Single top quark production in \( t \)-channel at the LHC in Noncommutative Space-Time

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Abstract

We study the production cross section of the \( t \)-channel single top quark at the LHC in the noncommutative space-time. It is shown that the deviation of the \( t \)-channel single top cross section from the Standard Model value because of noncommutativity is significant when \( |\vec{\theta}| \gtrsim 10^{-4} \) GeV\(^{-2} \). Using the present experimental precision in measurement of the \( t \)-channel cross section, we apply upper limit on the noncommutative parameter. When a single top quark decays, there is a significant amount of angular correlation, in the top quark rest frame between the top spin direction and the direction of the charged lepton momentum from its decay. We study the effect of noncommutativity on the spin correlation and we find that depending on the noncommutative scale, the angular correlation can enhance considerably. Then, we provide limits on the noncommutative scale for various possible relative uncertainties on the spin correlation measurement.

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1 Introduction

The study of single top quark processes at hadron colliders provides the opportunities to investigate the electroweak properties of the top quark, direct measurement of the \( V_{tb} \) CKM matrix element, and more importantly it provides the possibility to search for new physics. The standard model (SM) has been found to be in a good agreement with the present experimental measurements in many of its aspects. In the framework of the SM, top quark is the heaviest particle with the mass at the order of the electroweak symmetry breaking scale, \( v \sim 246 \) GeV. This large mass might be a hint that the top quark plays an essential role in the electroweak symmetry breaking. On the other hand, the reported experimental data from Tevatron and LHC on the top quark properties are still limited and no significant deviations from the standard model predictions has been observed yet.

Top quarks are mainly produced through two independent mechanisms at hadron colliders: The main production mechanism is via strong interactions where top quarks are produced in pair \((gg \rightarrow t\bar{t}, q\bar{q} \rightarrow t\bar{t})\)\(^{[1]}\). The production cross section of \( t\bar{t} \) at 7 TeV center-of-mass energy at the LHC is 157 pb at next to leading order \(^{[3]}\). Top quark can be produced singly via electroweak interaction. It occurs through three different processes: \( t \)-channel (the involved \( W \)-boson is space-like, \( ub \rightarrow dt \)), \( s \)-channel (the involved \( W \)-boson is time-like, \( ud \rightarrow \bar{b}t \)) and \( tW \)-channel (the involved \( W \)-boson is real, \( gb \rightarrow W^-t \)). The \( t \)-channel with the cross section of 60 pb is the largest source of single top at the LHC \(^{[1]}\).

The cross sections of \( t\bar{t} \) and single top production, the top quark mass, the helicity of \( W \) boson in top decay, the search for flavor changing neutral current, and many other properties of the top quark have been already studied \(^{[1],[2]}\). However, it is expected that top quark properties such as single top quark cross section measurement are going to be measured with high precision at the LHC due to very large statistics \(^{[1]}\).

The space-time noncommutativity is a generalization of the usual quantum mechanics and quantum field theory which may describe the physics at short distances of the order of the Planck length, since the nature of the space-time could change at these distances. There are motivations coming from string theory, quantum gravity, Lorentz breaking \(^{[4],[5],[6],[7]}\) to construct models on noncommutative space-time. The noncommutativity in space-time can be described by a set of constant c-number parameters \( \theta^{\mu\nu} \) or equivalently by an energy scale \( \Lambda_{NC} \) and dimensionless
parameters $C^{\mu\nu}$:

\[
[\hat{x}_\mu, \hat{x}_\nu] = i \theta_{\mu\nu} = \frac{i}{\Lambda_{NC}^2} C_{\mu\nu} \begin{pmatrix}
0 & -E_1 & -E_2 & -E_3 \\
E_1 & 0 & -B_3 & B_2 \\
E_2 & B_3 & 0 & -B_1 \\
E_3 & -B_2 & B_1 & 0
\end{pmatrix}
\]  

(1)

where $\theta_{\mu\nu}$ is a real anti-symmetric tensor which has the dimension of $[M]^{-2}$. Dimensionless electric and magnetic parameters ($\vec{E}, \vec{B}$) have been defined for convenience. It is notable that a space-time noncommutativity, $\theta_{0i} \neq 0$, might cause some problems with unitarity and causality [8],[9]. It has been shown that the unitarity can be satisfied for the case of $\theta_{0i} \neq 0$ provided that $\theta^{\mu\nu} \theta_{\mu\nu} > 0$ [10]. However for simplicity, in this article we take $\theta_{0i} = 0$ or equivalently $\vec{E} = 0$.

