Duality and Massless Monopoles*

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Abstract

Duality arguments suggest the existence of massless magnetic monopoles in gauge theories with the symmetry broken to a non-Abelian subgroup. I discuss how these arise and show how they are manifested as clouds of massless fields surrounding massive monopoles. The dynamics of these clouds is discussed, and the scattering of massless monopole clouds and massive monopoles is described.

1 Introduction

A quarter century ago 't Hooft and Polyakov showed [1] that magnetic monopoles could arise as topologically stable classical solutions in certain spontaneously broken gauge theories. The fact that these included all grand unified theories led to a considerable theoretical effort to study the detailed properties and the astrophysical and cosmological implications of these GUT monopoles, as well as to a number of experimental searches. However, the failure of these searches to detect any monopoles, together with astrophysical arguments that place stringent bounds on the monopole flux at the Earth’s surface, has quite understandably led to a decreased interest from the phenomenological point of view.

Nevertheless, these monopole solutions remain valuable as tools for probing the properties of quantum field theory. In particular, the properties of monopoles in the Bogomol’nyi-Prasad-Sommerfield (BPS) limit [2] has led to the conjecture [3] that the electric-magnetic duality of Maxwell’s equations might find a quantum field theoretic generalization that exchanges magnetically charged solitons with electrically charged elementary particles. The

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prime candidate for this is \( N = 4 \) supersymmetric Yang-Mills theory. In the case of \( SU(2) \) broken to \( U(1) \), the spectrum of massive particles is invariant under the simultaneous interchange of weak and strong coupling and of electric and magnetic charges. This result generalizes fairly easily to larger gauge groups, as long as the subgroup that remains unbroken is Abelian. However, if the unbroken subgroup is Abelian, the elementary particle sector contains massless particles with electric-type charge (the “gluons” and their superpartners). Duality would then require the existence of massless magnetically charged particles. However, one can easily show that these theories have no massless classical soliton solutions. In this talk, I will argue that, nevertheless, one can find evidence for the required massless monopoles by studying the dynamics of the massive monopoles.

2 BPS monopoles in \( SU(2) \) and larger groups

Throughout this talk I will work in the BPS limit, with monopoles obeying

\[
B_i = D_i \Phi. \tag{2.1}
\]

While this limit can be obtained by taking a limit of coupling constants, it is most naturally understood in the context of Yang-Mills theory with extended supersymmetry, where Eq. (2.1) is equivalent at the classical level to requiring that the soliton preserve half of the supersymmetry. This ensures that the BPS mass formula

\[
M = Q_M v = \int dS_i \text{Tr} B_i \Phi \tag{2.2}
\]

is preserved by quantum corrections.

One important feature of the BPS limit is that the Higgs scalar becomes massless and so can mediate a long-range attractive force between two monopoles. It turns out that this can exactly balance the magnetic repulsion between static monopoles, thus allowing static multimonopole solutions to exist. In fact, for the case of \( SU(2) \) broken to \( U(1) \), all higher charged BPS solutions can be understood as multimonopole solutions of this sort. Not only does a solution with \( n \) units of magnetic charge have \( n \) times

\footnote{Although I have \( N = 4 \) supersymmetric Yang-Mills in mind, the only feature of this theory that I will use explicitly is the fact that the Higgs field is in the adjoint representation. Note also that there is no confinement in this theory, even if there is an unbroken non-Abelian subgroup.}
the mass of the unit monopole, but index theory methods show [5] that the number collective coordinates needed to quantize the solution is precisely $4n$, corresponding to a three-dimensional position and a U(1) phase for each of the $n$ constituent monopoles; excitation of these phase coordinates gives independent dyonic electric charges to the individual monopoles. 

In order to have the possibility of symmetry breaking to a non-Abelian subgroup, we must start with a gauge group of rank $r \geq 2$. The generators of this gauge group can be chosen to be a set of $r$ commuting $H_i$ that form a basis for the Cartan subalgebra, together with a set of raising and lowering operators associated with the roots $\alpha$. The asymptotic adjoint Higgs field in some fixed direction can always be brought into the form

$$\Phi_0 = h \cdot H.$$  

(2.3)

If the $r$-component vector $h$ has nonzero inner products with all of the roots, then the gauge symmetry is broken maximally, to U(1)$^r$; if not, then the roots orthogonal to $h$ yield the root diagram of an unbroken non-Abelian group $K$. 

