A Quantitative Analysis of the Available Multicolor Photometry for Rapidly Pulsating Hot B Subdwarfs

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ABSTRACT

We present a quantitative and homogeneous analysis of the broadband multicolor photometric data sets gathered so far on rapidly pulsating hot B subdwarf stars. This concerns seven distinct data sets related to six different stars. Our analysis is carried out within the theoretical framework developed by Randall et al., which includes full nonadiabatic effects. The goal of this analysis is partial mode identification, i.e., the determination of the degree index $l$ of each of the observed pulsation modes. We assume possible values of $l$ from 0 to 5 in our calculations. For each target star, we compute a specific model atmosphere and a specific pulsation model using estimates of the atmospheric parameters coming from time-averaged optical spectroscopy. For every assumed value of $l$, we use a formal $\chi^2$ approach to model the observed amplitude-wavelength distribution of each mode, and we compute a quality-of-fit $Q$ probability to quantify the derived fit and to discriminate objectively between the various solutions. We find that no completely convincing and unambiguous $l$ identification is possible on the basis of the available data, although partial mode discrimination has been reached for 25 out of the 41 modes studied. A brief statistical study of these results suggests that a majority of the modes must have $l$ values of 0, 1, and 2, but also that modes with $l = 4$ could very well be present while modes with $l = 3$ appear to be rarer. This is in line with recent results showing that $l = 4$ modes in rapidly pulsating B subdwarfs have a higher visibility in the optical domain than modes with $l = 3$. Although somewhat disappointing in terms of mode discrimination,
our results still suggest that the full potential of multicolor photometry for $l$ identification in pulsating subdwarfs is within reach. It will be a matter of gathering higher S/N ratio observations than has been done up to now.

Subject headings: stars: horizontal-branch — stars: interiors — stars: oscillations — subdwarfs

1. INTRODUCTION

It is well established that an analysis of multicolor photometric data, whereby one compares oscillation amplitudes and/or phase differences in two or more wavebands, can lead to partial mode identification in pulsating stars (see, e.g., Heynderickx, Waenkens, & Smeyers 1994). The technique has been used successfully in the past to infer the values of the degree index $l$ of pulsation modes for many types of oscillating stars. To name just a few, let us mention that it has been applied to $\delta$ Scuti stars (Garrido, Garícia-Lobo, & Rodriguez 1990), $\beta$ Cepheids (Cugier, Dziembowski, & Pamyatnykh 1994), ZZ Ceti white dwarfs (Fontaine et al. 1996), $\gamma$ Doradus stars (Breger et al. 1997), and Slowly Pulsating main sequence B stars (Dupret et al. 2003). Attempts to understand the observed period spectrum on the basis of multicolor photometry have also been made for EC 14026 stars, first by Koen (1998) and more recently by Jeffery et al. (2004; 2005). The former study gives a qualitative interpretation of the periods observed for KPD 2109+4401 based on the theory of Watson (1988), while the latter assert to have provisionally identified or constrained the $l$ values for some of the modes detected for the fast oscillators KPD 2109+4401, HS 0039+4302, and PG 0014+067. In these cases, the $l$ identification is based on a qualitative comparison with amplitude ratios computed by Ramachandran, Jeffery, & Townsend (2004) in the adiabatic approximation for representative models.

A theoretical framework for the quantitative exploitation of multicolor photometry for pulsating sdB stars has been put forward recently by some of us (Randall et al. 2005). In that paper, the potential of the technique as applied to both types of pulsating sdB stars (the short-period $p$-mode pulsators of the EC 14026 type and the slowly oscillating $g$-mode variables of the PG 1716 type) has been explored in detail. The method of Randall et al. (2005) features a full nonadiabatic treatment of the atmospheric layers and uses a designated model atmosphere code which automatically incorporates the wavelength dependence of the limb darkening — thus avoiding the need for approximate parameterized limb darkening coefficients as used in most other multicolor photometric studies with the notable exception of Ramachandran et al. (2004). Of central importance, the method of Randall et al. (2005) has led to the first (and, so far, only) unambiguous determination of the $l$ index of a pulsation
mode in a oscillating sdB star using multicolor photometry, thus demonstrating the feasibility of the approach.

Indeed, at the time of the writing of the Randall et al. paper, the best available amplitude estimates of oscillation modes in a pulsating EC 14026 star were those based on the superb $u'g'r'$ data set gathered by Jeffery et al. (2004) on KPD 2109+4401 ($V = 13.38$) with ULTRACAM mounted on the 4.2-m William Herschel Telescope (WHT). Considering the largest amplitude mode reported with a period of 182.42 s, and adopting at face value the three amplitudes and their quoted uncertainties ($u' = 8.87 \pm 0.04$ mmag, $g' = 6.58 \pm 0.04$ mmag, $r' = 6.15 \pm 0.04$ mmag), Randall et al. (2005) were able to demonstrate that this mode must be a radial mode ($l = 0$) as first indicated by Jeffery et al. (2004).

The quality of these unique observations has remained unsurpassed so far, but there are nevertheless other available multicolor data sets that certainly deserve to be analyzed in the same fashion now that there is a proven theoretical framework to do that. While it is known that it is generally difficult to discriminate between modes with $l = 0, 1, $ or 2 on the basis of multicolor photometry for EC 14026 stars (Ramachandran et al. 2004; Randall et al. 2005), one may hope to distinguish between modes of that group from modes with $l = 3$ or $l = 4$ or possibly $l = 5$. Our survey of the published work in the field has revealed the following available data sets 1) the $UBVR$ photometry of Koen (1998; hereafter K98) on KPD 2109+4401, 2) the $UBV$ photometry of Silvotti et al. (2000; hereafter S00) on PG 1628+563B, 3) the BUSCA photometry of Falter et al. (2003; hereafter F03) on PG 1605+072, 4) the $u'g'r'/ULTRACAM$ photometry of Jeffery et al. (2004; hereafter J04) on HS 0039+4309, 5) the $u'g'r'/ULTRACAM$ photometry of Jeffery et al. (2004) on KPD 2109+4401, 6) the Strömgren photometry of Oreiro et al. (2005; hereafter O05) on BAL 090100001, 7) the $UBVR$ photometry of Baran et al. (2005; hereafter B05) on BAL 090100001, 8) the $u'g'r'/ULTRACAM$ photometry of Jeffery et al. (2005; hereafter J05) on PG 0014+067, and 9) the $u'g'r'/ULTRACAM$ photometry of Aerts et al. (2006) on SDSS J171722.08+58055.8. Except for cases 5) and 9) (see below), we analyze all of the other data sets in the present paper.

Partial mode identification (the identification of the degree index $l$) through multicolor photometry is a worthwhile venture in itself, but it has become particularly important for EC 14026 pulsators now that specific asteroseismic models have been proposed for a few of them. Indeed, by combining the forward method with high S/N ratio spectroscopy, it has been possible to carry out complete asteroseismological analyses for four EC 14026 pulsators so far: PG 0014+067 (Brassard et al. 2001), PG 1047+003 (Charpinet et al. 2003), PG 1219+534 (Charpinet et al. 2005a), and Feige 48 (Charpinet et al. 2005b). The ultimate product of these analyses is the determination of the global structural properties of the stars,
but complete mode identification (i.e., a determination of the radial order $k$ and the degree index $l$ of each pulsation mode) is a byproduct of the method. Multicolor photometry can thus be used as testing grounds for the proposed seismic models by checking if the $l$ assignments are correct. Of the four EC 14026 pulsators with a proposed specific seismic model, only PG 0014+067 has been observed so far in several filters simultaneously (J05). We will discuss this interesting case below as part of our analysis of that data set.

