Local Discrimination of Quantum Measurement without Assistance of Classical Information

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Abstract
The discrimination of quantum operations is an important subject of quantum information processes. For the local distinction, existing researches pointed out that, since any operation performed on a quantum system must be compatible with no-signaling constraint, local discrimination between quantum operations of two spacelike separated parties cannot be realized. We found that, however, local discrimination of quantum measurements may be not restricted by the no-signaling if more multi-qubit entanglement and selective measurements were employed. In this paper, we report that local quantum measurement discrimination (LQMD) can be completed via selective projective measurements and numerous seven-qubit GHZ states without help of classical communication if both two observers agreed in advance that one of them should measure her/his qubits before an appointed time. As an application, it is shown that the teleportation can be completed via the LQMD without classical information. This means that the superluminal communication can be realized by using the LQMD.

Keywords
Local Quantum Measurement Discrimination, Multi-Qubit Entanglement Systems, Superluminal Communication

1. Introduction
Quantum state discrimination is an important subject in quantum information theory [1]. It is well-known that two pure states cannot be perfectly discriminated unless they are orthogonal. An object closely related to quantum state discrimination is the discrimination of quantum operation, including unitary operations, quantum
channels, and quantum measurements, etc. Quantum measurement is an inevitable part of all quantum information processing tasks. The purpose of discriminating quantum measurements is to find out the identity of an unknown device secretly chosen from two known quantum measurements. Therefore, studying the distinguishability of quantum measurements is an interesting and important task. Ji et al. [2] have shown that perfect discrimination between two different projective quantum measurements is possible with finite number of uses of the measuring apparatus. Since then, various kinds of quantum measurement discrimination (QMD) schemes [2]-[5] have been put forward. It is easy found that, up to now, employing the local operations and classical communication is necessary in all the previous works (e.g. [2]-[5]) for discrimination of quantum measurements. For two spacelike separated parties, however, it has been shown theoretically [6]-[9] that since the operators at distance commute, the averages of the observable at distant site remain the same and do not depend on the operations employed by the other distant party, i.e., the no-signaling constraint [10] holds that entanglement cannot be used for transmission of information without help of classical communication. This constraint says that, consider two observers, Alice and Bob, who perform measurements in separated locations, the marginal probabilities for Alice’s observations of results for her measurements are independent of Bob’s choice of measurement setting. For example, assume that Alice and Bob have a bipartite quantum system in a known state $\rho$, they perform local quantum projection operators, with elements $\sum_{\mu} A_{\mu} A_{\mu} = I$, $\sum_{\nu} B_{\nu} B_{\nu} = I$ and $[A_{\mu}, B_{\nu}] = 0$, on the subsystem respectively, where $A_{\mu}$ and $B_{\nu}$ are the “detector operator” associated to the elements of POVM (positive operator valued measurement) for the observation of results $\mu$ by Alice and $\nu$ by Bob. If Bob is not informed that Alice got result $\mu$, the probability that he gets $\nu$ is $p_{\nu} = tr\left(\sum_{\nu} B_{\nu} \rho B_{\nu}^{\dagger}\right)$. This result does not depend on Alice’s operations, Bob cannot decide what measurements Alice did [11].

By a careful analysis, however, it may be found that, for multi-qubit entanglement systems shared by Alice and Bob, if Alice only announced publicly that she had completed her measurement and did not declare the result of her measurement after her operations, the local discrimination of quantum measurements can be completed without help of classical information. In this paper, we present a theoretical scheme for local quantum measurement discrimination (LQMD) by using selective projective measurement with numerous seven-qubit Greenberger-Horne-Zeilinger (GHZ) states. It is shown that, in this scheme, if both two observers (Alice and Bob) agreed in advance that one of them (e.g. Alice) should measure her qubits before an appointed time (it is equivalent that, after her measurement, Alice only announced publicly that she had completed the measurement, and did not declare the result of her measurement), the local discrimination of two different kinds of measurement can be completed by using a series of single-qubit correlative measuring basis without help of classical communication, that is to say, our LQMD scheme may be not restricted by the no-signaling. As an application, we discuss the teleportation [12] of an unknown single-qubit state via the LQMD, and show that the unknown original state can be recovered without help of classical information. This means that the superluminal communication can be realized by using the LQMD scheme.

