Experimental Correlation-Boosted Quantum Engine

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We design and experimentally implement a two-qubit quantum correlated SWAP heat engine that allows to achieve an efficiency above the standard Carnot limit, and to boost the amount of extractable work, in a wider qubit energy-gap ratio window, with respect to engine’s cycle in the absence of initial qubit correlations. The boosted efficiency arises from a trade-off between the entropy production and the consumption of quantum correlations during the full thermodynamic cycle. We derive a generalized second-law limit and experimentally demonstrate the engine efficiency enhancement by tailoring the qubits effective energy gap and temperature, using an IBM quantum processor.

Recent years have witnessed the rise of quantum thermodynamics (QTD), which has rapidly become an arena to test and debate fundamental concepts such as the laws of thermodynamics and the related fluctuation theorems at the quantum scale [1–12]. In parallel, QTD is facilitating the extension of practical concepts such as heat and work towards the design [13–20] and, more recently, the implementation [2, 21–27] of thermal machines based on a handful of quantum degrees of freedom. Particular attention is being given to the concept of entropy production for systems out of equilibrium [28–32] and to the second-law of thermodynamics [6, 33, 34], as well as to the role of quantum correlations in both fundamental concepts [35–39] and in the functioning of thermal devices [40–45]. Quantum features can be considered as extra resources. Squeezed states, for instance, can enhance the performance of a microscopic engine above its classical limits [17, 46]. Coherence with a dynamical interference [47–49], quantum measurements [50–56], and quantum operations causal order [57–62] also play nontrivial roles in the performance of thermodynamic tasks [40, 41].

Through the design and experimental implementation of a two-qubit thermal machine, here, we demonstrate that the use of quantum correlations as an extra resource can indeed lead to a generalized second law of thermodynamics encompassing regimes with efficiency larger than the standard Carnot limit. As an added bonus, quantum correlations increase the amount of extractable work, as well as extend the parameter region corresponding to useful work extraction for the proposed cycle.

Design and functioning of a correlated quantum heat engine.—Taking advantage of initial non-classical correlations in a two-qubit working substance, we design a quantum heat-engine cycle based on the concept of a partial SWAP operation. We prove that such an engine can exceed the conventional classical limits, with an output performance that is well described by an information-to-energy trade-off relation for the cycle efficiency, which can be seen as a quantum generalized efficiency limit for a two-stroke cycle in the presence of non-classical correlations. Based on the ancilla-assisted two-point-measurement method developed in Ref. [26], we use a quantum processor (ibmq_manila [63]) to experimentally implement a proof-of-principle of our design.

The quantum engine is composed by two qubits with energy gaps $\varepsilon_A$ and $\varepsilon_B$. Each of them is initially coupled to an effective heat environment, as schematized in Fig. 1(a), stroke 1. After a complete thermalization, the two qubits get correlated through stroke 1.2 (Fig. 1(a)) in a state given by

$$\rho_{AB}^0 = \rho_A^0 \otimes \rho_B^0 + \chi_{AB},$$

where the reduced local state for each qubit $\rho_i^0 = \exp (-\beta_i \mathcal{H}_i) / Z_i$ is a thermal equilibrium state at the inverse temperature $\beta_i = (k_B T_i)^{-1}$, $i = A, B$; $Z_i = \text{Tr}_i \exp (-\beta_i \mathcal{H}_i)$ is the corresponding partition function, and $\chi_{AB} = \alpha |01\rangle \langle 10| + \alpha^* |10\rangle \langle 01|$ gives the relevant correlation term, with Tr$_i \chi_{AB} = 0$. Here, the states $|0\rangle$ and $|1\rangle$ denote the ground and excited eigenstates of the qubit Hamiltonian $\mathcal{H}_i = -\frac{1}{2} \varepsilon_i \sigma_i^Z$; the total two-qubit Hamiltonian $\mathcal{H}_{AB} = \mathcal{H}_A + \mathcal{H}_B$. Qubit A is assumed to be hotter than the qubit B, that is $\beta_A < \beta_B$. We remark that the correlation term $\chi_{AB}$ does not contribute to the energy of qubits A and B. The energy exchange between the two qubits is determined by means of a partial SWAP operation, as illustrated in Fig 1(a), stroke 2, where work can be extracted from or performed on the quantum system. Physically, this can be implemented by an effective unitary Heisenberg exchange Hamiltonian evolution $\mathcal{U}_t = \exp \left( -\frac{i}{2} \sum_j \sigma_j^A \otimes \sigma_j^B \right) j = X, Y, Z$. At $t = 0$ there is no interaction at all; a complete SWAP operation [64, 65] takes place at $t = \pi/(2J)$. 


