Propagation of second-order moments of general truncated beams in atmospheric turbulence

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Abstract. Based on the partial-coherence theory and the method of the window function being expanded into a finite sum of complex-valued Gaussian functions, the analytic expressions for second-order moments of general truncated beams propagating through atmospheric turbulence are derived, from which some important characteristic parameters, such as the mean-squared beam width, the angular spread, the beam propagation factor (i.e. $M^2$-factor), the Rayleigh range and the effective radius of curvature are also derived. It is shown that general truncated beams may have the same directionality as a fully coherent Gaussian beam if a certain condition is satisfied. Taking a truncated sinh–Gaussian beam as an example of general truncated beams, some numerical calculations are performed to illustrate the general results obtained in this paper. The analytic results obtained in this paper are general and very useful in studying the propagation property and the beam quality of laser beams.

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1. Introduction

In recent years, the propagation of laser beams through atmospheric turbulence has attracted much attention because of its wide applications in many domains, such as in connection with remote sensing, imaging and communication systems [1–3]. In past years, much work was carried out concerning the influence of turbulence on the beam propagation property [4–17]. Meanwhile, the influence of turbulence on the beam quality was also investigated [18–21]. However, the above-mentioned studies are restricted to the untruncated case. It is very important to study the propagation of truncated laser beams because the laser beam emitted from a laser system is more or less truncated in practice. So far, only a few papers have dealt with the propagation of truncated laser beams through atmospheric turbulence. For example, the effects of turbulence on the average intensity distribution of truncated flattened Gaussian beams were studied in [22]. Recently, the effects of turbulence on the beam quality of truncated partially coherent beams were investigated, where the mean-squared beam width, power in the bucket (i.e. PIB, which indicates how much fraction of the total beam power is within a given bucket), $\beta$-parameter and the Strehl ratio $S_R$ were taken as the characteristic parameters of laser beam quality [23]. Very recently, the average intensity, PIB and $S_R$ of truncated laser beams with amplitude modulations and phase fluctuations through atmospheric turbulence were studied in detail [24]. So far, studies have dealt exclusively with certain characteristics of certain truncated laser beams in atmospheric turbulence just as mentioned above [22–24].

It is known that a more detailed characterization of laser beams requires the determination of second-order moments. The aim of this paper is to study the propagation of second-order moments of general truncated beams through atmospheric turbulence. However, for truncated laser beams, the divergence problem of second-order moments due to the high spatial frequencies is difficult to handle mathematically and is excluded from the ISO document [25]. So far, several approaches have been suggested in order to remedy this problem, such as the generalized truncated second-order moments method [26, 27], the asymptotic analysis method [28] and the self-convergent beam width method [29]. In this paper, the method of the window function being expanded into a finite sum of complex-valued Gaussian functions [30] is employed to overcome the divergence problem. Furthermore, the partial-coherence theory is used to characterize general beams in this paper. In section 2, the analytic expressions for second-order moments of general truncated beams propagating through atmospheric turbulence are derived, from which the analytic expressions for the mean-squared beam width, the angular spread, the $M^2$-factor, the Rayleigh range and the effective radius of curvature of general truncated beams in turbulence are also derived. The analytic expressions for second-order moments for untruncated and free space cases can be treated as special cases of our general
results, which are also discussed in section 2. In section 3, the general results obtained in this paper are illustrated by using numerical examples, where truncated sinh–Gaussian (shG) beams are considered. In section 4, the main results obtained in this paper are summarized.

2. Theoretical formulae

For simplicity, the analysis is based on two dimensions. We assume that a general beam is incident upon a slit with full width 2b oriented along the x-axis at the source plane z = 0. The window function of the slit is described by the rectangular function of the form

\[ T(x) = \begin{cases} 
1, & |x| \leq b, \\
0, & |x| > b.
\end{cases} \] (1)

According to Wen and Breazeale [30], \( T(x) \) can be expanded into a finite sum of complex-valued Gaussian functions

\[ T(x) = \sum_{m=1}^{M} A_m \exp\left(-\frac{B_m x^2}{b^2}\right), \] (2)

where \( A_m \) and \( B_m \) are the expansion and Gaussian coefficients, respectively. The coefficients \( A_m \), \( B_m \) and the number \( M \) can be evaluated by a computer optimization. For \( M = 10 \) the coefficients \( A_m \) and \( B_m \) are given in table 1 of [30], which are omitted here to save space.

