Structure factor of interacting one-dimensional helical systems

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We calculate the dynamical structure factor $S(q, \omega)$ of a weakly interacting helical edge state in the presence of a magnetic field $B$. The latter opens a gap of width $2B$ in the single-particle spectrum, which becomes strongly nonlinear near the Dirac point. For chemical potentials $|\mu| > B$, the system then behaves as a nonlinear helical Luttinger liquid, and a mobile-impurity analysis reveals interaction-dependent power-law singularities in $S(q, \omega)$. For $|\mu| < B$, the low-energy excitations are gapped, and we determine $S(q, \omega)$ by using an analogy to exciton physics.

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The edge states of two-dimensional topological insulators (quantum spin Hall insulators) are gapless one-dimensional (1D) eigenmodes of the helicity operator, in which the spin orientation of a particle is correlated with its momentum $|\mathbf{p}|$. Electron-electron interactions within the edge modes are typically described using helical Luttinger liquid (HLL) theory [3,9]. Experimental evidence for helical modes was found via the observation of a quantized conductance $2e^2/h$ in HgTe/CdTe quantum wells [10] and InAs/GaSb heterostructures [11,12]. A crucial aspect of these edge states is that they host a nonlinearity in the single-particle spectrum. In contrast, the magnetic field opens a gap and thus inelastic scattering channels have been explored [21,22].

In this work we study the interplay of interactions and broken TRS on the dynamical structure factor (DSF) $S(q, \omega)$ (i.e., the Fourier transform of the density-density correlation function) of 1D helical fermions at zero temperature. In the absence of magnetic fields, the single-particle spectrum is linear, so $S(q, \omega)$ exhibits delta-function singularities at the mass shell $\omega = u|q|$, where the sound velocity $u$ depends on the interaction strength. In contrast, the magnetic field opens a gap and thus introduces a nonlinearity in the single-particle spectrum. This leads to changes in $S(q, \omega)$ which are not captured by the conventional HLL approach. These modifications are particularly nontrivial in the presence of interactions and manifest themselves as power-law singularities whose origin can be traced back to the Fermi edge singularity problem [23–32].

We begin our study by first discussing the DSF for the noninteracting case. The Hamiltonian density in the presence of a magnetic field $\tilde{B} = (-B, 0, 0)$ perpendicular to the spin quantization axis of helical edge modes reads

$$\mathcal{H} = -i\hbar v_F \partial_x \sigma_3 + B\sigma_1 - \mu,$$  \hspace{1cm} (1)

where $v_F$ (henceforth we set $v_F = 1$ and $\hbar = 1$) is the Fermi velocity, $\sigma_{1,3}$ are Pauli matrices, and $\mu > 0$ is the chemical potential. The magnetic field $B$ (henceforth we will assume $B > 0$) gaps out the single-particle spectrum given by $\varepsilon_{\pm}(p) = \pm \sqrt{p^2 + B^2}$, where $p$ is the momentum. The DSF $S(q, \omega)$ is a measure of the rate of formation of particle-hole pairs due to the absorption of an external excitation with momentum $q$ and frequency $\omega$. It is convenient to calculate the imaginary part of the retarded polarization operator, $\text{Im} \Pi^R(q, \omega)$, which is related to $S(q, \omega)$ via the fluctuation-dissipation theorem, $S(q, \omega > 0) = -2\text{Im} \Pi^R(q, \omega)$. The energy-momentum representation of the zero-temperature polarization operator in the Matsubara formalism is given by

$$\Pi^M(q, \omega) = \int \frac{dp}{(2\pi)^2} \text{Tr} \left[ \mathcal{G}(p, i\epsilon) \mathcal{G}(p + q, i\epsilon + i\omega) \right],$$  \hspace{1cm} (2)

where $\text{Tr}$ denotes the trace, the single-particle Matsubara Green’s function is $\mathcal{G}(p, i\epsilon) = (1/2) \sum_{\beta = \pm} (1 + \beta \sigma_r)/(i\epsilon - \beta |\mathbf{r}'| + \mu)$, and $\sigma_r = \mathbf{r} \cdot \sigma$. The projection of the effective magnetic field $\mathbf{r}' = B\hat{\mathbf{e}}_z + p\hat{\mathbf{e}}_x$ on the vector of Pauli matrices $\mathcal{S}$. Performing the frequency integration and analytic continuation to real frequencies yields

$$S(q, \omega) = \frac{1}{2} \int_{-\infty}^{\infty} dp \sum_{\beta, \beta' = \pm} \left( 1 + \beta \beta' \frac{\mathbf{r}' \cdot \mathbf{s}}{|\mathbf{r}'| |\mathbf{s}|} \right) \times \left[ \Theta(\mu - |\mathbf{r}'|) - \Theta(\mu - \beta'|\mathbf{s}|) \right] \delta(\omega + |\mathbf{r}'| - \beta'|\mathbf{s}|),$$  \hspace{1cm} (3)

where $\mathbf{s} = B\hat{\mathbf{e}}_z + (p + q)\hat{\mathbf{e}}_x$ and $\Theta(x)$ is the Heaviside function.

