Static analysis for magneto-electro-elastic plates based on the scaled boundary finite element method

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Abstract. The scaled boundary finite element method (SBFEM) and the precise integration algorithm (PIA) are utilized to analyze the extended displacement field in clamped or simple-supported magneto-electro-elastic plates produced by external transverse loadings. There are no limitation on boundary conditions and types of external forces. Only the in-plane dimensions are divided into 2D elements. By introducing a set of scaled boundary local coordinates, 3D governing partial differential equations are converted into the second order ordinary differential matrix equation. By means of the internal nodal force, a first order ordinary differential equation is obtained and its general solution is a matrix exponential. The PIA is introduced to calculate the matrix exponential and any desired accuracy can be obtained. Finally, several numerical examples are provided to validate the versatility of the proposed technique.

1. Introduction
In the recent decade, magneto-electro-elastic plates due to exceptional nature to convert one form of energy into another have been drawn considerable discussions. With the increase in the usage of magneto-electro-elastic plates in engineering practices, an excellent understanding of their structural behavior are needed.

Over the past few years, mechanics of magneto-electro-elastic plates have received considerable research effort. Phoenix et al. [1] developed a finite element model based on the Reissner mixed variational theorem to analyze the static and dynamic analysis of magneto-electro-elastic plate problem. Built on the third-order shear deformation theory, Moita et al. [2] proposed a finite element model to investigate the static and free vibration problem of thin and thick magneto-electro-elastic plates. Liu [3] made use of the Kirchhoff hypothesis and classical thin-plate theory to study the bending problem of the magneto-electro-elastic rectangular plate subjected to certain type of applied loads acting on the top or bottom surfaces.

The purpose of this paper is to introduce the scaled boundary finite element (SBFEM) developed Wolf and Song [4] and Song and Wolf [5] to explore the static response of magneto-electro-elastic plates. The SBFEM method has its own attractive features. Recently, this method is extended to discuss the plate bending problem [6, 7].
The paper is outlined as follows. The solution procedure of the fundamental equations in SBFEM is presented in section 2. Subsequently, three numerical examples are provided to validate verify the proposed approach as well as to highlight its performance in Section 3.

2. Solution procedure

In this section, only the necessary equations are introduced and further details are listed in Ref. [6, 7]. The 3D geometry of a magneto-electro-elastic plate is shown in figure 1.

![Figure 1. The 3D geometry of a magneto-electro-elastic plate.](image)

With the internal nodal force, the matrix exponential can be expressed as

\[
\begin{align*}
\{u_\theta\} &= \{c_1\} \\
\{F_\theta\} &= \{-c_2\}
\end{align*}
\]

\[
\begin{align*}
\{u_r\} &= \exp(-[Z] \tau) \{c_1\} \\
\{F_r\} &= \exp(-[Z] \tau) \{c_2\}
\end{align*}
\]

(1)

The PIA proposed by Zhong et al. [8] is utilized to solve the matrix exponential. The total thickness \( t \) is divided into \( 2^N \) mini-layers of equal thickness.

\[
\{X_\tau\} = \exp\left(-[Z] \tau/2^N\right)^{2^N} \{c\} = \exp\left(-[Z] \tau \right)^{2^N} \{c\} = T^{2^N} \{c\} = \tilde{T} \{c\}
\]

(2)

Since \( \tau \) is an extremely small thickness, \( T \) is calculated as

\[
\begin{align*}
T &= \exp\left( \mathbf{H} \tau \right) \approx \mathbf{I} + \mathbf{T}_a \\
\mathbf{H} &= -[Z]
\end{align*}
\]

(3)

The estimation of \( \tilde{T} \) is performed in the following way:

\[
\tilde{T} = (\mathbf{I} + \mathbf{T}_a)^{2^N} = \left( (\mathbf{I} + \mathbf{T}_a)^{2^N} \right)^2 = \mathbf{I} + T^{2^N}
\]

(4)

in which

\[
\mathbf{T}_i = 2T_{i-1}^{2^N+1} + T_{i-1}^{2^N} \times T_{i-1}^{2^N+1} \quad (i = 1, 2, ..., N) \\
\mathbf{T}_0 = \mathbf{T}_a
\]

(5)

Substituting equation (4) into equation (1) and grouping the extended displacement field and external forces, the stiffness equation can be acquired.

\[
[S] \begin{bmatrix} \{u_\theta\} \\ \{u_r\} \end{bmatrix} = \begin{bmatrix} \{F_\theta\} \\ \{F_r\} \end{bmatrix}
\]

(6)

in which

\[
[S] = \begin{bmatrix} T_{11}^{-1} & T_{12}^{-1} & -T_{12}^{-1} \\ T_{21} & T_{22} & T_{22} - T_{12}^{-1} T_{11} \end{bmatrix}
\]

(7)

By applying boundary conditions, the extended displacement field of the magneto-electro-elastic plate is obtained.
3. Numerical examples
In this section, all examples are conducted with a material \( BF_{50\%} \), whose material properties are listed in Table 1 of Ref. [9].

