Can quantum mechanics be considered consistent? a discussion of Frauchiger and Renner’s argument.

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July 10, 2018

Abstract

We discuss the argument proposed in Ref. [1], and show that it does not particularly illustrate any inconsistency in quantum mechanics, but rather the well known difficulty often described as the “shifty split”: the exact point at which the von Neumann reduction postulate should be applied is ill defined. This is the origin of the famous Schrödinger’s cat or Wigner’s friend paradoxes.

We investigate the argument of Ref. [1] and show that it combines statements obtained by different agents assuming very different positions of the shifty split, and therefore applying the reduction postulate in different ways. This results in the introduction of several different state vectors, while such descriptions are considered as incompatible in standard quantum mechanics. To our knowledge, no interpretation of quantum mechanics includes this possibility; the argument thus refers to a new form of quantum mechanics that should be specified more precisely.

In Ref. [1], the authors propose a generalization of the “Wigner friend paradox” with a thought experiment illustrating the possible existence of inconsistencies in quantum mechanics. The purpose of the present note is to study their argument in more detail and, in particular, to write all relevant state vectors explicitly. This provides a useful guide to obtain a clearer view of the conclusions that can be drawn from the argument.

A well-known fundamental difficulty of quantum mechanics is that the fundamental equation of its dynamics (the Schrödinger equation giving the evolution of the state vector) allows the appearance of superpositions of macroscopically distinct states; since these superpositions are not observed, one needs to introduce a limit to the validity of the equation[1]. This difficulty is illustrated by the famous Schrödinger cat paradox [2] where, at the end of the scenario, the cat reaches a superposition state that is obviously meaningless (a “ridiculous case” in the words of Schrödinger in 1935). Another example is the “Wigner friend paradox” introduced by Wigner in 1961 [3], where a friend and his whole laboratory are supposed to reach a superposition of macroscopically distinct states – another illustration of the fact that the linear Schrödinger dynamics cannot be obeyed too far.

Actually, this problem was already well identified by von Neumann long before, in 1932 [4]. Treating the whole measurement process quantum mechanically, he pointed out that the consistency of the theory requires that measurement should not be treated with the usual deterministic dynamics. He therefore introduced the “projection postulate” which, under certain conditions, attributes a second rule for the evolution to the state vector. This solves the problem, but a real difficulty subsists: since the borders between the two different dynamics of the state vector are not precisely specified, the theory becomes ill defined. No-one knows exactly where to put this border, and physicists must resort to rules that are valid

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1 Unless one gives up macroscopic uniqueness, as in the Everett interpretation, where the difficulties are different.
only FAPP (for all practical purposes). In the abundant literature on the subject, it has often be named as the “shifty split” problem. We note in passing that the subject is not purely academic: the present progress of experimental techniques is such that the borders between the microscopic and macroscopic world are becoming more and more accessible.

Since the very essence of the Wigner paradox lies in possible contradictory evolutions of the state vector, we will complete the argument of [1] with calculations of the state vector in various situations - this reference evokes the calculation but never really completes it. Physics makes predictions with the help of equations, and in this case the equations contain the state vector (or the density operator); studying its exact expression is therefore necessary, independently of considerations on the interpretation of the theory. Since the question is to decide if and where state vector reduction should be applied, we will not make a selection between the various possible choices: we will perform the calculations in various scenarios, applying the reduction at different steps of the experiment, and trying in this way to exhaust the various possibilities to apply quantum mechanics consistently. As expected, all these different ways to apply quantum mechanics lead to different predictions, but we will check that none of these calculations leads to any inconsistency.

To facilitate the discussion, we use exactly the same notation as Ref. [1] and, for the sake of simplicity, we ignore the time evolution between the different measurements of the protocol described by these authors. This amounts to assuming that the whole experiment lasts a short time. Nevertheless, if this is not the case, it is straightforward to add unitary evolution operators wherever needed; this does not change the structure of the argument and its conclusions.

1 A first experiment

In §1.1 we first review the successive steps of the protocol and calculate the evolution of the state vector, without applying any von Neumann projection (this can be seen as the Everett calculation). To be sure that no entanglement effect has been missed, we write the state of all systems and all agents explicitly, even if this may lead to a somewhat cumbersome notation. The reader who is not particularly interested in these technicalities is invited to skip them and to go directly to the discussions of §§1.5 and 4.

Without state vector projection, the expression of the final state vector shows that all the statements of table II of Ref. [1] are not recovered together. We will then try to apply state vector projection and, since standard quantum mechanics is not very specific about when it should be applied, we will try different possibilities. In §1.2 we examine the case where agent \( F \) projects the state vector, in §1.3 where two agents project it, and in §1.4 the case where all agents project the state vector.

1.1 No projection of the state vector

We begin by a standard calculation including no state vector projection at all.

1.1.1 Successive expressions of the state vector

The first measurement is performed by an agent \( F \) (the letter F is for “friend”) with a qubit that is initially in a coherent superposition:

\[
|\Psi\rangle = \frac{1}{\sqrt{3}} |\text{heads}\rangle + \sqrt{\frac{2}{3}} |\text{tails}\rangle
\]  

(1)

If \( |F : h\rangle \) is the state of \( F \) (and her laboratory \( L \)) if she has observed a heads, and \( |F : t\rangle \) the state if she has observed a tails, after the first measurement the state of the whole system is:

\[
|\Psi_1\rangle = \frac{1}{\sqrt{3}} |\text{heads}\rangle |F : h\rangle + \sqrt{\frac{2}{3}} |\text{tails}\rangle |F : t\rangle
\]  

(2)
According to the scenario of Ref. [1], $\mathcal{F}$ then sends a spin to $F$ in a state that depends on her result of measurement: a state $|S : \uparrow \rangle$ down polarized along axis $Oz$ if she observes a heads, a state polarized in the orthogonal direction $Ox$ if she observes a tails. This leads to the state:

$$|\Psi_2 \rangle = \frac{1}{\sqrt{3}} |\text{heads} \rangle |\mathcal{F} : h \rangle |S : \downarrow \rangle + \sqrt{\frac{2}{3}} |\text{tails} \rangle |\mathcal{F} : t \rangle \left[ |S : \downarrow \rangle + \frac{1}{\sqrt{2}} |S : \uparrow \rangle \right]$$

$$= \frac{1}{\sqrt{3}} \left[ |\text{heads} \rangle |\mathcal{F} : h \rangle + |\text{tails} \rangle |\mathcal{F} : t \rangle \right] |S : \downarrow \rangle + \frac{1}{\sqrt{3}} |\text{tails} \rangle |\mathcal{F} : t \rangle |S : \uparrow \rangle$$

