Analysis of lateral seismic response internal force of circular shield tunnel

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Abstract: With the acceleration of urbanization in China, large-scale tunnel engineering has developed rapidly. The anti-seismic problem of tunnel engineering has become a research hotspot in the field of geotechnical seismic engineering. In this paper, the basic solution method of the orifice problem in the elastic theory is used to solve the internal force of the tunnel earthquake. The results show that the internal force of the tunnel structure is distributed along the circular tunnel in the shape of a goose egg. The larger the peak acceleration of the input seismic wave, the stronger the internal force of the tunnel structure. The conclusion provides a basis for anti-seismic design calculation of shield tunnel.

1. Introduction
With the acceleration of China's social and economic development and urbanization, the shortage of land resources has become increasingly prominent. Rational use and development of underground space resources is an important means to achieve sustainable urban development. Among them, shield tunnels have become an important way to solve urban traffic problems due to their unique construction advantages. Some major strategic tunnel projects have also been built or planned. However, China is located between the Pacific Rim seismic zone and the Eurasian seismic zone. Due to the compression of the Pacific plate, the Indian plate and the Philippine sea plate, earthquakes occur frequently, making China one of the countries in the world most greatly affected by earthquake disasters. According to historical and modern seismic data, there have been 86 major earthquakes of magnitude 7 or above in China, many of which occurred near large urban areas. Once an earthquake occurs in these big cities, it will cause damage to structures including the tunnel project, resulting in major casualties and property losses, affecting economic development and social stability.

Kiyomiya(1995)[1] analyzed the longitudinal elastic-plastic seismic response of the undersea tunnel in Japan by using the response displacement method, and proposed new tunnel design methods through experimental tests and field observations, and provided experience for longitudinal anti-seismic analysis of tunnels. Vanzi (2000) [2] proposed the dynamic solution of the shield tunnel under linear condition, and studied the longitudinal response of the shield tunnel under the traveling wave input by establishing a theoretical model. He and Koizumi (2001) [3] proposed a pseudo-static analysis method based on the composite finite element model of the underground tunnel and the beam spring element model, and verified the effectiveness of the analysis method, which can be directly used as the lateral anti-seismic...
design method of shield tunnel. Liu Jingbo et al. (2013, 2014) [4] based on the basic principle of response displacement method, established a monolithic response displacement method suitable for seismic response analysis of underground structures with complex sections, which provides a simple and highly applicable calculation method for anti-seismic analysis of underground structures. Yuan Yong et al. (2015) [5] established a multi-rigid-elastic damping hinge-damping hinge dynamic model of tunnel based on reasonable assumptions, and applied discrete time transfer matrix method (MS-DT-TMM) in multi-body dynamics theory to deducing its mathematical model and expression. Pelli and Sofianos (2018) [6] proposed an analytical solution method for the half-space stress field of tunnels under SV wave earthquakes. The proposed analytical method is compared with the relaxed boundary method, and it is found that the stress and displacement calculated by the former method are larger than that by the latter method.

These theoretical analysis methods have simple mechanical models, strict derivation, and efficient calculation efficiency, which provide ideas and research methods for anti-seismic analysis of tunnels. In this paper, the typical cross-section of the Suai subsea shield tunnel in Miao et al. (2018) [7] is analyzed using analytical and numerical analysis, and the internal force analysis of the lateral seismic response of the shield tunnel is carried out.

2. Analysis of analytical solution of lateral seismic in shield tunnel

2.1. Derivation of internal force formula of tunnel structure when soil-circular tunnel does not slip

Based on the basic solution of the orifice problem in elastic mechanics, it is studied that the peripheral shear force can be equivalent to the action of uniform distributed force under the action of s-wave. According to the elastic center method, it is decomposed into uniform distributed load in both horizontal and vertical directions for solution, and the formula for calculating the internal force of circular shield tunnel lining structure is obtained. As shown in Fig. 1, where: \( \gamma_s \) is the maximum shear strain at the position of the shield tunnel in the free field, \( \sigma_i \) is the shear modulus corresponding to \( \gamma_s \), \( \tau \) is the interaction force between the soil and the tunnel, and \( \sigma_1, \sigma_2 \) are the two principal stresses at the infinite distance of the soil.

