


table

**TABLE I**

| Representation  | Solution Space       | Packing Time | Flexibility |
|-----------------|----------------------|--------------|-------------|
| Normalized Polish Expression | \(O(n^{2/3} \cdot n^{1/3})\) | \(O(n)\) | slicing |
| Corner Block List \([3]\) | \(O(n^{2/3} / n^{1/3})\) | \(O(n)\) | Mosaic |
| Twin Binary Sequence \([4]\) | \(O(n^{2/3} / n^{1/3})\) | \(O(n)\) | Mosaic |
| O-tree \([5]\) | \(O(n^{2/3} / n^{1/3})\) | \(O(n)\) | compacted |
| B*-tree \([6]\) | \(O(n^{2/3} / n^{1/3})\) | \(O(n)\) | compacted |
| Corner Sequence \([7]\) | \(< \{w\}^k\) | \(O(n)\) | compacted |
| Sequence Pair \([8]\) | \(\{w\}^k\) | \(O(n^2)\) | general |
| TCGS \([9]\) | \(\{n\}^2\) | \(O(n \log n)\) | general |
| ACG \([10]\) | \(O(n^4)\) | \(O(n^6)\) | general |

modules. Floorplanning usually includes two types of circuit modules: 1) hard modules and 2) soft modules. The width and height of a soft module can be changed within a certain range meanwhile maintaining a certain area. So far, works for fixed-outline floorplanning can be divided into three categories: 1) heuristic methods for a small scale problem; 2) multilevel heuristic methods for a large-scale problem; and 3) analytical approaches.

Heuristic methods for small-scale fixed-outline floorplanning problem first adopt a floorplan representation, then use heuristic algorithms to search for an optimal solution in the representation space, and finally decode the optimal representation to the corresponding floorplan. Floorplan representation method also determines the complexity of transforming a floorplan into its representation, and the complexity of decoding a representation into its corresponding floorplan. Floorplan representation methods can be summarized in Table I.

In Table I, O-tree \([5]\) and B*-tree \([6]\) are two commonly used floorplan representations, followed by the most preferred simulated annealing algorithm. Parquet \([11]\) used sequence pair as the representation, on which a floorplan is also optimized using simulated annealing. Since the running time of this kind of algorithms grows very fast with the problem scale, they cannot be applied to large-scale floorplanning directly. A methodology to cope with this issue is the multilevel framework with the heuristic search methods.

The core idea of multilevel heuristic search methods is reducing the size of solution space multilevel by clustering or partitioning, and thus can handle the large-scale problem. Clustering-based floorplanning methods, e.g., MB*-tree \([12]\), merge modules from the bottom to up, until the number of modules is small enough, and then solve the floorplanning problem. The solution is refined in each level of de-clustering, leading finally to a solution of the original problem. A disadvantage of clustering-based floorplanning is that, modules cannot be clustered from a global perspective in the early
that, AR uses a relative position matrix, while UFO uses legal solutions in the legalization stage. The difference is them use second-order cone programming (SOCP) to obtain els are proposed for global floorplanning, respectively. Both of them as circles. Then, Attractor-Repeller and Push-Pull mod-
al soft and hard modules as squares, and further abstract and building an ILP model to achieve legalization of modules. polygon to approximate the curve of area of every soft module, legalization stages of the above methods, respectively.

Floorplanners analytical [18], F-FM [1], and Lin et al. [2] take soft modules as hard ones in global floorplanning. Moreover, they must be incorporated in the multilevel framework to handle large-scale floorplanning problem. Apparently, these will affect the solution quality.

In this article, we focus on the fundamental fixed-outline floorplanning problem. This is because algorithms for floorplanning with complex issues are extended from those for the fundamental floorplanning. Notice that Poisson’s equation-based VLSI global placement methods [24], [25] do not use multilevel framework. They are flat while achieve state-of-the-art results. In this article, we will propose a high-performance floorplanning engine by extending this type of methods to large-scale fixed-outline floorplanning with hard and soft modules.

To achieve this goal, we must not only consider optimizing the shapes of soft modules in global floorplanning, but also design a high-performance legalization algorithm correspondingly. Contributions of this article are summarized as follows.

1) We propose a novel mathematical model to characterize nonoverlapping of modules. Then under the concept of electrostatic system [24], we redefine the potential energy of modules based on the analytical solution of Poisson’s equation in [25]. This allows the widths of soft modules to appear as variables in the established fixed-outline global floorplanning model, and the derivative with respect to the widths of soft modules can be easily obtained.

2) We give a fast approximate computation scheme for the partial derivative of the potential energy, and design a fixed-outline global floorplanning algorithm based on the global placement method using Poisson’s equation.

3) By defining horizontal and vertical constraint graphs, we present a fast legalization algorithm to remove overlaps between modules after global floorplanning. The algorithm modifies relative positions of modules and compresses the floorplan to obtain finally a legal solution.

4) Within acceptable running time, our floorplanning algo-



| method       | global floorplanning | legalization          |
|--------------|----------------------|-----------------------|
| Analytical [18] | SOCP based on relative position matrix | SOCP |
| F-FM [1]     | placement model based on NTPlace [21] | SOCP |
| Lin et al. [2] | placement model based on NTPlace [21] | SAINT |
| AR [19]      | Attractor-Repeller model | SOCP based on constraint graph |
| UFO [20]     | Push-Pull model       | SOCP based on constraint graph |

TABLE II

SUMMARY OF ANALYTICAL FLOORPLANNING METHODS

earlier floorplanners AR [19] and UFO [20] directly regard all soft and hard modules as squares, and further abstract them as circles. Then, Attractor-Repeller and Push-Pull models are proposed for global floorplanning, respectively. Both of them use second-order cone programming (SOCP) to obtain legal solutions in the legalization stage. The difference is that, AR uses a relative position matrix, while UFO uses constraint graph. It must be remarked that solving SOCP is rather time-consuming when the problem size is moderate or large.

Note that floorplanning problem contains hard and soft modules. However, state-of-the-art analytical floorplanners analytical [18], F-FM [1], and Lin et al. [2] take soft modules as hard ones in global floorplanning. The remainder of this article is organized as follows. Section II is the preliminaries. Section III proposes a novel mathematical model to characterize nonoverlapping of modules, gives our global floorplanning model based on Poisson’s equation, and provides a fast computation scheme for the partial derivatives. In Section IV, we detail our fixed-outline global floorplanning algorithm and legalization algorithm. Experimental results and comparisons are presented in Section V, and conclusion is summarized in Section VI.
II. PRELIMINARIES

This section presents the problem of fixed-outline floorplanning, introduces the wirelength model and principle of Poisson’s equation method for handling modules overlap in VLSI global placement.

A. Fixed-Outline Floorplanning Problem

The floorplanning area is a rectangle with four edges parallel to the coordinate axes, with the lower left corner at the origin, and the upper right corner at \((W, H)\). Let \(V = V_s \cup V_h\) be the set of rectangular modules, where \(V_s\) and \(V_h\) represent the sets of soft and hard modules, respectively. For each module \(v_i \in V\), its width, height, area, and the coordinate of the center point are \(w_i, h_i, A_i\), and \((x_i, y_i)\), respectively. For each module \(v_i \in V_s\), the aspect ratio, calculated as the ratio of height to width, is between \(AR_u\) and \(AR_l\). Moreover, let \(E\) be the set of nets \(e_i\), \(i = 1, 2, \ldots, m\), for which \(WL(e_i)\) is the half-perimeter wirelength. Let \(WL(E) = \sum_{e_i \in E} WL(e_i)\). Then the fixed-outline floorplanning problem can be described as follows:

\[
\begin{align*}
\min & \quad WL(E) \\
\text{s.t.} & \quad \text{overlap} = 0 \\
& \quad \text{aspect ratio is suitable} \\
& \quad \text{all modules are within the fixed-outline.} \quad (1)
\end{align*}
\]

In this article, the objective of model (1) adopts the Log-Sum-Exp (LSE) wirelength function. Moreover, the first constraint in (1) ensures that all modules do not overlap. The second constraint means that the aspect ratio of every soft module meets the upper and lower bounds. The third constraint indicates that all modules are located within the fixed outline.

