Cost-Optimal Algorithms for Planning with Procedural Control Knowledge

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Abstract. There is an impressive body of work on developing heuristics and other reasoning algorithms to guide search in optimal and anytime planning algorithms for classical planning. However, very little effort has been directed towards developing analogous techniques to guide search towards high-quality solutions in hierarchical planning formalisms like HTN planning, which allows using additional domain-specific procedural control knowledge. In lieu of such techniques, this control knowledge often needs to provide the necessary search guidance to the planning algorithm, which imposes a substantial burden on the domain author and can yield brittle or error-prone domain models. We address this gap by extending recent work on a new hierarchical goal-based planning formalism called Hierarchical Goal Network (HGN) Planning to develop the Hierarchically-Optimal Goal Decomposition Planner (HOpGDP), an HGN planning algorithm that computes hierarchically-optimal plans. HOpGDP is guided by $h_{HL}$, a new HGN planning heuristic that extends existing admissible landmark-based heuristics from classical planning to compute admissible cost estimates for HGN planning problems. Our experimental evaluation across three benchmark planning domains shows that HOpGDP compares favorably to both optimal classical planners due to its ability to use domain-specific procedural knowledge, and a blind-search version of HOpGDP due to the search guidance provided by $h_{HL}$.

1 Motivation and Background

Formalisms for automated planning (to represent and solve planning problems) broadly fall into either domain-independent planning or domain-configurable planning. Domain-independent planning formalisms, such as classical planning requires that the users only provide models of the base actions executable in the domain. In contrast, domain-configurable planning formalisms (e.g., Hierarchical Task Network (HTN) planning) allow users to supplement action models with additional domain-specific knowledge structures that increases the expressivity and scalability of planning systems.

An impressive body of work exploring search heuristics has been developed for classical planning that has helped speed up generation of high-quality solutions. More specifically, search heuristics such as the relaxed planning graph heuristic [9], landmark generation algorithms [10, 17], and landmark-based heuristics [17, 11] dramatically improved optimal and anytime planning algorithms by guiding search towards (near-) optimal solutions to planning problems.

Yet relatively little effort has been devoted to develop analogous techniques to guide search towards high-quality solutions in domain-configurable planning systems. In lieu of such search heuristics, domain-configurable planners often require additional domain-specific knowledge to provide the necessary search guidance. This requirement not only imposes a significant burden on the user, but also sometimes leads to brittle or error-prone domain models. In fact, getting the best of heuristic search and hierarchical procedural knowledge (to decomposes planning tasks) has remained an unsolved problem since planning competitions first focused on heuristic search at AIPS-98 [12].

In this paper, we address this gap by developing the Hierarchically-Optimal Goal Decomposition Planner (HOpGDP), a hierarchical planning algorithm that uses admissible heuristic estimates to generate hierarchically-optimal plans (i.e., plans that are valid and optimal with respect to the given hierarchical knowledge). HOpGDP leverages recent work on a new hierarchical planning formalism called Hierarchical Goal Network (HGN) Planning [19, 18], which combines the hierarchical structure of HTN planning with the goal-based nature of classical planning.

In particular, our contributions are as follows:

- **Admissible Heuristic:** We present an HGN planning heuristic – $h_{HL}$ (HGN landmark heuristic) – that extends landmark-based admissible classical planning heuristics to derive admissible cost estimates for HGN planning problems. To the best of our knowledge, $h_{HL}$ is the first non-trivial admissible hierarchical planning heuristic.
- **Optimal Planning Algorithm:** We introduce HOpGDP, an $A^*$ search algorithm that uses $h_{HL}$ to generate hierarchically-optimal plans.
- **Experimental Evaluation:** We describe an empirical study on three benchmark planning domains in which HOpGDP outperforms optimal classical planners due to its ability to exploit hierarchical knowledge. We also found that $h_{HL}$ provides useful search guidance; despite substantial computational overhead, it compares favorably in terms of runtime and nodes explored to HOpGDPblind, using the trivial heuristic $h = 0$.

2 Preliminaries

In this section we detail the classical planning model, review how landmarks are constructed for classical planning and an admissible landmark-based heuristic $h_L$, and describe HGN planning using examples from assembly planning.
2.1 Classical Planning

We define a classical planning domain $D_{\text{classical}}$ as a finite-state transition system in which each state $s$ is a finite set of ground atoms of a first-order language $L$, and each action $a$ is a ground instance of a planning operator $o$. A planning operator is a 4-tuple $o = (\text{head}(o), \text{precond}(o), \text{effects}(o), \text{cost}(o))$, where $\text{precond}(o)$ and $\text{effects}(o)$ are conjuncts of literals called $o$'s preconditions and effects, and $\text{head}(o)$ includes $o$'s name and argument list (a list of the variables in $\text{precond}(o)$ and $\text{effects}(o)$). $\text{cost}(o)$ represents the non-negative cost of applying operator $o$.

Actions. An action $a$ is executable in a state $s$ if $s \models \text{precond}(a)$, in which case the resulting state is $\gamma(a) = (s - \text{effects}^{-}(a)) \cup \text{effects}^{+}(a)$, where effects$^{+}$ (a) and effects$^{-}$ (a) are the atoms and negated atoms, respectively, in effects(a). A plan $\pi = (a_1, \ldots, a_n)$ is executable in $s$ if each $a_i$ is executable in the state produced by $a_{i-1}$; and in this case $\gamma(s, \pi)$ is the state produced by executing $\pi$. If $\pi$ and $\pi'$ are plans or actions, then their concatenation is $\pi \circ \pi'$.

We define the cost of $\pi = (a_1, \ldots, a_n)$ as the sum of the costs of the actions in the plan, i.e. $\text{cost}(\pi) = \sum_{i \in \{1\ldots n\}} a_i$.

2.2 Generating Landmarks for Classical Planning

There are several landmark generation algorithms suggested in the literature [10, 17]. The general approach used in generating sound landmarks is to relax the planning problem, generate sound landmarks for the relaxed version, and then use those for the original planning problem. In this paper, we use LAMA’s landmark generation algorithm [17], which uses relaxed planning graphs and domain-transition graphs in tandem to generate landmarks.

2.3 $h_L$: an Admissible Landmark-based Heuristic for Classical Planning

We provide some background on $h_L$, the landmark-based admissible heuristic for classical planning problems proposed by Karpas and Domshlak [11] that we will be using in our heuristic.

