To the dynamically loaded reinforced-concrete elements’ calculation in the absence of adhesion between concrete and reinforcement

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Abstract. This work is devoted to the methodology for calculating the compressed reinforced concrete elements under dynamic loading, which takes into account the lack of adhesion of tensile reinforcement to concrete. As experiments have shown, the lack of adhesion between concrete and reinforcement leads not only to a decrease in the bearing capacity, but also to a qualitative change in the strain diagrams’ shape in the sections. The calculations according to the current regulatory documents do not take into account the fact that the reinforcement does not adhere to concrete, as a result, it is difficult to determine the actual bearing capacity of the element. The dynamic impact of loads adds complexity to these types of calculations. This approach makes it possible to perform the calculation in the elastic, elastoplastic and plastic stages of work, while taking into account the absence of reinforcement adhesion with concrete in determining the height of the concrete compressed zone.

Introduction
Concrete and reinforced concrete structures are exposed not only to operational, but also to various aggressive environments. Power loads and the effects of aggressive environments, the real properties of materials and, often, design flaws can lead to the appearance and accumulation of geometric imperfections in the system, elements, their connections, joints, as well as damage to sections with the reinforcement exposure and corrosion. This is due to the fact that due to age-related wear and the inevitable damage accumulation to concrete and reinforcement following a long stay in a real, especially in an aggressive environment, the resource of their strength resistance decreases [2, 0, 0, 0, 0].

At the present stage of scientific activity, to study the actual stress-strain state of elements, it is not enough to know the influence of only power loads, it is necessary to carry out the experimental and theoretical studies of reinforced concrete structures with no force [0, 0, 0, 0, 0].

The impact of aggressive environments leads not only to a change in the physical and mechanical materials’ properties, but also to a reduction in the area of concrete and reinforcement in the section of the element, along with these obvious features that should be taken into account in the calculation, a criterion for the lack of adhesion between reinforcement and concrete arises, which is an important parameter when calculating the compressed elements’ bearing capacity.
Therefore, there is a need to create the methods for dynamic loading of structures that take into account corrosion damage to materials, the lack of adhesion between reinforcement and concrete, describe the specificity and nature of dynamic processes.

**Main part**

The calculation of the reinforced concrete structures for dynamic loads is carried out according to the first group of limit states (limit state method) and is performed in three stages of work [0] element elastic, elastoplastic and plastic stages, while the determining criterion for the element destruction is the complete opening angle in the plastic hinge.

When calculating the reinforced concrete structures for dynamic effects, various laws of changes in loads over time can be used. The most closely corresponding to the nature of the dynamic load application on the elements is the exponential function (Figure 1).

![Figure 1](image)

**Figure 1.** The dynamic change law increasing load in time

The change in dynamic load over time is represented by an exponential function, having the form:

\[
N(t) = N_d e^{\frac{1}{E} \left( t - t_{ef} \right)^2}
\]  

where \(E\) – is the coefficient reflecting the intensity of the increase in dynamic load; \(t_{ef}\) – is the effective dynamic load exposure time; \(N_d\) – is the dynamic load on an element.

The equation for the column motion in the elastic stage has the form:

\[
-\frac{\partial^2 M_1}{\partial z^2} + N(t) \frac{\partial^2 y_1}{\partial z^2} + m \frac{\partial^2 y_1}{\partial t^2} = 0.
\]  

where \(N(t)\) – is the longitudinal dynamic load; \(y_1\) – is the column deflection; \(m\) – is the linear column mass; \(z\) – is the section coordinate along the column length.

Equation solution (2) can be represented as sums of deflection functions from the static and dynamic loads application:

\[
y_1(z; t) = y_{st} + y_d.
\]  

Finding the deflection in the sections of the column from the external load action can be represented as follows:
\[ y_1(z,t) = \frac{N_{e\eta} l}{2B_1} (lz - z^2) e^{\frac{1}{E} \left(\frac{t-t_{d1}}{B}\right)^2} + \sum_{n=1,3,5} T_{nl}(t) \sin \frac{\pi n}{l} z. \]  

(4)

where \( T_{nl}(t) \) – is the dynamism function obtained using the Bubnov-Galerkin method in the form:

\[ T_{nl}(t) = \frac{N_d e_o \eta l^2}{8B_1} e^{\frac{1}{E} \left(\frac{t-t_{d1}}{B}\right)^2} (\cos \omega_{nl} t - \frac{2t_{d1}}{EB^2 \omega_{nl}} \sin \omega_{nl} t). \]  

(5)

The bending moment in sections of the column elastic stage is determined by the formula:

\[ M(z,t) = Ne_o \eta e^{\frac{1}{E} \left(\frac{t-t_{d1}}{B}\right)^2} + B_1 \frac{\pi^2}{l^2} \sum_{n=1,3,5} n^2 T_{nl}(t) \sin \frac{\pi n}{l} z. \]  

