Spin-polarized electron transport in diluted magnetic semiconductors (DMS) in the paramagnetic phase is described within the thermoballistic transport model. In this (semiclassical) model, the ballistic and diffusive transport mechanisms are unified in terms of a thermoballistic current in which electrons move ballistically across intervals enclosed between arbitrarily distributed points of local thermal equilibrium. The contribution of each interval to the current is governed by the momentum relaxation length. Spin relaxation is assumed to take place during the ballistic electron motion. In paramagnetic DMS exposed to an external magnetic field, the conduction band is spin-split due to the giant Zeeman effect. In order to deal with this situation, we extend our previous formulation of thermoballistic spin-polarized transport so as to take into account an arbitrary (position-dependent) spin splitting of the conduction band. The current and density spin polarizations as well as the magnetoresistance are each obtained as the sum of an equilibrium term determined by the spin-relaxed chemical potential, and an off-equilibrium contribution expressed in terms of a spin transport function that is related to the splitting of the spin-resolved chemical potentials. The procedures for the calculation of the spin-relaxed chemical potential and of the spin transport function are outlined. As an illustrative example, we apply the thermoballistic description to spin-polarized transport in DMS/NMS/DMS heterostructures formed of a nonmagnetic semiconducting sample (NMS) sandwiched between two DMS layers. We evaluate the current spin polarization and the magnetoresistance for this case and, in the limit of small momentum relaxation length, find our results to agree with those of the standard drift-diffusion approach to electron transport.

PACS numbers: 72.20.Dp, 72.25.Dc, 75.50.Pp

I. INTRODUCTION

The electrical injection of spin-polarized carriers from magnetic contacts into, and their subsequent transport across, nonmagnetic semiconductors (NMS) constitute outstanding issues in the field of semiconductor spintronics. While spin-based semiconductor devices still await realization, major progress has been achieved over the past years in the understanding of the conditions and mechanisms governing polarized-carrier injection and transport.

The low injection efficiency observed for ferromagnetic metal (FM) contacts found an explanation in the large conductivity mismatch between the contacts and the semiconducting sample. To circumvent this obstacle, the introduction of spin-selective interface resistances was suggested and analyzed theoretically in some detail. A considerable enhancement of the injection efficiency due to externally applied electric fields was predicted. On the other hand, the mismatch problem is mitigated from the outset if contact layers made up of magnetic semiconductors are used. Promisingly high values of the spin injection efficiency were obtained in experiments using paramagnetic (II,Mn)VI DMS contacts. In the latter, the conduction band is spin-split due to the giant Zeeman effect in an external magnetic field, thereby making possible contact spin polarizations close to 100%. For this kind of spin injector, a novel magnetoresistance effect was observed and theoretically analyzed within the standard drift-diffusion theory of electron transport.

Spin-polarized transport in semiconducting structures formed of NMS layers and paramagnetic (II,Mn)VI DMS layers means electron transmission across spin-dependent potential profiles. These profiles depend on position via the position dependence of the electrostatic potential (composed of the conduction band edge potential and the external potential) and of the magnetic-field-induced spin splitting of the conduction band, which changes abruptly at the NMS/DMS interfaces. Transport across spin-dependent potential profiles corresponding to specific combinations of NMS and DMS layers has been studied theoretically in a number of cases in the (quantum-coherent) ballistic limit. Recently, the transport properties of II-VI resonant tunneling devices coupled to paramagnetic DMS contacts were found to depend strongly on the magnitude of the applied magnetic field.

In the present work, we consider arbitrarily shaped potential profiles including internal and external electrostatic potentials and exhibiting arbitrary, position-dependent spin splitting, thereby covering any combination of NMS layers and paramagnetic DMS layers that may occur in semiconducting structures. We study spin-polarized electron transport within the (semiclassical) thermoballistic description of carrier transport in nondegenerate semiconductors.
The basic element of this description is the thermoballistic current, in which electrons move ballistically across intervals enclosed between points of local thermal equilibrium. The contribution to the current of each such "ballistic interval" is governed by the momentum relaxation length ("mean free path"). The thermoballistic transport mechanism is intermediate between the diffusive (Drude) process, where the electrons move from one state of equilibrium to another, infinitesimally close-lying state (mean free path tending toward zero), and, on the other hand, the ballistic, collision-free motion across the electrostatic potential profile (mean free path tending toward infinity). In the thermoballistic current, diffusive and ballistic transport are thus unified in a way that allows the effect of these two aspects of the transport mechanism to be studied. In Ref. [34], we have introduced spin relaxation into the thermoballistic description for the case of electron transport in NMS. There, we have disregarded any spin splitting of the conduction band. Applications[34,35] have dealt with spin-polarized transport in heterostructures formed of an NMS layer sandwiched between two ferromagnetic metal contacts. In the present paper, we put forward the systematic extension of the thermoballistic approach to spin-polarized electron transport across spin-split potential profiles. While we are mainly concerned here with the general formulation of this extension, we also treat, within a simplified picture, spin-polarized transport in heterostructures formed of an NMS layer enclosed between two DMS layers. Applications to specific experimental configurations will be deferred to future work.

As a prerequisite to the thermoballistic description, we formulate, in the next two sections, the details of ballastic spin-polarized electron transport across spin-split potential profiles. In Sec. II, we introduce the electron densities at the points of local thermal equilibrium enclosing a ballistic interval, and construct the currents injected from these points into that interval. In Sec. III, we continue the injected currents and the densities from the points of local thermal equilibrium into the ballistic interval, and introduce the balance equation that describes spin relaxation inside the interval. The off-equilibrium ballistic spin-polarized current and density are expressed in terms of a spin transport function that is related to the splitting of the spin-resolved chemical potentials. In Sec. IV, the thermoballistic currents and densities are constructed by summing up the contributions from all ballistic intervals. The magnitude of these contributions is governed by the momentum relaxation length. The current and density spin polarizations as well as the magnetoresistance are each expressed as the sum of an equilibrium term determined by the spin-relaxed chemical potential, and an off-equilibrium contribution given in terms of the spin transport function. The spin-relaxed chemical potential is described through a resistance function. The latter, as well as the spin transport function, are each calculated from an integral equation. Section V deals with the thermoballistic description of spin-polarized electron transport in DMS/NMS/DMS heterostructures. We derive explicit expressions for the current spin polarization and the magnetoresistance and compare these, in the limit of small momentum relaxation length, to those of the standard drift-diffusion approach to electron transport. Numerical results for DMS/NMS/DMS heterostructures are presented and discussed in Sec. VI. In Sec. VII, we summarize the contents of this paper and make some concluding remarks.

II. POINTS OF THERMAL EQUILIBRIUM: DENSITIES AND INJECTED CURRENTS

The thermoballistic description of spin-polarized electron transport in NMS[34] makes use, in a one-dimensional geometry, of ballistic electron currents and densities in "ballistic intervals" between two points of local thermal equilibrium, $x'$ and $x''$ ($x_1 \leq x' < x'' \leq x_2$); see Fig. [1]. The ballistic electron transport across such an interval is determined by the densities at the points of local thermal equilibrium and by the shape of the potential profile inside the interval. For the purpose of generalizing the thermoballistic approach to spin-polarized electron transport in paramagnetic DMS, we introduce the thermal-equilibrium densities $n_{1,1}(x')$ and $n_{1,1}(x'')$ for spin-up ($\uparrow$) and spin-down ($\downarrow$) conduction band states, as well as the spin-dependent potential profiles

$$\epsilon_{\uparrow,\downarrow}(x) = E_c(x) \pm \Delta(x)/2$$

(1)

at position $x \in [x_1, x_2]$. Here, the (spin-independent) potential $E_c(x)$ comprises the conduction band edge potential and the external electrostatic potential, and $\Delta(x)$ is the Zeeman splitting of the conduction band due to an external magnetic field[22] [we restrict ourselves to considering a single Landau level whose energy is assumed to be included in $E_c(x)$]. In developing our formalism, we assume both $E_c(x)$ and $\Delta(x)$, and hence $\epsilon_{\uparrow,\downarrow}(x)$, to be continuous functions of $x$ in the interval $[x_1, x_2]$. Abrupt changes in one or the other of these functions, which occur at the interfaces in heterostructures, may be described, in a simplified picture, in terms of discontinuous functions. This aspect will be illustrated in Sec. V below for the case of DMS/NMS/DMS heterostructures.

In the thermoballistic transport mechanism, momentum relaxation takes place exclusively and instantaneously at the points of local thermal equilibrium. Spin relaxation via spin-flip scattering processes, on the other hand, occurs during the electron motion across the ballistic intervals (since the electrons spend only an infinitesimally short time span at the points of local thermal equilibrium, it is only inside the ballistic intervals that they can experience spin relaxation). This separation of momentum and spin relaxation is in accordance with the D’yakonov-Perel’ relaxation
FIG. 1: Schematic diagram showing, in a one-dimensional geometry, a paramagnetic semiconducting sample of length \( S \) enclosed between two nonmagnetic metal contacts. The spin splitting \( \Delta(x) \) of the potential profile \( E_c(x) \) gives rise to the spin-dependent potential profiles \( \epsilon_{\uparrow}(x) \) and \( \epsilon_{\downarrow}(x) \). The coordinates \( x', x'' \) denote points of local thermal equilibrium. Electrons injected at \( x'(x'') \) toward the right (left) move ballistically across the profiles \( \epsilon_{\uparrow}(x), \epsilon_{\downarrow}(x) \) to reach the end-point \( x''(x') \) of the ballistic transport interval \([x', x'']\). The thermoballistic equilibrium electron currents \( J(x) \) and \( \tilde{J}_-(x) \) are constructed from the common spin-relaxed chemical potential \( \tilde{\mu}(x') \) [with boundary values \( \tilde{\mu}_{1,2} \) at the contact/semiconductor interfaces] according to Eq. (77). Correspondingly, the off-equilibrium thermoballistic current \( \tilde{J}_-(x) \) is constructed from the spin transport function \( A(x') \) [with boundary values \( A_{1,2} \)].

mechanism\(^{36,37}\), which is known\(^{3,5}\) to be the dominant relaxation mechanism at large donor doping levels and at high temperatures. Therefore, our approach covers a broad range of cases interesting from the point of view of experiment.

A. Electron densities

The electron motion in a ballistic interval is activated at a point of local thermal equilibrium at one end and terminated at another such point at the other end. In the semiclassical model, the points of local thermal equilibrium are characterized by a local chemical potential ("quasi-Fermi level"). In a nondegenerate system, the spin-resolved electron densities, \( n_{\uparrow,\downarrow}(x') \), are given by

\[
n_{\uparrow,\downarrow}(x') = \frac{N_c}{2} e^{-\beta \epsilon_{\uparrow,\downarrow}(x') - \mu_{\uparrow,\downarrow}(x')},
\]

where \( \mu_{\uparrow,\downarrow}(x') \) are the spin-resolved chemical potentials, \( N_c/2 = (2\pi m^*/\beta h^2)^{3/2} \) is the effective density of states of either spin at the conduction band edge, \( m^* \) is the effective mass of the electrons (which, for simplicity, is assumed to be independent of position and of the external magnetic field), and \( \beta = (k_B T)^{-1} \). In the spin-relaxed state, the potentials \( \mu_{\uparrow,\downarrow}(x') \) become equal to the common "spin-relaxed chemical potential" \( \tilde{\mu}(x') \). Thus, the spin-relaxed densities at the point of thermal equilibrium \( x' \), denoted by \( \tilde{n}_{\uparrow,\downarrow}(x') \), are

\[
\tilde{n}_{\uparrow,\downarrow}(x') = \frac{N_c}{2} e^{-\beta \epsilon_{\uparrow,\downarrow}(x') - \tilde{\mu}(x')},
\]
For the Boltzmann factors $e^{-\beta E_{1,1}(x')}$, we introduce the notation
\[ \vartheta_{1,1}(x') = e^{-\beta E_{1,1}(x')} \] (4)
and define
\[ \vartheta_{\pm}(x') = \vartheta_{1}(x') \pm \vartheta_{1}(x') \] (5)
(the “±” notation will be used *mutatis mutandis* for various other quantities appearing further below), so that we have
\[ \tilde{n}_{\pm}(x') = \frac{N_e}{2} \vartheta_{\pm}(x') e^{\beta\tilde{\mu}(x')} . \] (6)
The total (i.e., spin-summed) spin-relaxed density, $\tilde{n}(x') \equiv \tilde{n}_{+}(x')$, is
\[ \tilde{n}(x') = \frac{N_e}{2} \vartheta_{+}(x') e^{\beta\tilde{\mu}(x')} , \] (7)
and for the “static” spin polarization $P(x')$ of the conduction band electrons, i.e., the polarization in the spin-relaxed state at zero external electric field, we have
\[ P(x') \equiv \frac{\tilde{n}_{-}(x')}{\tilde{n}(x')} = \frac{\vartheta_{-}(x')}{\vartheta_{+}(x')} = -\tanh(\beta\Delta(x')/2) . \] (8)
We note the relation
\[ \vartheta_{1,1}(x') = \frac{1}{2} [1 \pm P(x')] \vartheta_{+}(x') \] (9)
expressing the Boltzmann factors $\vartheta_{1,1}(x')$ in terms of $P(x')$.

