Particle Swarm Optimization-Based Fuzzy PID Controller for Stable Control of Active Magnetic Bearing System

Wang Bo, Geng Haipeng, Li Hao, Zheng Wei
Xi'an Jiaotong University, 28 Xianning Xi Lu, Xi'an City, Shaanxi Province, China
genghaipeng@mail.xjtu.edu.cn

Abstract. In this paper, the application of particle swarm optimization (PSO) algorithm based fuzzy PID control for the stable control of an active magnetic bearing (AMB) system is studied. Active magnetic bearings are known to be highly nonlinear multivariable systems. An AMB system is used in motors, turbines and various other machineries in different industries to provide an active suspension to the rotor shafts. The heuristic PSO algorithm is applied to optimize the parameters of the PID controller offline. The fuzzy PID controller based on PSO algorithm is designed to adjust the control parameters of AMB system online. The comparison of controlled responses of closed-loop systems resulting from the use of conventional PID, Incomplete differential PID and particle swarm optimization-based fuzzy logic PID control strategies is discussed. The proposed controller performance is superior as compared to PID and incomplete differential PID controllers in improving the system response of an AMB under normal conditions as well as under external disturbances.

1. Introduction
Active magnetic bearings are electromagnetic devices that provide support to actively suspend a rotor’s shaft in the radial directions without any physical contact of the shaft with a surface. AMB has been broadly used in flywheel energy storage systems, turbo compressors, vacuum pumps, vehicle gyroscopes, and so on [1]. An AMB has several advantages over conventional bearings in terms of reduction of friction, heat losses and noise-free operation.

The interest in AMBs can be seen growing widely in both research and industry. Among them, the research of control theory is particularly critical. Different control approaches have been reported for regulating the rotor position of AMB systems. PID control in [2] is a typical and efficient method to stabilize the rotor. Least Quadratic Regulator (LQR) control is designed and realized in a small size prototype AMB [3]. It is discovered in [4] that the LQR based controller has better performance than PID controller.

Over the past few decades, many heuristic algorithms have been proposed. Literature [5] presents the design of a PID controller based on multi-objective genetic optimization strategy. In Literature [6], the PSO algorithm is applied to the tuning of PID controller.

Many modern control methods can achieve better control effect in theory, but the system is complicated and the cost is high. PID is widely used, but it is difficult to set the parameters. In view of this situation, this paper combines intelligent optimization algorithm, intelligent control algorithm and classical PID control to realize the stability control of active magnetic bearing system.

This paper has been subdivided as follows: Sect. 2 describes the mathematical model of an AMB system, and Sect. 3 describes particle swarm optimization technique to optimize PID parameters. In
Sect. 4, we design a fuzzy-PID controller and Sect. 5 shows the result of performance of the three controllers. Finally, we draw the conclusions in Sect. 6.

2. Mathematical model of active magnetic bearing (AMB) system

Active magnetic bearing system is mainly composed of electromagnet, rotor, displacement sensor, power amplifier and controller. From the control point of view, it is a control problem of five degrees of freedom. Because magnetic bearings work similarly on five degrees of freedom, this paper studies the control system of single degree of freedom magnetic bearings. The schematic diagram of the control system is shown in Figure 1. The structure of magnetic bearing system is shown in Figure 2.

![Figure 1. Control system schematic diagram.](image)

Considering that the magnetic bearing adopts the way of differential excitation and the direction of force, the electromagnetic force received by the rotor in the direction of gravity is the difference of the attraction of upper and lower magnets, i.e.

\[
F = k \left[ \frac{(i_0 + i)^2}{(y_0 - y)^2} - \frac{(i_0 - i)^2}{(y_0 + y)^2} \right] \cos \beta
\]  

(1)

Where \( F \) is the total electromagnetic force, and the direction of \( F \) is the same as \( y \); \( i_0 \) is the bias current; \( i \) is the control current; \( y_0 \) is the air gap at the equilibrium position; \( y \) is the displacement of the rotor relative to the equilibrium position, in the direction of vertical upward.

\[
k = \frac{\mu_0 AN^2}{4}
\]

(2)

