Atom interferometry with ultracold Mg atoms: frequency standard and quantum sensors

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Abstract.
The results of theoretical and experimental studies aimed at the creation of matter wave interferometers with Mg atoms are presented. Atom-optical interferometers based on the Ramsey-Bordé scheme are of great interest for the development of optical frequency standards. Ultracold Mg atoms are promising for the development of an optical frequency standard with relative uncertainty and long-term frequency instability at a level of $10^{-17}$–$10^{-18}$. A long-term frequency stability of $3\cdot10^{-15}$ is obtained at an averaging time $\tau = 10^3$ s while stabilizing the frequency of a ‘clock’ laser at 457 nm ($^1S_0 \rightarrow ^3P_1$ transition) to narrow Ramsey-Bordé resonances of Mg atoms cooled and localized in a magneto-optical trap. The measured frequency stability is determined by the stability of the measurement system based on an optical frequency comb stabilized to the optical frequency of a Yb:YAG/I₂ standard. We also present the results of theoretical studies aimed at the use of Mg atom interferometers based on Bragg diffraction for quantum sensing.

1. Introduction
The mechanical action of light on atoms allows efficient cooling of neutral atoms to temperatures of several microkelvins. The progress in laser cooling techniques has given rise to various research areas, such as the study of quantum degenerate Bose and Fermi gases, the development of quantum sensors based on atomic interferometers (gyroscopes, gravimeters, accelerometers), and the use of cold atoms for quantum holography.

One of the most important applications of cold atoms is their use for the creation of optical frequency standards of a new generation with extremely low frequency uncertainty and instability, $\Delta \nu/\nu < 10^{-18}$. Such parameters are necessary both for basic physical research and various applications, for example, in navigation and metrology. Currently the most promising frequency standards are those based on neutral atoms trapped in an optical lattice operating at the magic wavelength: $^{87}$Sr [1–3], $^{199}$Hg [2,4], $^{171}$Yb [5,6], $^{24}$Mg [7,8], $^{169}$Tm [9], and on single ions trapped in a Pauli radio frequency trap: $^{199}$Hg⁺ [10], $^{27}$Al⁺ [11], $^{88}$Sr⁺ [12], $^{171}$Yb⁺ [13]. The best results on frequency stability and accuracy were obtained for Sr and Yb atoms.

The magnesium atom is one of the promising elements for creating an optical frequency standard. The simple electronic structure of the atom with two electrons on the outer shell makes it easy to calculate the Stark, Zeeman, and other frequency shifts (Figure 1). The presence of
a strong closed $^1S_0 \rightarrow ^1P_1$ transition with a natural width of 79 MHz allows Doppler cooling of atoms to temperatures of the order of a few millikelvins. The second cooling stage can be implemented at the $^3P_2 \rightarrow ^3D_3$ transition [14], or using the $^1S_0 \rightarrow ^3P_1$ intercombination transition in the power broadening regime [15]. The $^1S_0 \rightarrow ^3P_1$ transition that can be used as a ‘clock’ transition with a natural width of 36 Hz is the narrowest intercombination transition among the alkaline-earth atoms for which laser cooling was realized. The frequency shift of the clock transition of Mg atoms due to thermal radiation is an order of magnitude less than for Sr and Yb atoms [16]. At the same time, the $^1S_0 \rightarrow ^3P_0$ transition has an extremely small width, $4 \times 10^{-17}$ Hz, and is of interest for creating a frequency standard based on magnesium atoms localized in an optical lattice using magnetic field-induced spectroscopy [17].

![Figure 1. Level structure of Mg atom.](image)

Despite difficulties of using magnesium atoms in the creation of a frequency standard associated with implementation of the sub-Doppler cooling of atoms to temperatures of the order of $10 \mu K$ [18–20], the high recoil energy, and the Zeeman effect of the second order, the magnesium atom is considered as a promising candidate for creating an optical frequency standard with a relative uncertainty of less than $10^{-17} - 10^{-18}$ [21].

