Heavy flavour contributions to the spin structure function $g_1(x, Q^2)$ and the Bjørken sum rule

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Abstract

We discuss the order $\alpha_s^2$ corrections to the longitudinal spin structure function $g_1(x, Q^2, m^2)$ which are due to heavy flavour contributions. Here $Q$ denotes the virtuality of the photon and $m$ stands for the heavy flavour mass. Since the exact heavy quark coefficient functions are not known yet we have used the asymptotic forms which are strictly speaking only valid in the region $Q^2 \gg m^2$. However an analysis of the exact and asymptotic expressions for $F_2^{\text{NLO}}(x, Q^2, m^2)$ and $g_1^{\text{LO}}(x, Q^2, m^2)$ reveals that the asymptotic forms can be also used at smaller $Q^2$. It appears that for the region $0.01 < x \leq 0.1$ the NLO charm quark component can become twice as large as in LO. However it is still much smaller than $g_1(x, Q^2)$ due to light parton contributions. This is in contrast to the observations made for $F_2(x, Q^2)$ at small $x$. Also the charm quark contribution to the Bjørken sum rule turns out to be very small.

1 Introduction

Heavy quark production in deep inelastic electron-proton scattering proceeds via the following reaction (see Fig. 1)

$$e^- (l_1) + P(p) \rightarrow e^- (l_2) + Q(p_1) (\bar{Q}(p_1)) +'X'. $$  \hfill (1)

Where '$X'$ denotes any inclusive hadronic final state and $V$ stands for the neutral intermediate vector bosons given by $\gamma, Z$. In this paper we treat the case that $Q^2 \ll M_Z^2$ so that the process in Fig. 1 is dominated by the one photon exchange.

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mechanism only. If we also integrate over the momentum $p_1$ of the heavy (anti) quark $Q$ the unpolarised cross section is given by

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{(Q^2)^2} S_{eP} \left[ \{1 + (1 - y)^2\}F_2(x, Q^2, m^2) - y^2F_L(x, Q^2, m^2) \right]. \quad (2)$$

Here $S_{eP}$ denotes the centre of mass energy of the electron-proton system and the heavy quark component of the spin averaged structure functions are given by $F_i(x, Q^2, m^2)$ ($i = 2, L$). In the case the proton is polarised parallel ($\rightarrow$) or anti-parallel ($\leftarrow$) with respect to the spin of the incoming electron we obtain the cross section

$$\frac{d^2\sigma(\rightarrow)}{dx\,dy} - \frac{d^2\sigma(\leftarrow)}{dx\,dy} = \frac{8\pi\alpha^2}{Q^2} \left[ \{2 - y\}g_1(x, Q^2, m^2) \right], \quad (3)$$

where $g_1(x, Q^2, m^2)$ denotes the heavy quark component of the longitudinal spin structure function. Further we have defined the scaling variables

$$x = \frac{Q^2}{2pq}, \quad y = \frac{pq}{p_1}, \quad q^2 = -Q^2 < 0. \quad (4)$$

In QCD the twist two contributions to the charm component of the structure functions defined above can be written as

$$\frac{1}{n_f} \sum_{k=1}^{n_f} \epsilon_k^2 \int_x^{x_{max}} \frac{dz}{z} \left[ f_q^S \left( \frac{x}{z}, \mu^2 \right) \otimes L_{i,q}^S \left( z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) + f_g^S \left( \frac{x}{z}, \mu^2 \right) \otimes L_{i,g}^S \left( z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \right] + \epsilon_Q^2 \left[ f_q^{PS} \left( \frac{x}{z}, \mu^2 \right) \otimes H_{i,q}^{PS} \left( z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) + f_g^{PS} \left( \frac{x}{z}, \mu^2 \right) \otimes H_{i,g}^{PS} \left( z, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \right], \quad (5)$$

Figure 1: Kinematics of charm production in deep inelastic electron-proton scattering
with
\[ z_{\text{max}} = \frac{Q^2}{Q^2 + 4m^2} \]
which follows from
\[ s = \frac{(1-z)}{z} Q^2 \geq 4m^2. \] (6)