3.1. Verification
Comparisons provided by Alaimo et al. [9] are under condition. From table 1, it is obvious that the present results agree well with that from the reference.

3.2. Circular plate
It is meaningful to investigate the deformation of a circular magneto-electro-elastic plate as illustrated in figure 2 under uniform electric displacement load on the top plane. Thickness to length ratios \( t/l=0.01, 0.02 \) and 0.05 are selected, in which the characterized length is \( l=2R \). Clamped boundary conditions are given at the edge of the circular plate. The central transverse displacements under different kinds of external loadings are demonstrated in table 2. Figure 3 delineates variations of the electric and magnetic potential along the normalized thickness. From all tables and figures, a conclusion can be drawn that the thickness-to-length ratio greatly influences the distribution of electric and magnetic potential.

Table 1. Results of a square magneto-electro-elastic plate.

| Element order | \( t/l=0.01 \) | \( t/l=0.02 \) | \( t/l=0.05 \) |
|---------------|---------------|---------------|---------------|
| 4             | -1.574×10^{-9} | -1.072×10^{-2} | 9.106×10^{-6} |
| 6             | -1.570×10^{-9} | -1.069×10^{-2} | 9.033×10^{-6} |
| 8             | -1.567×10^{-9} | -1.065×10^{-2} | 8.818×10^{-6} |
| 10            | -1.560×10^{-9} | -1.045×10^{-2} | 8.228×10^{-6} |
| Ref. [9]      | -1.507×10^{-9} | -9.820×10^{-3} | 8.365×10^{-6} |

Figure 2. A circular plate meshed with fourth order spectral elements.

Table 2. Central transverse displacement for the circular plate.

| Element order | \( t/l=0.01 \) | \( t/l=0.02 \) | \( t/l=0.05 \) |
|---------------|---------------|---------------|---------------|
| 4             | 1.287×10^{-11} | -7.982×10^{-12} | 4.366×10^{-10} |
| 6             | 1.687×10^{-11} | -1.505×10^{-11} | 4.592×10^{-10} |
| 8             | 2.204×10^{-11} | -1.545×10^{-11} | 4.822×10^{-10} |
| 10            | 2.238×10^{-11} | -1.612×10^{-11} | 5.220×10^{-10} |
3.3. Square plates with four holes

In this example, a square plate with four holes are selected as a model as demonstrated in figure 4 under a sinusoidal mechanical load on the top plane. Simply supported boundary conditions are prescribed as all four outer edges while the inner edges are free. Additionally, zero electric potential and magnetic potential are applied on both the upper and lower plate surfaces. Table 3 reveals deflections of an open-hole square plate at the center. Variations of the electric and magnetic potential are displayed in figure 5. Similar findings on variations of the deflection in the previous example can also be found in Table 3. It is obvious that central transverse displacements in the case of $t/l=0.05$ are largest. Moreover, curves of the electric potential and magnetic potential are approximately parabolic in figure 5.

![Figure 4. An square plate model with open-holes.](image)

**Table 3.** Deflections of an simply supported square plate at the center.

| Element order | $t/l=0.01$ | $t/l=0.02$ | $t/l=0.05$ |
|---------------|------------|------------|------------|
| 4             | $8.838\times10^{-9}$ | $1.318\times10^{-7}$ | $1.041\times10^{-6}$ |
| 6             | $9.719\times10^{-9}$ | $1.457\times10^{-7}$ | $1.155\times10^{-6}$ |
| 8             | $9.888\times10^{-9}$ | $1.475\times10^{-7}$ | $1.165\times10^{-6}$ |
| 10            | $9.954\times10^{-9}$ | $1.483\times10^{-7}$ | $1.169\times10^{-6}$ |
4. Conclusion
Based on the SBFEM and PIA, the magnitude and distribution of the transverse deflection, electric potential and magnetic potential in a magneto-electro-elastic plate is discussed in the paper. Several numerical examples are given to verify the proposed approach. In the circular plate, a conclusion can be drawn that the thickness-to-length ratio plays an important role in determining the distribution of electric and magnetic potential. As to the open-hole square plate, curves of the electric potential and magnetic potential are approximately parabolic induced by mechanical loadings.

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