New natural convection heat transfer correlations in enclosures for building performance simulation

A. Rincón-Casadoa, F.J. Sánchez de la Florb, E. Chacón Verac and J. Sánchez Ramosc

aMechanical Engineering, University of Cadiz, Cadiz, Spain; bDepartment of Machines and Thermal Motors, University of Cadiz, Cadiz, Spain; cDepartment of Mathematics, Faculty of Mathematics, University of Murcia, Murcia, Spain

ABSTRACT

This paper presents new correlations to calculate natural convection heat transfer coefficients (CHTC) in enclosures for building performance simulation. Current work related to the development of correlations is not oriented to building enclosures, and the influence of high numbers of Rayleigh (Ra) and aspect ratio on the CHTC has not been studied in detail. In this work, two new correlations have been developed for a vertical wall, one in laminar regime ($Ra < 10^7$) and another one in turbulent regime ($10^7 < Ra < 10^{11}$). Moreover, two new correlations have been developed for floor and ceiling, one in laminar regime ($Ra < 10^6$) and another one in turbulent regime ($Ra > 10^6$). All correlations have been developed as function of the aspect ratio ($H/L$) and Ra numbers calculated from the enclosure average air temperature and wall surface temperature. By contrast, in previous works the Ra numbers have been calculated from the temperature difference of opposite walls. Computer Fluid Dynamics (CFD) simulations for different aspect ratios ($H/L = 0.5–1–2$) and Ra numbers ($10^3–10^{11}$) have been carried out in order to obtain these correlations. The SIMPLE algorithm has been used for the solution of the Navier–Stokes equations and the realizable turbulence $k$-$\varepsilon$ model with an enhanced wall-function treatment has been used. The correlations developed follow the expected trend for the low number of Ra in comparison with the expressions developed by other authors. For a high number of Ra, our correlations improve the previous correlations, because it is a function of the aspect ratio of the enclosure and the average air temperature of the enclosure. This approach is simple to implement in the construction of thermal simulation programs with low computational cost.

Nomenclature

| Symbol | Description |
|--------|-------------|
| $L$    | Room length [m] |
| $H$    | Room height [m] |
| $A$    | Area $[m^2]$ |
| $g$    | Gravitational acceleration $[m \cdot s^{-2}]$ |
| $T$    | Temperature $[^\circ C]$ |
| $\bar{T}_{air}$ | Mean temperature air in enclosure $[^\circ C] = (T_h + T_c)/2$ |
| $h$    | Heat transfer coefficient $[W \cdot m^{-2} \cdot K^{-1}]$ |
| $\overline{Nu}$ | Average Nusselt number $[-]$ |
| $Pr$   | Prandtl number $[-]$ |
| $Ra$   | Rayleigh number $[-]$ |
| $\varphi$ | CFD Streamlines $[-]$ |
| $k$    | Turbulent kinetic energy $[m^2 \cdot s^{-2}]$ or thermal conductivity $[W \cdot m^{-1} \cdot K^{-1}]$ |
| $Lc$   | Characteristic length $[m]$ |
| $k$    | Thermal conductivity of air $[W/mK]$ |
| $Cp$   | Specific heat capacity $[J \cdot kg^{-1} \cdot K^{-1}]$ |
| $\nu$  | Horizontal velocity component $[m \cdot s^{-1}]$ |
| $q$    | Density heat flux $[W \cdot m^{-2}]$ |
| $Q$    | Heat flux $[W \cdot m^{-2}]$ |

Greek symbols

| Symbol | Description |
|--------|-------------|
| $\nu$  | Kinematic viscosity $[m^2 \cdot s^{-1}]$ |
| $\beta$ | Coefficient of thermal expansion $[K^{-1}]$ |
| $\rho$ | Density $[kg \cdot m^{-3}]$ |
| $\delta$ | Distance of boundary node from wall $[mm]$ |
| $\varepsilon$ | Dissipation rate of turbulent kinetic energy $[kg \cdot m^2 \cdot s^{-3}]$ |
| $\Omega$ | Direction vector |
| $\mu$  | Dynamic viscosity $[kg \cdot m^{-2} \cdot s^{-1}]$ |

Subscripts

| Symbol | Description |
|--------|-------------|
| $h$    | Hot wall |
| $c$    | Cold wall |
| $\infty$ | Unperturbed fluid |

CONTACT A. Rincón-Casado alejandro.rincon@uca.es

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1. Introduction

Natural convection in closed enclosures has been widely studied both experimentally and numerically due to the special interest in many engineering applications: solar collectors, nuclear reactors, refrigeration in electronic components. However, current studies on enclosures of buildings are less detailed. In addition, specific natural convection correlations for buildings thermal simulation programs are not reported in the literature.

In the last four decades, numerous theoretical and experimental studies on natural convection in enclosures have been carried out. As a result, the heat and fluid flow due to natural convection has received considerable attention from many researchers. Recent work shows that research is continuing on these issues, as presented by Altaç and Uğurlubilek (2016) with regard to three-dimensional effect and various turbulence models. Another recent study carried out by Obyn and Van Moeseke (2015) focuses on the impact of convection heat transfer coefficient (CHTMC) expressions used for the calculation of heating and cooling loads of buildings.

Fluids, such as air, in the absence of external forces like fans or exterior wind, move due to density variations in their bulk. These density variations are caused by temperature gradients and buoyancy forces that appear in the presence of gravity. Consequently, the rising of low-density particles occurs along with the falling of high-density particles; this phenomenon is known as natural or free convection. Inside buildings where natural convection and forced convection coexist, the common practice is to evaluate the importance of each convection type separately, to determine whether either is dominant with respect to the other or if they must be considered simultaneously (Bejan, 2004).

In thermal simulation programs, such as ESP-r (2005), Energy Plus (2010), DOE-2 (2003), and TRNSYS (2012), the CHTC is fixed as a constant value; or, at most, the programs make the coefficients depend on the velocity and temperature difference between the surfaces. In such cases, a flat-plate correlation is used, or another empirical correlation is applied as obtained from Awbi and Hatton (1999), or Novoselac, Burley, and Srebric (2006). However, in many cases, the flow pattern, which is one of the most important factors in calculating such heat transfer coefficients, is not taken into account. This flow pattern directly depends on the geometry of the problem, such as the enclosure aspect ratio. In the case of natural convection, in general, existing correlations have been developed for small enclosures, where the fluid regime is laminar (Corcione, 2003). However, in typical building enclosures, due to their large dimensions, the regime is turbulent and the aspect ratios exert a non-negligible influence.

1.1. Three-dimensional effects

Three-dimensional laminar natural convection was studied by Mallinson and Davis (1977), Lee, Son, and Lee (1988), and Lankhorst and Hoogendoorn (1988) for enclosures with $1 \leq A \leq 2$ and for $10^5 \leq Ra \leq 10^6$. Fusegi, Hyun, Kuwahara, and Farouk (1991) investigated 3D natural convection in a differentially heated cavity for $10^3 \leq Ra \leq 10^6$. Tric, Labrosse, and Betrouni (2000) studied laminar natural convection of an air-filled cubic cavity for $10^3 \leq Ra \leq 10^7$. Wakashima and Saitoh (2004) studied the same problem for $10^3 \leq Ra \leq 10^6$.

A numerical benchmark study for $10^5 \leq Ra \leq 10^8$ was conducted by Pepper and Hollands (2002). Ravnik, Skerget, and Zunic (2008) examined 3D natural convection in an inclined enclosure using the boundary element method for $10^3 \leq Ra \leq 10^5$. The results for an inclined enclosure with $A = 2$ are also presented. Three-dimensional effects are less pronounced with a reduction of 2–4% compared to a 2D model for $Ra \leq 10^{12}$ and an aspect ratio of $H/L = 1$, according to results reported by Altaç and Uğurlubilek (2016). This is due to the fact that the 3D effect of the boundary layer is not significant in the average Nusselt in high Rayleigh ($Ra$) numbers. For that reason, this work considers negligible 3D effects, which produces a saving in computational cost and a focus on the influence of the enclosure aspect ratio.

