Nuclei in Core-Collapse Supernovae Engine

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Abstract

Herein, we review the nuclear equations of state (EOSs) and the constituent nuclei of core-collapse supernovae (CCSNe) and their roles in CCSN simulations. Various nuclei such as deuterons, iron, and extremely neutron-rich nuclei compose in the central engines of CCSNe. The center of a collapsing core is dominated by neutron-rich heavy nuclei prior to the occurrence of core bounce. Their weak interactions significantly affect the neutrino emission and the size of the produced proto-neutron star. After a core bounce, heavy nuclei are dissolved to protons, neutrons, and light nuclei between the expanding shock wave and the newly formed neutron star (NS). Some of the key components in determining the shock-wave dynamics and supernova explosion of outer envelopes are neutrino interactions of nucleons and light nuclei such as deuterons. An EOS provides the relations between thermodynamical properties and the nuclear composition, and is needed to simulate this explosion. Further investigations on uniform and non-uniform nuclear matter are needed to improve the understanding of the mechanism of CCSNe and the properties of supernova nuclei. The knowledge of the EOS for uniform nuclear matter is being continually improved by a combination of microscopic calculations, terrestrial experiments, and NS observations. With reference to various nuclear experiments and current theories, the finite temperature effects on heavy nuclei, formation of light nuclei in dilute nuclear matter, and transition to uniform nuclear matter should be improved in the model of the EOS for non-uniform nuclear matter.

1 Introduction

Core-collapse supernovae (CCSNe) are energetic events that occur at the end of the evolution of massive stars with masses exceeding $\sim 10M_\odot$. Their typical explosion energy is approximately $10^{51}$ erg, and a black hole or a neutron star (NS) is generated as a by-product. Currently, the explosion mechanism is not understood owing to its complexity. CCSNe are also excellent accelerators of cosmic rays, major sites for nucleosynthesis, and excellent emitters of neutrinos and gravitational waves [1, 2].

The central engines of CCSNe are iron-group cores, which are formed at the center of massive stars during their chemical evolution. The gravitational collapse of a core is stopped by repulsive nuclear interactions, and shock waves are generated at the surface of a hot new-born prott-NS (PNS) on core bounce occurrence. Numerous neutrinos with energies approximately $10^{53}$ erg are emitted and facilitate the thermal expansion of the shock matter.
Neutrinos, nuclear matter, electrons, muons, and photons form the supernova matter of the central engine. Nuclear matter is composed of nucleons (protons and neutrons) and nuclei. The thermodynamic conditions of supernova matter vary over a wide range of density, temperature, and neutron (or proton) fraction. The equation of state (EOS) of nuclear matter determines not only its thermodynamic properties but also its composition in chemical equilibrium for a given condition.

The theory of uniform nuclear matter based on a nuclear interaction model is a critical component in the construction of the nuclear EOS. In most general-purpose EOSs for CCSN and NS merger simulations, Skyrme-type interactions [3, 4, 5, 6] or relativistic mean field (RMF) theories [7, 8, 9, 10, 11, 12, 13, 14, 15, 16] are adopted. Both are based on phenomenological medium-dependent interactions and the model parameters are determined to reproduce some nuclear properties such as the nuclear saturation density. Some EOSs for supernova simulations have been formulated using microscopic models considering realistic interactions determined using nucleon–nucleon scattering data [17, 18, 19].

For non-uniform nuclear matter calculations, classic EOSs [3, 7, 8, 9] adopt single-nucleus approximation (SNA), in which the full distribution of nuclei is represented by a single nucleus with optimized properties. Optimization of the nuclear structure such as nuclear decompression and nuclear pastas can be considered in such a calculation; however, a realistic nuclear composition cannot be obtained [20, 21]. An extended nuclear statistical equilibrium (NSE) model is a modern approach to describe non-uniform nuclear matter in supernova EOSs, which yields the number densities of all nuclei and nucleons in a thermodynamical state [10, 22, 23]. In addition, hybrid models of the NSE and the SNA are available for low densities and high densities, respectively [4, 12, 13].

The nuclear composition determined by the EOS, weak interaction rates of non-uniform nuclear matter at subsaturation densities, and the EOS of uniform nuclear matter at supra-nuclear densities affect the dynamics of supernova explosion and the neutrino emission from CCSNe [24, 25]. However, in a hot, dense, and/or neutron-rich environment, the free energies of nuclei and uniform nuclear matter are not constrained well. Consequently, the models of nuclei and uniform nuclear matter for the EOSs currently used in supernova simulations show a wide range of variations. The reader is referred to a review of the EOSs for CCSNe and NSs [26] as well as to a detailed comparison of general-purpose EOSs [27]. Moreover, an online service CompOSE summarizes data tables of various EOSs [28, 26]. Herein, we review the EOSs for CCSN simulations, with a focus on nuclei. We discuss the nuclei present in a CCSN, their roles in simulations, and their behavior.

In Sec. 2, we introduce the mechanism of CCSNe, related neutrino interactions, and thermodynamic conditions for supernova simulations. Theories of uniform nuclear matter are presented in Sec. 3. Non-uniform nuclear matter and the EOS for supernova simulations are described in Sec. 4. The nuclei considered in supernova simulations are discussed in Sec. 5. Finally, a summary discussion is provided in Sec. 6.

2 Core-Collapse Supernovae

In this section, we briefly review a standard scenario for a CCSN explosion and the status of numerical simulations of the CCSN engine. For more detailed explanations, refer to the reviews in [29, 30].

2.1 Explosion scenario

Main-sequence stars are supported by the thermal pressure generated by the nuclear burning reactions occurring around the center of a star. As the core shrinks and the temperature rises, \( \alpha \) nuclei, which are even–even nuclei containing equal number of protons and neutrons, are produced by nuclear fusion and accumulate in the center as ash. The ash-\( \alpha \) nuclei form a new nuclear fuel, producing \( \alpha \) nuclei with larger mass numbers. By this cycle, massive stars in their final evolution stages have an onion-like
structure with an iron core surrounded by layers of $^{28}\text{Si}$, $(^{16}\text{O}, ^{20}\text{Ne}, \text{and} ^{24}\text{Mg})$, $(^{12}\text{C} \text{and} ^{16}\text{O})$, $^4\text{He}$ and $^1\text{H}$. An iron core is formed of iron-group nuclei with proton number $Z \sim 26$, and it does not burn because these nuclei have larger binding energies per baryon than the other nuclei.

The mass of an iron core is approximately $1.5 M_{\odot}$. The increase in the temperature and density of a core induce iron photodissociation and electron capture, respectively. The iron core becomes unstable and therefore, it starts to collapse. When the density increases and the chemical energy of electrons exceeds the mass difference between parent and daughter nuclei, electron capture occurs. Reduction in the electron degeneracy pressure leads to gravitational collapse, inducing further electron capture as $e^- + (N, Z) \rightarrow \nu_e + (N + 1, Z - 1)$, where $(N, Z)$ denotes nucleus with neutron number $N$ and proton number $Z$, $\nu_e$ is an electron-type neutrino, and $e$ is an electron. In contrast, when the temperatures exceed 0.4 MeV, photodissociation of the iron-group nuclei into protons and neutrons occurs as follows:

$$\begin{align*}
(N, Z) &\rightarrow \frac{Z}{2} \alpha + (N - Z)n, & N \geq Z, \\
(N, Z) &\rightarrow \frac{Z}{2} \alpha + (Z - N)p, & N < Z, \\
\alpha &\rightarrow 2p + 2n,
\end{align*}$$

where $\alpha$ is an $\alpha$ particle ($^4\text{He}$ nucleus), $n$ is a neutron, and $p$ is a proton. This endothermic reaction suppresses the increase in the thermal pressure caused by the core contraction, thereby accelerating the collapse. This photodissociation typically occurs before the electron capture, and gravitational collapse begins. For relatively light stars of approximately $10 M_{\odot}$, electron capture may occur first because of the low central temperature of the iron core.

In the early stages of a core collapse, the majority of neutrinos produced by the electron capture can escape from the core. When the density exceeds $10^{12}$ g/cm$^3$, the time scale for diffusion becomes shorter than the dynamical time scale, $O(1 \text{ ms})$, and the neutrinos are trapped and degenerated. The core during collapse is divided into two parts: the inner core, which falls with a speed proportional to its radius, and the outer core, which is free falling from the outer layers. When the density of the inner core exceeds the nuclear saturation density ($\rho_0 \sim 2.7 \times 10^{14}$ g/cm$^3$ or $n_0 \sim 0.16$ fm$^{-3}$), the nuclear interaction becomes repulsive and the EOS becomes stiff. The gravitational collapse is halted and shock waves are created. This is called a core bounce, and the core-collapse phase takes $O(10 \text{ ms})$ (from the beginning of the collapse to the core bounce). The region through which the shock waves pass is heated, and numerous neutrinos are emitted via electron capture by protons, $\nu_e + n \longleftrightarrow e^- + p$, after the reactions expressed in Eqs. (1–3). However, the neutrinos are almost trapped before the shock waves cross the neutrino sphere, which is the surface above which they can escape from the star. A large neutrino flux is emitted outward after the shock waves cross the neutrino sphere, which is called a neutronization burst, and it occurs after the bounce on the time scale of $O(1 \text{ ms})$.

The shock waves travel to the outer boundary of the iron core in $O(10 \text{ ms})$ and cause explosion of the outer layers in $O(10 \text{ h})$, resulting in a CCSN explosion. Parts of the outer layers are hit by the shock waves and accrete on the surface of the produced PNS. The lepton number of PNS composed of the above-mentioned inner core and the accreting matter is still large. The neutrinos carry away many leptons and the internal energy of the PNS on the time scale of $O(1 \text{ min})$, and the PNS becomes an NS. A black hole may remain instead of an NS, depending on the final stellar structure; however, the details are unknown.

Neutrinos, unlike electrons, neutrons, protons, and nuclei, are unaffected by electromagnetic or strong interactions. Consequently, the produced neutrinos have a long mean free path and carry away approximately 99% energy released by the gravitational collapse and the contraction of the PNS. The remaining approximately 1% energy is considered to be used for the kinetic energy of the explosion. The neutrinos also provide information about the core, from which electromagnetic waves cannot escape.
2.2 Supernova simulations

Twice decades ago, many groups conducted one-dimensional (1D) numerical simulations assuming spherical symmetry, demonstrating that the shock waves formed by the bounce of collapsing cores are decelerated and stalled by the energy losses due to the nuclear dissociations and the neutrino emission [24]. Since then, several two-dimensional (2D) and three-dimensional (3D) numerical simulations have been conducted with the neutrinos emitted from a PNS reinvigorating stalled shocks and allowing it to propagate outward again. This neutrino-heating mechanism is the most promising scenario for a shock revival in a CCSN engine.

Multi-dimensional (multi-D) effects such as convection and the standing accretion shock instability (SASI) are essential in increasing the efficiency of neutrino heating in the central engine of an explosion [32]. Recently, multi-D numerical simulations have successfully modeled the relaunch of a stalled shock wave, which may eventually produce supernova explosions [33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46]. However, most CCSN simulations are for less than \( O(1 \text{ s}) \), making it impossible to determine whether they can reproduce a canonical explosion energy of \( 10^{51} \text{ erg} \) and \( ^{56}\text{Ni} \) mass as well as how nuclear burning in accreting matter and ejecta occurs [47, 48, 49]. The progenitor that provides the initial condition of supernova simulations and is one of the key components of the dynamics of CCSNe is also uncertain [50, 51, 52, 53, 54].

The difficulties in the numerical simulations of CCSNe arise from two factors: multiple physics involvements in the system and the difficulty in using approximations. The compactness of a PNS is governed by strong interactions, weak interactions determine neutrino reactions, electromagnetic interactions affect the size of the nuclei in supernova matter, and gravitational interactions influence the dynamics of explosions and the structure of a PNS. In numerical simulations, the hydrodynamics and neutrino transport calculations must be solved in 3D and 6D phase spaces, respectively. Furthermore, relativistic effects should not be ignored [55, 56]. However, various approximations cannot be used. Supernova simulations with spherical symmetry approximation (1D simulations) cannot reproduce explosions, as previously mentioned. For neutrino transport, the diffusion approximation may be applied in the center, where scattering occurs frequently, and the free-streaming approximation may be adopted in the outer regions, where neutrinos are rarely scattered. In the middle region, where the shock waves expand owing to neutrino energy depositions, no approximations are available for neutrino transport.

At present, there is no complete supernova simulations and different approximations are employed in the simulations. In 3D simulations, neutrino momentum distributions in the phase space are not completely solved. For some 2D simulations, axial symmetry is assumed, whereas the full Boltzmann neutrino transport is solved in the phase space [38]. Based on Fig. 1, the full Boltzmann neutrino transport 2D simulations of a massive star using 11.2 \( M_\odot \) with the FYSS (VM) EOS [18] show the expansion of its shock waves. The shock radii depend on the polar angle, as shown subsequently in Fig. 5 and their minimum, average, and maximum values are presented in Fig. 1. The average and maximum radii of the shock waves are increased owing to the neutrino heating. The details of the EOS and the simulation are presented in the following sections.

