Quark Stars in 4D Einstein-Gauss-Bonnet gravity with an Interacting Quark Equation of State

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Abstract: The detection of gravitational waves (GWs) from the binary neutron star (BNS) has opened a new window on the gravitational wave astronomy. With current sensitivities, detectable signals coming from compact objects like neutron stars turn out to be a crucial ingredient for probing their structure, composition and evolution. Moreover, the astronomical observations on the pulsars and their mass-radius relations put important constraints on the dense matter equation of state (EoS). In this paper, we consider an homogeneous and unpaired charge neutral 3-flavor interacting quark matter with $O(m_s^4)$ corrections that account for the moderately heavy strange quark instead of the naive MIT bag model. Importantly, we perform a detailed analysis about strange quark stars in the context of recently developed 4D Einstein-Gauss-Bonnet (EGB) gravity. We pay particular attention to the possible existence of massive neutron stars of mass compatible with $M \sim 2M_\odot$. Our findings suggest that the fourth-order corrections parameter ($a_4$) of the QCD perturbation and coupling constant $\alpha$ of the GB term play an important role on the mass-radius relation as well as the stability of the quark star. Finally, we compare the results with the well-measured limits of the pulsars and their mass and radius extracted from the spectra of several X-ray compact sources.

Keywords: 4D EGB gravity; Interacting Quark Equation of State; Quark stars
1 Introduction

Over the past few years there have been a lot of interest in higher derivative gravity (HDG) theories. Although, many approaches have been introduced in order to modify GR and perhaps construct HDG theories appear in an effective level. In fact, this theory has been proposed in an expectation that higher order corrections to Einstein’s GR might solve the singularity problem of black holes, avoids causality problems at the classical level and so on. Among the higher derivative gravity theories, Lovelock gravity (LG) [1, 2] is a natural generalization of Einstein’s general relativity. Interestingly, the equations of motion, which depends only on the Riemann tensor and not on its derivatives, remain second order. Moreover, it obeys generalized Bianchi identities which ensure energy conservation, and unique ghost free when expanded on a flat space, avoiding problems with unitarity [3, 4]. For spacetime dimension \( d > 4 \), a Gauss-Bonnet term can be added to the Einstein-Hilbert action. In this vein, Einstein-Gauss-Bonnet (EGB) gravity [5] is considered as a special case of Lovelock’s theory of gravitation, which naturally appears in the low energy effective action of heterotic string theory [6–8].

However, in 4\( D \), the Gauss-Bonnet (GB) term becomes a topological invariant and does not contribute to the gravitational dynamics. Recently, Glavan and Lin have proposed a dimensional regularization of the Gauss-Bonnet equations and obtain a 4\( D \) metric theory in [9], which bypasses the conclusions of Lovelock’s theorem and avoids Ostrogradsky instability. The approach has been formulated in \( D \)-dimensions, by rescaling the coupling constant \( \alpha \to \alpha/(D - 4) \), and then taking the limit \( D \to 4 \). Thus, the GB term shows a nontrivial contribution to the gravitational dynamics, which is referred to as the 4\( D \) Einstein-Gauss-Bonnet (EGB) gravity. This process is referred to as \textit{regularization}, which was first considered by Tomozawa [10] with finite one-loop quantum corrections to

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Einstein gravity. In particular, rotating and non-rotating black hole solutions and their physical properties have been found, see [11–19]. Nevertheless, a number of interesting results in support of this ideas such as geodesics motion and shadow [20, 21], the strong/weak gravitational lensing by black hole [22–25], spinning test particle [26], thermodynamics AdS black hole [27], Hawking radiation [28, 29], quasinormal modes [30–32], and wormhole solutions [33, 34], were extensively analyzed. Additionally, new quark stars in the context of 4D Einstein-Gauss-Bonnet gravity have been recently proposed in Refs.[35, 36] with various equations of state. On the other hand, it is crucial for the possible existence of thermal phase transition between AdS to dS asymptotic geometries. This problem was discussed in [37]. Many aspects of the 4D EGB gravity were discussed in the literature, see [38–41] for instance.

Neutron stars (NSs) are dense, compact astrophysical objects which are the remains of very massive stars (1030 M⊙) that ended their lives in supernova explosions [42, 43]. However, the discovery of neutron stars with masses around 2 M⊙ [44, 45], put forward a strong on the EoS of matter in neutron stars. But in the interior of these objects determining the true state of the matter is still an open question, which is the greatest importance for particle physics as well as stellar astrophysics alike. Moreover, the composition and structure of compact stars depend on the nature of strong interaction. Under such conditions, the presence of different exotic matter with large strangeness fraction such as hyperon matter, Bose-Einstein condensates of strange mesons and quark matter may occur in neutron star interior. Other theories suggest that each exotic component of dense matter makes the EoS soft, and soft EoS generally gave rise to a compact star with smaller maximum mass and radius than those of a stiffer EoS [46].

However, the mass measurements of the massive neutron star J0348+0432 [45] with 2.01 ± 0.04 M⊙, and PSR J1614-2230 [47] with M = 1.97 ± 0.04 M⊙, has set rigid constraints on the theoretical models of dense nuclear matter. The existence of such massive stars has important implications for dense matter in Quantum Chromodynamics (QCD), where a phase transition from hadronic matter to a deconfined quark phase should occur in neutron star interior. Even more intriguing the existence of a quark core in a neutron star, is the possible existence of a new family of compact stars composed of the three lightest quark flavor states (up, down, and strange quarks) satisfying the Bodmer-Witten hypothesis [48, 49]. Despite all the advances in our understanding of QCD, most of the analysis for quark stars still continues to be performed in the context of the MIT bag model [50–52]. In MIT bag model, quarks in the bag are considered as a free Fermi gas and provide mechanism of quark confinement.