(3)

We now assume that the second friend $F$ performs a measurement of the $S_z$ component of the spin; $|F : \uparrow \rangle$ and $|F : \downarrow \rangle$ are the states reached by $F$ (and her laboratory $L$) corresponding to the two results of measurement. The state vector then becomes:

$$|\Psi_3 \rangle = \frac{1}{\sqrt{3}} \left[ |\text{heads} \rangle |\mathcal{F} : h \rangle + |\text{tails} \rangle |\mathcal{F} : t \rangle \right] |S : \downarrow \rangle |F : \downarrow \rangle + \frac{1}{\sqrt{3}} |\text{tails} \rangle |\mathcal{F} : t \rangle |S : \uparrow \rangle |F : \uparrow \rangle$$

(4)

At this stage, Ref. [1] introduces an external operator $\mathcal{W}$ (the letter $W$ emphasizes that this operator plays Wigner’s role in the usual paradox) who performs a measurement on $\mathcal{L}$ with eigenstates\(^2\) associated respectively to results “OK” and “fail”:

$$|\text{OK} \rangle_{\mathcal{L}} = \frac{1}{\sqrt{2}} \left[ |\text{heads} \rangle |\mathcal{F} : h \rangle - |\text{tails} \rangle |\mathcal{F} : t \rangle \right]$$

$$|\text{fail} \rangle_{\mathcal{L}} = \frac{1}{\sqrt{2}} \left[ |\text{heads} \rangle |\mathcal{F} : h \rangle + |\text{tails} \rangle |\mathcal{F} : t \rangle \right]$$

(5)

As remarked by Bub [5], the ket $|\Psi_3 \rangle$ has three other equivalent expressions:

$$|\Psi_3 \rangle = \frac{1}{\sqrt{3}} \left[ |\text{heads} \rangle |\mathcal{F} : h \rangle |S : \downarrow \rangle |F : \downarrow \rangle + |\text{tails} \rangle |\mathcal{F} : t \rangle |S : \downarrow \rangle |F : \downarrow \rangle + |\text{tails} \rangle |\mathcal{F} : t \rangle |S : \uparrow \rangle |F : \uparrow \rangle \right]$$

$$= \sqrt{\frac{2}{3}} |\text{fail} \rangle_{\mathcal{L}} |S : \downarrow \rangle |F : \downarrow \rangle + \frac{1}{\sqrt{3}} |\text{tails} \rangle |\mathcal{F} : t \rangle |S : \uparrow \rangle |F : \uparrow \rangle$$

$$= \frac{1}{\sqrt{3}} |\text{heads} \rangle |\mathcal{F} : h \rangle |S : \downarrow \rangle |F : \downarrow \rangle + \sqrt{\frac{2}{3}} |\text{tails} \rangle |\mathcal{F} : t \rangle |\text{fail} \rangle_{\mathcal{L}}$$

(6)

(the ket $|\text{fail} \rangle_{\mathcal{L}}$ in the third line will be defined below, in (8)). After $\mathcal{W}$’s measurement, the state vector written in (4) becomes:

$$|\Psi_4 \rangle = \sqrt{\frac{2}{3}} |\text{fail} \rangle_{\mathcal{L}} |\mathcal{W} : \text{fail} \rangle |S : \downarrow \rangle |F : \downarrow \rangle + \frac{1}{\sqrt{6}} \left[ |\text{fail} \rangle_{\mathcal{L}} |\mathcal{W} : \text{fail} \rangle - |\text{OK} \rangle_{\mathcal{L}} |\mathcal{W} : \text{OK} \rangle \right] |S : \uparrow \rangle |F : \uparrow \rangle$$

(7)

where $|\mathcal{W} : \text{OK} \rangle$ (and $|\mathcal{W} : \text{fail} \rangle$) are the states describing $\mathcal{W}$ (and his laboratory) having registered result $\text{OK}$ (and fail) respectively. At this stage, we check that $|\mathcal{W} : \text{OK} \rangle$ is perfectly correlated with $|S : \uparrow \rangle$, which shows that the third statement $S^Q_{\text{OK}}$ of Table II is correct.

This perfect correlation occurs because, in (1), the component of $|\Psi_3 \rangle$ on a down spin state is exactly orthogonal to $|\text{OK} \rangle_{\mathcal{L}}$; this is a destructive interference effect between the $|\text{heads} \rangle$ and $|\text{tails} \rangle$ components of $|\Psi_3 \rangle$ (opposite results obtained by $\mathcal{F}$). Of course, if the $|\text{heads} \rangle$ component of (1) was removed (because $\mathcal{F}$ has observed result “tails”), this effect would non longer occur and the correlation would vanish.

\(^2\)Of course, assuming the existence of these states is not usual, since they are coherent superpositions of macroscopically distinct states (states where $\mathcal{F}$ has registered different results in her memory register). Creating a pure state involving a coherent superposition of entire laboratories, such as those written below in (8), seems totally unrealistic. In fact, one generally assumes that such states do not exist, which means that the measurement procedure in question is impossible. But it is precisely the point of Wigner’s friend paradox and of Ref. [1] to discuss the possible effects of such macroscopic superpositions, and to introduce pure thought experiments; here we follow the same approach.
Finally, the external operator \( W \) performs a measurement on \( L \) with eigenstates:

\[
\begin{align*}
|\text{OK}\rangle_L &= \frac{1}{\sqrt{2}} \left[ |S:\uparrow\rangle |F:\downarrow\rangle - |S:\downarrow\rangle |F:\uparrow\rangle \right] \\
|\text{fail}\rangle_L &= \frac{1}{\sqrt{2}} \left[ |S:\downarrow\rangle |F:\downarrow\rangle + |S:\uparrow\rangle |F:\uparrow\rangle \right]
\end{align*}
\]

(8)

In the corresponding basis, \( |\Psi_4\rangle \) becomes:

\[
|\Psi_4\rangle = \frac{1}{\sqrt{12}} (|\text{OK}\rangle_T |W: \text{OK}\rangle |\text{OK}\rangle_L |W: \text{OK}\rangle - |\text{OK}\rangle_T |W: \text{OK}\rangle |\text{fail}\rangle_L |W: \text{fail}\rangle \\
+ \frac{1}{\sqrt{12}} |\text{fail}\rangle_T |W: \text{fail}\rangle |\text{OK}\rangle_L |W: \text{OK}\rangle + \frac{\sqrt{3}}{2} |\text{fail}\rangle_T |W: \text{fail}\rangle |\text{fail}\rangle_L |W: \text{fail}\rangle)
\]

