Radial Mixing in Galactic Disks: The Effects of Disk Structure and Satellite Bombardment

Jonathan C. Bird*, Stelios Kazantzidis, & David H. Weinberg
Department of Astronomy and the Center for Cosmology and Astro-Particle Physics, The Ohio State University, Columbus, OH 43210

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ABSTRACT
We use a suite of numerical simulations to investigate the mechanisms and effects of radial migration of stars in disk galaxies like the Milky Way (MW). An isolated, collisionless stellar disk with a MW-like scale-height shows only the radial “blurring” expected from epicyclic orbits. Reducing the disk thickness or adding gas to the disk substantially increases the level of radial migration, induced by interaction with transient spiral arms and/or a central bar. We also examine collisionless disks subjected to gravitational perturbations from a cosmologically motivated satellite accretion history. In the perturbed disk that best reproduces the observed properties of the MW, 20% of stars that end up in the solar annulus \(7 \text{ kpc} < R < 9 \text{ kpc}\) started at \(R < 6 \text{ kpc}\), and 7% started at \(R > 10 \text{ kpc}\). This level of migration would add considerable dispersion to the age-metallicity relation of solar neighborhood stars. In the isolated disk models, the probability of migration traces the disk’s radial mass profile, but in perturbed disks migration occurs preferentially at large radii, where the disk is more weakly bound. The orbital dynamics of migrating particles are also different in isolated and perturbed disks: satellite perturbations drive particles to lower angular momentum for a given change in radius. Thus, satellite perturbations appear to be a distinct mechanism for inducing radial migration, which can operate in concert with migration induced by bars and spiral structure. We investigate correlations between changes in radius and changes in orbital circularity or vertical energy, identifying signatures that might be used to test models and distinguish radial migration mechanisms in chemo-dynamical surveys of the MW disk.

Key words: methods: numerical; Galaxy: kinematics and dynamics; Galaxy: disc; galaxies: formation; galaxies: evolution

1 INTRODUCTION
Under the influence of gravity, disk galaxies are expected to assemble in an “inside-out” fashion: stars form first from high-density gas in the central region of the galaxy where the potential is deepest, and subsequently at increasing galacto-centric radii (e.g. Larson 1976). An immediate consequence of this formation scenario is that stars born at the same time and in the same region of a galaxy should have similar chemical compositions. However, observations in our Galaxy suggest that these initial conditions are not maintained. Wielen et al. (1996) argued that the Sun was substantially more metal rich than nearby solar age stars and the local interstellar medium (ISM). A recent recalibration of the Geneva Copenhagen Survey (GCS) using the infrared flux method finds no discrepancy with solar age stars (Casagrande et al. 2011), but even modern studies confirm that the age-metallicity relationships (AMRs) of field and solar neighborhood stars are characterized by higher dispersions than expected (Edvardsson et al. 1993; Nordström et al. 2004; Holmberg et al. 2007; Casagrande et al. 2011). In addition, simple chemical evolution models that divide the Galaxy into independently evolving concentric annuli predict many more low metallicity G-dwarfs in our region of the disk compared to those observed, a discrepancy known as “the local G-dwarf problem” (van den Bergh 1962; Schmidt 1963). Evidence seemingly in contradiction to standard galaxy chemical evolution theory is not limited to our own Galaxy. Metallicity gradients in disk galaxies are shallower than predicted by classical models (e.g., Magrini et al. 2007; Ferguson & Johnson 2001) and Ferguson et al. (2007) find unexpectedly old stellar populations on nearly circular orbits in the outskirts of M31 and M33, respectively. The outermost regions of NGC300 and NGC7739 show flattened or positive abundance gradients with radius (Vlajić et al. 2009, 2011). These perplexing observations cannot be readily explained within the confines of classic galaxy formation models.

A natural explanation for the observational challenges above arises if the present day radii of many stars could be significantly different from their birth radii. One difficulty in establishing radial migration as a common phenomenon, from a dynamical standpoint, lies in finding a mechanism that can cause a substantial fraction of stars to migrate several kiloparsecs while retain-
ing the observed approximately circular orbits. In a seminal paper, Sellwood & Binney (2002, hereafter SB02) investigated the relationship between changes in stellar angular momentum and disk heating. They found that radial migration is a ubiquitous process in spiral galaxies; stars naturally migrate (change angular momentum) as they resonantly interact with transient spiral waves. Stars in corotational resonance (CR) with said waves are scattered without heating the disk and maintain their nearly circular orbits (unlike Lindblad resonance (LR) scattering). In the present paper, we examine radial migration in simulations of disk galaxies. Our experiments include galactic disks evolved both in isolation and under the action of infalling satellites of the type expected in the currently favored cold dark matter (CDM) paradigm of hierarchical structure formation (e.g., Peebles 1983; Blumenthal et al. 1984). The latter set of experiments were presented in the studies of Kazantzidis et al. (2008, hereafter K08) and Kazantzidis et al. (2009) and were utilized to investigate the generic dynamical and morphological signatures of galactic disks subject to bombardment by CDM substructure.

Inspired by SB02, several groups have recently investigated the potential role of radial mixing in the chemical and dynamical evolution of disk galaxies. Schönrich & Binney (2009) presented the first chemical evolution model to incorporate radial migration. The rate at which stars migrate via the SB02 mechanism is left as a free parameter constrained by the metallicity distribution function (MDF) of solar neighborhood stars in the GCS (Nordström et al. 2004). Their model successfully reproduced, within systematic uncertainties, the observed age-metallicity distribution of stars in the GCS (Holmberg et al. 2007) and the observed correlation between tangential velocity and abundance pattern described by Haywood (2008). However, there is partial degeneracy between the magnitude of radial migration and other parameters in the model such as star-formation rates and gas inflow characteristics. Furthermore, it is unclear whether the level of migration required to fit the data is consistent with theoretical expectations.

Numerical simulations have confirmed the occurrence of radial migration under a variety of conditions. Roškar et al. (2008a,b) studied the migration of stars in a simulation of an isolated Milky Way (MW)-sized stellar disk formed from the cooling of a pressure-supported gas cloud in a $10^{12} M_\odot$ dark matter halo. In their simulations, some older stars radially migrated to the outskirts of the disk while maintaining nearly circular orbits, forming a population akin to that observed in M31 and M33 (Ferguson & Johnson 2001; Ferguson et al. 2007). Roškar et al. (2008b) found that ~50% of all stars in the solar neighborhood were not born in situ; this is a natural explanation for the observed dispersion in the AMR and solar neighborhood metallicity distribution function (MDF).

More recently, Quillen et al. (2009) investigated radial migration in a stellar disk perturbed by a low-mass ($\sim 5 \times 10^5 M_\odot$) orbiting satellite. Their numerical simulations integrated test particle orbits in a static galactic potential and highlighted the fact that mergers and perturbations from satellite galaxies and subhalos can induce stellar radial mixing. Although informative, test particle simulations in a static isothermal potential will not capture all the relevant physics of the process of stellar radial migration in disk galaxies; the interactions between the gravitational perturbations and the self-gravity of the disk are essential to a detailed analysis of the phenomenon. In our paper, we expand on the analysis of Quillen et al. (2009) by investigating radial migration using fully self-consistent numerical simulations both with and without satellite bombardment.