However, MIT bag model has some limitations that violates chiral symmetry even in the limit of massless quark. Moreover it was found that this EOS is not sufficiently reliable to characterize a system with interacting quarks or more complex structures. Thus, some authors have suggested some modified models, for example, the three-flavor quark matter with the particular symmetry is called the color-flavor locked (CFL) matter [53]. It is widely believed that the CFL matter is a real ground state of QCD at asymptotically large densities [54]. However, at extremely high density, the phase of matter is less certain. In these proceedings a 2-component model for quark stars have been reported [55] that can
produce stars as heavy as $2M_\odot$.

Motivated by the newly proposed EoS which is homogeneously confined matter in the stellar interior with 3-flavour neutral charge and a fixed strange quark mass [55], we propose a simple model for quark star in 4D EGB gravity. This accumulation will lead to various changes in the mass-radius relation of a quark star whose results were compared with compact stars candidates like J0348+0432 [45], PSR J1614-2230 [47], J1903+0327 [56], 4U 1608-52 [57]. Present article is organised as follows: We take a short recap of 4D EGB gravity and derive field equations in Sec.2. The main point of the section is to derive the so-called TOV equations in 4D EGB gravity. In Sec.3 we introduce a QCD motivated EoS. In Sec.4, we perform a detailed numerical analysis and present mass-radius relations for quark matter stars by solving the customized TOV equations. We demonstrate the physical properties of a constructed quark star in Sec.5. Finally, we summarize our findings and discuss our results in Sec.6.

2 Field equations and TOV equations in 4D EGB gravity

We start by considering the general action of Einstein-Gauss-Bonnet (EGB) gravity in D-dimensions and also deriving the equations of motion of the underlying theory. The action takes the form

$$S_{\text{EGB}} = \frac{c^4}{16\pi G_D} \int d^Dx \sqrt{-g} \left[ R + \alpha L_{\text{GB}} \right] + S_m, \quad (2.1)$$

where $g$ denotes the determinant of the metric $g_{\mu\nu}$ and $\alpha$ is the Gauss-Bonnet coupling constant, $G_D$ is the D-dimensional Newtons gravitational constant and $S_m$ is the matter field action. The Einstein-Gauss-Bonnet Lagrangian $L_{\text{GB}}$ is given by

$$L_{\text{GB}} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2. \quad (2.2)$$

Note that adding the matter action $S_{\text{matter}}$ induces the energy momentum tensor $T_{\mu\nu}$. Using the standard technique, we take a variation of the above action with respect to the metric $g_{\mu\nu}$ to obtain the field equation:

$$G_{\mu\nu} + \alpha H_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad \text{where} \quad T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} S_m)}{\delta g^{\mu\nu}}, \quad (2.3)$$

with $G_{\mu\nu}$ is the Einstein tensor and $H_{\mu\nu}$ is a tensor carrying the contributions from the Gauss-Bonnet (GB) term given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu},$$

$$H_{\mu\nu} = 2 \left( RR_{\mu\nu} - 2 R_{\mu\rho} R^\rho_\nu - 2 R_{\mu\nu\rho\sigma} R^{\rho\sigma} - R_{\mu\sigma\rho\delta} R^{\sigma\rho\delta} \right) - \frac{1}{2} g_{\mu\nu} L_{\text{GB}}, \quad (2.4)$$

where $R_{\mu\nu}$ is the Ricci tensor, $R$ and $R_{\mu\sigma\rho\nu}$ are the Ricci scalar and the Riemann tensor, respectively. It is well known that for the case of $D = 4$, the GB term (2.2) is a topological invariant, and thus it does not affect the Einsteins equations. While it gives nontrivial contributions when the dimension of spacetime is larger than four. Remarkably, many
nontrivial effects from the GB term in four dimensions were revealed by redefining the GB coupling constant as \( \alpha \to \alpha/(D - 4) \) \[9\]. Taking the variation of the Gauss-Bonnet \[58\] yields
\[
\frac{g_{\mu\sigma}}{\sqrt{-g}} \frac{\delta L_{\text{GB}}}{\delta g_{\mu\sigma}} = \frac{\alpha(D - 2)(D - 3)}{2(D - 1)} K^2 \delta \nu.
\] (2.5)

Notice that Eq.(2.5) does not vanish in \( D = 4 \). For solution describing stellar objects, we use the regularization process (see Refs. [9, 59]) in which the spherically symmetric solutions are also exactly same as those of other regularised theories [41, 60–62].

