Minimal Radiative Neutrino Mass Mechanism for Inverse Seesaw Models

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ABSTRACT

We study a minimal one-loop radiative mechanism for generating small Majorana neutrino masses in inverse seesaw extensions of the Standard Model with two singlet fermions per family. The new feature of this radiative mechanism is that the one-loop induced left-handed neutrino mass matrix is directly proportional to the Majorana mass matrix of the right-handed neutrinos. This is a very economical scenario without necessitating the existence of non-standard scalar or gauge fields.

I. INTRODUCTION

The understanding of the extra-ordinary smallness of neutrino masses, along with their observed large mixings [1], remains one of the major puzzles in the Standard Model (SM), and so a potential portal to New Physics. The simplest theoretical scenario which may explain this neutrino puzzle would be to introduce singlet Majorana masses that break electroweak symmetry breaking via the vacuum expectation value (VEV) of the Higgs field, \( \langle \Phi \rangle = (0, v)^T \), the SU(2)\(_L\)-doublet neutrinos \( \nu_{iL} \) receive a non-zero mass matrix of the form \( (M_{\nu_{iL}})_{ij} = \lambda_{ij}v^2/\Lambda \).

There exist three tree-level realizations [3–5] of this effective dimension-5 operator using only renormalizable interactions. The simplest one, widely known as the type-I seesaw [3], requires the extension of the SM by a number of extra-CHARGED leptons. The simplest one, widely known as the type-I seesaw [3], requires the extension of the SM by a number of extra-leptons, \( \nu_L \), which in turn implies the following (3 + \( n_R \)) complex symmetric matrix in the flavor basis \( \{(\nu_L)^C, \nu_{\alpha R}\} \):

\[
\mathcal{L}_\nu = \left(\begin{array}{c}
\nu_L \Phi \nu_{\alpha R} + \frac{1}{2} \bar{\nu}_{\alpha R}(M_R)_{\alpha \beta} \nu_{\beta R} + \text{H.c. ,}
\end{array}\right)
\]

which in turn implies the following \( (3 + n_R) \times (3 + n_R) \) complex symmetric matrix in the flavor basis \( \{(\nu_L)^C, \nu_{\alpha R}\} \):

\[
\mathcal{M}_\nu = \left(\begin{array}{cc}
0 & M_D \\
M_D^T & M_R
\end{array}\right),
\]

where \( M_D = y_{\nu_L}v \) is the Dirac mass matrix for the neutrinos and \( M_R \) is the \( (B - L) \)-breaking Majorana mass matrix of the RH neutrinos. In the usual seesaw approximation \( (M_D)_{\alpha \rho}/(M_R)_{\alpha \beta} \ll 1 \), the physical light neutrino masses are given by

\[
M_{\nu_L} \approx -M_DM_R^{-1}M_D^T.
\]

Evidently, the seesaw scale \( M_R \) is correlated with the magnitude of the Yukawa coupling \( y_{\nu_L} \). For the experimentally observed sub-eV light neutrinos [1], \( M_R \) is usually constrained to be much larger than the electroweak scale \( m_W \), unless \( y_{\nu_L} \) is very small \( (\lesssim 10^{-6}) \) [cf. (3)] or there are cancellations in the mass matrix structure given by (4) due to specific flavor symmetric patterns of \( y_{\nu_L} \) and \( M_R \).

An interesting realization for the seesaw scale to be in the TeV range is the so-called inverse seesaw model [7], where in addition to RH neutrinos \( \{\nu_{R\alpha}\} \), another set of SM singlet fermions \( \{S_{\rho L}\} \) are introduced. Here we consider a symmetric extension of the SM with three pairs of singlet neutrinos (i.e. \( \alpha, \rho = 1, 2, 3 \)). One of the salient features of the inverse seesaw mechanism is that the small neutrino masses are generated by a small lepton-number breaking \( 3 \times 3 \) mass matrix \( \mu_S \). In contrast to the type-I seesaw case, the mass matrix of the observed left-handed neutrinos

\[
\mu_S = \left(\begin{array}{ccc}
\mu_S & 0 & 0 \\
0 & M_D & M_R
\end{array}\right).
\]
(ν_μ, μ, τ)_L vanishes in the limit of lepton-number conservation: μ_μ → 0. More explicitly, the neutrino Yukawa sector of a general inverse seesaw model is described by the Lagrangian

$$-\mathcal{L}_Y = (y_\nu)_{\alpha\beta} \bar{L}_\alpha \Phi_\alpha R + \bar{S}_{\rho L}(M_N)_{\rho\alpha}\nu_{\alpha R} + \frac{1}{2}\bar{\nu}_{\alpha R}(\mu_R)_{\alpha\beta}\nu_{\beta R} + \frac{1}{2}S_{\rho}L(\mu_S)_{\rho\lambda}S^{C}_{\lambda L} + \text{H.c.} \ ,$$

