The wind profile up to 300 meters over flat terrain

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Abstract. Analysis of profiles of meteorological measurements from a 160 m high mast at the National Test Site for wind turbines at Høvsøre (rural, Denmark) shows that the wind profile based on surface-layer theory and Monin-Obukhov scaling is valid up to a height of 50 to 80 m. At higher levels deviations from the measurements progressively occur. The measurements also indicated that the height of the boundary layer influences the wind profile even in the lowest hundreds of meters. A parameterization of the wind profile for the entire boundary layer is formulated, with emphasis on the lowest 200 - 300 meters.

1. Theory
A detailed derivation of the wind profile is given in [1]. The starting point for the analysis is the general expression for the wind profile for the homogeneous, stationary boundary layer [2]:

\[
\frac{du}{dz} = \frac{u_\infty}{\kappa l}
\]

where \( u \) and \( z \) is wind velocity and height above ground, \( u_\infty \) is the local friction velocity, \( \kappa \) the von Karman constant and \( l \) is the local length scale. In the surface layer the local friction velocity can be considered constant

\[
u_\infty = u_{\infty_0}
\]

where \( u_{\infty_0} \) is the friction velocity near the ground. Above the surface layer the friction velocity diminishes and becomes small at the top of the boundary layer where it is here approximated as

\[
u_\infty(z) = u_{\infty_0}(1 - z/z_i)
\]

where \( z_i \) is the boundary-layer depth.

The wind profile length scale, \( l \), is composed of three terms. In the surface layer the first length scale (I) is taken to increase linearly with height with a stability correction following Monin-Obukhov
similarity. Above the surface layer the second (II) length scale \( L_{MBL} \) becomes independent of height but not of stability, and at the top of the boundary layer the third length scale (III) is assumed to be negligible. A simple model for the combined length scale that controls the wind profile and its stability dependence is formulated by inverse summation [1]:

\[
\frac{1}{l} = \frac{1}{L_{SL}} + \frac{1}{L_{MBL}} + \frac{1}{L_{UWL}}
\]

where \( L_{SL} \) represents the length scale in the surface layer, \( L_{MBL} \) in the middle of the boundary layer and \( L_{UWL} \) the upper part of the boundary layer.

Figure 1 illustrates the behaviour of the length scales for a 1000 meter deep neutral boundary layer. It can be seen that surface layer scaling \( L_{SL} = \kappa z \) (term I) is applicable up to about 50 metres, where the influence of \( L_{MBL} \) (term II) becomes noticeable. The height of the boundary layer (term III) can already be seen to influence the length scale at about 150 metres height.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Profiles of the length scale for neutral conditions with \( z_i = 1000 \ m \) and \( z_0 = 0.05 \ m \). The dashed-dotted line corresponds to the surface layer scaling only. The dashed line includes the effect of \( L_{MBL} = 150 \ m \) and the full line all three terms in the formulation of the length scale. The value \( L_{MBL} = 150m \) is taken from the analysis of data presented in section 4.2.}
\end{figure}

2. Wind profile

Inserting Eq. (3) into Eq. (1) using the expression Eq. (4) for the wind profile length scale \( l \), and integrating along \( z \) between the roughness length \( z_0 \) and height \( z \) yields for the neutral wind profile [1]:

\[
u(z) = \frac{u^*}{\kappa} \left( \ln \left( \frac{z}{z_0} \right) + \frac{z}{L_{MBL,N}} - \frac{z}{z_i} \left( \frac{z}{2 L_{MBL,N}} \right) \right)
\]

The function \( L_{MBL} \) and its parameterization are discussed in section 3 and 4.2. For atmospheric stable conditions the functional form of the length scale with stability correction for the surface layer reads:
\[ L_{SL} = \left(1 + b z / L\right) / z \]  \hspace{1cm} (6)

where \( b \approx 5 \) is an empirical constant and \( L \) is the Obukhov length scale:

\[ L = \frac{-u_{z0}^3}{\kappa (g/T) w'T_0} \]  \hspace{1cm} (7)

where \( T \) is temperature, \( g/T \) is the buoyancy parameter and \( w'T_0 \) is the kinematic heat flux at the surface. The corresponding wind profile for stable conditions reads:

\[ u(z) = \frac{1}{\kappa} \ln \left( \frac{z}{z_0} \right) + \frac{b z}{L} \left( 1 - \frac{z}{2 z_i} \right) + \frac{z}{L_{MBL}} - \frac{z}{L_{MBL}} \left( \frac{z}{2 L_{MBL}} \right) \]  \hspace{1cm} (8)

For atmospheric unstable conditions the wind profile can be expressed as [1]:

\[ u(z) = \frac{1}{\kappa} \ln \left( \frac{z}{z_0} \right) - \psi \left( \frac{z}{L} \right) + \frac{z}{L_{MBL}} - \frac{z}{L_{MBL}} \left( \frac{z}{2 L_{MBL}} \right) \]  \hspace{1cm} (9)

where the stability correction for the surface boundary layer is

\[ \psi \left( \frac{z}{L} \right) = \frac{3}{2} \ln \left( 1 + x + \frac{x^2}{3} \right) - \sqrt{3} \arctan \left( \frac{1 + 2x}{\sqrt{3}} \right) + \frac{\pi}{\sqrt{3}} \]  \hspace{1cm} (10)

and \( x = (1 - 12 z / L)^{1/3} \).

