THREE NOVEL OBSERVATIONAL TESTS OF GENERAL RELATIVITY

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ABSTRACT

I propose three novel observational tests of general relativity. First, I show that a gravitational wave pulse from a major merger of massive black holes at the Galactic center induces a permanent increase in the Earth-Moon separation. For black holes of mass $\sim 10^6 M_\odot$, the shift in the local gravitational potential is comparable to the Earth-Moon potential, leading to the Moon being perturbed relative to the Earth during the passage of the pulse. The permanent increase in the Earth-Moon separation is a fraction of a millimeter, measurable by lunar ranging for future merger events. Second, I show that General Relativity sets an absolute upper limit on the energy flux observed from a cosmological source as a function of its redshift. Detecting a brighter source in gravitational waves, neutrinos or light, would flag new physics. The derived flux limit can also be used to determine the maximum redshift possible for any source with an unknown origin. Third, I consider the implications of modified inertia at low accelerations for rockets. An attractive interpretation of MOdified Newtonian Dynamics (MOND) as an alternative to dark matter, changes the inertia of matter at accelerations $a \lesssim a_0 \approx 1.2 \times 10^{-8} \text{ cm s}^{-2}$. I show that if inertia is modified at low accelerations, this suppresses the exponential factor for the required fuel mass in low acceleration journeys. Rockets operating at $a \ll a_0$ might allow intergalactic travel with a modest fuel-to-payload mass ratio.
DETECTING THE MEMORY EFFECT FROM A MASSIVE BLACK HOLE MERGER AT THE GALACTIC CENTER THROUGH LUNAR RANGING

The black hole at the Galactic Center, SgrA*, grows in part through mergers of black holes of mass $M_{BH} \sim 10^6 M_\odot$ (Micic et al. 2011; Greene et al. 2020). Here we calculate the imprint of such mergers on the Earth-Moon separation.

A merger between black holes of the above mass results in a gravitational wave pulse of a characteristic duration,

$$\Delta t_{GW} \sim \left(\frac{GM_{BH}}{c^3}\right) \sim 5 \text{ s}.$$  \hspace{1cm} (1)

The mass equivalent of the radiated energy, $(\Delta M)_{GW}$, changes the near-Earth gravitational potential by an amount,

$$\Delta \phi_{GW} \sim \frac{G(\Delta M)_{GW}}{d_{GC}} = 0.6 \times 10^{-11} c^2 \left[\frac{(\Delta M)_{GW}}{M_{BH}}\right],$$  \hspace{1cm} (2)

where $c$ is the speed-of-light, $d_{GC} \approx 8 \text{ kpc}$ is the distance of the Galactic center from the Sun (Reid et al. 2019), and typically $(\Delta M)_{GW} \lesssim 0.1 M_{BH}$ (Healy et al. 2014).

Coincidentally, this shift in gravitational potential as a result of the energy carried by the pulse happens to be comparable to the gravitational potential that binds the Moon to Earth,

$$\phi_\oplus = \frac{GM_\oplus}{d_\text{Moon}} = 10^{-11} c^2,$$  \hspace{1cm} (3)

where $M_\oplus = 6 \times 10^{27} \text{ g}$ is the mass of the Earth and $d_\text{Moon} \approx 4 \times 10^{10} \text{ cm}$ is the Earth-Moon distance.

The gravitational radiation pulse traverses the Earth-Moon system over a light-crossing time $(\Delta t)_{\text{cross}} \sim (d_\text{Moon}/c) \sim 1.3 \text{ s}$, during which the gravitational-potential change affects one of the objects before the other. For $(\Delta \phi)_{GW} \lesssim \phi_\oplus$, the temporary weakening of the gravitational binding between the Earth and the Moon during the passage period $(\Delta t)_{\text{cross}}$ leads to an increase in the Earth-Moon separation by an amount,

$$\left(\frac{\Delta d_\text{Moon}}{d_\text{Moon}}\right) \sim \left(\frac{(\Delta \phi)_{GW}}{\phi_\oplus}\right) \times \frac{1}{2} \left[\frac{v_\text{Moon}(\Delta t)_{\text{cross}}}{d_\text{Moon}}\right]^2 \sim 0.3 \times 10^{-11} \left[\frac{(\Delta M)_{GW}}{M_{BH}}\right],$$  \hspace{1cm} (4)

where $v_\text{Moon} \approx 1 \text{ km s}^{-1}$ is the Moon’s orbital speed, and the geometric calculation ignored the small eccentricity in the Moon’s orbit.

