DYNAMICAL OUTCOMES OF PLANET-PLANET SCATTERING
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ABSTRACT
Observations in the past decade have revealed extrasolar planets with a wide range of orbital semimajor axes and eccentricities. Based on the present understanding of planet formation via core accretion and oligarchic growth, we expect that giant planets often form in closely packed configurations. While the protoplanets are embedded in a protoplanetary gas disk, dissipation can prevent eccentricity growth and suppress instabilities from becoming manifest. However, once the disk dissipates, eccentricities can grow rapidly, leading to close encounters between planets. Strong planet-planet gravitational scattering could produce both high eccentricities and, after tidal circularization, very short period planets, as observed in the exoplanet population. We present new results for this scenario based on extensive dynamical integrations of systems containing three giant planets, both with and without residual gas disks. We assign the initial planetary masses and orbits in a realistic manner following the core accretion model of planet formation. We show that, with realistic initial conditions, planet-planet scattering can reproduce quite well the observed eccentricity distribution. Our results also make testable predictions for the orbital inclinations of short-period giant planets formed via strong planet scattering followed by tidal circularization.

Subject headings: instabilities — methods: numerical — planetary systems — planetary systems: formation — scattering

Online material: color figures

1. INTRODUCTION
The study of extrasolar planets and their properties has become a very exciting area of research over the past decade. Since the detection of the planet 51 Peg b, more than 200 new planets (Butler et al. 2006) have been detected, and the large sky surveys planned for the near future can potentially detect many more. These detections have raised many questions about the formation and dynamical evolution of planetary systems. The extrasolar planet population covers a much greater portion of the semimajor axis and eccentricity plane than was expected based on the planets in our solar system (Lissauer 1995; Fig. 1 in this article). The presence of many giant planets in highly eccentric orbits or in very short period orbits (the “hot Jupiters”) is particularly puzzling.

Different scenarios have been proposed to explain the high eccentricities. The presence of a distant companion in a highly inclined orbit can increase the eccentricities of the planets around a star through Kozai oscillations (Mazeh et al. 1997; Holman et al. 1997). However, this alone cannot explain the observed eccentricity distribution (Takeda & Rasio 2005). Interaction with the protoplanetary gas disk could either excite or damp the eccentricities depending on the properties of the disk and the orbits. However, the combined effects typically result in eccentricity damping (Artyomowicz 1992; Papaloizou & Terquem 2001; Goldreich & Sari 2003; Ogilvie & Lubow 2003). Migration of two planets and trapping in a mean motion resonance (MMR) can also pump up the eccentricities efficiently, but this mechanism requires strong damping at the end or termination of migration right after trapping in resonance (Lee & Peale 2002) or else it leads to planet scattering (Sándor & Kley 2006). Zakamska & Tremaine (2004) proposed inward propagation of eccentricity after the outer planets are excited to high eccentricities following a close encounter with a passing star. Using typical values for such interactions with field stars in the solar neighborhood, however, they do not get very high eccentricities. Papaloizou & Terquem (2001), Terquem & Papaloizou (2002), and Black (1997) propose a very different formation scenario for planets from protostellar collapse in which both hot Jupiters and eccentric planets at higher semimajor axes are formed naturally; this scenario, however, cannot form sub-Jupiter-mass planets.

In this paper we explore another promising way to create high eccentricities: strong gravitational scattering between planets in a multiplanet system undergoing dynamical instability (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997). According to the model of oligarchic growth, planetesimals form in a nearly maximally packed configuration in the protoplanetary disk, followed by gas accretion (Goldreich et al. 2004; Ida & Lin 2004b; Kokubo & Ida 2002). Once the disk dissipates, mutual planetary perturbations (“viscous stirring”) of the planetesimals will lead to eccentricity growth, orbit crossing, and eventually close encounters between the big bodies in the disk (Ford & Chiang 2007; Levison & Morbidelli 2007). While planetary systems with more than two planets cannot be provably stable, they can remain stable for very long timescales depending on their initial separations (Chambers et al. 1996; Marzari & Weidenschilling 2002). A sufficiently massive disk can prevent interacting planets from acquiring large eccentricities and developing crossing orbits. However, once the gas disk is sufficiently dissipated and the planetesimal disk depleted, the eccentricities of the planets can grow to high values, possibly leading to strong planet-planet scattering and a phase of chaotic evolution that dramatically alters the orbital structure of the system (Lin & Ida 1997; Ford & Chiang 2007; Levison & Morbidelli 2007).

The detection of close-in planets with orbital periods as short as ~1 day, the so-called hot Jupiters (and, more recently, hot...
Neptunes and super-Earths), was another major surprise. Giant planets are most likely to form at much larger separations, beyond the ice line of the star where there can be enhanced dust production (Kokubo & Ida 2002; Ida & Lin 2004b). It is widely believed that the giant planets form beyond the ice line and then migrate inward to form the hot Jupiters we observe today. Different stopping mechanisms of inward migration have been proposed to explain the hot Jupiters, but it is unclear why they pile up at just a few solar radii around the star, rather than continue migrating and eventually accrete onto the star.

Strong gravitational scattering between planets in a multi-planet system may provide another way to create these close-in planets (Rasio & Ford 1996). A few of the planets scattered into very highly eccentric orbits could have sufficiently small peri-astron distances that tidal circularization takes place, giving rise to the hot Jupiters. The currently observed edge in the mass-period diagram is very nearly at the ideal circularization radius (Twinstead et al. 2004; see a detailed analysis for the dependence of various statistics on the sample size in Appendix A; see also Adams & Laughlin 2003). Each system has to be integrated for a long time, so that it reaches the orbit-crossing unstable phase and later evolves into a new, stable configuration. Given the rapid increase in computing power, we are now able to perform significantly more and longer integrations to obtain much better statistical results than was possible just a few years ago.

In addition, we also present the results of new simulations for systems of three giant planets still embedded in a residual gas disk. Here our goal is different: we focus on the transition from gas-dominated to gas-free systems, in the hope of better justifying the (gas-free) initial conditions adopted in the first part of our work. However, implementing the physics of planet-disk interactions implies a considerably higher computational cost, preventing us from doing a complete statistical study of outcomes at this point.

2. GAS-FREE SYSTEMS WITH THREE GIANT PLANETS

In this section we consider systems with three unequal-mass giant planets orbiting a central star of mass $1 M_\odot$ at distances of several AU. The planets interact with each other through gravity and physical collisions only. We first present our assumptions and initial conditions, with particular emphasis on realistic mass distributions for the planets, and then we describe our numerical results and their implications.
2.1. Initial Orbits

For all systems the initial semimajor axis of the closest planet is always set at \( a_1 = 3 \) AU. The other two planets are placed using the spacing law introduced by MW02,

\[
a_{i+1} = a_i + KR_{H,i},
\]

where \( R_{H,i} \) is the Hill radius of the \( i \)th planet and we set \( K = 4.4 \) for all runs in this section. These choices are somewhat arbitrary but are guided by the following considerations. For a solar-mass central star, the ice line is around 3 AU (Kokubo & Ida 2002) and it is difficult to form giant planets closer to the star (see, e.g., Kokubo & Ida 2002). Although inward type II migration (see, e.g., Goldreich & Tremaine 1980) can bring the giant planets closer to the star, we avoid putting the planets initially very close to the star since very small initial semimajor axis will lead to predominantly collisional outcomes. Furthermore, we would like to minimize the computing time, which leads us to consider closely spaced systems, while avoiding MMRs.

Since our simulations in this section do not include the effects of gas, they are not intended to model the early phases of planet formation. Instead, at \( t = 0 \), we begin integrating fully formed planetary systems with a disk sufficiently depleted that the planets are free to interact with each other without significant dissipation from the disk. In § 2.4 we show that the time until instability within a particular set of initial conditions does not affect the statistical properties of final outcomes. Indeed, we expect that the chaotic dynamics, both before and after the first close encounter, results in the distribution of final outcomes being independent of the instability timescale. This justifies our choice of a very compact initial configuration with short instability timescale (\( \sim 10^4 \) yr), which minimizes the computational cost. See MW02 and Appendix B for further discussion of the dependence of the instability timescale on \( K \).

Initial eccentricities are drawn from a uniform distribution between 0 and 0.1, and orbital inclinations are drawn from a uniform distribution between \( 0^\circ \) and \( 10^\circ \) (with respect to the initial orbital plane of the innermost planet). To make sure that we could discern any inclination-dependent effects, we used a slightly broader range of inclinations than seen in our solar system. However, in § 2.6 we show numerically that the choice of initial inclinations does not affect the distribution of final inclinations significantly. All initial phase angles are assigned random values between \( 0^\circ \) and \( 360^\circ \).

2.2. Planetary Mass Distributions

Our current understanding of planet formation remains full of uncertainties, and no single prescription can claim to predict a correct planet mass distribution. For this reason, we consider three different prescriptions to construct plausible initial mass distributions for Jupiter-like planets. In all cases we adopt the standard core accretion paradigm and closely follow the simple planet formation model described in Kokubo & Ida (2002). Planet masses depend on the distance of the planet from the central star through the gas surface density profile of the protoplanetary disk.

