Numerical Simulation of The Interaction Between Electromagnetic Wave and Plasma

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Abstract. With a view to the potential application on stealth design of inductively coupled plasma (ICP), the simulation model of ICP scattering parameters was established by using COMSOL and ZT-FDTD method. Some detail of the interaction between electromagnetic wave and plasma in time domain which was difficult to be observed in experiment was revealed in the simulation. Wave source points composed of continuous reflection waves in the plasma were observed. The position of the wave source depends on the spatial distribution of the electron density peak. The influence of gas pressure (10 mTorr~1 Torr) and power (300~700 W) on attenuation efficiency was studied. The wave attenuation in plasma is characterized by band attenuation. The attenuation significantly improved with the increase of pressure and thickness monotonically. As the power increased, both the band width and amplitude of wave attenuation increased.

1. Introduction

The interaction between electromagnetic wave and plasma is a critics focus of researches on plasma stealth. At macro level, there are diverse propagation such as reflection, refraction and attenuation when the wave hits on plasma. At micro level, the respond of free electrons in plasma to the electric field component of electromagnetic wave will change the propagation characteristic. In return, the motion equations of free electrons are also affected by electromagnetic wave. In a word, the interaction is extremely complicated. So it is far from adequate to study the interaction by experiments only, because a lot of micro phenomenon and transient reactions are difficult to be presented in experiments. Recently, the rapid development of simulation technology, which not only reveals more details of the interaction but also cuts down the cost, provides highly effective assistance for the research.

Amount of work has been done on the simulation of interaction between electromagnetic wave and plasma. The propagation characteristic of electromagnetic wave in inhomogeneous plasma was studied by using WKB (Wentael-Kramers-Brillouin) method and layered approximation in [1]. And the reflection and attenuation coefficient were calculated. For the same problem, SMM (Scattering Matrix Method) method was used and compared with FDTD (Finite-Difference Time-Domain) method in [2] and [3]. In [4], the scattering of magnetized and unmagnetized plasma was calculated by using SO (Shift-Operator) FDTD.

It was a pity that in most of the studies, the spatial distribution of plasma parameters was assumptive. But in fact, there is always drastic space gradient difference in electron density of plasma. Moreover, the variation is consecutive and uneven in both tangential and normal direction. In this case, layered approximation of electron density is unfaithful.
In this paper, a joint simulation of finite element method in COMSOL and ZT(Z-translation)-FDTD was introduced to calculate the scattering parameters of inductively coupled plasma (ICP) in a broad frequency range. By means of well coupling of plasma discharging and electromagnetic scattering calculation, the interaction detail of electromagnetic wave and plasma was revealed more deeply and visually.

2. Method
At present, WKB, SMM and FDTD are the main simulation method in the field of electromagnetic scattering. Layered approximation are used in WKB and SMM when the scattering body is heterogeneous plasma. That is dividing the plasma into a number of lamellas along the propagation path of wave. And every lamella is taken as homogeneous. When the propagation is one-dimensional and the electron density of plasma varies slowly, the error is small and acceptable. However, to more complex electromagnetic calculation in two-dimensional or three-dimensional space, layered approximation is disabled due to large error. FDTD method is different that the Maxwell curl equations in differential form are transformed into difference equations with second-order accuracy. The propagation progress of electromagnetic wave is simulated directly by recursion of time steps and iteration of space steps. FDTD method is appropriate for solving the electromagnetic scattering of arbitrary shapes dispersive medium in ultra wide band.

FDTD method has the following advantages relative to other simulation methods[5]. Firstly, the electromagnetic parameters of every mesh and cell are defined so that every subtle change in the medium parameters and shape of objects will be reflected exactly. This is a huge advantage when dealing with the scattering of complex-shape objects covered with dispersive medium. Secondly, the time-domain characteristic of electromagnetic field is expressed more directly and clearly. So some complicated physical processes such as the details of wave propagation in plasma are visual. Thirdly, pulse function in time-domain can be directly used as the exciting source. Then information of propagation progress in ultra wide band will be obtained after Fourier transform. Fourthly, FDTD method is appropriate for parallel computing because no matrix inversion is needed.

