This work analyzes the Einstein–Gauss–Bonnet gravity of charged black hole solutions through Newman–Janis approach. The Hawking temperature for corresponding black hole is also computed. The solution depends upon rotation parameter $\alpha$, black hole mass, charge and horizon. Moreover, the graphical behavior of temperature via event horizon to analyze the stability of black hole under the effects of rotation parameter is discussed. The graphs are plotted in the presence/absence of rotation parameter and charge. Furthermore, the Hawking temperature under gravity effects is studied by using the semi-classical method. It is also observed that the maximum temperature at non-zero horizon depicts the BH remnant. Finally, the logarithmic corrected entropy for given black hole is computed and the logarithmic corrected entropy under effects of rotation and correction parameter are studied.

1. Introduction

In order to study the gravity theories of higher-order curvature,\cite{1,2} a few modifications have been introduced in general relativity (GR) theory, a well-known theory is called Einstein–Gauss–Bonnet (EGB) gravity theory. This theory is actually a rare case of Lovelock gravity and also a modification of GR. The EGB gravity Lagrangian is a special type of the Lovelock gravity Lagrangian.\cite{3} One interesting feature of EGB is that it maintains the second-order equation of motion in self-assertive gravity Lagrangian.\cite{4} One of the significant features of EGB is that it modifies the second-order equation of motion in self-assertive number of dimensions $D \geq 5$. This significant feature gives the premise to research on gravity in the other structure. Although, because of the metric definition in dimensions $D > 4$, the extent of exploration is restricted to space-time with higher dimensions. By re-scaling the coupling constant $\alpha$, Glavan and Lin constructed 4-dimensions of EGB gravity.\cite{5} They showed if we take the product of GB action with a factor $\alpha$ rather than $\alpha$, then we will consider an unchanged contribution to Einstein’s equations in dimension $D = 4$. After this presented method, several investigations about the EGB gravity and its thermodynamics have been done. Many investigations about the GB gravity as well as its properties have been made after this valuable method is presented.\cite{6} The topological BH solutions of GB gravity have been derived using two kinds of non-linear electrodynamics\cite{7} and also discussed the stability of heat capacity. Clifton and his colleagues\cite{8} searched for the observational constraints on regular EGB gravity in 4 dimensions and find out the range of coupling constant. The following features about black holes (BHs) that is, radiation, instability, shadows, quasinormal modes, and gray-body factors in 4 dimensions EGB gravity have been studied in refs. \cite{9–13}. The most recent 20 years had a great deal of treat for theoretical cosmologists, coming from both cosmological scale information and furthermore from astrophysical scales occasions. Especially, the perception of the right now speeding up Universe coming from the standard candles SNe Ia,\cite{14} has completely changed our discernment of how the Universe evolves. The gravitational wave recognition coming from the neutron star consolidating GW170817 event,\cite{15} has totally influenced modified speculations of gravity, barring a few of these from being reasonable depictions of our Universe at astrophysical scales. Especially, the GW170817 occasion showed that the proliferating rate of the gravitational waves $c_1$ is equivalent to unity to that of light, specifically $c_1 \equiv 1$, in standard units. This factor as we specified, avoided quickly numerous elective speculations of gravity. Although, there exist many altered gravity speculations that actually stay strong against the GW170817 event results. Oikonomou and his colleagues\cite{16–20} have explored how the EGB theories can be rendered reasonable and viable with the GW170817 event. The gravitational wave speed for an EGB theory is equal to, $c_1 = 1 - \frac{Q_f}{Q_t}$ with $Q_t = 8c_1(\xi - H\xi)$, while $c_1$ represents the dimensionless constant multiplication factor that work in the Lagrangian with GB term, and the function $Q_t$ is $Q_t = \frac{1}{2}8c_1\xi H$, where $H$ stands for Hubble rate. So as indicated by our contemplations, if $Q_t = 0$ the gravitational wave speed would be equivalent to one. This infers that the GB coupling must hold the differential condition $\xi - H\xi = 0$, which suggests that $\xi = e^{\nu}$, here $N$ denotes the e-foldings number. This technique empowered us...
to communicate all the slow-roll as well as the observational indices as elements of the e-foldings number, and in the long run concentrate on the phenomenology of the demonstrate.

Semiclassical techniques based on Hawking radiation as a tunneling impact were introduced in the course of the last two decades and have gained a great deal of interest. This approach has been presented by Hawking at the point when he proposed his interesting theoretical disclosure named “Hawking radiation”[21] and after this, it has been clarified by Parikh and Wilczek[22,23] that how Hawking radiation occurs. The physical significance of this emission action shows that vacuum fluctuations accelerate particle (positive mass) anti-particle (negative mass) pairs toward the BH horizon. Hawking studied that positive mass particles have enough energy to radiate from the BH, while negative mass particles have no capability to emit from the BH. Parikh and Wilczek provided a complete path to this theoretical discovery in the form of WKB approximation. This method utilizes geometric optic estimation which is another perspective of eikonal estimation in wave explanation.[24] Particles are set in front of a boundary, when particles tunnel from this boundary the BH mass reduces in the form of particle energy. In spite of the fact that with the Parikh–Wilczek approach, a more precise strategy of the tunneling was given, but there were as yet unsolved issues like information loss, temperature divergence, and unitary. Numerous investigations have been done on the radiation and tunneling approach from the different BH horizons; some of these most significant investigations can be seen in refs. [25–55]. The tunneling radiation for different BHs has been studied and also examined the tunneling radiation with the effects of the BH geometry and various parameters. It is possible to discuss quantum-corrected tunneling features of BH by integrating generalized uncertainty principle (GUP) effects.[36] The GUP gives high-energy solution to BH thermodynamics, allowing quantum gravity theory to have a minimal length. By taking into account the GUP effects, it is very feasible to study the quantum modified thermodynamics of BHs. The GUP relation satisfies the following expression[50,57]

$$\Delta x \Delta p \geq \frac{1}{2} (1 + \alpha (\Delta p)^2)$$  \hspace{1cm} (1)

here $\alpha = \frac{\alpha_s}{M_p^2}$, $M_p^2$ shows the Plank’s mass and $\alpha_s < 10^{-14}$ represents the dimensionless parameter.

