Quantum noise in phase-sensitive heterodyne detection with a bichromatic local oscillator

Heng Fan, Dechao He, and Sheng Feng

MOE Key Laboratory of Fundamental Quantities Measurement, School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China
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The development of the quantum theory of optical heterodyne detection depended essentially on the concept of image sideband vacuum mode. However, with the usual understanding of the quantum noise produced by image sideband vacuum modes, we find a theoretical dilemma in the case of phase-sensitive heterodyne detection with a bichromatic local oscillator. We report the results of an experiment designed towards a solution to the problem. If assumed is that the image sideband vacuum modes contribute no extra quantum noise in the detection, the dilemma can be easily circumvented and agreement between theory and experiment will be reached. Furthermore, under the same assumption, we discover a paradox in the studied case of optical detection, which is verified by experimental data as well. Further investigation is encouraged to understand the mechanism through which the expected extra quantum noise of the image sideband vacuum modes is absent in the phase-sensitive heterodyne detection.

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I. INTRODUCTION

The quantum theory of optical detection [1], developed in consistency with the uncertainty principle, is commonly considered as a correct and complete description of the detection characteristics of non-classical light. However, our confidence about the perfection of the detection theory may be shaken by the new achievements in the study of the uncertainty principle: Heisenberg’s measurement uncertainty relation may not hold in full generality [14–22]. If this turns out to be true, doubt may be cast onto a famous prediction, made by the detection theory in accordance with the measurement uncertainty relation, that a 3 dB extra quantum noise takes place in joint measurement of conjugate quadratures of light field with heterodyne detectors [1 5 7 11 13].

Bearing the above understanding in mind, we focus, in this work, on another potential problem about the detection theory when it is applied to the special case of a heterodyne detector with a bichromatic local oscillator. As is well known, the quantum theory of detection deals with optical heterodyning essentially relying on the concept of image sideband vacuum mode [1 5 7 8 11 12]. A traditional heterodyne detector is a phase-insensitive device and suffers a 3 dB noise penalty due to the presence of the image sideband vacuum modes [1 4 5 7 11]. If the image sideband vacuum mode is excited into a coherent state to the vacuum mode [1, 4, 5, 7, 11], then the heterodyne detector becomes phase sensitive and is free of the noise penalty [8 25].

Although little attention has been given to the potential problem with the detection theory, it naturally appears when one considers a phase-sensitive heterodyne detector with a bichromatic local oscillator: On one hand, a 3 dB noise penalty is expected for the detector due to the presence of the image sideband vacuum modes. On the other hand, however, the detector should be noise free at the quantum level because of its phase-sensitive nature [25]. The dilemma reflects our lack of full understanding of the origin of the quantum noise in optical detection, especially the physics relevant to image sideband vacuum mode.

In what follows, we summarize in Sec. II the past works focusing on how the concept of image sideband vacuum mode was introduced in the detection theory to explain the quantum noise in optical heterodyning. In Sec. III, we turn to the special case of heterodyne detection with a bichromatic local oscillator. With the orthodox understanding about image sideband vacuum mode, we show that a 3 dB noise penalty should occur in the detection, in agreement with a previous work [26] studying heterodyne detection in a similar configuration. Then we show that the heterodyne detector in the studied configuration is indeed phase sensitive, and, hence, should be free of noise penalty on the contrary, according to the quantum theory of linear amplifier [23]. We report in Sec. IV an experiment designed towards a solution for the theoretical dilemma. The experimental results show that the 3 dB noise penalty was absent in the heterodyne detector. In Sec. V, with some theoretical analysis, we show that the theoretical dilemma can be circumvented and the experiment will agree with theory, if one assumes that the image sideband vacuum modes contributed no extra quantum noise in the heterodyne detection. Under the same assumption, we further discover a paradox about the phase-sensitive heterodyne detector. That the paradox is verified by the experimental data further justifies the above assumption. Finally, we conclude in Sec. VI that, to correctly describe in theory the quantum behavior of the phase-sensitive heterodyne detector.
with a bichromatic local oscillator, we must know the mechanism that prevented the extra quantum noise of the image sideband vacuum mode from being observed in the experiment. Further investigation should be able to deepen our current understanding of the origin of the quantum noise in optical detection.