2.3 Matter in supernovae

Supernova matter in the central engine of a CCSN consists of neutrinos, nuclear matter with electrons, muons, and photons. Neutrinos are not always in thermal or chemical equilibrium with nuclear matter, and thus, cannot be included in the EOS. Their non-equilibrium distributions should be computed using transport equations. Electrons are treated as ideal Fermi gases, whereas photons as ideal Bose gases. The presence of muons in a PNS has been reported [57]. However, they are located at the center of a PNS because of their larger masses than electrons, and thus, are not considered in this review. In addition, hyperons, quark matter, and pion- and kaon-condensates may drastically alter the dynamics
Figure 1: Time evolutions of maximum (dashed line), minimum (dotted line) and average (solid line) shock radii of the 2D numerical simulation [38] of model of a 11.2 $M_\odot$ star using FYSS (VM) EOS [18].

of CCSNe [58]. In this review, we focus on the nuclei and nuclear matter present below and around the nuclear saturation density, and the details of such new degrees of freedom at high densities are not discussed.

2.3.1 Hadronic matter

Depending on the density, temperature, and proton fraction, nuclear matter is formed of various phases. As shown in Fig. 2, matter with free nucleons are available at sub-nuclear densities and at high temperatures. At low temperatures, a mixture of nuclei and free nucleons are formed. The mass numbers of nuclei are larger at higher densities and at lower temperatures. Nuclear pastas may be seen at low temperatures (below about 3 MeV) and just below nuclear saturation densities [59, 60, 61]. Strongly interacting nucleons appear at supra-nuclear densities. The proton fraction, $Y_p$, is defined as the total proton number density, $n_p^{\text{total}}$, per baryon number density, $n_B$ as follows: $Y_p = n_p^{\text{total}} / n_B$. The details of nuclear matter are discussed in subsequent sections (Secs. 3–5).

2.3.2 Photon and lepton

The Helmholtz free energy density of supernova matter without neutrinos, $f_t$, is expressed as

$$f_t = f_\gamma + f_e + f_B,$$  \hspace{1cm} (4)

where $f_\gamma$ is the free energy of photons and $f_e$ is that of electrons and positrons. The free energy of nuclear matter, $f_B$, is explained subsequently in Sec. 4.

Photons are ideal bosons and their free energy densities are calculated as

$$f_\gamma = -\left(\frac{\pi^2}{45\hbar^3c^3}\right) T^4,$$  \hspace{1cm} (5)
Figure 2: Nuclear phases in typical environment of CCSNe are schematically shown in the plane of number density-temperature. The details depend on the models of nuclear matter and nuclei and proton fraction.

where $\hbar$ is the reduced Planck’s constant and $c$ is the speed of light. Electrons and positrons are ideal fermions that have no interactions \[62\]. The net electron number density is defined as the difference between the number density of electrons, $n_{e^-}$, and that of positrons, $n_{e^+}$, as

$$n_e = n_{e^-} - n_{e^+},$$  \hspace{1cm} (6)$$

$$n_{e^-} = \int_0^{+\infty} F_F(\mu_e, p) \frac{2d^3p}{(2\pi \hbar)^3},$$  \hspace{1cm} (7)$$

$$n_{e^+} = \int_0^{+\infty} F_F(-\mu_e, p) \frac{2d^3p}{(2\pi \hbar)^3},$$  \hspace{1cm} (8)$$

$$F_F(\mu_e, p) = \frac{1}{\left\{1 + \exp\left(\frac{\sqrt{p^2 + m_e} - \mu_e}{T}\right)\right\}},$$  \hspace{1cm} (9)$$

where $F_F$ is the Fermi distribution function. The factor of 2 in the denominator in Eq. (6) originates from the spin degree of freedom. The total proton fraction and the baryon number density are related to the net electron number density by the charge neutrality of the system as

$$n_e = Y_p n_B.$$ \hspace{1cm} (10)$$

The chemical potential of electrons, $\mu_e$, is obtained by solving Eq. (6) for a given $n_e$. The pressure, $p_e$, and free energy of electrons and positrons are respectively expressed as

$$p_e = \int_0^{+\infty} F_F(\mu_e, p) \frac{2\sqrt{p^2 + m_e}d^3p}{(2\pi \hbar)^3} + \int_0^{+\infty} F_F(-\mu_e, p) \frac{2\sqrt{p^2 + m_e}d^3p}{(2\pi \hbar)^3},$$ \hspace{1cm} (11)$$

$$f_e = n_e \mu_e - p_e.$$ \hspace{1cm} (12)$$

Note that the Coulomb interactions between electrons and between electrons and protons are considered in the Coulomb energies of nuclei, $f(N, Z)$. In uniform nuclear matter without nuclei, the local charge density is zero everywhere, because $n_e = n_p$, where $n_p$ is the proton number density. Hence, the Coulomb energies do not have to be considered.
Figure 3: Temperature versus density variation in the supernova simulation [38]. Color represents corresponding proton fraction. Values of meshes at several times before or the core bounce are superposed: $t = -160$ ms (circles, initial phase), 0 ms (squares, core bounce), 100 ms (triangles), 200 ms (crosses), and 300 (pluses) ms.

2.3.3 Thermodynamic conditions

Density $\rho_B$, temperature $T$, and proton fraction $Y_p$ at each time step in the above-mentioned 2D supernova simulation [38] are superposed in Fig. 3. Here, the density is defined as $\rho_B = m_u n_B$, where $m_u = 931$ MeV is the atomic mass unit [8]. In the final phase of the massive star evolution or in the initial phase of the core collapse, the thermodynamic conditions at the center of the core are $(\rho_B, T, Y_p) = (1.6 \times 10^{10}$ g/cm$^3$, 0.58 MeV, 0.42). During the core collapse, $\rho_B$ and $T$ increase almost adiabatically. The baryonic entropies are approximately $1 k_B$, as shown subsequently in Fig. 16.

Figure 4 presents the temperature and the proton fraction as functions of the central density of a collapsing core and a PNS. The proton fraction decreases to 0.24 owing to electron capture by heavy nuclei, and the temperature rises up to 14 MeV at the core bounce ($\rho_B = 3.2 \times 10^{14}$ g/cm$^3$ and $t = 0$). At densities exceeding $\rho_B \sim 10^{11} - 10^{12}$ g/cm$^3$, neutrinos are almost trapped, and the proton fraction is barely changed, whereas the emission and absorption of the trapped neutrinos slightly change $Y_p$.

After the core bounce, shock heating increases the entropy and temperature of the matter above the surface of the produced PNS. The large neutrino emission via electron capture by protons significantly reduces $Y_p$. Around the neutrino sphere at $\rho_B \sim 10^{11} - 10^{12}$ g/cm$^3$, the temperature is approximately 5.0 MeV and the proton fraction is approximately 0.09. The distributions of $\rho_B$, $T$, and $Y_p$ in the star at $t = 100$ ms and 200 ms after core bounce are shown in Figs. 5 and 6 respectively. The values around the shock waves are $T \sim 1$ MeV, $\rho_B \sim 10^9$ g/cm$^3$, and $Y_p = 0.5$. During the shock-revival phase, the temperature around the surface of the PNS reach close to 30 MeV and the central density increases up
Figure 4: Temperature (red solid line) and proton fraction (blue dashed line) as functions of central density in core-collapse phase. 

The central density increases to approximately $5.0 \times 10^{14}$ g/cm$^3$. 


Figure 5: Density (left) and temperature (right) distributions at 100 ms (top panel) and 200 ms (bottom panel) after core bounce in the supernova simulation [38] using FYSS (VM) EOS [18].
Figure 6: Distributions of total proton fractions (left) and average nucleon numbers of nucleons and nuclei (right) at 100 ms (top panel) and 200 ms (bottom panel) after core bounce in the supernova simulation [38] using FYSS (VM) EOS [18].
2.4 Weak interactions

Consideration of weak interactions in CCSN simulations plays an important role in both pre- and post-bounce stages, as noted in Sec. 2.1. In some simulations, an open-source neutrino interaction library Nulib is utilized, which is based on some studies [63, 64]. Many-body corrections may have to be considered in the calculations of weak rates [65, 66, 67]. In this section, we introduce the neutrino reactions included in the supernova simulation [38]; they are listed in Tab. 1. Referring to Tab. 1, the electron capture by free protons (reaction (i)) has a small impact on the evolution of $Y_p$ before the bounce. Free nucleons are the main sources of the neutrino emission and absorption in the post-bounce phase, which affect the cooling and heating rates via reactions (i) and (ii). The neutrino scattering by nucleons (reaction (iii)) is important for the mean free path of neutrinos, whereas electrons play a minor role in the total neutrino opacity. However, they are important in decreasing the neutrino energies through an inelastic reaction in reaction (iv).

Heavy nuclei have a major influence on the evolution of $Y_p$ in collapsing cores in the pre-bounce phase via reactions (vii) and (viii). The former determines the number of many neutrinos that are emitted. The latter affects the deleptonization by the neutrino trapping, although it does not directly change $Y_p$. For some nuclei, the calculated electron capture rates are presented in data tables [68, 69, 70, 71]. These tables are based on shell model calculations or the Monte Carlo approach with a random phase approximation. For the nuclei where no data are available, the following approximation formula as a function of the $Q$ value [72, 71] is frequently adopted.

\[
\begin{align*}
\lambda_i & = \frac{(\ln 2)B}{K} \left( \frac{T}{m_e c^2} \right)^5 \left[ F_4(\eta(N, Z)) - 2\chi(N, Z)F_3(\eta(N, Z)) + \chi^2 F_2(\eta(N, Z)) \right], \\
F_k(\eta) & = \int_0^\infty \frac{u^k du}{1 + \exp(u - \eta)},
\end{align*}
\]

where $K = 6146$ s, $\chi(N, Z) = (Q(N, Z) - \Delta E)/T$, and $\eta(N, Z) = (\mu_e + Q(N, Z) - \Delta E)/T$, where the electron chemical potential is denoted as $\mu_e$. The parameters of a typical matrix element ($B = 4.6$) and the transition energy from an excited state in a parent nucleus to a daughter state ($\Delta E = 2.5$ MeV) are fitted to the results of shell-model calculations for the pf-shell nuclei [71]. The $Q$ value of each nucleus, $Q(N, Z)$, is calculated as $Q(N, Z) = m(N, Z) - m(N + 1, Z - 1)$, where $m(N, Z)$ is the nuclear mass considering the Coulomb energy shifts [21, 73].

We note that the calculation of electron capture rates of the heavy nuclei such as $^{78}_{28}$Ni and $^{80}_{30}$Zn, is still controversial [74], which are important nuclei for deleptonization of core-collapse as discussed later. In actual, shell gaps are not taken into consideration the fitting formula of Eq. (13), which may overestimate the reaction rates [75, 76]. On the other hand, the nuclear shell structures at zero temperature may smear out by thermalization and the reaction rates increase by thermal unblocking effects at finite temperatures [77, 78, 79].
A cross-section of the neutrino-nucleus scattering rate (reaction (viii)) is evaluated as

$$\sigma_i(E_{\nu_e}) = \frac{G_W^2}{8\pi (\hbar c)^4} E_{\nu_e}^2 A^2 \left\{ 1 - \frac{2Z}{A} (1 - \sin^2 \theta_W) \right\} ^{2} \frac{2y + \exp(2y) - 1}{y^2},$$

(15)

where $A = N + Z$, $y = 1.92 \times 10^{-5} A^{2/3} E_{\nu_e}$, $G_W$ and $\theta_W$ are the weak coupling constant and the Weinberg angle, respectively. A non-degenerated nucleus and an isoenergetic zero-momentum transfer are assumed [80].

Inelastic weak interactions of light nuclei are considered in some simulations [81, 25]. For neutrino absorptions on deuterons, (ix) and (x), the data of vacuum cross-section are used [82]. The neutrino absorption rate (ix) is expressed as

$$\frac{1}{\lambda(E_{\nu})} = n_d \int dp_e \left[ \frac{d\sigma_{ud}}{dp_e}(E_{\nu}^*) (1 - F_F(E_e)) \right],$$

(16)

where $n_d$ is the deuteron number density and $F_F$ denotes the Fermi-Dirac distribution of electrons (see Eq. (9)). Electron capture by deuterons (reactions (xi) and (xii)) is estimated by assuming that the matrix elements of electron and positron capture are equivalent to those of neutrino absorption for reactions (ix) and (x) as

$$\frac{d\sigma_e^{2H}}{dp_{\nu}} \sim \frac{1}{2} \frac{d\sigma_{e^{2H}}}{dp_e}.$$  

(17)

Factor 1/2 originates from the difference between the spin degrees of freedom of neutrinos and electrons. The energy deposition by the relative motion of two nucleons is negligible because of the small injection energies of leptons.

Nuclei with three bound nucleons, $^3$H and $^3$He, interact with neutrinos via breakup or charge exchange, the latter of which is the dominant neutrino opacity source and is reaction (xiii) or (xiv). The rate of charge exchange in reaction (xiii) is calculated as

$$\frac{1}{\lambda(E_{\nu})} = n_{^3H} \left[ \frac{G_W^2 V_{ud}^2}{\pi (\hbar c)^4} \right] p_e E_e [1 - f_e(E_e)] B(GT),$$

(18)

where $n_{^3H}$ is the triton number density, $B(GT) = 5.87$, and $V_{ud} = 0.967$. All reverse reactions can be evaluated by a detailed balance with the rate of absorption.

