The gravo-magneto disc instability with a viscous dead zone

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ABSTRACT

We consider the evolution of accretion discs that contain some turbulence within a disc dead zone, a region about the disc mid-plane that is not sufficiently ionized for the magneto-rotational instability (MRI) to drive turbulence. In particular, we determine whether additional sources of turbulence within a dead zone are capable of suppressing gravo-magneto disc outbursts that arise from a rapid transition from gravitationally produced to MRI-produced turbulence. With viscous $\alpha$ disc models, we consider two mechanisms that may drive turbulence within the dead zone. First, we examine a constant $\alpha$ parameter within the dead zone. This may be applicable to hydrodynamical instability, such as baroclinic instability, where the turbulence level is independent of the MRI active surface layer properties. In this case, we find that the disc will not become stable to the outbursts unless the dead zone turbulent viscosity is comparable to that in the MRI active surface layers. Under such conditions, the accretion rate through the dead zone must be larger than that through the MRI active layers. In a second model, we assume that the accretion flow through the dead zone is a constant fraction (less than unity) of that through the active layers. This may be applicable to turbulence driven by hydrodynamic waves that are excited by the MRI active surface layers. We find that the instability is hardly affected by the viscous dead zone. In both cases however, we find that the triggering of the MRI during the outburst may be due to the heating from the viscosity in the dead zone, rather than self-gravity. Thus, neither mechanism for generating turbulence within the dead zone can significantly stabilize a disc or the resulting outburst behaviour.

Key words: accretion, accretion disc – MHD – turbulence – planets and satellites: formation – stars: pre-main sequence.

1 INTRODUCTION

Accretion discs transport angular momentum outwards allowing material to accrete on to the central object (Pringle 1981). In a fully ionized disc, the gas is well coupled to the magnetic field and the magneto rotational instability (MRI) drives turbulence and thus angular momentum transport (Balbus & Hawley 1991). However, protoplanetary discs are thought to have a sufficiently low-ionization fraction that a dead zone forms about the mid-plane where the MRI cannot efficiently transport angular momentum (Gammie 1996; Gammie & Menou 1998; Turner & Sano 2008). When a dead zone forms, for a range of accretion rates, the disc becomes unstable to the gravo-magneto (GM) disc instability, where the turbulence cycles between magnetic and gravitational (Armitage, Livio & Pringle 2001; Zhu, Hartmann & Gammie 2009, 2010b; Zhu et al. 2010a; Martin & Lubow 2011a, 2013).

Consider the case that a dead zone has zero viscosity. In that situation, a steady-state disc cannot contain a dead zone. Instead, there are three possible steady-state solutions at each radius in the disc. First, if the mid-plane temperature is sufficiently high there is a thermally ionized MRI steady state. Secondly, if the surface density is sufficiently low there is an externally ionized MRI steady state. The surface layers of the disc may be ionized by external sources such as cosmic rays or X-rays from the central star (Glassgold, Najita & Igea 2004). Turbulence in the dead zone may be driven by self-gravity if the disc becomes sufficiently massive (Paczynski 1978; Lodato & Rice 2004) and this can drive the third steady state which has a self-gravitating dead zone with MRI active surface layers. However, there may be a range of radii in the disc for which there exists no steady state and then the disc is unstable to the GM disc instability (Martin & Lubow 2011a, 2013).

We have explained the GM disc instability as transitions between these steady-state solutions by plotting a state diagram of the accretion rate through the disc against the surface density for a fixed radius (Martin & Lubow 2011a, 2013). This is similar to the
thermal-viscous instability ‘S-curve’ used to explain dwarf nova outbursts (Bath & Pringle 1982; Faulkner, Lin & Papaloizou 1983). In our previous work on the GM disc instability, we assumed that the viscosity in the dead zone is zero (Martin & Lubow 2011a; Martin et al. 2012a,b; Martin & Lubow 2013), but this is somewhat uncertain.