![Figure 1. Quasi-static method model.](image)

2.1.1. Radial displacement of soil orifice

In order to facilitate the determination of these functions according to the known conditions of each complex variable function on the boundary of the soil, this paper uses the conformal transformation \( z = \omega(\zeta) \) to transform the area occupied by the soil on the z-plane (i.e. the x-y plane) into the area on the \( \zeta \) plane. In this way, on the \( \zeta \) plane, we have:

\[
\zeta = \rho (\cos \theta + i \sin \theta) = \rho e^{i\theta}
\]  

Then \( \rho \) and \( \theta \) are the polar coordinates of the \( \zeta \) point on the \( \zeta \) plane (not the polar coordinates of the z point). A circumference \( \rho = \text{const} \) and a radial line \( \theta = \text{const} \) on the \( \zeta \) plane correspond to a curve on the z plane, respectively. These two curves can also be represented by \( \rho = \text{const} \) and \( \theta = \text{const} \), as shown in
Figure 2. Then \( \rho \) and \( \theta \) can be seen as the curve coordinates of a point on the z-plane. Due to the conformality of the transformation, this curve coordinate is always orthogonal to the curvilinear coordinates, and the relative direction of the coordinate axes \( \rho \) and \( \theta \) is the same as the relative direction of the coordinate axes x and y.

![Figure 2. Plane conformal transformation of soil.](image)

The area occupied by the soil in the z-plane is transformed into a so-called "central unit circle" on the \( \zeta \) plane (its center is at the origin of the coordinate \( \zeta = 0 \), and the radius is equal to 1), and the boundary of the orifice is transformed into the perimeter of the unit circle, and the transformation is \( z = \omega(\zeta) = R / \zeta \).

The complex variable potential function and the stress boundary conditions after transformation are equations (2) and (3):

\[
\begin{align*}
\varphi(\zeta) &= \frac{1+\mu}{8\pi} (\overline{F_x} + i\overline{F_y}) \ln \zeta + B \omega(\zeta) + \phi_0(\zeta) \\
\psi(\zeta) &= \frac{3-\mu}{8\pi} (\overline{F_x} - i\overline{F_y}) \ln \zeta + (B' + iC') \omega(\zeta) + \psi_0(\zeta) \\
\phi_0(\zeta) &= \sum_{i=1}^{N} \alpha_i \zeta^i \\
\psi_0(\zeta) &= \sum_{i=1}^{N} \beta_i \zeta^i
\end{align*}
\]

(2)

\[
f_0 = i \left( \int_{\zeta}^{B'} (\overline{F_x} + i\overline{F_y}) \ln \zeta - \frac{1+\mu}{8\pi} (\overline{F_x} - i\overline{F_y}) \frac{\omega(\sigma)}{\omega'(\sigma)} \sigma - 2B \omega(\sigma) - (B' - iC') \omega(\sigma) \right) d\sigma
\]

(3)

Where: \( B, B', \) and \( C' \) are real constants, \( F_x \) and \( F_y \) are the known surface force components on the orifice, \( \overline{F_x} \) and \( \overline{F_y} \) are the sum of the surface forces along the x and y directions on the inner boundary.

The soil is affected by shear stress \( \tau \) at infinity, and the boundary condition is equation (4):

\[
\begin{align*}
\sigma_1 &= \tau, \sigma_2 = -\tau, \alpha = \pi / 4 \\
B &= (\sigma_1 + \sigma_2) / 4 = 0 \\
B' - iC' &= -(\sigma_1 - \sigma_2) \omega^{2n} / 2 = -i \tau
\end{align*}
\]

(4)

The soil orifice is subject to the interaction of shear stress \( \tau \), and the surface force component and its projection on the coordinate axis are equation (5):

\[
\begin{align*}
\overline{F_x} &= -\frac{dx}{ds} \tau_x, \overline{F_x} = 0 \\
\overline{F_y} &= -\frac{dy}{ds} \tau_y, \overline{F_y} = 0
\end{align*}
\]

(5)

