Critic PI2: Master Continuous Planning via Policy Improvement with Path Integrals and Deep Actor-Critic Reinforcement Learning

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Abstract—Constructing agents with planning capabilities has long been one of the main challenges in the pursuit of artificial intelligence. Tree-based planning methods from AlphaGo to MuZero have enjoyed huge success in discrete domains, such as chess and Go. Unfortunately, in real-world applications like robot control and inverted pendulum, whose action space is normally continuous, those tree-based planning techniques will be struggling. To address those limitations, in this paper, we present a novel model-based reinforcement learning framework called Critic PI2, which combines the benefits from trajectory optimization, deep actor-critic learning, and model-based reinforcement learning. Our method is evaluated for inverted pendulum models with applicability to many continuous control systems. Extensive experiments demonstrate that Critic PI2 achieved a new state of the art in a range of challenging continuous domains. Furthermore, we show that planning with a critic significantly increases the sample efficiency and real-time performance. Our work opens a new direction toward learning the components of a model-based planning system and how to use them.

I. INTRODUCTION

Deep reinforcement learning (DRL) methods have shown recent success on continuous control tasks in robotics systems in simulation \cite{1} \cite{2} \cite{3}. Reinforcement learning (RL) algorithms generally fall into one of two categories: model-free techniques, which learn a direct mapping from states to actions, and model-based approaches, which build a predictive model of an environment and derive a controller from it. Model-free methods have shown promise as a general-purpose tool for learning complex policies from raw state inputs \cite{4} \cite{5} \cite{6}, and such methods are applied using no prior knowledge of the systems, leading to problematic sample complexity and thus long training times. However, when dealing with real-world physical systems, for which data collection can be an arduous process, model-free methods become struggling due to the sample complexity and model-based approaches are appealing due to their comparatively fast learning. Unfortunately, model accuracy acts as a bottleneck to policy quality, often causing model-based approaches to perform worse asymptotically than their model-free counterparts. What’s worse, little can be said about the stability or robustness of these resulting control policies, even if more traditional model-based optimal control solutions exist for these same systems.

Different from traditional RL, path-integral-based RL, such as policy improvement with path integrals (PI2), within the framework of stochastic optimal control, requires far fewer iterations and guarantees optimality and reliable training convergence \cite{7} \cite{8} \cite{9}. As a matter of fact, when training the traditional RL using approximated dynamics, instead of the authentic one of the studied practical system, which thus causing an approximation error. Unfortunately, this inaccuracy often degrades the performance of the system and thus needs to be carefully addressed. Inspired by this fact, the model-based Lyapunov method, which can ensure the robustness and stability of a nonlinear system even in the presence of various uncertainties, is introduced into the original model-based RL to solve a range of control problems. A similar model-based RL approach that can notably increase the learning speed is proposed in \cite{10}; however, its application scope is limited to video games.

In this paper, we investigate how to most effectively combine the superiority of deep actor-critic learning, policy optimization and model-based reinforcement learning in continuous domains. Specifically, we first introduce the Path-Integrals-based trajectory optimization algorithm into model-based reinforcement learning, thus obtaining the Deep PI2 (DPI2) algorithm, which enables agents the ability to plan in continuous space. However, the model predictive error caused by model approximation using deep neural networks hinders the further improvement of the performance. To outbreak this limitation, we draw insights from model-free reinforcement learning to introduce the Deep Actor-Critic Learning into our DPI2 frameworks, thus obtaining the Critic PI2 algorithm (summarized in Fig. 1), which introduces a critic to assist the planner and thus reduce the influence of model approximation error.

Our core contribution is a practical framework called Critic PI2 built on these insights, which can ease the burden of the predictive dynamic model by one-step inference and simultaneously improve the effectiveness of the planning algorithm by using a novel path-integrals-based trajectory optimization algorithm. The experiment results can support the conclusion that using a critic to assist the predictive model can effectively ease the misleading of incorrect predictive models, thus guaranteeing a good performance.

The remaining of the paper is organized as follows. In Section II we introduce some prior work in related domains. In Section III some preliminaries of our algorithm will be introduced. More details on the whole framework will be proposed in Section IV and adequate simulation experiments, reasonable analysis and credible results of the proposed
algorithm will be demonstrated in Section VI. Finally, Section VI concludes.