The main purpose of this paper is to investigate the charged BH in EGB gravity for rotating case and then to discuss its thermodynamics that is, Hawking temperature. Moreover, to check the stability condition of charged rotating BH in EGB gravity by graphical interpretation of Hawking temperature with horizon. In order to meet our aim, we attempt to construct the rotating charged BH solutions of EGB gravity with the help of Newman–Janis algorithm and then investigate their Hawking temperature and study the effects of rotation parameter with the help of plots. Furthermore, by using the quantum tunneling approach, we also study their quantum corrected Hawking temperature by using the semi-classical method with the help of WKB approximation. At last, we analyze the gravity effects by plotting the graphs of corrected temperature with horizon, which depicts the stability of BH in rotating case. This article is arranged as follows, Section 2 contains the space–time information of D-dimensional BHs. Subsection 2.1, investigates the solution of rotating charged BH in EGB gravity by using the Newman–Janis method and study the temperature for the associated BH. Section 3, gives the graphical temperature behavior with respect to event horizon and analyze the effects of rotation parameter. Section 4, checks the quantum corrected temperature for EGB gravity of charged rotating BH by using quantum tunneling method for boson particles in four dimension spaces. In Section 5, we study the quantum gravity graphical effects on corrected Hawking temperature for boson particles. Section 6 describes the logarithmic corrected entropy for EGB gravity of rotating charged BH. In Section 7, we summarize the results of our work.

2. Einstein–Gauss–Bonnet Gravity Theory of Black Holes

In this section, we examine the EGB D-dimension charged BHs gravity theory. To do so, we consider the EGB BHs gravity theory with Lagrangian equation

$$L = (R^2 + R_{abcd}R^{abcd} - 4R_{ab}R^{ab}) + R$$  \hspace{1cm} (2)

The solution for spherically symmetric charged BH with this gravity theory in D-dimension space-time can be given as[58]

$$ds^2 = -G(r)dt^2 + G^{-1}(r)dr^2 + r^2 d\Omega_{d-2}^2$$  \hspace{1cm} (3)

where

$$G(r) = 1 + \left[1 - \sqrt{1 + \frac{16\beta M}{r^{d-3}} - \frac{8q^2}{r^{2(d-4)}}} \right] r^{d-4} \frac{4\beta}{r^{d-4}}$$  \hspace{1cm} (4)

where $M$, $q$, and $\beta$ are BH mass, BH charge, and the BH coupling constant $\beta$ as $\beta = (d-3)(d-4)\beta/2$, respectively. We study the space–time by $d\Omega_{d-2}^2$ of the unit $(d - 2)$ area of sphere $A_{d-2}$. The only nonzero component of the electromagnetic potential has the following form

$$A_{t}(r) = - \frac{q}{r^{d-3}}$$  \hspace{1cm} (5)

Here, $M$ and $q$ are associated to the BH mass $M$ and Arnowitt–Deser–Misner BH charge as

$$q^2 = \frac{2q^2(d-3)}{d-2}, \quad M = \frac{\pi A_{d-2}(d-2)}{8\pi} \dot{M}$$  \hspace{1cm} (6)

The horizon solution at $r = r_h$ can be obtained from the zeros of the function by setting $G(r_h) = 0$

$$r_h^{2(d-3)} + 2\beta r_h^{2(d-4)} - 2Mr_h^{d-3} + q^2 = 0$$  \hspace{1cm} (7)

The above equation gives the two real root ($r_+ = r_h$), the positive sign denotes the largest root as well as negative sign describes the lowest root.
2.1. Hawking Temperature for Rotating Charged Black Hole

The spherically symmetric charged BH solution in 4D spacetime has the following form

\[ ds^2 = -G(r)dt^2 + G^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \] (8)

By the coordinate transformation \((t, r, \theta, \phi)\) to \((u, r, \theta, \phi)\), we have

\[ du = dt - \frac{dr}{G(r)} \] (9)

After applying the transformation to the metric Equation (8), we get

\[ ds^2 = -G(r)du^2 - 2du dr + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \] (10)

The metric in the null (EF) frame can be expressed as

\[ g^{\mu \nu} = -l^\mu l^\nu - l^\mu n^\nu + m^\mu \tilde{m}^\nu + m^\mu \tilde{m}^\nu \] (11)

The corresponding components are given as

\[ l^\nu = \delta^\nu_\mu \quad n^\nu = \delta^\mu_\nu - \frac{1}{2} G \delta^\mu_\nu \]

\[ m^\nu = \frac{1}{\sqrt{2r}} \delta^\nu_\phi + \frac{i}{\sqrt{2r} \sin \theta} \delta^\nu_\theta \]

\[ \tilde{m}^\nu = \frac{1}{\sqrt{2r}} \delta^\nu_\phi - \frac{i}{\sqrt{2r} \sin \theta} \delta^\nu_\theta \] (12)