2.1. Principle of FDTD
The differential equations of TM wave in time domain is:

\[
\begin{align*}
\frac{\partial E_x}{\partial t} & = -\mu \frac{\partial H_y}{\partial x} - \gamma \mu H_y \\
-\frac{\partial E_y}{\partial x} & = -\mu \frac{\partial H_x}{\partial t} - \gamma \mu H_x \\
\frac{\partial E_z}{\partial x} - \frac{\partial H_x}{\partial y} & = \varepsilon \frac{\partial E_x}{\partial t} + \gamma E_z \\
\end{align*}
\]

(1)

Here E is the electric field intensity. H is the magnetic field intensity. \(\varepsilon\) is the permittivity. \(\mu\) is the permeability. \(\gamma\) is the propagation constant. The principle of FDTD is to discrete formula (1) by sampling from the electromagnetic field discretely in space and time. Then the difference equations of Maxwell curl with second-order accuracy can be got. So the first step is grid division of computing space. In this paper, the classical Yee cell is employed as shown in Figure 1[6-7].
Yee cell is a kind of typical central difference approximation. Every electric field component is encircled by four magnetic field component and vice versa. Adjacent electric field component and magnetic field component differ by half a step. Such configuration of field quantity can make the geometry construction accord with Ampere law and Faraday law. And the boundary conditions of medium are consecutive.

Next, number the Yee cells in the discrete computing space according to the matchup: \((x, y) \Rightarrow (i\Delta x, i\Delta y)\). Here \(\Delta x\) and \(\Delta y\) represent the time step in x and y direction respectively. \(i\) and \(j\) are the numbers of Yee cells. The time step at moment \(n\) is \(\Delta t_{n-1/2} = (n-1/2) \Delta t\). \(\Delta t\) is the step of time. After discretization, the field component at arbitrary point is:

\[
\mathbf{F}^n(i, j) = \mathbf{F}^n(i\Delta x, j\Delta y)
\]

Based on this, the recursion formula of FDTD in vacuum is obtained.

\[
\begin{align*}
H_{x(i+1/2, j)}^{n+1} &= H_{x(i+1/2, j)}^n + \frac{\Delta t}{\Delta x} \frac{1}{\varepsilon_0 \mu_0} \left[ E_{z(i+1/2, j+1/2)}^n - E_{z(i+1/2, j-1/2)}^n \right] \\
H_{x(i, j+1/2)}^{n+1} &= H_{x(i, j+1/2)}^n + \frac{\Delta t}{\Delta x} \frac{1}{\varepsilon_0 \mu_0} \left[ E_{z(i+1/2, j+1/2)}^n - E_{z(i-1/2, j+1/2)}^n \right] \\
E_{z(i+1/2, j+1/2)}^{n+1} &= E_{z(i+1/2, j+1/2)}^n + \cdots \\
\Delta t \frac{\varepsilon_0}{\Delta x} \sqrt{\frac{\varepsilon_0}{\mu_0}} \left[ H_{x(i+1/2, j)}^n - H_{x(i+1/2, j+1)}^n + H_{x(i+1/2, j)}^n - H_{x(i+1/2, j+1)}^n \right]
\end{align*}
\]

What needs to be emphasized is that the result of FDTD is significant only when the simulation is convergent. So the value of time step and space step needs to meet the stability constraints. For two-dimension simulation, the constraints is[8]:

\[
c\Delta t \leq \frac{1}{\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}}}
\]

When \(\Delta x = \Delta y\), that is \(c\Delta t / \Delta x \leq 1/\sqrt{2}\).

