The lepton flavor violating decays $Z \to l_i l_j$

in the simplest little Higgs model

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Abstract

In the simplest little Higgs model the new flavor-changing interactions between heavy neutrinos and the Standard Model leptons can generate contributions to some lepton flavor violating decays of $Z$-boson at one-loop level, such as $Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$, and $Z \to \mu^\pm e^\mp$. We examine the decay modes, and find that the branching ratios can reach $10^{-7}$ for the three decays, which should be accessible at the GigaZ option of the ILC.

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I. INTRODUCTION

Little Higgs theory [1] has been proposed as an interesting solution to the hierarchy problem. So far various realizations of the little Higgs symmetry structure have been proposed [2–5], which can be categorized generally into two classes [6]. One class use the product group, represented by the littlest Higgs model [3], in which the SM $SU(2)_L$ gauge group is from the diagonal breaking of two (or more) gauge groups. The other class use the simple group, represented by the simplest little Higgs model (SLHM) [4], in which a single larger gauge group is broken down to the SM $SU(2)_L$. The flavor sector of little Higgs models based on product groups, notably the littlest Higgs model with T-parity (LHT) [5], has been extensively studied [7]. Recently, some attentions have been paid to the flavor sector of SLHM [8–10].

The lepton flavor violating (LFV) decays of $Z$-boson can be a sensitive probe for new physics because they are extremely suppressed in the SM but can be greatly enhanced in new physics models [11–13]. The experimental limits obtained at LEP [14] are

$$BR(Z \rightarrow \tau^\pm \mu^\mp) < 1.2 \times 10^{-5},$$

$$BR(Z \rightarrow \tau^\pm e^\mp) < 9.8 \times 10^{-6},$$

$$BR(Z \rightarrow \mu^\pm e^\mp) < 1.7 \times 10^{-6}.$$  \hfill (1)

The next generation $Z$ factory can be realized in the Giga$Z$ option of the International Linear Collider (ILC) [15]. About $2 \times 10^9$ $Z$ events can be generated in an operational year of $10^7$ s of Giga$Z$. Thus the expected sensitivity of Giga$Z$ to the LFV decays of $Z$-boson could reach [16]

$$BR(Z \rightarrow \tau^\pm \mu^\mp) \sim \kappa \times 2.2 \times 10^{-8},$$

$$BR(Z \rightarrow \tau^\pm e^\mp) \sim \kappa \times 6.5 \times 10^{-8},$$

$$BR(Z \rightarrow \mu^\pm e^\mp) \sim 2.0 \times 10^{-9}.$$  \hfill (2)

with the factor $\kappa$ ranging from 0.2 to 1.0. Therefore, Giga$Z$ can offer an important opportunity to probe the new physics via the LFV decays of $Z$-boson.

The SLHM predicts the existence of heavy neutrinos, which have flavor-changing couplings with the SM leptons mediated respectively by the SM gauge boson $W^\pm$ and the new heavy gauge boson $X^\pm$. These couplings can give great contributions to $Z$-boson decays
$Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$, and $Z \to \mu^\pm e^\mp$ at one-loop level. In this paper, we will calculate the branching ratios of these decay modes, and compare the results with the sensitivity of GigaZ and the present experimental bounds, respectively.

This work is organized as follows. In Sec. II we recapitulate the SLHM. In Sec. III we study respectively the decays $Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$ and $Z \to \mu^\pm e^\mp$. Finally, we give our conclusion in Sec. IV.

II. SIMPLEST LITTLE HIGGS MODEL

The SLHM is based on $[SU(3) \times U(1)_X]^2$ global symmetry [4]. The gauge symmetry $SU(3) \times U(1)_X$ is broken down to the SM electroweak gauge group by two copies of scalar fields $\Phi_1$ and $\Phi_2$, which are triplets under the $SU(3)$ with aligned VEVs $f_1$ and $f_2$. The uneaten five pseudo-Goldstone bosons can be parameterized as

$$
\Phi_1 = e^{i t_{\beta} \Theta} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \Phi_2 = e^{-i t_{\beta} \Theta} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix},
$$

(3)

where

$$
\Theta = \frac{1}{f} \begin{pmatrix} 0 & 0 & H \\ 0 & 0 & 0 \\ H^\dagger & 0 & 0 \end{pmatrix} + \frac{\eta}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

(4)

