Direct N-body Simulations of Tidal Disruption Rate Evolution in Unequal-mass Galaxy Mergers

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Abstract

The hierarchical galaxy formation model predicts supermassive black hole binaries (SMBHBs) in galactic nuclei. Due to the gas poor environment and the limited spatial resolution in observations they may hide in the center of a galaxy. However, a close encounter of a star with one of the supermassive black holes (SMBHs) may tidally disrupt it to produce a tidal disruption event (TDE) and temporarily light up the SMBH. In a previous work, we investigated direct N-body simulations with the evolution of TDE rates of SMBHB systems in galaxy mergers of equal mass. In this work we extend the investigation to unequal-mass mergers. Our results show that, when two SMBHs are far away from each other, the TDE rate of each host galaxy is similar as in an isolated galaxy. As the two galaxies and their SMBHs separation shrink, the TDE rate increases gradually until it reaches a maximum shortly after the two SMBHs become bound. In this stage, the averaged TDE rate can be enhanced by several times to an order of magnitude relative to isolated single galaxies. Our simulations show that the dependence of the TDE accretion rate on the mass ratio in this stage can be well fitted by power-law relations for both SMBHs. After the bound SMBHB forms, the TDE rate decreases with its further evolution. We also find that in minor mergers TDEs of the secondary SMBH during and after the bound binary formation are mainly contributed by stars from the other galaxy.

Key words: galaxies: evolution – galaxies: interactions – galaxies: kinematics and dynamics – galaxies: nuclei – methods: numerical

1. Introduction

Supermassive black hole binaries (SMBHBs) may form as a consequence of hierarchical galaxy mergers in the Λ cold dark matter cosmology (Begelman et al. 1980; Volonteri et al. 2003). Observations indicate that many massive galaxies harbor a supermassive black hole (SMBH) in the center, and there are close correlations between central SMBHs and their host galaxies (Magerrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002; Kormendy & Ho 2013). Because most of the massive galaxies may have experienced at least one major merger and many minor mergers in their evolution history, the existence of two or even multiple SMBHs in merger remnants should be expected. Taking into account the close link between SMBH and its host galaxy, the evolution of the SMBH pairs/SMBHBs in merging galaxies is of fundamental importance.

Whether two SMBHs in merging galaxies can sink down to the center of the remnant to form a bound binary, and how the binary evolves has been debated for many years. After a galaxy merger two SMBHs with their nuclear star clusters around them approach each other first through dynamical friction, until they form a gravitationally bound binary. But how the SMBHB evolves after that is not quite certain. If an SMBHB shrinks its orbit to several hundreds of Schwarzschild radii, it will finally coalesce due to energy loss by gravitational wave (GW) radiation in less than a Hubble time. Since dynamical friction is inefficient after the SMBHB became bound, additional processes such as few-body interactions between the SMBHB and surrounding stars or interaction with its gaseous environment are required to extract sufficient amounts of energy and angular momentum from the SMBHB to lead to the final coalescence (Begelman et al. 1980). Otherwise the SMBHB may stall at a relatively large separation for more than the Hubble time. In a gaseous environment, a SMBHB could coalesce more easily (Gould & Rix 2000), while in a gas poor environment, it has been argued that few-body superelastic scatterings between stars and the SMBHB are not efficient enough to lead to coalescence in less than a Hubble time (Begelman et al. 1980; Milosavljević & Merritt 2003; Berczik et al. 2005). This has been named the “final parsec problem” in some publications; if true in general, SMBHBs should be ubiquitous in most galaxies. But it has been shown that the so-called final parsec problem is more like an artifact of special conditions in the model, such as strict spherical symmetry, no rotation, no tidal perturbations, and no phases of gas inflow. However, both semi-analytical calculations and direct N-body simulations later indicated that under realistic conditions (e.g., rotating or triaxial galactic nuclei) the star-SMBHB scattering process is efficient enough to avoid the final parsec problem and to let many SMBHB coalesce in a relatively short time (Yu 2002; Merritt & Poon 2004; Berczik et al. 2006; Khan et al. 2011; Preto et al. 2011).

Observational evidence for SMBHBs, in particular statistical data on their occurrence, could tell how efficiently they would shrink and coalesce. Currently only a few candidates for SMBHB are known through indirect electromagnetic
detections such as periodic variabilities, double-peaked emission line profiles, and so on (Lehto & Valtonen 1996; Shen et al. 2013; Liu et al. 2014; Yan et al. 2015). Getting to sub-parsec separations in distant galaxies, especially in gas poor environments, is a considerable challenge for spatial resolution. But compact SMBHBs could be GW sources, which may be detected by the planned space based Laser Interferometer Space Antenna\(^7\) and ongoing Pulsar Timing Array (PTA) (Foster & Backer 1999; Verbiest et al. 2016). However, there is still no detection from PTA even after the characteristic amplitude of the GW background has been constrained to \(\sim 10^{-15}\) (Shannon et al. 2015; Arzoumanian et al. 2016; Babak et al. 2016). That could be caused by overestimation of either the GW characteristic amplitude of SMBHBs or the event rates (Shannon et al. 2015; Sesana et al. 2016). Actually, how many SMBHBs can be detected by GW instruments is still uncertain. The detection rate depends on many unclear factors, such as the merger rate, the mass ratio distribution, the lifetime of the binary system, etc. In another words, it is susceptible to the evolution of SMBHBs. Therefore observations with statistical data on SMBHB before coalescence are needed to reveal the internal properties of the evolution of SMBHB.

Particularly in gas poor environments effective probes that can help us to investigate SMBHBs are needed. Tidal disruption events (TDEs) are potentially such a probe. A star can be torn up by the tidal force when it closely encounters an SMBH. The critical distance for that to happen is defined as the tidal radius (Hills 1975; Rees 1988; Evans & Kochanek 1989; Guillochon & Ramirez-Ruiz 2013)

\[
r_t \approx \mu r_s (M_{\text{BH}}/m_*)^{1/3},
\]

where \(\mu\) is a dimensionless parameter of order unity (depending on the stellar structure), \(r_s\), \(m_*\), and \(M_{\text{BH}}\) are the stellar radius, the stellar mass, and the mass of the BH, respectively. The bound fraction of the debris of the disrupted star forms an accretion disk around the dormant SMBH and temporarily illuminates it, accompanied by a flare from \(\gamma\)-ray to radio bands, which has been confirmed by many observations (Gezari 2013; Komossa 2015, and references therein). Similarly, there will be TDEs from SMBHBs too. Due to the gravitational perturbation from the companion BH, the TDE light curve of the SMBHB may have characteristic drops, which has been confirmed by both theoretical and numerical studies (Liu et al. 2009; Coughlin et al. 2017; Vigneron et al. 2018), and a candidate also has been found in observation (Liu et al. 2014).

Though there are only tens of candidates and one candidate for TDEs in single SMBHs and in SMBHBs, respectively, theoretical estimates of tidal disruption rates (TDRs) have existed for long time and by many authors, for both the single SMBH and the SMBHB. The theoretical TDR of a single SMBH in isolated spherical galaxies is \(\sim 10^{-6}–10^{-4}\) yr\(^{-1}\) per galaxy, and can be enhanced a few times in flattened systems (Magorrian & Tremaine 1999; Syer & Ulmer 1999; Wang & Merritt 2004; Vasilyev & Merritt 2013; Stone & Metzger 2016). TDRs of SMBHB models, on the other hand, can be orders of magnitude lower if the binary is compact and has been embedded in a spherical isotropic galactic core dominated by two-body relaxation (Chen et al. 2008). However, the perturbation from the companion SMBH and the triaxial stellar distribution induced by the galaxy merger can actually enhance the TDR significantly (Ivanov et al. 2005; Chen et al. 2009; Wegg & Nate Bode 2011). According to the theoretical analysis made by Liu & Chen (2013), in merging galaxies before the formation of a bound SMBHB, the TDR can be enhanced up to \(\sim 10^{-2}\) yr\(^{-1}\). However, during and after the binary formed, the interaction between the stellar environment and the SMBHB is so rapid and presumably chaotic that both analytical estimations and scattering experiments are of limited value to reveal the underlying physical processes.

