nc}^T &= \begin{pmatrix}
R_{1c}^T \\
\vdots \\
R_{nc}^T 
\end{pmatrix} \\
S_{Nc}^C &= \begin{pmatrix}
S_{11}^c & S_{12}^c & \cdots & S_{1k}^c \\
S_{21}^c & S_{22}^c & \cdots & S_{2k}^c \\
\vdots & \vdots & \ddots & \vdots \\
S_{nc1}^c & S_{nc2}^c & \cdots & S_{ncn}^c 
\end{pmatrix} \\
B_{Nc}^C &= \begin{pmatrix}
B_{1c}^c \\
B_{2c}^c \\
\vdots \\
B_{nc}^c 
\end{pmatrix} = \begin{pmatrix}
C_1^c \\
C_2^c \\
\vdots \\
C_n^c 
\end{pmatrix}
\end{align}

\begin{align}
\text{Cell Sector } m=1,2,3 & \Rightarrow \\
R_{Nm}^T &= \begin{pmatrix}
R_{1m}^T \\
\vdots \\
R_{Nm}^T 
\end{pmatrix} \\
S_{Nm}^C &= \begin{pmatrix}
S_{11}^m & S_{12}^m & \cdots & S_{1k}^m \\
S_{21}^m & S_{22}^m & \cdots & S_{2k}^m \\
\vdots & \vdots & \ddots & \vdots \\
S_{nm1}^m & S_{nm2}^m & \cdots & S_{nmn}^m 
\end{pmatrix} \\
B_{Nm}^C &= \begin{pmatrix}
B_{1m}^m \\
B_{2m}^m \\
\vdots \\
B_{nm}^m 
\end{pmatrix} = \begin{pmatrix}
C_1^m \\
C_2^m \\
\vdots \\
C_n^m 
\end{pmatrix}
\end{align}

The BS decides the sub-carrier allocation based on the received bids. Once the bids are received, the BS determines the winner according to the set auction rules. The winner determination is elaborated in the next section.

### 3.2. Winner Determination

To solve the winner determination problem given by Equations (8) and (9), a computationally efficient yet sub-optimal method is applied here for the sub-carrier allocation. The AMS-FFR scheme adopts an approximation mechanism with greedy allocation [30]. To determine the winner bids, received from $N$ users, the BS ranks all the received bids, based
on the approximate value per sub-carrier. That is, for every received bid, the approximate value $a(n)$ is determined by using the following equation

$$a(n) = \frac{V_n}{\sqrt{|C_c|}}$$

(13)

where $\sqrt{|C_c|}$ represents the size of the bid submitted by a user; size means here the number of requested sub-carriers. The BS determines the approximate values of all $N$ received bids and places them in descending order in the list $A(N)$. If $N$ is the number of received bids, then this sorting phase takes time of the order of $N \cdot \log(N)$. For individual sub-regions, $A(N)$ is concluded for all the bids received by the BS.

$$A(N) = \begin{cases} A^c(N^c) = \{a^c(1), a^c(2), \ldots, a^c(n)\}, & a^c \in \{1, 2, \ldots, N^c\} \\ A^1(N^1) = \{a^1(1), a^1(2), \ldots, a^1(n), \ldots, a^1(N^1)\} \\ A^2(N^2) = \{a^2(1), a^2(2), \ldots, a^2(n), \ldots, a^2(N^2)\} \\ A^3(N^3) = \{a^3(1), a^3(2), \ldots, a^3(n), \ldots, a^3(N^3)\} \end{cases}$$

(14)

where $A^c(N^c)$ is a sorted list of approximate values for the bids submitted by the users of the cell-center region and calculated via Equation (14). Similarly, $A^1(N^1)$, $A^2(N^2)$, and $A^3(N^3)$ are the sorted lists of approximate values for sector-1, sector-2, and sector-3 of the cell-edge region, respectively. The user with the first value in the sorted list of each sub-region ($A^c(N^c)$, $A^1(N^1)$, $A^2(N^2)$, $A^3(N^3)$) is declared as the winner. The rest of the users are determined as either winners or losers, based on the conflict in the requested sub-carriers with previously declared winners. The bids are considered loser bids in the case of conflicts. Two bids $B_n$ and $B_n'$ are said to be in conflict if $C_n \cap C_n' \neq \emptyset$.