The paper is organized as follows. In Section 2, two different kinds of projective quantum measurements with a seven-qubit GHZ state are described. In Section 3, we present an explicit scheme for local discrimination between two different kinds of quantum measurements. In Section 4, quantum teleportation via the LQMD scheme is discussed. In Section 5, the obtained results are summarized.

## 2. Two Different Kinds of Projective Measurement

In order to present our ideas more clearly, let us first review the quantum measurement with entanglement system. Suppose Alice and Bob share an entangled pair of qubits in the state $(|01\rangle + |10\rangle)/\sqrt{2}$. Let Alice measure her qubit $A$ in the computational basis, the post-measurement states will be $|01\rangle$ with probability 1/2, and $|10\rangle$ with probability 1/2. We now consider two cases as follows. In the first case, after her measurement, Alice did not tell Bob anything. In this case, the reduced density operator of Bob’s system should be $\rho^B = 1/2$, thus Bob cannot know what measurements Alice did, and he cannot even tell if she had measured or not. In the second case, after her measurement, Alice only told Bob she had completed the measurement by the classical channel, and did not tell the result of her measurement. In this case, after Alice’s measurement, Bob can measure his qubit $B$ under the computational basis and the result of his measurement must be in the state corresponded to Alice’s outcome of measurement. As mentioned above, the viewpoint we emphasized is that, in the
second case, since Bob knows that Alice had completed the measurement by her announcement, he can affirm that the qubit $B$ must be collapsed into the state corresponded to Alice’s result of measurement, although he did not know Alice’s result of measurement. Our scheme is just based on the viewpoint.

Now let us consider two observers, Alice and Bob, who share a seven-qubit GHZ state

$$|G⟩ = \frac{1}{\sqrt{2}}(|0000000⟩ + |1111111⟩)_{A_1A_2A_3A_4A_5A_6B}$$  \hspace{1cm} (1)$$

where qubits $A_1, A_2, \cdots, A_6$ belong to Alice and $B$ to Bob, respectively. Assume that Alice and Bob agreed in advance that Alice should measure her qubits before an appointed time. Now, let Alice employ two different kinds of measurement on the state $|G⟩$. In the first kind of measurement, Alice performs in turn common projective measurements (CPMs) on the qubits $A_1, A_2, \cdots, A_6$ under the basis $\{|+,−⟩\}$, where

$$|+⟩ = \frac{1}{\sqrt{2}}(|0⟩ + |1⟩), \quad |−⟩ = \frac{1}{\sqrt{2}}(|0⟩ − |1⟩).$$

It is easy found that, after Alice’s measurements, 64 possible final collapsed states of the qubit $B$ will always be $\frac{1}{8}|+⟩_B$ or $\frac{1}{8}|−⟩_B$. Now let us turn to the second kind of measurement. To complete the LQMD, Alice will employ a novel kind of projective measurements, which we refer to as selective projective measurements (SPMs), with a series of single-qubit correlative measuring basis, on her qubits. Firstly, Alice measures the qubit $A_1$ in the state $|G⟩$ under the basis $\{|ν⟩, |ν⊥⟩\}$, where

$$|ν⟩ = x|0⟩ + y|1⟩, \quad |ν⊥⟩ = y|0⟩ − x|1⟩, \quad x \text{ and } y \text{ are real}, \quad x^2 + y^2 = 1, \quad \text{and let } x = \sqrt{6}/3, \quad y = \sqrt{3}/3.$$ If the result of Alice’s measurement is $|ν⟩_{A_1}$, the qubits $A_2, A_3, \cdots, A_6$ and $B$ will be collapsed into the state

$$|φ⟩ = \frac{1}{\sqrt{2}}(x|0000000⟩ + y|1111111⟩)_{A_1A_2A_3A_4A_5A_6B}.$$  \hspace{1cm} (2)$$