B. VLSI Global Placement Based on Poisson’s Equation

ePlace [24] is a representative VLSI global placement method. In ePlace, each module \(v_i\) is regarded as a positively charged particle with electric quantity \(q_i\), which is proportional to its area \(A_i\). Thus, the placement problem is modeled as a 2-D electrostatic system, in which the electric potential \(\psi(x, y)\) and the electric field \(\xi(x, y)\) satisfy that \(\xi(x, y) = (\xi_x, \xi_y) = -\nabla \psi(x, y)\). ePlace [24] characterizes \(\psi(x, y)\) as the solution of Poisson’s equation

\[
\begin{align*}
\nabla^2 \psi(x, y) &= -\rho(x, y), \quad (x, y) \in R \quad (2a) \\
\hat{n} \nabla \psi(x, y) &= 0, \quad (x, y) \in \partial R \quad (2b) \\
\iint_R \rho(x, y)dxdy &= \iint_R \psi(x, y)dxdy = 0 \quad (2c)
\end{align*}
\]

where \(R\) is the rectangular placement area. Equation (2a) is Poisson’s equation, \(\rho(x, y)\) is the charge density function on \(R\). Equation (2b) is the Neumann boundary condition, where \(\partial R\) and \(\hat{n}\) represent the boundary of area \(R\) and the outer normal vector of the boundary, respectively. Equation (2c) is the compatibility condition to ensure that the equation has a unique solution.

The electric forces \(F_i = q_i \xi(x_i, y_i), \quad i = 1, 2, \ldots, n\), guide the modules movement and henceforth reduce modules overlaps. Define the system potential energy \(N(V) = \sum_{v_i \in V} N(v_i)\), where \(N(v_i) = q_i \psi(v_i)\) is the potential energy of module \(v_i\). Then by ePlace [24], a necessary condition for \(\text{overlap} = 0\) is that the system potential energy \(N(V)\) is the smallest. This allows to use the regularizing term \(\lambda N(V)\) to transform the VLSI global placement into

\[
\begin{align*}
\min & \quad \text{LSE}(E) + \lambda N(V) \\
\text{s.t.} & \quad \text{all modules are in the placement area.} \quad (3)
\end{align*}
\]

Zhu et al. [25] defines the density function

\[
\rho(x, y) = \sum_{v_i \in V} \rho_i(x, y), \quad (x, y) \in [0, W] \times [0, H]
\]

\[
\rho_i(x, y) = \begin{cases} 
1, & \text{if } (x, y) \in R_i \\
0, & \text{else}
\end{cases} \quad (4)
\]

where \(R_i = [x_i - (w_i/2), x_i + (w_i/2)] \times [y_i - (h_i/2), y_i + (h_i/2)]\), \(h_i = (A_i/w_i)\). Then, by redefining

\[
\hat{\rho}(i, j) = \rho(x, y) - \frac{1}{WH} \iint_R \rho(x, y)dxdy.
\]

Zhu et al. [25] obtains an analytical solution of (2). The truncated version is

\[
\psi(x, y) = \sum_{u=0}^{K} \sum_{p=0}^{K} a_{up} \cos\left(\frac{u\pi}{W}x\right) \cos\left(\frac{p\pi}{H}y\right) \quad (5)
\]

where \(K\) is the truncation factor that controls the precision.

For fast global placement, assume that the placement area is divided in to \(K \times K\) bins, and the density of each grid is

\[
\hat{\rho}(i, j) = \frac{\text{Area}(\text{bin}_{i,j} \cap R_k)}{\text{Area}(\text{bin}_{i,j})}, \quad i, j = 0, 1, \ldots, K - 1. \quad (6)
\]

By a similar redefinition, Zhu et al. [25] obtained an analytical solution of (2), for which the truncated solution value at \((i + (1/2), j + (1/2))\) is

\[
\hat{\psi}(i, j) = \sum_{u=0}^{K-1} \sum_{p=0}^{K-1} a_{up} \cos\left(\frac{u(i + \frac{1}{2})\pi}{K}\right) \cos\left(\frac{p(j + \frac{1}{2})\pi}{K}\right). \quad (7)
\]

The coefficient calculations for (5) and (7) can be found in [25].

III. GLOBAL FLOORPLANNING MODEL AND GRADIENT CALCULATION

In this article, the fixed-outline floorplanning is decomposed into two stages: 1) global floorplanning and 2) legalization. In global floorplanning, partial overlaps between modules are allowed and a rough floorplanning is built. In legalization, the overlaps are eliminated and the final legal floorplan is obtained. To be able to optimize the shapes of soft modules in global floorplanning, we propose in this section a novel mathematical model to characterize nonoverlapping of modules, and then modify the VLSI global placement model (3) for fixed-outline global floorplanning. Based on the global floorplanning model, we present a fast approximate scheme for calculating partial derivatives of potential energy.
A. Global Floorplanning Model

First, we present a sufficient and necessary condition for nonoverlapping of modules. According to (4), we can prove the following results. The details will be clarified in the Appendix.

Lemma 1: Any two different modules \( v_i \) and \( v_j \) in \( V \) are nonoverlapping if and only if \( \int_{R_i} \rho_i(u, v)dv < 0 \).

Let Area\((R_i)\) be the area of module \( v_i \), we have the following lemma.

Lemma 2: \( \int_{R_i} \rho_i(u, v)dv = \text{Area}(R_i) \) for all modules \( v_i \in V \).

Theorem 1: Any module \( v_i \in V \) is nonoverlapping with other modules if and only if

\[
\int_{R_i} \sum_{v_j \in V} \rho_j(u, v)dv = \text{Area}(R_i).
\]

According to Lemma 2 and Theorem 1, it is obvious that

\[
\int_{R_i} \sum_{v_j \in V} \rho_j(u, v)dv \geq \text{Area}(R_i).
\]

Moreover, by Theorem 1, the nonoverlapping constraint in (1) is equivalent to (8). Therefore, by the regularization technique in machine learning [26], the fixed-outline floorplanning problem (1) can be transformed to

\[
\min_{v_i \in V} |\text{LSE}\{\theta\}| + \lambda \sum_{v_i \in V} \left( \int_{R_i} \sum_{v_j \in V} \rho_j(u, v)dv \right)
\]

s.t. aspect ratio is suitable

all modules are within the fixed-outline.

In the floorplanning model (9), \( \sum_{v_j \in V} \rho_j(u, v) \) is interpreted as the density function within the floorplanning region. According to VLSI global placement, there are three optional methods to relax the density function for facilitating optimization as follows.

1) Divide the floorplanning area into bins, smooth the corresponding density function with the Bell-shaped function, and then make integration.

2) Divide the floorplanning area into bins, smooth the corresponding density function with Poisson’s equation, and then make integration.

3) Make integration of \( \sum_{v_j \in V} \rho_j(u, v) \) directly.

In this article, we use scheme 2 since Poisson’s equation method has achieved state-of-the-art results in VLSI global placement. Let \( \psi(u, v) \) be the function given by Poisson’s equation (2) to smooth the density function \( \sum_{v_j \in V} \rho_j(u, v) \) defined in (5). Suggested by (9), for each module \( v_i \in V \), we define the potential energy \( N_i \) as follows:

\[
N_i = \int_{R_i} \psi(u, v)dv
\]

where the integration area \( R_i = [x_i - (w_i/2), x_i + (w_i/2)] \times [y_i - (h_i/2), y_i + (h_i/2)] \), \( h_i = (A_i/w_i) \).

Observing (10), for a soft module, the potential energy \( N_i \) is a function of variables \( x_i, y_i, \) and \( w_i \). While, for a hard module, the potential energy \( N_i \) is a function of variables \( x_i \) and \( y_i \). Therefore, for convenience we denote

\[
g_i(x_i, y_i, w_i) = N_i = \int_{R_i} \psi(u, v)dv \quad \forall v_i \in V_s
\]

\[
g_i(x_i, y_i) = N_i = \int_{R_i} \psi(u, v)dv \quad \forall v_i \in V_h.
\]

According to (11) and the fixed-outline floorplanning problem (9), we can establish the global floorplanning model

\[
\begin{align*}
\min_{x_i, y_i, w_i} \text{LSE}\{\theta\} + \lambda \left( \sum_{v_i \in V_s} g_i(x_i, y_i, w_i) + \sum_{v_i \in V_h} g_i(x_i, y_i) \right) & \\
\text{s.t.} \quad A_i = \frac{\lambda_i}{\lambda S_i} \quad \forall v_i \in V_s & \\
\frac{w_j}{2} \leq x_i \leq W - \frac{w_j}{2} & \forall v_i \in V_s & \\
\frac{h_i}{2} \leq y_i \leq H - \frac{h_i}{2} & \forall v_i \in V_s & \\
\end{align*}
\]

In (12), the first constraint ensures the aspect ratio of each soft module be between the upper and lower bounds. The last two constraints indicate that all modules are located within the floorplanning area.

