Enhancement of oscillation amplitude of cavitation bubble due to acoustic wake effect in multibubble environment

Takuya Yamamoto\textsuperscript{a,b,*}, Sergey V. Komarov\textsuperscript{a,b}

\textsuperscript{a} Graduate School of Environmental Studies, Tohoku University, Miyagi 980-8579, Japan
\textsuperscript{b} Department of Metallurgy, Graduate School of Engineering, Tohoku University, Miyagi 980-8579, Japan

\textbf{ARTICLE INFO}

\textbf{Keywords:}
Acoustic cavitation bubble
Ultrasound
Volume of fluid method
OpenFOAM
Acoustic wake effect
Multi-bubble system

\textbf{ABSTRACT}

Acoustic cavitation occurs in ultrasonic treatment causing various phenomena such as chemical synthesis, chemical decomposition, and emulsification. Nonlinear oscillations of cavitation bubbles are assumed to be responsible for these phenomena, and the neighboring bubbles may interact each other. In the present study, we numerically investigated the dynamic behavior of cavitation bubbles in multi-bubble systems. The results reveal that the oscillation amplitude of a cavitation bubble surrounded by other bubbles in a multi-bubble system becomes larger compared with that in the single-bubble case. It is found that this is caused by an acoustic wake effect, which reduces the pressure near a bubble surrounded by other bubbles and increases the time delay between the bubble contraction/expansion cycles and sound pressure oscillations. A new parameter, called “cover ratio” is introduced to quantitatively evaluate the variation in the bubble oscillation amplitude, the time delay, and the maximum bubble radius.

1. Introduction

Ultrasound irradiation into liquid can produce acoustic cavitation, which causes a number of interesting phenomena such as organic and inorganic chemical synthesis \cite{1-4}, ultrasonic cleaning \cite{5-8}, atomicization \cite{9}, and emulsification \cite{10-14}. These phenomena are caused by nonlinear oscillations of cavitation bubble. Therefore, the bubble oscillation is considered the most important phenomenon in the above-mentioned areas and that is why it has been the subject of numerous studies.

Oscillation of acoustic cavitation bubble has been studied especially for single bubble. These studies have been summarized by Lauterborn \cite{15} and Yasui \cite{16}. The spherical bubble oscillations can be described by Rayleigh-Plesset or Keller equation suggesting that the bubble oscillates nonlinearly in high-intense ultrasound fields. The bubble is collapsed during the nonlinear oscillations, and temperature and pressure inside the bubble becomes as high as 5000 K and 100 atm., respectively \cite{16,17}. During the collapse, the water vapor in the bubble is thermally decomposed into some radicals such as OH and H, which is considered to be the main contributors to sonochemical reactions. Also, if a bubble is located near a wall, its cavitation causes formation of high-speed liquid jet toward the wall \cite{18-20}. The bubble dynamic behavior and chemical reactions have been analyzed experimentally in single-bubble systems to clarify the above-mentioned phenomena both numerically and experimentally. On the other hand, in the above ultrasonic applications, many bubbles exist simultaneously which can interact each other. Therefore, it is important to elucidate the bubble behavior in a multi-bubble system under ultrasound irradiation.

A number of studies have been carried out using multi-bubble systems. According to the experimental results \cite{21-26}, many unique phenomena occur during the multi-bubble cavitation. For example, it has been found that a bubble cluster is formed due to the secondary Bjerknes force, and the bubble oscillation pattern in the multi-bubble system is different from that of single bubble cavitation \cite{21}. In the multi-bubble system, the efficiency of sonochemical reaction is drastically reduced when the sound pressure amplitude exceeds a specific value \cite{22}. The subharmonic acoustic spectra are generated in the multi-bubble system \cite{23,25}, and this phenomenon is considered to be caused by shockwave generation in a half cycle of ultrasonic wave \cite{26}. In addition, the bubble shape becomes planar during the oscillation in a multi-bubble system \cite{24}. Nevertheless, although some of these phenomena have been reported, the underlying mechanisms of bubble oscillations in multi-bubble environment have not yet been clarified. In addition, it is almost impossible to directly investigate the bubble–bubble interaction...
experimentally in the presence of a large number of bubbles. For this reason, numerical analysis has been attempted to clarify the related phenomena.

Numerical studies related to the multi-bubble cavitation can be classified into three categories: (1) analysis of bubble dynamic behavior for few bubbles by simulating the fluid flow near the bubbles directly [27–33], (2) elucidation of single bubble oscillations by solving the Rayleigh-Plesset or Keller equation taking into account the interaction between the cavitation bubbles [34–41], and (3) prediction of bubble spatial distribution and ultrasound propagation by solving the sound propagation equation and Rayleigh-Plesset or Keller equation using an Euler-Lagrange framework [42–45]. In the first case, less approximations are necessary in the numerical model, and the hydrodynamic interaction between cavitation bubbles is directly solved. Although the numerical simulation predicts the related phenomena accurately, the calculation becomes rather costly. In the second case, although the computational cost is smaller, many approximations are to be used to build the numerical model. For this reason, the solutions obtained with each numerical model are different, and do not guarantee that the actual phenomena predict accurately. To improve the numerical predictions, attempts have been made to develop models using a Euler-Lagrange framework according to the third category of studies. This method using Euler-Lagrange framework can be applied to simulate not only widely spread sound fields but also the nonlinear bubble oscillations. However, the interaction between the bubbles cannot be represented accurately with this approach. Therefore, the models should be further improved.

Although the bubble oscillations in multi-bubble systems have been numerically modeled and investigated as reviewed above, the interaction between cavitation bubbles has not been predicted accurately in these cases. To shed light on the mechanism of bubble oscillations and interactions in cavitation cloud, we investigated the behavior of cavitation bubble surrounded by other bubbles during ultrasound irradiation through numerical simulation using the Volume of Fluid (VOF) method.