2.2.1. Mass Distribution 1

In this prescription, we first assign the planetary core masses \( M_{\text{core}} \) assuming a uniform distribution between 1 and 10 \( M_J \). We assume that the cores accrete all gas within 4 Hill radii (Kokubo & Ida 2002) to reach a total mass \( M \) at a semimajor axis \( a \) given by

\[
M = 2\pi a \Delta \Sigma_{\text{gas}} + M_{\text{core}},
\]

where \( \Delta = 8 r_H \) is the feeding zone of the planet core and \( r_H \) is the Hill radius of the planet core, given by

\[
r_H = \left( \frac{1}{3} \frac{M_{\text{core}}}{M_*} \right)^{1/3} a.
\]

Here \( M_* \) is the mass of the central star and \( a \) is the orbital radius of the core (assumed to be on a circular orbit). The gas surface density in the disk is given by

\[
\Sigma_{\text{gas}} = f_g \Sigma_1 \left( \frac{a}{1 \text{ AU}} \right)^{-3/2} \text{ g cm}^{-2},
\]

where the coefficient \( f_g = 240 \) is the assumed gas-to-dust ratio (taken from Kokubo & Ida 2002), \( \Sigma_1 \) is the surface density at 1 AU, and the exponent comes from the minimum-mass disk model. We use \( \Sigma_1 = 10 \) in this case, which is a little higher than the minimum-mass solar nebula value of 7. The choice of \( \Sigma_1 \) is somewhat arbitrary and motivated to produce roughly Jupiter-mass planets. The initial masses of the planets obtained with this procedure are between about 0.4 and 1.2 \( M_J \).

For this mass distribution we performed a set of 1000 independent dynamical runs.

2.2.2. Mass Distribution 2

This is a slight refinement on the previous case in which we adopt an alternative prescription for accretion that takes explicitly into account the growing mass of the initial planetary core. The initial core masses are chosen as in § 2.2.1, but the final mass of each planet is now determined using the following equations. Assuming that an infinitesimal mass \( dm \) accreted by a planet of mass \( m \) decreases the disk density by \( d\Sigma \), we can write

\[
dm = -2\pi m_{\text{H}} r_H d\Sigma,
\]

where \( m_{\text{H}} \) is the number of Hill radii over which the mass is accreted. The final mass \( M \) of a planet at a distance \( a \) from the star, starting with a core mass \( M_{\text{core}} \), can be obtained by integrating equation (5) as follows:

\[
\int_{M_{\text{min}}}^{M} m^{-1/3} dm = -2\pi m_{\text{H}} a^2 \int_{\Sigma_1}^{0} d\Sigma,
\]

where \( \Sigma_i \) is the initial disk surface mass density. Solving equation (6) and replacing \( \Sigma_i \) from equation (4), we find

\[
M = \left[ \frac{4\pi m_{\text{H}} a^2}{M_{\text{core}}^{1/3} \Sigma_1^{3/4}} + m_{\text{e}}^{2/3} \right]^{3/2}.
\]

Here we use \( m_{\text{H}} = 8 \); i.e., we assume that the core accretes all mass within 4 Hill radii on either side. We use the same values for \( f_g \) and \( \Sigma_1 \) from § 2.2.1.

For this mass distribution we have integrated a smaller set of 224 systems.

2.2.3. Mass Distribution 3

We expect that the final distributions of different orbital properties may vary significantly with different initial mass distribution.
To further test this mass dependence, we created a third set of systems with a broader planet mass distribution. Here we assign planetary masses exactly as in § 2.2.1, but the initial core masses are chosen differently. We sample $M_{\text{core}}$ from a distribution of masses between 1 and 100 $M_{\oplus}$ uniform in $M_{\text{core}}^{1/3}$, while assuming again that these cores accrete all gas within 8 Hill radii. The exponent in the core mass distribution and the surface density at 1 AU, $\Sigma_1 = 15$, are chosen somewhat arbitrarily with the goal to obtain an initial mass distribution that peaks around a Jupiter mass but with a tail extending up to several Jupiter masses. The choices above produce initial masses spanning about an order of magnitude, in the range $0.4 - 4 M_{J}$. Moreover, the distribution for higher mass planets resembles the mass distribution of observed exoplanets (see § 2.8).

For this mass distribution we have integrated a set of 500 systems.

2.3. Numerical Integrations

We integrate each system for $10^7$ yr, which is $2 \times 10^6$ times the initial period of the closest planet ($T_{1,i}$) and typically much longer than the timescale for the onset of instability. We use the hybrid integrator of MERCURY 6.2 (Chambers 1999) and integrate the orbits symplectically while there is no close encounter, with a time step of 10 days, but switching to a Bulirsch-Stoer (B-S) integration as soon as two planets have a close approach (defined to be closer than 3 Hill radii). Runs with poor energy conservation ($|\Delta E/E| \geq 0.001$) with the hybrid integrator are repeated using the B-S integrator throughout with the same $|\Delta E/E|$ tolerance. This happens in $\sim 30\%$ of all runs, but our conclusions are not affected even if we reject these systems. We find that, in all systems, at least one planet is eventually ejected. Note that, for three-planet systems, following an ejection the remaining two planets may or may not be dynamically unstable. Therefore, we do not stop the integration following an ejection. Instead, we continue all integrations for two planets until a fixed stopping time of $10^7$ yr. For systems with two remaining planets we check for Hill stability using the known semianalytic criterion (Gladman 1993). In our simulations about $9\%$ of systems were not provably Hill stable at the integration stopping time. We discard those from our analysis. When a single planet remains (following a second ejection or when a collision took place), the integration is of course stopped immediately.

For systems with two remaining dynamically stable planets the orbits can still evolve on a secular timescale (typically $\sim 10^5 - 10^6$ yr for our simulated systems) much larger than the dynamical instability timescale (Adams & Laughlin 2006a; see also Murray & Dermott 2000, p. 274). We study these systems with two remaining stable planets by integrating the secular perturbation equations for a further $10^6$ yr with the analytical formalism developed in Ford et al. (2000). Note that the more standard formulation for solar system dynamics (Murray & Dermott 2000) is not appropriate for these planetary systems because a significant fraction present very high eccentricities and inclinations. We find that for most of our simulated systems our chosen integration stopping time effectively sampled the full parameter space (see detailed discussion in § 2.9).

We treat collisions between planets in the following simple way (“sticky sphere” approximation). A collision is assumed to happen when the distance between two planets becomes less than the sum of their physical radii. We assume Jupiter’s density ($1.33$ g cm$^{-3}$) for all planets when determining the radius from the mass. After a collision the two planets are replaced by a single one conserving mass and linear momentum. Because we account for collisions, our results are not strictly scale-free.

![Fig. 2. Time evolution of semimajor axes and eccentricities for two randomly chosen typical simulations. The solid, dotted, and dashed lines show the orbital elements for the initially closest ($a_1, e_1$), middle ($a_2, e_2$), and farthest ($a_3, e_3$) planets, respectively. The top two panels show a realization where the first planet is ejected at $\sim 4.1 \times 10^4 T_{1,i}$, and the integration concludes with two planets in provably stable orbits. The semimajor axes for both $P_1$ and $P_2$ remain constant, and the eccentricities oscillate stably on a secular timescale. The bottom two panels show another realization where $P_2$ collides with $P_1$ at $\sim 4.2 \times 10^3 T_{1,i}$; $e_2$ keeps increasing until, a little before $10^4 T_{1,i}$, $P_2$ gets ejected, leaving a single planet in the system. Since a single orbit is always stable, we stop the integration following this ejection. Numbers in the subscript represent the positional sequence of the planets starting from the star, and letters “1” and “2” mean initial and final values, respectively, in all plots. [See the electronic edition of the Journal for a color version of this figure.]

However, we find that collisions are relatively rare for our choice of initial conditions, so we still present all results with lengths scaled to $a_{1,i}$ and times scaled to $T_{1,i}$.

Since mass distribution 1 corresponds to our largest set of runs, we first show our results from this set in detail in the following subsections (§§ 2.4–2.7). Results for the other two sets are summarized in § 2.8.

2.4. Overview of Results

In Figure 2 we show a couple of randomly selected, representative examples of the dynamical evolution of these systems, showing both chaotic phases and stable final configurations. Note the order-of-magnitude difference in timescale to first orbit crossing, illustrating the broad range of instability timescales (see also the discussion in Appendix B). We find that strong scattering between planets increases the eccentricities very efficiently (Fig. 3). The median of the eccentricity distribution for the final inner planets is 0.4. The median eccentricity for the final outer planets is 0.37, and that for all simulated planets in their final stable orbits is $\sim 0.38$.

We compare our results with the observed eccentricity distribution of detected exoplanets in Figure 4. For a more meaningful comparison we restrict our attention to observed planets with masses greater than $0.4 M_{J}$, similar to the lower mass cutoff in our simulated systems. We also place an upper limit on the semimajor axis at 10 AU for the simulated final planet population to address the observational selection effects against discovering
planets with large orbital periods. Similarly, since planets close to the central star can be affected by additional physics beyond the scope of this study (e.g., tides, general relativistic effects; see Adams & Laughlin 2006b), we also omit observed close-in planets with semimajor axes below 0.1\(a_{1,i}\).

