Electronic materials with nanoscale curved geometries

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As the dimensions of a material shrink from an extended bulk solid to a nanoscale structure, size and quantum confinement effects become dominant, altering the properties of the material. Materials with nanoscale curved geometries, such as rolled-up nanomembranes and zigzag-shaped nanowires, have recently been found to exhibit a number of intriguing electronic and magnetic properties due to shape-driven modifications of charge motion or confinement effects. Local strain generated by curvature can also lead to changes in material properties due to electromechanical coupling. Here we review the development of electronic materials with nanoscale curved geometries. We examine the origin of shape-, confinement- and strain-induced effects and explore how to exploit these in electronic, spintronic and superconducting devices. We also consider the methods required to synthesize and characterize curvilinear nanostructures, and highlight key areas for the future development of curved electronics.

The electronic, magnetic and optical properties of a material change when confined to two-dimensional (2D) sheets, 1D nanowires and 0D quantum dots. Advances in nanostructuring techniques now also allow electronic materials with 3D nanoscale architectures to be constructed from 2D and 1D materials. The pursuit of these structures has largely been driven by the need to create electronic memory and logic devices with improved performance and reliability, although other applications—including 3D magnetic sensing and complementary metal–oxide–semiconductor (CMOS)-compatible magneto-impedance sensorics—can also benefit from such architectures.

The fabrication of architectures with curved geometric shapes, such as spiralling tubes or helices, is of particular interest. A curved geometric shape introduces a new length scale—the characteristic radius of curvature—which is crucial to the properties of a material. For example, when a channel with diffusive electron transport is geometrically deformed, the charge and spin transport characteristics can be affected when the radius of curvature becomes comparable to the electron mean free path. This is due to electron trajectories being bent by the curved shape of the channel. Curvature can also cause deviations in the intrinsic electronic properties of a material due to confinement and electromechanical coupling to inhomogeneous strain fields. This occurs when the curvature radius approaches the de Broglie wavelength of electrons near the Fermi level. Similarly, the ground state and elementary excitations of materials that exhibit long-range order, such as magnets and superconductors, can be geometrically tuned when the radius of curvature is comparable to the magnetic length or the superconducting coherence length.

The physical properties of essentially all electronic materials can thus be influenced by nanoscale geometry-induced effects and be endowed with characteristics that would be difficult to achieve otherwise. In this Review we explore the origin of shape-, confinement- and strain-induced functionalities, and consider how to exploit these in electronic devices, including spin and superconducting systems. We also consider the methods required to fabricate and characterize such nanostructures, and also highlight key areas for the future development of the field of curved electronics.

Classical shape effects
In electronic systems with Fermi wavelengths much smaller than the curvature radii of the material structures, geometry-induced effects are purely classical and are a direct consequence of the geometric shape.

Curved spintronics. A first example of such geometry-induced effects results from the structural inhomogeneities that can be engineered in curved electronic channels. Fabrication of curved channels using a (non-planar) shaped substrate that acts as a template yields thicknesses with a local profile in one-to-one correspondence with the geometry. Specifically, as demonstrated in Fig. 1a, the thickness of a curved channel is strongly reduced in regions of large curvature gradient. This characteristic can be used to efficiently tune the electrical properties of the system when the nanoscale thickness is comparable to the electronic mean free path. Even if quantum effects are negligible due to the smallness of the de Broglie wavelength, classical size effects consisting of an increase of the resistivity due to diffusive scattering at the channel and grain boundaries can transform the shape-driven thickness inhomogeneities into an enhanced local nanoscale resistivity (Fig. 1a).

In addition, the local sheet resistance \( \rho(t)/t \) decreases more quickly than the resistivity \( \rho(t) \propto 1/t \) as the thickness \( t \) increases (Fig. 1a). This different scaling can be exploited in curved metallic nanochannels with engineered local thickness of a few tens of nanometres when used as pure spin transport channels. Metallic materials form the basis of current spintronic technologies. In addition, the dominant spin relaxation mechanism corresponds to the so-called Elliot–Yafet mechanism. This dictates that the spin-
diffusion length is strictly locked to the resistivity of the metallic channel. The different scaling between local sheet resistance and resistivity consequently allows the charge and spin transport properties of a curved metallic channel to be controlled independently. Such independent tuning has been demonstrated in lateral non-local spin valves (Fig. 1b). A given spin-valve signal—defined as the difference in the non-local resistance between parallel and antiparallel magnetic states of the injector and detector electrodes—can be obtained for different values of the channel charge resistance (Fig. 1b) and vice versa, in contrast with conventional ‘flat’ channels where spin and charge resistances are locked to each other. In addition, for devices with the same lateral footprint and at a given charge resistance, the spin signal of a flat channel is always smaller than the electric response obtained using a curved channel. This is encoded in a curvature factor (Fig. 1c) that quantifies the gain in the spin signal that results from the intrinsic inhomogeneity of the curved channel. This generalized advantage, together with the independent tuning of charge and spin responses, is of immediate relevance when considering the practical implementation of spintronics. The use of geometric curvature to control on-demand spin and charge impedances in multiple-terminal devices adds a novel approach for their efficient integration with CMOS transistors.

Geometric nonlinear Hall effect. Geometric curvature can also lead to classical shape effects that directly derive from the charge motion in the tangential curved direction of the channel and can therefore appear even in planar structures. These are therefore completely different from the boundary scattering effect in the thickness direction that is the basis of curved spintronics. In curved channels, charge carriers are forced to follow paths that in conventional flat channels typically require the presence of external electromagnetic fields. Consider, for example, a planar curved wire taking the shape of a semicircular annulus. Injection of a current in this channel necessarily leads to the appearance of a transverse electric potential, and the current has to be accelerated radially to follow the circular path. This creates surface charges similar in nature to those of the classical Hall effect (Fig. 1d), even if a perpendicular magnetic field is absent. The ensuing transverse potential is quadratic in the current density, as required by time-reversal invariance, and can be regarded as a purely geometric nonlinear ‘Hall’ effect. The quadratic dependence on the injected current implies that an a.c. current with frequency will yield a transverse potential of the form.

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**Fig. 1** Classical shape effects. a, Schematic of an electronic channel grown on a trenched substrate with given height (A) and full-width at half-maximum (FWHMs). The thickness inhomogeneity along the channel creates a local resistivity distribution. There is a different scaling between the sheet resistivity  and the inverse of the spin diffusion length , and the current has to be accelerated radially to follow the circular path. This creates surface charges similar in nature to those of the classical Hall effect (Fig. 1d), even if a perpendicular magnetic field is absent. The ensuing transverse potential is quadratic in the current density, as required by time-reversal invariance, and can be regarded as a purely geometric nonlinear ‘Hall’ effect. The quadratic dependence on the injected current implies that an a.c. current with frequency will yield a transverse potential of the form.

