Improvement of a quantum broadcasting multiple blind signature scheme based on quantum teleportation

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Abstract Recently, a broadcasting multiple blind signature scheme based on quantum teleportation has been proposed for the first time. It is claimed to have unconditional security and properties of quantum multiple signature and quantum blind signature. In this paper, we analyze the security of the protocol and show that each signatory can learn the signed message by a single-particle measurement and the signed message can be modified at random by any attacker according to the scheme. Furthermore, there are some participant attacks and external attacks existing in the scheme. Finally, we present an improved scheme and show that it can resist all of the mentioned attacks. Additionally, the secret keys can be used again and again, making it more efficient and practical.

Keywords Security analysis · Quantum broadcasting multiple blind signature · Quantum teleportation

1 Introduction

Quantum signature is the counterpart in the quantum world of classical digital signature. Most classical digital signature schemes are based on public key cryptography
which can be broken by Shor’s algorithm [1]. Quantum signature, which is based on the laws of quantum physics, can provide us unconditional security. Many different quantum signature models are proposed for different application demands, such as arbitrated quantum signature [2–7], quantum proxy signature [8–11], quantum group signature [12–15], quantum blind signature [16–18] and quantum multiple signature [19,20].

A secure quantum signature scheme should satisfy the following two basic requirements: (1) No forgery. Specifically, the signature cannot be forged by any illegal signatory. (2) No disavowal. The signatory cannot disavow his signature, and the receiver cannot disavow his receiving it. Furthermore, the receiver cannot disavow the integrity of the signature [4].

As quantum cryptography has developed, many cryptanalyses of existing protocols have been presented [21–27]. Some effective attack strategies also have been proposed to eavesdrop in the existing quantum cryptography protocols [28], such as intercept-resend attacks [29], entanglement swapping attacks [30–32], teleportation attacks [33,34], dense-coding attacks [35–37], channel-loss attacks [38,39], denial-of-service attacks [40,41], correlation—extractability attacks [42–44], Trojan horse attacks [45–48], participant attacks [49] and collaborate attacks [50]. Understanding these attacks is very important for designing quantum signature schemes with higher security. It also advances the research in quantum signature. Zou and Qiu [4] analyzed the arbitrated quantum signatures based on GHZ states and Bell states, finding that the receiver Bob can successfully reject the signature by disavowing its integrity. Then, they proposed a new scheme by using a public board to fix this security loophole in which the entanglement was not needed any more.

Gao et al. [21] gave a perfect cryptanalysis on existing arbitrated quantum signature. They found that the signature can be forged by the receiver at will in almost all the existing AQS schemes and the sender can disavow the signature just by an intercept-resend method. Due to the existence of serious loopholes, it is imperative to reexamine the security of other quantum signature protocols.

Recently, a broadcasting multiple blind quantum signature scheme based on quantum teleportation has been proposed in Ref. [51]. It is said to have the properties of both quantum multiple signature and quantum blind signature. Here we show that it is not a real blind signature because the signatory can get the content of the signed message. In addition, the signed message can be modified at random by any attacker. Moreover, there are some participant attacks and external attacks existing in the scheme. For instance, the message sender Alice can impersonate $U_i$ successfully as she can get the content of the signature and $U_i$’s secret key $K_{CU_i}$. Moreover, Alice can sign arbitrary message at will. The signature collector Charlie can counterfeit the signature optionally. With respect to the external attacks, the eavesdropper Eve can forge $U_i$’s signature at will without knowing the secret key $K_{CU_i}$.

All the attack strategies are described in detail, and finally, we present an improved scheme which can resist all the mentioned attacks. Meanwhile, since all the secret keys can be reused, it may greatly increase the scheme’s efficiency and make it more practical.

The rest of this paper is organized as follows. First, in Sect. 2 we review the original protocol briefly. In Sect. 3, we present the security analysis of the original protocol
and describe the attack strategies in detail. In Sect. 4, we present an improved scheme and analyze its security by showing that the improved one can resist all the attacks mentioned above and that the keys can be used again and again. In Sect. 5, a short conclusion is given and an issue worthy of further research is proposed.

2 Review of the original protocol

The protocol in Ref. [51] involves the following four characters: (1) Alice is the message sender. (2) \( U_i \) is the \( i \)-th member of broadcasting multiple signatory. (3) Charlie is the signature collector. (4) Bob is the receiver and the verifier of the broadcasting multiple blind signature.

The scheme is composed of three parts: the initial phase, the individual blind signature generation and verification phase, and the combined multiple blind signature verification phase.

In this scheme, Alice sends \( t \) copies of \( n \)-bit classical message \( m \) to \( t \) signatories \( U_i \) (\( i = 1, 2, \ldots, t \)), respectively, then \( U_i \) signs the message \( m \) to get the blind signature \( S_i \) and sends \( S_i \) to Charlie. Charlie collects and verifies these blind signatures; then, he constructs a multiple signature and sends it to Bob. Finally, Bob verifies the multiple signature by confirming the message.

(1) Initial phase

(1.1) Alice transforms the classical message \( m \) into \( n \)-bit as

\[
m = m(1)||m(2)||\cdots||m(j)||\cdots||m(n), \quad m(j) = 0 \text{ or } m(j) = 1, \quad j = 1, 2, \ldots, n.
\]

(1.2) Quantum key distribution

Alice shares a secret key \( K_{AB} \) with Bob, a secret key \( K_{AC} \) with Charlie, and secret keys \( K_{AUi} \) (\( i = 1, 2, \ldots, t \)) with each signatory \( U_i \), respectively. Bob shares a secret key \( K_{BC} \) with Charlie, and Charlie shares secret keys \( K_{CUi} \) (\( i = 1, 2, \ldots, t \)) with each signatory \( U_i \), respectively. To obtain unconditional security, all these keys are distributed via QKD protocols.

(1.3) Alice sends \( E_{K_{AB}}^{C}(m) \) to Bob

Here \( E^{C} \) means classical one-time pad algorithm,

\[
E_{K_{AB}}^{C}(m) = K_{AB} \oplus m.
\]

(2) \( E_{K}^{Q} \) in the later means quantum one-time pad

\[
E_{K}^{Q}(|P\rangle) = \bigotimes_{i=1}^{n} \sigma_{x}^{k_{2i-1}} \sigma_{z}^{k_{2i}} |P_{i}\rangle,
\]

\( K \) is a secret key with \( |K| = 2n \), \( K_i \) is the \( K \)'s \( i \)-bit. \( |P\rangle \) is an \( n \)-bit quantum message, and \( |P_{i}\rangle \) is its \( i \)-bit. \( \sigma_{x} \) and \( \sigma_{z} \) are two Pauli operators.
The individual blind signature generation and verification phase

In this phase, we pick one of the signatory $U_i$ as the representative who signs the message.

