Ballistic spin currents in mesoscopic metal/In(Ga)As/metal junctions

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We investigate the ballistic spin transport through a two-dimensional mesoscopic metal/semiconductor/metal double junctions in the presence of spin-orbit interactions. It is shown that real longitudinal and/or transverse spin currents can flow in the presence of the Rashba and Dresselhaus terms.

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I. INTRODUCTION

Since the advent of “spintronics” to utilize electron’s spin rather than its charge for information processing and storage,\textsuperscript{1,2,3,4,5,6} there has been growing interest in generating spin currents in diverse ways.\textsuperscript{2,3,4,5,6} Though injecting spin-polarized carriers electrically remains a challenge,\textsuperscript{2} various kinds of all-semiconductor devices using ferromagnetic semiconductor heterostructures\textsuperscript{3} or spin-orbit (SO) interactions\textsuperscript{4} have been proposed. The so-called extrinsic spin Hall effect due to SO dependent scattering from magnetic impurities manifests a spin current because of a transverse spin imbalance generated from a charge current circulating in a paramagnetic metal.\textsuperscript{4} In weak impurity scattering limit but with substantial SO couplings, on the other hand, it was suggested that the intrinsic spin Hall effect gives rise to a dissipationless spin current perpendicular to the external electric field.\textsuperscript{5,6} Moreover, the spin Hall conductance has a universal value. However, it was demonstrated that the dissipationless (unreal) spin current does not vanish even in thermodynamic equilibrium in the absence of external fields, putting the interpretation of intrinsic spin Hall effect in a controversy.

In this paper we study the ballistic spin transport through a mesoscopic double-junction system consisting of a semiconductor stripe sandwiched by two normal metal leads; see Fig. 1. We use the coherent scattering theory and show that in the presence of SO couplings, both longitudinal and transverse spin currents can flow in the semiconductor. It is stressed that these currents are real; see Ref. \textsuperscript{7}.

II. MODEL AND SCATTERING THEORY

We consider a two-dimensional electron system (2DES) of semiconductor (S) between two normal (N) metal leads. We choose such a coordinate system that the x-axis (y-axis) is perpendicular (parallel) to the N/S interfaces and z-axis is perpendicular to the 2D plane; see Fig. 1. The length (width) of the semiconductor is $L$ ($W$); we will consider the limit $W \to \infty$. Within the effective-mass approximation the Hamiltonian reads as

$$\mathcal{H} = -\frac{\hbar^2}{2} \nabla \cdot \frac{1}{m(x)} \nabla + V(x, y) + \mathcal{H}_R(x) + \mathcal{H}_D(x).$$

The position-dependent effective mass $m(x)$ has values of $m_e$ and $m_* \equiv \epsilon m_e$ in the normal metals and the semiconductor ($-L/2 < x < L/2$), respectively. The confinement potential has a potential barrier of height $V_0$ inside the semiconductor:

$$V(x, y) = V_0 [\Theta(x + L/2) - \Theta(x - L/2)] + V(y),$$

where $\Theta(x)$ is the Heaviside step function and $V(y)$ accounts for the finite width $W$. The potential barrier height $V_0$ is lower than the Fermi energy $E_F$ in the normal metals so that $E_F^* = E_F - V_0 > 0$. The Rashba\textsuperscript{8,9} and Dresselhaus\textsuperscript{10,11} SO coupling terms are given by

$$\mathcal{H}_R = \frac{\alpha}{\hbar} (\sigma_x p_y - \sigma_y p_x) \quad \text{and} \quad \mathcal{H}_D = \frac{\beta}{\hbar} (\sigma_y p_y - \sigma_x p_x),$$

respectively, inside the semiconductors while they vanish in the normal metal sides. In Eq. \textsuperscript{12}, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices.

The Rashba term $\mathcal{H}_R$ arises when the confining potential of the quantum well lacks the inversion symmetry, while the Dresselhaus term $\mathcal{H}_D$ is due to the bulk inversion asymmetry. In some semiconductor heterostructures (e.g., InAs quantum wells) $\mathcal{H}_R$ dominates, and in others (e.g., GaAs quantum wells) $\mathcal{H}_D$ is comparable to (or even dominant over) $\mathcal{H}_R$. The coupling constants may range around $\alpha \sim 0.1 \text{eV} \cdot \text{Å}$ and $\beta \sim 0.09 \text{eV} \cdot \text{Å}$, respectively, depending on the structure and material.
where $k = \sqrt{\epsilon \mu}$ and $\mu$ is the electric charge. The eigenstates with spin parallel ($\pm$) or anti-parallel ($\mp$) to the wave vector $k$ are:

$$E_{\mu}(k) = \frac{\hbar^2}{2m_e} \left( k^2 - 2\mu k_{so}(\phi) \right),$$

where $k_{so}(\phi) = \sqrt{\alpha^2 + \beta^2 - 2\alpha \beta \sin 2\phi}$. From the continuity equation for the charge density, one can get the expression for the charge current associated with a given wave function $\Psi(r)$:

$$J_c = e \Re \left[ \Psi^+(r) \mathbf{v} \Psi(r) \right],$$

where $\mathbf{v}$ is the velocity operator defined by

$$\mathbf{v} = \frac{\mathbf{p}}{m_e} - \frac{\alpha}{\hbar} (\sigma_y \mathbf{\hat{x}} - \sigma_z \mathbf{\hat{y}}) - \frac{\beta}{\hbar} (\sigma_x \mathbf{\hat{x}} - \sigma_y \mathbf{\hat{y}}).$$

In the same manner, we define the spin current:

$$J_s(\mathbf{n}) = \frac{\hbar}{2} \Psi^+(r) \frac{\mathbf{v}(\mathbf{n} \cdot \sigma)}{2} \Psi(r)$$

according to the continuity equation

$$\partial_t Q_s + \nabla \cdot J_s = S_s$$

for the spin density (with respect to the spin direction $\mathbf{n}$)

$$Q_s(\mathbf{n}) = \frac{\hbar}{2} \left[ \Psi^+(r) (\mathbf{n} \cdot \sigma) \Psi(r) \right].$$

and the spin source

$$S_s(\mathbf{n}) = \frac{\hbar}{2} \Re \left[ \frac{\Psi^+(r)}{\hbar} [\mathbf{H}, \mathbf{n} \cdot \sigma] \Psi(r) \right].$$

The appearance of the spin source term in Eq. (10) is not surprising because the spin-orbit couplings break spin conservation inside the semiconductor.

Before going further, it will be useful to understand the origin of the spin current in physical terms. As illustrated in Fig. 2 for $\alpha, \beta \neq 0$ the Fermi contours

$$k_{F}^\mu(\phi) = \mu k_{so}(\phi) + \sqrt{k_{so}^2(\phi) + \epsilon_{F}^2}$$

with $k_{F}^\mu \equiv \sqrt{2m_e E_{F}/\hbar}$, are no longer isotropic, and the group velocities $v_{F}^\mu_k = \Psi\Psi^*_{k} \Psi_{k}^\mu$ of the eigenstates in Eq. (5) are not parallel to the wave vector $k$. Nevertheless, Eq. (10) reveals an important symmetry property of the group velocities: $v_{F}^+_{k} = v_{F}^-_{k}$. It means that the two eigenstates with opposite spin orientations make the same contributions to the charge transport along the $k$ direction (and opposite contributions along the perpendicular direction). On the contrary, for the spin transport with $\mathbf{n} = \mathbf{n}_k$, the inverse occurs: two eigenstates make opposite contributions along the $k$ direction and same ones in the perpendicular direction. This implies the possibility of the observation of the net spin current flowing perpendicular to the charge current. Particularly interesting are the cases of $\alpha = \pm \beta$, where all the spin orientations, $\pm \mathbf{n}_k$, for different wave vectors are parallel or anti-parallel to each other ($\varphi_k = \pi/4$); see Fig. 2(b). It results from conservation of $(\sigma_x \pm \sigma_y)/\sqrt{2}$, and the spin state become independent of the wave vector.

Now we study charge and spin transport in N/S/N double junction structures. Coherent scattering formalism at the N/S interfaces has been thoroughly developed in the previous studies considering the Rashba SO effect and appropriate boundary conditions requiring the conservation of probability current normal to the interface. It is straightforward to extend the scattering theory to incorporate the Dresselhaus effect. We have used the transfer-matrix formalism to calculate the conductance through and inside the semiconductor: for details refer to Refs. 15, 16.

We will consider electrons incident from the left lead and reflected from the junction interfaces or transmitted through them to the right lead. The wave vector of the incident electron is at angle $\theta$ with respect to the normal to the interface; see Fig. 1. Contrary to the Rashba effect, the Dresselhaus effect is not invariant under the rotation, leading to anisotropic transport. Hence the relative orientation, $\xi$, of the crystal symmetry axes and the interface (see Fig. 1) affects especially the spin current significantly. Below we will calculate the charge conductance $G^{(c)}(\nu)(\nu = x, y)$ in the $\nu$-direction for a definite incident angle $\theta$ as well as the angle-averaged quantity $G^{(c)} = \int_{-\pi/2}^{\pi/2} d\theta G^{(c)}(\theta)$. Also calculated are the
(analogously defined) spin conductances \( G_{x}^{(s,\hat{n})}(\theta) \) and \( G_{y}^{(s,\hat{n})} \) polarized in the direction \( \hat{n} \). The typical values for the parameters we will use below are \( E_{F} = 4.2 \) eV, \( \epsilon_{m} = 0.063, \beta = 0.1 \) eV\AA, \( L = 200 \) nm, and \( W = 1 \) \( \mu \)m. \( \alpha \) ranges from \(-2\beta\) to \(+2\beta\), and \( E_{F} \) from 0 to 20 meV. We assume sufficiently low temperatures (\( k_{B}T \ll E_{F} \)).