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**Panel b** Plot of room-temperature spin resistance measured in a non-local spin valve against the charge resistance of a curved aluminium channel grown on trenched substrates of different FWHMs. The charge resistance is measured in units of the curvature factor (Fig. 1c) that quantifies the gain in the spin signal relative to the charge resistance. D, Schematic of the surface charges induced in the classical Hall effect, and the surface charges in a curved wire that produce the electric field accelerating the carriers centripetally. E, Schematic of the surface charges induced in the classical Hall effect, and the surface charges in a curved wire that produce the electric field accelerating the carriers centripetally. F, In carbon nanoscrolls with open geometries, a purely transversal magnetic field yields a strongly anisotropic magnetoresistance (AMR). SEM image in b reproduced with permission from ref. 16, American Chemical Society. Panels adapted with permission from: a (right), ref. 21, Springer Nature Ltd; e, ref. 16, American Chemical Society.

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Directional-dependent magnetotransport. The complexity of electronic trajectories in curved channels becomes even greater in the presence of external magnetic fields. This can be shown by considering carbon nanotubes—one of the most common examples of a nanostructure with curved geometry—in the presence of a transversal magnetic field. Charge carriers in carbon nanotubes respond primarily to the normal component of the externally applied magnetic field, which, as shown in Fig. 1e, changes sign at the opposite side of the tube and averages out. When the cyclotron radius \( R_{\text{cy}} = \frac{m^* v_f (eB)}{B} \) associated with the externally applied magnetic field \( B \), where \( v_f \) is the Fermi velocity and \( m^* \) a density-dependent dynamical mass, is larger than the nanotube radius, the electronic trajectories correspond to helix-like orbits completely wrapping the tube. Computation of the magnetoconductance in the diffusive regime has shown that these classical helical orbits yield, even in a single-channel model, a quadratic longitudinal magnetoresistance that has been experimentally observed in nanotube bundles and multi-walled carbon nanotubes. In the regime where the cyclotron radius is smaller than the carbon-nanotube radius, the nature of the classical electron trajectories changes qualitatively. The externally applied magnetic field is indeed large enough to allow for the formation of cyclotron orbits completely localized in the regions where the surface normal is parallel to the magnetic field (Fig. 1e). These cyclotron orbits do not contribute to the magnetoconductance, in contrast to the snake orbits that are instead naturally formed in nanotube regions where the normal component of the magnetic field changes its sign. The contribution of snake orbits to the magnetoconductance is characterized by the \( \sqrt{B} \) power-law dependence explicitly shown in transport measurements. A similarly unconventional linear magnetoconductance has also been observed recently in a different material system: SnS\(_2\)/WSe\(_2\) van der Waals heterostructures rolled up into tubes. In this radial superlattice, the magnetoconductance strongly decreases by tilting the magnetic field towards the axial direction (Fig. 1e, right), suggesting that the formation of snake orbits is responsible for this phenomenon.

The imprint of snake orbits on the magnetotransport properties of tubular nanostructures becomes even more apparent when considering the open geometry of carbon nanoscrolls—spirally wrapped graphite layers that, unlike carbon nanotubes, have overlapping fringes. In carbon nanoscrolls, the density of snake orbits depends crucially on the direction of the transversal magnetic field. When the external magnetic field is directed towards the open edges there is a proliferation of snake orbits due to the charge carriers experiencing an additional sign change of the effective normal magnetic field, as compared to the case in which the external magnetic field is directed orthogonal to the open edges. For example, in a single-winding rolled-up open tube, the normal component of the magnetic field changes sign once or twice depending on the transversal magnetic field direction (Fig. 1f). Consequently, carbon nanoscrolls can display a strongly directional-dependent magnetoresistance similar to its functional dependence to the anisotropic magnetoconductance of spin–orbit-coupled magnetic materials. The magnitude of this effect, which is the immediate result of the broken rotation symmetry of the tubular structure, has been predicted to be remarkably large in single-winding carbon nanoscrolls, as it can reach 80% (Fig. 1f).

Confinement-induced curvature effects

When the quantum nature of the carriers becomes relevant, the bent electron trajectories result in the appearance of certain quantum states that are directly linked to the curved geometry of the material structures.

Ballistic transport. As a specific example, snake states can be encountered in the ballistic transport regime of traditional semiconducting materials with mean free paths much larger than the curvature radii. An important example is found in core–shell nanowires—systems consisting of a conducting shell surrounding an insulating core, such as InAs surrounding GaAs. In their electronic bandstructure, the inhomogeneous radial component of the magnetic field leads to Landau states that are condensed at small wavenumbers. These states are connected for larger momenta to dispersive states corresponding in a semiclassical analysis to the snake orbits. Such snake states are situated at the bottom of the energy spectrum. Consequently, they are the main actors in quantum transport at low chemical potentials. Numerical calculations have shown that snake states result in peaks in the quantum conductance of core–shell nanowires, with a peak amplitude that can be tailored by adjusting the position of the metallic contacts with respect to the alignment of the transversal magnetic field. Similar conductance oscillations due to snake states have also been reported in graphene p–n junctions in the ballistic regime.

Semiconducting core–shell nanowires are often grown with hexagonal cross-sections and with edges between the different crystallographic facets that are rounded to form regions with finite curvature. In these regions, the corresponding local radius of curvature can be small enough to be comparable to the de Broglie wavelength of the carriers, thus allowing us to probe the quantum geometric potential arising from confinement on curved surfaces (Box 1). This quantum geometric potential consists of a series of square wells, making the rounded edges of a prismatic core–shell nanowire regions of preferred localization. Such intrinsic curvature-induced localization has a strong interplay with the formation of snake orbits in the presence of a transversal magnetic field. Consider first a transversal magnetic field orthogonal to one of the facets of the core–shell nanowire. This will favour the formation of snake states on the two perpendicular edges, enhancing their charge localization as compared to the other edges. On the other hand, for a magnetic field pointing towards one edge, snake states will form along the facets and counteract the tendency to localize the electronic charge at the nanowire edges. This different charge localization mechanism is then reflected in a ballistic direction-dependent magnetoconductance.

