Topological Symmetry, Background Independence and Matrix Models

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Abstract

We illustrate a physical situation in which topological symmetry, its breakdown, space-time uncertainty principle, and background independence may play an important role in constructing and understanding matrix models. First, we show that the space-time uncertainty principle of string may be understood as a manifestation of the breakdown of the topological symmetry in the large $N$ matrix model. Next, we construct a new type of matrix models which is a matrix model analog of the topological Chern-Simons and BF theories. It is of interest that these topological matrix models are not only completely independent of the background metric but also have nontrivial ”p-brane” solutions as well as commuting classical space-time as the classical solutions. In this paper, we would like to point out some elementary and unsolved problems associated to the matrix models, whose resolution would lead to the more satisfying matrix model in future.

1 Invited paper to appear in the special issue of the Journal of Chaos, Solitons and Fractals on ”Superstrings, M, F, S, . . . Theory”, edited by M.S. El Naschie and C. Castro.

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3 Supported in part by Grant-Aid for Scientific Research from Ministry of Education, Science and Culture No.09740212.
1 Introduction

It seems that we are now in a new era of developments of quantum field theories since recent discovery of various remarkable ideas and tractable techniques for understanding the strong coupling and the non-perturbative regimes of the quantum field theories. So far the study of quantum field theories has been mainly restricted to the standard perturbative analyses of weakly coupled field theories. But in the last few years important progress was made in the study of the strongly coupled dynamics in a class of gauge theories and string theory. Such a progress is quite impressive in that the physics dealing with the strong coupling phase and the non-perturbative regime is certainly expected to provide a totally new insight about what would be the content of the strong coupling phase where the conventional perturbative analyses are out of control.

For instance, the discovery of three types of dualities, what we now call, S, T and U dualities has recently made it possible to clarify that five superstring theories and M-theory are in fact non-perturbatively equivalent in the sense that each of them is nothing but a perturbative expansion of a single underlying theory about a distinct point in the space of quantum vacua \[ \text{[1]} \]. As a second example, we can also list recent progress on the understanding of the phase structure of supersymmetric gauge theories in terms of rather simple properties of M 5-brane in eleven dimensions \[ \text{[2]} \]. In this respect, it is quite interesting that the results of the strongly coupled gauge theory are also best understood as string theory and M-theory phenomena. Moreover, the more recent conjecture \[ \text{[3]} \] that Type IIB superstring on \( AdS_5 \times S^5 \) is equivalent to \( N = 4 \) super Yang-Mills theory with gauge group \( SU(N) \) gives rise to a great deal of interests in the study of physics on the anti-de Sitter space. This is because according to the above conjecture the strong coupling regime of \( N = 4 \) super Yang-Mills theory with gauge group \( SU(N) \) should be described in terms of the weak coupling physics of Type IIB superstring on \( AdS_5 \times S^5 \).

On the other hand, another striking feature of recent developments is a fruitful interplay between superstring theory and a quantum theory of black holes. It is physically reasonable to imagine that a black hole plays a critical role in superstring theory since two concepts of black holes and elementary particles would merge at the Planck mass scale \[ \text{[4]} \] which is relevant to superstring theory. In other words, we expect that black holes may play a role similar to the hydrogen atom in quantum mechanics in the search of a quantum theory containing gravity. Referring to a connection with the strong coupling physics, black hole quantum mechanics provides us with a window into strong coupling quantum physics by raising several puzzles to which a quantum theory of gravity must answer. This is because in the region of parameter space where elementary particles become black holes, we inevitably go into strong coupling region.

Among many of the recent remarkable developments in quantum field theories maybe the most exciting one might be the discovery of matrix models. It is expected that the matrix models may be candidates for the non-perturbative formulation of M-theory \[ \text{[5]} \] and

\[ 4 \text{See the reference } \] \[ \text{[6]} \] for old fashioned dualities.
IIB superstring \[7, 8\] so that a priori they may determine uniquely the true vacuum among many perturbative vacua of superstring theory. However, it is a pity that although the matrix models have surprisingly passed a lot of nontrivial tests to date, it is far from being complete and further work is being carried out in a number of directions. For example, it is unclear how they yield our real four dimensional space-time and the plausible gauge group e.t.c. in the low energy region through some natural compactification mechanism. \[7\] In such a situation, one of interesting directions of study is to ask ourselves whether the matrix models at hand in fact equip with desirable characteristic features in themselves as Theory of Everything. In this context, we would like to ask the following elementary questions which have not been understood so well, but it is worthwhile to keep in mind that they should be resolved to get a more satisfactory matrix model in future:

- What are the fundamental principles behind matrix theories?
- What are the underlying gauge symmetries?
- Is it possible to construct matrix models which do not depend on the background fields?
- Why do matrix models involve gravity? In other words, why is gravity induced from gauge theory?
- What is a possible mechanism to realize four dimensional flat space-time?

Let us explain the above questions in order in more detail. In order to explain the first and second questions about the fundamental principles and the underlying gauge symmetries behind the matrix models, it is useful to compare the present status of the matrix models with general relativity by Einstein \[10\]. General relativity is built from only two basic concepts, namely, equivalence principle as the fundamental principle and general coordinate invariance as the gauge symmetry in the framework of Riemannian geometry. These basic concepts have played a very important role not only in establishing a complete form of general relativity but also in providing a unified picture of gravity and geometry. On the other hand, in the matrix models, we have not yet succeeded in finding such basic concepts. However, from the successful formulation of the matrix models one may have a glimpse of a hint that the final theory may be constructed based on the non-commutative geometry \[11\] which essentially describes the uncertainty principle of space-time at the Planck scale, which will be also discussed later.

