Lepton pair photoproduction in peripheral relativistic heavy-ion collisions

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Abstract

We study the lepton pair photoproduction in peripheral heavy-ion collisions based on the formalism in our previous work [1]. We present the numerical results for the distributions of the transverse momentum, azimuthal angle and invariant mass for $e^+e^−$ and $\mu^+\mu^−$ pairs as functions of the impact parameter and other kinematic variables in Au+Au collisions. Our calculation incorporates the information on the transverse momentum and polarization of photons which is essential to describe the experimental data. We observe a broadening effect in the transverse momentum for lepton pairs with and without smear effects. We also observe a significant enhancement in the distribution of $\cos(2\varphi)$ for $\mu^+\mu^−$ pairs. Our results provide a baseline for future studies of other higher order corrections beyond Born approximation and medium effects in the lepton pair production.

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I. INTRODUCTION

Extremely strong electromagnetic fields are generated in relativistic heavy-ion collisions [2–15] and induce many novel quantum transport phenomena such as the chiral magnetic effect, the chiral separation effect [2, 16, 17], the chiral electric separation effect [18–20], and other nonlinear chiral effects [21–25]. For more discussions of these chiral effects, we refer readers to recent reviews [26–34] and references therein. Meanwhile, some quantum electrodynamics (QED) effects have also been widely discussed, such as the light-by-light scattering [35], matter generation directly from photons [36], the vacuum birefringence [36–39] and the Schwinger mechanism [40–43].

According to the equivalent photon approximation (EPA) or the Weizsaecker-Williams method [44, 45], strong electromagnetic fields generated by a fast moving nucleus can be treated as the quasi-real photons. Recently, the lepton pair production through the fusion of two quasi-real photons, the named Breit-Wheeler process [46], has been measured in several experiments of relativistic heavy-ion collisions. The STAR Collaboration at Relativistic heavy-ion Collider (RHIC) has measured the Breit-Wheeler process in ultra-peripheral collisions (UPC) [36, 47] and peripheral collisions [48]. A significant enhancement of the lepton pair production at low transverse momenta of dileptons \( P_T < 0.15 \text{ GeV} \) in peripheral collisions has been measured in the STAR experiment [48] in comparison with the hadronic cocktails. The azimuthal asymmetry of the lepton pair originated from the linear polarization of incoming photons has also been observed by STAR [36] in connection with the vacuum birefringence phenomena [49]. Furthermore the broadening effects are seen in peripheral collisions by STAR [48] at RHIC, ATLAS [50] and CMS at the Large Hadron Collider [51].

We note that the EPA fails to describe experimental data [48, 52] due to missing the essential information of the transverse momentum and polarization of photons. To better describe the data, several theoretical methods have been proposed. These include a generalized EPA (gEPA) or a QED-based method in the background field approach [53–59], the transverse momentum dependent (TMD) parton distribution functions [60, 61], the factorization formalism with the photon Wigner functions [62, 63], and the wave-packet method based on the classical field approximation in QED [1].

Although these theoretical methods can account for most experimental data, the broadening effect for the transverse momentum of the dilepton pair in peripheral collisions has not
been fully understood. There are two extra sources to this effect, the higher order correction beyond the Born approximation and the medium effect. The Sudakov factor as a sum over the soft photon emission is a typical higher order correction beyond the Born approximation [60–62, 64, 65]. There are other effects from the medium, such as the effect from the Lorentz force [48], multiple scatterings of lepton pairs in the medium [50, 62, 64], that may also lead to the broadening of transverse momenta.

Meanwhile, although the azimuthal angle distribution of $e^+e^-$ pairs has been predicted and confirmed in experiments [60, 61], a systematic study of other lepton species such as $\mu^+\mu^-$ pairs is still needed for future experiments [66].

In this paper, we extend our previous study of the UPC [1] to peripheral collisions. In our approach, we will calculate the broadening effect and the azimuthal angle distribution for $e^+e^-$ and $\mu^+\mu^-$ pairs for different centralities in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. We expect to observe the enhancement of the $\cos(2\varphi)$ modulation in $\mu^+\mu^-$ pairs due to the mass effect as proposed in Ref. [60, 61]. Similar azimuthal asymmetry have also found in vector meson photoproduction [67–70]. For the broadening effect, we will consider a smearing of the lepton momentum from the finite resolution of the measured momentum and multiple scatterings in the detector as well as from the bremsstrahlung radiation of electrons. All these effects have been usually considered in experiments. In this work, as a first attempt, we add the smearing effect extracted from the measurement of $J/\psi \rightarrow l\bar{l}$ by STAR [71] to the transverse momentum distribution.