2. Numerical analysis

2.1. Calculation conditions

The goal of the present study is to numerically simulate dynamic behavior of a group of cavitation bubbles with one bubble surrounded by several others. The calculation conditions were similar to those for the collapse of cavitation bubble [46]. Fig. 1 shows is a schematic drawing of calculation domain, which was set to be a regular hexahedron with the side length of 400 μm. The cavitation bubbles and the liquid are set to be air and water, respectively. The physical properties used in the present simulation are shown in Table 1. One cavitation bubble was placed at the central area of calculation domain, while the other bubbles were located at a distance of 100 ~ 150 μm from the central bubble in such a way that lines connecting the bubbles form a regular tetrahedron (a), hexahedron (b) or octahedron (c), as shown in Fig. 2. The initial diameter of all bubbles was set to be 20 μm. The calculation conditions for the bubble arrangement are presented in Table 2. In the present study, the distance between the central and the surrounding bubbles, R, and the face number of regular polyhedrons was changed in order to evaluate their effect on the oscillations of the central bubble. A sound wave of sinusoidally changed pressure at a frequency of 20 kHz was imposed at the calculation domain boundary.

Table 1

| Variable                   | Value     | Unit     |
|----------------------------|-----------|----------|
| Liquid density, \( \rho \) | 1.00 \times 10^3 | kg/m^3   |
| Liquid viscosity, \( \mu_l \) | 1.00 \times 10^{-3} | Pas      |
| Gas viscosity, \( \mu_g \) | 1.84 \times 10^{-5} | Pas      |
| Liquid heat capacity, \( c_{v,l} \) | 4.20 \times 10^5 | J/(kg·K) |
| Gas heat capacity, \( c_{v,g} \) | 1.01 \times 10^5 | J/(kg·K) |
| Liquid thermal conductivity, \( k_l \) | 5.98 \times 10^{-1} | W/(m·K)  |
| Gas thermal conductivity, \( k_g \) | 2.64 \times 10^{-2} | W/(m·K)  |
| Gas constant, \( R_g \) | 8.31 \times 10^0 | J·K^−1·mol^−1 |
| Surface tension, \( \sigma \) | 7.20 \times 10^{-2} | N/m      |

To construct the numerical model, we introduced the following assumptions:

- The fluids are Newtonian and compressible.
- The evaporation and condensation are neglected.
- The density variation due to pressure oscillations follows the equations of state for ideal gas.
- The buoyancy force is neglected because the terminal velocity of 20-μm bubble is as small as 0.2 mm/s.

All the numerical models used in the present study were almost the same as those used in our previous studies [47,48]. Hence, the numerical models will be explained very briefly below, and the detailed

Fig. 1. Schematic drawing of calculation domain: A cavitation bubble is surrounded by other cavitation bubbles located at the vertices of (a) regular tetrahedron, (b) regular hexahedron, and (c) regular octahedron.
2.2. Governing equations

In the present study, the following conservation equations for the fluid momentum, mass, and energy were solved simultaneously.

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{u}) + \mathbf{F}_\text{ext},
\]

(1)

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0,
\]

(2)

\[
\frac{\partial (\rho T)}{\partial t} + \nabla \cdot (\rho \mathbf{u} T) = \nabla \left( \frac{\rho K}{c_v} \nabla T \right) - \frac{1}{c_v} \left( \nabla \cdot (\rho \mathbf{u}^2) + \nabla \cdot (\rho K \mathbf{u}) \right),
\]

(3)

where \(\rho\) is the fluid density, \(\mathbf{u}\) is the fluid velocity, \(t\) is time, \(p\) is the pressure, \(\mu\) is the viscosity, \(S_\text{ext}\) is the other external force terms, \(c_v\) is the heat capacity, \(T\) is the temperature, \(K\) is the thermal conductivity, and \(K\) is the specific kinetic energy. The gas density is calculated by the following equations of state for ideal gas as

\[
\rho_g = \frac{1}{R_g T}
\]

(4)

where the subscript \(g\) indicates liquid and gas, and \(R_g\) is the gas constant. The liquid density, \(\rho_l\), was set to be constant. These governing equations were solved by coupling with the multiphase flow model, which is explained in the subsequent section.

2.3. Multiphase flow model

The multiphase flow was modeled applying an algebraic Volume of Fluids (VOF) method [49,50]. In this model, the gas–liquid interface is captured using a function of liquid volume ratio, \(\alpha\), which is defined as

\[
\alpha = \begin{cases} 
0 & \text{Gas} \\
0 < \alpha < 1 & \text{Interface} \\
1 & \text{Liquid}
\end{cases}
\]

(5)

The volume fraction of liquid is advected through the following conservation equation of volume fraction.

\[
\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = -\alpha \frac{\partial \rho_g}{\partial t} / \rho_g
\]

(6)

where \(\mathbf{u}\) is the relative velocity between the liquid and gas phases, which is numerically modeled as a compressive term between the liquid and gas phases [50]. Details on modeling and discretization of compressive term have been given in earlier studies [48,49].

The surface tension was calculated by continuum surface force model [51], which is given as

\[
F_s = \sigma \nabla \alpha
\]

(7)

where \(\sigma\) is the surface tension, \(K\) is the surface curvature, and \(\alpha\) is the Laplace-filtered volume fraction of liquid. This surface tension term was added to the external source term of momentum balance equation (Eq. (1)). To reduce the calculation error, which is called spurious current, observed in a VOF method [52], the Laplace filter was imposed on the liquid volume fraction [53]. In the present study, the number of filtering operation was set to 1. When the surface curvature was calculated, we also imposed the Laplace filtering in the same way as that in our previous study [47].

