CP and T violation in non-perturbative chiral gauge theories

Werner Kerler

Institut für Physik, Humboldt-Universität, D-12489 Berlin, Germany

Abstract

We give a completely general derivation revealing the precise origin and the quantitative effects of CP and T violations in chiral gauge theories on the lattice.

1 Introduction

Imposing the Ginsparg-Wilson (GW) relation \( \{ \gamma_5, D \} = D\gamma_5D \) on the Dirac operator and considering a special form of the chiral projections Hasenfratz [2] has pointed out that the usual CP symmetry does not hold on the lattice. In our notation, writing the chiral projections as 

\[
P_- = \frac{1}{2}(1 - \gamma_5G), \quad \bar{P}_+ = \frac{1}{2}(1 + \bar{G}\gamma_5),
\]

this form is given by

\[
G = (1 - sD)/\mathcal{N}, \quad \bar{G} = (1 - (1-s)D)/\mathcal{N},
\]

with a real parameter \( s \) and \( \mathcal{N} = \sqrt{1 - s(1-s)DD^\dagger} \). CP violation then has been traced back to the singularity of (1.1) for \( s = \frac{1}{2} \) which does not allow to accommodate the interchange of \( s \) and \( 1 - s \) under the transformation in a symmetric way. This has been extended to some more general Dirac operators in Ref. [3], which in (1.1) amounts to a replacement of \( D \) by \( DF \) with a Hermitian function \( F(DD^\dagger) \).

It should be noted here that the chiral projections for GW fermions implicit in Ref. [4] and used in Ref. [5] in the present notation correspond to the choice \( G = 1 - D, \bar{G} = 1 \). We also note that the generalized chiral symmetry proposed in Ref. [6] can be more generally formulated in terms of \( G \) and \( \bar{G} \) as \( e^{ie\gamma_5G}De^{ie\gamma_5\bar{G}} = D \) (the non-unitary choice \( 1 - \frac{1}{2}D \) in the GW case in Ref. [6], however, being not a legitimate one for \( G \) and \( \bar{G} \)).

Recently Hasenfratz and Bissegger [7], again using the special form of Ref. [2] for the chiral projections of GW fermions, have also considered T transformations. They find violation of the symmetry quite similarly as in the CP case. In this context they also point out that CPT symmetry remains intact.
Obviously the form of the chiral projections based on \([1.1]\) represents a rather special case. Thus firstly the question arises whether the observed symmetry violations really persist in general. Secondly instead of only tracing the violations back to a parameter singularity to reveal the precise reason for them is preferable. Thirdly then to get quantitative hold of the violations is desirable.

In a more general approach \([8]\) it has been shown that the symmetric situation of continuum theory in the CP case is not admitted due to certain operator properties. The operators \(G, \bar{G}\) and \(D\) there have been functions of a basic unitary operator. Though this formulation includes all chiral operators discussed so far as special cases \([9]\), relying on the mentioned unitary operator introduces unnecessary restrictions and forms an obstacle for a more thorough investigation of the indicated symmetries.

To investigate CP, T and CPT symmetries in a general way, we here first analyze the possible properties of the chiral projections starting from the Dirac operator and imposing only minimal conditions. We find that due to a contribution which inevitably comes with opposite sign in \(G\) and \(\bar{G}\) one generally gets \(\bar{G} \neq G\). Furthermore, since the overall sign of the respective contribution remains open, it becomes obvious that in the construction of the chiral projections one is confronted with two distinct possibilities, of which one must be chosen to describe physics.

We next show that CP transformations as well as T transformations interchange the roles of \(G\) and \(\bar{G}\). This together with the fact that one generally has \(\bar{G} \neq G\) then is seen to constitute the origin of the symmetry violations. With respect to the need of choosing one of the mentioned two possibilities in the construction the interchange under CP and under T transformations means to violate the original choice. On the other hand, CPT symmetry is seen to be generally there and not to be affected by \(\bar{G} \neq G\).

Finally, considering correlation functions for any value of the index, we point out that the symmetry violation effects enter them via the bases involved. To get quantitative hold of such effects we note that if the related interchange of \(G\) and \(\bar{G}\) would be supplemented by a change of the respective sign in the construction one would get symmetry. Thus the effect of the violation turns out to be given by the difference of the results for the two sign choices and is seen to become manifest in entirely different subsets of bases contributing to the correlation functions.

In Section 2 we introduce basic relations and analyze the possibilities for the chiral projections. In Section 3 we derive the properties for CP, T and CPT transformations. In Section 4 we consider the effects caused in correlation functions. Section 5 contains our conclusions.