The long range part of the magnetic field must lie in the unbroken part of the gauge group. Hence, in the direction used to define $\Phi_0$, the leading part of asymptotic magnetic field can also be brought into the Cartan subalgebra and written in the form

$$B_i = g \cdot \frac{H_{\hat{r}_i}}{r^2}.$$  

(2.4)

In the case of maximal symmetry breaking there are $r$ topological charges, one for each of the unbroken U(1) factors. The connection between these and the magnetic charge is most easily seen by choosing a set of $r$ simple roots $\beta_a$. These form a basis for the root lattice of the Lie group with the property that any root can be written as a linear combination of simple roots with coefficients all of the same sign. There are many possible choices for the simple roots, but the vector $h$ associated with the Higgs field can be used to pick out a unique set satisfying $h \cdot \beta_a > 0$. The topological quantization condition [6] then takes the form

$$g = \frac{4\pi}{e} \sum_a n_a \frac{\beta_a}{\beta_a^2}$$  

(2.5)

where the integers $n_a$ are the topological charges. The BPS mass formula,
Eq. (2.2), can be written as

\[ M = g \cdot h = \sum a n_a \left( \frac{4\pi}{e} h \cdot \beta_a \right) \equiv \sum a n_a m_a , \]  

(2.6)

while the number of collective coordinates is

\[ N = 4 \sum a n_a . \]  

(2.7)

These results suggest that, just as in the SU(2) case, all higher charged solutions should be viewed as multimonopole configurations. Now, however, there are \( r \) species of fundamental monopoles, one for each U(1) factor, with the \( a \)th species having mass \( m_a \), topological charges \( n_b = \delta_{ab} \), and four degrees of freedom (three for center-of-mass motion and one U(1) phase). These fundamental monopoles can be explicitly obtained by embedding the SU(2) unit monopole in the SU(2) subgroups defined by the various \( \beta_a \).

When the symmetry breaking is nonmaximal, to \( K \times U(1)^{r-k} \), some of the simple roots, which I denote by \( \gamma_i \), are orthogonal to \( h \) and form a complete set of simple roots for \( K \). The remainder, denoted \( \tilde{\beta}_a \), can required, as before, to obey \( h \cdot \tilde{\beta}_a > 0 \). Equation (2.5) is replaced by

\[ g = 4 \pi e \sum a \tilde{n}_a \tilde{\beta}_a \tilde{\beta}_a^2 + \sum i q_i \gamma_i \gamma_i^2 . \]  

(2.8)

The \( \tilde{n}_a \) and \( q_i \) are integers, with the former being the conserved topological charges.\(^2\)

In general, the corresponding magnetic field has both Abelian and non-Abelian components. In order to avoid certain pathologies\(^3\) associated with non-Abelian magnetic charges, I will assume for the remainder of this talk that \( g \cdot \gamma_i = 0 \) for all \( i \), so that the long-range magnetic field is purely Abelian. (There is little loss of generality in this assumption since, given a configuration with nonzero \( g \cdot \gamma_i \), the additional monopoles needed to cancel the non-Abelian part of the total magnetic charge can be placed at an arbitrarily large distance.) With this assumption, Eqs. (2.6) and (2.7) are replaced by

\[ M = \sum a \tilde{n}_a m_a \]  

(2.9)

\(^2\)In contrast with the maximally broken case, the simple roots are not uniquely determined by the requirement that their inner product with \( h \) be positive. The various allowed sets are related by gauge transformations of the unbroken group. The \( \tilde{n}_a \) are invariant under these transformations, but the \( q_i \) are not.
\[ N = 4 \sum_a \tilde{n}_a + 4 \sum_i q_i. \] (2.10)

Analogy with the maximally broken case would then suggest that there is one fundamental monopole, with mass \( m_a \) and four degrees of freedom, associated each of the \( \tilde{\beta}_a \), and one massless monopole, also with four degrees of freedom, associated with each of the \( \gamma_i \). Indeed, the embedding construction for the massive fundamental monopoles goes through pretty much as before. The massless monopoles, on the other hand, cannot be constructed in this manner: applying the embedding construction to the SU(2) subgroup corresponding to one of the \( \gamma_i \) simply yields a pure vacuum solution. In fact, it is easy to show that there are no localized classical solutions with zero energy.

