Exchange effects on electron transport through single-electron spin-valve transistors

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We study electron transport through single-electron spin-valve transistors in the presence of non-local exchange between the ferromagnetic leads and the central normal-metal island. The Coulomb interaction is described with the “orthodox model” for Coulomb blockade and we allow for non-collinear lead magnetization directions. Two distinct exchange mechanisms that have been discussed in the literature are shown to be of comparable strength and are taken into account on equal footing. We present results for the linear conductance as a function of gate voltage and magnetic configuration, and discuss the response of the system to applied magnetic fields.

PACS numbers: 85.75.-d, 72.25.Mk, 73.23.Hk

I. INTRODUCTION

Downscaling magnetoelectronic devices to the nanometer regime implies that electron-electron interaction effects become prominent, as has been amply demonstrated by many experimental studies on the Coulomb blockade in double tunnel junctions with ferromagnetic elements. Much of the theoretical work focusses on F|N|F spin valves, in which the island is a normal metal (N) and the contacts are ferromagnets (F) with variable magnetization directions. Initially, the interest was mainly focussed on the giant magnetoresistance, i.e. the difference in the transport properties for parallel or antiparallel magnetizations. More recently, the interplay between spin and interaction effects for non-collinear magnetization configurations has attracted quite some interest.

A single-electron spin-valve transistor (SV-SET) is an F|N|F spin-valve with a sufficiently small normal-metal (N) island that is coupled to the ferromagnetic leads by tunnel barriers. When the electrostatic charging energy of the island is larger than the thermal energy, charge transport can be controlled at the level of single electron charges by varying bias and gate voltage, as is well known for non-magnetic SET’s. With spin-dependent electron tunneling rates and sufficiently long spin-decay lifetimes, a spin accumulation (or non-equilibrium magnetization) that strongly affects electron transport may build up in the nonmagnetic island.

In this article, we discuss the transport characteristics of metallic SV-SETs in the Coulomb blockade regime, allowing for arbitrary, noncollinear magnetization directions. In particular, we examine the influence of exchange effects through F|N tunnel contacts on the spin accumulation in the center island, presenting a more complete discussion compared to that in Ref. We argue that two separate exchange effects have to be taken into account. On one hand, there is the non-local interface exchange, which we call it “X1” in the following. In scattering theory for non-interacting systems it is described by the imaginary part of the spin-mixing conductance while in the context of current-induced magnetization dynamics X1 acts as an “effective field”. Such an effective field has been found experimentally to strongly affect the transport dynamics in spin valves with MgO tunnel junctions. This effect has recently also been involved to explain magnetoresistance effects in carbon nanotube spin valves and called spin-dependent interface phase shifts. The second exchange term (“X2”) is an interaction-dependent exchange effect due to virtual tunneling processes that is absent in non-interacting systems. It has been considered for islands in the electric quantum limit, in which transport is carried by a single quantized level only. The X2 effect is potentially attractive for quantum information processing, since it allows to switch on and off effective magnetic fields in arbitrary directions just by a gate electric potential. We compute here X2 for a metallic island in which size quantization is not important. We find that both exchange effects are of comparable magnitude and affect the transport properties in a characteristic way, but can be separated in principle by employing the gate dependence of X2.

The paper is organized as follows. In Sec. we introduce the model system for the SV-SET. In Sec. the two relevant types of exchange processes are discussed. Charge and spin transfer rates are determined in Sec. Finally, we present results for the transport characteristics as a function of magnetic configuration, gate voltage and applied magnetic field in Sec.
An SV-SET (see Fig. 1b) is composed of a small metallic cluster in contact with two large ferromagnetic electron reservoirs in thermal equilibrium characterized by magnetization directions \( \vec{m}_1 \) and \( \vec{m}_2 \) with \( \vec{m}_\alpha = (\sin \theta_\alpha, 0, \cos \theta_\alpha) \) (for \( \alpha = 1, 2 \)), where \( \theta_1 = \theta/2 \) and \( \theta_2 = -\theta/2 \).

\[
\begin{align*}
\begin{array}{ccc}
(a) & F_1 & N \quad C_1 \quad C_2 \quad F_2 \\
\hline
m_1 & \overrightarrow{s} & m_2 \\
V/2 & \quad C_G \quad -V/2 \\
\end{array}
\end{align*}
\]

\( \text{FIG. 1: (a) The spin-valve single-electron transistor: A small normal-metal island tunnel-coupled to two large ferromagnetic leads. The unpaired spin angular momentum on the island is denoted by } \overrightarrow{s}. \) (b) The magnetization directions in the leads define an angle } \theta.

The F\(_\alpha\)N contacts are tunneling barriers with conductances that depend on the electron spin, \( G_{\alpha}^{\uparrow} \) for the majority and \( G_{\alpha}^{\downarrow} \) for the minority spin in the ferromagnet. The total conductance for contact \( \alpha \) is then given by \( G_\alpha \equiv (G_{\alpha}^{\uparrow} + G_{\alpha}^{\downarrow}) \) and the contact polarization is defined as \( P_\alpha \equiv (G_{\alpha}^{\uparrow} - G_{\alpha}^{\downarrow}) / (G_{\alpha}^{\uparrow} + G_{\alpha}^{\downarrow}) \). The resistances \( R_\alpha = 1/G_\alpha \) are taken to be much larger than the resistance quantum \( R_Q = h/e^2 \), which, at low enough temperatures and bias voltages, allows us to study the blockade of transport by the Coulomb interaction. The electron tunneling rates are governed by the change of electrostatic energy of the whole circuit upon transfer of an electron. The capacitances of the junctions \( C_\alpha \) determine the charging energy of the island.

We limit our considerations to islands formed by metallic clusters for which the thermal energy \( (k_B T) \) is much larger than the average single-particle energy separation (reciprocal density of states) \( \delta = 1/\rho_N \), but much smaller than the single-electron charging energy. Therefore, many levels on the island participate in the transport and we may treat the electronic spectrum as continuous. For a gold cluster with a diameter of 10 nm, \( \delta \) approximately corresponds to a temperature of 2 K. The Kondo physics of quantum dots connected to ferromagnetic leads\(^{4,31,32,33} \) is suppressed in this regime.

