ALIGNMENTS OF BLACK HOLES WITH THEIR WARPED ACCRETION DISKS AND EPISODIC LIFETIMES OF ACTIVE GALACTIC NUCLEI

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ABSTRACT

Warped accretion disks have attracted intense attention because of their critical role in shaping the spin of supermassive massive black holes (SMBHs) through the Bardeen–Petterson effect, a general relativistic effect that leads to final alignments or anti-alignments between black holes and warped accretion disks. We study such alignment processes by explicitly taking into account the finite sizes of accretion disks and the episodic lifetimes of active galactic nuclei (AGNs) that delineate the duration of gas fueling onto accretion disks. We employ an approximate global model to simulate the evolution of accretion disks, allowing us to determine the gravitomagnetic torque that drives the alignments in a simple way. We then track down the evolutionary paths for mass and spin of black holes both in a single activity episode and over a series of episodes. Given with randomly and isotropically oriented gas fueling over episodes, we calculate the spin evolution with different episodic lifetimes and find that it is quite sensitive to the lifetimes. We therefore propose that the spin distribution of SMBHs can place constraints on the episodic lifetimes of AGNs and vice versa. The applications of our results on the observed spin distributions of SMBHs and the observed episodic lifetimes of AGNs are discussed, although both measurements at present are too ambiguous for us to draw a firm conclusion. Our prescription can be easily incorporated into semi-analytic models for black hole growth and spin evolution.

Key words: accretion, accretion disks – black hole physics – galaxies: active – quasars: general

1. INTRODUCTION

Accretion disks surrounding supermassive black holes (SMBHs) are believed to be the engines of the enormous radiative power of active galactic nuclei (AGNs) in the universe (e.g., Salpeter 1964; Lynden-Bell 1969). The shapes of accretion disks—flat or warped—depend on the orientation of the angular momentum of gas fueled from the circumnuclear region with respect to the rotating orientation of the central black hole. A wide variety of evidence has indicated that in many circumstances these two orientations are misaligned.

First, the orientation of AGNs seems uncorrelated with that of their host galaxies, which has been well established by observations (Kinney et al. 2000; Schmitt et al. 2003; Gallimore et al. 2006; Muñoz Marín et al. 2007; Shen et al. 2010; Lagos et al. 2011) and also confirmed by numerical simulations (e.g., Hopkins et al. 2012 and references therein). This suggests that gas channeled onto the central SMBH either loses the memory of its initial direction in the host galaxy or comes from outside the galaxy (e.g., Dubois et al. 2012; Nayakshin et al. 2012; Gaspari et al. 2013), most likely due to galaxy mergers (Kendall et al. 2003). Second, the current hierarchical framework of galaxy formation and evolution predicts that repeat galaxy mergers, in addition to secular evolution, trigger SMBH activities (e.g., Benson & Bower 2010) and gas accretion plausibly proceeds at episodic and random phases (in particular for minor mergers; e.g., King & Pringle 2006; Wang et al. 2006). Third, recent studies on SMBH spin through quantifying the radiative efficiency of AGN populations under the thin accretion-disks model shows that SMBH spin undergoes a decline with cosmic time s~2 (Wang et al. 2009; Li et al. 2012), potentially implying that partially random accretion takes place (King et al. 2008). Such evolutionary behavior has also been reproduced by Volonteri et al. (2013) in their semi-analytic model of SMBH growth and evolution (see their Figure 10).

Once gas fueling is inclined with respect to the central spinning black hole, a warped accretion disk forms inevitably. The Lense–Thirring torque arising from the frame-dragging effect will lead to precessions of the inclined accretion disk and the black hole around each other. The detailed dynamics of warped accretion disks has been studied extensively through theoretical analysis (e.g., Pringle 1992; Papaloizou & Lin 1995; Ogilvie 1999; Lubow et al. 2002) and numerical simulations (e.g., Nelson & Papaloizou 1999, 2000; Lodato & Pringle 2006, 2007; Fragile 2009; Zhuravlev et al. 2014). The intensive applications of warped accretion disks lie at their critical role in shaping the spin of the central SMBHs through the Bardeen–Petterson effect (Bardeen & Petterson 1975). In this effect, the presence of strong viscosity, most likely the case for normal AGNs, damps out the differential Lense–Thirring precession and induces the inner portion of the disk to align or anti-align with the spin axis of the hole. Accompanied with warp propagation, the entire disk will tend to align or anti-align with the hole (Scheuer & Feiler 1996; King et al. 2005; Martin et al. 2007).

Previous studies on the alignments between black holes and their warped accretion disks mainly focus on two aspects. The first assumes that accretion disks have infinite extensions and constant profiles. Black holes are always driven toward the disks and the black-hole–disk systems end up aligning (Scheuer & Feiler 1996; Martin et al. 2007; Perego et al. 2009). On the contrary, the second assumes that accretion disks are finite in both size and mass and the total angular momentum of the system is conserved (King et al. 2005, 2008), which means that there is no gas supply to the disks. Such systems can end up with a configuration of either alignment or anti-alignment.
the entire disk. Detailed theoretical analysis show that the dynamics of warped accretion disks is characterized into two regimes depending on the importance of the pressure force and viscous force (Papaloizou & Lin 1995). If we adopt the standard \(\alpha\)-prescription of viscosity, warps propagate as bending waves for inviscid or sufficiently thick disks \((\alpha < H/R)\), whereas they propagate in a diffusive fashion for strongly viscous disks \((\alpha > H/R)\). Here \(H\) is the semi-thickness of the disks. For simplicity, we only consider thin and viscous accretion disks with Keplerian rotation, in which \(\alpha > H/R\) is generally satisfied. This is the most likely case for accretion disks in AGNs with the Eddington ratio at around 0.01–1, the largest population in AGN surveys (e.g., McLure & Dunlop 2004; Shen et al. 2008).