Rapid development of computing technology in hardware and software, notably the use of general purpose graphics processing units for numerical computations in science, has promoted direct \(N\)-body simulations of star clusters and galactic nuclei. This has led to many models based on direct \(N\)-body simulations about TDR in spherical or axisymmetric nuclear stellar clusters around a single SMBH (Baumgardt et al. 2004; Zhong et al. 2014, 2015; Panamarev et al. 2018). While numerical investigations also exist about the dynamical evolution of two SMBHs in merging galaxies (Berczik et al. 2006; Khan et al. 2011, 2018; Preto et al. 2011), models studying the TDE evolution in merging galaxies and for SMBHB are very rare. (Li et al. 2017, hereafter Paper I) has investigated the TDR evolution of two SMBHs and SMBHBs in equal-mass mergers, and found that the TDR can be boosted significantly by up to two orders of magnitude during the phase when two SMBHs form a binary. But their results are only valid for equal-mass mergers, which should be rare compared to the more common minor mergers (Fakhouri & Ma 2008; Hopkins et al. 2010). Just recently, Pfister et al. (2019) studied the TDR evolution in gaseous minor merger systems through hydrodynamical simulations, and found that the TDR of the secondary BH will be temporarily enhanced. However, their estimation of TDR is based on analytical models with a spherical density distribution assumption, which is not the case when SMBHB is forming, and it cannot resolve scattering between stars and SMBHB.

Khan et al. (2012) have shown that the so-called “final parsec problem” does not exist even for minor mergers, due to the merger-induced triaxiality. But they found that compared with major mergers, it takes a longer time to form a compact bound SMBHB in a minor merger system. We showed in Paper I for major mergers that the TDR can be boosted significantly during the stage of the formation of a bound SMBHB (which we called “phase II” in Paper I), but only for a very short period of time. Hence, in total the phase II in our previous work only contributed a quarter of all TDEs in the entire evolution. In this paper we study the TDR in minor mergers, which has not been considered in the previously cited papers. Minor mergers are more frequent, and there is evidence from previous work that the phase II with enhanced TDR may last longer. Therefore we speculate at this stage that minor mergers could contribute a significant fraction of TDEs in total, comparable to or even more than for major mergers. So, in this paper, we focus on unequal-mass mergers.

This paper is organized as follows. In Section 2, we describe our simulation method and models. The results for the evolution of the disruption rate are given in Section 3. In Section 4, we briefly introduce how to estimate the disruption rate analytically in unequal-mass mergers. Then we discuss how to extrapolate our simulation results to a few typical cases.

\(^7\)https://www.elisascience.org/
realistic minor mergers, and provide a short discussion about
the implications of the observations in Section 5. A short
summary is presented in Section 6.

2. Numerical Simulations

All stars that can be disrupted by an SMBH will have orbits
inside a cone region in the phase space, which is the so-called
"loss cone." In principle, how many stars are inside the loss
cone and how fast stars outside the loss cone can be scattered
inwards will directly determine the TDR. Therefore, as
demonstrated in Paper I, since different phases have different
dominated loss cone refilling mechanisms, the TDR evolution
in equal-mass mergers can be estimated by combining
analytical models with numerical N-body simulations. Simi-
larly, this strategy also works for minor mergers. Here we
briefly introduce it. More details can be found in Paper I.

Our model assumes that there are two unequal-mass galaxies
or nuclei, and each of them has an SMBH in the center. Here
$M_{\text{BH1}}$ and $M_{\text{BH2}}$ represent the mass of the primary and the
secondary SMBH, respectively, $N_1$ and $N_2$ represent the
number of stars in the larger and the smaller galaxy/nucleus,
respectively, and $q = M_{\text{BH2}}/M_{\text{BH1}}$ is the mass ratio between
the two SMBHs. For simplicity, we assume that all the stars in
merging galaxies have the same mass, and the mass ratio
between a SMBH and its host galaxy/nucleus $f = M_{\text{BH}}$
$/M_G = 0.01$ is the same for both galaxies/nuclei, namely,
the mass ratio of two galaxies/nuclei is $M_{G2}/M_{G1} = N_2/N_1$
$= M_{\text{BH2}}/M_{\text{BH1}} = q$. In our models, we fix the mass of the primary
galaxies/SMBHs, and only vary the mass of the secondary
galaxies/SMBHs.

For simplicity, a spherical Dehnen model has been adopted
to represent the stellar distribution of each of the galaxies/
nuclei (Dehnen 1993). And the density profile of this Dehnen
model is

$$\rho(r) = \frac{3 - \gamma}{4\pi} \frac{M_a}{r^\gamma (r + a)^{4-\gamma}},$$

(2)

where $a$, $M$, and $\gamma$ denote the scaling radius, the total mass of
galaxy/nucleus, and the density profile index, respectively. In
accordance with Paper I, we adopt the units $G = M = M_{G1} = a = 1$
hereafter. Similar to Paper I, the relation between numerical and physical quantities can be derived as

$$[T] = \left(\frac{GM}{a^3}\right)^{-1/2} = 1.491 \times 10^7 (2^{1/\gamma} - 1)^{3/2}$$

$$\times \left(\frac{M}{10^9M_\odot}\right)^{-1/2} \left(\frac{r_{1/2}}{1 \text{ kpc}}\right)^{3/2} \text{ yr},$$

(3)

$$[V] = \left(\frac{GM}{a}\right)^{1/2} = 65.58 \times (2^{1/\gamma} - 1)^{-1/2}$$

$$\times \left(\frac{M}{10^9M_\odot}\right)^{1/2} \left(\frac{r_{1/2}}{1 \text{ kpc}}\right)^{-1/2} \text{ km s}^{-1},$$

(4)

$$[R] = a = (2^{1/\gamma} - 1) \left(\frac{r_{1/2}}{1 \text{ kpc}}\right) \text{ kpc},$$

(5)

$$[M] = M/[T] = 67.07 \times (2^{1/\gamma} - 1)^{-3/2}$$

$$\times \left(\frac{M}{10^9M_\odot}\right)^{3/2} \left(\frac{r_{1/2}}{1 \text{ kpc}}\right)^{-3/2} M_\odot \text{ yr}^{-1}. \quad (6)$$

Since we want to focus on how TDR evolves in unequal-
mass mergers with different mass ratios, here we fix the density
profile index to $\gamma = 1.0$. We select a fiducial model with
$M_{G1} = 4 \times 10^9 M_\odot$, $M_{\text{BH1}} = 4 \times 10^7 M_\odot$, half mass-radius
of primary galaxy/nucleus $r_{1/2} = 1 \text{ kpc}$, and $q = 0.25$.

Due to the huge number of stars in a typical galaxy, even
with the most powerful computer in the world, it is still
impossible to do a one-to-one direct N-body simulation. The
maximum particle number that can be managed at a reasonable
cost nowadays is only a few million. That means we have to
use millions of particles to simulate a galaxy, and then make
extrapolations. With this limited particle resolution, there are
some artificial effects that should be taken into account. For
instance, the two-body relaxation timescale $T_r$ is roughly
proportional to the particle number. Smaller particle numbers
will result in an artificially shorter two-body relaxation
timescale, which cannot be avoided in direct N-body simula-
tions. Besides, the mass ratio between the SMBH and solar-
type star should be approximate $10^6$–$10^9$ in real galaxies. While
in direct N-body simulations, if $f = 0.001$ and $q = 0.1$, with a
1 million particle number for the primary galaxy/nucleus, the
mass ratio of secondary SMBH to stellar particles should only
be 100. With this small mass ratio, stellar particles can
contribute significantly larger gravitational influences to
the BH than the situation in typical galaxies. In order to
reduce those influences we have to set the particle number and BH
to star mass ratio as large as we can. As a result, the particle
number of the primary galaxy/nucleus has been set to 1 million
in most of our models. And the mass ratio between the SMBH
and the host galaxy has been fixed to $f = 0.01$, which is
relatively larger than the popular ratio of $\sim 10^{-3}$ based on
observation. However, it is still reasonable in many ellipticals
(Kormendy & Ho 2013). With this configuration, and taking
into account that the total stellar mass of the primary galaxy is
fixed to one in our model, the BH to star mass ratio in
simulation can achieve $10^6$ for the primary SMBH, and $10^5$
for the secondary SMBH in minor mergers with the largest
disparities. Despite being two or three orders of magnitude
lower than the ratio between a $10^9 M_\odot$ SMBH and a solar-type
star, it is already close to the reality. And this is the highest
particle resolution we can achieve with reasonable computation
resources available nowadays.

