Creation of wormholes during the cosmological bounce

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Abstract

In this work we demonstrate that wormholes can in principle be naturally created during the cosmological bounce without the need for the exotic matter or any kind of additional modifications of the gravitational sector, apart from the one enabling the cosmological bounce. This result is general and does not depend on the details of the modifications of gravitational equations needed to support the bounce. To study the possible existence of wormholes around the cosmological bounce we introduce general modifications of Einstein’s field equations need to support the bouncing solutions. In this regime we show that it is possible to construct a cosmological wormhole solution supported by matter, radiation and vacuum energy, satisfying the Weak Energy Condition (WEC), which asymptotically approaches the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. However, at a specific cosmological time, which depends on the parameters of the bouncing cosmological model, the WEC describing the matter needed to support such wormholes is spontaneously violated. This means that such wormholes could potentially exist in large numbers during some period around the bounce, significantly changing the causal structure of space-time, and then vanish afterwards.

1 Introduction

Our current understanding of the cosmological evolution is based on Einstein’s general theory of relativity, which is one of the most successful physical theories.
General theory of relativity was so far verified by various types of experiments – from the light deflection and the perihelion advance of Mercury to the recent detection of gravitational waves [1, 2, 3, 4]. One of the consequences of this theory is the necessary existence of singularities if the space-time is respecting some usual causal properties and if the matter is respecting the usual energy conditions (there are different variants of this result including the Strong, Null or Weak Energy Condition), as proven by the singularity theorems of Hawking [5, 6, 7]. One consequence of this result is the necessary existence of singularity in the evolution of our Universe, called the big bang singularity, if general theory of relativity is correct. However, the physical relevance of this result is highly questionable – since it is precisely in such strong gravity regimes that we should doubt the validity of Einstein’s general relativity as the correct description of gravity. First of all, for strong gravitational fields both quantum behaviour of matter fields and space-time itself will probably become important and significantly change the field equations for gravity. The proper understanding of such regimes therefore requires a proper knowledge of quantum theory of gravity, which is, of course, still currently not available. On the other hand, assuming the actual physical existence of singularities would mean the capitulation with the respect to the fundamental goal of physics – namely, the complete, non-divergent and consistent description of reality, including the evolution of the Universe. For all this reasons, we should view the Hawking singularity theorems more as a signal of incompleteness of Einstein’s general theory of relativity, than the proof for the actual physical existence of singularities. Furthermore, we believe that the demand for singularity-free solutions constitutes one of the most important criteria for the future quantum theory of gravity, and therefore also for the effective theories which are being investigated in order to overcome the current gap between the quantum physics and description of gravity as a geometry of space-time.

Various investigations in the past decades have demonstrated that even simple modifications of gravitational Lagrangian with respect to the standard Einstein-Hilbert action, while leaving all other physical assumptions of Einstein’s general relativity intact, can prevent the appearance of the big-bang singularity [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Also, in higher curvature gravity theories some important non-singular investigations have been done in Gauss-Bonnet higher curvature gravity [22, 23, 24] and in \( f(R) \) gravity theories [25, 26, 27]. In such models the big bang singularity is then replaced by the cosmological bounce, in which the Universe undergoes a transition from contraction to expansion. It is worth remembering that there are in principle no real physical reasons for favoring Einstein-Hilbert action in comparison to higher curvature modifications, such as \( f(R) \), \( f(T) \) or higher derivative gravity theories, as long as they lead to the same observable weak field limit and have no theoretical pathologies. As a matter of fact, the Einstein-Hilbert action was historically introduced as the simplest action leading to the Newtonian limit, out of the infinitely many other equally possible options. The investigation of possible modifications of gravitational action, and their consequences on the
existence of the big bang singularity, are therefore important not just for trying
to overcome the limitations of general relativity - such as the existence of singu-
larities, but also as the path for better understanding the structure of potential quantum theory of gravity.

There were also numerous works which demonstrated the possibility of the cos-
mological bounce, if some new hypothetical additions to the standard cosmo-
logical model are added, such as specific scalar fields, extra dimensions or new
types of fluids [28, 29, 30, 31, 32, 33, 34, 35]. Although such investigations
can lead to some important insights regarding the problem of initial singularity
in the cosmological evolution, we think that, from a methodological point of
view, the approach based on the modification of field equations, assuming no
new ingredients, should be viewed as superior. This is because the addition of
new hypothetical structures and substances should be disfavored with respect
to explanation which assumes no new unobserved and yet unverified forms of
matter-energy or spacetime structure. To put it in different words, it is always
possible to obtain the desired physical goal by increasing the number of param-
eters and invoking various types of \textit{ad hoc} entities, but by doing so, the physical
theory looses its simplicity, necessity and integrity.

