Stochastic Gravitational-wave Background from Binary Black Holes and Binary Neutron Stars and Implications for \textit{LISA}

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Received 2018 October 10; revised 2018 November 30; accepted 2018 November 30; published 2019 January 24

Abstract

The advent of gravitational-wave and multimessenger astronomy has stimulated research on the formation mechanisms of binary black holes (BBHs) observed by the Laser Interferometer Gravitational-Wave Observatory (LIGO)/Virgo. In the literature, the progenitors of these BBHs could be stellar-origin black holes (sBHs) or primordial black holes (PBHs). In this paper, we calculate the stochastic gravitational-wave background (SGWB) from BBHs, covering the astrophysical and primordial scenarios separately, together with the one from binary neutron stars (BNSs). Our results indicate that PBHs contribute a stronger SGWB than that from sBHs, and the total SGWB from both BBHs and BNSs has a high possibility of being detected by the future observing runs of LIGO/Virgo and the \textit{Laser Interferometer Space Antenna (LISA)}.

\textbf{Key words:} gravitational waves – instrumentation: detectors – methods: data analysis – stars: black holes – stars: neutron

1. Introduction

The detections of gravitational waves (GWs) from binary black hole (BBH) and binary neutron star (BNS) coalescences by the Laser Interferometer Gravitational-Wave Observatory (LIGO)/Virgo (Abbott et al. 2016b, 2016c, 2016e, 2016f, 2017a, 2017b, 2017c, 2017d) have led us to the eras of GW and multimessenger astronomy. Up until now, there were several BBH merger events reported, of which the masses and redshifts are summarized in Table 1. The progenitors of these BBHs, however, are still under debate. Different formation mechanisms exist in the literature to account for the BBHs observed by LIGO/Virgo. Under the assumption that all the BBH mergers are of astrophysical origin, the local merger rate of stellar-mass BBHs is constrained to be $12 - 213 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (Abbott et al. 2017b). Besides, the rate of BNS mergers is estimated to be $1540^{+3200}_{-1220} \text{ Gpc}^{-3} \text{ yr}^{-1}$, utilizing the only so far observed BNS event, GW170817 (Abbott et al. 2017a).

Meanwhile, Table 1 indicates that the masses of BBHs extend over a relatively narrow range around $30 \text{ M}_\odot$ with source redshifts of $z \lesssim 0.2$, due to the detection ability of the current generation of ground-based detectors (the recent LIGO can measure the redshift of BBH mergers up to $z \sim 1$; Abbott et al. 2016a, 2018b). Hence, there are many more unresolved BBH merger events, along with other sources, emitting energies, which can be incoherently superposed to constitute a stochastic gravitational-wave background (SGWB; Christensen 1992).

Different formation channels for BBHs, in general, predict distinct mass and redshift distributions for BBH merger rates, and thus different energy spectra of SGWBs. Therefore, the probing of an SGWB may serve as a way to discriminate various formation mechanisms of BBHs.

Assuming all black holes (BHs) are of stellar origin (Belczynski et al. 2010, 2016; Abbott et al. 2016a; Coleman Miller 2016; Stevenson et al. 2017), the SGWB from BBHs was calculated in Abbott et al. (2016d, 2017e) and further updated to include BNSs (Abbott et al. 2018a), indicating that this background would likely be detectable, even before reaching LIGO/Virgo’s final design sensitivity, in the most optimistic case. In addition to astrophysical origin, there is another possibility that the detected BBHs are of primordial origin and (partially) play the role of cold dark matter (CDM).

In the early universe, sufficiently dense regions could undergo gravitational collapse by the primordial density inhomogeneity and form primordial black holes (PBHs; Hawking 1971; Carr & Hawking 1974). In the literature, two scenarios for PBHs to form BBHs exist (see e.g., García-Bellido et al. 2017 and Sasaki et al. 2018 for recent reviews). The first one is that PBHs in a DM halo interact with each other through gravitational radiation and occasionally bind to form BBHs in the late universe (Quinlan & Shapiro 1989; Mouru & Taniguchi 2002; Bird et al. 2016; Clesse & García-Bellido 2017a, 2017b). The resulting SGWB for the monochromatic mass function is significantly lower than that from the stellar origin and is unlikely to be measured by LIGO/Virgo (Mandic et al. 2016), while the one for a broad mass function could be potentially enhanced (Clesse & García-Bellido 2017a). The second one is that two nearby PBHs form a BBH due to the tidal torques from other PBHs in the early universe (Nakamura et al. 1997; Ioka et al. 1998; Sasaki et al. 2016). The SGWB was investigated in Wang et al. (2018) and Raidal et al. (2017), showing that it is comparable to that from the stellar-origin BBHs (SBBHs) and could serve as a new probe to constrain the fraction of PBHs in CDM. However, Raidal et al. (2017) only considered the tidal...
torque due to the nearest PBH, while Wang et al. (2018) assumed that all PBHs have the same mass.

