Determination of oscillation frequencies and stellar properties of three Delta Scuti variable stars using Kepler data

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Abstract: Asteroseismology is the science that deals with the oscillations in the stars with the goal of understanding the stellar structure. In this study, photometric data of Kepler mission have been used for the asteroseismic study of Delta Scuti type pulsating stars. The light curve analysis of three short cadences Kepler datasets of Delta Scuti stars has been presented. The pulsation frequencies were determined by applying a discrete Fourier transformation with the pre-whitening process. The full light curve of Kepler target KIC 9700322 revealed dominant radial fundamental frequency at 9.792 ± 0.002 d⁻¹ and 1st overtone frequency at 12.569 ± 0.002 d⁻¹ accompanied by a large number of frequency multiples. Non-radial frequencies of mode \( l = 2 \) quintuplet were detected with the frequency spacing of 0.134 d⁻¹ and it provided a rotation period of the star as 6.19 days, which was detected as the peak of 0.159 d⁻¹ in frequency spectra. Using the fundamental period, a value for relative density of the star was estimated as 0.10 ± 0.04 relative to the sun. Furthermore, the full dataset of KIC 11754974 was subjected to frequency analysis and the radial fundamental pulsation frequency and its 1st overtone were detected at 16.3451 ± 0.0002 d⁻¹ and 21.3997 ± 0.0002 d⁻¹, respectively. \( l = 2 \) frequency quintuplet was detected with a frequency spacing of 0.22 d⁻¹ and determined the rotation period and relative density of the star as 3.83 days and 0.29 ± 0.01, respectively. KIC 9845907 was identified as a purely radial pulsator. The fundamental pulsation frequency was recovered as 17.597 ± 0.002 d⁻¹ with two other dominant modes of overtones at 22.850 ± 0.002 d⁻¹ and 28.210 ± 0.002 d⁻¹. The relative density of the star was determined as 0.34 ± 0.06.

Keywords: Delta Scuti, oscillation modes, photometry, stellar pulsations.

INTRODUCTION

Astrophysics is the science that deals with understanding the physical and chemical phenomena related to celestial objects such as stars, galaxies and other celestial objects. Asteroseismology is an area in astrophysics devoted to the study of stellar oscillations, which probes the extreme status of temperature, density and pressure in stellar interiors (Guzik, 1997). Delta Scuti are a class of intrinsic variable stars, which consists of pure radial or multiple non-radial pulsation modes (Breger, 1976; Breger, 2020; Pakstiene et al., 2018).

Stellar oscillations mainly depend on the resonant frequencies of standing waves of sound inside a star. The speed of sound depends on the temperature and chemical composition of the gas. By ‘listening’ to these sounds intrinsic stellar properties such as temperature, gravity, chemical composition etc, can be determined. Asteroseismology is equivalent to identifying musical notes by listening to the audio response. This field is developed with the advent of helioseismology, which studies the internal structure of the sun through the observed oscillations. Most importantly, the solar interior structure was determined by observing oscillation modes using photometry (Buldgen et al., 2019).

The field of asteroseismology has been in rapid development because of the space telescopes, such as
COROT and Kepler. Unlike ground-based observatories affected by telluric turbulence and weather, space-based observatories can receive high quality, photometrically stable and temporally reliable data. Asteroseismic investigations are not limited to only solar-like stars but it could also be applied to many different types of stars.

In this investigation, asteroseismic techniques were applied to δ Scuti type variable stars. δ Scuti stars (DSS) are located where the classical instability strip crosses the main sequence of the Hertzsprung-Russell (H-R) diagram. They are very short period pulsators; typically the pulsation period is less than one day (Breger, 2000). The radial pulsations are more general in DSS while non-radial oscillations are also present. In brief, non-radial pulsation modes can be described by three quantised numbers $n$, $l$ and $m$, where $n$ is the radial nodes between centre and surface, $l$ is the number of nodes on the surface, and $m$ represents the surface nodes which pass through the pulsation axis (Aerts et al., 2010). DSS have been discovered with short (periods of a few minutes) and long (periods of an hour or two) periods associated with pressure (p-modes) and gravity (g-modes) modes, respectively. Gravity mode pulsations are deep interior modes, which exhibit asymptotic period spacing for radial nodes with $l \ll n$ (Reed et al., 2011).