### 3. Parameterization of \( L_{MBL} \)

At the top of the boundary layer the wind profile conforms to the geostrophic wind. A parametrization of \( L_{MBL} \) was achieved [1] by use of the geostrophic drag law (Rossby similarity theory) that relates the wind speed at the top of the boundary layer to the friction velocity near the ground. In barotropic, stationary conditions, the geostrophic drag law can be written

\[ \frac{G}{u_{*0}} = \frac{1}{\kappa} \sqrt{\left[ \ln \left( \frac{u_{*0}}{f z_0} \right) - B(\mu) \right]^2 + A^2(\mu)} \]  \hspace{1cm} (11)

where \( G \) is the geostrophic wind speed, and \( A \) and \( B \) are the resistance law functions depending on the dimensionless stability parameter \( \mu = u_{*0} / f L \) where \( f \) is the Coriolis parameter. For the neutral atmosphere the values \( A \approx 4.9 \) and \( B \approx 1.9 \) are rather well established but their stability dependence is still a matter of discussion. Applying the wind profiles Eqs. (5), (8) and (9) at the top of the boundary layer, \( z_i \), and eliminating \( G/u_{*0} \) by use of Eq. (11) a parameterisation of \( L_{MBL} \) can be obtained.

Owing to ambiguity in the formulation of the A and B functions of the geostrophic drag law, an empirical fit of the dependence between \( u_{*0} / f L_{MBL} \), \( u_{*0} / f z_0 \) and \( u_{*0} / f L \) will be devised. The roughness and atmospheric stability dependence of \( u_{*0} / f L_{MBL} \) can be approximated [1] as:
\[
\frac{\mu_0}{fL_{MBL}} = \left( -2 \ln \left( \frac{\mu_0}{fz_0} \right) + 55 \right) \exp \left( -\frac{(\mu_0/fL)^2}{400} \right). \tag{12}
\]

It is noted in [1] that the data fit to Eq. (12) is better for stable than for unstable conditions. Knowing \(\mu_0\), \(z_0\) and \(L\) as well as \(f\) allows \(L_{MBL}\) to be determined from Eq. (12) and the wind profile can then be estimated from Eqs. (5), (8) and (9). The height of the boundary layer can be taken from measurements, from an estimate from a validated meteorological pre-processor, e.g. [3] or it can be approximated by \(z_i \approx 0.1\mu_0/f\).

4. Measurements

Wind profiles are analyzed from the National Test Station for Wind Turbines at Høvsøre, Denmark, a rural area in the western part of Jutland. The Høvsøre site is located in a rural area near the west coast of Jutland. Measurements used for this analysis were taken from the sector 30 to 90° where the upwind land area is flat and homogeneous. Measurements were performed at a 116 meter meteorological mast and at 160 meter level at a nearby light-mast, figure 2. Wind speed was measured by cup-anemometers at 2, 10, 40, 60 80, 100, 116 and 160 meters height, wind direction at 10, 60, 100 and 160 meter height by wind vanes. Atmospheric turbulent fluxes of heat and momentum were derived from measurements sonic anemometers at 10, 20, 40, 60, 80, 116 and 160 meters height.

![Figure 2](image_url)

**Figure 2.** The surrounding area at the National Test Station for Wind Turbines at Høvsøre and the 60° sector used in the analysis. Water is dotted. The position of the meteorological mast is indicated by a circle and the light-mast by a diamond. The co-ordinate system refers to UTM32.

The data were classified based on the wind direction at 60 m and atmospheric stability at the lowest measuring level. Mean wind and turbulence profiles were calculated according to the stability classification, the characteristic surface parameters are given in Table 1.
Table 1 Mean meteorological parameters at 10 meters at Høvsøre, respectively. The period and wind direction sectors are indicated.

| Interval of the Monin-Obukhov length, L (m) | Mean Monin-Obukhov length, L (m) | Mean $u_*$ (ms$^{-1}$) | Mean $w'T$ (mKs$^{-1}$) | Mean wind speed (ms$^{-1}$) | Number of measurements |
|------------------------------------------|----------------------------------|------------------------|-------------------------|-----------------------------|------------------------|
| Høvsøre (15 March 2004 to 24 November 2005, sector 30 to 90º) | 10 to 50 | 28 | 0.152 | -0.012 | 3.85 | 434 |
|  | 50 to 200 | 108 | 0.249 | -0.013 | 4.69 | 514 |
|  | 200 to 500 | 323 | 0.358 | -0.013 | 6.15 | 404 |
|  | L>500; L<-500 (neutral) | 3910 | 0.388 | -0.001 | 6.39 | 578 |
|  | -500 to -200 | -275 | 0.405 | 0.023 | 6.63 | 148 |
|  | -200 to -100 | -142 | 0.367 | 0.032 | 5.71 | 161 |
|  | -100 to -50 | -71 | 0.340 | 0.051 | 4.91 | 168 |

