Learning function using a thermometer

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Abstract. This research aimed to produce a learning trajectory that can help students in 8th grade in learning functions. The learning approach used in this study was PMRI approach by using the context of the thermometer. This research used design research through three stages: preparing for the experiment, the design of experiments, and the retrospective analysis. Data collection techniques used were interviews, video footage and photographs, written tests, and field notes. The study involved six students in the 8th grade, which were composed of students with high, medium and low of capability in the first cycle, and involved 20 students in second cycle. The research conducted to produce learning trajectories which consists of a series of learning activity in the process. The results of this study are to produce a learning path consisting of exploring the context of the thermometer, presenting functions, determining (domains, codomains and ranges), finding function formulas and function values, and being able to solve the function problems.

1. Introduction

Function is considered a unifying concept both in the mathematics curriculum and in real life [1-9]. The definition of a function from a set $A$ to set $B$ is a special relationship that connects each $a \in A$ to exactly one $b \in B$ written $f:A \rightarrow B$. The set $A$ is called the domain of function $f$ and the set $B$ is called the codomain while the subset of set $B$ which is a map of set $A$ is called range [10]. The actual formula is a function and the concept of the function is shown by a developing pattern [11]. The developing pattern has five representations namely; (1) the pattern, that is, as context; (2) charts or tables; (3) symbolic equation; (4) graphs; and (5) languages. Each of the five representations has the same functional and recursive relationship. The interrelationship between a number in a row of numbers is an example of a function [12]. In this case, the relationship can be interpreted as a rule, formula, relationship, correspondence, simple format, not exaggerating [13].

When teaching functions, students must be given definitions according to where they can form their own functions [1, 8, 14]. On average, only 43% of international students are able to correctly answer questions involving functions, whereas only 24% of Indonesian students are able to correctly answer questions involving functions [15]. In addition, the percentage of mastery of the material in the 2016/2017 National Examination, students throughout Indonesia only 48.96% were able to correctly answer questions involving functions [16].

Based on the background and problems that have been explained, the problems in this study can be formulated into 3 main questions: (1) how students can find functions using a thermometer; (2) how students can present functions; (3) How students can solve problems about function. Thus, this study aims to produce learning trajectories consisting of exploring the context of the thermometer, presenting functions, determining (domains, domain codes and ranges), finding function formulas and function values, and being able to solve function problems.
2. Method

This study uses a design research method which is a form of qualitative approach. Design research is a systematic study of designing, developing and evaluating educational inventions (such as programs, strategies and learning materials, products and systems) as a solution to solving complex problems in educational practice [17]. Design research aims to develop local instructional theory which is based on existing theory (theory-driven) and empirically based experiments through collaboration between researchers and teachers to improve the relevance of research to educational policies and practices [18]. The stages in this study consisted of preparing for the experiment, the design experiment and retrospective analysis.

The stages in this research are as follows:

a. Stage I: Preparing for the experiment and preliminary design

At this stage the literature review is carried out on the subject matter of mathematical functions, approaches used, alleged strategies and thoughts and conducts discussions with the mathematics teacher about class conditions, research needs, selects observers, adjusts the schedule and the way research is conducted with the teacher concerned.

At this stage hypothetical learning and learning pathways are also designed. Hypothetical instructional theories are formulated consisting of (learning objectives, learning activities, and tools). Allegations are dynamic so they can be arranged and revised during the actual learning process (teaching experiments). The results of this stage are lesson plans, student worksheets, and teacher guides.

b. Stage II: Design Experiments.

At this stage, a series of learning activities are carried out then the researcher observes and analyses what happens during the learning process. This process aims to evaluate the conjectures contained in learning activities. Teaching trials are recorded using photo and video documentation.

c. Stage III: Retrospective Analysis.

After the experiment, data is obtained from learning activities in class and then analysed and the results are used to plan activities or to develop designs for further learning activities. The purpose of retrospective analysis is to develop local learning theories. At this stage, HLT is compared with actual student learning and answers are obtained from the problem formulation.

This study involved twenty students from 8th grade and a mathematics teacher as a model teacher and conducted in the academic year 2018/2019. The subjects of the study were 8th grade students at Xaverius Maria Palembang Middle School.

