Phenomenological Meaning of a Neutrino Mass Matrix
Related to Up-Quark Masses

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Abstract

Recently, a curious neutrino mass matrix has been proposed: it is related to up-quark masses, and it can excellently give a nearly tribimaxial mixing. It is pointed out that, in order to obtain such successful results, three phenomenological relations among masses and CKM parameters must be simultaneously satisfied. This suggests that there must be a specific flavor-basis in which down-quark and charged lepton mass matrices are simultaneously diagonalized.

1 Introduction

Recently, a curious neutrino mass matrix has been proposed by the author [1]: the mass matrix is related to up-quark masses as follows:

\[ M_{\nu} = M_D M_R^{-1} M_D^T , \]  

(1.1)

where the neutrino Dirac mass matrix \( M_D \) is given by \( M_D \propto M_e \) (\( M_e \) is a charged lepton mass matrix), and the right-handed neutrino Majorana mass matrix \( M_R \) is given by

\[ M_R \propto M_e M_{1/2}^{\alpha} + M_{1/2}^{\alpha} M_e. \]  

(1.2)

The mass matrix (1.1) with (1.2) has been derived from an idea that the origin of the mass spectra (i.e. effective Yukawa coupling constants) is due to vacuum expectation values (VEV) structures of gauge singlet scalars \( \Phi_{ij} \). (The details are reviewed in the next section.) In order to obtain the lepton mixing matrix \( U \), one must know forms of \( M_D \) and \( M_{u1/2}^{\alpha} \) in the “e-basis” (we refer to a diagonal basis of the mass matrix \( M_f \) as “f-basis”). The form \( M_D = M_e \) is given by \( M_e = \text{diag}(m_e, m_{\mu}, m_{\tau}) \) in the e-basis. For the form \( M_u^{1/2} \), by analogy with the relation \( M_u = V^T D_u V \) in the d-basis, where \( D_u = \text{diag}(m_u, m_c, m_t) \) and \( V \) is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix (and note that a mass matrix \( M_f \) is diagonalized as a form \( U_f^T M_f U_f = D_f \) in the present model because we assume an O(3) flavor symmetry as we mention it in Sec.2), we assume that \( M_u^{1/2} \) in the e-basis is given by a form

\[ M_u^{1/2} = V^T(\delta) D_u^{1/2} V(\delta), \]  

(1.3)

where we have adopted the standard expression \( V S D \) [2] as a phase convention of the CKM matrix \( V(\delta) \). In order to estimate the form \( M_\nu \), we use the following observed up-quark masse values at an energy scale of the weak interactions \( \mu = m_Z \) [3], \( m_u = 0.00127 \) GeV, \( m_c = 0.619 \) GeV, \( m_t = 171.7 \) GeV, and the observed CKM mixing parameters (best-fit values) [4] \( |V_{us}| = \)
Table 1: \( \delta \) dependence of predicted values in the standard phase convention of \( V(\delta) \). Here, \(|V_{us}|\), \(|V_{cb}|\) and \(|V_{ub}|\) have been used as three input values of the four independent parameters of \( V(\delta) \). The best-fit value of \( \delta \) in the quark sector is \( \delta_q = 69.8^\circ \) from the observed CKM matrix data.

| \( \delta \) | \( \sin^2 2\theta_{23} \) | \( \tan^2 \theta_{12} \) | \( |U_{13}| \) | \( \Delta m^2_{21}/\Delta m^2_{32} \) |
|---|---|---|---|---|
| 0 | 0.4803 | 0.4745 | 0.01042 | 0.00196 |
| 60° | 0.7631 | 0.4801 | 0.00844 | 0.00139 |
| 69.8° | 0.8127 | 0.4851 | 0.00781 | 0.00127 |
| 90° | 0.9028 | 0.5017 | 0.00615 | 0.00102 |
| 120° | 0.9688 | 0.5277 | 0.00386 | 0.00081 |
| 180° | 0.9952 | 0.5525 | 0.00094 | 0.00068 |

0.2257, \(|V_{cb}| = 0.0415\) and \(|V_{ub}| = 0.00359\) together with the observed charged lepton masses. (Here, since we use the values at \( \mu = m_Z \) for the CKM matrix parameters, we also use the running mass values at \( \mu = m_Z \).) Then, one can successfully obtain a nearly tribimaximal mixing [5],

\[
U = \begin{pmatrix}
+0.8026 & -0.5966 & -0.0009 \\
-0.4356 & -0.5871 & +0.6823 \\
+0.4076 & +0.5472 & +0.7311
\end{pmatrix},
\]

(1.4)

for \( \delta = \pi \), i.e.

\[
\sin^2 2\theta_{23} = 0.9952, \quad \tan^2 \theta = 0.5525, \quad |U_{13}| = 0.00094.
\]

(1.5)

For reference, we give phase-dependence of the numerical results in Table 1. The best-fit values [4] of the CKM mixing parameters show \( \delta = 69.8^\circ \). However, as seen in Table 1, the predicted value of \( \sin^2 2\theta_{23} \) at \( \delta \approx 69.8^\circ \) is in poor agreement with the observed value \( \sin^2 2\theta_{23} = 1.00_{-0.13}^{+0.06} \) [6], although the predicted value of \( \tan^2 \theta_{12} \) is roughly in agreement with the observed value \( \tan^2 \theta_{12} = 0.47_{-0.08}^{+0.06} \) [7]. As stated in the next section, since the flavor-basis transformation matrix is confined to an orthogonal matrix because the present model is based on an O(3) flavor symmetry, the phase parameter \( \delta \) must be 0 or \( \pi \).

We also list numerical results for the original Kobayashi-Maskawa phase convention [8] in Table 2. As seen in Table 2, not only the both cases, \( \delta = 0 \) and \( \delta = \pi \), but also any values of \( \delta \) cannot give a reasonable value of \( \sin^2 2\theta_{23} \). Thus, we find that the phenomenological success is only for the case of \( V(\delta) = V_{SD}(\delta) \) (not for the original KM phase convention of CKM matrix).

