SEISMIC STUDY OF THE CHEMICAL COMPOSITION OF THE SOLAR CONVECTION ZONE

CHIA-HSIEN LIN
Astronomy Department, Yale University, New Haven, CT 06520-8101

H. M. ANTIA
Tata Institute of Fundamental Research, Mumbai 400005, India

AND

SARBANI BASU
Astronomy Department, Yale University, New Haven, CT 06520-8101

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ABSTRACT

Recent downward revision of solar heavy-element abundances using a three-dimensional atmospheric model has introduced serious discrepancies between standard solar models and helioseismic inferences about solar structure. In this paper we investigate the possibility of determining the heavy-element abundances using helioseismic inversion techniques with the hope of providing an independent estimate. We use the adiabatic index \( \Gamma_1 \equiv (\partial \ln P/\partial \ln \rho) \), as a probe to examine the effects of the total heavy-element abundance, as well as the effects due to the abundance of individual elements. Our inversion results show that the new, lower, abundance increases the discrepancy between the Sun and the solar models.

Subject headings: Sun: abundances — Sun: interior — Sun: oscillations

1. INTRODUCTION

Until recently, various analyses had shown that the current standard solar models are in good agreement with the real solar structure to the order of \( 10^{-3} \) (see, e.g., Bahcall et al. 1995, 2001; Gough et al. 1996; Christensen-Dalsgaard 2002; Couvidat et al. 2003; Boothroyd & Sackmann 2003; Richard et al. 2004). These standard solar models were usually constructed with the OPAL equation of state (EOS; Rogers et al. 1996; Rogers & Nayfonov 2002), the OPAL opacity (Iglesias & Rogers 1996), and the heavy-element abundances from Grevesse & Sauval (1998, hereafter GS98) or Grevesse & Noels (1993). Recently, by using a three-dimensional hydrodynamical model atmosphere and improved atomic and molecular data, Allende Prieto et al. (2002) and Asplund et al. (2004, 2005) revised the photospheric abundances for the most abundant heavy elements. The new abundances are markedly lower than the earlier values of Grevesse & Sauval (1998) or Grevesse & Noels (1993) and result in a total metallicity \( Z = 0.0122 \), which is about 30% lower than the GS98 value of \( Z = 0.0169 \). The standard solar models implemented with the revised abundances deviate significantly from the solar structure as determined by the helioseismic analysis. In particular, the convection zone is too shallow and the helium abundance in the convection zone too low, which leads to a poor match of the sound speed and density profiles in the Sun (e.g., Basu & Antia 2004; Bahcall et al. 2005a; Guzik et al. 2005; Turk-Chièze et al. 2004). These models also have a different core-structure than the Sun (Basu et al. 2007). The discrepancy has triggered a great deal of interest in reexamining solar models. Various groups have explored the possibility of adjusting the opacity, diffusion rate, and neon abundance of the models to see if the discrepancy caused by the low abundances might be resolved. However, the attempts have not been successful (e.g., Basu & Antia 2004; Bahcall et al. 2005a; Guzik 2006; Turk-Chièze et al. 2004; Antia & Basu 2006; Basu et al. 2007). Even invoking a different composition in the radiative interior through accretion, Castro et al. (2007) fail to remove the discrepancy.

An independent spectroscopic analysis by Ayres et al. (2006), which uses a thermal profile different from the one in Asplund et al. (2004), suggests an oxygen abundance that is closer to the previous value (i.e., GS98) than the most recent value (Asplund et al. 2005, hereafter AGS05). On the other hand, using three-dimensional models with free parameters, Sotoc-Navarro & Norton (2007) find an oxygen abundance that is slightly lower than that of Asplund et al. (2004). Thus, there appears to be considerable uncertainty in spectroscopic determinations of abundances, making it interesting to attempt an independent determination of abundances using the seismic data. Antia & Basu (2006) carried out a helioseismic analysis to determine the heavy-element abundance. Their results suggest that \( Z = 0.0172 \pm 0.002 \), which is in good agreement with the value of GS98. In that study the abundance was estimated by using a dimensionless sound speed gradient \( W(r) = (r^2 G m) \frac{dc^2}{dr} \), which deviates from the ideal-gas value of \(-\frac{1}{5}\) in the ionization zones of elements. The deviation can then be used to measure the heavy-element abundances. However, because the ionization zones of different heavy elements overlap, it is difficult to estimate their abundances separately. Hence, this analysis was used to estimate only the total heavy-element abundance \( Z \).

