Control of probe response and dispersion in a three level closed $\Lambda$ system: Interplay between spontaneously generated coherence and dynamically induced coherence

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Abstract. The dispersion and the absorption properties of a closed three-level $\Lambda$ system with incoherent pumping have been studied when both dynamically induced coherence (DIC) and the spontaneously generated coherence (SGC) play a significant role. We analyze the interplay between the dynamically induced coherence (DIC) and the spontaneously generated coherence (SGC) on the absorption spectrum to induce different nonlinear processes like amplification without population inversion (AWI), electromagnetically induced transparency (EIT) and electromagnetically induced absorption (EIA). By changing the probe field Rabi frequency the response of the system can switch over from one nonlinear process to other and hence the probe field Rabi frequency can be used as a knob for switching different nonlinear responses. By changing the relative phase of two fields the response of the system switches over from EIA to EIT or EIT to AWI which can be used as optical switch. Choosing appropriate value of the relative phase and the probe field Rabi frequency, the dispersion and hence the propagation of the probe pulse can be controlled efficiently. Therefore these parameters can be used as a knob to manipulate light propagation from subluminal to superluminal. Exact analytical expressions for the coherences and populations in the steady state limit have been derived (keeping all orders of system parameters) to generalize the analysis. Any restrictions over the system parameters have been avoided to make it applicable to various atomic and molecular systems.

Keywords: Spontaneously Generated Coherence, EIT, EIA, Subluminal, Superluminal.

1. Introduction

Recently great interest has been directed to the modification of dispersion [1-5] and absorption [6-13] properties of an absorbing medium from the interference of spontaneous emissions of an atom to two closely lying levels. There has been renewed interest in the magnitude and the phase control of the group velocity in a transparent medium. The manipulation of the light speed is realized by changing the dispersive properties of the medium. A series of experiments have demonstrated both subluminal [14-18] and superluminal [19-22] propagation of light in a dispersive medium. The most important key to successful experiments on subluminal and superluminal light propagations lies in the ability to control the optical properties of a medium.
with laser field. The early experiments [14,15] produced the values of the group index $n_g = c/v_g$ in the range $10^2 - 10^3$. Harris [23] suggested that electromagnetically induced transparency (EIT) [2,3,13,24-30] can be used to slow down the group velocity. Hau et al measured a group velocity $v_g$ as low as $17m/s$ in an ultracold gas of sodium atoms [16]. Later, group velocities in hot gases were reduced to $90m/s$ [17], and $8m/s$ [18], and in a solid to $45m/s$ [31]. Based on an early proposal of Chiao and co-workers [32-35], Wang et al [19-20] demonstrated superluminal light propagation using the region of lossless anomalous dispersion between two closely spaced gain lines in a double-peaked Raman gain medium. The gain doublet is created by applying two intense detuned cw pumps with slightly different frequencies to one transition of a $\Lambda$-type three-level system in atomic cesium. Another important consequence of quantum coherence is that the so called electromagnetically induced absorption (EIA) can lead to a highly anomalous dispersion where enhanced absorption occurs [36,37]. Akulshin et al [38] and Kim et al [39] reported experiments where negative group velocities of $v_g = -c/14400$ in Cs vapor cells were obtained. The negative velocity means that the peak of the pulse emerges from the medium before its peak enters into the medium. Agarwal and Menon [40] proposed a scheme where by changing a knob i.e an additional microwave field coupling lower levels, one can switch over from subluminal to superluminal light propagation in $\Lambda$ system. Gonzalo et al. [41] showed theoretically the possibility of light propagation from subluminal to superluminal in $\Lambda$ system considering SGC and squeezed field. The effects of spontaneously generated coherence (SGC) on the group velocity by investigating the dispersion properties have been studied in $V$ system [42,43]. Sahrai et al [44] proposed a scheme using two lasers and one microwave field in a double $\Lambda$ system. They showed that the group velocity can be controlled by changing the relative phase between different fields. Agarwal et al [45] showed the possibility of superluminal propagation through coherent manipulation of a Raman process in $N$ configuration. The experimental demonstration was realized by Kang et al [46]. X-M Hu et al [47] showed that the change from positive to negative dispersion with increasing intensity of the driving field is achievable in the $V$ system, which is same as in the $\Lambda$ system. They presented a theoretical analysis of double switching from normal to anomalous dispersion via trichromatic phase manipulation of electromagnetic induced transparency [48]. Recently Hou et al [49] investigated the absorption and the dispersion properties with SGC in four-level tripod system. Mahmoudi et al [50] studied the effects of the incoherent pumping field and the SGC on the phase control of group velocity. They showed that for the weak probe field, and in the presence of SGC, the existence of the incoherent pumping is a necessary condition for the phase control of the dispersion, the absorption and the group index.

