Power-law to Power-law Mapping of Blazar Spectra from Intergalactic Absorption

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Abstract.
We have derived a useful analytic approximation for determining the effect of intergalactic absorption on the \(\gamma\)-ray spectra of TeV blazars in the energy range \(0.2\) TeV < \(E_\gamma\) < \(2\) TeV and the redshift range \(0.05 < z < 0.4\). In these ranges, the form of the absorption coefficient \(\tau(E_\gamma)\) is approximately logarithmic. The effect of this energy dependence is to steepen intrinsic source spectra such that a source with an approximate power-law spectral index \(\Gamma_s\) is converted to one with an observed spectral index \(\Gamma_o \simeq \Gamma_s + \Delta \Gamma(z)\) where \(\Delta \Gamma(z)\) is a linear function of \(z\) in the redshift range \(0.05 – 0.4\). We apply this approximation to the spectra of 7 TeV blazars.

1. Introduction
Stecker, Malkan & Scully [2] (SMS) have used recent Spitzer observations [3], [4] along with other data on galaxy luminosity functions and redshift evolution in order to make a detailed evaluation of the intergalactic photon density as a function of both energy and redshift for \(0 < z < 6\) for photon energies from .003 eV to the Lyman limit cutoff at 13.6 eV in a \(\Lambda\)CDM universe with \(\Omega_\Lambda = 0.7\) and \(\Omega_m = 0.3\). They then used their calculated intergalactic photon densities to calculate the optical depth of the universe, \(\tau\), for \(\gamma\)-rays having energies from 4 GeV to 100 TeV emitted by sources at redshifts from 0 to 5. They also gave a parametric fit with numerical coefficients for \(\tau(E_\gamma, z)\).

As an example of the application of the detailed numerical determination of the optical depth, SMS calculated the absorbed spectrum of the blazar PKS 2155-304 at \(z = 0.117\) and noted that an \(E^{-2}\) power-law differential photon source spectrum for this source would be steepened by approximately one power to \(E^{-3}\) as would be produced by a \(\gamma\)-ray opacity with a logarithmic energy dependence.

The purpose of this letter is to generalize this result by determining fits for approximating \(\tau(E_\gamma, z)\) by logarithmic functions in order to predict power-law steepenings in assumed blazar source spectra of the form \(E^{-\Gamma_s}\) to observed spectra of the form \(E^{-\Gamma_o}\) where \(\Gamma_o \simeq \Gamma_s + \Delta \Gamma\) and \(\Delta \Gamma\) is determined to be a linear function of \(z\).

2. Calculation
We fit the results from SMS for \(\tau(E_\gamma, z)\) to a form which is assumed to be logarithmic in \(E_\gamma\) in the energy range \(0.2\) TeV < \(E_\gamma\) < \(2\) TeV and which has a linear dependence on \(z\) over the range \(0.05 < z < 0.4\). We can then choose the form of this fitting function to be
\[ \tau(E_\gamma, z) = (A + Bz) + (C + Dz) \ln[E_\gamma/(1 \text{ TeV})]. \]

**Table 1. Optical Depth Parameters**

| Evolution Model | A       | B   | C      | D   |
|-----------------|---------|-----|--------|-----|
| Fast Evolution  | -0.475  | 21.6| -0.0972| 10.6|
| Baseline        | -0.346  | 16.3| -0.0675| 7.99|

If we then postulate an intrinsic source spectrum which can be approximated by a power law over this limited energy range of one decade,

\[ \Phi_s(E_\gamma) \simeq KE_{\gamma}^{-\Gamma_s}, \]

the spectrum which will be observed at the Earth following intergalactic absorption will be of the power-law form

\[ \Phi_o(E_\gamma) = Ke^{-(A+Bz)}E_{\gamma}^{-\Gamma_s+(C+Dz)}. \]

This can be compared with the empirically observed spectrum which is usually presented in the literature to be of the power-law form. The observed spectral index, \( \Gamma_o \), will then be given by

\[ \Gamma_o = \Gamma_s + \Delta \Gamma(z) \]

where the intrinsic spectral index of the source is steepened by \( \Delta \Gamma(z) = C + Dz \).