(4)

which gives rise to the following 9 × 9 neutrino mass matrix in the basis \{(ν_L)^C, ν_{α R}, (S_{ρ L})^C\}:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & \mu_R & M_N^T \\ 0 & M_N & \mu_S \end{pmatrix} \ .$$

(5)

Note that we have not included in \mathcal{L}_Y the dimension-4 lepton-number breaking term \bar{L}\Phi S^C_L which appears, for instance, in linear seesaw models \[5, 6\], since the resulting neutrino mass matrix in presence of this term can always be rotated to the form given in \[5, 9\]. Observe that the standard inverse seesaw model discussed originally in \[7\] is recovered, once we set the RH neutrino Majorana mass \(\mu_R = 0\) in \(\mathcal{L}_Y\). The smallness of the 3 × 3 symmetric matrices \(\mu_{R, S}\) are ‘technically natural’ in the ‘t Hooft sense. In other words, in the limit of \(\mu_{R, S} \to 0\), lepton number symmetry is restored and the light neutrinos ν_{1, 2, 3L} are massless to all orders in perturbation, whereas \{(ν_{1, 2, 3})_R, (S_{1, 2, 3})_L\} form three heavy singlet Dirac neutrinos of approximate mass \(M_{N, 1, 2, 3}\).

In this paper, we analyze another interesting realization of inverse seesaw models, where \(\mu_R \neq 0\), but \(\mu_S = 0\). In this case, the light neutrinos are massless at the tree level, but acquire a small mass at the one-loop level. We show that this one-loop induced light neutrino mass is directly proportional to the Majorana mass matrix \(\mu_R\) and results from well-known SM radiative corrections that involve the \(Z^-\) and Higgs bosons \[10\]. We call this scenario, the Minimal Radiative Inverse Seesaw Model. This is a rather economical scenario as it does not require the existence of other non-standard scalar or gauge fields or other fermionic matter beyond the singlet neutrinos \{ν_{α R}, S_{ρ L}\}. Finally, we also analyze the general case, where both \(\mu_R\) and \(\mu_S\) are non-vanishing and present numerical estimates regarding the typical relative size of the two Majorana mass matrices \(\mu_R\) and \(\mu_S\), while maintaining agreement with neutrino oscillation data.

This paper is organized as follows: In Section II, we discuss the tree-level neutrino mass matrix in inverse seesaw models. In Section III, we present the one-loop neutrino masses and mixings in the Minimal Radiative Inverse Seesaw Model. In Section IV, we analyze the general inverse seesaw model at one-loop level. Finally, our conclusions are given in Section V.

II. TREE-LEVEL NEUTRINO MASSES

To obtain the physical neutrino masses, we cast the mass matrix in \(\mathcal{M}_D\) into a form similar to the type-I seesaw case given in \[2\] by defining a 3 × 6 mass matrix \(\mathcal{M}_D = (M_D, 0)\) and a 6 × 6 mass matrix \(\mathcal{M}_S\) in the weak basis \{(ν_{1, 2, 3})_R, (S_{1, 2, 3})_L\}:

$$\mathcal{M}_S = \begin{pmatrix} \mu_R & M_N^T \\ M_N & \mu_S \end{pmatrix} \ .$$

(6)

We may now perform a block diagonalization of the 9 × 9 neutrino mass matrix \(\mathcal{M}_D\) by a unitary transformation \(\mathcal{V}\) to cast it into the form:

$$\mathcal{V}^T \mathcal{M}_\nu \mathcal{V} = \begin{pmatrix} M_{ν_L} & 0_{3×6} \\ 0_{6×3} & M_{ν_R} \end{pmatrix} \ ,$$

