On Catastrophic Forgetting in Generative Adversarial Networks

Hoang Thanh-Tung, Truyen Tran
Deakin University
hoangtha@deakin.edu.au

Abstract

In this paper, we view the training of Generative Adversarial Networks (GANs) [7] as a continual learning problem. The sequence of generated distributions is considered as the sequence of tasks. We show that catastrophic forgetting is present in GANs. We show how catastrophic forgetting can make the training of GANs non-convergent and provide a theoretical analysis of the problem. To alleviate catastrophic forgetting, we propose a way to adapt continual learning (CL) techniques to GANs. Our method is orthogonal to existing GANs training techniques and can be added to existing GANs without any architectural modification. Experiments on synthetic and real-world datasets confirm that the proposed method alleviates the catastrophic forgetting problem and improves the convergence of GANs.

1 Introduction

Generative Adversarial Networks (GANs) [7] are one of the most common tools for learning complex distributions. However, the original GAN is tricky to train and often suffer from problems such as mode collapse and non-convergence. In this paper, we show that catastrophic forgetting [6] is a cause of the problem and propose a solution to that.

Catastrophic forgetting in neural networks [6] is the phenomenon where the learning of a new skill catastrophically damage the performance of the previously learned skills. In machine learning literatures, mode collapse and catastrophic forgetting are usually studied independently. A number of methods have been proposed to address mode collapse [1] [8] [20] [24], non-convergence [10] [16] [24], and catastrophic forgetting [3] [13] [23] [25] as independent problems.

We view GANs training as a continual learning problem and show that catastrophic forgetting is present in GANs. The combined effect of catastrophic forgetting and mode collapse can make the training non-convergent. Our view allows the application of continual learning techniques to GANs. We show that continual learning techniques help GANs to converge to better equilibriums, i.e. equilibriums with less mode collapse.

Our contributions are:

- We propose a novel view of GANs training as a continual learning problem.
- We propose a sufficient condition for GANs to converge. The condition explains the effectiveness of a number of methods in stabilizing GANs.
- We show how catastrophic forgetting can make the discriminator violate our convergence condition, making the training of GANs non-convergent.
- We propose a way to apply continual learning algorithms to GANs. Our proposed method effectively prevents catastrophic forgetting, helping GANs to converge.

2 Preliminaries

2.1 Analyzing the training dynamic of GANs in the data space

[16] [17] analyzed the convergence of GANs in the parameter space with simplified models where the generator/the discriminator are linear, single parameter functions. In this paper, we take a different approach where we examine the convergence in the data space. This allows us to work with more complicated networks of higher capacity.
In reality the training of GANs only takes a finite number of iterations, the set of noise vectors, therefore, is a finite set. Let $G$ be the generator, $D$ be the discriminator. We denote the set of noise vectors as

$$\mathcal{D}_z = \{z_1, ..., z_m\},$$

the set of generated samples as

$$\mathcal{D}_g = \{y_1, ..., y_m\} = \{G(z_1), ..., G(z_m)\},$$

and the set of real samples as

$$\mathcal{D}_r = \{x_1, ..., x_n\}.$$

Let $y = G(z)$ be a fake sample and $\nabla \mathcal{L}_y^G = \partial \mathcal{L}_y^G / \partial y$ be the gradient of the generator’s loss w.r.t. $y$. Note that when the gradient back-propagate from $\mathcal{L}_y^G$ to $y$, it has only passed through the discriminator, the generator has not been involved yet. $\nabla \mathcal{L}_y^G$ only depends on the discriminator’s parameters, not the generator’s parameters. Upgrading the generator using gradient descent with small enough learning rate will move $y$ in the direction of $-\nabla \mathcal{L}_y^G$, by a distance proportional to $\|\nabla \mathcal{L}_y^G\|$. In Fig. 1, the opposite of the gradient at each datapoint is shown by an arrow. If the discriminator is fixed, then updating the generator with $\nabla \mathcal{L}_y^G$ will move $y$ along the integral curve associated with that fake sample. For commonly used losses such as the cross entropy loss or the Wasserstein loss, traveling along the integral curve in the direction of the arrow will increase the value of $D(y)$. The goal of the generator is to move fake samples toward samples from the target distribution, so we want the arrows to point toward real samples.