The inelastic interactions between neutrinos and other nuclei with $A > 3$ have been neglected in current supernova simulations of the neutrino heating. Haxton [83] pointed out the importance of these reactions as

$$\nu + (N, Z) \rightarrow \nu' + (N, Z)^*,$$

(19)

where $(N, Z)^*$ denotes an excited nucleus. Langanke et al. [84] revealed that the reactions of heavy nuclei with $Z = 24$–28 [85] reduce the high energy tail of the neutrino energy spectrum. The $\alpha$ particles are available around shock waves, as shown subsequently in Fig. 19. Although the effects of the reactions for the $\alpha$ particles on the dynamics of shock waves and neutrino observations are expected to be minor, they should be further investigated [86, 87].

3 Uniform Nuclear Matter

The free energies of the nucleons dripping out of nuclei at a sub-nuclear density or strongly interacting nucleons at a supra-nuclear density are calculated using theories of uniform nuclear matter. In some EOSs, nuclear bulk energies of non-uniform nuclear matter are also provided theoretically.
3.1 Models

In the following sections, we briefly outline some theoretical approaches. We refer the reader to a more comprehensive review [26] of nuclear matter theories. In this review, a general-purpose EOS is named as XX(YY). XX represents the initials of the authors in the original publication of the EOS model, which is discussed in Sec. 4. YY denotes the name of the uniform nuclear matter, which is introduced in this section (i.e., the name of the parameter set such as TM1 and SLy4) or the initials of the theory, such as VM for the variational method.

3.1.1 Skyrme-type interactions

Skyrme-type interactions, which are represented as expansions of the effective interaction in powers of momenta and density-dependent three-body contributions, are the most well-known choice for calculating the effective interaction energy. There are many Skyrme interaction parameter sets, and 240 were compared by Dutra et al. [SS]. The first general-purpose EOS with Skyrme interactions was built by Lattimer and Swesty (LS) [3]. The values of incompressibility, the definition of which is introduced in section 3.2, are set to $K = 180, 220$, and $375$, which are represented as LS $(180, 220, 375)$. Recently, Skyrme SLy4 parameters [89] have been commonly used in studies on NSs and CCSNe, and have been fitted to produce nuclear binding energies and radii (summarized subsequently in Sec. 3.2) as well as pure neutron matter calculated from nucleon-nucleon interaction data [90]. Raduta et al. used these parameters to construct a general-purpose EOS, called RG EOS [6]. Schneider et al. [4, 5] also used SLy4 and other parameter sets for some general-purpose EOSs: SRO (SLy4, APR, NRAPR, SkAPR, LS220, KDE0v1, LNS).

In the Skyrme-type interactions of the SRO EOS, the internal energy density, $\epsilon_B(n_B, x, T)$, with $n_B, x$, and temperature $T$ is expressed as

$$
\epsilon_B(n_B, x, T) = \frac{h^2 \tau_n}{2m_n^*} + \frac{h^2 \tau_p}{2m_p^*} + (a + 4bx(1-x)) n_B^2 + \sum_{j=0,1,2} (c_j + 4d_j x(1-x)) n_B^{1+\delta_j} + (1-x) n_B m_n + x n_B m_p , 
$$

(20)

where $a, b, c_j, d_j$, and $\delta_j$ are parameters of the Skyrme forces, $\tau_n$ and $\tau_p$ are the kinetic energy densities of neutrons and protons, respectively, and $m_n$ and $m_p$ are the masses of neutrons and protons, respectively. The first and second terms correspond to the non-relativistic kinetic energy density of neutrons and protons, respectively, and the third term represents two-body nucleon interactions, and the summation over $j$ approximates the effects of many-body or density-dependent interactions. The last two terms express the rest masses of neutrons and protons, respectively.

The effective masses, $m_n^*$ and $m_p^*$, are given by

$$
\frac{h^2}{2m_n^*} = \frac{h^2}{2m_n} + \alpha_1 n_n + \alpha_2 n_p ,
$$

(21)

$$
\frac{h^2}{2m_p^*} = \frac{h^2}{2m_p} + \alpha_1 n_p + \alpha_2 n_n .
$$

(22)

Parameters $\alpha_1$ and $\alpha_2$ are chosen to reproduce observables of uniform nuclear matter together with $a, b, c_j, d_j$, and $\delta_j$ [SS]. For example, for the SLy4 parameter set, $\alpha_1 = 81.8$ MeV fm$^5$ and $\alpha_2 = 32.5$ MeV fm$^5$. The number density of nucleons, $n_i$ ($i$ denotes neutron $n$ or proton $p$), is expressed as $n_p = x n_B$ and $n_n = (1-x) n_B$. It determines the degeneracy parameters, $\eta_i$, leading to the kinetic terms in Eq. (20). They are expressed as

$$
n_i = \frac{1}{2\pi^2} \left( \frac{2m_i^* T}{h^2} \right)^{\frac{3}{2}} F_{1/2}(\eta_i) ,
$$

(23)
where $F_k$ is the relativistic Fermi integral of order $k$, as expressed in Eq. (14). The entropy density and the free energy density are obtained by

$$
s_B = \frac{1}{n} \sum_{i=\text{n,p}} \left( \frac{5\hbar^2\tau_i}{6m_i^*T} - n_i\eta_i \right),
$$

$$
f_B = \epsilon_B - s_B T.
$$

In the LS EOS, the summation over many-body interactions in Eq. (20) is replaced by a simple term $cn_B^3$ and the effective masses are set as the rest masses; $\alpha_1 = \alpha_2 = 0$.

### 3.1.2 Relativistic mean-field models

The second general-purpose EOS, STOS (TM1), is based on the RMF for the free energies of nucleons [7, 8] with the TM1 parameter set [91]. In the RMF, nuclear interactions are described by the exchange of mesons. The employed Lagrangian is as follows:

$$
\mathcal{L}_{\text{RMF}} = \bar{\psi} [i\gamma_\mu \partial^\mu - M - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \tau_\alpha \rho^{\alpha\mu}] \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 - \frac{1}{4} R_{\mu\nu} R^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^{\mu\nu},
$$

where $\psi$, $\sigma$, $\omega$, and $\rho$ denote nucleons, scalar-isoscalar mesons, vector-isoscalar mesons, and vector-isovector mesons, respectively, and $W_{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu$ and $R_{\mu\nu} = \partial^\mu \rho^{\nu\alpha} - \partial^\nu \rho^{\mu\alpha} + g_\rho \epsilon^{\alpha\beta\gamma} \rho^{\beta\mu} \rho^{\gamma\nu}$. Nucleon-meson interactions are expressed as Yukawa couplings, and isoscalar mesons ($\sigma$ and $\omega$) interact with themselves. $M$ is the mass of nucleons and is assumed to be 938 MeV. In the TM1 parameter set, the masses of mesons—$m_\sigma$, $m_\omega$, and $m_\rho$—and the coupling constants—$g_\sigma$, $g_\omega$, $g_\rho$, $g_2$, $g_3$, and $c_3$—are determined; therefore not only the saturation of uniform nuclear matter but also the properties of finite nuclei can be best reproduced [91].

In the mean field theory, mesons are assumed to be classical and replaced by their ensemble averages. The Dirac equation for nucleons is quantized, and the free energies are evaluated based on their energy spectrum. The nucleon number density is expressed as

$$
n_i = \frac{1}{\pi^2} \int_0^\infty dk \frac{k^2}{2} \left( F_F (\nu_i, k) - F_F (-\nu_i, k) \right),
$$

where $F_F$ is the Fermi distribution function of Eq. (9) and $\nu_i$ is the kinetic part of the chemical potential. The equations of motion of meson fields are written as

$$
\sigma_0 = -\frac{g_\sigma}{m_\sigma^2} \sum_i \frac{\gamma}{2\pi^2} \int_0^\infty dk \frac{M^*}{k^2 + M^*} \left( F_F (\nu_i, k) + F_F (-\nu_i, k) \right) - \frac{1}{m_\sigma^2} \left( g_2 \sigma_0^2 + g_3 \sigma_0^3 \right),
$$

$$
\omega_0 = g_\omega \frac{m_\omega}{m_p} (n_p + n_n) - \frac{c_3}{m_\omega^2} \omega_0,
$$

$$
\rho_0 = g_\rho \frac{m_\rho}{m_p} (n_p - n_n).
$$
The effective mass is defined as $M^* = M + g_\rho \sigma_0$. In RMF calculations, $M^*$ and $\sigma_0$ are solved self-consistently for a given $n_i$. Note that the effective mass in the relativistic framework—Dirac mass—differs from the non-relativistic ones (see Eqs. (21) and (22)). The former is defined using the scalar part of the nucleon self-energy in the Dirac field equation and can only be determined from relativistic approaches, whereas the latter parameterizes the momentum dependence of a single-particle potential [92, 93].

The chemical potentials including nuclear interactions are expressed as
\[
\mu_p = \nu_p + g_\omega \omega + g_\rho \rho, \\
\mu_n = \nu_n + g_\omega \omega - g_\rho \rho.
\] (32)

where $\nu_p$ and $\nu_n$ are given by Eq. (28). The energy density and the entropy density are obtained by
\[
\epsilon_B = \frac{1}{2\pi^2} \sum_{i=0}^{\infty} dk k^2 \sqrt{k^2 + M^*^2} \left( F_F(\nu_i, k) + F_F(-\nu_i, k) \right) + \frac{1}{2} m_\sigma^2 \sigma_i^2 + \frac{1}{3} g_2 \sigma_i^3 + \frac{1}{4} g_3 \sigma_i^4
\] (34)

\[
\frac{1}{2} m_\rho^2 \omega_i^2 - \frac{1}{4} c_3 \omega_i^4
\]

\[
\epsilon_B = \frac{1}{2\pi^2} \sum_{i=0}^{\infty} dk k^2 \left[ -F_F(\nu_i, k) \ln F_F(\nu_i, k) + (1 - F_F(\nu_i, k)) \ln (1 - F_F(\nu_i, k)) \right]
\]

\[
-s_B = \frac{1}{2\pi^2} \sum_{i=0}^{\infty} dk k^2 \left[ \ln F_F(\nu_i, k) - (1 - F_F(-\nu_i, k)) \ln (1 - F_F(-\nu_i, k)) \right].
\] (35)

The HS (TM1) EOS [10] and the FYSS (TM1) EOSs [23] are also constructed using the TM1 parameter set. The RMF with the TM1 parameter set exhibits stiff properties of neutron-rich nuclear matter as listed in Tab. 2 and discussed in Sec. 3.2.

Therefore, Shen et al. [94] updated the parameter set and constructed the STOS (TM1e) EOS by introducing an additional $\omega$-$\rho$ coupling term in the Lagrangian. Hempel et al. adopted many parameter sets in the RMF to construct EOSs: HS (NL3, TMA, DD2, FSUgold, IUF, SFHx, SFHo) [10, 11]. Togashi et al. also formulated the GRDF1 and GRDF2 EOSs using parameter DD2 by introducing density-dependent parameters [16]. Shen et al. developed EOSs with two parameter sets: SHO with FSUgold and SHT with NL3 [12, 13].

### 3.1.3 Variational method

Skyrme-type interactions and RMF are phenomenological medium-dependent interactions. Their model parameters are determined to reproduce some nuclear properties such as the nuclear saturation density. Togashi et al. [17] constructed a general purpose EOS, TNTYST (VM) EOS, based on a realistic two-body nuclear potential, Argonne v18 [95], and a three-body potential, UIX [96, 97], employing the VM [98]. The VM is also used for the FYSS (VM) EOS [18].

Uniform nuclear matter is derived from nuclear potentials supplemented by three-body forces, which reproduce the saturation properties of nuclear matter [98]. The variational calculations of the free energies of free nucleons are based on Refs. [99, 100]. The nuclear Hamiltonian is composed of two-body potentials $V_{ij}$ and three-body potentials $V_{ijk}$, as in the Fermi hypernetted chain variational calculations [90] expressed as follows:
\[
H = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij} + \sum_{i<j<k}^{N} V_{ijk},
\] (36)

where $m$ is set as the mass of a neutron. The free energies derived from two-body interactions are obtained by combining the VM extension of Schmidt and Pandharipande [101, 102] with the AV18
two-body potential \cite{95} and the healing distance condition. The latter reproduces the internal energy per baryon of symmetric nuclear matter and neutron matter at zero temperature of Akmal et al. \cite{90}. The internal energies of three-body interactions are based on the UIX three-body potential \cite{96, 97}. The total free energy per baryon is calculated by minimizing it in relation to the effective masses of nucleons. The optimized free energies agree reasonably with those of Akmal et al. \cite{90} at zero temperature and those of Mukherjee \cite{103} at finite temperature.

### 3.1.4 Dirac Brückner Hartree Fock

The Dirac Brückner Hartree–Fock (DBHF) theory also employs the bare nuclear interaction adjusted to account for nucleon–nucleon scattering data. In contrast to non-relativistic many-body theories with a three-body potential such as the VM EOS, the DBHF theory reproduces nuclear saturation properties starting from two-body forces. In the calculations of the DBHF theory, three integral equations are solved: Bethe–Salpeter equation, single-particle self-energy, and Dyson’s equation. In the DBHF calculation \cite{104}, the Bonn A potential for two-body interactions \cite{105} and the subtracted T-matrix representation \cite{106} are utilized.

