DEEP HUBBLE SPACE TELESCOPE WFPC2 PHOTOMETRY OF NGC 288. II. THE MAIN-SEQUENCE LUMINOSITY FUNCTION

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ABSTRACT

The main-sequence luminosity function (LF) of the Galactic globular cluster NGC 288 has been obtained using deep Wide Field Planetary Camera 2 photometry. We have employed a new method to correct for completeness and fully account for bin-to-bin migration due to blending and/or observational scatter. The effect of the presence of binary systems in the final LF is quantified and is found to be negligible. There is a strong indication of the mass segregation of unevolved single stars and clear signs of a depletion of low-mass stars in NGC 288 with respect to other clusters. The results are in good agreement with the prediction of theoretical models of the dynamical evolution of NGC 288 that take into account the extreme orbital properties of this cluster.

Key words: globular clusters: individual (NGC 288) — stars: luminosity function, mass function

1. INTRODUCTION

After a decade of Hubble Space Telescope (HST) observations, the luminosity function (LF) of unevolved main-sequence (MS) stars has been derived for several Galactic globular clusters (GGCs). In many cases the LF extends down to the faintest stellar objects \( M \approx 0.1 \, M_\odot \); see, e.g., King et al. 1998; Bedin et al. 2001. For unevolved stars, the luminosity directly tracks the mass, the most fundamental parameter of a star. The determination of the initial mass function from the observed LF of main-sequence stars, as well as the study of the variation of the LF within a given cluster and among different clusters, is now a well-established field of research that has provided important insights about the dynamical evolution of GGCs (see, e.g., Ferraro et al. 1997; King et al. 1998; Marconi et al. 1998; Piotto & Zoccali 2000; Paresce & De Marchi 2000, and references therein). In particular, the signature of mass segregation, due to the energy equipartition established by two-body encounters among clusters stars, has been detected in many clusters by comparing the LFs obtained at different distances from the cluster center. Also, the effect of the pruning of the outer extremities of the clusters by the Galactic tidal field or by the violent interaction with the Galactic bulge and disc have been shown with the analysis of the LF of MS stars (see, e.g., Andreuzzi et al. 2001 and references therein).

Here we present the MS LF of the globular cluster NGC 288 down to \( V \approx 24.5 \). The LF is based on HST Wide Field Planetary Camera 2 (WFPC2) observations described in detail in a companion paper (Bellazzini et al. 2002, hereafter Paper I), which was devoted to the study of the binary population of this cluster. In Paper I we used a method very similar to the one adopted by Rubenstein & Bailyn (1997), based on extensive sets of artificial-star experiments, to estimate the global binary fraction \( (fb) \) of the cluster \( (7\% \leq fb \leq 30\%) \) and to show that binary systems are spatially segregated within the cluster. These same artificial-star experiments can be used to obtain LFs, corrected for all observational effects, in different regions of the cluster. In § 2 we briefly recall the characteristics of the adopted sample and of the artificial-star experiments, as well as the results of Paper I that are relevant in the present context. In § 3 we introduce a new method to correct the observed LF. The effect of the presence of binary systems on the final LF is also discussed and quantified with the support of the analysis performed in Paper I. In § 4 we present the final LF of NGC 288 and compare it with the LFs of other clusters, and finally in § 5 we summarize our results.

1.1. Globular Cluster NGC 288

NGC 288 is a loose \( \log \rho_0 = 1.80 \, L_\odot \text{yr}^{-3} \); Djorgovski 1993) cluster. It is a classical old GGC (Rosenberg et al. 1999) with a blue horizontal branch morphology (see Bellazzini et al. 2001; Catelan et al. 2001, and references therein). Its stars are not exceedingly metal-deficient \( ([\text{Fe}/\text{H}] = -1.39) \) and have the abundance pattern typical of halo stars (Shetrone & Keane 2000). The cluster lies on a
very inclined and eccentric orbit, which carries it into the inner, denser region of the Galaxy, where the effects of Galactic tides and disc or bulge shocks are more disruptive. According to Dinescu, Girard, & van Altena (1999) the perigalactic distance is \( R_p \approx 1.8 \) kpc, the eccentricity is \( e \approx 0.7 \), the orbital period is \( P \approx 220 \times 10^6 \) yr, and the “destruction rate” is one of the highest in the GGC system. The observed distance from the Galactic center (Harris 1996; \( R_{GC} \approx 11 \) kpc) indicates that the cluster is presently near the apogalactic point of its orbit (see also Pasquali, Brugas, & De Marchi 2000, hereafter PBD00).

2. THE DATA SET AND THE ARTIFICIAL-STAR EXPERIMENT

The observations, the data reduction process, and the artificial-star experiments are described in detail in Paper I. Here we summarize the points that are relevant in the present context.

2.1. Sample

Two \( HST \) WFPC2 fields were observed in the F555W and F814W passbands: one with the PC camera nearly centered on the center of the cluster (Int field) and one with the WF4 camera partially overlapping the WF3-Int field (Ext field; see Fig. 1 of Paper I). The Int field is almost completely enclosed in the region within 1 half-light radius (\( r_h \approx 1.6 r_c \), where \( r_c = 85'' \) is the core radius; Djorgovski 1993), while the Ext field samples a region contained in the annulus \( r_h < r < 2r_h \), where \( r \) is the distance from the cluster center. The data were transformed to the standard Johnson-Kron-Cousins photometric system by calibration to the ground-based data of Rosenberg et al. (2000). The Int sample contains 5766 sources from \( V \approx 13 \) to \( V \approx 25 \) and the Ext one contains 2013 sources between \( V \approx 16 \) and \( V \approx 25.5 \). The large majority of the observed stars are fainter than the turnoff point (\( V \approx 19 \)) and hence are unevolved MS stars (and/or binary systems containing two unevolved MS stars; see Paper I). The F336W and F225W observations of the Int field, which has been briefly discussed in Paper I, are not used in the present paper.

