Magneto-Optics of type-II superconductors

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The magneto-optical activity of superconducting YBa$_2$Cu$_3$O$_7$ observed by Karrai et al. is not present in many commonly employed models of vortex dynamics. Here we propose a simple, unifying picture for the frequency dependent magneto-optic response of type-II superconductors at low temperatures. We bring together Kohn’s theorem, vortex core excitations, and vortex pinning and damping into a single expression for the conductivity tensor. The theory describes magneto-optical activity observed in infrared transmission measurements of thin films of YBa$_2$Cu$_3$O$_7$.

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For applications of superconductivity vortex motion and pinning is of great importance. Many models for vortex dynamics have been developed such as those of Bardeen and Stephen, Nozieres and Vinen, and Clem and Coffey. One way of studying vortex dynamics is to go to frequencies that are large compared with the characteristic frequencies associated with pinning and damping to observe the free inertial response of the vortices. This regime is not well studied and moreover a microscopic approach is needed. We note that, for high temperature superconductors, estimates of the frequency range over which one expects electromagnetic absorption ($10^{10} - 10^{14}$ s$^{-1}$) overlaps with the microscopic energy scale associated with quantized quasiparticle states in vortex cores. Because of the short coherence length in high temperature superconductors, the energy scale of quasi-particle states localized at the vortex core is large.

These considerations have prompted a re-examination of vortex core states. Karrai et al. examined the magnetic field and frequency dependent infrared transmission coefficient of thin films of YBa$_2$Cu$_3$O$_7$ and found evidence for dipole transitions in vortex cores. In addition, magneto-optical activity of the superconducting state was observed in these experiments. For frequencies above the vortex resonance this chiral response is consistent with the cyclotron resonance of the mixed state. According to Kohn’s theorem, for an isotropic, homogeneous electron system in a uniform applied magnetic field $H$, the only excitation produced by a uniform electrodynamic field is the cyclotron resonance at the frequency $\omega_c = eH/mc$, where $m$ is the bare band mass. Thus, for an ideal, pure superconductor, cyclotron resonance is expected. From these considerations, one of the requirements of a theory of vortex dynamics is that it be consistent with Kohn’s theorem. Commonly employed theories fail this test since they are non-chiral.

The matrix elements and selection rules for the dipole transitions in vortex cores have been studied by Jankó and Shore and Zhu, Zhang and Drew. Kopnin proposed a peak in the electromagnetic absorption of pure superconductors due to these transitions. However Hsu showed that these intravortex transitions are not excited by long wavelength probes in the clean limit. Instead the collective cyclotron motion of the center of mass is excited. The ($q = 0$) conductivity is $\sigma_{xx} = (n e^2/m) (\delta(\omega - \omega_c) + \delta(\omega + \omega_c))/2$. The cyclotron resonance exhausts the sum rule $\int_0^\infty d\omega \text{Re} \sigma_{xx} = (\pi/2) n e^2/m$ so that there is no spectral weight at the bare dipole transition frequency in the vortex core, $\Delta^2/E_F \approx \omega_c(H_{c2}/H)$, which is distinct from $\omega_c$. Therefore, this theory is consistent with the Kohn’s theorem.

According to Hsu pinning activates the otherwise invisible vortex core excitation. Pinning also produces a zero frequency delta function in Re $\sigma_{xx}$. These features result from breaking translation invariance, which invalidates Kohn’s theorem. Consistent with this picture is the observation that films of YBa$_2$Cu$_3$O$_7$ grown on LaAlO$_3$ substrates have a much smaller signal at the vortex resonance than those grown on Si (with a YSZ buffer). Si/YSZ is a poorer lattice match, and more conducive to defect formation.

Our conductivity is based on identifying coherent excited states of a vortex core with translation of the core. With this identification we relate certain low-energy excitations with the velocity $v_L$ of the vortex core. The calculation of the $q = 0$ conductivity is based on three equations derived for the clean limit

$$\bar{J} = ne(v_S + \Phi(v_L - v_S)),$$

$$\frac{eF}{m} = \dot{v}_S - N_c(h/2e)v_L \times \hat{z} = \dot{v}_S - \omega_c v_L \times \hat{z},$$

$$\dot{v}_L = v_S - (1 - \Phi)Q_0(v_L - v_S) \times \hat{z} - \frac{v_L}{\tau_e} - \alpha^2 r.$$ 

