GAUGE INVARIANCE AND
ANOMALOUS GAUGE BOSON COUPLINGS

JOANNIS PAPAVASSILIOU
Department of Physics, New York University, 4 Washington Place
New York, NY 10003, USA
E-mail: papavass@mafalda.physics.nyu.edu

and

KOSTAS PHILIPPIDES
Department of Physics, New York University, 4 Washington Place
New York, NY 10003, USA
E-mail: kostas@mafalda.physics.nyu.edu

ABSTRACT
Using the $S$–matrix pinch technique we obtain to one loop order, gauge independent $\gamma W^- W^+$ and $ZW^- W^+$ vertices in the context of the standard model, with all incoming momenta off–shell. We show that the vertices so constructed satisfy simple QED–like Ward identities. These gauge invariant vertices give rise to expressions for the magnetic dipole and electric quadrupole form factors of the $W$ gauge boson, which, unlike previous treatments, satisfy the crucial properties of infrared finiteness and perturbative unitarity.

1. Introduction
A new and largely unexplored frontier on which the ongoing search for new physics will soon focus is the study of the structure of the three-boson couplings. A general parametrization of the trilinear gauge boson vertex for on–shell $W$s and off–shell $V = \gamma, Z$ is

$$
\Gamma_{\mu\alpha\beta}^V = -ig_V \left[ f \left( 2g_{\alpha\beta}\Delta_\mu + 4(g_{\alpha\mu}Q_\beta - g_{\beta\mu}Q_\alpha) \right) + 2\Delta_\kappa_V (g_{\alpha\mu}Q_\beta - g_{\beta\mu}Q_\alpha) + 4\frac{\Delta Q_V}{M_W^2}(\Delta_\mu Q_\alpha Q_\beta - \frac{1}{2}Q^2 g_{\alpha\beta}\Delta_\mu) \right] + \ldots ,
$$

(1)

with $g_\gamma = gs$, $g_Z = gc$, where $g$ is the $SU(2)$ gauge coupling, $s \equiv \sin\theta_W$ and $c \equiv \cos\theta_W$, and the ellipses denote omission of $C$, $P$, or $T$ violating terms. The four-momenta $Q$ and $\Delta$ are related to the incoming momenta $q$, $p_1$ and $p_2$ of the gauge bosons $V$, $W^-$ and $W^+$ respectively, by $q = 2Q$, $p_1 = \Delta - Q$ and $p_2 = -\Delta - Q$. The form factors $\Delta_\kappa_V$ and $\Delta Q_V$, also defined as $\Delta_\kappa_V = \kappa_V + \lambda_V - 1$, and $\Delta Q_V = -2\lambda_V$, are compatible with $C$, $P$, and $T$ invariance, and are related to the magnetic dipole moment $\mu_W$ and the electric quadrupole moment $Q_W$, by the following expressions:

$$
\mu_W = \frac{e}{2M_W}(2 + \Delta_\kappa_\gamma) \ , \ Q_W = -\frac{e}{M_W^2}(1 + \Delta_\kappa_\gamma + \Delta Q_\gamma) .
$$

(2)
In the context of the standard model, their canonical, tree level values are $f = 1$ and $\Delta \kappa_V = \Delta Q_V = 0$. To determine the radiative corrections to these quantities one must cast the resulting one–loop expressions in the following form:

$$\Gamma_{\mu \alpha \beta}^V = -ig_V [a_1^V g_{\alpha \beta} \Delta_{\mu} + a_2^V (g_{\alpha \mu} Q_{\beta} - g_{\beta \mu} Q_{\alpha}) + a_3^V \Delta_{\mu} Q_{\alpha} Q_{\beta}] ,$$  

(3)

where $a_1^V$, $a_2^V$, and $a_3^V$ are complicated functions of the momentum transfer $Q^2$, and the masses of the particles appearing in the loops. It then follows that $\Delta \kappa_V$ and $\Delta Q_V$ are given by the following expressions:

$$\Delta \kappa_V = \frac{1}{2} (a_2^V - 2a_1^V - Q^2 a_3^V) , \quad \Delta Q_V = \frac{M_W^2}{4} a_3^V .$$  

(4)