An ideal 3D topological insulator (TI) nanowire can be thought of as another material structure consisting of a conducting curved shell surrounding an insulating bulk. In the diffusive regime, the presence of an axial magnetic field gives rise to periodic Altshuler, Aronov and Spivak (AAS) magnetoconductance oscillations originating from weak anti-localization. These AAS oscillations, with period \( h/(2e) \), are substituted in the (quasi)ballistic transport regime by \( h/e \)-periodic Aharonov–Bohm (AB) oscillations that are instead the result of the existence of the so-called perfectly transmitted mode. In curved geometries, single (spin-momentum-locked) Dirac cone theory for the surface of a TI acquires a spin connection term that yields a Berry phase and consequently a gapped spectrum. However, a magnetic flux of half a flux quantum identically cancels the Berry phase and restores a gapless spectrum with an odd number of modes. Accordingly, a perfectly transmitted mode occurs with conductance \( e^2/h \) (ref. 30). AB magnetoconductance oscillations have been observed experimentally in Bi\(_2\)Se\(_3\) (ref. 31) and HgTe nanowires. Superimposing a local variation of the nanowire curvature radius, that is, considering nanocones, has recently been predicted to lead to other intriguing mesoscopic transport phenomena, such as resonant transmission through Dirac Landau levels.

Spin–orbit-coupled semiconducting materials. Semiconducting nanomaterials also exhibit curvature effects that are due to relativistic corrections. The interplay between spin–orbit coupling and curvature leads to the appearance of complex spin textures without counterparts in conventional ‘flat’ nanostructures. This is because in the
Let us first illustrate the emergence of curvature effects resulting from the quantum dynamics of charge carriers in the non-relativistic regime. Consider an electron entering a curved channel. For its mean trajectory to follow the geometry of the structure, the electron must be subject to a usual (harmonic) confining potential with the addition of an electric field directed along the normal that forces its mean velocity to change direction. Clearly, the strength of this electric field grows linearly with the local curvature of the channel. To monitor its effect, it is possible to employ an adiabatic separation between fast and slow quantum degrees of freedom. In particular, the quantum motion in the strongly confined fast normal direction can be obtained by solving at each tangential position a 1D Schrödinger equation. Owing to the presence of the electric field, the energy levels depart from those of the quantum harmonic potential. This deviation is of second order, because the first-order correction vanishes by symmetry. The local energy correction is thus negative and builds up an attractive potential for the slow degrees of freedom that is proportional to $\hbar^2k^2/(2m^*l^2)$, where $k(s)$ is the local geometric curvature and $m^*$ is the electronic effective mass. The existence of this quantum geometric potential has been rigorously proved by Jensen, Koppe\textsuperscript{137} and da Costa\textsuperscript{138} by employing a thin-wall quantization procedure. It starts with the Schrödinger equation for the quantum geometric potential has been rigorously proved by Jensen, Koppe\textsuperscript{137} and da Costa\textsuperscript{138} by employing a thin-wall quantization procedure.

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Where the effective field $\mathbf{h}_{\text{eff}}$ lies in the normal–binormal plane and has an explicit geometric component given by the local curvature $k(s)$, because $\mathbf{h}_{\text{eff}} = \{0, \lambda_s, k(s)\}$. For non-zero curvature the electron spin acquires a finite out-of-plane binormal component. Additionally, and as discussed in the main text, for inhomogeneous curvature there is a finite torque that yields an unconventional tangential spin component. This curvature control of the spin textures is introduced in the real-space geometry control of a quantum geometric phase. There is, in fact, a direct relation linking the spin texture to the Aharonov–Anandan (AA) geometric phase. We recall that the AA phase is the non-adiabatic analogue of the Berry phase, which can be defined as $\gamma = \oint \mathbf{A} \cdot d\mathbf{r}$, where $\mathbf{A}$ is the Berry connection $\mathbf{A} = -i \langle \mathbf{\psi}(x) | \nabla | \mathbf{\psi}(x) \rangle$, $\mathbf{r}$ is a vector parameter that varies with time and $\mathcal{C}$ is a closed path. The Berry curvature is the field strength with components $B_{\alpha\beta} = \epsilon_{\alpha\beta\gamma} \mathbf{A}_{\gamma}$, where $\epsilon_{\alpha\beta\gamma}$ is the Levi–Civita antisymmetric tensor, and transposes as a pseudovector in 3D and as a pseudoscalar in 2D systems.

In ferromagnetic materials, confinement effects on the exchange magnetic energy yield a curvature-induced Dzyaloshinskii–Moriya interaction (DMI) and magnetic anisotropy. Precisely as for the quantum geometric potential, one starts with the exchange energy in a thin curved shell with the energy density written as usual, $\mathcal{E}_{\text{ex}} = \nabla \mathbf{m} \cdot \nabla \mathbf{m}$, where $\mathbf{m}$ is the magnetization. The zero thickness limit of this 3D exchange magnetic energy yields a surface energy consisting of three different terms\textsuperscript{15}: $\mathcal{E}_{\text{ex}} = \xi_{\text{ex}}^0 + \xi_{\text{ex}}^D + \xi_{\text{ex}}^A$. For a generic curved surface and assuming that an orthonormal curvilinear local basis has been found, the three different contributions can be written as follows

\[
\begin{align*}
\xi_{\text{ex}}^0 &= \nabla_a m_\beta \nabla_a m_\beta + \nabla_a m_\alpha \nabla_a m_\alpha \\
\xi_{\text{ex}}^D &= 2\hbar \sigma_{\alpha\beta} (m_\beta \nabla_a m_\alpha - m_\alpha \nabla_a m_\beta) + 2\epsilon_{\alpha\beta\gamma} \Omega_{\gamma} m_\beta \nabla_\gamma m_\alpha \\
\xi_{\text{ex}}^A &= (\hbar \gamma_{\alpha\beta} + \Omega^2 \delta_{\alpha\beta}) m_\alpha m_\beta + (M^2 - 2K) m_\alpha^2 \\
&\quad + 2\epsilon_{\alpha\beta} \hbar \gamma_{\alpha\beta} \Omega_{\gamma} m_\alpha m_\beta.
\end{align*}
\]

In the above equations we introduce the Weingarten curvature tensor $\gamma_{\alpha\beta}$, the spin connection $\Omega$ associated with the 2D curved surface, and the 2D Levi–Civita tensor $\epsilon_{\alpha\beta}$. Additionally, $m_\alpha$ is the component of the magnetization in the direction normal to the curved surface. The emergence of the curvature-induced DMI interaction, in particular, reflects the fact that bending of a curved thin magnetic layer breaks the centrosymmetry.