In order to use a nonlinear optimization method to solve problem (12), the partial derivative of the objective function with respect to each variable needs to be calculated. Since the wirelength function LSE\(\{\theta\}\) is in analytical form, the partial derivative is easy to calculate. Moreover, the difference between the potential energy functions of soft modules and hard modules is only in the variable \( w_i \). Therefore, we only show calculations of the partial derivatives of potential energy functions of soft modules in the next section.

B. Partial Derivative of Potential Energy

In order to calculate the partial derivative of \( g_i(x_i, y_i, w_i) \), first we need to calculate \( g_i(x_i, y_i, w_i) \) itself. According to Section II-B, Poisson’s equation has the solution as in (5). Substituting it into (11) can get

\[
g_i(x_i, y_i, w_i) = \int_{R_i} \psi(z, v)dv
\]

\[
= \sum_{u=0}^{K} \sum_{p=0}^{K} a_{u,p} \left[ \int_{x_i - \frac{w_i}{2}}^{x_i + \frac{w_i}{2}} \cos\left( \frac{\pi u z}{W} \right) dz \right] \int_{y_i - \frac{h_i}{2}}^{y_i + \frac{h_i}{2}} \cos\left( \frac{\pi v z}{H} \right) dv
\]

\[
= \sum_{u=0}^{K} \sum_{p=0}^{K} a_{u,p} WH \left[ \sin \left( \frac{\pi u (x_i + \frac{w_i}{2})}{W} \right) - \sin \left( \frac{\pi u (x_i - \frac{w_i}{2})}{W} \right) \right] \left[ \sin \left( \frac{\pi v (y_i + \frac{h_i}{2})}{H} \right) - \sin \left( \frac{\pi v (y_i - \frac{h_i}{2})}{H} \right) \right]
\]

\[
+ \sum_{u=0}^{K} a_{u,0} WH \left[ \sin \left( \frac{\pi u (x_i + \frac{w_i}{2})}{W} \right) - \sin \left( \frac{\pi u (x_i - \frac{w_i}{2})}{W} \right) \right] \left[ \sin \left( \frac{\pi v (y_i + \frac{h_i}{2})}{H} \right) - \sin \left( \frac{\pi v (y_i - \frac{h_i}{2})}{H} \right) \right]
\]

\[
+ \sum_{p=0}^{K} a_{0,p} WH \left[ \sin \left( \frac{\pi u (x_i + \frac{w_i}{2})}{W} \right) - \sin \left( \frac{\pi u (x_i - \frac{w_i}{2})}{W} \right) \right] \left[ \sin \left( \frac{\pi v (y_i + \frac{h_i}{2})}{H} \right) - \sin \left( \frac{\pi v (y_i - \frac{h_i}{2})}{H} \right) \right].
\]

Next, calculate the partial derivatives of (13) with respect to \( x_i, y_i, \) and \( w_i \), respectively

\[
\frac{\partial g_i(x_i, y_i, w_i)}{\partial x_i} = \sum_{u=1}^{K} \sum_{p=1}^{K} a_{u,p} \frac{H}{W} \left[ \sin \left( \frac{\pi u (x_i + \frac{w_i}{2})}{W} \right) - \sin \left( \frac{\pi u (x_i - \frac{w_i}{2})}{W} \right) \right] \left[ \frac{\pi u (y_i + \frac{h_i}{2})}{H} - \frac{\pi u (y_i - \frac{h_i}{2})}{H} \right]
\]
\[
+ \sum_{i=1}^{K} a_{i,0} \frac{W_h}{W} \left\{ \frac{\cos \left( \frac{x_i + \frac{w_i}{2}}{W} \right)}{W} + \cos \left( \frac{x_i - \frac{w_i}{2}}{W} \right) \right\} - \frac{p A_i}{H w_i} \left\{ \frac{\sin \left( \frac{y_i + \frac{b_i}{2}}{H} \right)}{H} - \sin \left( \frac{y_i - \frac{b_i}{2}}{H} \right) \right\} - \frac{p A_i}{H w_i} \left\{ \frac{\sin \left( \frac{x_i + \frac{w_i}{2}}{W} \right)}{W} - \sin \left( \frac{x_i - \frac{w_i}{2}}{W} \right) \right\} - \frac{p A_i}{H w_i} \left\{ \frac{\sin \left( \frac{y_i + \frac{b_i}{2}}{H} \right)}{H} - \sin \left( \frac{y_i - \frac{b_i}{2}}{H} \right) \right\} \right\}.
\]

At this point, the partial derivative calculations are completed.

For each module \( v_i \), according to (14)–(16), the time complexity required to calculate the partial derivatives with respect to the variables \( x_i, y_i, \) and \( w_i \) are \( O(K^2) \), when the coefficients are given. Here, \( K \) is the truncation constant in (5), which is used to control the accuracy of the solution. Therefore, the time complexity of calculating partial derivatives for all \( n \) modules is \( O(n K^2) \), which is linear with \( n \). However, experiments show that the constant \( K^2 \) is very large. Thus, even if the run-time increases linearly with the scale of the problem, it still takes a large run-time to solve a large-scale problem. Based on this, the next section looks for a way to further reduce the run-time complexity.

C. Fast Approximate Calculation of Partial Derivatives of Potential Energy

From (6) and (7), the floorplanning area \([0, W] \times [0, H]\) has been discretized into \( K \times K \) bins. The width and height of each bin are \( w_b = (W/K) \) and \( h_b = (H/K) \), respectively. Note that \( R_i \) is the rectangular area occupied by module \( v_i \). Let \( B_q \) be the set of bins occupied by \( R_i \), and denote the right, left, upper, and lower boundaries of \( B_q \) as \( X_i', Y_i', X_i'' \), and \( Y_i'' \), respectively. That is, \( B_q = \{ \text{bin} \mid x \leq p \leq X_i', Y_i' \leq q \leq Y_i'' \} \).

First, we use auxiliary functions to determine the minimum number of bins covering the rectangular area \( R_i \). The auxiliary functions are defined as follows:

\[
\begin{aligned}
\text{up}(x) &= \begin{cases} 0, & \text{if } x = 0 \\ \left\lfloor x \right\rfloor + 1, & \text{else } x > 0 \end{cases} \\
\text{down}(x) &= \left\lceil x \right\rceil, \quad x \geq 0.
\end{aligned}
\]

Then, the four boundaries of the minimum bins covering the rectangular area \( R_i \) can be calculated by

\[
X_i' = \text{down} \left( \frac{x_i - \frac{w_i}{2}}{w_b} \right), \quad X_i'' = \text{up} \left( \frac{x_i + \frac{w_i}{2}}{w_b} \right),
\]

\[
Y_i' = \text{down} \left( \frac{y_i - \frac{b_i}{2}}{h_b} \right), \quad Y_i'' = \text{up} \left( \frac{y_i + \frac{b_i}{2}}{h_b} \right).
\]

And, the bins enclosed in the rectangle with lower left corner \((X_i', w_b, Y_i', h_b)\) and upper right corner \((X_i'' + 1, w_b, Y_i'' + 1, h_b)\) is exactly the set \( B_q \) of bins occupied by module \( v_i \). Further, let \( X_i'' = X_i' + 1, \quad Y_i'' = Y_i' + 1 \).

