The influence of noise on electron dynamics in semiconductors driven by a periodic electric field

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Abstract. Studies concerning the constructive aspects of noise and fluctuations in different non-linear systems have shown that the addition of external noise to systems with an intrinsic noise may result in a less noisy response. Recently, the possibility of reducing the diffusion noise in semiconductor bulk materials by adding a random fluctuating contribution to the driving static electric field has been tested. The present work extends the previous theories by considering the noise-induced effects on the electron transport dynamics in low-doped n-type GaAs samples driven by a high-frequency periodic electric field (cyclostationary conditions). By means of Monte Carlo simulations, we calculate the changes in the spectral density of the electron velocity fluctuations caused by the addition of an external correlated noise source. The results reported in this paper confirm that, under specific conditions, the presence of a fluctuating component added to an oscillating electric field can reduce the total noise power. Furthermore, we find a non-linear behaviour of the spectral density with the noise intensity. Our study reveals that, critically depending on the external noise correlation time, the dynamical response of electrons driven by a periodic electric field benefits from the constructive interplay between the fluctuating field and the intrinsic noise of the system.

Keywords: classical Monte Carlo simulations, fluctuations (theory), transport properties (theory)
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1. Introduction

The presence of noise in experiments is generally considered a disturbance, especially in studying the efficiency of semiconductor-based devices, where strong fluctuations could affect their performance. Recently, however, an increasing interest has been directed towards the constructive aspects of noise in the dynamical response of non-linear systems. A counterintuitive enhancement of the stability can be induced in systems containing metastable states by the addition of noise [1]–[4]. Some excitable systems may achieve more order in the presence of noise, even in the absence of an external signal, such as in phenomena of coherent resonance where an optimal level of noise leads to regular excursions from the ground state [5]. The effect of interaction between an external source of fluctuations and an intrinsically noisy system was analytically investigated, for the first time, by Vilar and Rubí in 2001. They have demonstrated that the spectral intensity of the output signal in a low-frequency domain can be reduced by the addition of small amplitude noise on the input of the system [6]. Their analytical theory was developed by using very general statistical properties of non-linear systems and by assuming internal and external sources of noise completely uncorrelated. The main limitation of this theory is the assumption of an intrinsic white noise. These restrictions persist in the recent (2004) work of Walton and Visscher, in which they extended the previous theory by considering the total power spectrum of the output signal for greater intensities of the input fluctuations [7]. In realistic cases, however, there always exists some correlation during the characteristic relaxation time of the system and the noise spectra are not strictly white [8].

For semiconductor bulk materials, the possibility of reducing the diffusion noise by adding a correlated random contribution to a driving static electric field was tested by Varani and collaborators in 2005 [9]. Their numerical results, obtained by including energetic considerations in the theoretical analysis, have shown that, under specific conditions for the fluctuating electric field, it is possible to suppress the intrinsic noise in n-type GaAs bulk [9].

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Recent studies of the electron velocity fluctuations in GaAs bulks driven by periodic electric fields have shown that the spectral density strongly depends on the frequency of the applied field and critical modifications are observed when two mixed high-frequency large amplitude periodic electric fields are used [10]. This means that the total power spectrum of the intrinsic noise is dependent on both the amplitude and the frequency of the excitation signals [10]. In the wake of these results, we have found that the opportunity to suppress the diffusion noise in semiconductor bulk materials exists also under cyclostationary conditions, by adding a Gaussian fluctuating contribution to the driving electric field [11].

In this paper we investigate the noise-induced effects on the intrinsic carrier noise spectral density in low-doped n-type GaAs semiconductor driven by a high-frequency periodic electric field. The electron dynamics is simulated by a Monte Carlo procedure which takes into account all the possible scattering phenomena of the hot electrons in the medium. The semiconductor intrinsic noise is obtained by computing both the velocity fluctuation correlation function and the spectral density [12, 13] and directly calculating the variance of the electron velocity fluctuations. The effects caused by the addition of an external source of correlated noise are investigated by analysing (i) the noise spectral density at the same frequency of the external driving field and (ii) the integrated spectral density (ISD), which coincides with the variance of the electron velocity fluctuations. Our results confirm that the presence of a random contribution to a high-frequency periodic electric field can reduce the total noise power. Furthermore, we find a non-linear behaviour of both the noise spectral density at the driving frequency and the ISD with the noise intensity. In particular, the ISD shows a minimum which critically depends on the value of the noise correlation time. Detailed investigations of the electron transport dynamics in the semiconductor reveal that the system benefits from the constructive interplay between the random fluctuating electric field and the intrinsic noise. The paper is organized as follows. In section 2 we present the details of the Monte Carlo procedure, the statistical quantities used to investigate the electronic noise and a physical model describing the dependence of the noise spectra on the frequency of the electric field; in section 3 the results of our calculations are given and discussed. Final comments and conclusions are given in section 4.