For all points in the BH metric the null vectors of the null tetrad satisfy the conditions of \(l, l^\nu = n^\mu n^\nu = m^\mu m^\nu = l^\mu m^\nu = m^\mu l^\nu = 0\) and \(l^\nu n^\nu = -m^\mu \tilde{m}^\mu = 1\) in the place of \((u, r)\) coordinate transformation are \(u \to u - ia \cos \theta, r \to r + ia \cos \theta\), then we perform the transformation \(G(r) \to \tilde{G}(r, a, \theta)\) and \(\Sigma = a^2 \cos^2 \theta + r^2\). The null vectors in the \((u, r)\) space can be written as

\[ l^\nu = \delta^\nu_\mu \quad n^\nu = \delta^\mu_\nu - \frac{1}{2} \tilde{G} \delta^\mu_\nu \] (13)

\[ m^\nu = \frac{1}{\sqrt{2\Sigma}} \left( \delta^\nu_\mu + ia \sin \theta (\delta^\nu_\phi - \delta^\nu_\theta) + \frac{i}{\sin \theta} \delta^\nu_\phi \right) \]

\[ \tilde{m}^\nu = \frac{1}{\sqrt{2\Sigma}} \left( \delta^\nu_\mu - ia \sin \theta (\delta^\nu_\phi - \delta^\nu_\theta) - \frac{i}{\sin \theta} \delta^\nu_\phi \right) \]

From the null tetrad definition of the metric tensor \(g^{\mu \nu}\) are given by

\[ g^{tu} = \frac{a^2 \sin^2 \theta}{\Sigma^2}, \quad g^{ru} = g^{ur} = -\frac{1 - a^2 \sin^2 \theta}{\Sigma^2}, \]

\[ g^{rr} = \tilde{G} + \frac{a^2 \sin^2 \theta}{\Sigma^2}, \quad g^{\theta \theta} = \frac{1}{\Sigma^2}, \]

\[ g^{\phi \phi} = \frac{1}{\Sigma^2 \sin^2 \theta}, \quad g^{\phi u} = g^{\phi r} = \frac{a}{\Sigma^2}, \quad g^{\phi \theta} = g^{\phi \psi} = -\frac{a}{\Sigma^2} \] (14)

The new metric according to null tetrad can be obtained as

\[ ds^2 = -\tilde{G}(r)du^2 - 2du dr - 2a \sin \theta^2 (1 - \tilde{G}) d\phi^2 \\
+ 2a \sin \theta^2 d\phi^2 + \Sigma^2 d\theta^2 + \sin \theta^2 (\Sigma^2 - a^2 \sin \theta^2 (\tilde{G} - 2)) d\phi^2 \] (15)

Now, we consider the coordinate transformation from the Eddington Finkelstein (EF) to Boyer Lindquist (BL) coordinates as

\[ du = dt + \gamma(r) dr, \quad d\phi = d\phi + F(r) dr \] (16)

where

\[ \gamma(r) = a^2 + \frac{r^2}{r^4 \tilde{G} + a^2}; \quad F(r) = -\frac{a}{r^2 \tilde{G} + a^2} \]

\[ \tilde{G}(r, \theta) = \frac{a^2 \cos^2 \theta + r^2 \tilde{G}(r)}{\Sigma^2} \] (17)

Finally, we get the BH spacetime in the form

\[ ds^2 = \left[ 1 + \frac{\Delta'}{4r^4} \left( 1 - \sqrt{\frac{8(q^2 - 16m^2)}{r^4} + \frac{16Gm}{r^2}} \right) \right] dt^2 + \frac{\Sigma^2}{\Delta'} dr^2 \\
-2a \sin \theta d\theta^2 + a^2 \sin^2 \theta d\phi^2 \\
+ \Sigma^2 d\theta^2 + a^2 \sin^2 \theta \left[ \frac{1}{\Sigma^2} - \frac{\Delta'}{4r^4} \left( 1 - \sqrt{\frac{8(q^2 - 16m^2)}{r^4} + \frac{16Gm}{r^2}} \right) \right] d\phi^2 \] (18)

here

\[ \Delta_r = r^2 + a^2 + r^4 \left( 1 - \sqrt{\frac{8(q^2 - 16m^2)}{r^4} + \frac{16Gm}{r^2}} \right) \] (19)

Now, we discuss the thermodynamical property (i.e., Hawking temperature) for corresponding BH. In order to derive the Hawking temperature, we use the following formula for spherically symmetric BHs

\[ T_H = \frac{\tilde{G}'(r_+)}{4\pi} \] (20)

The Hawking temperature for
Figure 1. \( \Delta \) via horizon \( r_+ \) for \( M = 1, \beta = 0.01 \). Left: \( q = 1, a = 10 \) (solid green), \( a = 11 \) (solid blue), \( a = 12 \) (solid red), \( a = 13 \) (solid cyan). Right: \( a = 0, q = 15 \) (solid cyan), \( q = 16 \) (solid red), \( q = 17 \) (solid blue), \( q = 18 \) (solid green).

Figure 2. \( T_\text{H} \) via \( r_+ \) for \( M = 10 \). Left: \( \beta = 1, q = 0.5, a = 3 \) (dashed), \( a = 4 \) (dot-dashed), \( a = 5 \) (small-dashed). Right: \( \beta = 0.1, a = 1, q = 5 \) (dashed), \( q = 5.5 \) (dot-dashed), \( q = 6 \) (small-dashed).

\[
T_\text{H} = r_+^2 \left( Y - r_+^2 \right) - 4M r_+^4 \beta + 2a^2 r_+ \left( 4q^2 \beta + r_+^4 \left( Y - r_+^2 \right) - 10M r_+ \beta \right) \quad \frac{8\pi \gamma \beta}{r_+^2 + a^2}
\]

(21)

where

\[
Y = \sqrt{r_+^2 - 8q^2 \beta + 16Mr_+ \beta}
\]

(22)

The Hawking temperature depends upon the BH mass \( M \), arbitrary constant \( \beta \), BH charge \( q \), spin parameter \( a \), and BH horizon radius \( r_+ \).