In the FYSS (DBHF) EOS, analytical formulas of the interaction energy of homogeneous nuclear matter and the effective masses of nucleons are used, which are fitted to the DBHF calculation in the zero-temperature limit \cite{104}. The free energy of a baryon of homogeneous matter consists of kinetic and interaction parts expressed as

\[
F(n_B, x, T) = F_{\text{kin}}(n_B, x, T) + E_{\text{int}}(n_B, x),
\]

where

\[
F_{\text{kin}}(n_B, x, T) = \frac{2}{\pi^2 n_B} \sum_{i=p,n} \int_0^\infty dk k^2 (F_F(\nu_i, k) + F_F(-\nu_i, k)) \times [E_i^*(k) + (M_i - M_i^*)] - \frac{sT}{n_B},
\]

\[
E_{\text{int}}(n_B, x) = \{4x(1-x)E_{\text{snm}}(n_B) + (1-2x)^2E_{\text{nm}}(n_B)\}/n_B - F_{\text{kin}}(n_B, x, T = 0).
\]

Although this quadratic expression is an approximation, the obtained results reproduce well the exact calculations for asymmetric nuclear matter \cite{104}.

### 3.1.5 Chiral effective field theory

The chiral effective field theory (χEFT) also describes the nuclear interaction microscopically. It is based on the effective chiral Lagrangian, which respects the required symmetries and is expanded in powers of \( q/\Lambda_\chi \). Here, \( q \) denotes a (small) momentum or pion mass, and \( \Lambda_\chi \sim 1 \text{ GeV} \) corresponds to
the scale of chiral symmetry breaking. Pions appear naturally as degrees of freedom at low energies, because they are Goldstone bosons related to the spontaneous breaking of the chiral symmetry in quantum chromodynamics (QCD). The classification of different contributions to the interaction and connection \[109\] are advantages of the \(\chi\)EFT. In addition, at each order, nucleonic contact operators are obtained, which correspond to short-range interactions, whose strength is controlled by low-energy constants (LECs) fitted to experimental data. For example, in a recent calculation \[110\], LECs were fitted to the binding energy and lifetime of a triton. The energies per baryon with the rest mass subtracted, \(\omega(n_B, x)\), is approximately divided between the symmetric nuclear matter energy and the symmetry energy as

\[
\omega(n_B, x) \approx \omega(n_B, 0.5) + \{\omega(n_B, 0) - \omega(n_B, 0.5)\}(1 - 2x)^2. \tag{40}
\]

Their leading terms or derivatives of the wave number at \(n_B = n_0\) are the following parameters: energy per baryon of symmetric nuclear matter \(E_0\), incompressibility \(K_0\), nuclear symmetry energy \(J_0\), and slope of the symmetry energy \(L_0\).

\[
\omega(n_B, 0.5) = E_0 + \frac{K_0}{2} \left(\frac{n_B - n_0}{3n_0}\right)^2 + \ldots, \tag{41}
\]

\[
\omega(n_B, 0) - \omega(n_B, 0.5) = J_0 + L_0 \left(\frac{n_B - n_0}{3n_0}\right) + \ldots. \tag{42}
\]

Measurements of density distributions and nuclear masses \[114\] \[115\] \[116\] \[117\] yield \(n_0 = 0.15 - 0.16\) fm\(^3\) and \(E_0 = -16 \pm 1\) MeV; however, their extractions are ambiguous. \(K_0\) is estimated using experimental data from isoscalar giant monopole resonances in heavy nuclei: \(K_0 = 240 \pm 10\) MeV \[118\], and \(K = 248 \pm 8\) MeV \[119\]. Isospin diffusion measurements, giant and pygmy resonances, isoscaling, isobaric analog states, pion and kaon production, and measurements of the neutron skin thickness in heavy nuclei, can constrain the values of \(S_0\) and \(L_0\). This can also be achieved by astrophysical observations of the radius, mass, and tidal deformability of NSs (see a recent review of NSs and EOSs \[120\]). These data led to some constraints: \(29.0 < J_0 < 32.7\) MeV and \(40.5 < L < 61.9\) MeV \[121\], \(30.2 < J_0 < 33.7\) MeV and \(35 < L < 70\) MeV \[122\], \(25 < L < 50\) MeV \[123\], \(42 < L < 117\) MeV \[124\], and \(J_0 = 38.1 \pm 4.7\) MeV and \(L = 106 \pm 37\) MeV \[125\]. The last constraints are based on a recent experiment on the neutron skin thickness of nuclei \[126\]. The isovector part of nuclear matter is still uncertain, as evidenced by the diversity of the constraints.

The uniform nuclear matter theory in \(\beta\)-equilibrium is also characterized by NS properties. A set of the masses and radii of NSs corresponds to a \(\beta\)-equilibrium EOS. The maximum mass of NSs, \(M_{\text{max}}\), has to be greater than \(2.14^{+1.5}_{-1.0}\) \(M_{\odot}\), which is the mass of one of the heaviest observed pulsars \[127\]. A gravitational wave observation of the NS merger, GW170817 \[128\] \[129\], also estimated that the radius of an NS with \(1.4M_{\odot}\), \(R_{1.4}\) would be smaller than \(12.9\) km \[130\] or \(13.4\) km \[131\], based on observed tidal deformability.
The nuclear saturation properties and NS properties of some EOSs of uniform nuclear matter are summarized in Tab. 2. Most nuclear and NS properties are satisfied using the Skyrme interaction with the SLy4 parameter set and the VM model. Subsequently, they are utilized to construct general-purpose EOSs, such as the SRO, RG, TNTYST, and FYSS EOSs. The symmetric matter of the DBHF EOS is soft, and it yields the highest nuclear saturation density of symmetric nuclear matter, $n_0$. This deep effective potential for $x = 0.5$ (low $E_0$ and large $n_0$) is one of the characteristics of the DBHF theory. For a long time, the TM1 parameter set was used; however, in this decade, it has been discovered that its NS matter is extremely stiff (large values of $J_0$, $L_0$, and $R_{1.4}$). However, a recent experiment may revive it \cite{125,126}, and we cannot conclude which EOS for uniform nuclear matter is the best at present.

KOKO

4 Non-uniform Nuclear Matter

This section reviews non-uniform nuclear matter, which is a mixture of nucleons and nuclei as illustrated in Fig. 7. The total Helmholtz free energy density for all ingredients is expressed as:

$$ f_B = f_{pn} + \sum_{N,Z} f(N,Z), \tag{43} $$

where $f_{pn}$ is the free energy density of the nucleons outside the nuclei and $f(N,Z)$ is that of individual nuclei with neutron number $N$ and proton number $Z$. For uniform nuclear matter, there are no
contributions of nuclei, and \( f_B = f_{pn} \).

### 4.1 Single nucleus approximation

In the first two general EOSs, LS and STOS EOSs, the SNA is assumed; the entire ensemble of nuclei is replaced by a single representative nucleus as illustrated in Fig. 7. The free energy density is expressed as:

\[
    f_B = f_{pn} + f_\alpha + f_{rep},
\]

where \( f_\alpha \) and \( f_{rep} \) are free energy densities of \( \alpha \) particles and representative nuclei, respectively. In SNA EOSs, the ensemble of light nuclei is frequently represented by \( \alpha \) particles.

#### 4.1.1 Compressible liquid drop model

In the LS EOS, the compressible liquid drop model (CLDM) is adopted [3]. The free energy of the representative nucleus is expressed by

\[
    f_{rep} = A n_{rep} \{ \omega(n_B^{in}, x^{in}, T) + F_C(n_B^{in}, x^{in}, u) + F_S(n_B^{in}, x^{in}, u) + F_t(n_B^{in}, u) \} \tag{45}
\]

where \( n_B^{in} \) and \( x^{in}_p \) are the density and proton fraction of the nucleons inside nuclei, respectively. The uniform nuclear matter theory, Skyrme interactions, is used to calculate the bulk energy, \( \omega \). The other Coulomb, surface, and translational energies—\( F_C \), \( F_S \), and \( F_t \)—and the volume fraction in the cell, \( u \), are described in Sec. 4.2. In the calculation, \( n_B^{in}, x_p^{in}, u, \alpha \)-particle density \( n_\alpha \) for \( f_\alpha \), density and charge fraction of dripped nucleons \( n_B^{out} \) and \( x_p^{out} \) for \( f_{pn} \), as well as radius of the cell \( r \) (a total of seven quantities) are optimized to reduce the total free energy density under charge and baryon conservations. The optimization of \( n_B^{in} \) corresponds to compression or decompression of nuclei.

#### 4.1.2 Thomas-Fermi approximation

The STOS and TNTYST EOSs [7, 17] use the Thomas-Fermi (TF) approximation for finite nuclei with dripped nucleons and \( \alpha \) particles [133]. Thus, the TNTYST EOS can be represented as STOS (VM). The Wigner-Seitz cell is assumed to be a sphere whose volume is the same as the unit cell in a body centered cubic lattice. The nucleon distribution in the cell, \( n_{n/p}(r) \), where \( n \) and \( p \) denote neutrons and protons, respectively, is defined as

\[
    n_{n/p}(r) = \begin{cases} 
    (n_{n/p}^{in} - n_{n/p}^{out}) \left[ 1 - \left( \frac{r}{R_{n/p}} \right)^{t_{n/p}} \right]^3 + n_{n/p}^{out}, & 0 \leq r \leq R_{n/p}, \\
    n_{n/p}^{out}, & R_{n/p} \leq r \leq R_{cell}, 
    \end{cases}
\]

where \( r \) is the distance from the center of the nucleus and \( R_{cell} \) denotes the radius of the Wigner-Seitz (WS) cell; \( n_{n/p}^{in} \) and \( n_{n/p}^{out} \) are the densities at \( r = 0 \) and \( r \geq R_{n/p} \); and \( R_{n/p} \) and \( t_{n/p} \) are the boundary and the relative surface thickness of the representative nucleus. The distribution function of \( \alpha \) particles, \( n_\alpha(r) \), is assumed as

\[
    n_\alpha(r) = \begin{cases} 
    -n_\alpha^{out} \left[ 1 - \left( \frac{r}{R_p} \right)^{t_p} \right]^3 + n_\alpha^{out}, & 0 \leq r \leq R_p, \\
    n_\alpha^{out}, & R_p \leq r \leq R_{cell}. 
    \end{cases}
\]

The local free energy density is calculated using the uniform nuclear matter theory (the RMF theory with the TM1 or TM1e parameter set in STOS EOSs or the VM in the TNTYST EOS).
Figure 8: Distributions of neutrons, protons, and $\alpha$ particles in WS cell of STOS EOS [8] at $T = 10$ MeV, $Y_p = 0.3$, and $\rho_B = 10^{13.5}\text{g/cm}^3$ are plotted as solid, dashed, and dot-dashed curves, respectively. To show $\alpha$ particle distribution clearly, it is enlarged by factor of 10.

Figure 8 (Fig. 1 in [8]) shows the distributions of nucleons and $\alpha$ particles at $T = 10$ MeV, $Y_p = 0.3$, and $\rho_B = 10^{13.5}\text{g/cm}^3$. There are two constraints for the given $Y_p$ and $n_B$, which are expressed as follows:

$$n_B = \frac{3}{4\pi R_{\text{cell}}^3} \int_{\text{cell}} [n_n (r) + n_p (r) + 4n_\alpha (r)] \, d^3r , \quad (48)$$

$$Y_p n_B = \frac{3}{4\pi R_{\text{cell}}^3} \int_{\text{cell}} [n_p (r) + 2n_\alpha (r)] \, d^3r . \quad (49)$$

The equilibrium state is obtained by minimizing the free energy density with respect to the ten variables under the two constraints: $R_{\text{cell}}, n_{n}^{\text{in}}, n_{n}^{\text{out}}, R_n, n_{p}^{\text{in}}, n_{p}^{\text{out}}, R_p, t_n, n_{\alpha}^{\text{in}}, n_{\alpha}^{\text{out}},$ and $n_{\alpha}^{\text{out}}$.

Thus, in the SNA, the structure of a representative nucleus in the WS cell is solved to reproduce the minimum free energy density under a given condition. In the other SNA EOS (LS EOS [3]), the thickness of the nuclear surface between $n_{p/n}^{\text{in}}$ and $n_{p/n}^{\text{out}}$ in the cell is not considered, which corresponds to the limit of $t_{n/p} = \infty$ in Eq. (48).

4.2 Nuclear statistical equilibrium and extensions

Nuclear EOSs present some relations among thermodynamical properties, such as the pressure as a function of $\rho_B$, $T$, and $Y_p$. In addition, the composition of nuclear matter (number densities of nucleons
and all nuclei) is determined by the NSE EOS, in which the free energy of a model is minimized using the parameters of the free energy. In the following, first the standard NSE is introduced and subsequently several extended NSE EOSs are discussed. The free energy of nuclei, \( f(N, Z) \), is expressed as

\[
f(N, Z) = n(N, Z) \{ F_g(N, Z) + F_t(N, Z) \}
\]  

where \( n(N, Z) \) represents the nuclear number density, \( F_g(N, Z) \) represents the nuclear gross energy corresponding to the nuclear mass (Secs. 4.2.2.2 and 4.2.2.3), and \( F_t(N, Z) \) represents the translational energy (Sec. 4.2.2.4).