(2.1) Message transformation

Assume that Alice is to send the message $m$. She prepares $n$-qubit state $|\psi(m)\rangle_M$ as

$$|\psi(m)\rangle_M = \bigotimes_{j=1}^{n} |\psi(j)\rangle_M,$$

where

$$|\psi(j)\rangle_M = \begin{cases} \frac{1}{\sqrt{2}}(|0\rangle_M + |1\rangle_M) & \text{if } m(j) = 1 \\ \frac{1}{\sqrt{2}}(|0\rangle_M - |1\rangle_M) & \text{if } m(j) = 0. \end{cases}$$

(2.2) Quantum channel setup

Alice prepares $n$ EPR pairs. Each pair is denoted as

$$|a(j)\rangle_{AC} = \frac{1}{\sqrt{2}} (|00\rangle_{AC} + |11\rangle_{AC}), j = 1, 2, 3, \ldots, n + l.$$  

(2.3) Signature Phase

(2.3.1) Alice picks up her $n$ EPR particles denoted as $\{|\varphi(N)\rangle_A\}$, i.e.,

$$\{|\varphi(N)\rangle_A\} = \{|\varphi(1)\rangle_A, |\varphi(2)\rangle_A, \ldots, |\varphi(j)\rangle_A, \ldots, |\varphi(n)\rangle_A\},$$

and the other $n$ EPR particles denoted as $\{|\varphi(N)\rangle_C\}$, i.e.,

$$\{|\varphi(N)\rangle_C\} = \{|\varphi(1)\rangle_C, |\varphi(2)\rangle_C, \ldots, |\varphi(j)\rangle_C, \ldots, |\varphi(n)\rangle_C\}.$$  

(2.3.2) To distinguish each signatory, Alice creates a unique serial number which is denoted as $SN$ attaching to $\{|\varphi(N)\rangle_A\}$. Since $SN$ is a classical string, Alice transfers it to a quantum state sequence $|SN\rangle$ with the basis $B_Z = \{|0\rangle, |1\rangle\}$. Then, she sends $E_{K_{AU_i}}^Q (|\psi(N)\rangle_{MA}, |SN\rangle)$ to $U_i$. Here

$$|\psi\rangle_{MA} = \bigotimes_{j=1}^{n} |\psi(j)\rangle_M \otimes |\varphi(j)\rangle_A.$$  

After that, Alice sends $E_{K_{AC}}^Q (\{|\varphi(N)\rangle_C\}, |SN\rangle)$ to Charlie.

(2.3.3) $U_i$ decrypts $E_{K_{AU_i}}^Q (|\psi\rangle_{MA}, |SN\rangle)$ to get $|\psi\rangle_{MA}$ and $|SN\rangle$, then he performs Bell-basis measurement to get the outcomes $\{\beta_{MA}(j) | j = 1, 2, \ldots, n\}$. Each $\beta_{MA}(j) = \beta_{kl}(k, l \in \{0, 1\})$ is expressed by 2-bit string $kl$ according to $|\beta_{kl}\rangle \mapsto kl$. Then, he gets $S_i$ as

$$S_i = \beta_{MA}(1)||\beta_{MA}(2)|| \cdots ||\beta_{MA}(j)|| \cdots ||\beta_{MA}(n).$$
(2.3.4) $U_i$ sends $E^C_{K_{CU_i}}(S_i, SN)$ to Charlie.

(2.4) Verification Phase
(2.4.1) Charlie decrypts $E^C_{K_{CU_i}}(S_i, SN)$ to get the signature $S_i$ and $SN$.
(2.4.2) According to $S_i$, $SN$ and quantum teleportation, Charlie performs one of the corresponding reverse transformation $(I, X, Y, Z)$ on each particle $|\psi(j)\rangle_C$ in his hand to get $|\psi'(j)\rangle_C$. He obtains $|\psi'(m)\rangle_C$ as

$$|\psi'(m)\rangle_C = \bigotimes_{j=1}^{n} |\psi'(j)\rangle_C. \quad \text{(11)}$$

(2.4.3) Charlie gets $m'$ by measuring each $|\psi'(j)\rangle$ in the basis of $\{\frac{1}{\sqrt{2}}(|0\rangle_M + |1\rangle_M), \frac{1}{\sqrt{2}}(|0\rangle_M - |1\rangle_M)\}$. Then, he sends $E^C_{K_{BC}}(m')$ to Bob.

(2.4.4) Bob decrypts $E^C_{K_{BC}}(m'), E^C_{K_{AB}}(m)$ by the secret keys $K_{BC}$ and $K_{AB}$, respectively, and compares $m$ with $m'$. If they are the same, $S_i$ is accepted. Otherwise, it is rejected.

(3) The combined multiple signature generation and verification phase
(3.1) Charlie collects all individual signatures to generate the multiple signature $S = \{S_i | i = 1, 2, \ldots, t\}$ and generates the message $\{m'_i | i = 1, 2, \ldots, t\}$. If $m'_i$ is equal to $m'_{i+1}$ ($i = 1, 2, \ldots, t-1$), he confirms the message and sends $E^C_{K_{BC}}(m'_1)$ to Bob. If it is not equal, the process is terminated.
(3.2) After Bob decrypts $E^C_{K_{BC}}(m'_1)$ and $E^C_{K_{AB}}(m)$, he accepts $S$ if $m'_1$ is equal to $m$; otherwise, he terminates the process.

3 Cryptanalysis of the original protocol

In this section, we point out that there are some security loopholes in the scheme in Ref. [51] and describe the corresponding attack strategies in detail.

3.1 Each $U_i$ can learn the signed message $m$

The scheme is claimed to have properties of quantum blind signature so that the signatory cannot learn the signed message. Here we show that each signatory $U_i$ can get the message just by a single-particle measurement.

Suppose Alice wants to send an $n$-bit classical message $m$ to get $U_i$’s signature, according to the scheme, she will transform it into $n$-qubit state $|\psi(m)\rangle_M$ according to Eqs. (4) and (5). Because $\frac{1}{\sqrt{2}}(|0\rangle_M + |1\rangle_M)$ and $\frac{1}{\sqrt{2}}(|0\rangle_M - |1\rangle_M)$ are orthogonal to each other, they can form an orthonormal basis of the two dimensional Hilbert Space. When $U_i$ gets $K_{AU_i}^{O}(|\psi(m)\rangle_M)$ from Alice in the signature phase, he can decrypt it and perform a single-particle measurement in the basis of $\{\frac{1}{\sqrt{2}}(|0\rangle_M + |1\rangle_M), \frac{1}{\sqrt{2}}(|0\rangle_M - |1\rangle_M)\}$ on the first state to get the message $m$, which has no effect on the following process. From this problem, we can see the original scheme is not a real blind signature scheme.
3.2 Any attacker can modify message \( m \) at random

Here we show the signed message \( m \) can be modified at random through the intercept-resend method by any attacker, including participant attackers or external attackers.

In the original scheme, messages \( m \) and \( m' \) are encrypted according to the one-time pad encryption algorithm during their transmission. Any attacker can intercept \( E_{K_{AB}}^C(m) \) and resend \( E_{K_{AB}}^C(m) \oplus m_0 \) to Bob in Step (1.3). According to the scheme, Bob will get \( m \oplus m_0 \) instead of \( m \), here \( m_0 \) is an arbitrary \( n \)-bit random binary string. At the same time, he intercepts \( E_{K_{BC}}^C(m') \) and resends \( E_{K_{BC}}^C(m') \oplus m_0 \) in Step (2.4.3). According to the scheme, \( m \oplus m_0 \) can pass the following verification process. Because \( m_0 \) is arbitrary, \( m \) can be modified at random by any attacker through intercept-resend method.

