Effect of Accretion of Phantom Energy on Initial Mass of a Primordial Black Hole

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Abstract. Hawking had shown that black holes radiate with a temperature inversely proportional to their mass, thereby losing energy and hence mass. For sufficiently small masses (less than $10^{15}$ g) the black hole would evaporate today and hence has a “life” equal to the present age of the universe. One explanation of the observed acceleration of the universe is by phantom energy. In 2010 Jamil and Qadir have showed that this energy enhances the rate of evaporation. Thus, to have a primordial black hole evaporating today, its initial mass should be larger than $10^{15}$ g or the primordial black holes of mass less than $10^{15}$ g should evaporate earlier. In fact, it was claimed that the black holes would be ten orders of magnitude larger! This effect is revisited and its dependence on the value of equation of state parameter is studied. It is found that the effect of phantom energy for the $10^{15}$ g black hole is negligible but for a $10^{22}$ g black hole would be significant. In that case, though, the black hole would not be now exploding. The mass at which the effect of phantom energy equals the effect of Hawking radiation has been called the transition mass. The transition mass has been discussed and the correction term in the lifetime for primordial black holes is computed.

1. Introduction
In an attempt to construct a theory of quantum gravity, as a “half-way-house”, Hawking tried quantizing scalar fields in a classical curved space-time background of a Schwarzschild black hole. He found that there would be radiation at a temperature inversely proportional to the mass [1]. Consequently the black hole loses mass and therefore the temperature rises. The more it loses the faster is the rise, leading to an explosion burst of radiation in which the black hole loses its mass totally. As such sufficiently small mass Primordial Black Holes (the black holes thought to be formed within the first 100s after the big bang [2], PBHs hereafter) could evaporate today [1]. If PBHs are to evaporate today due to Hawking radiation (HR hereafter) only, then their initial mass would have been less than $10^{15}$ g and they must have formed within the first $10^{-23}$ s [3]. Although there is no evidence of their formation, their study is of great interest because their formation could provide a unique probe of the early Universe (if $M_{PBH} < 10^{15}$ g), gravitational collapse (if $M_{PBH} > 10^{15}$ g), high energy physics (if $M_{PBH} \sim 10^{15}$ g), quantum gravity (if $M_{PBH} < 10^{-5}$ g) and may place constraints on the physics relevant to these areas even if they never existed [3].

The universe is generally believed to have an accelerated expansion [4] despite claims that the observations could be accounted for by the inhomogeneities in the universe [5, 6]. The accelerated expansion is consistent with a cosmological constant but is generally taken to be due
to some field that arises some time after the Plank era [7]. This field has been variously taken to be quintessence, dark energy [8] (DE hereafter) or phantom energy [9]. These exotic forms of energy have an equation of state (EoS) \( \omega \equiv p/\rho \). For phantom energy \( \omega < -1 \), hence violating the dominant energy condition, a notion that helps prohibit time machine and wormholes [9].

Phantom energy is found to be compatible with most classical tests of cosmology based on current data as well as cosmic microwave background (CMB) anisotropy and mass power spectrum. The energy density of phantom energy increases with time [9]. It becomes infinite in finite time overcoming all other forms of matter and ripping apart the fabric of the universe [10]. It was shown that the accretion of phantom energy on a black hole effectively decreases its mass [11, 12, 13, 14]. In particular, Jamil and Qadir [13] have discussed the decrease in mass of a PBH due to the combined effect of accretion of phantom energy and Hawking evaporation. A relation between the time and mass of a PBH (which evaporates at \( t = t_f \)) was derived and it was said that the accretion of phantom energy on a black hole decreases its life or in other words we have to increase its initial mass to keep the lifetime of a black hole (BH hereafter) the same. The evolution of PBHs under the combined effect of HR, accretion of phantom energy and radiation accretion is studied by Guariento et al in 2008 [14]. In 2011 Nayak and Singh have discussed the increase in lifetime of PBHs when radiation accretion is taken into account in addition to HR [15]. Also in 2011, Nayak and Jamil have discussed the accretion of matter, radiation and vacuum energy on PBHs [16].