In the presence of high viscosity, the differential precession in the inner disk will be damped out rapidly, giving rising to an inner flat region (the so-called Bardeen–Petterson effect; Bardeen & Petterson 1975). This region extends to the radius where the precessing timescale is comparable with the warp-diffusion timescale. The precessing timescale reads \(c^2R^3/2G\) and the warp-diffusion timescale reads \(R^2/\nu_2\), leading to the warp radius as

\[
R_w \sim \frac{2G\alpha}{\nu_2 c^2},
\]

where \(\nu_2\) is the vertical shear viscosity governing the warp diffusion through the accretion disks.

Integrating over all the annuli yields the total torque exerted on the accretion disk:

\[
T_{LT} = \frac{4\pi G}{c^2} \mathbf{J}_h \times \int \frac{L(R)}{R^2} dR.
\]

As a reaction, the black hole, in the meantime, suffers an equal but opposite torque that causes its angular momentum to process as well. By taking into account the angular momentum carried by mass accretion, the angular momentum of the black hole evolves following

\[
\frac{d\mathbf{J}_h}{dt} = \dot{M}_h \ell_{ms} \mathbf{J}_h - T_{LT},
\]

where \(\dot{M}_h\) is the mass-accretion rate onto the black hole and \(\ell_{ms}\) is the specific angular momentum at the marginal stable orbit \(R_{ms}\). Hereafter a hat symbol on top of a vector represents the corresponding direction vector.

Similarly, the angular momentum of the disks changes with time as

\[
\frac{d\mathbf{J}_d}{dt} = -\dot{M}_h \ell_{ms} \mathbf{J}_h + \mathbf{T}_{LT} + \mathbf{J}_{\mathbf{t}},
\]

where \(\mathbf{J}_d\) is angular momentum summed over all the annuli and \(\mathbf{J}_{\mathbf{t}}\) is angular momentum change due to gas fueling. Note that \(\mathbf{T}_{LT}\) is always orthogonal to \(\mathbf{J}_h\); therefore, it can be expressed in the form of (King et al. 2005)

\[
\mathbf{T}_{LT} = K_1\mathbf{J}_h \times \mathbf{J}_d + K_2\mathbf{J}_h \times (\mathbf{J}_h \times \mathbf{J}_d),
\]

where the first term on the right-hand side corresponds to the precession and the second term corresponds to the change in the angle between the hole and the disk. We keep in mind that
both $K_1$ and $K_2$ are generally positive as shown in our previous numerical analysis (Li et al. 2013).

Considering that the inner region of the disk is aligned with the black hole and the Lense–Thirring torque drops off rapidly with radius, the major contribution to $T_{LT}$ in Equation (3) comes from the integral around the warp radius $R_w$. We therefore approximate $K_2$ as

$$K_2 \approx \frac{2\pi G J_0 L(R_w)}{c^2 R_w}. \quad (7)$$

Below, we will show that such an approximation of $K_2$ yields results that exactly agree with the previous analytical studies for small warps (Scheuer & Feiler 1996; Martin et al. 2007). Note that $K_1$ only governs the precession of the black-hole–disk system and therefore is not important for the present purpose.

3. BASIC EVOLUTION EQUATIONS

3.1. Accretion Disks and Inclination Angles

From the above equations, we can now determine the evolution of the angular momenta $J_\theta$ and $J_\phi$ and the angle $\theta$ between the black hole and the accretion disk. We first define a lifetime of gas feeding $\Delta T_{f}$ as that during which the accretion disk is continuously fed at a mass rate limited by the Eddington rate $\dot{M}_{\text{edd}} = L_{\text{edd}}/\eta c^2 = 2.2 M_\odot\eta^{-1} M_\odot\text{yr}^{-1}$, where $L_{\text{edd}}$ is the Eddington luminosity, $\dot{M}_{\text{edd}} = M_{\odot}/10^8 M_\odot$, and $\eta = 0.1\eta_{-1}$ is the radiative efficiency for mass accretion (see below in Section 3.2 for definition).

Note again that the inner region of the disk is aligned with the black hole. Multiplying $\dot{J}_\theta$ to Equation (4) yields

$$\frac{dJ_\theta}{dt} = M_\ell \ell_{ms}, \quad (8)$$

and multiplying $\dot{J}_\phi$ by Equation (5) yields

$$\frac{dJ_\phi}{dt} = -M_\ell \ell_{ms} \cos \theta - K_2 \sin^2 \theta + J_\ell \cos \theta, \quad (9)$$

where we assume that the angular momentum of the gas fueling is aligned with $J_\ell$. The angle between the black hole and the accretion disk changes with time as

$$J_\phi J_\theta \frac{d\cos \theta}{dt} = \frac{dJ_\phi}{dt} \cdot \frac{1}{J_\theta} - J_\theta \cos \theta \frac{dJ_\phi}{dt} - J_\phi \cos \theta \frac{dJ_\theta}{dt}. \quad (10)$$

We distinguish the evolution of $\theta$ into two phases as below.

