Effect of Axial Profile Modification on the Characteristics of a Finite Length Misaligned Journal Bearing

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Abstract. Misalignment in journal bearings is a common industrial problem which affects the general performance of the bearing. This includes a significant effect on the load carrying capacity of the bearing and the change in the level and shape of the pressure distribution. There are many causes for the presence of misalignment in the practical uses of the journal bearing system, such as deflection of the shaft under high levels of load, manufacturing errors, installation error, and others. In the case of severe misalignment levels, a sharp drop in the lubricant film thickness or even a direct contact may occur between the shaft and the bearing at the edges of the bearing. This paper considers such extreme cases of misalignment, where the introduction of bearing axial profile modification to prevent edge contacts is investigated in detail. A general 3D model of misalignment for the case of a finite length bearing is considered in the analyses, where the Reynolds equation is solved numerically based on the finite difference method. The results reveal that the use of profile modification leads to an improvement in the levels of film thickness and an increase in the load-carrying capacity of the bearing.

Key words: Journal bearing, 3D Misalignment, axial profile modification

1. Introduction

The journal bearing is considered an essential part of the rotary machines that are used in wide range of applications. It comprises the shaft (journal), which rotates inside the bearing in order to carry radial load. Journal bearings are usually operated under hydrodynamic lubrication, in which the bearing surface is separated from the journal surface by lubricant film generated by the journal rotation. One of the main problems that affects the performance of the journal bearing system is misalignment.

Where there is correct alignment, the axes of both shaft and bush are parallel during the operation under the applied load and rotational speed. In the practical use of this type of bearing, such an ideal situation rarely exists, and the shaft will suffer from some degree of misalignment while rotating inside the bearings. This is the general concept of the misalignment journal bearing. The formation of the film thickness in both circumferential and axial directions is responsible for protecting the surfaces from direct contact. In severe misalignment cases, the film thickness decreases significantly, which may lead to sudden contact between the shaft and the bearing at the point of maximum value of misalignment. This makes the problem of misalignment in bearings very important in the design and installation of the bearing systems. The main challenge...
in the bearing system design is to maintain an adequate amount of lubricant to avoid metal-to-metal contact that may be followed by a failure of the bearing.

Journal bearing performance with particular consideration of misalignment has drawn the attention of many researchers, due to the wide application of this bearing type.

Czyzewski and Titus (1987) [1] presented a solution for non-linear Reynolds equation for arbitrarily misaligned gas-lubricated full journal bearing of finite length. Their results revealed asymmetric pressure distribution, which was affected by the misalignment parameters.

Elkotb et al. (1989) [2] calculated the performance characteristics for a misaligned full journal bearing by the modified Reynolds equation, under turbulent regime. They found that the power consumption increased with the increasing of degree of misalignment for the same load capacity.

Qiu and Tieu (1995) [3] presented a numerical solution using a finite difference method in a fixed coordinate system for Reynolds equation, to obtain the static and dynamic performance characteristics of a horizontally-grooved bearing under different eccentricity and misalignment conditions. The presented results were compared with experimental data. These scholars found that the misalignment decreased the load-carrying capacity and the parallel bearings had the biggest load capacity.

Nacy (1997) [4] explored another idea to be utilized in journal bearing without considering the misalignment effect. A chamfer was machined circumferentially at both sides of the bearing’s internal surface, subsequently limiting or almost maintaining the side-leakage flow rate in lightly loaded journal bearings. This concept eliminates the requirement for fixing and collecting this leakage. The circumferential flow was found to be sufficient to achieve cooling of the bearing. Solutions were given for chamfered bearings with different length to diameter ratios, different chamfer length to bearing length ratios and finally different chamfer angles. It was found that the chamfer can be employed with a lightly loaded condition and the best result was at angle \(20^\circ\) for all ratio \(L/D\) and that the load carrying capacity is increased proportional to the length of chamfer.

Jun Sun and Changlin (2004) [5] analyzed hydrodynamic lubrication characteristics of a journal bearing with misalignment, caused by shaft deformation. The characteristics were calculated for a journal bearing with various values of misalignment degree and eccentricity ratio. The results showed obvious changes in pressure distribution where high value of pressure and relatively low film thickness were found when shaft deformation was taken into consideration. Therefore, it is necessary to consider misalignment caused by shaft deformation when analyzing the lubrication of journal bearing.