(9)

If \( |W: \text{OK}\rangle \) and \( |W: \text{fail}\rangle \) are the states describing \( W \) (and his laboratory) after this measurement, the state vector finally becomes:

\[
|\Psi_5\rangle = \frac{1}{\sqrt{12}} (|\text{OK}\rangle_T |W: \text{OK}\rangle |\text{OK}\rangle_L |W: \text{OK}\rangle - |\text{OK}\rangle_T |W: \text{OK}\rangle |\text{fail}\rangle_L |W: \text{fail}\rangle \\
+ \frac{1}{\sqrt{12}} |\text{fail}\rangle_T |W: \text{fail}\rangle |\text{OK}\rangle_L |W: \text{OK}\rangle + \frac{\sqrt{3}}{2} |\text{fail}\rangle_T |W: \text{fail}\rangle |\text{fail}\rangle_L |W: \text{fail}\rangle)
\]

(10)

Relation (10) provides what could be called the “Everett view” of the experiment, where no state vector projection ever occurs and all observers are entangled. In particular we remark that, because of \( W \)'s entanglement, the state has now acquired a non-zero component over \( (|\text{OK}\rangle_T |W: \text{OK}\rangle |S: \downarrow\rangle |F: \downarrow\rangle) \); if we insert (8) into (10) instead of (9), the component no longer vanishes. Therefore \( W \) observing \( \overline{\text{OK}} \) does not mean that the spin is necessarily up.

1.1.2 Correlations between measurement results

We first notice on (10) that, one times out of 12, the two results \( \overline{\text{OK}} \) and OK are obtained, in agreement with the fourth statement \( s^W_Q \) of Table II of Ref. [1].

(i) The \( r=\text{tails} \) component of (10) is obtained by using relations (5):

\[
|\Psi_5\rangle_{\text{tails}} = |\text{tails}\rangle_T \left[ 1 \right] ( |\text{fail}\rangle_T |W: \text{fail}\rangle |\text{OK}\rangle_L |W: \text{OK}\rangle + \frac{\sqrt{3}}{2} |\text{fail}\rangle_T |W: \text{fail}\rangle |\text{fail}\rangle_L |W: \text{fail}\rangle) \]

(11)

which shows that results \( r=\text{tails} \) and OK are compatible; the first statement \( s^T_Q \) of Table II of Ref. [1] is not valid.

If, nevertheless, one suppresses the kets describing the state of \( \overline{W} \) from (11), the first and the third term in the right hand side of this expression cancel each other; the component on \( |W: \text{OK}\rangle \) disappears by a destructive interference effect – exactly as in the third line of (9), where \( |\text{tails}\rangle \) is associated with \( |\text{fail}\rangle_L \) only. This (arbitrary) operation restores the first statement of Table II. But, since \( W \) is actually entangled with \( T \)'s laboratory in the full state vector (11), this destructive interference effect does not occur.

(ii) The \( r=\text{heads} \) component is obtained in the same way as (11); it also contains 4 orthogonal components, which restores the \( |\text{heads}\rangle_T |\overline{F}: \text{h}\rangle |S: \uparrow\rangle |F: \uparrow\rangle \) component that had disappeared from the first line of (9). From this discussion we conclude that, if one wishes to study the correlations between the results of \( F \) and \( W \), one must take into account the perturbation introduced by \( \overline{W} \)'s measurement: \( F \)'s initial statement is valid only if she ignores the future perturbations introduced by this measurement.
(iii) We can also start from (11) and expand the ket $|\text{OK}(L,W)\rangle |\text{OK}(L,W)\rangle$ by inserting the definitions (5) and (6) of the kets in the product; we then obtain coherent superpositions of two possible states of the coin, of the two operators $F$ and $F$, and of the spin:

$$
|\text{OK}(L,W)\rangle |S;o\rangle |F;o\rangle = \frac{1}{\sqrt{2}} \left[ |\text{OK}(L,W)\rangle |S;\rangle - |\text{OK}(L,W)\rangle |S;\rangle \right] |S;o\rangle |F;o\rangle
$$

The first line shows again that result OK is possible after result $r$=tails. The second line shows that the spin can be in any state if $\text{OK}$ is obtained, in contradiction with statement $s^W$. The same is true for other combinations of results obtained by $\text{W}$ and $W$.

(iv) The conclusion of this subsection is that, in terms of assumption (S) of Ref. [1], the average value of the projector $P^H$ considered in Assumption (Q) is not equal to unity. It would certainly be the case only if the perturbations introduced by $\text{W}$ at an intermediate time were ignored, but this would have no justification: in quantum mechanics, measurements do introduce perturbations. As we see in the next subsection, recovering statement $s^W$ is indeed possible, but requires that $F$ should apply the postulate of state vector reduction, which then changes other predictions.

1.2 Operator $F$ projects the state vector

We can try to improve the correlations by using state vector projection. If operator $F$ projects the state vector under the effect of her measurement, and if she observes “tails”, relation (2) then becomes:

$$
|\Psi_1'\rangle = |\text{tails}\rangle |F;o\rangle
$$

and (3) is replaced by:

$$
|\Psi_2'\rangle = |\text{tails}\rangle |F;o\rangle \left[ \frac{1}{\sqrt{2}} |S;\rangle + \frac{1}{\sqrt{2}} |S;\rangle \right]
$$

The state vector just after the second measurement by $F$ now becomes, instead of (11):

$$
|\Psi_3'\rangle = |\text{tails}\rangle |F;o\rangle \left[ \frac{1}{\sqrt{2}} |S;\rangle |F;\rangle + \frac{1}{\sqrt{2}} |S;\rangle |F;\rangle \right]
$$

which, in the basis of the eigenstates (8) performed by $W$ is nothing but:

$$
|\Psi_4'\rangle = \frac{1}{2} \left[ |\text{fail}\rangle |W;\rangle - |\text{OK}\rangle |W;\rangle \right] \left[ |S;\rangle |F;\rangle + |S;\rangle |F;\rangle \right]
$$

(i) After $W$’s measurement, this state becomes:

$$
|\Psi_4'\rangle = \frac{1}{2} \left[ |\text{fail}\rangle |W;\rangle - |\text{OK}\rangle |W;\rangle \right] \left[ |S;\rangle |F;\rangle + |S;\rangle |F;\rangle \right]
$$

which, in the basis if the eigenstates (8) performed by $W$, reads:

$$
|\Psi_4'\rangle = \frac{1}{\sqrt{2}} \left[ |\text{fail}\rangle |W;\rangle - |\text{OK}\rangle |W;\rangle \right] |\text{fail}\rangle
$$