Since the currents flowing into and out of the cluster are spin-polarized, the island may become magnetized. The number of unpaired spins on the island is limited by spin-flip scattering, which we parametrize by a spin-flip relaxation time \( \tau_{sf} \). There is evidence from several experiments that the spin-flip times in metallic nanoparticles can be much longer than in bulk systems, which implies that the effects of a spin accumulation on the island should be taken into account. For later convenience we introduce the spin-flip conductance parameter \( G_{sf} \equiv \rho_N e^2 / (2\delta) \). We assume that the energy relaxation on the island is fast compared to the electron dwell time.

The total Hamiltonian for the SV-SET is

\[
H = H_N + \sum_{\alpha=1,2} (H_{F\alpha} + H_{T\alpha} + H_{exa}),
\]

where \( H_N \) is the Hamiltonian for the normal metal cluster in the “orthodox model” for Coulomb blockade

\[
H_N = \sum_{k_s} \varepsilon_k c_k^\dagger c_k + \frac{e^2}{2C} (n_N - C_G V_G / e)^2.
\]

Here \( c_k^\dagger \) is a creation operator for an electron state with orbital index \( k \) and spin \( s \in \{\uparrow, \downarrow\} \), where the \( z \)-axis is chosen as spin quantization axis. The Hamiltonian includes an electrostatic interaction energy which depends on the junction capacitances \( C_\alpha \), the gate voltage \( V_G \) and the excess number of electrons on the island \( n_N \). The gate voltage shifts the potential and induces a charge \( C_G V_G \). We assume that the gate capacitance \( C_G \ll C_1, C_2 \), and in the following we use \( C_1 = C_2 = C/2 \). The energy levels in the two ferromagnetic leads (denoted by \( \alpha = 1, 2 \)) are spin-dependent:

\[
H_{F\alpha} = \sum_{k_s} \varepsilon_{\alpha ks} a_{\alpha ks}^\dagger a_{\alpha ks}.
\]

The operators \( a_{\alpha ks}^\dagger \) create electrons with spin \( s \) in the spin-quantization axis along \( \vec{m}_\alpha \).
It is convenient to introduce annihilation operators $c_{\alpha ks}$ for electrons in the normal metal defined for a quantization axis in the direction of $\vec{m}_\alpha$. The relation between operators in the two bases is then $c_{\alpha ks} = \hat{U}_{ss'}(\theta_\alpha) c_{ks'}$, expressed in terms of the spin $\frac{1}{2}$ rotation matrix

$$\hat{U}(\theta_\alpha) = e^{i\sigma_3 \theta_\alpha/2} = \begin{pmatrix} \cos \theta_\alpha/2 & \sin \theta_\alpha/2 \\ -\sin \theta_\alpha/2 & \cos \theta_\alpha/2 \end{pmatrix}.$$  

Then, for each contact, a tunneling Hamiltonian

$$H_{T\alpha} = \sum_{kqs} T_{kq\alpha}^\alpha a_{\alpha ks}^\dagger a_{\alpha qs} + \text{h.c.}$$  

describes the coupling to the island. The tunneling coefficients are assumed to not significantly depend on energy on the scale of the charging energy. We discuss the exchange contribution, represented by the Hamiltonian $H_{ex\alpha}$, in the next section.

### III. EXCHANGE EFFECTS

Here we discuss two different exchange effects that affect the electrons in the normal metal island attached to magnetic contacts. These two flavors arise when the transport properties for an SV-SET are determined to lowest order in the tunnelling probabilities.

#### A. Nonlocal interface exchange (X1)

The non-local exchange coupling between ferromagnetic films through a normal metal spacer is an important effect that determines the ground state of magnetic multilayers (see Ref. [24] for a recent review). Electrons in a normal metal that are reflected at a contact to a ferromagnet, pick up a phase depending on the electron spin relative to the magnetization direction. In sufficiently clean and narrow F|N|F structures, quantum well states are formed in N whose energy depend on the magnetic configuration through the spin-dependent phase. By a rotation of the magnetization directions the energy spectrum and Fermi energy varies, causing the ground state energy to depend on the relative angle $\theta$. In metallic multilayers with a suitable spacer thickness, this can lead to an antiparallel ground state, which displays the celebrated giant magnetoresistance when the magnetizations are forced into a parallel direction by a magnetic field. Even when the ground state energies are not significantly affected by the exchange coupling, configuration-dependent quantized states can still be observed in transport. This has been shown for high-quality planar tunnel junctions as well as spin valves in which the node is formed by single carbon nanotubes with a quantized energy spectrum. In Ref. [30] the effect of interfacial phase shifts on the magnetoresistance of ballistic quantum wires between ferromagnetic leads was calculated. The spin-dependent phase shifts give rise to a slightly different quantization condition, which can spin-split the energy levels. Since we are here interested in classical islands with a continuous electron spectrum, we calculate energy shifts for a semiclassical island using the Bohr-Sommerfeld quantization rule in Appendix A.

Here we consider the limit of tunnel junctions between a normal metallic island and ferromagnetic electrodes. The torques on the ferromagnets are then very small. The exchange coupling does not significantly disturb the ferromagnets in this limit, but persists to affect transport. The present study focusses on the charge transport properties in the limit of small tunneling matrix elements, thus from the outset excluding resonant tunneling, co-tunneling or Kondo type physics. The states on the island may be size quantized, i.e. the energy level spacing exceeds the thermal energy (“quantum dot”), or, in the opposite limit, better described by a semicontinuous density of states (“classical dot”). Here we concentrate on the latter, i.e. semiclassical, diffuse, or chaotic islands, for which it can be shown quite generally that equilibrium spin currents are suppressed. The state of the island is then characterized by a semiclassical charge and spin distribution function that has to be determined self-consistently as a function of the junction parameters and the applied voltages. For non-interacting systems, the spin and charge currents through an F|N interface are determined not only by the conventional conductances $G_{\alpha\uparrow\downarrow}^{\uparrow\downarrow}$ and $G_{\alpha\downarrow\uparrow}^{\downarrow\uparrow}$ introduced above, but also by the complex spin-mixing conductances $G_{\alpha\uparrow\downarrow}^{\uparrow\downarrow}$ which are discussed in Sec. [13]. The real part Re $G_{\alpha\uparrow\downarrow}^{\uparrow\downarrow}$ is the material parameter that is proportional to the spin-transfer torque. The imaginary part Im $G_{\alpha\uparrow\downarrow}^{\uparrow\downarrow}$ reflects the spin-dependent interface phase shifts and affects the magnetization and spin accumulation dynamics as an effective exchange magnetic field parallel to the magnetization direction. Im $G_{\alpha\uparrow\downarrow}^{\uparrow\downarrow}$ is relatively small for intermetallic interfaces, but is in general comparable in magnitude to the other conductance parameters. The non-local interface exchange has been discussed in similar terms for spin valves consisting of Luttinger liquids with ferromagnetic contacts.
The blocking of transport by the Coulomb charging is usually described by Fermi’s Golden Rule (see below), which employs a probability (squared matrix elements) and energy conservation. As long as the charging energy is much smaller than atomic energy scales (like the Fermi energy), the junction parameters such as the interface transparency and spin-mixing conductance are unaffected and the Coulomb blockade is governed by the energy conservation criterion only. This implies that the exchange effect can be described by the $\Im G_{\alpha}^{\uparrow\downarrow}$ of the bare junction.