The main purpose of the paper is to present a quantitative and homogeneous analysis based on the method of Randall et al. (2005) for each of the available multicolor data sets that we found in the published literature on pulsating EC 14026 stars. Considerable observational efforts went into the gathering of these data sets and we feel that it is most worthwhile to attempt extracting the maximum information from them. Partial mode identification is the goal. We note that the exact same approach has been used recently by Aerts et al. (2006) for analyzing their ULTRACAM/WHT data of the very faint EC 14026 pulsator SDSS J171722.08+58055.8 ($B \simeq 16.7$), so it is not necessary to reconsider their data here. Unfortunately, in that case, the identification of $l$ was not possible for the two pulsation modes uncovered because the uncertainties on the observed amplitudes turned out to be too large due to insufficient S/N ratio. In contrast, some interesting mode identifications appear possible for the Jeffery et al. (2004) data set on KPD 2109+4401 (beyond that for the 182.42 s mode discussed in Randall et al. 2005), but we defer our analysis to a separate publication that will include unpublished additional observations that we gathered at the Canada-France-Hawaii 3.6-m Telescope on that star. We note also that two-color ($U$ and $R$) photometry has been gathered and analyzed for two PG 1716 sdB pulsators in order to constrain the $l$ index (see the paper of Randall et al. 2006a on PG 1627+017 and that of Randall et al. 2006b on PG 1338+481).

2. METHOD

2.1. The computations of theoretical amplitudes

To analyze multicolor data for pulsating sdB stars, we follow the modeling method developed by Randall et al. (2005). While we refer the reader to that paper for more information, we include here a brief recap of the most relevant aspects for convenience. The technique employed is based on the well-known fact that the observed wavelength dependence of an oscillation’s amplitude and phase bears the signature of the mode’s degree index $l$ and its period, as well as the star’s atmospheric parameters, the intrinsic amplitude of the periodicity, and its viewing aspect. To first order, the influence of the latter two (unknown) parameters can be eliminated by computing the ratio of pulsational amplitudes (and the
difference between phases) as measured in two or more bandpasses. Given a target’s effective temperature and surface gravity, the observed mode’s degree index can thus, in principle, be inferred from multicolor photometry, leading to partial mode identification.

The method proposed by Randall et al. (2005) incorporates a full nonadiabatic description of the atmospheric layers in the computations of theoretical pulsation observables. As in main sequence stars (e.g., Dupret et al. 2003), such description is found to be quite important in sdB stars as well. In particular, while the predicted phase shifts between various bandpasses generally remain quite small — they would all be identical to zero in the adiabatic approximation — Randall et al. (2005) found that amplitude ratios cannot, in that approximation, be computed with enough accuracy for quantitative studies. Furthermore, they found that the amplitude ratios do depend sensitively on the atmospheric parameters of the target, so for the purposes of our present analysis, we need to compute a specific detailed model atmosphere as well as a full pulsation model for each one of our targets.

For each value of \( l \), the brightness variation expected across the visible disk during a pulsation cycle can be expressed in terms of temperature, radius, and surface gravity perturbations to the emergent flux. These in turn are dependent on quantities obtainable from model atmospheres and nonadiabatic pulsation theory, as well as on the period of the mode in question (see, e.g., equations 34 – 38 of Randall et al. 2005). The model atmosphere parameters are made up of the logarithmic derivative of the emergent flux with respect to the effective temperature / surface gravity and the weighted limb darkening integral together with its derivatives. Initially computed as monochromatic quantities (\( \alpha_{T \nu}, \alpha_{g\nu}, b_{\nu}, b_{\nu,T}, \) and \( b_{\nu,g} \) in the notation of Randall et al. 2005) from a specially modified sdB atmosphere code, they are subsequently integrated over the bandpasses of interest (the quantities \( \alpha_{T x}, \alpha_{g x}, b_{lx}, b_{lx,T}, \) and \( b_{lx,g} \), where \( x \) symbolizes the appropriate bandpass, again in the notation of Randall et al. 2005) to allow for comparison with observations.

The second set of parameters includes the nonadiabatic quantities \( R \) and \( \Psi_{T} \), which are related to an oscillation’s departure from adiabaticity in amplitude and phase respectively. Together with the adiabatic gradient, \( \nabla_{ad} \), they are computed on the basis of the so-called “second generation” static stellar models described in Charpinet et al. (2001). These full models are submitted to adiabatic and nonadiabatic pulsation calculations that yield, among other things, the relative behaviour of the radius (gravity) and temperature perturbations in the stellar atmosphere for each mode considered. Integrated over the atmospheric layers contributing most to the emergent flux (taken to lie at optical depths \( \tau = 0.1–10 \)), the ratio of the two perturbations’ moduli and their phase lag yield the atmosphere-averaged values of \( R \) and \( \Psi_{T} \) describing the departure from adiabaticity of the observed brightness variations for each mode. The atmospheric value of \( \nabla_{ad} \) is obtained in a similar way.
For each reported mode in each observed target, we calculate the predicted amplitude ratios and phase differences from the available bandpasses for values of the degree index from \( l = 0 \) to \( l = 5 \). Since, as it turns out, there is very little information content in the phase shifts (the predicted differences are indeed too small in all cases of interest to be detectable at the accuracy achieved in the available data sets), we concentrate solely in what follows on comparing predicted and observed amplitudes. To provide a quantitative framework for such a comparison, we contrast for each mode the predicted multicolor amplitudes with those observed using a \( \chi^2 \) minimization procedure following Fontaine et al. (1996). For every degree index \( l \), the theoretical amplitudes \( a_{\text{theo}}^i \) in each of the \( N \) available bandpasses \( i \) are multiplied by a scale factor \( f_l \) chosen in such a way as to minimize

\[
\chi^2(l) = \sum_{i=1}^{N} \left( \frac{f_l a_{\text{theo}}^i - a_{\text{obs}}^i}{\sigma^i} \right)^2 ,
\]

where \( a_{\text{obs}}^i \) is the amplitude observed in a given waveband and \( \sigma^i \) is the error on the measurement. Compared to the standard normalization of all amplitudes to one particular waveband, this is a more objective way of determining the quality of a match, since the data from all bandpasses are treated on the same footing.

Finally, since our procedure is a standard \( \chi^2 \) approach, it is possible, following Fontaine et al. (1996), to quantify the value of a given \( \chi^2 \) solution by computing explicitly the quality-of-fit \( Q \) as described in Press et al. (1986). The quantity \( Q \) depends on the value of \( \chi^2 \) for each solution and the number of degrees of freedom (\( N \) fitted points minus the free parameter \( f_l \) gives \( N - 1 \) degrees of freedom in the present case). We adopt the canonical notion suggested by Press et al. (1986) that a fit is acceptable if its quality-of-fit \( Q > 0.001 \).