3.1.1. Phase I: Gas Fueling

During the phase with gas fueling, the accretion disk is continuously fed so that it maintains a preferred steady angular momentum distribution. As a result, one shall expect that the alignment between the black hole and the accretion disk becomes conserved. The disk is subjected to both alignment and viscous diffusion. The former decreases the angular momentum of the disk ($K_2 > 0$ in Equation (9)), whereas the latter transfers the angular momentum outward. Substituting Equations (8) and (9) into Equation (10) and with some simple mathematical implementations, we arrive at

$$\frac{d\cos \theta}{dt} = -\frac{M_\ell \ell_{ms}}{J_\ell} \sin^2 \theta + \frac{K_2}{J_\ell} \sin^2 \theta \left(1 + \frac{J_\phi}{J_\ell \cos \theta}\right). \quad (14)$$

Following Kumar et al. (2008), we employ an approximate global model to simulate the influence of mass accretion on the disk properties so that accretion rate $\dot{M}$ can be determined in a quite simple fashion. To this end, we define the effective radius of the disk $R_d$ as

$$\frac{J_\ell}{\dot{M} R_d} = \ell (R_d) = (G M_h R_d)^{1/2}. \quad (15)$$

The mean global accretion rate is then given by

$$M_h = -\frac{dM_d}{dt} = \frac{M_d}{t_{\text{acc}}}, \quad (16)$$

where the accretion timescale reads

$$t_{\text{acc}} \sim \frac{R_d^2}{\nu_1 R_d}, \quad (17)$$

where $\nu_1$ is the normal viscosity associated to the azimuthal shear due to the differential rotation of the disk. With the

3 Note that Scheuer & Feiler (1996) omitted a factor of $\sqrt{2}$ in their Equation (11).

4 We stress that the definition of $R_d$ aims to simulate the disk evolution in a global way. It has the same order of magnitude as, but is not necessarily identical to, the disk’s physical outer truncated radius.
standard $\alpha$-prescription, $\nu_1$ is expressed by

$$\nu_1 = \alpha \Omega_k H^2 = \alpha h^2 \Omega_k R^2,$$

where $h = HR$ is the disk’s aspect ratio and $\Omega_k$ is the Keplerian angular momentum. Combining the above equations, we can obtain

$$M_h = \frac{\alpha h^2 G^2 M_*^2 M_\bullet^4}{J_d^3}.$$  \hfill (19)

This equation apparently indicates the influences of alignment and accretion on $M_h$. From Equation (9), alignment processes reduce the disk angular momentum $J_d$ at a rate of $\sim K^2$ but leave the disk mass $M_d$ unchanged, therefore increasing the accretion rate $M_h$. On the contrary, mass accretion reduces the disk mass but approximately keeps the disk angular momentum unchanged, considering that angular momentum loss due to the accretion inflow through the marginal stable orbit is usually negligible, therefore decreasing $M_h.$

Indeed, if only considering the influence of accretion rate, an asymptotic solution to Equation (16) exists (see Kumar et al. 2008):

$$M_d(t) = M_d(t = 0) \left[1 + \frac{3t}{t_{acc}'}\right]^{-1/3},$$

and

$$M_h(t) = M_h(t = 0) \left[1 + \frac{3t}{t_{acc}'}\right]^{-4/3},$$

where $t_{acc}' = t_{acc}(0)$ is the accretion timescale at $t = 0$. For small warp cases, we expect that the mass-accretion rate approaches the above asymptotic behaviors, i.e., $M_h \propto t^{-4/3}$ for $t \gg t_{acc}'$.

With mass-accretion rate, we now rewrite $K_2$ in a more convenient form. Previous studies showed that the vertical viscosity $\nu_2$ is related to the normal azimuthal viscosity $\nu_1$ by (Ogilvie 1999; Lodato & Pringle 2007; see also Perego et al. 2009)

$$\nu_2 = \frac{f_\nu}{2\alpha^2},$$

where $f_\nu$ is a coefficient of the order of unity. As a zeroth-order approximation, we neglect the detailed shape of warped accretion disks and make use of the estimate for surface density $\Sigma \approx M_d/3\pi h_1$ and for the angular momentum $L(R) \approx M_h/\sqrt{GM_h R}/3\pi h_1$ as in flat disks. This is reasonable since accretion disks become maximally warped around the warp radius $R_w$ and then rapidly approach flat geometry beyond $R_w$ (Scheuer & Feiler 1996; Martin et al. 2007; Chen et al. 2009; Li et al. 2013). Combining Equations (2) and (7) gives

$$K_2 \approx \frac{f_\nu M_d}{6\alpha^2 \sqrt{GM_h R_w}}.$$ \hfill (23)

For small warp amplitude, $f_\nu = 1$, whereas for large warp amplitude, $f_\nu$ moderately decreases below unity, depending on the warp amplitude (Ogilvie 1999; Lodato & Price 2010).

Ogilvie (1999) developed a prescription to determine $f_\nu$ for any warp amplitude. However, this requires knowing the detailed warp profile as a function of radius and is therefore beyond the scope of the present work. We simply set $f_\nu = 1$ throughout our calculations and below we will show the reliability of our results in light of this simplification.