The paper is organized as follows. In Sec. II, we give a brief overview on our previous work [1]. After introducing parameters used in our calculation in Sec. IIIA, we compute the transverse momentum broadening effect in Sec. IIIB, the azimuthal angle distributions for $e^+e^-$ and $\mu^+\mu^-$ pairs in Sec. IIIC, and the invariant mass distribution in Sec. IIID. We make a summary of our results in Sec. IV. Throughout this paper, we use the sign convention for the metric tensor $g_{\mu\nu} = \text{diag}\{+,-,-,-\}$.

II. CROSS SECTIONS FOR LEPTON PAIR PHOTOPRODUCTION

In this section, we give a brief overview on differential cross sections for lepton pair photoproduction at Born level derived in our previous work [1].

We consider head-on collisions of two identical nuclei $A_1$ and $A_2$ moving in $\pm z$ direction
in which a pair of leptons (\( l \) and \( \bar{l} \)) are generated accompanied by other particles \( X_1, \cdots, X_f \). The process can be expressed by

\[
A_1(P_1) + A_2(P_2) \rightarrow l(k_1) + \bar{l}(k_2) + \sum_f X_f(K_f),
\]

where the particles’ four-momenta are given in parentheses. The on-shell momenta of two nuclei are denoted as \( P_i^\mu = (E_{P_i}, P_i) = M u_i^\mu \) for \( i = 1, 2 \), where \( E_{P_i} = \sqrt{P_i^2 + M^2} \), \( u_i^\mu = \gamma (1, 0, 0, \pm v) \) with \( \gamma = 1/\sqrt{1 - v^2} \), and \( M \) are the energies, four-velocities and mass of two nuclei. The three-momenta of two nuclei are \( P_1 = (0, 0, P_1^z) \) and \( P_2 = (0, 0, -P_2^z) \) in the center of mass frame of the collision. We consider the photon fusion as the sub-process of the collision

\[
\gamma(p_1) + \gamma(p_2) \rightarrow l(k_1) + \bar{l}(k_2).
\]

Note that each photon does not need to be emitted from the center of a nucleus.

In our previous work [1], we employ the narrow wave-packet to describe two colliding nuclei. The differential cross section for the photoproduction of lepton pairs can be put into a compact form

\[
\frac{d\sigma}{d^3k_1 d^3k_2} = \frac{1}{32(2\pi)^6} \frac{1}{v E_{P_1} E_{P_2} E_{k_1} E_{k_2}} \int d^2b_T d^2b_1T d^2b_2T \int d^4p_1 d^4p_2 \times \delta^{(2)}(b_T - b_{1T} + b_{2T}) (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \times S_{\sigma\mu}(p_1, b_{1T}) S_{\rho\nu}(p_2, b_{2T}) \times \sum_{\text{spin of } lT} L^{\mu\nu}(p_1, p_2; k_1, k_2) L^{\rho\sigma}(p_1', p_2'; k_1, k_2),
\]

where we have introduced the transverse position \( b_{iT} \) of the photon emission in the nucleus \( A_i \), \( b_T = b_{1T} - b_{2T} \) is the impact parameter of the colliding nuclei, the lepton tensor \( L^{\mu\nu} \) at the tree or Born level is given by

\[
L^{\mu\nu}(p_1, p_2; k_1, k_2) = -ie^2 \bar{\pi}(k_1) \left[ \gamma^\mu \gamma^\nu \left( \frac{k_1 - p_1}{(k_1 - p_1)^2 - m^2 + i\epsilon} \right) \left( \frac{k_1 - p_1}{(k_1 - p_1)^2 - m^2 + i\epsilon} \right) \right] v(k_2),
\]

and \( S_{\sigma\mu}(p_i, b_{iT}) \) denotes the Born-level Wigner function for photons [62, 63]

\[
S_{\sigma\mu}(p_i, b_{iT}) \equiv \int \frac{d^2\Delta_{IT}}{(2\pi)^2} \int \frac{d^4y_i}{(2\pi)^4} e^{i p_i \cdot y_i} \langle P'_i | A_\sigma^\dagger(0) A_\mu (y_i) | P_i \rangle e^{-ib_{iT} \cdot \Delta_{iT}}.
\]

