COMPARATIVE ANALYSIS OF NONHYPERBOLIC MULTIPARAMETRIC TRAVEL-TIME APPROXIMATIONS OF MULTICOMPONENT SEISMIC DATA CONSIDERING DIFFERENCE OF DEPTH BETWEEN SOURCE AND RECEIVER USING OBN TECHNOLOGY

Nelson Ricardo Coelho Flores Zuniga, Eder Cassola Molina and Renato Luiz Prado

ABSTRACT. The processing of multicomponent seismic data is already a challenge concerning the velocity analysis. When it is performed for offshore survey, the difficulty increases a lot more with the use of OBN (Ocean Bottom Nodes) technology. The ray tracing asymmetry generated by the wave conversion and the difference of datum between source and receptor are not the only factors which contribute for a strongly nonhyperbolic travel-time event. The layered subsurface models and the large offsets employed in the offshore surveys make the nonhyperbolicity even stronger. Aiming to solve this problem, eight approximations to perform the velocity analysis were tested for two models. The complexity analysis of each nonhyperbolic multiparametric approximation was also studied to understand their behaviors during the optimization process. The relative error between the observed curve and the calculated curve with each approximation was computed for PP and PS reflection events of two models. With these information, it was possible to determine which approximation is the most reliable one for this kind of models.

Keywords: multicomponent, OBN, nonhyperbolic, multiparametric.

RESUMO. O processamento de dados sísmicos multicomponentes já é um desafio com relação à análise de velocidades. Quando realizado para levantamentos marítimos, a dificuldade aumenta muito mais com o uso da tecnologia OBN (Ocean Bottom Nodes). A assimetria no traçado de raios gerada pela conversão de onda e pela diferença de profundidade entre fonte e receptor não são os únicos fatores que contribuem para um evento de tempos de trânsito fortemente não-hiperbólico. Os modelos estratificados de subsuperfície e os grandes afastamentos aplicados nos levantamentos marítimos tornam a não-hiperbolicidade ainda mais forte. Visando resolver este problema, oito aproximações para realizar a análise de velocidades foram testadas para dois modelos. A análise de complexidade de cada aproximação não-hiperbólica multiparamétrica também foi estudada para entender seus comportamentos durante o processo de otimização. Os erros relativos entre as curvas observadas e calculadas com cada aproximação foram calculados para os eventos de reflexão PP e PS dos dois modelos. Com estas informações, foi possível determinar qual aproximação é a mais confiável para estes tipos de modelos.

Palavras-chave: multicomponente, OBN, não-hiperbólico, multiparamétrico.

Corresponding author: Nelson Ricardo Coelho Flores Zuniga
Universidade de São Paulo, Instituto de Astronomia, Geofísica e Ciências Atmosféricas (IAG-USP), Departamento de Geofísica. Rua do Matão 1226 - Cidade Universitária 05508-090 São Paulo, SP, Brazil – Phone: +55 11 3091-4755 - Fax: 3091-5034 – E-mails: nelson.zuniga@iag.usp.br, eder.molina@iag.usp.br, renato.prado@iag.usp.br
INTRODUCTION
The travel-time approximations are significantly important in the reflection seismic processing, more specifically when they are applied to moveout correction and velocity analysis (Yilmaz, 2000).

For homogeneous isotropic media with short offsets and no difference of datum between source and receptor, and PP-wave reflection events, it is used the hyperbolic approximation proposed by Dix (1955). However, this approximation is not valid for PS-wave reflection events, for data with difference of datum between the source and the receptor, large offsets and for layered media. The nonhyperbolicity generated by these factors can be overcome using approximations that consider this effect to perform a reliable determination of the parameters.

In the last decades, several characteristics of the nonhyperbolic behavior were studied and some approximations have been shown in different works aiming to control some nonhyperbolic effect (Malovichko, 1978; Blias, 1983 and 2009; Muir & Dellinger, 1985; Castle, 1988 and 1994; Slotboom, 1990; Tsvankin & Thomsen, 1994; Alkhalifah & Tsvankin, 1995; Li & Yuan, 1999 and 2001; Cheret et al., 2000; Causse et al., 2000; Tsvankin & Grechka, 2000a,b; Fomel & Grechka, 1994 and 2001; Leiderman et al., 2003; Li, 2003; Silva et al., 2003; Ursin & Stovas, 2006; Aleixo & Schleicher, 2010; Golikov & Stovas, 2012). However, most of them were not proposed to be used for multicomponent seismic data, and none of them for a reflection seismic data that considers difference of datum between the source and the receptor.

