Integral Action in Sliding Mode Control for Reduction of Chattering in Speed Regulation of a Synchronous Motor

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Abstract. In this paper a sliding mode controller for speed regulation of a PMSM is evaluated. Here, the oscillation attenuation due to the nonlinear discontinuous control action is addressed by means of low-pass filters based on iterated integrals, where the undesirable chattering problem is considered as high-frequency noise.

1. Introduction

Recently, the permanent magnet synchronous motor (PMSM) has achieved notoriety in industrial applications (e.g., electric vehicles, computer numeric control machines, industrial robots, etc.). Among its, the most relevant features are fast dynamic response, compact size, high power-density, high torque capability, and low Joules losses, which make it highly efficient. The PMSM performance can be affected during operation by the dynamic system nonlinearities, parametric variations, and load torque disturbances. Thus, the use of nonlinear control strategies is necessary to achieve high dynamic performance indicators.

Nowadays, sliding mode control (SMC) has become popular as an efficient and powerful strategy for controlling complex, nonlinear and high order dynamic systems. The main advantages over conventional controllers are low sensitivity to bounded disturbances and reasonable parametric variations. Therefore, an accurate mathematical model is unnecessary, decreasing in this fashion the complexity of state feedback control design, through order reduction and decoupling of the systems in smaller independent subsystems [1-3]. This scheme is based on a nonlinear discontinuous control action that leads the controlled system states to a specified sliding surface, in finite time. Once the system is on the sliding surface it becomes immune to uncertainties and disturbances [4].

In relay or switched systems, there exists an undesired oscillation or fluctuation that has a low amplitude and high frequency, named chattering [5, 6]. This harmful phenomenon is generated by the presence of unmodeled or parasitic dynamics (sensors, actuators, and data not included in the ideal model) and switching imperfections. Chattering is considered as an inherent feature of SMC systems.
and can be visualized as an infinite number of commutations in a finite time interval about the sliding manifold, which causes degradation in the performance of the systems, e.g., high heat losses in electrical power circuits, high wear of mechanical parts and low control accuracy [3]. The chattering is a major obstacle in implementation of SMC, this must be solved to improve the control performance.

Recent works are focused on eliminating and reducing the chattering effects [7-13]. In the boundary layer scheme [3, 7, 8] a saturation function is used to approximate and replace the discontinuous sign function, which allows the SMC design to be used as a continuous control, due to the trajectories are adjusted to a short proximity of the sliding manifold, where the method has a drawback that the width of the boundary layer leads to lower control accuracy and an increase in error. One way for counteracting the chattering phenomenon is to reduce the gain magnitude $M$ of the discontinuous control while maintaining the sliding mode, because this magnitude is proportional to the amplitude of the chattering [3, 5, 6]. In the method "State-dependent control" $M$ is selected as a function of the system states, decreasing along with the vector of the states. Similarly, in the "Equivalent control dependent gain" approach, there exists a dependence of $M$ related to the equivalent control $U_{eq}$. Here, the gain magnitude decreases meanwhile the slide mode occurs along the sliding surface, the gain is composed of a positive constant part and a variable part that represents the mean value of the sign function, in these methods significantly reduce the amplitude of the control input, so the control effort decreases as the controlled state is stabilized, leading to chattering at a lower level, but in some cases are not applicable since control system actuators are power converters with a finite number of output voltage values. In [13-17] state observers are used for attenuating the chattering. The asymptotic observer acts as a bypass for high-frequency components, where the discontinuous control is the observer input and the state estimation is the output. Observers based controllers use estimated states instead of real measures directly from the plant, in order to exclude unmodeled dynamics. Although the control action becomes continuous, it is a function of discontinuous time, where a sliding mode can be forced. The use of state observers represents an important effort in the control design since the parameters of the plant must be known, so the observation strategy is adequate. Finally, higher order sliding modes (HOSM) [10, 15] is able to deal with the chattering while maintaining the features of the sliding mode. In this control scheme, the discontinuous control effect appears in the upper derivatives of the sliding variable. Nevertheless, the HOSMC exhibits limitations in the implementation, since it depends on detailed system information (i.e., the state measurements used in the design of the slide surfaces) and requires complex calculations. The most widely used HOSMC is the Second Order Sliding Mode Control (SOSMC) due to their simple structure and low dependence of the system information [17-19].

