Visualization of density variations produced by alternating-current dielectric-barrier-discharge plasma actuators using the background-oriented schlieren method

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Abstract
Gas density perturbations generated by an alternating-current dielectric-barrier-discharge plasma actuator (ac-DBDPA) are quantitatively visualised using the background-oriented schlieren (BOS) method. A method of setting the optimum boundary condition for solving the Poisson equation in the BOS method is studied, and an integration method for the boundary condition in the vicinity of the plasma where the density change is steep is proposed. The BOS method is applied in two cases with different voltage amplitudes, and the variation in the absolute value of the density is discussed with the discharge properties. The results show a decrease in density in the synthetic jet induced by the ac-DBDPA and a spatiotemporal variation indicating a step-wise gas-heating phenomenon due to plasma discharge.

Keywords: plasma actuator, dielectric barrier discharge, flow visualization, background oriented schlieren

1. Introduction
Airflow control using a dielectric-barrier-discharge plasma actuator (DBDPA) is a promising technique because it can actively and instantly change the airflow around the surface of an object using an electric signal [1–3]. It is thus expected that the DBDPA can be used in many applications, such as a vertical-axis wind turbine [4, 5], leading-edge separation control [1], and for the drag reduction of vehicles [6]. The initial research on the DBDPA comprised the use of an alternating-current (AC) power source for the DBDPA; such a device is called an ac-DBDPA. Recently, a nanosecond-pulse power source has been used as an alternative to the AC power source in what is called the ns-DBDPA [7, 8]. It is believed that the control principles of the ac- and ns-DBDPAs are different. Furthermore, the control principle of the ac-DBDPA consists of a surface jet induced by the ac-DBDPA, which adds momentum to the flow and results in flow control. In contrast, the control principle of the ns-DBDPA consists of the fast-gas heating mechanism, which produces a strong density gradient in the flow and acts as a disturbance in the flow [8, 9]. Although the density gradient generated by the ns-DBDPA has been widely investigated, few investigations have been conducted on the density gradient generated by the ac-DBDPA. Leonov et al [10] demonstrated that the
temperature on the surface of the dielectric barrier is high after 30 s of ac-DBDPA actuation. They indicated that this steady-state surface heating may increase the viscosity of the gas, thus promoting the flow separation and lowering the gas density, deflecting the stream lines, and changing the effective shape of the aerodynamic profile. Meanwhile, instantaneous heating in the ac-DBDPA has been implicitly observed by many researchers through schlieren visualisation of the induced flow [3, 10]. Opalits et al [11] demonstrated through their numerical study that the air density in the starting vortex induced by the dielectric barrier discharge is reduced by the heating effect of the plasma discharge. Such an instantaneous heating of the starting vortex induced by the ac-DBDPA has the potential to affect the flow-control effects, and the density and temperature variation in the ac-DBDPA should thus be investigated.

Schlieren visualisation is useful for detecting the density gradient of the air, and it has therefore been adopted by many researchers for flow diagnostics [12, 13] and plasma diagnostics [14–17]. In contrast, background-oriented schlieren (BOS) visualisation has been developed to quantitatively measure the absolute density, especially in the research field of supersonic flow [18]. The main difference between schlieren and BOS visualisations is the measurement parameters. Schlieren visualisation measures the variation in light intensity, which is mainly due to the variation in the refractive index resulting from the presence of the air density gradient, whereas BOS visualisation measures the displacement of the background pattern. Hence, schlieren visualisation can typically be used to obtain the density gradient in only one direction whereas BOS visualisation can be used to obtain the density gradient in two directions, which aids in solving the elliptic partial differential equation for density. Ramanah et al [19] investigated scramjet flows in a hypersonic impulse facility and compared their obtained results with conventional schlieren images. Kirmse et al [20] performed combined particle image velocimetry (PIV) and BOS measurements in a high-enthalpy shock tunnel at Mach 8. Biganzoli et al [21] proposed the possibility of applying BOS visualisation to the induced flow in plasma actuators (PAs) and showed that the refraction index around the starting vortex induced by the ac-DBDPA is slightly reduced by the heating effects of the plasma. However, the air density distribution immediately above the board on which the DBDPA is mounted has not yet been obtained. One difficulty in measuring the air density in the vicinity of the electrodes is the determination of the boundary conditions in the vicinity of the wall. Although the wall-boundary condition is required for solving the elliptic partial differential equation of the density in the BOS method, it is difficult to determine because many specific parameters, such as the density, pressure, and temperature in the vicinity of the board, are unknown—especially during the transient period of the initial stage of the induced flow formation. The consideration of the boundary condition is one of the critical issues that greatly affects the calculation result of the elliptic partial differential equation. Therefore, many studies on have been conducted on this subject thus far [22–24]. Furthermore, the comparison between the results calculated using different sets of boundary conditions is important if the boundary condition is unknown or not fixed. Thus, it is essential to specifically consider the boundary condition for a measurement object that has a temporally and spatially sharp change in density near the boundary—such as in the case of the DBDPA—through a comparison of the results calculated using the different sets of boundary conditions.