she can measure the qubits $A_2, A_3, \cdots, A_6$ under the basis $\{|ν⟩, |ν⊥⟩\}$, successively. After that, the qubit $B$ will be in the state $\frac{1}{2\sqrt{2}}|ν⟩_B$ or $\frac{1}{2\sqrt{2}}|ν⊥⟩_B$, here $|ν⟩ = \frac{1}{\sqrt{2}}(|0⟩ + |1⟩)$ and $|ν⊥⟩ = \frac{1}{\sqrt{2}}(|0⟩ − |1⟩)$. If Alice’s measurement outcome is $|ν⟩_{A_1}$, the state of the qubits $A_2, A_3, \cdots, A_6$ and $B$ will be

$$|φ⟩ = \frac{1}{\sqrt{2}}(y|0000000⟩ − x|1111111⟩)_{A_1A_2A_3A_4A_5A_6B}.$$  \hspace{1cm} (3)$$

Then Alice measures the qubit $A_2$ under the basis $\{|λ⟩, |λ⊥⟩\}$, which is given by

$$|λ⟩ = \frac{1}{F_2}(x|0⟩ + y|1⟩), \quad |λ⊥⟩ = \frac{1}{F_2}(y|0⟩ − x|1⟩),$$  \hspace{1cm} (4)$$

where $F_2 = [(x/y)^2 + (y/x)^2]^{1/2}$. Corresponding to Alice’s measurement outcome $|λ⟩_{A_2}$ or $|λ⊥⟩_{A_2}$, the qubits $A_3, \cdots, A_6$ and $B$ will be collapsed into the state $|φ⟩$ or $|φ′⟩$ with probability 1/2 each, which are given by

$$|φ⟩ = \frac{1}{\sqrt{2}F_2}(x|0000000⟩ − y|1111111⟩)_{A_1A_2A_3A_4A_5A_6B},$$  \hspace{1cm} (5)$$

$$|φ′⟩ = \frac{1}{\sqrt{2}F_2}(y^2|0000000⟩ + x^2|1111111⟩)_{A_1A_2A_3A_4A_5A_6B}.$$

As mentioned above, one can easily see that the goal of our SPMs is as much as possible to make the qubit $B$ collapsed into the state $\frac{1}{R}|ν⟩$ or $\frac{1}{R}|ν⊥⟩$ after all, where $R$ is a constant or a coefficient related to $x$ and $y$.

To present the SPMs more clearly, by described above, we can give the general approach of the SPMs as follows:
After Alice measured her qubit $A_*$, if the qubits $A_{n+1}, A_{n+2}, \cdots, A_m$ and $B$ are collapsed into the state

$$\phi_* = \frac{1}{\sqrt{2T_n}} \left( \frac{y^{p_n}}{x^{p_n+1}} |00\cdots00\rangle + \frac{x^{p_n}}{y^{p_n+1}} |11\cdots11\rangle \right)_{A_{n+1}A_{n+2}\cdots A_mB}$$

where $n = 1, 2, \cdots, m$, $T_n = F_1F_2\cdots F_n$, $p_n = 2^{n-1}$, $F_n = \left[ (x/y)^{p_n} + (y/x)^{p_n} \right]^{1/2}$, and let $F_1 = 1$, Alice should employ a new single-qubit projective measurement on the qubit $A_{n+1}$ under the basis \{\ket{\lambda}, \ket{\lambda}^\perp\}, which is given by

$$|\lambda\rangle = \frac{1}{F_{n+1}} \left[ \left( \frac{x}{y} \right)^{p_n} |0\rangle + \left( \frac{y}{x} \right)^{p_n} |1\rangle \right], \quad |\lambda^\perp\rangle = \frac{1}{F_{n+1}} \left[ \left( \frac{x}{y} \right)^{p_n} |0\rangle - \left( \frac{y}{x} \right)^{p_n} |1\rangle \right].$$

