Estuary Classification Revisited

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Studies over a period of several decades have resulted in a relatively simple set of equations describing the tidally and width-averaged balances of momentum and salt in a rectangular estuary. We rewrite these equations in a fully non-dimensional form that yields two non-dimensional variables: (i) the estuarine Froude number; and (ii) a modified tidal Froude number. The latter is the product of the tidal Froude number and the square root of the estuarine aspect ratio. These two variables are used to define a prognostic estuary classification scheme, which compares favourably with published estuarine data.
1. Introduction

Since the introduction of stratification-circulation diagram by Hansen and Rattray (1966), numerous estuarine classification schemes have been proposed. The reader might ask - why revisit this topic? Our motivation for pursuing a new classification scheme stems from notable recent advances in estuarine physics, many of which are reviewed in MacCready and Geyer (2010). These advances led us to hypothesize that there might be a simple means to determine the conditions under which a sufficiently well behaved estuary will be well mixed, partially mixed, or highly stratified. We start by outlining the classical tidally averaged model as presented by MacCready and Geyer (2010). We then rewrite the equations of this model in non-dimensional form. Using this new set of equations we develop our classification scheme, and then compare its predictions with field observations.

2. Classical Tidally Averaged Model

The physics of estuarine circulation is governed by the competing influences of river and oceanic flows. While the former adds fresh water, the latter adds denser salt water which moves landward due to the combined effect of tides and gravitational circulation (or exchange flow). The complicated balance between the river, the exchange flow and the tides determines the estuarine velocity and salinity structure.

We consider an idealized rectangular estuary of depth $H$ and width $B$. The origin of the coordinate system is at the free surface at the mouth of the estuary with the horizontal ($x$) axis pointing seawards and the vertical ($z$) axis pointing upwards. Therefore, both the horizontal and vertical distances within the estuary are negative quantities. To obtain the width-averaged and tidally-averaged horizontal velocity ($u$), and salinity ($s$) distribution in the estuary, these quantities are first decomposed into depth averaged (overbar) and depth varying (prime) components: $u = \bar{u}(x,t) + u'(x,z,t)$, $s = \bar{s}(x,t) + s'(x,z,t)$. The quantity $\bar{u} = Q_R/A$ is the cross-sectionally averaged river velocity, where $Q_R$ is the mean river flow
rate and $A = BH$. The solution for both partial and well mixed estuaries was given by Hansen and Rattray (1965) (for recent review, see MacCready and Geyer (2010)):

$$
u = \bar{\nu} + \nu' = \bar{\nu}P_1 + \nu_E P_2$$ (1)

$$s = \bar{s} + s' = \bar{s} + \frac{H^2}{K_S} \bar{s}_x (\bar{\nu}P_3 + \nu_E P_4)$$ (2)

where

$$P_1 = \frac{3}{2} - \frac{3}{2} \xi^2$$

$$P_2 = 1 - 9 \xi^2 - 8 \xi^3$$

$$P_3 = -\frac{7}{120} + \frac{1}{4} \xi^2 - \frac{1}{8} \xi^4$$

$$P_4 = -\frac{1}{12} + \frac{1}{2} \xi^2 - \frac{3}{4} \xi^4 - \frac{2}{5} \xi^5$$ (3)

In (3), $\xi = z/H \in [-1, 0]$ is the normalized vertical coordinate. The subscript $x$ implies $\partial/\partial x$ where $x$ is dimensional. $K_S$ is the vertical eddy diffusivity. For exchange dominated estuaries, an important parameter is the exchange velocity scale:

$$u_E = c^2 H^2 \Sigma_x / (48 K_M)$$ (4)

Here $c = \sqrt{g \beta s_{ocn} H}$ is twice the speed of the fastest internal wave that can be supported in an estuary (MacCready and Geyer 2010). $K_M$ is the vertical eddy viscosity and $\beta \approx 7.7 \times 10^{-4}$ psu$^{-1}$. The non-dimensional salinity is defined as $\Sigma = s/s_{ocn}$, where $s_{ocn}$ is the ocean salinity. Equations (1)-(2) were derived under the assumption that the density field is governed by the linear equation of state: $\rho = \rho_0 (1 + \beta s)$ where $\rho_0$ is the density of fresh water. The details of the derivation are well documented in MacCready (1999, 2004).