As seen in Figures 3 and 4, our simulations slightly overestimate the eccentricities of the planetary orbits. However, the slopes of the cumulative eccentricity distributions at higher eccentricity values are similar. In a realistic planetary system, there might be damping effects from lingering gas, dust, or planetsimals in a protoplanetary disk. While our simplified models already come close to matching the eccentricity distribution of observed planets, including damping may further improve this agreement. To be more quantitative, we perform a Kolmogorov-Smirnov (K-S) test and find that we cannot rule out the null hypothesis (that the two populations are drawn from the same distribution) at the 85% level (Table 1). In §2.8 we show that a broader initial distribution of planet masses results in an even better match to the observed eccentricity distribution.

The top and bottom panels in Figure 5 show the cumulative distributions of the initial versus final semimajor axes for the planets. The planet that is closest to the star initially may not remain closest at the end of the dynamical evolution. In fact, all three planets, independent of their initial positions, have roughly equal probability of becoming the innermost planet in the final stable configuration when the planet masses are not very different. In 20% of the final stable systems, we find a single planet around the central star, two planets having been lost from the system through some combination of collisions and dynamical ejection. The other systems have two giant planets remaining in stable orbits. We find that the planets in the outer orbits show a tendency for higher eccentricities correlating with larger semimajor axes (Fig. 6). We now know that many of the current observed exoplanets may have other planets in distant orbits (Wright et al. 2007). From our results we expect that planets scattered into very distant bound orbits will have higher eccentricities. Long-term radial velocity monitoring should be able to test this prediction.

Next, we investigate to what extent the final orbital properties depend on the instability timescale (equivalently, on how closely packed the initial configuration was). For each system we integrated in §2.2.1, we noted the first time when the semimajor axis of any one of the planets in the system changed by at least 10%. We use this as a measure of the dynamical instability growth timescale. Then, we divide the set into two subgroups, based on whether this growth time was below (group 1) or above (group 2) its median value (so 50% of the integrated systems are in each group). Figure 7 compares the final eccentricity and semimajor-axis

![Comparison of Eccentricity Distributions](image)

**Figure 3.** Cumulative distributions showing initial and final eccentricities of the planets. Top and bottom panels show the initial and final cumulative eccentricity distributions, respectively. In the top panel solid, dotted, and dashed lines represent the closest, middle, and farthest planets, respectively. They are on top of each other because the initial eccentricity distribution is the same for all of the planets. In the bottom panel solid and dotted lines represent the final inner and outer planets, respectively. The dashed line shows all remaining planets in final stable orbits. [See the electronic edition of the Journal for a color version of this figure.]

**Figure 4.** Comparison between the simulated and observed exoplanet populations. The solid line shows the cumulative distribution of the eccentricities of the remaining planets in their final stable orbits. The dashed line is that for the observed population. For this comparison we employ a lower mass cutoff of 0.4\(M_J\) on the observed population, addressing the fact that we do not have lower mass planets in our simulations. We also consider only the simulated planets that are finally within 10 AU from the star to address the fact that in the observed population we do not have planets farther out. We also employ a lower semimajor-axis cutoff of 0.1\(a_{1,i}\) on the observed population. [See the electronic edition of the Journal for a color version of this figure.]

**Table 1**

| Mass Distribution | \(D\) | \(P\) |
|-------------------|------|------|
| 1                 | 0.113| 0.15 |
| 2                 | 0.171| 0.01 |
| 3                 | 0.087| 0.32 |

**Notes.** For each mass distribution, we compare the final eccentricity distribution of the simulated population with the observed exoplanet population (Figs. 4 and 18). Using the kstwo function in Numerical Recipes, we calculate the two-sample K-S statistic, \(D\), and the corresponding probability, \(P\). In each case, the high value of \(P\) indicates that we cannot reject the null hypothesis that both samples were drawn from the same population.
distributions between the two groups. We find that the distributions are indistinguishable, demonstrating that the final (observable) orbital properties are not sensitive to when exactly a particular system became dynamically unstable, as long as the dynamics was sufficiently active (ensuring that close encounters occur) and avoiding initial conditions so closely packed that physical collisions would become dominant. This result is hardly surprising since we expect the chaotic evolution to efficiently erase any memory of the initial orbital parameters. Our results can therefore be taken as representative of the dynamical outcome for analogous systems with an even larger initial spacing between planets (but avoiding MMRs; see Appendix B). In practice, performing a large number of numerical integrations for these more widely spaced initial configurations would be prohibitively expensive (see Appendix B).

2.5. Hot Jupiters from Planet-Planet Scattering

We find that a significant fraction of systems emerge with planets in orbits having very small periastron distances. Figure 6 shows the final positions of the planets that are still bound to the central star in the \(a-e\) plane. The solid lines represent different constant pericenter distances. Note that the planets show weak correlations between the eccentricity and the semimajor axis. For the inner planets, a lower semimajor axis tends to imply higher eccentricity, while the outer planets show an opposite trend. The final inner and outer planets form two clearly separated clusters of points in the \(a-e\) plane due to stability considerations.

Figure 8 shows the cumulative distribution of the periastron distances of the final bound planets around the star. For the sake of comparison, we also show the pericenter distribution of all observed exoplanets in Figure 8. We see that 10% of the systems harbor planets with periastron distances \(\leq 0.05a_{1,i}\) and a few \(\sim 2\%\) harbor planets with periastron distances \(\leq 0.01a_{1,i}\). Since we do not include tidal effects, we cannot compare this quantitatively with the observed population. However, this is consistent with the \(\sim 5\%\) of observed planets with semimajor axes within ordinary
However, recall that systems with much smaller values would also lead to more physical collisions than in our simulations. Moreover, a full numerical study of this scenario should include tidal dissipation as part of the dynamical integrations and possibly also include additional physics such as GR effects, etc. (Nagasawa et al. 2008).

2.6. Planets on High-Inclination Orbits

Since the star and planets get their angular momenta from the same source, planetary orbits are generally expected to form in a coplanar disk perpendicular to the stellar spin axis. In Figure 9, we compare the distributions of the final inclination angles. Here each angle reported is the absolute value of the orbital inclination measured with respect to the initial invariable plane, defined as the plane perpendicular to the initial total angular momentum vector of the planetary orbits. Note that the direction of the initial total angular momentum can differ from the direction of the total angular momentum of the bound planets at the end of a simulation, since planets are frequently ejected from the system, carrying away angular momentum.

Strong scattering between planets often increases inclinations of the orbits, leading to a higher final rms value of planet inclinations compared to the initial configuration (Fig. 9, top panel). In general, the inclinations tend to increase for all planets. The middle and bottom panels in Figure 9 show the initial and final inclinations of the orbits of individual planets, respectively. The inclination of the final inner planet is typically larger than that of the final outer planet (Fig. 9, bottom panel).

Our results show that strong planet-planet scattering can dramatically affect the coplanarity of some planetary systems (Fig. 9, bottom panel). Since the timescale for tidal damping of inclinations is usually much greater than the age of the stars (Winn et al. 2005), significantly increased inclinations could be found in some planetary systems that have gone through strong gravitational scattering phases in their lifetimes. Measuring a poor degree of alignment among the planetary orbits in multiple-planet systems, or between the angular momentum of one planet and the spin axis of its host star, could be used to identify systems that have undergone a particularly tumultuous dynamical history.

If a system were initially assigned to a strictly coplanar configuration, then angular momentum conservation dictates that it would remain coplanar always. However, away from this trivial limit, we expect little correlation between the initial and final inclinations, given the chaotic nature of the dynamics. We test this hypothesis here by investigating the correlation between the initial and final inclinations of all planets in our simulations. We find that the final inclination of the inner planet indeed does not depend on the initial rms inclination (Fig. 10). We can quantify the amount of correlation between the initial and final inclinations using the bivariate correlation coefficient. The bivariate correlation coefficient ($r_{xy}$) for two variables $x$ and $y$ is given by the following equation:

$$r_{xy} = \frac{\text{Cov}(x, y)}{\text{sd}(x)\text{sd}(y)}.$$
where Cov(x, y) is the covariance of x and y and sd(x) or sd(y) is the standard deviation of x or y. We find that the correlation coefficient between the initial rms and the final inner planet orbital inclinations is $r_{\text{rms,close}} = 0.05$. The low value of $r$ confirms that the high final inclinations are not merely a reflection of the initial conditions. As long as the planetary system is not strictly coplanar initially, strong planet-planet scattering can increase the orbital inclinations of some systems significantly.

The final inclination of the inner planet, which is the most easily observable, shows a weak anticorrelation with the pericenter distance of its orbit (Fig. 11): lower pericenter orbits tend to have higher inclinations. The correlation coefficient in this case is $r_{\text{rp,close}} = -0.13$ (eq. [8]).

