The gaps separating two different states widely exist in various physical systems: from the electrons in periodic lattices \[1\] to the analogs in photonic \[2\], phononic \[3\], and plasmonic \[4\] systems even the quasi-crystals \[5\]. Recently, a thermalization gap was proposed \[6\] for light in disordered structures, which is intrinsically induced by the disorder-immune chiral symmetry \[7\–10\] and can be reflected by the photon statistics. The lattice topology was further identified a decisive role in determining the photon statistics when the chiral symmetry is satisfied \[11\]. Being very distinct with one-dimensional lattices, the photon statistics in ring lattices are dictated by its parity, i.e. odd- or even-sited. Here, we for the first time experimentally observe the parity-induced thermalization gap in strongly disordered ring photonic structures. In limited scale, though the light tends to be localized, we are still able to find clear evidences of the parity-dependent disorder-immune chiral symmetry and the resulted thermalization gap by measuring photon statistics, while strong disorder-induced Anderson localization overwhelm such phenomenon in larger-scale structure. Our results shed new light on the relation among symmetry, disorder and localization, and may inspire new resources and artificial devices for information processing and quantum control on a photonic chip.

Symmetry is crucial for establishing of energy gap for electrons in lattices. Disorder often decrease such effect \[1\], but not for all cases. Certain disorder-immune chiral symmetries would emerge in random matrix theory \[12\], such as chiral \[9\] and particle-hole symmetric ensembles \[13\], and play a key role in the fields ranging from superconductivity \[14\] to quantum chromodynamics \[15\]. A distinctive characteristic of disorder-immune chiral symmetry \[12\–16\] is that the system Hamiltonian can be transformed into a block off-diagonal matrix form—a separate bipartite sub-lattices \[17\].

The tight-binding lattices have been suggested to study the disorder-immune chiral symmetry \[6\]. The resulted thermalization gap, not by parity, is circuitously confirmed in one-dimensional disordered lattices \[18\]. The parity-dependent disorder-immune chiral symmetry is predicted in ring lattices \[11\], the chiral symmetry for the even-sited ring lattices and chiral-symmetry-breaking for the odd-sited one. The resulted thermalization gap can be revealed by observing differential photon statistics.

In this letter, we demonstrate experimental observation of the parity-induced thermalization gap on a photonic chip. By using femtosecond laser direct writing, we are able to freely prototype waveguides as sites of lattices and introduce disorder by precise coupling control in three dimensions. We measure the evolved light distribution out of 120 lattices with randomly picked coupling parameters in a fixed disorder level to obtain photon statistics. We observe a thermalization gap where

![Figure 1](https://example.com/figure1.png)

**FIG. 1.** Hamiltonian form and intensity correlation of disordered ring lattices.

\( a, b \) The Hamiltonian of off-diagonal disordered ring lattices with different site number. The Hamiltonian for even-sited ring lattices could be rearranged into block off-diagonal matrix form \((a)\), but not for odd-sited \((b)\). \( c \) There exists a significant difference of intensity correlation \(g^{(2)}\) between odd- and even-sited ring lattices. The difference becomes smaller and even disappears when the disorder level and the site number increase.
the cross correlation in even-sited lattices is significantly larger than the one in odd-sited lattices. This work may deepen the understanding of the relation among symmetry, disorder and localization, and inspire applications for quantum integrated photonics.

The dynamics of light in photonic lattices can be described by a set of coupled discrete Schrödinger equations \[ \frac{-i}{\partial z} \psi_n = c_{n-1} \psi_{n-1} + c_{n+1} \psi_{n+1} \] (1)
where \( \psi_n \) is the complex field amplitude of site \( n \), \( z \) is the propagation distance along the waveguides and maps the time variable, coefficient \( c \) represents the coupling strength between the neighboring waveguides. The equations could also be rewritten in matrix form: \[ -i \frac{\partial}{\partial z} \psi = H \psi, \] where the Hamiltonian \( H \) is coupling coefficient matrix.