Where \( \mu_0 \) is the permeability in vacuum, \( A \) is the magnetic pole area of the electromagnet, and \( N \) is the number of coil turns. Taylor expansion of Equation (1) around \( y=0 \) and \( i=0 \) and omitting higher order infinitesimal quantities, the relation is obtained:

\[
F = k_y y + k_i i
\]

(3)

where

\[
k_y = \frac{\mu_0 AN^2 i_0^2 \cos \beta}{y_0^3}
\]

(4)

\[
k_i = \frac{\mu_0 AN^2 i_0 \cos \beta}{y_0^2}
\]

(5)
\( k_y \) is the displacement stiffness and \( k_i \) is the current stiffness.

\[
y = k_y y + k_i i - mg
\]  

(6)

The transfer function model of magnetic bearing with displacement \( Y \) as output and current \( I \) as input on a single degree of freedom can be obtained by using the Laplace transform of Equation (6).

\[
G_p(s) = \frac{Y(s)}{I(s)} = \frac{k_i}{ms^2 - k_y}
\]  

(7)

In this paper, the relevant parameters of AMB-rotor system are shown in Table1 and Table2.

**Table1.** The related parameters of radial AMB.

| Parameters                      | Value   |
|---------------------------------|---------|
| Coil number of single-pole \( N \) | 160     |
| Magnetic pole area \( A/mm^2 \) | 223.2   |
| Bias current \( I_0/A \)        | 2.5     |
| Air gap \( x_0/mm \)           | 0.3     |
| Permeability of vacuum \( \mu_0 \) | \( 4\pi \times 10^{-7} \) |
| Pole angle \( \beta \)         | 22.5    |

**Table2.** Parameters of the AMB-rotor system.

| Parameters                      | Value   |
|---------------------------------|---------|
| Rotor shaft mass \( m/kg \)     | 1.7     |
| Current stiffness \( k_i/N/A \) | 184.3   |
| Displacement stiffness \( k_y/N/m \) | \( 1.536 \times 10^6 \) |
| Power amplifier gain \( k_o/A/V \) | 0.4     |
| Displacement sensor gain \( k_s/V/m \) | 5000    |
3. Particle swarm optimization of PID parameters

3.1. Particle swarm optimization
Particle Swarm Optimization is famous for its nonlinear optimization. It is a global random search algorithm proposed by simulating the foraging of birds. The particles adjust their speed according to their own flight experience and the flight experience of their companions. Each particle will take into account the best position reached so far, $p_{i,\text{best}}$. It also considers the global optimal position reached by any particle, $p_{g,\text{best}}$. Speed and position are updated according to the following law.

$$v_{i+1} = \omega \cdot v_i + c_1 r_1 (p_{i,\text{best}} - x_i) + c_2 r_2 (p_{g,\text{best}} - x_i)$$

$$x_{i+1} = x_i + v_{i+1}$$  \hspace{1cm} (8)

Where $v_i$ is the present velocity of a particle at $i$th iteration; $\omega$ is weight factor; $c_1$, $c_2$ are two positive constant; $r_1$, $r_2$ are two random number between 0-1; $x_i$ is current position of particle at $i$th iteration; $v_{i+1}$ is updated velocity of particle at $i$th iteration; $x_{i+1}$ is updated position of particle at $i$th iteration; $\omega = 0.9 - \text{iter} \times 0.8 / \text{MaxIter}$, iter is current iteration, MaxIter is the maximum iteration.

3.2. PSO selects PID parameters
When the controller adopts PID controller, mathematically, it is described as [7]:

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{de(t)}{dt}$$  \hspace{1cm} (10)

![Flowchart of particle swarm optimization (PSO).](image)
Where $K_p$ and $K_i$ and $K_d$ are proportional, integral and derivative gains of the PID controller respectively. Corresponding transfer function is as follows:

$$G_c(s) = K_p + K_i / s + K_d s$$

Combined with the data in Table 2 and the transfer function of the closed-loop system, the range of PD parameter is calculated, $K_p$ (8.3–41.6), $K_d$ (0.0009–0.1331). According to the determined parameter range, the PSO algorithm adopted is shown in Figure 3 [8]. The particle swarm size is set as 30 and the number of iterations is set as 30.