Recently, the Laser Interferometer Space Antenna (LISA), which aims for a much lower frequency regime, roughly $10^{-4} \sim 10^{-1}$ Hz, than that of LIGO/Virgo, has been approved (Audley et al. 2017). In this paper, we will revisit the SGWB produced by BBHs and BNSs, covering both the LIGO/Virgo and LISA frequency band. The impacts of the SGWB on LISA’s detection abilities are also investigated. For sBHs, we adopt the widely accepted “Vangioni” model (Dvorkin et al. 2016) to calculate the corresponding SGWB. For PBHs, we only consider the early universe scenario. The merger rate for PBHs, taking into account the torques by all PBHs and linear density perturbations, was considered in Ali-Haïmoud et al. (2017) and later improved to encompass the case with a general density spectrum of a GW background can be described by the dimensionless quantity (Allen & Romano 1999),

$$\Omega_{GW}(\nu) = \frac{\nu d\rho_{GW}}{\rho_c} d\nu,$$

where $d\rho_{GW}$ is the energy density in the frequency interval $\nu$ to $\nu + d\nu$, $\rho_c = 3H_0^2 c^2/(8\pi G)$ is the critical energy density of the universe, and $H_0 = 67.74$ km s$^{-1}$ Mpc$^{-1}$ is the Hubble constant taken from Planck (Ade et al. 2016). For the binary mergers, the magnitude of an SGWB can be further transformed to (Phinney 2001; Regimbau & Mandic 2008; Zhu et al. 2011, 2013)

$$\Omega_{GW}(\nu) = \frac{\nu}{\rho_c H_0} \int_0^{z_{max}} dz \int dm_1 dm_2 R(z, m_1, m_2) \frac{dE_{GW}}{d\nu}(\nu_1, m_1, m_2) \times \frac{(1 + z) E(\Omega_r, \Omega_m, \Omega_\Lambda, z)}{\sqrt{\Omega_r(1 + z)^4 + \Omega_m(1 + z)^3 + \Omega_\Lambda}},$$

where $\nu_1 = (1 + z)\nu$ is the frequency in the source frame, and $E(\Omega_r, \Omega_m, \Omega_\Lambda, z) = \sqrt{\Omega_r(1 + z)^4 + \Omega_m(1 + z)^3 + \Omega_\Lambda}$ accounts for the dependence of the comoving volume on redshift $z$. We adopt the best-fit results from Planck (Ade et al. 2016) where $\Omega_r = 9.15 \times 10^{-5}$, $\Omega_m = 0.3089$, and $\Omega_\Lambda = 1 - \Omega_r - \Omega_m$. For the cutoff redshift $z_{max}$, we choose $z_{max} = 10$ for SOBBHs (Abbott et al. 2016d) and $z_{max} = \nu_3/\nu - 1$ for POBBHs (Wang et al. 2018), in which $\nu_3$ is given by Equation (3) below. The energy spectrum emitted by a single BBH $dE_{GW}/d\nu_1$ is well approximated by (Chernoff & Finn 1993; Cutler et al. 1993; Zhu et al. 2011)

$$dE_{GW} = \frac{(\pi G)^{3/2} M^{5/3} \eta}{3} \frac{\nu_1^2}{\nu_1^{2/3} (4(\nu_1 - \nu_2)^2 + \nu_2^2)},$$

where $\nu_1 = (a_1\eta^2 + b_1\eta + c_1)/(\pi GM/c^3)$, $M = m_1 + m_2$ is the total mass of the binary, and $\eta = m_1 m_2/M^2$. The coefficients $a_1$, $b_1$, and $c_1$ can be found in Table 1 of Ajith et al. (2008). Since the frequency band of nonzero eccentricity during the inspiral phase is below $10^{-4}$ Hz (Dvorkin et al. 2016), which is beyond the frequency range of LISA, we hence only consider the circular orbit during the inspiral phase. A careful discussion of the impact of eccentricity on the SGWB can be found in D’Orazio & Samsing (2018).

We will follow the widely accepted “Vangioni” model (Dvorkin et al. 2016) to estimate the SGWB from SOBBHs and BNSs in the universe. The merger rate density, $R(z, m_1, m_2)$, in Equation (2) for the SOBBHs or BNSs is a convolution of the sBHs or neutron stars’ (NSs) formation rate, $R_{\text{birth}}(z, m_1)$, with the distribution of the time delays $P_d(t_d)$ between the formation and merger of SOBBHs or BNSs,

$$R = N \int_{t_{min}}^{t_{max}} \frac{R_{\text{birth}}(t(z) - t_d, m_1) \times P_d(t_d)}{\min(m_1, m_{max} - m_1) - m_{min}} dt_d,$$

where $N$ is a normalization constant and $t(z)$ is the age of the universe at merger. Here, $P_d \propto t_d^{-1}$ is the distribution of delay time, $t_d$ with $t_{min} < t_d < t_{max}$ (Abbott et al. 2018a). The minimum delay time of a massive binary system to evolve until coalescence are set to $t_{min} = 50$ Myr for SOBBHs, and $t_{min} = 20$ Myr for BNSs. Meanwhile, the maximum delay time, $t_{max}$, is set to the Hubble time. In order to comply with the previous studies (Abbott et al. 2017b, 2018a), we restrict the component masses of BBHs to the range $m_{min} \leq m_2 \leq m_1$ and $m_1 + m_2 \leq m_{max}$, with $m_{min} = 5M_\odot$ and $m_{max} = 100M_\odot$. We note that the merger rate density of POBBHs (see Equation (12))

| Events      | Primary Mass | Secondary Mass | Redshift |
|-------------|--------------|----------------|----------|
| GW150914    | 36.2±3.2 $M_\odot$ | 29.1±2.1 $M_\odot$ | 0.09±0.04 |
| LVT151012   | 23.4±1.6 $M_\odot$ | 13.6±0.9 $M_\odot$ | 0.20±0.09 |
| GW151226    | 14.2±3.3 $M_\odot$ | 7.5±1.3 $M_\odot$ | 0.09±0.03 |
| GW170104    | 31.2±4.3 $M_\odot$ | 19.4±3.5 $M_\odot$ | 0.18±0.08 |
| GW170608    | 12.3±3.2 $M_\odot$ | 7.2±2.2 $M_\odot$ | 0.07±0.03 |
| GW170814    | 30.5±3.2 $M_\odot$ | 25.3±3.2 $M_\odot$ | 0.11±0.04 |
below) is quite different from Equation (4), due to the distinct formation mechanisms of POBBHs and SOBBHs.