In practice, gradients are averaged over a mini-batch so an individual fake sample in the mini-batch will not move along its integral curve. However, if the generator has large enough capacity, it can move each individual fake sample along an independent path which is close to the integral curve. For simplicity, we assume that mini-batches of size 1 are used in training and updating the generator with the gradient from a fake sample does not affect other fake samples.

### 2.2 Continual Learning methods

We review here “prior focused” continual learning methods, methods that use intermediate models as priors for learning new models for new tasks [5].

Let us consider a continual learning problem with $k$ tasks $\mathcal{T}_{1:k} = \{\mathcal{T}_1, ..., \mathcal{T}_k\}$ and learnable parameter $\theta$. The goal is to prevent catastrophic forgetting so that when new tasks are introduced, the performance of the model on old tasks is not damaged catastrophically.

#### 2.2.1 Elastic Weight Consolidation

Elastic Weight Consolidation (EWC) [13] approaches the problem of continual learning from a Bayesian perspective where it tries to approximate the posterior over $\theta$. The posterior of $\theta$ is:

$$p(\theta | \mathcal{T}_{1:k}) \propto p(\theta) \prod_{i=1}^{k} p(\mathcal{T}_i | \theta)$$

$$\propto p(\theta) p(\mathcal{T}_{1:k-1} | \theta) p(\mathcal{T}_k | \theta)$$

The $k$-th posterior can be computed sequentially using Eqn. 2. To make the computation tractable, the likelihood $p(\mathcal{T}_i | \theta)$ is approximated using a Gaussian distribution:

$$p(\mathcal{T}_i | \theta) \approx \mathcal{N}(\theta; \theta^*_i, F^{-1}_i)$$

where $\theta^*_i$ is the maximum a posteriori (MAP) parameter when learning task $i$ and $F_i$ is the Fisher information matrix evaluated at $\theta^*_i$. See [13] for how to compute $F_i$. Using this approximation and applying the negative log to Eqn. 2 result in the following loss for the $i$-th task

$$-\log p(\mathcal{T}_i | \theta) + \frac{1}{2} \sum_{j=0}^{i-1} \| \theta - \theta^*_j \|^2_{F_j}$$

where the norm is the Mahalanobis norm. Intuitively, the regularization term $\| \theta - \theta^*_j \|^2_{F_j}$ prevents weights that are important to the task $j$ from changing while allowing less important weights to change more freely. When combined, regularization terms prevent the model from forgetting any of the previously learned tasks.

One drawback of EWC is that the number of regularization terms, and therefore, the amount of memory and time needed for evaluating a sample, grow linearly with the number of tasks. Online EWC [21] overcomes that drawback by applying the approximation to the whole posterior in Eqn. 2 resulting in the following loss:

$$-\log p(\mathcal{T}_i | \theta) + \frac{1}{2} \| \theta - \theta^*_0 \|^2_{\sum_{j=0}^{i-1} F_j}$$

In online EWC, only the latest MAP parameter and the running sum of Fisher information matrices are kept.

#### 2.2.2 Synaptic Intelligence

EWC computes the importance of each parameter at the end of a task. That could limit the use of EWC. Synaptic Intelligence (SI) [25] computes the importance of each parameter in an online manner as the training progresses. SI performs similarly to EWC while being more biologically plausible.
2.2.3 Variational Continual Learning

Variational Continual Learning (VCL) [18] uses Bayesian Deep Learning to alleviate catastrophic forgetting problem. VCL also computes the posterior sequentially using Eqn. 2 but it approximates the posterior \( p(\theta|T_{1:k-1}) \) with a variational distribution \( q_{k-1}(\theta) \). VCL tries to balance between the model’s performance on the current task and the KL divergence between the new variational posterior \( q_k(\theta) \) and the previous variational posterior \( q_{k-1}(\theta) \). One advantage of VCL over EWC is that in theory, it can learn without task boundaries. In practice, VCL performs better than EWC and its variants.

