Imaging and Visualization of Multiphase Fluid Flow by different reconstruction method for Electrical Computer Tomography

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Abstract. A new reconstruction method called Functional Weighted Inverse Matrix (FWIM) has been applied to an ill-posed inverse problem involving an electrical computer tomography for multiphase flow. Four image reconstruction algorithms—FWIM, Landweber (LW), Iterative Tikhonov regularization (ITR) and Generalized Vector Sampled Pattern Matching (GVSPM) methods—are tested using four types of pseudo particle distribution images with different dispersion concentration rates (DCR). The reconstructed images, as shown in figure, are compared in terms of capacitance residual, image error and image correlation. Overall, the accuracy is strongly dependent upon the image type and the dispersion concentration rates (DCR). However, the FWIM method proved superior to the LW, the ITR and the GVSPM methods in terms of the annular and bottom pseudo images.

1. INTRODUCTION
Recently, the application of non-invasive monitor, such as computed tomography (CT), in multiphase flow measurement has become increasingly popular. The electrical capacitance CT has been investigated as a visualization technique for the solid behavior in solid-air two-phase flow (Dyakowski et al. 1999; Halow et al. 1992; Huang et al. 1989). In capacitance CT, a sensor containing several electrodes is wrapped around the circumference of a pipeline, and the capacitances between the electrodes are measured. The particle concentration distribution, which is based on the permittivity distribution in a cross section, is obtained from the experimental capacitances. This is performed using an image reconstruction technique that is based on an ill-posed mathematical inverse problem. Because inverse problems are heavily dependent upon the system equation, a suitable image reconstruction technique for this capacitance CT is necessary. A variety of image reconstruction techniques have already been proposed (Isaksen 1996). Recently, the iterative techniques such as the Landweber method and the Iterative Tikhonov regularization method have been widely used because of the relatively high accuracy of their reconstruction images (Yang et al. 2003). These conventional reconstruction techniques, however, have several drawbacks with regard to empirical value setting and convergence at an infinitive number of iterations. For example, the Landweber method requires an empirical gain value in order to converge upon the image, and it strongly depends upon the iteration number. It indicates that an image that has been processed over a suitable number of iterations will become extremely distorted (Liu et al. 2001). Therefore, the Landweber method requires advance knowledge of the empirical gain value and the empirical iteration number. The Iterative Tikhonov regularization method, on the other hand, needs an empirical value to use the singular value
decomposition of the system equation to produce a pseudo inverse matrix. These conventional reconstruction methods are not able to accurately reconstruct an image with empirical values because they do not use an objective function to confirm the stability of the solution during the iterative process.

In order to overcome the drawbacks of these iterative methods, a novel solution strategy called the Generalized Vector Sampled Pattern Matching (GVSPM) method was proposed, and has already been applied to the optimization of electron beam dosing (Saotome et al. 1995; Yoda et al. 1997). However, as a kind iterative methods, the solve time of GVSPM method is too long for real time visualization. Therefore, a new non iterative method called Functional Weighted Inverse Matrix (FWIM) method was put forward for the real time visualization of multiphase flow.

In this study, the new FWIM method is applied to the reconstruction of particle distribution images. This paper details the characteristics of this method that were examined using a simulation for pseudo particle concentration distribution images. The results of this simulation were then compared to those from the conventional Landweber, Iterative Tikhonov regularization and GVSPM methods. The results in three areas were compared: capacitance residual, image error and image correlation.