It has been shown that good agreement between the ten-term Gaussian beam solution and the results of numerical integration is obtained throughout the beam field, and discrepancies exist only in the extreme near field (<0.12 times the Fresnel distance) [30–32].

Within the framework of partial-coherence theory and under the quasi-monochromatic assumptions, the cross-spectral density of general beams is expressed as [33]

\[ W^{(0)}(x'_1, x'_2) = \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} \lambda_{i,j} \phi_i(x'_1) \phi_j^*(x'_2), \] (3)

where \( \lambda_{i,j} \) are the corresponding mode coherence coefficients (MCCs) and \( i \) and \( j \) are the mode indices. \( \phi(x) \) represents the field distribution, which can be expressed as a series of orthogonal basis modes, e.g. the Hermite–Gaussian (H–G) modes at the \( z = 0 \) plane in the rectangular coordinate system, i.e. [33]

\[ \phi_l(x) = \left[2^{l/2}/(\pi^{1/4}w_{l0}!)\right]^{1/2}\exp(-x^2/w_{l0}^2)H_l(2^{1/2}x/w_{l0}) \quad (l = i, j), \] (4)

with \( w_{l0} \) being the waist width of the basis Gaussian mode and \( H_l \) denoting the \( l \)th order Hermite polynomial.

By using the orthogonality of the H–G series, the MCCs are obtained from equation (3), i.e.

\[ \lambda_{i,j} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \phi_i^*(x'_1) W^{(0)}(x'_1, x'_2) \phi_j(x'_2) dx'_1 dx'_2. \] (5)
Based on the extended Huygens–Fresnel principle, the intensity of general truncated beams in free space reads as

\[
I(x, z) = \frac{k}{2\pi z} \int_{-b}^{b} \int_{-b}^{b} dx_1' dx_2' W^{(0)}(x_1', x_2') \exp \left\{ \frac{ik}{2z} \left[ (x_1'^2 - x_2'^2) - 2(x_1' - x_2')x \right] \right\},
\]

where \( k \) is the wave number related to the wave length \( \lambda \) by \( k = 2\pi/\lambda \).

Considering equations (1) and (2), equation (6) can be rewritten as

\[
I(x, z) = \frac{k}{2\pi z} \sum_{m=1}^{M} \sum_{n=1}^{M} A_m A_n \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx_1' dx_2' W^{(0)}(x_1', x_2') \exp \left\{ \frac{ik}{2z} \left( x_1'^2 - x_2'^2 - 2(x_1' - x_2')x \right) \right\}.
\]

The second-order moment \( \langle x^2 \rangle \) is defined as [34]

\[
\langle x^2 \rangle = \frac{\int_{-\infty}^{+\infty} x^2 I(x, z)dx}{\int_{-\infty}^{+\infty} I(x, z)dx}.
\]

On substituting from equation (7) into equation (8), recalling the integral formulae [35]

\[
\int_{-\infty}^{+\infty} x^2 \exp(-i2\pi sx) dx = \frac{\delta''(s)}{4\pi^2},
\]

\[
\int_{-\infty}^{+\infty} f(x)\delta''(x) dx = f''(0),
\]

\[
\int_{-\infty}^{+\infty} \exp[-(x - y)^2]H_m(\alpha x)H_n(\beta x) dx
\]

\[
= \left( \frac{\beta}{\alpha} \right)^n \sqrt{\pi} \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{\binom{m}{l} n!}{k!(n-2k-l)!} \frac{1}{(\beta^2 - \alpha^2)^{l-1/2}} \left( 1 - \frac{1}{(\beta^2 - \alpha^2)^{l+1/2}} \right)^k,
\]

where \( \delta \) denotes the Dirac delta function and \( \delta'' \) is its second derivative, \( f \) is an arbitrary function and \( f'' \) is its second derivative, \( \lfloor n/2 \rfloor \) represents the integer of \( n/2 \), \( \min(m, n-2k) \) denotes the minimum of \( m \) and \( n-2k \), and performing very tedious integral calculations, we obtain the second-order moment \( \langle x^2 \rangle \) of general truncated beams in free space, i.e.