For a system with a single degree of freedom, this value of \( Q \) corresponds to the case where the predicted amplitudes fall simultaneously in all bandpasses of interest within \( 3\sigma^i \). Hence, we shall use the criterion \( Q > 0.001 \) to determine if a given \( \chi^2 \) fit is acceptable and to discriminate quantitatively between possible \( l \) solutions.

### 2.2. The available data sets

The vital characteristics of each of the seven data sets used in this study are summarized in Table 1. The first column gives the name of the EC 14026 pulsator, the second one gives its \( V \) magnitude (\( B \) if \( V \) is unavailable), the third one lists the total length of the data set, the fourth column refers to the bandpasses of interest, the fifth one indicates the instrument used, and this is followed by the identification of the telescope used and its location, and finally a reference to the original paper is given.
In the calculations of theoretical amplitudes for each target star, it is necessary to specify the atmospheric parameters of the star, $\log g$ and $T_{\text{eff}}$. To do this, we rely on the recent and ongoing spectroscopic efforts of Green, Fontaine, & Chayer (2006, in preparation) to derive reliable and homogeneous atmospheric parameters for a large sample of sdB stars. For each of our target stars, we used estimates for the atmospheric parameters coming from that study. These are given in Table 2.

In addition, we indicate in the same table the three ingredients that we used in the convolution process transforming monochromatic quantities into waveband-integrated quantities suitable for comparison with the observational data. These include the transmission curve of each filter, the response curve of the detector, and the extinction curve of the site. Tests indicate that, by far, the transmission curves of the filters are the most important component, and that, at the other extreme, the choice of the extinction curve is a sophistication that has little impact for sites located at roughly the same altitude. In this connection, we picked the atmospheric transparency curve of a representative medium altitude site (in this case, Kitt Peak National Observatory), a choice that should be appropriate for all but perhaps Mt. Suhora Observatory in Poland located at an altitude of about 1000 m. Further tests indicate that the largest sources of uncertainty for the computed amplitudes are associated with the uncertainties on $T_{\text{eff}}$ and $\log g$, assuming that the filters used by the observers have indeed the claimed transmission properties. For typical uncertainties on the atmospheric parameters of $\Delta T_{\text{eff}} = \pm 400$ K and $\Delta \log g = \pm 0.05$ (this excludes possible systematic effects associated with the model atmospheres used by Green et al. 2006), the real effects on the theoretical amplitudes remain quite small and generally not significant enough to affect the implications for mode identification (given the observational uncertainties on the observed amplitudes).

For each data set, we generally accepted the data, i.e., the observed multicolor amplitudes and their associated uncertainties at face value. This implicitly implies that each reported mode was fully resolved and that its amplitudes were free of spectral contamination from neighboring peaks in the Fourier domain. This is an important issue: if a mode is not sufficiently “monochromatic” in frequency, attempts to model its $l$ signature as a function of wavelength are usually frustrated. One exception to this is the case of an unresolved frequency multiplet due to rotational splitting. In that case, since the amplitudes do not depend on the azimuthal order $m$, the modeling remains valid for such an unresolved multiplet in a slowly rotating star.

An equally important concern is the question of the uncertainties on the observed amplitudes. Since our approach is based on a $\chi^2$ statistics, it is absolutely essential to have realistic and accurate estimates of these uncertainties. The quality of the fit, as measured by
the quantity $Q$, and which gives the discriminatory power of the method, depends directly on the values of these uncertainties. We found, in this connection, that our failure sometimes to model in an acceptable way a given pulsation mode could be traced back to the fact that the errors on the amplitudes were probably underestimated.

3. RESULTS

3.1. KPD 2109+4401 (K98)

The paper of K98 is the first one reporting on multicolor photometry for a rapidly pulsating sdB star. It was a key source of motivation and inspiration for the theoretical work that we presented in Randall et al. (2005). In K98, the author reported, among other things, on some 27 h of $UBVR$ photometry on KPD 2109+4401 ($V = 13.38$) gathered with a 4-channel photometer mounted on the McDonald Observatory’s 2.1-m Struve Telescope. Seven distinct pulsation modes were uncovered, and $UBVR$ amplitudes and phases were provided for each one of them. The author, however, was clearly reluctant to provide estimates of the uncertainties on the amplitudes and phases, being quite aware, as he argued, that formal errors coming from least-squares fits of light curves tend to be unreliable. Reading between the lines, and using the information on the relative efficiency of each bandpass of the Steining photometer that we had from previous work (Fontaine et al. 1996), we established that reasonable estimates of the observational uncertainties on the amplitudes would be

\[ \sigma(U) = 0.105 \text{ mmag}, \sigma(B) = 0.105 \text{ mmag}, \sigma(V) = 0.085 \text{ mmag}, \text{ and } \sigma(R) = 0.155 \text{ mmag}, \]

irrespective of the amplitude of a given mode.

Table 3 summarizes the results of our modeling effort for the K98 data set.\(^1\) For each observed mode, the table gives a block of data. The first line in each block gives the period of the mode of interest, and this is followed by the observed amplitude and its uncertainty in each of the available bandpass. The next line gives the best-fit theoretical amplitude in each of the bandpass for an assumed degree index $l = 0$, and this is followed by the value of $\chi^2$ obtained in the minimization procedure (eq. 1), and the value of the quality-of-fit $Q$. The next five lines show similar data, but for assumed values of $l$ from 1 to 5, respectively.

Figure 1 is a graphical representation of our results for the 182.42 s mode (the first one listed in Table 3 and the second largest amplitude one in the K98 data). The

\(^{1}\text{Note that we explicitly used the transmission curves of the }UBVR\text{ filters of the Steining photometer as published in Robinson et al. (1995) for this analysis. These are similar but not exactly the same as the standard Johnson-Cousins filters.}\)
behavior of that mode is well modeled for values of $l = 0$, $l = 1$, or $l = 2$ as can be seen in the figure. The plot is consistent, of course, with the values of $Q$ all larger than 0.001 for those three possibilities, while it also clearly shows the poor fits ($Q \ll 0.001$; see Table 3) obtained for models assuming $l = 3$, $l = 4$, or $l = 5$. We thus find that, while the uncertainties on the observed amplitudes are too large for us to discriminate between the values of $l$ from 0 to 2, we can safely discard the possibilities that the 182.42 s mode in KPD 2109+4401 has a degree index of 3, 4, or 5. We note that this result is consistent with the determination of $l = 0$ proposed by Randall et al. (2005) for that mode on the basis of the the much higher S/N ratio data of Jeffery et al. (2004).

The highest amplitude mode in the K98 data is the 196.31 s pulsation which, interestingly enough but not uncommonly in sdB pulsators, showed smaller amplitudes than the 182.42 s mode when KPD 2109+4401 was observed by Jeffery et al. (2004). Our modeling of the K98 data shows a very similar situation to the previous mode as can be seen in Figure 2. Again, we cannot formally discriminate between $l = 0$, $l = 1$, or $l = 2$, but we can safely exclude the possibilities that the 196.31 s mode in KPD 2109+4401 has a value of $l = 3$, $l = 4$, or $l = 5$. The very small values of $Q$ associated with these solutions (Table 3) certainly confirm this conclusion.