2.2. Artificial-Star Experiments

Simulations containing more than 80,000 artificial stars per chip have been made on the \( V \) and \( I \) frames. In each run of the artificial-star experiments \( \approx 100 \) artificial stars per chip were added at the same position in the median image (which we used as a master frame to obtain a list of positions of bona fide stars; see Paper I) and in each of the single frames (which were used to obtain accurate photometry; see Paper I). The set of experiments for a given chip (subfield) was completed with \( \approx 800 \) independent runs. The final total number of artificial-star experiments is greater than 1,500,000. For each artificial star a random input \( V \) magnitude is drawn from a distribution similar to the observed LF extrapolated to magnitudes slightly below the observed lower limit. This fainter limit was required so that the simulations would contain a large number of stars fainter than the observed limiting magnitude to correctly sample the magnitude bins in which the incompleteness is expected to be highest (see Paper I).

We ultimately derive the true LF in a way that does not depend on the assumed artificial-star LF (see § 3.2). The input \( I \) is obtained from the input \( V \) by using the cluster ridgeline. Each simulated star thus had the appropriate color and was simultaneously added to the corresponding \( V \) and \( I \) frames at the same position. The simulated stars were randomly distributed on the frames with an additional constraint (see Paper I and also Tosi et al. 2001 for a detailed description of the method) preventing any interference between simulated stars. In practice, for each run of the artificial-star experiment, only one artificial star is added in each \( 80 \times 80 \) pixel portion of a chip. The whole process of data reduction has been repeated on all the frames with artificial stars added, and the output magnitudes have been recorded (see Paper I for details).

Figure 1 shows the input and output color-magnitude diagrams (CMDs) for the stars simulated in the PC-Int, PC-Ext, WF2-int, and WF2-Ext frames. The WF2 samples have been chosen to be representative of the WF cameras for the Int and Ext fields. Within a given field (Int and Ext) the observations with the different WF cameras have very similar properties (see Paper I). Note the different response to the same input color-magnitude distribution in the four cases represented. This arises because the larger pixels of the WF cameras allow them to reach fainter limiting magnitudes than those of the PC camera and because the Int and Ext fields have different average degrees of crowding.

All the effects of the whole process of observation and data reduction on a large sample of stars representing a pure simple stellar population (SSP) are shown in Figure 1. These effects can be imagined to be the results of the application of a multidimensional function, which we will call the observation and measure function (OMF) to the true stars of the clusters, here represented by the input artificial stars lying on the ridgeline. In Paper I we have used the empirical knowledge of the effects of the OMF to correct the observed LF from all the spurious deformations due to the same factors.

2.3. Completeness and Division in Subsamples

The artificial-star experiments have been carried out independently for each camera of each observed field. The completeness functions for each camera have been presented in Figure 5 of Paper I. The crowding conditions of the data set are never particularly critical, and the completeness is rather similar everywhere. The completeness factor, \( C_f \), is larger than 80% for \( V \leq 23.5 \) in all the samples except the PC-Int, the innermost one, where \( C_f \geq 80\% \) for \( V \leq 22 \). It has been shown that the spatial variation of completeness within a single camera are negligible. For the present purpose, we take advantage of the great similarity of crowding conditions in the WF cameras of the Int and Ext samples to divide the total sample into the following subsamples: (1) PC-Int, (2) WF-Int, containing the WF2-Int, WF3-Int and WF4-Int samples, (3) PC-Ext, and (4) WF-Ext, containing the WF2-Ext, WF3-Ext, and WF4-Ext samples. The four subsamples are internally very homogeneous and cover different radial regions of the cluster. In this sense, the difference between the PC-Ext and the WF-Ext samples is small but we preferred to keep them separate because of the different limiting magnitude.
3. CORRECTION OF THE OBSERVED LF

The effects of the OMF on the true properties of stars that may affect the LF derived from an observed sample are the following:

1. Star loss.—Stars may be lost in the observation and measurement process. This occurs for two different reasons:
   
   (a) Near the limiting magnitude of the observations a star can be successfully detected if it falls on a positive fluctuation of the background that brings it above the detection threshold and can be lost if it falls on a negative fluctuation. This effect is purely stochastic; it does not depend on the crowding. It just depends on the amplitude of the random fluctuations of the background. There is a level of magnitude beyond which no possible positive fluctuation can bring a star above the detection threshold. All the stars fainter than this limit are lost (unobserved) and just contribute to the diffuse luminosity of the unresolved background.

