Numerical simulation of strong expansion and collapse of cavitation bubbles with centers located in a straight line

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Abstract. A technique for numerical investigation of strong expansion and collapse of cavitation bubbles is presented. The bubbles are located on a straight line, radially convergent shock waves are supposed to appear inside the bubbles in the final stage of their collapse. The efficiency of the technique is achieved by using different models for simulating the expansion of the bubbles and the beginning of their collapse where the interaction between the bubbles is essential, and for simulating the end of the collapse of the bubbles where their interaction is insignificant. The first model is that of joint dynamics of weakly-nonspherical bubbles. In that model, it is assumed that the liquid is weakly compressible, its motion is potential, the vapor in the bubbles is homobaric, the pressure inside the bubbles is equal to the saturated vapor pressure at the ambient liquid temperature. The second model is that of dynamics of a single axisymmetric bubble. It takes into account the liquid compressibility, the nonstationary heat conductivity of liquid and vapor, the nonequilibrium evaporation and condensation at the interphase boundaries, the considerable nonsphericity of the bubble and the strong non-uniformity of the vapor in the final stage of the bubble collapse. It utilizes realistic equations of state for liquid and vapor. An example illustrating the effect of peripheral bubbles on the dynamics of the central one is given in the case of three bubbles located on a straight line.

1. Introduction

It is experimentally found [1] that in the final stage of bubble collapse it is possible for a short time to reach inside a bubble a temperature of more than 16 000 K. Realization of even higher temperatures in bubbles is reported in a number of publications by Taleyarkhan and co-authors [2, 3]. The possibility of achieving such high temperatures is of great interest for science and applications. In particular, it can be used for the intensification of chemical reactions in sonochemistry [4]. The highest parameters inside a bubble are attained when the strong collapse of the bubble and the compression of its contents are close to spherical. In such a case nearly-spherical radially-converging shock waves appear in the bubble in the final stage of its collapse [5]. The convergence and focusing of those waves lead to very high temperatures, densities, and pressures in a small central area of the bubble. It is believed [6] that the highest parameters are achieved in the case of a bubble located in the central region of a cluster of bubbles. At the same time, the presence of neighboring bubbles can also reduce those parameters due to, for example, deflecting the bubble shape from the spherical one. To get first estimate of the influence of the neighboring bubbles on strong collapse of a bubble, one can use a linear streamer of bubbles in which the bubbles are located on the same straight line. Such a streamer is the simplest
model of a cluster. This paper presents an economical technique for calculating the dynamics of cavitation bubbles during their strong expansion and collapse in the central region of the linear streamer in the case the deformations of the bubble at the final stage of its collapse are not small.

2. Problem statement

Strong expansion and collapse of cavitation bubbles with centers located in a straight line is considered. At the initial time moment \( t = 0 \) the bubbles are spherical, equal, equidistant, filled with the saturated vapor of the surrounding liquid (Fig. 1). The pressure in the liquid \( p_x \) oscillates harmonically: \( p_x = p_0 - p_a \cos \omega t \) (\( p_a \) is the oscillation amplitude, \( \omega \) is the oscillation frequency, \( p_0 \) is the static pressure). In the negative acoustic pressure phase the bubbles expand, whereas in the positive one their expansion stops and they collapse. The main attention is paid to the central bubbles, which, in the streamer with a sufficiently large number of bubbles, remain practically motionless relative to the other bubbles due to the fact that the action of the neighboring bubbles located on their different sides is equal in value but opposite in direction.

![Figure 1. A linear streamer of identical equidistant bubbles](image)

\( R_i \) is the radius of the \( i \)-th bubble, \( d \) is the distance between the bubble centers.

3. Numerical technique

The present numerical technique of calculating the dynamics of bubbles in a linear streamer is in fact a combination of two techniques previously developed by the authors for computing the coupled dynamics of two slightly nonspherical bubbles [7] and the dynamics of a single axisymmetric bubble [8]. At the expansion of the bubbles in the streamer and at the low-speed beginning of their collapse the technique of [7], slightly generalized to the case of the linear streamer, is used. That technique assumes that the liquid is weakly compressible, its motion is potential, the vapor in the bubbles is homobaric, the pressure inside the bubbles is constant and equal to the pressure of the saturated vapor at the ambient liquid temperature. The bubble sphericity distortions are assumed small so that the problem is split into spherical and non-spherical components. As a result, the dynamics of interacting bubbles is described by a system of ordinary differential equations in radii of the bubbles, the coordinates of their centers and the amplitudes of the deviations of their shapes from the spherical one in the form of spherical harmonics.

At the high-speed stage of collapse of the bubbles their interaction becomes insignificant due to their small size. At the same time, it becomes necessary to take into account the non-uniformity of the vapor inside the bubbles, the heat conductivity of liquid and vapor, the nonequilibrium evaporation and condensation on the interphase surfaces, the considerable nonsphericity of the bubbles, the nonsphericity of dynamics of vapor in the bubbles, including the shock waves that arise in them. Therefore, the technique of computing the dynamics of a single axisymmetric bubble of work [8] is used at this stage for the motionless central bubbles in the streamer. For every bubble it solves a system of gas dynamics equations governing the liquid and vapor motion presented in moving coordinates with utilising spherical or cylindrical co-ordinates as a fixed reference system. The spherical reference system is first applied together with a radially divergent computational grid. When the shape of the shock wave in the bubble significantly deviates from the spherical one, that system is changed by the cylindrical reference system, and the radially divergent grid in the central part of the bubble is changed by a grid close to the Cartesian one. The numerical solution is found by an original modification of the Godunov method of second order of accuracy.
The moment \( t^* \) of transition from the low-speed stage of the bubble collapse to its high-speed stage is determined by a series of computational experiments with varying the moment of switching-off the bubble interaction. The switching-off moment is increased until the dynamics of the considered central bubble in the streamer with bubble interaction being turned on and off is sufficiently close. The proximity is estimated by comparing the time dependences of the radius of the central bubble and the amplitudes of the deviation of its shape from the spherical one.