There are at least two mechanisms that may drive turbulence in the dead zone. First, it is possible that turbulence within the dead zone may be driven by hydrodynamic instabilities such as the baroclinic instability (e.g. Klahr & Bodenheimer 2003; Petersen, Julien & Stewart 2007; Lesur & Ogilvie 2010). In a baroclinic state, the pressure varies over surfaces of constant density. The non-axisymmetric misalignment between surfaces of constant density and surfaces of constant pressure generates vorticity. Vortices within a dead zone could be associated with a viscosity parameter as high as $\alpha = 5 \times 10^{-3}$ (Lyra & Klahr 2011). However, there is still some uncertainty over the viability of such a mechanism for driving turbulence within a protoplanetary disc. The conditions under which the baroclinic instability can drive turbulence are not well constrained (see e.g. Armitage 2011). The dissipation from the baroclinic instability is poorly known as vortices behave very differently to turbulence. To model the effects of such hydrodynamic instabilities, we take a constant $\alpha$-parameter in the dead zone of a layered disc model.

Secondly, shearing box simulations suggest that the magnetohydrodynamic (MHD) turbulence generated in the disc surface layers may produce some hydrodynamic turbulence in the dead zone layer that may produce a small but non-zero viscosity (e.g. Fleming & Stone 2003; Simon, Armitage & Beckwith 2011; Gressel, Nelson & Turner 2012). Hydrodynamic waves are excited by the turbulence in the active surface layers. These can penetrate to the mid-plane and exert a non-zero Reynolds stress. To model this, we choose a Reynolds stress based on the stratified local simulations of Fleming & Stone (2003) and the mid-plane Reynolds stress in the dead zone is that of the fully turbulent disc. However, those simulations used an isothermal equation of state, and the strength of Reynolds stresses communicated to the mid-plane may be sensitive to that assumption and to the thickness of the active surface layer (Bai & Goodman 2009; Armitage 2011). Thus, there is still some uncertainty associated with the strength of such turbulence. In this work, we model the turbulence driven in the dead zone from the active layer with an $\alpha$-parameter that varies such that the accretion rate through the dead zone is a constant fraction of that through the active layers.

Bae et al. (2013) considered the time-dependent evolution of a disc with a small viscosity in the dead zone that is GM unstable. In this work, we consider how large the viscosity in the dead zone can be before the disc reaches a fourth steady state, a viscous dead zone state, thus stabilizing the outburst behaviour. We consider a protoplanetary disc model but we note that the GM disc instability can occur in discs on a range of scales. For example, dead zones and the GM instability also likely occur in circumplanetary discs (Martin & Lubow 2011b; Lubow & Martin 2012, 2013).

In Section 2, we describe the layered disc model. In Section 3, we present the results for disc solutions with a constant viscosity in the dead zone. These may be appropriate for a dead zone viscosity driven by the baroclinic instability, for example. In Section 4, we consider solutions with an accretion rate through the dead zone that is a fixed fraction of that through the active layers. These may be more appropriate for dead zone viscosities driven from the turbulence in the active layers.

2 LAYERED DISC MODEL

We follow the disc model of Martin & Lubow (2011a) that was initially developed by Armitage et al. (2001). Material in the disc orbits the central star of mass $M = 1 M_\odot$ at radius $R$ with Keplerian velocity, $\Omega = \sqrt{GM/R^3}$. If the temperature of the disc is greater than the critical value, $T > T_{\text{crit}}$, then the disc is thermally ionized and fully MRI active. The value of the critical temperature is thought to be around $T_{\text{crit}} = 800 K$ (Umebayashi 1983). Similarly if the surface density of the disc is smaller than the critical that can be ionized by external sources (such as cosmic rays and X-rays), $\Sigma < \Sigma_{\text{crit}}$, the disc has a fully MRI active solution. However, if neither of these conditions are satisfied, then there is a dead zone layer about the mid-plane.

There remains some uncertainty in the critical surface density that may be ionized by external sources, $\Sigma_{\text{crit}}$. If cosmic rays are the dominant ionization source, $\Sigma_{\text{crit}} \approx 200 g$ cm$^{-2}$ (Gammie 1996; Fromang, Terquem & Balbus 2002). However, without cosmic rays, X-rays from the central star may dominate and in this case the active layer surface density is much smaller (Matsumura & Padirtz 2003). Ionization balance calculations determine $\Sigma_{\text{crit}}$ in a disc subject to various processes such as ambipolar diffusion and the presence of polycyclic aromatic hydrocarbon and dust and find that they suppress the ionization fraction further (e.g. Bai & Goodman 2009; Bai 2011; Perez-Becker & Chiang 2011; Simon et al. 2013). However, these calculations predict accretion rates much lower than those observed in typical T Tauri stars that suggest $\Sigma_{\text{crit}} > 10 g$ cm$^{-2}$ (Perez-Becker & Chiang 2011). In this work, we consider two values, $\Sigma_{\text{crit}} = 200 g$ cm$^{-2}$ and $\Sigma_{\text{crit}} = 20 g$ cm$^{-2}$.

