Disorder-induced Effects in Noisy Dynamics of Quantum Bose-Hubbard and Fermi-Hubbard Glasses

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We address the effects of quenched disorder averaging in the time-evolution of systems of ultracold atoms in optical lattices in the presence of noise, imposed by the presence of an environment. For bosonic systems governed by the Bose-Hubbard Hamiltonian, we numerically quantify the result of the presence of disorder in the Hamiltonian parameters in terms of physical observables, including bipartite entanglement in the ground state and report the existence of disorder-induced enhancement in weakly interacting cases. For systems of two-species fermions described by the Fermi-Hubbard Hamiltonian, we find similar results. In both cases, our dynamical calculations show no appreciable change in the effects of disorder from that of the initial state of the evolution. We explain our findings in terms the statistics of the disorder in the parameters and the behaviour of the observables with the parameters.

I. INTRODUCTION

Over the past few decades there has been an exceptional development in the experimental realisation of quantum systems that has led to substantial enrichment of our understanding of strongly correlated quantum many-body systems [1][2]. Quantum simulators have been successfully implemented in different physical systems including optical lattices [3][4], photons [5][6], ion-traps [7][8] and superconducting qubits [9][10]. This enables one to study several interesting physical observables in a many-body scenario such as magnetic order, entanglement [11] etc. Presence of disorder in the physical realisations is supposed to suppress such properties but in special cases counterintuitive effects like disorder-induced enhancements have also been observed [12][36]. The other interesting disorder-induced phenomena include existence of phases like spin glass [37][40] and Bose glass [11][42]. Advances in the experimental techniques in recent times can be used to study in detail the effect of disorder in such systems in well-controlled conditions.

In particular, the experiments with cold atoms in optical lattices have proved to be an exceptional tool that provide with the ability to realise these quantum systems and carry out precise measurements on them, due to the remarkable control over the system parameters. Efficient isolation from the environment enables observation of coherent dynamics as well as non-equilibrium dynamics [43]. Nevertheless, in a real experimental setup, noise cannot be fully avoided. Technical noise such as the amplitude fluctuations of the optical lattice potential causes fluctuation in the Hamiltonian parameters. On the other hand, a major source of noise of quantum nature arises from the spontaneous emission events due to coupling to the vacuum modes of the electromagnetic radiations.

In this work, we address the effect of quenched disorder in terms of the quenched averaged ground state properties as well as their dynamical evolution in the presence of noise, imposed by the presence of an environment. We specifically look at two different systems, viz. of bosons and of two-species fermions, loaded in the lowest band of an 1D optical lattice, described by the single-band Bose-Hubbard [44] and Fermi-Hubbard Hamiltonians [45] respectively. Each of these Hamiltonians has only two parameters: the tunnelling amplitude to adjacent sites and an on-site interaction energy. The fluctuations in these parameters introduce disorder in the system. We numerically investigate the effects in the ground state properties for disorders imposed individually in these two parameters. The effect we seek to find is a disorder-induced enhancement in the properties and we do find it for particular parameter ranges. We then go on to study the dynamics of these effects in presence of spontaneous emission events which we do by evolving the corresponding master equation for the system density operator [46][52]. The disorder-induced effects are found to be sustaining their outcomes, at least in terms of short-time behaviours. For the numerical results on large systems, where exact diagonalisation is not possible, we use a combination of density matrix renormalisation group (DMRG) algorithm [55][57] with the quantum trajectory methods [58][61] to compute the dynamics.

The article is organised as follows. In Sec. II we discuss the model Hamiltonians and lay out the physical observables we compute. In Sec. III we explain the quenched disordered systems and the process of quenched averaging. We show our findings in the ground states of our model Hamiltonians in Sec. IV In Sec. V we show the dynamical results in an open quantum system. We present our concluding remarks in Sec. VI.

II. MODEL HAMILTONIANS AND OBSERVABLES

The Bose-Hubbard Hamiltonian in 1D that describes bosons in the lowest band of an optical lattice is given by,

\[ H_{BH} = -J \sum_{\langle i,j \rangle,S} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1), \]

(1)
where $J$ is the tunnelling amplitude between adjacent sites, $b_i$ ($b_i^\dagger$) are the bosonic annihilation (creation) operators for $i$-th site, $U$ is the on-site interaction strength and $n_i$ is the number operator ($b_i^\dagger b_i$). On the other hand, the single-band two-species (with spins $\uparrow$ and $\downarrow$) Fermi-Hubbard Hamiltonian in 1D we choose to work with is given by,

$$H_{FH} = -J \sum_{\langle i,j \rangle,s} c_{i,s}^\dagger c_{j,s} + U \sum_i n_{i,\uparrow} n_{i,\downarrow},$$

where $J$ is again the tunnelling probability between two neighbouring sites and $U$ is the on-site interaction energy that comes into play when two fermions with different spins are on the same site. The annihilation (creation) operators $c_{i,s}$ ($c_{i,s}^\dagger$) for site $i$ and spin $s$ obey the fermionic anti-commutation relations. The number operator is, $n_{i,s} = c_{i,s}^\dagger c_{i,s}$, and for each spin species it has eigenvalues 0 and 1, due to Pauli exclusion principle.