In the present study, the spatiotemporal changes in the absolute density in the plasma-induced flow is measured with a high spatial resolution (0.17 mm) and using the BOS method. The discharge properties of the ac-DBDPA are first investigated, and two experimental conditions with different voltage amplitudes are selected as the test cases. The spatiotemporal variation of the absolute density and the obtained discharge power characteristics are then discussed. In addition, a method is presented for obtaining a boundary condition that is suitable for a measurement object that has an abrupt density change near the boundary, such as the DBDPA.

2. Measurement method

2.1. Principle of BOS method

Figure 1 shows a schematic view of the BOS visualisation. The principle of the method is similar to that of conventional schlieren visualisation, which evaluates the variation in the refractive index due to the density gradient from the light intensity. In the BOS method, the variation in the refractive index is evaluated based on the displacement of the background image. If the refractive index between the background and camera changes owing to a change in density, the background image is captured by the camera with a displacement Δy because of the refraction of light passing through the density gradient. The relation between Δy and the refractive index is expressed as [25]

$$\varepsilon = \frac{1}{n_0} \int_{Z_D - \Delta Z_D}^{Z_D} \frac{\varepsilon n}{\delta y} \mathrm{d}z$$

where ε is the deflection angle, $Z_D$ is the distance between the background and measurement object, $\Delta Z_D$ is the length of the measurement object, and $n_0$ is the refractive index of the atmosphere.

If the density gradient of the measurement object is two-dimensional and constant along the line of sight, equation (1) can be rewritten as

$$\varepsilon = \frac{1}{n_0 \delta y} \Delta Z_D$$

Moreover, the relation between ε and Δy is geometrically described for a small value of ε as

$$\varepsilon = \frac{Z_B}{Z_{tot} \Delta y}$$

where $Z_B$ is the distance between the lens and background, $Z_D$ is the distance between the measurement object and background, and $f$ is the focal length.
On combining equations (2) and (3), we obtain
\[ \Delta y = \frac{Z_B n_0 \Delta Z_D}{Z_D f} \frac{\partial n}{\partial y} \Delta Z_D. \] (4)

The relation between the density \( \rho \) and refractive index \( n \) is given by the Gladstone–Dale equation:
\[ \frac{n - 1}{\rho} = G, \] (5)
where \( G \) is the Gladstone–Dale constant. On substituting \( n \) in equation (5) into (4), we obtain
\[ \frac{\delta \rho}{\delta y} = \frac{Z_B n_0}{Z_D f \Delta Z_D G} \Delta y. \] (6)

On differentiating equation (6) for \( y \) and repeating the procedure (equations (1)–(6)) for the \( x \) direction, we obtain an elliptic partial differential equation, which is referred to as the Poisson equation:
\[ \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = \left( \frac{Z_B}{Z_D f} \right) \left( \frac{n_0}{n} \right) \left( \frac{\delta \Delta y}{\delta y} + \frac{\delta \Delta x}{\delta x} \right). \] (7)