\[
\langle x^2 \rangle = A + B \frac{z^2}{k^2},
\]

where

\[
A = \sum_{m=1}^{M} \sum_{n=1}^{M} \sum_{i=0}^{\infty} \sum_{j=0}^{\lfloor i/2 \rfloor} \sum_{l=0}^{\min(i,l-2r)} \sum_{r=0}^{\infty} \sum_{s=0}^{\lfloor l/2 \rfloor} \frac{Q}{4P^2} \left[ \frac{8\alpha(\alpha - 1)}{w_0^2 P^2 - 2}H_{\alpha-2}(0) + 2H_{\alpha}(0) \right] H_{\beta}(0)/S,
\]

\[
B = \sum_{m=1}^{M} \sum_{n=1}^{M} \sum_{i=0}^{\infty} \sum_{j=0}^{\lfloor i/2 \rfloor} \sum_{l=0}^{\min(i,l-2r)} \sum_{r=0}^{\infty} \sum_{s=0}^{\lfloor l/2 \rfloor} QR/S,
\]

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\[ P = \sqrt{\frac{2}{w_{0h}^2} + \frac{B_m + B_n^*}{b^2}}, \]  
\[ Q = \frac{A_m A_n^* \lambda_{i,j} 2^{-[(i-j)/2] - j - r + s}!}{P(i! j!)^{1/2} r!(l - 2r - s)!} \left( \begin{array}{c} j \\ i \\ l \end{array} \right) \left( \begin{array}{c} i \\ l \\ s \end{array} \right) \left( 1 - \frac{2}{w_{0h}^2 P^2} \right)^{[(i+j)/2] - r - s}, \]
\[ R = \left[ \frac{1}{w_{0h}^2} + \frac{B_m + B_n^*}{2b^2} - \frac{(B_m - B_n^*)^2}{2P^2b^4} \right] H_\alpha(0) H_\beta(0) \]
\[ + \frac{16\sqrt{2}\alpha\beta}{w_{0h}\sqrt{w_{0h}^2 P^2 - 2}} \left( \frac{P}{2} - \frac{B_m - B_n^*}{2Pb^2} \right) H_{\alpha-1}(0) H_{\beta-1}(0) \]
\[ + \frac{4\alpha(\alpha - 1)}{w_{0h}^2 P^2 - 2} \left[ \frac{B_m - B_n^*}{b^2} - \frac{P^2}{2} - \frac{(B_m - B_n^*)^2}{2P^2b^4} \right] H_{\alpha-2}(0) H_{\beta}(0) \]
\[ - \frac{16}{w_{0h}^2} \beta(\beta - 1) H_\alpha(0) H_{\beta-2}(0), \]
\[ S = \sum_{m=1}^{M} \sum_{n=1}^{M} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \frac{A_m A_n^* \lambda_{i,j} 2^{-[(i+j)/2] - j + s}!}{P(i! j!)^{1/2} r!(l - 2r - s)!} \left( \begin{array}{c} j \\ i \\ l \end{array} \right) \left( \begin{array}{c} i \\ l \\ s \end{array} \right) \left( 1 - \frac{2}{w_{0h}^2 P^2} \right)^{[(i+j)/2] - l} H_{i+j-2l}(0), \]
\[ \alpha = i + l - 2r - 2s, \quad \beta = j - l. \]

The formulae of the second-order moments \( \langle x^2 \rangle, \langle \theta^2 \rangle \) and \( \langle x\theta \rangle \) of partially coherent beams propagating through atmospheric turbulence are expressed as \([36]\)
\[ \langle x^2 \rangle = \langle x^2 \rangle_0 + 2 \langle x\theta \rangle_0 z + \langle \theta^2 \rangle_0 z^2 + \frac{2}{3} T z^3, \]
\[ \langle \theta^2 \rangle = \langle \theta^2 \rangle_0 + 2Tz, \]
\[ \langle x\theta \rangle = \langle x\theta \rangle_0 + \langle \theta^2 \rangle_0 z + Tz^2, \]
where the angular brackets with subscript 0 denote the second-order moments in the \( z = 0 \) plane, and
\[ T = \pi^2 \int_0^\infty \kappa^3 \Phi_n(\kappa) \, d\kappa, \] with \( \Phi_n \) being the spatial power spectrum of the refractive index fluctuations of the turbulent atmosphere.

