Research on Double-track Railway Dispatching Considering the Deadline of the Trains

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Abstract: Train dispatching plays an important role in ensuring trains operation normally and efficiently. We endue different weights to the trains to indicate its importance and priority. Considering the deadline of the freight trains, we build the adjustment model which regard the satisfaction degree as the objective function, which include the exceeded time and the number of late trains. Matrix method is introduced to analyze the situation of train dispatching and overtaking, and the maximal algebraic method and immune genetic algorithm are used to solve the problem. Finally, we set the train dispatching situation of Suining-Sanhuizhen as an example, and verify the validity and feasibility of the two algorithms.

1. Introduction

Train dispatching is one of the key links of transportation organization, and the adjustment results directly affect the recovery speed. It is necessary to improve the quality of freight services, and ensure the punctuality rate of freight trains, and reduce the delay of trains. The current train dispatching is a priority to ensure the punctual operation of passenger trains, but less attention is paid to the punctuality of freight trains, and it is difficult to ensure the overall benefits of transport production. Because the redemption time of the freight train's arrival time is long, it is considered to be on time that the train arrives within the delivery deadline. At the same time, the number of late trains directly reflects the level of train punctuality and it is an important evaluation indicator for transport production. Therefore, it is necessary to consider passenger and freight trains as a whole and consider how to optimize efficient freight services and study efficient train operation adjustment algorithms.

The existing research results 1-7 mainly studied the tactics and constraints of train operation adjustment, and used different optimization algorithms to obtain adjustment results. The main methods are: The earliest conflict method, genetic algorithm, rough set algorithm, topological graph solving method, ordinal optimization theory and algorithm. Therefore, according to the characteristics of the train operation of the double-track railway, this paper establishes the train dispatching model considering the arrival time, and uses the Max-plus algebra method and immune genetic algorithm to solve the calculation, in order to compare the calculation speed and calculation results of the two algorithms.

2. Train operation adjustment method

2.1 Train Operation Adjustment Process

First, considering the basic constraints of train operation and the constraints of overriding, the model is established with the goal of maximum train overrun deadline and late train number. When trains are
delayed, the affected trains and stations are selected as adjustment targets. Different adjustment strategies are selected according to different late conditions, and the adjusted arrival time of the train at each station is output as a result, and an optimal adjustment scheme is obtained.

2.2 Train Operation Adjustment Model Construction

The following parameters are defined: $E_i$ denotes the interval within the segment; $S_k$ denotes the station on the segment; $P_l$ denotes the running train; $L(i)$ denotes the level of the $i$-train; $t^d_{ij}, t^a_{ij}$ indicate the departure and the arriving, the departure and arriving, between arriving, and between departure of the train at the station $S_k$; $s_{i,k}$ represents the minimum stop time of train $P_l$ at station $S_k$; $t^d_{i,j}$ represents the minimum running time of train $P_l$ in section $E_i$; $\tau_{i,k}, \mu_{i,k}$ denote respectively The start and stop time of the train on the interval $E_i$; $t^d_{i,k}$ indicates the maximum allowable delay time; $t_w$ is the delay time of the first train; $\delta_{i,k}$ is the stopping station identifier. If the station stops, then $\delta_{i,k} = 1$, otherwise $\delta_{i,k} = 0$; $\Theta$ is the operation symbol, $a\Theta b = \begin{cases} a - b & \text{if } a \geq b \\ a - b + T & \text{if } a < b \end{cases}$, $T$ denotes the adjustment period of the train diagram; $x_{i,k}^0, y_{i,k}^0$ indicate the scheduled departure and arrival time of the train $P_l$ at the station $S_k$.

The decision variable parameters: $x_{i,k}, y_{i,k}$ represent the actual departure and arrival time of the train $P_l$ at the station $S_k$.