3.3 Alice’s attack

To illustrate Alice’s attack, here take a 1-bit message \( m(j) \) to make a demonstration.

3.3.1 Alice can get the signature

Suppose Alice sends the message \( m(j) \) to get \( U_i \)’s blind signature \( S_i(j) \). From the scheme, we can see that \( U_i \) signs \( m(j) \) by measuring \( |\psi(j)\rangle_{MA} \) in the Bell basis, which is sent from Alice in Step(2.3.3). Alice can get \( S_i(j) = \beta_{kl} \) by measuring \( |\psi(j)\rangle_{MA} \) on Bell basis and recording the outcome \( \beta_{kl} \) before she sends it to \( U_i \). Instead, Alice sends the two-particle state \( |\beta_{kl}\rangle_{MA} \) to \( U_i \). Then, \( U_i \)’s measurement outcome is \( \beta_{kl} \). Then, Alice can get each \( S_i(j) \).

3.3.2 Alice can get \( U_i \)’s secret key \( K_{CU_i} \)

It has been illustrated that Alice can get each \( S_i(j) \) and then Alice can get \( U_i \)’s signature \( S_i \). Alice can intercept \( E_{K_{CU_i}}^C(S_i)||SN \) when it is sent from \( U_i \) to Charlie in Step (2.3.4). Because Alice knows \( S_i \), she can extract \( U_i \)’s secret key \( K_{CU_i} \) by adding \( S_i \) to \( E_{K_{CU_i}}^C(S_i) \) as \( K_{CU_i} = S_i \oplus E_{K_{CU_i}}^C(S_i) \). Then, she resends \( E_{K_{CU_i}}^C(S_i)||SN \) to Charlie. All of these cannot be discovered.

3.3.3 Alice can sign the message at will

We can see that Alice can completely replace the signatory \( U_i \) to sign the message. In order to illustrate Alice can sign arbitrary message at will, we will demonstrate the quantum teleportation process of the above protocol as follows:

Suppose that the particle \( M \) carry a 1-bit classical information \( m(j) \) and the state of particle \( M \) are denoted as

\[
|\psi(j)\rangle_M = \frac{1}{\sqrt{2}}(|0\rangle_M + d|1\rangle_M), \quad d = \pm 1.
\]
The EPR pairs shared between Alice and Charlie are denoted as
\[ |a(j)\rangle_{AC} = \frac{1}{\sqrt{2}} (|00\rangle_{AC} + |11\rangle_{AC}). \] (13)

The two states are combined to form a three particle state \( |\Phi(j)\rangle \) as
\[
|\Phi(j)\rangle = |\psi(j)\rangle_M \otimes |a(j)\rangle_{AC} \\
= \left( \frac{|0\rangle_M + d|1\rangle_M}{\sqrt{2}} \right) \left( \frac{|00\rangle_{AC} + |11\rangle_{AC}}{\sqrt{2}} \right) \\
= \frac{1}{2} \left[ |\beta_{00}\rangle_{MA} \left( \frac{|0\rangle_C + d|1\rangle_C}{\sqrt{2}} \right) + |\beta_{01}\rangle_{MA} \left( \frac{|1\rangle_C + d|0\rangle_C}{\sqrt{2}} \right) \right. \\
\left. \quad + |\beta_{10}\rangle_{MA} \left( \frac{|0\rangle_C - d|1\rangle_C}{\sqrt{2}} \right) + |\beta_{11}\rangle_{MA} \left( \frac{|1\rangle_C - d|0\rangle_C}{\sqrt{2}} \right) \right], \] (14)

where
\[
|\beta_{00}\rangle_{MA} = \frac{|00\rangle_{MA} + |11\rangle_{MA}}{\sqrt{2}}, \quad (15) \\
|\beta_{01}\rangle_{MA} = \frac{|01\rangle_{MA} + |10\rangle_{MA}}{\sqrt{2}}, \quad (16) \\
|\beta_{10}\rangle_{MA} = \frac{|00\rangle_{MA} - |11\rangle_{MA}}{\sqrt{2}}, \quad (17) \\
\text{and} \quad |\beta_{11}\rangle_{MA} = \frac{|01\rangle_{MA} - |10\rangle_{MA}}{\sqrt{2}}. \quad (18)
\]

From Eq. (14), we can see if the measurement outcome is \( \beta_{00} \), the state of the particle \( C \) is just the information state \( |\psi(j)\rangle_M \). Then, we take operation \( I \) on the state of \( C \). If the measurement outcome is \( \beta_{01} \), then we perform operation \( X \) on \( C \) to recover it to the information state. If the outcomes are \( \beta_{10} \) and \( \beta_{11} \), then we take the operation \( Z \) and \( Y \), respectively.

Here, we show Alice can modify the signature \( S_i(j) \) at random as follows: When Alice prepares \( |\psi(j)\rangle_M \) and \( |a(j)\rangle_{AC} \) in Step (2.1) and Step (2.2), she does not send \( |\varphi(j)\rangle_C \) to Charlie and \( |\psi(j)\rangle_{MA} \) to \( U_i \) immediately. Instead, she performs a Bell-basis measurement on \( |\psi(j)\rangle_{MA} \) to get the outcome \( \beta_{kl} \) and she sends another Bell state \( |\beta_{kl'}\rangle_{MA} \) to \( U_i \). Then, the signature \( S_i'(j) = \beta_{kl} \) has been changed into \( S_i'(j) = \beta_{kl'} \). In order to make sure \( S_i'(j) \) can pass the verification, Alice performs a corresponding operator \( V \) on \( |\varphi(j)\rangle_C \) before sending it to Charlie. Alice can derive the corresponding operator \( V \) according to Eq. (14). Assume \( |\psi(j)\rangle_M \) is teleported from Alice to Charlie and the measurement outcome is \( \beta_{kl} \). Also using this equation, Charlie will performs a Pauli operator \( V_1 \) on \( |\varphi(j)\rangle_C \) to make sure
\[ V_1 |\varphi(j)\rangle_C = |\psi(j)\rangle_M. \] (19)
Table 1 Alice’s attack strategies: first, Alice measures $|\psi(j)\rangle_{MA}$ to get $S_i(j) = \beta_{kl}$, but she sends $|\beta_{k'l'}\rangle_{MA}$ to $U_i$ instead.

