Spin–isospin properties of $N = Z$ odd–odd nuclei from a core+ $pn$ three-body model including core excitations

Futoshi Minato$^{1,2,a}$, Yusuke Tanimura$^{3,4}$

$^1$ Nuclear Data Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan
$^2$ NSCL/FRIB Laboratory, Michigan State University, East Lansing, Michigan 48824, USA
$^3$ Department of Physics, Tohoku University, Sendai 980-8578, Japan
$^4$ Graduate Program on Physics for the Universe, Tohoku University, Sendai, 980-8578, Japan

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Abstract For $N = Z$ odd–odd nuclei, a three-body model assuming two valence particles and an inert core can provide insight into pairing correlations in the ground state and spin–isospin excitations. However, since residual core–nucleon interactions can have a significant impact, the inclusion of core excitations in the model is essential for a detailed understanding of both the ground state and spin–isospin properties. To quantitatively understand the effect of core excitation on the ground state and spin–isospin excitations of $N = Z$ odd–odd nuclei, we solve the three-body core–nucleon–nucleon problem including core vibrational states described by the random-phase approximation. We pay a particular attention to the magnetic dipole and Gamow–Teller transitions between $0^+$ and $1^+$ states of $N = Z$ nuclei and discuss it in terms of the SU(4) multiplet. We also discuss the effect of coupling to the core vibration on low energy spectra of even–odd nuclei, the spatial distribution, core contributions to the ground state, and possible uncertainties in the present framework.

1 Introduction

The nuclear many-body problem remains a complicated and difficult task to solve due to the large number of degrees of freedom involved. However, schematic models simplifying the nuclear many-body problem have succeeded in explaining experimental observables theoretically. The typical method is a three-body model, which assumes two particles and a core nucleus (or three particles) in the system. The core is usually either a closed-shell nucleus or one which can be virtually regarded as structureless.

This assumption greatly reduces the model space and allows us to solve the many-body problem rather simply. Thus, the three-body model has been applied to mainly light nuclei near the drip lines. In the last decade, new experimental facilities using a highly intensive radioactive beam have enabled us to explore nuclei near the drip-line of the nuclear chart, where the three-body model becomes an effective approach to studying correlations between valence nucleons in neutron-rich [1–18] and proton-rich nuclei [19–23] including quasi-bound systems [4–8,12–15,19,24]. More recently, three-body models are employed also in the context of Efimov physics in $^{19}$B [25] and $^{72}$Ca [26]. A three-body model has also been applied to one-proton radioactivity of $^6_3$Li hypernucleus [27].

The three-body model also has an advantage to focus on the two-body subsystem consisting of two valence particles and provides us a beneficial understanding of pairing correlations in the ground state and spin–isospin excitations. For this reason, in the previous work [28,29], $N = Z$ odd–odd nuclei, which are made up of one proton, one neutron, and an even–even core nucleus with mass $A = 14–100$, were investigated in order to study the effect of isospin $T = 0$ and $T = 1$ pairing correlations between the proton and neutron. It was found that the SU(4) multiplets [30], which are characterized by the nucleon spin–isospin plane, are notably present in $^{18}$F and $^{42}$Sc and result in strong magnetic dipole (M1) and Gamow–Teller (GT) transitions.

Intuitively, the assumption of an inert core in the three-body model looks reasonable as long as the energy region in question is below the first excited state of the core nucleus. However, it has been seen that a residual two-body interaction significantly breaks the stationary structure of a core nucleus even for spherical magic nuclei [31,32]. It is also known that a core polarization due to particle–vibration cou-
pling causes the parity inversion of 1/2− and 1/2+ states in 11Be [33,34]. Thus, the assumption of the inert core might be too simple. In fact, low-lying Gamow–Teller strengths were not reproduced well in the previous work [28,29]. One is able to consider the effect of core breaking by extending the active model space to include one or more lower shells and performing a large scale shell model calculation. However, such an approach sacrifices the simple picture of three-body model. As an alternative approach, one can assume the core breaking as a core collective excitation. This approach keeps the concept of a three-body model and allows us to focus on the subsystem consisting of valence nucleons in the same manner as the three-body model with an inert core.

The core collective excitations are characterized as rotation and vibration. The coupling to rotationally excited states have been investigated in the two-body system, i.e. core nucleus + one valence particle [35–39] and the three-body system such as 12Be [40,41] and 31Li [42]. The coupling to surface vibrations can be similarly described by particle–vibration coupling [43,44] and has been applied to various systems, such as light nuclei [45], neighbors of magic nuclei [46–54], three-body systems [55–58], open shell nuclei [59], and deformed systems [60]. The valence particle(s) may couple with a pairing vibration, however, its contribution is considered to be small [61].

We discuss in this paper spin–isospin properties of N = Z nuclei from the viewpoint of one step beyond Refs. [28,29] by allowing the core nucleus to vibrationally excite. The same nuclei of Ref. [28] are chosen as the target of this work, that is, 14N( 12C+pn), 18F( 16O+pn), 30P( 28Si+pn), 34Cl( 32S+pn), 42Sc( 40Ca+pn), and 58Cu( 56Ni+pn) N = Z odd–odd nuclei. The main focus of this paper is to see the SU(4) multiplet, which explains the large B(M1) and B(GT) of 18F and 42Sc [28] and discuss the difference from the result obtained in the case of the inert core assumption.

The core vibrational degree of freedom is included in the same manner as used in the particle–vibration coupling of the two-body system [43,50,60,62]. Phonons are estimated by the random-phase approximation (RPA) in this work. The obtained result is compared with that calculated by the three-body model using an inert core and try to understand the core vibration effect qualitatively. Note that 14N and 30P have the deformed cores of 12C and 28Si, respectively. In these nuclei, rotational coupling will also be present, and so our results are thus a minimum estimation of the coupling with core excitations for those nuclei.

The inclusion of the core vibration will generate some changes in the nuclear structure, which has not been seen in the three-body model with an inert core. In addition to the spin–isospin properties and SU(4) multiplet, we therefore discuss mean values of spatial distribution of valence particles. We also examine which phonon state of the core nucleus plays an important role in the ground state wave function in the framework used in this paper.

Section 2 describes the framework of three-body model calculation with core excitation and the model space used, and Sect. 3 provides results of low energy spectra of core + n systems, magnetic moments, spin–isospin excitations, low-lying GT transitions, core contribution, and mean values of spatial distributions calculated in order to see the effect of core vibrations. We also discuss a possible uncertainty of the model used in this work. The results are compared with those calculated by the three-body model using an inert core. We summarize our discussion in Sect. 4.

2 Theoretical framework

2.1 Three-body system with core vibrational excitations

The formalism of the three-body model with an inert core can be found in Refs. [1,4,5,63]. We now extend it to the case with core vibrational excitations. For the general purpose, we label one of the valence nucleons as 1 and the other as 2 (1 and 2 are either proton or neutron). In the rest frame of the core nucleus, the Hamiltonian is given by

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V_{1\nu}(\mathbf{r}_1) + V_{2\nu}(\mathbf{r}_2) + V_p(\mathbf{r}_1, \mathbf{r}_2) + \frac{(p_1 + p_2)^2}{2M_{\text{core}}} + H_{\text{core}}. \tag{1}$$

The first two terms in Eq. (1) are the kinetic energies, and $m_1$ and $m_2$ take either neutron mass, $m_n$, or proton mass, $m_p$. The function $V_{1\nu}(\mathbf{r}_1)$ is a core–nucleon effective interaction working between the valence nucleon labeled by $i$ and the core nucleus, and $V_p(\mathbf{r}_1, \mathbf{r}_2)$ is a pairing interaction between the valence nucleons (see Eq. (12)). The sixth term is the recoil kinetic energy term and $M_{\text{core}} = N_c m_n + Z_c m_p$ is the mass of the core nucleus, where $N_c$ and $Z_c$ are the number of neutron and proton of the core nucleus. The last term in Eq. (1) is the Hamiltonian for the intrinsic degree of freedom of the core nucleus.