(1) The normalized train exceeds the delivery deadline function

The difference between the actual arrival time of each train at the station along the train and the time limit of the scheduled arrival time of the train is beyond the arrival time of the train.

$$ Z_a = \sum_{k=1}^{q} \sum_{i=1}^{p} (y_{i,k} - (t^d_{i,k} + y^0_{i,k})) \quad (1) $$

To normalize, the normalized train exceeds the delivery deadline and the satisfaction function is

$$ F_a = e^{-\theta a Z_a} \quad (2) $$

In the formula, $\theta_a$ is the weight, which means that the train exceeds the adjustment factor of the expiry time index. If all trains arrive within the delivery time limit, the $F_a$ value is “1”.

(2) The normalized late train quantity function

The mathematical symbol function $Sgn(x)$ is introduced.

$$ Sgn(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \quad (3) $$

The late train quantity function $Z_b$ has the following expression:

$$ Z_b = \sum_{k=1}^{q} \sum_{i=1}^{p} [Sgn(y_{i,k} - y^0_{i,k})] \quad (4) $$

It is also normalized using a negative exponential function. The normalized number of train delays is

$$ F_b = e^{-\theta b Z_b} \quad (5) $$

In the formula, $\theta_b$ is the weight, which indicates the adjustment factor of the late train number index. If all trains are running on time, the $F_b$ value is “1”.

(3) The normalized comprehensive satisfaction function

After the above two satisfaction levels, after the normalized weighted summation, the overall satisfaction degree can be

$$ F = \omega_a \cdot F_a + \omega_b \cdot F_b \quad (6) $$

In the formula, $F$ is the comprehensive satisfaction degree, $\omega_a$ is the weight of the satisfaction of the train beyond the delivery deadline, $\omega_b$ is the weight of the late train number satisfaction, and $\omega_a + \omega_b = 1$, $\omega_a \geq 0$, $\omega_b \geq 0$.

The train operation constraints are mainly basic physical constraints, and the train interval running time constraints represent the minimum running time of the train in the interval, such as formula (10); the train stop time constraints indicate the minimum stopping time of the train at the station, such as formula (11); station interval The time constraint indicates that the two trains arrive, depart, and pass
the minimum interval time at the station, as shown in equations (12)-(15); the train arrival time constraint indicates that the actual arrival time of the train cannot be earlier than the scheduled arrival time, as shown in (16).); trains pass the rank constraint that the different classes of trains between the restrictions, such as formula (17); Yue Hang on the amendment of the restraint station that the occurrence of the deviant, you need to stop the original stop station marker correction, as in equation (18). The following is a model for the operation adjustment of a double-track train that considers the arrival time.

$$\max \ F = \omega_a \cdot F_a + \omega_b \cdot F_b$$  \hspace{1cm} (7) \\
$$F_a = e^{-\theta_a z_a}$$  \hspace{1cm} (8) \\
$$F_b = e^{-\theta_b z_b}$$  \hspace{1cm} (9)

s.t  
\[ y_{i,k+1} \theta x_{i,k} \geq t_{i,k} + \delta_{i,k} \cdot \tau_{i,k} + \delta_{i,k+1} \cdot \mu_{i,k+1} (i = 1,2,\ldots,p; k = 1,2,\ldots,q-1) \]  \hspace{1cm} (10) \\
\[ x_{i,k} \theta y_{i,k} \geq \delta_{i,k} \cdot s_{i,k},(i = 1,2,\ldots,p; k = 2,3,\ldots,q-1) \]  \hspace{1cm} (11) \\
\[ y_{j,k} \theta y_{j,k} \geq l_{i,j}^k, (i,j,k = 1,2,\ldots,q-1;i > j) \]  \hspace{1cm} (12) \\
\[ x_{i,k} \theta y_{j,k} \geq t_{d}^k (1 - \varphi_{i,j}^k), (i,j,k = 1,2,\ldots,q-1;i > j) \]  \hspace{1cm} (13) \\
\[ (1 - 2\varphi_{i,j}^k)(y_{j,k} \theta x_{i,k}) \geq l_{i,j}^k (1 - \varphi_{i,j}^k), (i,j,k = 1,2,\ldots,q-1;i > j) \]  \hspace{1cm} (14) \\
\[ x_{i,k} \theta x_{i,k}^0 \geq 0, (i = 1,2,\ldots,p; k = 1,\ldots,q-1) \]  \hspace{1cm} (15) \\
\[ y_{i,k} \theta y_{i,k}^0 \geq 0, (i = 1,2,\ldots,p; k = 2,3,\ldots,q) \]  \hspace{1cm} (16) \\
\[ L(i) - L(j) \leq M (1 - \varphi_{i,j}^k), (i,j,k = 1,2,\ldots,q-1;i > j) \]  \hspace{1cm} (17) \\
\[ 2\delta_{i,k} \geq \delta_{i,k} \cdot \varphi_{i,j}^k, (i,j,k = 1,2,\ldots,q-1;i > j) \]  \hspace{1cm} (18)