Formula (6) can be obtained by applying Cauchy integral formula in formula (2) ~ (5).
The radial displacement of the soil orifice can be solved by the complex variable displacement formula:

$$\phi(\zeta) = \phi_s(\zeta) = i(\tau - \tau_s)R\zeta$$

$$\psi(\zeta) = iR\zeta + i(\tau - \tau_s)R\zeta^2, \psi_s(\zeta) = i(\tau - \tau_s)R\zeta^3$$

The radial displacement of the soil orifice can be solved by the complex variable displacement formula:

$$\mu_s(\theta) = \frac{1 + \mu}{E} \left[ 4(1 - \mu)(3 - 4\mu)\tau_s \right] R\sin(2\theta)$$

$$\Delta D_s = \mu_s(\theta) + \mu_s(\theta + \pi) = \frac{2(1 + \mu)}{E} \left[ 4(1 - \mu)(3 - 4\mu)\tau_s \right] R\sin(2\theta)$$

$$\Delta D_{max} = \frac{2(1 + \mu)}{E} \left[ 4(1 - \mu)(3 - 4\mu)\tau_s \right] R$$

2.1.2. Radial displacement of the tunnel structure

The cross-section force of the tunnel and the sign of the internal force are shown in Figure 3. The structural force is equivalent to the normal and tangential distribution forces:

$$p_s(R, \theta) = p\cos(2\theta), \quad p_s(R, \theta) = p\sin(2\theta)$$

Solved by the elastic center method of structural mechanics:

$$N(\theta) = -pR\cos(2\theta)$$

$$V(\theta) = -pR\sin(2\theta)$$

$$M(\theta) = -pR^2\cos(2\theta) / 2$$

The radial displacement of the tunnel structure is shown in equation (12):

$$\Delta D_{max} (R, 0) = \frac{pR^2}{3E_l}(1 - \nu^2)$$

The internal force of the structure in pure shear state is obtained by replacing $$\theta$$ with $$\theta + \pi / 4$$ in the above formula.

The coordination equation of force and displacement between soil-circular tunnel is:

$$\begin{cases}
\rho = \tau_s \\
\Delta D_{max} = \Delta D_{max}
\end{cases}$$

According to equations (10) and (13), the internal force of the tunnel structure can be obtained:

$$\begin{cases}
N(\theta) = \frac{3E_lI_s \Delta D_{max}}{R^2(1 - \nu^2)} \cos[2(\theta + \frac{\pi}{4})] \\
V(\theta) = \frac{3E_lI_s \Delta D_{max}}{R^2(1 - \nu^2)} \sin[2(\theta + \frac{\pi}{4})] \\
M(\theta) = \frac{3E_lI_s \Delta D_{max}}{2R^2(1 - \nu^2)} \cos[2(\theta + \frac{\pi}{4})] \\
\Delta D_{max} = \frac{4(1 - \mu)\tau_s}{1 + \mu} - \frac{6E_lI_s(1 + \mu)(3 - 4\mu)}{R^2E(1 - \nu^2)}
\end{cases}$$

Where, $$N(\theta), V(\theta), M(\theta)$$ are the axial force, shear force and bending moment of the tunnel lining respectively, $$E$$ is the tunnel lining elastic modulus (pa), $$E$$ is the surrounding rock soil elastic modulus
\( I \) is the unit moment of inertia \( m^4/m \) of transverse (along longitudinal) of tunnel lining. \( \nu_s \) is the Poisson's ratio of the tunnel lining and \( \mu \) is the Poisson's ratio of the surrounding rock.

### 2.2. Analytical solution of internal force of shield tunnel structure

The Suai subsea shield tunnel in Shantou City, Guangdong Province is taken as an example of the tunnel structure. The section of the tunnel borehole LZC-02 is taken as the research object and the soil layer distribution at the cross-section is shown in Table 1. The tunnel lining has an outer diameter of 14.5m, an inner diameter of 13.3m, a tube thickness of 0.6m and a minimum buried depth of 20m. The two sides of the tunnel take 6 times the aperture width, the calculated width is 188.5m, and the calculated depth is 100m. The tunnel structure and the soil are bound by Tie connection. The tunnel structure adopts the linear elastic constitutive structure. The bottom and sides of the model adopt viscoelastic artificial boundary [8]. The constitutive model is the modified Davidenkov viscoelastic dynamic constitutive model [9], of which the parameters are shown in Table 2. The ground motion input uses artificial wave 1 and Iwate wave, as shown in Figure 4. The corresponding maximum peak acceleration is 0.4g, which is only horizontal input. The calculation model is shown in Figure 5.

