LEP Limits on CP-Violating Non-Minimal Higgs Sectors

J.F. Gunion\textsuperscript{a}, B. Grzadkowski\textsuperscript{b}, H.E. Haber\textsuperscript{c} and J. Kalinowski\textsuperscript{b,d}

\textsuperscript{a} Davis Institute for High Energy Physics, University of California, Davis, CA, USA
\textsuperscript{b} Institute of Theoretical Physics, Warsaw University, Warsaw, Poland
\textsuperscript{c} University of California, Santa Cruz, CA, USA
\textsuperscript{d} Deutsches Elektronen-Synchrotron, DESY, Hamburg, Germany

Abstract

We derive a sum rule which shows how to extend LEP limits on the masses of the lightest CP-even and CP-odd Higgs bosons of a CP-conserving two-Higgs-doublet model to any two Higgs bosons of a general CP-violating two-Higgs-doublet model. We generalize the analysis to a Higgs sector consisting of an arbitrary number of Higgs doublets and singlets, giving explicit limits for the CP-conserving and CP-violating two-doublet plus one-singlet Higgs sectors.

Models of electroweak symmetry breaking driven by elementary scalar dynamics predict the existence of one or more physical Higgs bosons. One can use LEP data to place significant bounds on Higgs boson masses. The minimal model consists of a one-doublet Higgs sector as employed in the Standard Model (SM), which gives rise to a single CP-even scalar Higgs, \( h_{\text{SM}} \). The absence of any \( e^+e^- \to Z h_{\text{SM}} \) signal in LEP1 data (where the \( Z \) is virtual) and LEP2 data (where the \( Z \) is real) translates into a lower limit on \( m_{h_{\text{SM}}} \) which has been increasing as higher energy data becomes available. For example, the latest ALEPH data implies \( m_{h_{\text{SM}}} \gtrsim 70.7 \) GeV \cite{1}. Ultimately, the strongest limit will be obtained by combining the ALEPH data with similar results from the L3, OPAL and DELPHI experiments. The simplest and most attractive generalization of the SM Higgs sector is a two-Higgs-doublet model (2HDM), the CP-conserving version of which has received considerable attention, especially in the context of the minimal supersymmetric model (MSSM) \cite{2}. A CP-conserving two-Higgs-doublet model predicts the existence of two neutral CP-even Higgs bosons (\( h^0 \) and \( H^0 \), with \( m_h \leq m_H \)), one neutral CP-odd Higgs (\( A^0 \)) and a charged Higgs pair (\( H^\pm \)). The negative results of Higgs boson searches at LEP can be formulated as restrictions on the parameter space of this and more general Higgs sector models. For the most general CP-conserving two-doublet model one can
exclude the \((m_{h^0}, m_{A^0})\) and \((m_{h^0}, m_{H^0})\) regions shown in Fig. 1 on the basis of \(e^+e^- \rightarrow Zh^0,\) \(e^+e^- \rightarrow ZH^0\) and \(e^+e^- \rightarrow h^0A^0\) event rate limits\(^1\) as explained in the Appendix. The ability to exclude the illustrated \((m_{h^0}, m_{A^0})\) region derives from a coupling constant sum rule which implies that the two production processes, \(e^+e^- \rightarrow Zh^0\) and \(e^+e^- \rightarrow h^0A^0\), cannot both be simultaneously suppressed when kinematically allowed. Similarly, the illustrated \((m_{h^0}, m_{H^0})\) region is excluded by virtue of a second sum rule which implies that the couplings responsible for the \(e^+e^- \rightarrow ZH^0\) and \(e^+e^- \rightarrow ZH^0\) processes cannot be simultaneously suppressed.

However, there is no reason to assume that the Higgs sector is CP-conserving; CP-violation is still very much a mystery from both an experimental and theoretical point of view, and need not be entirely a consequence of the complex Yukawa couplings built into the Kobayashi-Maskawa matrix \(^3\). The possibility that CP-violation derives largely from the Higgs sector is especially intriguing \(^4\). In a general 2HDM, CP-violation can arise either explicitly or via the CKM matrix \(^3\). The possibility that CP-violation derives largely from the Higgs sector is still very much a mystery from both an experimental and theoretical point of view, and need not be entirely a consequence of the complex Yukawa couplings built into the Kobayashi-Maskawa matrix \(^3\). The possibility that CP-violation derives largely from the Higgs sector is especially intriguing \(^4\).