Further \( n_f \) denotes the number of light flavours and \( \mu \) stands for the mass factorization scale which is put to be equal to the renormalization scale. The charges of the light quark \( k \) and the heavy quark \( Q \) are given by \( e_k \) and \( e_Q \) respectively. Notice that \( n_f \) is determined by all quarks which are lighter than the heavy quark \( Q \) and Eq. (6) has to be understood in the fixed flavour number scheme. In Eq. (6) the gluon density is denoted by \( f_S^g(z, \mu^2) \) and the singlet (S) and non singlet (NS) combination of quark densities are defined by
\[ f_S^q(z, \mu^2) = \sum_{k=1}^{n_f} [f_k(z, \mu^2) + f_{\bar{k}}(z, \mu^2)], \] (7)
\[ f_{\text{NS}}^q(z, \mu^2) = f_k(z, \mu^2) + f_{\bar{k}}(z, \mu^2) - \frac{1}{n_f} f_S^q(z, \mu^2). \] (8)

The heavy quark coefficient functions \( L_{i,k} \) and \( H_{i,k} \) \((i = 1, 2, L; \ k = q, g)\) are distinguished according to the production mechanisms from which they originate. The quantity \( L_{i,k} \) is given by the processes where the virtual photon couples to the light quark whereas \( H_{i,k} \) originates from the reactions where the virtual photon is attached to the heavy quark. Hence these coefficient functions in Eq. (5) are multiplied by \( e_k^2 \) and \( e_Q^2 \) respectively.

Up to order \( \alpha_s^2 \) the heavy quark coefficient functions are given by the following parton subprocesses. In order \( \alpha_s \) we have the virtual photon gluon fusion mechanism given by
\[ \gamma^* + g \rightarrow Q + \bar{Q}, \] (9)
from which one obtains the heavy quark coefficient function \( H_{i,g}^{S(1)} \). Here the superscript denotes the order in the perturbation series. In next-to-leading order (NLO) one has the following processes. First one has to include the virtual gluon corrections to reaction (9) which have to be added to the gluon bremsstrahlung process
\[ \gamma^* + g \rightarrow Q + \bar{Q} + g. \] (10)
This leads to the coefficient function \( H_{i,g}^{S(2)} \). Finally we also have the subprocess
\[ \gamma^* + q(\bar{q}) \rightarrow Q + \bar{Q} + q(\bar{q}). \] (11)

The above reaction has two different production mechanisms. The first one is the Bethe-Heitler process where the virtual photon is coupled to the heavy quark
Figure 2: The function $x H_{1,g}^{\text{asymp},(2)}(x, Q^2/m_c^2, 1) (m_c = 1.5 \text{ GeV}/c)$ for $Q^2 = 10$ (lower solid line), $Q^2 = 100$ (lower dotted line), $Q^2 = 10^3$ (upper solid line), $Q^2 = 10^4$ (upper dotted line). All units are in $(\text{GeV}/c)^2$.
which leads to the coefficient function $H^{PS,(2)}_{i,q}$. Here PS means purely singlet since this function originates from the partonic process where a gluon (flavour singlet) is exchanged in the $t$-channel between the light quark $q$ and the heavy quark $Q$.

The second one is the Compton process where the virtual photon is coupled to the light quark from which one obtains $L^{NS,(2)}_{i,q}$. The expressions for the order $\alpha_s$ contribution $H^{S,(1)}_{i,g}$ can be found in [1] ($i = L, 2$; unpolarised) and [2] ($i = 1$; polarised). The exact analytic expressions for $L^{NS,(2)}_{i,q}$ are calculated in [3] ($i = L, 2$; unpolarised) and [4] ($i = 1$; polarised). The exact heavy quark coefficient functions $H^{PS,(2)}_{i,q}$ and $H^{S,(2)}_{i,g}$ have been only calculated in the unpolarised case ($i = L, 2$) (see [5]) but the exact expressions in polarised scattering ($i = 1$) are not known yet. However in [3] and [4] we could derive analytic formulae for the asymptotic heavy quark coefficient functions up to order $\alpha_s^2$ for unpolarised and polarised electroproduction respectively. The asymptotic heavy quark coefficient functions are defined by

$$H_{i,k}^{\text{asymp}}(x, Q^2, m^2, \mu^2) = \lim_{Q^2 \gg m^2} \left[ H_{i,k}^{\text{exact}}(x, Q^2, m^2, \mu^2) \right]. \quad (12)$$

These asymptotic expressions are very useful because

1. They provide us with a check on the exact calculation.

2. The exact calculation of $H^{PS,(2)}_{i,q}$ and $H^{S,(2)}_{i,g}$ ($i = 2, L$) is semi-analytic and they are only available in the form of long computer programs (see [5]). The latter show numerical instabilities at $Q^2 \gg m^2$ which are remedied by using in this region $H_{i,k}^{\text{asymp}}$.

