Newtonian motion as origin of anisotropy of the local velocity field of galaxies

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Abstract

The origins of recently reported anisotropy of the local velocity field of nearby galaxies (velocities < 500 km/s corresponding to the distance less than 8 Mpc) are studied. The exact solution of the Newtonian equation for the expanding Universe is obtained. This solution allows us to separate the Newtonian motion of nearby galaxies from the Hubble flow by the transition to the conformal coordinates. The relation between the Hubble flow and the Newtonian motion is established. We show that the anisotropic local velocity field of nearby galaxies can be formed by such a Newtonian motion in the expanding Universe, if at the moment of the capture of galaxies by the central gravitational field.

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I. INTRODUCTION

Recent observation of the local velocity field of galaxies gives a three-dimensional ellipsoid with different values of the Hubble parameter, clearly showing its anisotropic character [3, 4].

In this paper, we present a possible point of view that this local velocity field of galaxies can be explained by their Newtonian motions.

The analysis of the observational data will be based on the radial velocities of nearby galaxies, belonging to the Local Group. Our paper is organized in the following manner. In Section II, the cosmic evolution is described. In Section III, we introduce the Newtonian motion and separate this motion from the cosmic one. In Section IV, the initial data of the galaxies capture by a central gravitational field is considered. In Sections V, the simplest example is given to elucidate our results. The paper ends with the conclusions.

II. COSMIC EVOLUTION OF A FREE PARTICLE

Effects of cosmic evolutions are considered in the Friedmann–Lemâïtre–Robertson–Walker (FLRW) metrics

\[ (ds^2) = (dt)^2 - a(t)^2(dx^i)^2. \]  

(1)

The formulation of the Newtonian problem in this metrics proposes a choice of physical variables and coordinates. The modern cosmology uses two choices of such variables: the conformal time (\(\eta\)) and coordinate distance \(x^i\) with the interval

\[ (ds^2) = a(\eta)^2 [(d\eta)^2 - (dx^i)^2], \]  

(2)

and the Friedmann time \(t\) and distance \(X^i = ax^i\) in terms of which the interval (1) takes the form

\[ (ds^2) = (dt)^2 - \left[dX^i - H(t)X^idt \right]^2, \]  

(3)

where \(H(t) = \dot{a}(t)/a(t)\) is the Hubble parameter, we have used here the formula of differential calculus \(adx = d(ax) - xda\). Both the sets are mathematically equivalent\(^1\).

In terms of the Friedmann variables \(X^i = ax^i\) the Newton action in the space with the interval takes the form

\[ S_A = \int_{t_i}^{t_0} dt \left[ P_i(\dot{X}^i - HX^i) - \frac{P_i^2}{2m_0} \right]. \]  

(4)

\(^1\) Note that \(a(t) = a(\eta(t))\) is connected with \(a(\eta)\) by equation \(dt = a(\eta)d\eta\).
The equations of motion

\[ \dot{X}^i - HX^i = \frac{P_i}{m_0}, \quad \dot{P}^i + HP^i = 0 \]  

(5)

have the simplest solution \( X^i = ax_0^i \), where \( x_0^i \) is a constant.

The transition to the conformal variables gives the action

\[ S_A = \int_{\eta_1}^{\eta_0} d\eta \left[ p_i \frac{dx^i}{d\eta} - \frac{P_i^2}{2m_0 a(\eta)} \right] \]  

(6)

for a particle with the running mass \( m_0 a(\eta) \) (with the present–day value \( a(\eta_0) = a_0 = 1 \)). This transition is just the main idea of our paper to separate the Hubble velocity field

\[ H_{\text{tot}} = \frac{1}{R} \frac{dR}{dt} \left( R = \sqrt{X_1^2 + X_2^2 + X_3^2} \right) \]  

(7)

from possible Newtonian motion. Substituting \( R = ar \), where \( r = \sqrt{x_1^2 + x_2^2 + x_3^2} \), in the definition of the total Hubble flow (7) we get this Hubble flow in the following form

\[ H_{\text{tot}} = \frac{1}{a} \frac{da}{dt} + \frac{1}{r} \frac{dr}{dt} = H + \Delta H. \]  

(8)