In view of many potential applications of subluminal and superluminal light propagation, a natural question is how one can have a controlling parameter for switching from one to the other regime of light propagation. In this paper we show theoretically the effect of both SGC and dynamically induced coherence (DIC) on the absorption and the dispersion properties of a closed three level $\Lambda$ system in presence of incoherent pumping. When the contributions from both the coherences (DIC and SGC) are prominent, the response of a closed three level $\Lambda$ system with incoherent pumping will change dramatically from the responses obtained either from SGC or from DIC separately. This is because of the fact that these two coherences interfere constructively or destructively depending on their relative magnitude and sign [13]. This aspect of control of dispersion due to interplay between SGC and DIC has not been studied before. It will be shown here that under the weak probe field condition, the response of a three-level $\Lambda$ system is controlled predominantly by SGC. Therefore in the weak probe field condition, one may lose some dramatic effects on the non-linear probe response and dispersion, which are caused by the interplay between these two types of coherences in presence of incoherent pumping. In our previous work [13], we have shown that the probe field Rabi frequency can be used as a switch to change the response of the system from AWI to EIT. Here we will show that by changing the
probes field Rabi frequency, one can control the response so that it switches over from EIA to AWI (with Raman gain) through EIT. As a result the control over the dispersion of the system can be achieved by controlling the probe field Rabi frequency and the phase difference of the two coupling fields. By tuning the relative phase between two applied fields, the desired response of the system can be achieved around the resonance of the two fields. We have analyzed our results from the exact analytical expressions for coherences and populations given here.

This paper is organized as follows: In Section 2.1, we have presented the density matrix equations for a system and in 2.2 we have given the analytical steady state solutions of the polarizations. In Section 3, we have shown the effect of SGC and DIC on the dispersion and absorption (or emission). In Section 3.1, we have discussed the effect of relative phase between two applied fields. Finally conclusions are drawn in Section 4.

2. Theory

2.1. The System and Density Matrix Equations

We consider a closed Λ-type three level system with the ground state $|3\rangle$ and two excited states $|2\rangle$ and $|1\rangle$, as shown in fig.1a. The transition $|2\rangle \leftrightarrow |1\rangle$ of energy difference $\hbar \omega_{12}$ is driven by a coherent coupling laser of frequency $\omega_c$ with Rabi frequency $\Omega_0$. The transition $|3\rangle \leftrightarrow |1\rangle$ of energy difference $\hbar \omega_{13}$ is pumped with a rate $2\Lambda$ by a bidirectional incoherent pumping. $2\gamma_{13}$ $(2\gamma_{12})$ is the spontaneous decay width from the state $|1\rangle$ to the state $|3\rangle$ $(|2\rangle)$. A probe laser of frequency $\omega_p$ with Rabi frequency $G_0$ is applied to the transition $|1\rangle \leftrightarrow |3\rangle$. Since the existence of SGC effect depends on the nonorthogonality of the dipole moments $\vec{d}_{13}$ and $\vec{d}_{12}$, we have to consider an arrangement shown in fig. 1b, that one field acts on only one transition, where $\theta$ represents the angle between the two induced dipole moments.

The density matrix equation of motion [51, 52] in the rotating-wave approximation and
electric dipole approximation can be written as:

\[
\dot{\rho}_{11} = i\Omega_c\rho_{21} - i\Omega^*_c\rho_{12} + iG_p\rho_{31} - iG^*_p\rho_{13} - 2(\gamma_{12} + \gamma_{13} + \Lambda)\rho_{11} + 2\Lambda\rho_{33} \tag{1}
\]

\[
\dot{\rho}_{22} = i\Omega^*_c\rho_{12} - i\Omega_c\rho_{21} + 2\gamma_{12}\rho_{11} \tag{2}
\]

\[
\dot{\rho}_{33} = iG^*_p\rho_{13} - iG_p\rho_{31} + 2(\gamma_{13} + \Lambda)\rho_{11} - 2\Lambda\rho_{33} \tag{3}
\]

\[
\dot{\rho}_{12} = i\Omega_c(\rho_{22} - \rho_{11}) + iG_p\rho_{32} - (\gamma_{12} + \gamma_{13} + \Lambda - i\Delta_c)\rho_{12} \tag{4}
\]

\[
\dot{\rho}_{13} = iG_p(\rho_{33} - \rho_{11}) + i\Omega_c\rho_{23} - (\gamma_{12} + \gamma_{13} + 2\Lambda - i\Delta_p)\rho_{13} \tag{5}
\]

\[
\dot{\rho}_{23} = i\Omega^*_c\rho_{13} - iG_p\rho_{21} + [i(\Delta_p - \Delta_c) - \Lambda]\rho_{23} + 2\sqrt{\gamma_{13}\gamma_{12}}\cos\theta\eta_0\rho_{11} \tag{6}
\]

The above equations are constrained by \(\rho_{ij} = \rho^*_{ji}\) and \(\rho_{11} + \rho_{22} + \rho_{33} = 1\) and \(\rho\) satisfies the positivity condition, i.e. real eigenvalues are non-negative [53]. Here Rabi frequencies are denoted as

\[
\Omega_c = \frac{\vec{d}_{12}.\vec{E}_c}{\hbar} = \Omega_0 \sin \theta
\]

\[
G_p = \frac{\vec{d}_{13}.\vec{E}_p}{\hbar} = G_0 \sin \theta
\]

where \(\Omega_0\) and \(G_0\) are the Rabi frequencies without SGC. The detunings are \(\Delta_c = \omega_c - \omega_{12}\) and \(\Delta_p = \omega_p - \omega_{13}\). Here \(\eta_0\) will be zero (one) if the spontaneously generated coherence effect is ignored (considered). It should be noted that only for small energy spacing between the two lower levels, SGC effect becomes remarkable. For large energy spacing, the rapid oscillation in \(\rho_{23}\) will average out such effect [6, 51].