3. Result and Discussion

The initial stage carried out in this research is designing hypothetical instructional theory (HLT) regarding the function material to be taught which includes learning objectives, learning activities, and students' conjectures of thought. The main objective of this stage is to develop learning sequences and design instruments to evaluate the learning process [19]. The preparation of the HLT is also based on the results of an initial survey of students' abilities and discussions conducted by researchers with mathematics teachers in Xaverius Maria Palembang.

The HLT was compiled based on a literature review and the results of the observations of the students' initial abilities, then discussed again with the mathematics subject teacher in the 8th grade of Xaverius Maria Palembang Middle School and with the supervisor. The results of this discussion are then used as input to improve HLT. The revised HLT has 4 activities namely exploring the context of the thermometer, presenting functions and determining domains, codomains and ranges, finding function equations and values, and being able to solve problems about functions.

The researcher tried out the learning activities in the pilot experiment to see students' abilities and make adjustments to HLT. The test aims to explore and hypothesize students' strategies and thoughts during the actual learning process [19]. Based on the trials that have been carried out, then the HLT is revised again. The revised learning trajectory was then trialled in a teaching experiment to see the learning summary that had been compiled on the learning function in the 8th grade of junior high school. The learning trajectory made in this study uses the PMRI approach. The following activities are described using the PMRI approach in the learning process.
3.1. Activity 1: Exploring The Context of The Thermometer

The teacher opens the learning process with apperception activities, conveys basic competencies, learning objectives and activities to be carried out. Furthermore, the teacher asks students to sit in groups that have been determined. Then the teacher distributes activity sheet 1. Next, the teacher guides the students to discuss the answers to the questions with their respective group friends.

For activity 1, students are given a question that directs students to find out the scale of the two thermometers, able to distinguish between the two thermometer scales. After that students conclude that the relationship between temperature at degrees Celsius and Fahrenheit is a function. The following pictures and transcripts on this activity.

![Figure 1](image1)

** Observer:** For number 4, is it sorted according to the picture?

**Student:** Yes, I only write down the numbers listed according to the thermometer image.

**Observer:** For number 5, why is the relationship between thermometer temperatures called functions?

**Student:** Yes, sir, because it matches the characteristics of its function. Only have one partner, sir. So, for example, 20 to 68 can no longer be 20 to 40.

From the transcript and Figure 1, students are able to explain that the relationship between Celsius and Fahrenheit is an example of a function. After students finish working on activity sheet 1, the teacher asks representatives of one group to present the results of their discussion, and other groups respond to it. Then the teacher concludes the results of the discussion.

3.2. Activity 2: Present functions (tables, arrow diagrams and sequential pairs of sets) and specify domains, codes and ranges

In this activity, students are asked to explore the scale on the thermometer, then presented in tables, arrow diagrams and sequential pairs of sets. The following figure and transcripts of this activity.

![Figure 2](image2)

**Figure 2.** Student’s answers to questions number 1 and 2 worksheets 2.
Observer: To determine table 0 to 32, how do you get it?
Student: count, sir, if each Fahrenheit increases to two, 20, 22, 24, 26, 28, 30, 32
Observer: For the arrow diagram?
Student: according to the table above
Observer: For a set of sequential pairs, is it the same as the table?
Student: yes sir.

From the transcript and Figure 2, students are able to determine the scale on Fahrenheit if the centigrade is known, and can present it back to the arrow diagram and the set of sequential pairs. In the next activity, students determine the domain, codomain and range, along with a figure and transcripts of this activity.

![Figure 3](image)

**Figure 3.** Student’s answers to question number 3 worksheets 2.

Observer: For domain, codomain and range, look at the arrow diagram, which one is the codomain?
Student: which is the right domain.
Observer: domain?
Student: on the left
Observer: what is the range?
Student: Range is the result area that has a partner and is in the codomain.

From the transcript and Figure 3, students are able to determine the domain, code and range in the arrow diagram. After students finish working on worksheet 2, the teacher asks representatives of one of the groups to present the results of the discussion, and the other groups respond to it. Then the teacher concludes the results of the discussion.