One can see that the total Hubble flow differs from the classical one \( H = \dot{a}/a \) by the value of the local flow

\[ \Delta H = \frac{1}{r} \frac{dr}{dt}. \]  

(9)

III. NEWTONIAN MOTION IN AN EXPANDING UNIVERSE

Let us consider the action

\[ S_A = \int_{t_1}^{t_0} dt \left[ P_i (\dot{X}^i - HX^i) - \frac{P_i^2}{2m_0} + \frac{\alpha}{R} \right], \]  

(10)

where \( \alpha = M_O m_0 G \) is a constant of a Newtonian interaction of a galaxy with a mass \( m_0 \) in a gravitational field of a central mass \( M_O \). Action (10) for radial momentum \( P_R \) and orbital moment \( P_\theta \) in the cylindrical coordinates

\[ X^1 = R \cos \theta, \quad X^2 = R \sin \theta, \quad X^3 = 0 \]  

(11)

\footnote{The action (11) and (6) differ from the one (7.4) in monograph [1], that is not compatible with quantum field theory on the conformal flat metric (2) [2].}
takes the form

\[ S_A = \int_{t_i}^{t_0} dt \left[ P_R(\dot{R} - H R) + P_\theta \dot{\theta} - \frac{P_R^2}{2m_0} - \frac{P_\theta^2}{2m_0 R^2} + \frac{\alpha}{R} \right]. \]  

(12)

To separate the Newtonian motion from the Hubble velocity field, we use the conformal variables

\[ p_r = P_R a(t), \quad r = R/a(t), \quad d\eta = dt/a(t). \]

In terms of these variables the action takes the form

\[ S_A = \int_{\eta_I}^{\eta_0} d\eta \left[ p_r r' + P_\theta \theta' - \frac{p_r^2}{2m_0 a(\eta)} - \frac{P_\theta^2}{2m_0 a(\eta)r^2} + \frac{\alpha}{r} \right]. \]  

(13)

IV. THE CAPTURE OF GALAXIES BY THE CENTRAL GRAVITATIONAL FIELD

The energy of a particle with the running mass \( m(\eta) = a(\eta)m_0 \) described by the action (13)

\[ E(\eta) = \frac{p_r^2}{2m_0 a(\eta)} + \frac{P_\theta^2}{2m_0 a(\eta)r^2} - \frac{\alpha}{r} \]  

is not conserved in the contrast to the energy of particle with a constant mass in the Newtonian mechanics. In our case (14), if the scale factor \( a(\eta) \) increases, the energy (14) runs from its positive values to negatives ones. There is a moment of a time \( \eta_I = \eta_0 \) when the energy (14) is equal to zero:

\[ \frac{E(\eta_I)}{m_0 a(\eta_I)} = \frac{(r'_I)^2 + v_I^2}{2} - w_I^2 = 0, \]  

(15)

where \( r'_I = p_r(\eta_I)/m_I r_I \) is radial initial velocity, \( m_I = m_0 a(\eta_I), \quad r_I = r(\eta_I) \) are the initial conformal mass and coordinate distance, and

\[ v_I = \frac{P_\theta}{m_I r_I}, \quad w_I = \frac{\alpha}{m_I r_I} \]  

are the orbital velocity and Newtonian one, respectively. It is known that the change of a sign of the energy means the change of an unrestricted motion of a particle by a finite motion in the central field. Therefore, the time \( \eta_I \) can be treated as the time of the capture of a particle (cosmic object) by the central gravitational field.

If the initial radial velocity is too equal to zero \( r'_I = 0 \), the zero–energy constraint (15)

\[ v_I^2 = 2w_I^2 \]  

(17)

becomes the equation for the initial data \( m_0 a(\eta_I) r(\eta_I) \equiv m_I r_I \). The solution of this equation \( m_I r_I = P_\theta^2/(2\alpha) \) can give a orbital velocity for all trajectories of the captured cosmic objects

\[ v_I = \frac{2\alpha}{P_\theta} = \text{constant} \]  

(18)
The fact of the universality of the orbital velocity (18) for all ellipsoidal trajectories (due to the zero energy initial data of the formation of a local universe) gives us a possibility of expanding both the numerous observational data on the law of the constant orbital velocity [4, 5, 6, 7] and the anisotropy of the local velocity field [3, 4].