The effective interaction $V_{1\nu}(\mathbf{r}_1)$ is generally density-dependent. Since the amplitude of the core surface vibration is small, we can expand the interaction up to the first order with respect to the density,

$$V_{1\nu}(\mathbf{r}) \simeq V_{1\nu}^0(\mathbf{r}) + \sum_\nu \int_\mathbb{R}^3 \frac{\delta V_{1\nu}(\mathbf{r})}{\delta \rho(\mathbf{r}')} \delta \rho_i(\mathbf{r}') d\mathbf{r}', \tag{2}$$

The first term is the core–nucleon interaction at the static density of core nucleus, $\delta V_{1\nu}(\mathbf{r})/\delta \rho(\mathbf{r}')$ in the second term is the residual interaction generated by the core surface vibration, and $\delta \rho_i(\mathbf{r})$ is the transition density operator of the vibra-
tional phonon state $v$, acting on a core intrinsic wave function. Spherical harmonics altering the orbital angular momentum of valence nucleons and core nucleus are included in the transition density operator. The residual interaction can be classified by isoscalar (IS) and isovector (IV) components [50,62] as 

$$\int \frac{\delta V_{ic}(r)}{\delta \rho_v(r')} \delta \hat{\rho}_v(r') d\mathbf{r}' = \int \left( v_{res}^{IS}(r, r') \delta \rho_v^{IS} + v_{res}^{IV}(r, r') \delta \rho_v^{IV} \right) d\mathbf{r}' .$$

(3)

The first term of the right hand side of Eq. (3) does not change the isospin of either the $pn$-subsystem or core nucleus, while the second term of the isovector component may change their isospin by $\Delta T = 1$. Due to this, for example, the $1^+$ ground state has a component of isospin $T = 1$ in both the $pn$-subsystem and core nucleus in addition to the $T = 0$ component.

We neglect the recoil kinetic energy of the core nucleus in the sixth term of Eq. (1) as done in Ref. [28] to purely discuss the effect of core vibration. We have confirmed that some slight improvements are obtained when including the sixth term. However, the effect of the recoil kinetic energy does not change our discussion significantly.

From Eqs. (1), (2), and (3), the effective Hamiltonian then becomes 

$$H = H_1 + H_2 + V_p(r_1, r_2) + H_{core} + \sum_{ij} \int \left( v_{res}^\beta(r_1, r) + v_{res}^\beta(r_2, r) \right) \delta \hat{\rho}_\nu^\beta(r) d\mathbf{r},$$

(4)

($\beta =$IS and IV), and we define 

$$H_i = \frac{\mathbf{P}_i^2}{2m_i} + V_{ic}^0(r_i).$$

(5)

Equation (5) is identical to the single-particle shell model Hamiltonian for the valence nucleon $i(= 1, 2)$ in the static core. By imposing spherical symmetry on the system, it satisfies the relation; 

$$H_i \psi_{njli}(r_i) = E_{njli} \psi_{njli}(r_i),$$

(6)

where $\psi_{njli}$ and $E_{njli}$ are the eigenfunction and the eigenvalue of the single-particle states (principal quantum number $n$, total angular momentum $j$, and orbital angular momentum $l$), respectively.

We diagonalize the effective Hamiltonian of Eq. (4) by the superposed wave functions of two-body and intrinsic core bases coupled with total spin $I$ and its projection on the $z$-axis $K$, defined as 

$$\psi^{IK}(r_1, r_2, \xi) = \sum_{\alpha J} \sum_{\alpha J'} c_{\alpha J} \left[ \Theta_{\alpha}^J(r_1, r_2) \Phi^{0}_\alpha(\xi) \right]^{IK} + \sum_{\alpha J' V} c_{\alpha J'} \left[ \Theta_{\alpha}^{J'}(r_1, r_2) \Phi^{L \nu}_{\alpha}(\xi) \right]^{IK},$$

(7)

where $\Theta_{\alpha}^{J M}$ is the uncorrelated two-body wave function of valence nucleons coupled with angular momentum $J$ and its projection on the $z$-axis $M$, and $\Phi^{L \nu}_{\alpha}(\xi)$ is the core intrinsic wave function with multipolarity $L$ and its projection on the $z$-axis $M$. The subscript $\alpha$ means the set of good quantum numbers of particles 1 and 2, namely $(n_1, j_1, l_1)$ and $(n_2, j_2, l_2)$. The single-particle levels that are occupied by the core nucleus at the mean-field level are excluded for the states of particles 1 and 2. The two-body wave function, $\Theta_{\alpha}^{J M}$, is given in an anti-symmetrized form of two single-particle wave functions of $\psi_{njli}$. The variable $\xi$ indicates the core intrinsic coordinate. The first term of Eq. (7) is the superposition of the two-body wave function and the core ground state ($\Phi^0_{\alpha}$), and the second term is that of the two-body wave function and core excited states ($\nu \neq 0$). The coefficients $c_{\alpha 0}$ and $c_{\alpha V}$ are determined by the diagonalization of Hamiltonian of Eq. (4).

The core ground state is defined as 

$$\hat{Q}_v \phi^{00}_\nu(\xi) = 0, \tag{8}$$

and the excited states are 

$$\nu = \nu_{L \nu_{M \nu}}(\xi) = \hat{Q}_v^\dagger \phi^{00}_\nu(\xi), \tag{9}$$

where $\hat{Q}_v$ and $\hat{Q}_v$ are the phonon creation and annihilation operators of core vibrational states, respectively. The definition of $\hat{Q}_v^\dagger$ and $\hat{Q}_v$ in the case of the RPA are given in Ref. [64]. The function $\Phi^{L \nu_{M \nu}}_{\alpha}$ is the eigenstate of $H_{core}$.

$$H_{core} \psi^{L \nu_{M \nu}}_{\alpha}(\xi) = E_v \psi^{L \nu_{M \nu}}_{\alpha}(\xi), \tag{10}$$

where $E_v$ is the excitation energy of core nucleus.

The transition density operator of Eqs. (2), (3), (4) is expressed by the phonon creation and annihilation operators [43], 

$$\delta \hat{\rho}_\nu^\beta(r) = \delta \rho_\nu^\beta(r) \hat{Q}_v^\dagger + (\delta \rho_\nu^\beta)^*(r) \hat{Q}_v, \tag{11}$$

where $\delta \rho_\nu^\beta(r) \propto Y_{L \nu_{M \nu}}$ is the transition density of $\nu$ state.

The zero-range type interaction is used as the pairing interaction of $V_p(r_1, r_2)$ in this work. It is separated into the spin-singlet and spin-triplet components as [28]

$$V_p(r_1, r_2) = \delta(r_1 - r_2) \times \left\{ \hat{P}_1 v_1 \left( 1 + x_{1s} f^{as}(r_1) \right) + \hat{P}_2 v_2 \left( 1 + x_{1s} f^{as}(r_1) \right) \right\}, \tag{12}$$

$$\hat{P}_i = \frac{\mathbf{P}_i^2}{2m_i} + V_{ic}^0(r_i).$$

(13)
where \( f(r) = (1 + \exp[(r - R_n)/a])^{-1} \). The operators \( \hat{P}_x \) and \( \hat{P}_y \) project the uncorrelated two-body wave function on the spin-singlet and spin-triplet channels, respectively. The parameters \( v_x, x_s, x_t, v_t / a, a \), and \( \alpha \) are determined from experimental data, the details of which are presented in the next section.

As in Refs. [43,50,60,62], this work includes a polarization effect, but does not take into account a correlation effect which appears when the correlated ground state of the core nucleus is properly included [44,49,65]. The correlation effect is weaker than the polarization effect for the case of particle–core coupling [49], although it plays an important role in light systems [33,55]. We should note that a similar correlation effect, but does not take into account a correlation between \( \Theta \) function of Eq (7) violates the Pauli principle because the core nucleus is properly included [44,49,65]. The correlation effect, but does not take into account a correlation in light systems [33,55]. We should note that a similar framework to the present model, but with the correlation effect, has been already applied to study pairing correlations of \(^{12}\text{Be} [55], ^{120}\text{Sn} [56,57] \), and also for the nuclear matter [66–68]. There is also a remark that the model wave function of Eq (7) violates the Pauli principle because the anti-symmetrization between \( \Theta^d \) and \( \Phi_v^L \) is not taken into account. Therefore, Eq. (7) assumes that valence particles are not blocked significantly by particles excited from core nuclei. To the contrary, the three-body model with inert core does not break the Pauli principle. However, we would like to note that the three-body model with inert core is also an approximation as the one with vibrational core. The anti-symmetrization would be taken into account by considering the commutation relation of particle creation/annihilation operator and phonon creation/annihilation operator defined in Eqs. (8) and (9). We consider it as the future work.