We then see that result OK is never obtained. Physically this is because, if result “tails” only is selected, $F$ sends to $L$ a transversely polarized spin, which after measurement by $F$ projects $F$ and $L$ into a state that is orthogonal to $|\text{OK}\rangle |L;\rangle$: a destructive quantum interference effect then forbids result OK. Moreover, $W$’s measurement performed on laboratory $L$ does not change the state of $L$, so that

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3. Agents $W$ and $W$ perform their measurements on two different systems, $L$ and $L$, which can be at an arbitrary distance. Quantum mechanics does not allow superluminal communication, and it obeys the so called NS conditions [6, 7]. What is nevertheless important is the order in which measurements are performed on one site, since the corresponding projectors do not commute. In §3 we study what happens if the order of measurements is reversed.
cannot reintroduce result OK. Result “fail” is therefore always obtained by $W$, and the first statement $s_{Q}^{F}$ in Table II of Ref. [1] is now correct.

(ii) Nevertheless, the third statement $s_{Q}^{W}$ of Table II is now not correct: in [16], the correlation between results associated with $|\bar{W} : \bar{OK}\rangle$ and $|S:|\rangle$, which was perfect in [17], has disappeared. Therefore, $\bar{W}$ obtaining result $\bar{OK}$ no longer means that $\bar{W}$ can sometimes also obtain result OK.

This is because the initial projection of the state vector performed after $F$’s measurement has destroyed a $|\text{heads}\rangle$ component of the state vector that interfered destructively with another component, and was the origin of the perfect correlation. To understand how a second component can reappear after the first one has been cancelled, we can expand the bracket of (18) by inserting relations (5) and obtain:

$$|\Psi_{4}\rangle = \frac{1}{2} \left[ \left( |\text{heads}\rangle |\bar{F}: h\rangle + |\text{tails}\rangle |\bar{F}: t\rangle \right) |\bar{W}: \text{fail}\rangle - \left( |\text{heads}\rangle |\bar{F}: h\rangle - |\text{tails}\rangle |\bar{F}: t\rangle \right) |\bar{W}: \text{OK}\rangle \right] |\text{fail}\rangle_{L},$$

which indeed has a non-zero component on $|\text{heads}\rangle$, even if this is not the case in the previous states $|\Psi_{2}\rangle$ and $|\Psi_{3}\rangle$. The properties of (18) are similar to those of (11): the component result $|\text{heads}\rangle$ vanishes if the state of $\bar{W}$ is ignored (the entanglement with $\bar{W}$ is ignored), but no longer does when entanglement is included.

This shows that, as above, it is the perturbing effect of $\bar{W}$’s measurement on system $\bar{F}$ that changes the correlations; here, it reintroduces a component on $|\text{heads}\rangle$ into a ket that had no such component. In other words, agent $\bar{W}$ changes the content of the memory of $\bar{F}$, who thought that she has observed r=tails, but once perturbed no longer remembers any specific result.

### 1.3 $\bar{F}$ and $F$ project the state vector

If $F$ observes $|S:|\rangle$, relation (15) is replaced by:

$$|\Psi_{2}\rangle = |\text{tails}\rangle |\bar{F}: t\rangle |S:|\rangle |F :|\rangle = \frac{1}{2} \sqrt{2} \left[ |\text{tails}\rangle |\bar{W}: \text{fail}\rangle - |\bar{OK}\rangle_{\bar{F}} |\bar{W}: \bar{OK}\rangle \right] |S:|\rangle |F :|\rangle$$

$$= \frac{1}{2} \left[ |\text{tails}\rangle_{\bar{W}} - |\bar{OK}\rangle_{\bar{F}} \right] \left[ |\text{tails}\rangle_{\bar{W}} - |\bar{OK}\rangle_{\bar{F}} \right]$$

If $F$ observes $|S:\downarrow\rangle$, the only difference is that a plus sign replaces the minus sign in the second bracket. In any case, the state is a product: whatever $\bar{W}$’s result is, no information is obtained on the spin. Actually, neither the first statement $s_{Q}^{\bar{F}}$ nor the third $s_{Q}^{\bar{W}}$ of Table II are then correct. Nevertheless, $F$’s measurement has changed the state of his laboratory $L$ and restored the possibility of $\bar{W}$ observing result OK.

### 1.4 Every agent projects the state vector

We now assume that each of the four agents projects the state vector. The last two measurements, by $W$ and $\bar{W}$, determine the final form of the state vector, which is nothing but one of the four components of (11). If, for instance, these two agents obtained results $\bar{OK}$ and OK, we retain only the first component, and obtain the final state vector (after normalization) as:

$$|\Psi_{5}^{F}\rangle = |\bar{OK}\rangle_{\bar{F}} |\text{OK}\rangle_{L} |\bar{W}: \bar{OK}\rangle |W : \text{OK}\rangle$$

$$= \frac{1}{2} \left[ |\text{heads}\rangle |\bar{F}: h\rangle - |\text{tails}\rangle |\bar{F}: t\rangle \right] \left[ |S:|\rangle |F :|\rangle - |S:\downarrow\rangle |F :\downarrow\rangle \right] |\bar{W}: \bar{OK}\rangle |W : \text{OK}\rangle$$

The observation of result $\bar{OK}$ provides no information on the present state of the spin; the observation of result OK is not incompatible with tails.
1.5 Discussion

Assume now that a “superoperator” \( O \) decides to build a logical reasoning by combining the statements made by the various agents who perform measurements, including of course the two friends \( F \) and \( W \). The superoperator attempts to find a logical framework in which he can obtain all lines of Table II of Ref. \([1]\). For this purpose, he can take different points of view, depending whether he considers that \( F \) and \( W \) (and their laboratories), just after they perform a measurement, should necessarily be described by quantum states associated with one single result (as one usually does in quantum mechanics), or can also be described by superposition of such states (as in Wigner’s friend original paradox). Since several agents play a role in the experiment, different choices are possible.

(i) The most natural point of view is to consider that that \( F \) can communicate valid statements about her results to \( O \) only when she and her laboratory (position of the pointer of her apparatus, etc.) are described by an eigenstate associated with her measurement, for instance \(|\text{tails}\rangle\). Otherwise her “memory register” (in Everett’s terms) is in a state containing at the same time different results; under these conditions, one may wonder how she could express any statement with certainty to \( O \). But then, since \( F \) is not in a quantum superposition when \( W \) starts his measurement, we have seen in §1.2 that result \( \underline{\text{OK}} \) does not necessarily implies that the spin is in state \(|S\uparrow\rangle\); statement \( s_{Q}^{\underline{\text{OK}}} \) is not correct.

