Theory of Spin Hall Magnetoresistance from a Microscopic Perspective

X. P. Zhang, 1, 2 V. N. Golovach, 1, 2, 3 and F. S. Bergeret 1, 2

1 Donostia International Physics Center (DIPC), Manuel de Lardizabal, 4, 20018, San Sebastian, Spain
2 Centro de Fisica de Materiales (CFM-MPC), Centro Mixto CSIC-UPV/EHU, 20018 Donostia-San Sebastian, Basque Country, Spain
3 IKERBASQUE, Basque Foundation for Science, 48013 Bilbao, Basque Country, Spain

We present a theory of the spin Hall magnetoresistance of metals in contact with magnetic insulators. We express the spin-mixing conductances, which govern the phenomenology of the effect, in terms of the microscopic parameters of the interface and the spin-spin correlation functions of the local moments on the surface of the magnetic insulator. The magnetic-field and temperature dependence of the spin-mixing conductances leads to a rich behavior of the resistance due to an interplay between the Hanle effect and spin mixing at the interface. Our theory provides a useful tool for understanding the experiments on heavy metals in contact with magnetic insulators of different kinds.

Introduction

The spin-orbit coupling (SOC) in metals and semiconductors leads to a conversion between the charge and spin currents, which results in the spin Hall effect (SHE) and its inverse effect [1–13].

A manifestation of the SHE in a normal metal (NM) is a modulation of the magnetoresistance (MR) with respect to the direction of the applied magnetic field when the metal is in contact to an adjacent magnetic insulator (MI) in NM/MI structures [14–16]. This effect, called the spin Hall magneto-resistance (SMR), has been observed in several experiments [17–20]. The origin of the SMR is the spin-dependent scattering at the NM/MI interface which depends on the angle between the polarization of spin Hall current and the magnetization of the MI [18, 21]. The latter can be controlled by magnetic fields.

Although the theory of SMR [18, 21] is well established and provides a qualitative description of the effect, it does not describe the dependence of the resistivity on the strength of the applied magnetic field $B$, or on the temperature $T$. The spin mixing conductances, which are at the heart of the SMR effect, have traditionally been regarded as phenomenological parameters in every experiment, because their computation was thought to be a formidable task which could only be carried out by ab initio methods [22–24]. Furthermore, the magnetic field alone leads to the Hanle magnetoresistance (HMR) [25, 26], which is identical by angular dependence to SMR [26], but not requiring a MI. Despite the fact that SMR and HMR have different origins, they cannot always be easily separated in an experiment, which adds onto the uncertainties in interpreting the experimental data. It is, therefore, desirable to have a theory of SMR which has predictive power about the dependence of the spin mixing conductances on $B$ and $T$ and is able to cover a wide range of magnetic system, from classical to quantum magnets.

In this letter, we present a general theory of the electronic transport in NM/MI structures. We describe the spin-dependent scattering at the NM/MI interface via a microscopic model based on the sd-coupling between the localized magnetic moments of the MI and the conduction electrons in the NM. The temperature and magnetic-field dependence of the interfacial scattering coefficients is obtained by expressing them in terms of spin-spin correlations functions. The latter are determine by the magnetic behaviour of the MI layer. As examples we study the MR of a metallic film adjacent to a paramagnet, and a Weiss ferromagnet. At enough low temperatures we find a striking non-monotonic behaviour of the MR. We also discuss how our model can be straightforwardly adapted to the study of the MR in single NM layer with magnetic impurities. In general, our model provides a tool to reveal, by MR measurements, magnetic properties of the NM/MI interface.

The Model

We consider a NM in contact with a MI, as shown in Fig. [1]. We assume that both layers are translation invariant in the $(x, y)$ plane and that the NM/MI interface is located at $z = 0$. We model this system by the Hamiltonian $H = H_{\text{NM}} + V_{sd} + H_{\text{MI}}$. Here $H_{\text{NM}}$ is Hamiltonian of NM which is assumed to have a sizable...
The Greek indices, $\epsilon$, correct the current. and repeated indices are implicitly summed over.