DSS are at the stellar evolutionary stage of core or shell hydrogen burning. Therefore, DSS are in the transition phase of convective to radiative energy transport and assists in understanding the physics of the transition phase of stars (Aerts et al., 2010).

DSS were selected from the database of Kepler Asteroseismic Science Operations Center (KASOC). KASOC provides asteroseismological data from the Kepler mission to astronomers who are members of the Kepler Asteroseismic Science Consortium (KASC). Three Delta Scuti stars, KIC 9700322, KIC 11754974 and KIC 9845907 (KIC = Kepler Input Catalogue, Brown et al., 2011) observed by Kepler mission were selected for this analysis. KIC 9700322, KIC 11754974 and KIC 9845907 stars were mainly selected based on the presence of many frequencies in their power spectra as well as complex frequency combinations and spacing. The frequency spacing was explained as simple combinations of frequencies arising from non-linearity of the oscillation and hence provides information of physical properties of the stars. The observational details of these stars are listed in Table 1. These targets were selected based on higher coverage of observation and the availability of oscillation frequencies in their light curves.

| Star          | R.A.     | Dec.     | $K_p$ | Kepler quarters | Data points |
|---------------|----------|----------|-------|----------------|-------------|
| KIC 9700322   | 19h 07m 50.7s | 46° 29 12 | 12.685 | Q11.1          | 45184       |
|               |          |          |       | Q11.2          | 43543       |
|               |          |          |       | Q11.3          | 46311       |
|               |          |          |       | Q12.1          | 34037       |
|               |          |          |       | Q12.2          | 39984       |
|               |          |          |       | Q12.3          | 32946       |
| KIC 11754974  | 19h 08m 15.9s | 49° 57 15 | 12.678 | Q6.1           | 39630       |
|               |          |          |       | Q6.2           | 45255       |
|               |          |          |       | Q6.3           | 43996       |
|               |          |          |       | Q7.1           | 44036       |
| KIC 9845907   | 19h 49m 30.4s | 46° 40 02 | 11.640 | Q8.1           | 45184       |
|               |          |          |       | Q8.2           | 38648       |
|               |          |          |       | Q8.3           | 29390       |
|               |          |          |       | Q9.1           | 53282       |
|               |          |          |       | Q9.2           | 41711       |
|               |          |          |       | Q9.3           | 45616       |

Table 1: Summary of observational parameters of selected δ Scuti stars. $K_p$ is the Kepler magnitude.
Oscillation frequencies of Delta Scuti stars

Asteroseismology of a star highly depends on the precise determination of oscillation frequencies among artifacts generated by different sources such as frequency aliasing in ground-based data and periodic changes of the space observatories such as reorientations, safe-modes, etc (Garcia et al., 2011). Signal-to-Noise (S/N) ratio of the frequency spectrum is another major issue of demarcation of stellar frequencies.

METHODOLOGY

Kepler light curves

The Kepler space telescope was launched in 2009 to search Earth-like planets. The period of Earth trailing orbit of the satellite is 372.5 days. The observations were focused on 115 square degrees field of view in the constellation of Cygnus and Lira (Garcia et al., 2011). The stars were observed in Kepler magnitude (Kp) in the wavelength range of 430 – 890 nm. The Kp is defined as AB magnitudes (Oke, 1974, Smith et al., 2002) derived from each target’s calibrated g, r, i magnitudes. Details of the Kp is available in Brown et al. (2011). Kepler observations are made in two different operating modes: Long-Cadence (LC) targets are sampled every 29.5 min (Nyquist frequency of 283.45 µHz) and Short-Cadence (SC) every 58.5 s of exposure (Nyquist frequency of ~8.5 mHz) (Garcia et al., 2011). SC light curves of DSS were used allowing the high resolution of power spectra which can resolve frequencies up to 100 cycles per day (1.16 µHz). We used the corrected flux by the KASOC database and performed further corrections of eliminating photometric jumps and outliers, and linear trends for every quarter in SC data. The full light curve of quarter 11 and 12 of KIC9700322 is shown in Figure 1. The time stamp of the light curves is in Barycentric Julian Date (BJD). The mean flux levels of observed stars; specifically, the third or fourth order polynomials that best represented the baseline of the shifted quarters. Polynomials were tested with order in the range 2–10 that best represented the baseline of the shifted quarters. It was found that the third or fourth order polynomials sufficiently fit the overall baseline and provide a better fit than a linear baseline. A part of a light curve after the conversion to ppm is shown in Figure 2. This conversion process of flux improves the S/N ratio of the power spectrum and hence possible to determine the weak frequency signals.