One can obtain a noncommutative version of an ordinary field theory by replacing all ordinary products among fields with Moyal $\star$ product defined as [11]:

\[
(f \star g)(x) = \exp \left( \frac{i}{2} \theta^{\mu\nu} \partial_\mu f(x) \partial_\nu g(x) \right) \bigg|_{y=z=x}
\]

\[
= f(x)g(x) + \frac{i}{2} \theta^{\mu\nu} (\partial_\mu f(x)) (\partial_\nu g(x)) + O(\theta^2).
\]  

(2)

The approach to the noncommutative field theory based on the Moyal product and Seiberg-Witten maps allows the generalization of the standard model to the case of noncommutative space-time, keeping the original gauge group and particle content [12],[13],[14],[15],[16],[17]. Seiberg-Witten maps relate the noncommutative gauge fields and ordinary fields in commutative theory via a power series expansion in $\theta$. Indeed the noncommutative version of the Standard Model is a Lorentz violating theory, but the Seiberg Witten map shows that the zeroth order of the theory is the Lorentz invariant Standard Model. The effects of noncommutative space-time on some rare decay, collider processes, leptonic decay of the $W$ and $Z$ bosons and additional phenomenological results have been presented in [18],[19],[20],[21],[22],[23],[24],[25],[26],[28],[29],[30] and some limits have been set on noncommutative scale.

In this article, we calculate the contributions that the $t$-channel single top quark cross section receive from the noncommutativity in space-time at the LHC in the center-of-mass energy of 7 TeV. Then, we estimate a bound on the noncommutative parameter $\theta$ by comparing the
recent measurement of the CMS collaboration of the t-channel cross section with the theoretical calculations.

In Section 2 of this article, a short introduction for the noncommutative standard model (NCSM) is given. Section 3 presents the calculations of the noncommutative effects on the single top quark cross section and limit on $\theta$ from current measured single top production rate. Finally, Section 4 concludes the paper.

2 The Noncommutative Standard Model (NCSM)

The NCSM action is obtained by replacing the ordinary products in the action of the classical Standard Model by the Moyal products and then matter and gauge fields are replaced by the appropriate Seiberg-Witten expansions. The action of NCSM can be written as:

$$S_{NCSM} = S_{fermions} + S_{gauge} + S_{Higgs} + S_{Yukawa},$$  \hspace{1cm} (3)

This action has the same structure group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the same fields number of coupling parameters as the ordinary SM. The approach which has been used in [13], [14], [15], [16] to build the NCSM is the only known approach that allows to build models of electroweak sector directly based on the structure group $SU(2)_L \times U(1)_Y$ in a noncommutative background. The NCSM is an effective, anomaly free, noncommutative field theory [31], [32]. We just consider the fermions. The fermionic part of the action in a very compact way is:

$$S_{fermions} = \int d^4x \sum_{i=1}^{3} \left( \tilde{\Psi}_L^{(i)} \star (i \tilde{\mathcal{D}} \tilde{\Psi}_L^{(i)}) \right) + \int d^4x \sum_{i=1}^{3} \left( \tilde{\Psi}_R^{(i)} \star (i \tilde{\mathcal{D}} \tilde{\Psi}_R^{(i)}) \right),$$  \hspace{1cm} (4)

where $i$ is generation index and $\Psi_{L,R}^{i}$ are:

$$\Psi_{L}^{(i)} = \begin{pmatrix} L_{L}^{i} \\ Q_{L}^{i} \end{pmatrix}, \hspace{0.5cm} \Psi_{R}^{(i)} = \begin{pmatrix} e_{R}^{i} \\ u_{R}^{i} \\ d_{R}^{i} \end{pmatrix},$$  \hspace{1cm} (5)

where $L_{L}^{i}$ and $Q_{L}^{i}$ are the well-known lepton and quark doublets, respectively. The Seiberg-Witten maps for the noncommutative fermion and vector fields yield:

$$\tilde{\psi} = \tilde{\psi}[V] = \psi - \frac{1}{2} \theta^{\mu \nu} V_{\mu} \partial_{\nu} \psi + \frac{i}{8} \theta^{\mu \nu} [V_{\mu}, V_{\nu}] \psi + O(\theta^2),$$

$$\tilde{V}_{\alpha} = \tilde{V}_{\alpha}[V] = V_{\alpha} + \frac{1}{4} \theta^{\mu \nu} \{ \partial_{\mu} V_{\alpha} + F_{\mu \alpha}, V_{\nu} \} + O(\theta^2),$$  \hspace{1cm} (6)
where $\psi$ and $V_\mu$ are ordinary fermion and gauge fields, respectively. Noncommutative fields are denoted by a hat. For a full description and review of the NCSM, see [13],[14],[15],[16].

