Quantum key distribution with no shared reference frame

F. Rezazadeh · A. Mani · V. Karimipour

Abstract

Any quantum communication task requires a common reference frame (i.e., phase, coordinate system). In particular, quantum key distribution requires different bases for preparation and measurements of states which are obviously based on the existence of a common frame of reference. Here, we show how QKD can be achieved in the absence of any common frame of reference. We study the coordinate reference frame, where the two parties do not even share a single direction, but the method can be generalized to other general frames of reference, pertaining to other groups of transformations.

Keywords Shared reference frame · Quantum key distribution · Shared coordinate system

1 Introduction

Any quantum communication protocol requires that the parties involved do have a shared reference frame (SRF). Sharing a reference frame enables the parties to use the same coordinates (same phase coordinates or direction coordinates) to characterize quantum states and operations, i.e., states and operations have the same mathematical representations for the players. The absence of such a common reference frame is equivalent to the existence of an almost depolarizing channel between the two parties. The reason is that any state $\rho_A$ sent by $A$ is seen by $B$ to have been transmitted through a channel

$$\rho_A \rightarrow \rho_B = \int dg P(g)U(g)\rho_A U^\dagger(g), \quad (1)$$

where $g \in G$, and $G$ is the group of transformations connecting the two reference frames, and $U(g)$ is a unitary (not necessarily irreducible) representation of the group, and $P(g)$ represents the partial knowledge that the parties have on the misalignment of
their frames \[1\]. The group measure is also left and right invariant, i.e., \(dg = d(gh) = d(hg)\) \(\forall h \in G\). When the state \(\rho\) carries an irreducible representation of the group \(G\), and \(P(g)\) is uniform, the state \(\rho_B\) will be a completely mixed state \(\rho_B \propto I\), due to the Schur’s lemma. The group \(G\) depends on the variable which the state encodes, i.e., it can be \(U(1)\) if the variable is a phase or \(SO(3) \sim SU(2)/Z_2\) if the variable is a direction in space. In practice, the players may align their frames by other possibly classical means, but it is a theoretically interesting question to ask, how they can do quantum communication in the complete absence of common reference frames. There has been intensive interest on the subject of reference frames \[1–7\], and there has been reports on different protocols ranging from tests of Bell inequality in the absence of reference frames \[8–10\] to quantum key distribution when the two players only share a common direction \[11–13\]. Nevertheless, the problem of quantum key distribution, as one of the most important quantum communication tasks, has not been studied in its generality. This is the subject of the present paper.

Our motivation stems from the unique features of this task, namely: the necessity of using different random bases for preparation of states on one side and different random bases for measurements on the other, both of which depend crucially on the existence of common reference frames. It is thus a pressing question that in the absence of any common reference frame, when the two players do not share even a single direction, how such a protocol can be run.

In the following, although we emphasize on one type of reference frame, namely a coordinate system, it is fairly clear how our considerations can be generalized to other types of frames, whose transformations belong to a different group. The question we ask is how the two players (conventionally called Alice and Bob) can run a QKD protocol when they do not share any coordinate system nor even a single direction. A QKD scheme is essentially based on the notion that a state which has no coherence in one basis (i.e., \(|z_+\rangle\) in the case of spin 1/2 particles) is maximally coherent in another basis \((\frac{1}{\sqrt{2}}(|x_+\rangle + |x_-\rangle))\) \[14–19\]. Therefore, if measured in the wrong basis, such a state will produce completely random and hence uncorrelated results with those of the other player. However, in the absence of frames of reference, not all superpositions are well defined, as they are forbidden by superselection rules \[20,21\]. Here, we propose a scheme which solves this problem and explains it in the context of coordinate reference frames, when the degrees of freedom are those of spin 1/2 particles or photons. The basic idea can be generalized to other frames and their corresponding groups of transformations. Essentially, it is based on encoding the variable of interest in the fusion space of representations of the group, as we will discuss at the end of the paper.