Consider a classical planning problem $P = (D_{\text{classical}}, a_0, g)$ and a landmark graph $L_G = (L, \text{Ord})$ computed using any of the off-the-shelf landmark generation algorithms mentioned in Section 2.2. Then, we can define $\text{Unreached}(L, s, \pi) \subseteq L$ to be the set of landmarks that need to be achieved from $s$ onwards, assuming we got to $s$ using $\pi$. Note that $\text{Unreached}(L, s, \pi)$ is path-dependent: it can vary for the same state when reached by different paths. It can be computed as follows:

$$\text{Unreached}(L, s, \pi) = L \setminus (\text{Accepted}(L, s, \pi) \setminus \text{ReqAgain}(L, s, \pi))$$

where $\text{Accepted}(L, s, \pi) \subseteq L$ is the set of landmarks that were true at some point along $\pi$, $\text{ReqAgain}(L, s, \pi) \subseteq L$ is the set of landmarks that were accepted but are required again; an accepted landmark is required again if (1) it does not hold true in $s$, and (2) it is greedily-necessarily ordered before another landmark $l'$ in $L$ that is not accepted.

Karpas and Domshlak show that it is possible to partition the costs of the actions $A$ in $D_{\text{classical}}$ over the landmarks in $\text{Unreached}(L, s, \pi)$ to derive an admissible cost estimate for the state $s$ as follows: let $\text{cost}(\phi)$ be the cost assigned to the landmark $\phi$, and $\text{cost}(a, \phi)$ be the portion of $a$'s cost assigned to $\phi$. Furthermore, let us suppose these costs satisfy the following set of inequations:

$$\forall a \in A : \sum_{\phi \in \text{Unreached}(a \mid L, s, \pi)} \text{cost}(a, \phi) \leq \text{cost}(a)$$

$$\forall \phi \in \text{Unreached}(L, s, \pi) : \text{cost}(\phi) \leq \min_{a \in \text{ach}(\phi \mid s, \pi)} \text{cost}(a, \phi)$$

where $\text{ach}(\phi \mid s, \pi) \subseteq A$ is the set of possible achievers of $\phi$ along any suffix of $\pi$, and $\text{ach}(a \mid L, s, \pi) = \{ \phi \in \text{Unreached}(L, s, \pi) \mid a \in \text{ach}(\phi \mid s, \pi) \}$.

Informally, what these equations are encoding is a scheme to partition the cost of each action across all the landmarks it could possibly achieve, and assigns to each landmark $\phi$ a cost no more than the minimum cost assigned to $\phi$ by all its achievers. Given this, they prove the following useful theorem:  

**Theorem 1.** Given a set of action-to-landmark and landmark-to-action costs satisfying Eqn. (1) $h_L(L, s, \pi) = \text{cost}(|\text{Unreached}(L, s, \pi)|) = \sum_{\phi \in \text{Unreached}(L, s, \pi)} \text{cost}(\phi)$ is an admissible estimate of the optimal plan cost from $s$.

Note that the choice of exactly how to do the cost-partitioning is left open. One of the schemes Karpas and Domshlak propose is an optimal cost-partitioning scheme that uses an LP solver to solve the constraints in Eqn. (1) with the objective function $\max \sum_{\phi \in L} \text{cost}(\phi)$. This has the useful property that given two sets of landmarks $L$ and $L'$, if $L \subseteq L'$, then $h_L(L, s, \pi) \leq h_L(L', s, \pi)$. In other words, the more landmarks you provide to $h_L$, the more informed the heuristic estimate.

2.4 Goal Networks and HGN Methods

We extend the definitions of HGN planning [19] to work with partially-ordered sets of goals, which we call a goal network.

A goal network is a way to represent the objective of satisfying a partially ordered multiset of goals. Formally, it is a pair $gn = (T, \prec)$ such that:

- $T$ is a finite nonempty set of nodes;
- each node $t \in T$ contains a goal $g_t$ that is a DNF (disjunctive normal form) formula over ground literals;
- $\prec$ is a partial order over $T$.

![Figure 1: Three generic goal networks we use for examples of the various relationships within a goal network.](image)

We will provide examples of both generic and concrete goal networks. Figure shows three generic goal networks. Each subfigure
is itself a goal network denoted $g_{m0}, g_{n0}, g_{e}$. Directed arcs indicate a subgoal pair (e.g., $(g_k, g_j)$) from $g_{m0}$ such that the first goal must be satisfied before the second. Consider the network $g_{n0}$ where $g_k$ is a subgoal of $g_j$, then $g_{n0} = \{(g_j, g_k), (g_k, g_j)\}$. Network $g_{n0}$ shows a partial ordering, where $\{(g_m, g_n) \prec g_i\}$. Similarly, $(\{g_2, g_3\} \prec g_1)$ and this implies both must occur before $g_1$. Consider a network $g_{n_e}$ that is composed of $g_{n0}$ and $g_{n0}$. Then $g_{n_e} = \{(g_1, g_2, g_3), (g_2, g_3, g_1)\}$. Note that $g_{n_e}$ is a partially ordered forest of goal networks.

Figure 2: Goal Network for an Automated Manufacturing domain

Figure 2 shows a concrete goal network for an automated manufacturing domain. $\text{joined}(x, y)$ denotes the goal of assembling the parts $x$ and $y$ together, while $\text{at}(x, \text{loc})$ represents the goal of getting $x$ to location $\text{loc}$. In this goal network, $\text{joined}(p_2, p_1)$ and $\text{joined}(p_3, p_1)$ are unordered with respect to one another. Furthermore, $\text{joined}(p_2, p_1)$ has three subgoals that need to be achieved before achieving it, i.e., the goals of getting the parts $p_1, p_2$ and the tool to the assembly table. These subgoals are also unordered with respect to one another, indicating that the goals can be accomplished in any order.

HGN Methods

An HGN method $m$ is a 4-tuple $(\text{head}(m), \text{goal}(m), \text{precond}(m), \text{network}(m))$ where the head $\text{head}(m)$ and preconditions $\text{precond}(m)$ are similar to those of a planning operator. goal $(m)$ is a conjunct of literals representing the goal $m$ decomposes. network $(m)$ is the goal network that $m$ decomposes into. By convention, network $(m)$ has a last node $t_g$ containing the goal $(m)$ to ensure that $m$ accomplishes its own goal.

