How multiple supernovae overlap to form superbubbles

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21 June 2017

ABSTRACT
We explore the formation of superbubbles through energy deposition by multiple supernovae (SNe) in a uniform medium. We use total energy conserving, 3-D hydrodynamic simulations to study how SNe correlated in space and time create superbubbles. While isolated SNe fizzle out completely by $\sim 1$ Myr due to radiative losses, for a realistic cluster size it is likely that subsequent SNe go off within the hot/dilute bubble and sustain the shock till the cluster lifetime. For realistic cluster sizes, we find that the bubble remains overpressured only if, for a given $n_{g0}$, $N_{OB}$ is sufficiently large.

1 INTRODUCTION
HI holes, shells, rings, expanding cavities, galactic chimneys, and filaments are ubiquitous structures which are embedded in the large scale gas distribution of a galaxy. Heiles (1979) identified large cavities in the local interstellar medium (ISM) with energy requirement of $\gtrsim 3 \times 10^{52}$ erg as superbubbles. Our solar system is itself embedded in such a cavity (radius $\sim 100$ pc) filled with hot ($\sim 10^6$ K) and tenuous ($n \sim 5 \times 10^{-3}$ cm$^{-3}$) plasma (Sanders et al. 1977; McCammon et al. 1983) known as the local hot bubble (LHB). When the size of a superbubble becomes comparable to the galactic HI scale height, it may break out of the galactic disk if the shell is sufficiently fast (e.g., Mac Low & McCray 1988; Roy et al. 2013) and inject energy and metals into the galactic halo. The widely accepted model of galaxy-scale superwinds involves injection of mechanical energy by massive stars in the form of radiation ($L_\star$), stellar winds ($L_\nu$) and supernova (SN) explosions ($E_{SN} \sim 10^{51}$ erg). Clearly, such large cavities cannot be created by either the wind from a single massive star or by the supernova explosion of a single star. Further it is known from observations of O-type stars in the Galaxy that $\sim 70\%$ of them are associated with clusters and OB associations and a very small fraction of the known O-stars are isolated (Chu & Gruendl 2008). Out of the remaining 30\%, more than one-third are runaway stars which have been ejected in close gravitational encounters (Gies 1987). Hence the most plausible mechanism for the formation of large superbubbles is quasi-continuous energy injection from multiple stars. The expanding shells of each individual star/SN merge to form a large scale bubble known as a superbubble.

Pikel’Ner (1968); Avedisova (1972) studied the interaction of a strong stellar wind with the interstellar medium (ISM). The circumstellar shell enters the snowplow phase when the radiative cooling timescale for the swept gas becomes equal to the dynamical age of the shell. Weaver et al. (1977) calculated the detailed structure for interaction of a strong stellar wind with the interstellar medium. Castor et al. (1975) obtained a solution for the case of continuous energy injection (at a point) inside a homogeneous medium by a stellar wind ($L_\nu = Mv_\nu^2/2$) in the absence of radiative energy losses and found the presence of a transition region dominated by thermal conduction between the cold outer layer of the shell (shocked ISM) and the hot inner layer of the shell (shocked stellar wind). Weaver et al. (1977) analytically calculated detailed structure of the bubble in various phases of evolution, including the effects of radiative cooling. McCray & Kafatos (1987) highlighted that the stellar initial mass function and stel-
lar ages are such that the impact of mechanical energy input from supernovae (SNe) within a star cluster can be well modeled as a constant luminosity driven superbubble.

Chevalier & Clegg (1985) obtained the steady wind solution driven by a constant rate of mass and thermal energy injection within a small spherical volume. Their solution is subsonic within the injection radius, and beyond that reaches a constant supersonic speed. They applied their wind solution to understand the observations of the galactic outflow in M82. Tomisaka et al. (1981) performed 1-D calculations in a medium with constant particle density. In their calculations all explosions occur at the same point in space sequentially inside the cavity created by previous SNe. Vasiliev et al. (2015) have recently carried out 3-D simulations in which SNe are uniformly distributed throughout the simulation box. Durier & Dalla Vecchia (2012) studied the concordance of supernovae feedback methods based on thermal energy deposition and kinetic energy deposition. Sharma et al. (2014) (hereafter, SRNS14) show that isolated supernova, in typical ISM conditions, lose almost all their mechanical energy by radiative losses by \( \lesssim 0.1 \) Myr, whereas a sequence of explosions occurring inside the cavity blown by previous SNe can retain up to \(-40\%\) of the injected mechanical energy for few tens of Myr (of order the galactic dynamical time \(-50\) Myr). Krause et al. (2013, 2014) have studied the evolution of interacting interstellar bubbles of three massive stars in a uniform medium. Their key finding is that a larger fraction of energy is retained in the ISM for more closely packed stars. The hot bubble mostly emits in soft X-rays below 1 keV.

Understanding the impact of massive stars, via their radiation, winds, and SNe, on the ISM is essential for star and galaxy formation. Observed star formation is inefficient, both locally on pc scales and references therein). Because of several complex processes involved, there is no consensus on the relative contribution of these different mechanisms acting on molecular cloud scales. The situation is slightly better on galactic scales (\( \gtrsim 1 \) kpc) at which thermal supernova feedback seems to be the dominant mechanism for regulating star formation (e.g., Strickland et al. 2004; De Avillez & Breitschwerdt 2005; Joung & Mac Low 2006; Creasey et al. 2013; Hennebelle & #39;Iffrig 2014; Li et al. 2015). It is well recognized that isolated SNe suffer catastrophic cooling losses in high density clouds in which they are born (e.g., Thornton et al. 1998). In this case, almost all of the injected energy is lost rather than coupling to the ISM, especially over global dynamical timescales (\(-10s\) of Myr). Even when SNe coalesce before each of them suffer radiative losses (i.e., if supernova rate density is high enough), they only retain \(-10\%\) of the injected energy (Sharma et al. 2012). Even such a small efficiency of mechanical energy coupling to the ISM appears more than enough to significantly suppress star formation on global scales for Milky Way and lower mass galaxies (e.g., Eftastion 2000; red dot-dashed line in Fig. 4 of Sharma et al. 2012).

Shocks generated by supersonic turbulence (expected within the dense shell) enhance density perturbations and gravitational instability locally (e.g., McCray & Kafatos 1987), but turbulence and magnetic fields in the dense shell, in all likelihood, prevent efficient global star formation (e.g., Stone & Norman 1992; Mac Low & Klessen 2004). Since turbulence can only be faithfully captured in 3-D, it is necessary to study the ISM using 3-D simulations.

The problem of star-ISM interaction involves complex chemical, ionization/recombination, thermal, and dynamical processes, and it is necessary to begin with understanding the most important processes in some detail. Multi-physics simulations (including gravity, chemistry, photoelectric heating, molecular physics and supernovae feedback) of ISM have been done by many authors (e.g., Gatto et al. 2015; Martuzzi et al. 2015; Walsh & Naab 2015; Walsh et al. 2015). In this paper we ignore all these processes except for idealized dynamical and thermal processes associated with SNe resulting from the death of massive stars. We also ignore magnetic fields and thermal conduction, which can greatly modify the structures with large temperature gradients (e.g., Fig. 9 of SRNS14).

We only consider the hot and warm phases of the ISM by turning off cooling below 10^4 K, corresponding to the thermally stable warm neutral medium of the ISM. We do not consider the dense cold neutral phase because: (i) the stable cold phase exists globally only for a large enough ISM pressure, and hence is unlikely to be present in substantial amounts in galaxies less massive than Milky Way (Wolfire et al. 1995); (ii) our focus is on feedback at scales larger than molecular clouds, and we assume that a good fraction of supernova energy is able to leak out (aided by low density channels formed due to stellar winds and radiation) into the more uniformly spread and geometrically thicker warm neutral disk. Thus, this paper is a generalization of 1-D simulations of SRNS14, with a realistic spatial distribution of SNe in 3-D. Unlike that work, we also use a total energy conserving code so that the value of mechanical efficiency is more accurate.

In this paper we study the formation of superbubbles using idealized 3-D hydrodynamic numerical simulations of SNe exploding in an initially homogeneous, isotropic ISM. Ours are among the highest resolution uniform-grid 3-D simulations of their kind. In section 2 we describe the physical setup and numerical simulations. In section 3.1 we describe the key results from our simulations. Section 4 discusses analytic estimates and implications of our work. In section 5 we conclude.

## 2 PHYSICAL SETUP

We choose an idealized physical setup of a uniform ISM at 10^4 K, corresponding to the warm neutral medium (WNM) maintained in thermal balance by photoelectric/photoionization and cosmic ray heating (Wolfire et al. 1995). The Milky Way Giant Molecular Clouds (GMCs) have gas (H2) densities ranging from 10–1000 cm^-3 and mean size around \(10–20\) pc, as shown in Roman-Duval et al. (2010). Our scales of interest are much bigger (\(100\) pc), corresponding to the WNM. Thermal energy is injected by SNe going off at random locations inside a spherical ‘star cluster’ and plasma is allowed to cool due to free-free and line emission till 10^4 K. The key aim is to study the dynamics and thermodynamics of SNe coalescing in the WNM, and to study the conditions for the formation of overpressured superbubbles.