The second term on the right hand side describes the function $\omega$. The function in Fig. 1, shown in Fig. 1, is the longitudinal (transverse) spin relaxation times within the b-region. Owing to the diffusive motion of the electron in the metal, the interaction events along the electron trajectory are to a good approximation uncorrelated, justifying the use of the Born-Markov approximation:

$\frac{1}{\tau} = \frac{2\pi}{\hbar} \nu_{imp} n_{B} \omega_{m} \beta L_{\parallel} |n_{B}(\omega_{m})| (\langle \hat{S}_{\parallel} \rangle)$,

where $n_{B}(\omega_{m}) = 1/(\epsilon_{B} \omega_{m} - 1)$ is the Bose-Einstein distribution function at temperature $\beta = 1/k_{B} T$ for magnon with frequency $\omega_{m}$, and $\hat{S}_{\parallel}$ is the component of the localized spin operator $\hat{S}$, parallel to the applied magnetic field. In the above expression, we have used Weiss theory, i.e., mean-field approximation, where the interaction between localized magnetic moments only amounts to a renormalization of the Zeeman term. Thus, Weiss field can be written as $\omega_{m}^{\parallel} = \omega_{B} - 6 \langle \hat{S}_{\parallel} \rangle (J_{m}/\hbar)$. As we will show, the SMR has its origins in (i) a reduction of symmetry (from spherical to uniaxial) of the electron spin relaxation tensor at the interface, Eq. (4) and (ii) a renormalization of the Larmor precession frequency of the electron spin at the interface, Eq. (3). The spin-mixing conductances $G_{r}$ and $G_{i}$ are related to (i) and (ii), respectively. Both (i) and (ii) occur under the influence of the local moments, as sketched in the inset of Fig. 1.

The spin current in the NM can be written as:

$J_{s,i}^{\nu}(r) = \frac{\sigma_{D}}{2e} \delta_{i}(\nu_{S}) \hat{S}_{\parallel} \hat{S}_{\parallel} E_{k} \beta L_{\parallel}$,

where the first term is the usual diffusive current, and the second one, proportional to the electric field $E$, is the contribution stemming from the SHE parameterized by the spin Hall angle $\theta_{SH}$. For the geometry shown in Fig. 1, $E$ points in the x-direction. By substitution of Eq. (7) into Eq. (2) one obtains the spin-diffusion equation in the NM $\nabla_{\tau}^{2} \mu_{s}^{\nu}(r) = \frac{\eu}{\tau_{S}} \delta_{\nu} n_{B}^{\nu}(z) - \frac{\eu}{\tau_{S}} \nu_{imp} n_{B}^{\nu}(r)$.

Here $\tau_{S}$ is the isotropic part of spin relaxation induced by SOC or magnetic impurities in the NM, whereas $\tau_{\parallel}$ (or $\tau_{\perp}$) is the longitudinal (transverse) spin relaxation times within the b-region. Owing to the diffusive motion of the electron in the metal, the interaction events along the electron trajectory are to a good approximation uncorrelated, justifying the use of the Born-Markov approximation: 

$\frac{1}{\tau_{\parallel}} = \frac{2\pi}{\hbar} \nu_{imp} n_{B} \omega_{m} \beta L_{\parallel} |n_{B}(\omega_{m})| (\langle \hat{S}_{\parallel} \rangle)$,

$\frac{1}{\tau_{\perp}} = \frac{\pi}{\hbar} \nu_{imp} n_{B} \omega_{m} \beta L_{\parallel} |n_{B}(\omega_{m})| (\langle \hat{S}_{\parallel} \rangle)$,

where $n_{B}(\omega_{m}) = 1/(\epsilon_{B} \omega_{m} - 1)$ is the Bose-Einstein distribution function at temperature $\beta = 1/k_{B} T$ for magnon with frequency $\omega_{m}$, and $\hat{S}_{\parallel}$ is the component of the localized spin operator $\hat{S}$, parallel to the applied magnetic field. In the above expression, we have used Weiss theory, i.e., mean-field approximation, where the interaction between localized magnetic moments only amounts to a renormalization of the Zeeman term. Thus, Weiss field can be written as $\omega_{m}^{\parallel} = \omega_{B} - 6 \langle \hat{S}_{\parallel} \rangle (J_{m}/\hbar)$. As we will show, the SMR has its origins in (i) a reduction of symmetry (from spherical to uniaxial) of the electron spin relaxation tensor at the interface, Eq. (4) and (ii) a renormalization of the Larmor precession frequency of the electron spin at the interface, Eq. (3). The spin-mixing conductances $G_{r}$ and $G_{i}$ are related to (i) and (ii), respectively. Both (i) and (ii) occur under the influence of the local moments, as sketched in the inset of Fig. 1.