Figure 3 describes the goal network that the $\text{deliver-obj}$ method, a method responsible for solving problems related to delivering parts and tools to their destinations, decomposes a goal into. This method is relevant to $\text{at}(x, \text{loc})$ goals (since that’s the last node), and its preconditions are $\text{precond}(	ext{deliver-obj}) = \{\neg\text{reserved}(\text{agent}), \text{can-carry}(\text{agent}, p), \ldots\}$. $\text{at}(p, \text{loc}), \text{at}(\text{agent}, p), \text{with}(p, \text{agent}), \text{reserved}(\text{agent})$.

Figure 3: Subgoal network of $\text{deliver-obj}(p, \text{loc}, \text{agent})$, an HGN method to deliver the part $p$ to $\text{loc}$ using agent $\text{agent}$.

Whether a node has predecessors impacts the kinds of operations we allow. We refer to any node in a goal network $g_n$ having no predecessors as an unconstrained node of $g_n$, otherwise the node is constrained. The constrained nodes of Figure 3 include $g_k, g_l, g_n$, and the remaining are unconstrained. The unconstrained nodes in Figure 2 include all the $\text{at}$ nodes as well as the $\text{joined}(p_3, p_1)$ node.

We define the following operations over any goal network $g_n = (T, \prec)$:

1. **Goal Release**: Let $t \in T$ be an unconstrained node. Then the removal of $t$ from $g_n$, denoted by $g_n - t$, results in the goal network $g_n' = (T', \prec')$ where $T' = T \setminus \{t\}$ and $\prec'$ is the restriction of $\prec$ to $T'$.

2. **Method Application**: Let $t \in T$ be an unconstrained node. Also, let $m$ be a method applied to $t$ with network $(m) = (T_m, \prec_m)$.

   - **Unconstrained Goal Satisfaction**: In the following cases, let $t \in T$ be an unconstrained node. Also, let $m$ be a method applicable in $s$ and relevant to $g_t$, then any solution to $P' = (D, s_0, g_n_{0} \circ t, m)$ is also a solution for $P$.

   - **Unconstrained Goal Satisfaction**: If $s_0 = g_t$, then any solution for $P' = (D, s_0, g_n_{0} - t)$ is also a solution for $P$.

   - **Action Application**: If action $a$ is applicable in $s_0$ and $a$ is relevant to $g_t$, then a solution for $P' = (D, g_t(s_0), g_n_{0})$, then $a \circ \pi$ is a solution for $P$.

   - **Method Decomposition**: If $m$ is a method applicable in $s$ and relevant to $g_t$, then any solution to $P' = (D, s_0, g_n_{0} \circ t, m)$ is also a solution for $P$.

Note that HGN planning allows an action to be applied only if it is relevant to an unconstrained node in $g_n$; this prevents unrestricted chaining of applicable actions as done in classical planning and allows for tighter control of solutions in HTN planning.

Let us denote $S(P)$ as the set of solutions to an HGN planning problem $P$ as allowed by Definition 2. Then we can define what it means for a solution $\pi$ to be hierarchically optimal with respect to $P$ as follows:

**Definition 3** (Hierarchically Optimal Solutions). A solution $\pi^{h,*}$ is hierarchically optimal with respect to $P$ if $\pi^{h,*} = \arg\min_{\pi \in S(P)} \text{cost}(\pi)$.

**3 The HOpGDP Algorithm**

Algorithm 1 describes HOpGDP. It takes as input an HGN domain $D = (D', M)$, the initial state $s_0$ and the initial goal network $g_{n0}$. It does an $A^*$ search using the admissible HGN heuristic $h_{HL}$ described in Section 2 to compute a hierarchically optimal solution to the problem; it either returns a plan if it finds one, or failure if the problem is unsolvable.
Initialization. It starts off by initializing open (Line 2), which is a priority queue that sorts the HGN search nodes yet to be expanded by their \( f \)-value, where \( f(s, gn, \pi) = \text{cost}(\pi) + h_{HL}(s, gn) \). open initially contains the initial search node \((s_0, gn_0, \emptyset)\). It also initializes searchSpace (Line 3), the set of all nodes seen during the search process. This data structure keeps track of the best known path for each \((\text{state, goal-network})\) pair, and is thus helpful to detect when we find a cheaper path to a previously seen HGN search node.

**Algorithm 1** Pseudocode of HOpGDP. It takes as arguments the domain description \( D = (\text{Domains,} M) \), the initial state \( s_0 \), and the initial goal network \( gn_0 \). It either returns a plan if it finds one, or failure if it doesn’t.

```plaintext
1: function HOpGDP(D, s_0, gn_0)
2: open ← (s_0, gn_0, { })
3: searchSpace ← (s_0, gn_0, { })
4: while open is not empty do
5:   rem. (s, gn, \pi) with lowest \( f \)-value from open
6:   if gn is empty then return \pi
7:   successors ← getSuccessors(D, s, gn, \pi)
8:   for \((s’, gn’, \pi’)\) ∈ successors do
9:     if \( \exists (s’, gn’, \eta) \) ∈ searchSpace then
10:        if cost(\pi’) < cost(\eta) then
11:           replace \((s’, gn’, \eta)\) with \((s’, gn’, \pi’)\) in searchSpace
12:        else continue
13:     else add \((s’, gn’, \pi’)\) to searchSpace
14:     eval. \( f \)-value of \((s’, gn’, \pi’)\) and add to open
15: return failure
16:
17: function getSuccessors(D, s, gn, \pi)
18:   successors ← \emptyset
19:   for unconstrained \( g \in gn \) satisfied in \( s \) do
20:     add the node \((s, gn - \{g\}, \pi)\) to successors
21:   \( A \) ← actions in \( D \) applicable in \( s \) and relevant to an unconstrained goal in \( gn \)
22:   for \( a \in A \) do
23:     add the node \((\gamma(s, a), gn, \pi o a)\) to successors
24:   \( M \) ← \{ (m, g) s.t. m ∈ M is applicable in \( s \) and relevant to an unconstrained goal \( g \) in \( gn \) \}
25:   for \((m, g)\) ∈ \( M \) do
26:     add the node \((s, gn o g, m, \pi)\) to successors
27: return successors
```

Search. HOpGDP now proceeds to do an \( A^* \) search in the space of HGN search nodes starting from the initial node. While open is not empty, it does the following (Lines 3–14); it removes the HGN search node \( N = (s, gn, \pi) \) with the best \( f \)-value from open (Line 5), and first checks if \( gn \) is empty (Line 9). If this is true, this means that all the goals in \( gn_0 \) have been solved, and \( \pi \) is the optimal solution to the HGN planning problem.

