The interplay between one-dimensional confinement and two-dimensional crystallographic anisotropy effects in ballistic hole quantum wires

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\textbf{Abstract.} In this paper, we study the Zeeman spin-splitting in hole quantum wires aligned along the $[\bar{2}33]$ and $[0\bar{1}1]$ crystallographic axes of a high-mobility (311)A undoped AlGaAs/GaAs heterostructure. We obtained measurements of the effective $g$-factor $g^*$ as a function of the wire width for in-plane magnetic fields aligned both parallel and perpendicular to the wire axis. We interpret our data in terms of a qualitative model that involves the interplay of two effects—1D confinement and 2D crystalline anisotropy.

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Spintronics aspires to enhance conventional electronics by using spin rather than charge for processing information [1]. The spin–orbit interaction in semiconductors [2, 3] has been the focus of intense interest recently due to the exciting possibility of using electrostatic spin manipulation to realize the spintronic equivalent of a field-effect transistor [4]. Narrow band-gap semiconductors such as InGaAs have been used for many years to study spin–orbit effects in semiconductor devices [5, 6]. However, hole systems in p-type GaAs heterostructures are attracting increasing attention due to three key advantages. Firstly, because holes originate from p-like valence band states with orbital angular momentum $l = 1$, they have a much stronger spin–orbit interaction than electrons [7]. Secondly, the wider band gap of GaAs results in a larger Schottky barrier and hence more stable gates that are less prone to current leakage. Thirdly, the high mobility in GaAs ($\sim 10^6$ cm$^2$ V$^{-1}$ s$^{-1}$) leads to very long ballistic transport lengths [8, 9].

One of the most striking features of holes is their total angular momentum (spin) $j = l + s = \frac{3}{2}$, which gives them some remarkable spin properties compared with equivalent electron systems. These properties have been extensively studied in high mobility two-dimensional hole systems (2DHSs) in (311) AlGaAs/GaAs heterostructures with two notable findings. Firstly, the hole spin-splitting is anisotropic, with different effective Landé $g$-factors $g^*$ for in-plane magnetic fields $B$ oriented along the [\(\overline{2}33\)] and [01\(\overline{1}\)] crystallographic directions [10, 11]. Secondly, an anomalous out-of-plane spin polarization is observed for an in-plane field applied along the lower symmetry [\(\overline{2}33\)] direction [12].

Recently, it has become possible to make high-quality hole quantum wires by imposing additional 1D confinement on a 2DHS [13, 14]. Anisotropic spin-splitting was also observed for these quantum wires, with a $g$-factor that increased with the strength of the 1D confinement [15], hinting that the 2D crystallographic anisotropy [10, 11] and 1D confinement [16] combine to give a rich spin-splitting behaviour for hole quantum wires. This raises an interesting
question: how does the spin-splitting depend on the relative orientations of the wire, magnetic field and the crystallographic axes?

To answer this question, we have studied undoped (311)A AlGaAs/GaAs heterojunction devices with two orthogonal, 400 nm long, 1D hole quantum wires, aligned along the [233] and [011] crystal axes. The lack of modulation doping and high hole mobility in our devices ($\mu_{\text{peak}} > 600,000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$) combine to give highly stable and reproducible 1D conductance plateaus [14]. This allows us to clearly resolve the Zeeman spin-splitting of the 1D subbands and accurately determine the hole $g$-factors for different orientations of the quantum wire and magnetic field. The low disorder in our devices and high quality of the data allow us to resolve a complex dependence of the $g$-factor on the magnetic field, wire direction and crystal axes, reflecting the interplay between 1D confinement and 2D crystal anisotropy effects.