Frequency analysis

The amplitude spectrum of each star as a function of time, t, was expressed using a Fourier representation based on the harmonic term sum in equation 2.

\[ m = m_0 + \sum_{i=1}^{N} A_i \sin[2\pi(f_it + \phi_i)] \]  

where \( m_0 \) is the zero-point and \( A_i, f_i \) and \( \phi_i \) are the amplitude, frequency and phase of the \( i^{th} \) term, respectively.

Period04 (Lenz & Breger, 2005) software package was used to compute the discrete Fourier transformation (DFT) power spectrum of the light curves as shown in Figures 3, 4 and 5 respectively. Period04 detects dominant frequencies in the amplitude spectra and calculates the amplitude and the phase of the detected frequencies. The frequencies were searched with the step size of 0.0098 µHz and pre-whitening process applied, which subtracts the detected frequencies from the light curve before searching the next frequency. The ‘a’, ‘b’, ‘c’ and ‘d’ subplots in Figures 3, 4 and 5 consecutively represent the frequencies determined by the pre-whitening process after removing the previous strongest peak. The pre-whitening process recovers frequencies from the residual light curve. The process of pre-whitening is very useful to extract the frequencies of the star from the amplitude spectrum, and particularly the unwanted frequency peaks can be avoided, which otherwise arise due to aliases.
Figure 1: Upper: Kepler flux of KIC9700322 for the quarters of 11 and 12. The blue and red lines indicate the linear best fits of the quarter 11 and 12, respectively. Lower: The two quarters shifted to the median of each dataset. The red line indicates the best fit polynomial used for the conversion of ppm.

Figure 2: Part of the light curve of KIC 9700322 after converting to ppm
The amplitude detection threshold of the frequency spectrum was calculated using the region of free form pulsations. When the S/N ratio is less than 4 magnitudes (Baran et al., 2012), the signal is no longer considered statistically significant. This criterion was followed to terminate the frequency detection process so that the successive pre-whitening stops when the peak of the highest-amplitude in the periodogram has a value, which is considered too small to be significant.

**Frequency combinations and the equidistant frequency patterns**

Most of the overtones and linear combinations of the frequencies in the power spectrum can be expressed in a very simple way through a general equation \( f = mf_1 \pm nf_2 \) where \( m \) and \( n \) are small integers and \( f_1 \) and \( f_2 \) are two frequencies with the highest amplitudes in the power spectrum. These frequencies are called dominant.

**Table 2:** Frequencies of KIC 9700322 given in cycles per day and \( \mu \)Hz. Amplitudes are in parts per million (ppm). The major modes are also identified.

