Supernovae evidence for an accelerating expansion of the universe

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(submitted June 16, 2000)

Recent experimental results find strong indications that the universe is flat, while other experimental results from supernovae Ia observations have been interpreted to show that, not only that there is an accelerating expansion of the universe, but also that the universe is strongly curved. By means of a recently proposed metric, I am able to show that the experimental results which had previously been analyzed to give a strong curvature are quite consistent with a flat universe, thus resolving the apparent mismatch. The conclusion of these latter authors which indicated an accelerating expansion of the universe is unchanged.

96.30Sf, 97.60Bw, 95.85Bh, 04.26-q

Recently de Bernardis et al. [1] reported that very careful examination of the cosmic microwave background provides, as further explained by Hu [2], very strong evidence that the universe is flat!

On the other hand, data has recently been reported by Riess et al. [3], and Perlmutter et al. [4] on measurements of the luminosity and the redshift of a number of high-redshift supernovae of type Ia. The authors’ best fit to their data involves a strongly curved spacetime. The curvature constant \( \Omega_k \) (defined below) that they find is about -1.05, which corresponds to a strongly curved, open universe. These authors have analyzed their data under the assumption of the Friedman-Lemaître line element at large distances and the resulting cosmological relation between the rate of expansion, the mean mass density, the radius of curvature of space, and the cosmological constant. This relation follows from Einstein’s field equations. [3] The authors have concluded, in terms of this model of the universe, that instead of the rate of expansion decreasing, as many workers had thought, their data is best fit by a model in which the rate of expansion is increasing. Riess et al. [3] find that the deceleration/acceleration parameter \( q_0 < 0 \) (acceleration) with better than 90% confidence, and Perlmutter et al. [4] find the same at the 2.86 standard deviation level of confidence. Similar error limits apply to \( \Omega_k \).

I have recently found [5] that the commonly used “Swiss cheese model,” in which a static, exterior, Schwarzschild solution is inserted in an expanding Friedmann-Lemaître metric, possesses the unphysical property that some of the trajectories are discontinuous functions of their initial conditions. This feature is, I think, unacceptable. I have proposed an alternate metric which avoids this problem. In this letter, I apply this alternate metric to the analysis of the above cited supernovae data. I find that, the principal conclusion, that an accelerating universe best fits the data remains unchanged. However the mean density and cosmological constant parameters describing the best fit change significantly, and are now quite consistent with a flat universe.

The alternate line element is, [6]

\[
ds^2 = -e^\mu (dr^2 + r^2 d\theta^2 + \hat{r}^2 \sin^2 \theta d\phi^2) + e^\nu dt^2, \tag{1}
\]

where

\[
e^\mu = \frac{a(t)^2}{1 + \left(\frac{a(t)\hat{r}}{2a(t)R}\right)^2} \left(1 + \frac{m}{2a(t)\hat{r}}\right)^4,
\]

\[
e^\nu = e^2 \left(\frac{1 - \frac{m}{2a(t)\hat{r}}}{\frac{m}{2a(t)\hat{r}^2}}\right)^2, \quad m = \frac{GM}{c^2} \left[1 + \left(\frac{\hat{r}}{2R}\right)^2\right]^{1/2}
\]

where \( G \) is Newton’s constant of gravitation, and \( M \) is the mass concentrated at the origin. This metric has dynamics which are undetectably different from those of the Schwarzschild metric in the solar system, and the metric is asymptotically equal to the Friedmann-Lemaître metric at great distances. This metric was developed in the context of gravitationally bound systems, and, as is the case for the Schwarzschild metric, it was only considered valid for \( m/2r \ll 1 \). This being said, it makes physical sense. I think, for the effects of the long range gravitational force not to be cut-off at some finite distance well short of the event horizon, as is the case when the Friedmann-Lemaître metric is used to analysis high redshift objects. This view provides a physical justification to use my alternate metric in this analysis.

In order to apply this result to the present case, we make the further assumption that, on the scale of the distance to the high redshift supernovae under discussion, the universe is homogeneous and isotropic. Birkhoff’s theorem [3] says in a homogeneous, zero pressure cosmology that outside a sphere, within which the mass is spherically symmetrically distributed, one finds that on the surface of the sphere (and outside as well), the effects are the same as though the total mass was concentrated at the origin. This theorem suggests that when considering a particular supernova, we can take \( M \) to be the total mass inside the sphere centered on that supernova and which has the observer (us) on its surface.
We may take as a helpful guide to the proper *modus operandi*, Carroll, Press, and Turner.\[6\] The relation between the rate of expansion of the universe, given by the universal multiplier \(a(t)\) defined by \(\dot{r}(t) = a(t)\dot{r}(t_0)/a(t_0)\) is \[6\]