2. Optical frequency standard based on cold magnesium atoms

Earlier we created an experimental setup including a laser system for cooling and trapping magnesium atoms in a magneto-optical trap (MOT), a laser system for high-resolution spectroscopy, and a frequency stabilization system [22]. The strong $^1S_0 \rightarrow ^1P_1$ transition at a wavelength of 285 nm allows efficient cooling of the atoms to a temperature of $\sim$3 mK and their trapping in the MOT. The atoms trapped in the MOT are studied by a ‘clock’ laser tuned to the $^1S_0 \rightarrow ^3P_1$ intercombination transition. The clock laser frequency is stabilized to the $^1S_0 \rightarrow ^3P_1$ transition in Mg atoms by locking to a time-domain Ramsey-Bordé atom interferometer. The clock laser frequency is measured using an optical frequency comb based on a femtosecond titanium-sapphire laser. A schematic representation of the frequency standard based on neutral $^{24}$Mg atoms is shown in Figure 2.

2.1. Cooling laser system and magneto-optical trap

The cooling laser system is a continuous ring R6G dye laser with frequency stabilization by a Fabry-Pérot interferometer and frequency doubling in a nonlinear BBO ($\beta$-barium borate) crystal in the enhancement cavity assembled in a ‘butterfly’ configuration. The pump laser is a Coherent Verdi V8 laser at 532 nm. With a pump power of 6.5 W, the dye laser generates a power of about 1 W at 570 nm. The estimated linewidth of the laser stabilized by the Fabry-Pérot interferometer is 100 kHz. The dye laser frequency is stabilized by the Fabry-Pérot etalon using the Pound-Drever-Hall technique [23] and an active electronic control system: one of the interferometer mirrors is automatically piezo-controlled using signal from the Angstrom WS-7
Figure 2. Schematic of the frequency standard based on cold magnesium atoms. Mg atoms are captured from the thermal beam in the MOT formed by three pairs of laser beams. For cooling the Mg atoms, frequency-doubled radiation of the R6G dye laser stabilized by the Fabry-Pérot interferometer (etalon) is used. The clock transition spectroscopy is performed with the frequency-doubled titanium-sapphire (Ti:Sa) laser stabilized by an ultra-stable Fabry-Pérot etalon. Laser pulses for recording Ramsey-Bordé resonances are created by acousto-optical modulators (AOMs). The frequency of the Ti:Sa laser is compared with a reference frequency source using an optical frequency comb (OFC) based on a femtosecond titanium-sapphire laser. The frequency of the clock laser system is tuned by using a computer and an Agilent N5181A frequency synthesizer, which controls the AOM frequency in the frequency stabilization system by the Fabry-Pérot etalon.

wavelength meter. To reduce the laser frequency drift, the Fabry-Pérot interferometer is placed in a vacuum chamber with a residual gas pressure of less than $10^{-6}$ torr and thermally stabilized. The power of ultraviolet radiation at 285 nm at the enhancement cavity output reaches 100 mW with a laser power at the input of 600 mW. This makes possible trapping of about $\sim 10^6$–$10^7$ atoms in the magneto-optical trap.

The MOT for magnesium atoms is formed by the intersection of three orthogonal pairs of counterpropagating laser beams with the corresponding circular polarizations ($\sigma^+$ and $\sigma^-$ configuration), red detuned by 40–50 MHz ($\sim 0.5 \Gamma$) from the cooling transition frequency, and a quadrupole magnetic field with a gradient at the center of the trap of 150 G/cm in the direction of the $z$-axis and 75 G/cm in the radial direction $r$. The high-vacuum chamber of the magneto-optical trap is pumped to a residual gas pressure of $10^{-8}$ Pa. The gradient of the quadrupole magnetic field of the trap is generated by two copper rings in an anti-Helmholtz configuration located inside the vacuum chamber. Turning on and off the magnetic field takes less than 10 $\mu$s.