$f = \sqrt{f_1^2 + f_2^2}$ and $t_{\beta} \equiv \tan \beta = f_2/f_1$. Under the $SU(2)_L$ SM gauge group, $\eta$ is a real scalar, while $H$ transforms as a doublet and can be identified as the SM Higgs doublet. The kinetic term in the non-linear sigma model is

$$
\mathcal{L}_\Phi = \sum_{j=1,2} \left| \partial^\mu + i g A^a_\mu T^a - i \frac{g_x}{3} B^a_\mu \right| \Phi_j^2,
$$

(5)

where $g_x = g t_W / \sqrt{1 - t_W^2 / 3}$, and $t_W = \tan \theta_W$ with $\theta_W$ being the electroweak mixing angle. As $\Phi_1$ and $\Phi_2$ develop their VEVs, the new heavy gauge bosons $Z', Y^0$, $Y^{0i}$ and $X^\pm$ get
their masses after eating five Goldstone bosons,

\[
M_X = \frac{g f}{\sqrt{2}} \left( 1 - \frac{v^2}{4f^2} \right),
\]

\[
M_{Z'} = \frac{\sqrt{2} g f}{\sqrt{3 - t_W^2}} \left( 1 - \frac{3 - t_W^2}{c_W^2} \frac{v^2}{16f^2} \right),
\]

\[
M_Y = \frac{g f}{\sqrt{2}}.
\]

(6)

The gauged $SU(3)$ symmetry promotes the SM fermion doublets into $SU(3)$ triplets. For each generation of lepton, a heavy neutrino is added, whose mass is

\[
m_{N_i} = f s_\beta \lambda^i_{N_i}.
\]

(7)

Where \( i = 1, 2, 3 \) is the generation index and \( \lambda^i_N \) is the Yukawa coupling constant.

After the EWSB the light and the heavy neutrino of the same family have the mixing, which is parameterized by \( \delta_v = -\frac{v}{\sqrt{2}f s_\beta} \). The mixing angle \( \delta_v \) is experimentally constrained to be small \[17\], and taken as a typical upper limit \( \delta_v < 0.05 \) following the ref. \[8\]. Besides, there is family mixing as long as the Yukawa matrix of heavy neutrinos and that of leptons are not aligned. This can induce the lepton flavor-changing interactions of charged currents proportional to \( V_{\ell}^{ij} \tilde{N}_i \gamma^\mu X^{+\mu} \ell_j \) and \( \delta_v V_{\ell}^{ij} \tilde{N}_i \gamma^\mu W^{+\mu} \ell_j \), where \( V_{\ell}^{ij} \) is the mixing matrix \[6, 8, 9\].

III. THE LFV DECAYS \( Z \to \tau^\pm \mu^\mp \), \( Z \to \tau^\pm e^\mp \) AND \( Z \to \mu^\pm e^\mp \)

In the SLHM, the Feynman diagrams for \( Z \to \mu^\pm e^\mp \) can be depicted by the Fig. 1, and the diagrams for \( Z \to \tau^\pm \mu^\mp \), \( Z \to \tau^\pm e^\mp \) are same as Fig.1, but replacing \( \mu \) and \( e \) with the corresponding final particles. For the ’t Hooft-Feynman gauge, the flavor-changing interactions between the heavy neutrino and lepton, mediated by the gauge bosons (Goldstone bosons) \( X^{\pm} (x^{\pm}) \) and \( W^\pm (\phi^\pm) \), can contribute to these decays. The relevant Feynman rules can be found in \[8\].

The calculations of the loop diagrams in Fig. 1 are straightforward. Each loop diagram is composed of some scalar loop functions \[18\] which are calculated by using LoopTools \[19\]. The analytic expressions from our calculation are presented in Appendix A.

The SM input parameters relevant in our study are taken as ref. \[20\]. The free SLHM parameters involved are \( f \), \( t_\beta \), the heavy neutrino mass \( m_{N_i} \) (\( i = 1, 2, 3 \)), and the mixing
matrix $V_\ell$ which can be parameterized with standard form. To simply our calculations, we take the parameters 

$$s_{12} = \sqrt{0.3}, \quad s_{13} = \sqrt{0.03}, \quad s_{23} = \frac{1}{\sqrt{2}}, \quad \delta_{13} = 65^\circ, \quad (8)$$

which is consistent with the experimental constraints on the PMNS matrix \[22\], and $\delta_{13}$ is taken to be equal to the CKM phase. To satisfy the present experimental bounds of $Br(\mu \to e\gamma)$ and $Br(\mu \to eee)$, the mass splitting of the first and the second heavy neutrinos must be very small \[8\]. So in this paper we will take $m_{N_1} = m_{N_2} = m_1 = 400$ GeV and $m_{N_3} = m_3$ in the range of 500 GeV-3000 GeV. Ref. \[4\] shows that the LEP-II data requires $f > 2$ TeV. In our numerical calculation we will take several values of $t_\beta$ for $f = 2$ TeV, $f = 4$ TeV and $f = 5.6$ TeV, respectively.

