Is a Deterministic and Local Interpretation of Quantum Mechanics Possible?... First Steps

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Abstract. Quantum states in superposition cannot be observed, suggesting that they merely embody information on possible measurement results. Yet, we think of them as describing physical systems that evolve in time according to given mathematical equations. Furthermore, evolution takes place in physical space-time, and local causality is imposed on space-time by Special Relativity (a sequence of cause and effect that constitutes a fundamental principle by which we think about and do scientific work).

It is therefore strange that there is non-locality and violation of causality embedded in Quantum Mechanics. We try to give here a first step to see whether a deterministic and local interpretation is possible. We will show that Bell’s inequality is not conclusive about the non-local nature of Quantum Mechanics, in that there exists an interpretation of reality (which we put forward) which is local and in which Bell’s inequality cannot be derived. Assuming determinism and locality a new Bell-like inequality is derived, which is always satisfied by Quantum Mechanics and which allows for local hidden variables [1].

1. Introduction

In spite of the great successes of Special/General Relativity and Quantum Mechanics, not everything has been smooth and kind. While Special Relativity (SR) forces a causal structure of spacetime in the form of light cones that prevents an event to influence another event which lies outside its future light cone (cf. Fig. 1), entanglement emerges in Quantum Mechanics (QM) when a set of two or more particles can only be described as a whole, and not as individual entities, which seems to force the state of each particle to depend on the state of the others even when they are widely separated (i.e., space-like related). Furthermore, if one were able to perform the same experiment repeatedly, QM does not always give the same results (in contrast with a deterministic theory): it only provides a statistical (though consistent) interpretation of natural phenomena.

The idea of Einstein, Podolsky and Rosen (EPR) [2] of completing QM with other (“hidden”) variables was later met by the work of Bell [3], who constructed an inequality which every theory of (local) hidden variables must satisfy, and demonstrated that QM violated this inequality, thus concluding the impossibility to explain the non-local phenomena of Quantum Mechanics through local hidden variables.
Thus, strange as it may seem, QM tells us that matter (or any quantum system) in some planet of the universe might know how to “behave”, through instructions given by matter (or any other quantum system) in some other distant planet of the universe; and, furthermore, instantly (!), when these two are entangled.

However, Bell made a few assumptions to build his inequality that are not true in a deterministic scenario; when adding true determinism to the evolution of particles Bell’s inequality cannot be constructed. In this work, we try to give a first step towards a deterministic and local interpretation of QM.

We will show that Bell’s inequality is not conclusive about the non–local nature of Quantum Mechanics, in that there exists an interpretation of reality (which we put forward) which is local and in which Bell’s inequality cannot be derived. Assuming determinism and locality a new Bell-like inequality is derived, which is always satisfied by Quantum Mechanics and which allows for local hidden variables [1].

2. Brief Account of the Incompleteness of QM

For the sake of completeness, the EPR argument runs as follows:

Consider 2 particles prepared together in the state

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{ik(x_1-x_2+x_0)} \, dp,$$  

(1)

where $x_0$ is some constant and $k = p/\hbar$. The eigenfunctions of momentum for each particle are

$$u_p(x_1) = e^{ikx_1} \quad \text{and} \quad \psi_p(x_2) = e^{-ik(x_2-x_0)},$$  

(2)

with eigenvalues $p$ and $-p$ respectively, so that

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} u_p(x_1)\psi_p(x_2) \, dp.$$  

(3)
The state (1) can also be expanded in eigenfunctions of position:

Write

\[ \delta(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ips} dp, \]

set \( s = 2\pi(x - x_2 + x_0)/\hbar \), and use \( \delta(ax) = \frac{1}{|a|}\delta(x) \) to get

\[ \hbar \delta(x - x_2 + x_0) = \int_{-\infty}^{\infty} e^{ik(x - x_2 + x_0)} dp. \]

Then, \( \varphi_x(x_2) = \hbar \delta(x - x_2 + x_0) \) is an eigenfunction of position of the second particle. Since \( v_x(x_1) = \delta(x - x_1) \) is an eigenfunction of position of the first particle, we have

\[ \Psi(x_1, x_2) = \int_{-\infty}^{\infty} v_x(x_1)\varphi_x(x_2) dx. \]

Now the EPR argument goes as follows:

Measure position of particle 1; get result \( q \). By Eq. (6), particle 2 is in state \( \psi_r(x_2) \) with position \(-q + x_0\).