If \( gn \) is not empty, then the algorithm proceeds by using the getSuccessors subroutine to compute \( N \)'s successor nodes (Line 7). For each successor node \((s’, gn’, \pi’)\), it proceeds to do the following: it checks to see if another path \( \eta \) to \((s’, gn’)\) exists in searchSpace (Line 9). If this is the case and if \( \eta \) is costlier than \( \pi’ \) (Line 10), it updates searchSpace with the new path; and reopens the search node (Line 11); if \( \eta \) is cheaper than the new plan \( \pi’ \), it simply skips this successor (Line 12).

If \((s’, gn’)\) has not been seen before, it adds \( N’ = (s’, gn’, \pi’) \) to searchSpace to track the currently best-known plan \( \pi’ \) to \((s’, gn’)\) (Line 13). It also evaluates the \( f \)-value of \( N’ \) (note that this is where \( h_{HL} \) is called) and adds it to open (Line 14).

If there are no more nodes left in open, this implies that it has exhausted the search space without finding a solution, and therefore returns failure (Line 15).

**Computing Successors.** The procedure getSuccessors computes the successors of a given HGN search node \((s, gn, \pi)\) in accordance with Definition 2. First, we check to see if there are any unconstrained goals \( g \) in \( gn \) that are satisfied in the current state \( s \). We then proceed to create new HGN search nodes by removing all such goals from \( gn \) (Line 9–20). Next, we compute all actions applicable in \( s \) and relevant to an unconstrained goal \( g \) in \( gn \) (Line 21) and create new search nodes by progressing \( s \) using these actions (Line 22–23). We compute all pairs \((m, g)\) such that \( m \) is an HGN method applicable in \( s \) and relevant to an unconstrained goal \( g \) in \( gn \) (Line 24) and create new search nodes by decomposing \( g \) in \( gn \) using \( m \) (Line 25–26). Finally, we return the set of generated successor nodes (Line 27).

4 \( h_{HL} \): An Admissible Heuristic for HGN Planning

**Algorithm 2** Procedure for computing landmarks for relaxed HGN planning problems.

```plaintext
1: function computeHGNLandmarks(s, gn)
2: queueSeeds ← gn
3: queue ← \emptyset
4: while queueSeeds is not empty do
5:   choose a g w/o successors from queueSeeds, and remove it along with all associated orderings
6:   addLM(g), add g to queue
7:   add any orderings \( g \) shares with other goals from \( gn \) already added to LG
8:   while queue is not empty do
9:     pop landmark \( \psi \) from queue and use LMGENC to generate the new set of landmarks \( \Phi \)
10:    for \( \phi \in \Phi \) do ADDLM(\( \phi, \phi \rightarrow gn \) \( \psi \))
11:  return LG
12:
13: function addLM(\( \phi \))
14:  if \( \phi \) is a fact and \( \exists \phi’ \in LG : \phi’ \neq \phi \land \phi \models \phi’ \) then
15:    remove \( \phi’ \) from LG and all orderings it is part of
16:  if \( \exists \phi’ \in LG : \phi’ \models \phi’ \) then return \( \phi’ \)
17:  if \( \phi \notin LG \) then add \( \phi \) to queue and return \( \phi \)
18:
19: function addLMandOrdering(\( \phi, \phi \rightarrow gn \) \( \psi \))
20: \( \eta \leftarrow \text{addLM}(\phi) \)
21: add ordering \( \eta \rightarrow x, \psi \) to LG
```

As mentioned in Section 3, HOpGDP uses \( h_{HL} \) to compute the \( h \)-values (and thus, the \( f \)-values) of search nodes. In this section, we will proceed to describe how to construct \( h_{HL} \) as follows:

1. We define a relaxation of HGN planning that ignores the provided methods and allows unrestricted action chaining as in classical planning, which expands the set of allowed solutions.
2. We will extend landmark generation algorithms for classical planning problems to compute sound landmark graphs for the relaxed
HGN planning problems, which in turn are sound with respect to the original HGN planning problems as well, and finally

3. We will use admissible classical planning heuristics like $h_L$ on these landmark graphs to compute admissible cost estimates for HGN planning problems.

4.1 Relaxed HGN Planning

Definition 4 (Relaxed HGN Planning). A relaxed HGN planning problem is a triple $P = (D_{\text{classical}}, s_0, g_{n0})$ where $D$ is a classical planning domain, $s_0$ is the initial state, and $g_{n0}$ is the initial goal network. Any sequence of actions $\pi$ that is executable in state $s_0$ and achieves the goals in $g_{n0}$ in an order consistent with the constraints in $g_{n0}$ is a valid solution to $P$.

Relaxed HGN planning can thus be viewed as an extension of classical planning to solve for goal networks, where there are no HGN methods and the objective is to generate sequences of actions that satisfy the goals in $g_{n0}$ in an order consistent with $g_{n0}$. In fact, it is easy to show that relaxed HGN planning, in contrast to HGN planning, is no more expressive than classical planning, and relaxed HGN planning problems can be compiled into classical planning problems quite easily.

Next, we will show how to leverage landmark generation algorithms for classical planning to generate landmark graphs for relaxed HGN planning.

4.2 Generating Landmarks for Relaxed HGN Planning

This section describes a landmark discovery technique that can use any landmark discovery technique for classical planning (referred to as LMGEN$_C$ here) to compute landmarks for relaxed HGN planning problems. The main difference here is that while classical planning problems are (state, goal) pairs, relaxed HGN planning problems are (state, goal-network) pairs; every goal in the goal network can be thought of as a landmark. Therefore, there is now a partially ordered set of goals to compute landmarks from, as opposed to a single goal in classical planning.

We therefore need to generalize classical planning landmark generation techniques to work for relaxed HGN planning problems. The computeHGNLandmarks algorithm (Algorithm 2) describes one such generalization. At a high level, computeHGNLandmarks proceeds by computing landmark graphs for each goal $g$ in $g_{n0}$ (which in fact is a classical planning problem) and merging them all together to create the final landmark graph $LG$.

computeHGNLandmarks takes as input a relaxed HGN planning problem $(s, g_{n0})$ and generates $LG$, a graph of landmarks. First, sequenceSeeds is initialized with a copy of $g_{n0}$ (Line 2). This is because unlike in classical planning where we generate landmarks for a single goal, in HGN planning we have a partially ordered set of goals to seed landmark generation; sequenceSeeds stores these seeds. We also initialize queue, theopenslist of landmarks, to $\emptyset$.