For the dynamical description of spin-polarized transport, we now introduce the ratios $\alpha_{1,1}(x')$ of the densities to their spin-relaxed counterparts \[2\]
\[ \alpha_{1,1}(x') \equiv \frac{n_{1,1}(x')}{\tilde{n}_{1,1}(x')} = e^{\beta[\mu_{1,1}(x')-\tilde{\mu}(x')]} ; \] (10)
working with these quantities will provide us with a description in terms of linear equations, instead of the nonlinear description that we would encounter when employing the chemical potentials $\mu_{1,1}(x')$ themselves. This aspect has been emphasized previously\[13\] within the diffusive approach to spin polarization in nondegenerate semiconductors. [We note that the quantity $\alpha_{1,1}(x')$ introduced in Eq. (10) is different from that of Eq. (3.8) in Ref. 34 in the spin-relaxed state, we have in the present case $\alpha_{1,1}(x') = 1$, while in Ref. 34 we have $\alpha_{1,1}(x') = 1/2$.] Using Eqs. (4), (7), (9), and (10), we can write the densities $n_{1,1}(x')$ in the form
\[ n_{1,1}(x') = \frac{1}{2} \tilde{n}(x') [1 \pm P(x')] \alpha_{1,1}(x') , \] (11)
so that
\[ n_{\pm}(x') = \frac{1}{2} \tilde{n}(x') [\alpha_{\pm}(x') + P(x') \alpha_{\pm}(x')] . \] (12)
Here, the functions $\alpha_{+}(x')$ and $\alpha_{-}(x')$ appear as independent dynamical quantities.

We now require the total density, $n(x') \equiv n_{+}(x')$, to be unaffected by spin relaxation, i.e., we set
\[ n(x') \equiv \tilde{n}(x') . \] (13)
This requirement is justified\[12\] for n-doped (unipolar) systems, for which spin-flip scattering processes involving valence band states can be disregarded. Substituting Eq. (13) in the upper Eq. (12), we find $\alpha_{+}(x')$ and $\alpha_{-}(x')$ related by
\[ \alpha_{+}(x') = 2 - P(x')\alpha_{-}(x') , \] (14)
so that we can eliminate $\alpha_{+}(x')$ from the lower Eq. (12) to obtain the spin-polarized density, $n_{-}(x')$, as
\[ n_{-}(x') = n(x') \left\{ P(x') + \frac{1}{2}[Q(x')]^{2} \alpha_{-}(x') \right\} , \] (15)
where

\[ |Q(x')|^2 = 1 - |P(x')|^2 = \frac{4\vartheta_1(x')\vartheta_1(x')}{|\vartheta_1(x')|^2}, \]

In the spin-relaxed state, we have

\[ \alpha_{\uparrow,\downarrow}(x') = 1, \]

and hence

\[ \alpha_-(x') = 0, \quad \alpha_+(x') = 2. \]

Then, from Eq. (15), the equilibrium spin-polarized density at the point of thermal equilibrium \( x' \), \( \tilde{n}_-(x') \), is

\[ \tilde{n}_-(x') = n(x') P(x'), \]

[see Eq. (8)], and the off-equilibrium spin-polarized density, \( \tilde{n}_-(x') \), is there

\[ \tilde{n}_-(x') \equiv n_-(x') - \tilde{n}_-(x') = \frac{1}{\beta} n(x') |Q(x')|^2 \alpha_-(x'). \]

The off-equilibrium spin-polarized density is proportional to the quantity \( \alpha_-(x') \), i.e., to the difference of the spin-resolved densities relative to their spin-relaxed values [see Eq. (13)], but also contains, via the factor \( |Q(x')|^2 \), the effect of the static polarization \( P(x') \).

For later use, we establish the relation between the difference of the spin-resolved chemical potentials, \( \mu_-(x') \), and the quantity \( \alpha_-(x') \) which will appear as the key element determining the spin dynamics of off-equilibrium spin-polarized transport in the thermoballistic approach (see Secs. III.B and IV.C). From Eq. (16), we obtain, using Eqs. (2), (4), (7), and (9),

\[ \vartheta_1(x')\alpha_1(x') + \vartheta_1(x')\alpha_1(x') = \vartheta_+(x'), \]

and hence, using Eq. (10),

\[ \alpha_{1,\downarrow}(x') = 1 \pm \frac{1}{2}[1 + P(x')]|\alpha_-(x')|. \]

With the help of Eq. (10), we then have

\[ \mu_-(x') = \frac{1}{\beta} \ln \left( \frac{\alpha_1(x')}{\alpha_1(x')} \right) = \frac{1}{\beta} \ln \left( \frac{1 + \frac{1}{2}[1 - P(x')]\alpha_-(x')}{1 - \frac{1}{2}[1 + P(x')]\alpha_-(x')} \right), \]

and, reversely,

\[ \alpha_-(x') = \frac{2 \tanh(\beta \mu_-(x')/2)}{1 + P(x') \tanh(\beta \mu_-(x')/2)}. \]

For the mean chemical potential \( \bar{\mu}(x') \equiv \frac{1}{2} \mu_+(x') \), we find

\[ \bar{\mu}(x') = \frac{1}{2} \mu'(x') \equiv \frac{1}{2} \ln(\alpha_1(x')\alpha_1(x')) \]

\[ \bar{\mu}(x') + \frac{1}{2} \ln(\{1 + \frac{1}{2}[1 - P(x')]\alpha_-(x')\{1 - \frac{1}{2}[1 + P(x')]\alpha_-(x')\}) \]

\[ \bar{\mu}(x') + \frac{1}{2} \ln(1 - P(x')\alpha_-(x') - \frac{1}{4}\frac{1}{2}[Q(x')|^2[\alpha_-(x')|^2) \right. \}

Finally, we quote the relation between the spin-resolved chemical potentials \( \mu_{\uparrow,\downarrow}(x') \) and the common spin-relaxed chemical potential \( \bar{\mu}(x') \),

\[ \frac{1}{2}[1 + P(x')]=e^{\beta \mu_+(x')} + \frac{1}{2}[1 - P(x')]=e^{\beta \mu_+(x')}, \]

which, like Eq. (21), follows from Eq. (13), but this time using Eqs. (2), (4), (7), and (9).
B. Current injection

In the thermoballistic description, a point of thermal equilibrium lies between two ballistic intervals (except at the ends of the sample, which will be considered further below), and electrons from the left- and right-lying intervals enter into it to be equilibrated instantaneously. At the same time, equilibrated electrons, which have no preferred direction of motion, are injected symmetrically (i.e., “half-and-half”) into either ballistic interval, forming the spin-resolved electron current densities (electron currents, for short) \( v_e n_{\uparrow, \downarrow}(x') \), where \( v_e = (2\pi m^* \beta)^{-1/2} \) is the emission velocity. If, however, there lies a potential barrier \( \epsilon_{\uparrow, \downarrow}(x', x'') - \epsilon_{\uparrow, \downarrow}(x') > 0 \) \( (\epsilon_{\uparrow, \downarrow}(x', x'') \) is the maximum of the spin-dependent potential profile in the interval \([x', x'']\)) between, say, the left point of current injection, \( x' \), and the opposite point of current absorption, \( x'' \), part of the current will be reflected at this barrier (and equilibrated while it travels back and forth between \( x' \) and the left side of the barrier, – we may call it the “confined current”). The part that surmounts the barrier (henceforth called the “injected current”) is given by

\[
J_{\uparrow, \downarrow}^I(x', x'') = \frac{N_e}{2} (2\beta/\pi m^*)^{1/2} \int_0^\infty dp \frac{p}{m^*} e^{-\beta |p^2/2m^*+\epsilon_{\uparrow, \downarrow}(x')-\epsilon_{\uparrow, \downarrow}(x'')|} \Theta(p^2/2m^*+\epsilon_{\uparrow, \downarrow}(x')-\epsilon_{\uparrow, \downarrow}(x', x''))
\]

(27)

where

\[
T_{\uparrow, \downarrow}^I(x', x'') \equiv e^{-\beta \epsilon_{\uparrow, \downarrow}(x', x'')-\epsilon_{\uparrow, \downarrow}(x')} = \frac{\partial_{\uparrow, \downarrow}^m(x', x'')}{\partial_{\uparrow, \downarrow}^m(x')}
\]

(28)

is the (classical) transmission probability for the spin-resolved current injected at the left end-point \( x' \) of the ballistic interval to reach the opposite right end-point \( x'' \). Here, we have introduced the notation

\[
\partial_{\uparrow, \downarrow}^m(x', x'') = e^{-\beta \epsilon_{\uparrow, \downarrow}(x', x'')},
\]

(29)

which corresponds to the definition (1) for the Boltzmann factors \( \partial_{\uparrow, \downarrow}(x') \), with \( \epsilon_{\uparrow, \downarrow}(x') \) replaced with \( \epsilon_{\uparrow, \downarrow}(x', x'') \) [note that the factors \( \partial_{\uparrow}^m(x', x'') \) and \( \partial_{\downarrow}^m(x', x'') \) are local functions depending, for a given ballistic interval \([x', x'']\), each on the position of the maximum of the potential profiles \( \epsilon_\uparrow(x) \) and \( \epsilon_\downarrow(x) \), respectively]. If the potential profile is constant along the interval or if its maximum lies at the injection point \( x' \) itself, then \( \epsilon_{\uparrow, \downarrow}(x', x'') = \epsilon_{\uparrow, \downarrow}(x') \) and \( T_{\uparrow, \downarrow}^I(x', x'') = 1 \).

In analogy to Eq. (11) for the spin-resolved densities, we now write the currents (27) in the form

\[
J_{\uparrow, \downarrow}^I(x', x'') = \frac{1}{2} J_{\uparrow}^I(x', x'') [1 \pm \Theta(x'-x'')] \alpha_{\uparrow, \downarrow}(x')
\]

(30)

where

\[
J_{\uparrow}^I(x', x'') \equiv J_{\uparrow}^I(x', x'') = \frac{v_e N_e}{2} \Theta_{\uparrow}^m(x', x'') e^{\beta \mu(x')}
\]

(31)

is the total spin-relaxed current injected at \( x' \) into the interval \([x', x'']\) from the left, and

\[
P_{m}^m(x', x'') = \frac{\partial_{m}^m(x', x'')}{\partial_{m}^m(x', x'')}
\]

(32)

is a static, “nonlocal” spin polarization depending on the (generally different) positions of the maximum of the potential profile for the spin-up state and for the spin-down state, respectively [compare with the analogous relation (8) for the local spin polarization \( P(x') \)].

From expression (30), we obtain, in analogy to Eq. (12),

\[
J_{\pm}^I(x', x'') = \frac{1}{2} J_{\uparrow}^I(x', x'') [\alpha_{\pm}(x') + P_{m}^m(x', x'') \alpha_{\pm}(x')].
\]

(33)

In line with the condition (13) imposed on the total density \( n(x') \), we now require the total current \( J_{\uparrow}^I(x', x'') \) to equal its spin-relaxed limit,

\[
J_{\uparrow}^I(x', x'') = \tilde{J}_{\uparrow}^I(x', x'').
\]

(34)

Following the argument leading to expression (15) for the spin-polarized density \( n_-(x') \), we then find

\[
J_{-}^I(x', x'') = J_{\uparrow}^I(x', x'') \{P_{m}^m(x', x'') + \frac{1}{2} (Q_{m}^m(x', x'')) \alpha_{-}(x') \},
\]

(35)
where

\[ [Q^m(x', x'')]^2 = 1 - [P^m(x', x'')]^2 = \frac{4\theta_m^m(x', x'')\theta_m^m(x', x'')}{[\theta_m^m(x', x'')]^2} \]  

(36)

(see the analogous relation (16) for \([Q(x')]^2\)).

In the spin-relaxed state [see Eq. (18)], Eq. (33) yields for the injected equilibrium spin-polarized current

\[ \bar{J}_l^m(x', x'') = J_l^m(x', x'') P^m(x', x''), \]

so that the injected off-equilibrium spin-polarized current is

\[ \bar{J}_{\pm}^m(x', x'') = \bar{J}_{\pm}^m(x', x'') - \bar{J}_{\pm}^m(x', x'') = \frac{1}{2} J_{\pm}^m(x', x'') [Q^m(x', x'')]\alpha_{\pm}(x'), \]

(38)
in parallel to Eqs. (19) and (20), respectively.

All the preceding formulas hold also in the case where the left end-point of the ballistic interval coincides with the left end-point of the sample, \(x' = x_1\), and similarly for the right end-points, \(x'' = x_2\), with the understanding that the chemical potentials \(\bar{\mu}(x_1) = \bar{\mu}_l\) and \(\bar{\mu}(x_2) = \bar{\mu}_r\) have fixed values given by the chemical potentials in the contacts bordering on the semiconducting sample at either end.

### III. SPIN-POLARIZED TRANSPORT ACROSS A BALLISTIC INTERVAL

The electron currents discussed up to now are the currents injected into the ballistic interval at the point of thermal equilibrium as they leave the latter. Once they have entered this interval, they become transmitted ballistic currents, which will be considered in the following.