| $S_i(j)$ | $S'_i(j) = \beta_{k'l'}$ | $V_1$ | $V_2$ | $V$ |
|---------|------------------|------|------|-----|
| $S_i(j) = \beta_{00}$ | $S'_i(j) = \beta_{01}$ | $I$ | $X$ | $X$ |
| $S_i(j) = \beta_{00}$ | $S'_i(j) = \beta_{10}$ | $I$ | $Z$ | $Z$ |
| $S_i(j) = \beta_{00}$ | $S'_i(j) = \beta_{11}$ | $I$ | $Y$ | $Y$ |
| $S_i(j) = \beta_{01}$ | $S'_i(j) = \beta_{00}$ | $X$ | $I$ | $X$ |
| $S_i(j) = \beta_{01}$ | $S'_i(j) = \beta_{10}$ | $X$ | $Z$ | $Y$ |
| $S_i(j) = \beta_{01}$ | $S'_i(j) = \beta_{11}$ | $X$ | $Y$ | $Z$ |
| $S_i(j) = \beta_{10}$ | $S'_i(j) = \beta_{00}$ | $Z$ | $I$ | $Z$ |
| $S_i(j) = \beta_{10}$ | $S'_i(j) = \beta_{11}$ | $Z$ | $X$ | $Y$ |
| $S_i(j) = \beta_{11}$ | $S'_i(j) = \beta_{00}$ | $Y$ | $I$ | $Y$ |
| $S_i(j) = \beta_{11}$ | $S'_i(j) = \beta_{01}$ | $Y$ | $X$ | $Z$ |
| $S_i(j) = \beta_{11}$ | $S'_i(j) = \beta_{10}$ | $Y$ | $Z$ | $X$ |

Then, the signature $S_i(j)$ has been changed into $S'_i(j)$. At the same time, Alice performs a corresponding unitary operation $V$ on $|\varphi(j)\rangle_C$ before sending it to Charlie.

In other words, the particle $C$ is in the state

$$|\varphi(j)\rangle_C \equiv V_1|\psi(j)\rangle_M. \quad (20)$$

Here $A \equiv B$ means $A$ is equivalent to $B$ except for a global phase. Alice performs the corresponding $V$ on $|\varphi(j)\rangle_C$, so

$$|\varphi(j)\rangle_C \equiv V^\dagger_1 V_1|\psi(j)\rangle_M. \quad (21)$$

Here $V^\dagger$ is the conjugate transpose of $V$. When $S_i(j)$ is changed into $S'_i(j)$, and Charlie will take another Pauli operator $V_2$ on $|\varphi(j)\rangle_C$ to return it to the information state $|\psi(j)\rangle_M$, then

$$|\varphi(j)\rangle_C \equiv V_2 V^\dagger_1 V_1|\psi(j)\rangle_M \equiv |\psi(j)\rangle_M. \quad (22)$$

From Eq. (22), we can conclude that

$$V_2 V^\dagger_1 V_1 = I \quad \text{or} \quad V = V_1 V_2. \quad (23)$$

We take a simple example to make an illustration. Suppose Alice gets the measurement outcome $\beta_{00}$, but she sends $|\beta_{01}\rangle_{AM}$ to $U_i$, according to Eq. (14), $V_1 = I$, $V_2 = X$, then $V = X$ according to Eq. (23). We list Alice’s attack strategies in Table 1.

3.4 Charlie’s attack

In the original scheme, Charlie can also attack the program by modifying the signature $S$ at will.
Charlie is the signature collector whose duty is to collect all the individual signature $S_i (i = 1, 2, \ldots, t)$ and extract $m_i'$ by first recovering each $|\varphi(j)\rangle_C$ to $|\psi'(j)\rangle_C$ according to Eq. (14) and then measuring $|\psi'(m)\rangle_C$ in the basis of $\{\frac{1}{\sqrt{2}}(|0\rangle_M + |1\rangle_M), \frac{1}{\sqrt{2}}(|0\rangle_M - |1\rangle_M)\}$. Charlie can modify the signature $S$ into arbitrary $S'$ and keep the message $m_i'$ unchanged after confirming the message. Because Bob just verifies whether $m$ is equal to $m_1'$ or not, $S'$ can pass the verification without being discovered.

3.5 Eavesdropper Eve’s forgery attack

In Ref. [51], it is declared that the eavesdropper Eve cannot forge $U_i$’s signature on the assumption that she can get $U_i$’s secret key $K_{CU_i}$ because of the quantum teleportation. Here we show that Eve can forge $U_i$’s signature at will even though she knows nothing about the $U_i$’s key $K_{CU_i}$.

Here we take a 1-bit message $m(j)$ to make a demonstration. This is an incomplete message $m(j)$ whose signature is $S_i(j)$. Eve replaced $S_i(j)$ with another $S'_i(j)$ when it is sent from $U_i$ to Charlie. Then, Charlie will recover the message according to $S'_i(j)$ based on teleportation. Suppose the signature $S_i(j)$ is $\beta_{00}$. It is changed into $S'_i(j) = \beta_{01}$ under Eve’s attack. We take an illustration as follows:

1) Without Eve’s attack

Suppose the signature $S_i(j)$ is $\beta_{00}$ and Charlie’s particle $C$ is in the state

$$|\varphi(j)\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle_C + d|1\rangle_C), d = \pm 1. \quad (24)$$

Then, Charlie will perform $I$ on his particle to recover it to the information state $|\psi'(j)\rangle_C$ according to Eq. (14), here

$$|\psi'(j)\rangle_C = I|\varphi(j)\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle_C + d|1\rangle_C). \quad (25)$$

After that Charlie measures it and extracts the message $m'(j)$ as

$$m'(j) = \begin{cases} 1 & \text{if} \quad d = 1 \\ 0 & \text{if} \quad d = 0. \end{cases} \quad (26)$$

2) With Eve’s attack

The signature is tampered with $S'_i(j) = \beta_{01}$ with Eve’s attack, and Charlie will perform $X$ on his particle $C$ which is still in the state of Eq. (24). Then, the state of $C$ will be

$$|\psi'(j)\rangle_C = X|\varphi(j)\rangle_C = \frac{1}{\sqrt{2}}(|1\rangle_C + d|0\rangle_C). \quad (27)$$
After that, Charlie measures $|\psi'(j)_C|$ to extract the message $m''(j)$ as

$$m''(j) = \begin{cases} 1 & \text{if } d = 1 \\ 0 & \text{if } d = 0. \end{cases}$$  \hspace{1cm} (28)$$

From Eqs. (26) and (28), we can see Charlie will get the same messages, i.e., $m''(j) = m'(j)$. See the list of all the cases in Table 2.

From the second column of Table 2, we can see $\beta_{00}$ and $\beta_{01}$ are interchangeable and so is the $\beta_{10}$ and $\beta_{11}$. Specifically, when Eve tampered with $S_i(j) = \beta_{00}(\beta_{10})$ by $S'_i(j) = \beta_{01}(\beta_{11})$ or vice versa, Charlie will extract the same message. Precisely, $m'(j)$ is equal to $m''(j)$. Accordingly, $S'_i(j)$ can always pass the verification. Eve’s other modification of the signature is not interchangeable. We can see all the other cases get different message $m''(j) \neq m'(j)$, but they all satisfy $m''(j) = m'(j) \oplus 1$ where $\oplus$ is modulo 2 addition.

