Towards Metric-Affine Quantum Gravity

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I review here some motivations to consider a theory of gravity based on independent metric and connection, and its status as a quantum theory.

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1. Why Metric-Affine Gravity?

Metric-Affine Gravity (henceforth MAG) is a class of theories of gravity with independent metric and connection. There are phenomenological motivations to study modified theories of gravity like MAG, but here we shall only be concerned with the relation between gravity and the other interactions. Unlike Einstein’s General Relativity (GR), in the case of the electroweak and strong forces, the mediator of the interaction is a connection. This is a good motivation for alternative theories of gravity where also the gravitational connection is dynamical.

GR can be formulated in many inequivalent ways. One choice is the set of fields one works with. Unimodular metric, metric, tetrad are just some of the possibilities, with increasing number of fields. When one increases the number of fields, however, one also enlarges the gauge group, in such a way that the number of physical degrees of freedom remains always the same. Thus, these different formulations are all equivalent, independently of the form of the action: they just amount to different partial gauge fixings of the same theory.

Another way to increase the number of dynamical fields, without actually changing the physics, is to allow the connection to be independent of the metric, or tetrad, as in Palatini and Einstein-Cartan formulations of GR. In these cases the elimination of the additional fields is the result of the equations of motion. This, however,

\[ \text{See [1] for a comprehensive overview.}\]

\[ \text{Alternatively, one can think of them as the result of applying the Stückelberg trick to unimodular gravity, which is the formulation with the least number of fields that is still local [2].}\]

\[ \text{For simplicity we consider here pure gravity without matter.}\]
is a peculiarity of theories with a Lagrangian that is linear in curvature. When we allow Lagrangians quadratic in curvature, the theory is no longer equivalent to a purely metric theory. We are then in the realm of MAGs in the proper sense.

Just as gauge theories can be in different phases, also gravity can, although our understanding is much more limited in this case. We shall see that MAGs look like gauge theories in a Higgs phase. This provides a partial answer to the question “what is the origin of the Planck scale”, that is analogous to the answer that the Standard Model (SM) gives to the question “what is the origin of the Fermi scale”. At the core of our arguments lies a dynamical understanding of the conditions of metricity and torsionlessness, that in GR are merely postulated.

2. Gauge theories in the Higgs phase

We start by considering the paradigmatic example of a superconductor. It is well-known that the macroscopic properties of a superconductor follow simply from the assumption that the electromagnetic $U(1)$ gauge group is in the Higgs phase \cite{Higgs}. There is a complex scalar field $\phi = \rho e^{i\varphi}$ with nonzero VEV. The radial mode $\rho$ has a mass and can be ignored at sufficiently low energy. Infrared physics only depends on the phase $\varphi$, which behaves like a Goldstone boson. It transforms under $U(1)$ as $\varphi(x) \rightarrow \varphi(x) + \alpha(x)$ and has a covariant derivative $D_\mu \varphi = \partial_\mu \varphi - A_\mu$. It is natural to assume that the Lagrangian for $\varphi$ contains a term quadratic in $D\varphi$, so, in a static situation, its energy will be minimized if the vacuum is such that

$$D_i \varphi = 0 , \quad (1)$$

This is the main property of the superconducting state. It implies that the magnetic field vanishes in the bulk of the material. This is known as the Meissner effect. The vanishing of resistivity can also be derived from this property. If one chooses the gauge so that $\varphi$ is constant, the kinetic term of $\varphi$ is seen to become a mass for the longitudinal part of the photon: this is the essence of the Higgs phenomenon.

