The effects of Galactic model uncertainties on LISA observations of double neutron stars

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

Observations of binaries containing pairs of neutron stars using the upcoming space-based gravitational wave observatory, LISA, have the potential to improve our understanding of neutron star physics and binary evolution. In this work we assess the effect of changing the model of the Milky Way’s kinematics and star formation history on predictions of the population of double neutron stars that will be detected and resolved by LISA. We conclude that the spatial distribution of these binaries is insensitive to the choice of galactic models, compared to the stochastic variation induced by the small sample size. In particular, the time-consuming computation of the binaries’ Galactic orbits is not necessary. The distributions of eccentricity and gravitational-wave frequency are, however, affected by the choice of star-formation history. Binaries with eccentricities $e > 0.1$, which can be measured by LISA observations, are mostly younger than $100$ Myr. We caution that comparisons between different predictions for LISA observations need to use consistent star formation histories, and that the Galactic star formation history should be taken into account in the analysis of the observations themselves. The lack of strong dependency on Galactic models means that LISA detection of double neutron star binaries may provide a relatively clean probe of massive binary star evolution.

Key words: Gravitational waves – instrumentation: detectors – stars: neutron – Galaxy: structure

1 INTRODUCTION

The Laser Interferometer Space Antennae (LISA) is an upcoming space-based gravitational wave (GW) detector. LISA is set to launch sometime in the next decade and will observe a diversity of mHz frequency gravitational sources that includes Galactic compact binaries in the mHz GW regime (Amaro-Seoane et al. 2022). The most numerous such systems will be binaries containing two white dwarfs, but binaries containing neutron stars will also be observed and offer the possibility to constrain binary evolution, neutron-star formation and the equation of state of neutron-star material. This work focuses on double neutron stars (DNSs).

We currently know of approximately a dozen DNS candidates, as summarised in Tauris et al. (2017). DNSs are of particular astrophysical interest because short gamma-ray bursts can originate from the coalescence of DNSs: this model was corroborated by the observation of GRB 170817A and its associated kilonova shortly following the first DNS merger observed in gravitational waves, GW 170817 (Goldstein et al. 2017; Abbott et al. 2017; Smartt et al. 2017; Metzger 2019). Studying DNS systems is challenging, as at least one of the neutron stars needs to be a pulsar to permit detection. DNSs are particularly bright sources of GWs, and are expected to be detected by LISA millions of years before their coalescence. LISA observations of DNSs will provide insight into the structure and post-formation behaviour of neutron stars (see Özel & Freire (2016) for a review), formation channels of DNS systems (Andrews et al. 2020), detection of binary pulsars (Kyutoku et al. 2019), and short-duration gamma-ray bursts.

In this work, we explore how various models for the Milky Way affect the distribution of LISA observable properties of DNSs. Theoretical predictions for the population of DNSs visible to LISA import uncertainties from both binary evolution and modelling of the stellar populations and dynamics of the Milky Way. The largest uncertainties likely lie in the binary evolution – for example treatment of common-envelope evolution and the winds of low-metallicity stars (Vigna-Gómez et al. 2018) – and this will allow LISA observations to constrain the astrophysics of DNS formation. However, in order to quantify the impact of different binary evolution prescriptions it is necessary to understand the effect of the model adopted for the Galactic environment. Past works on LISA-resolvable DNSs have used a range of analytic models for the Galaxy (see e.g. Lau et al. (2020) and Andrews et al. (2020)) and local-group galaxies (Seto 2019). Lamberts et al. (2019) used cosmological simulations from the FIRE project (Hopkins et al. 2014) to predict the LISA white dwarf population. Yu & Jeffery (2015) studied the effect of varying the galactic star formation history and binary evolution physics on the number and properties of the expected population. None of these works, however, systematically study the effect of varying both their Galactic models and star-formation rates. Understanding the uncertainty in the LISA DNS detection rate caused by uncertainties in galactic models is valuable for understanding LISA’s utility to study DNS formation and evolution.

We take a single prescription for the formation of DNSs and investigate the impact on the expected LISA-observed population of chang-
ing the Galactic potential, stellar distribution and star-formation history. In Section 2 we describe our Fiducial model for the Milky Way, and the resulting LISA-visible binary population is presented in Section 2.4. In Section 3, alternate models are proposed for the Milky Way disc, Galactic potential, and star-formation rate. The effects of these alternate models on the observable population of DNSs are presented in Section 4, and discussed in Section 5.