Fig. 1 shows the specific meaning of the above notations. Based on the rectangle rule in numerical integration, the integration in (11) can be approximated as follows:

\[
\int_{R_i} \psi(z, v) \, dz \, dv \approx \sum_{\text{bin} \in B_q} \text{Area(\text{bin} \cap R_i)} \frac{\psi(p, q)}{\text{Area}(\text{bin})}.
\]

Expanding the right term of the above equation, we get

\[
g_i(x_i, y_i, w_i) \approx \sum_{\text{bin} \in B_i} \text{Area(\text{bin} \cap R_i)} \frac{\psi(p, q)}{\text{Area}(\text{bin})}
\]

\[
= \sum_{p=q=X_i'}^{X_i''} \sum_{q=Y_i'}^{Y_i''} w_b h_b \frac{\psi(p, q)}{\text{Area}(\text{bin})}
\]

\[
- \sum_{p=q=Y_i'}^{Y_i''} h_b \left[ X_i'' w_b - \left( x_i + \frac{w_i}{2} \right) \right] \psi(X_i', q)
\]

\[
- \sum_{q=q=X_i'}^{X_i''} w_b \left[ y_i'' h_b - \left( y_i + \frac{b_i}{2} \right) \right] \psi(X_i', q)
\]
In the above equation, $w_i$ appears as a variable explicitly. Moreover, $h_i = (A_i/w_i)$. Hence, by taking the partial derivative of the last equality with respect to $w_i$, we get

$$\frac{\partial g_i(x_i, y_i, w_i)}{\partial w_i} \approx \sum_{q=Y_i}^{y_i} h_l \left[ -\hat{\psi}(X_i^i, q) + \hat{\psi}(X_{i+l}, q) \right]$$

Similarly, the partial derivatives of $g_i(x_i, y_i, w_i)$ with respect to $x_i$ and $y_i$ can be approximated by

$$\frac{\partial g_i(x_i, y_i, w_i)}{\partial x_i} \approx \sum_{q=Y_i}^{y_i} h_l \left[ -\hat{\psi}(X_i^i, q) + \hat{\psi}(X_{i+l}, q) \right]$$

$$\frac{\partial g_i(x_i, y_i, w_i)}{\partial y_i} \approx \sum_{p=X_i}^{x_i} w_h \left[ -\hat{\psi}(X_i^i, q) + \hat{\psi}(X_{i+l}, q) \right]$$

According to (18), (19), and (20), the computing cost for $\left(\frac{\partial g_i(x_i, y_i, w_i)}{\partial x_i}\right)$, $\left(\frac{\partial g_i(x_i, y_i, w_i)}{\partial y_i}\right)$, and $\left(\frac{\partial g_i(x_i, y_i, w_i)}{\partial w_i}\right)$ is a linear function of $(x_i - X_i^i) + (Y_i - Y_i^i)$, if all $\hat{\psi}(p, q)$ have been computed. Therefore, the time complexity of calculating the partial derivatives for all modules is $O(n(\text{AVG}/n))$, where $\text{AVG} = \sum_{i=1}^{n} (X_i^i - X_i^i) + (Y_i^i - Y_i^i)$, if all $\hat{\psi}(p, q)$ have been computed.

According to (7), we first invoke the fast Fourier transform (FFT) one time to calculate all the coefficients $a_{pq}$, and then invoke the FFT another time to calculate all values of $\hat{\psi}(p, q)$. Since the time complexity of one FFT is $O(K^2 \log K)$, all the values $\hat{\psi}(p, q)$ can be calculated in time $O(K^2 \log K)$ by the FFT on (7). Hence, the total time complexity of calculating the partial derivatives for all modules is $O(K^2 \log K) + O(n(\text{AVG}/n))$. It must be remarked that, in ePlace [24] each iteration requires invoking the FFT three times for calculating the partial derivatives for all modules [see (21) and (24) in [24]].

Further, in our fast approximate calculation of partial derivatives, it is apparently that the average number $(\text{AVG}/n)$ of squares occupied by all modules is much smaller than the total number $K^2$ of squares. So the time complexity of this fast approximate calculation of partial derivatives still has a linear relationship with the total number of modules, but the difference is that the constant $(\text{AVG}/n)$ is no longer related to the truncation constant $K^2$ as in (14)–(16). Therefore, the cost of calculating all partial derivatives is reduced.

For the convenience of description, we will call the method in Section III-B the partial derivative calculation method before acceleration (PDPA), and the method in this section the accelerated partial derivative calculation method.

**IV. FIXED-OUTLINE FLOORPLANNING ALGORITHM**

In this section, we present our algorithm for fixed-outline floorplanning, as shown in Fig. 2. It contains three phases: 1) initial floorplanning; 2) global floorplanning; and 3) legalization. We directly adopt the QP method in ePlace [24] to produce an initial floorplan. Hence, we mainly introduce in this section the global floorplanning and legalization algorithms.

**A. Global Floorplanning Algorithm**

In the global floorplanning phase of Fig. 2, Charge Density Calculation, Solve Poisson’s Equation, Potential Energy Calculation, Partial derivative in $x, y$ directions, Partial derivative in $w$ direction, and Wirelength Gradient have been explained in Section III. In this section, we present other parts of the global floorplanning algorithm for problem (12), as in Algorithm 1.
Algorithm 1 Global Floorplanning Algorithm

Input: initial solution \( u_0 = (x_0, y_0, w_0) \), \( \lambda_0 \), maximum iterations \( k_{\text{max}} \), minimum overlap \( O_{\text{min}} \);

Output: solution \( u_{\text{gp}} \);
1: for \( k = 0 \) to \( k_{\text{max}} \) do
2: \( f_k = \text{LSE}(E) + \lambda_k \left( \sum_{i,j} V_i g(x_i, y_j, w_i) + \sum_{i,j} V_i g(x_i, y_j) \right) \);
3: calculate partial derivatives \( \nabla f_k \), \( \nabla w_k \) and \( \nabla f_k \) of \( f_k \);
4: \( (u_{k+1}, y_{k+1}) = \text{Nesterov-Solver}(u_k, y_k, f_k) \);
5: \( v_{k+1} = \text{Normal-Solver}(w_k, 1, f_k) \);
6: \( u_{\text{gp}} = u_{k+1} = (x_{k+1}, y_{k+1}, w_{k+1}) \);
7: project \( u_{k+1} \) onto the feasible region;
8: calculate overlap \( O_{k+1} \) and update \( \lambda_k \) to \( \lambda_{k+1} \);
9: if \( O_{k+1} \geq O_{\text{min}} \) then break;
10: return \( u_{\text{gp}} \);

Algorithm 2 Nesterov-Solver(\( u_k, f_k \)) // Nesterov’s Method at the 4th Iteration [24]

Input: major solution \( u_k \), reference solution \( v_k \), optimization parameter \( \lambda_k \) and objective function \( f_k = f(v_k) \);

Output: \( u_{k+1}, v_{k+1} \);
1: calculate the gradient in the \( x, y \) direction: \( \nabla x f_k = \nabla x f_k(v_k) \);
2: steplength \( a_k = \arg \max_a \{ f_k - f(v_k) - a \nabla x f_k \} \geq 0.5a\|\nabla x f_k\| \);
3: new solution \( u_{k+1} = v_k - a_k \nabla x f_k \);
4: parameter update \( \lambda_{k+1} = 1 + \frac{a_k}{\sqrt{2a_k+1}} \);
5: new reference solution \( v_{k+1} = u_{k+1} + \frac{(a_k-1)(a_{k+1}-a_k)}{a_{k+1}} \);
6: return \( u_{k+1}, v_{k+1} \).

In Algorithm 1, we use projected gradient method to solve problem (12). Line 2 constructs the objective function \( f_k \) required in the 4th iteration. Line 3 calculates the partial derivative of the objective function in each direction, which will be used in lines 4 and 5.

We use Nesterov’s method [24] to optimize the module positions \( (x_i, y_i) \). In line 4 of Algorithm 1, we first fix the widths of soft modules and call Nesterov’s method to optimize module positions (Algorithm 2), which was first adopted for VLSI placement in ePlace [24]. Nesterov’s method is the first-order optimization algorithm with convergence rate \( O(1/k^2) \), where \( k \) is the number of iterations. We set \( a_0 = 1, u_0 = v_0 \), and \( v_0 \) is the solution given by the initial floorplanning.

In Nesterov’s method, since we have fixed the widths of soft modules when we call the Nesterov-Solver, we use the same diagonal Jacobi preconditioner as ePlace [24] to get faster convergence and a better solution.

Line 5 of Algorithm 1 uses normalized gradient descent method to optimize the widths of soft modules. By fixing the positions of modules obtained in line 4, we calculate \( \nabla w f_k = \nabla w f_k(u_k) \). Then, we set \( u_{k+1} = u_k - \text{step} \nabla w f_k \), in which step \( a = 1/\|\nabla w f_k\| \), and, here, we set the constant \( a = 1 \). Note that, after optimizing the widths \( w_i \) of soft modules, the positions of pins will be affected. Hence, in order to ensure the optimization accuracy, the positions of pins will be updated proportionally before the next time we call the Nesterov-Solver.