From Table 2, we can make a law of the message and its corresponding signature as follows:

If the signature $S_i(j) = \beta_{kl}$ is changed into $S'_i(j) = \beta_{k'l'}$, then their corresponding messages $m'(j)$ and $m''(j)$ ($k, k', l, l' \in \{0, 1\}$ $j = 1, 2, \ldots, n$) will satisfy

$$m''(j) = \begin{cases} m'(j) & \text{if } k = k' \\ m'(j) \oplus 1 & \text{if } k \neq k'. \end{cases}$$  \hspace{1cm} (29)$$

Next, we show Eve can forge each signature $S_i$ by the intercept-resend method. Eve can intercept $E^C_{K_{CUi}}(S_i)$ when it is sent from $U_i$ to Charlie. She adds a $2n$-bit binary string

$$l = i_1i_2 \ldots i_{2n}$$  \hspace{1cm} (30)$$
Improvement of a quantum broadcasting multiple blind... to $E_{KC_{1i}}^C(S_i)$ and sends it to Charlie. Then, Charlie will get

$$S_i' = S_i \oplus l.$$  \hspace{1cm} \text{(31)}

Charlie will recover the information $m''$ based on $S_i'$ according to the teleportation rather than $S_i$. Then, Charlie will get

$$m'' = m' \oplus l',$$  \hspace{1cm} \text{(32)}

where

$$l' = j_1 j_2 \ldots j_n.$$  \hspace{1cm} \text{(33)}

According to Eq. (29), $l'$ must satisfy

$$j_k = \begin{cases} 
0 & \text{if } i_{2k-1} = 0 \\
1 & \text{if } i_{2k-1} = 1.
\end{cases} \hspace{1cm} \text{(34)}$$

At the same time, Eve intercepts $E_{K_{AB}}^C(m)$ in the Step (1.3) and resends $E_{K_{AB}}^C(m) \oplus l'$ to Bob. Then, Bob will get $m \oplus l'$ instead of $m$. $S_i'$ will be accepted for the signature of $m \oplus l'$. We can see that Eve’s forgery attack can get successful.

4 An improved scheme

Here we present an improved scheme and show it can resist all the attacks mentioned above. Also the secret keys can be reused which can provide efficiency and practicality.

Before we present the improved scheme, it is necessary to introduce the QOTP algorithm it uses. Suppose a quantum message

$$|P\rangle = \bigotimes_{j=1}^n |P_j\rangle$$  \hspace{1cm} \text{(35)}

is composed of $n$ qubits

$$|P_j\rangle = \alpha_j |0\rangle + \beta_j |1\rangle,$$  \hspace{1cm} \text{(36)}

where

$$|\alpha_j|^2 + |\beta_j|^2 = 1$$  \hspace{1cm} \text{(37)}

and the encryption key $K \in \{0, 1\}^{4n}$. The QOTP encryption $E_K$ used in this scheme on the quantum message can be described as
\[ E_K(|P\rangle) = \bigotimes_{j=1}^{n} \sigma_x^{K_{4j-3}} \sigma_z^{K_{4j-2}} \sigma_x^{K_{4j-1}} T \sigma_x^{K_{4j}} |P_j\rangle \]  

(38)

where

\[ T = \frac{i}{\sqrt{3}} (\sigma_x - \sigma_y + \sigma_z). \]  

(39)

This QOTP encryption \( E_K \) is the improved one introduced in Ref. [52] for the first time. The assistant operator \( T \) can make sure the encrypted message cannot be forged in the scheme. Distinctly, for arbitrary message \(|P\rangle\), there are no nonidentity unitary \( V \) and unitary \( U \) such that

\[ E_K^+ V E_K |P\rangle \equiv U |P\rangle. \]  

(40)

In order to make the secret keys reusable in the improved scheme, we use a one-way hash function here [7]:

\[ H(x) : \{0, 1\}^n \rightarrow \{0, 1\}^{4n}. \]  

(41)

This scheme also contains four factors: (1) Alice is the message sender. (2) \( U_i \) \((i = 1, 2, \ldots, t)\) is the \( i \)-th member of broadcasting multiple signatory. (3) Charlie is the signature collector. (4) Bob is the receiver and the verifier of the broadcasting multiple blind signature.

The improved scheme is also composed of three parts: the initial phase, the individual blind signature generation and verification phase, and the combined multiple blind signature verification phase.

(1) Initial Phase

1.1 Quantum key distribution

Alice shares the secret key \( K_{AB} \) with Bob, \( K_{AC} \) with Charlie, and \( K_{AU_i} \) \((i = 1, 2, \ldots, t)\) with each signatory \( U_i \); Bob shares a secret key \( K_{BC} \) with Charlie; Charlie shares secret keys \( K_{CU_i} \) \((i = 1, 2, \ldots, t)\) with each signatory \( U_i \). All the secret keys are 4\( n \)-bit. To obtain unconditional security, all these keys are distributed via QKD protocols.

1.2 Message concealing and message transformation

Alice gets \( m' = m \oplus r \) where \( m \) is an \( n \)-bit classical message and \( r \) an \( n \)-bit random binary string. Alice transforms the classical message \( m' \) into \( n \)-qubit state

\[ |\psi(m')_M\rangle = \bigotimes_{j=1}^{n} |\psi(j)_M\rangle, \]  

(42)

where

\[ |\psi(j)_M\rangle = \begin{cases} b|0\rangle_M + c|1\rangle_M & \text{if } m'(j) = 1 \\ c|0\rangle_M - b|1\rangle_M & \text{if } m'(j) = 0, \end{cases} \]  

(43)

\( b, c \) are different real constants.
(1.3) Alice sends the message \( m \) to Bob

Alice transforms \( m \) into \( |m\rangle \) as

\[
|m\rangle = \bigotimes_{j=1}^{n} |m(j)\rangle,
\]

where

\[
|m(j)\rangle = \begin{cases} 
|0\rangle & \text{if } m(j) = 0 \\
|1\rangle & \text{if } m(j) = 1.
\end{cases}
\]

Alice randomly chooses a 4\( n \)-bit binary sequence \( r_0 \), computes \( H(K_{AB}||r_0) \), and sends \( [E_{H(K_{AB}||r_0)}(|m\rangle)] \otimes |r_0\rangle \) to Bob where

\[
|r_0\rangle = \bigotimes_{j=1}^{4n} |r_0(j)\rangle,
\]

and

\[
|r_0(j)\rangle = \begin{cases} 
|0\rangle & \text{if } r_0(j) = 0 \\
|1\rangle & \text{if } r_0(j) = 1.
\end{cases}
\]

Bob extracts \( r_0 \) by measuring each particle of \( |r_0\rangle \) in the basis \( \{|0\rangle, |1\rangle\} \). Then, he can compute \( H(K_{AB}||r_0) \) and decrypt \( E_{H(K_{AB}||r_0)}(|m\rangle) \) to get \( |m\rangle \). After that he can get \( m \) by performing a measurement in the basis \( \{|0\rangle, |1\rangle\} \).

(1.4) Alice sends \( r \) to Charlie

\( |r\rangle \) is generated as Eqs. (46) and (47). Further more, all the \( |r_k\rangle \)’s generation in the rest of the scheme is always the same. Alice sends \( E_{H(K_{AC}||r_1)}(|r\rangle) \otimes |r_1\rangle \) to Charlie. Then, Charlie extracts \( r \) by performing a measurement in the computational basis.

(2) The individual blind signature generation and verification phase

In this phase, we pick one of the signatory \( U_i \) as the representative who signs the message.