2. GOVERNING EQUATION FOR ECT

The capacitance CT sensor is shown in Fig. 1(A) and (B). The twelve sensor electrodes are separated by insulation material (Yang 1996). The relationship between capacitance and permittivity in a static-electro field is expressed by:

$$C_{i,j} = -\frac{\varepsilon_0}{V_c} \int \varepsilon(r) \nabla V_i(r) \cdot dr$$  \hspace{1cm} (1)

where \(i\) is the standard electrode number that ranges from 1 to 11, and \(j\) is the reference electrode number, which ranges from \(i+1\) to 12. \(C_{i,j}\) is the measured capacitance between the standard electrode \(i\) and the reference electrode \(j\), \(\varepsilon_0\) is the known vacuum permittivity, \(\varepsilon(r)\) is the relative permittivity distribution on the cross section, \(r\) is a position vector on the cross section: \(r = (x, y)\), \(V_i\) is the known voltage to the \(i\)th electrode, \(\Gamma_j\) is the area affected by the electric line of force, and \(V_i(r)\) is the potential distribution on the cross section between the \(i\)th and \(j\)th electrodes. Even though the values of \(\varepsilon(r)\) and \(V_i(r)\) in Eq. (1) are unknown, \(\varepsilon(r)\) can be approximated by assuming that the electric charge’s linear coupling at a position \(r\) with a weight of sensitivity in \(\Gamma_j\) area is the total capacitance. As detailed in a previous work (Polydorides et al. 2002; Williams et al. 1995), when a particle exists solely in an infinitesimal small \(\Delta x \times \Delta y\) area at the center point \(r = r_0\) between the \(i\) and \(j\) electrodes, and air exists at the remaining positions, the Laplace equation

$$\nabla [\varepsilon(r_0) \nabla V_i(r_0)] = 0$$  \hspace{1cm} (2)

is assumed to hold in the cross section. A Finite Element Method can be used to discretize Eq. (2), and the distribution of \(V_i(r)\) can be obtained by substituting the boundary conditions into Eq. (1). Then, Eq. (1) can be rewritten as the following matrix expression:

$$C = S \cdot E$$  \hspace{1cm} (3)

In other words, the capacitance CT can be used to obtain the permittivity distribution of particles \(E\) in the cross section from both the known sensitivity map matrix \(S\) and the measured capacitance matrix \(C\). In the case of 12 electrodes and \(32 \times 32 = 1024\) pixels in the pipe cross section, as in Fig. 1(C), the sensitivity matrix \(S\) in Eq. (3) is a \(66 \times 1024\) matrix, the capacitance matrix \(C\) is a \(66 \times 1\) column vector, and the permittivity distribution matrix \(E\) is a \(1024 \times 1\) column vector. The mathematical method used to obtain the permittivity matrix \(E\) from both the capacitance matrix \(C\) and the sensitivity matrix \(S\) is an ill-posed inverse problem because the inverse matrix \(S^{-1}\) does not exist. The raw values of \(S\) have an extremely wide range; the sensitivity to an adjacent electrode pair is more than 100 times larger than that to an opposing electrode pair.
3. FWIM METHOD THEORY FOR INVERSE PROBLEM

An inverse problem is the task that often occurs in solving the system where the values of some model parameter(s) must be obtained from the observed data. Inverse problems are typically ill posed, it means that the number of unknown is more than the number of formula. As addressed above, the ill posed problem need to be solve in this research is shown as Equation 3. Here, the solution vector \( E \) could be assumed to be the product of a weighted \( 1024 \times 66 \) determinant \( W \) and a \( 66 \times 1 \) column vector \( S \) as follow,

\[
E = WS \tag{4}
\]

If the determinant \( CW \) is the non-singular, Equation 3 can be reduced to

\[
E = W[CW]^{-1}Y \tag{5}
\]

In Equation 5, the key point is the way to decide the weight determinant \( W \). If the solution \( e \) is the function of the solution space \( \alpha \), it could be given as,

\[
e(\alpha) = s_0 + s_1 \cos \alpha + s_2 \sin \alpha + s_3 \cos 2\alpha + s_4 \sin 2\alpha + \ldots \tag{6}
\]

Equation 6 can be rewrite using complex numbers as,

\[
e(\alpha) = s_0 + s_1 \text{Re}^{e^{i\alpha}} + s_2 \text{Im}^{e^{i\alpha}} + s_3 \text{Re}^{e^{i\alpha}} + s_4 \text{Im}^{e^{i\alpha}} + \ldots \tag{7}
\]