In order to obtain the phenomenological success, it is essential to assume not only the neutrino mass matrix form (1.1) with (1.2), but also forms of flavor-basis transformation matrices \( U_{ud} \) and \( U_{ue} \)

\[
U_{ud} = V_{SD}(\delta_q) \quad (\delta_q \approx 70^\circ),
U_{ue} = V_{SD}(\delta_R) \quad (\delta_R = 180^\circ),
\]

(1.6)

where \( U_{ff'} \) transforms a matrix in an \( f \)-basis into that in an \( f' \)-basis, and \( V_{SD}(\delta) \) is the standard
Table 2: $\delta$ dependence of predicted values in the original Kobayashi-Maskawa phase convention of $V(\delta)$. Here, $|V_{us}|$, $|V_{td}|$ and $|V_{ub}|$ have been used as three input values of the four independent parameters of $V(\delta)$. The best-fit value of $\delta$ in the quark sector is $\delta_q = 90.8^\circ$ from the observed CKM matrix data.

| $\delta$  | $\sin^2 2\theta_{23}$ | $\tan^2 \theta_{12}$ | $|U_{13}|$ | $\Delta m_{21}^2/\Delta m_{32}^2$ |
|-----------|------------------|----------------|---------|-------------------------------|
| 0         | 0.7821           | 0.5074         | 0.00769 | 0.00093                       |
| 60$^\circ$| 0.8088           | 0.3587         | 0.0303  | 0.0052                        |
| 90$^\circ$| 0.8781           | 0.1862         | 0.0614  | 0.04269                       |
| 120$^\circ$| 0.8482          | 0.3523         | 0.03303 | 0.00752                       |
| 180$^\circ$| 0.8369          | 0.5028         | 0.00329 | 0.00169                       |

phase convention of the CKM matrix with the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ as three input values of the four independent parameters of $V_{SD}$.

In Sec.2, we give a short review of the model which leads to the mass matrix (1.1) with (1.2). In Sec.3, we investigate relations between conditions for tribimaximal mixing and the empirical neutrino mass matrix (1.1) with (1.2) and (1.3) from the phenomenological point of view. One will find that three phenomenological relations among the masses and CKM matrix parameters must simultaneously be satisfied in order to get a nearly tribimaximal mixing. As we state in Sec.3, it is hard to consider that such the simultaneous coincidence are accidental, so that it should be considered that such phenomenological relations originate in a common law. In Sec.4, we speculate possible forms of the mass matrices $M_d$ and $M_e$. It is concluded that there is a flavor basis in which the down-quark and charged lepton mass matrices, $M_d$ and $M_e$, are simultaneously diagonalized. Finally, Sec.5 is devoted to summary and remarks.

2 Model

The neutrino mass matrix related to up-quark masses has first been derived on the basis of a U(3) flavor symmetry model [9], and then, the form (1.1) with (1.2) has been derived on the basis of an O(3) flavor symmetry model [1]. In this section, we give a short review of the O(3) model.

It is assumed that effective Yukawa coupling constants $Y_f^{eff}$ are given by VEVs $\langle Y_f \rangle$ of gauge singlet scalars $Y_f$ (for convenience, we refer to those fields as “Yukawons”) which belong to $(3 \times 3)_S = 1 + 5$ of an O(3) flavor symmetry:

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} U_i(Y_u)_{ij} Q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} D_i(Y_d)_{ij} Q_j H_d$$

$$+ \sum_{i,j} \frac{y_e}{\Lambda} L_i(Y_e)_{ij} N_j H_u + \sum_{i,j} \frac{y_e}{\Lambda} L_i(Y_e)_{ij} E_j H_d + h.c. + \sum_{i,j} y_R N_i(Y_R)_{ij} N_j,$$

(2.1)

where $Q$ and $L$ are quark and lepton SU(2)$_L$ doublet fields of O(3)$_F$ triplets, and $U$, $D$, $N$, and $E$ are SU(2)$_L$ singlet matter fields of O(3)$_F$ triplets, and $\Lambda$ is an energy scale of an effective theory.
Since we assume the O(3) flavor symmetry, the Yukawaons $Y_f \ (f = u, d, e, \nu, R)$ are symmetric. Under this definition of $(Y_f)_{ij}$ given by Eq.(1.1), the VEV matrix $(Y_f^T)_{ij}$ are diagonalized as $U_f^T (Y_f)_{ij} U_f = (Y_f^T)^D$, where the index $D$ means that the matrix is diagonal, and the quark and lepton mixing matrices $V$ and $U$ are given by $V = U_u^T U_d$ and $U = U_e^T U_\nu$, respectively. In order to distinguish the Yukawaons $Y_f$ from each other, the following U(1)$_X$ charges are assigned: $Q_X(Y_f) = x_f \ (f = u, d, \nu, e)$, $Q_X(U) = -x_u, Q_X(E) = -x_e$, and so on. The field $Y_R$ has a charge $Q_X(Y_R) = 2x_\nu$.

One writes a superpotential $W$ under the following conditions: (i) Terms consist of, at most, holomorphic cubic terms of the fields, and do not contain higher dimensional terms, except for the Yukawa interaction terms $W_Y$; (ii) Those are invariant under the O(3)$_F$ and U(1)$_X$ symmetries. (iii) Yukawaons $Y_f$ always behave as a combination of $1 + 5$, so that, for example, 5 alone never appears in the interaction terms.

The VEV spectra $\langle Y_f \rangle$ are evaluated from supersymmetric (SUSY) vacuum conditions for a superpotential $W = W_Y + W_u + W_d + W_e + W_\nu + W_R$, where $W_f \ (f = u, d, \nu, e, R)$ play a role in fixing the VEV structures $\langle Y_f \rangle$. (Since one can easily show $\langle Q \rangle = \langle L \rangle = \langle U \rangle = \langle D \rangle = \langle N \rangle = \langle E \rangle = 0$, hereafter, the term $W_Y$ is dropped from the superpotential $W$ as far as the VEV structures of $Y_f$ are investigated.) For example, a spectrum of $\langle Y_u \rangle$ is obtained from the following superpotential terms $W_u$:

$$W_u = \lambda_u \text{Tr}[\Phi_u \Phi_u A_u] + \mu_u \text{Tr}[Y_u A_u] + W_{\Phi_u},$$