### 4.2.1 Standard NSE

The standard NSE is used in various calculations: the final phase of stellar evolution, which is the initial condition of CCSN simulations, initial condition for nucleosynthesis of the cooling ejecta from supernova explosions or NS mergers, low-density part in hybrid EOSs [4] (see Sec. 4.3), and weak rate data table [134]. As already noted, in the NSE at temperatures above \( T \sim 0.4 \) MeV, nuclear reactions via strong and electromagnetic interactions reach equilibrium as follows:

\[
\begin{align*}
(N, Z) & \leftrightarrow (N, Z - 1) + p , \\
(N, Z) & \leftrightarrow (N - 1, Z) + n .
\end{align*}
\]  

The abundances of nuclei under NSE for a given \( \rho_B, T, \) and \( Y_p \) are obtained by minimizing the model free energy with respect to the many parameters under the two constraints of mass and charge conservations as

\[
\begin{align*}
n_p + n_n + \sum_{N,Z} (N + Z)n(N, Z) & = n_B = \rho_B/m_u, \\
n_p + \sum_{N,Z} Zn(N, Z) & = Y_p n_B ,
\end{align*}
\]

where \( m_u \) is the atomic mass unit. The parameters are \( n(N, Z) \) and number densities of free protons \( n_p \) and neutrons \( n_n \). By introducing Lagrange multipliers \( \alpha \) and \( \beta \) for these constraints, the minimization conditions are expressed as

\[
\begin{align*}
\frac{\partial}{\partial n_p} \{ f_B - \alpha(n_p + n_n + \sum_{N,Z} (N + Z)n(N, Z) - n_B) - \beta(n_p + \sum_{N,Z} Zn(N, Z) - Y_p n_B) \} & = 0, \quad (55) \\
\frac{\partial}{\partial n_n} \{ f_B - \alpha(n_p + n_n + \sum_{N,Z} (N + Z)n(N, Z) - n_B) - \beta(n_p + \sum_{N,Z} Zn(N, Z) - Y_p n_B) \} & = 0. \quad (56)
\end{align*}
\]

The differentials yield the relations of Lagrange multipliers \( \alpha \) and \( \beta \) and the chemical potentials of protons and neutrons as follows:

\[
\begin{align*}
\alpha & = \frac{\partial f_B}{\partial n_n} = \mu_n, \\
\beta & = \frac{\partial f_B}{\partial n_p} - \frac{\partial f}{\partial n_n} = \mu_p - \mu_n.
\end{align*}
\]

In contrast, differentiation with respect to the number densities of nuclei yields the relations between the chemical potentials of nuclei, \( \mu(N, Z) = \partial f_B/\partial n(N, Z) \), and those of protons and neutrons:

\[
\mu(N, Z) = N\mu_n + Z\mu_p.
\]
Here, we assume nucleons and nuclei as ideal Boltzmann gases with constant masses as follows:

\[
    f(N, Z) = n(N, Z) \left[ m(N, Z) + T \ln \left( \frac{n(N, Z)}{g(N, Z)n_Q(N, Z)} \right) - T \right]
\]

(60)

\[
    n_Q(N, Z) = \left( \frac{m(N, Z)k_B T}{2\pi\hbar^2} \right)^{3/2},
\]

(61)

where \( n_Q(N, Z) = \lambda_T(N, Z)^{-3} \) with thermal wavelength \( \lambda_T = \left( \frac{2\pi\hbar^2}{mk_B T} \right)^{1/2} \). When \( n(N, Z) > n_Q(N, Z) \), quantum statistical properties such as degeneracy of Fermi particles or condensation of Bose particles emerge, although such states at high densities and low temperatures do not appear in supernova matter \[135\]. The partial differentials of Eq. (60) with respect to \( n(N, Z) \) and Eq. (59) lead to \( n(N, Z) \) as follows:

\[
    n(N, Z) = g(N, Z) n_Q(N, Z) \exp \left( \frac{Z\mu_p + N\mu_n - m(N, Z)}{T} \right),
\]

(62)

where \( g(N, Z) \) is the degeneracy factor of the nucleus. In the standard NSE EOS, only the spin degree of freedom is considered as \( g(N, Z) = g_0(N, Z) \), because the nuclei are assumed to be unexcited. For \( m(N, Z) \), some experimentally known masses in vacuum \[116\] \[136\] \[137\] are utilized. Eq. (62) is a transformed expression of the Saha equation.

4.2.2 Extended NSE

4.2.2.1 Free energies of dripped nucleons

The extended NSE EOS is complex and \( f_{pn} \) is not simply the free energies of Boltzmann gases, different from the standard NSE EOS. In some extended NSE EOSs such as the HS and FYSS EOSs, the excluded volume is considered in the free energy density of free nucleons, which is expressed as

\[
    f_{pn} = \xi(n'_p + n'_n)\omega(n'_p + n'_n, \frac{n'_p}{n'_p + n'_n}, T),
\]

(63)

where \( \xi = 1 - V_{ex}/V \), \( V \) is the total volume, \( V_{ex} \) is the excluded volume that is occupied by nuclei and expressed as \( V_{ex} = \sum_{N,Z} V_N(N, Z) \), \( V_N \) is the nuclear volume as defined as \( V_N = A/n_s(N, Z, 0) \) with saturation density \( n_s(N, Z, T = 0) \) (explained in Sec. 4.2.2.2), \( n'_{p/n} \) is the local number density of protons and neutrons in the unoccupied volume \((V - V_{ex})\) for nucleons and is defined as \( n'_{p/n} = (N_{p/n})/(V - V_{ex}) \), \( N_{p/n} \) is the number of free protons and neutrons, and \( \omega \) is the free energy per baryon of uniform matter of nucleons. \( \omega \) as a function of \( n_B, Y_p, \) and \( T \) is calculated using a model for uniform nuclear matter. For instance, the models in the FYSS (TM1, VM, DBHF) EOSs \[15\] \[18\] \[19\] are the RMF theory with the TM1 parameter set \[91\], VM \[88\], and DBHF approach \[104\], respectively. Combinations of known uniform and non-uniform matter models are summarized in Table 3.

4.2.2.2 Mass of heavy nuclei

In most EOSs such as the HS and RG EOSs, the nuclear gross energy is not explicitly dependent on the temperature and expressed as an experimentally determined or a theoretically-predicted nuclear mass isolated in vacuum with Coulomb energy shifts.

\[
    F_g(N, Z) = m(N, Z) + \Delta_{Coul}(N, Z),
\]

(64)

\[
    \Delta_{Coul}(N, Z) = \frac{3}{5} \left( \frac{4\pi}{3} \right)^{1/3} e^2 n_0^2 \left( \frac{Z}{A} \right)^2 V_N^{5/3} \left( -\frac{3}{2} u^{1/3} + \frac{1}{2} u \right),
\]

(65)

where \( e \) is the elementary charge and \( u = V_N/V_C \) is the filling factor defined using nuclear volume \( V_N = A/n_0 \) and cell volume \( V_C = Z/n_e \). Temperature dependence of nuclear energy is included in the internal degree of freedom, \( g(N, Z, T) \), which is explained subsequently in Sec. 4.2.2.4.
In the GRDF1 and GRDF2 EOSs [16], the mass shift of heavy nuclei by Pauli blocking is introduced as

\[ F_g(N, Z) = m(N, Z) + \Delta_{\text{Coul}}(N, Z) + \Delta_{\text{Pauli}}(N, Z). \]  

(66)

In the EOSs, the excluded volume effects introduced in Sec. [2.4] are not considered and the dissolution density at which nuclei disappear is determined by a simplified function of a mass number. The dissolution density for \( \Delta_{\text{Pauli}} \) is expressed as \( n_{\text{diss}}(A) = \frac{n_0}{3+28/A} \) in GRDF1 EOS and \( n_{\text{diss}}(A) = \frac{n_0}{2+32/A} \) in GRDF2 EOS, and in both, \( n_{\text{diss}}(4) = n_0/10 \) is obtained.

In the FYSS EOS [19], the nuclear gross energy is dependent on the temperature, and that of heavy nuclei with \( 6 \leq Z \) consist of the bulk, Coulomb, surface, and shell energies as follows:

\[ F_g(N, Z) = F_{\text{bulk}}(N, Z) + F_{\text{Coul}}(N, Z) + F_{\text{surf}}(N, Z) + F_{\text{shell}}(N, Z). \]

(67)

The nuclear bulk energy is evaluated using the same model for dripping nucleons and uniform nuclear matter as \( F_{\text{bulk}}(N, Z) = A \{ \omega (n_s, Z/A, T) \} \), where \( n_s(N, Z, T) \) is defined as the density at which the free energy per baryon, \( \omega (n_B, Z/A, T) \), reaches its local minimum value around \( n_0 \). For nuclei, when experimental or theoretical mass data are available ([115] [138] [116] [117], the shell energies are

\[ \left. \begin{align*}
F_{\text{Coul}}(N, Z) & = \frac{(36\pi)^{1/3}}{5} e^2 n_s^2 \left( \frac{Z}{A} - n_p'/n_s \right)^2 V_{CAZ}^5 D_C(u), \\
D_C(u) & = \frac{u^{5/3}(1-u)^2 D(u) + u^2(1-u)^{5/3} D(1-u)}{u^2 + (1-u)^2 + C_{sp} u^2(1-u)^2}, \\
D(u) & = 1 - \frac{3}{2} u^{1/3} + \frac{1}{2} u, \\
F_{\text{surf}}(N, Z) & = 4\pi \left( \frac{3V_N}{4\pi} \right)^{2/3} \sigma(T, n_n', n_p') D_S(u), \\
D_S(u) & = \frac{u^2(1-u)^{2/3} + u^{2/3}(1-u)^2}{u^2 + (1-u)^2 + C_{sp} u^2(1-u)^2}, \\
\sigma(T, n_n', n_p') & = \sigma_0 \left( \frac{16 + C_{st}}{(1-Z/A)^{-3} + (Z/A)^{-3} + C_{st}} \right) \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} \left( 1 - \frac{n_p' + n_n'}{n_s} \right)^2.
\end{align*} \]

The expressions for \( D_C \) and \( D_S \) asymptotically approach the factors for nuclear droplet and bubble phases. Coefficients \( C_{cp} = -0.863 \) and \( C_{sp} = 4.19 \) are set to reproduce the Coulomb and surface energies of a nuclear slab phase at \( u = 0.5 \). The values of \( \sigma_0 \) and \( C_{st} \) are optimized to minimize the sum of the positive shell energies per baryon in vacuum. For instance, \( (\sigma_0, C_{st}) = (1.01 \text{ MeV/fm}^2, 42.5 \text{ MeV}) \) for the FYSS (VM) EOS. The critical temperature, \( T_c(N, Z) \), is defined as the temperature at which \( \left( \partial P_{\text{bulk}}/\partial n_B \right)|_{x=Z/A} = 0 \) and \( \left( \partial^2 P_{\text{bulk}}/\partial n_B^2 \right)|_{x=Z/A} = 0 \), where \( P_{\text{bulk}} = n_B^2 \partial \omega (n_B, x, T)/\partial n_B \) [141].
Thus, the modeling of the free energy density of nuclei differs from model to model. However, in all above EOSs, the change in the Coulomb energies of nuclei at high densities is included. The in-medium effects during a core collapse [142] are introduced subsequently in Sec. 5.1.

4.2.2.3 Mass of light nuclei The treatment of the gross energy of light nuclei also differs with the EOS. In the HS and RG EOSs, their free energy models are the same as those of heavy nuclei (mass data with Coulomb energy shifts). In the GRDF1 and GRDF2 EOSs [16], Pauli blocking and coupling of nucleons inside the nuclei to meson fields are calculated based on predictions from microscopic quantum statistical calculation [143]. The Pauli energy shifts [132] for deuterons, \(^3\)H, \(^3\)He, and \(\alpha\) particles, which are fitted to the results of quantum statistical calculations [143], are expressed as

\[
\Delta_{\text{Pauli}}(n_{pl}, n_{nl}, T) = -\tilde{n} \left[1 + \frac{\tilde{n}}{2\bar{n}_0(T)}\right] \delta B(T),
\]

(73)

\[
\delta B(T) = \begin{cases} 
    a_1/T^{3/2} \left[1/\sqrt{\eta} - \sqrt{\eta} a_3 \exp (a_3^2 y) \text{erfc} (a_3 \sqrt{\eta}) \right] & \text{for } d, \\
    a_1/(Ty)^{3/2} & \text{for } t, h, \alpha,
\end{cases}
\]

(74)

where \(\tilde{n} = 2(Z n_{pl} + N n_{nl})/A, y = 1 + a_2/T, n_{pl}/n_{nl}\) represents the local proton and neutron number densities that include light nuclei as well as free nucleons, and \(\bar{n}_0(T) = B^0/\delta B(T)\), where \(B^0\) is the binding energy in vacuum.