II. CONVENTIONAL HETERODYNE DETECTION

According to the quantum theory of optical detection, a conventional heterodyne detector, classified as a phase-insensitive device in the theory of linear amplifier [23], possesses a 3 dB extra quantum noise in comparison with a homodyne detector, caused by the image sideband vacuum mode involved in the detection [11, 12]. Although there are different models for theoretically treating the problem of heterodyne detection, most models adopt the imageband-mode concept [1, 2, 4, 5, 7, 8, 11, 13] and agree with one another on the 3 dB heterodyne noise penalty.

Let consider balanced heterodyne detection, in a traditional configuration (Fig. 1.), of an optical signal at frequency $\omega_s$, $\hat{E}_s^{(+)}(t) = a_s e^{-i\omega_0 t + i\phi}$, where $\hat{a}_s$ is the photon annihilation operator with $\hat{a}_s\dagger$ $a_s$ in units of photons per second. The signal enters the input port of a 50-50 beamsplitter together with a vacuum mode $\hat{E}_l^{(+)}(t) = \hat{a}_l e^{-i\omega_0 t + i\phi}$. The light entering the signal port of the beamsplitter is combined with a much more powerful coherent light at frequency $\omega_0 = \frac{1}{2}(\omega_s + \omega_l)$, $\hat{a}_l^{(+)}(t) = \hat{a}_l e^{-i\omega_0 t + i\phi}$. Here the amplitude $\hat{a}_l$ is a real number. Both outputs of the beamsplitter are directed onto photodiodes, whose output photocurrent is differenced and then filtered to pick out the beatnote signal at frequency $\Omega = \omega_s - \omega_l$ ($\omega_s > \omega_l$ is assumed for simplicity). Then the result is detection of the quantity $\hat{X} = \hat{a}_s e^{-i\phi} + \hat{a}_l e^{i\phi}$ and $\hat{J}_- = e^{i\phi} e^{-i\Omega t + i\phi} \hat{X} + h.c.$.

$$\hat{J}_- = \hat{J}_1(t) - \hat{J}_2(t)$$

Here $\phi = \phi_0 - (\phi_s + \phi_l)/2$, $\Delta\phi = (\phi_s - \phi_l)/2$, and $e$ is the charge on the electron. The quantum efficiency is assumed perfect for the moment.

To quantitatively describe the noise penalty in optical detection, one usually makes use of the quantity of noise figure (NF), which is defined as the ratio of the signal-to-noise ratio (SNR) at the input of a detector to that at its output. According to this definition, NF = 3 dB means a 3 dB noise penalty.

If the detector senses an optical signal in a coherent state, a special kind of quantum state, the SNR at its input is

$$\text{SNR}_{\text{in}} = \frac{\tilde{N}}{(\Delta N)^2} = \tilde{N}_\gamma, \quad (2)$$

where $\tilde{N}_\gamma$ stands for the average photon number received by the detector and $\Delta N_s^2 = \tilde{N}_\gamma$ is the corresponding photon-number fluctuation for coherent light. The SNR at the detector’s output is

$$\text{SNR}_{\text{out}} = \frac{P_1}{P_n}, \quad (3)$$

in which $P_1$ is the average power of the output photoelectric signal and $P_n$ the quantum-noise power of the output signal. One can easily calculate the classical quantity $P_1$ as

$$P_1 = \langle e^{\delta_1^*} e^{-i\Omega t + i\phi} \hat{X} \rangle > = (e\alpha_s \delta_1)^2/2, \quad (4)$$