Our emphasis here is mostly theoretical. Evidently, when it comes to practical matters, complications arise which may not be so easy to overcome. Moreover, in real experimental situations, we do have much partial information about the degree of misalignment of the two frames which can be remedied by other possibly classical means. We briefly discuss these issues at the end. Nevertheless, it is worth to study this scheme in view of its generality (as we will see) and mathematical beauty when expressed for general groups.

The structure of this paper is as follows: In Sect. 2, we introduce a four-state protocol which is meant to be the SRF-free version of the BB84 protocol, and in
Sect. 3, we present the SRF-free version of the six-state protocol. We end the paper with a discussion in which we also discuss generalizations to other groups or other reference frames and also briefly discuss some practical and experimental issues.

2 A four-state protocol without shared reference frame

To suggest an SRF-free analog of the BB84 protocol, we briefly remind the basic points of the BB84 protocol. To share a secret key, the players of BB84 choose a two-dimensional Hilbert space (for example the Hilbert space of a qubit). Then, Alice chooses two different preparation bases in the mentioned space and Bob chooses two different measurement bases. The scheme is based on two important facts: first is that a prepared state which has no coherence in one measurement basis is (maximally) coherent in another basis, and the second is that the two measurement bases of Bob are independent, i.e., they do not commute.

In the absence of a SRF, in accord with Eq. (1), Bob will receive the transformed version of the states sent by Alice. Hence, to design a SRF-free analog of the BB84, Alice and Bob should use some states that do not have any reference to any coordinate axis of the two players. Such states are invariant under the map (1). If we take one spin $\frac{1}{2}$ particle, the only state which is invariant under the map (1) is the identity matrix and there will be no correlations between the states sent by Alice and the measurement results of Bob. In the next stage, if we take two spin 1/2 particles, their total spin can be 0 or 1. At first glance, it seems that we can take the states of these two subspaces to define the required two-dimensional Hilbert space. However, there are two basic problems which prevent the possibility of designing a SRF-independent QKD scheme by using two-qubit states. The first is that a superselection rule allows superposition of only those basis states which have the same total spin. More generally and concretely, suppose that Alice and Bob have frames which are connected by a group of transformations $G$ and let Alice prepare a state $|\psi\rangle_A = a|\mu\rangle + b|\nu\rangle$ where $|\mu\rangle$ and $|\nu\rangle$ are two states which transform under two inequivalent irreducible representations of the group $G$. We now use (1), and the Schur’s two lemmas according to which if for a matrix $M$, $U_\mu(h)MU_\nu^\dagger(h) = M \quad \forall h \in G$, then $M \propto I$ for $\mu = \nu$ and $M = 0$ for $\mu$ inequivalent to $\nu$. This immediately leads to the following state for Bob

$$\rho_B = |a|^2|\mu\rangle\langle\mu| + |b|^2|\nu\rangle\langle\nu|, \quad (2)$$

which has completely lost its original coherence. The second problem is about the measurements that Bob should perform. Since Bob does not have any shared reference frame with Alice, he can only perform the measurements which are linear expansions of total spin projectors [22]. Hence, choosing two different independent measurement bases (measurement bases that do not commute) is impossible for him.

The simplest solution is to take two states of three particles both of which have the same total spin, but have different internal spins. That is, we consider the two basis states of a qubit to be the two states which have total spin equal to $\frac{1}{2}$, while the spin of the pair (1, 2) is 0 and 1, Fig. 1. They can be written as
Fig. 1 Two different bases for the subspace of three particles with total spin 1/2, Eqs. (5) and (6). Alice randomly encodes her bits into one of these two bases, and Bob also measures in one of these bases (Color online)

\[ |\phi_0^m\rangle := \frac{1}{\sqrt{2}} |1^{123}, 0^{12}, m\rangle, \quad |\phi_1^m\rangle := \frac{1}{\sqrt{2}} |1^{123}, 1^{12}, m\rangle, \]

where \( m = \pm \) is the z-component of the total spin. This is reminiscent of what we have in topological quantum computation, when the Hilbert space of anyons corresponds to the various ways that a specific number of anyons can fuse to get a total charge with a specific value [23–25]. For the same space, one can choose another basis, denoted by \( |\psi_0\rangle \) and \( |\psi_1\rangle \), where this time, the spin of the last two particles, namely the pair \((2, 3)\), is either 0 or 1, Fig. 1.