2. Semiconductor model and noise calculation methods

The electron dynamics in a GaAs bulk semiconductor, driven by an oscillating electric field, is simulated by a Monte Carlo method. The motion of electrons is characterized by an average velocity, which depends on the external parameters of the system, such as the amplitude of the applied electric field and its frequency. The fluctuations of electron velocity around its mean value correspond to the intrinsic noise of the system.

2.1. Monte Carlo procedure

The Monte Carlo algorithm, developed for simulating the motion of electrons in a GaAs semiconductor, follows the standard procedure described in [14]. The conduction bands of GaAs are the Γ valley, four equivalent L valleys and three equivalent X valleys. The parameters of the band structure and scattering mechanisms are also

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taken from [14]. Our computations include the effects of the intravalley and intervalley
scattering of the electrons, in multiple energy valleys, and of the non-parabolicity of
the band structure. Electron scatterings due to ionized impurities, acoustic and polar
optical phonons in each valley as well as all intervalley transitions between the equivalent
and non-equivalent valleys are accounted for. We assume field-independent scattering
probabilities; accordingly, the influence of the external fields is only indirect through the
field-modified electron velocities. All simulations are obtained in a GaAs bulk with a free
electron concentration $n = 10^{13}$ cm$^{-3}$. To neglect the thermal noise contribution and to
highlight the partition noise effects we have chosen a lattice temperature $T = 80$ K. We
have assumed that all donors are ionized and that the free electron concentration is equal
to the doping concentration.

The semiconductor bulk is driven by a fluctuating periodic electric field
\[ E(t) = E_0 \cos(\omega t + \phi) + \eta(t) \] (1)
with frequency $f = \omega / 2\pi$ and amplitude $E_0$. The random component of the electric field
is modelled with an Ornstein–Uhlenbeck (OU) stochastic process $\eta(t)$, which obeys the
following stochastic differential equation:
\[ \frac{d\eta(t)}{dt} = -\frac{\eta(t)}{\tau_c} + \sqrt{\frac{2D}{\tau_c}} \xi(t) \] (2)
where $\tau_c$ and $D$ are, respectively, the correlation time and the variance of the OU process,
and $\xi(t)$ is the Gaussian white noise with the autocorrelation $\langle \xi(t)\xi(t') \rangle = \delta(t - t')$. The
OU correlation function is $\langle \eta(t)\eta(t') \rangle = D \exp(-|t - t'|/\tau_c)$.

2.2. Semiconductor noise calculation

The changes on intrinsic noise properties are investigated by the statistical analysis of
the autocorrelation function of the velocity fluctuations and of its mean spectral density.
When the system is driven by a periodic electric field (cyclostationary conditions), the
correlation function $C_{\delta v\delta v}(t, \tau)$ of the velocity fluctuations $\delta v(t) = v(t) - \langle v(t) \rangle$ can be
calculated [13] as
\[ C_{\delta v\delta v}(t, \tau) = \left\langle v \left( t - \frac{\tau}{2} \right) v \left( t + \frac{\tau}{2} \right) \right\rangle - \left\langle v \left( t - \frac{\tau}{2} \right) \right\rangle \left\langle v \left( t + \frac{\tau}{2} \right) \right\rangle \] (3)
in which $\tau$ is the correlation time and the average is over a sequence of equivalent time
moments $t = s + mT$, with $s$ belonging to the time interval $[0, T]$ ($T$ is the field period)
and $m$ is an integer [13]. This two-time symmetric correlation function eliminates any
regular contribution and describes only the fluctuating part of $v(t)$. By averaging over the
whole set of values of $t$ within the period $T$, the velocity autocorrelation function becomes
\[ C_{\delta v\delta v}(\tau) = \frac{1}{T} \int_0^T C_{\delta v\delta v}(t, \tau) dt \] (4)
and the spectral density can be calculated as the Fourier transform of $C_{\delta v\delta v}(\tau)$. In the
computations of the autocorrelation function we have considered $10^3$ possible initial values
of $s$ and a total number of equivalent time moments $m \cong 10^6$.

