Research on Time-Varying Mesh Stiffness of Double Involute Gears

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Abstract. The tooth of double involute gear (DIG) is considered as a cantilever beam on the tooth root circle, considering the real tooth profile of DIG, an improved energy method is used to solve the time-varying mesh stiffness. The influence of tooth waist order parameters and the difference of time-varying mesh stiffness between DIG and common involute gear (CIG) are studied. The results show that the time-varying mesh stiffness of the single tooth pair is less than that of CIG with the same parameters when the contact line passing through the tooth waist grading position, and the comprehensive time-varying mesh stiffness of DIG is less than that of CIG. The tooth waist order parameters will affect the time-varying mesh stiffness of DIG, and compared with the tooth waist altitude coefficient (\( l^* \)), the tooth waist tangential modification coefficient (\( y^* \)) has little effect on the time-varying mesh stiffness.

1. Introduction

As an important nonlinear excitation in gear transmission system, time-varying mesh stiffness excitation has a great impact on the vibration and transmission accuracy of gears, and this can determine the application and stability of gear transmission on a certain extent[1]. Therefore, the calculation of time-varying meshing stiffness excitation is of great significance to the study of gear dynamic characteristics.

Double involute gear is a new kind of gear which combines the advantages of involute gear and double circular arc gear[2]. Its basic tooth profile is shown in figure 1. In figure, \( l_a^*, l_d^*, y_a^*, \) and \( y_d^* \) are the tooth waist order parameters of double involute gears, and they are the main characteristics of DIG different from CIG, and these characteristics are also the core research on DIG. For ease of calculation, \( l_a^* = l_d^* = l^* \), and \( y_a^* = y_d^* = y^* \) are chosen in this paper[3].

At present, the finite element method is mostly used in the research of time-varying mesh stiffness, and the method adopts quasi-static simulation, which is not easy to obtain the accurate gear geometry, and consuming the calculation time, and the stiffness value obtained is too small [4]. In the energy analysis method, the tooth is assumed to be a cantilever beam on the base circle of the gear, and the coincidence between the base circle and the root circle of the gear is not taken into account, resulting in large errors in the calculation results [5]. In this paper, based on the related research in Ref. [6], combined with the real tooth profile of DIG, the time-varying mesh stiffness is solved by the improved energy method, and the accuracy of the method is verified. At the same time, the influence of tooth waist order parameters, and the difference of time-varying mesh stiffness between DIG and CIG are studied.
2. Meshing stiffness calculation model of double involute gear based on improved energy method

2.1. Accurate modeling of tooth profile curve of double involute gear

Figure 2 is a structure diagram of equivalent cantilever beam of a tooth of double involute gear, in which green tooth profile represents common involute gear profile and black tooth profile represents double involute gear profile. It can be seen from the figure that the tooth profile curve of DIG is composed of five parts, tooth top profile AB, tooth top profile BC, tooth waist profile CD, tooth root profile DE, and tooth root profile EF, which is more complex than CIG. In this paper, according to the characteristics of DIG tooth profile, the accurate tooth profile curve of DIG is carried out through the CIG profile curve. In addition, the double involute gear transmission does not contact at the tooth waist grading position, so the tooth waist profile CD is not calculated in this paper.

The expression of tooth top profile BC is as follows

\[
\begin{align*}
x &= r_i \sin \phi \\
y &= r_i \cos \phi - (y_{a}^{*}m_{n} - l_{a}^{*}m_{n} \tan \alpha_{a})
\end{align*}
\]

(1)

The expression of root profile DE is as follows

\[
\begin{align*}
x &= r_i \sin \phi \\
y &= r_i \cos \phi + (y_{a}^{*}m_{n} - l_{a}^{*}m_{n} \tan \alpha_{a})
\end{align*}
\]

(2)

Where \( r_i \) is the gear center distance at any point on the tooth profile of CIG. \( \phi = \pi / (2n_i) - (\text{inv}\alpha - \text{inv}\alpha_0) \). \( n_i \) is the number of teeth. \( \text{inv}\alpha \) is the involute development angle on the radius circle. \( \text{inv}\alpha_0 \) is the involute development angle on the reference circle.

The expression of tooth root transition profile EF is as follows

\[
\begin{align*}
x &= r \sin \phi - (a_i / \sin \gamma + r_{\rho}) \cos(\gamma - \phi) \\
y &= r \cos \phi - (b_i / \sin \gamma + r_{\rho}) \cos(\gamma - \phi) + (y_{a}^{*}m_{n} - l_{a}^{*}m_{n} \tan \alpha_{a})
\end{align*}
\]

(3)

Where \( r_{\rho} \) is the radius of the reference circle. \( \phi = (a_i / \tan \gamma + b_i) / r \), \( a_i \) is the distance between the tool tip fillet and the center line, \( a_i = m(h_{a}^{*} + c^{*}) - r_{\rho} \), \( r_{\rho} \) is the radius of the tool top fillet, \( r_{\rho} = c^{*}m / (1 - \sin \alpha_{a}) \). \( b_i \) is the distance between the center of the tool tip fillet and the center line of
the tooth slot, $h_1 = \pi m \times h_0^* \tan \alpha_0 + r_p \cos \alpha_0$. $m$ is the modulus. $h_0^*$ is the addendum coefficient, and $c^*$ is the tip clearance coefficient.

Figure 2. Structure diagram of equivalent cantilever beam of DIG.

2.2. Solving meshing stiffness by improved potential energy method
In order to ensure the stability of the gear transmission, the DIG must be made into the form of helical gear. For facilitate calculation, the "subsection method" is used to divide the contact lines in the meshing plane into $m$ equal parts, as shown in figure 3. Assuming that the load and meshing stiffness acting on the gear change uniformly along the direction of the contact line, the average meshing stiffness of each equal part is calculated respectively, and the average value is taken as the average meshing stiffness of the moment. The formula can be expressed as

$$k(t) = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} k_{ij}^j(t)$$

(4)

Where $k_{ij}^j(t)$ is the average meshing stiffness of the $j$ section of the $i$ th contact line at time $t$.