which encodes the information of transverse phase space for photons and leads to the transverse momentum broadening. In Eq. (5), \( P'_i \equiv (E_{P_i}, P_{iT} + \Delta_{iT}, P_{iz}) \) is connected to \( p'_i \) in
Eq. (3) by $p_i + p'_i = P_i + P'_i$, and the transverse momentum dependent (TMD) distribution function for photons at Born level [60, 61] can be obtained from $S_{\sigma \mu}(p_i, b_{iT})$ after integration over $b_{iT}$. Equation (3) is very close to the differential cross section in the framework of TMD factorization [60–64]. For other discussions on the geometric relations for ions or protons can also be found in Ref. [52, 57, 62, 72–76].

We adopt the classical field approximation for $S_{\sigma \mu}(p_i, b_{iT})$, then the cross section (3) can be put into the form

$$\sigma = \frac{Z^4 e^4}{2\gamma^4 v^8} \int d^2 b_T d^2 b_{1T} d^2 b_{2T} \int \frac{d\omega_1 d^2 p_{1T} d\omega_2 d^2 p_{2T}}{(2\pi)^3}$$

$$\times \int \frac{d^2 p'_{1T} e^{-ib_{1T} \cdot (p'_{1T} - p_{1T})}}{(2\pi)^2} \frac{F^*(\bar{p}^2_{1T})}{\bar{p}^2_{1T}} \frac{F(\bar{p}^2_{1T})}{\bar{p}^2_{1T}}$$

$$\times \int \frac{d^2 p'_{2T} e^{-ib_{2T} \cdot (p'_{2T} - p_{2T})}}{(2\pi)^2} \frac{F^*(\bar{p}^2_{2T})}{\bar{p}^2_{2T}} \frac{F(\bar{p}^2_{2T})}{\bar{p}^2_{2T}}$$

$$\times \left(2\pi\right)^4 \delta^{(4)}(\bar{p}_1 + \bar{p}_2 - k_1 - k_2) \delta^{(2)}(b_T - b_{1T} + b_{2T}),$$

where $F(p)$ is the charge form factor of the nucleus, i.e. the Fourier transformation of nuclear charge density, $\bar{p}_i$ and $\bar{p}'_i$ satisfying $\bar{p}_i \cdot u_i = \bar{p}'_i \cdot u_i = 0$ are photon momenta in the classical field approximation which can further be written as

$$\bar{p}^\mu_i = \left(\omega_i, p^\mu_{iT}, (-1)^{i+1} \frac{\omega_i}{v}\right), \quad \bar{p}'^\mu_i = \left(\omega_i, p'^\mu_{iT}, (-1)^{i+1} \frac{\omega_i}{v}\right).$$

Equation (6) incorporates the information of the position, momentum and polarization of the photons. If we integrate over $b_{iT}$, Eq. (6) reduces to the differential cross section used in recent studies [56–59] based on QED models in the classical background field approach [53–55]. In the ultra-relativistic limit, Eq. (6) gives the same expression as EPA [52, 57, 73] or gEPA [56–59]. In the twist expansion of the photon’s Wigner function, Eq. (6) is equivalent to the differential cross section at Born level in the TMD framework up to the twist-2 [60–64].

**III. NUMERICAL RESULTS**

In this section, we present numerical results based on Eq. (6) for the spectra of the transverse momentum, azimuthal angle and invariant mass for lepton pairs in Au+Au collisions.
Table I: Centralities and impact parameters in Au+Au collisions from Ref. [74].

| Centralities          | Impact parameters          |
|-----------------------|---------------------------|
| UPC RHIC Au+Au        | > 2R_A                    |
| 60-80% RHIC Au+Au     | 11.4 – 13.2 fm            |
| 40-60% RHIC Au+Au     | 9.4 – 11.6 fm             |
| 10-40% RHIC Au+Au     | 4.8 – 9.4 fm              |

at 200 GeV. The results are compared with the experimental data in Refs. [36, 48].