In order to study the stellar objects, we consider static spherically symmetric \( D \)-dimensional metric anstaz with two independent functions of radial coordinate which takes the form:
\[
d s^2_D = -e^{2\Phi(r)} c^2 dt^2 + e^{2\Lambda(r)} dr^2 + r^2 d\Omega^2_{D-2},
\] (2.6)

where \( d\Omega^2_{D-2} \) is the metric on the unit \((D - 2)\)-dimensional sphere and \( \Phi(r) \) and \( \Lambda(r) \) are functions of \( r \), solely. We assume that the energy momentum tensor \( T_{\mu\nu} \) is a perfect fluid matter source and describes the interior of a star, which in this study is written as
\[
T_{\mu\nu} = (\epsilon + P)u_\nu u_\nu + Pg_{\mu\nu},
\] (2.7)

where \( P = P(r) \) is the pressure, \( \epsilon = \epsilon(r) \) is the energy density of matter, and \( u_\nu \) is a \( D \)-velocity. Invoking the metric (2.6) with the energy momentum tensor (2.7), in the limit \( D \to 4 \), the \( tt, rr \) and hydrostatic continuity equations (2.3) yield
\[
\frac{2}{r} \frac{d\Lambda}{dr} = e^{2\Lambda} \left[ \frac{8\pi G}{c^4} \left( 1 - \frac{1 - e^{-2\Lambda}}{r^2} \right) \right] \left[ 1 + \frac{2\alpha(1 - e^{-2\Lambda})}{r^2} \right]^{-1},
\] (2.8)
\[
\frac{2}{r} \frac{d\Phi}{dr} = e^{2\Lambda} \left[ \frac{8\pi G}{c^4} P + \frac{1 - e^{-2\Lambda}}{r^2} \left( 1 - \frac{\alpha(1 - e^{-2\Lambda})}{r^2} \right) \right] \left[ 1 + \frac{2\alpha(1 - e^{-2\Lambda})}{r^2} \right]^{-1},
\] (2.9)
\[
\frac{dP}{dr} = -(\epsilon + P) \frac{d\Phi}{dr}.
\] (2.10)

As usual, the asymptotic flatness implies \( \Phi(\infty) = \Lambda(\infty) = 0 \) and the regularity at the center imposes the condition \( \Lambda(0) = 0 \).

Here we can define the gravitational mass within the sphere of radius \( r \) given by \( e^{-2\Lambda} = 1 - 2Gm(r)/c^2r \). It is straightforward to derive the Tolman-Oppenheimer-Volkoff (TOV) equations based on the underlying \( 4D \) EBG theory and we write by using (2.9-2.10)
\[
\frac{dP}{dr} = -Ge(r)m(r) \left[ \frac{1 + \frac{P(r)}{\epsilon(r)}}{1 + \frac{4\pi r^3 P(r)}{c^4 m(r)}} \right] \left[ 1 - \frac{2Gm(r)}{c^2 r^3} \right] \frac{1}{1 + \frac{4Gm(r)}{c^2 r^3}}.
\] (2.11)

Notice that if we take the \( \alpha \to 0 \) limit, the above equation reduces to the TOV equation of the standard GR. From the last equality of Eq. (2.8), we obtain the gravitational mass
\[
m'(r) = \frac{6\alpha Gm(r)^2}{4\alpha Gm(r) + c^2 r^4},
\] (2.12)
We will use the initial condition of \( m(r) = 0 \). It is more convenient to work with the dimensionless variables. Therefore in the present analysis, we take \( \bar{P}(r) = \epsilon_0 \bar{P}(r) \) and \( \epsilon(r) = \epsilon_0 \bar{\epsilon}(r) \) and \( m(r) = M_\odot \bar{M}(r) \), with \( \epsilon_0 = 1 \text{ MeV/fm}^3 \). As a result, the above two equations become

\[
\frac{d\bar{P}(r)}{dr} = -c_1 \frac{\bar{\epsilon}(r) \bar{M}(r)}{r^2} \left[ 1 + \frac{P(r)}{\bar{\epsilon}(r)} \right] \frac{1 + c_2 r^3 \bar{P}(r)}{M(r)} - \frac{2c_1 \alpha \bar{M}(r)}{r^4},
\]

and

\[
M_\odot \frac{d\bar{M}(r)}{dr} = \frac{6c_1 \alpha \bar{M}(r)^2 + c_2 r^6 \bar{\epsilon}(r)}{4c_1 \alpha \bar{M}(r) + r^4},
\]

where \( c_1 \equiv \frac{GM_\odot}{c^2} = 1.474 \text{ km} \) and \( c_2 \equiv \frac{4\pi \epsilon_0}{M_\odot c^2} = 1.125 \times 10^{-5} \text{ km}^{-3} \). The relationship between mass \( M \) and radius \( R \) can be straightforwardly quantified using Eq. (2.14) with a given EoS. As a result, the final two Eqs. (2.13) and (2.14) can be numerically solved for a given EoS \( P = P(\epsilon) \). In the next section, we will discuss the equation of state based on an interacting quark matter.