Concerning the third question, it is useful to cite the words of Witten \[12\]: "Finding the right framework for an intrinsic, background independent formulation of string theory is one of the main problems in the subject, and so far has remained out of reach." "Though gauge invariant open-string and closed-string field theories are now known, the problem of background dependence of string field theory has not been successfully addressed. This problem is fundamental because it is here that one really has to address the question of what kind of geometrical object the string represents." In other words, string theory and M-theory, as

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5 See the reference \[9\] for recent development to this problem.
a theory including quantum gravity, should pick up its own space-time background in a dynamical manner and should not be a priori formulated on the basis of a special background field. This problem, of course, is closely related to the one of how the theory finds a unique vacuum state which describes our realistic world. However, the actions of the matrix models [6, 7] involve the flat background metric in the kinetic and/or the potential parts, which is not allowed from the viewpoint of Witten. We will discuss how to construct the background independent matrix model in section 3. Maybe in theories dealing with quantum gravity the dynamical background metric may be induced from some quantum effect or through a mechanism of spontaneous symmetry breakdown of topological symmetry [13]. This issue will be also argued in this paper.

Next, we would like to comment on the fourth question of the relation between the matrix models and general relativity. We should notice that we have at present no clear understanding of how the matrix theories are connected with Einstein’s general relativity. Even if there is circumstantial evidence that the low energy theory of the matrix models contains general relativity (or supergravity), it is quite obscure how general relativity is derived from the matrix models in a comprehensive manner [14]. But the recent progress on AdS/CFT correspondence [4] seems to suggest that the supergravity in a bulk theory could be described by the corresponding gauge theory on its boundary. Such a viewpoint is physically plausible from the following two reasons. One reason comes from the Bekenstein-Hawking entropy formula of black holes, which strongly suggests that if quantum gravity couples to the theory one space dimension is effectively reduced and only the boundary theory is relevant to the physics. The other reason is that as is well known all observables in quantum gravity such as mass and charge e.t.c. are defined on the boundary (usually, at spatial infinity).

Finally, let us consider the problem of Kaluza-Klein compactification in the matrix models. Since we wish to regard the matrix models as promising candidates of the final theory, they should provide a natural mechanism for compactification yielding the four dimensional flat space-time from eleven or ten dimensional space-time. With respect to this point, I have a conjecture although this conjecture is not limited to the matrix models but connected with a universal feature of string theory and M-theory. For sake of simplicity, we confine ourselves to the eleven dimensional M-theory. M-theory is usually defined as the theory which possesses $M2$-brane and its EM-dual, $M5$-brane as its classical solutions. Suppose that $M2$-brane and $M5$-brane occupy the directions of space-time coordinates along $(t, x_4, x_5)$ and $(t, x_6, x_7, x_8, x_9, x_{10})$, respectively. Here the key observation is to recall that all extended objects except string are unstable quantum mechanically as well as classically. The reason is that roughly speaking we cannot balance the gravitational attraction with the centrifugal repulsion in all directions on $p$-branes ($p \geq 2$). Then it is natural to make a guess that the instability associated with $M2$-brane and $M5$-brane in M-theory plays an important role in the spontaneous compactification of the excessive space-time dimensions. If we assume that the space directions tangential to $M2$-brane and $M5$-brane are compactified as a result of their instability, we are then left with the four dimensional space-time $x_\mu$ ($\mu = 0, 1, 2, 3$). This conjecture is quite speculative (in fact, it is based on a simple arithmetic $11 - (2 + 5) = 4$), but
it seems to be surprising at least for me that M-theory has a natural seed for compactification in the form of classical solutions in its own right according to the above conjecture.

The paper is organized as follows. In section 2, we construct the Schild action for general bosonic p-brane that is classically equivalent to the Nambu-Goto action except some singular configurations, where special attention is paid to the meaning of the constraints in the Schild action for string. In section 3, we derive a stronger form of the space-time uncertainty principle from the topological field theory where the classical action is trivially zero. The key idea here is the breakdown of the topological symmetry in passing from the continuous field theory to the discrete matrix model. In section 4, we incorporate the spinors in the above theory and construct a new matrix model. If we require this theory to be invariant under $N = 2$ supersymmetric transformations in ten dimensions, it turns out that this new matrix model becomes the IKKT model or the Yoneya model for Type IIB superstring. This choice is dependent on the form of a classical solution for a scalar function. In section 5, we study two types of background independent matrix model and examine some intriguing problems such as its classical solutions and local symmetries. Moreover, we incorporate the spinors in BF matrix model in a background independent way and construct a new matrix model with BRST-like supersymmetry whose partition function yields the Casson invariants. The final section is devoted to conclusions.

## 2 The Schild action for general p-brane

In this section, we construct the Schild action for general bosonic p-brane that is equivalent to the Nambu-Goto action for p-brane and then analyse the structure of the constraints in the Hamiltonian formalism.

First of all, let us recall the Schild action for bosonic string ($p = 1$), which is of the form

$$S_{n}^{p=1} = -\frac{1}{n} \int d^2 \xi \ e \left[ \frac{1}{e^n} \left\{ -\frac{1}{2\lambda_1^2} \right\}^{\frac{n}{2}} + n - 1 \right],$$

(1)

where $e(\xi)$ is a positive definite scalar density defined on the string world sheet parametrized by $\xi^0$ and $\xi^1$, $\lambda_1 = 2\pi\alpha'$, and $\sigma^{\mu_1\mu_2}$ is defined as $\varepsilon^{\alpha_1\alpha_2} \partial_{\alpha_1} X^{\mu_1} \partial_{\alpha_2} X^{\mu_2}$. Here $X^\mu(\xi)$ ($\mu = 0, 1, \ldots, D - 1$) are space-time coordinates and the index $\alpha$ runs over the world sheet indices $0$ and $1$. Throughout this paper, we assume that the flat space-time metric takes the form defined as $\eta_{\mu\nu} = diag(- + + \cdots +)$.

