Conditional implementation of asymmetrical universal quantum cloning machine

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We propose two feasible experimental implementations of an optimal asymmetric 1 → 2 quantum cloning of a polarization state of a photon. Both implementations are based on a partial and optimal reverse of recent conditional symmetrical quantum cloning experiments. The reverse procedure is performed only by a local measurement of one from the clones and ancilla followed by a local operation on the other clone. The local measurement consists only of a single unbalanced beam splitter followed in one output by a single photon detector and the asymmetry of fidelities in the cloning is controlled by a reflectivity of the beam splitter.

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Copying the quantum states is apparently dissimilar to classical information processing since it is impossible to precisely duplicate an unknown quantum state as a consequence of a linearity of quantum mechanics [1]. To clone an unknown quantum state at least approximately, universal quantum cloning machines (UQCM) were developed [2]. The UQCM is a device that universally and optimally produces a copy ρS from an unknown quantum state |Ψ⟩S of the original. Specifically, an optimal symmetrical 1 → 2 UQCM (SUQCM) for qubits creates a copy ρS′ with a maximal state-independent fidelity F2 = ⟨Ψ|ρS|Ψ⟩S = 5/6. Simultaneously, a pure state of original changes to mixed state ρS exhibiting maximally the same fidelity Fs = ⟨Ψ|ρS|Ψ⟩S = 5/6 as the clone. To optimally control the fidelities Fs and F2, a concept of asymmetrical UQCM (AUQCM) has been theoretically developed [3, 4]. The optimal 1 → 2 AUQCM produces the copies having state-independent fidelities controlled by a parameter R in such a way that for a given fidelity of the copy F2(R), the fidelity Fs(R) of the original is maximal. More specifically, assigning a qubit in an unknown state |Ψ⟩ then the original S and clone S′ leaving 1 → 2 AUQCM can be represented by the following density matrices [3, 4, 5, 6]

ρS,S′ = F2|Ψ⟩⟨Ψ| + (1 − F2)|Ψ⟩⟨Ψ|, (1)

where the fidelities 5/6 ≤ F2 ≤ 1 and 1/2 ≤ Fs ≤ 5/6 satisfy the cloning relation

(1 − F2)(1 − F2) ≥ (1/2 − (1 − F2) − (1 − F2))2, (2)

where the equality corresponds to an optimal AUQCM, in the sense that for a larger F2 cannot be obtained for given F2. The Eq. (2) is the tightest no-cloning bound for the fidelities of the 1 → 2 cloner which copies an unknown qubit state to the another with an isotropic noise. The recent experimental effort to build different quantum cloners is mainly stimulated by their use as individual attacks in quantum communication and cryptography [5]. More information about this practical application of the asymmetrical universal cloning as optimal attack for a cryptographic protocol can be found in Ref. [6].

To build quantum cloners, quantum networks using CNOT gates for both optimal SUQCM and AUQCM were proposed [10]. However, a strength of the state-of-the-art nonlinear interaction at a single photon level is unfortunately too weak to produce a deterministic and efficient CNOT operation only by a direct interaction between photons. For this reason, the deterministic SUQCM and AUQCM still have not been experimentally implemented in quantum optics. Netherless, stimulated or spontaneous parametric down-conversions were used to realize a conditional implementation of the SUQCM for a polarization state of photon [11, 12, 13]. However, to the best of our knowledge, no feasible experimental setup for an optimal 1 → 2 AUQCM has been presented yet. On the other hand, an experimental realization of optimal asymmetrical cloning machine has already been proposed for coherent states [15].