3 Interacting Quark Matter Equation of State

The high density and relatively low temperature required to produce color superconducting quark matter may be attained in compact stars (hybrid neutron stars or strange stars). Even though much effort to explore the EoS and other properties of matter in the interior of such compact stars, the problem remains unsolved [63]. This scenario has been corroborated by the determinations of the masses of PSR J1614-2230 [64, 65] and PSR J0348+0432 [45] have set an observational bound on the maximum mass of a NS not lower than about \( 2 M_\odot \). In addition to these astrophysical observations of the pulsar can be employed to constrain the composition and behaviour of the theoretical models of the EOS. There are some strange stars [66–68] (at the moment hypothetical objects) that can be viewed as an ultra-compact NSs (neutron stars), where it is possible to fit the EoS associated with these types of objects. Thus, the discovery of pulsars may not adjust their masses and radius to the NSs models, but set a lower limit to the maximum mass and mass-radius relation that could have led to an alternative to typical NSs. Then NSs may be converted to quark stars (QSs) [69, 70], which consists of a deconfined mixture of up (u), down (d) and strange (s) quarks (together with an appropriate number of electrons to guarantee electrical neutrality) satisfying the Bodmer-Witten hypothesis [48, 49]. Such compact stars are referred as strange quark stars or shortly strange stars (SS). A widely accepted and easy-to-handle quark star model is the so-called thermodynamic bag model. The most prominent bag model is known as the MIT bag model [71], which is the simplest and frequently used form to illustrate the interior a quark star. But, the reliable existence of the QSs, whose hypothesis cannot be conclusively ruled out depending on the bag constant \( B \), which explicitly violated the chiral symmetry of quantum chromodynamics (QCD). Incidentally, there are many other models based on QCD corrections of second and fourth order with the aim of giving an approximate characterization of confined quarks, see [72].
Here we discuss the EoS that used in modeling the strange star. The EoS is assumed to be homogeneous and unpaired charge neutral 3-flavor interacting quark matter, which we describe using the simple thermodynamic Bag model EoS [73] with $O(m_s^4)$ corrections that account for the moderately heavy strange quark. According to Ref. [55], an interacting quark EoS is given by

$$P = \frac{1}{3} (\epsilon - 4B) - \frac{m_s^2}{3\pi} \sqrt{\frac{\epsilon - B}{a_4}} + \frac{m_s^4}{12\pi^2} \left[ 1 - \frac{1}{a_4} + 3 \ln \left( \frac{8\pi}{3m_s^2} \sqrt{\frac{\epsilon - B}{a_4}} \right) \right] ,$$

(3.1)

where $\epsilon$ is the energy density of homogeneous quark matter (also to $O(m_s^4)$ in the Bag model). Coming back to the EoS (3.1), the mass $M$ and radius $R$ are determined by solving the TOV equations (2.13) and (2.14). To illustrate our approach in a simple setting, we consider the boundary conditions $P(r_0) = P_c$ and $M(R) = M$, and integrates Eq. (2.13) outwards to a radius $r = R$ in which fluid pressure $P$ vanishes for $P(R) = 0$. Corrections this one can obtain the quark star radius $R$ and mass $M = m(R)$. At this stage we set a very small numbers with initial radius $r_0 = 10^{-5}$ and mass $m(r_0) = 10^{-30}$ rather than zero to avoid discontinuities which appears in denominators within the equations.

It is worth noting that a unit conversion 1 fm = 197.3 MeV is used in order to synchonize the unit of each term given in Eq. (3.1). Introducing this conversion, we find MeV$^4 = 197.3^{-3}$MeV fm$^{-3}$. Therefore, Eq. (3.1) becomes

$$P = \frac{1}{3} (\epsilon - 4B) - \frac{1}{3\pi} \sqrt{\frac{1}{197.3^3} m_s^2 \sqrt{\frac{\epsilon - B}{a_4}}} + \frac{1}{12\pi^2} 197.3^3 \left[ 1 - \frac{1}{a_4} + 3 \ln \left( \frac{8\pi}{3m_s^2} \sqrt{\frac{\epsilon - B}{a_4}} \right) \right].$$

(3.2)

The strange quark mass $m_s$ will be assumed to be 100 MeV [74], and $B$ is the Bag constant whose standard accepted range is around $57 \leq B \leq 92$ MeV/fm$^3$ determined by the stability condition with respect to iron nuclei for 2-flavour and the 3-flavour quark matter [75], respectively. Finally, the parameter $a_4$ comes from the QCD corrections on the pressure of
the quark free Fermi sea, which is related to the maximum mass of the star around $2M_\odot$ at $a_4 \approx 0.7$ as suggested in [76]. For the study of quark matter with $\mathcal{O}(m_4)$ corrections, we demonstrate how the pressure and energy density do distribute by using a median value of the bag constant range such that $B \sim 70$ MeV/fm$^3$. In Fig.1, we plot the variation of the pressure and density with radius of the star. At that same time, we quantify the variation of mass versus central density and the variation of mass with radius are shown in Fig.2. For $\alpha > 0$ the mass of star for given radius increases with fixed value of $B$. In all the presented cases, one can note that there are significantly different for positive and negative values of $\alpha$, but $\alpha = 0$ case is equivalent to standard general relativity.

![Figure 2](image)

**Figure 2.** For the interacting EoS with different values of $\alpha = 0, \pm 2.5, \pm 5$, where we set $P(r_0) = 700.00$ MeV/fm$^3$, $B = 70.00$ MeV/fm$^3$, we display the variation of mass versus central energy density (left panel) and the variation of mass with radius (right panel).

### 4 Numerical details and analysis of mass-radius relation

In this section, we present the detailed results for the EoS (3.2), and show all relevant outcomes for isotropic QSs in the 4D EGB gravity. To start with, we consider a certain value of central pressure, $P(r_0) = 700$ MeV/fm$^3$ and the radius of the star is identified when the pressure vanishes or drops to a very small value. Due to the long range effects of confinement of quarks, the stability of strange QSs is represented by the bag constant, $B$. We then consider the engineered TOV equations Eq. (2.13) and mass function Eq. (2.14). It is important to note that the mass is measured in the solar mass unit ($M_\odot$), radius in km, while energy density and pressure are in unit of MeV/fm$^3$. The bag constant $B$ is also in MeV/fm$^3$. In the present analysis, we treat the values of $B$ and $\alpha$ as free constant parameters. Since, the parameter $B$ can vary from 57 to 92 MeV/fm$^3$ [49]. In the following, numerical values of the GB coupling $\alpha$ are given in km$^2$ unit.