| Frequency (cycles/day) ±0.002 | Frequency (\( \mu \)Hz) | Amplitude (ppm) | Identification | Comment       |
|-----------------------------|------------------------|-----------------|----------------|---------------|
| Main frequencies            |                        |                 |                |               |
| 9.792                       | 113.34                 | 1227651.58      | \( f_1 \)     | Fundamental   |
| 12.569                      | 145.48                 | 1319764.29      | \( f_2 \)     | 1st Overtone  |
| 0.159                       | 1.85                   | 4583.00         | \( f_3 \)     | Rotation      |
| 0.0121                      | 0.14                   | 46826.81        | \( f_4 \)     |               |
| 11.316                      | 131.02                 | 6348.07         | \( f_5 \)     | Quintuplet    |
| 11.455                      | 132.58                 | 5775.80         | \( f_6 \)     | Quintuplet    |
| 11.589                      | 134.14                 | 22978.14        | \( f_7 \)     | Quintuplet    |
| 11.719                      | 135.64                 | 10042.14        | \( f_8 \)     | Quintuplet    |
| 11.858                      | 137.25                 | 5230.76         | \( f_9 \)     | Quintuplet    |
| Frequency combinations      |                        |                 |                |               |
| 0.026                       | 0.31                   | 11464.82        | \( 2f_4 \)    |               |
| 2.775                       | 32.13                  | 122446.58       | \( f_2-f_1 \) |               |
| 5.550                       | 64.24                  | 5982.86         | \( 2f_2-2f_1 \)|               |
| 7.016                       | 81.21                  | 28803.27        | \( 2f_1-f_2 \)|               |
| 9.781                       | 113.21                 | 13594.57        | \( f_1+f_2-f_3 \)|               |
| 9.792                       | 113.48                 | 8693.03         | \( f_1+f_2-f_3+2f_4 \)|               |
| 12.545                      | 145.19                 | 96857.34        | \( 2f_2-2f_4 \)|               |
| 12.569                      | 145.35                 | 67945.97        | \( f_2-f_4 \) |               |
| 12.582                      | 145.63                 | 14856.26        | \( f_2+f_4 \) |               |
| 15.345                      | 177.61                 | 23618.59        | \( 2f_1-f_2 \) |               |
| 19.585                      | 226.68                 | 102097.16       | \( 2f_1 \)   |               |
| 22.361                      | 258.81                 | 225373.91       | \( f_1+f_2 \) |               |
| 25.137                      | 290.94                 | 126096.23       | \( 2f_2 \)   |               |
| 27.914                      | 323.08                 | 9578.32         | \( 3f_2-f_1 \) |               |
| 29.377                      | 340.01                 | 5754.46         | \( 3f_2 \)   |               |
| 32.153                      | 372.15                 | 23401.58        | \( 2f_1+f_2 \) |               |
| 34.929                      | 404.28                 | 24567.54        | \( f_1+2f_2 \) |               |
| 37.705                      | 436.41                 | 14988.67        | \( 3f_2 \)   |               |
| 44.723                      | 517.63                 | 6819.39         | \( 2f_1+2f_2 \) |               |
| 62.541                      | 723.86                 | 5842.42         | \( 5f_2 \)   |               |
frequencies. Additional patterns are also identified using the rotational frequency of the star. Most of the detected frequencies can be identified as part of regular patterns. By using this methodology, the dominant frequencies in the data were identified and are given in Tables 2, 3 and 4, respectively for KIC 9700322, KIC 11754974 and KIC 9845907.

### Table 3: Frequencies of KIC 11754974 given in cycles per day and µHz. Amplitudes are in parts per million (ppm). The major modes are also identified.