Let us first discuss the case $\mu > B$. In that case, the absorption spectrum ($\omega > 0$) receives contributions from the following values of $(\beta, \beta')$: $(+, +)$ which corresponds to (intra-band) transitions within the upper band, and $(-, +)$ which signifies (inter-band) transitions from the lower to the upper band.

For $B = 0$ the electronic spectrum is linear and spin is a good quantum number. Therefore, only spin-conserving transitions are allowed and Eq. (3) becomes a delta-function, $S(q, \omega) = \delta(|q| + (|q| - |\mathbf{r}'|) \Theta(\mu - |\mathbf{r}'|) \delta(\omega - |q|)).$
On the other hand, a nonzero magnetic field $B$ relaxes the constraints on the transitions and additional regions of support emerge in the $(\omega, q)$ plane. These non-overlapping regions can be classified as those acquiring contributions from either intra-band or inter-band transitions, see Figs. 1A and 1B. The total DSF is a sum of contributions from these two regions, $S = S_1 + S_2$.

**Intra-band transitions:** The contribution to the DSF from the intra-band transitions is

$$S_1(q, \omega) = R_1(q, \omega)I(q, \omega)\cos^2[\delta(q, \omega, B)],$$

where

$$I(q, \omega) = \frac{2[(\omega^2 - q^2)^2 + 4B^2q^2]}{(\omega^2 - q^2)^2\sqrt{1 - 4B^2/(\omega^2 - q^2)}}$$

$$\delta(q, \omega, B) = \frac{1}{2}\left[\tan^{-1}\left(\frac{p_+}{2B}\right) + \tan^{-1}\left(\frac{p_-}{2B}\right)\right],$$

with $p_{\pm} = |q| \pm \omega\sqrt{1 - 4B^2/(\omega^2 - q^2)}$. The kinematic thresholds are encoded in the function

$$R_1(q, \omega) = \Theta(\omega_{\text{Intra}}^U - \omega)\Theta(\omega - \max[\omega_{\text{Intra}}^L, \omega_{\text{Intra}}^U]),$$

where $\omega_{\text{Intra}}^U = \epsilon_+(k_F + |q|) - \mu$ is the upper threshold for intra-band processes, and $\omega_{\text{Intra}}^L = \pm(\epsilon_+(k_F - |q|) - \mu)$ are the lower thresholds for $|q| \geq 2k_F$, as shown in Fig. 1a.

Typical intra-band excitations involve transitions of a particle with momentum $p_i$ and energy $\epsilon_+(p_i)$ from within the Fermi sea $(p_i < k_F = \sqrt{\mu^2 - B^2})$ to a state with momentum $p_f = p_i + q$ and energy $\epsilon_+(p_f) = \epsilon_+(p_i) + \omega$ above the Fermi sea. For $|q| < k_F$ the final state momentum $p_f$ will be parallel to the initial momentum $(p_iB_f > 0)$; see for example Figs. 2A and 2B, whereas for $|q| > k_F$, $p_f$ can be either parallel or anti-parallel to $p_i$ depending on whether $\omega \gtrless \sqrt{k_F^2 + B^2} - B$, respectively. Note that the anti-parallel option is absent for $B = 0$, because the spin-states of the left and right moving fermions are then orthogonal.

It is known that for a quadratic spectrum $\epsilon = p^2/2m$, the noninteracting DSF at fixed $|q| < 2k_F$ is constant, $S(q, \omega) = m/|q|$, between the thresholds $\omega_{\pm}(q) = k_F|q|/m \pm q^2/(2m)$. In contrast, for the nonparabolic spectrum $\epsilon_{\pm}(p)$, the DSF exhibits power-law behavior, see Fig. 4B. For $B < \mu$ and $|q| < k_F$ it can be approximated as $S(q, \omega) \approx 4Bq^2/(q^2 - \omega^2)^{3/2}$ with the width of the support given by $\delta\omega = \omega_{\text{Intra}}^U - \omega_{\text{Intra}}^L \approx B^2q^2/\mu^3$.

