Nonequilibrium Noise in Metals at Mesoscopic Scales

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We review a semiclassical theory of high-field current noise in strongly degenerate conductors, based on propagator solutions to the Boltzmann transport equation for current fluctuations. The theory provides a microscopic description of correlation-induced suppression of noise in high-mobility heterojunction devices, and is also applicable to diffusive mesoscopic structures. We discuss the behaviour of thermal noise in a mesoscopic wire, from equilibrium to the high-field limit.

Keywords: nonequilibrium fluctuations, noise, electron correlations, mesoscopics

I. FORMALISM

A. Transport

Our method generalises the purely classical Boltzmann-Green-function theory of Stanton and Wilkins. We start with the one-body transport equation. For simplicity consider a uniform conductor. The semiclassical one-electron Boltzmann equation is

$$\left[ \frac{\partial}{\partial t} - \frac{eE}{h} \frac{\partial}{\partial k} \right] f_{s,k}(t) = C_{s,k}[f(t)],$$

where the time-dependent distribution $f_{s,k}(t)$ is labelled by band-and-spin index $s$ and wave vector $k$; the local driving field (uniform here) is $E$. The collision operator $C_{s,k}[f]$ carries, at the semiclassical level, the effects of microscopic scattering from all sources. In a degenerate system, $C$ is inherently nonlinear in $f$. Normalisation to the density $n$ is such that $n = \sum_{s,k} f_{s,k}/\Omega$ in a system of volume $\Omega$.

In the steady state, Eq. (1) maps the equilibrium Fermi-Dirac distribution $f^{eq}$ to $f$. Introducing the difference function $g \equiv f - f^{eq}$, we have

$$- \frac{eE}{h} \frac{\partial g_{s,k}}{\partial k} = - \frac{eE}{h} \frac{\partial f_{s,k}^{eq}}{\partial k} + C_{s,k}[f^{eq} + g].$$

(2)

B. Steady-State Fluctuations

The equilibrium density-density fluctuation for free electrons is $\Delta f^{eq} \equiv k_{B}T \partial f^{eq}/\partial \epsilon_{F}$, where $k_{B}T$ is the thermal energy and $\epsilon_{F}$ is the chemical potential. Eq. (2), linearised, maps $\Delta f^{eq}$ adiabatically to its nonequilibrium counterpart. Solving for the fluctuation difference $\Delta g$ according to

$$- \frac{eE}{h} \frac{\partial \Delta g_{s,k}}{\partial k} = - \frac{eE}{h} \frac{\partial \Delta f_{s,k}^{eq}}{\partial k} + \sum_{s',k'} \delta C_{s,k} \left( \Delta f_{s',k'}^{eq} + \Delta g_{s',k'} \right),$$

(3)

the complete steady-state fluctuation can be constructed as $\Delta f \equiv \Delta f^{eq} + \Delta g$. The effect of degeneracy on $\Delta g$ is explicit, since $\Delta f^{eq} = f^{eq}(1 - \rho^{eq})$. Moreover, the leading right-hand term of Eq. (3) for $g$ itself contains $\partial f^{eq}/\partial k \propto \Delta f^{eq}$. Thus the very nature of the equilibrium state imposes a connection between the nonequilibrium one- and two-particle structures, $g$ and $\Delta f$.}

C. Dynamic Fluctuations

These are obtained from the linearised form of Eq. (1) by calculating its retarded resolvent $R(t)$ with which one then constructs the transient current autocorrelation $C_{xx}(t)$:

$$R_{ss':kk'}(t - t') \equiv \theta(t - t') \frac{\delta f_{s,k}(t)}{\delta f_{s',k'}(t')}$$

(4)

and
where \((v_{s})_{s,k}\) is the group velocity in the direction of \(-E\),
and \(l\) is the sample length. Eq. (5) extends the classical
definition of Stanton and Wilkinson to degenerate systems,
with the following interpretation. At \(t = 0\) a spontaneous
fluctuation \(v_{s}' \Delta f'\) perturbs the flux in its steady
state. Its subsequent fate is determined by the propagator \(R(t)\).
After removal of the steady-state asymptote at \(t \to \infty\), the product \((-ev_{s}/l)R(-ev_{s}/l)\Delta f'\) gives the dynamical
current fluctuations. The density governs \(\varepsilon_{0}\) because the mean-field
electronic potential in the well is self-consistent. As a result \(\Delta f'\)
is renormalised to \(\gamma \Delta f'\) by the suppression factor
\((\varepsilon_{0}) = 1 + (\delta \varepsilon_{0}/\varepsilon_{0})\Delta n/k_{B}T\)^{-1}.