\[ |\psi_0^m\rangle := \frac{1}{\sqrt{2}} |1^{123}, 0^{23}, m\rangle, \quad |\psi_1^m\rangle := \frac{1}{\sqrt{2}} |1^{123}, 1^{23}, m\rangle, \]

The explicit form of these states is

\[ |\phi_0^+\rangle := \frac{1}{\sqrt{2}} (|+, -, +\rangle - |-, +, +\rangle), \]
\[ |\phi_1^+\rangle := \frac{1}{\sqrt{6}} (|+, -, +\rangle + |-, +, +\rangle - 2|+, +, -\rangle), \]

and

\[ |\psi_0^+\rangle := \frac{1}{\sqrt{2}} (|+, +, -\rangle - |+, -, +\rangle), \]
\[ |\psi_1^+\rangle := \frac{1}{\sqrt{6}} (|+, +, -\rangle + |+, -, +\rangle - 2|-, +, +\rangle), \]

with \( |\phi^-\rangle \) and \( |\psi^-\rangle \) obtained from the above states by flipping all the spins.
The states have the following inner products:

\[ \langle \phi^\pm_i | \psi^\mp_j \rangle = S_{i,j}, \quad \langle \phi^\pm_i | \psi^\mp_j \rangle = 0, \]

where \( S_{i,j} \) are the elements of the following matrix

\[ S = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}. \]

(8)

Suppose now that Alice sends the states \(|\phi_0^+\rangle\) and \(|\phi_0^-\rangle\) with equal probability to Bob. The statistics of measurements by Bob is, as if, Alice is sending him the state

\[ \rho_0 := \frac{1}{2} \left( |\phi_0^+\rangle \langle \phi_0^+| + |\phi_0^-\rangle \langle \phi_0^-| \right). \]

(9)

This state being the projector to spin 0 of the pair (1,2) is invariant under the channel (1) and when Bob measures the total spin of the pair (1,2), he obtains with probability 1, the spin 0. The other states sent by Alice are as follows:

\[ \rho_1 = \frac{1}{2} \left( |\phi_1^+\rangle \langle \phi_1^+| + |\phi_1^-\rangle \langle \phi_1^-| \right), \]

\[ \sigma_0 = \frac{1}{2} \left( |\psi_0^+\rangle \langle \psi_0^+| + |\psi_0^-\rangle \langle \psi_0^-| \right), \]

\[ \sigma_1 = \frac{1}{2} \left( |\psi_1^+\rangle \langle \psi_1^+| + |\psi_1^-\rangle \langle \psi_1^-| \right). \]

(10)

Remark The states that Alice sends are always pure and of the form \(|\phi_i^\pm\rangle\) (with equal probability) or \(|\psi_i^\pm\rangle\) (with equal probability). The statistics produced corresponds to the above mixed states.

The states \( \rho_i \) have total spin \( S_{1,2} = i \) for the pair (1, 2), and the states \( \sigma_i \) have total spin \( S_{2,3} = i \) for the pair (2, 3). From (7), one can infer the inner product of these states:

\[ Tr(\rho_i \rho_j) = Tr(\sigma_i \sigma_j) = \frac{1}{2} \delta_{i,j} \quad Tr(\rho_i \sigma_j) = \frac{1}{2} (S_{i,j})^2. \]

(11)

It is also important to note that any rotation in the coordinate system of Alice does not change the total values of spins for the three particles or the pairs (1,2) and (2,3), and henceforth, the symmetry or antisymmetry of these states with respect to the aforementioned interchanges. More precisely, suppose that the frames of Bob and Alice are not perfectly aligned and we have only partial information about their alignment in the form of a probability distribution \( P(R) \), where \( R \in SO(3) \) is the rotation necessary to align them. Then, any state \( \rho \) which is sent by Alice seems to go through a channel

\[ \rho_A \rightarrow \rho_B = \int dR P(R) U(R) \otimes^3 \rho_A U^\dagger(R) \otimes^3, \]

(12)
and is received by Bob. The point is that the states given in (9) and (10) are all invariant states of this quantum map since they are eigenstates of the total spin operator.