3. Stability Analysis via Graphical Interpretation

This section investigates the stability of charged rotating BH with EGB gravity in the presence and absence of spin parameter \( a \). We observe the horizon structure of BH by plotting the graphs of \( \Delta \), with respect to horizon \( r_+ \), as well as discuss the stability condition for BH under the effects of different parameters that is, BH charge \( q \) and rotation parameter \( a \) in the range \( 0 \leq r_+ \leq 15 \).

Figure 1 shows the behavior of \( \Delta \), with \( r_+ \) for fixed value of \( M = 1 \) (BH mass) and \( \beta = 0.01 \) (arbitrary parameter) in the presence and absence of spin parameter. In plot (i) the different values of spin parameter \( a \) for fixed values of BH charge \( q = 1 \) satisfy the relation of \( \Delta \), with horizon equation by showing the positive behavior. In plot (ii) one can observe that in the absence of rotation parameter \( a = 0 \) and for different values of charge \( q \), the behavior also satisfies the horizon relation.

Figure 2 represents the temperature \( T_\text{H} \) behavior with respect to horizon \( r_+ \) for fixed value of BH mass \( M = 10 \). In plot (i) the \( T_\text{H} \) shows the behavior for fixed values of BH charge \( q = 0.5 \), arbitrary parameter \( \beta = 1 \) and different values of rotation parameter \( a \). One can observe that the temperature slowly goes on increasing with the increasing values of horizon but after a height it gradually goes on decreasing with the increasing horizon which satisfies the Hawking’s phenomenon, so guarantees the physical and stable form of BH. One can also observe that with the increasing value of spin parameter the values of temperature increase.

The plot (ii) depicts the behavior of \( T_\text{H} \) for constant values of spin parameter \( a = 1 \), arbitrary parameter \( \beta = 0.1 \), and different values of charge \( q \). We can observe at initial the BH is not stable but as time goes on BH attains its stable form, when it eventually drops down from a height and obtains an asymptotically flat state.
till \( r_+ \to \infty \). The decreasing temperature with increasing horizon also shows the stable condition of BH. One can observe with the increasing values of charge the values of temperature decreases.

**Figure 3** actually represents the behavior of temperature with event horizon for special case in the absence of spin parameter \( a \) or charge \( q \). Plot (i) shows the behavior of \( T_{\text{th}} \) for constant values of mass \( M = 15 \), arbitrary parameter \( \beta = 0.01 \), and different values of \( q \) (charge) in the absence of \( a \) (rotation parameter). One can analyze in the absence of rotation BH parameter also shows its stable form after passing some time. From Figures 2 and 3, we can observe in the presence of rotation BH parameter the temperature increases as compare in the absence of \( a \).

Plot (ii) depicts the \( T_{\text{th}} \) behavior for changing values of rotation BH parameter \( a \) and fixed the parameter of \( \beta = 1 \), \( M = 25 \) in the absence of charge \( q \). One can observe that in the absence of charge the BH is also in its stable form. We can also see that as compared to Figure 2(ii) the temperature is higher in Figure 3(ii) in the absence of charge.

### 4. Corrected Tunneling

The metric Equation (18) can be re-expressed as

\[
ds^2 = -G dt^2 + f dr^2 + X d\theta^2 + Y dp^2 + 2Z dt d\phi
\]

where

\[
G = \left[ 1 + \frac{r^4}{4p} \left( 1 - \sqrt{1 - \frac{8dp}{r^2} + \frac{16M}{r^4}} \right) \right],
\]

\[
Y = a^2 \sin^2 \theta \left[ \frac{r^4}{4p} \left( 1 - \sqrt{1 - \frac{8dp}{r^2} + \frac{16M}{r^4}} \right) \right],
\]

\[
Z = a \sin^2 \theta \left[ \frac{r^4}{4p} \left( \sqrt{1 - \frac{8dp}{r^2} + \frac{16M}{r^4}} \right) - 1 \right],
\]

\[
\hat{t} = 1, \quad \hat{X} = \frac{\Sigma}{\Delta r}, \quad \hat{t} = \frac{\Sigma}{\Delta r}
\]

\[
\phi^0 = \frac{-\hat{Y} \phi_0 + \hat{Z} \phi_2}{\hat{X}}, \quad \phi^1 = \hat{C} \phi_1, \quad \phi^2 = \frac{1}{\hat{X}} \phi_2,
\]

\[
\phi^3 = \frac{\hat{Z} \phi_0 + \hat{C} \phi_2}{\hat{X}}, \quad \phi^0 = \frac{\hat{C}(-D \phi_0 + Z \phi_{13})}{(\hat{Y} + \hat{Z})^2},
\]

\[
\phi^2 = \frac{-\hat{Y} \phi_{02}}{X(\hat{Y} + \hat{Z})}, \quad \phi^3 = \frac{-\hat{C}(-2 \hat{Y} \phi_{02} + \hat{C} \phi_{03})}{(\hat{Y} + \hat{Z})^2},
\]

\[
\phi^2 = \frac{\hat{C} \phi_{12}}{X(\hat{Y} + \hat{Z})}, \quad \phi^3 = \frac{\hat{C}(-2 \hat{Y} \phi_{02} + \hat{C} \phi_{03})}{X(\hat{Y} + \hat{Z})^2}
\]

