Single Cooper pair tunneling induced by non-classical microwaves

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Abstract

A mesoscopic Josephson junction interacting with a mode of non-classical microwaves with frequency $\omega$ is considered. Squeezing of the electromagnetic field drastically affects the dynamics of Cooper tunneling. In particular, Bloch steps can be observed even when the microwaves are in the squeezed vacuum state with zero average amplitude of the field $\langle E(t) \rangle = 0$. The interval between these steps is double in size in comparison to the conventional Bloch steps.

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There has been a lot of work on single charge tunneling in small junctions with current bias. The Bloch oscillations of the voltage with fundamental frequency $f = I/2e$ has been observed. This phenomenon is dual to the traditional AC Josephson effect. The same junctions biased with a current $I_{DC} + I_{AC}\sin(2\pi ft)$ produce a voltage that has a DC component when $I_{DC} = 2enf$ (so called Bloch steps which are dual to well known Shapiro steps).

Non-classical electromagnetic fields (squeezed states) have been studied extensively in the last few years. They are sinusoidal signals in their average value of $<E>, <B>$ but they differ from each other in the quantum fluctuations $\Delta E, \Delta B$. At optical frequencies they are experimentally available in many laboratories, but they have also been produced at microwave frequencies. In recent work the AC Josephson effect has been studied in the presence of non-classical electromagnetic fields. In this paper we study the effect of quantum electromagnetic fields on a dual phenomenon - the Bloch oscillations.

We consider a small Josephson junction in a resonant cavity connected to a quantum source of microwaves. A coupling of the junction to the microwaves can be achieved, for example, by extending the electrodes of the junction in the direction of the electric field (see inset of Fig. 1). The field polarizes a charge

$$\hat{Q}_p = Cd\hat{E}$$

at the electrodes of the junction (here $C$ is the total capacitance between the electrodes and $d$ is a coefficient of proportionality of the order of the length $L$ of the electrodes). The Hamiltonian of the Josephson junction has the form,

$$H_J = \frac{(\hat{Q} - \hat{Q}_p)^2}{2C} - E_J \cos \hat{\phi},$$

where the charge $\hat{Q} = 2ne$ and the Josephson phase difference $\hat{\phi}$ across the junction are canonically conjugated operators, $[\hat{Q}, \hat{\phi}] = -2ei$.

We assume that the quantum source excites a single mode of the microwaves with the frequency $\omega$. In order to test a dynamics of the junction, it is instructive to impose in addition a classical time-dependent component $E_{cl}(t)$ so that the total electric field is:
\[ \hat{E} = E_{\text{cl}}(t) + i E_\xi(a - a^\dagger). \] (3)

Here \( a^\dagger, a \) are creation and annihilation operators for the external non-classical electromagnetic field and \( E_\xi \) is a constant of the order of \( (\hbar \omega/\epsilon_0 V)^{1/2} \), \( V \) being the volume of the resonator. Note that in Eqs. (1), (2) we have neglected the dynamical redistribution of the charge on the junction (characterized by relaxation time \( \tau_{\text{rel}} \)) and used adiabatic description of the Josephson tunneling. For this reason, our model is valid at low frequencies \( \omega \ll \min(\tau_{\text{rel}}^{-1}, \Delta/\hbar) \), \( \Delta \) being the superconducting gap.

The eigenvalues \( E_n(\hat{Q}_p) \) and eigenfunctions \( \Psi_{n,\hat{Q}_p}(\phi) \) of the Hamiltonian \( H_J \) can be readily found in the basis of states, in which the operator \( \hat{Q}_p \) is diagonal. The energies \( E_n(\hat{Q}_p) \) are \( 2e \)-periodic functions of the polarization charge \( \hat{Q}_p \) (which is analogous to a quasimomentum in the standard band theory). We will assume that both classical \( (E_{\text{cl}}(t)) \) and quantum \( (i E_\xi(a - a^\dagger)) \) parts of the electromagnetic field change slowly enough in time (see the condition (3) below). For this reason, we will consider only the lowest energy band \( E_0(\hat{Q}_p) \). The Hamiltonian of the system acquires a simple form,

\[ H = E_0(\hat{Q}_p) + \hbar \omega(a^\dagger a + 1/2). \] (4)

For strong Josephson coupling \( E_J \gg E_C \equiv e^2/2C \) a dispersion relation for \( E_0(\hat{Q}_p) \) has the form

\[ E_0(\hat{Q}_p) = \frac{\delta}{2} - \frac{w_0}{2} \cos \left( 2\pi \frac{\hat{Q}_p}{2e} \right), \] (5)

where \( w_0 = 32 \cdot 2^{-1/4} \pi^{-1/2} E_C^{1/4} E_J^{3/4} \exp[-(8E_J/E_C)^{1/2}] \) is the widths of the band and the energy \( \delta = (8E_C E_J)^{1/2} \) corresponds to the gap between the lowest and the next energy bands.