For \( T = 0 \), equation (20) reduces to the second-order moments \( \langle x^2 \rangle \) in free space, which should be equal to equation (12) and yields \( \langle x^2 \rangle_0 = A, \langle x\theta \rangle_0 = 0 \) and \( \langle \theta^2 \rangle_0 = B/K^2 \). Therefore, we obtain the second-order moments \( \langle x^2 \rangle, \langle \theta^2 \rangle \) and \( \langle x\theta \rangle \) of general truncated beams in
turbulence, i.e.

\[ \langle x^2 \rangle = A + \frac{B}{k^2} z^2 + \frac{2}{3} T z^3, \quad (24) \]

\[ \langle \theta^2 \rangle = \frac{B}{k^2} + 2 T z, \quad (25) \]

\[ \langle x \theta \rangle = \frac{B}{k^2} z + T z^2. \quad (26) \]

According to [34], the physical meaning in second-order moments is clear, e.g. \( \langle x^2 \rangle \), \( \langle \theta^2 \rangle \) and \( \langle x \theta \rangle \) are relative to the beam width, the divergence angle and the radius of curvature, respectively. Based on the second-order moments (i.e. equations (24)–(26) together with equations (13)–(19)), the analytic expressions for the mean-squared beam width, the angular spread, the \( M^2 \)-factor, the Rayleigh range and the effective radius of curvature of general truncated beams in turbulence can be derived, which are given as follows:

(i) The mean-squared beam width [34]

\[ w(z) = 2 \sqrt{\langle x^2 \rangle} = 2 \sqrt{A + \frac{B}{k^2} z^2 + \frac{2}{3} T z^3}. \quad (27) \]

(ii) The angular spread [34]

\[ \theta_{sp}(z) = 2 \sqrt{\langle \theta^2 \rangle} = 2 \sqrt{\frac{B}{k^2} + 2 T z}. \quad (28) \]

The first term on the right-hand side in equation (28) represents the angular spread of general truncated beams in free space; the second term describes the effect of turbulence on the angular spread, which is independent of the beam parameters.

It is shown that the angular spread of a fully coherent Gaussian beam specified by subscripts ‘Gs’ is expressed as [9]

\[ \theta_{sp}(z) \big|_{Gs} = 2 \left( \frac{1}{k_{Gs}^2 w_{0Gs}^2} + 2 T z \right)^{1/2}. \quad (29) \]

From a comparison of equations (28) and (29), we conclude that a general truncated beam will generate the same angular spread as a fully coherent Gaussian beam both in free space and in turbulence if the condition

\[ \frac{B}{k^2} = \frac{1}{k_{Gs}^2 w_{0Gs}^2} \quad (30) \]

is satisfied. The result is independent of the spatial power spectrum \( \Phi_\nu(\kappa) \). Such general truncated beams are called equivalent general truncated beams [9], which have the same directionality as a fully coherent Gaussian beam if the angular spread is taken as the characteristic parameter.

(iii) The \( M^2 \)-factor [34]

\[ M^2 = 2k \left[ \langle x^2 \rangle \langle \theta^2 \rangle - \langle x \theta \rangle^2 \right]^{1/2} \]

\[ = \left( 4AB + 8k^2 AT z + \frac{2}{3} BT z^3 + \frac{4}{3} k^2 T^2 z^4 \right)^{1/2}. \quad (31) \]
(iv) The effective radius of curvature \[ R = \frac{\langle x^2 \rangle}{\langle x\theta \rangle} = \frac{A + \frac{B}{k^2} \langle z^2 \rangle + \frac{2}{3} T \langle z^3 \rangle}{\frac{B}{k^2} + T \langle z^2 \rangle}. \] (32)