A non-abelian version of this occurs in the Weinberg-Salam electroweak theory, and more generally whenever a gauge theory with gauge group $G$ is in the Higgs phase, with unbroken subgroup $H$ \cite{Higgs}. There is an order parameter, a scalar field $\phi$ in some representation of $G$, and a potential that is minimized when $\phi$ lies in an orbit of $G$ that is diffeomorphic to $G/H$. The potential gives a mass to the radial degrees of freedom of $\phi$. At sufficiently low energy these massive degrees of freedom can be ignored and one only has the fields $\varphi^\alpha$ with values in the coset space $G/H$. These are the Goldstone bosons. Their dynamics is given by the Lagrangian

$$L_g = -\frac{f^2}{2} D_\mu \varphi^\alpha D^\mu \varphi^\beta h_{\alpha\beta} \quad \text{where} \quad D\varphi = \partial \varphi^\alpha + g A^a K^\alpha_a (\varphi) , \quad (2)$$

and $h_{\alpha\beta}$ is an invariant metric in $G/H$. The Lie algebra of $G$ is the direct sum of the Lie algebra of $H$ and a space $\mathcal{P}$. Thus we can decompose the Yang-Mills field $A = A|_{L(H)} + A|_{\mathcal{P}}$. In the unitary gauge where $\varphi = \varphi_0$ is constant,

$$D\varphi^\alpha_0 = g A^a|_{\mathcal{P}} K^\alpha_a (\varphi_0) \quad (3)$$
and the kinetic term of the Goldstone bosons becomes a mass term for the $P$-component of the gauge field:
\[
L_g = -\frac{1}{2} m_A^2 \sum_{a \in P} A_{\mu}^a A^{\mu a}, \quad \text{where } m_A = gf.
\]

We call this a “Higgsless Higgs mechanism”, because there is no Higgs field left over. As in superconductivity, the basic low energy property of the vacuum is that the Goldstone bosons are covariantly constant. This is a gauge-invariant statement; in the unitary gauge it becomes the statement that the $P$-components of the gauge field vanish. Only the $L(H)$ gauge field remains massless and is dynamical.

The standard Higgs mechanism is obtained by going back to the field $\phi$ carrying a linear representation of $G$ (i.e., in geometrical language, isometrically embedding $G/H$ in a vectorspace). For example, if the potential is $V = \frac{\lambda}{4}(\rho^2 - f^2)^2$ with $\rho = |\phi|$, the Higgs scalar $\rho$ has a mass $m_\rho = \sqrt{\lambda f}$ and the Higgsless Higgs mechanism is a good description of physics at energy $E \ll m_\rho$. In addition, at energies $E \ll m_A$, one can set to zero the $P$-components of the gauge field. This is a property of the electroweak vacuum at energies below approximately 100 GeV.

3. The Higgs phenomenon in MAG

3.1. Fields and gauges

Before introducing a metric, parallel transport in gravity is given by a linear connection, so, a priori, the analog of the gauge group $G$ for gravity is the linear group $GL(4)$. However, in the standard metric and tetrad formalisms, invariance under $GL(4)$ is not manifest. This is because one usually works with special classes of linear bases in the tangent bundle: either coordinate bases $\{\partial_\mu\}$ or orthonormal bases (tetrads) $\{e_a\}$. For our purposes it is convenient to have manifest $GL(4)$ invariance \cite{718}, so we shall work with generic linear bases, or “frames”, $\{\theta_a\}$, where $\theta_a = \theta_a^\mu \partial_\mu$, or equivalently the dual co-frames $\theta_a = \theta_a^\mu dx^\mu$ that shall also be called the “soldering form”. The frame (or co-frame) field will be a dynamical variable, generalizing the tetrad. In addition we have a dynamical metric, whose components in the frame $\{\theta_a\}$ are $\gamma_{ab}$. Finally, there is a dynamical connection with components $A_{\mu}^a b = \theta_c^\mu A_c^{a b}$.

Let us point out right away that, already at the kinematical level, the frame and the metric are non-linear variables, because of the constraints:

- the metric $\gamma_{ab}$ has signature $-,+,+,+$; \hspace{1cm} (5)
- the frame field $\theta^a_\mu$ is non-degenerate: $\det \theta \neq 0$. \hspace{1cm} (6)

\textsuperscript{4}These statements about local physics have well-known geometrical counterparts in the following theorems: a principal bundle $P$ has an $H$ structure iff there exists a global section $\sigma$ of the associated bundle $P \times_G G/H$. Furthermore, a connection $A$ in $P$ is an $H$ connection iff $\sigma$ is covariantly constant. See \cite{6}.
In particular, also $\gamma$ is non-degenerate. This means that, locally, the metric is a field with values in the coset space $GL(4)/O(1,3)$ and the frame has values in $GL(4)$. For this reason we will refer to these fields as gravitational Goldstone bosons.