(7)

where the unitary matrix \(\mathcal{V}\) has an exact representation in terms of a 3 × 6 arbitrary matrix \(\zeta\) \[11\]:

$$\mathcal{V} = \begin{pmatrix} (1_3 + \zeta^T \zeta^T)^{-1/2} & \zeta^T (1_6 + \zeta^T \zeta^T)^{-1/2} \\ -\zeta^T (1_3 + \zeta^T \zeta^T)^{-1/2} & (1_6 + \zeta^T \zeta^T)^{-1/2} \end{pmatrix} \ ,$$

(8)
giving rise to the exact analytic expressions for the block mass eigenmatrices \(^{12}\):

\[
M_{\nu R} = (1 + \zeta^T \zeta)^{-1/2} (M_S + M_D \zeta^T) \zeta^*(1 + \zeta^T \zeta)^{-1/2},
\]

\[
M_{\nu L} = -(1 + \zeta^T \zeta)^{-1/2} (M_D \zeta^T + \zeta^T M_D - \zeta M_S \zeta^T) (1 + \zeta^T \zeta)^{-1/2} = -\zeta M_{\nu R} \zeta^T.
\]

The block diagonalization condition implies that the \((1, 2)\) block element of \(V^T M_{\nu} V\) in \(^{11}\) vanishes, i.e.,

\[
M_D - \zeta M_S - \zeta M_D^T \zeta^* = 0_{13 \times 6},
\]

which can be solved for \(\zeta\) in terms of \(M_D\) and \(M_S^{-1}\). Moreover, \(M_S^{-1}\) is obtained by inverting the mass matrix given in \(^{10}\):

\[
M_S^{-1} = \begin{pmatrix}
(\mu_R - M_N \mu_S^{-1} M_N)^{-1} & -(\mu_R - M_N \mu_S^{-1} M_N)^{-1} M_N^{T} \mu_S^{-1} \\
-M_N^{T} \mu_S^{-1} M_N^{-1} & (\mu_R - M_N \mu_S^{-1} M_N)^{-1} M_N^{T} \mu_S^{-1}
\end{pmatrix}.
\]

Note that \(M_S^{-1}\) given by \(^{12}\) is symmetric, since it is easy to show that for symmetric invertible matrices \(\mu_R, \mu_S\),

\[
\left[(\mu_R - M_N \mu_R^{-1} M_N^{T})^{-1} M_N \mu_S^{-1}\right]^{T} = (\mu_R - M_N \mu_S^{-1} M_N)^{-1} M_N^{T} \mu_S^{-1}.
\]

The physical neutrino mass matrices given by \(^{9}\) and \(^{10}\) can be expanded in a converging Taylor series in \(\zeta\), provided its norm is much smaller than unity, i.e., \(||\zeta|| \equiv \sqrt{\text{Tr}(\zeta^T \zeta)} \ll 1\), which is naturally satisfied within the generic seesaw framework. To leading order in \(||\zeta||\), \(^{11}\) implies \(\zeta = M_D M_S^{-1}\), and hence the light neutrino mass matrix given by \(^{10}\) simplifies to

\[
M_{\nu L} = -M_D M_S^{-1} M_D^T + \mathcal{O}(||\zeta||^2)
\]

\[
= -M_D \left( M_S^{-1}\right)^{\nu_R \nu_R^C} M_D^T + \mathcal{O}(||\zeta||^2).
\]

This is analogous to the type-I seesaw formula \(^{9}\), which is shown diagrammatically in Figure 1. Notice that it is only the \(\nu_R \nu_R^C\) component of \(M_S^{-1}\) that contributes to the light neutrino mass matrix at the tree level.

\[\Phi \rangle \quad \text{FIG. 1: The tree-level diagram for the light neutrino mass in the Inverse Seesaw Model.} \]

In the limit \(||\mu_R, \mu_S|| \ll ||M_N||\), the \((1, 1)\) block of \(M_S^{-1}\) given by \(^{12}\) can be expanded in powers of \(\mu_S\). We then obtain from \(^{14}\) the tree-level light neutrino mass matrix \(^{2}\):

\[
M_{\nu L}^{\text{tree}} = M_D M_N^{-1} \mu_S (M_N^T)^{-1} M_D^T + M_D M_N^{-1} \mu_S (M_N^T)^{-1} \mu_R M_N^{-1} \mu_S (M_N^T)^{-1} M_D^T + \mathcal{O}(\mu_S^3),
\]

which identically vanishes in the limit \(\mu_S \to 0\). This can also be deduced from the exact expression in \(^{14}\), since in this limit, \(\zeta M_D^T = 0\) which implies \(\zeta = M_D M_S^{-1}\) from \(^{11}\), and hence, \(M_{\nu L} = 0\) in \(^{10}\). Therefore, in this limit, the full neutrino mass matrix \(M_{\nu}\) in \(^{13}\) has rank 6 and the light neutrinos remain massless at the tree level, even if \(\mu_R \neq 0\).