3 The Noncommutative Corrections to the $t$-Channel Cross Section

The $q_1(p) \rightarrow W(q) + q_2(k)$ vertex in the NCSM up to the order of $\theta^2$ can be written as [22]:

$$ \Gamma_{\mu,NC} = \frac{gV_{tb}}{\sqrt{2}} \left[ \gamma_\mu + \frac{1}{2} (\theta_{\mu\nu} \gamma_\alpha + \theta_{\alpha\mu} \gamma_\nu + \theta_{\nu\alpha} \gamma_\mu) q^\nu p^\alpha \right. $n\nonumber $\left. - \frac{i}{8} (\theta_{\mu\nu} \gamma_\alpha + \theta_{\alpha\mu} \gamma_\nu + \theta_{\nu\alpha} \gamma_\mu) (q\theta p) q^\alpha p^\nu \right] P_L. $$

where $P_L = \frac{1 - \gamma_5}{2}$ and $q\theta p \equiv q^\mu \theta_{\mu\nu} p^\nu$. This vertex is similar to the vertex of $W$ decays into a lepton and anti-neutrino [26]. However, one should note that due to the ambiguities in the SW maps there are additional terms in the above vertex. Since they will not affect the results, we have ignored them [27].

After some algebra the noncommutative corrections to the squared matrix element for the $t$-channel process ($u(p_1) + b(p_3) \rightarrow d(p_2) + t(p_4)$) is as follows:

$$ |M_{NC}|^2 = 4G_F^2 m_W^4 |V_{tb}|^2 |V_{ud}|^2 \times \left\{ q_3 p_3 [(s^2 + u^2) - m_t^2 (s + u)] \right\} \theta_{ij} $$

where, $m_t$ is the top quark mass, $m_b$ is the $b$-quark mass and $m_W$ is the mass of $W$-boson. $V_{tb}, V_{ud}$ are the CKM matrix elements. In Eq. 8 the contributions coming from $O(\theta^2)$ and higher have been ignored as well as the masses of light quarks. $s, t, u$ are the Mandelstam variables.

The total cross section of the $t$-channel signal top production in proton-proton collisions at the LHC is given by:

$$ \sigma = \sum_{a,b} \int dx_1 \int dx_2 f_a(x_1, Q^2) f_b(x_2, Q^2) \hat{\sigma}_{ab} $$

where $\hat{\sigma}_{ab}$ is the partonic level cross section for the process $u + b \rightarrow t + d$. The calculation is performed at $Q = m_W$. $f_a(x, Q^2)$ are the parton distribution functions. CTEQ6 [35] is used as for the proton parton distribution functions. Fig.1 shows the dependence of the single top cross section as a function of noncommutative parameter $\theta$ as well as the experimental precision.
Figure 1: The $t$-channel single top quark production cross section as a function of noncommutative parameter $|\vec{\theta}|$ as well as the experimental precision band from the recent LHC measurement. The solid red line is the standard model value for the signal top cross section. According to Fig. 1, the noncommutativity has constructive effect on the cross section.

Using an integrated luminosity of $36 \text{ pb}^{-1}$ collected with the CMS detector at the LHC, the value of the single top cross section found to be $83.6 \pm 29.8 (\text{stat.} + \text{syst.}) \pm 3.3 (\text{lumi.}) \text{ pb}$ \cite{30}. This measurement is consistent with the standard model expectation.

Comparing the CMS experimental results on single top cross section measurement with the single top cross section in the noncommutative space-time, we get an upper value on the non-commutative parameter $\theta$ of $\mathcal{O}(0.001) \text{ GeV}^{-2}$. If we assume $|\vec{B}| = 1$, this limit can be translated into $\Lambda \gtrsim 100 \text{ GeV}$. However, for smaller values of $|\vec{\theta}|$, the noncommutative corrections get smaller and smaller. Since the LHC experiments are able to measure the $t$–channel cross section with high precision with larger amount of data \cite{37}, this limit can be higher. For example, a measurement with 10% uncertainty leads to the bound of $\Lambda \gtrsim 500 \text{ GeV}$. 