2.2. ZT-FDTD

In ZT-FDTD, to avoid the complex integral term generated by Fourier inversion, the electromagnetic equations of Drude model are represented as discrete difference equations in Z domain[9-11]. The constitutive relation of plasma medium in Z-domain is:

\[
\mathbf{D}(z) = \mathbf{E}(z) + \mathbf{S}(z)
\]

Here \(\mathbf{D}(z)\) is the electric displacement vector in Z-domain. \(\mathbf{S}(z)\) is defined as

\[
\mathbf{S}(z) = z^{-s}\mathbf{S}(z)[1 - \exp(-\nu_m \Delta t)] - z^{-s}\mathbf{S}(z)\exp(-\nu_m \Delta t) + \frac{\partial}{\partial z} \mathbf{E}(z)z^{-s}\Delta t[1 - \exp(-\nu_m \Delta t)]
\]
Here the first two polynomials on right of the equation are respectively the value of $S(z)$ in the first two time steps. $\nu_m$ is the collision frequency of plasma. $\omega_p$ is the oscillation frequency of plasma. The recursion relation of field expressions of electromagnetic wave in plasma are shown in (7)–(10).

$$E^n = D^n - S^n$$  \hspace{1cm} (7)

$$S^n = S^{n-1}[1 - \exp(-\nu_m \Delta t)] - S^{n-2} \exp(-\nu_m \Delta t) + \frac{\partial S^n}{\nu_m} \Delta t[1 - \exp(-\nu_m \Delta t)]$$  \hspace{1cm} (8)

$$H^{n+\frac{1}{2}} = H^{n-\frac{1}{2}} - \frac{\Delta t}{\mu} [\nabla \times E]^n$$  \hspace{1cm} (9)

$$D^{n+1} = D^n + \Delta t[\nabla \times H]^{n+1}$$  \hspace{1cm} (10)

3. Simulation calculation

3.1. Model Setup

Thanks to the advantages of high electron density, steady state of discharge, and ease of conformity to the aircraft surface, ICP is very appropriate for stealth design of aircraft local. The distribution of electron density ($n_e$) and $\nu_m$ in discharging space of plasma need to be obtained before calculating the scattering of ICP by FDTD. So the ZT-FDTD model in MATLAB was combined with the Boltzmann solver and plasma fluid dynamics model in COMSOL. Data exchanged among different platform as shown in Figure 2. M-function of MATLAB was used to design calculating program of wave propagation in plasma. The joint simulation model is shown in Figure 3. ICP was in a quartz chamber. The space is divided into plasma area, total field area and scattering area. The spatial distribution of $n_e$ and $\nu_m$ in plasma area was calculated in COMSOL. Plane wave which was in the form of Gaussian pulse was used as excitation source. The influence of quartz chamber on wave propagation was not considered.
It is a key problem to define the size of Yee cell and choose the dielectric parameter of plasma in data exchanging between COMSOL and MATLAB. The sampling points of electromagnetic wave in time domain need to reach a certain amount. A general principle is $\Delta x, \Delta y < \lambda/8$. $\lambda$ is the wavelength of incident wave. In this model, the variation of relative dielectric constant of plasma will lead to change of wave length. The shortest wave length in the pulse spectrum was selected specific to Gaussian pulse. Considering the huge gradient difference of $n_e$ among the ICP space, the wave length $\lambda$ is subdivided into 60 parts, that is $\Delta x=\Delta y=\lambda_{\text{max}}/60$. The frequency band selected in this paper is 0.5GHz to 20GHz, which is common used in radar. So $\Delta x=0.25$ mm. According to the Courant stability condition in function (4), $\Delta t=0.589$ps.

Boundary conditions must be considered because the differential Maxwell equations will be disabled at the abrupt interface of medium parameters. To Yee cells at the border of different medium, the electric field component at surface is tangential and the magnetic field component is normal. As shown in Figure 4, the whole calculation space was discretized according to the size of Yee cell. At last, the total number of Yee cell was $N \times M$. The sampling number of dielectric parameter in COMSOL was 4 times the number of Yee cells, and every Yee cell was corresponding to 9 sampling points. In this way, the dielectric parameter at the boundary of Yee cell in MATLAB and the sampling points of electromagnetic parameters in COMSOL were coordinating.

