\[ \gamma \gamma \rightarrow \pi \pi \pi \] TO ONE LOOP IN CHIRAL PERTURBATION THEORY

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ABSTRACT

The \[ \gamma \gamma \rightarrow \pi^0 \pi^0 \pi^0 \] and \[ \gamma \gamma \rightarrow \pi^+ \pi^- \pi^0 \] amplitudes are discussed in the general context of Chiral Perturbation Theory (ChPT) to \( O(p^6) \). Chiral loops are found to play a major role. This makes these processes a good test of ChPT, mainly in its anomalous sector. We correct earlier numerical results at tree level and determine the one-loop results as well.
1 INTRODUCTION

The purpose of the present paper is to discuss the $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$ transitions at the one loop level in Chiral Perturbation Theory (ChPT) [1]. The experimental interest on these transitions centers in projected high-luminosity $e^+e^-$-machines (such as the DAPHNE $\phi$-factory under construction in Frascati) from which data can be expected in a near future. In the past, these two processes were treated in the theoretical context of Current Algebra (CA). Adler et al [2] solved a preexisting controversy on the $\gamma\gamma \rightarrow \pi\pi\pi$ amplitudes and, more recently, their CA results for the $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$ amplitude have been confirmed by Bos et al [3] working with ChPT at tree level, i.e., in a context which is essentially equivalent to CA. The situation, however, is not satisfactory. First, because there are good reasons to believe that one loop corrections to the lowest order amplitudes (which vanish in a specific chiral limit [2]) can be very important. Secondly, because the predictions in Ref. [3] for the $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$ cross section are in sharp disagreement with (roughly, one order of magnitude smaller than) those coming from previous work using equivalent lowest order amplitudes [4].

In the context of 2 flavor ChPT, the triplet of pseudoscalar mesons $P$, i.e., the pions, is described in terms of the Hermitian matrix

$$P = \begin{pmatrix} \pi^0 & \pi^+ \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \pi^- \\ \pi^- & \frac{\sqrt{2}}{2} \pi^0 \end{pmatrix},$$

which appears in the various pieces of the ChPT lagrangian through the conventional parameterization $\Sigma \equiv \exp \left(2iP/f\right)$, with $f$ at lowest order equal to the charged pion decay constant $f = f_\pi = 132$ MeV [4, 5].

The lowest order lagrangian of ChPT (order two in particle four–momenta or masses, $O(p^2)$) is

$$\mathcal{L}_2 = \frac{f^2}{8} tr(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi \Sigma^\dagger + \chi^\dagger \Sigma).$$

The covariant derivative, $D_\mu \Sigma \equiv \partial_\mu \Sigma + ieA_\mu [Q, \Sigma]$, contains the photon field $A_\mu$ and the quark charge matrix $Q \equiv \text{diag}(2/3, -1/3)$, and the non–derivative terms are proportional to the quark mass matrix via the identification $\chi = \chi^\dagger = B \mathcal{M}$, with $\mathcal{M} = \text{diag}(m_u, m_d)$. The parameter $B$ relates at lowest order the pion mass with the quark masses: $B = 2m_\pi^2/(m_u + m_d)$.

The next order lagrangian, $O(p^4)$, can be divided in two sectors: the Wess–Zumino term [6] and a series of seven terms identified and studied by Gasser and Leutwyler [1].

$$\mathcal{L}_4 = \mathcal{L}_{WZ} + \sum_1^7 l_i \mathcal{L}_i^{(4)}.$$
The only pieces of $L_{WZ}$ relevant for the $\gamma\gamma \rightarrow \pi\pi\pi$ amplitudes are

$$L_{WZ} = - \frac{e}{16\pi^2} \epsilon_{\mu\nu\alpha\beta} A_\mu tr(Q\partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \partial_\beta \Sigma \Sigma^\dagger - Q\partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \partial_\beta \Sigma - Q\partial_\nu \Sigma \partial_\alpha \Sigma^\dagger \partial_\beta \Sigma + \frac{1}{2} Q\partial_\nu Q\partial_\alpha \Sigma^\dagger \Sigma + \frac{1}{2} Q\Sigma^\dagger Q\partial_\alpha \Sigma).$$