The most interesting case in the K98 data is that of the 198.19 s mode (the third largest amplitude mode), which shows an amplitude-wavelength behavior characteristic of a $l = 4$ mode as can be appreciated in Figure 3. In fact, formally speaking, the $l = 4$ solution is the only one acceptable, with a value of $Q > 0.001$ in Table 3. While this remains highly suggestive, we caution that one should not jump too hastily to the conclusion that mode discrimination has been achieved beyond any doubt for that mode. The reason for our cautionary remark is that the next best fit, the one corresponding to $l = 2$ and $Q = 2.77 \times 10^{-4}$ in Table 3, does not show a quality-of-fit value that is much smaller than the passage criterion of $Q = 0.001$. For instance, if we were to increase the uncertainties on the observed amplitudes by some 10%, the solution with $l = 2$ would become formally acceptable. So here is a case where realistic and accurate estimates of the amplitude uncertainties become critical.

For the other modes uncovered in K98, we formally constrain the value of $l$ to 0, 1, 2, or 4 for the 184.72 s pulsation, to 2 or 4 for the 184.75 s pulsation, and to 1, 2, or 4 for the 191.85 s pulsation. No mode discrimination was possible for the lowest amplitude mode at 196.69 s, and all values of $l$ from 0 to 5 provide acceptable fits. We note that the provisional $l$ assignments made by Jeffery et al. (2004) on the basis of their ULTRACAM/WHT observations are consistent with our more quantitative results here for all modes, except perhaps for the 198.19 s pulsation for which we may have isolated a unique value at $l = 4$, although,
as discussed just above, the $l = 2$ solution (which is the suggestion of Jeffery et al. 2004 for that mode) should not be dismissed too hastily.

### 3.2. PG 1618+563B (S00)

Amplitude and phase data were reported for two modes (139.3 s and 143.9 s) observed in PG 1618+563B ($V = 13.52$) by S00. Those were derived from $UBV$ observations using the 3-channel Tromso photometer attached to the 2.5-m NOT Telescope. Unfortunately, only 1.5 h of data were gathered and, consequently, the S/N ratio was not large enough to allow $l$ index discrimination. Our results are summarized in Table 4 (the format is similar to that of Table 3) and clearly show that all possible $l$ values from 0 to 5 provide acceptable fits for both modes. Figure 4 gives a graphical representation of our results for the 139.3 s mode.

### 3.3. PG 1605+072 (F03)

The relatively bright ($V = 12.92$), large-amplitude sdB pulsator PG 1605+072 has been observed by F03 who provided 12.3 h of $BUSCA$ photometry, a 4-channel system allowing the simultaneous observations in bandpasses centered on 3600 Å($UV$), 4800 Å($B$), 6300 Å($R$), and 8000 Å($NIR$). These data were gathered at the 2.2-m telescope at Calar Alto. F03 isolated 11 modes in their light curves and provided measurements of amplitudes and phases in their Tables 2 and 3. However, only 5 of those modes have simultaneous measurements in all the 4 $BUSCA$ bandpasses. We have restricted our analysis to these 5 modes.

In our modeling effort, we were careful to convolve the transmission curves of the 4 filters with the response curve of the $BUSCA$ instrument as provided to us by O. Cordes (2005, private communication). We also point out that the estimates of the atmospheric parameters that we used for PG 1605+072 (see Table 2) are remarkably consistent with the independent values obtained by Heber, Reid, & Werner (1999), which are $T_{\text{eff}} = 32,300 \pm 300$ K and $\log g = 5.25 \pm 0.05$ dex.

Despite careful modeling, we were unable to reproduce satisfactorily the amplitude-wavelength behaviors of all 5 modes when using the amplitudes and their uncertainties given in Tables 2 and 3 of F03. In all 5 cases, and for all assumed values of $l$ from 0 to 5, we obtained unacceptable quality-of-fit values of $Q \ll 0.001$. While we cannot completely rule out the possibility that our modeling is inadequate for this particular star, we believe instead that this problem is related to the fact that the amplitude uncertainties quoted
by F03 most likely underestimate the true errors. In fact, the authors of F03 seem to be aware of that and provide two other different estimates of the amplitude uncertainties. For instance, they explicitly write that the quoted uncertainties on the reported amplitudes in their Tables 2 and 3 are formal fit errors from a least-squares sine fit procedure, while the (much larger) uncertainties shown in their Figure 5 for 4 different modes have been estimated in another way. Furthermore, F03 write in their subsection 3.2 that the 1σ noise level in their photometry corresponds to 1.52 mmag in the UV bandpass, 1.53 mmag in B, 1.12 mmag in R, and 1.37 mmag in NIR, which is clearly at odds with the much smaller formal fit errors reported in their tables.

We used these last figures to derive more realistic estimates of the amplitude uncertainties for the F03 data set. To this end, we adopted the recipe of Montgomery & O’Donoghue (1999), which suggests that the amplitude uncertainty should be equal to about 0.8 times the 1σ noise level, irrespective of the actual amplitude of a mode. This leads to \( \sigma(\text{UV}) = 1.22 \) mmag, \( \sigma(B) = 1.22 \) mmag, \( \sigma(R) = 0.90 \) mmag, and \( \sigma(\text{NIR}) = 1.10 \) mmag. We note that these correspond to values 10 to 25 times larger than the formal fit errors quoted by F03 in their Tables 2 and 3, a huge difference perhaps in line with the reluctance shown by K98 to quote realistic errors for this type of data. While the approach of Montgomery & O’Donoghue (1999) is conservative, we believe that it leads to more realistic estimates of the amplitude uncertainties in this case, and we have consequently redone our analysis of the F03 data set with these revised values.

Our results are summarized in Table 5. Partial mode discrimination is possible for the three highest amplitude modes considered, and we show the corresponding fits in Figures 5, 6, and 7. The best result is obtained for the largest amplitude mode observed by F03 (481.75 s), for which, according to our analysis, the \( l \) value is equal to either 0, 1, or 2.

3.4. HS 0039+4302 (J04)

HS 0039+4302 has been observed by J04 using the 3-channel ULTRACAM camera attached to the 4.2-m William Herschel Telescope. Some 16.2 h of \( u'g'r' \) Sloan photometry has been acquired on this relatively faint EC 14026 pulsator at \( V = 15.5 \). Six modes with \( u'g'r' \) amplitudes have been isolated by the authors. While they have quoted an average amplitude uncertainty of 0.10 mmag for the 3 bandpasses of interest for all the 6 modes in their Table 3, we can use the results of J05 and Aerts et al. (2006) based on the same detector/telescope combination to infer wavelength-dependent amplitude uncertainties of \( \sigma(u') = 0.156 \) mmag, \( \sigma(g') = 0.063 \) mmag, and \( \sigma(r') = 0.081 \) mmag. These are the values that we used in our modeling of the 6 different modes. Note that during this modeling
exercise, we convolved the transmission curves of the Sloan filters $u'$, $g'$, and $r'$ with the response curves of the CCD's used in ULTRACAM kindly provided to us by V. Dhillon (2005, private communication).

