Removal of Closed Timelike Curves in Kerr-Newman Spacetime

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Abstract

A simple yet systematic new algorithm to investigate the global structure of Kerr-Newman spacetime is suggested. Namely, the global structures of $\theta = \text{const.}$ timelike submanifolds of Kerr-Newman metric are studied by introducing a new time coordinate slightly different from the usual Boyer-Lindquist time coordinate. In addition, it is demonstrated that the possible causality violation thus far regarded to occur near the ring singularity via the development of closed timelike curves there is not really an unavoidable pathology which has plagued Kerr-Newman solution but simply a gauge (coordinate) artifact as it disappears upon transforming from Boyer-Lindquist to the new time coordinate. This last point appears to lend support to the fact that, indeed, the Kerr-Newman spacetime is a legitimate solution to represent the interior as well as the exterior regions of a rotating, charged black hole spacetime.

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I. Introduction

In the present work, we would like to suggest a new algorithm to investigate the global structure and maximal analytic extension of the Kerr-Newman spacetime [1,2] which is simpler yet more systematic than the existing ones. To our knowledge, the first serious attempts to uncover them were made long ago by Carter [5,7] and by Boyer and Lindquist [6]. Along the line of procedures suggested earlier by Finkelstein [8] and by Kruskal [9] and by Graves and Brill [4], Boyer and Lindquist [6] and independently Carter [5] were able to perform a detailed study of the global structure and the maximal analytic extension of the Kerr-Newman spacetime. Working in the Kerr coordinates [6], which can be thought of as the axisymmetric generalization of Eddington-Finkelstein null coordinates, Carter [5], however, had to confine himself to the study particularly on the symmetry axis $\theta = 0$ (where $\theta$ denotes the polar angle) for technical limitations. And then from the conclusion derived from this study, he conjectured that the other $\theta = \text{const.}$ hypersurfaces would presumably have the same global structures as that of the symmetry axis. The new algorithm we are about to present here allows us to “complete” the status of development in this direction. Namely, we shall be able to explore the global structures of the “$\theta = \text{const.}$” timelike submanifolds of Kerr-Newman spacetime from the “symmetry axis” ($\theta = 0$) all the way to the “equatorial plane” ($\theta = \pi/2$) one by one. Our strategy is first to scan the full, 4-dim. Kerr-Newman manifold by slicing it into pieces of $\theta = \text{const.}$ timelike submanifolds and next to examine the global structure of each one of them. And to this end, we shall introduce a “new” time coordinate different from the usual Boyer-Lindquist time coordinate $t$ [6] which can be defined only on the $\theta = \text{const.}$ timelike submanifolds. Now let us be more specific. Since the geometry of $\theta = \theta_0$ (with $0 < \theta_0 \leq \pi/2$) submanifolds is essentially 3-dim. whereas that of the symmetry axis ($\theta_0 = 0$) is effectively 2-dim. (since it has a degeneracy along $\phi$ direction), things get more involved compared to the case of the symmetry axis discussed by Carter [5]. Therefore in order to make the analysis tractable, we shall introduce a “new” time coordinate $\tilde{t}$ slightly different from the usual Boyer-Lindquist time coordinate $t$ [6] based on
the philosophy that the global structure remains unaffected under coordinate changes. From there one can then apply the same methods of Finkelstein [8] and Kruskal [9] to obtain the maximal analytic extensions of the conformal diagrams of the $\theta = \theta_0$ submanifolds. Then it is straightforward to see that the maximally extended conformal diagram of the $\theta = \theta_0$ (with $0 < \theta_0 < \pi/2$) submanifolds of Kerr-Newman spacetime remain the same, i.e., still take the same structure as that of the symmetry axis [5]. The extended conformal diagram of the equatorial plane, on the other hand, takes exactly the same structure as that of the Reissner-Nordstrom (RN) solution [3]. As a result, it does have the ring singularity at $\Sigma = 0$ (or more precisely at $r = 0$ since $\theta_0 = \pi/2$) and one cannot, on the equatorial plane, extend to negative values of $r$ while one can do so on other $\theta = \theta_0$ submanifolds. As we shall discuss in detail in the text shortly, in addition to the clearer overview of the global structure of the full Kerr-Newman spacetime that our algorithm provides, there is another, even more crucial role played by the “new” time coordinate. Recall, as was first pointed out by Carter [7], that there is an issue of possible occurrence of closed timelike curves and hence the possibility of causality violation near the ring singularity if one analyzes the Kerr-Newman metric in Boyer-Lindquist coordinates. As we shall see in a moment, this occurrence of closed timelike curves disappears by transforming to the “new” time coordinate. This, of course, implies that the seemingly possible violation of causality near the ring singularity of Kerr-Newman spacetime is not an unavoidable pathology but simply a gauge (coordinate) artifact whose appearance can be attributed to the poor choice of (Boyer-Lindquist) coordinates. In short, this elimination of the possibility of causality violation near the ring singularity via a transformation to the new time coordinate appears to lead us to believe that the Kerr-Newman solution [1,2] might be really valid as the spacetime of a rotating charged black hole. And our study, particularly, of the global topology of the equatorial plane confirms the existence of the ring singularity in a more transparent manner and supports the general belief that the spacetime produced by physically realistic collapse of even nonspherical bodies would be qualitatively similar to the spherical case, i.e., the RN geometry [3].
II. Introduction of a “New” time coordinate.