Although the values of the terms in the first set of brackets on the right-hand side of equation (7) can be obtained geometrically, it is difficult to obtain \( Z_B \) and \( Z_D \) precisely, and this affects the accuracy of the measured absolute value of the density. In the present study, therefore, the values of the terms in the first set of brackets are experimentally obtained for the calibration. The calibration method is described in the following section. The values of the terms in the second set of brackets are obtained as \( \Delta Z_D = 0.1 \) m, which is determined based on the length of the actuator electrodes, and as \( G = 2.3 \times 10^{-4} \) m\(^3\) kg\(^{-1}\), which has been used in the literature [25]. The values of the terms in the third set of brackets are obtained using the BOS method. The experimental procedure of the BOS method is illustrated in figure 2. The BOS method provides a reference image and measurement image. These images are obtained for both with and without the DBDPA actuation. The displacement of the dot pattern is calculated by comparing the reference and measured images using the cross-correlation algorithm commonly used in the PIV technique. The commercial PIV analysis software Dynamic Studio (Dantec Co.) is used in the present study. The derivative of the obtained density gradients is an elliptic partial differential equation of the form shown in equation (7). The equation is solved using the successive over-relaxation method, which can be considered as an improvement over the Jacobi method. The acceleration factor is set as 1.9. The iterations were stopped when the relative error was less than \( 10^{-8} \).

2.2. Experimental setup
Experiments were conducted in quiescent air. The experimental setup is shown in figure 3. The optical system for the BOS measurement consisted of a green light-emitting diode (LED) light source (LE-TQ9WP; OSRAM Opto Semiconductors Co., Ltd), a digital single-lens reflex camera.
with a microfocus lens (Ai AF Micro Nikkor ED 200 mm; Nikkor), and two plano-convex lenses with a diameter and focal length of 50.8 and 1000 mm, respectively. A random dot pattern was printed on a transparent film and placed 100 mm behind the PA electrodes. The camera was focused on the background pattern. The field of view of the camera was set at 64.5 mm × 43 mm, and the final interrogation window for obtaining the cross-correlation was 16 × 16 pixels with a 50% overlap; the resulting vector spacing was 0.172 mm, and the overall measurement was on a grid of 375 × 250 vectors.

This optical system could be easily changed to an optical system for schlieren imaging when the background image is removed and the knife edge is placed at the second focal point in front of the camera. This is done in order to cut out the lower half of the focused LED light, and the obtained schlieren images are therefore sensitive to the density gradient in the longitudinal direction.

Figure 4 shows the electrode configuration comprising silicone of 0.4 mm thickness as a dielectric barrier and gold flash materials of 0.018 mm thickness as electrodes. The covered ground electrode was 15 mm wide while the exposed high-voltage electrode was 5 mm wide. The effective length of the PA discharge was approximately 100 mm. The maximum voltage applied to the electrodes was \( V_{pp} = 16 \) kV, where \( V_{pp} \) is the peak-to-peak voltage of the applied voltage, and the dielectric breakdown has not been observed during the experiments. Therefore, the dielectric strength of the silicone used as the dielectric barrier in this experiment is at least over \( V_{pp} = 40 \) kV mm\(^{-1}\).

A high-voltage AC power source was used for the experiments. This power source comprised three units, the charge unit (PSI-PW0500, PSI), amplifier unit (PSI-PA1050, PSI), and transformer unit (PSI-TR15, PSI), and the input voltage signal generated by the function generator (WF1974, NF Corp) can be amplified by 1500 times. The peak output power of the amplifier unit is 500 W for 3 s. The voltage waveform applied to the PA electrode was measured using a high-voltage probe (Tektronix, P6015A). The discharge energy was measured by adopting the \( V-Q \) Lissajous method [26], wherein the charge was measured by placing a capacitor of capacitance \( C \) between the grounded electrode and earth. Figure 5 shows the measured \( V-Q \) Lissajous curve for \( V_{pp} = 8 \) and 12 kV. The area of this curve corresponds to the energy per period; the electrical power is then obtained by multiplying this value with the waveform frequency [26].

Figure 6 shows the timing chart of the PA voltage, camera exposure, and LED lighting. The camera exposure time was set as 1 s, and the LED was operated in pulse mode with a pulse width of 0.1 ms to obtain instantaneous images. The power source and LED were synchronised using a function generator. The camera shutter was first opened, and the voltage was almost simultaneously applied to the electrodes. The LED was then flashed at a predetermined time \( t' \), thus resulting in an instantaneous image captured at \( t = t' \).