It remains to parametrize the exchange in the limit of the tunneling Hamiltonian, i.e., to lowest order in the interface transmission. We show below that this is achieved by adding the following exchange term $H_{\text{ex}}$ to the Hamiltonian for the two leads:

$$H_{\text{ex}} = \sum_{ks} \Delta\epsilon_{\alpha ks} c_{\alpha ks}^\dagger c_{\alpha ks}. \quad (6)$$

The energy shifts $\Delta\epsilon_{\alpha ks}$, see Eq. (A2), are proportional to the inverse density of states, but they remain relevant for small level splitting because the dwell time is inversely proportional to the average energy level separation or inverse density of states $\delta = \rho^{-1}_N$. This Hamiltonian is an effective Zeeman splitting caused by an exchange magnetic field in the direction of the magnetization, see Sec. IV B.

### B. Virtual tunneling processes (X2)

The interface exchange term $X1$ is a property of the separate interfaces and they contribute independently. The second type of exchange (X2) felt by the spins on the island is a property of the entire device. It originates from virtual tunneling processes, corresponding to single-electron transfer from and to the cluster. In the tunneling regime, this process can be treated and understood in terms of perturbation theory. In the absence of tunneling, the number of electrons on the island is a good quantum number. The perturbation by the contact to the electrodes allows mixing in of states in which the number of electrons on the island is changed by unity, at the cost of the charging energy. In second order perturbation theory this results in an energy gain represented by a sum over (virtually) excited states in which the Coulomb energy appears in the denominator and the tunneling probability in the numerator. When the leads are non-magnetic, these virtual processes correspond to a quantum correction to the average charge on the central electrode. This effect depends strongly on the applied gate voltage. When the unperturbed $N+1$ ($N-1$) particle ground state is tuned in energy just above the $\tilde{N}$ particle state, the quantum correction will be large and positive (negative). At the degeneracy point, perturbation theory breaks down, but the ensuing divergence can be controlled by taking into account finite temperatures.

When the tunneling probabilities to the ferromagnetic contacts are spin dependent, the deviations from the exact quantized charge on the island become spin-dependent, and therefore lead to a net excess of spins in the ground state that depends on the configuration of the contact magnetizations. For a symmetric spin valve it is easy to see that the island ground state magnetization due to these virtual processes X2 is maximal for parallel magnetizations and vanishes for antiparallel ones.

The additional exchange affects non-equilibrium electron transport, in contrast to higher-order so-called co-tunneling processes, to the same order as the in- and out-tunneling processes. For a quantum-dot island with a single quantized level, König and Martinek showed that in the case of non-collinear magnetizations the non-equilibrium spins on an island injected by a finite source-drain voltage are dephased by precessing around the effective exchange field. This effect was also discussed for few-level quantum dots. Since X1 discussed in Sec. III A is a material constant, the gate voltage dependence of X2 provides a handle for an experimental discrimination of the two effects. We derive an expression for the effective X2 exchange field for a classical SV-SET in Sec. IV B.

### IV. CHARGE AND SPIN TRANSPORT

We compute the transport characteristics of the SV-SET in lowest-order perturbation theory for a diffusive or chaotic island in the sequential tunneling regime. The rate equations lead to a probability distribution for the excess number of charges $n_N$. The excess spin accumulation $\vec{s}$ contains a large number of spins and we are interested in its average value in the steady state that is found from the condition $\langle d\vec{s}/dt \rangle = 0$.

#### A. Charge transfer

The operators for the excess number of electrons on the island and on the two leads are $n_N = \sum_{ks} c_{ks}^\dagger c_{ks}$ and $n_{F\alpha} = \sum_{ks} a_{\alpha ks}^\dagger a_{\alpha ks}$, respectively. The unpaired spin angular momentum on the cluster is written as
\[ \vec{s} = (\hbar/2) \sum_{kk's's'} c_k \hat{\sigma}_{ss'} c_{k's'} \], where \( \hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) is the vector of Pauli spin matrices. It is convenient to introduce a vector chemical potential \( \Delta \mu \) in the island, with size \( |\Delta \mu| = 2 |\langle \vec{s} \rangle| / (\rho_N \hbar) \) (see also Ref. [12]), where \( \rho_N \) is the density of states at the Fermi energy. We can take into account Stoner enhancement intra-island exchange effects in terms of the static susceptibility \( \chi_s \), and we may also write \( \Delta \mu = 2 \mu_B^2 |\langle \vec{s} \rangle| / (\chi_s \hbar) \). We denote the unit vector in the direction of the spin accumulation by \( \vec{s} \).

The charge current is equal to the expectation values for the rate of change of \( n_N \). In terms of the tunneling Hamiltonian \( H_T = H_{T1} + H_{T2} \), the time-evolution is given by

\[
\frac{dn_N}{dt} = \frac{i}{\hbar} [H_T, n_N] = \frac{i}{\hbar} \sum_{akqs's'} T_{kqs'} c_{akqs'}^\dagger c_{akqs'} + h.c.,
\]

We use the interaction representation, and write the total Hamiltonian as \( H = H' + H_T \). To second order in \( H_T \) we have

\[
\left\langle \frac{dn_N(t)}{dt} \right\rangle = \frac{i}{\hbar} \int_{-\infty}^{t} dt' \left\langle \left[ \frac{dn_N(t)}{dt}, H_T(t') \right] \right\rangle ,
\]

where \( \left\langle . \right\rangle \) denotes an expectation value with respect to Hamiltonian \( H' \). The electrochemical potentials of the two reservoirs are \( \mu_{cF1} = eV/2 \) and \( \mu_{cF2} = -eV/2 \). It is convenient to introduce grand canonical Hamiltonians including the chemical potentials as\(^{15,44}\)

\[
K_N = H_N - \hbar^{-1} \Delta \mu \cdot \vec{s},
\]

\[
K_{F_n} = H_{F_n} - \mu_{cF_n} n_{F_n},
\]