While there is a goal $g$ from $g_{n0}$ that we have not yet computed landmarks for (Line 3), we do the following: we remove it from sequenceSeeds along with all induced orderings and add it to queue (Lines 5–6). We also add $g$ to $LG$ using addLM; we also add any ordering constraints it might have with other elements of $g_{n0}$ that have already been added to $LG$. This queue is then used as a starting point by LMGEN$_C$ to begin landmark generation. We iteratively use LMGEN$_C$ to top landmarks off the queue and generate new landmarks by backchaining until we can no longer generate any more landmarks (Lines 8–10). Each new landmark is added to $LG$ by the addLM procedure. Once all goals in $g_{n0}$ have been handled, the landmark generation process is completed and the algorithm returns $LG$.

The addLM procedure takes as input a computed landmark $\phi$, adds it to $LG$ and returns a landmark $\eta$. There are three cases to consider:

- \( \phi \) subsumes another landmark $\phi'$ in $LG$, implying we can remove $\phi'$ and replace it with $\phi$ (since $\phi$ is a stronger version of $\phi'$), and return $\phi$ (Line 14).
- \( \phi \) is subsumed by another landmark $\phi'$ in $LG$, implying we can ignore $\phi$ (Line 16). In this case, we don’t add any new landmark to $LG$ and simply return $\phi'$.
- \( \phi \) is a new landmark, in which case we can simply add it to $LG$ and return $\phi$ (Line 17).

The addLM procedure takes as input a landmark $\phi$ and an ordering constraint $\phi \rightarrow_{s} \psi$ and adds them to $LG$. More precisely, it adds $\phi$ to $LG$ using addLM, which returns the added landmark $\eta$. It then adds the ordering constraint between $\eta$ and $\psi$ in $LG$.

**Figure 4:** (a) LM graph on goal network containing a single goal $at(p_1, table)$. (b) LM graph after decomposing $at(p_1, table)$ with deliverObj($p_1, table, A1$). The double-circled landmarks represent new landmarks inferred after the method decomposition, while the landmarks colored gray are new landmarks that subsume an existing one in (a).

**LM graph computation example.** Figure 4 illustrates the working of computeHGNLandmarks. Let us assume the goal network $g_{n0}$ contains only one goal $g = at(p_1, table)$. Figure 4a illustrates the output of computeHGNLandmarks on $g$. This is identical to what LMGEN$_C$ would generate, since $g_{n0}$ contains only one goal, making the relaxed HGN problem equivalent to a classical planning problem.

Now, let us assume that we decompose $g_{n0}$ using the $m = deliverObj(p_1, table, A1)$, and get the new goal network $g_{n'}$, which essentially looks like an instantiated version of the network in Figure 4a. Now if we run computeHGNLandmarks on $g_{n'}$, we end up generating the landmark graph in Figure 4b, which is a more focused version of the first landmark graph. This is because the goals in $g_{n'}$ are landmarks that must be accomplished, which constrains the set of valid solutions that can be generated. For instance, since we’ve committed to agent $A1$, every solution we can generate from $g_{n'}$ will involve the use of $A1$. We can, as a result, generate more focused landmarks than we otherwise could have from just the top-level goal $g$. This includes fact landmarks that replace disjunctive landmarks (the ones in gray in Fig. 4b) as well as completely new landmarks that arise as a result of the method; e.g. $reserved(A1)$ is not a valid landmark for $g_{n0}$, but is one for $g_{n'}$. 


An important point to note at this point is that the subgoals in \( gn' \) are not true landmarks for \( g \); they are landmarks once we commit to applying method \( m \). However, this actually ends up being useful to us, since it allows us to generate different landmark graphs for different methods; for instance, if we had committed to \( A2 \), we would have obtained a different set of landmarks specific to \( A2 \). Now, landmark-based heuristics when applied to these two graphs would get us different heuristic estimates, thus allowing to differentiate between these two methods by using the specific subgoals each method introduces.

It is easy to show that computeHGNLandmarks generates sound landmark graphs for relaxed HGN planning problems:

**Claim 5.** Given a relaxed HGN planning problem \( P = (D_{\text{classical}}, s_0, gn_0) \), \( LG = \text{computeHGNLandmarks}(s_0, gn_0) \) is a sound landmark graph for \( P \).

Let \( P = ((D_{\text{classical}}, M), s_0, gn_0) \) be an HGN planning problem, and let \( P' = (D_{\text{classical}}, s_0, gn_0) \) be the corresponding relaxed version. Then by definition, any solution to \( P \) is a solution to \( P' \). Therefore, it is easy to see that a landmark of \( P' \) is also a sound landmark of \( P \). More generally, a landmark graph generated for \( P' \) is going to be sound with respect to \( P \) as well:

**Claim 6.** Given an HGN planning problem \( P \), then \( LG = \text{computeHGNLandmarks}(s_0, gn_0) \) is a sound landmark graph for \( P \).

### 4.3 Computing \( h_{HL} \)

The main insight behind \( h_{HL} \) is the following: since the computeHGNLandmarks algorithm generates sound landmarks and orderings for relaxed (and therefore regular) HGN planning problems, we can use any admissible landmark-based heuristic from classical planning to derive an admissible cost estimate for HGN planning problems.

In particular, \( h_{HL} \) uses \( h_L \) as follows: given an HGN search node \((s, gn)\), the landmark graph is given by \( LG_{HGN} = \text{computeHGNLandmarks}(s, gn) \). Then

\[
h_{HL}(s, gn, \pi) = h_L(LG_{HGN}, s, \pi)
\]

where \( \pi \) is the plan generated to get to \((s, gn)\).

A couple of important implementation details: when using \( h_L \) to guide classical planners, it is sufficient to compute the landmark graph just once upfront since it can be reused in every state along the plan due to the goal staying the same. This isn’t the case in HGN planning; method decomposition can change the goal network. So, \( h_{HL} \) requires re-computing the landmark graph each node. In our implementation, we try to optimize this process by computing landmark graphs for each goal network we encounter from the initial state and caching them for use in future nodes containing the same goal network. Section 4.2 discusses the impact of this overhead in the experiments. Secondly, while the optimal cost partitioning scheme in \( h_L \) provides more informed heuristic estimates, we chose to use the uniform cost partitioning scheme in our implementation since the former requires solving an LP at each search node, which is costly.