If the result of Alice’s measurement is $|\lambda\rangle$, the qubits $A_{n+2}, A_{n+3}, \cdots, A_m$ and $B$ will be collapsed into the state $|\phi_{n+1}\rangle$, which is given by

$$|\phi_{n+1}\rangle = \frac{1}{\sqrt{2T_{n+1}}} \left( x|00\cdots00\rangle + y|11\cdots11\rangle \right)_{A_{n+2}A_{n+3}\cdots A_mB}$$

she can measure the qubits $A_{n+2}, A_{n+3}, \cdots, A_m$ under the basis \{\ket{+}, \ket{-}\} successively, the qubit $B$ will be collapsed into the state $|\mu\rangle$ or $|\mu^\perp\rangle$, here $d = (m-n-1)/2$. If the outcome of Alice’s measurement is $|\lambda^\perp\rangle$, the qubits $A_{n+2}, A_{n+3}, \cdots, A_m$ and $B$ will be collapsed into the state $|\phi_{n+1}\rangle$, which is given by

$$|\phi'_{n+1}\rangle = \frac{1}{\sqrt{2T_{n+1}}} \left( \frac{y^{p_n}}{x^{p_n+1}} |00\cdots00\rangle - \frac{x^{p_n}}{y^{p_n+1}} |11\cdots11\rangle \right)_{A_{n+2}A_{n+3}\cdots A_mB}$$

she should repeat above similar approach, until the result of measurement $|\lambda_{m-1}\rangle$ or $|\lambda_{m-1}^\perp\rangle$ in the basis \{\ket{\lambda_{m-1}}, \ket{\lambda_{m-1}}^\perp\} has been obtained, and the qubit $B$ has been collapsed into the state $|\mu^\perp\rangle_B$ or $\frac{1}{T_m} |\mu^\perp\rangle_B$ after all.

By above general approach, after Alice’s measurements, 64 possible final collapsed states of the qubit $B$ can be obtained. The relation of the results of Alice’s measurement and the possible final collapsed states of the qubit $B$ can be expressed as follows:

$$|\nu\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{4\sqrt{2}} |\mu^+\rangle_B$$ (32 terms)

$$|\lambda_{d_2}\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{4T_2} |\mu^+\rangle_B$$ (16 terms)

$$|\lambda_{d_3}\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{2\sqrt{2T_3}} |\mu^+\rangle_B$$ (8 terms)

$$|\lambda_{d_4}\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{2T_4} |\mu^+\rangle_B$$ (4 terms)

$$|\lambda_{d_5}\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{\sqrt{2T_5}} |\mu^+\rangle_B$$ (2 terms)

$$|\phi_{n+1}\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{T_6} |\mu^+\rangle_B$$ (1 term)

$$|\phi'_{n+1}\rangle_A \rightarrow |\psi^+\rangle_B = \frac{1}{2\sqrt{2T_6}} \left( \frac{y^{32}}{x^{32}} |0\rangle + \frac{x^{32}}{y^{32}} |1\rangle \right)_B$$ (1 term).
Thus much Alice’s selective measurements have been completed. From Equation (10), it is easy found that, after Alice performing the SPMs on her all qubits, the probability of the qubit B being in the state \( \frac{1}{g_n T_n} |\mu^k\rangle \) \( (g_n = 2^{(6-n)/2}, \ n = 1, 2, \cdots, 6) \) is \( \frac{63}{64} \).

Clearly, after Alice performing the CPMs or SPMs on her qubits respectively, the final collapsed states of the qubit B are obvious different. If Alice employs the CPMs on her qubits, after Alice’s measurement, 64 possible final collapsed states of the qubit B will always be always \( \frac{1}{8} |+\rangle_B \) or \( \frac{1}{8} |-\rangle_B \). If Alice performs the SPMs on her qubits, after Alice’s measurement, 64 possible final collapsed states of the qubit B can be given by Equation (10). It must be emphasized that, whether Alice’s measurements are the CPMs or SPMs, since Alice and Bob agreed in advance that Alice should measure her qubits before an appointed time, Bob can know that the qubit B must be collapsed into the state corresponded to one of Alice’s 64 results of measurement after Alice’s measurements.