3. Further if these asymptotic heavy quark coefficient functions are substituted in Eq. (3) it turns out that (see [3], [4])

$$R_2(x, Q^2, m^2) = \frac{F_{2,Q}^{\text{asymp}}(x, Q^2, m^2)}{F_{2,Q}^{\text{exact}}(x, Q^2, m^2)}, \quad (13)$$

tends to unity as $Q^2 > 10 m^2$ and $x < 0.01$. In the case of charm production ($Q = c$ and $m_c = 1.5$ GeV/c) this happens for $Q^2 > 20$ (GeV/c)$^2$. This implies that above a rather low value of $Q^2$ it does not make any difference whether we use the asymptotic or the exact expressions for $H_{i,k}$.

4. The asymptotic forms of the heavy quark coefficient functions also play a major role in the derivation of the variable number flavour scheme (VFNS) from the fixed flavour number sheme (FFNS) (see [4], [8]).

The asymptotic heavy quark coefficient functions get the typical form

$$H_{i,k}^{\text{asymp},(l)}(x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) \sim \alpha_s^l \sum_{n+j \leq l} a_{nj}(x) \ln^n \left( \frac{\mu^2}{m^2} \right) \ln^j \left( \frac{Q^2}{m^2} \right), \quad (14)$$
with an analogous expression for \( I_{i,k}^{\text{asymp}} \). There are two ways to determine Eq. (14).

1. Evaluate the phase space integrals and the Feynman integrals in the usual way for \( Q^2 \gg m^2 \).

2. Use operator product expansion techniques and the theorem of mass factorization.

The last method was applied in [3] and [4] to compute the asymptotic forms of the heavy quark coefficient functions for unpolarised and polarised scattering respectively.

2. Order \( \alpha_s^2 \) contributions to \( g_1(x, Q^2, m^2) \)

In this section we will analyse the charm component of the longitudinal spin structure function \( g_1 \). To compute the latter in NLO the following coefficient functions are available: \( H_{1,g}^{\text{exact},(1)} \) (Born), \( L_{1,q}^{\text{exact},(2)} \) (Compton process), \( H_{1,g}^{\text{asymp},(2)} \) (gluon bremsstrahlung), \( H_{1,q}^{\text{asymp},(2)} \) (Bethe-Heitler). Like in unpolarised scattering the gluonic coefficient functions indicated by \( H_{1,g}^{(l)} \) are the most important ones. Therefore deep inelastic electroproduction of charm quarks provide us with an excellent way to determine the gluon density in polarised scattering (see Eq. (5)). However as we will show below the charm component in the spin case is much smaller with respect to the light parton contribution to the structure function as observed for \( F_2 \) in unpolarised scattering. This is revealed by \( H_{1,g}^{\text{asymp},(2)} \) in Fig. 2 where we observe a strongly oscillatory behaviour which is in contrast to its unpolarised analogue shown in Fig. 8 of [9]. Further it satisfies the sum rule

\[
\int_0^1 dx \, H_{1,g}^{(l)}(x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) = 0 ,
\]

which holds in any scheme. Notice that the Born contribution \( H_{1,g}^{(1)} \) also satisfies the above relation and we may safely assume that Eq. (15) holds in all orders for general \( Q^2 \) and \( m^2 \). Further it turns out that the coefficient functions \( H_{1,k}^{(l)} \) \((k = q, g)\) are much smaller than their unpolarised analogues. In particular the large logarithmic terms of the type \( \ln \frac{m^2}{x} \) which are characteristic of the latter do not show up in the spin case. The absence of these corrections and the oscillatory behaviour leads to much smaller charm quark contributions to \( g_1 \) than observed for \( F_2 \) or \( F_L \). To give a better estimate of the order \( \alpha_s^2 \) corrections we have to improve the asymptotic form of the coefficient functions \( H_{1,g}^{\text{asymp},(2)} \) and \( H_{1,q}^{\text{asymp},(2)} \) a little bit. In [4] we multiplied them by \( \sqrt{1 - \frac{4m^2}{s}} \) and added to \( H_{1,g}^{\text{asymp},(2)} \) the soft gluon bremsstrahlung term which is universal (see Eqs. (5.1)-(5.3) in [4]). These approximations were tested for \( F_2^{\text{NLO}} \) and \( g_1^{\text{LO}} \) and we found a fairly good agreement with the exact results as long as \( x < 0.1 \) and \( Q^2 > 10 \) (GeV/c)^2.
Figure 3: The charm component of $x g_1(x, Q^2, m^2_c)$ at $Q^2 = 50$ (GeV/c)$^2$.
Dotted line: $x g_1^{\text{exact}}$(Born); solid line: $x g_1^{\text{approx}}$(NLO).