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this region an almost constant spin density. This results in
\[ -\frac{1}{e\nu_F} J_{s,z}^{\nu} \big|_{z=b} = b \omega_L \ell_{\nu} n_{\nu} \mu_s^s = \left( \frac{b}{\tau_L} + \frac{1}{\tau_L} \right) \mu_s^s \]
(9)
where \( \omega_L \) is defined in Eq. (3). This boundary condition is rather general and applies to any NM/MI interface with arbitrary magnetic configuration. In the limit \( b \to 0 \), one can write the boundary condition in a more customary way by introducing the spin-mixing conductance terms and adopting a vector notation for the spin
\[ -e J_{s,z}(0) = G_s \mu_s + G_r n \times (n \times \mu_s) + G_i n \times \mu_s, \]
(10)
where we have assumed that the spin-current vanishes at \( z = -b \), i.e., \( J_{s,z}^\nu(z = -b) = 0 \). The different components of the spin-mixing conductance, \( G_{r,i,s} \) can be expressed in terms of the isotropic, transverse and longitudinal spin-relaxation times:
\[ G_s = -e^2 \nu_F \frac{1}{\tau_L}, \]
(11)
\[ G_r = e^2 \nu_F \left( \frac{1}{\tau_L} - \frac{1}{\tau_L} \right), \]
(12)
\[ G_i = -\frac{e^2}{h} n_{imp}^2 \nu_F J_{sd}(S^s_\parallel). \]
(13)

Importantly, according to Eqs. (9-13) these three interface parameters are related to the spin averages which in turn depend on the temperature and magnetic field. Examples for their behaviour as a function of \( T \) and \( B \) are shown in Fig. 2, and in examples in Ref. [30].

Eqs. (9-13) is one of the main results of this letter. The magnetic state of the MI (para-, ferro, antiferromagnetic, etc) is encoded in the spin-operator averages which can be calculated independently within any suitable approach. Thus, the effect of the temperature and external magnetic field is included in our model via the boundary condition Eq. (10) together with the relations (9) and (11-13).

For a typical experimental geometry as the one shown Fig. 1, the spin accumulation only depends on the \( z \)-coordinate and one has to solve the one dimensional diffusion equation, Eq. (3), by imposing zero current at \( z = d \) and the boundary condition Eq. (10) at \( z = 0 \). From the knowledge of \( \mu_s(z) \) one can compute the currents and the longitudinal (along \( x \))/transverse (along \( y \)) resistivity (LR/TR). The expressions have the same form as in Ref. [21]:
\[ \rho_L \approx \rho_D + \Delta \rho_0 + \Delta \rho_1 \left( 1 - n_y^2 \right), \]
(14)
\[ \rho_T \approx -\rho_D \omega_c \tau_D \hat{n}_z + \Delta \rho_1 \hat{n}_x \hat{n}_y + \Delta \rho_2 \hat{n}_z, \]
(15)
but now the different contributions are given by
\[ \frac{\Delta \rho_0}{\rho_D} = \theta_{SH}^2 \left[ 2 - \mathcal{R}(G_s, \ell_S) \right], \]
(16)
\[ \frac{\Delta \rho_1}{\rho_D} = \theta_{SH}^2 \left[ \mathcal{R}(G_s, \ell_S) - \text{Re}[\mathcal{R}(G_s - G_{\perp\perp}, \ell_+)] \right], \]
(17)
\[ \frac{\Delta \rho_2}{\rho_D} = \theta_{SH}^2 \text{Im}[\mathcal{R}(G_s - G_{\perp\perp}, \ell_+) \mathcal{R}(G_s, \ell_S)], \]
(18)
where \( G_{\perp\perp} = G_x + iG_y, \ell_+^2 = \ell_x^2 - \ell_y^2 = (i\omega_c)^2 \), \( \omega_c \) is cyclotron frequency, generated by perpendicular component of magnetic field. \( \rho_D \) and \( \tau_D \) are Drude resistivity and relaxation time, respectively. The dimensionless coefficient \( \mathcal{R} \) can be written as
\[ \mathcal{R}(G, \ell) = \mathcal{R}_{HMR}(\ell) \mathcal{R}_{SMR}(G, \ell), \]
(19)
with
\[ \mathcal{R}_{HMR}(\ell) = \frac{2\ell}{d_N} \coth \left( \frac{d_N}{2\ell} \right), \]
(20)
\[ \mathcal{R}_{SMR}(G, \ell) = 1 - \frac{\rho_D G}{2\rho_D G} \coth \left( \frac{4\ell}{d_N} \right) \]
(21)
The effect of the NM/MI interface, the origin of SMR, is described by the function \( \mathcal{R}_{SMR}(G, \ell) \). Whereas \( \mathcal{R}_{HMR}(\ell) \) does not depends on the interface properties. For example, in a single NM without any magnetic interface, the SHE generates a spin current flowing in the \( \hat{z} \)-direction and polarized along the \( y \)-axis. A magnetic field applied in \( y \)-direction clearly does not lead to spin precession and hence does not affect the LR (cf. Eqs. 14 and 10). But with \( G_s = 0 \), if the field is applied along the \( x \)- or \( z \)-direction, the Haule precession takes place. The changes of \( \rho_L \) due to such precession is determined by the third term Eqs. (14) which results in \( \rho_L \sim \{ 2 - \text{Re}[\mathcal{R}_{HMR}(\ell_+)] \} \). For small fields this leads to the characteristic quadratic HMR behaviour \( \rho_L \propto B_x^2(B_y^2) \). [25,26].