The anisotropy of the local velocity field \( \Delta H = 0 \) [1, 2] is not compatible with the class of the isotropic circular trajectories \( r(\eta_0) \equiv r_0, r' = r'' = 0 \) with the equation of motion

\[
S_A = r_0 m_0 \int_{a_I} da \left\{ p_y \frac{dy}{da} + v_0 \frac{d\theta}{da} - \frac{1}{c_0} \left[ p_y^2 + v_0^2/y^2 - aw_0^2 \right] \right\},
\]

(21)

where \( w_0 = \sqrt{\alpha/m_0 r_0} \), \( c_0 = H_0 r_0 \) are the Newtonian velocity and the Hubble one, respectively, and \( a_I = 1/(1 + z_I) \) is determined with the redshift \( z_I \) at the moment of formation \( \eta = \eta_I \). The total energy of the system (14) in terms of the new variable takes the form

\[
E(a) = \frac{1}{c_0} \left[ p_y^2 + v_0^2/y^2 - aw_0^2 \right].
\]

(22)

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3 Remind that the hypothesis of the Cold Dark Matter was proposed in [7, 8] to explain the law of the constant orbital velocity in the class of the circular trajectories in the nonexpanding Universe where (19) is valid.
It is easy to see that $v_0$ is a constant of the motion $dv_0/da = 0$. The equations of the radial motion

$$p_y = c_0 \frac{dy}{da}, \quad c_0 \frac{dp_y}{da} = \frac{v_0^2}{y^3} - \frac{av_0^2}{y^2}$$

(23)

can be written in the Lagrangian form

$$\frac{d^2y}{da^2} = \left( \frac{w_0}{c_0} \right)^2 \left[ \left( \frac{v_0}{w_0} \right)^2 \frac{1}{y^3} - \frac{a}{y^2} \right].$$

(24)

At the present-day time $a = a_0 = 1$, $y_0 = 1$ this equation determines the second derivative

$$\left[ \frac{d^2y}{da^2} \right]_{a=1} = \left( \frac{w_0}{c_0} \right)^2 \left[ \left( \frac{v_0}{w_0} \right)^2 - 1 \right]$$

(25)

in terms of three velocities $w_0, v_0, c_0$, and it allows us to choose any relations between $w_0$ and $v_0$, in particular $v_0^2 = 2w_0^2$, in contrast to the circular trajectory where $v_0^2 = w_0^2$.

The general solution of this equation (24) can be obtained in the parametrical form:

$$a(\tau) = c_1 \frac{N_2(\tau)}{\tau^{2/3}N(\tau)}, \quad y(\tau) = c_2 \tau^{2/3}N(\tau),$$

(26)

where $\tau$ is the parameter of solution with the initial date $r_0 = r(\eta_0)$, and

$$N(\tau) = \alpha_1 U^2(\tau) + \beta_1 U(\tau)V(\tau) + \gamma_1 V^2(\tau),$$

$$N_2(\tau) = \left( \frac{dN(\tau)}{d\tau} + \frac{2}{3}N(\tau) \right)^2 \pm 4\tau^2 N^2(\tau) + \omega^2 \Delta,$$

$$\Delta = 4\alpha_1 \gamma_1 - \beta_1^2,$$

$$c_1 = \left( \frac{v_0}{w_0} \right) \left( \frac{3c_0}{4w_0} \right)^{1/3} \frac{1}{2\omega \Delta^{1/2}},$$

$$c_2 = \left( \frac{v_0}{w_0} \right) \left( \frac{4w_0}{3c_0} \right)^{1/3} \frac{1}{\omega \Delta^{1/2}}.$$  

(27)  

(28)  

(29)  

(30)  

(31)  

(32)  

Three constants $\alpha_1$, $\beta_1$, $\gamma_1 = \text{const}$ can be found from the following system of three equations:

$$a|_{\tau=0} = a|_{\eta=\eta_I} = a_I = \frac{1}{1 + z_I}, \quad y|_{\tau=0} = \frac{r_I}{r_0} = 1 + z_I, \quad \frac{dy}{da}|_{\tau=0} = 0;$$