The most complicated part of Equation (4) is the computation of the birth rate of sBHs or NSs, which is given by (Dvorkin et al. 2016)

$$R_{\text{birth}}(t, m_{\text{rem}}) = \int \psi(t - \tau(m_*)[\phi(m_*) \times \delta(m_* - g^{-1}_{\text{rem}}(m_{\text{rem}}))] dm_*,$$  \hspace{1cm} (5)

where $m_*$ is the mass of the progenitor star, $m_{\text{rem}}$ is the mass of remnant, and $\tau(m_*)$ is the lifetime of a progenitor star, which can be ignored (Schaerer 2002). Here, $\phi(m_*)$ is the so-called initial mass function, which is a uniform distribution ranging from 1 $M_\odot$ to 2 $M_\odot$ for NSs and $\phi(m_*) \propto m_*^{-2.35}$ for sBHs. In addition, $\psi(t)$ is the star formation rate (SFR), which is given by (Nagamine et al. 2004)

$$\psi(z) = k \frac{a \exp[b(z - z_m)]}{a - b + b \exp[a(z - z_m)].}$$ \hspace{1cm} (6)

We will use the fit parameters given by the “fiducial+PopIII” model from Dvorkin et al. (2016): namely, the sum of Fiducial SFR (with $k = 0.178 M_\odot$ yr$^{-1}$ Mpc$^{-3}$, $z_m = 2$, $a = 23.7$, $b = 1.8$) and PopIII SFR (with $k = 0.002 M_\odot$ yr$^{-1}$ Mpc$^{-3}$, $z_m = 11.87$, $a = 13.8$, $b = 13.36$). Dirac delta function in Equation (5) relates to the process of BH formation. For NSs, $g^{-1}_{\text{rem}}(m_*) = m_{\text{rem}}$, one obtains a relatively simple form of the birth rate. However, for sBHs, the masses of the progenitor star and the remnant are related by some function of $m_{\text{bh}} = g_{\text{bh}}(m_*)$, which is model-dependent and still unclear yet. In this paper, we consider the WWp model (Woosley & Weaver 1995) of sBH formation, which is simple and indistinguishable from the widely used Fryer model at a low redshift (Dvorkin et al. 2016). For the progenitor with an initial mass, $m_*$, the mass of the remnant BH, $m_{\text{bh}}$, is extrapolated as

$$\frac{m_{\text{bh}}}{m_*} = A \left( \frac{m_*}{40 M_\odot} \right)^{1 + \beta} \left[ \frac{Z(z)}{0.01 Z_{\odot}} \right]^\gamma + 1,$$ \hspace{1cm} (7)

where $Z(z)$ is the metallicity, and an explicit functional form can be found in Belczynski et al. (2016). The fiducial values of this extrapolation are $A = 0.3$, $\beta = 0.8$ and $\gamma = 0.2$ (Dvorkin et al. 2016). Solving the equation above yields the function $m_* = g_{\text{bh}}^{-1}(m_{\text{bh}})$.

Integrating over the component masses in the merger rate density results in the merger rate as a function of the redshift

$$R(z) = \int R(z, m_1, m_2) dm_1 dm_2.$$ \hspace{1cm} (8)

The local merger rate, $R \equiv R(z = 0)$, is inferred to be $R = 103^{+110}_{-63}$ Gpc$^{-3}$ yr$^{-1}$ for SOBBHs (Abbott et al. 2017b) and $R = 1540^{+2200}_{-1220}$ Gpc$^{-3}$ yr$^{-1}$ for BNSs (Abbott et al. 2017a). Utilizing Equation (2), we then calculate the SGWB from SOBBHs and BNSs. In Figure 1, we show the corresponding SGWBs, as well as the power-law integrated (PI) curves of LIGO (Abbott et al. 2017a) and LISA (Cornish & Robson 2017, 2018), indicating that the total SGWB from both BBHs and BNSs has a high possibility of being detected by the future observing runs of LIGO/Virgo and LISA. The energy spectra from both the SOBBHs and BNSs are well approximated by $\Omega_{\text{GW}} \propto v^{2/3}$ at low frequencies covering both LISA and LIGO’s bands, where the dominant contribution is from the inspiral phase. We also summarize the background energy densities $\Omega_{\text{GW}}(\nu)$ at the most sensitive frequencies of LIGO (near 25 Hz) and LISA (near $3 \times 10^{-3}$ Hz) for each of the BNS, SOBBH, and Total Background Contributions, along with the 90% Poisson Error Bounds.

|            | $\Omega_{\text{GW}}(25 \text{ Hz})$ | $\Omega_{\text{GW}}(3 \times 10^{-3} \text{ Hz})$ |
|------------|----------------------------------|-----------------------------------------------|
| BNS        | $0.7^{+1.5}_{-0.4} \times 10^{-9}$| $1.7^{+5.7}_{-1.4} \times 10^{-12}$           |
| BBH        | $1.1^{+1.9}_{-0.5} \times 10^{-9}$| $2.7^{+8.6}_{-2.2} \times 10^{-12}$           |
| Total      | $1.8^{+2.7}_{-1.2} \times 10^{-9}$| $4.4^{+5.3}_{-1.6} \times 10^{-12}$           |