A. Parameters

In this subsection, we show the parameters used in the numerical calculation. The charge form factor for the nucleus is chosen to be

\[ F(p) = \frac{4\pi \rho^0}{p^3 A} \left[ \sin(pR_A) - pR_A \cos(pR_A) \right] \frac{1}{a^2 p^2 + 1}, \]

(8)

where \( a = 0.7 \) fm, \( A \) is the number of nucleons in the nucleus, \( R_A = 1.2A^{1/3} \) fm is the nucleus radius, and \( \rho^0 = 3A/(4\pi R_A^3) \) is the nucleon number density which makes \( F(p = 0) = 1 \) [77]. The form factor (8) is very close to the Fourier transformation of the Woods-Saxon distribution that has been used in the calculation of photoproduction and other processes [52, 60, 61, 77].

When the impact parameter \( b_T \) is larger than \( \sim 13.2 \) fm corresponding to \( \geq 80\% \) centrality, two colliding nuclei may undergo mutual Coulomb excitation and emit neutrons [58, 61]. To account for the effect of such processes, one needs to multiply an extra pre-factor \( \mathcal{P}^2(b_T) \) to the cross section, where \( \mathcal{P}(b_T) \) is the probability of emitting neutrons from an excited nucleus. It can be parametrized as [78],

\[ \mathcal{P}(b_T) = \sum_{N_{\gamma},=1}^{\infty} \frac{1}{N_{\gamma}!} w^{N_{\gamma}} \exp(-w) = 1 - \exp(-w), \]

(9)

where \( N_{\gamma} \) denotes the number of photons that can be absorbed by a nucleus. In our previous work [1], we have chosen the \( w \) from the Giant Dipole Resonance model [55, 61, 73, 78, 79],

\[ w = 5.45 \times 10^{-5} Z^2(A - Z)/[A^{2/3}b_T^2] \]

In the comparison with the experimental measurement, one needs to consider the case of emitting multiple neutrons and rewrite the \( w \) as [58, 73, 80–...
Figure 1: The spectra of \((P^{ee}_T)^2\) in 60-80% centrality. (a) The STAR data \cite{48} and results from some models with the invariant mass \(M_{ee}\) in the range \([0.40,0.76]\) GeV; (b) The results of this work with \(M_{ee}\) in other ranges. The rapidity \(y_{ee}\) and pseudo-rapidity \(\eta_e\) are in the range \([-1,1]\) and the transverse momentum of the single electron or positron is larger than 0.2 GeV.

\[ w_{Xn}(b_T) = \int d\omega n(\omega, b_T)\sigma_{\gamma+A\rightarrow A'+Xn}(\omega), \quad (10) \]

where the \(X \geq 1\) stands for the number of neutrons emitted by a nuclei and \(n(\omega, b_T)\) is the photon flux \cite{1, 53, 84} and the photon-nucleus cross section \(\sigma_{\gamma+A\rightarrow A'+Xn}\) is given by the fixed-target experiments \cite{80, 85, 86}. One can also approximates \(\mathcal{P}(b_T) \approx w\) \cite{55} or \(\mathcal{P}(b_T) \approx w \exp(-w)\) \cite{58, 78, 79, 87} for small \(w\) or \(N_\gamma \approx 1\) respectively. In the current work, we have chosen \(\mathcal{P}(b_T)\) in Eq. (9) with \(w\) given by Eq. (10) to describe the multiple neutrons emission processes in the experiments, when the impact parameter is larger than \(\sim 13.2\) fm.

Since the differential cross section (6) involves a high-dimension integration, we employ the algorithm of ZMCintegral \cite{88, 89} to handle such a high-dimension integral. Other applications of ZMCintegral to relativistic heavy-ion collisions can be found in Ref. \cite{90?}.

For the sake of clarity, we define

\[ P^\parallel_T = k_{1T} + k_{2T}, \]
\[ K^\parallel_T = \frac{1}{2}(k_{2T} - k_{1T}), \quad (11) \]

where \(k_{1T}\) and \(k_{2T}\) are the transverse momentum of the lepton and antilepton in the collision (2), respectively. We can further define \(k^\parallel_{1T} \equiv |k_{1T}|, \ k^\parallel_{2T} \equiv |k_{2T}|, \ P^\parallel_T \equiv |P^\parallel_T|, \ K^\parallel_T \equiv |K^\parallel_T|, \]
Figure 2: The distributions of $\sqrt{\langle (P_{T}^{e})^2 \rangle}$ as functions of $M_{ee}$. (a) The results computed from Eq. (6) with and without smear corrections. The results from EPA [48] and QED model [59] as well as STAR data [48] are also shown; (b) Comparison of results in different centralities. Other parameters are chosen to be the same as in Fig. 1.