Now consider the stationary axisymmetric Kerr-Newman solution of the Einstein-Maxwell equations. The Kerr-Newman metric is given in Boyer-Lindquist coordinates as
\[ ds^2 = -\left(\Delta - a^2 \sin^2 \theta\right) dt^2 - \frac{2a \sin^2 \theta(r^2 + a^2 - \Delta)}{\Sigma} dt d\phi + \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \] (1)
where \( \Sigma \equiv r^2 + a^2 \cos^2 \theta \) and \( \Delta \equiv r^2 - 2Mr + a^2 + e^2 \) with \( M \) being the mass, \( a \) being the angular momentum per unit mass and \( e \) being the total \( U(1) \) charge of the hole. We are now particularly interested in the \( \theta = \text{const.} \) timelike surfaces as submanifolds of this Kerr-Newman spacetime. Namely, consider the \( \theta = \theta_0 \) (\( 0 \leq \theta_0 \leq \pi/2 \)) timelike submanifolds of Kerr-Newman spacetime with the metric
\[ ds^2 = -\left(\Delta - a^2 \sin^2 \theta_0\right) dt^2 - \frac{2a \sin^2 \theta_0(r^2 + a^2 - \Delta)}{\Sigma} dt d\phi + \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta_0}{\Sigma} \sin^2 \theta_0 d\phi^2 + \frac{\Sigma}{\Delta} dr^2 \] (2)
where \( \Sigma = r^2 + a^2 \cos^2 \theta_0 \) now. These \( \theta = \theta_0 \) surfaces have metrics which are literally 3-dim. in structure and possess an off-diagonal component in an intricate way. Thus in order to make the study of global structure of \( \theta = \theta_0 \) submanifolds tractable, here we consider a coordinate transformation which is defined only on the \( \theta = \text{const.} \) timelike submanifolds. Namely consider a transformation from the Boyer-Lindquist time coordinate “\( t \)” to a new time coordinate “\( \tilde{t} \)” given by
\[ \tilde{t} = t - (a \sin^2 \theta_0) \phi \] (3)
with other spatial coordinates \( (r, \phi) \) remaining unchanged. Note that the new time coordinate \( \tilde{t} \) is different from the old one \( t \) only for “rotating” case \( (a \neq 0) \) and even then only for \( \theta_0 \neq 0 \). In terms of this “new” time coordinate \( \tilde{t} \), the metric of \( \theta = \theta_0 \) submanifolds becomes
\[ ds^2 = -\frac{\Delta}{\Sigma} d\tilde{t}^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma \sin^2 \theta_0(d\phi - \frac{a}{\Sigma} d\tilde{t})^2 \]
\[ = -N^2(r)d\tilde{t}^2 + h_{rr}(r)dr^2 + h_{\phi\phi}(r)(d\phi + N^\phi(r)d\tilde{t})^2. \]
Namely, the metric now takes on the structure of a remarkably simple ADM’s \((2+1)\) space-
plus-time split form with the lapse, shift functions and the spatial metric components being
given respectively by