In this paper, it is proposed to perform the complexity analysis of the objective function and the velocity analysis of several nonhyperbolic multiparametric approximations for two stratified offshore models. A multicomponent seismic data is used, with focus on PP and converted PS events, with the use of the OBN (Ocean Bottom Nodes) technology considering long offsets between source and receptors.

METHODOLOGY
Travel-Time Approximations
Eight approximations were chosen for this study. The first one is the hyperbola equation (Eq. 1), proposed by Dix (1955).

\[ t = \sqrt{t_0^2 + \frac{x^2}{v^2}} \]  

(1)

Where \( x \) is the vector of offsets, \( t_0 \) is the zero-offset travel-time and \( v \) is the RMS (Root Mean Square) velocity.

However, this approximation is treated here only as a reference, due to the fact that it is insufficient for the strong nonhyperbolic conditions studied here.

Malovichko (1978) proposed an approximation known as shifted hyperbola (Eq. 2) which was also studied and derived by Castle (1988 and 1994) and de Bazelaire (1988). This approximation was proposed to control the effect of large offsets in inhomogeneous media using the \( S \) parameter.

\[ t = t_0 \left(1 - \frac{1}{S}\right) + \frac{1}{S} \sqrt{t_0^2 + \frac{Sx^2}{v^2}} \]  

(2)

The \( S \) parameter depends on the \( \mu_1 \) and \( \mu_2 \) by the relation \( S = \mu_1/\mu_2^2 \), where \( \mu_j (j = 2, 4) \) is the \( j \)-th velocity momentum (Eq. 3).

\[ \mu_j = \sum_{k=1}^{n} \frac{v_{k}}{t_{k}} / \sum_{k=1}^{n} t_{k} \]  

(3)

and \( v_k \) is the interval velocity of the \( k \)-th layer and \( t_k \) is the travel-time of the \( k \)-th layer.

An approximation aiming to analyze the nonhyperbolic behavior concerning the wave conversion was proposed by Slotboom (1990). However, this approximation (Eq. 4) has only the \( t_0 \) and \( v \) as the unknown parameters, which make it simpler than most of the nonhyperbolic approximations.

\[ t = \frac{t_0}{2} + \sqrt{\frac{t_0^2}{2} + \frac{x^2}{2v^2}} \]  

(4)

Alkhalifah & Tsvankin (1995) proposed an approximation (Eq. 5) that uses the \( \eta \) parameter. This parameter (Eq. 6) is a function of the anisotropic parameters of Thomsen (1986).

\[ t = \sqrt{\frac{t_0^2}{v^2} + \frac{2\eta x^2}{v^2 [t_0/v^2 + (1 + 2\eta)x^2]}} \]  

(5)

\[ \eta = \frac{e - \delta}{1 + 2\delta} \]  

(6)

where \( e \) quantifies the difference between the wave velocities along the symmetry axis and perpendicular to the symmetry axis, and \( \delta \) represents the propagation of P-wave for angles close to the symmetry axis.

An approximation that also uses the \( S \) parameter was proposed by Ursin & Stovas (2006). However, for this approximation (Eq. 7), the \( S \) parameter is expressed in a quasi-acoustic case as a function of the anisotropic parameters of Thomsen (1986).
Another approximation that uses the $S$ parameter was proposed by Blias (2009). To develop this approximation (Eq. 8), the author performed several numerical tests related to the walkway vertical seismic profile (VSP).

\[
t = \frac{1}{2} \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{(S - 1)x^4}{4v^4 \left( t_0^2 + \frac{(S - 1)x^2}{2} \right)}} + \frac{1}{2} \sqrt{t_0^2 + \frac{1 + \sqrt{S - 1}}{v^2} x^2}
\]  

(7)

Muir & Dellinger (1985) proposed an approximation (Eq. 9) which uses the anellipticity parameter $f$, which describes how much the wavefront differs from the spherical shape and tends to the elliptical shape.