To find the steady-state disc solutions, we solve the accretion disc equation

$$M = 3\pi (v_m \Sigma_m + v_d \Sigma_d)$$

(Pringle 1981), where $M$ is the steady accretion rate through the disc, $\Sigma_m$ is the surface density in the MRI active surface layers, $\Sigma_d$ is the surface density in the dead zone layer, the total surface density is $\Sigma = \Sigma_m + \Sigma_d$. The accretion rate through the MRI active surface layers is $M_m = 3\pi v_m \Sigma_m$ and the accretion rate through the dead zone is $M_d = 3\pi v_d \Sigma_d$. The viscosity in the MRI active surface layers is

$$v_m = \frac{\alpha_m c_m^2}{\Omega},$$

where $c_m = \sqrt{KT_m/\mu}$ is the sound speed, $T_m$ is the temperature and $\alpha_m$ is the Shakura & Sunyaev (1973) viscosity parameter that we take to be $\alpha_m = 0.01$ (see e.g. Hartmann et al. 1998).

The dead zone may become self-gravitating if the Toomre (1964) parameter becomes less than the critical,

$$Q = \frac{c_s \Omega}{\pi G \Sigma} < Q_{\text{crit}},$$

where $c_s = \sqrt{KT_s/\mu}$ is the sound speed in the mid-plane layer of temperature $T_s$ and we take $Q_{\text{crit}} = 2$. The dead zone layer has viscosity

$$v_d = (\alpha_d + \alpha_g) \frac{c_d^2}{\Omega},$$

where the part due to self-gravity is

$$\alpha_g = \alpha_m \left[ (\frac{Q_{\text{crit}}}{Q})^2 - 1 \right],$$

if $Q < Q_{\text{crit}}$ and zero otherwise (Lin & Pringle 1987, 1990). The form of the $Q$ dependence in equation (5) does not affect the disc...
evolution provided it is a strongly decreasing function (see also Zhu et al. 2010a; Martin & Lubow 2013). In fact, we find that the addition of the residual dead zone viscosity term, \( \alpha_d \), also does not affect the viscosity in the dead zone, if the disc is self-gravitating (unless it is close to the fully MRI active value, \( \alpha_d \approx \alpha_m \), see the discussion section in Martin & Lubow 2013, for more details). We consider two different models for \( \alpha_d \). First, we consider \( \alpha_d = \) constant (see Section 3) and then we consider \( \alpha_d \) that satisfies \( M_d = f M_m \), where \( f = \) constant (see Section 4). All solutions with a dead zone have MRI active surface layers with surface density \( \Sigma_m = \Sigma_{\text{crit}} \) that are ionized by the external sources.

We also consider the restriction that the accretion rate through the dead zone, \( M_d \), is less than the accretion rate through the active layer, \( M_m \). If the dead zone viscosity is driven from the active layer, it is unlikely that a small amount of turbulence in the surface layers can drive a large amount of turbulence in a more massive dead zone region (Zhu et al. 2009). On the other hand, if the baroclinic instability is driving the turbulence, it is possible that the viscosity in the dead zone may be higher than that in the active layers because it is not determined by the turbulence in the surface layers.

The mass conservation equation is solved coupled with a steady energy equation for the surface temperature, \( T_c \).

\[
\frac{\sigma T_c^4}{8} = \Omega^2 (v_m \Sigma_m + v_0 \Sigma_0)
\]

(Pringle, Verbunt & Wade 1986, Cannizzo 1993). There are three temperatures defined in the disc, the mid-plane temperature, \( T_c \), the surface layer temperature, \( T_m \), and the surface temperature, \( T_e \). These are related to each other by considering the energy balance in a layered disc model in thermal equilibrium to find

\[
T_m^2 = \tau_m T_c^4 \tag{7}
\]

\[
\frac{\sigma T_c^4}{8} = \Omega^2 (v_m \Sigma_m \tau_m + v_0 \Sigma_0 \tau_d^4)
\]

(Martin & Lubow 2011a), where \( \tau_m \) and \( \tau_d \) are the optical depths of the magnetic surface layers and dead zone, respectively, and \( \tau = \tau_m + \tau_d \). We have found that the effects of irradiation from the central star are negligible for the outburst model presented here. The characteristic temperature for outbursts is of the order of \( T_{\text{crit}} = 800 \) K. Stellar irradiation cannot produce such high mid-plane disc temperatures on the AU scale where GM outbursts occur. By solving equations (1) and (6) with (7) and (8) we find the steady-state disc structure. In the following two sections we analyse such solutions.