## 2 Operator properties

### 2.1 Basic relations

Introducing chiral projections \(P_{\pm}\) and \(\bar{P}_{\pm}\) with \(P_{+} + P_{-} = \bar{P}_{+} + \bar{P}_{-} = 1\) and requiring that they satisfy

\[
\bar{P}_{\pm} D = DP_{\pm}
\]  (2.1)
we get the decomposition of the Dirac operator into Weyl operators

\[ D = \bar{P}_+ D P_+ + \bar{P}_- D P_. \] (2.2)

Considering \( P_- \) and \( \bar{P}_+ \) in the following, we write them in the form

\[ P_- = \frac{1}{2}(1 - \gamma_5 G), \quad \bar{P}_+ = \frac{1}{2}(1 + \bar{G} \gamma_5), \] (2.3)

which because of \( P_-^\dagger = P_- = P_-^2 \) and \( \bar{P}_+^\dagger = \bar{P}_+ = P_+^2 \) implies unitarity and \( \gamma_5 \)-Hermiticity,

\[ G^{-1} = G^\dagger = \gamma_5 G \gamma_5, \quad \bar{G}^{-1} = \bar{G}^\dagger = \gamma_5 \bar{G} \gamma_5. \] (2.4)

According to (2.1) the operators \( G \) and \( \bar{G} \) are subject to

\[ D + \bar{G} D^\dagger G = 0. \] (2.5)

### 2.2 Spectral representations

We consider a finite lattice and require the Dirac operator to be normal, \([D^\dagger, D] = 0\), and \( \gamma_5 \)-Hermitian, \( D^\dagger = \gamma_5 D \gamma_5 \). It then has the spectral representation

\[ D = \sum_j \hat{\lambda}_j P_j + \sum_k (\lambda_k P^I_k + \lambda_k^* P^II_k), \] (2.6)

where the eigenvalues are all different and satisfy \( \text{Im} \hat{\lambda}_j = 0 \) and \( \text{Im} \lambda_k > 0 \). For the orthogonal projections the relations \( \gamma_5 P_j = P_j \gamma_5 \) and \( \gamma_5 P^I_k = P^I_k \gamma_5 \) hold and we have

\[ 1 = \sum_j P_j + \sum_k (P^I_k + P^{II}_k), \] (2.7)

where we associate \( P_0 \) to \( \hat{\lambda}_0 = 0 \), i.e. \( j = 0 \) to the zero modes of \( D \).

We require that \( G \) and \( \bar{G} \) are functions of \( D \), which means that their eigenvalues are functions of those of \( D \). Using this and imposing (2.5) we obtain for \( G \) and \( \bar{G} \) the spectral representations

\[ G = \eta_0 P_0 - \sum_{j \neq 0} \eta_j P_j + \sum_k (e^{i\varphi_k} P^I_k + e^{-i\varphi_k} P^{II}_k), \]

\[ \bar{G} = \bar{\eta}_0 P_0 + \sum_{j \neq 0} \bar{\eta}_j P_j + \sum_k (e^{i\bar{\varphi}_k} P^I_k + e^{-i\bar{\varphi}_k} P^{II}_k), \] (2.8)

with \( \eta_j \) and \( \bar{\eta}_0 \) taking the values \( \pm 1 \) and phases \( \varphi_k, \bar{\varphi}_k \) being subject to

\[ e^{i(\varphi_k + \bar{\varphi}_k - 2\alpha_k)} = -1 \quad \text{where} \quad e^{i\alpha_k} = \lambda_k/|\lambda_k|, \quad 0 < \alpha_k < \pi. \] (2.9)

At this point we make the important observation that because of the opposite signs of the \( j \)-sums in (2.8) one generally has \( \bar{G} \neq G \).
2.3 Chiral features