   (b) Stars may be lost if they fall on a brighter source (star or galaxy) and/or if their images are badly corrupted by defects of the chip or the hit of a cosmic ray (these two latter effects are expected to have a negligible impact in

![Fig. 1.—CMDs of the simulated stars obtained with the input magnitudes (V_in, I_in; left), and with the output magnitudes (V_out, I_out; right). The plot shows the effect of the observation and reduction process on the PC-Int and PC-Ext frames, while WF2-Int and WF2-Ext are shown as representative of the WF cameras in the Int and Ext samples, respectively. N_s is the total number of simulated stars, while N_R is the total number of recovered stars. Note that the ratio between these two numbers is not directly comparable at face value, since the limiting magnitude of the LF from which the artificial stars were drawn was different in the different cases. Since the observed limiting magnitude was different in the various cases, the limiting magnitude of the artificial-star LF was varied accordingly to exclude unnecessarily faint stars (e.g., more than ~1 mag below than the observed limiting magnitude) from the simulations.](image)
the present case because of the adopted observation–plus–data reduction strategy; see Paper I). This effect depends on the local degree of crowding, as well as on the magnitude of the star.

2. Blending.—A star may be blended with a fainter source and its measured magnitude will be brighter than the true one because of the flux contributed by the fainter blended source. Considered from the perspective of artificial-star experiments the maximum possible effect of a blending is to change the observed magnitude of a star by \( \pm 0.75 \) mag. In fact, if we recover an artificial star by its (known) position in a frame with an output magnitude more than 0.75 mag brighter than its input magnitude, this means that it has fallen on a real star brighter than the simulated star. Thus, we are not recovering or measuring the simulated star but the real one instead, i.e., thus the artificial star has been lost.

3. Observational scatter.—The magnitude of the true star is altered by all the sources of stochastic noise associated with the process of observation or data reduction (e.g., photon noise, imperfect point-spread function fit, etc.). The maximum possible effect of this factor is magnitude dependent and may be roughly evaluated on the basis of the typical error in a given measure. For instance, in the present case we included in the final samples only the stars with errors\(^4\) lower than 0.2 mag in both \( I \) and \( V \). Hence a \( \approx 3 \) \( \sigma \) fluctuation can change the magnitude of an observed star by at most \( \approx 0.6 \) mag near the limiting magnitude level.

The derived LFs are in practice histograms of the number of stars as a function of magnitude. The factors above change the true LF histogram by depopulating the bins as a function of (increasing) magnitude (mainly factor 1) and by moving stars from one bin to another (migration) because of the magnitude changes induced by factor 2 and/or 3. However, factors 2 and 3 are strictly interwoven and their effect cannot be separated. For example, a given star may be moved, say, 0.4 mag brighter because it is blended with a fainter unresolved source, but it might simultaneously be affected by a negative fluctuation of the random observational scatter, so its final magnitude may differ from the true one by just 0.2 mag. A blending between two unresolved sources can bring them above the detection threshold, injecting into the sample two stars that should have been lost if not blended.\(^5\) The possibilities are too numerous to list here. The real contribution of the three effects cannot be known for a specific case. On the other hand, it is clear that we can obtain only a statistical knowledge of the actual LF.

The traditional way to recover the true LF from an observed one consists in multiplying the numbers in each bin of the observed LF by the inverse of the completeness factor \( C_i = N_R/N_S \), where \( N_R \) is the number of recovered stars and \( N_S \) is the number of the simulated ones) at the center of the bins. It is clear that such a procedure corrects at first order only for the effects of star loss (factor 1), while the effects of the other factors are ignored. A popular misconception is that the observational scatter cannot produce bin migration if the width of the bin is larger than the typical error in the estimate of the magnitude. This is obviously not true since a star whose true magnitude is near the edge of a bin can migrate because of an arbitrarily small fluctuation. The effects of migration may be amplified by the completeness correction, especially in the faintest bins of the LF.

Furthermore the completeness factor may be obtained from the artificial-star experiments only as a function of input magnitude (say, \( V_{\text{in}} \)), while the observed LF is (obviously) a function of the observed magnitude, whose corresponding quantity in the artificial-star experiment\(^6\) is \( V_{\text{out}} \). Thus when an observed LF is corrected by multiplying it by \( 1/C(V_{\text{in}}) \), it is implicitly assumed that \( V_{\text{in}} \) and \( V_{\text{obs}} \) are the same quantity, which in general is not true.

There have been few attempts to correct the observed LFs for all the effects above (Drukier et al. 1988; Stetson & Harris 1988). The proposed methods require a complex mathematical approach and may rely on the a priori assumption of an analytical form of the true LF (Stetson & Harris 1988) or on approximate treatments of the propagation of errors under matrix inversions (see Drukier et al. 1988; Mighell 1990, for references and discussion).

Here we propose a new method that is fully nonparametric and extremely simple in nature and also takes advantage of the huge enhancement in computational power of workstations and PCs that took place since the attempts quoted above. As we will show below (1) the proposed method takes full account of the effects of bin migration and (2) the completeness correction is consistently applied to an LF that is a function of \( V_{\text{in}} \) as the completeness factor. The method is particularly appropriate for the application to LFs of SSPs, but it can probably be generalized to other cases. The prerequisite is the availability of huge sets of realistic artificial-star experiments that sample well the whole spatial, luminosity, and color range of the observations.

