Gravitational waves from SGRs and AXPs as fast-spinning white dwarfs

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Abstract. Gravitational waves (GWs) emission due to magnetic deformation mechanism is applied for Soft Gamma Repeaters (SGRs) and Anomalous X-Ray Pulsars (AXPs), described as fast-spinning and magnetized white dwarfs (WDs). The emission is caused by the asymmetry around the rotation axis of the star generated by its own intense magnetic field. Thus, for the first time in the literature, it is estimated the GWs counterpart for SGRs/AXPs described as WD pulsars. We find that some SGRs/AXPs can be observed by the space detectors BBO and DECIGO. In particular, 1E 1547.0-5408 and SGR 1806-20 could be detected in 1 year of observation, whereas SGR 1900+14, CXOU J171405.7-381031, Swift J1834.9-0846 and SGR 1627-41 could be observed with a 5-year observation time. We also found that SGRs/AXPs as highly magnetized neutron stars are far below the sensitivity curves of BBO and DECIGO. This result indicates that a possible detection of continuous GWs originated from these objects would corroborate the WD pulsar model.
1 Introduction

Over the last decade, there has been an increasing interest of the astrophysics community on highly magnetized white dwarfs (HMWDs) both from the theoretical and observational points of view. These sources constitute at least 10% of the white dwarfs (WDs) if observational biases are considered [1]. These WDs with surface magnetic fields ranging from $10^6$ G to $10^9$ G have been confirmed by the recent results of the Sloan Digital Sky Survey (SDSS) [2–6]. Besides their high magnetic fields, most of them have been shown to be massive, and responsible for the high-mass peak at $1\,M_\odot$ of the WD mass distribution; for instance: REJ 0317–853 has $M \approx 1.35\,M_\odot$ and $B \approx (1.7–6.6) \times 10^8$ G [7, 8]; PG 1658+441 has $M \approx 1.31\,M_\odot$ and $B \approx 2.3 \times 10^6$ G [9, 10]; and PG 1031+234 has the highest magnetic field $B \approx 10^9$ G [2, 11]. The existence of ultra-massive WDs has been revealed in several studies [12–19].

Typically, WDs rotate with periods of days or even years. Recently, a WD pulsar known as AR Scorpii was discovered with a period of $1.97$ min, emitting radiation in a broad range of frequencies, typically of neutron star (NS) pulsars [20]. The spindown power is an order of magnitude larger than the observed luminosity (dominated by the X-rays), which, together with an absence of obvious signs of accretion, suggests that AR Sco is primarily rotation-powered. The AR Sco Å's broadband spectrum is characteristic of synchrotron radiation, requiring relativistic electrons, possibly originated from the neighborhood of the WD and accelerated to almost the speed of light [21]. Furthermore, other sources have been proposed as candidates of WD pulsars. A specific example is AE Aquarii, the first WD pulsar identified, with a short rotation period of $P = 33.08$ s [22] and spinning down at a rate $P = 5.64 \times 10^{-14}$ s/s. The rapid braking of the WD and the nature of hard X-ray pulses detected with SUZAKU space telescope [23] can be explained in terms of spin-powered pulsar mechanism [see 24]. On the other hand, the X-ray Multimirror Mission (XMM) - Newton satellite has observed a WD faster than AE Aquarii. Mereghetti et al. [25] showed that the X-ray pulsator RX J0648.0-4418 is a massive WD with mass $M = 1.28\,M_\odot$ and radius $R = 3000$ km [see 26, 27, for derived mass-radius relations for massive oxygen-neon WDs that predict this radius], with a very fast spin period of $P = 13.2$ s, that belongs to the binary system HD 49798/RX J0648.0-4418.

Notwithstanding, several currently studies of fast-rotating and magnetized WDs have been done, in particular the one involving WD pulsars in an alternative description for Soft Gamma Repeaters (SGRs) and Anomalous X-Ray Pulsars (AXPs) [see 28–32, and references therein]. From this perspective, a canonical spin-powered pulsar model can explain the process
of energy emission released by dipole radiation in a WD, since they share quite similar aspects [29, 33]. In addition, these sources could also be candidates for GW emission, since the huge magnetic field can deform the star in a non-symmetrical way, thus generating a variation in the quadrupolar moment of the star.