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\(^{1}\) Some approximate seesaw expressions are also studied in \(^{13}\).

\(^{2}\) Notice that \(||\mu_R, \mu_S||\) could be bigger than \(||M_D||\) in this inverse seesaw approximation.
III. RADIATIVE NEUTRINO MASSES AND MIXINGS AT THE ONE-LOOP LEVEL

At the one-loop level, the light neutrino mass matrix \( M_{\nu_L} \) acquires a small radiative mass from the self-energy diagrams shown in Figure 2. We have drawn the figures in the flavor basis to explicitly show the lepton number violating mass insertions. Note here that the scalar propagator \( \Phi \) includes both the neutral Higgs boson (\( H \)) and the neutral Goldstone boson (\( G^0 \)) contributions. In the on-shell renormalization scheme and in the Feynman gauge, these are the only one-loop diagrams contributing to the neutrino mass, as in the type-I seesaw case [10, 14, 15].

In a general \( R_\xi \)-gauge parameterized by the gauge-fixing parameter \( \xi_Z \), the \( \xi_Z \) dependence of the self-energy for the \( Z \)-loop diagram is exactly canceled by the \( G^0 \)-loop diagram, and the final result is independent of \( \xi_Z \) [14].

The one-loop induced light neutrino masses are obtained from the momentum-dependent self-energy as follows:

\[
-M_{\nu_L}^{1-\text{loop}} P_R = P_R \Sigma(p) P_R \big|_{p=0},
\]

where \( P_R = (1 + \gamma^5)/2 \) is the right-chirality projection operator. Here \( \Sigma(p) \) is the sum of the self-energies of the diagrams shown in Figure 2 and is given by the following gauge-invariant Feynman amplitude:

\[
P_R[i\Sigma(p)]_{PR} = \frac{g^2 m_W^2}{4m_W^2} M_D \int \frac{d^4 k}{(2\pi)^d} P_R \left[ \frac{(\not p - \not k)_{16} + \not M_S}{(p - k)^2 - M_S^2 + i\epsilon} \right] \left[ \frac{1}{(k^2 - m_H^2 + i\epsilon)^2} \right] P_R M_D^T
\]

\[
+ \frac{g^2}{4 \cos^2 \theta_w} M_D \int \frac{d^4 k}{(2\pi)^d} P_R \gamma^\mu \left[ \frac{(\not p - \not k)}{(p - k)^2 + i\epsilon} \right] \left[ \frac{(\not p - \not k)_{16} + \not M_S}{(p - k)^2 - M_S^2 + i\epsilon} \right] \gamma^\nu P_R \left[ \frac{g_{\mu\nu}}{k^2 - m_Z^2 + i\epsilon} \right] M_D^T,
\]

where \( g \) is the weak gauge coupling, \( \theta_w \) is the weak mixing angle, and \( d = 4 - \varepsilon \) is the dimensionality of space-time in the dimensional regularization scheme. Notice that the integrands in (17) vanish in the infra-red (IR) limit \( k^\mu \to 0 \), because of the tree-level vanishing condition on light neutrino masses. However, the loop momentum \( k \) is integrated over all possible values, and so we get a non-zero contribution to the light-neutrino mass matrix at the one-loop level. A straightforward evaluation of the ultra-violet (UV) finite Feynman integrals appearing in (17) leads to the following neutrino mass matrix in (16):

\[
M_{\nu_L}^{1-\text{loop}} = \frac{\alpha_W}{16\pi m_W^2} M_D M_S \left[ \frac{m_H^2}{M_S^2 - m_H^2} \ln \left( \frac{M_S^2}{m_H^2} \right) + \frac{3m_Z^2}{M_Z^2 - m_Z^2} \ln \left( \frac{M_Z^2}{m_Z^2} \right) \right] M_D^T,
\]
FIG. 3: The analytic dependence of the one-loop function $f(x_N)$ [cf. (20)] and $x_N f(x_N)$ on the heavy neutrino mass scale $x_N$.