Note that Bob, having no shared reference frame with Alice, can only perform measurements which are total spin projectors of either the pair (1,2) or the pair (2,3). For ease of notation and to make the scheme parallel to the BB84 protocol, we call these the two bases of measurements of Bob, and simply call them the $\rho$ basis and the $\sigma$ basis, respectively. In other words, Bob randomly uses one of the following two sets of projective measurements:

$$E_{\rho} = \{(\Pi_0)_{12}, (\Pi_1)_{12}\}$$
$$E_{\sigma} = \{(\Pi_0)_{23}, (\Pi_1)_{23}\}.$$  \hfill (13)$$

Interestingly, in view of Eqs. (9) and (10), these two projectors are proportional to the mixed states $\rho$ and $\sigma$. Therefore, we have

$$E_{\rho} = \{2\rho_0, 2\rho_1\},$$
$$E_{\sigma} = \{2\sigma_0, 2\sigma_1\}.$$  \hfill (14)$$

Let us remind the reader of the basics of the protocol. Alice encodes the bit $i = 0, 1$ randomly into the mixed states $\rho_i$ or $\sigma_i$. In practice, she sends pure states of the form $|\phi_i^{\pm}\rangle$ or $|\psi_i^{\pm}\rangle$ with equal probability. Bob randomly chooses one of the POVM’s $E_{\rho}$ and $E_{\sigma}$. For simplicity, we call these bases both for preparation and for measurements the $\rho$ and the $\sigma$ bases.

Let $P_{\rho,\rho}(B = i|A = j)$ denote the probability that Bob obtains the value $i$ when Alice sends the bit value $j$, in case that both use the $\rho$ bases, with similar definition for similar expressions. Then, it is clear from (11) that

$$P_{\rho,\rho}(B = i|A = j) = Tr((\Pi_i)_{12}\rho_j) = 2Tr(\rho_i\rho_j) = \delta_{i,i}$$
$$P_{\sigma,\sigma}(B = i|A = j) = Tr((\Pi_i)_{23}\sigma_j) = 2Tr(\sigma_i\sigma_j) = \delta_{i,i}.$$  \hfill (15)$$

Therefore, in those rounds where they use the same bases, we have perfect correlation between the bits sent by Alice and measured by Bob. On the other hand, if they use different bases, then we will have

$$P_{\rho,\sigma}(B = i|A = j) = Tr((\Pi_i)_{23}\rho_j) = 2Tr(\sigma_i\rho_j) = (S_i,j)^2$$
$$P_{\sigma,\rho}(B = i|A = j) = Tr((\Pi_i)_{12}\sigma_j) = 2Tr(\rho_i\sigma_j) = (S_i,j)^2.$$  \hfill (16)$$

In those rounds where the bases are not the same, perfect correlation is lost between the bits, and these rounds are discarded after public announcement of the bases by the two parties. The important point is that the basic concept and methodology of the BB84 protocol are also at work here, i.e., random preparation and measurements in two bases, public announcement of bases and discarding those rounds where the bases do not match.

**Remark** Furthermore, note that Alice and Bob can arbitrarily rotate their coordinate systems, without affecting the performance of the protocol. In this way, they prevent

Springer
Eve from aligning her coordinate system with those of Alice or Bob and possibly doing non-invariant measurements on single particles which leak information to her.