The power of each of the six MOT laser beams is about 6 mW. The shape and intensity of a cloud of cold atoms in the magneto-optical trap is detected by a UV CCD camera, and the fluorescence signal of atoms at the $^1S_0 \rightarrow ^1P_1$ cooling transition is detected by a photomultiplier tube. The temperature of the atoms in the cloud was determined by the ‘release-capture’ method [24] and was equal to 3 mK.
2.2. Clock laser and frequency stabilization

The laser system for spectroscopy is based on a cw ring titanium-sapphire (Ti:Sa) laser pumped by a Coherent Verdi V18 solid-state laser and frequency stabilized by two highly stable cavities and frequency doubling in the nonlinear KNbO$_3$ crystal in an enhancement cavity (Figure 3). The titanium-sapphire laser generates about 1 W at 914 nm with a pump power of 12.5 W.

**Figure 3.** Scheme of the laser setup for spectroscopy of the $^{1}S_0 \rightarrow ^3P_1$ clock transition in Mg atoms. Ti:Sa laser — titanium-sapphire laser, PDH lock — Pound-Drever-Hall lock, FR — Faraday rotator, AOM — acousto-optical modulator, Enh. SHG in KNbO$_3$ — second harmonic generation of radiation in the KNbO$_3$ crystal in the enhancement cavity, PM — photomultiplier, CCD — camera with a CCD matrix, DDS — direct digital synthesizer, PC — computer.

The laser frequency is stabilized by two highly stable Fabry-Pérot interferometers with the Pound-Drever-Hall technique. The first interferometer provides a preliminary narrowing of the linewidth of the titanium-sapphire laser, and the second one ($F = 10^5$) is used to correct low-frequency disturbances of the first interferometer by adjusting its length with a piezo-controlled mirror. The laser radiation is directed into the second etalon via a two-pass acousto-optical modulator. Both interferometers are placed in vacuum chambers, thermally stabilized and vibration-isolated. The laser frequency drift associated both with the temperature fluctuations in the laboratory and with the aging of the second interferometer base material (sitall CO-115M) is $\approx 0.2–0.4$ Hz/s. The radiation from the titanium-sapphire laser at 914 nm is frequency doubled in the KNbO$_3$ crystal in the enhancement cavity, which outputs about 150 mW of power at 457 nm with a radiation linewidth of less than 100 Hz.

The clock transition spectroscopy of magnesium atoms is performed using Ramsey-Bordé spectroscopy in time-separated laser fields [25]. A cloud of magnesium atoms is sequentially irradiated by 2 pairs of counterpropagating light pulses formed from cw radiation of a clock laser system at 457 nm using AOMs and directed to the MOT through two single-mode polarization-maintaining fibers. The waists of the laser beams in the MOT are $w_0 = 2$ mm, the pulse durations are $\tau = 5$ μs, which corresponds to the $\pi/2$-pulse ($\Omega_{\text{res}} \tau = \pi/2$, where $\Omega_{\text{res}}$ is the Rabi frequency) with a laser beam power of 25 mW. The time delay between the pairs of pulses $T$ is 20 μs. The pulse duration $\tau$, the delay between the pulses $T$, and the duration of the cooling/testing cycle are controlled by a multi-channel timer, a multiplexer, and a 4-channel direct digital synthesizer (DDS). The delay time $T$ for a pair of codirectional pulses can be varied depending on the desired spectral resolution $\Delta = 1/[4 \left(T + \frac{\pi}{\tau}\right)]$. The best spectral resolution of resonances...
achieved so far was obtained at $T = 310 \mu s$ and corresponds to half width at half maximum $\Delta \nu = 390$ Hz. To record narrow Ramsey-Bordé resonances, the laser system frequency is tuned using the Agilent frequency synthesizer, which controls the frequency of the two-pass AOM in the frequency stabilization system by the second etalon.