4 The Noncommutative Effect on the Spin Correlation

One of the important features of $t$-channel single top quark production is the large polarization for a suitable choice of spin quantization axis [38]. Because of the large mass of the top quark, it decays before hadronizations via weak interactions and no hadronic bound state can be formed. On the other hand, in the SM because of the $V-A$ (Vector-Axial) structure of the top quark couplings to the $W$ boson, the decay products of a polarized top quark have a particular structure of angular correlations. The angular distributions of the decay products of the top quark have the following form [40]:

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d(\cos \theta_i)} = \frac{1}{2} \left( 1 + \alpha_i \cos \theta_i \right) .
\]  

(10)

where $\theta_i$ is defined as the angle between the momentum of the $i$th decay product and the top quark spin quantization axis in the top quark rest frame. The $\alpha_i$ coefficients are called spin correlation coefficients and shows the degree of correlation with the spin direction of the top quark. In the leptonic decay of the top quark ($t \rightarrow l^+ + \nu_l + b$), with $\alpha_l = 1$, the charged lepton is maximally correlated with the top quark spin direction. Please notice that $\alpha_\nu = -0.32$ and $\alpha_b = -0.40$.

In [22], the effects of noncommutative space-time on the charged lepton spin correlation coefficient $\alpha_l$ has been calculated. We showed that depending on the value of the noncommutative characteristic scale $\Lambda$, $\alpha_l$ can deviate significantly from its SM value.

In the $t-$channel single top quark production ($u + b \rightarrow d + t$), the direction of the spectator jet ($d-$type quark) is optimal for the spin quantization axis [38]. More than 96% of the top quarks are produced with spin directions aligned with the momenta of the $d-$type quark in the top quark rest frame. In the leptonic decay of the top quark, the charged lepton has the strongest correlation with the top quark spin, $\alpha_l = 1$. Accordingly, the largest correlations appear for the case of measurement of the angle between the charged lepton and the spectator jet in the top quark rest frame. Fig[2] shows the distribution of the cosine of the angle between the charged lepton momentum and the $d-$type quark momentum in the top quark rest frame for the SM case and in the noncommutative SM with different values of the noncommutative scale $\Lambda$. According to Fig[2], the angular distribution is significantly sensitive to the noncommutative space-time when $\Lambda \lesssim 1$ TeV.
Figure 2: The angular correlation in \( t \)-channel single top quark production for the SM case with different values of the noncommutative scale \( \Lambda \) at the LHC.

| Relative Uncertainty \( \Delta\alpha/\alpha \) | Lower limit on \( \Lambda \) in GeV |
|---------------------------------------------|----------------------------------|
| 5\%                                        | 980                              |
| 10\%                                       | 844                              |
| 15\%                                       | 742                              |
| 20\%                                       | 708                              |

Table 1: The lower limit on \( \Lambda \) in GeV assuming various relative uncertainties on measurement of \( \alpha_l \).

A bin by bin precise comparison of the angular distribution with real data could provide much better limit on \( \Lambda \) with respect to the limit obtained from the total cross section measurement. Fig. 3 depict the lower bound on the noncommutative characteristic scale \( \Lambda \) in GeV as a function of the charged lepton spin correlation coefficient. In this Figure, the lower limit on \( \Lambda \) has been shown depending on different uncertainties which the LHC experiments could measure. For example, in Fig. 3 the end point of the thick red curve is corresponding to the lower limit on \( \Lambda \) axis assuming the relative uncertainty on 5\% on \( \alpha_l \). Table 1 shows the lower limit on \( \Lambda \) in GeV assuming various relative uncertainties on measurement of \( \alpha_l \). As you can see, the best lower limit that we can achieve via spin correlation from single top events is \( \Lambda \gtrsim 980 \) GeV.
Figure 3: The lower limits on $\Lambda$ in GeV assuming various relative uncertainties on the measurement of $\alpha_l$.

5 Conclusions

In conclusion, the noncommutative effects on the single top quark production cross section at the LHC are very small for most of the parameter space (less than $\mathcal{O}(0.001)$ GeV$^{-2}$). Therefore, it seems that there is not much hope of determining possible noncommutative effects directly from the single top quark cross section. However, the distribution of the cosine of the angle between the charged lepton momentum from the top decay and the momentum of the spectator quark in the top quark rest frame is highly sensitive to the noncommutative space-time when $\Lambda \sim 1$ TeV. A precise comparison of the angular distribution with real data could provide much better limit on $\Lambda$. We find the lower limit on $\Lambda$ depending on different relative uncertainties on measurement of $\alpha$, if the experiments could measure $\alpha$ with the precision of 5%, the lower limit on $\Lambda$ will be 980 GeV.

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