The WKB approximation is

\[
\phi_{e} = k_{e} \exp \left[ \frac{\imath}{h} I_{0}(t, r, \phi, \theta) + \Sigma h^\nu I_{\nu}(t, r, \phi, \theta) \right]
\]
By putting all the components in Equation (25), we get the following set of field equations in the form

\[
\begin{align*}
\tilde{G}Y & \left[ (\partial_0 I_0)(\partial_1 I_0)k_1 - (\partial_1 I_0)^2 k_0 - a(\partial_1 I_0)^4 k_0 + eA_0(\partial_1 I_0)k_1 + a(\partial_0 I_0)^3 (\partial_1 I_0)k_1 \right] \\
+ eA_0(\partial_0 I_0)^2 (\partial_1 I_0)k_1 & - \frac{\tilde{G}Z}{(GY + Z^2)} \left[ a(\partial_1 I_0)^4 k_3 + (\partial_1 I_0)^2 k_3 - (\partial_1 I_0)(\partial_3 I_0)k_1 + (\partial_1 I_0)(\partial_3 I_0)^2 ak_1 \right] \\
+ \frac{Y}{X(GY + Z^2)} & \left[ (\partial_0 I_0)(\partial_2 I_0)k_2 + (\partial_0 I_0)(\partial_2 I_0)ak_2 - (\partial_2 I_0)^3 k_0 + (\partial_2 I_0)^4 ak_0 + eA_0(\partial_2 I_0)k_2 \right] \\
+ eA_0a(\partial_0 I_0)^3 (\partial_1 I_0)k_2 & + \frac{\tilde{G}}{(GY + Z^2)} \left[ (\partial_0 I_0)(\partial_1 I_0)k_1 + (\partial_0 I_0)(\partial_1 I_0)ak_1 - (\partial_1 I_0)(\partial_3 I_0)k_1 + (\partial_1 I_0)(\partial_3 I_0)^2 ak_1 \right] \\
+ eA_0(\partial_3 I_0)k_1 & + eA_0(\partial_0 I_0)^2 (\partial_1 I_0)k_1 - m^2 \left( \frac{\tilde{G}k_0 - Zk_3}{GY + Z^2} \right) = 0 \\
\end{align*}
\]
\[\frac{\partial Y}{\partial (GY + Z^2)} \left[ (\partial_0 I_0)^2 k_1 + (\partial_0 I_0)^4 ak_3 - (\partial_0 I_0)(\partial_1 I_0) k_0 - (\partial_1 I_0)(\partial_0 I_0) \right] ^3 ak_0 + cA_0 (\partial_0 I_0) k_3 \]

\[+ eA_0 (\partial_0 I_0) ak_3 - \frac{Y}{X(GY + Z^2)} \left[ (\partial_0 I_0)^2 k_1 + (\partial_1 I_0)^4 ak_3 - (\partial_0 I_0)(\partial_1 I_0) k_0 - (\partial_1 I_0)(\partial_0 I_0) \right] ^3 ak_1 \]

\[- \frac{Z}{X(GY + Z^2)} \left[ (\partial_0 I_0)(\partial_0 I_0) k_2 + (\partial_1 I_0)^3 (\partial_0 I_0) ak_2 - (\partial_0 I_0)(\partial_0 I_0) k_0 + (\partial_0 I_0)^4 ak_0 + cA_0 (\partial_0 I_0) k_2 + cA_0 \right] \]

\[= 0 \] (32)

By utilizing the technique of separation of variables, we obtain

\[I_0 = \left( E - \sum_{i=1}^{2} j_i \Omega \right) t + W(r) + j\phi + n(\theta) \] (33)

where \(E\) and \(J\) represent the particle energy, the particle angular momentum corresponding angle \(\phi\). After using the Equation (33) into Equations (29)–(32), we get a matrix of order 4 \(\times\) 4 labeled as "\(U_n\)"

\[U(k_0, k_1, k_2, k_3)^T = 0 \] (34)

The non-zero elements are expressed as follows

\[U_{00} = -\frac{\partial Y}{\partial (GY + Z^2)} \left[ W_1^2 + aW_1^4 \right] - \frac{Y}{X(GY + Z^2)} \left[ U_1^2 + aU_1^4 \right] - \frac{\partial Y}{\partial (GY + Z^2)} \left[ v_1^2 + a\nu_1^4 \right] - m^2 \frac{Y}{GY + Z^2} \]

\[U_{01} = \frac{\partial Y}{\partial (GY + Z^2)} \left[ \dot{E} + a\dot{E}^3 + cA_0 + aeA_0 \dot{E} \right] W_1 + \frac{\partial Z}{\partial (GY + Z^2)} \left[ v_1 + a\nu_1 \right] \]

\[U_{02} = \frac{\partial Z}{\partial (GY + Z^2)} \left[ \dot{E} + a\dot{E}^3 + cA_0 + aeA_0 \dot{E} \right] J, \quad K_{12} = \frac{\gamma}{X^3} [W_1 + aW_1^3] J. \]

\[H_{01} = \frac{\partial E}{\partial (GY + Z^2)} \left[ W_1 + aW_1^3 \right] - \frac{\partial Y}{\partial (GY + Z^2)} \left[ \dot{E} + a\dot{E}^3 + cA_0 + aeA_0 \dot{E} \right] v_1 + m^2 \frac{Z}{GY + Z^2} \]

\[U_{10} = \frac{\partial D}{\partial (GY + Z^2)} \left[ EW_1 + aEW_1^3 \right] - Gm^2 - \frac{eA_0 \dot{Y}}{(GY + Z^2)} \left[ W_1 + aW_1^3 \right] \]

\[U_{11} = \frac{\partial D}{\partial (GY + Z^2)} \left[ E^2 + aE^4 - eA_0 E - aeA_0 E W_1^3 \right] + \frac{\partial Z}{\partial (GY + Z^2)} \left[ v_1 + a\nu_1 \right] \dot{E} - \frac{\dot{Y}}{X} \left[ J^2 + aJ^4 \right] \]

\[- \frac{G}{GY + Z^2} \left[ v_1 + a\nu_1 \right] - m^2 \frac{G}{GY + Z^2} - \frac{eA_0 \dot{Y}}{(GY + Z^2)} \left[ E + aE^3 - eA_0 - aeA_0 \dot{E} \right] + \frac{eA_0 \dot{Z}}{(GY + Z^2)} \left[ v_1 + a\nu_1 \right] \]