In Fig. 2, Fig. 3 and Fig. 4 we plot the decay branching ratios of $Z \to \tau^\pm \mu^\mp$, $Z \to \tau^\pm e^\mp$

FIG. 1: Feynman diagrams for $Z \to \mu^+ e^-$ in the SLHM.
and $Z \rightarrow \mu^{\pm}e^{\mp}$ versus $m_3$ for $f = 2$ TeV, $f = 4$ TeV and $f = 5.6$ TeV, respectively. We find that the branching ratios increase with the mass of the third generation heavy neutrino. The reason is that the decays are enhanced by the large mass splitting $m_3 - m_1$, which increases as $m_3$ gets large since we have fixed the value of $m_1$. Besides, the branching ratios drop as the scale $f$ or $t_\beta$ get large, and the reason is that the lepton flavor-changing couplings $\tilde{N}_Li\gamma^\mu W^{\pm}\mu_{Lj}$ and $\tilde{N}_i\phi^+\ell_j$ are proportional to $\delta_v = -\frac{v}{\sqrt{2}t_\beta}$.

Fig. 2, Fig. 3 and Fig. 4 show the branching ratios of $Z \rightarrow \tau^{\pm}\mu^{\mp}$, $Z \rightarrow \tau^{\pm}e^{\mp}$ and $Z \rightarrow \mu^{\pm}e^{\mp}$ are below the present experimental upper bounds, respectively. However, the ratios can be enhanced to reach the sensitivity of the GigaZ. For $f = 2$ TeV, $t_\beta = 4$ and $m_3 = 2$ TeV, the branching ratios can reach $10^{-7}$ for $Z \rightarrow \tau^{\pm}\mu^{\mp}$, $Z \rightarrow \tau^{\pm}e^{\mp}$ and $Z \rightarrow \mu^{\pm}e^{\mp}$, which exceed much the sensitivity of GigaZ. In the LHT, all the three ratios can reach $10^{-6}$ [12]. Therefore, the LFV decays of $Z$-boson may be accessible at GigaZ, and thus may serve as a probe of the little Higgs models.
FIG. 4: The branching ratios of $Z \rightarrow \mu^\pm e^\mp$ versus $m_3$.

IV. CONCLUSION

In the framework of the simplest little Higgs model, we studied the LFV decays $Z \rightarrow \tau^\pm \mu^\mp$, $Z \rightarrow \tau^\pm e^\mp$ and $Z \rightarrow \mu^\pm e^\mp$. In the parameter space allowed by current experiments, the branching ratios of the three decays can exceed respectively much the sensitivity of GigaZ, which should be accessible at the GigaZ option of the ILC. Therefore, the measurement of these rare decays at the GigaZ may serve as a probe of the simplest little Higgs model.

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Appendix A: The effective coupling of $Z\mu^+e^-$

Here we take the effective coupling of $Z\mu^+e^-$ for example. The other two couplings $Z\tau^+\mu^-$ and $Z\tau^+e^-$ can be obtained via some corresponding replacement of the analytic
expressions for $Z\mu^+e^-$. The effective coupling of $Z\mu^+e^-$ is given by

$$
\Gamma_{Z\mu e} = \Gamma_{V_{F_1F_2}}^\alpha [X(W), N_i, \nu_j] + \Gamma_{SF_{F_1F_2}}^\alpha [X(W), N_i, \nu_j] + \Gamma_{S_{F_1F_2}}^\alpha [x(\phi), N_i, \nu_j] + \Gamma_{SF_{F_1F_2}}^\alpha [x(\phi), N_i, \nu_j]
$$

where the particles in the square brackets represent the particles which contribute to the vertex, and $\Gamma_{self(k-n)}^\alpha$ correspond to the vertexes in Fig. 1 $(k-n)$. The self-energy and vertex contributions in the above equation are given by