$\bar{J}$ is the spatially averaged current density, $n$ is the carrier density, $v_S$ is the uniform background superfluid velocity, $h\Omega_0 \equiv \Delta^2/E_F \approx \hbar^2/2m_\xi^2$ is the spacing between
quasiparticle levels in the core \[13\], \(\Delta\) is the bulk gap energy, \(E_F\) is the Fermi energy, \(\xi\) is the coherence length, \(\Phi \equiv \omega_c/\Omega_0 \approx H/H_c\) is the magnetic field, \(\omega_c\) is the cyclotron frequency, \(\mathcal{E}\) is the spatially averaged electric field, \(N_v\) is the areal density of vortices, \(\tau_v\) is a vortex damping rate, and \(\alpha\) is a pinning frequency.

Eq. (1) is obtained by evaluating the expectation value of the current operator for a vortex core of velocity \(\mathbf{v}_L\) and then applying Galilean invariance. This ensures that when \(\mathbf{v}_L = \mathbf{v}_S\) the current is simply that of all carriers moving at \(\mathbf{v}_S\). The \(\mathbf{v}_L - \mathbf{v}_S\) part of the current, not present in conventional theories, comes from coherent excitations of the core \[18\]. In our calculation we have focused on the polarizability of the core which couples to an applied electric field. We have neglected modifications due to screening.

Eq. (2) says that the total electric field is the sum of a gauge term from the uniform acceleration of the superfluid background plus the average Josephson electric field due to transverse motion of the vortices. The sign of the latter term corresponds to a magnetic field pointing in the +\(\hat{z}\) direction and positive charge carriers.

Eq. (3) is an equation of motion for a vortex core. It was derived in Ref. \[13\] by looking at the microscopic equation of motion for low energy quasiparticles in the core and applying the gap equation. \(\tau_v\) is identified as the quasiparticle relaxation time in the vortex core. It is the analogue of the Drude relaxation time for itinerant electrons. Note that the damping in Eq. (3) is inversely proportional to \(\tau_v\) whereas in theories based on the Bardeen-Stephen model \[1, 2\] it is proportional to the normal state electron transport lifetime \(\tau\). This difference is due to the clean limit \((\Omega_0 \tau_v \gg 1)\) in our theory and the effectively dirty limit \((\Omega_0 \tau \ll 1)\) conditions of the hydrodynamic models. The Bardeen-Stephen model breaks down when \(\tau \to \infty\). Nevertheless, as we note below, the two approaches agree in certain cases. The harmonic pinning term, \(-\alpha^2 \mathbf{r}\), can be derived from a simple single-particle short-range repulsive potential \[13\]. It has an appealing and generally expected form (for low vortex densities).

Notice that in the limit of small dissipation and pinning the steady state solution of Eq. (3) is \(\mathbf{v}_L = \mathbf{v}_S\) as required by Galilean invariance.

Eq. (4) contains a term corresponding to a Magnus force. The existence of a Magnus force has been controversial in type II superconductors. \[3\]. Recently, however, very general arguments have been given \[17\] for a Magnus force even in the presence of pinning and viscous drag. A comparison \[13\] of Eq. (3) to the Magnus force gives an effective inertial “mass” of the core \(M_c = \hbar n / 2\Omega_0 \approx n\ell^2 m\) per unit length in the low vortex density limit. This mass, determined microscopically from the core energy level spacing, is not equal to the Suhl value \[18, 19\] for the vortex mass. Vortex dynamics studies by Kopnin in superconductors \[11\] and superfluid \(^3\)He \[20\] and by Baym and Chandler \[21\] in superfluid \(^4\)He find a vortex mass similar to ours. The different mass obtained by Suhl appears to be a consequence of the neglect of quantized core excitations in the Landau-Ginzburg theory. The electrodynamic response we are considering corresponds to a polarization of the vortex core and not the motion of a current pattern across an otherwise uniform charge density as discussed by Suhl.

The factor \((1 - \Phi)\) in Eq. (3) was not derived microscopically but included in order that Re \(\sigma_{xx}(\omega) > 0\) \[3\]. This is ensured because Eqs. (2) and (3) lead to

\[
\frac{n}{2} m \frac{d}{dt} \mathbf{v}_L = (1 - \Phi) \mathbf{v}_S^2 + \frac{\alpha^2}{\tau} \mathbf{v}_L.
\]

Taking the time average over one period shows that \(\langle \mathbf{J} \cdot \mathbf{E} \rangle > 0\). We note that other plausible generalizations of conventional vortex dynamics theories to include the vortex mass generally do not satisfy this condition. A decreasing \(\Omega_0\) with magnetic field is expected on general grounds. It can come from a renormalization of the Magnus force due to the presence of nearby vortices. Also the magnetic field increases as the vortices overlap and will affect the dynamics. These effects were ignored in our single vortex calculation.