\[
t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{f(1-f)x^4}{v^2(v^2t_0^2 + fx^2)}}
\]  

(9)

In the Li & Yuan (2001) approximation (Eq. 10), $\gamma$ is used as the nonhyperbolic parameter. This approximation aims to consider the CP (Conversion Point), which helps to control the effects of a nonhyperbolicity associated to the wave conversion.

\[
t = \sqrt{t_0^2 + \frac{x^2}{v^2} - \frac{(\gamma - 1)x^4}{\gamma v^2} - \frac{(\gamma - 1)x^4}{4t_0^2 v^2 + (\gamma - 1)x^2}}
\]  

(8)

Where $\gamma$ is the ratio between the squared P-wave stacking velocity $V_p$ and the squared converted wave stacking velocity $V_{c2}$ (Eq. 11).

\[
\gamma = \frac{V_{c2}^2}{V_p^2} = \frac{\gamma_{eff}(1 + \gamma_s)}{(1 + \gamma_{eff})}
\]  

(11)

The relation $\gamma_{eff}$ is expressed by $\gamma_{eff} = \gamma_s^2 / \gamma_s$, where $\gamma_s$ is the ratio between the stacking P-wave and stacking S-wave, and $\gamma_s$ is the ratio between P-wave velocity and S-wave velocity which travel along the normal component.

### Offshore Models Studied

The models studied in this paper are offshore and presented a layered structure (Fig. 1). They were based on stratigraphic informations obtained from well logs from the Santos Basin, Brazil. The models present a large salt structure sealing the carbonate reservoir.

The use of multicomponent seismic data is fundamental to obtain more accurate results. And considering an offshore investigation, there is a necessity of using the OBN technology to reach the S-wave information. The maximum offset used for these models was 15000 meters. The reflection events analyzed are concerning the base of the salt structure and, therefore, the top of the reservoir.

In Model 1 (Table 1), the 3rd, 4th and 5th layers are part of the salt structure and the carbonate reservoir ($V_p = 4010$ m/s and $V_S = 2012$ m/s) is beneath this structure.

### Table 1 – The parameters of the Model 1: Layer thickness ($\Delta z$), P-wave velocity ($V_p$), S-wave velocity ($V_S$) and $V_p / V_S$ ratio.

| Layer | $\Delta z$ (m) | $V_p$ (m/s) | $V_S$ (m/s) | $V_p / V_S$ |
|-------|---------------|------------|------------|-------------|
| Water | 2157          | 1500       | 0          | -           |
| 1     | 496           | 2875       | 1200       | 2.40        |
| 2     | 108           | 3505       | 1628       | 2.15        |
| 3     | 664           | 4030       | 2190       | 1.84        |
| 4     | 262           | 5005       | 2662       | 1.88        |
| 5     | 1485          | 4220       | 2210       | 1.91        |

In the Table 2, the carbonate reservoir ($V_p = 3599$ m/s and $V_S = 1800$ m/s) is present under the 6th layer. The 4th, 5th and 6th layers are the salt structure.

### Table 2 – The parameters of the Model 2: Layer thickness ($\Delta z$), P-wave velocity ($V_p$), S-wave velocity ($V_S$) and $V_p / V_S$ ratio.

| Layer | $\Delta z$ (m) | $V_p$ (m/s) | $V_S$ (m/s) | $V_p / V_S$ |
|-------|---------------|------------|------------|-------------|
| Water | 2101          | 1500       | 0          | -           |
| 1     | 431           | 2852       | 1190       | 2.40        |
| 2     | 82            | 3390       | 1512       | 2.24        |
| 3     | 525           | 3461       | 1590       | 2.18        |
| 4     | 212           | 3801       | 1885       | 2.02        |
| 5     | 1151          | 4321       | 2219       | 1.95        |
| 6     | 503           | 3820       | 1899       | 2.01        |

### Complexity Analysis

As the numerical analysis was treated as an inverse problem by an optimization criterion, the complexity analysis was important to determine the behavior of each approximation, which one
is unimodal or multimodal, to figure out the complexity of the optimization for each approximation (Kurt, 2007).