Figure 4. The relationship of sampling number and Yee cell number.
3.2. Analysis of Results

Figure 5. The propagation of wave in ICP at 10 mTorr.

The simulated process of interaction between sine wave pulse and ICP by ZT-FDTD is shown in Figure 5. Pressure in the discharging chamber of ICP was 10 mTorr. The distribution of $n_e$ at the power of 700 W is shown in Figure 5 (a). The frequency of incident wave was 10 GHz and the incident angle was 5°. As can be seen in Figure 5 (b), because the electron density was not too high, there was not strong reflection when the wave entered into the border area of plasma. As time went on, the closer to the center of the plasma, the higher the electron density and the stronger the reflection of electromagnetic wave. Obvious and continuous reflected wave was observed in Figure 5 (c) and (d). The wave source point was coincident with the peak point of electron density. Moreover, when the reflected wave came into air again, the amplitude was decreasing and the energy was lower than the incident wave. It can also be observed that the reflected wave was comprised by two portions, one was reflected by the plasma and the other was by the metal reflector. And continuous refraction at low amplitude was caused by the change of wave vector. So the reflected wave was no longer plane.

Increase the pressure of ICP to 1 Torr, the simulated result is shown in Figure 6. ICP was compressed in higher pressure. There were two peak points of electron density symmetrically located beside the center of chamber. Consequently, two wave source points of reflected wave were generated. So the scattering by higher pressure ICP was more complicated.
Figure 6. The propagation of wave in ICP at 1 Torr. The waveform of incident and reflected wave in time domain is shown in Figure 7. It can be seen that as time went on, the intensity of reflected wave gradually declined, and the reflected wave became lower when the pressure rose to 100 mTorr. So it may be effective to reduce the reflected wave by increasing the thickness of plasma and the pressure moderately.

Figure 7. The intensity of electric field at the sampling point in time domain.
The reflectivity of ICP at different pressure simulated from ZT-FDTD is shown in Figure 8. The reflectivity was defined as

$$R_{ef} = 20 \log \frac{E(\omega, z)}{E_0}$$

Here $E(\omega, z)$ and $E_0$ are the electric field amplitude of reflected wave and incident wave respectively.

At the pressure of 10 mTorr, $\nu_m$ is in a magnitude of 108 Hz. The wave frequency corresponding to $\omega_p$ is in a magnitude of 1010 Hz, $\nu_m << \omega_p$. So the attenuation to wave by collision was generally weak, except in partly narrow band. The valley value of reflectivity moved to high frequency as the power rose. This was because the resonance attenuation got stronger when the wave frequency got closer to $\omega_p$. $\nu_m$ was ten times higher when the pressure rose to 100 mTorr. It was seen that the attenuation was promoted. Compared with the pressure of 10 mTorr, the band width in which the attenuation was more than 10 dB and the maximum of attenuation both substantially increased. The pressure rose continually to 1 Torr and $\nu_m$ came to the magnitude of 1010 Hz, approximate to $\omega_p$. The attenuation increased further in a wider band. There was only one reflection valley at lower power (<400 W). As the power rose, there were more valley values. The frequency band in which attenuation was more than 20 dB was from 3.4 GHz to 10.2 GHz at the power of 700 W.

4. Conclusion

Compared with other simulation methods using layered approximation, ZT-FDTD method has advantages in accuracy and visualization. A joint simulation of finite element method in COMSOL and ZT(Z-translation)-FDTD was established to study the scattering characteristic of ICP. It was observed that the reflected wave was no longer plane because the wave vector was change due to the
drastic change of plasma electromagnetic parameters. Wave source points composed of continuous reflection waves in the plasma were observed. The wave source point was coincided with the peak point of electron density at power pressure. However, there were two wave sources when pressure rose to 1 Torr. The attenuation significantly improved with the increase of pressure and thickness monotonically. As the power increased, both the band width and amplitude of wave attenuation increased. So it is suggested that the attenuation band can be adjusted by plasma parameters.

Acknowledgments
This paper was supported by the National Natural Science Foundation of China and the Natural Science Foundation of Shaanxi Province.

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