wherein $\langle \cdot \rangle_s$ means statistical average, $\alpha_s = <\hat{a}_s>$, and the load (spectrum analyzer) resistance was, and hereafter, set as one without loss of generality. The quantum-noise power $P_n$ in a measurement time of one second is

$$P_n = \langle (\Delta \hat{J}_-) > > /2 = \langle (\hat{J}_2^2 - <\hat{J}_->) > s /2 = (e\delta_1)^2, \quad (5)$$

for input light in a coherent state. Here the factor of $1/2$ comes from averaging the power of AC signals over time. If the input light is in a two-mode squeezed state, a quantum correlation may occur between the signal mode and the image sideband mode. In this case, the quantum-noise power $P_n$ becomes

$$P_n = (e\delta_1)^2 < e^{2s \cos^2 \phi + e^{-2s \sin^2 \phi} \hat{X} > s, \quad (6)$$

which agrees with Eq. (5) when the degree of squeezing reduces down to zero, i.e., the squeezing parameter $s = 0$. Therefore, the SNR at the output of a conventional heterodyne detector sensing coherent light is, according to Eq. (3),

$$\text{SNR}_{\text{out}} = \frac{P_1}{P_n} = \frac{(e\alpha_s \delta_1)^2/2}{(e\delta_1)^2} = \alpha_s^2/2. \quad (7)$$

Since $\alpha_s^2$ is the photon number per second, $\alpha_s^2 = \tilde{N}_\gamma$ for a measurement time of one second. Therefore, the noise
where only the terms at the heterodyne frequency Ω remain in the last step. Accordingly, for an optical signal in a coherent state, the average power of the output photoelectric signal at frequency Ω is

\[
P_i = \left(\frac{e\delta_i}{\sqrt{2}}\right)^2 \text{Re}(e^{-\alpha \Omega t} < \hat{X}^2 >) \rightarrow_s \frac{(e\alpha e\delta_i)^2}{2}. \tag{9}
\]

And the quantum-noise power of the photoelectric signal, similar to Eq. (4), reads

\[
P_n = \langle (\Delta \hat{J}_-)^2 \rangle_s \rightarrow_s \frac{1}{2} \left[ \left\langle \hat{J}_+^2 \right\rangle_s + \left\langle \hat{J}_-^2 \right\rangle_s \right] = \left(\frac{e\delta_i}{2}\right)^2 <4 + 2 \cos(2\Omega t + \phi_2 - \phi_1)\rangle_s = (e\delta_i)^2. \tag{10}
\]

Then, with Eq. (10), it is straightforward to show SNR_{out} = \alpha_s^2/2, the same as Eq. (7). Together with Eq. (2), one can easily show that the NF of the heterodyne detector NF = 3 dB, the nature of a phase-insensitive device [23], resulted from the image sideband vacuum modes involved in the detection. Similar theoretical results have been obtained previously for such a heterodyne detector that senses light in two-mode squeezed states [26].

Nonetheless, as we will show in the following, the heterodyne detector with a bichromatic local oscillator is a phase-sensitive device and is supposed to be free of the 3 dB noise penalty, i.e., NF = 0 dB [8, 25].

The 3 dB degradation of the SNR at the output of a traditional heterodyne detector has been theoretically obtained previously for such a heterodyne detector [26] with no doubt at all, whereas the latter case deserves further investigation.

We consider an optical signal, \( \hat{E}_s^+(t) = \hat{a}_s e^{-i\omega_1 t + i\phi_s} \), to be detected by a heterodyne detector with a bichromatic local oscillator \( \hat{\delta}_i^q(t) = (\hat{d}_1 e^{-i\omega_1 t + i\phi_1} + \hat{d}_2 e^{-i\omega_2 t + i\phi_2})/\sqrt{2} \), where \( \omega_1 - \omega_s = \omega_s - \omega_2 = \Omega \) (Fig. 3). In this case, two image sideband vacuum modes, \( \hat{E}_{i1}^+(t) = \hat{a}_{i1} e^{-i\omega_1 t + i\phi_1} \) and \( \hat{E}_{i2}^+(t) = \hat{a}_{i2} e^{-i\omega_2 t + i\phi_2} \), are involved in the detection [26]. Obviously, \( \omega_1 - \omega_1 = \omega_2 - \omega_2 = \Omega \).