Quantum-engine nonequilibrium-thermodynamics quantifiers.—For the aforementioned cycle, we calculate the mean energies involved in the whole process: the average values for the work, $\langle W \rangle = \text{Tr}[(\rho_{AB}^0 - \rho_{AB})\mathcal{H}_{AB}]$, (that takes place in stroke 2) and the heat contributions from the hot ($\langle Q_A \rangle$) and cold ($\langle Q_B \rangle$) environments, $\langle Q_A \rangle = -\text{Tr}[\rho_A^1 - \rho_A^0] \mathcal{H}_A$, where $\rho_A^0 = \text{Tr}_B(\rho_{AB}^0)$ is the final out-of-equilibrium reduced state for qubit $A$ ($B$) after stroke 2. We obtain:

\begin{equation}
\langle W \rangle = 2 (\varepsilon_B - \varepsilon_A) f(\Delta \nu, \lambda, \alpha),
\end{equation}

\begin{equation}
\langle Q_A \rangle = 2 \varepsilon_A f(\Delta \nu, \lambda, \alpha), \quad \langle Q_B \rangle = -2 \varepsilon_B f(\Delta \nu, \lambda, \alpha),
\end{equation}

with $f(\Delta \nu, \lambda, \alpha) = \sinh(\Delta \nu) / \sqrt{\Delta \nu^2 + 4 \alpha^2}$, where $\Delta \nu = (\varepsilon_B - \varepsilon_A)^2 / 2$. Since the total energy is conserved, Eq. (2) to (3) fulfill energy conservation, i.e., $\langle W \rangle = -\langle Q_A \rangle - \langle Q_B \rangle$. These results for the correlated case ($\alpha \neq 0$) are plotted in Fig. 2(b) and as the continuous curves in Fig. 2(d) (see also Fig. S3, Supplementary Material). For comparison, in Fig. 2(a) and (c) we plot the corresponding energies in the absence of initial correlations ($\alpha = 0$). From Eq. (2) and (3) it is straightforward to obtain the SWAP engine efficiency, $\eta = 1 - \langle W \rangle / \langle Q_A \rangle = 1 - \frac{\varepsilon_B - \varepsilon_A}{\varepsilon_A}$. For qubit energies such that $\frac{\varepsilon_B}{\varepsilon_A} = \frac{\beta_A}{\beta_B}$, the quantum engine achieves the standard Carnot limit, $\eta_{\text{Carnot}} = 1 - \frac{\beta_A}{\beta_B}$.

Boosting quantum engine efficiency by quantum correlations.—The mutual information $I(A:B) = S_A + S_B - S_{AB}$ gives a measure of the total correlations between systems $A$ and $B$, where $S_i = -\text{Tr}[\rho_i \ln \rho_i]$ is the von Neumann entropy of state $\rho_i$. We next derive an analytical expression for the SWAP engine efficiency. Such efficiency involves an information-to-energy tradeoff relation written in terms of the single-cycle variation of the mutual information between qubits $A$ and $B$, $\Delta I(A:B) = \Delta S_A + \Delta S_B$ (where $\Delta S_i = S(\rho_i^f) - S(\rho_i^0)$) and of the entropy production cycle, $\Sigma_{\text{eng}} = D[\rho_i^f || \rho_i^0]$. 