In the VOF method, the physical properties are updated depending on the volume fraction as

\[
X = X_\rho \alpha + X_\mu (1 - \alpha), X = \mu, \rho, c_v, k
\]

(8)

where \(X\) is the physical properties, namely viscosity, density, heat capacity and thermal conductivity.

2.4. Boundary and initial conditions

At all the calculation boundaries, free-inlet and free-outlet conditions were adopted for fluid flow velocity, which was calculated based on the pressure gradient at the boundary. The pressure at all the boundaries was set to be the following sinusoidal function to compute the sound pressure.

\[
p = p' - P \sin(\omega t)
\]

(9)

where \(p'\) is the ambient pressure, \(P\) is the sound pressure amplitude, and
ω is the angular frequency. In the present study, the frequency of ultrasound was set to 20 kHz, and the following three amplitudes of sound pressure were used − 0.5, 0.7, 0.9 atm. The advective boundary condition was imposed on the temperature at the area where the flow is outward while at the other area temperature was set to equal to 300 K. The heat and fluid can flow in and out of computational domain freely according to these boundary condition, and the flow and temperature fields are not affected by the size of computational domain. On the other hand, although the pressure wave can be reflected at the boundary using Eq. (9), we confirmed that no pressure wave was reflected at all the boundaries at any time.

As for the initial conditions, the temperature was set to constant and equal to 300 K, and the fluid was set to be stationary. The pressure outside the bubble was set to 1 atm., and that in the bubble was set to $1.144 \times 10^5$ Pa taking into account the Laplace pressure.

2.5. Other calculation conditions

In the present study, the cavitation bubble decreases in size during its contraction. In such a case, the grid resolution must be high enough especially in the bubble area. To simulate the bubble oscillations accurately and reduce the calculation cost, adaptive mesh refinement (AMR) was adopted. The numerical grid near and in the bubble zone was split, and new octree grid cells were created. The based numerical grid was structured and uniformly distributed. The side length of each cubic grid cell was set to 2 μm. Therefore, the grid spacing was 1 μm in the refinement zone through the AMR. The time step was controlled by a local Courant number, which was set to be lower than 0.05. The total number of computational grid cells was approximately 8,200,000.

The spatial derivatives in the governing equations were discretized by a linear interpolation scheme except for the convective term in Eq. (6), which was discretized by van Leer model [54]. The time derivatives were discretized by a backward scheme except for that in Eq. (6), which was discretized by the multi-dimensional limiter for an explicit solution (MULES) scheme. The velocity and pressure fields were implicitly coupled through a pressure implicit with splitting operator (PISO) algorithm [55]. All the numerical models were incorporated into an open-source software, OpenFOAM of v1812 version. Since the validation of calculation code has been carried out in our previous study [47,48], description of validation procedure is omitted in the main body of present article. The validation for the solver is shortly explained in appendix B. The calculation time was strongly dependent on the calculation condition. We carried out these numerical simulations using the supercomputing system in Kyoto university (system B, Cray CS400 2820XT). This system used the Intel Xeon Broadwell processors, and their clock frequency was 2.1 GHz. It took approximately 150 h to compute one cycle of ultrasound oscillation using 72 cores at the maximum. The number of time step iterations was approximately 25,000.

3. Results and discussion

3.1. Bubble oscillations

The bubble oscillation amplitude was investigated first. Fig. 2 shows the time variation of pressure at the bubble central location and bubble shape in the single-bubble system. The central of bubble was always located in the central point of calculation domain. Fig. 3 shows the time variation of equivalent radius of spherical bubble in the case of single bubble. The equivalent radius, $R_e$ was calculated as

$$R_e = \left( \frac{3}{4\pi} \int (1 - \alpha) d\Omega \right)^{\frac{1}{3}}$$

where, $Q$ is the volume of calculation domain. At the large pressure amplitudes, the bubble shape becomes non-spherical. Therefore, we used the equivalent radius instead. In the present calculation, the initial conditions were found to slightly affect the calculation results. Therefore, two cycles of bubble oscillation were computed, and the oscillation amplitude was evaluated in the second cycle at two sound pressure amplitudes, 0.5 and 0.7 atm. It is to be noted that the bubble shape becomes non-spherical under the pressure amplitude of 0.9 atm., as shown in Fig. 2, and such a bubble continued its expansion in the second oscillation cycle. Also, non-spherical oscillation of bubble enlarges the bubble size at the initial stage of the second cycle as shown in Fig. 3. Therefore, at the amplitude of 0.9 atm., the bubble oscillations were evaluated in the first cycle.

As reported in a number of studies, the bubble oscillates linearly when the pressure amplitude is lower than approximately 0.5 atm. On the other hand, the bubble oscillates non-linearly at the pressure amplitude higher than 0.7 atm. This is caused by a phase shift between the sound pressure and bubble oscillations. The details of this phenomenon are not explained in the present study because it has been described in earlier studies [15,16].

Next, the effect of surrounding bubbles on the pressure oscillation was investigated. Fig. 4 shows the time variation of pressure in the central bubble under different arrangements of surrounding bubbles at the pressure amplitudes of 0.7 and 0.9 atm. In the latter case, the vertical axis is plotted in logarithmic scale. It is readily seen that the pressure oscillations in the central bubble are larger when surrounding bubbles exist compared with the single bubble oscillations in both cases. Also, the pressure oscillations increase in amplitude with number of surrounding bubbles regardless of pressure amplitude. Fig. 5 shows the time variation of equivalent bubble radius at 0.7 atm. in single- and multi-bubble systems. Similarly to the pressure variation, the bubble is expanded more, and its oscillation amplitude becomes larger in the case of multi-bubble system.