Now we show that the NF of a heterodyne detector in this configuration is 3 dB if the image sideband modes introduce extra quantum noise in the detection. The physical quantity detected by the heterodyne detector is \( \hat{X}' = \hat{a}_s^\dagger e^{-i\phi_s + i\phi_1} + \hat{a}_s e^{i\phi_s - i\phi_2} + \hat{a}_{i1} e^{i\phi_1 - i\phi_1} + \hat{a}_{i2} e^{-i\phi_2 + i\phi_2} \).

FIG. 2. (color online) Illustration of two ways to construct a phase-sensitive heterodyne detector out of traditional phase-insensitive heterodyne ones. The center-frequency mode in each case is labeled by red color. (a) A phase-sensitive heterodyne detector with a monochromatic local oscillator. All the frequency modes involved in the detection are excited into coherent states. (b) A phase-sensitive heterodyne detector with a bichromatic local oscillator. Two image sideband modes are in vacuum states in this case.

FIG. 3. (color online) Theoretical model for heterodyne detection with a bichromatic local oscillator.

The theoretical model for heterodyne detection with a bichromatic local oscillator is made out of conventional heterodyne detectors, which are phase-insensitive devices, in two different ways (Fig. 2): (1) To excite the image sideband mode into a state similar to the signal mode or (2) to utilize a bichromatic local oscillator instead of a monochromatic local oscillator. As already pointed out above, a phase-sensitive detector in the former case is free of 3 dB noise penalty [25] with no doubt at all, whereas the latter case deserves further investigation.

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FIG. 4. (color online) Experimental setup for the study of the quantum noise of a phase-sensitive optical heterodyne detector with a bichromatic local oscillator. PM: Single-mode polarization-maintaining fiber, used as spatial-mode cleaner. M: Mirror. LO: Local oscillator. S: Signal light. 50/50: 50-50 balanced beamsplitter. PD1 & PD2: Photodiodes. SA: Spectrum analyzer. PZT: Piezoelectric transducer. HV: High-voltage PZT driver.

A phase-insensitive device is one, whose average output signal is invariant under arbitrary phase transformations. The output photoelectric signal of the heterodyne detector is, according to Eq. (8),

$$< \hat{J}_- > = (e^{i\phi_1}/\sqrt{2}) \text{Re} \left[ e^{i\Omega t} < \hat{X}' > \right] = (\sqrt{2}e^{i\phi_1}\alpha_s \cos \phi') \cos(\Omega t + \delta\phi'),$$

(11)

where $\phi' = \phi_s - (\phi_1 + \phi_2)/2$ and $\delta\phi' = (\phi_2 - \phi_1)/2$. The amplitude of $< \hat{J}_- >$ is definitely not invariant under phase transformation. For instance, under the transformation $\phi' \rightarrow \phi' + \pi/2$, the amplitude of $< \hat{J}_- >$ may drop down to zero from its maximum, thereby showing the phase-sensitive property of the studied heterodyne detector.

Then we come to the theoretical dilemma for the heterodyne detector: On one hand, due to the existence of the image sideband vacuum modes, the detector should suffer a 3 dB noise penalty. On the other hand, as a phase-sensitive device, it should be noise free. At this point, pure theoretical investigation may just lead to fruitless debate and one needs to resort to experimental study for a verdict.

IV. EXPERIMENTAL STUDY OF A HETERODYNE DETECTOR WITH A BICHROMATIC LOCAL OSCILLATOR

In this section, we present an experiment on the quantum noise in phase-sensitive heterodyne detection of coherent light with a bichromatic local oscillator. The results of this experiment are crucial for finding a solution to the above problem of theoretical dilemma and for one to gain a full understanding of the origin of the quantum noise in optical detection.