In superconductors, an homogeneous spin–triplet pairing order parameter can be conveniently expressed by introducing the so-called $\mathbf{d}$ vector:

\[
\hat{\Delta} = \begin{pmatrix} \Delta^{\uparrow \uparrow} & \Delta^{\uparrow \downarrow} \\ \Delta^{\downarrow \uparrow} & \Delta^{\downarrow \downarrow} \end{pmatrix} = i (\mathbf{d} \cdot \mathbf{\sigma}) \mathbf{\sigma}_y,
\]

where we use the relation $\Delta^{\uparrow \downarrow} = \Delta^{\downarrow \uparrow}$. The complex components of the $\mathbf{d}$ vector are related to the pair amplitudes by

\[
\begin{align*}
\Delta^{\uparrow \uparrow} &= \langle \uparrow \uparrow | \hat{\Delta} | \uparrow \uparrow \rangle \\
\Delta^{\uparrow \downarrow} &= \langle \uparrow \downarrow | \hat{\Delta} | \uparrow \downarrow \rangle \\
\Delta^{\downarrow \uparrow} &= \langle \downarrow \uparrow | \hat{\Delta} | \downarrow \uparrow \rangle \\
\Delta^{\downarrow \downarrow} &= \langle \downarrow \downarrow | \hat{\Delta} | \downarrow \downarrow \rangle.
\end{align*}
\]
Box 1 | Origin of confinement-induced curvature effects (continued)

\[ d = \left( -\frac{\Delta^{1\downarrow}}{2}, \frac{\Delta^{1\uparrow}}{2}, \Delta^{2\downarrow}, \Delta^{2\uparrow}, \Delta^{3\uparrow} \right). \]

Hence, each component of the vector indicates the pair amplitude for the Cooper-pair spin perpendicular to the corresponding spin axis. Non-vanishing amplitudes of the vector components are mainly dictated by the structure of the pairing potential, and the orientation of the vector is determined, for example, by spin–orbit coupling, magnetic fields or intrinsic magnetism. Triplet pairing with a non-zero value of the product \( d \times d^* \) implies that the spins of the Cooper pairs are polarized. Additionally, the relative 0 (\( \pi \)) spin-phase difference appearing between the \( \Delta^{1\downarrow} \) and \( \Delta^{1\uparrow} \) matrix elements when only the \( d_1 \) (\( d_2 \)) component is present can be relevant for superconducting spintronic applications. One can generally show that there will be a non-trivial Josephson coupling, both in the charge and spin channel, which depends on the relative vector misalignment angle. For the case of a typical Josephson junction configuration with tunnel-coupled superconducting regions marked by \( d \) vectors with a misalignment angle \( \beta \), the Josephson current \( J_{\text{J}} \) for the charge and spin sector is proportional to \( \sin(\beta + \phi) + \sin(\beta - \phi) \) and \( \sin(\beta + \phi) - \sin(\beta - \phi) \), respectively, with \( \phi \) being the phase difference between the \( d \) vectors.

rest frame of the electrons, the effective spin–orbit field has a geometric component \( \vec{d}_{\text{geo}} \) (Box 1) governed by the local geometric curvature. This geometric spin–orbit field has been directly probed in semiconducting quantum rings with Rashba spin–orbit coupling\cite{40,41}. In these systems, the out-of-plane tilt of the spin textures (Fig. 2a) gives characteristic fingerprints in the conductance interference patterns\cite{42,43}. The conductance modulations are regulated by the AA geometric phase—the non-adiabatic analogue of the Berry phase—which is in one-to-one correspondence with the spin textures\cite{44}. Importantly, the quasi-periodic modulations can also be tuned by means of external planar magnetic fields. The geometric spin–orbit field has remarkable consequences in geometries with inhomogeneous curvature\cite{45}. In this case, the spin–orbit field exerts a torque on the electronic spin that then acquires a finite component parallel to the electron propagation direction (Fig. 2a). The ensuing complex 3D spin textures prove local curvature control of the AA geometric phase and can be directly probed in interferometric spintronic devices with unconventional geometries such as ellipses and squares\cite{46}.

The geometric spin torque also has an important role in zigzag-shaped nanowires where the local geometric curvature has a periodic profile\cite{47,48}. The miniband structure of this geometric superlattice is characterized by the presence of minigaps opened by spin–orbit coupling at unpinned points in the mini Brillouin zone. The periodic buckling of a nanowire therefore induces a metal–insulator transition defining a geometric transistor switch. Additionally, the insulating states generally display Tamm–Shockley in-gap end modes\cite{49}. The concomitant presence of the confinement-induced quantum geometric potential and the geometric spin torque can be exploited to design new solid-state electronic set-ups. For example, it has been theoretically proposed that a zigzag-shaped nanowire with strong Rashba spin–orbit coupling can operate as a topological charge pump (Fig. 2b) in the complete absence of superimposed oscillating local voltages\cite{50}, differently from the conventional pumping protocol in 1D systems originally introduced by Thouless\cite{51}. To operate the device, an external rotating planar magnetic field—for example, one generated by running current pulses in two perpendicular conductors with a \( \pi/2 \) phase shift—is used as the periodic a.c. perturbation driving the charge pumping. The time-dependent Zeeman coupling acting on the spin textures realized by the geometric spin–orbit torque results ultimately in a sliding superlattice charge potential. Combining the latter with the charge gap opening mechanism provided by the quantum geometric potential leads to states with a non-trivial Berry curvature in the synthetic 2D mini Brillouin zone, which, when integrated, yields a ‘dynamical’ non-zero Chern number\cite{52}. Hence, in each pumping period, the device pumps two electronic charges with quantization that is topologically protected against external perturbations, and can be relevant for metrological purposes.

Curvilinear magnetism. Curvature effects can also be exploited for novel spintronic device concepts relying on magnetic domain-wall (DW) motion. In magnetic wires with helical shapes, geometry offers unconventional means to control Rashba spin torque-driven\cite{53} as well as spin current-driven\cite{54} DW dynamics. This is because the magnetic equilibrium state in thin films and wires is directly influenced by geometric curvature\cite{55,56}. For example, in a buckled magnetic wire with tangential anisotropy, the magnetization vector generally displays local deviations from the tangential direction (Fig. 2c). These effects are due primarily to the magnetic exchange energy, which yields an effective DMI and an effective magnetic anisotropy (Box 1) as a result of the confinement on curved geometries. The existence of this effective antisymmetric exchange interaction also has important consequences for the existence and stability of modulated magnetic phases whenever the geometric curvature is comparable to the exchange magnetic length. Consider, for example, a flat ultrathin film with an intrinsic DMI. It is known that there exists a critical DMI strength separating homogeneous and periodic magnetization distributions\cite{57}. If, now, the ultrathin film is bent with a constant-curvature radius so as to create a magnetic nanotube, the effective curvature-induced DMI will renormalize the critical strength at which the modulated phase sets in\cite{58}. In particular, by decreasing the curvature radius, modulated phases can be stabilized by the geometry and appear even for vanishing intrinsic DMI strength (Fig. 2d). Similar features are encountered when considering skyrmions in magnetic nanotubes, which have already been predicted to exist stably in moderate magnetic field ranges\cite{59}.