3 Catastrophic Forgetting in GANs

In GANs, there are a number of scenarios where catastrophic forgetting can hurt the generative performance:

1. GANs are used to learn a set of distributions \( p_0, ..., p_n \) that are introduced sequentially.

2. GANs are used to learn a single distribution \( p_r \).

The first scenario is a standard continual learning problem. The discriminator \( D \) at task \( n \) does not have access to distributions \( p_0, ..., p_{n-1} \). \( D \), thus, forgets about previous target distributions and cannot teach the generator \( G \) to generate samples from these distributions. In order to maximally deceive \( D, G \) only produces samples from \( p_n \). [22] used Elastic Weight Consolidation (EWC) algorithm in [13] to solve catastrophic forgetting in GANs in this setting and achieved promising results.

The second scenario is the main subject of this paper. It can also be viewed as a continual learning problem in which \( D \) has to discriminate a sequence of model distributions \( p_0, ..., p_T \) (where \( T \) is the number of training iterations) from the target distribution \( p_r \), and \( G \) has to deceive a sequence of discriminators \( D_0, ..., D_T \). At step \( t \), \( D \) still has access to samples from the target distribution \( p_r \), but it cannot access to samples from previous model distributions \( p_i \), \( i = 0 : t - 1 \). As a result, \( D \) is biased toward discriminating the current model distribution \( p_T \) from the target distribution, forgetting previous model distributions. Furthermore, \( D \) could focus on separating fake samples from nearby real samples, ignoring distant real samples.

The first row in Fig. 1 demonstrates the problem on the 8 Gaussian toy dataset. \( D \) assigns higher score to datapoints that are further away from fake samples, regardless of the true labels of these points. It is interesting to note that datapoints on the right of red boxes in Fig. 1(c) have higher score than real datapoints located around coordinates \((0.0, 1.5)\) and \((0.0, -1.5)\). Moving from Fig. 1(a) to Fig. 1(c), we see that the direction of almost all vectors changes as the fake samples move. The phenomenon suggests that the discriminator has no memory of past (real and fake) samples.

Furthermore, gradients w.r.t. datapoints that \( D \) has forgotten point in wrong directions (top red box in Fig. 1(c)) or have small norms (blue box in Fig. 1(c)). That implies that \( D \) overemphasizes on the current fake samples and nearby real samples while lowering the importance of distant real samples. In other words, the performance of \( D \) on old and distant samples is catastrophically damaged by the learning of the current task. We note that distant real samples are present in every mini-batch as we used large mini-batches of size from 32 to 256 in our experiments. Catastrophic forgetting happens not only because the data of old tasks are no longer accessible, but also because of the way the network distribute its capacity.

If \( G \) has mode collapse, then \( D \) only focuses on a small set of modes of \( p_r \) that are close to samples produced by \( G \), and forgets other modes. That worsens catastrophic forgetting in \( D \), making \( D \) unable to guide \( G \) to other modes of \( p_r \). \( G \) also has the chance to fool \( D \) by turning back to an old state which \( D \) has forgotten. When this situation occurs, \( D \) and \( G \) could fall into a loop and will not settle at an equilibrium. The vector field/discriminator and the model distribution/generator at iteration 20000 (Fig. 1(e)) are very similar to these at iteration 3000 (Fig. 1(a)). For the experiment in Fig. 1, the loop continues for many cycles without any sign of breaking.

Formulating GANs learning as a continual learning problem allows us to borrow methods from continual learning literature to improve the stability of GANs.

4 When do GANs converge?

4.1 A sufficient condition for convergence

Consider the vector field generated by a converged discriminator in Fig. 1(i) we note that gradients w.r.t. datapoints near a real datapoint point toward that real datapoint. We call such point a sink of the discriminator. If a fake sample \( y \) falls into the basin of attraction of a sink \( x \), it will be attracted toward \( x \). Sinks are also local maxima of the discriminator’s function.