It can be observed that the topographic structure varies drastically with the used approximation. However, the variation with the kind of reflection event is merely concerning the displacement of the structure, which is associated to the difference of values of each parameter. Despite of this, the model influences slightly the topography of the function.

In Figures 2A and 2B, the approximation proposed by Malovichko (1978) showed a homogeneous topological structure concerning the objective function and also presented to be always unimodal (only the global minimum region) without dependence on the model (Figs. 2C and 2D).

Figures 3A and 3B showed that the Alkhalifah & Tsvankin (1995) approximation presents the same kind of variation between the PP and PS events. It also can be observed that between the Model 1 and Model 2 (Figs. 3C and 3D), this approximation presented only soft variations concerning the topological structure related to the model variation.

It can be observed, in Figures 4A to 4D and in Figure 5A to Figure 5D, that the approximation proposed, respectively, by Ursin & Stovas (2006) and the one proposed by Blias (2009) present the same kind of variations between PP and PS events and between the two models. Even presented to be unimodal for the models tested here, they presented multimodal (presents the global and one or more local minimum regions) characteristics in
Figure 2 – Residual function maps to demonstrate the complexity of the Malovichko (1978) approximation for the (A) PP and (B) PS event of the (C) Model 1 and (D) Model 2. Red dispersions represent the global minimum regions.

Figure 3 – Residual function maps to demonstrate the complexity of the Alkhalifah & Tsvankin (1995) approximation for the (A) PP and (B) PS event of the (C) Model 1 and (D) Model 2. Red dispersions represent the global minimum regions.
Figure 4 – Residual function maps to demonstrate the complexity of the Ursin & Stovas (2006) approximation for the (A) PP and (B) PS event of the (C) Model 1 and (D) Model 2. Red dispersions represent the global minimum regions.

Figure 5 – Residual function maps to demonstrate the complexity of the Blias (2009) approximation for the (A) PP and (B) PS event of the (C) Model 1 and (D) Model 2. Red dispersions represent the global minimum regions.
previous works (Zuniga et al., 2015 and 2018), what can increase the difficulty of the analysis for the kinds of models used here for these two approximations.

The approximation developed by Muir & Dellinger (1985) presented a multimodal behavior. However, the variation between the PP and the PS reflection events is different than the others (Figs. 6A and 6B). Its variation is not only concerning the displacement if the objective function structure, but there is also a variation of the position between the global and the local minimum region. This happens due to the local and global minimum region values being too close to each other. The variation between Model 1 and Model 2 (Figs. 6C and 6D) is subtle like the previous approximations.

The approximation proposed by Li & Yuan (2001) showed a multimodal behavior and only a structural variation between PP and PS reflection events (Figs. 7A and 7B). In Figures 7C and 7D, it can be observed the same kind of variations between the Model 1 and Model 2.

**Comparison of Travel-Time Approximations**

To compare the performance for different approximations, it was analyzed the difference between the observed curve and the calculated curve with each one of the eight approximations for the PP and PS reflection events of the two models. The relative errors in travel-times were computed to perform the accuracy analysis of the approximations.

The approximation proposed by Li & Yuan (2001) showed the best result for the PP reflection event of the Model 1 (Fig. 8). The second best result for this event was shown by Ursin & Stovas (2006) approximation. The third and the fourth best results for this event were respectively shown by Blias (2009) and Malovichko (1978).

The approximation proposed by Li & Yuan (2001) presented the best result for the PS reflection event of the Model 1 (Fig. 9). The approximations proposed by Blias (2009) and Malovichko (1978) presented the second and the third best results, respectively, with the fourth more accurate result being shown by the Ursin & Stovas (2006) approximation.

For the PP event of the Model 2 (Fig. 10), it is observed again the approximations proposed by Li & Yuan (2001) and by Ursin & Stovas (2006) presenting, respectively, the most accurate and the second most accurate results. For this event, Malovichko (1978) approximation presented the third best result, discreetly better than the Blias (2009) approximation.
Figure 7 – Residual function maps to demonstrate the complexity of the Li & Yuan (2001) approximation for the (A) PP and (B) PS event of the (C) Model 1 and (D) Model 2. Red dispersions represent the global minimum regions and the blue dispersions represent the local minimum regions.