Let us now analyze the density matrix equations (1-6) in a field dependent basis given by

\[
|1\rangle, |+\rangle = \frac{\Omega_c|2\rangle + G_p|3\rangle}{F}, |-\rangle = \frac{G_p|2\rangle - \Omega_c|3\rangle}{F}
\]

(6a)

Here \(F = \sqrt{\Omega_c^2 + G_p^2}\). We have considered the Coherent Population Trapping (CPT) condition \(\Delta_p = \Delta_c = 0\). In this basis we can get

\[
\rho_{++} = (\Omega_c^2\rho_{22} + G_p^2\rho_{33} + \Omega_cG_p\rho_{23} + \Omega_cG_p\rho_{32})/F^2
\]

(6b)

\[
\rho_{--} = (G_p^2\rho_{22} + \Omega_c^2\rho_{33} - \Omega_cG_p\rho_{23} - \Omega_cG_p\rho_{32})/F^2
\]

(6c)

with the relation \(\rho_{11} + \rho_{++} + \rho_{--} = 1\). Here \(\rho_{++}, (\rho_{--})\) is the population of the bright (dark) state. It is well known that all atoms populated in the dark state (i.e. \(\rho_{--} = 1\), which is called as CPT) corresponds to ideal EIT. We will explore later, whether EIT can be obtained even when CPT is not preserved.

2.2. The Analytical Solutions

Usually systems with well separated levels do not depend on the relative phase between the two applied fields. But in case of closely spaced levels, the \(\Lambda\) system becomes quite sensitive to phases of the two coupling fields due to existence of the SGC term [6, 7, 10, 11]. So we have to treat the Rabi frequencies as complex parameters. If \(\phi_p\) and \(\phi_c\) are the phases of the probe and the coherent field respectively, then we get \(\Omega_c = \Omega e^{-i\phi_c}, G_p = Ge^{-i\phi_p}, \rho_{ii} = \tilde{\rho}_{ii}, \tilde{\rho}_{12} = \rho_{12}e^{i\phi_c}, \tilde{\rho}_{13} = \rho_{13}e^{i\phi_p}\) and \(\tilde{\rho}_{23} = \rho_{23}e^{i\phi}\), where \(\phi = \phi_p - \phi_c\). Thus we obtain equations in redefined density matrix elements \(\tilde{\rho}_{ij}\) which are found to be identical to Eq. (1-6), with the SGC parameter \(\eta_0\) replaced by \(\eta_0 e^{i\phi}\), \(G_p\) replaced by \(G\) and \(\Omega_c\) replaced by \(\Omega\), where \(G\) and \(\Omega\) are treated as real parameters.

We have analytically solved the above density matrix equations in the steady state limit without any approximation i.e. keeping all the orders of both the probe field and coherent field Rabi frequencies, spontaneous decay rates and incoherent pumping.
Imaginary part of $\tilde{\rho}_{13}$ is given by

$$v_{13} = Im(\tilde{\rho}_{13})$$

$$= \frac{[A_1 + \Omega^2 G^3 c \Lambda (b + \Lambda) + \Omega^3 P(\sin \phi A_2 + \cos \phi A_3)] \Lambda b}{(D_1 + G^2 \gamma_{12} b - \Omega G \beta P \cos \phi) C_1 + [G(\Delta_p - \Delta_c) b - G \Delta_c \Lambda] (R_1 + P_0)}$$ (7)

The constants in the numerator are defined as

$$A_1 = \Omega^3 G cb \Lambda + \Omega^2 G A^2 b c (b + \Lambda) + \Omega^2 G b \Lambda \gamma_{12} + b \gamma_{13} + \Lambda \gamma_{13} (\Delta_c - \Delta_p)^2$$

$$+ \Omega^2 G A^2 \gamma_{12} \Delta_c^2 - 2 \Omega^2 G A^2 \gamma_{12} \Delta_c \Delta_p$$ (8)

$$A_2 = [(\Delta_c - \Delta_p)(b + \Lambda) - \Delta_p \Lambda] \Lambda b$$ (9)

$$A_3 = [\Lambda b (b + \Lambda) - \Delta_p (\Delta_p - \Delta_c) b + \Omega^2 b + G^2 (b + \Lambda) \Lambda]$$ (10)

$$P = 2 \sqrt{\gamma_{12} \gamma_{13} \cos \theta \eta_0}$$ (11)