In Fig. 1(i) real datapoints are sinks of the discriminator. If the discriminator is fixed (and therefore, the vector field), then a generator trained to maximize the score of fake samples, will converge to some of these sinks (real datapoints).
Proposition 1 (Sufficient condition for convergence). Given a GAN with generator \( G \) and discriminator \( D \). If \( D \) has some sinks at some fixed locations, and fake samples are located in the basin of attraction of these sinks, and \( G \) has large enough capacity so it could move any fake datapoint independently of all other fake datapoints, then \( G \) and \( D \) converge to an equilibrium.

Proof. We provide here a conceptual proof for the condition. Let \( S = \{s_1, ..., s_k\} \) be the set of fixed sinks of \( D \). Consider a fake datapoint \( y \) in the basin of attraction of a sink \( s \). Because the movement of \( y \) is independent of other fake datapoints, \( y \) can freely move toward \( s \). Training the generator with gradient descent will make \( y \) converge to \( s \). This is true for all fake datapoints in \( \mathcal{D}_g \). Because the locations of sinks are fixed during training, the generator and the discriminator will come to an equilibrium where the generator can generate some of the sinks.

We analyze how to make GANs to satisfy the condition:

1. **\( D \) has some sinks at some fixed locations**: this condition is the hardest to satisfy. As is shown in section 4.2, catastrophic forgetting removes sinks from the vector field, making GANs non-convergent. Because the goal of the generator is generating samples from the target distribution, we want sinks to be real samples from the target distribution. In section 5, we introduce and compare a number of ways to make real samples sinks.

2. **\( G \) has enough capacity to move any fake sample independently of all other fake samples**: such a generator can be created by associating a set of parameter for each fake sample. That generator, in turn, can be approximated using a MLP of \( O(mn^d) \) parameters. See appendix A for a way to construct such MLP based generator. In practice, large deep neural networks are used as generators. Because deep nets are more powerful than shallow ones, practical generators usually have enough capacity to move distant fake samples in independent paths.

3. **Fake samples are located near sinks**: As the generator tries to improve the score of fake samples, it will move fake samples toward local maxima/sinks of \( D \). Therefore, fake samples will be likely to fall into the basin of attraction of some sinks as the training progresses.

4.2 Catastrophic forgetting in the Dirac GAN

4.2.1 Revisiting the Dirac GAN

To see the effect of catastrophic forgetting on GANs and to motivate our solution, let us consider the Dirac GAN [16], a GAN that learns a 1 dimensional Dirac distribu-
Figure 2: The Dirac GAN. The blue line represents the discriminator’s function. The real datapoint \( x \) is shown in blue, the fake datapoint \( y = G(z) \) is shown in red. The value \( D(x) \) and \( D(G(z)) \) are shown in light blue and light red, respectively. First row: low capacity Dirac GAN. The discriminator is defined as \( D(x) = \psi_1 \sigma(\psi_0 x) \), where \( \sigma \) is the Leaky ReLU activation function. Second row: high capacity Dirac GAN trained with two fake samples. The old fake sample is on the left, the current fake sample is on the right. The discriminator has 2 hidden neurons: \( D(x) = \Psi_1^\top \sigma(\Psi_0 x) \) where \( \Psi_0, \Psi_1 \in \mathbb{R}^{2 \times 1} \).

The training objective is defined as

\[
\mathcal{L}(\theta, \psi) = \mathbb{E}_{p_x}[f(D(G(z)))] + \mathbb{E}_{p_r}[f(-D(x))] \quad (5)
\]

where \( f(\cdot) \) is a real valued function. For the original GAN, \( f(t) = -\log(1 + e^{-t}) \), and \( f(t) = -t \) for the Wasserstein GAN [1]. The set of real samples \( D_r \) contains a single datapoint \( x_0 = 0 \), the set of noise also contains a single vector \( D_z = \{z_0\} \), \( z_0 \neq 0 \). During training, the generator tries to minimize the objective \( \mathcal{L} \) while the discriminator tries to maximize it.