3 FIXED \( \alpha_d \) IN THE DEAD ZONE

Here, we consider steady-state disc solutions with a constant residual viscosity in the dead zone, \( \alpha_d = \) constant. This may be appropriate for modelling discs with a dead zone that is unstable to hydrodynamic instabilities, such as the baroclinic instability, for example.

A disc with a dead zone with zero residual viscosity can only be in steady state if the disc is sufficiently massive to be self-gravitating. However, with an additional small viscosity in the dead zone, further steady-state solutions are possible if the dead zone and active layer together can transport material at a rate equal to the infall accretion rate. Consider the case of a disc that has a very small level of residual turbulence (very small \( \alpha_d \)). In order to carry the required mass flux, the dead zone surface density must be very high. As a result, the disc optical depth and therefore mid-plane temperature must be high. However, the mid-plane disc temperature of such a steady state must be less than \( T_{\text{crit}} \), the critical temperature for the onset of MRI. Otherwise, the MRI disc turbulence would set in. The resulting mass flux would then be too high for a steady state and the disc would undergo outbursts by cycling between these levels of turbulence. Hence, a steady-state disc does not exist for all dead zone turbulence levels \( \alpha_d \). Hence, the steady solution does not exist for all disc parameters in the GM unstable region of the disc.

3.1 Locally steady models

We first consider a disc model with \( \Sigma_{\text{crit}} = 200 \) g cm\(^{-2} \). There are four steady-state disc solutions; the thermally ionized MRI, the externally ionized MRI, the GM and the viscous dead zone solution. The first three of these solutions are independent of \( \alpha_d \), as discussed in the previous section. The top-left plot in Fig. 1 shows where each of these three steady solutions exist for varying infall accretion rate and radius. The unshaded region shows the parameter space for which the disc is unstable to the GM instability with \( \alpha_d = 0 \). Note that we do not include the thermal-viscous instability in this work. Zhu et al. (2007) found that in order to fit the Spitzer Space Telescope IRS spectrum of the eponymous outbursting system FU Ori, the rapidly accreting, hot inner disc must extend out to about 1 au, inconsistent with a pure thermal instability model. We therefore concentrate on the GM instabilities that occur on the au scale.

In the remaining three plots, we show at radii \( R = 1, 3 \) and 5 au where the four steady solutions exist for varying infall accretion rate and dead zone viscosity. For the viscous dead zone solution, we find the critical \( \alpha_d \) for which a steady solution exists by setting \( T_c = T_{\text{crit}} \). This is shown in the thick lines. In the dark shaded regions, we show the range of \( \alpha_d \) for which a steady viscous dead zone solution exists (with \( T_c < T_{\text{crit}} \) and \( Q \geq Q_{\text{crit}} \)). In the dark shaded hatched region (shown at \( R = 1 \) and \( R = 3 \) au), the accretion rate through the dead zone is smaller than that through the active layer. In plots that show no hatched region there is no steady viscous dead zone solution that has an accretion rate through the dead zone less than that through the active layer. Especially for large radii, and high accretion rates, the viscous dead zone solution requires a large viscosity in the dead zone for a steady state to exist.

In Fig. 2, we show the same plots for a disc with a smaller active layer of \( \Sigma_{\text{crit}} = 20 \) g cm\(^{-2} \). With smaller active layer surface density there is a larger range of GM unstable accretion rates and similarly a slightly larger range of \( \alpha_d \) for which a steady solution exists. However, the region in which a steady viscous dead zone solution exists, with \( M_d < M_m \), is significantly smaller.