1.2. Previous experimental works on vertical surfaces

Advanced experimental techniques have been used in the past, but not without difficulty; accurate experimental work has been limited by the low absolute velocity values and the fact that the fluid flow and heat transfer inside a rectangular cavity are highly sensitive to the experimental and boundary conditions.
The majority of the most rigorous research in this field is oriented towards the study of rectangular enclosures, where heat flux is unidirectional; i.e., flotation is induced from the vertical walls or from the floor to the roof. This is reported in the general review works by Catton (1978) and Ostrach (1972) on the natural convection in closed cavities.

A significant number of experimental works have been carried out in the past decade in an attempt to understand turbulent flow in enclosures, heated and cooled from opposing isothermal faces. The increase in the Reynolds number has been gradual, beginning with $Ra = 10^4$ in previous years, to $Ra \geq 10^{10}$ in the current period. These experiments are needed to provide benchmark data for other studies, such as that developed by Leong, Hollands, and Brunger (1998), who reported the Nusselt number result for vertical walls for a cubical air-filled cavity in laminar flow ($Ra = 4 \cdot 10^4$), tilted at $0^\circ$, $45^\circ$, or $90^\circ$. Betts and Bokhari (2000) studied a tall enclosure in laminar natural convection for $Ra = 0.86 \cdot 10^6$ and $Ra = 1.43 \cdot 10^6$. Mamun, Leong, Hollands, and Johnson (2003) published an extension to previous work in which the mean Nusselt number for $10^4 < Ra < 3 \cdot 10^8$ was measured. The results are considered suitable for the testing of computational codes. Ampofo (2005) conducted an experimental benchmark data study of low-level turbulence natural convection in an air-filled vertical square cavity. The dimensions of the cavity were $0.75 \text{m} \times 0.75 \text{m} \times 1.5 \text{m}$, giving two-dimensional flow, with an $Ra$ number of $1.58 \cdot 10^9$. Salat et al. (2004) experimentally and numerically investigated the turbulent natural convection flow that develops in a differentially heated cavity of height ($H = 1 \text{m}$, width $W = H$, and depth $D = 0.32H$), submitted to a temperature difference between the active vertical walls equal to $15 \text{K}$, resulting in a characteristic $Ra$ number equal to $1.5 \cdot 10^9$. Baire (2008) experimentally and numerically studied the natural convection in air-filled 2D tilted square cavities. In his study, various geometrical and thermal configurations were performed for $10 \leq Ra \leq 10^4$ and tilt angles $0 \leq \alpha \leq 360$. Saury, Rouger, Djanna, and Penot (2011) presented an experimental work for large $Ra$ numbers in a 4 m-high cavity with a horizontal cross-section equal to $0.86 \times 1.00 \text{ m}^2 (4 \cdot 10^{10} \leq Ra \leq 1.2 \cdot 10^{11})$. İnan, Başaran, and Ezan (2016) numerically and experimentally studied the heat transfer in a rectangular cavity that simulated a double-skin façade and included natural convection. $Ra$ numbers ranging from $8.59 \cdot 10^9$ to $1.41 \cdot 10^{10}$ were studied, and a correlation for the Nusselt number was developed. However, the aspect ratio experimental enclosure was not studied due to difficulties in experimentation; therefore, the correlations from the work are not general and the applicability is limited.

1.3. Previous numerical works on vertical surfaces

In numerical and theoretical studies, strong coupling of the boundary layer and the core flow makes computation very difficult. In addition, direct simulation of turbulent natural convection in a cavity is still too costly. Numerical results from various $k$-$\varepsilon$ models are non-unique. In addition, none of the turbulence models can correctly predict whole velocity and temperature fields. The limitations of computer technologies have restricted the numerical studies to 2D models up until the last few decades. Among the works that have studied heat transfer on vertical walls of a square enclosure with an adiabatic floor and ceiling, it is worth highlighting the developments by Barakos, Mitsoulis, and Assimacopoulos (1994), Markatos and Pericleous (1984), De Vahl Davis and Jones (1983), and Lari, Baneshi, Gandelikan Nassab, Komiya, and Maruyama (2011), who studied the effects of radiative transfer in participating media.

These results are often used to validate methodologies and calculation models. More recently, Altaç and Uğurlu bilek (2016) used the results reported by other authors for comparison with various models of turbulence. However, one can observe in these works that the influence of the aspect ratio on rectangular enclosures has been poorly studied. Bejan (2004) provided further insight into the aspect ratio influence, although for small aspect ratios and only in the laminar regime. Osman, Poole, and Chakraborthy (2012) continued this work, but the turbulent regime was not investigated. Trias, Gorobets, Soria, and Oliva (2010) developed a set of direct numerical simulations of a differentially heated cavity of aspect ratio 4 with adiabatic horizontal walls in turbulent flow. Five configurations based on the cavity height were presented ($Ra = 6.4 \cdot 10^8, 2 \cdot 10^9, 10^{10}$ and $10^{11}$). These are valid configurations for façades of buildings, but are not suitable for rooms or spaces in which the aspect ratio is less than 1.

1.4. Previous numerical works on horizontal surfaces

Some other papers have dealt with heat transfer in closed enclosures with a hot floor, cold roof, and adiabatic vertical walls; for instance, in Anil, Velusamy, Balaji, and Venkateshan (2007), the natural convection in an enclosure with localized heating from below, and the radiation effect on all surfaces, was analyzed. However, Corcione (2003) performed more extensive work by studying the
effect of 2D heat flux in a rectangular enclosure and analyzing a range of configurations, temperature distributions, aspect ratios, and \( Ra \) numbers, though only under a laminar regime. From his work, it is concluded that for aspect ratios greater than 2 the influence of the temperature distribution across the different walls on the mean Nusselt number is negligible. Vasiliev et al. (2016) developed a numerical and experimental study in a cubic cell with turbulent Rayleigh–Bénard convection using two similar (but not identical) experimental setups in different laboratories. Both teams conducted experiments for the same set of Prandtl and \( Ra \) numbers (\( Pr = 3.5, Pr = 6.1, Ra = 2.0 \cdot 10^9, Ra = 6.0 \cdot 10^9, Ra = 1.6 \cdot 10^{10} \)). Sheard and King (2011) developed horizontal convection in a rectangular enclosure driven by a linear temperature profile along the bottom boundary, which was investigated numerically using a spectral-element discretization for velocity and temperature fields. They study geophysical and geological applications using water as a working fluid. A study of aspect ratio influence was developed for critical \( Ra \) numbers with different aspect ratios (2, 1, 0.625, 0.333, and 0.16).

Experimental data obtained independently by the two teams can be used as a benchmark for Computer Fluid Dynamics (CFD) codes. However, the authors did not develop any correlation and did not discuss the influence of 3D effects. Yang and Wu (2016) studied the effects of natural convection, wall thermal conduction, and thermal radiation on heat transfer uniformity at a heated plate located at the bottom of a 3D rectangular enclosure. This work applies to laminar flow and an aspect ratio of 0.5–1.5, and radiation effects. The authors developed new correlations, but the turbulent flow effect was not considered.

1.5. Motivation of the present work

Based on the current state of natural convection investigations, the knowledge gap lies in the calculation of CHTC oriented to building enclosures. In this case, the high \( Ra \) numbers are predominant, and the enclosure aspect ratio and enclosure average air temperature must be studied. In addition, the correlations should be simple to implement in the building’s thermal simulation programs.

In this work, a series of correlations have been developed as a function of the aspect ratio (\( H/L \)) and the \( Ra \) numbers, calculated from the enclosure average air temperature and wall surface temperature. The characteristic length used is the length of the surface, and not the separation between opposing surfaces as considered by other authors. All this information is available in buildings’ thermal simulation programs. To resolve this problem, first, vertical walls are studied when the horizontal walls are adiabatic; second, the floor is studied when verticals walls are adiabatic. The influence of the aspect ratio and high \( Ra \) numbers are studied in both configurations. The challenges of this work pertains to solving problems in natural convection and steady flow with high \( Ra \) numbers, where reaching a steady state is necessary to solve the transient state.

There are many studies in this field, but few study have focused on large enclosures (high \( Ra \) numbers) and the influence of different aspect ratios. The enclosures studied have a real dimensional enclosure in buildings (3 × 3 m, 3 × 6 m, or 3 × 1.5 m, for example).