(6)

Time \( t_1 \) normal cracking is determined from the condition:

\[ M_1(z,t_1) = Mcrc,e. \]  

(7)

To simplify the time \( t_1 \) determination it is possible to use the equation (8), where the time \( t_1 \) is determined by iterative selection.

\[ (t_1 - t_{d1})^2 = -\frac{1}{E} \ln\left(\frac{Mcrc,e}{N_d e_o \eta} + 13.5 \cdot e^{-\frac{t_{d1}}{EB}} \cdot \cos \omega_{nl} t_1 \right) \]  

(8)

The column motion equation in the elastoplastic stage has the form:

\[ -\frac{\partial^2 M_2}{\partial z^2} + N(t) \frac{\partial^2 y_1}{\partial z^2} + m \frac{\partial^2 y_2}{\partial t^2} - \frac{\partial^2 M_1}{\partial z^2} = 0. \]  

(9)

The deflection and bending moment in the column sections is determined by the formulas:

\[ y_2(z,t) = \sum_{n=1,3,5} T_{n2}(t) \sin \frac{\pi n}{l} z. \]  

(10)

\[ M_2(z,t) = B_2 \frac{\pi^2}{l^2} \sum_{n=1,3,5} n^2 T_{n2}(t) \sin \frac{\pi n}{l} z. \]  

(11)

The dynamic function in the column elastoplastic stage with the normal cracks’ manifestation in the stretched zone of concrete is determined by the formula:

\[ T_{n2}(t) = C_n \sin \omega_{n2} (t - t_1) + D_n \cos \omega_{n2} (t - t_1) \]  

(12)

Time \( t_2 \) of the column plastic stage beginning is determined from the condition:

\[ M_{crc,e} + N_d e_o \eta e^{\frac{1}{E} \left(\frac{t_2-t_{d1}}{B}\right)^2} + B_2 \frac{\pi^2}{l^2} T_2(t_2) = M_u. \]  

(13)

To determine the time \( t_2 \) it is possible to use the equation (14)
\[(t_2 - t_0)^3 = -EB^2 \ln \left( \frac{Mu - Mcrc, e}{N_d e_0 e} \right) - B_2 \frac{\pi^2}{l^3} \left[ C_{n_2} \sin \alpha_{n_2} (t_2 - t_0) + D \cos \alpha_{n_2} (t_2 - t_0) \right] \] (14)

The plastic stage of the column is formed under the condition:

\[M(z, t_m) > Mu. \] (15)

\[M(z, t_m) = Mcrc, e + B_2 \frac{\pi^2}{l^1} \sum_{n=1,3}^5 n^2 T_n z(t_m) \sin \frac{\pi n}{l} z. \] (16)

After the plasticity hinge formation, the column is two separate blocks connected by the plastic hinge.

\[ \text{Figure 2. The design scheme of an eccentrically compressed column in the plastic stage of work} \]

The column motion equation in the plastic stage is:

\[ \ddot{\varphi}(t) - \lambda^2 \varphi(t) = -M_{wo} \frac{24}{ml^3} \] (17)

where \(M_{wo} = N_d e\) – is the bending moment relative to the external load application point; \(\lambda\) is found by the formula:

\[ \lambda = \sqrt{\frac{24N_d e_0 e}{ml^3}} \] (18)

The equation solution (17) has the form:

\[ \varphi(t_m)_{\text{max}} = \frac{\dot{\varphi}_0}{\lambda} \cdot sh \lambda (t_m - t_2) + \frac{M_{wo}}{N_d e} \cdot ch \lambda (t_m - t_2) - \frac{M_{wo}}{N_d e} \] (19)
where \( e = e_0 n + h / 2 - a \); \( \dot{\phi}_0 \) - is the initial angular velocity, determined by the formula:
\[
\dot{\phi}_0 = - \frac{N_0 e_0 n l (t_2 - t_{ef})}{1.5 E_B B^2} e^{\frac{T}{\pi l}} \sum_{n=1,3,5} \frac{1 - T_{n2}(t_2)}{\pi l}.
\]

Time \( t_m \) reach the maximum angle of rotation \( \varphi_{max} \) and is determined by the formula:
\[
t_m = t_2 + \frac{1}{2\lambda} \ln \frac{1 + T}{1 - T}.
\]

The full opening angle in the plasticity hinge is determined by the formula:
\[
\psi_{max} = 2\varphi(t_m)_{max}
\]

The maximum opening angle in the plasticity hinge is determined by the formula:
\[
\psi_u = \left( \frac{\varepsilon_{sw} - \frac{k_{cd} R}{E_i (h_0 - x_d)}}{x_d} \right) l_{pl}
\]

The column strength is ensured with the condition:
\[
\psi_{max} \leq \psi_u.
\]

To determine the actual height of the compressed zone of an eccentrically compressed element, necessary for finding by the formula ((23) the maximum opening angle in the plasticity hinge of a corrosion - damaged section, it is necessary to take into account the lack of adhesion between the stretched reinforcement and concrete. On the basis of the obtained experimental data, when comparing the strain diagrams of eccentrically compressed prototypes with secured and impaired adhesion, a different nature of the inclination angle formation of the lines forming the strain diagram is revealed (Figure 3).