| Frequency (cycles/day) ± 0.0002 | Frequency (µHz) | Amplitude (ppm) | Identification | Comment |
|---------------------------------|-----------------|-----------------|----------------|---------|
| **Main frequencies**            |                 |                 |                |         |
| 19.7839                         | 228.98          | 85.08           | f6             | Quintuplet |
| 20.0194                         | 231.71          | 102.61          | f7             | Quintuplet |
| 20.2437                         | 234.30          | 125.27          | f8             | Quintuplet |
| 20.4562                         | 236.76          | 68.22           | f9             | Quintuplet |
| 20.6560                         | 239.07          | 51.71           | f10            | Quintuplet |
| **Frequency combinations**      |                 |                 |                |         |
| 0.4921                          | 5.70            | 102.97          | f2-f3          |         |
| 1.2605                          | 14.59           | 86.04           | f5-f2          |         |
| 4.5624                          | 52.81           | 559.64          | f3-f1          |         |
| 4.5990                          | 53.23           | 271.40          | f4-f1          |         |
| 5.0546                          | 58.50           | 532.20          | f2-f1          |         |
| 6.3159                          | 73.10           | 90.36           | f5-f1          |         |
| 11.2905                         | 130.68          | 251.95          | 2f1-f2        |         |
| 11.7461                         | 135.95          | 69.57           | 2f1-f4        |         |
| 11.7826                         | 136.37          | 224.09          | 2f1-f3        |         |
| 15.8530                         | 183.48          | 54.36           | f1-f3-f2      |         |
| 26.4542                         | 306.18          | 60.32           | 2f2-f1        |         |
| 27.6356                         | 319.86          | 56.36           | 3f1-f2        |         |
| 28.1277                         | 325.55          | 53.22           | 3f1 - f3      |         |
| 32.6902                         | 378.36          | 1505.81         | 2f1           |         |
| 37.2518                         | 431.16          | 467.73          | f1 + f3       |         |
| 37.2900                         | 431.60          | 391.97          | f1 + f4       |         |
| 37.7447                         | 436.86          | 695.09          | f1 + f2       |         |
| 39.0061                         | 451.46          | 63.36           | f1 + f5       |         |
| 41.8151                         | 483.97          | 58.96           | 2f3           |         |
| 41.8533                         | 484.41          | 92.22           | f4 + f3       |         |
| 42.3072                         | 489.67          | 137.90          | f2 + f3       |         |
| 42.7993                         | 495.36          | 102.06          | 2f2           |         |
| 49.0353                         | 567.54          | 329.11          | 3f1           |         |
| 53.5969                         | 620.34          | 149.79          | 2f1 + f3      |         |
| 53.6351                         | 620.78          | 96.23           | 2f1 + f4      |         |
| 54.0898                         | 626.04          | 158.53          | 2f1 + f2      |         |
| 59.1444                         | 684.54          | 83.70           | 2f2 + f1      |         |
| 65.3804                         | 756.72          | 74.84           | 4f1           |         |
Table 4: Frequencies of KIC 9845907 given in cycles per day and µHz. Amplitudes are in parts per million (ppm). The major modes are also identified.

| Frequency (cycles/day) | Frequency (µHz) | Amplitude (ppm) | Identification | Comment |
|------------------------|-----------------|-----------------|----------------|---------|
| 17.597 | 203.67 | 10496.36 | f1 | Fundamental |
| 22.850 | 264.51 | 2636.76 | f2 | 1st Overtone |
| 28.210 | 326.56 | 530.07 | f3 | 2nd Overtone |
| 0.065 | 0.75 | 450.92 | f4 | |
| 13.831 | 160.08 | 195.37 | f5 | |
| 27.010 | 312.62 | 685.87 | f6 | |

Frequency combinations

| 1.693 | 19.60 | 151.43 | f2-f1 |
| 13.898 | 160.85 | 136.75 | f3+f4 |
| 26.880 | 311.11 | 192.45 | f6-2f4 |
| 26.945 | 311.86 | 156.26 | f6-f4 |
| 29.736 | 344.17 | 448.16 | 2f5 -f6 + f4 |
| 29.797 | 344.88 | 163.88 | 2f5 -f6 + 2f4 |
| 31.363 | 363.00 | 1181.84 | f1+f5 -f4 |
| 31.428 | 363.75 | 2209.07 | f1 + f5 |
| 31.493 | 364.51 | 1993.26 | f1 + f5 + f4 |
| 35.195 | 407.35 | 572.34 | 2f1 |
| 49.025 | 567.42 | 176.29 | f1 + f2 |
| 49.090 | 568.17 | 125.97 | f1 + f2 + f4 |
| 59.471 | 688.32 | 393.34 | 4f5 -2f6 +2f4 |
| 61.164 | 707.92 | 324.04 | 2f1 + f2 - 2f4 |
| 62.856 | 727.50 | 315.41 | 2f1 + 2f5 |
| 77.070 | 892.00 | 163.59 | |