Intrinsic noise has been investigated also by estimating directly the electron velocity
variance. This calculation has been performed separately for each energy valley, following
the same method of equivalent time moments as was described above.

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2.3. Physical model of intrinsic noise

When the semiconductor is driven by a static electric field, the shape of the spectral density of electron velocity fluctuations is exclusively determined by the strength of the applied field. For amplitudes smaller than the threshold field (Gunn field) $E_G$ for intervalley transitions, the diffusion is the most relevant source of noise, while, for $E > E_G$, the complex structure of the semiconductor becomes relevant and random transitions of carriers among the available energy valleys must be taken into consideration. In this case, the intrinsic noise is mainly determined by a partition noise, caused by stochastic carrier transitions between regions characterized by different dynamical properties (intervalley transfers) in momentum space. The partition noise is characterized by a pronounced peak in the spectral density at a frequency $\nu_G$, which is defined as the ‘natural’ transition frequency of the system between the valleys [10].

Under cyclostationary conditions, the noise behaviour depends on both the amplitude and the frequency of the applied field. In particular, it is similar to that of the static field case only for very low-frequency fields ($f \ll \nu_G$). In contrast, for frequencies $f \gtrsim \nu_G$, the intervalley transfers are driven by the external field, the system enters a forced regime of oscillations and the velocity fluctuations become time correlated [10]. In this case, the spectral density exhibits: (i) a peak centred around the frequency of the periodic signal and (ii) a significant enhancement in the low-frequency region.

In this work we focus our attention to the noise-induced effects on the electron dynamics in a GaAs semiconductor, driven by a high-frequency oscillating field.

3. Numerical results and discussion

The spectral density of the electron velocity fluctuations has been studied by adopting a fluctuating periodic electric field with frequency $f = 500$ GHz. The amplitude of this field has been chosen on the basis of a preliminary analysis of the variance of velocity fluctuations and the spectral density $S_0(E)$ at zero frequency, as a function of the amplitude of the oscillating field. Following [6] and [9], the most favourable condition for obtaining a noise suppression effect in our system is reached when $d^2 S_0(E)/dE^2$ is negative and the variance of velocity fluctuations exhibits a maximum. In figures 1(a) and (b) we can see which range of amplitudes of electric field verifies these conditions. Accordingly, we have chosen a driving electric field with amplitude $E_0 = 10$ kV cm$^{-1}$ and frequency $f = 500$ GHz.

In the absence of external noise, the amplitude of the forcing field is large enough to switch on intervalley transitions from the $\Gamma$ valley to the L valleys and, since the frequency $f$ is of the same order as $\nu_G$, the electron velocity fluctuations are mainly determined by partition noise [15]. In this case, the spectrum is characterized by the features described in section 2.3. In figure 2(a) we show how the spectral density of electron velocity fluctuations is modified by the presence of noise. The addition of an external source of fluctuations to the driving electric field strongly changes the spectrum and, in particular, the height of the peak around 500 GHz, in a way that critically depends on the OU correlation time. In figure 2(b) we plot the maximum of the spectral density at the frequency of the driving field as a function of the external noise amplitude $D^{1/2}$, for five different values of $\tau_c$. An interesting non-linear behaviour of this quantity is observed for increasing noise intensities.
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Figure 1. (a) Spectral density $S_0(E)$ at zero frequency and (b) variance of electron velocity fluctuations as a function of the amplitude of the oscillating electric field.

Figure 2. (a) Spectral density of electron velocity fluctuations as a function of the frequency. The solid line shows the results obtained in the absence of external noise; the dotted line shows the results obtained with $D^{1/2} = 7 \text{ kV} \cdot \text{cm}^{-1}$ and $\tau_c = 1 \text{ ps} = 0.5T$; the dashed–dotted line is obtained with $D^{1/2} = 7 \text{ kV} \cdot \text{cm}^{-1}$ and $\tau_c = 16 \text{ ps} = 8T$. (b) Height of the peak in the spectral density of electron velocity fluctuations as a function of the external noise amplitude for five different values of the correlation time $\tau_c$ of the added noise source.

and correlation times. In particular, for values of $\tau_c$ smaller than or equal to the period $T$ of the oscillating electric field, the spectral density at 500 GHz shows a monotonic decreasing trend with increasing noise amplitude. For values of $\tau_c$ greater than $T$, the spectral density is reduced only for small amplitudes of the external noise, while an enhancement of the...
peak is observed for greater intensities. When the intrinsic noise is mainly due to the partition effect, the height of the peak in the spectral density depends on the population of the different valleys, reaching a maximum when the populations are nearly at the same level [15, 16]. Since the ‘effective’ electric field experienced by electrons in the presence of a fluctuating field is different, the number of intervalley transitions changes with respect to the case in which the external source of noise is absent. This could be responsible of the observed changes on the peak of the spectral density.