### 3. Local Discrimination of Two Different Kinds of Measurement

Now let us turn to depict the LQMD. Suppose that two spacelike separated observers, Alice and Bob, share 30 seven-qubit GHZ states, which are given by

\[
G^{(k)} = \frac{1}{\sqrt{2}} \left( |0000000\rangle + |1111111\rangle \right) \mu^{(k)}, \quad \text{where} \quad k = 1, 2, \cdots, 30,
\]

and the qubits \( A_1^{(k)}, A_2^{(k)}, \cdots, A_6^{(k)} \) belong to Alice and \( B^{(k)} \) to Bob, respectively. Different from previous quantum operation discrimination schemes, we assume that there is no classical channel between Alice and Bob. In this case, before the agreed time \( t \), Alice randomly performs two different kinds of measurements, CPMs or SPMs, on her qubits in the state \( G^{(k)} \) \( (k = 1, 2, \cdots, 30) \) respectively. If Alice employs the CPMs on her qubits, after Alice’s measurements, all qubits \( B^{(k)} \) will be in the states \( \frac{1}{8} |+\rangle_{B^{(k)}} \) or \( \frac{1}{8} |-\rangle_{B^{(k)}} \). At the appointed time \( t \), Bob measures his qubits \( B^{(k)} \) all in the basis \( \{|0\rangle, |1\rangle\} \). After Bob’s measurements, by statistics theory, the probability of all qubits \( B^{(k)} \) in the state \( |0\rangle \) or \( |1\rangle \) will be in the ratio of one to one. If Alice’s measurements are the SPMs, by described above, after Alice’s selective measurements, the probability of all qubits \( B^{(k)} \) in the states \( \frac{1}{g_n T_n} |\mu^k\rangle \) or \( \frac{1}{g_n T_n} |\mu^k\rangle \) \( (g_n = 2^{(6-n)/2}, \ n = 1, 2, \cdots, 6) \) is

\[
\left( \frac{63}{64} \right)^{30} = 0.62, \ \text{i.e.}, \ \text{the probability of at least one qubit} \ B^{(k)} \ \text{in the state} \ |\psi^+\rangle \ \text{is} \ 1 - \left( \frac{63}{64} \right)^{30} \approx 0.38. \ \text{At the appointed time} \ t, \ \text{Bob measures the qubits} \ B^{(k)} \ \text{all in the basis} \ \{|0\rangle, |1\rangle\}. \ \text{One can see that, after measurements of Bob, in the 38\% cases, the probability of the qubits} \ B^{(k)} \ \text{in the state} \ |0\rangle \ \text{or} \ |1\rangle \ \text{will be different from the case Alice employed the CPMs. To illustrate this clearly, without loss of generality, we first discuss the case in which only one qubit} \ B^{(k)} \ \text{in the state} \ |\psi^+\rangle \ \text{after Alice’s measurements. From the state} \ |\psi^+\rangle \ \text{in Equation (10), it is easily found that, after Bob’s measurements, the probability of the qubit} \ B^{(k)} \ \text{in the state} \ |0\rangle \ \text{or} \ |1\rangle \ \text{will be in the ratio of one to} 2.5 \ \text{after Bob’s measurements. For general cases in which the qubit} \ B^{(k)} \ \text{in the state} \ |\psi^+\rangle \ \text{and other 29 qubits} \ B^{(k)} \ \text{collapsed randomly into the states} \ \frac{1}{g_n T_n} |\mu^k\rangle \ (g_n = 2^{(6-n)/2}, \ n = 1, 2, \cdots, 6) \ \text{after}
Alice’s measurements, it is easily found that the probability of the 30 qubits $B^{(i)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $w_{(i)}$ (where $w_{(i)} > 2.5$) after Bob’s measurements. Now we consider the case in which there are two qubits $B^{(i)}$ and $B^{(j)}$ in the state $|\psi^+_k\rangle$ after Alice’s measurements. Similar to the above described, one can find that the probability of the 30 qubits $B^{(i)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $w_{(i)}$ (where $w_{(i)} > 2.5$) after Bob’s measurements. For the cases in which more qubits $B^{(i)}, B^{(j)}, \ldots, B^{(l)}$ ($l = 3, 4, \ldots, 30$) collapsed into the state $|\psi^+_k\rangle$ after Alice’s measurements, the probability of the 30 qubits $B^{(i)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $w_{(i)}$ (where $w_{(i)} > 2.5$, $l = 3, 4, \ldots, 30$) after Bob’s measurements.

As mentioned above, after Alice’s measurements, in the cases in which at least one qubit $B^{(i)}$ in the state $|\psi^+_k\rangle$ (i.e., in the 38% cases), the probability of the 30 qubits $B^{(i)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $w_{(i)}$ (where $w_{(i)} > 2.5$).