1.6. Problem definition

Calculating the heat transfer coefficient in 3D enclosures in natural convection through CFD techniques is an extremely complex problem in which it is difficult to obtain convergence, and that is associated with an elevated computational cost. Therefore, most authors simplify the problem through 2D planes, a transversal plane, and a longitudinal plane. This simplification is possible because air movement in natural convection can only occur in the direction of gravity, in a positive or negative sense; therefore, the flow pattern is similar in all cases (Altay & Uğurlubilek, 2016).

It is necessary to find a correlation to implement in thermal building simulation software. Extant studies have not clarified the correlation used in the case of a typical building room in natural convection. To simplify the 2D problem enclosure for high \( Ra \) numbers, the hypothesis is proposed that the heat transfer coefficients will be approximated to those of a flat plate for vertical walls and floor and ceiling.

With the objective to study 2D enclosures with different temperatures on the surfaces, two levels of approximation or initial hypotheses are proposed: first, the vertical or horizontal flat plate approximation, and second, the approximation to two enclosures, where the vertical and horizontal surfaces are studied separately.

Due to the large dimensions of typical building spaces, the hypothesis that the heat transfer coefficients of the walls are not influenced by the remainder of the walls is typically considered; therefore, the heat transfer coefficients can be calculated in each wall as an isolated vertical or horizontal flat plate without the need to solve the problem of the entire enclosure. Figure 1 shows a schematic of this hypothesis.

A better approximation is proposed in the present article and consists of approximating a 2D enclosure with different temperatures as two 2D enclosures of the square cavity-type problem, where the vertical and horizontal
Figure 1. Equivalence schematic between the enclosure or cavity in natural convection with isolated vertical and horizontal flat plates.

Figure 2. Scheme 2D cavity in natural convection (a) vertical wall; (b) horizontal wall, floor, and ceiling.

surfaces are studied separately. In the first enclosure, the roof and the floor are adiabatic, and the vertical walls are at different temperatures (Figure 2(a)). In the second enclosure, the vertical walls are adiabatic, and the floor and roof are at different temperatures (Figure 2(b)). In this latter case, there are two variants that depend on the floor and roof temperature (hot floor and cold roof or cold floor and hot roof). This methodology is more complex; however, there are many such problems that have been solved in literature, such as Barakos et al. (1994), Markatos and Pericleous (1984), and De Vahl Davis and Jones (1983), that serve as validation cases for the methodology used. Figure 1 shows the schematic of the proposed hypothesis.

In a 2D enclosure where all surfaces are at a different temperature and where the mean temperature of the enclosure is known, the heat transfer coefficients of the vertical surfaces can be calculated from a model with adiabatic horizontal surfaces and where the vertical surfaces are at different temperatures (Figure 2(a)). With these boundary conditions, the CFD problem is solved and the heat transfer coefficient for the vertical surface is obtained. In the same way, the heat transfer coefficients of floor and ceiling are calculated from the results CFD (Figure 2(b)).

2. Materials and methods

2.1. Computational model

In the computational model studied, steady state, 2D geometry, and Newtonian fluid are considered. All of the fluid properties remain constant except for the density, which depends on the temperature difference. The studied phenomenon is natural or free convection; thus, buoyancy effects are studied due to the gravity effect. The CFD results are obtained by solving the Navier–Stokes equations and the energy equation via finite volumes using commercial software (Fluent, 2015). The numerical algorithm used is semi-implicit method for pressure linked equations (SIMPLE), which was developed by Patankar and Spalding (1972) and recently Kengni Jotsa and Pennati (2015) using in a cost-effective Finite Element (FE) method 2D Navier–Stokes equations. The Q-QUICK scheme is used for convective flux in incompressible flow on unstructured grids, as per the validation developed by Hua, Xing, Chu, and Gu (2009). In the equations solution, the Boussinesq approximation is considered for buoyancy. Although the problem to be solved is a steady state problem, due to its computational complexity, it is necessary to solve it as a transient until a steady state solution is reached. The convergence criteria imposed on all of the equations is $10^{-5}$. To determine the time step size, the criteria $\Delta t = \left(\frac{L}{\beta g \Delta T} \right)^{1/2}$ with $20{\degree}C$ of difference temperature on the hot side and the cold side, as recommended by Fluent (2015), is used. In order to obtain accurate and meaningful numerical solution, meshing the computational domain is the crucial first step. This importance is more pronounced in fast-moving flows due to steep gradients occurring within the boundary layers. For this reason, in order to accurately resolve these steep gradients in the buoyancy-driven boundary layers and also satisfy the $y^+$ criteria, different grid configurations involving fine meshes near the walls are used.

2.2. Turbulence models

The turbulent model used is the Realizable $k$-$\varepsilon$ model. Realizable $k$-$\varepsilon$ model include an eddy viscosity model based on the positivity of normal Reynolds stresses and the Schwarz' inequality for turbulent shear stresses. This model was developed by Shih, Liou, Shabbir, Yang, and Zhu (1995) to model airflow in enclosures, which showed that it is reliable to predict the velocity and temperature mean fields. This model was applied by Teodosiu, Hohota, Rusaouën, and Woloszyn (2003) within a prediction of the indoor environment in a mechanically ventilated test room. Kuznik, Rusaouën, and Hohotă (2006) compared measurements with simulations using...
the realizable $k$-$\varepsilon$ model and the Large Eddy Simulation (LES) model. They concluded that all models predicted the flow relatively well, with best results for the realizable $k$-$\varepsilon$ model. Recently Altaç and Uğurlubilek (2016) published a paper for 2- and 3D unsteady-state continuity, where they studied six turbulence models: Standard $k$-$\varepsilon$, Re-Normalization Group $k$-$\varepsilon$, Realizable $k$-$\varepsilon$ (RKE), Reynolds Stress Model, Standard $k$-$u$, and Shear Stress Transport $k$-$\varepsilon$ RANS (Reynolds averaged Navier–Stokes) models are used in conjunction with the two-layer (or Enhanced Wall Treatment) wall model for Ra numbers ranging from $10^8$ to $10^{13}$. The results showed that the RKE model provides the best performance of all the $k$-$\varepsilon$ model versions for several validations of separated flows and flows with complex secondary flow features.

2.3. Mesh and wall functions

One of the most important aspects when using CFD tools is creating the computational grid for the domain. The grid used is structured with rectangular elements and with uniform growth from the wall towards the center of the enclosure. In this way, we have better convergence properties where the heat transfer coefficients are calculated because there is greater density of elements within the thermal boundary layer. The parameter that controls the correct solution of the viscous sublayer is $y^+$; this dimensionless parameter depends on the turbulence model used (Equation (8)–(12)). To ensure the correct solution inside the boundary layer, it is necessary to have two nodes in the viscous sublayer, and the growth rates towards the center must be 5–10%. Thus, for the case of the turbulence $k$-$\varepsilon$ model with enhanced wall treatment, the parameter $y^+$ must be close to 1.

$$y^+ = \frac{\rho \delta u_T}{\mu}$$

(1)

Because $y^+$ depends on the friction velocity ($u_T$), its value is known after the simulation once the problem reaches convergence. Therefore, to obtain the desired $y^+$, the value of the distance of the boundary node from the wall is varied, which is the distance measured from the wall to the first node of the grid until $y^+ = 1$ is reached. To build the mesh, ICEM meshing tool of ANSYS (2015) software is employed. The growth ratio is 1.05, and the first element is 5 mm. With this configuration, meshes with proportional growth are achieved along with smaller discretization errors. The number of elements in the X and Y direction are varied depending on the geometry of the enclosure.

2.4. Grid sensitivity analysis

To obtain a mesh configuration that offers a good trade-off between accuracy and computing costs, it is necessary to establish a mesh refinement process. The method chosen is that developed by Celik, Ghia, Roache, Freitas, & Coleman (2008). This process consists of selecting three different grids with various coarseness definitions; that is, a coarse grid, a medium grid and a fine grid. This mesh is refined near the plate surface so that the velocity and temperature gradient vary rapidly. The CFD results of the Nusselt number are compared with one another to derive the best compromise between accuracy and computational cost. The convergence criteria imposed on all of the residual equations is above $10^{-5}$.