The signal-to-noise ratio (S/N) for measuring the SGWB of LISA with observing time $T$ is given by (Thrane & Romano 2013; Caprini et al. 2016)

$$S/N = \sqrt{T \left[ \frac{\int d\nu \Omega_{\text{GW}}(\nu)}{\Omega_n(\nu)} \right]^{1/2}},$$ \hspace{1cm} (9)

where $\Omega_n(\nu) \equiv 2\pi^2 v^2 |S_n(\nu)|/(3H_0^2)$ and $S_n$ is the sensitivity of LISA. Figure 2 shows the expected accumulated $S/N$ of LISA as a function of the observation time. The predicted median total background from BBHs and BNSs may be identified with $S/N = 5$ after about 20 hr of observation. The total background could be identified with $S/N = 5$ within 3 hr of observation for the most optimistic case and after about 8 days for the most pessimistic case.

The total SGWB due to SOBBHs and BNSs is so strong that it may become an unresolved noise, affecting the ongoing missions of LISA. For instance, the detection of massive black

$\text{Figure 1. Predicted SGWB from the BNSs and SOBBHs. The red and green curves are backgrounds from the BNSs and BBHs, respectively. The total (BNS and BBH) background is shown in the blue curve, while its Poisson error bars are in the gray shaded region. Here, we adopt the local merger rate of $R = 103^{+110}_{-63}$ Gpc$^{-3}$ yr$^{-1}$ for SOBBHs (Abbott et al. 2017b) and $R = 1540^{+2200}_{-1220}$ Gpc$^{-3}$ yr$^{-1}$ for BNSs (Abbott et al. 2017a). We also show the expected PI curves for LISA with 4 yr of observation (dashed) and LIGO’s observing runs of O2 (black) and design sensitivity (dotted–dashed). The PI curves for LISA and LIGO’s design sensitivity cross the Poisson error region, indicating the possibility to detect this background.}$

$\text{Table 2}$

|                | $\Omega_{\text{GW}}(25 \text{ Hz})$ | $\Omega_{\text{GW}}(3 \times 10^{-3} \text{ Hz})$ |
|----------------|----------------------------------|-----------------------------------------------|
| BNS            | $0.7^{+1.5}_{-0.4} \times 10^{-9}$| $1.7^{+5.7}_{-1.4} \times 10^{-12}$           |
| BBH            | $1.1^{+1.9}_{-0.5} \times 10^{-9}$| $2.7^{+8.6}_{-2.2} \times 10^{-12}$           |
| Total          | $1.8^{+2.7}_{-1.2} \times 10^{-9}$| $4.4^{+5.3}_{-1.6} \times 10^{-12}$           |
hole binary (MBHB) coalescences is one of the key missions of LISA (Audley et al. 2017), and the largest detectable redshift of a MBHB merger may be significantly reduced by the additional noise.

Following Barack & Cutler (2004) and Cornish & Robson (2018), we define the noise strain sensitivity due to the SGWB as

$$S_{GW}(\nu) = \frac{3H_0^2 \Omega_{GW}(\nu)}{2\pi^2 \nu^3},$$

which can be added to the strain sensitivity of LISA $S_n(\nu)$ to obtain an effective full strain sensitivity of $S_{eff}(\nu) = S_n(\nu) + S_{GW}(\nu)$. The resulting strain sensitivity curves are shown in Figure 3. Additionally, the S/N of a single incoming GW strain signal (also called waveform), $h(t)$, has the following form:

$$S/N = 2 \left[ \int dt \frac{|\tilde{h}(\nu)|^2}{S_{eff}(\nu)} \right]^{1/2},$$

where $\tilde{h}(\nu)$ is the frequency domain representation of $h(t)$, and we adopt the phenomenological waveform provided by Ajith et al. (2008).

Studying the growth mechanism of massive black holes (MBHs) is an important science investigation (SI) of LISA (Audley et al. 2017). Among the observational requirements of that SI, being able to measure the dimensionless spin of the largest MBH with an absolute error less than 0.1 and to detect the misalignment of spins with the orbital angular momentum better than $10^4$, requires an accumulated $S/N$ of at least 200. The effect on the $S/N$ of MBHB coalescences due to an unsolved SGWB signal is shown in Figure 4, indicating that the largest detectable redshift (with a fixed $S/N = 200$ or $S/N = 1000$) will be reduced. This means that the total detectable region of LISA is suppressed, thus decreasing the event rate of LISA’s scientific missions. Currently, the studies of the origin of MBHs predict that masses of the seeds of MBHs lie in the range of about $10^3 M_\odot$ to several $10^5 M_\odot$, with the formation redshift around $10 \lesssim z \lesssim 15$ (Volonteri 2010). As shown in Figure 4, the precise measurement of those seeds above $10^5 M_\odot$ in a high formation redshift will be significantly affected by the confusion noise of the unsolved SGWB. Therefore, a further analysis is needed to subtract the SGWB signals from the data in order to improve the performance of the detectors.
3. SGWB from Binary PBHs