$$
\Gamma_{V_{F_1F_2}}^\alpha = \frac{i}{16\pi^2} [(d_1Z_R^fP_L + c_1Z_L^fP_R)(-2C_{\sigma\rho}\gamma^\sigma\gamma^\rho - 2\gamma^\alpha)(c_2P_L + d_2P_R)
$$

$$
-2(\not q_e + \not q_\mu)\gamma^\alpha C_\beta\gamma^\beta(c_1Z_L^fP_L + d_1d_2Z_R^fP_R) + 4m_{F_2}(c_2d_1Z_L^fP_L
$$

$$
+\gamma^\alpha(\not q_e + \not q_\mu - m_{F_2}\gamma^\alpha)(c_2P_L + d_2P_R)](q_e, q_\mu, m_{F_1}, m_V, m_{F_2}),
$$

$$
\Gamma_{SF_{F_1F_2}}^\alpha = \frac{i\gamma_{VV}^V}{16\pi^2} (d_1P_L + c_1P_R)\{ -4C_{\sigma\rho}\gamma^\sigma\gamma^\rho + \gamma^\alpha - 2C_\beta\gamma^\beta(q_e + q_\mu) + 2(4m_{F} - 2\not q_\mu)C_\alpha
$$

$$
+\gamma^\alpha(m_{F} - 2\not q_\mu)(q_e + q_\mu)C_0 - [C_{\sigma\rho}\gamma^\sigma\gamma^\rho - \frac{1}{2}]\gamma^\alpha - C_\beta\gamma^\beta(\not q_\mu + m_{F})\gamma^\alpha - \not q_Z C_\beta\gamma^\beta
$$

$$
+\gamma^\alpha(\not q_\mu + m_{F})(\not q_Z - \not q_e - \not q_\mu)C_0](c_2P_L + d_2P_R)(q_e, q_\mu, m_V, m_{F}, m_V),
$$

$$
\Gamma_{SF_{F_1F_2}}^\alpha = \frac{i}{16\pi^2} [C_{\sigma\rho}\gamma^\sigma\gamma^\rho(2a_1Z_R^fP_L + a_1b_2Z_L^fP_R) + \frac{1}{2}\gamma^\alpha(a_2b_1Z_R^fP_L + a_1b_2Z_L^fP_R)
$$

$$
+C_\beta\gamma^\alpha(a_1Z_L^fP_L + b_1Z_R^fP_R)(\not q_e + \not q_\mu + m_{F_2})(a_2P_L + b_2P_R)
$$

$$
+m_{F_2}\gamma^\alpha(b_2b_1Z_R^fP_L + a_1a_2Z_L^fP_R)C_\beta\gamma^\beta + m_{F_1}\gamma^\alpha(b_1Z_L^fP_L
$$

$$
+a_1Z_R^fP_R)(\not q_e + \not q_\mu + m_{F_2})(a_2P_L + b_2P_R)C_0](q_e, q_\mu, m_{F_1}, m_S, m_{F_2}),
$$

(A1)
\[ \Gamma_{\alpha}^{\text{FSS}} = \frac{ig_{\text{VSS}}}{16\pi^2} \left\{ -2C_\alpha \gamma^\beta (a_2 b_1 P_L + a_1 b_2 P_R) - (q_e + q_\mu)^\alpha C_\beta \gamma^\beta (a_2 b_1 P_L + a_1 b_2 P_R) + \left[ -2C_\alpha - (q_e + q_\mu)^\alpha C_0 \right] \phi_\mu (a_2 b_1 P_L + a_1 b_2 P_R) + m_F \left[ -2C_\alpha - (q_e + q_\mu)^\alpha C_0 \right] (a_1 a_2 P_L + b_1 b_2 P_R) \right\} \] (A5)

\[ \Gamma_{\alpha}^{\text{FVS}} = -\frac{ig_{\text{VVS}}}{16\pi^2} \gamma^\alpha (c_1 P_L + d_1 P_R) [C_\beta \gamma^\beta + (\phi_\mu + m_F) C_0] \times (a_2 P_L + b_2 P_R)(q_\mu, q_e, m_S, m_F, m_V), \] (A6)