#### A. Continuing the injected currents and densities into the ballistic interval

We again consider injection at the left point of thermal equilibrium, \(x'\), of the ballistic interval \([x', x'']\). If the system is in the spin-relaxed state at the injection point \(x'\), we have \(\mu_{\uparrow,1}(x') = \bar{\mu}(x')\) and \(\alpha_{\uparrow,1}(x') = 1\). Then the transmitted (in the following, we omit the attribute “transmitted”) ballistic spin-resolved current at \(x'\), given by expression (30), propagates along the ballistic interval without spin relaxation, i.e., it is conserved. If, however, \(\mu_{\uparrow,1}(x') \neq \bar{\mu}(x')\), the injected current will relax along the ballistic interval and hence will no longer be conserved. It is then natural to continue expression (30) into the ballistic interval \([x', x'']\) by replacing \(\alpha_{\uparrow,1}(x')\) with a more general quantity \(\alpha_{\uparrow,1}(x', x''; x)\) depending both on the end-points \(x'\) and \(x''\) and on the position \(x \in [x', x'']\), so that the ballistic spin-resolved current is obtained as

\[ J_{\uparrow,1}^l(x', x''; x) = \frac{1}{2} J_l^l(x', x'') [1 \pm P^m(x', x'')] \alpha_{\uparrow,1}(x', x''; x) = \frac{v_o N_c}{2} \bar{\vartheta}_{l,1}^m(x', x'') e^{\beta \bar{\mu}(x')} \alpha_{\uparrow,1}(x', x''; x), \]

(39)

with the initial condition

\[ \alpha_{\uparrow,1}(x', x''; x) = \alpha_{\uparrow,1}(x'), \]

(40)

which holds for all \(x' \geq x_1\).

In line with the density ratios \(\alpha_{\uparrow,1}(x')\) in Eq. (30), the generalized quantities \(\alpha_{\uparrow,1}(x', x''; x)\) express the deviation of the currents inside the ballistic interval from their spin-relaxed values, so that here the condition for spin equilibrium is again

\[ \alpha_{\uparrow,1}^l(x', x''; x) = 1 \]

(41)

[see Eq. (17)], and hence

\[ \alpha_{\uparrow,1}^l(x', x''; x) = 0, \quad \alpha_{\downarrow,1}(x', x''; x) = 2, \]

(42)
as in Eq. (18).

Extending condition (42) and the procedure ensuing therefrom, we obtain for the (conserved) total ballistic current inside the ballistic interval

\[ J_l^l(x', x''; x) \equiv J_{\uparrow,1}^l(x', x''; x) + J_{\downarrow,1}^l(x', x''; x) = \frac{v_o N_c}{2} \bar{\vartheta}_{l,1}^m(x', x'') e^{\beta \bar{\mu}(x')} \]

(43)
and, similarly, for the (conserved) ballistic equilibrium spin-polarized current

\[ \tilde{J}_1^l(x', x''; x) = J^l(x', x'') P^m(x', x'') = \frac{v_e N_c}{2} \vartheta_\alpha^m(x', x'') e^{{\beta \tilde{\mu}(x')}} \] (44)

[see Eq. (37)], while for the ballistic off-equilibrium spin-polarized current we have

\[ \tilde{J}_1^l(x', x''; x) = \frac{1}{2} J^l(x', x'') [Q^m(x', x'')]^2 \alpha_\alpha^l(x', x''; x) \]
\[ = \frac{v_e N_c}{2} \vartheta_\alpha^m(x', x'') e^{{\beta \tilde{\mu}(x')}} \alpha_\alpha^l(x', x''; x) \] (45)

[see Eq. (38)], where

\[ \vartheta_\alpha^m(x', x'') = \frac{2 \vartheta_\alpha^m(x', x'') \vartheta_\alpha^m(x', x'')}{\vartheta_\alpha^m(x', x'')} = \frac{1}{2} \vartheta_\alpha^m(x', x'') [Q^m(x', x'')]^2 . \] (46)

In Eq. (45), the spin dynamics of the ballistic off-equilibrium spin-polarized current \( \tilde{J}_1^l(x', x''; x) \) is determined by the factor \( \alpha_\alpha^l(x', x''; x) \), whose dependence on the coordinate \( x \) reflects the spin relaxation inside the ballistic interval \([x', x'']\). This dependence will be considered in Sec. III.B.

The density \( n_{1,1}^l(x', x''; x) \) associated with the ballistic spin-resolved current \( J_1^l(x', x''; x) \) will be called “ballistic spin-resolved density”. It is obtained from Eq. (39) by replacing \( v_e \) with \( C_{1,1}^l(x', x''; x)/2 \),

\[ n_{1,1}^l(x', x''; x) = \frac{N_e}{4} D_{1,1}^m(x', x''; x) e^{\beta \tilde{\mu}(x') \alpha_1^l(x', x''; x)} \] (47)

(see the analogous relation between the current (2.2) and the density (2.7) in Ref. 34). Here,

\[ D_{1,1}^m(x', x''; x) = C_{1,1}^l(x', x''; x) \vartheta_{1,1}^m(x', x'') \] (48)

and

\[ C_{1,1}^l(x', x''; x) = \frac{1}{T_{1,1}^l(x', x''; x)} \operatorname{erfc}([-\ln T_{1,1}^l(x', x''; x)]^{1/2}) \] (49)

[see Eq. (2.8) of Ref. 31], with

\[ T_{1,1}^l(x', x''; x) \equiv e^{-\beta (\epsilon_{1,1}^l(x', x'') - \epsilon_{1,1}(x))} = \frac{\vartheta_{1,1}^m(x', x'')}{\vartheta_{1,1}^l(x)} ; \] (50)

the latter quantity may be interpreted as the transmission probability corresponding to injection at the point \( x \) toward the region containing the maximum of the potential profile in the interval \([x', x'']\). The ballistic velocity \( v_{1,1}^l(x', x''; x) \) is given by

\[ v_{1,1}^l(x', x''; x) = \frac{J_{1,1}^l(x', x''; x)}{n_{1,1}^l(x', x''; x)} = \frac{2v_e}{C_{1,1}^l(x', x''; x)} . \] (51)

It is not affected by spin relaxation, since the spin-flip mechanism is assumed not to influence the kinematics of the electron motion.

From Eq. (47), the continuation of half (see the remarks at the beginning of Sec. II.B) the densities \( n_{1,1}^l(x', x''; x) \) into the ballistic interval now follows as

\[ n_{1,1}^l(x', x''; x) = \frac{4}{3} \tilde{n}_{1,1}^l(x', x''; x) \left[ \alpha_{1,1}^l(x', x''; x) + P_{C}^m(x', x''; x) \alpha_{1,1}^l(x', x''; x) \right] , \] (52)

where

\[ \tilde{n}_{1,1}^l(x', x''; x) \equiv \tilde{n}_{1,1}^l(x', x''; x) = \frac{N_e}{4} D_{1,1}^m(x', x''; x) e^{\beta \tilde{\mu}(x')} \] (53)

[see Eq. (11)] is the total ballistic spin-relaxed density, and, in generalization of expression (32) for \( P^m(x', x'') \),

\[ P_{C}^m(x', x''; x) \equiv P_{C}^m(x', x''; x) = \frac{D_{C}^m(x', x''; x)}{D_{1,1}^m(x', x''; x)} . \] (54)
Extending condition (13) into the ballistic interval by requiring
\[ n'(x',x'';x) \equiv n_+(x',x'';x) = \tilde{n}(x',x'';x), \] 
we find from Eqs. (52)
\[ \alpha_+(x',x'';x) = 2 - \mathcal{P}_+(x',x'';x) \alpha_-(x',x'';x) \]  
[see Eq. (14)], and hence for the ballistic spin-polarized density inside the interval \([x',x'']\), in parallel to Eq. (15),
\[ \tilde{n}_-(x',x'';x) = n_+(x',x'';x) \left\{ \mathcal{P}_+(x',x'';x) + \frac{1}{2} [\mathcal{Q}_+(x',x'';x)]^2 \alpha_-(x',x'';x) \right\}, \]
where
\[ [\mathcal{Q}_+(x',x'';x)]^2 = 1 - [\mathcal{P}_+(x',x'';x)]^2 = \frac{4D^m_+(x',x'';x)D^m_+(x',x'';x)}{[D^m_+(x',x'';x)]^2} \]
[see Eq. (56)]. Thus, we obtain for the ballistic equilibrium spin-polarized density
\[ \tilde{n}_-(x',x'';x) = \frac{N_c}{4} D^m_+(x',x'';x) e^{\tilde{\mu}(x')} \]  
[see Eq. (19)], and for the ballistic off-equilibrium spin-polarized density
\[ \tilde{n}_-(x',x'';x) = \frac{N_c}{4} D^m_+(x',x'';x) e^{\tilde{\mu}(x')} \alpha_-(x',x'';x) \]
[see Eq. (20)], where
\[ D^m_+(x',x'';x) = \frac{2D^m_+(x',x'';x) D^m_+(x',x'';x)}{D^m_+(x',x'';x)} = \frac{1}{2} D^m_+(x',x'';x)[\mathcal{Q}_+(x',x'';x)]^2, \]
in parallel to Eq. (46).

We remark that for constant potential profiles, \( \epsilon_{\uparrow \downarrow}(x) \equiv \epsilon_{\uparrow \downarrow}, \) when \( C_{\uparrow \downarrow}(x',x'';x) = 1 \) so that \( D^m_+(x',x'';x) = \tilde{\phi}^m_+(x',x'') \), Eq. (59) becomes \( \tilde{n}(x',x'';x) = \tilde{n}(x')/2 \). This reflects the fact that the left-hand side of this relation refers to the density associated with the ”half-sided” injected current, while \( \tilde{n}(x') \) is the total density at the point of thermal equilibrium \( x' \).

### B. Spatial behavior of the off-equilibrium spin-polarized current and density

The total ballistic current \( J_1(x',x'') \) and density \( n_1(x',x'';x) \), as well as the ballistic equilibrium spin-polarized current \( \tilde{J}_1(x',x'';x) \) and density \( \tilde{n}_1(x',x'';x) \), and \( n_1(x',x'';x) \), are determined by the quantities \( \tilde{\phi}^m_+(x',x'') \) and \( D^m_+(x',x'';x) \) [which, in turn, are completely determined by the potential profiles \( \epsilon_{\uparrow \downarrow}(x') \)], and, most importantly, by the spin-relaxed chemical potential \( \tilde{\mu}(x') \), which is the only dynamical quantity appearing. The latter is to be calculated according to the thermoballistic procedure presented in Ref. [34] (see below).

By contrast, the ballistic off-equilibrium spin-polarized current \( \tilde{J}_1(x',x'';x) \) and density \( \tilde{n}_1(x',x'';x) \) contain, in addition, the common factor \( \tilde{\phi}^m_+(x',x'') \), whose spatial behavior (x-dependence) is determined by the process of spin relaxation in the ballistic interval \([x',x'']\). This process is governed by the balance equation connecting the off-equilibrium ballistic spin-polarized current and density (see Ref. [34]),
\[ \frac{d}{dx} J_1(x',x'';x) + \frac{2v_c}{l_s} \tilde{n}_1(x',x'';x) = 0; \]  
(62)

here, \( l_s = 2v_c \tau_s \) is the (ballistic) spin relaxation length, which we assume, as in the case of the effective electron mass \( m^* \), to be independent of position and of the external magnetic field. The spin relaxation time \( \tau_s \) is given by \( 1/\tau_s = 1/\tau_1 + 1/\tau_1, \) where \( 1/\tau_1 \) (1/\( \tau_1 \)) is the rate for spin-flip scattering from spin-up (spin-down) to spin-down (spin-up) states. Inserting expressions (45) and (60) in Eq. (62), we now obtain a first-order differential equation for \( \alpha_-(x',x'';x) \),
\[ \frac{d}{dx} \alpha_-(x',x'';x) + \frac{C(x',x'';x)}{l_s} \alpha_-(x',x'';x) = 0, \]  
(63)
where
\[ C(x', x''; x) \equiv C_1(x', x''; x) C_1(x', x''; x) \frac{\partial m(x', x'')}{D_m(x', x''; x)} = \frac{[Q_m(x', x''; x)]^2}{[Q_m(x', x'')]^2} \frac{D_m(x', x''; x)}{\partial m(x', x'')}. \] (64)

The solution of Eq. (63) obeying the initial condition (10) is
\[ \alpha^c_-(x', x''; x) = \alpha_-(x') e^{-C(x', x'':x',x)/\ell_s}, \] (65)

where
\[ C(x', x''; z_1, z_2) = \int_{z_1}^{z_2} dz C(x', x''; z), \] (66)

with \( z_+ = \min(z_1, z_2) \), \( z_- = \max(z_1, z_2) \).