\[U_{11} = \frac{\partial E}{\partial (GY + Z^2)} \left[ W_1 + aW_1^3 \right] E + \frac{G}{GY + Z^2} \left[ W_1 + aW_1^3 \right] v_1 + \frac{\dot{Z}eA_0}{GY + Z^2} \left[ W_1 + aW_1^3 \right]. \] (35)
\[ U_{20} = \frac{\dot{Y}}{X(GY + Z^2)} [\dot{E} + a\dot{E}^1] + \frac{\dot{Z}}{X(GY + Z^2)} [\dot{E} + a\dot{E}^1 v_i] - \frac{\dot{Y} e A_0}{X(GY + Z^2)} [J + aJ^1], \]

\[ U_{21} = \frac{\dot{G}}{X} [J + aJ^1] W_i, \quad U_{23} = \frac{\dot{G}}{X(GY + Z^2)} [J + aJ^1] v_i, \]

\[ U_{22} = \frac{\dot{Y}}{X(GY + Z^2)} [E^2 + aE^1 - eA_0 E - eA_0 E] - \frac{\dot{Z}}{X(GY + Z^2)} [E + aE^1 - eA_0 - eA_0 E^1] v_i - \frac{m^2}{X} - \frac{\dot{G}}{X(GY + Z^2)} [v_i + a v_i^1] - \frac{e A_0 \dot{Y}}{X(GY + Z^2)} [E + aE^1 - eA_0 - eA_0 E^1]. \]

\[ U_{30} = \frac{(\dot{G}Y - \dot{A}^2)}{G(GY + Z^2)} [v_i + a v_i^1] + \frac{Z}{X(GY + Z^2)} [J^2 + aJ^1] - \frac{m^2}{G(GY + Z^2)} - \frac{e A_0 (\dot{G}Y - \dot{A}^2)}{G(GY + Z^2)} [v_i + a v_i]. \]

\[ U_{31} = \frac{\dot{G}}{G(Y + Z^2)} [v_i + a v_i^1] W_i, \quad U_{12} = \frac{Z}{X(GY + Z^2)} [J + aJ^1] \dot{E} + \frac{\dot{G}}{X(GY + Z^2)} [v_i + a v_i^1] J, \]

\[ U_{13} = \frac{(\dot{G}Y - \dot{A}^2)}{G(GY + Z^2)} [E^2 + aE^1 - eA_0 E - eA_0 E^1] - \frac{\dot{G}}{G(Y + Z^2)} [W_i^2 + a W_i^1] - \frac{\dot{G}}{X(GY + Z^2)} [J^2 + aJ^1] \]

\[ - \frac{m^2 \dot{G}}{G(Y + Z^2)} - e A_0 (\dot{G}Y - \dot{A}^2) [E + aE^1 - eA_0 E^1] \]

\[ (36) \]

where \( E = E - \sum J_i \omega_i \), \( f = \partial f_1 \), \( W_i = \partial W_i \) and \( v_i = \partial v_1 \). We take the solution for non-trivial \( |U| = 0 \) and get

\[ Im W^a = \pm \int \sqrt{\frac{(E - j\Omega - eA_0)^2 + X_i}{G(Y + Z^2)}} \left[ 1 + \frac{a \dot{X}_i}{X_i} \right] dr, \]

\[ = \pm i \sqrt{\frac{(E - j\Omega - eA_0)^2 + X_i}{2X(r_s)}} \left[ 1 + \frac{a \dot{X}_i}{X_i} \right] \]

\[ (37) \]

where

\[ X_i = \frac{Z}{G(GY + Z^2)} [E - eA_0] v_i + \frac{1}{(G(Y + Z^2)} [v_i^2] - \frac{1}{G} m^2, \]

\[ X_i = \frac{\dot{Y}}{G(GY + Z^2)} \left[ E^2 - eA_0 E^2 \right] \]

\[ + \frac{Z}{G(X(GY + Z^2)} [E^2 - eA_0 E^2] v_i - \frac{1}{(G(Y + Z^2)} [v_i^2] - W_i^1 \]

\[ (38) \]

We observe the tunneling radiation without back-reaction and self-gravity interaction and also explore the corrected tunneling probability rate. The tunneling phenomenon is computed under the consideration of charge–energy conservation and quantum gravity effects. The modified tunneling probability depends on the geometry of BHs as well as quantum gravity. The modified tunneling probability for charged rotating BH with EGB gravity can be defined as

\[ \Gamma = \frac{\Gamma_{\text{emission}}}{\Gamma_{\text{absorption}}} = \exp \left[ -2 \pi \frac{(E - j\Omega - eA_0)}{\kappa(r_s)} \right] [1 + a \Xi] \]

\[ (39) \]

here

\[ \kappa(r_s) = \frac{r_s^4 (Y - r_s^2) - 4Mr_s^4 - 2a^2 r_s (4q^2 \beta + r_s^4 (Y - r_s^2) - 10Mr_s^2)}{4\pi \beta (r_s^2 + a^2)}, \]

\[ \Xi = \sqrt{r_s^2 - 8q^2 \beta + 16Mr_s^2} \]

\[ (40) \]

In the presence of gravity terms, we compute the \( T_{st}^* \) of the 4D rotating charged BH in EGB gravity theory by considering the factor \( \Gamma_B = \exp[(E - j\omega - eA_0)/T_{st}^*] \) (Boltzmann factor) as

\[ T_{st}^* = \frac{r_s^4 (Y - r_s^2) - 4Mr_s^4 + 2a^2 r_s (4q^2 \beta + r_s^4 (Y - r_s^2) - 10Mr_s^2)}{8\pi \beta (r_s^2 + a^2)} \times [1 + a \Xi] \]

\[ (41) \]