Figure 8 – Relative errors in travel-time between the observed curve and the calculated curve with each approximation, for the PP reflection event of the Model 1.

For the converted wave event of the Model 2, the approximation proposed by Li & Yuan (2001) showed the best result (Fig. 11). The approximation proposed by Blias (2009) presented the second most accurate result. Ursin & Stovas (2006) and Malovichko (1978) approximations presented respectively the third and the fourth best results.

As expected due to the fact of being the simplest approximations tested here, the approximations proposed by Dix
(1955) and by Slotboom (1990) showed respectively the worst and the second worst sets of results for PP and PS reflection events for both models.

For the nonhyperbolic multiparametric travel-time approximations, the worst set of results was presented by the Alkhalifah & Tsvankin (1995) approximation. Muir & Dellinger (1985) approximation appeared as the second less accurate approximation with three parameters for all the events tested here.

In a general form, Li & Yuan (2001) approximation showed the best results for PP and converted events for both models. Furthermore, the approximation proposed by Ursin & Stovas (2006) presented the second best results for PP events of both models, while the approximation proposed by Blas (2009) reached the second best set of results for the converted wave events. The approximation proposed by Malovichko (1978) showed the fourth best set of results with the maximum error not exceeding 0.5%.
CONCLUSIONS

The approximation proposed by Li & Yuan (2001) showed, in a general form, the best set of results for each event observed. However, this approximation proved to be multimodal, which always demands the use of a global search algorithm or a multi-start procedure using a local search algorithm.

The approximations proposed by Ursin & Stovas (2006) and Blias (2009) showed the second best results respectively for PP events and converted PS events. The behavior of both approximations was unimodal for all events tested. However, both approximations have already shown a multimodal behavior (Zuniga et al., 2017). Thus, it is challenging to predict their behavior once it appears to depend on the characteristics of the model.

Even with the accuracy shown by the Malovichko (1978) approximation (error not higher than 0.5%), this equation presented the fourth best set of results. However, it proved to be unimodal, which does not require the use of a global search algorithm.

ACKNOWLEDGEMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brazil (CAPES) - Finance Code 001. This study was financed in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico – Brazil (CNPq).

REFERENCES

ALEIXO R & SCHLEICHER J. 2010. Traveltime approximations for q-P waves in vertical transversely isotropic media. Geophysical Prospecting, 58: 191-201.

ALKHALIFAH T & TSVANKIN I. 1995. Velocity analysis for transversely isotropic Media. Geophysics, 60: 1550-1566.

BLIAS E. 1983. Reflected waves travel-time curve in flat-bedded medium with transverse layers and their interpretation. Soviet Geology and Geophysics, N2: 91-95.

BLIAS E. 2009. Long-offset NMO approximations for a layered VTI model: Model study. In: 79th Annual International Meeting. Expanded Abstract, Houston, Texas, USA: Society of Exploration Geophysics, p. 3745-3749.

CASTLE RJ. 1988. Shifted hyperbolas and normal moveout. In: 58th Annual International Meeting. Expanded Abstract, California, USA: Society of Exploration Geophysics, pp. 894-896.

CASTLE RJ. 1994. A theory of normal moveout. Geophysics, 59: 983-999.

CAUSSE E, HAUGEN GU & ROMMEL BE. 2000. Large-offset approximation to seismic reflection traveltimes. Geophysical Prospecting, 48: 763-778.

CHERET T, BALE R & LEANEY S. 2000. Parameterization of polar anisotropic moveout for converted wave. In: 70th Annual International Meeting. Expanded Abstract, Calgary, Alberta, Canada: Society of Exploration Geophysics, p. 1181-1184.
DE BAZELAIRE E. 1988. Normal moveout revisited: Inhomogeneous media and curved interfaces. Geophysics, 53: 143-157.

DIX Ch. 1955. Seismic velocities from surface measurements. Geophysics, 20: 68-86.

FOMEL S & GRECHKA V. 1994. On nonhyperbolic reflection moveout in anisotropic media. Geophysics, 59: 617-640.

FOMEL S & GRECHKA V. 2001. Nonhyperbolic reflection moveout of P-waves: An overview and comparison of reasons. Colorado School of Report CWP-372. 23 pp.