A similar difficulty occurs with \( s_{Q}^{\underline{\text{W}}:\text{tail}} \) statement: after \( F \) has obtained “tails”, we have seen that she can be certain that result “fail” will necessarily be obtained by \( W \) only if \( F \) is allowed to go through a quantum superposition of \(|\text{heads}\rangle\) and \(|\text{tails}\rangle\), of \(|\text{heads}\uparrow\rangle\) and \(|\text{tails}\downarrow\rangle\). One can save both statements \( s_{Q}^{F} \) and \( s_{Q}^{\underline{\text{W}}} \), but at the price of treating the two friends \( F \) and \( W \) in a non symmetric way, and losing \( s_{Q}^{\underline{\text{W}}} \).

(ii) One could also envisage a “mirror” case where \( O \) allows \( F \) to be described after measurement by quantum superpositions of \(|\text{heads}\rangle\) and \(|\text{tails}\rangle\), but considers that \( F \) is restricted to quantum states associated with well-defined results. In this case, we have seen in §1.2 that result \( \underline{\text{OK}} \) observed by \( W \) means that the spin is in state \(|S\uparrow\rangle\); as a consequence, \( W \) can observe both results \( \underline{\text{OK}} \) and “fail”. But then \( F \) has been projected into another coherent superposition of \(|\text{heads}\rangle\) and \(|\text{tails}\rangle\), and cannot make any specific statement at any time.

(iii) If \( O \) allows \( F \) and \( W \) to be described by any superposition after they have performed a measurement, he does not get any certain statement from \( F \) at time \( t=0 \), and therefore cannot write the fourth line of Table II. At later times, we have seen in §1.3 that the perturbation introduced by \( W \)’s measurement destroys the perfect correlation between r=tails and w=“fail”. This is because the perturbation has erased any specific result contained in \( F \)’s memory, and put her “memory register” (in Everett’s terms) into a state containing at the same time different results; under these conditions, one may wonder how she could express any statement with certainty to \( O \).

(iv) Finally, if neither of the friends \( F \) and \( W \) is allowed to remain after measurement in quantum superpositions of states associated with different measurement results, we have seen in §1.3 that the successive projections destroy the correlations.

In conclusion of this discussion, if \( O \) wants to write all statements of Table II together, he must accept a statement made by \( F \) about a specific result she has observed one specific result even if, at the same time, \( F \) is actually described by a state vector where its memory contains several different results. This amounts to combining the properties arising from several state vectors, i.e. combining correlations properties associated with incompatible measurements (\( F \)’s and \( W \)’s measurements are associated with eigenstates that, for photons, would be at 45 degrees from each other, and therefore incompatible measurements). But consistency requires that he should use a single state vector: just after \( F \) performed her measurement, either she is in an eigenstate of measurement (and she can then issue a valid statement), or she is in a quantum superposition of states associated with several different results (she then cannot make any statement), but not both at the same time.

\(^4\) We have checked in §1.3 that, if this coherent superposition is destroyed, the possibility of result “OK” reappears.

\(^5\) Compare the third line of (6) and (11).
2 Reversing the order of measurements

Another possibility is to try changing the orders of the measurements, in order to explore if this provides a way to obtain all statements of table II at the same time.

2.1 $\overline{W}$ performs his experiment before $\overline{F}$

Assume now that $\overline{W}$ performs his experiment before $\overline{F}$: the order of measurements is now $F, \overline{W}$, and then only $\overline{F}$. When $\overline{W}$ performs his measurement, operator $\overline{F}$ and his laboratory still in their initial state, which we denote $|\overline{F} : 0\rangle$; the eigenstates of $W$’s measurement are obtained by replacing, in (5), both $|F : h\rangle$ and $|F : t\rangle$ by $|\overline{F} : 0\rangle$. When this substitution is made, the state vector $|\Psi^\prime_W\rangle$ after $\overline{W}$ has obtained result $\overline{OK}$ is obtained by using (7):

$$|\Psi^\prime_W\rangle = |\overline{OK}\rangle_{\overline{W}} |\overline{W} : \overline{OK}\rangle |S : \uparrow\rangle |F : \uparrow\rangle = \frac{1}{\sqrt{2}} \left( |\mathrm{heads}\rangle - |\mathrm{tails}\rangle \right) |\overline{F} : 0\rangle |\overline{W} : \overline{OK}\rangle |S : \uparrow\rangle |F : \uparrow\rangle$$

(22)

In this way, the correlation between the measurement result $\overline{OK}$ and $|S : \uparrow\rangle$ is obtained again. But then, if $\overline{F}$ performs his measurement and obtains result “tails”, the state vector becomes:

$$|\Psi^\prime\rangle = |\mathrm{tails}\rangle |\overline{F} : t\rangle \left[ |\overline{W} : \overline{OK}\rangle |S : \uparrow\rangle |F : \uparrow\rangle = |\mathrm{tails}\rangle |\overline{F} : t\rangle \left[ |\overline{OK}\rangle_L + |\mathrm{fail}\rangle_L \right] \right]$$

(23)

operator $\overline{F}$ can non longer be sure that observing “tails” implies that $W$ will obtain any specific result. Reversing the order of $\overline{W}$ and $\overline{F}$ experiments restores the perfect correlation between results $\overline{OK}$ and $|S : \uparrow\rangle$, but cancels that between “tails” and “fail”.

2.2 $W$ performs his measurement before $\overline{W}$

In order to better conserve the correlation between the result observed by $\overline{F}$ and that obtained by $W$, we now assume that $W$ performs his measurement before $\overline{W}$ does (or that $W$ performs no measurement at all).

2.2.1 Without projection the state vector

In this case, we need to directly expand the ket (1) onto the eigenstates (5), which provides:

$$|\Psi_3\rangle = \frac{1}{\sqrt{6}} \left[ |\mathrm{heads}\rangle |\overline{F} : h\rangle + |\mathrm{tails}\rangle |\overline{F} : t\rangle \right] \left[ |\overline{OK}\rangle_L + |\mathrm{fail}\rangle_L \right] + \frac{1}{\sqrt{6}} |\mathrm{tails}\rangle |\overline{F} : t\rangle \left[ |\mathrm{fail}\rangle_L - |\overline{OK}\rangle_L \right]$$

$$= \frac{1}{\sqrt{6}} \left[ |\mathrm{heads}\rangle |\overline{F} : h\rangle \right] \left[ |\overline{OK}\rangle_L + |\mathrm{fail}\rangle_L \right] + \frac{\sqrt{2}}{3} |\mathrm{tails}\rangle |\overline{F} : t\rangle |\mathrm{fail}\rangle_L$$

(24)

This provides the state vector just before $W$ performs his measurement.