In this paper we conduct a systematic study of the effects of both the total abundance and the element mixture by using another dimensionless quantity, the adiabatic index \( \Gamma_1 \equiv (\partial \ln P/\partial \ln \rho) \), where \( s \) is the entropy. Our aim is to first identify the signatures due to the variation of the element abundance and then use this knowledge to assess the abundance difference between the Sun and the models.

Since the normally used EOSs, such as OPAL (Rogers & Nayfonov 2002) and MHD (Däppen et al. 1987, 1988; Hummer & Mihalas 1988; Mihalas et al. 1988), do not allow the user to change the chemical element mixture and are currently available only with a fixed mixture, they are not suitable for our investigation. Instead, following Antia & Basu (2006), we use the Eggleton et al. (1973) EOS with Coulomb corrections (Guenther et al. 1992; Christensen-Dalsgaard & Däppen 1992). This EOS (referred to as CEFF, the “Coulomb-corrected” EFF) includes all ionization states of 20...
elements. The ionization fractions are calculated using only the ground-state partition function. With the CEFF EOS we can specify the relative heavy-element abundance that we want to use, and hence study the influence of different heavy-element mixtures. Studies have also shown that CEFF gives a reasonable description of the thermodynamic structure of the solar material.

The reason for choosing the adiabatic index \( \Gamma_1 \) to probe chemical composition is that it is sensitive to both the total abundance and the element mixture. Although \( \Gamma_1 \) also depends on temperature and density, these dependences can be separated from compositional variations if the EOS is known. The disadvantage of using \( \Gamma_1 \) is that the errors in the EOS also affect \( \Gamma_1 \). The effects of the EOS errors and the composition errors might be entangled and difficult to separate. To systematically study the relation among \( \Gamma_1 \), EOS, and chemical composition, we constructed a number of test models with different abundances and EOSs. The test models and solar data used in this study are described in §2, the inversion method is explained in §3, and the results from the theoretical study and from using solar data are presented in §4. Finally, §5 summarizes the results of this study.

2. SOLAR DATA AND MODELS

We use solar oscillation frequencies obtained from data collected by the Michelson Doppler Imager (MDI) on board SOHO during its first 360 days of observations, from 1996 May 1 to 1997 April 25. We refer to this as the MDI360 set (Schou et al. 1998). We have chosen this set because, among the available helioseismic data for intermediate-degree modes, it was analyzed from the longest time series. Other sets have been obtained from much shorter (72 or 108 days) time series. Solar oscillation frequencies are known to change with activity, and a longer time series would involve observations from periods of increasing solar activity, which would change the frequencies. It is a well established that solar oscillations frequencies increase with solar activity. However, it is also known that the increase is such that it does not reflect a change in structure of the solar interior (Basu 2002). The MDI360 set was obtained from a time series when solar activity was low. This helps minimize solar-cycle–related frequency shifts, while providing lower errors than the 72 day sets. From the estimated errors in our results it is clear that the errors are acceptable for our work. It may be possible to improve the results by individually inverting all 72 day sets available for the entire solar cycle and then averaging the results.