Since \( u_{k+1} \) may not satisfy the constraints in problem (12). In order to obtain a feasible solution, line 7 of Algorithm 1 projects \( u_{k+1} \) onto the feasible region of problem (12), such that it satisfies the constraints (12b)–(12d). First, to meet the constraint (12h), line 7 projects the widths \( w_i \) of soft modules in \( u_{k+1} \) as follows:

\[
\begin{align*}
    w_i = \begin{cases} 
        \frac{A_i}{\sqrt{\frac{A_i}{A_L}}}, & \text{if } \frac{A_i}{w_i} < \frac{A_L}{w_L} \\
        \frac{A_i}{\sqrt{\frac{A_i}{A_U}}}, & \text{if } \frac{A_i}{w_i} > \frac{A_U}{w_U} \\
        w_i, & \text{else.}
    \end{cases}
\end{align*}
\]

Next, line 7 of Algorithm 1 projects in the same way the positions of modules such that they satisfy the constraints (12c) and (12d):

\[
\begin{align*}
    x_i = \begin{cases} 
        \frac{w_i}{2}, & \text{if } x_i < \frac{w_i}{2} \\
        x_i, & \text{else}
    \end{cases}
\end{align*}
\]

\[
\begin{align*}
    y_i = \begin{cases} 
        \frac{h_i}{2}, & \text{if } y_i < \frac{h_i}{2} \\
        y_i, & \text{else}
    \end{cases}
\end{align*}
\]

Line 8 of Algorithm 1 updates the value of penalty parameter \( \lambda \) and calculates the overlaps between modules. Here, the method for updating the penalty parameter value is the same as that in ePlace [24]. First, set an expected increase of wirelength \( \Delta \text{HPWL}_{\text{ref}} \) for each iteration. Then, the penalty parameter is updated as follows:

\[
\lambda_{k+1} = \mu_k \lambda_k, \quad \mu_k = \mu_0 \frac{\Delta \text{HPWL}_{\text{ref}}}{\text{HPWL}_{\text{ref}} - \text{HPWL}_{\text{ref}}^0 + 1}
\]

where \( \Delta \text{HPWL}_{\text{ref}} = \text{HPWL}(u_k) - \text{HPWL}(u_{k-1}) \). The constant \( \mu_0 \) is set to 1.1, and \( \mu_k \) is usually truncated to the interval \([0.75, 1.1]\).

Finally, in lines 9 and 10, Algorithm 1 will stop if the current total overlap between modules is less than the threshold \( O_{\text{min}} \), which ensures that the overlapping area must be less than \( O_{\text{min}} \) before legalization begins.

B. Legalization

After global floorplanning, we can get a rough floorplanning of modules with a small overlap. However, there is still a little overlap in the floorplan (more details can be found in the experimental results section). In other words, it does not satisfy the constraint that there is no overlap between modules. Hence, legalization will be used to make the global floorplan be a legal floorplan. Therefore, this section will discuss how to eliminate the overlap between modules.

In legalization, we use the global floorplanning result as the initial solution, and treat all soft modules as hard ones. If a floorplan is small scale, then it is legalized using the \( p/2xp \) function in Parquet [11] directly; otherwise it is legalized using the method proposed in this section.

Abacus [27], SAINT [23], and Floorist [28] are state-of-the-art legalization methods to eliminate overlaps between modules. Abacus [27] is used in VLSI placement, in which standard cells can be aligned to rows. However, in fixed-outline floorplanning, the modules are of various heights. Hence, it is difficult to extend Abacus to legalize a floorplan. SAINT [23] uses polygon to approximate the curve of the area of every soft module, and solve an ILP for legalization, which is time consuming and is difficult to scale up for large-scale floorplanning problem. Floorist [28] is a floorplan repair method,
its main idea is as follows. First, a nonoverlapping floorplan is constructed based on the global floorplanning result. If it exceeds the floorplanning area, then the positional relationship of modules is modified and the floorplan is compressed, until all modules are located legally in the floorplanning area.

We propose a new legalization algorithm based on Floorist [28]. The biggest difference between our legalization algorithm and Floorist is as follows. Floorist considers how to insert constraints into the constraint graphs so that all modules finally are nonoverlapping, but does not delete constraints in the constraint graphs. During insertion, the fixed outline is satisfied but the nonoverlapping condition may be violated. Eventually, our algorithm will satisfy the fixed-outline condition by modifying the constraint graphs.

1) **Horizontal and Vertical Constraint Graphs and Related Definitions:** First, we construct horizontal and vertical constraint graphs HCG and VCG, according to the positional relationship of the modules after global floorplanning. Assume that the coordinate of the lower left corner of module $A$ is smaller than that of $B$ in both the horizontal and vertical directions, as Fig. 3 shows. Then, we have the following processing for modules $A$ and $B$.

1. If $A$ and $B$ do not overlap horizontally and vertically, then we have $A \not\rightarrow B \in \text{VCG}$ and $A \rightarrow B \in \text{HCG}$.
2. If $A$ and $B$ overlap vertically but do not overlap horizontally, then we have $A \rightarrow B \in \text{HCG}$.
3. Assume that $A$ and $B$ overlap both horizontally and vertically. If the horizontal overlap is greater than the vertical overlap. Then there is $A \not\rightarrow B \in \text{VCG}$, otherwise $A \rightarrow B \in \text{HCG}$.

Basically, our algorithm legalizes a floorplan alternatively in the horizontal and vertical directions. In the horizontal direction, our legalization algorithm modifies a floorplan by deleting an edge between two modules in HCG, then adding an edge between the two modules in VCG, and compress the floorplan. The vertical legalization operates in a similar way. Generally, we can arbitrarily delete or add an edge between two modules, if there is no contradiction. Moreover, it is not needed to ensure that HCG and VCG are transitive, for effectively reducing the number of edges in HCG and VCG.

However, choosing an edge for adjustment is an important step that affects the effectiveness of our legalization algorithm. During horizontal legalization, we tend to select an edge that has the least impact on VCG, which is achieved by comparing the weights of the edges in HCG defined in (24). The vertical legalization selects an edge in VCG similarly. The specific details will be given in Algorithm 4.

During horizontal legalization, according to the HCG, we first move the modules as far as possible to the left in the floorplanning area, while keeping the HCG unchanged. The method is, we calculate the abscissa $x_A$ of the lower left corner of every module $A$

$$x_A = \begin{cases} \max_{\forall B(B \rightarrow A)}(x_B + w_B), & \text{if } \exists B(B \rightarrow A) \\ 0, & \text{else.} \end{cases}$$

(21)

Similar to (21), the ordinate $y_A$ of the lower left corner of every module $A$ can be calculated. After the above calculations, the coordinate $(x_A, y_A)$ of the lower left corner of each module is determined, and all overlaps between modules are eliminated.

Next, let noRight $= \{A|\forall B, A \not\rightarrow B\}$, we calculate the width $W_d$ of the smallest rectangle enclosing all modules

$$W_d = \max_{A \in \text{noRight}}(x_A + w_A).$$

(22)

Similar to (22), we can calculate the height $H_d$ of the smallest rectangle enclosing all modules.

Then, we change temporarily the origin of the coordinate system to $(W_d, 0)$, and change the positive direction of the $x$-axis to left. In the temporary coordinate system, we construct the horizontal constraint graph $\text{HCG}'$ of the floorplan. According to the $\text{HCG}'$, by calculating (21), the lower left corner coordinate $x_A'$ of each module $A$ can be obtained. After the above calculations, the horizontal slack $S_x^A$ of a module $A$ is defined as follows:

$$S_x^A = W - (x_A + x_A' + w_A)$$

(23)

here $W$ is the width of the floorplanning area.

Similarly, we can define the vertical slack $S_y^A$ of module $A$. Then, we say a horizontal relationship $A \rightarrow B$ is critical if $S_x^A = 0$ and $S_y^B = 0$. Similarly, we say a vertical relationship $A \uparrow B$ is critical if $S_x^A = 0$ and $S_y^B = 0$.