In the denominator of eqn.7, the constants are defined as

$$C_1 = B_1 \Lambda b + G^2 b(b + \Lambda)(\Delta_c - \Delta_p)^2 + \Omega^2 G^2 \Lambda b + G^4 \Lambda b + G^2 \Lambda b (b + \Lambda)$$ (12)

$$D_1 = \Lambda \gamma_{12} b^2 + (3 \Lambda + \gamma_{13}) \Omega^2 b + \Delta_c^2 \Lambda \gamma_{12}$$ (13)

$$B_1 = [\Omega^2 + (\Delta_c - \Delta_p) \Delta_p]^2 + (b + \Lambda)^2 (\Delta_c - \Delta_p)^2 + 2 \Omega^2 \Lambda (b + \Lambda) + \Lambda^2 (b + \Lambda)^2 + \Lambda^2 \Delta_p^2$$ (14)

$$R_1 = \Omega^2 G cb (b + \Lambda)(\Delta_c - \Delta_p) - \Omega^2 G \Delta_p c \Lambda b + Q \gamma_{12}[G(\Delta_c - \Delta_p) b + G \Delta_c \Lambda]$$ (15)

$$P_0 = b P[R_2 \cos \phi + Q \Omega \Lambda \sin \phi]$$ (16)

$$Q = \Lambda (b + \Lambda)^2 + G^2 (b + \Lambda) + \Omega^2 (b + \Lambda) + \Delta_p^2 \Lambda$$ (17)

$$R_2 = \Omega^3 (b + \Lambda)(\Delta_c - \Delta_p) - \Omega^3 \Lambda \Delta_p - Q \Omega (\Delta_c - \Delta_p)$$ (18)

$$c = \gamma_{13} - \gamma_{12}$$ (19)

$$b = \gamma_{13} + \gamma_{12} + \Lambda$$ (20)

At the resonance condition i.e. when $\Delta_p = \Delta_c = 0$, eqn. 7 reduces to

$$v'_{13} = \frac{\Omega^2 \Lambda[G(\gamma_{13} - \gamma_{12}) + \Omega P \cos \phi]}{[\Omega^2 + G^2 + \Lambda (b + \Lambda)] [b \Lambda \gamma_{12} + (3 \Lambda + \gamma_{13}) \Omega^2 + G^2 \gamma_{12} - G \Omega P \cos \phi]}$$

$$= a_1 + a_2$$ (21)

Where

$$a_1 = \frac{\Omega^2 \Lambda G(\gamma_{13} - \gamma_{12})}{[\Omega^2 + G^2 + \Lambda (b + \Lambda)] [b \Lambda \gamma_{12} + (3 \Lambda + \gamma_{13}) \Omega^2 + G^2 \gamma_{12} - G \Omega P \cos \phi]}$$ (21a)

and

$$a_2 = \frac{\Omega^3 \Lambda P \cos \phi}{[\Omega^2 + G^2 + \Lambda (b + \Lambda)] [b \Lambda \gamma_{12} + (3 \Lambda + \gamma_{13}) \Omega^2 + G^2 \gamma_{12} - G \Omega P \cos \phi]}$$ (21b)

Imaginary part of $\tilde{\rho}_{23}$ is given by

$$v_{23} = Im(\tilde{\rho}_{23})$$
\[ \Omega \Delta b(R_1 + P_0) \]

At the resonance condition i.e. when \( \Delta_p = \Delta_c = 0 \), eqn. 22 reduces to

\[ v_{23}' = \frac{\Omega^2 \Lambda^2 \beta^2 P \sin \phi [\Lambda(b + \Lambda)^2 + G^2(b + \Lambda) + \Omega^2(b + \Lambda)]}{[\Omega^2 + G^2 + \Lambda(b + \Lambda)][b\Lambda \gamma_{12} + (3\Lambda + \gamma_{13})\Omega^2 + G^2\gamma_{12} - G\Omega P \cos \phi]} \] (23)

Imaginary part of \( \tilde{\rho}_{12} \) is given by

\[ v_{12} = \text{Im}(\tilde{\rho}_{12}) \]

\[ = \frac{\Omega C_1 \Lambda \gamma_{12} b}{(D_1 + G^2 \gamma_{12} - \Omega G b P \cos \phi) C_1 + [G(\Delta_p - \Delta_c) b - G \Delta_c \Lambda](R_1 + P_0)} \] (24)

At the resonance condition i.e. when \( \Delta_p = \Delta_c = 0 \), eqn. 24 reduces to

\[ v_{12}' = \frac{\Omega \Lambda b \gamma_{12} [2\Omega^2 \Lambda^2 b(b + \Lambda) + \Lambda^3 b(b + \Lambda)^2 + \Omega^2 G^2 b + G^4 \Lambda(b + \Lambda) + G^2 \Lambda^2 b(b + \Lambda)]}{[\Omega^2 + G^2 + \Lambda(b + \Lambda)][b\Lambda \gamma_{12} + (3\Lambda + \gamma_{13})\Omega^2 + G^2\gamma_{12} - G\Omega P \cos \phi]} \] (25)

The real parts of \( \tilde{\rho}_{12}, \tilde{\rho}_{13}, \tilde{\rho}_{23} \) are given below

\[ \text{Re}(\tilde{\rho}_{12}) = u_{12} = \frac{Gv_{23} - \Delta_c v_{12}}{\gamma_{12} + \gamma_{13} + \Lambda} \] (26)

\[ \text{Re}(\tilde{\rho}_{13}) = u_{13} = \frac{-\Delta_p v_{13} + \Omega v_{23}}{\gamma_{12} + \gamma_{13} + 2\Lambda} \] (27)

\[ \text{Re}(\tilde{\rho}_{23}) = u_{23} = -\frac{(\Delta_p - \Delta_c) v_{23} + \Omega v_{13} + G v_{12}}{\Lambda} + \frac{P\Omega v_{12} \cos \phi}{\gamma_{12} \Lambda} \] (28)