To study the effects of the chemical composition, we use a series of models constructed with different compositions. Specifically, we constructed several models that have the same relative mixture of heavy-element abundances (either GS98 or AGS05) but different \( Z/X \) values, ranging from 0.0165 to 0.0245, to examine the effect of metallicity. We also made models that have the same \( Z/X \), but with the abundance of some chemical elements either increased or decreased. It should be noted that the opacity tables or EOS for a solar model are calculated with a specified mixture of heavy elements that does not change with depth. The value of \( Z \), the on the other hand, is determined by specified value of \( Z/X \) at the surface and varies with depth due to diffusion. To increase the relative abundance of one element with respect to the rest without changing \( Z/X \), the abundances of other elements is reduced to account for this adjustment in the mixture. Another alternative is to increase the abundance of one element and increase \( Z/X \) appropriately to keep the abundance of other elements at standard values. In some cases (e.g., for neon) we have tried both options and the results can be compared. These models are static models calculated by solving the equations of stellar structure at a fixed age. Inputs needed are the hydrogen and heavy-element abundance profiles (although not the absolute abundances). We have used the hydrogen and heavy element composition profiles from a solar model of Brun et al. (2002). The model incorporates diffusion of helium and heavy elements and mixing in the tachocline region. The composition profiles are scaled to the required values of \( X \) and \( Z \).

The exact structure of the models is not crucial for our work, since we are looking at the intrinsic \( \Gamma_1 \) difference, which depends only on the EOS and heavy-element abundances. This difference does not depend on the difference in structure between the Sun and the model, and nor does it depend on the difference in the helium abundance. Consequently, we do not make any special effort to get models with correct sound speed profile or the convection zone depth or the helium abundances. Nevertheless, these quantities for all models used in this work are listed in Table 1. In the table \( r_o \) is the position of the base of the convection zone. These models use the OPAL opacity table computed for the appropriate heavy-element mixture and use low-temperature opacities from Kurucz (1991). The purpose of these models is to examine the signatures in the adiabatic index of differences in the relative mixture of heavy elements. Since \( \Gamma_1 \) reflects the effects of both the EOS and the chemical composition, we constructed two models implemented with the latest version of the OPAL EOS. These models are referred to as 245OPAL and 165OPAL, and have \( Z/X = 0.0245 \) and 0.0165, respectively. Because they are constructed with the OPAL EOS, the heavy-element mixture contains only four elements (C, N, O, and Ne). In order to examine the effects resulting from the difference between the CEFF and OPAL EOSs, we have constructed two CEFF models that use the same mixture and \( Z/X \) as the OPAL models. These models with OPAL mixtures use the standard OPAL opacity tables to ensure that the difference is only due to the EOS and not to opacities. The properties of all models used in this paper are summarized in Table 1.

The relative difference in \( \Gamma_1 \) can be determined by helioseismic inversion. Although the complete set of oscillation modes can be computed for these models, we only used the modes that are also present in the observed data for the inversions between models in order to get realistic results. For the purpose of assessing the effect of the errors in the observed frequencies, we added random errors that are consistent with those in the observed frequencies to the model frequencies. The observational data set used for our mode selection is the aforementioned MDI360.

3. INVERSION PROCEDURES

The frequencies of solar oscillation modes depend on the solar structure. The starting point of helioseismic inversions is the linearization of the oscillation equations around a known solar model (the so-called reference model) using the variational principle. The frequency differences can then be related to the relative variations in sound speed \( (c) \) and density \( (\rho) \) either between two models or between the Sun and the reference model. The relation between the differences in frequency and these two variables (i.e., \( c \) and \( \rho \)) can be written as (Dziembowski et al. 1990; Antia & Basu 1994)

\[
\frac{\delta \omega_i}{\omega_i} = \int_0^R K_{c, \rho}^j (r) \frac{\delta c^2}{c^2} (r) \, dr + \int_0^R K_{\rho, \rho}^i (r) \frac{\delta \rho}{\rho} (r) \, dr + \frac{F_{surf}(\omega_i)}{Q_i} + \epsilon_i,
\]

where \( c \) is the adiabatic sound speed, \( K' \) are the kernels, and \( \epsilon_i \) is the observational error in \( \delta \omega_i/\omega_i \). \( F_{surf}(\omega_i)/Q_i \) represents the effect of uncertainties in the model close to the surface and is usually
called the “surface term.” Here $Q_i$ is a measure of the mode inertia.