2.2 Model space

2.2.1 Vibrational states

The vibrational excited states of the core nucleus are calculated by the RPA with a Skyrme force. All of the residual interactions is taken into account in the RPA calculation, however, we omit the spin-orbit force because it is numerically time-consuming but has a minor role on resonance states. Continuum states are discretized by introducing a box boundary condition of \( r_{\text{max}} = 30 \) fm (a step size is equal to 0.1 fm). Cutoff energies of single-particle levels and unperturbed 1p–1h states are set to be 200 MeV. We used the SkM* effective force [69], which provides a reasonable value for the low-lying \( 2^+ \) state [70] (we have also tested with other effective forces, but the results of the three-body system did not change significantly). Although the RPA cannot always reproduce experimental data, especially of low-lying states, it would be reasonable to use it in order to examine the core vibration effect. The isoscalar dipole states are not included because it is nothing but an unphysical translation mode. The phonon states to be coupled with the valence nucleon(s) are restricted to natural parity states from \( L = 0 \) to 5 below \( E_v \leq 30 \) MeV with the fraction of the total isoscalar or isovector strength being larger than 5%. The strengths obtained exhaust the energy weighted sum rule above 99% for \( L = 0 \) to 5; contributions from \( L \geq 6 \) are negligibly small. We do not consider couplings to unnatural parity states and charge-exchange vibration modes of core nuclei to retain the dimension of diagonalization of Hamiltonian at a numerically feasible level. It is expected that their influences are not as strong as those of the natural parity states because of the small collectivities at low energies. However, it is reported that couplings to unnatural parity states generate a repulsive pairing interaction and quench pairing gaps [57,71]. In addition, a recent study reveals that couplings to the charge-exchange modes induces a redistribution of the GT strength of magic nuclei [72]. Therefore, the coupling to unnatural parity states and charge-exchange mode might give some variations to the present result. Including those effects in the three-body model is planned as our future work.

In RPA, the transition densities of Eq. (11) are calculated by

\[
\delta \rho_{v,q}(r) = \frac{1}{\sqrt{2L_v + 1}} \sum_{mi} \left( X^v_{mi} + (-1)^{L_v} Y^v_{mi} \right) \times u_{nu,lmn}(r)u_{ni,ji}(r)(f_{mj}\|Y_{L_v}\|ji)Y^{f}_{nu,LM_v}(\hat{r}).
\]

where \( u_{nij}(r) \) is the radial part of the single-particle wave function of \( \psi_{nij}(r), q \) takes the proton or neutron, and indices \( m \) and \( i \) specify particle and hole states, respectively. The coefficients \( X_{mi} \) and \( Y_{mi} \) in Eq. (13) are the RPA forward and backward amplitudes, respectively [64]. The isoscalar and isovector transition densities from Eq. (11) are given by

\[
\delta \rho^{1S}_{v,n} = \delta \rho_{v,n} + \delta \rho_{v,p} \quad \text{and} \quad \delta \rho^{1V}_{v} = \delta \rho_{v,n} - \delta \rho_{v,p},
\]

2.3 Two-body wave functions and interactions

Nucleon mass in Eq. (4) is set to \( m = m_n = m_p = 938 \) MeV/c\(^2\). The core nucleus is assumed to be a spherical even–even nucleus. The core-valence nucleon interaction in the static density, \( V^{0}_{v}(r) \), is replaced by a phenomenological Woods–Saxon potential of the same form as Eq. (3) of [28]. The spatial parameters for the Woods–Saxon potential are \( R_n = 1.27A^{1/3} \) fm, and \( a = 0.67 \) fm, where \( A_c = N_c + Z_c \). The Coulomb potential is calculated by a uniformly charged sphere of radius \( R_n \) and charge \( Z_c e \), and hyperfine structure is set to be \( e^2/\hbar c = 1/137.036 \). We include all the possible combination satisfying \( \epsilon_{n_1j_1l_1} + \epsilon_{n_2j_2l_2} \leq E_{\text{cut}} \) as a two-body wave function of the valence nucleons \( \Theta^{JM}(r_1, r_2) \), where \( E_{\text{cut}} \) is set to be 20 MeV.

The spin-orbit strength, \( v_{ls} \) (see Eq. (3) of [28]), is determined so as to reproduce the LS splitting between the \( 1d_{5/2} \) and \( 1d_{3/2} \) states of \(^{17}\text{O} \). The potential depth, \( v_0 \) (see Eq. (3)}

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Table 1 Parameters of the Woods–Saxon potential and pairing interaction. The values for the case of inert core \cite{28} are listed for comparison with those of vibrational core (vib. core).

| Woods–Saxon potential | Parameter | Nucleus | Vib. core | Inert core |
|-----------------------|-----------|---------|-----------|------------|
| \( v_0 \) (MeV)       | \(^{12}\)C | -39.67  | -40.63    |            |
|                       | \(^{16}\)O | -44.89  | -49.21    |            |
|                       | \(^{28}\)Si | -44.45  | -47.30    |            |
|                       | \(^{32}\)S | -45.61  | -46.53    |            |
|                       | \(^{40}\)Ca | -48.55  | -51.79    |            |
|                       | \(^{56}\)Ni | -48.43  | -50.95    |            |

| Pairing interaction  | Parameter | Vib. core | Inert core |
|----------------------|-----------|-----------|------------|
| \( s_x \)            | -1.138    | -1.229    |            |
| \( s_t \)            | -1.248    | -1.417    |            |
| \( \alpha_s, \alpha_t \) | 1.000     | 1.233     |            |

The strength parameters of the pairing interactions of Eq. \((12)\) are determined from the proton–neutron scattering length as \cite{63}

\[
v_{s,t} = \frac{2 \pi^2 \hbar^2}{m} \frac{2 \alpha_{pn}^{(s,t)}}{\pi - 2 \alpha_{pn}^{(s,t)} k_{\text{cut}}}, \tag{14}\]

where \( \alpha_{pn}^{(s)} = -23.749 \text{ fm} \) and \( \alpha_{pn}^{(t)} = 5.424 \text{ fm} \) \cite{73} are the empirical \( p–n \) scattering length in the spin-singlet and spin-triplet channels, respectively, and \( k_{\text{cut}} = \sqrt{m E_{\text{cut}}/\hbar^2} \). The other parameters of the pairing interactions are determined by the \( 0^+, 1^+ \) and \( 3^+ \) state of \(^{18}\)F nucleus as done in \cite{28}. The result of the pairing parameters are summarized in Table 1.

The isoscalar and isovector residual interactions of \( v_{\text{res}}^{\text{IS,IV}} \) can be derived from the second derivative of the Skyrme energy density with respect to densities. In order to simplify our numerical calculations, in this work, we used the momentum dependent terms by adopting the Landau–Migdal form \cite{50,62}, in which the corresponding parameters of \( t_1, x_1, \alpha, \) and \( k_F \) are taken from SkM* force.

The Hamiltonian matrix of Eq. \((4)\) is sparse and its dimension reaches about a few hundred thousand. The eigenvalue problem of such large sparse matrices is numerically solved by the JADAMILU code \cite{74}.

3 Numerical result

3.1 Low energy spectrum

We first begin with discussing the result of low energy spectra of \( N = Z \) core + \( n \) system. The calculation is carried out by omitting the second, third, and \( v_{\text{res}}^{\beta}(r_2, r) \) terms of Eq. \((4)\) and set the valence nucleon 1 as a neutron. Low energy levels for \(^{13}\)C, \(^{17}\)O, \(^{29}\)Si, \(^{33}\)S, \(^{41}\)Ca, and \(^{57}\)Ni are shown in Fig. 1. If the calculated level ordering is different from the experi-
mental one, we label the spin-parity state anew near the corresponding levels of inert core (inert) and vibrational cores (vib). Spectra with inert core are determined simply by the unperturbed single-particle energies. For $^{17}\text{O}$, the results of inert core and vibrational core are almost the same. This is because $v_0$ and $v_g$ parameters are fitted to experimental data in the same way for both cases. The core + $n$ model with an inert core extremely overestimates several states, e.g. $5/2^+$ and $3/2^+$ of $^{29}\text{Si}, 5/2^+$ and $1/2^+$ of $^{33}\text{S}, 3/2^+$ of $^{41}\text{Ca}$, and $7/2^-$ of $^{57}\text{Ni}$. Some of those deviations are moderated by considering the core vibration, e.g. $5/2^+$ of $^{29}\text{Si}, 1/2^+$ and $5/2^+$ of $^{33}\text{S}, 3/2^+$ of $^{41}\text{Sc}$, and $7/2^-$ of $^{57}\text{Ni}$.

