Remark on Z' limits at hadron colliders

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Abstract

Simple estimates for Z' exclusion limits and Z' model measurements in pp (p\bar{p}) collisions are derived. Due to properties of the structure functions, the Z' exclusion limits depend only logarithmically on the Z' couplings to fermions and on the integrated luminosity. The predicted scaling of Z' exclusion limits and errors of Z' measurements with the c.m. energy and luminosity allows an easy extrapolation of existing analyses to other colliders.

It is well known that Z' exclusion limits from e^+e^- collisions vary strongly with the Z' model, while the Z' constraints from pp and p\bar{p} collisions show only a weak model dependence [1] - [7]. The mechanisms leading to Z' limits in hadron collisions and in e^+e^- collisions are essentially different. The Z' is detected through indirect interference effects between the Z' contributions and the SM contributions in e^+e^- collisions. It is detected through direct production in pp or p\bar{p} collisions. However, this cannot be the origin of the weak model dependence of the Z' exclusion limits from hadron collisions because the cross section of direct Z' production depends on the fourth power of the model dependent Z' couplings to SM fermions, while the indirect Z' limits depend only on the square of these couplings.

In this paper, we show that the weak model dependence of the Z' exclusion limits from hadron collisions is due to properties of the structure functions, which lead to an effective logarithmic dependence of the Z' limits on the Z' couplings. The decay mode of the Z' does not influence this dependence. Therefore, we assume for simplicity that the Z' is detected through a muon pair. Our derivation is based on the assumption that the Z' is produced by a quark antiquark pair. The considered reaction is much less sensitive to ZZ' mixing than e^+e^- collisions at the Z peak. Therefore, ZZ' mixing effects can be neglected.

We now derive the scaling law of Z' limits with the c.m. energy \sqrt{s} and the integrated luminosity L approximating the Born cross section,

$$\sigma_T(pp(p\bar{p}) \rightarrow (\gamma, Z, Z')X \rightarrow \mu^+\mu^-X)$$

$$= \sum_q \int_0^1 dx_1 \int_0^1 dx_2 \sigma_T(sx_1x_2; q\bar{q} \rightarrow \mu^+\mu^-)G^q_T(x_1, x_2, M^2_{Z'})\theta(x_1x_2s - M^2_{\Sigma}),$$

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where \( G_T^q(x_1, x_2, M^2_{Z'}) \) depends on the structure functions of the partons \( q \) and \( q \),
\[
G_T^q(x_1, x_2, M^2_{Z'}) = q(x_1, M^2_{Z'})\bar{q}(x_2, M^2_{Z'}) + \bar{q}(x_1, M^2_{Z'})q(x_2, M^2_{Z'}), \tag{2}
\]
and \( M_\Sigma \) is the sum of the masses of the final particles. We assume that the \( Z' \) signal is well above the SM background in the appropriate region of the invariant energy of the muon pairs. This condition is fulfilled in the \( E_6 \) GUT, the left-right theory and in many other GUT's \([2]\).

The approximation (7) and the exact calculation (5) of \( \sigma_T(Q^2; q\bar{q} \to \mu^+\mu^-) \) can then be treated in the narrow width approximation,
\[
\frac{Q^4}{|Q^2 - M^2 + iM\Gamma|^2} \rightarrow \delta(Q^2 - M^2)\frac{\pi M^4}{M\Gamma}, \tag{3}
\]
leading to \([2]\)
\[
\sigma_T \left( pp(p\bar{p}) \rightarrow (\gamma, Z, Z')X \rightarrow \mu^+\mu^-X \right) = \frac{4\pi^2}{3s} \frac{\Gamma_{Z'}}{M_{Z'}} Br(Z' \rightarrow \mu^+\mu^-) \sum_q Br(Z' \rightarrow q\bar{q})f^q \left( \frac{\sqrt{s}}{M_{Z'}}, M^2_{Z'} \right), \tag{4}
\]
with
\[
f^q (r_z, M^2_{Z'}) = \int_{1/r_z^2}^1 \frac{dx}{x} G_T^q \left( x, \frac{1}{s xr_z^2}, M^2_{Z'} \right) \quad \text{and} \quad r_z = \sqrt{s} \frac{M_{Z'}}{M_{Z'}}.
\]