2 THE FIDUCIAL SIMULATION

In this section, we present our Fiducial model for DNS simulations in the Milky Way. We build up a population using the orbital parameters and space velocities of DNSs at the time of DNS formation from Church et al. (2011), and normalise the amount of DNSs to the Milky Way (§2.1). Birth times are assigned to the DNSs and their binary orbits\(^1\) are integrated to the present day. DNSs are included in the population if their initial eccentricity \(e_i\) and present day second-harmonic GW frequency \(f_{\text{gw,circ}} = 2/P_{\text{orb}}, \) where \(P_{\text{orb}}\) is the binary’s orbital period,\(^2\) meet the following conditions:

\[
\begin{align*}
    e_i &\geq 0 \quad \text{for } 1 > f_{\text{gw,circ}} > 2 \times 10^{-5} \text{ Hz} \\
    e_i &> 0.7 \quad \text{for } 2 \times 10^{-5} > f_{\text{gw,circ}} > 10^{-5} \text{ Hz} \\
    e_i &> 0.9 \quad \text{for } 10^{-5} > f_{\text{gw,circ}} > 10^{-6} \text{ Hz} \\
\end{align*}
\]

since empirically binaries that lie outside these cutoffs are never resolvable. We show the cutoff criteria in Figure 1. The remaining DNSs are assigned an initial position in the Milky Way disc and their Galactic orbits are integrated to the present time (§2.2). The signal-to-noise ratio is calculated for all binaries to determine the LISA-resolvable DNS population (§2.3).

2.1 Binary population synthesis and orbital evolution

The population of DNSs is taken from Church et al. (2011), who synthesised a population of DNSs using the rapid binary population synthesis code BSE (Hurley et al. 2002), modified according to the core mass–remnant mass relationship of Belczynski et al. (2008). They evolved a population of binaries from the zero-age main sequence at solar metallicity, with both masses drawn independently from the Kroupa et al. (1993) initial mass function above 3 \(M_\odot\) and the orbital semi-major axis \(a\) taken to be flat in log \(a\) over the range in which DNSs were produced. They drew the natal kicks of both neutron stars independently from the distribution of (Arzoumanian et al. 2002), which is on the stronger side of literature kick distributions; they found that this choice reproduced well the orbital properties of observed Galactic DNSs. Church et al. (2011) evolved \(4 \times 10^7\) galactic binaries which yielded 2857 bound DNSs; we take these as our input population. As our “neutron-star kick” we adopt the total effect of the natal kicks and mass-loss kicks on the Galactic velocity of the centre of mass of the DNS.

We normalise the population to the Milky Way by matching the inferred present-day DNS merger rate of \(\mathcal{R}_{\text{MW}} = 42 \text{ Myr}^{-1}\) in the whole Milky Way (Belczynski et al. 2018; Pol et al. 2019). This merger rate is the peak probability value of observational data from DNS merging within a Hubble time, with a confidence of 90% between 28 and 72 DNSs per Myr. To calculate the merger rate per Myr of a given simulated population, we assign birth times to each DNS and calculate how many DNSs merged during the last Myr. The merger rate varies across different realisations of the same population, so we take an average merger rate over 100 realisations.

The Fiducial model assumes a constant star-formation rate (SFR) for the Milky Way, with a maximum age of 10 Gyr. To obtain the correct present-day Galactic merger rate we scale the Church et al. (2011) population by a factor of 256 by randomly re-sampling.

We integrate the orbital parameters of the DNS population by following the orbital evolution due to quadrupole GW emission. The eccentricities and semi-major axes are evolved using the following equations from Peters (1964):

\[
\begin{align*}
    \dot{a} &= -\frac{64}{5} \frac{G^3}{a^3} \frac{m_1 m_2 (m_1 + m_2)}{\left(1 - e^2\right)^{7/2}} \left[1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right], \\
    \dot{e} &= -\frac{305}{15} \frac{G^3}{a^4} \frac{m_1 m_2 (m_1 + m_2)}{\left(1 - e^2\right)^{5/2}} \left[1 + \frac{121}{304} e^2\right],
\end{align*}
\]

where \(a\) is the semi-major axis, \(e\) is the eccentricity, \(m_1\) and \(m_2\) are the masses of the two neutron stars.
2.2 Galactic disc and potential

We assign positions to the DNS population by using the exponential density model of the Milky Way’s stellar disc (Gilmore & Reid 1983; McMillan 2017). DNSs are assumed to form in the disc’s mid-plane due to the short evolutionary time-scale of their progenitors. The stellar density follows

\[ \rho_d(R, z) = \frac{\Sigma_0}{2\pi z_d} \exp\left(-\frac{|z|}{z_d} - \frac{R}{R_d}\right), \]

(3)

with a central surface density \( \Sigma_0 = 886.7 \pm 116.2 M_\odot \text{pc}^{-2} \), scale height \( z_d = 300 \text{pc} \), and scale length \( R_d = 2.60 \pm 0.52 \text{kpc} \). We assume that all of the DNSs form in the thin disc.

The stellar surface density for the plane of the disc is obtained by integrating the stellar density for all \( z \).

\[ \Sigma(R) = \int_{-\infty}^{\infty} \rho_d(R, z) dz = \Sigma_0 \exp\left(-\frac{R}{R_d}\right). \]

(4)

We sample the distribution in Equation 4 to build up the initial positions of the DNS population.