The experiment utilized as the light source a laser (Mephisto, Innolight GmbH) emitting a continuous-wave single-frequency coherent light beam (spectral linewidth < 1 kHz for 0.1 s measurement time, $\lambda = 1064$ nm). The laser beam was split into two, each of which was sent through an AOM (Crystal Technology, LLC) for frequency shifting. One of the frequency-shifted beams served as the signal light to be detected, while the other one was used as the bichromatic local oscillator for the detector (Fig. 4). Two photodiodes (ETX 500, JDS Uniphase) collected light from the output ports of the 50-50 beamsplitter where optical heterodyning took place and the differenced photocurrent was fed into a spectrum analyzer (Agilent, N9320B) for data record.

As is shown by Eq. (11), the heterodyne detector is a phase-sensitive device. To experimentally demonstrate it, one may monitor the output photoelectric signal while scanning the relative phase between the signal beam and the local oscillator. The data presented in Fig. 5 indeed verify the phase-sensitive property of the heterodyne detector.

In what follows, we demonstrate the capability of the setup to detect the quantum-noise floors of light at appropriate power levels. To this end, we compared the power levels of the detected noises of light with theoretical expectations. The observed noise-power density was $-138 \pm 0.4$ dBm/Hz for a 1.0 mW optical oscillator, in which case the theoretical expectation for the quantum-noise power density was $-139$ dBm/Hz when a 70% quantum efficiency is taken into account for the detector. Similar result was produced for a 2.0 mW optical oscillator. Moreover, we observed that doubling the power of the optical oscillator resulted in a $2.9_{-0.6}^{+0.5}$ dB increase, which
TABLE I. Experimentally-determined noise figure (NF) of the heterodyne detector with a 2.0 mW bichromatic local oscillator. SNR\textsubscript{in} was obtained according to Eq. 2 with a quantum efficiency of 70% taken into account for the detector. SNR\textsubscript{out} was the ratio of the average power of the photoelectric signal to the corresponding shot-noise power, according to Eqs. 3-4. Technically, the average power of the photoelectric signal was obtained by averaging the data presented in Fig. 4.

| \(P_n\) (nW) | \(\text{SNR}_{\text{in}}\) (dB) | \(\text{SNR}_{\text{out}}\) (dB) | NF (dB) |
|--------------|-----------------|-----------------|--------|
| 0.5±0.1      | 62.7±0.9        | 63.5±1.2        | -0.8±1.2 |
| 1.0±0.1      | 65.7±0.5        | 66.5±0.4        | -0.8±1.2 |
| 2.0±0.1      | 68.8±0.4        | 69.8±0.4        | -1.0±0.4 |

V. DISCUSSIONS

The concept of image sideband mode plays a crucial role in the detection theory to describe the quantum nature of optical heterodyne detectors. The image sideband vacuum modes are supposed to introduce extra quantum noise whenever they are involved in optical detection. Although a consensus among the literature has been reached on the connection between the quantum noise in heterodyne detection and the image sideband vacuum modes, theoretical inconsistency does exist when a heterodyne detector with a bichromatic local oscillator is concerned. The above experimental results have provided important information for a solution to the problem of the theoretical dilemma: No extra quantum noise was observed due to the existence of the image sideband vacuum modes. This may indicate our lack of full understanding of the origin of the quantum noise in optical heterodyne detection.