In addition to that, smaller particle numbers will also lead to
less TDEs in the simulation, which makes the statistical
investigation have larger uncertainties. In our fiducial model,
we have $[T] \sim 3.97 \text{ Myr}$, $[V] \sim 102 \text{ km s}^{-1}$, $[R] \sim 0.4 \text{ kpc}$,
and $[M] \sim 252 M_\odot \text{ yr}^{-1}$. Thus the tidal radius of a solar-type
star should be $\sim 10^{-8}$. That means, even with millions of
particles adopted, we still do not have enough particle
resolution to collect enough TDEs for statistical analyze with
this tiny tidal radius.

In conclusion, a compromised solution is to adopt a set of
simulations with different larger tidal radii and limited particle
counts that can be managed by the direct N-body integration,
and then extrapolate to the condition close to real galaxies.
Since this scheme has been adopted in many studies
(Baumgardt et al. 2004; Brockamp et al. 2011; Li et al.
2012; Zhong et al. 2014, and references therein), we will not
provide the details here. Interested readers are referred to Paper I with more discussion.

Similar to Paper I, as the first step, we put an SMBH into the center of each galaxy/nucleus and make them dynamically relaxed. Then two nuclei are set with separation $d \sim 20$, and we make sure that they have parabolic orbits with the first pericenter $d \sim 1$. We adopt a direct $N$-body code $\varphi$-GRAPE/$\varphi$-GPU for integration. It is an accurate and efficient parallel code with fourth-order Hermite integrator and accelerated by GPU, which is the same code adopted in Paper I (Makino & Aarseth 1992; Berczik et al. 2005; Harfst et al. 2007). All integrations are calculated on the laohu GPU cluster at the National Astronomical Observatories of China (NAOC).

Table 1 lists all the models we have integrated. The columns (1)–(4) are, respectively, the model index, the mass ratio, the particle number of primary galaxy/nucleus, and the tidal radius. The main simulations concern different mass ratios from models Q10–Q1. And there are two sets of models focusing on the influence of different particle numbers and tidal radii with models N250, N500, and RT5-5 to RT1-3, respectively. Model Q4 is our fiducial model.

### 3. Results

In this section, we present our numerical simulation results. As mentioned in Paper I and the Appendix, the entire evolution of two SMBHs in merging galaxies can be roughly divided into three phases, which corresponds to two SMBHs far away from each other, forming a bound binary and evolving to a hard binary, respectively. And the evolution of the TDR in each phase is dominated by a different mechanism. However, how to divide different phases is an ambiguous concept without accurate criteria. In order to make detailed comparison among different models, a precise criteria in our numerical models is needed. Here we follow the same strategy adopted in Paper I to empirically divide the evolution into three phases. The criteria are based on the fluctuation of the TDE number rate relative to the averaged rate during the entire evolution. Here we define $\Delta N_{\text{TDE}}$ as the variation between the peak rate and the averaged rate. The start/end point of phase II corresponds to the time when the number of TDEs rapidly increase/decrease a large fraction of $\Delta N_{\text{TDE}}$. In this work we empirically fix the fraction to 0.4, to prevent misjudgement in some extreme conditions. However, it is not a finely tuned fraction. Slight changes to this value will not have a significant influence on our results. In short, the criteria is based on the TDR evolution. We have also tried to divide the three phases according to the dynamical evolution of two SMBHs, which means the separation evolution of two SMBHs, and found that they are roughly consistent. In principle, the latter is the most physical criteria. However, in practice the separation of two BHs varies rapidly, which often results in the wrong judgment. Nevertheless, in most cases, the phase II we have divided in our models roughly corresponds to a 1–0.05 influence on the radius of the SMBHB.

Figure 1 demonstrates the tidal disruption evolution of our fiducial mode Q4. The gray dotted line represents the separation of two SMBHs, and the gray solid line represents the TDE number count $N_{\text{TDE}}$ within one $N$-body time unit bin for the summation of two SMBHs. Red-dashed and blue-dashed–dotted lines are, respectively, the $N_{\text{TDE}}$ of the primary and the secondary SMBH, respectively. Two vertical solid lines divide the evolution into three phases.

Figure 1. Tidal disruption evolution of the fiducial model Q4. Here the $x$-axis and the left $y$-axis represent evolv time and $r_{\text{BH}}$, the separation of two SMBHs, respectively. The right $y$-axis gives corresponding tidal disruption number count $N_{\text{TDE}}$ within one $N$-body time unit bin. Gray dotted and solid lines represent the $r_{\text{BH}}$ and total $N_{\text{TDE}}$ of two SMBHs, respectively. Red-dashed and blue-dashed–dotted lines denote the $N_{\text{TDE}}$ of the primary and the secondary SMBH, respectively. Two vertical solid lines divide the evolution into three phases.
corresponding to pericenters in phase I. Besides, the TDR of the primary SMBH is obviously higher than the rate of the secondary SMBH. Especially in phase II, the primary SMBH dominates the contribution. Similar results can be found in all of the rest of the models.

Figure 2 gives the evolution of the separation and disruption rate for all models from Q10–Q1. As shown in the top left panel, all of the models have a similar separation evolution for SMBHBs. But the lighter the secondary SMBH is, the longer dynamical evolution it has. Because a more massive SMBH usually experiences a stronger influence from dynamical friction, two SMBHs could more efficiently sink to the galactic center. This effect is more significant for models with very small mass ratios, for instance, models Q10 and Q8. In order to finish the integration of such kind of systems and achieve enough data for statistical analyze in phase III, we have to extend the simulation time. For simplicity, we do not consider inspiral orbits here, which is too time consuming for small mass ratio models. We set the termination time for models Q4, Q3, Q2, and Q1 to \( t = 250 \), and extend it to \( t = 400 \), \( t = 350 \), and \( t = 300 \) for models Q10, Q8, and Q5, respectively.

The remaining three panels in Figure 2 represent the evolution of the tidal disruption accretion rate for the summation of two BHs, the primary SMBH, and the secondary SMBH, respectively. According to the bottom two panels, the peak rate of the primary SMBH does not significantly depend on \( q \), while the secondary SMBH is the opposite. Actually, as shown by Equation (12) in the following analysis, the
disruption rate of the secondary SMBH depends on the mass of itself in phase II, while the rate of the primary SMBH does not sensitively depend on \( q \), which is consistent with the results here.

Since the mass and particle number of two galaxies/nuclei are unequal, it is interesting to know how stars from different galaxies/nuclei contribute to the tidal disruption. Figure 3 presents similar evolution results as those shown in Figure 1 for models Q8, Q4, and Q2, which corresponds to the top, middle, and bottom panels, respectively. Here we separately record tidal disruption of two SMBHs, which gives the results of the primary SMBH and the secondary SMBH in the left and right columns individually. In addition to that, we also distinguish those disrupted stars according to their original host galaxies/nuclei. For reference, the separation of two SMBHs is marked as a dotted gray line and the total rate taking into account stars from both galaxies/nuclei is marked as a gray solid line. The red-dashed and blue-dashed–dotted lines are the contributions from the primary and the secondary galaxy/nucleus, respectively. As shown in the figure, the TDRs of two SMBHs are dominated by their own host galaxies in phase I for all of the mass ratios, which means their tidal disruption evolution are more or less the same as the situation in an isolated system. While in phase II when two SMBHs are close enough to form a binary, the contribution from the companion becomes more and more significant. For the primary SMBH, although most of disrupted stars come from the primary galaxy/nucleus for all models, there are still some contributions from the secondary galaxy/nucleus. The more massive the secondary SMBH is, the more stars from the secondary galaxy/nucleus are disrupted by the primary SMBH. The results for the secondary SMBH are more interesting. We find that the rates of the secondary SMBH in phases II and III are actually dominated by stars originated from the other galaxy/nucleus in minor mergers.

In models Q8 and Q4, the contribution of stars from the secondary galaxy/nucleus will sharply drop in phase II, while it does not appear in model Q2, which corresponds to a major merger. Though only three models have been shown in Figure 3, we actually have checked all of the other models. This effect is significant for all models with mass ratio \( q \leq 0.3 \). After the formation of the SMBHB in minor mergers, when the binary evolves to phase III, the disruption rate will be dominated by stars from the primary galaxy/nucleus, while there is still a significant contribution from the secondary galaxy/nucleus in major mergers.