The time dependence \( c_{ks}(t) = e^{iK_{Nt}} c_{ks} e^{-\frac{i}{\hbar} K_{Nt}} \) can be formulated in terms of the projection operators

\[
\hat{u}^\dagger (\vec{s}) = \frac{1}{2} (I + \vec{s} \cdot \vec{\sigma}),
\]

\[
\hat{u}^\dagger (\vec{s}) = \frac{1}{2} (I - \vec{s} \cdot \vec{\sigma}),
\]

where \( I \) is the unit matrix, by making use of the equality

\[
e^{iK_{Nt}} \Delta \mu \cdot \vec{s} \cdot \vec{c}_{ps's'} e^{-\frac{i}{\hbar} \Delta \mu \cdot \vec{s} \cdot \vec{c}_{ps's'}^\dagger} = \sum_{s''} \left[ e^{-\frac{i}{\hbar} \Delta \mu t} \hat{u}^\dagger (\vec{s}) + e^{\frac{i}{\hbar} \Delta \mu t} \hat{u}^\dagger (\vec{s}) \right] \cdot \vec{c}_{ps's''}. \]

The leads and the island are supposed to be in thermal equilibrium, so that \( \left\langle c_{ks} c_{ks'}^\dagger \right\rangle = \int f (\epsilon) \delta_{kk'} \delta_{s's'}, \) with Fermi-Dirac distribution \( f (\epsilon) \equiv (1 + e^{\beta \epsilon})^{-1} \), where \( \beta \) is the inverse temperature. Using the expression for the matrix elements

\[
U (\theta_\alpha) u_{s''} (\vec{s}) U (\theta_\alpha)^\dagger \bigg|_{s's'} = \frac{1}{2} (1 + s's'\vec{s} \cdot \vec{m}_\alpha),
\]

with \( s', s'' \in \{\uparrow, \downarrow\} = \{+, -\} \),

the rate of change of the number of electrons on the island reads

\[
\left\langle \frac{dn_N}{dt} \right\rangle_{n_N = m} = \sum_{s's''} \frac{1}{2e^2} (G_\alpha + s'' P_\alpha G_\alpha \delta \cdot \vec{m}_\alpha) \times \left[ -F \left( -E_{m-1} + E_m - \mu_{cF_\alpha} + s'' \Delta \mu \right) + F \left( E_m - E_{m+1} + \mu_{cF_\alpha} - s'' \Delta \mu \right) \right],
\]

where \( F (\epsilon) \equiv \epsilon (1 - e^{-\beta \epsilon})^{-1} \) and \( E_m \equiv e^2 (m - CGvG/e^2) / 2C \). The relation between the up and down spin conductances \( (G^{\uparrow}_\alpha \uparrow \alpha) \) and \( (G^{\downarrow}_\alpha \downarrow \alpha) \) and the tunneling coefficients is \( G^{ss}_\alpha = (\pi e^2 / \hbar) \rho_N \rho_{F_\alpha s} |T_s^{\alpha}|^2 \), where \( |T_s^{\alpha}|^2 \) is the value of \( |T_{kqs}^{\alpha}|^2 \) at the Fermi energy averaged over all the modes. \( \rho_{F_\alpha s} \) is the spin-dependent density of states in ferromagnet \( \alpha \).
In the low-bias regime considered here we can linearize Eq. (13) in $\Delta \mu$ and $\mu_{cFa}$. The resulting expression for the rate for electron tunneling through contact $\alpha$, increasing the excess number of electrons $n_N$ from “0” to “1”, is denoted by $\Gamma_{\alpha}^{0\to1}$. The analogous rate for removing one electron when $n_N$ is “1” is $\Gamma_{\alpha}^{1\to0}$. Explicitly, we find

$$
\Gamma_{\alpha}^{0\to1}(V,G,\Delta \mu) = \frac{G_{\alpha}}{e^2} F(E_0 - E_1) + \frac{G_{\alpha}}{e^2} F'(E_0 - E_1) \left(-\mu_{cFa} + \frac{\Delta \mu}{2} P_{\alpha} \hat{s} \cdot \vec{m}_{\alpha}\right), \quad (15)
$$

$$
\Gamma_{\alpha}^{1\to0}(V,G,\Delta \mu) = \frac{G_{\alpha}}{e^2} F(E_1 - E_0) - \frac{G_{\alpha}}{e^2} F'(E_1 - E_0) \left(-\mu_{cFa} + \frac{\Delta \mu}{2} P_{\alpha} \hat{s} \cdot \vec{m}_{\alpha}\right). \quad (16)
$$

Now that we have determined the tunneling rates we can write down the master equation for electron transport in the orthodox model. We consider a regime in which $\epsilon V \ll k_B T \ll e^2/2C$, and restrict ourselves to a gate voltage range for which the excess number of electrons $n_N$ alternates between “0” and “1” ($0 < C_G V_G < \epsilon$), knowing that the results will periodically repeat with this period. The center of the Coulomb oscillation for transitions between $n_N = “0”$ and “1” electrons is at $C_G V_G = \epsilon/2$.

The steady state on the island is characterized by a constant spin accumulation (to be determined below) and the probabilities $p_0$ and $p_1$ that there are “0” or “1” excess electrons. We have $p_0 + p_1 = 1$. The rate equation for the probabilities is

$$
dp{p}{n} = -p_n \left(\Gamma_{n\to n+1}^{n=0} + \Gamma_{n\to n-1}^{n=1}\right) + p_{n+1} \Gamma_{n+1\to n}^{n=1} + p_{n-1} \Gamma_{n-1\to n}^{n=1}. \quad (17)
$$

From the condition of detailed balance, $p_0 \Gamma_{1\to0}^{0\to1} = p_1 \Gamma_{1\to0}^{1\to0}$, we find

$$
p_0(V,G,\Delta \mu) = f(E_0 - E_1) + \frac{\beta f(E_0 - E_1)f(E_1 - E_0)}{G_1 + G_2} \sum_{\alpha} \left(G_{\alpha} \mu_{cFa} - F_{\alpha} G_{\alpha} \frac{\Delta \mu}{2} \hat{s} \cdot \vec{m}_{\alpha}\right). \quad (18)
$$