### 4.4 Admissibility of \( h_{HL} \)

Claim 6 shows that given an HGN problem \( P = (D, s_0, gn_0) \), \( LG = \text{computeHGNLandmarks}(s_0, gn_0) \) is a sound landmark graph with respect to \( P \). Furthermore, Lemma 1 shows that \( h_L(LG(s_0, \langle \rangle)) \) provides an admissible cost estimate of the optimal plan starting from \( s_0 \) that achieves all the landmarks in \( LG \). Since every solution to \( P \) has to achieve all the landmarks in \( LG \) in a consistent order, \( h_L(LG(s_0, \langle \rangle)) \) provides an admissible estimate of the optimal cost to \( P \) as well. However, from Eq. \( 2 \),

\[
h_{HL}(s_0, gn_0, \langle \rangle) = h_{HL}(s_0, gn_0, \langle \rangle)
\]

Therefore, we have the following theorem:

**Theorem 7 (Admissibility of \( h_{HL} \)).** Given an HGN planning domain \( D \), a search node \((s, gn, \pi)\) and its cost-optimal solution \( \pi_{s, gn, \pi} \), \( h_{HL}(s, gn, \pi) \leq \pi_{s, gn, \pi} \).

### 5 Experimental Evaluation

We implemented HOpGDP within the Fast-Downward codebase, and extended LAMA’s landmark generation code to develop \( h_{HL} \), our HGN planning heuristic.

We tested two hypotheses in our study:

**H1:** HOpGDP’s ability to exploit hierarchical planning knowledge enables it to outperform state-of-the-art optimal classical planners. To test this, we compared the performances of HOpGDP with \( A^*-h_L \) [11], the optimal classical planner whose heuristic we extended to develop \( h_{HL} \).

It might seem that \( H1 \) is obviously true due to the dominance of hierarchical planners (e.g., SHOP2 and GDP) over classical planners, but these are merely satisficing planners. It is not clear whether this advantage would carry over to optimal planning because HOpGDP needs to do an optimal search in the possibly larger space of \((\text{state}, \text{goal-network})\) pairs, in contrast to classical planners, which search in the space of states.

**H2:** The heuristic used by HOpGDP, \( h_{HL} \), provides useful search guidance. To test this, we compared the performances of HOpGDP with HOpGDP \( \text{blind} \), which is identical to HOpGDP except that it uses the trivial heuristic estimate of \( h = 0 \).

#### 5.1 Experimental Results

We evaluated HOpGDP, HOpGDP \( \text{blind} \), and \( A^*-h_L \) on three well-known planning benchmarks, Logistics, Blocks World and Depots. We chose these 3 domains because from a control-knowledge standpoint, these three domains capture a wide spectrum: Logistics consists only enough control-knowledge to define allowed solutions, Blocks-World is at the other extreme, defining sophisticated knowledge that significantly prunes the search space, and Depots incorporates elements of both.

For each domain, we randomly generated 25 problem instances per problem size. We ran all problems on a Xeon E5-2639 with a per problem limit of 4 GB of RAM and 25 minutes of planning time. Data points were discarded if the planner did not solve all of the corresponding problem instances within the time limit.

**Logistics.** We modified the standard PDDL Logistics model to limit the capacity of all vehicles to one to ensure the HGN and non-HGN planners compute the same solutions. We generated 25 random logistics problems for each problem size ranging from 4.6,...,14 packages. For HOpGDP and HOpGDP \( \text{blind} \), we provided the HGN methods used in GoDel’s experimental evaluation [13]. There are three methods in this knowledge base that together capture all the possible (minimal) solutions to a Logistics problem; these are (1) a method to move packages within the same city using trucks, (2) a method to move packages between airports using planes, and (3) a
method that combines the previous two to move packages across different cities.

Figures 5a and 5d show the performance of the three planners in terms of number of nodes expanded by the planners and overall planning time. Both HOpGDP and HOpGDP blindly could solve problems up to size 10 (i.e., within the time limit), while A*-h_L could solve problems only up to size 6.

In terms of nodes expanded, Figure 5a shows that the heuristic in HOpGDP helped to modestly decrease the number of nodes expanded; HOpGDP on average expanded 22% fewer nodes than HOpGDP blindly. We did not include A*-h_L because it expanded many orders of magnitude more nodes than either HOpGDP variant (e.g., for problems of size 6, A*-h_L on average expanded $12 \times 10^5$ nodes). With regard to running time, Figure 5a shows that the modest gain by h_L was outweighed by the computational overhead of running the heuristic (on average about 35% of the total running time). HOpGDP blindly, despite its blind search, was slightly faster than HOpGDP.

**Blocks World.** We generated 25 random blocks-world problems for problem sizes ranging from 4,6,. . . ,20 blocks. As in our study with Logistics, we use the same HGN methods used in GoDeL’s evaluation [13]. In contrast to our Logistics study, the methods encode sophisticated knowledge that allows the planners to prune search paths that don’t lead to good solutions (e.g., it contains a recursively defined axiom that checks if a block is in its final position and only then builds towers on top of it).

Figures 5b and 5e show the performance of the three planners on these blocks-world problems. A*-h_L could solve problems up to size 10, HOpGDP blindly to size 16, and HOpGDP could solve problems up to size 18.

Figure 5b displays the number of nodes expanded by the three planners. In this domain, the guidance provided by h_L helped substantially; HOpGDP on average expanded 76% fewer nodes than HOpGDP blindly. This savings far outweighed the heuristic computational overhead (on average about 48% of the total running time), resulting in smaller overall planning times for HOpGDP as can be seen in Figure 5c.

**Depots.** We generated 25 random depots problems for problem sizes ranging from 4,5,. . . ,10 crates. Since the Depots domain combines aspects of Logistics (moving cargo around) and Blocks-World (stacking them in a particular manner), the HGN methods for Depots is a combination of the HGNs used in Logistics and Blocks-World.

Figures 5c and 5f show the performance of the three planners on the generated problems. A*-h_L could solve problems up to only size 6, while both HOpGDP and HOpGDP blindly could solve problems up to size 9. Figure 5c shows the average number of nodes expanded by the three planners. The h_L heuristic in HOpGDP provides good search guidance, reducing the number of nodes expanded by about 46% when compared to HOpGDP blindly. As in Logistics, we didn’t show the nodes expanded by A*-h_L since it was many orders of magnitude more than either HOpGDP variant; for size 6 problems, on average, it expanded $3.5 \times 10^7$ nodes.

In terms of planning time (Figure 5f), the provided domain knowledge clearly helps both HOpGDP variants in scaling much better than A*-h_L. Furthermore, the additional search guidance provided by h_L results in overall lower runtimes for HOpGDP in comparison to HOpGDP blindly, even with the computation overhead of the heuristic (which is about 56% of the total time).