A numerical inspection of the function \( f^q (r_z, M^2_{Z'}) \) shows that it has only a very weak dependence on \( M^2_{Z'} \) in the region we are interested in, i.e. \( f^q (r_z, M^2_{Z'}) \approx f^q (r_z) \). Furthermore, the functions for different quarks \( q = u, d \) differ mainly by a constant factor. For our purposes, we can make the following replacement in equation (4),
\[
\sum_q Br(Z' \rightarrow q\bar{q})f^q \left( \frac{\sqrt{s}}{M_{Z'}}, M^2_{Z'} \right) = f^u \left( \frac{\sqrt{s}}{M_{Z'}} \right) \left[ Br(Z' \rightarrow u\bar{u}) + \frac{1}{C_{ud}} Br(Z' \rightarrow d\bar{d}) \right]. \tag{5}
\]
Remembering the \( pp \) and \( p\bar{p} \) colliders under discussion \([5]\), we see that the functions \( f^q (r_z) \) are needed only in a narrow interval of \( r_z \), i.e. \( 3 < r_z < 5 \) for \( pp \) collisions and \( 2 < r_z < 3.5 \) for \( p\bar{p} \) collisions. In these regions, we have \( C_{ud} = f^u (r_z)/f^d (r_z) \approx 2 \) (25) for \( pp \) (\( p\bar{p} \)) collisions. Therefore, the \( Z' \) search has a reduced sensitivity to \( Z' \bar{d}d \) couplings, especially in \( p\bar{p} \) collisions.

The integral defining the function \( f^u (r_z) \) could be approximated by the function \( r^a_z (r_z - 1)^b \), which takes into account the parametrization of the structure functions. For our purposes, we would prefer an approximation with a function, which can be inverted analytically. It turns out that \( f^u (r_z) \) can be fitted by an exponential function in the relevant interval of \( r_z \),
\[
f^u (r_z) \approx Ce^{-A/r_z}, \quad C = 600 \ (300), \quad A = 32 \ (20) \quad \text{for} \ \ pp \ \ (p\bar{p}) \ \ \text{collisions}. \tag{6}
\]
The approximation (5) and the exact calculation (6) of \( f^u (r_z) \) are shown in figure 1. Note that the fit works satisfactorily up to \( r_z = 10 \). We use the structure functions (8). The dependence of our results on this choice is negligible.

The expected number of \( Z' \) events can now be written as
\[
N_{Z'} = L\sigma_T \left( pp(p\bar{p}) \rightarrow (\gamma, Z, Z')X \rightarrow \mu^+\mu^-X \right) \approx \frac{L}{s} c_{Z'} C \exp \left\{ -\frac{A}{\sqrt{s}} M_{Z'} \right\},
\]
with
\[
c_{Z'} = \frac{4\pi^2}{3} \frac{\Gamma_{Z'}}{M_{Z'}} Br(Z' \rightarrow \mu^+\mu^-) \left[ Br(Z' \rightarrow u\bar{u}) + \frac{1}{C_{ud}} Br(Z' \rightarrow d\bar{d}) \right]. \tag{8}
\]
The function $f_u(\sqrt{s}/M_{Z'}, 25\text{TeV}^2)$ and the approximation (7). The curves of $f_u(\sqrt{s}/M_{Z'}, Q^2)$ for $Q^2 = 600\text{TeV}^2$ or $1\text{TeV}^2$ could not be distinguished from $f_u(\sqrt{s}/M_{Z'}, 25\text{TeV}^2)$.

All details of the $Z'$ model are collected in the constant $c_{Z'}$. The approximate exponential dependence of $N_{Z'}$ on $M_{Z'}$ can be recognized, for instance, from figures 1 to 5 in reference [5]. It also holds in associated $Z'$ production, $pp \rightarrow Z'W, pp \rightarrow Z'Z$, as can be seen from figure 3 of reference [9].