The expression for the conductance of the SV-SET as a function of the spin accumulation can now be calculated and reads

$$
G(V,G,\Delta \mu) = -e p_0 \Gamma_{1\to0}^{0\to1} + e p_1 \Gamma_{1\to0}^{1\to0}
= \frac{G_1 G_2}{G_1 + G_2} \frac{\beta (E_0 - E_1)}{2 \sinh \beta (E_0 - E_1)} \left[1 - \frac{\Delta \mu}{2 e \epsilon V} \hat{s} \cdot (P_1 \vec{m}_1 - P_2 \vec{m}_2)\right]. \quad (19)
$$

### B. Spin accumulation

The steady-state spin accumulation is found by setting the total rate of change of $\hat{s}$ to zero. There are several contributions to the dynamics of the spin accumulation:

$$
\left\langle \frac{d \hat{s}}{dt}\right\rangle = p_0 \left\langle \frac{d \hat{s}}{dt}\right\rangle_{n_N = 0} + p_1 \left\langle \frac{d \hat{s}}{dt}\right\rangle_{n_N = 1} + \sum_{\alpha} \left\langle \frac{d \hat{s}}{dt}\right\rangle_{exa} \left\langle \frac{d \hat{s}}{dt}\right\rangle_{magn} + \left\langle \frac{d \hat{s}}{dt}\right\rangle_{sf}. \quad (20)
$$

The first two terms are due to the tunneling processes, the remaining ones to exchange, external magnetic fields and spin flip. We start from

$$
\frac{d \hat{s}}{dt} = \frac{i}{\hbar} [H_T, \hat{s}], \quad (21)
$$

with an expectation value that to second order in $H_T$ reads

$$
\left\langle \frac{d \hat{s}(t)}{dt}\right\rangle = \frac{i}{\hbar} \int_{-\infty}^{t} dt' \left\langle \left[\frac{d \hat{s}(t)}{dt}, H_T (t')\right]\right\rangle. \quad (22)
$$
The spin current (rate of change of the spin angular momentum) due to tunneling when \( m \) excess electrons are on the island reads (cf. Eq. 14)

\[
\left\langle \frac{d\vec{s}}{dt} \right\rangle_{n,N=m} = \frac{\hbar}{4e^2} \sum_{\alpha s''} (G_\alpha s'' \hat{s} + P_\alpha G_\alpha \vec{m}_\alpha) \times \\
\left[ -F \left(-E_{m-1} + E_m - \mu_{cF\alpha} + s'' \frac{\Delta \mu}{2} \right) + F \left(-E_{m+1} + E_m + \mu_{cF\alpha} - s'' \frac{\Delta \mu}{2} \right) \right] \\
+ \frac{\hbar}{4\pi e^2} \sum_{\alpha s''} P_\alpha G_\alpha s'' (\vec{m}_\alpha \times \hat{s}) \times \\
\left( \int \frac{d\epsilon_1}{\epsilon_2 - \epsilon_1} \frac{f(\epsilon_1) (1 - f(\epsilon_2))}{(\epsilon_2 - \epsilon_1 + E_m - E_{m+1} + \mu_{cF\alpha} - s'' \frac{\Delta \mu}{2})} \\
- \int \frac{d\epsilon_1}{\epsilon_2 - \epsilon_1} \frac{f(\epsilon_2) (1 - f(\epsilon_1))}{(\epsilon_2 - \epsilon_1 + E_m - E_{m-1} + \mu_{cF\alpha} - s'' \frac{\Delta \mu}{2})} \right),
\]

(22)

where the prime denotes a principal value integral. Here we used the relation:

\[
\left[ U(\theta_\alpha) w''(\hat{s}) \sigma' U(\theta_\alpha)^\dagger \right]_{s,s''} = \frac{1}{2} s'' \hat{s} + \frac{1}{2} s' \vec{m}_\alpha + \frac{1}{2} i s' s'' (\vec{m}_\alpha \times \hat{s}),
\]

(23)

with \( s', s'' \in \{\uparrow, \downarrow\} = \{+, -\} \).

To first order in the small induced energy shifts the exchange Hamiltonian \( H_{\text{ex}} \) modifies the unpaired spins as

\[
\left. \frac{d\vec{s}}{dt} \right|_{\text{ex}} = \frac{i}{\hbar} [H_{\text{ex}}, \vec{s}(t)],
\]

(24)

which results in a precession:

\[
\left\langle \frac{d\vec{s}}{dt} \right\rangle = \frac{1}{\hbar M} \sum_{m} (\Delta \epsilon_{\alpha m\uparrow} - \Delta \epsilon_{\alpha m\downarrow}) (\hat{s}) \times \vec{m}_\alpha,
\]

(25)

where \( M \) is the number of transport channels in the normal metal and the energy shifts \( \Delta \epsilon \) are found in Eq. (A2). The conductance parameters of an F|N contact are given by

\[
G^{s\alpha s'} = \frac{e^2}{\hbar} \sum_{nm} (\delta_{nm} - r^{nm}_{s\alpha} (r^{nm}_{s'\alpha})^*), (s, s' \in \{\uparrow, \downarrow\}).
\]

(26)

Here \( n \) and \( m \) denote the transport channels in the normal metal and \( r^{nm}_{s\alpha} \) and \( r^{nm}_{s'\alpha} \) are the corresponding spin-dependent reflection coefficients. The contact conductances for spin-up and spin-down electrons are \( G^{\uparrow\alpha} \) and \( G^{\downarrow\alpha} \) and the mixing conductance \( G^{\uparrow\downarrow\alpha} \) governs the transverse spin currents that are absorbed and reflected by the ferromagnet \( \alpha \). The current polarized normal to the magnetization but in the plane of \( \hat{s} \) and \( \vec{m}_\alpha \) is proportional to \( \text{Re} G^{\uparrow\downarrow\alpha} \) and describes the spin-transfer to the magnet, thereby dissipating the spin accumulation. In the case of tunnel junctions \( \text{Re} G^{\uparrow\downarrow\alpha} \rightarrow G_\alpha/2 \). The out-of-the-plane \( \hat{s}, \vec{m}_\alpha \) plane spin current is caused by reflection processes that make spins precess around \( \vec{m}_\alpha \) and is proportional to \( \text{Im} G^{\uparrow\downarrow\alpha} \). This mixing conductance has been evaluated from first principles for various contact materials and is small for intermetallic interfaces because positive and negative contributions in the space spanned by the transport channels average out. However, there is no general reason that \( \text{Im} G^{\uparrow\downarrow} \) should be smaller than \( G \) or \( \text{Re} G^{\uparrow\downarrow} \). It is known to be quite large for the Fe|InAs interface and found to be very significant for the magnetization dynamics of MgO magnetic tunnel junctions. For a simple model barrier discussed in Appendix B we find the value \( \text{Im} G^{\uparrow\downarrow}/G = -0.26 \). Using the relation between the reflection phases and the energy shifts as derived in Eq. (A2) we can rewrite the contribution given in Eq. (26) in terms of the imaginary part of the mixing conductance as (cf. Ref. 23)

\[
\left\langle \frac{d\vec{s}}{dt} \right\rangle_{\text{ex}} = \frac{\text{Im} G^{\uparrow\downarrow\alpha}}{\rho N \epsilon^2} \vec{m}_\alpha \times \langle \hat{s} \rangle.
\]

