Power Output Prediction for LM Wind Turbine Blade using Blade Element Momentum Theory and GH Bladed Software

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Abstract. The blade element momentum theory with Prandtl’s tip loss and Glauert’s correction factors was utilized to compute the power coefficient and to predict the power output of LM rotor blade as a function of hub wind speed ranging from 3 m/s to 25 m/s. The blade length is 43.8 m and consists of five (5) different airfoils. The design tip speed ratio is 8.65 suitable for Class IIA wind turbine which can generate a capacity of 2.5 MW at rated speed of 16 rpm using permanent magnet direct-drive wind turbine generator. The thrust force and driving force profiles in terms of dimensionless blade length as well as the power coefficient and predicted power output were examined and compared with the theoretical equations derived from GH Bladed. Numerical results indicate that there are some degrees of similarities with GH Bladed software output having a maximum power coefficient of 0.49.

1. Introduction

Several investigators used blade element momentum (BEM) theory either to identify the airfoil characteristics from experimental data on horizontal axis wind turbine (HAWT) based on systematic approach or directly determine the wind turbine power output of a particular blade with known airfoil details. Bak et al. used BEM concept to investigate and to derive the airfoil characteristics using four different methods such as inverse momentum theory, actuator disc theory, numerical optimization and quasi-3D CFD computations [1]. The results of aeroelastic calculations indicate that good agreement was formed for power measurements and flap moments derived by actuator disc theory and inverse BEM method on a 41-m full scale rotor with LM19.1 blades. Manwell et al. proposed two solution methods that determine the flow conditions and forces at each blade section with twist angle at the blade tip approaches zero degree [2]. The first method calculates the lift coefficient and axial induction factor based on the measured airfoil characteristics and BEM equations. The second method is an iterative approach that computes the flow conditions and the induction factors. Ingram used the first method proposed by Manwell et al. to establish a wind turbine blade design and provided a step-by-step procedure in calculating the wind turbine power output with a blade length of 5 m using NACA 0012 airfoil [3].

The present study however, discussed the calculation method for power coefficient and power output prediction of LM wind turbine blade using the blade element momentum theory with Prandtl’s tip loss and Glauert’s correction factors. The results of computation were compared with the solutions derived from GH Bladed software [4]. To compare the numerical results of two BEM concepts, the power coefficient and the predicted power output of LM wind turbine blade are evaluated. GH Bladed is a commercial software that is widely used in the wind turbine industry which simulates the
performance of wind turbine for optimizing its design by satisfying different design load cases according to international standards such as IEC, DNV-GL, etc. The two BEM calculation methods were completed using spreadsheet with user defined-function in macro platform of MS Excel. In this study, the LM blade length is 43.8 m with a 1.3 m distance from hub centre to rotor shaft flange. The rotor blade consisted of five (5) different airfoils, hereafter called LM43.8_40, LM43.8_21.5, LM43.8_21, LM43.8_18 and LM43.8_15 with airfoil section distribution of 25.11%, 22.83%, 4.57%, 41.10% and 6.39%, respectively. The blade design tip speed ratio is 8.65 suitable for Class IIA horizontal axis wind turbine capable to generate 2.5 MW at rated rotor speed of 16 rpm using a permanent magnet direct-drive wind turbine generator. Figure 1 illustrates the LM43.8 blade chord profile as a function of dimensionless blade length. The LM43.8 blade twist angle profile in terms of dimensionless blade length is also shown in figure 2.

![Figure 1. LM43.8 blade chord profile as a function of dimensionless blade length.](image1)

![Figure 2. LM43.8 blade twist angle profile in terms of dimensionless blade length.](image2)

### 2. Momentum Theory

The momentum theory was adopted in analyzing the horizontal axis wind turbine blade that originated from the axial momentum concept by Rankine [5]. To understand the concept, the governing equations used are the following: mass density identification, mass conservation, and axial momentum.