The observable noise is therefore reduced by as much as 65% for sheet densities of \(10^{12}\text{cm}^{-2}\), typical in conductive channels. Under normal conditions this means that the effective noise temperature in a HEMT is \(100K\), not the 300K of a bulk conductor. Such suppression is unique to self-consistently quantised systems. Circuit-theoretical arguments show that, in production devices, it should lead to an extrinsic noise figure which is half
that of bulk field-effect transistors, of otherwise similar
performance.

## II. CONSEQUENCES

The simplest form of our theory is based on the Drude
model in a single parabolic conduction band, for which
\(\mathcal{C} = -g/\tau\); at room temperature the inelastic collision
time \(\tau\) is of order \(10^{-13}\). The noise spectral density is

\[
S(\omega) = \frac{4Gk_{B}T}{1 + \omega^{2} + \tau^{2}} \left[ 1 + \left( \frac{\Delta n}{n} \right) \frac{m^{*} \mu^{2} E^{2}}{k_{B}T} \right],
\]

in which \(G\) is the sample conductance, \(m^{*}\) the effective
mass, \(\mu = e\tau/m^{*}\) the mobility, and \(\Delta n = 2\Sigma_{k} \Delta f_{k}/\Omega\).
For \(E = 0\) in the static limit, \(S(0)\) is the Johnson-Nyquist
noise. At high fields the “hot-electron” term \(\propto E^{2}\) in Eq.
(6) is strongly suppressed by degeneracy; while a classical system has \(\Delta n/n = 1\), a \(\nu\)-dimensional degenerate
system has, instead, \(\Delta n/n = v_{k}Bt/2\pi \lambda \ll 1\).

Equations (4), (6), and (7) show \(S\) to be a linear functional
of \(\Delta f_{eq}\). Insofar as Coulomb and exchange interactions modify the free-particle form of the equilibrium
density-density fluctuation, it follows that thermal noise carries a signature of the internal correlations of the electron gas. Apart from its physical significance, this has immediate practical implications.

### A. Device Noise

An important consequence for microwave technology
is reduction of thermal noise in a two-dimensional (2D)
electron gas confined at the heterojunction of a high-electron-mobility transistor (HEMT). Basically, the occupation within a HEMT is \(f_{eq} = \{1 + \exp[(\varepsilon_{k} + \varepsilon_{0}(n) - \varepsilon_{F})/k_{B}T]\}^{-1}\), where \(\varepsilon_{k}\) is the 2D band energy and \(\varepsilon_{0}(n)\) is the ground-state energy in the heterojunction quantum
well. The density governs \(\varepsilon_{0}\) because the mean-field
electronic potential in the well is self-consistent. As a result \(\Delta f'\)
is renormalised to \(\gamma \Delta f'\) by the suppression factor
\((\varepsilon_{0}) = 1 + (\delta \varepsilon_{0}/\varepsilon_{0})\Delta n/k_{B}T\)^{-1}.

The observable noise is therefore reduced by as much as 65% for sheet densities of \(10^{12}\text{cm}^{-2}\), typical in conductive channels. Under normal conditions this means that the effective noise temperature in a HEMT is \(100K\), not the 300K of a bulk conductor. Such suppression is unique to self-consistently quantised systems. Circuit-theoretical arguments show that, in production devices, it should lead to an extrinsic noise figure which is half
that of bulk field-effect transistors, of otherwise similar
performance. This agrees well with real device comparisons.

## B. Mesoscopic Noise

Sample lengths approaching the mean free path \(\lambda\) take us into the mesoscopic regime.\[\\]

Even for an embedded slice of long, spatially uniform conductor, the propagator \(R(t)\) acquires spatial structure below \(\lambda\), satisfying

\[
\begin{align*}
\frac{\partial}{\partial t} + v_{s,k} \frac{\partial}{\partial r} - \frac{eE}{\hbar} \frac{\partial}{\partial k} \Bigg[ & \sum_{s',k'} R_{s',k'}(r - r', t - t') \\
& \Bigg] \equiv \Omega \delta_{kk'} \delta_{ss'} \eta(r - r') \delta(t - t') \\
& + \sum_{s',k'} \frac{\delta C_{s,k}}{\delta f_{s',k'}} R_{s',k'}(r - r', t - t').
\end{align*}
\]

The sums in Eq. (5) for \(C_{xx}\) now include integrals over the
mesoscopic slice, while the macroscopically homoge-
"nous propagator of Eq. (6) re-emerges as \(\int d^{2}r R/\Omega\).