The frequency of the clock laser system is stabilized by the central band of Ramsey-Bordé resonances. The optical frequency of the system is tuned using the Agilent N5181A synthesizer. To stabilize the frequency of the laser system, a digital analog of the third-harmonic stabilization method [26, 27] is used. To generate an error signal, the fluorescence signal of a cloud of cold atoms $S_i$ is recorded at four points on the slopes of the interferogram corresponding to the following frequency detunings from the central peak of the resonances: $S_1 = -(3/4)\Delta f_r$, $S_2 = -(1/4)\Delta f_r$, $S_3 = +(1/4)\Delta f_r$, $S_4 = +(3/4)\Delta f_r$, where $\Delta f_r = 1/[2(2 + \frac{1}{\pi}T)]$ is the period of the resonances. Frequency offsets are generated using the DDS and transmitted to the AOMs forming the laser pulses. The error signal $dS = (S_1 - 3S_2 + 3S_3 - S_4)$ adjusts the frequency of the two-pass AOM every second using the Agilent N5181A synthesizer.

2.3. Frequency stability measurement using optical frequency comb

The frequency stability of the clock laser stabilized by a time-domain Ramsey-Bordé atom interferometer of cold Mg atoms was measured by beating with an optical frequency comb [8,28] stabilized either by an optical frequency standard based on a Yb:YAG/I laser [29] or an etalon-stabilized clock laser for an optical frequency standard based on a single Yb$^+$ ion [30]. Long-term stability was measured in the first case, and short-term stability in the second case.

In Figure 4(a), the black line shows the Allan deviation of the frequency of the clock laser system, which characterizes the stability of the optical frequency standard based on magnesium atoms. As can be seen from the graph, the long-term stability of the clock laser system stabilized by magnesium atoms is $3 \times 10^{-15}$ at the averaging time $\tau = 10^3$ s. The continuous measurement time was 1.5 h. The short-term stability is $4.7 \times 10^{-15}$ at the averaging time $\tau = 1$ s.

![Figure 4](image)

**Figure 4.** (a) Allan standard deviation of the frequency measurement. $\circ$ — Allan standard deviation characterizing the frequency stability of the Mg optical frequency standard, $\square$ — measured when stabilizing the frequency comb using the Yb:YAG/I$_2$-standard, $\bigcirc$ — when stabilizing the frequency comb using the etalon-stabilized clock laser for the Yb$^+$-standard; (b) Allan standard deviation of the Mg optical frequency standard calculated from the residual signal in the feedback loop of the frequency stabilization system.

Currently the main factors limiting the frequency stability measurement of our Mg frequency
standard are the stability of the reference oscillators used to stabilize the pulse repetition rate of
the frequency comb and aging of the base material of the second interferometer and temperature
fluctuations in the laboratory that cause the frequency drift of the clock laser system. At long
averaging times, the measured stability equals the stability of the reference Yb:YAG/I standard,
which is confirmed by an analysis of the residual signal in the feedback loop of the frequency
stabilization system of the Mg frequency standard by Ramsey-Bordé resonances. The Allan
deviation (Figure 4(b)) calculated from the residual signal allows one to suggest the stability
of the Mg frequency standard to be $7 \times 10^{-16}$. For correct stability measurement, it is necessary to
use a more stable reference oscillator or an independent frequency standard. In our laboratory,
such measurements are planned in the near future using a second MOT for Mg atoms and a new
MOPA system at 457 nm based on the diode laser and tapered amplifier.

3. Mg quantum state manipulation for atomic interferometers

Narrow optical $^{1}S_0 \rightarrow ^{3}P_0$ and $^{1}S_0 \rightarrow ^{3}P_1$ transitions in alkaline-earth atoms are also of great
interest for creating highly sensitive atomic interferometers for detecting inertial forces. Over the
past decade, there have been several proposals on using such interferometers for the detection of
gravitational waves in the low-frequency spectral region ($f < 10$ Hz) [31,32]. In the next section
we will discuss the $^{24}$Mg atoms quantum state manipulation for atomic interferometers (AIs).