The seven non-anomalous terms of $L_4$ contain products of covariant derivatives and/or mass–terms similar to those in Eq.(2). Each one of the corresponding constants $l_i$ can be divided into two pieces: a divergent one and a finite, real constant. The divergent terms are needed to cancel the divergences appearing in one–loop calculations with vertices from $L_2$, thus rendering the results finite, while the renormalized real constants, $l^r_i$, have been fixed by experimental data [1]. Alternatively, the various $l^r_i$ can be deduced with a good approximation assuming that they are saturated by the exchange of known meson resonances as discussed in Refs. [7, 8]. Fixing the renormalization mass–scale around these resonance masses ($\mu = M_\rho$, for instance), the finite and renormalized values for $l^r_i$ are small enough to justify the convergence of the perturbative series. These remarks obviously do not apply to the $L_{WZ}$ term in Eq. (3). It generates only anomalous processes with coupling strengths derived from the anomaly. However, higher order (counter-) terms belonging to $L_6$ – which can be similarly separated into two pieces contributing to anomalous and non-anomalous processes, respectively – are in general expected to give smaller contributions than their corresponding lower order lagrangians in Eq. (3): $L_{WZ}$ and $l^r_i L_4^{(i)}$. Details on counterterms in the anomalous sector and the saturation of the corresponding free constants in terms of resonances can be found in Refs. [9, 10, 11, 12].

2 TREE-LEVEL AMPLITUDE FOR $\gamma\gamma \rightarrow \pi\pi\pi$

The lowest order contribution in ChPT (order four in particle four–momenta or masses) to the amplitude for $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ proceeds exclusively through the first diagram of Fig.1, where the propagator corresponds to a $\pi^0$. One can immediately read the two relevant vertices from the lagrangians $L_2$ and $L_{WZ}$ in Eqs. (2) and (4). If $p_{1,2,3}$ denote the three pion four–momenta and $k, k'$ and $\epsilon, \epsilon'$ the two photon four–momenta and polarizations, one easily obtains

$$A(\gamma\gamma \rightarrow \pi^0\pi^0\pi^0)_{\text{tree}} = \frac{e^2}{\sqrt{2\pi^2 f_\pi^2 s}} \frac{m_\pi^2}{s - m_\pi^2} \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu'} k^{\nu'} \epsilon'^{\alpha'} k'^{\beta'},$$

where $s = (k + k')^2 = (p_1 + p_2 + p_3)^2$ and $m_\pi^2 = p_i^2$. Notice that this amplitude is proportional to $m_\pi^2$, thus vanishing in the chiral limit. The corresponding cross section for $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ at this (lowest) order is shown as dotted (dashed line) in Fig.2.
The lowest order amplitude for $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$ receives contributions from all the diagrams in Fig. 1 and can similarly be obtained to be

$$A(\gamma\gamma \rightarrow \pi^+\pi^-\pi^0)_{\text{tree}} = \frac{e^2}{\sqrt{2} \pi^2 f^3} \epsilon_{\mu
u\alpha\beta} \epsilon^{\mu\nu} k^\alpha k^\beta \left[ \frac{1}{2} (-1 + \frac{p_+^2 - m_{\pi}^2}{s - m_{\pi}^2}) \epsilon^\alpha \epsilon^\beta \right] \epsilon^\mu \epsilon^\nu p_+ \cdot k^- p_+ \cdot k^\beta \left[ \epsilon^\mu \epsilon^\nu \right] \epsilon^\alpha \epsilon^\beta \right],$$

where now the three pion four–momenta are written as $p_+$, $p_-$ and $p_0$ and the notation $p_{ij}^2 \equiv (p_i + p_j)^2$ is used. Our tree-level amplitude coincides with those previously deduced in Refs. [2] and [3]. The corresponding cross section has been plotted in Fig. 3 (dashed line). It turns out to be one order of magnitude smaller than the cross section predicted by Bos et al. [3] and two orders of magnitude smaller than that in Ref. [4]. The $3\pi^0$ and $\pi^+\pi^-\pi^0$ cross-sections are similar near threshold but the one for the neutral pions is much smaller for large values of the center of mass energy. This is due to the $m_{\pi}^2$ proportionality of the amplitude in Eq. (5).