\[
N^2(r) = \frac{\Delta}{\Sigma}, \quad N^\phi(r) = -\frac{a}{\Sigma}, \quad N^\gamma(r) = 0
\]

\[
h_{rr}(r) = N^{-2}(r), \quad h_{\phi\phi}(r) = \Sigma \sin^2 \theta_0, \quad h_{r\phi}(r) = h_{\phi r}(r) = 0.
\]

In terms of this new time coordinate \(\tilde{t}\), therefore, it is now manifest that the \(\theta = \theta_0\) sub-
manifolds of Kerr-Newman spacetime have the topology of \(R^2 \times S^1\) (which was rather obscured
in the metric form eq.(2) given in the original Boyer-Lindquist time coordinate \(t\)) and hence
the new time coordinate is better-suited for the study of global structure. That is to say,
the global structure of the timelike 2-dimensional submanifolds \(T = R^2\) would mirror that
of the full \(\theta = \theta_0\) submanifolds since each point of \(T = R^2\) can be thought of as representing
\(S^1\). We would like to add a comment here. Of course it is true that it is the manifold itself
that has topology, not the metric. Therefore, regardless of the coordinates and hence the
metrics one chooses, they all describe the same manifold with a single topology. However,
the point we would like to make here is that the metric given in new time coordinate \(\tilde{t}\)
(eq.(4)) demonstrates more clearly that the \(\theta = \theta_0\) submanifolds it describes has the topol-
ogy of \(R^2 \times S^1\) than the metric in Boyer-Lindquist time coordinate \(t\) (eq.(2)) does.

Thus we now turn to the analysis of the global structures of the \(\theta = \theta_0\) timelike submani-
folds. Firstly, consider the \(\theta_0 = 0\) timelike submanifolds representing the “symmetry axis”
of the Kerr-Newman spacetime with the metric being given by

\[
ds^2 = -\left(\frac{\Delta}{r^2 + a^2}\right)dt^2 + \left(\frac{\Delta}{r^2 + a^2}\right)^{-1}dr^2.
\]

Note that this metric of the symmetry axis is effectively 2-dim. since it is \textit{degenerate} along
the \(\phi\) direction. And this diagonal, 2-dim. structure of the metric of the symmetry axis
allowed a complete analysis of its global structure as had been carried out by Carter [5].

Secondly, consider the \(\theta_0 = \pi/2\) surface which represents the “equatorial plane” of Kerr-
Newman spacetime. The metric of this submanifold is obtained in the new time coordinate
\(\tilde{t}\) by setting \(\theta_0 = \pi/2\) in eq.(4)
\[ ds^2 = -N^2(r)d\tilde{t}^2 + N^{-2}(r)dr^2 + r^2[d\phi + N^\phi(r)d\tilde{t}]^2 \]  

(7)

with the lapse \( N(r) \) and the shift \( N^\phi(r) \) in above \((2 + 1)\)-split form being given by

\[ N^2(r) = \frac{\Delta}{r^2} = \left[1 - \frac{2M}{r} + \frac{(a^2 + e^2)}{r^2}\right], \quad N^\phi(r) = -\frac{a}{r^2}. \]  

(8)

Finally, note that the metric of the \( \theta = \theta_0 \) (\( 0 < \theta_0 < \pi/2 \)) submanifolds given in eq.(4) are everywhere non-singular including \( r = 0 \) and possess exactly the same causal structure (except for the appearance of ergoregion) as that of the symmetry axis (\( \theta = 0 \)). Therefore, the maximal analytic extension of the \( \theta = \theta_0 \) (\( 0 < \theta_0 < \pi/2 \)) submanifolds representing their global structure is essentially the same as that of the symmetry axis first studied by Carter [5]. The metric of the equatorial plane (\( \theta_0 = \pi/2 \)) given in eqs.(7),(8), however, possesses a curvature singularity at \( r = 0 \) as expected (since it is the “ring singularity”, \( r = 0, \theta_0 = \pi/2 \)) whereas it exhibits almost the same causal structure (again except for the presence of the ergoregion) as that of the symmetry axis. As a result, the maximal analytic extension of the equatorial plane is identical to that of the RN spacetime [3]. Detailed analysis of the global structure and the maximal analytic extension will be presented in a separate publication.