The well-known schemes of atom interferometers use the stimulated Raman transitions
between two internal atomic states to separate cold atoms in momentum space. Usually, these are the hyperfine energy levels of alkali elements (see, for example, [33–37]). An alternative way is to use Bragg diffraction in a moving optical lattice for splitting the atomic clouds on the two
interferometer arms [38–41]. In contrast to Raman-transition AIs, the atoms in the two arms of
a Bragg diffraction interferometer remain in the same internal state, which makes it much less
sensitive to many systematic effects like ac-Stark and Zeeman shifts. Large momentum splitting
achieved in Bragg-type AIs may significantly enhance the interferometer sensitivity. Another
important factor that greatly affects the AI sensitivity is the number of atoms involved in the
interference signal. The Bragg transitions between two momentum states are very sensitive
to the atom velocity, and only the atoms with a narrow momentum distribution $\Delta p$ near the
resonant velocity are involved in optical coupling. This may significantly limit the interferometer
sensitivity. In the next section we will discuss the question of increasing the efficiency of the
Bragg transition by increasing the width of the multiphoton resonance in the momentum space
to attract more atoms into the interferometer signal.

![Figure 5](image_url)

Figure 5. Partial transition diagram for two-photon (a) and four-photon (b) transitions
caused by first- or second-order Bragg diffraction in a moving optical lattice formed by
counterpropagating light waves with detunings $\delta_1$ and $\delta_2$. 
The momentum width of the multiphoton transition resonance is estimated to be

$$\Delta p \simeq \hbar k \Omega_{\text{eff}}^{(2N)}/(2\omega_R),$$

(1)

where $\Omega_{\text{eff}}^{(2N)}$ is the effective Rabi frequency of the $2N$ multiphoton Bragg transition with the effective wavenumber $k_{\text{eff}} = N\hbar k$ ($N = 1, 2, \ldots$ is an integer number), and $\omega_R = \hbar k^2/(2M)$ is the recoil frequency. For the $^{24}\text{Mg}$ atoms intercombination transition $^1S_0 \rightarrow ^3P_1$ the recoil frequency $\omega_R \simeq 2\pi \times 40 \text{ kHz}$.

The $|g,p\rangle \rightarrow |g,p+2\hbar k\rangle$ transition between the external states of the ground energy level (g) related to the atom motion (Figure 5(b)) can be induced by a light field formed by two counterpropagating plane waves with detunings $\delta_1$ and $\delta_2$ satisfying the two-photon resonance condition

$$\delta_1 = \delta_2 + 2kp/M + 4\omega_R.$$

(2)

The optical coupling between these two states is determined by the effective two-photon Rabi frequency

$$\Omega_{\text{eff}}^{(2)} = \frac{\Omega^2}{2\Delta},$$

(3)

where $\Delta = \delta_1 - (pk/M + \omega_R) = \delta + \omega_R$, $\delta = (\delta_1 + \delta_2)/2$ is a one-photon detuning taking into account the Doppler and recoil effects. For the four-photon transition $|g,p\rangle \rightarrow |g,p+4\hbar k\rangle$ (Figure 5(b)) the detunings of counterpropagating waves should satisfy the modified resonance condition

$$\delta_1 = \delta_2 + 2kp/M + 8\omega_R,$$

(4)

and the effective Rabi frequency is

$$\Omega_{\text{eff}}^{(4)} = \left(\frac{\Omega^2}{2\Delta_2}\right)^2 \frac{1}{8\omega_R},$$