(2.2)

where a new filed $A_u$ has U(1)$_X$ charge $Q_X = -x_u$. Here, the term $W_{\Phi_u}$ has been introduced in order to fix eigenvalues of $\langle \Phi_u \rangle$. Since the purpose of the present paper is not to discuss quark and lepton mass spectra, an explicit form of $W_{\Phi_u}$ is given in Appendix A. Since $W_{\Phi_u}$ contains $Y_u$ and $\Phi_u$ as shown in Appendix A, SUSY vacuum conditions $\partial W/\partial Y_u = 0$ and $\partial W/\partial \Phi_u = 0$ will be discussed in Appendix A. From a SUSY vacuum condition $\partial W/\partial A_u = 0$ (for the moment, one regards $W_u$ as $W$), one obtains

$$\frac{\partial W}{\partial A_u} = 0 = \lambda_u \Phi_u \Phi_u + \mu_u Y_u,$$

(2.3)

so that one obtains a bilinear relation

$$\langle Y_u \rangle = -\frac{\lambda_u}{\mu_u} \langle \Phi_u \rangle \langle \Phi_u \rangle,$$

(2.4)

i.e. the field $\Phi_u$ plays a role of $M_u^{1/2}$ in Eq.(1.2). For convenience, we refer to $\Phi_f$ as “ur-Yukawaons”. The ur-Yukawaons $\Phi_f$ play a role in fixing VEV spectra of Yukawaons. Although we consider 5 Yukawaons $Y_f \ (f = u, d, e, \nu, R)$, we will consider only 2 ur-Yukawaons $\Phi_e$ and $\Phi_u$ in the present model. Note that, since the matrix $\langle \Phi_u \rangle$ is not Hermitian, the relation

$$U_u^T \langle Y_u \rangle U_u = \langle Y_u \rangle^D \propto \text{diag}(m_u, m_e, m_\nu),$$

(2.5)

does not always mean

$$U_u^T \langle \Phi_u \rangle U_u = \langle \Phi_u \rangle^D \propto \text{diag}(\sqrt{m_u}, \sqrt{m_e}, \sqrt{m_\nu}),$$

(2.6)
where $D$ denotes that the matrix is diagonal. As one sees later, one needs the relation (2.6). Therefore, we assume the field $\Phi_u$ (and also $Y_u$) is real, so that the matrix $U_u$ is orthogonal matrix.

For the charged lepton sector, we also assume superpotential term $W_e$ similar to the up-quark sector:

$$ W_e = \lambda_e \text{Tr}[\Phi_e \Phi_e A_e] + \mu_e \text{Tr}[Y_e A_e] + W_{\Phi_e}, \quad (2.7) $$

where $\Phi_e$, $Y_e$ and $A_e$ have U(1)$_X$ charges $\frac{1}{2} x_e$, $x_e$ and $-x_e$, respectively, so that one obtains relations

$$ Y_e = \frac{\lambda_e}{\mu_e} \Phi_e, \quad (2.8) $$

with $\Phi^D_e \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, where one has again assumed that the field $\Phi_e$ is real. (Here and hereafter, for simplicity, we will sometimes express VEV matrices $\langle M \rangle$ as simply $M$.)

Next, let us investigate a possible form of $W_R$. We introduce a new field $A_R$ with U(1)$_X$ charge $Q_X = -2x_\nu$. In order to obtain the relation (1.2), we assume the following form of $W_R$:

$$ W_R = \lambda_R \text{Tr}[(Y_e \Phi_u + \Phi_u Y_e)A_R] + \mu_R \text{Tr}[Y_R A_R] + \lambda_R \xi \text{Tr}[Y_\nu Y_\nu A_R], \quad (2.9) $$

where we have assumed a relation among the U(1)$_X$ charges,

$$ 2x_\nu = \frac{1}{2} x_u + x_e. \quad (2.10) $$

From SUSY vacuum conditions $\partial W/\partial Y_R = 0$, one obtains $A_R = 0$. Then, the requirement $\partial W/\partial Y_e = 0$ leads to the condition $\partial W_e/\partial Y_e = 0$, so that one obtains the relation (2.8). From $\partial W/\partial A_R = 0$, one obtains

$$ Y_R = -\frac{\lambda_R}{\mu_R} (Y_e \Phi_u + \Phi_u Y_e + \xi Y_\nu Y_\nu). \quad (2.11) $$

The third term ($\xi$-term) does not affect a form of the lepton mixing matrix $U$ because the term gives a constant term proportional to a unit matrix I as shown later. Thus, one can obtain the desirable form (1.2) of $Y_R$.

Next, we discuss how to obtain $\langle Y_\nu \rangle = \langle Y_e \rangle$. The simplest assumption to obtain a relation $M_D \propto M_e$ (i.e. $Y_\nu \propto Y_e$) is to assume that the fields $N$ and $E$ have the same U(1)$_X$ charges (i.e. $x_\nu = x_e$), and to consider a model without $Y_\nu$. However, then, one obtains $x_\nu = x_e = x_u/2$ from the relation (2.10), so that $Y_e$ and $\Phi_u$ (and also $Y_u$ and $Y_R$) have the same U(1)$_X$ charges. This brings some additional terms into $W_u$, $W_e$ and $W_R$ due to the mixings between $Y_e$ and $\Phi_u$ and between $Y_u$ and $Y_R$, so that one cannot obtain desirable relations without ad hoc selections of those terms. Therefore, in order to obtain the relation $Y_\nu \propto Y_e$ with $x_\nu \neq x_e$, we assume the following structure of $W_\nu$:

$$ W_\nu = \lambda_{\nu e} \phi_\nu \text{Tr}[Y_\nu A_\nu] + \lambda_{\nu e} \phi_e \text{Tr}[Y_e A_\nu], \quad (2.12) $$

5
where $\phi_\nu$ and $\phi_e$ are gauge- and flavor-singlet fields, and we assign $U(1)_X$ charges as $Q_X(A_\nu) = x_\nu$, $Q_X(\phi_\nu) = -(x_\nu + x_\nu)$ and $Q_X(\phi_e) = -(x_e + x_e)$. From $\partial W/\partial \phi_\nu = 0$ and $\partial W/\partial \phi_e = 0$, one obtains $A_\nu = 0$. From $\partial W/\partial A_\nu = 0$, one obtains

$$Y_\nu = -\frac{\lambda_{\nu e} \phi_e}{\lambda_{\nu \nu} \phi_\nu} Y_e. \quad (2.13)$$