In the FYSS EOSs, the free energy of light nuclei is based on the Pauli energy shift and the self-energy shift, expressed as

\[ F(Z, N) = M_{\text{data}}(Z, N) + \Delta_{\text{Pauli}} + \Delta_{\text{Self}} + \Delta_{\text{Coul}}. \]

The self-energy shift, \(\Delta_{\text{Self}}\), is the sum of the self-energy shifts of the individual nucleons composing the light nuclei, expressed as

\[ \Delta E_{n/p} = \Sigma_{n/p}(T, n_{p}, n_{n}') - \Sigma_{n/p}(T, n_{p}, n_{n}), \]

where \(\Sigma^0\) and \(\Sigma\) are the vector and scalar potentials of nucleons and the contribution from their effective masses \(\Delta_{\text{eff.mass}} = s (1 - m^*/m_B)\), where \(m^* = m_B - \Sigma_{n/p}(T, n_{p}', n_{n}')\) and \(m_B = 939 \text{ MeV}\):

\[ \Delta_{\text{Self}}(n_{p}', n_{n}', T) = (A - Z) \Delta E_n + Z \Delta E_p + \Delta_{\text{eff.mass}}. \]

(75)

For potentials \(\Sigma^0\) and \(\Sigma\), the parametric formula for the RMF with the DD2 parameter set is utilized (see Eqs. (A1) and (A2) in the Ref. [132]). The self-energy shift for the other light nuclei (\(Z \leq 5\)) are set as zero. The Pauli energy shifts for the other light nuclei are calculated similar as that for \(\alpha\) particles. Parameters \(a_1, a_2, a_3,\) and \(s\) are listed in Table I of Ref. [132].

4.2.2.4 Translational energies of nuclei In the HS, FYSS, and RG EOSs, the translational energies of nuclei are expressed as Boltzmann gases with excluded volume effects as follows:

\[
F_t(N, Z) = T \left\{ \log \left( \frac{n(N, Z)/\kappa}{g(N, Z, T) n_Q(N, Z)} \right) - 1 \right\},
\]

(76)

\[
n_Q(N, Z) = \left( F_g T / 2\pi \hbar^2 \right)^{3/2},
\]

(77)

where \(\kappa = 1 - n_B/n_0\) and \(g(T, N, Z)\) is a factor for internal degrees of freedom. At \(n_B = n_0\), the number density of nuclei, \(n(N, Z)\), becomes zero, as expressed subsequently in Eq. (79).

In the HS and RG, SRO EOSs, the temperature dependence of nuclear free energies is considered in the internal degree of freedom, \(g(Z, N, T)\). In the HS EOSs [10, 11], the following formula [144] is utilized:

\[
g(N, Z, T) = g_A^0 + \frac{c_1}{A^{5/3}} \int_0^{16.2 A} dE e^{-E/T} \exp \left( \sqrt{2a(A)/E} \right),
\]

(78)

where \(a(A) = (A/8)(1 - c_2 A^{-1/3}) \text{MeV}^{-1}, c_1 = 0.2 \text{MeV}^{-1}, c_2 = 0.8,\) and \(g_A^0 = 1\) for even nuclei and \(g_A^0 = 3\) for odd nuclei, and the upper bound of the integral is set as a typical bulk energy, 16.2\(A\). In the
RG EOS, the data table of level densities is employed \[135\]. Partition-function data \[146\] are used in the SRO EOS. Some of these temperature dependences were compared in a systematical study \[147\].

In the FYSS EOS, \( g(T, N, Z) = (g_0(N, Z) - 1) \frac{\tau}{\sinh \tau} + 1 \), where \( \tau = 2\pi^2 T / (41 A^{-1/3}) \) for heavy nuclei \((Z > 5)\). The temperature dependence of introduced in line with the washout of the shell energy, \( F_{\text{shell}} \), introduced in subsection 4.2.2 \[13\]. For light nuclei \((Z < 6)\), the excluded volume effects and the shell washout are not included because \( \kappa = 1 \) and \( g(T, N, Z) = g_0(N, Z) \).

### 4.2.3 Minimization of free energy and thermodynamical properties

The abundances of nuclei as a function of \( \rho_B, T, \) and \( Y_p \) is obtained by minimizing the derived model free energy with respect to the number densities of nuclei and nucleons under the constraints of baryon and charge conservations, as expressed in Eqs. (53) and (54). Differentiating the free energy for nuclei with respect to the number densities of nuclei, \( n(Z, N) \), they can be expressed as follows:

\[
n(Z, N) = \kappa g(Z, N, T) n_q(Z, N) \exp \left( \frac{\mu(Z, N) - F_g(Z, N)}{k_B T} \right) .
\] (79)

The chemical potential of nuclei are calculated using those of nucleons, \( \mu(Z, N) = Z \mu_p + N \mu_n \).

In most general-purpose EOSs, the number densities of nuclei depend only on \( \mu_p \) and \( \mu_n \). Two conservation equations, i.e., Eqs. (53) and (54), can be solved for \( \mu_p \) and \( \mu_n \) similar to the standard NSE discussed in Sec. 4.2.2.1. In the FYSS EOS, the gross energies of nuclei, \( F_g \), depend on the number densities of dripped nucleons, \( n'_p \) and \( n'_n \), and, hence, the number densities of nuclei depend on \( \mu_p, \mu_n, n'_p \) and \( n'_n \). Their relations are as follows:

\[
\mu_p = \frac{\partial f_B}{\partial n_p} = \mu'_p(n'_p, n'_n, T) + \sum_{N, Z} n(Z, N) \frac{\partial F_g(Z, A)}{\partial n_p} ,
\] (80)

\[
\mu_n = \frac{\partial f_B}{\partial n_n} = \mu'_n(n'_p, n'_n, T) + \sum_{N, Z} n(Z, N) \frac{\partial F_g(Z, A)}{\partial n_n} ,
\] (81)

where \( \mu'_p \) and \( \mu'_n \) are the chemical potentials of nucleons in a vapor of volume \( V' \), which are obtained from uniform matter calculation, \( \omega \). The number densities of the nucleons in the total volume, \( n_p \) and \( n_n \), are different from local number densities \( n'_p \) and \( n'_n \) as defined in Sec. 4.2.2.1. The second term originates from the dependences of \( F_g(N, Z) \) on \( n'_p \) and \( n'_n \) and is not included in the standard NSE EOS and other general-purpose EOSs. In the FYSS EOS, not only the two conservation equations, i.e., Eqs. (53) and (54), but also the two chemical potential in Eqs. (80) and (81) must be solved for the four variables: \( \mu_p, \mu_n, n'_p \) and \( n'_n \).

After minimization, the free energy density is obtained with the abundances of various nuclei and free nucleons as a function of \( \rho_B, T, \) and \( Y_p \). Once the free energy density, \( f_B \), is obtained, other physical properties are derived by its partial differentiation. The baryon pressure and the baryon entropy density are calculated using the following thermodynamic relations:

\[
\rho_B = \left[ n_B^2 \frac{\partial}{\partial n_B} \left( \frac{f_B}{n_B} \right) \right]_{T, Y_p} ,
\] (82)

\[
s_B = - \left( \frac{\partial f_B}{\partial T} \right)_{\rho_B, Y_p} .
\] (83)

The baryon internal energy density is calculated using the following thermodynamical relation:

\[
\epsilon_B = f_B - T s_B .
\] (84)
In some EOSs, the entropy and the internal energy are calculated and Eq. (84) yields the free energy. The consistency of the thermodynamic quantities is used to evaluate the numerical calculations.

$$\frac{f_B}{n_B} = \mu_n (1 - Y_p) + \mu_p Y_p - \frac{p}{n_B}.$$  (85)

The sound velocity of supernova matter is obtained using

$$c_s = \sqrt{\left( \frac{\partial p}{\partial \rho} \right)_{s,Y_p}} = \sqrt{\Gamma \rho},$$  (86)

where $p$ is the total pressure, $\rho$ is the total density, and $s$ is the total entropy. The included contributions of electrons are comparable to the baryonic part, particularly in the core-collapse phase. Although the sound speed must be lower than the light speed, some EOSs violate this condition at extremely high densities because of strong repulsion. The adiabatic index, $\Gamma$, which characterizes the stiffness of supernova matter is calculated as

$$\Gamma = \left( \frac{\partial \ln p}{\partial \ln \rho} \right)_{s,Y_p}.$$  (87)

### 4.3 Hybrid models

In the SHO and SHT EOSs, two different EOSs—NSE at low densities and SNA with the Hartree approximation at high densities—are employed. The critical density at which the two calculations are switched is determined by the free energy densities as

$$f_B = \min(f_{\text{SNA}}, f_{\text{NSE}}),$$  (88)

where $f_{\text{SNA}}$ and $f_{\text{NSE}}$ are the free energy densities of the EOSs based on the SNA and the NSE, respectively. The NSE shows the lower free energy densities at low densities and vice versa at high densities. In the SNA approach, the density distribution in a spherical cell is optimized without any assumptions about the nuclear shape. Thus, the description of the transition from non-uniform nuclear matter to uniform nuclear matter is the most complex part in general-purpose EOSs. In a spherical symmetry, some nuclear structures such as slab and rod phases cannot be obtained. However, a spherical shell structure is observed at intermediate densities between spherical and bubble phases, which is similar to the slab phase of nuclear pastas.

In the SRO EOSs, the CLDM model of the LS EOS (Sec. 4.) is utilized at high densities. The free energy densities of the SNA and EOSs at low densities are combined using a density-dependent function as

$$f_B = \chi(n_B)f_{\text{SNA}} + \left[1 - \chi(n_B)\right]f_{\text{NSE}},$$  (89)

$$\chi(n_B) = \frac{1}{2} \left[1 + \tanh \left( \frac{\log_{10}(n_B) - \log_{10}(n_t)}{n_\delta} \right) \right],$$  (90)

where the center of the transition is set $n_t = 10^{-4} \text{ fm}^{-3}$, and its dimensionless width is defined as $n_\delta = 0.33$. This approach ensure a more smooth connection between the NSE and SNA EOSs than that between the SHO and SRO EOSs.

### 4.4 Ambiguities in supernova EOSs

In the last two decades, various EOSs for supernova simulations have been formulated, which are listed in Tab. 3. Uniform nuclear matter theories, which are compared in Sec. 3. are one of their characteristics.
Table 3: Bulk properties of nuclear matter obtained using EOSs for uniform nuclear matter based on RMF theory using TM1 parameter set [91, 9], VM [98, 17], and DBHF [104].

The presence of exotic hadrons such as hyperons and quark phase is also an open question, which is discussed subsequently in Sec. 5.3.3.

For non-uniform nuclear matter, there is a major difference between the SNA and NSE EOSs. In terms of thermodynamical properties, this difference is insignificant; however, the nuclear composition, which determines weak interaction rates, differs [20, 149]. Hence, in the CCSN simulations, the weak interactions of nuclei should be calculated using an NSE EOS. At least, some weak rate data based on an NSE [134] should be employed, when the SNA EOS is used in CCSN simulations.

Coulomb energy density dependences are almost universal in NSE EOSs. Other energy shifts caused by the dense environment may affect the EOS only in the final stages of core collapses of massive stars with densities exceeding $\rho_B \sim 10^{13}$ g/cm$^3$ [19]. However, temperature dependence of nuclear energy such as the shell washout and the internal degree of freedom, exhibit different nuclear compositions and entropies even in the initial phase of the core-collapse at $T = 1$ MeV [147].

Transition to uniform nuclear matter is also a major task for the construction of the complete supernova EOS; however, its details may have little impact on the dynamics of CCSNe, owing to the narrow density region of the transition. In the FYSS and LS EOSs, nuclear bubble phases are based on liquid drop models, in which a nuclear structure is determined by simple functions of volume fraction $u$ in spherical cells. In the SHO and SHT EOSs, the structures of nuclear pastas with a spherical symmetry (droplet, spherical shell, or bubble phase) are obtained by optimization of the density distribution in a spherical cell. Although nuclear pastas are not considered, nuclei are diminished by the excluded volume effects in the HS and RG EOSs and by the Pauli energy shifts in the GRDF1 and GRDF2 EOSs [16]. More detailed studies such as a molecular dynamics simulations [150] and 3D finite-temperature Skyrme-Hartree-Fock studies [151], will provide more insight into the issue.

Regarding combinations of the SNA and NSE EOSs, some self-consistent approaches can describe both the ensemble of various nuclei and individual nuclear properties [152, 149, 141]. However, they necessitate significant computing power and are unsuitable for building a general-purpose EOS that can deal with a wide range of thermodynamic conditions. Readers are referred to some papers comparing these EOSs [23, 153, 147, 27, 154].
Figure 9: Mass fractions of nuclei in log$_{10}$ on $(N, Z)$ plane for $(\rho_B, T, Y_p) = (2.0 \times 10^{10} \text{ g/cm}^3, 0.64 \text{ MeV}, 0.42)$.

5 Nuclei in supernova simulations

In this section, we discuss the distribution of the nuclei that were included in the supernova simulation \cite{38} of an 11.2 $M_\odot$ progenitor using the FYSS (VM) EOS \cite{18}.