#### 2.1. Identification of mass density

The fluid static concept is primarily used to determine the mass density of air. As the hub height of permanent-magnet direct-drive wind turbine generator is about 80 m from the ground, it is important that the mass density is properly calculated to predict the wind turbine power output. Combining the pressure variation of static fluid to vertical position, the ideal gas equation of state, and assuming the temperature is decreasing linearly with elevation yields the air mass density ($\rho$) as a function of pressure ($p_0$), temperature ($T_0$), and elevation difference ($z - z_0$) as shown in equation (1).

$$\rho = \frac{p_0[T_0 - \beta(z - z_0)](\gamma/\beta R)}{R T_0^{\gamma/\beta R}}$$

(1)

#### 2.2. Mass conservation

The differential form of mass conservation or continuity equation can be expressed as

$$0 = \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{q}$$

(2)

where, $\mathbf{q}$ is the velocity vector. For any arbitrary control volume and considering air as an incompressible fluid (i.e., for $M < 0.3$) at steady-state condition, the above equation yields

$$\rho \nabla \cdot \mathbf{q} = 0.$$
Using mass conservation between upstream and downstream parts of wind turbine gives

\[ \rho AV = \rho A_T V_T = \rho A' V' \]  

(4)

where, A and V represent the surface area and mean velocity, respectively. The subscript T denotes the turbine blade. The control volume of stream tube around the wind turbine is shown in figure 3.

2.3. Momentum Equation

Applying the momentum equation along the horizontal line of a control volume determines the thrust force and by combining this with equation (4) the equation reduced to

\[ F_{\text{thrust}} = \rho A_T V_T (V - V'). \]  

(5)

The thrust force can also be defined as the difference of pressure between the front and rear portions of the rotor which is given by

\[ F_{\text{thrust}} = (p_b - p_c)A_T. \]  

(6)

However, using Bernoulli’s concept the pressure difference can also be expressed as

\[ (p_b - p_c) = \frac{\rho}{2} (V^2 - V'^2). \]  

(7)

Combining equations (5), (6) and (7) but eliminating the thrust force and pressure difference yields

\[ V_T = \frac{1}{2} (V + V'). \]  

(8)

The axial induction factor \((a)\) indicates the degree by which the fluid velocity in the upstream part of rotor decreases as it approaches the turbine. The fluid velocity at the rotor will be

\[ V_T = V(1 - a) \quad \text{and} \quad V' = V(1 - 2a). \]  

(9)

Substituting equation (9) to equation (5) and differentiating the reduced equation shows the thrust force of an annular element which can be expressed in terms of axial induction factor as

\[ \frac{dF_{\text{thrust}}}{dr} = 4\pi \rho a(1 - a) V^2 r dr. \]  

(10)

On the other hand, the tangential induction factor denoted as \((a')\) is the ratio of induced tangential angular flow velocity and rotor angular velocity \((\Omega)\) which can be written as
\[ a' = \frac{\omega}{2\Omega} \]  

(11)

Knowing equation (11) and considering the conservation of angular momentum in an annular stream tube, the torque on the annular element gives

\[ dT = 4a'(1-a)\rho V\Omega \pi r^3 \, dr. \]  

(12)

### 3. Blade Element Theory

In blade element theory, the rotor blade is divided into strips or elements in the span wise direction. The drag and lift forces developed by the element are calculated and unified over the total blade length. With known components of velocity, helps to predict the torque and power generated by the blade. Considering the number of blades could determine the overall torque and subsequently compute the rotor power. Figure 4 illustrates the airfoil profile showing the elemental lift and drag forces as well as the velocity components. The upstream velocity \( V \) decreases to \( V(1-a) \) as it reaches the rotor. A velocity of \( \Omega r (1+a') \) occurs due to blades rotation and the amount of wake that generates behind the rotor. \( W \) denotes the apparent wind velocity or resultant of these two velocities. \( \alpha \) and \( \beta \) are the angle of attack and blade setting angle, respectively. The sum of these two angles is \( \phi \) commonly known as the flow angle given by

\[ \phi = \tan^{-1}\left[ \frac{V(1-a)}{\Omega r (1+a')} \right] \]  