Then it is quite straightforward to build the Schild action for general bosonic p-brane by generalizing (1). The concrete expression is given by

$$S_{n}^{p} = -\frac{1}{n} \int d^{p+1} \xi \ e \left[ \frac{1}{e^n} \left\{ -\frac{1}{(p+1)!\lambda_1^2} \right\}^{\frac{n}{2}} + n - 1 \right],$$

(2)
where \( \sigma^{\mu_1 \cdots \mu_{p+1}} = \varepsilon^{\alpha_1 \cdots \alpha_{p+1}} \partial_{\alpha_1} X^{\mu_1} \cdots \partial_{\alpha_{p+1}} X^{\mu_{p+1}} \) and the world volume index \( \alpha \) now takes the values \( 0, 1, \cdots, p \).

In fact, we can demonstrate that (2) is equivalent to the Nambu-Goto action for \( p \)-brane as follows. Taking the variation with respect to the auxiliary field \( e(\xi) \), one obtains the constraint

\[
e(\xi) = \frac{1}{\lambda_p} \sqrt{- \frac{1}{(p + 1)!} (\sigma^{\mu_1 \cdots \mu_{p+1}})^2}.
\]

(3)

Plugging the constraint (3) into the Schild action (2), one obtains

\[
S_p^n = - \int d^{p+1} \xi \sqrt{-1} \lambda_p \frac{1}{\sqrt{- \frac{1}{(p + 1)!} (\sigma^{\mu_1 \cdots \mu_{p+1}})^2}} \int d^{p+1} \xi \sqrt{- \det \partial_\alpha X^\mu \partial_\beta X_\mu},
\]

(4)

where the identity

\[
\det \partial_\alpha X^\mu \partial_\beta X_\mu = \frac{1}{(p + 1)!} (\sigma^{\mu_1 \cdots \mu_{p+1}})^2
\]

was used. Hence the Schild action (2) becomes at least classically equivalent to the Nambu-Goto action (4) except some singular configurations.

In order to understand the constraint (3) more closely, it is useful to make use of the Hamiltonian formalism. The canonical conjugate momenta to the \( X^\mu \) are given by

\[
P_\mu = \frac{1}{\varepsilon^{n-1} p! \lambda_p^2} \left\{ - \frac{1}{(p + 1)! \lambda_p^2} (\sigma^{\mu_1 \cdots \mu_{p+1}})^2 \right\}^{\frac{1}{2}-1}
\]

\[
\times \sigma_{\mu_1 \cdots \mu_p} \varepsilon^{i_1 \cdots i_p} \partial_{i_1} X^{\mu_1} \cdots \partial_{i_p} X^{\mu_p},
\]

(6)

where the index \( i \) takes the values from 1 to \( p \). From (6), it is easy to see that the momenta satisfy the primary constraints

\[
P_\mu \partial_\nu X^\mu = 0,
\]

(7)

\[
P^2 + \frac{1}{\lambda_p^2} \det \partial_\alpha X^\mu \partial_\beta X_\mu = 0,
\]

(8)

where the lapse (Hamiltonian) constraint (8) is a consequence of the constraint (3) while the shift (momentum) constraints (7) come from the definition (3) trivially. In this sense, the constraint (3) encodes all the dynamical informations of the Schild action for \( p \)-brane.

Finally, it is valuable to point out that in the case of string theory the constraint (3) expresses the space-time uncertainty principle of string [17] when the Poisson bracket is replaced by a commutator in the large \( N \) matrix model, and was utilized as the first principle for constructing a type IIB supersymmetric matrix model [18]. (See the other construction of matrix models on the basis of the Schild action [19, 20].)
3 A matrix model from space-time uncertainty principle and breakdown of topological symmetry

In this section let us construct a bosonic matrix model which expresses an essential content of the space-time uncertainty principle of string [21]. Let us start by considering a topological theory [22] where the classical action is trivially zero but has a nontrivial dependence on the fields $X^\mu(\xi)$ and $e(\xi)$ as follows:

$$S_c = S_c(X^\mu(\xi), e(\xi)) = 0.$$  \hfill (9)

The BRST transformations corresponding to the topological symmetry are given by

$$\delta_B X^\mu = \alpha^\mu, \quad \delta_B \alpha^\mu = 0,$$

$$\delta_B e = e \eta, \quad \delta_B \eta = 0,$$

$$\delta_B \bar{c} = b, \quad \delta_B b = 0,$$  \hfill (10)

where $\psi^\mu$ and $\eta$ are ghosts, and $\bar{c}$ and $b$ are respectively an antighost and an auxiliary field. Note that these BRST transformations are obviously nilpotent. Also notice that the BRST transformation $\delta_B e$ shows the character as a scalar density of $e$.

As always in the analysis of a topological field theory, the first step is pick up a gauge which describes an interesting moduli space. The key idea in this paper, then, is to fix partially the topological symmetry corresponding to $\delta_B e$ by the "conformal" constraint (3) in the case of string. Consequently, the quantum action defined as $S_b = \int d^2 \xi \, eL_b$ becomes

$$L_b = \frac{1}{e} \delta_B \left[ e \left( \frac{1}{2} \{ X^\mu, X^\nu \}^2 + \lambda^2 \right) \right]$$

$$= b \left( \frac{1}{2} \{ X^\mu, X^\nu \}^2 + \lambda^2 \right) - \bar{c} \left( \frac{1}{2} \{ X^\mu, X^\nu \}^2 + \lambda^2 \right) + 2 \{ X^\mu, X^\nu \} \{ X^\mu, \alpha^\nu \},$$  \hfill (11)

where the BRST transformations (10) were used. Here for later convenience it is useful to redefine the auxiliary field $b$ by $b + \bar{c} \eta$. Then $L_b$ can be cast into a simpler form

$$L_b = b \left( \frac{1}{2} \{ X^\mu, X^\nu \}^2 + \lambda^2 \right) - 2\lambda^2 \bar{c} \eta - 2\bar{c} \{ X^\mu, X^\nu \} \{ X^\mu, \alpha^\nu \}.$$  \hfill (12)