There are now several established phenomena that can cause a star to populate a region of the disk different from its birth radii. Stars on elliptical orbits maintain their guiding center and angular momentum (modulo asymmetries in the potential) but can be found over the range in galacto-centric radius defined by their pericenter and apocenter. Changing a star’s angular momentum, and hence its guiding center, requires direct scattering or a resonant interaction with transient patterns in the disk. The local encounters of stars with molecular clouds (e.g., Spitzer & Schwarzchild 1953) or Lindblad resonance (LR) scattering between stars and spiral waves (e.g., SB02) both change stellar guiding centers (albeit to a relatively small degree) and increase the random motions of stars over time, “blurring” the disk. Stars scattered at CR with spiral waves can change their guiding centers by several kiloparsecs without increasing the amplitude of their radial motion. For any single spiral wave, SB02 predict that stars are scattered on each side of the CR, “churning” the contents of the disk. Stars may undergo several encounters with transient spiral waves throughout their lifetimes. While SB02 investigate all resonant interactions between spiral waves and stars, we will refer to this special case of CR as the “SB02 mechanism”.

As SB02 note, migration due to spiral waves can be described by blurring and churning regardless of how the waves arise (satellites, e.g., could induce spiral structure that would lead to migration described by SB02). Simulations have shown that other transient wave patterns internal to a galaxy, such as those resulting from bar propagation, can produce resonance overlap with existing spiral patterns and induce radial migration (Minchev & Famaey 2010; Minchev et al. 2011). Orbiting satellites, external to the galaxy and discussed above, will have a complex interaction with the disk as they provide a means of direct scattering over a large area and also induce spiral modes in the disk. In this work, we aim to characterize radial migration induced by satellite bombardment and compare its effects on the stellar disk with those observed in secularly evolved galaxies.

Our investigation complements earlier and ongoing radial migration studies. We perform a simulation campaign, including numerical experiments of isolated disk galaxies with different scale heights and gas fractions, which in turn lead to different levels of spiral structure. For the first time, we examine the effect of satellite bombardment on radial migration utilizing simulations where galactic disks are subjected to a cosmologically motivated satellite accretion history. Via a comparative approach, we determine how the magnitude and efficiency of radial migration depend on input physics, establish correlations between orbital parameters and migration, and present evidence that each of the three migration mechanisms is distinct in the examined parameter space. These characteristics lead to possible observational signatures that may constrain the relative importance of each migration mechanism in the Milky Way.

2 METHODS

2.1 Isolated Disk Models

We employ the method of Widrow & Dubinski (2005) to construct numerical realizations of self-consistent, multicomponent...
disk galaxies. These galaxy models consist of an exponential stellar disk, a Hernquist bulge (Hernquist 1993), and a Navarro et al. (1996, hereafter NFW) dark matter halo. They are characterized by 15 free parameters that may be tuned to fit a wide range of observational data for actual galaxies including the MW and M31. The Widrow & Dubinski (2005) models are derived from three-integral, composite distribution functions and thus represent self-consistent equilibrium solutions to the coupled Poisson and collisionless Boltzmann equations. Owing to their self-consistency, these galaxy models are ideally suited for investigating the complex dynamics involved in the process of radial mixing. The Widrow & Dubinski (2005) method has been recently used in a variety of numerical studies associated with instabilities in disk galaxies, including the dynamics of warps and bars (Dubinski & Chakrabarty 2003; Dubinski et al. 2009), the gravitational interaction between galactic disks and infalling satellites (Gauthier et al. 2006; K08; Purcell et al. 2009; Kazantzidis et al. 2009), and the transformation of disky dwarfs to dwarf spheroidal galaxies under the action of tidal forces from a massive host (Kazantzidis et al. 2011). We refer the reader to Widrow & Dubinski (2005) for an overview of all relevant parameters and a detailed description of this technique.

For the majority of numerical experiments in the present study, we employ model “MWb” in Widrow & Dubinski (2005), which satisfies a broad range of observational constraints on the MW galaxy. Specifically, the stellar disk has a mass of $M_{\text{disk}} = 3.35 \times 10^{10} M_\odot$, a radial scale length of $R_d = 2.82$ kpc, and a sech$^2$ scale height of $z_d = 400$ pc. We note that the adopted value for the scale height is consistent with that inferred for the old, thin stellar disk of the MW (e.g., Kent et al. 1999; Juric et al. 2008). The equivalent exponential scale height is approximately 200 pc, but the sech$^2$ vertical distribution is more accurate. The bulge has a mass and a scale radius of $M_b = 1.18 \times 10^{10} M_\odot$ and $a_b = 0.88$ kpc, respectively. The NFW dark matter halo has a tidal radius of $R_h = 244.5$ kpc to keep the total mass finite (K08), a mass of $M_h = 7.35 \times 10^{11} M_\odot$, and a scale radius of $r_h = 8.82$ kpc. The total circular velocity of the galaxy model at the solar radius, $R_\odot \simeq 8$ kpc, is $V_c(R_\odot) = 234.1$ km s$^{-1}$, and the Toomre disk stability parameter is equal to $Q = 2.2$ at $R = 2.5 R_d$. We note that direct numerical simulations of the evolution of model MWb in isolation confirm its stability against bar formation for 10 Gyr.

We wish to address the dependence of radial mixing in isolated disk galaxies upon initial disk thickness and the presence of gas in the galactic disk. Because of their smaller “effective” Toomre $Q$ stability parameter (e.g., Romeo & Wiegert 2011), thinner disks are more prone to gravitational instabilities and thus yield stronger and better defined spiral structure. Therefore, we might expect them to cause enhanced radial mixing compared to their thicker counterparts. Similarly, because radiative cooling in the gas damps its random velocities, the presence of gas is associated with long-lived spiral structure in disks (e.g., Carlberg & Freedman 1985), which may act to increase the amount of radial mixing. Correspondingly, we initialize several additional disk galaxy models.

The first modified galaxy model was constructed with the same parameter set as MWb but with a scale height 2 times smaller ($z_d = 200$ pc). Except for disk thickness and vertical velocity dispersion, all of the other gross properties of the three galactic components of this model are within a few percent of the corresponding ones for MWb. Not surprisingly, being more prone to gravitational instabilities, this model develops a bar inside $\sim 5$ kpc at time $t \sim 1.2$ Gyr. The second set of modified galaxy models are the same as MWb except for the fact that a fraction $f_g$ of the gas component is replaced by gas. Thus, the resulting gaseous component is constructed with the same initial density distribution as the stellar disk. In the present study, we employ two values for the gas fraction, $f_g = 0.2$ and $f_g = 0.4$, and the adopted methodology will be described in detail in a forthcoming paper (Kazantzidis et al. 2011, in preparation). Briefly, the construction of the gas disk is done by assuming that its vertical structure is governed by hydrostatic equilibrium and by computing the potential and the resulting force field for the radially varying density structure of the gas component. By specifying the polytropic index and the mean molecular weight of the gas and using a tree structure for the potential calculation, the gas azimuthal streaming velocity is determined by the balance between gravity and centrifugal and pressure support.