For the ballistic off-equilibrium spin-polarized current, we then have from Eq. (45)
\[ \tilde{J}^l_-(x', x''; x) = \frac{v_c N_e}{2} \vartheta^m(x', x'') A(x') e^{-C(x', x''; x', x)/\ell_s}, \] (67)

and for the ballistic off-equilibrium spin-polarized density from Eq. (60),
\[ \tilde{n}^l_-(x', x''; x) = \frac{N_e}{4} D^m(x', x''; x) A(x') e^{-C(x', x''; x', x)/\ell_s}. \] (68)

Here, we have introduced the spin transport function
\[ A(x') = e^{\beta \bar{\mu}(x')} \alpha_-(x') = e^{\beta \mu_+(x')} - e^{\beta \mu_-(x')}. \] (69)

[Owing to the different definitions of \( \alpha^c_+(x') \) in the present work and in Ref. 34, the quantities \( A(x') \) are not the same in the two papers.] In the thermoballistic approach, this function completely describes the dynamics of off-equilibrium spin-polarized transport. In terms of the difference of the spin-resolved chemical potentials, \( \mu_-(x') \), which commonly has been used as the basic dynamic variable in previous approaches to spin-polarized transport, it is expressed, owing to Eq. (24), as
\[ A(x') = \frac{2 e^{\beta \bar{\mu}(x')} \tanh(\beta \mu_-(x')/2)}{1 + P(x') \tanh(\beta \mu-(x')/2)}. \] (70)

The procedure for the calculation of the spin transport function will be outlined in Sec. IV.C.

The spatial behavior of the off-equilibrium spin-polarized current and density in the ballistic interval \([x', x'']\), i.e., their dependence on the coordinate \( x \), is determined by the factor \( e^{-C(x', x'':x',x)/\ell_s} \). Hence, spin relaxation in this interval departs from a purely exponential behavior unless the potential profiles \( \epsilon_{1,1}(x) \) are constant over the interval.

**C. Net ballistic currents and joint ballistic densities**

So far, we have only considered injection from the left end-point, \( x' \), of the ballistic interval \([x', x'']\). The expressions for the currents and densities obtained for that case are easily transcribed to the analogous expressions for injection from the right end-point \( x'' \), \( J^r(x', x'') \), \( J^l_-(x', x''; x) \), \( J^l_+(x', x''; x) \), \( J^r_-(x', x''; x) \), \( n^l_-(x', x''; x) \), \( n^l_+(x', x''; x) \), \( \tilde{n}^l_-(x', x''; x) \), \( \tilde{n}^l_+(x', x''; x) \), and \( \tilde{n}^r_-(x', x''; x) \), by replacing everywhere the superscript \( l \) with \( r \), and interchanging the arguments \( x' \) and \( x'' \) (owing to the symmetry of \( \epsilon_{1,1}(x', x'') \), the quantities \( \vartheta^m_\pm(x', x'') \), \( \vartheta^m_+(x', x'') \), \( \vartheta^m_-(x', x'') \), \( D^m_\pm(x', x''; x) \), and \( D^m_-(x', x''; x) \) are symmetric under the exchange of \( x' \) and \( x'' \)).

Now, combining the ballistic currents injected from the left and right, we have for the (conserved) net total ballistic current inside the ballistic interval, using Eq. (43),
\[ \tilde{J}(x', x'') = \tilde{J}^r(x', x'') - \tilde{J}^l(x', x'') = \frac{v_c N_e}{2} \vartheta^m(x', x'') \left[ e^{\beta \bar{\mu}(x')} - e^{\beta \bar{\mu}(x'')} \right]. \] (71)

Using Eq. (44), we find for the net ballistic equilibrium spin-polarized current
\[ \tilde{J}_-(x', x'') = \frac{\tilde{J}^r_-(x', x'') - \tilde{J}^l_-(x', x'')}{P(x', x'')} = J(x', x'') P(x', x'') \] (72)
while for the net ballistic off-equilibrium spin-polarized current, we have, using Eq. (77),
\[
\hat{J}_-(x', x''; x) = \hat{J}^l_-(x', x''; x) - \hat{J}^r_-(x', x''; x) = \frac{v_n N_e}{2} g^m(x', x'') [A(x') e^{\beta \mu(x', x'; x) / l_s} - A(x'') e^{-\beta \mu(x', x; x'') / l_s}].
\]
Similarly, using Eq. (53), we find for the joint total ballistic density associated with the net total ballistic current (74),
\[
n(x', x''; x) = n^l(x', x''; x) + n^r(x', x''; x) = \frac{N_e}{4} D^m(x', x''; x) [e^{\beta \mu(x)} + e^{\beta \mu(x'')}],
\]
while for the joint ballistic equilibrium spin-polarized density we obtain, using Eq. (59),
\[
\tilde{n}_-(x', x''; x) = \tilde{n}^l_-(x', x''; x) + \tilde{n}^r_-(x', x''; x) = n(x', x''; x) P^C_m (x', x''; x) = \frac{N_e}{4} D^m(x', x''; x) [e^{\beta \mu(x')} + e^{\beta \mu(x'')}].
\]
Finally, for the joint ballistic off-equilibrium spin-polarized density, we have, using Eq. (58),
\[
\tilde{n}_-(x', x''; x) = \tilde{n}^l_-(x', x''; x) + \tilde{n}^r_-(x', x''; x) = \frac{N_e}{4} D^m(x', x''; x) [A(x') e^{-\beta \mu(x', x'; x) / l_s} + A(x'') e^{-\beta \mu(x', x'') / l_s}].
\]
It becomes apparent from the preceding results that the role of the x-independent, local Boltzmann factors \(\vartheta^m_{11}(x', x'')\) in the currents is taken over, in the densities, by the x-dependent, nonlocal factors \(D^m_{11}(x', x''; x)\).

IV. THERMOBALLISTIC SPIN-POLARIZED TRANSPORT

The thermoballistic approach to semiclassical electron transport has been developed in Ref. 33, and generalized in Ref. 34 to describe spin-polarized electron transport across a spin-degenerate conduction band. In this approach, the net ballistic currents and joint ballistic densities defined in ballistic transport intervals form the building blocks for the construction of the corresponding thermoballistic currents and densities. In order to deal with the case of a spin-split conduction band, we have to modify and augment the formulation of Ref. 34. We construct the thermoballistic currents and densities in terms of the net ballistic currents and joint ballistic densities derived in Sec. III.C, and outline the procedures for calculating the equilibrium chemical potential and the spin transport function. From the thermoballistic currents and densities, the corresponding equilibrium and off-equilibrium spin polarizations are determined, and the magnetoresistance is obtained.

A. Thermoballistic currents and densities

Following Refs. 33 and 34, we introduce, for given position \(x\) inside the semiconducting sample extending from \(x_1\) to \(x_2\) (see Fig. 1), the set of ballistic intervals \([x', x'']\) that enclose \(x\), thus restricting \(x'\) and \(x''\) by \(x_1 \leq x' < x < x'' \leq x_2\). From the net ballistic currents and joint ballistic densities of Eqs. (71)–(76), here summarily represented by the symbol \(\mathcal{F}(x', x''; x)\), the corresponding thermoballistic currents and densities, \(\mathcal{F}(x)\), are constructed by summing up the currents and densities \(\mathcal{F}(x', x''; x)\) over all ballistic intervals. In the summation, the ballistic intervals are weighted (assuming one-dimensional transport) with the probability \(e^{-|x'-x''| / l}\) that the electrons traverse the interval without impurity or phonon scattering, multiplied by the probability \(dx'/l\) (\(dx''/l\)) of their being absorbed (or emitted) in an interval \(dx'\) (\(dx''\)) at an end-point \(x'\) (\(x''\)). At the ends of the semiconducting sample, \(x_{1,2}\), absorption and emission occur with unit probability. Here, the quantity \(l\) is the momentum relaxation length (mean free path), which governs the relative contribution of the ballistic and diffusive (“thermal”) transport mechanisms (see Ref. 33). Like the effective electron mass \(m^*\) and the ballistic spin relaxation length \(l_s\), the momentum relaxation length \(l\) is assumed to be independent of position and of the external magnetic field.

By this procedure, the thermoballistic currents and densities \(\mathcal{F}(x)\) are obtained in the form
\[
\mathcal{F}(x) = e^{-(x_2-x_1)/l} \mathcal{F}(x_1, x_2; x) + \int_{x_1}^{x} \frac{dx'}{l} e^{-(x_2-x')/l} \mathcal{F}(x', x_2; x)
+ \int_{x}^{x_2} \frac{dx''}{l} e^{-(x'-x_1)/l} \mathcal{F}(x_1, x''; x) + \int_{x_1}^{x} \frac{dx'}{l} \int_{x}^{x_2} \frac{dx''}{l} e^{-(x''-x')/l} \mathcal{F}(x', x''; x).
\]
Thus, the total thermoballistic current \( \mathcal{F}(x) \equiv J(x) \) is given by Eq. (77) with \( \mathcal{F}(x', x''); x \) replaced with \( J(x', x'') \) of Eq. (71). Comparing with Eq. (2.12) of Ref. 34 we see that the current \( J(x) \) of the present work corresponds to the current \( J(x) \) of Ref. 34 if one sets \( E_0^u = 0 \) and substitutes for the weights \( w_n(x', x''); l \) [\( n = 1, 2, 3 \)] the function

\[
w_+(x', x''; l) = \frac{1}{2} e^{-|x'| - |x''|/l} \vartheta^m(x', x''),
\]

(78)

which in the limit of zero spin splitting, \( \vartheta^m(x', x'') = 2 \), reduces to the (\( n \)-independent) weight \( w_n(x', x''); l \) for one-dimensional transport.

The equilibrium thermoballistic spin-polarized current \( \mathcal{F}(x) \equiv \tilde{J}_-(x) \) follows from Eq. (77) by identifying \( \mathcal{F}(x', x''; x) \) with \( \tilde{J}_-(x', x'') \) of Eq. (72). For the current \( \tilde{J}_-(x) \), there is no explicit counterpart in the formulation of Ref. 34, in which the equilibrium part of the thermoballistic spin-polarized current merely has been parameterized in terms of an arbitrary constant \( \tilde{\alpha}_- \) [see Eq. (3.27) of Ref. 34].

The off-equilibrium thermoballistic spin-polarized current \( \mathcal{F}(x) \equiv \tilde{J}_+(x) \) is obtained from Eq. (77) by replacing \( \mathcal{F}(x', x''; x) \) with \( \tilde{J}_+(x', x''; x) \) of Eq. (73). The resulting current \( \tilde{J}_+(x) \) is seen to correspond to the current given by Eq. (3.28) of Ref. 34 if the function

\[
\tilde{w}(x', x''; l) = \frac{1}{2} e^{-|x'| - |x''|/l} \vartheta^m(x', x'')
\]

(79)
is substituted for the weights \( w_n(x', x''); l \). Here again, in the limit of zero spin splitting, \( \vartheta^m(x', x'') = 1 \), the function \( \tilde{w}(x', x''; l) \) agrees with the weight \( w_n(x', x''); l \) for one-dimensional transport.

We note that the total and equilibrium spin-polarized currents, \( J(x) \) and \( \tilde{J}_-(x) \), both are linear functionals of the factors \( \vartheta^m_n(x', x'') \), and hence of the Boltzmann factors corresponding to the maxima of the potential profiles \( \epsilon_{1,1}(x) \) in the ballistic interval \( [x', x''] \). The off-equilibrium current \( \tilde{J}_+(x) \), on the other hand, is, via its dependence on the factor \( \vartheta^m(x', x'') \), a nonlinear functional of the Boltzmann factors.

The thermoballistic densities \( n(x) \), \( \tilde{n}_-(x) \), and \( \tilde{n}_+(x) \) corresponding to the thermoballistic currents \( J(x) \), \( \tilde{J}_-(x) \), and \( \tilde{J}_+(x) \), respectively, are evidently obtained by substituting the joint ballistic densities of Eqs. (74)–(76) for \( \mathcal{F}(x', x''; x) \) in Eq. (77). Comparing the thermoballistic densities of the present work to the densities given by Eqs. (2.17), (3.29), and (3.30) of Ref. 34 we find that the remarks made above in conjunction with the thermoballistic currents also apply in Eq. (77). Comparing the thermoballistic densities of the present work to the densities given by Eqs. (2.17), (3.29), and (3.30) of Ref. 34, in which the equilibrium part of the thermoballistic spin-polarized current merely has been parameterized in terms of an arbitrary constant \( \tilde{\alpha}_- \) [see Eq. (3.27) of Ref. 34].

**B. Spin-relaxed chemical potential and equilibrium spin polarizations**

The net total ballistic and ballistic equilibrium spin-polarized currents, Eqs. (71) and (72), respectively, as well as the joint total ballistic and ballistic equilibrium spin-polarized densities, Eqs. (74) and (75), respectively, and hence also the corresponding thermoballistic currents and densities, are dynamically determined by the spin-relaxed chemical potential \( \tilde{\mu}(x) \). The procedure for calculating the latter quantity is set up by employing the conditions

\[
\frac{1}{x_2 - x_1} \int_{x_1}^{x_2} dx \, J(x) = J,
\]

(80)

where \( J \) is the total physical current, and

\[
J(x_1^+) = J(x_2^-)
\]

(81)

for the total thermoballistic current \( J(x) \) given by Eq. (77) with \( \mathcal{F}(x) \equiv J(x) \).