3.3. Activity 3: Finding function formulas and function values
The teacher opens the learning process with apperception activities related to determining formulas in number patterns that have been studied previously. Next the teacher conveys basic competencies, learning objectives and activities to be carried out. Furthermore, the teacher asks students to sit in groups that have been determined. Then the teacher distributes worksheets 3. Next, the teacher guides the students to discuss the answers to the questions that are with their group friends.

For activity 3 in this activity the students write back to the large scale table at Celsius and Fahrenheit. Then students are assisted by the teacher, trying to find functional relationships. Then write down the equation of the function. Next, figure and transcripts of this activity.

![Figure 4](image)

**Figure 4.** Student’s answers to question number 1 worksheets 3.
Observer: where did you find $\frac{9}{5}$ From the table that was answered?
Student: $\frac{9}{5}$ Get from a comparison between Fahrenheit and Celsius.
Observer: where did the number 0 come from?
Student: 0 is obtained from Celsius
Observer: why plus 32?
Student: to produce Fahrenheit with number 32
Observer: So the formula that you got from us?
Student: $F = C + 32$

From the transcript and Figure 4, students can make functional equations by determining functional relationships first. In the next activity, students are directed to determine the value of functions using the equations they have created. The following figure and transcripts of this activity.

**Figure 5.** Student’s answers to questions number 2 and 3 worksheets 3.

Observer: For question number 2, how is the process?
Student: use the previous formula.
Observer: Why did you replace 78?
Student: because the Celsius thermometer shows the number 78 degrees
Observer: how about number 3?
Student: From the formula, Fahrenheit was changed, because Fahrenheit's temperature was 206.6

From the transcript and Figure 5, students can determine the value of the function using the existing formula. After students finish working on the activity sheet, the teacher asks representatives from one of the groups to present the results of the discussion, and the other groups respond to it. Then the teacher concludes the results of the discussion.

### 3.4. Activity 4: solve problems about functions
In this activity, students are asked to solve problems related to the function, determine the function equation and determine the value of the function. The following figure and transcripts of this activity.
Figure 6. Student’s answers to questions number 1 and 2 worksheets 4.

Observer : does this fill the table according to the graph?
Student : Yes
Observer : Where is this 7/2 from?
Student : from a comparison between rates and distance.
Observer : 7 from where?
Student : 7 of the price
Observer : then 2?
Student : from distance
Observer : from this price, what has changed?
Student : the distance changes
Observer : plus the number 3 what is it for?
Student : in order to get a price value
Observer : So if we enter the distance 2, the price?
Student : 10
Observer : so the formula?
Student : $y = \frac{7}{2}x + 3$

From the transcript and Figure 6, students are able to make functional equations by determining functional relationships first. In the next activity, students are directed to determine the value of the function using the equations they have made. The following figure and transcripts of this activity.

Figure 7. Student’s answers to question number 3 worksheets 4.

Observer : How about this one?
Student : what is asked is price
Observer : So, where was the 17 included?
Student : 17 it is entered at a distance

From the transcript and Figure 7, students are able to determine the value of the function if the formula is known. After students finish working on the activity sheet, the teacher asks representatives
of one of the groups to present the results of the discussion, and the other groups respond to it. Then the teacher concludes the results of the discussion. The final activity of the teacher guides students to conclude learning at this second meeting.

4. Conclusion
Based on the results of the research and discussion described previously, it can be concluded that this research has produced a learning trajectory that can assist 8\textsuperscript{th} grade students in solving problems related to the function. The things in this study that are able to support students to solve functional problems are (a) The activity explores a room thermometer that can help students get to know the function; (b) The activity of determining the relationship between scales on a room thermometer can help students to find the functional relationship which will later become a function formula and then determine the value of the function. (c) Using functional relationships in tables is important to be directed to students. If it is not directed towards determining functional relationships, only students with high ability are able to solve functional problems. Thus, the learning trajectory that has been implemented in this study is a form of positive contribution to the development of Local Institutional Theory (LIT) in learning functions using the thermometer context.

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