**III. Physical meaning of the “New” time coordinate.**

Note that both in the old Boyer-Lindquist coordinates and in the new coordinates, the \( \theta = \theta_0 \) submanifolds of Kerr-Newmann metric are stationary and axisymmetric and hence possess the associated time-translational and rotational Killing fields \( \{ \xi^\mu = (\partial/\partial t)^\mu, \psi^\mu = (\partial/\partial \phi)^\mu \} \) and \( \{ \tilde{\xi}^\mu = (\partial/\partial \tilde{t})^\mu, \tilde{\psi}^\mu = (\partial/\partial \tilde{\phi})^\mu \} \) respectively. Since the azimuthal angle coordinate \( \phi \) is not being transformed, we shall henceforth write \( \tilde{\phi} = \phi \) in all the expressions given in the new coordinates \( (\tilde{t}, r, \tilde{\phi}) \). Now, in order to have physical meaning of the new coordinate system \( (\tilde{t}, r, \tilde{\phi}) \) for each \( \theta = \theta_0 \) timelike submanifold given by eq.(3), we begin by considering how the associated time-translational and rotational Killing fields transform accordingly. Namely from:

\[ g_{\tilde{t}\tilde{t}} = g_{tt}, \quad g_{\tilde{t}\tilde{\phi}} = (a\sin^2 \theta_0)g_{tt} + g_{t\phi}, \]  

(9)

\[ g_{\tilde{\phi}\tilde{\phi}} = (a^2\sin^4 \theta_0)g_{tt} + 2(a\sin^2 \theta_0)g_{t\phi} + g_{\phi\phi}, \]
and $g_{t\mu} = \xi^\mu \xi_\mu$, $g_{t\phi} = \xi^\mu \psi_\mu$, $g_{\phi\phi} = \psi^\mu \psi_\mu$ and $g_{\check{t}\check{t}} = \check{\xi}^\mu \check{\xi}_\mu$, $g_{\check{t}\check{\phi}} = \check{\xi}^\mu \check{\psi}_\mu$, $g_{\check{\phi}\check{\phi}} = \check{\psi}^\mu \check{\psi}_\mu$, we can conclude
\[ \check{\xi}^\mu = \xi^\mu, \quad \check{\psi}^\mu = \psi^\mu + (a \sin^2 \theta_0) \xi^\mu. \] (10)

We are now ready to interpret the physical meaning of this coordinate transformation on each $\theta = \theta_0$ timelike submanifold in terms of this transformation law for the Killing fields. In the old Boyer-Lindquist coordinate $(t, r, \phi)$, the coordinate “$t$” is a timelike coordinate outside the event horizon with unbounded range $0 \leq t < \infty$ and the coordinate “$\phi$” is compactified with the period of $2\pi$, i.e., $\phi \sim \phi + 2\pi n \ (n \in \mathbb{Z})$ in order for the Kerr-Newman metric to be asymptotically-flat. And this amounts to identifying points along the “straight” orbits of the rotational Killing field $(\partial/\partial \phi)^\mu$. Now, notice that under the coordinate transformation given in eq.(3), the associated Killing fields transform as in eq.(10), namely
\[ \left(\partial/\partial \check{\phi}\right)^\mu = (\partial/\partial \phi)^\mu + a \sin^2 \theta_0 (\partial/\partial \check{t})^\mu, \quad \left(\partial/\partial \check{t}\right)^\mu = (\partial/\partial t)^\mu. \] (11)