The rotation angles are associated with the effective temperature of each qubit and with the correlation parameter $\alpha$, $\theta_A = \arccos \sqrt{p_\perp}$ and $\theta_B = \arccos \sqrt{p_\parallel}$, where $p_\perp = p_A + p_B \pm \sqrt{(p_B - p_A)^2 + 4\alpha^2}$ and $p_i = \exp(-\beta_i \varepsilon_i / Z_i)$. The qubits energy-gap ratio is given by $\varepsilon_B / \varepsilon_A = \beta_B / \beta_A$. The rotation angles are $\theta_A = \arccos \sqrt{p_\perp}$ and $\theta_B = \arccos \sqrt{p_\parallel}$, where $p_\perp = p_A + p_B \pm \sqrt{(p_B - p_A)^2 + 4\alpha^2}$ and $p_i = \exp(-\beta_i \varepsilon_i / Z_i)$. The qubits energy-gap ratio is given by $\varepsilon_B / \varepsilon_A = \beta_B / \beta_A$.
\( D[\rho^f_B]\|\rho_B^0] \). Here, \( D[\rho||\sigma] = \text{Tr} \left[ \rho \left( \ln \rho - \ln \sigma \right) \right] \) is the Kullback–Leibler divergence [1]. For the cycle, we arrive at the following generalized efficiency:

\[
\eta = \eta_{\text{Carnot}} - \frac{\Sigma_{\text{eng}} + \Delta I(A:B)}{\beta_B \langle Q_A \rangle}. \tag{4}
\]

We then introduce an efficiency booster quantifier \( B_{\epsilon} \),

\[
B_{\epsilon} \equiv \frac{\Sigma_{\text{eng}} + \Delta I}{\beta_B \langle Q_A \rangle}, \tag{5}
\]

which tracks the direct competition between entropy production and correlations consumption. Equation (4) implies that there may exist efficiencies above Carnot, \( \eta > \eta_{\text{Carnot}} \), depending on the sign of \( \Delta I \), with the following engine efficiency criterion arising

\[
B_{\epsilon} < 0, \quad \eta > \eta_{\text{Carnot}}, \tag{6}
\]

\[
B_{\epsilon} \geq 0, \quad \eta \leq \eta_{\text{Carnot}}. \tag{7}
\]

Equation (5) can be seen as a quantum generalization of the second law efficiency and expression (6) is the condition for performance over the classical limit to occur, and indeed it is satisfied in Fig. 3(a). This result arises since the variation of mutual information \( \Delta I \) is always negative, and there is a trade-off with the always positive entropy production \( \Sigma_{\text{eng}} \) (here given by the quantum discord [66]), and \( C \) represents classical correlations, demonstrates that there is a consumption of quantum correlations during the thermodynamic cycle. For \( \varepsilon_B/\varepsilon_A < 1/2 \), this makes \( \Sigma_{\text{eng}} < |\Delta I(A:B)| \) and hence \( B_{\epsilon} < 0 \), which in turn implies \( \eta > \eta_{\text{Carnot}} \) in Eq. (4).

The emergence of condition (6) depends on a proper choice of initially correlated states and of the driving Hamiltonian in the stroke 2. It only arises if \( B_{\epsilon} < 0 \). Otherwise, \( \eta \leq \eta_{\text{Carnot}} \) (Eq. (7)); \( B_{\epsilon} \geq 0 \) also describes engine operation for initially uncorrelated qubits (Fig. 2(a)): entropy production is always greater or equal than variation of mutual information. An experimental verification of the engine efficiency criterion Eqs. (6) and (7) (see Fig. 3; Fig. S2 and S5) is provided below.