Since $\gamma_5 P_j = P_j \gamma_5$ we get the decomposition $P_j = P_j^+ + P_j^-$ with $\gamma_5 P_j^\pm = P_j^\pm \gamma_5 = \pm P_j^\pm$. Furthermore $\gamma_5 P_k^\pm = P_k^\mp \gamma_5$ implies $\text{Tr}(\gamma_5 P_k^\pm) = \text{Tr}(\gamma_5 P_k^\mp) = 0$. For $N_j^\pm = \text{Tr} P_j^\pm$ then according to $\text{Tr}(\gamma_5 1) = 0$ and (2.7)

$$\sum_j (N_j^+ - N_j^-) = 0 \quad (2.10)$$

follows. The index of $D$ is given by $I = N_0^+ - N_0^-$. For the numbers of the Weyl and anti-Weyl degrees of freedom $N = \text{Tr} P_-$ and $\bar{N} = \text{Tr} \bar{P}_+$ we thus obtain

$$N = d + \frac{1}{2}(-\eta_0 I + K), \quad \bar{N} = d + \frac{1}{2}(\bar{\eta}_0 I + K), \quad (2.11)$$

where $d = \frac{1}{2} \text{Tr} 1$ and

$$K = \sum_{j \neq 0} \eta_j (N_j^+ - N_j^-). \quad (2.12)$$

These relations allow us to discuss (2.8) in more detail. Firstly we see that (2.11) gives $\bar{N} - N = \frac{1}{2}(\bar{\eta}_0 + \eta_0)I$. Therefore in order to have $\bar{N} - N = I$ we must put $\bar{\eta}_0 = \eta_0 = 1$. Next we note that $\bar{N} + N = 2d + K$. If there is only one term in the $j$-sums of (2.8), as has been the case for all operators discussed so far [9], using (2.10) we obtain $K = -\eta_1 I$. For $I = 0$ this quite reasonably implies $\bar{N} + N = 2d$. In the more general case admitted here putting $\eta_j = \eta$ for $j \neq 0$ we get $K = -\eta I$ and thus also $\bar{N} + N = 2d$ for $I = 0$. Finally we see that to minimize the differences between $G$ and $\bar{G}$ the $k$-sums in (2.8) can readily be made equal by requiring $\bar{\varphi} = \varphi$. We thus arrive at the form

$$G = P_0 - \eta \sum_{j \neq 0} P_j + \sum_k (e^{-i\varphi_k} P_k^1 + e^{+i\varphi_k} P_k^\mp),$$

$$\bar{G} = P_0 + \eta \sum_{j \neq 0} P_j + \sum_k (e^{+i\varphi_k} P_k^1 + e^{-i\varphi_k} P_k^\mp), \quad (2.13)$$

and remain with the two possible choices $\eta = 1$ or $\eta = -1$. The important observation here is that to describe physics we are forced to decide for one of such choices.

3 Transformation properties

3.1 CP transformations

With the charge conjugation matrix $C$ and with $R_{n' n}^{P} = \delta^4_{n' n} r^{P}$, $U_{4n}^{CP} = U_{4n}^{*}$, and $U_{4n}^{CP} = U_{k, n p - k}^{T}$ for $k = 1, 2, 3$, where $n^P = (-\vec{n}, n_4)$, we have for $D$ the CP transformation

$$D(U^{CP}) = W^{CP} D^{T}(U) W^{CP\dagger}, \quad W^{CP} = R^P \gamma_4 C^\dagger, \quad (3.1)$$
in which $T$ denotes transposition and where $W^{CP\dagger} = W^{CP-1}$ holds. Because $G$ and $\bar{G}$ are functions of $D$ they inherit its transformation properties so that

$$G(U^{CP}) = W^{CP}G^T(U)W^{CP\dagger}, \quad \bar{G}(U^{CP}) = W^{CP}\bar{G}^T(U)W^{CP\dagger}. \quad \text{(3.2)}$$

Using (3.2) it becomes obvious that the forms (2.3),

$$P_-(U) = \frac{1}{2}(I - \gamma_5G(U)), \quad \bar{P}_+(U) = \frac{1}{2}(I + \bar{G}(U)\gamma_5), \quad \text{(3.3)}$$

because of $\{\gamma_5, W^{CP}\} = 0$ transform to

$$P_-^{CP}(U^{CP}) = \frac{1}{2}(I - \gamma_5\bar{G}(U^{CP})), \quad \bar{P}_+^{CP}(U^{CP}) = \frac{1}{2}(I + G(U^{CP})\gamma_5), \quad \text{(3.4)}$$

with $P^{CP}(U^{CP}) = W^{CP}\bar{P}_+^{CP}(U)W^{CP\dagger}$ and $\bar{P}^{CP}(U^{CP}) = W^{CP}P_+^{CP}(U)W^{CP\dagger}$. Insertion into $I = \bar{N} - N = \text{Tr} \bar{P}_+ - \text{Tr} P_-$ shows that $I^{CP\dagger} = -I$.