Particularly, the transformation law for the rotational Killing field $\psi^\mu = (\partial/\partial \phi)^\mu$ indicates that now one should identify points along the “twisted” orbits of $\psi^\mu + (a \sin^2 \theta_0) \xi^\mu$. Namely, since the compactification direction now involves that of time coordinate, the corresponding identification of points should be
\[ (t, r, \phi) \equiv (t + 2\pi n (a \sin^2 \theta_0), r, \phi + 2\pi n), \quad n \in \mathbb{Z} \] (12)

which implies that, except along the poles, $t$ is periodic, namely, there are closed timelike curves. Then upon this change of coordinates, the metric of each $\theta = \theta_0$ timelike submanifold remarkably simplifies to the form given in eq.(4) or equivalently to
\[ ds^2 = -\left[ \frac{\Delta - a^2 \sin^2 \theta_0}{\Sigma} \right] d\tilde{t}^2 - 2a \sin^2 \theta_0 d\tilde{t} d\tilde{\phi} + \Sigma \sin^2 \theta_0 d\tilde{\phi}^2 + \frac{\Sigma}{\Delta} dr^2, \] (13)

now with the identification of points $(\tilde{t}, r, \tilde{\phi}) \equiv (\check{t}, r, \tilde{\phi} + 2\pi n)$ in a standard manner. The fact that one recovers standard way of point identification after the coordinate transformation given in eq.(3) can indeed be represented as follows: just as the old Boyer-Lindquist time
coordinate “\( \tilde{t} \)” is constant along the orbits of \( \psi^\mu \), i.e., \((\partial/\partial\phi) t = 0\), the new coordinate “\( \tilde{t} \)” is the “adapted” time coordinate which is constant along the orbits of \( \tilde{\psi}^\mu = \psi^\mu + (a \sin^2 \theta_0) \xi^\mu \), i.e.,

\[
\left( \frac{\partial}{\partial \tilde{\phi}} \right) \tilde{t} = \left[ \left( \frac{\partial}{\partial \phi} \right) + a \sin^2 \theta_0 \left( \frac{\partial}{\partial t} \right) \right] (t - a \sin^2 \theta_0 \phi) = 0. \tag{14}
\]

As we stressed earlier, in terms of new coordinates, it becomes much more transparent that the topology of each \( \theta = \theta_0 \) submanifold is \( R^2 \times S^1 \), which, although expected, was not so clear in the old Boyer-Lindquist coordinates. We now remark on some interesting features that emerge when we describe the \( \theta = \theta_0 \) submanifolds of the Kerr-Newman black hole spacetime in terms of this new coordinates aside from the advantage in studying their global structure.

(1) Gauge (Coordinate) dependence of the occurrence of closed timelike curves.

Indeed, this is the point of central importance we would like to make in the present work concerning the crucial role played by this new coordinate system. First, notice that the transformation from the Boyer-Lindquist time coordinate “\( t \)” to the new time coordinate “\( \tilde{t} \)” introduced in the present work involves the identification of points \( t \sim t + 2\pi n(a \sin^2 \theta_0) \) in addition to the usual \( \phi \sim \phi + 2\pi n \). As was pointed out, this implies that this particular transformation involves, along the way, the compactification of time coordinate or equivalently, the emergence of closed timelike curves away from the poles at \( \theta_0 = 0, \pi \). Once the transformation to the new time coordinate is completed, however, the \( \theta = \theta_0 \) submanifolds of the Kerr-Newman spacetime admits metrics free of the closed timelike curves since now \( \tilde{t} \) possesses usual, semi-infinite, unbounded range, \( 0 \leq \tilde{t} < \infty \). Namely, this coordinate transformation partly involves an implicit process in which the closed timelike curves are “gauged away”. One might suspect that the compactification of old time coordinate involved in this coordinate transformation is not relevant and hence physically unacceptable. It is, however, well-known that closed timelike curves indeed occur in the interior region of Kerr-Newman black hole spacetime represented in Boyer-Lindquist coordinates and hence the explicit emergence of the closed timelike curves during the course of this coordinate
transformation process presumably could reflect this fact. Thus in the following, we shall demonstrate that this is indeed the case and hence in the new time coordinate \( \tilde{t} \), the closed timelike curves are completely gauged away. To this end, we begin by recalling the argument first given by Carter [7] exhibiting the possible occurrence of causality violation in some interior region of Kerr-Newman spacetime.