3.2. Black Hole Evolution

The direction of the inner flat region of the disk depends on the inclinations of the incoming gas flow and the black hole: if $\theta \leq \pi/2$, it aligns with the hole, whereas if $\theta > \pi/2$, it anti-aligns with the hole. By taking into account the mass loss carried away by radiation, one obtains that the black hole grows as

$$\frac{dM_h}{dt} = (1 - \eta)M_h,$$ \hfill (24)

and by combining with Equation (8), the black hole spin evolves as

$$\frac{da}{dt} = (1 - \eta)\frac{M_h}{M_h} \left(\frac{1}{1 - \eta R_g c} - 2a\right),$$ \hfill (25)

where $\eta$ is the radiative efficiency depending on the spin parameter $a$ via

$$\eta = 1 - \sqrt{1 - \frac{2}{3r}},$$ \hfill (26)

and the specific angular momentum is calculated as

$$\ell_{ms} = \pm \sqrt{GM_h R_{ms}} \frac{1 \pm 2ar^{-3/2} + a^2r^{-2}}{(1 - 3r^{-1} \pm 2ar^{-3/2})^{1/2}},$$ \hfill (27)

for $a < 1$ and

$$\ell_{ms} = \pm \sqrt{GM_h R_{ms}} \frac{1 \pm r^{-1/2} + r^{-1} \pm r^{-3/2}}{(1 \pm 2r^{-1/2})^{1/2}},$$ \hfill (28)

for $a = 1$, where $r = R_{ms}/R_g R_g = GM_h/c^2$ is the gravitational radius, and the upper sign refers to prograde accretion while the lower sign refers to retrograde accretion (Bardeen et al. 1972).\footnote{Note that, according to the calculation of Thorne (1974), the radiation emitted by the geometrically thin disk and swallowed by the hole produces a counteracting torque that prevents the hole from spaning beyond an upper limit of $a \approx 0.998$. In our calculations, the accretion may proceed in a retrograde-to-prograde fashion over the episodes. This effectively means that the black hole spin may suffer a flip from $a = -1$ to $a = 1$. For the sake of simplicity, we neglect the counteracting torque and assume that black holes can carry any spin between $-1$ and $1$.}

4. RESULTS

According to the standard accretion-disk model, the disk’s aspect ratio $h = HR$ is on an order of $10^{-3} \sim 10^{-2}$ and weakly depends on radius in proportion to $R^{-1/20}$, on black hole mass in proportion to $M^{-1/20}$, and on mass-accretion rate in proportion to $M_h^{1/5}$ (Shakura & Sunyaev 1973; see also Collin-Souffrin & Dumont 1990; Natarajan & Pringle 1998; King et al. 2008). An accretion disk becomes self-gravitating when its mass reaches a factor $h$ of the black hole mass (e.g., Pringle 1981). We therefore adopt the initial disk mass $M_d \approx hM_h$. Given with $h$, the initial effective radius of the disk...
Our calculating procedure is as follows. We start with a black hole, and an angular momentum of \( J_d = M_d (GM_d R_d)^{1/2} \). The initial inclination angle between the black hole and the disk is \( \theta \). We then evolve Equations (9) and (16) for the angular momentum and mass of the disk, Equations (24) and (25) for the mass and spin of the black hole, and finally Equations (11) and (14) for the inclination angle. The mass-accretion rate \( \dot{M}_d \) is determined by Equation (19). We keep in mind that in the phase with gas fueling, the disk’s mass and angular momentum are unchanged. For a series of accretion events, we just repeat the above procedure but with the mass and spin of the black hole inherited from the previous episode.

We assume that in the beginning of an episode, the disk has already been formed, and we neglect the stage for the formation of the disk. It is expected that during the formation stage, the accretion rate must be very insignificant so that the influence on black hole spin evolution is negligible. Also, we implicitly assumed that the time lag between the successive episodes is long enough to let the disk be consumed on the accretion timescale before the next episode starts. This can be verified by the fact that the observed “duty cycle” is generally much less than unity (e.g., Shankar et al. 2009; Li et al. 2012). Duty cycle measures the fraction of active black holes to total black holes, and equivalently indicates the time fraction of active phases over total lifetime (active and inactive) for a single black hole (Wang et al. 2006).

In the following calculations, unless stated otherwise, we set \( \alpha = 0.1 \), \( h = 5 \times 10^{-3} \), \( R_d = 2.7 \times 10^{3} R_g \), \( \dot{M}_d = 10^{6} M_\odot \), and \( a \) as the fiducial values. It is worth mentioning that the choice of the dependence of \( R_d \) on black hole mass and the radiative efficiency aims to let the dimensionless accretion rate \( \dot{M}_d / \dot{M}_{\text{Edd}} \) at about 0.25, the typical value observed in luminous AGN surveys (Kollmeier et al. 2006; Shen et al. 2008; Li et al. 2011). We also set an upper limit of mass-accretion rate by \( \dot{M}_d \leq \dot{M}_{\text{Edd}} \) considering that beyond the Eddington limit, radiative feedback may be important to regulate the accretion rate at around the Eddington limit.