In a second proposed scenario, that of a WD pulsar, the optical/IR data are explained by the WD photosphere and by a disk [34]. Recently, a new scenario has been proposed to explain the spectral energy distribution (SED) of 4U 0142+61, from mid-infrared up to hard X-rays [35]. In this model, the persistent emission comes from an accreting isolated magnetic WD surrounded by a debris disk, having gas and dusty regions.

On the other hand, direct observations of GWs have recently been made by LIGO and Virgo. The first event was detected in 2015 by LIGO [36]. This event, named GW150914, came from the merging of two black holes of masses $\sim 35.6 M_\odot$ and $30.6 M_\odot$ that resulted in a black hole of mass $\sim 63.1 M_\odot$. Thereafter, LIGO in collaboration with Virgo observed 9 more such events [37–40]. In addition, the event GW170817 reports the first detection of GW from a binary NS inspiral [41]. All GW detections are within a frequency band ranging from 10 Hz to 1000 Hz, which is the operating band of LIGO and Virgo. As is well known, there are proposed missions for lower frequencies, such as LISA [42, 43], whose frequency band is of $(10^{-4} - 0.01)$ Hz, BBO [44, 45] and DECIGO [46, 47] in the frequency band ranging from 0.01 Hz to 10 Hz.

Different possibilities of generation of continuous GWs have already been proposed [see e.g., 48–56, and references therein]. More recently, Kalita and Mukhopadhyay [57] shows that continuous GWs can be emitted from rotating magnetized WDs and will possibly be detected by the upcoming GW detectors such as LISA, DECIGO and BBO. Here we explore the magnetic deformation mechanism of gravitational radiation emission in SGRs/AXPs as fast-spinning magnetized WD. In this case, the GW emission is generated by asymmetry around the rotation axis of the star due to the intense magnetic field.

This paper is organized as follows. In Sec. 2, we present the aspects of the model used for explain SGRs/AXPs. In Sec. 3, we describe the mechanism of GW emission by deducting the equations for the gravitational amplitude and luminosity. In Sec. 4 we present and discuss the calculations applied to SGRs/AXPs described as WD pulsars. Finally, in Sec. 5 we summarize the main conclusions and remarks.

2 SGRs/AXPs as white dwarf pulsars

SGRs and AXPs are a special class of pulsars that present distinct characteristics from radio pulsars and X-ray pulsars (see Table 1). They are described by the magnetar model, where they are considered strongly magnetized NS with magnetic field of the order of $10^{12} - 10^{15}$ G. Also, these objects are known as very slow rotating pulsars comparing to ordinary pulsars, with rotational periods in the range of $P \sim 2 - 12$ s and a high spindown rate of $\dot{P} \sim 10^{-13} - 10^{-10}$ s/s [see 58, and references therein].

Recently, three SGRs with low magnetic field ($B \sim 10^{12} - 10^{13}$ G) have been observed, namely, SGR 0418+5729, Swift J1822.3-1606 and 3XMM J185246.6+00331. These new discoveries open the question concerning the nature of SGRs/AXPs, emerging alternative scenarios, in particular the WD pulsar model. These astronomical observations have based an alternative description of the SGRs/AXPs, which are modeled as rotating highly magnetized

\footnote{For information about the SGRs/AXPs, we refer the reader to the McGill University’s online catalog available at: http://www.physics.mcgill.ca/~pulsar/magnetar/main.html}
Table 1. Observational quantities taken from McGill Pulsar Group’s online catalog for confirmed SGRs/AXPs: Period ($P$), spindown ($\dot{P}$), observed luminosity ($L_X$) and distance to the source ($r$).