(2.1) Quantum channel setup

Charlie prepares \( n+l \) pairs of EPR particles denoted as \( \{|a(1)\rangle_{U_iC}, |a(2)\rangle_{U_iC}, \ldots, |a(n+l)\rangle_{U_iC}\} \), where \( |a(j)\rangle_{U_iC} = \frac{1}{\sqrt{2}}(|00\rangle_{U_iC} + |11\rangle_{U_iC}) \), \( j = 1, 2, \ldots, n+l \). Then, he sends the first particle to \( U_i \) and keeps the other himself for every EPR pair. After \( U_i \) receives all particles, Charlie randomly chooses \( l \) particles to perform a measurement randomly in the basis of \( \{|0\rangle, |1\rangle\} \) or \( \{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\} \) and reports the position of the particles that he has measured and the basis that he has chosen to \( U_i \). \( U_i \) takes the same measurement on the corresponding particles and compares the measurement outcomes with Charlie. If there is no error, the channel is considered to be safe. Otherwise, they abandon the quantum channel and set it up again.

(2.2) Signature Phase

(2.2.1) Alice sends the information quantum state \( |\psi(m')\rangle_M \) to \( U_i \)

Alice randomly chooses a 4\( n \)-bit sequence \( r_2 \) and computes \( H(K_{AU_i}||r_2) \). Then, she sends \( [E_{H(K_{AU_i}||r_2)}(|\psi(m')\rangle_M)] \otimes |r_2\rangle \) to \( U_i \).

(2.2.2) \( U_i \) signs the message \( m \)
\( U_i \) decrypts \([E_{H(K_{AU_i}}]|r_2\rangle (|\psi(m')\rangle_M)\] to get \(|\psi(m')\rangle_M\) by first extracting \(r_2\) and then computing \(H(K_{AU_i}|r_2\rangle\). He generates \(|\psi(j)\rangle_{MU_i}\) \(j = 1, 2, \ldots, n\) by combining each \(|\psi(j)\rangle_M\) with his EPR particle. Then, \(U_i\) performs a Bell-basis measurement on \(|\psi(j)\rangle_{MU_i}\) \(j = 1, 2, \ldots, n\) to get the outcomes \(\{\beta_{MU_i}(j)| j = 1, 2, \ldots, n\}.\) According to Eq. (14), \(\beta_{MU_i}(j)\) is an Bell state \(|\beta_{kl}\rangle\) which can be expressed in 2-bit classical string according to \(|\beta_{kl}\rangle \rightarrow kl, k, l \in \{0, 1\}.\) By introducing a \(4n\)-bit random binary string \(R_i, U_i\) gets the blind signature \(S_i\) of \(m'\) as

\[
S_i = (\beta_i \oplus K_{CU_i})||H[(\beta_i \oplus K_{CU_i})||R_i]
\]

where

\[
\beta_i = \beta_{MU_i}(1)||\beta_{MU_i}(2)|| \cdots ||\beta_{MU_i}(j)|| \cdots ||\beta_{MU_i}(n).
\]

(2.2.3) \(U_i\) sends \(|S_i\rangle\) to Charlie.

\(U_i\) transforms \(S_i\) into quantum state \(|S_i\rangle\) as

\[
|S_i\rangle = \bigotimes_{j=1}^{6n} |S_i(j)\rangle
\]

where

\[
|S_i(j)\rangle = \begin{cases} 0 & \text{if } S_i(j) = 0 \\ 1 & \text{if } S_i(j) = 1. \end{cases}
\]

\(U_i\) randomly chooses a 6-dimensional \(4n\)-bit string vector \(r_3 = (r_3^1, r_3^2, r_3^3, r_3^4, r_3^5, r_3^6)\) and computes \(H(K_{CU_i}|r_3\rangle\). Then, he sends \([E_{H(K_{CU_i}}||r_3\rangle (|S_i\rangle) \otimes |r_3\rangle\) to Charlie where

\[
H(K_{CU_i}|r_3\rangle) = H(K_{CU_i}|r_3^1\rangle) || H(K_{CU_i}|r_3^2\rangle) || H(K_{CU_i}|r_3^3\rangle) || H(K_{CU_i}|r_3^4\rangle) || H(K_{CU_i}|r_3^5\rangle) || H(K_{CU_i}|r_3^6\rangle)
\]

and

\[
|r_3\rangle = |r_3^1\rangle \otimes |r_3^2\rangle \otimes |r_3^3\rangle \otimes |r_3^4\rangle \otimes |r_3^5\rangle \otimes |r_3^6\rangle.
\]

(2.3) Verification Phase

(2.3.1) Charlie decrypts \(E_{H(K_{CU_i}}||r_3\rangle(|S_i\rangle)\) to get \(|S_i\rangle\), and then, he can get \(S'_i\) by performing a measurement in computational basis. After that he further gets \(\beta'_i\) based on \(K_{CU_i}\) according to Eq. (48). Here

\[
S'_i = (\beta'_i \oplus K_{CU_i})||H[(\beta_i \oplus K_{CU_i})||R_i]\]

If there is no incorrection happened in the transmission and measurement, \(\beta'_i\) and \(S'_i\) will be equal to \(\beta_i\) and \(S_i\), respectively.
(2.3.2) According to $\beta_i'$ and quantum teleportation in Eq. (14), Charlie performs one of the corresponding reverse transformation ($I$, $X$, $Y$, $Z$) on each particle in his hand. He obtains the states of these particle denoted as $|\psi'(j)\rangle_C$, $j = 1, 2, \ldots, n$, which carry information of message $m''$.

(2.3.3) Charlie gets $m''$ by measuring each $|\psi'(j)\rangle_C$, $j = 1, 2, \ldots, n$ in the basis of $\{b|0\rangle_M + c|1\rangle_M, c|0\rangle_M - b|1\rangle_M\}$. Then, he computes $m^* = m'' \oplus r$ and sends $[E_H(K_{BC}\|r_4)(m^*))] \otimes |r_4\rangle$ to Bob.

(2.3.4) Bob decrypts $E_H(K_{BC}\|r_4)(m^*))$ to get $|m^*\rangle$ and further gets $m^*$ by performing a measurement in the basis of $\{|0\rangle, |1\rangle\}$. Then, he compares $m^*$ with $m$. If they are not equal, $S_i$ is rejected. Otherwise, Bob informs Charlie and $U_i$ to announce $S_i$ and $R_i$ on the public board, respectively. Then, he computes $H[\{\beta_i \oplus K_{CU_i}\}||R_i]\rangle$ and compares it to $[H[\{\beta_i \oplus K_{CU_i}\}||R_i]\rangle]$. If they are the same, $S_i$ is accepted. Otherwise, $S_i$ is considered to be compromised and it is rejected.

(3) The combined multiple signature generation and verification phase

(3.1) Charlie generates the message sequence $\{m^*_i|i = 1, 2, \ldots, t\}$ and collects all individual signature $\{S'_i|i = 1, 2, \ldots, t\}$. If $m^*_i$ is equal to $m^*_{i+1}$, $(i = 1, 2, \ldots, t-1)$, he confirms the message and generates the multiple signature $S = \{S'_i|i = 1, 2, \ldots, t\}$. If not, the process is terminated. After Charlie confirms the message, he sends $[E_H(K_{BC}\|r_5)(m^*_i)] \otimes |r_5\rangle$ to Bob.