5.1 Core-collapse phase

Snapshots of the nuclear mass fractions in the $(N, Z)$ plane at the center of the core-collapse simulation \cite{38} at $(\rho_B, T, Y_p) = (2.0 \times 10^{10} \text{ g/cm}^3, 0.64 \text{ MeV}, 0.42)$, at $(2.0 \times 10^{11} \text{ g/cm}^3, 1.1 \text{ MeV}, 0.34)$ and at $(2.0 \times 10^{12} \text{ g/cm}^3, 1.7 \text{ MeV}, 0.27)$ are shown in Figs. 9\textendash 11. The mass fraction, $X(N, Z)$, is defined as

$$X(N, Z) = \frac{A_n}{n_B}, \quad (91)$$

The initial state of the core collapse corresponds to the first one, in which the iron-group nuclei are most abundant. As the supernova matter becomes denser, electrons weaken the Coulomb repulsion between the protons inside the heavy nuclei, in turn, producing nuclei with larger mass numbers. In addition, the reduction in $Y_p$ increases the chemical potential difference, $\mu_n - \mu_p$, and, therefore, neutron-rich nuclei become abundant. At $\rho_B = 10^{11}\text{--}10^{12} \text{ g/cm}^3$, large electron-type neutrinos are emitted during the core collapse. Nuclei are abundant near the neutron magic numbers $(N = 28, 50, 82)$ such as $^{78}\text{Ni}$, $^{80}\text{Zn}$, and $^{122}\text{Zr}$. At $T = 1.7 \text{ MeV}$, the nuclear shell effects are partially reduced and non-magic nuclei become as abundant as magic ones \cite{147}. These nuclei with $Z = 20\text{--}50$ and $N = 40\text{--}90$ that emerge at densities $\rho_B = 10^{11}\text{--}10^{12}\text{ g/cm}^3$ are critical for determining the weak interaction rate and the subsequent core dynamics \cite{64,147}.

Figures 12 show the neutron mass fraction, $n_n/n_B$, and the total mass fractions of heavy $(Z \geq 6)$ and light nuclei $(Z \leq 5)$ as functions of the central density, which are calculated as $\sum_{Z \geq 6} X(N, Z)$ and $\sum_{Z \leq 5} X(N, Z)$, respectively. At the beginning of the core collapse, heavy nuclei are dominant in abundance. When $\rho_B$ increases and $Y_p$ decreases, neutrons start to drip. For comparison, those for...
the TNTYST (VM), HS (DD2), and HS (SFHx) EOSs under the same thermodynamical conditions \((\rho_B, T, Y_p)\) are displayed. However, this comparison is only perturbative. In the supernova simulations using the TNTYST (VM) or HS EOS, the thermodynamical conditions deviate from those using the FYSS (VM) EOS. Comparisons of different EOSs in supernova simulations are discussed in Sec. 5.3.

In the entire core-collapse phase, heavy nuclei practically dominate. The TNTYST (VM) EOS yields more dripped neutrons than the FYSS (VM) EOS, owing to the SNA. In the TNTYST (VM) EOS, there are fewer light nuclei (\(\alpha\) particles) than in other EOSs above \(\rho_B \sim 10^{12} \text{ g/cm}^3\), because of the lack of deuterons and \(^3\text{H}\). The differences in modeling the free energies of light and heavy nuclei causes a gap between the FYSS and HS EOSs.

The average mass numbers and proton numbers of heavy nuclei at the center of the core are shown in Fig. 13, which are expressed as

\[
\bar{A} = \frac{\sum_{Z \geq 6} A n(N, Z)}{\sum_{Z \geq 6} n(N, Z)},
\]

\[
\bar{Z} = \frac{\sum_{Z \geq 6} Z n(N, Z)}{\sum_{Z \geq 6} n(N, Z)}.
\]

The shell effect causes a stepwise growth, because nuclei with neutron magic numbers \(N = 28, 50,\) and 82 \((A \sim 60, 80,\) and 120) are stable in the HS EOSs. the FYSS (VM) EOS is essentially identical to the HS EOS models at low densities, because both are similar to the standard NSE EOS, which is introduced in Sec. 4.2.1. The mass and proton numbers from the FYSS (VM) EOS reduce above \(\rho_b \sim 10^{12} \text{ g/cm}^3\), because the reductions in shell and surface energies are more critical for nuclei with smaller mass numbers, as explained subsequently. The FYSS and TNTYST EOSs show significant differences in the nuclear composition, even with the same nuclear interaction for uniform nuclear matter (VM). By contrast, the HS (DD2 and SHFx) EOSs yield similar results owing to the same nuclear model.
Figure 11: Mass fractions of nuclei in log_{10} on (N, Z) plane for (\rho_B, T, Y_p) = (2.0 \times 10^{12} \text{ g/cm}^3, 1.7 \text{ MeV}, 0.27). The thermodynamical condition is available at the center of a collapsing core in the supernova simulation [38] using FYSS (VM) EOS.

The nuclear abundance during a core collapse is sensitive to the gross energy, \( F_g(N, Z) \). Figure 14 shows the average energies of nuclei and each contributions of the energy shifts as functions of the nuclear mass number at (\rho_B, T, Y_p) = (2.0 \times 10^{10} \text{ g/cm}^3, 0.64 \text{ MeV}, 0.42) and (2.0 \times 10^{12} \text{ g/cm}^3, 1.7 \text{ MeV}, 0.27). For example, the average gross and Coulomb energies are defined as

\[
\bar{F}_g(A) = \frac{\sum_{Z+N=A} F_g(Z, N)n(Z, N)}{\sum_{Z+N=A} n(Z, N)},
\]

\[
\bar{F}_{\text{Coul}}(A) = \frac{\sum_{Z+N=A} F_{\text{Coul}}(Z, N)n(Z, N)}{\sum_{Z+N=A} n(Z, N)}.
\]

The mass number at which the average gross energy is the lowest increases at a high density owing to the Coulomb energy reductions. Because the density increases during a core collapse, large nuclei can form. In contrast, the shell and surface energies are reduced as the temperature rises, making nuclei with small mass numbers more stable [15, 147]. As \( Y_p \) decreases, the average bulk and gross energies increases owing to the high neutron richness in the nuclei (the low value of \( Z/A \)).

Fig. 15 shows the absolute values of the baryonic pressure. In the initial phases, the pressure is negative, because the Coulomb energies of heavy nuclei decrease under the compression [23]. In addition, near narrow density region \( \rho_B \sim 10^{14} \text{ g/cm}^3 \), the EOSs other than the FYSS (VM) EOS yield the negative pressures. Note that a positive leptonic pressure is much larger than a baryonic one, below the nuclear saturation density. Hence, the total pressure never becomes negative. At \( \rho_B \sim 10^{13} - 10^{14} \text{ g/cm}^3 \), in the TNTYST (VM) EOS, the thermal pressures of nuclei and the baryonic pressures may be underestimated owing to the SNA, in which translational motions of various nuclei are not included. Above \( \rho_B \sim 10^{14} \text{ g/cm}^3 \), the HS (DD2) EOS yields larger baryon pressures owing to its stiffer bulk properties.

Baryonic entropies are sensitive to the temperature dependence of nuclear free energies [147]. In actual, there are difference between the entropies obtained with different EOSs, as shown in Fig. 16. In the FYSS (VM) EOS, heavy nuclei with small mass numbers are more abundant than in the other
Figure 12: Mass fractions of heavy nuclei (thin lines), light nuclei (middle lines), and neutrons (thin lines) for FYSS (VM) (black solid lines), TNTYST (VM) (red dashed lines), HS (DD2) (blue dotted lines), and HS (SHFx) (green dashed-dotted lines) EOSs at \((\rho_B, T, Y_p)\) of Fig. 4 in center of the collapsing core of the supernova simulation [38] using FYSS (VM) EOS [18].

Figure 13: Average mass numbers (thick lines) and average proton numbers (thin lines) of heavy nuclei for FYSS (VM) (black solid lines), TNTYST (VM) (red dashed lines), HS (DD2) (blue dotted lines), and HS (SHFx) (green dashed-dotted lines) EOSs at \((\rho_B, T, Y_p)\) of Fig. 4 in center of the collapsing core of the supernova simulation [38] using FYSS (VM) EOS [18].
Figure 14: Nuclear gross energy per baryon (black solid lines) and shell (cyan dashed lines), bulk (blue dotted dashed lines), surface (green double-dotted dashed lines), and Coulomb (red dotted lines) energies at \((\rho_B, T, Y_p) = (2.0 \times 10^{10} \text{ g/cm}^3, 0.64 \text{ MeV}, 0.42)\) (thin lines) and \((2.0 \times 10^{12} \text{ g/cm}^3, 1.7 \text{ MeV}, 0.27)\) (thick lines). The thermodynamical condition is available at the center of a collapsing core in the supernova simulation [38] using FYSS (VM) EOS. Rest masses are subtracted from gross and bulk energies. They are averaged over the nuclides with same mass numbers. Magenta dotted vertical lines display mass numbers of heavy nuclei averaged over entire ensemble of nuclei.

EOSs, because of the shell and surface energy washouts. Hence, the thermal entropy is larger in the former than in the other EOSs and, in addition, the entropy due to the temperature dependence of the nuclear gross energies contributes. The TNTYST (VM) EOS yields the lowest baryonic entropy, which may also be owing to the SNA.

Nuclear excited states are considered in various formulations in the EOSs [155]. The effect of excited states can be expressed as a partition function as

\[ g_{\text{exp}}(T) = \sum_i g_i \left(1 + \frac{M_0}{\Delta E_i} \right)^{3/2} \exp \left(-\frac{\Delta E_i}{T} \right), \]

where \(i\) denotes the sum over all known states. The average excitation energies, \(\Delta E\), in the degeneracy factor, \(g(T)\), similar to Eq. (78), is expressed as

\[ \Delta E = \frac{\partial g\ T^2}{\partial T\ g}. \]

The nuclear excitation energies per baryon, \(\Delta E/A\), approximately 0.03–0.06 MeV at \(T = 0.86\) MeV and around 2–6 MeV at \(T = 8.6\) MeV, depending on the nucleus and on the NSE EOS model [155]. In the
Figure 15: Absolute value of baryonic pressure using FYSS (VM) (black solid lines), TNTYST (VM) (red dashed lines), HS (DD2) (blue dotted lines), and HS (SHFx) (green dashed-dotted lines) EOSs at $(\rho_B, T, Y_p)$ of Fig. 4 in center of the collapsing core of the supernova simulation [38] using FYSS (VM) EOS [18]. Sign changes occur from positive to negative at $\rho_B \approx 2.0 \times 10^{11} \text{g/cm}^3$ and from negative to positive at approximately $\rho_B \approx 10^{14} \text{g/cm}^3$.

Figure 16: Entropies per baryon from FYSS (VM) (black solid lines), TNTYST (VM) (red dashed lines), HS (DD2) (blue dotted lines), and HS (SHFx) (green dashed-dotted lines) EOSs at $(\rho_B, T, Y_p)$ of Fig. 4 in center of the collapsing core of the supernova simulation [38] using FYSS (VM) EOS [18].
supernova simulations, the nuclei may be around 0.03–6 MeV per baryon excited during a core collapse. Nevertheless, the excitation of each nucleus in supernova matter, which is related to the temperature dependence in \( f(N, Z) \), is inadequately investigated.

### 5.2 Shock revival phase

After the core bounce, shock waves are generated, and can be seen by jumps in some properties such as the temperature \( (T \sim 1 \text{ MeV}) \), density \( (\rho_B \sim 10^9 \text{ g/cm}^3) \), and average nucleon number \( (\langle A \rangle \sim 10) \), which are shown in Figs. 5 and 6. The average nucleon number for all baryons is defined as \( \langle A \rangle = \sum_k A_k n_k/n_B \), where index \( k \) denotes a nucleon or a nucleus, \( A_k \) is the nucleon number of the particle (e.g., \( A_n = A_p = 1, A_A = 2, A_\alpha = 4, A_{^{56}\text{Fe}} = 56 \)), and \( n_k \) is the number density of the nucleons or nuclei. The distributions of the shock waves are close to spherical at \( t = 100 \text{ ms} \) after the core bounce. However, deformed shock wave are observed at \( t = 200 \text{ ms} \). In some areas—\((x, z) \sim (\pm 100 \text{ km}, 80 \text{ km}) \) at \( t = 100 \text{ ms} \), and \((x, z) \sim (0 \text{ km}, \pm 200 \text{ km}) \) at \( t = 200 \text{ ms} \)—inward flows with low entropies and large values of \( \langle A \rangle \) and \( Y_p \) occur.

Figures 17 and 19 show the distributions of the mass fractions of nucleons and nuclei in the shock expanding phase at \( t = 100 \text{ and } 200 \text{ ms} \) of the supernova simulation [18]. The central region is composed of interacting nucleons. The minimum, angle-averaged, and maximum shock radii are approximately 220, 250 and 290 km at \( t = 100 \text{ ms} \), and approximately 250, 310 and 550 km at \( t = 200 \text{ ms} \), respectively. In shocked matter, nucleons and light nuclei are dominant. Accreting matter consists of iron-group nuclei and \( \alpha \) particles.

As a temperature rise is caused by shock heating, the entropy contribution, \( S \), to the free energy, \( F = U - TS \), increases, relative to the internal energy, \( U \). Nucleons and light nuclei have larger entropies per baryon than heavy nuclei, and, hence, their number densities increase to lower the total free energy, even if their internal energies per baryon is larger than those of heavy nuclei. Along with inward flows, heavy nuclei and \( \alpha \) particles have large mass fractions, where nucleons and deuterons have small mass fractions. Around shock waves, \( \alpha \) particles are abundant, because of their large binding energies. In contrast, deuterons are populated just above the surface of the produced PNS. The mass fractions of nuclei with \( A = 3, ^3\text{H}, \) and \(^3\text{He} \) are also large around the PNS surface.