Following STAR experiments [36, 48], we set the momentum cutoffs: $k_{T}^{e-}, k_{T}^{e+} \geq 200$ MeV, $P_{T}^{ee} \leq 100$ and 150 MeV in UPC and peripheral collisions respectively, $k_{T}^{\mu-}, k_{T}^{\mu+} \in [180, 300]$ MeV, and $P_{T}^{\mu\mu} \leq 100$ MeV. The rapidity ranges for $y_{ee}$, $\eta_{e-}$ and $\eta_{e+}$ are set to $[-1, 1]$, and those for $y_{\mu\mu}$, $\eta_{\mu-}$ and $\eta_{\mu+}$ are set to $[-0.8, 0.8]$.

In Table I, we list the centralities and impact parameters in Au+Au collisions following Ref. [74].

**B. Transverse momentum distribution**

In Fig. 1, we plot the spectra of $(P_{T}^{e})^2$ in 60-80% centrality in Au+Au collisions at 200 GeV in different ranges of the invariant mass $M_{ee}$. In the figure we use $dN/dy_{ee}d(P_{T}^{e})^2$, it can also be expressed as $d\sigma/dy_{ee}d(P_{T}^{e})^2$ through the relation between the number of final particles $N$ and the cross section as
Figure 3: (a) $\sqrt{\langle (P_{\mu\mu}^T)^2 \rangle}$ as functions of $M_{\mu\mu}$ in different centralities. (b) $\sqrt{\langle (P_{ee}^T)^2 \rangle}$ and $\sqrt{\langle (P_{\mu\mu}^T)^2 \rangle}$ as functions of $b_T$ in different invariant mass ranges. The parameters for electrons are chosen to be the same as in Fig. 1. Both $y_{\mu\mu}$ and $\eta_\mu$ are integrated over the range $[-0.8, 0.8]$, and the ranges of $P_{ee}^T$ and $P_{\mu\mu}^T$ are set to $P_{ee}^T \in [0.00, 0.15] \text{GeV}$ and $P_{\mu\mu}^T \in [0.00, 0.10] \text{GeV}$.

In Fig. 1(a), we see that both our result from Eq. (6) and that based on QED model [56] match the experimental data well especially for small $(P_{ee}^T)^2$. We have numerically check that the difference between our results and the one based on QED model [56] comes from the choice of the radius of nuclei $R_A$ in the region $(P_{ee}^T)^2 \geq 0.004 \text{GeV}^2$. The result of STARLight [52] based on EPA and that from gEPA [56] cannot reproduce the data in the small $(P_{ee}^T)^2$ region because the information about the transverse momentum and polarization of photons is missing in STARLight and gEPA which is essential to reproduce the data. We can see in Fig. 1(b) that the spectra of $(P_{ee}^T)^2$ decrease with increasing $M_{ee}$, which agrees with the observation in our previous work [1] and experimental data [36, 48].

In Fig. 2, we plot the distribution of $\sqrt{\langle (P_{ee}^T)^2 \rangle}$ as functions of $M_{ee}$ in different centralities. In Fig. 2(a), we compare the results from Eq. (6) with and without smear corrections, those from EPA [48] and QED models [59] as well as experimental data in 60-80% centrality. The EPA (or STARLight) [48] does not include the information for the transverse momentum of photons and impact parameters, so it fails to give the broadening effects. Since the transverse momentum and impact parameter of photons are incorporated into Eq. (6), our results can describe the experimental data [48] and are consistent with recent calculations.

$$dN = \frac{d\sigma}{\pi(b_{T,max}^2 - b_{T,min}^2)}.$$ (12)
Another possible missing correction in our previous work [1] and in Eq. (6) is the smear correction [71]. The limitation of momentum resolution in the detectors and the Bremsstrahlung of leptons inside the detectors may change the final transverse momentum distribution of leptons. In experiments, one needs to add the smear corrections to adjust the $P_T$ distribution in the end. In the current work, we implement the smear corrections extracted from the measurements of $J/\psi \rightarrow \ell \bar{\ell}$ [71]. In Fig. 2(a), we find that the smear corrections lead to an enhancement of $\sqrt{\langle (P_{T}^e)^2 \rangle}$ when $M_{ee} > 2$ GeV and are almost negligible in small $M_{ee}$ region. However, it seems that the smear corrections are still insufficient to match the data, so higher order corrections beyond the Born level or medium effects need to be considered in the future. Since the smear corrections do not have significant effects, we do not include them into other quantities except $\sqrt{\langle (P_{T}^e)^2 \rangle}$.