We consider a photonic lattice model in which the sites are arranged in a closed ring structure with nearest-neighbor-only coupling. The off-diagonal disorder [23], especially, is introduced into the lattice via randomly varying coupling coefficient \( c \). Here, we consider \( c \) is uniform probability distributed between \( \bar{c} - \Delta c \) and \( \bar{c} + \Delta c \), where \( \bar{c} \) is the average value of the coupling coefficient, \( \Delta c \) is half-width of \( c \) range, and defines the disorder level as \( \eta = \Delta c / \bar{c} \).

In this model, the Hamiltonian \( H \) for even-sited ring lattice could be rearranged into a block off-diagonal matrix form (Fig.1(a)), while the \( H \) for odd-sited ring lattice cannot (Fig.1(b)). It means that the disorder-immune chiral symmetry emerges in even-sited ring lattices, but not in odd-sited ring lattices. The interplay between disorder and symmetry could be revealed through observing the high-order statistics: the normalized intensity correlation of light after evolution, corresponding to the photon statistics \( g^{(2)} \) in quantum optics theory, would be larger when the disorder-immune chiral symmetry is satisfied. The \( g^{(2)} \) can be obtained by measuring the light intensity [6]:
\[ g^{(2)} = \frac{\langle I^2 \rangle}{\langle I \rangle^2} \] (2)
where \( \langle \cdot \rangle \) denotes ensemble average. Apparently, the \( g^{(2)} \) is uniformly equal to 1 for all non-disordered system.

We investigated the dynamics under this lattice model, and the result shows there is indeed a thermalization gap between the even- and odd-sited ring lattices. In Fig.1(c), the simulation result of \( g^{(2)} \) for the excited sites is illustrated. At low disorder level, the \( g^{(2)} \) of even-sited ring lattices is substantially larger than the one of odd-sited ring lattices, representing an existence of thermalization gap between them. The thermalization gap tends to be smaller when the disorder level and the system scale increase, and even disappeared, see the upper right area in Fig.1(c). The physical mechanics behind is that the strong Anderson localization behavior prevents the light from spreading and therefore breaks the disorder-immune chiral symmetry.

The relation among system scale, disorder and local-
FIG. 3. Sketch of ring lattices and experimental observation of parity-induced thermalization gap. 

a. The ring lattice chips are fabricated by using femtosecond laser direct writing technique. The propagation distance \( z \) along waveguides in the ring lattices maps the time variable. 
b. Sketch of introducing uniform-probability-distributed disorder. The shade of gray denotes the probability of position distribution, which is uniform due to the exponential relation between the separation distance and coupling strength. 
c. Experimentally imaging the output intensity distributions for samples up to 120. 
d. The \( g^{(2)} \) of even-sited ring lattices is distinctly larger than the value of odd-sited ring lattices. The gray ribbon presents the thermalization gap centered at 1.4.

The localization level can be well visualized in Fig. 2(a). We define 
\[
\lambda = \frac{1}{2} \sum_{i=1}^{N} |I_i - 1/N|
\]
to quantify the localization level, where \( N \) is the site number. We identify a bound of 0.2 to divide the regions for being able to observe thermalization gap and localization respectively (see Methods). We can see that the disorder must be large enough to enable the observation of the two phenomena while the ultimate large value is not accessible in practice. Our simulation suggests that a disorder level of 0.8 is preferable since we can observe the thermalization gap for the site number under 6 and localization for the site number above 11.