In this paper, we propose optimal 1 → 2 AUQCM which is a simple and experimentally feasible extension of the recent experiments on the conditional symmetrical cloning of the polarization state of a photon. Our method is based on a partial optimal reverse of SUQCM by a specific controllable joint measurement on one of the copies and an auxiliary photon leaving the cloning process. By this partial reverse the quantum information between the disturbed original and copy can be redistributed posteriori only using the local operations and classical communication. It can be experimentally accomplished adding only a single unbalanced beam splitter followed by a single photon detector in the recent cloning experiments [11, 12, 13]. An experimental realization of the conditional 1→2 SUQCM [11, 12] was based on a nonlinear parametric down-conversion stimulated in a signal beam by a single photon prepared in an unknown polarization state |Ψ⟩S = a|V⟩S + b|H⟩S, where V and H denote the vertical and horizontal polarizations. The experimental arrangement is depicted in Fig. 1. The input single photon extracted from a laser pulse is prepared in the state |Ψ⟩S in a preparation device using λ/2 and λ/4 wave plates. A more intensive part of the laser pulse is frequency doubled...
and used to pump a BBO non-linear crystal. An action of a non-degenerate type II parametric down-conversion process in the crystal can be described by the Hamiltonian $H_{1} = i\hbar \chi (a_{H}^{\dagger} b_{V}^{\dagger} - a_{V}^{\dagger} b_{H}^{\dagger}) + \text{h.c.}$, where $\chi$ is proportional to a nonlinear susceptibility of the crystal, and h.c. denotes the hermitian conjugation. Here the annihilation operators $a, b$ are assumed to be acting on the selected signal mode $S$ and idler mode $I$, respectively. A short-time approximation of the evolution operator $U = \exp(-iH_{1}t/\hbar) \approx 1 - \frac{i}{\hbar}H_{1}t$ is used. This approximation is correct for this kind of experiments, since the gain is $\chi$ is usually very small ($|\chi| \ll 1$). Within the short-time approximation, polarization basis states $|H\rangle_{S} \equiv |1,0\rangle_{S}$ and $|V\rangle_{S} \equiv |0,1\rangle_{S}$ of the input photon evolve according to the following rules

$$U|1,0\rangle_{I} \approx |1,0\rangle_{I} + g(\sqrt{2}|2,0\rangle_{I} - |1,1\rangle_{I}),$$

$$U|0,1\rangle_{I} \approx |0,1\rangle_{I} + g(\sqrt{2}|0,2\rangle_{I} - |1,1\rangle_{I}).$$

(3)

Here, the produced states $|2,0\rangle$ and $|0,2\rangle$ represent an effect of a stimulated emission which is used to prepare the clone, and the state $|1,1\rangle$ corresponds to an unavoidable effect of a spontaneous emission. After the amplification, a balanced polarization-insensitive beam splitter $BS2$ in the signal mode separates two photons in the states $|2,0\rangle$ or $|0,2\rangle$ to two distinguishable spatial modes corresponding to the disturbed photon $S$ and clone $S'$. An action of the beam splitter $BS2$ on a pair of photons is as follows

$$|1,1\rangle_{S'} |0,0\rangle_{S} \rightarrow \frac{1}{2}(|1,1\rangle_{S'} |0,0\rangle_{S} + |0,0\rangle_{S} |1,1\rangle_{S} + |1,0\rangle_{S'} |0,1\rangle_{S} + |0,1\rangle_{S} |1,0\rangle_{S'}),$$

$$|2,0\rangle_{S'} |0,0\rangle_{S} \rightarrow \frac{1}{2}(|2,0\rangle_{S'} |0,0\rangle_{S} + |0,0\rangle_{S} |2,0\rangle_{S'} + |2,0\rangle_{S} |0,0\rangle_{S'}),$$

$$|0,2\rangle_{S'} |0,0\rangle_{S} \rightarrow \frac{1}{2}(|0,2\rangle_{S'} |0,0\rangle_{S} + |0,0\rangle_{S} |0,2\rangle_{S'} + |0,2\rangle_{S} |0,0\rangle_{S'}).$$

where $S$ and $S'$ are the signal modes. 

FIG. 1: Setup for conditional AUQCM based on stimulated parametric down-conversion: $L$ - laser, BS1-BS3 beam splitters, PBS – polarization beam splitter, $2\omega$ – frequency doubler, BBO – nonlinear BBO type II crystal, $\lambda/2, \lambda/4$ – wave plates, D1 – single photon detector.