Experimental demonstration of performance boosting in the correlated SWAP heat engine.—For the proof-of-principle of the quantum heat engine we collected data from a sample of \( N = 20000 \) experiments for each of the considered engine initial-state values. The implementation was performed on ibmq_manchila. We characterized the performance of the quantum engine by varying the energy gap ratio \( \varepsilon_B/\varepsilon_A \), setting \( \beta_B = 2\beta_A \), and fixing the SWAP parameter to \( \lambda = 0.6 \) (other values are considered in the Supplementary Material). For the correlated initial state we considered \( \alpha \)'s maximum value, \( \alpha_{\text{max}} = 1/(Z_A Z_B) \). Further details regarding the quantum circuit setup used for the quantum heat engine implementation (Fig. 1) are given in the Supplementary Material (see Figs. S1 and S4).

In Fig. 2(a) and (b) we plot the parameters' diagram for temperature and energy gap ratios \( \{\beta_B/\beta_A, \varepsilon_B/\varepsilon_A\} \) required for work extraction for both, initially uncorrelated, and correlated scenarios. The dashed curve in the diagram for the case without initial correlations, Fig. 2(a), separates the regions for which the system can work as a refrigerator (work injection) or as a heat engine (work extraction). In contrast, for initially correlated qubits (Fig. 2(b)), work injection gets suppressed in the whole gap ratio \( \varepsilon_B/\varepsilon_A < 1 \), for all \( \beta_A < \beta_B \), and work extraction becomes much higher; the maximal extracted work can be seen, e.g., for \( \beta_B \gtrsim 3\beta_A \) and \( 0.2 \lesssim \varepsilon_B/\varepsilon_A \lesssim 0.5 \) (Fig. 2(b)).

In Fig. 2(c) and (d) we plot the mean energy (re-scaled) quantities, work \((\langle W \rangle, \text{black dots})\) and heat from the hot \((\langle Q_A \rangle, \text{red dots})\) and cold \((\langle Q_B \rangle, \text{blue dots})\) environments obtained from the experiments. The error bars were estimated using the standard deviation of the measured data. The solid curves correspond to the theoretical prediction.
qubits scenario (Fig. 2(c)): refrigerator (0 < $\varepsilon_B/\varepsilon_A$ < 1/2), heat engine (1/2 < $\varepsilon_B/\varepsilon_A$ < 1), and accelerator ($\varepsilon_B/\varepsilon_A$ > 1). However, when the qubits are initially correlated, the partial SWAP engine only exhibits two operational modes: heat engine (0 < $\varepsilon_B/\varepsilon_A$ < 1) and accelerator ($\varepsilon_B/\varepsilon_A$ > 1) (Fig. 2(d)). In the engine operation mode, quantum correlations boost the amount of work that can be extracted, making it at its maximum about an order of magnitude larger than the one obtained in the absence of initial qubit correlations. This result has also been verified for other $\lambda$ values (see Fig. S2 and S3). In Fig. 2(c) and (d) we find a very good agreement between the experimental results and the corresponding theoretical prediction from Eqs. (2) and (3) (solid curves). Furthermore, quantum correlations enlarge the $\varepsilon_B/\varepsilon_A$ values’ window where work extraction is possible.

In Fig. 3, we plot the experimental results for the efficiency, the entropy and the generalized second law related quantities as a function of the energy gap ratio. These quantities have been obtained by using quantum state tomography (QST), see Fig. S4, Supplementary Material. The efficiency $\eta$ (black dots) and the efficiency booster $B_\varepsilon$ (blue dots) is displayed in Fig. 3(a). The plotted error bars were estimated as in Fig. 2. The booster $B_\varepsilon$ reaches negative values hence the efficiencies go above the Carnot limit, $\eta_{\text{Carnot}} = 0.5$, in agreement with our theoretical findings (solid lines). In Fig. 3(b), we plot the experimental entropy production (black dots), the variation of mutual information (red dots), and of quantum discord (blue dots). For $\varepsilon_B/\varepsilon_A < 0.5$, we obtain $|\Delta I(A : B)| > \Sigma_{\text{eng}}$ and since $\Delta I$ is always negative, $B_\varepsilon < 0$ and the SWAP engine efficiency surpasses the standard Carnot limit $\eta > \eta_{\text{Carnot}}$, which is in perfect agreement with Fig. 3(a) and with the criterion Eq. (6). This result has been experimentally verified for other $\lambda$ values (see Fig. S2 and S5).