In this work, the use of low-pass filters based on iterated integrals in a SMC is evaluated to reduce the negative effect caused by the chattering phenomenon in the speed regulation of a PMSM.

2. Mathematical model of PMSM
The PMSM is a nonlinear system, strong coupling between its mechanical and electrical variables. For the sake of simplicity on tasks of speed regulation, the Field Oriented Control (FOC) can be employed, in order to attain an independent control of flux and torque. This is possible due to this strategy is based on transformations between frame $abc$ and the $dq$ reference frames, which allows a dynamic motor decoupling. The mathematical model of the PMSM in the $dq$ reference frame is given as follows

\[
\frac{di_d}{dt} = \frac{R_s}{L_d} i_d + \frac{P L_i}{L_d} i_q \omega + \frac{1}{L_d} u_d
\]  
(1)

\[
\frac{di_q}{dt} = \frac{R_s}{L_q} i_q - \frac{P L_i}{L_q} i_d \omega - \frac{P L_m}{L_q} \omega + \frac{1}{L_q} u_q
\]  
(2)

\[
\frac{d\omega}{dt} = \frac{1.5 P L_m}{J} i_q - \frac{b}{J} \omega - \frac{1}{J} T_i
\]  
(3)
where $i_d$, $i_q$, $u_d$ and $u_q$, are $dq$ components of the current and the voltage in the stator respectively, $R_s$ is the stator resistance, $P$ is the number of pole pairs, $\lambda_m$ is the flow generated by the permanent magnet of the rotor, $L_d$ and $L_q$ are the $dq$ components of the stator winding inductance, $b$ is a viscous damping coefficient, $J$ is the inertia moment and $T_l$ is the load torque. In case the PMSM is surface mounted permanent magnet type, $L_q$ is equal $L_d$.

3. Controller design

This work evaluates the elimination or attenuation of chattering in speed regulation tasks of a PMSM, adding integral terms to SMC, chattering is approached from the focus of high-frequency noise. Chattering can be considered as a high-frequency oscillation. That conforms to the definition given by Fliess in [20], where the noise is described as rapid fluctuations around zero. The non-standard formalization of fast oscillations and fluctuations in [21] provides a mathematical framework that does not require the usual statistical analysis of the effect of noise, allowing the introduction of noise and its averages in a totally deterministic framework. A Lebesgue-integrable real-time function $f$ is said to have fast oscillations or fluctuations, if and only if, the value of integral $\int_{t_0}^{t} F d\tau$ over any finite interval is infinitesimal. High-frequency noise can be considered as an unstructured disturbance, which can be attenuated by using iterated integrals that function as a low-pass filter.

Speed tracking is divided into two parts, the inner loop contains the control for the currents $dq$, in this part exist two independents second-order sliding mode controllers one for each current, this loop provides control of the system’s electrical variables. The second part is the outer loop, this fixed the reference to the current that must follow the inner loop. The target currents are calculated using velocity desired, speed is regulated by the proportional and integral controller. The outer loop control the mechanics part, therefore its dynamics must be slower than the inner loop. To control the currents, two sliding surfaces are designed, so that the following sliding surface is chosen

$$S_d = e_d + \alpha_d e_d$$  \hspace{1cm} (4)

$$S_q = e_q + \alpha_q e_q$$  \hspace{1cm} (5)

An integral term is added to the sliding surface, which means that the trajectories of the system begin on the same sliding surface, causing the reach phase to be completely eliminated. The error in the currents is defined as the difference between the actual value and the reference value

$$e_d = i_d - i_d^*$$  \hspace{1cm} (6)

$$e_q = i_q - i_q^*$$  \hspace{1cm} (7)

Starting from the desired speed, the reference values of the $i_q$ current are found, it is relevant to know that the reference for the $i_d$ current must remain at zero. Therefore $i_q^*$ is given by

$$i_q^* = \frac{1}{\gamma_\omega} \left( \nu_\omega + \frac{b}{J} \alpha \right)$$  \hspace{1cm} (8)

where $\gamma_\omega$ and $\nu_\omega$ are

$$\gamma_\omega = \frac{1.5 P \lambda_m}{J}$$  \hspace{1cm} (9)

$$\nu_\omega = \omega^* - \alpha_\omega e_\omega - \alpha_0 \int_0^t e_\omega dt$$  \hspace{1cm} (10)