One of the important limitations of the usual strategy of investigating bouncing
cosmologies based on specific constructions (i.e. the specific type of modifica-
tions of Einstein’s general relativity or matter-energy content of the Universe)
is that the obtained results are highly dependent on the specific assumptions
which are taken to derive them. It is thus not easy, and sometimes simply not
possible, to see which of the properties of solutions are general and which are
the result of highly hypothetical and often not properly motivated new modifi-
cations and additions. This problem signifies the need for a model independent
study of bouncing cosmologies. We first tried to contribute to this research pro-
gram by studying the bouncing and cyclic solutions supported by a general type
of higher order curvature corrections [36]. This research was then extended and
generalized by studying the model independent dynamic properties of bouncing
cosmologies and then applying the results to different types of modified gravity
theories [37]. It was also demonstrated that the problem of magnetogenesis has
a simple possible solution in the model-independent approach to bouncing cos-
mology [38]. Recently, we proposed a model-independent approach to bouncing
cosmology in which we proposed a simple solution of the cosmological constant
problem and also studied the effects of quantum fluctuations of the spacetime
geometry [39]. We will use some of the obtained conclusion in the present study
of wormholes created during the cosmological bounce.

There is an obvious technical connection between cosmological bounce and
another type of hypothetical gravitational solutions - wormholes, that comes
from the fact that both types of solutions imply physics beyond standard gen-
eral relativity. Wormholes are solutions of field equations for gravity which
represent a shortcut through spacetime, a tube-like structure which is asymp-
totally flat at both ends, and connects two distant parts of the Universe. While such spacetime configuration is actually a solution of classical Einstein’s equations, it requires the violation of Weak Energy Condition (WEC) for its existence in the framework of Einstein’s gravity [40]. However, it is at the present time not known which kind of substance could lead to the needed violation of the usual WEC on macroscopic scales. Therefore, if WEC is assumed to be satisfied then wormholes can be realized only by virtue of modification of field equations for gravity. Numerous realizations of such wormhole solutions not requiring the WEC violation were extensively studied in the literature [41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60].

One special case of potential wormhole solutions supported by modified gravity are “small cosmological wormholes”, an approximate type of solution where for a large enough distance from the wormhole throat the spacetime geometry can be described by the standard cosmological FLRW metric [61]. Such type of solutions enable us to simply study wormholes which are contained in the expanding Universe.

As we discussed, both the bouncing cosmological solutions and wormholes can be supported by an effective violation of usual energy conditions coming from the additional terms in equations with respect to Einstein’s gravity, playing the role of effective pressures and energy densities, while preserving the energy conditions for the matter content of the Universe. This naturally leads to the question: what is the relationship between the cosmological bounce and potential existence of wormholes? We address this question in the present paper where we show that if the conditions for the cosmological bounce are established, then wormholes can exist without any further modification of field equations or without introducing any kind of exotic matter. To do this we first present a simple, very general and model-independent, description of bouncing cosmology and then show how cosmological wormholes can be further constructed on such spacetime. We then demonstrate that there is no violation of WEC in the matter sector implied for such a kind of solutions.

The paper is organized in the following manner: in Section II we discuss how to generally represent all possible types of cyclic cosmologies on FLRW spacetime under the assumption of the stress-energy tensor conservation, in Section III we discuss the wormhole geometry around the cosmological bounce, in Section IV we derive field equations for wormholes around the cosmological bounce, obtain the solutions and discuss the WEC violation, and we finally conclude in V.

## 2 General approach to bouncing cosmology

In order to study the possibility of wormhole existence during the cosmological bounce we first need to present a general (i.e. model independent) description of bouncing cosmological solutions. Here we follow the steps discussed in [39].
With this aim we introduce the following action for gravity \((c = 1)\):

\[
S_{\text{eff}} = \frac{1}{16\pi G} \int \sqrt{-g} R d^4 x + S_{\text{mod}},
\]

(1)

here \(S_{\text{mod}}\) is a general type of correction needed to support the cosmological bounce. We add the action for matter fields and then vary this action with respect to the metric, thus obtaining the effective field equation for gravity:

\[
G_{\mu\nu}^{\text{eff}} = G_{\mu\nu} + G_{\mu\nu}^{\text{mod}} = 8\pi G (T_{\mu\nu}^{\text{rad,mat}} + T_{\mu\nu}^{\text{vac}}),
\]