Ebrat et al. (2004) [6] used a new algorithm as an extension to the classical approach of evaluating film dynamic characteristics that depended on journal eccentricity perturbation. The analysis included the effect of journal misalignment and deformation of bearing structural in rotor dynamics. The finite difference method was solved the Reynolds equation using a successive over-relaxation (SOR) technique.

Sun et al. (2005) [7] developed a special test to study journal bearing lubrication affected by journal misalignment as a result of shaft deformation under load. The results showed clear variation in the journal bearing characteristics due to journal misalignment. The higher the load on the shaft, the larger the journal misalignment resulted from shaft deformation, the more obvious effect on lubrication performance of journal bearing.

Strzelecki (2005) [8] suggested the use of bearings with variable axial profile, e.g. hyperboloidal, convex profile in the axial cross section of bearing where only the whole bearing is modified and
no partial modification (chamfer) of the axial profile is considered. These bearings successfully carry the extreme load in conditions of misaligned axis of journal and the bush which eliminates the necessity of using self-aligning bearings.

Nikolakopoulos and Papadopoulos (2008) [9] solved the Reynolds equation numerically and developed an analytical model to find the relationships among the friction force, the misalignment angles and wear depth. The bearing was assumed to operate in the hydrodynamic region, at high eccentricities, wear depths, and angular misalignment. The results showed that the friction coefficient is increased with increasing wear depth as a result of misalignment.

PengHe et al. (2012) [10] studied the effects of journal misalignment on a journal bearing caused by an asymmetric rotor structure using a new model. Misalignment due to asymmetric structure affected the bearing performance. These researchers used a relatively simple stepped shaft to effectively represent a misaligned journal bearing. Consideration of the asymmetry deflection has a great effect on the lubrication of journal bearing.

Ajeel and Mohammed (2013) [11] presented a numerical solution to the Reynolds equation, which includes the effects of surface roughness and non-Newtonian behavior of the lubricant on the performance of misaligned journal bearing. It was found that for all surface roughness patterns that the load in rough misaligned journal bearing was lower than that in rough aligned journal bearing.

Jang and Khonsari (2015) [12] presented a study for the performance of journal bearing considering misalignment for the circular type of bearing. It was shown that the misalignment has significant effects on the static and dynamic characteristics of the system, especially at heavy loaded bearings.

Chin Jao et al. (2017) [13] modified the Reynolds equation to include the anisotropic slip and the film thickness with misaligned angle to analyze the hydrodynamic lubrication problem of journal bearing. They found that the slip boundary has consequences in terms of the effect of misalignment on journal bearing.

Tarasevych et al. (2017) [14] studied the effect of random change of main geometrical parameters of journal bearings where axes misalignment was considered in the analysis. The results emphasize the necessity of applying such an approach with consideration of both operating, as well as geometrical, parameters of bearing.

Jamali and A. Al-Hamood (2018) [15] solved the Reynolds equation numerically for a finite length bearing to study the problem of shaft misalignment. They presented a solution using a new model for the representation of the 3D misalignment geometry under hydrodynamic regime. The results showed that severe level of misalignment causes high difference in the levels of film thickness and the maximum pressure value in comparison with the corresponding aligned journal bearing.

2. Governing Equations

The main basic equations to obtain the pressure distribution in journal bearings under hydrodynamic lubrication regime are Reynolds equation, which is derived from the Navier-Stokes under classic assumptions, and the film thickness equation. At the steady state position of the finite journal, these two equations are given by [16].
\[
\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = 6\eta r\omega \frac{\partial h}{\partial x}
\]  
(1)

The film thickness equation around the journal is [15]:

\[h = c(1 + e \cos(\theta - \beta))\]  
(2)

Where, the lubricant viscosity, radius of bearing, radial clearance between shaft and the bearing and journal angular velocity are denoted by \(\eta\), \(r\), \(c\), and \(\omega\), respectively.

The positive x-axis is along the circumferential direction and the z-axis is along the axial direction.

