Investigation of critical activities in a network with point-to-point relations

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Abstract

Recently developed point-to-point relations have greatly extended the modeling capability of the Precedence Diagram Method. These relations can connect any inner (and extreme) points of the related activities. Furthermore, the definition of multiple relations is allowed between the same two activities. These features make point-to-point relations suitable for modeling overlapping activities more easily and with greater precision than it was possible before. However, there is a lot of work ahead. Although both proper mathematical modeling, and the algorithm that handles both minimal and maximal relations are available, re-investigation of well-known definitions for float, critical path, critical path metrics, classification of critical activities etc. are still missing. This paper deals with the investigation of the critical path. It describes the ways in which an activity can be part of the critical path. Collecting the possible cases provides the opportunity for classifying critical activities. The main result of the paper is that it provides a method for determining the type of the critical activity based on the relationships going in and out of the activity.

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Peer-review under responsibility of the organizing committee of the Creative Construction Conference 2015

Keywords: critical path; point-to-point relations; precedence diagramming.

1. Introduction

The Precedence Diagram Method (PDM) is the most widespread network technique due the fact that the network is easy to draw, there is a wide choice of relationships and an infinite number of lead/lag factors compared to the CPM network where only FS0 relationships can be defined. [1] The scheduling applications used today display the networks with a Gantt diagram complete with the precedence dependencies. PDM has barely changed since Roy [2,

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3] and Fondahl [4] published their results despite the critiques it has received about its modeling capabilities, especially modeling activity overlapping. Point-to-point relationship may be able to solve this problem and be more suitable for modeling overlapping.

In a PDM network, different activities of the critical path may behave in various ways. These categories have already been defined in case of networks with only end-point relationships. This paper aims to show how the changes of the activity duration affect the project duration and whether new categories should be introduced. In addition, it also discusses whether the total float has to be calculated in a different way.

2. Point-to-point relationships

Proper modeling of overlapping activities seems to be a never-ending debate when traditional precedence relations are used, [5] because traditional endpoint relations are simply not suitable for describing this kind of logic. Different solutions have been proposed using the traditional precedence relations; but the fragmentation of activities and the developments based on this idea [6] seem to be the best theoretical solution despite the arising practical problems, namely the multiplication of the number of activities and precedence relations. Probably the fragmentation technique has given the idea of connecting the inner points of the activities. These point-to-point relations connecting the internal points of the activities seem to be theoretically more suitable for modeling overlapping activities – especially if continuous activities are assumed – as multiple relations are allowed between the activities. To the best of our knowledge, four, partly parallel, works regarding point-to-point relations can be found: Kim [7, 8] calls his new relations bee-line relations and the graphical representation Bee-line Diagram (BDM), while Francis and Miresco [9, 10] call their new relations temporal functions and their graphical representation method chronographic approach. Plotnick [11] calls his method Relationship Diagramming method (RDM) using the term ‘event’ for an internal point, which is also an acceptable terminology. Ponce de Leon [12] uses the term Graphical Diagramming Method (GDM) and the connected internal points are called embedded nodes. Despite the minor differences in terminology and definitions, e.g. bee-line and RDM relation does not allow a lag between the connected inner points, maximal lags are defined only in the work of Francis and Miresco [9, 10], the concept behind all these works is the same. All these improvements regarding the relationships can be seen as the same new type of precedence relation that can substitute all traditional precedence relations (FS, FF, SS, SF). Figure 1 shows the representation of a point-to-point relationship. The internal points could be given in either the volume of work or in time units (e.g. days). [13] However, only time units can be used for lags.
A proper mathematical model and the algorithm using standardized technical terminology is introduced in Hajdu’s work. [14]

3. Critical activities

In a network the critical path (CP) is the longest path between the start and the finish of the project. (Resources, constraints and calendars are not allowed for the sake of simplicity.) The activities comprising the critical path are called critical activities. The total float of each of these activities is zero. The previous statements are valid for both CPM and PDM networks. However, in case of CPM networks, it is true that changing the activity duration of a critical activity results in a similar change of the project duration. In case of a PDM network, the result of such a change is more difficult to calculate. Due to the various relationship types and infinite options for the lag/lead times, the project duration could either increase (+), decrease (-), or not change at all (0). Figure 2 summarizes all the mathematically possible combinations.