Focusing on the first excited states, they are reproduced within 1 MeV except $^{13}\text{C}$ and $^{33}\text{S}$ both for the inert and vibrational core, and $^{57}\text{Ni}$ of the vibrational core. Several reasons of the deviations can be considered, e.g. the assumption of spherical shape of core nuclei and omission of the correlation effect. It should be noted that a work considering the correlation effect and phenomenologically fitted phonon states [45] and a work considering the deformation effect [81] reproduce reasonably the low energy levels of $^{15}\text{C}$. It is also reported that the low energy levels of $^{33}\text{S}$ can be reproduced by the coupling to vibrational cores with locally fitted parameters [82,83]. Underestimations of the transition strength associated with collective phonon states are another possibility that will be discussed in the case of $^{16}\text{O}$ and $^{40}\text{Sc}$ in Sect. 3.6. Looking at the level ordering, $1/2^-, 3/2^-$ and $5/2^-$ of $^{17}\text{O}$, and $3/2^+$ and $5/2^-$ of $^{41}\text{Ca}$ are inverted for inert core. The level ordering of $^{41}\text{Ca}$ is recovered by including the core vibration effect. However, the level ordering of $1/2^-, 3/2^-$ and $5/2^-$ of $^{17}\text{O}$ remain the same as the inert core, and $1/2^-$ and $5/2^-$ of $^{57}\text{Ni}$ are inverted for vibrational core although it is reproduced in the case of the inert core.

Calculations of the three-body model with a vibrational core are carried out within these low energy levels renormalized by phonons together with other higher levels that are not shown in Fig. 1. We discuss the result of the three-body model in the following sections.

3.2 Magnetic moments and spin–isospin excitations

In this section, we present the following: the energy difference between $1^+_1$ and $0^+_1$ states ($\Delta E$), the magnetic moment ($\mu$) and the magnetic dipole moment ($B(M1)$) from the first $0^+$ (1$^+$ for $^{34}\text{Cl}$ and $^{42}\text{Sc}$) excited state to the $1^+$ (0$^+$) ground state, and the low-lying GT strength ($B(GT)$). One of the interests of this work is to see the variance between our results and those using three-body model with an inert core, so that our outcomes are compared with the previous work of Ref. [28]. By doing so, we try to understand the core vibration effect. In calculating $\mu$, $B(M1)$, and $B(GT)$, we did not take into account the contributions from the core nucleus (the third term of Eq. (21) in the appendix) because they are expected to be small at a low energy.

The results of $\Delta E$, $\mu$, and $B(M1)$ are listed in Table 2. The three-body model calculations of vibrational core (vib) are compared with those using an inert core (inert) and experimental data (exp). We would like to remind the reader that $\Delta E$ of $^{18}\text{F}$ is reproduced perfectly because it is used to fit the parameters of the pairing interaction used in this work. Although the results of the three-body model using an inert core has been already discussed in Ref. [28], we briefly demonstrate the outcomes. The three-body model using an inert core is just to discuss the interplay between the isoscalar spin-triplet and the isovector spin-singlet pairings. In spite of the simple picture, we could explain several features. The main result is that the large $B(M1)$ of $^{18}\text{F}$ and $^{42}\text{Sc}$ and the relatively small $B(M1)$ for the other nuclei. This nature can be explained by considering the first $0^+$ and $1^+$ states of $^{18}\text{F}$ and $^{42}\text{Sc}$ as the members of the SU(4) multiplet in the spin–isospin space [28,30]. The spin–isospin operator $s \tau_z$, where $s = \frac{1}{2}\sigma$ is the spin operator and $\tau_z$ is the $z$-component of isospin operator, strongly connects the $0^+$ and $1^+$ states in the same SU(4) multiplet, and the $B(M1)$ value become so large in those nuclei as a result. It can be understood that the $B(GT)$ for $^{14}\text{N}$, $^{30}\text{P}$, and $^{34}\text{Cl}$ are quenched because their $0^+$ and $1^+$ states do not form the SU(4) multiplet. It is also found that the level orderings of $1^+$ and $0^+$ states of $N = Z$ nuclei, including the inversion observed for $^{34}\text{Cl}$ and $^{42}\text{Sc}$, are reproduced although the absolute values are different from the experimental data.

Let us see how the $B(M1)$ as well as $\Delta E$ and $\mu$ are changed by including the core vibration. The three-body model with a vibrational core works positively to be close to the experimental data for $\Delta E$ of $^{14}\text{N}$ and $\mu$ of $^{50}\text{Cu}$, while it works negatively for $\Delta E$ of $^{42}\text{Sc}$ and $B(M1)$ of $^{18}\text{F}$ and $^{42}\text{Sc}$. However, for the most part, the difference from the experimental data basically remains the same as the inert core. The $B(M1)$ of $^{18}\text{F}$ and $^{42}\text{Sc}$ are quenched as compared to the three-body model with an inert core, however, the relatively large values remain the same as the result of inert core. This indicates that $0^+$ and $1^+$ states of those nuclei meaningfully form the SU(4) multiplet even if we assume the core vibration.

Note that $^{14}\text{N}$ and $^{30}\text{P}$ have a deformed core, so that the deformation effect and the coupling with rotationally excited states would provide even further changes. The low-lying $0^+$ and $1^+$ states of $^{58}\text{Cu}$ are also discussed in the framework of proton–neutron particle–particle relativistic time blocking approximation [87], in which it is shown that the particle–vibration coupling gives a significant influence to the low-lying states. We should mention that a large discrepancy of the $B(M1)$ of $^{14}\text{N}$ from the experimental data is due to the three-body force [88,89] which is difficult to include in the present framework.
Table 2: Comparison of $\Delta E = E_Q - E_{1+}$, magnetic moment $\mu_{(1)}$, and magnetic dipole transition $B(M1)$ for $N = Z$ odd–odd nuclei calculated by the three-body model with a vibrational core (vib) and inert core (inert) with experimental data (exp). The nucleus in parentheses indicates the core nucleus. Experimental data of $\mu$ for $^{14}$N and $^{58}$Cu are taken from $^{[84,85]}$, respectively, and those of $\Delta E$ and $B(M1)$ from the National Nuclear Data Center $^{[86]}$. The numbers in parentheses give the experimental errors in the last digits. Errors of the experimental data of $\Delta E$ are omitted because they are very small.

| Observables | $^{14}$N ($^{12}$C) | $^{18}$F ($^{16}$O) | $^{30}$P ($^{28}$Si) | $^{34}$Cl ($^{32}$S) | $^{42}$Sc ($^{40}$Ca) | $^{58}$Cu ($^{56}$Ni) |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $\Delta E$  | Exp 2.31 1.04 0.68  | $-0.46$ $-0.61$ $0.20$ |                    |                    |                    |                    |
| (MeV)       | Inert 0.05 1.04 0.02 | $-0.69$ $-0.61$ 0.68 |                    |                    |                    |                    |
| $\mu$       | Exp 0.48 1.04 $-0.08$ | $-0.24$ $-0.13$ 0.02 |                    |                    |                    |                    |
| ($\mu_{(1)}$) | Inert 0.379 0.834 0.318 | 0.426 0.686 0.28 |                    |                    |                    |                    |
| $B(M1)$     | Exp 0.404 0.421 1.32 (14) | 1.0077 (6) 6.16 (265) |                    |                    |                    |                    |
| ($\mu_{(1)}$) | Inert 0.682 18.11 0.24 | 0.15 6.81 0.58 |                    |                    |                    |                    |
| Vib         | 0.630 15.75 3.40 | 0.13 5.33 7.35 |                    |                    |                    |                    |

Coupling to higher-order configuration causes quenching and damping of spin–isospin excitations, and the missing strengths are brought to a higher energy region or converted to a $\Delta$-hole excitation $^{[90,91]}$. The present three-body model also clarifies the damping of $B(M1)$ by allowing the core nuclei to vibrationally excite. We can see in Table 2 that the $B(M1)$ of the three-body model using a vibrational core are certainly smaller than that of the model using an inert core for $^{14}$N, $^{18}$F, $^{34}$Cl, and $^{42}$Sc. However, for $^{30}$P and $^{58}$Cu, the $B(M1)$ are enhanced rather than reduced. Those nuclei have a different configuration for the $1^+$ ground state between the three-body models of inert core and vibrational core, as discussed in Sect. 3.3.