Besides, the size of ensemble seriously impacts the width of the gap. As a statistical result, the thermalization gap appears only when the size of ensemble is large enough. As showed in Fig. 2(b,c), the \( g^{(2)} \) uncertainty range of odd- and even-sited ring lattice overlaps when...
FIG. 4. Experimental observation of Anderson localization.

a, b. The imaged output intensity distributions of 11- and 12-sited ring lattices. For each sample, the light tends to be localized in one or two sites.

c, d. The averaged results of 11- and 12-sited ring lattices show that the light is more likely to be localized in the excited site and nearest-neighboring sites.

e. The averaged intensity distributions of 3-, 4-, 5- and 6-sited ring lattices. The light tends to be equally localized to all the sites.

f. The measured localization level agrees well with theoretical prediction. The black/white arrows mark the excited sites.

the ensemble size is small, and significantly reduces as the ensemble size increasing. As long as the sample number larger than 100, the expected values of $g^{(2)}$ tends to stable and their uncertainty ranges are substantially suppressed so that the thermalization gaps become clearly distinguishable. The results imply that over hundreds of ring lattices have to be fabricated and measured for every fixed site number to confirm the disorder-immune chiral symmetry emerging, which is very experimentally challenging, not only the huge workload but also repeatability of fabrication and testing systematic parameters.

In our experiment, we construct the ring lattices in borosilicate glass wafer using femtosecond direct laser writing technique (Fig. 3(a)) [24–28]. We introduce $\eta = 0.8$ off-diagonal disorder into the ring lattices. The uniform-probability-distributed random coupling coefficients $c$ between each two sites are realized by precisely controlling the relative positions of waveguides (see Methods). As shown in Fig.3(b), the position distribution of each site is not uniform due to the exponential relation between the separation distance and coupling strength [29]. We achieve the aforementioned demanding requirement on repeatability by monitoring and locking the parameters of the laser, the chip carrier and the lab environment with close loop. We fabricate 120 and 200 ring lattices for each site number to observe the thermalization gap and the strong Anderson localization behavior respectively.

In Fig.3(c), we display part of the experimentally imaged output intensity distribution. We can see that the light tends to be localized but still spreads to all the sites. We can derive the intensity correlation $g^{(2)}$ with all these experimental data (see Methods). The obtained correlations of even-sited ring lattices in which the disorder-immune chiral symmetry is satisfied, $1.46 \pm 0.019$ for 4-sited and $1.50 \pm 0.024$ for 6-sited, are distinctly larger than the values of odd-sited ring lattices, $1.37 \pm 0.015$ for 3-sited and $1.32 \pm 0.013$ for 5-sited (see Fig.3(d)). We can see that there is a clear thermalization gap around 1.4, which implies the distinct difference of thermal behavior in photon statistics for odd- and even-sited ring lattices. The disorder-immune chiral symmetry and induced thermal gap are not directly visible from individual output density image, but can be revealed by probing the high-order statistics. Surprisingly, albeit the imperfection of fabrication and measurement may happen over 480 samples, the predicted photonic thermalization gap still can be successfully observed, revealing the robustness of the disorder-immune chiral symmetry.

As has been shown in Fig.2(a), with the same disorder level of 0.8, we will be able to observe that Anderson localization emerges while the thermalization gap disappears for the site number above 11. The intensity distributions of 11- and 12-sited ring lattices are shown in Fig.4(a,b). The light behaves in a strongly localized manner, and it is worth mentioning that the specifically lo-
calized position of the light can be whichever site, not necessarily at the excited site. After averaging all samples result, the strong disorder-induced Anderson localization is even more distinct. In a finite evolution time, the light is more likely to be localized in the excited site and nearest-neighboring sites (see Fig. 4c–d), for example, the proportions of the excited site are up to 0.24 and 0.25 for 11- and 12-sited ring lattices respectively.

Fig. 4e shows the averaged intensity distributions of 3-, 4-, 5- and 6-sited ring lattices. In contrast, the light spreads to all the sites and the proportion of every site is nearly uniform though the light tends to be localized in every individual sample. We calculate the localization level $\lambda$ for the experimental data, and the result agrees well with the simulation, $\lambda < 0.2$ for 3-, 4-, 5- and 6-sited ring lattices and $\lambda > 0.2$ for 11- and 12-sited ring lattices (see Fig. 4f). Interestingly, with same disorder level, Anderson localization is weak in limited-scale lattice structures, but becomes strong and breaks the disorder-immune chiral symmetry in large scale.

