String effects in Polyakov loop correlators.

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We compare the predictions of the effective string description of confinement in finite temperature gauge theories to high precision Monte Carlo data for the three-dimensional $Z_2$ gauge theory. We show that string interaction effects become more relevant as the temperature is increased towards the deconfinement one, and are well modeled by a Nambu-Goto string action.

1. INTRODUCTION

The effective string picture of confinement describes the quantum fluctuations of the color flux tube that joins confined color sources in gauge theories. By assuming that such fluctuations are described by a bosonic string theory one obtains quantitative predictions about gauge invariant correlation functions in the confined phase, that can be compared to the results of Monte Carlo simulations of gauge theories in the confining regime to obtain a stringent test of the validity of such an effective description (see Ref.\textsuperscript{[1]} for references to the original literature).

The three-dimensional $Z_2$ gauge theory is an ideal testing ground for the effective string description of confinement: on one hand, its configuration space is very small compared to the one of four dimensional Yang-Mills theories, so that high precision Monte Carlo results can be obtained with comparatively small computational effort; on the other hand it is widely believed that the effective string theory that describes confinement is to a large extent universal, that is independent from the gauge group and the dimensionality of spacetime. Therefore many results obtained in the context of $Z_2$ gauge theory are likely to be valid with little or no modification in Yang-Mills theories as well.

In this contribution we compare the predictions of the effective bosonic string theory of confinement to Monte Carlo results obtained in three dimensional $Z_2$ gauge theory at finite temperature. The gauge-invariant quantity of interest is the correlation function of two Polyakov loops in the confined phase. The effective string theory predicts the dependence of this correlation function from the temperature, the distance between the Polyakov loops, and the zero temperature string tension. Our aim is to compare such predictions to Monte Carlo results. First, however, a careful discussion of the physical regime in which the string picture is expected to hold is in order: in particular we will discuss what region in the parameter space is expected to be described by a free bosonic string theory, and where one should instead expect corrections to this picture.

For a more detailed discussion of all these issues, see Ref.\textsuperscript{[1]}. Note however that the numerical data reported in the present contribution are much more precise than the ones reported in Ref.\textsuperscript{[1]}, being obtained with a new algorithm for the computation of Polyakov loop correlators that will be described in a forthcoming publication.

2. FREE STRING PREDICTIONS AND THEIR RANGE OF VALIDITY

Suppose the quantum fluctuations of the color flux tube are modelled by a free bosonic string
theory. Then if we are interested in Polyakov loop correlation functions, such a string lives on a world sheet that is bounded by the two loops in the space-like direction, and periodic in the time-like direction. The partition function of such a string can be computed, for example, using $\zeta$-function regularization. The prediction for the Polyakov loop correlation function is

$$\langle P(0) P^1(R) \rangle = \exp \left[ - F(R, L) \right]$$

(1)

where $L = 1/T$ is the spatial size of the lattice, and $F(R, L)$, the free energy of the string, is made of a classical and a quantum contribution: in three spacetime dimensions

$$F_3(R, L) = \sigma_0 LR + k(L)$$

(2)

$$F_4(R, L) = \log \eta(\tau)$$

(3)

where $\sigma_0$ is the zero temperature string tension at the same coupling, $k(L)$ is a non universal constant depending on $L$ only, $\eta$ is the Dedekind function, and $\tau$ is the modular parameter $\tau \equiv iL/2R$.

Physical considerations allow us to determine the range of physical parameters where we expect the free string picture to hold: in particular, both distances $R$ and $L$ appearing in Eqs. (1) must be large in the following sense:

- **large $R$:** the picture of a thin, free string is certainly an idealization, since we know that the actual flux tube has a non zero width of order $1/T_c$ where $T_c$ is the deconfinement critical temperature. Therefore we do not expect the prediction embodied by Eqs. (1) to hold for distances $R \leq 1/T_c$.

- **large $L$:** if one assumes the free string picture to be valid at all temperatures up to $T_c$, one obtains a prediction for the value of such temperature which is very far from the actual value obtained from Monte Carlo simulations. The free string picture must break down for temperatures close to $T_c$.

3. **CORRECTIONS TO THE FREE STRING PICTURE: THE NAMBU-GOTO STRING**

The free string described in the previous section is the infrared limit of a large class of models having in common the bosonic character of the field describing the fluctuations of the color flux tube (see [2]). Among these various models, the Nambu-Goto string is the best candidate to describe the short distance corrections to the free string picture that, as discussed above, must be present in the effective description of confinement.

There are at least two reasons for choosing this particular model: first, it gives a prediction for the dimensionless ratio $T_c/\sqrt{\sigma_0}$ which is in good agreement with Monte Carlo results for several gauge models. Second, the Nambu-Goto string accurately describes interface fluctuations in the three-dimensional Ising model [2], which is related by duality to the $\mathbb{Z}_2$ gauge theory. For these reasons, we will use the Nambu-Goto string to model short distance corrections to the free string picture.

For Polyakov loop correlators, the Nambu-Goto string predicts a correction to the free string behavior that has been computed in [3] using $\zeta$-function regularization at two-loop level, (the loop expansion parameter being $1/(\sigma_0 LR)$) and is given, in three dimensions, by:

$$F_4^{(NLO)} = - \frac{\pi^2 L}{152 \sigma_0 R^3} \left[ 2E_4(\tau) - E_2^2(\tau) \right]$$

(4)

in terms of the Eisenstein functions $E_2$ and $E_4$. Note that these corrections do not involve any free parameters.

4. **RESULTS**

The comparison between the string predictions described above and Monte Carlo data for Polyakov loop correlation functions is shown in Figs. 1 and 2. They refer to the same value of the coupling $\beta = 0.75180$, corresponding to a critical temperature $T_c = 1/8a$ [4], where $a$ is the lattice spacing. The size $L$ of the lattice for the two figures is 24 and 16, so that the temperatures are respectively $T_c/3$ and $T_c/2$. The Monte Carlo data plotted correspond to the quantity

$$Q_q(R, L) = \log \left( \frac{G(R)}{G(R + 1)} \right) - \sigma_0 L$$

(5)

as a function of $z = 2R/L$; the zero-temperature string tension $\sigma_0$ is taken from previously pub-
Figure 1. Data at $T = T_c/3$: the solid line is the free string prediction; the dotted lines enclose the range of predictions given by the Nambu-Goto string (the uncertainty derives from the uncertainty on $\sigma_0$).

Figure 2. Same as Fig. 1 for $T = T_c/2$

string behavior are present, as expected, at high temperatures, and are well modeled by a Nambu-Goto string action.

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