APPLICATION OF EMPIRICAL MODE DECOMPOSITION METHOD FOR CHARACTERIZATION OF RANDOM VIBRATION SIGNALS

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Abstract
Characterization of finite measured signals is a great of importance in dynamical modeling and system identification. This paper addresses an approach for characterization of measured random vibration signals where the approach rests on a method called empirical mode decomposition (EMD). The applicability of proposed approach is tested in one numerical and experimental data from a structural system, namely spar platform. The results are three main signal components, comprising: noise embedded in the measured signal as the first component, first intrinsic mode function (IMF) called as the wave frequency response (WFR) as the second component and second IMF called as the low frequency response (LFR) as the third component while the residue is the trend. Band-pass filter (BPF) method is taken as benchmark for the results obtained from EMD method.

Keywords: EMD; BPF; IMF; vibration signals.

I. INTRODUCTION
Modeling and system identification either based on input-output or output-only models require finite measured signals. Characterization of signals is a great of importance in selecting identification models. In practice, there are three main components in raw signals; namely noise, dynamical characteristics of the system and trend. Noise with certain level of SNR distributes in Gaussian and non-Gaussian functions. If the noise has Gaussian function, zero-mean with finite variance and statistically uncorrelated variables, then it is called as Gaussian white noise. This type of noise is embedded in mostly random vibration signals in addition to non-Gaussian and non-white noise. Noise with high level of SNR has significant effect in time and frequency domains. In time domain, noise tends to lower the modeling accuracy. Further, in frequency domain, noise tends to hide the predominant frequency bands. Dynamical characteristics of the system cover the linear and non-linear behaviors. The latter appears in signals in terms of sub harmonics, super harmonics and frequency interactions [1-2]. If the behaviors change with respect to time, then the system is linear and non-linear time-varying systems. Identification models must accommodate the non-stationary of the system in terms of time-varying model coefficients. Recent researches in this area can be found in [3-4].

Trend is characterized as a long-term movement in signals. It has an upward or downward tendency and rate of change in a time series. In many cases, this trend leads to a direct component (DC) term in signals. These three main components must be characterized as a preliminary stage for modeling and system identification of system dynamic. The results can be taken as consideration for choosing the suitable model structure and model coefficient estimation method for identification models.

This paper proposes the application of empirical mode decomposition (EMD) method in de noising, de trending, and decomposing the measured random vibration signals into the three main components. The method is applied in one numerical and experimental data.
II. EMPIRICAL MODE DECOMPOSITION

The essence of empirical mode decomposition is to decompose signal into its oscillatory mode, called intrinsic mode function (IMF). This is achieved by sifting process. The overall EMD algorithm has been well documented in reference [5-8]. Details are available in those references, but it is revisited in this paper for a more concise notation. A step by step procedure of EMD method can be summarized as follows:

1. Identify all the local extreme, maxima and minima of, and then connect the local maxima and minima using the cubic spline to obtain the upper and lower envelope, respectively. Those envelopes should cover all the data of. Their mean is designated as, and the difference between \( y(n) \) and \( m_1 \) is defined by:

\[
H_1(t) = y(n) - m_1
\]  

Ideally, \( h_1 \) should be an IMF if it satisfies two conditions: i) in all data of \( y(n) \), the number of extrema and zero crossings must be either be equal or differ at most by one; and ii) at any point, the mean value of the envelope defined by the local maxima and minima is zero.

2. If \( h_1 \) does not satisfy the conditions, set \( h_1 \) as the original data and repeat the process in step 1 until the conditions are fulfilled and the first IMF is achieved.

3. Residue is then subtracted from step 2, taken as original signal and sifting process is repeated to obtain another IMF. The process is repeated until \( m \) IMF is obtained, where relationship between IMFs and the original data \( y(n) \) may be expressed as:

\[
y(n) = \sum_{i=1}^{m} C_i(n) + r_m(n)
\]  

Term \( C_i(n) \) contains the IMFs of the \( y(n) \), from high to low frequency components. Each \( C_i(n) \) also contains a different frequency component, while \( r_m(n) \) is the residual which is the trend of the data or a constant.

For an easy interpretation, all steps are depicted in Figure 1. The square dot (…) denotes the upper envelope while the dash line (---) denotes the lower envelope and the mean envelope is denoted with long dash dot line (-.-.). Since the residue (lower panel of Figure 1) is not a monotonic function, sifting process is then performed following step 1 until step 3.

In practice, sifting process produces an IMF which contains more than one natural frequency component. This is called as a mode mixing, because of some drawbacks in the EMD algorithm. The mode mixing must be avoided for the purpose of this paper. Rilling et al. [9] proposed intermittent frequency to avoid the mode mixing. In their work, the intermittency is based on the period length to separate the signals into different modes.

The criterion frequency is set as the upper limit of the period that can be included in any given IMF component, so that the resulting IMF will not contain any natural frequency components smaller than the intermittent frequency. However, the intermittent frequency as an additional criterion might not always guarantee the final expected results, since choosing the intermittent frequency is a subjective task. Rato et al. [10] solved this problem by proposing some modifications on the EMD algorithm. Their finding results show that some issues related to the drawbacks of EMD algorithm can be solved, such as mode mixing and tail effect. Hence, their EMD algorithm is adopted in this paper for decomposing the vibration signals.

III. NUMERICAL DATA

To validate the application of EMD method in filtering, detrending, and decomposing vibration signals, a deterministic discrete vibration signal is simply selected and described by Eq. (3),

\[
y(n) = A_1 \sin(2\pi f_1 n) + A_2 \sin(2\pi f_2 n), \tag{3}
\]

where \( A \) and \( f \) are amplitude and frequency, respectively. The chosen system parameters for Eq. (3) are as follows: \( A_1 = 1 \), \( A_2 = 2 \), \( f_1 = 8 \text{ Hz} \) and \( f_2 = 4 \text{ Hz} \), respectively.