The three equations in four unknowns \((\mathbf{J}, \mathbf{E}, \mathbf{v}_L, \mathbf{v}_S)\) may be used to eliminate \(\mathbf{v}_L\) and \(\mathbf{v}_S\) leaving \(\mathbf{J} = \sigma \mathbf{E}\) with

\[
\sigma_{\pm} = \frac{ie^2}{m \omega} \left[ \frac{\omega (\omega + \Omega_0) + (1 - \Phi)(i\omega/\tau_v - \alpha^2)}{(\omega + \Omega_0)(\omega \pm \omega_c) + i\omega/\tau_v - \alpha^2} \right],
\]

where \(\sigma_{\pm} = \sigma_{xx} \pm i\sigma_{xy}\) and \(\Omega_0 = (1 - \Phi)\Omega\). The \(\pm\) refers to cyclotron resonance active (−) or inactive (+) modes of circularly polarized light. This conductivity satisfies a number of important limits. First, when \(B = 0\) the London conductivity is recovered. For zero pinning and zero dissipation the conductivity contains simply the cyclotron resonance as given above. In the limit of no pinning, zero frequency, and low vortex density, \(\rho_{xx} = (m/ne^2\tau_v)\Phi = \rho_{xx}^0 \Phi\) \(\rho_{xx}^0\) is the normal state resistance since \(\tau_v\) is identified with \(\tau\). Therefore we obtain the same expression in the clean limit as was found in the hydrodynamic theories \[13\]. In this limit we also obtain \(\rho_{xy} = B/nec\) as expected from Galilean invariance. The sum rule is satisfied. For example pinning produces a delta function,

\[
\text{Re } \sigma_{xx}(\omega) = \frac{ne^2}{m} \left[ \frac{(1 - \Phi)\alpha^2}{\alpha^2 + \omega_c\Omega_0} \right] \pi \delta(\omega).
\]

This spectral weight is removed from the cyclotron resonance. The cyclotron resonance is shifted to \(\omega_c + \alpha^2/\Omega_0\)
to produce a hybrid cyclotron-pinning resonance. Spectral weight is also transferred between the cyclotron resonance and the vortex core resonance. In Fig. 1 we plot the conductivity and show how it is affected by pinning. For values of the pinning frequency that are consistent with experiments our theory predicts a very small optical response at $\Omega_0$ in the cyclotron active mode.

If we calculate the thin film surface impedance, $Z$, in the low frequency limit ($\omega \ll \alpha, \Omega_0$) and the limit where pinning dominates the magnus force ($\alpha^2 \gg \omega_0 \Omega_0$) we obtain the Gittleman-Rosenblum [22], Coffey-Clem [23] result for pinned vortices. Alternatively we could derive this by dropping the $\mathbf{v}_L$, $\mathbf{v}_S$, and $\mathbf{v}_L \times \xi$ terms in Eq. (3). Dropping the $\mathbf{v}_L$ term is equivalent to setting the mass to zero, and defining a drag coefficient $\eta = M_\alpha / \tau_v$, and a pinning force constant $\kappa = M_\alpha \alpha^2$,

$$Z \approx \frac{4\pi \lambda^2 \omega}{tc^2 (1 - \Phi)} + \frac{B\Phi_0}{tc^2 (\eta + i\kappa/\omega)}.$$  \hspace{1cm} (7)

The first term is the London reactance term. It differs from the Coffey-Clem result by the $1 - \Phi$ factor. This reduction of the effective superfluid density by $1 - (H/H_c)$ is not unexpected nor contradicted by experiment. Eq. (7) was obtained under the clean limit assumptions of our theory. Remarkably it has the same form as the dirty limit case. Therefore if the Magnus force were present in the dirty limit, as argued by Ao and Thouless [17], then the correct conductivity function in that limit might also resemble Eq. (3).