In order to obtain a neutrino mixing matrix form in the $e$-basis, one must know a matrix form of $\langle \Phi_u \rangle$ in the $e$-basis, although the form $\langle \Phi_u \rangle^D$ on the $a$-basis is given by Eq.(2.6). (Now, “$f$-basis” is defined as a flavor basis in which the VEV matrix $\langle Y_f \rangle$ is diagonal.) Let us define a transformation of a VEV matrix $\langle Y_f \rangle$ from a $b$-basis to an $a$-basis as

$$\langle Y_f \rangle_a = U_{ba}^T \langle Y_f \rangle_b U_{ba}, \quad (2.14)$$

where $U_{ab}$ are unitary matrices, and $\langle Y_f \rangle_a$ denotes a VEV matrix form on the $a$-basis. The unitary matrices $U_{ab}$ satisfy $U_{ab}^T = U_{ba}$ and $U_{ab}U_{bc} = U_{ac}$. (These operators $U_{ab}$ are not always members of $O(3)$ flavor-basis transformations.) Since $Y_f^T = Y_f$ in the present model, the VEV matrix $\langle Y_f \rangle$ are diagonalized as $U_f^T \langle Y_f \rangle U_f = (Y_f^f)^D \equiv \langle Y_f^f \rangle^D$. Therefore, $\langle Y_u \rangle_d$ is given by

$$\langle Y_u \rangle_d = V^T(\delta) \langle Y_u \rangle_u V(\delta), \quad (2.15)$$

where $V(\delta)$ is the standard expression of the CKM matrix. The simplest assumption is to consider that the $d$-basis is identical with the $e$-basis, and then, one can regard $U_{ue}$ as $U_{ue} = V$ because $U_{ud} = V$. However, since one has assumed that $Y_u$ and $Y_e$ are real, the flavor-basis transformation matrix $U_{ue}$ must be orthogonal, i.e. the phase parameter $\delta$ is 0 or $\pi$ even if one assumes the form $U_{ue} = V(\delta)$. As one has already seen in Table 1, the case with $\delta = \pi$ can give reasonable numerical results.

Anyhow, one assumes the form

$$\langle \Phi_u \rangle_e = U_{ue}^T \langle \Phi_u \rangle_u U_{ue} = V^T(\delta) \langle \Phi_u \rangle^D V(\delta), \quad (2.16)$$

one can obtain the following phenomenological neutrino mass matrix

$$\langle M_\nu \rangle_e = k_\nu Y_e^D \left[ V^T(\delta) \Phi_u^D V(\delta) Y_e^D + Y_e^D V^T(\delta) \Phi_u^D V(\delta) + \xi Y_u Y_u \right]^{-1} Y_e^D$$

$$= k_\nu \left[ (Y_e^D)^{-1} V^T(\delta) \Phi_u^D V(\delta) + V^T(\delta) \Phi_u^D V(\delta) (Y_e^D)^{-1} + \xi_0 1 \right]^{-1}, \quad (2.17)$$

where $Y_e^D \propto \text{diag}(m_e, m_\mu, m_\tau)$ and $\Phi_u^D \propto \text{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_s})$. The third term ($\xi_0$ term) does not affect the lepton mixing matrix $U$. Rather, the existence of the $\xi_0$ term is useful to adjust the value of $\Delta m_{21}^2/\Delta m_{32}^2$ because the predicted values in Table 1 were considerably small compared to the observed value $|R| = 0.028 \pm 0.004$, where one has used the observed values $|R| = 0.028 \pm 0.004$, where one has used the observed values.

### 3 Conditions for a tribimaximal mixing

In this section, we investigate what phenomenological relations are required for the mass matrix (2.17) in order to give a nearly tribimaximal mixing. Since one know [10] that a mixing
matrix for \((M_\nu)^{-1}\) is given by \(U^*\) when a mixing matrix for \(M_\nu\) is given by \(U\), for the purpose to obtain conditions for a tribimaximal mixing, one can investigate the following matrix

\[
M = (Y_e^D)^{-1}V^T(\delta)\Phi_u^D V(\delta) + V^T(\delta)\Phi_u^D V(\delta)(Y_e^D)^{-1} + \xi_0 1,
\]  
(3.1)
i.e.

\[
M_{ij} = \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}}\right) \sum_k \sqrt{m_{uk}V_{ki}V_{kj}},
\]  
(3.2)

instead of the mass matrix (2.17). Since the \(\xi_0\)-term is not essential for evaluating the mixing matrix \(U\), hereafter, we put \(\xi_0 = 0\). (Although a similar study has been done in Ref.[9] based on a \(U(3)\) flavor symmetry, where the VEV matrix \(\langle \Phi_u^e \rangle\) has been given by \(\langle \Phi_u^e \rangle = V^\dagger(\delta)\langle \Phi_u^u \rangle V(\delta)\), in the present \(O(3)\) model, the VEV matrix \(\langle \Phi_u^e \rangle\) is given by \(\langle \Phi_u^e \rangle = V^T(\delta)\langle \Phi_u^u \rangle V(\delta)\).

As shown in Appendix, the conditions to obtain the maximal \(2 \leftrightarrow 3\) mixing, i.e.

\[
\sin^2 2\theta_{23} \equiv 4|U_{23}|^2|U_{33}|^2 = 1, \quad |U_{13}|^2 = 0,
\]  
(3.3)

are

\[
|M_{12}| = |M_{13}|,
\]  
(3.4)

and

\[
|M_{22}| = |M_{33}|.
\]  
(3.5)

From Eq.(3.2), one obtains

\[
M_{12} \simeq \sqrt{m_e} \frac{V_{21}V_{22}}{m_e},
\]  
(3.6)
\[
M_{13} \simeq \sqrt{m_t} \frac{V_{31}V_{33}}{m_e},
\]  
(3.7)
\[
M_{22} \simeq 2\sqrt{m_t} \frac{V_{23}^2}{m_\mu},
\]  
(3.8)
\[
M_{33} \simeq 2\sqrt{m_t} \frac{V_{33}}{m_\tau},
\]  
(3.8)

where one has assumed a hierarchical structure of \(|V_{ij}|\) similar to the observed CKM matrix. The condition (3.5) requires

\[
\sqrt{\frac{m_e}{m_t}} \simeq \frac{m_\mu}{m_\tau},
\]  
(3.9)