The salt balance is given by:

$$\frac{d}{dt} \int \Sigma \, dx = -\bar{\nu} \bar{\Sigma}' + K_H \bar{\Sigma}_x - \bar{\nu} \bar{\Sigma}$$ (5)

where, $K_H$ is the horizontal diffusivity. This equation physically implies that the temporal salt accumulation in an estuary is due to the competition between salt addition and removal.
processes. While exchange (note that $u'\overline{\Sigma}'$ is negative) and tidal processes add salt, river inflow removes it. At steady state (5) can be rewritten as:

$$\overline{\Sigma}_x = (L_{E3}\overline{\Sigma}_x)^3 + (L_{E2}\overline{\Sigma}_x)^2 + L_{E1}\overline{\Sigma}_x + L_H\overline{\Sigma}_x$$

$R \quad E_3 \quad E_2 \quad E_1 \quad T$  (6)

where $L_H = K_H/\bar{u}$

$$L_{E1} = 0.019\bar{u}H^2/K_S$$

$$L_{E2} = 0.031cH^2/(K_SK_M)^{1/2}$$

$$L_{E3} = 0.024(c/\bar{u})^{1/3}cH^2/(K_SK_M^{2})^{1/3}$$  (7)

The different terms in (6) are as follows: $R$ is the river term, $T$ is the tidal term, while $E_1$, $E_2$ and $E_3$ are the different components of the exchange term. Hansen and Rattray (1965) presented (6) in a slightly different form, and MacCready (2004, 2007) introduced the length scales in (7).

The length scales in (7) depend upon the mixing co-efficients: $K_S$, $K_M$ and $K_H$. Making use of an extensive study of Willapa Bay, Banas et al. (2004) proposed:

$$K_H = a_1u_TB;$$  (8)

where $a_1 = 0.035$ and $u_T$ is the amplitude of the depth averaged tidal flow. Based on field studies and modeling of the Hudson River estuary, Ralston et al. (2008) obtained

$$K_M = a_0C_Du_TH \quad \text{and} \quad K_S = K_M/Sc;$$  (9)

where $a_0 = 0.028$, $C_D = 0.0026$ and $Sc = 2.2$ is a Schmidt number. We will use (8) and (9) in the development of a non-dimensional set of equations.

While the governing equations (1), (2) and (6) are elegant representations of the problem of estuarine circulation, they are sufficiently complicated that simplifications have been
sought after. Numerous investigators, including Hansen and Rattray (1965); MacCready (2004); MacCready and Geyer (2010) have assumed \( \bar{u} \ll u_E \), which yields:

\[
\bar{\Sigma} = \left( \frac{L_{E3}}{L_{E3}} \right)^3 + L_H \bar{\Sigma}_x
\]

\[
R \quad E_3 \quad T \quad \text{(10)}
\]

进一步简化(10)为两个简单情况，具有解析解，即交换主导的情况 \((T \to 0)\)，和潮汐主导的情况 \((E_3 \to 0)\)。虽然这些近似已被广泛使用，但似乎没有进行任何严重尝试来确定它们适用的条件。

3. Non-dimensional Tidally Averaged Model

在本节中，我们将 governing equations (1), (2) and (6) 重写为非维形式，以期揭示问题中非维数参数的最重要非维数参数，并有助于比较每项在(6)中的相对量。定义

\[
X = \frac{x}{L_{E3}}
\]

(6) 可以重写为:

\[
\bar{\Sigma} = \bar{\Sigma}_X^3 + \left( \frac{L_{E2}}{L_{E3}} \right)^2 \bar{\Sigma}_X^2 + \left( \frac{L_{E1}}{L_{E3}} \right) \bar{\Sigma}_X + \left( \frac{L_{H}}{L_{E3}} \right) \bar{\Sigma}_X
\]

\[
R \quad E_3 \quad E_2 \quad E_1 \quad T \quad \text{(11)}
\]