We make explicit the role of the measured correlations (both quantum and classical), by plotting $\Delta I = \Delta I_Q + \Delta C$ and $\Delta D_Q$, during the thermodynamic cycle (see the two lower curves of Fig. 3(b)). Here, the variation of quantum discord closely follows that of mutual information and both are always negative. This means that while entropy production increases, both variations in classical and quantum correlations are consumed during the cycle. The larger correlations consumption is of purely quantum origin, and come from the discord. These results have been further experimentally verified and plotted for $\Sigma_{\text{eng}}$, $\Delta I$ and $\Delta D_Q$ in Fig. S5, for other values of $\lambda$. The shaded areas in Fig. 3(b) represent the theoretical results calculated with the actual initial experimental states (instead of the ideal ones) of the various experimental runs. The theoretical predictions are in very good agreement with the experimental results.

Figure 3(c) shows the experimental results for a verification of the generalized efficiency formula as a function of the qubits’ energy-gap ratio. The black dots give the results for the sum of the engine efficiency and the booster $\eta + B_\varepsilon$, as averaged measurements following the qubits statistics from collected data from 20000 experiments. We find that the generalized second law limit measured with the *ibmq_malaga* quantum processor is at most within two standard deviations from the analytical result Eq. (4), thus verifying the quantum origin of the working thermodynamical principle for enhancing the efficiency of the correlated SWAP quantum heat engine.

In summary, the limits posed by the second law of thermodynamics may be affected by the presence of initial quantum correlations in the working fluid of a thermal machine, leading to efficiency higher than the Carnot standard limit and to a boost in the extractable work in each cycle. A criterion for the construction of such enhanced thermodynamic feature is given in terms of a
trade-off between entropy production and quantum correlations consumption during the implemented experimental thermal machine’s cycle. The design of thermal machines that use extra resources based on quantum correlations highlight the need for a revision of the standard thermodynamics limits. In this framework, the energetic cost of building initial correlations should not be included in the efficiency definition. This is in line with the practice of not including costs related to the production of (hot) heat sources in classical internal-combustion engines (e.g., fuel production/refining, etc.).

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SUPPLEMENTAL MATERIAL

Here we provide supplementary details about the experimental implementation of the quantum heat engine, the theoretical results and the data analysis.

QUANTUM HEAT ENGINE EXPERIMENTAL SETUP

For the implementation and characterization of the quantum heat engine we ran several experiments on the 5-qubit *ibmq_manila* quantum processor [1]. We were able to experimentally demonstrate the performance enhancement of a correlated SWAP heat engine and confirm our theoretical predictions.

For each experiment, we implemented the quantum circuit depicted in Fig. 1(b) (main text) and collected the qubits statistics over a sample of size 20000. We performed the experiment for different values of the partial SWAP parameter \( \lambda \). Additionally, for each \( \lambda \), we ran 10 experiments with the aforementioned sample size in order to see the fluctuations of the experimental setup. From these, we estimated the error propagation, using the standard deviation. Each circuit run is determined by the correlation (\( \alpha \)) and thermalization (\( \lambda \)) parameters (both described in the main text).