3. Max-Plus algorithm solving

3.1 Max-Plus method

Max-Plus is defined as: Let \( e = -\infty \), and \( R \) is a real number set, let \( R_{\max} = R \cup \{\varepsilon\} \), for any 2 scalars \( a, b \in R_{\max} \), \( a \oplus b = \max(a, b) \), \( a \otimes b = a + b \), \( D = \{R_{\max}, \oplus, \otimes\} \), then \( D \) is called maximal algebra, where \( \oplus \) and \( \otimes \) are the additions and multiplications of the maximal algebra respectively. \( \varepsilon \) is the unit of addition and multiplication.

3.2 Matrix representation of train operations

In order to facilitate the calculation of the model, each element in the train diagram needs to be represented in a mathematical form. The matrix row represents the train and the matrix row represents the station or interval. Train stop matrix \( S = [\delta_{i,k}]_{p \times q} \), train interval operation The time matrix \( R = [r_{i,j}]_{p \times (q-1)} \), Train The stop time matrix \( ST = [h_{i,k}]_{p \times q} \), the train moves The matrix \( O = [o_{i,j}]_{p \times (q-1)}, \) train to The time matrix \( T = [t_{i,2k-2}, t_{i,2k-1}]_{p \times 2q-2} \).

3.3 Algorithm steps

Step1: We determine the train and station range that needs to be adjusted.

Step2: We calculate the actual arrival time \( t(o(i,k-1), 2k-2), (k = m, m+1, \ldots, q-1) \) of the late train \( o(i, k-1), (i = n, n+1, \ldots, p) \) at the late station \( S_k \).

Step3: We calculate the predicted departure time \( t(o(i,k-1), 2k-1)^*, (k = m, m+1, \ldots, q-1) \) of the late train \( o(i, k-1) \) at the late station \( S_k \).

Step4: We calculate the estimated arrival time of the late train \( o(i,k-1) \) at the late station \( S_{k+1}, t(o(i,k-1), 2k)^* \).

Step5: We calculate the actual departure time \( t(o(l,k-1), 2k-1)(l = n, n+1, \ldots, p-1, j = l+1, l+2, \ldots, p) \) of the late train \( o(l,k-1) \) at the late departure station \( S_k \).

Step6: Judgment. If \( L(o(l,k)) \leq L(o(j,k-1)) \), that is, the rear vehicle level is not lower than the
preceding vehicle level, then the overtaking determination is made:

\[ t_{i,j}^{k+1} = t(o(j,k-1),2k-2) \Theta t(o(l,k),2k) \]

1. If \( t_{i,j}^{k+1} < 0 \), it belongs to the natural overtravel condition;
2. If \( t_{i,j}^{k+1} > 0 \), consider the value of \( t_{i,j}^{w} \):
   1. If \( 0 < t_{i,j}^{w} \leq t_{i,j}^{k+1} - I_{dd} \) and \( F_1 > F_2 \), then the line is not over; otherwise, the line is over;
   2. If \( t_{i,j}^{k+1} - I_{dd} < t_{i,j}^{w} \leq t_{i,j}^{k} - I_{dd} \) and \( F_1 > F_2 \), then the line is not crossed. Otherwise it will go over;
3. If \( t_{i,j}^{w} > t_{i,j}^{k} - I_{dd} \) and \( F_1 > F_2 \), do not go over the line, otherwise go over.