**III. NORMAL INCIDENCE**

Owing to the symmetry \( |v_{\uparrow}^{+}| = |v_{\downarrow}^{-}| \) [see the discussion below Eq. (12)], for normal incidence \((\theta = 0)\) the charge current is completely longitudinal; i.e., \( G_{y}^{(c)}(\theta = 0) = 0 \). For a single transverse mode, we obtain the longitudinal charge conductance

\[
G_{x}^{(c)}(\theta = 0) = \frac{e^{2}}{\hbar} \frac{32\hbar^{2}}{|(1+\kappa)^{2}-(1-\kappa)^{2}|^{2}2\pi|\Delta kL|^{2}},
\]

(14)

where \( \Delta k \equiv \sqrt{k_{s\alpha}^{2}(-\xi) + k_{F}^{2}}, \kappa \equiv \Delta k/\epsilon_{m}k_{F}, k_{F} \equiv \sqrt{2m_{e}E_{F}/\hbar} \). Moreover, the spin current has only transverse component and is polarized entirely in the \( xy \)-plane; i.e., \( G_{x}\hat{n}(\theta = 0) = 0 \) for any \( \hat{n} \) and \( G_{y}\hat{n}(\theta = 0) = 0 \). The \( \hat{t}_{x} \)-polarized spin conductance \( G_{y}^{(s,\hat{n})}(\theta = 0) \) is given by

\[
G_{y}^{(s,\hat{n})}(\theta = 0) = \frac{e}{4\pi} \frac{L}{W} \frac{32(m^{2}/\hbar^{4})\alpha\beta \cos 2\xi}{\epsilon_{m}k_{F}k_{s\alpha}(-\xi)} \times \frac{(1+\kappa^{2}) - (1-\kappa^{2}) \sin 2\Delta kL}{2\Delta kL^{2}},
\]

(15)

\( G_{x}^{(c)}(\theta = 0) \) and \( G_{y}^{(s,\hat{n})}(\theta = 0) \) are plotted in Fig. 3 as functions of \( E_{F}^{\ast} \) and \( \alpha/\beta \) for different crystal orientations \( \xi \). The peaks in \( G_{x}^{(c)}(\theta = 0) \) and \( G_{y}^{(s,\hat{n})}(\theta = 0) \) as a function of \( E_{F}^{\ast} \) come from the Fabry-Perot interference, which gives rise to resonances for

\[
\Delta kL = n\pi \quad (n = 0, 1, 2, \ldots).
\]

(16)

Unlike the (longitudinal) charge current, the spin current is very sensitive to the SO coupling strengths, \( \alpha \) and \( \beta \), and the crystal orientation, \( \xi \), as seen from the factor \( \alpha\beta \cos \xi \) in Eq. (15).

**IV. ANGLE-AVERAGED CONDUCTANCES**

For true one-dimensional (1D) leads \( (k_{F}W \ll 1) \), where only a single transverse mode is allowed, one has only to consider normal incidence \((\theta = 0)\); or at a certain fixed \( \hat{n} \). In the opposite limit (i.e., \( k_{F}W \rightarrow \infty \)), there are many transverse modes contributing to the transport. In this case, we should add up all the contributions from \( \theta \) in the range \((-\pi/2, \pi/2)\). It is quite complicated (even though possible) to find the scattering states for non-zero incidence angle \( \theta \), and more convenient to work numerically. Therefore, here we just present the numerical results.

**FIG. 3:** The charge conductance \( G_{x}^{(c)}(\theta = 0) \) [(a) and (b)] and the spin conductance \( G_{y}^{(s,\hat{n})}(\theta = 0) \) [(c) and (d)] for normal incidence as functions of \( E_{F}^{\ast} \) [(a) and (c)] and \( \alpha/\beta \) [(b) and (d)]. \( \alpha/\beta = 0.5 \) in (a) and (b), and \( E_{F}^{\ast} = 12 \) and 14 meV in (b) and (d). \( \xi \) has been chosen to be 0 (black line), \( \pi/2 \) (red solid line), \( \pi \) (green dashed line), and \( 3\pi/2 \) (blue dotted line). Notice that in (a) curves for different \( \xi \)'s overlap almost completely.