The results of our analysis for HS 0039+4302 are summarized in Table 6. They are interesting in that they show, despite the relative faintness of the target, that partial mode identification is possible. Except for the 134.44 s mode (the lowest amplitude one) for which mode discrimination is not feasible, partial identification is indeed possible. For instance, Figures 8, 9, and 10, corresponding to the three largest amplitude modes, indicate that these pulsations have values of the degree index $l$ of either 0, 1, or 2. We note that the preliminary $l$ assignments made by J04 for the 6 modes they observed in HS 0039+4302 are consistent with our quantitative results.

### 3.5. BAL 090100001 (O05)

Oreiro et al. (2004) reported the discovery of short-period luminosity variations in BAL 090100001, the brightest and largest amplitude EC 14026 pulsator so far discovered. From a strictly observational point of view, these characteristics make it an ideal target to attempt multicolor photometry. Hence, less than a year after the initial discovery of the variability of BAL 090100001, O05 presented follow-up observations, including 9.3 h of simultaneous Strömgren photometry on that star. This data set was gathered with the dedicated Strömgren photometer attached to the 0.9-m telescope at the Sierra Nevada Observatory.

Our attempts to model the amplitude-wavelength behaviors of the two pulsation modes reported by O05 (see their Table 5) were frustrated and no acceptable fits were found for any of the assumed $l$ values from 0 to 5. Table 7 summarizes our results and clearly indicates that our quality-of-fit values $Q$ are completely unacceptable. Figure 11 illustrates graphically the situation for the largest amplitude mode (356.3 s). Again, as in the case of PG 1605+072 discussed above, we cannot exclude completely the possibility that our modeling is inadequate for BAL 090100001, but we rather strongly suspect that the quoted amplitude uncertainties in Table 5 of O05 (most likely formal fit errors) underestimate largely the true errors. Unfortunately, unlike the case of PG 1605+072 in F03, not enough information is given in O05 that would have allowed us to obtain perhaps more realistic estimates of the amplitude uncertainties. Thus, we must conclude that we cannot model adequately the data of O05 as presented by them.
3.6. BAL 090100001 (B05)

BAL 090100001 was also observed by B05 who, in a major effort, gathered more than 126 h of $UBVR$ photometry on the 0.6-m telescope of the Mt. Suhora Observatory in Poland. The authors uncovered many periodicities, including not only short-period pulsations characteristic of EC 14026 stars, but also long-period oscillations most likely due to $g$-mode pulsations as found in the PG 1716 stars (and see also O05). We have concentrated our modeling effort on the 9 short-period modes (labelled $f_1$ through $f_9$ in B05) with reliable amplitude determinations in all four bandpasses. We note that the authors provided a very careful frequency analysis and that they adopted the approach of Montgomery & O’Donoghue (1999) for estimating the uncertainties on the reported amplitudes and phases. For the amplitudes, this leads to $\sigma(U) = 0.25 \text{ mmag}$, $\sigma(B) = 0.18 \text{ mmag}$, $\sigma(V) = 0.16 \text{ mmag}$, and $\sigma(R) = 0.17 \text{ mmag}$, irrespective of the actual amplitudes of a given mode.

The light curve of BAL 090100001, as observed by B05, is dominated by a very large amplitude mode (the largest ever observed so far in a sdB pulsator) with a period of 356.19 s. Its color amplitudes are $U = 75.23 \text{ mmag}$, $B = 57.71 \text{ mmag}$, $V = 53.34 \text{ mmag}$, and $R = 50.26 \text{ mmag}$ (see Table 1 of B05). Not surprisingly, B05 also report the detection of the first and second harmonic of that mode, the first with an amplitude larger than 6 of the 9 modes we retained for analysis. The dominant mode appears to be isolated, whereas the three next largest ones (354.20 s, 354.01 s, and 353.81 s) appear to form an almost perfectly symmetric triplet in frequency space, leading to the suggestion that this triplet could be due to rotational splitting. Considering that the main mode does not show an equivalent multiplet structure, this led B05 to suggest that the 356.19 s mode could be a $l = 0$ mode, while the triplet would correspond to a rotationally-split $l = 1$ pulsation. It is obviously of high interest to verify if these sensible suggestions could be proven true.

To model the data of B05, we adopted the atmospheric parameters given in Table 2 ($T_{\text{eff}} = 29,810 \text{ K}$ and $\log g = 5.58$), and we convolved our basic monochromatic quantities with the standard transmission curves of the Johnson/Cousins filters, the KPNO extinction curve, and a gray response for the CCD detector since, in the latter case, we had no information as to the exact response curve of the detector. The results of our effort for the 9 modes retained are summarized in Table 8. In addition, we include here Figures 12, 13, 14, and 15 that refer, respectively, to the main mode and the triplet mentioned above. The other modes have relatively low amplitudes and it becomes increasingly difficult to discriminate between the possible values of $l$ for them.

Figure 12 illustrates our results for the main mode. Formally speaking, all model fits shown in the figure for that mode must be rejected since $Q \ll 0.001$ for all values of $l$ considered in our simulations. At the same time, the figure also clearly illustrates that the
solution must be either \( l = 0 \) or \( l = 1 \). We note that if the reported amplitude uncertainties are multiplied by a factor of 3.5 (to mimic the possibility that these uncertainties have been perhaps underestimated), then the solutions \( l = 0 \) and \( l = 1 \) become formally acceptable, while the others can be safely discarded. However, we have no particularly good reason to believe that the uncertainties on the amplitudes would have been underestimated by such a relatively large amount in this data set. Let us consider instead the data at face value and examine if we can rise up to the challenge offered by this very high S/N observation.

The first obvious possibility to improve the match between the data points and one \((l = 0)\) or the other \((l = 1)\) of the possible solutions is to vary the model parameters within reasonable ranges, in particular the values of the atmospheric parameters \( T_{\text{eff}} \) and \( \log g \) that we assumed for BAL 090100001. The values of these parameters, as derived by Green et al. (2006), are \( T_{\text{eff}} = 29,810 \pm 400 \) K and \( \log g = 5.58 \pm 0.05 \). Explicit calculations within these bounds indicate that the situation is not changed at the qualitative level: the values of \( \chi^2 \) for the \( l = 0 \) and \( l = 1 \) solutions remain within a factor of 2 of each other as in Table 8, and the values of \( Q \) are not significantly improved.

The same is true by redoing the modeling exercise using, this time, the different set of atmospheric parameters derived by Oreiro et al. (2004), i.e., \( T_{\text{eff}} = 29,450 \pm 500 \) K and \( \log g = 5.33 \pm 0.10 \). In that case, the solution must again be either \( l = 0 \) or \( l = 1 \), and the respective \( \chi^2 \) values are \( 1.39 \times 10^2 \) and \( 1.52 \times 10^2 \), both comparable to each other, but still way too high to be formally acceptable as their associated \( Q \) values are much smaller than the criterion level of 0.001. We also experimented with changing the extinction curve from that of KPNO to one appropriate at sea level and found very little changes in our theoretical amplitudes. Likewise, assuming a specific CCD response, instead of using a flat gray response as we did in our initial calculations, also led to very little qualitative changes compared to the situation depicted in Figure 12 and Table 8. (In that latter experiment, we did not use the response curve of the CCD employed by B05 since it was not available to us, but that of CCD21 of Steward Observatory as a surrogate.) Thus, we are unable to find a formally acceptable \((Q > 0.001)\) \( l \) model for the main pulsation mode in BAL 090100001, but we still conclude that it must be either a \( l = 0 \) or a \( l = 1 \) oscillation.