This low field behaviour may change drastically with an adjacent MI. If we assume that the magnetic moment of the MI is saturated, by considering for example a low temperature regime, then the parameters \( G_{r,i,s} \) describing the spin-dependent scattering at the NM/MI interface do not depend on the \( B \)-field. In this case expansion of Eq. (14) for small fields gives a linear in \( B \) change of the MR. Specifically, \( \delta \rho_L \propto -\theta_{SH}^2 n_d^d B J_{sd}, \) for \( d_N \ll l_S, \) and \( \delta \rho_L \propto \theta_{SH}^2 B J_{sd}/d_N, \) for \( d_N >> l_S. \) This a remarkable result: There is a change of sign of the linear in \( B \) term, when going from the thin to the thick film limit (see Ref. [29]). The sign of the term also depends on the sign of the \( J_{sd} \) coupling. At this stage, one expects that the interplay between the linear in \( B \) term and the quadratic HMR contribution may lead to striking behaviours of \( \rho_L(B) \). In the next section we provide examples for paramagnetic and ferromagnetic interfaces.

It is worth mentioning that the above equations provide a description of the dependence of MR on the magnetic field and temperature, in terms of microscopic parameters. In this respect our results generalize those obtained in Ref. [21], by allowing for arbitrary magnetic
state of the MI and by taking into account the Hanle precession. The SMR results of Ref. [21] can be recovered by setting $R(G, \ell) = R(G, \ell_0).

There are two remarkable and novel results from our model: One is the the $B_y$-dependence of LR caused by $G_s$ (cf. Eqs. (11) and (5)), which goes beyond the original SMR theory [21]. The second one is the possibility to describe the so-called spin Hall MR gap $\Delta G_s \equiv \rho_L(B_{x(z)} \rightarrow 0) - \rho_L(B_y \rightarrow 0)$. Substituting Eq. (14), it reads:

$$\frac{\Delta G_s}{\rho_D} = \theta_{SH}^2 \left\{ R(G_s, \ell_S) - Re \left[ R(G_s - G_{1+}, \ell_+ \right) \right\} \right|_{B \rightarrow 0}. \tag{22}$$

Finally, our model can be easily be adapted to the case of a single NM layer with magnetic impurities: In such a case one should simply substitute the current expression Eq. (7) into Eq. (2) to obtain the diffusion equation with the corresponding precession and relaxation terms [31].

**Examples:** To illustrate how to apply the above derived expressions in a particular situation we assume that the MI can be described as a Weiss ferromagnetic insulator (PMI) that can exhibit an spontaneous finite average magnetization, $\langle \hat{S} \rangle \sim S$, at temperatures below the Curie-Weiss temperature $T_c$. The $B$- and $T$- dependence of $\langle \hat{S} \rangle$ is obtained by the transcendental equation,

$$\langle \hat{S} \rangle = -B_S [\beta S (g \mu_B B - 6\langle \hat{S} \rangle J_m)], \tag{23}$$

where $B_S[X]$ is Brillouin function. This expression is also valid for a paramagnetic insulator (PMI) after setting $J_m = 0$. Our model assumes that the magnetic configuration of the localized magnetic moments in MI is not affected by the spin of conduction electrons of the NM, such that it is determined only by Eq. (23).

The strength of the interface exchange interaction on the conduction electrons is described by the coupling parameter $J_{sd}$ that can be ferromagnetic ($J_{sd} > 0$) or antiferromagnetic ($J_{sd} < 0$). The sign of the interaction affects dramatically the behavior of MR as shown in Fig. 3 where we show the the $B$-dependence of $\rho_L$ for values of $J_{sd}$ of the order of those reported in the literature [32, 33].