In the right hand side of this result, we notice the absence of any term in $|\mathrm{tails}\rangle |\overline{F} : t\rangle |\overline{OK}\rangle_L$, due to a destructive interference between two terms in the first line of (24). Now that the perturbation introduced by $\overline{W}$’s measurement has disappeared, we see that $|\mathrm{tails}\rangle$ and $|\mathrm{fail}\rangle_L$ are indeed perfectly correlated. Statement $s^W_Q$ in Table II of Ref. [1] then becomes correct, but statement $s^W_Q$ vanishes.

2.2.2 With a projection the initial state vector

Another way to check that $|\mathrm{tails}\rangle$ and $|\mathrm{fail}\rangle_L$ are perfectly correlated is to assume that $\overline{F}$ measures “tails”, and applies the projection postulate to $|\Psi_1\rangle$ given by (2). Then $|\Psi_2\rangle$ in (3) is truncated to its $|\mathrm{tails}\rangle$ component (multiplied by $\sqrt{3}/2$ for normalization) and $|\Psi_3\rangle$ becomes:

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}} |\mathrm{tails}\rangle |\overline{F} : t\rangle \left[ |S : \downarrow\rangle |F : \downarrow\rangle + |S : \uparrow\rangle |F : \uparrow\rangle \right] = |\mathrm{tails}\rangle |\overline{F} : t\rangle |\mathrm{fail}\rangle_L$$

(25)
Clearly, $\mathcal{T}$ can then predict with certainty that $W$’s measurement will give the result “fail”. This is easy to understand physically: if $\mathcal{F}$ obtains a tails, she sends a spin in a transverse $Ox$ direction to $L$, so that $F$ and her laboratory $L$ reach a coherent superposition that is orthogonal to $|\text{OK}_L\rangle$. Result OK is then impossible, due to a destructive interference between the components $|S:\downarrow\rangle|F:\downarrow\rangle$ and $|S:\uparrow\rangle|F:\uparrow\rangle$. Note in passing that this assumes and interference between two macroscopically distinct states ($F$ is macroscopic): the measurement apparatus used by $W$ necessarily makes these states overlap again (in the dBB interpretation, one says that a macroscopic wave that was empty becomes effective again).

But, if $\mathcal{W}$ performs his measurement before $W$, this destructive interference effect no longer occurs: the coefficients are not the same in (10) and in (24). If $\mathcal{T}$ has obtained “tails”, $\mathcal{W}$ can then obtain both results. If for instance he obtains result $\text{OK}$, one has to project (10) onto states containing $|\text{OK}_T\rangle$, and one obtains (after normalization):

$$
|\Psi_5\rangle = \frac{1}{\sqrt{2}} |\text{OK}_T\rangle|W:\text{OK}\rangle |\text{OK}_L\rangle|W:\text{OK}\rangle - \frac{1}{\sqrt{2}} |\text{OK}_T\rangle|\text{OK}_T\rangle|\text{fail}_L\rangle|W:\text{fail}\rangle
$$

$$
= |\text{OK}_T\rangle|W:\text{OK}\rangle |\text{OK}_L\rangle|S:\downarrow\rangle|F:\downarrow\rangle
$$

(26)

Now state $|S:\uparrow\rangle|F:\uparrow\rangle$ has completely disappeared from this ket, so that the destructive interference effect can non longer take place; none of the final results, OK or fail is forbidden in the $|\text{tails}\rangle$ component. This is also visible in the expression:

$$
|\Psi_5\rangle = \frac{1}{\sqrt{2}} |\text{W}_{\text{OK}}\rangle |\text{OK}\rangle \left[ |\text{OK}_L\rangle + |\text{fail}_L\rangle \right]
$$

(27)

### 3 Adding a qubit to the experiment

It has also been suggested [8] to protect the information obtained by $\mathcal{T}$ from the perturbations created by the outside observers, and to assume that she uses a “secret qubit” to store her result of measurement. The hope is that this additional qubit may connect the situations of §1.1 and 1.2, by providing a which-path information at the level of $\mathcal{F}$ that remains perfectly stable. Alternatively, this qubit can be seen as representing a “friend of the friend”, which we name $\mathcal{G}$. In any case, the corresponding quantum system reaches state $|\mathcal{G}:h\rangle$ if $\mathcal{T}$ observes “heads”, and state $|\mathcal{G}:t\rangle$ if she observes “tails”. Then $|\Psi_3\rangle$ becomes:

$$
|\tilde{\Psi}_3\rangle = \frac{1}{\sqrt{3}} \left[ |\text{heads}\rangle |\mathcal{F}:h\rangle |\mathcal{G}:h\rangle + |\text{tails}\rangle |\mathcal{F}:t\rangle |\mathcal{G}:t\rangle \right] |S:\downarrow\rangle |F:\downarrow\rangle
$$

$$
+ \frac{1}{\sqrt{3}} |\text{tails}\rangle |\mathcal{F}:t\rangle |\mathcal{G}:t\rangle |S:\uparrow\rangle |F:\uparrow\rangle
$$

(28)

The external operator $\mathcal{W}$ now performs a measurement on $\mathcal{L}$ with unchanged eigenstates $|\mathcal{L}\rangle$. In this basis, $|\tilde{\Psi}_3\rangle$ becomes:

$$
|\hat{\Psi}_4\rangle = \frac{1}{\sqrt{6}} \left\{ \left[ |\text{OK}_T\rangle |\mathcal{W}:\text{OK}\rangle + |\text{fail}_T\rangle |\mathcal{W}:\text{fail}\rangle \right] |\mathcal{G}:h\rangle
$$

$$
+ \left[ |\text{fail}_T\rangle |\mathcal{W}:\text{fail}\rangle - |\text{OK}_T\rangle |\mathcal{W}:\text{OK}\rangle \right] |\mathcal{G}:t\rangle \right\} |S:\downarrow\rangle |F:\downarrow\rangle
$$

$$
+ \frac{1}{\sqrt{6}} \left[ |\text{fail}_T\rangle |\mathcal{W}:\text{fail}\rangle - |\text{OK}_T\rangle |\mathcal{W}:\text{OK}\rangle \right] |\mathcal{G}:t\rangle |S:\uparrow\rangle |F:\uparrow\rangle
$$

(29)