(27)
The spin accumulation can also be affected by a magnetic field $\vec{B}$, which can either be externally applied, a stray field from the ferromagnets, or an internal anisotropy field. The spin accumulation induced by this magnetic field can safely be neglected, but the induced precession of the spin accumulation is relevant, and is given by

$$\left\langle \frac{d\vec{s}}{dt} \right\rangle_{\text{magn}} = \frac{g\mu_B}{\hbar} \vec{B} \times \langle \vec{s} \rangle .$$

Finally, spin-flip relaxation in the normal metal is taken into account by spin-accumulation decay with a spin-flip relaxation time $\tau_{sf}$,

$$\left\langle \frac{d\vec{s}}{dt} \right\rangle_{\text{sf}} = -\frac{\langle \vec{s} \rangle}{\tau_{sf}} .$$

Combining the terms in Eq. (28), the spin accumulation should fulfill the stationary state condition:

$$\left\langle \frac{d\vec{s}}{dt} (V, V_G) \right\rangle = \frac{\hbar}{2e^2} \frac{\beta (E_0 - E_1)}{2 \sinh \beta (E_0 - E_1)} \times$$

$$\left[ \frac{G_1 G_2}{G_1 + G_2} eV (P_1 \vec{m}_1 - P_2 \vec{m}_2) - (G_1 + G_2) \frac{\Delta \mu}{2} \left( \vec{s} + \left( \vec{s} \cdot \vec{b} \right) \vec{b} \right) \right]$$

$$+ \frac{g\mu_B}{\hbar} \vec{B}_{\text{eff}} \times \langle \vec{s} \rangle - \frac{\langle \vec{s} \rangle}{\tau_{sf}} = 0,$$

where

$$\vec{b} \equiv \frac{P_1 G_1}{G_1 + G_2} \vec{m}_1 + \frac{P_2 G_2}{G_1 + G_2} \vec{m}_2 .$$

The total effective magnetic field $\vec{B}_{\text{eff}}$ consists of the external magnetic field and contributions from the exchange effects $X_1$ and $X_2$, and reads

$$\vec{B}_{\text{eff}} (V_G) = \vec{B} + \vec{B}_{X_1} + \vec{B}_{X_2} (V_G) ,$$

with

$$\vec{B}_{X_1} = \frac{h}{\rho_N g \mu_B e^2} \sum_\alpha \text{Im} G^{\uparrow \downarrow}_\alpha \vec{m}_\alpha ,$$

$$\vec{B}_{X_2} (V_G) = -\frac{h}{2 \rho_N g \mu_B e^2} (G_1 + G_2) \vec{b}$$

$$\left[ \frac{1}{\pi} f (E_0 - E_1) \int d\epsilon f' (\epsilon) \eta \left( \epsilon + E_0 - E_{-1}, \frac{e^2}{C} \right) \right]$$

$$+ \frac{1}{\pi} f (E_1 - E_0) \int d\epsilon f' (\epsilon) \eta \left( \epsilon + E_1 - E_0, \frac{e^2}{C} \right) \right] .$$

Here we introduced $\eta (\epsilon, U)$

$$\eta (\epsilon, U) \equiv \int' d\omega \left( \frac{1 - f (\omega)}{\omega - \epsilon} + \frac{f (\omega)}{\omega - \epsilon - U} \right)$$

$$= -\text{Re} \left[ \Psi \left( \frac{1}{2} + \frac{i \beta \epsilon}{2\pi} \right) - \Psi \left( \frac{1}{2} + \frac{i \beta (\epsilon + U)}{2\pi} \right) \right] ,$$

where $\Psi (z)$ is the Digamma function. In appendix $C$ we discuss the derivation of the expression for $\vec{B}_{X_2}$ in more detail and comment on the differences compared to the case of a single-level quantum dot.

V. RESULTS AND DISCUSSION

Fig. 2 shows the magnitude of the total effective magnetic field $\vec{B}_{\text{eff}}$ as a function of gate voltage (solid line) for a symmetric spin valve with parallel magnetizations (it vanishes for the antiparallel configuration) and a polarization
$P_1 = 0.7$. The X1 term is a constant that does not depend on gate voltage (dotted line). $\vec{B}_{X2}$ vanishes when $C_G V_G$ equals $0, e/2$ and $e$. At these points, contributions from incoming and outgoing electrons cancel each other (see appendix C). The curve repeats as a function of gate voltage with period $e/C_G$. The spin accumulation on the island found from Eq. (30) tends to suppress the current through the system. Spin-flip and exchange effects that dissipate or dephase the spin-accumulation therefore increase the conductance. As a reference we list here the conductance found from Eq. (30) tends to suppress the current through the system. Spin-flip and exchange effects that dissipate or dephase the spin-accumulation therefore increase the conductance. As a reference we list here the conductance $G(\theta)$ for a spin valve without interaction, with equal conductance parameters for the left and the right tunneling barrier $G_1 = G_2$, $P_1 = P_2$:

$$G(\theta) = \frac{G_1}{2} \left( 1 - \frac{P_1^2 G_1 (G_1 + 2G_{sf}) \sin^2 \theta/2}{G_1 + 2G_{sf} + \left(2 \cos \frac{\theta}{2} \Im G_{1+}^\dagger \right)^2} \right).$$

(36)

The final result for the symmetric spin-valve with interaction can be obtained simply from this expression by the substitutions:

$$G_1 \rightarrow \frac{\beta (E_0 - E_1)}{2e \sinh \beta (E_0 - E_1)} G_1,$$

(37)

$$\Im G_{1+}^\dagger \rightarrow \frac{\beta^2 \rho N g \mu \beta B_{\text{eff}}}{2 \cos \frac{\theta}{2}}.$$

(38)

For nonmagnetic contacts ($P_1 = 0$) this result reduces to the known expression for normal metal single-electron transistors.