This failure to model properly the main oscillation observed in BAL 090100001 is somewhat bothersome (assuming again that the reported amplitude uncertainties are realistic estimates). We remind the reader that Randall et al. (2005), using the exact same tools as those used in this paper, have been able to model successfully the 182.42 s mode UL-TRACAM data reported by Jeffery et al. (2004) for KPD 2109+4401. We can think of a potentially important difference between the two cases, however, and it is the fact that the amplitude of the dominant mode in BAL 090100001 is so large that nonlinear effects are
obviously present in the form of the first and second harmonics. No such nonlinear features have been observed in KPD 2109+4401. The theory used to determine amplitude ratios between different bandpasses is strictly linear, and it is possible, although this remains unproven, that it becomes inadequate to treat very large amplitude oscillations such as the 356.19 s mode in BAL 09010001.

Figures 13, 14, and 15 show our model fits for the triplet of modes which has been interpreted as a rotationally-split \( l = 1 \) mode by B05. Taking again the data at face value, we find (see Table 8) that the 354.20 s mode has \( l = 2 \), the 354.01 s mode has a value of \( l \) of either 1 or 2, and the 353.81 s component has a value of \( l \) of either 1, 2, or 4. If, however, the amplitude uncertainties are somewhat underestimated, then the \( l = 1 \) possibility for the 354.20 s mode should not be dismissed too hastily because its \( Q \) value would not be much smaller than the passage value of \( Q = 0.001 \). Because of this, we prefer to be conservative and conclude that if the observed triplet of modes is indeed due to rotational splitting, then it must have a value of either \( l = 1 \) or \( l = 2 \). In the latter case, an unfavorable observation angle or some other cause could perhaps hide two components of the quintuplet.

We have thus been unable to confirm the suggestion of B05 that the dominant 356.19 s mode in BAL 09010001 is a radial mode, and that the 354.20, 354.01, and 353.81 s modes are the components of a rotationally-split \( l = 1 \) mode. Our results are nevertheless consistent with that suggestion, but we found that the dominant mode could also have a degree index of \( l = 1 \) while the triplet could be three components of a split \( l = 2 \) mode.

### 3.7. PG 0014+067 (J05)

The case of PG 0014+067 is particularly interesting because a specific asteroseismic model has been proposed by Brassard et al. (2001) including, of course, complete mode identification. This model has been refined subsequently by Charpinet et al. (2005c) who exploited a higher quality (white light) data set and found the same mode identification. In both cases, in the search in parameter space for the optimal model, it was explicitly assumed that the observed modes had to belong to degree indices from \( l = 0 \) up to and including \( l = 3 \). The inclusion of \( l = 3 \) was deemed necessary because the observed mode density in PG 0014+067 is too large to be explained solely in terms of modes with \( l = 0, 1, \) and 2. In other words, there are more observed modes in the relevant period window of PG 0014+067 than there are theoretical periods with \( l = 0, 1, \) and 2, so there seemed to be no choice but conclude that some additional modes with \( l \geq 3 \) are excited to visible levels in that star. Except for the restriction to \( l \) values from 0 to 3, no a priori constraints were imposed in the asteroseismological exercises carried out by Brassard et al. (2001) and Charpinet et al.
(2005c), and the actual mode identification (the determination of both the radial order $k$ and the degree index $l$ for each observed pulsation) came out as a natural byproduct of the method.

The results of Brassard et al. (2001) correspond to the first claimed successful asteroseismological exercise for a pulsating sdB star, and the importance of testing the proposed seismic model of PG 0014+067 cannot be overstated. It is in this spirit that J05 decided to attempt testing the $l$ identification inferred in Brassard et al. (2001) using multicolor photometry. Even though the authors of J05 realized fully that, at $V \simeq 16$, PG 0014+067 was going to be a challenge, they thought the matter to be sufficiently important to invest almost 30 h of WHT time using the ULTRACAM camera. It would have been difficult to do better than they did on this from an observational point of view. As in the previous targets studied in this paper, the details of their data set are given in Table 1. J05 isolated some 10 distinct pulsation modes in PG 0014+067, and we modeled each one of these pulsations using the parameters listed in Table 2.

Not unexpectedly, our analysis reveals that most of the observed modes in PG 0014+067 cannot be tested for $l$ identification as the measured amplitudes usually do not have high enough S/N ratios. The details of our quantitative analysis are presented in Table 9 for the 10 modes of interest. At the same time, our work also indicates that partial $l$ discrimination is possible for the three largest amplitude modes uncovered by J05, and we show our model fits for these modes in Figure 16 ($141.01$ s), Figure 17 ($141.06$ s), and Figure 18 ($146.50$ s). We find that the observed amplitudes of the $141.01$ s mode can be accommodated if the degree index $l$ has a value of either 0, 1, 2, or 4. This agrees with the conclusion of J05 that this mode (the $f_{12}$ mode in their notation) cannot have a value of $l = 3$, and indeed we find that $Q = 1.2 \times 10^{-21}$ for that model, much too small to be acceptable. We also find that the $141.06$ s mode ($f_{11}$ in J05) has a value of $l$ of either 0, 1, or 2, again in agreement with the inference made in J05. Finally, we find that the observed amplitude-wavelength distribution of the $146.50$ s mode ($f_9$ in J05) can be quantitatively explained if the mode has a $l$ value of either 0, 1, or 2. This again is consistent with J05 who found that this mode cannot be a mode with $l = 3$ or $l = 4$.

The most interesting result of this analysis is the conclusion, first put forward by J05, that the $141.06$ s mode (observed as the $141.07$ s mode in the lower resolution white light data of Brassard et al. 2001) cannot have a degree index of $l = 3$. Given that Brassard et al. (2001) formally identified this mode as a $l = 3$ pulsation, this implies that a reevaluation of their seismic model is warranted. It should be pointed out that at the time of the analysis of Brassard et al. (2001), the theory of multicolor photometry had not yet been applied to realistic models of pulsating sdB stars. Using an Eddington limb darkening law (instead
of an exact form coming from detailed sdB model atmospheres), Brassard et al. (2001) estimated that the visibility factor would be equal to 1.0000, 0.7083, 0.3250, 0.0625, and 0.0208 respectively for modes with \( l = 0 \), 1, 2, 3, and 4. Hence, the \( l = 3 \) modes would be more than three times “more visible” than their \( l = 4 \) counterparts, and it was deemed natural to limit the search to modes with \( l = 3 \) beyond those with \( l = 0 \), 1, and 2 since this was now sufficient to account for the relatively high density of observed modes.