| Critical activity types                  | Effect on project duration, if activity duration |
|-----------------------------------------|--------------------------------------------------|
|                                         | decreases | increases |
| normal critical                         | -         | +         |
| neutral critical                        | 0         | 0         |
| reverse critical                        | +         | -         |
| bi-critical                             | +         | +         |
| decreasing neutral-increasing normal    | 0         | +         |
| decreasing reverse-increasing neutral   | +         | 0         |
| decreasing normal-increasing neutral    | -         | 0         |
| decreasing neutral-increasing reverse   | 0         | -         |
| decreasing normal-increasing reverse    | -         | -         |

Figure 2. Effects of the changes of activity durations on the critical path

Wiest [1] was the first to classify the critical activities in a PDM network. His categories are the following:

- normal: Increasing the activity duration will cause the project duration to increase as well, while decreasing it will make the project shorter. This is what we would normally expect.
- reverse: Lengthening the activity duration will result in a shorter project duration, while crashing the activity will make the critical path longer. This is an anomalous case.
- neutral: Changing the activity duration in either direction will not change the project duration.
- perverse: Changing the activity duration in either direction will increase the project duration.

Moder et al. [15] and Moder and Crandall [16] divided the neutral activities into two groups: start-critical and finish-critical ones depending on whether the critical path passed through the start or the finish of the activity. Essentially only the start or the finish of the neutral activity is critical. They also renamed the perverse activities as bi-critical activities.

Hajdu [17] added two new categories to the above-mentioned ones. These were the following:

- decreasing neutral, increasing normal: This is basically a combination of the neutral and normal types. If the activity duration is decreased, the project duration does not change. However, if it is increased, the project duration also increases.
- decreasing reverse, increasing neutral: This is a combination of the reverse and neutral categories. If the activity is crashed, the project duration increases, while it does not change, if the activity duration is lengthened.

Now there are six categories altogether. We can divide them into two groups. One contains the simple types (normal, neutral and reverse), while the complex types are the combinations of the simple ones. In Figure 2, the two groups are separated by a green dashed line. The combinations listed after the red line are only theoretical outcomes which practically never arise. This matter will be discussed later.

Figure 3 summarizes the above-mentioned six categories in case of endpoint relationships (FS, FF, SS, SF). It is important to note that our basic assumption is that the activities are not splittable. In addition, the observed changes are only true for 1-day differences, since greater changes can cause other paths to become critical instead. The columns in the middle show how the critical path (or paths) passes through the activities.

![Figure 3. Classification of critical activities in case of endpoint relationships](image-url)
4. Critical activities in networks with point-to-point relationships

4.1. Simple example

Figure 4 shows a simple network consisting of only three activities and two relationships.

In Figure 5 a possible model of Figure 4’s network can be seen. Owing to the fact that the scheduling software cannot handle point-to-point relationships, Activity 2 is broken down into 10 1-day long segments, which are connected by FS0 and maxFS0 relationships to ensure that they “stay” together. The project duration is 12 days. There is only one path, which is critical, of course. Therefore all the activities of the network are critical.

The next step is to determine in which class Activity 2 belongs. For this, let us see what happens, if the activity duration is shortened by 1 day, or increased by 1 day. Figure 6 shows the results.
Based on the results shown in Figure 6, it could be stated that Activity 2 is normal critical. The critical path passes through the activity in a left to right direction, meaning that it comes out later than it goes in. This is the same arrangement that is drawn in Figure 3 for the normal critical activity.

Based on the position of the second relationships start, Activity 2 could be either reverse, neutral or normal. Figure 7 summarizes the three cases.

![Figure 6. How the project duration changes if the activity duration of Activity 2 changes](image)

In summary, it could be stated that the same categories exist for networks with point-to-point relationships as for ones with endpoint relationships. The latter could be seen as extreme cases of the former.

4.2. Complex example

In case there is more than one critical path, the network can be seen as a combination of simple networks. The project duration has to be calculated in each one, and the longest one has to be chosen. Let us see an example for a “complex” network. (Figure 8)
Figure 8. Network with four critical paths

Figure 9 shows how Figure 8’s network can be broken down into simple cases.

Now each of the cases listed in Figure 9 has to be examined and the one that determine the longest project duration for both increasing and decreasing have to be found. In case the activity duration of Activity 2 is decreased by one day, the reverse case gives the longest project duration, while in case of increasing the activity duration, the normal case is the winner. Table 1 gives the order of the simple cases. So if there is a reverse case, then that will give the longest project duration, if the activity duration is decreased. If there is no reverse case, then the neutral one is the winner. The normal case gives the shortest project duration. In case the activity duration is increased by one day, then the normal case gives the longest project duration, while the neutral comes in second.