As shown in Fig. 20, various heavy nuclei emerge around a shock wave; however their mass and proton numbers are small. Even in shocked matter, not all iron-group nuclei are dissolved into nucleons and light nuclei, specifically at \( t = 200 \text{ ms} \) (e.g., \( x \sim 0 \text{ km} \) and \( z \sim 200 \text{ and } 300 \text{ km} \) at the polar angle of \( \theta = 0 \)), because they have inward flows with low entropies. Fig. 21 shows the radial profiles of the mass fractions of nucleons and some nuclei at some polar angles \( \theta = 0 (x = 0 \text{ and } z \geq 0), \frac{1}{3} \pi, \frac{2}{3} \pi, \), and \( \pi (x = 0 \text{ and } z \leq 0) \). The radial distributions of the nuclear compositions differ with the angle, because the shock wave is deformed. Above the shock wave, heavy nuclei are dominant, whereas below it, \( \alpha \) particles are the most abundant. Particularly, in the low-entropy flows (\( \theta = 0 \text{ or } \pi \) and \( r \geq 200 \text{ km} \)), \( \alpha \) particles dominate. The radial profiles around the PNS (\( r < 50 \text{ km} \)) are similar to each other; however, the mass fractions of nucleons, deuterons, and \(^3\text{He} \) are large.

Figs. 22 and 23 show the radial profiles of the mass fractions of light nuclei and the baryonic entropies at polar angle \( \theta = \pi \), respectively. Those for the TNTYST (VM), HS (DD2), and HS (SFx) EOSs under the same thermodynamical conditions \( (\rho_B, T, Y_p) \) are also presented. The FYSS and HS EOSs present similar results, because the nuclear masses of light nuclei are similar. At \( r \sim 10 \text{ km} \), a slight difference is due to the in-medium effects. In the TNTYST EOS, the lack of deuterons and tritons decreases the light nuclei mass fractions and baryonic entropies at \( r < 50 \text{ km} \), whereas the underestimation of the mass fractions of heavy nuclei [20, 21] increases them around and above shock radius \( r \sim 330 \text{ km} \).

As discussed in some studies [25, 86, 156] and Sec. 2.4, light nuclei are never dominant targets in neutrino emission and absorption. However, their emergence reduces the number of nucleons, which
Figure 17: Distributions of free proton fractions (left) and free neutron fractions (right) at 100 ms (top panel) and 200 ms (bottom panel) after core bounce in the supernova simulation [38] using FYSS (VM) EOS [18].
Figure 18: Distributions of mass fractions for light nuclei (left) and heavy nuclei (right) at 100 ms (top panel) and 200 ms (bottom panel) after core bounce in the supernova simulation [38] using FYSS (VM) EOS [18].
Figure 19: Distributions of deuteron mass fractions (left) and $\alpha$ particle mass fractions (right) at 100 ms (top panel) and 200 ms (bottom panel) after core bounce in the supernova simulation [38] using FYSS (VM) EOS [18].
Figure 20: Distributions of average proton numbers (left) and average mass numbers (right) for heavy nuclei \((Z \geq 6)\) at 100 ms (top panel) and 200 ms (bottom panel) after core bounce in the supernova simulation [38] using FYSS (VM) EOS [18].
Figure 21: Mass fractions for $A = 1$: protons and neutrons (blue solid lines), $A = 2$: deuterons (green dashed lines), $A = 3$: $^3$H and $^3$He (red dotted lines), $A = 4$: $\alpha$ particles (magenta dotted dashed lines), and $A > 4$: other nuclei (black double-dotted dashed lines) at time after the core bounce, $t_b = 200$ ms and in polar angles of $\theta = 0$ (left upper panel), $\theta = \frac{1}{3}\pi$ (right upper panel), $\theta = \frac{2}{3}\pi$ (left bottom panel), and $\theta = \pi$ (right bottom panel) in the supernova simulation [38] using FYSS (VM) EOS [18].

are dominant neutrino heating and cooling sources. In addition, deuteron heating and cooling reactions such as reactions (ix) and (x) are non-negligible, and their contribution to the total neutrino reaction rate is up to $\sim 10\%$ depending on the time and the location [86, 25]. The heating of the matter around an expanding shock wave may also be influenced by the neutrino absorption and the inelastic scattering of $\alpha$ particles [86, 87].

5.3 Effects of EOSs on simulations

Many EOSs have been adopted in astrophysical calculations. In this review, we introduce some studies related to CCSNe.

5.3.1 Uniform nuclear matter

Several EOSs used in CCSN simulations have been compared, in CCSN simulations and it has been found that soft EOSs are preferred for neutrino-driven supernova explosions, e.g. [157, 158]. Sumiyoshi et al. [24] compared the LS (220) and STOS (TM1) EOSs in 1D simulations. The lower $J_0$ of the LS
Figure 22: Mass fractions of light nuclei using FYSS (VM) (black solid lines), TNTYST (VM) (red dashed lines), HS (DD2) (blue dotted lines), and HS (SHFx) (green dashed-dotted lines) EOSs for thermodynamical conditions at polar angle $\theta = \pi$ in the supernova simulation [38] using FYSS (VM) EOS [18].

Figure 23: Baryonic entropy using FYSS (VM) (black solid lines), TNTYST (VM) (red dashed lines), HS (DD2) (blue dotted lines), and HS (SHFx) (green dashed-dotted lines) EOSs for thermodynamical conditions at polar angle $\theta = \pi$ in the supernova simulation [38] using FYSS (VM) EOS [18].
EOS allows for more protons to be dripped than that by the STOS EOS. The dripped protons capture electrons, and thus, the decrease in electron degenerate pressure causes the mass of the PNS to be approximately 0.1 \( M_\odot \) lower than that from the STOS (TM1) EOS. The smaller mass of a PNS leads to smaller kinetic energy of the shock waves at bounce, which is negative for shock revival. However, the central density of the PNS in a simulation using the LS EOS is higher than that from the STOS EOS, because of the softness (low values of \( K_0 \), \( J_0 \), and \( L_0 \)) in the LS EOS. The higher luminosity and higher average energy of the neutrinos due to the higher central densities of the PNS are advantages for shock revival.

Suwa et al. \[159\] shows similar results for 2D simulations by comparing the LS (180, 375) and STOS (TM1) EOSs. The simulation using the STOS (TM1) EOS did not reproduce a shock revival. The electron antineutrino luminosity in the LS EOS models was high, and the mass enclosed within the gain region was large. In addition, they also found more aspherical downward flows of the accretion matter on the PNS surface with the model using the LS EOS than with those with the STOS EOS. In other 2D simulations comparing the LS (220) and FYSS (TM1) EOSs, a PNS calculated by the softer EOS (LS EOS) shrinks faster and emits more neutrinos, leading to a larger radius of the shock wave \[160\] [161]. Fully relativistic 3D CCSN simulations were performed using the HS (TM1, DD2, and SFHx) EOSs \[162\]. They showed that in supernova models with softer EOS, the development of the SASI is more active, affecting the gravitational wave observations.

The SRO EOSs have been systematically compared in supernova simulations using approximately 100 parameter sets \[3\]. Using them in 1D simulations were implemented, whereas six 3D simulations were conducted using some selected parameters. It was found that the effective masses of nucleons, i.e., \( m_p^* \) and \( m_n^* \), in Eq. (20) at \( n_B \geq n_0 \) are the most influential uncertainties in Skyrme-type EOSs affecting neutrino emission and dynamics. The peak frequency of the gravitational waves is also sensitive to the effective masses, which is related to the contraction of the PNS \[163\]. It has also been pointed out that PNS convection and supernova explosion are dependent on the entropy of the PNS, which is determined by the EOS \[164\].

Neutrino signals from the PNS cooling after a shock revival are also dependent on the EOS \[165\]. It is known that the time between a core-bounce and the formation of a black hole is short for a soft EOS \[166\], \[167\]. Gravitational wave signals and the possibility of a shock revival along with the collapse of the PNS to a black hole are also dependent on the EOS \[168\].

It is noted that the thermodynamical conditions of supernova matter (see Fig. 3) are quite different of NS matter. NS properties are sensitive to the EOS at \( Y_p \sim 0.1 \) and \( T = 0 \) MeV. Supernova matter has a wider range of thermodynamic conditions than NS matter: \( 0.1 < Y_p < 0.5 \) and \( T > 0.5 \) MeV, as shown in Figs. 3, 5, and 6. In addition, the effective mass, which determines the dynamics of CCSN simulations, does not play a key role in NS properties \[5\]. Therefore, a general-purpose EOS has many aspect as functions of \( \rho_B \), \( T \), and \( Y_p \). Regarding the constraints on NS matter, simulations and observations of neutron star mergers \[128\], \[129\] are more promising than those of CCSNe introduced in Sec. 3.2.

### 5.3.2 Non-uniform nuclear matter

Hempel et al. \[169\] performed 1D supernova simulations using the STOS (TM1), LS (180, 220), and HS (TMA, FSUgold). The differences between the nuclear interactions (180 and 220, or TMA and FSUgold) are smaller than the differences between the nuclear models (STOS, LS and HS). In 2D simulations \[159\] comparing the LS (180, 375) and STOS (TM1) EOSs, similar results were obtained. Despite the large difference in the incompressibility, both 2D simulations of a15 \( M_\odot \) progenitor using the LS (180) and LS (375) EOSs show shock revival after a shock stall. In a simple analysis \[170\], the chemical potentials of protons and neutrons, which are directly related to the weak interaction rates around shock waves, are more sensitive to non-uniform nuclear matter than to uniform nuclear matter.
Nagakura et al. [25] systematically compared not only the EOSs but also the weak interaction consistencies with the EOSs, and found that the weak rate consistency affects the structure of the PNS and the time evolution of shock-waves to the same degree as the EOS difference. The weak rate difference originating from the nuclear composition of the EOSs is quite large. For example, in an early phase of the core collapse, electron capture rates of heavy nuclei in the SNA EOS may be underestimated by approximately 80% true rates [149]. In addition to the nuclear composition given by the EOS, weak interaction rates themselves, particularly electron capture rates of heavy nuclei, have major influences on the dynamics [64].

5.3.3 Heavy leptons and exotic hadrons

Additional degrees of freedom due to heavy leptons and exotic hadrons at high densities and high temperatures affect the dynamics of supernovae and the final fate of PNSs. The muons in the central part of a PNS, soften the EOS, and, hence, the luminosity and mean energy of neutrinos increase [57]. It is also pointed out that the second collapse of the core with a phase transition from hadronic to quark matter may induce shock revivals for massive progenitors [58, 171].

The time from the core-bounce of a PNS to the black hole formation is dependent on the maximum mass of the PNS. The transition to quark matter reduces the maximum mass, life time of the PNS, and duration of the neutrino emission [172, 173]. The emergence of hyperons also decreases the time for a PNS to become a black hole [174, 175, 176]. Central densities of the PNS for the typical progenitors of massive stars (e.g., 11.2 M⊙ [31]) are up to a few times nuclear saturation density. Populations of hyperons and quark phases in the EOS are more likely to be realized in the core collapse of highly massive stars (e.g., 40 M⊙).

6 Summary

We reviewed the EOSs for the hot and dense stellar matter and nuclei in the central engines of CCSNe. Dripped neutrons and dense electrons allow populations of neutron-rich heavy nuclei to form during the core-collapse stage, owing to the reductions in the Coulomb energy and the total proton fraction. The nuclei with Z =20–50 and N =40–90 specifically determine the core-deleptonization and the subsequent core dynamics [64, 147]. Nucleons and light nuclei dominate shock matter, and their mass fractions and weak interactions have significant effects on shock wave dynamics. α particles are available around shock waves and in inward flows with low entropies, whereas deuterons exist immediately above the surface of the PNS.

Recently, numerous EOSs have been developed. Numerous constrains on the EOS for uniform nuclear matter are derived from ab-initio calculations of neutron matter, NS observations, and terrestrial experiments. Consequently, some phenomenological EOSs have been updated, e.g., the SLy4 parameter set for Skyrme-type interactions and the TM1e parameter sets for the RMF theory [4, 6, 94]. Recent experiments on PREX-II [126, 125], in which the neutron skin thickness of heavy nuclei is measured, also help to constrain and improve the calculations of asymmetric nuclear matter. We should also study the EOSs based on microscopic calculations [17, 19].

There are three major ambiguities in modeling the EOS of non-uniform nuclear matter: (1) the temperature dependence of the free energies of neutron-rich heavy nuclei; (2) calculations of mixtures of nucleons and light nuclei at high entropies; and (3) the transition from a mixture of various nuclei (non-uniform nuclear matter) to uniform nuclear matter below nuclear saturation density, ρB ∼ 10^{14} g/cm^3. The first one should be addressed with nuclear experiments and up-to-date calculations of heavy nuclei [177, 178]. Investigation on few-nucleon systems and experiments would help to improve the second task [132, 143, 154]. For the third task, molecular dynamics [150] and precise calculations of nuclear pastas [151, 179] would be helpful.
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