In summary, we experimentally observe a parity-induced thermalization gap in strongly-disordered ring photonic lattices. We identified the existence of disorder-immune chiral symmetry in even-sited disordered ring lattices by measuring photon statistics, manifesting an intensity correlation dependence on the parity of site number.

Our results have interpreted a delicate relation among symmetry, disorder and localization. The parity-induced thermalization gap is clear in a limited-scale structure, while the disorder-induced Anderson localization becomes dominant in a larger-scale structure. Though the symmetry is normally sensitive to the disorder, but can become more robust when the chiral symmetry is satisfied. The chiral symmetry still can be broken by a large disorder, especially in large-scale structure.

With the successful experimental implementation here, many fascinating questions related to entropy generation can be explored in the future [6], for example, the potential impact of nonlinearity induced in the lattice, the dynamics with a varied time dimension, and the evolution of non-classical light in activated chiral lattices. It is also intriguing to investigate the existence of disorder-immune symmetry and the associated thermalization gap in quasi-crystals [5, 30] and Hubbard model [31, 33].

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**Methods**

**The bound of observing thermalization gap and Anderson localization** It should be noticed that the localization level defined in previous work [21] can not be directly applied to this work. Their definition of localization level is not sensitive to the site number. With each fixed site number, the averaged distributions of light and their trend as a function of disorder level can be visually seen in Supplementary Fig. S1 corresponding to the results presented in Fig. 2(a). To identify the bound of dividing the regions for being able to observe thermalization gap and localization respectively, we present the results of the simulated $g^{(2)}$ and the scale of localization level together. As shown in Supplementary Fig. S2(a), we mark the critical positions with red dots where the gap tends to disappear. We then mark the corresponding position in Supplementary Fig. S2(b) which give a bound around 0.2.

**Introducing uniform-probability-distributed disorder:** Fig. 3b illustrates how to introduce uniform-probability-distributed disorder. We determine each site’s position before performing fabrication. Firstly we determine the separation distance between each two nearest-neighboring sites according to characterized coupling coefficient, then calculate and fix the circumscribed circle since all sites supposed to stay on a circle, then we will be able to fix the position of every site. The average separation distance is 9.76 $\mu$m corresponding to $\bar{c} \approx 0.5 \text{ mm}^{-1}$ at the light wavelength of $\lambda = 852 \text{ nm}$. The position of each site is specifically determined so that the coupling strength is uniformly distributed with the disorder level $\eta = 0.8$. It should be noticed that the relation between the separation distance and the coupling strength is not linear. In every lattice, the waveguides are designed to be 34.5 $\mu$m long, corresponding to the normalized propagation distance $\bar{z} \approx 17.25$, to ensure that the light has hoped sufficiently before being measured.

**Deriving the intensity correlation $g^{(2)}$:** To faithfully retrieve the intensity correlation $g^{(2)}$, the condition of statistical stationarity needs to be satisfied: the total density at the output facet of every sample is uniform, which means it is necessary to normalize the output density before calculating $g^{(2)}$. The next challenge is to give a preciser statistical uncertainty from 120 samples. We process the experiment data with Monte Carlo method. We randomly select $N$ samples from 120 experiment results, then calculate the $g^{(2)}$ of the $N$ samples selected according to the equation (2). We repeat the steps above for $M$ times. After those operations, there are $M$ results: $g^{(2)}_1, g^{(2)}_2, \ldots, g^{(2)}_i, \ldots, g^{(2)}_M$. We set the average value $\bar{g}^{(2)} = \frac{1}{M} \sum_i g^{(2)}_i$ as the final experiment $g^{(2)}$ result and the standard deviation is the uncertainty.
I. SUPPLEMENTARY FIGURES - THE BOUND OF OBSERVING THERMALIZATION GAP AND ANDERSON LOCALIZATION
FIG. 5. Averaged distribution of light varying with the disorder level and site number.
FIG. 6. The derivation of the bound of observing thermalization gap and Anderson localization.