For our solar system, the angle between the spin axis of the Sun and the invariable plane is $\approx 6^\circ$. The angle between the stellar rotation axis and the orbital angular momentum of a transiting planet ($\lambda$) can be constrained via the Rossiter-McLaughlin effect. Observations have measured $\lambda \sin i$ for five systems (Winn et al. 2006): $-4.4^\circ \pm 1.4^\circ$ for HD 209458b (Winn et al. 2005), $-1.4^\circ \pm 1.1^\circ$ for HD 189733b (Winn et al. 2006), $11^\circ \pm 15^\circ$ for HD 149026b (Wolf et al. 2007), $30^\circ \pm 21^\circ$ for TrES-1b (Narita et al. 2007), and, most recently, $62^\circ \pm 25^\circ$ for HD 17156b (Narita et al. 2008). Our study implies that planetary systems with a tumultuous dynamical history will sometimes show a large $\lambda$. Therefore, we look forward to precise measurements of $\lambda$ for many planetary systems to determine the fraction of planets among the exoplanet population with a significant inclination. In particular, HD 17156b is very interesting in this regard, since the potentially high $\lambda$ together with the high eccentricity ($e = 0.67$) strongly indicates a dynamical scattering origin for this planet. Measurements of $\lambda$ would be particularly interesting for the massive short-period planets ($m > M_J$), the very short period giant planets ($P < 2.5$ days), or the eccentric short-period planets, since these planets might have a different formation history than the more common short-period planets with $m \approx 0.5 M_J$ in nearly circular orbits.

The radial velocity planet population currently includes 20 multiplanet systems, and at least 5 of those systems are in MMR (4 appear to be in a 2:1 MMR). MMRs can have strong effects on the dynamical evolution and stability of planetary systems. The 2:1 MMR is particularly interesting given the proximity of the two orbits and the increased possibility for close encounters that could result in strong gravitational scattering between the two planets (Sándor & Kley 2006; Sándor et al. 2007).

It is widely believed that MMRs between two or more planets in a planetary system arise naturally from migration. Convergent migration in a dissipative disk can lead to resonant capture into a stable MMR, particularly the 2:1 MMR (Lee & Peale 2002). Simulations including an empirical dissipative force show that planetary orbits predominantly get trapped in 2:1 MMR (Moorhead & Adams 2005; Nagasawa et al. 2008).

While we regard differential migration as a natural way to trap planets into MMRs, we did explore the possibility of trapping two planets into a 2:1 MMR using only the mutual gravitational perturbations and without any damping. We certainly expect this to be more difficult than with damping. Finding even a few systems trapped in MMR without any dissipation would be both surprising and interesting. In a three-planet system it is possible that one planet acts as a source or sink of energy to let the other two planets dynamically evolve into or out of an MMR. If pure dynamical trapping into MMRs were efficient, then this would open up interesting possibilities. For one, it does not require a common disk origin, as is a requirement for the migratory origin of MMRs. In addition, this mechanism could operate in a planetary system at a much later time after the protoplanetary disk has been dissipated.

To look for possible 2:1 MMR candidates, we isolate systems that have two remaining planets with their final periods close to a
Fig. 12.—Time evolution plots for the two resonance angles $\theta_1$ and $\theta_2$, the semimajor axes, and the eccentricities of the planets. From top to bottom the panels show the time evolutions of $\theta_1, \theta_2$, semimajor axes, and eccentricities, respectively. The time axis is in units of the initial orbital period of the initially closest planet ($T_{1,i}$). For the panels showing semimajor axes and eccentricity, the solid and dotted lines show the evolutions of the two planets that enter a 2:1 MMR. Note that a little before $1.88 \times 10^9 T_{1,i}$, both $\theta_1$ and $\theta_2$ start librating. [See the electronic edition of the Journal for a color version of this figure.]

2:1 ratio. Then we calculate the two resonance angles $\theta_1$ and $\theta_2$ over the full time of their dynamical evolution. Here the two resonance angles are given by

$$\theta_{1,2} = \phi_1 - 2\phi_2 + \omega_{1,2},$$

where $\phi_1$ and $\phi_2$ are the mean longitudes of the inner and outer planets and $\omega_1$ and $\omega_2$ are the longitudes of periastron for the inner and outer planets, respectively. When the planets are not in an MMR, $\theta_{1,2}$ circulate through $2\pi$. When trapped in an MMR, the angles librate around two values (Lee 2004). Finally, we check whether the periodic ratio and libration of the resonant angles are long lived or just a transient stage in their dynamical evolution.

We find one system where two planets are clearly caught into a 2:1 MMR (Fig. 12). The top two panels show the time evolution of the resonant arguments $\theta_1$ and $\theta_2$. The two resonant angles go from the circulating phase to the librating phase at around $1.88 \times 10^9 T_{1,i}$. The two bottom panels show the evolution of the semimajor axes and the eccentricities of the two planets in MMR. Note that the semimajor axes are nearly constant and the eccentricities oscillate stably. Since there is no damping in the system, the somewhat large libration amplitude of the resonant angles is to be expected. In principle, the presence of even a little damping (due to some residual gas or dust in the disk) might reduce the amplitudes of libration and eccentricity oscillations for systems such as this one. A case like the one illustrated in Figure 12 is clearly not a typical outcome of purely dynamical evolution. We found a few other systems ($\sim 1\%$) showing similar librations of $\theta_{1,2}$ at different times during their dynamical evolution, but only for a brief phase never exceeding $\sim 10^8 T_{1,i}$. However, if our simulations had included even some weak dissipation, the frequency of such resonances might have increased significantly. We encourage future investigation of this possibility.

2.8. Mass Dependencies

Our simulations show the effects of mass segregation, as heavier planets preferentially end with smaller semimajor axes. This trend can be easily seen by comparing the initial and final mass distributions of the planets in Figure 13. The mass distribution clearly shifts toward higher values in the final inner planet mass histograms, whereas the outer planet mass more closely reflects the initial mass distribution (compare the top and bottom panels of Fig. 13). We do not find a strong effect of mass on eccentricity, but we note that collisions tend to reduce the fraction of highly eccentric systems (Fig. 14). The collision products can be seen in the cluster around and above $1.5 M_J$. We find no other significant mass-dependent effect in the final orbital parameters for our set of runs using mass distribution 1.

We now describe briefly the results obtained with the two alternative initial mass distributions for the three planets. Figure 15 shows correlation between semimajor axis and mass for both mass distribution 1 ($\S$ 2.2.1) and mass distribution 2 ($\S$ 2.2.2). Somewhat surprisingly, for mass distribution 2, we find no significant differences from the results obtained with the much simpler prescription of mass distribution 1. This is possibly because in both mass distributions 1 and 2, the mass range and distribution are similar (mass distribution 2 is only shifted toward slightly higher values).

For this reason we also studied a third choice of mass distribution, mass distribution 3 ($\S$ 2.2.3), with a much larger range of planetary masses, enabling us to observe mass-dependent effects more clearly. For example, we now see that the tendency for
higher mass planets preferentially to become the final inner planets (Fig. 16) is more prominent than in our other simulations. Similarly, the effect of a mass distribution on the final eccentricities of the remaining planets is more prominent with this broader mass distribution. The higher mass planets preferentially excite the eccentricities of the lower mass counterparts, often to the point of ejection. This effectively reduces the overall eccentricities of the final stable orbits (Fig. 17). The median value of the final inner orbit eccentricities is 0.24, and that for the outer orbit is 0.23 in this case. The final cumulative distribution of eccentricities matches the observations even more closely with mass distribution 3 than with mass distribution 1 or 2 (Fig. 18; Table 1). We employ similar selection criteria as described in § 2.5. The final semimajor-axis distribution is statistically indistinguishable from the one obtained with mass distribution 1. We also clearly see that the lower mass planets get scattered around preferentially while the heavier counterparts do not move much and stay mostly near their initial positions (Fig. 19). This is in accord with the observation that close-in planets are often of lower mass than planets with moderate semimajor axes (Cumming et al. 2008; Naef et al. 2005). At present, the correlation between planet mass and orbital period for radial velocity planets is consistent with a population of systems where the less massive planets have been scattered inward. We predict that planet searches sensitive to longer period planets will eventually find a population of sub-Jupiter planets that have been scattered outward. Furthermore, our simulations predict a negative correlation between mass and orbital period among such long-period planets, if they are launched into their current orbits via strong gravitational scattering. We find that mass and eccentricity have a weak anticorrelation (Fig. 20). We do not find any systems with two planets trapped in 2:1 MMR for this case.

2.9. Secular Evolution

It is known from numerous previous studies that secular perturbations of one planet on another in a multiplanet system can modify the planets’ orbital properties on a timescale much longer...
than the relevant dynamical (orbital, or strong dynamical instability) timescales (Adams & Laughlin 2006a; see also Murray & Dermott 2000). Since secular timescales can be orders of magnitude longer than the orbital timescales, one might obtain results biased toward the initial part of the oscillations if at least a full secular period is not sampled properly. Figure 21 shows a dramatic example where the eccentricities of both planets and the relative inclinations between the planetary orbits oscillate secularly with a very long period ($\gtrsim 100$ Myr) compared to the orbital timescale and the observed eccentricities and inclinations can be very different from what would be expected right after dynamical stabilization of the system. Hence, any study of orbital properties of planets after dynamical interactions should also worry about the secular evolution of the orbital properties that follows the orders of magnitude quicker dynamical phase. Nevertheless, we should point out that in our simulated systems this is not typical. For most cases the secular time period is typically $\sim 10^5 - 10^6$ yr. For our simulated systems containing two provably stable planets at the end of our common integration stopping time...
we study the evolution of the eccentricities for a further 10^9 yr to confirm that the orbital properties at the end of our integration correctly represent the true final distribution.

To evaluate the secular evolution of these planets, we use the octupole-order formalism presented by Ford et al. (2000). Note that the more standard formulation in terms of the Laplace coefficients (Murray & Dermott 2000) is not appropriate for these planetary systems because a significant fraction of these systems contain orbits with very high eccentricities and inclinations.