Next, we define the compressible relationship between modules. Modules $A$ and $B$ has a compressible relationship in the horizontal direction, denoted as $A \cap_H B = \emptyset$, if $A \rightarrow B$ and $[y_A, y_A + h_A] \cap [y_B, y_B + h_B] = \emptyset$. This definition means that, if there are two modules $A$ and $B$ such that $A \cap_H B = \emptyset$, then $A$ or $B$ may be moved horizontally such that the floorplan is more compact. Similarly, we can define the vertical compressible relationship, and denote it as $A \cap_V B = \emptyset$. Moreover, for a horizontal critical relationship $A \rightarrow B$, we define the weight of $A \rightarrow B$ as follows:

$$\text{Weight}(A \rightarrow B) = \begin{cases} S_x^A - h_B, \text{ if } y_A \leq y_B \\ S_y^B - h_A, \text{ else.} \end{cases}$$

(24)

For $y_A \leq y_B$, if we delete the horizontal critical relationship $A \rightarrow B$ in HCG and add the vertical relationship $A \uparrow B$ in VCG, then the vertical slack of the module $A$ is $S_y^A - h_B$. Similarly, for $y_A > y_B$, after deleting $A \rightarrow B$ in HCG and adding $B \uparrow A$ in VCG, the vertical slack of the module $B$ is about $S_y^B - h_A$. By setting this weight, when we need to delete an edge in the HCG and add a new edge in the VCG, the algorithm will preferentially select the edge with the smallest increase of the height $H_d$ of the smallest rectangle enclosing all modules. Similarly, we can define the weight of a vertical critical relationship in the vertical constraint graph.
Algorithm 3 Legalization Algorithm

**Input:** width and height of floorplanning area \((W, H)\), horizontal and vertical constraint graphs \((HCG, VCG)\), maximum number of iterations \(N\);

**Output:** coordinate \((\hat{x}_A, \hat{y}_A)\) of lower left corner of each module.

1. for \(i = 1 \rightarrow N\) do
2. \((HCG, VCG) = LG_i(W, HCG, VCG)\);
3. \((HCG, VCG) = LG_i(H, HCG, VCG)\);
4. calculate the smallest rectangle \((W_d, H_d)\) enclosing all modules according to \((HCG, VCG)\);
5. if \(W_d \leq W\) and \(H_d \leq H\) then
   6. break;
7. return \((\hat{x}_A, \hat{y}_A)\).

Algorithm 4 \(LG_x(W, HCG, VCG)\) // Horizontal Legalization

**Input:** width \(W\) of floorplanning area, horizontal and vertical constraint graphs \((HCG, VCG)\);

**Output:** \((HCG, VCG)\) after horizontal legalization.

1. calculate \((\hat{x}_A, \hat{y}_A)\) and \((S_A^x, S_A^y)\) of all modules and the width \(W_d\) based on the current \((HCG, VCG)\);
2. while \(W_g > W\) do
3. let \(S = \{A \rightarrow B \in HCG | S_A^x = 0 \text{ and } S_B^y = 0\}\) be the set of horizontal critical relationships;
4. if \(\exists (A \rightarrow B) \in S \text{ and } A \cap B = \emptyset\) then
   5. delete relationship \(A \rightarrow B\) from the current \(HCG\);
6. else
7. for \(\forall (A \rightarrow B) \in S\) do
8. calculate \(\text{Weight}(A \rightarrow B)\);
9. find modules \(A\) and \(B\) with largest \(\text{Weight}(A \rightarrow B)\);
10. delete relationship \(A \rightarrow B\) from the current \(HCG\);
11. if \(y_A \leq y_B\) then
12. add relationship \(A \uparrow B\) to the current \(VCG\);
13. else
14. add relationship \(B \uparrow A\) to the current \(VCG\);
15. do line 1 again;
16. return \((HCG, VCG)\).

2) **Legalization Algorithm:** Algorithm 3 shows the overall process of our legalization algorithm, which finds a legalized floorplan within a given number of iterations.

Lines 2 and 3 of Algorithm 3 legalize the floorplan in the horizontal and vertical directions alternatively, which will be introduced in Algorithm 4. According to the current HCG and VCG, line 4 uses (21) to calculate the coordinate \((\hat{x}_A, \hat{y}_A)\) of the lower left corner of each module, and uses (22) to calculate the smallest rectangle enclosing these modules. Line 5 of Algorithm 3 determines whether the smallest rectangle is within the floorplanning area. If so, exit the iteration.

Algorithm 4 presents our legalization algorithm in the horizontal direction. The vertical legalization algorithm is similar, and will not be given here.

According to the current HCG and VCG, lines 1 and 15 use (21) to move all modules to the left and bottom in the floorplanning area, and make them nonoverlapping. Then, lines 1 and 15 use (23) and (22) to calculate horizontal and vertical slacks \((S_A^x, S_A^y)\) of all modules, and the width \(W_d\) of the smallest rectangle enclosing these modules, respectively.

Line 2 determines whether the floorplan obtained from lines 1 or 15 is already in the floorplanning area. If so, a feasible solution in the horizontal direction has been obtained, and the algorithm stops. Line 3 of Algorithm 4 finds all critical relationships in the HCG. Lines 4 and 5 determine whether there is a compressible relationship \(A \rightarrow B\) in the set of horizontal critical relationships. If so, it indicates that module \(B\) can be moved left so that \(A\) and \(B\) still do not overlap. Therefore, we delete relationship \(A \rightarrow B\) from the current HCG.

Lines 7–14 deal with the situation that the floorplan cannot be compressed directly in the horizontal direction. In this situation, we use (24) to calculate \(\text{Weight}(A \rightarrow B)\) of all horizontal critical relationships, and find the largest one between lines 7 and 9. Then we delete the \(A \rightarrow B\) corresponding to the largest \(\text{Weight}(A \rightarrow B)\) in HCG at line 10, and finally add a new corresponding VCG edge between lines 11–14. The purpose of this is to minimize the increase in the height \(H_d\) of the floorplan after changing the relationship.

Let \(HCG\) be \(G(V, E)\). According to the topological-sort and (21)–(23), the time complexity of lines 1 and 15 in Algorithm 4 is \(O(|E| + |V|)\). Note that, \(|S| \leq |E|\) holds for the set of horizontal critical relationships. So, the time complexity of lines 4, 5, and 7–9 of Algorithm 4 are all \(O(|E|)\). Therefore, the time complexity of each iteration of Algorithm 4 is \(O(|E| + |V|)\).

3) **Example of Horizontal Legalization:** To illustrate the effectiveness of our legalization algorithm, we implement it on the example of XDP [29] as shown in Fig. 4(a), which was used in [29] for mixed-size placement to show that it is nontrivial to identify a right set of edges for adjustment.

Fig. 4(a) presents the horizontal constraint graph corresponding to the floorplan in Fig. 4(a). First, according to the current HCG and VCG, line 1 of Algorithm 4 will obtain a floorplan without overlapping that is compressed in the horizontal and vertical directions, which is shown in Fig. 4(b).

Then because \(W_d > W\), we have to legalize the modules in the horizontal direction, such that they are all in the horizontal outline. First, we need to extract critical relationships from HCG through line 3 of Algorithm 4, and line 4 will delete all compressible relationships until \(W_d \leq W\). In this case, the floorplan obtained from lines 1 and 15 cannot be compressed directly in the horizontal direction. Then we use (24) to calculate \(\text{Weight}(A \rightarrow B)\) of all horizontal critical relationships, and find the largest one between lines 7 and 9. Then we delete the \(A \rightarrow B\) corresponding to the largest \(\text{Weight}(A \rightarrow B)\) in HCG at line 10, and finally add a new corresponding VCG edge between lines 11–14. The purpose of this is to minimize the increase in the height \(H_d\) of the floorplan after changing the relationship.

Therefore, the time complexity of each iteration of Algorithm 4 is \(O(|E| + |V|)\).
example, since there are no compressible relationships, we use (24) to calculate the edge weights. Fig. 5(b) is the corresponding critical relationships of HCG and the weights of the edges.

Now we need to select an edge from HCG with the largest weight to delete, and add a new vertical edge in VCG. We sequentially delete \( m_1 \to m_2 \) and then add \( m_1 \uparrow m_2 \), delete \( m_1 \to m_3 \) and then add \( m_1 \uparrow m_3 \). It is worth to note that when we delete a relationship and create a new relationship, the weights of related edges will be updated. Finally, after executing line 15 of Algorithm 4, we get a horizontally legal floorplan, which is shown in Fig. 4(c).