These two equations can be written in a dimensionless form as:

\[
\frac{\partial}{\partial \theta} \left( H^3 \frac{\partial P}{\partial \theta} \right) + \left( \frac{r}{L} \right)^2 \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P}{\partial Z} \right) = \frac{\partial H}{\partial \theta}
\]  
(3)

\[H = (1 + \varepsilon \cos(\theta - \beta))\]  
(4)

Where,

\[P = \frac{P - P_o}{6\eta \omega} \left( \frac{c^2}{r^2} \right)\]

\(x = r\theta; H = h/c; Z = z/c\) and

\(P\): the dimensionless pressure

\(P\): pressure, \(P_o\) the atmospheric pressure

\(H\): the dimensionless gap between the rotating shaft and the bushing

\(\varepsilon\): eccentricity ratio

\(\beta\): attitude angle

3. 3D misalignment in journal bearing

Misalignment results from deviation of the journal axis from the bearing axis. This deviation may occur in the horizontal plane, vertical plane or both. The more general case is denoted by 3D misalignment, which will be discussed in the next section. It can be seen in figure 1 that the line \(O_1O_2\) is deviated due to 3D misalignment to the new position as explained by the line \(O_1'O_2'\). The vertical and horizontal deviations are defined by \(s_v\) and \(s_h\) respectively. These deviations vary linearly from a maximum value at the edge to zero at mid length (or axial center) of the bearing. The maximum values at the edges are defined by \(s_{vmax}\) and \(s_{hmax}\) for the vertical and horizontal misalignment respectively.
Figure 2 illustrates the deviation of the journal center for the general 3D misalignment case that is shown in Figure 2a, which represents the deviation for \( z \leq L/2 \) while Figure 2b illustrates the deviation when \( z > L/2 \).

Both of the vertical and horizontal misalignments, \( s_v \) and \( s_h \), are functions of the \( z \) axis. The deviation at the edges \( s_{v_{\text{max}}} \) and \( s_{h_{\text{max}}} \) are chosen as input parameters to examine the effect of the misalignment on the performance of journal bearing.

It is easy to derive the following equations for the misalignment at any position in the axial direction, \( z \) due to the linear variation of the misalignment along this direction as explained previously. [15]

\[
\begin{align*}
\delta_v(z) &= \delta_{v_{\text{max}}} \left( 1 - 2z/L \right) \text{ for } z \leq L/2 \\
\delta_v(z) &= \delta_{v_{\text{max}}} \left( 2z/L - 1 \right) \text{ for } z > L/2 \\
\delta_h(z) &= \delta_{h_{\text{max}}} \left( 1 - 2z/L \right) \text{ for } z \leq L/2 \\
\delta_h(z) &= \delta_{h_{\text{max}}} \left( 2z/L - 1 \right) \text{ for } z > L/2
\end{align*}
\]

(5)
Equations 5 can be written in a dimensionless form as

\[ e(z) = \sqrt{(e \cos \beta - \delta_v(z))^2 + (e \sin \beta + \delta_h(z))^2} \]  
\[ \beta(z) = \text{Arc tan} \left( \frac{e \sin \beta + \delta_h(z)}{e \cos \beta - \delta_v(z)} \right) \]

for \( z \leq L/2 \)

\[ e(z) = \sqrt{(e \cos \beta + \delta_v(z))^2 + (e \sin \beta - \delta_h(z))^2} \]  
\[ \beta(z) = \text{Arc tan} \left( \frac{e \sin \beta - \delta_h(z)}{e \cos \beta + \delta_v(z)} \right) \]

for \( z > L/2 \)

Equations 5 can be written in a dimensionless form as

\[
\begin{align*}
\Delta_v(z) &= \Delta_{v \text{max}} (1 - 2z/L) \text{ for } z \leq L/2 \\
\Delta_v(z) &= \Delta_{v \text{max}} (2z/L - 1) \text{ for } z > L/2 \\
\Delta_h(z) &= \Delta_{h \text{max}} (1 - 2z/L) \text{ for } z \leq L/2 \\
\Delta_h(z) &= \Delta_{h \text{max}} (2z/L - 1) \text{ for } z > L/2 
\end{align*}
\]

Where \( \Delta = \frac{\delta}{c} \) is the non-dimensional form of the misalignment, which means the misalignment scaled to the clearance of the system.