Apparently, a trivial result in consistency with the experiment can be generated if one assumes that image sideband vacuum modes did not produce extra quantum noise in the phase-sensitive heterodyne detection: The quantum-noise power of the detector at its output, previously described by Eq. (10) with the extra quantum noise due to the image sideband vacuum modes taken into account, should be

\[
P_n = \frac{\langle (\Delta \hat{J}_-)^2 \rangle_s}{2}\]

\[
x + 2 + 2 \cos(2\Omega t + \phi_2 - \phi_1) > s
\]

\[
= \langle e\delta t_i \rangle^2 / 2.
\]

If this is the case, the NF of the phase-sensitive detector would be 0 dB, instead of 3 dB, in a good agreement with the experimental observations. Then the aforementioned theoretical dilemma naturally disappears.

What is more interesting will come into sight when one considers the NF of the heterodyne detector, provided that the relative phase \(\phi = \phi_s - (\phi_1 + \phi_2) / 2\) between the signal light and the bichromatic oscillator is locked for maximal output signal. In this case, the average power of the output photocurrent is, instead of Eq. (9),

\[
P_i = \left[ \frac{e\delta t_i}{\sqrt{2}} \text{Re}(e^{-i\hat{t}} \hat{X} \hat{y})|_{\theta = k\gamma} \right]^2 > s = (e\alpha t_s \delta t_i)^2
\]

Here \(k\) is an integer. In combination with Eq. (12), the SNR at the output of the detector reads

\[
\text{SNR}_{\text{out}} = \frac{P_i}{P_n} = \frac{(e\alpha t_s \delta t_i)^2}{(e\delta t_i)^2 / 2} = 2\alpha_s^2 = 2\tilde{N}_s.
\]
Nevertheless, the predicted -3 dB NF for the heterodyne detector for $\phi = k\pi$ is in agreement with the experiment.

As a matter of fact, the data presented in Fig. 5 and Fig. 6 provide one the SNR information of the detector when the relative phase $\phi$ was being scanned. For a 0.5 nW signal light at the input and a 2.0 mW bichromatic local oscillator for the detector, the average power of the output photoelectric signal at its maximum value was -39.6 dBm (Fig. 5) with the noise power of $-105.8 \pm 0.4$ dB (Fig. 6). It is not difficult to calculate the SNR of the output signal as $\text{SNR}_{\text{out}} = 66.2 \pm 0.5$ dB. The SNR of the corresponding input signal was $\text{SNR}_{\text{in}} = 62.7 \pm 0.9$ dB (Table 1). So the NF of the detector was $\text{NF} = -3.5 \pm 0.9$ dB, indicating an SNR increase in the output signal for some chosen values of the relative phase $\phi$ and, thereby, verifying the above paradox about the negative NF for the heterodyne detector.

Similar paradox can also be found in Eq. (8) of the early work of Haus and Townes [23] for homodyne detectors. The key to understand the physics of the paradox is to be aware of that the relative phase $\phi$ must be locked to some preferred values, otherwise the paradox would not have come into being. Technically, the locking scheme assumes that the input light is time-stationary in power. Notwithstanding, the power of a real signal-carrying light varies rapidly with time, which prevents any locking scheme from normal function. In other words, in practical applications of signal communication, it is impossible for one to run a phase-sensitive detector with a -3 dB NF.

The experiment and its great agreement with Eq. (12) strongly suggest that the image sideband vacuum modes should have introduced no extra quantum noise in the phase-sensitive heterodyne detector. This, on one hand, may challenge our current understanding about the origin of the quantum noise in optical heterodyne detection, and, on the other hand, urges one to make effort to uncover the mechanism that led to the disappearance of the expected extra quantum noise of the image sideband vacuum modes in the heterodyne detection.

VI. CONCLUSIONS

We have studied the quantum noise of a phase-sensitive heterodyne detector with a bichromatic local oscillator. We have first addressed a theoretical dilemma inherent to the phase-sensitive device about its quantum noise. Then we have reported an experiment designed for the problem of the inconsistency. Further theoretical analysis, in combination with the experimental results, indicates that the image sideband vacuum modes contributed no extra quantum noise in the heterodyne detection. More study should be carried out to reveal the mechanism through which the extra quantum noise of the image sideband vacuum modes disappeared in the experiment.

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