Our results are in contrast to an earlier study of the $g$-factor anisotropy in AFM-defined hole quantum wires on a (311)A modulation doped heterostructure [17]. In [17], it was found that the 1D confinement enhanced the spin-splitting anisotropy relative to that found in a 2DHS (consistent with previous studies of high-quality 1D hole quantum wires aligned along [233] [15]), but the authors concluded that the anisotropy of the 1D hole spin-splitting was primarily due to the crystallographic anisotropy of the spin–orbit interactions rather than the 1D confinement. Unfortunately the devices studied in [17] show numerous artefacts, such as resonances appearing at low conductance and the suppression of the conductance plateaus beneath their integer quantized values at higher conductance. Similar artefacts have previously been observed in disordered and quasi-ballistic 1D electron wires [18]–[20], suggesting that disorder is disrupting the ballistic transport in the AFM-defined devices. The disorder is presumably caused by the use of a heterostructure with a very shallow 2DHS ($\sim 35 \text{ nm}$) [21], which is required by the local anodic oxidation technique used to make these devices [22]. These effects of disorder make it very difficult to reliably determine the hole $g$-factor (for example the measurements and analysis in [17] suggest that the energy separation of the 1D subbands decreases with an increase in 1D confinement, which is non-physical). The present study clarifies the roles of spin–orbit coupling, 1D confinement and 2D crystallographic anisotropy.

2. Experimental details

We have studied two devices, each with two orthogonal 1D hole wires on a single Hall bar made from an undoped (311)A AlGaAs/GaAs heterojunction [14, 23]. Two devices were studied (four quantum wires in total), and both gave similar results; data are presented here from device no. 19. Electron micrographs of the two 400 nm long wires are shown in the insets to figure 1, aligned so that the current flows along the [233] and [011] directions, which we refer to as QW233 and QW011, respectively. The devices were measured in a dilution refrigerator with a base temperature of 20 mK using standard ac lock-in techniques with an excitation voltage of $\sim 20 \mu$V at a frequency of 17 Hz. All measurements were obtained with a top-gate voltage $V_{\text{TG}} = -0.48 \text{ V}$ corresponding to a 2D hole density of $p = 1.69 \times 10^{11} \text{ cm}^{-2}$. The width of the wire and its conductance can be gradually reduced by applying a positive voltage $V_{\text{SG}}$ to the two side-gates (SGs), as shown in figure 1. For both wires, we observe the classic ‘staircase’ of quantized conductance plateaus [24] as the wire is made narrower with increasing $V_{\text{SG}}$, until the wire is ‘pinched off’ at $V_{\text{SG}} \sim 1.2 \text{ V}$. The similar pinch-off voltages for the two orthogonal wires indicate that they have similar dimensions and lateral confinement potentials. The accurate quantization of the plateaus, at $G = n \times 2e^2 / h$, where $n$ is
Figure 1. The measured wire conductance $G$ versus SG voltage $V_{SG}$ for QW233 (solid red line) and QW011 (dashed blue line). The inset shows the SEM micrographs of QW233 (top) and QW011 (bottom), defined by electron-beam lithography (EBL) and wet etching.

the number of occupied 1D subbands, indicates that transport is ballistic through the wires [14, 24]. Moving corresponds to strengthening the 1D confinement, taking the device from being quasi-2D (large $n$ and large $G$) to quasi-1D (small $n$ and small $G$).

We study the spin properties of the 1D holes by measuring the Zeeman spin-splitting for different orientations of the wire and magnetic field with respect to the crystallographic axes. To obtain the $g$-factor for the various 1D subbands $n$, we use a technique that compares the 1D subband splitting due to an applied dc source–drain bias [25] (see figure 2) and in-plane magnetic field [26] (see figure 3). These two sets of measurements are repeated in two cool-downs (to rotate the sample) for the four different combinations of wire and magnetic field orientation with respect to the crystallographic axes.