In Fig. 2(b), we observe the broadening of $P_{T}^e$ characterized by $\sqrt{\langle (P_{T}^{ee})^2 \rangle}$ grows with decreasing of the impact parameter or the centrality [48]. We note that a similar broadening effect also exists for $P_{T}^{\mu\mu}$ as shown in Fig. 3(a). In Fig. 3(b), we plot $\sqrt{\langle (P_{T}^{ee})^2 \rangle}$ and $\sqrt{\langle (P_{T}^{\mu\mu})^2 \rangle}$ as functions of $b_T$ at different values of $M_{ee}$ and $M_{\mu\mu}$. Again, we observe that $\sqrt{\langle (P_{T}^{\mu\mu})^2 \rangle}$ increases as $b_T$ decreases or $M_{\mu\mu}$ increases. We see in Fig. 3 that the results for $e^+e^-$ and $\mu^+\mu^-$ are quite similar except that $\sqrt{\langle (P_{T}^{\mu\mu})^2 \rangle}$ is less than $\sqrt{\langle (P_{T}^{ee})^2 \rangle}$ at fixed $b_T$ and $M_{\mu\mu}$ due to the mass effect.

C. Azimuthal angle distributions

In Fig. 4, we plot the azimuthal angle $\varphi$ distributions for electron pairs in Au+Au collisions. We show the results for UPC and 60-80% centrality at 200 GeV and those for 80-100% centrality at 54.4 GeV in comparison with STAR data [36].

In Fig. 4(a), we find that our results agree with experimental data and also close to the results of the QED model [56]. Again, due to the lack of the essential information for the transverse momentum and polarization of photons, the STARLight results [36] do not match the data. It is another piece of evidence that our key formula (6) which includes the spectra of transverse momentum and polarization of photons can better describe the experiments than EPA.

We see in Fig. 4 that there is a $\cos(4\varphi)$ modulation behavior [60, 61]. Such a modulation
Figure 4: The distributions of $\varphi$ in Au+Au collisions. The invariant mass of electron pairs is in the range $M_{ee} \in [0.45, 0.76]$ GeV. (a) UPC and 60-80% centrality at 200 GeV. The dark-blue-solid, dark-blue-dashed, green-dashed, red-dash-dotted and brown-dash-dotted lines denote the results from Eq. (6) for emission of $X > 1$ neutrons, those for emission of one neutron by each colliding nucleus, STARLight [36], QED model [56] and SupreChic program [36], respectively. The data points are from STAR measurements [36]. (b) Prediction for 80-100% centrality in Au+Au collisions at 54.4 GeV.

Figure 5: The average of $\cos(4\varphi)$ for electron pairs as functions of $P_T^{ee}$ in Au+Au collisions at 200 GeV. (a) In different centrality bins; (b) In 60-80% centrality with different nucleus radius $R_A$. The ranges for $y_{e^-}$ and $y_{e^+}$ are $[-1, 1]$, and the range for $K_T^{ee}$ is $[0.20, 0.40]$ GeV.

behavior is a signal for the linear polarization of incoming photons and is connected with the vacuum birefringence [36]. The same phenomenon also exists in gluons [91–93]. In Fig. 4(b), we present the results for the modulation behavior in Au+Au collisions at 54.4 GeV and 80-100% centrality. Our results from Eq. (6) show that that the modulation behavior
Figure 6: The average of $\cos(4\varphi)$ for electron pairs as functions of $b_T$ in Au+Au collisions at 200 GeV within different $K_T^{ee}$ bins. The ranges for $y_{ee}$ and $P_T^{ee}$ are $[-1, 1]$ and $[0.00, 0.15]$ GeV, respectively.

Figure 7: The distributions of $\varphi$ in Au+Au collisions at 200 GeV and 60-80% centrality. The range of $M_{\mu\mu}$ is set to $[0.40, 0.64]$ GeV. The dark-blue-solid and red-dashed lines are the results from Eq.(6) and QED models [56, 66], respectively.

at 54.4GeV also satisfies the function $1 + A_{4\varphi} \cos(4\varphi)$ ($A_{4\varphi}$ is the coefficient) similar to the one at 200GeV.