### 2.2 Perturbed Disk Models

In the standard CDM paradigm of hierarchical structure formation, galaxies grow via continuous accretion of smaller satellite systems. To investigate the effect of accretion events on radial mixing in galactic disks, we have also analyzed $N$-body simulations of the gravitational interaction between a population of dark matter subhalos and disk galaxies. These simulations were first presented by K08, and we summarize them briefly here.

K08 performed high-resolution, collisionless $N$-body simulations to study the response of the galactic model MWb discussed above to a typical $\Lambda$CDM-motivated satellite accretion history. The specific merger history was derived from cosmological simulations of the formation of Galaxy-sized CDM halos and involved six dark matter satellites with masses, orbital pericenters, and tidal radii of $7.4 \times 10^8 \lesssim M_{\text{sat}}/M_\odot \lesssim 2 \times 10^{11}$, $r_{\text{peri}} \lesssim 20$ kpc, and $r_{\text{tid}} \gtrsim 20$ kpc, respectively, crossing the disk in the past $\sim 8$ Gyr. As a comparison, the Large Magellanic Cloud has an estimated total present mass of $\sim 2 \times 10^{10} M_\odot$ (e.g., Schommer et al. 1992; Mastropietro et al. 2005), so the masses of the perturbing dark matter satellites correspond to the upper limit of the mass function of observed satellites in the Local Group. K08 modeled satellite impacts as a sequence of encounters. Specifically, starting with the first subhalo, they included subsequent systems at the epoch when they were recorded in the cosmological simulation. We note that although K08 followed the accretion histories of host halos since $z \sim 1$, when time intervals between subhalo passages were larger than the timescale needed for the disk to relax after the previous interaction, the next satellite was introduced immediately after the disk had settled from the previous encounter. Each satellite was removed from the simulation once it reached its maximum distance from the disk after crossing. This approach was dictated by the desire to minimize computational time and resulted in a total simulation time of $\sim 2.5$ Gyr instead of $\sim 8$ Gyr that would formally correspond to $z = 1$.

K08 and Kazantzidis et al. (2005) found that these accretion events severely perturbed the galactic disk of model MWb without destroying it and produced a wealth of distinctive morphological and dynamical signatures on its structure and kinematics. In this paper, we will investigate the magnitude of radial mixing induced in disk model MWb by these accretion events. As in the case of isolated disk models, it is worthwhile to examine the dependence of radial mixing upon disk thickness. For this purpose, we employed the thinner galaxy model with a scale height of $z_d = 200$ pc described above and repeated the satellite-disk encounter simulations of K08.
2.3 Numerical Parameters

All collisionless numerical simulations discussed in this paper were carried out with the multi-stepping, parallel, tree $N$-body code PKDGRAV (Stadel 2001). The hydrodynamical simulations were performed with the parallel TreeSPH $N$-body code GASOLINE (Wadsley et al. 2004). In the gasdynamical experiments, we include atomic cooling for a primordial mixture of hydrogen and helium, star formation and (thermal) feedback from supernovae. Our star formation recipe follows that of Stinson et al. (2006), which is based on that of Katz (1992). Gas particles in cold and dense regions which are simultaneously parts of converging flows spawn star particles with a given efficiency $c^*$ at a rate proportional to the local dynamical time. Feedback from supernovae is treated using the blast-wave model described in Stinson et al. (2006), which is based on the analytic treatment of blastwaves described in McKee & Ostriker (1977). In our particular applications, gas particles are eligible to form stars if their density exceeds 0.1 atoms/cm$^3$ and their temperature drops below $T_{\text{max}} = 1.5 \times 10^4$ K, and the energy deposited by each Type-II supernova into the surrounding gas is $4 \times 10^{50}$ erg. We note that this choice of parameters and numerical techniques is shown to produce realistic disk galaxies in cosmological simulations (Governato et al. 2007). Lastly, in an attempt to investigate the effect of star formation efficiency on radial mixing, for each of the gas fractions $f_g$ above, we adopted two different values for $c^*$, namely $c^* = 0.05$ (which reproduces the slope and normalization of the observed Schmidt law in isolated disk galaxies) and a lower value of $c^* = 0.01$.

All $N$-body realizations of the disk galaxy models contain $N_d = 10^6$ particles of $m_d = 3.53 \times 10^4 M_{\odot}$ in the disk; $N_b = 5 \times 10^5$, $m_b = 2.36 \times 10^4 M_{\odot}$ in the bulge; and $N_h = 2 \times 10^6$, $m_h = 3.675 \times 10^5 M_{\odot}$ in the dark matter halo. The gravitational softening lengths for the three components were set to $\epsilon_d = 50$ pc, $\epsilon_b = 50$ pc, and $\epsilon_h = 100$ pc, respectively. These are “equivalent Plummer” softenings; the force softening is a cubic spline. In the hydrodynamical simulations of isolated galaxies, gaseous disks were represented with $N_g = 2 \times 10^5$ and $N_g = 4 \times 10^5$ for gas fractions of $f_g = 0.2$ and $f_g = 0.4$, respectively. In these cases, the gravitational softening length for the gas particles was set to $\epsilon_g = 50$ pc. All numerical simulations of isolated and perturbed disks were analyzed at 2.5 Gyr. This time-scale corresponds to ap-
3 RESULTS

Figure 1 presents surface density maps of the stellar distributions of our four collisionless galactic disks with different initial scale-heights \( z_0 \). Each simulation exhibits a distinctive combination of morphological features that could affect radial migration. The initial smooth disks are unstable to spiral instabilities arising from the swing amplification of particle shot noise, an effect that leads to emerging spiral structure as seen in the final isolated disks (e.g., Julian & Toomre 1966). Owing to its smaller “effective” Toomre \( Q \) stability parameter (e.g., Romeo & Wiegert 2011), the \( z_d = 200 \) pc disk (hereafter, disks with this initial scale height will be referred to as “thin”) develops prominent spiral structure and a strong bar. Conversely, the \( z_d = 400 \) pc disk (hereafter, disks with \( z_d = 400 \) pc will be referred to as “thick”) has relatively little spiral structure and does not form a bar in isolation. We emphasize that the “thick”, \( z_d = 400 \) pc disk is the one in best agreement with the old, thin disk of the MW, while \( z_d = 200 \) pc is too thin (see Section 2.1).