**Inter-band transitions:** An inter-band process involves the excitation of a particle with momentum $p_i$ from the lower band to a state with momentum $p_f = p_i + q$ in the upper band. The energy difference is $\omega = \epsilon_+(p_f) - \epsilon_-(p_i)$. The corresponding contribution to the DSF is

$$S_2(q, \omega) = R_2(q, \omega)I(q, \omega)\sin^2[\delta(q, \omega, B)],$$

where

$$R_2(q, \omega) = \Theta(\omega - \omega_{\text{Inter}}^L) + \Theta(\omega - \omega_{\text{Inter}}^U)$$

$$+ 2\Theta(q - 2k_F)\Theta(\omega_{\text{Inter}}^L - \omega)\Theta(\omega - \sqrt{q^2 + 4B^2}).$$

Here the threshold frequencies are at $\omega_{\text{Inter}}^L = \mu + \epsilon(k_F \pm |q|)$ (see Fig. 1 and Figs. 2C,D for the threshold transitions corresponding to $\omega_{\text{Inter}}^L$ and $\omega_{\text{Inter}}^U$, respectively).

For $|q| > k_F$ such transitions are allowed for frequencies within $\omega_{\text{Inter}} < \omega < \omega_{\text{Inter}}^U$ an additional channel involving anti-parallel momenta $p_iq < 0$ (however, $p_ip_f > 0$) opens up. Hence, the DSF acquires a discontinuity at $\omega = \omega_{\text{Inter}}^U$. For small momenta $|q| \ll k_F$ and energies $\omega \approx 2\mu$, $S(q, \omega) \propto q^2B^2/(\omega^3\sqrt{\omega^2 - 4B^2})$.

On the other hand, transitions with anti-parallel initial and final momenta $p_ip_f < 0$ are possible for $|q| > k_F$. For $2k_F > |q| > k_F$ such transitions are allowed for frequencies within $\omega_{\text{Inter}}^L < \omega < B + \sqrt{B^2 + q^2}$. However, for $|q| > 2k_F$ the frequency regime as given in the second line of Eq. 10, i.e., $\sqrt{q^2 + 4B^2} < \omega < \omega_{\text{Inter}}^U$, also supports $p_ip_f < 0$ transitions. We would like to point out that in this regime the frequency $\omega = \sqrt{(p_i + q)^2 + B^2}$ also supports $p_ip_f < 0$ transitions. The consequence of this is that $S(q, \omega)$ develops a square-root singularity at the frequency $\omega = \sqrt{q^2 + 4B^2}$.

**Interactions:** The edge features of the noninteracting structure factor are strongly modified even for weak interactions, since they generally lead to power-law singu-
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tion of the correlation function also modifies the fields in the correlation function, but
\[ \partial \] terms linear in information on the Hamiltonian \( H \) in order to remove the
in Fourier space. To this end, we perform a unitary transformation on the Hamiltonian \( H \) in order to remove the terms linear in \( \partial_x \phi \) and \( \partial_x \theta \).

Determining \( S(q, \omega) \) for \( \omega \approx \omega_{\text{L}} \) requires a calculation of the correlation function \( \langle R_\omega(x) R_{\omega'}(0) R_{\omega}(0) R_{\omega'}(0) \rangle \) in Fourier space. To this end, we perform a unitary transformation on the Hamiltonian in order to remove the terms linear in \( \partial_x \phi \) and \( \partial_x \theta \).

The transformation also modifies the fields in the correlation function, but evaluating it is straightforward and we obtain
\[
S(q, \omega) \propto \int \frac{dte^{i(\omega-\omega)t}}{(\epsilon + iut - i\delta\epsilon)^{\delta_L/4\pi}(\epsilon + iut + i\delta\epsilon)^{\delta_L/4\pi}},
\]
where \( \epsilon = 0^+, \delta_L = \sqrt{\pi K - \sqrt{\pi K}} \) and \( \delta_R = \sqrt{\pi K + \sqrt{\pi K}} \).

Since \( u > \bar{v} \) the integral is non-zero only for \( \omega > \bar{\omega} \). Therefore, we obtain for \( \omega \approx \bar{\omega} \equiv \omega_{\text{L}} \),
\[
S(q, \omega) \propto \frac{\theta[\omega - \bar{\omega}(q)]}{[\omega - \bar{\omega}(q)]^\nu},
\]
The exponent \( \nu = 1 - (\delta_R^2 + \delta_L^2)/4\pi \) becomes for weak interactions,
\[
\nu = \frac{V(0) - V(q) \cos^2[(\gamma_{k_F} - \gamma_{k_F})/2]}{\pi(u - \bar{v})} > 0.
\]

where \( p = k_F - q \). Therefore the structure factor diverges for frequencies above \( \omega_{\text{L}} \), and vanishes below this threshold.