Using the integrator developed by McMillan (2017), we integrate the positions in the Milky Way of the DNS population from their positions of the DNS population. We sample the distribution in Equation 4 to build up the initial positions of the DNS population. The positions in the Milky Way of the DNS population from their positions of the DNS population. We sample the distribution in Equation 4 to build up the initial positions of the DNS population.

2.3 Resolvability to LISA

The LISA sensitivity curve is expressed as the effective noise power spectral density \( S_n \) from Robson et al. (2019):

\[ S_n(f) = \frac{10}{3L^2} \left[ P_{\text{OMS}} + 2 \int \cos^2\left(\frac{f}{f_c}\right) \frac{P_{\text{acc}}}{(2\pi f)^4} \left(1 + \frac{3}{5} \left(\frac{f}{f_c}\right)\right) \right], \]

(5)

\[ P_{\text{OMS}} = (1.5 \times 10^{-11} \text{m}^2)(1 + \left(\frac{2 \text{mHz}}{f}\right)^4) \text{Hz}^{-1}, \]

(6)

\[ P_{\text{acc}} = (3 \times 10^{-15} \text{m}^2 \text{s}^{-2})^2 \left(1 + \left(\frac{0.4 \text{mHz}}{f}\right)^2\right)^2 \left(1 + \left(\frac{f}{8 \text{mHz}}\right)^4\right) \text{Hz}^{-1}. \]

(7)

where \( L = 2.5 \times 10^9 \text{m} \) is the length of the LISA arms, \( f_c = 19.09 \times 10^{-3} \text{Hz} \) is the transfer frequency, \( P_{\text{OMS}} \) is the single-link optical metrology noise, and \( P_{\text{acc}} \) is the single proof mass acceleration noise. Both \( P_{\text{OMS}} \) and \( P_{\text{acc}} \) are expressed in power spectral density.

The number of DNSs per frequency bin increases with decreasing \( f_{gw,\text{circ}} \) – this is true for all Galactic binaries, particularly double white dwarfs which become so highly concentrated below 1 mHz that LISA is unable to distinguish between individual sources (e.g. Nelemans et al. 2001; Korol et al. 2017; Lamberts et al. 2019). This effect manifests as a galactic confusion noise \( S_c \) contribution to the LISA sensitivity curve (Babak et al. 2021).

\[ S_c(f) = Af^{-7/3}e^{-\left(\frac{f}{f_{gw,\text{circ}}}\right)^{12}} \left(1 + \tanh\left(-\frac{f-f_k}{f_1}\right)\right), \]

(8)

where the parameter values are given in Table 1. The complete LISA curve is then the sum of \( S_n \) and \( S_c \). We compare the number of resolvable DNSs for a realisation of the Fiducial model calculated with the galactic background of Babak et al. (2021), henceforth the Babak background to the galactic background from Robson et al. (2019), the Robson background:

\[ S_c(f) = Af^{-7/3}e^{-\left(\frac{f}{f_{gw,\text{circ}}}\right)^{12}} \left(1 + \tanh\left(\gamma\left(f-f_k\right)\right)\right), \]

(9)

where the parameter values are given in Table 1. The LISA sensitivity curves resulting from the two backgrounds are shown in Figure 3.

Following Lau et al. (2020), we define a binary to be resolvable to LISA if it has a SNR greater than 8. The sky-averaged, inclination averaged, and polarisation-averaged SNR \( \rho \) is calculated for each DNS. We use the python package legwork (Wagg et al. 2022b) to facilitate SNR calculations of binaries with non-negligible eccentricities. We also use legwork to determine the GW harmonic at which the signal has the highest SNR, which we denote by \( f_{gw} \).

The amplitude spectral density \( \sqrt{\bar{N}} \) is calculated from the SNR of the DNS and PSD of LISA at the dominant frequency \( f_N \)

\[ \sqrt{\bar{N}} = \rho \sqrt{\bar{N}_n(f_N) + S_c(f_N)}. \]

2.4 Fiducial Results

We computed fifty realisations of the Fiducial model. The resolvable binaries from all these realisations are plotted as triangles in Figure 1, coloured by their ages. Young DNSs at high eccentricity can contribute even at relatively large semi-major axes since the higher harmonics of their GW emission fall in the frequency region where LISA is most sensitive. Older resolvable binaries have had time to circularise and evolve from wide to closer orbits. This dichotomy can clearly be seen in the upper panel of Figure 2, where the young, high-eccentricity binaries are systematically at larger values of \( f_{gw} \) despite being wider than their low-eccentricity, old counterparts. The lower panel of Figure 2 shows the frequency harmonic which contributes most strongly to the binary’s detectability. This illustrates the importance of including eccentricity when calculating gravitational-wave observables for non-circular binaries. The background colourmap shows the population of binaries that are not resolvable from a single run of the Fiducial model. This population increases in number towards larger orbital semi-major axis and hence we cut out the wider binaries that empirically never become resolvable to reduce the computational cost of evaluating the model; the cuts are shown as dashed cyan lines in the Figure.