We have fit measurements of the infrared transmission of thin films of YBa$_2$Cu$_3$O$_7$ with this theoretical conductivity. The experimental technique is described in Refs. [6-14]. The transmission coefficient is $T^\pm = 4N/|N + 1 + (4\pi/c)\sigma \pm t|^2$. $N$ is the refractive index of the substrate. In Fig. 2 we plot the ratio of the transmission coefficients $T^+$ and $T^-$. This ratio mostly cancels out non-chiral components of the transmission. The figure shows a definite optical activity in the sample which at high frequencies is well described by a lossless free electron conductivity [19]. At frequencies below $\sim 50$ cm$^{-1}$, however, the chiral response cannot be well represented even if damping is included in the free electron conductivity as can be seen in Fig. 3. In particular there is a steep drop in $T^+/T^-$ at low frequencies which corresponds to the onset of the hybrid cyclotron-pinning resonance. This feature is well fit by our theoretical conductivity. The free parameters in our fits are $\tau_v$, $\alpha$, and $\Omega_0$. We note that the conventional conductivity function [18], even when generalized to finite vortex mass, is non-chiral and would predict simply $T^+/T^- = 1$. That $T^+/T^-$ falls below unity at 30 cm$^{-1}$ is particularly noteworthy as it implies a sign reversal of the a.c. Hall effect at low frequencies.

Another observation from the data displayed in Fig. 2 is that the chiral signal associated with the vortex core resonance is very small compared with the cyclotron-pinning resonance. Our theory also predicts a very small chiral signal but does not agree in detail with the measured signal. The measured resonance is larger than the theory predicts. Moreover, there are substantial non-chiral features observed in the transmission spectrum around 65 cm$^{-1}$. These features are brought out in unpolarized transmission as can be seen in Fig. 3. The peak around 65 cm$^{-1}$ is substantially larger in the unpolarized spectrum than in the $T^+/T^-$ spectrum. This figure also shows the theoretical unpolarized transmission. The corresponding small broad peak in the calculated curve is chiral and has been adjusted to have the maximum amplitude consistent with the low frequency rise in the transmission. Our conductivity does not contain the non-chiral response observed in the experiments. Nevertheless, we believe [14] that this non-chiral response and the enhanced chiral response may be due to vortex core excitations. The effects of defects beyond harmonic pinning induce optical transitions which, because of the breakdown of the cylindrical symmetry, have less restrictive optical selection rules.

The presence or absence of polarization dependence is controversial. The considerations of Zhu, Zhang and Drew [10] and Jankó and Shore [9] would predict that the vortex resonance is in the cyclotron active mode. The conductivity here leads to the opposite effect: a weak vortex resonance in the cyclotron resonance inactive mode.

In conclusion, we have presented a conductivity function which produces magneto-optical activity at high frequencies and explains how it is diminished by vortex pinning and damping at low frequencies. The theory predicts that the dipole-allowed vortex resonance is very weak due to strong screening by the vortex motion even in the presence of pinning. Thus, the Kohn theorem is very robust in this system. The conventional theories of vortex dynamics are not consistent with Kohn’s theorem and do not contain optical activity. Therefore our conductivity function provides a better starting point for studying the interaction of vortices with impurities and the excitation of vortex core states.

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FIG. 1. The frequency dependence of the conductivity function, Eq. (5), is plotted for cyclotron inactive (+) and active (−) modes. The parameters are, $\omega_c = 10$ cm$^{-1}$, $\alpha = 20$ cm$^{-1}$, $\Omega_0 = 60$ cm$^{-1}$, and $1/\tau_v = 10$ cm$^{-1}$. There is also a delta function at zero frequency whose strength is given by Eq. (6).

FIG. 2. The chiral response for the YBa$_2$Cu$_3$O$_7$/Si sample at $H = \pm 12$ T and 2.2 K as determined from measurements with a polarizer with a 0.9 mm quartz wave plate. The solid curve is the best fit to our conductivity function. The best fit parameters are $\alpha = 50 \pm 2$ cm$^{-1}$, $1/\tau_v = 40 \pm 5$ cm$^{-1}$, when $\Omega_0$ was taken as 60 cm$^{-1}$ and the cyclotron mass $m$ was taken to be 3.1 electron masses [7]. The dashed curve is the best fit to the damped free electron model. The corresponding damping parameter is $1/\tau = 56$ cm$^{-1}$.

FIG. 3. Transmission ratio at 2.2 K for the YBa$_2$Cu$_3$O$_7$/Si sample taken at 12 T and 2.2 K in unpolarized light. The solid line is the theoretical result using the best fit parameters from the analysis of the chiral data in Fig. 2.