In this section, we will calculate the SGWB from PBHs assuming all BHs observed by LIGO/Virgo so far are of primordial origin. Here, we adopt the merger rate for POBBHs presented in Chen & Huang (2018), which takes into account the torques by both all PBHs and linear density perturbations. For a general normalized mass function with parameters, \( \theta \), or the probability distribution function (PDF) for PBHs, \( P(m|\theta) \), the comoving merger rate density in units of Gpc\(^{-3}\)yr\(^{-1}\) is given by (Chen & Huang 2018)

\[
\mathcal{R}_{12}(t|\theta) \approx 3.9 \times 10^{-6} \times \left( \frac{1}{t_0} \right)^{\frac{34}{37}} f^2 (f^2 + \sigma_{eq}^2)^{\frac{34}{77}} \times \left( \frac{P(m_1|\theta)}{m_1} + \frac{P(m_2|\theta)}{m_2} \right) \times \left( m_1 m_2 \right)^{\frac{3}{37}} (m_1 + m_2)^{\frac{36}{37}},
\]

where \( t_0 \) is the age of our universe, and \( \sigma_{eq} \) is the variance of density perturbations of the rest DM on scale of the order of \( \mathcal{O}(10^3 \sim 10^4)M_\odot \) at radiation-matter equality. The component masses of a POBBH, \( m_1 \) and \( m_2 \), are in units of \( M_\odot \). Similar to Ali-Haïmoud et al. (2017) and Chen & Huang (2018), we take \( \sigma_{eq} \approx 0.005 \). Here, \( f \) is the total abundance of PBHs in nonrelativistic matter, and the fraction of PBHs in CDM is related to \( f \) by \( f_{\text{PBH}} = \Omega_{\text{PBH}}/\Omega_{\text{CDM}} \approx f/0.85 \). Integrating over the component masses yields the merger rate

\[
\mathcal{R}(t|\theta) = \int \mathcal{R}_{12}(t|\theta) \, dm_1 \, dm_2,
\]

which is time (or redshift) dependent. The local merger rate density distribution then follows

\[
\mathcal{R}_{12}(t_0|\theta) = Rp(m_1, m_2|\theta),
\]

where \( p(m_1, m_2|\theta) \) is the distribution of BH masses in coalescing binaries. The local merger rate, \( R \equiv \mathcal{R}(t_0|\theta) \), is a normalization constant, such that the population distribution, \( p(m_1, m_2|\theta) \), is normalized. Note that all masses are source-frame masses.

We are then interested in extracting the population parameters \( \{\theta, R\} \) from the merger events observed by LIGO/Virgo. This is accomplished by performing the hierarchical Bayesian inference on the BBH’s mass distribution (Abbott et al. 2016b, 2016g, 2016h; Fishbach et al. 2018; Mandel et al. 2018; Thrane & Talbot 2018; Wysocki et al. 2018). Given the data for \( N \) detections, \( d = (d_1, \ldots, d_N) \), the likelihood for an inhomogeneous Poisson process (Fishbach et al. 2018; Mandel et al. 2018; Thrane & Talbot 2018; Wysocki et al. 2018) follows as

\[
p(d|\theta, R) \propto R^N e^{-R} \beta(\theta)^N \prod_i \int d\lambda \, p(d_i|\lambda) \, p(\lambda|\theta),
\]

where \( \lambda \equiv \{m_1, m_2\} \), and \( p(d_i|\lambda) \) is the likelihood of an individual event with data \( d_i \) given the binary parameters, \( \lambda \). Since the standard priors on masses for each event in the LIGO/Virgo analysis are taken to be uniform, one has \( p(d_i|\lambda) \propto p(\lambda|d_i) \), and we can use the announced posterior samples (Vallisneri et al. 2015; Abbott et al. 2016b; Biwer et al. 2018) to evaluate the integral in Equation (15). Meanwhile, \( \beta(\theta) \) is defined as

\[
\beta(\theta) \equiv \int d\lambda \, V(\lambda) \, p(\lambda|\theta),
\]

where \( V(\lambda) \) is the sensitive spacetime volume of LIGO. We adopt the semi-analytical approximation from Abbott et al. (2016g, 2016h) to calculate \( V \). Specifically, we neglect the effect of spins for BHs and use a LIGO “Early High Sensitivity” scenario to approximate the power spectral density curve. We also consider a single-detector \( S/N \) threshold of \( \rho_{th} = 8 \) for detection, which roughly corresponds to a network threshold of 12.

The posterior probability function, \( p(\theta, R|d) \), of the population parameters, \( \{\theta, R\} \), can be computed using some assumed prior, \( p(\theta, R) \). The local merger rate \( R \) can be easily obtained:

\[
p(\theta, R|d) \propto p(d|\theta, R) \propto \frac{1}{R}
\]

With this prior in hand, the posterior marginalized over \( R \) could be easily obtained:

\[
p(\theta|d) \propto [\beta(\theta)]^{-N} \prod_i \int d\lambda \, p(d_i|\lambda) \, p(\lambda|\theta).
\]

This posterior has been used in previous population inferences (Abbott et al. 2016b, 2016g, 2016h, 2017b; Fishbach & Holz 2017). We will follow the same procedure as in Abbott et al. (2016b, 2016g, 2016h, 2017b), by first using Equation (19) to constrain the parameters \( \theta \), and then fixing \( \theta \) to their best-fit values in Equation (17) to infer the local merger rate \( R \). As done in the Section 2, we also restrict the component masses of BBHs to the range 5 \( M_\odot \leq m_1 \leq m_1 \) and \( m_1 + m_2 \leq 100 \ M_\odot \). At the time of writing, the data analysis for LIGO’s O2 observing run is still ongoing, we therefore only use the 3 events from LIGO’s O1 observing run, which contains 48.6 days of observing time (Abbott et al. 2016b). An update analysis could be performed until the final release of LIGO’s O2 samples. In the following subsections, we will consider two distinct mass functions for PBHs. We will first constrain the population parameters \( \{\theta, R\} \) using LIGO’s O1 events, and then calculate the corresponding SGWB from the inferred results.