This solution shows that the \( T_{st}^* \) depends on the BH geometry. Also, it is alike form that of the other scalar and fermion particles. The modified temperature depends on the correction
parameter $a$, spin parameter $a$, BH charge $q$, arbitrary constant $\beta$, BH mass $M$, and BH radius $r_*$. Quantum corrections decelerate the increase in $T'_{\text{H}}$ during the radiation phenomenon. These corrections make the radiation cease at some particular Hawking temperature, leaving the remnant mass. When this consideration holds, the temperature stops rising \cite{59}

\begin{equation}
(M - dM)(1 + a q) \approx M
\end{equation}

where $\omega = dM$, $a_0\left(\frac{1}{M_{\text{Planks}}}\right) = a$, and $M_{\text{Planks}} = \omega$. Here, $a_0$ and $M_{\text{Planks}}$ represent the dimensionless parameter and Planck mass and quantum gravity effects for $a_0 < 10^5$ in refs. \[50, 60, 61\].

5. Graphical $T'_{\text{H}}$ Analysis

This section describes the stability conditions of rotating charged BH with EBG gravity with the quantum gravity effects. We analyze the physical behavior and stability of the corresponding BH by plotting the graphs of corrected temperature $T'_{\text{H}}$ via horizon $r_*$ for fixed value of arbitrary parameter $\Sigma = 1$.

**Figure 4** shows the corrected temperature $T'_{\text{H}}$ behavior with respect to horizon $r_*$ under the effects of quantum gravity.

For the different values of the mass ($M = 5$), arbitrary parameter and charge ($\beta = 0.5 = q$), and spin parameter ($a = 5$), we can observe the physical behavior (positive temperature) of BH under the effects of correction parameter. Moreover, the $T'_{\text{H}}$ decreases as $r_*$ increases, which also satisfies the Hawking’s phenomenon.

Plot (ii) depicts the $T'_{\text{H}}$ behavior for constant values of mass $M = 1$, arbitrary parameter $\beta = 0.5$, charge $q = 1$, correction parameter $a = 0.1$ and for different values of rotation parameter $a$. At initial, the BH shows the non-physical behavior (negative temperature) but as time runs out and after a maximum height the temperature slowly decreases as $r_*$ increases as well as obtains an asymptotically flat condition to represent the complete stable form till $r_* \to \infty$. **Figure 5** represents the $T'_{\text{H}}$ behavior in the presence of correction parameter $a$ for fixed value of mass $M = 1$ and in the absence of charge and spin parameter ($q = 0 = a$), respectively.

Plot (i) in **Figure 5** gives the temperature behavior in the absence of charge $q = 0$ (charge) for constant values of $\beta = 0.5$, $a = 10$ and different values of gravity parameter $\omega$. One can observe that the temperature gradually increases and then slowly decreases with the increasing $r_*$ that depicts the stable form of BH.

Plot (ii) represents the $T'_{\text{H}}$ behavior in the absence of $a = 0$ (spin parameter) for fixed $\beta = 0.1$, $q = 5$, and for varying $\omega$. We can examine that the $T'_{\text{H}}$ behavior slowly decreases as horizon increases and gets an asymptotically flat condition that shows the stability of BH.

We can conclude from both plots that the BH is stable in the presence as well as absence of charge and spin parameter under the influence of quantum gravity parameter effects.

6. Logarithmic Correction of Entropy for Rotating Charged BH

In this section, the Bekenstein–Hawking entropy is considered to be corrected by particular correction terms in the quantum loop expansion. The logarithmic terms in BH entropy occur as
a one-loop addition to the classical BH entropy. The BH entropy plays a significant role in the account of primordial BHs.\cite{62} It imposes a minimum mass of the primordial BHs so the small ones have enough time to evaporate totally until the present form. The modified entropy with the logarithmic terms demonstrates, this conclusion is no more in place and the BH evaporation decompresses as soon as mass decreases below the critical mass (BH mass $\approx \sqrt{\frac{c}{\hbar}}$ (planks mass)) that extends to much longer BHs lifetime. In this form the small primordial BHs do not evaporate totally. Now, we analyze the corrections of entropy for EGB gravity of charged rotating BH. Banerjee and Majhi\cite{63-65} have studied the corrections of Hawking temperature and entropy by taking into account the back-reaction effects via null geodesic method. We calculate the corrections of entropy for EGB gravity of charged rotating BH by considering the Bekenstein–Hawking entropy formula for first order corrections.\cite{66} We derive the logarithmic corrections of entropy by using corrected temperature $T_H$ and standard entropy $S_o$ for EGB gravity of charged rotating BH. The corrections of entropy can be computed by the following formula

$$S = S_o - \frac{1}{2} \ln \left| T_H^2, S_o \right| + \ldots$$ \tag{43}$$

The standard entropy for EGB gravity of rotating charged BH can be calculated by the following formula

$$S_o = \frac{A_o}{4}$$ \tag{44}$$

where

$$A_o = \int_0^{2\pi} \int_0^\infty \sqrt{\text{BohKde}} d\theta d\phi,$$

$$= \pi \left[ 2(r_o^2 + a^2)^2 - a^2 q^2 + r_o Ma^2 \right] (r_o^2 + a^2)$$ \tag{45}$$

So the entropy term for standard can be derived as

$$S_o = \frac{\pi \left[ 2(r_o^2 + a^2)^2 - a^2 q^2 + r_o Ma^2 \right]}{4(r_o^2 + a^2)}$$ \tag{46}$$

After substituting the values from Equations (41) and (46) into Equation (43), we compute the correction of entropy as

$$S = \frac{\pi \left[ 2(r_o^2 + a^2)^2 - a^2 q^2 + r_o Ma^2 \right]}{4(r_o^2 + a^2)} - \frac{1}{2} \ln \left| \left[ \left( r_o^2 (Y - r_o^2) - 4Mr_o^2 \beta + 2a^2 r_o \left( 4q^2 \beta + r_o^2 (Y - r_o^2) - 10Mr_o \beta \right) \right) (1 - \alpha \Xi) \right]^2 \chi \right| + \ldots$$ \tag{47}$$

The Equation (47) represents the correction of entropy for EGB gravity of rotating charged BH.