The geometric factor of the BOS measurement system is calibrated using equation (3), which expresses the relation between the deflection angle, \( \varepsilon \), and \( \Delta y \). \( \varepsilon \) is artificially changed using a pair of wedge windows. Figure 7 shows the schematic view of the optical path through a pair of wedge windows. The deflection angle is expressed by the following
set of equations.

\begin{align}
\phi_0 &= \sin^{-1}\left\{\frac{n_g}{n_{air}} \sin \omega_1\right\} - \omega_1, \quad (8) \\
\phi_p &= \sin^{-1}\left\{\frac{n_{air}}{n_g} \sin(\phi_0 + \omega_2)\right\}, \quad (9) \\
\end{align}

and

\begin{equation}
\phi_{out} = \sin^{-1}\left\{\frac{n_g}{n_{air}} \sin(\phi_p - \omega_2)\right\}. \quad (10)
\end{equation}

The deflection angle of the transmitted light through a pair of the wedge windows, \(\phi_{out}\), can be changed by altering the two wedge angles \(\omega_1\) and \(\omega_2\). In the present study, a pair of wedge windows is used for which the wedge angle \(\omega_1\) is 0.5°, and the effective wedge angle \(\omega_2\) is changed by rotating window 2 from \(-20°\) to \(20°\). The obtained slope is 0.178, as shown in figure 8. The geometrically obtained value is \(0.2\), which is a 12% overestimate relative to the experimentally obtained value.

### 2.3. Data processing method

The data processing of the BOS method is examined. The BOS method comprises two methods for calculating the absolute density. In one method, the Poisson equation in equation (7) is solved, while in the other, equation (6) is spatially integrated [27]. The boundary condition is required to be determined for solving the Poisson equation. Figure 9 presents a schematic view of the boundary condition for the Poisson equation. The equation of Case 3 is derived from equation (6).
because the knife edge is placed at the second focal point in order to cut out the lower half of the focused LED light. The direction of spatial integration of equation \((6)\) was examined for \(x, -x, y,\) and \(-y\) directions, and integration in the \(-y\) direction was adopted because the error was comparatively small for this direction. The density calculated using the integration method is therefore expressed as

\[
\rho(x, y) = C \int_{-\infty}^{y} \Delta y(x, y) \, dy,
\]

where \(C = \frac{Z_{\text{m}} a_0}{Z_{\text{f}} G}\). As compared with the data in figures 10(c), (e), and (g), the structure of the starting vortex is confirmed in both images although the density around the starting vortex in (c) is greater than that in (e) and (g). On comparing the data in figures 10(d), (f), and (h), it is observed that the density in (d) is much greater than those in (f) and (h), and it is inconsistent with the schlieren image in (b) because the schlieren signal—which is proportional to the density gradient—is small around \((x, y) = (10-30, 5-10)\), and the density gradient change observed in (d) cannot be recognised. Meanwhile, in figures 10(f) and (h), the structure of the plasma-induced flow can be recognised. It is observed that the density decreases. This is consistent with the schlieren image.

The reason for this would be the boundary condition at the board on which the DBDPA is mounted. Although the density variation near the electrode surface is estimated to be extremely steep owing to excessive heating in the plasma, the boundary values of the density are \(\rho/\rho_0 = 1\) in Case 1, which increases the value of the entire density distribution. The Dirichlet boundary condition (Case 1) is, thus, inadequate for application to the boundary of the board on which the DBDPA is mounted. Moreover, although the density distributions calculated using the Poisson equation with the boundary condition of Case 2 appear to be consistent with the corresponding schlieren images, the Neumann boundary condition is also inadequate for the boundary condition at the