The structure variables $c^2$ and $\rho$ in equation (1) can be converted into any other pair of independent variables. Specifically, for our study, we convert the variables $c^2$ and $\rho$ to $\Gamma_1$ and $\rho$ by using the relation $c^2 = \Gamma_1 P/\rho$ and the equation of hydrostatic equilibrium, which links the pressure and density. While the variable differences in equation (1) are the differences calculated at the same fractional depth, we are interested in the difference of $\Gamma_1$ compared at the same $P$ and $\rho$ (or any pair of thermodynamic variables). It is because comparing $\Gamma_1$ at the same $P$ and $\rho$ removes the effects due to the discrepancies in the main macrophysical properties (e.g., the solar structure variables $P$ and $\rho$), and hence such $\delta \Gamma_1/\Gamma_1$ reflects only the microphysical (i.e., EOS) and composition discrepancies. To calculate the $\delta \Gamma_1/\Gamma_1$ that represents only the microphysical discrepancies, which we call the “intrinsic” $\Gamma_1$ difference, we followed the technique proposed by Basu & Christensen-Dalsgaard (1997):

$$
\frac{\delta \Gamma_1}{\Gamma_1} = \left( \frac{\partial \ln \Gamma_1}{\partial \ln P} \right)_{Y, \rho} \frac{\delta P}{P} + \left( \frac{\partial \ln \Gamma_1}{\partial \ln \rho} \right)_{Y, \rho} \frac{\delta \rho}{\rho} + \left( \frac{\partial \ln \Gamma_1}{\partial Y} \right)_{p, \rho} \delta Y + \frac{\delta \Gamma_{1, \text{int}}}{\Gamma_1},
$$

where $Y$ is the helium abundance by mass, and $\delta \Gamma_{1, \text{int}}/\Gamma_1$ is the relative difference caused by the microphysical discrepancies. Consequently, the linear relation between $\delta \omega_i/\omega_i$ and $\delta \Gamma_{1, \text{int}}/\Gamma_1$ becomes

$$
\frac{\delta \omega_i}{\omega_i} = \int_0^R K_{i, u}(r) \frac{\delta u}{u} (r) dr + \int_0^R K_{i, \rho}(r) \delta \rho (r) Y(r) dr + \int_0^R K_{i, \Gamma_{1, \text{int}}}(r) \frac{\delta \Gamma_{1, \text{int}}}{\Gamma_1} (r) dr + F_{\text{surf}}(\omega_i) + \epsilon_i,
$$

where $\mu = P/\rho$ is the isothermal sound speed. The functions $K'(r)$ are known functions of the reference model.

We use the method of optimally localized averages (OLA) to determine $\delta \Gamma_{1, \text{int}}/\Gamma_1$ by “inverting” equation (3). The OLA technique (Backus & Gilbert 1968) attempts to produce a linear combination of data such that the resulting resolution kernel, or “averaging kernel,” is suitably localized, while simultaneously controlling the error estimates. Rabello-Soares et al. (1999) discuss the formulation of OLA inversion to study solar structures and how one may go about determining the inversion parameters for this type of an inversion.

The $\delta \Gamma_{1, \text{int}}/\Gamma_1$ in the formulation above has two contributions, one from differences in the EOS and another from differences in $Z$:

$$
\frac{\delta \Gamma_{1, \text{int}}}{\Gamma_1} = \frac{\delta \Gamma_{1, \text{EOS}}}{\Gamma_1} + \frac{\partial \ln \Gamma_1}{\partial Z} \delta Z.
$$

In principle, it should be possible to separate out the two parts, although in practice we find that the limited number of mode frequencies makes it difficult to get a reliable separation of the two quantities by inversions. Hence, we limit ourselves to $\delta \Gamma_{1, \text{int}}/\Gamma_1$ in this work, rather than its two components.