II. PRIOR WORK

Model-free reinforcement learning algorithms based on Q-learning [11] [12] [13], actor-critic methods [14] [15] [16] and policy gradients [17] [18] like Deep Deterministic Policy Gradient (DDPG) have been shown to learn very complex skills in high-dimensional state spaces, including simulated robotic locomotion, driving, video game playing, and navigation. However, the high sample complexity of purely model-free algorithms has made them difficult to be implemented in the real world, where sample collection is limited by the constraints of real-time operation.

Model-based algorithms are known in general to outperform model-free learners in terms of sample complexity [19], and in practice have been applied successfully to control robotic systems both in simulation and in the real world, such as pendulums [20], legged robots [21], swimmers [22], and manipulators [23]. However, the most efficient model-based algorithms have used relatively simple function approximators, such as Gaussian processes [24] [25], time-varying linear models [26] [27], and mixtures of Gaussians [28], and the model error caused by the approximation tends to cripple the performance of model-based approaches, which is also known as model-bias. As is discovered previously, even a small model error can severely degrade multi-step rollouts since the model error will compound as the steps increases. Hence the predicted states will move out of the region where the model has high accuracy after a few steps.

Several previous works have been proposed to alleviate the influence of compounding model error in different ways. For example, some work [29] hybrids model-based and model-free algorithm by initializing a model-free learner with a model-based algorithm like model predictive control (MPC).

However, the hybrid algorithm can only achieve better final performance at the expense of much higher sample complexity. Although the model-based part of this kind of method [29] is far more efficient in sampling and more flexible than task-specific policies learned with model-free reinforcement learning, their asymptotic performance is usually worse than model-free learners due to the influence of model bias.

III. PRELIMINARIES

A. Reinforcement Learning

We consider a Markov decision process (MDP), defined by the tuple $(S, A, p, r, \gamma, \rho_0)$. $S$ and $A$ are the state and action spaces, respectively, and $\gamma \in (0, 1)$ is the discount factor. The dynamics or transition distribution are denoted as $p(s' | s, a)$, the initial state distribution as $\rho_0(s)$, and the reward function as $r(s, a)$. The goal of reinforcement learning is to find the optimal policy $\pi^*$ that maximizes the expected sum of discounted rewards, denoted by $\eta$:

$$\pi^* = \operatorname{argmax}_\pi \eta(\pi) = \operatorname{argmax}_\pi \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \right] \quad (1)$$

The dynamics $p(s' | s, a)$ are assumed to be unknown. Model-based reinforcement learning methods aim to construct a model of the transition distribution, $f_\theta(s' | s, a)$, with the MDP data collected from interaction, typically using imitation learning.

B. Stochastic Optimal Control

The general stochastic dynamical system is expressed as follows:

$$\dot{x}_t = f(x_t, t) + G(x_t) (u_t + \varepsilon_t)$$

$$= f_t + G_t (u_t + \varepsilon_t)$$

(2)

where $u_t \in \mathbb{R}^{p \times 1}$ represents the control vector, $f_t \in \mathbb{R}^{n \times 1}$ stands for the system dynamics, $G_t \in \mathbb{R}^{n \times p}$ donates the
control matrix, \( x_t \in \mathbb{R}^{n \times 1} \) is the system’s state, and \( \epsilon_t \in \mathbb{R}^{p \times 1} \) is the Gaussian noise submitting to \( N(0, \Sigma_e) \).

The performance criterion function for a path \( \tau \) starting at time \( t_i \) in state \( x_t \) and ending at time \( t_f \) is defined as follows [30]

\[
J(\tau) = \phi_{t_f} + \int_{t_i}^{t_f} L[x_t, u_t, t] \, dt
\]  

(3)

wherein \( \phi_{t_f} \) is terminal value, and \( \int_{t_i}^{t_f} L[x_t, u_t, t] \, dt \) stands for process value. The immediate cost is defined as follows

\[
L_t = L[x_t, u_t, t] = q_t + \frac{1}{2} u_t^T R u_t
\]  

(4)

where \( q_t = q(x_t, t) \) is the cost function related to states, \( R \) represents the coefficient matrix. The goal of stochastic optimal control is to seek out the control \( u_t \) to minimize the following performance criterion function

\[
\min_{V(x_t)} V(x_t) = V_t = \min_{t_i \leq t \leq t_f} E_{\tau} [J(\tau)]
\]  

(5)

where \( E_{\tau}[\cdot] \) is the expectation of all trajectories’ performance criterion function starting from \( x_t \).