3.1. Power-law Mass Function

We now consider a power-law PBH mass function (Carr 1975),

\[
P(m) \approx \frac{\alpha - 1}{M_{\text{min}}} \left( \frac{m}{M_{\text{min}}} \right)^{-\alpha}
\]

for \( m \geq M_{\text{min}} = 5 \ M_\odot \), and \( \alpha > 1 \) is the power-law slope. In this case, the free parameters are \( \{\alpha, R\} \).

Accounting for the selection effects and using three events from LIGO’s O1 run, we find the best-fit result for \( \alpha \) is \( \alpha = 1.61 \). Fixing \( \alpha \) to this best-fit value, we obtain the median
value and the 90% equal tailed credible interval for the local merger rate, \( R = 80_{-56}^{+108} \, \text{Gpc}^{-3} \, \text{yr}^{-1} \). The posterior distributions are shown in Figure 5. From the posterior distribution of local merger rate \( R \), we then infer the fraction of PBHs in CDM as \( f_{\text{pbh}} = 3.8_{-1.8}^{+2.3} \times 10^{-3} \). Such an abundance of PBHs is consistent with previous estimations that \( 10^{-3} \lesssim f_{\text{pbh}} \lesssim 10^{-2} \), confirming that the dominant fraction of CDM should not originate from the stellar-mass PBHs (Sasaki et al. 2016; Ali-Haimoud et al. 2017; Raidal et al. 2017; Chen & Huang 2018; Kocsis et al. 2018).

Utilizing Equation (2), we then calculate the corresponding SGWB. The result is shown in Figure 6, indicating that the total SGWB from both POBBHs (with a power-law PDF) and BNSs has a high possibility of being detected by the future observing runs of LIGO/Virgo and LISA. In general, a variation of the PBH mass function will affect the profile, e.g., the cutoff frequency and the magnitude, of the energy spectrum, \( \Omega_{\text{GW}} \). To illustrate this impact, we plot a dotted line in Figure 6, showing the background with \( M_{\text{min}} = 0.5 \, M_\odot \), by fixing \( \{ \alpha, R \} \) to their best-fit values as well. The result indicates that the decreasing of \( M_{\text{min}} \) will increase the population of the lighter PBHs and hence raise the cutoff frequency. The enhancement of \( \Omega_{\text{GW}} \) is mainly due to the extra contribution from the POBBHs with a mass range of \( 0.5 \sim M_\odot \). Note that LIGO’s O2 result implies \( M_{\text{min}} \) may not be too small; otherwise, SGWB will exceed the upper bound from LIGO’s O2.

The energy spectra from both the POBBHs (with a power-law PDF) and BNSs are well approximated by \( \Omega_{\text{GW}} \propto \nu^{5/3} \) at low frequencies, covering both LISA and LIGO’s bands, where the dominant contribution is from the inspiral phase. We also summarize the background energy densities \( \Omega_{\text{GW}}(\nu) \) at the most sensitive frequencies of LIGO (near 25 Hz) and LISA (near \( 3 \times 10^{-3} \, \text{Hz} \)) in Table 3.

Figure 7 shows the expected accumulated S/N of LISA as a function of the observing time. The predicted median total background from POBBHs (with a power-law PDF) and BNSs may be identified with \( S/N = 5 \) after about 10 hr of observation. The total background could be identified with \( S/N = 5 \) within 2 hr of observation for the most optimistic case, and after about 5 days for the most pessimistic case. The strain sensitivity curves for LISA are shown in Figure 8. The effect on the S/N of MBHB coalescences due to the unsolved SGWB signal is shown in Figure 9, indicating the precise measurement of the seeds of MBHs above \( 10^5 \, M_\odot \) in a high formation redshift will be significantly affected by the confusion noise of the unsolved SGWB.

![Figure 5. Posterior distributions for \( \{ \theta, R \} = \{ \alpha, R \} \) for the power-law mass function of PBHs, using three events from LIGO’s O1 observing run.](image)

![Figure 6. Predicted SGWB from the BNSs and POBBHs with a power-law mass function. The red and green curves are backgrounds from the BNSs and BBHs, respectively. The total (BNS and BBH) background is shown in the blue curve, while its Poisson error bars are in the gray shaded region. For BBHs, we adopt the best-fit value for \( \alpha = 1.61 \) and the inferred local merger rate, \( R = 80_{-56}^{+108} \, \text{Gpc}^{-3} \, \text{yr}^{-1} \), which corresponds to \( f_{\text{pbh}} = 3.8_{-1.8}^{+2.3} \times 10^{-3} \). For BNSs, we adopt \( R = 1540_{-120}^{+180} \, \text{Gpc}^{-3} \, \text{yr}^{-1} \) (Abbott et al. 2017a). The dotted line shows the background from BBHs with \( M_{\text{min}} = 0.5 \, M_\odot \), by fixing \( \alpha = 1.61 \) and \( R = 80 \, \text{Gpc}^{-3} \, \text{yr}^{-1} \). We also show the expected PI curves for LISA with 4 years of observation (dashed) and LIGO’s observing runs of O2 (black) and design sensitivity (dotted–dashed). The PI curves for LISA and LIGO’s design sensitivity cross the Poisson error region, indicating the possibility to detect this background or set upper limits on the population parameters \( \{ \alpha, R \} \).](image)