If a trip occurs, we calculate the late train \( o(l,k-1) \) at the late departure station \( S_k \) and actually depart \( t(o(l,k),2k-1) \).

Step 7: We determine whether all trains and stations have been adjusted. If yes, go to Step 8, otherwise go to Step 2.

Step 8: We calculate the final arrival time of the train \( o(i,q-1) \) late to get the optimal solution.

The Max-Plus algebraic solution flow is shown in Figure 1.

4. Immune genetic algorithm

4.1 Immune Genetic Algorithm
The immune genetic algorithm introduces the immune and memory functions in the organism into the genetic algorithm. This algorithm not only retains the search characteristics of the genetic algorithm, but also uses the multiple mechanisms of the immune algorithm to solve the adaptive characteristics of the optimal solution of the multi-objective function. To a great extent, it avoids "premature" and converges to local extremes.

4.2 Algorithm recursive steps
(1) Coding method. The starting sequence of the trains at each station is used as an antibody, and a real-coded method is used to define the sequence of train departures from each station as the encoding
gene. The encoding format is $x_k = (x_1^k, x_2^k, \ldots, x_p^k)$. 

(2) Fitness function. The fitness function $f$ can be the same as the objective function, and the greater the individual fitness, the better the performance of the solution.

(3) Antibody concentration. The affinity between antibodies reflects the degree of similarity between antibodies and can be expressed by formula (20).

$$S_{ij} = \frac{x_{i,j}}{L} \quad (20)$$

$x_{i,j}$ is the same part of antibodies $i$ and $j$; $L$ is the length of the antibody.

When the concentration reaches a certain value, the production of this antibody should be suppressed. On the contrary, the probability of producing a low-concentration antibody should be increased. The concentration $C_i$ is defined as the ratio of antibodies in the population that are similar to the fitness of the $i$th individual, as shown in (21).

$$C_i = \frac{1}{N} \sum_{j=1}^{N} S_{ij} \quad (21)$$

$$S_{ij} = \begin{cases} 1, & S_{ij} > \beta \\ 0, & \text{others} \end{cases}, \quad \beta \text{ is the threshold value, the general value range is } [0.9,1].$$

(4) Memory cell renewal. The individual population with the highest fitness function is added to the cell bank.

(5) Promotion and inhibition of antibody production. Find the antibody with higher concentration by formula (21) and write them as individual 1, 2, ⋯, $t$, then define the concentration density of the $t$ individuals as in (22).

$$P_d = \frac{1}{N} \left(1 - \frac{1}{N}\right) \quad (22)$$

$1 < t < N$, and the concentration probabilities of other $N-t$ individuals are shown in Formula (23).

$$P_d = \frac{1}{N} \left(1 + \frac{t^2}{N^2 - N} \times t \right) \quad (23)$$

The individual fitness probability $P_f$ is calculated using the roulette selection method, and the individual's selection rate $P$ is represented in (24), where $\alpha$ is a constant.

$$P = \alpha P_f + (1 - \alpha) P_d \quad (24)$$
6. Crossover operation. We select two intersection points, the progeny individuals are generated as follows: 1) Select the intersection position; 2) Combine the first half of the first generation with the second half of the second generation to generate children; 3) Combine the second half of the first generation with the first. The first half of the second generation generates children. It can be expressed as (25) and (26), where $a$ is a random number:

\[ x_{ik} = (1-a)x_{ik} + bx_{jk} \]  
\[ x_{jk} = (1-a)x_{ik} + bx_{jk} \]

7. Variation operation. In this paper, the variation is mainly the value of the interaction between the two genes. For example, $x_k = (x_1^1, x_2^1, \ldots, x_q^p)$, assuming that the mutation positions are 1 and 2, the mutated individuals are $x_k = (x_2^1, x_1^1, \ldots, x_q^p)$.

8. Vaccination and selection. Immune selection is the detection of individuals who have been vaccinated, comparing the fitness of the father and offspring, and choosing the best among them to enter the next generation.

9. Train arrival time is determined. Determine the arrival time of the train is based on the order of departure of the train, in order to directly determine the arrival time of the train at each station.