Next, we discuss the damping and quenching of $B(GT)$. The GT transitions are frequently studied in a framework beyond the standard particle-1 hole RPA in order to understand the effect of higher-order configurations on quenching and damping $^{[95–97]}$. Because the GT transition is a major contributor to $\beta$-decay for most nuclei, a thorough understanding of it is important for various fields, for example, nucleosynthesis in astrophysics and nuclear engineering. We study the GT transition from the $0^+$ ground state of $(N_e + 2, Z_e)$ even–even nucleus to $1^+$ states of $(N_e + 1, Z_e + 1)$ odd–odd nucleus with the three-body model. Parent $(N_e + 2, Z_e)$ nuclei, i.e. the system of core plus two neutrons, are calculated in the same manner as that of core plus proton and neutron. It should be mentioned that the studies of quenching and damping of GT strengths for even–even nuclei have been carried out with the particle–vibration coupling in a Skyrme energy density functional $^{[98–100]}$ and a covariant density functional $^{[72,101,102]}$. This work complements the results obtained in those works through the three-body model and discusses the low-lying GT transition in terms of SU(4) symmetry (the details concerning the SU(4) symmetry in the nuclei studied in this work will be discussed in Sect. 3.3).

The low-lying $1^+$ excitation energies and the $B(GT)$ (in units of $g_A^2/4\pi$, where $g_A$ is the axial vector coupling constant of a free nucleon) for $^{18}$O $\rightarrow ^{18}$F, $^{42}$Ca $\rightarrow ^{42}$Sc, and $^{58}$Ni $\rightarrow ^{58}$Cu transitions calculated by the three-body model with an inert core (inert) and vibrational core (vib). The $B(GT)$ of the three-body model with an inert core are taken from Ref. $^{[28]}$ and are corrected by a factor of two. Experimental data are taken from Refs. $^{[92–94]}$.

Table 3: Low-lying GT strengths (in units of $g_A^2/4\pi$) for $^{18}$O $\rightarrow ^{18}$F, $^{42}$Ca $\rightarrow ^{42}$Sc, and $^{58}$Ni $\rightarrow ^{58}$Cu transitions calculated by the three-body model with an inert core (inert) and vibrational core (vib). The $B(GT)$ of the three-body model with an inert core are taken from Ref. $^{[28]}$ and are corrected by a factor of two. Experimental data are taken from Refs. $^{[92–94]}$.

| #  | $E_{1+}$ (MeV) | $B(GT)$ ($g_A^2/4\pi$) |
|---|----------------|------------------------|
| 1 | 4.96 3.99 3.11 ± 0.03 | Inert 0.0 0.0 0.0 |
| 2 | 4.79 5.27 0.056 0.102 | Vib 0.61 0.25 0.61 |
| 3 | 6.87 6.96 0.072 0.147 | Exp 1.0 1.0 1.0 |
| 4 | $^{42}$Ca(g.s.) $\rightarrow ^{42}$Sc (1$^+$) | Inert 0.0 1.0 1.0 |
| 5 | 3.71 4.71 0.79 0.54 | Vib 0.0 0.0 0.0 |
| 6 | $^{58}$Ni(g.s.) $\rightarrow ^{58}$Cu (1$^+$) | Exp 1.0 1.0 1.0 |
first $1^+$ state. On the other hand, $B$(GT) of $^{58}$Cu are enhanced by the core vibration for the first $1^+$ states, and it is one order larger than the experimental data. It is expected at least, from the experimental data, that the $1^+$ state of $^{58}$Cu and the $0^+$ state of $^{58}$Ni do not form the SU(4) multiplet. The overestimation of experimental data is discussed in Sect. 3.3.

To demonstrate the quenching of $B$(GT), we plot cumulative $B$(GT) defined as

$$\text{Cum}(E) = \sum_{\nu \in E} B(\text{GT}, \nu)$$

(15)

in Fig. 2 as a function of excitation energy $E$ of the $1^+$ state up to 12 MeV. Note that the results shown in Fig. 2 are the GT transitions between the valence nucleons and those of the core nucleus (the third to fifth terms of Eq. (21) in Appendix) is omitted. We can observe in Fig. 2 that Cum($E$) at $E = 12$ MeV for $^{18}$F, $^{30}$P, $^{42}$Sc and $^{58}$Cu are approximately 6, which is a reasonable value with respect to the Ikeda sum rule, while those of $^{14}$N and $^{34}$Cl are only about 1. The missing strengths for $^{14}$N ($^{34}$Cl) are formed by the transition of the core nucleus from $\nu 1p_3/2 (1d_5/2)$ to $\pi 1p_1/2 (1d_5/2)$ states. We have carried out an estimation by RPA with the SGII force [107] and found that the first significant GT transitions of core nuclei are expected to be at about 2 (7) MeV for $^{14}$N ($^{34}$Cl). For $^{58}$Cu, the giant GT strength, which is mainly formed by the transition of the core nucleus from $\nu 1f_7/2$ to $\pi 1f_5/2$ states, appears around 10 MeV in the RPA calculation [108] in addition to the Cum($E$) shown in Fig. 2.

From Fig. 2, the quenching of $B$(GT) can be clearly observed for $^{18}$F, $^{30}$P, and $^{42}$Sc. The ratio of Cum($E$) at $E = 12$ MeV for the three-body model using a vibrational core to that using an inert core are $0.83 - 0.91$ and is consistent with the results of particle–vibration coupling [99] and the second Tamm–Dancoff approximation [97]. The quenching effect for $^{14}$N, $^{34}$Cl, and $^{58}$Cu is not as obvious as it is with the other nuclei. It will appear in the giant GT components coming from the core nuclei.

For $^{42}$Sc and $^{58}$Cu, Cum($E$) of the three-body models overestimates the experimental data. However, for those pf-shell nuclei, the quenching factor has to be commonly introduced in the shell model calculation in order to reproduce the experimental data [109, 110]. In the present models, we determine the quenching factor so as to reproduce the experimental $B$(GT) for the first $1^+$ state of $^{42}$Sc and obtain $(0.77 \pm 0.03)^2$ and $(0.84 \pm 0.03)^2$ for the three-body model using an inert core and vibrational core, respectively. These values are consistent with studies used in Refs. [111–113]. The result with the quenching factors is shown in Fig. 3. We can see the three-body models reasonably reproduce the overall $B$(GT) distribution of $^{42}$Sc up to 8(6) MeV for the three-body model using a vibrational (an inert) core. On the other hand, the three-body models for $^{58}$Cu overestimate the $B$(GT) strengths in low energy regions. This problem may be due to the approximations used in this work and further discussion is given in the next section.

### 3.3 SU(4) multiplet

With the limit of no spin-orbit and Coulomb forces, proton and neutron states in the same orbit are degenerate and form the SU(4) multiplets [30]. In these circumstances, as briefly mentioned in Sect. 3.2, the spin–isospin operator $\sigma \tau$ connects the SU(4) multiplets and the transition energies between $0^-$ and $1^+$ states appear at zero with notably strong
$B(M1)$ and $B(GT)$. Even though the spin-orbit and Coulomb forces work in nuclei, it is expected that the SU(4) symmetry appears for some nuclei. The picture of core plus valence nucleons in the three-body model is suitable for discussing the SU(4) symmetry of two valence nucleons in nuclei. It was seen in three-body model using an inert core [28] and other calculations [114–116] that $^{18}\text{F}$ and $^{42}\text{Sc}$ have a SU(4) multiplet because the $0^+$ and $1^+$ states are largely dominated by spin $S = 0$ and $S = 1$ components, respectively, and they show a large $B(M1)$ and $B(GT)$. This begs the following question: what is the effect of core vibration on the SU(4) symmetry?

To discuss the SU(4) multiplet for $N = Z$ nuclei, it is convenient to consider the fraction of spin-singlet ($S = 0$) and spin-triplet ($S = 1$) components in the valence nucleon subsystem, which are defined as $P(S)$. It is calculated by using the $LS$–$jj$ coupling coefficient:

$$| \langle j_\pi j_\nu J \rangle \rangle = \sum_{LS} \left\{ \frac{1}{2} \frac{1}{2} l_\pi l_\nu L \right\} \hat{S} \hat{j}_\pi \hat{j}_\nu (l_\pi l_\nu LS; J).$$

The probability of a particle–particle configuration with total angular momentum $j$ in the lowest particle state, defined as $P(j^\pi, j^\nu) = |c_{\alpha\nu}|^2 + \sum_{\alpha} |c_{\alpha\nu}|^2$, where $\alpha \in (j^\pi, j^\nu)$, is also useful in the following discussion. The results of $P(S)$ for the first $0^+$ and $1^+$ states for core+pn (i.e. $N = Z$ odd–odd nuclei), and the $0^+$ ground state for core+nn (i.e. $N = Z + 2$ even–even nuclei) are shown in Fig. 4, and those of $P(j^\pi, j^\nu)$ are plotted in Fig. 5.