(32)

here for the upper sign (restricted solution at infinity, $\tau = +\infty$) in (28)

$$U(\tau) = J_{1/3}(\tau), \quad V(\tau) = Y_{1/3}(\tau), \quad \text{and} \quad \omega = \frac{2}{\pi};$$

(33)

where $J_{1/3}(\tau)$ and $Y_{1/3}(\tau)$ are the Bessel functions of the first and second (or Niemann function) kind, while for the lower sign (unrestricted solution at infinity, $\tau = +\infty$) in (28)

$$U(\tau) = I_{1/3}(\tau), \quad V(\tau) = K_{1/3}(\tau), \quad \text{and} \quad \omega = -1,$$

(34)

where $I_{1/3}(\tau)$ and $K_{1/3}(\tau)$ are the modified Bessel functions of the first and second kind, respectively.
where \( I_{1/3}(\tau) \) and \( K_{1/3}(\tau) \) are the modified Bessel functions of the first and second (or MacDonald function) kind (see, e.g. [14]).

A solution of the equation of motion following from the action (13) is given in Fig. 1 and Fig. 2. We use this solution for construction of two plots.

Figure 3 gives the values of the correction (9) to the Friedmann Hubble flow resulting from taking into account eq. (9). It is clearly seen that the corrections are dumped with time and have a quasi–periodical character.

In Fig. 4 the angular distribution of the Hubble flow correction, as given by (9), is presented. The correction is anisotropic. We consider the 2-dimensional case while Karachentsev’s anisotropy is observed in 3-dimensions. Nevertheless, our 2-dimensional analysis allows one to see the anisotropy of the Hubble flow and to estimate the order of the magnitude of the anisotropy.

We have considered the case when the formation of a galaxy began from the zero–energy state (the initial data \( E_0 = 0 \), and velocity \( y'_0 = 0 \)). These data correspond to the relation \( v_0^2 = 2w_0^2 \).

VI. CONCLUSION

Our paper was motivated by the finding that in the local Universe the velocity field is anisotropic [3, 4]. This effect is difficult for explanation. The only possible suggestion, but rejected by Karachentsev, was rotation [4]. We are trying to find the origin of this anisotropy. In order to do this, we consider the general uniform expansion of the Universe. Since this is the nearby (less than 8 Mpc) part of the Universe around us, we use the Newtonian approach. We studied the motion of the test massive particle in the central gravitational field on the background of cosmic evolution of the type of FLRW space–time with uniform expansion. We assume the rigid state of the matter when densities of energy and pressure are equal and which corresponds to conformal cosmology [10] compatibles with Supernova data [11]. We obtained the exact solution of the above–mentioned Kepler problem, to find the difference between the uniform Hubble flow and our case. We have shown that this difference was anisotropic. In such a way we explained the anisotropy of the local velocity field by the Newtonian motion of galaxies in the central field. Of course, our 2–D consideration shows a possible mechanism of the observed 3–D anisotropy.

Having the solution of the Kepler problem we admit the rotation of galaxies around the center of the local Universe. This local Universe must be regarded as the Local Group of galaxies. In such a way, we support the picture in which galaxies rotate around the center of the Local Group in the
class of ellipsoidal trajectories.

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Figure 1: Graph of function \( y(\tau) = a(\tau)r(\tau)/r_0 \) at \( v_0^2/c_0^2 = 0.2 \) and \( w_0^2/c_0^2 = 0.1 \).

Figure 2: Graph of functions product \( a(\tau)y(\tau) \) at \( v_0^2/c_0^2 = 0.2 \) and \( w_0^2/c_0^2 = 0.1 \) for \(-3 < a(\tau) < 14\).

Figure 3: Graph of function \( \Delta H \) in units \( H_0 \) at \( v_0^2/c_0^2 = 0.2 \) and \( w_0^2/c_0^2 = 0.1 \).

Figure 4: Graph of function \( \Delta H \) in units \( H_0 \) at \( v_0^2/c_0^2 = 0.2 \) and \( w_0^2/c_0^2 = 0.1 \) for \( 0.8 < a(\tau) < 14 \).