Curvature effects also yield peculiar features in the dynamics of DWs in magnetic nanotubes. The configuration of a DW in a magnetic nanotube is characterized by the presence of a coreless vortex structure in the region separating the two oppositely magnetized domains. The two possible vorticities then distinguish two different DW configurations, which are energetically degenerate but have different dynamical properties. Because of the presence of a finite radial magnetization component, an applied magnetic field exerts a torque on the DW. This, in turn, affects the radial magnetization itself. Whether the DW is distorted with a compression or enhancement of the radial magnetization depends on the handedness of the system defined by combining the vorticity of the DW with the magnetic field vector direction\cite{60}. This chiral-dependent distortion ultimately modulates the motion of the DWs. Micromagnetic calculations based on the Landau–Lifshitz–Gilbert equation show that the velocity of the DWs is strongly dependent on the chirality. Importantly, the stability of the DW also displays chiral-dependent features. Certain chiral DWs have a completely suppressed Walker breakdown\cite{61}: the collapse of the DW structure at a critical velocity, and one of the main complications for the optimization of memory device and logic gates based on fast and controlled DW motion. This is primarily due to the fact that breakdown of a DW in a tube involves a vortex–antivortex pair creation contrary to the single vortex-mediated breakdown of DW in flat thin films. Based on this topological constraint, DWs generally have a strongly enhanced stability when guided in tubular nanostructures.
Chiral symmetry breaking effects have been predicted and more recently observed in magnonics. Spin waves—magnetic excitations that hold potential in information processing—acquire a peculiar asymmetric dispersion in magnetic nanotubes with an equilibrium state in which the magnetization rotates around the circumference of the tube. Specifically, spin waves of the same frequency propagating in different directions are characterized by different wavelengths. Because the rotating magnetization together with the propagation direction defines a handedness, the occurrence of this phenomenon implies a chiral symmetry breaking. Importantly, this effect, occurring in conventional thin-film geometries only in the presence of an intrinsic DMI, does not originate from the interaction but rather from the non-local dipole–dipole interaction.

Superconducting electronics. Recent theoretical studies have suggested that the effective spin torques activated by geometry might also affect electronic pairing and possibly lead to new functionalities in superconducting spintronics. One of the paradigms of superconducting spintronics is to exploit spin–triplet Cooper pairs because they can carry angular momentum without energy dissipation, and thus can be functionalized to yield spin-polarized supercurrents for ultrahigh energy-efficient storage and transfer of information. Spin–triplet pairing has a vectorial nature and can be typically encoded in the so-called $d$ vector, the components of which correspond to the zero spin projections of the triplet state along the symmetry axes (Box 1). A crucial challenge in the area of superconducting spintronics is to achieve control mechanisms and devise systems that are able to convert spin–singlet into spin–triplet electron pairs because spin–singlet superconductors are more abundant in nature. This issue has been largely investigated by engineering superconductor–magnet heterostructures with suitably designed non-collinear magnetic patterns and more recently by considering spin–orbit coupling without breaking time-reversal symmetry. In this context, systems with Rashba spin–orbit coupling due to structural inversion asymmetry are particularly appealing.

In 2D, the lack of inversion symmetry removes the spin degeneracy of the Bloch states and a spin texture develops in the Brillouin zone due to the Rashba interaction that couples the electron spin with the crystal wavevector. For Cooper pairs, this symmetry reduction forces the occurrence of a mixing of spin–singlet and spin–triplet pairing at the Fermi level. Moreover, due to the spin anisotropy, the spin–triplet $d$ vector follows the electron spin orientation, resulting in a helical pattern (Fig. 2e). If at a given momentum $k$ at the Fermi level the electron spin points along the $y$ direction, $\uparrow_y$, with the time-reversal partner $\downarrow_y$ at $-k$, the resulting spin–triplet configuration can only be with vanishing total spin projection along the $y$ orientation, thus corresponding to the $d_y$ component. In analogy to the spin–triplet texture occurring in momentum space along the Fermi line, a variation of the $d$-vector orientation occurs in real space when geometric curvature is present. As discussed above, the geometric spin–orbit torque yields local variations of the electron spin orientation. Hence, to avoid pair breaking, the spin–triplet configuration has to follow the spin anisotropy by twisting the $d$ vector according to the curvature of the electronic channel (Fig. 2e). Crucially, the reconstruction depends on the ratio between the geometric curvature and the superconducting coherence length. Then, the combined presence of geometric curvature and spin–orbit coupling has the effect to generate spin–triplet pairs...
with an orientation that is substantially dictated by the profile of the confining potential. These mechanisms can lead to striking effects in superconducting electronics. As a direct example, the Josephson effect can be mechanically controlled both in the amplitude and phase of the supercurrent by geometrically curving the superconducting electrodes.66

### Strain-driven curvature effects

In nanosystems with curved geometries, electromechanical coupling leads to effects that often coexist and amplify the confinement-induced curvature effects discussed above. A key property of bent nanostructures fabricated using strain engineering methods is the presence of local strain fields varying on the nanoscale.73

#### Strain-induced geometric potential

In insulators, the presence of these strain gradients is the basis of the flexoelectric effect, an electromechanical coupling that, contrary to piezoelectricity, is universal and symmetry-allowed in all solids.66 Strain gradients spontaneously generated in curved nanostructures also yield remarkable effects in semiconducting materials. According to the linear potential deformation theory,67, a local strain yields a shift of the conduction (valence) band edges, and therefore corresponds to a local potential attracting the charge carriers towards the maximally strained regions (Fig. 3a).