Stability classification: 10<L<50 (very stable); 50<L<200 (stable); 200<L<500 (near neutral/stable); L>500 or L<-500 (neutral); -500<L<-200 (near neutral/unstable); -200<L<-100 (unstable) and -100<L<-50 (very unstable)

4.1. Boundary-layer height

The height of the atmospheric boundary layer was estimated from the mean profiles of friction velocity $u_*(z)$, and kinematic heat flux, $w'T(z)$ by fitting empirical profiles to the measurements. For the momentum flux a profile adapted from [4] has been used based on an empirical fit to data from LES simulations

$$
\left( \frac{u_*(z)}{u_*(20)} \right)^2 = \exp \left( -3 \left( \frac{z}{z_i} \right)^2 \right).
$$

In the formulation used here the height of the boundary layer is reached when the momentum flux becomes 5% of its value near the ground. The expression deviates from the traditional linear fit for $u_*(z)$ in the sense that the friction velocity does not become zero at the top of the boundary layer. For the kinematic heat flux expressions from [5] have been applied:

$$
\frac{w'T(z)}{w'T(10)} = \left( 1 - \frac{z}{z_i} \right) \quad \text{stable conditions,} \quad (14)
$$

$$
\frac{w'T(z)}{w'T(10)} = \left( 1 - 1.2 \frac{z}{z_i} \right) \quad \text{unstable conditions} \quad (15)
$$

where the coefficient 1.2 in Eq. (15) accounts for the effect of the entrainment zone. Below, figures 3 and 4 show mean profiles and the best empirical fits for the Høvsøre site according to the above formulae.
Figure 3. Normalized profiles of $u^2$ (left panels) and $\overline{w' T'}$ for stable and neutral conditions.
The height of the boundary layer is determined from the fitted profiles, Table 2. It can be seen that Eq. (13) provides a good fit of the momentum flux profile. For the heat flux profile the linear profile fits well. It can also be seen that the boundary layer height estimated from the momentum flux profile is somewhat larger than the height that is determined from the heat flux profile. Generally the agreement is best for the stable conditions, most likely because the stable boundary layer is shallower and better covered by the measurements. The value of the boundary layer height that will be used in the analysis of the wind profile is composed as a rounded mean of the best fit profiles for momentum and heat.
Table 2  Boundary layer height estimated from the profiles of momentum and kinematic heat fluxes

| Mean Monin-Obukhov length, L (m) | Momentum flux profile, Eq.(13) (m) | Kinematic heat flux profile Eqs. (14) & (15) (m) | Mean (m) | Applied value (m) |
|---------------------------------|-----------------------------------|---------------------------------|----------|------------------|
| Høvsøre (19 May 2005 to 24 November 2005, sector 30 to 90°) |
| 24 231 173 202 200              | 202                               | 202                             | 200      | 200              |
| 93 207 204 205 200              | 205                               | 205                             | 200      | 200              |
| 310 279                           | 279                               | 279                             | 280      | 280              |
| Neutral 316                      | 316                               | 316                             | 320      | 320              |
| -284 614 433 524 500             | 524                               | 524                             | 500      | 500              |
| -140 459 447 453 500             | 453                               | 453                             | 500      | 500              |
| -75 612                           | 612                               | 612                             | 508      | 500              |

4.2. Wind profiles

The measured wind speed profiles are distributed in Monin-Obukhov length classes according to Table 1 and normalized with the friction velocity at the lowest level (10 meters height). Using the average Monin-Obukhov length and boundary layer height, lines of best fit based on Eqs. (5), (8) and (9) are plotted. First the roughness length $z_0$ is determined by forcing the theoretical wind profile to coincide with the measurements at the lowest level. Then the length scale $L_{MBL}$ that accounts for the correction to the mean wind profile in the middle of the boundary layer is by subjective fitting determined from the measured wind profile. Values are given in [1] where it can be seen that $L_{MBL}$ has a minimum at neutral conditions of about 150 m. Figure 5 shows the mean normalized wind profiles from Høvsøre. Generally the theoretical expressions Eqs. (5), (8) and (9) can be seen to provide a good fit for all stabilities. The widely used expressions [6] for the wind profile in the surface layer are shown in figure 6 to provide a poor approximation to the wind profile beyond the surface boundary layer for all stabilities.

![Figure 5](image_url). Normalized wind profiles as function of stability.
5. Discussion
The model presented here can be an alternative to the frequent use of the logarithmic profile beyond the surface layer at high wind speeds in different applications. The wind speed profiles based on surface layer theory and Monin-Obukhov scaling are compared with measurements and the new parameterization suggested here, figure 6. The wind profile based on surface layer theory and Monin-Obukhov scaling is valid up to a height of about 80 meters for neutral conditions and somewhat less for the diabatic atmosphere. Higher up the wind profile is not logarithmic in neutral conditions but increases faster in response to the non-linearity of the length scale. It is interesting to note that even when limiting ourselves to the wind profile in the lowest hundreds of meters the effect of the height of the boundary layer is important, especially during stable conditions

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