In our calculation we impose quench disorder on both the Hamiltonian parameters and examine the cases where the disorder is only in the interaction term or only in the tunnelling term. In each of these cases the physical quantities we compute are the following. For the Bose-Hubbard Hamiltonian a quantum phase transition from the superfluid phase to the Mott insulator phase [62] occurs at $U/J = 3.37$ for unity filling in the 1D case [63]. This phase transition can be quantised by the looking at the single particle density matrix (spdm), $\langle b_i^\dagger b_j \rangle$. In the superfluid phase spdm elements show polynomial decay with distance between the sites, $|i - j|$, whereas in the Mott insulator phase, the spdm elements fall off exponentially with distance. For our work we define two Mott orders, averaged over the lattice sites,

$$M_1 = \frac{1}{N} \sum_{i=1}^N \left| \frac{\langle b_i^\dagger b_i \rangle}{\langle b_i^\dagger b_{i+1} \rangle} \right|,$$

and,

$$M_2 = \frac{1}{N} \sum_{i=1}^N \left| \frac{\langle b_i^\dagger b_i \rangle}{\langle b_i^\dagger b_{i+2} \rangle} \right|.$$  

Now for the fermions, since the two-species Fermi-Hubbard Hamiltonian is a paradigmatic system to study antiferromagnetic order for repulsive interaction ($U > 0$) [3, 4], we look at two measures of spin correlation functions that quantify magnetic order. Defining the $z$ component of spin at the $i$-th site, $S_i^z = (n_{i,\uparrow} - n_{i,\downarrow})/2$, we write down the on-site spin correlation function, averaged over all the lattice sites,

$$S_0 = \frac{1}{N} \sum_{i=1}^N \left< (S_i^z)^2 \right>.$$  

The other quantity we look at is the spin correlation between two adjacent sites in the middle of the chain,

$$S_1 = \left< S_{\frac{N}{2}}^z S_{\frac{N}{2}+1}^z \right>.$$  

As a measure of the bipartite entanglement we calculate the logarithmic negativity, $\mathcal{LN}$ [64,67]. The negativity $\mathcal{N}$ of a bipartite system $\rho_{AB}$, comprising of sub-systems $A$ and $B$, is defined as the absolute value of the sum of negative eigenvalues of $\rho_{AB}^T$, which is the partial transpose of $\rho_{AB}$, with respect to $A$. In terms of the trace norm of $\rho_{AB}$, defined as, $||\rho_{AB}^T||_1 = \text{Tr} \sqrt{\rho_{AB}^T \rho_{AB}}$, negativity can be rewritten as

$$\mathcal{N} = \frac{||\rho_{AB}^T||_1 - 1}{2},$$

and logarithmic negativity is then defined as

$$\mathcal{LN} = \log_2 (2\mathcal{N} + 1).$$

We evaluate these quantities to examine the effect of the quenched disorder averaging process which we describe next.

### III. QUENCHED DISORDER AND QUENCHED AVERAGING

In the ordered system the Hamiltonian parameters have constant values throughout. For the disordered case, the Hamiltonian parameters are ‘quenched’ in the sense that the typical timescales required for the parameters to equilibrate are much larger than the timescales of the system dynamics that is experimentally relevant. Therefore, for all our purposes, the value of a particular disorder parameter remain constant during the time considered for observation of the dynamics. For example, if we want the disorder only to be in the tunnelling term we take,

$$J = J_0 + \delta J(\mu, \sigma),$$

where in this work we take $\delta J(\mu, \sigma)$ from a set of identical and independent distributed Gaussian random variables with mean $\mu$ and standard deviation $\sigma$. In order to investigate the effect of different $\sigma$ values while $\mu$ is set to be zero we look at different $\sigma$-s. The results are found to be qualitatively similar where strengths of the effect increase with the $\sigma$ values. In this work we show the results for $\sigma = 0.5$. For the Gaussian random variables the values in $\delta J(\mu, \sigma)$ typically fall between $\mu - 3\sigma$ and $\mu + 3\sigma$. For the tunnelling term we discard the negative values for the tunnelling term is negative, which is around 1.25% of the total realisations. To do so we first determine the ground state for a given set of values of $U/J_0$. For the bosons this is done at unity filling and for the fermions, at half-filling for each of the spin-species.
For small enough system \((N = 8)\) we use exact diagonalisation to find the ground state. For a system with larger number of lattice sites \((N = 32)\) the ground state is found with imaginary time evolution method using time evolving block decimation (TEBD) algorithm \([53]\) under the framework of DMRG methods. The quenched disorder averaging is done by taking the average of a particular observable \(\mathcal{O}\) over all the disorder realisations. The effect of the disorder is quantified by the relative difference (scaled up by 100) of the average value with respect to its value in the ordered case, which we denote by \(\Delta(\mathcal{O})\), a convergence plot of which is shown in Fig. 1. The typical number of realisations that need to be considered to attain convergence up to third decimal place is found to be a few thousands. This plot shows the result obtained for the Fermi-Hubbard model with a system size \(N = 32\) computed using DMRG with bond dimension of 256. In the following sections we report our findings for 5000 realisations of disorder values.