Table 1. Order of cases

| ORDER | DECREASE | INCREASE |
|-------|----------|----------|
| 1.    | reverse  | normal   |
| 2.    | neutral  | neutral  |
| 3.    | normal   | reverse  |

Figure 10 shows the project duration in case the activity duration of Activity 2 is decreased by 1 day, and increased by 1 day. It could be seen that no matter how the activity duration is changed, the project duration increases. This is a bi-critical case. (See the classification in Figure 3.)
Based on the above, it could be understood why the last three mathematically possible combinations (see Figure 2) are not actually possible outcomes. For example, if there is a normal and a neutral case, the latter gives a longer project duration than the former, if the activity duration is decreased. In case of increasing the activity duration, the situation is the opposite. Therefore the activity is a decreasing neutral, increasing normal critical and not vice versa. Also the last combination in Figure 2 would be the opposite of bi-critical, but it does not exist due to similar reasons. In conclusion, it could be stated that only six categories exist no matter if the network only contains endpoint or there are point-to-point relationships as well.

4.3. Changes of the project duration

Now we know in which direction the project duration changes owing to the increase or decrease of the activity duration. However, the rule that is true for networks with endpoint relationships, that is a 1-day change in the activity duration means a zero or a 1-day change in the project duration, cannot be applied here.

The value with which the project duration changes depends on the positions where the relationship is entering and where it is leaving the activity and the activity duration. Equation 1 shows the formula which could be used.

\[
\Delta PD = \Delta P / t
\]

where
- \(\Delta PD\): change in project duration
- \(\Delta P\): difference between the ingoing and outgoing relationships’ positions
- \(t\): activity duration

When defining \(\Delta P\), only the determining relationships have to be taken into account, in other words, the critical relationships, which are part of the critical path. For example, in case of the network shown in Figure 8, if we would like to calculate the project duration, when Activity 2’s duration is 9 days, the reverse case is the one that will determine the project duration (see Table 1 and Figure 10) because it gives the greatest result. The incoming relationship enters at position 8, while the outgoing leaves at position 2. The difference is 6 units (5.4 days), while the activity duration is divided into 10 (9 days). It means that the project duration will change with 0.6 days. Due to the fact that this is a reverse case, the project duration will increase by this amount. Therefore the original 12 days becomes 12.6 days. The project duration for the 11-day-long Activity 2 case could be calculated by using the same logic.

If a neutral case is the one that determines the project duration, the formula still works, because then the difference of the positions is 0, so the project duration does not change.

The formula works for endpoint relationships as well, since there the difference of the positions is either equal to the activity duration or 0, which means that the result is either 1 or 0.
5. Floats

In case there is a non-critical activity in the network, its float can be determined. Due to the fact that the activities are not splittable, they cannot be made longer or interrupted, the principles of the calculation are the same as the ones set for networks with only endpoint relationships. The fact that early and late start and finish dates are slightly more complicated to determine does not affect the formula used for determining the floats, because what they mean is essentially the same.

For instance, in case of the total float it still means the number of days with which the activity could be delayed so that the project duration remains unchanged. It can be calculated as the difference between the late and early start or finish dates of the activity.

See Figure 11 for an example. Here Activity 3 is non-critical. It could start on day 1 at the earliest. Then it finishes on day 7. It has to finish on day 11 at the latest, then it starts on day 5. Consequently, the total float is 5-1=11-7=4 days. The result is also shown in the Float column in Figure 11.

![Figure 11. Network with one non-critical activity](image)

The other types of floats (free, independent, conditional) could also be calculated the same way as before. Hajdu’s work [17] contains the formulae.

6. Summary

Based on the previous chapters, it can be concluded that although the use of point-to-point relationships in a network may make calculating the project duration and the rest of the dates related to the activities slightly more complicated, the original classification of the critical activities true for the networks with only endpoint relationships is valid for networks with point-to-point relationships as well. There are still three simple categories: normal, neutral and reverse. The complex classes are the combinations of the simple ones. Here we have the bi-critical, the decreasing neutral, increasing normal and the decreasing reverse, increasing neutral categories. If the activity duration changes, the project duration may decrease, increase or not change at all. The only difference is that with the endpoint relationships a 1-day change could induce a zero or 1-day change, with the point-to-point relationships, this value depends on the ratio of the difference between where the relationships are going in and out and the activity duration.

Floats of non-critical activities can also be calculated in the same way as before, in case of networks with only endpoint relationships.
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