The radial and vertical morphology of the perturbed disks is distinct from their isolated counterparts. Both perturbed disks develop bars and are characterized by prominent flaring and much larger scale heights than those of the isolated disks (K08; Kazantzidis et al. 2009). There is evidence that some spiral structure and does not form a bar in isolation. Throughout this section, we will investigate where particles move throughout each simulation. Figure 2 presents the distribution of \( \Delta r \) for the six isolated disk simulations, where \( r_f \) and \( r_i \) refer respectively to each particle’s final and initial projected distance from the galactic center. Each panel contains the \( \Delta r \) distribution, the median \( |\Delta r| \) (denoted \( \Delta r_{\text{med}} \)), and the 80th percentile (denoted \( \Delta r_{80} \)) for two of the six isolated galaxies in our simulation suite.

The isolated, collisionless simulations (left panel) clearly demonstrate that disk scale height and gas content affect the radial migration process. The thin disk’s \( \Delta r \) distribution (black line) is considerably broader than the thick disk’s (gray hatch). Both \( \Delta r_{\text{med}} = 0.91 \) kpc and \( \Delta r_{80} = 2.03 \) kpc of particles in the thin disk are approximately double those found in the thick disk.

The remaining panels of Figure 2 compare the \( \Delta r \) distributions of the hydrodynamical simulations of our sample. We examine four isolated disks with two initial gas fractions: \( f_g = 20\% \) (middle panel) and \( f_g = 40\% \) (right panel). The two histograms in each panel represent star formation efficiencies \( (\epsilon^*) \) of 1% (solid line) or 5% (gray, hatched histogram). Gas has a strong impact on particle radial migration: \( \Delta r_{\text{med}} \) increases from 0.47 kpc in the collisionless case (all hydrodynamical simulations are of “thick” disks) to \( \sim 0.70 \) kpc when \( f_g = 20\% \) and \( \sim 0.80 \) kpc when \( f_g = 40\% \). The extent of radial migration is less dependent on star formation efficiency; there are only minor differences amongst each pair of histograms in the middle and right panels. However, in both panels, \( \Delta r_{\text{med}} \) and \( \Delta r_{80} \) are highest when \( \epsilon^* = 1\% \). In these four hydrodynamical simulations, the time-averaged gas content of the disk is directly correlated with the percentage of particles that migrate and their typical displacement relative to their formation radius.

Decreasing the scale height of the stellar disk, increasing the fraction of the initial disk mass in gas, and lowering the star formation efficiency all increase the midplane density of the disk. This enhanced density increases the coherence and self-gravity of spiral structure, supporting it against the dissipative nature of individual particles’ random motions (as described by, e.g., Toomre 1977). The correlation of spiral structure strength with the fraction of particles that migrate away from their birth radii and the median migratory distance traversed suggests that the SB02 mechanism, intimately linked with spiral waves, plays a major role in the migration of particles in the isolated systems. The symmetric shapes (within 0.4% about 0 kpc) of the \( \Delta r \) distributions are also consistent with migration via the SB02 mechanism. Resonances between the bar and spiral structure could also influence the timescale for migration in the thin disk models (Minchev & Famaey 2010), but the \( z_d = 400 \) pc models do not develop bars, regardless of whether they include gas.

Figure 3 compares the \( \Delta r \) distributions of the isolated collisionless disks to those of their perturbed counterparts. The \( \Delta r_{\text{med}} \) and \( \Delta r_{80} \) of each distribution are labeled and color-coded to match the corresponding histogram. Each panel, the most sharply peaked histogram (red) shows the expected \( \Delta r \) distribution from epicyclic motion alone, which we compute given the particle’s initial phase space coordinates. We model the epicycle as a simple harmonic oscillator about the particle’s guiding center radius \( R_g \) with an amplitude set by its initial radial energy \( (E_r) \) (Chapter 3, Binney & Tremaine 2008). The energy associated with the circular component of the particle’s orbit \( (E_{\text{circ}}) \) is set by \( R_c \), which is the radius of a circular orbit with angular momentum equal to the midplane component of the particle’s angular momentum. The radial
energy of the orbit is \( E_r = E_{\text{tot}} - E_z - E_{\text{circ}} \) where \( E_z \) is the vertical energy component and \( E_{\text{tot}} \) is the total energy. As \( E_r \) sets the amplitude of the epicycle, we can solve for the position of the particle as function of time with respect to its guiding center radius. We choose two random phases of the epicycle oscillation, designating the first as \( r_1 \) and the second as \( r_2 \), and compute \( \Delta r \). The resulting “baseline” distribution is shown in red and is a result of observing the two random phases (akin to our “initial” and “final” snapshots) of the initially elliptical orbits. It does not take into account the potential heating of these initial orbits and the subsequent increase in amplitude of their radial motion (blurring). The shape of the distribution does not change if we change our random seed or increase in amplitude of their radial motion (blurring). The shape of the distribution dramatically, nearly doubling in each panel. We report the fraction of all particles in non-overlapping 500 pc bins of \( \Delta r \). The two collisionless simulations (left panel) illustrate the effect of disk structure on radial mixing, i.e., there is more radial migration in the smaller scale height (\( z_4 = 200 \) pc, black line) disk than in its thicker counterpart (\( z_4 = 400 \) pc, hatched histogram). The four hydrodynamical simulations are in the middle (\( f_g = 20\% \)) and right (\( f_g = 40\% \)) panels. In both of these panels, the grey, hatched histograms represent the simulations with higher star formation efficiency (\( c^* = 0.05 \)); those with \( c^* = 0.01 \) are shown in black. Both increased gas fractions and smaller star formation efficiencies yield greater radial mixing.

![Figure 2](image-url)
Radial Mixing in Galactic Disks

Figure 3. The $\Delta r$ distribution for all disk particles in the four collisionless simulations. Each histogram show the fraction of particles in non-overlapping 500 pc bins of $\Delta r$. The isolated (hatched, gray histogram) and perturbed (black line) $z_d = 200$ pc (left panel) and $z_d = 400$ pc (right panel) disks are shown. The perturbed disks show greater dispersion in $\Delta r$ compared to isolated galaxies with the same geometry. The red histograms denote the expected $\Delta r$ distribution from epicyclic motion alone, given the initial orbital configurations of the particles in each disk. $\Delta r_{\text{isol}}$ and $\Delta r_{\text{80}}$ are labeled and color-coded for the three distributions in each panel.

gular momentum projected along the axis of symmetry ($J_z$) versus specific binding energy ($E$). Particles are grouped into 50 distinct linear bins of $E$. We calculate the median $J_z$ in each bin. Red, yellow, and orange regions connect the $68^{th}$, $95^{th}$, and $99^{th}$ percentile $J_z$ intervals (centered on each median $J_z$) respectively, across all energy bins. The $1\%$ most discrepant particles in $J_z$ for a given $E$ are plotted as individual points. Using each disk’s rotation curve, we plot $J_z$ and $E$ for circular orbits in the midplane of the disk (green line). Here, $J_z = v_c(r) \times r$, where $v_c(r)$ is the circular velocity at radius $r$ and $E = \frac{1}{2} v_c^2(r) + \Phi(r)$, where $\Phi(r)$ is the potential in the disk midplane at radius $r$. Particles on a circular orbit have the maximum $J_z$ allowed given their energy.