At the resonant condition,

\[ u_{12}' = \frac{Gv_{23}'}{b} \] (29)

\[ u_{13}' = -\frac{\Omega v_{23}'}{b + \Lambda} \] (30)

The general expression of the steady state populations are given

\[ \rho_{11} = \frac{\Omega v_{12}}{\gamma_{12}} \] (31)

\[ \rho_{33} = (\gamma_{13} + \Lambda)\rho_{11} - G v_{13} \] (32)

\[ \rho_{22} = 1 - \rho_{11} - \rho_{33} \] (33)

From Eqn.(23) we can infer that at the resonant condition, \( \text{Im}(\rho_{23}) \neq 0 \) when \( \phi \neq 0 \) or \( \pi \). But when \( \phi = 0 \) or \( \pi \) in presence of SGC or in absence of SGC (i.e. \( P = 0 \)), \( \text{Im}(\rho_{23}) = 0 \) and hence \( \rho_{23} = \text{Re}(\rho_{23}) \) [54-57]. Similarly \( \text{Re}(\rho_{12}) \) and \( \text{Re}(\rho_{13}) \) are non-zero at resonance when \( \phi \neq 0 \) or \( \pi \). Thus we can control the dispersion of the probe signal i.e \( \text{Re}(\rho_{13}) \) by controlling the value of \( P \) and \( \phi \).

The gain coefficient for the probe field coupled to the transition \( |3\rangle \leftrightarrow |1\rangle \) is proportional to \( \text{Im}(\tilde{\rho}_{13}) \). If \( \text{Im}(\tilde{\rho}_{13}) > 0 \), the probe laser will be attenuated, whereas if \( \text{Im}(\tilde{\rho}_{13}) < 0 \), the probe field is amplified. The interplay between SGC and DIC determining the value and the sign of \( v_{13} \) can be shown more clearly if we consider a simplified case where detunings for both the fields are zero i.e. at the resonances of both the fields (see equation 21). In presence of
incoherent pumping, the first two terms in the numerator of Eqn.21 is proportional to G and the third term is proportional to the product of Ω, P and cos φ, other parameters remaining the same. When G is much less than Ω the third term which contains the SGC parameter P will dominate and hence the sign of $v_{13}$ will be determined by the sign of the product $P \cos \phi$ i.e. on both the sign of P and cos φ. Therefore under the weak probe field condition, the magnitude and the mode of probe response/dispersion will be determined predominantly by the contribution from SGC. But when the value of G becomes comparable to the value of Ω, two terms in the numerator may interfere destructively or constructively to give EIT, EIA and AWI depending on the value of P, cos φ and on the relative values of two spontaneous decay widths. It is clear that the sign of $a_1$ will depend on the relative magnitude of the spontaneous decay widths, whereas that of $a_2$ will depend on the sign of P (i.e. sign of cos θ) and also on the sign of cos φ as mentioned before. Hence the nature of interference will also depend on the relative phase of probe and coherent fields. Hence in this system one can achieve net emission (AWI), electromagnetically induced transparency (EIT) or maximum absorption at resonance of both the fields (EIA), depending on the relative values and the sign of the system parameters. Therefore the absorption or gain profiles obtained under the weak probe field approximation (i.e. when SGC is dominant) may change dramatically when the contribution from the first two terms (i.e. DIC) becomes comparable to that from the third term (i.e. SGC). It can be shown that by putting $P = 0$ in equation (7), right hand side will reduce to the expression for $v_{13}$ obtained only from DIC [54, 55]. It has been shown in our previous work [54, 55] that under the weak probe field approximation the magnitude of gain/absorption which is proportional to the value of $v_{13}$ is orders of magnitude smaller than that obtained when the Rabi frequency for both the fields are comparable. It can be noted that the value of Ω need not be much larger than the spontaneous decay width on the probe transition and hence by choosing moderate values of Ω and G one can enhance the AWI process.

The group velocity in a dispersive medium can written as

$$v_g = \frac{c}{1 + 2\pi \chi'(\omega_p) + 2\pi \omega_p \left(\frac{\partial \chi'(\omega_p)}{\partial \omega_p}\right)_{\omega=\omega_p}}$$

(34)

where $\chi'(\omega_p)$ is the real part of the susceptibility $\chi(\omega_p)$. The relation of the susceptibility and the density matrix element $\rho_{13}$ can be easily derived as [40]

$$\chi(\omega_p) = \frac{2N|\tilde{d}_{13}|^2}{\hbar \epsilon_0 G} \rho_{13}$$

(35)

It is clear from the expression of the group velocity that when $\chi'(\omega_p)$ is zero and the dispersion is very steep and positive, the group velocity is significantly reduced, leading to subluminal light propagation. On the other hand, strong negative dispersion can lead to an increase in the group velocity and even to its becoming negative. Hence the superluminal light propagation can be achieved.