Calculating the one-loop expressions for $\Delta \kappa_V$ and $\Delta Q_V$ is a non-trivial task, both from the technical and the conceptual point of view. If one calculates just the Feynman diagrams contributing to the $\gamma W^+ W^-$ vertex and then extracts from them the contributions to $\Delta \kappa_{\gamma}$ and $\Delta Q_{\gamma}$, one arrives at expressions that are plagued with several pathologies, gauge-dependence being one of them. Indeed, even if the two $W$ are considered to be on shell, since the incoming photon is not, there is no a priori reason why a gauge-independent (g.i.) answer should emerge. In the context of the renormalizable $R_\xi$ gauges the final answer depends on the choice of the gauge fixing parameter $\xi$, which enters into the one-loop calculations through the gauge-boson propagators ( $W, Z, \gamma$, and unphysical Higgs particles). In addition, as shown by an explicit calculation performed in the Feynman gauge ($\xi = 1$), the answer for $\Delta \kappa_{\gamma}$ is infrared divergent and violates perturbative unitarity, e.g. it grows monotonically for $Q^2 \to \infty$. All the above pathologies may be circumvented if one adopts the pinch technique (PT). The application of this method gives rise to new expressions, $\hat{\Delta} \kappa_{\gamma}$ and $\hat{\Delta} Q_{\gamma}$, which are gauge fixing parameter ($\xi$) independent, ultraviolet and infrared finite, and well behaved for large momentum transfers $Q^2$.

2. The pinch technique

The S-matrix pinch technique is an algorithm that allows the construction of modified g.i. $n$-point functions, through the order by order rearrangement of Feynman graphs, contributing to a certain physical and therefore ostensibly g.i. amplitude, (an S-matrix in our case). This rearrangement separates the S–matrix into kinematically distinct pieces, akin to self–energies, vertices and boxes, from which the new effective $n$-point functions can be extracted. The resulting expressions are independent of the initial gauge choice and of the specific process one employs. For the case of vertices, the PT amounts into identifying and appending to the usual vertex–graphs those parts of the box graphs that exhibit a vertex–like structure. These parts are called pinch parts; they emerge every time a gauge–boson propagator, a three gauge boson vertex or a scalar–scalar–gauge boson vertex contributes an integration momentum $k_\mu$.
to the original graph’s numerator. Such momenta, when contracted with a $\gamma$ matrix, trigger an elementary identity of the form

$$k_\mu \gamma^\mu P_L = \# P_L = S_1^{-1}(p + k) P_L - P_R S_1^{-1}(p) + m_i P_L - m_j P_R.$$  \hspace{1cm} (5)

The first term on the r.h.s. of Eq.(5) removes the internal fermion propagator and generates a pinch term, while the second vanishes on shell. Collecting all vertex-like pinch parts and appending them to the usual vertex graphs gives rise to a g.i. sub–amplitude, with the same kinematical properties as a vertex.

3. Gauge–invariant gauge boson vertices and their Ward identities

We consider the S-matrix element for the process

$$e^- + \nu + e^- \rightarrow e^- + e^- + \gamma.$$  \hspace{1cm} (6)

and isolate the part $T(q, p_1, p_2)$ of the S–matrix which depends only on the momentum transfers $q, p_1,$ and $p_2$. Since the final result (with pinch contributions included) is g.i., we choose to work in the Feynman gauge ($\xi_i = 1$); this particular gauge simplifies the calculations because it removes all longitudinal parts from the tree-level gauge boson propagators. So, pinch contributions can only originate from appropriate momenta furnished by the tree–level gauge boson vertices. Applying the pinch technique algorithm we isolate all vertex–like parts contained in the box diagrams and allot them to the usual vertex graphs. The final expressions for one loop g.i. trilinear gauge boson vertices are:

$$\frac{1}{g^3 s} \hat{\Gamma}_{\mu \alpha \beta}^{\gamma W^- W^+} = \Gamma_{\mu \alpha \beta}^{\gamma W^- W^+} |_{\xi_i = 1} + q^2 B_{\mu \alpha \beta} + U_W^{-1}(p_1)^\rho B_{\mu \rho \beta}^+ + U_W^{-1}(p_2)^\rho B_{\mu \rho \beta}^-$$