**Figure 10.** Comparison among (a), (b) schlieren images, (c), (d) BOS images calculated using the Poisson equation and the boundary condition of Case 1, (e), (f) BOS images calculated with the Poisson equation and the boundary condition of Case 2, and BOS images calculated by integrating in the \(-y\) direction (g), (h) at \(t = 10\) and 100 ms. \(V_{\text{p-p}} = 12\) kV and \(f = 9.5\) kHz.
board because there is no supporting evidence that the density gradient at the boundary must be zero. In an atmospheric-pressure plasma, it is reported that the temperature in the plasma increases instantaneously over 100 K for the point-to-plane discharge \[27\] and for the surface-barrier discharge \[28\]. This gas-heating phenomenon in the plasma and the properties of the wall heat transfer could be closely related to the distribution of the density near the wall, and therefore, the instantaneous distributions of the density near the wall in this case cannot be simply assumed to be represented by the Neumann boundary condition. To avoid this inconsistency, the boundary values at the surface of the electrodes are substituted with the values calculated using equation (11). The entire density is recalculated using the Poisson equation and the boundary condition of Case 3. The obtained results are shown in figure 11. Figure 11(a) shows the density distribution obtained when the Poisson equation is solved using the boundary condition of Case 3 in figure 11 for \(V_{p-p} = 12\) kV at \(t = 10\) ms. As compared with the data in figure 10(c), there is no area in which the density unnaturally exceeds \(\rho/\rho_0 = 1\). For a quantitative comparison, figures 11(b) and (c) present plots of the density distributions along lines (i) and (ii) in figure 11(a)—which are calculated using four different solutions of the Poisson equation with the boundary condition of Cases 1–3—and the integral solution obtained using equation (11). In the density distribution solved with the boundary condition of Case 3, there is no region wherein the density is unnaturally larger than \(\rho/\rho_0 = 1\), as observed in Case 1. As compared with the density distribution solved with the integral solution shown in figure 10(g) and that solved with the boundary condition of Case 3 shown in figure 11(a), the obtained density distribution in figure 11(a) is smoother than the distribution in figure 10(g). This indicates that the integral solution is easily affected by the measurement error in the integration path. The obtained results reveal that it is appropriate to solve the Poisson equation with the calculated boundary condition using the integration method if an appropriate boundary condition is not predetermined. Hereafter, we show and discuss the density distributions solved using the Poisson equation with the boundary condition of Case 3.

3. Results

Figure 12 shows the dependence of the discharge energy—which is calculated from the \(V-Q\) Lissajous curve—on \(V_{p-p}\) for various discharge frequencies \(f = 6–12\) kHz. The discharge energy is approximately proportional to \(V_{p-p}^{3.94}\) and is

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**Figure 11.** (a) Spatial distributions of the normalised density, \(\rho/\rho_0\), with \(V_{p-p} = 12\) kV at \(t = 10\) ms calculated for BOS Case 3 (see text). Distributions of the normalised density, \(\rho/\rho_0\), along (b) line (i) and (c) line (ii) shown in (a) calculated with four different solutions.
independent of \( f \) at \( V_{p-p} < 14 \text{ kV} \). A previous study showed that the streamer propagation length of the ac-DBDPA and, thus, the discharge power depends on both \( V_{p-p} \) and \( f \) \([28]\). As the frequency range that we investigated is lower and narrower than that investigated previously, the frequency dependence could not be observed. Therefore, in the frequency range that we investigated, the discharge power is proportional to the frequency if \( V_{p-p} \) is the same. Pons \textit{et al} \([29]\) and Dong \textit{et al} \([30]\) showed that the relation between the voltage, energy, and frequency is described by a quadratic proportionality, which is different from that proposed in our study. This difference is caused by the actuator geometry, dielectric material, and environmental conditions \([28]\). In particular, the effect of the thickness of the dielectrics on the discharge energy is extensive. Roth \textit{et al} \([31]\) and Enloe \textit{et al} \([32]\) reported that the discharge power is proportional to \( V_{p-p}^n \) with \( n \) ranging from 2 to 3.5 in the case of thin dielectrics. At \( V_{p-p} > 14 \text{ kV} \), the plotted data does not follow the fitting curve of \( \sim V_{p-p}^{3.94} \). This is possibly because the discharge becomes filamentary when the applied voltage increases, thus resulting in an increased power consumption \([28]\). Moreau \([26]\) indicated that if the voltage and frequency are increased to a great extent, the electrical charge on the dielectric is built-up to a very high value, and therefore, filaments appear between different points on the surface where the space charge has not been relaxed. Boeuf \textit{et al} \([33]\) also shows via numerical simulations that the discharge current consists of large-amplitude short current pulses during which a filamentary plasma spreads along the surface. In order to verify this, we obtained the discharge emission of light for various voltage amplitudes, as shown in figure 13.