Therefore, the controllers are

$$u_q = L_q \nu_q - \nu_q$$  \hspace{1cm} (11)
\[ u_d = L_d \nu_d - \nu_{d_f} \]  
(12)

where

\[ \nu_{q_f} = -R_i q - P L_d i_d \omega - P \lambda_{m} \omega \]  
(13)

\[ \nu_{d_f} = -R_i q + P L_q i_q \omega \]  
(14)

The control variable to be applied to the system is obtained by integrating the following equations (15-16), the resulting control is continuous, smooth and free of the harmful high-frequency chattering

\[ \dot{\nu}_q = -W_q \text{sign}(S_q) + \varphi_q \]  
(15)

\[ \dot{\nu}_d = -W_d \text{sign}(S_d) \]  
(16)

where

\[ \varphi_q = -e_q a_q + \frac{d^2 i^*_q}{dt^2} \]  
(17)

The control objective is for the sliding mode to converge to zero in a finite time so that both the error and the derived error must be equal to zero. The resulting control acts as a disturbance compensator and preserves the invariant property of the sliding mode.

4. Simulation results

Some simulation results are presented verifying the proposed control scheme performance. In this assessment, a suitable attenuation of the chattering phenomenon effects is achieved by means of introduced iterated integrals in SMC. Moreover, a proper control for the PMSM current and speed is observed. The parameters of the PMSM surface mounted permanent magnet type are considered, its values can be summarized in table 1.

| Parameter                              | Value     |
|----------------------------------------|-----------|
| Number of pole pairs                   | 2         |
| Stator resistance                      | 2.6Ω      |
| Flux linkage (rotor)                   | 0.319Wb   |
| d-axis inductance                      | 6.73e-3H  |
| q-axis inductance                      | 6.73 e-3H |
| Rotational inertia                     | 3.5e-5kg.m² |
| Viscous friction coefficient           | 0.0005Nm.s/rad |

The simulation is performed considering the following conditions: the behavior of the load torque is defined by \( T_l = 2^* (1 - e^{-3t}) \), the reference speed is given by a Bézier polynomial \( \psi \) for providing a sufficiently smooth transfer between the actual and desired speed reference values, within a specific time interval. Then, the reference trajectory is as follows

\[ \omega^* = \begin{cases} 
\omega_1 + (\omega_2 - \omega_1) \psi(t, T_1, T_2) & \text{for } T_1 \leq t \leq T_2 \\
\omega_1 & \text{for } T_2 < t \leq T_3 \\
\omega_1 + (\omega_2 - \omega_1) \psi(t, T_3, T_4) & \text{for } T_3 < t \leq T_4 \\
\omega_2 & \text{for } t > T_4 
\end{cases} \]

where \( \omega_1 = 190 \text{ rad/sec}, \omega_2 = 140 \text{ rad/sec}, T_1 = 0 \text{ s}, T_2 = 2.7 \text{ s}, T_3 = 6 \text{ s} \) and , \( T_4 = 8.5 \text{ s} \). The gains for the PI controller and the sliding surface of the two current control loops are: \( \alpha_1 = 200, \alpha_0 = 25, \)
\[ \alpha_q = 150 \text{ and } \alpha_d = 1. \] With an initial speed condition equal to 5 rad/s and a current on the d axis of 0.01 A.

In figure 1 the current response on the q-axis is portrayed. It can be seen that although a small error persists in a stationary state, the reference is followed quickly. The behavior of the current on d axis is presented in the figure. Notice, the current convergence to zero in finite time. Figure 3 shows the electromagnetic torque generated by the motor, which complies with the load torque behavior established for the simulation. Finally, in figure 4 the speed profile shows how the desired speed is reached smoothly. The graphs above show the behavior of the PMSM system variables proving the attenuation of the chattering phenomenon.

5. Conclusions
An integral action in Sliding Mode Control technique is developed for the current control of a PMSM. The integral action was used to remove the chattering phenomenon in the dynamical tracking for desired velocity profiles. The proposed method ensures that the speed and current tracking errors converge to zero asymptotically. The simulation results based on Matlab/Simulink prove that the proposed algorithm can diminish the chattering and improve the performance of the PMSM control system such as fast response, high robustness and precise tracking speed.
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