3.1. Equivalent Sample

Consider a large set of artificial-star experiments performed in a region of uniform crowding conditions, e.g., the WF2-Ext set presented in Figure 1. Suppose that the input magnitudes of the artificial stars are distributed the same way as the “true” stars of the parent population (in this case the stars of NGC 288; the reasons for this hypothesis will become clear at the end of this section). From this set it is possible to extract a subsample of recovered stars with exactly the same dimension and the same distribution as a function of \( V_{\text{out}} \) as the observed sample. Call this subsample an equivalent sample\(^7\) (ES). It is important to note that the ES contains only stars that have been successfully recovered in the artificial-star experiments process, hence that have suffered only the effects of bin migration (i.e., blending and/or observational scatter).

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\(^4\) The error associated with each magnitude entry is the error in the mean over repeated measures; see Paper I.

\(^5\) Note that this phenomenon may significantly alter the star counts in the bins near the limiting magnitude, at least in cases in which the crowding conditions are critical; see Tosi et al. 2001, for discussion.

\(^6\) The input magnitude \( V_{\text{in}} \) of artificial stars corresponds to the true magnitude of real stars \( V_{\text{true}} \), while the output magnitude \( V_{\text{out}} \) corresponds to the observed one \( V_{\text{obs}} \) under the extant conditions.

\(^7\) The operational procedure to extract an ES is the following: Consider the histogram representing the observed LF. Each bin \( i \) contains \( N_i \) stars. From the artificial-star set we extract at random \( N_i \) stars whose output magnitudes lie in the \( i \)th bin. Repeating this step for each bin of the observed LF we obtain a sample of recovered artificial stars that have an output LF equal to the observed one. Note that the same procedure can be applied on a star-by-star basis, by randomly extracting an artificial sister (see § 3.4) for each observed star.
By definition the ES has the same output (e.g., observed) properties of the observed sample. It is one of the many possible realizations that are mapped by the OMF from the input or true domain to the output or observed one. Thus the input (e.g., true) properties of the ES are one of the possible true sets of properties that may have been mapped into the observed sample by the OMF. It is immediately apparent that the LF of the input magnitudes of the ES is equivalent to the observed LF once it is corrected for all the effects of bin migration. Thus the identification of an ES allows (1) the removal of all the effects of bin migration and (2) the expression of the observed LF as a function of input magnitude, consistently with the completeness factor (see § 3). If we apply the completeness correction to the input LF of the ES we find one possible realization of the final LF, corrected for all the observational effects. Since different ESs may be mapped into the same observed sample by the OMF, repeating the process for a number of (randomly extracted) different ESs and taking the average will allow us to obtain a final LF that is more robust to random fluctuations, as well as a direct estimate of the uncertainty associated with the whole process.

However, there is another important factor we have to take into account. While the completeness correction is entirely independent of the distribution in magnitude of the artificial-star set, the derived correction for bin migration is not. Consider a given bin of the observed LF. Such a bin will be primarily filled by stars with $V_{\text{out}} \approx V_{\text{in}}$ but also by a certain fraction of stars that have migrated from the nearby bins because of blending and photometric errors. Increasing the number of artificial stars in the nearby bins with respect to the considered bin will enhance the probability of extracting stars that enter the bin because of migration, hence increasing the final “migration correction.” The right correction would be obtained only if the artificial-star subset is distributed the same way as the true LF of the population, which is unknown, being the very target of our analysis. We circumvent this problem by iteratively adjusting (a posteriori) the distribution of the artificial-star set until the best approximation is obtained (see below for details). The approach has proved to be successful since we obtain the same final LF independently of the initial distribution of the artificial-star set. In the following section the adopted operational procedure is described in detail.

### 3.2. Operational Procedure

For each of the four observed subsamples (PC-Int, WF-Int, PC-Ext, and WF-Ext) we adopted the following procedure:

1. An initial distribution of input magnitudes is assumed for the artificial stars, and the largest possible subset accordingly distributed is extracted from the whole set of artificial stars.
2. Twenty different ESs are extracted at random from the above subset and their LF($V_{\text{in}}$) values are corrected for completeness.
3. The (bin per bin) average of the 20 obtained LFs is adopted as the “current corrected LF,” and the standard deviation is adopted as the corresponding uncertainty.

4. The “current corrected LF” is assumed to be the distribution of the artificial stars, and a new subset of artificial stars accordingly distributed is extracted from the whole set of artificial stars as in step 1.

5. Steps 2, 3, and 4 are repeated 20 times. The twentieth “current corrected LF” is adopted as the final LF with the corresponding uncertainties.

Step 4 ensures that at each new iteration the adopted set of artificial stars will become more and more similar to the “true” one, giving increasingly appropriate rates of bin migration.

In all the cases considered here the process converged in $\leq 6$ iterations, e.g., the “current corrected LFs” become very stable after the sixth iteration. Furthermore, the result is independent of the initial distribution of input magnitudes assumed for the artificial-star subset. We have tried many different distributions (some of which are shown in Fig. 2), and in all cases the process converged to the same final LF in few iterations.

Some examples of the procedure are shown in Figure 2, applied to the WF-Ext sample. In Figure 2a the artificial-star subset was initially distributed the same way as the observed LF down to $V \leq 25.5$ and uniformly from this limit down to $V = 27$. A scaled version of the adopted distribution is plotted as a thick dotted line. The thin dotted histogram shows the “current corrected LF” after the first iteration of the process. The thin continuous histogram shows the “current corrected LF” after the sixth iteration, and the circles represent the final LF, corresponding to the twentieth iteration of the process, with the current standard deviation of the 20 realizations of the corrected LFs shown as error bars. The double circle represents the first bin of the LF for which $C_f < 0.5$; i.e., the completeness correction is larger than a factor of 2. The fast convergence and the stability of the final solution can be readily appreciated.