The plan of the paper is as follows. In section 2, we have repeated the calculations of [13] to correct a few mistakes and found that the results (formula, graphs and hence their interpretation) are different; increasing the ratio of the phantom energy mass reduction coefficient to that of the HR does not lead to any odd features. In the next section we calculate the transition masses of PBHs, i.e., the values of mass below which HR becomes more effective than phantom energy [14]. This depends on the time \( t_o \) when DE starts dominating over matter. Since at present the dark energy is more than twice the matter energy, it is reasonable to take \( t_o \) to be somewhere between 1 and 10 billion years. We have calculated the transition masses at the two extreme ends of this range. Taking into account the increased evaporation rate of PBHs, we compute the decrease in the “life” of PBHs of mass \( 10^{15} \) g in section 4 and find that this decrease is insignificant. However the effect is significant for larger mass BHs (\( \sim 10^{22} \) g). We conclude with a brief summary.

2. Mass Evolution due to Hawking Radiation and Phantom Energy

Decrease in mass of a BH due to the combined effect of Hawking Radiation and phantom energy is given by [13]

\[
\frac{dM}{dt} = \left( \frac{dM}{dt} \right)_{hr} + \left( \frac{dM}{dt} \right)_p ,
\]

(1)

\[
\frac{dM}{dt} = -\frac{\hbar c^4}{G^2 M^2} + \frac{8G M^2}{3c^3 t_o^2} (1 + \omega) ,
\]

(2)

where \( t_o \) is the time of domination of DE. Rewriting Eq. (2) as

\[
\frac{dM}{dt} = -aM^2 - \frac{b}{M^2} ,
\]

(3)

where

\[
a = -\frac{8G}{3c^3 t_o^2} (1 + \omega) \equiv \frac{8\epsilon G}{3c^3 t_o^2} ,
\]

(4)

\[
b = \frac{\hbar c^4 \alpha}{G^2} = 7.6804 \times 10^{25} g^3 s^{-1} .
\]

(5)
Eq. (3) can be written as
\[-\int dt = \frac{1}{b} \int \frac{M^2}{1 + \frac{a}{b} M^4} dM ,\]  
(6)

\[t_f - t = \frac{1}{4\sqrt{2}(a^3b)^{\frac{1}{4}}} \left[ \ln \left| \frac{1 - \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m + (\frac{a'}{b'})^{\frac{1}{2}} m^2}{1 + \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m + (\frac{a'}{b'})^{\frac{1}{2}} m^2} \right| \right.\]
\[+ \left. 2\tan^{-1}(1 + \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m) - 2\tan^{-1}(1 - \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m) \right],\]  
(7)

where we have used the initial condition that black hole mass vanishes at \(t_f\). By comparison of Eq. (7) with Eq. (24) of [13], the difference between the two can be seen. Here \(t = \) age of universe when mass of black hole is \(M, M = mM_i\) (where \(m = [0, 1]\)). Thus Eq. (3) becomes
\[\frac{dm}{dt} = -a' m^2 - \frac{b'}{m^2},\]  
(8)

where \(a' = aM\) and \(b' = \frac{b}{M_i}\). Thus Eq. (7) becomes
\[t = t_f - \frac{1}{4\sqrt{2}(a^3b)^{\frac{1}{4}}} \left[ \ln \left| \frac{1 - \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m + (\frac{a'}{b'})^{\frac{1}{2}} m^2}{1 + \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m + (\frac{a'}{b'})^{\frac{1}{2}} m^2} \right| \right.\]
\[+ \left. 2\tan^{-1}(1 + \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m) - 2\tan^{-1}(1 - \sqrt{2}(\frac{a'}{b'})^{\frac{1}{4}} m) \right].\]  
(9)