In order to clarify the mechanism of enhancement in the bubble expansion due to the surrounding bubbles, the dynamic behavior of bubbles was investigated. Fig. 6 shows the time variation of cavitation bubble shape during their contraction half-period. In this case, eight cavitation bubbles are located around the central bubble. The two bubbles appear to be overlapped in this figure. The size of central bubble is slightly decreased despite the surrounding bubbles are compressed rapidly. This reveals that the time delay due to the phase shift between sound pressure and bubble oscillations is larger for the central bubble when compared to that for the surrounding bubbles. This time delay is
considered to enhance the amplitude of bubble oscillations in the multi-bubble system.

To explain the reason why the time delay becomes larger in the multi-bubble system, the time variation in pressure distribution near the bubbles was calculated. Fig. 7 shows the pressure distribution in a cross-sectional plane for the case of pressure amplitude of 0.9 atm., and distance between the center and surrounding bubbles of 100 μm. The white lines indicate the gas–liquid interface. Under this condition, the bubbles start to compress at approximately 19 μs. The pictures shown correspond to the initial stage of contraction phase. At the moment when the bubble size reaches its maximum (19 μs), the ambient pressure has already begun to increase due to the above-mentioned phase shift between the sound pressure and bubble oscillations. Therefore, the central bubble is in the shade of surrounding bubble in terms of sound wave, and thus, the surrounding bubbles impede the propagation of sound pressure wave lowering the pressure near the central bubble. This phenomenon is called acoustic wake effect [56,57]. This acoustic wake results in increase of the phase shift between bubble and pressure oscillations, and only the central bubble remains expanded at 20 μs, as shown in Fig. 7 (b). The delay of central bubble compression causes the compressed velocity to increase. Also, the surrounding bubbles generate liquid jet toward the central bubble as shown in Fig. 7 (d). This phenomenon was also observed by Huang et al. (2018) [29].

In the other calculation conditions, the similar phenomena were observed. The acoustic wake results in enhancement of the bubble oscillation amplitude, and the liquid jet occurs at the high-pressure amplitude. In the next section, the influence of acoustic wake effect on the bubble dynamic behavior was investigated quantitatively.

3.2. Quantification of acoustic wake effect

As explained in the previous section, the acoustic wake affects the oscillation amplitude of bubble and the phase shift between the bubble...
and sound pressure oscillations. The acoustic wake occurs in the shade zone of sound wave. Therefore, the cross-sectional area of surrounding bubbles should be of significant importance. Therefore, to evaluate the influence of acoustic wake on the bubble oscillation, we introduce a new parameter which can be called "cover ratio", \( r_{\text{cov}} \), defined as
\[
 r_{\text{cov}} = \frac{\int S_{\text{cross}} \, dS}{S_s} = \frac{4 \pi R_{\text{bubble}}^2}{4 \pi R^2} = \frac{n R_{\text{bubble}}^2}{R^2} \tag{10}
\]
where \( S_{\text{cross}} \) is the cross-sectional surface area of surrounding bubbles, \( S_s \) is the surface area of a sphere on which the surrounding bubbles are located, \( R \) is the distance between the central and the surrounding bubbles, \( n \) is the number of surrounding bubbles, \( R_{\text{bubble}} \) is the radius of surrounding bubbles. Table 3 shows the cover ratio, \( r_{\text{cov}} \), obtained under the present calculation conditions.

To calculate the oscillation amplitude of bubble, the minimum pressure in the bubble, \( p_{b,\text{min}} \), was determined according to the cover ratio. The bubble oscillation amplitude can be indirectly calculated from the minimum pressure. Fig. 8 shows the relationship between the minimum pressure in the central bubble and cover ratio at different pressure amplitudes (a), and difference in minimum pressures between the multi-bubble and the single-bubble systems. It is seen that the minimum pressure is decreased with increase of cover ratio and pressure amplitude. The figure indicates that there is a linear relationship between the minimum pressure and cover ratio at all the pressure amplitudes, which can be described as:
\[
p_{b,\text{min}} \propto r_{\text{cov}} \tag{11}
\]
It means that the cover ratio can be an evaluation parameter for the bubble oscillation in the multi-bubble system.

The relationship between the maximum equivalent radius of bubble and cover ratio is shown in Fig. 9. The difference in the equivalent radius between the single bubble and multi bubble systems is shown in Fig. 9 (b). It is seen that the maximum bubble radius increases in proportion to the cover ratio.
\[
 R_{e,\text{max}} \propto r_{\text{cov}} \tag{12}
\]

According to the previous study [15], the pressure of the first collapse event during bubble oscillation is determined by the maximum radius before the collapse. Hence, the collapse pressure is increased with the covered ratio as seen in Fig. 4 because the maximum bubble radius is increased with the covered ratio. The increase in the maximum bubble radius can be explained by the acoustic wake effect and the time delay due to phase shift between the bubble and sound pressure oscillations, which is described below.

The time delay due to phase shift between the bubble and pressure oscillations was investigated. The time delay, \( \tau_d \), was defined as time lag

Table 3

| \( R/\mu m \) | Polyhedron | \( r_{\text{cov}} \)-/– |
|---|---|---|
| 100 | 4 | 0.04000 |
| 150 | 4 | 0.01778 |
| 100 | 6 | 0.08000 |
| 150 | 6 | 0.03556 |
| 100 | 8 | 0.06000 |
| 150 | 8 | 0.02667 |