By construction, particles are initially (left column of Figure 5) on nearly circular orbits with a small radial velocity component. The red region, falling close to the circular orbit curve in both initial disks, confirms that $v_r$ is small. Particles significantly displaced from the green curve in the initial states either have extreme $v_r$ or are on highly inclined orbits ($J_z$ is projected along the $z$ axis). Due to this inclination effect, the initial thick disk has a lower median $J_z$ as a function of $E$ than the thin disk despite having the same $v_c$ distribution.

In the isolated thin disk (upper middle panel of Figure 5), the range of $J_z$ grows relative to the initial values at every energy $E$. The change is largest at lower energies. Recall from Figure 1 that the isolated thin disk develops a bar, which is composed of particles on radial orbits with relatively low $J_z$, in the most bound region of the disk. Thus, this relatively large change towards less circular orbits at low energies can be associated with bar formation. In contrast, the distribution of $J_z$ in the isolated thick disk is basically equivalent to its initial state. In the isolated thick disk there is no bar, relatively little spiral structure develops, and we find the least radial migration among the four simulations. Ignoring the region of the Lindblad diagram influenced by members of the bar, the isolated disk diagrams suggest that the extensive radial migration seen in the thin disk and associated with guiding center modification decreases the angular momentum of particles.

The two perturbed disks (right column of Figure 5) show larger changes in their Lindblad diagrams. Bar formation in both simulations can explain the significantly lower median $J_z$ at lower energies. At higher energies, corresponding to less bound particles further out in the disk, both disks show a much larger dispersion in $J_z$ than their isolated counterparts. Additionally, there is substructure in the Lindblad diagrams (groups of relatively low $J_z$ in a narrow range of energy) not seen in the isolated disks. Satellite bombardment in the perturbed disk simulations has a qualitatively discernible impact on the angular momentum distribution of the disk.

To quantify changes in $J_z$, we introduce the circularity ($\epsilon$) quantity (e.g., Abadi et al. 2003). For a particle of energy $E_i$ and angular momentum $J_z$ in the $z$-direction, we define circularity as $\epsilon = J_z / J_{\text{circ}}(E_i)$, the ratio of $J_z$ to the specific angular momentum of the particle would have if it were on a circular orbit with energy $E_i$ (obtained using the circular orbit curve). Circular orbits have $\epsilon = 1$ and radial orbits have $\epsilon = 0$. Negative circularities correspond to retrograde orbits. We note that this method is formally different from that of Abadi et al. (2003). The circular orbit curve may not represent the highest $J_z$ for a given $E$ due to shot noise in the potential or gravitational potential asymmetries. Strictly defining the maximum $J_z$ in each energy bin (as in Abadi et al. 2003) results in the same qualitative trends discussed later. Our method benefits from the use of a mathematically constructed and reproducible rotation curve and systematically decreases $\Delta \epsilon = \epsilon_i - \epsilon_i$ at the 0.01
Figure 5. Lindblad diagrams for all collisionless simulations. Each column shows the initial (left); final, isolated (middle); and final, perturbed (right) particle angular momentum projected along the $z$ axis ($J_z$, kpc km s$^{-1}$) vs. specific energy ($E$, km$^2$ s$^{-2}$). The red, yellow, and orange regions encompass 68%, 95%, and 99%, respectively, of particles centered on the median $J_z$ as a function of $E$. $J_z > 99\%$ outliers are plotted as points. The green line denotes the theoretical angular momentum - energy curve for circular orbits. Each panel includes a histogram illustrating the distribution of circularity ($\epsilon$) for all disk particles (see text for details). The isolated thin disk shows evolution towards slightly non-circular orbits. There is little change in $\epsilon$ in the isolated $z_d = 400$ pc case. Simulations with satellite bombardment show a pronounced shift towards less circular orbits. For reference, the total energy of midplane orbits at 2, 5, 10, and 15 kpc from the galactic center is approximately $-2.0$, $-1.6$, $-1.2$, and $-1.0$ km$^2$ s$^{-2}$, respectively.

Returning to Figure 5, we see that the two isolated disks show markedly different evolution of their circularity distributions. In the isolated thin disk, the fraction of stars with $\epsilon \sim 1$ (rightmost bin) drops from 0.38 to 0.26, and the median $\epsilon$ drops from 0.980 to 0.955. Particles in the bar predominantly populate the newly
formed low circularity tail. In the thick isolated disk, on the other hand, the $\epsilon$ distribution is nearly identical to the initial disk’s, with median $\epsilon$ dropping only 0.002. This lack of evolution in the circularity distribution is fully consistent with CR scattering. As described by SB02, individual stars may exchange angular momentum across CRs (churning) while the overall distribution would remain unchanged. However, our results from Section 3.1 show that the Delta R distribution of this model, which reflects individual particles and their orbits, is nearly identical to that expected from observing the particles’ initially elliptical orbits. The circularity distribution of the isolated, thick disk combined with its Delta R distribution imply that, on average, individual particle guiding centers are not significantly modified.

The circularity distributions of the perturbed disks are demonstrably altered from their isolated counterparts but are similar to one another. The most common circularities are now in the range $0.96 \leq \epsilon \leq 0.98$ in both disks, with a decrease for $\epsilon > 0.98$. The thin disk starts with more $\epsilon \sim 1$ particles because of its smaller orbital inclinations, and its circularity distribution evolves more strongly, with median $\epsilon$ dropping from 0.980 to 0.936 vs. 0.963 to 0.929 for the perturbed thick disk. Notably, the perturbed thin and thick disks evolve to similar $\Delta r$ (Figure 4) and $\epsilon$ distributions despite starting with different scale heights.

Figure 7 tracks the changes of selected individual particles in the $(E, J_z)$ space of the Lindblad diagram. While $E$ and $J_z$ are not individually conserved in the presence of a non-axisymmetric perturbation, the Jacobi invariant $I = E - \Omega_b J_z$ is, where $\Omega_b$ is the pattern speed of the perturbation, assumed to be static and small (SB02, Sellwood 2010). If $\Delta I = 0$, then $\Delta J_z / \Delta E \approx \Omega_b$. The SB02 mechanism operates at the corotation resonance of the star/particle and the spiral wave, requiring that $\Omega_b = \Omega_{\text{rot}}$. Thus, in the galaxy, particles should move parallel to the line that is tangent to the circular orbit curve at their binding energy prior to scattering. In other words, the SB02 mechanism requires that changes in energy be accompanied by changes in angular momentum that preserve the orbital shape, modulo differences in the slope of the circular orbit curve over the range $[E_i, E_f]$ of a given particle.