Defining
\[p = \frac{a'}{b'},\]  
(10)
where \( a' = b' \) indicates equal contribution from phantom energy and HR in the evaporation of PBHs and normalizing Eq. (9), we get

\[
t = t_f\left[1 - \frac{\ln\left|\frac{1 - \sqrt{2p^\frac{1}{4}m + p^\frac{1}{2}m^2}}{1 + \sqrt{2p^\frac{1}{4}m + p^\frac{1}{2}m^2}}\right| + 2\tan^{-1}\left(1 + \sqrt{2p^\frac{1}{4}m}\right) - 2\tan^{-1}\left(1 - \sqrt{2p^\frac{1}{4}m}\right)}{\ln\left|\frac{1 - \sqrt{2p^\frac{1}{4}m + p^\frac{1}{2}m^2}}{1 + \sqrt{2p^\frac{1}{4}m + p^\frac{1}{2}m^2}}\right| + 2\tan^{-1}\left(1 + \sqrt{2p^\frac{1}{4}m}\right) - 2\tan^{-1}\left(1 - \sqrt{2p^\frac{1}{4}m}\right)}\right].
\]

To re-plot the graphs of [13], we take the normalized time \( \tau = t/t_f \) and plot it against dimensionless mass parameter \( m \). For \( p \leq 1 \) in Eq. (11), we get similar results, i.e., graphs as in [13]. But for \( p > 1 \), we get different graphs and hence their different interpretation. Figures 1 – 2 are the plots for \( p = 5 \) and \( p = 100000 \) respectively. These graphs do not contain any non-physical or redundant part unlike figures 7 and 9 in [13].

3. Calculation of Transition Mass of PBHs

Eq. (10) can be written as

\[
p = \frac{a}{b}M^4.
\]

\( M \mid_p=1 \equiv M_t \) is called the transition mass, i.e., the value of the mass of a PBH below which the evaporation due to HR dominates over the evaporation due to the phantom energy [14]. Using Eqs. (4) and (5), we get the transition mass

\[
M_t = 3.4637 \times 10^{15} \frac{t_o^{\frac{1}{2}}}{\epsilon^{\frac{1}{4}}} \ g,
\]

where \( t_o \) is the DE domination time measured in seconds. For \( p < 1 \), \( M \) is the corresponding upper bound for mass below which phantom energy is never dominant over HR; \( p = 1 \) gives the value of transition mass and \( p > 1 \) is the case when evaporation rate due to phantom energy is relatively more than that due to HR. Thus, we infer from here that figures 1 – 2 have a part of the curve in the start which represents domination of phantom energy over the HR in the evaporation rate in a certain time interval.

It has been claimed on the basis of a double beta decay experiment that \( 0.05 < \epsilon < 0.67 \) [17]. As such we evaluate \( M_t \) for the extreme and middle values and then include a much larger value \( \epsilon = 10 \) to check the sensitivity of the transition mass to \( \epsilon \) (in case the claim is invalid). For \( t_o \) to be 10 billion years, the results are given in table 1 for different values of \( \epsilon \).

Table 1. Mass of a PBH for Different Contribution from Phantom Energy for Different Values of \( \epsilon \), taking \( t_o = 10 \) billion years.