Now we take a look at the average of $\cos(4\varphi)$ on independent kinematic variables in Fig. 5. We follow Refs. [60, 61] to define the average of $\cos(4\varphi)$ and $\cos(2\varphi)$ as

$$\langle \cos(n\varphi) \rangle = \frac{\int d\mathcal{P} \cdot \mathcal{S} \cdot \cos(n\varphi) d\mathcal{P} \cdot \mathcal{S}}{\int d\mathcal{P} \cdot \mathcal{S} \cdot d\mathcal{P} \cdot \mathcal{S}}, \quad n = 2, 4. \quad (13)$$

where $\mathcal{P} \cdot \mathcal{S}$ denotes the independent kinematic variables in collisions. For example, if we want to compute $\langle \cos(n\varphi) \rangle$ as a function of $P_T^n$, then $\mathcal{P} \cdot \mathcal{S}$ stands for all independent kinematic variables except $P_T^n$, such as the impact parameter, invariant mass, rapidity, and so on.
Figure 8: The distributions of (a) $-\cos(4\varphi)$ and (b) $\cos(2\varphi)$ for muon pairs as functions of $P_T^{\mu\mu}$ in Au+Au collisions at 200 GeV in different centrality bins. The ranges for $y_\mu^-$ and $y_\mu^+$ are $[-0.8, 0.8]$, and the range for $K_{T}\mu\mu$ is $[0.2, 0.4]$ GeV.

Figure 9: The distributions of (a) $-\cos(4\varphi)$ and (b) $\cos(2\varphi)$ for muon pairs as functions of $b_T$ in Au+Au collisions at 200 GeV for different $K_{T}\mu\mu$. The ranges for $y_\mu^-$ and $y_\mu^+$ are $[-0.8, 0.8]$, and the range for $P_T^{\mu\mu}$ is $[0.00, 0.10]$ GeV.

In Fig. 5(a), we observe that the magnitude of $\langle \cos(4\varphi) \rangle_e$ decreases with the centrality. In UPC, we find that $\langle \cos(4\varphi) \rangle_e$ oscillates slowly as a function of $P_T^{ee}$. In collisions of 80-100% and 60-80% centralities, it reaches the maximum value at about $P_T^{ee} \sim 0.02-0.03$ GeV and then decreases. When the centrality is less than 40%, $-\langle \cos(4\varphi) \rangle_e$ always increases slowly with $P_T^{ee}$. These shapes are also observed in early works [60, 61]. In Fig. 5(b), we present the $R_A$ dependence of $\langle \cos(4\varphi) \rangle_e$. The nuclear radius $R_A$ we use is different from $R_A = 1.1A^{1/3}$ used in Refs. [60, 61]. We find that $\langle \cos(4\varphi) \rangle_e$ is sensitive to $R_A$ when $P_T^{ee} \geq 0.02$ GeV/c. Therefore, it may be possible to measure $R_A$ through $\langle \cos(4\varphi) \rangle_e$ in experiments.
As a comparison with the results of Fig. 5, we plot the impact parameter dependence of \( \langle \cos(4\varphi) \rangle_e \) in Fig. 6. We see that the magnitude of \( \langle \cos(4\varphi) \rangle_e \) decreases with increasing \( b_T \), consistent with Fig. 5(a). We also see that it decreases slightly with increasing \( K_{ee}^T \).

We also study the azimuthal angle \( \varphi \) distributions for muon pairs. In Fig. 7, we can see that our results from Eq. (6) agree with those based on QED models [56]. But our results and those of QED models for muon pairs are similar to those for electron pairs but the magnitudes are smaller than the STAR data [66].

In Fig. 8(a), we plot \( -\langle \cos(4\varphi) \rangle_\mu \) as functions of \( P_T^\mu \) at different centralities. We find that the shape of \( -\langle \cos(4\varphi) \rangle_\mu \) is similar to \( -\langle \cos(4\varphi) \rangle_e \) in Fig. 5(a), except in the region of small \( P_T^\mu \). When \( P_T^\mu < 0.015 \) GeV, \( -\langle \cos(4\varphi) \rangle_\mu \) increases with the centrality while \( -\langle \cos(4\varphi) \rangle_e \) decreases with the centrality.