IV. EFFECT OF DISORDER IN THE GROUND STATE

To investigate the effect of the quenched disordered averaging we indulge in the two scenarios in context with the two different terms in the Hamiltonians. When the disorder is only realised in the interaction term of the Hamiltonians (Eq. (1) and Eq. (2)), we do not observe any noticeable disorder induced order. The observable values for the random Gaussian distribution of the disorder values also appear to be Gaussian in nature with the average value close enough to the ordered value of the observable, resulting in absence of an enhancement. We describe the second scenario with disorder in the tunnelling term in the following.

A. Bose-Hubbard Hamiltonian

When the disorder is in the tunnelling term of the Bose-Hubbard Hamiltonian, we detect presence of enhancement in suitable parameter range. Here, for the ordered state with \(J = J_0\) and for the disordered states we first measure the Mott orders and the bipartite entanglement in the ground state. We find existence of disorder induced enhancement (i.e. positive \(\Delta\) values) for small enough values of \(U/J_0\) which becomes smaller as we increase \(U/J_0\) and eventually \(\Delta\) values become negative. This result is shown in Fig. 2(a) where \(\Delta(M_1)\), the Mott order in terms of adjacent sites for a 8-site system is displayed with \(U/J_0\). We have taken \(N_D = 5000\) to have good convergence. The error bars are omitted as they tend to be quite small. Similarly we have looked for the Mott order in terms of next-neighbour sites, the findings of which are shown in the inset of Fig. 2(a) where we plot \(\Delta(M_2)\) with \(U/J_0\). We additionally look into the log negativity, \(\mathcal{LN}\) as a measure of entanglement, taking two adjacent sites. The disorder effect can be found in Fig. 2(c) where \(\Delta(\mathcal{LN})\) is plotted as a function of \(U/J_0\). Here the enhancement can be found to be present for small enough values of \(U/J_0\). The sta-
tistical distribution of the observables for the quenched disorder realisations are displayed in Fig. 2(b), Fig. 2(c) inset and Fig. 2(d) inset for $|\langle b_i^\dagger b_{i+1}\rangle|$ (in connection to $M_1$), $|\langle b_i^\dagger b_{i+2}\rangle|$ (in connection to $M_2$) and $\Delta(\mathcal{LN})$ respectively. We have chosen $U/J_0 = 2$ for these plots but the shape of these asymmetric distributions do not qualitatively change much for various $U/J_0$ values. For the cases where $\Delta$ values are positive the averages of the distribution are greater than the corresponding ordered values. In order to explain the effect of disorder in the tunnelling term on the physical observables, e.g. $M_1$, it is instructive to look at the behaviour of $|\langle b_i^\dagger b_{i+1}\rangle|$ as a function of $U/J_0$, shown in Fig. 2(d). For $U/J_0 = 2$, in the superfluid regime, the value of $|\langle b_i^\dagger b_{i+1}\rangle|$ falls fast as one increases $U/J_0$. The statistics of $U/J$, shown in the inset of Fig. 2(d), show a similar asymmetric distribution with median at $U/J_0 = 2$. This distribution along with the rate of change in $|\langle b_i^\dagger b_{i+1}\rangle|$ value with $U/J_0$ cause more weightage from the larger side of $|\langle b_i^\dagger b_{i+1}\rangle|$. Due to the fact that $|\langle b_i^\dagger b_i\rangle| = 1$ (uniform number density for periodic boundary condition), the quenched disordered average of $M_1$ therefore is bigger than the ordered value.

On the other hand, as one goes deeper in the Mott insulator regime, the $|\langle b_i^\dagger b_{i+1}\rangle|$ value falls at much slower rate with increase in $U/J_0$. This results in less weightage from the larger side of $|\langle b_i^\dagger b_{i+1}\rangle|$, and hence the quenched disordered average of $M_1$ is smaller than the ordered value. This similar explanation holds for the other observables as well.