In Figure 3 we plot the amplitude spectral density \( \sqrt{\bar{N}} \) as a function of \( f_{gw} \) for a typical realisation of our Fiducial model. The number of resolvable binaries is sensitive to the SNR cut for resolvability, which in turn sensitive to the galactic background. The Robson background peaks at lower frequencies than that of the Babak background, towards the protrusion of DNSs going above the LISA curve. This in turn means that the Robson background removes more of the DNSs as the curve is higher. The number of resolvable binaries for this particular run was 61 using the Babak background background, and 55 using the Robson background. These numbers are similar to Lau et al. (2020) but distributed differently in \( f_{gw} \). In particular, they are

| Background | \( A \) [10^{-44}] | \( \alpha \) [10^{-3}] | \( f_k \) [10^{-3}] | \( \gamma/f_k \) [10^{-3}] | \( f_1 \) [10^{-3}] | \( \beta \) |
|------------|----------------|----------------|----------------|----------------|----------------|--------|
| Babak      | 1.14           | 1.8            | 2.33           | 0.31           | 1.41           | -221   |
| Robson     | 0.9            | 0.138          | 1.13           | 1.68           | -                 |        |

Table 1. Values of the confusion noise parameters after a 4 yr mission (Robson et al. 2019; Babak et al. 2021).
Figure 2. GW frequency of resolvable DNSs from the *Fiducial* model across all realisations as a function of present day eccentricity. (top) The DNSs are coloured according to their ages. Two distinct populations emerge: young binaries with higher eccentricities than old binaries. (bottom) The DNSs are coloured according to the GW harmonic which corresponds to the highest SNR.

Figure 3. Amplitude spectral density \( \sqrt{S_n} \) as a function of gravitational-wave frequency \( f_{gw} \). The blue line is the power-spectral density of noise in LISA including the Robson background confusion model; the red line is the same but for the Babak background. The blue triangles are DNSs for one realisation of the *Fiducial* model that are resolved by LISA (i.e. with SNR $> 8$) using the Robson sensitivity curve, while the red triangles are the resolved DNSs using the Babak sensitivity curve. The DNSs are plotted according to the GW harmonic with the largest SNR. The gray dashed line is the strain amplitude of a DNS located at the distance of the Galactic Centre, 8.3 kpc from LISA (McMillan 2017). The binary is taken to be circular, with both neutron stars having masses of $1.4 M_\odot$.

### 3 ALTERNATE MODELS

A summary of the set of galactic models that we consider can be found in Table 3; the details of the models are presented in the following sections.

#### 3.1 Stellar density in the disc

##### 3.1.1 Miyamoto distribution

Instead of using a standalone stellar density for the disc, we can also obtain a density profile from a model potential by making use of Poisson’s equation \( \nabla^2 \Phi_d = 4 \pi G \rho_d \), henceforth the *Miyadens* model. Following Lau et al. (2020), the resulting stellar density from the Miyamoto & Nagai (1975) potential is

\[
\rho_d(R, z) = \frac{b_d^2 M_d}{4 \pi} \frac{a_d R^2 + \left(a_d + 3 \sqrt{z^2 + b_d^2}\right)^2}{\left(z^2 + b_d^2\right)^{3/2}} \left[R^2 + \left(a_d + \sqrt{z^2 + b_d^2}\right)^2\right]^{5/2}. \tag{10}
\]

where the constants \( M_d, a_d, \) and \( b_d \) are the same as used for the potential in Section 3.2.1 and can be found in Table 2. In the same way as section 2.2, the stellar surface density is obtained by integrating the density over all \( z \) (in this case, the integration is done numerically). The stellar distribution calculated from the Miyamoto & Nagai (1975) potential is more extended than Gilmore & Reid (1983), with about 10% of stars beyond 20 kpc from the Galactic Centre.

##### 3.1.2 Spiral arms

The Milky Way’s disc is not homogeneous; in particular, star formation is concentrated in the spiral arms. We consider a model where all DNSs are born in the spiral arms, henceforth the *Spiral* model. We
use the same radial stellar density as the Fiducial model but place the binaries along the two-armed bared spiral following Pichardo et al. (2003). The initial angle of a DNS in the Fiducial model is changed to:

$$\phi_{\text{spiral}}(R) = \frac{1}{N \tan i_p} \ln \left[ 1 + \left( \frac{R}{R_s} \right)^N \right] - \Omega_p t_{\text{birth}} - \frac{\pi}{6},$$

(11)

where $N = 100$ determines the prominence of the bar, $i_p = 11^\circ$ is the winding angle, $R_s = 3.3$ kpc is the length of the bar, $\Omega_p = 0.0204402$ km s$^{-1}$ kpc$^{-1}$ is the angular velocity of the spiral, and $t_{\text{birth}}$ is the birth time of a DNS. The spiral is rotated $\pi/6$ from the Sun-Galactic line as it is the estimated direction of the bar.