What is necessary to obtain a stronger form of the space-time uncertainty relation is to move to the large $N$ matrix theory where we have the following correspondence

$$\int d^2 \xi \, e \longleftrightarrow \text{Trace},$$

$$\int De \longleftrightarrow \sum_{n=1}^{\infty},$$  \hfill (13)

where the trace is taken over $SU(n)$ group. These correspondence can be justified by expanding the hermitian matrices by $SU(n)$ generators in the large $N$ limit as is reviewed by the reference [23].
Now in the large $N$ limit, we have

$$S_b = Tr \left( b \left( \frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right) - 2 \lambda^2 \bar{c} \eta - 2 \bar{c} [X^\mu, X^\nu] [X^\mu, X^\nu] \right).$$  \hspace{1cm} (14)$$

Then the partition function is defined as

$$Z = \int D X^\mu D \alpha^\mu D e D \eta D \bar{c} Db e^{-S_b}$$
$$= \sum_{n=1}^{\infty} \int D X^\mu D \alpha^\mu D \eta D \bar{c} Db e^{-S_b}.$$ \hspace{1cm} (15)

At this stage, it is straightforward to perform the path integration over $\eta$ and $\bar{c}$. Consequently, one obtains

$$Z = \sum_{n=1}^{\infty} \int D X^\mu D \alpha^\mu D b e^{-Tr b \left( \frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right)}.$$ \hspace{1cm} (16)

In (16) since the quantum action does not depend on $\alpha^\mu$ it is obvious that there remains the gauge symmetry

$$\delta \alpha^\mu = \omega^\mu,$$ \hspace{1cm} (17)

which is of course nothing but the remaining topological symmetry. Now let us factor out this gauge volume or equivalently fix this gauge symmetry by the gauge condition $\alpha^\mu = 0$, so that the partition function is finally given by

$$Z = \sum_{n=1}^{\infty} \int D X^\mu D b e^{-Tr b \left( \frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right)}.$$ \hspace{1cm} (18)

It is remarkable that the variation of the action with respect to the auxiliary variable $b$ in (18) gives a stronger form of the space-time uncertainty relation and the theory is ”dynamical” in the sense that the ghosts have completely been decoupled in (18). In other words, we have shown how to derive the space-time uncertainty principle from a topological theory through the breakdown of the topological symmetry in the large $N$ matrix model. Why has the topological theory yielded the nontrivial ”dynamical” theory? The technical reason is very much simple. In the passage from the continuous theory (13) to the matrix theory (14), the dynamical degree of freedom associated with $e(\xi)$ was replaced by the discrete sum over $n$ while the corresponding BRST partner $\eta$ remains the continuous variable. This distinct treatment of the BRST doublet leads to the breakdown of the topological symmetry giving rise to a ”dynamical” matrix theory. In this respect, it is worthwhile to point out that while the topological symmetry is ”spontaneously” broken in this process, the other gauge symmetries never be violated. Moreover, notice that the above-examined phenomenon is a peculiar feature in the matrix model with the scalar density $e(\xi)$, which means that an existence of the gravitational degree of freedom is an essential ingredient since the generators
of the world-sheet reparametrizations, the Virasoro operators, provide the Ward-identities associated with the target space general covariance.

A rather unexpected appearance of the topological field theory also seems to be plausible from the following intuitive arguments. Suppose that we live in the world where the topological symmetry is exactly valid. In such a world we have no means of measuring any distance owing to lack of the metric tensor field so that there is neither concept of distance nor the space-time uncertainty principle. But once the topological symmetry which is particularly connected with the gravitational degrees of freedom, is spontaneously broken by some dynamical mechanism, an existence of the dynamical metric together with a string having the minimum length would give us both concepts of the distance and the space-time uncertainty principle. Our bosonic matrix model actually realizes this scenario in a concrete way.

4 Supersymmetric matrix models

Having obtained a bosonic matrix model, we now turn our attention to a more interesting model, i.e., its generalization to a supersymmetric matrix model [21]. Actually, non-perturbative formulations of both M-theory [3] and IIB superstring [4, 5] are based on the supersymmetry. Here we should emphasize that our philosophy in constructing a supersymmetric matrix model is rather different from that in the bosonic case in the previous section although we will go along a similar path of procedure in what follows. Namely, so far by starting with the topological field theory [22], we have tried to derive the space-time uncertainty principle proposed by Yoneya [17, 18]. In this section, we promote the space-time uncertainty principle to one of the basic principles for construction of a supersymmetric matrix model. In other words, on the basis of only two basic principles which are the space-time uncertainty principle of string and the topological symmetry, we attempt to construct a new supersymmetric matrix model. Of course, in the process of the model building, we will furthermore demand a strict invariance under the supersymmetric transformation although the topological symmetry is broken (in some case even the space-time uncertainty principle is not explicit) at the final stage.

As a first step for constructing a supersymmetric matrix model, one has to require the classical action to depend on the Majorana spinor field \( \psi_\alpha(\xi) \) as well as the bosonic fields \( X^\mu(\xi) \) and \( e(\xi) \)

\[
S_c = S_c(X^\mu(\xi), \psi_\alpha(\xi), e(\xi)) = 0,
\]

where the subscript \( \alpha \) stands for spinor index which should not be confused with the topological ghost \( \alpha^\mu(\xi) \) corresponding to \( X^\mu(\xi) \). The reason why we consider only the Majorana spinor will be explained later. This time, in addition to the BRST transformations (10) one has to add the following BRST transformations for fermions:

\[
\delta_B \psi_\alpha = \beta_\alpha, \quad \delta_B \beta_\alpha = 0.
\]
Next let us set up the gauge condition for $\delta_B e$. Instead of the bosonic case

$$\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 = 0,$$

(21)

we shall set up its natural extension involving the spinor field

$$\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 + \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} = 0.$$