10. Termination conditions. The termination condition is one of the following two conditions: 1) The maximum evolutionary generation is reached; 2) The maximum computation time is reached.

The immune genetic algorithm flow chart shown in Figure 2.

**Figure 2 Flow chart of immune genetic algorithm**

5. Example analysis

5.1 Basic data
We select the train operation diagram from the upstream section of Suining to Sanhui town on the 2016 completion line as a case. The basic parameters are: the parking additional time is $\mu_{i,k} = 2\text{ min}$, the additional starting time is $\tau^p_{i,k} = 2\text{ min}$, the passenger car $\tau^p_{i,k} = 1\text{ min}$, the train does not start at the same time. The job interval time $I_{d_d}^k = 4\text{ min}$. The train does not work at the same time $I_{d_f}^k =$
The immune genetic algorithm operating parameters are set out in Table 1.

### Table 1: Immune Genetic Algorithm Parameter Settings

| Parameter     | Value |
|---------------|-------|
| Population size | 30    |
| Crossover probability | 0.8   |
| Variation probability | 0.01  |
| Maximum algebra capacity | 100   |
| Cell bank capacity | 10    |
| Concentration threshold β | 0.9   |
| Parameter α | 0.9   |

5.2 Late Assumptions and Operational Adjustment Results

Assume that the number of late trains is $n=13$, the late time is $t_w = 30$, and the late stations are $m=2$, 3, 4, 5, 6, and 7, respectively. Table 2 shows the adjustment results of the two algorithms. Figure 3 shows respectively Maximum addition algebra method and immune genetic algorithm adjust the result graph.

### Table 2: Numerical results of train operation adjustment

| Late trains | Late time | Late station | before fixing | Max-Plus algebra method | Immune genetic algorithm |
|-------------|-----------|--------------|---------------|-------------------------|--------------------------|
|             |           |              | $Z_a$ | $Z_b$ | $Z_a$ | $Z_b$ | calculating time | $Z_a$ | $Z_b$ | calculating time |
| 13          | 30        | 2            | 55    | 23    | 48    | 24    | $<2s$         | 55    | 22    | 10min         |
| 13          | 30        | 3            | 45    | 21    | 43    | 21    | $<2s$         | 40    | 20    | 8.5min        |
| 13          | 30        | 4            | 47    | 17    | 57    | 15    | $<2s$         | 44    | 16    | 8min          |
| 13          | 30        | 5            | 52    | 12    | 48    | 11    | $<2s$         | 48    | 11    | 8min          |
| 13          | 30        | 6            | 34    | 7     | 32    | 7     | $<2s$         | 32    | 7     | 7.5min        |
| 13          | 30        | 7            | 16    | 2     | 16    | 2     | $<2s$         | 16    | 2     | 7.5min        |

Figure 3: Results of Max-plus algebra method (left) and Immune genetic algorithm (right)

From the above figure, we can see that in the time when the train exceeds the delivery deadline, the use of the immune genetic algorithm can obtain better results than the maximum plus algebra method. In the late train number, the immune genetic algorithm is used to obtain slightly better results than the maximum additive algebra method. In terms of time, the use of maximal additive algebra can significantly reduce the computation time. Therefore, the immune genetic algorithm can obtain better results than the maximum additive algebra method, but the running time is too long to be suitable for the adjustment of small-scale train operation. The calculation time and calculation results are comprehensively considered, and the maximum addition algebra method is more effective.

6. Conclusion

In this paper, we use the Max-Plus algebra method and immune genetic algorithm to study the adjustment model and algorithm of the existing double-track train. Firstly, on the basis of considering the arrival time of cargo trains, an adjustment model is established to maximize the overall satisfaction of the trains beyond the delivery deadline and the number of late trains. The maximum additive algebra method and the immune genetic algorithm are used to solve the problem. Update the status of the train operation process to get the train operation adjustment program. Finally, through the actual case analysis, verify the effectiveness and feasibility of the two algorithms. The maximum plus...
algebraic algorithm has certain advantages in practical applications.

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