4. RESULTS

4.1. What the Models Tell Us

The first part of our study involves pairs of models and allows us to investigate the individual as well as combined effects on $\Gamma_{1, \text{int}}$ caused by differences in $Z/X$, the relative abundances of different elements, and the EOS. This approach also allows us to test our inversion technique. In order to do so, we use the frequency differences between two known models to infer $\delta \Gamma_{1, \text{int}}/\Gamma_1$ between them and then compare it with the known value. In this way we can systematically investigate features resulting from different factors (e.g., differences in EOS, metallicity, relative composition, etc.).
we therefore concentrate only on the results for large errors or very poor resolution or both. In the rest of the paper depth in the two models differs significantly, which results in either in this region. This problem is more severe if the convection zone







differences caused by Z/X. We can only probe the smaller parallel shifts at \( r < 0.92 \, R_s \). We find that for small differences in Z/X, as in the case of the models GS245 and GS230, \( \delta \Gamma_{1,\text{int}}/\Gamma_1 \) between the models below \( r \approx 0.92 \, R_s \) is comparable to the estimated errors. In other words, the inversion cannot distinguish between two models if the difference in Z/X is less than \( \pm 6\% \).

The EOSs used to construct our solar models are not perfect and are known to have differences with respect to the actual EOS of the solar matter (see, e.g., Basu & Christensen-Dalsgaard 1997; Basu et al. 1999). Hence, to interpret results obtained with solar data, we need to know what type of differences are seen in \( \Gamma_{1,\text{int}} \) because of EOS differences alone. For this purpose, we looked at the difference in \( \Gamma_{1,\text{int}} \) between models constructed with the OPAL EOS and CEFF EOS. To ensure that the revealed features are solely due to the differences in the EOSs, we use CEFF models OM245 and OM165 that have the same heavy-element mixture as that implemented in OPAL EOS tables. Figure 2b shows that for the same difference in Z/X between a pair of models, the difference in \( \Gamma_{1,\text{int}} \) does not depend too much on the EOS used to construct the pair. The difference between OPAL and CEFF model pairs is of the order of the inversion errors. Figure 2c shows the difference in \( \Gamma_{1,\text{int}} \) between different EOSs for models with the same composition. We can see that although the difference between the two EOSs causes very large features near the surface (\( r > 0.93 \, R_s \)), the two EOSs are almost indistinguishable below \( r \approx 0.90 \, R_s \). We can very easily invert the \( \Gamma_{1,\text{int}} \) differences between the two models in the region that is permitted by our mode sets. The fact that the two EOSs are indistinguishable throughout
most of the convection zone also means that any solar result obtained using these CEFF models will be close to the ones obtained using OPAL models, as long as we do not study regions very close to the surface.