3. Results

3.1. 1D subband spacings and source–drain bias measurements

We first discuss the source–drain bias measurements shown in figure 2. The data were obtained with a dc bias $V_{SD}$ added to the 20 $\mu$V ac bias used to measure the conductance. We take the derivative of the conductance, $dg/dV_{SG}$ (the transconductance), and plot it as a greyscale against $V_{SG}$ and the bias across the wire $V_{SD}$ (after correcting for the voltage drop in the leads and contacts). Moving upwards along the centre of figures 2(a) and (b) corresponds to the traces in figure 1. The black regions correspond to high transconductance (the risers between conductance plateaus) and white regions correspond to low transconductance (the conductance plateaus themselves). Thus the black regions indicate when a particular 1D subband crosses the Fermi energy. As $V_{SD}$ is increased, the plateaus at multiples of $2e^2/h$ evolve into plateaus at odd multiples of $e^2/h$. The subband spacing $\Delta E_{n,n+1} = eV_{SD}$ is obtained from the source–drain bias.
Figure 2. Measuring the 1D subband spacing: greyscale map of the transconductance versus \( V_{\text{SD}} \) on the \( x \)-axis and \( V_{\text{SG}} \) on the \( y \)-axis for (a) QW233 and (b) QW01\( \bar{1} \). White areas mark plateaus in conductance (low transconductance), and black areas mark risers between conductance plateaus (high transconductance). The superimposed numbers in (a) indicate the conductance \( G \) of the corresponding plateau (see figure 1) in units of \( 2e^2/h \).

\( V_{\text{SD}} \) at which the adjacent transconductance peaks cross (i.e. from the dark regions at finite \( V_{\text{SD}} \)). The subband spacings for QW233 vary from 365 \( \mu eV \) \( (n=1) \) to 140 \( \mu eV \) \( (n=7) \). The subband spacings for QW01\( \bar{1} \) are slightly smaller, ranging from 282 to 98 \( \mu eV \). Our measurements of both wires show that the subband spacing decreases monotonically as the 1D confinement is weakened (i.e. for increasing \( n \)), as expected. We repeated the subband spacing measurements on the second cool-down and obtained identical results to within 10 \( \mu eV \), confirming the stability and reproducibility of measurements obtained from these devices [14].

3.2. Zeeman spin-splitting measurements

The effect of an in-plane magnetic field \( B \) on the 1D subbands is shown in figure 3 for different orientations of the quantum wire and magnetic field. The transconductance \( \text{d}G/\text{d}V_{\text{SG}} \) is plotted as a greyscale versus \( B \) and \( V_{\text{SG}} \). Again, the black regions mark the 1D subband edges (high transconductance corresponding to the risers between conductance plateaus). For most of the orientations measured, the applied field causes spin-splitting of the 1D subbands, as in figure 3(c): initially, as \( B \) is increased, the subband edges (black lines) move apart, and the conductance plateaus occur at multiples of \( e^2/h \) [15, 26]. If \( B \) is increased further, 1D subbands with different spin orientations can cross, as seen in the right-hand side of figure 3(c), and the conductance plateaus then occur at odd multiples of \( e^2/h \). We were unable to observe this crossing in all orientations as the ohmic contacts degrade rapidly at \( B \gtrsim 4 \text{T} \).

We now examine the spin-splitting for different orientations, starting with the wire aligned along [\( \bar{2}33 \)]. In figure 3(a), \( B \) is aligned along the wire and spin-splitting is clearly observed.
Figure 3. Measuring the 1D spin-splitting: greyscale map of the transconductance versus in-plane magnetic field $B$ and SG voltage $V_{SG}$ for QW233 with (a) $B \parallel [233]$ and (b) $B \parallel [01\bar{1}]$. Data for QW01$\bar{1}$ with (c) $B \parallel [233]$ and (d) $B \parallel [01\bar{1}]$. The superimposed numbers in (a) indicate the conductance $G$ of the corresponding plateau (see figure 1) in units of $2e^2/h$. The two white dashed lines in (c) are guides to the eye that track the Zeeman spin-splitting of the $n = 2$ subband with magnetic field.