4. Axial profile modification of bearing

It should be noted that the increase in the misalignment produces a significant reduction in the minimum oil film thickness of the bearing. This section illustrates the required equations for modifying the design of the bearing, by adding chamfer at the edges of the inner surface of the bearing in order to reduce the relatively high levels of pressure due to misalignment and also increase the minimum oil film thickness. Figure 3 explains a 3D representation of a modified bearing where Figure 3a is a representation for the whole modified bearing, Figure 3b illustrates a longitudinal section of the bearing and the two parameters of the chamfer.
4.1 The equation of axial profile modification

A second order equation is used for the profile modification in the axial direction. Regarding figure 4, this equation can be given by

\[ f(Z) = AZ^2 + BZ + D \]  \hspace{1cm} (11)

Where A, B and D are constant to be determined regarding the following conditions:

For the Left side \((Z \leq z_0)\)

\[ \text{at } Z = 0 \rightarrow f(Z) = cr \]
\[ \text{at } Z = z_0 \rightarrow f(Z) = \frac{df(Z)}{dZ} = 0 \]

Substituting these conditions in equation (11) yields

\[ A = \frac{cr}{z_0^2}, \quad B = -2\frac{cr}{z_0}, \quad D = cr \]

Therefore, the general equation of modification when \(z_0 \leq L/2\) is

\[ f(Z) = cr \left( \frac{1}{z_0^2} Z^2 - \frac{2}{z_0} Z + 1 \right) \]  \hspace{1cm} (12)

Similarly, for the Right side \((Z \geq L - z_0)\)
Substituting these conditions in equation (11) yields

$$f(Z) = \frac{cr}{z^2}$$

at $$Z = L \rightarrow f(Z) = cr$$

at $$Z = (L - zo) \rightarrow f(Z) = \frac{df(Z)}{dZ} = 0$$

Substituting these conditions in equation (11) yields

$$A = \frac{cr}{z^2}, \ B = -2 \frac{cr}{z^2}(l - zo), \ D = \frac{cr(l - zo)^2}{z^2}$$

And the general equation of modification for the right side becomes,

$$f(Z) = \frac{cr}{z^2}(Z^2 - 2(l - zo)Z + (l - zo)^2)$$

(13)

5. Numerical solution

The Reynolds equation can be solved with appropriate boundary conditions to obtain the pressure distribution, the discretization of the pressure gradient part in the circumferential direction (using central difference, as shown in Figure 5) can be given by:

![Figure 5](image_url)

**Figure 5.** Representation of a part from the solution domain

$$\frac{\partial}{\partial \theta} \left( H^2 \frac{\partial P}{\partial \theta} \right) = \frac{H^2 \frac{\partial P}{\partial \theta} b - H^2 \frac{\partial P}{\partial \theta} a}{d\theta}$$

(14)

$$\frac{\partial P}{\partial \theta} |a = \frac{P_{(i,j)} - P_{(i+1,j)}}{d\theta}$$

(14-a)

$$\frac{\partial P}{\partial \theta} |b = \frac{P_{(i+1,j)} - P_{(i,j)}}{d\theta}$$

(14-b)

$$H^2 |a = \left[ \frac{H(i, j) + H(i-1, j)}{2} \right]^3$$

(14-c)
\[ H^3|b| = \left[ \frac{H(i+1,j)+H(i,j)}{2} \right]^3 \]  

(14-d)

\[ \frac{\partial H}{\partial \theta} = \frac{H(i+1,j)-H(i+1,j)}{2d\theta} \]  

(15)

Similarly, the gradient in the z direction is

\[ \frac{R^2}{L^2} \frac{\partial}{\partial Z} \left( H^3 \frac{\partial P}{\partial Z} \right) = \left[ H^3 \frac{\partial P}{\partial \theta} \right] c - \left[ H^3 \frac{\partial P}{\partial \theta} \right] d \]  

(16)

\[ \frac{\partial P}{\partial Z} = \frac{P(i,j)-P(i,j)}{dZ} \]  

(16-a)

\[ \frac{\partial P}{\partial Z} = \frac{P(i,j)-P(i,j)}{dZ} \]  