(2)

here \(G_{\mu\nu}\) is Einstein’s tensor, \(G_{\mu\nu}^{\text{mod}}\) simply denotes the collection of terms which arise after variation of \(S_{\text{mod}}\) with respect to the metric, \(T_{\mu\nu}^{\text{rad,mat}}\) is stress-energy tensor for matter and radiation and \(T_{\mu\nu}^{\text{vac}}\), is the stress-energy contribution of the quantum vacuum. At this point we make no restrictions on the properties of \(G_{\mu\nu}^{\text{mod}}\), apart from assuming that it is a proper tensor and that the total stress-energy tensor is conserved, which leads to \(\nabla_{\mu} G_{\nu}^{\text{eff}} = \nabla_{\mu} G_{\nu}^{\text{mod}} = 0\). We now focus on the spherically symmetric and isotropic type of cosmological spacetime, that is we turn our attention on the Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime, given by the following form

\[
ds^2 = -dt^2 + a(t)^2 \left( dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right).
\]

(3)

Although the collection of terms \(G_{\mu\nu}^{\text{mod}}\) can in principle be some arbitrary complicated function of the curvature invariants and their combinations (such as Ricci scalar, contractions of Riemann and Ricci tensors, their higher powers and derivatives), the only cosmological coordinate on which components of this collection on terms can depend is - due to the homogeneous and isotropic nature of the considered spacetime – the time coordinate. Therefore, despite the arbitrary nature of such corrections, on the FLRW spacetime the field equations must take the following form [39]:

\[
3H^2 = 8\pi G \left( \rho_{\text{rad}}^0 a^{-4} + \rho_{\text{mat}}^0 a^{-3} + \rho_{\text{vac}} \right) + S(t).
\]

(4)

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho_{\text{mat}} + \rho_{\text{rad}} + 3p_{\text{rad}} + \rho_{\text{vac}} + 3p_{\text{vac}} \right) + \frac{G_{rr}^{\text{mod}}(\text{FLRW})}{2a^2} - \frac{S(t)}{6}
\]

(5)

Here \(S(t) = -G_{00}^{\text{mod}}(\text{FLRW})\), \(T_{00}^{\text{vac}} = -p_{\text{vac}} g_{00}\) and \(T_{ij}^{\text{vac}} = -p_{\text{vac}} g_{ij}\), while also using the standard description of matter and radiation modelled by the ideal fluid with the equation of state \(w = 0\) and \(w = 1/3\) respectively. From the demand for the conservation of the stress-energy tensor it follows that \(\nabla_{\mu} G_{\nu}^{\text{mod}} = 0\), and from that we obtain the following identity for \(G_{rr}^{\text{mod}}\) on FLRW spacetime:

\[
3H a^2 G_{rr}^{\text{mod}}(\text{FLRW}) = \dot{S}(t) + 3HS(t).
\]

(6)
It is simple to check that (5) is, under these assumptions, just a time derivative of (4), and thus we will in further analysis inspect mostly equation (4). Note that (4) and (5) are the most general form of Friedmann equations coming from the purely mathematical modification of the action for gravity with respect to general relativity – limited only by the symmetries of the spacetime and the total stress-energy conservation. Of course, when additional physical degrees of freedom are included in the action, such as scalar fields or various forms of non-minimal coupling, the modified equations will not have the form of equations (4) and (5), but such theories are beyond our interest here. In other words, in this paper we are only concerned with type of theories we could call minimally physically modified with respect to general relativity – i.e. involving only the mathematical generalizations of the action for gravity and no extra physical degrees of freedom.

Now we need to fix the correction function $S(t)$ in order that it leads to a bouncing cosmology. Here also, as we are interested in reaching general conclusions regarding the relationship between wormholes and cosmological bounce, we do not want to subscribe to some specific bouncing scenario, but to remain as general as possible. For any bouncing solution it necessarily follows that, at the time of the bounce, $t = t_b$, where the scale factor reaches its minimal value, $a = a_{\text{min}}$, we have $H = 0$ and therefore using (4) we obtain the following condition:

$$S(t_b) = -8\pi G \left( \rho_{\text{rad}}^0 a_{\text{min}}^{-4} + \rho_{\text{mat}}^0 a_{\text{min}}^{-3} + \rho_{\text{vac}} \right).$$

(7)

Furthermore, since $t = t_b$ represents the minimum of the scale factor, around the bounce $a(t) \approx a_{\text{min}} + (c/2)(t - t_b)^2$, where $c$ is a positive constant. From this, one easily obtains the approximation for $H(t)$ around the bounce valid for any possible type of bouncing cosmology. This approximate form of $S(t)$ around the bounce is given by

$$S(t)_{\text{bounce}} \approx 3 \left( \frac{c(t - t_b)}{a_{\text{min}} + c(t - t_b)^2/2} \right)^2 - 8\pi G \left[ \rho_{\text{rad}}^0 \left( a_{\text{min}} + c(t - t_b)^2/2 \right)^{-4} + \rho_{\text{mat}}^0 \left( a_{\text{min}} + c(t - t_b)^2/2 \right)^{-3} + \rho_{\text{vac}} \right].$$