### C. Trajectory Optimization With Path Integral

It is very difficult to obtain the analytical solution for optimization problem as (5), instead, with path integral algorithm, which is a numerical method used to solve stochastic optimal control problems, for which the goal is to minimize a performance criterion for a stochastic dynamical system [30], we can propose its numerical solutions after iteration convergence of (6).

\[
u_t^* = \int P(\tau_i) u(\tau_i) \, d\tau_i
\]  

(6)

where \( P(\tau_i) \) is defined [31] as follows

\[
P(\tau_i) = \frac{e^{-\frac{1}{2} S(\tau_i)}}{\int e^{-\frac{1}{2} S(\tau_i)} \, d\tau_i}
\]  

(7)

Equation (7) can be approximated to (5) for the convenience of implementation.

\[
P(\tau_i) = \frac{e^{-\frac{1}{2} S(\tau_i)}}{\sum_{i=1}^{N} e^{-\frac{1}{2} S(\tau_i)}}
\]  

(8)

wherein \( S(\tau_i) \) is a normalized version of the path cost defined as follows

\[
S(\tau_i) = \frac{C(\tau_i) - \min(C)}{\max(C) - \min(C)}
\]  

(9)

where \( C(\tau_i) \) represents the cost function similar to \( J(\tau_i) \), \( C \) represents the cost function vector for \( N \) trajectories.

It’s obvious that we can unite the representations of reinforcement learning and stochastic optimal control by making the reward function equal to the negative of the cost function. For ease of demonstration, we will use the representations of RL manners in the following sections.

### IV. CRITIC PI2 FRAMEWORKS

In this section, more implementation details of the Critic PI2 will be revealed. We introduce how to learn the model predictive network in [IV-A] detail how to train actor-critic architecture in Sec [IV-B] and demonstrate the whole frameworks of Critic PI2 in Sec [IV-C].

#### A. Dynamics Network

Traditional model-based reinforcement learning algorithms always learn dynamic network via imitation learning, whose structure can be found in Fig. 2(a) which maps from current state \( s_t \) and current action \( a_t \) to next state \( s_{t+1} \). Different from that approach, we choose to learn a differential target \( s_{t+1} - s_t \) as (10), which can reduce the influence of the environment noise and improve the learning effect.

\[
s_{t+1} - s_t = \tilde{f}_\theta(s_t, a_t)
\]  

(10)

In the normal manner of PI2 [32], the discounted cumulative reward \( R(\tau) \) is used to guide the improvement direction of the policy. However, [33] proves the fact that even a small model error will cause great approximation bias of the value function obtained by simulating \( k \) steps in the incorrect predictive dynamics shown as (11).

\[
e^\eta_{\text{predictive}}[\pi] - e^\eta[\pi] \leq \text{bias}
\]  

(11)
bias = 2σr_{max} \left[ \frac{γ^{k+1}ε_π}{(1-γ)^2} + \frac{γ^k+2}{(1-γ)}ε_π + \frac{k}{1-γ}(ε_m + 2ε_π) \right]

ε_m = \max_t E_{s \sim π_D, t} [DTV(p(s', r | s, a)||p_θ(s', r | s, a))],

which can be estimated in practice by measuring the validation loss of the model on the time-dependent state distribution of the data-collecting policy π_D, and the distribution shift by the maximum total-variation distance is remembered as ε_π, which means \( max_s D_{TV}(π||π_D) \leq ε_π \).

It reminds us of fact that the estimation error of the \( R(τ) \) is positive correlated with the length of trajectory inference. Thus, we can use the N step time difference (TD) methods to estimate the \( R(τ) \) as \( (12) \) instead of using Monte Carlo estimation as \( (13) \) directly.

\[
R^{(n)}(τ) = \sum_{k=0}^{n} γ^k r_{t+k+1} + γ^n V_π(s_{t+n+1})
\]

\[
R(τ) = \sum_{t=0}^{∞} γ^k r_{t+k+1}
\]

To apply the N step TD method, we must construct an approximator to estimate the state value function \( V_π(s) \), which will be proposed in the next Section.

B. Actor Critic Network

Deep neural networks have been used to represent the value function \( V_π(s) \) like Fig 2(b). With the help of it, we can directly get the approximation of \( η(π) \) by using the one-step TD method as \( (12) \), which just need to infer only one step in the incorrect dynamics. After combining the superiority of model-free reinforcement learning and model-based reinforcement learning, even with extremely little sample the agent can still achieve a remarkable score in a range of continuous control tasks on the mujoco platform.