**5.2 Interpretation of Results**

There are two main takeaways from this empirical study:

**Support for H1.** Hierarchical planning knowledge helps in scaling up solving of optimal planning problems. In all three benchmark domains, both of the HOpGDP variants solved more problems while requiring less time and expanding fewer nodes than A*-h_L, showing that the additional overhead of searching through
the space of \((\text{state}, \text{goal-network})\) pairs was outweighed by the benefit that hierarchical planning knowledge can provide in terms of more focused search.

**Support for H2.** The HGN heuristic \(h_{HL}\) provides useful guidance when searching for hierarchically optimal plans. We can conclude this from the decrease in the number of nodes expanded in HOpGDP as compared to HOpGDP\(_{blind}\) in all three benchmark domains. The Logistics results only weakly support this due to only a modest decrease in the number of nodes expanded (22%), while the results from Blocks-World and Depots are more conclusive, registering large savings in number of nodes expanded (76% and 46% respectively).

We posit that the reduction in number of nodes expanded by \(h_{HL}\) is a function of the input HGN knowledge. For instance, the Logistics methods do not encode any expert knowledge and instead only model the minimum knowledge required to capture the three ways to move a package: by truck, by plane, and by a combination of the two. Therefore, the goal networks always contain landmarks or morefocused versions of landmarks (e.g. \(\text{airplane-at}(ap_1,loc_1)\) instead of \(\text{airplane-at}(ap_1,loc_1)\)) that can be detected by landmark generation algorithms. This means that the landmarks generated do not change much after a method application, implying that the heuristic estimates are unlikely to change much either. In contrast, methods in both Blocks-World and Depots contain specialized knowledge that, when applied, yield goal networks containing subgoals and orderings that cannot be detected by landmark generation algorithms. That is, when landmark generation is run on these goal networks, because the subgoals in the goal network serve as seeds for landmark generation, a richer set of landmarks will be generated, resulting in more informed heuristic estimates.

Another important takeaway from the experiments is the following: the current implementation of \(h_{HL}\) imposes a substantial overhead on HOpGDP. On average, it uses 35%, 48% and 56% of the total planning time in Logistics, Blocks-World, and Depots respectively. This is partly due to the current implementation not being optimized. For instance, unlike landmark-based classical planners where the landmark graph needs to be computed only once for the final goal, HOpGDP needs to compute landmark graphs for every goal network it generates during search. Reusing the computed landmark graphs more effectively can potentially help in substantially reducing planning times.

### 6 Related Work

HTN planners solve planning problems by (1) forward state-space search, such as in the SHOP\(_1\) and SHOP2\(_2\) HTN planners, or (2) partial-order causal-link planning (POCL) techniques, such as in UMCP\(_3\) and in the hybrid planning literature\(_4,5\).

HGN planning can be translated to HTN planning in a plan-preserving manner\(_5\), meaning we can, in theory, use any optimal HTN planner for optimal HGN planning. However, there is little research on search heuristics for forward-search HTN planning\(_6,7\). Therefore, planners often provide other domain-specific mechanisms for users to encode search strategies. For example, SHOP2 allows domain-specific knowledge, known as \(HTN\) methods, to be specified in a ‘good’ order according to the user, and attempts to apply them in the same order. SHOP2 also provides support for external function calls\(_8\) that can call arbitrary code to perform intensive computations, thus minimizing the choices that need to be made during search. For example, in the 2002 Planning Competition for hand-tailored planners, the authors of SHOP2 used a graph-algorithm library that SHOP2 could call externally to generate shortest paths\(_9\).

Waibrodt\(_10\) developed \(H2O\), a HTN planner that augments SHOP2 with classical planning heuristics to make local decisions on which method to apply next by estimating how close the method’s goal is to the current state. However, \(H2O\) retains the depth-first search structure of SHOP2, making it difficult to generate high-quality plans.

Marthi et al\(_11,12\) propose an HTN-like formalism called \(an\)-genic hierarchical planning that allows users to annotate abstract tasks with additional domain-specific information (i.e., lower and upper bounds on the costs of the possible plans they can be used to generate). They then use this information to compute hierarchically-optimal plans. In contrast, we require the costs of only the primitive actions and use domain-independent search heuristics to compute hierarchically-optimal plans.

There has been recent work on developing search heuristics for POCL HTN planners\(_7,6\). However, these heuristics typically provide estimates on how many more plan refinement steps need to be taken from a search node to obtain a solution. This differs from planning quality estimates, which is our focus in this paper.

**Hierarchical Goal Network (HGN) Planning** combines the hierarchical structure of HTN planning with the goal-based nature of classical planning. It therefore allows for easier infusion of techniques from classical planning into hierarchical planning, such as adapting the FF heuristic for \(method\ ordering\) in the GDP planner\(_19\), and using landmark-based techniques to plan with partial amounts of domain knowledge in GoDeL\(_18\). However, both planners use depth-first search and inadmissible heuristics, so they cannot provide any guarantees of plan quality.

Another less-related domain-configurable planning formalism is Planning with Control Rules\(_4\), where domain-specific knowledge is encoded as linear-temporal logic (LTL) formulas. TLPlan, one of the earliest planners developed under this formalism, used control rules written in LTL to prune trajectories deemed suboptimal by the user. There have also been attempts to develop heuristic search planners that can plan with LTL-f, a simplified version of LTL that works with finite traces. This has been used to incorporate search heuristics to solve for temporally extended goals written in LTL\(_f\)\(_5\), planning for preferences\(_23\), as well as to express landmark-based heuristics that guide classical planners\(_21\).

### 7 Conclusion

Despite the popularity of hierarchical planning techniques in theory and practice, little effort has been devoted to developing domain-independent search heuristics that can provide useful search guidance towards high-quality solutions. As a result, end-users need to encode domain-specific heuristics into the domain models, which can make the domain-modeling process tedious and error-prone.