### 4.1. Alignments and Anti-alignments

In Figure 1, we demonstrate evolution of the inclination angle between the disk and the black hole for different lifetimes of gas fueling: \( \Delta T_i = (0, 1, 2, 4, 10) \times 10^5 \) yr. The initial value of \( \theta \) is set to be \( \theta_0 = \pi/6 \) and \( \theta_0 = 5\pi/6 \) in the left and right panels, respectively. As can be seen, for the case of \( \theta_0 = \pi/6 \), the inclination angle rapidly decreases on a timescale of \( 10^5 \) yr, which is the typical estimate in several previous studies (see, e.g., Perego et al. 2009; Li et al. 2013 and...
As the lifetime of gas fueling $\Delta T_f$ increases, the decay rate of $\theta$ is slightly reduced. This is because during the gas fueling, the disk maintains its orientation and only the black hole is driven to align to the disk, whereas after the gas fueling is quenched, both the disk and the hole change their orientations toward the direction of the total angular momentum of the system. This can be further verified from Equations (11) and (14). Interestingly, the behavior of $\theta$ for $\Delta T_f = 10^6$ yr, namely, the disk being continuously fed throughout the time range of calculations, is exactly identical to the analytic solution found in Scheuer & Pringle (1996) and Martin et al. (2007).

The evolution of $\theta$ is much more complicated for the case of $\theta_0 = 5\pi/6$ as shown in the right panel of Figure 1. With no gas fueling ($\Delta T_f = 0$), the inclination angle rises up to $\pi$ all the way. Indeed, the instantaneous behavior of $\theta$ is governed by the ratio $J_d/J_h$ in Equation (14): for $J_d/J_h > -\cos \theta$, the inclination $\theta$ decreases, while for $J_d/J_h < -\cos \theta$, the inclination $\theta$ increases (see also Li et al. 2013). As $\Delta T_f$ increases, the final configuration of the system transits from anti-alignment ($\theta = \pi$) to co-alignment ($\theta = 0$). We note that there is a steep decreasing/increasing trend of $\theta$ just after the gas fueling is quenched, which we ascribe to the rapid reduce of the angular momentum of the disk and hence the enhancement of the mass-accretion rate due to the viscous dissipation arising from the alignments. See below for a detailed explanation.

Figure 2 shows the corresponding mass-accretion rates. For the sake of comparison, the asymptotic evolution of accretion rate by Equation (21) is superposed in the left panel. If there is no gas fueling, as the inclination angle tends toward zero, the angular momentum of the disk approaches a constant value. As a result, the mass-accretion rate evolves asymptotically following Equation (21). In the right panel, the initial value of $\theta$ is $\theta_0 = 5\pi/6$. Instead of a gradual decay, there exists a notable peak of accretion rate, which even reaches the Eddington limit $\dot{M}_{\text{edd}}$ in some cases. The similar behavior of an enhancement of the mass-accretion rate also appears in the previous numerical simulations on warped accretion disks (see, e.g., Lodato & Pringle 2006 and Nixon et al. 2012). This can be easily understood in terms of Equations (9), (15), and (16): strong dissipation arising from alignments leads to an intense reduction in the magnitude of $J_d$: the effective radius $R_d$ accordingly shrinks to maintain a Keplerian rotating disk. As a consequence, the accretion timescale $t_{\text{acc}}$ is shortened and the mass-accretion rate $\dot{M}_h$ is significantly enhanced. In reality, the emergence of the enhancement of accretion rate depends on competition between the viscous-dissipation-driven deduction of $J_d$ (due to warp alignments) and mass-accretion-driven deduction of $\dot{M}_d$. This is the reason why there is no accretion-rate enhancement for some curves in Figure 2.

By assuming the angular momentum conservation of the system, King et al. (2005) derived a condition for the occurrence of anti-alignments: the initial angle between the disk and the hole satisfies $\cos \theta_0 < -J_d/2J_h$. This equation is intensively used in subsequent studies on the cosmological evolution of SMBH spins (e.g., Volonteri et al. 2007; King et al. 2008; Lagos et al. 2009; Fanidakis et al. 2011; Dotti et al. 2013; Dubois et al. 2014; Sesana et al. 2014). We relax the assumption of the conservation of angular momentum and let the accretion disk be steadily fueled for a lifetime of $\Delta T_f$. Since the total angular momentum of the system is not conserved, it is inappropriate to directly use the King et al. (2005) condition. In the right panel of Figure 1, the initial ratios $J_d/J_h = 0.40$ and $\cos \theta_0 = -0.87$, and therefore the condition $\cos \theta_0 < -J_d/2J_h$ is well established. However, for $\Delta T_f \gtrsim 2 \times 10^5$ yr, the system ends up as an alignment instead of an anti-alignment.

Nevertheless, the ratio $J_d/J_h$ remains an important indicator of alignments or anti-alignments (if there is no gas fueling). Small ratios mean that the angular momentum of black holes dominates over that of disks, therefore it is comparatively easier to drive the disks toward alignment or anti-alignment. Conversely, if the angular momentum of the disks is dominated, the holes are easier to change their orientations. This is somehow similar to the cases in which the disks are continuously fueled: the holes tend to align toward the disks (see also the discussions of King et al. 2005 and Lodato & Pringle 2006).
4.2. Spin Evolution

4.2.1. A Single Accretion Event

In Figures 3 and 4, we show the evolution of the black hole properties, i.e., mass and spin, respectively, for initial inclination angles $q_0 = \pi/6$ and $q_0 = 5\pi/6$ with a set of gas fueling lifetimes $\Delta T_f$. Compared to the case of $q_0 = \pi/6$, we find that the black hole mass growth for $q_0 = 5\pi/6$ is much more efficient. This is because the Eddington limit is larger with lower radiative efficiency due to retrograde accretion for $q > \pi/2$ and also because the accretion rate is enhanced as plotted in Figure 2. The spin evolution in Figure 4 clearly illustrates the important influences of gas fueling. As the lifetime of gas fueling increases, the black hole is gradually driven to align with the disk regardless of the initial inclination. As a result, in the right panel of Figure 4, there exists a trend for $\Delta T_f \gtrsim 2 \times 10^5$ yr in which the spin first declines due to retrograde accretion and then increases when the inclination angle transits to $q < \pi/2$. In addition, we find that the spin changes are relatively more significant for black holes with $\Delta T_f \lesssim 2 \times 10^5$ yr because these holes undergo retrograde accretion throughout the episode, which carries larger angular momentum compared with the prograde accretion (see Equations (27) and (28)).