These results indicate that different mechanisms dominate the loss cone refilling in different phases. In phase I, since two SMBHs are far away from each other, the tidal disruption will be dominated by stars in their own host galaxies, while in phase II, when the secondary SMBH get closer to the primary SMBH, more and more stars around the former will be tidally stripped. If the secondary SMBH is significantly light than the primary SMBH, most of its stars will be finally stripped. As a result, the loss cone refilling of the secondary SMBH in the late stage of phase II will be like a scatter process, which means the tidal disruption accretion rate should be proportional to the cross section. Thus we find in minor mergers that stars disrupted by the secondary SMBH are mainly from the secondary galaxy/nucleus in phase I, and then suddenly switch to the other galaxy in phases II and III. Understanding this transition is important for our later derivation of the extrapolation relations.

With divided three phases we can calculate the corresponding averaged tidal disruption accretion rate of every model. Table 2 lists the demarcation points of different phases, and also the time \( t_p \) corresponding to the peak rate. In order to simplify the comparison, we assume that the duration of phase III is 70 time units, and phase I starts from \( t = 20 \) to avoid some numerical artificial effects at the beginning of the integration. Corresponding results can be found in Table 3. The first column in the table is the model index, and the next three columns, columns (2)–(4), are the averaged rates of phase I for the summation of two SMBHs, the primary SMBHs, and the secondary SMBHs, respectively. The next three columns, columns (5)–(7), are the rates of phase II and the last three columns are the rates of phase III.

Figure 4 shows how the tidal disruption accretion rate depends on the mass ratio. The top left, bottom left, and bottom right panels are the results for phases I, II, and III, respectively. The top right panel is the dependence of the accretion rate ratio on the mass ratio in phase I. Here the red solid squares and blue solid circles represent the averaged results based on simulations of models Q10–Q1. The red solid lines and blue-dashed lines are fitting results based on the analytical predictions in Section 4.

In phase I, since two SMBHs are relatively far away from each other, their TDR should not be significantly perturbed by their companions. Thus the primary SMBH should keep a nearly constant TDR for different mass ratios, while the simulation results indicate a slightly increasing trend. We will discuss this in Section 5.1.1. On the other hand, since larger mass ratio corresponds to heavier secondary SMBH and higher TDR, the secondary SMBH should have a monotonically increasing rate, which is consistent with the simulation results in Figure 4. With the assumption that the loss cone is almost full in phase II, the TDR of two SMBHs will be dominated by the perturbations from each other. As a result, the TDRs of both SMBHs should be increasing for larger mass ratio models. As demonstrated in the bottom left panel of Figure 4, the results of the secondary SMBH are well consistent with that prediction, and the primary SMBH also shows a slightly increasing relation. In the last phase, as presented by the bottom right panel of Figure 4, the TDRs of both SMBHs monotonically increase related to the mass ratio. In Sections 4.3 and 5.1 we present an in-depth analysis and give more detailed explanations and fitting formulae to the results here.

4. Analytical Model

In Paper I, by assuming that the loss cone refilling is dominated by two-body relaxation and perturbation in phases I and II, respectively, the TDR in equal-mass mergers can be estimated analytically. Most of their assumptions and results are still valid for minor mergers. In this section we derive the TDR of minor mergers based on some basic assumptions and results in Paper I, and more details can be found in the Appendix.

4.1. Phase I

With the assumption that the loss cone refilling is dominated by two-body relaxation in phase I, we can easily derive the accretion rate for both SMBHs. In Paper I, the mass accretion
Figure 3. How stars from different galaxies/nuclei contribute to the tidal disruption of the primary and the secondary SMBH. Similar to Figure 1, the x-axis and the left y-axis represent evolved time and $r_{BH}$, respectively. The right y-axis gives the corresponding tidal disruption accretion rate. Gray dotted and solid lines represent $r_{BH}$ and the total accretion rate from both galaxies/nuclei. Red-dashed and blue-dashed–dotted lines denote the contribution from the primary and the secondary galaxy/nucleus, respectively. The top, middle, and bottom panels are the results of models Q8, Q4, and Q2, respectively. The left and right columns are the results of the primary and the secondary SMBHs, respectively. Similar to Figure 1, two vertical solid lines in each panel divide the evolution into three phases.
rate of the SMBH due to tidal disruption can be estimated by

\[
\ln M_1 \approx \ln A - \frac{2\gamma - 1}{8 - 2\gamma} \ln \left( \frac{N_1}{\ln A} \right) + \frac{9 - 4\gamma}{8 - 2\gamma} \ln r_c. \tag{7}
\]

The definition of \( A \) and the Coulomb logarithm \( \ln A \) can be found in Appendix A.1. Similarly, with \( M_{\text{BH2}} = M_{\text{BH1}} q \) and \( N_2 = N_1 q \), the mass accretion rate of the secondary SMBH can be derived as

\[
\ln M_2 \approx \ln A - \frac{2\gamma - 1}{8 - 2\gamma} \ln N_1 + \frac{7 - 7\gamma}{8 - 2\gamma} \ln q + \frac{9 - 4\gamma}{8 - 2\gamma} \ln r_c. \tag{8}
\]

As a result

\[
\frac{M_2}{M_1} = \left( 1 + \frac{\ln q}{\ln A} \right)^{\frac{2\gamma - 1}{8 - 2\gamma}} \frac{q^{\frac{9 - 4\gamma}{8 - 2\gamma}}}{q^{\frac{7 - 7\gamma}{8 - 2\gamma}}}.	ag{9}
\]

As indicated by Equation (7), the rate of the primary SMBH does not depend on \( q \). The averaged rate hereafter should be constant.

4.2. Phase II

In phase II, as demonstrated in Paper I and Appendix A.2, we can consider the secondary SMBH as the perturber, which induced the replenishing process of the loss cone. Thus according to the assumption that the mass of the perturber \( M_p \propto M_{\text{BH1}} q \) and Equation (33), we have

\[
M_1 \propto q^{a_{\beta} - a_{\beta}} r_{t_1}^{12 - 12}. \tag{10}
\]

Analogously, the rate of the secondary SMBH in phase II is dominated by the perturbation from the primary BH and stars around it. Since \( M_{\text{BH2}} = M_{\text{BH1}} q \), it can be derived as

\[
M_2 \propto q^{a_{\beta} - a_{\beta}} r_{t_1}^{12 - 12}. \tag{11}
\]

It is important to note that the \( r_t \) in Equations (10) and (11) is the corresponding tidal radius of each BH.

In order to get the results above, it assumes that most of stars disrupted by the secondary BH are from the inner region with \( r \approx r_h \), and they are relatively far away from the perturber, which may not be the case for the entire phase II. Our simulation results in Figure 3 show that, for minor mergers, the TDEs of the secondary SMBH in phase II are dominated by stars from its host galaxy at the beginning and then switch to stars from the other galaxy. This can be explained by the following. At the beginning of phase II, the separation of two SMBHs is significantly larger than the influence radius of the secondary SMBH. The disruption should be mainly contributed by perturbed stars originally bound to the secondary SMBH, while at the end of phase II, the separation of two SMBHs has been shrunk rapidly, which is much more smaller than the influence radii of both BHs. Most of the bound stars around the secondary BH have been stripped away during this process. Thus stars outside the influence radius could significantly contribute to the disruption rate.

Actually, for a minor merger, the secondary SMBH in the late stage of phase II can be considered as embedded in a stellar background that is bound to the primary BH. The tidal disruption accretion rate for such kind of systems will be proportional to the cross section, which gives \( M_2 \propto M_{\text{BH2}}^{\alpha_{\beta}} r_{t_1}^{\beta} \propto q^{\alpha_{\beta} r_{t_1}^{\beta}} \) (Li et al. 2012). On average, we can assume that the average rate of the secondary SMBH in phase II follows

\[
M_2 \propto M_{\text{BH2}}^{\alpha_{\beta}} r_{t_1}^{\beta} \propto q^{\alpha_{\beta} r_{t_1}^{\beta}}, \tag{12}
\]

where \( \alpha \) and \( \beta \) are undetermined indices. This is consistent with the results of Merritt & Poon (2004) with similar triaxial but no evolving stellar distribution, which found that \( M \propto r_t \) for \( \gamma = 1 \) and \( M \propto r_t^{0.5} \) for \( \gamma = 2 \).