As shown in Fig. 3(a), changing the relative strengths of X1 and X2, or, since the X2 contribution is proportional to the polarization of the leads, $\Im G_{1+}^\dagger / P_0 G_0$, qualitatively modifies the current profile of the Coulomb oscillations. The constant offset given by $B_{X1}$ skews the exchange field around $C_G V_G = e/2$, causing asymmetric conductance curves. When the offset starts to dominate the symmetry gets restored. The X2 contribution vanishes when the Coulomb blockade is lifted ($C_G V_G = e/2$), so the angular dependence of the conductance for different values of $\Im G_{1+}^\dagger / G_1$ in Fig. 3(b) reflects only the X1 effect. The curve is a simple cosine for $\Im G_{1+}^\dagger = 0$, but is sharpened for larger $\Im G_{1+}^\dagger$ because of the dephasing of the spin accumulation occurring for noncollinear angles. In Fig. 3(c) $\Im G_{1+}^\dagger / G_1$ is fixed to 0.25 and curves are plotted for different values of the gate voltage. It can be seen that the angular dependence differs because the X2 depends on $V_G$ in an asymmetric way around $C_G V_G = e/2$.

As can be seen in Fig. 4 the shape of the Coulomb oscillation can develop minima when the polarization is high and the magnetizations are nearly antiparallel. At the values of gate voltage where the X1 and X2 exchange effects cancel, the spin accumulation is not dephased and the conductance is suppressed.

Fig. 5 shows results for the conductance and spin accumulation as a function of applied magnetic field in the $x$ (solid line), $y$ (dashed) and $z$ (dotted) directions. The spin valve is again symmetric with $P_1 = 0.7$ and $\Im G_{1+}^\dagger = G_1/4$. 

![FIG. 2: The effective magnetic field strength $|\vec{B}_{\text{eff}}|$ as a function of gate voltage (solid line) for a spin valve in the parallel configuration. The parameters are $G_1 = G_2$, $P_1 = P_2 = 0.7$, $\Im G_1^\dagger = \Im G_2^\dagger = G_1/4$ and $e^2/(2C) = 10 k_B T$. The imaginary part of the mixing conductance gives a constant offset (dotted). The dot-dashed/dashed curves show the effective field for zero and one excess electron on the island.](image-url)
FIG. 3: (a) Coulomb oscillations at fixed angle $\theta = \pi/2$ for a symmetric SV-SET with ratio $\text{Im} G_1^{\uparrow\downarrow}/G_1 = 0$ (solid), 0.25 (dashed) and 1 (dotted) in units of $G_1$. The polarization $P$ is 0.7. (b) Conductance as a function of the angle for the same parameters as in a), with $C_G V_G$ fixed to 0.5. (c) Conductance as a function of $\theta$ with $\text{Im} G_1^{\uparrow\downarrow}/G_1 = 0.25$. Results are shown for $C_G V_G$ equal to 0.45 (dashed), 0.5 (solid) and 0.55 (dotted).

The angle $\theta$ is fixed to $\pi/2$ and $C_G V_G = e/2$. The conductance then depends only on the $x$-component of the spin accumulation (see Eq. 14). Without applied magnetic field, the spin accumulation has components in the $x$ and $y$ directions, while $\vec{B}_{\text{eff}}$ is in the $y$-direction. The results can be understood in terms of the dephasing of the spin accumulation by the magnetic-field induced precession that, for sufficiently large and non-collinear magnetic fields, quenches the spin accumulation. This “Hanle” effect is responsible for the conductance minimum at negative applied magnetic field in the $y$ direction. In (c) only two curves are visible because the curves for magnetic fields in the $x$ and $y$ direction overlap.

VI. SUMMARY

We studied the transport properties of single-electron spin valve transistors as a function of the magnetization configurations in the orthodox model of the Coulomb blockade. Two types of exchange effects between the spin accumulation on the island and the lead magnetizations play a role: a nonlocal interface exchange effect ($X_1$) and exchange due to virtual tunneling processes ($X_2$). For metallic dots these two effects are found to be of comparable magnitude. We predict that a line shape analysis of the Coulomb oscillation peaks should help to experimentally disentangle the two contributions. Additional information can be obtained by the Hanle effect.
VII. ACKNOWLEDGEMENTS

We would like to thank J. König, J. Martinek, M. Braun and Y.V. Nazarov for useful discussions. This research was supported by the NWO and by the DFG via the SFB 689, and supported in part by the National Science Foundation under Grant No. PHY99-07949.

APPENDIX A: ENERGY SHIFTS

Let us consider a normal metal island in contact to a ferromagnet by a tunnel barrier (See Fig. 6) without Coulomb interaction. We wish to calculate the spin-dependent shifts of the energy levels due to the presence of the F|N contact. In Ref. 30 an analogous calculation was done for a ballistic one-dimensional quantum wire. Here we consider an island in the quasi-classical regime, i.e. the de Broglie wavelength is much smaller than the size of the island. The Bohr-Sommerfeld quantization rule\(^47\)

\[
\frac{1}{\hbar} \oint p^m(x) \, dx + \phi_0^m + \phi_s^m = 2\pi \left( n + \frac{1}{2} \right)
\]

\[(A1)\]

can be used to find the energy shifts, where \(p^m(x)\) is the classical momentum for an electron in mode \(m\), and \(n\) is an integer. The integral is over a whole period of the classical motion in the quasi-classical region. The total phase shift due to the reflections at the turning points is \(\phi_0^m + \phi_s^m\), where \(\phi_0^m\) is the spin-independent phase shift picked up during the reflections from the boundaries for an isolated island without contact to the ferromagnet. The small spin-dependent phase shift \(\phi_s^m \ll 1\) arises from the weak coupling to the ferromagnet. The phase shifts have to be computed quantum mechanically via the spin-dependent reflection coefficients \(r_s^mm\) for mode \(m\) at an interface that is assumed to be specular (see also Appendix B).