The spiral arm’s angle is retroactively applied to the Fiducial model $\phi_{\text{spiral, f}} = \phi_{\text{spiral, i}} + (\phi_{\text{Fiducial, f}} - \phi_{\text{Fiducial, i}})$. This method is valid since our adopted Galactic potential is axisymmetric, hence the angle difference between birth and present time is independent of starting angle. The effect of applying spiral arms to our population is plotted in Figure 5.

### 3.2 Integration models

#### 3.2.1 Miyamoto and Nagai potential

We investigate a potential that has components from the bulge, disc, and dark matter halo (Miyamoto & Nagai 1975; Paczynski 1990), henceforth the MiyaPot model. The potential is given in cylindrical coordinates and is axisymmetric. We choose this potential because...
of its analytical form and extensive use in the literature.

\[
\Phi_s(R,z) = -\frac{GM_s}{\sqrt{R^2 + (a_s + \sqrt{z^2 + b_s^2})^2}},
\]

(12)

\[
\Phi_d(R,z) = -\frac{GM_d}{\sqrt{R^2 + (a_d + \sqrt{z^2 + b_d^2})^2}},
\]

(13)

\[
\Phi_b(r) = \frac{GM_s}{r_c} \left[ \frac{1}{2} \ln \left( 1 + \frac{r^2}{r_c^2} \right) + r_c \arctan \left( \frac{r}{r_c} \right) \right],
\]

(14)

where \( r = \sqrt{R^2 + z^2} \). The full Miyamoto potential is the sum of the three potentials: \( \Phi = \Phi_s + \Phi_d + \Phi_b \). Values for the parameters can be found in Table 2. It is shallower than the McMillan (2017) potential used for the Fiducial model.

### 3.2.2 No Integration

Following Lau et al. (2020), who do not integrate the positions of their DNS population but rather set their positions at the birth site of their progenitors, we include a model where the final positions of the DNS population are sampled directly from the stellar disc density given in Equation 4. This NoInt model tests whether integrating the trajectories of DNSs has a significant effect on the population of LISA-resolvable binaries.

### 3.3 Star formation rate of the Milky Way

We investigate two alternative models for the star formation history. Both models follow an exponentially decreasing star formation rate, with one of the models including a late starburst phase.

#### 3.3.1 Exponential SFR profile

We take Model A from Just & Jahreiß (2010) (henceforth the J&J model), which uses data from the HIPPARCOS mission. The model is formulated as:

\[
\text{SFR}(t) = \text{SFR}_0 \left( \frac{t + t_0}{t_1 + t_0} \right)^{4/3} \left( \frac{t + t_0}{t_1 + t_0} \right)^{2/3}
\]

(15)

where \( \text{SFR} = 3.75 M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \), \( t_0 = 5.6 \text{ Gyr} \), \( t_1 = 8.2 \text{ Gyr} \), and \( t_n = 9.9 \text{ Gyr} \).

This model has an initially high SFR and exponentially decreases to present time, and assumes the Milky Way formed 12 Gyr ago. To reach \( \mathcal{R}_{\text{MW}} = 42 \text{ Myr}^{-1} \) (Belczynski et al. 2018; Pol et al. 2019), we scale the DNS population from Church et al. (2011) by a factor of 617.

#### 3.3.2 Exponential SFR profile (with starburst)

For our starburst model we take the SFH of Mor et al. (2019), henceforth the Mor model, which uses data from Gaia DR2. This model has an initially high SFR which decreases exponentially, with a starburst occurring around 2 Gyr from the present day. Mor et al. (2019) assume that the Milky Way formed 10 Gyr ago. The DNSs formed using the Mor model are much older than for a constant SFR, with the average resolvable DNS having a birth time of 4 Gyr. To reach our normalisation criterion of \( \mathcal{R}_{\text{MW}} = 42 \text{ Myr}^{-1} \) (Belczynski et al. 2018; Pol et al. 2019) we scale the DNS population from Church et al. (2011) by a factor of 525.

### 3.4 SFR with radial-temporal correlation

In this model, we present a time-space coupled SFR based on an analytical inside-out model of the Milky Way from Schönrich & McMillan (2017). We use this model to probe how a radial dependence on the SFR affects the characteristics of the LISA resolvable DNSs. An initial gas mass for the MW disc of \( 1 \times 10^{9} M_\odot \) is set at 12 Gyr ago. Two gas accretion inflow components feed the galactic disc, with one component accreting \( 5 \times 10^{10} M_\odot \) and the other accreting \( 1 \times 10^{11} M_\odot \), supporting star formation. Both accretion components are exponentially decreasing, with exponential decay times of 1 Gyr and 9 Gyr respectively.