The components of the metric and connection in coordinate basis are given by

$$g_{\mu\nu} = \theta^a_\mu \theta^b_\nu \gamma_{ab},$$
$$A^{\lambda}_{\mu\nu} = \theta^{-1}_a \Lambda^a_{\lambda b} \theta^b_\nu + \theta^{-1}_a \partial_\lambda \theta^a_\nu.$$ (7)

They can be viewed as composite fields of the fundamental variables $\theta$, $\gamma$ and $A$.

The gauge group of the theory consists of automorphisms of the bundle of linear frames. Locally, they are given by diffeomorphisms and local changes of frame. The latter act on the fields as follows:

$$A_{\mu a b} = \Lambda^{-1}_{- c d} A_{\mu c d} + \Lambda^{-1}_{- c} \partial_\mu \Lambda^c_{b},$$ (8)
$$\theta^a_\mu = \Lambda^{-1}_{- c} \theta^c_\mu,$$ (9)
$$\gamma'_{ab} = \gamma_{cd} \Lambda^c_a \Lambda^d_b.$$ (10)

In particular, choosing $\Lambda = \theta$, we see that we can enforce the gauge condition $\theta^a_\mu = \delta^a_\mu$. This brings us to a coordinate basis and completely breaks $GL(4)$. In this gauge (7) shows that there is no difference between latin and greek indices. This corresponds to the standard formulation of gravity in terms of a metric, possibly with an independent connection. This will be called the metric gauge.

Alternatively one can choose the gauge so that $\gamma_{ab} = \eta_{ab} = \text{diag}(-1,1,1,1)$, leaving an unbroken $O(1,3)$ gauge group. This means that we are using an orthonormal frame. Equations (7) are the usual relations holding in the tetrad formalism, relating the components of the metric and connection in a coordinate frame to those in an orthonormal frame. This will be called the tetrad gauge.

It is crucial that there is not enough gauge freedom to fix both gauges simultaneously. Therefore, unlike the theories of Section 2, where the Goldstone bosons were completely absorbed in the longitudinal part of the gauge field, one of the two Goldstone bosons remains as a physical dynamical variable at low energy.

### 3.2. Lagrangian and VEVs

We can now introduce torsion and non-metricity as the covariant derivatives of the Goldstone bosons:

$$\Theta^{\alpha}_{\mu \nu} = \partial_\mu \theta^a_\nu - \partial_\nu \theta^a_\mu + A^{\alpha}_{a b} \theta^b_\nu - A^{\alpha}_{b \nu} \theta^b_\mu,$$ (11)
$$Q_{\lambda a b} = -\partial_\lambda \gamma_{a b} + A^{c}_{a \lambda b} \gamma_{c b} + A^{c}_{a c} \gamma_{b b}.$$ (12)

Note that the meaning of these tensors is obscured if one works with coordinate or orthonormal frames. Indeed, in the metric gauge (coordinate frames) torsion

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\*One could try to see them as the result of breaking $GL(4)$ in a model with global linear invariance, but this interpretation is not so useful. To justify the terminology, it is enough that the gauge group acts transitively on these variables, as we see in [10]."
appears to be just an algebraic combination of connection components

\[ \Theta_\mu^\rho_\nu = A_\mu^\rho_\nu - A_\nu^\rho_\mu , \]

whereas in the tetrad gauge (orthonormal frames) it is the non-metricity that appears to be a purely algebraic combination of connection components:

\[ Q_{\lambda ab} = A_{\lambda ab} + A_{\lambda ba} . \]

In order to recognize that these tensors play very similar roles, it is necessary to work with generic frames.