According to the discussion in Paper I, the system in phase II is rapidly evolving, and the loss cone is almost full for both BHs in an equal-mass merger. This conclusion is applicable for the secondary BHs but may not valid for the primary BHs with very light companions in unequal-mass mergers. For the primary BH, a very light perturber may not be heavy enough to dominate the loss cone refilling compared with two-body relaxation. As a result, the system will correspond to an intermediate state, which is hard to describe analytically. Thus we focus on the mass ratio \( q \geq 0.1 \) in this work, wherein the secondary SMBH can still be considered as a relatively large perturber, and the assumption of full loss cone for both BHs is still acceptable.

4.3. Phase III

In the last stage phase III, all of the three factors in Equation (30) could make a non-negligible contribution. The stellar distribution around an SMBHB in this stage could be triaxial (e.g., Bortolas et al. 2018). And both the SMBHB and the stellar distribution are evolving. Chen et al. (2008) tried to investigate tidal disruptions in compact SMBHBs with surrounding unbound isotropic stellar distribution by a numerical scattering experiment. They estimated the cross section of stars to be scattered to the tidal radii of both SMBHs, while the loss cone refilling rate of the SMBHB is still uncertain. In brief, the TDR is very difficult to estimate analytically. We can only give some fitting results based on numerical simulations, which may be not convincing.
5. Discussion

5.1. Extrapolation

As mentioned in Section 2, the results of direct N-body simulations cannot be simply applied to real galaxies without extrapolation because the limited particle resolution, which is actually limited by the computer technology today, may introduce some artificial effects (a detailed discussion can be found in Paper I). In order to extrapolate the simulation results to reality, we need to find out how the simulation results depend on mass ratio, particle number, and tidal radius. Related simulations have been done with model N500 and N250, and RT5-5 to RT1-3, respectively. Combining simulation results for different mass ratios offers the chance to extrapolate our results to more realistic galaxies.

5.1.1. Phase I

As demonstrated by Equation (7), the TDR of the primary SMBH in phase I should be independent of mass ratio. However, our result in the top left panel of Figure 4 shows a weak $q$ dependence. A smaller $q$ corresponds to a smaller rate. That may because the situation in phase I is not exactly the same as an SMBH in an isolated galaxy. Actually, as shown in Figures 1 and 3, every time two SMBHs approach their pericenters in phase I, there is a small burst of the tidal disruption attendant. The loss cone refilling at those short stages should be contributed both from two-body relaxation and the companion perturbation. A model with smaller $q$ corresponds to a smaller perturber, which corresponds to smaller rate.

Taking into account contributions from all the effects discussed above, we can simply assume that the rate here is the superposition of a constant value and Equation (10). That gives fitting results $M_\text{f} \sim 1.8 \times 10^{-5} \times q^{0.23} + 1.0 \times 10^{-5}$, which are denoted by the red-dashed line in the top left panel of Figure 4. Nevertheless, since the periods around the pericenter usually are short here, this effect can only slightly change the result. In order to simplify the problem, we adopt the averaged rate $M_\text{f} \sim 2.4 \times 10^{-5}$ of all seven models, which is indicated by the red solid line.

On the other hand, the rate of the secondary SMBH has a significant $q$ dependence, which can be well fitted by Equation (8) with $M_\text{s} \sim 2.65 \times 10^{-5} \times q^{0.44} (12.9 + \ln q)^{0.0047}$. And there is also a good fitting result given in Equation (9) in the top right panel, which gives $M_\text{s} \sim 0.44 (1 + \ln q)^{12.9/0.035}$.

In addition to mass ratio, according to Equations (7) and (8), if we fix other parameters, the tidal disruption accretion rate in phase I should be proportional to the power law of $N/\ln \Lambda$. That has been confirmed in the top panels of Figure 5. The top left and top right panels are, respectively, the rates for different particle numbers of the primary and the secondary SMBHs. The filled squares and circles represent our simulation results of the primary and the secondary SMBHs, respectively. The solid-dashed lines are the fitting results based on Equations (7) and (8). For the primary SMBH, the fitting result gives $M_\text{p} \sim 0.011 \times (N/\ln \Lambda)^{-0.54}$ in phase I, and the secondary SMBH has $M_\text{s} \sim 9.4 \times 10^{-4} \times (N/\ln \Lambda)^{-0.42}$, where $\Lambda = \ln(0.4N_\text{t})$ is the Coulomb logarithm for the secondary galaxy.

The bottom left panel of Figure 5 represents the results of models RT5-5-RT1-3 and Q4 in phase I, which shows how the averaged accretion rate depends on tidal radius. It is clear that the rates in phase I of both SMBHs have significant power-law dependence on $r_t$. According to Equations (7) and (8), there are good fitting results with $M_\text{f} \sim 7.0 \times 10^{-4} \times r_t^{0.44}$ and $M_\text{s} \sim 4.1 \times 10^{-4} \times r_t^{0.44}$. And the same fitted power-law index for both BHs is consistent with the prediction of Equations (7) and (8).

The results above indicate that our analysis in Section 4.1 for phase I can be confirmed by direct N-body simulations. For orders of magnitude estimation, combined with simulation results, it is possible to derive the averaged tidal disruption accretion rate in phase I with different mass ratios, particle numbers, and tidal radii. According to Equation (7) and relevant simulation results, the rate of the primary SMBH in phase I can be estimated by a two-dimensional fitting

$$\ln M_\text{f} \sim -0.97 - 0.56 \ln \frac{N_\text{t}}{\ln \Lambda} + 0.44 \ln r_t,$$

Table 3: Tidal Disruption Accretion Rates in Different Phases

| Model  | $M_{\text{f1}} \times 10^{-5}$ | $M_{\text{f1}} \times 10^{-4}$ | $M_{\text{f1}} \times 10^{-3}$ | $M_{\text{f1}} \times 10^{-5}$ | $M_{\text{f1}} \times 10^{-4}$ | $M_{\text{f1}} \times 10^{-3}$ | $M_{\text{f1}} \times 10^{-5}$ | $M_{\text{f1}} \times 10^{-4}$ | $M_{\text{f1}} \times 10^{-3}$ | $M_{\text{f1}} \times 10^{-5}$ |
|--------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Q10    | 2.66                          | 0.73                          | 1.30                          | 1.00                          | 0.30                          | 1.55                          | 1.03                          | 0.51                          | 0.23                          | 0.01                          |
| Q8     | 2.97                          | 0.91                          | 1.44                          | 1.09                          | 0.35                          | 1.74                          | 1.18                          | 0.56                          | 0.27                          | 0.04                          |
| Q5     | 3.51                          | 1.16                          | 1.46                          | 1.07                          | 0.39                          | 2.07                          | 1.37                          | 0.70                          | 0.40                          | 0.11                          |
| Q4     | 3.86                          | 1.43                          | 1.55                          | 1.10                          | 0.44                          | 2.36                          | 1.57                          | 0.79                          | 0.44                          | 0.16                          |
| Q3     | 4.21                          | 1.69                          | 1.52                          | 1.02                          | 0.49                          | 2.60                          | 1.70                          | 0.90                          | 0.49                          | 0.20                          |
| Q2     | 4.79                          | 2.09                          | 1.84                          | 1.12                          | 0.73                          | 3.99                          | 2.43                          | 1.56                          | 0.79                          | 0.30                          |
| Q1     | 5.22                          | 2.56                          | 2.16                          | 1.04                          | 1.12                          | 5.77                          | 2.91                          | 1.56                          | 0.79                          | 0.30                          |
| N250   | 7.26                          | 2.42                          | 1.66                          | 1.24                          | 0.42                          | 2.76                          | 1.75                          | 1.01                          | 0.79                          | 0.30                          |
| N500   | 5.64                          | 1.94                          | 1.70                          | 1.19                          | 0.51                          | 2.54                          | 1.75                          | 1.01                          | 0.79                          | 0.30                          |
| RT5-5  | 1.40                          | 0.52                          | 0.29                          | 0.20                          | 0.09                          | 0.45                          | 0.30                          | 0.11                          | 0.05                          | 0.01                          |
| RT1-4  | 1.96                          | 0.74                          | 0.47                          | 0.34                          | 0.13                          | 0.75                          | 0.52                          | 0.23                          | 0.11                          | 0.05                          |
| RT1-3  | 5.34                          | 1.98                          | 2.68                          | 1.84                          | 0.83                          | 3.59                          | 2.27                          | 1.32                          | 0.83                          | 0.44                          |