(16-b)

\[ H^3|c| = \left[ \frac{H(i,j+1)+H(i,j)}{2} \right]^3 \]  

(16-c)

\[ H^3|d| = \left[ \frac{H(i,j)+H(i,j-1)}{2} \right]^3 \]  

(16-d)

Substituting the above equations in Equation 3 yields:

\[ P(i,j) = \frac{1}{\gamma} \left[ H_b^3 P(i+1,j) + H_\alpha^3 P(i-1,j) + B_1 B_2 H_c^3 P(i+j) + B_1 B_2 H_d^3 P(i,j) - C_1 H(i+1,j) + C_1 H(i-1,j) \right] \]  

(17)

Where,

\[ B_1 = \frac{R^2}{L^2}, \quad B_2 = \frac{d\theta^2}{dZ}, \quad C_1 = \frac{d\theta}{2} \]

\[ \gamma = H_b^3 + H_\alpha^3 + B_1 B_2 H_c^3 + B_1 B_2 H_d^3 \]

The load components can be obtained by integrating the pressure, as given by the following equations:

\[ wr = \int_0^\theta \! P \cos \theta \, dx \, dz \]

\[ wt = \int_0^\theta \! P \sin \theta \, dx \, dz \]

\[ w = \sqrt{wr^2 + wt^2} \]  

(18)
The load components in dimensionless form as:

\[
W_t = \int_0^\theta P \sin \theta R \left[ L d\theta \right] \quad \quad W_r = \int_0^\theta P \sin \theta R \left[ L d\theta \right] \quad \quad W = \sqrt{W_r^2 + W_t^2}
\]  

(19)

The angle between the vertical load \( W \) and the line of centers is called the attitude angle, which can be given by:

\[
\beta = \tan^{-1}\left( \frac{W_t}{W_r} \right)
\]

(20)

6. Results

The effect of mesh density on the maximum pressure and minimum film thickness levels is shown in Figure 6. The total number of mesh points (k) in the longitudinal and circumferential direction is increased from 50 to 25600, as illustrated in this figure, to ensure the independence of the results on the mesh density. It can be seen in this figure that when \( k > 300 \), the calculated maximum pressure and minimum oil film are not affected significantly by changing the number of nodes.

The results presented in this paper were firstly verified with the available analytical solution for the two special cases of journal bearing. This includes comparison with the analytical solution of short and long bearings. A series of tests have been performed, using the full Sommerfeld condition, to calculate pressure distribution. An example of this test is shown in Figure 7. The results of the numerical solution are very close to the results of the corresponding analytical solution, whereby the maximum difference is only 0.019%. This excellent agreement between the two set of results can be attributed to many causes, such as the use of a central difference method in the discretizing of the governing equations, the use of very fine mesh and the convergence criteria.
The type of boundary conditions adopted in this paper is Reynolds boundary conditions, which assumes $p = 0$ and $\frac{\partial p}{\partial x} = 0$ at a position in the circumferential direction. However, this position is unknown and therefore an iterative procedure is required to accurately identify the point when the pressure gradient as well as pressure value fall to zero. Other types, such as the full Sommerfeld method, which assumes negative pressure, and the half Sommerfeld method, which assumes a sharp drop in the value of pressure from the highest value to $p = 0$ at angle=180, are considered nonrealistic from the continuity of fluid flow point of view.

The pressure distribution resulting from the successive iterations using the Reynolds boundary conditions method is explained in Figures 8 and 9. Figure 8 explains a sectional comparison while Figure 9 illustrates a full 3D pressure distribution for some iterations. It can be seen that iterations are continued until the value of pressure at all positions in the solution space become positive. The first iteration corresponds to the full Sommerfeld assumption while the last iteration represents the solution based on the Reynolds boundary condition method, where the pressure distribution extends beyond 180.
Two important parameters that affect the performance of journal bearing are the maximum pressure distribution and the minimum oil film thickness; therefore, this section will focus on the variation in these two parameters for 3D misalignment and profile modifications of bearings.