其中

\[
\left( \frac{L_{E2}}{L_{E3}} \right)^2 = \left( \frac{0.031}{0.024} \right)^2 Sc^{1/3} F_R^{2/3} = 2.17 F_R^{2/3}
\]

\[
\frac{L_{E1}}{L_{E3}} = \left( \frac{0.019}{0.024} \right) Sc^{2/3} F_R^{4/3} = 1.34 F_R^{4/3}
\]

\[
\frac{L_{H}}{L_{E3}} = \left( \frac{a_0 a_1 CD}{0.024} \right) Sc^{-1/3} (B/H) F_T^2 F_R^{-2/3}
\]

(12)

The velocity \( \bar{u} \) and \( u_T \) have been non-dimensionalized by \( c \) to obtain the densimetric estuarine Froude number \( F_R = \bar{u}/c \) and the tidal Froude number \( F_T = u_T/c \)。Substituting (12) into (11) yields:

\[
\bar{\Sigma} = \bar{\Sigma}_X^3 + C_1 F_R^{2/3} \bar{\Sigma}_X^2 + C_2 F_R^{4/3} \bar{\Sigma}_X + C_3 F_T^2 F_R^{-2/3} \bar{\Sigma}_X
\]

\[
R \quad E_3 \quad E_2 \quad E_1 \quad T \quad \text{(13)}
\]
where \( C_1 = 2.17 \), \( C_2 = 1.34 \), \( C_3 = 8.16 \times 10^{-5} \), and the modified tidal Froude number, 
\[ \tilde{F}_T = F_T \sqrt{B/H} \]. Typically the estuarine aspect ratio \( B/H \sim O(10^2 - 10^3) \), see Table 1. The magnitude of different terms in (13) can be easily compared by noting that 
\[ 0 \leq O \left( F_r^{4/3} \right) < O \left( F_r^{2/3} \right) < O(1) < O \left( F_r^{-2/3} \right) \]. The tidal term (T) however depends on an additional parameter \( \tilde{F}_T \), whose (order of) magnitude needs to be known for making the comparison.

Like the salt balance equation, the momentum and salinity equations, i.e. (1) and (2) can also be expressed in non-dimensional form as follows:

\[
U = C_4 F_R^{1/3} \Sigma_X P_2 + F_R P_1 \tag{14}
\]

\[
\Sigma = \Sigma + C_5 F_R^{2/3} \Sigma_X^2 P_4 + C_6 F_R^{4/3} \Sigma_X P_3 \tag{15}
\]

The constants \( C_4 = 0.667 \), \( C_5 = 47.0 \) and \( C_6 = 70.5 \). In (14), the quantity \( U = u/c \) is the non-dimensional horizontal velocity (not to be confused with \( F_r \), which is \( \bar{u}/c \)). Equations (13)-(15) are the non-dimensional governing equations for our idealized estuary.

Eq. (13) poses a non-linear initial value problem which can only be solved numerically. For that, the conditions at the estuary mouth have to be determined. One such condition is \( \Sigma(0, -1) = 1 \); meaning the salinity at the bed of the estuary at its mouth has to be the same as the ocean salinity. Substituting (15) into (13) and making use of this condition, we obtain

\[
\left( \Sigma_X |_0 \right)^3 + C_7 F_R^{2/3} \left( \Sigma_X |_0 \right)^2 + \left( C_8 F_R^{4/3} + C_3 \tilde{F}_T^{-2} \right) \Sigma_X |_0 = 1; \tag{16}
\]

where \( C_7 = 5.31 \) and \( C_8 = 6.04 \). Eq. (16) is actually the non-dimensional version of Eq. (19) of MacCready (2004). Being a cubic equation, it can be solved analytically to evaluate the salinity gradient at the estuary mouth, \( \Sigma_X |_0 \). Additionally, (16) indicates that \( \Sigma_X |_0 \) is only a function of \( F_R \) and \( \tilde{F}_T \). The variation of \( \Sigma_X |_0 \) with these two Froude numbers is depicted in Fig. 1. The figure shows that \( 0 < \Sigma_X |_0 < 1 \) over the entire parameter space.
4. **Estuary Classification**