To generalize this method to those cases where there is no common frame of reference (be it a phase or a coordinate reference or any other reference pertaining to a group \( G \)), one should encode the bits in the internal degrees of freedoms (or internal charge, in the language of topological quantum computation \([23–25]\)) of composite systems, where the states of these composite systems are themselves invariant under the twirling operation of the group. Here, the internal charge is the total spin of pairs (1,2) or (2,3) and Eq. (12) is the twirling operator of the rotation group. In other words, the states which encode the bits are the basis states of the fusion space of the three particles. This internal charge (in our case, spin) is hidden from the adversaries who do not know in which basis to measure the charge. In this way, Alice and Bob establish a shared random key between themselves which turns out to be secure against attacks by Eve in the same way that BB84 is secure \([26–28]\), as long as the resources available to Eve are strong enough to manipulate the package of three particles. It should be noted that any adversary cannot access to bits of the key, since she cannot align her reference frame with both frames in possessions of the two parties. In fact, Alice and Bob can even arbitrarily rotate their frames in different rounds without affecting the performance of the protocol. In the absence of such a common frame, the only measurement which leads to meaningful results for Eve is the total spin measurement of pairs of spins. In the same way as in the BB84 protocol, she has to measure the total spins of the pairs in random. In this way, in half of those rounds where the so-called bases of Alice and Bob do agree, Eve chooses the wrong basis for measurement. It is then seen that in total she incurs error on bits shared between Alice and Bob at a rate of 3/16. To see this, consider as an example the case where Alice and Bob both choose the basis \( \rho \) but Alice sends 0 and Bob receives 1 due to Eve’s intervention. The probability of this is given by

\[
P_{\rho,\rho}^{\text{error}}(B = 1|A = 0) = \frac{1}{2} P_{\rho \sigma}(B = 1|E = 0) P_{\sigma \rho}(E = 0|A = 0) + \frac{1}{2} P_{\rho \sigma}(B = 1|E = 1) P_{\sigma \rho}(E = 1|A = 0) = \frac{1}{2} \left( |S_{10}|^2 |S_{00}|^2 + |S_{11}|^2 |S_{10}|^2 \right) = \frac{3}{16}. \tag{17}
\]

A similar analysis leads to the same value for the other cases, when the two parties are using the basis \( \sigma \). Thus, in this protocol, the intervention of Eve introduces an error rate of \( \frac{3}{16} \), which can lead to detection of an adversary, when the two parties compare only a subsequence of the bits.

**Remark** Using tools from information theory, the absolute security of the BB84 protocol has been proved in \([26–28]\). Our emphasis here is on the theoretical possibility of an SRF-free version of this protocol. Therefore, as far as information theory is concerned and even if we assume strong enough resources for Eve for manipulation of packages of three particles, the scheme presented here is also absolutely secure.
course practical implementation of this protocol is a different problem which naturally has its own complications like preparation and measurements of entangled states, lower bit rate by a factor of 3, particle losses, etc.

3 A six-state protocol without shared reference frame

Having three different bases, it is only natural to consider the analog of the six-state protocol [16], by including the third pair of spins, namely the pair (1, 3) and encode the qubit in the charge of this pair. The protocol is based on the use of three sets of preparation and measurements rather than two sets. That is, Alice uses the total spin of (1, 2), (2, 3) or (1, 3) for encoding, and Bob correspondingly has three sets of POVM’s. More precisely, in addition to the two sets of states given in (3) and (4), Alice can also encode her bits 0 and 1 into the following set of states, whose spin of the pair (1, 3) is 0 or 1.

\[
|\chi_0^+\rangle := \frac{1}{\sqrt{2}}((+,-,-) - (-,+,+)), \\
|\chi_1^+\rangle := \frac{1}{\sqrt{6}}((+,-,-) + (-,+,+) - 2|+,-,+\rangle).
\] (18)

As in the previous cases, Alice encodes a bit \(i\) with equal probability into \(|\chi_i^+\rangle\) and \(|\chi_i^-\rangle\). The new states by Alice are described by the following density matrices

\[
\gamma_0 = \frac{1}{2} \left( |\chi_0^+\rangle\langle\chi_0^+| + |\chi_0^-\rangle\langle\chi_0^-| \right), \\
\gamma_1 = \frac{1}{2} \left( |\chi_1^+\rangle\langle\chi_1^+| + |\chi_1^-\rangle\langle\chi_1^-| \right),
\] (19)

and the new POVM used by Bob (measuring the total spin of the pair (1,3)) is given by

\[
E_\gamma = \{2\gamma_0, 2\gamma_1\}.
\] (20)

Alongside Eq. (7), we also have the following inner products:

\[
\langle \phi_i^\pm | \chi_j^\pm \rangle = \langle \psi_i^\pm | \chi_j^\pm \rangle = S_{ij},
\] (21)

with the same matrix elements as in (8). This leads to the following relation in addition to (11)

\[
Tr(\gamma_i \gamma_j) = \frac{1}{2} \delta_{i,j}, \\
Tr(\gamma_i \rho_j) = Tr(\gamma_i \sigma_j) = \frac{1}{2}(S_{i,j})^2.
\] (22)