(22)

When transforming to the matrix theory later, this gauge condition becomes a generalized stronger form of the space-time uncertainty principle. Although this generalized form is different from the original one proposed by Yoneya [17, 18] by the spinor part, in the ground state they are obviously equivalent so we take the above gauge condition (22). Interestingly enough, it will be shown later that the gauge choice (22) leads to the same theory as Yoneya’s one if a suitable solution for the auxiliary variable is chosen. Incidentally, the spinor part in (22) is adopted from an analogy with the supersymmetric Yang-Mills theory. Thus we have the quantum action $S_q = \int d^2 \xi \ e (L_b + L_f)$ with the bosonic contribution $L_b$ (11) and the fermionic one $L_f$ given by

$$L_f = \frac{1}{e} \delta_B \left( \bar{e} e \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} \right) = b \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} - \bar{e} \frac{1}{2} \left( \beta \Gamma_\mu \{X^\mu, \psi\} - \bar{\psi} \Gamma_\mu \{\alpha^\mu, \psi\} - \bar{\psi} \Gamma_\mu \{X^\mu, \beta\} \right).$$

(23)

Here in a similar way to the bosonic case, let us redefine the auxiliary field $b$ and the ghost $\beta$ by $\bar{b} + \bar{e} \eta$ and $\beta - \frac{1}{2} \bar{\psi} \eta$, respectively. As a result, $L_b$ is given by (12), on the other hand, $L_f$ takes the same form as (23). When we rewrite the fermionic part $L_f$ in this process, we need the famous Majorana identity $\bar{\psi} \Gamma_\mu \psi = 0$, for which we have confined ourselves to the Majorana spinor in this paper.

As before, at this stage let us pass to the matrix model. Again it is straightforward to carry out the path integration over $\bar{e}$ and $\eta$ in a perfect analogous way to the bosonic theory. Accordingly, we arrive at the following partition function

$$Z = \sum_{n=1}^{\infty} \int D X^\mu D \alpha^\mu D \beta_\alpha D b e^{-Tr \left( b \left( \frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 + \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} \right) \right)}.$$ 

(24)

In this expression since the quantum action is independent of $\alpha^\mu$ and $\beta_\alpha$ we have the remaining topological symmetries given by

$$\delta \alpha^\mu = \omega^\mu, \quad \delta \beta_\alpha = \rho_\alpha.$$

(25)

After factoring these gauge volumes out, the partition function is finally cast to be

$$Z = \sum_{n=1}^{\infty} \int D X^\mu D \psi_\alpha D b e^{-S_q} = \sum_{n=1}^{\infty} \int D X^\mu D \psi_\alpha D b e^{-Tr \left( b \left( \frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 + \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} \right) \right)}.$$ 

(26)
Of course, the action $S_q$ still possesses the zero volume reduction of the usual gauge symmetry
\[
\begin{align*}
\delta \psi_\alpha &= i [X_\mu, \Lambda], \\
\delta X_\mu &= i [\psi, \Lambda], \\
\delta b &= i [b, \Lambda].
\end{align*}
\]

In this way, we have constructed a new matrix model with the Majorana spinor variable on the basis of the space-time uncertainty principle and the topological symmetry. Although the action contains the spinor variable in addition to the bosonic variable, it is not always supersymmetric. The supersymmetry plays the most critical role in the matrix models for M-theory [4] and IIB superstring theory [7], so we should require the invariance under the supersymmetry for the action $S_q$ obtained in (26). The most natural form of $N=2$ supersymmetric transformations is motivated by a supersymmetric Yang-Mills theory whose (0+0)-dimensional reduction is given by
\[
\begin{align*}
\delta \psi^{ab}_\alpha &= i [X_\mu, X_\nu]^{ab} (\Gamma^{\mu\nu}\varepsilon)_\alpha + \zeta_\alpha \delta^{ab}, \\
\delta X^{ab}_\mu &= i\bar{\varepsilon} \Gamma_\mu \psi^{ab}, \\
\delta b^{ab} &= 0,
\end{align*}
\]
where we have explicitly written down the matrix indices to clarify that $\varepsilon_\alpha$ and $\zeta_\alpha$ are the Majorana spinor parameters. These supersymmetric transformations are of the same form as in IKKT model [7]. At this stage, we assume the space-time dimensions to be ten in order to make contact with IIB superstring.

To make the action $S_q$ in (26) invariant under the $N=2$ supersymmetry (28), it is easy to check that $b^{ab}$ must take the diagonal form with respect to the hermitian matrix indices. There are two interesting solutions. One of them is to select the auxiliary variable $b^{ab}$ to be proportional to $\delta^{ab}$ up to a constant. Without generality we take the proportional constant to be $-\frac{1}{2}$, therefore
\[
b^{ab} = -\frac{1}{2} \delta^{ab}.
\]
Here if we redefine $X^\mu$, $\psi$, and $-\frac{1}{2} \lambda^2$ in terms of $\alpha \frac{i}{2} X^\mu$, $\sqrt{2} \alpha \frac{i}{4} \psi$, and $\beta$, respectively, the action $S_q$ can be rewritten to be
\[
S_q = \alpha \left( -\frac{1}{4} Tr [X^\mu, X^\nu]^2 - \frac{1}{2} Tr \bar{\psi} \Gamma_\mu [X^\mu, \psi] \right) + \beta Tr 1.
\]

Note that this action is completely equivalent to the action in the IKKT model [7]. In this case, we cannot derive the space-time uncertainty relation from the equation of motion, but this relation might be encoded implicitly in the matrix character of the model.