We now rewrite \( J(x) \), with the help of Eq. (77), in terms of the (spin-relaxed) “resistance function” \( \chi(x) \)

\[
\chi(x) = \frac{v_e N_0}{J} [e^{\beta \tilde{\mu}(x)} - e^{\beta \tilde{\mu}(x)}].
\]

(82)

Inserting the result in Eq. (80) and replacing the parameter \( x_2 \) with the variable \( x \), we obtain a linear, inhomogeneous, Volterra-type integral equation for \( \chi(x) \),

\[
\frac{x - x_1}{l} - f_+(x; x_1; l) \chi(x) + \int_{x_1}^{x} \frac{dx'}{l} K_+(x, x'; x_1; l) \chi(x') = 0,
\]

(83)

where

\[
f_+(x; x_1; l) = u_+(x_1, x; l) + \int_{x_1}^{x} \frac{dx'}{l} u_+(x', x; l)
\]

(84)
and
\[ K_+(x, x'; x_1; l) = u_+(x', x; l) - u_+(x_1, x'; l) + \int_{x_1}^{x} \frac{dx''}{l} u_+(x', x''; l), \tag{85} \]
with
\[ u_+(x', x''; l) = \frac{x'' - x'}{l} w_+(x', x''; l), \tag{86} \]
where \( w_+(x', x''; l) \) is given by Eq. (78). At \( x = x_1 \), the resistance function \( \chi(x) \) is discontinuous,
\[ \chi(x_1) = 0, \tag{87} \]
\[ \chi(x_1^+) = \frac{2}{\varphi_+(x_1, x_1^+)} = \frac{2}{\varphi_+(x_1, x_1)} \tag{88} \]
(note that we assume the potential profiles \( \epsilon_{1,1}(x) \) to be continuous in the interval \([x_1, x_2]\)). The function \( f_+(x; x_1; l) \) and the kernel \( K_+(x, x'; x_1; l) \) are seen to depend on the profiles \( \epsilon_{1,1}(x) \) solely via the quantity \( \varphi_+(x', x'') \). The solution of Eq. (83) is obtained, in general, by numerical propagation.

In order to construct a unique chemical potential \( \tilde{\mu}(x) \), following Ref. 34, we have to solve Eq. (83) twice: first, with the original profiles \( \epsilon_{1,1}(x) \) to obtain the solution \( \chi_1(x) \), and second, with the spatially reversed profiles \( \epsilon_{1,1}^*(x) = \epsilon_{1,1}(x_1 + x_2 - x) \) to obtain the solution \( \chi_1^*(x) \). Defining
\[ \chi_2(x) = \chi_1^*(x_1 + x_2 - x), \tag{89} \]
we introduce the function
\[ \chi_-(x) = \hat{a}_1[\chi_1(x) - \frac{1}{2} \chi_1(x_2)] - \hat{a}_2[\chi_2(x) - \frac{1}{2} \chi_2(x_1)] \tag{90} \]
and the constant
\[ \chi = \hat{a}_1 \chi_1(x_2) + \hat{a}_2 \chi_2(x_1). \tag{91} \]

The coefficients \( \hat{a}_{1,2} \) are given34 by
\[ \hat{a}_{1,2} = \frac{a_{1,2}}{a}, \tag{92} \]
with
\[ a_1 = \int_{x_1}^{x_2} \frac{dx'}{l} \left\{ w_+(x_1, x'; l) \left[ \chi_2(x') - \chi_2(x_1) \right] + w_+(x', x_2; l) \chi_2(x') \right\}, \tag{93} \]
\[ a_2 = \int_{x_1}^{x_2} \frac{dx'}{l} \left\{ w_+(x_1, x'; l) \chi_1(x') + w_+(x', x_2; l) \left[ \chi_1(x') - \chi_1(x_2) \right] \right\}, \tag{94} \]
and \( a = a_1 + a_2 \), so that \( \hat{a}_1 + \hat{a}_2 = 1 \). For symmetric potential profiles, \( \epsilon_{1,1}^*(x) = \epsilon_{1,1}(x) \), we have \( \chi_2(x) = \chi_1(x_1 + x_2 - x) \), and hence \( \hat{a}_1 = \hat{a}_2 = \frac{1}{2} \).

In terms of \( \chi_-(x) \) and \( \chi \), the thermoballistic spin-relaxed chemical potential \( \tilde{\mu}(x) \) is given34 by
\[ e^{\beta \tilde{\mu}(x)} = \eta_+ - 2 \frac{\chi_-(x)}{\chi} \eta_- \tag{95} \]
\((x_1 \leq x \leq x_2)\), where
\[ \eta_\pm = \frac{1}{2} (e^{\beta \tilde{\mu}_1} \pm e^{\beta \tilde{\mu}_2}) \tag{96} \]
and \( \tilde{\mu}_{1,2} = \tilde{\mu}(x_{1,2}) \). The chemical potential \( \tilde{\mu}(x) \) thus obtained in the thermoballistic approach characterizes the thermal-equilibrium attribute of the thermoballistic system, which co-exists with its ballistic attribute. It immediately
leads, via Eqs. (7) and (19), to the total spin-relaxed density \( n(x) \) and the equilibrium spin-polarized density \( \tilde{n}_-(x) \), respectively.

By analogy with Eq. (2.44) of Ref. 34, we introduce the (dimensionless) reduced resistance

\[
\tilde{\chi} = \frac{1}{2} \vartheta_+^m(x_1, x_2) \chi,
\]

which fixes, for given total current \( J \), via the current-voltage characteristic

\[
J = \frac{1}{e} v_e N_e \vartheta_+^m(x_1, x_2) e^{\beta \tilde{\mu}_1} \frac{1}{\tilde{\chi}} \left( 1 - e^{-\beta \tilde{V}} \right)
\]

[see Eq. (2.42) of Ref. 34] the ”spin-relaxed” voltage bias

\[
\tilde{V} = \frac{\tilde{\mu}_1 - \tilde{\mu}_2}{e}
\]

between the contacts bordering the semiconducting sample. In the zero-bias limit, \( \tilde{\chi} \) determines, via its dependence on the Zeeman splitting \( \Delta(x) \), the magnetic-field-dependent equilibrium resistance

\[
\tilde{R} = \frac{\tilde{V}}{e J} \bigg|_{J=0} = \frac{\tilde{\chi} e^{-\beta E_c(x_1)} e^{2v_e n_1(0)}}{\beta e^2 v_e n_1(0) \vartheta_+^m(x_1, x_2)} \frac{2}{\vartheta_+^m(x_1, x_2)}, \tag{100}
\]

where \( n_1(0) = N_e e^{-\beta [E_c(x_1) - \tilde{\mu}_1]} \) is the total (spin-relaxed) electron density at \( x_1 \) for zero external magnetic field, i.e., for \( \Delta(x) = 0 \). [Note that, owing to its definition in terms of a current density, \( \tilde{R} \) has the dimension of an interface resistance.] It is notationally convenient, and physically meaningful, to introduce, instead of the density \( n_1(0) \), the Sharvin interface conductance

\[
\varrho_1(0) = \beta e^2 v_e n_1(0), \tag{101}
\]

so that

\[
\tilde{R} = \frac{\tilde{\chi} e^{-\beta E_c(x_1)}}{\varrho_1(0)} \frac{2}{\vartheta_+^m(x_1, x_2)}. \tag{102}
\]

Adding to the spin-relaxed bias \( \tilde{V} \) the off-equilibrium bias, one obtains the magnetoresistance proper (see Sec. IV.D).

The total thermoballistic current \( J(x) \) is obtained in terms of \( \tilde{\mu}(x) \) or, equivalently, \( \chi_-(x) \) and \( \tilde{\chi} \) by substituting expression (74) in Eq. (77) with \( F(x) \equiv J(x) \). In an analogous manner, the total thermoballistic density \( n(x) \) associated with the total thermoballistic current is obtained by referring to Eqs. (72) and (77).

We define the equilibrium current spin polarization \( \tilde{P}_J(x) \) and the equilibrium density spin polarization \( \tilde{P}_n(x) \) in terms of the thermoballistic currents and densities, respectively, as

\[
\tilde{P}_J(x) = \frac{\tilde{J}_-(x)}{\tilde{J}(x)} \tag{103}
\]

and

\[
\tilde{P}_n(x) = \frac{\tilde{n}_-(x)}{n(x)}. \tag{104}
\]

The thermoballistic currents and densities are, in general, not equal to the corresponding physical currents and densities. However, the relative spin content is the same in both, so that the “thermoballistic” ratio in Eqs. (103) and (104), respectively, is equal to the corresponding “physical” ratio.

In view of Eq. (95) and the first of Eqs. (98), the term in brackets on the right-hand side of Eq. (74) for the net total ballistic current \( J(x', x'') \) is proportional to the total physical current \( J \), since the function \( \chi_-(x) \) is independent of \( J \). From expression (74), the total thermoballistic current \( J(x) \) is then again found to be proportional to \( J \), and the same holds for the equilibrium thermoballistic spin-polarized current \( \tilde{J}_-(x) \) [see Eqs. (72) and (77)]. The equilibrium current spin polarization \( \tilde{P}_J(x) \) is therefore independent of \( J \) (it is an intrinsic property determined exclusively by the spin splitting of the conduction band inside the semiconductor). The equilibrium density spin polarization \( \tilde{P}_n(x) \), on the other hand, is found to depend, in general, on the bias \( \tilde{V} \), i.e., on \( J \) [see Eq. (2.53) of Ref. 34].
C. Spin transport function and off-equilibrium spin polarizations

The off-equilibrium thermoballistic spin-polarized current $\mathbf{J}^\text{\(\perp\)}(x)$ is obtained by substituting expression (73) in Eq. (77) with $\mathcal{F}(x) \equiv \mathbf{J}^\text{\(\perp\)}(x)$, and analogously for the corresponding density (76). Following Ref. 34, we invoke the spin balance equation connecting these two quantities and arrive at an integral equation for the spin transport function $\mathcal{A}(x)$ of the form [see Eqs. (3.35)–(3.37) of Ref. 34]

$$\mathbf{W}(x_1, x; l, l_a) A_1 + \mathbf{W}(x, x_1; l, l_a) A_2 - \mathbf{W}(x_1, x; l, l_a) A_1 + \int_{x_1}^{x_2} \frac{dx'}{l} \mathbf{W}(x', x; l, l_a) A(x') = 0 \ ,$$  

(105)

where

$$\mathbf{W}(x', x''; l, l_a) = \mathbf{w}(x', x''; l) e^{-\mathcal{C}(x', x''; x', x'')} / l,$$

(106)

and

$$\mathbf{W}(x; x_1, x_2; l) = \mathbf{w}(x; x_1; l) + \mathbf{w}(x; x_2; l) + \int_{x_1}^{x_2} \frac{dx'}{l} \mathbf{w}(x'; x; l) \ .$$  

(107)

Here, $A_{1,2} = A(x_{1,2})$ are the values of the spin transport function at the contact side of the interfaces with the left and right contacts, respectively; they are determined by the chemical potentials $\mu_{1,1}(x_{1,2})$ at these positions [see Eq. (69)]. The exponent $\mathcal{C}(x', x''; x', x'')$ is given by Eq. (66), and $\mathbf{w}(x', x''; l)$ by Eq. (75). Equation (105) is a linear, inhomogeneous, Fredholm-type integral equation for the spin transport function $A(x)$. The corresponding homogeneous equation is solved by $A(x) \equiv 0$ only, so that the solution $A(x)$ of Eq. (105) for $x_1 < x < x_2$ is linear and homogeneous in $A_1$ and $A_2$. The function $A(x)$ is not, in general, continuous at the interfaces with the contacts, $A(x_{1,1}^+ \neq A_1, A(x_{2,1}^- \neq A_2)$, as a consequence of analogous discontinuities of the spin-resolved chemical potentials $\mu_{1,1}(x)$ [“Sharvin effect”]. In general, Eq. (105) can be solved numerically by using matrix methods after discretization.

The off-equilibrium current spin polarization is given by

$$\mathbf{P}_J(x) = \frac{\mathbf{J}^\text{\(\perp\)}(x)}{J(x)},$$  

(108)

and similarly, the off-equilibrium density spin polarization, by

$$\mathbf{P}_n(x) = \frac{\mathbf{n}^\text{\(\perp\)}(x)}{n(x)} .$$  

(109)

Here again, as in the case of the spin-relaxed polarizations, we calculate the polarizations in terms of thermoballistic currents and densities, respectively.

These off-equilibrium polarizations are determined, via Eq. (77), by the current $\mathbf{J}^\text{\(\perp\)}(x', x''; x)$ and density $\mathbf{n}^\text{\(\perp\)}(x', x''; x)$, Eqs. (73) and (76), respectively, which are linearly connected with the spin transport function $A(x)$, and therefore, linear in the boundary values $A_{1,2}$.

$$\frac{\mathbf{J}^\text{\(\perp\)}(x)}{\sigma_{\text{th}} N_c} = F_1(x) A_1 + F_2(x) A_2 ,$$  

(110)

with “formfactors” $F_1(x)$ and $F_2(x)$.