The left- and right-hand sides of Eq.(3.9) give values [3] 0.060 and 0.059, respectively. Therefore, the condition (3.5) is phenomenologically well satisfied. On the other hand, the condition (3.4) requires

\[
\sqrt{\frac{m_e}{m_t}} \simeq \frac{|V_{31}|}{|V_{21}|}.
\]  
(3.10)
In order to evaluate the relation (3.10), one uses a relation

$$\frac{V_{31}}{V_{21}} = -\left(\frac{V_{23}^*}{V_{33}^*} + \frac{V_{11}^* V_{13}^*}{V_{21}^* V_{33}^*}\right), \quad (3.11)$$

from the unitary relation $V_{11} V_{13}^* + V_{21} V_{23}^* + V_{31} V_{33}^* = 0$. For a standard expression of the CKM matrix, Eq.(3.11) leads to

$$\frac{V_{31}}{V_{21}} \simeq -\left(|V_{cb}| - \frac{|V_{ub}|}{|V_{us}|} e^{i\delta}\right). \quad (3.12)$$

The left-hand side of Eq.(3.10) is 0.060, and the right-hand side is 0.0412 + 0.0174 = 0.0586 for $\delta = \pi$. Thus, the condition is also well satisfied. Note that if the observed value of $|V_{td}|$, $|V_{td}| = 0.00874$ [4], as the value $|V_{31}|$ is used, the condition (3.10) cannot be satisfied. This is a reason for that when one used the original Kobayashi-Maskawa phase convention instead of the standard CKM matrix expression, one could not give a nearly tribimaximal mixing as seen in Table 2.

Next, we check the condition to give $\tan^2 \theta_{12} = 1/2$,

$$\eta^2 \left((M_{22} M_{33})^{1/2} + M_{23}\right) - M_{11} = \eta(M_{12} M_{13})^{1/2}, \quad (3.13)$$

(see (B.16) in Appendix B). From Eqs.(3.6) - (3.8) and

$$M_{11} \simeq 2\sqrt{m_t} \left(\sqrt{\frac{m_e}{m_t}} V_{21}^2 + \sqrt{\frac{m_u}{m_t}} V_{11}^2\right), \quad (3.14)$$

$$M_{23} \simeq \sqrt{\frac{m_t}{m_u}} V_{32} V_{33}, \quad (3.15)$$

one finds $|M_{22} + M_{23}| \ll |M_{11}|$, so that the condition (3.13) requires $M_{23} \simeq M_{11}$ ($\eta = -1$ in the present case). The condition $M_{23} \simeq M_{11}$ requires

$$|V_{us}| + \frac{1}{|V_{us}|} \sqrt{\frac{m_u}{m_t}} \simeq \frac{1}{2}. \quad (3.16)$$

The left-hand side of Eq.(3.16) gives 0.2257 + 02007 = 0.4264. Considering the present rough approximation, one may consider that the condition (3.13) is roughly satisfied.

In conclusion, in order to obtain a tribimaximal mixing, the three phenomenological relations (3.9), (3.10) and (3.16) must simultaneously be satisfied. It is hard to consider that such the simultaneous coincidences are accidental. Rather, it should be considered that such the phenomenological relations originate in a common law. Also, one must note that, in order to satisfy the condition (3.10), one must take the standard expression of the CKM matrix and use the observed values $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ in order to fix the three rotation angles in the CKM matrix. This suggests that the down-quark mass matrix $M_d$ has a similar structure with
the charged lepton mass matrix $M_e$. In the next section, we will investigate a possible relation between $M_d$ and $M_e$.

4 Possible structures of $M_d$ and $M_e$

In this section, we speculate possible mass matrix forms of the down-quark and charged lepton mass matrices $M_d$ and $M_e$ which lead to the assumption (1.6).

Generally, there are 9 phase conventions of the CKM matrix $V$ [11]:

$$V(m, n) = R_m P_l R_n \quad (m \neq \ell \neq n), \quad (4.1)$$

where $m, n, \ell = 1, 2, 3$, and

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (4.2)$$

$$P_1 = \text{diag}(e^{i\delta}, 1, 1), \quad P_2 = \text{diag}(1, e^{i\delta}, 1), \quad P_3 = \text{diag}(1, 1, e^{i\delta}). \quad (4.3)$$

($c = \cos \theta$ and $s = \sin \theta$). For example, the standard expression $V_{SD}$ of the CKM matrix

$$V_{SD}(\delta) = R_1(\theta_{23}) P_3(\delta) R_2(\theta_{13}) P_3(-\delta) R_3(\theta_{12}), \quad (4.4)$$

is rewritten as

$$V_{SD}(\delta) = e^{i\delta} P_1(-\delta) R_1(\theta_{23}) P_2(-\delta) R_2(\theta_{13}) R_3(\theta_{12}) P_3(-\delta), \quad (4.5)$$

because $P_3(\delta) = e^{i\delta} P_1(-\delta) P_2(-\delta)$. Since the factors $e^{i\delta} P_1(-\delta)$ and $P_3(-\delta)$ in the left- and right-hand sides can be absorbed into the unobservable phases of up- and down-quarks, respectively, the standard expression $V_{SD}$ corresponds to the expression $V(1, 3)$ defined in (4.1).

In the O(3) model, where the mass matrices are symmetric, the mass matrices $M_u$ and $M_d$ are diagonalized as

$$U_u^T M_u U_u = D_u, \quad U_d^T M_d U_d = D_d, \quad (4.6)$$

and the CKM matrix $V$ is given by

$$V = U_u^T U_d. \quad (4.7)$$

As seen in the general expressions of $V$ given in (4.1), one can always find a flavor basis (we refer to it as a “$x$-basis”) in which the $CP$-violating phases are factorized as

$$\langle Y_u \rangle_x = P_u(\delta_u) \langle \tilde{Y}_u \rangle_x P_u(\delta_u), \quad \langle Y_d \rangle_x = P_u(\delta_d) \langle \tilde{Y}_d \rangle_x P_u(\delta_d), \quad (4.8)$$

where $\langle \tilde{Y}_u \rangle_x$ and $\langle \tilde{Y}_d \rangle_x$ are real matrices, and they are diagonalized by rotation matrices $R_u$ and $R_d$ as