In order to check our results and to understand the origin of the discrepancies with [3] and [4], we have performed an analytic calculation of the tree-level cross sections for $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$ in the non–relativistic approximation (NR). We take $m_{\pi^\pm} = m_{\pi^0}$ for simplicity and for illustration purposes. For both cross sections one has

$$\sigma(\gamma\gamma \rightarrow \pi\pi\pi)^{NR} = \frac{1}{27 \sqrt{3} \pi^2} \left( 1 - 6 \frac{m_{\pi}^2}{s} + 9 \frac{m_{\pi}^2}{s} \right) |T|^2,$$

where the squared matrix elements are

$$|T_{000}|^2 = \frac{1}{3!} \Sigma |A(\gamma\gamma \rightarrow \pi^0\pi^0\pi^0)|^2 = \frac{1}{3!} \left( \frac{\alpha m_{\pi}^2}{\pi f^3} \right)^2 \left( \frac{s}{s - m_{\pi}^2} \right)^2 \rightarrow \frac{1}{3!} \left( \frac{9 \alpha m_{\pi}^2}{8 \pi f^3} \right)^2,$$

for the neutral pion case, and

$$|T_{+0}|^2 = \frac{1}{4} \Sigma |A(\gamma\gamma \rightarrow \pi^+\pi^-\pi^0)|^2 \rightarrow 18 \left( \frac{\alpha m_{\pi}^2}{\pi f^3} \right)^2 \left[ \left( \frac{7}{4} + \frac{1}{128} \right) + 2 - \left( 4 - \frac{1}{4} \right) \right] = \left( \frac{3 \alpha m_{\pi}^2}{8 \pi f^3} \right)^2,$$

for the charged one. In both cases, the arrow ($\rightarrow$) indicates that we have restricted to values at threshold, $s = 9m_{\pi}^2$. This allows for several tests. From Eq. (7) and the $s$ dependent values in Eq. (8) one obtains a reasonable approximation to the $3\pi^0$ cross section at the tree-level for the whole range of relevant energies, as shown (dot-dashed line) in Fig. 2. Similarly, from Eq. (6) and the threshold value in Eq. (4) one obtains the short solid line drawn near threshold in Fig. 3, in good agreement with our tree-level prediction for this charged pion case. Notice also that the threshold value for the $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ amplitude is found to be three times larger than that for $\gamma\gamma \rightarrow \pi^+\pi^-\pi^0$, in agreement with Ref. [3].
A plausible explanation for the huge disagreement between our predictions for $\sigma(\gamma\gamma \to \pi^+\pi^-\pi^0)$ and those from previous work (Refs. [4] and [3]) is offered by Eq. (9). There is a drastic destructive interference between the two different amplitudes shown in Eq. (6) as seen inside the brackets in Eq. (9), where the first two terms refer to the two squared moduli and the third, negative one, to the interference. To see that this drastic effect is also valid for the whole range of energies, the contribution to the cross-section from each independent amplitude has been plotted in Fig. 3 (two almost coincident higher lines) compared to the total cross-section. A precise and numerically accurate treatment is required to extract the correct values for the tree-level cross section from the difference between these two large and destructive contributions.

3 $O(p^6)$ CORRECTIONS

The one-loop contributions in ChPT (order six in particle four–momenta or masses) to the amplitudes for $\gamma\gamma \to \pi\pi\pi$ proceed through various (not–shown) diagrams and from wave function, mass and decay constant renormalization. We have neglected the effects of the $\eta$ and kaon loops, since they are expected to be small as in $\gamma\gamma \to \pi\pi$ [13].