Consider the norm of the Killing field \( \psi^\mu = (\partial/\partial \phi)^\mu \) evaluated on a \( \theta = \theta_0 \) \( (0 \leq \theta_0 \leq \pi/2) \) submanifold with metric given in Boyer-Lindquist coordinates;

\[
\psi^\mu \psi_\mu = g_{\phi\phi} = \left[ r^2 + a^2 + \frac{(2Mr - e^2)a^2 \sin^2 \theta_0}{\Sigma} \right] \sin^2 \theta_0.
\]  

(15)

For “negative” values of \( r \) of sufficiently small magnitude and for \( \theta_0 \) sufficiently close to \( \pi/2 \), \( \psi^\mu \psi_\mu < 0 \), i.e., \( \psi^\mu = (\partial/\partial \phi)^\mu \) can go “timelike” near the ring singularity at \( r = 0, \theta_0 = \pi/2 \). However, since the orbit of \( \psi^\mu \) must be, as stated, closed with period of \( 2\pi \) in order for the Kerr-Newman spacetime to be asymptotically-flat as \( r \to \infty \), this timelike-going behavior of \( \psi^\mu \) near the ring singularity indicates the possible occurrence of closed timelike curves and hence the possible violation of causality there. On the other hand, consider this time the norm of \( \tilde{\psi}^\mu = (\partial/\partial \tilde{\phi})^\mu \) evaluated again on the same \( \theta = \theta_0 \) hypersurface with the metric given in new coordinates;

\[
\tilde{\psi}^\mu \tilde{\psi}_\mu = g_{\tilde{\phi}\tilde{\phi}} = (r^2 + a^2 \cos^2 \theta_0) \sin^2 \theta_0 > 0.
\]  

(16)

Namely, it is always positive-definite meaning that \( \tilde{\psi}^\mu \) is everywhere spacelike and can never go timelike even near the ring singularity. Therefore, if one works in this new coordinates \( (\tilde{t}, r, \tilde{\phi}) \), no possibility of causality violation is encountered anywhere he/she goes in the Kerr-Newman spacetime! To conclude, the possible timelike-going behavior of the rotational Killing field and hence the seemingly possible causality violation near the ring singularity turns out to be just a gauge artifact, i.e., it can be attributed to the poor choice of (Boyer-Lindquist) coordinates and can be eliminated by transforming to the new coordinates introduced in the present work in much the same way as the coordinate singularity at the event horizon of the Kerr-Newman metric can be gauged away by transforming to null
coordinates such as Kerr coordinates or Kruskal-Szekers coordinates. As is well understood, since the gauge, or equivalently, coordinate transformation from one to another in gravity amounts to the shift of observer's "state of motion" from one to another in curved spacetime, this apparent gauge dependence of the occurrence of closed timelike curves indicates that they, like the appearance of event horizon, may be present or absent depending on the observer's state of motion. In fact, there is a simple observation that might provide a clue to the nature of relative motion between the two observers representing the two coordinate systems at each $\theta = \theta_0$ hypersurface. To this end, we begin with the computation of the angular velocity of zero-angular-momentum-observer (ZAMO) \cite{10} in each of the two coordinates. Generally, the orthonormal tetrad is a set of four mutually orthogonal unit vectors at each point in a given spacetime that give the directions of the four axes of locally Minkowskian coordinate system. And the ZAMO frame (or the locally-non-rotating-frame (LNRF)) is one such orthonormal tetrad that singles out since an observer at rest in it has zero angular momentum with respect to the rotating Kerr-Newman black hole. Also, ZAMO is a fiducial observer (FIDO) moving with the 4-velocity which is orthogonal to spacelike hypersurfaces $t = \text{const}$. Namely, the ZAMO geodesic is defined by $ds^2 = -N^2 dt^2$ or equivalently \{dr = 0, d\theta = 0, (d\phi + N^\phi dt) = 0\} for Kerr-Newman spacetime given in Boyer-Lindquist coordinates and as a result, ZAMO angular velocity is given by $\Omega = d\phi/dt = -N^\phi = -g_{t\phi}/g_{\phi\phi}$.