Mescheder et al. showed that although an equilibrium exists at \( \theta = \psi = 0 \), Dirac GAN could not converge and oscillate around the equilibrium. This is because gradient descent updates of \( D \) and \( G \) push \( \psi \) and \( \theta \) in different directions: when \( \theta \) is pushed toward 0, \( \psi \) is pushed away from 0 and vice versa. The phenomenon is shown in the first row of Fig. 2 (although the discriminator there has two weights \( \psi_0, \psi_1 \) and uses Leaky ReLU activation function, the discriminator’s function is similar to the discriminator’s function the original Dirac GAN which is a monotonic function with no local extrema).

The discriminator in Dirac GAN is overly simplified, it has no extrema so it cannot satisfy the condition in proposition [1]. In practice, discriminators are usually deep neural networks with non-linear activation functions. Such discriminators’ functions are likely to contain many local extrema.

### 4.2.2 Catastrophic forgetting in high capacity GANs

The original Dirac GAN cannot converge because the discriminator is always a monotonic function with no local extrema. We consider here a higher capacity discriminator of the form: \( D(x) = \Psi_1^\top \sigma(\Psi_0 x) \) where \( \Psi_0, \Psi_1 \in \mathbb{R}^{2 \times 1} \) and \( \sigma \) is the Leaky ReLU activation function. This function is not always a monotonic function. An example of the function with a local extremum is shown in Fig. 2(g).

However, if the training dataset contains only 1 fake datapoint, then unless the fake datapoint is exactly the same as the real datapoint, the optimal discriminator will be a monotonic function (see Fig. 3). This is because the discriminator is not optimized for other fake datapoints from other (old) model distributions and overfits to the current distribution. In other words, the discriminator catastrophically forgets other model distributions while maximizing its performance on the current task. Catsa-
trophic forgetting removes extrema from the discriminator.

GANs trained on higher dimensional data suffer from the same problem. In Fig. 1(a), 1(c) and 1(e) because of catastrophic forgetting, there are no sinks in the vector field. If the discriminator is fixed, the generator will diverge.

4.2.3 Old data helps GANs to converge

Let \( x \) be a real datapoint in the blue box in Fig. 1(a). \( x \) is not a local maximum because the discriminator does not force the score of its neighboring datapoints to be lower than its score. The reason is that the discriminator no longer has access to the (old) fake datapoints around \( x \) and focuses its capacity on discriminating the current fake samples from the real samples.

A solution is to keep fake samples from previous training iterations and reintroduce them to the current discriminator [19]. As the discriminator maximizes the score of the real sample and minimizes scores of neighboring (old) fake samples, the real sample will become a local maximum. The effect of using old fake samples on Dirac GAN is shown in the second row of Fig. 2. When there are two fake samples on different sides of the real sample, the optimal discriminator must be the one with a local maximum at the real datapoint.

Although this method improves the convergence of GANs, it requires additional memory to keep fake samples from previous iterations. For one dimensional data, in order to make the real sample a local maximum, the discriminator only requires one old fake datapoint. Because of the curse of dimensionality, the number of fake samples needed to make a real sample a local maximum would grow exponentially with the dimensionality of a sample. This method, therefore, is not very effective for high dimensional data. In section 5, we show how to use continual techniques to achieve similar to this method without having to store old fake data.

5 Improving GANs with Continual Learning

5.1 Creating sinks

Consider the 8 Gaussian dataset in Fig. 1(a), 1(c) and 1(e) we note that for most of the time, the gradient w.r.t. a neighbor of a real datapoint either points toward the real datapoint and has large norm, or points away from the real datapoint and has small norm. In other words, for most of the time, the real datapoint has higher score than its neighboring datapoints. The gradient point away from a real datapoint and has large norm only when a fake datapoint approaches the real datapoint (bottom red box in Fig. 1(c)). We can thus make the gradient to point toward real datapoints by taking the average of the vector field at different time steps. For the 8 Gaussian dataset, we use the running average of the gradient:

\[
\nabla \bar{L}_y^{G,t} = \gamma \nabla L_y^{G,(t-1)} + (1 - \gamma) \nabla L_y^{G,t}
\]

where \( \tau \) is the interval between samples of the vector field. Because consecutive discriminators/vector fields are very similar, using larger \( \tau \) helps to improve the diversity of the vector fields in the ensemble.