The lowest band approximation requires that both classical and non-classical parts of the polarization charge \( \hat{Q}_p \) change slowly on the time-scale determined by the gap between the two lowest bands,

\[ \langle (d\hat{Q}_p/dt)^2 \rangle^{1/2}/e \ll \delta/\hbar \] (6)
(the average is taken over the state the electromagnetic field, see below). In what follows we will consider the simplest case when the Josephson junction perturbs the electromagnetic mode weakly, \( w_0 \ll \hbar \omega \). Clearly, this condition can be matched with the condition of adiabaticity (3) at least in the limit of strong Josephson coupling (when \( w_0 \ll \delta \)).

In this limit the operator \( \hat{V} = dE_0(\hat{Q}_p)/d\hat{Q}_p \) of the voltage across the junction is a harmonic function of \( \hat{Q}_p \) (generally, it can be presented as a sum over the Fourier harmonics). The voltage \( V(t) \) is given by the expectation value of the operator \( \hat{V}(t) \) in the pure quantum state \( |\Psi\rangle \) of the electromagnetic mode,

\[
V(t) = V_0 \Im \{ e^{i q_{cl}(t)} \langle \Psi | \exp[\xi(a^\dagger e^{i\omega t} - ae^{-i\omega t})]|\Psi\rangle \},
\]

where \( V_0 = \pi w_0/2e \), \( q_{cl}(t) = \pi Q_{cl}(t)/e = \pi dE_{cl}(t)/(e/C) \) and \( \xi = \pi dE_\xi/(e/C) \). We use the interaction representation in which the time evolution of the operators is governed by a free field Hamiltonian (second term in Eq.(4)).

Specifically, we will consider squeezed states, \( |\Psi\rangle = \hat{D}\hat{S}|0\rangle \), where squeezing (\( \hat{S} \)) and displacement (\( \hat{D} \)) operators are given by

\[
\hat{S} = \exp\left(\frac{r}{4} e^{-i\gamma} a^2 - \frac{r}{4} e^{i\gamma} a^\dagger a^2\right),
\]

\[
\hat{D} = \exp(\alpha a^\dagger - \alpha^* a).
\]

Using the transformation properties of \( \hat{D} \) and \( \hat{S} \) (namely, \( \hat{D}^{-1} f(a, a^\dagger) \hat{D} = f(a + \alpha, a^\dagger + \alpha^*) \)) and analogous relation for \( \hat{S} \) together with the relation \( \langle e^B \rangle = \exp(\langle B^2 \rangle/2) \) for an arbitrary linear combination \( B \) of Bose operators, we obtain from Eq.(7),

\[
V(t) = V_0 \sin\{q_{cl}(t) + 2\xi A \sin(\omega t - \chi)\}
\]

\[
\times \exp\{-\langle \xi^2 \rangle/2[\cosh r + \sinh r \cos(2\omega t - \gamma)]\}
\]

where \( A \) and \( \chi \) are the amplitude and the phase of the squeezed state, \( \alpha = Ae^{i\chi} \).

We consider now several applications of general formula (10). First we assume that the classical component of the field increases linearly in time, \( q_{cl}(t) = \omega_{cl}t \), which corresponds to
a bias of the junction by the DC current $I = 2e\omega_{cl}/2\pi$. One can see from Eq. (10) that the voltage has a DC component $\bar{V}$ if $\omega_{cl} = n\omega$ with integer $n$. This corresponds to a transfer of $n$ Cooper pairs through the junction during the period $T = 2\pi/\omega$ of the electromagnetic field oscillations. One can say that the transfer of Cooper pairs is phase-locked by oscillations of the electromagnetic field.