4.2.2. A Series of Accretion Events

The spin of black holes will reach an equilibrium value after a series of accretion events, depending on the fraction of prograde and retrograde accretion (King et al. 2008; Sesana et al. 2014). Evidently, this fraction is intimately relevant to the degrees of anisotropy in the orientations of accretion disks with respect to the holes. If all the accretion events occur with the initial inclination angles $q_0 < \pi/2$, the black holes accrete in a coherent fashion and their spin will be spun up all the way to the maximum ($a = 1$). On the other hand, the inclination angles $q_0$ may distribute randomly over the whole space ($0 \sim \pi$) (e.g., King et al. 2008; Wang et al. 2009; Hopkins et al. 2012; Li et al. 2012). For simplicity, we only focus on the random distribution of $q_0$ in proportion to $\sin q_0$ over $0 \sim \pi$, so that both alignments and anti-alignments can occur in our calculations. We first generate a random inclination angle and then apply the prescription described above to track down the
mass growth and spin evolution of black holes. The calculation ends when the accretion rate decreases to one-tenth of its initial value, i.e., $0.025 M_{\text{edd}}$.

In Figure 5, we illustrate how black hole spin evolves alongside mass growth during a series of accretion events. We stress that we adopt the effective radius of accretion disks dependent on black hole mass as $R_d = 2.7 \times 10^3 M_h^{-2/3} \eta_n^{-1/3} R_s$, in order to guarantee the initial dimensionless accretion rate at 0.25 in Equation (29). Once there is no gas fueling, irrespective of the initial spin values, black holes lose the memory of their initial status and approach a relatively low spin roughly after doubling their mass. Then as mass growth, there is a weak trend in which their spin gradually falls off. We provide an intuitive explanation for such weakly negative dependence of spin on mass through the ratio $J_d / J_h \approx 1$. Since we adopt $R_d \propto M_h^{-2/3}$, Equation (31) yields $a \propto M_h^{1/3}$. Of course, the real situation is much more complicated, but the negative dependence is retained (see also below for further discussions).

Once the disks are fueled for a lifetime longer that the typical alignment timescale (e.g., $\Delta T_I = 10^6 \text{ yr}$), as expected, the holes are always aligned to the disks and are rapidly spun up to fast rotating. However, we note that the final spin never reaches the maximal value $a = 1$ because during the alignments, there exists a period with retrograde accretion that spins down the holes. This indicates that even though the final configuration in an accretion event is alignment instead of anti-alignment, the spin decrease during the time with retrograde accretion still contributes to the final spin status and therefore should be appropriately included.

5. IMPLICATIONS FOR SPIN DISTRIBUTION AND EPISODIC LIFETIMES OF AGNs

5.1. Monte-Carlo Simulations

We perform Monte-Carlo simulations to explore the spin distribution with the above-described prescription. The free parameters $\alpha$, $h$, and $R_d$ are set by their fiducial values as in Section 4. Initially, we assign to a black hole a mass drawn from a uniform distribution in logarithm over $(10^6 \sim 10^9) M_\odot$ and a spin drawn uniformly over $(0 \sim 1)$. For this newly generated black hole, we first let it grow with 100 accretion episodes, to remove the memory of its initial conditions. Afterward, this hole continues to accrete for a series of episodes. We randomly assign the number of episodes between $(0 \sim 900)$ so that on average the hole undergoes 500 iterations. We then record the final mass and spin information of the hole. In each accretion episode, the inclination $\theta_0$ between the hole and its accretion disk follows a uniform distribution of $\cos \theta_0$ over $(-1 \sim 1)$. An individual event is regarded to be terminated once the accretion rate decreases below one-tenth of its initial value. We repeat this procedure 500 times and are finally left with a sample of 500 black holes.

In Figure 6, we plot the spin distribution with black hole mass for different gas fueling lifetime $\Delta T_I = 0, 10^5 \text{ yr}, 10^6 M_\odot^{-1} \text{ yr}$, and $10^8 M_\odot^{-1} \text{ yr}$ in panels (a)–(d), respectively. In panel (a), there is no gas fueling and the disks will be consumed within an accretion timescale $t_{\text{acc}} \sim 8.8 \times 10^5 \text{ yr}$ (see Equation (30)). As the ratio $J_d / J_h$ decreases with black hole mass, it becomes easier to change the orientations of the massive black hole accretion disks and thus anti-alignments become more probable. This results in a negative correlation between spin and mass. A similar trend was also found by (Dotti et al. 2013, corresponding to $F = 0$ in their model). When we switch on the gas fueling for a time of $\Delta T_I = 10^5 \text{ yr}$ in panel (b), black holes are always driven to align toward the disks during this period. Therefore, the fraction of retrograde accretion is significantly reduced and all the black holes carry a spin of $|a| \gtrsim 0.5$.