The notable difference between \( \varphi \) distributions of electron pairs and muon pairs is in the distribution of \( \cos(2\varphi) \). As pointed out in Refs. [60, 61], \( \langle \cos(2\varphi) \rangle_l \) is proportional to \( (m_l/P_T^l)^2 \) with \( m_l \) being the lepton mass. For electrons, we have \( \langle \cos(2\varphi) \rangle_e \propto (m_e/P_T^e)^2 \rightarrow 0 \) since \( m_e/P_T^e \ll 1 \), while for muons, which are much heavier than electrons, we have \( \langle \cos(2\varphi) \rangle_\mu \propto (m_\mu/P_T^\mu)^2 \) which is not negligible. We have numerically checked that \( \langle \cos(2\varphi) \rangle_e \) is very close to zero, but \( \langle \cos(2\varphi) \rangle_\mu \) is sizable as shown in Fig. 8(b). We observe that \( \langle \cos(2\varphi) \rangle_\mu \) as functions of \( P_T^\mu \) has minimal values in the range \( P_T^\mu \sim 0.005-0.015 \) GeV at different centralities. For large \( P_T^\mu \), \( \langle \cos(2\varphi) \rangle_\mu \) becomes saturated at centralities larger than 60% and in UPC.

In Fig. 9, we present the distributions of \( -\langle \cos(4\varphi) \rangle_\mu \) and \( \langle \cos(2\varphi) \rangle_\mu \) as functions of \( b_T \) for different \( K_T^\mu \). We see \( -\langle \cos(4\varphi) \rangle_\mu \) decreases and \( \langle \cos(2\varphi) \rangle_\mu \) increases with increasing \( b_T \), which is consistent with the centrality dependence in Fig. 8. Compared with the results for electron pairs in Fig. 6, we find that the magnitude of \( \langle \cos(4\varphi) \rangle_\mu \) is smaller than that of \( \langle \cos(4\varphi) \rangle_e \) due to the mass effect.

**D. Invariant mass distribution**

In this subsection, we present the invariant mass distributions for electron and muon pairs in Au+Au collisions at 200 GeV.

We plot in Fig. 10(a) the results from Eq. (6) for electron pairs which are in good agreement with the experimental data [48]. The invariant mass distributions for electron pairs...
decrease with increasing $M_{ee}$ when $M_{ee} \geq 0.5$ GeV and are not very sensitive to centralities (slightly decrease with the centrality). In Fig. 10(b), we present our results for muon pairs which are also in good agreement with the experimental data [66]. We find a similar centrality dependence in the invariant mass distribution for muon pairs. The difference between our results and the experimental data may come from higher order corrections, e.g. a possible contribution from Sudakov factors.

IV. CONCLUSION AND OUTLOOK

We study the lepton pair photoproduction in peripheral heavy-ion collisions based on our previous work in Ref. [1]. We calculate the distributions of the transverse momentum $P_{Tl}$, the azimuthal angle $\varphi$ and invariant mass $M_{ll}$ for lepton pairs as functions of the impact parameter $b_T$ (or centrality equivalently) and other kinematic variables in Au+Au collisions.

Our results from Eq. (6) are in good agreement with the experimental data for electron pairs. The information on the transverse momentum and polarization for photons is essential to describe the experimental data. Our results for the azimuthal angle distribution of muon pairs do not match the experimental data very well but agree with other models based on QED. Such differences between theoretical results and experimental data require further
studies in the muon pair production.

Although our results for $\sqrt{\langle P_{\mu\bar{\nu}}^2 \rangle}$ in Fig. 2(a) are roughly consistent with the STAR measurement, there is space to allow for improvements. In the current work, we have included the smear corrections, which can give an enhancement in the large $M_{ee}$ region. The higher order corrections beyond the Born level, such as Sudakov factor [60–62, 64, 65], and possible medium effects [48, 50, 62, 64] may be necessary.

In Fig. 5(b), we find that $\langle \cos(4\varphi) \rangle_e$ is sensitive to the nuclear radius. It is possible to extract the nuclear radius through $\langle \cos(4\varphi) \rangle_e$ in experiments. We also obtain a significant enhancement of $\cos(2\varphi)$ for muon pairs proposed in Ref. [60, 61].

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