Figure 3a shows what happens when the abundance of any one element is increased. As can be seen, the different elements leave a distinct pattern. The changes shown in the figure are large enough to be seen in the inverted results. It should be noted that while for the model shown the abundances were changed by a factor of 2, the oxygen abundance in the AGS05 and GS98 tables differs only by a factor of about 1.5, and the carbon abundance differs by a factor of about 1.35. The curve for neon is of interest, since Antia & Basu (2005) had suggested that increasing the neon abundance by a factor of about 4 may cause the solar models constructed with the AGS05 abundances to have the correct position of the convection-zone base. Bahcall et al. (2005b) suggested a change by factor of between 2.5 and 3.5 to bring the convection-zone base and the convection zone helium abundance back into agreement with solar values. Similar results have been obtained by Zaatri et al. (2007) using the scaled small-frequency separation in low-degree modes. We see that adding neon makes the $\delta\Gamma_{1\text{,int}}$ positive below 0.9 $R_\odot$ and negative between 0.9 and 0.95 $R_\odot$. The results shown in Figure 3a are, however, somewhat disappointing in that they suggest that we could wipe out detectable changes in $\Gamma_{1\text{,int}}$ by, e.g., increasing the abundance of oxygen and carbon simultaneously. We could also increase the signal by reducing one element and increasing another; in other words, there may be multiple chemical compositions (with the same $Z/X$) that may produce almost indistinguishable $\Gamma_1$ in the convection zone. This indicates that the chemical composition of the solar convection zone may not be uniquely determined from $\Gamma_{1\text{,int}}$. The cancellation effects may be seen in the $\Gamma_{1\text{,int}}$ differences between AGS05 and GS98 models with the same $Z/X$ (Fig. 3b). Another reason that the AGS05 and GS98 models with the same $Z/X$ have such similar $\Gamma_{1\text{,int}}$ is that the relative abundances of the important elements such as C, N, O, and Ne between the GS98 and AGS05 abundance tables are almost the same, since these elements are reduced by almost identical factors in the AGS05 tables relative to the GS98 values. In general, the EOS is determined by the number density of different ions, and the relative abundance by number of heavy elements with respect to hydrogen is of the order of $10^{-4}$. Hence, the typical differences in $\Gamma_1$ due to variations in $Z$ are of this order, as can be seen from Figures 1 and 2. In the case of a difference between GS98 and AGS05 mixtures with the same $Z$, the differences in relative abundances are much smaller, and hence the effect is also very small.

4.2. Results with Solar Data

The inversion results of $\delta\Gamma_{1\text{,int}}/\Gamma_1$ between the Sun and various reference models are presented in the Figures 4, 5, and 6. The inversion results using the solar data MDI360 and various models

![Fig. 3.— Effect of changes in the relative heavy-element abundances keeping $Z/X$ the same. (a) Effect when one of the elements is enhanced by a factor of 2. (b) $\Gamma_{1\text{,int}}$ differences between two models, both with $Z/X = 0.0165$, but one constructed with the GS98 relative heavy-element abundance and the other the AGS05 relative heavy-element abundance.](image1)

![Fig. 4.— Differences in $\Gamma_{1\text{,int}}$ between the Sun and different models obtained by inverting the frequency differences between the Sun and the models. The models are described in Table 1.](image2)

![Fig. 5.— $\Gamma_{1\text{,int}}$ differences between the Sun and models with the abundance of some elements changed. The value of $Z/X$ is the same for all models.](image3)
enhanced models are the ones proposed by Antia & Basu (2005) to get models that show that the Sun has a higher total metallicity (i.e., the total metallicity of AGS05) are shown in Figure 4. The first notable inference is that models with $Z/X = 0.0165$ are more discrepant with respect to the Sun than models with $Z/X = 0.0245$ or 0.023. This is yet another result that shows that the Sun has a higher $Z/X$ than that given by AGS05. The difference between the Sun and models with $Z/X = 0.0165$ appears to be the same regardless of whether the models were constructed with the GS98 or AGS05 relative heavy-element abundances. This is not surprising given the results in Figure 3b discussed above. Thus, using our inversions we cannot say whether the GS98 or AGS05 relative heavy-element mixture is closer to that in the Sun. We can only say that the AGS05 model with $Z/X = 0.0165$ does not match the solar $\Gamma_1$. For OPAL models too the agreement with the Sun is worsened for $Z/X = 0.0165$. It may be noted that in this case the OPAL models use a very different heavy-element mixture as compared to other models, which would also account for some of the differences.

Even though $\delta \Gamma_{1,\text{int}}/\Gamma_1$ between the Sun and the models with $Z/X = 0.0245$ and 0.023 are visibly different, the difference in the region $0.77 R_\odot < r < 0.90 R_\odot$ is comparable to the magnitude of the inversion errors. Despite the difference between CEFF and OPAL, Figure 4 shows that the solar models implemented with either the OPAL or CEFF EOS favor the previous heavy-element abundance (i.e., $Z/X = 0.0245$ and 0.0230) over the latest, lower value $Z/X = 0.0165$. Hence, based on the precision of current helioseismic data, models with either $Z/X = 0.0245$ or $Z/X = 0.0230$ are equally plausible.