The signal is corrupted with a Gaussian noise having SNR of 40 dB and a polynomial trend. The results are depicted in Figure 2. It is seen in Figure 2(a) that the noise can be accordingly extracted with SNR of 41.05 dB and has Gaussian distribution. Sinusoidal signal with respective frequency of \( f_1 = 8 \text{ Hz} \) and \( f_2 = 4 \text{ Hz} \)
can be also extracted with insignificant discrepancy between the actual result and that of EMD method (Figures 2(b) – 2(c)). Similar result also can be observed for extracted polynomial trend as shown by Figure 2(d). Overall results in Figure 2 show that the decomposition process results in signals from high frequency to low frequency. This finding result is also reported in the previous researches [7-8].

IV. EXPERIMENTAL DATA

The experimental scheme is shown in Figure 3. Details of setup have been described in [4] and briefly summarized here. The model test was tested in the wave tank, excited with excitation function (random waves) and measured with wave probes. The motion response in horizontal plane was measured by an optical tracking camera.

A. Raw Data Processing

Raw time series of motion response as a random vibration signal is displayed in Figure 4(a). By observing the motion response in Figure 4(a), it can be seen that the amplitudes distribution appears to be drifted upward. Its probability density is unsymmetrical distributed, suggesting that the motion response is non-linear random waves and non-Gaussian-type. This result is validated by using normality test under Kolmogorov-Smirnov test, Lillie test, and Jarque-Bera test. The motion response is also non stationary which is confirmed by Kwiatkowski-Phillips-Schmidt-Shin test, Phillips-Perron test,
and Augmented Dickey-Fuller test. As additional information, the motion response is non-white noise as validated with Ljung-Box Q-statistic.

The signal is then converted into frequency domain with FFT length of 1024. It can be observed in Figure 4(b) that the motion response has two principal frequency peaks. The first peak (low frequency response, LFR) is hidden around the low frequency. The second peak (wave frequency response, WFR) is clearly around 0.077 Hz, corresponds to the frequency of random waves. It is seen that the first peak is hidden and is difficult to be identified although the FFT length is higher than 1024 and only produces ripples. These results suggest that the signal of measured motion response is contaminated with noise and it might be due to a polynomial trend as proved by the next section. Modeling and system identification using this data will lead to a biased estimation. Hence, EMD method is applied for characterization purpose so that noise and trend can be accordingly extracted from the signal.

**B. Application of EMD Method**

Based on result in Figure 4(b), EMD method is applied to the raw signal of motion response in the Figure 4(a). This method simultaneously performs denoising, decomposition, and detrending in time domain. Results in time domain are then converted into frequency domain using FFT and depicted in Figures 5-8. The first, second and third components are successfully decomposed by using FFT length of 1024 for all components. As benchmark for the EMD method, band-pass filter (BPF) method is chosen.

As seen in those figures, decomposition results can be classified into noise, two IMFs and a trend. Each can be interpreted as follows:

i) The first component is identified as measurement noise embedded in the raw signal of motion responses confirmed by FFT result.

ii) The first component is the first IMF, identified as WFR as confirmed by FFT result. The frequency is similar with the second peak in the lower panel of Figure 4.

iii) The second component is the second IMF, identified as LFR as confirmed by FFT result. The frequency is around 0.005 Hz and looks clearer than the first peak in the lower panel of Figure 4.

iv) The last component is the trend of the motion response, which is found as a polynomial trend. The polynomial trend obtained from

![Figure 4. Raw data of motion response (a) time series; (b) spectrum](image)

![Figure 5. Decomposition result: noise (a) time series; (b) spectrum](image)
both methods has similar pattern where both trends increase with respect to time.

When BPF method is applied as filter for the measured signal, the noise embedded in the signal is comparable with the first component, followed by the first and second IMFs. Minor differences are found between those two methods due to different resolution bandwidth. This finding result confirms that the EMD method may be used as an alternative filter and decomposition tool for noisy signals besides the conventional BPF method. Scale separation of EMD method is adjusted to decompose the signal into the WFR and LFR according to the results shown in Figure 4(b). With some trials, it is found that the best resolution for the separation scale is found to be around 7 - 10 dB. For BPF method, after several trials, it is found out that the suitable band pass is between 0.04 Hz and 0.1 Hz for the WFR and between 0.001 Hz and 0.03 Hz for the LFR.

Based on results in Figures 5-8, statistics of the signal are performed and listed in Table 1. Overall, it is shown that the mean, variance skewness and kurtosis values decrease after signal are treated with EMD method.

Table 2 shows the properties of signal before and after EMD method is applied. It can be seen that properties of signal are still preserved in this

| Data      | Property              |
|-----------|-----------------------|
| Motion    | Non-Gaussian | Non-stationary | Non-white noise |
| Response  | "Non-Gaussian" | Non-stationary | Non-white noise |

*denoised, decomposed and detrended, **raw
case, only its statistics change. It is noted that all the statistical tests are run under Matlab statistics toolbox.

V. CONCLUSION

EMD method is able to decompose random vibration signals in terms of noise, IMFs, and trend. BPF method as benchmark produces comparable results with minor differences due to different resolution bandwidth. Selection of pass band range for decomposition process indicates that the EMD method is more flexible than the BPF method. The noise and the trend can be included or excluded according to the purpose of identification process. As such, identification models in terms of model structure and model coefficient estimation method can be accordingly selected. As a recommendation, current work can be extended to the experimental modal analysis by combining the EMD method with respective modal analysis method.

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