(although the spin-splitting is not uniform for all 1D subbands, which we will discuss later). In contrast, for $B$ perpendicular to the wire, no splitting is observed up to the highest field in any of the 1D subbands (figure 3(b)). These results are consistent with previous studies of quantum wires aligned along $[233]$ at higher magnetic fields ($B = 9$ T), where 1D subband splitting was only visible for $B$ parallel to the wire [15]. However, for QW01$\bar{1}$, when $B$ is applied perpendicular to the wire, strong spin-splitting is observed for all subbands, as shown in figure 3(c). For $B$ parallel to the wire (figure 3(d)), spin-splitting is still observed, albeit weaker than in figure 3(c). Therefore, the anisotropy of the spin-splitting for QW01$\bar{1}$ is actually opposite to that of QW233; whereas for QW233 a perpendicular field causes almost no spin-splitting [15], for QW01$\bar{1}$ it is instead a parallel field that results in almost no spin-splitting.
Figure 4. (a)–(d) Schematic diagrams of the expected behaviour of the 1D hole $g$-factor for different orientations of the wire and the applied $B$ as a function of the 1D subband index $n$. The red dashed line shows the 2D $g^*$ due to anisotropy of the (311) crystal. The solid blue line shows the effect of a 1D confining potential in the spherical approximation, showing the enhancement of $g^*$ when $B$ is applied along the axis of the wire. (e)–(h) The $g$-factors measured from the data shown in figures 2 and 3. The black lines are guides to the eye.

3.3. $g$-factor measurements for the four field and wire orientations

The stability of our devices and the high quality of the data allow us to combine the results in figures 2 and 3 to calculate $g^*$ as a function of the subband index $n$ for the four different combinations of wire and field orientations, as shown in figures 4(e)–(h). The values of $g^*$ are obtained in two different ways [15]: firstly, for integer $n$, we combine the subband splitting rate due to an applied dc bias, $\frac{\partial V_{SG}}{\partial V_{SD}}$ from figure 2, with the splitting rate due to an applied field, $\frac{\partial V_{SG}}{\partial B}$ from figure 3, to obtain:

$$g^*_n = \frac{e}{\mu_B} \frac{\partial V_{SD}}{\partial V_{SG}} \frac{\partial V_{SG}}{\partial B}.$$  \hspace{1cm} (1)

The $g^*$ values obtained are plotted as solid symbols, with error bars marking the uncertainty in measured $g^*$. The second method is to measure the average $g$-factor for two adjacent subbands, using the 1D subband spacing and the field $B^c$ at which levels with different spin orientations from the $n$th and $(n+1)$th subbands cross, giving

$$(g^*_n, g^*_{n+1}) = \frac{e V_{SD}^c}{\mu_B B^c},$$  \hspace{1cm} (2)

where $V_{SD}^c$ is the dc bias at the $(n, n+1)$ subband crossing in figure 2, and $B^c$ is the magnetic field at which the two subbands cross in figure 3. Data obtained in this way are plotted as solid symbols at half-integer values of $n$ in figure 4. Finally, if the spin-splitting is small it is difficult to extract $g^*$. In such cases, we can only obtain an upper bound on $g^*$, i.e. if $g^*$ were larger than...
this upper bound we would be able to resolve the spin-splitting in our measurements. Thus the actual \( g \)-factor must lie between zero and this upper bound, as indicated by the shaded region in figure 4(e).

4. Discussion

4.1. Qualitative picture of the observed \( g \)-factor behaviour

The anisotropic spin-splitting in hole systems arises due to strong spin–orbit coupling, which means that these systems are best described by the total angular momentum \( \hat{J} \). In 2D systems, strong spin–orbit coupling forces the quantization axis for the angular momentum to point perpendicular to the 2D plane, such that the \( g \)-factor takes different values depending on the relative orientation of \( B \) and \( \hat{J} \). For a 2DHS grown on a high-symmetry crystal plane, such as (100), this means that there is a spin-splitting if the magnetic field is applied perpendicular to the 2D plane, and no spin-splitting if \( B \) is applied in the 2D plane (\( g^* = 0 \)). Further confinement of the holes to a 1D wire causes the quantization axis for \( \hat{J} \) to lie along the wire in the 1D limit. In this case one expects the \( g \)-factor will be suppressed to the lowest order unless \( B \) is applied parallel to the wire. This expected behaviour of \( g^* \) is shown schematically in figures 4(a)–(d) as a function of the 1D subband index \( n \), for the different orientations of the wire and \( B \). The solid blue lines show the effect of the 1D confinement to the lowest order (ignoring cubic crystal anisotropies). In this spherical approximation, appropriate for high symmetry crystals such as (100), the in-plane \( g \)-factor is zero in all orientations in the 2D limit, and only becomes nonzero if \( B \) is aligned along the wire (figures 4(b) and (c)) [27, 28]. For \( B \) parallel to the wire, \( g^* \) increases as the system becomes more 1D (lower \( n \)) and the quantization axis aligns with the applied \( B \).