### 2.1 Simulation Setup

We solve the hydrodynamic equations for the evolution of density, velocity and pressure in 3-D Cartesian coordinates using the static grid version of the finite volume, conservative, Godunov Eulerian code PLUTO (Mignone et al. 2007). The mass and energy injected due to SNe are added as source terms. The grid spacing is taken to be uniform in \(x, y\) and \(z\) directions. We numerically solve the following equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = -\dot{\rho}_S(t, x),
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla \pi + \rho \mathbf{g},
\]

\[
\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho e \mathbf{v}) = \Lambda, (3)
\]

where symbols have their usual meanings, \(c_s = (\gamma p/\rho)^{1/2}\) is the sound speed, \(\dot{\rho}_S\) is the mass density source term, \(e_{SN}\) is the thermal energy source term mimicking supernova feedback (see section 2.2), \(\epsilon_{rad} = \epsilon_{rad}(n_e)\) is electron number density, \(n_e\) is...
ion number density and $\Delta[T]$ is the temperature-dependent cooling function) is the rate of energy loss per unit volume due to radiative cooling. We use the ideal gas equation

$$\rho v = \frac{p}{(\gamma - 1)}$$

with $\gamma = 5/3$ ($\epsilon$ is internal energy per unit mass).

PLUTO solves the system of conservation laws which can be written as

$$\frac{\partial \mathbf{u}}{\partial t} = -\nabla \cdot \mathbf{F} + \mathbf{S},$$

where $\mathbf{u}$ is a vector of conserved quantities, $\mathbf{F}$ is the flux tensor and $\mathbf{S}$ is the source term. The system of equations is integrated using finite volume methods. The temporal evolution of Eq. 5 is carried by explicit methods and the time step is limited by the Courant-Friedrichs-Lewy (CFL; Courant et al. 1928) condition.

Sedov-Taylor stage results are independent of whether SN energy is deposited as kinetic or thermal energy (see their Figs. 2 & 3), so we simply deposit thermal energy.

Each SN deposits a mass of $M_{\text{SN}} = 5 M_\odot$ and internal energy of $E_{\text{SN}} = 10^{51}$ erg over a sphere of size $r_{\text{SN}} = 5$ pc; the SN energy injection radius is chosen to prevent artificial cooling losses (see Eq. 7 in SRNS14, corresponding to their thermal explosion model). SRNS14 found that the late time (after a SN enters the Sedov-Taylor stage) results are independent of whether SN energy is deposited as kinetic or thermal energy (see their Figs. 2 & 3), so we simply deposit thermal energy.

Mass and energy injection from each SN is spread in space and time using a Gaussian kernel, such that the mass and internal energy source terms ($\rho_{\text{SN}}$ in Eq. 1 and $E_{\text{SN}}$ in Eq. 3) are proportional to $\exp\left[-(t-t_i)^2/\delta t_i^2 \right] \times \exp\left[-(x-x_i)^2/\delta x_i^2 \right]$, where $\delta t_i$ is the time between successive SNe given by

$$\delta t_i = \frac{\Delta t_{\text{OB}}}{N_{\text{SN}}}$$

where $\Delta t_{\text{OB}}$ (chosen to be 30 Myr) is the life time of the OB association and $N_{\text{OB}}$ is the total number of SNe (which equals the total number of O and B stars). Ferrand & Marovitch 2010 have shown that statistically the supernova rate is uniform. McCray & Kafatos 1987 also show that a constant mechanical luminosity is a good approximation to supernova energy injection. Also, it helps to understand the numerical results with simple analytic calculations.

In this section we describe in detail the morphology and evolution of a superbubble for number of SNe $N_{\text{SN}} = 100$, initial gas density $n_{\text{g0}} = 1$ cm$^{-3}$, and cluster radius $r_c = 100$ pc, which we choose as our fiducial run. The assumed parameters are typical of supershells (e.g., Heiles 1979; Suad et al. 2014; Bagetakos et al. 2011), but as mentioned earlier, $r_c$ is larger than typical cluster sizes. Our spatial resolution is $\delta L = 2.54$ pc (run R2.5 in Table B1). Simulations with different $N_{\text{OB}}$ and $n_{\text{g0}}$ evolve in a qualitatively similar fashion, the differences being highlighted in section 3.4. Numerical resolution quantitatively affects our results, although the qualitative trends remain similar. Strict convergence is not expected because thermal and viscous diffusion are required to resolve the turbulent boundary layers connecting hot and warm phases (e.g., Koyama & Inutsuka 2004). A detailed convergence study is presented in Appendix B. Fig. 1 shows the gas density and pressure slices in the midplane of the simulation domain at times when SNe are effectively isolated (1.27 Myr) and when they have coalesced (9.55 Myr) to form an overpressured superbubble. Since the evolution of a single SN is well known (see, e.g., Figs. 1 & 2 in Kim & Ostriker 2015), in order to compare with superbubble evolution we just briefly review the different phases of SN evolution. A SN shock starts in the free-expansion phase, moving...
Figure 1. Gas density (left panels; $\log_{10} n_g$ [cm$^{-3}$]) and pressure (right panels) snapshots in the $z = 0$ plane from our fiducial run shown before (top panels) and after (bottom panels) SNe coalesce. The yellow dots mark the projected location of SNe in the $z = 0$ plane, with four SNe having exploded by 1.27 Myr and 31 by 9.55 Myr. Top panels show that the SNe at 1.27 Myr are effectively isolated and even at this short time (say, compared to a galaxy’s dynamical time) the pressure within their individual bubbles is smaller than the ISM pressure. The bottom panels show the formation of a superbubble due to the overlap of several SNe. The pressure inside most of the bubble volume, except at the center, is larger than the ISM value. Note that a SN has gone off just before 9.55 Myr, and it creates a high pressure sphere right at the center. Also note that while the density scale is logarithmic, the pressure scale is linear.

ballistically till the ejecta sweeps up its own mass in the ISM. The next phase is the well-known adiabatic Sedov-Taylor (ST) phase, which transitions to a radiative snowplow phase with a thin radiative shell. The radius at which a supernova enters the ST phase can be written as

$$r_{ST} = 4.3 M_{SN,5}^{1/3} n_{g0,1}^{-1/3} \text{ pc},$$  \hspace{1cm} (7)$$

which in all cases is more than twice the grid resolution. Therefore, in our fiducial run we barely resolve the ST phase of the first few SNe. The corresponding ST timescale is

$$t_{ST} = 6 \times 10^{-4} M_{SN,5}^{5/6} n_{g0,1}^{-1/2} \text{ Myr}.$$  \hspace{1cm} (8)

For supernovae going off inside a rarified bubble (in which most subsequent SNe explode) $r_{ST}$ is larger and $t_{ST}$ is longer. In the ST
Figure 2. Energy (thermal + kinetic) and mass injected in the simulation box (their value at a given time minus the initial value, normalized appropriately) due to SNe as a function of time for the fiducial run. Injected mass and energy are normalized ($5 \, M_\odot$ for mass and $10^{51}$ erg for energy) such that every SN adds 1 unit. Total energy injected is larger than just the thermal energy put in due to SNe by $\approx 30\%$ because kinetic energy is injected in addition to the dominant thermal energy. The insets at top left and bottom right show a zoom-in of injected energy and mass, respectively. One can clearly see a unit step in the injected mass and energy for each SN that goes off.

Figure 3. The fraction (percentage) of injected energy retained as kinetic energy and thermal energy of gas inside the simulation box. At the end of the simulation the gas retains a small fraction, $\approx 1\%$ and $\approx 5\%$ of the total injected energy as kinetic and thermal energy respectively. The periodic spikes in energies correspond to individual SNe going off. In the legend, $KE$ stands for the kinetic energy and $\Delta TE$ for the change in thermal energy within the computational domain.

3.1.1 Global mass and energy budget
A key advantage of using a total energy conserving code like PLUTO is that energy is conserved to a very high accuracy and we can faithfully calculate the (typically small) mechanical efficiency of superbubbles. Fig. 2 demonstrates that our mass injection (mimicking SNe) adds $100M_{SN}$ by 30 Myr, the intended amount. The energy added is higher by $\approx 30\%$ because, as mentioned earlier, the mass added by the density source term (Eq. 1) is added at the local velocity, and hence mass addition leads to the addition of kinetic energy. Fig. 3 shows thermal, kinetic, and total energy efficiency as a function of time for the fiducial run. Energy efficiency is defined as the ratio of excess energy (current minus initial) in the simulation domain and the total energy injected by SNe. The energy efficiency that is higher at early times, decreases and asymptotes to a small value. Due to efficient cooling, most ($\approx 95\%$ by 30 Myr) of the deposited energy is lost radiatively. Out of the remaining $5\%$, $\approx 4\%$ is retained as the thermal energy and $1\%$ is retained as the kinetic energy of the gas. In terms of the energy deposited by a single supernova, the total (kinetic+thermal) energy retained is $\approx 6 \, E_{SN}$.