**B. Fermi-Hubbard Hamiltonian**

Now for the Fermi-Hubbard Hamiltonian we also find disorder induced enhancement that exists for small enough values of $U/J_0$ and becomes smaller as $U/J_0$ is increased. The $\Delta$ values become negative for $U/J_0 > 3$. This result is shown in Fig. 3(a) where $\Delta(S_0)$ for a 32-site system is displayed with $U/J_0$. Here also $N_D = 5000$ that gives good convergence and all the DMRG results are computed with bond dimension 256. We again discard the very small error bars. The anti-ferromagnetic spin order is examined further by looking at the spin correlation between two adjacent sites in the middle of the chain the findings of which is shown in the inset of Fig. 3(a) where we plot $\Delta(S_1)$ with $U/J_0$. As a measure of bipartite entanglement, the log negativity $\mathcal{LN}$ is computed in the middle of the chain. The disorder effect can be found in Fig. 3(c) where $\Delta(\mathcal{LN})$ is plotted as a function of $U/J_0$ where the enhancement can be found to be present for $U/J_0 < 2.5$. As before we report the asymmetric statistical distributions of $S_0$, $S_1$ and $\mathcal{LN}$ in Fig. 3(b), Fig. 3(b) inset and Fig. 3(c) inset respectively. For these distributions $U/J_0 = 2$, although their shapes do not qualitatively change much for different $U/J_0$ values. The presence of disorder induced enhancement for small $U/J_0$ values and its absence for large $U/J_0$ values can again be explained as before, in terms of the behaviour of the physical observables with $U/J$ and the distribution of $U/J$ itself. They are shown in Fig. 3(d) and its inset respectively. The rate of increase of $S_0$ is much larger for small $U/J_0$ values and therefore, due to the asymmetric distribution of $U/J$ values results in more weightage from the larger side of $S_0$. This causes the quenched disordered average of $S_0$ to be bigger than the ordered value. Similarly, for larger $U/J_0$ values the smaller rate of increase of $S_0$ causes the quenched disordered average of $S_0$ to be smaller than the ordered value.

In both cases of bosons and fermions we therefore have an intuitive idea about the behaviour of the quenched disordered average. Quantising them, however, is only possible with the aid of numerical analysis.

**V. TIME EVOLUTION**

After looking at the ground state properties the next step is to evolve the system in time in the presence of noise that a real experimental set-up would suffer from. In the context of realising Bose-Hubbard or Fermi-Hubbard Hamiltonian with cold atoms in optical lattices, an often unavoidable noise source is the spontaneous emissions of the atoms due to the coupling to the vacuum.
These two species are practically identical due to the large
with fermions, the two spin species are usually realised
ment of the lattice. On the other hand, in experiments
the ultracold temperature and typical tight confine-
site [51]. The lowest approximation works very well due
the ultracold temperature and typical tight confine-
mer displaying an undulating behaviour.
For the fermionic case, the decoherence causes the spin
correlation function to diminish as a function of time.
The value of $\Delta(S_i)$, however, is also found to be not
changing much as a function of time as can be seen in
Fig. 4(c), which is computed on a 8-site lattice with 2000
trajectories for the ordered state with $U/J_0 = 2$ and $\gamma = 0.1J_0$.
The number of trajectories used for it is 2000, making the error bars very small.
The quenched disordered average is plotted as the solid
blue line which is computed with 2000 disorder values
with 50 trajectories for each value. The difference be-
tween them is the disorder induced enhancement which
does not change much over time. The same features are
found for $M_2$ which is shown in the inset. Fig. 4(b) shows
the same result for $\Delta(N)$ which is found to be decreasing
over time in a similar fashion for both the ordered case
and the quenched disordered average case, with the for-
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VI. CONCLUSION

We have looked at one-dimensional systems of ultra-cold atoms loaded on optical lattices, governed by Bose-Hubbard and Fermi-Hubbard Hamiltonians for bosons and fermions respectively. We have numerically computed the effects of quenched disorder averaging in these systems, where Gaussian random disorder has been used for the Hamiltonian parameters. For bosons, we looked at the single particle density matrix elements that characterise the many-body ground state as well as logarithmic negativity as a measure of bipartite entanglement. For fermions, we looked at the spin correlation functions as measures of magnetic order along with logarithmic negativity. In both cases, we find disorder-induced enhancement of the observables in the ground state for weak interactions with the disorder in the tunnelling term that cease to exist when the interaction grows stronger. We attribute this behaviour to the statistics of the disorder realisations and the dependence of the observables on the Hamiltonian parameters. We then consider the dynamics of these systems under the decohering action of spontaneous emission events and find the effects of the quenched disorder averaging to not change noticeably with respect to what we observe at the initial instance. Our results thus provide with a means to quantify the effects of such quenched disorder averaging and can be used to benchmark ongoing optical lattice experiments probing into the effects of disorder.

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