$$- 2 \Gamma_{\mu \alpha \beta} \left[ I_{WW}(q) + s^2 I_{W \gamma}(p_1) + c^2 I_{WZ}(p_1) + s^2 I_{W \gamma}(p_2) + c^2 I_{WZ}(p_2) \right]$$

$$+ p_{2 \beta} g_{\mu \alpha} M^- + p_{1 \alpha} g_{\mu \beta} M^+,$$  \hspace{1cm} (7)

$$\frac{1}{g^3 c} \hat{\Gamma}_{\mu \alpha \beta}^{Z W^- W^+} = \Gamma_{\mu \alpha \beta}^{Z W^- W^+} |_{\xi_i = 1} + U_Z^{-1}(q)^\rho B_{\rho \alpha \beta} + U_W^{-1}(p_1)^\rho B_{\mu \rho \beta}^+ + U_W^{-1}(p_2)^\rho B_{\mu \rho \beta}^-$$

$$- 2 \Gamma_{\mu \alpha \beta} \left[ I_{WW}(q) + s^2 I_{W \gamma}(p_1) + c^2 I_{WZ}(p_1) + s^2 I_{W \gamma}(p_2) + c^2 I_{WZ}(p_2) \right]$$

$$+ q_{\mu} g_{\alpha \beta} M_Z^2 M^- + p_{2 \beta} g_{\mu \alpha} M_W^2 M^- + p_{1 \alpha} g_{\mu \beta} M_W^2 M^+.$$  \hspace{1cm} (8)

The quantities $I_{ij}$ and $M^\pm$ are integrals with two and three scalar propagators, respectively; their explicit expressions have been reported elsewhere 4.

The g.i. vertices satisfy the following simple Ward identities (WI), relating them to the g.i. $W$ self energy and $\chi_{WW}$ vertex constructed also via the PT:

$$q^\mu \hat{\Gamma}_{\mu \alpha \beta}^{Z W^- W^+} + i M_Z \hat{\Gamma}_{\alpha \beta}^{\gamma W^- W^+} = gc \left[ \hat{\Pi}_{\alpha \beta}^W(1) - \hat{\Pi}_{\alpha \beta}^W(2) \right],$$  \hspace{1cm} (9)
q^\mu \Gamma^{\gamma W^-}_{\mu \beta} = g_s \left[ \tilde{\Theta}^W_{\alpha \beta}(1) - \tilde{\Theta}^W_{\alpha \beta}(2) \right]. (10)

These WI are the one–loop generalizations of the respective tree level WI; their validity is crucial for the gauge independence of the S–matrix. It is important to emphasize that they make no reference to ghost terms, unlike the corresponding Slavnov-Taylor identities satisfied by the conventional, gauge–dependent vertices.

For the case of on–shell Ws one sets $p_1^2 = p_2^2 = M_W^2$ and neglects all terms proportional to $p_{1\alpha}$ and $p_{2\beta}$, as well as the left over pinch terms of the W legs. Then the $\gamma W W$ vertex reduces to the form

$$\frac{1}{g^3 s} \Gamma^{\gamma W^- W^+}_{\mu \alpha \beta} = \Gamma^{\gamma W^- W^+}_{\mu \alpha \beta} |_{\xi_i = 1} + g^2 B_{\mu \alpha \beta}(q, p_1, p_2) - 2\Gamma_{\mu \alpha \beta} I_{WW}(q). (11)$$

This is of course the same answer one obtains by applying the PT directly to the S–matrix of $e^+ e^- \rightarrow W^+ W^-$. Thus for the form factors $\Delta \kappa_\gamma$, $\Delta Q_\gamma$ the only function we need is $B_{\mu \alpha \beta}$, given below

$$g^2 B_{\mu \alpha \beta} = \sum_{V = \gamma, Z} g_V^2 \int \frac{d^4 k}{i(2\pi)^4} \frac{g_{\alpha \beta} (k - \frac{3}{2}(p_1 - p_2))_\mu - g_{\alpha \mu} (3k + 2q)_\beta - g_{\beta \mu} (3k - 2q)_\alpha}{[(k + p_1)^2 - M_W^2][(k - p_2)^2 - M_W^2][k^2 - M_V^2]}.$$ (12)