Let us now come to the dynamics. The Goldstone bosons can be viewed as a special type of matter, and as in any gauge theory coupled to matter, the Lagrangian has terms quadratic in curvature and terms quadratic in the covariant derivatives of the matter fields. Thus the action of MAG is

\[ S_{MAG} = \int d^4 x \sqrt{-g} L_{MAG} , \quad L_{MAG} = L_P + L_F + L_{TQ} . \]

Here \( L_P = -m_P^2 F_{ab}^{\mu \nu} \) is the Palatini Lagrangian,

\[ L_F = F^{\mu \nu \rho \sigma} (c_1 F_{\mu \rho \sigma} + c_2 F_{\mu \sigma \rho} + c_3 F_{\rho \sigma \mu} + c_4 F_{\mu \rho \sigma} + c_5 F_{\rho \sigma \mu}) \]
\[ + L^{\mu \nu} (c_7 L_{\mu \nu} + c_8 L_{\nu \mu}) + K^{\mu \nu} (c_9 K_{\mu \nu} + c_{10} K_{\nu \mu}) + K^{\mu \nu} (c_{11} L_{\mu \nu} + c_{12} L_{\nu \mu}) \]
\[ + F^{\mu \nu} (c_{13} F_{\mu \nu} + c_{14} L_{\mu \nu} + c_{15} K_{\mu \nu}) + c_{16} L^2 , \]

\[ L_{TQ} = T^{\mu \nu \rho} (a_1 T_{\mu \rho \nu} + a_2 T_{\mu \nu \rho}) + a_3 T^{\mu \nu} T_\mu + Q^{\mu \nu} (a_4 Q_{\mu \nu} + a_5 Q_{\nu \mu}) \]
\[ + a_6 Q^\mu Q_\mu + a_7 \tilde{Q}^\mu \tilde{Q}_\mu + a_8 Q^\mu \tilde{Q}_\mu + a_9 T^{\mu \nu} Q_{\rho \mu 
u} + a_{10} T^{\mu \nu} Q_\mu + a_{11} T^\mu \tilde{Q}_\mu , \]

where \( T_\mu = T^\lambda_\mu , Q_\mu = Q^\lambda_\mu , \tilde{Q}_\mu = Q_\lambda^\mu \).

Here \( L_F \) is a generalization of the Yang-Mills Lagrangian and \( L_{TQ} \) are the kinetic terms of the Goldstone bosons. The Palatini term has no analog and diffeomorphism invariance forbids a potential term for \( \gamma \) and \( \theta \), except for a cosmological term, that we shall ignore.

As a “vacuum” we choose flat Minkowski space: \( F_{abcd} = 0, T_{abc} = 0, Q_{abc} = 0 \). In a suitable gauge, it can be represented as \( A_{\mu}^{ab} = 0, \gamma^{ab} = \eta_{ab}, \theta^{\mu} = \delta^{\mu}_0 \).

Denoting \( a_{\mu}^{ab} \) the fluctuation of the connection, we see that

\[ \Theta_\mu^{\rho_\nu} = a_{\mu}^{\rho_\nu} - a_{\nu}^{\rho_\mu} , \quad Q_{\mu ab} = a_{\mu ab} + a_{\mu ba} . \]

These equations are analogous to (3). Then \( L_{TQ} \) just becomes a mass term for the connection. The curvature also contains a term quadratic in the gauge field, so that the Palatini lagrangian also contributes to the mass term \(-m_P^2 (a_9 a^{ac} a_{bc} - a_{bac}^{a abc})\).

It is natural to assume that all the coefficients \( a_1, \ldots a_{11} \) are of the order of \( m_P^2 \).

For generic values of the coefficients the mass matrix will be non-degenerate and all components of the connection will become massive, with a mass of order of the Planck mass. This is a gravitational version of the “Higgsless” Higgs phenomenon, because it involves only Goldstone bosons and no “Higgs” particle \cite{S9, H10}. 

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3.3. The low energy effective field theory

The Higgs phenomenon removes the connection degrees of freedom from the spectrum below the Planck mass. However, we have already observed that it is not possible to remove both Goldstone bosons by a choice of gauge. One of them remains as a low energy massless degree of freedom. Let us now consider the low energy dynamics of this Goldstone boson.