In this Letter, we show that the coupling constant sum rules appropriate in the CP-violating case to yield a single sum rule that requires at least one of the \(Zh_h\) couplings to be substantial in size. The LEP 95% confidence level exclusion region in the \((m_{h^i}, m_{h^j})\) plane that results from the general sum rule is quite significant, as illustrated in Fig. 1 using \(i = 1, j = 2\) notation.

In this Letter, we also present an analysis of both the CP-conserving and the CP-violating versions of the two-doublet plus one-singlet (2D1S) extension of the 2HDM. A 2D1S model yields five neutral Higgs bosons. We derive sum rules that can be used to demonstrate that LEP data excludes the possibility that three of these neutral Higgs bosons can be light. (There is no sum rule which allows exclusion of the possibility that only two of the neutral Higgs bosons of the 2D1S model are light.) The 95% confidence level boundaries in three-Higgs-boson mass space for the CP-conserving and CP-violating cases, based on the procedures described in the Appendix, are presented in Figs. 2 and 3, respectively.

We now turn to a derivation of the sum rules required and a discussion of how they lead to the experimental constraints outlined above. In the 2HDM, the two complex neutral Higgs fields contain four neutral degrees of freedom. One is eaten by the \(Z\) gauge boson; the others mix to yield three physical neutral Higgs bosons, \(h_i\) \((i = 1, 2, 3)\). We shall denote their couplings to the \(Z\) boson by

\[g_{Zh_i} = \frac{g_{mZ}}{c_W}C_i,\]

\[g_{Zh_i} = \frac{g}{2c_W}C_{ij},\]

\[g_{Zhh} = \frac{g^2}{2c_W^2}\delta_{ij},\]

where \(C_i\) and \(C_{ij} = C_{ji}\) \(^i\) are model-dependent coupling strengths and \(c_W \equiv \cos \theta_W\).

In the CP-conserving 2HDM, the \(h^0\) and \(H^0\) are mixtures of the real parts of the neutral Higgs fields (the diagonalizing mixing angle is denoted by \(\alpha\)) while the CP-odd state, \(A^0\),

\(^1\) Stronger limits are possible in models such as the MSSM where there are relations between the Higgs masses and their couplings.

\(^2\) For \(i = j\), \(C_{ij} = 0\) by Bose symmetry.
derives from the imaginary components not eaten by the $Z$. One finds $C_{h^0} = C_{H^0A^0} = \sin(\beta - \alpha)$, $C_{H^0} = C_{h^0A^0} = \cos(\beta - \alpha)$ and $C_{A^0} = C_{h^0H^0} = 0$; $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values for the neutral components of the two Higgs doublet fields. For any two Higgs bosons, we wish to obtain an excluded mass region that does not depend upon knowledge of the third Higgs boson. There are two pairs of interest: (a) $h^0, H^0$ and (b) $h^0, A^0$. In case (a) the excluded region in $(m_{h^0}, m_{H^0})$ parameter space is illustrated by the dotted curve in Fig. [1]. Inside the excluded region, failure to observe $e^+e^- \rightarrow Zh^0, ZH^0$ at LEP implies 95% CL limits on $C_{h^0}^2$ and $C_{H^0}^2$ such that $C_{h^0}^2 + C_{H^0}^2 < 1$, whereas the relevant couplings obey the sum rule

$$C_{h^0}^2 + C_{H^0}^2 = \sin^2(\beta - \alpha) + \cos^2(\beta - \alpha) = 1. \quad (2)$$

In case (b) the excluded region in $(m_{h^0}, m_{A^0})$ parameter space is indicated by the dashed line in Fig. [1]. Inside the excluded region, failure to observe $e^+e^- \rightarrow Zh^0$ and $e^+e^- \rightarrow h^0A^0$ events implies 95% CL upper limits on $C_{h^0}^2$ and $C_{h^0A^0}^2$, respectively, such that $C_{h^0}^2 + C_{h^0A^0}^2 < 1$, whereas these couplings obey the sum rule

$$C_{h^0}^2 + C_{h^0A^0}^2 = \sin^2(\beta - \alpha) + \cos^2(\beta - \alpha) = 1. \quad (3)$$

Thus, LEP data exclude the possibility that both the scalar and pseudoscalar Higgs bosons of a CP-conserving 2HDM are light. The asymmetry of the $(m_{h^0}, m_{A^0})$ excluded region arises as follows. If $m_{h^0}$ is small, then non-observation of $Zh^0$ events implies that $C_{h^0}^2$ is limited to very small values, implying via Eq. (3) that $C_{h^0}^2$ is close to (or above) $\sqrt{s} - m_Z$, then $Zh^0$ events would not be observed even if $C_{h^0}^2 = 1$: at the same time, $C_{h^0}^2 = 1$ implies via Eq. (3) that $C_{h^0A^0}^2 = 0$ so that there is no constraint from non-observation of $h^0A^0$ events.