(8)

In order to describe the general solutions for cosmological wormholes on bouncing spacetime we need to match the geometry of the cosmological spacetime possessing a bounce with a wormhole geometry. We will do this in the following section by considering the wormhole geometries which asymptotically approach the bouncing cosmological solutions.
3 The geometry of wormholes around the bounce

Firstly, we introduce the geometry of a static spherically symmetric wormhole

\[ ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2), \]  

(9)

where \( \Phi(r) \) is sometimes called the redshift function and \( b(r) \) is the shape function, respecting the condition \( 1 - \frac{b(r)}{r} \geq 0 \). In order to represent a traversable wormhole the redshift function must be finite everywhere, therefore there are no horizons. Also, if (9) is a wormhole geometry then the shape function must satisfy the flaring-out condition

\[ \frac{b(r) - b'(r)r}{b(r)^2} > 0. \]

(10)

This condition tells us that there exist a throat of a wormhole which represents the minimal radius of the geometry given by (9).

In a cosmological setting we are interested in wormholes which evolve in the cosmological time, whose evolution is governed by the scale factor \( a(t) \). In that case the static spherically symmetric wormhole can be generalised to a spherically symmetric cosmological wormhole

\[ ds^2 = -e^{2\Phi(r,t)}dt^2 + a(t)^2 \left( \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right), \]  

(11)

where again the shape function is given by \( b(r) \), and the redshift is \( \Phi(r, t) \) but now it can also be a function of time \( t \), and finally \( a(t) \) is the scale factor.

To asymptotically match such type of the solution to the bouncing FLRW cosmology we use the considerations presented in [61]. We use the approximation of a small cosmological wormhole: we suppose that the spacetime geometry of the wormhole is such that the redshift and shape functions are negligible for \( r > r_c \), where the value of \( r_c \) is chosen in such a manner that \( r_cH \ll 1 \) for all considered times. Under such approximation we can treat the wormhole as being effectively conﬁned within the region \( r < r_c \) with no inﬂuence on the global geometry of FLRW spacetime. In other words, such wormhole can be treated as if its asymptotical inﬁnity is actually placed at \( r = r_c \). We furthermore assume that all the dynamical properties of such wormholes are determined only by the expansion of the Universe, so that the dynamics of the scale factor, \( a(t) \), for the wormhole is the same as for the Universe. Thus, considering the case of bouncing cosmology, the scale factor of the cosmological wormhole appearing in (11) can be computed from modiﬁed Friedmann equations (4)-(5), taking into account the Taylor expansion of the correction factor around the bounce, given by (5).

The components of the stress-energy tensor will in general not be the same
in the wormhole region, \( r < r_c \), and in the cosmological region, \( r > r_c \). In the cosmological region the matter-energy is modelled by the ideal fluid with the equation of state \( p_{\text{cosmo}} = w\rho_{\text{cosmo}} \), with the EOS parameter \( w = 0 \) for matter, \( w = 1/3 \) for radiation and \( w = -1 \) for quantum vacuum. Here we labeled the components of the stress-energy tensor in the cosmological region with subscript “cosmo” to distinguish them from the components in the wormhole region. In the region \( r < r_c \) matter supporting the wormhole will be described by anisotropic fluid, given by

\[
T^\mu_\nu = \text{diag}\left( -\rho(r, t), p_r(r, t), p_t(r, t), p_t(r, t) \right),
\]

(12)

where the tensor components are density, radial pressure and tangential pressure respectively. Since the anisotropic fluid supporting the wormhole must approach the isotropic fluid of the bouncing Universe the following boundary conditions need to be satisfied:

\[
\rho(r_c, t) = \rho_{\text{cosmo}}(t),
\]

(13)

\[
p_r(r_c, t) = p_{\text{cosmo}}(t),
\]

(14)

\[
p_t(r_c, t) = p_{\text{cosmo}}(t),
\]

(15)

also, at \( r = r_c \) the relationships between the density and pressure of the fluid supporting the cosmological wormholes needs to go to the cosmological equation of state, for matter, radiation and quantum vacuum:

\[
p_r(r_c, t) = p_t(r_c, t) = w\rho(r_c, t).
\]

(16)