In order to obtain accurate and meaningful numerical solution, meshing the computational domain is again the crucial first step, and again this importance is more pronounced in fast-moving flows due to steep gradients occurring within the boundary layers. For this reason, to solve the boundary layers gradients it is necessary satisfy the $y^+$ criteria, and select an independent solution mesh size. Figure 3 shows the results for $Ra_H = 10^9, 10^{10}$ and $10^{11}$ and an aspect ratio $H/L = 1$ with a growth rate mesh of 1.1. The black symbols show the optimum mesh where there is a balance between accuracy and computational cost.

Santhosh, Suresh, and Das (2009) also used the full approximation scheme multi-grid method. The solutions computed exhibit a very good representation of secondary and tertiary vortices, but this methodology is not used in turbulent flow.

2.5. Governing equations

To calculate the heat-transfer coefficients, the velocity, temperature, and pressure fields of the room must be determined. Thus, we employed the Navier–Stokes

![Figure 3. Variation of Nusselt number with the grid size.](image-url)
equations, which describe the fluid motion for a given set of boundary conditions. These equations, along with the turbulence model and energy equation, are solved at each node of the mesh.

The turbulence model employed here is the realizable \( k-\varepsilon \) model, implemented in Fluent (2015). This model is a low-\( k-\varepsilon \) model, and differs from the standard \( k-\varepsilon \) model through a new formulation of turbulent viscosity and transport equation for \( \varepsilon \). The equations for the 3D model are provided in tensor notation, where \( x_i \) represents the variables X, Y, and Z and \( u_i \) represents the corresponding velocity components. These are listed below:

Continuity equation:
\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \tag{2}
\]

Equation for conservation of momentum:
\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \right) \right] + \rho g_i \tag{3}
\]

Energy equation:
\[
\frac{\partial (\rho T)}{\partial t} + \frac{\partial (\rho u_i T)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[ \left( \mu + \frac{\mu_t}{Pr_T} \right) \frac{\partial T}{\partial x_i} \right] \tag{4}
\]

Shear viscosity equation:
\[
\mu_t = \frac{\rho C_\mu k^2}{\varepsilon} \tag{5}
\]

Here, the variable \( C_\mu \) is calculated as follows:
\[
C_\mu = \frac{1}{4.04 + A_i (k U^*/\varepsilon)} \tag{6}
\]

where
\[
U^* = \sqrt{S_{ij} S_{ij}} + \Omega_{ij} \Omega_{ij} ; \Omega_{ij} = \Omega_{ij} - 2 \varepsilon_{ijk} \omega_k ;
A_i = \sqrt{6} \cos \phi_i ; S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{7}
\]

The “k” transport equation in the turbulence model is:
\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho k u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \nonumber \\
+ \frac{\partial G_k}{\partial x_i} + G_b - \rho \varepsilon + S_k \tag{8}
\]

where \( G_k \) represents the production of turbulent kinetic energy, which is common to all \( k-\varepsilon \) turbulence models and is given by
\[
G_k = -u_i \frac{\partial u_j}{\partial x_i} \tag{9}
\]

The term \( G_b \) represents the generation of turbulent kinetic energy because of buoyant forces when the system is under a gravitational field, and it is calculated as follows:
\[
G_b = \beta g_i \frac{\mu_T}{Pr_T} \frac{T}{\partial x_i} \tag{10}
\]

where \( Pr_T = 0.72 \) is the Prandtl number for energy and \( \beta \) is the thermal expansion coefficient, which is calculated as follows:
\[
\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial x_i} \right) \tag{11}
\]

The transport equation for \( \varepsilon \) from the turbulence model is:
\[
\frac{\partial (\varepsilon)}{\partial t} + \frac{\partial (\rho u_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_T}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] \nonumber \\
+ \rho C_\varepsilon S_\varepsilon - \rho C_2 \frac{e^2}{k + \sqrt{\nu \varepsilon}} + C_3 \varepsilon \frac{e}{k} G_b + S_\varepsilon \tag{12}
\]

The source terms \( S_\varepsilon \) and \( S_k \) can be defined for each case, and are optional.

The coefficient \( C_{3\varepsilon} \) is calculated as follows:
\[
C_{3\varepsilon} = \tan h \left( \frac{|u|}{U^*} \right) \tag{13}
\]

The constants used in the realizable \( k-\varepsilon \) model are as follows:
\[
\sigma_k = 1.0, \sigma_\varepsilon = 1.3, C_{1\varepsilon} = 1.44, 
C_{2\varepsilon} = 1.92, C_2 = 0.43, C_2 = 1.9 \tag{14}
\]

Newton’s law of cooling provides the convective heat transfer coefficient between the surface of the wall and the fluid in motion at different temperatures. This coefficient depends on the total heat flux of the wall, the transfer area, and the temperature difference between the temperature surface and temperature of the unperturbed fluid:
\[
\bar{h}_i = \frac{Q_i}{A_i (T_{si} - T_\infty)} \tag{15}
\]

The total heat flux transfers between the fluid and the wall is calculated from the temperature gradient produced inside of the thermal boundary layer of the fluid through an integration of Fourier’s law at each wall.
Because the heat flux depends on the temperature gradient, it is solved by using CFD. Thus, the heat flux at each wall is obtained by integrating the temperature gradient along the wall, and it is affected by the fluid’s conductivity, as shown in Equation (16).

$$Q_{\text{CFD}}|_n = \int_0^{L_n} k \frac{\partial T}{\partial n} \, dn|_{n=0} \quad (16)$$

Therefore, at the fluid–solid interface, the heat transfer by conduction equals the heat transfer by convection, and the average heat-transfer coefficients are as follows:

$$\bar{h}_n = \frac{\int_0^{L_n} k \frac{\partial T}{\partial n} \, dn|_{n=0}}{A_i(T_h - T_{\infty})} \quad (17)$$

Equation (17) shows that the temperature difference between the wall ($T_h$) and free fluid ($T_{\infty}$) is proportional to the temperature gradient of the thermal boundary layer. Thus, increases in the temperature transferred between the wall and fluid correspond to increases in the gradient within the boundary layer, which maintains a constant ratio and indicates that the heat-transfer coefficient is temperature independent. This finding is valid for flat plates where the fluid is not perturbed by other walls and for rooms with walls whose temperatures are all equal. However, this finding is not valid for cases in which the walls have different temperatures.

3. Results

3.1. Correlation for vertical surfaces in rectangular enclosures

To obtain the correlations in the vertical walls of an enclosure in natural convection and for different aspect ratios, the correlations in the scientific literature are analyzed and compared with the results of the present work. In addition, the correlations obtained along with those of other authors are compared with existing correlations for a vertical flat plate.

The square enclosure or cavity with two vertical walls at different temperatures, where the other two walls are adiabatic, is a reference problem from literature (Arnold, Catton, & Edwards, 1976) that serves as the validation model for numerical and empirical simulations. The problem consists of a square cavity in which the $Ra$ number increases with the enclosure size and maintains the hot wall temperature at 303.15 K and the cold wall temperature at 283.15 K.

Table 1 shows the results obtained by different authors in the study of a square cavity ($H/L = 1$) with lateral walls at different temperatures; the $Ra$ number is varied through the cavity size and the variable of interest is the mean Nusselt number. As validation, the results of the different cases are compared, and indicate the characteristics of the grid used. When $Ra \geq 10^8$, the turbulent model is used because turbulent effects are important in the Nusselt number. From Table 1, it can be observed that the mean Nusselt numbers obtained in our research are in agreement with those in the literature. The mean deviation is less than 15%, which therefore validates the CFD methodology used.