Below, we shall demonstrate that if CP is not conserved, the sum rules of Eqs. (2) and (3) can be generalized in the 2HDM case to

$$C_i^2 + C_j^2 + C_{ij}^2 = 1, \quad (4)$$

where $i \neq j$ are any two of the three possible indices. The power of the Eq. (4) sum rule derives from the facts that it involves only two of the neutral Higgs bosons and that the experimental upper limit on any one $C_i^2$ derived from $e^+e^- \rightarrow Zh_i$ data is very strong: $C_i^2 \leq 0.1$ for $m_{h_i} \leq 50$ GeV. Thus, if $h_i$ and $h_j$ are both below about 50 GeV in mass then Eq. (4) requires that $C_{ij}^2 \sim 1$, whereas for such masses limits on $e^+e^- \rightarrow h_ih_j$ from $\sqrt{s} = 161$ GeV and 172 GeV data require $C_{ij}^2 \ll 1$. The excluded region in the $(m_{h_i}, m_{h_j})$ plane that results is illustrated in Fig. [1] — there cannot be two light Higgs bosons even in the general CP-violating case.

The sum rule of Eq. (4) is required in order to have unitary high energy behavior at tree-level for the $W^+W^- \rightarrow W^+W^-$, $ZZ \rightarrow W^+W^-$ and $ZZ \rightarrow h_ih_j$ scattering amplitudes; see

\[\text{We have plotted this region symmetrically, even though by definition we should only consider } m_{h^0} \geq m_{h^0}.\]

\[\text{Note that Eq. (4) reduces to Eq. (3) in the CP-conserving limit when we identify } h_i = h^0 \text{ and } h_j = A^0 \text{ and use } C_{A^0} = 0. \text{ Similarly, Eq. (4) reduces to Eq. (2) in the CP-conserving limit when we identify } h_i = h^0 \text{ and } h_j = H^0 \text{ and use } C_{h^0H^0} = 0.\]
Eqs. (4.1), (4.2) and (A.18) of Ref. [5], respectively. In the context of a Higgs sector containing only doublet and singlet fields, the cited equations of Ref. [5] reduce to the requirements

\[
\sum_i C_i^2 = 1
\]

\[
C_i^2 + \sum_{k \neq i} C_{ik}^2 = 1,
\]

where the 1 on the right hand side of Eq. (6) arises from the \(ZZh_ih_i\) 4-point interaction of Eq. (1) contributing to \(ZZ \rightarrow h_ih_i\) scattering. To derive Eq. (4) in the 2HDM [for which \(i, k = 1, 2, 3\) in Eqs. (5) and (6)], we sum the \(i = 1, 2\) and the \(i = 1, 2, 3\) cases of Eq. (6), respectively, to obtain

\[
C_1^2 + C_2^2 + C_{12}^2 = 2 - (C_{12}^2 + C_{13}^2 + C_{23}^2),
\]

\[
C_1^2 + C_2^2 + C_3^2 = 3 - 2(C_{12}^2 + C_{13}^2 + C_{23}^2),
\]

respectively. We then employ the relation \(C_1^2 + C_2^2 + C_3^2 = 1\) from Eq. (5) in Eq. (8) to show that \(C_{12}^2 + C_{13}^2 + C_{23}^2 = 1\) and substitute this result into Eq. (7) to obtain \(C_1^2 + C_2^2 + C_{12}^2 = 1\). Cyclic permutation gives the other cases of Eq. (4). Eq. (4) is much more useful for obtaining experimental limits than either Eq. (5) or (6) since the latter two sum rules involve three Higgs bosons (in the 2HDM), whereas the former refers to just two. This distinction only arises in the CP-violating case. In the CP-conserving limit, Eqs. (5) and (6) can be used to derive Eqs. (2) and (3), respectively, while Eq. (4) implies both (as described earlier).