### 4 Wormhole solutions and WEC around the bounce

We can proceed by deriving the equations of motion for wormholes around the bounce. Here we are not interested in supporting the wormhole with some specific type of modification of gravity crafted in such a way so that the WEC for matter fields is satisfied on the wormhole spacetime. We rather want to show that even a minimal modification of gravity, supporting the cosmological bounce, can on its own support the wormhole solutions contained in the FRWL spacetime without the WEC violation. For this reason, while analysing the wormhole solutions, we consider only the type of modification responsible for supporting the cosmological bounce and that is \( S(t) \). Of course, in general, modifications of action for gravity will lead to correction functions that will in the wormhole region show also a radial dependence, i.e. \( S(t, r) \). However, this radial dependence of modifications if not of interest for the analysis presented here. Adding an extra degree of freedom, coming from the presence of radially dependent correction terms, just makes it additionally easier to achieve the absence of WEC violation, while blurring the question of supporting the wormholes by the existence of the cosmological bounce alone. For this reason, we work in a setting where we neglect any additional effects of the modification of gravity on the wormhole, apart from the correction needed to support the
bounce. Physically speaking, this assumption corresponds to considering such theories for which modifications with respect to general relativity appear only for very high values of curvature reached around the bounce, while leading to negligible departures with respect to general relativity for the changes on the spatial scales of the wormhole (thus making $S$ practically constant with respect to $r$).

Equations of motions \[(4)\] and \[(5)\] are valid for the isotropic case, however in our case the fluid becomes anisotropic as generally $p_r(r, t) \neq p_t(r, t)$ for the wormhole geometry. Therefore the modified Einstein’s equations are slightly different in the anisotropic case. Going back to equation \[(2)\] we define $G^\mu_\nu,\text{mod} = -8\pi GT^\mu_\nu,\text{mod}$ where

\[
T^\mu_\nu,\text{mod} = \text{diag}\left(-\rho^\text{mod}(t), p^\text{mod}_r(t), p^\text{mod}_t(t), p^\text{mod}_t(t)\right),
\]

\[
\rho^\text{mod}(t) = \frac{S(t)}{8\pi G},
\]

\[
p^\text{mod}_r(t) = -\frac{1}{8\pi G}\left(\frac{a(t)\dot{S}(t)}{\dot{a}(t)(3 + r\Phi(t, r))} + S(t)\right),
\]

\[
p^\text{mod}_t(t) = -\frac{1}{8\pi G}\left(\frac{a(t)\dot{S}(t)(2 + r\Phi(t, r))}{2\dot{a}(t)(3 + r\Phi(t, r))} + S(t)\right),
\]

where the condition $\nabla^\mu T^\mu_\nu = 0$ is satisfied from the definition above. In this case the field equations are

\[
H_{\mu\nu} = 8\pi G(T^\mu_\nu,\text{mod} + T^\mu_\nu) - G_{\mu\nu} = 0.
\]

By putting the wormhole geometry \[(11)\] in field equation \[(21)\] we obtain the following equations of motion

\[
H^t_t = 8\pi G\rho(t, r) + S(t) - \frac{3\dot{a}(t)^2 e^{-2\Phi(t, r)}}{a(t)^2} + \frac{b'(r)}{r^2} = 0,
\]

\[
H^r_r = \frac{a(t)\dot{S}(t)}{\dot{a}(t)(3 + r\Phi(t, r))} + S(t) - \frac{r^3 \dot{a}(t)^2 e^{-2\Phi(t, r)} + 2r b(t)\Phi'(t, r) - 2r^2 \Phi'(t, r) + b(r)}{r^3 a(t)^2} - \frac{2 e^{-2\Phi(t, r)} (\dot{a}(t) - \dot{a}(t)\Phi(t, r))}{a(t)} - 8\pi G p_r(t, r) = 0,
\]
\[ H^0 = \frac{1}{2a(t)^2} \left( -2\dot{a}(t)^2 e^{-2\Phi(t,r)} + \frac{2}{r^2} (r\Phi'(t,r)^2 + \Phi'(t,r) + r\Phi''(t,r)) - b'(r) (r\Phi'(t,r) + 1) - \right. \]
\[ \left. 2r (r\Phi'(t,r)^2 + \Phi'(t,r) + r\Phi''(t,r)) - b'(r) (r\Phi'(t,r) + 1) - \right. \]
\[ \left. b(r) \left( 2r^2 \Phi'(t,r)^2 + 2r^2 \Phi''(t,r) + r\Phi'(t,r) - 1 \right) \right) \frac{a(t)\dot{S}(t) (r\Phi'(t,r) + 2)}{2\dot{a}(t) (r\Phi'(t,r) + 3) +} \]
\[ S(t) - \frac{2e^{-2\Phi(t,r)} \left( \ddot{a}(t) - \dot{a}(t)\Phi'(t,r) \right)}{a(t)} - 8\pi G p(t, r) = 0, \quad (24) \]
\[ H^t = -\frac{2\dot{a}(t)e^{-2\Phi(t,r)}\Phi'(t,r)}{a(t)} = 0, \quad (25) \]

where the dot denotes the time derivative and the prime the derivative with respect to the radial coordinate \( r \). From the off diagonal equation (25) it appears that either \( \Phi'(t,r) = 0 \) or \( \dot{a} = 0 \), we choose the condition \( \Phi'(t,r) = 0 \) in order to support the idea of a cosmological wormhole. Also, for simplicity and in order to be sure that the wormhole is free from horizons we choose \( \Phi(t,r) = 0 \).