To ensure the result of Bob’s measurements more reliable, it can be further supposed that Alice and Bob share 40 entangled states groups (ESGs), each consisting of 30 seven-qubit GHZ states $|G^{(i)}\rangle$ (see Equation (11)). If Alice’s measurements are the CPMS, it is easy found that, after Alice’s and Bob’s measurements, the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be still in the ratio of one to one. If Alice’s measurements are the SPMs, by statistics theory, after Alice’s and Bob’s measurements, in 15 ESGs the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one. If Alice’s measurements are the SPMs, after Bob’s measurements, in 15 of the 40 ESGs the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one. If Alice’s measurements are the CPMS, after Bob’s measurements, in 15 of the 40 ESGs the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one. If Alice’s measurements are the CPMS, after Bob’s measurements, in 15 of the 40 ESGs the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one. If Alice’s measurements are the CPMS, after Bob’s measurements, in 15 of the 40 ESGs the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one.

As described above, one can see that, in this scheme, at the appointed time $t$, Bob should measure his qubits $B^{(i)}$ all in the basis $\{0\}, \{1\}\}$. If Alice employs the CPMS on her qubits, after Bob’s measurements, the probability of all qubits $B^{(i)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one. If Alice’s measurements are the SPMs, after Bob’s measurements, in 15 of the 40 ESGs the probability of all qubits $B^{(i)}$ of each ESG in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one. In accordance with these outcomes, Bob can discriminate that the measurements performed by Alice are CPMS or SPMs. Thus, the LQMD is completed successfully.

It should be pointed out that, in the present LQMD scheme, Bob did not obtain Alice’s quantum information, i.e., if Alice’s measurements are SPMs, Bob couldn’t have learned the coefficients $x$ and $y$ in the measuring basis performed by Alice since he is not informed that Alice got result of measurement. In fact, Bob doesn’t need to know Alice’s quantum information (e.g. the coefficients $x$ and $y$). As mentioned above, after his measurements, Bob can determine that the measurements performed by Alice are CPMS or SPMs respectively. In accordance with these outcomes, Bob can discriminate that the measurements performed by Alice are CPMS or SPMs respectively without assistance of classical communication.  

4. Applications: Quantum Teleportation via the LQMD

As an example of application, we present a quantum teleportation protocol (QTP) by using the LQMD. Suppose that Alice wants to teleport an unknown arbitrary single-qubit state $|\xi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob, with $|\alpha|^2 + |\beta|^2 = 1$, and the state shared by Alice and Bob as the quantum channel is an EPR pair $|\xi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$, where qubit 2 belongs to Alice and 3 to Bob. The state of whole system can be expressed as

$$|\Xi\rangle = \frac{1}{2} \left[ |\Phi^+\rangle_{12} |\xi\rangle_3 + |\Phi^-\rangle_{12} \sigma_x |\xi\rangle_3 + |\Psi^+\rangle_{12} \sigma_y |\xi\rangle_3 + |\Psi^-\rangle_{12} (-i\sigma_x) |\xi\rangle_3 \right]$$

(12)

where $|\Phi^+\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Phi^-\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are four Bell states, and $\sigma_i$ ($i = x, y, z$) is the Pauli matrix. Different from previous protocols of teleportation, we suppose that there is no classical channel between Alice and Bob, instead, another quantum channel, called informing quantum channel (IQC), which consists of two encoding-decoding sets (EDSs), either EDS is composed of 40 ESGs and each ESG consisting of 30 seven-qubit GHZ states, which are given by

$$|G^{(i)}\rangle = \frac{1}{\sqrt{2}} \left( |0000000\rangle \pm |1111111\rangle \right)_{\sigma_{(i)}, \sigma_{(i)}},$$

(13)
where $i = 1, 2, \ j = 1, 2, \cdots, 40, \text{ and } k = 1, 2, \cdots, 30, \text{ and qubits } A_{(i,j)}^{(k)}, A_{(i,j)}^{(k)}, \cdots, A_{(i,j)}^{(k)} \text{ belong to Alice and } B_{(i,j)}^{(k)} \text{ to Bob, respectively. To complete the teleportation, Alice and Bob should make three agreements beforehand as follows:}

1) The informing messages 0 and 1 are represented by two different kinds of measurement CPMs and SPMs respectively.