To further improve the learning efficiency, we use the V-trace target inspired by [34] as our value function’s target, which can be computed as \( (14) \):

\[
v_s = V(x_s) + \sum_{t=s}^{s+n-1} γ^{t-s} \left( \prod_{i=s}^{t-1} c_i \right) δ_t V
\]

\[
δ_t V = ρ_t (r_t + γ V(s_{t+1} - V(s_t)))
\]

\[
ρ_t = \min(π, \frac{π(a_t | s_t)}{μ(a_t | s_t)})
\]

\[
c_i = \min(ε, \frac{π(a_i | s_i)}{μ(a_i | s_i)})
\]

where π is the behavior policy and \( μ \) is the target policy.

The policy is constructed as a multi-variate Gaussian distribution approximated by a deep neural network, whose variance is normally fixed and the mean is predicted by deep neural networks described in Fig. 2(c), which is learned via imitation learning.

C. Critic PI2

After showing how to learn the dynamics, actor and critic, there is still a core role in Fig. 1(a) waits to be revealed, which is called planner.

Different from traditional PI2 [32], we apply a greedy strategy into the construction of the planner during optimization to further improve the optimality of the planning algorithm. Specially, the previous PI2 method will select K actions from the Gaussian distribution and calculate their corresponding cumulative reward. Then it will calculate the PI2 optimal action \( a_\text{opt} \) with (13). After generating the optimal action, it will use the optimal action as the mean of the Gaussian Distribution to generate K actions and then repeat the previous procedures several times. Finally, it will use the final optimal action as the expert action \( a^*_\text{opt} \). However, with the assistance of the critic network, our method can easily calculate a trajectory’s reward with (12), which is much faster, much more accurate, and has low variance. Hence, we propose a greedy strategy. Our agent will remember the best expert action. Extensive experiments prove the fact that the greedy strategy is critical to performance.

After agents completely learn the four core components of Critic PI2 (Actor, Critic, Planner, Dynamics), we now can demonstrate our whole frameworks in Fig 1. More implementation details can be found in Algorithm 1.

Algorithm 1 Critic Policy Improvement with Path Integrals (Critic PI2)

1: Initialize policy network \( π_φ \), dynamic network \( f_θ \), value network \( v_θ \), replay buffer \( D \)
2: for \( N \) episodes do
3: Draw an observation \( o_t \) from the environment
4: for \( T \) time steps do
5: for \( M \) Iterations do
6: Generate \( K \) trajectories from dynamics with \( K \) actors
7: Obtain \( R(τ_i) \) using (12)
8: Generate an expert action from planner with the assistance of critic
9: Actor learns from the expert
10: end for
11: Obtain the expert action with greedy strategy
12: Take the expert action in the environment, draw next observation \( o_{t+1} \) and reward \( r_t \)
13: Add experience to replay buffer
14: end for
15: for \( E \) Epochs do
16: Update Actor, Dynamics, Critic centrally
17: end for
18: end for

Return: \( f_θ^*, v_θ^*, π_φ^* \)

V. EXPERIMENTS

Our experiments aim to answer the following three questions:
1) How does Critic PI2 perform compared with model-free RL methods and previous state-of-the-art model-based RL methods using model predictive control?

2) Whether Critic PI2 obtain the optimality at the cost of real-time performance?

3) What are the critical components of our overall algorithm?

A. Critic PI2 Approach on Benchmark Tasks

To provide a comparative evaluation against prior methods, we evaluate Critic PI2 on two tasks (InvertedDoublePendulum, InvertedPendulum) from OpenAI Gym [36], and more details on the experiment platform can be found in Appendix A. We compare our method to the following state-of-the-art model-based and model-free algorithms:

1) MPC [29] This is a widely used model-based reinforcement learning planning algorithm, which represents a comparison to state-of-the-art model-based reinforcement learning.

2) PI2 [32] Our method builds on top of this model-based algorithm, which makes it a perfect baseline to emphasize the superiority of Critic PI2.

3) DDPG [35] This is an off-policy actor-critic algorithm, which represents a comparison to state-of-the-art model-free reinforcement learning.

Massive experiments have been carried out on the Mujoco platform [37], and relevant results can be found in Fig. 3, which illustrates the fact that the sample efficiency of Critic PI2 is far better than both model-based and model-free alternatives. This indicates that overcoming the representation learning bottleneck, coupled with efficient off-policy Critic, provides for fast learning similar to model-based methods, while attaining final performance comparable to fully model-free techniques that learn from state. Critic PI2 also substantially outperforms MPC. This difference can be explained in part by the use of an efficient off-policy Critic, which can better take advantage of the learned representation and the guarantee of stability obtained by using a trajectory optimization algorithm from stochastic optimal control.