3.3. Sub-Carrier Allocation

The set of requested sub-carriers $C_n$ is allocated to the user $n$ with the first value $a(1)$ of the sorted list of any particular sub-region. For the rest of the users, the algorithm examines each bid of the sorted lists ($A^c(N^c)$, $A^1(N^1)$, $A^2(N^2)$, $A^3(N^3)$). The requested sub-carriers are allocated to the users if there is no conflict with the previously allocated sub-carriers.

3.4. Payment Calculation

The BS calculates the payments or price for each bidder, after collecting the single-minded bids from all the users $N$ for the available sub-carriers. A straightforward way to allocate the sub-carriers is to declare the highest bidder as a winner and charge him according to his bid or valuation. Such allocation is simple; however, the game would have no dominant strategy solution and the user can submit false valuation ($s_j \neq v_i$) to improve his throughput. To design an incentive-compatible mechanism for the maximization problem, it is sufficient to consider a mechanism in which the payments are decided such that, for each bidder, truth-telling is the dominant strategy [30]. The AMS-FFR payment scheme is based on Vickery’s second-price auction, which ensures that the users must submit their true valuation. In Vickery’s second-price auction, the bidder with the highest bid wins the auction and the price paid is equal to the second-highest bid. The dominant strategy of a user is a strategy that maximizes the throughput of that user, regardless of the other bidders’ strategies. If $s_i$ is the strategy of the user, then, for any $s_{-i} \neq s_i$ and any strategy profile of other $s_{-i}$, we have

$$u_i(s_i, s_{-i}) \geq u_i(s'_{-i}, s_i)$$

(15)
In the case of Vickery’s second-price auction mechanism, if \( i \) is the highest bidder 
\[ s_i > \max_{j \neq i} (s_j), \]
its utility can be calculated as follows, whereas the other user would pay nothing while having the utility of zero.
\[
    u_i(s_i) = \begin{cases} 
    v_i - \max_{j \neq i} s_j & \text{if } s_i > \max_{j \neq i} s_j \\
    0 & \text{otherwise}
    \end{cases}
\]

(16)

In line with the approximation mechanism of the AMS-FFR scheme for the sub-carrier allocation, presented in the previous section, where the average amount per sub-carrier is used, the payment scheme must also adopt the same criterion. Therefore, a user pays, per sub-carrier, the average price submitted by the first bid in the list \( A(N) \) that is rejected because of his bid. The payment scheme can be expressed as follows:
\[
    P_i(C_n) = \begin{cases} 
    0 & \text{if } i \text{ is denied} \\
    |C_n|a(j) & \text{no } a(j) \text{ in list after } i \\
    |C_n|a(j) & \text{exist in the list}
    \end{cases}
\]

(17)

Equation (17) shows that the bidder \( i \) would pay nothing if it has been rejected or if there is no \( a(j) \) that is denied because of him. Moreover, if there is an \( a(j) \) in the list and \( i \)’s bid \( \langle C_n, V_n \rangle \) is granted, then the \( i \) will pay \( |C_n|a(j) \), the average amount per sub-carrier of the bid \( a(j) \) that is next in the list. After the price calculation for each winning user, the BS allocates the bundle of sub-carriers \( C_n \) to the respective user. The bidders’ list and the remaining sub-carriers are updated. The BS checks if there are still sub-carriers available to auction, bids are collected from the updated bidders only, until each sub-carrier is allocated, and the process is repeated. The total complexity of AMS-FFR is \((2N + N\log(N))\)—that is, the sorting process takes the time of the order of \( N\log(N) \), the sub-carrier allocation phase requires time linear in \( N \), and the payment calculation also requires time linear in \( N \).

4. Performance Analysis

The performance study of the offered AMS-FFR scheme is presented in this section. Monte Carlo simulations are executed to precisely evaluate the performance of the proposed AMS-FFR scheme. The proposed scheme is compared with the basic FFR-3 scheme along with the Dynamic-FFR3 [24] for the same simulation setup, presented in the next subsection. The frequency resources are partitioned into four fixed-size sub-bands in the basic FFR-3 scheme, with each sub-band allotted to the users of the respective sub-region. The spectrum resources in the Dynamic-FFR3 are dynamically partitioned and allocated based on the traffic demand or load of the individual sub-regions of the corresponding cell.