Our goal is to develop a simple classification scheme that distinguishes between well-mixed, partially mixed and highly stratified estuaries. A relevant parameter for classifying estuaries is the non-dimensional salinity stratification at the estuary mouth, \( \Phi_0 \). It is defined as follows:

\[
\Phi_0 = \Sigma (0, -1) - \Sigma (0, 0)
\]  

This parameter ranges between 0 and 1. While the lower limit implies a very well mixed estuary, the upper limit indicates the transition to salt wedge. Substituting (15) into (17) yields:

\[
\Phi_0 = C_9 F_R^{2/3} (\bar{\Sigma}_X|_0)^2 + C_{10} F_R^{4/3} \bar{\Sigma}_X|_0; 
\]  

where \( C_9 = 7.06 \) and \( C_{10} = 8.82 \). If \( F_R \) and \( \widetilde{F}_T \) are known, then \( \bar{\Sigma}_X|_0 \) can be directly obtained by solving (16). Consequently, \( \Phi_0 \) can be evaluated from (18), yielding Fig. 2.

We follow Hansen and Rattray (1966) and use the condition \( \Phi_0 = 0.1 \) to define the transition between well mixed and partially mixed estuaries. To distinguish between partially mixed and highly stratified estuaries we use the condition \( \Phi_0 = 1.0 \), corresponding to fresh surface water extending to the mouth of the estuary. Our classification scheme is obtained by plotting these transitional criteria on Fig. 2.

When \( \widetilde{F}_T = 0 \), the transition between well-mixed and partially-mixed estuaries is predicted to occur at \( F_R = 0.0017 \), and from partially-mixed to highly stratified at \( F_R = 0.113 \). The value of \( F_R \) for both transitions increases as \( \widetilde{F}_T \) increases, the increase being more rapid for the transition from partially-mixed to highly stratified estuaries. These results are in qualitative agreement with Fig. 2.7 of Geyer (2010).
5. Discussion

Together (16) and (18) provide new insight into estuarine physics. Apart from broadly classifying estuaries into three categories, viz. highly stratified, partially mixed and well mixed, the equation set identifies $F_R$ and $\tilde{F}_T$ to be the only two parameters determining the stratification at the estuary mouth, $\Phi_0$. The new non-dimensional parameter $\tilde{F}_T = F_T \sqrt{B/H}$ reveals that “tidal effect” is not simply represented by the tidal Froude number $F_T$, but the latter combined with the square-root of the estuarine aspect ratio $B/H$. Moreover the equation set predicts $\Phi_0$, given $F_R$ and $\tilde{F}_T$. If estuarine condition changes, e.g. river flow changes from low to high, tidal flow changes from spring to neap, or estuary depth changes due to dredging, the parameters $F_R$ and $\tilde{F}_T$ will change correspondingly. These newly obtained Froude numbers will produce a new $\Phi_0$, which reflects the response of estuarine circulation and mixing to variability.

To test the applicability of our classification scheme we made use of the field data presented in [Prandle (1985)]. Using these data we have computed $F_R$, $F_T$, $B/H$, and $\tilde{F}_T$ directly, and $\Phi_0$ from (16) and (18); see Table 1. We have compared the computed value of $\Phi_0$ with the measured value in Fig. 3. The comparison is good considering the accuracy to which $F_R$ and $\tilde{F}_T$ can be determined from field data.