2) The four Bell states $|\Phi^+\rangle, |\Psi^+\rangle, |\Psi^-\rangle$ and $|\Phi^-\rangle$ are represented by the informing messages 00, 01, 10 and 11 respectively.

3) Before an agreed time $T$, Alice can perform a Bell-state measurement on her qubits 1 and 2, and then according to her result of measurement, she should encode the messages of her result of measurement on the IQC. At the agreed time $T$, Bob should check Alice’s informing messages in the IQC.

Our QTP can be expressed as follows. Before the agreed time $T$, Alice performs a Bell-state measurement on her qubits 1 and 2. Without loss of generality, assume her result of measurement is $|\Psi^\prime\rangle_{12}$. Then Alice encodes her informing messages 01 on the IQC, i.e., she makes the CPMs on the set $G_{1,j}^{(k)}$ and the SPMs on the set $G_{2,j}^{(k)}$, successively. At the agreed time $T$, Bob should check the states of qubits $B_{(i,j)}^{(k)} (i = 1, 2)$, that is, he measures in turn his qubits $B_{(i,j)}^{(k)}$ and $B_{(i,j)}^{(k)}$ all in the basis $\{ |0\rangle, |1\rangle \}$. As described above, after that, Bob can extract that Alice employed the CPMs on the set $G_{1,j}^{(k)}$ and the SPMs on the set $G_{2,j}^{(k)}$, this means that Alice’s informing massages are 01 and the result of Bell-state measurement is $|\Psi^\prime\rangle_{12}$. By Equation (12), Bob knows that the qubit 3 has been collapsed into the state $\frac{1}{2}(\beta |0\rangle + \alpha |1\rangle)$, then he can perform $\sigma_x$ on his qubit 3 and the original state $|\xi\rangle$ can be recovered. Thus, the teleportation is completed successfully.

Compared with previous teleportation protocols, in our QTP there is no classical channel. It is just because of this, the speed of teleporting quantum information is no longer limited by the speed of light, but it depends on the speed of quantum state collapse (or speed of quantum information) [13]. In recent years, the results of some EPR experiments [13]-[16] set a lower bound on this speed of $10^5 - 10^7$ times the speed of light. Obviously, in our QTP, if Alice and Bob are spaced far enough, the required time completing the quantum information transmission (including the time completed all measurements by Alice and Bob) will be less than the required time by the classical communication. In other words, the speed of teleporting an unknown quantum state in the QTP will be faster than the speed of light. It must be pointed out that, however, our QTP is not in contradiction with the special relativity, because, in the present scheme of LQMD, only multipartite entanglement systems were used and none of real material objects were transmitted between Alice and Bob.

5. Conclusion

In conclusion, we have proposed a theoretical scheme for local discrimination of two different kinds of measurement by using selective measurement and numerous seven-qubit GHZ states. To realize the scheme, a series of single-qubit correlative measuring basis has been employed. It is shown that, in this scheme, if both two observers agreed in advance that one of them (e.g. Alice) should measure her qubits before an appointed time, local discrimination of quantum measurement can be completed successfully without help of classical communication. Compared with previous LQMD protocol [11], the advantage of the present scheme is that our LQMD may be not restricted by the no-signaling, i.e., the entanglement can be used for transmission of information without assistance of classical communication. As an application, we have shown that the teleportation of an arbitrary single-qubit state can be completed via the present LQMD scheme. Compared with previous teleportation protocols, the advantage of our QTP is that it does not require help of classical information. This means that the superluminal communication can be realized by using the present LQMD scheme. Obviously, our LQMD scheme can also be applied to other quantum communication protocols such as remote state preparation [17], quantum secret sharing [18], controlled remote operation [19], and so on. Furthermore, we should emphasize that our work has been completed in the framework of standard quantum mechanics. So far there has been experiment implementing the eight-qubit GHZ state [20], hence, we hope our work can be experimentally realized in the near future and stimulate further research on quantum communication and quantum information processing.
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