Figure 7 zooms in on the area of the Lindblad diagram designated by the two small boxes drawn on the initial state diagrams in Figure 5. We randomly select ten particles within the same initial $E$ and $J_z$ range (filled, dark squares in Figure 7) and plot their final $E$ and $J_z$. We require $E_f - E_i = \Delta E < -0.05 \times 10^5$ km$^2$ s$^{-2}$ to ensure that the individual tracks are visible. In the isolated thin disk (top left), most particles lose total energy in the isolated thick disk (bottom left), their $J_z$ typically remains closer to the circular orbit curve than in the isolated thick disk. The changes in energy and angular momentum are relatively small, indicating that the guiding centers are modified to a modest extent (Binney & Tremaine 2008). Note that these particles are in the top 1% of $|\Delta J_z|$; most particles in this particular experiment have $\Delta J_z \approx 0$. The randomly selected particles in both perturbed disks have slopes $\Delta J_z / \Delta E$ that are steeper than the slope of the tangent to the circular orbit curve at corotation. This distinct coupling of $\Delta E$ and $\Delta J_z$, combined with our results concerning the migration probability as a function of $R_{\text{corr}}$ (Section 3.1), offer compelling evidence that migration in the perturbed disks is driven, at least in part, by a mechanism that does not operate in the isolated systems. Figure 7 shows that the orbital characteristics of many particles in the perturbed disks are modified in
a fashion inconsistent with either epicyclic motion or a single spiral wave scattering event.

We now examine the correlation between migration and changes in orbital properties. Since the metallicity of star-forming gas increases with time and decreases with radius, any such correlations also imply observable correlations between the present orbital parameters and metallicities of stars as a function of age and Galacto-centric radius. Figure 8 shows the median change in circularity, $\Delta \epsilon = \epsilon_f - \epsilon_i$, as a function of radial migration distance $\Delta r$ in the four collisionless simulations (open squares). In the isolated thick disk, changes in circularity are tiny (median $|\Delta \epsilon| < 0.005$) at any $\Delta r$; Figure 8 shows that particles in this simulation stay close to the circular velocity curve even if they change energy. In the other three simulations, particles with negative $\Delta \epsilon$ show substantial drops in circularity (typical median $\Delta \epsilon < -0.06$), which are almost certainly associated with bar formation. The bars that form in these three simulations increase the orbital eccentricities of their members. To focus on behavior in the disk proper, the filled squares in each panel show the median $\Delta \epsilon$ vs. $\Delta r$ for those particles that start and end the simulation at $r \geq 4$ kpc, beyond the extent of the bar. Error bars mark the inter-quartile range (25th to 75th percentile) at each $\Delta r$.

In the isolated thin disk, particles that migrate inwards ($\Delta r < 0$) experience a modest decrease in circularity, stronger for more negative $\Delta r$ (and $\Delta E$), consistent with the tracks shown in Figure 7. Particles that migrate outwards ($\Delta r > 0$) increase their energy and typically traverse areas of the Lindblad diagram with relatively little change in the slope of the circular orbit curve. Following the tangent to the circular orbit curve, particles will not significantly change their circularity in such a scenario (note the relative lack of low circularity particles at $E > -1.5 \, \text{km}^2 \, \text{s}^{-2}$ in Figure 5). The perturbed disks show a decrease in median circularity at every $\Delta r$, and a much wider inter-quartile range indicating a greater range of orbital inclinations and eccentricities. Circularity drops more strongly for particles that have experienced strong radial migration, either inward or outward. The minimum in these curves is slightly offset to positive $\Delta r$ because dynamical heating slightly puffs up the disk radially, decreasing the potential at a given radius, thereby increasing its total energy, moving particles to the right on the Lindblad diagram, and lowering the circularity for particles with $\Delta r = 0$. Figure 8 indicates that stars with anomalous chemistry for their age and current position should have preferentially more eccentric orbits. The trend is smaller than the inter-quartile range, but similar in magnitude.

Like Figure 8, but instead of $\Delta \epsilon$ it plots the change in vertical energy, quantified by the maximum distance $z_{\text{max}}$ that a particle can reach from the disk plane. We compute $z_{\text{max}}$ approximately from each particle’s vertical velocity component assuming that the final potential of each simulation is static, namely $z_{\text{max}} = |z| + \frac{v_z^2}{2 \, G \, \Sigma(r)}$, where $z$ is the vertical position of the particle at the final output, $v_z$ is the velocity along the $z$ axis, $G$ is the gravitational constant, and $\Sigma(r)$ is the surface density of the disk at radius $r$ assuming all the mass of the disk is in the midplane. The change in $z_{\text{max}}$ is $\Delta z_{\text{max}} = z_{\text{max},f} - z_{\text{max},i}$ where the subscripts $f$ and $i$ refer to the initial and final simulation snapshots, respectively.

The two isolated simulations show a shallow linear trend between $\Delta z_{\text{max}}$ and $\Delta r$ (open squares for all particles; filled squares for those with $r_1, r_f > 4$ kpc). When particles move outwards (inwards) through the disk, they experience a weaker (stronger) gravitational potential, thus increasing (decreasing) $z_{\text{max}}$. However, changes in $z_{\text{max}}$ are small, less than 300 pc even when we consider the quartile range at the extremes of $\Delta r$. The perturbed disks, by contrast, show larger changes in the median $z_{\text{max}}$ and a dramatic increase in the inter-quartile range at all $\Delta r$. K08 show that the perturbed 400 pc disk develops a two-component vertical structure in quantitative agreement with the observed thin/thick disk structure of the MW. The perturbed 200 pc disk increases its scale height (to $\approx 500$ pc at the solar annulus), but it can still be described by a single component model. This vertical heating by satellite perturbations is evident in Figure 9. Particles that have large positive $\Delta r$ have the largest increase in $z_{\text{max}}$, which is plausibly a consequence of moving outwards towards regions of lower disk surface density.
Radial Velocity Experiment (RA VE, Steinmetz 2006) concentrate earlier analysis specifically for stars that reside in the solar annulus on the solar neighborhood. In this section, we repeat some of our accurate at smaller distances. Some of the most detailed chemodynamic simulations are more accurate in the solar annulus throughout the simulation. Both thin disks and the perturbed thick disk show a broad range of $R_{\text{form}}$ at the solar annulus. The gray histogram in each panel is the $R_{\text{form}}$ distribution for those particles that are in the solar annulus and at large heights above the plane ($1.0 < |z| < 1.5 \text{kpc}$) in the final simulation output.

and thus weaker vertical restoring force. For $\Delta r < 0$, the trend of median $\Delta z_{\text{max}}$ with $\Delta r$ is approximately flat, suggesting a cancellation between the effects of increased $\Sigma(r)$ at smaller $r$ and direct satellite-induced heating of those particles with the largest excursions. Figure 7 implies that stars with high metallicity for their age and present location should have preferentially larger $z_{\text{max}}$, though the scatter is larger than the trend.