Thus the ZAMO angular velocity when evaluated in each of the two coordinates is given by

$$\Omega = -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{a(r^2 + a^2 - \Delta)}{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta_0}, \quad \tilde{\Omega} = -\frac{\tilde{g}_{t\tilde{\phi}}}{\tilde{g}_{\tilde{\phi}\tilde{\phi}}} = \frac{a}{r^2 + a^2 \cos^2 \theta_0}.$$  \hspace{1cm} (17)

Next, the angular velocity of the horizon (and hence the black hole) is nothing but that of ZAMO at $r = r_+$ and again in each of the two coordinates, it is

$$\Omega_H = -\frac{g_{t\phi}}{g_{\phi\phi}}|_{r_+} = \frac{a}{r_+^2 + a^2}, \quad \tilde{\Omega}_H = -\frac{\tilde{g}_{t\tilde{\phi}}}{\tilde{g}_{\tilde{\phi}\tilde{\phi}}}|_{r_+} = \frac{a}{r_+^2 + a^2 \cos^2 \theta_0}.$$  \hspace{1cm} (18)

where $r_+ = M + (M^2 - a^2 - e^2)^{1/2}$ denotes the location of the event horizon which turns out to be precisely the same (we shall come back to this issue shortly) in the two coordinates. As we shall see in a moment, the static limit, i.e., the outer boundary of the ergoregion,
turns out to occur exactly at the same point \( r_s = M + (M^2 - a^2 \cos^2 \theta_0 - e^2)^{1/2} \) in the two coordinates as well. It is now straightforward to see that \( \tilde{\Omega} > \Omega \) and \( \tilde{\Omega}_H > \Omega_H \) for \( r \geq r_+ \) and \( \theta_0 \neq 0, \pi \). Thus if we combine these observations, i.e., the invariance of the causal structure and \( \tilde{\Omega} > \Omega \) and hence \( \tilde{\Omega}_H > \Omega_H \) (for \( \theta_0 \neq 0, \pi \)), we can conclude that the two observers associated with the two coordinate systems have no relative radial motions but just relative azimuthal angular motions and the new coordinates (and the observer in it) appears to rotate in opposite direction around the hole’s rotation axis with respect to the Boyer-Lindquist coordinates (and the observer in it) at the angular velocity that changes with \( \theta_0 \) such that it gets maximized at the equatorial plane \( \theta_0 = \pi/2 \) while becomes zero on the poles \( \theta_0 = 0, \pi \). To summarize, the closed timelike curves and the event horizon are on *equal footing* in that they have *apparent gauge dependence* and hence should be distinguished from the unavoidable curvature singularity, i.e., the ring singularity at \( r = 0, \theta_0 = \pi/2 \) that clearly is the drawback of the Kerr-Newman solution as a classical solution to the Einstein-Maxwell equations. Indeed, due to the occurrence of closed timelike curves and hence the emergence of causality violation near the ring singularity, the Kerr-Newman solution has been regarded as not being able to represent the interior region of charged, rotating black holes thus far. In this regard, the demonstration of the possible elimination of closed timelike curves near the ring singularity (and elsewhere) via the transformation to the new coordinates discovered in the present work appears to lend support to the viability of the Kerr-Newman solution as a legitimate, physical metric being able to represent both the exterior and interior of charged, rotating black hole spacetime.

(2) *Invariance of the causal structure*

Note that it is the linear combinations

\[
\chi^\mu = \xi^\mu + \Omega_H \psi^\mu, \quad \tilde{\chi}^\mu = \tilde{\xi}^\mu + \tilde{\Omega}_H \tilde{\psi}^\mu
\]