| SGR/AXP          | $P$ (s) | $\dot{P}$ ($10^{-11}$ s/s) | $L_X$ ($10^{33}$ erg/s) | $r$ (kpc) |
|------------------|---------|----------------------------|--------------------------|-----------|
| CXOU J010043.1-721134 | 8.020392 | 1.88                       | 65                       | 62.4      |
| 4U 0142+61       | 8.688692 | 0.2022                     | 105                      | 3.6       |
| SGR 0418+5729    | 9.078388 | 0.0004                     | 0.00096                  | 2         |
| SGR 0501+4516    | 5.76207  | 0.594                      | 0.81                     | 2         |
| SGR 0526-66      | 8.0544   | 3.8                        | 189                      | 53.6      |
| 1E 1048.1-5937   | 6.457875 | 2.25                       | 49                       | 9         |
| 1E 1547.0-5408   | 2.072126 | 4.77                       | 1.3                      | 4.5       |
| PSR J1622-4950   | 4.3261   | 1.7                        | 0.44                     | 9         |
| SGR 1627-41      | 2.594578 | 1.9                        | 3.6                      | 11        |
| CXOU J164710.2-455216 | $10.61064 \leq 0.04$ | 0.45                       | 3.9                     |
| 1RXS J170849.0-400910 | 11.00502 | 1.9455                     | 42                       | 3.8       |
| CXOU J171405.7-381031 | 3.825352 | 6.40                       | 56                       | 13.2      |
| SGR J1745-2900   | 3.763638 | 1.385                      | $\leq 0.11$              | 8.3       |
| SGR 1806-20      | 7.54773  | 49.5                       | 163                      | 8.7       |
| XTE J1810-197    | 5.540354 | 0.777                      | 0.043                    | 3.5       |
| Swift J1822.3-1606 | 8.437721 | 0.0021                     | $\leq 0.00040$           | 1.6       |
| SGR 1833-0832    | 7.565408 | 0.35                       | ...                      | $\leq 10^a$ |
| Swift J1834.9-0846 | 2.482302 | 0.796                      | $\leq 0.0084$            | 4.2       |
| 1E 1841-045      | 11.78898 | 4.092                      | 184                      | 8.5       |
| J185246.6+003317 | 11.55871 | $\leq 0.014$               | $\leq 0.0060$            | 7.1       |
| SGR 1900+14      | 5.19987  | 9.2                        | 90                       | 12.5      |
| SGR 1935+2154    | 3.245065 | 1.43                       | ...                      | $\leq 10^b$ |
| 1E 2259+586      | 6.979043 | 0.0483                     | 17                       | 3.2       |

$^a$see [59]. $^b$see [60].

and very massive WDs [see 28, 29, for further details]. From this perspective, a canonical spin-powered pulsar model can explain the process of energy emission released by a dipole radiation in a WD, since they share quite similar aspects [see 33]. In this new description, several observational properties are explained as a consequence of the large radius of a massive WD that manifests a new scale of mass density, moment of inertia, rotational energy, and magnetic dipole moment in comparison with the case of NSs [see e.g., 29, 31, and references therein].

In the canonical pulsar model, a rotating star with magnetic dipole moment misaligned to the axis of rotation converts rotational energy into electromagnetic energy. Thus, the system emits radiation due to the variation of the magnetic dipole and the pulsar rotation becomes slower. If we consider that all rotational energy loss is converted to electromagnetic energy, we can infer the magnetic field on the star’s surface, $B_s$, as a function of the period $P = 1/f_{\text{rot}}$ and its derivative $\dot{P} = dP/dt$ [29]

$$B_s = \left(\frac{3e^3 I}{8\pi^2 R^6} P \dot{P}\right)^{1/2}.$$  \hfill (2.1)
where $I$ is the moment of inertia, $R$ is the star radius and $c$ is the speed of light.

Note that because the moment of inertia values for a NS and a WD are different, the magnetic fields required in each model are also different. For example, for a NS with mass $M = 1.4M_\odot$ and radius $R = 10^6$ cm, the magnetic field on the star’s surface are in the range of $10^{13} - 10^{15}$ G. For a massive WD with mass $M = 1.4M_\odot$ and radius $R = 3 \times 10^8$ cm [see 29], the magnetic field has smaller values and is in a range around $10^8 - 10^{10}$ G, comparable to the inferred values of known HMWDs [2–6]. Thus, these values of mass and radius generating the moment of inertia $I \sim 1.26 \times 10^{30}$ g.cm$^2$, will be adopted hereafter in this work as the fiducial WD model parameters. These results clearly show that the scale of the magnetic field in WD is $10^5$ times larger than for NSs.