Specifically, Fig. 3 shows both, the paramagnetic (PM, zero Curie-Weiss temperature $T_c$) and ferromagnetic (FM, $T_c = 100$ K) cases. As mentioned above, the dependence of $\rho_L$ on $B_y$ is caused by that of $G_s$ and it is more pronounced at temperatures close to $T_c$, when $G_s$ depends strong on $B_y$. It is also interesting to notice the spin Hall MR gap, $\Delta G_s$, in panels b,d of FIG. 3. According to Eq. (22) this gap appears when $G_{1+} = G_r + iG_i \neq 0$, or in other words, when the system is in the FM state and exhibits an spontaneous finite average magnetization, $\langle \hat{S} \rangle \sim S$, even a zero field.

The solid lines in Fig. 3 show the dependence of $\rho_L$ on fields parallel to $x(z)$-axis. For enough low temperatures the shape of the MR curves not only strongly depends on the sign of $J_{sd}$ but also on the thickness $d_N$ of the NM (see Ref. [30]). One striking result is the non-monotonic behaviour of $\rho_L(B)$ that occurs for enough low temperatures (see for example blue curves in FIG. 3-d). Such a behaviour can be attributed to the interplay between different MR effects. At low fields, HMR is negligible small and the $B$-dependence of $\rho_L$ is mainly due to the change of $G_{r,i,s}$ (cf. Fig. 2b). At larger fields the interface coefficients start to saturate and the shape of the $\rho_L(B)$ curve is determine by both SMR and HMR. For the particular thickness studied here, and for $J_{sd} < 0$, $\rho_L$ decreases as a function of $B$ for these intermediate values.

FIG. 2. (Color online) (a) Spin-mixing conductance, $G_s(x = s, r, i)$, as a function of magnetic field $B$, is plotted for paramagnetic insulator at temperature $T = 1$ K. (b) Spin-mixing conductance, $G_s(x = s, r, i)$, as a function of temperature $T$, is plotted for ferromagnetic insulator with magnetic field $B \rightarrow 0$ T. Parameters: $S = 2$, $\ell_s = 3.0$ nm, $E_F = 1.0$ eV, $a_e = 0.4$ nm, $n_{imp}^{2a_c} = 0.5$, $J_{sd}a_c^{-3} = +0.1$ eV.

FIG. 3. (Color online) Longitudinal resistivity, $\rho_L(B) = \rho_L(B_y \rightarrow 0)/\rho_D$, as a function of magnetic field $B$, are shown as solid lines ($B = B_x$) and dash lines ($B = B_y$). Figures (a) and (b) correspond to positive coupling constant ($J_{sd}a_c^{-3} = +0.1$ eV) with (a) paramagnetic insulator (PMI) insulator ($T_c = 0$ K) (b) ferromagnetic insulator (FMI) ($T_c = 100$ K), respectively; while, Figures (c) and (d) correspond to negative coupling constant ($J_{sd}a_c^{-3} = -0.1$ eV) with (c) PMI (d) FMI, respectively. Parameters: $\theta_{SH} = 0.1$, $d_N = 3$ nm, $\rho_D = 1.0 \times 10^{-6}$ $\Omega \cdot m$. $\theta_{SH} = 0.1$.
of the field. Finally, at larger fields, HMR dominates and \( \rho_L \) increase monotonically. Notice that in the FM state (Fig. 3) this non-monotonic behaviour can be observed over large ranges of temperatures below \( T_c \). It is important to emphasize that the non-monotonic behaviour of \( \rho_L(B) \) for \( J_{sd} < 0 \) can also be observed for larger values of \( d_N \). In contrast if \( d_N < l_\perp \) our theory predicts that non-monotonicity occurs for \( J_{sd} > 0 \) (see Ref. [30]).

**Conclusions** We present a complete theoretical model to investigate the effect of a magnetic field and temperature on the MR of a normal metal adjacent to a magnetic insulator with arbitrary magnetic ordering. We apply our theory to a paramagnetic and a Weiss ferromagnetic MI and found and interesting interplay between the SMR and HMR that depends dramatically on the sign of the \( s-d \) coupling and the thickness of the normal metal. Our model provides a tool to investigate the magnetic properties of MIs by performing relative simple resistance measurements.

**Acknowledgement** This work was supported by Spanish Ministerio de Economía y Competitividad (MINECO) through the Projects No. FIS2014-55987-P and FIS2017-82804-P, and EU’s Horizon 2020 research and innovation program under Grant Agreement No. 800923 (SUPERTED).