We find that for the same value of $Z/X = 0.0245$, enhancing the carbon abundance of a model brings it in closer agreement with the Sun, as is shown in Figure 5. As can also be seen, enhancing the oxygen abundance actually worsens the agreement, particularly when compared to a model with a normal GS98 relative heavy-element abundance. Given that one of the suggestions to improve models constructed with AGS05 abundances was to increase the neon abundance (Antia & Basu 2005; Bahcall et al. 2005b), we turn our attention to models with enhanced neon abundance. Antia & Basu (2005) had suggested a neon enhancement by a factor of about 4. Keeping all other elemental abundances at the AGS05 level, this would imply a $Z/X$ of 0.0209. We first look at models with $Z/X = 0.0209$ with and without neon enhancement to disentangle the effects of neon enhancement from those of increased $Z/X$. The results are shown in Figure 6a. What we see is that enhancing neon actually worsens the agreement with the Sun in the deeper part of the convection zone but makes the agreement better above about 0.90 $R_\odot$. However, the neon-enhanced models are better than models with the unmodified AGS05 relative abundances and the AGS05 total metallicity of $Z/X = 0.0165$, as can be seen in Figure 6b. We can also see that the model with neon enhanced by a factor of 2 and C, N, and O increased by an amount corresponding to their 1 $\sigma$ errors in the AGS05 tables (which was another suggestion of Antia & Basu 2005) also results in a better agreement than the plain AGS05 model. However, neither of these models fares as well as the model with GS98 abundances and abundance ratios. Thus, it appears that the improvement relative to AGS05 models is more due to the increased $Z/X$ in the neon-enhanced models than to the enhancement of neon. Thus, while enhancing neon brings the position of the convection-zone base of the solar models in agreement with that of the Sun, the discrepancy in the adiabatic index remains to a large extent.

5. SUMMARY

In this work we examine the difference in the adiabatic index $\Gamma_1$ between the Sun and various solar models. Our aim is to determine whether this index can be used to determine the heavy-element abundances in the convection zone. In order to examine the effect of heavy-element abundances on $\Gamma_1$, we use the so-called intrinsic $\Gamma_1$ differences between the Sun and the models, i.e., differences in $\Gamma_1$ at the same pressure, density, and helium abundance. What remains when these dependences are removed are $\Delta_1$ differences caused by differences in the EOS and by differences in the heavy-element abundances. For convenience we call these the $\Gamma_{1,\text{int}}$ differences. These differences do not depend on differences in structure or helium abundance. We first use a series of models to test the sensitivity of the inversions and also to determine separately what type of $\Gamma_{1,\text{int}}$ differences are caused by differences in the total metallicity (i.e., $Z/X$) differences in the EOS, and differences in the relative abundances of heavy elements. From the study of models we find that our inversion results are reliable in the region $0.77 R_\odot < r < 0.92 R_\odot$. In this region $Z/X$ differences cause an almost parallel shift of the $\delta \Gamma_{1,\text{int}}$ curve. EOS differences appear only for $r > 0.90 R_\odot$. Enhancing the abundance of one element gives humps at different radii, although if two or more elements are changed, the features can largely cancel out in some cases. Because of these effects, it is not possible to distinguish between the relative heavy-element abundances of the GS98 and the AGS05 mixtures.