3.1.2 Density-pressure phase diagram
A bubble (associated with both an individual SN and a superbubble) remains hot and dilute for a long time (several Myr) but is not overpressured with respect to the ISM for a similar duration. The strength of the bubble pressure compared to that of the ambient medium is a good indicator of its strength. As pressure decreases with the expansion of the bubble, it will no longer be able to sustain a strong forward shock and will eventually degenerate into a sound wave. Fig. 4 shows the volume distribution of pressure at all times for the fiducial run. At $t = 0$ all the gas is at the ambient ISM pressure (indicated by the vertical red line at $1.38 \times 10^{12}$ dyne cm$^{-2}$). Because of a very short-lived high pressure (Sedov-Taylor) phase and a small volume occupied by the very overpressured gas, the volume fraction of gas with pressure...
The white plus (+) at Fig. 4, till grow in size they start overlapping and create a superbubble. In first few SNe behave as if they are isolated, and as their remnants

At this stage the shell propagates as a sound wave. In short, the isolated supernova (isolated SNe) becomes comparable to the ambient pressure. The superbubble weakens after the outer shock speed becomes comparable to the sound speed; i.e., $\eta_{\text{mech}}$. The superbubble radius ($r_{\text{sb}}$) and velocity ($v_{\text{sb}} = dr_{\text{sb}}/dt$) evolves with time as (Eq. 5 of Weaver et al. 1977)

$$r_{\text{sb}} = 58 \text{ pc} \eta_{\text{mech},-1}^{1/3} \left( \frac{E_{51,SN,51} N_{OB,2}}{10^{50} \text{ erg} \, N_{OB,2}} \right)^{1/3} \left( \frac{10^{-21} \text{ M}_{\odot} \text{ Myr}^{-1}}{M_{\odot} \text{ Myr}^{-1}} \right)^{1/5}$$

$$v_{\text{sb}} = 34 \text{ km s}^{-1} \eta_{\text{mech},-1}^{1/3} \left( \frac{E_{51,SN,51} N_{OB,2}}{10^{50} \text{ erg} \, N_{OB,2}} \right)^{1/3} \left( \frac{10^{-21} \text{ M}_{\odot} \text{ Myr}^{-1}}{M_{\odot} \text{ Myr}^{-1}} \right)^{2/5},$$

where $E_{51,SN}$ is the SN energy scaled to $10^{51}$ erg, $N_{OB}$ is the number of OB stars in units of 100, $n_{g0}$ is the age of OB association in units of 30 Myr, $n_{g0}$ is the ambient gas density in cm$^{-3}$, and $\Delta t_{\text{mech}}$ is time in Myr. The mechanical energy retention efficiency $\eta_{\text{mech},-1}$ is scaled to 0.1. The superbubble velocity can be expressed in terms of its radius as

$$v_{\text{sb}} = 34 \text{ km s}^{-1} \eta_{\text{mech},-1}^{1/3} \left( \frac{E_{51,SN,51} N_{OB,2}}{10^{50} \text{ erg} \, N_{OB,2}} \right)^{1/3} \left( \frac{10^{-21} \text{ M}_{\odot} \text{ Myr}^{-1}}{M_{\odot} \text{ Myr}^{-1}} \right)^{1/3}.$$

The superbubble weakens after the outer shock speed becomes comparable to the sound speed; i.e., $v_{\text{sb}} = c_0$ ($c_0 \approx [y k T_0/\mu m_p]^{1/2}$ is the sound speed in the ambient ISM). Thus, using Eq. 10, the fizzle-out time is

$$t_{\text{fizzle}} = 21.3 \text{ Myr} \eta_{\text{mech},-1}^{1/3} \left( \frac{E_{51,SN,51} N_{OB,2}}{10^{50} \text{ erg} \, N_{OB,2}} \right)^{1/5} \left( \frac{10^{-21} \text{ M}_{\odot} \text{ Myr}^{-1}}{M_{\odot} \text{ Myr}^{-1}} \right)^{5/2} c_0^{-1}.$$

The radius and velocity evolution of bubbles is critically dependent on the presence of radiative losses (encapsulated by $\eta_{\text{mech}}$).
In order to assess the strength of a superbubble, it is useful to define an overpressure volume fraction ($\eta_0$) as

$$
\eta_0 = \frac{V_{\text{g}}}{V_{\text{g}} + V_{\text{c}}},
$$

where $V_{\text{g}}$ is the volume occupied by gas at pressure $p > 5p_0$ and $V_{\text{c}}$ is the volume occupied by gas at $p < p_0/1.5$ ($p_0$ is the ambient ISM pressure; the choice of 1.5 is somewhat arbitrary). Thus $\eta_0$ gives the fraction of volume occupied by high pressure, hot and dilute bubble gas. Since we exclude gas close to the ambient ISM pressure in its definition, $\eta_0$ is independent of the computational domain and characterizes the bubble pressure. In Fig. 7 we also show the evolution of the hot volume fraction ($\eta_0$) as a function of time for the fiducial run. The hot volume fraction drops initially when SNe have not overlapped, but reaches unity after $\approx 3$ Myr, and starts decreasing rapidly after radiative losses become significant and the bubble pressure become comparable to the ISM pressure (or equivalently, the shock velocity becomes comparable to the ISM sound speed). The nature of the hot volume fraction evolution is discussed in more detail in sections 3.4.2 and 4.1.

### 3.2 Effects of thermal conduction

We have done the fiducial run with the isotropic thermal conduction module in PLUTO code, which implements Spitzer and saturated thermal conduction based on super time stepping (STS, Alexiades et al. 1996; $\tau_{\text{STS}} = 0.01$). Matter evaporates from the cold shell to the interior of the hot bubble (made of shocked SNe) due to thermal conduction, as shown analytically by Castor et al. 1975 (see also the right panel of Fig. 9 in SNRS14). Fig. 8 shows the density snapshots and projected velocity unit vectors for the fiducial run with (right panel) and without (left panel) conduction. The density in the hot bubble is much higher with conduction due to the evaporative flow from the dense shell to the hot bubble, as indicated by the velocity unit vectors in the right panel. Such a flow is absent in the run without conduction. The maximum temperature reached by the gas with conduction is much smaller than without it (c.f. Fig. 19). Overall, we find that thermal conduction does not affect the dynamics of the shell (e.g., its radius and velocity) but affects the temperature distribution of gas within the shell, which can influence its emission/absorption.
signatures. Since supernova bubbles are unaffected by thermal conduction, we do not include it in the rest of our simulations.

### 3.3 Comparison with 1-D simulations

Most of supernova and superbubble studies are carried out in spherical 1-D geometry because these systems are spherical (although only crudely) and very high resolution runs can be done. We want to compare our more realistic 3-D simulations (albeit with much lower resolution compared to the modern 1-D simulations) with 1-D runs to highlight the similarities and differences between the two.

For a realistic comparison of 1-D spherical and 3-D Cartesian runs, we run a 3-D simulation in which we explode all SNe at the origin (i.e., \( r_\delta = 0 \)). Both the 1-D and 3-D runs have the same resolution as the fiducial run \((\delta L = \delta r = 2.54 \text{ pc}; \text{ the only difference between this 3-D simulation and the fiducial run is that here } r_\delta = 0)\). As discussed in section 3.1.1, the amount of total mechanical energy injected in the box is slightly larger than \(N_{\Omega B} \times E_{SN}\) because of extra kinetic energy that we put in due to mass addition at the local velocity. For an exact comparison of our 1-D and 3-D runs we match the total energy injected in our 1-D and 3-D runs (by slightly scaling \(E_{SN}\) for the 1-D run). Three panels of Fig. 9 (except the bottom-right one) compare the time evolution of 1-D and 3-D simulations. The top two panels show that the total radiative losses are slightly higher (by \(\approx 3\%\)) for the 1-D spherical run (correspondingly, mechanical energy in the box is slightly smaller), and they are similar for the 3-D Cartesian simulations with \(r_\delta = 0\) and \(r_\delta = 100 \text{ pc}\). The overpressure fraction \(\eta_0\) evolution is also very similar for the spherical 1-D and the 3-D simulation with \(r_\delta = 0\). The rapid fluctuations in \(\eta_0\) at late times show that the bubble pressure is close to the ISM value and jumps above 1.5\(n_0\) after every new SN explodes inside it. The outer (inner) shell radius for the 3-D simulation (with \(r_\delta = 0\)) is only slightly larger (smaller) than the 1-D run. To conclude, 1-D spherical simulations capture the correct evolution of global (or volume-averaged) quantities such as mechanical efficiency.

The bottom right panel of Fig. 9 shows the radial distribution of emissivity for the three runs. For 3-D runs, average pressure and density are obtained by averaging over radial shells of size \(\delta L\) and emissivity \((n_\nu n_1 A(T))\) is calculated. The almost discontinuous rise in emissivity corresponds to the contact discontinuity between the shocked SN ejecta and the shocked ISM. While the 1-D emissivity...
Figure 6. Gas density, pressure and x−velocity profiles along the x−axis (y = z = 0) for the fiducial run at various times. The swept-up shell density decreases with time as the superbubble weakens and eventually the shell propagates at the sound speed in the ambient medium (c_0 = 15 km s^{-1}). As seen in Fig. 1, the bubble density is ~ 4 orders of magnitude smaller than the ambient value. The main bubble pressure decreases with time, except during SN injection, during which a high pressure core and an adiabatic wind with large velocity and small pressure (similar to Chevalier & Clegg 1985) forms (the streaks seen in some panels of Fig. 5 are also a signature of this). The inset in the lowest panel shows that the dense shell propagates at about half the sound speed in the ambient ISM, but the velocities in the low density bubble are much higher.

Figure 7. The inner (green line) and outer (red line) radius of the superbubble shell as a function of time for the fiducial run. The blue line shows the overpressure fraction (p_O) as a function of time. The superbubble starts to fizzle out when the overpressure fraction starts falling from 1, which happens around 15 Myr. The average outer shell velocity is comparable to the ISM sound speed; the inner shell speed is smaller. The bottom panel of Fig. 6 shows that at late times the shell material moves at v_o/v_I, similar to the inner shell speed, The outer shell velocity is higher, v_o ≈ 16 km s^{-1}, consistent with the shell density decreasing in time.

3.4 Effects of cluster & ISM properties

After discussing the fiducial run in detail, in this section we study the influence of cluster and ISM parameters (cluster radius r_{cl}, number of OB stars N_{OB}, and ISM density n_{g0}).

profile is very sharp, the transition for 3-D runs (particularly with r_{cl} = 100 pc) is smoother. This smoothing is due to deviation from sphericity, in particular the crinkling of the contact surface seen in the bottom panels of Fig. 1. This also makes the shell in Cartesian simulations slightly thicker compared to the spherical 1-D run. Radiative losses for 3-D runs are spread almost throughout the spherical run (see Fig. 5 in SRNS14).

Both the 1-D and 3-D simulations show that the bubbles are smaller than the analytic estimates because of radiative cooling. Even in a uniform medium the shell can be unstable to various 3-D instabilities such as 'Vishniac instability' (Vishniac 1983), which affect the morphology of supershells (c.f. Fig. 13; see also Krause et al. 2013).