We have also investigated the effects of changing various parameters. The functional form for \( \alpha_d \) has little effect on the disc stability (see Martin & Lubow 2013, for a discussion). The value of the critical Toomre parameter below which gravitational instabilities operate, \( Q_{\text{crit}} \), may be as small as 1.7 (see e.g. Boley et al. 2006; Durisen et al. 2007). However, we find the effects of such a change to be negligible to the work presented here. The critical temperature above which the MRI operates, \( T_{\text{crit}} \), on the other hand does change the results quantitatively. For example, the instability region described in the top-left plot of Fig. 1 is shifted a small amount when we double \( T_{\text{crit}} \). However, qualitatively, this does not affect our conclusions on the disc stability.

3.2 Local limit cycle

We consider the limit cycle in the \( \Sigma - \dot{M} \) plane described in Martin & Lubow (2011a) in Fig. 3. This limit cycle explains the GM disc...
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Figure 1. A disc with $\Sigma_{\text{crit}} = 200 \, \text{g cm}^{-2}$, $T_{\text{crit}} = 800 \, \text{K}$ and $\alpha_{\text{in}} = 0.01$. Top Left: radii in the disc for which steady-state MRI and GM disc solutions exist for $\alpha_d = 0$ for varying infall accretion rate. The shaded region shows where such a steady state exists. The unshaded region has no steady state and thus if the accretion rate lies within this region the disc may be GM unstable. The remaining three plots show the steady solutions including the viscous dead zone solution for varying dead zone viscosity for $R = 1 \, \text{au}$ (top right), $R = 3 \, \text{au}$ (bottom left) and $R = 5 \, \text{au}$ (bottom right). The steady thermally and externally ionized MRI solutions and GM solutions exist for accretion rates in the pale shaded regions and the long-dashed vertical lines denote the boundaries between these regions. In the dark shaded region, there is a viscous dead zone steady state with viscosity parameter $\alpha_d$. In the dark hatched region (where it exists), the mass flow through the dead zone layer is less than that through the active layer ($\dot{M}_d < \dot{M}_m$). Similarly, in the dark unhatched region, the accretion rate through the dead zone is greater than that through the active layer ($\dot{M}_d > \dot{M}_m$). The surface temperature is related to the accretion rate through equations (1) and (6) with

$$T_e = \left( \frac{3M \Omega^2}{8\pi\sigma} \right)^{\frac{1}{4}}.$$

The disc has $\Sigma_{\text{crit}} = 200 \, \text{g cm}^{-2}$ and we show a radius $R = 3 \, \text{au}$. The steady-state solutions with $\alpha_d = 0$ (from Martin & Lubow 2011a) are shown in the thick black lines. Steady-state solutions with $\alpha_d > 0$ are shown in the grey/white lines for three different values of $\alpha_d$. The white part shows where the flow through the dead zone is larger than that through the active layer and the grey part shows the solutions where the flow through the dead zone is smaller than that through the active layer. The upper ends of the white lines are where $T_e = T_{\text{crit}}$. In the case of $\alpha_d = 10^{-5}$, the grey line ends where the solution becomes self-gravitating and there is no white part.

If there remains a range of accretion rates for which no steady solution exists, the disc will be unstable. As shown in Martin & Lubow (2011a), if the accretion rate on to the disc lies in the unstable range, the disc will exhibit a limit cycle as it transitions between the two steady states. For the GM limit cycle (with zero viscosity in the dead zone), the two steady states are the thermal MRI and the self-gravitating solution. However, with a non-zero $\alpha_d > 10^{-5}$ in the dead zone, a similar limit cycle exists where the disc transitions between the thermal MRI and the viscous dead zone steady state. The viscosity in the dead zone drives extra turbulence and heating within the dead zone that eventually triggers the thermal MRI and an outburst ensues.

In order to stop the outburst cycle, the steady-state viscous dead zone solutions would need to cover the whole of the unstable region. Thus, to stabilize the disc to the GM disc instability requires a...
Figure 2. Same as Fig. 1 but with $\Sigma_{\text{crit}} = 20$ g cm$^{-2}$. The dark shaded hatched region for the viscous dead zone solutions with $M_d < M_m$ exists only in a very small region of parameter space at $R = 1$ au and does not exist for $R = 3$ or $R = 5$ au.