We also choose a mass-dependent lifetime of gas fueling for the sake of comparison with observations. In panel (c), the gas fueling lifetime is $\Delta T_I = 10^5 M_\odot^{-1} \text{ yr}$, indicating that the disks surrounding $10^5 M_\odot$ black holes will be fueled for $10^6 \text{ yr}$, far longer than the alignment timescale. Accordingly, smaller black holes are overall spun up to maximum rotating. There is a strong anti-correlation between spin and mass. On the other hand, in panel (d), we reverse the mass dependence of fueling lifetime to $\Delta T_I = 10^8 M_\odot^{-1} \text{ yr}$. As expected, massive black holes at $\sim 10^8 M_\odot$ have maximum spin ($a = 1$) whereas lower massive black holes in this case are not affected (since $T_I$ is much less than the accretion timescale and can be neglected) and just follow the distribution in panel (a).

5.2. Comparison with Observations

We now compare our calculated spin distributions with observations. Notwithstanding that measuring the spin of massive black holes is still challenging, there are several developing techniques that provide useful insight (e.g., Wang et al. 2009; Davis & Laor 2011; Czerny et al. 2011; Li et al. 2012; Brenneman 2013; Reynolds 2014; Done et al. 2013; Wu et al. 2013). Among these techniques, we focus on the direct measurements using the broad iron K line and the indirect estimates using the radiative efficiency of thin accretion disks as a surrogate of the black hole spin (see Equation (26); Davis & Laor 2011; Li et al. 2012). Both these methods had been applied to a sample or a population of AGNs, allowing us to perform straightforward comparison. Unfortunately, each method bears a large source of uncertainties that are not well understood and the obtained results seem incompatible at this stage. Therefore, we perform a separate comparison as follows.

In panel (c) of Figure 6, we superpose the spin of 19 sources measured from iron Kα lines, most of which are Seyfert I or narrow line Seyfert I AGNs residing in spirals or lenticular
galaxies (see Sesana et al. 2014). Reynolds (2014) proposed that in this sample of low-mass black holes (\(\sim 10^7 M_\odot\)) are spinning fast while black holes with mass \(>10^8 M_\odot\) may have moderate spin. Our calculations show that a lifetime of gas fueling as \(D_L = T M^{-5} \times 10^8\) yr can generally reproduce such observed spin distribution. This means low-mass black holes require longer gas fueling to reach alignments and to accrete coherently.

On the other hand, Davis & Laor (2011) reconstructed the spectral energy distributions for a sample of 80 Palomar-Green quasars and estimated their individual radiative efficiency of accretion flows based on the thin disk model. They found a tight correlation between the efficiency with black hole mass approximately as \(\eta \propto M_{bh}^{1/2}\) (but see also Raimundo et al. 2012), plausibly implying an increase in the black hole spin with mass. Meanwhile, Li et al. (2012) applied Soltan’s argument (Soltan 1982) to solve the continuity equation of SMBH demography in active and quiescent galaxies (Li et al. 2011) and quantified the radiative efficiency with redshift and black hole mass. Their results suggested that the efficiency undergoes cosmological evolution and generally there is an increasing trend in the efficiency with black hole mass at redshift \(z \gtrsim 1\) (see also Cao & Li 2008; Volonteri et al. 2013; Ueda et al. 2014). Figure 6 compares the radiative efficiency from our calculations by adopting the gas fueling lifetime \(\Delta T_L = T M^{-5} \times 10^8\) yr with those from the observational constraints of Davis & Laor (2011) and Li et al. (2012, at \(z \sim 1\)). Here, the radiative efficiency is calculated from black hole spin as Equation (26) by assuming that the thin disk model applies. As stated above, in the context of chaotic accretion without gas fueling, high-mass black holes (\(\sim 10^9 M_\odot\)) have low spin because of net spin-down by retrograde accretion with anti-alignments. Therefore, to
maintain the observed high radiative efficiency (and high spin), sufficient prolonged gas fueling is in need for these holes.

Longer gas fueling means that there is plenty of gas reservoir progressively channeled into the accretion disks and the black holes are accordingly long-lived. There are several techniques developed for measuring or estimating the episodic lifetimes of AGNs (see Martini 2004 for a review). In Table 1, we summarize the observational constraints on episodic lifetimes of AGNs through the proximity effect and the fluorescent Lyα emission. Note that both these constraints are presently only realizable for high-redshift, luminous quasars, indicating that there reside high-mass black holes (plausibly \(>10^9 M_\odot\)). It seems that the observed AGN episodic lifetimes are generally \(\sim10^7\) yr but with large uncertainties. Such long lifetimes strongly imply that high-mass black holes are rapidly spinning at high redshift, consistent with previous studies probed through the radiative efficiency (e.g., Wang et al. 2009; Li et al. 2012; Trakhtenbrot 2014). Our calculations in the present work further show that if the correlation between the efficiency and the black hole mass is reliable, low-mass black holes must accrete in a more chaotic way and a gas fueling lifetime as \(\Delta t\propto M_{\bullet}^{-0.5}\) can roughly reproduce the observed slope \(\eta\propto M_{\bullet}^{1/2}\). This gives rise to an episodic lifetime of \(>3 \times 10^6\) yr for SMBHs with \(M_{\bullet} > 10^9 M_\odot\), coincident with the observations.