Figure 2b–2c has the same symbols as Figure 2a but shows the convergence of the process starting from (very) different initial distributions of the artificial-star subset. The circles represent the same final LF shown in Figure 2a. Thus Figure 2a–2b shows that the process converges to the same final LF independently of the assumed initial distributions of the artificial-star subset.

Finally, Figure 2d shows in detail the results of the 20th (i.e., final) iteration of the process shown in Figure 2a. The 20 histograms are the corrected LFs obtained by the 20 different ESs extracted from the finally adopted subset of artificial stars. The filled dots and error bars represent their means and standard deviations.

### 3.2.1. Final LFs

Since the obtained LFs are independent from the initial distribution of the adopted subset of artificial stars, we derived our final (corrected) LFs for all the considered samples by assuming the same initial distribution of the artificial stars shown in Figure 2a. The final LFs are given in Table 1 (PC-Int and WF-Int samples) and in Table 2 (PC-Ext and WF-Ext samples). The tables give the $V_{\text{in}}$ (e.g., $V_{\text{true}}$) of the center of the bins and, for each sample, the observed LF, the completeness factor, $C_f$, the final LF, and the total error in the final LF (see below).

The final LF is given down to the first bin that has $C_f < 0.50$. In the following plots this point is circled to serve as a reminder that it is has been derived by applying a large
correction. We retain these points as educated guesses about the behavior of the LFs beyond the range in which moderate and safe corrections can be made. We consider as fully reliable only the points of the LF for which \( C_f \geq 0.50 \). We note that the last fully reliable points, according to the above criterion, have \( C_f = 0.607, 0.769, 0.806, \) and 0.842 for the PC-Int, WF-Int, PC-Ext, and WF-Ext samples, respectively. Thus the completeness is quite high for the whole considered range.

The final total error reported in Tables 1 and 2 has been obtained by summing in quadrature the error in the completeness factor (estimated according to eq. [18] by Mighell 1990) to the standard deviation of the 20 different realizations of the LF that are obtained in the last step of the procedure (see § 3.2 and Fig. 2d). In all the following plots the error bars associated with the LFs of NGC 288 represent this final total error. The final LFs (open circles) are shown with the corresponding observed LF (filled circles) in Figure 3.

As a further check we compared our final LFs with those obtained by dividing the observed LF(\( V_{\text{obs}} \)) by \( C_f(V_{\text{in}}) \). We note that the differences are within 1 \( \sigma \) in most bins and less than 2 \( \sigma \) in any bin. As expected, in this particular application the effects of bin migration are not large. This would not be the case if much more crowded fields were considered (see § 3; Tosi et al. 2001).

### 3.3. Effect of Binary Systems on the LF

There is an intrinsic kind of blending that cannot be directly tackled with artificial-star experiments. Physical binary systems are strictly equivalent to chance superposition bandlings. The LF for single stars cannot be obtained if an independent estimate of the incidence of binary systems
is not available. Further, since the actual binary fraction \(f_b\) depends on the (unknown) distribution of mass ratios \(P(q)\); see Paper I], any correction would be model dependent. Indeed, a direct estimate of the impact of binary systems on the LF of a globular cluster has been possible only for NGC 6752 (Rubenstein & Bailyn 1999).

Now we can add a second cluster by using the results of Paper I. We measured \(f_b\) in the Int and Ext regions of NGC 288. In the Int region we found that the observations are compatible with \(8\% \leq f_b \leq 38\%\) and the most probable range is \(10\% \leq f_b \leq 20\%\). In the Ext region \(f_b \leq 10\%\) and the most probable value of \(f_b\) is zero, independently of the assumed \(P(q)\). Thus, a sizeable fraction of binary systems is present in NGC 288, at least in the Int sample, and may affect the derived LF of single stars.

To quantify the effect we computed the mean bin-by-bin ratio between the LFs obtained from five ESs with \(f_b = 20\%\) and from five ESs with \(f_b = 0\%\). This number gives (approximately) the fractional variation of the star content of the bins due to binary systems. Furthermore, by dividing the final LF by the quoted ratio, the LF is corrected for the binary content and mapped to the “pure single stars” case.