(5)

where $\Delta_2 = \delta + 3\omega_R$ is the one-photon detuning. The effective Rabi frequency for the N-th-order diffraction is proportional to a power of $2N$ of the one-photon Rabi frequency $\Omega$ [42]: $\Omega_{\text{eff}}^{(2N)} \sim \Omega^{2N}/(\Delta_1\Delta_2\ldots\Delta_{2N-1})$ (where $\Delta_N$ corresponds to one-photon detunings). Note that all the above formulas for the effective Rabi frequency of multi-photon transitions are valid only for the sufficiently small one-photon Rabi frequency $\Omega$, i.e. for $\Omega \ll \Delta, \Delta_2$ to avoid the atom excitation and spontaneous losses. This may significantly limit the absolute value of $\Omega_{\text{eff}}^{(2N)}$, i.e. the momentum width (1) of the atoms involved in the multi-photon transition.

Thus, the resulting efficiency of the multi-photon transition for an atom cloud with a width in the momentum space larger than $\hbar k$ might be significantly reduced if the laser power available in the experiment is limited. Below we will estimate the laser power required for two-photon and four-photon transitions with the width $\Delta p \simeq \hbar k$ near the resonant velocity.

As an example, let us consider the ratio of the momentum widths of atoms involved in two- and four-photon transitions

$$\frac{\Delta p^{(4\hbar k)}}{\Delta p^{(2\hbar k)}} = \frac{\Omega_{\text{eff}}^{(4)}}{\Omega_{\text{eff}}^{(2)}} = \alpha^2 \frac{\Delta_2 - 2\omega_R}{16\omega_R}.$$ 

(6)

Here $\alpha = \Omega/\Delta_2 \ll 1$ is a small parameter. Figure 6 shows the evolution of the momentum distribution of $^{24}\text{Mg}$ atoms due to two-photon (a) and four-photon (b) transitions induced by a $\pi$-pulse laser field. In Figure 6, we choose a one-photon Rabi frequency $\Omega$ close to a minimum and an appropriate value of one-photon detuning $\Delta$ to have a sufficiently wide momentum width $\Delta p^{(2\hbar k)}$.
close to \( h k \) without significant excitation of the excited state \((e)\). For the parameters \( \Omega = 5 \omega_R \) being considered (the intensity of each counterpropagating light wave is \( I = 3.5 \text{ W/cm}^2 \) for the \( ^{24}\text{Mg} \) intercombination transition) and the one-photon detuning \( \Delta = -15 \omega_R \), the momentum width of the two-photon resonance \( \Delta_p^{(2h k)} \approx 0.4 \hbar k \ (\Omega_{_{\text{eff}}}^{(2)} = 0.83 \omega_R) \) (Figure 6(a)) is much larger than the momentum width of the four-photon resonance \( \Delta_p^{(4h k)} \approx 0.05 \hbar k \ (\Omega_{_{\text{eff}}}^{(4)} = 0.1 \omega_R) \) (Figure 6(b)). The \( \pi \)-pulse durations for the two- and four-photon transitions are different and correspond to \( \tau^{(2h k)} \approx 3.7 \omega_R^{-1} \approx 15 \mu s \) and \( \tau^{(4h k)} \approx 27 \omega_R^{-1} \approx 110 \mu s \) for the field parameters being considered. The parameter \( \alpha \approx 0.3 \) is not extremely small as well, which results in some excitation of the \((e)\) state (red lines in Figure 6) that was taken into account in the simulations.

Additionally, one can see that the four-photon resonance condition (4) for the optical coupling of the \((g, p)\) and \((g, p + 4 h k)\) states also satisfy the coupling condition of two-photon resonance \((g, p + h k) \rightarrow (g, p + 3 h k)\) for atoms with the momentum \( p_0 \) shifted to \( h k \), i.e. \( p_0 = p + h k \) (2). These two-photon transitions result in modulation of the momentum distribution near \( p = 4 h k \) and \( p = 6 h k \) in Figure 6(b).

