Spinor Operator Giving Both Angular Momentum and Parity

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In heavy quark effective theory, heavy mesons which contain a heavy quark (or antiquark) are classified by \( s_\pi^\ell \), i.e., the total angular momentum \( s_\ell \) and the parity \( \pi_\ell \) of the light quark degrees of freedom around a static heavy quark. In this case, however, one needs to separately estimate the parity other than the angular momentum of a light quark to describe heavy mesons.

A new operator \( K \) was proposed some time ago by two of us (T.M. and T.M.). In this Letter, we show that the quantum number \( k \) of this operator is enough to describe both the total angular momentum of the light quark degrees of freedom and the parity of a heavy meson, and derive a simple relation between \( k \) and \( s_\pi^\ell \).

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Recent discovery of narrow meson states \( D_sJ(2317) \) and \( D_sJ(2460) \) by BaBar and the following confirmation by CLEO and Belle [1] has triggered a series of study on spectroscopy of heavy mesons again. Though \( D_sJ(2317) \) and \( D_sJ(2460) \) are assigned as \( j^P = 0^+ \) and \( 1^+ \), respectively, their masses are significantly smaller than the predictions based on many of potential models [2]. To explain these masses, Bardeen, Eichten, Hill and others [3, 4] proposed an interesting idea of an effective Lagrangian with chiral symmetries of light quarks and heavy quark symmetry. The heavy meson states with the total angular momentum \( j = 0 \) and \( j = 1 \) related to \( s_\ell \) (the total angular momentum of the light quark degrees of freedom) = 1/2 make the parity doublets \((0^-,0^+)\) and \((1^-,1^+)\), respectively, and the members in these doublets degenerate in the limit of chiral symmetry. Furthermore, the two states \((0^-,1^-)\) degenerate in the limit of heavy quark symmetry, as well as \((0^+,1^+)\). These doublets are called the heavy spin multiplets.

These newly discovered states are well classified in heavy quark effective theory, i.e., in terms of \( s_\pi^\ell \), where \( s_\ell \) and \( \pi_\ell \) represent the total angular momentum and the parity of the light quark degrees of freedom around a static heavy quark, respectively. In this case, however, one has to separately estimate the parity and the angular momentum of a light quark for each heavy meson state.

Some time ago, two of the authors (T.M. and T.M.) proposed a new bound state equation for atomlike mesons, i.e., heavy mesons composed of a heavy quark and a light antiquark, and they also proposed a new operator \( K \) which can classify heavy mesons well [5]. In this Letter, we show that this operator \( K \), given by Eq. (5) below, has the information about not only \( s_\ell \) but also the parity of heavy mesons and naturally explains the heavy spin multiplets. That is, only the quantum number \( k \) corresponding to the operator \( K \) can reproduce both the total angular momentum of the light quark degrees of freedom and the parity of a heavy meson. We also discuss the relation between \( k \) and \( s_\pi^\ell \).

Let us consider a heavy meson composed of a heavy quark \( Q \) and a light antiquark \( \bar{q} \). The effective Hamiltonian of this system is obtained by applying the Foldy-Wouthuysen-Tani (FWT) transformation to the heavy quark \( Q \). One can formulate the equation so that we can cast the structure of the eigenvalue equation into a simple form and make the Dirac-like equation in the large limit of the heavy quark mass \( m_Q \) [5]. In order to show why we can introduce a new operator \( K \) for heavy mesons, we consider the equation with \( 1/m_Q \) corrections neglected, whose contribution should be important in numerical analysis of spectroscopy.

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The lowest energy for the \( Q\bar{q} \) bound state is given by \( m_Q + E_0^a \) after solving the equation [5]

\[
H_0 \otimes \psi_0^a = E_0^a \psi_0^a, \quad H_0 = \vec{a}_q \cdot \vec{p}_q + \beta_q (m_q + S(r)) + V(r),
\]

where \( a \) expresses all the quantum numbers and quantities with the subscript \( q \) mean those for a light antiquark. \( S(r) \) is a confining scalar potential and \( V(r) \) is a Coulombic vector potential at short distances. Both potentials have dependence only on \( r \), the relative distance between \( Q \) and \( \bar{q} \). With a symbol \( \otimes \), one should note that gamma matrices for a light antiquark be multiplied from left with the wave function while those for a heavy quark from right.

Using the \( 2 \times 2 \) matrix eigenfunctions \( y_j^m \) of angular part defined below and the radial functions \( f_k \) and \( g_k \), the \( 4 \times 4 \) matrix solution to Eq. (1) is given by [5]

\[
\psi_0^a = \begin{pmatrix} 0 & \Psi_{j m}^k (\vec{r}) \end{pmatrix},
\Psi_{j m}^k (\vec{r}) = \frac{1}{r} \begin{pmatrix} f_k (r) y_j^m & ig_k (r) \bar{y}_j^m \end{pmatrix},
\]

where \( j \) and \( m \) are the total angular momentum of a heavy meson and its z-component, respectively. The total angular momentum of a heavy meson is the sum of the total angular momentum of the light quark degrees of freedom \( \vec{S}_t \) and the heavy quark spin \( \frac{1}{2} \vec{\Sigma}_Q \):

\[
\vec{J} = \vec{S}_t + \frac{1}{2} \vec{\Sigma}_Q \quad \text{with} \quad \vec{S}_t = \vec{L} + \frac{1}{2} \vec{\Sigma}_q,
\]

where \( \frac{1}{2} \vec{\Sigma}_q \) (= \( \frac{1}{2} \vec{\gamma}_q 1_{2 \times 2} \) ) and \( \vec{L} \) are the 4-component spin and the orbital angular momentum of a light antiquark, respectively. Furthermore, \( k \) is the quantum number of the spinor operator \( K \), which was introduced in Eq. (20) of Ref. [5], defined by

\[
K = -\beta_q \left( \vec{\Sigma}_q \cdot \vec{L} + 1 \right), \quad K \Psi_{j m}^k = k \Psi_{j m}^k.
\]

It is interesting to note that the same form of the operator \( K \) is defined in the case of a single Dirac particle in a central potential [6]. It is remarkable that in our approach \( K \) can be defined even for a heavy meson which is a two-body bound system composed of a heavy quark and a light antiquark.