### 3.1.1. Nusselt and Rayleigh number in vertical surfaces

In this work, the characteristic length used to calculate Nusselt number on the enclosure vertical walls is the enclosure height ($H$). The convective heat transfer is calculated using Fourier’s law with the temperature difference between the hot wall and the enclosure average air temperature (Equation (18)). Therefore, the Nusselt number is a function of $H$ and $Th-T_{\text{air}}$, as shown in
Equation (19).

\[ Q_{\text{CFD}}^H = h \cdot H(T_H - \bar{T}_{\text{air}}) \]  

\[ \overline{\text{Nu}}_H = \frac{h \cdot H}{k} \cdot \frac{Q_{\text{CFD}}^H}{k \cdot (T_H - \bar{T}_{\text{air}})} \]  

In this work, all developed correlations depend on the temperature difference between the wall and the enclosure average air temperature, which is due to the fact that most buildings’ simulation thermal programs make this information available. Other authors have used the temperature difference between the hot and cold wall (\(T_{\text{h}}-T_{\text{c}}\)). In this case, the Nusselt number is defined using an asterisk (*), as shown in Equation (20). Knowing that the average air temperature of the enclosure is \(T_{\text{air}} = (T_{\text{h}}-T_{\text{c}})/2\), the relationship between both Nusselt numbers is shown in Equation (21).

\[ \overline{\text{Nu}}_H^* = \frac{Q_{\text{CFD}}^H}{k \cdot (T_{\text{h}} - T_{\text{c}})} \]  

\[ \overline{\text{Nu}}_H^* = \frac{1}{2} \cdot \overline{\text{Nu}}_H \]  

In the same way, the \(Ra_H\) number is defined with \(H\) as the characteristic length and the temperature difference between the hot wall and the enclosure average air temperature (\(T_{\text{h}}-T_{\text{air}}\)). However, many authors have used the length (\(L\)) as the characteristic length, and \(T_{\text{h}}-T_{\text{c}}\) as the temperature difference. In this case, the \(Ra\) number is defined as \(Ra^*_L\); the relationship between both \(Ra\) numbers is shown in Equation (22).

\[ Ra_H = \frac{1}{2} Ra^*_H = \frac{1}{2} \left(\frac{L}{H}\right)^3 Ra^*_L \]  

### 3.1.2. Aspect ratio influence

In cavities heated by a vertical wall, it is necessary to analyze the influence of the enclosure aspect ratio. For such analysis, the cavities are classified according to the aspect ratio (\(H/L\)); based on this ratio, there are enclosures with \(H/L < 1\), \(H/L = 1\), or \(H/L > 1\). For aspect ratios greater than 1, \(H/L = 2\) is taken as the reference, and for aspect ratios less than 1, \(H/L\) is considered equal to 0.5; most building spaces are within this range. The case of \(H/L = 1\) was analyzed in the previous section and compared with the correlations proposed by other authors; we now deal with cases \(H/L > 1\) and \(H/L < 1\). As before, the existing correlations are analyzed and compared with those obtained in our computations.

In the case of \(H/L > 1\), Bejan (1979) proposes a correlation in which the \(\overline{\text{Nu}}_H^*\) depends on the \(Ra^*_H\) and \(L/H\):

\[ \overline{\text{Nu}}_H^* = 0.364 \left(\frac{L}{H}\right) L^{1/4} Ra^*_H^{1/4} \quad H/L > 1; \quad \frac{L}{H} Ra^*_H^{1/4} > 5 \]  

Berkovsky and Polevikov (1977) proposed the following correlations:

For: \(1 < (H/L) < 2\).

\[ \overline{\text{Nu}}_H^* = 0.18 \left(\frac{Pr}{0.2} + Pr Ra^*_H\right)^{0.29} \left(\frac{L}{H}\right)^{-0.13} 10^{-3} < Pr < 10^{13} \]  

\[ \frac{Pr}{0.2} + Pr Ra^*_H (\frac{L}{H})^{3} > 10^{3} \]  

For: \(2 < (H/L) < 10\).

\[ \overline{\text{Nu}}_H^* = 0.22 \left(\frac{Pr}{0.2} + Pr Ra^*_H\right)^{0.28} \left(\frac{L}{H}\right)^{0.09} Ra^*_H < 10^{13} \]  

\[ Pr < 10^{5} \]  

### Cases with \(H<L\).

Table 2. Numerical values of Nusselt numbers according to the graph from Bejan (1980).

| \(Ra^*_H\) | \(\overline{\text{Nu}}_H^*\) |
|-----------|-----------------|
| 1.00E+04  | 1.8             |
| 1.00E+05  | 4.0             |
| 1.00E+06  | 9.0             |
| 1.00E+07  | 17.0            |
| 1.00E+08  | 28.0            |

In the case of enclosures whose height is less than the enclosure length, there are a few correlations that calculate the mean Nusselt number; the most representative one is that proposed by Bejan (1980), who developed a calculation through correlations that fit well with experimental data. Bejan provided a graph that relates the aspect ratio, \(Ra\) number, and mean Nusselt number; thus, for an aspect ratio of \(H/L = 0.5\), the values obtained are as shown in Table 2. Although the values in Table 2 are valid for laminar flow, there are no correlations for cases of turbulent flow that address this range of application. Due to the diversity of the existing correlations and their different forms, it is necessary to find our own correlation that takes into account the influence of the aspect ratio, and compare this with those obtained by other authors. Therefore, different cases are solved by varying the \(Ra\) number through the space dimensions and
Table 3. Compilation of cases studied on rectangular enclosure with vertical walls at different temperatures and adiabatic horizontal surfaces.

| $Ra_H$  | $Nu_H (H/L = 0.5)$ | $Nu_H (H/L = 1)$ | $Nu_H (H/L = 2)$ |
|---------|--------------------|------------------|------------------|
| 5.00E+02 | 1.21               | 2.24             | 4.01             |
| 5.00E+03 | 3.41               | 4.48             | 5.04             |
| 5.00E+04 | 8.14               | 9.04             | 10.00            |
| 5.00E+05 | 16.66              | 17.63            | 18.22            |
| 5.00E+06 | 32.05              | 33.05            | 33.54            |
| 5.00E+07 | 59.24              | 60.45            | 61.05            |
| 5.00E+08 | 109.71             | 110.83           | 110.83           |
| 5.00E+09 | 204.41             | 204.41           | 204.41           |
| 5.00E+10 | 393.12             | 393.12           | 393.12           |
| 5.00E+11 | 739.61             | 739.61           | 739.61           |

Note: ($TH = 30°C$ and $TC = 10°C$) Air properties: $T = 20°C$, $\nu = 1.509 \cdot 10^{-5} kg/m/s$; $Cp = 1006.1 W/kgK$; $k = 0.02564 W/mK$; $Pr = 0.7129$.

The aspect ratio, where the values taken are 0.5, 1 and 2. Table 3 shows the results in terms of the Nusselt number. For low $Ra$ numbers ($10^3$ to $10^5$), the Nusselt number depends on the aspect ratio; however, for high $Ra$ numbers ($10^5$ to $10^{11}$), the Nusselt number does not change significantly. This phenomenon was also observed by Altaç and Uğurlubilek (2016), although these authors did not analyze the aspect ratio $H/L < 1$.

Figure 4 demonstrates a comparison of the streamlines and isotherms for $Ra = 10^3 – 10^6 – 10^{10}$ and $H/L = 0.5$, while Figure 5 shows streamlines and isotherms for $Ra = 10^3 – 10^6 – 10^{10}$ and $H/L = 2$. Figure 4 illustrate that in enclosures of buildings with a high $Ra$ number and low aspect ratio ($H/L$), the temperature influence of the opposing walls is low. Figure 5 shows that in enclosures with a high aspect ratio ($H/L$), the temperature influence of the opposing walls is high.

To analyze the validity of the initial hypothesis, which claims that in typical building enclosures, the vertical walls behave as isolated flat plates, the previous results are compared with those of a vertical flat plate in free convection. For that comparison, correlations reported by Tsuji and Nagano (1988) for the laminar regime (Equation (26)) and for turbulent flow are used (Equation (27)).

$$Nu_H = 0.59 Ra_H^{1/4} \text{for } 10^4 < Ra_H < 10^9 \text{(laminar)}$$  \hspace{1cm} (26)

$$Nu_H = 0.10 Ra_H^{1/3} \text{for } 10^9 < Ra_H < 10^{13} \text{(turbulent)}$$ \hspace{1cm} (27)

The $Ra$ number is determined by the following equation:

$$Ra_H = \frac{g \beta (Th - T_{air}) H^3 Pr}{\nu^2}$$ \hspace{1cm} (28)

Figure 4. Streamlines (right) and isotherms (left) for $Ra = 10^3 – 10^6 – 10^{10}$ and $H/L = 0.5$. Adiabatic horizontal surfaces. $Th = 30°$, $Tc = 20°$. 
Figure 5. Streamlines (right) and isotherms (left) for \( Ra = 10^3 - 10^6 - 10^{10} \) and \( H/L = 2 \). Adiabatic horizontal surfaces. \( Th = 30°, Tc = 20° \).