This potential ultimately yields a strain-induced geometric potential of the same functional form as the confinement-induced quantum geometric potential, but strongly, often gigantically, boosting it. Such a large enhancement of curvature effects renders the phenomena predicted in curved nanostructures, ranging from winding-generated bound states in rolled-up nanotubes71 to topological band structures in systems constrained to periodic minimal surfaces72, observable above the sub-kelvin energy scale. In micrometre-sized wrinkled ribbons of GaAs, for example, curvature-induced localized end states have been predicted to be observable up to a few kelvin if local strain fields are explicitly taken into account.73

### Gauge fields in Dirac materials

Strain fields also yield remarkable effects in Dirac materials, such as graphene. There are two different strain-induced effects that can be distinguished. First, there is a renormalization of the Fermi velocity directly proportional to the local geometric curvature.74. This space-dependent Fermi velocity is expected to induce spatial oscillations of the local density of states near the ripples that are naturally formed in suspended graphene samples.75. The correlation between the morphology of the graphene samples and their electronic properties could then explain the local variations of the charge compressibility observed in scanning single-electron transistor experiments.66

In Dirac materials, in-plane mechanical deformations76 lead to another curvature-induced effect. Specifically, electrons can react to strain as if external electromagnetic fields were applied. Spin fields indeed result in effective gauge fields (Box 2) that are opposite in the two graphene valleys. These gauge fields lead to a complete reorganization of the spectrum when they generate a 'pseudo'-magnetic field.

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**Fig. 3 | Strain-induced effects.** a. Bending a flat nanostructure creates a strain gradient with regions under compressive and tensile strain separated by the mechanical neutral plane \( q_z = 0 \). b. Graphene grown on silicon carbide substrates displays nanoprisms, as evidenced in atomic force microscopy tomography, where one Si layer is missing. The formation of pseudo-Landau levels in the strained nanoprisms can be observed even at room temperature using angle-resolved photoemission spectroscopy. c. Left: STM topography of a buckling-induced 2D triangular superstructure in graphene deposited on \( \mathrm{NbSe}_2 \). Right: measured \( dV/dI \) maps at energy corresponding to one of the flat-band regions created by the buckling-induced periodic pseudomagnetic field. d. Top: artificially corrugated bilayer graphene (BLG) on a patterned hexagonal boron nitride (hBN) substrate. Bottom: top view of the regions where bilayer graphene is strained. A nonlinear Hall current \( J_{nh} \) is generated under the application of an electric field \( E \) parallel to the dipole \( D \). Panels adapted with permission from: b, ref. 83, AAAS; c, ref. 84, Springer Nature Ltd. Panel d reproduced with permission from: top, ref. 85, Springer Nature Ltd; bottom, ref. 90, APS.
Box 2 | Gauge fields in strained nanostructures

We illustrate the emergence of gauge fields in a strained nanosystem with Dirac electrons by considering the specific example of graphene, and start with the simplest tight-binding model Hamiltonian, that is, considering only hopping processes between nearest-neighbour atomic sites, which reads

$$\mathcal{H}_{MLG} = - \sum_{i\neq j} a^i \mathbf{b}_{i-j} + \text{c.c.}$$

where $a^i$ and $b^j$ ($a, b$) are creation (annihilation) operators on the A and B sublattices, respectively. In the equation above, the subscript $i$ runs over all unit cell positions, and we introduced the three nearest-neighbour vectors

$$\mathbf{\delta}_1 = \frac{a}{\sqrt{3}} \left\{ \frac{\sqrt{3}}{2}, 1, -\frac{1}{2} \right\}, \quad \mathbf{\delta}_2 = \frac{a}{\sqrt{3}} \left\{ -\frac{\sqrt{3}}{2}, 1, 1 \right\}, \quad \mathbf{\delta}_3 = \frac{a}{\sqrt{3}} \{ 0, 0, -1 \}.$$

The presence of strain implies that the hopping amplitudes $t_{ij}$ explicitly depend on the nearest-neighbour vectors. Specifically, $t_{ij} = t_0 (1 - \beta \delta_{ij})$, where $t_0$ is the hopping amplitude for the pristine three-fold rotation symmetric honeycomb lattice. The lattice parameter $\beta$ can be determined by Raman spectroscopy, whereas the relative distance changes $\delta_{ij}$ can be expressed in terms of the strain tensor components $\epsilon_{ij}$ as

$$\delta_{ij} = \frac{\delta_{ij}}{a^2} \epsilon_{ij}. $$

Next, we go to momentum space and write the Bloch Hamiltonian as

$$\mathcal{H}_{MLG} = - \sum_{n} t_n \begin{pmatrix} 0 & e^{-i(\mathbf{k'} + \mathbf{q}) \cdot \delta_n} \\ e^{i(\mathbf{k'} + \mathbf{q}) \cdot \delta_n} & 0 \end{pmatrix},$$

where we have rewritten the momenta as $\mathbf{k'} + \mathbf{q}$, because we are interested in the electronic properties close to the K or K' valleys of the Brillouin zone given by $\mathbf{K} = \left\{ \frac{\pi}{\sqrt{3} a_0}, 0 \right\}$ and $\mathbf{K'} = \left\{ -\frac{4\pi}{\sqrt{3} a_0}, 0 \right\}$. The Bloch Hamiltonian can then be expanded to linear order in the small momenta $\mathbf{q}$. Using simple vector identities and assuming for simplicity an anisotropic biaxial strain with $\epsilon_{xx} \neq \epsilon_{yy} \neq 0$ and $\epsilon_{xy} = 0$, the continuum low-energy Hamiltonian near the two valleys of the Brillouin zone can be rewritten as

$$\mathcal{H}_{eff}(\mathbf{q}) = \xi \mathbf{v}_s \mathbf{q} \sigma_x + \nu_F \mathbf{A}_x \sigma_x + \nu_F \mathbf{q} \sigma_y,$$

where $\xi = \pm 1$ is the valley index. Here, $\mathbf{A}_x = \sqrt{3} \beta (\epsilon_{xx} - \epsilon_{yy})/(2a)$ is a strain-induced ’pseudo’-gauge field, and $\mathbf{v}_F = \sqrt{3} t_0 a/2$ is the Fermi velocity of the Dirac carriers in unstrained samples. In addition, $\nu_x = \nu_x [1 - \beta (\epsilon_{xx} + \epsilon_{yy})]/4$ and $\nu_y = \nu_y [1 - \beta (\epsilon_{xx} + 3\epsilon_{yy})]/4$ are renormalized Fermi velocities that become anisotropic due to momentum–strain coupling.

that nanobubbles are generally present in van der Waals heterostructures, but are typically avoided in high-quality devices because in-plane strains represent a dominant factor limiting the electronic mobility\(^1\). However, nanobubbles have been recently used as active elements for the formation of nanometre-scale lateral p–n junctions in a charge-transfer graphene-based heterostructure\(^2\).