Figure 9. 3D Pressure distribution for some iterations
The journal bearing operates under one of the following three cases: perfectly aligned, misaligned and misalignment with modified axial profile cases. A 3D pressure distribution is explained in Figure 10, for the three cases of journal bearing system. The compared cases are perfectly aligned (Figure 10a), 3D misaligned (Figure 10b) and 3D misaligned with the consideration of axial profile (Figure 10c). The maximum pressure for the first case (aligned) is 0.8556 while for the second case (misaligned) a significant change can be seen in pressure distribution where \( P_{\text{max}} \) is 1.6858. In the third case, the introduction of profile modification returns the maximum pressure to a level very close to that of perfectly aligned bearing, where the maximum pressure decreased from 1.6858 to 1.1304.

These results, and the corresponding minimum film thickness for the three cases, are shown in Table 1. In the first, perfectly aligned, case, the maximum pressure is the lowest value as explained previously and the oil film thickness is the highest value compared with the other cases. It is worth mentioning that in all cases, Reynolds boundary conditions are used and the results converged to the same value of applied load. The maximum pressure distribution \( P_{\text{max}} \) is 0.8556 and minimum oil film thickness \( H_{\text{min}} \) is 0.4003. However, this perfect case is far removed from real practical operating conditions for many reasons, as explained previously.

In the second case, the 3D misalignment considers the occurrence of misalignment in the vertical plane and horizontal plane, where the value of the maximum vertical and horizontal displacement is 0.55. This value corresponds to the case when the journal is very close to the bearing surface. In this case, \( P_{\text{max}} = 1.6858 \) and \( H_{\text{min}} = 0.0667 \). This means that an increasing in maximum pressure
of 96.8 % and a reduction in the film thickness of 83.3 %, in comparison with the first, perfectly aligned case.

The last case considers the modification in bearing axial profile. The chamfer in inner surface of bearing gives significant improvement in both $P_{\text{max}}$ and $H_{\text{min}}$. This modification in the design of bearing decreases $P_{\text{max}}$ from 1.6858 in 3D misalignment case to 1.1304 and increases $H_{\text{min}}$ from 0.0667 in 3D misalignment case to 0.2204. That means in other words about one third reduction in $P_{\text{max}}$ and about three times increasing in $H_{\text{min}}$.

It is obvious from the data given in Table 1 that modification of the bearing gives significant increase in the minimum oil film thickness, which leads to avoidance of the direct contact between the journal and bearing. In other words, it means that a significant rising in the load-carrying capacity of the bearing (as it is related to the minimum film thickness of the lubricant between the surfaces) is detected. This important outcome is further investigated in the next section in terms of the modification parameters, which are $cr$ and $zo$.

| Case                          | $P_{\text{max}}$ | $H_{\text{min}}$ |
|-------------------------------|------------------|------------------|
| Perfectly aligned             | 0.8556           | 0.4003           |
| 3D misalignment               | 1.6858           | 0.0667           |
| Without chamfer               |                  |                  |
| 3D misalignment with chamfer  | 1.1304           | 0.2204           |

Table 1. Effect of chamfer on $P_{\text{max}}$ and $H_{\text{min}}$

Figure 11 shows the variation in $P_{\text{max}}$ for a wide range of $cr$ and $zo$. The range of $zo$ is from 0 to 0.5 where $zo=0$ corresponds to the case of no modification while $zo=0.5$ represents the case of whole bearing profile modification. The values of $cr$ are: 0.1, 0.2, 0.3, 0.4, 0.5, 1 and 2. For the case of $zo=0$ (3D misalignment without modification), the maximum pressure is 1.6858 and the axial profile modification is performed in order to reduce this level of pressure which results from the 3D misalignment. It can be seen that the maximum pressure continues to drop below this value for all values of $cr$ when $zo\leq0.2$ and also a reduction in $P_{\text{max}}$ is obtained for the whole range of $zo$ when $cr\leq0.4$. When $cr>0.5$ the maximum pressure starts to increase and sharp spikes are obtained when $cr\geq1$. So that high value of $cr$ must be avoided. It can be concluded from this figure that the satisfactory values of $cr$ and $zo$ are $cr\leq0.5$ and $zo\leq0.2$. For such range $P_{\text{max}}$ drops from 1.6858 to 1.1304.