We rescale the parameters in order to have a dimensionless system of equations in the following manner

\[ \Omega = \frac{\rho}{\rho_c}, \quad \tilde{p} = \frac{p}{\rho_c}, \quad \tilde{t} = H_0 t, \quad \tilde{b} = H_0 b, \quad \tilde{r} = \frac{r}{r_0}, \quad (26) \]

where \( H_0^2 = 8\pi G \rho_c / 3 \) and \( r_0 \) is the radius of the throat. In principle \( H_0 \) and \( r_0 \) are independent quantities, however, as mentioned before the critical case is \( r_0 = 1/H_0 \). This point represents the upper limit for the dimension of wormholes (the throat radius is \( r_0 \sim 10^{26} \) m for the current measured \( H_0 \)), and in this regime the assumption of a small cosmological wormhole, \( H_0 r_0 << 1 \), obviously does not hold anymore. Interestingly, in our calculations there are no significant qualitative differences for wormholes with throats smaller or bigger than \( r_0 = 1/H_0 \) with 10^\pm10 orders of magnitude due to a highly suppressed radial functions in the field equations with respect to the scale factor in the early Universe. In other words, as will be discussed below, for the considered realistic values of parameters the fluid supporting the wormholes approaches the limit of ideal cosmological fluid very fast away from the throat. This confirms the robustness of our discussion.

Now we need to inspect the Weak Energy Condition (WEC) at the throat \( (r = r_0) \) in order to support a wormhole. The WEC in our rescaled quantities for the stress-energy tensor given by \( [12] \) reads \( \[23, 24\]

\[ \Omega \geq 0, \quad (27) \]
\[ \Omega + \tilde{p}_{radial} \geq 0, \quad (28) \]
\[ \Omega + \tilde{p}_{tangential} \geq 0. \quad (29) \]

By combining equations of motion \( \[22, 24\] \) with the re-scaled quantities \( \[26\] \)
we can deduce the following useful expressions for the WEC analysis

\[ \Omega(r, t) = -\frac{3\tilde{r}^2 \dot{a}(t)^2 + \tilde{r}^2 a(t)^2 \tilde{S}(t) - \tilde{b}'(r)}{3\tilde{r}^2 a(t)^2}, \]  

(30)

\[ \Omega(r, t) + \tilde{p}_r(r, t) = \frac{1}{9\tilde{r}^3 a(t)^2 \dot{a}(t)} \left( 3\tilde{r}^2 \dot{a}(t) \tilde{b}'(r) - 3\tilde{b}(r) \dot{a}(t) + 6\tilde{r}^3 \dot{a}(t)^3 - 6\tilde{r}^3 a(t) \dot{a}(t) \ddot{a}(t) + \tilde{r}^3 a(t)^3 \tilde{S}(t) \right), \]  

(31)

\[ \Omega(r, t) + \tilde{p}_t(r, t) = \frac{1}{18\tilde{r}^3 a(t)^2 \dot{a}(t)} \left( 3\tilde{r}^2 \dot{a}(t) \tilde{b}'(r) + 3\tilde{b}(r) \dot{a}(t) + 12\tilde{r}^3 \dot{a}(t)^3 - 12\tilde{r}^3 a(t) \dot{a}(t) \ddot{a}(t) + 2\tilde{r}^3 a(t)^3 \tilde{S}(t) \right), \]  

(32)

where \( \tilde{S}(t) = H_0^{-2} S(t) \), \( \dot{a} = da/d\tilde{t} \), \( \ddot{a} = da^2/d\tilde{t}^2 \) and we set \( \Phi(r, t) = 0 \), based on the previously mentioned considerations.

In order to inspect the bouncing region in the cosmological epoch we will use the approximate form of \( S(t) \) given by equation (8) \( S(t) = S(t)_{\text{bounce}} \) where we fixed \( a(t) = a_{\text{min}} + c(t - t_b)^2/2 \). In that case we can proceed with an example of a wormhole, where we choose the shape function as

\[ b(r) = \frac{r_0^3}{r^2}, \]  

(33)

which is one of the simplest function to preserve the flaring-out condition and one of the most common form used in literature [61, 65].