Figure 6 shows all the simulation results that we obtain for the different aspect ratios and using the flat plate correlation and the correlations developed by other authors. We observe that Bejan (1980) and Berkovsky and Polevikov (1977) studied enclosures with low \( Ra \) numbers \( (Ra_H < 10^7) \), wherein the fluid has a laminar behavior. In addition, the correlations presented depend on the aspect ratio. Barakos et al. (1994) and Markatos and Pericleous (1984) studied enclosures with \( 10^5 < Ra_H < 10^{11} \), but only for square enclosures \( (H/L = 1) \). The results of this work show that for \( Ra_H < 10^5 \), the Nusselt number depends on the aspect ratio \( (H/L) \), whereas for \( Ra_H > 10^5 \), the Nusselt number does not depend on the aspect ratio. The results reported by Altaç and Ugurlubilek (2016) show good accuracy in the range \( 5 \cdot 10^6 < Ra_H < 10^{12} \), for square enclosures \( (H/L = 1) \). Likewise, when compared with the correlation flat plate developed by Tsuji and Nagano (1988) for the laminar zone and turbulent zone, it can be seen that the results of this work approach the flat plate as the \( Ra \) increases.

To calculate the mean Nusselt number in a rectangular enclosure, a correlation is obtained that depends on the aspect ratio and the \( Ra \) number. The correlation coefficients are obtained through a multiple regression of the nonlinear functions that minimizes the mean quadratic error. The software used is Datafit (2014), and the correlation obtained is shown in Table 4. Figure 7 shows the goodness of fit for this correlation. The Nusselt numbers obtained through the simulation are presented against those estimated by the correlation and fitting a 45° line.

### 3.2. Correlations of horizontal surfaces in rectangular enclosures

We performed a literature review for horizontal surfaces in rectangular enclosures. The results found have considered hot floor, cold roof, and adiabatic lateral walls on a laminar regime. We did not find any reference to the turbulent regime. Corcione (2003) considered different hot floor configurations in the laminar regime; this is the closest line of research to our aims and is useful as a reference case. Corcione provided a correlation obtained from numerical simulation data, which is compared with our results. The proposed correlation is as follows:

\[
Nu^*_H = 0.21 \cdot \left( \frac{L}{H} \right)^{0.09} \cdot Ra^*_H^{0.25} \quad \text{for } 10^4 \leq Ra^*_H \leq 10^6 \text{ and } 0.66 \leq \frac{L}{H} \leq 8 \quad (29)
\]

In this work, the characteristic length used to calculate the Nusselt number on the enclosure horizontal walls is the enclosure length \( (L) \). The convective heat transfer is calculated using Fourier’s law with the temperature difference between the hot wall and the enclosure average air temperature (Equation (30)). Therefore, the Nusselt number is a function of \( H \) and \( T_h-T_{air} \), as shown in Equation (31).

\[
\frac{QL_{CFD}}{k} = hL(T_h - T_{air}) \quad (30)
\]

\[
\frac{Nu_L}{L} = \frac{h \cdot L}{k} = \frac{QL_{CFD}}{k(T_h - T_{air})} \quad (31)
\]

Other authors have used the temperature difference between the hot and cold wall \( (Th-Tc) \). In this case, the Nusselt number is defined with an asterisk (*), as shown in Equation (32). Knowing that the average air temperature of the enclosure is \( T_{air} = (T_h+T_c)/2 \), the
Figure 6. Comparison of the mean Nusselt numbers for different aspect ratios with those derived by other authors.

Table 4. Proposed enclosure vertical wall correlation in the laminar and turbulent regime.

| Proposed enclosure vertical wall correlation | Laminar | Turbulent |
|--------------------------------------------|---------|-----------|
| $\overline{N_u_H} = C (H/L)^m R_a^m_H$    |         |           |
| $R^2$                                      | 0.998   | 0.9978    |
| Coefficients                               | 0.4458  | 0.433     |
| $m$                                        | 0.192   | 0         |
| $n$                                        | 0.277   | 0.276     |
| $H/L$                                      | $0.5 < H/L < 2$ | $0.5 < H/L < 2$ |
| Applicability                              | $R_a_H < 10^5$ | $10^7 < R_a_H < 10^{11}$ |

relationship between both Nusselt numbers is shown in Equation (33).

$$\overline{N_u_H} = \frac{Q_{C_{F_{D}}}^L}{k(H - T_c)}$$

$$\overline{N_u_L} = \frac{2}{H \overline{N_u_H}}$$

In the same way, the $Ra$ number ($R_a_L$) is defined with $L$ as the characteristic length and the temperature difference between the hot wall and the enclosure average air temperature ($T_h - T_{air}$). However, many authors have used the length ($H$) as the characteristic length, and $T_h - T_c$ as the temperature difference. In this case, the $Ra$ number is defined as $R_a^*H$; the relationship between both $Ra$ numbers is shown in Equation (34). In our study, different cases with a hot floor ($30^\circ C$) and cold roof ($10^\circ C$) are simulated, so that our aspect ratio is $H/L$, and this parameter is varied using the $L$ dimension ($H/L = 0.5, 1,$ and $2$). In addition, the $Ra$ number used is $R_a_L$, where the characteristic length is $L$ and the temperature difference is $T_h - T_{air}$. The studied cases are shown in Table 5.

$$Ra_L = \frac{1}{2} Ra^*_L = \frac{1}{2} \left( \frac{H}{L} \right)^3 Ra^*_H$$

Once all cases have been simulated to convergence, the heat flux provided by the CFD program is obtained and the mean Nusselt number calculated. In this case, the conduction heat flux is in the vertical direction and the Nusselt number is calculated from Equation (30).

Figure 8 demonstrates a comparison of the streamlines and isotherms for $Ra = 10^5-10^9-10^{11}$ and $H/L = 0.5$, and Figure 9 shows the streamlines and isotherms for $Ra = 10^5-10^9-10^{11}$ and $H/L = 2$. Figure 8 shows that in enclosures of buildings for all $Ra$ numbers and a low aspect ratio ($H/L$), two of the Benard cells appear in a horizontal direction; however, Figure 9 shows the Benard cells in a vertical direction for all $Ra$ numbers. To compare our results with those reported by Corcione (2003), a few parameter changes have to be made. First, it is necessary to relate the Nusselt number as a function of $T_h - T_c$. 

Figure 7. Fit of the correlation with respect to the simulation data. Correlation of enclosure with a vertical wall.
Table 5. Cases studied for hot floor and cold roof in a rectangular enclosure and adiabatic vertical walls ($T_h = 30^\circ C$ and $T_c = 10^\circ C$).

| F | H/L | $Ra_L$ | $Nu_L$ |
|---|-----|--------|--------|
| 1 | 0.5 | 4.00E + 03 | 1.00 |
| 2 | 0.5 | 4.00E + 04 | 2.40 |
| 3 | 0.5 | 4.00E + 05 | 4.43 |
| 4 | 0.5 | 4.00E + 06 | 7.55 |
| 5 | 0.5 | 4.00E + 07 | 12.79 |
| 6 | 0.5 | 4.00E + 08 | 23.38 |
| 7 | 0.5 | 4.00E + 09 | 39.59 |
| 8 | 0.5 | 4.00E + 10 | 191.24 |
| 9 | 1   | 5.00E + 02 | 1.00 |
| 10| 1   | 5.00E + 03 | 2.15 |
| 11| 1   | 5.00E + 04 | 3.91 |
| 12| 1   | 5.00E + 05 | 6.28 |
| 13| 1   | 5.00E + 06 | 12.18 |
| 14| 1   | 5.00E + 07 | 21.11 |
| 15| 1   | 5.00E + 08 | 36.63 |
| 16| 1   | 5.00E + 09 | 85.30 |
| 17| 1   | 5.00E + 10 | 196.16 |
| 18| 2   | 6.25E + 01 | 1.00 |
| 19| 2   | 6.25E + 02 | 2.17 |
| 20| 2   | 6.25E + 03 | 4.20 |
| 21| 2   | 6.25E + 04 | 9.59 |
| 22| 2   | 6.25E + 05 | 18.50 |
| 23| 2   | 6.25E + 06 | 31.75 |
| 24| 2   | 6.25E + 07 | 59.77 |
| 25| 2   | 6.25E + 08 | 126.31 |
| 26| 2   | 6.25E + 09 | 233.10 |

Note: Air properties: $T = 20^\circ C$, $\nu = 1.509 \cdot 10^{-5}$ kg/m/s; $C_p = 1006.1$ W/kgK; $k = 0.02564$ W/mK; $C_p = 1006.1$ W/kgK; $Pr = 0.7129$.