Fig. 7. Time variation of pressure distribution near the central bubble in the sliced plane. The pressure amplitude is 0.9 atm., the distance between the center and surrounding bubbles is 100 \( \mu m \), and the bubble arrangement is regular octahedron. Time is counted from the beginning of simulation. The time is (a) 19, (b) 20, (c) 21, and (d) 22 \( \mu s \).
between the time when the pressure in a bubble reaches its minimum and the time when the sound pressure near the bubble becomes minimal. Fig. 10 (a) shows the relationship between the cover ratio and the time delay, \( t_d \). Fig. 10 (b) presents the difference in time delay between in the single-bubble and multi-bubble systems. It is readily seen that the time delay is increased with increase of cover ratio and pressure amplitude. Besides, the relationship between the time delay and cover ratio is linear. In the case of 0.5 atm. pressure amplitude, the values of time delay are slightly deviated from the linear relationship probably because the time delay is within the computational error. In the case of 0.5 atm., the averaged calculation time step was 0.02–0.03 \( \mu \text{s} \), although the time delay is less than 0.1 \( \mu \text{s} \) when the cover ratio is less than 0.04. In this range, the calculation time step is too large to evaluate the time delay accurately even when one uses the maximum Courant number of as small as 0.05. The results also indicate that the cover ratio can be an evaluation parameter for the bubble oscillation behavior in the multi-bubble system.

As a result, the maximum size of bubble becomes larger at this time...
delay causing a higher collapse pressure.

To understand the acoustic wake effect carefully, we focused on the time variation of pressure inside and outside of central bubble when its radius becomes close to the maximum because the bubble oscillation amplitude is determined by the maximum radius of oscillating bubble as explained above. Fig. 11 shows the time variation of pressure inside and outside the cavitation bubble when the bubble radius is the largest. To evaluate the effect of surrounding bubble on the pressure variation, the both results of single- and multi-bubble systems are shown in this figure. The location of surrounding bubbles is at the vertices of regular hexahedron with the distance of 100 μm. The monitoring location is the center point of spherical bubble in the inside case, and at a distance of 20 μm from the center point in the outside case. It is seen that, there is no difference in the pressure variation between in single- and multi-bubble systems within the first 10 μs. The result also indicates that there is no delay of ultrasound within this period. Within the first 10 μs, the surrounding bubbles are not expanded largely, resulting in small ultrasound disturbance. Meanwhile, the difference in the pressure between in the single-and multi-bubble systems becomes large in both inside and outside the bubble. The pressure outside the bubble starts to be increased approximately at 13–14 μs causing the corresponding increase in pressure inside the bubble in both the systems. Also, the time shift of pressure outside the bubble becomes larger in the multi-bubble system. This shift is an evidence of lowered ultrasound velocity due to the surrounding bubbles.

Fig. 12 shows the time variation of equivalent radius of cavitation bubble during its expansion. The surrounding bubbles are located at the vertices of regular hexahedron at a distance of 100 μm from the center bubble, and the pressure amplitude is 0.7 atm. Similarly to the time variation of pressure, the bubble radius remains almost the same within the first 10 μs. After that, the bubble is expanded and the expansion extent is larger in the multi-bubble system. The largely expanded bubble is compressed more violently, and its radius is largely fluctuated in the compression phase as seen in Fig. 12. By comparison of Figs. 11 and 12, we can conclude that the cavitation bubble in the multi-bubble system is more largely expanded due to the delay of ultrasound propagation especially during expansion of the surrounding bubbles. After that, the largely expanded bubble is compressed and collapsed more violently. In the case of other bubble arrangement in the multi-bubble system, the same phenomena were observed.

Finally, we also evaluated the difference in the oscillation behavior of central and surrounding bubbles. Figs. 13 and 14 show the time variations of pressure in the cavitation bubble and equivalent radius of cavitation bubble. The selected multi-bubble system is the regular hexahedron where the bubbles are located at a distance of 100 μm from the center bubble, and the ultrasound amplitude is 0.7 atm. The time variation of equivalent radius and pressure at the center of bubble in a single-bubble system are also shown for comparison. Before 10 μs, there is no difference in the pressure variation between the surrounding and center bubbles. Meanwhile, the oscillation amplitude of pressure variation becomes larger in the central bubble compared with the surrounding bubbles. As explained above, the acoustic wake effect enhances the oscillation amplitude of pressure and bubble radius. This acoustic wake effect is remarkable at the central bubble. It should be noted that the oscillation amplitude of pressure is larger even in the surrounding bubble than that in the single-bubble cavitation. As shown in Fig. 7, the pressure near the surrounding bubble is also reduced due to the acoustic wake effect, and consequently the expansion of surrounding bubbles becomes larger compared with that in the single-bubble system as shown in Fig. 14. This expansion in the radius of surrounding bubble increases the shade area in the central bubble, resulting in the enhancement of oscillation amplitude of central bubble. Also, this
phenomenon was observed in the other bubble arrangement in the multi-bubble system. We also investigated the effect of emitted pressure waves due to compression of surrounding bubbles on the oscillation of central bubble. Fig. 15 shows the time variation of cavitation bubbles and pressure distribution in the multi-bubble system when the cavitation bubbles are being compressed. The regular hexahedron was selected where the surrounding bubbles were located at a distance of 100 μm from the central one, and the pressure amplitude was 0.9 atm. The surrounding bubbles are first compressed non-spherically, and the pressure around the surrounding bubbles increases due to the bubble compression. Then, the center bubble is rapidly compressed before the pressure wave, which is emitted by the surrounding bubbles, reaches to the central bubble, and the high-pressure zone is created around the center bubble due to the compression of central bubble. Therefore, the pressure wave, emitted by the surrounding bubbles, affects the oscillation of center bubble in a lesser extent even though the acoustic wake effect is the largest for the regular hexahedron where the bubbles are located at a distance of 100 μm and the ultrasound amplitude is 0.9 atm in the present calculation condition. After completion of the first bubble compression, the surrounding bubbles move toward the central bubble due to the secondary Bjerknes force.