FIG. 2: Setup of conditional AUQCM based on spontaneous parametric down-conversion: $L$ - laser, BS1-BS3 beam splitters, PBS – polarization beam splitter, $2\omega$ – frequency doubler, BBO – nonlinear BBO type II crystal, $\lambda/2, \lambda/4$ – wave plates, D1-D2 – single photon detectors.

where $S$ and $S'$ are the signal modes. In the next procedure only such cases when a single photon is present in the mode $S$ are considered. Returning to the previous notation $|1,0\rangle_{i} = |H\rangle_{i}$ and $|0,1\rangle_{i} = |V\rangle_{i}$, the SUQCM transformation

$$|\Psi\rangle_{S} \rightarrow \frac{1}{\sqrt{3}}(|\Psi\rangle_{S} + |\Psi_{\perp}\rangle_{S} + |\Psi_{\perp}\rangle_{S}),$$

(5)

where $|\Psi_{\perp}\rangle = \frac{1}{\sqrt{2}}(|\Psi\rangle_{S} + |\Psi_{\perp}\rangle_{S})$ and $|\Psi_{\perp}\rangle = a^{\dagger}|H\rangle - b^{\dagger}|V\rangle$ is the orthogonal state to $|\Psi\rangle$, is actually performed. This SUQCM is optimal and transforms an unknown state $|\Psi\rangle_{S}$ of the original to the disturbed one and a copy, with the fidelities $F_{S} = F'_{S} = 5/6$. Both the output states of photons $S$ and $S'$ have to be measured using the state analyzer composed from the $\lambda/2$-wave plate and $\lambda/4$-wave plate, the polarization beam splitter PBS and a pair of single photon detectors $D3, D4$, as depicted in Fig. 1. Note, in the cloning experiment $[12]$, the fidelities were approximately $F_{S}, F'_{S} \approx 0.81$ which are really close to the optimal value of $5/6 = 0.833$.

Recently, a different implementation of the $1 \rightarrow 2$ SUQCM was experimentally performed $[13]$. It is based on the following joint projection

$$\Pi_{S} = (1_{SS'} - |\Psi_{\perp}\rangle_{SS'}\langle\Psi_{\perp}|- 1_{I})$$

(6)

of an unknown polarization state $|\Psi\rangle_{S'} = a|V\rangle + b|H\rangle$ of the input photon and the antisymmetric polarization Bell state $|\Psi_{\perp}\rangle_{SI} = \frac{1}{\sqrt{2}}(|VH\rangle - |HV\rangle)$ of two photons produced by the spontaneous parametric down-conversion from the same BBO nonlinear crystal as in the previous experiment. The corresponding experimental setup is depicted in Fig. 2. The initial state $|\Psi\rangle_{S'} = a|V\rangle_{S'} + b|H\rangle_{S'}$ is prepared by the same method. The projection $\Pi_{S}$ to the symmetric subspace $[14]$ on the state $|\Psi\rangle_{S'}|\Psi_{\perp}\rangle_{SI}$ can be accomplished by a sequence of two beam splitters.
BS1, BS2 and a single-photon detector D1. If two input photons in the same state $|0,1\rangle_S|0,1\rangle_{S'}$ or $|1,0\rangle_S|1,0\rangle_{S'}$ constructively interfere on the first balanced beam splitter BS1 and no photon is detected by the detector D1, then a twin of photons in the state $|0,2\rangle_S$ or $|2,0\rangle_S$ is produced in the mode $S$ with a success probability 1/2. The twins are further divided on the second balanced beam splitter BS2 to separate the photons to the different spatial modes and if we selected only such cases when exactly a single photon is in every mode $S, S', I$ the symmetric states $|0,1\rangle_S|0,1\rangle_{S'}$ or $|1,0\rangle_S|1,0\rangle_{S'}$ with probability 1/4 are obtained at a result. On the other hand, if two orthogonal basis states $|0,1\rangle_S|1,0\rangle_{S'}$, $|1,0\rangle_S|0,1\rangle_{S'}$ are mixed at the beam splitter BS1, they do not mutually interfere and in addition, if no photon is registered on the detector D1, then the state $|1,1\rangle_S$ with the success probability 1/2 is within the mode $S$. Thus after splitting the photons by BS2 to separate spatial modes, a symmetric state $\sqrt{2}/2(|1,0\rangle_S|0,1\rangle_{S'} + |0,1\rangle_S|1,0\rangle_{S'})$ is obtained with the total success probability 1/4. Thus, with the probability 1/4 the following transformation