Figure 13 shows the discharge emission of light for \( V_{p-p} = 8 \text{ kV} \) (a), 12 kV (b) and 15 kV (c). The discharge frequency is 9.5 kHz and the camera exposure time is 0.25 s. The inlet graphs show the emission intensity profiles of the discharge along the \( x \) direction, which were spatially averaged over 10 cm along the spanwise direction. Figure 14 shows the spatiotemporal variations of the schlieren images and density distributions for \( V_{p-p} = 8 \text{ and 12 kV} \). It is observed from the schlieren images that the starting vortex, which is generated by the plasma-induced flow, forms around the edge of the electrode and propagates in the \( x \) direction. After the starting vortex is diffused, a steady synthetic jet forms along the wall surface. As \( V_{p-p} \) increases from 8 to 12 kV, the structure of the vortex becomes larger, and the propagation speed of the starting vortex generation of nonlinear effects of the discharge energy, as shown in figure 12. At \( V_{p-p} = 8 \text{ and 12 kV} \), the discharge morphology is almost uniform, and these voltages are, therefore, used in the present study as test cases for BOS and schlieren measurements. The discharge energies at \( V_{p-p} = 8 \text{ and 12 kV} \) are 0.34 and 1.7 mJ, respectively, while the corresponding discharge powers are 3.26 and 16.15 W, respectively, at \( f = 9.5 \text{ kHz} \).
increases. It is observed from the BOS images that the densities in the starting vortex and the synthetic jet decrease, with distributions corresponding to the schlieren images. The density decreases as \( V_{p-p} \) increases. As compared with the data at \( t = 100 \) and 1000 ms for \( V_{p-p} = 12 \) kV, the density inside the synthetic jet temporarily decreases owing to the continuous heating by the discharge during \( t = 100–1000 \) ms.

The difference in the density distribution around the centre of the starting vortex is compared for \( V_{p-p} = 8 \) and 12 kV. Figure 15 shows the density distributions along the \( y \)-axis through the centre of the starting vortex at (a) \( V_{p-p} = 8 \) kV and (b) \( V_{p-p} = 12 \) kV. The minimum values of the density at the centre of the starting vortex for \( V_{p-p} = 8 \) kV are \( \rho/\rho_0 = 0.994 \) at \( t = 10 \) ms and \( \rho/\rho_0 = 0.998 \) at \( t = 20 \) ms, while those for \( V_{p-p} = 12 \) kV are \( \rho/\rho_0 = 0.985 \) at \( t = 10 \) ms and \( \rho/\rho_0 = 0.990 \) at \( t = 20 \) ms. In the period \( t = 20–30 \) ms, the minimum density value remains nearly constant while the area in which the density decreases grows in both cases. Such a nonlinear change in air density around the starting vortex would correspond to the combined effects of the continuous additional heating due to the discharge and the resulting thermal diffusion.

Figure 16 shows the density distributions along the \( y \)-axis for (b) \( V_{p-p} = 8 \) kV and (d) \( V_{p-p} = 12 \) kV at \( t = 1000 \) ms. The distributions are plotted every 5 mm from \( x = 0 \) mm, as indicated by the dotted line in figures 16(a) and (c). For \( V_{p-p} = 8 \) kV, the...
density has a minimum value of approximately \( \rho/\rho_0 \approx 0.992 \) around \((x, y) = (5 \text{ mm}, 0 \text{ mm})\), and the height of the profiles increases as \( x \) increases. A similar profile change for the \( x \) values was previously obtained \[34\] for the spatial distribution of the velocity of the induced flow and was explained by the results of the diffusion and momentum transfer from plasma to air. For \( V_{p-p} = 12 \text{ kV} \), the density has a minimum value of approximately \( \rho/\rho_0 \approx 0.978 \) around \((x, y) = (15 \text{ mm}, 0.82 \text{ mm})\), and the height of the profiles increases as \( x \) increases. It is interesting to note that the minimum value is obtained slightly above the board at \( y = 0.82 \text{ mm} \) when \( V_{p-p} = 12 \text{ kV} \), whereas it is obtained on the board at \( y = 0.0 \text{ mm} \) when \( V_{p-p} = 8 \text{ kV} \). This is possibly due to the effect of the heat transfer between the plasma and board on which the DBDPA is mounted. This implies that it is not the case that there is always a certain boundary condition on the board; i.e. it is not possible to predetermine the boundary condition, and the boundary condition obtained using the integral solution that we proposed is therefore useful for a measurement object having an abrupt density change near the boundary, as in the case of the DBDPA. The density distribution at \( t = 1000 \text{ ms} \) is discussed in detail in the following section.