We have performed the calculation of the diagrams contributing to the $\gamma\gamma \to \pi^0\pi^0\pi^0$ amplitude both by hand and using the algebraic manipulation program FORM [14]. One obtains the same result which turns out to be divergent and requires appropriate counterterms. However, inspecting the $O(p^6)$ counterterms to anomalous processes given in Ref. [9] one realizes that there is none contributing to this process. Thus, the divergences (generated only by loops with non-anomalous vertices) must be cancelled by the $l_i$ counterterms in Eq. (3).

A convenient expression is given in terms of the scale independent $\bar{l}$'s, introduced by Gasser and Leutwyler [1] in $SU(2)_L \times SU(2)_R$ ChPT. The matrix element at $O(p^6)$ reads

$$A(\gamma\gamma \to \pi^0\pi^0\pi^0) = \frac{e^2}{2\sqrt{2}\pi^2 f^3_{\pi}} \frac{m^2_{\pi}}{s - m^2_{\pi}} \epsilon_{\mu\nu\alpha\beta}$$

$$\left\{ \epsilon^\mu k^\nu \epsilon^\alpha k'^\beta \left( \frac{1}{3} + \frac{1}{16\pi^2 f^2_{\pi}} \left( (2\bar{l}_1 + 4\bar{l}_2 - 6)(\frac{3p^4_{12} - s^2 - 3m^4_{\pi}}{9m^2_{\pi}}) \right. \right. \right.$$

$$\left. \left. - \frac{2}{3}(\bar{l}_4 - 1)(s - 3m^2_{\pi}) - (\bar{l}_3 - 1)m^2_{\pi} \right) + (2(s - p^2_{12})(p^2_{12} - m^2_{\pi}) - 1)N(p^2_{12} - 8(s - m^2_{\pi})(p^2_{12} - m^2_{\pi}) - 1)R(p^2_{12}, k' \cdot p_{12}) \right] +$$

$$+ \epsilon'' k'' \frac{1}{16\pi^2 f^2_{\pi}} \left( 8(s - m^2_{\pi})(p^2_{12} - m^2_{\pi}) - 1 \right) R(p^2_{12}, k' \cdot p_{12}) (\epsilon'^\alpha - \epsilon'^\nu \cdot p^\nu_{12}) p^\beta_{12}$$

$$+ \left( p_{12} \leftrightarrow p_{13} \right) + \left( p_{12} \leftrightarrow p_{23} \right) + \left[ \left( \bar{k}' \right) \leftrightarrow \left( \bar{k}' \right) \right] +$$

$$+ \left( \bar{p}_{12} \leftrightarrow \bar{p}_{13} \right) + \left( \bar{p}_{12} \leftrightarrow \bar{p}_{23} \right), \quad \text{(10)}$$
where

\[
N(p^2) \equiv -\beta \ln \frac{\beta - 1}{\beta + 1} - 2
\]

\[
R(p^2, k \cdot p) \equiv \frac{I(\lambda^2) - I(\lambda'^2)}{\lambda^2 - \lambda'^2} + \frac{1}{2} N(p^2)
\]  \tag{11}

with

\[
\lambda^2 \equiv p^2/m^2, \quad \lambda'^2 \equiv (p^2 - 2p \cdot k)/m^2, \quad \beta \equiv \sqrt{1 - 4/\lambda^2}
\]

\[
I(\lambda^2) \equiv \frac{1}{2} \ln^2 \frac{\beta - 1}{\beta + 1} + \frac{3}{2} \lambda^2 + \frac{\beta \lambda^2}{2} \ln \frac{\beta - 1}{\beta + 1}.
\]  \tag{12}

Notice that there are three independent gauge invariant amplitudes. The first one is of the tree-type \(\mathcal{B}\) and it is the only one that requires the introduction of the counterterms, reflected in the presence of the finite constants \(\bar{L}_i\). There is also no contribution from the VMD estimate of the “anomalous” \(O(p^6)\) counterterms. Moreover, it should be stressed that, contrary to the tree level amplitude, the \(O(p^6)\) result is no longer proportional to \(m^2\). There is thus no reason to expect small corrections to the lowest order cross-section when including the \(O(p^6)\) contribution.