The gas disc is assumed to always follow an exponential profile, with scale length determined by:

\[
R_d(t) = R_{d,0} + (R_{d,e} - R_{d,0}) \times \left\{ \arctan \left( \frac{t - t_0}{t_g} \right) + \arctan \left( \frac{t_0}{t_g} \right) \right\} \times N,
\]

where \( R_{d,0} = 0.75 \text{ kpc} \) is the initial scale length, \( R_{d,e} = 3.75 \text{ kpc} \) is the final scale length after 12 Gyr, and \( N \) is a normalisation condition such that \( R_d(t = 12 \text{ Gyr}) = 3.75 \text{ kpc} \). \( t_g = 2 \text{ Gyr} \) is the growth time scale and \( t_0 = 1 \text{ Gyr} \) is the offset time.

For star formation, we follow the Kennicutt (1998) law. The stellar surface density \( \Sigma_\star \) changes according to the current gas density \( \Sigma_g \):

\[
\Sigma_\star = 0.15 \begin{cases} \Sigma_g^{1.4} & \text{if } \Sigma_g > \Sigma_{\text{crit}} \\ \Sigma_g^{2.6} \Sigma_{\text{crit}}^{.4} & \text{if } \Sigma_g \leq \Sigma_{\text{crit}} \end{cases}
\]

where \( \Sigma_{\text{crit}} = 4 M_\odot \text{ pc}^{-2} \) is the critical density for star formation. We assume the gas density decreases at the same rate as the stellar density increases (all of the gas is transformed into stars.)

The resulting SFH is one which resembles the J&J model introduced in the previous section, with an initially large SFR that decreases exponentially to the present day. We make a decoupled version of the model, where we sample the radial and temporal distributions independently, to compare with our original SFH. The coupled and decoupled models are named \( \text{TRadCoup} \) and \( \text{TRadDecoup} \) respectively. For both coupled and decoupled models, we scale our population by a factor of 670 to reach the normalisation criterion of \( \mathcal{R}_{\text{MW}} = 42 \text{ Myr}^{-1} \) (Belczynski et al. 2018; Pol et al. 2019).

### 4 RESULTS

Fifty realisations were computed for each alternative model. The Robson background was used to compute the SNR. Synthetic observations are presented in the subsequent sections. We compare the distributions of distances, both radially from LISA and distance out of the disc, as well as gravitational wave frequencies and eccentricities for the resolvable DNS population across each model.
| Stellar distribution | Fiducial | MiyaPot | NoInt | MiyaDens | Spiral | J&J | Mor | TRad |
|----------------------|----------|---------|-------|----------|--------|-----|-----|------|
| Equation 4           |          | //      | //    | Obtained numerically | //     | //  | //  | Obtained numerically |

| Galactic potential   | PJM_17   | Equation 13 | None | // | // | // | // | // |
|----------------------|----------|--------------|------|----|----|----|----|----|

| SFR distribution     | Uniform | // | // | // | // | // | Exponential | +Starburst | Obtained numerically |

4.1 Number of resolvable binaries

The average number of resolvable binaries across all realisations of the various models is given in Table 4. The number of resolvable binaries scales directly with the present-day DNS merger rate. Alternative determinations of the DNS merger rate show that it can be much lower than $R_{\text{MW}} = 42 \text{ Myr}^{-1}$ (Kim et al. 2015; Chrulsinska et al. 2018; Kruckow et al. 2018); On the other hand, the empirical merger rate of the MW from the observation of two DNS mergers by LIGO is $R_{\text{MW}} = 210 \text{ Myr}^{-1}$ (The LIGO Scientific Collaboration et al. 2019). This means that the number of DNS system resolvable to LISA, while influenced by changing galactic models as shown in Table 4, is dominated by the uncertainty on $R_{\text{MW}}$.

The average number of resolvable binaries produced varies by model, especially for models considering alternate stellar densities. The MiyaDens model has the lowest number of resolvable binaries per run. The stellar density is lower between the Galactic Centre and LISA, compared to the Fiducial model, yielding fewer binaries. Binaries near the Galactic Centre make up the majority of high-frequency systems, whereas lower-frequency binaries must be closer to LISA in order to be observed. The binaries lost from this change in stellar density are those whose frequencies are around $10^{-3}$ Hz since their resolvability is more sensitive to changes in distance, compared to binaries with $f_{\text{gw}} > 6 \times 10^{-3}$ Hz.