As shown in Fig. 1(b), 1(d), 1(f), the averaged vector field is robust to changes in the generator’s distribution. The averaged vector field has much nicer pattern than individual vector field where gradients w.r.t. to datapoints inside the circle always point toward real samples.

The averaged vector field also has some sinks (in Fig. 1(f), the blue box is a sink, the red box is almost a sink). If the discriminator is fixed and the model distribution is initialized at the right location, then the model distribution could converge to some of the modes of the target distribution.

The averaged vector field is produced by a discriminator whose output is the average of outputs of discriminators at different time steps. The naive implementation of the idea would require us to store many copies of the discriminator at different time steps. The memory and the time required to evaluate a sample grows linearly with the number of models. We could use distillation [11] to produce a single discriminator whose output is similar to that of the ensemble. However, distillation still requires all models in the ensemble to be present. To remove that requirement, we use continual learning techniques. Our method can be seen as an online distillation process where the knowledge from old discriminators is continually distilled to the current discriminator.

5.2 Method

The discriminator \( D_t \) at step \( t \) is likely to be the best discriminator to separate \( p_{\theta} \) from \( p_{r} \). The ensemble \( D_{1:t} \) will likely to have good performance on all \( t \) tasks. We would like to detect parameters that are crucial to the performance of \( D_t \) on the \( t \)-th task and transfer that knowledge to subsequent discriminators. The Fisher information matrix captures how importance each parameter is to the current task. We could use continual learning algorithms such as EWC to help the discriminator to preserve important information about old tasks.

Naive application of continual learning algorithms such as EWC to GANs would require us to compute the Fisher
information matrix at every discriminator’s iteration and the number of regularization terms is equal to the number of iterations. To reduce computation cost, we use the online EWC algorithm presented in section 2.3. We also note that too old fake samples have lower quality than more recent ones and could add noise to the training. A simple solution is to exponentially forget old samples. In our method, this is implemented by exponentially forgetting old Fisher information matrices. We have the following loss for the discriminator at iteration $k$:

$$L_{EWC}^{D,k} = L^D + \lambda \| \theta^{D,k} - \theta^{D,tr} \|_F^2$$  \hspace{1cm} (7)

where $t = \lfloor \frac{k}{\tau} \rfloor$, and

$$F_{t\tau} = \gamma F_{(t-1)\tau} + (1 - \gamma) F_{t\tau}$$  \hspace{1cm} (8)

$F_{t\tau}$ is computed every $\tau$ iterations, using real and fake samples used in iterations from $(t-1)\tau + 1$ to $t\tau$. As stated in the previous subsection, $\tau$ controls the diversity of discriminators in the ensemble. $\gamma$ controls how fast old Fisher matrices are forgotten. The smaller $\gamma$ is, the faster the old information is forgotten. $\lambda$ controls the balance between the current task (mini-batch) and old tasks. $L^G$ could be any of the standard loss functions for GANs such as the cross entropy loss or the Wasserstein loss.

5.3 Comparison to other methods

5.3.1 Zero centered Gradient Penalties

Zero centered Gradient Penalty on training examples only:

To reduce the oscillation in Dirac GAN, [16] proposed the zero-centered gradient penalty on training examples (0-GP-sample) which pushes the gradient at real and/or fake datapoints toward $0$. The 0-GP-sample loss is defined as follows

$$L_{0-GP-sample} = L + \lambda \mathbb{E}_{v \in D_r} \| (\nabla D)_v \|^2$$  \hspace{1cm} (9)

where $L$ is the loss in Eqn. 5. The penalty encourages the generator and the discriminator to reach an equilibrium where the generator can generate the real datapoint $x_0$.

When trained with 0-GP-sample, the discriminator forces the gradient w.r.t. real datapoints to be $0$, encouraging real datapoints to be local extrema of the discriminator’s function. Because the discriminator also tries to maximize the score of real datapoints, they are likely to be local maxima of the discriminator. 0-GP-sample encourages real datapoints in the training dataset to be sinks of the discriminator. Because the location of real datapoints are fixed, 0-GP-sample helps the discriminator to satisfy condition 1.