### 4. Magnetic dipole and electric quadrupole form factors for the W

Having constructed the g.i. $\gamma W W$ vertex we proceed to extract its contributions to the magnetic dipole and electric quadrupole form factors of the W. We use carets to denote the g.i. one–loop contributions. Clearly,

$$\hat{\Delta} \kappa_\gamma = \Delta \kappa_\gamma^{(\xi = 1)} + \Delta \kappa_\gamma^P,$$ (13)

and

$$\hat{\Delta} Q_\gamma = \Delta Q_\gamma^{(\xi = 1)} + \Delta Q_\gamma^P,$$ (14)

where $\Delta Q_\gamma^{(\xi = 1)}$ and $\Delta Q_\gamma^{(\xi = 1)}$ are the contributions of the usual vertex diagrams in the Feynman gauge $\tilde{\Gamma}$, whereas $\Delta Q_\gamma^P$ and $\Delta Q_\gamma^P$ the analogous contributions from the pinch parts. By performing the momentum integration in $B_{\mu \alpha \beta}$, we find for $p_1^2 = p_2^2 = M_W^2$

$$B_{\mu \alpha \beta} = -\frac{Q^2}{8\pi^2 M_W^2} \sum_{V = \gamma, Z} g_V^2 \int_0^1 da \int_0^1 (2\pi dt) \frac{F_{\mu \alpha \beta}}{L_V^2}, (15)$$

where

$$F_{\mu \alpha \beta} = 2(\frac{3}{2} + at)g_{\alpha \beta}\Delta \mu + 2(3at + 2)[g_{\alpha \mu}Q_\beta - g_{\beta \mu}Q_\alpha], (16)$$

and

$$L_V^2 = t^2 - t^2 a(1 - a)\left(\frac{4Q^2}{M_W^2} - (1 - t)\right)\frac{M_W^2}{M_V^2}, (17)$$
from which follows that

$$\Delta \kappa_\gamma^P = -\frac{1}{2} \frac{Q^2}{M_W^2} \sum_V \frac{\alpha_V}{\pi} \int_0^1 da \int_0^1 (2t dt) \frac{(at - 1)}{L_V^2},$$  \hspace{1cm} (18)

and

$$\Delta Q_\gamma^P = 0.$$  \hspace{1cm} (19)

We observe that $\Delta \kappa_\gamma^P$ contains an infrared divergent term, stemming from the double integral shown above, when $V = \gamma$. This term cancels exactly against a similar infrared divergent piece contained in $\Delta \kappa_\gamma^{(\xi = 1)}$, thus rendering $\hat{\Delta} \kappa_\gamma$ infrared finite. After the infrared pieces have been cancelled, one notices that the remaining contribution of $\Delta \kappa_\gamma^P$ decreases monotonically as $Q^2 \to \pm \infty$; due to the difference in relative signs this contribution cancels asymptotically against the monotonically increasing contribution from $\Delta \kappa_\gamma^{(\xi = 1)}$. Thus by including the pinch part the unitarity of $\hat{\Delta} \kappa_\gamma$ is restored and $\hat{\Delta} \kappa_\gamma \to 0$ for large values of $Q^2$. It would be interesting to determine how these quantities could be directly extracted from future $e^+ e^-$ experiments.

5. Conclusions

We showed how to use the PT in order to construct g.i. gauge–boson vertices, which satisfy naive QED–like Ward identities. These vertices give rise to magnetic dipole and electric quadrupole form factors for the $W$, which can, at least in principle, be promoted to physical observables.

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7. References

1. W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys.; B 46 (1972) 319.
2. E.N. Argyres, G. Katsilieris, A.B. Lahanas, C.G. Papadopoulos, and V.C. Spanos, Nucl. Phys. B 391 (1993) 23.
3. J. M. Cornwall, in Proceedings of the French-American Seminar on Theoretical Aspects of Quantum Chromodynamics, Marseille, France, 1981, edited J. W. Dash (Centre de Physique Théorique, Marseille, 1982).
4. J. Papavassiliou and K. Philippides, Phys. Rev. D 48 (1993) 4255.