Viscous fluid flow in a wide annular gap of two rotating coaxial cylinders

I V Morenko
Institute of Mechanics and Engineering, FRC Kazan Scientific Center, Russian Academy of Sciences, Russian Federation
E-mail: morenko@imm.knc.ru

Abstract. The Couette-Taylor flow is considered in a wide gap between coaxial cylinders in the case when the outer cylinder is at rest and the inner one rotates at a constant speed. Unsteady motion of a viscous fluid is described by the system of Navier-Stokes equations. The problem is solved by the control volume method in the OpenFOAM software. Two stable states were obtained depending on the surface rotation speed: the cylindrical Couette flow and the Taylor vortex flow. The calculation results are compared with the known data of other authors. The dependence of the dimensionless shear rate on the Taylor number is obtained.

1. Introduction
The fluid flow between two rotating coaxial cylinders is observed in many rotating mechanisms, in oil, mixing and heat transfer equipment. The relevance of the research topic is also due to the fact that the flow of conductive media, including liquid metals and plasma, in the presence of a magnetic field are the basis of various industrial technologies, for example, flowmeters of electrically conductive liquids, electromagnetic pumps, actively used in metallurgy and cooling systems of nuclear reactors. In practice, three cases are possible: the inner cylinder rotates when the outer one is stationary, the outer cylinder rotates when the inner one is stationary, and both cylinders rotate. The cylindrical flow is also interesting in connection with the development of the theory of hydrodynamic stability. Taylor in 1923 [1] established that when the rotation speed of the inner cylinder increases to a certain critical value, the Couette flow loses stability and a new stable state of the flow with toroidal vortices is formed. The axes of the Taylor vortices are parallel to the direction of the circumferential velocity of the rotating cylinder. In this case, the direction of rotation of the vortices alternates. Later, other modes of fluid motion in a cylindrical channel were established, including wavy vortices, modulated wavy vortices, spirals, wavy spirals, spiral turbulence, turbulent Taylor vortices, ripple, and a map of flow regimes was compiled [2].

It is known [3] that the hydrodynamic parameters of fluid flow in the gap between coaxial cylinders are depended on the Taylor number $Ta = \Omega R_i^5 d^{1.5} / \nu$, the radius ratio $\eta = R_i / R_o$, and aspect ratio of the channel $\Gamma = H / (R_o - R_i)$, where $R_o, R_i$ is the radii of the outer and inner cylinders, $H$ is the length of the cylinder, $\Omega$ is the angular velocity of rotation of the inner surface of the cylinder, $\nu$ is the kinematic viscosity of the fluid.

It should be noted that most theoretical and experimental studies of the Couette-Taylor flow have been performed for relatively narrow channels ($\eta > 0.67$). This paper presents the results of a
numerical study of viscous fluid flow in a wide gap ($\eta = 0.5$) between two coaxial cylinders in the
absence of an axial gradient of overpressure in the case when the inner cylinder rotates and the outer
one is stationary.

2. Mathematical model
The laminar flow of a viscous incompressible fluid in an annular channel is considered. Note that, as a
rule, in theoretical studies of the Couette-Taylor flow, the cylinders were assumed to be infinite. In
practice, however, the cylinders are of finite length. In order to minimize the edge effects, the
influence of the end boundaries of the region on the fluid flow, the length of the cylinders $H$ should
significantly exceed the width of the channel $R_o - R_i$.

We place the origin of the rectangular Cartesian coordinate system $x_1x_2x_3$ in the end face of the
region so that the abscissa axis coincides with the axis of the channel. The laminar motion of a viscous
incompressible fluid, in the absence of gravity, is described by the system of Navier-Stokes equations
in the form:

$$\frac{\partial u_i}{\partial t} + \rho \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right), \quad i = 1, 2, 3,$$

where $t$ is time; $u_i$, $u_j$ is the velocity vector components; $p$ is pressure, $\rho$ is the fluid density, $\mu$
is the coefficient of dynamic viscosity.

The initial and boundary conditions for problem (1)-(2) are defined as follows. It is believed that at
the initial moment of time the liquid is at rest. No-slip conditions are specified on the channel surfaces.
In the study of flows in axisymmetric channels such as annular, a fixed computational domain can be
used. The rotation of the surface can be simulated by setting the appropriate boundary conditions. At
the same time, at the boundary of the outer cylinder and the end boundaries $u \equiv 0$; at the rotating
boundary of the inner cylinder, the normal component of the fluid velocity $u_n = 0$, the tangent
component $u_\tau = \Omega R$. Similar mathematical models were used to calculate viscous crossflow around a
rotating circular cylinder in an unlimited region [4-6] and other problems [7].