4.2 Spatial distribution of resolvable binaries

A cumulative distribution function (CDF) of the distances from LISA to the resolvable binaries in our various models is shown in Figure 6. Most of the alternative models are consistent with the Fiducial model given the spread of individual runs, with the exception of the MiyaDens model. The MiyaDens model extends to larger galactic radii than the Gilmore & Reid (1983) distribution while the TRad models show the largest deviation towards a flatter disc, apart from the NoInt model in which $z = 0$ for all DNSs.)
Table 4. The mean $\mu$ and standard deviation $\sigma$ of the number of resolvable binaries across all runs in each model.

|                | Fiducial | MiyaPot | NoInt | MiyaDens | Spiral | J&J   | Mot   | TRadCoup | TRadDecoup |
|----------------|----------|---------|-------|----------|--------|-------|-------|----------|------------|
| $\mu$          | 51       | 51      | 56    | 40       | 51     | 50    | 49    | 48       | 49         |
| $\sigma$       | 7        | 7       | 7     | 6        | 8      | 8     | 7     | 8        | 8          |

Figure 8. CDF of the resolvable DNS population as a function of their present day eccentricities. CDFs for individual realisations of the Fiducial model are plotted as dashed gray lines. The residuals of alternate models to the Fiducial model are plotted below the CDF. Alternate SFH models show a substantial increase in low-eccentricity binaries due to having systematically older populations (see Figure 9).

Figure 9. CDF of birth times for the resolvable DNS population. CDFs for individual realisations of the Fiducial model are plotted as dashed gray lines. The residuals of alternate models to the Fiducial model are plotted below the CDF. Alternate SFH models employ some type of exponentially decreasing SFR, producing older populations compared to the Fiducial model.

4.3 Eccentricities of resolvable binaries

The distributions of eccentricities of resolvable binaries are plotted in Figure 8. The observed distributions are model-independent with the exception of models which change the adopted Galactic star formation history (Mor, J&J, and TRad). The different star formation histories lead to different distributions of birth times for resolvable binaries: see Figure 9. In particular, the exponentially decaying SFRs at the present day in the Mor, J&J, and TRad models lead to a much lower amount of resolved DNSs with ages between 100 Myr and 1 Gyr because their high eccentricities increase the rate at which their orbits contract. Binaries that are born wider and evolve into visibility on ~Gyr timescales circularise in the process of doing so and hence have $e < 0.1$ when observed. Models with a constant SFR, in which there is a significant number of young binaries, produce systems with measurable eccentricities ($e > 0.1$). Finally, for low eccentricities ($e < 0.1$) the evolution of $f_{gw}$ is largely independent of $e$, and hence of the birth time distribution. For the models which have a constant SFR, ~45% of DNSs have eccentricities greater than 0.1. This percentage drops down to ~40% for the TRad and J&J models. The Mor model has the lowest percentage of high-eccentricity binaries, with less than 35% of resolvable DNSs having eccentricities greater than 0.1.

4.4 Frequency distribution of resolvable binaries

The GW frequency distributions of most models (Figure 10) are distributed the same as for the Fiducial model to within the spread of individual Fiducial runs, with the exception of the Mor model. All SFH
models exhibit a trend towards lower frequency binaries, which is a direct consequence of their lower number of high eccentricity, young DNSs. A higher eccentricity will increase the loudest harmonic up to larger frequencies (see Figure 2). The effect is most apparent in the Mor model which has 10% less binaries with $f_{\text{gw}} > 2 \times 10^{-3}$ Hz. Small deviations from the Fiducial model in alternate density and potential models occur due to the difference in spatial distributions discussed in section 5.2. Low-frequency binaries ($f_{\text{gw}} < 10^{-3}$ Hz) must be at distances of $d < d_{\text{GC}}$ to have sufficient SNR to be resolvable; see Figure 3. Hence the frequency distribution depends on the relative ratio of local and distant binaries.

5 DISCUSSION

We investigate the observable consequences of using different galactic models when studying the LISA-resolvable population of DNSs. Our goal is to understand how sophisticated the Galactic modelling needs to be when studying the likely populations of double neutron star binaries that LISA will see.

The observable properties – spatial distribution, gravitational wave frequency, and eccentricity – are largely insensitive to a reasonable choice of models. Changing the initial positions of DNSs does not affect the frequency or eccentricity of resolvable binaries. An extended stellar distribution as presented in Section 3.1.1 will bias the spatial distribution of binaries with $d > d_{\text{GC}}$ to larger galactic radii, in turn decreasing the total number of resolvable binaries; however, the overall number of resolvable binaries is a weak probe of binary evolution physics due to its normalisation to a highly uncertain empirical merger rate (The LIGO Scientific Collaboration et al. 2019). An alternative normalisation to the theoretically predicted merger rate derived from binary evolution and the Galactic star formation history would also come with a large uncertainty (Lamberts et al. 2019). Therefore, alternative observables such as the distribution of eccentricities, are better candidates for constraining the physics of DNS formation.

Integrating the DNSs’ galactic orbits is not necessary to predict the LISA-resolved DNS population, and the resolved population does not depend strongly on the DNS Galactic orbit calculation. The dominant population of LISA-resolvable binaries lives near the Galactic Centre, where $|\Phi| \gg v_{\text{kick}}^2$, so retention of binaries in the Galaxy is likely. The choice of galactic model does not strongly impact the distribution of distance above the Galactic plane ($z$), which suggests that this observable may provide a useful constraint on the distribution of neutron-star kicks in DNSs. The spiral pattern from Section 3.1.2 is only visible in binaries younger than 20 Myr, which is a very small fraction of observed binaries. The pattern is smeared out by differential rotation, so complete randomisation of the spiral is expected on a Galactic orbital timescale.