where $\alpha_w \equiv g^2/4\pi$ is the weak fine structure constant. In the limit $||\mu_{R,S}|| \ll ||M_N||$ and assuming that $M_N = m_N 1_3$, for simplicity, we derive the simpler expression

$$M^{1-\text{loop}}_{\nu_L} \simeq \frac{f(x_N)}{m^2_W} M_D \mu_R M^T_D,$$

(19)

where the one-loop function $f(x_N)$ is given by

$$f(x_N) = \frac{\alpha_W}{16\pi} \left[ \frac{x_H}{x_N - x_H} \ln \left( \frac{x_N}{x_H} \right) + \frac{3x_Z}{x_N - x_Z} \ln \left( \frac{x_N}{x_Z} \right) \right],$$

(20)

with $x_N = m^2_N/m^2_W$, $x_H = m^2_H/m^2_W$, $x_Z = m^2_Z/m^2_W \equiv 1/\cos^2 \theta_w$. From (19), we clearly see that at the one-loop level, the light neutrino mass depends linearly on the lepton number breaking term $\mu_R$ (of dimension-3), and hence it does not vanish in the limit $\mu_S = 0$, unlike at the tree level.

The one-loop function $f(x_N)$ given by (20) is plotted in Figure 3 for $x_H = 1.88$ (corresponding to $m_H = 125$ GeV), $x_Z = 1.30$ and $\alpha_W = 0.033$. Note that in the limit $x_N \gg 1$ (as in the canonical type-I seesaw), the one-loop function stated in (20) becomes

$$f(x_N) \simeq \frac{\alpha_W}{16\pi} \frac{x_H + 3x_Z}{x_N} \ln(x_N).$$

(21)

Hence the light neutrino masses given by (19) are suppressed by $1/m^2_W$:

$$M^{1-\text{loop}}_{\nu_L} \simeq \frac{\alpha_W}{16\pi} \frac{M_D \mu_R M^T_D}{m^2_W} \frac{m^2_H + 3m^2_Z}{m^2_W} \ln \left( \frac{m^2_N}{m^2_W} \right).$$

(22)

On the other hand, it is interesting to note that in the limit $x_N \ll 1$, the one-loop function takes on the form:

$$f(x_N) \simeq \frac{\alpha_W}{16\pi} \ln \left( \frac{x_H x^3_Z}{x_N^4} \right).$$

(23)

This leads to a light neutrino mass which is only logarithmically dependent on the heavy neutrino mass:

$$M^{1-\text{loop}}_{\nu_L} \simeq \frac{\alpha_W}{16\pi} \frac{M_D \mu_R M^T_D}{m^2_W} \ln \left( \frac{m_H m^2_Z}{m^2_N} \right).$$

(24)
It is important to stress here that this $\ln(m_N)$ dependence of the light neutrino mass is a unique feature of the Minimal Radiative Inverse Seesaw Model we have been studying here.

Finally, we wish to offer some comments on the validity of our one-loop analysis presented here. In a general study, one should also consider the loop corrections to the other mass terms appearing in the full inverse seesaw mass matrix in (3), as well as wave-function and mixing renormalizations \cite{14,16}. However, in the MS renormalization scheme, the UV divergences of the Dirac mass in the $\nu_L-\nu_R$ sector can be absorbed in the renormalized $M_D$. On the other hand, all singlet Majorana and Dirac mass matrices in the $\nu_R-S_L$ sector do not get any UV-divergent contributions, at least at the one-loop level.

IV. THE GENERAL INVERSE SEESAW MODEL

Let us now consider the general inverse seesaw scenario, where both the Majorana mass matrices $\mu_S$ and $\mu_R$ do not vanish, i.e. $\mu_S \neq 0, \mu_R \neq 0$. To leading order in $\mu_{R,S}$, the light neutrino mass matrix is given by

$$M_{\nu_L} = M_D \left( \mu_S + x_N f(x_N) \mu_R \right) M_D^T + \mathcal{O}(\mu_S^2, \mu_R^2),$$

where the one-loop function $f(x_N)$ is defined in (20).