We depict the mass-radius curves obtained from the of the QSs as a function of the radius $R$ shown in Fig.3 with two values of the bag constant $B$ and various values of the GB coupling $\alpha$. Moreover, in comparison our results with the data, we have used the observational constraints of the NS mass from four pulsar measurements explained in the following. The upper limit NS mass is given by Ref. [45] with mass $2.01 \pm 0.04M_\odot$. Next, the mass from the binary pulsar J1903+0327 of $1.667 \pm 0.021 M_\odot$ [56]. The NS mass is predicted
1.4408 ± 0.008 \( M_\odot \) form the data collection and analysis of thirty years of observations of PSR B1913+16 \[77\]. Lastly, the NS mass measurements of the relativistic binary pulsar PSR J1141-6545 \[78\] is given by 1.3 ± 0.02 \( M_\odot \). These mass values have been utilized to compare with the mass-radius results of an anisotropic QSs with the interacting quark EoS in Ref. \[79\]. In Fig.3, note that the mass-radius relation of the QSs in GR and 4D EGB cases is represented by setting \( \alpha = 0 \) and \( \alpha \neq 0 \), respectively. As the results, the existence of a two solar mass compact star is found in the case when \( \alpha > 0 \) for the lowest values of the bag constant \( B = 57 \text{MeV}/\text{fm}^3 \), while such a star in the GR case, i.e. \( \alpha = 0 \), can not be obtained for all possible bag constant values. On one hand, furthermore, all solutions of the TOV equation for the maximum values of the bag constant \( B = 92 \text{MeV}/\text{fm}^3 \) are located in the range of the mass constraint around \( 1.2 \, M_\odot < M < 1.7 \, M_\odot \) from pulsars J1141-6545 and J1903+0327 constrained region. On the other hand, the minimum bag constant \( B = 57 \text{MeV}/\text{fm}^3 \) gives the mass solution around \( 1.6 \, M_\odot < M < 2.1 \, M_\odot \) which are in the pulsars J1903+0327 and J0348+0432 mass constrained region.

![Figure 3](image)

**Figure 3.** Figure displays the mass-radius relation where the bag constant is set to \( B = 92 \text{MeV}/\text{fm}^3 \) (dashed lines) and \( B = 57 \text{MeV}/\text{fm}^3 \) (solid lines). The later represents the smallest value that the bag parameter can take. The parameters \( a_4 = 0.7 \) and the GB coupling \( \alpha \) take several values. The horizontal bands show the observational constraints from various pulsar measurements: J0348+0432 (green) \[45\], J1903+0327 (blue) \[56\], B1913+16 (black) \[77\] and J1141-6545 (orange) \[78\].

We also further investigate the maximum values of the QS mass (in the solar mass unit) and its radius (km unit) from the TOV equation in the 4D EGB gravity with the interacting quark EoS. The numerical results can be represented in the contour plots for all possible bag constant values and the range of the GB coupling \(-5 \text{km}^2 \leq \alpha \leq 5 \text{km}^2\) with three values of \( a_4 = 0.4, 0.7 \) and 0.9 and they are displayed in Fig.(4-6). The results show that the GB gravity coupling \( \alpha \) plays an important role for enhancing or reducing
Figure 4. Figure shows the maximum masses (upper panel) and their corresponding radii (lower panel) for values of $-5 \leq \alpha \leq 5$ and $57 \text{ MeV/fm}^3 \leq B \leq 92 \text{ MeV/fm}^3$. We have considered a particular value of the fourth-order-corrected parameter $a_4 = 0.4$. The white lines are equipped masses and radii lines.

Figure 5. Maximum masses and their corresponding radii have been plotted. Same as of Fig. 4 for $a_4 = 0.7$.

the maximum mass of the QS masses as well as the radii with respect to the relative signs of the GB coupling. While, the enhancement of the bag constant reduces the masses and radii of the QSs. In addition, according to the results in the anisotropic QS case, Ref. [79] has speculated that more interacting quarks lead to less values of the maximum masses, and vice versa. We observe that our results are also compatible with the speculation in Ref. [79].
Figure 6. Maximum masses and their corresponding radii have been plotted. Same as of Fig. 4 for \( a_4 = 0.9 \).

5 Structural properties of strange stars

For completeness, we would also like to explore the physical properties in the interior of the fluid sphere.

Figure 7. Plots for adiabatic index, \( \gamma \), of the stars using the interacting EoS. The parameters are the same as used in Fig. 1.

5.1 The stability criterion and the adiabatic indices

We begin our consideration of stability in stars by examining adiabatic index (\( \gamma \)) based on our EoS concerning quark matter models. Since, the adiabatic index is a basic ingredient of the instability criterion, and is related to the thermodynamical quantity. This method was introduced by Chandrasekhar [80] for dynamical stability based on the variational method. For an adiabatic perturbation, the adiabatic index, which appears in the stability formula,
as described by the equation [80, 81]

$$\gamma \equiv \left(1 + \frac{\epsilon}{P}\right) \left(\frac{dP}{d\epsilon}\right)_S,$$

(5.1)

where $dP/d\epsilon$ is the speed of sound in units of speed of light and the subscript $S$ indicates the derivation at constant entropy. Note that the above equation is a dimensionless quantity measuring the stiffness of the EoS.