| Object          | Redshift | Episodic Lifetime (Myr) | Telescope | Reference |
|-----------------|----------|--------------------------|-----------|-----------|
| Q 0302-003      | 3.285    | 10–30                    | VLT       | 1, 2      |
| HE 2347-4342    | 2.885    | \(\sim 25\)              | VLT       | 3         |
| KP 76           | 2.466    | \(\geq 25\)              | Keck, HIRES | 4         |
| KP 77           | 2.535    | 16–33                    | Keck, HIRES | 4         |
| HS 1700+6416    | 2.748    | \(\geq 20\)              | HST/COS   | 5         |
| Quasar Pair Sample | \~2.2 | \~1                       | Keck, Lick, NOAO, SDSS | 6 |

Fluorescent Lyα Emission

| Object          | Redshift | Episodic Lifetime (Myr) | Telescope | Reference |
|-----------------|----------|--------------------------|-----------|-----------|
| HS 1549+1919    | \~2.84   | \(\geq 1.3\)             | Keck      | 7         |
| QSO 0420-388    | \~3.1    | \(\geq 60\)              | VLT       | 8         |
| Quasar Sample   | \~2.7    | 1–20                     | Keck      | 9         |

**References.** (1) Worseck & Witoszki (2006); (2) Jakobsen et al. (2003); (3) Worseck et al. (2007); (4) Gonçalves et al. (2008); (5) McQuinn & Worseck (2014); (6) Kirkman & Tytler (2008); (7) Adelberger et al. (2006); (8) Cantalupo et al. (2007); (9) Trainor & Steidel (2013).
velocity to the velocity dispersion of the galaxies mirrors the degree of anisotropy of the gas component channeled into the nuclear regions. They argued that the rotation velocity measures the bulk rotation of the galaxies while the velocity dispersion measures how chaotic the galaxies are. This seems in line with the recent finding of Ho & Kim (2014) that the virial factor in reverberation mapping, which encodes the geometric and kinetic properties of the broad-line regions of AGNs, is related to the large-scale morphology of the host galaxy bulges. However, numerical simulations showed that the angular momentum of gas inflowing into the nuclear regions (~pc) will lose its memory of the initial direction on larger scales (~kpc; Barnes & Hernquist 1996; Hopkins et al. 2012). This is well supported by the observations that AGNs are misaligned with the host galaxies (e.g., Kinney et al. 2000; Gallimore et al. 2006; Shen et al. 2010; Lagos et al. 2011). These conflicts reflect our poor understanding of the detailed processes in black hole fueling from the galaxy scale to the central disk scale. We await future observations to justify these conflicts and we are content with our present assumption.

We consider only thin accretion disks with the disk aspect ratio $H/R \ll \alpha$ to guarantee that the viscosity is strong enough to validate the Bardeen–Peterson effect (Papaloizou & Lin 1995). Therefore, our approach only applies to the accretion disks with dimensionless accretion rate $M_\ast/M_{\rm Edd}$ at $10^{-2} \sim 1$, which are believed to correspond to the thin disk regime. Beyond this range, accretion disks transit to be thick and the Bardeen–Peterson effect may be weak or even disappear, as confirmed by recent MHD numerical simulations (e.g., Fragile 2009; Zhuravlev et al. 2014). However, the heuristic MHD simulations by McKinney et al. (2013) revealed a new “magneto-spin alignment” mechanism that works in thick disks with strong magnetic forces. Unlike the Bardeen–Peterson effect, this mechanism arises from the magnetic torque of the black hole’s magnetosphere, which is already aligned with the black hole spin axis due to frame-dragging forces. It is unknown yet how the spin evolves for thick warped disks and more investigations would be highly worthwhile.

In conclusion, we study the alignments of black holes and their accretion disks by taking into account the finite sizes of disks and continuous gas fueling. Our results show that, with fiducial values for the free parameters, the gas fueling lifetime is crucial to the alignments/anti-alignments and therefore to the spin evolution. By applying our prescription to a series of accretion activities and assuming that the disk orientations are chaotically distributed over episodes, we compared our calculated spin distribution with the spin measurements through iron Kα line fitting and through the radiative efficiency, and we made an attempt to place constraints on the gas fueling lifetime. Unfortunately, the spin measurements through these two techniques at this stage seem incompatible with allowing us to draw a firm conclusion. We find that generally a gas fueling lifetime of $\Delta T_f = 10^5 M_\ast^{1.5}$ yr can reproduce the spin distribution reported by the iron Kα line fitting, whereas a lifetime of $\Delta T_f = 10^5 M_\ast^{3.5}$ yr can reproduce these constraints through radiative efficiency. However, the latter case seems consistent with the observed episodic lifetime as long as $\sim 10^7$ yr for very luminous AGNs at high redshift, which plausibly harbor $>10^7 M_\odot$ SMBHs. Since the lifetimes of gas fueling are linked to the episodic lifetimes of AGNs, we propose that the episodic lifetimes should be regarded as a new ingredient for the semi-analytic models of SMBH growth and spin evolution.

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Figure 8. Influence of $f_\ell$ on spin distribution. Blue points represent the case $f_\ell = 1$ for all the $\theta$s and red points represent the case $f_\ell = 0.1$ for $\theta \in (\pi/3 - 2\pi/3)$. The gas fueling lifetime is $\Delta T_f = 10^5 M_\ast^{1.5}$ yr.