3. Results and Discussions

Here, we will present the analytical results by plotting the dispersion spectrum $Re(\tilde{\rho}_{13})$ and the absorption spectrum $Im(\tilde{\rho}_{13})$. In fig.2, choosing $\gamma_{13} = 1$, $\gamma_{12} = 4$, $\Lambda = 0.1$, $\eta = \eta_0 \cos \theta = 0.5$ and $\Delta_c = 0 \Omega = 3 \sin \theta$ and $G = 2 \sin \theta$, we get EIT (see solid curve). Here EIT occurs due to the destructive interference of DIC and SGC term since DIC and SGC are of opposite sign and of equal magnitude (see table 1). In this figure we have also shown that by changing the value of G, the response of the system switches over to AWI (with Raman inversion) or to EIA through EIT. When G is much less than Ω i.e. under weak probe field condition, the
Figure 2. Dependence of $\text{Im}(\rho_{13})$ upon probe detuning $\Delta_p$ with $\gamma_{13} = 1$, $\gamma_{12} = 4$, $\Lambda = 0.1$, $\cos \theta = 0.5$, $\Delta_c = 0$, $G = 2 \sin \theta$, $\Omega = 3 \sin \theta$, $\phi = 0$ (—). The value of $G$ for $- - -$ $= 0.1 \sin \theta$, $\cdots \cdots$ $= \sin \theta$ and $- - - = 3 \sin \theta$. The right inset represents dependence of $\text{Re}(\rho_{13})$ upon the probe detuning $\Delta_p$.

Figure 3. Dependence of $\text{Re}(\rho_{13})$ upon probe detuning $\Delta_p$ with $\gamma_{13} = 1$, $\gamma_{12} = 4$, $\Lambda = 0.1$, $\cos \theta = 0.5$, $\Delta_c = 0$, $G = 0.1 \sin \theta$, $\Omega = 3 \sin \theta$, $\phi = 0$ (—). The value of $G$ for $- - -$ $= \sin \theta$, $\Box = 0.5 \sin \theta$ and $\cdots \cdots$ $= 0.4 \sin \theta$

Figure 4. Dependence of $\text{Im}(\rho_{13})$ (——) and $\text{Re}(\rho_{13})$ (——) upon probe field Rabi frequency ($G_0$). The value of $\Delta_p$ for $- - -$ $= 0$ and $- - - - = 0.1$. Other parameters are kept same as the fig.2. The inset shows the variation of $\text{Im}(\rho_{13})$ against the probe field Rabi frequency ($G_0$) with $\Delta_p = 0.1$. 
Figure 5. The variation of populations in the bare states against the probe detuning. The chosen parameters are kept same as fig.2, but the value of $G_0$ is different. For the first (second) graph in the upper channel, the value of $G_0 = 0.1$ ($G_0 = 1$). For the first (second) graph in the lower channel, the value of $G_0 = 2$ ($G_0 = 3$).

Figure 6. The variation of trapped populations as a function of probe detuning. For the left panel: $\gamma_{13} = 1$, $\gamma_{12} = 4$, $\Lambda = 0.1$, $\cos \theta = 0.5$, $\Delta_e = 0$, $G = 2 \sin \theta$, $\Omega = 3 \sin \theta$, $\phi = 0$. For the right panel: $G = 0.1 \sin \theta$, other parameters remaining the same.
Table 1. Analytical value of $\text{Im}(\hat{\rho}_{13})$ at the resonance of both the fields for different value of the probe field Rabi frequency $G_0$. The second column represents the value of $\text{Im}(\hat{\rho}_{13})$ in absence of SGC effect. The third and the fourth column represent the contribution from $a_1$ and $a_2$ respectively (see Eqn.21). The last column represents the value of $\text{Im}(\hat{\rho}_{13})$ combining both the terms ( Eqn.21).

| $G_0$ | $\text{Im}(\hat{\rho}_{13})$ (P=0) | contribution from DIC term ($a_1$) | contribution from SGC term ($a_2$) | Total $\text{Im}(\hat{\rho}_{13})$ ($a_1 + a_2$) |
|-------|----------------|-----------------|----------------|----------------|
| 3     | $-0.915826(02)$ | $-0.136581(01)$ | $0.910540(02)$ | $-0.455271(02)$ |
| 2     | $-0.131480(01)$ | $-0.201190(01)$ | $0.201190(01)$ | $0$ |
| 1     | $-0.128817(01)$ | $-0.175283(01)$ | $0.350567(01)$ | $0.175283(01)$ |
| 0.8   | $-0.113889(01)$ | $-0.147236(01)$ | $0.368091(01)$ | $0.220855(01)$ |
| 0.5   | $-0.798504(02)$ | $-0.942523(02)$ | $0.377009(01)$ | $0.282757(01)$ |
| 0.1   | $-1.172064(02)$ | $-0.177776(02)$ | $0.355553(01)$ | $0.337775(01)$ |

The analytical value of $\text{Im}(\hat{\rho}_{13})$ at the resonance of both the fields for different value of the probe field Rabi frequency $G_0$. The second column represents the value of $\text{Im}(\hat{\rho}_{13})$ in absence of SGC effect. The third and the fourth column represent the contribution from $a_1$ and $a_2$ respectively (see Eqn.21). The last column represents the value of $\text{Im}(\hat{\rho}_{13})$ combining both the terms ( Eqn.21).