In general, the EoS related to neutron star matter, $\gamma$ lies between 2 to 4 [82]. The analysis in [83] shows that adiabatic index on the instability conditions are also applicable to describe compact objects including white dwarfs, neutron stars and supermassive stars. Since the value of $\gamma$ should exceed $4/3$ for relativistic polytropes depending on the ratio $\epsilon/P$ at the centre of the star [84]. In support of $\gamma > 4/3$, authors in [85] have found for stability of an extended cluster with $\rho_c/\rho_0 \ll 1$ in Newtonian gravity. Finally, our results are shown in Fig. 7, where we plot $\gamma$ as a function of radius. From Fig. 7, the resulting $\gamma > 4/3 \sim 1.33$ shows that our model is stable against the radial adiabatic infinitesimal perturbations and increasing values of $\gamma$ mean the growth of pressure for a given increase in energy density, i.e. a stiffer EoS.

5.2 Compactness and Binding energy

Our next step is to calculated the emission produced by the photons from the star surface through the gravitational redshift [82]

$$Z_{\text{surf}} = (1 - r_g/R)^{-1/2} - 1,$$

(5.2)

where $r_g = 2GM/c^2$, and $R$ is the radius of the star. From this point of view, compactness $2MG/Rc^2$ leads to the redshift value for the given EoS (3.1). For clarity, we display the
We summarize the parameters of the quark stars using various values of the 4D EGB coupling constant, $\alpha$. We show the maximum mass of the stars $M$ in a unit of the solar mass $M_\odot$ with their radius $R$ in km and the central energy density $\epsilon_c$.

| $\alpha$ (km$^2$) | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) | $\epsilon_c$ (MeV/fm$^3$) | $v_s/c$ | $B_{\text{bind}}^{\text{max}}$ ($M_{\text{max}}$) |
|-----------------|-----------------|-----------|-----------------|-------|-----------------|
| $-5.0$          | $1.697$         | $10.551$  | $2 \times 10^3$ | $0.546$ | $0.163$         |
| $0$             | $1.900$         | $10.913$  | $2 \times 10^3$ | $0.546$ | $0.182$         |
| $5.0$           | $2.105$         | $11.256$  | $2 \times 10^3$ | $0.546$ | $0.202$         |

| $\alpha$ (km$^2$) | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) | $\epsilon_c$ (MeV/fm$^3$) | $v_s/c$ | $B_{\text{bind}}^{\text{max}}$ ($M_{\text{max}}$) |
|-----------------|-----------------|-----------|-----------------|-------|-----------------|
| $-5.0$          | $1.261$         | $8.219$   | $3.20 \times 10^3$ | $0.552$ | $0.155$         |
| $0$             | $1.515$         | $8.683$   | $3.20 \times 10^3$ | $0.552$ | $0.174$         |
| $5.0$           | $1.773$         | $9.107$   | $3.20 \times 10^3$ | $0.552$ | $0.210$         |

Table 1. We summarize the parameters of the quark stars using various values of the 4D EGB coupling constant, $\alpha$. We show the maximum mass of the stars $M$ in a unit of the solar mass $M_\odot$ with their radius $R$ in km and the central energy density $\epsilon_c$.

The compactness parameter $r_g/R$, where $r_g = 2GM/c^2$ in Fig. 8 for particular values of the bag parameter $B = 57$ and $92$ MeV/fm$^3$ with different GB coupling constant $\alpha = 0, \pm 5$ km$^2$.

As pointed in Ref. [82], there exists a universal relation between the total binding energy and the stellar mass of the neutron star. The binding energy ($B_{\text{bind}}$) of a stable neutron star correlates with its gravitational mass. A more precise formula of the binding energy, containing the compactness parameter $\beta = r_g/R$, was proposed by Lattimer & Prakash [46]. It is formulated via the following relation:

$$B_{\text{bind}} \simeq 1.6 \times 10^{53} \left( \frac{M}{M_\odot} \right) \left( \frac{\beta}{0.3} \right) \frac{1}{1 - 0.25\beta} \text{erg.}$$  \hspace{1cm} (5.3)

In terms of the radius dependence, it is given by

$$\frac{B_{\text{bind}}}{M} \simeq \frac{0.298 \beta}{1 - 0.25\beta}.$$  \hspace{1cm} (5.4)

An approximated value of $B_{\text{bind}} = B_{\text{bind}}^{\text{max}}$ for $M = M_{\text{max}}$ is shown in the last column of Table 1 for its corresponding radius, $r = R$.