\[ \Gamma_{\alpha}^{\text{FSV}} = \frac{ig_{\text{VSV}}}{16\pi^2} (a_1 P_L + b_1 P_R) [C_\beta \gamma^\beta + (\phi_\mu + m_F) C_0] \gamma^\alpha \times (c_2 P_L + d_2 P_R)(q_\mu, q_e, m_V, m_F, m_S), \] (A7)

\[ \Gamma_{\alpha}^{\text{self}(k)} = -\frac{ig}{16\pi^2 \epsilon W (q_\mu^2 - m_e^2)^2} \gamma^\alpha \left[ (-\frac{1}{2} + s_W^2) P_L + s_W^2 P_R \right] (\phi_\mu + m_e) \left[ (2B_\beta \gamma^\beta + (2B_0 - 1) \phi_e) (c_1 c_2 P_L + d_1 d_2 P_R) - 2m_F (2B_0 - 1) (c_1 d_1 P_L + c_1 d_2 P_R) \right] (q_\mu, m_V, m_F), \] (A8)

\[ \Gamma_{\alpha}^{\text{self}(l)} = -\frac{ig}{16\pi^2 \epsilon W (p_\mu^2 - m_e^2)^2} \gamma^\alpha \left[ (-\frac{1}{2} + s_W^2) P_L + s_W^2 P_R \right] (\phi_\mu + m_e) \left[ (B_\beta \gamma^\beta + \phi_e B_0) (a_2 b_1 P_L + a_1 b_2 P_R) + m_F B_0 (a_1 a_2 P_L + b_1 b_2 P_R) \right] (q_\mu, m_S, m_F), \] (A9)

\[ \Gamma_{\alpha}^{\text{self}(m)} = \frac{ig}{16\pi^2 \epsilon W (p_\mu^2 - m_e^2)^2} \gamma^\alpha \left[ (-\frac{1}{2} + s_W^2) P_L + s_W^2 P_R \right] (\phi_\mu + m_e) \left[ (B_\beta \gamma^\beta + \phi_e B_0) (a_2 b_1 P_L + a_1 b_2 P_R) + m_F B_0 (a_1 a_2 P_L + b_1 b_2 P_R) \right] (q_\mu, m_S, m_F), \] (A10)

\[ \Gamma_{\alpha}^{\text{self}(n)} = -\frac{ig}{16\pi^2 \epsilon W (p_\mu^2 - m_e^2)^2} \gamma^\alpha \left[ (-\frac{1}{2} + s_W^2) P_L + s_W^2 P_R \right] (\phi_\mu + m_e) \left[ (B_\beta \gamma^\beta + \phi_e B_0) (a_2 b_1 P_L + a_1 b_2 P_R) + m_F B_0 (a_1 a_2 P_L + b_1 b_2 P_R) \right] (q_\mu, m_S, m_F), \] (A11)

where \( q_\mu = -p_\mu, q_e = -p_e \) and \( P_{L,R} = (1 \mp \gamma_5)/2 \). The functions \( B \) and \( C \) are 2- and 3-point Feynman integrals [19], and their functional dependence is indicated in the bracket following them. The tensor loop functions can be expanded as the scalar functions [19]. In our calculation the contraction of Lorentz indices is performed numerically. The parameters appearing above are from

\[ V \bar{e} f : i \gamma^\mu (c_1 P_L + d_1 P_R), \]

\[ S \bar{e} f : a_1 P_L + b_1 P_R, \]

\[ Z S^+ S^- : i g_{\text{VSS}} (p_{S+}^\mu - p_{S-}^\mu), \]

\[ Z \bar{V}^+ V^- : -ig_{\text{VVV}} \left[ (p_{V+} - p_{V-})^\mu g^\mu \nu + (p_Z - p_{V+})^\nu g^\mu \nu + (p_{V-} - p_Z)^\mu g^{\mu \nu} \right], \]

\[ Z \bar{f}_1 f_2 : i \gamma^\mu (Z_{F1}^\mu P_L + Z_{F2}^\mu P_R), \]

where \( V \) represents gauge bosons and \( S \) represents scalar particles. These couplings represent the seven different classes of vertices involved in our calculation. In each class of vertices,
the parameters $a_1$, $b_1$, $a_2$, $b_2$, $c_1$, $d_1$, $c_2$, $d_2$, $g_{VSS}$, $g_{VVV}$, $Z^f_L$ and $Z^f_R$ take different values for different concrete coupling. The analytic expressions of these parameters can be found in [8].

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