The second column represents the value of $\text{Im}(\hat{\rho}_{13})$ in absence of SGC effect. The third and the fourth column represent the contribution from $a_1$ and $a_2$ respectively (see Eqn.21). The last column represents the value of $\text{Im}(\hat{\rho}_{13})$ combining both the terms ( Eqn.21).

The contribution from the SGC term determines the probe response and hence absorption maximum occurs at resonance (EIA) (see the dashed curve). When $G = \sin \theta$, the contribution from SGC term ($a_2$) will be still dominating and we get absorption but with lesser magnitude than the previous case (EIA) at resonance (see dotted curve). When $G = \Omega$, the contribution from the DIC term ($a_1$) is dominant and we get emission at resonance (see dash-dot curve). The right inset in the figure shows the variation of $\text{Re}(\hat{\rho}_{13})$ as a function of probe detuning for the probe response showing EIT and as expected for EIT, the value of $\text{Re}(\hat{\rho}_{13})$ is zero at resonance i.e. at $\Delta_p = \Delta_c = 0$. The dispersion is positive, indicating lossless subluminal propagation of light around the resonance. The dispersion behavior against the probe detuning for different value of $G_0$ are shown in fig.3 with other parameters same as in fig.2. The solid line with $G_0 = 0.1$ displays that dispersion at the resonance is zero and the slope of the curve around the resonance is negative, which implies that the probe laser propagates at superluminal group velocity with maximum absorption (EIA, see fig.2). In case of $G_0 = 1$ (dashed curve), it is found that the dispersion at the resonance is zero, and its slope around the resonance is positive, which implies that the probe laser propagates at a subluminal group velocity with absorption. The dotted and the squares represent the dispersion for $G_0 = 0.4$ and 0.5 respectively. In these two curves, the kink around the resonance implies probe propagation with negligible dispersion. The probe response in these two cases will show absorption since $G \ll \Omega$. In fig.4, choosing $\Delta_c = 0$, $\Delta_p = 0$ and keeping all the other parameters same as in fig.2, we have plotted the dependence of $\text{Im}(\hat{\rho}_{13})$ against the probe field Rabi frequency ($G_0$) (see solid curve). At $G_0 = 2$, $\text{Im}(\hat{\rho}_{13}) = 0$. When $G_0 > 2$, we get emission and for $G_0 < 2$, we get absorption. The dashed line in this figure represents the variation of $\text{Re}(\hat{\rho}_{13})$ against the probe field Rabi frequency ($G_0$) with $\Delta_p = 0.1$ (other parameters are kept same as in fig.2). At the resonance, we know $\text{Re}(\hat{\rho}_{13}) = 0$ as $\phi = 0$ [see Eqn. 23 and 30]. Thus for $0 < G_0 \leq 0.42$, we get negative dispersion, but $0.42 < G_0 \leq 3$, we get positive dispersion. The inset in the figure represents the variation of $\text{Im}(\hat{\rho}_{13})$ against the probe field Rabi frequency ($G_0$) with $\Delta_p = 0.1$. Thus for a particular value of SGC parameter and the phase difference ($\phi$), the response and the dispersion of the probe can be changed only by controlling the probe field Rabi frequency. The response of the system changes significantly with $G$ when the contributions from both the coherences are prominent and $G$ acts as a knob for this switching. In fig.5 the population distribution as a function of probe detuning for different values of the probe field Rabi frequency have been plotted. For $G_0 = 2$ and 3, there is Raman inversion of population between $|2\rangle \leftrightarrow |3\rangle$ levels and for $G_0 = 0.1$ and 1, non-inversion of population is
preserved. For $G_0 = 2$ (EIT), the non-inversion of populations is preserved at resonance. In fig.6, we have plotted the trapped populations in dark (solid) and bright (dashed) states [6, 7] as a function of probe detuning (see eqns.6b and 6c). The figures in the right (left) panel show the trapped populations for the EIA (EIT) curve in fig.2. The atoms are partly trapped in the dark state (i.e. $\rho_{\ldots \ldots} < 1$), thus we get EIT although CPT is not preserved.

3.1. Effect of relative phase between the two coherent fields on $\text{Im}(\tilde{\rho}_{13})$ and $\text{Re}(\tilde{\rho}_{13})$

In usual EIT setup with well separated ground levels in a Λ system, the absorption and the dispersion do not depend on the relative phase between two applied fields. However, in case of closely spaced levels SGC makes the system quite sensitive to the relative phase between the two applied fields. So far dependence of AWI on the phase difference of two fields has been studied [11] under weak probe field condition (i.e. contribution only from SGC). In another paper [50], the effects of relative phase between probe and coupling fields on the absorption and the dispersion have been discussed under weak probe field condition. Here we will show that when the contributions from DIC becomes comparable to SGC, one can get significantly different nonlinear response of the system by changing phase difference between two fields.