The above increase in distance as a result of the motion of the Moon relative to Earth is of the same magnitude as the known “memory effect” (Zel’dovich & Polnarev 1974; Braginskii & Grishchuk 1985; Christodoulou 1991; Bieri et al. 2012), for which the permanent change in separation between free-floating objects of negligible mass initially at rest relative to each other, is also of order, $(\Delta d_{\text{Moon}}/d_{\text{Moon}}) \sim [(\Delta \phi)_{GW}/c^2]$. 
The resulting permanent change $\Delta d_{\text{Moon}} \sim 1 \text{ mm}[(\Delta M)_{\text{GW}}/M_{\text{BH}}]$ is above the ultimate sensitivity threshold of lunar ranging (Murphy et al. 2012), $(\Delta d_{\text{Moon}}/d_{\text{Moon}}) \sim 10^{-14}$, and could be measured for future merger events.

A tight binary of black holes with individual masses $\sim 2 \times 10^6 M_\odot$ and a separation $a$ would merge on a timescale of $\sim 40$ yr ($a/10^{14}$ cm)$^4$ (Peters 1964). The existence of such a binary is not ruled out by the orbits of the S-stars which are observed at much larger distances, $\gtrsim 10^{15}$ cm (Gualandris et al. 2010).

The permanent displacement from the memory effect would increase slightly the eccentricity of stellar binaries at wide separations $\gtrsim 10^{16}$ cm, but this imprint is not detectable at the precision enabled by astronomical surveys such as Gaia (Hwang et al. 2022), even when considering the increase in its amplitude with decreasing Galactocentric distance.

2. LIMITING FLUX VERSUS REDSHIFT AS A FLAG OF NEW PHYSICS

According to General Relativity, the maximum possible luminosity of a source which is bound by its own gravity, equal to its total rest-mass energy, $Mc^2$, divided by the light crossing-time of its gravitational radius, $GM/c^2$. This limit applies to all possible carriers of energy, including gravitational waves, elementary particles such as neutrinos, or electromagnetic radiation. Packing the energy to a smaller scale would result in an implosion to a black hole according to the hoop conjecture (Peng 2021), and a shorter emission time would require faster than light travel (Schiller 2021; Jowsey & Visser 2021; Cardoso et al. 2018; Hogan 1999). These considerations are purely classical and do not involve quantum mechanics.

The ratio between the maximum emission energy and the minimum emission time is independent of mass $M$, implying that the maximum luminosity is a universal constant, combining the speed of light $c$ and Newton’s constant $G$,

$$L_{\text{max}} = c^5 \frac{G}{c} = 3.64 \times 10^{59} \text{ erg s}^{-1}. \quad (5)$$

A similar argument can be applied to any self-gravitating system of size $r$, where the characteristic velocity $v$ is dictated by the virial theorem, $v^2 \sim GM/r$, and the characteristic energy release time is limited by the crossing-time, $\sim r/v$, so that the limiting output power is $\sim v^5/G$, smaller by a factor of $\sim (v/c)^5$ than the universal limit in equation (5).

Given the luminosity distance as a function of redshift, $d_L(z)$, the above limit gives a maximum energy flux that can be observed from a cosmological source which emits isotropically any form of radiation or relativistic particles,

$$f_{\text{max}}(z) = \frac{L_{\text{max}}}{4\pi d_L^2(z)}. \quad (6)$$

To quantify this universal flux limit, I use an analytic approximation to $d_L(z)$ which is accurate to a sub-percent level (Adachi & Kasai 2012) for the standard flat
cosmology with a matter density parameter $\Omega_m = 0.32$ and a Hubble constant of 70 km s$^{-1}$ Mpc$^{-1}$ (Planck Collaboration et al. 2020). This gives,

$$f_{\text{max}}(z) = \frac{13.26 \text{ erg s}^{-1} \text{ cm}^{-2}}{(1 + z)^2 \{\phi(2.13) - (1 + z)^{-1/2}\phi[2.13(1 + z)^{-3}]\}^2},$$

where,

$$\phi(x) \equiv \frac{1 + 1.32x + 0.4415x^2 + 0.02656x^3}{1 + 1.392x + 0.5121x^2 + 0.03944x^3},$$

and $\phi(2.13) = 0.91$.

In the high-redshift limit, $z \gg 1$, I get the simple result,

$$f_{\text{max}}(z) = \frac{15.93 \text{ erg s}^{-1} \text{ cm}^{-2}}{(1 + z)^2[1 - \sqrt{1.21/(1 + z)^2}]}.$$

This flux limit can be used to set an upper limit on the redshift of a source with an unknown origin.