**Figure 3.** The deformations plots of the normal section in the experimental columns with unbroken (a) and broken (b) adhesion between reinforcement and concrete

So, for the elements with traction provided, the inclination angle of the line corresponding to the compressed zone is less than the inclination angle of the line corresponding to the extended zone. For the elements with impaired adhesion, the inclination angles of the lines forming the compressed and
stretched zone on the strain diagram are inverse, that is, the inclination angle of the line in the compressed zone is always greater than the inclination angle of the line in the stretched zone.

Based on the data of Figure 3, it is possible to conclude that reinforced concrete eccentrically compressed elements with no adhesion between reinforcement and concrete do not adequately correspond to the flat sections’ hypothesis concept and, as a result, to the real analysis of the stress-strain state according to generally accepted methods. This fact indicates a certain redistribution of stresses and deformations in the cross section of the element with a broken adhesion between reinforcement and concrete.

When conducting the experimental studies in [0] it is fixed that the diagrams of deformations and, accordingly, the stresses in the compressed concrete zone are non-linear and are S-shaped. This nonlinearity is caused by the feature of the element when the reinforcement is unevenly bonded to concrete (Figure 4).

The reinforced concrete elements’ calculation is based on the plane section deformation hypothesis, in order to preserve the existing methods for analyzing the stress-strain state, taking into account the lack of adhesion between reinforcement and concrete, it is necessary to adapt the existing calculation methods. One of the basics of adapting the calculation model, taking into account the lack of adhesion between reinforcement and concrete, is to change the method for determining the height of the compressed zone based on finding the average height of the compressed zone $x_m$.

Based on this stress-strain state of the block, it is possible to assume that the neighboring blocks are rotated around the zero point on the average deformations’ diagram. The position of this point is the distance $x_m$ to the upper face of the compressed zone and is determined by the average deformation of concrete $\varepsilon_{bm}$ and fittings $\varepsilon_{bs}$.

4 shows the normal section deformations’ plots of an eccentrically compressed reinforced concrete element with the actual $x_a$, middle $x_m$ and trapezoid $x=\beta x_t$ height of the compressed zone.

To simplify the calculations, the actual S–shaped diagram of a compressed zone with a height $x_f$ is replaced by trapezoid with height $x=\beta x_t$.

$$A_t = A_a$$

(25)

$A_t$ – defines the deformations’ trapezoidal diagram area;

$A_a$ – is the actual triangular strain diagram area;
Find the area of the trapezoidal deformation plot \( A_t \), can be expressed by \( \varepsilon_b \), \( x_t \) and \( \beta \) and the equation can be obtained:

\[
[\varepsilon_b + (1 - \beta)\varepsilon_b] \beta x_t / 2 = A_t
\]  

(26)

Expressing from the equation \( \beta \), we get

\[
\beta = 1 - \sqrt{1 - (2A_t / \varepsilon_b x_t)}
\]  

(27)

Wherein \( A_t \) – the actual triangular plot area is determined by the experimental tests’ results.

Based on the experimental studies \([0]\] the dependence for determining the coefficient \( \beta \) is proposed as:

\[
\beta = \beta^* - \left[1 - (\eta_{crc})^{\phi / \lambda}\right] \quad \text{npu } \eta_{crc} > 1.0
\]  

where \( \eta_{crc} = l_{crc} / 2h \) – is the relative distance between cracks; \( \beta^* \) - is the coefficient value \( \beta \), appropriate to \( \eta_{crc}=1.0 \) and is found from the expression:

\[
\beta^* = (0.264 + 4.04\mu)(0.883 + 0.013\alpha)[(1 - 0.25(\mu' / \mu)^{1.5}] \leq 1.0
\]  

(29)

\( \mu' \) - is the percentage of reinforcement with compressed reinforcement; \( \mu \) – is the percentage of reinforcement by stretched reinforcement; \( \alpha = E_s / E_b \).