In Figure S1, we outline the transpiled circuit that has been implemented following the topology and optimization of *ibmq_manila*. This corresponds to the engine thermodynamic cycle portrayed in the circuit of Fig. 1(b) (main text). The correlated SWAP engine implementation parameters (see Table SI) have energy-gap ratio \( \varepsilon_B / \varepsilon_A = \beta_A \ln (p_B / p_A) / [\beta_B \ln (p_A / p_A)] \), and angles \( \theta_A = \arccos \sqrt{B} - \theta_B = \arccos \sqrt{A} \), where \( p_B = p_B + p_A(\sqrt{B} - \sqrt{A})^2 + 4a^2 \) and \( p_B = \exp(-\beta_B \varepsilon_B) / \sqrt{B} \). \( \phi_A = \arcsin \sqrt{A} \), with \( \lambda(\alpha) = (p_A - p_B) / (p_B + 1) \), and \( \phi_B = \phi_A - \arcsin \sqrt{A} \). Here, \( \lambda(\alpha) \) and \( \lambda \) give the correlation and thermalization parameters, respectively. In Table SI we display the experimental angles used in all the experiments here reported.

In the transpiled circuit (Fig. S1), three basic gates are used; \( R_z \) rotations, \( \sqrt{X} \) and CNOT, in order to parallel implement the original quantum circuit of Fig. 1(b) (main text). We resort to the gating and decoherence times for *ibmq_manila* [1] as figures of merit to check that the coherence properties required in the implementation of the quantum thermodynamic cycle are guaranteed. Clearly, relaxation (\( T_1 \)) and decoherence (\( T_2 \)) times are about three orders of magnitude longer than the qubits’ gating times. These times are reported in Table SII. The last column gives the execution times of the ‘i,j’ two-qubit gates according to the *ibmq_manila* topology (\( i,j = 0, 1, \ldots, 4 \)). By assuming that each gate takes its longest possible execution time (\( \sim 500 \, \text{ns} \)), we can overestimate the runtime of the full circuit to approximately 11 \( \mu \)s, which is shorter than the shortest \( T_2 \) time reported in Table SII.

In Fig. S2 we plot the average work and efficiency for (a) and (c) initially uncorrelated qubits, (b) and (d) initially correlated qubits. The blue dots give the results of 10 experiments for each energy gap ratio value shown in the figure. Each dot is obtained by running the circuit Fig. S1 with 20000 shots, using the parameters given in Table SI. Figures S2 (a) and (b) demonstrate that the extractable work (negative work) under initially correlated qubits is larger (about an order of magnitude larger at its maximum) than the one obtained in the absence of initial qubit correlations. In Fig. S2 (c) and (d) we show the efficiency obtained for the experimental implementation of the thermal machine. This is in agreement with the criterion given in Eqs. (6) and (7) of main text: efficiency

| ibmq_manila calibration data | Qubit | \( T_1 (\mu s) \) | \( T_2 (\mu s) \) | Gate time (ns) |
|-----------------------------|-------|----------------|----------------|--------------|
| \( q_0 \)                   | 177.13 | 78.73          | 0:1            | 277.33       |
| \( q_1 \)                   | 186.02 | 75.55          | 1:2            | 469.33       |
|                            |       |                | 1:0            | 312.89       |
| \( q_2 \)                   | 136.19 | 22.30          | 2:3            | 355.56       |
|                            |       |                | 2:1            | 504.88       |
| \( q_3 \)                   | 184.82 | 46.64          | 3:4            | 334.22       |
|                            |       |                | 3:2            | 391.11       |
| \( q_4 \)                   | 122.91 | 43.53          | 4:3            | 298.67       |

Table SII. Calibration data for the qubits involved in the generation of the heat engine at the 5-qubit *ibmq_manila* quantum processor (Fig. S1).
for uncorrelated initial qubits remain below the standard Carnot limit, while initially correlated qubits may allow for a boost in efficiency, with values well above the Carnot limit.