The next step is the identification of radial modes and non-radial modes. In radial pulsating stars the period ratios of the modes are good approximations of mode identification. For DSS, the theoretically expected period ratio of fundamental radial (P₁) and first radial overtone (P₂/P₁) pulsation is 0.756 – 0.789 (Stellingwerf, 1979). The ratio of second overtone period (P₃) to the fundamental (P₃/P₁) is 0.611 – 0.632 (Stellingwerf, 1979). However, the period ratios also depend on metallicity, stellar rotation period, stellar mass and density (Mirosh et al., 2019). The higher metallicity and higher rotational velocity increase the period ratios around 10⁻³ (Breger, 2000; Paparo et al., 2017). By using the period ratios, the radial modes of fundamental, first harmonic and second harmonic were identified, which fall well within the above mentioned ranges and shown in Tables 2, 3 and 4. The frequency multiplets can be used to identify the degree, l, as 2l + 1 produce azimuthal modes m, so that l = 1 produces triplets with -1, 0 and +1, l = 2 produces quintuplets of -2, -1, 0, +1, and +2. These frequency separations depend on the value of degree l. Therefore, evenly spaced peaks in frequency may help to identify l values (Paparo et al., 2013). In order to recover the frequency separations all detected frequencies were arranged in ascending order for the three targets. The identified equidistant frequency patterns for KIC 9700322 and KIC 11754974 are given in Tables 2 and 3, respectively.
Determination of stellar properties

The stellar rotation period \( P_{\text{rot}} \) of a star can be written as,

\[
P_{\text{rot}} = \frac{1 - C_n l}{\nu_{n,l,m} - \nu_{n,l,0}} \quad \text{(3)}
\]

Where \( \nu_{n,l,m} \) is the observed frequency and \( \nu_{n,l,0} \) is the unperturbed central frequency of the multiplet for \( m = 0 \), which is unaffected by the rotation. \( C_{n,l} \) expresses the Ledoux constant (Ledoux, 1951) which determines the usual equidistance splitting valid in the limit of slow rotation (Goupil et al., 2000) and \( C_{n,l} = \frac{2}{l(l+1)} \) for g-modes and negligibly small for p-modes. By using this equation the stellar rotation period was determined.

By knowing the splitting, \( m \), and the Ledoux constant, it is possible to calculate the stellar rotation period. The frequency splitting \( \Delta \nu_{n,l,m} \) is the difference of \( \nu_{n,l,m} \) and \( \nu_{n,l,0} \). The frequency splitting observed from spectra for KIC 9700322 is 0.134 \( \text{d}^{-1} \) for \( l = 2 \) quintuplet and for KIC 11754974 it is 0.218 \( \text{d}^{-1} \) for the same degree of \( l = 2 \). The Ledoux constant \( C_{n,l} \) for \( l = 2 \) quintuplet is 0.164 (Kurtz, 2014). By substituting these values in the above equation, the value of rotation period \( (P_{\text{rot}}) \) was calculated.

Furthermore, the pulsation constant \( Q \) is defined as;

\[
Q = \frac{P_1}{\sqrt{\rho}} \quad \text{(4)}
\]

\( P_1 \) is the fundamental pulsation period in days, and \( \rho \) is the mean stellar density, which is normalised to the solar value. Substitution of \( Q \) for the radial fundamental mode, 0.033 \( \text{d} \) (Breger & Bregman, 1975; Balona et al., 2012) with the determined fundamental periods in equation 4 provides the relative mean densities of three DSS. The error estimation of the computation was taken from the propagation error of the frequency. The frequency error was taken as the standard deviation of hundred iterations of Monte Carlo simulation in Period04.

Figure 3: Amplitude spectra of KIC 9700322. The oscillation frequencies are seen as sharp peaks. The panels a, b, c and d show the remaining frequencies after pre-whitening the dominant frequency.
Figure 4: Amplitude spectra of KIC 9845907. The oscillation frequencies are seen as sharp peaks. The panels a, b, c and d show the remaining frequencies after pre-whitening the dominant frequency.

Figure 5: Amplitude spectra of KIC 11754974. The oscillation frequencies are seen as sharp peaks. The panels a, b, c and d show the remaining frequencies after pre-whitening the dominant frequency.
RESULTS AND DISCUSSION