We find that indeed individual eccentricities of these planetary orbits can change significantly. Figure 22 shows a scatter plot of the final eccentricities after secular evolution for 10^9 yr as a function of the eccentricities after our integration stopping time for both planets. It is clear that the individual eccentricities can change significantly, especially for the inner planet. However, the overall distribution does not change significantly from the distribution obtained right after our integration stopping time in § 2.3. Figure 23 shows that the eccentricity distributions before and after secular evolution for the outer planet, in particular, are statistically identical. For the inner planets we find that, after secular evolution, there is a little overabundance of very high eccentricity (e ≥ 0.8) orbits (Fig. 23). In order to quantify the likeness of the two distributions before and after secular evolution, we perform K-S tests for both the inner and outer planetary orbits. We find that we cannot rule out the null hypothesis (that the distributions before and after secular evolution are drawn from the same distribution) at 62% and 1% significance level for the inner and outer planetary orbits, respectively. The very low values of the significance level along with the large ensemble essentially mean that the two distributions are very similar. We perform the same test with the relative inclination of the planetary orbits in the subset of our systems with two dynamically stable remaining planets (Fig. 24). For these distributions the significance level for the K-S test with the same null hypothesis is 27%. This confirms that our choice of integration stopping time already sampled the full parameter space for the secular evolution.

3. EFFECTS OF A RESIDUAL GAS DISK

In the previous section we considered the dynamical evolution of three-planet systems with fully formed planets on initially
near-circular orbits and no gas disk. Implicit assumptions are that sufficiently massive disks damp planetary eccentricities and that residual gas disks dissipate quickly enough to allow the later chaotic evolution of planetary systems. Here we verify these assumptions by simulating three-planet systems within residual gas disks.

3.1. Photoevaporation

The final stage of disk dissipation remains poorly understood. Since viscous evolution alone cannot explain the observed rapid dispersal of disks (~10^7 yr; see, e.g., Simon & Prato 1995), some other mechanism must be responsible for removing a residual disk. The most likely is photoevaporation (e.g., Shu et al. 1993; Hollenbach et al. 1994). Clarke et al. (2001) proposed that, once the viscous accretion rate drops to a level comparable to the wind mass-loss rate, photoevaporation takes over the disk evolution. When this limit is reached, surface layers of the disk beyond the gravitational radius (\( R_g = GM/c_s^2 \)), where the sound speed \( c_s \) exceeds the disk’s escape speed, start removing disk mass faster than it is being replenished by viscous evolution. As a result, the disk is divided into inner and outer parts: the inner disk drains onto the central star on a short viscous timescale, while the outer disk evaporates on longer timescales (e.g., Clarke et al. 2001; Alexander et al. 2006). Alexander et al. (2006) showed that the disk clearing by this mechanism takes about 10^5 yr, which is comparable to the observed dissipation time.

The viscous evolution time at semimajor axis \( a \) is defined as

\[
t_{\text{vis}}(a) = \frac{M_{\text{disk}}(\leq a)}{\dot{M}_{\text{disk}}(a)},
\]

where \( \nu \) and \( \Sigma \) are the viscosity and surface mass density, respectively.

On the other hand, the photoevaporation time at \( a \) is

\[
t_{\text{photo}}(a) = \frac{M_{\text{disk}}(\leq a)}{\dot{M}_{\text{wind}}(a)},
\]

where the wind mass-loss rate for an optically thick disk is (Clarke et al. 2001)

\[
\dot{M}_{\text{wind}} = 4.4 \times 10^{-10} \left( \frac{\Phi}{10^{41} \text{ s}^{-1}} \right)^{1/2} \left( \frac{M_\odot}{M_d} \right)^{1/2} M_\odot \text{ yr}^{-1},
\]

and for an optically thin disk (Alexander et al. 2006)

\[
\dot{M}_{\text{wind}} = 9.68 \times 10^{-10} \mu \left( \frac{\Phi}{10^{41} \text{ s}^{-1}} \right)^{1/2} \left( \frac{h/a}{0.05} \right)^{-1/2} \times \left( \frac{a_{\text{in}}}{3 \text{ AU}} \right)^{1/2} \left( 1 - \left( \frac{a_{\text{in}}}{a_{\text{out}}} \right)^{0.42} \right) M_\odot \text{ yr}^{-1}.
\]

Here \( \Phi \) is the ionizing flux from the central star, \( h \) is the pressure scale height of the disk, and \( a_{\text{in}} \) and \( a_{\text{out}} \) are the inner and outer disk radii, respectively.

Photoevaporation becomes effective when \( t_{\text{vis}} \geq t_{\text{photo}} \). For typical disks, this corresponds to a disk mass of a few Jupiter masses. When a disk mass drops below this critical value, planets are likely to become dynamically unstable if the photoevaporation time is shorter than the dynamical instability growth time (\( t_{\text{photo}} < t_{\text{dyn}} \)). In this section we investigate this further by simulating three-planet systems with various disk masses.

3.2. Numerical Method and Assumptions

For this study we use a hybrid \( N \)-body and one-dimensional (1D) gas dynamics code to follow the evolution of three-planet systems for several different disk masses. Our hybrid code in its current form combines an existing \( N \)-body integrator with a 1D implementation of a viscous, nearly Keplerian gas disk (Thommes 2005). The \( N \)-body code is based on SyMBA (Duncan et al. 1998). It is fast for near-Keplerian systems, requiring only \( 10^2 \) time steps per shortest orbit, while undergoing no secular growth in energy error. In addition, it makes use of an adaptive time step to resolve close encounters between pairs of bodies.

The gas disk is divided into radial bins, each of which represents an annulus whose properties (surface density, viscosity, temperature, etc.) are azimuthally and vertically averaged, following the general approach of Lin & Papaloizou (1986). Arbitrary viscosities can be specified through a standard \( \alpha \)-parameterization (Shakura & Syunyaev 1973). Although this disk is explicitly 1D, the vertical and azimuthal structures are implicitly included in the model. For the former, a scale height is assigned to every annulus. The latter is key to the planet-disk interactions, which result from the raising of azimuthally asymmetric structure (spiral density waves) in the disk by the planet. This effect is added in the form of the torque density prescription of Goldreich & Tremaine (1980) as modified by Ward (1997), which describes the disk-planet angular momentum exchange taking place as waves are launched. Planetary eccentricities are damped on timescales as in Ward (1993) and Artymowicz (1993). Since we do not take account of the saturation of corotation resonances, which could lead to the eccentricity excitation by Lindblad resonances (Goldreich & Sari 2003; Moorhead & Adams 2008), the eccentricity damping considered here is an upper limit.
3.3. Results: Onset of Dynamical Instability

For initial conditions, we randomly choose 30 three-planet systems from the set using mass distribution 1 in § 2.2.1 and study their orbital evolution within nine different disk masses. The surface mass density profiles of these disks are shown in Figure 25. These are obtained by evolving a minimum-mass solar nebula disk model with a viscosity parameter \( \alpha = 0.005 \) for various times (without planets). Disk properties are summarized in Table 2, and we refer to our models as DISK1–DISK9 from here on. We assume that each of these disks is inviscid for dynamical runs with planets, meaning that type II planet migration is not taken into account. However, this should not affect our results significantly since even the most massive disk (DISK1) contains only 3.7 \( M_J \), which is comparable to the planetary masses used in our simulations. Most of our disks are therefore too small to affect planet migration.

The dynamical instability is commonly characterized by the orbital crossings of planets. Figure 26 shows the first orbital crossing time of each system for each disk mass. Diagonal lines are disk clearing timescales by photoevaporation for optically thick disks (gray lines) and optically thin disks (black lines). Also plotted is the viscous evolution times at the gravitational radius. This figure indicates that photoevaporation takes over disk evolution for disks with a few to several Jupiter masses, depending on the photoevaporation models.

It appears that the range of first orbital crossing time \( t_{\text{dyn}} \) is relatively independent of disk masses and around \( \sim 10^{7} \) yr. Note, however, that the number of systems going through orbital crossings decreases for larger disk masses. Excluding mergers, 19/24, 14/28, and 4/28 systems for DISK1, DISK2, and DISK3, respectively, do not experience any orbital crossings during the simulation time (\( 10^{7} \) yr), while all systems with lighter disk masses go through at least one crossing within the run time. This is concordant to expectations that planets become dynamically unstable more readily in a less massive gas disk and the same planetary system that remained stable in a sufficiently massive disk can become unstable once the disk dissipates.