Figure 4 shows the fitness curve of the optimal individual in each iteration. After 10 iterations, the fitness value of the optimal individual has been relatively low and tends to be stable.

In the iteration process, the optimization curve of the three parameters of the PID controller is shown in Figure 5. Finally, a set of parameter values obtained through particle swarm optimization algorithm are: $K_p$ = 41.6, $K_i$ = 1368.3, $K_d$ = 0.0393.
4. Fuzzy PID controller design

PID controller and its transformation are widely used in the standard control of industrial processes and systems. Fuzzy logic has the ability to understand the imprecision and uncertainty of the real world. It also improves the right balance between meaning and precision, which allows it to be used in complex systems that many classical controllers cannot cover [9].

A fuzzy logic system mainly consists of four parts: fuzzification, fuzzy rule base, inference machine and defuzzification. It can be used as an intelligent method to tune the parameters of PID controller online. Fuzzy logic control (FLC) gives the fuzzy PID controller nonlinear characteristics, because there is a nonlinear mapping between the input and output.

In this article, a Mamdani-type fuzzy logic system is designed that auto-tunes the parameters of a PID controller. In order to design fuzzy PID controller, two inputs are used, namely error and error change. The output of the fuzzy logic controller is the parameters of the PID controller, namely Kp, Ki and Kd. Here, the inputs use Gaussian membership function, gaussmf. The outputs use a triangular membership function, trimf.

Table 2, 3 and 4 are fuzzy rule base prepared for FLC, which are used to determine approximations of Kp, Ki and Kd respectively. Where, NB denotes “negative large”, NM denotes “negative medium”, NS denotes “negative small”, ZO denotes “zero”, PS denotes “positive small”, PM denotes “positive medium”, and PL denotes “positive large”.

| Table 3. Rule base for $\square$ Kp. |
|---|---|---|---|---|---|---|---|
| | NB | NM | NS | ZO | PS | PM | PB |
| e | | | | | | | |
| e | | | | | | | |
| NB | PB | PB | PM | PM | PS | PS | ZO |
| NM | PB | PB | PM | NM | PS | ZO | ZO |
| NS | PM | PM | PM | PS | ZO | NS | NM |
| ZO | PM | PS | PS | ZO | NS | NM | NM |
| PS | PS | PS | ZO | NS | NS | NM | NM |
| PM | ZO | ZO | NS | NM | NM | NM | NB |
| PB | ZO | NS | NS | NM | NM | NB | NB |

| Table 4. Rule base for $\square$ Ki. |
|---|---|---|---|---|---|---|
| | NB | NM | NS | ZO | PS | PM | PB |
| e | | | | | | | |
| e | | | | | | | |
| NB | NB | NB | NB | NM | NM | ZO | ZO |
| NM | NB | NB | NM | NS | NS | ZO | ZO |
| NS | NM | NM | NS | NS | ZO | PS | PS |
| ZO | NM | NS | NS | ZO | PS | PS | PM |
| PS | NS | NS | ZO | PS | PS | PM | PM |
| PM | ZO | ZO | PS | PM | PM | PB | PB |
| PB | ZO | ZO | PS | PM | PB | PB | PB |

| Table 5. Rule base for $\square$ Kd. |
|---|---|---|---|---|---|---|
| | NB | NM | NS | ZO | PS | PM | PB |
| e | | | | | | | |
| e | | | | | | | |
| NB | PS | NS | ZO | ZO | ZO | PB | PB |
| NM | NS | NS | NS | NS | ZO | NS | PM |
| NS | NB | NB | NM | NS | ZO |(NS | PM |
| ZO | NB | NM | NM | NS | ZO | PS | PM |
| PS | NB | NM | NS | NS | ZO | PS | PS |
| PM | NM | NS | NS | NS | ZO | PS | PS |
| PB | PS | ZO | ZO | ZO | ZO | PB | PB |
Figure 6 shows the surface view plot of $K_p$, $K_i$ and $K_d$, which indicates the relationship between the inputs and the outputs, respectively.