The percentage of valid rounds (e.g., the rounds which are not discarded) now drops from 1/2 to 1/3, but the error rate induced by intervention of an adversary increases from \(\frac{3}{16}\) to \(\frac{4}{16} = \frac{1}{4}\). This is simply seen by following the same steps that led to Eq. (17).
The fusion space of three spin 1 particles can be used to encode the values 0, 1 and 2 and perform an SRF-free version of the BB84 QKD system with qutrits. In principle, the scheme can be generalized to every other group.

All these considerations can be generalized to other representations of $SU(2)$ and other non-abelian groups. For example, if the particles have spin 1, then we have the decomposition rule

$$1 \otimes 1 \otimes 1 = 0 \oplus 1^3 \oplus 2^2 \oplus 3,$$

where the superscripts show the multiplicities of these representations. It shows for example that there are three different paths of fusion, all leading to the total spin 1. Each path corresponds to a state with specific internal spins (charge) for the pair of particles (1,2), Fig. 2. Thus, in the same way that we have shown above, these states and the pair of particles which are chosen for measurements can act as a QKD system with three-level states (qutrits) in the complete absence of reference frames. The same method can be used if instead of the rotation group, $G$ is any other non-abelian group. It is true that in practical situations, there are more feasible ways, either by actively aligning the coordinate systems [29,30] or by using other degrees of freedom, like angular momentum of light [31]. It is also true that we may have partial information about the two coordinate systems, like one common direction [11–13], which alleviates the need for these schemes. Nevertheless, exploring the theoretical possibility of achieving such general schemes for any group of transformation is interesting. Finally, with advances in control and manipulation of entangled states of photons [35], the scheme proposed in this paper may also find practical applications.

### 4 Conclusion

We have discussed QKD protocol in the absence of shared reference frame from a general point of view, where the two frames are related to each other by elements of a general group $G$. To define the versions of BB84 or six-state protocols for a general group $G$, we need to define reference-frame-free states and measurements. In the minimal scheme, three particles should be used and the states are those which have identical total charge but different patterns of internal charges for creating that total charge. In the terminology used for anyons in the context of topological quantum computation [23–25], these different patterns are called fusion paths, Fig. (2). For
each total charge, the number of different paths determines the dimension of the Hilbert space, i.e., QKD with qubits or qudits. Where all these considerations apply to any group relating two different reference frames, we have used the case of the coordinate reference frame and the rotation group for definiteness and probably for its possible practical relevance, i.e., in those situations where the two stations are rotating or where noises in optical fibers or turbulence in free space communication is too high. Of course we do not emphasize this practical aspect since, needless to say, implementation of these protocol is a different problem which naturally has its own complications like preparation and measurements of entangled states, lower bit rate by a factor of 3, particle losses, etc. Furthermore, in practical situations, there may be more feasible ways, for active alignment of coordinate systems [32–34], or use other more specific protocol for coordinate reference frames [36]. Nevertheless, for the sake of completeness, we note that while our work considers the theoretical possibility of a SRF-free QKD, the optical preparation of the states (5) and (6) may not be out of reach. The left hand side states of (5) and (6) are two-photon Bell state accompanied by a single photon, and the Bell state can be realized by the methods used in SPDC (spontaneous parametric down conversion) [37]. The right hand side states of (5) and (6) are three-photon entangled states from the W-type class [38]. There are numbers of proposals for the preparation of W-states in optical systems [39–42] which may be modified for preparing a state from W-class with unequal amplitudes.

Acknowledgements The authors thank Marzieh Bathaee and Sadegh Raeisi for valuable comments. Their special thanks also go to Mauro Paternostro for a series of very instructive discussions through email. This work was partially supported by the Grant No. G950222 from the vice-chancellor of Sharif University of Technology. The works of A. Mani and F. Rezazadeh were supported by a Grant No. 96011347 from Iran National Science Foundation. Vahid Karimipour also thanks Abdus Salam ICTP where the final stages of this work were done and the Simons Foundation for financial support through the ICTP associate program.