| \( p \) | \( M \) (\( \epsilon = 0.05 \)) | \( M \) (\( \epsilon = 0.36 \)) | \( M \) (\( \epsilon = 0.67 \)) | \( M \) (\( \epsilon = 10 \)) |
|---|---|---|---|---|
| 0.01 | \(1.3011 \times 10^{24}\) | \(0.7944 \times 10^{24}\) | \(0.6801 \times 10^{24}\) | \(0.3460 \times 10^{24}\) |
| 0.1 | \(2.3138 \times 10^{24}\) | \(1.4126 \times 10^{24}\) | \(1.2094 \times 10^{24}\) | \(0.6153 \times 10^{24}\) |
| 0.5 | \(3.4599 \times 10^{24}\) | \(2.1123 \times 10^{24}\) | \(1.8085 \times 10^{24}\) | \(0.9200 \times 10^{24}\) |
| 1.0 | \(4.1146 \times 10^{24}\) | \(2.5120 \times 10^{24}\) | \(2.1508 \times 10^{24}\) | \(1.0941 \times 10^{24}\) |
| 5 | \(6.1527 \times 10^{24}\) | \(3.7562 \times 10^{24}\) | \(3.2161 \times 10^{24}\) | \(1.6362 \times 10^{24}\) |
| 10 | \(7.3168 \times 10^{24}\) | \(4.4670 \times 10^{24}\) | \(3.8246 \times 10^{24}\) | \(1.9458 \times 10^{24}\) |
The values of $M_t$ show that HR dominates over phantom energy for a sufficiently higher value of mass. Thus, the evaporation of a BH will not slow down towards the end as might have been expected from $dM/dt \propto M^2$, but will explode precisely as in the case of only HR. The only way to have the phantom energy effect significant is to reduce the value of $M_t$ sufficiently, which is impossible since neither can we take much higher values for $\epsilon$ nor can we take much smaller values of $t_o$. Also, this implies that for a BH of initial mass less than $M_t$ the dominant evaporation process is always HR.

For $t_o = 1$ billion years, we get the results shown in table 2. These results have a similar physical interpretation as the results of table 1. By comparing the results of both tables, we find that for a sufficiently early domination of DE, the HR will dominate over phantom energy at smaller values of mass (transition mass).

### Table 2. Mass of a PBH for Different Contribution from Phantom Energy for Different Values of $\epsilon$, taking $t_o = 1$ billion years.

| $p$  | $M (\ g \ (\epsilon = 0.05))$ | $M (\ g \ (\epsilon = 0.36))$ | $M (\ g \ (\epsilon = 0.67))$ | $M (\ g \ (\epsilon = 10))$ |
|------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 0.01 | $0.4141 \times 10^{24}$       | $0.2512 \times 10^{24}$       | $0.2150 \times 10^{24}$       | $0.1094 \times 10^{24}$       |
| 0.1  | $0.7317 \times 10^{24}$       | $0.4467 \times 10^{24}$       | $0.3825 \times 10^{24}$       | $0.1946 \times 10^{24}$       |
| 0.5  | $1.0941 \times 10^{24}$       | $0.6680 \times 10^{24}$       | $0.5719 \times 10^{24}$       | $0.2910 \times 10^{24}$       |
| 1.0  | $1.3012 \times 10^{24}$       | $0.7944 \times 10^{24}$       | $0.6801 \times 10^{24}$       | $0.3460 \times 10^{24}$       |
| 5    | $1.9457 \times 10^{24}$       | $1.1878 \times 10^{24}$       | $1.0170 \times 10^{24}$       | $0.5174 \times 10^{24}$       |
| 10   | $2.3138 \times 10^{24}$       | $1.4126 \times 10^{24}$       | $1.2095 \times 10^{24}$       | $0.6153 \times 10^{24}$       |

### 4. Phantom Energy Accretion and Initial Mass of PBHs Evaporating Today

If we consider the evaporation due to HR only then by integrating the first term in Eq. (1) we get [1]

$$T = \frac{M_i^3}{3 \ b} ,$$

where $M_i$ is the initial mass and $T$ is the lifetime of a BH. This equation gives initial mass of a BH evaporating today (i.e., a lifetime of $4.3392 \times 10^{17} \ s$) due to HR to be $M_i = 4.64 \times 10^{14} \ g \equiv M_{hr}$. Since the accretion of phantom energy accelerates the evaporation rate, a PBH of mass $\sim 10^{15} \ g$ will vanish earlier than today. If we take $t = time of formation of a PBH \equiv t_i$ then $M = M_i$. Thus Eq. (7) becomes

$$T \equiv t_f - t_i = \frac{1}{4\sqrt{2}(a^3 b)^{3/4}} \left[ ln \left| 1 - \frac{\sqrt{2}(a/b)^{3/4} M_i + \left(\frac{a}{b}\right)^{3/4} M_i^2}{1 + \sqrt{2}(a/b)^{3/4} M_i + \left(\frac{a}{b}\right)^{3/4} M_i^2} \right| \right] + 2\tan^{-1}(1 + \sqrt{2}\left(\frac{a}{b}\right)^{3/4} M_i) - 2\tan^{-1}(1 - \sqrt{2}\left(\frac{a}{b}\right)^{3/4} M_i) ,$$