6 Conclusion

In this article, we have theoretically constructed the ultra-dense compact objects called ‘neutron stars’. The recent discovery of pulsars by radio telescopes and X-ray satellites has imposed restrictions on the EoS that need to describe matter inside compact objects. Here, we represent the so-called quark stars by considering quark matter EoS in the context of recently proposed 4D Einstein-Gauss-Bonnet gravity. As mentioned in the introduction that regularized 4D EGB gravity has nontrivial dynamics and free from the Ostrogradsky instability.
Astronomical observations in favour of the possible existence of compact stars are partially or totally made up of quark matter. But the existence of quark stars is still controversial and its EoS is also uncertain. Here, we have solved the TOV equation in the 4D EGB gravity with the interacting quark EoS. To be more specific, we have studied millisecond pulsars modelled as quark stars with interacting quark EoS. As the results, several solutions of the mass-radius relation are compatible with four pulsar constraints of the NS mass with upper and lower limits of the bag constant values. We obtained the two solar mass NSs in the 4D EGB gravity with the lowest value of the bag constant while it is not possible for the standard GR theory. More importantly, increase and decrease of the QS masses are controlled by the plus and minus signs of the GB coupling, $\alpha$, respectively where as the enhancement of the bag constant reduces the NS masses. In addition, we further investigate the maximum values of the masses and radii of the QSs by varying the GB gravity coupling and the bag constant with the three values of the $a_4$. We found that more interacting quarks reduce the maximum mass of the QSs, and vice versa. On one hand, furthermore, the stability of the QSs can be achieved by the given values of the parameters in the theory. On the other hand, the binding energies of the QSs in 4D EGB gravity are calculated and they provided reasonable values for the observation data.

In the conclusion, the novel 4D EGB gravity provided a good result in the analysis of the mass of the QSs. In particular for the two solar mass of the observed NSs, it has been well known that it is difficult to obtain the two solar mass of the NSs in the standard GR gravity with several models of the EoS. However, the appearance of the 4D EGB gravity come to rescue for this problem. The mass of the NSs or QSs can be increased or decreased depending on the magnitudes of the higher order gravity coupling as shown in this work. To gain more and deeper understanding of the compact objects in the framework of the 4D EGB gravity, an extended analysis is worth for further study related to astrophysical observables, for instances, gravitational waves signal from binary system, pulsar timing array, accretion disk analysis of the NSs and etc. We leave them for further investigation.