References

1. Bartlett, S.D., Rudolph, T., Spekkens, R.W.: Classical and quantum communication without a shared reference frame. Phys. Rev. Lett. 91, 027901 (2003)
2. Bagan, E., Baig, M., Muñoz-Tapia, R.: Aligning reference frames with quantum states. Phys. Rev. Lett. 87, 257903 (2001)
3. Chiribella, G., D’Ariano, G.M., Perinotti, P., Sacchi, M.F.: Efficient use of quantum resources for the transmission of a reference frame. Phys. Rev. Lett. 93, 180503 (2004)
4. Peres, A., Scudo, P.F.: Entangled quantum states as direction indicators. Phys. Rev. Lett. 86, 4160 (2001)
5. Peres, A., Scudo, P.F.: Transmission of a Cartesian frame by a quantum system. Phys. Rev. Lett. 87, 167901 (2001)
6. Aolita, L., Walborn, S.: Quantum communication without alignment using multiple-qubit single-photon states. Phys. Rev. Lett. 98, 100501 (2007)
7. Gour, G., Marvian, I., Spekkens, R.W.: Measuring the quality of a quantum reference frame. The relative entropy of frameness. Phys. Rev. A 80, 012307 (2009)
8. Senel, C.F., Lawson, T., Kaplan, M., Markham, D., Diamanti, E.: Demonstrating genuine multipartite entanglement and nonseparability without shared reference frames. Phys. Rev. A 91, 052118 (2015)
9. Wallman, J.J., Bartlett, S.D.: Observers can always generate nonlocal correlations without aligning measurements by covering all their bases. Phys. Rev. A 85, 024101 (2012)
10. Verstraete, F., Cirac, J.I.: Quantum nonlocality in the presence of superselection rules and data hiding protocols. Phys. Rev. Lett. 91, 10404 (2003)
11. Laing, A., Scarani, V., Rarity, J.G., O’Brien, J.L.: Reference-frame-independent quantum key distribution. Phys. Rev. A 82, 012304 (2010)
12. Souza, C.E.R., Borges, C.V.S., Khoury, A.Z., Huguenin, J.A.O., Aolita, L., Walborn, S.P.: Quantum key distribution without a shared reference frame. Phys. Rev. A 77, 032345 (2008)
13. Slater, J.A., Branciard, C., Brunner, N., Tittel, W.: Device-dependent and device-independent quantum key distribution without a shared reference frame. New J. Phys. 16, 043002 (2014)
14. Bennett, C.H., Brassard, G.: Quantum cryptography: Public key distribution and coin tossing. In: Proceedings IEEE International Conference on Computers, Systems, and Signal Processing, Bangalore, India, p. 175. IEEE, New York (1984)
15. Ekert, A.: Quantum cryptography based on Bell’s theorem. Phys. Rev. Lett. 67, 661–663 (1991)
16. Bruss, D.: Optimal eavesdropping in quantum cryptography with six states. Phys. Rev. Lett. 81, 3018 (1998)
17. Cerf, N.J., Bourennane, M., Karlsson, A., Gisin, N.: Security of quantum key distribution using d-level systems. Phys. Rev. Lett. 88, 127902 (2002)
18. Bourennane, M., Karlsson, A., Bjork, G., Gisin, N., Cerf, N.: Quantum key distribution using multilevel encoding: security analysis. J. Phys. A 35, 10065 (2002)
19. Karimipour, V., Bahraminasab, A., Bagherinezhad, S.: Quantum key distribution for d-level systems with generalized Bell states. Phys. Rev. A 65, 052331 (2002)
20. Gour, G., Spekkens, R.W.: The resource theory of quantum reference frames: manipulations and monotones. New J. Phys. 10, 033023 (2008)
21. Bartlett, S.D., Rudolph, T., Spekkens, R.W.: Reference frames, superselection rules, and quantum information. Rev. Mod. Phys. 79(2), 555–609 (2007)
22. Bartlett, S.D., Rudolph, T., Spekkens, R.W.: Optimal measurements for relative quantum information. Phys. Rev. A 70, 032321 (2004)