It remains to determine the quantities $A_{1,2}$, or rather $\alpha_{1,2} = \alpha_{\text{\(\perp\)}(x_{1,2})}$ [see Eq. (69)]. We do this by making use of the continuity of the total current spin polarization

$$\mathbf{P}_J(x) = \mathbf{P}_J(x) + \mathbf{P}_J(x) ,$$  

(111)

closely following Sec. IV of Ref. 34. At the interfaces with the semiconductor at $x = x_1$ and $x = x_2$, respectively, the current polarizations $P_J(x_{1,2})$ in the left and right (semi-infinite) external contacts, which we take as nonmagnetic metals, are found as

$$P_J(x_{1,2}) = + \frac{G_{1,2}}{2\beta e J} \ln \left( \frac{1 + \frac{1}{2} [1 - P_{1,2} \alpha_{1,2}]}{1 - \frac{1}{2} [1 + P_{1,2} \alpha_{1,2}]} \right) ;$$  

(112)

here,

$$G_{1,2} = \frac{\sigma_{1,2}}{L_{1,2}} \ ,$$  

(113)
where \( \sigma_{1,2} \) are the conductivities of the two contacts, \( L^{(1,2)}_s \) the spin diffusion lengths, and \( P_{1,2} = P(x_{1,2}) \). We note that Eq. (112) differs from Eqs. (4.7) and (4.8) of Ref. 34 with respect to the role of the polarizations \( P_{1,2} \). In the latter equations they refer to the ferromagnetic leads (note also that the normalization of the quantities \( \alpha_{1,2} \) used here differs from that used in Ref. 34). The corresponding values of the polarization in the semiconductor are given by

\[
P_J(x_{1,2}) = \tilde{P}_J(x_{1,2}) + \frac{v_\ell N_c}{\kappa_{1,2} J} [F_1(x_{1,2}) A_1 + F_2(x_{1,2}) A_2],
\]

with \( \kappa_{1,2} = J(x_{1,2})/J \). Equating the right-hand sides of Eqs. (112) and (114), we obtain a pair of nonlinear equations which determine the quantities \( \alpha_{1,2} = A_{1,2} e^{-\beta \tilde{P}(x_{1,2})} \), and thereby, the off-equilibrium current spin polarization \( \tilde{P}_J(x) \), in terms of the equilibrium polarization \( P_J(x) \), i.e., of the Zeeman splitting \( \Delta(x) \), and the material parameters of the metal contacts and the semiconductor.

In the zero-bias limit, we have \( |\alpha_{1,2}| \ll 1 \), and the pair of equations for \( \alpha_{1,2} \) become linear,

\[
\pm \frac{G_{1,2}}{2\beta e^2} \alpha_{1,2} - \frac{v_\ell \tilde{n}(x_{1,2})}{\kappa_{1,2}} [F_1(x_{1,2}) \alpha_1 + F_2(x_{1,2}) \alpha_2] = \tilde{P}_J(x_{1,2}) J .
\]

As solutions of Eq. (115), the quantities \( \alpha_{1,2} \) are proportional to the current \( J \), so that the current spin polarization \( P_J(x) \) is independent of \( J \).

Summing up, we obtain the current spin polarization along the entire heterostructure, \( P_J(x) \), as follows. In the nonmagnetic contacts, it decays exponentially away from the interfaces at \( x_{1,2} \) with decay length \( L^{(1,2)}_s \),

\[
P_J(x) = \frac{G_{1,2}}{2\beta e^2 J} \mu_-(x_{1,2}) e^{-|x_{1,2}-x|/L^{(1,2)}_s}
\]

[see Eqs. (4.3) and (4.5) of Ref. 34 with \( P_{1,2} = 0 \)]. In the semiconductor, it is given by expression (111), with the formfactors \( F_{1,2}(x) \) determined by the solution of the integral equation (110). The explicit form of the latter depends on the structure of the semiconductor.

\[ \text{D. Magnetoresistance} \]

Once the quantities \( \alpha_{1,2} \) are determined, we can obtain the magnetoresistance \( R = [V/eJ]_{J=0} \), where \( V \) is the voltage bias related to the mean chemical potentials \( \bar{\mu}_{1,2} = \bar{\mu}(x_{1,2}) \) at the contact side of the contact-semiconductor interfaces,

\[
V = \frac{1}{e} (\bar{\mu}_1 - \bar{\mu}_2)
\]

(here, we assume the metal contacts to have infinitely high conductivity, so that there is no voltage drop inside the contacts). From Eq. (25), we have

\[
\bar{\mu}_{1,2} = \bar{\mu}_{1,2} + \frac{1}{2\beta} \ln(\alpha_\uparrow(x_{1,2})\alpha_\downarrow(x_{1,2})) ,
\]

which in the zero-bias case reduces to

\[
\bar{\mu}_{1,2} = \bar{\mu}_{1,2} - \frac{1}{2\beta} P_{1,2} \alpha_{1,2} .
\]

Subtracting the two equations (119) from one another, we obtain the magnetoresistance as

\[
R = \tilde{R} + \dot{R} ,
\]

where the equilibrium contribution \( \tilde{R} \) is given by Eq. (112), and

\[
\dot{R} = -\frac{1}{2\beta e^2 J} (P_1 \alpha_1 - P_2 \alpha_2 )
\]

is the off-equilibrium contribution.
V. DMS/NMS/DMS HETEROSTRUCTURES: THEORY

As an illustrative example, we now consider the thermoballistic description of spin-polarized electron transport in a heterostructure formed of an NMS layer sandwiched between two DMS layers. As a structure of this kind is composed entirely of semiconducting material, one should ideally treat it as a single sample, so that the ballistic intervals \([x', x'']\) may enclose one or both of the DMS/NMS interfaces. In this case, one has to consider carefully the effect of the interfaces on the electron motion. The interfaces are characterized by (i) abrupt changes in the material parameters, reflecting changes in impurity and phonon scattering and in magnetic scattering, (ii) high structural disorder in their vicinity, and (iii) an abrupt change in the electrostatic potential due to the change in spin splitting. Taking into account the combined effect of these features in a consistent way is beyond the scope of the present work.

Since, here, we place emphasis on the general formulation of the theory, we adopt a pragmatic point of view: we assume the different layers in a DMS/NMS/DMS heterostructure to be homogeneous and require the interfaces to act as fixed points of thermal equilibrium. That is, we apply the thermoballistic description separately to the different layers (thus dealing with conduction band potentials and spin splittings that are, in general, discontinuous at the interfaces), evaluating for each layer the spin transport function and the current spin polarization for a homogeneous semiconductor, and subsequently connect these functions across the interfaces to obtain the position dependence of the spin polarization as well as the magnetoresistance. In a previous publication\cite{39}, we have applied this concept within a heuristic approach.

A. Homogeneous semiconductor

Inside a homogeneous semiconducting layer, at zero bias, we have a flat spin-independent potential profile, \(E_c(x) \equiv E_c\), and a constant spin splitting, \(\Delta(x) \equiv \Delta\), so that the profiles \(E_{\pm,1}(x)\) are given by

\[
\epsilon_{\pm,1}(x) \equiv E_c \pm \Delta/2
\]

\((x_1 \leq x \leq x_2)\). For the functions \(\vartheta^m_\pm(x', x'')\) and \(\vartheta^m_-(x', x'')\), respectively, we then have from Eq. (88)

\[
\vartheta^m_+(x', x'') \equiv 2e^{-\beta E_c} \cosh(\beta \Delta/2) = \frac{2}{Q} e^{-\beta E_c}
\]

and

\[
\vartheta^m_-(x', x'') \equiv -2e^{-\beta E_c} \sinh(\beta \Delta/2) = \frac{2P}{Q} e^{-\beta E_c},
\]

where \(Q = (1 - P^2)^{1/2}\), and \(P = -\tanh(\beta \Delta/2)\) is the static spin polarization. Now, using Eqs. (78) and (123) in Eq. (88), we easily see from Eqs. (83)–(85) that the resistance functions \(\chi^1_m(x)\) and \(\chi^2_m(x)\) for the homogeneous semiconductor are given by the corresponding functions for \(E_c(x) \equiv \Delta(x) \equiv 0\) [see Eq. (2.68) of Ref. 34], multiplied by \(e^{\beta E_c}Q\). Hence, the function \(\chi^2_m(x)/\chi^1_m\) is independent of \(E_c\) and \(\Delta\). Equations (95) and (96) show that, for given \(\mu_{1,2}\), the spin-relaxed chemical potential \(\tilde{\mu}_m(x)\) is also independent of \(E_c\) and \(\Delta\). Using Eqs. (71) and (123) in Eq. (77), we then find the total thermoballistic current in the semiconductor as

\[
J_h(x) \equiv J_h = \frac{e^{-\beta E_c}}{Q} J_h^{(0)},
\]

where \(J_h^{(0)}\) is the (conserved) total physical current for \(E_c = \Delta = 0\). Similarly, using Eqs. (72) and (124) in Eq. (77), we have for the thermoballistic equilibrium spin-polarized current

\[
\hat{j}^h(x) \equiv \hat{j}_h = \frac{e^{-\beta E_c} P}{Q} J_h^{(0)},
\]

and hence for the zero-bias equilibrium current spin polarization of the homogeneous semiconductor

\[
\hat{P}_j^h(x) = \frac{\hat{j}^h(x)}{J_h(x)} \equiv \frac{\hat{j}_h}{J_h} = P,
\]

thereby retrieving the static spin polarization, as was to be expected.
As to the off-equilibrium current spin polarization, we have from Eq. (46)

\[ \vartheta^m(x', x'') = \frac{e^{-\beta E_c}}{\cosh(\beta \Delta/2)} = Qe^{-\beta E_c}; \]

(128)

this factor drops out from the integral equation (105), and as its solution, the spin transport function \( A(x) \) is seen to be equal to the function for \( E_c = \Delta = 0 \), given by Eq. (3.56) of Ref. [34],

\[ A(x) = C_1e^{-(x-x_1)/L} + C_2e^{-(x-x_2)/L}, \]

(129)

where the generalized spin diffusion length \( L \) is given by

\[ L = \sqrt{\bar{l}l_s} = \frac{L_s}{\sqrt{1 + l/l_s}}, \]

(130)

with

\[ \frac{1}{\bar{l}} = \frac{1}{l} + \frac{1}{l_s}, \]

(131)

and

\[ L_s = \sqrt{\bar{l}l_s} \]

(132)

is the spin diffusion length proper. The coefficients \( C_{1,2} \) are linearly connected with the boundary values \( A_{1,2} \) [see Eqs. (3.57) and (3.58) of Ref. [34]].

Using Eq. (73) with Eq. (128) in Eq. (77), we find for the thermo ballistic off-equilibrium spin-polarized current

\[ j^h_-(x) = Qe^{-\beta E_c} j^h_{-0}(x), \]

(133)

where \( j^h_{-0}(x) \) is given by

\[ j^h_{-0}(x) = -v_e N_c \int dA(x) \frac{dx}{dx} \]

(134)

[see Eq. (4.11) of Ref. [34]]. The factor 2 appearing in the right-hand side of the latter equation reflects the fact that the normalization of \( A(x) \) used there differs from that used in the present paper]. Combining expressions (133) and (125), we obtain the zero-bias off-equilibrium current spin polarization of the homogeneous semiconductor as

\[ \tilde{P}^h_j(x) = \frac{J^h_j(x)}{J^h_0(x)} = Q^2 \frac{J^h_{-0}(x)}{J^h_{-0}(0)}, \]

(135)

and hence

\[ P^h_j(x) = \tilde{P}^h_j(x) + \tilde{R}^h_j(x) = P + Q^2 \frac{J^h_{-0}(x)}{J^h_{-0}(0)} \]

(136)

for the total (equilibrium plus off-equilibrium) current spin polarization.

Turning now to the magnetoresistance of a homogeneous semiconductor, we find from Eq. (97) that the reduced resistance \( \tilde{\chi}_h \) equals that for \( E_c(x) = \Delta(x) = 0 \),

\[ \tilde{\chi}_h = \frac{2l + S}{2l}, \]

(137)

[see Eq. (2.69) of Ref. [34]]. Therefore, we have, from Eq. (102),

\[ \tilde{R}_h = \frac{Q}{G^h_0} \frac{2l + S}{2l} \]

(138)

for the equilibrium resistance, and, from Eq. (121),

\[ \tilde{R}_h = \frac{P}{2\beta e^2 J} (\alpha_1 - \alpha_2) \]

(139)

for the off-equilibrium resistance.
FIG. 2: Schematic energy diagram for a DMS/NMS/DMS heterostructure composed of three homogeneous layers.

B. Current spin polarization in DMS/NMS/DMS heterostructures

For parameterizing a DMS/NMS/DMS heterostructure, we attach labels $j = 1, 2, 3$ to quantities referring to the left DMS layer, the NMS layer, and the right DMS layer, respectively, and denote the positions of the interfaces (including the DMS/contact interfaces) by $x_k (k = 1, 2, 3, 4)$ [see Fig. 2], thereby deviating from the notation used in the main body of the paper. Quantities referring to the left and right contact are labelled "l" and "r", respectively.

The position dependence of the polarization inside the left and right contacts (there labelled 1 and 2) is given by Eq. (116), with $\mu_{-}(x_{1,4}) = \alpha_{1,4}/\beta$. The current spin polarization $P_j(x)$ in the three layers is found in terms of the boundary values of the spin transport function $A(x)$ at the interface positions $x_k$, using Eq. (136) together with Eqs. (129) and (134), separately in each layer. In each of the layers $j$, the function $A(x)$ has the Sharvin discontinuities $\Delta A_j$ and $\Delta A_{j+1}$ at the interface positions $x_j$ and $x_{j+1}$, respectively [see Eqs. (3.61) and (3.62) of Ref. 34]. However, since in the zero-bias limit, $J \to 0$, these discontinuities vanish with $J$, we may regard the spin transport function $A(x)$ as continuous all across the heterostructure. Recalling that in the zero-bias limit the spin-relaxed chemical potential $\tilde{\mu}(x)$ may be taken as constant, $\tilde{\mu}(x) \equiv \tilde{\mu}_0$, it is seen from Eq. (69) that the function $\alpha_{-}(x)$ is also continuous across the entire heterostructure.