Here we recall that given $\theta, \gamma$, there is a unique connection $\Gamma$ such that $\bar{\Theta} = 0$, $\bar{Q} = 0$. It is called the Levi–Civita Connection and its components in a general frame are

$$
\Gamma_{abc} = \frac{1}{2} (E_{cab} + E_{abc} - E_{bac}) + \frac{1}{2} (C_{abc} + C_{bac} - C_{cab}) ,
$$

(18)

where $E_{cab} = \theta^{-1} c^\lambda \partial_\lambda \gamma_{ab}$ and $C_{abc} = \gamma_{ad} \theta^d \lambda (\theta^{-1} b^\mu \partial_\mu \theta^{-1} c^\lambda - \theta^{-1} c^\mu \partial_\mu \theta^{-1} b^\lambda)$. In a coordinate frame $C_{abc} = 0$ and this formula reduces to the Christoffel symbols, while in an orthonormal frame $E_{abc} = 0$. Any connection $A$ can be split uniquely $A = \Gamma + \Phi$, where $\Phi$ is a tensor. Then, in the action we may replace $A$ by $\Phi$. Thus we can write $S(A, \theta, \gamma) = S(\Gamma(\theta, \gamma) + \Phi, \theta, \gamma) = S'(\Phi, \theta, \gamma)$. Now we note that

$$
\Theta_{\mu \nu} = \Phi_{\mu \nu} - \Phi_{\nu \mu}, \quad Q_{\mu ab} = \Phi_{\mu ab} + \Phi_{\nu ba}
$$

(19)

and therefore, without making any assumptions about the nature of the vacuum, $\mathcal{L}_{TQ}$ is seen to be a mass term for $\Phi$. The mass matrix is the same as the mass matrix for $a$ when we expanded about the flat space. We assume that it is generic and therefore nondegenerate. This implies that, for any $\theta$ and $\gamma$, the deviation of $A$ from the Levi–Civita connection is massive, with a mass that one may reasonably presume to be of the order of the Planck mass. For all sub-Planckian physics it is therefore a very good approximation to assume that $\Phi = 0$, or in other words $\Theta = 0$ and $Q = 0$. In this way we understand that these conditions, which in GR are simply postulated, are natural properties of MAG at low energy.

If we impose that $\Theta = 0$ and $Q = 0$, the curvature tensor $F$ reduces to the Riemann tensor constructed with $\theta$ and $\gamma$ (or equivalently with the metric $g$), and the Lagrangian $\mathcal{L}_F$ reduces to the Lagrangian of Higher Derivative Gravity (HDG), whereas $\mathcal{L}_P$ reduces to the Hilbert Lagrangian. In this context, the explanation for the stiffness of spacetime - its resistance to being curved - is due to the large value of the Planck density, compared to ordinary energy density. Indeed, at low energy, the HDG action is negligible with respect to the Hilbert action and the equations of motion say that “curvature $\approx \frac{1}{2} m_P^2 T$”, where $T$ is the energy density of matter. It takes an energy density of order of the Planck density to produce a curvature of the order of $m_P^2$. From the point of view of MAG, the stiffness of spacetime is very reminiscent of the Meissner effect. The vacuum has no curvature and due to

\footnote{This is because the Palatini term already gives a Planck mass to some components of $A$. It would be highly interesting is some components of $A$ had a mass that is much below the Planck scale, but we shall not discuss this possibility here.}
the mass of the gauge field, a point-like disturbance will generate a curvature that
decays exponentially within a Planck length.

MAG is much more similar to the theories of the electroweak and strong interaction
than GR, and this somewhat shifts the focus of the questions that one would
like to be answered by quantum gravity. In QCD, crucial issues are confinement and
chiral symmetry breaking. Even though a detailed explanation is lacking, they are
believed to be due to the complicated, strong dynamics at low energy. In particular,
the strong scale emerges from the phenomenon of “dimensional transmutation”. [2]
The central issue of the electroweak theory is the origin of “electroweak symmetry
breaking”. As long as the Higgs particle had not been discovered, one could simply
have modelled it by the Higgsless theory described in Section 2, where the order
parameter carries a nonlinear realization of the gauge group. This theory has the
drawback of not being renormalizable. Even worse, it becomes strongly coupled
and breaks down at energies comparable to the Higgs VEV. In the SM, the order
parameter carries a linear realization and the theory is renormalizable. For the time
being, this description in terms of an elementary scalar field is completely satisfac-
tory, but this may not remain the case indefinitely. In fact, we have the example of
superconductivity, where the Landau-Ginzburg description in terms of a complex
order parameter is known to be only an effective low energy approximation of BCS
theory, where the scalar is viewed as a condensate of pairs of electrons.