It is interesting to note that if both $F_R$ and $\tilde{F}_T$ are small then (16) reduces to $\Sigma X\mid_0 = 1$ and (18) reduces to:

$$\Phi_0 \approx 7F_R^{2/3};$$

which is exactly the same as Eq. (19) of [MacCready and Geyer (2010)]. They have found this equation by combining Knudsen’s relations [Knudsen (1900)] with [9]. Since Knudsen’s relations are derived from mass and salt balances and do not consider momentum balance, (19) provides a rather simplistic prediction. We compare (19) with the exact solution of (18) for both $\tilde{F}_T = 0$ and $\tilde{F}_T = 30$ in Fig. 4. When $\tilde{F}_T = 0$, (19) is very accurate up to the transition between well-mixed and partially mixed estuaries ($\Phi_0 = 0.1$). The deviation
between \(19\) and the exact solution increases with increasing \(F_R\). Eq. \(19\) predicts the transition between partially mixed and highly stratified estuaries \((\Phi_0 = 1.0)\) at \(F_R = 0.054\) rather than at \(F_R = 0.113\). When \(\tilde{F}_T = 30\), \(19\) is always less than the exact solution in terms of \(\Phi_0\).

We also compare the theoretical results with the field data of Prandle \(1985\) in Table 1 and Fig. 2.6 of Geyer \(2010\) in Fig. 4. This comparison is mainly intended to provide a qualitative estimate. We have chosen to plot \(18\) for \(\tilde{F}_T = 0\) and 30, since estuaries mostly have \(\tilde{F}_T\) within this range. Ideally, most of the partially and well mixed estuaries should cluster within the grey region bounded by the lines \(\tilde{F}_T = 0\) and 30, which is indeed the case. The most important aspect of this comparison is that the theoretical curves follow the overall trend of the field data. However these curves grossly over-predict \(\Phi_0\), therefore they under-predict vertical mixing. This discrepancy may arise if the values of \(\Phi_0\) were measured at an upstream location, rather than at the mouth (which might be the case for the data points of Geyer \(2010\)). Surprisingly, disagreement between theory and field data does not appear in Fig. 2.6 of Geyer \(2010\). The latter figure shows that the line, referred to as “Eq. (2.22)”, matches very well with the data points. Although Eq. (2.22) is actually \(\Phi_0 = 8.73F_R^{2/3}\) (we calculated the coefficient from the associated text in Geyer \(2010\)), it is mistakenly plotted as \(\Phi_0 \approx 3F_R^{2/3}\).

Finally we refer to the assumptions behind our theoretical analyses and their consequences. We have simplified the problem by assuming a tidally averaged estuary with rectangular geometry. In real estuaries bathymetry can play a crucial role in determining the estuarine circulation. Moreover the appearance of just two parameters \((F_R\) and \(\tilde{F}_T\)) in our equations is a consequence of the empirical equations \((8)\) and \((9)\). These two equations are also used in determining the coefficients \(C_1, C_2, \ldots, C_{10}\). All these coefficients are found to depend upon \(Sc\), making it the most important parameter in this regard; see Table 2. Following Ralston et al. \(2008\), \(Sc = 2.2\) in all our calculations. Although \((8)\) and \((9)\) are simple and elegant, they may not be very realistic. In real estuaries both \(K_M\) and \(K_S\)
are variables. Moreover, other empirical parameterizations have shown that $K_M$ depends upon Richardson number (MacCready and Geyer 2010). While the inclusion of any relevant third parameter might improve the predictability of the classification scheme, the value of this improvement would have to be weighed against the added complexity of the resulting classification scheme.

6. Conclusions

The equations governing the physics of estuarine circulation have been presented in non-dimensional form. The two resulting non-dimensional parameters are the estuarine Froude number, $F_R$, and the modified tidal Froude number, $\bar{F}_T$. Given these parameters the non-dimensional salinity gradient at the estuary mouth, $\bar{\Sigma}_X|_0$, and the non-dimensional salinity stratification (also at the estuary mouth), $\Phi_0$, can be computed. The latter result forms the basis of a classification scheme that can be used to predict whether an estuary is fully or partially mixed, or highly stratified. The predictions of this classification scheme compare well with estuarine data.
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2. List of coefficients used in different equations.
Table 1. Estimates of estuarine parameters calculated using the data of Prandle (1985) and values of $B$ obtained from maps. Eq. (18) is used to obtain $\Phi_0$-theory.