To predict $M_{Z'}^{\text{lim}}$, we have to invert equation (8),

$$M_{Z'}^{\text{lim}} \approx \sqrt{s} \frac{\ln \left( \frac{L c_{Z'} C}{N_{Z'}} \right)}{A}, \quad (9)$$

where now $N_{Z'}$ is the number of detected $Z'$ events demanded for a $Z'$ signal. Relation (8) describes the scaling of $M_{Z'}^{\text{lim}}$ with the c.m. energy and the integrated luminosity. $M_{Z'}^{\text{lim}}$ depends on $L$ only logarithmically. Therefore, $M_{Z'}^{\text{lim}}$ depends only marginally on detector efficiencies or event losses due to background suppression. The model dependent constant $c_{Z'}$ enters (8) only under the logarithm leading to the weak model dependence of $Z'$ exclusion limits in $pp$ and $p\bar{p}$ collisions. The physical origin of this effect is hidden in the properties of the structure functions entering the definition (4) of $f(r_z)$. Therefore, relation (8) obtained for $\sigma_T(pp(p\bar{p}) \rightarrow (\gamma, Z, Z')X \rightarrow \mu^+\mu^-X)$ is qualitatively true for other observables too.

Radiative corrections lead to deviations of $N_{Z'}$ from the Born prediction, which can be taken into account by a multiplication with a $K$ factor, $N_{Z'} \rightarrow KN_{Z'}$. The effect on $M_{Z'}^{\text{lim}}$ is expected to be moderate because also $N_{Z'}$ enters this limit only under the logarithm.

The scaling (8) can be compared with the scaling law known for $e^+e^-$ collisions [4, 10],

$$M_{Z'}^{\text{lim}} \approx (s L)^{1/4}. \quad (10)$$

Not shown in (10) is the direct proportionality to the square of the coupling constants of the $Z'$ to SM fermions [7], which is the origin of the strong model dependence of $M_{Z'}^{\text{lim}}$. 
For practical purposes, it is useful to write equation (9) in the form
\[ \frac{M_{Z'}^{lim}(s, L)}{M_{Z'}^{lim}(s_0, L_0)} \approx \frac{\sqrt{s}}{\sqrt{s_0}} \left( 1 + \xi \ln \frac{s L}{s_0 L_0} \right), \quad \xi = \left[ \ln \frac{L_0 c_{Z'} C}{s_0 N_{Z'}} \right]^{-1}, \tag{11} \]
where now all model dependence is hidden in the constant \( \xi \). Normalizing at one collider, equation (11) predicts the limits for colliders with different energy and luminosity.

All \( Z' \) exclusion limits published in figure 1 of reference [3] can be reproduced by equation (11) with an accuracy of 10% for \( \xi = 0.13 \) (0.10) for \( pp \) (\( pp \)) collisions. The logarithmic dependence of \( M_{Z'}^{lim} \) on \( L \) can also be recognized in figure 3 of reference [4]. The reduction of \( M_{Z'}^{lim} \) due to a decrease of the event rate by a factor two is predicted by relation (11) to be 9% (7%) for \( pp \) (n\( p \)) collisions. These numbers, which do not discriminate between \( Z' \) models and colliders are in agreement with the last line of table 2 in reference [4].

The analyses [3] and [4] report the search limits for different future colliders for \( N_{Z'} = 10 \), where it is assumed that the \( Z' \) does not decay to exotic fermions. The analysis [3] is based on detected muon pairs only, while in [4] the electron pairs are included too. We selected two scenarios from every paper giving \( \sqrt{s} \) and \( L \) in table 1. The numbers are produced from figure 1 of [3] and taken from table 2 of [4]. The \( Z' \) models \( \chi, \psi \) and \( \eta \) in table 1 belong to the \( E_6 \) theory, \( LR \) is a \( Z' \) in the left-right symmetric model, while \( SSM \) is the \( Z' \) in the sequential standard model. We see that the prediction (11) agrees with the exact results within 10% in a wide range of \( L \) and \( s \). At fixed \( s \) and \( L \), equation (3) predicts for \( pp \) collisions \( M_{Z'}^{lim} / M_{SSM}^{lim} = .91 \) (0.82, 0.85, 0.93) for \( Z' = \chi(\psi, \eta, LR) \). This prediction is in good agreement with the numbers quoted in table 1.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Analysis} & \frac{\sqrt{s}}{\text{TeV}} & \frac{L}{\text{fb}^{-1}} & \chi & \psi & \eta & \text{LR, SSM, estimate [4]} \\
\hline
[3] & 2 (pp) & 10 & 1.05 & 1.05 & 1.08 & 1.10 & 1.15 & 1.06 \\
[3] & 14 (pp) & 100 & 4.46 & 4.15 & 4.30 & 4.54 & 4.80 & 4.47 \\
[4] & 60 (pp) & 100 & 13.3 & 12.0 & 12.3 & 13.5 & 14.4 & 15.0 \\
[4] & 200 (pp) & 1000 & 43.6 & 39.2 & 40.1 & 43.2 & 45.9 & 49.3 \\
\hline
1000 \cdot c_{Z'} (pp) & 1.17 & 0.572 & 0.712 & 1.35 & 2.27 & - \\
1000 \cdot c_{Z'} (pp) & 0.40 & 0.437 & 0.556 & 0.77 & 1.41 & - \\
\hline
\end{array}
\]