The dependence of the intrinsic noise suppression effect on the amplitude and the correlation time of the external source of fluctuations has been investigated also by studying the integrated spectral density (ISD), i.e. the total noise power, as a function of the OU noise amplitude, for three different values of $\tau_c$, namely 0.5, 2 and 8$T$. In figure 3 we show a clear reduction of the ISD in the presence of external noise. In particular, for each value of the correlation time we find a range of $D^{1/2}$ in which the electric field fluctuations reduce the semiconductor intrinsic noise. This effect is more evident for higher correlation times.

From a microscopic point of view, this suppression can arise from the fact that the fluctuating electric field forces the carriers to visit regions of the momentum space characterized by a smaller variance with respect to the case for zero noise [7]. We have investigated the details of the electron dynamics under the fluctuating electric field by analysing the relative occupation time and the velocity variance separately in different valleys, for different correlation times. In figure 4 (left panels) we show that, when the noise intensity increases, the electron occupation time of the $\Gamma$ valley decreases and the corresponding times calculated for the L and X valleys increase. This behaviour is expected because the addition of fluctuations to the driving electric field leads to an
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Figure 4. Relative occupation time (left panels) and variance of the electron velocity fluctuations (right panels) as a function of the external noise amplitude, for three different correlation times.

increase of scattering events which are responsible for an increase of transitions from the Γ valley to valleys at higher energy. Moreover, this behaviour depends on the correlation time of the external noise source. In particular, for any fixed value of the external noise amplitude, the effect of reduction of the relative occupation time for the Γ valley and
the corresponding increase for the L valleys is more pronounced for shorter correlation times.

Less obvious is the behaviour of the electron velocity variance evidenced in figure 4 (right panels). In fact, while common experience would suggest an increase of the velocity variance when the external noise intensity grows, we find that the velocity variance in the Γ valley can be reduced in a specific range of the noise amplitude, depending on the value of the correlation time. An increasing trend is instead observed for the L and X valleys. The reduction of the electron velocity variance observed in the Γ valley for \( \tau_c = 2T \) and \( D^{1/2} \) between 1 and 6 and, even more so, for \( \tau_c = 8T \) and \( D^{1/2} \) between 1 and 8, represents an intrinsic effect of the dynamics of electrons in the Γ valley without taking into account any transfer to valleys characterized by different dynamical properties. This effect of noise-induced stability can explain the longer residence times of electrons in the Γ valley at higher correlation times.

### 4. Conclusions

The results reported in this work confirm that the intrinsic noise in an n-type GaAs semiconductor can be reduced by the addition of external fluctuations to the driving high-frequency periodic electric field. These findings have been obtained by investigating the noise-induced modifications of the spectrum of electron velocity fluctuations and the ISD. The reduction of the characteristic peak (at the driving frequency) of the noise spectral density has been observed for a wide range of noise amplitudes and for OU correlation times shorter than the period of the driving electric field. For higher values of \( \tau_c \), a non-monotonic behaviour of this maximum is clearly evident. A less noisy response in the presence of a driving periodic electric field containing time-correlated fluctuations is observed. This interpretation is confirmed by our study on the electron velocity variance, calculated separately for every single energy valley of the semiconductor. Previous studies ascribe the reduction of the electron velocity fluctuations to an overall effect of intervalley transfers. In this work, we have shown that the velocity variance of an electron moving in the Γ valley is reduced by the presence of correlated noise, independently of the transitions to upper valleys, bringing longer residence times. This effect of noise enhanced stability (NES) arises from the fact that the transport dynamics of electrons in the semiconductor benefits from the constructive interplay between the fluctuating electric field and the intrinsic noise of the system.

To conclude, both the amplitude and the correlation time of the electric field fluctuations are important for the intrinsic noise reduction effect. Further studies are needed to investigate the existence of a relationship between the semiconductor characteristic timescales, which essentially depend on the scattering probabilities, the noise correlation time and the period of the oscillating driving field.

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