3.4.1 Effects of ISM density

The gas density in which SNe explode is a crucial parameter that determines their subsequent evolution, both in adiabatic (\rho_{\text{sh}} \propto \rho^{-1/3}) and radiative (radiative losses are higher for a larger density) regimes. The left panel of Fig. 10 shows that the overpressure fraction at early times (< 5 Myr) both falls and rises slowly for a higher density ISM. The overlap of SNe at higher densities takes longer because the individual bubble radius is smaller for a higher density and one needs to wait longer to fill the whole cluster with hot gas. At late times, the overpressure fraction drops earlier for higher densities because of larger radiative losses (although the bubble pressure scales as n_{g0}^{-2/5} according to Weaver et al. 1977 adiabatic scaling).

The right panel of Fig. 10 shows that the bubble expands more rapidly in the lower density medium. It also shows that although the shell in a higher density ISM expands slowly, it sweeps up more mass. An adiabatically expanding strong bubble in a uniform medium is expected to sweep up gas at a rate \propto n_{g0}^{-2/5} \rho^{1/5}. Therefore, the ratio of mass swept by the shells with n_{g0} = 0.5, 0.8 cm^{-3} shown in Fig. 10 is expected to be (0.5/0.8)^{2/5} = 0.8, whereas the actual value is = 0.9. This is because the bubble expanding in a denser ISM is slower than the adiabatic model due to radiative losses; moreover, shells in a higher density medium suffer larger radiative losses. The shell for the highest density run (n_{g0} = 2 cm^{-3}) sweeps up an increasingly larger mass at later times because R_{DO} \propto c t \rho at late times, when the shell moves close to the ISM sound speed.
### 3.4.2 Effects of cluster radius

The key difference of this work from SRNS14 is that we are doing 3-D simulations, which are necessary to study a realistic spatial distribution of SNe. In 1-D spherical setup all SNe can only explode at the origin because of spherical symmetry. Fig. 11 shows the evolution of overpressure volume fraction \(n_{\text{g}}\) as a function of time for simulations with \(N_{\text{SN}} = 10^5\), \(n_{\text{g}} = 10^{-3}\), and different star cluster radii.\(^1\) The plot has a characteristic shape with an initial fall, a rise and saturation, and an eventual fall. The initial fall occurs as isolated SNe, without overlapping, fizzle out due to radiative losses (the top panels of Fig. 1 show density and pressure in this stage). The rise happens as SNe overlap and form a superbubble. Eventually, the overpressure volume fraction drops as the volume of the superbubble becomes too large and the outer shock weakens due to adiabatic and radiative losses.

We can estimate the time when SNe start to overlap. The radius of an isolated SN remnant is given by \(r_{\text{SN}} = (E_{\text{SN}}/\rho)^{1/3}\). Suppose \(n\) SNe have gone off independently by some time \(t\). The volume occupied by the non-overlapping SN remnants is \(\sum_{i=1}^n (4\pi/3)E_{\text{SN}}^2\delta t_{\text{SN}}^2\rho_i^{2/3}\). \(\delta t_{\text{SN}}\) is the rise happens as SNe overlap and form a superbubble. Eventually, the overpressure volume fraction drops as the volume of the superbubble becomes too large and the outer shock weakens due to adiabatic and radiative losses.

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We can make another estimate for the SN overlap timescale by assuming that SNe overlap only after they have become radiative. In this case, by a similar argument as that of the last paragraph, the overlap time \(t_{\text{rad}}\) is given by \(t_{\text{rad}} = \tau_{\text{OB}}/t_{\text{SN} \text{ad}}\), where \(t_{\text{rad}} \sim 37 \text{ Myr} E_{\text{SN}15}^{-1/3} n_{\text{g}0}^{1/3} (1/4)\) (Eq. 2 in Roy et al. 2013) is the hot/dilute bubble radius when the remnant becomes radiative. Note that the bubble radius does not increase by more than a factor of 2 after this time (e.g., Fig. 2 in Kim & Ostriker 2015). Thus, the overlap time, assuming a radiative bubble, is given by \(t_{\text{rad}} = 0.6 \text{ Myr} \tau_{\text{OB},15} N_{\text{SN},15}^{-1/3} E_{\text{SN},15}^{-1/3} n_{\text{g}0}^{1/3} r_{\text{cl},2}^{3/2}\) (15)

The evolution seen in Fig. 11 lies somewhere in between Eqs. 14 & 15.

The time for the overpressure volume to saturate after overlap of SNe and transition to a superbubble evolution is given by (using Weaver et al. 1977 scaling and setting the superbubble shell radius equal to the cluster radius),\(t_{\text{rad}} = 1.2 \text{ Myr} r_{\text{cl},2}^{1/3} \tau_{\text{OB},15}^{1/3} E_{\text{SN},15}^{1/3} n_{\text{g}0}^{1/3} r_{\text{cl},2}^{1/2}\) (16) where we have scaled the result with a mechanical efficiency \(\eta_{\text{mech}}\) of 0.1 (i.e., only \(\sim 10\%\) of the input SN energy goes into blowing the superbubble; \(\sim 90\%\) is lost radiatively). This estimate for the time of superbubble formation roughly matches the results in Fig. 11. Finally, the time when the superbubble pressure \(\sim 0.75 P_{\text{ad}}\) falls to \(\sim 1.5\) the ISM pressure is given by (apart from factors of order unity, this is essentially the same as Eq. 12)

\(t_{\text{rad}} = 10.3 \text{ Myr} T_{\text{g},15}^{1/2} n_{\text{g}0}^{-1/2} E_{\text{SN},15}^{1/2} \tau_{\text{OB},15}^{1/2} r_{\text{cl},2}^{-1/2}\) (17)

(\(T_{\text{g}}\) is the ISM temperature in units of \(10^4\) K) which is only slightly lower than the time corresponding to the late time drop in the overpressure volume fraction in Fig. 11. Note that unless the cluster size \(r_{\text{cl}}\) is unrealistically large, overlap of supernovae is likely

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\(^1\) Here we choose parameters \((N_{\text{SN}}, n_{\text{g}})\) different from the fiducial run because the different stages of evolution are nicely separated in time for this choice. The temporal behaviour is expected to be qualitatively similar for different choice of parameters.
to occur. In this state the time for a superbubble to fizzle out is independent of the cluster size.

3.4.3 Effects of supernova rate: formation of a steady wind

Chevalier & Clegg 1985 found a solution (hereafter CC85) for the wind driven by internal energy and mass deposited uniformly within an injection radius ($r < R$). This was applied to the galactic outflow in M82. For a large number of SNe (i.e., a large $N_{OB}$),
the mechanical energy injection can be approximated as a constant luminosity wind, \( L_w = N_{OB} E_{SN} \epsilon_{SN}/\dot{r}_{OB} \). According to CC85, within the injection radius \( r \leq R \) the mass density is constant, whereas at large radii (wind region, \( r \geq R \)) density is expected to be \( \propto r^{-2} \). A termination shock is expected at the radius where the wind ram pressure balances the pressure inside the shocked ISM. For small \( N_{OB} \), however, the individual SNe ejecta does not thermalize within the termination shock radius \( r_{TS} \) as the SN occurs inside a low density bubble (the bubble density is low in the absence of significant mass loading as most of the ambient gas is swept up in the outer shell) created by the previous SNe. For a large SN rate the solution should approach the steady state described by Chevalier & Clegg, 1985. SRNS14 derived analytic constraints on \( N_{OB} \) required for the existence of a smooth CC85 wind inside the superbubble (see their Eq. 11) as,

\[
\delta \sin,\text{CC85} \gtrsim 0.008 \text{ Myr} E_{\text{SN,51}}^{-9/26} \epsilon_{\text{SN,51}}^{-1/13} M_{\text{SN,50}}^{1/5/2} \tag{18}
\]

where \( M_{SN,50} \) is the SN ejecta mass and \( t_{Myr} \) is the age of the starburst in Myr. This time between SNe corresponds to a required of \( N_{OB} \geq 4 \times 10^{5} \) for a smooth CC85 wind to appear by 1 Myr. Using the standard stellar mass function, this corresponds to a star formation rate of \( \sim 0.01 M_{\odot} \text{ yr}^{-1} \). This is a lower limit because thermalization just before the termination shock does not lead to a high density/ emissivity core, the characteristic feature of a CC85 wind. Fig. 12 shows the density profiles for a range of \( N_{OB} \) (\( N_{OB} = 10^{5} \) corresponds to a SN rate of \( \sim 0.003 \text{ yr}^{-1} \)). As expected from thermalization of a SN within the ejecta of all previous SNe (Eq. 18), a smooth CC85-type wind with density \( \propto r^{-2} \) at 30 Myr only forms for \( N_{OB} \geq 10^{6} \). Since SNe form in OB associations, they are expected to overlap and form superbubbles. For a sufficiently large number of SNe (\( \gtrsim 10^{5} \), e.g., in the super star clusters powering a galactic wind in M82) a strong termination shock (with Mach number \( \gg 1 \)) exists till late times, which may accelerate majorite of Galactic and extragalactic high energy cosmic rays (e.g., Parizot et al. 2004). In contrast, strong shocks (especially the reverse shock; McKee 1974) in isolated SNe exist only at early times (\( \lesssim 10^{3} \) yr), after which the reverse shock crushes the central neutron star and the outer shock weakens with time (in fact catastrophically after it becomes radiative). Fig. 13 shows the 2-D density snapshots of the 3-D runs shown in Fig. 12, albeit at an earlier time. As expected, the shell is much thinner for a larger number of SNe. Also, a dense injection region and a clear termination shock are visible for the runs with \( N_{OB} \gtrsim 10^{4} \). Crinkling of the contact discontinuity and the thin shell is the key difference of 3-D runs as compared to the spherical 1-D simulations.