In order to increase the atom fraction involved in the four-photon transition, one has to increase \( \Omega_{_{\text{eff}}}^{(4)} \) by increasing the one-photon detuning \( \Delta \) and the Rabi frequency \( \Omega \) simultaneously to avoid transition to the excited state and the subsequent spontaneous decay. Moreover, for the condition

\[
\Omega_{_{\text{eff}}}^{(2)}/ \left( 2 \Omega_{_{\text{eff}}}^{(4)} \right) = K,
\]

where \( K \) is an integer number, the use of the \( \pi \)-pulse for the four-photon transition results in suppression of the population transfer of the two-photon transition (Figure 7). In fact, in the following it becomes equivalent to the \( 2 \pi \)-pulse for this transition.

Finally, in this section we have discussed the light field parameters and the conditions for the realization of effective two-photon and four-photon Bragg transitions when the initial momentum distribution of the atomic cloud is not extremely narrow in the momentum space \( \sigma_p \geq h k \). We have made numerical analysis taking into account the excited state \((e)\). To avoid one-photon excitation the one-photon detuning should be large enough, so as for a limited power of laser sources to limit the width of multi-photon transitions in the momentum space \( \Delta p \ll h k \), and,
Figure 7. Evolution of the $^{24}$Mg momentum distribution due to the $\pi$-pulse for a four-photon transition. The initial momentum distribution of atoms (---) is a Gaussian function with $\sigma_p = \hbar k$ centered at $p_0 = 3\hbar k$, --- is the final momentum distribution of atoms in the ground state, and ---, in the excited state. The one-photon Rabi frequency $\Omega = 20\omega_R (I \approx 55 \text{ W/cm}^2)$, the detunings $\delta_1 = -48\omega_R$ and $\delta_2 = -68\omega_R$. Here $\Omega_{eff}^{(2)}/(2\Omega_{eff}^{(4)}) \approx 1$.

thus, to limit the number of atoms involved in resonance for an atomic cloud with the width in the momentum space $\sigma_p \geq \hbar k$. For the $^1S_0 \rightarrow ^3P_1$ transition in $^{24}$Mg atoms the typical light field intensity required for the one-photon and four-photon transitions of atoms to have the resonance width in the momentum space $\Delta p \approx \hbar k$ is estimated to be $I \approx 1 \text{ W/cm}^2$ for the two-photon transition and $I \approx 50 \text{ W/cm}^2$ for the four-photon transition.

4. Conclusions
In summary, we have considered frequency stabilization of a laser system by a Ramsey-Bordé atom interferometer at the $^1S_0 \rightarrow ^3P_1$ intercombination transition in ultracold $^{24}$Mg atoms localized in a MOT with a frequency stability of $3 \times 10^{-15}$ at the averaging time $\tau = 10^3$ s. Nevertheless, the stability measurement is limited by the reference frequency oscillators, and for correct measurement of stability a second Mg optical frequency standard is being developed. We plan to improve the frequency stability of the standard by implementing sub-Doppler cooling of magnesium atoms [15,18–20] and localizing Mg atoms in an optical lattice. We have analyzed the Mg atom motional quantum states manipulation in a moving optical lattice. The light field parameters for effective two-photon and four-photon transitions without one-photon excitation were estimated for the $^{24}$Mg $^1S_0 \rightarrow ^3P_1$ intercombination transition.

Acknowledgments
The research was carried out within the state assignment of the Ministry of Science and Higher Education of the Russian Federation (theme no. AAAA-A19-119102890006-5). This work was supported in part by RFBR grant 19-29-11014. The work on measuring the frequency stability using a frequency comb was supported by RFBR grant 19-02-00514. M. A. Tropnikov’s work on improving frequency stabilization system was supported by the grant of the Russian Science Foundation, RSF (project no. 17-72-20089). O. N. Prudnikov’s work on analysis of magnesium quantum state manipulation was supported by RFBR and the government of Novosibirsk Region within project 18-42-540003. The authors thank A. A. Lugovoy for assistance with measurements.

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