The number of one-loop diagrams contributing to the \(\gamma\gamma \rightarrow \pi^+\pi^-\pi^0\) amplitude is larger than in the neutral channel. Thus, we have performed this longer calculation using FORM \(\mathcal{F}\). Some partial checks, however, have been done using partial subsets of diagrams. We obtained the \(\pi\pi\) scattering amplitudes as given by Gasser and Leutwyler \(\mathcal{G}\) and the \(\gamma\pi\pi\pi\) amplitude derived in \(\mathcal{H}\). The loops give a gauge invariant result, with divergent contributions which have to be cancelled by appropriate counterterms. Contrary to the neutral process, where the non-anomalous \(O(p^4)\) \(\bar{L}_i\) were the only counterterms needed, the charged process needs additional, genuine, \(O(p^6)\) counterterms. These have been discussed in general in Ref. \(\mathcal{I}, \mathcal{J}, \mathcal{K}\). We have explicitly checked that the divergences appearing in the one-loop calculation cancel with the known counterterms from the previous references. The terms of \(O(p^6)\) from the lagrangian also contribute to the amplitude via tree diagrams that contain two free constants. These constants have been fixed assuming their saturation by the vector meson contribution \(\mathcal{L}\).

The final expression for the amplitude is very long and will be given elsewhere \(\mathcal{M}\). We will only mention here that it contains 10 independent amplitudes (actually, using Schouten identities it can be shown that this is the maximum number of allowed independent amplitudes), whereas only 3 of them appeared at lowest order.
4 NUMERICAL RESULTS

The $O(p^6)$ cross-sections for $\gamma\gamma \to \pi^0\pi^0\pi^0$ and $\gamma\gamma \to \pi^+\pi^-\pi^0$ have been plotted (solid lines) in Figs. 2 and 3 respectively. We have used $m_{\pi^\pm} = m_{\pi^0}$ in the amplitude but the experimental values in the phase space and we fixed the renormalization scale $\mu = M_{\rho}$, according to our assumption of the saturation of the free constants in the lagrangian at $O(p^6)$ by the vector meson resonances. The values used for the constants in $L_4$ that contribute to our processes are the central values quoted in [1],

$$\bar{l}_1 = -2.3 \quad \bar{l}_2 = 6.0 \quad \bar{l}_3 = 2.9 \quad \bar{l}_4 = 4.3.$$  \hspace{1cm} (13)

The corrections are very large in both channels. In the neutral channel the corrections increase the cross-section up to two orders of magnitude! This is due to the vanishing of the lowest order amplitude in the chiral limit, which no longer occurs at $O(p^6)$. Since the $O(p^4)$ amplitude is proportional to $m_{\pi^0}^2$, which is very small compared to the momenta involved in the process, the corrections are very large. In the charged channel the reason for the small cross-section at lowest order is different. Here there is a large cancellation between the two gauge invariant amplitudes contributing to this process. The $O(p^6)$ corrections modify both amplitudes, thus spoiling the almost perfect cancellation, and adds new gauge invariant amplitudes.

The $\gamma\gamma \to \pi\pi\pi$ cross sections obtained at one loop in ChPT are significantly larger than the lowest order predictions. However, they are still smaller than the ones for other interesting $\gamma\gamma$ processes as, for instance, $\gamma\gamma \to \pi^0\pi^0$ [13, 18, 17]. In any case, they have some chances of being measured at Daphne. We have estimated that, working with the optimal projected machine luminosity, about 180 $\pi^+\pi^-\pi^0$ and 23 $\pi^0\pi^0\pi^0$ events per year should originate from photon–photon collisions. An eventual experimental confirmation of these processes would be a clear indication of the important role played by the chiral loops in the anomalous WZ sector.