3.3 Globally steady models

Here, we consider the range of accretion rates for which a steady solution exists at all radii in the disc. In Martin & Lubow (2013), we found the radius of the inner edge of the GM unstable region to be

$$R_{\text{crit}} = 1.87 \frac{M'^{4/9} M^{1/3}}{\alpha'^{2/9} T_{\text{crit}}^{1/3}} \text{ au},$$

(10)

where $M' = M/10^{-6} M_\odot \text{ yr}^{-1}$, $\alpha' = \alpha/0.01$, $M' = M/M_\odot$ and $T_{\text{crit}} = 800$ K. This is the radius where $T_c = T_{\text{crit}}$ for the fully MRI active solution and is shown as the line under which a steady thermally ionized MRI solution exists in the top-left plot of Fig. 1. Outside of this radius, there is not a thermally ionized MRI steady-state solution.

If there was no limit on the temperature of a steady viscous dead zone solution (it must satisfy $T_c < T_{\text{crit}}$, otherwise the MRI is triggered), we could find a steady viscous dead zone solution for all disc parameters. The solutions would fill the unstable region in the top-left plot of Fig. 1. Because the temperature of the disc decreases with radius, if there is a viscous dead zone solution at $R = R_{\text{crit}}$ with $T_c < T_{\text{crit}}$, then there is a solution throughout the unstable region. In this case, there is no unstable region and there is a steady solution throughout the disc. Similarly, if there is another type of steady solution at $R = R_{\text{crit}}$ (an externally ionized MRI or GM steady solution), then there is no GM unstable region and there is a steady solution throughout the disc. This can be seen in Fig. 1. At $R = R_{\text{crit}}(M)$, if $\Sigma < \Sigma_{\text{crit}}$ then the disc has an externally ionized fully MRI active steady solution. This occurs for $M < 3.01 \times 10^{-8} M_\odot \text{ yr}^{-1}$. If the accretion rate is sufficiently high ($M > 1.10 \times 10^{-4} M_\odot \text{ yr}^{-1}$) there is a GM steady-state solution at $R = R_{\text{crit}}(M)$. For accretion rates between these two values, we determine possible GM stable discs that result from having a non-zero $\alpha_d$.

The critical $\alpha_d$ above which a steady viscous dead zone solution occurs where $T_c = T_{\text{crit}}$. We show where the steady-state solutions exist in Fig. 4. This is similar to the plots in Figs 1 and 2 but the radius is not constant with accretion rate, we take $R = R_{\text{crit}}(M)$. This represents the globally steady disc solutions. A globally steady
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3.4 Summary

If the viscosity in the dead zone is driven by hydrodynamic instabilities such as the baroclinic instability, there is no requirement on the accretion flow through the dead zone being less than that through the active layer. In this section, we showed that a high value of $\alpha_d \lesssim 10^{-3}$ is needed to stabilize the disc to outbursts. The amount of turbulence that can be generated by hydrodynamic instabilities is unlikely to be this high. If the baroclinic instability can drive turbulence, it will be significantly smaller than turbulence generated by the MRI (e.g. Armitage 2011). Thus, we find it is unlikely that hydrodynamic instabilities in the dead zone can stabilize the disc to the outburst behaviour. However, outbursts may be triggered by heating from the viscosity in the dead zone, rather than the heating from self-gravity.

4 FIXED FRACTIONAL ACCRETION RATE IN THE DEAD ZONE

In this section, we consider steady-state solutions where $\alpha_d$ is not a fixed number but the accretion rate through the dead zone is a constant fraction, $f$, of that through the active layer,

$$M_d = f M_m. \quad (11)$$

This is equivalent to

$$\alpha_d = \alpha_m \min\left(1, \frac{f T_m}{T_c} \frac{\Sigma_m}{\Sigma_d}\right). \quad (12)$$

The viscosity is limited so that a larger $\alpha$ is not driven in the dead zone compared to that in the active layer even when the dead zone layer has a very small surface density.

This viscosity prescription is applicable to dead zone viscosities driven by hydrodynamic waves that are excited in the dead zone by the turbulence from the active layer. Thus, its magnitude depends upon the depth of the active layer compared to the dead zone layer (Fleming & Stone 2003). The MHD calculations in Turner & Sano (2008) find that the dead zone transports material at a rate with

![Figure 3. $\Sigma - M$ or $T_c$ plane at $R = 3$ au for a disc with $\Sigma_{crit} = 200$ g cm$^{-2}$.](image)

The thick black lines show the steady-state solutions with no viscosity in the dead zone. These are the thermal MRI, external MRI and GM solutions. The dashed line shows where there is a dead zone solution with $\alpha_d = 0$, that is not steady. The shaded region shows the GM unstable region with $\alpha_d = 0$. The grey/white lines show steady-state viscous dead zone solutions for $\alpha_d = 10^{-3}$, $10^{-4}$ and $10^{-5}$. The grey part denotes where $\dot{M}_d < \dot{M}_m$ and the white part where $\dot{M}_d > \dot{M}_m$.