After $\mathcal{W}$’s measurement, we obtain:

$$
|\tilde{\Psi}_4\rangle = \frac{1}{\sqrt{6}} \left\{ \left[ |\text{OK}_T\rangle |\mathcal{W}:\text{OK}\rangle + |\text{fail}_T\rangle |\mathcal{W}:\text{fail}\rangle \right] |\mathcal{G}:h\rangle
$$

$$
+ \left[ |\text{fail}_T\rangle |\mathcal{W}:\text{fail}\rangle - |\text{OK}_T\rangle |\mathcal{W}:\text{OK}\rangle \right] |\mathcal{G}:t\rangle \right\} |S:\downarrow\rangle |F:\downarrow\rangle
$$

$$
+ \frac{1}{\sqrt{6}} \left[ |\text{fail}_T\rangle |\mathcal{W}:\text{fail}\rangle - |\text{OK}_T\rangle |\mathcal{W}:\text{OK}\rangle \right] |\mathcal{G}:t\rangle |S:\uparrow\rangle |F:\uparrow\rangle
$$

(30)
Finally, $W$ performs his measurements with unchanged eigenstates $\psi$. The expansion of $\psi$ on these eigenstates finally leads to the complicated expression:

$$
\psi = \frac{1}{\sqrt{12}} \left\{ \left( |\text{OK}\rangle_L |\text{W} : |\text{OK}\rangle + |\text{fail}\rangle_L |\text{W} : |\text{fail}\rangle \right) |G : h\rangle \\
+ \left( |\text{fail}\rangle_L |\text{W} : |\text{fail}\rangle - |\text{OK}\rangle_L |\text{W} : |\text{OK}\rangle \right) |G : t\rangle \right\} \left[ |\text{OK}\rangle_L + |\text{fail}\rangle_L \right] \\
+ \frac{1}{\sqrt{12}} \left( |\text{fail}\rangle_L |\text{W} : |\text{fail}\rangle - |\text{OK}\rangle_L |\text{W} : |\text{OK}\rangle \right) |G : t\rangle \left[ |\text{fail}\rangle_L - |\text{OK}\rangle_L \right]
$$

or:

$$
\psi = \left( |\text{OK}\rangle_L |\text{W} : |\text{OK}\rangle \right) |\text{OK}\rangle_L \frac{1}{\sqrt{12}} |G : h\rangle \\
+ \left( |\text{OK}\rangle_L |\text{W} : |\text{OK}\rangle \right) |\text{fail}\rangle_L \left[ \frac{1}{\sqrt{12}} |G : h\rangle - \frac{2}{\sqrt{12}} |G : t\rangle \right] \\
+ \left( |\text{fail}\rangle_L |\text{W} : |\text{fail}\rangle \right) |\text{OK}\rangle_L \frac{1}{\sqrt{12}} |G : h\rangle \\
+ \left( |\text{fail}\rangle_L |\text{W} : |\text{fail}\rangle \right) |\text{fail}\rangle_L \left[ \frac{1}{\sqrt{12}} |G : h\rangle + \frac{2}{\sqrt{12}} |G : t\rangle \right]
$$

As expected, the state $|G : t\rangle$ is correlated with only one result of measurement performed by $W$, namely result “fail”. This is because $|G : t\rangle$ keeps a memory of a past event where $F$ sent a transversely polarized spin to $F$, which projects $L$ into a state that is orthogonal to $|\text{OK}\rangle_L$, and therefore necessarily corresponds to result “fail”. But $|\text{OK}\rangle$ is not correlated to any specific spin orientation or state of agent $F$. Replacing $F$ by $G$ therefore does restore the validity of the first statement of Table II of Ref. [1], but cancels others. Not surprisingly, we recover the same predictions as in §1.2, since the selection of one quantum state of the added qubit is equivalent to a projection of the state vector. Similarly, if another hidden qubit was introduced to store $F$’s result, we would recover the results of §1.3.

## 4 Conclusion

Because of the “shifty split” problem, most authors agree that standard quantum mechanics is, to some extent, fundamentally an ill defined theory – even if, in practice, physicists know how to use it perfectly well. This is the reason why we have attempted to recover all statements of Table II of Ref. [1] by applying the projection postulate in different ways (or apply no projection at all); we have also tried to reverse the order of experiments in order to change the correlations. The conclusion is that there seems to be no way to obtain all statements of Table II together with a single state vector; depending on the method of calculation, some hold and some do not, but no calculation leads to all statements at the same time.

For instance, they require that a statement made by $F$ after having observed a specific result should be considered as valid even if, at the same time, $F$ is described by a state vector where its memory contains several different results. This amounts to combining conclusions obtained by putting the “shifty split” at completely different places. Bohr would probably have seen this situation as one more illustration of complementarity: statements that are relative to incompatible experimental descriptions or devices (or different time evolutions) should not be combined together. This can also be seen, for instance, in Ref. [5], which explicitly gives the three expressions of the state vector expanded on three different basis associated with three incompatible couple of measurements. So, even if ill defined to some extent, standard quantum mechanics remains consistent.

The reasoning of Ref. [1] requires combining the observations and deductions of several agents, who use assumption (Q). Nevertheless this assumption is ambiguous, and actually can be understood in different ways. It may indeed be seen as an invitation for each agent to make predictions by using his own state vector (or density matrix), and by calculating the average value of projectors that correspond only to the
measurements she/he has made (or might have made). Each agent would then take into account only the specific result of her/his measurement by projecting his state vector (or density matrix) independently of the others, and ignoring the effects of other measurements in the past or of the future. For instance, as we have seen in §§1.5 and 2.1, \( W \) can predict that his result \( \Omega \) necessarily corresponds to a spin \( |\uparrow\rangle \) if he ignores the effects of \( F \)’s measurement, and takes into account only his own projector.

But, in standard quantum mechanics (independently of its interpretation), this is not the way a probability (or a certainty) should be calculated. If several measurements are performed, the probability must be calculated by calculating the average value of products of symmetrically embedded projectors (this is sometimes called the Wigner probability formula). For instance, if \( F \) makes the first measurement, and if the information on the result leaks outside (to the superoperator), the corresponding projector must be applied first to the state vector; the discussion of the hidden qubit in §3 also illustrates the role of this information. These successive projections automatically prevents the profusion of different state vectors. Generally speaking, if one remains within standard quantum theory, one should not combine the predictions obtained from two different quantum states, even if they are states describing the same physical system at different times (for instance \( |\Psi_3\rangle \) and \( |\Psi_5\rangle \) before and after \( W \) performed his measurement). In other words, if a physical system has at a given time certain correlation properties, there is no reason in standard theory why these properties should remain at all times, when these correlations might have been perturbed. Nothing of course forbids changing the rules of quantum mechanics to invent a new theory\(^6\), but the inconsistencies that may then be obtained should not be ascribed to quantum mechanics. In his discussion of the same subject [10], A. Sudbury comes to very similar conclusions, and analyzes it in terms of the BBB theory (Bell version of the de Broglie-Bohm field theory).