The SNe driven wind is able to maintain a strong non-radiative termination shock that is able to power the outward motion of the outer shock. The CC85 model has two parameters: the efficiency with which star formation is converted into thermal energy (\( \alpha \equiv E/\dot{M} \), SFR), and the mass loading factor (\( \beta \equiv M/\dot{M} \text{ SFR} \)), which determine the properties of galactic outflows (e.g., Sarkar et al. 2016). From our setup we can determine the mass-loading for large \( N_{OB} \) simulations by calculating the mass loss rate from the cluster measured at radii where the mass outflow rate \( M(t) \equiv 4\pi r^2 \dot{m} \) is roughly constant. The mass loading factor for our \( N_{OB} \geq 10^{5} \) run is \( \sim 1 \) as most of the SN injected mass flows out in a roughly steady wind. For much larger \( N_{OB} \) (or equivalently, SFR) valid for starbursts, the mass loading factor can be reduced because of radiative cooling and mass drop-out from the dense ejecta of SNe (e.g., Wüsch et al. 2007, 2008, 2011). Girichidis et al. 2016 have investigated launching of galactic outflows based on multi-physics simulations which include variation in SN rate and various strategies for placing supernovae (random, or at density peaks or isolated). Palmor et al. 2016; Simpson et al. 2016 investigate the effect of cosmic ray diffusion on dynamics of galactic outflows. We will investigate the effect of additional processes in our future work.
Figure 11. The evolution of overpressure fraction as a function of time for $n_\text{g0} = 10 \text{ cm}^{-3}$ and $N_\text{OB} = 10^4$, but with different star-cluster sizes ($r_\text{cl}$). The overpressure fraction plummets initially as SNe are effectively isolated and cool catastrophically within 1 Myr. After that, as more SNe go off, they start to overlap and create an overpressed bubble. As expected, the transition to overlap happens later for a larger star-cluster. The late time drop in overpressure fraction, occurring due to adiabatic and radiative losses, is similar for different $r_\text{cl}$. This suggests that the superbubble evolution is independent of the cluster size, once the coherent overlap of SNe occurs.

4 DISCUSSION

In this section we discuss the astrophysical implications of our work, focusing on radiative losses, comparison the observed HI ble evolution is independent of the cluster size, once the coherent evolution is independent of the cluster size, once the coherent overlap of SNe occurs.

4.1 Mechanical efficiency & critical supernova rate for forming a superbubble

While isolated SNe lose all their energy by $\sim$ 1 Myr, even overlapping SNe forming superbubbles lose majorities of energy injected by SNe. The mechanical efficiency of superbubbles is defined as $\eta_\text{mech} = (KE + \Delta TE) / E_\text{inj}$, where $KE$ is the total kinetic energy of the box, $\Delta TE$ is the increase in the box thermal energy, and $E_\text{inj}$ is the energy injected by SNe (which is slightly larger than $N_\text{OB} E_\text{SN}$ because mass is added at the local velocity). By energy conservation (the computational box is large enough that energy is not transported in or out of it), $\eta_\text{mech} = 1 - RL / E_\text{inj}$, where $RL$ are cumulative radiative losses. Fig. 14 shows the mechanical efficiency (Eq. 19) as a function of the initial gas density ($n_\text{g0}$) at various times for runs with different $N_\text{OB}$. One immediately sees that mechanical efficiency decreases with an increasing ISM density ($n_\text{g0}$). Efficiency also decreases with time (by almost a factor of 10 from 5 to 30 Myr), especially for higher densities. The maximum efficiency is $\sim$ 20%, occurring at early times. Our simulations show that the mechanical efficiency of 3-D and 1-D simulations are comparable and almost independent of the cluster size ($r_\text{cl}$, see section 3.3 & Fig. 9), provided that SNe overlap before fizzling out. A rough scaling of $\eta_\text{mech} \propto n_\text{g0}^{2/3}$ is expected scaling ($\eta_\text{mech}^{1/5} = n_\text{g0}^{1/5}$) with various $N_\text{OB}$, and $E_\text{inj}$ is the kinetic energy injected in units of $10^{51}$ erg; see Table 1. The thin vertical lines mark the cluster radius ($r_\text{cl}$) in the scaled unit. The density profile attains a smooth, steady CC85 profile (its signature is the $\rho \propto r^{-2}$ profile beyond a core region) within the bubble for a larger $N_\text{OB}$ $\geq 10^4$, consistent with the analytic considerations in section 4.3 of SRNS14 (see also Fig. 3 in their paper). The radiative shell in 1-D run (with the same resolution as the 3-D run) is much thinner as compared to 3-D because the 3-D shell is not perfectly spherical and the contact discontinuity is crinkled (see Fig. 13). The outer shock is weaker for a smaller $N_\text{OB}$ but its location scales with the analytic scaling ($\propto N_\text{OB}^{1/5}$).

Fig. 14 shows mechanical efficiencies that are about an order of magnitude smaller than the values quoted in SRNS14. For example, the efficiency (which equals 1− fractional radiative losses; see the right panel of Fig. 8 in SRNS14) for $N_\text{OB} = 10^5$ and $n_\text{g0} = 1$ cm$^{-3}$ in SRNS14 at 30 Myr is $\approx 40\%$. The value for the same choice of parameters from Fig. 14 is $\approx 6\%$, smaller by a factor of $\approx 7$. This discrepancy is mainly due to the much higher resolution in the 1-D simulations of SRNS14 (see section 4.4).
out or not. As described earlier, we consider a superbubble fizzled out if the average overpressure fraction falls below 0.5 at late times (25 to 30 Myr). Fig. 15 shows the plot of critical number of SNe required to produce an average overpressure volume fraction of 0.5 at late times (25 to 30 Myr), for a given gas density. We vary the ISM density for a given NOB, such that the late-time overpressure fraction is close to 0.5. The critical N$_{OB}$ roughly scales as $n_{g0}$.  

Now we turn to analytic arguments to understand the scaling of critical N$_{OB}$ for a given ISM density ($n_{g0}$). The superbubble pressure as a function of time, according to the adiabatic model of Weaver et al. (1977), is $\frac{P_{\text{b,late}}}{k_B} = 1.7 \times 10^8$ K cm$^{-3}$ $n_{\text{mech}}^{2/3}$ $n_{OB,30}^{-2} n_{g0}^{1/3}$ $T_r^{4/3}$, which at the end of cluster lifetime becomes

$$\frac{P_{\text{b,late}}}{k_B} = 1.7 \times 10^8 \text{K cm}^{-3} n_{\text{mech}}^{2/3} n_{OB,30}^{-2} n_{g0}^{1/3} T_r^{4/3},$$

where mechanical efficiency has been scaled to 0.01. Equating this to 1.5 times the ambient ISM pressure $P_{\text{ISM}}/k_B = 10^4 n_{g0} 1 R_e$, gives

$$n_{\text{OB,crit}} = 7.3 \times 10^4 n_{\text{mech}}^{-1} n_{OB,30}^{-2} n_{g0}^{1/3} T_r^{2/3}. \quad (21)$$

This estimate of the critical number of OB stars to maintain an overpressured bubble at late times agrees with Fig. 15 in that the critical N$_{OB}$ for $n_{g0} = 10$ cm$^{-3}$ is about $10^4$. From Eq. 21, we get the scaling of critical N$_{OB}$ as $n_{OB,\text{crit}} \propto n_{g0}^{-2/3}$, which when we use the dependence of $n_{\text{mech}}$ on $n_{g0}$ from Fig. 14 ($n_{\text{mech}} \propto n_{g0}^{-2/3}$), gives $n_{\text{OB,\text{crit}}} \propto n_{g0}^{-1/3}$. This scaling is similar to the scaling of critical N$_{OB}$ observed in Fig. 15; namely,

$$n_{\text{OB,\text{crit}}} \propto 200 n_{g0}^{1/3} R_{OB,30}^{-4/3}.$$  \(22\)

A steeper $n_{\text{mech}}$ versus $n_{g0}$, which is not inconsistent with Fig. 14, will give an even better match. The important point to note is that a decreasing mechanical efficiency with an increasing ISM density, is required to explain the critical N$_{OB}$ curve.

The scaling between N$_{OB}$ and gas density (hereafter critical curve) in Fig. 15 can be compared with the empirical relation between star formation rate (SFR) and gas density. The Kennicutt-Schmidt (hereafter KS) relation (Schmidt 1959; Kennicutt 1998) between gas surface density and SFR surface density is

$$\Sigma_{\text{SFR}} = \frac{3 \times 10^{-3}}{10M_\odot \text{pc}^{-2}} \left( \frac{\Sigma_g}{10M_\odot \text{pc}^{-2}} \right)^{1.4}, \quad (23)$$

which is valid for $\Sigma_g \geq 10 M_\odot \text{pc}^{-2}$, below which a much steeper relation holds (Bigiel et al. 2008). Consider a scale height ($H$) of 100 pc and a disk radius ($R_d$) of 1 kpc. For each OB star, the total stellar mass is $\sim 100 M_\odot$ for Kroupa/Chabrier initial mass function (Kroupa 2002; Chabrier 2003). Then for a star formation time scale of 30 Myr, we have $\Sigma_{\text{SFR}} = 10^{-6} n_{OB} M_\odot$ yr$^{-1}$ kpc$^{-2}$ $r_{OB,30}^{-1} R_{OB,30}^2$. For a gas density of $n_{g0}$ cm$^{-3}$, we also have $\Sigma_g = 3 n_{g0} H_{100} M_\odot \text{pc}^{-2}$ for mean molecular weight $\mu = 1.3$ (assuming neutral/molecular disk). Therefore, the KS relation can be rewritten in terms of the parameters used in this paper as

$$n_{OB,\text{KS}} = 550 n_{g0}^{1.4} H_{100}^{0.4} R_{OB,30}^{2.4}.$$  \(24\)

An important point to note is that the scaling of N$_{OB,\text{KS}}$ with gas density is somewhat shallower than the scaling for the critical curve (Eq. 22), but comparable in magnitude (this depends on the assumed scale height and disk radius). In the case of starbursts, the normalization for the KS relation can be larger (Kennicutt...
number of supernovae, \( N_{\text{OB}} \)

While this is much smaller than the Heckman limit, it is comparable to the lower limit on SFR density for the appearance of radio halos in spiral disks, \(-10^{-5}\) erg cm\(^{-2}\) s\(^{-1}\) (equivalent to SFR density of \(10^{-3}\) M\(_{\odot}\) yr\(^{-1}\) kpc\(^{-2}\); Dahlem et al. 1995), Roy et al. (2013) argue that to form a galactic superwind the superbubble has to break out with a sufficiently high Mach number (\(\gtrsim 5\)), but our critical curve (Eq. 21) is based on a Mach number of unity.