Figure 3. Sectional drawing of meshing area of double involute gear transmission.

In the traditional potential energy method, the gear tooth is simplified as a cantilever beam on the base circle of the gear. However, if the base circle of the gear does not coincide with the root circle of the gear, the meshing stiffness will produce some errors in the process of solving. According to the improved potential energy method proposed in Ref. [6], the tooth of double involute gear is considered as cantilever beam from the tooth root circle, and combined with the accurate tooth profile model of double involute gear, the meshing stiffness on a certain section of a contact line is obtained, and then the time-varying meshing stiffness of double involute gear is calculated.

2.3. Verification of time-varying mesh stiffness calculation model of DIG
The basic parameters of double involute gear are shown in table 1.
Table 1. Main parameters of double involute gear system.

| Number of teeth | Normal modulus (mm) | Pressure angle (°) | Helix angle (°) | Tooth width (mm) | Speed (r/min) | Torque (N.m) |
|-----------------|---------------------|--------------------|-----------------|------------------|---------------|--------------|
| 23/32           | $m_n = 4$           | $\alpha_n = 20$   | $\beta = 16°57'42''$ | $B = 50$         | $n_i = 2000$   | $T_p = 160$  |

In reference [7], the calculation method of time-varying mesh stiffness is approximately replaced by the change of contact line length of gear pair, and the accuracy of the calculation model is proved by comparing with the finite element method. In order to further verify the reliability of the calculation results in this paper, the time-varying mesh stiffness of DIG obtained by this method is compared with that obtained by the calculation method in reference [7], as shown in figure 4. The comparison shows that the change trend of the comprehensive time-varying meshing stiffness of DIG obtained by the two methods is similar, and the maximum deviation of the two calculation methods is less than 10%, which indicates the reliability of the method proposed in this paper.

![Figure 4](image.png)

Figure 4. Comparison of calculation results of time-varying mesh stiffness of DIG.

3. Comparative analysis of time-varying mesh stiffness between DIG and CIG

Under the same working conditions and parameters, the time-varying mesh stiffness of DIG has different from that of CIG, as shown in figure 5. It can be seen from figure 5 (a) that the single tooth pair time-varying mesh stiffness of DIG and CIG has little difference at meshing in and meshing out position. When the contact line of DIG passing through its tooth waist grading position, the length of contact line becomes shorter, which reduces the load acting on the contact line, resulting in the time-varying mesh stiffness less than that of CIG. It can be seen from figure 5 (b) that the change trend of comprehensive time-varying mesh stiffness of DIG is similar to that of CIG, both of which show periodic change. However, the comprehensive time-varying mesh stiffness of DIG is smaller than that of CIG, which indicates that the vibration and noise reduction of DIG is better than that of CIG under the same working conditions.
4. Influence of gear waist order parameters on time-varying meshing stiffness of double involute gears

4.1. Influence of tooth waist altitude coefficient ($l^*$)

Figure 6 shows the influence of $l^*$ on the time-varying mesh stiffness of double involute gear transmission when $y^* = 0.05$. It can be seen from figure 6(a) that the change of $l^*$ has little effect on the time-varying mesh stiffness at meshing in and meshing out position. When the contact line passing through the tooth waist grading position, the time-varying mesh stiffness of single tooth pair decreases gradually with the increase of $l^*$. This is due to the fact that the length of contact line is shortened and the load is reduced with the increase of $l^*$. As shown in figure 6(b), the change trend of comprehensive time-varying stiffness is the same under different $l^*$, and with increasing $l^*$, the comprehensive time-varying stiffness of DIG decreases gradually.

4.2. Influence of tooth waist tangential modification coefficient ($y^*$)

Figure 7 shows the influence of $y^*$ on the time-varying mesh stiffness of double involute gear transmission when $l^* = 0.05$. As shown in figure 7(a), the change of $y^*$ have a certain impact on the time-varying mesh stiffness of the single tooth pair of the DIG transmission. With increasing $y^*$, the maximum value of the single tooth on the time-varying mesh stiffness decreases slightly, but the
change of the value is small. Because the change of $y^*$ has little effect on the length of the contact lines, and has little effect on the force condition. Figure 7(b) shows that with increasing $y^*$, the maximum value of comprehensive time-varying mesh stiffness decreases slightly. Compared with $l^*$, $y^*$ has less influence on time-varying mesh stiffness of DIG.

Figure 7. Influence of $y^*$ on time-varying mesh stiffness of DIG. (a) Meshing stiffness of single tooth pair. (b) Comprehensive meshing stiffness.

5. Conclusions
(1) According to the characteristics of the tooth profile of DIG, the tooth is regarded as a cantilever beam on the tooth root circle, and combined with the improved energy method to solve the time-varying mesh stiffness of DIG. By comparing with the calculation method of the existing literature, the accuracy of the calculation method proposed in this paper is verified.

(2) The comprehensive time-varying mesh stiffness of DIG is smaller than that of CIG, but shows the same trend. The time-varying mesh stiffness of single tooth pair of DIG has less difference from that of CIG at the meshing in and meshing out position, but smaller than that of CIG when the contact lines passing through the tooth waist grading position.

(3) The tooth waist order parameters has certain influence on the time-varying mesh stiffness of DIG. In a certain range, the larger the $l^*$ is, the smaller the time-varying mesh stiffness of DIG transmission is. Compared with $l^*$, $y^*$ has little effect on the time-varying mesh stiffness of DIG.

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