Firstly, we present the time evolution of the throat \( (r = r_0) \) around the cosmological bounce for the density parameter \( \Omega(r = r_0, t) \) on Fig. 1
Figure 1: Here we present the density parameter, \( \Omega \) divided by factor \( 10^{124} \), at the throat, \( r = r_0 = 10^{-2}/H_0 \), for different values of the parameter \( \tilde{c} \). We also assumed \( \Phi(r,t) = 0 \), \( b(r) = r_0^3/r^2 \), \( S(t) = S(t)_{\text{bounce}} \), \( a_{\text{min}} = 10^{-32} \) and the cosmological parameters \( \Omega_{\text{vac}}^0 = 0.692 \), \( \Omega_{\text{mat}}^0 = 0.308 \) and \( \Omega_{\text{rad}}^0 = 10^{-4} \).

The other two conditions for the WEC are depicted in Fig. 2 and Fig. 3.
Figure 2: Here we present the component $\Omega + p_r$, divided by factor $10^{124}$, at the throat, $r = r_0 = 10^{-2}/H_0$, for different values of the parameter $\tilde{c}$. We also assumed $\Phi(r, t) = 0$, $b(r) = r_0^3/r^2$, $S(t) = S(t)_{\text{bounce}}$, $a_{\min} = 10^{-32}$ and the cosmological parameters $\Omega_{\text{vac}}^0 = 0.692$, $\Omega_{\text{mat}}^0 = 0.308$ and $\Omega_{\text{rad}}^0 = 10^{-4}$.

Figure 3: Here we present the component $\Omega + p_t$, divided by factor $10^{123}$, at the throat, $r = r_0 = 10^{-2}/H_0$, for different values of the parameter $\tilde{c}$. We also assumed $\Phi(r, t) = 0$, $b(r) = r_0^3/r^2$, $S(t) = S(t)_{\text{bounce}}$, $a_{\min} = 10^{-32}$ and the cosmological parameters $\Omega_{\text{vac}}^0 = 0.692$, $\Omega_{\text{mat}}^0 = 0.308$ and $\Omega_{\text{rad}}^0 = 10^{-4}$.
It can be seen that at the cosmological bounce the WEC is “maximally” satisfied at the throat in the sense that the inequalities for the WEC take maximally positive values. The value of the parameter $c$ dictates how further in time will the WEC be satisfied. Namely, with the higher value of $c$ the scale factor more rapidly increases, leading to a higher acceleration of the Universe. This claim can also be supported by a simple analysis of equation (31). Firstly, the equation (31) can be rewritten in a more general (and dimensionless) form

$$H_0^{-2} S(t)_{bounce} = \tilde{S}(t)_{bounce} = 3 \left( \frac{\dot{a}}{a} \right)^2 - 3 \left( \Omega_{\text{rad}}^0 a^{-4} + \Omega_{\text{mat}}^0 a^{-3} - \Omega_{\text{vac}}^0 \right),$$

by taking the time derivative, $d/d\tilde{t}$, of the above equation we obtain

$$\dot{\tilde{S}}(t)_{bounce} = \frac{3\ddot{a}}{a^5} \left( 4\Omega_{\text{rad}}^0 + 3\Omega_{\text{mat}}^0 a - 2a^2 \dot{a}^2 + 2a^3 \ddot{a} \right).$$

Finally, by using $\dot{\tilde{S}}(t)_{bounce}$ from equation (35) and putting it in equation (31) we get the following, remarkably simple, result:

$$\Omega(r, t) + \tilde{p}_r(r, t) = \frac{4}{3} \frac{\Omega_{\text{rad}}^0}{a^4} + \frac{\Omega_{\text{mat}}^0}{a^3} - \frac{\ddot{b} - \tilde{r} \ddot{b}'}{3\ddot{a} a^2},$$

The first two positive contributions decrease with the power of $a^4$ and $a^3$ but the last negative contribution ($b - r\dot{b}'$, which is nothing else than the flaring-out condition) decrease with the power of $a^2$. This explains why at the bounce where $a$ is minimal the WEC is satisfied. Similarly, but in a bit more relaxed way, the third condition (32) behaves like

$$\Omega(r, t) + \tilde{p}_t(r, t) = \frac{4}{3} \frac{\Omega_{\text{rad}}^0}{a^4} + \frac{\Omega_{\text{mat}}^0}{a^3} + \frac{4}{9} \left( \frac{H}{H_0} \right)^2 + \frac{\ddot{b} + \tilde{r} \ddot{b}'}{6a^2} + \frac{4}{9} \frac{\ddot{a}}{a},$$

where the positive acceleration $\ddot{a}$ dominates at late times.