The result (3.1) obviously differs from the untransformed relation (3.3) by an interchange of $G$ and $\bar{G}$. This together with fact that, due to the opposite signs of the $j$-sums in (2.8), one generally has $\bar{G} \neq G$ means violation of the symmetry.

With respect to the need that to describe physics one has to choose a definite value of $\eta$ in (2.13), the interchange of $G$ and $\bar{G}$ is seen to cause a change violating the original choice of $\eta$.

### 3.2 $T$ transformations

With $R^{T\dagger}_{n'n} = \delta^T_{n'n'}, U^T_{4n} = U^T_{4n'r - 4}$ and $U^T_{kn} = U^*_{kn'}$ for $k = 1, 2, 3$, where $n^T = (\bar{n}, -n_4)$, one has for $D$ the $T$ transformation

$$D(U^T) = W^TD^T(U)W^{T\dagger}, \quad W^T = R^T\gamma_5C\gamma_4, \quad \text{(3.5)}$$

where $W^{T\dagger} = W^{-1}$. Therefore we can proceed quite analogously to the CP case using

$$G(U^T) = W^TG^T(U)W^{T\dagger}, \quad \bar{G}(U^T) = W^T\bar{G}^T(U)W^{T\dagger}. \quad \text{(3.6)}$$

Because of $\{\gamma_5, W^{CP}\} = 0$ this leads to

$$P^T(U^T) = \frac{1}{2}(I - \gamma_5G(U^T)), \quad \bar{P}^T(U^T) = \frac{1}{2}(I + G(U^T)\gamma_5), \quad \text{(3.7)}$$

with $P^T(U^T) = W^T\bar{P}_+^T(U)W^{T\dagger}$, $\bar{P}^T(U^T) = W^TP_+^T(U)W^{T\dagger}$ and $I^T = -I$.

The result (3.7) obviously differs from the untransformed relation (3.3) by an interchange of $G$ and $\bar{G}$. Thus again the general mechanism of symmetry violation takes place which has been described above in the CP case.

---

\footnote{With $\gamma^\dagger_\mu = \gamma_\mu$ and $\gamma^T_\mu = (-1)^{\mu}\gamma_\mu$ for $\mu = 1, \ldots, 4$ and $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ we have $\gamma_5^T = \gamma_5$ and get $C^T = -C = C^\dagger = C^{-1}$ and $[\gamma_5, C] = 0$ for $C = \gamma_2\gamma_4$.}
3.3 CPT transformations

With $R^{\text{CPT}}_{n'n} = \delta_{n',-n}$ and $U^{\text{CPT}}_{\mu'n} = U^\dagger_{\mu',-n-\bar{\mu}}$ we get for $D$ the CPT transformation

$$D(U^{\text{CPT}}) = W^{\text{CPT}} D(U) W^{\text{CPT}} \dagger = R^{\text{CPT}} D(U) R^{\text{CPT}} \gamma_5,$$  
(3.8)

and the analogous relations for $G$ and $\bar{G}$. Using them we obtain

$$P^{\text{CPT}}_-(U^{\text{CPT}}) = \frac{1}{2} (1 - \gamma_5 G(U^{\text{CPT}})),$$  
$$\bar{P}^{\text{CPT}}_+(U^{\text{CPT}}) = \frac{1}{2} (1 + \bar{G}(U^{\text{CPT}}) \gamma_5).$$  
(3.9)

Obviously (3.9) has the same form as (3.3) which means that there is CPT symmetry. It is seen that this symmetry is not affected by $\bar{G} \neq G$.

4 Correlation functions

4.1 Basic relations

We consider basic fermionic correlation functions in a form which also applies in the presence of zero modes and for any value of the index. Integrating out the Grassmann variables we have

$$\langle \psi_{\sigma_{r+1}} \ldots \psi_{\sigma_N} \bar{\psi}_{\bar{\sigma}_{r+1}} \ldots \bar{\psi}_{\bar{\sigma}_N} \rangle_f = \frac{1}{r!} \sum_{\sigma_1 \ldots \sigma_r} \sum_{\bar{\sigma}_1 \ldots \bar{\sigma}_r} \bar{\Upsilon}^{\ast}_{\bar{\sigma}_1 \ldots \bar{\sigma}_N} \Upsilon_{\sigma_1 \ldots \sigma_N} D_{\sigma_1 \sigma_1} \ldots D_{\sigma_r \sigma_r}$$  
(4.1)

with the alternating multilinear forms

$$\Upsilon_{\sigma_1 \ldots \sigma_N} = \sum_{i_1, \ldots, i_N = 1}^{N} \epsilon_{i_1, \ldots, i_N} u_{\sigma_1 i_1} \ldots u_{\sigma_N i_N}, \quad \bar{\Upsilon}_{\bar{\sigma}_1 \ldots \bar{\sigma}_N} = \sum_{j_1, \ldots, j_N = 1}^{N} \epsilon_{j_1, \ldots, j_N} \bar{u}_{\bar{\sigma}_1 j_1} \ldots \bar{u}_{\bar{\sigma}_N j_N}.$$  
(4.2)