We may also choose to measure momentum of particle 1; get result \( r \). By Eq. (3), particle 2 is in state \( \psi_r(x_2) \) with momentum \(-r\).

That is, without disturbing system 2 we can predict with certainty either its position or momentum. Therefore, position and momentum must have simultaneous reality. QM does not account for these two properties simultaneously, thus EPR conclude that it is therefore incomplete.

It is customary, for ease of experimentation, to use Bohm’s version of EPR system, with spin as variable instead of momentum. In this case one considers a singlet state

\[ |\psi\rangle = \frac{1}{\sqrt{2}} [ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle], \]

for which the measurement of one sub-system completely determines the second sub-system. In this case, particles going through two Stern-Gerlach apparatus oriented at angles \( a \) and \( b \) with respect to, say, the \( z \)-axis, deviate according to quantum-mechanical probabilities

\[ P(\text{up, up}) = P(\text{down, down}) = \frac{1}{2} \sin^2 \left( \frac{a - b}{2} \right), \]

\[ P(\text{up, down}) = P(\text{down, up}) = \frac{1}{2} - \frac{1}{2} \sin^2 \left( \frac{a - b}{2} \right), \]

and not according to classical probabilities. This has been explained through statements such as “the observer produces the results when measuring”, or equivalently, “the properties are not there before measurement”. But if this were so, how does one of the particles know what to choose when looked at?

We believe that the events on different places may be correlated: no problem; but the intervention at one place cannot immediately influence the other! The root of the problem is, thus, the non-local causality.
3. Assumptions in Bell’s Theorem

Let \( P(A,a) \) denote the probability of obtaining result \( A \) (up, down; or +1, -1) with the apparatus set at angle \( a \), that is, \( P(A,a) = \text{probability of } \sigma \cdot a = \pm 1 \), where \( \sigma \) denotes the spin-operator vector. The anti-correlation of particles implies, of course, that \( \sigma_1 \cdot a = 1 \leftrightarrow \sigma_2 \cdot a = -1 \).

Let \( P(A,B | a,b) \) denote the conditional probability of results \( A \) and \( B \) given angles \( a \) and \( b \) respectively at the two ends. Note that, in general \( P(A,B | a,b) \neq P_1(A | a) \cdot P_2(B | b) \) because the subsystems are correlated. Since \( |\psi\rangle \) in Eq. (7) does not determine the result of an individual measurement on one subsystem, the possibility of a complete and local specification calls for extra hidden variables \( \lambda \). They can be of any nature, scalar, vectorial, or tensorial, and by introducing them \( P(A | a) \) then becomes \( P(A | a,\lambda) \) and the correlation is lost

\[
P(A,B | a,b,\lambda) = P_1(A | a,\lambda) \cdot P_2(B | b,\lambda). \tag{10}
\]

If \( \rho(\lambda) \) is the probability distribution of \( \lambda \), i.e., \( \int_\Lambda \rho(\lambda) \, d\lambda = 1 \) where \( \Lambda \) is the set where \( \lambda \) takes its values, then the expectation value of the correlation between \( A \) measured at angle \( a \) and \( B \) measured at angle \( b \), is, from Eq. (10),

\[
E(a,b) = \int_\Lambda A(a,\lambda)B(b,\lambda)\,\rho(\lambda)\,d\lambda. \tag{11}
\]

Bell considers 3 measurement directions \( a, b, c \) and derives the inequality

\[
|E(a,b) - E(a,c)| \leq 1 + E(b,c), \tag{12}
\]

which does not necessarily hold in QM, thus concluding that QM cannot be completed by local hidden variables.

We have stated at the beginning that the evolution of states takes place in physical space-time, and local causality is imposed on space-time by Special Relativity. By assuming then determinism and locality, each individual ontological pair particle-apparatus will evolve according to a function of a hidden variable \( \lambda \) and time \( t \),

\[
\mathcal{F} : \Lambda \times \mathbb{R} \rightarrow \mathbb{R}^3 \times \mathbb{R}^3, \quad (\lambda, t_0) \mapsto (o_A, o_B),
\]

where \( o_A(\lambda, t_1) \) and \( o_B(\lambda, t_1) \) are the spin projection orientations of each component of the pair.