**FIG. 4:** Angle-averaged charge conductance \( G_{x}^{(c)} \) as a function of (a) \( E_{F}^{\ast} \) and (b) \( \alpha/\beta \). \( \xi = 0 \) has been chosen, but \( G_{x}^{(c)} \) is not sensitive to \( \xi \). Values of other parameters are indicated in the figures.

Apparently, the longitudinal charge current has a main contribution from the normal incidence. Consequently, as shown in Fig. 4 the \( \theta \)-averaged longitudinal conductance \( G_{x}^{(c)} \) is rather similar to the normal incidence case \( G_{x}^{(c)}(\theta = 0) \), although the peaks are rounded off.

This is not the case for the spin transport. Figure 5 shows the \( \theta \)-dependence of the spin conductances polarized in the \( \hat{n}_{x} \) and \( \hat{z} \), respectively. Again, the peaks cor-
respond to the Fabry-Perot-type resonances. When summing up, the contributions to the $\hat{n}_k^\alpha$-polarized spin current from different angles are mostly canceled with each other, and hence the angle-averaged spin conductance $G^{(\alpha, \xi)}_y$ becomes very small compared with the longitudinal charge conductance $G^{(c)}_x$. On the other hand, the $\hat{z}$-polarized spin current is not subject to such cancellations, and remains rather large (still smaller than the longitudinal charge current); see Fig. 6. Especially for $\xi = 0$, the spin conductance $G_{y}^{(\alpha, \hat{z})}$ remains almost constant in the region $|\alpha/\beta| < 1$ and changes its sign abruptly at $\alpha/\beta = \pm 1$. This behavior is reminiscent of the intrinsic spin Hall conductances in the previous works.\footnote{G.A. Prinz, Science 282, 1660 (1998); S.A. Wolf, D.D. Awschalom, R.A. Buhrman, J.M. Daughton, S. von Molnár, M.L. Roukes, A.Y. Chtchelkanova, and D.M. Treger, Science 294, 1488 (2001).} However, in our case $G^{(\alpha, \xi)}_y$ depends on the strength of the SO couplings, the potential barrier, crystal orientation, and the channel length, showing no universal characteristics.

Here we stress the differences between origins in the spin Hall conductance of ours and the intrinsic spin Hall effect. The intrinsic spin Hall effect is an (semi-classical) effect driven by external electric field penetrating the (infinitely large) system.\footnote{P.R. Hammar, B.R. Bennett, M.J. Yang, and M. Johnson, PRL 83, 203 (1999).} In our case, the external bias voltage merely shifts the relative chemical potentials of the “contacts” (or reservoirs) attached to the metallic leads where the electrons undergo ballistic transport and does not feel an electric field.\footnote{Y. Ohno, D.K. Young, B. Beschoten, F. Matsukura, H. Ohno, D.D. Awschalom, Nature 402, 790 (1999).} Moreover, it has been pointed out that the spin current in the intrinsic spin Hall effect is an equilibrium background current and is not real.\footnote{M.I. Dyakonov and V.I. Perel, Zh. Eksp. Ter. Fiz. 13, 657 (1971) [JETP 33, 467 (1971)]; J.E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999); S. Zhang, Phys. Rev. Lett. 85, 393 (2000); L. Hu, J. Gao, and S.Q. Shen, Phys. Rev. B 68, 115302 (2003); 68, 153303 (2003).} To the contrary, our spin currents originate from non-equilibrium properties of the system, and are real.

We also note that when electrons are incident oblique to the junction interface, the transverse charge current and the longitudinal spin current do not vanish any longer. Therefore, the angle-averaged conductances $G^{(\alpha)}_y$ and $G^{(\alpha, \hat{z})}_y$ are finite, even though quite small compared to $G^{(c)}_x$. Finite $G^{(\alpha)}_y$ in the semiconductor can be attributed to the anisotropy introduced by the Dresselhaus effect. It distorts the group velocity, which thus prefers one of $\pm y$ directions so that the current has same sign as $\alpha/\beta$. Nonzero $G^{(\alpha, \hat{z})}_y$ reflects the breaking of spin conservation inside the semiconductor. For oblique incidence, there exists no direction consistent with the boundary conditions along which spin state is stationary (for example, the spin parallel to the direction $\hat{n}_k$ for a given wave vector $k = k_x \hat{x} + k_y \hat{y}$ is not stationary any longer for $k' = -k_x \hat{x} + k_y \hat{y}$ after reflection from the junction interface). This means that an electron with any spin polarization experiences precession during transmission through the semiconductor.

V. CONCLUSION

Ballistic spin currents with different spin polarizations through mesoscopic metal/2DES/metal double junctions have been investigated in the presence of spin-orbit interactions. Using the coherent scattering theory we showed that longitudinal and/or transverse spin currents flow through 2DES. It was argued that arising from the non-equilibrium distribution of electrons, the spin Hall currents observed are real.

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