The bases $\bar{u}_{\sigma' j}$ and $u_{\sigma i}$ in (4.2) satisfy

$$P_- = uu^\dagger, \quad u^\dagger u = 1_w, \quad \bar{P}_+ = \bar{u}\bar{u}^\dagger, \quad \bar{u}^\dagger \bar{u} = 1_{\bar{w}}.$$  
(4.3)

Since by (4.3) the bases are only fixed up to unitary transformations, $u[S] = uS$, $\bar{u}[\bar{S}] = \bar{u}\bar{S}$, we impose the condition

$$\det_w S \cdot \det_{\bar{w}} \bar{S}^\dagger = 1$$  
(4.4)

on such transformations in order that general correlation functions remain invariant.
4.2 Transformations

We now address the case of CP transformations, noting that analogous relations hold for T transformations. With conditions (4.3) and (4.1) being satisfied by \( u, \bar{v}, \bar{S}, \bar{S} \) as well as by \( u^{CP}, \bar{u}^{CP}, S^{CP}, \bar{S}^{CP} \), the (equivalence classes of pairs of) bases transform as

\[
u^{CP} S^{CP} = W^{CP} \bar{u}^{*} \bar{S}^{*} \bar{S}_{\zeta}, \quad u^{CP} S^{CP} = W^{CP} u^{*} S^{*} \bar{S}_{\zeta},
\]

(4.5)

where the additional unitary operators \( S_{\zeta} \) and \( \bar{S}_{\zeta} \) have been introduced for full generality. Inserting (4.5) into (4.1) gives for the correlation functions

\[
\langle \psi_{\sigma_1}^{CP} \cdots \psi_{\sigma_R}^{CP} \bar{\psi}_{\sigma_1}^{CP} \cdots \bar{\psi}_{\sigma_R}^{CP} \rangle_{CP}^{CP} = e^{i\theta_{CP}} \sum_{\sigma_1, \ldots, \sigma_R} \bar{W}_{\sigma_1, \sigma_2}^{CP} \cdots \bar{W}_{\sigma_R, \sigma_1}^{CP} \langle \psi_{\bar{\sigma}_1} \cdots \psi_{\bar{\sigma}_R} \bar{\psi}_{\bar{\sigma}_1} \cdots \bar{\psi}_{\bar{\sigma}_R} \rangle_{\bar{CP}}^{CP} \bar{W}_{\bar{\sigma}_1, \bar{\sigma}_2}^{CP} \cdots \bar{W}_{\bar{\sigma}_R, \bar{\sigma}_1}^{CP}
\]

(4.6)

where \( e^{i\theta_{CP}} = \det \bar{S}_{\zeta} \cdot \det \bar{S}_{\zeta}^{*} \). Since repetition of the transformation must lead back, \( S_{\zeta} \) and \( \bar{S}_{\zeta} \) are restricted to choices for which \( \theta_{CP} \) is a universal constant. Accordingly the factor \( e^{i\theta_{CP}} \) gets irrelevant in full correlation functions and, without restricting generality, we can put \( \theta_{CP} = 0 \).

Though (4.6) then superficially looks “CP covariant”, it is affected by the missing CP symmetry of the chiral projections. Indeed, while the pair \( u, \bar{v} \) is related to one choice of \( \eta \) in (2.13), the pair \( u^{CP}, \bar{u}^{CP} \) is related to the other one.

4.3 Symmetry-violation effects

To get quantitative hold of the symmetry violations we note that for CP transformations as well as for T transformations an additional change of the value of \( \eta \) in (2.13) would lead to symmetry. Thus the symmetry-violation effect is given by the difference of the results for the two choices of \( \eta \).