In Fig. S3 the average work and heat from the hot and cold environments, for different \( \lambda \) values are plotted. In Fig. S3(a) we consider initially uncorrelated qubits (\( \lambda = 0 \)); in this case we find three different modes of operation of the SWAP quantum engine. The engine mode occurs for \( 0.5 < \varepsilon_B/\varepsilon_A < 1 \). Otherwise, for initially correlated qubits, the engine mode expands its range to \( 0.0 < \varepsilon_B/\varepsilon_A < 1 \), as it is shown in Fig. S3(b)-(d), \( \lambda = 0.8, 0.6, 0.2 \), respectively. The lower the \( \lambda \) parameter the greater the amount of extracted work. In fact, executing a full SWAP (\( \lambda = 1 \)) in the engine would erase the advantage due to its quantum correlations hence obtaining the same result as in the absence of initial correlations.

**QUANTUM STATE TOMOGRAPHY AND ENTROPY RELATED QUANTITIES**

For the quantum state tomography (QST) implementation we have used a module in the qiskit-ignis library [2]. For a complete QST of a two-qubits state, 9 circuits are needed. The result for each circuit is averaged over 20000 shots and, additionally, in order to average over the system fluctuations, we repeat the process 5 times for the initial state and 5 times for the final one. In Figure S4(a)-(e) we show the QST result for the initially correlated state \( \rho_{AB}^{0} \), considering \( \varepsilon_B/\varepsilon_A = 0.10, 0.30, 0.55, 0.70, 0.90 \), respectively. In the same way, in Fig. S4(f)-(j) we give the QST result for the final state, with \( \lambda = 0.6 \) and \( \varepsilon_B/\varepsilon_A = 0.10, 0.30, 0.55, 0.70, 0.90 \), respectively. Similar results were obtained for \( \lambda = 0.2, 0.8 \) (not shown). The qubits density matrix can be approximated as (in the computational basis):

\[
\rho = \begin{pmatrix}
a & 0 & 0 & 0 \\
0 & b & z & 0 \\
0 & z & c & 0 \\
0 & 0 & 0 & d \\
\end{pmatrix}
\] (S1)

In Fig S5 we plot the variation of entropic and correlation quantities as function of \( \varepsilon_B/\varepsilon_A \), namely the variation of the engine entropy production \( \Sigma_{eng} \), of the mutual information \( \Delta I \) and of the quantum discord \( D_Q \), for (a) \( \lambda = 0.8 \), (b) \( \lambda = 0.6 \), and (c) \( \lambda = 0.2 \). For the cal-
Figure S3. Experimental results for the average work, and heat from the hot ($\langle Q_A \rangle$) and cold ($\langle Q_B \rangle$) reservoirs for (a) initially uncorrelated qubits ($\lambda = 0$), and initially correlated qubits: (b) $\lambda = 0.8$, (c) $\lambda = 0.6$, and (d) $\lambda = 0.2$. In all the experiments, $\beta_B = 2\beta_A$ and $\alpha_{\text{max}} = 1/(\mathcal{Z}_A\mathcal{Z}_B)$. The error bars were estimated using the standard deviation of the measured data. The solid curves are obtained from our theoretical predictions from Eqs. (2) and (3) (main text) and numerical simulations.

Figure S4. Quantum state tomography of the two-bit working substance for the initially correlated state (upper row) and corresponding final state (lower row) for $\lambda = 0.6$ and for the following qubits energy gap ratios $\varepsilon_B/\varepsilon_A$: (a) and (f) 0.10; (b) and (g) 0.30; (c) and (h) 0.55; (d) and (i) 0.70; (e) and (j) 0.90. Data shown represent the result over 20000 shots for each circuit used in one of the QST implementations.