Since the order parameter for the gravitational Higgs phenomenon is the met-
ric, the central question for a quantum theory of spacetime is: why is the metric
nondegenerate, and more precisely, why does it have signature $- + ++$? [12]. This
question cannot be answered within the present formulation of MAG, which is based
on nonlinear (Goldstone) fields. One may begin to answer it by extending the va-

didity of the theory towards higher energies. In Landau-Ginzburg theory and in the
SM, a more fundamental description is given in terms of the linearly transforming
field $\phi$, and going from $\varphi$ to $\phi$ one has to enlarge the number of dynamical fields. In
the case of gravity, this is not necessary, because the constrains $[0]$ are formulated
as inequalities rather than equalities. To go from the nonlinearly realized theory to
the linear one, in gravity, it is enough to relax those constraints. No new degrees of
freedom are needed. Unfortunately, when one relaxes these constraints, the metric
may become degenerate or even zero, and it is very difficult to formulate dynam-

ics under these circumstances. Some attempts to derive the constrains $[1]$ from a
self-consistent bi-metric dynamics were made in $[13,14]$.

4. Towards Quantum MAG

Let us discuss a little more the properties of MAG. In order to minimize the number
of independent variables, we can work in the metric gauge (i.e. with coordinate frames),
where the dynamical variables are the metric $g_{\mu\nu}$ and the independent

8See $[11]$ for possible analogies between QCD and gravity.
linear connection \( A_\lambda^\mu \). When linearized around flat space, it is found to propagate many different degrees of freedom, depending on the values of the coefficients \( c_i \) and \( a_i \). In general, the metric fluctuation \( h_{\mu\nu} \) contains one \( 2^+ \), one \( 1^- \) and two \( 0^+ \) states (where \( J^P \) denotes a state with spin \( J \) and parity \( P \)). The fluctuation of the connection \( a_\lambda^\mu \) contains one \( 3^- \) state, three \( 2^+ \), two \( 2^- \), six \( 1^- \), three \( 1^+ \), four \( 0^+ \) and one \( 0^- \). In contrast to the Yang-Mills Lagrangian, that uses a positive definite inner product in the Lie algebra, the Lagrangian \( L_{MAG} \) contains in general ghosts and tachyons. (This is what also happens, but for different reasons, in HDG). It is therefore interesting to find ranges of values of coefficients for which there are no ghosts and tachyons. Given that the general MAG action depends on 28 parameters, this is an extremely complicated issue. It has been partly solved in two special cases, namely when one imposes \( Q = 0 \) \[15,16\] or \( T = 0 \) \[17\]. Using \( F = R + \nabla \Phi + \Phi^2 \) we can rewrite \( S(g, A) = S'(g, \Phi) \) where, schematically,

\[
S' = \int d^4x \sqrt{|g|} \left[ R + \Phi^2 + R^2 + R\nabla \Phi + R\Phi^2 + (\nabla \Phi)^2 + \nabla \Phi \Phi^2 + \Phi^4 \right].
\]

It is interesting to observe that when one rewrites the action in this way, the ghost- and tachyon-free actions turn out not to contain any terms \( \sim R^2 \). Thus, the origin of the ghosts and tachyons in MAG and in HDG are, at least in some cases, related. Unfortunately, there appears to be no reason for these special conditions to be radiatively stable, so that the issue of the ghosts and tachyons remains open. We will discuss this further below.

Classical scale invariance is achieved by putting \( m_P = a_1 = a_2 = \ldots = a_{11} = 0 \). This leaves the free parameters \( c_1, c_2 \ldots c_{16} \). In fact, if we postulate \( \delta A_\rho^\nu = 0 \), as in Yang-Mills theory, \( L_F \) is even Weyl-invariant. Scale invariance is in any case broken in the quantum theory because of the running of couplings under the Renormalization Group (RG). However, there may be Fixed Points (FP), where the beta functions vanish and one achieves quantum scale invariance \[18,19\]. If a RG trajectory reaches a FP in the UV, it describes a UV complete theory. There can be free fixed points, leading to asymptotic freedom (AF) and interacting fixed points, leading to asymptotic safety (AS).