In order to get an order of magnitude estimate of the relative size of $\mu_R$ with respect to $\mu_S$ required to fit the neutrino oscillation data, let us choose a basis in which the charged lepton mass matrix is diagonal. Then we may write the light neutrino mass matrix as

$$M_{\nu_L} = U^T \tilde{\lambda}^\text{diag} U,$$

where $U$ is the PMNS mixing matrix\cite{3} given in terms of the three mixing angles, one Dirac and two Majorana CP phases \cite{1}.

$$U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{23} & s_{12}c_{23} \exp(i\delta) \\
-s_{12}c_{23} & c_{12}c_{23} & c_{12}c_{23} \exp(i\delta) \\
s_{12}s_{23} & -s_{12}s_{23} & c_{13} \exp(-i\delta)
\end{pmatrix} \times \text{diag}(\exp(i\alpha_1/2, \exp(i\alpha_2/2, 1),$$

with $c_{ij} \equiv \cos \theta_{ij}, s_{ij} \equiv \sin \theta_{ij}$. For a normal hierarchy, one has

$$\tilde{\lambda}_{\nu_L}^\text{diag} = \text{diag}(m_1, \sqrt{\Delta m_{\text{sol}}^2}, \sqrt{\Delta m_{\text{atm}}^2}).$$

Using the central values of a recent global-fit analysis of the neutrino oscillation parameters \cite{18}:

$$\Delta m_{\text{sol}}^2 = 7.62 \times 10^{-5} \text{ eV}^2, \ \Delta m_{\text{atm}}^2 = 2.55 \times 10^{-3} \text{ eV}^2, \ \theta_{12} = 34.4^\circ, \ \theta_{23} = 40.8^\circ, \ \theta_{13} = 9.0^\circ, \ \delta = 0.8\pi,$$

and assuming $m_1 = 0, \alpha_1 = \alpha_2 = 0$, we obtain from (26):

$$M_{\nu_L} = (10^{-2} \text{ eV}) \times \begin{pmatrix}
0.31 - 0.12i & -0.09 + 0.32i & -0.72 + 0.37i \\
-0.09 + 0.32i & 2.53 + 0.04i & 2.19 + 0.01i \\
-0.72 + 0.37i & 2.19 + 0.01i & 3.07 - 0.03i
\end{pmatrix}. $$

For illustration, let us absorb the flavor structure of $M_D$ in (25) into the matrices $\mu_{R,S}$, by rewriting the light-neutrino mass matrix as

$$M_{\nu_L} = \lambda_D \left( \bar{\mu}_S + x_N f(x_N) \bar{\mu}_R \right) \equiv \lambda_D \bar{\mu}_{\text{eff}},$$

\text{3 For simplicity, we have not considered here the leptonic non-unitarity effects which are generally of order $||\xi||$ in inverse seesaw models \cite{17}.}
where \( \lambda_D = \frac{m_{D_{\text{max}}}^2}{m_N^2} \) and \( m_{D_{\text{max}}} = \max \left( |(M_D)_{ij}| \right) \) is the largest element of \( M_D \) (in absolute-value terms). In addition, we have re-defined \( \mu_{S,R} \) as

\[
\bar{\mu}_S = K\mu_SK^T, \quad \bar{\mu}_R = K\mu_RK^T, \tag{32}
\]

where \( K = \frac{M_D}{m_{D_{\text{max}}}} \) is in general a dimensionless \( 3 \times 3 \) complex matrix. With these definitions, we can now make a one-to-one mapping between the elements of \( M_{\nu_L} \) and \( \bar{\mu}_{R,S} \). This is shown in Figure 4 where we have plotted against

![Graph showing the magnitudes of the largest element of \( \bar{\mu}_S \) (dashed line) and \( \bar{\mu}_R \) (solid lines for various values of \( x_N \)) as a function of the ratio \( \lambda_D \).](image)

FIG. 4: The magnitudes of the largest element of \( \bar{\mu}_S \) (dashed line) and \( \bar{\mu}_R \) (solid lines for various values of \( x_N \)) as a function of the ratio \( \lambda_D \).

the ratio \( \lambda_D \): (i) the magnitude of the largest entry in \( \bar{\mu}_S \) (denoted as \( \bar{\mu}_{S_{\text{max}}} \)) for \( \bar{\mu}_R = 0 \) (the dashed line); (ii) the magnitude of the largest entry in \( \bar{\mu}_R \) (denoted as \( \bar{\mu}_{R_{\text{max}}} \)) for \( \bar{\mu}_S = 0 \) and for \( x_N = 0,1,100 \) (the solid lines). We note that the mass scale of \( \bar{\mu}_{R_{\text{max}}} \) is roughly 2–4 orders of magnitude larger than that of \( \bar{\mu}_{S_{\text{max}}} \). Thus, if \( \mu_R \neq 0 \), a much milder hierarchy can be realized between the lepton-number breaking scale \( \bar{\mu}_{R_{\text{max}}} \) and the electroweak scale \( m_W \) in the Minimal Radiative Inverse Seesaw Model.