What we learned in Randall et al. (2005), among many other things, is that the visibility of a \( p \)-mode with \( l = 4 \) is, in fact, substantially larger than that of a \( l = 3 \) mode in a sdB pulsator in the optical domain. The discussion of Figure 1 in Randall et al. (2005) is quite explicit about this. Hence, it will be necessary in future asteroseismological exercises concerning EC 14026 pulsators (especially those showing more modes than could be accommodated by invoking the presence of only modes with \( l = 0 \), 1, and 2) to include the possibility that the detected modes could also have values of \( l = 4 \) as they have a higher probability of being detected than the \( l = 3 \) modes. In particular, a reanalysis of the data presented by Brassard et al. (2001) and Charpinet et al. (2005c) on PG 0014+067 including this possibility must be carried out. This is being done and will be reported in due time.

4. DISCUSSION

We have presented in this paper a quantitative and homogeneous analysis of the broadband multicolor photometric data sets gathered so far on rapidly pulsating sdB (EC 14026) stars. This analysis was carried out within the theoretical framework developed by Randall et al. (2005) with the goal of partial mode identification, i.e., the determination of the degree index \( l \) of each of the observed pulsation modes. With the exception of the very high S/N ratio \( u'g'r'/ULTRACAM \) data set obtained by Jeffery et al. (2004) on KPD 2109+4401 (an analysis of which will be presented in a separate publication), we considered all available data that we could find in the literature. This consists of 7 distinct data sets pertaining to 6 different EC 14016 stars and involving 41 pulsation modes.

Our final results are summarized in Table 10 where we list the name of the target star (1st column), the period of the mode of interest (2nd column), the acceptable values of \( l \) for that mode (3rd column), and a qualifier as to the level of mode discrimination achieved (4th column). It can be seen that no completely convincing and unambiguous \( l \) identification has been possible on the basis of the available data, although we came close to that goal in the case of two modes in KPD 2109+4401 (K98) and three modes in BAL 090100001 (B05) for which only two possible values of \( l \) were found to be consistent with the observed amplitude-wavelength distributions. At the same time, we found that no mode discrimination (between
values from \( l = 0 \) to \( l = 5 \) was possible for 16 out of 41 modes. There is no doubt that these disappointing (but perhaps not totally unexpected) results are due to the fact that the available data sets were of insufficient sensitivity. The exception is the data set of B05 which features a remarkably high S/N value for the largest amplitude mode (356.19 s) uncovered in that study. We speculate here that nonlinear effects (observed in the form of the first and second harmonic of that mode) may have undermined our ability to model properly the amplitude-wavelength behavior of that particular mode. The latter shows an exceptionally large amplitude by EC 14026 star standards. We recall, in this context, that the feasibility of our approach has been demonstrated beyond any doubt by Randall et al. (2005) in their analysis of the 182.42 s mode measured by Jeffery et al. (2004) in KPD 2109+4401. Hence, the case of the 356.19 s mode in BAL 090100001 remains enigmatic.

It is interesting to examine the statistics of the numbers shown in Table 10. Considering only those cases where partial mode discrimination has been possible, we find that the \( l = 0 \) solution comes up 19 times, the \( l = 1 \) solution 23 times, the \( l = 2 \) solution 24 times, the \( l = 3 \) solution 3 times, the \( l = 4 \) solution 13 times, and the \( l = 5 \) solution once. While no single \( l \) index identification has been achieved in our study, one can argue, from a strictly statistical point of view, that these results are certainly consistent with the view that the pulsations seen in rapidly pulsating sdB stars are low-degree modes, mostly with values of \( l = 0, 1, \) and 2. This is not a great surprise by any means, but it confirms the early interpretation of the EC 14026 phenomenon in terms of low-order, low-degree \( p \)-mode pulsations as presented by Charpinet et al. (1997). Moreover, the higher frequency of \( l = 4 \) solutions compared to that of \( l = 3 \) solutions is consistent with the finding of Randall et al. (2005) that the former modes are more visible than the latter (in the optical domain).

In conclusion, we find that in order to exploit the full potential of multicolor photometry for EC 14026 pulsators, it will be necessary in the future to gather higher S/N ratio observations than has been done up to now. This is certainly within reach, however, as our results clearly imply. Furthermore, it will be necessary in future asteroseismological exercises such as those carried out by Brassard et al. (2001) and Charpinet et al. (2005c) to include, as required, theoretical modes with \( l = 4 \) since those have a higher visibility factor in sdB stars than \( l = 3 \) modes.

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Table 1. Basic characteristics of the data sets used in this paper

| Object         | $V$   | Hours | Filters | Instrument          | Telescope | Location       | Reference |
|----------------|-------|-------|---------|---------------------|-----------|----------------|-----------|
| KPD 2109+4401  | 13.38 | 26.6  | $UBVR$  | 4-channel Steining  | Struve    | Mt. Locke      | K98       |
| PG 1618+563B   | 13.52 | 1.5   | $UBV$   | 3-channel Tromso    | NOT       | La Palma       | S00       |
| PG 1605+072    | 12.92 | 12.3  | $UV, B, R, NIR$ | 4-channel BUSCA | 2.2 m     | Calar Alto     | F03       |
| HS 0039+4302   | 15.5  | 16.2  | $u'g'r'$ | ULTRACAM           | WHT       | La Palma       | J04       |
| BAL 090100001  | 11.8($B$) | 9.3 | $uvby$  | Strömgren           | 0.9 m     | Sierra Nevada  | O05       |
| BAL 090100001  | 11.8($B$) | 126 | $UBVR$  | CCD                 | 0.6 m     | Mt. Suhora     | B05       |
| PG 0014+067    | 15.9  | 29.5  | $u'g'r'$ | ULTRACAM           | WHT       | La Palma       | J05       |
|                |       |       |         |                     |           | Spain          |           |
Table 2. Atmospheric parameters used in the computations of the theoretical amplitudes

| Object         | $T_{\text{eff}}$ (K) | log $g$ | Filters       | Instrument response | Extinction |
|----------------|----------------------|---------|---------------|---------------------|------------|
| KPD 2109+4401  | 31380                | 5.65    | $UBVR$        | gray                | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
| PG 1618+563B   | 34320                | 5.79    | $UBV$         | gray                | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
| PG 1605+072    | 32520                | 5.27    | $UV, B, R, NIR$ | BUSCA               | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
| HS 0039+4302   | 32320                | 5.68    | $u'g'r'$      | ULTRACAM            | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
| BAL 090100001  | 29810                | 5.58    | $uvby$        | gray                | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
| BAL 090100001  | 29810                | 5.58    | $UBVR$        | gray                | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
| PG 0014+067    | 34130                | 5.77    | $u'g'r''$     | ULTRACAM            | KPNO       |
| ...            | ...                  | ...     | ...           | ...                 | ...        |
Table 3. Fits of predicted $UBVR$ amplitudes to those observed for the pulsation modes of KPD 2109+4401 reported by K98

| Period (s) | $U$ (mmag) | $B$ (mmag) | $V$ (mmag) | $R$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|-------------|-------------|-------------|-------------|-----|---------|-----|
| 182.42     | 5.300±0.105 | 4.200±0.105 | 3.900±0.085 | 3.700±0.155 | ... | ...     | ... |
| 184.72     | 3.000±0.105 | 2.600±0.105 | 2.400±0.085 | 2.200±0.155 | ... | ...     | ... |
| 184.75     | 3.900±0.105 | 3.400±0.105 | 3.300±0.085 | 3.100±0.155 | ... | ...     | ... |
| 191.85     | 2.700±0.105 | 2.300±0.105 | 2.400±0.085 | 2.200±0.155 | ... | ...     | ... |
Table 3—Continued