B. Real Time Performance

As we all know, real-time performance is critical in real-world continues control task, in this section we will investigate Whether Critic PI2 obtain the optimality at the cost of real-time performance.

Different algorithms cost different time while planning an expert policy, and thus affect the real-time performance. Ode The predictive horizon \(H\) of MPC and PI2 is 50. We carried out those experiments using a single GTX1660Ti GPU, and an i7-9750H CPU. The results shown in Table I prove the fact that Critic PI2 can obtain comparable final performance and real-time performance to model-free algorithms like DDPG and simultaneously the comparable sample efficiency to other model-based reinforcement learning algorithm.

C. Ablation Experiment

Fig. 4. Learning curves of the Ablation Experiment on Critic PI2 (ours), Critic PI2 without greedy strategy while trajectory optimization, Critic PI2 without a Critic to assist the trajectory optimization and planning from scratch using Critic PI2 without training the actor. Each algorithm is evaluated every 700 environment episodes in InvertedDoublePendulu, where each evaluation reports the average return over every episode. The actor, critic and the dynamics are simultaneously trained 100 times every two episodes. The dashed reference lines are the asymptotic performance of Critic PI2.

| Algorithm | Mean Planning Cost(s) |
|-----------|-----------------------|
| Critic PI2 | 0.0139s               |
| PI2       | 1.09s                 |
| MPC       | 1.22s                 |
| DDPG      | 0.001s                |

Table I: Time Cost for Planning
We further carry out an ablation experiment to characterize the importance of three main components of our algorithm:

1) **Without Critic**: No critic to assist the trajectory optimization, just using the Monte Carlo method as in [13].

2) **Without Greedy**: Not adopting a greedy strategy while trajectory optimization, but still use the critic.

3) **Without Training Actor**: In this settings, the parameters of Actor Network in Fig. 2(c) will not be updated during central training.

The results are shown in Fig. 4. As we can see, Critic PI2 can obtain a good final performance while ablating the Critic will lead to an extreme performance dropping, which proves the fact that using a critic to assist the predictive model can effectively ease the misleading of the incorrect predictive model, thus guaranteeing good performance. From Fig. 4 we can also find out that ablating the greedy strategy will lead to an extreme performance dropping, which demonstrates the fact that only if the planning strategy and the critic work together can the algorithm guarantee a good performance and superior sample efficiency. It’s worthy to mention that Critic PI2 algorithm without training the actor can still obtain a good performance at the cost of more sample episodes, which proves the fact that Critic PI2 share a reliable training convergence and optimality of stochastic optimal control and thus can guide the agent to seek out an optimal policy even planning from the scratch.

In conclusions, these experiments can prove the fact that Critic PI2 can plan an expert action even from the scratch and thus perform a new state-of-the-art in both final performance, sample efficiency and real-time performance (compared with other algorithms in Table I), whose success can be attributed to both the approximation ability of deep critic network and the dynamic network and the superior planning ability of the PI2 with greedy strategy.

VI. CONCLUSIONS

In this paper, we present a novel model-based reinforcement learning frameworks, namely Policy Improvement With Path Integrals Using Critic (Critic PI2), simultaneously obtaining the superior learning effect and real-time performance of deep Q learning, great sample efficiency of model-based reinforcement learning and robustness of the stochastic optimal control. Empirical experiment results show asymptotic performance and higher sample efficiency than previous model-based reinforcement learning algorithms on several benchmark continuous control tasks. For future work, we will investigate the usage of Critic PI2 in other model-based RL frameworks and study how to leverage frameworks better.

APPENDIX

A. Experiment Platform

1) **InvertedPendulum**: This environment has a cart sliding on a rail. A pole is connected to the cart. The action is the force applied on the cart along the rail. The actuator force is a real number. The observation includes the angle of the pole away from the upright position $\theta_1$, the position of the cart away from the centre of the rail $x_t$ and their first derivative with respect to time (velocity).

2) **InvertedDoublePendulum**: This environment has a cart sliding on a rail. A pole is connected to the cart while another pole is connected to the pole. The action is the force applied on the cart along the rail. The actuator force is a real number. The observation includes the sine and cosine of the angle of the two poles away from the upright vertical position $\sin \theta_1$, $\sin \theta_2$, $\cos \theta_1$, $\cos \theta_2$, the position of the cart away from the centre of the rail $x_1$ and the position the top of the second pole away from the centre of the rail $x_2$, their first derivative with respect to time (velocity) $x_1, \dot{x}_2, \theta_1, \theta_2$ and constrain force.

We present our environment settings used in our experiments in Table II.

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