### I. SUPPLEMENTARY INFORMATION

#### I. USEFUL LIMITING CASES

Here we explore the behaviour of the longitudinal resistance expanding it with respect to the magnetic field in \( x(z) \) direction. Read from Eq. (14) of main text, it is given by

\[
\frac{\rho_L}{\rho_D} \simeq 1 + \theta_{SH}^2 \{ 2 - \Re [\mathcal{R}(G_s - G_{\uparrow\downarrow}, \ell_\perp) \} \},
\]

(S.1)

with \( G_{\uparrow\downarrow} = G_\uparrow + iG_\downarrow, \ell_\perp = \ell_\parallel^2 + i\ell_\perp^2 \), where \( \ell_\parallel = \sqrt{\mathcal{D} \tau_\parallel} \) and \( \ell_\perp = \sqrt{\mathcal{D}/\omega_B} \). \( \tau_\parallel \) is spin relaxation time, \( \mathcal{D} = v_F^2 \tau/2 \) is the diffusion coefficient, \( v_F \) the Fermi velocity and \( \tau \) the momentum relaxation time. \( \omega_B \) is Larmor frequency. \( \rho_D \) is Drude resistivity. The dimensionless coefficient \( \mathcal{R} \) can be written as \( \mathcal{R}(\mathcal{G}, \ell) = \mathcal{R}_{HMR}(\ell) \mathcal{R}_{SMR}(\mathcal{G}, \ell) \) with

\[
\mathcal{R}_{HMR}(\ell) = \frac{2\ell}{d_N} \tanh \left( \frac{d_N}{2\ell} \right),
\]

(S.2)

\[
\mathcal{R}_{SMR}(\mathcal{G}, \ell) = \frac{1 - \rho_D \mathcal{G} \ell \coth \left( \frac{d_N}{2\ell} \right)}{1 - 2\rho_D \mathcal{G} \ell \coth \left( \frac{d_N}{2\ell} \right)}.
\]

(S.3)

The effect of the NM/MI interface, the origin of SMR, is described by the function \( \mathcal{R}_{SMR}(\mathcal{G}, \ell) \). Whereas \( \mathcal{R}_{HMR}(\ell) \) does not depends on the interface properties. The different components of the spin-mixing conductance, \( G_{r,s} \), can be expressed in terms of the transverse and longitudinal spin-relaxation times:

\[
G_\perp = -e^2 v_F \frac{1}{\tau_\parallel},
\]

(S.4)

\[
G_\parallel = e^2 v_F \left( \frac{1}{\tau_\parallel} - \frac{1}{\tau_\perp} \right),
\]

(S.5)

\[
G_s = -\frac{e^2 \nu_F}{\hbar} n_{imp}^2 \nu_F J_{sd} \langle \hat{S}_s^2 \rangle.
\]

(S.6)

The longitudinal (transverse) spin relaxation times, \( \tau_\parallel (\tau_\perp) \) within the \( b \)-region can be written as

\[
\frac{1}{\tau_\parallel} = \frac{2\pi}{\hbar} n_{imp}^2 \nu_F J_{sd} \beta \hbar \omega_B^m n_B(\omega_B) \left[ 1 + n_B(\omega_B^m) \right] \langle \langle \hat{S}_s\rangle \rangle,
\]

(S.7)

\[
\frac{1}{\tau_\perp} = \frac{1}{2\tau_{\parallel \perp}} + \frac{\beta}{2} \frac{\nu_F^2 n_{imp}^2 J_{sd}^2 \langle \hat{S}_s^2 \rangle}{\hbar},
\]

(S.8)

where \( n_B(\omega_B^m) = 1/(e^{\beta \hbar \omega_B^m} - 1) \) is the Bose-Einstein distribution function at temperature \( \beta = 1/k_B T \) for magnon with frequency \( \omega_B^m \), and \( \hat{S}_s \) is the component of the localized spin operator \( \hat{S} \) parallel to the applied magnetic field. The Weiss field can be written as \( \omega_B^m = \omega_B - 6\langle \hat{S}_s \rangle (J_\perp/h) \). Importantly, these three interface parameters, \( G_{r,s} \) in Eqs. (S.4) (S.5) and (S.6), are related to the spin averages. Their behaviour as a function of \( T \) and \( B \) are shown in Fig. 4.