3 Low energy monopole dynamics

Although we cannot obtain classical solutions corresponding to isolated massless monopoles, one can find evidence of these monopoles in the dynamics of multimonopole systems. The moduli space approximation \[^{10}\] is a convenient tool for studying such systems at low energy. The essential idea is to approximate solutions with slowly moving monopoles \[^{3}\] as being motion on the moduli space of static BPS multimonopole solutions. More precisely, let \( \{ A_i^{BPS}(r, z), \Phi^{BPS}(r, z) \} \) be a family of gauge-inequivalent BPS solutions parameterized by a set of collective coordinates \( z_a \). In the moduli space approximation one adopts the Ansatz

\[
\begin{align*}
A_0(r, t) &= 0 \\
A_i(r, t) &= U^{-1}(r, t)A_i^{BPS}(r, z(t))U(r, t) - iU^{-1}(r, t)\partial_i U(r, t) \\
\Phi(r, t) &= U^{-1}(r, t)\Phi^{BPS}(r, z(t))U(r, t).
\end{align*}
\] (3.1)

With this Ansatz, the time derivatives of the fields are of the form

\[
\begin{align*}
\dot{A}_i &= \dot{z}_j \left[ \frac{\partial A_i}{\partial z_j} + D_i \epsilon_j \right] \equiv \dot{z}_j \delta_j A_i \\
\dot{\Phi} &= \dot{z}_j \left[ \frac{\partial \Phi}{\partial z_j} + [\Phi, \epsilon_j] \right] \equiv \dot{z}_j \delta_j \Phi.
\end{align*}
\] (3.2)

\[^{3}\] These includes dyons with small electric charges, since these correspond to slowly varying U(1) phases.
The terms involving $\epsilon_j$ arise from differentiating the gauge function $U(r,t)$; they are fixed uniquely by Gauss’s law, which turns out to be equivalent to imposing a background gauge condition on $\dot{A}_i$ and $\dot{\Phi}$.

Substituting this Ansatz into the Yang-Mills Lagrangian gives

$$L_{MS} = \frac{1}{2} \int d^3 r \text{Tr} \left[ \dot{A}^2_i + \dot{\Phi}^2 + B^2_i + D_i \Phi^2 \right] = \frac{1}{2} g_{ij}(z) \dot{z}_i \dot{z}_j + M$$  \hspace{1cm} (3.3)

where the static energy $M$ is constant on the moduli space and

$$g_{ij}(z) = \int d^3 r \left[ \delta_i A_k \delta_j A_k + \delta_i \Phi \delta_j \Phi \right].$$  \hspace{1cm} (3.4)

If we interpret $g_{ij}$ as a metric on the moduli space, then the solutions to $L_{MS}$ are simply geodesic motion on the moduli space. In most cases, it turns out not to be practicable to use Eq. (3.4) directly to determine $g_{ij}(z)$. However, by using indirect methods the moduli space metrics for a number of interesting cases have been by more indirect methods. I will make use of some of these results in the next two sections.

4 An SO(5) example

The simplest example where one finds evidence of massless monopoles arises with the gauge group SO(5), whose root diagram is shown in Fig. 1. With the Higgs field vector $h$ is in the direction shown on the left, the symmetry is broken to $U(1) \times U(1)$. There are two species of massive fundamental monopoles, corresponding to the simple roots $\beta$ and $\gamma$. If instead the Higgs vector is vertical, as shown on the right, the unbroken gauge group is $SU(2) \times U(1)$, with the $SU(2)$ having roots $\pm \gamma$. The fundamental $\beta$-monopole remains massive, but now $\gamma$ corresponds to an elusive massless monopole.

I will focus on solutions with

$$g = \frac{4\pi}{e} \left( \frac{\beta}{\beta^2} + \frac{\gamma}{\gamma^2} \right).$$  \hspace{1cm} (4.1)

In the maximally broken case, these are composed of two distinct massive fundamental monopoles, and have an eight-dimensional moduli space whose metric is [11].
Figure 1: The root diagram of $SO(5)$. With the Higgs vector $h$ oriented as in (a) the gauge symmetry is broken to $U(1) \times U(1)$, while with the orientation in (b) the breaking is to $SU(2) \times U(1)$.

\[
\begin{align*}
 ds^2 &= Md_{\text{cm}}^2 + \frac{16\pi^2}{M} d\chi_{\text{tot}}^2 + \left(\mu + \frac{k}{r}\right) \left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \\
 &+ k^2 \left(\mu + \frac{k}{r}\right)^{-1} (d\psi + d\cos\theta d\phi)^2.
\end{align*}
\]

Here $X_{\text{cm}}$ is the center-of-mass position of the two monopoles; $r$, $\theta$, and $\phi$ are their relative coordinates; and $\chi_{\text{tot}}$ and $\psi$ are overall and relative $U(1)$ phases. $M$ and $\mu$ are the total and reduced masses of the monopole pair, and $k$ is a numerical constant related to the normalization of roots.