**Tab. 1:** The lower bound on \( Z' \) masses \( M_{Z'}^{lim} \) in TeV, which could be excluded by the different colliders. The estimate [4] for \( M_{SSM}^{lim} \) is added in the last column. The last two lines contain the values of 1000 \( c_{Z'} \), for convenience. The \( Z' \) is assumed to decay into SM fermions only.

If a \( Z' \) signal is found in hadron collisions, one would like to measure some details of the \( Z' \) model. This can be done [2], [4]-[8] by measurements of different asymmetries \( A_X \). A reasonable model measurement requires enough events to assume that they are gaussian distributed. The one-\( \sigma \) statistical errors can then be estimated as \( \Delta A_X \approx 1/\sqrt{N_{Z'}} \). From equation (9), we deduce the following estimate of \( \Delta A_X \),
\[
\Delta A_X \approx N_{Z'}^{-1/2} \approx \sqrt{\frac{s}{L c_{Z'} C}} \exp \left\{ \frac{A M_{Z'}}{2 \sqrt{s}} \right\}.
\tag{12}
\]
Relation (12) relies on the approximation (7), which becomes inaccurate for too large \( \sqrt{s} / M_{Z'} \). One should therefore be careful in interpolating to \( \sqrt{s} / M_{Z'} > 10 \). Compared to \( M_{Z'}^{lim} \), the
error of the asymmetry measurement, $\Delta A_X$, is much more dependent on the $Z'$ model because the constant $c_{Z'}$ enters not under the logarithm. The estimate (12) can be confronted with the results quoted in table 2 of reference [9], which are obtained for $\sqrt{s} = 14 TeV$, $L = 100 fb^{-1}$ and $M_{Z'} = 1 TeV$:

$$
\begin{array}{cccccc}
\text{Model:} & \chi & \psi & \eta & LR & SSM \\
\Delta A_X \text{ from (12):} & 0.008 & 0.012 & 0.011 & 0.008 & 0.006 \\
\Delta A_{FB}^e \text{ from [9]:} & 0.007 & 0.016 & 0.014 & 0.006 & - \\
\end{array}
$$

Having in mind the crude approximations, which lead to the estimate (12), the agreement is good.

Combining equations (9) and (12), we can predict the precision of the measurement of $A_X$ for a given $M_{Z'} < M_{Z'}^{\text{lim}}$ if we know only $M_{Z'}^{\text{lim}}$ for the same collider,

$$
\Delta A_X \approx \left( \frac{\sqrt{s}}{L c_{Z'} C} \right)^{1-M_{Z'}^{\text{lim}}/M_{Z'}^{\text{lim}}} \left( \sqrt{N_{Z'}} \right)^{-M_{Z'}^{\text{lim}}/M_{Z'}^{\text{lim}}}.
$$

The dependence (14) is illustrated in figure 2. It is similar for different colliders and $Z'$ models.

The approximations (12) and (14) relying on the statistical errors only do not hold for measurements of $M_{Z'}$ and $\Gamma_{Z'}$, where the systematic errors become important. See reference [9] for details.

To summarize, we have derived simple scaling laws for $Z'$ exclusion limits and for statistical errors of $Z'$ asymmetry measurements in $pp$ and $p\bar{p}$ collisions. Our estimates were confronted with existing exact results of $Z'$ analyses and found to be in good agreement with them. The estimates make the dependence of $Z'$ limits on the c.m. energy, the luminosity and the $Z'$ model parameters transparent. They are useful rules to extrapolate existing $Z'$ limits to other colliders and $Z'$ models.

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