Because of the saturated regions in activation functions such as tanh, sigmoid, and ReLU, the gradient around a training datapoint could vanish although the datapoint is not a local extremum. In Fig. 1(a), although the gradient around the real datapoint in the blue box is close to $0$, that real datapoint is not a sink of the vector field. A more extreme scenario is shown in Fig. 1(b) where the gradient w.r.t. all of the real and fake datapoints are close to $0$ while none of them are sinks and the two networks collapse to a bad local equilibrium. The phenomenon suggests that forcing the gradient at real datapoints toward $0$ is not enough to make them sinks.

Our method creates sinks by distilling the knowledge from an ensemble of discriminators. If the distillation is perfect then our method only fails where the ensemble fails. Therefore, the diversity of discriminators in the ensemble is crucial to our method. Our method is orthogonal to 0-GP-sample and the two methods can be

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1 Although 0-GP-sample significantly improves the quality of generated samples, [24] suggested that it might encourage GANs to remember the real samples in the training dataset and exhibit poor generalization capability.

2 The dying ReLU problem has been studied in the context of general neural networks [9]. Our analysis here focuses on the problem of activation functions with saturated regions in GANs and it justifies the use of Leaky ReLU [15] in GANs [4].
Figure 4: Result on MNIST. The networks are 3 hidden layer MLP with 512 hidden neurons. Adam optimizer with learning rate of 0.03 and $\beta_1 = 0.5, \beta_2 = 0.9$ was used. (a), (b) Standard GAN at iteration 10000 and 20000. (c), (d) EWC-GAN with $\tau = 1, \lambda = 10, \gamma = 0.9$ at iteration 10000 and 20000. (e), (f) EWC-GAN with $\tau = 10, \lambda = 10, \gamma = 0.9$ at iteration 10000 and 20000. (g), (h) EWC-GAN with $\tau = 100, \lambda = 10, \gamma = 0.9$ at iteration 10000 and 20000.

combined to create a better regularizer for GANs.

Zero centered Gradient Penalty on interpolated samples:

[24] studied the generalization capability of GANs and showed that 0-GP-sample and non-zero centered gradient penalties do not improve the generalization of GANs. The authors proposed to improve the generalization of GANs using a gradient penalty of the following form:

$$L_{0-GP} = L + \lambda E_{v \in C}[(\nabla D)_v]^2$$

(10)

where $C \in supp(p_g) \cup supp(p_r)$ is a path from a fake sample to a real sample. $C$ is approximated by a straight line segment in practice. [24] showed that 0-GP helps to distribute the capacity of the discriminator more equally between regions of the space. Situations similar to the blue boxes in Fig. 1(a) and 1(c) are less likely to happen when 0-GP is used. 0-GP, therefore, can be seen as a method to avoid catastrophic forgetting.

5.3.2 Momentum based optimizers

In Eqn. 6 we take the running average of the gradient. That equation is similar to the equation used to calculate the gradient of a momentum based optimizer. Momentum based optimizers, therefore, can also alleviate catastrophic forgetting problem. The fact partly explains the success of momentum based optimizers such as SGD+momentum and Adam [12] in training GANs. Fig. 1(j) and 1(k) show the effect of Adam optimizer on the 8 Gaussian dataset. As can be seen from the figures, when fake samples move, the vector field/the discriminator does not change as much as before. More interestingly, gradients in the red box in Fig. 1(k) still point toward the real sample although fake samples are located near by. See Fig. 6 in appendix C for an evolution sequence of GAN+Adam.

5.3.3 Mixture of discriminators

[2] showed that an infinite mixture of generators and an infinite mixture of discriminators converge to an equilibrium. The authors experimentally verified that finite mixtures well approximate infinite mixtures and result in improved stability and sample quality. From our point of view, if each discriminator in the mixture remembers a different region of the data space, then using a mixture of discriminators also helps preventing catastrophic forgetting.