3.3. Interpretation of enhancement of bubble oscillation amplitude

From the above simulation data, the possible mechanism of enhancement in oscillation amplitude of cavitation bubble can be explained as follows:

- All bubbles perform oscillations in accordance with the ultrasound pressure variation.
- Due to ultrasonic wave disturbance at the surrounding bubbles, the ultrasound propagation is delayed when these bubbles become largely expanded.
- The compression of central cavitation bubble is delayed as shown in Fig. 6.
- Then, the central bubble is largely expanded due to the delay of ultrasound propagation, and more violent compression occurs.

To illustrate that the above hypothesis is correct, we also evaluated the sound speed change and sound attenuation due to the surrounding bubbles. The sound speed in a bubbly fluid can be described as [58–60]...
In the previous study carried out by Shi et al. (2020) [46], the collapse time of cavitation bubble was shown to be proportional to the number of surrounding bubbles. It is our opinion that the number alone is insufficient to describe the cavitation bubble behavior if for no other reason, because the influence of distance between the central and surrounding bubbles is ignored in this case. On the other hand, the cover ratio proposed in the present study is generally capable in evaluating the bubble dynamic behavior even when the distance between the central and surrounding bubbles is different.

In the previous studies, the Rayleigh-Plesset or Keller equation has introduced the effect of pressure wave generated by the surrounding bubbles, as reviewed in the introduction section. However, to the author’s best knowledge, the effect of acoustic wake on the bubble dynamic behavior has not been considered in these equations so far. Ida et al. (2009) found that the expansion rate of cavitation bubble is decreased in a multi-bubble system during cavitation inception [35], and it was confirmed in their experiments [62]. However, this model describes cavitation inception without sinusoidal ultrasonic waves. Yasui et al. (2008) [34] introduced the secondary Bjerknes force into Keller equation, and found that the bubble oscillation amplitude is decreased in the cavitation cloud. It is well known that the secondary Bjerknes force is similar to the acoustic wake effect. Although the above-
cited authors consider the secondary Bjerknes force and the sound pressure emitted by the surrounding bubbles, the models proposed do not take into account the phenomena of pressure wave shading.

The time delay of pressure wave should be introduced into the Rayleigh-Plesset equation to model the bubble behavior accurately in the multi-bubble system in order to reduce the simulation time. However, it is difficult because the velocity change and the attenuation of sound are determined by the nonlinear oscillation of cavitation bubble. To calculate the change in sound velocity and attenuation accurately and with reasonable computational cost, one has to solve the Rayleigh-Plesset or related equation for each bubble, and the attenuation rate and the velocity change must be calculated from the solution over the whole field of computational domain. In other words, any accurate prediction, requires at least the Rayleigh-Plesset coupling as explained in the introduction. Hence, in the future, the Euler-Lagrange coupling may be one of the solutions to simulate the sound field and dynamic behavior of cavitation bubble in a multi-bubble system and to incorporate the acoustic wake effect with a reasonable computational cost.

Finally, some remarks should be made on the difference between the actual applications and the present simulation. In the present study, the pressure amplitude was set below 0.9 atm. Meanwhile, in actual ultrasound applications, the pressure amplitude may be much higher especially in those applications which use sonotrodes. We expect that the same effect will be observed under higher-pressure amplitudes. The main reason why the acoustic wake effect enhances the oscillation amplitude of bubble is shading of ultrasound pressure wave. Under a higher-pressure amplitude, a more remarkable acoustic wake effect is expected because the larger the size of surrounding bubbles, the wider is the shadow area and the greater is the phase shift between the oscillations of surrounding bubbles and imposed ultrasonic pressure. Also, although the condensation and evaporation were neglected in the present simulation, these two phenomena may occur in actual applications. As reported by Yasui [63], number of H2O molecules in a bubble is increased during expansion period due to evaporation, and it is decreased in the compression period. In a cycle, the total number of molecules in a bubble remains constant. It means that the bubble expansion should become larger in the expansion period due to evaporation, and the bubble contraction should become larger in the compression period due to condensation. Even in this case, the same acoustic wake effect can be expected.

4. Conclusion

In the present study, we numerically investigated the dynamic behavior of cavitation bubbles during ultrasound propagation in a multi-bubble system. The multiphase flow containing acoustic cavitation bubbles has been numerically modeled using a compressible algebraic volume of fluid method. The main findings of the present study can be summarized as follows:

- The oscillation amplitude of cavitation bubble becomes larger in the multi-bubble system than in the single-bubble system due to the acoustic wake effect.
- The ultrasound propagation is delayed due to the presence of surrounding bubbles according to the acoustic wake effect, and this delay causes a greater expansion of cavitation bubble and its larger oscillation amplitude.
- The enhancement of bubble oscillation amplitude can be quantitatively evaluated by the cover ratio, which is defined as the ratio of projection area of surrounding bubbles to that of the central bubble.
- The minimum pressure in cavitation bubble is proportional to the cover ratio.
- The time delay due to phase shift between the sound pressure and bubble oscillations is also proportional to the cover ratio.
- The maximum radius of cavitation bubble is found to be proportional to the cover ratio.
• In the multi-bubble system, the oscillation amplitude of central cavitation bubble is larger than that of surrounding bubbles. Also, the oscillation amplitude of surrounding bubbles in the multi-bubble system is larger than the oscillation amplitude of center bubble in the single-bubble system.

• The pressure wave emitted by the surrounding bubble has no influence on the oscillation of center bubble in the present simulation condition.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Acknowledgements**

The present research is supported partly by JSPS KAKENHI, Grant-in-Aid for Young Scientists Grant Number JP20K15079 and Grant-in-Aid for Challenging Exploratory Research Grant Number JP17K18969, and partly by Collaborative Research Program for Young Scientists of ACCMS and IIMV, Kyoto University.