| Estuary Name                | $F_R$ | $F_T$ | $B/H$ | $F_H$ | $\Phi_0$ | $\Phi_0$-theory |
|-----------------------------|-------|-------|-------|-------|----------|----------------|
| Vellar                      | 1.27  | 0.64  | 200   | 9.0   | 1.00     | 1.00          |
| Columbia                    | 0.026 | 0.43  | 150   | 5.3   | 0.40     | 0.50          |
| James                       | 0.004 | 0.25  | 360   | 4.7   | 0.22     | 0.17          |
| Tees                        | 0.014 | 1.03  | 75    | 8.9   | 0.18     | 0.33          |
| Southampton Waterway        | 0.0012| 0.37  | 200   | 5.2   | 0.10     | 0.06          |
| Tay                         | 0.014 | 1.38  | 400   | 27    | 0.10     | 0.17          |
| Narrows of the Mersey       | 0.0009| 0.83  | 65    | 6.7   | 0.05     | 0.05          |
| Bristol Channel             | 0.006 | 1.59  | 300   | 27    | 0.02     | 0.06          |
Table 2. List of coefficients used in different equations.

| Coefficient | Value                      |
|-------------|----------------------------|
| $C_1$       | $1.67Sc^{1/3}$             |
| $C_2$       | $0.792Sc^{2/3}$            |
| $C_3$       | $41.7a_0a_1CDSc^{-1/3}$    |
| $C_4$       | $0.868Sc^{-1/3}$           |
| $C_5$       | $36.2Sc^{1/3}$             |
| $C_6$       | $41.7Sc^{2/3}$             |
| $C_7$       | $4.08Sc^{1/3}$             |
| $C_8$       | $3.57Sc^{2/3}$             |
| $C_9$       | $5.43Sc^{1/3}$             |
| $C_{10}$    | $5.21Sc^{2/3}$             |
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1. The variation of salinity gradient at the estuary mouth ($\bar{\Sigma}_X|_0$) with the estuary control variables - $F_R$ and $\widetilde{F}_T$. The solid lines represent isocontours of $\bar{\Sigma}_X|_0$. [17]

2. Estuary Classification Diagram. The lines represent isocontours of $\Phi_0$. The three regions represent three types of estuaries: (a) light grey - well mixed, (b) white - partially mixed, and (c) dark grey - highly stratified or salt wedge. The letters denote estuaries: C - Columbia, J - James, Te - Tees, SW - Southampton Waterway, Ta - Tay, NM - Narrows of the Mersey and B - Bristol Channel. For data, see Table 1. [18]

3. Comparison between stratification at the estuary mouth obtained from theory with field data. [19]

4. Comparison between our estuary classification scheme and the approximation $\Phi_0 = 7F_R^{2/3}$ in (19). The grey area indicates the region where estuaries should ideally cluster. Field data from Geyer (2010) and Prandle (1985) are plotted for comparison with the theoretical predictions. [20]
Fig. 1. The variation of salinity gradient at the estuary mouth ($\Sigma X|_0$) with the estuary control variables - $F_R$ and $\tilde{F}_T$. The solid lines represent isocontours of $\Sigma X|_0$. 

\[ F_T \sqrt{B/H} \]

\[ F_R \]

- $0.1$  
- $0.3$  
- $0.5$  
- $0.7$  
- $0.9$  

- $0$  
- $10^{-3}$  
- $10^{-2}$  
- $10^{-1}$  

- $0$  
- $10$  
- $20$  
- $30$  

- $0.3$  
- $0.5$  
- $0.7$  
- $0.9$  


Fig. 2. Estuary Classification Diagram. The lines represent isocontours of $\Phi_0$. The three regions represent three types of estuaries: (a) light Grey- well mixed, (b) White - partially mixed, and (c) dark Grey- highly stratified or salt wedge. The letters denote estuaries: C - Columbia, J - James, Te - Tees, SW - Southampton Waterway, Ta - Tay, NM - Narrows of the Mersey and B - Bristol Channel. For data, see Table 1.
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