The plot completeness coefficient of the compressed fiber concrete \( \omega_m \), which is then needed to determine the average strain \( \varepsilon_{bm} \) can be represented as:

\[
\omega_m = \omega_m^*(\eta_{crc})^{-0.22} \quad \text{npu } \eta_{crc} > 1.0
\]  

(33)

\( \omega_m^* \) - is the coefficient value \( \omega \), appropriate to \( \eta_{crc}=1.0 \) and can be found from the expression:

\[
\omega_m^* = (1.4 \sqrt{\mu} - 0.94\mu + 0.02) \sqrt{\alpha} \leq 1.0
\]  

(34)

\[
\gamma_{\omega} = \begin{cases} 
1 - (\eta_{crc} - 0.2)(1 - 2.7\omega_m^*) / 0.8 & \text{npu } 0.27\omega_m^* \leq 1.0 \\
1 + (\eta_{crc} - 0.2)(2.7\omega_m^* - 1) / 0.8 & \text{npu } 0.27\omega_m^* > 1.0 
\end{cases}
\]  

(35)

The coefficient value \( \xi_m \) can be found through the average deformations’ values of concrete and reinforcement:

\[
\xi_m = \varepsilon_{bm} / (\varepsilon_{bm} + \varepsilon_{sm})
\]  

(37)

where \( \varepsilon_{bm} \) – is the average in two blocks of the extreme concrete fiber deformation; \( \varepsilon_{sm} \) - medium in two deformation blocks of tensile reinforcement

In this case, the concrete average deformation is:

\[
\varepsilon_{bm} = \omega_b \varepsilon_b
\]  

(38)
where \( \omega_m \) – is the coefficient of the plot completeness in the compressed zone along the length of the blocks and is determined by the formula (33).

The average deformation of the tensile reinforcement we find and accept as:

\[
\varepsilon_{sm} = \varepsilon_s
\]

(39)

It is possible to find the average deformation of concrete and reinforcement taking into account (38), (39) and the transformations:

\[
\varepsilon_{lm} = \omega_s \sigma_b / E_p
\]

(40)

\[
\varepsilon_m = \sigma_b / \mu
\]

(41)

\( n'_s = (\omega_s \sigma_b) \leq \mu \sigma_{sc} / \sigma_b \) – is the coefficient taking into account the presence of compressed reinforcement and \( \omega_s \) is taken equal to 0.75; \( \sigma_{sc} \) – is the stress in the compressed reinforcement corresponding to ultimate concrete strain \( \varepsilon_b = 0.002 \), i.e. 400MPa.

Substituting the resulting expressions \( \varepsilon_{lm} \) and \( \varepsilon_m \) in (37) and cancelling out by \( \sigma_b \) we get:

\[
\tilde{\varepsilon}_m = \frac{V_r \mu \alpha \omega_m}{V_r \mu \alpha \omega_m + (V_b \omega_b \tilde{\varepsilon}_m + n'_s) - V_b \beta \mu},
\]

(42)

After the conversion (42) we get:

\[
\tilde{\varepsilon}_m = -D/T + \sqrt{\frac{(D/T)^2 + K/T}{}}
\]

(43)

where, \( D = V_r \mu (\omega_m \alpha + n'_s) / 2 \); \( K = V_b \omega_m \beta \).

Formula (43) allows to determine the relative height of the conditional compressed zone \( x_m = \tilde{\varepsilon}_m h_0 \) and as a result, the compressed zone height in the section with a crack \( x = x_m \beta \) taking into account the lack of adhesion between reinforcement and concrete.

The compressed zone \( x_a \) actual height calculation is performed by the formula:

\[
x_a = \beta_0 \tilde{\varepsilon}_m h_0
\]

(44)

\( \beta_0 \) – is the coefficient for the transition from the average height of the compressed zone \( x_m \) to the actual height of the compressed zone \( x_a \).

To determine the coefficient \( \beta_0 \), considering the S—figurative nature of the strain diagram, it can be taken into consideration that the area of the real curved diagram \( A_0 = 0.5 \varepsilon_b \beta_0 x_m \) after substitution \( A_0 \) in (27) we get:

\[
\beta = 1 - \sqrt{1 - ( \varepsilon_b \beta_0 \varepsilon_m / \varepsilon_b x_m )}
\]

(45)

Expressing \( \beta_0 \), we will have:

\[
\beta_0 = 1 - (1 - \beta)^2
\]

(46)

\( \beta \) – is a conversion factor from the conditional height of the compressed zone \( x_m \) to the height of the compressed zone \( x \) (Figure 4) is determined by (28).

After finding (44) the actual height of the compressed zone, we determine the limiting opening angle in the plasticity hinge by (23). And then we check the condition of the column strength by (24).

**Summary**

Based on the experimental and theoretical studies, a method for calculating a dynamically loaded corrosion-damaged compressed reinforced concrete elements is proposed, taking into account the lack
of adhesion between reinforcement and concrete. Thus, it becomes possible to perform a correct analysis of the actual element’s operation and, accordingly, find the value of the limit theoretical load.

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