(13)

The elemental lift force and drag force can be expressed as follows:

\[ dL = \frac{1}{2} \rho c C_L W^2 \, dr \quad \text{and} \quad dD = \frac{1}{2} \rho c C_D W^2 \, dr \]  

(14)

where, \( c \), \( C_L \) and \( C_D \) are the chord length, lift coefficient, and drag coefficient, respectively. The thrust force, driving force, and torque evolved by the elemental blade length \( dr \) positioned at a radius of \( r \) are given by

\[ dF_{\text{thrust}} = \frac{1}{2} \rho c W^2 \,(C_L \cos \phi + C_D \sin \phi)\, dr \]  

(15)

\[ dF_{\text{driving}} = \frac{1}{2} \rho c W^2 \,(C_L \sin \phi - C_D \cos \phi)\, dr \]  

(16)

\[ dT = \frac{1}{2} \rho c W^2 \,(C_L \sin \phi - C_D \cos \phi)\, r \, dr. \]  

(17)

If the number of blades (B) will also be considered in the equation, then the total thrust force and total torque would be

\[ dF_T = \frac{1}{2} \rho c BW^2 \,(C_L \cos \phi + C_D \sin \phi)\, dr \]  

(18)
\[ \text{dT}_r = \frac{1}{2} \rho c B W^2 (C_L \sin \phi - C_D \cos \phi) r \, dr. \]  

4. Blade Element Momentum Theory by DNV/Risø

The blade element momentum concept is a combination of axial momentum and blade element theory that contributes more understanding to the design parameters of wind turbine. Combining equations (10) and (18) but eliminating the elemental thrust force and considering the apparent wind velocity be in the form of flow angle, the new axial induction factor gives

\[ a = \frac{1}{4 \sin^2 \phi \left[ \sigma_r (C_L \cos \phi + C_D \sin \phi) \right] + 1} \quad \text{and} \quad \sigma_r = \frac{Bc}{2nr} \]  

where, \( \sigma_r \) is the local solidity factor. Concurrently, combining equations (12) and (19) but eliminating the elemental torque and considering the apparent wind velocity be in terms of flow angle, the new tangential induction factor yields

\[ a' = \frac{1}{4 \sin \phi \cos \phi \left[ \sigma_r (C_L \sin \phi - C_D \cos \phi) \right] + 1} \]  

With Prandtl’s tip loss factor (F) as defined in equation (22), the axial and tangential induction factors become

\[ F = \frac{2}{\pi} \cos^{-1} \left\{ \exp \left[ \frac{B}{2} \frac{R-r}{r \sin \phi} \right] \right\} \]  

\[ a = \frac{1}{4 F \sin^2 \phi \left[ \sigma_r (C_L \cos \phi + C_D \sin \phi) \right] + 1} \quad \text{and} \quad a' = \frac{1}{4 F \sin \phi \cos \phi \left[ \sigma_r (C_L \sin \phi - C_D \cos \phi) \right] + 1} \]  

When the axial induction factor is greater than \( a_c \approx 0.2 \), then the Glauert’s correction can be applied as shown in equation (24). This is used to properly calculate the induced velocities for small wind speeds [6].

\[ a = \frac{1}{2} \left\{ 2 + K(1 - 2a_c) - \sqrt{[2 + K(1 - 2a_c)]^2 + 4[Ka_c^2 - 1]} \right\} \quad \text{and} \quad K = \frac{4 F \sin^2 \phi}{\sigma_r (C_L \cos \phi + C_D \sin \phi)} \]  

5. Blade Element Momentum Theory by GH Bladed

In GH Bladed, the annular axial and tangential induction factors may be expressed as

\[ a = \frac{1}{4 F \sin^2 \phi \left[ \sigma_r (C_L \cos \phi + C_D \sin \phi) \right] + 1} \quad \text{and} \quad a' = \frac{1}{4 F \sin \phi \cos \phi \left[ \sigma_r (C_L \sin \phi - C_D \cos \phi) \right] + 1} \]  