& Evans 2012), but this normalization is also consistent with the critical curve in this paper. Also note that the critical number of OB stars (Eq. 22) depends sensitively on the ISM temperature and can be much smaller for a cooler (say 100 K) disk. There-fore, a comparison of Eqs. 22 & 24 should be made only after using appropriate disk/ISM parameters. A steeper slope for critical \(N_{\text{OB}}\) as compared to the KS relation (despite the dependence on other parameters as disk radius and ISM temperature) implies that SNe can disrupt the star-forming regions more easily in weak/moderate star-forming regions but not in dense starbursts. This may explain the observed higher efficiency of star-formation (or a larger normalization of KS relation; Eq. 23) in starbursts relative to moderate star forming regions. The key uncertain step in this argument (which is beyond the scope of this paper) is how the maintenance of an overpressured bubble translates into suppression of star formation.

We can also compare our critical \(N_{\text{OB}} = n_{g0}\) curve with the observed threshold of SFR \(= 0.1\) M\(_{\odot}\) yr\(^{-1}\) kpc\(^{-2}\) for galactic superwinds (e.g., Heckman 2002). Using similar arguments used to derive Eq. 24, the critical SFR density corresponding to our critical curve is

\[
\Sigma_{\text{SFR, crit}} = 2 \times 10^{-4} M_\odot \text{yr}^{-1} \text{kpc}^{-2} n_{g0}^{1.89} R_d^{-2}. \tag{25}\]

While this is much smaller than the Heckman limit, it is comparable to the lower limit on SFR density for the appearance of radio halos in spiral disks, \(-10^{-5}\) erg cm\(^{-2}\) s\(^{-1}\) (equivalent to SFR density of \(10^{-3}\) M\(_{\odot}\) yr\(^{-1}\) kpc\(^{-2}\); Dahlem et al. 1995), Roy et al. (2013) argue that to form a galactic superwind the superbubble has to break out with a sufficiently high Mach number (\(\gtrsim 5\)), but our critical curve (Eq. 21) is based on a Mach number of unity.

\[\frac{L_{\text{w,38}}}{n_{g0}} = \left( \frac{r_{sb}}{38 \text{ pc}} \right)^2 \left( \frac{v_{sb}}{34 \text{ km s}^{-1}} \right)^3 \] \tag{26}\]

The vertical colorbar in Fig. 16 shows the contours of constant \(L_{\text{w,38}}/n_{g0}\). The \(\bigotimes\) symbols represent the radius and velocity obtained from our simulations (\(N_{\text{OB}}, n_{g0}\) ranges from 100 to 10\(^3\)); it is very encouraging that the observed distribution of \(r_{sb} - v_{sb}\) is similar to our simulations, which correspond to reasonable star cluster parameters. The solid red color track marks the evolution of a bubble with \(N_{\text{OB}} = 10^3\) and \(n_{g0} = 1.0\) cm\(^{-3}\), which corresponds to \(L_{\text{w,38}}/n_{g0} = 10\). But the track lies close to the analytic contours of
$L_{\text{w,38}}/n_{\text{c0}} \approx 0.5 - 1.0$. It means that, for a given $r_b - r_a$, the adiabatic theory overestimates $L_{\text{w,38}}/n_{\text{c0}}$ by a factor of $\sim 10 - 20$. This discrepancy is primarily due to large radiative losses; mechanical efficiency in Fig. 14 $\lesssim 10\%$ is consistent with the evolution in the $r_b - r_a$ space. Also, we note that some of the simulation points (below the $t = 30$ Myr line) have a dynamical age ($\approx 5 r_b/3 v_b$) longer than the simulation time.

If SNe are the dominant cause of bubble formation, then we require large OB associations for the creation of the observed HI supershells. In order to quantify the size of OB associations, we also need to evaluate the mechanical energy injection from stellar winds and radiation. However, even without accounting for these additional energy/momentum sources, the observed shells are much smaller and slower compared to what is expected from the predictions of adiabatic theory applied to the observed stellar population—the so-called power problem in superbubbles (Oey 2009 and references therein). Our simulations show that radiative losses can account for the power problem.

### 4.3 Gas removal from clusters

Due to the presence of feedback from OB stars (radiative, stellar winds and SNe) the star forming regions clear gas on timescales $\sim 10^9$ yr (Lada & Lada 2003). For clusters simulated by us the $m_{\text{tens}}/M_{\text{cl}} = 0.17 N_{\text{OB}} \zeta /n_{\text{c0}} v_b^2 \lesssim 1$ (for each OB star, the total stellar mass $\approx 100 M_\odot$ for Kroupa (Chabrier initial mass function), therefore the gravitational well is largely provided by the cluster gas (this is also true for embedded clusters buried in their natal molecular clumps). As a result of gas expulsion the cluster potential becomes shallower and the cluster may become unbound depending on the ratio of gas removal timescale and dynamical timescale of the cluster (Lada & Lada 2003). If the gas expulsion time is long compared to the dynamical time, the stars can adiabatically attain new virial equilibrium without being unbound. However, in the opposite regime because of a suddenly reduced gravity majority of stars become unbound. The timescale of gas expulsion is also important to account for multiple populations observed in globular clusters (e.g., Krause et al. 2016 and references therein).

While our simulations do not account for gravity that holds the star cluster together (inclusion of gravity is important for strongly bound massive clouds, not so much for smaller clumps with lower gravitational binding energies), we can qualitatively understand the action of supernova/stellar wind energy injection in gas expulsion from star clusters. Fig. 17 shows the mass fraction $m_{\text{cl}}(r)/m_{\text{c0}}$ of the mass inside the cluster radius, $r < r_{\text{cl}}$, as a function of time for various values of $N_{\text{OB}}$ and $n_{\text{c0}}$. Since the ratio of energy injected by SNe to the gravitational potential energy $-N_{\text{OB}} E_{\text{SN}}/\left(G M_{\text{cl}}^2 m_{\text{c0}}^2 n_{\text{c0}}^2 \lesssim 5 \times 10^7 N_{\text{OB}} E_{\text{SN}} /m_{\text{c0}}^2 n_{\text{c0}}^2 \right)$ is large, the effect of neglecting gravity is negligible for the choice of our parameters (for simulations with gravity, see Calura et al. 2015; Krause et al. 2016). We find that the clusters are evacuated due to the formation of a superbubble within $\lesssim 10$ Myr (Fig. 17). As expected, lower ISM density and higher $N_{\text{OB}}$ evacuate the cluster gas in a shorter time. An estimate of evacuation timescale is given in Eq. 16. The estimate depends strongly on the cluster radius ($r_{\text{cl}}$) but is weakly sensitive to parameters such as ISM density, $N_{\text{OB}}$, $n_{\text{c0}}$, etc. The results in Fig. 17 are consistent with the timescale in Eq. 16; therefore, for different parameters our numerical results can be scaled according to the theoretical scaling.

### 4.4 Convergence of $\eta_{\text{mech}}$ & temperature distribution of radiative losses

One of the key questions is whether our results are converged. Convergence of the fiducial 3-D simulation is discussed in Appendix B. Fig. B1 clearly shows that the higher resolution simulations show finer features. What about the convergence of volume-averaged quantities such as mechanical efficiency ($\eta_{\text{mech}}$)? Fig. 18 shows mechanical efficiency (Eq. 19) measured at 30 Myr for the fiducial 3-D and 1-D runs at various resolutions. Even average quantities like $\eta_{\text{mech}}$ do not show perfect convergence (we get a higher value of $\eta_{\text{mech}}$ with increasing resolution). The 1-D simulations can be carried out at a much higher resolution than the 3-D ones, and yet $\eta_{\text{mech}}$ increases with an increasing resolution. In section 3.3 we show that at the same resolution the radiative losses are comparable in 3-D and 1-D (top-right panel of Fig. 9).

From this, we expect that even the very high resolution 3-D simulations (which are beyond the capabilities of current computational resources) will not show convergence. Recent very high resolution 1-D simulations (Gentry et al. 2016; Gupta et al. 2016) have highlighted the importance of very high resolution to obtain mechanical efficiency and momentum delivered to the ISM by supernovae. However, our Fig. 18 clearly shows the lack of convergence even at the highest resolutions. The cooling losses in any simulation with unresolved boundary layers (radiative relaxation layer and contact discontinuity) will keep on decreasing with an increasing resolution because the volume of cooling layers (and hence radiative loss rate) decreases with an increasing resolution. This means that convergence can only be achieved by explicitly including diffusive processes such as thermal conduction and/or viscosity, which can numerically resolve the radiative layers. Moreover, the values of physical conductivity and viscosity are too small (especially for the dense phases) to be resolved on the grid. Therefore, artificially large numerical diffusivities (which may crudely mimic small-scale turbulent transport) must be used. The importance of resolving cooling layers via explicit thermal conduction to obtain convergence in thermal instability simulations is highlighted in Koyama & Inutsuka (2004). Similarly Fromang & Papaloizou (2007); Lesur & Longaretti (2007) show that explicit resistivity and viscosity are required to get converged results for angular momentum transport due to magnetorotational instability (MRI) in unstratified shearing boxes.