We first remove contribution of LR which is not dependent on magnetic field, by defining

\[
\Delta \rho_L = \rho_L - \rho_D - 2\theta_{SH}^2 \rho_D.
\]

(S.9)

Obviously, \( \Delta \rho_L \) is proportional to \( \theta_{SH}^2 \). For the sake of simplicity, we here introduce a dimensionless

\[
\mathcal{R}_L = \Delta \rho_L / \rho_D / \theta_{SH}^2.
\]

(S.10)

In the following, we explore the magnetic field dependence of \( \mathcal{R}_L \) in all kinds of limits.
with it is instructive to expand the HMR effect in powers of $B$ magnetic (FM) insulator, \[ \text{FIG. 5. (Color online) (a,b) Longitudinal resistivity of ferromagnetic (FM) insulator, } [\rho_L(B)]/[\rho_D], \text{ as a function of magnetic field } B, \text{ for (a) positive } (J_{sd}a_c^{-3} = +0.1 \text{ eV}) \text{ and (b) negative coupling constant } (J_{sd}a_c^{-3} = -0.1 \text{ eV}). \text{ Parameters: } T_c = 100 \text{ K, } T = 10 \text{ K}. \]

A. The Hanle magneto-resistance

In the absence of SMR, i.e., $G_{s,t,\uparrow\downarrow} = 0$ and $\mathcal{R}_{SMR} = 1$, we obtain the LR

\[ \mathcal{R}_L = -\Re \left[ \mathcal{R}_{HMR}(\ell_+^0) \right]. \quad (S.11) \]

Using the following relation

\[ \frac{\ell_S}{\ell_+} = \sqrt{1 + i\omega_B\tau_S}, \quad (S.12) \]

it is instructive to expand the HMR effect in powers of $\omega_B\tau_S$,

\[ \mathcal{R}_L \approx -\frac{2\ell_S}{d_N} \tan \left( \frac{d_N}{2\ell_S} \right) + C_2\omega_B^2\tau_S + \mathcal{O}(\omega_B^3\tau_S^3), \quad (S.13) \]

with

\[ C_2 = \frac{1}{8} \left[ \frac{d_N^3}{\ell_S^3} \tan \left( \frac{d_N^2}{2\ell_S} \right) + 3 \tan^2 \left( \frac{d_N}{2\ell_S} \right) \right] \]

\[ + \left( \frac{6\ell_S}{d_N} - \frac{d_N}{\ell_S} \right) \tan \left( \frac{d_N}{2\ell_S} \right) - 3 \],

where $C_2$ is always positive. For most experimental needs, expansion of HMR to the second order in $\omega_B\tau_S$ is sufficient. The linear in $\omega_B$ term drops out when expanding and taking the real part in Eq. (S.1). Then, only the quadratic term remains and it leads to the characteristic quadratic dependence of the HMR effect.

B. The Spin-Hall magneto-resistance

In the absence of HMR, i.e., $\ell_+ = \ell_S$ and $\mathcal{R} = \mathcal{R}_{HMR}(\ell_S)\mathcal{R}_{SMR}(G_s - G_{\uparrow\downarrow}, \ell_S)$, we obtain the LR

\[ \mathcal{R}_L = -\mathcal{R}_{HMR}(\ell_S)\Re [\mathcal{R}_{SMR}(\eta - \eta_0 + i\eta_l, \ell_S)], \quad (S.15) \]

with

\[ \mathcal{R}_{SMR}(\eta, \ell) = \frac{1 + \frac{1}{2} \tan \left( \frac{d_N}{2\ell} \right) \coth \left( \frac{d_N}{2\ell} \right) \eta}{1 + \eta}, \quad (S.16) \]

where we define three dimensionless parameters:

\[ \eta_s = 2G_s^r\rho_D\ell_s \coth \left( \frac{d_N}{\ell_S} \right), \quad (S.17) \]

\[ \eta_r = 2G_r^r\rho_D\ell_s \coth \left( \frac{d_N}{\ell_S} \right), \quad (S.18) \]

\[ \eta_l = 2G_l^r\rho_D\ell_s \coth \left( \frac{d_N}{\ell_S} \right), \quad (S.19) \]

The slope of LR with magnetic field is given by

\[ \mathcal{R}_L' = D_1 \Re \{[1 + \eta_r - \eta_s - i\eta_l] [\eta_r' - \eta_s' + i\eta_l'] \}. \quad (S.20) \]

with

\[ D_1 = \left. \frac{\frac{\ell_S}{d_N} \left( 2\tan \left( \frac{d_N}{2\ell_S} \right) - \tan \left( \frac{d_N}{2\ell_S} \right) \right) \right|}{1 + \eta_r - \eta_s + i\eta_l |^4}, \quad (S.21) \]

In the limit of $\eta_l \ll 1$, Eq. (S.20) reduces into

\[ \mathcal{R}_L' \approx D_1 (1 + \eta_r - \eta_s)^2 (\eta_r' - \eta_s'). \quad (S.22) \]

The sign of the first term depends on the slope of spin-mixing conductance $G_{s,\uparrow\downarrow} = G_s - G_{\uparrow\downarrow} (\text{or } \tau_{\uparrow\downarrow}^{-1})$, while the second one is always positive. Obviously, if we change the sign of $J_{sd}$ (or $\eta_l$), the real part of $\mathcal{R}_{SMR}$ in Eq. (S.16) does not change. Thus, the behavior of SMR is not dependent on the sign of $J_{sd}$.