At the transition moment \( t^* \) the pressure, density, temperature, and velocity fields are presented as a sum of their spherical components and their small non-spherical perturbations. The spherical components are determined by computing the considered central bubble collapse using a one-dimensional (spherically symmetric) version of the two-dimensional (axially symmetric) single bubble dynamics model [8]. Their non-spherical perturbations are derived based on the results of computing the expansion and collapse of the considered central bubble using the model of interacting bubbles [7]. It is taken into account that the interaction between the bubbles during their collapse may influence their dynamics. Therefore, the dynamics of the central bubbles is approximated by the dynamics of single bubbles. For this purpose, the liquid pressure on the external boundary of the numerical domain (it is assumed constant throughout the collapse) is varied until the time dependencies of the bubble radii in the cases of the single and interacting bubbles are close to each other. The found liquid pressure is later used for calculating the final high-speed stage of the bubble collapse using the single axisymmetric bubble dynamics model.

4. Application illustration
The strong expansion and collapse of three cavitation bubbles in liquid acetone with centers located on a straight line is considered. The liquid pressure oscillates with \( p_a = 15 \text{ bar}, \omega = 2\pi \times 19.3 \text{ kHz} \) about \( p_0 = 1 \text{ bar} \), the liquid temperature is 273 K. At \( t = 0 \), the radius of the bubbles is 5 \( \mu \text{m} \), their internal pressure is 0.09 bar, the distance between the bubbles is \( 11 R_{\text{max}} \) where \( R_{\text{max}} = 438 \mu\text{m} \) is the radius of the central bubble at the moment of its maximum expansion.

Computational experiments show that in the considered case, the moment at which the radius of the central bubble during its collapse is equal to 150 \( \mu\text{m} \) can be taken as the time instant \( t^* \) separating the low-speed and high-speed stages of the bubble collapse. To determine the spherical components of the pressure, density, temperature, and velocity fields at the moment \( t^* \) one can use the results of calculations of single spherically symmetric bubble collapse at a liquid pressure of 12.9 bar.

Computations show that the central bubble first expands to a radius of \( R_{\text{max}} = 438 \mu\text{m} \) and then collapses to a radius (average in volume) of \( R_{\text{min}} \approx 19 \mu\text{m} \). Due to the influence of the neighboring bubbles, its originally spherical shape changes with time. During the expansion and the most part of the collapse, the shape of the central bubble is determined by the second surface harmonic. During the expansion, the relative amplitude (referred to the current radius of the bubble) of its non-sphericity in the form of the second surface harmonic grows monotonically to 0.00086 (the bubble becomes increasingly flattened along the axis passing through the centers of the interacting bubbles). During the collapse, the central bubble first becomes spherical, then slightly elongated, after that again spherical, and finally flattened. At the end of collapse, the dimensionless amplitude of its non-sphericity in the form of the second and fourth harmonics increases to 0.08 and 0.015, respectively.

Figure 2 shows the bubble shape variation, the change of the pressure and temperature fields inside the central bubble and in the surrounding liquid layer in the final stage of collapse. A radially convergent shock wave is seen to appear in the bubble. At the moment of its formation (moment 1), the shock wave and the pressure and temperature fields in vapor are close to spherical. As the shock wave converges to the center of the bubble (moments 1-3), their nonsphericity grows quite rapidly and strongly. By moment 3 the bubble is slightly flattened, and the region with high pressures and temperatures is located in the vicinity of the most convex part of the shock wave front.
Figure 2. The pressure and temperature fields in the central bubble and the surrounding liquid at five consecutive time moments (indicated by numbers 1-5) in the final stage of the bubble collapse. The black thick solid line is the surface of the bubble, the black thin dotted line is the boundary of the central region of the bubble of 16 μm in diameter. The upper row and the left side of the middle row correspond to a region of 35 μm in diameter, the right side of the middle row and the lower row correspond to a region of 16 μm.

Then (moments 4 and 5) the maximum pressure and temperature levels increase until the beginning of the interaction of the upper and lower parts of the shock wave. The interaction is realized in the form of collision of two near-planar shock waves. The cross-sectional size of the shock wave at the moment of the collision is 6 μm, which is much larger than the super-hot nucleus size in the case of a single spherical bubble (∼ 100 nm) [2, 3]. This means that the presence of the peripheral bubbles leads to a decrease in the energy focusing within the bubble.
5. Application illustration
An efficient technique for calculating the strong expansion and collapse of cavitation bubbles in the central region of a linear streamer (the centers of the bubbles are located on a straight line) is presented. The initially small non-sphericity of the bubbles can significantly increase to the end of their collapse. The technique is based on the joint application of models (previously created by the authors) of dynamics of interacting weakly-spherical bubbles and the dynamics of a single non-spherical bubble. The efficiency of such a combination is achieved by taking into account the features of the considered phenomenon. At the low-speed expansion of the bubbles and at the stage of low-speed beginning of their collapse the hydrodynamic interaction between the bubbles significantly influences the evolution of their shape, whereas the bubbles remain close to homobaric, and the non-spherical deformations of their surfaces are small. In the final high-speed stage of the bubble collapse it is important to take into account the liquid compressibility, the non-uniformity of the vapor in the bubbles, the evaporation and condensation on the interphase surfaces, the formation and focusing of the non-spherical shock waves in the central region of the bubbles, whereas the interaction of the bubbles is inessential. An example of the use of the presented technique for calculating strong expansion and collapse of a central bubble in the simplest streamer consisting of three bubbles is given.

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