\[
8\pi G \rho \frac{c^2}{2} = 8\pi T_4^4 = \frac{3}{a(t)^2 R_0^2 \left(1 + \frac{m}{2a(t)\dot{r}}\right)} + 3 \dot{a}^2 \frac{c^2}{a^2} - \Lambda,
\]

where \(c\) is the velocity of light, \(\rho\) is the mean density, \(T\) is the stress-energy tensor, and \(\dot{r}\) is the proper distance with respect to \(t\). We define the standard parameters,

\[
\Omega_M = \frac{8\pi G \rho_0}{3H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}, \quad \Omega_k = -\frac{c^2}{R_0^2 H_0^2},
\]

where \(H_0 = \dot{a}(t_0)/a(t_0)\) is the Hubble constant at the present time, \(\Lambda\) is the cosmological constant, and \(R_0\) is the radius of curvature at the present time. Note that as usual, \(R_0^2\) is negative in an open universe. In these terms, Eq. \[3\] becomes

\[
\left(\frac{\dot{a}}{a H_0}\right)^2 = \Omega_M \left(\frac{\rho}{\rho_0}\right) + \frac{R_0^2 \Omega_k}{R^2 \left(1 + \frac{m}{2a(\dot{r})}\right)} + \Omega_\Lambda
\]

(5)

Since at the present time, the look-back time is, of course, zero, as explained above the radius of the sphere enclosing total mass \(M\) will be zero. Thus, as the mass inside the sphere vanishes like the cube of the radius, the term \(m/2a\dot{r} \propto \dot{r}^2 \to 0\). Hence, from Eq. \[3\] at the present time, we have

\[
1 = \Omega_M + \Omega_k + \Omega_\Lambda.
\]

(6)

Using this result, we may eliminate \(\Omega_k\) and rewrite Eq. \[3\] as,

\[
\left[\frac{\dot{a}(t)}{H_0}\right]^2 = \left(1 + \frac{m}{2a(t)\dot{r}}\right)^{-5} + \Omega_M \left[\frac{1}{a} - \left(1 + \frac{m}{2a(t)\dot{r}}\right)^{-5}\right] + \Omega_\Lambda \left[a^2 - \left(1 + \frac{m}{2a(t)\dot{r}}\right)^{-5}\right].
\]

(7)

The next quantity that we need is the distance as a function of the observed luminosity and the redshift for our line element. As is well known \[3\] \(a(t)/a(t_0) = 1/(1 + z)\). Thus from Eqs. \[3\], \[6\] and \[8\] we may deduce

\[
\dot{z} = H_0(1 + z)\left((1 + z)^2 \left(1 + \frac{m}{2a\dot{r}}\right)^{-5} + \Omega_M \left(1 + z)^3 -(1 + z)^2 \left(1 + \frac{m}{2a\dot{r}}\right)^{-5}\right) + \Omega_\Lambda \left[1 - (1 + z)^2 \left(1 + \frac{m}{2a\dot{r}}\right)^{-5}\right]\right)^{0.5}.
\]

The proper distance is just \(c\) times the proper lookback time which gives

\[
ev^{0.5} d\dot{r} = ev^{0.5} \frac{dz}{z},
\]

\[
\sqrt{1 - \left(\frac{r}{R}\right)^2} = c \left[1 - \frac{m}{2a\dot{r}}\right] \frac{(1 + z) dz}{z},
\]

(9)

where we have used the standard change of variables,

\[
r = \frac{\dot{r}}{\left(\frac{\dot{r}}{2R}\right)^2}.
\]

(10)

If we now integrate Eq. \[8\], we get, following Carroll *et al.*, the lookback time and the luminosity distance,

\[
t_0 - t_1 = \int_0^{z_1} \frac{1 - \frac{m}{2a\dot{r}}}{1 + \frac{m}{2a\dot{r}}} \frac{dz}{z} = \frac{D_L}{cH_0} - 1 + \Omega_k = (1 + z) \left[\frac{1 - \frac{m}{2a\dot{r}}}{1 + \frac{m}{2a\dot{r}}}\right] \frac{(1 + z) dz}{z}
\]

(11)

where \(z\) is given by Eq. \[8\] and “sinn” is defined as sinh when \(\Omega_k > 0\) (open universe) and as sin when \(\Omega_k < 0\) (closed universe). These equations coincide with the results of Carroll *et al.*, when \(m = 0\), as expected.