(v) The Rayleigh range

It is known that the Rayleigh range is defined as the propagation distance at which the cross-sectional area of a beam doubles [34], i.e. \( 2 \langle x^2 \rangle_0 = \langle x^2 \rangle_{z_R} \), where \( \langle x^2 \rangle_{z_R} \) represents the second-order moment \( \langle x^2 \rangle \) in the \( z = z_R \) plane. From equation (24) the Rayleigh range \( z_R \) of general truncated beams in turbulence is determined by the cubic equation, i.e. \( (2/3) T z_R^3 + (B/k^2) z_R^2 - A = 0 \). Among the three solutions of this cubic equation, there is only one real solution, which represents the Rayleigh range \( z_R \), i.e.

\[ z_R = \frac{1}{2T} \left( -\frac{B}{k^2} + \frac{B^2}{k^4} C + C \right), \] (33)

where

\[ C = \left\{ 6AT^2 - \frac{B^2}{k^6} + \left[ 12A \left( 3AT^2 - \frac{B^2}{k^6} \right) \right]^{1/2} T \right\}^{1/3}. \] (34)

Equations (27), (28), (30)–(33) together with equations (13)–(19) and (34) are the general results obtained in this paper, which include some interesting results given as follows:

(i) For \( b \to \infty \) (the untruncated case), equations (13) and (14) reduce to

\[ A_{\text{untr}} = \left( w_{0b}^2 / 4 \right) \sum_{i=0}^{+\infty} \left\{ (1 + 2i) \beta_{i,i} + 2 \left[ (i + 1)(i + 2) \right]^{1/2} \beta_{i,i+2} \right\}, \] (35)

\[ B_{\text{untr}} = \left( z^2 / w_{0b}^2 \right) \sum_{i=0}^{+\infty} \left\{ (1 + 2i) \beta_{i,i} - 2 \left[ (i + 1)(i + 2) \right]^{1/2} \beta_{i,i+2} \right\}, \] (36)

where \( \beta_{i,i} = \lambda_{i,i} / \sum_{i=0}^{+\infty} \lambda_{i,i} \) and \( \beta_{i,i+2} = \lambda_{i,i+2} / \sum_{i=0}^{+\infty} \lambda_{i,i} \) are the weighting factors of all basis modes and the weighting factors of the MCCs of the \( i \)th and the \( (i + 2) \)th, respectively. If \( A_{\text{untr}} \) and \( B_{\text{untr}} \) (i.e. equations (35) and (36)) replace \( A \) and \( B \), equations (27), (28) and (31)–(33) reduce to the analytic expressions for the mean-squared beam width, the angular spread, the \( M^2 \)-factor, the Rayleigh range and the effective radius of curvature of general untruncated beams in turbulence, and equation (30) reduces to the condition that a general untruncated beam will generate the same directionality as a fully coherent Gaussian beam, which are all omitted here in order to save space.

(ii) For \( T = 0 \) (the free space case), equations (27), (28), (31) and (32) reduce to the corresponding formulae of general truncated beams in free space, which are given by

\[ w(z)_{\text{free}} = 2 \sqrt{A + \frac{B}{k^2} z^2}, \] (37)

\[ \theta_{sp}(z)_{\text{free}} = 2 \sqrt{\frac{B}{k^2}}, \] (38)

\[ M^2_{\text{free}} = 2 \sqrt{AB}, \] (39)
Figure 1. The mean-squared beam width $w(z)$ versus the propagation distance $z$.

$$R_{\text{free}} = \frac{A + \frac{B}{k^2} z^2}{\frac{B}{k^2} z}.$$  \hfill (40)

However, equation (33) is invalid when $T = 0$ (i.e. free space). For the free space case, from equation (12) the Rayleigh range of general truncated beams can be derived, i.e.

$$z_{R_{\text{free}}} = k \sqrt{\frac{A}{B}}.$$  \hfill (41)

3. Numerical examples

Taking a truncated shG beam as an example of general truncated beams, some numerical calculations are performed to illustrate the results obtained above. The cross-spectral density function of shG beams at the $z = 0$ plane is expressed as

$$W^{(0)}(x'_1, x'_2) = \exp\left(-\frac{x'^2_1}{w^2_0}\right) \sinh\left(\Omega_0 x'_1\right) \exp\left(-\frac{x'^2_2}{w^2_0}\right) \sinh\left(\Omega_0 x'_2\right),$$  \hfill (42)

where $w_0$ is the waist width of the Gaussian part and $\Omega_0$ is the parameter associated with the sinh part.