KIC 9700322

The full dataset of KIC 9700322 from quarter 11.1 (Q11.1) through quarter 12.3 (Q12.3) was obtained from the KASOC. For a total of 242005 data points spanning a total time of 180 days after normalisation, the pulsation frequencies were determined by applying discrete Fourier transformation (DFT) signal processing using Period04 software. The oscillation frequencies were searched in the range of 0 µHz to 1157.41 µHz in 0.0093 µHz step size. The pulsation frequencies of KIC 9700322 are listed in Table 2. This target was also observed and analysed for frequencies by Breger, et al. (2011). Both analyses derived the same set of prominent frequencies with \( l = 2 \) quintuplet. In addition, this analysis confirmed the fundamental frequency as 9.792 d\(^{-1}\) and first overtone as 12.569 d\(^{-1}\) using the period ratio method. Furthermore, the relative density of KIC 9700322 \((\frac{\rho}{\rho_\odot})\) was determined as 0.10 ± 0.04. The period ratios of \( P/P_\odot \) of this star is 0.779 which is quite high compared to the 0.687 (Böhm-Vitense, 1992) for more realistic polytrope of \( n = 3 \), where \( n \) is the polytrope index. For higher \( n \), there should be higher mass concentration towards the centre of a star, which represents a real star. Therefore, the higher \( P/P_\odot \) ratio of KIC 9700322 implies higher central density concentration. In 2014, Suarez et al. (2014), showed the relationship of large frequency separation and mean density of DSS. According to this prediction the relative mean density of 0.10 ± 0.04 is compatible with the fundamental frequency of 9.792 d\(^{-1}\) (113 µHz). In general, the upper limit of mass of DSS is 2.5 \( M_\odot \) (Aerts et al., 2010). Therefore, the upper limit of the radius of KIC 9700322 can be estimated as 2.9 ± 0.3 \( R_\odot \).

KIC 11754974

The full dataset of KIC 11754974 from quarter 6.1 (Q6.1) through quarter 7.1 (Q7.1) was obtained from the KASOC. For 172917 data points, spanning a total time of 120 days after normalisation, the pulsation frequencies were determined by the same method as in KIC 9700322. The oscillation frequencies were searched in the range of 0 µHz to 1157.41 µHz in the step size of 0.0098 µHz. The pulsation frequencies of KIC 11754974 are listed in Table 3. The determined relative density using the period-density relationship of KIC 11754974 is 0.29 ± 0.01. The upper limit of the radius is 1.86 ± 0.02 for the standard upper mass of 2.5 of DSS stars.

KIC 9845907

The full dataset of KIC 9845907 from quarter 8.1 (Q8.1) through quarter 9.3 (Q9.3) was obtained from the KASOC. This star was observed in 171 days resulting 253831 flux values. The frequency analysis was done using Period04 in the range of 0 µHz to 1157.41 µHz in the step size of 0.0098 µHz. The pulsation frequencies of the KIC 9845907 are listed in Table 4. A mean relative density \((\frac{\rho}{\rho_\odot})\) of 0.34 ± 0.06 revealed that this star has a maximum radius of 1.94 ± 0.09 \( R_\odot \) for the upper ceiling of mass 2.5 \( M_\odot \).

The dominant radial modes of KIC 9700322, KIC 11754974 and KIC 9845907

The frequency ratio of fundamental \((f_1)\) and 1\(^{st}\) harmonic \((f_2)\) were calculated for all three stars. The ratios, \( f_2/f_1 \), for KIC 9700322, KIC 11754974 and KIC 9845907 are 0.779, 0.763 and 0.779, respectively. These values are close to the expected frequency ratio of fundamental and first overtone radial pulsation of DSS (Stellingwerf, 1979). Furthermore, the frequency ratio \( f_2/f_1 \) 1\(^{st}\) overtone (\( f_1 = 22.854 \text{ day}^{-1} \)) and 2\(^{nd}\) overtone (\( f_1 = 28.214 \text{ day}^{-1} \)) of KIC 9845907 is 0.81, which is close to the expected frequency ratio of 1\(^{st}\) overtone and 2\(^{nd}\) overtone for radial pulsation of DSS.

In mode identification, the simplest modes are the radial modes, which is represented by \( l = 0 \) and integer values of \( n \) for fundamental and its overtones. The frequency of fundamental radial mode \((n = 1)\) is known as \( f_1 \). In this mode, the star swells and contracts, heats and cools in spherical symmetry with the core as a node and the surface as a moving anti-node.

The frequency of the first overtone radial mode \((f_2)\) which corresponds to \( n = 2 \), has one radial node that is a concentric shell within the star. As we are thinking in terms of the radial displacement, that shell is a non-moving node; the motions above and below the node move in anti-phase.