In our model for $S(t) = S(t)_{bounce}$ it is possible to find at which times the WEC is broken. The most difficult condition to satisfy is the condition given by equation (36), by solving it with $a(t) = a_{min} + ct^2/2$, ($t_b = 0$) we get four solutions

$$\tilde{\ell} = \pm \frac{2a_{min} \tilde{c} (\tilde{r} \tilde{b}' - \tilde{b})}{\sqrt{\tilde{c}^2 (16\Omega_{\text{rad}}^0 (\tilde{b} - \tilde{r} \tilde{b}') + 9\tilde{r}^3 (\Omega_{\text{mat}}^0)^2) + 3\tilde{r}^3 \Omega_{\text{mat}}^0}}^{1/2},$$

from which the real solutions, in the case $b(r) = r_0^3/r^2$, $\tilde{c} = 10^5$ with the cosmological parameters $\Omega_{\text{rad}}^0 = 10^{-4}$ and $\Omega_{\text{mat}}^0 = 0.308$, results in $t \sim 0.00248/H_0$. Again, the result highly depends on the parameter $\tilde{c}$, on the acceleration of $a$ around the bounce, where for example with $\tilde{c} \sim 0.617$ the time approaches to $1/H_0$. However, we should remember that such calculations can be assumed to be valid only around the bounce, so high values of the time interval for the
existence of wormholes, comparable to the age of the Universe, obviously have no physical interest – since after the bounce is finished the functional forms for \(a(t)\) and \(S(t)\) assumed here are no longer valid. We can thus conclude from this discussion that for proper values of the acceleration of \(a(t)\), cosmological wormholes can exist as long as the bouncing phase. Interestingly, the solutions do not depend on the constant vacuum energy \(\Omega^0_{\text{vac}}\) as only the derivative \(\dot{S}(t)\) appears in the equation (31).

We can also analyze the asymptotic, \(r \to \infty\), behaviour of the wormhole. Firstly, it can be seen that from the flaring-out condition \(b(r) - b'(r)r > 0\) the critical asymptotic case \((b(r) - b'(r)r = 0)\) is given by

\[
\lim_{r \to +\infty} b(r) \sim r, \quad (39)
\]

therefore, functions \(b(r)\) which have the asymptotic behaviour with greater power in \(r\) are not allowed as a representation of a wormhole. By taking this limit in the field equations (22)-(25) we indeed recover the cosmological equations given in [39] with the properties given by (16) when \(r \to \infty\). Therefore, the fluid supporting the wormhole for large \(r\) approaches the values of pressures and densities of the cosmological fluid fast, thus confirming the consistency of the approximation of a small cosmological wormhole set up in a FLRW spacetime and supported by the cosmological bounce. It can also be easily seen that equations are everywhere regular for all the values of \(r\) at and away from the throat, and that they approach the homogeneous and isotropic limit fast for \(r > r_0\). Since the plots for \(r\) dependence of the fluid densities and pressures thus do not reveal any new information, while WEC is simply satisfied away from the throat, and the differences in values with respect to the cosmological fluid values are minimal, we do not show those plots here.

5 Discussion and conclusion

In this work we have discussed the existence of wormholes during the cosmological bounce without the need for additional exotic matter supporting the wormhole. We have considered the approximation of a small cosmological wormhole, whose fluid components needed for supporting its geometry, for large enough values of \(r\), approach the components of the ideal cosmological fluid and whose dynamics is dictated by the time evolution of the cosmological scale factor, \(a(t)\). Thus, this geometry can be treated as a dynamical wormhole situated in the FLRW spacetime. By considering the most general form of modified gravity on FLRW spacetime coming from the mathematical generalizations of action, needed for the existence of bounce replacing the big bang singularity, we have obtained the cosmological wormhole solutions which do not need any additional exotic matter, that is, which satisfy WEC. This is possible since the same modification of gravity which supports the bounce can at the same time support the existence of wormholes, and no additional exotic matter is thus needed. The
effect leading to simultaneous support of a bounce and a small wormhole geometry is described by the function $S(t)$, and is arising from the generalization of action for gravity, while all the matter fields, $\Omega$, are completely standard and do not contain any new exotic component. We have thus demonstrated that the cosmological bounce in general represents an ideal environment for the natural creation of wormholes. Such wormholes could then serve as tunnels through spacetime in the early Universe, connecting very distant points that would otherwise be causally disconnected. Therefore, the existence of wormholes around the bounce can further help to solve the horizon problem, relaxing the necessary duration of the bouncing phase in order to obtain the causal picture implied by the CMB. We have also demonstrated that such wormholes spontaneously start to violate WEC after a certain time, which strongly depends on the rate of acceleration of the cosmological scale factor during the bounce. This can mean that after such critical time wormholes can no longer be supported in the Universe and vanish. This opens an interesting problem of detailed physical description of “death of a dynamic wormhole”, which is left for future work. Solving this problem could also be helpful for our better understanding of how to construct wormholes artificially (which is just the inverse of the mentioned problem in time).

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