4.1. Simulation Setup

A multi-cellular network of eight irregular geometry Voronoi-shaped cells of random coverage area is considered, as shown in the system model. The BSs are assumed to be distributed according to PHCP. The resultant inter-BS distance is random, unlike the fixed distance of the regular geometry hexagon grid model. The random inter-BS distance lies in the range of 600 to 1000 m. Therefore, the nearest BS posing the ICI to the serving BS is positioned at a minimum distance of 600 m. Moreover, 100 mobile users are assumed to be randomly distributed in the coverage region of each BS. The users have diverse traffic demands and hence a heterogeneous traffic load is considered. A random distribution function in the range of \((30 \text{ kbps}, R^{\max}_{d})\) is utilized to generate the user demand. To experience the conventional web-browsing requirement of each user, the lower bound is fixed to 30 kbps, whereas the upper bound is \( R^{\max}_{d} \in 100, 200, \ldots, 1000 \text{ kbps} \). The network load can be calculated by using the following expression.

\[
    \text{Load} = \left( \frac{N \ast (R^{\text{mean}})}{N \ast R^{\text{max}}_{d}} \right)
\]

(18)
The user traffic demand is generated randomly; therefore, the load calculation considers the current mean range \( R_{\text{mean}} = (30 + R_{\text{max}}) / 2 \) kbps and the maximum mean range \( R_{\text{max}} = (30 + 1000) / 2 \) kbps. To maintain the user load distribution during the simulation, user mobility is not considered. Moreover, the general parameters used in the simulation are listed in Table 1. The attainable throughput is calculated using Shannon’s theorem, which gives the user throughput in bits per second as an upper bound. The expression for the cell throughput \( R_{\text{cell}}^{(C)} \) is given as follows:

\[
R_{\text{cell}}^{(C)} = R_{\text{center}} + \sum_{s=1}^{S} R_s
\]  

(19)

where \( R_{\text{center}} \) represents the achievable rate or throughput of the cell-center region, which can be calculated as

\[
R_{\text{center}} = B^C \sum_{n=1}^{N^C} \log_2 (1 + \text{SINR}_n)
\]  

(20)

where \( N^C \) is the number of users in the cell-center region, and \( B^C \) is the bandwidth allocated to the cell-center region. In terms of sub-carrier allocation, the above expression can be written as

\[
R_{\text{center}} = \Delta f \sum_{n=1}^{N^C} \sum_{k=1}^{K^C} \log_2 (1 + \text{SINR}_{n,k})
\]  

(21)

In the above expression, \( K^C \) is the maximum sub-carriers dedicated to the cell-center region, and \( R_s \) represents the achievable rate of the sector \( s \) of the cell-edge region and can be computed as follows

\[
R_s = B_s^T \sum_{n=1}^{N^S} \log_2 (1 + \text{SINR}_n)
\]  

(22)

where \( N^S \) is the number of users in the cell-center region, and \( B_s^T \) is the bandwidth allocated to sector \( S \) of the cell-edge region. The total number of sub-carriers in sub-band \( B_s^T \) is \( K^S \) and can be found using Equation (32) of [24]. Therefore, in terms of sub-carrier allocation, Equation (22) can be written as

\[
R_s = \Delta f \sum_{n=1}^{N^S} \sum_{k=1}^{K^S} \log_2 (1 + \text{SINR}_{n,k})
\]  

(23)

### Table 1. Simulation parameters.

| Network Parameter                        | Value                      |
|------------------------------------------|----------------------------|
| Cellular layout                         | Irregular (Voronoi) Geometry |
| Network size                            | 8 cells                    |
| Number of user per cell                 | 100                        |
| Inter-cell distance                     | Random                     |
| Carrier frequency                       | 2 GHz                      |
| System bandwidth                        | 10 MHz (LTE)               |
| Number of sub-carriers                  | 600                        |
| Sub-carrier spacing                     | 15 KHz                     |
| BS transmit power                       | 46 dBm                     |
| White noise power density               | $-174$ dBm/Hz              |
| Shadowing standard deviation             | 8 dB                       |
| Multipath fading                        | Rayleigh fading            |

### 4.2. Achievable Throughput

The attainable throughput is determined according to Shannon’s theorem. In Figure 4, the Cumulative Distribution Function (CDF) of the users’ throughput is demonstrated. The results reveal that the suggested AMS-FFR scheme greatly boosts users’ possible throughput
when compared to FFR-3 and Dynamic-FFR3. Specifically, 46% and 65% improvement is recorded through the AMS-FFR scheme compared to the Dynamic-FFR3 and conventional FFR-3 schemes, respectively. The static sub-band partition in the basic FFR-3 scheme leads to the worst performance. In the AMS-FFR, the multiuser diversity is exploited to allow the user to bid for the sub-carrier of their choice. Based on the channel condition information, the users bid for multiple sub-carriers according to their traffic demand.