**TABLE 1**

| \(V\)  | \(N_{\text{obs}}\) | \(C_f\) | \(N_c\) | \(\sigma_{N_c}\) | \(N_{\text{obs}}\) | \(C_f\) | \(N_c\) | \(\sigma_{N_c}\) |
|-------|-----------------|-------|--------|----------------|-----------------|-------|--------|----------------|
| 18.25  | 15              | 0.978 | 14.8   | 3.9            | 126             | 0.956 | 135.1  | 12.2          |
| 18.75  | 29              | 0.963 | 30.2   | 5.7            | 264             | 0.981 | 270.2  | 17.1          |
| 19.25  | 44              | 0.957 | 46.3   | 7.1            | 376             | 0.975 | 388.6  | 20.8          |
| 19.75  | 70              | 0.946 | 73.0   | 8.9            | 433             | 0.972 | 452.1  | 22.8          |
| 20.25  | 56              | 0.920 | 60.7   | 8.3            | 519             | 0.969 | 544.5  | 25.1          |
| 20.75  | 51              | 0.898 | 58.5   | 8.5            | 522             | 0.956 | 551.5  | 26.2          |
| 21.25  | 54              | 0.832 | 65.1   | 9.1            | 528             | 0.947 | 568.6  | 26.9          |
| 21.75  | 52              | 0.767 | 67.8   | 9.8            | 471             | 0.935 | 508.3  | 26.7          |
| 22.25  | 44              | 0.695 | 63.1   | 10.0           | 465             | 0.927 | 504.9  | 27.1          |
| 22.75  | 41              | 0.607 | 66.2   | 10.9           | 439             | 0.913 | 479.9  | 26.3          |
| 23.25  | 35              | 0.490 | 73.4   | 13.3           | 460             | 0.899 | 521.2  | 28.8          |
| 23.75  | 31              | 0.203 | ...    | ...            | 464             | 0.796 | 539.6  | 27.5          |
| 24.25  | 2               | 0.016 | ...    | ...            | 322             | 0.410 | 813.1  | 52.3          |
| 24.75  | 0               | 0.000 | ...    | ...            | 75              | 0.096 | ...    | ...            |
| 25.25  | 0               | 0.000 | ...    | ...            | 8               | 0.011 | ...    | ...            |

**TABLE 2**

| \(V\)  | \(N_{\text{obs}}\) | \(C_f\) | \(N_c\) | \(\sigma_{N_c}\) | \(N_{\text{obs}}\) | \(C_f\) | \(N_c\) | \(\sigma_{N_c}\) |
|-------|-----------------|-------|--------|----------------|-----------------|-------|--------|----------------|
| 18.25  | 4               | 0.996 | 3.9   | 2.0           | 19              | 0.945 | 19.4  | 4.5           |
| 18.75  | 13              | 0.989 | 12.7  | 3.6           | 70              | 0.991 | 70.8  | 8.6           |
| 19.25  | 10              | 0.987 | 10.5  | 3.4           | 96              | 0.984 | 97.4  | 10.2          |
| 19.75  | 18              | 0.980 | 18.5  | 4.4           | 116             | 0.982 | 118.6 | 11.4          |
| 20.25  | 16              | 0.980 | 16.4  | 4.2           | 123             | 0.983 | 124.2 | 11.6          |
| 20.75  | 21              | 0.980 | 21.8  | 4.9           | 162             | 0.975 | 168.9 | 14.0          |
| 21.25  | 22              | 0.964 | 22.4  | 5.0           | 153             | 0.971 | 157.8 | 13.1          |
| 21.75  | 24              | 0.947 | 25.3  | 5.3           | 144             | 0.961 | 149.2 | 12.8          |
| 22.25  | 13              | 0.915 | 14.2  | 4.1           | 134             | 0.954 | 143.1 | 12.9          |
| 22.75  | 15              | 0.895 | 16.3  | 4.3           | 132             | 0.953 | 137.3 | 12.7          |
| 23.25  | 16              | 0.879 | 19.9  | 5.2           | 180             | 0.943 | 192.2 | 15.6          |
| 23.75  | 23              | 0.806 | 25.6  | 5.8           | 177             | 0.938 | 173.4 | 15.5          |
| 24.25  | 25              | 0.369 | 63.5  | 14.0          | 263             | 0.842 | 299.7 | 20.5          |
| 24.75  | 2              | 0.069 | ...   | ...           | 204             | 0.466 | 419.9 | 32.2          |
| 25.25  | 0               | 0.004 | ...   | ...           | 51              | 0.116 | ...   | ...           |

**Note.**—The \(V\) columns report the \(V_{\text{true}}\) magnitude of the center of the 0.5 mag bins. \(N_{\text{obs}}\) is the observed number of stars in the bin. \(C_f\) is the completeness factor in the center of the bin. \(N_c\) is the corrected number of stars in the bin (i.e., this column contains the final corrected LF), and \(\sigma_{N_c}\) is the error in \(N_c\). The final corrected LFs are reported down only to the first bin with \(C_f < 50\%\).
In Figure 4 the final LF of the WF-Int sample not corrected for the binary content (circles with error bars) is compared with its binary-corrected versions on the assumption that $f_b = 20\%$ and for three different $F(q)$ (see caption and Paper I for details). The comparison is performed in the magnitude range in which the binary estimate of Paper I has been performed ($20 \leq V \leq 23.5$).

There are several noteworthy features in Figure 4: (1) The maximum correction is $\leq 3\%$. (2) All the corrected LFs lie within the uncertainties of the uncorrected one. (3) The PLMR (peaked at low mass ratios) $F(q)$ case is most similar to the uncorrected case (see Paper I). This was expected, since in this case the large majority of binary systems has a very low mass secondary. This means that the migration of the primary is small and that most of the secondary stars are so faint that they fall in an unobserved region of the LF. (4) The correction for the PHMR (peaked at high mass ratios) case depletes the intermediate bins ($21.5 \leq V \leq 23$; because it corrects for the migration of the primary) and enhances the last bin because of the recovery of some secondary stars as single stars. (5) No bin is exclusively depleted or enhanced by the redistribution of the binary components as single stars.

From the above results and discussion we conclude that

1. The correction for binary systems can be safely neglected for the Ext samples where the binary fraction is much lower than $f_b = 20\%$ and is possibly null.
2. The correction can be neglected also for the Int samples, at least for $V < 23.5$, since it is smaller than the uncertainties of the final LF. However, it cannot be excluded that the faintest bins of the single-star LF need a significant enhancement to take into account the effect of binaries, particularly if the actual $F(q)$ favors high mass ratios.