Strain-induced Landau levels have also been generated in triangular nanoprisms of a silicon carbide (SiC) substrate (Fig. 3b) and observed by angle-resolved photoemission spectroscopy\(^3\). A periodic arrangement of pseudomagnetic fields with periods on the scale of tens of nanometres has been realized in buckled graphene superlattices\(^4\). The pseudo-Landau levels in this case form weakly dispersive bands that are strongly localized in real space (Fig. 3c). Because the kinetic energy is quenched, the system can also develop a correlated phase characterized by a pseudogap-like depletion of the density of states, similar to the situation encountered in magic-angle twisted bilayer graphene\(^5\). Buckling instabilities can thus be exploited to investigate interaction phenomena, with important advantages in ease of fabrication and scalability when compared to twotronics.

Quantum nonlinear Hall effect. Strain-induced gauge fields trigger curvature-induced phenomena not strictly related to the presence of pseudo-Landau levels. These occur in non-centrosymmetric materials with Berry curvature—the quantity that encodes the geometric properties associated with the quantum electronic wavefunctions. More specifically, these effects originate from the interplay between the intrinsic crystalline anisotropies of the Dirac material and structural anisotropies induced by certain mechanical deformations. In their pristine form, 2D Dirac materials have a trigonal crystalline structure. Now suppose the nanomembrane is deposited onto an anisotropic pre-patterned substrate to induce a highly anisotropic 1D buckling instability (Fig. 3d). The end product is a superstructured material with an unusually low crystalline symmetry, where all rotational symmetries are broken.

Electronic systems with substantial Berry curvatures and very low-symmetry crystalline content can exhibit a segregation of positive and negative regions of Berry curvature in momentum space, leading to a net dipole moment. This Berry curvature dipole yields a quantum nonlinear Hall effect in time-reversal symmetry conditions\(^6\). A nonlinear electrodynamical phenomenon relevant for high-frequency rectification and long-wavelength photodetection\(^7\). Creating anisotropic mechanical deformations in gated bilayer graphene satisfies all the requirements for the existence of a sizable Berry curvature dipole. Inversion symmetry breaking is achieved with an external electric field perpendicular to the layers. Moreover, anisotropic strains\(^8\) already in their simplest homogeneous form have been predicted to trigger Berry curvature dipoles (Fig. 3d) comparable to those of transition-metal dichalcogenides. The observation of a large Berry curvature dipole in corrugated bilayer graphene\(^9\) has brought to reality the concept of such a curvature-induced quantum nonlinear Hall effect. Importantly, this also opens a new approach to electronic devices that can be used as energy harvesters and terahertz detectors via geometric design.

Synthesis and characterization methods

The creation and exploration of curvature effects strongly relies on the ability to synthesize nanoscale objects with a targeted geometric shape. Curvilinear 1D nano-objects in planar structures can be fabricated using state-of-the-art thin-film technology processes, including electron-beam lithography and ion-beam etching. These approaches have been widely used for the fabrication of, for example, quantum rings\(^{10,11,12}\), curved parabolic stripes\(^{13}\), or square loops\(^{14}\) of different materials. The main advantage of this approach is that the functional curvilinear nanostructure can be easily supplemented with stable electrical contacts for charge injection and gate
Furthermore, the equilibrium spin textures of magnetic materials can be retrieved using a number of techniques, including Lorentz transmission electron microscopy and X-ray magnetic circular dichroism photoelectron emission (XMCD-PEEM). Imaging the magnetization states of \( \text{Ni}_8\text{Fe}_{12} \) (permalloy) parabolic stripes using XMCD-PEEM provided the first experimental

**Fig. 4 | Fabrication of curvilinear nano-objects.**

- **a.** Schematic of the fabrication process for the formation of a high-order rolled-up van der Waals heterostructure (vdWH). The cross-section of the SnS\(_2\)/WSe\(_2\) superlattice was imaged by scanning transmission electron microscopy.
- **b.** Schematics of nanopillars fabrication by means of charged aerosol particles guided with electrostatic lenses and the resulting nanopillar formed with various elements and geometrical compositions, imaged by means of SEM with energy-dispersive X-ray spectroscopy.
- **c.** Layer-by-layer focused electron-beam-induced deposition of various self-standing geometries with complex topographies and topologies.
- **d.** Integration of a ferromagnetic 3D nanobridge in a microelectronic circuit by means of a FEBID direct-writing technique. Panels adapted with permission from: **a,** ref. 21, Springer Nature Ltd; **b,** ref. 110, Springer Nature Ltd; **c,** ref. 113 under a Creative Commons licence CC BY 4.0; **d,** ref. 114 under a Creative Commons licence CC BY 4.0.
Another advantage of planar structures is that a unique sample can consist of arrays of nano-objects. This configuration is ideal in electronic transport, because it naturally provides ensemble-averaging, which filters out device-to-device variations\(^{14,15}\). The use of InGaAs square-loop arrays\(^5\), for example, has been essential to identify the distinctive signatures of the geometric spin–orbit torque in quantum conductance experiments.

The use of planar structures is ideally suited to identifying the signatures of confinement-induced curvature effects. However, it does not give access to a number of classical shape effects and strain-driven effects that rely on the synthesis of 3D nanoarchitectures. There are different routes to the fabrication of these complex nanosystems. The first is based on strain engineering and involves the synthesis, at some critical stage, of a free-standing nanomembrane released from its substrate by a specialized anisotropic etching procedure\(^6\). A spatial distribution of strain, engineered in the nanomembrane, for example, by heteroepitaxial growth of materials with different lattice constants, then produces a tendency to roll the layer up to create nanotubes\(^7\) or coils\(^8\). More elaborate architectures can be formed using additional lithographic patterning. The rolling-up process can also be induced externally. For example, van der Waals planar heterostructures and monolayers can be driven into a capillary-driven roll-up\(^9,10\) by inserting or dropping ethanol solution (Fig. 4a). The formation of these nanoarchitectures has led to the observation of distinctive signatures of shape-driven curvature effects, including the linear transverse magnetoresistance mentioned above. Moreover, compact 3D nanoarchitectures fabricated by rolled-up nanotechnology have footprints that are orders of magnitude smaller than conventional planar structures. This additional advantage has triggered interest in realizing various functional devices ranging from semiconductor electronic field-effect transistors\(^11\) and high-sensitive sensors for magnetic fields\(^12,13\), to rolled-up Josephson junctions\(^14\).