### 3.3 Solar Annulus

The solar neighborhood is easier to study than other regions of the Galaxy, since high-precision spectroscopy is easier for brighter stars and parallax and proper motion measurements are more accurate at smaller distances. Some of the most detailed chemodynamic surveys, such as the GCS (Nordström et al. 2004) and the Radial Velocity Experiment (RAVE, Steinmetz 2006), concentrate on the solar neighborhood. In this section, we repeat some of our earlier analysis specifically for stars that reside in the solar annulus ($7 \text{kpc} \leq r \leq 9 \text{kpc}$) at the end of each simulation. This focus on the solar annulus also removes much of the impact of the bars that dominate evolution of the inner disk ($r < 3 \text{kpc}$) in three of our simulations, though some particles from the bar region can migrate as far as the solar radius, and resonances between the bar and spiral structure may increase migration frequency (Minchev & Famaey 2010).

Figure 10 shows the radius of formation ($R_{\text{form}}$) distribution of particles residing in the solar annulus, marked by the vertical dashed lines. Only the isolated thick disk simulation predicts a final solar annulus dominated by stars born in the solar annulus, with tails extending 1–2 kpc on either side. The broad $\Delta r$ distributions of the other three simulations show that their stars migrate to the solar annulus from a wide range of formation radii. The global $\Delta r$ (Figure 8) and solar annulus $R_{\text{form}}$ distributions of the isolated and perturbed thin disks are remarkably similar, despite the differences in migration mechanisms discussed in Section 3.2.

In the isolated thin disk, 32% of solar annulus particles originated at $R_{\text{form}} \leq 6 \text{kpc}$ and 4% at $R_{\text{form}} \geq 10 \text{kpc}$. Corresponding numbers for the perturbed thin disk are 36% and 3%. The $R_{\text{form}}$ distribution of the perturbed thick disk is slightly narrower, but it still broad with respect to the isolated thick disk.

Figure 11 shows the correlations between $\Delta \epsilon$ and $\Delta r$ for particles that end in the solar annulus. Consistent with results for the full disk (Figure 8), the solar annulus particles in the isolated thick disk show no significant change in median $\epsilon$ regardless of $\Delta r$. In the isolated thin disk, the range of $\Delta \epsilon$ is much larger, with a modest decrease in median $\epsilon$. The median $\Delta \epsilon$ drops at large positive $\Delta r$ because particles move to less inclined orbits as they migrate outward and disk heating has slightly modified the galaxy’s circular velocity curve, allowing outward moving particles to potentially increase circularity. In the perturbed thick disk, there is a strong and nearly linear trend between $\Delta \epsilon$ and $\Delta r$. Particles that migrated to the solar annulus from the outer disk have experienced substantial drops in circularity, while particles migrating from the inner disk show only modest decreases. The range of $\Delta \epsilon$ is large at all $\Delta r$. Results for the perturbed thin disk are intermediate between those of the isolated thin and perturbed thick disks: quasi-linear trends at large $|\Delta r|$ but a flat plateau at intermediate $|\Delta r|$. This behavior is plausibly a consequence of two different mechanisms contributing to migration, with spiral-induced mixing dominating at intermediate $\Delta r$ and satellite-induced mixing dominating at the extremes.

Figure 12 shows the solar annulus correlations of $\Delta z_{\text{max}}$ with $\Delta r$, analogous to Figure 8 for the full disk. For the two isolated disks there is a clear linear trend of median $\Delta z_{\text{max}}$ with $\Delta r$ as expected from the arguments in Section 3.2. Particles that migrate outward move to a region of lower disk surface density, so if their vertical velocities are not systematically changed by the radial migration they will attain higher $z_{\text{max}}$. Both perturbed disks show signs of the strong vertical heating induced by satellite accretion. The median $\Delta z_{\text{max}}$ is $\leq 0.2 \text{kpc}$ almost independent of $\Delta r$, and the range of $\Delta z_{\text{max}}$ is much larger than in the isolated disks. Since inwardly migrating particles experience a higher vertical potential at $r_{\text{f}}$ than $r_{\text{i}}$, they must experience more vertical heating than outwardly migrating particles to keep the $\Delta z_{\text{max}}$ trend flat.
Returning to Figure 10, gray histograms represent the \( R_{\text{form}} \) distributions of particles that end the simulations in the solar annulus at high \( z, 1.0 < |z| < 1.5 \) kpc. In the isolated thick disk and both perturbed disks, the \( R_{\text{form}} \) distribution of high \( z \) particles resembles that of all solar annulus particles. In these three models, therefore, selecting high- \( z \) particles does not isolate a population with atypical radial migration. In the isolated thin disk, on the other hand, the \( R_{\text{form}} \) distribution of high- \( z \) particles is strongly skewed towards low formation radii, with a peak at \( R_{\text{form}} = 5 \) kpc. Here, only initially hot particles that migrate significantly outwards via churning or interactions with the bar and experience a weaker potential have enough vertical energy to overcome the local restoring force and reach high \( z \). Only \( 0.1\% \) of the solar annulus is at high \( z \) in the isolated thick disk (truly the tail of the initial vertical velocity dispersion); this number rises to \( 3.2\% \) and \( 7.5\% \) in the thin and thick perturbed disks, respectively. The radial migration mechanisms in the perturbed disks ensure that even the high \( z \) population of the solar annulus comes from a broad range of \( R_{\text{form}} \). Measurements of the age-metallicity relation for high- \( z \) stars could be a valuable diagnostic for distinguishing models of radial migration and vertical heating.

4 SUMMARY AND DISCUSSION

Observations suggest that radial migration plays an important role in the chemical evolution of the MW disk [Wielen et al. 1993; Schönrich & Binney 2003]. SB02 described a mechanism by which stars at corotation resonance with spiral waves can scatter, producing large changes in guiding center radius while keeping stars on nearly circular orbits. Previous simulations have shown that migration over several kiloparsecs can occur in isolated disks grown by smooth accretion [Roskar et al. 2008] and that encounters with satellites can also induce migration [Quillen et al. 2009]. Here we have carried out a systematic investigation of a variety of simulations to characterize the role of stellar disk properties, gas fractions, and satellite perturbations in producing radial migration. Most importantly, our suite of simulations includes experiments with a level of satellite bombardment expected in LCDM models of galaxy formation, which earlier investigations [K08; Kazantzidis et al. 2009] have shown to produce vertical and in-plane structure resembling that seen in the MW.