The solution vector \( E \) can be write as,

\[
E = \begin{pmatrix}
e_1 \\
e_2 \\
\vdots \\
e_m
\end{pmatrix} = \begin{pmatrix}
1 & 1 & 0 & 1 & \ldots \\
1 & \cos \alpha & \cos \alpha & \cos 2\alpha & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & \cos[(n-1)\alpha] & \sin[(n-1)\alpha] & \cos[2(n-1)\alpha] & \ldots
\end{pmatrix}\begin{pmatrix}
s_0 \\
s_1 \\
\vdots \\
s_{n-1}
\end{pmatrix} = WS \tag{8}
\]

Therefore, the weight determinant \( W \) can be given by,

\[
W = \begin{pmatrix}
1 & 1 & 0 & 1 & \ldots \\
1 & \cos \alpha & \cos \alpha & \cos 2\alpha & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
1 & \cos[(n-1)\alpha] & \sin[(n-1)\alpha] & \cos[2(n-1)\alpha] & \ldots
\end{pmatrix} \tag{9}
\]

4. IMAGE RECONSTRUCTION AND DISCUSSION

4.1 Reconstruction conditions and example images

In this section, the Landweber (LW), Iterative Tikhonov regularization (ITR), GVSPM and FWIM methods are tested using pseudo particle concentration distribution simulation images of solid-air two-phase flow. The following guidelines are established to compare each method impartially.

1) The iterative number of LW, ITR and GVSPM method is 30.
2) The GVSPM’s normalized sensitivity matrix is normalized with the norm of the column vector.
3) The first capacitance, which is obtained by multiplying the sensitivity matrix and the original pseudo image, is normalized with a minimum value of 0.0 and a maximum value of 1.0. The first
capacitance is the measured capacitance in the real experiments.

4) In both the original pseudo images and the final reconstructed images, all of the pixels outside the pipe are ignored because the area outside the pipe is not relevant to the image comparisons.

5) After per iteration, the capacitances for the LW, ITR and GVSPM method are normalized with a minimum value of 0.0 and a maximum value of 1.0. The reconstructed image is not normalized by itself.

6) Because the results are normalized by the norm after each iteration, the final images of the GVSPM method provide only relative values. Therefore, the final images and final capacitances must be normalized with a minimum value of 0.0 and a maximum value of 1.0.

7) The gain $\alpha$ of LW method is fixed at 2.0 and the $\gamma$ of ITR method is fixed at 0.01 for all of the pseudo particle images because of the relatively high correlation value.

Figure 2 shows the simulation images for four types of pseudo particle distributions. Because the actual solid-air two-phase flow image consists of main particle bulk and dispersion particles around it, many different dispersion concentration rates (DCR) between 0% and 100% are considered. For example, a 20% DCR correlates to a random white noise value between 0.0 and 0.2, which highlights the dispersion particles. These values are assigned to each pixel in the original 0% DCR image. Values over 1.0 replace all of the pixels with the original pseudo particle images.

Figure 2(a) displays the pseudo particle images with a 0% DCR and Figure 2(b) shows the representative images for a 100% DCR. In these figures, (*-1), (*-2), (*-3) and (*-4) are the annular, the center, the bottom and the four-bulk pseudo particles image within the pipe, respectively. In these pseudo images, the red pixels indicate the highest particle concentration, 1.0, and the blue pixels indicate air.

The representative reconstructed results of annular pseudo image that were obtained using each method are presented in Figure 3 as a reference. From these figures it is evident that the ring is reconstructed precision well using FWIM and GVSPM method, however, the reconstructed results using LW and ITR method are not ring and become circle.