V. EXPERIMENTAL RESULTS

In this section, we compare our fixed-outline floorplanning algorithm with state-of-the-arts experimentally on MCNC [30], GSRC [31], HB+ [32], and ami49_x [12], [15] benchmarks. Our algorithm was implemented in C++ and run on a Linux machine with 3.60-GHz Intel Core i3-9100 CPU and 4-GB memory. We focus on floorplanning with hard and soft modules. In comparisons, wirelength is calculated using the HPWL, and all compared data are cited from relevant literatures directly.

In experiments of our floorplanning algorithm, parameters were set as follows: \( k_{\text{max}} = 1200 \) and \( \Theta_{\text{min}} \) is set to about 1% in Algorithm 1, and \( N = 10 \) in Algorithm 3.

A. MCNC and GSRC Benchmarks

These benchmarks are ami33, ami49, n100, n200, and n300. The numbers of modules are 33, 49, 100, 200, and 300, respectively, and all modules are soft. We conducted two experiments. The first experiment fixed the whitespace at 15%, and changed the aspect ratio of floorplanning area from 1:1 to 4:1. The purpose is to test the adaptability of the algorithms to aspect ratio. The second experiment reduced the whitespace to 10%, and fixed the aspect ratio as 1:1. In this experiment, none of the three floorplanners uses a multilevel framework. For all test benchmarks, the I/O pads are fixed at the location given originally. The detailed test results are put in Table III.

In the table, the first and second columns present the names of the test benchmarks and the floorplanning algorithms, respectively. Columns 3–6 give the test results of respective floorplanners. The last three rows of the table show the average results of the two floorplanners proportional to that of our floorplanning algorithm, respectively. From Table III, it can be seen that for different aspect ratio, the average HPWL of our algorithm is at least 16% less than that of analytical [18], and at least 31% less than that of Parquet-4 [11].

In the second experiment, for each benchmark, the aspect ratio was set to \( \gamma = 1 \), and the I/O pads were shifted to the boundary of the floorplanning area. Compared floorplanning algorithms are: analytical-based methods AR [19] and Lin et al. [2] without using multilevel framework, Capo 10.2 [13], IMF [15], and IMF + AMF [12] based on a multilevel framework, DeFer [14], and Parquet-4 [11] based on simulated annealing.

Table IV lists the HPWL results obtained by respective algorithms. The data of DeFer are cited from [14], and the data of the other compared algorithms are cited from [2] and [19] directly. On average, our HPWL result is 6%, 6%, 7%, 13%, and 29% less than that of DeFer [14], IMF [15], IMF + AMF [12], Capo 10.2 [13], and Parquet-4 [11], respectively. Compared to the analytical algorithms Lin et al. [2] and AR [19], our HPWL result is 2% and 5% less.
TABLE V
RESULTS ON THE HB+ BENCHMARKS

| Name | Hard | Soft | L/O | white-space | QinFer | Ref. [2] | ours | ours | QinFer | Ref. [2] | ours | ours |
|------|------|------|-----|--------------|--------|---------|------|------|--------|---------|------|------|
| HB+1 | 665  | 246  | 246 | 26%         | 2.91   | 3.18    | 3.09 | 3.03 | 3.00   | 5.0     | 150  | 18.9 |
| HB+2 | 1200 | 271  | 259 | 25%         | 6.47   | 6.80    | 6.17 | 6.53 | 6.02   | 11.3    | 338  | 15.3 |
| HB+3 | 999  | 290  | 283 | 30%         | 9.17   | 9.68    | 9.19 | 8.27 | 8.17   | 8.6     | 262  | 4.0  |
| HB+4 | 1289 | 295  | 287 | 25%         | 11.16  | 9.87    | 10.26| 10.03| 9.72   | 12.4    | 209  | 14.2 |
| HB+5 | 571  | 176  | 166 | 25%         | 8.03   | 8.50    | 8.78 | 7.95 | 7.91   | 7.8     | 125  | 7.8  |
| HB+6 | 829  | 291  | 287 | 25%         | 14.88  | 15.10   | 15.48| 14.73| 14.07  | 8.5     | 230  | 4.6  |
| HB+7 | 968  | 301  | 286 | 25%         | 16.49  | 17.60   | 18.73| 18.35| 17.19  | 22.8    | 324  | 19.3 |
| HB+8 | 869  | 253  | 285 | 25%         | 17.24  | 18.30   | 16.66| 16.32| 15.81  | 8.7     | 259  | 4.2  |
| HB+10| 809  | 786  | 744 | 20%         | 42.33  | 46.70   | 45.12| 41.58| 40.61  | 11.4    | 191  | 6.3  |
| HB+11| 1124 | 373  | 406 | 25%         | 25.7   | 28.20   | 26.99| 25.38| 24.58  | 10.1    | 494  | 7.1  |
| HB+12| 582  | 651  | 637 | 26%         | 52.83  | 53.60   | 50.17| 49.26| 48.96  | 16.6    | 136  | 5.5  |
| HB+13| 830  | 424  | 490 | 25%         | 35.44  | 35.40   | 35.51| 32.51| 32.65  | 23.7    | 90   | 5.9  |
| HB+14| 1021 | 614  | 517 | 25%         | 60.68  | 63.40   | 64.50| 61.57| 59.62  | 14.9    | 208  | 12.0 |
| HB+15| 1019 | 393  | 383 | 25%         | 77.48  | 79.10   | 84.29| 74.97| 73.93  | 37.1    | 532  | 14.7 |
| HB+16| 633  | 458  | 504 | 25%         | 98.53  | 92.90   | 98.66| 87.66| 87.32  | 11.2    | 160  | 8.1  |
| HB+17| 682  | 760  | 743 | 25%         | 140.84 | 140.00  | 144.56| 136.37| 138.44 | 15.2    | 169  | 14.7 |
| HB+18| 658  | 285  | 272 | 25%         | 70.36  | 70.70   | 71.86| 68.76| 68.05  | 12.4    | 66   | 11.3 |
| Ratio | 1.05 | 1.08 | 1.08 | 1.02 | 1.00 | 1.23 | 24.43 | 0.89 | 7.06 | 1.00 |

B. HB+ Benchmarks

HB+ benchmarks were generated from HB benchmarks, with the largest hard macros being enlarged by 100% and the area of remaining soft macros reduced to preserve the total cell area. Therefore, HB+ benchmarks are very hard to handle. In this experiment, we compare the results by our algorithm to multilevel-based floorplanners DeFer [14], QinFer [16], and Lin et al. [2].

DeFer [14] has the characteristics of fast speed and high solution quality. It can solve large-scale fixed-outline slicing floorplanning problems. QinFer [16] is a newly published partitioning-based fixed-outline floorplanning algorithm with high solution quality. Ref. [2] is an analytical floorplanner based on multilevel framework. Note that we have two versions of calculating partial derivatives of potential energy. One is the PDBA presented in Section III-B, and another is the accelerated partial derivative calculation method presented in Section III-C. Hence, we have two versions of the floorplanning algorithm, respectively. "Ours PDBA" means our floorplanning algorithm using the PDBA, and ours indicates our floorplanning algorithm using the accelerated partial derivative calculation method.

Table V gives the basic information of the HB+ benchmarks and the test results of compared floorplanners. The first 5 columns of Table V present the basic information of the benchmarks, including the names of benchmarks, the numbers of hard modules and soft modules, the number of terminals (I/O pads), and the whitespace fractions. The other information of each benchmark, such as the width and height of the floorplanning area, the upper and lower bounds of aspect ratios of soft modules, can be found in the corresponding literature. In the table, columns 6–10 are the HPWL results obtained by the respective algorithms. According to the last row of Table V, the average HPWL of our algorithm is 5%, 8%, and 8% less than that of QinFer, DeFer, and Lin et al. [2], respectively.

C. ami49_x Benchmarks

In this experiment, we test our floorplanning algorithm on ami49_x benchmarks [12], [15] to see its performance on large-scale fixed-outline floorplanning. The benchmarks ami49_x were designed as follows. First, duplicate each module vi and net x times, and get new modules vi,1, vi,2, ..., vi,x. Then, add x − 1 nets (vi,1, vi,2), (vi,1, vi,3), ..., (vi,1, vi,x). Finally, reduce the width and height of each module by 5 1.86-GHz CPU with 2-GB memory, QinFer was Intel core i5-7500 3.40 GHz with 8-GB memory, and [2] was Intel Xeon 2.00-GHz CPU. On average, the run times of DeFer and QinFer are 89% and 123% of our algorithm, respectively. However, the run-time of Lin et al. [2] is quite long. Although the computing environments are different, the speed of our floorplanning algorithm can still be compared roughly.