For disks evolved in isolation, the degree of migration correlates with the degree of disk self-gravity, susceptibility to bar formation, and spiral structure. The collisionless stellar disk with \( z_\text{d} = 400 \) pc, chosen to match that of the thin disk in the MW, has no bar and minimal spiral structure. It exhibits limited radial migration; the median value of radial change \( |r_\text{f} - r_\text{i}| \) is \( \Delta r_{\text{med}} = 0.47 \) kpc, consistent with the level expected from epicyclic motion (Figure 6). The collisionless \( z_\text{d} = 200 \) pc disk is much more unstable, develops a bar and spiral structure (Figure 1), and has a higher degree of radial migration, with \( \Delta r_{\text{med}} = 0.91 \) kpc and \( \Delta r_{90} = 2.03 \) kpc. The presence of gas is a catalyst for radial migration in the \( z_\text{d} = 400 \) pc case. Both \( \Delta r_{\text{med}} \) and \( \Delta r_{90} \) increase in models with larger gas fractions or lower star formation efficiency (which consumes gas more slowly). The galactic bar strongly influences, and perhaps dominates, the resultant individual particle dynamics in the inner galaxy. Substantial radial migration, consistent with the SB02 mechanism or being satellite-induced, occurs in the outer portion of the disk in all of our collisionless experiments except for the isolated \( z_\text{d} = 400 \) pc disk. In the \( z_\text{d} = 200 \) pc disk, stars that finish the simulation in the solar annulus (\( 7 < R < 9 \) kpc) have \( \Delta r_{\text{med}} = 1.36 \) kpc, \( \Delta r_{90} = 2.68 \) kpc.

Adding satellite perturbations dramatically increases radial migration in the \( z_\text{d} = 400 \) pc disks, raising \( \Delta r_{\text{med}} \) from 0.47 kpc to 0.89 kpc globally and from 0.60 kpc to 1.15 kpc in the solar annulus. Perturbations do not change the \( \Delta r \) distribution so drastically in the \( z_\text{d} = 200 \) pc disks, but there are other indications that the nature of radial migration is different. In the isolated galaxies, migrating particles in the outer disk follow tracks in \( F, J_z \) space that parallel the tangent to the local circular velocity curve, as expected for the SB02 mechanism (see Section 2.3). However, inwardly migrating particles in the perturbed disks lose more angular momentum for a given change in energy, causing migration associated with heating and thus different from the SB02 mechanism in this respect. The relationship between \( \Delta E \) and \( J_z \) found in the perturbed experiments is broadly more consistent with scattering at LR (see SB02), but a detailed decomposition of the modes in the disk and satellites (beyond the scope of this paper) is necessary to confirm LR scattering on a particle by particle basis. The radial distribution of migrating particles presents an even clearer distinction, one with important implications for chemical evolution models (Figure 4). In the isolated disks, the probability that a particle with a formation radius \( R_{\text{form}} \) undergoes significant migration (\( |\Delta r| > 1 \) kpc) is approximately proportional to the disk surface density at \( R_{\text{form}} \); i.e., migration follows mass. In the perturbed disks, the probability of migration is flat or increasing with radius, so a much larger fraction of migration comes from the tenuous outer disk. Particles in the outer disk are overall more likely to migrate relative to the inner disk as 1) they feel a weaker potential and are thus more susceptible to direct heating by satellites and 2) their circular frequencies make them more likely to resonantly interact with satellites and migrate. Thus, satellite-induced migration patterns are distinct from those produced by purely secular evolution.

Satellite bombardment heats the disk both vertically and radially. Compared to the isolated disks, the perturbed disks experience a greater drop in median circularity (hence growth of median eccentricity) during their evolution, and individual particles experience a wider range of circularity changes. The circularity change is moderately correlated with radial migration distance and direction, though the trend is smaller than the inter-quartile range at a given \( \Delta r \) (Figure 8). If we restrict our analysis to the solar annulus, the correlation between \( \Delta r \) and \( \Delta e \) is more significant. In particular,
particles in the solar annulus of the $z_d = 400 \text{ pc}$ disk that migrate 2–4 kpc inwards experience substantial drops in circularity, while those that migrate the same distance outwards nearly maintain their initial circularity (Figure 11). The large inward excursions from the outer disk, driven by satellite perturbations, systematically remove angular momentum from particle orbits.

As shown by K08, vertical heating by satellite bombardment produces (in the case of the $z_d = 400 \text{ pc}$ disk) a two-component vertical structure in quantitative agreement with the thin and thick disk profiles observed in the MW. In the isolated disks, we find the expected trend that as particles migrate outwards, they experience a weaker vertical potential and increase their vertical energy (characterized by $z_{\text{max}}$, the maximum distance from the plane that a particle can reach given its current location and velocity). However, in our simulations, this effect is not sufficient to produce a second, “thick-disk” component in the isolated systems even if they have substantial radial migration. The satellite perturbations increase the median and range of $z_{\text{max}}$ considerably at all $\Delta r$. The resulting overall trend of $\Delta z_{\text{max}}$ with $\Delta r$ is fairly flat (especially at a fixed $r_I$, such as the solar annulus, see Figures 9 and 12), suggesting that systems with low vertical velocity dispersion and subjected to weaker potentials (as in the outer disk) are more susceptible to vertical heating (similar to the radial heating trends seen in Section 3.1). Particles with $R_{\text{form}}$ interior or exterior to a given $r_I$ can be found at significant distances above the plane in the perturbed disks, while increases in $z_{\text{max}}$ are smaller in the isolated systems and rely on particles moving outwards (Figure 10).

Our results confirm earlier findings that spiral structure development in isolated disks (Roškar et al. 2008; Loebman et al. 2010) and perturbations by satellites (Quillen et al. 2009) can produce significant radial mixing of stellar populations while retaining reasonable orbital structure for disk stars. They strongly support the view (Wielen et al. 1976; SB02; Schönrich & Binney 2009) that radial mixing is an essential ingredient in understanding the chemical evolution of the MW and disk galaxies in general. While a combination of metallicity and age can be used to estimate a star’s formation radius given a chemical evolution model, the uncertainties in this approach (including the difficulty of estimating ages for typical stars in a spectroscopic survey) will make it difficult to reconstruct formation radius distributions for observed stars, even in the solar neighborhood. However, our analysis provides theoretical guidance for chemical evolution models that incorporate radial mixing (e.g. Schönrich & Binney 2009) and suggestions for the correlations one might search for between chemical abundances and orbital properties (though the predicted trends are fairly weak). We regard our perturbed $z_d = 400 \text{ pc}$ disk as the most relevant simulation, as it includes the satellite accretion events expected in ΛCDM and produces a final vertical structure similar to that measured in the MW. In this simulation 41% of particles in the 7 kpc < $R < 9$ kpc solar annulus were “born” in that annulus, 20% migrated there from $R < 6$ kpc, and 7% migrated there from $R > 10$ kpc.

Radial mixing and orbital dynamics changes are sensitive to several different aspects of disk modeling, as shown by the variety of our results. Robust predictions should therefore come from calculations that self-consistently include star formation, chemical enrichment, gas accretion, and accretion events, and the interplay among these elements. We will move in this direction with our future simulations. Giant spectroscopic surveys such as SEGUE, RAVE, APOGEE, LAMOST, and HERMES offer an extraordinary opportunity to unravel the formation history of the MW, and they offer an exciting new challenge to theoretical models of galaxy formation.

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