GOLIKOV P & STOVAS A. 2012. Accuracy comparison of nonhyperbolic moveout approximations for qP-waves in VTI media. Journal of Geophysics and Engineering, 9: 428–432.

KURT H. 2007. Joint inversion of AVA data for elastic parameters by bootstrapping. Computers & Geosciences, 33(3): 367-382.

LEIDERMAN R, ARTOLA FAV, SILVA MBC & FONTOURA SAB. 2003. Exemplo de modelagem sísmica dos modos PP e PS a partir de dados de poço e perfil litológico. In: 2º Congresso Brasileiro de P&D em Petróleo & Gás. Expanded Abstract. Brazil. Rio de Janeiro, RJ, Brazil.

LI XY. 2003. Converted-wave moveout analysis revisited: The search for a standard approach. In: 73rd Annual International Meeting. Expanded Abstract, Dallas, Texas, USA: Society of Exploration Geophysics, p. 805-808.

LI XY & YUAN J. 1999. Converted-wave moveout and parameter estimation for transverse isotropy. In: 61st EAGE Conference. Expanded Abstract. Helsinki, Finland: EAGE. v. 1, p. 4-35.

LI XY & YUAN J. 2001. Converted-wave imaging in inhomogeneous, anisotropic media: Part I. Parameter estimation. In: 63rd EAGE Conference. Expanded Abstract, Amsterdam, The Netherlands: EAGE. v. 1, p. 109.

MALOVICHKO AA. 1978. A new representation of the travelt ime curves of reflected waves in horizontally layered media. Applied Geophysics (in Russian), 91(1): 47-53.

MUIR F & DELLINGER J. 1985. A practical anisotropic system. In: SEP-44. Stanford Exploration Project, p. 55-58.

SILVA MBC, KELLER FILHO O, GUIMARÃES MAG, WINDELS R, LEIDERMAN R, ARTOLA FAV, FONTOURA SAB & SOBREIRA JFF. 2003. 4-C seismic data processing experience in Brazil. In: 8th International Congress of the Brazilian Geophysical Society. Expanded Abstract. Rio de Janeiro, RJ, Brazil.

SLOTBOOM RT. 1990. Converted wave (P-SV) moveout estimation. In: 60th Annual International Meeting. Expanded Abstract, San Francisco, California, USA: Society of Exploration Geophysics, p. 1104-1106.

THOMSEN L. 1986. Weak elastic anisotropy. Geophysics, 51: 1954–1966.

TSVANKIN I & GRECHKA V. 2000a. Dip moveout of converted waves and parameter estimation in transversely isotropic media. Geophysical Prospecting, 48: 257-292.

TSVANKIN I & GRECHKA V. 2000b. Two approaches to anisotropic velocity analysis of converted waves. In: 70th Annual International Meeting. Expanded Abstract, Calgary, Alberta, Canada: Society of Exploration Geophysics, p. 1193-1196.

TSVANKIN I & THOMSEN L. 1994. Nonhyperbolic reflection moveout in anisotropic media. Geophysics, 59: 1290-1304.

URSIN B & STOVAS A. 2006. Traveltime approximations for a layered transversely isotropic medium. Geophysics, 71: D23-D33.

YILMAZ O. 2000. Seismic Data Analysis. Society of Exploration Geophysics, Tulsa. 2065 pp.

ZUNIGA NRCF, BOKHONOK O & DIOGO LA. 2015. Comparison of nonhyperbolic travel-time approximations for multicomponent seismic data. In: 14th SBGF Congress. Expanded Abstract, Rio de Janeiro, RJ, Brazil. p. 1176-1181.

ZUNIGA NRCF, MOLINA EC & PRADO RL. 2017. Comparison of travel-time approximations for unconventional reservoirs from Santos Basin, Brazil. Revista Brasileira de Geofísica, 35(4): 273-286.

ZUNIGA NRCF, RIBEIRO FB & PRIIMENKO VI. 2018. Relation between the model and the topography of the objective function in a velocity analysis using a nonhyperbolic multicomponent travel-time approximation. Brazilian Journal of Geophysics, 36(4): 375-384.