In terms of the quantities $\alpha_k = \alpha_{-}(x_k)$ and $A_k = A(x_k) = e^{\beta\tilde{\mu}_0}\alpha_k$, the complete position dependence of the current spin polarization can then be written as

$$P_j(x) = \mp \hat{G}_{l,r} \frac{\nu e N_{1,4}}{2J} e^{-|x_{1,4} - x|/L_{l,r}},$$

if $x < x_1$ and $x > x_4$, respectively, and as

$$P_j(x) = P_j + \frac{\nu e N_{1,4} Q^2}{J_j} \gamma_j [C_{j1} e^{-(x-x_j)/L_j} - C_{j2} e^{-(x_{j+1}-x)/L_j}],$$

if $x_j \leq x \leq x_{j+1}$. Here,

$$\hat{G}_{l,r} = \frac{G_{l,r}}{G_{l,r}^{(0)}},$$

where $G_{l,r}$ is given by Eq. (118), and $G_{l,r}^{(0)}$ is the Sharvin interface conductance (101). For the coefficients $C_{j1}$ and $C_{j2}$, we have

$$C_{j1} = \frac{1}{D_j} [(1 + \gamma_j) e^{S_j/L_j} A_j - (1 - \gamma_j) A_{j+1}]$$

(143)
and
\[
C_{j2} = -\frac{1}{D_j}[(1 - \gamma_j)A_j - (1 + \gamma_j)e^{S_j/L_j}A_{j+1}],
\]
respectively, where
\[
D_j = 2[(1 + \gamma_j^2) \sinh(S_j/L_j) + 2\gamma_j \cosh(S_j/L_j)]
\]
and
\[
\gamma_j = \frac{\bar{l}_j}{L_j} = \frac{L_j}{\bar{l}_j},
\]
with \(L_j\) and \(\bar{l}_j\) defined by Eqs. (130) and (131), respectively, and \(S_j = x_{j+1} - x_j\) is the thickness of layer \(j\).

Conservation of the total current \(J\) across the heterostructure takes the form
\[
J_j = \frac{e^{-\beta E_c^{(j)}}}{Q_j} J_j^{(0)} = J ; \ j = 1, 2, 3.
\]

We now require the current spin polarization \(P_j(x)\) to be continuous at all interfaces, setting \(P_2 = 0\) (so that \(Q_2 = 1\)). This leads to the following set of four coupled linear equations for the quantities \(\alpha_k\),
\[
(G_l + Q_1 g_1)\alpha_1 - Q_1 h_1 \alpha_2 = -\frac{2J}{v_e n_1^{(0)}} P_1,
\]
\[
- Q_1 h_1 \alpha_1 + (Q_1 g_1 + e^{\delta_{12}} g_2) \alpha_2 - e^{\delta_{12}} h_2 \alpha_3 = \frac{2J}{v_e n_1^{(0)}} P_1,
\]
\[
- e^{\delta_{12}} h_2 \alpha_2 + (e^{\delta_{12}} g_2 + Q_3 g_3) \alpha_3 - Q_3 h_3 \alpha_4 = -\frac{2J}{v_e n_4^{(0)}} P_3,
\]
\[
- Q_3 h_3 \alpha_3 + (G_r + Q_3 g_3) \alpha_4 = \frac{2J}{v_e n_4^{(0)}} P_3.
\]

Here,
\[
g_j = h_j \left[ \cosh(S_j/L_j) + \gamma_j \sinh(S_j/L_j) \right]
\]
and
\[
h_j = \frac{4\gamma_j}{D_j},
\]
and we have introduced the band offsets
\[
\delta_{jj'} = E_c^{(j)} - E_c^{(j')}.
\]

The system of equations (148)-(151) is easily solved, so that the complete position dependence of the current spin polarization is obtained in explicit form.

Specializing to the case of a symmetric heterostructure, where the parameters of the right DMS layer and contact are identical to those of the left DMS layer and contact, and assuming infinitely high conductivity of the metal contacts, \(\sigma_{l,r} \to \infty\), so that \(G_{l,r} \to \infty\), we find
\[
\alpha_1 = \alpha_4 = 0,
\]
\[
\alpha_2 = -\alpha_3 = \frac{2J}{v_e n_D^{(0)}} \frac{P}{QgD + (g_N + h_N)e^{\delta_{DN}}},
\]
\[ \dot{G}_1 \alpha_1 = -\dot{G}_4 \alpha_4 = -\frac{2J}{v_s n_D^{(0)}} P + Q h_D \alpha_2. \] (157)

Here, the DMS parameters have been labelled by "D", the NMS parameters by "N", and \( P = P_1 = P_3 \).

Thus, in the symmetric case, the current spin polarization \( P_J(x) \) is completely determined by the quantity \( \alpha_2 \). Explicitly, we have, setting \( L_s^{(c)} = L_s^{(l)} = L_s^{(r)} \),

\[ P_J(x) = P \left[ 1 - \frac{Q h_D}{Q g_D + (g_N + h_N) e^{\beta_D} e^{\beta_{DN}}} \right] e^{-\frac{|x-x_{1,4}|}{L_s^{(c)}}}, \] (158)

if \( x < x_1 \) and \( x > x_4 \), respectively, while

\[ P_J(x) = P \left\{ 1 - \frac{Q h_D}{Q g_D + (g_N + h_N) e^{\beta_D} e^{\beta_{DN}}} \left[ \cosh\left(\frac{|x-x_{1,4}|}{L_D}\right) + \gamma_D \sinh\left(\frac{|x-x_{1,4}|}{L_D}\right) \right] \right\}, \] (159)

if \( x_1 \leq x \leq x_2 \) and \( x_3 \leq x \leq x_4 \), respectively, and

\[ P_J(x) = \frac{2 P h_N}{Q g_D e^{-\beta_{DN}} + g_N + h_N} \left[ \cosh\left(S_N/2L_N\right) + \gamma_N \sinh\left(S_N/2L_N\right) \right] \cosh\left(\frac{|x-(x_2+x_3)/2|}{L_N}\right), \] (160)

if \( x_2 \leq x \leq x_3 \). Note that, as a consequence of our assumption of infinitely high conductivity of the contacts, the polarization \( P_J(x) \) is independent of the effective electron mass \( m^* \) and of the densities \( n_D^{(0)}, n_N^{(0)} \).

C. Magnetoresistance of DMS/NMS/DMS heterostructures

The total magnetoresistance of a DMS/NMS/DMS heterostructure, \( R = \tilde{R} + \dot{R} \), is obtained by adding the equilibrium, \( \tilde{R} \), and off-equilibrium, \( \dot{R} \), contributions corresponding to the three layers [see Eqs. (138) and (139)],

\[ \tilde{R} = \sum_{j=1}^{3} \frac{Q_j}{\dot{g}_j^{(0)}} \frac{2l_j + S_j}{2l_j} \] (161)

and

\[ \dot{R} = -\frac{1}{2\beta e^2 J} [P_1(\alpha_1 - \alpha_2) + P_3(\alpha_3 - \alpha_4)]. \] (162)

For a symmetric structure and infinite conductivity in the contacts, we have

\[ \tilde{R} = \frac{2Q}{\dot{g}_D^{(0)}} \frac{2l_D + S_D}{2l_D} + \frac{1}{\dot{g}_N^{(0)}} \frac{2l_N + S_N}{2l_N} \] (163)

and, using Eqs. (155) and (156),

\[ \dot{R} = \frac{P \alpha_2}{\beta e^2 J} = \frac{1}{\dot{g}_D^{(0)}} \frac{2P^2}{Q g_D + (g_N + h_N) e^{\beta_D} e^{\beta_{DN}}}. \] (164)

We then have

\[ \frac{\tilde{R}}{\dot{R}} = \frac{2P^2}{Q g_D + (g_N + h_N) e^{\beta_D} e^{\beta_{DN}}} \frac{1}{2Q(2l_D + S_D)/2l_D + e^{-\beta_{DN}}(2l_N + S_N)/2l_N} \] (165)

for the relative magnetoresistance \( \tilde{R}/\dot{R} \equiv (R - \tilde{R})/\dot{R} \). As we have assumed \( l_D^{(N)} \) and \( l_s^{(D,N)} \) to be independent of the external magnetic field, expression (165) reflects solely the "spin accumulation part" of the magnetoresistance. While \( \tilde{R} \) and \( \dot{R} \) depend on the effective electron mass \( m^* \) and the densities \( n_D^{(0)}, n_N^{(0)} \) via the Sharvin interface conductances \( \dot{g}_D^{(0)}, \dot{g}_N^{(0)} \), the relative magnetoresistance \( \tilde{R}/\dot{R} \) is independent of \( m^* \) and \( n_D^{(0)}, n_N^{(0)} \).
D. Diffusive limit

It is instructive to consider in some detail the diffusive limit of the thermoballistic description of spin-polarized transport in DMS/NMS/DMS heterostructures as it allows comparison to the results of Ref. 26 obtained within the standard drift-diffusion approach to electron transport.

In the diffusive limit, where \( l_j \ll S_j \) and \( l_j \ll \lambda^{(j)}_S \), we have \( L_j \rightarrow L^{(j)}_S = \sqrt{l_j \lambda^{(j)}_S} \) and \( \gamma_j \rightarrow l_j/L^{(j)}_S = \sqrt{l_j/\lambda^{(j)}_S} \ll 1 \) [see Eqs. (130)–(132) and (146)], and hence from Eqs. (152) and (153)

\[
g_j = h_j \cosh(S_j/L^{(j)}_S) \tag{166}
\]

and

\[
h_j = \frac{2l_j}{L^{(j)}_S} \frac{1}{\sinh(S_j/L^{(j)}_S)}, \tag{167}
\]

respectively. Equation (166) for the symmetric heterostructure then reduces to

\[
\alpha_2 = \frac{2\beta e^2 J P}{Q^2 G_D \coth(S_D/L^{(D)}_S) + G_N \coth(S_N/2L^{(N)}_S) e^{\delta_{DN}}} , \tag{168}
\]

where

\[
G_j = \frac{\sigma_j}{L^{(j)}_S} \tag{169}
\]

\((j = D, N)\), and \(\sigma_j\) is the (spin-summed) conductivity,

\[
\sigma_j = 2\beta e^2 v_\alpha n_j l_j = \frac{2}{Q_j} \hat{G}^{(0)}_j l_j . \tag{170}
\]

Now, using Eq. (168) with \(\delta_{DN} = 0\) in Eq. (141), we find that the values of the current spin polarization \(P_j(x)\) at \(x_1, x_2,\) and \((x_2 + x_3)/2\), respectively, agree with those given by Eqs. (16)–(18) of Ref. 26, if we identify in the latter equations the spin-flip lengths \(\lambda_D\) and \(\lambda_N\) with \(L^{(D)}_S\) and \(L^{(N)}_S\), respectively, and the layer thicknesses \(d\) and \(2x_0\) with \(S_D\) and \(S_N\), respectively.

Similarly, from Eq. (168) and from Eq. (164), using Eq. (168), we find the equilibrium contribution to the magnetoresistance, \(\tilde{R}\), and the off-equilibrium contribution, \(\hat{R}\), respectively, to agree with the corresponding results of Ref. 26, viz., the first and second term on the right-hand side of Eq. (10) of that reference. Then, trivially, we find agreement of the relative magnetoresistance \(\tilde{R}/\hat{R}\) given by Eq. (165) with the first term on the right-hand side of Eq. (13) of Ref. 26.

E. Ballistic limit

In the ballistic limit, where \(l_j \gg S_j \) and \( l_j \gg \lambda^{(j)}_S \), we have \( L_j \rightarrow l^{(j)}_S \) and \( \gamma_j \rightarrow 1 \) [see Eqs. (130)–(132) and (146)], so that \( g_j = 1 \) and \( h_j = e^{-S_j/l^{(j)}_S} \).

For the symmetric heterostructure, we then have from Eq. (156)

\[
\alpha_2 = \frac{2J}{n^{(0)}_N} \frac{P}{Q e^{-\delta_{DN}} + 1 + e^{-S_N/l^{(N)}_S}} . \tag{171}
\]

Using this expression for \(\alpha_2\), we obtain the ballistic limit for the current spin polarization and the magnetoresistance. The equilibrium contribution to the magnetoresistance, in particular, follows from Eq. (163) in the form

\[
\tilde{R} = 2 \frac{Q}{\hat{G}^{(0)}_D} + \frac{1}{\hat{G}^{(0)}_N} \tag{172}
\]

in which the Sharvin interface resistances \(1/\hat{G}^{(0)}_{D,N}\) characterize, via their dependence on the constant densities \(n^{(0)}_{D,N}\), the (homogeneous) DMS and NMS layers. Notably, owing to the presence of the factor \(Q\), the contribution to \(\tilde{R}\) from the DMS layers vanishes when \(P \rightarrow 1\). If we had not assumed the interfaces at \(x = x_{2,3}\) to be fixed points of thermal equilibrium, the ballistic equilibrium magnetoresistance would be given in terms of the Sharvin interface resistances at the end-points \(x_{1,4}\) [see Eq. (2.51) of Ref. 34 and of the potential barriers provided by the profiles (1)].
FIG. 3: Relative zero-bias current spin polarization \( P_J(x)/P \) across a symmetric DMS/NMS/DMS heterostructure enclosed between metal contacts with infinitely high conductivity, calculated from Eqs. (158)–(160) for zero band offset, \( \delta = 0 \). The different curves correspond to different values of the momentum relaxation length \( l = l_D = l_N \). The remaining parameter values are: \( S_D = 1 \, \mu \text{m}, S_N = 2 \, \mu \text{m}, l_s^{(D)} = l_s^{(N)} = 2.5 \, \mu \text{m}, P = 0.8, L_c^{(c)} = 60 \, \text{nm} \).