Figure 12 illustrates the corresponding results for the dimensionless minimum film thickness. For the case of $zo=0$ (3D misalignment without modification) the minimum film thickness is 0.0667 and the bearing profile modification is used in order to increase this level of film thickness which results from the 3D misalignment. Similar results are also obtained for the minimum film thickness where the satisfactory range of $cr$ and $zo$ are $cr\leq0.5$ and $zo\leq0.2$ where for such range $H_{\text{min}}$ increases from 0.0667 to 0.22.
Figure 13 shows more detailed results for the case when $zo=0.2$ and a range of $cr$ 0 to 5. It shows that the maximum pressure is $< 1.1666$ when $cr$ less than 0.5, after that, any increase in $cr$ increases the maximum pressure. Similar results are obtained for the corresponding minimum film values, where $H_{min}$ decreases significantly for the range of $cr>0.5$.

7. Conclusions

The results presented in this work reveal that the parameters of chamfer, which are chamfer height, $cr$ and the length of chamfer in the axial direction, $zo$, are extremely important. Increasing the chamfer height over a certain limit causes negative consequences in terms of pressure levels. On the other hand, when $(cr \leq 0.5)$ and $(zo \leq 0.2)$, significant reduction in the level of maximum pressure and significant improvement in the levels of film thickness have been obtained. This means increasing the load-carrying capacity of the bearing, which is one of the most important
features in the designing of bearings. Further investigation is required to incorporate the thermal effects into the analysis when the chamfer is taken into consideration.

**Nomenclature**

| Symbol | Description | Units |
|--------|-------------|-------|
| c      | Bearing radial clearance | mm |
| cr     | Chamfer high | mm |
| e      | Eccentricity of journal | m |
| h      | Oil film thickness | m |
| $H$    | Non-dimensional oil film thickness, $H = \frac{h}{c}$ | - |
| $H_{min}$ | Minimum Oil film thickness | mm |
| K      | Total number of mesh $k = m \times n$ | - |
| L      | width of the bearing | m |
| $M$    | Number of mesh point in the longitudinal direction (z) | - |
| $N$    | Number of mesh point in the circumferential direction (θ). | - |
| $P$    | Oil film pressure | N/m² |
| p      | Non-dimensional oil film pressure, $P = \frac{P}{6\pi\omega (\frac{c}{R})^2}$ | - |
| $P_{max}$ | Maximum pressure | N/m² |
| r      | Bearing radius | m |
| $S_h$  | The horizontal misalignment | mm |
| $S_{h_{max}}$ | The maximum horizontal misalignment at the edge of bearing | mm |
| $S_v$  | The vertical misalignment | mm |
| $S_{v_{max}}$ | The maximum vertical misalignment at the edge of bearing | m |
| W      | Total load of journal bearing Non-dimensional | - |
| Symbol | Description | Units |
|--------|-------------|-------|
| $W_r$ | Load in the radial-direction Non-dimensional | N |
| $W_t$ | Load in the tangential-direction Non-dimensional | N |
| $W$ | Load carrying capacity | N |
| $w_r$ | Load in the radial direction | N |
| $w_t$ | Load in the tangential direction | N |
| $x, y$ | Horizontal and vertical components of journal bearing center | m |
| $z$ | Axial coordinate, $0 \leq z \leq L$ | m |
| $z_0$ | Chamfer length | mm |
| $Z$ | Non-dimensional axial co-ordinate, $Z = \frac{Z}{L}$ | - |

**Greek symbols**

| Symbol | Description | Units |
|--------|-------------|-------|
| $\varepsilon_r$ | Eccentricity Ratio, $\varepsilon_r = \frac{e}{c}$ | - |
| $\eta$ | Lubrication viscosity | Pa. s |
| $\theta$ | Angle in the circumferential direction | degree |
| $\theta_c$ | Cavitation angle | degree |
| $\beta$ | Attitude angle | degree |
| $\omega$ | Journal Angular velocity, $\omega = \frac{2\pi N}{60}$ | rad/sec |
| $\Delta$ | Dimensionless misalignment $\Delta = \frac{\delta}{c}$ | - |
| $\Delta \theta$ | Step in the circumferential direction | degree |
| $\Delta z$ | Step in the longitudinal direction | m |

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