Due to the technical complication, no systematic analysis of the RG of MAG has been undertaken so far. The literature on AS of gravity focuses almost entirely on metric theories. At one loop HDG can be AF (depending on the sign of some parameter) \[20\]. This result has been reproduced using the Functional RG, but in this case Newton’s coupling is nonzero at the FP \[21,22,23\]. Whether the coefficients of \( R^2 \) are AF or AS in a more sophisticated approximation, is a question that is currently still being studied, but the expectation is that there should exist a FP where all couplings are nonzero \[24\]. There have been some calculations of beta functions with the Functional RG in a theory with independent connection \[25,26,27,28\] but none in the full MAG with running couplings \( c_i \). Altogether it seems that quantum scale invariance in MAG is possible and even likely, but much more work is needed to establish this.

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If AS can be achieved in MAG, we remain with the issue of ghosts, and more generally of unitarity. Over time, there have been many proposals on how to circumvent the ghost problem, see [30,31,32] for some recent proposals. In the literature on AS gravity, where the connection is always constrained to be the Levi-Civita connection, but higher derivative terms are present, one common point of view is that ghosts are artifacts of a finite truncation: if one could keep all terms in the action, the propagator would be an entire function with only one pole at zero. Another possibility is that ghosts are an artifact of the expansion around flat space [33]. In the context of MAG, where the Planck scale appears as the mass of the connection, the following mechanism may be at work: the physical (pole) mass of a state is defined as the running mass evaluated at the pole mass itself: \( m_{\text{phys}} = m(k = m_{\text{phys}}) \).

If the theory achieves scale invariance at high energy, the running mass runs like the cutoff \( k \) itself, and this implicit equation may well not have any solution [34]. We note that this mechanism is actually blind to the sign of the residue, so that it may eliminate both ghosts and healthy states. In fact it suggests that no particle could exist with mass comparable or higher than the Planck mass. Interestingly, a similar conclusion has been reached also in a context of non-commutative geometry [35]. If scale invariance at high energy removes all heavy particles from the physical spectrum, the whole phenomenology of MAG could be very different from what is normally envisaged [36].

Lastly, it is worth recalling that MAG can be easily generalized to construct unified theories of gravity and Yang-Mills interactions [8,37,38]. To this end, let us consider the subclass of MAGs with \( Q = 0 \). Then, the gauge group of the theory is the Lorentz group \( SO(1,3) \). All other interactions can be unified in the orthogonal gauge group \( SO(10) \). It is natural to imagine a unification within a larger pseudo-orthogonal group - either \( SO(1,13) \) or \( SO(3,11) \) or, more exotically \( SO(7,7) \). We shall call such a unified theory a “graviGUT” and its main order parameter is now an extended soldering form \( \theta^a_{\mu} \), with \( a = 1 \ldots 14 \). We will not review this in detail (see [39] for a recent review of unified theories) except to recall the following points. The fermions of the SM are at the same time spinors of the Lorentz group and of the GUT group \( SO(10) \). Such fields can be viewed as forming an irreducible representation (a 64-real-dimensional Majorana-Weyl spinor) of the graviGUT group \( SO(3,11) \) or \( SO(7,7) \). The group \( SO(1,13) \) has Weyl representation, but no Majorana-Weyl, and the decomposition of a 64-complex-dimensional Weyl gives both a family and an antifamily, that is hard to get rid of. It is possible to write an action for the fermions, coupled to the graviGUT gauge field, that reduces to the correct action of \( SO(10) \) fermions coupled to gravity if one postulates the existence of suitable VEVs. Natural generalizations of the torsion squared terms, in the presence of a VEV for the extended soldering form, give a mass to all components of the graviGUT gauge field except for \( SO(10) \). The breaking of \( SO(10) \) to the SM gauge group, requires separate order parameters.

Altogether, one can say that some ingredients of a unified theory are present and work as expected, but several aspects remain to be properly understood.
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