![Surface view plot of $K_p$, $K_i$, $K_d$ against the inputs.](image)

**Figure 6.** Surface view plot of $K_p$, $K_i$, $K_d$ against the inputs.

5. **Simulation results and discussion**

In order to know the effectiveness of the proposed strategy and the designed fuzzy adaptive PID controller, a simulation model of magnetic bearing system was built in MATLAB/Simulink. The relevant parameters of the magnetic bearing-rotor system used for simulation are shown in Table 2. Parameters optimized by particle swarm optimization algorithm are used as initial values. Figure 7 is the simulation block diagram of the three controllers in MATLAB/Simulink.

![Simulink block diagram of three kinds of controllers.](image)

**Figure 7.** Simulink block diagram of three kinds of controllers.

The control performance of PID controller, incomplete differential PID controller and fuzzy PID controller is simulated and compared. The tracking control and disturbance rejection control are carried out respectively. Figure 8 is the displacement response comparison diagram of the three controllers.

It can be seen from Figure 8(a) that incomplete differential PID has a better control effect than PID, and the fuzzy PID controller has the best performance. Compared with PID control, the overshoot is reduced by 43.75% and the adjustment time is shorter. As can be seen from Figure 8(b), when the magnetic bearing system is affected by external interference, the incomplete differential PID controller...
is slightly better than the PID controller. The anti-interference performance of the fuzzy PID controller is very superior. The overshoot is reduced by 59.2% and the adjustment time is reduced by 14.8%. The simulation results show that the fuzzy adaptive PID controller designed based on the optimized parameters of particle swarm optimization algorithm as the initial values can improve the response of the AMB-rotor system under external disturbances.

![Comparison of tracking of three controllers](image)

![Comparison of anti-interference of three controllers](image)

**Figure 8.** Displacement response comparison of the three controllers. (a) Comparison of tracking of three controllers; (b) Comparison of anti-interference of three controllers.

### 6. Conclusion

This paper firstly uses Particle Swarm Optimization to optimize the three parameters of PID controller off-line to obtain a group of better control parameters. Then, the optimized parameters are taken as the initial values of the online control, and the fuzzy adaptive PID controller is designed to control the magnetic bearing system. The PID controller, incomplete differential PID controller and fuzzy PID controller are compared with the tracking control and disturbance rejection control in MATLAB environment. The proposed controller is found to be more efficient than PID and incomplete differential PID controllers in improving the system response of an AMB under normal conditions as well as under external disturbances. It provides a lesser overshoot and settling time in both the cases. Therefore, the combination of offline optimization and online regulation control methodology for the control of an AMB system is highly efficient and easy to implement in real-time situation.

### References

[1] Dong, L. and L. You, Adaptive control of an active magnetic bearing with external disturbance. Isa Transactions, 2014. 53(5): p. 1410-1419.

[2] Arredondo I, Jugo J, Etxebarria V. Modeling and control of a flexible rotor system with AMB based sustentation. ISA Trans 2008;47(1):101–12.

[3] Kim Ha-Yong, Lee Chong-Won. Design and control of active magnetic bearing system with Lorentz force type axial actuator. Mechatronics 2006;16 (1):13–20.

[4] Jastrzebski Rafal Piotr, Pollanen Riku. Centralized optimal position control for active magnetic bearings: comparison with decentralized control. Electr Eng 2009;91(2):101–14.

[5] Wei, C. and D. Soeffker, Optimization Strategy for PID-Controller Design of AMB Rotor Systems. Ieee Transactions on Control Systems Technology, 2016. 24(3): p. 788-803.

[6] Stimac, G., S. Braut, and R. Zigulic, Comparative Analysis of PSO Algorithms for PID Controller Tuning. Chinese Journal of Mechanical Engineering, 2014. 27(5): p. 928-936.

[7] G. Stimac , S. Braut , R. Žigulić , Comparative analysis of PSO algorithms for PID controller tuning, Chin. J. Mech. Eng. 27 (5) (2014) 928–936.
[8] Acharya, D. and D.K. Das, Swarm optimization approach to design PID controller for artificially ventilated human respiratory system. Computer Methods and Programs in Biomedicine, 2021. 198.

[9] Dhyani, A., M.K. Panda, and B. Jha, Moth-Flame Optimization-Based Fuzzy-PID Controller for Optimal Control of Active Magnetic Bearing System. Iranian Journal of Science and Technology-Transactions of Electrical Engineering, 2018. 42(4): p. 451-463.