23. Freedman, M.H., Kitaev, AYu., Wang, Z.: Simulation of topological field theories by quantum computers. Commun. Math. Phys. 227, 587–603 (2002)
24. Freedman, M., Larsen, M., Wang, Z.: A modular functor which is universal for quantum computation. Commun. Math. Phys. 267, 605–622 (2002)
25. Freedman, M.H., Kitaev, A., Larsen, M.J., Wang, Z.: Topological quantum computation. Math. Soc. 40, 31–38 (2003)
26. Lo, H.K., Chau, H.F.: Security of quantum key distribution. Science 283, 2050–2056 (1999)
27. Mayers, D.: Unconditional security in quantum cryptography. JACM 48(3), 351–406 (2001)
28. Biham, E., Boyer, M., Oscar Boykin, P., Mor, T., Roychowdhury, V.: A proof of the security of quantum key distribution. J. Cryptol. Arch. 19(4), 381–439 (1999)
29. Yin, H.L., Chen, T.Y., Yu, Z.W., Liu, H., You, L.X., Zhou, Y.H., Chen, S.J., Mao, Y., Huang, M.Q., Zhang, W.J., Chen, H., Li, M.J., Nolan, D., Zhou, F., Jiang, X., Wang, Z., Zhang, Q., Wang, X.B., Pan, J.W.: Measurement-device-independent quantum key distribution over a 404 km optical fiber. Phys. Rev. Lett. 117, 190501 (2016)
30. Comandar, L.C., et al.: Quantum key distribution without detector vulnerabilities using optically seeded lasers. Nat. Photon 10(5), 312–316 (2016)
31. Spedalieri, F.M.: Quantum key distribution without reference frame alignment: exploiting photon orbital angular momentum. Opt. Commun. 260, 340 (2006)
32. Rezazadeh, F., Mani, A., Karimipour, V.: Secure alignment of coordinate systems using quantum correlation. Phys. Rev. A 96, 022310 (2017)
33. Rezazadeh, F., Mani, A., Karimipour, V.: Power of a shared singlet state in comparison to a shared reference frame. Phys. Rev. A 100, 022329 (2019)
34. Beheshti, A., Raeisi, S., Karimipour, V.: Entanglement-assisted communication in the absence of shared reference frame. Phys. Rev. A 99, 042330 (2019)
35. Zeilinger, A.: Light for the quantum. Entangled photons and their applications: a very personal perspective. Phys. Scr. 92(7), 072501 (2017)
36. Tabia, G., Englert, B.-G.: Efficient quantum key distribution with trines of reference-frame-free qubits. Phys. Lett. A 375, 817–822 (2011)
37. Kwiat, P.G., Mattle, K., Weinfurter, H., Zeilinger, A., Sergienko, A.V., Shih, Y.H.: New high-intensity source of polarization-entangled photon pairs. Phys. Rev. Lett. 75, 4337 (1995)
38. Duer, W., Vidal, G., Cirac, J.I.: Three qubits can be entangled in two inequivalent ways. Phys. Rev. A 62, 062314 (2000)
39. Mikami, H., Li, Y., Fukuoka, K., Kobayashi, T.: New high-efficiency source of a three-photon state and its full characterization using quantum state tomography. Phys. Rev. Lett. 95, 150404 (2005)

40. Yamamoto, T., Tamaki, K., Koashi, M., Imoto, N.: Polarization-entangled W state using parametric down-conversion. Phys. Rev. A 66, 064301 (2002)

41. Tashima, T., Wakatsuki, T., Özdemir, S.k., Yamamoto, T., Koashi, M., Imoto, N.: Local transformation of two EPR photon pairs into a three-photon W state using a polarization dependent beamsplitter. In: International Conference on Quantum Communication and Quantum Networking, pp. 39–45. Springer, Berlin, Heidelberg (2009)

42. Zou, X.B., Pahlke, K., Mathis, W.: Generation of entangled photon states by using linear optical elements. Phys. Rev. A 66, 014102 (2002)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.