4. Discussion

The magnitude of the decrease in density in the ac-DBDPA is discussed in this section. Figures 16(b) and (c) show that the minimum density in the induced flow is approximately \( \rho/\rho_0 \approx 0.992 \) for \( V_{p-p} = 8 \text{ kV} \) and \( \rho/\rho_0 \approx 0.978 \) for \( V_{p-p} = 12 \text{ kV} \). Assuming that the pressure in the induced flow is constant and the room temperature is 293 K, the maximum temperature rise calculated using the equation of state for a perfect gas is \( \Delta T = (1/0.992 - 1) \times 293 = 2.36 \text{ K} \) for \( V_{p-p} = 8 \text{ kV} \) and \( \Delta T = (1/0.978 - 1) \times 293 = 6.59 \text{ K} \) for \( V_{p-p} = 12 \text{ kV} \). Jukes et al \[35\] conducted cold-wire anemometry for measuring the temperature of the induced flow of an ac-DBDPA and found a weak increase in the flow temperature, on the order of 2°C, in the plasma-induced flow. An accurate comparison of their result cannot be made with the results of the present study because the corresponding discharge powers and shapes of the electrodes differ greatly. However, the results of the two studies exhibit the same trend, in that, an increase in temperature of several degrees Kelvin in the plasma-induced flow is observed. Ukai et al \[36\] showed that the density reduces to a value as low...
as $\rho_p/\rho_0 = 0.95$ at $t = 100$ ms at 1 kHz for discharges in the ns-DBDPA. Although performing an accurate comparison of these results with the results of the present study is difficult because the corresponding discharge powers differ, their result suggests a greater decrease in density for the ns-DBDPA than for the ac-DBDPA, as reported in many previous studies. For obtaining an accurate comparison of the decrease in gas density between ac- and ns-DBDPAs, the measurement of the density under the same conditions with the same discharge power is required.