Apart from \( t_{\text{dyn}} \), the eccentricity damping timescale \( (t_{\text{damp}}) \) is very important to know, since \( t_{\text{damp}} \) determines whether the disk can damp the eccentricities back to near zero after one (or more) orbit-crossing episode(s), before the disk is depleted. We define \( t_{\text{damp}} \) as the time taken to damp the planetary eccentricities from \( e > 0.1 \) to \( e < 0.1 \). For 30 different systems for four disk masses...
(DISK1–DISK4) we find the median $t_{\text{damp}}$ to be $2 \times 10^5, 4 \times 10^5, 4 \times 10^6$, and $7 \times 10^6$ yr, respectively. For less massive disks we do not find significant damping. While the evolution of disks is dominated by viscous evolution and $t_{\text{damp}} < t_{\text{vis}}$ (DISK1 and DISK2), planetary orbits are expected to remain nearly circular since after an instability there is enough time to damp the eccentricities before the disk is depleted. The nature of evolution can change drastically once photoevaporation dominates the disk evolution and starts depleting the disk more rapidly. We find that most systems reach at least one orbit-crossing episode for the least massive disks (DISK5 and DISK9) since $t_{\text{dyn}} > t_{\text{photo}}$. Some planetary systems in more massive disks have $t_{\text{dyn}} < t_{\text{photo}}$ (Fig. 26). For these more massive disks eccentricities excited via planet–planet interaction may be damped if $t_{\text{damp}} < t_{\text{photo}}$. However, since the median $t_{\text{damp}}$ is longer than $t_{\text{photo}}$ for these disks, planetary eccentricities excited via planet–planet interaction do not have time to be damped before the gas disk is depleted, once photoevaporation is efficient.

In summary, we expect that planetary systems will remain stable with nearly circular orbits while the planets are embedded in a sufficiently massive disk. Even if there is an occasional orbital crossing or merger, the eccentricities and inclinations will rapidly damp in such a disk, so that the system returns to nearly circular orbits. Then eccentricities will evolve more freely once photoevaporation takes over the disk evolution, and the disk clearing time becomes short compared to the instability growth time ($t_{\text{dyn}} > t_{\text{photo}}$). Even when planets become unstable before the disk is completely depleted ($t_{\text{dyn}} < t_{\text{photo}}$), it is unlikely that their eccentricities are damped, since the eccentricity damping times of these disks tend to be longer than the disk dissipation time ($t_{\text{damp}} > t_{\text{photo}}$). Therefore, we expect that most planetary systems become dynamically unstable when a gas disk dissipates. This further justifies our initial conditions in § 2. In a future paper we will further investigate the evolution of multiple-planet systems within an evolving gas disk (S. Matsumura et al. 2008, in preparation).

4. COMPARISON WITH PREVIOUS STUDIES

The previous work most similar to ours was the pioneering study by MW02 on (gas-free) three-planet systems. MW02 also studied the orbital properties of planetary systems following a dynamically active phase of their evolution. However, their study was computationally limited and their systems were rather idealized in terms of assumed planetary masses and initial orbits. Our results are in good qualitative agreement with those of MW02. For example, they showed for the first time with three-planet systems how scattering can produce large eccentricities. However, our more realistic and generalized initial conditions enable us to explore a larger parameter space and to study in more detail the most interesting phenomena such as the generation of large, potentially observable inclinations. We also find that the final stable planets can be scattered at even smaller semimajor axes than they predicted. Since these very low semimajor-axis planets are in the tail of the distribution, it is expected that a simulation of a smaller sample size will miss some of them (see Appendix A). Moreover, our much larger simulated sets and improved statistics on dynamical outcomes allow us to better compare our theoretical predictions to observations (see Appendix A).

In addition to the orbital properties of remaining planets, MW02 also presented a stability timescale analysis for planetary systems with three giant planets. In verifying these results, we realized the importance of this study, especially for our choice of initial spacing, and we therefore decided to perform a much more detailed timescale analysis, with significant improvements over MW02 made possible by the dramatically increased speed of present-day computers. The results of this analysis are presented in Appendix B.

Moorhead & Adams (2005) studied in detail two-planet systems with an empirical dissipation arising from a residual disk. They found that, even in initially well-separated two-planet systems, migration can bring the planets close enough for dynamical instability. In their study they accounted for a disk outside both planets with their empirical formula, whereas we immerse the three planets in a protoplanetary disk with varying disk masses (§ 3). Another major difference between their study and ours is the number of planets considered. The dynamical evolution of two-planet systems can be very different from that of systems with three or more planets (see § 1). Keeping these differences in mind, we compare key points between the two studies. For example, for sufficiently massive disks we find that the eccentricity damping timescale is less than the disk dissipation timescale. However, as the disk mass is diminished, the timescale for eccentricity damping and the number of unstable systems increases. They also find that scattering fills up the a-e plane for the inner planet orbit. Due to the setup of their initial conditions and the dynamical limitations of two-planet systems, they do not find planets with large orbital periods, normally produced by strong scattering between planets. They also stop integrating after 1 Myr or when the system has only one planet left. One should remember that in cases where two planets are remaining, the planetary properties can still change either through dynamical scattering (see discussion in § 2.3) or even for dynamically stable systems, through long-term secular perturbations (for a detailed discussion see § 2.9).

More recently Juric & Tremaine (2008) perform an interesting study as an extension of the pioneering work by Lin & Ida (1997). They study the dynamical evolution of generic $N$-planet systems with $N \geq 3$ and a wide range of initial conditions. Although their three-planet systems were dynamically inactive as a result of their choice of initial separations and integration stopping time, their other runs with higher $N$ bear very relevant results for our study. In particular, they find a similar final eccentricity distribution, suggesting that this distribution may be universal. One of the most interesting results in their study is that the final number of surviving planets following a dynamically active phase is almost always $2 \pm 3$, independent of the initial number. Since these systems are chaotic, the properties of any planetary system emerging out of a dynamically active phase will have little memory of the initial number of planets or the exact initial conditions (also see discussion in §§ 2.4 and 2.6). One can imagine a situation where a system started with $N > 3$ and, followed by many collisions and ejections, reaches a stage with $N = 3$. If dissipation circularizes orbits after each ejection or collision, then such a system could reach a state similar to the initial conditions for our three-planet simulations. Thus, our results may be representative of even more generic multiplanet systems.

Nagasawa et al. (2008) study the dynamics of three equal-mass planets including dynamical tides. Since they can apply tides while the three-planet dynamical scattering phase is still active, they find increased efficiency to tidally isolate planets that would otherwise still actively take part in three-planet scattering. In this study we did not include tides. However, we find that 8% of these systems have one planet accreted onto the star. We also find that ~40% of these systems contained at least one planet that, during the three-planet violent scattering phase, reached a pericenter distance within 0.01 AU of the star. In most of the
systems these close-in planets end up being ejected, while some of them collide with the star or another planet. None of these systems remain stable at the end of the run. The addition of tides during these scattering phases may stabilize some systems by isolating the close-in planet from the other planets dynamically. Of course, the circularization process needs to be very efficient and quick so that the planet gets circularized and decoupled from the others before it can be ejected. We should also point out that Nagasawa et al. (2008) study equal-mass planets. We find that the dynamical evolution of planetary systems with unequal masses is very different than for equal-mass systems (see discussion in §§ 2.2 and 2.8). In particular, the lower mass planets preferentially get scattered inward or outward while the heavier counterparts remain near their initial positions throughout the whole evolution (see, e.g., Fig. 15). One should also remember that the tidal circularization timescale depends on the mass and radius of these planets (e.g., Ivanov & Papaloizou 2004). This can affect the efficiency of tidal circularization for high-eccentricity planets in their setup. At present their study actually produces too many (30%) “hot” planets compared to the current observed population of ~5% within 0.03 AU (at 0.03 AU the tidal circularization timescale is ~10^8 yr for Jovian planets; see Nagasawa et al. 2008). Note that the selection biases of radial velocity surveys can only reduce the fraction of hot Jovian planets in the future. It will be interesting to see results of similar studies with a more realistic mass distribution.

5. SUMMARY AND CONCLUSIONS

We have studied in detail how the orbital properties evolve through strong gravitational scattering between multiple giant planets in a planetary system containing three giant planets around a solar-mass star. We perform a detailed study for gas-free generic planetary systems. We focus on the final orbital properties of the planets that remain bound to the central star in stable orbits after chaotic evolution due to strong mutual interactions, followed by a prolonged secular evolution (~10^9 yr) when two planets remain after the scattering phase. We perform the experiments with realistic planetary systems containing three giant planets (§ 2). In all of these systems at least one planet is eventually ejected before reaching a stable configuration. This supports models of planet formation that predict that planetary systems initially form several closely spaced planets, but instabilities reduce the number of planets until the stability timescale exceeds the age of the planetary system. In 20% of the cases, two planets are lost through ejections or collisions, leaving the system with only one giant planet. Thus, the planet scattering model predicts the existence of many systems with a single eccentric giant planet, as well as many free-floating planets (depending on how many planets are formed before the planet scattering phase of evolution).

We find that strong gravitational scattering between giant planets can naturally create high-eccentricity orbits. The exact distribution of eccentricities for the final remaining planets in stable orbits depends on the choice and range of the initial mass distribution. When the initial mass distribution spans a broad range of masses, the less massive planets typically start to acquire larger eccentricities. However, these planets with highly excited orbits are often ejected, reducing the overall eccentricities of the remaining dynamically stable planetary orbits. Although the first two sets of our models (with a narrower range of initial planet masses) predict eccentric planets to be slightly more common than observed (Fig. 4), a wider initial mass distribution can result in remarkable similarity with the observed distribution (Fig. 18). Recently, a similar trend in eccentricities was found independently by Juric & Tremaine (2008) for generic dynamically active multiplanet systems independent of the details of the initial conditions or the initial number of planets.