The CDF of the cell-center region users’ throughput is plotted in Figure 5 for the proposed AMS-FFR scheme, along with the FFR-3 scheme and Dynamic-FFR3 schemes. The graph shows that the AMS-FFR scheme enhances the cell-center users’ throughput by 43.65% and 25.5% compared with the FFR-3 and Dynamic-FFR3. Similarly, the CDF plot of the users’ throughput for the cell-edge region shows a significant improvement of 89% and 58.3% for the proposed AMS-FFR compared to the FFR-3 and Dynamic-FFR3 algorithms, as shown in Figure 6.

![Figure 4. CDF of the users’ achievable throughput.](image1)

![Figure 5. PDF of the cell-center users’ achievable throughput.](image2)
4.3. Average Sum Rate

The average sum rates of users at the cell, along with users at the cell-center and cell-edge regions, are plotted in Figure 7 for the proposed AMS-FFR, basic FFR-3, and Dynamic-FFR3 schemes. All three schemes divide the cell into cell-center and cell-edge regions and, to realize full spectrum utilization, further divide the cell-edge region into three sectors. However, the basic FFR-3 approach does not consider the spectrum requirement of each sub-region and fixedly segregates the available spectrum. The Dynamic-FFR3 scheme divides the spectrum according to the demand of each sub-region; however, it does not consider the spectrum requirement of an individual user. Meanwhile, the proposed AMS-FFR scheme, after the dynamic spectrum partition, optimally allocates the spectrum resources to the individual users of each sub-region while considering heterogeneous traffic demand. Therefore, the findings demonstrate that the AMS-FFR scheme enhances the cell’s average sum rate by 44.9% and 64.15%, respectively, when compared to the Dynamic-FFR3 and FFR-3 approaches.
4.4. Achievable Throughput with Respect to Load

To evaluate the proposed scheme for the attainable throughput to load, the diverse traffic demand of the individual user of the cell is first converted into load, as given by Equation (18). Figure 8 shows the cell’s attainable throughput to the load. When the load value approaches 1, it means that the cell is loaded at 100%. The AMS-FFR scheme improves upon the Dynamic-FFR3 and FFR-3 approaches by 41.6% and 58.6%, respectively. The improvement in the system performance is achieved since the proposed AMS-FFR algorithm optimally distributes the dynamically divided spectrum of each sub-region to the individual users while factoring in the heterogeneous traffic load.

![Achievable throughput with respect to load.](image)

**Figure 8.** Achievable throughput with respect to load.

4.5. Users’ Satisfaction

The users’ satisfaction is plotted in Figure 9. User satisfaction represents the relative throughput of the user compared to the throughput of the users in the same cell. Moreover, it shows how close the user throughput of a user is to the maximum throughput of a user in the cell. User satisfaction can be computed as the sum of the user throughput divided by the total number of users into maximum user throughput [24]. It ranges between 0 and 1; when user satisfaction approaches 1, it means that all the users in the cells are experiencing the same throughput. Meanwhile, when user satisfaction approaches 0, it means that there is a large variation in the achievable throughput of the users in the cell. In the proposed AMS-FFR scheme, the users are allowed to bid for the sub-carrier of their choice, which results in improved individual throughput, and, therefore, the users’ satisfaction is improved by 20.3% and 45.4% for the AMS-FFR scheme when compared to the Dynamic-FFR3 and FFR-3.

![Users’ satisfaction plot.](image)
5. Conclusions

The AMS-FFR scheme is designed in this research as an ICI mitigation scheme for OFDMA-based multicellular networks with irregular geometry. To capture the realistic scenario, a PHCP-based irregular geometry-based network model is considered. Cell partitioning and sectoring of the irregular geometry cells produce sub-regions with distinct coverage areas and, consequently, diverse user densities. Moreover, non-cooperative selfish users are considered to compete for radio resources. The proposed AMS-FFR scheme is developed based on a combinational auction mechanism, where the users are allowed to bid for any combination of the sub-carriers in a distributive fashion and according to their heterogeneous traffic demands. The proposed AMS-FFR is analyzed in terms of the CDF of users’ attainable throughput, average sum throughput, achievable throughput for different load conditions, and users’ satisfaction. The collected findings demonstrate that the proposed AMS-FFR scheme significantly outperforms the FFR-3 and Dynamic-FFR3 techniques in terms of users’ achievable throughput, average sum rate, average throughput to load, and user satisfaction.

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