Generalized Second Law Limit: Boosting the Engine Performance

Here, we give a demonstration for the efficiency result (Eq. (4)) of the main text. The entropy production of the correlated SWAP quantum engine can be expressed as the sum of two relative entropies that reads

\[
D \left[ \rho_f^i || \rho^0_i \right] + D \left[ \rho_f^j || \rho^0_j \right] = -\Delta S_A - \Delta S_B + \beta_A \text{Tr} \left[ \left( \rho_f^i - \rho^0_i \right) \mathcal{H}_A \right] + \beta_B \text{Tr} \left[ \left( \rho_f^j - \rho^0_j \right) \mathcal{H}_B \right],
\]

where $\rho_f^i$ is the final out-of-equilibrium state for the qubit $i = A, B$. We simplify this equation by introducing calculation of the quantum correlations we have used the geometrical quantum discord $D_Q$ for a two-qubit $X$ state [3, 4], which can be written in terms of the elements of the density matrix as:

\[
D_Q = \left[ k_1 - 2k_2 + 4z^2 - \max \left( k_1 - 2k_2, 2z^2 \right) \right],
\]

where $k_1 = a^2 + b^2 + c^2 + d^2$ and $k_2 = ac + bd$. Figure S5 demonstrates that the variation of mutual information is mostly due to the consumption of quantum correlations between the qubits. It also illustrates that for initially correlated systems $|\Delta I|$ may be greater than $\Sigma_{\text{eng}}$, leading to a correlation boosting of the engine efficiency.
Figure S5. Entropy and correlation quantities during the thermodynamic cycle: (a) $\lambda = 0.8$, (b) $\lambda = 0.6$, and (c) $\lambda = 0.2$. Experimental data in black, blue, and red correspond to the entropy production, variation of quantum discord and variation of mutual information, respectively. In all the experiments, the temperature relation $\beta_B = 2\beta_A$ and the correlation factor $\alpha_{max} = 1/(Z_A Z_B)$. The shaded areas denote the theoretical results calculated with the actual initial experimental states (instead of the ideal one), see e.g. Fig. S4. The error bars were estimated using the standard deviation of the measured data.

\[ \langle Q_i \rangle = -\text{Tr} \left[ \left( \rho_i^f - \rho_i^0 \right) H_i \right], \]

hence

\[ D \left[ \rho_A^f || \rho_A^0 \right] + D \left[ \rho_B^f || \rho_B^0 \right] = -\Delta S_A - \Delta S_B - \beta_A \langle Q_A \rangle - \beta_B \langle Q_B \rangle. \tag{S4} \]

We use the fact that $\Delta I(A : B) = \Delta S_A + \Delta S_B$ to rewrite Eq. (S4) as

\[ D \left[ \rho_A^f || \rho_A^0 \right] + D \left[ \rho_B^f || \rho_B^0 \right] = -\beta_A \langle Q_A \rangle - \beta_B \langle Q_B \rangle. \]

This is equivalent to

\[ D \left[ \rho_A^f || \rho_A^0 \right] + D \left[ \rho_B^f || \rho_B^0 \right] + \Delta I(A : B) = -\beta_A \langle Q_A \rangle - \beta_B \langle Q_B \rangle, \tag{S5} \]

Energy conservation implies that the average values for the heat and work, $\langle Q_A \rangle + \langle Q_B \rangle + \langle W \rangle = 0$. The extracted work $\langle W_{ext} \rangle = -\langle W \rangle = \langle Q_A \rangle + \langle Q_B \rangle$ and the quantum heat engine efficiency $\eta = \langle W_{ext} \rangle / \langle Q_A \rangle$ reads

\[ \eta = 1 - \frac{\beta_A}{\beta_B} D \left[ \rho_A^f || \rho_A^0 \right] + D \left[ \rho_B^f || \rho_B^0 \right] + \Delta I(A : B) \frac{\beta_B \langle Q_A \rangle}{\beta_B \langle Q_A \rangle}, \tag{S6} \]

where $1 - \beta_A/\beta_B$ is the standard Carnot limit. Equation (S6) defines a generalized second law limit for bipartite quantum engine in the presence of initial correlations. As discussed in the main text, the efficiency booster $B_E \equiv (\Sigma_{eng} + \Delta I)/\beta_B \langle Q_A \rangle$ sets a criterion (Eq. (6) of the main text) for the enhancement of the engine’s efficiency and extractable work (see Fig. S2, S3 and S5).

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