C. Interplay between HMR and SMR

In the presence of both HMR and SMR, we obtain the LR

\[ \mathcal{R}_L = -\Re \left[ \mathcal{R}_{HMR}(\ell_+)\mathcal{R}_{SMR}(G_s - G_{\uparrow\downarrow}, \ell_+) \right]. \quad (S.23) \]

The slope of LR with magnetic field is given by

\[ \mathcal{R}_L' = \left. -\frac{\partial \mathcal{R}_{HMR}}{\partial \ell_+} \frac{\partial \mathcal{R}_{SMR}}{\partial \ell_+} \mathcal{R}_{HMR} \left( \frac{\partial \mathcal{R}_{SMR}}{\partial \ell_+} \frac{\partial \mathcal{R}_{HMR}}{\partial \ell_+} \frac{\partial \mathcal{R}_{SMR}}{\partial \ell_+} \right) \right|_{\theta = G_s - G_{\uparrow\downarrow}}. \quad (S.24) \]

The interplay of HMR and SMR becomes interested in low temperature. The $B$-dependence of LR in low temperature in small magnetic field is dominated by SMR, which has been discuss in Sec. [13]. Here we focus on a bit strong magnetic field, where all spin-mixing conductance almost saturate. Hence, we can omit the third term of the right hand side of Eq. (S.24), which reduce into

\[ \frac{\rho_L'}{\rho_D} = -\partial \mathcal{R}_{HMR} \frac{\partial \mathcal{R}_{SMR}}{\partial \ell_+} \mathcal{R}_{HMR}. \quad (S.25) \]

In the presence of $G_i$, the liner in $\omega_B$ term does not drop out when taking the real part in Eq. (S.1). Since
the expansion coefficient is rather cumbersome, we further expand it for $d_N \ll \ell_S$ and $d_N \gg \ell_S$. However, before that we write out the zeroth order in $\omega_B$ term by replacing $\ell_+ \to \ell_S$ in Eq. (S.1):

$$\Delta \rho_1(0) = \frac{\theta_{SH}^2 \{ R(G_s, \ell_S) - \text{Re} [ R(G_s - G_{1\perp}, \ell_S) \} \}}{4d_0^2 [ G_i^2 + (G_s - G_i)^2 ]}$$

(S.26)

which represents SMR only without any HMR. The linear in $\omega_B$ term for $d_N \ll \ell_S$ reads

$$\Delta \rho_1 \approx \Delta \rho_1(0) - \frac{\theta_{SH}^2 d_N^3 G_i \omega_B}{4d_0^2 [ G_i^2 + (G_s - G_i)^2 ]}$$.  

(S.27)

Interestingly, the parameter $\sigma_D$ drops out from the absolute correction in this limit. In the limit $d_N \gg \ell_S$ we obtain

$$\Delta \rho_1 \approx \Delta \rho_1(0) + \frac{2\theta_{SH}^2 \ell_S^4 G_i \omega_B \left( 1 + 2\ell_S^2 \frac{G_s - G_i}{\sigma_D} \right)}{D_0 d_N \sigma_D^2 \left( 1 + 2\ell_S^2 \frac{G_s - G_i}{\sigma_D} \right)^2 + 4\ell_S^4 G_i^2}$$

(S.28)

Interestingly, there is a change of sign of the linear in $\omega_B$ term, when going from the limit $d_N \ll \ell_S$ to the limit $d_N \gg \ell_S$. Also, in this limit, the correction depends on $\sigma_D$. The sign change of the slope for different thicknesses $d_N$ are shown in Fig. 5 which plots LR as a function of magnetic field in $x$-axis direction for (a) positive and (b) negative $J_{sd}(G_i \propto J_{sd})$. As depicted in Fig. 5(a), a transition from negative to positive MR can be observed in small magnetic field regime, when thicknesses, $d_N$ changes from ($d_N = 2$ nm) to ($d_N = 5$ nm) for positive $J_{sd}(G_i)$. While, the opposite transition happens for negative $J_{sd}(G_i)$ (see Fig. 5(b)).

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Generalization of our model to an anisotropic MI is straightforward since we are neglecting the influence of the conducting electrons on the MI magnetic configuration.

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See Supplementary Material.

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