The synthesis of 3D nanoarchitectures with complex shapes necessitates other strategies. For example, curved templates such as lithographically patterned substrates, spherical nanoparticles, nanocylinders or ion-beam-induced cones can be coated with a functional material. This generally yields geometric shapes with strongly inhomogeneous thicknesses relevant for shape effects in metallic materials\(^15\). Patterned substrates can be used to create buckled structures with periodic strain fields that can exist over macroscopic regions\(^16\). Spherical geometries obtained with the use of SiO\(_2\) nanoparticles has allowed us to map curvature effects on the magnetic vortex states of magnetic permalloy caps\(^17\). Another method involves the use of pre-stretched elastomeric substrates in which strain relaxation imparts forces at a collection of lithographically patterned locations of precursive planar structures. This results in a process of compressive buckling\(^18\) that extends the structures in the third dimension with broad geometric diversity.

Complex 3D nanosystems can also be fabricated using direct-growing and direct-writing methods. Glancing angle deposition, for example, has been employed to create various functional nanoarchitectures\(^19,20\). By tuning the rotation speed of the substrate, the vapour source flux intensity and the angle of incidence, nanopillars, nanoflowers and nanohelix arrays have been synthesized. A direct coupling between the chirality of the helices and magnetism\(^21\) has been directly proven by the occurrence of magnetochiral dichroism of light. Recent direct-writing techniques include two-photon lithography combined with deposition postprocessing\(^22,23\), charged aerojet nanoprinting\(^24\), as well as focused electron and ion-beam-induced depositions\(^21\) (Fig. 4b,c). Direct-writing techniques allow to directly integrate complex 3D architectures into microelectronic circuitry with lithographically patterned contacts (Fig. 4d). Using this method, magnetotransport studies of a ferromagnetic 3D nanobridge have brought to light unusual angular dependences in magnetoresistive effects, including the anomalous Hall effect\(^14\). For 3D nanomagnets, a full understanding of curvature-induced effects can be achieved by putting in a one-to-one correspondence between the geometric shape and the magnetization distribution. This can be addressed using the recently developed soft X-ray magnetic tomography\(^25\) and holographic vector field electron tomography\(^26\).

### Outlook

Advances in synthesis methods will be crucial for the exploration of new geometry-induced nanoscale effects. For example, 3D nanomagnets with curved geometries are expected to exhibit a torsion-induced asymmetric spin–wave dispersion\(^9\) that can be detected using time-resolved scanning transmission X-ray microscopy. If these 3D structures are made using helimagnetic materials, magnetoelastic devices could be enriched that consist of a nanomagnet embedded in a piezoelectric matrix. Because the magnetic states are controlled by changing the geometry of the curved helimagnetic wire, the piezoelectric matrix allows electric-field-induced switching between different magnetic states\(^7\) that can then be assigned a logical ‘1’ or ‘0’.

Three-dimensional geometries allow both intra-structure magnetization textures\(^27\) and magnetic-field nanotextures to be tuned\(^28\). Curvilinear architectures with reconfigurable magnetic-field textures are of use in DW-based memory and logic devices: the sensor readout of racetrack memories can potentially be optimized by pinning magnetic DWs in curvilinear nanowires and by tailoring stray fields. At a later stage, networks of curvilinear nanowires could also be used for 3D magnonics and to achieve the related concept of reservoir computing\(^29\). Anti-ferromagnets with curvilinear geometries will be central in antiferromagnetic spintronics and it has recently been predicted\(^1\) that curvilinear 1D anti-ferromagnets can control the magneto-chiral response, induce weak ferromagnetism and tailor magnonic bandgaps.

The relationship that exists between strain fields and pseudomagnetic fields in Dirac materials provides a route to exploit effective gauge fields in nanoscale electronic devices via spin or pseudospin control\(^30\), and the main challenges here are in material synthesis. Promising future directions include the robotic assembly of twisted van der Waals solids\(^31\) and the controlled formation of nanobubbles via irradiation\(^32\) or by trapping of substances\(^33\) (where curvature radii of a single nanometre could be achieved\(^34\)). Progress in the fabrication of rolled-up tubular structures made of van der Waals materials may allow direction-dependent magnetotransport properties that are intrinsically related to the broken rotational symmetry of spiral structures to be explored. The anisotropy in the transversal magnetoresistance has been predicted\(^35\) to decrease as 1/w, where w is the number of windings of the roll. Thus, in multiple-winding nanostructures, the anisotropy becomes practically undetectable. In addition, other material structures could be designed to realize the classical geometric nonlinear Hall effect\(^36\)—because it has general requirements, this phenomenon is expected to appear strongly in traditional semiconducting materials with curved channels that can be fabricated in planar structures.

The quantum geometric potential, which has so far only been reported in metamaterials\(^37,38\), could be revealed by unique electronic ‘fingerprints’ in material structures. The quantum geometric potential in thin films with an inhomogeneous curvature profile should result in an in-plane internal electric field, which can potentially drive a linear Hall-like effect at zero magnetic field, called the Magnus Hall effect, in materials with substantial Berry curvature such as transition-metal dichalcogenides\(^39\). This could therefore provide experimental confirmation of the existence of the quantum geometric potential. Geometry is expected to play an increasingly important role when the quantum Hall regime is reached. In synthetic structures with locally non-flat geometries, the electronic...
density of a quantum Hall fluid is directly coupled to the Gaussian curvature via the mean orbital spin. Lattice disclinations can lead to an additional intrinsic rotation of the electronic fluid caused by the gravitational anomaly. Future experimental efforts in the fabrication of appropriate material structures engineered with, for example, nanodomes will, however, need to rely on new theoretical directions to understand how to disentangle physical phenomena related to confinement-induced curvature effects and geometric transport properties.

A relation between the real-space geometry of a nanoscale system and the internal geometry of the electronic wavefunction has been demonstrated in corrugated bilayer graphene. Research into similar relations in ultrathin films of magnetic complex oxides, such as SrRuO$_3$, which exhibit strong Berry curvature should be pursued in the future. Isolated monocristalline nanomembranes of SrRuO$_3$, which could then be transferred to curved templates, have been obtained, as have periodically curved layers of the material by using a ferroelectric-metal superlattice. Thus, oxide electronic devices are an ideal material platform for the exploration of nanoscale geometry effects in quantum phenomena. The use of nanoscale curvature for the synthesis of superstructures with reduced crystalline symmetry is likely to play a vital role in searching for unconventional superconducting phases, such as orbital combinations of pair density waves, which could be relevant for superconducting orbitronics. Such nanoscale geometry-induced effects are also likely to be crucial to designing topological superconducting phases with higher resilience, or to turn a typical superconductor into a topological superconductor.

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**Author contributions**

C.O. coordinated the project. P.G. produced the original illustrations. All authors wrote and commented on the manuscript.

**Competing interests**

The authors declare no competing interests.

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