Note. Column (1): Model index. Column (2): Averaged total accretion rate during phase I. Column (3): Averaged rate of the primary SMBH in phase I. Column (4): Averaged rate of the secondary SMBH in phase I. Column (5): Averaged total accretion rate in phase II. Column (6): Averaged rate of the primary SMBH in phase II. Column (7): Averaged rate of the secondary SMBH in phase II. Column (8): Averaged total accretion rate in phase III. Column (9): Averaged rate of the primary SMBH in phase III. Column (10): Averaged rate of the secondary SMBH in phase III.
and the rate of the secondary SMBH in phase I, according to Equation (8), can be estimated by

\[
\ln M_2 \sim -1.93 - 0.48 \ln \left( \frac{N_1}{\ln \Lambda + \ln q} \right) + 0.51 \ln q + 0.41 \ln n, \\
\] (14)

which is consistent with previous fitting results.

Combined with Equation (6), the tidal disruption accretion rate in phase I for real galaxies can be derived as

\[
\dot{M}_1 \sim 1.5 \times 10^{-2} \times \left( \frac{M}{10^5 M_\odot} \right)^{3/2} \left( \frac{r_1/2}{1 \text{ kpc}} \right)^{-1.94} \times \left( \frac{N_1}{\ln \Lambda} \right)^{-0.56} \left( \frac{n}{10^{-6} \text{ pc}} \right)^{0.44} M_\odot \text{ yr}^{-1}. \\
\] (15)
The results above, especially the index of \( r_t \) for the primary SMBH, however, is not the same as the Equation (26) in Paper I. As discussed at the beginning of this subsection, the TDR for each SMBH in phase I is not exactly equivalent to the rate in an isolate galaxy because every close encounter of two nuclei could bring a small boost of tidal disruption. That means this effect weakly depends on the mass ratio. Consequently, the above results for unequal-mass mergers with different mass ratios considered are not surprisingly different from Paper I based on equal-mass models.

\[
M_2 \sim 1.1 \times 10^{-2} \times q^{0.51} \times \left( \frac{N_1}{\ln \Lambda + \ln q} \right)^{-0.48} \\
\times \left( \frac{M}{10^6 M_{\odot}} \right)^{3/2} \left( \frac{r_{t/2}}{1 \text{ kpc}} \right)^{-1.91} \left( \frac{r_1}{10^{-6} \text{ pc}} \right)^{0.41} M_{\odot} \text{ yr}^{-1}. \tag{16}
\]

Once the physical parameters of the system have been decided, combined with Equation (1), we can make the extrapolation. In our fiducial model, the extrapolated disruption rates of the primary and the secondary SMBH are \( \sim 7.0 \times 10^{-6} M_{\odot} \text{ yr}^{-1} \) and \( \sim 1.1 \times 10^{-5} M_{\odot} \text{ yr}^{-1} \) for \( q = 0.25 \) and \( \sim 7.0 \times 10^{-6} M_{\odot} \text{ yr}^{-1} \) and \( \sim 6.5 \times 10^{-6} M_{\odot} \text{ yr}^{-1} \) for \( q = 0.1 \), respectively. It seems that the rate difference between the primary and the secondary SMBHs are not significant, which is actually not unexpected. In our models the secondary galaxy always corresponds to smaller particle numbers due to the fixed configuration \( N_2/N_1 = q \), while as illustrated by Figure 5, the rate is a monotone decreasing function of particle number. As a result, though the less massive secondary BH should correspond to a significantly lower rate compared with the primary BH, the gap between two final rates after extrapolations will be narrowed because of the lower star number of the secondary galaxy/nucleus.
Liu & Chen (2013) analytically estimated the TDR for different minor mergers. We have compared our result of the fiducial model in phase I with their model. Since Liu & Chen (2013) did not present their results for the mass ratio \( q = 0.25 \), we follow their model to have a new integration. With similar parameters adopted in our fiducial model, their model gives the TDE rates for the primary SMBH and the secondary SMBH are \( 1.9 \times 10^{-5} \text{ yr}^{-1} \) and \( 3.4 \times 10^{-5} \text{ yr}^{-1} \), respectively. Considering that our extrapolation is just an orders of magnitude estimation, there is a very good consistency when two galaxies are far away from each other. However, their analytical estimation tends to give a higher rate when two SMBHs are getting close in phase I. We believe there are two reasons. First of all, Liu & Chen (2013) assumed spiral-in orbits, which corresponds to long-term gradual evolution, while in our models two SMBHs with their star clusters are in parabolic orbits with relatively rapid orbital evolution, corresponding to much shorter interaction periods. Second, we assumed that the loss cone refilling is dominated by two-body relaxation in phase I and by perturbation in phase II. However, due to limited particle resolution, the mass ratio between SMBHs and stars in our simulation models are significantly smaller than they should be in real galaxies. Thus the two-body relaxation is artificially enhanced in our simulation. Consequently, the transition from phase I to phase II in our simulation tends to be delayed compared with reality. Then two SMBHs need to get more closer to enter the perturbation dominate phase in our simulations. Though the above extrapolations could avoid the influence of limited particle number and tidal radius to averaged accretion rates, it is still insufficient to derive an accurate transition boundary between phases I and II. Nevertheless, on orders of magnitude, our extrapolations on typical averaged accretion rates in phase I above and phase II bellow should be still valid.

5.1.2. Phase II

In phase II, the rate of the primary SMBH should weakly depend on \( q \) according to Equation (10), while the rate of the secondary SMBH should be proportional to \( q^{-5} \). Our results in the bottom left panel of Figure 4 confirm that prediction. The dependence of the primary SMBH on \( q \) is very weak, which gives a fitting result of \( M_1 \sim 1.1 \times 10^{-4} \times q^{0.009} \), as indicated by the red solid line. And the relation between the rate of the secondary SMBH and the mass ratio can be fitted by \( M_2 \sim 1.04 \times 10^{-4} \times q^{0.57} \), which is indicated by the blue-dashed line. As shown in Table 3, since the loss cone of both SMBHs is relatively full, the rates of two SMBHs do not significantly depend on particle numbers in phase II, while as represented in the bottom right panel of Figure 5, the results of the primary and the secondary SMBHs can also be fitted quite well by Equations (10) and (12), respectively. For the primary SMBH it is \( M_1 \sim 3.1 \times 10^{-2} \times \eta^{-0.94} \), and the secondary SMBH has \( M_2 \sim 1.6 \times 10^{-2} \times \eta^{-0.76} \).

Analogously to phase I, we can make similar extrapolations for phase II. Since the rates of both SMBHs are not \( N \) dependent, they can be estimated through Equations (10) and (12) and the fitting results above as

\[
\ln M_1 \sim -3.61 + 0.007 \ln q + 0.73 \ln \eta, \tag{17}
\]

\[
\ln M_2 \sim -3.30 + 0.56 \ln q + 0.77 \ln \eta. \tag{18}
\]

And the corresponding estimation for real galaxies are

\[
M_1 \sim 3.5 \times 10^{-6} q^{0.007} \left( \frac{M}{10^6 M_\odot} \right)^{3/2} \left( \frac{r_{\text{in}}}{1 \text{ kpc}} \right)^{2.23} \times \left( \frac{\eta}{10^{-6} \text{ pc}} \right)^{0.73} M_\odot \text{ yr}^{-1}. \tag{19}
\]

\[
M_2 \sim 2.1 \times 10^{-6} q^{0.56} \left( \frac{M}{10^6 M_\odot} \right)^{3/2} \left( \frac{r_{\text{in}}}{1 \text{ kpc}} \right)^{-2.27} \times \left( \frac{\eta}{10^{-6} \text{ pc}} \right)^{0.77} M_\odot \text{ yr}^{-1}. \tag{20}
\]

In our fiducial model, the averaged disruption rates in phase II for the primary and the secondary SMBHs are \( \sim 1.3 \times 10^{-4} M_\odot \text{ yr}^{-1} \) and \( \sim 3.9 \times 10^{-5} M_\odot \text{ yr}^{-1} \) for \( q = 0.25 \), and \( \sim 1.3 \times 10^{-4} M_\odot \text{ yr}^{-1} \) and \( \sim 2.3 \times 10^{-5} M_\odot \text{ yr}^{-1} \) for \( q = 0.1 \), respectively. The corresponding peak rates at \( q = 0.25 \) are \( \sim 3.6 \times 10^{-4} M_\odot \text{ yr}^{-1} \) and \( \sim 8.3 \times 10^{-5} M_\odot \text{ yr}^{-1} \), respectively, for the primary and the secondary SMBHs. And the peak rates at \( q = 0.1 \) are \( \sim 3.6 \times 10^{-7} M_\odot \text{ yr}^{-1} \) and \( \sim 6.1 \times 10^{-8} M_\odot \text{ yr}^{-1} \), respectively, for the primary and the secondary SMBHs. In general, the averaged TDR in phase II can be enhanced several times to an order of magnitude relative to phase I. And the peak rate in phase II can be boosted dozens of times.