In summary, we have estimated the cross sections for the processes $\gamma\gamma \to \pi^+\pi^-\pi^0$ and $\gamma\gamma \to \pi^0\pi^0\pi^0$ at lowest order, $O(p^4)$, and at one loop, $O(p^6)$, in Chiral Perturbation Theory. The corrections are extremely large due to the smallness of the lowest order cross-sections. Since the reasons for these small values at lowest order are understood and disappear at $O(p^6)$ in both channels, it is not expected that the $O(p^8)$ corrections will modify our results in an important way.

ACKNOWLEDGEMENTS
We are pleased to thank D. Espriu, Ll. Garrido, J. Gasser, J. Matias and J. Taron for discussions. This work was partially supported by CICYT under contracts: AEN94-0936 and AEN95-0815, and by EURODAPHNE, HCMP, EEC Contract #CHRX-CT920026.

REFERENCES

[1] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158 (1984) 142; the extension to the 3-flavor case is in J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.

[2] S. L. Adler, B.W. Lee, S.B. Treiman and A. Zee, Phys. Rev. D4 (1971) 3497 and references therein.

[3] J. W. Bos, Y. C. Lin and H. H. Shih, Phys. Lett. B337 (1994) 152.

[4] M. Pratap et al, Phys. Rev. D5 (1972) 269.

[5] Particle Data Group, Phys. Rev. D50 (1994) 1173.

[6] J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95.
   E. Witten, Nucl Phys. B233 (1983) 422.

[7] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B321 (1989) 311.

[8] J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. D39 (1989) 147.

[9] J. Bijnens, A. Bramon and F. Cornet, Z. fur Phys. C46 (1990) 595.

[10] J. Bijnens, A. Bramon and F. Cornet, Phys. Lett. B237 (1990) 488.

[11] J. Bijnens, Int. Journal Mod. Phys. A8 (1993) 3045.

[12] R. Akhoury and A. Alfakih, Ann Phys. 210 (1991) 81.

[13] J. Bijnens and F. Cornet, Nucl. Phys. B296 (1988) 557.

[14] J.A.M. Vermaseren, “The symbolic manipulation program FORM”. KEK-TH-326 (1992).

[15] H.W. Fearing and S. Scherer, TRIUMF report TRI-PP-94-68 and hep-ph/9408346 (1994).

[16] Ll. Ametller et al. (in preparation).

[17] S. Bellucci, J. Gasser and M. E. Sainio, Nucl. Phys. B423 (1994) 80; ibid. B431 (1994) 413 (erratum).

[18] J. Donoghue, B. Holstein and Y.C. Lin, Phys. Rev. D37 (1988) 2423.
List of Figures

1. Tree level diagrams in ChPT for $\gamma \gamma \rightarrow \pi \pi \pi$ ............................................. 9
2. $\gamma \gamma \rightarrow \pi^0 \pi^0 \pi^0$ cross section ................................................................. 10
3. $\gamma \gamma \rightarrow \pi^+ \pi^- \pi^0$ cross section ................................................................. 11
Figure 1: Tree level diagrams in ChPT for $\gamma \gamma \rightarrow \pi \pi \pi$. 

(a) (b) (c) (d)
Figure 2: $\gamma\gamma \rightarrow \pi^0\pi^0\pi^0$ cross section at tree level (dashed line) as a function of $\sqrt{s}$. The dot-dashed line corresponds to the non-relativistic tree level approximation. The solid line corresponds to the $O(p^6)$ result.
Figure 3: $\gamma\gamma \to \pi^+\pi^-\pi^0$ cross section as a function of $\sqrt{s}$. The dashed line corresponds to the tree level ChPT prediction. The short solid line near threshold is the non-relativistic tree level approximation. The two higher dotted curves show how would the cross section be for each one of the two gauge invariant amplitudes in Eq.(6), and illustrate their (largely destructive) interference effects. The solid line corresponds to the $O(p^6)$ result.