To study this in detail we note that using (2.13) we obtain

\[
P_{-} = P_{0}^{-} + \sum_{j \neq 0} P_{j}^{+} + \sum_{k} P_{k}^{X}, \quad \bar{P}_{-} = P_{0}^{+} + \sum_{j \neq 0} P_{j}^{+} + \sum_{k} \bar{P}_{k}^{X}
\]

(4.7)

for \( \eta = \pm 1 \),

where \( P_{k}^{X} = f_{k}(0) - \gamma_{5} f_{k}(\varphi) \), \( \bar{P}_{k}^{X} = f_{k}(0) + f_{k}(\varphi) \gamma_{5} \) with \( f_{k}(\varphi) = \frac{1}{2}(e^{i\varphi} P_{k}^{I} + e^{-i\varphi} P_{k}^{II}) \).

From the representation (4.7) it becomes obvious that for the two choices of \( \eta \) either \( \sum_{j \neq 0} P_{j}^{+} \) or \( \sum_{j \neq 0} P_{j}^{-} \) is involved in the chiral projections.

The crucial point now is that this implies the occurrence of the entirely different contributions \( \sum_{j \neq 0} P_{j}^{+} = \sum_{l=1}^{L^{+}} u_{l}^{+} u_{l}^{+\dagger} \) and \( \sum_{j \neq 0} P_{j}^{-} = \sum_{l=1}^{L^{-}} u_{l}^{-} u_{l}^{-\dagger} \) to (1.3) (where since \( L^{-} - L^{+} = 1 \) even the numbers of terms can differ). In the multilinear forms (4.2) then the subsets \( u_{l}^{+} \) and \( u_{l}^{-} \) of bases, being related to entirely different projections, clearly lead to different results. This in turn causes differences in the correlation functions which give the effects of the symmetry violations.

There remains obviously the question which one of the two choices for \( \eta \) is the appropriate one for the description of physics. However, at present no theoretical principle is in sight to decide about this.
5 Conclusions

We have investigated CP, T and CPT symmetries in a general way, imposing only minimal conditions, namely normality and $\gamma_5$-Hermiticity of the Dirac operator and that it has a general decomposition into Weyl operators.

We first have analyzed the possible properties of the chiral projections starting from the Dirac operator. It has turned out that due to a contribution in the spectral representations which inevitably comes with opposite sign in the operators $G$ and $\bar{G}$, which enter the chiral projections, one generally gets $\bar{G} \neq G$. Furthermore, because the overall sign of the respective contribution remains open, it has become obvious that in the construction of the chiral projections one is confronted with two distinct possibilities, of which one must be chosen to describe physics.

We next have shown that CP transformations as well as T transformations cause an interchange of the rôles of $G$ and $\bar{G}$. This together with the observation that one generally gets $\bar{G} \neq G$ has been seen to constitute the origin of the symmetry violations. With respect to the need of choosing one of the mentioned two possibilities in the construction it has become obvious that the interchange of $G$ and $\bar{G}$ under CP and under T transformations means violation of the original choice. On the other hand, CPT symmetry has been seen to be generally there and not to be affected by $\bar{G} \neq G$.

Finally, using a form of the correlation functions which applies also in the presence of zero modes and for any value of the index, we have pointed out that the symmetry-violation effects enter them via the bases involved. To get quantitative hold of such effects we have noted that if the related interchange of $G$ and $\bar{G}$ would be supplemented by a change of the respective sign in the construction one would get symmetry. Thus the effects of the violations have turned out to be given by the difference of the results for the two choices in the construction of the chiral projections. This has been seen to become manifest in entirely different subsets of bases appearing in the correlation functions.

Acknowledgement

I wish to thank Michael Müller-Preussker and his group for their kind hospitality.

References

[1] P.H. Ginsparg, K.G. Wilson, Phys. Rev. D 25 (1982) 2649.
[2] P. Hasenfratz, Nucl. Phys. B (Proc. Suppl.) 106 (2002) 159.
[3] K. Fujikawa, M. Ishibashi, H. Suzuki, Phys. Lett. B 538 (2002) 197; JHEP 0204 (2002) 046.
[4] R. Narayanan, H. Neuberger, Phys. Rev. Lett. 71 (1993) 3251; Nucl. Phys. B 412 (1994) 574; Nucl. Phys. B 443 (1995) 305.

[5] M. Lüscher, Nucl. Phys. B 549 (1999) 295; Nucl. Phys. B 568 (2000) 162.

[6] M. Lüscher, Phys. Lett. B 428 (1998) 342.

[7] P. Hasenfratz, M. Bissegger, hep-lat/0501010

[8] W. Kerler, Nucl. Phys. B 680 (2004) 51.

[9] W. Kerler, Nucl. Phys. B 646 (2002) 201.