In phase II, the separation of two SMBHs shrinks from several times of the primary SMBH’s influence radius to a few percent within a very short time. The estimations made by Liu & Chen (2013) are not valid here because they integrate the tidal disruption evolution to the separation around twice the influence radius, which only corresponds to the beginning of phase II here. Chen et al. (2009, 2011) investigated the TDR evolution for close SMBBHs. They found that the tidal disruption was contributed by both the Kozai–Lidov effect (Kozai 1962; Lidov 1962) and chaotic loss cone repopulation. For the binary with high eccentricity, which is the situation we have here, the latter is dominated. However, since they focused on systems with very small mass ratio, which neglected the stellar distribution distortion induced by the two SMBHs, comparison between their results and ours is not straightforward.

5.1.3. Phase III

We have tried to numerically fit the \( q \) dependence results and find that the rate of the primary SMBH can be fitted by a power law \( M_1 \sim 3.0 \times 10^{-3} \times q^{0.47} \), and the fit to the rate of the secondary SMBH prefers a linear relation, which gives \( M_2 \sim 2.7 \times 10^{-5} \times q + 1.8 \times 10^{-6} \). Our assumption of a scattering process of loss cone refilling of the secondary SMBH in phase II is probably more or less still valid for phase III.

Similar to phase II, we cannot find significant \( N \) dependence for both SMBHs. However, due to our limited knowledge about how the loss cone refilling process depends on simulation particle resolution in phase III, it should be noted that the particle dependence results here may not be an universal answer. On the other hand, as presented by Figure 6, though the dependence of TDRs on \( r_\text{t} \) is not analytically clear in phase III, we still found that the tidal radius dependence of the primary SMBH can be roughly fitted by a power law with \( M_1 \sim 3.6 \times 10^{-3} \times r_\text{t}^{0.73} \), while the secondary SMBH can be
As mentioned in Paper I, the boosted TDR during phase II usually are an order of magnitude higher than those of phase I, which may indicate that the contribution in phase II should be significant. But as demonstrated in Figure 2 and Table 2, those smaller mass ratio models usually correspond to significantly extended periods of phase I, while periods of phase II for all the models are roughly the same. If we assume that the evolution of two SMBHs before phase II are dominated by dynamical friction, and the corresponding TDR is approximately equal to the averaged TDR in phase I, then the total TDE number will be proportional to $\Gamma \times t_{df}$, where $\Gamma$ is the TDE, and $t_{df} \propto q^{-1}$ is dynamical friction timescale (Liu & Chen 2013). For a minor merger with $q \lesssim 0.25$, compared with an equal-mass merger, it may take several times longer duration to evolve to phase II. As a result, the contribution of phase II in minor mergers could be submerged by the earlier and later stages, with lower rates but longer periods. In other words, in major mergers a detected TDE still has a relatively high possibility to occur in phase II, while the possibility of a TDE detected in a minor merger contributed by phase II should be small. However, the minor mergers are more common. According to Lotz et al. (2011), the minor merger rate $(0.1 \lesssim q < 0.25)$ in a local universe is $\sim 3$ times that of major mergers. Actually, based on detailed predictions about the SMBHB-induced TDE detection rate made by Thorp et al. (2019), the Large Synoptic Survey Telescope (LSST) and the extended Roentgen Survey with an Imaging Array (eROSITA) may detect tens of TDEs induced by SMBHBs every year. That means that we may still have a chance to detect a few TDEs corresponding to the phase II stage of minor mergers every year. Another planned X-ray space transient survey, the Einstein probe, can also detect several tens to hundreds TDEs per year (Yuan et al. 2015), which corresponds to a few detections relevant to phase II of minor mergers within the lifetime of the satellite. It should be noted that according to the discussion in Section 5.1, our results trend to underestimate the duration of phase II. Thus above estimation of TDEs in phase II can only give a lower limit.

An interesting result is that the TDR of the secondary SMBH in phase I can be close to the primary SMBH for most minor merger models. As mentioned in Section 5.1, smaller galaxies with less stars correspond to shorter two-body relaxation timescales, which results in relatively higher TDRs in phase I. In other words, the contribution of both SMBHs in a minor merger is similar in the early stage. In phase II, though the contribution is dominated by the primary SMBH, there is still a significant fraction of TDEs contributed by the secondary SMBH. Furthermore, there is another special but not rare condition in which the primary SMBH mass exceeds $\sim 10^6 M_\odot$. In that case the tidal radius of the SMBH for solar-type stars is likely smaller than its Schwarzschild radius, which means the tidal disruptions from the primary SMBH may do not have emission at all. Then the TDEs of the secondary SMBH will dominate the contribution.

Besides, according to Figure 3, most of tidal disruptions of the secondary SMBH are contributed by stars from the other galaxy in phases II and III. That indicates that many of those disrupted stars could have parabolic or even hyperbolic orbits before tidal disruption. Most of observations on tidal disruption right now, however, have assumed that disrupted stars are in parabolic orbits. In reality, the light curve of a TDE from an...
eccentric or hyperbolic star can be significantly different from the “classic” parabolic orbits (Hayasaki et al. 2013, 2018). As extreme cases, all the debris from disrupted stars with strongly bound eccentric orbits will fall back to the SMBH, leading to a significant deviation from the “canonical” $r^{-5/3}$ fallback rate. And there will be no debris fallback from those disrupted stars with extreme hyperbolic orbits, which corresponds to invisible TDEs. Recently, Hayasaki et al. (2018) found that most disrupted stars are in parabolic orbits for a single SMBH in a spherical isolate galaxy, while our results here tend to indicate that the disrupted stars with hyperbolic orbits may occupy a large fraction in merging galaxies. Actually we also find a significant fraction of stars with eccentric orbits in our simulation results. A detailed discussion about this issue is beyond the scope of this paper. We will provide an in-depth investigation in our next paper.

6. Summary

We have done direct N-body simulations of unequal-mass merging galaxies with SMBHs in their centers, with a detailed study of the TDRs of stars near the SMBH; the evolution is divided into three phases, first “phase I,” where both SMBH and their nuclear star clusters are largely independent of each other, “phase II” where a bound SMBHB forms, and there is a strong interaction of both nuclear star clusters, rapid and presumably chaotic evolution with a significantly enhanced TDR, and a “phase III” during which the system has settled. We find that the TDR during the formation of bound SMBHBs, can be magnified several times to dozens of times, and there are power-law relations between the TDR and the mass ratio of two SMBHs/galaxies. In addition, we also find that in a minor merger, after two SMBHs become bound, the TDEs of both SMBHs are mainly contributed by stars from the more massive galaxy.

Based on the method and results in Paper I, here we have extended our research on tidal disruption in galaxy mergers to more popular and realistic unequal-mass mergers. Though the dynamical evolution timescales are longer, their TDR evolutions are quite similar to equal-mass mergers. There will be significantly enhanced disruption rates in phase II, with maximum rates boosted to dozens of times relative to normal galaxies. The rate of the primary SMBH weakly or barely depends on the mass ratio before and during the formation of the SMBHB, while the rate of the secondary SMBH has a significant mass ratio dependence that gives $M_2 \sim 2.65 \times 10^{-5} \times q^{0.49}(12.9 + \ln q)^{0.0047}$ in phase I and $M_2 \sim 1.04 \times 10^{-4} \times q^{0.57}$ in phase II. The rates for both SMBHs obviously depend on mass ratio after a compact SMBHB has been formed. More interestingly, during the late stage of phase II and the early stage of phase III, most of stars disrupted by the secondary SMBH actually originate from the host galaxy/nucleus of the other SMBH. That means that there should be some unbound disrupted stars in hyperbolic orbits, which have significantly different light curves compared with classical tidal disruptions with parabolic orbits assumed.