Note that both of our floorplanning algorithm and Lin et al. [2] are modified from VLSI placement algorithms, respectively, and both of them do not use a multilevel framework for the small-scale benchmarks in Table IV. According to Table IV, the average HPWL of our algorithm is 2% less than that of Lin et al. [2]. However, for large-scale benchmarks in Table V, our floorplanning algorithm still does not use a multilevel framework, but the algorithm in [2] does. According to Table V, the HPWL of our algorithm is 8% less than that of [2]. This further improvement is obviously due to that our floorplanning algorithm is flat, i.e., does not use the multilevel framework.

Theoretically, the results of “our PDBA” should be better than ours, since it calculates exactly the partial derivatives of the potential energy. However, according to Table V, the average HPWL result of our PDBA has a slight increase of 2% over ours. This seems to be inconsistent with the theory. In fact, the fixed-outline floorplanning is a nonconvex optimization problem, and its objective function has multiple local minima. Therefore, a better solution may not be obtained by searching in the exact gradient descent direction. While, each iteration in ours has a certain error of partial derivative calculation, which makes our algorithm have limited ability to jump out of local optima, and find a better solution.
D. Run-Time Analysis

Next, we make the running time component analysis of our floorplanning algorithm with an accelerated partial derivative calculation method. Note that, the total running time of our algorithm = initial solution generation time by QP + run-time of our global floorplanning + run-time of our legalization.

E. Results of Global Floorplanning and Legalization on HB+ and ami49_x Benchmarks

Next, we analyze the overlap rate after global floorplanning and the wire-length increase after the legalization of our floorplanning algorithm. Table VII gives the overlap rates...
TABLE VII
RESULTS OF OVERLAP RATE AFTER GLOBAL FLOORPLANNING AND HPWL INCREASE AFTER LEGALIZATION

| Name          | Overlap Rate Before Legalization | Rate of change | Name          | Overlap Rate After Legalization | Rate of change |
|---------------|---------------------------------|----------------|---------------|---------------------------------|----------------|
| HB_0         | 0.8%                            | 2.39           | HB_15        | 0.9%                            | 2.14           |
| HB_1         | 1.00%                           | 4.05           | HB_16        | 1.00%                           | 26.54          |
| HB_2         | 0.95%                           | 5.77           | HB_17        | 2.05%                           | 41.31          |
| HB_3         | 0.95%                           | 7.27           | HB_18        | 0.95%                           | 28.09          |
| HB_4         | 0.95%                           | 8.63           | HB_19        | 0.95%                           | 23.40          |
| HB_5         | 0.95%                           | 10.00          | HB_20        | 0.90%                           | 18.40          |
| HB_6         | 0.95%                           | 11.57          | HB_21        | 0.95%                           | 13.99          |
| HB_7         | 0.95%                           | 13.11          | HB_22        | 1.00%                           | 9.88           |
| HB_8         | 0.95%                           | 14.66          | HB_23        | 1.00%                           | 7.81           |
| HB_9         | 0.95%                           | 16.22          | HB_24        | 1.00%                           | 6.35           |
| HB_10        | 0.95%                           | 17.78          | HB_25        | 0.95%                           | 5.08           |
| HB_11        | 0.95%                           | 19.34          | HB_26        | 0.95%                           | 4.00           |
| HB_12        | 0.95%                           | 20.90          | HB_27        | 0.95%                           | 3.02           |
| HB_13        | 0.95%                           | 22.46          | HB_28        | 0.95%                           | 2.05           |
| HB_14        | 0.95%                           | 24.02          | HB_29        | 0.95%                           | 1.00           |
| HB_15        | 0.95%                           | 25.58          | HB_30        | 0.95%                           | 0.95           |

The wirelength increase of HPWL is calculated using (columns 2 and 7) and HPWLs (columns 3 and 8) before legalization, the HPWLs after legalization (columns 4 and 9), and the wirelength increase of HPWL (columns 5 and 10) after our legalization algorithm for each benchmark, respectively. Here, the overlap rate is calculated by

\[
\text{Overlap Rate} = \prod_{i=0}^{K-1} \sum_{j=0}^{K-1} \max(\text{OR}(i, j), 0) \times 100\%
\]

in which

\[
\text{OR}(i, j) = \frac{\sum_{i \in V} \text{Area}(\text{bin}_{i}\cap R_j)}{\text{Area}(\text{bin}_{i})} - 1.
\]

The wirelength increase of HPWL is calculated using

\[
\frac{\text{HPWL After Legal.} - \text{HPWL Before Legal.}}{\text{HPWL Before Legal.}} \times 100\%
\]

for which a positive number means that the legalization has increased wirelength, and a negative number means that wirelength has been decreased.

From Table VII, it can be seen that our global floorplanning optimizes the wirelength while effectively reduces the overlap. On average, the average overlap rate is 0.96% after global floorplanning. Further, our legalization reduces on average the wirelength by 0.34%. The reduction in wirelength in fact is due to compressing the floorplan. Note that, there is no significant change in wirelength before and after legalization. This is due to that our legalization is designed as a local modification algorithm which does not modify massively the global floorplanning result.

VI. CONCLUSION

Based on a novel mathematical model for characterizing nonoverlapping of modules and an analytical solution of Poisson’s equation, this article has proposed an algorithm without a multilevel framework for fixed-outline floorplanning. The algorithm consists of global floorplanning and legalization phases. In global floorplanning, it uses redefined potential energy based on the novel mathematical model and the analytical solution of Poisson’s equation, to realize spreading of modules and optimizing the widths of soft modules. In legalization, the algorithm uses constraint graphs to alternatively eliminate overlaps between modules in the horizontal and vertical directions. Experimental results on the MCNC, GSRC, HB+, and ami49_x benchmarks show that the average HPWLs by our algorithm are at least 2% and 5% less than compared state-of-the-arts on small and large-scale benchmarks, respectively, within acceptable run-time. Considering other issues in our floorplanning algorithm, such as routability, timing, and thermal, are under investigation.

APPENDIX

Lemma 3: Any two different modules \(v_i\) and \(v_j\) in \(V\) are nonoverlapping if and only if \(\int_{R_i} \rho_j(u,v) dv du = 0\).

Proof: Any module \(v_i\) in \(V\) is nonoverlapping with other modules if and only if \(\int_{R_i} \rho_j(u,v) dv du = 0\) for all \(v_j \in V\).

\[
\int_{R_i} \rho_j(u,v) dv du = \int_{R_i} 1 dv du = \text{Area}(R_i).
\]

Based on Lemmas 3 and 4, the following theorem can be proved.

Theorem 2: Any module \(v_i\) in \(V\) is nonoverlapping with other modules if and only if

\[
\int_{R_i} \sum_{v_j \in V} \rho_j(u,v) dv du = \text{Area}(R_i).
\]

Proof: If module \(v_i\) in \(V\) is nonoverlapping with other modules, then by Lemmas 1 and 2,

\[
\int_{R_i} \sum_{v_j \in V} \rho_j(u,v) dv du = \sum_{v_j \in V} \int_{R_i} \rho_j(u,v) dv du
\]

\[
= \int_{R_i} \rho_i(u,v) dv du + \sum_{v_j \in V \setminus v_i} \int_{R_i} \rho_j(u,v) dv du
\]

\[
= \text{Area}(R_i) + 0 = \text{Area}(R_i).
\]

Conversely, if (25) holds, we can have

\[
\text{Area}(R_i) = \int_{R_i} \sum_{v_j \in V} \rho_j(u,v) dv du
\]

\[
= \int_{R_i} \rho_i(u,v) dv du + \sum_{v_j \in V \setminus v_i} \int_{R_i} \rho_j(u,v) dv du
\]

\[
= \text{Area}(R_i) + \sum_{v_j \in V \setminus v_i} \int_{R_i} \rho_j(u,v) dv du
\]

where the last equality comes from Lemma 2. Then

\[
\sum_{v_j \in V \setminus v_i} \int_{R_i} \rho_j(u,v) dv du = 0.
\]

Since \(\rho_j(u,v) \geq 0\), the above equality implies that \(\int_{R_i} \rho_j(u,v) dv du = 0\) holds for modules \(v_j \neq v_i\). Hence by Lemma 1, the conclusion holds.
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