(3.2) Bob decrypts $E_H(K_{BC}\|r_5)(m^*_i)$ to get $|m^*_i\rangle$ and further gets $m^*_i$ by performing a measurement in the basis of $\{|0\rangle, |1\rangle\}$. Then, he compares $m^*_i$ with $m$. If they are not equal, $S$ is rejected. Otherwise, Bob informs Charlie to announce $S$ and each $U_i(i = 1, 2, \ldots, t)$ to announce $R_i$ on the public board, respectively. Then, he computes $F$ and compares it to $F'$ where

\begin{align}
F &= \{H[\{\beta_i \oplus K_{CU_i}\}||R_i]\}, i, i^* = 1, 2, \ldots, t\} \\
F' &= \{H[\{\beta_i \oplus K_{CU_i}\}||R_i]\}', i, i^* = 1, 2, \ldots, t\}.
\end{align}

If $F' \subseteq F$, $S$ is accepted. If not, $S$ is rejected.

Let’s use Fig. 1 to illustrate our quantum signature model as follows:

Next, we list the improvements of our new scheme compared to the original one:

(1) Introducing the improved QOTP encryption algorithm.
(2) Bringing in a hash function to authenticate the originality of the signature.
(3) The secret keys become reusable by introducing some random strings.
(4) Bringing in public boards.
(5) Classical message is concealed before turning into quantum message. Meanwhile, the transformation method in the improved scheme is according to Eq. (43) rather than Eq. (5) in the original one.
(6) The entangled quantum channel between Charlie and each $U_i(i = 1, 2, \ldots, t)$ is set up by Charlie rather than Alice. At the same time, a channel checking process is added to make sure it is secure.
(7) Classical message from Alice to Bob is transmitted through quantum method in Step (1.3).
5 Cryptanalysis of the improved scheme

In this section, we present the security analysis of the improved scheme. We show there is no disavowal and forgery in the improved scheme. Meanwhile, we also point out the signatory cannot learn the signed message and the signed message cannot be modified by attackers in the improved scheme.

5.1 No disavowal

5.1.1 Each signatory $U_i$ cannot disavow his signature $S_i$

From Eq. (48), we can see that because each $S_i$ contains $U_i$’s secret key $K_{CU_i}$ in the improved scheme, $U_i$ cannot disavow his signature $S_i$. Meanwhile, $U_i$ cannot disavow $S_i$ by the intercept-resend method mentioned in Ref. [21]. Because in Step (2.3.4) Charlie announces $S_i$ on the public board instead of sending it to Bob, which is only for reading on the public board, $U_i$ cannot disavow his signature by intercept-resend method.

5.1.2 The receiver Bob cannot disavow the signature

In the improved scheme, the signature $S$ is announced on the public board by Charlie when Bob informs him $m = m_1^*$ in Step (3.2) so that everyone can witness Bob has received the signature, so Bob cannot disavow his receiving the signature. Also, Bob cannot disavow the integrity of the signature by claiming $m \neq m_1^*$ as in Ref. [4]. Assume $m = m_1^*$ and Bob lies to claim $m \neq m_1^*$ for his own benefit in Step (3.2). We can ask Alice, Charlie and Bob to public announce the message, respectively. Then, the dishonest behavior of Bob can be caught according to the voting rule, on the assumption that there is no collaborate attack.
5.2 No forgery

According to Eq. (48), each $S_i$ is composed of $\beta_i$, $K_{CU_i}$ and $R_i$. Here we show there is no participant forgery and external forgery in the improved scheme.

5.2.1 Alice cannot forge the signature

In the improved scheme, Charlie prepares the EPR pairs $\{ |a(j)\rangle_{CU_i} | j = 1, 2, \ldots, n + l \}$, Alice just prepares the information state $|\psi(m')\rangle_{M}$ so that she cannot get each $\beta_i$ by measuring each $|\psi(j)\rangle_{MU_i}$ directly. Next, we show Alice cannot get $\beta_i$ by intercept-resend method either. Assume Alice intercepts each $|\varphi(j)\rangle_{U_i}$ and resends the measurement outcome $|\beta_{kl}\rangle_{MU_i}$ to $U_i$. Because Charlie and $U_i$ perform a checking process in Step (2.1), Alice’s intercept-resend attack will be discovered. According to the improved scheme, Alice cannot get $\beta_i$. $K_{CU_i}$ is shared between Charlie and $U_i$ via QKD protocol so that Alice has no chance to get it. Moreover, $R_i$ is chosen by $U_i$ randomly and it is not acquired by anyone else until it is announced on the public board so that Alice cannot get it in the signing phase. Therefore, Alice cannot forge the signature.

5.2.2 Charlie cannot forge the signature

Charlie, the signature collector who can get each $S'_i$ and the secret key $K_{CU_i}$ in the improved scheme, is considered to be most likely to forge the signature successfully. Here we show he cannot forge the signature either. Charlie can modify the signature at random and keep the message unchanged when he has confirmed the message, which has no influence on the following message comparison. Charlie cannot learn each $U_i$’s random string $R_i$. Since he does not know how to modify $H'( (\beta_i \oplus K_{CU_i}) || R_i)$ to fit his modification deterministically, Charlie’s forgery attack can definitely be discovered in Step (3.2).

5.2.3 Bob cannot forge the signature

Bob is the receiver of the scheme, he cannot get the signature $S$ until Charlie announced it on the public board. Therefore, the only way Bob can modify the signature is to perform a unitary operator $V$ on $E_{H(K_{CU_i} || r_3)} (|S_i\rangle)$ in Step (2.2.3). According to Eq. (40), Bob’s modification cannot follow his will, and then, it will be definitely discovered in the verification process. As a consequence, Bob cannot forge the signature.

5.2.4 The eavesdropper Eve cannot forge the signature

In the improved scheme, the classical bit $m'(j)$ is transformed into quantum state according to Eqs. (42) and (43). Here we take a 1-bit message $m'(j)$ to illustrate any of Eve’s modification on $S_i(j)$ can be detected.

Supposed $S_i(j) = \beta_{01}$ is replaced with $S'_i(j) = \beta_{00}$ by Eve, the corresponding message are $m'(j)$ and $m''(j)$, respectively. We present this case as follows:

1. Without Eve’s attack:
(1.1) Assume Alice prepares the state $|\psi(j)\rangle_{M} = b|0\rangle_{M} + c|1\rangle_{M}$, and its signature is $S_{i}(j) = \beta_{01}$, according to the teleportation, Charlie’s particle will be in the state $|\varphi(j)\rangle_{C} = b|1\rangle_{C} + c|0\rangle_{C}$. Then, Charlie performs operation $X$ on $|\varphi(j)\rangle_{C}$ to get the state $|\psi'(j)\rangle_{C} = b|0\rangle_{C} + c|1\rangle_{C}$. After that Charlie performs a measurement in the basis of $\{b|0\rangle_{M} + c|1\rangle_{M}, c|0\rangle_{M} - b|1\rangle_{M}\}$ to extract $m'(j) = 1$.