The other interesting solution would be of the form
\[
b^{ab} = c \delta^{ab},
\]
with some additional auxiliary variable $c$. With this choice, the partition function (26) can be reduced to be

$$Z = \sum_{n=1}^{\infty} \int D\psi_\alpha Dc e^{-S_q}$$

$$= \sum_{n=1}^{\infty} \int D\psi_\alpha Dc e^{-c \text{ Tr} \left( \frac{1}{2}[X^\mu, X^\nu]^2 + \lambda^2 + \frac{1}{2}\nabla_\mu [X^\mu, \psi] \right)}.$$  

(32)

At first sight, it seems that we have obtained a new supersymmetric matrix model, but this is an illusion. We shall show that the above model is entirely equivalent to the Yoneya model [18] in what follows. Provided that we take account of the stronger form of the space-time uncertainty principle instead of the weaker form, the Yoneya model can be expressed in terms of the partition function

$$Z = \sum_{n=1}^{\infty} \int D\psi_\alpha Dc e^{-S_y}$$

$$= \sum_{n=1}^{\infty} \int D\psi_\alpha Dc e^{-c \text{ Tr} \left( \frac{1}{2}[X^\mu, X^\nu]^2 + \lambda^2 \right) - \text{ Tr} \frac{1}{2}\nabla_\mu [X^\mu, \psi]}.$$  

(33)

This partition in the Yoneya model does not look like the partition (32). But Yoneya has defined the supersymmetric transformations in a slightly different manner compared to ours (28). His supersymmetry is

$$\delta\psi_{\alpha}^{ab} = i c [X^\mu, X_\nu]^{ab} (\Gamma^{\mu\nu} \varepsilon)_\alpha + \zeta_\alpha \delta^{ab},$$

$$\delta X_\mu^{ab} = i \varepsilon \Gamma\mu \psi^{ab},$$

$$\delta c = 0.$$  

(34)

Note that there exists $c$ variable in the first term of the right-handed side in the first equation while it is absent in our formula (28) (Of course, in (28) we should replace $\delta b^{ab} = 0$ with $\delta c = 0$ for present consideration). Then it is easy to show that if we redefine $\psi_\alpha, \varepsilon$ and $\zeta$ by $c\frac{1}{2} \psi_\alpha, c\frac{1}{2} \varepsilon$ and $c\frac{1}{2} \zeta$, respectively in the Yoneya model, his action $S_y$ and supersymmetric transformations (34) conform to our action $S_q$ and supersymmetric transformations (28), respectively. To demonstrate a complete equivalence, we have to consider the functional measure. From these redefinitions the functional measure receives a contribution of an additional factor $c^2$, but this change is absorbed into a definition of the functional measure $Dc$ since the variable $c$ is the supersymmetrically invariant non-dynamical auxiliary variable in the model at hand. In this way, we can show that the solution (31) gives rise to the Yoneya model. It is surprising that depending on a choice of the scalar function $b$ our model leads to the IKKT model [7] and the Yoneya model [18], which on reflection clarifies the difference between both the matrix models.

Our approach heavily relies on the mechanism of the breakdown of the topological symmetry, so we should examine more closely the reason why our model gives rise to the nontrivial
"dynamical" theory from at least classically trivial topological theory. As mentioned in section 3, the technical reason lies in asymmetric treatment between the BRST doublet \( e \) and \( \eta \). However, there exists a deeper reason behind it. To make our arguments clear, it is useful to compare the present approach with the previous studies about the topological (pregauge-) pregeometric models \([23, 20]\) whose essential ideas will be recapitulated in what follows.

For generality, we consider an arbitrary dimension of space-time. We take the Nambu-Goto action as a classical action where we restrict ourselves to the case that the dimension is equal between world-volume and space-time. Then we can prove that this classical action becomes topological because we can eliminate all the dynamical degrees of freedom by means of the world-volume reparametrizations. Let us rewrite it to the Polyakov form

\[
S = -\frac{1}{\lambda} \int d^D \xi \sqrt{-\det \partial_a X \cdot \partial_b X} = \int d^D \xi \sqrt{-g} \left( g^{ab} \partial_a X^\mu \partial_b X^\nu + \lambda \right). \tag{35}
\]

In spite of lack of proof, the above two actions might be equivalent even in the quantum level owing to the topological character where there is no anomaly. Next work is to evaluate the effective action for the metric \( g^{ab} \) due to the quantum fluctuation of the "matter" fields \( X^\mu \) whose result is given by \([27]\)

\[
S_{\text{eff}} = i \, \text{Tr} \log \left( \left( \partial_a \sqrt{-g} g^{ab} \partial_b \right) \right) + \lambda \int d^D \xi \sqrt{-g}. \tag{36}
\]

When the curvature is small, it reduces to the Einstein-Hilbert action with the cosmological constant

\[
S_{\text{eff}} = \int d^D \xi \sqrt{-g} \left( \tilde{\lambda} + \frac{1}{16\pi G} R + O(R^2, \log \Lambda^2) \right), \tag{37}
\]

with

\[
\tilde{\lambda} = \frac{DA^4}{8(4\pi)^2} + \lambda,
\]

\[
\frac{1}{16\pi G} = \frac{DA^2}{24(4\pi)^2}, \tag{38}
\]

where we have introduced the momentum cutoff \( \Lambda \) of the Pauli-Villars type. Note that \([38]\) shows that we can choose the effective cosmological constant \( \tilde{\lambda} \) as small as we want, and the cutoff \( \Lambda \) is of the order the Planck mass. It is quite interesting to ask why the topological action has produced the Einstein-Hilbert action. This is because the momentum cutoff \( \Lambda \) breaks the topological symmetry with keeping the general covariance. In other words, we have secretly introduced a seed for breaking the topological symmetry by the form of the cutoff. Of course, it is an interesting idea to make a conjecture that renormalization may induce such a scale, but it seems to be quite difficult to prove this conjecture.