$$|\Psi\rangle_{SS'} \rightarrow \sqrt{2}|\Psi\rangle_{SS'},$$

$$|\Psi\rangle_{\perp SS'} \rightarrow \frac{1}{\sqrt{2}}(|\Psi\rangle_{\perp SS'} + |\Psi\rangle_{SS'}),$$

of the states of the photons $S, S'$ is in fact conditionally implemented. Assuming that a state of the idler photon $I$ is selected only if this procedure is successful, the total projection (6) transforms the input state $|\Psi\rangle_S|\Psi\rangle_{S'}_S$ to (7). Thus the optimal SUQCM is conditionally accomplished however now a spontaneous emission of maximally entangled pairs is used rather than a stimulated emission in the previous experiment. In the experiment based on this idea (9), the fidelities of the clone and disturbed original are $F_S, F_{S'} \approx 0.826$ which are even more closer to the theoretical value 5/6 = 0.833 than it has been in the previous case.

An extension of both setups to achieve the optimal AUSQCM can be presented. It is known that the symmetrical quantum cloning is LOCC reversible (10). If a projective measurement $\Pi_\perp = |\Psi\rangle\langle\Psi|_{SS'}$ on one clone and ancilla is performed, the other clone returns back to the initial state $|\Psi\rangle$. Thus we can guess that if an appropriate projection in a form $a|1\rangle + b|\Pi\rangle_{SS'}$ is applied on the one clone and the ancilla, an intermediate case corresponding to the optimal asymmetrical cloning machine could be obtained.

Now we show that this projection can be conditionally implemented if one mixes a pair of photons in the idler $I$ and signal $S'$ modes on unbalanced beam splitter BS3 having a variable reflectivity $0 \leq R \leq 1/2$ and select only the cases when both photons leaving the beam splitter are separated. Then the beam splitter can be simply described by transformation

$$|VV\rangle_{S'1} \rightarrow (T - R)|VV\rangle_{S'1},$$

$$|HH\rangle_{S'1} \rightarrow (T - R)|HH\rangle_{S'1},$$

$$|HV\rangle_{S'1} \rightarrow T|HV\rangle_{S'1} - R|VV\rangle_{S'1},$$

$$|VH\rangle_{S'1} \rightarrow T|VH\rangle_{S'1} - R|HH\rangle_{S'1},$$

where $T + R = 1$. It can be simply proved that this transformation can be expressed in a covariant way

$$|\Psi\rangle_{S'1} \rightarrow |(T - R)|\Psi\rangle_{S'1},$$

$$|\Psi\rangle_{\perp S'1} \rightarrow T|\Psi\rangle_{\perp S'1} - R|\Psi_{\perp}\rangle_{S'1}.$$  

(9)

It is an asymmetrical projection controlled by the parameter $R$, in a contrast to the symmetrizing projection (7). If an output state is selected only when there is a single photon in each mode $S, S', I$, we obtain the following transformation for the polarization states of the photon

$$|H\rangle \rightarrow \frac{1}{\sqrt{N(R)}}((2 - R)|HH\rangle_{S'1} -$$

$$(1 + R)|HV\rangle_{S'1} - (1 - 2R)|VV\rangle_{S'1}),$$

$$|V\rangle \rightarrow \frac{1}{\sqrt{N(R)}}((2 - R)|VV\rangle_{S'1} -$$

$$(1 + R)|HV\rangle_{S'1} - (1 - 2R)|HH\rangle_{S'1})$$  

(10)

where $N(R) = 6(1 - R(1 - R))$. In a real experiment, a detection of the photon in the mode $I$ can be performed destructively by a single-photon detector $D2$ whereas the signal photons from total cloning operation are detected in the state analyzers. It can be simply proved that the transformation (14) is covariant and it can be written in a form

$$|\Psi\rangle \rightarrow \frac{1}{\sqrt{N(R)}}((2 - R)|\Psi\rangle_{\perp S'1} -$$

$$(1 + R)|\Psi_{\perp}\rangle_{S'1} - (1 - 2R)|\Psi\rangle_{S'1})$$  

(11)

which corresponds to the following projection

$$\Pi_A(R) = ((1 - 2R)|S'\rangle \otimes I_I + 2|R\rangle|\Psi\rangle_{S'1}(|\Psi_{\perp}\rangle \otimes 1_I),$$  