The parameters \( A, B, C, \) and \( D \) obtained by fitting the optical depths derived for the fast evolution (FE) and baseline (B) models of SMS are given in Table 1.

**3. Spectral Indices of Individual Sources**

Table 2 gives a list of blazars which have been detected at TeV energies and for which spectral indices (\( \Gamma_o \)) have been measured in the energy range 0.2 – 2 TeV. We also give the observed redshifts of these sources and the intrinsic spectral indices of the sources (\( \Gamma_s \)) derived from the baseline (B) and fast evolution (FE) models using our analytic expressions for \( \Delta \Gamma(z) \). A spectral index of less than 2 indicates that the energy range of the observation would be below the Compton peak energy in the spectral energy distribution, \( E_{\gamma}^2 \Phi(E_\gamma) \) of a synchrotron self-Compton (SSC) model [5]. In the case of the blazar PG 1553+113, the energy range of the observation is from 0.09 to 0.4 TeV which is somewhat below the energy range we have used in deriving our approximation.

**4. Conclusions**

We have derived a useful analytic approximation for determining the effect of intergalactic absorption on the spectra of \( \gamma \)-ray sources in the energy range 0.2 TeV \( < E_\gamma < 2 \) TeV and the redshift range 0.05 \( < z < 0.4 \), where the absorption is primarily from interactions with intergalactic photons in the optical to near infrared wavelength range. In these energy and redshift ranges, the form of the absorption coefficient \( \tau(E_\gamma) \) is approximately logarithmic. The effect of this energy dependence is to steepen the intrinsic source spectrum such that a source with an approximate power-law spectral index \( \Gamma_s \) is converted to one with an observed spectral
Table 2. Spectral Indices for Seven Observed Blazars in the 0.2 – 2 TeV Energy Range

| Source     | z      | $\Gamma_o$ | $\Gamma_s$ (B) | $\Gamma_s$ (FE) |
|------------|--------|------------|----------------|-----------------|
| Mkn 180    | 0.045  | 3.3 [6]    | 3.0            | 2.9             |
| PKS 2005-489| 0.071  | 4.0 [7]    | 3.5            | 3.4             |
| PKS 2155-304| 0.117  | 3.3 [8]    | 2.4            | 2.2             |
| H 2356-309 | 0.165  | 3.1 [9]    | 1.9            | 1.5             |
| 1ES 1218+30 | 0.182  | 3.0 [10]   | 1.6            | 1.2             |
| 1ES 1101-232| 0.186  | 2.9 [11]   | 1.5            | 1.0             |
| PG 1553+113| 0.36   | 4.2 [12]   | 1.4            | 0.5             |

The index $\Gamma_o \simeq \Gamma_s + \Delta \Gamma$ in the energy range 0.2 – 2 TeV, where $\Delta \Gamma(z)$ is a linear function of $z$ in the redshift range 0.05 – 0.4. We have applied this approximation to the spectra of seven TeV blazars. These power-law approximations, both observational and theoretical, are only useful over the limited energy range indicated, viz. 0.2 – 2 TeV. The actual spectra should exhibit some curvature. At higher energies the spectra should cut off either because of a natural upper limit to the source spectra or, more likely in the case of high frequency peaked BL Lac objects (HBLs), because of the increase in optical depth with energy owing to interactions with the much more numerous intergalactic far infrared photons. A full numerical treatment of absorption can be made using the exact form of the optical depth as a function of redshift and energy as calculated in the detailed work of SMS. Examples of more detailed treatments of effects of intergalactic $\gamma$-ray absorption are given for Mkn 501 and Mkn 421 by Konopelko et al (2003) and for PKS 2155-304 by SMS. However, because of the limited energy range of empirical TeV spectral determinations, and because of the significant statistical and systematic uncertainties in the empirical determinations of spectral indices of TeV sources, our approximate linear relations can be quite useful in making rapid and simple analyses of TeV source spectra.

Acknowledgments
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