With $SU(2) \times U(1)$ breaking, Eq. (4.1) corresponds to a combination of one massive and one massless monopole. It turns out to be relatively straightforward to solve the field equations explicitly \[12\]. The solutions are spherically symmetric and depend on eight parameters: three position coordinates $X$, four $SU(2) \times U(1)$ phase angles $\alpha$, $\beta$, $\gamma$, and $\chi$ and one last parameter, $a$, that can take on any real positive value and whose interpretation will become clear shortly. The fields can be decomposed according their transformations under the unbroken $SU(2)$. The singlet terms are just what would be obtained by embedding the unit $SU(2)$ monopole using the composite root $2\beta + \gamma$; they have no dependence on $a$. By themselves, they would describe an object with
a massive core surrounded by a simple Coulomb magnetic field. The doublet terms fall exponentially fast outside the monopole core and are relatively uninteresting. Finally, the triplet parts of the gauge and Higgs fields are given by invariant tensors multiplied by the function

\[ G(r, a) = \left[ \frac{v}{\sinh(erv)} - \frac{1}{er} \right] \left[ 1 + \frac{r}{a} \coth(vr/2) \right]^{-1}. \]  

For \( r \lesssim a \), the second factor on the right hand side is approximately unity, so \( G \), and hence the corresponding \( A_i \), falls as \( 1/r \). This produces a Coulomb magnetic field in the unbroken SU(2) subgroup. However, for \( r \gtrsim a \), the last factor gives an additional \( 1/r \), so the triplet part of \( A_i \) falls as \( 1/r^2 \), and the non-Abelian component of the magnetic field fall more rapidly.

Hence, we may view the solution as being composed of a massive core of radius \( \sim (ev)^{-1} \) surrounded by a “cloud” of non-Abelian fields of radius \( \sim a \). Inside this cloud one finds the magnetic field appropriate to a monopole with both Abelian and non-Abelian magnetic charges; i.e., the charge appropriate to an isolated massive \( \beta \)-monopole. Outside the cloud, only the Abelian charge is evident. Thus, the massless \( \gamma \)-monopole can be viewed as a shell of radius \( \sim a \) surrounding the massive monopole. Curiously, the energy of the solution is independent of \( a \), despite the fact that this parameter is not associated with any symmetry of the system.

We can proceed further and examine the moduli space metric. Using the explicit form of the solutions and Eq. (2.5), one obtains [13]

\[ ds^2 = MdX^2 + \frac{16\pi^2}{M}d\chi^2 + k \left[ \frac{da^2}{a} + a \left( d\alpha^2 + \sin^2\alpha d\beta^2 + (d\gamma + \cos\alpha d\beta)^2 \right) \right] \]  

where \( k \) is the same constant as in Eq. (4.2).

Now let us return for a minute to the maximally broken case and imagine approaching the SU(2)×U(1) case by “rotating” the Higgs vector until it is vertical. This corresponds to taking the \( \gamma \)-monopole mass, and thus the reduced mass \( \mu \), to zero. Taking this limit in the metric of Eq. (4.2) gives precisely Eq. (4.4), except for a curious change in notation: The monopole separation \( r \) becomes the cloud parameter \( a \), while the spatial angles \( \theta \) and \( \phi \) combine with the relative U(1) phase \( \psi \) to give the SU(2) Euler angles.
5 A more complex system

An example with somewhat more structure is obtained by considering the case of \( SU(N+2) \) broken to \( U(1) \times SU(N) \times U(1) \), with the unbroken \( SU(N) \) lying in the middle \( N \times N \) block in a basis where the eigenvalues of the Higgs field decrease monotonically along the diagonal. A purely Abelian asymptotic magnetic field can be obtained by setting all of the \( \tilde{n}_j \) and \( q_i \) in Eq. (2.8) equal to unity; i.e., by combining one each of the two species of massive and \( N-1 \) species of massless monopoles. The moduli space of such solutions is \( 4(N+1) \)-dimensional, with the collective coordinates including a position and \( U(1) \) phase for each of the massive monopoles, a number of \( SU(N) \) orientation angles, and a single cloud parameter, \( b \), that can take on any value greater than or equal to \( r \), the separation between the two massive monopoles.