For 2D holes grown on lower symmetry substrates, such as the (311)A crystal used in our experiments, cubic terms in the Hamiltonian due to crystallographic anisotropy result in a finite in-plane 2D \( g \)-factor [7] with \( g^* = 0.6 \) for \( B \parallel [\overline{2}33] \) and \( g^* = 0.2 \) for \( B \parallel [01\overline{1}] \). The dashed red lines in figures 4(a)–(d) show the theoretical \( g \)-factor in the 2D limit (large \( n \)) taking into account cubic crystal anisotropy. In our experiments, we thus expect the \( g \)-factor to show anisotropies that result from a combination of the underlying properties of the (311) crystal and the 1D confining potential.

4.2. Spin-splitting for the magnetic field aligned along the higher symmetry [01\overline{1}] direction

The data in figure 4 indicate a complex dependence on wire and field orientation: we begin by considering QW\( 233 \) with \( B \parallel [01\overline{1}] \) (figure 4(e)). In the 2D (large \( n \)) limit, the quantization axis points perpendicular to \( B \), and \( g^* \sim 0.2 \), as indicated by the dashed red line. In the 1D (small \( n \)) limit, the quantization axis is perpendicular to \( B \), strongly suppressing the Zeeman splitting, again giving very small \( g \)-factors. This is consistent with our measurements, where the splitting is so small that we can only determine an upper bound for \( g^* \).

Figure 4(f) shows the results from the other wire, QW\( 01\overline{1} \), with the same orientation of magnetic field, \( B \parallel [01\overline{1}] \). At large \( n \), \( g^* \) takes its expected 2D value of \( \sim 0.2 \). The measured \( g \)-factor increases as the wire is made more 1D (decreasing \( n \)). However, unlike in 1D electron wires, this enhancement of the \( g \)-factor with a decrease in the 1D subband index is unlikely to arise from exchange interactions, since these are strongly suppressed for \( B \parallel [01\overline{1}] \) [12].
Instead it can be explained by the spin–orbit interaction: making the wire more 1D should change the quantization axis from being out-of-plane to lying along the axis of the wire, giving a corresponding enhancement of the spin-splitting and an increase in \( g^* \), as observed.

### 4.3. Spin-splitting for the magnetic field aligned along the lower symmetry \([\overline{2}33]\) direction

For the remaining two cases, where \( B \) lies along \([\overline{2}33]\), the physics is more complex. For 2D holes in (311) heterostructures, higher order crystallographic anisotropies mean that the Zeeman term for the topmost heavy-hole subband contains three terms: \( B_x \sigma_x, B_y \sigma_y \) and \( B_z \sigma_z \), where the \( x-, y- \) and \( z- \)axes are the crystallographic directions \([\overline{2}33]\), \([01\overline{1}]\) and \([311]\), and \( \sigma \) are the relevant Pauli spin matrices (see equation (7.13) of [7] for details). The \( B_x \sigma_x \) term gives an anomalous out-of-plane spin polarization in response to \( B \parallel [\overline{2}33] \) (which does not occur for \( B \parallel [01\overline{1}] \)) [12]. It is this \( B_x \sigma_x \) term that leads to the well-known in-plane \( g \)-factor anisotropy for (311) 2DHSs, by causing an out-of-plane spin polarization when \( B \) is applied along \([\overline{2}33]\). This causes the calculated 2D \( g \)-factor anisotropy indicated by the red dashed lines in figure 4, where \( g^* \sim 0.6 \) for \( B \parallel [\overline{2}33] \) and \( g^* \sim 0.2 \) for \( B \parallel [01\overline{1}] \) [10].