**Appendix A. Detailed descriptions about the numerical model**

Especially for the simulation of multiphase flow, an inevitable error flow is generated due to the calculation error of interfacial curvature at the gas-liquid interface. This flow is called spurious current. As argued by Popinet and Zaleski (1999), and Lafaurie et al. (1994) [64,65], the Reynolds (Re) number of spurious current is proportional to the Laplace (La) number, which is described as

\[ \text{La} = \frac{\sigma \rho D}{\mu} \]  

(A1)

where \( D \) is the diameter of bubble. From the relationship between the Re and La numbers, one can get the following relationship

\[ \frac{U \sigma}{\mu} \]  

(A2)

where \( U \) is the flow velocity of spurious current. From these equations, the flow velocity of spurious current is independent of bubble size. A smaller bubble is more easily moved due to the spurious current. In the case of acoustic cavitation, the bubble size is of micron order, and, therefore, the spurious current affects the calculation accuracy. To reduce the calculation error, many methods have been proposed such as CLSVOF method [66,67], S-CLSVOF method [68,69], and etc. In the present study, we used the Laplace filtering for the calculation of interfacial curvature. The detailed description about the discretization and modeling of multiphase flow is given below.

The advection equation of volume fraction was discretized by finite volume method in the framework of OpenFOAM. The advection and compressive terms are discretized as

\[ \int_{\Omega} [\nabla \cdot (\alpha \mathbf{u})] + \nabla \cdot (\mathbf{S}_{\alpha}^f) \, d\Omega = \sum_{j} \left[ (\alpha \mathbf{u})_{j} + \left( (1 - \alpha) a \mathbf{u} \right)_{j} \right] \cdot \mathbf{S}_{j} = \sum_{j} \left( \phi_{j} \right)_{j} \]  

(A3)

where \( \phi_{j} \) is the volumetric flux at the grid cell face and \( S_{j} \) is the surface area vector of grid face. The term in the right-hand side is modeled as a compressive term of gas-liquid interface zone. The relative volumetric flux at the grid cell face \( \left( \phi_{j} \right)_{j} \) is described as:

\[ \left( \phi_{j} \right)_{j} = \min \left( C_{a} \left[ \phi_{j} \right] \max \left( \left[ \phi_{j} \right] \right) \right) \]  

(A4)

where \( C_{a} \) is the compressive parameter and \( n_{j} \) is the surface normal flux to the free surface at the grid cell face. The surface normal flux compresses the gas-liquid interface zone algebraically. In the model of multiphase flow, solver implemented in OpenFOAM, the compression strength is controlled by \( C_{a} \). The selection of \( C_{a} \) value is of significantly importance. The effect of \( C_{a} \) on the calculation accuracy is evaluated in Appendix B. The compressive parameter in the present study was set to be 2.

The surface normal flux to the free surface at the grid cell face is calculated as

\[ n_{j} = n_{j} \cdot S_{j} \]  

(A5)

where \( n_{j} \) is the normal vector to the free surface at the grid cell face, which is calculated as

\[ n_{j} = \frac{\left( \nabla \alpha \right)_{j}}{|| \nabla \alpha_{j} || + \delta_{j}} \]  

(A8)

where \( \alpha \) is the Laplace-filtered liquid volume fraction and \( \delta_{j} \) is the numerical stabilizing parameter. The calculation of normal vector to the gas-liquid interface includes large numerical error when one uses VOF method. To reduce the calculation error, the gradient of Laplace filtered volume fraction was used. The Laplace-filtered volume fraction was calculated by

\[ \frac{\sum \alpha_{i} S_{i}}{\sum S_{i}} \]  

(A9)
where $S_i$ is the surface area at the grid cell face. By applying the Laplace filtering, the volume fraction becomes flatter and smoothed. In the present calculation, once filtering was applied. The stabilizing parameter $\delta_i$ is calculated as

$$\delta_i = \frac{1 \times 10^{-8}}{\sqrt{V/N}}$$

where $V$ is the volume of grid cell and $N$ is the total number of grid cell.

### Appendix B. Validation of simulation program

Influence of calculation model and calculation parameters on the spurious current and Laplace pressure was investigated for a 20-micron static bubble. The calculation domain, calculation condition and initial condition were the same as those used in the main part. The sinusoidal sound pressure was not applied at the boundary, and a single bubble was located in the center of calculation domain in this test case. Also, the AMR grid refinement was not adopted in this simulation. We evaluated the effect of maximum Courant number, $C_{\text{max}}$, and Laplace filter on the magnitude of spurious current, Laplace pressure, and bubble displacement. The calculation error of Laplace pressure is defined as

$$E_{\text{Laplace}} = \frac{|P_{\text{calc}} - P_{\text{theory}}|}{P_{\text{theory}}} \times 100$$

(B1)

The bubble displacement error due to spurious current is defined as

$$E_{\text{disp}} = \frac{\sum x(1-\alpha)d\Omega}{\int (1-\alpha)d\Omega}$$

(B2)

where $\alpha$ is the coordinate vector and $\Omega$ is the volume of calculation domain.

Table B1 shows the calculation results of evaluation parameters at 0.0001 s. The most important factor in this case is the bubble displacement error, $E_{\text{disp}}$, because this error changes the distance between bubbles and intensity of bubble interaction. By considering the results shown in Table B1, the optimum $C_{\text{max}}$ is 2 though the spurious current is fast. In the case of $C_{\text{max}} = 0.5$, the calculation errors seem to be small in the present static test case, although small $C_{\text{max}}$ value is not appropriate for the dynamic behavior of cavitation bubble because the gas-liquid interfacial zone is broadened, and the bubble is artificially fragmented easier. Therefore, we fixed the $C_{\text{max}}$ value to 2. The Laplace filtering can reduce the calculation error remarkably especially for the velocity of spurious current and bubble displacement error. Therefore, the Laplace filtering is necessary in the present simulation. The maximum Courant number does not influence the calculation accuracy. Meanwhile, the extremely high time resolution is required especially when the liquid jet and bubble collapse occur as seen in Fig. 10. Therefore, we set $C_{\text{max}}$ as small as 0.05 in the present simulation.