In the case of the density distribution along the $x$-direction at $y = 0$ mm shown in figure 16, there was no appreciable decrease in density immediately below the exposed electrode ($x = 0$ mm), but there was an appreciable decrease in density from $x = 2.5$ mm at $V_{p-p} = 8$ kV and from $x = 5$ mm at $V_{p-p} = 12$ kV. As compared with the distribution of the discharge emission of light shown in figure 13, a strong discharge emission was observed up to approximately $x = 2.5$ mm at $V_{p-p} = 8$ kV and up to approximately $x = 5.0$ mm at $V_{p-p} = 12$ kV, which is consistent with the region in which there was no appreciable decrease in density. Although it is difficult to clarify the cause for this in the present study, the following two reasons are conceivable. The first is that the resolution of the BOS measurement in this study might not be sufficient for capturing the extremely thin layer of the plasma-induced flow in the acceleration region near the exposed electrode. As it is estimated that the major-acceleration region roughly corresponds to the region wherein the discharge emission is observed, a very thin layer of induced flow may have formed in the acceleration region near the exposed electrode. If the thickness of the layer of the induced flow is below the spatial resolution of 0.172 mm of this study, the measurement in this region might include some errors. However, as indicated by many velocity field measurement results [37–40], the height of the layer of major acceleration and dynamics related to the induced flow is in the order of a few millimetres from the wall. Although the possibility of the insufficiency of the resolution of the BOS measurement is small, it might be necessary to conduct a simultaneous measurement of a highly spatially resolved velocity field measurement and density field measurement for a further discussion regarding the validity of the measured density in the acceleration region near the exposed electrode. The second plausible reason is the effect of two-step gas heating due to plasma discharge, which is considered as the detailed description of the Joule heating phenomenon while considering the elastic and inelastic energy loss processes with a subsequent energy transfer process. It has been reported that the translational energy increases in two steps as an isochoric process and isobaric process [7, 41–43]. The isochoric process in an atmospheric-pressure discharge is known as the fast gas heating process and is induced by the quenching of electronically excited molecules, such as $N_2(C^1Σ_u^+)$ and $N_2(B^3Σ_u^+)$, within the time constant of the relaxation of the pressure. The isobaric process that occurs in a streamer discharge is known as the slow gas heating process [17] and is caused by the energy relaxation of vibrationally excited molecules. As the time constant of the vibrationally excited molecules is less than the sound of speed, the slow gas heating process slowly increases the translational temperature at an almost constant pressure. It is reported that more than 70% of the electric energy is used to produce the vibrationally excited molecules, whereas only a small amount of electric energy is used to increase the translational temperature in atmospheric-pressure pulsed discharge [43]. Therefore, the impact of the energy deposition caused by the slow gas heating process on the flow temperature may be greater than that caused by the fast gas heating process if the time constant of the energy relaxation time of vibrational energy is shorter than the characteristic time of the flow induced by ac-DBDPA. The time constant of the vibrational energy relaxation is reported as approximately 0.1–1 ms at atmospheric-pressure [44], which is equivalent to the time scale of the formation of the plasma-induced flow. Therefore, there exists a possibility that an increase in the gas temperature caused by the vibrational relaxation occurs after the heated air is shed from near the electrodes and away from the electrodes area. Although such a stepwise heating phenomenon has been investigated in detail with respect to the ns-DBDPA, there are few reports on the ac-DBDPA. Neretti et al. [45] reported that the vibrational temperature is one order of magnitude higher than the rotational temperature in the ac-DBDPA. In addition, in the ac-DBDPA, the rate of accumulation of charge on the dielectric surface is higher than the rate of the voltage increase, and the internal electric field generated by the accumulation of charged particles on the dielectric surface weakens the local field, such that the net weakened electric field $E/N$ during the discharge is as strong as the electric field for the discharge without a dielectric barrier [46, 47]. As the fraction of energy that is spent on the fast and slow gas heating depends on $E/N$, during the discharge [48, 49], there is a possibility that the effect of the slow gas heating is appreciable owing to the repetitive low $E/N$ discharges in the ac-DBDPA. Furthermore, such slow gas heating due to vibration relaxation occurs in our study, and as a result, the density may decrease in a region slightly away from the electrode. It has also been previously suggested that this slow gas heating plays an important role in leading-edge separation control around an airfoil in ns-DBDPA experiments [7, 8, 50]. As for the ns-DBDPA, there exists the possibility that the slow gas heating generated by the ac-DBDPA plays an important role in flow separation control, in addition to the conventional induced flow. For further clarification, the simultaneous measurement of the translational and vibrational temperatures around the synthetic jet induced by the ac-DBDPA is required.

5. Conclusions

The variation in the gas density in the ac-DBDPA was investigated using the BOS method, which is a quantitative method used for measuring absolute density. A condition that is necessary for solving the Poisson equation was newly proposed for the boundary at which the DBDPA is mounted. The evaluation of the four different solutions shows that the boundary condition obtained using the integral solution that we proposed is useful in the measurement of an object having an abrupt density change near the boundary, such as in the case of the DBDPA. The results of the density measurement indicate that as the discharge voltage increased from $V_{p-p} = 8$ to 12 kV corresponding to the discharge power of 3.26–16.15 W, the
density decreased in both the starting vortex and synthetic jet. The density had a minimum value at different locations for $V_{p-p} = 8$–12 kV possibly because of the combined effect of continuous heating due to the discharge and the resulting thermal diffusion. The distribution of the density along the direction in which the discharge propagates indicates slow gas heating due to vibration relaxation. As a result, the density may decrease in a region slightly away from the electrode. The quantitative measurement of the density obtained on adopting the method of setting the boundary condition that is proposed in this study is useful for elucidating the principle of airflow control in the case of the DBDPA and for improving the performance of the DBDPA.

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