On substituting from equations (4) and (42) into equation (5), after very tedious integral calculations, the analytic expression for the MCCs can be derived as

$$\lambda_{i,j} = \left(\frac{\pi}{2}\right)^{1/2} \gamma w_{0h} \left(2 - \gamma\right)^{(i+j)/2} \exp\left(\frac{\xi}{1+\gamma}\right) H_i\left[\frac{\gamma \xi}{(4-\gamma^2)^{1/2}}\right] H_j\left[\frac{\gamma \xi}{(4-\gamma^2)^{1/2}}\right],$$  \hfill (43)

where $i, j$ are odd numbers, $\xi = w^3_0 \Omega^2_0$ and $\gamma = 2(w_0/w_{0h})^2$. 

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Figure 2. The angular spread $\theta_{sp}$ versus the truncation parameter $b'$; $z = 10$ km.

Figure 3. $M^2$-factor versus the propagation distance $z$.

The modified von Kármán spectrum is adopted as a model of the atmospheric turbulence [1], i.e.

$$
\Phi_n(\kappa) = 0.033C_n^2 \left( \kappa^2 + 1/L_0^2 \right)^{-11/6} \exp\left(-\kappa^2/\kappa_m^2\right),
$$

(44)
Figure 4. The effective radius of curvature $R$ versus the propagation distance $z$; $b' = 2.5$.

Figure 5. The Rayleigh range $z_R$ versus the truncation parameter $b'$. 
where \( \kappa_m = 5.92/l_0 \) and \( l_0 \) and \( L_0 \) are the turbulence inner scale and outer scale, respectively. \( C_n^2 \) is the refraction index structure constant, which describes how strong the turbulence is. Letting \( l_0 = 0.01 \) m and \( L_0 = 10 \) m, equation (23) reduces to \( T = 7.067C_n^2 \).

Some numerical calculation results for the truncated shG beam are given in figures 1–5, where \( w_0 = 4 \) mm, \( \gamma = 0.5 \), \( \gamma = 6 \) and \( \lambda = 0.6328 \) \( \mu \)m are taken. Figure 1 shows that the mean-squared beam width \( w(z) \) increases with decreasing truncation parameter \( b' = b/w_0 \), but the difference of \( w(z) \) between the two cases (in free space and in turbulence) increases with increasing \( b' \).

From figure 2 it can be seen that angular spread \( \theta_{sp} \) decreases as \( b' \) increases, but the difference of \( \theta_{sp} \) between the solid line (in free space) and the dashed line (in turbulence) increases as \( b' \) increases. Figure 3 indicates that the \( M^2 \)-factor is propagation invariant in free space, but in turbulence \( M^2 \)-factor increases with increasing propagation distance \( z \). Furthermore, in turbulence \( M^2 \)-factor increases faster for \( b' = 1 \) than for \( b' = 2.5 \). From figure 4 it can be seen that there exists a minimum of the effective radius of curvature \( R \) both in free space and in turbulence. But \( R \) decreases due to turbulence. Figure 5 shows that the Rayleigh range \( z_R \) increases as \( b' \) increases both in free space and in turbulence, and the turbulence results in a decrease of \( z_R \). However, the difference of \( z_R \) between the two cases (in free space and in turbulence) increases with increasing \( b' \). Therefore, truncated laser beams with smaller truncated parameter \( b' \) are less affected by turbulence than those with larger \( b' \). In particular, truncated laser beams are less sensitive to turbulence than untruncated laser beams.

4. Conclusions

In this paper, based on the partial-coherence theory and the method of the window function being expanded into a finite sum of complex-valued Gaussian functions, the analytic expressions for second-order moments of general truncated beams propagating through atmospheric turbulence have been derived, from which some important characteristic parameters, such as the mean-squared beam width, the angular spread, the \( M^2 \)-factor, the Rayleigh range and the effective radius of curvature have been obtained. In addition, the condition that equivalent general truncated beams may have the same directionality as a fully coherent Gaussian beam both in free space and in turbulence has also been given.