$$R_u^T \langle \tilde{Y}_u \rangle_x R_u = D_u, \quad R_d^T \langle \tilde{Y}_d \rangle_x R_d = D_d, \quad (4.9)$$
respectively. Then, since $U_u = P_n(-\delta_u)R_u$ and $U_d = P_n(-\delta_d)R_d$, one obtains the expression of the flavor-basis transformation operator $U_{ud}$

$$U_{ud} = V = R_u^T P_n(\delta_u - \delta_d)R_d. \quad (4.10)$$

Similarly, one can obtain an expression of $U_{ue}$ as follows:

$$U_{ue} = R_u^T P_n(\delta_u - \delta_e)R_e. \quad (4.11)$$

Now, let us return to our model. As seen in Sec.3, the requirement (1.6) for a nearly tribimaximal mixing is rewritten as

$$U_{ud} = R_1(\theta_{23})R_3(\theta_q)R_2(\theta_{13})P_3(-\delta_1)R_3(\theta_{12}),$$
$$U_{ue} = R_1(\theta_{23})R_3(\theta_q)R_2(\theta_{13})P_3(-\delta_\ell)R_3(\theta_{12}), \quad (4.12)$$

where the rotation angles are fixed by the observed CKM mixing data as

$$\theta_{13} = \sin^{-1}|V_{ub}|,$$
$$\theta_{23} = \sin^{-1}(|V_{cb}|/\sqrt{1-|V_{ub}|^2}),$$
$$\theta_{12} = \sin^{-1}(|V_{us}|/\sqrt{1-|V_{ub}|^2}), \quad (4.13)$$

and the phase parameters are taken as $\delta_q \simeq 70^\circ$ and $\delta_\ell = 180^\circ$. This suggests that the mass matrices $M_d$ and $M_e$ in the $x$-basis are diagonalized by the same rotation matrix

$$R_d = R_2(\theta_{13}^d)R_3(\theta_{12}), \quad (4.14)$$

while the up-quark mass matrix $M_u$ in the $x$-basis is diagonalized by

$$R_u = R_2^T(\theta_{13}^u)R_1^T(\theta_{23}), \quad (4.15)$$

where $\theta_{13} = \theta_{13}^d - \theta_{13}^u$, $\theta_{23}$ and $\theta_{12}$ are given by (4.13), and the phase parameters are given by

$$\delta_q = \delta_d - \delta_u \simeq 70^\circ, \quad \delta_\ell = \delta_e - \delta_u = 180^\circ. \quad (4.16)$$

(Since we have assumed that $Y_u$ and $Y_e$ are real in the present model, the phase factors $\delta_u$ and $\delta_e$ must be 0 or $\pi$.) Therefore, forms of the mass matrices $M_u$, $M_d$ and $M_e$ in the $x$-basis are given by

$$\langle Y_u \rangle_x = P_2(\delta_u)R_2^T(\theta_{13}^u)R_1^T(\theta_{23})D_uR_3^T(\theta_{12})R_2(\theta_{13})P_2(\delta_d),$$
$$\langle Y_d \rangle_x = P_2(\delta_d)R_2(\theta_{13}^d)R_3(\theta_{12})D_uR_3^T(\theta_{12})R_2^T(\theta_{13}^d)P_2(\delta_d),$$
$$\langle Y_e \rangle_x = P_2(\delta_e)R_2(\theta_{13}^e)R_3(\theta_{12})D_eR_3^T(\theta_{12})R_2^T(\theta_{13}^e)P_2(\delta_e). \quad (4.17)$$

In other words, one can choose such a $x$-basis in which the mass matrices $M_d$ and $M_e$ are diagonalized simultaneously, and CP-violating phase factors are factorized as shown in (4.17).

5 Concluding remarks
When one consider a neutrino mass matrix form
\[ M_\nu \propto (\langle \Phi_e \rangle^m \langle \Phi_u \rangle^n + \langle \Phi_u \rangle^n \langle \Phi_e \rangle^m)^{-1}, \] (5.1)
one can find that a case which can give a reasonable lepton mixing is only a case with \( m = -2 \) and \( n = 1 \), even if one consider any form of \( U_{ue} \). (This is related to the observed fact \( \sqrt{m_e/m_\tau} \approx m_\mu/m_\tau \).) One also find that the case with \( m = -2 \) and \( n = 1 \) can lead to a nearly tribimaximal mixing only when one assume \( U_{ue} = V_{SD}(\pi) \), where \( V_{SD}(\delta) \) is the standard expression of the CKM matrix with the inputs \( |V_{us}|, |V_{cb}| \) and \( |V_{ub}| \). Therefore, in the present paper, it has been investigated what structure of the neutrino mass matrix form (1.2) play an essential role in giving a nearly tribimaximal mixing. We have found that, in order to obtain such a nearly tribimaximal mixing, we need to accept the three phenomenological relations (3.9), (3.10) and (3.16). It is hard to consider that such the relations accidentally hold, so that we consider that the ad hoc assumption \( U_{ue} = V(\delta_\ell) \) has an underlying meaning. In Sec.4, we have investigated possible structures of the down-quark and charged lepton mass matrices. We have concluded that there must be a specific flavor basis in which the down-quark and charged mass matrices are simultaneously diagonalized.

In the present model, an \( O(3) \) flavor symmetry has been assumed. Relations which are obtained from the \( O(3)_F \) invariant superpotential by using SUSY vacuum conditions hold only in flavor bases which are connected by an orthogonal transformation. Therefore, in order to use those relations in the \( e \)-basis and/or \( u \)-basis, it has been assumed that \( \langle \Phi_e \rangle \) and \( \langle \Phi_u \rangle \) are real and the \( e \)-basis and \( u \)-basis can be connected by an orthogonal transformation \( U_{ue} \). On the other hand, one knows that \( \langle Y_d \rangle \) cannot be real because of the observation of \( CP \) violating phenomena in the quark sector. Therefore, one cannot use the relations from the SUSY vacuum conditions in the \( d \)-basis. (However, this does not mean that one cannot build a down-quark mass matrix model. Relations including Yukawaon \( Y_d \) still hold in the \( u \)-basis.)