For comparison, a flux of $\sim 15$ erg s$^{-1}$ cm$^{-2}$ is generated by local blackbody radiation at a temperature of 23 K, about ten times hotter than the cosmic microwave background today.

The highest luminosities for astrophysical sources are expected to occur during the formation of compact objects, in the form of gravitational waves or a $\gamma$-ray burst for a black hole or neutrinos for a neutron star. A violation of the limit in equation (8) for an isotropic, self-gravitating source with a known redshift $z$, would flag new physics.

The redshift of a cosmological source can be inferred from the spectral lines of its host galaxy or the Ly$\alpha$ absorption imprinted by the intergalactic medium.

The limit in equation (8) should be multiplied by a correction factor, $f_\Omega = (4\pi/\Delta\Omega)$, for a source radiating its energy into a limited solid angle $\Delta\Omega$.

3. IMPLICATIONS OF MODIFIED INERTIA AT LOW ACCELERATIONS FOR ROCKETS

MOdified Newtonian Dynamics (MOND) was proposed by Milgrom four decades ago (Milgrom 1983) to explain the flat rotation curves of galaxies and the baryonic Tully-Fisher relation (Milgrom 2020; McGaugh 2012; McGaugh et al. 2021). An attractive interpretation of MOND is that at accelerations of magnitude, $a \ll a_0 = 1.2 \times 10^{-8}$ cm$^{-2}$, the inertia of an object of mass $m$ satisfies a modified equation of motion in response to a force $F$ (Milgrom 2011, 2015),

$$m \frac{a^2}{a_0} = F.$$  

Below I consider the implications of modified inertia for a rocket whose fuel burns so as to produce a steady low-acceleration, $a \ll a_0$. 

The force (momentum delivered per unit time) acting on a rocket, is given by the mass ablation rate, $\dot{m}$, times the exhaust speed of the ablated gas relative to the rocket, $v_{\text{exh}}$:

$$F = -\dot{m}v_{\text{exh}}. \quad (11)$$

For a constant acceleration, the solution to equations (10) and (11) is,

$$\left( \frac{m_{\text{initial}}}{m_{\text{final}}} \right) = \exp \left\{ \left( \frac{a}{a_0} \right) \left( \frac{v_{\text{final}} - v_{\text{initial}}}{v_{\text{exh}}} \right) \right\}, \quad (12)$$

where the subscripts ‘initial’ and ‘final’ refer to the initial and final values of the rocket mass and speed. This result differs from the standard Tsiolkovsky solution to the rocket equation (Tsiolkovsky 2000) by the suppression factor $(a/a_0)$ in the exponent. Whereas the amount of fuel that needs to be carried grows exponentially with terminal speed in the standard Tsiolkovsky solution, a modified inertia offers the prospects of reaching high speeds by carrying much less fuel. This allows for intergalactic travel at a modest fuel-to-payload mass ratio.

As a concrete example, consider an intergalactic journey at a final speed of $v_{\text{final}} \sim 300$ km s$^{-1}$, an order of magnitude faster than the rockets launched so far by humans. For standard chemical fuel, this terminal speed exceeds the exhaust speed by a factor $(v_{\text{final}}/v_{\text{exh}}) \sim 10^2$ (Gilster 2004). Thus, in order to achieve this terminal speed through an average acceleration magnitude $(a/a_0) \sim 0.01$ in free space, the required fuel mass would be comparable to the payload mass, $(m_{\text{initial}} - m_{\text{final}}) \sim 1.7m_{\text{final}}$. At this acceleration, the above terminal speed is obtained over a timescale, $t \sim 8$ Gyr, comparable to the remaining lifetime of the Sun. During this time, the rocket would be able to traverse a distance $\frac{1}{2}at^2 \sim 1.2$ Mpc, all the way to the edge of the Local Group of galaxies. Of course, additional fuel would be needed to overcome the binding energy of the Earth, the Sun and the Milky-Way galaxy.

The validity of the modified rocket equation can be tested by launching our own low-acceleration rocket or by finding low-acceleration rockets which arrived to our vicinity from great distances. It is unclear which approach is more likely to bear fruit as the first direct test of the modified inertia interpretation of MOND.

While escaping from the Earth, the Sun and the local Galactic environment, a rocket would need to overcome gravitational accelerations in excess of $a_0$. However, it is the net acceleration that counts in MOND, and so the rocket engine can be designed to produce just a little above what is needed to escape and stay in the MOND acceleration regime. This requires fine tuning of the rocket thrust, but is possible.

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