Even if one decides to give up the standard rules, it is not clear how one could reason on experiments where observers project each other into different quantum states, mutually changing the contents of their memories. Even if the agent had in mind a specific result she observed before, she may be put into another state where her memory now contains different measurement results at the same time, a state where they no longer know what she had observed. Once the experiment is completed, all the observers are supposed to exchange information and to build a reasoning together, or to send the information to the superoperator \( O \). But what is the meaning of a discussion between observers that are sorts of Schrödinger cats, remembering opposite facts at the same time? It seems more reasonable to consider an agent as a reliable source of information only if his/her memory contains a result that is certain. One could of course always invoke a “freedom of choice of the agents”, and consider that the memories of each agent belong each to their own world, which is described by their own private state vector; but the agents then belong to parallel, incompatible, worlds, so that their observations should not be combined in a single reasoning. Mathematically, certainties are a special case of probabilities, and it is known that combining them requires that they should all belong to the same space of probabilities. The exact nature of this space should be defined precisely before building a logical scheme.

Our conclusion is that quantum mechanics may be ill defined, but that the argument of Ref. [1] does not particularly point to any specific possible internal inconsistency. Rather, it shows that logical inconsistencies may appear if one changes the rules by describing of the same physical evolution with several different independent state vectors, and then combines the corresponding predictions. Since none of the interpretations discussed in Ref. [1] includes this possibility, none of them leads to all 4 statements of Table II, which means that they do not necessarily have to violate one of the 3 assumptions (Q),(C) and (S) of this reference. Of course, examining their compatibility with the different interpretations of quantum mechanics is an interesting question in itself. Nevertheless, it would also be interesting to clarify precisely which sort of modified quantum theory could lead to all conclusions of Table II.

Acknowledgments

The author is very grateful to Philippe Grangier for drawing his attention to Ref. [1], as well as for many interesting discussions and useful comments.

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\(^6\)The modal interpretation of quantum mechanics introduces several states vectors to describe the same quantum system, but in a way that is carefully designed not to change the predictions of quantum mechanics.
Appendix

It is instructive to investigate how some of the interpretations of quantum mechanics describe the succession of events taking place in the Frauchiger-Renner experiment. We will examine two of them: modified Schrödinger dynamics \[11, 12\] and dBB (de Broglie-Bohm) theory \[13, 14\].

(i) Modified Schrödinger dynamics introduces a stochastic reduction process of the state vector. Any macroscopic superposition of states localized in different regions of space is quickly resolved into one of its components by this process. Since every agent is indeed macroscopic, state vector reduction occurs at macroscopic superposition of states localized in different regions of space is quickly resolved into one of the two possible results; as in §1.2, result OK is not correlated only with a \(|\Psi_4\rangle\) spin state, so that \(|\Psi_4\rangle\) is readily obtained from the ket \(|\Psi'_4\rangle\) written in \((18)\) and \((19)\):

\[
|\Psi'_4\rangle = \frac{1}{\sqrt{2}} \left[ (\text{fail})_L |W : \text{fail}\rangle - (\text{OK})_L |W : \text{OK}\rangle \right] |\text{fail}\rangle_L |W : \text{fail}\rangle \\
= \frac{1}{2} \left[ \left( |\text{heads}\rangle |F : h\rangle + |\text{tails}\rangle |F : t\rangle \right) |W : \text{fail}\rangle \\
- \left( |\text{heads}\rangle |F : h\rangle - |\text{tails}\rangle |F : t\rangle \right) |W : \text{OK}\rangle \right] |\text{fail}\rangle_L |W : \text{fail}\rangle
\]  

(33)

The Bohmian position of the pointer of \(W\)’s apparatus may be in two regions of space, indicating one of the two possible results; as in §1.2, result OK is not correlated only with a \(|S : \uparrow\rangle\) spin state, so that the third statement \(s_Q^F\) of Table II vanishes. Nevertheless, the pointer of \(W\)’s measurement apparatus is necessarily in a region of space where it indicates a result “fail”, in agreement with the first statement \(s_Q^F\). We see that the pointer of \(F\)’s measurement apparatus may have changed its position under the effect of \(W\)’s measurement; it may now indicate “heads”, while it initially indicated “tails”. This is nothing but a direct consequence of the re-appearance of the \(|\text{heads}\rangle\) component discussed in (ii) of §1.2.

This curious property can be understood in terms of the existence of the so called “surrealistic Bohmian trajectories” \[15\]. The effect of \(W\)’s measurement on \(F\)’s laboratory is to recombine two components of the wave function associated with different results obtained before, heads or tails. What happens is similar to the recombination of the two components of the wave function studied in \[16\], where the interference between these components may reverse the motion of a pointer. In the dBB view, the pointers constantly indicate a specific result by their position, but this indication may change in time if the whole
apparatus is subject to the perturbation of an external measurement. So, when speaking of the indication of pointers and results of measurement, one should carefully specify at which stage of the experiments these indications are taken into account, as already mentioned above.

Finally, let us come back to the full state vector of §1.1 without initial projection resulting from $\mathcal{F}$’s measurement. She observes “tails” if the Bohmian position of her laboratory falls in the region of configuration space where the $|\text{tails}\rangle$ component of the ket (3) does not vanish; the $|\text{heads}\rangle$ component is then empty. Moreover, since only this empty component can lead to $W$ obtaining result OK in the future, it seems that this result should be impossible. But we have seen that $\overline{W}$’s measurement may bring back the Bohmian position of $\overline{\mathcal{F}}$’s laboratory into the region of configuration space associated with this component, which therefore becomes active again: result OK can then be observed. In the Bohmian description, it is the effect of $\overline{W}$’s measurement that reactivates a wave that was empty and restores the possibility of obtaining result OK. Agent $\mathcal{F}$ should therefore not conclude that OK is impossible if she obtains “tails”, because this would amount to ignoring the perturbations induced by $\overline{W}$’s measurement.

In conclusion, as expected, none of these interpretations leads to all statements of Table II of Ref. [1]. Therefore, their consistency does not require that they should necessarily reject one of the three assumptions (Q), (C) and (S) of this reference – even if of course this does not mean either that they should satisfy all of them.

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