3.4. Further Developments and Other Applications

The case under study is particularly simple. NGC 288 is a very loose and relatively nearby cluster, so our WFPC2 observations well resolve the large majority of the stars down to the limiting magnitude of each subsample, with a very high degree of completeness. Further, the density gradient is a very fortuitous match to WFPC2, allowing a few large subsamples to nicely probe radial variations.

It is important to note that the equivalent sample concept does not require such simplifications and may be successfully applied under any condition. If a strictly rigorous approach is adopted the ES method may have far-reaching
applications in the study of stellar populations. Here we provide two possible examples to show the potential of the method.

Extremely crowded field with strong density gradient.— The method can tackle samples with arbitrarily large density gradient, for instance by subdividing the observed sample in subsamples with nearly uniform stellar density (as is done in the present application). If the dimension of the observed sample is not sufficiently large to allow this approach, the ES method can also be implemented on a star-by-star basis. It will suffice to assemble the ESs by associating an artificial-star analogue for each observed star, randomly extracted from a set of artificial stars having (approximately) the same $V_{\text{out}}$ and position in the frame of the considered observed star (artificial sisters). The only basic requirement to successfully follow this approach is to have a very large set of artificial stars, sampling with large multiplicity the whole range of magnitudes and positions covered by the observed sample.

Mapping synthetic CMDs into the observational plane.— The standard tool used to derive the star formation history from the CMD of a resolved galaxy consists in the reproduction of the observed CMD by a synthetic CMD in which the stars are extracted from theoretical evolutionary tracks (see Lejeune & Fernandes 2001 for references and discussion of the various adopted techniques). To transform the synthetic CMD to the observational plane the effects of incompleteness, blending, and observational scatter must be added to the synthetic stars. This task is not easy and it is usually done with simplified approaches. For instance, the observational scatter is usually introduced by randomly extracting “errors” from Gaussian distributions with standard deviations equal to the typical $1 \sigma$ photometric error. However, the distribution of the observational scatter is not Gaussian, in general, and it is not easy to parameterize. Furthermore, the effects of blending are not separable from the effects of observational scatter, and Gaussian distributions do not provide a good reproduction of the real distribution, introducing undesired noise in the comparisons.

With the ES approach, one can easily map the input magnitudes (e.g., “true” magnitudes) extracted by the evolutionary track into the observational plane with the correct OMF by “asking” the artificial-star experiments what is the probability that such a star is successfully recovered or not. If a star passes the “incompleteness barrier,” it can be correctly mapped into the observational plane, for instance by assigning to it the output properties of an artificial analog extracted at random from a set of artificial sisters.

In this way, the whole effects of the OMF would be correctly reproduced without dangerous approximations and/or parameterizations.

4. COMPARISON OF LFs

Once the observed LFs have been corrected for all the observational effects and the impact of binary systems has been quantified, we can use the final LF to study the dynamical status of NGC 288, either by comparing the LFs in the Int and Ext regions of the cluster or by comparing the LF of NGC 288 with those of other clusters. In the following we will normalize all the samples to be compared with the number of stars in the range $18.5 \leq V \leq 20.5$, following the approach of Piotto & Zoccali (1999, hereafter PZ99).

NGC 288 is not sufficiently nearby to allow a safe derivation of the LF down to the hydrogen-burning limit even with $HST$. The comparison with other state-of-the-art LFs of more nearby clusters (see Fig. 6) shows that such a limit would occur a couple of magnitudes below our faintest valid point (the faintest bin of the WF-Ext LF). The mass-luminosity relation adopted in Paper I indicates that the lowest mass efficiently sampled by our deepest LF is $M \approx 0.35 M_\odot$.

Usually MFs are compared according to their slope in the log $N – log M$ plane for $M \leq 0.5 M_\odot$ (see PZ99 and references therein). Our best LF has just three points below this limit, thus the slope of the MF would be poorly constrained. Because of this limitation we avoid any conversion of our LFs to mass functions, since that would necessarily be somewhat model dependent and uncertain (see, e.g., Bedin et al. 2001). Instead we will make direct comparisons between LFs in the allowed range.

4.1. Mass Segregation within Single Stars in NGC 288

The LFs of the PC fields are identical (to within the errors) to those of the respective WF samples, in the range in which the comparison is possible. Since the WF samples are much larger and reach fainter magnitudes we will limit our analysis to these homogeneous data sets in the following.

In Figure 5 the LFs from the WF-Int and the WF-Ext samples are compared. It is evident at a first glance that the LF of the WF-Int sample is flatter than that of WF-Ext. Given the adopted normalization, this is a clear indication of a depletion of low-mass stars in the region within $r_1$ (Int) with respect to the Ext field. If the brightest bin is included in the normalization the effect is significantly enhanced. We
also note that any correction for binaries would be much smaller than the observed difference (see Fig. 4). On the other hand, if fainter stars ($20.5 < V < 24$) are used for normalization, one would infer a substantial excess of the more massive stars in the Int sample.

Independently of the assumed normalization it is worth establishing whether the difference between the two LFs is statistically significant. To check this hypothesis we performed a $\chi^2$ test, adopting the appropriate definition of the statistic according to equation (14.3.3) by Press et al. (1992). The comparison has been performed in the safely comparable range $18.5 < V < 24$. The resulting reduced $\chi^2$ is 2.31 with 12 degrees of freedom. The difference between the two LFs is found to be significant at the 99.1% confidence level, i.e., highly significant.