From this point of view, it is valuable to reconsider why the present formulation has produced the nontrivial matrix models from the topological field theory. Originally, in membrane
world, the matrix model has appeared to regularize the lightcone supermembrane action with area-preserving diffeomorphisms where it has been remarkably shown that the action becomes exactly that of ten dimensional $SU(n)$ supersymmetric Yang-Mills theory reduced to $(0 + 1)$-dimensions [28]. Similarly, in our models, passing from the continuous topological field theory to the discrete matrix model is equal to an introduction of the regularization where the regularization parameter corresponds to the size of the matrices. This type of the regularization breaks only the topological symmetry, from which we can obtain the nontrivial "dynamical" matrix models. It is very interesting that the matrix model is equipped with such a natural regularization scheme in itself. If the topological symmetry is truly broken by some mechanism in order to make the topological field theory a physically vital theory, we believe that theories equipped with some natural regularization scheme such as matrix model and induced gravity (pregeometry) would play an important role.

5 Background independent matrix models

In this section, we shall construct two different kinds of topological matrix models, which we call, Chern-Simons matrix model [29] and BF matrix model [30].

Let us start with a background independent matrix model which consists of only the hermitian matrices $X_\mu (\mu = 0, 1, \cdots , D - 1)$.

$$S^{D}_{CS} = \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} Tr X_{\mu_1} X_{\mu_2} \cdots X_{\mu_D}.$$ (39)

Interestingly enough, we can construct such an action only in the case that $D$ is odd numbers since an action with even numbers of $X_\mu$ is identically zero because of the cyclic property of trace and the totally antisymmetric property of the Levi-Civita tensor density. Thus we will set $D$ to be $2d + 1$ with $d \in \mathbb{Z}_+ \cup \{0\}$ in this section. Incidentally, the topological matrix model with any number of $X_\mu$ will be built later.

The equations of motion derived from the action (39) read

$$\varepsilon^{\mu_1 \mu_2 \cdots \mu_{2d}} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_{2d}} = 0.$$ (40)

Note that (40) does not include the background metric tensor in comparison with the equations of motion derived from IIB matrix models [7], whose formal expression is provided by

$$\eta^{\mu \nu} [X_\mu, [X_\nu, X_\rho]] = 0$$ (41)

with the flat Minkowskian metric $\eta^{\mu \nu}$. At this stage, it is useful to find the classical solutions satisfying the equations of motion (40). One obvious solution is the one satisfying the equation $[X_\mu, X_\nu] = 0$, that is, this solution has the form of the diagonal $N \times N$ matrix

$$X_\mu = \begin{pmatrix} X^{(1)}_\mu \\ \vdots \\ X^{(N)}_\mu \end{pmatrix},$$ (42)
which we call "classical space-time" in this paper. Next nontrivial solution is "string" solution given by

$$X_\mu = (X_0, X_1, 0, \cdots, 0),$$

(43)

where we have considered the string along 1st axis without losing generality. Similarly, "membrane" solution stretched out in the direction of 1st and 2nd axes reads

$$X_\mu = (X_0, X_1, X_2, 0, \cdots, 0).$$

(44)

It is obvious that this kind of solutions continues to exist until "(2d - 1)-brane"

$$X_\mu = (X_0, X_1, X_2, \cdots, X_{2d-1}, 0).$$

(45)

Moreover, a solution associated with several ",k-branes" (1 \leq k \leq 2d - 1) can be built out of the above solution for single "k-brane" in a perfectly similar way to the case of IIB matrix model [4]. For instance, the solution for two "strings" separated by the distance b along 2nd axis is given by

$$X_0 = \begin{pmatrix} x_0 & 0 \\ 0 & x_0 \end{pmatrix}, \quad X_1 = \begin{pmatrix} x_1 & 0 \\ 0 & x_1 \end{pmatrix},
X_2 = \frac{b}{2} \begin{pmatrix} 0 & 0 \\ 0 & -\frac{b}{2} \end{pmatrix}, \quad X_3 = \cdots = X_{2d} = 0,$$

(46)

where $$x_0$$ and $$x_1$$ are certain nonzero elements.

Now let us turn our attention to the symmetries in the action (43). It is remarkable that as well as the conventional gauge symmetry

$$X_\mu \to X'_\mu = UX_\mu U^{-1}$$

(47)

with $$U \in U(N)$$, the action (43) is invariant under the local translation of the diagonal element

$$X_\mu \to X'_\mu = X_\mu + V_\mu(X) \cdot 1$$

(48)

with $$V_\mu(X)$$ being not a matrix but a c-number function of $$X_\mu$$. This symmetry is in sharp contrast with the matrix models [3, 7] where $$V_\mu$$ is a global parameter of c-number. Namely, the global translation in [3, 7] is now promoted to the local translation. In this respect, it is of interest to recall the following things. Firstly, in the matrix models [3, 7] the diagonal matrix like (42) corresponds to the classical space-time coordinates while the non-diagonal matrix describes the interactions. Hence the local symmetry (48) coincides with the local space-time translation at the classical level. Secondly, it is well known that general relativity is the gauge theory with the local translation as the gauge symmetry, so the existence of this symmetry might be a signal of the existence of general relativity in this matrix model though we need more studies to confirm this conjecture in future.
Thus far, we have considered the Chern-Simons matrix model, but this model has some problems. In particular, it is quite unsatisfactory that we cannot construct the matrix model in even space-time dimensions. Furthermore, it seems to be difficult to make a supersymmetric extension of the Chern-Simons matrix model without introducing the background metric. Finally, it is at present unclear that the Chern-Simons matrix model has a relationship with general gravity. Luckily, we have already met a similar situation to this in topological quantum field theories where the Chern-Simons theory is replaced with the BF theory \[31, 32, 33, 34, 35\] in order to overcome these impasse. In the case of the matrix model at hand we also proceed with the same line of argument as the topological quantum field theories.