Under these assumptions, these functions are absolutely deterministic, and a direct consequence of this is the fact that the orientation of the detectors is also encoded in \( \lambda \), i.e., a detector in a different orientation will have different values of \( \lambda \) at earlier times. All we are saying by this is that for any given function, different outcomes of the function must come from different inputs. If there are non-local correlations, these must emerge from the deterministic evolution of a shared hidden variable between two components of an ontological pair; entanglement thus arises every time two (or more) physical entities share hidden variables.

When the 3 scenarios pictured by Bell to derive his inequality (cf. Fig. 2) are seen deterministically, each set of outcomes must come from a different set of hidden variable values, that is

\[
\mathcal{F}(\lambda, t_1) = (\pm a, \pm b) \leftrightarrow \lambda \in \Lambda_1, \\
\mathcal{F}(\lambda, t_1) = (\pm a, \pm c) \leftrightarrow \lambda \in \Lambda_2, \\
\mathcal{F}(\lambda, t_1) = (\pm b, \pm c) \leftrightarrow \lambda \in \Lambda_3.
\]

So, if we were to follow Bell to derive his inequality, we would start by comparing the expectation values,

\[
E(a,b) - E(a,c) = \int_{\Lambda_1} A(a,\lambda)B(b,\lambda)\rho(\lambda)\,d\lambda - \int_{\Lambda_2} A(a,\lambda)B(c,\lambda)\rho(\lambda)\,d\lambda, \tag{13}
\]
and we cannot carry on to Bell’s next steps, which include factorisation of various terms as well as the assumptions

\[
\begin{align*}
A_1(a, \lambda) &= A_2(a, \lambda), \\
B_1(b, \lambda) &= -A_3(b, \lambda), \\
B_2(c, \lambda) &= B_3(c, \lambda).
\end{align*}
\]

That is, in a local deterministic scenario Bell’s inequality cannot be derived, therefore the violation of this inequality by experiments does not show that the assumption of locality in this scenario is incorrect.

4. Derivation of a New Inequality

In order to build Bell’s inequality it is required that the set of values of the hidden variables that lie behind the three different scenarios be one and the same (\(\lambda \in \Lambda\)). Furthermore, the assumptions (14) place constraints on the functions \(F_i(\lambda, t)\) that have to be met in order for Bell’s inequality to be derived. These constraints, in turn, tamper with the probabilities of getting (\(\pm a\)), (\(\pm b\)) or (\(\pm c\)) in the measurements performed, resulting in [1]

\[
\begin{align*}
E(a, b) &= -\cos \theta_{ab}, \\
E(a, c) &= -\cos \theta_{ac}, \\
\text{but } E(b, c) &= -\cos \theta_{ab} \cos \theta_{ac}.
\end{align*}
\]

The expectation values of the correlation between \(A\) and \(B\) measured at different angles analysed within Bell’s assumptions lead to Eqs. (15), where that for \(E(b, c)\) differs in an important way, arriving at Eq. (12) being equivalent to

\[
| -\cos \theta_{ab} + \cos \theta_{ac} | \leq 1 - \cos \theta_{ab} \cos \theta_{ac}
\]

and this inequality is always satisfied.

When the analysis is done without Bell’s assumptions (see [1] for full details), one arrives to the same inequality (16), but now \(\cos \theta_{ab} \cos \theta_{ac}\) is just a quantity, not an expectation value of a specific scenario.

The above suggests that... if the experiments were to satisfy the constraints necessary to build Bell’s inequality, then the expectation values would be such that when plugged into the inequality one would get \(| -\cos \theta_{ab} + \cos \theta_{ac} | \leq 1 - \cos \theta_{ab} \cos \theta_{ac}\) which is always satisfied. Set in stronger words, the violation of Bell’s inequality by the experiments does not show that reality cannot behave in a local deterministic way.
5. Conclusions

Local realism can be recovered for Quantum Mechanics. The usual application of Bell’s inequality to experiments is not a proof of the non–local nature of reality, in that in a local, deterministic universe Bell’s inequality cannot be derived. There is a Bell-like inequality, however, that can be derived. This inequality is always satisfied by Quantum Mechanics’ predictions, and thus by the known experimental results.

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References

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