When mixtures are used, the memory requirement grows linearly with the number of components in the mixture. That prevents this method from scaling up to large networks and datasets. Our method does not require additional discriminators and generators and has the same memory requirement as standard GAN. Furthermore, the memory stays the same as we vary the interval $\tau$. Our method, therefore, is applicable to massive datasets and
6 Related work

[14] independently came up with the same idea about using continual learning methods for improving GANs. The authors proposed a slightly different way of using continual learning methods in GANs. The paper showed that GANs with EWC and SI perform similarly on various datasets. The paper, however, does not provide a detailed analysis of the catastrophic forgetting problem. Our paper, on the other hand, focuses on the theoretical aspect of the problem. We provide a theoretical analysis of the problem and its effects. We also show that continual learning techniques help to improve the convergence of GANs by making the discriminator to satisfy our convergence condition.

7 Experiments

The code for all experiments will be released after the review process.

Although our regularizer can be added to arbitrary loss functions, we perform experiments on the cross entropy loss. Our GAN is denoted as EWC-GAN. We note that Synaptic Intelligence and similar algorithms can also be adapted to GANs.

The result of EWC-GAN on the 8 Gaussian dataset is shown in Fig. 1(g), 1(h), and 1(i). From the figures, we can see that continual learning effectively reduces the catastrophic forgetting problem. The generator and the discriminator then converge to a good equilibrium where the generator can generate all modes in the target distribution.

Fig. 4 shows the result on MNIST dataset. We note the convergence in Fig. 4(g) and Fig. 4(h), going from iteration 10000 to iteration 20000. For most images, the digits stay the same, only the quality get improved. Without EWC, the GAN in Fig. 4(a) and 4(b) does not exhibit convergence: the digits keep changing as the training continues. EWC-GANs with small $\tau$ also do not converge. When $\tau$ is small, the discriminators in the ensemble are very similar so they are likely to forget the same set of samples. As a result, the distilled discriminator also forgets that same set of samples. Larger $\tau$ improves the diversity of discriminators in the ensemble, resulting in less forgetting and more diverse images. In Fig. 4 the diversity of generated images increases as $\tau$ increases.

8 Conclusion

In this paper, we study the catastrophic forgetting problem in GANs. We show that catastrophic forgetting is a reason for non-convergence. We propose a sufficient condition for convergence and show that the condition is violated when catastrophic forgetting happens. From that insight, we propose to apply continual learning techniques to GANs to alleviate the catastrophic forgetting problem. Experiments on synthetic and MNIST datasets confirm that continual learning techniques improve the convergence of GANs and the diversity of generated samples.

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A Constructing powerful generators

Given a dataset

\[ D_g = \{ y_1, \ldots, y_m \} = \{ G(z_1), \ldots, G(z_m) \} \]

of \( d \)-dimensional fake samples which are created using \( d_z \)-dimensional noise. For simplicity, we assume that noise vectors are normalized: \( \| z_i \| = 1, \forall i \). A generator that approximately satisfies the requirement in condition 1 can be constructed as a 1 hidden layer MLP as follow:

\[ G(z) = W_2^\top \times \sigma(W_1 \times z) \]

where \( W_1 \in \mathbb{R}^{m \times d_z}, W_2 \in \mathbb{R}^{m \times d}, \) and \( \sigma \) is the softmax function. \( W_1 \) is defined as

\[ W_1 = k \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} \]

For large enough \( k \), \( \sigma(W_1 z_i) \) will become an one-hot vector with the \( i \)-th element being 1. The output of \( G(z_i) \) is the \( i \)-th row of \( W_2 \) and gradient update to \( G \) will affect that row only. Such MLP-generator can move any individual fake sample in a path independent of all other fake samples.

B Catastrophic forgetting in high capacity Dirac GAN

C Experiments
Figure 5: Catastrophic forgetting in high capacity Dirac GAN. The discriminator is a 1 hidden layer neural network with Leaky ReLU activation function and 2 hidden neurons. Although the discriminator has enough capacity to become a non-monotonic function, catastrophic forgetting makes it a monotonic function. High capacity Dirac GAN still oscillates around the equilibrium.

Figure 6: Result of GAN+Adam on the 8 Gaussian dataset.