In the general inverse seesaw scenario where \( \mu_{R,S} \neq 0 \), the mass matrix \( \bar{\mu}_{\text{eff}} \) defined by (31):

\[
\bar{\mu}_{\text{eff}} = \bar{\mu}_S + x_Nf(x_N)\bar{\mu}_R, \tag{33}
\]

is fixed by the neutrino oscillation data for a given value of \( \lambda_D \). The dependence of \( \bar{\mu}_{\text{eff}} \) on \( \lambda_D \) is exactly the same as that of \( \bar{\mu}_{\text{max}} \) (with \( \bar{\mu}_R = 0 \)), as shown by the dashed line in Figure 4. However, the relative size between \( \mu_R \) and \( \mu_S \) depends on their relative sign, for a fixed given ratio \( x_N \). For instance, when both \( \bar{\mu}_{S_{\text{max}}} \), \( \bar{\mu}_{R_{\text{max}}} > 0 \), the constant contours of \( \bar{\mu}_{\text{eff}} \) (in keV) are shown in Figure 5 for \( x_N = 1 \). For \( \bar{\mu}_{S_{\text{max}}} < 0 \) and \( \bar{\mu}_{R_{\text{max}}} > 0 \), the corresponding contours are shown in Figure 6, where we have the cancellation regions (\( \bar{\mu}_{\text{eff}} \rightarrow 0 \)) for certain combinations of \( (\bar{\mu}_{S_{\text{max}}}, \bar{\mu}_{R_{\text{max}}}) \).

We conclude this section by commenting that our numerical results only depend on the largest element of the light neutrino mass matrix which is always of the order of \( \sqrt{\Delta m^2_{\text{atm}}} \), irrespective of the neutrino mass hierarchy. Hence our order of magnitude estimates will be valid for an inverted hierarchical light neutrino mass spectrum as well.

V. CONCLUSIONS

We have presented a minimal radiative mechanism for generating light neutrino masses in inverse seesaw models. The radiative neutrino masses arise at the one-loop level from known SM electroweak quantum effects involving
the $Z$ and Higgs bosons. Unlike in other radiative inverse seesaw mechanisms existing in the literature \cite{19}, the implementation of our radiative mechanism does not require any extra non-standard fields other than usual SM singlet neutrinos.

In the Minimal Radiative Inverse Seesaw Model, where the dimension-3 lepton-number breaking mass matrix $\mu_R$ of the right-handed neutrinos is non-zero, we have found that the light neutrino masses generated at one-loop level are UV-finite and are directly proportional to $\mu_R$. We also showed that the 3-by-3 Majorana mass matrix $\mu_R$ could be $2-4$ orders of magnitude larger than the other 3-by-3 Majorana mass matrix $\mu_S$ present in the standard inverse seesaw models. Hence, this could alleviate the hierarchy between the lepton number breaking scale and the electroweak scale in these models. In a supersymmetric version of this mechanism (e.g., \cite{20}), one might expect to ameliorate this hierarchy even further, in addition to having a scalar dark matter candidate in the form of the lightest sneutrino \cite{21}. 

**FIG. 5:** Contours of constant $\bar{\mu}_{\text{eff}}^{\text{max}}$ (in keV) for $x_N = 1$ and for $\bar{\mu}_R^{\text{max}} > 0$.

**FIG. 6:** Contours of constant $\bar{\mu}_{\text{eff}}^{\text{max}}$ (in keV) for $x_N = 1$ and for $\bar{\mu}_S^{\text{max}} < 0$, $\bar{\mu}_R^{\text{max}} > 0$. 

\[ \text{Log}_{10} \left( \frac{\mu^{\text{max}}}{\text{eV}} \right) \]
We hope to return to these issues in a future communication.

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