One observationally important diagnostic is the temperature distribution of cooling losses in superbubbles; this determines the wavebands in which they emit. Fig. 19 shows the temperature distribution of the radiative loss rate for the fiducial 3-D run with and without thermal conduction. Both with and without conduction, the radiative losses occur primarily at $\sim 10^4$ K; the fractional radiative losses for $T < 10^6$ K are 99.6% and 99.8% with and without conduction, respectively. This result is consistent with the recent superbubble simulations in dense molecular gas (Gupta et al. 2016), which show that the cooling losses at $\sim 10^4$ K are about two order of magnitude larger than X-ray ($\sim 10^{6-7}$ K) and molecular ($\sim 100$ K) losses. Thermal conduction reduces the maximum temperature in the hot bubble due to the evaporation of mass from the dense shell to the hot bubble, as shown in Fig. 8.

### 5 CONCLUSIONS

We have carried out 3-D hydrodynamic simulations of supernovae (SNe) in an OB association that creates and drives a superbubble. Our aim has been to study the effect of multiple SNe distributed over a limited region of a cluster, on the ambient material far outside the cluster, and derive the dependence of fundamental parameters such as the efficiency of energy deposition and the critical number of SN required to create overpressured bubbles. Our settings have been admittedly, and intentionally, kept idealized so that we can perform controlled numerical experiments. Physical effects such as magnetic fields, thermal conduction, stratification, and inhomogeneities in the ambient gas, which we have...
Figure 17. The fraction of original cluster gas remaining within the cluster ($r < r_{cl}$; $r_{cl} = 100$ pc for all these runs) as a function of time for various SN counts ($N_{OB}$). Each of the sub-panels show curves corresponding to different values of gas density $n_{g0}$, the initial ISM density inside the cluster. It is noted that even a small $N_{OB}$ makes the cluster lose all its gas by about $8 - 10$ Myr. The evacuation time increases with density, as expected.

Figure 18. Mechanical efficiency measured at 30 Myr as a function of grid resolution for various 1-D and 3-D superbubble simulations. The grid parameters are $N_{OB} = 100$, $n_{g0} = 1$ cm$^{-3}$ and $r_{cl} = 0$ (for 1-D runs) and $r_{cl} = 100$ pc (for 3-D runs). The blue solid line is the best least squares power-law fit to the data points. The inset shows the evolution of mechanical efficiency for the high resolution 1-D runs. Mechanical efficiency does not converge even for the highest resolution 1-D simulations.

Figure 19. Radiative loss rate per temperature bin ($dE_{rad}/dt dT$) as a function of gas temperature at different times for our fiducial simulation with (TC) and without (NC) thermal conduction. We calculate the radiative loss rates in logarithmically spaced temperature bins with $\Delta \log_{10} T = 0.1$. Thermal conduction reduces the maximum gas temperature in the box because of evaporation of matter into the hot bubble.

not included here, presumably do play important roles in superbubble formation and evolution, and will be the focus of our future studies.

The broad astrophysical implications of our results are discussed in section 4. Our key results can be summarized as follows:

- While isolated SNe fizzle out by $\sim 1$ Myr due to radiation losses, for a realistic cluster size it is likely that subsequent SNe go off in a hot and tenuous medium and sustain a shock lasting for the cluster lifetime $\sim 30$ Myr, comparable to the galactic dynamical timescale. 1-D numerical simulations faithfully capture the global energetics but cannot, by construction, capture morphological features such as the crinkling of the contact discontinuity seen in 3-D.

- While most of the input energy is lost via radiative cooling, the superbubble retains a fraction $\eta_{mech}$ of the input energy,
and this fraction scales as $\eta_{\text{mech}} \propto n_g^{2/3}$, being of order $\sim 6\%$ for $n_g \sim 1$ cm$^{-3}$ over a time period of $\sim 30$ Myr. We note that the mechanical efficiency increases with an increasing resolution, and that converged result can only be obtained by resolving cooling layers using explicit diffusion.

- We have explored the parameter space of ISM density ($n_g$), number of SNe ($N_{\text{OB}}$) and star cluster radius ($r_{\text{cl}}$) to study the conditions for the formation of an overpressured superbubble. For realistic cluster sizes, we find that the bubble remains overpressured only if, for a given $n_g$, $N_{\text{OB}}$ is larger than a threshold value. Our results show that threshold condition can be roughly expressed as $N_{\text{OB, crit}} \sim 200 n_g^{1.9} r_{\text{cl}}^{-1}$ where $n_g$ is the particle density in cm$^{-3}$.

- Classical adiabatic superbubble evolution overestimates the ratio of the wind luminosity and the ISM density ($L_w/n_g$) by a factor of $\sim 10 - 20$, by not taking radiation losses into account. This explains the ‘power problem’ of the observed size and speed of superclusters, and our simulations confirm that radiative losses are the reason for discrepancies between the size-speed distribution of HI supershells and the sizes of OB associations driving them.

- We confirm that a minimum value of $N_{\text{OB}}( \gtrsim 10^3)$ is needed to produce a steady wind and a strong termination shock within the cluster region. For a smaller number of SNe, all the supernova energy is deposited at the radiative dense shell.

**ACKNOWLEDGMENTS**

We are grateful to the Supercomputing Education and Research Centre (SERC) at IISC for facilitating our use of Cray XC40-SahasarT cluster, without which these challenging simulations could not be carried out. This work is partly supported by the DST-India grant no. Sr/S2/HEP-048/2012 and an India-Israel joint research grant (6-10/2014\[IC\]; which also supports Naveen Yadav). We wish to thank Kartick Sarkar, Arpita Roy, Deovrat Yadav, Mukherjee, Sharma, & Nath.

**REFERENCES**

Aleixades V., Amiez G., Gremaud P.-A., 1996, Communications in Numerical Methods in Engineering, 12, 31

Avedisova V. S., 1972, Soviet Ast., 15, 708

Bagetakos I., Brinks E., Walter F., de Blok W. J. G., Usero A., 2015, ApJ, 814, L14

Bigiel F., Leroy A., Walter F., Brinks E., de Blok W. J. G., Madore B., Thornley M. D., 2008, AJ, 136, 2846

Calura F., Few C. G., Romano D., D’Ercole A., 2015, ApJ, 814, L14

Castor J., McCray R., Weaver R., 1975, ApJ, 200, L107

Chabrier G., 2003, PASP, 115, 763

Chevalier R. A., Clegg A. W., 1985, Nature, 317, 44

Chabrier G., 2003, PASP, 115, 763

Chen Y., Gruendl R. A., 2008, in Beuther H., Linz H., Henning T., eds, Astronomical Society of the Pacific Conference Series Vol. 387, Massive Star Formation: Observations Confront Theory, p. 415 (arXiv:0712.1871)

Courant R., Friedrichs K., Lewy H., 1928, Mathematische Annalen, 100, 32

Cox D. P., 1972, ApJ, 178, 159

Creasey P., Theuns T., Bower R. G., 2013, MNRAS, 429, 1922

Dahlem M., Lisfenfeld U., Golia G., 1995, ApJ, 444, 119

De Avillez M. A., Breitschwerdt D., 2005, A&A, 436, 585

DURER F., Dalla Vecchia C., 2012, MNRAS, 419, 465

Efstathiou G., 2000, MNRAS, 317, 697

Ferrand G., Marcowith A., 2010, A&A, 510, A101

Fromang S., Papaloizou J., 2007, A&A, 476, 1113

Gatto A., et al., 2015, MNRAS, 449, 1057

Gentry E. S., Krumholz M. R., Dekel A., Madau P., 2016, preprint, (arXiv:1606.01243)

Gies D. R., 1987, ApJS, 64, 545

Girichidis P., et al., 2016, MNRAS, 456, 3432

Gupta S., Nath B. B., Sharma P., Schekinin Y., 2016, MNRAS, 462, 4532

Heckman T. M., 2002, in Mulchaey J. S., Stocke J. T., eds, Astronomical Society of the Pacific Conference Series Vol. 254, Extragalactic Gas at Low Redshift. p. 292 (arXiv:astro-ph/0107438)

Heiles C., 1979, ApJ, 229, 533

Hennebelle P., Iffrig O., 2014, A&A, 570, A81

Houng M. K. R., Mac Low M.-M., 2006, ApJ, 653, 1266

Kauffmann G., et al., 2003, MNRAS, 341, 54

Kenneicutt Jr. R. C., 1998, ApJ, 498, 541

Kenneicutt R. C., Evans N. J., 2012, ARA&A, 50, 531

Kim C.-G., Ostriker E. C., 2015, ApJ, 802, 99

Koyama H., Inutsuka S.-i., 2004, ApJ, 602, L25

Krause M., Fierlinger K., Diehl R., Burbert A., Voss R., Ziegler U., 2013, A&A, 550, A49