Choosing the parton density set in [10] (Standard Scenario) with $n_f = 3$ and $\Lambda = 200$ MeV we have plotted the charm component of $g_1^{\text{exact}}$(Born) and $g_1^{\text{approx}}$(NLO) at $Q^2 = 50$ (GeV/c)$^2$ in Fig. 3. From this figure we infer that the charm component becomes maximal in the region $0.01 < x < 0.1$. Further we observe that in NLO the result becomes twice as large than in LO. We also made a comparison in Fig. 4 with the quantity $g_1^{\text{light}}$(NLO) which is due to light partons ($u, d, s$ and $g$) only. Here we see that the charm component amounts to about 4% of $g_1^{\text{light}}$(NLO). This is in contrast to the observation made for the structure function $F_2$ in unpolarised scattering (see [11], [12]). In the latter case the charm component becomes large at very small $x$ (i.e. $x = 10^{-4}$) and it amounts to about 25% with respect to the total structure function. In [4] we also studied the spin structure functions plotted at $Q^2 = 10$ (GeV/c)$^2$ and $Q^2 = 100$ (GeV/c)$^2$. These plots show the same features as the ones made for $Q^2 = 50$ (GeV/c)$^2$ so that our conclusions are unaltered.
Figure 4: The charm component and the light parton contribution to $xg_1(x, Q^2, m_c^2)$ at $Q^2 = 50$ (GeV/c)$^2$. Dotted line: $xg_1^{\text{approx}}$(charm, NLO); solid line: $xg_1^\text{light}$(NLO).

3 Order $\alpha_s^2$ contributions to the Bjørken sum rule

Since we know the exact result for $L_{1,q}^{\text{NS},(2)}$ \cite{1} we can compute the order $\alpha_s^2$ correction to the Bjørken sum rule due to heavy flavour contributions. The latter is given by

$$\int_0^1 dx \left[ g_1^P(x, Q^2, m_c^2) - g_1^N(x, Q^2, m_c^2) \right] = \frac{1}{6} \left| \frac{G_A}{G_V} \right| \left( L_{1,q}^{\text{NS}} \right)^{(1)},$$

(16)

where the superscript (1) denotes the first moment (Mellin transform). In order $\alpha_s^2$ we have the following properties

$$Q^2 \ll m^2 \quad \Rightarrow \quad \left( L_{1,q}^{\text{NS},(2)} \right)^{(1)} \sim \frac{Q^2}{m^2},$$

(17)

which shows the decoupling of the heavy quark when $Q^2 \ll m^2$.

$$Q^2 \gg m^2 \quad \Rightarrow \quad \left( L_{1,q}^{\text{NS},(2)} \right)^{(1)} = C_F T_f \left( \frac{\alpha_s(n_f, \mu^2)}{4\pi} \right)^2 \left[ -4 \ln\left( \frac{Q^2}{m^2} \right) + 8 \right].$$

(18)

The light parton contribution to the Bjørken sum rule is given by

$$\int_0^1 dx \left[ g_1^P(x, Q^2) - g_1^N(x, Q^2) \right] = \frac{1}{6} \left| \frac{G_A}{G_V} \right| \left( C_{1,q}^{\text{NS}} \right)^{(1)},$$

(19)
where \( C_{1,q}^{\text{NS}} \) denotes the massless quark coefficient function. The first moment of the latter quantity has been calculated up to order \( \alpha_s^3 \) in [13] and the order \( \alpha_s^4 \) term has been estimated too (see e.g. [14]). It turns out that the charm contribution to the Bjorken sum rule is even smaller than the estimated order \( \alpha_s^4 \) correction (see [15]). The contributions coming from the bottom and top quark are even smaller which can be attributed to the decoupling of heavy quarks shown in Eq. (17). Finally we have the relation

\[
C_{1,q}^{\text{NS}}(n_f, x, \frac{Q^2}{\mu^2}) + \lim_{Q^2 \gg m^2} L_{1,q}^{\text{NS}}(x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2}) = C_{1,q}^{\text{NS}}(n_f + 1, x, \frac{Q^2}{\mu^2}),
\]

so that for \( Q^2 \gg m^2 \) the large logarithm \( \ln \frac{Q^2}{m^2} \) in the heavy quark coefficient function \( L_{1,q}^{\text{NS}} \) (Eq. (19)) can be absorbed in the light quark coefficient function \( C_{1,q}^{\text{NS}} \). Hence the number of light flavours in the running coupling constant and in \( C_{1,q}^{\text{NS}} \) on the righthand side of Eq. (20) will be enhanced by one unit.

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