### Table 3

Estimates of the Background Energy Density \( \Omega_{\text{GW}}(\nu) \) at the Most Sensitive Frequencies of LIGO (near 25 Hz) and LISA (near \( 3 \times 10^{-3} \, \text{Hz} \)) for Each of the BNS, POBBH (with a Power-law PDF), and Total Background Contributions, along with the 90% Poisson Error Bounds

| \( \Omega_{\text{GW}}(25 \, \text{Hz}) \) | \( \Omega_{\text{GW}}(3 \times 10^{-3} \, \text{Hz}) \) |
|-------------------------------|-------------------------------|
| BNS | 0.7_{-0.4}^{+1.4} \times 10^{-9} | 1.7_{-0.9}^{+1.9} \times 10^{-12} |
| BBH | 1.8_{-1.0}^{+4.2} \times 10^{-9} | 4.3_{-2.5}^{+7.2} \times 10^{-12} |
| Total | 2.5_{-1.0}^{+1.8} \times 10^{-9} | 6.0_{-2.4}^{+4.4} \times 10^{-12} |

### 3.2. Lognormal Mass Function

We now consider another mass distribution, which has a lognormal form (Dolgov & Silk 1993),

\[
P(m) = \frac{1}{\sqrt{2\pi \sigma m}} \exp \left( -\frac{\ln^2(m/m_c)}{2\sigma^2} \right),
\]

where \( m_c \) and \( \sigma \) give the peak mass of \( mP(m) \) and the width of mass spectrum, respectively. In this model, the free parameters are \( \{ \theta, R \} = \{ m_c, \sigma, R \} \).

Accounting for the selection effects and using 3 events from LIGO’s O1 run, we find the best-fit results for \( \theta = \{ m_c, \sigma \} = \{ 14.8 \, M_\odot, 0.65 \} \). Fixing \( \theta \) to their best-fit values, we obtain the median value and the 90% equal tailed credible interval for the local merger rate, \( R = 55_{-13}^{+36} \, \text{Gpc}^{-3} \, \text{yr}^{-1} \). From the posterior distribution of local merger rate \( R \), we then infer the fraction of PBHs in CDM \( f_{\text{pbh}} = 2.8_{-1.3}^{+1.6} \times 10^{-3} \). Such an abundance of PBHs is consistent with previous estimations that \( 10^{-3} \lesssim f_{\text{pbh}} \lesssim 10^{-2} \), confirming that the dominant fraction of CDM should not originate from the stellar-mass PBHs.
The posterior distributions are shown in Figure 10. Compared to the results given in Raidal et al. (2017), we see that, with the sensitivity of LIGO considered, more restrictive constraints on the PDFs could be achieved.

Utilizing Equation (2), we then calculate the corresponding SGWB. The result is shown in Figure 11, indicating that the total SGWB from both POBBHs (with a power-law PDF) and BNSs has a high possibility of being detected by the future observing runs of LIGO/Virgo and LISA. To illustrate the impact of the mass function on the profile of $\Omega_{GW}$, we also plot a dotted line in Figure 11, showing the background with $m_\ast = 30 M_\odot$, by fixing $\{\sigma, R\}$ to their best-fit values. The result indicates that the shifting of the central mass $m_\ast$ to a larger value will decrease the cutoff frequency and increase the magnitude of $\Omega_{GW}$, and vice versa.

The SGWB for the lognormal mass function was earlier calculated in Raidal et al. (2017; see Figure 2 therein). They obtain a larger $\Omega_{GW}$ than ours, indicating that LIGO’s O1 and O2 have the possibility to detect the SGWB from POBBHs. There are a few reasons for this discrepancy. First, Raidal et al. (2017) inferred the parameters of the lognormal PDF by fitting the mass function instead of the merger rate distribution with LIGO’s events and estimate the sensitivity of LIGO by restricting the mass range to $\sim 50 M_\odot$. Here, we should note that the events of LIGO may not represent the intrinsic mass function due to the selection bias, of which the impact was ignored by Raidal et al. (2017). Their best-fits are $\{m_\ast, \sigma\} = \{33 M_\odot, 0.8\}$, which is quite different from ours.
Second, Raidal et al. (2017) used the local merger rate $12 \sim 213$ Gpc$^{-3}$ yr$^{-1}$ derived from SOBBHs (Abbott et al. 2017b), which might serve as a good conservative estimation, to infer the fraction of PBHs, $f_{\text{pbh}}$. In this paper, however, we improve their results by fitting the merger rate distribution using a full hierarchical Bayesian analysis and obtain $R = 55^{+12}_{-38}$ Gpc$^{-3}$ yr$^{-1}$ for POBBHs.

The energy spectra from both the POBBHs (with a lognormal PDF) and BNSs are well approximated by $\Omega_{\text{GW}} \propto \nu^{2/3}$ at low frequencies, covering both LISA and LIGO’s bands, where the dominant contribution is from the inspiral phase. We also summarize the background energy densities $\Omega_{\text{GW}}(\nu)$ at the most sensitive frequencies of LIGO (near 25 Hz) and LISA (near $3 \times 10^{-3}$ Hz) in Table 4.

Figure 12 shows the expected accumulated $S/N$ of LISA as a function of the observing time. The predicted median total background from POBBHs (with a lognormal PDF) and BNSs may be identified with $S/N = 5$ after about 5 hr of observation. The total background could be identified with $S/N = 5$ within 1 hr of observation for the most optimistic case, and after about 3 days for the most pessimistic case. The strain sensitivity curves for LISA are shown in Figure 13. The effect on $S/N$ of MBHB coalescences due to unsolved SGWB signal is shown in Figure 14, indicating the precise measurement of the seeds of MBHs above $10^3 M_\odot$ in high formation redshift will be significantly affected by the confusion noise of the unsolved SGWB.