These considerations can be generalized to extensions of the 2HDM. The simplest extension is to add one complex singlet (neutral) Higgs field. In this case, it is no longer possible to place restrictions on two Higgs bosons. The best that one can do is to write the sum rules in such a way as to demonstrate that there cannot be three light neutral Higgs bosons. Consider first the case where CP is conserved. In the 2D1S model, there will then be three CP-even Higgs bosons (labelled 1,2,3 in order of increasing mass) and two CP-odd Higgs bosons (labelled 4,5 in order of increasing mass). Using the sum rules of Eqs. (5) and (6), with \(C_4 = C_5 = C_{12} = C_{13} = C_{23} = C_{45} = 0\), we easily derive the three crucial sum rules:

\[
C_1^2 + C_2^2 + C_3^2 = 1,
\]

\[
C_1^2 + C_2^2 + C_{14}^2 + C_{24}^2 = 1 + C_{35}^2,
\]

\[
C_1^2 + C_{14}^2 + C_{15}^2 = 1.
\]

Eqs. (9), (10), and (11) can be used to show that there cannot be three CP-even, two CP-even plus one CP-odd, and one CP-even plus two CP-odd Higgs bosons, respectively, that are all light. (In the second sum rule, since \(C_{35}^2 \geq 0\) the region within which exclusion is certain is obtained by setting \(C_{35}^2 = 0\).) The boundaries in the three-dimensional mass spaces that follow from these sum rules [with \(C_{35}^2 = 0\) in Eq. (11)] are shown in Fig. 2.

In the case that CP is violated, the required sum rule for the 2D1S is obtained by generalizing the procedure sketched for the derivation of Eq. (4) in the 2HDM case. Focusing on Higgs bosons numbers 1, 2 and 3, one finds:

\[
C_1^2 + C_2^2 + C_3^2 + C_{12}^2 + C_{13}^2 + C_{23}^2 = 1 + C_{45}^2.
\]
This sum rule implies a lower bound (obtained with $C_{45}^2 = 0$) for the sum of all the couplings squared responsible for production of $Zh_1$, $Zh_2$, $Zh_3$, $h_1h_2$, $h_1h_3$ and $h_2h_3$ in $e^+e^-$ collisions. The portion of the $(m_1, m_2, m_3)$ mass space excluded by LEP data in the CP-violating case, as implied by the sum rule of Eq. (12), is shown in Fig. 3.

The above considerations can be further generalized to a Higgs sector that contains $\ell$ doublets and $m$ neutral complex singlets; the number of physical neutral Higgs mass eigenstates is $2(\ell + m) - 1$. The general sum rule will apply to any subset containing $n = (\ell + m)$ of these Higgs bosons. Let us label the members of the subset with indices $i = 1, \ldots, n$. Following the techniques illustrated in the 2D1S case, and assuming that CP violation is present, we derive the coupling constant sum rule

$$ \sum_{i=1}^{n} C_i^2 + \sum_{i<j}^{n} C_{ij}^2 = 1 + \sum_{i<j}^{2n-1} C_{ij}^2, $$

where the most conservative bounds on the subset would be obtained by setting all the $C_{ij}^2 = 0$ on the right hand side. If CP violation is not present in the Higgs sector, then the above sum rule will reduce to a simpler form that depends upon the CP nature of the Higgs bosons included in the subset.

In this Letter, we have derived a new coupling constant sum rule which makes it possible to use LEP data to exclude a portion of $(m_{h_1}, m_{h_2})$ mass space for the lightest two neutral Higgs bosons of the most general CP-violating two-Higgs-doublet model. Although this region is not as large as that excluded in the $(m_{h^0}, m_{A^0})$ mass parameter space in the CP-conserving case, it is still very substantial. Thus, LEP data implies that it is not possible for two of the three neutral Higgs bosons of a general two-Higgs-doublet model to be light. We have further shown how to extend this type of analysis to both CP-conserving and CP-violating Higgs sectors with an arbitrary number of doublets and singlets. In the two-doublet plus one-singlet Higgs model, LEP data already excludes the possibility that three of the five neutral Higgs bosons are light.