![Figure 2 Pseudo images](image-url)
Figure 3 Reconstructed images of different DCR (Annular)

4.2 Estimation of reconstructed images

In order to estimate each method quantitatively, the capacitance residual \( C_R \), the image error \( I_E \) and the image correlation \( I_C \) are calculated using the following:

\[
C_R = \frac{\sum_{i=1}^{n} (c_i^{(k)} - c_i^{\text{exp}})^2}{\left( \sum_{i=1}^{n} (c_i^{(k)})^2 \right)^{1/2}}, \quad I_E = \frac{\sqrt{\sum_{i=1}^{n} (c_i^{(k)} - c_i^{\text{original}})^2}}{\sqrt{\sum_{i=1}^{n} (c_i^{\text{original}})^2}}, \quad I_C = \frac{\sum_{i=1}^{n} \left[ c_i^{(k)} - \bar{E}^{(k)} \right] \left[ c_i^{\text{original}} - \bar{E}^{\text{original}} \right]}{\sqrt{\sum_{i=1}^{n} (c_i^{(k)} - \bar{E}^{(k)})^2} \sqrt{\sum_{i=1}^{n} (c_i^{\text{original}} - \bar{E}^{\text{original}})^2}}
\]

(10)

In these equations, \( c_i^{(k)} \) is \( i \)th element of the final reconstructed image \( E^{(k)} \), \( \bar{E}^{(k)} \) is the special mean pixel value of \( E^{(k)} \), \( c_i^{\text{original}} \) is \( i \)th element of the original pseudo image matrix \( E^{\text{original}} \), and \( c_i^{(k)} \) is \( i \)th element of the final capacitance \( C^{(k)} \) calculated from the final reconstructed image \( E^{(k)} \). The low values of \( C_R \) and \( I_E \), the high value of \( I_C \) mean accurate reconstructed images.

Figure 4 to 7 show the estimation categories of the annular, the center, the bottom and the four-bulk pseudo particles images including the dispersion concentration rate DCR. In general, \( C_R \) and \( I_E \) increase and \( I_C \) decreases as DCR increases. The superior method is highly dependent upon the image type and the DCR. In the case of the annular image in Figure 4, the FWIM method proves superior to
the LW, ITR and GVSPM methods. Figure 4(a) shows the $I_C$ value for all the four methods is decrease as DCR increases. $I_C$ for FWIM method is much higher than that for LW, ITR and GVSPM method, the $I_C$ for LW, ITR method are nearly the same and always the lowest. Figure 4(b) displays that the $I_E$ for FWIM method is lower than LW, ITR and GVSPM, specifically, when DCR<20% and DCR>80%, $I_E$ for GVSPM is the lowest. The $C_R$ value for FWIM and GVSPM are the same and the lowest when DCR>30% as shown in Figures 4(c), however, when DCR<30%, $C_R$ for LW and ITR are the lowest.

The reconstructed images in the case of the center image of Figure 5 are highly dependent on the reconstructed method and the DCR. More specifically, in Figure 5(a), the $I_C$ for GVSPM are higher than those of FWIM, LW and ITR in the case of DCR<20%; however, when 20%<DCR<80%, $I_C$ for FWIM are the highest; when DCR>80%, $I_C$ for FWIM and GVSPM become superior to LW and ITR, whose $I_C$ value becomes extremely low. Figure 5(b) shows that the $I_E$ for FWIM is nearly same as that for GVSPM, however, when 20%<DCR<80%, $I_E$ for LW and ITR are the lowest when DCR>70%, $C_R$ for all these four method nearly become the same.

Next, the estimation result of the bottom image is shown in Figure 6. Considering the three estimation categories as a whole, FWIM is also superior to LW, ITR and GVSPM method. As shown in Figure 6(a), the $I_C$ for FWIM is nearly same as that for GVSPM, and always higher than LW and ITR. In Figure 6(b), it is clear that $I_E$ for GWIM is the lowest and that for LW and ITR are the highest. In terms of $C_R$ as shown in Figure 6(c), LW and ITR is lower than FWIM and GVSPM, and when DCR>70%, $C_R$ for all these four method nearly become the same.