4. Summary and Discussion

In this paper, we compute the total SGWB arising from both the BBH and BNS mergers. The influences of this SGWB on $LISA$’s detection abilities is also investigated. Two mechanisms for BBH formation, the astrophysical and primordial origins, are considered separately.
For sBHs, we adopt the widely accepted “Vangioni” model (Dvorkin et al. 2016). For the PBHs, we consider two popular but distinctive mass functions: the power-law and lognormal PDFs, respectively. For the power-law case, we infer the local merger rate to be $R = 80.3^{+108}_{-56} \times 10^{-3}$, which corresponds to $f_{\text{pbh}} = 3.8^{+2.3}_{-1.5} 	imes 10^{-3}$; while for the lognormal case, we infer the local merger rate to be $R = 55^{+74}_{-38} \times 10^{-3}$ yr$^{-1}$ and $f_{\text{pbh}} = 2.8^{+1.6}_{-1.3} 	imes 10^{-3}$. Comparing to the lognormal mass function, the power-law one implies a relatively lighter BBH mass distribution and is compensated by a larger local merger rate for consistency with the event rate of LIGO/Virgo. Note that for both PDFs of PBHs, the inferred abundance of PBHs, $f_{\text{pbh}}$, is consistent with previous estimations that $10^{-3} \lesssim f_{\text{pbh}} \lesssim 10^{-2}$, confirming that the dominant fraction of CDM should not originate from the stellar-mass PBHs (Sasaki et al. 2016; Ali-Haïmoud et al. 2017; Raidal et al. 2017; Chen & Huang 2018; Kocis et al. 2018).

The resulting amplitude of the SGWB from PBHs is significantly larger than the previous estimation in Mandic et al. (2016), which adopted the late universe scenario and assumed that all PBHs are of the same mass. There are two reasons to account for this discrepancy. One is that the early universe scenario predicts a much larger local merger rate than the late universe case ($R = 16 \times 10^{-3}$ yr$^{-1}$ in Mandic et al. 2016). Another one is that the merger rate (see Equation (12)) of the early universe model is strongly dependent on the redshift and sharply increases with the redshift. However, the merger rate of the late universe model is weakly dependent on the redshift and slightly increases with the redshift. We refer to Mandic et al. (2016) for more details on the late universe model. We should emphasize that the above discussion applies only to the late universe scenario with a monochromatic mass function. For the late universe scenario with a general mass function, the merger rate could be significantly enhanced, and the amplitude of SGWB could be greatly increased (Clesse & García-Bellido 2017a).

Furthermore, PBHs contribute a stronger (at least comparable if we consider the uncertainties on the formation models of sBHs) SGWB than that from the sBHs (see Figures 1, 6, 11, and also Tables 2–4). This is because merger rate densities from PBHs and sBHs have quite different dependencies on the BH masses and redshift. In particular, the merger rate of PBHs sharply increases with the redshift, while the merger rate of sBHs first increases, then peaks around $z \sim 1 – 2$, and last rapidly decreases with redshift.

In addition, the background energy densities from primordial and astrophysical BBH mechanisms both show no clear deviation from the power-law spectrum, $\Omega_{\text{GW}} \propto \nu^{-2/3}$, within the LIGO/Virgo and LISA sensitivity band. Thanks to their similar effects on the spectra, distinguishing between the backgrounds of POBBHs (the early universe scenario) and SOBBHs will be challenging. However, Clesse & García-Bellido (2017a) claimed that the SGWB of POBBHs from the late universe could potentially deviate $\Omega_{\text{GW}} \propto \nu^{-2/3}$ at the pulsar timing arrays frequencies and even at the frequencies high enough to be probed by LISA. The feature presented in Clesse & García-Bellido (2017a) may be used to distinguish between different formation channels of BBHs.

Finally, the total SGWB from both BBHs (whether of astrophysical or primordial origin) and BNSs has a high possibility of being detected by the future observing runs of LIGO/Virgo and LISA, as could be seen from Figures 1, 2, 6, 7, 11, and 12. This SGWB also contributes an additional source of confusion noise to LISA’s total noise curve (see Figures 3, 8, and 13), and hence weakens LISA’s detection abilities. For instance, the detection of MBHB coalescences is one of the key missions of LISA, and the largest detectable redshift of MBHB mergers can be significantly reduced (see Figures 4, 9, and 14). Therefore, a further analysis is needed to subtract the SGWB signals from the data in order to improve the performance of the detectors.

We thank the anonymous referee for valuable suggestions and comments. We would also like to thank Yun-Kau Lau, Zheng-Cheng Liang, Lang Liu, Hao Wei, Wen Zhao, Yuetong Zhao, and Xiao-Bo Zou for useful conversations. We acknowledge the use of HPC Cluster of ITP-CAS. This work is supported by grants from NSFC (grant No. 11335012, 11575271, 11690021, 11747601), the Strategic Priority Research Program of Chinese Academy of Sciences (grant No. XDB23000000), Top-Notch Young Talents Program of China, and Key Research Program of Frontier Sciences of CAS. This research has made use of data, software and/or web tools obtained from the Gravitational Wave Open Science Center (https://www.gw-openscience.org), a service of the LIGO Laboratory, the LIGO Scientific Collaboration, and the Virgo Collaboration. LIGO is funded by the U.S. National Science Foundation. Virgo is funded by the French Centre National de Recherche Scientifique (CNRS), the Italian Istituto Nazionale della Fisica Nucleare (INFN), and the Dutch Nikhef, with contributions by Polish and Hungarian institutes.
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