In spite of such disadvantage of the \( O(3)_F \) model, the reason that one consider \( O(3) \) flavor symmetry is as follows: If we consider a \( U(3) \) flavor symmetry, the Yukawaon \( Y_R \) (and also \( Y_u \) in a grand unification scenario) must be a \( 6 \) of \( U(3)_F \). It is difficult to build a \( U(3)_F \) invariant superpotential for \( Y_R \) without considering higher dimensional terms. (For example, a Yukawaon model based on a \( U(3)_F \) symmetry is found in Ref.[9]. However, the superpotential term for \( Y_R \) in the \( U(3)_F \) model is somewhat intricate.) In order to build a simpler model for \( Y_R \), one will be obliged to adopt an \( O(3)_F \) model.

In the present scenario, it is assumed that there are no higher dimensional terms with \( (1/\Lambda)^n \) \( (n \geq 1) \) in the superpotential except for the effective Yukawa interaction terms \( W_Y \), Eq.(2.1). Although we want to build a model of \( W_Y \) without any higher dimensional terms, at present, we have no idea for such a model. It is a future task to us.

So far, we have not discussed a structure of \( W_d \) which gives a down-quark mass matrix \( \langle Y_d \rangle \), although an attempt to give such \( W_d \) has been proposed in Ref.[1]. Since this is not the question of the moment in the present paper, we did not discuss. We will discuss a possible structure of \( W_d \) elsewhere.

Although the present approach to the masses and mixings of quarks and leptons is not conventional and not yet established, this approach will become one of the promising approaches....
because one can treat the masses and mixings without discussing explicit forms of the Yukawa coupling constants.

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Appendix A: An example of $W_{\phi f}$

The superpotential term $W_{\phi u}$ in Eq.(2.2) has been introduced to fix the VEV spectrum of the ur-Yukawaon $\Phi_u$. In this appendix, we demonstrate an example of $W_{\phi u}$.

When one introduces a further new field $B_u$ with a U(1)$_X$ charge $Q_X = -(3/2)x_u$, one can have a term $\text{Tr}[\Phi_u Y_u B_u]$. However, of course, if one has only this term, one cannot fix the eigenvalues of $\langle \Phi_u \rangle$, because one needs a cubic equation in $\langle \Phi_u \rangle$. Therefore, one assume existence of $\text{Tr}[A]\text{Tr}[BC], \text{Tr}[B]\text{Tr}[CA]$ and $\text{Tr}[C]\text{Tr}[AB]$ in addition to the term $\text{Tr}[ABC]$ only for the term $W_{\phi u}$. Then, the superpotential $W_u$ for the up-quark sector is given by

$$W_u = \lambda_u \text{Tr}[\Phi_u \Phi_u A_u] + \mu_u \text{Tr}[Y_u A_u] + W_{\phi u}, \quad (A.1)$$

$$W_{\phi u} = y_u \text{Tr}[(\Phi_u Y_u + Y_u \Phi_u)B_u] + 2y_{1u} \text{Tr}[\Phi_u \text{Tr}[Y_u B_u]] + 2y_{2u} \text{Tr}[Y_u \text{Tr}[\Phi_u B_u]] + 2y_{3u} \text{Tr}[B_u \text{Tr}[\Phi_u Y_u]]. \quad (A.2)$$

The SUSY vacuum condition $\partial W/\partial A_u = 0$ has already been investigated in Sec.2. In this appendix, we will investigate $\partial W/\partial Y_u = 0$, $\partial W/\partial \Phi_u = 0$ and $\partial W/\partial B_u = 0$.

From the conditions $\partial W/\partial Y_u = 0$ and $\partial W/\partial \Phi_u = 0$, one obtains

$$\frac{\partial W}{\partial Y_u} = 0 = \mu_u A_u + y_u (\Phi_u B_u + B_u \Phi_u) + 2y_{1u} \text{Tr}[\Phi] B_u + 2y_{2u} \text{Tr}[\Phi B_u] 1 + 2y_{3u} \text{Tr}[B_u] \Phi_u, \quad (A.3)$$

$$\frac{\partial W}{\partial \Phi_u} = 0 = \lambda_u (\Phi_u A_u + A_u \Phi_u) + y_u (Y_u B_u + B_u Y_u) + y_{1u} \text{Tr}[Y_u B_u] 1 + y_{2u} \text{Tr}[Y_u B_u] \Phi_u + y_{3u} \text{Tr}[B_u] Y_u. \quad (A.4)$$

Since one searches a vacuum with $\Phi_u \neq 0$ and $Y_u \neq 0$, one can obtain

$$A_u = B_u = 0, \quad (A.5)$$

by requiring Eqs.(A.3) and (A.4) simultaneously. On the other hand, from $\partial W/\partial B_u = 0$, one obtains

$$\frac{\partial W}{\partial B_u} = 0 = y_u (\Phi_u Y_u + Y_u \Phi_u) + 2y_{1u} \text{Tr}[\Phi_u] Y_u + 2y_{2u} \text{Tr}[Y_u \Phi_u] + 2y_{3u} \text{Tr}[\Phi_u Y_u] 1. \quad (A.6)$$

By substituting $Y_u \propto \Phi_u \Phi_u$, Eq.(2.4), one obtains a cubic equation in $\Phi_u$:

$$y_u \Phi_u^3 + y_{1u} \text{Tr}[\Phi_u] \Phi_u^2 + y_{2u} \text{Tr}[\Phi_u^2] \Phi_u + y_{3u} \text{Tr}[\Phi_u^3] 1 = 0. \quad (A.7)$$
Since the coefficient of $\Phi_u$, $y_{1u}\text{Tr}[^2\Phi_u]/2y_u$, in a cubic equation (A.7) must be equal to $-\text{Tr}[\Phi_u]$, one obtains a restriction

$$y_{1u} = -y_u.$$  \hspace{1cm} (A.8)

Also, from constraints for the coefficients of $\Phi$ and 1 in the cubic equation, one obtains

$$\frac{y_{2u}}{y_u}\text{Tr}[\Phi_u^2] = \frac{1}{2}\left(\text{Tr}[\Phi_u]^2 - \text{Tr}[\Phi_u^2]\right),$$  \hspace{1cm} (A.9)

and

$$\frac{y_{1u}}{y_u}\text{Tr}[\Phi_u^3] = -\det\Phi_u,$$  \hspace{1cm} (A.10)

respectively. The constraints (A.9) and (A.10) lead to formulas

$$\frac{\text{Tr}[\Phi_u^2]}{\text{Tr}[\Phi_u]^2} = \frac{1}{1 + 2y_{2u}/y_u},$$  \hspace{1cm} (A.11)

and

$$\det\Phi_u = \frac{y_{3u}/y_u}{2(1 + 3y_{3u}/y_u)}\text{Tr}[\Phi_u]\left(\text{Tr}[\Phi_u]^2 - 3\text{Tr}[\Phi_u^2]\right),$$  \hspace{1cm} (A.12)

respectively. Thus, the VEV spectrum can completely be determined by the coefficients $y_{1u}/y_u$, $y_{2u}/y_u$ and $y_{3u}/y_u$.