Another way of interpretation is the following. In the absence of electromagnetic field ($\hat{Q}_p = 0$ in Eq. (4)) the states of the Hamiltonian (2) which differ by an integer number $k$ of the flux quanta ($\hat{\phi} \rightarrow \hat{\phi} + 2\pi k$) are degenerate. In the presence of the classical field the part $Q_p^{cl} = 2e\xi(t)/2\pi = e\omega_{cl}t/\pi$ of the Hamiltonian (3) can be transformed into an additional term $-\hbar\omega_{cl}\hat{\phi}/2\pi$ of the Hamiltonian by a gauge transformation. The states of the Hamiltonian are no more degenerate: they form instead an equidistant ladder with energy spacing $\hbar\omega_{cl}$. The interaction with quantum-coherent component of electromagnetic field of frequency $\omega$ effectively restores the degeneracy of the states with different number $k$ of the flux quanta whenever the energy $n\hbar\omega$ matches with the level spacing $\hbar\omega_{cl}$. A dissipationless tunneling of the Josephson phase between these states gives rise to a DC voltage $\bar{V}$ across the junction.

The voltage $\bar{V}$ can be computed numerically by integrating Eq. (10) over the period $T$. The DC voltage is a function of the phases $\chi$ and $\gamma$. The maximum $\bar{V}_{max}$ of the voltage with respect to these phases determines the maximum amplitude of the Bloch steps. For a pure coherent state ($r = 0$) we obtain,

$$\bar{V}_{max}^{(n)} = V_0 \exp(-\xi^2/2)J_n(2\xi A),$$

(11)

for $\omega_{cl} = n\omega$ and $\bar{V} = 0$ otherwise (here $J_n(x)$ are the Bessel functions). This expression differs from the result for a classical field only by a constant factor $\exp(-\xi^2/2)$ which describes the effect of quantum fluctuations.

The situation is different for squeezed states ($r \neq 0$). Figure 1 shows the voltages $\bar{V}_{max}^{(n)}$ for the first four steps ($n = 1, 2, 3, 4$) as functions of the coherent amplitude $A$ of a squeezed state. One sees that the even steps $n = 2, 4$ exist even when the amplitude $A$ of the field is
zero. Indeed, for a squeezed vacuum \((A = 0)\) Eq. (10) gives

\[
\bar{V}_{\text{max}}^{(2m)} = V_0 \exp \left( -\frac{\xi^2}{2} \cosh r \right) I_m \left( \frac{\xi^2}{2} \sinh r \right),
\]

(12)

for \(\omega_{cl} = 2m\omega\) and \(\bar{V} = 0\) otherwise (here \(I_m(x)\) are the Bessel functions of imaginary argument). This doubling of the interval between the steps is related to the fact that squeezed vacua are superpositions of eigenstates with even number of photons. The amplitudes of the first four steps \((n = 2m = 2, 4, 6, 8)\) as functions of the squeezing parameter \(r\) are shown in Fig. 2.

Finally we consider the interaction of a classical AC field \(q_{cl}(t) = A_{cl} \sin(\Omega t)\) with quantum field in a squeezed vacuum state. The voltage across the junction has a DC component whenever the frequencies \(\Omega\) and \(2\omega\) are commensurable, (i.e. \(\Omega/2\omega = p/q\), where \(p\) and \(q\) are coprime integers). The amplitude of the Bloch step is given by:

\[
\bar{V}_{\text{max}}^{(p,q)} = V_0 \exp \left( -\frac{\xi^2}{2} \cosh r \right) \sum_m J_{mq}(A_{cl}) I_{mp} \left( \frac{\xi^2}{2} \sinh r \right).
\]

(13)

We conclude with a comment on the relation between the present work and previous work on the electromagnetic environment (see Ch. 2 of Part 3 of Ref. 1 for a review). The results of previous papers were obtained for an environment in thermal equilibrium. In this work the external electromagnetic field is assumed to be in a particular quantum state; and the results depend on this state. The objective here is to use a carefully prepared quantum state of the electromagnetic field to influence the dynamics of the junction.

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7 Note that the definition of squeezed states here is different from that of Ref.6; both definitions are equivalent, see e.g. part 1 of Ref.4.
FIGURES

FIG. 1. Maximum amplitudes of the Bloch steps \( n = 1, 2, 3, 4 \) as functions of the amplitude \( A \) of the squeezed state. We chose the parameters \( \xi = 1, r = 1 \). Inset: layout of the system.

FIG. 2. Maximum amplitudes of the Bloch steps \( n = 2, 4, 6, 8 \) as functions of the squeezing parameter for a squeezed vacuum state. We chose \( \xi = 1 \).