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References

[1] D. Lovelock, J. Math. Phys. 12, 498 (1971).
[2] D. Lovelock, J. Math. Phys. 13, 874 (1972).
[3] B. Zumino, Phys. Rep. 137, 109 (1986).
[4] B. Zwiebach, Phys. Lett. B156, 315 (1985).
[5] C. Lanczos, Annals Math. 39 842 (1938).
[6] D. L. Wiltshire, Phys. Lett. B 169, 36 (1986).
[7] D.G. Boulware and S. Deser, Phys. Rev. Lett. 55, 2656 (1985).
[8] J.T. Wheeler, Nucl. Phys. B 268, 737 (1986).
[9] D. Glavan and C. Lin, Phys. Rev. Lett. 124, 081301 (2020).
[10] Y. Tomozawa, arXiv:1107.1424 [gr-qc].
[11] S. G. Ghosh and R. Kumar, arXiv:2003.12291 [gr-qc].
[12] R. A. Konoplya and A. Zhidenko, arXiv:2003.12492 [gr-qc].
[13] A. Kumar and R. Kumar, arXiv:2003.13104 [gr-qc].
[14] A. Kumar and S. G. Ghosh, arXiv:2004.01131 [gr-qc].
[15] C. Y. Zhang, S. J. Zhang, P. C. Li and M. Guo, arXiv:2004.03141 [gr-qc].
[16] C. Liu, T. Zhu and Q. Wu, arXiv:2004.01662 [gr-qc].
[17] P. Liu, C. Niu and C.-Y. Zhang, arXiv:2004.10620v1 [gr-qc].
[18] P. Liu, C. Niu and C.-Y. Zhang, arXiv:2005.01507v1 [gr-qc].
[19] S.-W. Wei, Y.-X. Liu, arXiv:2003.07769v2 [gr-qc].
[20] X. X. Zeng, H. Q. Zhang and H. Zhang, arXiv:2004.12074 [gr-qc].
[21] M. Guo and P. C. Li, arXiv:2003.02523 [gr-qc].
[22] S. U. Islam, R. Kumar and S. G. Ghosh, arXiv:2004.01038 [gr-qc].
[23] R. Kumar, S. U. Islam and S. G. Ghosh, arXiv:2004.12970 [gr-qc].
[24] M. Heydari-Fard, M. Heydari-Fard and H. R. Sepangi, arXiv:2004.02140 [gr-qc].
[25] X. H. Jin, Y. X. Gao and D. J. Liu, arXiv:2004.02261 [gr-qc].
[26] Y.-P. Zhang, S.-W. Wei and Y.-X. Liu, arxiv:2003.10960v2 [gr-qc].
[27] S. Ali and H. Mansoori, arXiv:2003.13382v1 [gr-qc].
[28] C. Y. Zhang, P. C. Li and M. Guo, arXiv:2003.13068 [hep-th].
[29] R. A. Konoplya and A. F. Zinhailo, arXiv:2004.02248 [gr-qc].
[30] M. S. Churlilova, arXiv:2004.00513 [gr-qc].
[31] A. K. Mishra, arXiv:2004.01243 [gr-qc].
[32] A. Aragon, Ramon Becar, P. A. Gonzalez, and Yerko Vasquez, arXiv:2004.05632v2 [gr-qc].
[33] K. Jusufi, A. Banerjee and S. G. Ghosh, arXiv:2004.10750 [gr-qc].
[34] P. Liu, C. Niu, X. Wang and C.-Y. Zhang, arXiv:2004.14267v2 [gr-qc].
[35] A. Banerjee and K. N. Singh, arXiv:2005.04028 [gr-qc].
[36] A. Banerjee, T. Tungphati and P. Chanmuie, arXiv:2006.00479 [gr-qc].
[37] D. Samart and P. Chanmuie, arXiv:2005.02826 [gr-qc].
[38] K. Jusufi, arXiv:2006.03606 [gr-qc].
[39] K. Yang, B. M. Gu, S. W. Wei and Y. X. Liu, arXiv:2004.14468 [gr-qc].
[40] S.-J. Yang, J.-J. Wan, J. Chen, J. Yang and Y.-Q. Wang, arXiv:2004.07934v1 [gr-qc].
[41] L. Ma and H. Lu, arXiv:2004.14738 [gr-qc].
[42] S. E. Woosley, A. Heger, and T. A. Weaver Rev. Mod. Phys. 74, 1015 (2002).
[43] A. Heger, C. L. Fryer, S. E. Woosley, N. Langer and D. H. Hartmann, Astrophys. J. 591, 288 (2003).
[44] P. Demorest, T. Pennucci, S. Ransom, M. Roberts and J. Hessels, Nature 467, 1081 (2010).
[45] J. Antoniadis, et al Science 340, 6131 (2013).
[46] J. Lattimer and M. Prakash, Astrophys. J. 550, 426 (2001).
[47] N. B. Zhang and B. A. Li, Astrophys. J. 879, 99 (2019).
[48] A. R. Bodmer Phys. Rev. D 4, 1601 (1971).
[49] E. Witten Phys. Rev. D 30, 272 (1984).
[50] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf Phys. Rev. D 9, 3471 (1974).
[51] A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn Phys. Rev. D 10, 2599 (1974).
[52] A. Peshier, B. Kampfer, and G. Soff Phys. Rev. C 61, 045203 (2000).
[53] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537, 443 (1999).
[54] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B 422, 247 (1998).
[55] J. Asbell and P. Jaikumar, J. Phys. Conf. Ser. 861, 012029 (2017).
[56] P. C. C. Freire, et al. Mon.Not.Roy.Astron.Soc. 412, 2763 (2011).
[57] T. Guver, F. Ozel, A. Cabrera-Lavers and P. Wroblewski, Astrophys. J. 712, 964 (2010).
[58] S. G. Ghosh and S. D. Maharaj, arXiv:2003.09841 [gr-qc].
[59] G. Cognola, R. Myrzakulov, L. Sebastiani and S. Zerbini, Phys. Rev. D 88, 024006 (2013).
[60] R. A. Hennigar, D. Kubiznak, R. B. Mann and C. Pollack, arXiv:2004.09472 [gr-qc].
[61] H. Lu and Y. Pang, arXiv:2003.11552 [gr-qc].
[62] A. Casalino, A. Colleaux, M. Rinaldi and S. Vicentini, arXiv:2003.07068 [gr-qc].
[63] J. M. Lattimer and M. Prakash, Phys. Rept. 621, 127 (2016).
[64] P. Demorest, T. Pennucci, S. Ransom, et al. Nature 467, 1081 (2010).
[65] E. Fonseca, T. T. Pennucci, J. A. Ellis, et al. ApJ, 832, 167 (2016).
[66] I. Bombaci, Phys. Rev. C 55, 1587-1590 (1997).
[67] X.-D. Li, Z.-G. Dai, and Z.-R. Wang, Astron. Astrophys. 303, L1 (1995).
[68] X. Li, I. Bombaci, M. Dey, J. Dey and E. van den Heuvel, Phys. Rev. Lett. 83, 3776 (1999).
[69] I. Bombaci, I. Parenti and I. Vidana, Astrophys. J. 614, 314 (2004).
[70] J. Staff, R. Ouyed and M. Bagchi, Astrophys. J. 667, 340 (2007).
[71] A. Chodos, R. Jaffe, K. Johnson, C. B. Thorn and V. Weisskopf, Phys. Rev. D 9, 3471 (1974).
[72] C. V. Flores, Z. B. Hall, II and P. Jaikumar, Phys. Rev. C 96, 065803 (2017).
[73] M. Alford, M. Braby, M. Paris and S. Reddy, Astrophys. J. 629, 969 (2005).
[74] J. Beringer et al. [Particle Data Group], Phys. Rev. D 86, 010001 (2012).
[75] D. Blaschke and N. Chamel, Astrophys. Space Sci. Libr. 457, 337 (2018).
[76] E. S. Fraga, R. D. Pisarski and J. Schaffner-Bielich, Phys. Rev. D 63, 121702 (2001).
[77] J. M. Weisberg and J. H. Taylor, ASP Conf. Ser. 328, 25 (2005).
[78] M. Bailes, S. M. Ord, H. S. Knight and A. W. Hotan, Astrophys. J. Lett. 595, L49 (2003).
[79] E. A. Becerra-Vergara, S. Mojica, F. D. Lora-Clavijo and A. Cruz-Osorio, Phys. Rev. D 100, 103006 (2019).
[80] S. Chandrasekhar, Astrophys. J. 140, 417 (1964).
[81] M. Merafina and R. Ruffini, Astron. Astrophys. 221, 4 (1989).
[82] P. Haensel, A. Y. Potekhin, and D. G. Yakovlev, Neutron Stars 1: Equation of State and Structure, Astrophysics and Space Science Library Vol. 326 (Springer-Verlag, New York, 2007).
[83] C. C. Moustakidis, Gen. Rel. Grav. 49, 68 (2017).
[84] E. N. Glass and A. Harpaz, Mon. Not. Roy. Astron. Soc., 202, 1, (1983).
[85] P. H. Chavanis Astron. Astrophys. 381, 709 (2002).