Using solar data we find that models constructed with $Z/X = 0.0165$ (i.e., the total metallicity of AGS05) are not consistent with the Sun. The $\Gamma_{1,\text{int}}$ for these models are lower than that of the Sun in the region of the convection zone that we can study. Models with $Z/X = 0.023$ (i.e., GS98 total metallicity) fare much better and confirm the results of Antia & Basu (2006), who used
the dimensionless sound speed gradient inside the convection zone. There are, however, discrepancies between the $Z/X = 0.023$ models and the Sun close to the surface, which probably point to differences in the EOS between our models and the Sun. These results thus add to the list of discrepancies between models with AGS05 metallicities and the seismic data. As mentioned earlier, the discrepancy between the position of the convection-zone base was the first to be noted. Since then we have known that the models are also deficient in the core (Basu et al. 2007; Zaatri et al. 2007), as well as in the dimensionless gradient of the sound speed in the convection zone (Antia & Basu 2006). Now we can add the discrepancy in $\Gamma_1$ in the convection zone to that list. It may be noted that the discrepancy in the depth of the convection zone and the helium abundance in the convection zone or in the core structure are due to differences in opacity caused by reduction in $Z$. In principle, these differences can be resolved if the opacities are revised upward by 10%–20% (Basu & Antia 2004; Turck-Chize et al. 2004; Bahcall et al. 2005a). However, the discrepancies in the dimensionless sound speed gradient or $\Gamma_1$ will not be affected by increasing the opacity. Thus, it is clear that revising the opacity of solar material will not resolve all discrepancies. While the dimensionless sound speed gradient does depend weakly on the density profile or convection zone depth, the difference $\delta\Gamma_{1,\text{int}}$ studied in this work is obtained by correcting for these differences and hence depends only on the EOS and heavy-element abundances. It may be argued that changing $Z$ would also change $X$ or $Y$, which could indirectly affect $\Gamma_1$. This effect is not expected, since in the region we are studying, $r < 0.92 R_\odot$, hydrogen and helium are fully ionized and will not contribute to departure of $\Gamma_1$ from 5/3. The exact location of ionization zones of various elements depends on treatment of the EOS, and there could be some error in these due to uncertainties in the EOS. However, since two independent EOSs give similar results, we expect that these errors are not too large. Judging by the nature of the $\Gamma_{1,\text{int}}$ differences between AGS05 models and the Sun, EOS errors are unlikely to explain the difference. From our results we can estimate the $Z/X$ value in the Sun to be around 0.023.

Looking at the differences in intrinsic $\Gamma_1$ (Fig. 4) it appears that the discrepancy in the region $r > 0.85 R_\odot$ is probably due to the EOS. We may expect the EOS discrepancy to reduce in deeper layers as we move away from the He ii ionization zone. In view of this discrepancy it may be difficult to determine the value of $Z/X$, but a value close to 0.023 appears to be preferred over smaller values. It is possible to further decompose $\delta\Gamma_{1,\text{int}}$ in terms of EOS contribution and another contribution from $Z$ or even $\Gamma_1$, the abundances of each heavy element. Because of the discrepancy in the EOS and the fact that the ionization zones of different heavy elements overlap, it may be difficult to determine individual $Z_i$, although it may still be possible to determine total $Z$.

Since an increased neon abundance has been suggested (Antia & Basu 2005; Bahcall et al. 2005b) as a possible solution to resolve the discrepancy between solar models and seismic data, we examine the models with enhanced neon abundance. We find that the models with the abundance of neon enhanced with respect to the AGS05 values do better than the AGS05 models, but the improvement appears to be a result of an increase in $Z/X$ rather than an increase in the neon abundance—increasing neon while keeping $Z/X$ fixed makes the agreement between the Sun and models worse. Thus, it appears that increasing neon abundance alone does not help in resolving the discrepancy. It is necessary to increase the abundances of other elements such as oxygen too. Increasing Ne abundance over the GS98 abundances may lead to better agreement with seismic data. But it is difficult to make any definitive statement, as it will also depend on the EOS. It may help if more sophisticated EOS tables are available for different element mixtures.

Our investigation is handicapped by the lack of high-degree modes in the set. Differences in $\Gamma_{1,\text{int}}$ caused by errors in the EOS or heavy-element abundances are largest above 0.95 $R_\odot$. High-precision frequencies of high-degree modes are needed to investigate the effects of EOS and abundances on the adiabatic index properly. However, the study indicates that the heavy-element abundance of the Sun is significantly higher than that suggested by Asplund et al. (2005) and close to that of Grevesse & Sauval (1998).

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