Eq. (15) gives the lifetime of a PBH of some initial mass evaporating under the effect of HR and phantom energy. For physically reasonable values of $t_o \sim 1$ billion years, $a \sim 10^{-72} \ g^{-1} s^{-1}$ ($[a/b]^{3/4} \sim 10^{-26} \ g^{-1}$). In fact, even for $t_o \sim 1 \ s$, $a \sim 10^{-39} \ g^{-1} s^{-1}$ ($[a/b]^{3/4} \sim 10^{-16} \ g^{-1}$). As such, we can use a Taylor expansion for small $[a/b]^{3/4} M_i$. Thus we get

$$T = \frac{M_i^3}{3 \ b} \left[ 1 - \frac{1}{10} \left(\frac{a}{b}\right)^{3/4} M_i^2 + O\left(\frac{a}{b} M_i^4\right) \right] .$$


We then see that for $t_0 = 10$ billion years, $\epsilon = 0.1$ and $M_i = M_{hr}$, the correction factor is $\sim 10^{-22}$ as compared to 1 which corresponds to a decrease of $\sim 10^{-5} \text{s}$ in a lifetime of $\sim 10^{17} \text{s}$! For $t_o \sim 1$ billion years, the correction factor is $\sim 10^{-21}$ which is also negligible as opposed to the claim in [13].

The lifetime of BHs of initial masses $10^{23} \text{g}$, $10^{24} \text{g}$ and $10^{25} \text{g}$ are $4.34005 \times 10^{42} \text{s}$, $4.34005 \times 10^{45} \text{s}$ and $4.34005 \times 10^{48} \text{s}$ respectively if evaporation only due to HR is considered. When effect of phantom energy accretion is also considered in evaporation process (taking $t_o = 10$ billion years), Eq. (15) gives the lifetime to be $4.34005 \times 10^{42} \text{s}$, $4.33998 \times 10^{45} \text{s}$ and $3.28317 \times 10^{48} \text{s}$ in the same order. These results show that the effect of phantom energy accretion in evaporation process is significant for such higher masses.

Also, for $\epsilon = 0.1$ the transition mass is $\sim 10^{24} \text{g}$ which is greater than $M_{hr}$ by 10 orders of magnitude. We conclude that for a PBH evaporating today under the combined effect of HR and phantom energy, the dominant evaporation process is HR even for a very early domination of phantom energy.

5. Summary
There were algebraic errors in [13] that have been corrected in this paper. The correction makes a significant difference. First, there was nonphysical behavior appearing in the graphs for the variation of mass with time that caused severe problems of interpretation. With the correction that nonphysical behavior has disappeared.

More importantly, it is found that due to the accretion of phantom energy the lifetime of a BH is decreased, but not by much. More substantial analytical work has been done to find the first order correction term in the lifetime of a PBH of some initial mass evaporating under the combined effect of phantom energy accretion and HR. There had been a claim of an enormous effect of phantom energy [13] whereby the required initial mass changed by 10 orders of magnitude if the initial phantom energy is 10 times the HR. We found that the increase in initial mass of a PBH evaporating today under the combined effect of phantom energy accretion and HR is insignificant.

Further, we have discussed the transition mass and studied its variation with EoS parameter ($\omega < -1$) numerically. We found that $M_i$ came out to be larger than $M_{hr}$ by approximately 10 orders of magnitude. This shows that phantom energy cannot be more significant in the evaporation process of a PBH evaporating today than Hawking radiation. In fact, for it to become equally significant we would have to wait $10^{31}$ times the present age of the Universe!

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