To address this issue, we leverage recent work on HGN planning, which allows tighter integration of hierarchical and classical planning, to develop \(h_{HL}\), an admissible HGN planning heuristic, and \(H\)OpGDP, an \(A^*\) search algorithm guided by \(h_{HL}\) to compute hierarchically-optimal plans. Our experimental study showed that \(H\)OpGDP outperforms optimal heuristic search classical planners (due to its ability to exploit domain-specific planning knowledge) and optimal blind search HGN planners (due to the search guidance provided by \(h_{HL}\)).
There are several directions for future work, such as:

- **Extension to Anytime Planning:** An obvious and a practically useful extension of this work is to extend HOpGDP to work in an *anytime* manner (i.e., generate a solution quickly such that a solution is available at any time during execution and then iteratively/continuously improve the plan’s quality over time) instead of trying to compute the optimal solution up-front. We can of course adapt techniques used in anytime classical planners like LAMA, which runs a series of weighted-‘A’ searches. However, we also plan to explore the use of block-deordering [29], a technique for continual plan improvement that seems to lend itself well to plans that are hierarchically structured.

- **Extension to Temporal Planning:** We also plan on investigating temporal extensions of HGN planning and HOpGDP to develop search heuristics and hierarchical planners that can leverage procedural knowledge to find high-quality plans and schedules.

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**REFERENCES**

[1] Ron Alford, Gregor Behnke, Daniel Höller, Susanne Biundo, Pascal Bercher, and David W. Aha, ‘Bound to plan: Exploiting classical heuristics via automatic translations of tail-recursion in HTN problems’, in Proc. of the 26th Int. Conf. on Automated Planning and Scheduling (ICAPS). AAAI Press, (2016).

[2] Ron Alford, Vikas Shivashankar, Ugur Kuter, and Dana S. Nau, ‘On the feasibility of planning graph style heuristics for HTN planning’, in Proc. of the 24th Int. Conf. on Automated Planning and Scheduling (ICAPS), pp. 2–10. AAAI Press, (2014).

[3] Ron Alford, Vikas Shivashankar, Mark Roberts, Jeremy Frank, and David W. Aha, ‘Hierarchical planning: relating task and goal decomposition with task sharing’, in Proc. of the 25th Int. Joint Conf. on Artificial Intelligence (IJCAI). AAAI Press, (2016).

[4] Fahiem Bacchus and Fréduald Kabanza, ‘Using temporal logics to express search control knowledge for planning’, Artif. Intell., 116, 123–191, (2000).

[5] Jorge A. Baier and Sheila A. McIlraith, ‘Planning with first-order temporally extended goals using heuristic search’, in AAAI Conference on Artificial Intelligence, (2006).

[6] Pascal Bercher, Shawn Keen, and Susanne Biundo, ‘Hybrid planning heuristics based on task decomposition graphs’, in Proc. of the Seventh Annual Symposium on Combinatorial Search (SoCS), pp. 35–43. AAAI Press, (2014).

[7] Mohamed Elkalwagy, Pascal Bercher, Bernd Schattenberg, and Susanne Biundo, ‘Improving hierarchical planning performance by the use of landmarks’, in AAAI Conference on Artificial Intelligence, pp. 1763–1769, (2012).

[8] Kutluhan Erol, James Hendler, and Dana S. Nau, ‘UMCP: A sound and complete procedure for hierarchical task-network planning’, pp. 249–254. (June 1994). ICAPS 2009 influential paper honorable mention.

[9] J. Hoffmann and Bernhard Nebel, ‘The FF planning system’, Journal of Artificial Intelligence Research, 14, 253–302, (2001).

[10] Jorg Hoffmann, Julie Porteous, and Laura Sebastian, ‘Ordered landmarks in planning’, Journal of Artificial Intelligence Research, 22, 215–278, (2004).

[11] Erez Karpas and Carmel Domshlak, ‘Cost-optimal planning with landmarks’, in IJCAI 2009, Proceedings of the 21st International Joint Conference on Artificial Intelligence, Pasadena, California, USA, July 11-17, 2009, ed. Craig Boutilier, pp. 1728–1733, (2009).

[12] Derek Long, Henry A. Kautz, Bart Selman, Blai Bonet, Hector Geffner, Jana Koehler, Michael Brenner, Jorg Hoffmann, Frank Ritter, Corin R. Anderson, Daniel S. Weld, David E. Smith, and Maria Fox, ‘The aips-98 planning competition’, AI Magazine, 21, 13–33, (2000).

[13] B. Marthi, S.J. Russell, and J. Wolfe, ‘Angelic semantics for high-level actions’, in International Conference on Automated Planning and Scheduling, (2007).

[14] B. Marthi, S.J. Russell, and J. Wolfe, ‘Angelic hierarchical planning: Optimal and online algorithms’, in International Conference on Automated Planning and Scheduling, pp. 222–231, (2008).

[15] Dana S. Nau, Tsz-Chiu Au, Okhtay Ilghami, Ugur Kuter, J William Murdock, Dan Wu, and Fasun Yaman, ‘SHOP2: An HTN planning system’, Journal of Artificial Intelligence Research, 20, 379–404, (2003).

[16] Dana S. Nau, Yue Cao, Amnon Lotem, and Héctor Muñoz-Avila, ‘SHOP: Simple hierarchical ordered planner’, in International Joint Conference on Artificial Intelligence, ed., Thomas Dean, pp. 968–973, (August 1999).

[17] Silvia Richter and Matthias Westphal, ‘The LAMA planner: Guiding cost-based anytime planning with landmarks’, J. Artif. Intell. Res. (JAIR), 39, 127–177, (2010).

[18] Vikas Shivashankar, Ron Alford, Ugur Kuter, and Dana Nau, ‘The GoDel planning system: a more perfect union of domain-independent and hierarchical planning’, in Proc. of the 23rd Int. Joint Conf. on Artificial Intelligence (IJCAI), pp. 2380–2386. AAAI Press, (2013).

[19] Vikas Shivashankar, Ugur Kuter, Dana Nau, and Ron Alford, ‘A hierarchical goal-based formalism and algorithm for single-agent planning’, in Proc. of the 11th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS), volume 2, pp. 981–988. Int. Foundation for Autonomous Agents and Multiagent Systems, (June 2012).

[20] Fazlul Hasan Siddiqui and Patrik Haslum, ‘Continuing plan quality optimisation’, J. Artif. Intell. Res. (JAIR), 54, 369–435, (2015).

[21] Salome Simon and Gabriele Roger, ‘Finding and exploiting lit trajectory constraints in heuristic search’, in Symposium on Combinatorial Search, (2015).

[22] Shirin Sohrabi, Jorge Baier, and Sheila McIlraith, ‘Htn planning with preferences’, in IJCAI, (2009).

[23] Nathaniel Waisbrot, Ugur Kuter, and Tolga Konik, ‘Combining heuristic search with hierarchical task-network planning: A preliminary report’, in International Conference of the Florida Artificial Intelligence Research Society, (2008).