![Figure 4. The globally steady disc solutions for $\Sigma_{crit} = 200$ g cm$^{-2}$ (left) and $\Sigma_{crit} = 20$ g cm$^{-2}$ (right). Same as the top-right panels in Figs 1 and 2, for example, but with $R = R_{crit}(M)$.](image)
fraction, \( f \), in the range 0.04–0.61 depending on disc parameters such as the presence of dust grains and the ionizing sources. Simon et al. (2011) find a more constant value of \( \alpha_d = 10^{-4} \) in the dead zone but consider only radii of 4 au and 10 au in a minimum mass solar nebula (MMSN). Similarly, all of these calculations assume an MMSN surface density that may be much smaller than that in a real time-dependent disc. Thus, in this work we consider a range of values for \( f \).

In Fig. 5, we show the local limit cycle for such solutions with different \( f \) at a radius \( R = 3 \) au. As the surface density of a steady solution increases, the viscosity in the dead zone decreases. Thus, on the far right of the diagram, the GM branch is not noticeably affected by the addition of a dead zone viscosity. Because a larger amount of material flows through the disc compared to the zero viscosity dead zone solution, the solutions form a branch above the dead zone branch. For \( f < 0.56 \), the new viscous dead zone branch connects to the GM branch on the right-hand side and the range of GM unstable accretion rates is unaffected. The outburst will be triggered by the heating from self-gravity. However, for \( f > 0.56 \), the branch is above the top of the GM branch and ends where \( T_e = T_{crit} \). The range of unstable accretion rates is slightly decreased, as shown in the shaded region for \( f = 1 \) compared to the shaded region for \( f = 0 \). In this case, the outburst will be triggered by the heating from the viscosity in the dead zone rather than the heating from self-gravity.

With the flow through the dead zone restricted to be smaller than that through the active layer (\( f < 1 \)), the additional steady states do not increase the range of unstable accretion rates significantly. The dead zone viscosity can only produce an increase in the lowest unstable accretion rate by a fraction of the accretion rate through the active layer. To prevent the outburst cycle, the steady-state viscous dead zone solutions would need to cover the whole of the unstable region. Thus, we find that even with these additional steady-state solutions, closure of the gap is not likely unless the gap is small already with \( \alpha_d = 0 \). Thus, hydrodynamic turbulence in the dead zone, generated from the turbulence in the active layers is unlikely to stabilize a disc to the GM disc instability.

5 CONCLUSIONS

With zero viscosity in the dead zone, for a range of accretion rates, a disc is unstable to the GM disc instability (where the turbulence cycles from gravitationally produced to magnetically produced). We have examined a further class of disc solutions where the dead zone has a small residual viscosity that allows the possibility of a steady state. We have considered two types of turbulence in the dead zone. First, we took a constant \( \alpha \)-viscosity. This may be applicable to hydrodynamic instabilities such as the baroclinic instability. We found that a steady solution may only be found if the turbulence in the dead zone is very high, comparable to that in the MRI active layers, but this is unlikely. The second type of turbulence we considered generates an accretion flow through the dead zone that is a fixed fraction of that through the MRI active layers. This may be applicable to hydrodynamic turbulence generated from the MRI active layers. The range of unstable accretion rates is hardly affected by the additional dead zone viscosity. However, the triggering mechanism for the outbursts may be the heating from the viscosity in the dead zone, rather than the self-gravity. Thus, we find it is unlikely that either type of turbulence within the dead zone can stabilize a disc and prevent the outbursting behaviour.

ACKNOWLEDGEMENTS

We thank an anonymous referee and Jacob Simon for useful comments. RGM’s support was provided under contract with the California Institute of Technology (Caltech) funded by NASA through the Sagan Fellowship Programme. SHL acknowledges support from NASA grant NNX11AK61G.

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