The anomalous spin polarization for \( B \parallel [\overline{2}33] \) also explains the difference in the \( g \)-factor anisotropies between the two wires studied here. For QW01\overline{1} with \( B \parallel [\overline{2}33] \) (figure 4(h)), to first order we expect that since the quantization axis points out-of-plane in the 2D limit, \( g^* \) should be small for large \( n \) (as happens for the same wire with \( B \parallel [01\overline{1}] \) in figure 4(f)). However, in 2D, the \( B_x \sigma_x \) term results in an out-of-plane spin polarization that enhances the Zeeman splitting to give \( g^* \sim 0.6 \). This is consistent with the measured data at large \( n \) in figure 4(h). There is also a gradual increase in \( g^* \) with a decrease in \( n \) in this orientation, which cannot be due to the 1D confinement, because even in the 1D limit, the quantization axis is perpendicular to the field. The increased \( g^* \) may be due to an exchange enhancement as the wire becomes more 1D a similar behaviour is observed in 1D electron systems [29, 30]. We suggest this because in 2D, exchange enhancement can only occur for (311) holes with \( B \parallel [\overline{2}33] \) [12].

Finally, we come to QW233 and \( B \parallel [\overline{2}33] \) (figure 4(g)), which is the most complex and interesting case. The data clearly show a striking non-monotonic behaviour, with large \( g^* \) for both large and small \( n \), and \( g^* \) dropping sharply for intermediate \( n \). This is quite different from the behaviour reported in [17]. One possible interpretation of the large \( n \) (2D) and small \( n \) (1D) limits is as follows: for large \( n \), the spin-splitting is enhanced and \( g^* \sim 0.6 \) due to the \( B_x \sigma_x \) term caused by crystallographic anisotropy, as in figure 4(h). For small \( n \), the spin-splitting is also enhanced, but this time the 1D confinement tries to point the quantization axis along the wire and parallel to \( B \), in the same way as in figure 4(f). The interesting behaviour occurs at intermediate \( n \): whereas \( g^* \) gradually increases as \( n \) is reduced when the wire and \( B \) are both aligned along [01\overline{1}] \( (g^* \sim 0.6 \) due to the \( B_x \sigma_x \) term caused by crystallographic anisotropy, as in figure 4(h)); the wire and \( B \) are both aligned along [\overline{2}33] \( (g^* \sim 0.6 \) due to the \( B_x \sigma_x \) term instead drops almost to zero and then rises again with a decrease in \( n \). Our data strongly suggest that the transition between the quasi-2D state at large \( n \) and the quasi-1D state at small \( n \) is made via an intermediate state with near zero spin polarization \( (g^* \simeq 0) \). Thus the non-monotonic behaviour is due to competition between the crystal anisotropy at large \( n \) and the 1D confinement at by small \( n \). While orbital effects may play a role in determining \( g^* \), since the shape of the wavefunction is affected both by the 1D confinement and by the field applied along the wire, we do not expect it to be the central cause of the non-monotonic behaviour that we observe in figure 4(g).
5. Conclusions

We have measured the Zeeman spin-splitting of hole quantum wires fabricated in an undoped (311)A AlGaAs/GaAs heterostructure for the four different combinations of magnetic field and wire orientation along the two key crystallographic directions [01\bar{1}] and [\bar{2}33]. Our use of an undoped heterostructure with a relatively deep and high mobility 2DHS allows us to make high-quality wires with well quantized conductance plateaus, in which Zeeman splitting can be clearly resolved. We find two quite different behaviours depending on the orientation of the magnetic field with respect to the crystal axes. When \( B \) is aligned along the higher symmetry [01\bar{1}] direction, the \( g \)-factor is enhanced if the wire is parallel to \( B \) and diminished to almost zero if the wire is perpendicular to \( B \). In contrast, when \( B \) is aligned along the lower symmetry [\bar{2}33] direction, we observe a complex interplay between the 1D confinement and 2D crystal anisotropy, which causes an unusual non-monotonic behaviour of the \( g \)-factor for the [\bar{2}33] wire as it is made narrower. Although we provide a phenomenological model to explain this behaviour, there is clearly a need for more detailed theoretical calculations to fully understand the rich nature of spin–orbit coupling and anisotropic spin-splitting in 1D hole systems.

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