A similar analysis yields the behavior near the threshold \( \omega_{\text{L}} \). Now the velocity \( \bar{v} \) at \( p = k_F + q \) is greater than \( u \). This has two main consequences. On the one hand, the exponent \( \nu \), which is still formally given by Eq. (17), now has its sign reversed, \( \nu < 0 \), so the DSF is convergent at this threshold. On the other hand, the integral (15) corresponding to \( \bar{\omega} = \omega_{\text{L}}(+) \) is non-zero on either side of \( \omega_{\text{L}}(+) \).

The DSF near the lower threshold for inter-branch processes, \( \omega_{\text{L}}(+) \), exhibits a one-sided divergence, \( S(q, \omega) \propto \theta[\omega - \omega_{\text{L}}(+)])/[\omega - \omega_{\text{L}}(+) \nu'] \). The exponent is now modified to \( \nu' = (V(0) - V(q) \sin^2[(\gamma_{k_F} - \gamma_{k_F})/2])/\pi(u - \bar{v}) \), where \( \bar{v} \) is the velocity of the fermion in the lower band. Near the threshold \( \omega_{\text{L}}(+) \) (due to transition of a fermion with momentum \( k_F + q \) from the lower band to the Fermi level \( k_F \) in the upper band) the exponent still has the above \( \nu' \) form, but the response function exhibits divergences from both sides of the threshold frequency since in this regime \( |\bar{v}| > u \).

So far we have considered the case \( \mu > B \). Due to the particle-hole symmetry, the structure factor exhibits identical behavior for \( \mu < -B \). On the other hand, a different scenario emerges for \( |\mu| < B \). The chemical potential now lies in the gap and the non-interacting response function exhibits square-root singularity at the edge \( \omega = \sqrt{q^2 + 4B^2} \).

As the chemical potential is in the gap, interactions can no longer be treated via the Luttinger liquid theory. However, exploiting the similarity to the problem of Mahan excitons in semiconductors [19], we used a ladder diagram resummation (see Fig. 3) for a generic interaction potential \( V(q) \) to map the problem on a single-particle Schrödinger equation. As a result, one finds sharp (delta-function type) subgap resonances in the DSF due to the formation of two-particle bound states.

To simplify the result, we will assume a contact potential between electrons, \( V(q) = V(0) \), in the following. In this case the polarization operator can be evaluated by directly summing up the ladder series in Fig. 3. We obtain
\[
S(q, \omega) = \frac{\pi q^2}{2B^{3/2} \omega - \omega_q + \pi^2 V(0)^2 B} \Theta(\omega - \omega_q) + \frac{\pi^3 q^2 V(0)}{B} \delta[\omega - \omega_q + \pi^2 V(0)^2 B],
\]
where \( \omega_q = 2B + q^2/4B \) (note for \( |q| < B, \sqrt{4B^2 + q^2} \approx \omega_q \)). From Eq. (15), we can draw two important conclusions.

First, the interactions modify the square root divergence at \( \omega = \omega_q \) (present in the non-interacting limit) into a square-root suppression. The second nontrivial effect is the emergence of a single bound-state resonance, which manifests itself as a sharp peak in the structure factor at sub-gap energies, \( \omega = \omega_q - \pi^2 V(0)^2 B \).

The structure factor and its accompanying singular features can in principle be extracted based on the recently proposed technique involving a source-probe setup [18]. The proposal would be to use time-dependent electric field at the source point to create charge excita-
tions in the 1D helical modes which would then induce currents at the probe point, thus yielding information on the spatially and temporally resolved response function. In addition, Coulomb drag measurements can serve as a useful probe for the DSF [35, 36].

To summarize, we have studied the dynamical structure factor \( S(q, \omega) \) of a helical liquid, and the role of magnetic-field induced nonlinear spectrum. We explicitly considered the contributions from the intra- and interband transitions. We found that the thresholds present in the noninteracting \( S(q, \omega) \) turn into power-law singularities upon the introduction of interactions. The edge exponents \( \nu \) depend on the momentum \( q \), the interaction strength, and the curvature of the spectrum. As a consequence of the nonparabolic spectrum and the nontrivial spin texture of the edge states, the DSF differs strongly from that of conventional spinless and spinful fermion systems.

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