The ratio of the first overtone to the fundamental in DSS is a direct consequence of the sound speed gradient in them, hence of the temperature and (in some places) of chemical composition gradients. Thus, just by observing two pulsation frequencies we have had our first look into the interiors of KIC 9700322, KIC 11754974 and KIC 9845907.
The combination frequencies and the quintuplet

We can clearly identify that the detected frequencies of KIC 9700322 (Breger et al., 2010), KIC 11754974 and KIC 9845907 show simple linear combinations of \( f_1 \) and \( f_3 \) and the rotational frequency. These combinations help in identifying the radial modes. For non-radial modes, combination frequencies are not allowed.

In the frequency spectrum of KIC 9700322 and KIC 11754974 we can clearly identify the equidistant frequency pattern of peaks with a number of components, which depends on the value of \( l \). These five peaks (quintuplet) for \( l = 2 \) is given by the equation \( m = 2l + 1 \). Therefore, in \( l = 2 \) mode, there are five peaks possible as -2, -1, 0, +1 and +2 where the central peak is denoted as \( m = 0 \).

In KIC 9700322, quintuplet was found (\( f_1, f_3, f_{-1}, f_{+1} \) and \( f_5 \) in Table 2) around the two dominant modes and the average spacing between the frequencies in the quintuplet was 0.134 day\(^{-1}\). In KIC 11754974, quintuplet was found (\( f_1, f_3, f_{-1}, f_{+1} \) and \( f_5 \) in Table 3) around the two dominant modes and the average spacing between the frequencies in quintuplet was 0.2180 day\(^{-1}\). This value is identified as the frequency splitting (\( \Delta \nu_{\text{rot}} \)) in equation 3. The rotation period (\( P_{\text{rot}} \)) and rotation frequencies can be calculated by using this frequency splitting.

**The stellar rotation period (\( P_{\text{rot}} \))**

For the star KIC 9700322, the frequency splitting of \( \Delta \nu_{\text{rot}} \) was substituted in equation 3 and the value of rotation period (\( P_{\text{rot}} \)) was estimated. The rotation period was 6.19 days, which means the rotational frequency is 0.1614 day\(^{-1}\). We observed a frequency value \( f_1 = 0.159 \) d\(^{-1}\) in Table 2 which is closer to the value calculated above. Therefore, it can be determined that \( f_1 \) is the rotational frequency of KIC 9700322. This result also confirmed that the rotational frequency is much smaller than the pulsation frequency of DDS. Similarly for KIC 11754974, the value \( \Delta \nu_{\text{rot}} \) was 0.218 d\(^{-1}\) and the rotation period (\( P_{\text{rot}} \)) is 3.8348 days. The rotational frequency was 0.260 d\(^{-1}\). However, this frequency could not be recovered from the power spectra of KIC 11754974.

**CONCLUSIONS**

The frequency analysis of three high-amplitude \( \delta \) Scuti stars observed with the Kepler spacecraft named, KIC 9700322, KIC 11754974 and KIC 9845907 was presented. More than 180 days of continuous observations of short cadence Kepler data was used to detect 41 independent frequencies in KIC 9700322, 50 frequencies in KIC 11754974 and 40 frequencies in KIC 9845907. KIC 9700322, KIC 11754974 and KIC 9845907 stars were mainly selected based on the presence of many frequencies in their power spectra as well as complex frequency combinations and spacing. The frequency spacing was explained as simple combinations of frequencies arising from non-linearity of the oscillation. Combination frequencies, the quintuplets (\( l=2 \) mode) and the estimations of relative densities and radii for all three stars were presented. In addition, the rotation periods were determined as 6.19 days and 3.83 days for KIC 9700322 and KIC 11754974, respectively. The values of relative densities \( \rho/\rho_{\odot} \) were determined as 0.10 ± 0.04, 0.29 ± 0.01 and 0.34 ± 0.06 for KIC 9700322, KIC 11754974 and KIC 9845907, respectively. KIC 9700322 and KIC 11754974 both contain radial and non-radial pulsation while KIC 9845907 is only a radial pulsator.

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