Using Eqs. (5), (6), and (7) we have computed the thermal-noise spectrum for an embedded uniform 1D
wire, in the Drude model previously studied by Stanton
within the classical limit \(k_{B}T \gg \varepsilon_{F}\). The example, al-
though suggestive, is artificial not least because “wire”
and “leads” are operationally indistinguishable.

We examine the degenerate limit, for which \(\lambda = \tau v_{F}\)
in terms of the Fermi velocity. At zero frequency the equilibrium
noise is given by

\[
S_{eq}(0) = 4Gk_{B}T \left[ 1 - \frac{\lambda}{\tau} (1 - e^{-1/\lambda}) \right],
\]

where \(l\) is the length of the wire segment, much smaller than the enclosing 1D “volume” \(\Omega\). In the limit \(l \ll \lambda\)
Eq. (8) reduces to \(S_{eq}(0) = (4e^{2} / \pi \hbar) k_{B}T\). Since this is the ballistic regime we note that the thermal noise scales with \(e^{2}/\hbar\), the universal unit of conductance.

On the other hand the nonequilibrium ballistic noise is
\[ S(0) = \frac{4e^2}{\pi \hbar} k_B T \left[ 1 + \left( 1 + \frac{\mu E}{v_F} \right) \exp \left( \frac{-2v_F}{\mu E} \right) \right]. \quad (9) \]

The expression is nonperturbative in \( E \) owing to nonanalyticity of the solutions to Eqs. (\ref{eq:appin1}) and (\ref{eq:appin2}) for a uniform nonequilibrium system. Our exact, if simple, model calculation shows that one cannot take for granted the existence of an expansion for the fluctuations near the equilibrium state, in powers of the field.

At large fields the ballistic noise becomes linear:

\[
S(0) = \frac{4e^2}{\pi \hbar} k_B T \left( \frac{\mu E}{v_F} \right) + \mathcal{O}(E^{-2})
\]

\[
\rightarrow 4Gk_B T \left( \frac{eV}{4v_F} \right),
\quad (10)
\]

where \( V = EL \) is the voltage across the wire (still with \( l \ll \lambda \)), and we have reinstated the Johnson-Nyquist normalisation. Two remarks can be made on this equation:

(a) the linear dependence on \( E \) is kinematic, reflecting the shift of the centroid of the fluctuation distribution in \( k \)-space by \( eE \tau / \hbar \). In contrast, the quadratic dependence of the macroscopic noise in Eq. (\ref{eq:appin1}) is dissipative, and sensitive to thermal broadening of \( \Delta f \) over the bulk.

(b) The second form of Eq. (\ref{eq:appin1}) is equivalent to \( S(0) = 2eI(\Delta n/n) \), where \( I = GV \) is the current through the wire. Thus, ballistic thermal noise in the 1D Drude approximation is given by the classical shot-noise formula[7][8], attenuated by the degeneracy, a result valid for all densities and temperatures.

While thermal noise must go to zero with the temperature, this is not the case for true shot noise, whose non-thermal origin is the random transit of individual carriers across the sample. It is beyond our present scope to discuss applications of the Boltzmann-Green function formalism to diffusive mesoscopic shot noise[7][8][9], which we are actively investigating.

### III. SUMMARY

We have outlined a semiclassical framework for calculating thermal fluctuations in metallic electron systems far from equilibrium. Our approach also describes how Coulomb and exchange correlations, present at equilibrium, appear in the nonequilibrium current noise. The consequences for device design are exemplified by the physics of correlation-induced noise suppression in heterojunction field-effect transistors.

The same formalism can be applied to diffusive mesoscopic noise. An illustrative 1D model provides evidence that mesoscopic noise may not always have a perturbation expansion at low fields, if the underlying distributions are nonanalytic in the equilibrium limit. At high fields the model recovers the shot-noise-like behaviour of ballistic thermal noise[9][10]. For a metallic wire we find that this is attenuated in proportion to the degeneracy of the system.

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