The Milky Way star formation history, however, matters when constructing models of the LISA-resolvable DNSs. From the $\text{T}_{\text{Rad}}$ models, we see that coupling the SFR of the MW to its stellar distribution resulted in no change to the resolvable DNS CDFs. An exponentially decreasing SFH, even including a late starburst, heavily biases the resolvable DNS population to older systems (see Figure 9). Gravitational wave emission then results in orbital circularization at present day (see Figures 2, 8.) This means that, when predicting LISA-visible populations of DNSs, attention should be paid to the choice of star formation history. Similarly, when comparing models with different binary population synthesis approaches, care must be taken to adopt the same star formation history to avoid introducing spurious differences into the observed population. Finally, LISA observations of high-eccentricity DNSs may be a good (if noisy) probe of the total current rate of star formation Galaxy-wide, including star-forming regions that are heavily obscured by dust.

Our results are consistent with the earlier work of Yu & Jeffery (2015). They studied the effect of constant and exponentially decaying star formation histories, and found that they led to very similar numbers and frequency distributions for the resolvable DNS population (see e.g. their fig. 9). They also considered a starburst SFH which, while not realistic for the Milky Way is instructive, and found that such a SFH would lead to fewer or no resolvable DNS binaries in the present-day Milky Way, depending on the binary evolution physics. This is broadly consistent with our Figure 9 for the Mor and J&J models, where most of the star formation is early. They find that differences in the frequency distribution from changing the binary evolution physics are much larger than the differences produced by changing the SFH, which implies that LISA measurements of DNS binaries have the potential to be a useful probe of binary evolution physics.

Our conclusions come with some caveats. Despite keeping the DNS population unchanged we are unable to isolate ourselves from binary evolution models (for e.g. SeBa (Portegies Zwart & Verbunt 1996; Toonen & Nelemans 2013), BSE (Hurley et al. 2002), COSMIC (Breivik et al. 2020), MOBSE (Mapelli et al. 2017; Giacobbo et al. 2018; Giacobbo & Mapelli 2020), and COMPAS (Stevenson et al. 2017; Vigna-Gómez et al. 2018)), which yield various DNS birth velocity distributions. If the Church et al. (2011) DNSs shown in Figure 1 have too small $v_{\text{kick}}$, our results may not hold. The natal kicks from Arzoumanian et al. (2002) are relatively strong so this is unlikely.

In this work we assume the ability to distinguish resolvable DNS systems from other compact binaries, specifically double white dwarfs. The chirp mass and eccentricity measurement of these systems will likely reveal the type of system if their frequency is $> 1.75 \times 10^{-3}$ Hz (Nelemans et al. 2001; Andrews et al. 2020).
Follow up observations may be a viable option for nearby binaries since their isotropic sky positions reduce the confusion with other sources in the Galactic disc (see Figure 4).

6 CONCLUSIONS

We have built a model for the Milky Way and evolved a population of double neutron stars from formation to the present day. We have changed the stellar spatial distribution, galactic potential, and star formation histories of our models in order to investigate the observable properties of LISA-resolvable systems.

We show that varying the radial density model for the Galactic disc has a small effect on the spatial distribution of the resolvable DNS. That in turn translates to a small shift in their frequency distribution, with no discernible change to the eccentricity CDFs. Changing the potential of the Milky Way has little effect on the eccentricity distributions. Lastly, we show that varying the stellar spatial density model for the Galactic disc changes the expected eccentricity and frequency distributions. A low present-day SFR in the MW significantly reduces the number of young detectable DNS systems. LISA will be able to distinguish binaries with $e > 0.1$ from circular binaries, and we show that essentially all such binaries are the result of recent star formation.

Our results show that, when comparing different predictions for LISA DNS observations, the choice of Galactic models is relatively unimportant. However, the choice of star formation history does have a significant impact on the predicted eccentricity and frequency distributions. This should be taken into account e.g. when comparing the differences in expected LISA populations from different binary evolution prescriptions. These results suggest that LISA observations of DNS will help constrain massive stellar evolution due to the lack of effect from varying galactic models on resolvable DNS populations.

ACKNOWLEDGEMENTS

The authors would like to thank the anonymous referee of the original manuscript for their thoughtful and constructive comments, and Quentin Baghi and Thomas Wagg for assistance with LISA SNR calculations, for which we used LEGWORK (Wagg et al. 2022a). The authors would also like to thank Abbas Askar, Nerea Gurrutxaga, Paul McMillan, Florent Renaud, and Álvaro Segovia for their helpful comments. This work was funded in part by the Swedish Research Council through the grant 2017-04217.

DATA AVAILABILITY

The data used in this paper can be obtained on request to the corresponding author.

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This paper has been typeset from a TeX/LATEX file prepared by the author.