#### Table 1. Distribution of soil in Section.

| Soil layer code | Name of the soil     | Orifice height(m) | Depth(m)   |
|-----------------|----------------------|-------------------|------------|
| ②1              | Lean clay            | -6.36             | 2.01-9.50  |
| ②3              | Clayey sand          | -6.36             | 9.50-10.70 |
| ③4              | Silty sand           | -6.36             | 10.70-22.30|
| ④1              | Lean clay            | -6.36             | 22.30-30.40|
| ④4              | Silty sand           | -6.36             | 30.40-31.30|
| ④1              | Lean clay            | -6.36             | 31.30-33.30|
| ④4              | Silty sand           | -6.36             | 33.30-34.20|
| ④1              | Lean clay            | -6.36             | 34.20-43.60|
| ⑥2              | Weathered bedrock    | -6.36             | 43.60-infinity|

#### Table 2. The parameters of the constitutive model for different soils in Suai Bay.

| Soils          | Parameters of the Davidenkov backbone curve | \( A \) | \( B \) | \( \gamma \times 10^4 \) |
|----------------|--------------------------------------------|--------|--------|--------------------------|
| Lean clay      |                                            | 1.03   | 0.38   | 4.2                      |
| Clayey sand    |                                            | 1.07   | 0.37   | 5.2                      |
| Silty sand     |                                            | 1.2    | 0.38   | 6.5                      |
| Weathered bedrock |                                      | 1.26   | 0.45   | 10.9                     |

Figure 4. Bedrock acceleration time history curve.
The analytical solution is based on the software PRO SHAKE to establish a horizontal layered site. At the same time, the following assumptions are made for the free space: (1) the soil layer is infinitely extended, and the soil is isotropic linear viscoelastic body; (2) the soil bedrock is semi-infinite space, capable of absorbing all reflected waves under bedrock interfaces. Based on the above two assumptions, the one-dimensional wave theory frequency domain analysis can be applied to the bedrock. The maximum shear strain at the location of shield tunnel in the free field is solved, and the series of shear strains obtained are substituted into formula (14) for solution. The solution method is programmed based on MATLAB software.

Figures 6 and 7 are the internal force diagrams of the cross-section of the tunnel under the action of artificial wave 1 and Iwate wave. It can be seen that different ground motions have a great influence on the structure. The internal force under the action of Iwate wave is much smaller than that of artificial waves. The maximum axial force, maximum shear force and maximum bending moment of the structure under the action of artificial waves are 352 kN, 355 kN and 1279 kN·m, respectively. The maximum axial force, maximum shear force and maximum bending moment of the structure under the action of Iwate wave are 31 kN, respectively. 31 kN and 115 kN·m. From the perspective of the circumferential distribution of the internal force of the structure along the circular tunnel, the internal force is distributed in a goose-egg shape, and the maximum axial force and bending moment are distributed at the position of the arch shoulder and the arch.
3. Conclusion
In this paper, the seismic response analysis of the tunnel is carried out by using the basic solution of the orifice problem in the elastic theory. The conclusions are as follows:

Analytical solution analysis of the internal force of the shield tunnel structure shows that the internal force of the structure along the circumferential distribution of the circular tunnel is distributed in a goose-egg shape, which meets the requirements of engineering precision, and also shows the practicality and convenience of the analytical solution.

When the peak acceleration of the input ground motion is the same, the calculated seismic response of the underground structure is also different because of the large difference in the spectrum of different ground motions.

As the peak acceleration of the input seismic wave increases, on the one hand, the seismic action gradually increases. On the other hand, the stiffness of the soil around the structure becomes smaller, and the constraint on the underground structure is weakened, so the seismic response of the structure is enhanced.

Acknowledgment
Natural science research Project in Jiangsu higher education institutions(No.17KJD610003). The study was supported by the Qing Lan Project in Jiangsu Province.

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