The results obtained in this paper are general, e.g., the general results reduce to those for general untruncated beams in turbulence when the truncation parameter approaches infinity, to those (except the Rayleigh range) for general truncated beams in free space when the strength of turbulence is zero and to those for a certain truncated beam in turbulence if the expression for the MCCs of the corresponding beam is adopted. On the other hand, in this paper it has been shown that the method (i.e. the window function being expanded into a finite sum of complex-valued Gaussian functions) is useful in overcoming the divergence problem of second-order moments of general truncated beams. The main advantage of the method is that analytic formulae of second-order moments of general truncated beams in turbulence can be derived. Our analytic results provide an effective and convenient way of studying the propagation property and the beam quality of laser beams both in turbulence and in free space, which is useful in many practical laser applications.
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References

[1] Andrews L C and Phillips R L 2005 Laser Beam Propagation Through Random Media 2nd edn (Bellingham, WA: SPIE Optical Engineering Press)
[2] Fante R L 1985 Progress in Optics XXII ed E Wolf (Amsterdam: Elsevier) chapter VI
[3] Ricklin J and Davidson F 2002 J. Opt. Soc. Am. A 19 1794
[4] Gbur G and Wolf E 2002 J. Opt. Soc. Am. A 19 1592
[5] Dogariu A and Amarande S 2003 Opt. Lett. 28 10
[6] Cai Y and He S 2006 Appl. Phys. Lett. 89 041117
[7] Zhu Y, Zhao D and Du X 2008 Opt. Express 16 18437
[8] Pu J and Korotkova O 2009 Opt. Commun. 282 1691
[9] Ji X and Li X 2009 J. Opt. Soc. Am. A 26 236
[10] Baykal Y, Eyyuboğlu H and Cai Y 2009 Appl. Opt. 48 1943
[11] Ji X and Li X 2010 J. Opt. 12 035403
[12] Wu G, Guo H, Yu S and Luo B 2010 Opt. Lett. 35 715
[13] Ji X, Eyyuboğlu H and Baykal Y 2010 Opt. Express 18 6922
[14] Mao H and Zhao D 2010 Opt. Express 18 1741
[15] Li X, Ji X, Eyyuboğlu H and Baykal Y 2010 Appl. Phys. B 98 557
[16] Eyyuboğlu H, Baykal Y and Ji X 2010 Appl. Phys. B 98 857
[17] Zhou G 2011 Opt. Express 19 3945
[18] Korotkova O and Wolf E 2007 Opt. Lett. 32 2137
[19] Dan Y and Zhang B 2008 Opt. Express 16 15563
[20] Yuan Y, Cai Y, Qu J, Eyyuboğlu H, Baykal Y and Korotkova O 2009 Opt. Express 17 17344
[21] Ji X and Shao X 2010 Opt. Commun. 283 869
[22] Chu X, Ni Y and Zhou G 2007 Opt. Commun. 274 274
[23] Ji X and Ji G 2008 J. Opt. Soc. Am. A 25 1246
[24] Ji X and Li X 2011 Appl. Phys. B 104 207
[25] 1995 Terminology and test methods for lasers ISO Document ISO/TC172/SC9/WG1N80 (Geneva: ISO)
[26] Martínez-Herrero R and Mejías P M 1993 Opt. Lett. 18 1669–71
[27] Zhou G 2010 J. Opt. 12 015701
[28] Belanger P A, Champagne Y and Pare C 1994 Opt. Commun. 105 233–42
[29] Amarande S, Giesen A and Hügel H 2000 Appl. Opt. 39 3914–24
[30] Wunderlich J and Breazeale M 1974 J. Acoust. Soc. Am. 83 1752
[31] Ding D and Liu X 1999 J. Opt. Soc. Am. A 16 1286
[32] Lü B and Ji X 2004 J. Opt. A: Pure Appl. Opt. 6 161
[33] Du K, Herziger G, Loosen P and Rühl F 1992 Opt. Quantum Electron. 24 1081
[34] Weber H 1992 Opt. Quantum Electron. 24 1027
[35] Gradsteyn I and Ryzhik I 1980 Tables of Integrals, Series and Products (New York: Academic)
[36] Dan Y and Zhang B 2009 Opt. Lett. 34 563