In fig.7, choosing $\gamma_{13} = 4$, $\gamma_{12} = 1$, $\Lambda = 2$, $\eta = \eta_0 \cos \theta = 0.5$ and $\Delta_c = 0$, $\Omega = 3 \sin \theta$, $G = 2 \sin \theta$, we have plotted the dependence of $\text{Im}(\tilde{\rho}_{13})$ (solid curve) and $\text{Re}(\tilde{\rho}_{13})$ (dotted curve) against the probe detuning. An absorption peak appears at zero detuning (i.e EIA). This EIA occurs due to the constructive interference effect of SGC and DIC. In this case contribution from both the coherences is positive (since $\gamma_{13} > \gamma_{12}$ and $P \cos \phi > 0$). Around the resonance, the dispersion becomes negative and the superluminal light propagation is accompanied with an absorption maxima (EIA). Away from the resonance, the negative dispersion smoothly changes to positive dispersion with smaller magnitude of absorption. This negative dispersion around the resonance does not occur when SGC is ignored. The right inset shows the variation of $\text{Im}(\tilde{\rho}_{13})$ (solid curve) and $\text{Re}(\tilde{\rho}_{13})$ (dotted curve) for $\eta = 0$ (keeping other parameters constant).
inset shows that the dispersion is positive around the resonance, corresponding to subluminal light propagation accompanied with an absorption. The left inset shows the variation of $\text{Im}(\hat{p}_{13})$ (solid curve) and $\text{Re}(\hat{p}_{13})$ (dotted curve) for $\eta = 1$ and $G = 0.1 \sin \theta$ (keeping other parameters constant). A maximum absorption occurs at zero detuning due to the contribution only from the SGC as $G << \Omega$. Around the resonance, the dispersion is negative hence the light propagates with superluminal group velocity. Away from the resonance, the negative dispersion changes to positive dispersion with AWI. With the change of the relative phase from $\phi = 0$ to $\phi = \pi$, the response of the system changes significantly. The solid curve in fig.8 represents the variation of $\text{Im}(\hat{p}_{13})$ against probe detuning showing EIT. Around the resonance, the slope of the dispersion becomes positive and we get subluminal light propagation (see dashed curve). The right inset shows the variation of $\text{Im}(\hat{p}_{13})$ (solid curve) and $\text{Re}(\hat{p}_{13})$ (dotted curve) for $\phi = \pi/2$ (keeping other parameters constant). At the resonance $\text{Re}(\hat{p}_{13}) \neq 0$ as $\phi$ is different from 0 or $\pi$, which is in good agreement with the analytical findings (see Eqn. 30).

In fig.9, we plot $\delta \text{Re}(\hat{p}_{13})/\delta \Delta_p$ as a function of probe detuning (keeping the same parameters as in fig.7) which gives negative value around the resonance. From the dispersion curves in right (when SGC is absent) and left (in case of EIT of fig.8) insets, it is found that there is a sharp change in dispersion around the resonance. It changes from negative to positive and then to negative in both the cases. In order to get a deeper insight into the modulation effect of the relative phase $\phi$ on the absorption spectrum and the dispersion spectrum of the probe, we plot $\text{Im}(\hat{p}_{13})$ (solid curve) and $\text{Re}(\hat{p}_{13})$ (dashed curve) as function of $\phi$ for $\Delta_p = 2$ in fig.10 (keeping other parameters same as in fig.7). The $\text{Im}(\hat{p}_{13})$ shows oscillation with phase difference $\phi$. From the slope of oscillating $\text{Re}(\hat{p}_{13})$ curve, it can be noticed that the dispersion oscillates between negative to positive value, leading to oscillation between superluminal and subluminal light propagation with the change in phase difference $\phi$. Thus the response of the system changes significantly with different value of $\phi$. Hence the relative phase can be used as a knob to change light propagation from subluminal to superluminal and also the nonlinear response from
Figure 9. Dependence of $\delta \text{Re}(\tilde{\rho}_{13})/\delta \Delta_p$ as a function of probe detuning (keeping the same parameters of fig.7). The right inset represents the same with $\phi = \pi$ (i.e. with the same parameters as in fig.8). The left inset represents the same when $\eta_0 = 0$, other parameters remaining the same as in the main figure.

Figure 10. Dependence of $\text{Im}(\tilde{\rho}_{13})$ (solid line) and $\text{Re}(\tilde{\rho}_{13})$ (dashed line) on $\phi$ with $\gamma_{13} = 4$, $\gamma_{12} = 1$, $\Lambda = 2$, $\cos \theta = 0.5$, $G = 2 \sin \theta$, $\Omega = 3 \sin \theta$, $\Delta_c = 0$ and $\Delta_p = 2$. 
EIT to EIA. Thus we can efficiently control the absorption and the dispersion by an appropriate choice of the relative phase.

4. Conclusion
In this paper, we have investigated the dispersion and the absorption spectrum of a three level closed system with incoherent pumping due to the combined effect of DIC and SGC. We have shown that the probe response and the dispersion can be controlled by changing probe field Rabi frequency and the relative phase between the probe and the coherent field. The probe field Rabi frequency can be used as a knob for switching different nonlinear responses and also for switching the dispersion from positive to negative value, which leads to the change of the velocity of the probe field from subluminal to superluminal. Moreover by tuning the relative phase, one can also control the light propagation velocity from subluminal to superluminal and the nonlinear response from EIA to EIT.

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