Krause M., Diehl R., Böhringer H., Freyberg M., Lubos D., 2014, A&A, 566, A94

Krause M. G. H., Charbonnel C., Bastian N., Diehl R., 2016, A&A, 587, A53

Kroupa P., 2002, Science, 295, 82

Krumholz M. R., Tan J. C., 2007, ApJ, 654, 304

Kumhoolz M. R., et al., 2014, Protostars and Planets VI, pp 234–266

Lada C. J., Lada E. A., 2003, ARA&A, 41, 57

Larsen S. S., 1999, A&AS, 139, 293

Lesar G., Longaretti P.-Y., 2007, MNRAS, 378, 1471

Li M., Ostriker J. P., Cen R., Bryan G. L., Naab T., 2015, ApJ, 814, 4

Mac Low M.-M., Klessen R. S., 2004, Reviews of Modern Physics, 76, 125

Mac Low M.-M., McCray R., 1988, ApJ, 324, 776

Martizzii D., Faucher-Giguère C.-A., Quataert E., 2015, MNRAS, 450, 504

McCnammar D., Burrows D. N., Sanders W. T., Kraushaar W. L., 1983, ApJ, 269, 107

McCray R., Kafaton M., 1987, ApJ, 317, 190

McKee C. F., 1974, ApJ, 188, 335

Mignone A., Bodo G., Massaglia S., Matsakos T., Tesileanu O., Zanni C., Ferrari A., 2007, ApJS, 170, 228

O’Connell R. W., Gallagher III J. S., Hunter D. A., Colley W. N., 1995, ApJ, 446, L1

Oey M. S., 2009, in Smith R. K., Snowden S. L., Kunz K. D., eds, American Institute of Physics Conference Series Vol. 1156, American Institute of Physics Conference Series, pp 295–304, doi:10.1063/1.3211829

Pakmor R., Pfrommer C., Simpson C. M., Springel V., 2016, ApJ, 824, L30

Parizot E., Marcowith A., van der Swaluw E., Bykov A. M., Tatischev O., 2004, A&A, 424, 747

Pikel’Ner S. B., 1968, Astrophys. Lett., 2, 97

Press W. H., Flannery B. P., Teukolsky S. A., 1986, Numerical recipes. The art of scientific computing

Roman-Duval J., Jackson J. M., Heyer M., Rathborne J., Simon R., 2010, ApJ, 723, 492

Roy A., Nath B. B., Sharma P., Schekinin Y., 2013, MNRAS, 434, 3572

Sanders W. T., Kraushaar W. L., Nousek J. A., Fried M. P., 1977, ApJ, 217, L87
Sarkar K. C., Nath B. B., Sharma P., Shchekinov Y., 2016, ApJ, 818, L24
Schmidt M., 1959, ApJ, 129, 243
Sharma P., McCourt M., Parrish I. J., Quataert E., 2012, MNRAS, 427, 1219
Sharma P., Roy A., Nath B. B., Shchekinov Y., 2014, MNRAS, 443, 3463
Simpson C. M., Pakmor R., Marinacci F., Pfrommer C., Springel V., Glover S. C. O., Clark P. C., Smith R. J., 2016, ApJ, 827, L29
Stone J. M., Norman M. L., 1992, ApJ, 390, L17
Strickland D. K., Heckman T. M., Colbert E. J. M., Hoopes C. G., Weaver K. A., 2004, ApJ, 606, 829
Suad L. A., Caiafa C. F., Arnal E. M., Cichowolski S., 2014, A&A, 564, A116
Sutherland R. S., Dopita M. A., 1993, ApJS, 88, 253
Thornton K., Gaudlitz M., Janka H.-T., Steinmetz M., 1998, ApJ, 500, 95
Tomisaka K., Habe A., Ikeuchi S., 1981, Ap&SS, 78, 273
Toro E. F., Spruce M., Speares W., 1994, Shock Waves, 4, 25
Vishniac E. T., 1983, ApJ, 274, 152
Walch S., Naab T., 2015, MNRAS, 451, 2757
Walch S., et al., 2015, MNRAS, 454, 238
Wang Q. D., 2014, in Ray A., McCray R. A., eds, IAU Symposium Vol. 296, Supernova Environmental Impacts. pp 273–281 (arXiv:1404.6289), doi:10.1017/S1743921313009587
Winkler R., McCray R., Castor J., Shapiro P., Moore R., 1977, ApJ, 218, 377
Wolfire M. G., Hollenbach D., McKee C. F., Tielens A. G. G. M., Wakelam V., 2003, ApJ, 585, 266
Wang Q. D., 2014, in Ray A., McCray R. A., eds, IAU Symposium Vol. 237, Supernovae to Superbubbles. (arXiv:1401.6209), doi:10.1017/S1743921313009587
Winkler R., McCray R., Castor J., Shapiro P., Moore R., 1977, ApJ, 218, 377
Wolfire M. G., Hollenbach D., McKee C. F., Tielens A. G. G. M., Bakes E. L. O., 1995, ApJ, 443, 152
Wünsch R., Palouš J., Tenorio-Tagle G., Silich S., 2007, in Elmegreen B. G., Palouš J., eds, IAU Symposium Vol. 237, Triggered Star Formation in a Turbulent ISM. pp 497–497, doi:10.1017/S1743921307002852
Wünsch R., Tenorio-Tagle G., Palouš J., Silich S., 2008, ApJ, 683, 683
Wünsch R., Silich S., Palouš J., Tenorio-Tagle G., Muñoz-Tuñón C., 2011, ApJ, 740, 75

APPENDIX A: RADIUS DETERMINATION OF THE SHELL

For 3-D simulations the dense shell is not perfectly spherical. Some figures (e.g., Figs. 7, 16) show the evolution of the shell radius with time. Fig. A1 shows how we determine the inner and outer radii of the supershells. We construct angle-averaged radial density profiles by dividing the simulation box into spherical shells of thickness $\delta r = \Delta r$, and averaging over all the grid cells contained within the shell. The inner shell radius is taken at the radius where $n_g = 0.98n_{g0}$ and the outer shell radius has $n_g = 1.02n_{g0}$.

APPENDIX B: CONVERGENCE

In order to ensure the convergence of the results, we carried out our fiducial run ($n_{g0} = 1 \text{ cm}^{-3}$, $T_0 = 10^4$ K, $N_{\text{grid}} = 100$, $\Delta t = 100 \text{ pc}$) with different grid resolutions (see Table B1). The timestep is shorter for a higher grid resolution as $\Delta t \propto N^{-1}$, where $N$ is the number of grid points along any direction. Hence, the total computational cost scales $\sim N^3$, which becomes prohibitive for a large number of grid-points. An optimum resolution, large enough to capture key physical features but computationally feasible, needs to be chosen.

Figure A1. Determining the radius of supershells: first we calculate the angle-averaged density profiles in spherical shells of size $\delta r = \Delta r$. The outer shell radius ($R_O$) corresponds to the radius at which the average density is larger than 1.02 times the ambient ISM density ($n_{g0}$) and the inner radius ($R_I$) corresponds to the radius at which the average density falls below 0.98$n_{g0}$.

Fig. B1 compares the evolution of volume integrated quantities and the shell radius for various grid resolutions. A larger resolution is required to resolve gradients, but the difference is small for the highest resolution ($\Delta L = 1.27$, 2.54 pc). The evolution of the inner and outer shell radii are also similar.

Fig. B2 shows the density snapshots of four simulations with the grid resolution of 1.27 pc, 2.54 pc and 3.57 pc and 4.53 pc at 9.55 Myr. The simulations with higher resolution better resolve the internal structures within the bubble. Strict convergence is only expected with explicit viscosity and thermal conductivity. Since molecular transport is negligible, we do not include these in our simulations. The run with $\Delta L = 2.54$ pc looks morphologically very similar to the run with $\Delta L = 1.27$ pc, but is $16$ times faster. Since simulations of the cluster over its typical lifetime (~ 30 Myr) is computationally expensive, we have chosen a resolution close to $\Delta L = 2.54$ (corresponding to run R2.5 in Table B1) for most of our simulations (see Table 1).
Figure B1. Various volume-averaged quantities (kinetic energy, change in thermal energy, overpressure fraction, and inner and outer radii of the shell) for the fiducial parameters ($N_{\text{OB}} = 100$, $n_g = 1$ cm$^{-3}$, $r_{\text{cl}} = 100$ pc) as a function of time for different grid resolutions, $\delta L = 1.27, 2.54, 3.57, 4.53$ pc. The top two and the bottom-left panels show binned data with a bin-size of 0.18 Myr. The results show convergence with an increasing resolution.

Table B1. Convergence runs for fiducial parameters

| Label | $L$ (pc) | $N$ | $\delta L$ (pc) | $R_O$ (pc) | $R_I$ (pc) | $KE$ ($10^{51}$ erg) | $\Delta T E$ ($10^{51}$ erg) | $E_{\text{inj}}$ ($10^{51}$ erg) | $\eta_{\text{mech}}$ (%) | $\eta_O$ |
|-------|---------|-----|-----------------|------------|-----------|-----------------|--------------------------|------------------|-------------|--------|
| R4.5  | 714     | 315 | 4.54            | 281        | 490       | 0.69            | 3.84                     | 100.38           | 4.51        | 0.40   |
| R3.6  | 714     | 400 | 3.57            | 293        | 496       | 0.79            | 4.37                     | 103.05           | 5.01        | 0.59   |
| R2.5  | 649     | 512 | 2.54            | 299        | 505       | 0.98            | 5.29                     | 105.02           | 5.97        | 0.68   |
| R1.3  | 649     | 1024| 1.27            | 308        | 512       | 1.19            | 6.32                     | 106.11           | 7.08        | 0.72   |

$^{\dagger}R_O$ ($R_I$) is the outer (inner) radius of the shell at $\approx 25$ Myr.

$^{\dagger}$ Kinetic and thermal energy added to the simulation box by SNe.
Figure B2. Density snapshots in the \(x-y\) plane at 9.55 Myr for the fiducial parameters but with various grid resolutions (see Table B1). The runs with higher resolution better resolve the features at the bubble-shell interface. Yellow filled circles indicate the projected locations of SNe that have gone off.