7. Conclusions

In this article, we have used the Newman–Janis algorithm to study the charged rotating BH with EGB gravity solution. We have derived the thermodynamics (Hawking temperature) at the outer horizon for the corresponding BH by using a general formula for symmetric space–time. The Hawking temperature depends upon rotation parameter $a$ (occurs due to Newman–Janis algorithm), BH mass $M$, BH charge $q$, BH horizon $r_o$, and arbitrary constant $\beta$. Moreover, we analyzed the graphical behavior of temperature with respect to horizon to analyze the stability of BH under the effects of spin parameter. We have also investigated the stability of BH in the presence and absence of charge and spin parameter $q = 0 = a$, respectively. We have also investigated the physical significance of our plots. We conclude the main results from the graphical analysis of Hawking temperature via horizon as follows:

- For different values of spin parameter $a$ the $T_H$ slowly goes on increasing with the increasing values of horizon but after a height it gradually goes on decreasing with the increasing horizon which satisfies the Hawking’s phenomenon, so guarantees the physical and stable form of BH. The temperature $T_H$ also increases with the increase in $a$.

- For varying values of charge $q$ the BH at first shows the nonphysical form with negative temperature but as time goes on BH attains its stable form, when it eventually drops down from a height and obtains an asymptotically flat state till $r_o \rightarrow \infty$. The decreasing temperature with increasing horizon also shows the stable condition of BH. The temperature decreases with the increasing values of charge.

- In the absence of spin parameter $a$, after a maximum height the $T_H$ behavior slowly decreases as horizon increases and gets an asymptotically flat condition till $r_o \rightarrow \infty$ that shows the stability of BH. From Figures 2 and 3, it is clear that in the presence of the rotation parameter the temperature increases as compared to the absence of $a$.

- In the absence of charge the temperature $T_H$ gradually increases and then slowly decreases with the increasing $r_o$ that depicts the stable form of BH. But it can be observed from the
plots that BH is in more stable form in the absence of charge $q$.

In all plots, the BH depicts its stable condition by satisfying the Hawking’s idea (the size of BH reduces when more radiations emit), the temperature is high at small horizon. We observe maximum temperature at minimum value of horizon. Furthermore, we have calculated the Hawking temperature under the influence of quantum gravity by using the semi-classical method and also examined the graphical corrected temperature $T_H$ behavior with respect to horizon $r_+$ , to check the physical and stable conditions of BH under the effects of $a$ (quantum gravity parameter). We also discussed the stability conditions of BH in the absence of charge and spin parameter with or without gravity effects. We have concluded that with or without quantum gravity the BH represents its stable form. In our analysis we have found that the quantum corrections decelerate the increase in temperature during the radiation process. This correction causes the radiation ceased at some specific temperature, leaving the remnant mass. The remnant will obtain at specific condition

$$M_{\text{Rem}} = \frac{M_p^2}{\beta_0 \omega} \geq \frac{M_p}{\beta_0}$$

The results from the graphical analysis of corrected Hawking temperatures with respect to the horizon for the given BH is summarized as follows:

- The $T_H$ decreases with the increasing horizon and BH reflects the stable state for varying values of rotation parameter $a$ and correction parameter $\alpha$. The corrected temperature $T'_H$ also increases with the increase in $a$ and $\alpha$. The BH remnant can be obtained at nonzero horizon with maximum temperature in the domain $0 \leq r_+ \leq 15$.
- In the absence of charge the corrected temperature $T'_H$ gradually increases and then slowly decreases with the increasing $r_+$ that depicts the stable form of BH.
- In the absence of spin parameter $a$ the $T'_H$ behavior slowly decreases as horizon increases and gets an asymptotically flat condition till $r_+ \rightarrow \infty$ that shows the stability of BH.

From our plots, we conclude that the tunneling emission rate increases (gives high temperature) with the increasing values of $a$. The corrected temperature of charged rotating BH with EGB gravity satisfies both GUP and Hawking conditions that guarantee the physical and stable states of BH. According to GUP condition the next order corrections must be small as compared to the standard term of uncertainty relation. The positive value of temperature in these plots also satisfies the GUP relation, when GUP conditions do not satisfy the temperature becomes negative (shows non-physical state of BH). Now by comparing the plots in the presence/absence of quantum gravity parameter, we can observe that the corrected temperature is lower than the original temperature. In conclusion, the Newman–Janis algorithm gravity has attracted more attention for different space–time. In this paper, we have evaluated the BH space–time in Newman–Janis algorithm and computed the Hawking temperature with the effects of the rotation parameter and correction parameter. Finally, we computed the logarithmic corrected entropy for EGB gravity of rotating charged BH and also checked the logarithmic corrected entropy with the effects of the rotation parameter and correction parameter.

In future, we will study on the other objects (i.e., wormholes and black rings) in the Newman–Janis algorithm.

**Conflict of Interest**

The authors declare no conflict of interest.

**Data Availability Statement**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Keywords**

black holes, Einstein–Gauss–Bonnet gravity theory, Hawking temperature, Lagrangian equation, logarithmic corrected entropy, Newman–Janis algorithm.

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