Now we would like to present BF matrix model \[30\] which has the form

\[ S_n^D = \varepsilon^\mu_1\mu_2\cdots\mu_D \text{Tr} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_n} B_{\mu_{n+1}\cdots\mu_D}, \]  

(49)

where a totally antisymmetric tensor matrix \(B\) is introduced. In this respect let us recall that the original form of topological BF theory \[31, 32, 33, 34, 35\] is

\[ S_{BF} = \int \varepsilon^{\mu_1\mu_2\cdots\mu_D} \text{Tr} F_{\mu_1\mu_2} B_{\mu_3\cdots\mu_D}, \]  

(50)

where the 2-form field strength \(F\) is defined as \(F = dA + A^2\). Thus, precisely speaking, the straightforward generalization of the topological BF theory \[50\] to the matrix model corresponds to the case of \(n = 2\) in \[49\]. Of course, owing to the introduction of the matrix \(B\) the action \[49\] makes sense in arbitrary space-time dimension.

The classical equations of motion derived from the BF matrix model \[49\] read

\[ \varepsilon^{\mu_1\mu_2\cdots\mu_D} X_{\mu_1} X_{\mu_2} \cdots X_{\mu_n} = 0, \]  

(51)

\[ \sum_{i=1}^{n} (-1)^{i-1} \varepsilon^{\mu_1\cdots\hat{\mu}_i\cdots\mu_D} X_{\mu_i+1} \cdots X_{\mu_n} B_{\mu_{n+1}\cdots\mu_D} X_{\mu_1} \cdots X_{\mu_{i-1}} = 0, \]  

(52)

where \(\hat{\mu}_i\) denotes that the index \(\mu_i\) is excluded. Note that apart from the number of \(X_\mu\), Eq.\[51\] accords with \[40\] in the Chern-Simons matrix theory. Thus the structure of the solutions with respect to \(X_\mu\) is almost the same as that case. On the other hand, it is Eq.\[52\] that appears for the first time in the BF matrix model. In fact, we can show that this equation has an important implication in relating the model at hand to general relativity \[30\].

As for the gauge symmetries, besides the usual \(U(N)\) gauge symmetry, at first glance the action \[49\] looks like it might be invariant under the following natural generalization of the local translation symmetry \[48\]

\[ X_\mu \rightarrow X'_\mu = X_\mu + V_\mu(X) \mathbf{1}, \]  

\[ B_{\mu_{n+1}\cdots\mu_D} \rightarrow B'_{\mu_{n+1}\cdots\mu_D} = B_{\mu_{n+1}\cdots\mu_D} + W_{\mu_{n+1}\cdots\mu_D}(X) \mathbf{1}. \]  

(53)

However, it is interesting to notice that only the action \[49\] with \(n\) being even integers has such a local translation symmetry while the action \[49\] with odd \(n\) has neither the local nor the global translation symmetry.
Now we will discuss some generalizations of the BF model to include fermionic symmetry. Indeed, in the matrix models [6, 7] the fermionic symmetry, in particular, the supersymmetry, was needed to guarantee the cluster and BPS properties of instantons. One possibility is to add fermions of integer spin to achieve a BRST-like symmetry. It is known that the partition function of the BF theory is related to the Ray-Singer torsion [36] while that of the BF theory with such a BRST-like symmetry corresponds to the Casson invariants. We think that this statement is valid even in the BF matrix model treated in this paper. Let us start by the following BRST-like fermionic symmetry:

\[
\delta X_\mu = \eta \psi_\mu, \quad \delta \psi_\mu = 0, \\
\delta X_{\mu_1 \cdots \mu_D} = -\eta B_{\mu_1 \cdots \mu_D}, \quad \delta B_{\mu_1 \cdots \mu_D} = 0. \tag{54}
\]

We can check explicitly the following action to be invariant under the fermionic symmetry (54):

\[
S_D^{\psi} = \varepsilon^{\mu_1 \mu_2 \cdots \mu_D} \text{Tr}(X_{\mu_1} X_{\mu_2} \cdots X_{\mu_n} B_{\mu_{n+1} \cdots \mu_D} \\
- \sum_{i=1}^n X_{\mu_1} X_{\mu_2} \cdots X_{\mu_{i-1}} \psi_{\mu_i} X_{\mu_{i+1}} \cdots X_{\mu_n} \chi_{\mu_{n+1} \cdots \mu_D} ). \tag{55}
\]

For even integers \(n\), this action is still invariant under the enlarged local translation which constitutes of Eq.(53) and

\[
\psi_\mu \rightarrow \psi'_\mu = \psi_\mu + v_\mu(X), \\
\chi_{\mu_{n+1} \cdots \mu_D} \rightarrow \chi'_{\mu_{n+1} \cdots \mu_D} = \chi_{\mu_{n+1} \cdots \mu_D} + w_{\mu_{n+1} \cdots \mu_D}(X). \tag{56}
\]

A more interesting possibility of incorporating fermions of half integer spin would be to twist the action (55) like the topological quantum field theory [22]. Here note that even if the bosonic action (39) is nontrivial its BRST-like generalization (53) is BRST-exact form so that we can use the twisting technique developed in the reference [22].

6 Conclusions

This work has explored various topological approaches that are useful in constructing and understanding the matrix theory. Making use of the Schild action explained in section 2, in sections 3 and 4 we have showed that the space-time uncertainty principle of string and the matrix models by Yoneya and IKKT are interpreted as being induced from the spontaneous symmetry breakdown of the topological symmetry.

In section 5, we have presented an alternative form of topological matrix models, which we have called the Chern-Simons and the BF matrix models. This construction has a close parallel with that of the conventional topological field theory which is based on the continuous fields. These matrix models are independent of the background metric, but give us the nontrivial field equations such that the classical solutions include the commuting space-time
coordinates as well as interesting p-brane solutions. We think it is worthwhile to push forward line of inquiry of the models in more detail in future. In particular, we would like to clarify how the $SL(2,\mathbb{Z})$ duality is realized in these matrix models by following a similar procedure adopted in the reference [37].

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