We have already demonstrated that mass segregation exists in NGC 288, since both binary systems and their by-products, the blue straggler stars, are also more centrally concentrated than the single stars. Although the evidence is not as strong (because of the possible ambiguity in the interpretation associated with the choice of the normalization range), we feel that we now find some evidence for further mass segregation in the difference between the LFs of unevolved stars in the Int and Ext regions of the cluster.

### 4.2. Low-Mass Star Depletion in NGC 288

In Figure 6 the WF-Int and the WF-Ext LFs of NGC 288 are compared with the LFs of M10, M22, and M55 by PZ99. The transformation from apparent to absolute magnitude has been obtained by assuming $(m-M)_0 = 14.67$ and $E(B-V) = 0.03$, according to Ferraro et al. (1999). The faintest point of the WF-Ext LF reaches $M_V \approx 10$, while the LF of the (more nearby) PZ99 clusters reaches $M_V \approx 12–13$. The faintest reported point appears to coincide with the turnover point found in the LFs of the PZ99 clusters, so this possible feature in the LF of NGC 288 is not detectable in our data (but apparently it has been detected by PBD00).

On the other hand, the comparison in the range $6 < M_V < 10$ shows that the LF of NGC 288 is significantly flatter than those of M10, M22, and M55. Note that the LFs of these clusters were obtained near the half-mass radius (usually approximated with the half-light radius $r_h$; see Djorgovski 1993). This is the region where the mass distribution of the cluster population is expected to be fairly representative of the (present day) global population (see Meylan & Heggie 1997; Vesperini & Heggie 1997, and references therein). Since the Int and Ext sample are separated at $r_h$, the LF at the half-mass radius must be intermediate between the WF-Int and the WF-Ext and the derived LFs should be representative of the global LF of NGC 288.

The comparison presented in Figure 6 strongly suggests that the global population of NGC 288 is significantly depleted of low-mass stars with respect to that of M10, M22, and M55 (see PZ99 for an indirect comparison with other clusters). NGC 288 has a mass ranging from $1/2$ to $1/6$ that of the comparison clusters. Its central density ranges from $1/2$ to $1/60$ (Pryor & Meylan 1993) that of the comparison clusters. Thus NGC 288 is intrinsically less resistant to harassment and heating by external forces. Furthermore, among the clusters considered here, it moves on the orbit having the smallest perigalactic distance, the highest eccentricity, and the highest inclination to the Galactic plane. Hence, it is expected to suffer strong bulge and disk shocks and indeed may be one of the clusters with the highest disruption rates (Dinescu et al. 1999; Gnedin & Ostriker 1997). The coupled effects of internal energy equipartition (mass segregation), pushing the lighter stars to outer, weakly bound orbits, and of the tidal field of the Galaxy, removing the less bound stars venturing in the outer parts of the cluster halo, may well be at the origin of the depletion of lighter stars in NGC 288. We regard this result as a reassur-
ing confirmation of the theoretical predictions that account for the orbital characteristics of actual GGCs (Dinescu et al. 1999; Gnedin & Ostriker 1997).

Recently, PBD00 presented an LF in the J and H near-infrared bands obtained with the HST NIC3-NICMOS camera (parallel observations). The sample is small but reaches fainter magnitudes (lighter masses) than our data. PBD00 conclude that the mass function of NGC 288 is very similar to that of all other studied clusters and that it shows no sign of the strong harassment predicted by Dinescu et al. (1999) and Gnedin & Ostriker (1997). We regard the results (and the conclusions) of the present paper as more robust than those by PBD00 since (1) the LF by PBD00 is based on a sample of 75 stars, a very small sample, which may be subject to strong statistical fluctuations and which is not supported by artificial-star experiments, while our LF is based on a sample of more than 6000 stars corrected by the artificial-star experiments for all the observational biases, and (2) the field observed by PBD00 is a small stamp (51′2 × 51′2, about 15 times smaller than the field observed here) located near the field observed by PBD00 is a small stamp (51′2 × 51′2, about 15 times smaller than the field observed here) located at \( r \sim 2.4r_h \), far outside the region of our sample, which is expected to be representative of the global cluster LF.

5. SUMMARY AND CONCLUSIONS

The LF of the globular cluster NGC 288 (down to \( M_V \sim 10 \)) has been obtained from a field covering a region comprised between the center of the cluster and \( r \sim 2r_h \).

A new method to correct the observed LFs for all the observational effects has been introduced and applied. The method is based on the equivalent sample concept, which may have many interesting applications in the study of stellar populations. The effect of the presence of binary systems in the final LF has been quantified, and it has been found negligible in the considered magnitude range.

The comparison of the LFs obtained in different regions of the clusters indicates the presence of mass segregation within NGC 288. Independently of the assumed normalization the LFs of the inner (WF-Int) and outer (WF-Ext) sample turn out to be different at the 99.1% confidence level.

The comparison with the LFs of other clusters strongly suggests that the global population of unevolved stars in NGC 288 is significantly depleted of low-mass stars \((M < 0.5–0.6 ~ M_\odot)\), in general agreement with recent theoretical predictions that also take into account the orbital properties of the cluster.

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