In addition, since the high magnetic field can deform the star in a non-symmetrical way, new values for the ellipticity are expected and, consequently, new values for the GW amplitude as well (see section 3). In the next section, we describe the magnetic deformation mechanism and deduce the GW amplitude and luminosity emitted by this process.

3 Magnetic deformation mechanism: basic equations

WDs might generate GWs whether they are not perfectly symmetric around their rotation axes. This asymmetry can occur, for example, due to the huge dipole magnetic field that can make the star become oblate [61]. In this work, we analyze the emission of gravitational radiation from SGRs/AXPs as fast magnetized WDs by this mechanism.

Thus, in this section we consider the deformation of the WDs induced by their own huge magnetic fields. Due to the combination of magnetic field and rotation, a WD can become triaxial, presenting therefore a triaxial moment of inertia. In order to investigate the effect arising from the magnetic stress on the equilibrium configuration of the stars, let us introduce the equatorial ellipticity, defined as follows [62, 63]

\[
\epsilon = \frac{I_1 - I_2}{I_3},
\]

(3.1)

where $I_1$, $I_2$ and $I_3$ are main moments of inertia with respect to the $(x, y, z)$ axes, respectively.

If the star rotates around the $z$–axis, then it will emit monochromatic GWs with a frequency twice the rotation frequency, $f_{\text{rot}}$, and amplitude given by [62, 63]

\[
h_0_{\text{eff}} = \frac{16\pi^2 G I_3 f_{\text{rot}}^2}{c^4} \epsilon,
\]

(3.2)

and the rotational energy of the star decreases at a rate given by [62, 63]

\[
L_{\text{GW}_{\text{eff}}} = -\frac{2048\pi^6 G}{5 c^5} I_3^2 \epsilon^2 f_{\text{rot}}^6.
\]

(3.3)

On the other hand, recall that the ellipticity of magnetic origin can be written as follows [61, 64]

\[
\epsilon = \frac{35 B_s^2 R^4}{24 GM^2},
\]

(3.4)

where $B_s$ is the dipole magnetic field, $R$ and $M$ are the radius and the mass of the star, respectively.

Finally, substituting this last equation into Eqs. (3.2) e (3.3) and considering $I_3 = 2MR^2/5$, one immediately obtains that
and
\[ L_{GW} = \frac{6272\pi^6 B_2^4 R_6^2 f_{rot}^3}{45e5 GM^2}. \tag{3.6} \]

Thereby, we find equations for the gravitational luminosity and the GW amplitude which depends on the rotation frequency and the magnetic field strength.

Now, we are ready to calculate the GW amplitude and luminosity for SGRs/AXPs as massive fast-spinning WDs. The next section is devoted to this issue as well as the corresponding discussion of the results.

4 Results and Discussions

Here we consider that SGRs/AXPs are fast-spinning and magnetized WDs which emit GWs due to the deformation caused by their own intense magnetic field. For this study, we use the magnetic field values inferred from the canonical pulsar model (see Eq. (2.1)), where it is considered that all the spindown luminosity of the star is converted to electromagnetic luminosity.

Thus, using Eq. (3.6), we calculate the GW luminosity for several SGRs/AXPs, considering these objects as a massive WD of \( M_{WD} = 1.4 M_\odot \) and radius \( R_{WD} = 3.0 \times 10^8 \text{ cm} \) [28]. The result of this calculation is presented in Table 2, which also displays the ellipticity \( \epsilon \), the spindown luminosity \( L_{sd} = 4\pi^2 I_3 f_{rot} f_{rot} \) and the efficiency \( \eta_{df} = L_{GW}/L_{sd} \). Notice that the efficiencies are around \( 10^{-10} \) to \( 10^{-12} \). This implies that the gravitational luminosity is much smaller than the spindown luminosity when considering the magnetic fields inferred by the dipole model. Thus, we see that we can apply these magnetic field values to calculate the GW amplitude, since the emission of gravitational energy is negligible as compared to the rotational energy loss rate, not changing significantly the inferred magnetic fields.