The parameter \( H \) is defined as follows: (1) for \( a \leq 0.3539 \), then \( H \) is equal to unity, and (2) for \( a > 0.3539 \), then

\[ H = \frac{4a(1-a)}{(0.6+0.61a+0.79a^2)} \]  

When the axial induction factor is greater than 0.3539, then the denominator of \( H \) becomes the empirical equation for thrust coefficient [4]. The above equations for \( a \) and \( a' \) can be solved iteratively with both initial values of 0.0. The procedure requires to compute the flow angle, angle of attack, lift and drag coefficients, solidity factor and using the equations above to update \( a \) and \( a' \). This process continues until the axial and tangential induction factors have converged satisfying the tolerance value of \( 1.0 \times 10^{-6} \). Numerical integration such as trapezoidal rule was used to compute the total torque and by multiplying it with angular velocity yields the power output. The power coefficient was obtained by getting the ratio of rotor power output with the available kinetic energy of upstream wind speed. Figure 5 illustrates the logical structure of computation for flow angle, lift, drag, normal and torque coefficients as well as induction factors, power coefficient and predicted power output of LM wind turbine blade using the BEM concept derived from GH Bladed.
6. Discussion of Results

The axial and tangential induction factors based on blade element momentum theory proposed by DNV/Ris0 were compared with the equations derived from GH Bladed. The input parameters are mass density of 1.225 kg/m$^3$, upstream velocity of 11.26 m/s, rotor speed of 16 rpm and aerodynamic blade profiles of LM43.8. The thrust force and driving force profiles were generated and plotted for comparison as shown in figure 6 and figure 7, respectively. Statistical results indicate that no significant changes have been made for both figures having correlation coefficients equal to unity.

![Figure 5. Logical structure of calculation method for airfoil characteristics and power output using BEM Theory from GH Bladed.](image)

**Figure 5.** Logical structure of calculation method for airfoil characteristics and power output using BEM Theory from GH Bladed.

**Figure 6.** Elemental thrust force profiles in terms of dimensionless blade length of LM43.8 based on DNV/Ris0 and GH Bladed.

**Figure 7.** Elemental driving force profiles in terms of dimensionless blade length of LM43.8 based on DNV/Ris0 and GH Bladed.
However, when the predicted rotor power output and power coefficient were computed and compared based on DNV/Risø and GH Bladed, Student t-tests dictate otherwise with correlation coefficients of 0.99991 and 0.93227, respectively. Values of power output and power coefficient became identical for larger wind speeds above 13.0 m/s as shown in figure 8 and figure 9. But the discrepancies can be seen at lower wind speeds due to the presence of hub loss factor in GH Bladed that takes place at the blade root in which the bound circulation must fall to zero. The hub loss factor is denoted as $F_h = 1/H$.

![Figure 8. Power output of LM43.8 blade at different wind speeds without pitch control.](image)

![Figure 9. Power coefficient of LM43.8 blade as a function of hub wind speed.](image)

### 7. Conclusions
The blade element momentum theory with Prandtl’s tip loss and Glauert’s correction factors was compared with the theoretical equations derived from GH Bladed. With upstream velocity of 11.26 m/s and rated rotational speed of 16 rpm, the calculated rated power output based on DNV/Risø and GH Bladed yields 2.516 MW and 2.501 MW, respectively. The elemental thrust force and driving force profiles as a function of dimensionless blade length of LM43.8 using the BEM concept derived from DNV/Risø and GH Bladed were generated and compared. Statistical results indicate that there are some degrees of similarities with correlation coefficients equal to unity.

However, it was found that the power output and power coefficient of these two BEM concepts became identical only for hub wind speeds larger than 13.0 m/s. The differences can be seen at lower wind speeds due to the presence of hub loss factor in GH Bladed that takes place at the blade root where the bound circulation approaches zero. Maximum power coefficient of 0.49 was obtained when the hub wind speed is 9.0 m/s using the theoretical equations derived from GH Bladed.

### References

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