Knowing that $T_{air} = (Th - Tc)/2$. Second, Corcione uses the characteristic length $H$, but it is necessary to relate the Nusselt number as a function of $L$, from the aspect ratio $H/L$, as follows shown in Equation (33). The $Ra$ number employed is $Ra^*_L$, which depends on the characteristic length, $L$, and the temperature difference, $Th - Tc$. This number equals the number of $Ra_L$ (Equation 34).

Figure 10 shows the good agreement of our results in the simulation compared to the correlation by Corcione (2003) in the range of $10^3 – 10^6$ (laminar region); however, there are no results for the turbulent regime. For the case of a hot horizontal flat plate, Bergman, Incropera, and Lavine (2011) proposed two empirical correlations for two different ranges of $Ra$ number:

$$\overline{Nu}_L = 0.54Ra_L^{1/4}$$ for $10^4 < Ra_L < 10^7$ (laminar) (35)

$$\overline{Nu}_L = 0.15Ra_L^{1/3}$$ for $10^7 < Ra_L < 10^{11}$ (turbulent) (36)

Figure 11 shows all of the simulated cases along with the flat plate correlation proposed by Bergman et al. (2011). Moreover, laminar and turbulent flows with the results reported by Corcione (2003) for laminar flow of enclosure, the horizontal flat plate, and the results of this work, are compared. It is observed that the lower limit is
pure conduction ($Nu = 1$); i.e., the air has no movement, or movement is minimal, and the heat is transferred via conduction along the height of the enclosure ($H$), thus producing temperature stratification. In the laminar zone ($Ra < 10^7$), the dependence of the Nusselt number with respect to the aspect ratio is strong. However, in the turbulent zone it is observed that the flat plate is the upper limit, where the Nusselt number in turbulent regime is higher than in an enclosure because the air circulates freely and is not affected by other walls. Furthermore, in the turbulent zone, where there are high $Ra$ numbers, the difference between the Nusselt numbers for different aspect ratios decreases and converges to the same result.

Similarly, in the case of vertical walls, a correlation is developed from the CFD results, one for the laminar zone and another one for the turbulent zone. Table 6 shows the best fit coefficients to CFD results for “$C$”, “$m$” and “$n$”. The heat transfer coefficient ($h$) is calculated from (Equation (37)).

$$h = \frac{k \cdot Nu_L}{L}$$ (37)

Figure 12 shows the Nusselt number obtained through the simulation and the Nusselt number obtained through the correlation for the laminar and turbulent regime. According to Figure 12, the square of the correlation coefficient ($R^2$) is extremely good in both regions; however, there is greater deviation in the laminar case. Furthermore, it is worth noting that the most common building cases are in the turbulent region; therefore, the second correlation would be the one used in most cases.

4. Discussion

The present paper shows new correlations for laminar and turbulent regimes in enclosures with natural convection of $10^3 \leq Ra \leq 5 \cdot 10^{11}$ and an aspect ratio of $0.5 \leq H/L \leq 2$. These correlations are oriented to building enclosures and implemented in thermal building simulation programs. The correlations are developed using a high number of CFD simulations for different aspect ratios ($H/L$) and surface temperatures. Both vertical walls and the horizontal floor are studied separately. Thus, two correlations for vertical walls and two others for horizontal surfaces are presented.

This study shows that in vertical walls the Nusselt number depends on the aspect ratio in the laminar flow ($Ra < 10^7$); however, in the transition and turbulent regime ($Ra > 10^7$), there is no dependence on the aspect ratio. The results reported by other authors do not indicate the turbulent zone for different aspect ratios, which
Figure 11. Comparison between an enclosure with a hot floor and a hot horizontal flat plate.

Table 6. Hot floor–cold roof correlation in the laminar and turbulent regime.

| Proposed Hot floor–Cold roof correlation | Laminar | Turbulent |
|-----------------------------------------|---------|-----------|
| $Nu_L = C(H/L)^{m}Ra_L^n$               | $R^2$   | $-0.995$  |
| $R^2$                                   |         | $-0.992$  |
| Coefficients                            | $C$     | $-0.4378$ |
|                                          | $m$     | $-0.498$ |
|                                          | $n$     | $-0.258$ |
| Applicability                           | $H/L$   | $0.5 < H/L < 2$ |
|                                          | $Ra_L$  | $10^3 < Ra_L < 10^7$ |

Figure 12. Representation of the results obtained through CFD simulation and through the correlation proposed in the laminar and turbulent regime.

may be due to the computational complexity involved in solving this problem using CFD techniques.

Regarding enclosures with a hot floor and cold roof, it is demonstrated that the beginning of the convection mechanism depends on the enclosure aspect ratio. However, for low $Ra$ numbers ($Ra < 10^7$), the Nusselt number depends on the aspect ratio, while for high $Ra$ numbers ($Ra > 10^7$), the dependency is low, and is close to the horizontal flat plate results. Therefore, two correlations are developed, one for the laminar regime and another for the transition and turbulent regimes.

In this paper, all developed correlations depend on the temperature difference between the wall and the enclosure average air temperature, which is due to the fact that most buildings' simulation thermal programs make this information available. However, many works have used the different temperature between opposite surfaces. In addition, we use the characteristic length used in the development of correlations, along with the wall length, as opposed to the separation between opposing walls, as used by most researchers. The correlation developed can be implemented in thermal building simulation programs.

The main limitation of this study lies in the high aspect ratio enclosure ($H/L > 2$), and the 3D effects for the high $Ra$ numbers. Future studies can be focused in this direction. Another interesting line of study in this field is to analyze the influence of convective heat transfer coefficients on the annual demands of conditioning in the building.

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References
Ampofo, F. (2005). Turbulent natural convection of air in a non-partitioned or partitioned cavity with differentially heated vertical and conducting horizontal walls. Experimental Thermal and Fluid Science, 29, 137–157. doi:10.1016/j.expthermflusci.2004.02.005
Anil, K. S., Velusamy, K., Balaji, C., & Venkateshan, S. P. (2007). Conjugate turbulent natural convection with surface radiation in air filled rectangular enclosures. International Journal of Heat and Mass Transfer, 50, 625–639. doi:10.1016/j.ijheatmasstransfer.2006.07.022
Arnold, J. N., Catton, I. L., & Edwards, D. K. (1976). Experimental investigation of natural convection in inclined rectangular regions of differing aspect ratios. Journal of Heat Transfer, 98, 67–71. doi:10.1115/1.3450472
Awbi, H., & Hatton, A. (1999). Natural convection from heated room surfaces. Energy and Buildings, 30, 233–244. doi:10.1016/S0378-7788(99)00004-3
Baïri, A. (2008). Nusselt-Rayleigh correlations for design of industrial elements: Experimental and numerical investigation of natural convection in tilted square air filled enclosures. Energy Conversion and Management, 49, 771–782. doi:10.1016/j.enconman.2007.07.030
Barakos, G., Mitsoulis, E., & Assimacopoulos, D. (1994). Natural convection flow in a square cavity revisited: Laminar and turbulent models with wall functions. International Journal for Numerical Methods in Fluids, 18, 695–719. doi:10.1002/fld.1650180705
Bejan, A. (1979). Note on Gill’s solution for free convection in a vertical enclosure. Journal of Fluid Mechanics, 90, 561–568. doi:10.1017/S0022112079002391
Bejan, A. (1980). A synthesis of analytical results for natural convection heat transfer across rectangular enclosures. International Journal of Heat and Mass Transfer, 23, 723–726. doi:10.1016/0017-9310(80)90017-4
Bejan, A. (2004). This is a book title: Convection heat transfer. New York: John Wiley & Sons.
Bergman, T. L., Incropera, F. P., & Lavine, A. S. (2011). Fundamentals of heat and mass transfer. Hoboken, NJ: John Wiley & Sons.
Berkovsky, B. M., & Polevikov, V. K. (1977). Numerical study of problems on high-intensive free convection. In Heat Transfer and Turbulent Buoyant Convection (Eds.), Proceedings of International Turbulent Buoyant Convection Seminar, 443–455. Washington.
Kuznik, F., Rusauën, G., & Hohotă, R. (2006). Experimental and numerical study of a mechanically ventilated enclosure with thermal effects. *Energy and Buildings*, 38, 931–938. doi:10.1016/j.enbuild.2005.08.016

Lankhorst, A. M., & Hoogendoorn, C. J. (1988). Three-dimensional numerical calculations of high Rayleigh number natural convective flows in enclosed cavities. *Proc National Heat Transfer Conf ASME HTD-96*, 3, 463–470.