VI. DMS/NMS/DMS HETEROSTRUCTURES: APPLICATION

In the thermoballistic description of spin-polarized electron transport in DMS/NMS/DMS heterostructures, the momentum and ballistic spin relaxation lengths, \( l \) and \( l_s \), respectively, are the fundamental dynamical quantities. These, apart from the potential profiles \( E^{(D,N)}_c \) and the geometric dimensions \( S_{D,N} \), determine the current spin polarization and the magnetoresistance of the structure as a function of the Zeeman splitting \( \Delta \), i.e., of the strength of the external magnetic field. Here, we illustrate the dependence of polarization and magnetoresistance on \( l \) by specific numerical examples. (We recall that, when varying \( l \), we understand the different values to represent a class of semiconductors which have similar material properties, but differ in the strength of impurity and phonon scattering.) Furthermore, we point out the possibility of determining (experimentally) the quantities \( l \) and \( l_s \) from their effect on the (experimentally accessible) spin polarization and magnetoresistance.

In Fig. 3 the relative current spin polarization, i.e., the ratio of current spin polarization and static DMS polarization, \( P_J(x)/P \), across a symmetric DMS/NMS/DMS heterostructure at zero bias is shown for different values of the momentum relaxation length \( l = l_D = l_N \) at a fixed value of the ballistic spin relaxation length \( l_s = l_s^{(D)} = l_s^{(N)} \). The behavior of \( P_J(x)/P \) exhibits two features. First, the injected spin polarization, i.e., the value of the polarization at the DMS/NMS interfaces, remains on a high, weakly \( l \)-dependent level ranging between 0.6 and 0.7 when \( l \) varies by three orders of magnitude from the diffusive to the ballistic regime. This behavior contrasts with that calculated within the thermoballistic treatment of spin injection at the interfaces of the metallic ferromagnets and the semiconductor in a ferromagnet/NMS/ferromagnet structure\(^{34} \). The results for the latter case are shown in Fig. 5 of Ref. \(^{34} \). These have been obtained with an \( l_s \)-value equal to that used in Fig. 3 of the present paper, and with the bulk polarization of the ferromagnets equal to the static DMS polarization. The injected polarization at the ferromagnet/semiconductor interfaces (for zero interface resistance) is seen to be very small in the diffusive limit, and rapidly rising with increasing \( l \). The strong dependence on \( l \) reflects the conductivity mismatch\(^{6} \) between metallic ferromagnets and semiconductor, which is very large in the diffusive (low-\( l \)) regime where the conductivity of the semiconductor vanishes with \( l \). No such mismatch occurs at the
DMS/NMS interfaces in a DMS/NMS/DMS heterostructure.

Second, the behavior of the polarization inside the NMS layer is governed by the \( l \)-dependence of the generalized spin diffusion length \( L = [l_s/(1 + l/l_s)]^{1/2} \). In the diffusive limit, where \( L \approx \sqrt{l_s} \), the polarization dies out rapidly; in the ballistic limit, we have \( L \approx l_s \), i.e., the decay is determined solely by the ballistic spin relaxation length. Qualitatively, the behavior of the polarization inside the NMS layer does not depend on the kind of spin injector (ferromagnet or DMS). However, due to the larger injected polarization, the magnitude of the polarization inside the NMS layer is larger for the DMS injector.

In Fig. 4 (solid curves), the relative magnetoresistance \( \Delta R/\bar{R} \) of a symmetric DMS/NMS/DMS heterostructure is shown as a function of the static DMS polarization \( P \) for the parameter values of Fig. 3. For \( P \) not too close to unity, the qualitative behavior of the curves is determined by the overall factor \( P^2 \) in Eq. (165). When \( P \) approaches unity, the curves rise sharply owing to the \( P \)-dependence of the terms proportional to \( Q \) in the denominator of the expression for \( \Delta R/\bar{R} \). As a function of \( l \), \( \Delta R/\bar{R} \) rises, over the full \( P \)-range, by about one order of magnitude when \( l \) varies from the diffusive to the ballistic regime.

In the example of Fig. 4, \( \Delta R/\bar{R} \) tends to increase with decreasing thickness \( S_D \) of the DMS layers, as is seen by comparing the solid and long-dashed curves for \( l = 10 \) nm (the magnitude of this increase becomes progressively smaller with increasing \( l \)). In the diffusive calculation of Ref. 26, the relative magnetoresistance was found, for a specific parameter choice, to increase with increasing \( S_D \). We have reproduced this behavior quantitatively by choosing the parameter values in Eq. (165) so as to conform to those underlying the results shown in Fig. 3b of Ref. 26. In particular, we took \( l_D = 10 \) nm and used Eqs. (132) and (170), respectively, to relate the ballistic spin relaxation lengths \( l_s^{(D,N)} \) to the spin diffusion lengths \( \lambda_{D,N} \) of Ref. 26 and the ratio \( l_D/l_N \) to the ratio \( \sigma_D/\sigma_N \). A detailed study of the full parameter dependence of the magnetoresistance is clearly desirable.

For spin-polarized electron transport in DMS/NMS/DMS heterostructures, the thermoballistic description in the simplified form presented in this paper leads to transparent explicit expressions for the current spin polarization [Eqs. (158)–(160)] and for the magnetoresistance [Eqs. (163) and (164)]. If these turn out to reproduce, albeit approximately, the results of actual measurements for different values of \( P \) and various choices of \( S_D \) and \( S_N \), one should be able to determine the values of the parameters \( l \) and \( l_s \) for magnetic and nonmagnetic materials from experiment in a quite direct manner.
VII. SUMMARY AND CONCLUSIONS

We have presented the systematic extension of the thermoballistic description of spin-polarized electron transport in semiconductors to the case of a spin-split conduction band, allowing us to envisage applications to spin-polarized transport in paramagnetic DMS and in DMS/NMS/DMS heterostructures.

Assuming arbitrarily shaped potential profiles exhibiting arbitrary, position-dependent spin splitting, we have constructed the thermoballistic currents and densities by starting from the spin-resolved densities at points of local thermal equilibrium, at which electron currents across ballistic transport intervals are activated. These currents are subject to spin relaxation. Dividing the ballistic currents and densities into their equilibrium (spin-relaxed) and off-equilibrium parts, we have obtained a description of the equilibrium parts in terms of the spin-relaxed chemical potential, and of the off-equilibrium parts in terms of a spin transport function that is related to the splitting of the spin-resolved chemical potentials. From the ballistic currents and densities, the corresponding thermoballistic quantities are constructed by weighted summation over all ballistic intervals. The procedures for calculating the spin-relaxed chemical potential and the spin transport function are outlined.

The thermoballistic description of spin-polarized electron transport has been applied to the calculation of the current spin polarization and magnetoresistance in DMS/NMS/DMS heterostructures. The results have been compared to those of the customary drift-diffusion approach. The important role of the fundamental momentum and spin relaxation lengths as well as the possibility of their experimental determination are pointed out.

The emphasis in the present paper has been on the systematic development of the formalism. In future work, this formalism will have to be implemented in detail. To this end, a careful study dealing with the role of the interfaces in electron transport in heterostructures, as well as their adequate modelling within the thermoballistic description, is required. Furthermore, efficient algorithms for the solution of the integral equations for the resistance function and the spin transport function are to be developed. Then, by performing exploratory calculations sampling the full parameter space, one may be able to identify novel features and trends in spin-polarized electron transport in paramagnetic semiconductors.

\* Electronic address: wille@helmholtz-berlin.de

References:

1. G. Schmidt and L. W. Molenkamp, Semicond. Sci. Technol. \textbf{17}, 310 (2002).
2. I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. \textbf{76}, 323 (2004).
3. G. Schmidt, J. Phys. D: Appl. Phys. \textbf{38}, R107 (2005).
4. Spin Physics in Semiconductors, edited by M. I. Dyakonov (Springer, Berlin, 2008).
5. G. Schmidt, D. Ferrand, L. W. Molenkamp, A. T. Filip, and B. J. van Wees, Phys. Rev. B \textbf{62}, R4790 (2000).
6. A. T. Filip, B. H. Hoving, F. J. Jedema, B. J. van Wees, B. Dutta, and S. Borghs, Phys. Rev. B \textbf{62}, 9996 (2000).
7. E. I. Rashba, Phys. Rev. B \textbf{62}, R16267 (2000).
8. D. L. Smith and R. N. Silver, Phys. Rev. B \textbf{64}, 045323 (2001).
9. A. Fert and H. Jaffrès, Phys. Rev. B \textbf{64}, 184420 (2001).
10. G. Schmidt and L. W. Molenkamp, Semicond. Sci. Technol. \textbf{17}, 310 (2002).
11. Z. G. Yu and M. E. Flatté, Phys. Rev. B \textbf{66}, 045323 (2001).
12. Z. G. Yu and M. E. Flatté, Phys. Rev. B \textbf{66}, 235302 (2002).
13. J. D. Albrecht and D. L. Smith, Phys. Rev. B \textbf{66}, 113303 (2002).
14. J. D. Albrecht and D. L. Smith, Phys. Rev. B \textbf{68}, 035340 (2003).
15. V. Ya. Kravchenko and E. I. Rashba, Phys. Rev. B \textbf{67}, 121310(R) (2003).
16. W. Van Roy, P. Van Dorpe, J. De Boeck, and G. Borghs, Mater. Sci. Eng. B \textbf{126}, 155 (2006).
17. M. Oestreich, J. Hübner, D. Hägele, P. J. Klar, W. Heimbroidt, W. W. Rühle, D. E. Ashenford, and B. Lunn, Appl. Phys. Lett. \textbf{74}, 1251 (1999).
18. B. T. Jonker, Y. D. Park, B. R. Bennett, H. D. Cheong, G. Kioseoglou, and A. Petrou, Phys. Rev. B \textbf{62}, 8180 (2000).
19. J. K. Furddy, J. Appl. Phys. \textbf{64}, R29 (1988).
20. T. Dietl, in: Handbook on Semiconductors, edited by T. S. Moss (North-Holland, Amsterdam, 1994), Vol. 3b, p. 1251.
21. J. Cibert and D. Scalbert, in Ref.\textsuperscript{8}, chap. 13.
22. G. Schmidt, G. Richter, P. Grabs, C. Gould, D. Ferrand, and L. W. Molenkamp, Phys. Rev. Lett. \textbf{87}, 227203 (2001).
23. G. Schmidt, C. Gould, P. Grabs, A. M. Lunde, G. Richter, A. Slobodskyy, and L. W. Molenkamp, Phys. Rev. Lett. \textbf{92}, 226602 (2004).
24. A. Khaetskii, J. C. Egues, D. Loss, C. Gould, G. Schmidt, and L. W. Molenkamp, Phys. Rev. B \textbf{71}, 235327 (2005).
25. J. C. Egues, Phys. Rev. Lett. \textbf{80}, 4578 (1998).
28 Y. Guo, H. Wang, B. L. Gu, and Y. Kawazoe, J. Appl. Phys. 88, 6614 (2000).
29 J. C. Egues, C. Gould, G. Richter, and L. W. Molenkamp, Phys. Rev. B 64, 195319 (2001).
30 K. Chang and F. M. Peeters, Solid State Commun. 120, 181 (2001).
31 D. Sánchez, C. Gould, G. Schmidt, and L. W. Molenkamp, IEEE Trans. Electron Devices 54, 984 (2007).
32 A. Slobodskyy, C. Gould, T. Slobodskyy, G. Schmidt, L. W. Molenkamp, and D. Sánchez, Appl. Phys. Lett. 90, 122109 (2007).
33 R. Lipperheide and U. Wille, Phys. Rev. B 68, 115315 (2003).
34 R. Lipperheide and U. Wille, Phys. Rev. B 72, 165322 (2005).
35 R. Lipperheide and U. Wille, Mater. Sci. Eng. B 126, 245 (2006).
36 M. I. D’yakonov and V. I. Perel’, Fiz. Tverd. Tela (Leningrad) 13, 3581 (1971) [Sov. Phys. Solid State 13, 3023 (1971)].
37 M. I. Dyakonov, in Ref. [28], chap. 1.
38 Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. 48, 984 (1965) [Sov. Phys. JETP 21, 655 (1965)].
39 R. Lipperheide and U. Wille, AIP Conf. Proc. 893, 1279 (2007).