We can see in Fig. 4a, c that $P(S = 0)$ for the core+pn and core+nn systems calculated by the three-body model using a vibrational core are slightly smaller than those by inert core, but the variations are not large. This indicates that the core vibration has an effect disturbing the spin anti-parallel alignment of the valence nucleons; however, its influence is small. Similarly, in Fig. 4b, $P(S = 1)$ for the core+pn system calculated by the three-body model of vibrational core are almost the same as those by inert core. However, we can see an exception in $^{58}\text{Cu}$.

The effect of core vibration is also limited for $P(j^\pi, j^\nu)$. In Fig. 5a, c, $P(j^\pi, j^\nu)$ values for the $0^+$ core+pn and core+nn systems calculated by the three-body model using a vibrational core and inert core show almost the same results, although the vibrational core gives little smaller values than the inert core. In Fig. 5b, $P(j^\pi, j^\nu)$ values for the $1^+$ core+pn system show a similar result between the vibrational core and the inert core, however, $^{30}\text{P}$ and $^{58}\text{Cu}$ exhibit a large difference between them. We will discuss this abnormality later.

For $^{18}\text{F}$ ($^{16}\text{O}+\text{pn}$ system) and $^{42}\text{Sc}$ ($^{40}\text{Ca}+\text{pn}$ system), the $0^+$ and $1^+$ core+pn states are dominantly made up of $S = 0$ and $S = 1$ components, respectively as shown in Fig. 4, and the valence nucleons are in the same orbit ($1d_{5/2}$ for $^{18}\text{F}$ and $1f_{7/2}$ for $^{42}\text{Sc}$) as displayed in Fig. 5. As a result, this system forms a good SU(4) symmetry in the subsystem made up of valence nucleons and a significantly large value of $B(M1)$ appears. Similarly, because the $0^+$ core+nn system is dominated by the $S = 0$ component as seen in Fig. 4c and the valence neutrons stay the same orbit as the $1^+$ core+nn system as displayed in Fig. 5b, c, large $B(GT)$ appears for the $^{18}\text{O}\rightarrow^{18}\text{F}$ and $^{42}\text{Ca}\rightarrow^{42}\text{Sc}$ transitions, as seen in Table 3. As for the $^{12}\text{C}+\text{pn}$ and $^{32}\text{S}+\text{pn}$ systems, the valence nucleons occupy the same orbit between different systems as displayed in Fig. 5. However, $P(S = 0)$ for the $0^+$ core+nn system shown in Fig. 4a are small. As a result, $B(M1)$ at a low energy are suppressed for those nuclei. Similarly, because $P(S = 0)$ of $0^+$ core+nn system exhibit small values, $B(GT)$ at a low energy are suppressed for those nuclei.

On the other hand, a small $B(M1)$ of $^{30}\text{P}$ ($^{28}\text{Si}+\text{pn}$ system) and $^{58}\text{Cu}$ ($^{56}\text{Ni}+\text{pn}$ system) is attributed to the fact that the valence nucleons occupy a different orbit in $1^+$ core+pn system [28]. We can see that $P(j^\pi, j^\nu)$ val-
ues for the $1^+ 28\text{Si}+pn$ and $56\text{Ni}+pn$ systems are small in the case of the inert core approximation, as seen in Fig. 5b. The actual major particle–particle configurations of the $1^+ 28\text{Si}+pn$ and $56\text{Ni}+pn$ systems are $(1d_3/2, 2s_1/2)$ and $(2p_3/2, 1f_5/2)$, respectively. The situation is changed by including core vibration. In the three-body model using a vibrational core, $P(2s_1/2, 2s_1/2)$ values for the $1^+ 28\text{Si}+pn$ system and $P(2p_3/2, 2p_3/2)$ values for the $1^+ 56\text{Ni}+pn$ system increase to 70.1% and 65.0%, respectively. This increment supports forming SU(4) multiplet and causes the enlargement of $B(M1)$ and $B(GT)$ for $30\text{P}$ and $58\text{Cu}$ shown in Tables 2 and 3.

The changes of the configuration in the $1^+ 28\text{Si}+pn$ and $56\text{Ni}+pn$ systems can be understood by an interchange of the ground state and the other $1^+$ excited state. To explain it, we draw the probabilities of configuration consisting of the ground state (the first $1^+$ state) and the second $1^+$ state for $30\text{P}$ in Fig. 6. In the case of an inert core, the ground state (the upper left) is mainly made up of the $(d_3/2, s_1/2)$ and $(s_1/2, d_3/2)$ states with a total probability 84.5%. The second $1^+$ state appearing at 1.14 MeV (the upper right) is mainly composed of the $(s_1/2, s_1/2)$ state with 90.5%. In the vibrational core case, the ground state (the bottom left) is mainly made up of the $(s_1/2, s_1/2)$ state with 71.2%. The second $1^+$ state (the bottom right) is mainly made up of the $(d_3/2, s_1/2)$ and $(s_1/2, d_3/2)$ states with a total probability 71.5%. The configuration of the ground state (the $1^+$ state) in the case of an inert core resembles the second $1^+$ state (the ground state) in the case of core vibration. It is qualitatively understood that the ground state configuration and the second $1^+$ state configuration are inverted by the effect of core vibration.

The configurations of the ground state and excited $1^+$ state for $58\text{Cu}$ are shown in Fig. 7. For the inert core, the ground state (the upper left) is mainly made up of the $(p_3/2, f_5/2)$ (36.8%) and $(f_5/2, p_3/2)$ (33.1%) states, and the second $1^+$ state (the upper right) is mainly made up of the $(p_3/2, p_3/2)$ state (81.4%). For the vibrational core, the ground state (the bottom left) is mainly made up of the $(p_3/2, p_3/2)$ state (65.8%), and the fourth $1^+$ state (the bottom right) is made up of the $(p_3/2, f_5/2)$ (24.0%) and $(f_5/2, p_3/2)$ states (27.3%). As seen in $30\text{P}$, the configuration of the ground state and the excited $1^+$ state looks interchanged.

Because the experimental data for $30\text{P}$ has a relatively large $B(M1)$ of $1.32\pm0.14(\mu_\text{N}^2)$, the SU(4) symmetry between $0^+$ and $1^+$ states might emerge weakly. The three-body model using an inert core underestimates the $B(M1)$, while that using a vibrational core overestimates it. Therefore, it is considered that the true probability of the particle–particle
Fig. 7 Configurations of the ground state (the first $1^+$) and excited $1^+$ states for $^{58}\text{Cu}$ calculated by the three-body model using an inert core and vibrational core. The upper left and right panels are the results of the ground state and the second $1^+$ state for the three-body model using an inert core, respectively. The bottom left and right panels are the results of the ground state and the fourth $1^+$ state for the three-body model using a vibrational core, respectively.

configuration $P(2s_{1/2}, 2s_{1/2})$ will be between the results of the three-body model using an inert core and vibrational core. Since the core nucleus $^{28}\text{Si}$ is a deformed nucleus, it is expected that the $B(M1)$ is improved by including the deformation effect in addition to the vibration effect. In case of $^{58}\text{Cu}$, the experimental data of the first two $B(GT)$ are small (0.455 in total) as shown in Table 3, so that the SU(4) symmetry will not be formed. However, the three-body model using a vibrational core as well as inert core shows larger $B(GT)$ than the experimental data at this low energy region. We carried out a calculation for $^{58}\text{Cu}$ with alternative parameters optimized to the neutron separation energy of $^{57}\text{Ni}$ and $1^+$ and $0^+$ states of $^{58}\text{Cu}$, but the $B(GT)$ was not reproduced. We suppose that there are several reasons why we could not reproduce the result: (a) the momentum dependent terms of the residual core–nucleus interaction are approximated by the Landau–Migdal form, (b) the tensor force between the valence particles and the core–nucleus interaction is ignored. It may play a role because the tensor force changes the relative orbital angular momentum of two scattering particles by $\Delta L = 2$ and disturbs the SU(4) symmetry. Using the zero-range force in the pairing interaction might also be inappropriate, and (c) the subsystem $^{57}\text{Cu}$ (core plus proton) is a weakly bound system ($S_p = 690$ keV), so that the exact treatment of the continuum states might be also important. Further study of this will be done in future work.