In addition, we adopt two additional values for the masses of the WDs \( M_{WD} \) as well as their corresponding radii, namely, 0.6 \( M_\odot \) \( (R_{WD} = 7.5 \times 10^8 \text{ cm}) \) and 1.0 \( M_\odot \) \( (R_{WD} = 5.5 \times 10^8 \text{ cm}) \). For these two masses, the efficiencies are also very small, namely, around \( 10^{-8} \) to \( 10^{-10} \) and \( 10^{-9} \) to \( 10^{-11} \) for \( M_{WD} = 0.6 M_\odot \) and \( M_{WD} = 1.0 M_\odot \), respectively. Therefore, since \( \eta_{df} \ll 1 \), we can use the inferred values for the magnetic field to calculate the amplitude of the GWs. Figure 1 shows the GW amplitudes versus the magnetic fields for the 23 confirmed SGRs/AXPs (see also Table 1).

Notice that some SGRs/AXPs produce GWs with amplitudes that can be detected by BBO and DECIGO. For example, 1E 1547.0-5408 and SGR 1806-20 could well be detected.
Table 2. Parameters for SGRs/AXPs assuming these objects are massive WDs of $M_{WD} = 1.4 M_\odot$ and radius $R_{WD} = 3.0 \times 10^8$ cm.

| SGRs/AXPs     | $B_\star$ $(10^9 \text{ G})$ | $\epsilon$ $(10^{-7})$ | $L_{sd}$ $(10^{38} \text{ erg/s})$ | $L_{GWdf}$ $(\text{erg/s})$ | $\eta_{df}$ $(10^{-11})$ |
|---------------|------------------|-----------------|------------------|------------------|------------------|
| CXOU J010043.1-721134 | 4.62 | 4.82 | 1.45 | 9.62 $\times 10^{27}$ | 6.63 |
| 4U 0142+61 | 1.58 | 0.562 | 0.122 | 8.06 $\times 10^{25}$ | 0.658 |
| SGR 0418+5729 | 0.072 | 0.00116 | 0.000212 | 2.66 $\times 10^{20}$ | 0.00124 |
| SGR 0501+4516 | 2.20 | 1.09 | 1.23 | 3.61 $\times 10^{27}$ | 2.92 |
| SGR 0526-66 | 6.59 | 9.80 | 2.89 | 3.87 $\times 10^{28}$ | 13.36 |
| 1E 1048.1-5937 | 4.54 | 4.65 | 3.32 | 3.28 $\times 10^{28}$ | 9.87 |
| 1E 1547.0-5408 | 3.74 | 3.17 | 213.34 | 1.39 $\times 10^{31}$ | 65.19 |
| PSR J1622-4950 | 3.23 | 2.36 | 8.36 | 9.29 $\times 10^{28}$ | 11.12 |
| SGR 1627-41 | 2.64 | 1.58 | 43.29 | 8.98 $\times 10^{29}$ | 20.74 |
| CXOU J164710.2-455216 | 0.776 | 0.136 | 0.0133 | 1.42 $\times 10^{24}$ | 0.106 |
| 1RXS J170849.0-400910 | 5.52 | 6.87 | 0.582 | 2.92 $\times 10^{27}$ | 5.02 |
| CXOU J171405.7-381031 | 5.89 | 7.84 | 45.49 | 2.16 $\times 10^{30}$ | 47.38 |
| SGR J1745-2900 | 2.72 | 1.68 | 10.37 | 1.09 $\times 10^{29}$ | 10.46 |
| SGR 1806-20 | 23.02 | 119.6 | 45.81 | 8.51 $\times 10^{30}$ | 185.72 |
| XTE J1810-197 | 2.47 | 1.38 | 1.82 | 7.22 $\times 10^{27}$ | 3.97 |
| Swift J1822.3-1606 | 0.158 | 0.00567 | 0.00139 | 9.80 $\times 10^{21}$ | 0.00705 |
| SGR 1833-0832 | 1.94 | 0.847 | 0.322 | 4.21 $\times 10^{26}$ | 1.31 |
| Swift J1834.9-0846 | 1.67 | 0.632 | 0.0207 | 1.88 $\times 10^{29}$ | 9.08 |
| 1E 1841-045 | 8.27 | 15.44 | 0.993 | 9.76 $\times 10^{27}$ | 9.82 |
| 3XMM J185246.6+003317 | 0.479 | 0.0518 | 0.00361 | 1.24 $\times 10^{23}$ | 0.0343 |
| SGR 1900+14 | 8.23 | 15.31 | 26.04 | 1.30 $\times 10^{30}$ | 50.10 |
| SGR 1935+2154 | 2.56 | 1.48 | 16.65 | 2.08 $\times 10^{29}$ | 12.48 |
| 1E 2259+586 | 0.692 | 0.108 | 0.057 | 1.11 $\times 10^{25}$ | 0.196 |