**Acknowledgments**

This work was supported in part by U.S. Department of Energy under grants DE-FG03-91ER40674 and DE-FG03-92ER40689, by the Davis Institute for High Energy Physics, by the Committee for Scientific Research (Poland) under grant No. 2 P03B 180 09, and by Maria Sklodowska-Curie Joint Fund II (Poland-USA) under grant No. MEN/NSF-96-252. BG and JK would like to thank the Davis Institute for High Energy Physics for hospitality.
Appendix. The limits presented in Figs. 1-3 have been obtained from LEP1 and LEP2 data on $e^+e^- \rightarrow Zh_i$ and $e^+e^- \rightarrow h_ih_j$ production using the following procedures. Consider first any given $Zh_i$ channel. For $m_{h_i} < 50$ GeV, the 95% CL upper limit on $C_i^2$ [defined in Eq. (1)] is obtained by using the smaller of the values shown in Fig. 5b of Ref. [6] and Fig. 29 from Ref. [7]. For $m_{h_i} \geq 50$ GeV, the 95% CL upper limit on $C_i^2$ is obtained as the ratio of the 95% CL upper limit on the number of events as observed by ALEPH, taken to be $\sim 3$ events from the graph in Ref. [1] (which includes data at $\sqrt{s} = m_Z$, $\sqrt{s} = 161$ GeV and $\sqrt{s} \sim 172$ GeV), to the number of events expected at the given Higgs mass in the SM, as plotted in the same graph. If the assumed $C_i^2$ exceeds the 95% CL as defined above at the input $m_{h_i}$, then the parameter choice is taken to be excluded at the 95% CL. For a two-Higgs channel, $h_ih_j$, we approximate the ALEPH 95% CL limits implicit in Ref. [1] by employing integrated luminosities of $L = 11.08$ pb$^{-1}$ at $\sqrt{s} = 161$ GeV and $L = 10.5$ pb$^{-1}$ at $\sqrt{s} = 172$ GeV and the quoted efficiencies and branching ratios of $\epsilon = 0.55$, $BR = 0.83$ for the $4b$ channel and $\epsilon = 0.45$, $BR = 0.16$ for the $2b2\tau$ channel. We compute the expected number of events (combining the $4b$ and $2b2\tau$ channels) for the input value of the $Zh_ih_j$ coupling strength and then evaluate the Poisson probability that no events are observed. If this probability is below 5% then the chosen parameter set for the $h_i$ and $h_j$ is said to be excluded at 95% CL.

References

[1] We employ the results presented by G. Cowan, CERN Seminar, February 25, 1997, posted at [http://alephwww.cern.ch/ALPUB/seminar/Cowan-172-jan/01.html](http://alephwww.cern.ch/ALPUB/seminar/Cowan-172-jan/01.html).

[2] For a review and references, see: J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, *The Higgs Hunters Guide* (Addison-Wesley Publishing Company, Redwood City, CA, 1990); J.F. Gunion, A. Stange, and S. Willenbrock, *Weakly-Coupled Higgs Bosons*, preprint UCD-95-28 (1995), to be published in *Electroweak Physics and Beyond the Standard Model*, World Scientific Publishing Co., eds. T. Barklow, S. Dawson, H.E. Haber, and J. Siegrist.

[3] K. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).

[4] T.D. Lee, *Phys. Rev.* **D8**, 1226 (1973). S. Weinberg, *Phys. Rev.* **D42**, 860 (1990).

[5] J.G. Gunion, H.E. Haber and J. Wudka, *Phys. Rev.* **D43**, 904 (1991).

[6] M. Acciarri et al. (L3 Collaboration), *Phys. Lett.* **B385**, 454 (1996).

[7] A. Sopczak, CERN-PPE/95-46 (1995) [hep-ph/9504300].
Figure 1: The LEP 95% confidence level boundaries (based on our use of the latest ALEPH results [4] and earlier LEP1 results as contained in Refs. [6, 7]) in the $(m_1, m_2)$ plane for three two-Higgs-doublet model cases: (i) the CP-conserving (CP C) model where $h_1 = h^0$, $h_2 = H^0$ (dotted curve); (ii) the CPC model where $h_1 = h^0$, $h_2 = A^0$ (dashed curve); (iii) the CP-violating (CPV) model where $h_1$ and $h_2$ are the two lightest neutral Higgs bosons (solid curve). This figure is based on including just two constraints: a particular choice of masses is excluded (a) if some $ZZh_i$ coupling must lie above the 95% CL upper limit for the assumed $m_{h_i}$ or (b) if one or more events are expected in $e^+e^- \rightarrow h_ih_j$ production at the 95% CL for some choice of $i \neq j$. For details, see the Appendix.
Figure 2: The LEP 95% confidence level boundaries for the two-doublet plus one-singlet CP-conserving Higgs sector in the $(m_1, m_2, m_3)$, $(m_1, m_2, m_4)$ and $(m_4, m_5, m_1)$ spaces for three CP-even, two CP-even plus one CP-odd, and two CP-odd plus one CP-even Higgs bosons, respectively. Mass axes are in GeV units. Constraints are as in Fig. 1.
Figure 3: The LEP 95% confidence level boundary for the two-doublet plus one-singlet CP-violating Higgs sector in the \((m_1, m_2, m_3)\) parameter space. Mass axes are in GeV units. Constraints are as in Fig. [1].