(12)

on the state (5) produced by the SUQCM. An interpretation of this projective measurement is straightforward: the asymmetrical cloning is obtained as a partial optimal reverse of the symmetrical one. For $R = 0$ the SUQCM is obtained and for $R = 1/2$ the SUQCM is reversed. An additional state of the original is precisely restored. An optimal total reverse of the state after the symmetrical cloning was previously theoretically already analyzed (10). After the total reverse, any input state is deterministically revealed by the complete Bell-state measurement on the clone and ancilla. Thus this obtained result also represents a solution of the problem of a partial but still optimal reverse of the symmetrical cloning. Further, this reverse is also obtained only using the local operations on the clones and classical communication between them. It enables the redistribution of the quantum information encoded in symmetric clones at a distance without an additional quantum channel.
Since the photon in the idler mode is detected in such a way that no information about its polarization is acquired, we trace over the idler mode and obtain the final output states of the modes $S$ and $S'$.

\[
\rho_{SS'} = \frac{1}{N(R)} \left[ (2 - R)^2 |\Psi\rangle \langle \Psi|_{SS'} + (1 + R)|\Psi\rangle \langle \Psi|_{SS} + (1 - 2R)|\Psi\rangle \langle \Psi|_{SS'} \right]
\]

This state carries both the disturbed original and clone

\[
\rho_S = \frac{1}{N(R)} \left( (2 - R)^2 + (1 + R)^2 \right) |\Psi\rangle \langle \Psi| + (1 - 2R) |\Psi\rangle \langle \Psi|_I,
\]

\[
\rho_{S'} = \frac{1}{N(R)} \left( (2 - R)^2 + (1 - 2R)^2 \right) |\Psi\rangle \langle \Psi| + (1 + R)^2 |\Psi\rangle \langle \Psi|_I
\]

and is conditioned by a simultaneous detection of a single photon in each of the output modes $S, S', I$. The marginal states of the disturbed original and clone in the selected sub-ensemble have the following fidelities

\[
F_S = \frac{(2 - R)^2 + (1 + R)^2}{6(1 - R)(1 - R)}
\]

\[
F_{S'} = \frac{(2 - R)^2 + (1 - 2R)^2}{6(1 - R)(1 - R)}
\]

with an initial state $|\Psi\rangle$ which vary with increasing $R \in (0, 1/2)$ from a perfect SUQCM ($R = 0$) to the trivial non-cloning case ($R = 1/2$), as depicted in Fig. 3. The output states of the original and clone can be measured by the state analyzers analogically as it was discussed for the experiments with the SUQCMs. Inserting the fidelities \[15\] to the cloning inequality \[2\] which restricts all the possible AUQCM, the equality is obtained in Eq. \[2\] as can be straightforwardly proved.

In this paper an extension of the recent conditional cloning experiment for a polarization state of photon toward the optimal asymmetrical $1 \rightarrow 2$ quantum cloning machine is proposed. Our method is based on a conditional partial optimal reverse of the SUQCM controlled by an experimental parameter $R$. We have applied this method in the recent symmetrical cloning experiments \[3\] \[4\] \[5\] to obtain the optimal asymmetrical cloning. In summary, the AUQCM can be described as the projection $\Pi(R) = \Pi_A(R)\Pi_S$, given explicitly by

\[
\Pi(R) = ((2 - R)1_S \otimes 1_S - 2(1 - 2R)|\Psi\rangle \langle \Psi|_S \otimes 1_I,
\]

on the state $|\Psi\rangle_0 \otimes |\Psi\rangle_1$ composed from an initial unknown state and the antisymmetric Bell state produced from the spontaneous parametric down-conversion. Since the fidelities obtained in these experiments are very close from the spontaneous parametric down-conversion, the AUQCMs is feasible and it could be straightforwardly realized.

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