### Table B1

| $(C_{\text{max}}, C_{\text{Rlaplace}})$ | 0.05, 1, 0 | 0.05, 1, 1 | 0.05, 1, 2 | 0.1, 1, 1 | 0.05, 0.5, 1 | 0.05, 2, 1 |
|-------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $E_{\text{spurious, max}}$ (x10^5) | 4.911     | 2.388     | 1.712     | 2.029     | 1.402     | 2.641     |
| $E_{\text{Laplace}}$ (x10^5)      | 12.89     | 14.14     | 14.18     | 14.30     | 15.92     | 9.382     |
| $E_{\text{disp}}$ (x10^5)         | 4.757     | 2.504     | 1.156     | 2.481     | 7.969     | 7.888     |

References

[1] K.S. Suslick, S.B. Choe, A.A. Chichowlas, M.W. Grinstaff, Sonochemical synthesis of amorphous iron, Nature 353 (1991) 414-416, https://doi.org/10.1038/353414a0.
[2] K. Okitsu, M. Ashokkumar, F. Grieser, Sonochemical synthesis of gold nanoparticles: effects of ultrasound frequency, J. Phys. Chem. B 109 (2005) 20673-20675, https://doi.org/10.1021/jp0549374.
[3] Kumar, R., Kumar, V.B., Gedanken, A., 2020. Sonochemical synthesis of carbon dots, mechanism, effects of parameters, and catalytic, energy, biomedical and tissue engineering applications, Ultrason. Sonochem. 64, 105059. X10.1016/j.ultsonch.2020.105059.
[4] S. Zinatloo-Ajabshir, M. Baladi, O. Amir, M. Salavati-Niasari, Sonochemical synthesis and characterization of silver tungstate nanostructures as visible-light-driven photocatalyst for waste-water treatment, Sep. Purif. Technol. 248 (2020), 117062, https://doi.org/10.1016/j.seppur.2020.117062.
[5] T.J. Mason, Ultrasonic cleaning: An historical perspective, Ultrason. Sonochem. 29 (2016) 519–523, https://doi.org/10.1016/j.ultsonch.2015.05.004.
[6] W. Tangaopa, J. Thongori, Development of an industrial ultrasonic cleaning tank based on harmonic response analysis, Ultrasonics 91 (2019) 68–76, https://doi.org/10.1016/j.ultras.2018.07.013.
[7] Yamashita, T., Ando, K., Low-intensity ultrasound induced cavitation and streaming in oxygen-supersaturated water: Role of cavitation bubbles as physical cleaning agents, Ultrason. Sonochem. 52, 268-279. 10.1016/j.ultsonch.2018.11.025.
[8] Uchida, T., 2021. Quantitative evaluation of ultrasonic cleaning ability using acoustic cavitation signal, Jpn. J. Appl. Phys. 60, S20004. 10.3848/1347-4605/abec5d.
[9] R.J. Lang, Ultrasonic atomization of liquids, J. Acout. Soc. Am. 34 (1962) 6–8, https://doi.org/10.1121/1.1909020.
[10] M.K. Li, H.S. Fogler, Acoustic emulsification. Part 2. Breakup of the large primary oil droplets in a water medium, J. Fluid Mech. 13 (1978) 513–528, https://doi.org/10.1017/S0022112078000244.
[11] J.P. Canselier, H. Delmas, A.M. Wilhelm, B. Bismail, Ultrasound emulsification – an overview, J. Disper. Sci. Technol. 23 (2002) 333–349, https://doi.org/10.1080/0193269029842095.
[12] T.S. Perdik, M. Zapanc, M. Dular, Revision of the mechanisms behind oil-water (O/W) emulsion preparation by ultrasound and cavitation, Ultrason. Sonochem. 51 (2019) 298–304, https://doi.org/10.1016/j.ultsonch.2018.10.003.
[13] T. Yamamoto, S.V. Komarov, Liquid jet directionality and droplet behavior during emulsification of two liquids due to acoustic cavitation, Ultrason. Sonochem. 62 (2020), 104874, https://doi.org/10.1016/j.ultsonch.2019.104874.
[14] T. Yamamoto, R. Matsutaka, S.V. Komarov, High-speed imaging of ultrasonic emulsification using a water-gallium system, Ultrason. Sonochem. 71 (2021), 105387, https://doi.org/10.1016/j.ultsonch.2020.105387.
[15] W. Lauterborn, T. Kurz, Physics of bubble oscillations, Rep. Prog. Phys. 73 (2010), 106501, https://doi.org/10.1088/0034-4885/73/10/106501.
[16] K. Yasui, Acoustic Caviation And Bubble Dynamics, Springer briefs in Molecular Science, Ultrasound and Sonochemistry, Springer, 2017.
[17] K.S. Suslick, D.A. Hammernton, R.E. Cline, Sonochemical hot spot, J. Am. Chem. Soc. 108 (1986) 5641–5642, https://doi.org/10.1021/ja00278a005.
[18] C.D. Ohl, M. Arora, R. Dijkink, V. Janve, D. Lohse, Surface cleaning from laser-induced cavitation bubbles, Appl. Phys. Lett. 89 (2006), 074102, https://doi.org/10.1063/1.2337506.
[19] V. Minsier, J.D. Wilde, J. Proost, Simulation of the effect of viscosity on jet penetration into a single cavitation bubble, J. Appl. Phys. 106 (2009), 084906, https://doi.org/10.1063/1.3242288.