We conclude that planet-planet scattering could easily account for the observed distribution of eccentricities exceeding 0.2. However, our simulations slightly underestimate systems with eccentricities less than 0.2. This may suggest that some observed systems are affected by late-stage giant collisions. Alternatively, the presence of a residual gas or planetesimal disk could lead to eccentricity damping. We find the latter explanation particularly attractive given the observed correlation between planet mass and eccentricity (Butler et al. 2006). While our simulations suggest that high eccentricities are most common among less massive giant planets, the known population of extrasolar planets suggest that high eccentricities are more common among the more massive planets (Ford & Rasio 2008). This apparent discrepancy could be resolved if a modest disk often remains after the final major planet-planet scattering event. Less massive planets would be more strongly affected by the remaining disk, so their eccentricities could be damped, while more massive planets would typically be immune to eccentricity damping.

We find that it is possible to scatter some planets into orbits with low perihelion distances (Fig. 8). Approximately 10% of the systems obtain perihelion distances less than 0.05a_1, whereas a smaller fraction (~2%) can reach within 0.01a_1. If the initial semimajor axes are small enough, then strong gravitational scattering could result in planet orbits with sufficiently small perihelion distances, such that tidal effects could circularize their orbits at small orbital distances.

We find that the inclination distribution of such planets could be significantly broadened. If we assume that the angular momentum of the host star is aligned with that of the initial orbital angular momentum of the planets, then measurements of \( \lambda \) (the angle between stellar spin axis and planet’s orbital angular momentum) should typically be small in the absence of perturbations from other planetary or stellar companions (§ 2.6). We find that strong gravitational scattering between the giant planets can naturally increase the inclinations of the final planetary orbits with respect to the initial total orbital angular momentum plane (Fig. 9). Since the timescale to tidally align the stellar spin and the planetary angular momentum is much greater than the age of the star (~10^{12} yr; Greenberg 1974; Hut 1980; Winn et al. 2005), inclinations excited by planet-planet scattering after the disk had dispersed could be maintained for the entire stellar lifetime. Observations of a hot Jupiter with a significantly nonzero \( \lambda \) would be suggestive of previous planet-planet scattering. However, caution would be necessary if the star had a binary stellar companion (Wu & Murray 2003; Takeda & Rasio 2005; Fabrycky & Tremaine 2007). On the other hand, observations of many hot Jupiters with orbital angular momenta closely aligned with their stellar rotation axis would suggest a formation mechanism other than strong gravitational scattering followed by tidal circularization. Unfortunately, current observations measure this angle for only a few systems, and some measurements have uncertainties comparable to the dispersion of inclinations found in our simulations. We encourage observers to improve both the number and precision of Rossiter-McLaughlin observations.

We find that the relative inclinations between planetary orbits in the systems with two remaining planets in their final stable orbits also increase via planet-planet scattering. Future observations using astrometry or transit timing could possibly measure relative inclinations between planetary orbits in multiplanet systems. Furthermore, we find that in ~20% of the systems having two giant planets in their final dynamically stable configurations, the relative inclination between the two planets is higher.
than 40°. For these systems it is possible for the planets to go through Kozai-type oscillations (Nagasawa et al. 2008). Although effects of a debris disk on planetary dynamics and vice versa are beyond the scope of this study, the warped disk observed in β Pictoris could be one interesting example where inclined planetary orbits and the debris disk exchange torques, resulting in a warped debris disk (Smith & Terrile 1984; Heap et al. 2000). Mouillet et al. (1997) suggest that the observed asymmetry in the debris disk can be explained by the presence of a planetary companion in an inclined orbit. Strong planetary scattering, as we find, can be a natural way to create planetary orbits with large semimajor axes and highly inclined orbits.

Less massive planets are more likely to be scattered far away from the site of their formation. Our simulations show that both the close-in and farther out planets should have lower mass than the planets with moderate semimajor axes (Figs. 15 and 19). This trend can be verified in future observations using adaptive optics to detect and image giant planets farther out (40–100 AU) from the central star (Lafrenière et al. 2007). We find that a few percent of the simulated population has very high semimajor axes in the final stable configuration (e.g., Fig. 6). Such giant planets are extremely unlikely to be created in situ, since the timescale for planet formation greatly exceeds the age of the star (Veras & Armitage 2004). In addition, there is simply insufficient disk mass to form a giant planet at such large orbital distances (Kokubo & Ida 2002; Ida & Lin 2004a, 2004b). Strong scattering between planets in multiplanet systems can be a natural mechanism to create such long-period planets (a > 50 AU). Our simulations suggest that this population of high semimajor-axis planets will have high eccentricities and inclinations (Fig. 6). Future planet searches using astrometry or direct detection can test these predictions.

We have also presented a preliminary study of the effects of a residual gas disk on planetary dynamics (§3). We compare the importance of dynamics for nine different disk models with different disk surface densities, keeping the initial orbital properties of the embedded planets the same for all cases. We identify important timescales for the dynamical evolution of these systems. In particular, we characterize the transitional stage of the dynamical evolution from the stable, eccentricity-damped phase, where the planets are embedded in a massive disk, to the unstable free eccentricity evolution stage following disk depletion. We show that it is possible to understand the overall evolution after planets are fully formed as an interplay between four different timescales, namely, the viscous timescale ($t_{vis}$) of the disk, the photoevaporation timescale ($t_{photo}$) of the disk, the dynamical instability timescale ($t_{inst}$) for the planetary orbits, and the eccentricity damping timescale ($t_{damp}$) for the planetary orbits in a disk (§3.3). Our study clearly shows that planets will remain stable on nearly circular orbits while a sufficient amount of gas remains present in the disk, while with the same initial orbits without gas the system would become unstable. The boundary between these two different phases can be characterized by $t_{damp}$, $t_{vis}$, and $t_{photo}$. The unstable phase starts when gas mass is sufficiently depleted so that $t_{damp} > t_{photo/vis}$. We find that the transition within a disk from a small to a large $t_{damp}$ can be fairly quick once $t_{photo} < t_{vis}$ (Fig. 26). After photoevaporation takes over the disk evolution, the system undergoes a quick transition. Until the critical mass for photoevaporation is reached, planetary eccentricities remain close to zero independent of the disk mass and previous dynamical history. Then, once the critical density is reached, the system behaves as if it had started with near-circular initial planetary orbits in a gas-free environment. Thus, apart from highlighting the relative importance of these timescales for the evolution of planetary systems, our results justify typical initial conditions used in most studies of gas-free multiplanet systems, including our own (§2). The initial properties for a gas-free system would be the orbital properties of the system as found at the boundary where $t_{damp} > t_{photo/vis}$ in this context.

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**APPENDIX A**

**SAMPLE SIZE NEEDED TO ACCURATELY CHARACTERIZE THE ECCENTRICITY DISTRIBUTION**

Due to the chaotic nature of $N$-body integrations and finite precision of computer arithmetic, the long-term integration of an exact system is impossible. Instead, we must perform ensembles of $N$-body integrations and interpret the results in terms of the statistical properties of the outcomes of a set of similar $N$-body systems. Given that any study is based on a finite sample size, it is important to recognize the limitations on the precision of various statistics due to the limited number of integrations. In the context of this paper (and similar works) many simulations of similar planetary systems are performed to generate a random sample of outcomes, each of which can be compared to the observable properties, such as the eccentricity and the semimajor axis of extrasolar planetary systems. Here we present an analysis of precision of several statistics describing the eccentricity distribution as a function of the number of $N$-body integrations performed. Formally, our estimated precision is applicable only to our specific choice for the distribution of initial conditions. Since this and previous studies have shown that many variations of the planet-planet scattering model result in similar eccentricity distributions, we expect that our results can be applied to many similar studies to estimate the accuracy of various statistical properties. Our results should also give a quantitative way to decide the required number of simulations to estimate various statistical properties to a given precision, and we expect that this will be of use to other researchers when formulating research plans. Since the eccentricities of the planetary systems are one of the most interesting properties, we focus our attention on statistics describing the distribution of final eccentricities. However, the basic idea can be applied to any statistic describing the masses or orbital properties of the simulated planetary systems.

We estimate the precision of several interesting statistics (the mean, the standard deviation, and the 5th [P5] and the 95th [P95] percentiles of the distribution of final eccentricities). We estimate the “true” value for the underlying population based on estimates obtained making use of our full sample of $N = 1515$ simulations (see §2). We then estimate the same statistics based on $m$ subsets of the full sample, where each subset is a random sample of $n$ systems. Next, we compare the statistic estimated from each subset of
moments of the eccentricity distribution can come within 10% of the population moments based on only sample size is required to characterize the tails of the eccentricity distribution accurately. For example, the errors in estimating the error in the estimation of the standard deviation of the eccentricity distribution is.