From Eq. (A1), we see that increasing the quantum number \(n\) by one corresponds to introducing an extra phase period that increases the kinetic energy by \(M/\rho_N\), where \(\rho_N\) is the density of states of the island and \(M\) is the number of modes. The energy shift for an electron in mode \(m\) is therefore, to linear order in \(\phi_s^m\),

\[
\Delta\epsilon_{ms} = -\frac{M}{\rho_N} \frac{\phi_s^m}{2\pi}.
\]

\[(A2)\]

The effect of the interface on the island states can be taken into account by introducing an effective Hamiltonian as in Eq. (6). In the case of a spin-independent tunneling barrier to a ferromagnet, the spin-splitting of the energy levels is small, of the same order as the transmission probability (see App. B).
FIG. 5: (a) Conductance as a function of a magnetic field applied along the x (solid), y (dashed) or z (dotted) direction in units of $G_1$. The SV-SET has symmetric junction parameters, with polarizations $P_1 = 0.7$ and $\text{Im} G_1^{\uparrow\downarrow} = G_1/4$. The magnetizations are fixed to $\vec{m}_{1/2} = (\pm 1, 0, 1)/\sqrt{2}$, yielding an angle $\theta = \pi/2$ and $C_G V_G = e/2$. (b,c,d) The x, y and z components of the spin accumulation for the same parameters. The curves in (c) for magnetic fields in the x and y directions overlap.

FIG. 6: A normal metal island with tunnel contact to a ferromagnetic lead.

APPENDIX B: RECTANGULAR BARRIERS

Here we evaluate the spin-mixing conductance $G^{\uparrow\downarrow}$ for a model barrier, giving more details of the results of Ref. 23. We consider a smooth rectangular barrier between a normal metal and a Stoner-model ferromagnet. The solution of the Schrödinger equation for spin $s$ in the normal metal, $\psi_m^s(x, y, z)$ can be used to determine the reflection coefficients $r_{ss}^{mm}$ for each mode $m$. It reads

$$\psi_s^m(x, y, z) = \frac{\chi_s^m(x, y, z)}{\sqrt{k_{N}^m}} \left( e^{i k_{N}^m x} + r_{ss}^{mm} e^{-i k_{N}^m x} \right),$$

where $\chi_s^m(y, z)$ is the transverse wave function and $k_{N}^m$ is the longitudinal wave number for mode $m$ in the normal metal. In terms of the wave numbers in the normal metal $k_{N}^m$, barrier $k_{B}^m$ and ferromagnet $k_{F}^m$ for a given energy, the
reflection coefficient for mode $m$ at the barrier reads
\[ r_s^{mm} = \rho(k_N^m, k_B^m) + e^{2i\alpha k_B^m} \tau(k_N^m, k_B^m) \rho(k_B^m, k_F^m) \tau(k_F^m, k_N^m), \]
where $a$ is the barrier thickness and
\[ \tau(k_1, k_2) \equiv \frac{2\sqrt{k_1k_2}}{k_1 + k_2}, \]
\[ \rho(k_1, k_2) \equiv \frac{k_1 - k_2}{k_1 + k_2}. \]
For a tunneling barrier, $k_B^m$ is imaginary and the spin dependent correction to the reflection coefficient is exponentially small in the barrier thickness.

For a numerical estimate we use a Fermi energy in the normal metal of 2.6 eV, a barrier height of 3 eV and barrier thickness of $a = 1$ nm. The Fermi momenta in the ferromagnet are taken to be $k_{F\uparrow} = 1.09\,\text{Å}^{-1}$ and $k_{F\downarrow} = 0.42\,\text{Å}^{-1}$ (characteristic for Fe, see Ref. [39]).

For the spin-mixing conductance $G_{\uparrow\downarrow}$, Eq. (26), we find that $\text{Im} G_{\uparrow\downarrow}/G = -0.26$ for this choice of parameters. The effective field due the interface exchange effect is therefore not negligible compared to the conductance parameters. More realistic electronic structure calculations should be carried out to obtain better estimates.

**APPENDIX C: X2 EXCHANGE IN CLASSICAL DOTS**

Here we present more details concerning the derivation of Eq. (34) for the effective exchange field X2 in classical SV-SET’s, that complement the derivation in Refs. [17] for single-level quantum dots. Since the model is periodic in the gate voltage with period $e/C_G$, we restrict our discussion to the range $0 < C_GV_G < e$. From Eq. (20), the contributions from virtual tunneling processes to the rate of change of $\vec{s}$ then read:
\[ \langle \frac{d\vec{s}}{dt} \rangle_{X2} = p_0 \langle \frac{d\vec{s}}{dt} \rangle_{X2,n=0} + p_1 \langle \frac{d\vec{s}}{dt} \rangle_{X2,n=1}. \]

Using the spin currents from Eq. (22), we obtain, e.g.:
\[ \langle \frac{d\vec{s}}{dt} \rangle_{X2,n=0} = \frac{h}{4\pi e^2} \sum_{\alpha\alpha'} P_{\alpha} G_{\alpha\alpha'} \langle \vec{m}_{\alpha} \times \hat{s} \rangle \times \left[ \int d\epsilon_1 \int d\epsilon_2 \frac{f(\epsilon_1)(1-f(\epsilon_2))}{(\epsilon_2 - \epsilon_1 - E_{-1} - E_0 + \mu_{cF\alpha} - s'\Delta\mu)/2} \right. \]
\[ \left. - \int d\epsilon_1 \int d\epsilon_2 \frac{f(\epsilon_2)(1-f(\epsilon_1))}{(\epsilon_2 - \epsilon_1 + E_0 - E_1 + \mu_{cF\alpha} - s''\Delta\mu)/2} \right]. \]

The first term in brackets describes virtual processes in which an electron tunnels out of the island, and the second term corresponds to incoming electrons. The expressions for the energy differences are given by
\[ E_{-1} - E_0 = (C_GV_G + e/2)e/C, \]
\[ E_0 - E_1 = (C_GV_G - e/2)e/C. \]

Because of the periodicity in the gate voltage
\[ \langle \frac{d\vec{s}}{dt} \rangle_{X2,n=1} = \langle \frac{d\vec{s}}{dt} \rangle_{X2,n=0} \text{ with } V_G \rightarrow V_G - e/C_G. \]

We can now rewrite Eq. (C1) in terms of the function $\eta(\epsilon, U)$, defined in Eq. (33). The probabilities $p_0$ and $p_1$ are taken from Eq. (18). After linearization in $V$ and $\Delta\mu$, we arrive at the expression Eq. (34).

We note the differences with the results for single-level quantum dots[17] our expression includes an additional integral over the island states. For a single-level quantum dot, X2 is active only when exactly one electron resides on the dot, since there is no unpaired spin in an empty or doubly occupied dot. In contrast, a net spin accumulation
can reside on our classical dot for all numbers of electrons. The effective magnetic field is a sum weighted with the probabilities for “0” and “1” electrons on the dot, which leads to a partial cancellation of the contributions for different $n_N$, as is shown in Fig. 2.

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