The computational domain is covered by an uneven grid of hexahedral elements (figure 1). A dense
distribution of mesh elements is implemented in the boundary layer. The mesh was created using
blockMesh utility. The number of grid nodes in the axial, radial and tangential directions is,
respectively, 150, 32, 128.

Figure 1. Computational grid.
To solve the problem, we use the finite volume method implemented on the open integrable platform OpenFOAM (http://www.openfoam.com). The utility PisoFOAM is involved. Visualization of the results of numerical simulation is carried out in the ParaView software.

3. Results and Discussions

The calculations are performed at the following values of the initial parameters: the radius of the inner cylinder is \( R_1 = 0.05 \text{ m} \), the radius of the outer cylinder is \( R_2 = 0.1 \text{ m} \), the radius ratio is \( \eta = 0.5 \), the length of the channel is \( H = 1.5 \text{ m} \), respectively, dimensionless parameter is \( \Gamma = 30 \); the density of the liquid is \( \rho = 1.2 \text{ kg/m}^3 \), the coefficient of kinematic viscosity is \( \nu = 1.46 \cdot 10^{-5} \text{ m}^2/\text{s} \). During numerical experiments, the rotation speed of the surface of the inner cylinder changes, so that the Taylor number varies from 60 to 800.

At small Taylor numbers less than the critical value \( \text{Ta} \leq \text{Ta}_c \), a cylindrical Couette flow is observed, which occurs under the action of viscous friction forces acting on the fluid. In the case \( \text{Ta} > \text{Ta}_c \), the critical value of the Taylor number is \( \text{Ta}_c = 68.2 \) according to [3]. After reaching the critical value of the rotation speed of the surface of the inner cylinder, the one-dimensional layered flow is reconstructed. A system of paired toroidal vortices appears, which rotate in opposite directions. It was shown that vortices first appear near the end boundaries of the channel. Then they evolve from both ends of the channel to its center and subsequently fill the entire channel. The formed spatial structures of the flow in a cylindrical channel are illustrated in figure 2.

![Figure 2. The isosurface pressure at Ta=200.](image)

Subsequently, with an increase in the Taylor number, the vortices begin to oscillate, azimuthal waves are excited.

The moments of forces acting from the liquid side on the lateral surface of the outer and inner cylinders are written as

\[
T_o = 2\pi R_o^2 H \tau_{w,o}, \quad T_i = 2\pi R_i^2 H \tau_{w,i},
\]

where \( \tau_{w,o}, \tau_{w,i} \) is viscous shear stresses on the respective surfaces of the cylinder.

It is established that in the case under consideration, when the inner cylinder rotates, and the outer one is at rest, the average viscous shear stress on the inner cylinder is higher than on the outer one. In addition, with an increase in the Taylor number, the values of the tangential stresses on the surfaces of both cylinders increase.

Dimensionless torques are determined by the formulas

\[
g_o = \frac{T_o}{H\Omega R_o^2}, \quad g_i = \frac{T_i}{H\Omega R_i^2}.
\]

To verify the numerical model, the dimensionless torque is calculated at \( \text{Ta} = 600 \). According to the calculation results the dimensionless moment is 45.49 on the inner cylinder, according to [8] – 46.36, and according to Batchelor’s theory [9] – 44.09. Satisfactory agreement of the data is noted.
Let $\gamma$ be the average tangential component of the shear rate. The dimensionless shear rate $\gamma/\Omega$ can be expressed in terms of the dimensionless moment $g_o$ as follows

$$\frac{\gamma}{\Omega} = \frac{g_o \eta^2}{2\pi}.$$  

The results of mathematical modeling are compared with the known theoretical solutions and experimental data of other authors in figure 3.

Figure 3. The dependence of dimensionless shear rate on Taylor number. Comparison of calculation results (solid line) with data of other authors: [3] (■); [8] (♦); [10] (●).

On the figure 3 it can be seen that the results of the numerical simulation are in good agreement with the data of other authors.

As a result of mathematical modeling, depending on the Taylor number, two stable states are obtained: the Couette flow and the Taylor vortex flow in the gap of coaxial cylinders in the case when the outer cylinder is stationary and the inner cylinder rotates at a constant angular speed.

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