In Figure 27 we show each estimate for a given statistic as a single tick mark. Each panel presents results for a different statistic: mean (top left), standard deviation (bottom left), P95 (top right), and P5 (bottom right) of the eccentricity distribution. For each of the above statistics, we show the mean plus and minus the standard deviation (short-dashed lines) and the 5th and 95th percentiles (long-dashed lines). Hence, the upper long-dashed line of the top left panel shows the P95 for the estimate of the mean eccentricity based on a sample of \( m = N/n \) estimates of the mean eccentricity each using a sample size of \( n \). We find that for \( n = 50 \) the standard error in estimating the population mean from the sample can have a standard error of 37%, whereas for \( n = 100 \) the precision improves to 5% (Fig. 27). In estimating the standard deviation of the eccentricity from the small subsamples, we find that with \( n = 100 \) the standard error in the estimation of the standard deviation of the eccentricity distribution is \( \sim 7\% \) (Fig. 27). Although estimates for the first two moments of the eccentricity distribution can come within 10% of the population moments based on only \( n \geq 100 \), a much larger sample size is required to characterize the tails of the eccentricity distribution accurately. For example, the errors in estimating the P5 and the P95 of the underlying eccentricity distribution using sample sizes of \( n = 100 \) are 30% and 7%, respectively. If the sample size is increased to \( n \sim 1000 \), then P5 and P95 can be estimated within 4% and 1% error, respectively.

Both previous and future studies often generate predictions for the eccentricity distribution of planetary systems based on various theoretical models. For the sake of comparing the precision with which these studies estimated the predicted eccentricity distribution, we have used the above result to obtain an empirical relation between the standard deviation of the estimates of various summary statistics describing the eccentricity distribution and the number of simulations used to estimate the statistic (Fig. 28). We expect that this relation will also be useful for planning future studies, where researchers will want to make a deliberate choice regarding the number of simulations and other simulation parameters such as length of integration time, number of particles, and inclusion of additional physics. We find that the standard deviation in the deviation of the estimated mean eccentricity from the population mean eccentricity decreases as a power law of sample size \( n \) with an index of \( -1.586 \pm 0.004 \). The standard deviation estimating P5 and P95 decreases less steeply with \( n \); here the power-law indices are \( -0.58 \pm 0.05 \) and \( 0.51 \pm 0.03 \), respectively. (These empirical relations are valid only for \( n \geq 5 \).) For example, a study that uses 100 simulations would typically estimate the mean of the predicted eccentricity distribution to within \( \pm 0.008 \). However, a larger number of simulations becomes increasingly important for estimating the tails of the eccentricity distribution precisely. For example, an ensemble of 100 simulations typically estimates the P5 or the P95 with a precision of \( \pm 0.025 \) or \( \pm 0.045 \), respectively.

**APPENDIX B**

**STABILITY TIMESCALE**

According to the core accretion model of planet formation, planets form in a protoplanetary disk separated by a small number of Hill radii away from each other (Kokubo & Ida 1998, 2002). Hence, it is very interesting to have a good and statistically reliable investigation of the stability timescales, as well as the distributions of the timescales as a function of the planet-planet distances in
multiples \((K)\) of their mutual Hill radii. A similar timescale study was also performed by Chambers et al. (1996). However, their study covers a very different range of planetary masses. The large ensembles used by our study not only produce a better statistical characterization of these timescales as a function of \(K\), but they also enable us for the first time to show the actual nontrivial shapes of these distributions. The actual distributions of these timescales for a given \(K\)-value are particularly interesting for anyone performing a similar study and trying to decide on a reasonable initial planetary separation, since, due to the broad range, the computational effort needed will be determined by the few unusually stable realizations rather than the more frequent ones where instability can grow orders of magnitude quicker. For better comparison with MW02 we put the planet closest to the star at 5 AU and then determine the semimajor axes of the other two planets as follows:

\[
a_{i+1} = a_i + KR_{Hi,i+1},
\]

where \(K\) is the spacing measured in terms of \(R_{Hi,i+1}\), the mutual Hill radius for the \(i\)th and \((i+1)\)th planets,

\[
R_{Hi,i+1} = \left(\frac{M_i + M_{i+1}}{3M_\odot}\right)^{1/3} \frac{a_i + a_{i+1}}{2},
\]

following their prescription. Here \(M_i\) is the mass of the \(i\)th planet, \(M_\odot\) is the mass of the central star, and \(a_i\) is the semimajor axis of the \(i\)th planet. Note that we use a different definition of Hill radius from that in § 2.2, following MW02 for easier comparison.

We integrate a number of three-planet systems with different initial conditions: 1000 for \(K \leq 4.3\), 500 for \(4.3 < K \leq 5.0\), and 200 for \(K \geq 5.0\). Apart from the large number of realizations, we use a more general distribution of the initial eccentricities and orbital inclinations as described in § 2.2.

Figure 29 shows the results as a function of \(K\). The filled circles show the median, and the vertical bars above and below represent \(\pm 34\%\) around the median. Note that the vertical bars are not error bars, but they are representative of the actual distributions of the
We also show the mean of each distribution to compare it with the median. In each case the mean overestimates the timescale and lies often outside the 34\% bars around the median. Our results are consistent with the findings of MW02 qualitatively. We see similar trends near an MMR. However, we find that a simple linear fit as was tried by MW02 does not work well. A better empirical fit is given as follows:

\[
\log_{10} t_m(K) = a + b \exp(cK), \tag{B3}
\]

where \(a, b,\) and \(c\) are constants (henceforth called fit 1). The best-fit values for \(a, b,\) and \(c\) (Table 3) can predict the median timescales with fractional error less than 10\% away from MMR. We also tried to find a simpler linear fit \(t_{m,\text{linear}}(K)\) for our data away from MMR, following MW02, writing

\[
\log_{10} t_{m,\text{linear}}(K) = a + bK, \tag{B4}
\]

where \(a\) and \(b\) are fitting parameters (henceforth called fit 2). The best-fit values for \(a\) and \(b\) are also given in Table 3. We find that fit 1 is much better than fit 2. In particular, we find that the linear fitting formula for the instability timescale as a function of initial spacing (as suggested by MW02) is inaccurate by over 3 orders of magnitude for initial spacings such that the planets are beyond the 2:1 MMR.

We also present the first study of the actual shapes of the timescale distributions. In particular, in cases of broad or skewed distributions, knowing only the median (or mean) timescale cannot provide a complete description of the distribution of timescales to instability. We find that the shapes of the distributions of the timescales are essentially the same for any \(K\)-value away from a major MMR, whereas near a major MMR the shape is qualitatively different with a much slower decay above the median timescale (Figs. 30 and 31). Both systems near and away from MMR show a similar exponential part in the stability timescale distribution. However, due to the MMR configuration, some of the systems enjoy increased stability manifested as a broader distribution to the higher time end. Note that the histograms of the timescale distributions are normalized such that \(\sum n_i \Delta t_i = 1\), where \(\Delta t_i\) is the bin

![Image](image.jpg)

**TABLE 3**

| Fit | \(a\) | \(b\) | \(c\) | Max Error (%) |
|-----|------|------|------|--------------|
| 1   | 1.07 | 0.03 | 1.10 | 10           |
| 2   | -1.74| 1.29 | ...  | 50           |

*Note.*—The best-fit values of the fitting parameters for the empirical fits for the median stability timescale of the systems as a function of their initial spacing parameter \(K\).*
size in logarithm of time. The normalized number distribution for times lower than the median timescale (henceforth denoted as \(n_L\)) has an exponential shape; above the median timescale (henceforth denoted as \(n_R\)) the number distribution has a linear decay for all \(K\) away from major MMRs. The fitting formulae for \(n_L\) and \(n_R\) are given by

\[
\begin{align*}
\text{(B5)} & & n_L & = N_L \exp\left\{ \frac{\log_{10} t - \log_{10} t_{\text{m}}(K)}{t_L} \right\}, \\
\text{(B6)} & & n_R & = N_R - t_R \log_{10} t.
\end{align*}
\]

Fig. 30.—Histograms for the timescale distributions at two different \(K\)-values both away from MMR. Note that times are shown in log scale. Each histogram shown here corresponds to \(10^3\) runs for that \(K\)-value. The number distributions are normalized such that \(\sum_i n_i \Delta t_i = 1\), where \(\Delta t_i\) is the bin size in logarithm of time. This normalization essentially makes the area under each histogram normalized to 1. The solid histogram corresponds to \(K = 3.9\), and the dashed histogram corresponds to \(K = 2.3\). Both these \(K\)-values are away from any major MMR. The two histograms have essentially the same shape. The dotted lines show the analytical fitting curves for timescale distributions at the left and the right sides of the mode of the distributions. For systems with stability, timescales less than the median of the distribution show an exponential shape, whereas those with timescales higher than the median show a linear drop-off (see eqs. [B5] and [B6]; Table 4). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 31.—Histograms for the timescale distributions near and away from an MMR. Each histogram corresponds to \(10^3\) runs for that \(K\)-value. We follow the same normalization scheme as mentioned earlier. \(K = 3.3\) is near the \(K\)-value for a 3:2 commensurability between the periods of the first and the second as well as the second and the third planetary orbits (dashed line). \(K = 3.9\) is away from MMR (solid line). The distributions near and away from MMR have somewhat similar shapes for times lower than the medians of the distributions. However, for times higher than the medians the decay is not as sharp near an MMR as for systems far from an MMR.
Here $N_l$ and $N_R$ are the normalization constants for the peak amplitudes of the distributions, $t_m(K)$ is the median of the timescale distribution as a function of $K$, and $t_L$ and $t_R$ are fitting constants characterizing the exponential index and the slope of the two curves, respectively. The best-fit values for $N_l$, $N_R$, $t_L$, and $t_R$ are listed in Table 4. For a given $K$-value, the median timescale can be estimated using equation (B3), and then using the median timescale, the shapes of the distributions can be obtained using equations (B5) and (B6).

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