constructed out of the time-translational and rotational Killing fields \( \{\xi^\mu, \psi^\mu\} \) and \( \{\tilde{\xi}^\mu, \tilde{\psi}^\mu\} \) in Boyer-Lindquist and new coordinates respectively which are tangential to the null geodesic generator of the horizon. And \( \Omega_H \) and \( \tilde{\Omega}_H \) above are as given earlier in eq.(18). Now, the
event horizon is a Killing horizon and generally speaking, Killing horizons occur at points where the Killing field $\chi^\mu$ or $\tilde{\chi}^\mu$ above becomes null. As is well-known, $\chi^\mu \chi_\mu = 0$ occurs for $\Delta = r^2 - 2Mr + a^2 + e^2 = 0$, namely at $r_\pm = M \pm (M^2 - a^2 - e^2)^{1/2}$. Interestingly, but as expected to some extent, even in the new coordinates, $\tilde{\chi}^\mu \tilde{\chi}_\mu = 0$ occurs again for $\Delta = 0$. Next, we turn to the static limit, the outer boundary of the ergoregion. It occurs at the point where the time-translational Killing field $\xi^\mu$ or $\tilde{\xi}^\mu$ becomes null. Thus again, it is amusing to note both in the old Boyer-Lindquist and in the new coordinates, the static limit occurs exactly at the same point $r_s = M + (M^2 - a^2 \cos^2 \theta_0 - e^2)^{1/2}$ since $\xi^\mu \xi_\mu = \tilde{\xi}^\mu \tilde{\xi}_\mu = g_{tt} = - \left[ \frac{\Delta - a^2 \sin^2 \theta_0}{\Sigma} \right] = 0$ occurs for $(\Delta - a^2 \sin^2 \theta_0) = 0$. To summarize, the Killing horizons and the static limit occur respectively at precisely the same locations and hence the causal structure of the Kerr-Newman black hole spacetime remains the same regardless of the coordinate systems (old or new) used to represent the metrics of $\theta = \theta_0$ submanifolds. And of course, it can be attributed to the nature of the coordinate transformation in which the radial coordinate “$r$” remains unaffected.

IV. Concluding Remarks

We now conclude with some comments worth mentioning. First, we would like to point out the complementary roles played by the two alternative time coordinates $t$ and $\tilde{t}$. The Boyer-Lindquist time coordinate $t$ is the usual Killing time coordinate with which one can obtain the rotating hole’s characteristics such as the angular velocity of the event horizon or the surface gravity as measured by an outside observer who is “static” with respect to, say, a distant star. The “new” time coordinate $\tilde{t}$, on the other hand, is a kind of an unusual one in that it can be identified with the time coordinate of a non-static frame which rotates around the axis of the Kerr-Newman black hole in opposite direction to that of the hole (of course outside the ergoregion) with an angular velocity that increases with the polar angle. This new time coordinate, however, is particularly advantageous in exploring the global structure of the $\theta = \text{const.}$ submanifolds of Kerr-Newman black hole since it allows one to transform to Kruskal-type coordinates and hence eventually allows one to draw the Carter-Penrose...
diagrams much more easily than the case when one employs the usual Boyer-Lindquist time coordinate. Yet, however, the most remarkable point to be emphasized concerning the important roles played by the new time coordinate $\tilde{t}$ is that in terms of which the seemingly possible causality violation near the ring singularity can be eliminated as we pointed out earlier. Namely, consider the norm of the rotational Killing field $\tilde{\psi}^\mu = (\partial/\partial \tilde{\phi})^\mu$ evaluated on a $\theta = \theta_0$ ($0 \leq \theta_0 \leq \pi/2$) timelike submanifold. Unlike the case when one employs the usual Boyer-Lindquist time coordinate $t$, if one employs the new time coordinate $\tilde{t}$, then $\tilde{\psi}^\mu \tilde{\psi}_\mu = g_{\tilde{\phi}\tilde{\phi}} > 0$, i.e., $\tilde{\psi}^\mu$ is everywhere spacelike and can never go timelike even near the ring singularity. This observation implies that the possible occurrence of closed timelike curves and hence the possibility of causality violation in the vicinity of ring singularity is just a gauge artifact as it disappears upon a gauge (coordinate) transformation. This last point appears to be a big discovery and hence the introduction of the “new” time coordinate $\tilde{t}$ leads us to regard the Kerr-Newman spacetime as a valid solution to describe both the exterior and the interior regions of a rotating charged black hole with ever more credibility.

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