Finally, it is difficult to determine the best method from the estimation result of four-bulk reconstructed images as shown in Figure 7. In Figure 7(a), When DCR<20%, the $I_C$ for GVSPM is superior to other method, however, when DCR>20%, $I_C$ for FWIM become the highest. As shown in Figure 7(b), when DCR<20%, the $I_E$ for GVSPM is the lowest; when 20%<DCR<90%, $I_E$ for FWIM is lower than other method, however, when DCR>90%, it become to the highest. In terms of $C_R$ as shown in Figure 7(c), LW and ITR is lower than FWIM and GVSPM, and when DCR>40%, $C_R$ for all these four method nearly become the same.
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Figure 5 $I_C$, $I_E$ and $C_R$ in the case of center image

Figure 6 $I_C$, $I_E$ and $C_R$ in the case of the bottom image
5. CONCLUSIONS

The Functional Weighted Inverse Matrix (FWIM) has been applied to an ill-posed inverse problem involving the electrical capacitance for multiphase flow. Four types of pseudo particle distribution images with dispersion concentration rates (DCR) were used to compare the FWIM method to the conventional Landweber (LW), Iterative Tikhonov regularization (ITR) and Generalized Vector Sampled Pattern Matching (GVSPM) method. The results of these comparisons are detailed below:

1) In terms of the annular pseudo particle images, the superior method depends on the DCR. However, considering the three estimation categories as a whole, FWIM method is a little better.

2) For the centre image cases, it is difficult to determine the best method.

3) FWIM is superior to the LW, ITR and GVSPM method for bottom pseudo particle image in terms of capacitance residual, image error, and image correlation, even though the FWIM method is not an iterative method.

4) For the four-bulk image cases, the superior method depends on the DCR. When 20%<DCR<90%, FWIM is proved superior to other method, however, when DCR<20% and DCR>90%, it is also difficult to determine the best method.

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REFERENCES

Dyakowski,T. et al. (1999). "On-line monitoring of dense phase flow using real-time dielectric imaging", Powder Technology, 104, pp. 287-295.

Halow, J. S. et al. (1992). "Observations of fluidized bed coalescence using capacitance imaging", Powder Technology, 69, pp.255-277.
Huang,S.M. et al. (1989). "Tomographic imaging of two-component flow using capacitance sensors", J.Phys, E:Sci, Instrum, 22, pp.173-177.

Isaksen,O. (1996). "A review of reconstruction techniques for capacitance tomography", Meas. Sci. Technol, 7(3), pp.325-337.

Yang,W.Q. et al. (2003). "Image reconstruction algorithms for electrical capacitance tomography", Meas. Sci. Technol, 14, pp.R1-R13.

Liu,S. et al. (2001). "Comparison of three image reconstruction algorithms for electrical capacitance tomography", Proc. Second world congress on industrial process tomography, Hanover, Germany, pp.29-34.

Saotome,J., et al. (1995). "Magnetic core shape design by the sampled pattern matching method", IEEE Trans.Magn., 31, pp.1976-1979.

Yoda,K., et al. (1997). "Dose optimization of proton and heavy ion therapy using generalized sampled pattern matching", Phys.Med.Biol., 42, pp. 2411-2420.

Yang.W.Q., (1996). "Hardware design of electrical capacitance tomography systems", Meas. Sci. Technol, 7(3), pp.225-232.

Polydorides,N., et al. (2002). "A Matlab toolkit for three-dimensional electrical impedance tomography: a contribution to the Electrical Impedance and Diffuse Optical Reconstruction Software project", Meas. Sci. Technol., 13(12), pp.1871-1883.

Williams,R.A. et al. (1995). Process tomography principles, techniques and applications, Butterworth Heineman, Chapter 15, pp.301-312.