To complete the computation, it remains to supply \(m/2a\dot{r}\) as a function of the redshift. For notational convenience we introduce the following dimensionless variables,

\[
\mathcal{R} = H_0 \dot{r}/c, \quad \mathcal{M} = H_0^3 M(\dot{r})/c^3 \rho_0,
\]

\[
v(z) = \frac{m}{2a\dot{r}} = \frac{GM(\dot{r})}{2c^2 a\dot{r}} = \frac{3(1 + z)\Omega_M \mathcal{M}}{16 \pi \mathcal{R}} \left[1 - 0.25\Omega_k \mathcal{R}^2\right]^{0.5},
\]

\[
q(z) = \{((1 + z)^2 + \Omega_M \left[(1 + z)^3(1 + v)^5 - (1 + z)^2\right] + \Omega_\Lambda \left[(1 + v)^5 - (1 + z)^2\right]\}^{0.5}.
\]

If \(l\) is the proper distance, then by Hubble’s law, \(z = H_0 l/c\). By use of Eqs. \[1\], \[2\], \[6\], and Hubble’s law, we may deduce the following two non-linear differential equations,
\[ \frac{d\mathcal{R}}{dz} = -(1 + z) \left( 1 - 0.25\Omega_k \mathcal{R}^2 \right) (1 + v)^{0.5} \frac{q(z)}{R} \]

\[ \frac{d\mathcal{M}}{dz} = -4\pi(1 + z)(1 + v)^{0.5} \mathcal{R}^2 \frac{q(z)}{(1 - 0.25\Omega_k \mathcal{R}^2)^2}. \] (13)

The initial conditions are \( \mathcal{R} = M = 0, z = z_1 \) and we integrate the equations from \( z = z_1 \) to \( z = 0 \).

FIG. 1 The metric correction term \( v(z) = m(\hat{r})/2a\hat{r} \) for the five models A – E. The intersection of these curves with the dotted line corresponds to the special point \( z_c \) in each of these models. The end of each curve corresponds to the last integration point before \( v \) blows up.

I have solved these equations by the Runge-Kutta method. To do so I have rewritten the appropriate routine in Press et al. [8] in double precision. Some sample cases, Models A - E, are shown in Fig. 1. In this figure I plot the potential, \( v(0, z_1) \), through which light must climb to reach an observer who sees a redshift of \( z_1 \). The five models are as follows Model A, \( \Omega_M = 1, \Omega_\Lambda = 0; \) Model B, \( \Omega_M = 0.1, \Omega_\Lambda = 0; \) Model C, \( \Omega_M = 0.1, \Omega_\Lambda = 0.9; \) Model D, \( \Omega_M = 0.01, \Omega_\Lambda = 0; \) Model E, \( \Omega_M = 0.01, \Omega_\Lambda = 0.99. \) These are all either flat or open models. One will note that the principal effect at small \( z_1 \) is the mass density. It is to be further noted that the point \( v(z_c) = 1 \) is special. At this point, \( dD_L/dz \) would change sign, as can be seen in Eq. 13. In Fig. 2 we display the lookback time as a function of \( z \) for these same models. The point \( z_c \) is marked by a large dot. It is to be noticed that the lookback time begins to decrease with \( z \) before this special point is reached as a foreshadowing of the special point. In addition, \( \mathcal{R}(0, z_1) \) also begins to decrease before the special point is reached, but \( M(0, z_1) \) and \( v(0, z_1) \) both increase right through this point and blow up shortly after, as indicated by the end of the lines in Figs. 1 and 2. These results are numerical as the proper analysis of Eq. 13 is beyond the scope of this letter.

FIG. 2 The lookback time for the five models A – E. The large dot on each of these curves correspond to the intersection points in Fig. 1.

It is not my purpose to obtain a full statistical analysis of the redshift – luminosity data of 8 and 9. Rather I have worked with the best fit of 4, \( \Omega_M = 0.73, \Omega_\Lambda = 1.32 \). Specifically I find that I can fit this best fit for the luminosity distance as a function of redshift to within \( \pm 0.02 \) magnitudes over the range of the data used. This is, I think, at least as close as that curve accurately represents the data.

My best fit is \( \Omega_M = 0.22 \pm 0.03, \Omega_\Lambda = 0.91 \pm 0.15 \) for the accuracy quoted above. My best fit for flat space is \( \Omega_M = 0.19, \Omega_\Lambda = 0.81 \). The magnitude of \( v(0, z_1) \) is roughly proportional to \( z^2 \), and its largest value in both cases is less than \( 0.07 \). When the statistical and systematic errors are taken into account the uncertainties are probably about 3 to 4 times larger than those quoted above. My result for flat space is almost the same as Model C of Carroll et al. 8. That is to say that it is flat, has a plausible amount of matter and involves the use of the cosmological constant. Note is also taken that for the special case of flat space, Eq. 9 reduces to Eq. 9 of reference 8; however, our Eq. 9 still reflects a change in the metric. Our results predict that the acceleration param-
eter \( q_0 \approx -0.79 \), which is strongly negative and therefore agrees with the main conclusion of references [3] and [4] that the expansion of the universe is accelerating. We have shown however that the disagreement between their best-fit, curvature results and those of references [1] and [2] can be resolved by employing my alternate metric.

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