Lari, K., Baneshi, M., Gandjalikhan Nassab, S. A., Komiya, A., Lankhorst, A. M., & Hoogendoorn, C. J. (1988). Three-dimensional numerical calculations of high Rayleigh number convection in a square cavity containing participating gases. *International Journal of Heat and Mass Transfer*, 31, 3053–3069. doi:10.1016/0017-9310(88)90284-0

Lee, T. S., Son, G. H., & Lee, J. S. (1988). Numerical predictions of three-dimensional natural convection in a box. *Proc 1st KSME-JSME Thermal and Fluids Engng Conf*, 2, 278–283.

Leong, W. H., Hollands, K. G. T., & Bruner, A. P. (1998). On a physically-realizable benchmark problem in internal natural convection. *International Journal of Heat and Mass Transfer*, 41, 3817–3828. doi:10.1016/S0017-9310(98)00095-7

Mallinson, G. D., & Davis, G. D. V. (1977). Three-dimensional natural convection in a box: A numerical study. *Journal of Fluid Mechanics*, 83, 1–31. doi:10.1017/S0022112077001013

Mamun, M. A. H., Leong, W. H., Hollands, K. G. T., & Johnson, D. A. (2003). Cubical-cavity natural-convection benchmark experiments: An extension. *International Journal of Heat and Mass Transfer*, 46, 3655–3660. doi:10.1016/S0017-9310(03)00155-8

Markatos, N. C., & Pericleous, K. A. (1984). Laminar and turbulent natural convection in an enclosed cavity. *International Journal of Heat and Mass Transfer*, 27, 755–772. doi:10.1016/0017-9310(84)90145-5

Novoselac, A., Burley, B., & Srebric, J. (2006). Development of new and validated of existing convection correlations for rooms with displacement ventilation systems. *Energy and Buildings*, 38, 163–173. doi:10.1016/j.enbuild.2005.04.005

Obyn, S., & Van Moeiske, G. (2015). Variability and impact of internal surfaces convective heat transfer coefficients in the thermal evaluation of office buildings. *Applied Thermal Engineering*, 87, 258–272. doi:10.1016/j.applthermaleng.2015.05.030

Osman, T., Poole, R. J., & Chakraborty, N. (2012). Influences of boundary conditions on laminar natural convection in rectangular enclosures with differentially heated side walls. *International Journal of Heat and Fluid Flow*, 33, 131–146. doi:10.1016/j.ijheatfluidflow.2011.10.009

Ostrach, S. (1972). Natural convection in enclosures. *Advances in Heat Transfer*, 8, 161–227. doi:10.1016/S0065-2717(08)70039-X

Patankar, S. V., & Spalding, D. B. (1972). A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows. *International Journal of Heat and Mass Transfer*, 15, 1787–1806. doi:10.1016/0017-9310(72)90054-3

Pepper, D. W., & Hollands, K. G. T. (2002). Summary of benchmark numerical studies for 3-D natural convection in an air-filled enclosure. *Numerical Heat Transfer, Part A: Applications*, 42, 1–11. doi:10.1080/10407780290059396

Ravnik, J., Skerget, L., & Zunic, Z. (2008). Velocity–vorticity formulation for 3D natural convection in an inclined enclosure by BEM. *International Journal of Heat and Mass Transfer*, 51, 4517–4527. doi:10.1016/j.ijheatmasstransfer.2008.01.018

Salat, J., Xin, S., Joubert, P., Sergent, A., Penot, F., & Le Quere, P. (2004). Experimental and numerical investigation of turbulent natural convection in a large air-filled cavity. *International Journal of Heat and Fluid Flow*, 25, 824–832. doi:10.1016/j.ijheatfluidflow.2004.04.003

Santhosh, D. K., Suresh, K. K., & Das, M. K. (2009). A fine grid solution for a lid-driven cavity flow using multigrid method. *Engineering Applications of Computational Fluid Mechanics*, 3, 336–354. doi:10.1016/j.icheatmasstransfer.2009.11051275

Saury, D., Rouger, N., Djanna, F., & Penot, F. (2011). Natural convection in an air-filled cavity: Experimental results at large Rayleigh numbers. *International Communications in Heat and Mass Transfer*, 38, 679–687. doi:10.1016/j.ijheatmasstransfer.2011.03.019

Sheard, G. J., & King, M. P. (2011). Horizontal convection: Effect of aspect ratio on Rayleigh number scaling and stability. *Applied Mathematical Modelling*, 35, 1647–1655. doi:10.1016/j.apm.2010.09.041

Shih, T., Liou, W. W., Shabbir, A., Yang, Z., & Zhu, J. (1995). A new k-ε eddy viscosity model for high Reynolds number turbulent flows. *Computers & Fluids*, 24, 227–238. doi:10.1016/0017-9310(94)90032-T

Teodosiu, C., Hobota, R., Rusauën, G., & Wołoszyn, M. (2003). Numerical prediction of indoor air humidity and its effect on indoor environment. *Building and Environment*, 38, 655–664, 360—1323. doi:10.1016/j.buildenv.2002.02.0111

Trias, F. X., Gorobets, A., Soria, M., & Oliva, A. (2010). Direct numerical simulation of a differentially heated cavity of aspect ratio 4 with Rayleigh numbers up to 1011 – Part I: Numerical methods and time-averaged flow. *International Journal of Heat and Mass Transfer*, 53, 665–673. doi:10.1016/j.ijheatmasstransfer.2009.10.026

Tric, E., Labrosse, G., & Betrouni, M. (2000). A first incursion into the 3D structure of natural convection of air in a differentially heated cubic cavity, from accurate numerical solutions. *International Journal of Heat and Mass Transfer*, 43, 4043–4056. doi:10.1016/S0017-9310(00)00307-5

TRNSYS. (2012). Version 17. Location. Retrieved from http://www.trnsys.com/

Tsui, T., & Nagano, Y. (1988). Characteristics of a turbulent natural convection boundary layer along a vertical flat plate. *International Journal of Heat and Mass Transfer*, 31, 1723–1734. doi:10.1016/0017-9310(88)90284-0

Vasiliev, A., Sukhanovskii, A., Frick, P., Budnikov, A., Fomichev, V., Bolshukhin, M., & Romanov, R. (2016). High Rayleigh number convection in a cubic cell with adiabatic side-walls. *International Journal of Heat and Mass Transfer*, 102, 201–212. doi:10.1016/j.ijheatmasstransfer.2016.06.015

Wakashima, S., & Saitoh, T. S. (2004). Benchmark solutions for natural convection in a cubic cavity using the high-order time–space method. *International Journal of Heat and Mass Transfer*, 47, 853–864. doi:10.1016/j.ijheatmasstransfer.2003.08.008

Yang, G., & Wu, J. Y. (2016). Effects of natural convection, wall thermal conduction, and thermal radiation on heat transfer uniformity at a heated plate located at the bottom of a three-dimensional rectangular enclosure. *Numerical Heat Transfer, Part A: Applications*, 69, 589–606. doi:10.1080/10407782.2015.1090238