Here we show that there is a relation between \( k \) and \( s_\ell \), being often used in heavy quark effective theory. Let us calculate the square of \( K \).

\[
K^2 = (\Sigma_q)_j (\Sigma_q)_j L_i L_j + 2 \vec{\Sigma}_q \cdot \vec{L} + 1 = \vec{L}^2 + \vec{\Sigma}_q \cdot \vec{L} + 1 = \vec{S}_t^2 + \frac{1}{4}.
\]

Therefore, the operator \( K^2 \) is equivalent to \( \vec{S}_t^2 \) and it holds

\[
k = \pm \left( s_\ell + \frac{1}{2} \right) \quad \text{or} \quad s_\ell = |k| - \frac{1}{2}.
\]

Now, let us briefly summarize the properties of the eigenfunctions \( y_j^m \), whose details are given in [5]. To begin with, we need to introduce the so-called vector spherical harmonics which are defined by [7]

\[
Y_{j m}^{(L)} = -i \vec{\sigma} \times \vec{Y}_{j m}^{(E)} = \frac{r}{\sqrt{j(j+1)}} \vec{\omega} Y_{j m}^{(E)}, \quad Y_{j m}^{(M)} = -i \vec{\sigma} \times \vec{Y}_{j m}^{(E)},
\]

where \( Y_j^m \) are the spherical polynomials and \( \vec{\omega} = \vec{r}/r \). These vector spherical harmonics are nothing but a set of eigenfunctions for a spin-1 particle. \( \vec{Y}_{j m}^{(A)} \) (A=L, M, E) are eigenfunctions of \( \vec{J}^2 \) and \( J_z \), having the eigenvalues \( j(j+1) \) and \( m \). The parities are assigned as \( (-)^{j+1} \), \( (-)^j \), \( (-)^{j+1} \) for A=L, M, E, respectively, since \( Y_j^m \) has a parity \( (-)^j \).

In order to diagonalize the leading Hamiltonian of Eq. (1) in the \( k \) space, it is necessary to make \( \vec{Y}_{j m}^{(A)} \) and \( Y_j^m \) into the spinor representation \( y_j^m \) by the following unitary transformation

\[
\begin{pmatrix} y_{j m}^{-(j+1)} \\ y_{j m}^{j+1} \\ \sigma_y Y_{j m}^{(L)} \\ \sigma_y Y_{j m}^{(M)} \end{pmatrix} = U \begin{pmatrix} Y_{j m}^m \\ \sigma \cdot \vec{Y}_{j m}^{(E)} \end{pmatrix}, \quad \begin{pmatrix} y_{j m}^m \\ y_{j m}^m \\ \sigma_y Y_{j m}^{(L)} \\ \sigma_y Y_{j m}^{(E)} \end{pmatrix} = U \begin{pmatrix} \sigma \cdot \vec{Y}_{j m}^{(L)} \\ \sigma \cdot \vec{Y}_{j m}^{(M)} \end{pmatrix},
\]
Thus, using the relations of Eqs. (13) and (15) and taking into account the intrinsic parity of the light antiquark, one can simply write the parity of a heavy meson as

\[ P = -P' = \frac{k}{|k|} (-1)^{|k|+1} \]

Notice that the parity \( P \) of the whole system is equal to the parity \( \pi_\ell \) of the light quark degrees of freedom, as can be seen in TABLE I, since the intrinsic parity of a heavy quark is +1.

In heavy quark effective theory, heavy mesons are normally classified in terms of \( s_t^\ell \), since at the lowest order heavy quarks in these mesons are considered to be static, namely it stays rest at the center of a heavy meson system. In this work, we have found that (i) the parity of a heavy meson and (ii) the total angular momentum of the light quark degrees of freedom can be reproduced in terms of \( k \) alone as seen from Eqs. (16) and (7), respectively. We have also found that the degeneracy between members in each heavy spin multiplet, \((0^-, 1^-)\) and \((0^+, 1^+)\), is automatic in our approach [5], while the method using the effective Lagrangian with heavy-quark as well as chiral symmetries must force degeneracy among parity doublets to construct such a Lagrangian [3, 4]. These are the main results of this paper.

As our summary, several states are classified by various quantum numbers in TABLE I. The states with different \( j \) but with the same parity \( P \) make a heavy spin multiplet of heavy mesons, which corresponds to heavy quark symmetry in heavy quark effective theory. One can see that \( k \) naturally explains the heavy spin doublets.
Before closing our discussions, we comment about $k$ from the phenomenological point of view. The lowest order solution satisfies degeneracy in $k$ since the energy depends only on $k$, i.e., $j^P = 0^-$ and $1^-$ states have the same mass, so are the $0^+$ and $1^+$ states. This degeneracy is resolved by including higher order terms in $1/m_Q$ [5] and one can phenomenologically discuss mass spectra of these heavy mesons even though some objections [8] for using a potential model exist. A comprehensive analysis on mass spectra of heavy mesons including $D_{sJ}(2317)$ and $D_{sJ}(2460)$ is in progress.

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