for the entire mass range considered, while SGR 1900 + 14, CXOU J171405.7-381031, Swift J1834.9-0846 and SGR 1627-41 would be detected either they do not have large masses or with an integration time of $T = 5$ years.

SGRs/AXPs described as WDs, which have moments of inertia five orders of magnitude greater than a NS, would generate GW amplitudes much larger than SGRs/AXPs described as NSs (see Figure 3). Consequently, if these sources are NSs the GW amplitudes generated are far below the sensitivity curves of BBO and DECIGO. Thus, if these space based instruments observe continuous GWs from these SGRs/AXPs, this would corroborate the model of fast-spinning and magnetic WDs. This supports the description of SGR and AXPs as belonging to a class of very fast and magnetic massive WDs in perfect accord with recent astronomical observations of HMWDs.

5 Summary

Besides the search and detection of GWs from the merger events [40, 41], the search for continuous GWs has been of great interest in the scientific community. It is well known that,
besides compact binaries, rapidly rotating NSs are promising sources of GWs which could be detected in a near future by Advanced LIGO (aLIGO) and Advanced Virgo (AdV), and also by the planned Einstein Telescope (ET) and the space-based LISA, BBO and DECIGO. These sources generate continuous GWs whether they are not perfectly symmetric around their rotation axis, i.e. if they present some equatorial ellipticity. Undoubtedly, SGRs and AXPs are also good candidates in this context. Here we investigate the gravitational radiation from these objects described as fast-spinning WDs using the magnetic deformation mechanism. It is worth stressing that these putative uncommon WDs are known to have a high rotation (a few seconds to minutes) and a huge magnetic field (\(10^6\) G to \(\sim 10^{10}\) G).

Then, by describing SGRs/AXPs as rotation-powered WD pulsars, we consider the role
played by the magnetic dipole field on the deformation of these objects and its consequences as regards the generation of GWs considering a mass range $0.6M_\odot \leq M_{WD} \leq 1.4M_\odot$ for these sources. It is worth mentioning that, this is the first time in the literature that the GW counterpart for SGRs/AXPs are modeled as fast-spinning and magnetized WDs.

We note that some SGRs/AXPs, described as WD pulsars, emit GWs with amplitudes that could be detected by BBO and DECIGO, namely: 1E 1547.0-5408 and SGR 1806-20 can be observed for the entire considered mass range for one year of observation time, while SGR 1900+14, CXOU J171405.7-381031, Swift J1834.9-0846 and SGR 1627-41 can be detected either they do not have large masses or with an integration time of 5 years.

Last but not least, it is worth mentioning that recent astronomical observations suggest that we should revisit the real nature of AXP/SGRs: are they really magnetars or fast-spinning and magnetized WDs? Thereby, a possible detection of continuous GWs coming from SGRs/AXPs would be a good indication that could corroborate the WD model, because for the NSs description, they are far below the BBO and DECIGO sensitivity curves. We also encourage future observational campaigns to determine the radii and the magnetic field of these sources to elucidate the real nature of SGRs and AXPs.

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