We also assume the same structure $W_e$ as $W_u$ for the charged lepton sector. Then, if one takes $y_{2u}/y_u = 1/4$, one obtains $\text{Tr}[\Phi_e^2]/\text{Tr}[\Phi_e]^2 = 2/3$, so that one can obtain an interesting charged lepton mass relation [12]. However, since the purpose of the present paper is not to discuss the mass spectra of quarks and leptons, we do not touch this problem.

**Appendix B: Mass matrix form for a tribimaximal mixing**

A general mass matrix form which gives a tribimaximal mixing [5] has been given by He and Zee [13]. We summarize the general form for a case of the tribimaximal mixing matrix with phases, and we discuss conditions for $\sin^2 2\theta_{23} = 1$ and $\tan^2 \theta_{12} = 1/2$ separately.

An orthogonal mixing matrix $U$ which gives a maximal $2 \leftrightarrow 3$ mixing

$$\sin^2 2\theta_{23} = 1 \text{ and } U_{13} = 0,$$  \hspace{1cm} (B.1)

is given by a form

$$\tilde{U} = \begin{pmatrix} c & s & 0 \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s & \frac{1}{\sqrt{2}}c & \frac{1}{\sqrt{2}} \end{pmatrix},$$  \hspace{1cm} (B.2)

where $c = \cos \theta$ and $s = \sin \theta$. Since a mixing matrix $U$ with $U_{13} = 0$ cannot contain a $CP$ violating phase, an extended form $U$ from the orthogonal mixing matrix $\tilde{U}$ to a unitary mixing matrix is given by

$$U = P(\alpha)\tilde{U}P(\beta),$$  \hspace{1cm} (B.3)
where
\[ P(\delta) = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}). \]  
(B.4)

When one defines a mass matrix \( M \) with \( M^T = M \) which is diagonalized by \( U \) as follows:
\[ U^T M U = D \equiv \text{diag}(m_1, m_2, m_3), \]  
(B.5)
one can obtain
\[ \tilde{U}^T \tilde{M} \tilde{U} = P^2(-\beta) D \equiv \text{diag}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3) \equiv \tilde{D}, \]  
(B.6)
where \[ \tilde{M} = P(\alpha) M P(\alpha). \]  
(B.7)
The matrix \( \tilde{M} \) which is diagonalized by an orthogonal matrix is real except for a common phase factor, so that the eigenvalues \( \tilde{m}_i \) are also real. As seen in Eq.(B.6), the phases \( \beta_i \) in \( \tilde{m}_i = m_i e^{-2i\beta_i} \) are the so-called Majorana phases, they are unobservable in neutrino oscillation experiments. Hereafter, for convenience, we denote \( \tilde{m}_i \) as \( m_i \) simply. Then, one can obtain the explicit form of \( \tilde{M} \) from \( \tilde{M} = \tilde{U} \tilde{D} \tilde{U}^T \) as
\[ \begin{align*}
\tilde{M}_{11} &= \frac{1}{2}(m_2 + m_1) - \frac{1}{2}(m_2 - m_1) \cos 2\theta, \\
\tilde{M}_{22} &= \tilde{M}_{33} = \frac{1}{2}m_3 + \frac{1}{4}(m_2 + m_1) + \frac{1}{4}(m_2 - m_1) \cos 2\theta, \\
\tilde{M}_{12} &= \tilde{M}_{13} = \frac{1}{2\sqrt{2}} (m_2 - m_1) \sin 2\theta, \\
\tilde{M}_{23} &= -\frac{1}{2}m_3 + \frac{1}{4}(m_2 + m_1) + \frac{1}{4}(m_2 - m_1) \cos 2\theta. 
\end{align*} \]  
(B.8)

Therefore, the conditions that the mass matrix \( \tilde{M} \) gives the maximal 2 ↔ 3 mixing (B.1) are
\[ \tilde{M}_{12} = \tilde{M}_{13} \text{ and } \tilde{M}_{22} = \tilde{M}_{33}, \]  
(B.9)
i.e.
\[ M_{12} e^{i\alpha_2} = M_{13} e^{i\alpha_3} \text{ and } M_{22} e^{2i\alpha_2} = M_{33} e^{2i\alpha_3}. \]  
(B.10)
The conditions (B.10) are rewritten as
\[ \left( \frac{M_{12}}{M_{13}} \right)^2 = \frac{M_{22}}{M_{33}} = e^{2i(\alpha_3 - \alpha_2)}. \]  
(B.11)

On the other hand, the mixing angle \( \theta \equiv \theta_{12} \) is obtained from
\[ \tan 2\theta = \frac{2\sqrt{2} \tilde{M}_{12}}{\tilde{M}_{33} + \tilde{M}_{23} - \tilde{M}_{11}}, \]  
(B.12)
i.e.
\[ \tan 2\theta = \frac{2\sqrt{2} \eta (M_{12} M_{13})^{1/2}}{\eta^2 ((M_{22} M_{33})^{1/2} + M_{23}) - M_{11}}, \]  
(B.13)
where
\[ \eta = \exp i \left( -\alpha_1 + \frac{\alpha_2 + \alpha_3}{2} \right). \]  
(B.14)

Therefore, the conditions for a tribimaximal mixing, i.e. constraints (B.1) and

\[ \tan^2 \theta = \frac{1}{2}, \]  
(B.15)

require the conditions (B.11) and

\[ \eta^2 \left( (M_{22}M_{33})^{1/2} + M_{21} \right) - M_{11} = \eta (M_{12}M_{13})^{1/2}, \]  
(B.16)

respectively.

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