The solutions can be obtained explicitly [14] by using the Nahm construction [15]. Although their detailed structure is somewhat complex, their behavior well outside the monopole cores is fairly simple. The asymptotic Higgs field can be written as

\[
\Phi_\infty(r) = U^{-1}(r) \text{diag}(v_3, v_2, \ldots, v_2, v_1) U(r)
\]

where \( U(r) \) is a gauge transformation whose form will not concern us and the Higgs eigenvalues satisfy \( v_3 > v_2 > v_1 \). The unbroken \( SU(N) \) lies in the middle \( N \times N \) block corresponding to the repeated eigenvalue \( v_2 \). The form of the magnetic field depends on how the distances \( y_L \) and \( y_R \) from the two massive monopoles compare to \( b \). For \( y_L, y_R \ll b \),

\[
B(r) = U^{-1}(r) \text{diag} \left( \frac{\hat{y}_R}{2y_R^2}, -\frac{\hat{y}_R}{2y_R^2}, \frac{\hat{y}_L}{2y_L^2}, 0, \ldots, 0, -\frac{\hat{y}_L}{2y_L^2} \right) U(r) + \cdots
\]

where the dots represent terms that fall off more rapidly with distance, while for \( y \equiv (y_L + y_R)/2 \gg b \)

\[
B(r) = U^{-1}(r) \text{diag} \left( \frac{\hat{y}}{2y^2}, 0, 0, \ldots, 0, -\frac{\hat{y}}{2y^2} \right) U(r) + \cdots
\]

Thus, at distances smaller than \( b \) one sees fields corresponding to both Abelian and non-Abelian magnetic charges, but at larger distances only the Abelian component survives, just as in the SO(5) example.
It is instructive to study the scattering of the two massive monopoles in these solutions. Because these monopoles are associated with orthogonal roots [or, equivalently, because they correspond to embeddings into commuting $2 \times 2$ blocks of the SU($N$) matrices], there is no direct interaction between them. Instead, they must interact through the massless monopole cloud. In the low energy limit, we can use the moduli space approximation to study this process [10]. The moduli space metric for the maximally broken case with $N+1$ distinct massive fundamental monopoles is known [11]. The SO(5) example of the previous section suggests that this metric is still valid when $N-1$ of these monopoles become massless, although, just as in that case, the physical interpretations of the collective coordinates change in the massless limit, with the positions of the massless monopoles being transformed into gauge orientations and a single gauge invariant parameter, $b$. The various gauge orientation and angular variables are most easily handled by expressing them in terms of the conserved charges and angular momentum. This still leads to a fairly complicated dynamics, but matters simplify considerably if the electric-type charges in the unbroken U(1) $\times$ SU($N$) $\times$ U(1) all vanish. After separating out the total center-of-mass motion, one is then left with an effective Lagrangian

$$L = \left[ \frac{\mu}{2} + \frac{kb}{4(b^2 - r^2)} \right] r^2 + \frac{kb}{4(b^2 - r^2)} b^2 + \frac{br}{b^2 - r^2} \dot{b}^2 - \frac{bJ^2}{2r^2(k + \mu b)}$$

(5.4)

governing the time development of $b$ and $r$. Here $J$ is the angular momentum, $\mu$ is the reduced mass of the massive monopoles and, as in the previous section, $k$ is a numerical constant.

Analysis of the Euler-Lagrange equations that follow from Eq. (5.4) shows that $r$ and $b$ decouple at large times. Thus, at large $|t|$ the energy of the system is approximately the sum of a massive monopole kinetic energy $E_r = \mu r^2/2$ and a cloud energy $E_b = \dot{b}^2/4b$, with $r$ and $b$ varying asymptotically as

$$r \sim v|t| + \cdots$$
$$b \sim ct^2 + \cdots$$

(5.5)

with $v$ and $c$ constants. In a typical scattering process the cloud initially decreases in size while simultaneously the two massive monopoles approach one another. Eventually $b$ and $r$ reach their minimum values (generally not at the same time) and then begin to increase again according to Eq. (5.5).
In the course of this process, energy is exchanged between the cloud and the massive monopoles, so that the final splitting between $E_r$ and $E_b$ at $t \to \infty$ is different from the initial one at $t \to -\infty$.

One surprising feature of Eq. (5.5) is the quadratic growth of $b$ with time. It seems likely that this is an artifact of the moduli space approximation, and that radiation of massless gluons will have the effect of reducing this to a linear growth.

6 Summary and conclusion

Electric-magnetic duality in gauge theories with the symmetry spontaneously broken to a non-Abelian subgroup requires the existence of massless magnetically charged objects that would be the duals to the electrically charged massless gauge bosons. These massless monopoles cannot be exhibited as isolated classical soliton solutions. However, as I have shown, their existence is manifested through clouds of non-Abelian field surrounding one or more massive monopoles. These clouds are described by a small number of collective coordinates, which correspond to the massless monopole degrees of freedom; for static solutions, the energy is independent of the values of these parameters.

Duality suggests that these cloud parameters should have counterparts in the perturbative sector. It would be instructive to understand more precisely what these are, and to look in perturbative sector scattering for analogies with the low-energy scattering of massive monopoles and massless monopole clouds. Investigations in this direction would certainly lead to deeper insights into the properties of non-Abelian gauge theories.

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