(1.2) Assume Alice prepares the state $|\psi(j)\rangle_{M} = (c|0\rangle_{M} - b|1\rangle_{M})$ and its signature is $S_{i}(j) = \beta_{01}$, according to the teleportation, Charlie’s particle will be in the state $|\varphi(j)\rangle_{C} = c|1\rangle_{C} - b|0\rangle_{C}$. Then, Charlie performs operation $X$ on $|\varphi(j)\rangle_{C}$ to get the state $|\psi'(j)\rangle_{C} = c|0\rangle_{C} - b|1\rangle_{C}$. After that Charlie performs a measurement in the basis of $\{b|0\rangle_{M} + c|1\rangle_{M}, c|0\rangle_{M} - b|1\rangle_{M}\}$ to extract $m'(j) = 0$.

(2) With Eve’s attack:

(2.1) Assume Alice prepares the state $|\psi(j)\rangle_{M} = b|0\rangle_{M} + c|1\rangle_{M}$ and Eve replaces $S_{i}(j) = \beta_{01}$ with $S_{i}'(j) = \beta_{00}$; then, Charlie’s particle is still in the state $|\varphi(j)\rangle_{C} = b|1\rangle_{C} + c|0\rangle_{C}$ as Eve’s attack has no effect on it. According to the teleportation, Charlie will perform operation $I$ on $|\varphi(j)\rangle_{C}$ to get $|\psi'(j)\rangle_{C} = b|1\rangle_{C} + c|0\rangle_{C}$. After that Charlie performs a measurement in the basis of $\{b|0\rangle_{M} + c|1\rangle_{M}, c|0\rangle_{M} - b|1\rangle_{M}\}$ to extract $m''(j)$, and it will have a probability of $4b^{2}c_{2}$ to get $m''(j) = 1$ and a probability of $c^{4} + b^{4} - 2b^{2}c^{2}$ to get $m''(j) = 0$.

(2.2) Assume Alice prepares the state $|\psi(j)\rangle_{M} = (c|0\rangle_{M} - b|1\rangle_{M})$ and Eve replaces $S_{i}(j) = \beta_{01}$ with $S_{i}'(j) = \beta_{00}$; then, Charlie’s particle is still in the state $|\varphi(j)\rangle_{C} = c|1\rangle_{C} - b|0\rangle_{C}$. According to the teleportation, Charlie will perform operation $I$ on $|\varphi(j)\rangle_{C}$ to get $|\psi'(j)\rangle_{C} = c|1\rangle_{C} - b|0\rangle_{C}$. After that Charlie performs a measurement in the basis of $\{b|0\rangle_{M} + c|1\rangle_{M}, c|0\rangle_{M} - b|1\rangle_{M}\}$ to extract $m''(j)$, and it will have a probability of $c^{4} + b^{4} - 2b^{2}c^{2}$ to get $m''(j) = 1$ and a probability of $4b^{2}c_{2}$ to get $m''(j) = 0$.

From (1) and (2), we can see that Eve’s modification of the signature can be discovered in Step (2.3.4) with a nonzero probability. Other cases can be presented similarly. We can see Eve cannot forge the signature.

5.3 Each $U_{i}$ cannot learn the signed message

In the improved scheme, the classical message $m$ is turned into $m' = m \oplus r$ before it is transformed into quantum states. If $U_{i}$ performs a measurement in the basis of $\{b|0\rangle_{M} + c|1\rangle_{M}, c|0\rangle_{M} - b|1\rangle_{M}\}$ on the information quantum sequence $\{|\psi(j)\rangle_{M}| j = 1, 2, \ldots, n\}$, he will just get $m'$ and has no chance to get the signed message $m$. If he wants to learn $m$, he has to know $r$ which will be sent from Alice to Charlie in Step (1.4). Because $r$ is turned into $|r\rangle$ and encrypted by $K_{AC}$ according to the quantum one-time pad algorithm during its transmission, $U_{i}$ cannot get the random string $r$ without the key $K_{AC}$. Therefore, we know $U_{i}$ cannot learn the signed message $m$.

5.4 The signed message $m$ cannot be modified

In the improved scheme, the signed message $m$ is turned into quantum state $|m\rangle$ according to Eqs. (44) and (45) before it is sent from Alice to Bob in Step (1.3). It is encrypted by $H(K_{AB}|r_{0})$ according to Eq. (38). Here we show any attacker without the key $K_{AB}$ cannot modify the message $m$ by the intercept-resend method. Suppose the
attacker wants to modify the signed message $m$. He intercepts the $[E_H(K_{AB}||r_0)(|m\rangle)] \otimes |r_0\rangle$. Because he does not get the secret key $K_{AB}$, the attacker cannot decrypt it directly. Then, he can choose to perform a unitary operator $V$ on $[E_H(K_{AB}||r_0)(|m\rangle)]$ and send $V[E_H(K_{AB}||r_0)(|m\rangle)] \otimes |r_0\rangle$ to Bob. In order to make sure this modification can pass the verification, there must exist a nonidentity unitary operator $U$ to satisfy Eq. (40). Because $|m\rangle$ is from classical message $m$ according to Eqs. (44) and (45), here $U$ can be restricted to Pauli operators. Exactly

$$V[E_H(K_{AB}||r_0)(|m\rangle)] \equiv E_H(K_{AB}||r_0)(U|m\rangle)$$

$$U = \bigotimes_{j=1}^{n} Q_j$$

$$Q_j \in \{I, \sigma_x, \sigma_y, \sigma_z\}$$

The question the attacker has to face now is whether there exist such nonidentity unitary operators $U$ and $V$ to satisfy Eq. (57). Unfortunately, it is pointed out that there does not exist such $U$ and $V$ for any message $|m\rangle$ in Ref. [52]. Then, the modification mentioned above will be discovered in the verification process.

Next, we show Bob can modify the signed message $m$ at random in the improved scheme, but we can refute it by the voting rule when this dispute takes place. Because Bob has got $K_{AB}$ and $K_{BC}$, according to the improved scheme, he can modify $m$ at random and make this modification pass the verification. When the dispute on the message $m$ happens between Alice and Bob, we can ask Alice, Bob and Charlie to public their message and arbitrate it according to the voting rule on the assumption that they are all just loyal to themself. From above, we can see the signed message cannot be modified in the improved scheme.

6 Conclusion

In this paper, we have given a security analysis on the quantum broadcasting multiple blind signature scheme based on teleportation in Ref. [51], which was recently proposed. We have pointed out that there are some security loopholes in the protocol and described the attack strategies in detail. Then, we have presented an improved scheme by introducing hash function, public board, and the improved QOTP encryption algorithm proposed in Ref. [52]. After that, we have shown the improved scheme can resist all the mentioned attacks and that the secret keys can be reusable by bringing in some random strings. The improved scheme may be more practical and secure, and it may have some potential applications to E-payment system, E-business and E-government.

The improved quantum broadcasting multiple blind signature can only sign classical message. So it is worthwhile to design a scheme for quantum messages in future.

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