As direct N-body simulations, the results were inevitably limited by the particle resolution. Due to the high accuracy requirement of the integration, none of the high-performance clusters in the world can afford a direct N-body simulation with a particle number close to the number of stars in a normal galaxy. An alternative solution is to simulate a galaxy with much less particles and then extrapolate the results to the situation in reality, which needs to be done very carefully because many physical processes, for instance, two-body relaxation, significantly depend on the particle number. In Paper I, by dividing the whole evolution into three different phases, we analytically assumed some dominate processes in each phase, and derived corresponding $N$ and $r_t$ dependence. With these guidelines, combined with the simulation results in different parameters, we made convincing extrapolations and derive the TDRs for real equal-mass mergers. As demonstrated in Section 5.1, this strategy is also valid for minor mergers. We analytically estimate the $N$, $r_t$ and $q$ dependence of TDR in phases I and II, and found that they are consistent with our simulation results. Therefore, combined with the theoretical analyses and direct N-body simulation results, we can estimate the TDRs of minor mergers for the first time. The only deficiency is that the rate in phase III is still not clear because it is contributed by many processes and none of them is dominated.

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Appendix

Analytical Model for Equal-mass Mergers

As demonstrated in Paper I, it is possible to analytically estimate the TDR evolution for equal-mass mergers. A single star that is approaching a single BH will be tidally disrupted within one orbital period if its specific angular momentum is less than $J_c \approx (2GM_{\text{BH}}r_t)^{1/2}$. All of such kind of possible orbits in phase space are inside a cone region, which is the so-called “loss cone.” The velocity vectors corresponding to a loss cone have an opening angle (Lightman & Shapiro 1977; Merritt 2013, and references therein)

$$\theta_c = \frac{J_c}{J_c},$$

where $J_c$ is the specific circular angular momentum with same energy of the star.

In a spherical isotropic system with equal stellar mass, $\theta_c$ can be estimated inside the influence radius $r_{\text{in}}$ of the BH

$$\theta_c^2 \approx \frac{2}{\frac{3}{2}} \frac{r_{\text{in}}}{r}.$$
(Frank & Rees 1976; Baumgardt et al. 2004). In a more realistic system with lots of stars, the angular momentum of every star changes with time because of gravitational interactions between them. If the effective deflection angle is small enough with \( \theta_D(r) \ll \theta_c(r) \), which means the orbital averaged deflection angle of velocity is very small, all the stars inside the loss cone will be disrupted by a BH within one orbital period (Frank & Rees 1976; Shapiro & Lightman 1976; Lightman & Shapiro 1977). As a result, the loss cone will be empty because of the inefficient replenishment of stars through diffusion. Conversely, if \( \theta_D(r) \gg \theta_c(r) \), the replenishment of stars is so efficient that the loss cone can remain full. Both theoretical and numerical evaluations find that there will be a critical radius \( r_{\text{crit}} \) corresponding to \( \theta_D(r_{\text{crit}}) \sim \theta_{\text{th}}(r_{\text{crit}}) \) (Frank & Rees 1976; Lightman & Shapiro 1977; Amaro-Seoane et al. 2004; Zhong et al. 2014).

In a spherical system, the TDE rate within a shell can be estimated on the order of magnitude by

\[
d\Gamma \approx \frac{4\pi r^3 dr \rho(r) \theta^2(r)}{m_\ast t_d(r)} (27)
\]

(Frank & Rees 1976; Syer & Ulmer 1999; Liu & Chen 2013), where \( t_d(r) \) is the dynamical timescale of the star, \( \rho \) is the stellar mass density, and

\[
\theta^2 = \min\left(\theta_{\text{th}}^2, \frac{\theta_D^2}{\ln \theta_{\text{th}}^2}\right) (28)
\]

is a dimensionless coefficient for stars depleting into the loss cone (Syer & Ulmer 1999). In order to precisely calculate the total disruption rate, one should integrate this formula. However, for order of magnitude estimation, we can assume that TDEs are mainly contributed by stars around the critical radius, which gives

\[
\Gamma \sim \frac{r^2 \theta_D^2 \rho(r)}{t_d m_\ast} \bigg|_{r=r_{\text{crit}}} (29)
\]

(Frank & Rees 1976; Baumgardt et al. 2004), where for simplicity all stars are assumed to have equal mass \( (m_\ast = M_\odot) \).

In order to analytically estimate the evolution of TDE in merging galaxies, Liu & Chen (2013) assumed that the effective deflection angle is mainly contributed by stellar two-body relaxation, massive perturbation from another SMBH with surrounding stars, and triaxial stellar distribution

\[
\theta_D^2 = \theta_2^2 + \theta_p^2 + \theta_c^2, (30)
\]

where \( \theta_2, \theta_p, \) and \( \theta_c \) are, respectively, the contribution from two-body relaxation, massive perturber, and triaxial gravitational potential. According to the separation of two SMBHs, and results based on direct N-body simulation for equal-mass mergers in Paper I, the evolution of merging systems can be divided into three phases. In phase I, two merging stellar systems still maintain a relatively large distance, which means the perturbation from one system to the other is not significant. For order of magnitude estimation, we can assume most of the TDEs in this stage are contributed by two-body relaxation, which corresponds to \( \theta_2 \sim \theta_t \). In phase II, two systems are so close that two SMBHs will slip into the influence radii of each other and finally form a binary system. The period of this phase is relatively short, and the system contains lots of stars with chaotic orbits. For this reason, the perturbation dominates the contribution to TDEs. After an SMBHB is formed, it will continuously harden in phase III. The estimation of the TDE in this stage is very complicated because all three factors listed above may have non-negligible contributions.

A.1. Phase I

In phase I, as mentioned in Paper I, due to the relatively large separation, the TDE of each galaxy is more or less similar to isolate galaxy, which means two-body relaxation dominates the loss cone filling. For this reason, with the assumption \( \theta_2 \sim \theta_t \), the mass accretion rate of each SMBH due to tidal disruption can be described by following equations, which are the same as Equations (23) and (24) in Paper I

\[
\ln M_t \approx \ln A - \frac{2\gamma - 1}{8 - 2\gamma} \ln \left( \frac{N_t}{\ln A} \right) + \frac{9 - 4\gamma}{8 - 2\gamma} \ln \eta, (31)
\]

and

\[
A = 2.94 \times 0.23 \frac{\eta_{\text{th}}}{\eta} G^2 (\rho \eta_{\text{th}}^2) \frac{\eta^2}{\eta_{\text{BH}}^2}, (32)
\]

where \( \ln A \approx \ln(0.4N_t) \) is the Coulomb logarithm for the primary galaxy (Spitzer 1987). Here we assume that the mass of the SMBH will not be significantly changed by mass accretion due to tidal disruption, and \( \rho(r) = \rho_0(r/r_0)^{-7} \).

A.2. Phase II

In phase II, which roughly corresponds to the stage where the companion BH enters the influence radius of the primary BH, the perturbation of the companion BH dominates the replenishing process of the loss cone. Thus the estimation in Paper I is still valid. According to Equation (31) in Paper I, and neglecting the \( q \) dependence for \( t_d \) in that equation, the tidal disruption accretion rate of the primary BH is

\[
M_t \propto M_p^{\frac{4 \gamma - 2}{3}} M_{\text{BH}}^{\frac{1}{3} - \frac{2 \gamma}{3}} r^{-\gamma/2}, (33)
\]

where \( M_p \) is the total mass of the perturber.

A.3. Phase III

Since all of the three factors may have significant contributions, the estimation of the TDR in phase III is quite uncertain. Therefore we will not give any analytical estimations. However, it is possible to get some dependencies by purely numerical fittings. As shown in Figure 6, there are significant \( \eta \) dependencies for both SMBHs. More details can be found in Section 5.1.3.

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