3.4 Core contribution

In the previous sections, we have seen that the core vibration has a considerable influence on spin–isospin properties. Now let us see how much the core vibration contributes to the wave function of Eq. (7). Figure 8 shows the probabilities of core vibration consisting of the ground state of the (a) core$+pn$ and (b) core$+n$ systems. We also list in Table 4 the resonance energies and $B(EL, 0 \rightarrow L)$ values of the first $2^+$ and $3^-$ states as a reference. Note that the $2^+_1$ states of $^{16}\text{O}$ and $^{40}\text{Ca}$ are not considered as the phonon that couples with the valence nucleons because their strengths are too small to satisfy the condition assumed in Sect. 2.2.1. We can see from Fig. 8a that the main core contribution is from the $2^+$ states, while the contribution from the $1^-$ and $5^-$ states are negligibly small. The total of the core contributions of the core$+pn$ system for $^{28}\text{Si}$, $^{32}\text{S}$, $^{40}\text{Ca}$, and $^{56}\text{Ni}$ are about 15%. In particular, the $2^+_1$ state contributes significantly for $^{28}\text{Si}$, $^{32}\text{S}$, and $^{56}\text{Ni}$. This is because the low-lying strong isoscalar $2^+_1$ states appear at 2.4, 3.1, and 3.9 MeV for $^{28}\text{Si}$, $^{32}\text{S}$, and $^{56}\text{Ni}$, respectively, as seen in Table 4, and those states become major contributors to the core polarization effect. Similarly, the high probability of core vibration of $^{40}\text{Ca}$ results from the $3^-_1$ state appearing at 3.7 MeV. For the core$+pn$ systems of...
12C and 16O, the total of the core contributions is only about 6 and 7%, respectively, because their first 2+ and 3− states. The RPA results are calculated with SkM* force and the experimental data are taken from [117,118] and that between proton and neutron, \( \langle r_{pn}^2 \rangle^{1/2} \) are calculated, which are defined as

\[
r_{C−pn}^2 = \frac{1}{2I+1} \sum_K \left| \psi_K \right|^2 \left( \frac{r_1 + r_2}{4} \right)^2 \psi_K \tag{17}
\]

and

\[
r_{pn}^2 = \frac{1}{2I+1} \sum_K \left| \psi_K \right|^2 \left( \frac{r_1 - r_2}{2} \right)^2 \psi_K \tag{18}
\]

respectively.

The results of the rms of \( r_{C−pn} \) for the three-body model and the HF limit are shown in Fig. 9a. The three-body model using an inert core and the HF limit are almost identical except for 18F. The large \( (r_{C−pn})^{1/2} \) of 18F is due to the scattering of the valence nucleons to the bound s-wave state, namely the 2s1/2 state (\( \epsilon_{2s} = -0.65 \) and \( \epsilon_{2p} = -3.27 \) MeV) from 2d5/2 state by the pairing interaction. The proton 2s1/2 state is weakly bound so that the tail of density distribution extends largely outside of nucleus. As the angular momentum increases, the leakage of the density is hindered because of the centrifugal barrier. Namely, the pairing force does not affect significantly the distance between the center of mass of proton and neutron and the core nucleus unless they are scattered into a weakly bound state with a low angular momentum. If the core vibration is considered, \( (r_{C−pn})^{1/2} \) becomes systematically large. Compared with the inert core, \( (r_{C−pn})^{1/2} \) increase by 0.7, 0.5, 0.4, 0.3, 0.4, and 0.2 fm for 14N, 18F, 30P, 34Cl, 42Sc, and 58Cu, respectively.

Table 4 Resonance energies and \( B(E.L, 0 \to L) \) of the 2+ and 3− states. The RPA results are calculated with SkM* force and the experimental data are taken from [117,118]

| Nuclei | RPA E(MeV) | E2fm2L | Experiment E(MeV) | E2fm2L |
|--------|------------|--------|-------------------|--------|
| 12C    | 2+         | 7.512  | 20.5              | 4.438  |
|        | 3−         | 13.635 | 186               | 9.641  |
| 16O    | 2+         | 11.364 | 0.07              | 6.917  |
|        | 3−         | 5.895  | 446               | 6.129  |
| 28Si   | 2+         | 2.385  | 87                | 1.779  |
|        | 3−         | 11.696 | 1703              | 6.879  |
| 32S    | 2+         | 3.098  | 94                | 2.230  |
|        | 3−         | 6.184  | 3254              | 5.006  |
| 40Ca   | 2+         | 9.376  | 0.04              | 3.904  |
|        | 3−         | 3.704  | 5323              | 3.737  |
| 56Ni   | 2+         | 3.904  | 265               | 2.701  |
|        | 3−         | 9.489  | 9161              | 4.932  |

12C and 16O, the total of the core contributions is only about 6 and 7%, respectively, because their first 2+ and 3− states calculated by RPA are high and the transition strengths are weak. However, the effect of core vibration cannot be ignored in either the ground state or spin–isospin excitations as we have seen in the previous sections.

The result of the core+n system is shown in Fig. 8b. We can see that the core contributions are smaller than the core+pn system because the polarization effect is caused by only one neutron. However, the contributions of each vibrational state look similar to the core+pn system. We plot the total core contribution of the core+pn system divided by a factor of 2 in the same panel, which is indicated by the small circle (the thin line). The result looks similar to the total core contribution of the core+n system. It indicates that the core polarization is almost proportional to the number of nucleons, at least up to two particles.

3.5 Mean value of spatial distribution of valence particles

In the Hartree–Fock (HF) limit in which there is neither pairing nor residual core–nucleus forces, the valence nucleons stay in the first lowest orbitals above the Fermi surface. If there exists a pairing interaction, the valence nucleons collide with each other and scatter into other orbitals; this is exactly what the three-body model using an inert core can calculate. If the residual core–nucleon interaction generated by core vibration exists in addition, it also brings the valence particle to other orbitals. As a result, the geometrical structure of valence nucleons will change. To investigate this, the root mean square (rms) distance between the core nucleus and the center of mass of two valence nucleons, \( (r_{C−pn})^{1/2} \)
within the 1p–1h picture. However, it is not necessarily able to reproduce phonon states correctly, especially of low-lying states as seen in Table 4. For the $2_1^+$ and $3_1^-$ states of $^{16}$O and $^{40}$Ca, which originates from a more complex configuration mixing than 1p–1h excitations realized by the RPA [119, 120], the resonance energies are overestimated and the $B(E.L, 0 \rightarrow L)$ are poorly reproduced. These results would lead us to think about the oversight of the core vibrational effects. We therefore discuss the possible uncertainties arising from phonon states calculated by the present RPA framework in this section.

Because the vibrational state and its coupling to nucleons are complicated, we set the following two criteria to discuss the uncertainties. First, $^{18}$F and $^{42}$Sc nuclei are chosen because they have spherical cores, and the deformation effect can be meaningfully removed. In addition, they are not significantly affected by the change of the ground state configuration as seen in $^{36}$P and $^{58}$Cu (see Sect. 3.3). Second, we assume that the major phonons that couple with valence nucleons are the $2_1^+$ and $3_1^-$ states and discuss uncertainties arising only from them. Therefore, we consider a minimum uncertainty. We mimicked the experimental $2_1^+$ and $3_1^-$ states by replacing the RPA resonance energies to the experimental ones, and scaled the RPA transition densities to reproduce the experimental $B(E.L, 0 \rightarrow L)$ for the $2_1^+$ and $3_1^-$ states. The RPA transition densities would not be exactly the same forms as the experimentally observed ones, however, we expect that this approximation is not so bad. The procedures to determine the interaction parameters are the same as given in Sect. 2.3 and the obtained results are $v_0 = -42.73$ for $^{16}$O and $-46.47$ for $^{40}$Ca, $v_{ls} = 34.2$, $x_s = -1.199$, $x_t = 1.323$, and $\alpha_s = \alpha_t = 1.191$.

The result of $\Delta E$, $\mu$, and $B(M1)$ and $B(GT)$ for the first excited state is shown in Table 5. We here refer to the result of the three-body model using the modified $2_1^+$ and $3_1^-$ RPA phonons as “modified.” To quantitatively evaluate the variations from the result of the three-body model using the original RPA phonons, we take the ratio of the modified RPA phonons to the original RPA phonons. Let us recall again that $\Delta E$ of $^{18}$F is used as the parameter fitting and the ratio is unity. We can see that the ratio of $\mu$, $B(M1)$ and $B(GT)$ to the original RPA phonons is about 90% for $^{18}$F and 79 to 88% for $^{42}$Sc. For $\Delta E$ of $^{42}$Sc, it is reduced by the modified RPA phonons and this tendency is positive to be close to the experimental data.

In Table 5, $\langle r_{C-pn}^2 \rangle^{1/2}$ and $\langle r_{pn}^2 \rangle^{1/2}$ of $^{18}$F and $^{42}$Sc are also listed together with the result of the three-body model using the original RPA phonons (original). The mean value of the spatial distribution increases by about 6% for $^{18}$F and 9%
for $^{42}$Sc. Although we do not show in Table 5, we also check the core contribution to the ground state for $^{18}$O and $^{42}$Sc. The probability of $2^+$ state changes to 4.1% (4.3% for the original RPA phonons) and that of $3^-$ state is 5.5% (2.5%) for $^{18}$F. Similarly, in the case of $^{42}$Sc the probability of $2^+$ state is 3.8% (4.2% for the original RPA phonons) and that of $3^-$ state is 9.7% (4.5%). The contributions from the $3^-$ state almost double one hand, those from $2^+$ state slightly decrease. This is because the effect of the modified first $3^-$ phonon is more significant than that of the first $2^+$ state, the contributions from the $2^+$ states are relatively reduced. We also plot the low-lying spectra of $^{17}$O and $^{41}$Ca in Fig. 10. The energy levels for $^{17}$O almost remain the same as the result of the original RPA phonon, and the inverted level ordering is not recovered. From this result including the findings obtained in Sect. 3.1, the parameter fitting used in the present work might strongly constraint the level structure of $^{17}$O. Similar to $^{17}$O, the energy levels for $^{41}$Ca also do not change significantly except the $3/2^+$ state, which increases by about 600 keV due to the modified RPA phonons.

Let us summarize this section. We estimated the possible uncertainties arising from the low-lying RPA $2^+$ and $3^-$ states, and there existed 10% uncertainties in $\mu$, $B(M1)$, and $B(GT)$, and about 6% uncertainties in the mean spatial distribution for $^{18}$O, while there existed about 12 to 20% uncertainties in $\mu$, $B(M1)$, and $B(GT)$, and about 9% uncertainties in the mean spatial distribution for $^{42}$Sc. The low energy spectra were less sensitive to the modified RPA phonons than the other quantities, however, some states might be varied by a few hundred to 600 keV.

4 Summary

We have studied the spin–isospin properties with the three-body model including a vibrational core. The nuclei studied in this work were $N = Z$ odd–odd nuclei which consist of an even–even core nucleus plus proton and neutron. We paid a particular attention to the magnetic dipole and GT transitions between $0^+$ and $1^+$ states of $N = Z$ nuclei. As for the result of $\Delta E$, $\mu$, and $B(M1)$, the three-body model using a vibrational core reproduce the most part of the experimental data at a nearly equal level to the three-body model using an inert core. We demonstrated the large $B(M1)$ and $B(GT)$ observed for $^{18}$F and $^{42}$Ca in terms of the SU(4) multiplet. We found that the $0^+$ and $1^+$ states stay forming the SU(4) multiplet even if we assume the core vibration. The damping of the low-lying $B(GT)$ was observed in the three-body model using a vibrational core and the result was close to the experimental data.

Even when core excitation is considered, the situation remained almost the same. For $^{30}$P and $^{58}$Cu, it was found that the ground state of the first $1^+$ state and excited $1^+$ state were interchanged by the effect of core vibration. As a result, the SU(4) symmetries, which were hindered in the case of the inert core, turned out to have an impact, and induced significant changes in the $B(M1)$ and $B(GT)$. However, the present result was not consistent with experimental data. Several issues in the present calculation of $^{30}$P and $^{58}$Cu were discussed. Further investigation considering a more general expression of the pairing and the core–nucleon forces, the core deformation, and the exact treatment of continuum states is our next perspective.

The core contribution for the ground state configurations were about 15% for $^{30}$P, $^{34}$Cl, $^{42}$Sc, and $^{58}$Cu and a significant effect of the core vibration was observed in those nuclei. While the core contributions in the ground state wave function for $^{14}$N and $^{18}$F were only 6 to 7%, the effect of core vibration for those nuclei cannot also be ignored in either the ground state or spin–isospin excitations. The core excitation is thus important for a detailed understanding of nuclear structure. We also found that the core contribution of the two-body system, namely core plus neutron system, was approximately half that of the three-body system.

We discussed that $\langle r_{C-pn} \rangle_{1/2}$ and $\langle r_{pn} \rangle_{1/2}$ systematically increased if core vibration was considered. This was due to the fact that the residual core–nucleon interactions induce the valence nucleons to scatter to higher orbits in addition to the pairing interaction. We also tried to estimate the possible uncertainties arising from the RPA $2^+_1$ and $3^+_1$ states.

Experimental study of the $B(M1)$ for $^{58}$Cu and the magnetic moment of $^{30}$P, which have not been measured, is helpful for further interpretation of the SU(4) multiplets for those nuclei. Because the $B(M1)$ for $^{18}$F and $^{42}$Sc have large uncertainties, an accurate measurement will be also important in understanding the effect of core contribution and SU(4) multiplets sufficiently. Including the anti-symmetrization between valence particles and core is the our next study.
A Transition amplitude

A one-body operator $\mathcal{O}$ with rank $L$ can be separated into the valence particles and core nucleus as,

$$\mathcal{O} = \mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_\xi,$$

where $\mathcal{O}_i (i = 1, 2, \xi)$ operates the coordinate $\mathbf{r}_1, \mathbf{r}_2,$ and $\xi$, respectively. The reduced transition matrix of the one-body operator is calculated as using Eq. (7),

$$\langle \Psi' \rangle |\mathcal{O}| \langle \Psi \rangle = \left( \langle \Psi' \rangle |\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_\xi \right) |\langle \Psi \rangle \rangle = \sum_{aa'} c_{a_0}^* c_{a_0} \left( \langle \Theta_{a_0}^{J_0} \Phi_{a_0}^{I_0} | \mathcal{O}_1 \mathcal{O}_2 + \mathcal{O}_\xi \Phi_{a_0}^{I_0} \rangle \right)$$

$$+ \sum_{a J_1 a' J_1} c_{a_1}^* c_{a_1} \left( \langle \Theta_{a_1}^{J_1} \Phi_{a_1}^{I_1} | \mathcal{O}_1 + \mathcal{O}_2 \rangle \right) \left( \langle \Theta_{a_1}^{J_1} \Phi_{a_1}^{I_1} | \mathcal{O}_\xi \rangle \right)$$

$$+ \sum_{a' J_1 a J_1} c_{a_1}^* c_{a_1} \left( \langle \Theta_{a_1}^{J_1} \Phi_{a_1}^{I_1} | \mathcal{O}_1 \rangle \right) \left( \langle \Theta_{a_1}^{J_1} \Phi_{a_1}^{I_1} | \mathcal{O}_\xi \rangle \right)$$

$$+ \sum_{a J_1 v' J_1} c_{a_1}^* c_{a_1} \left( \langle \Theta_{a_1}^{J_1} \Phi_{a_1}^{I_1} | \mathcal{O}_\xi \rangle \right) \left( \langle \Theta_{a_1}^{J_1} \Phi_{a_1}^{I_1} | \mathcal{O}_\xi \rangle \right).$$

The first term of the right hand side in Eq. (20) is the contribution from the valence two nucleons when the core nucleus is inert and the second term corresponds to the higher-order correlation due to the core excitation. The third to fifth terms are the contributions from the core nucleus directly excited by the one-body external field. Using some formulas concerning angular momentum, Eq. (20) becomes

$$\langle \Psi' \rangle |\mathcal{O}| \langle \Psi \rangle = \sum_{aa'} c_{a_0}^* c_{a_0} \left( \langle \Theta_{a_0}^{J_0} | \mathcal{O}_1 \rangle + \mathcal{O}_2 |\Theta_{a_0}^{J_0} \rangle \right)$$

$$+ \sum_{a J_1 J_1} c_{a_1}^* c_{a_1} \left( \langle \Theta_{a_1}^{J_1} | \mathcal{O}_1 \rangle \right) \left( \langle \Theta_{a_1}^{J_1} | \mathcal{O}_\xi \rangle \right)$$

$$+ \sum_{a J_1 J_1} c_{a_1}^* c_{a_1} \left( \langle \Theta_{a_1}^{J_1} | \mathcal{O}_2 \rangle \right) \left( \langle \Theta_{a_1}^{J_1} | \mathcal{O}_\xi \rangle \right)$$

$$+ \sum_{a J_1 J_1} c_{a_1}^* c_{a_1} \left( \langle \Theta_{a_1}^{J_1} | \mathcal{O}_\xi \rangle \right) \left( \langle \Theta_{a_1}^{J_1} | \mathcal{O}_\xi \rangle \right).$$

In this work, we did not consider the contributions of core nucleus expressed by the third to fifth terms of Eq. (21) in Sect. 3 to focus on the two-body subsystem.

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