| Period (s) | $U$ (mmag) | $B$ (mmag) | $V$ (mmag) | $R$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|------------|------------|------------|------------|----|----------|----|
| 196.31     | 7.200±0.105| 5.600±0.105| 5.200±0.085| 4.800±0.155| ...|...      |... |
| ...        | 7.306      | 5.481      | 5.131      | 4.978      | 0  | 4.29e+00 | 2.31e−01 |
| ...        | 7.170      | 5.518      | 5.207      | 5.070      | 1  | 3.74e+00 | 2.91e−01 |
| ...        | 7.004      | 5.557      | 5.294      | 5.160      | 2  | 1.03e+01 | 1.65e−02 |
| ...        | 8.084      | 5.215      | 4.685      | 4.016      | 3  | 1.47e+02 | 1.38e−31 |
| ...        | 6.294      | 5.693      | 5.577      | 5.569      | 4  | 1.20e+02 | 9.75e−26 |
| ...        | 5.246      | 5.505      | 5.849      | 6.473      | 5  | 5.22e+02 | 8.49−113 |
| 196.69     | 0.800±0.105| 0.700±0.105| 0.700±0.085| 0.800±0.155| ...|...      |... |
| ...        | 0.920      | 0.690      | 0.646      | 0.627      | 0  | 2.97e+00 | 3.97e−01 |
| ...        | 0.904      | 0.696      | 0.657      | 0.639      | 1  | 2.32e+00 | 5.09e−01 |
| ...        | 0.884      | 0.701      | 0.668      | 0.651      | 2  | 1.70e+00 | 6.37e−01 |
| ...        | 1.006      | 0.649      | 0.583      | 0.500      | 3  | 9.72e+00 | 2.11e−02 |
| ...        | 0.802      | 0.726      | 0.711      | 0.710      | 4  | 4.15e−01 | 9.37e−01 |
| ...        | 0.676      | 0.709      | 0.754      | 0.834      | 5  | 1.85e+00 | 6.03e−01 |
| 198.19     | 4.800±0.105| 4.000±0.105| 4.100±0.085| 4.000±0.155| ...|...      |... |
| ...        | 5.318      | 3.989      | 3.733      | 3.622      | 0  | 4.90e+01 | 1.33e−10 |
| ...        | 5.224      | 4.021      | 3.794      | 3.694      | 1  | 3.32e+01 | 2.91e−07 |
| ...        | 5.108      | 4.055      | 3.863      | 3.765      | 2  | 1.90e+01 | 2.77e−04 |
| ...        | 5.832      | 3.771      | 3.387      | 2.902      | 3  | 2.22e+02 | 7.82e−48 |
| ...        | 4.614      | 4.177      | 4.092      | 4.087      | 4  | 6.29e+00 | 9.85e−02 |
| ...        | 3.880      | 4.073      | 4.330      | 4.795      | 5  | 1.11e+02 | 7.10e−24 |
Table 4. Fits of predicted $UBV$ amplitudes to those observed for the pulsation modes of PG 1618+563B reported by S00

| Period (s) | $U$ (mmag) | $B$ (mmag) | $V$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|------------|------------|------------|----|----------|----|
| 139.30     | 1.500±0.500| 1.200±0.600| 0.600±0.500| ...| ...      | ...|
| ...        | 1.292      | 1.050      | 0.983      | 0  | 8.23e−01 | 6.63e−01 |
| ...        | 1.276      | 1.056      | 0.995      | 1  | 8.82e−01 | 6.43e−01 |
| ...        | 1.254      | 1.062      | 1.008      | 2  | 9.61e−01 | 6.19e−01 |
| ...        | 1.442      | 1.007      | 0.853      | 3  | 3.74e−01 | 8.30e−01 |
| ...        | 1.168      | 1.079      | 1.055      | 4  | 1.31e+00 | 5.19e−01 |
| ...        | 1.078      | 1.063      | 1.105      | 5  | 1.79e+00 | 4.09e−01 |
| 143.90     | 4.600±0.700| 4.300±0.800| 2.600±1.000| ...| ...      | ...|
| ...        | 4.608      | 3.745      | 3.504      | 0  | 1.30e+00 | 5.23e−01 |
| ...        | 4.566      | 3.778      | 3.559      | 1  | 1.35e+00 | 5.10e−01 |
| ...        | 4.508      | 3.820      | 3.627      | 2  | 1.43e+00 | 4.89e−01 |
| ...        | 4.940      | 3.478      | 2.949      | 3  | 1.41e+00 | 4.93e−01 |
| ...        | 4.278      | 3.958      | 3.872      | 4  | 2.01e+00 | 3.66e−01 |
| ...        | 4.048      | 3.994      | 4.159      | 5  | 3.20e+00 | 2.02e−01 |
Table 5. Fits of predicted BUSCA amplitudes to those observed for the pulsation modes of PG 1605+072 reported by F03

| Period (s) | $UV$ (mmag) | $B$ (mmag) | $R$ (mmag) | $NIR$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|-------------|-------------|-------------|--------------|-----|----------|-----|
| 440.51     | 8.92±1.22   | 6.85±1.22   | 6.85±0.90   | 6.66±1.10    | ...| ...      | ...|
| ...        | 8.80        | 7.15        | 6.99        | 6.29         | 0  | 2.10e−01 | 9.76e−01|
| ...        | 8.62        | 7.18        | 7.03        | 6.41         | 1  | 2.28e−01 | 9.73e−01|
| ...        | 8.38        | 7.22        | 7.08        | 6.51         | 2  | 3.72e−01 | 9.46e−01|
| ...        | 10.21       | 7.55        | 6.44        | 3.11         | 3  | 1.21e+01 | 6.97e−03|
| ...        | 7.55        | 7.15        | 7.16        | 7.12         | 4  | 1.62e+00 | 6.55e−01|
| ...        | 6.04        | 6.40        | 6.99        | 8.44         | 5  | 8.40e+00 | 3.84e−02|
| 475.82     | 19.76±1.22  | 16.94±1.22  | 16.68±0.90  | 17.18±1.10   | ...| ...      | ...|
| ...        | 21.21       | 17.19       | 16.80       | 15.10        | 0  | 5.09e+00 | 1.66e−01|
| ...        | 20.73       | 17.27       | 16.92       | 15.42        | 1  | 3.36e+00 | 3.40e−01|
| ...        | 20.17       | 17.41       | 17.07       | 15.70        | 2  | 2.26e+00 | 5.19e−01|
| ...        | 24.16       | 18.05       | 15.40       | 7.58         | 3  | 9.26e+01 | 5.92e−20|
| ...        | 18.20       | 17.28       | 17.32       | 17.27        | 4  | 2.26e+00 | 5.20e−01|
| ...        | 14.58       | 15.52       | 17.00       | 20.62        | 5  | 2.95e+01 | 1.79e−06|
| 481.75     | 36.88±1.22  | 29.27±1.22  | 28.45±0.90  | 28.72±1.10   | ...| ...      | ...|
| ...        | 37.05       | 30.03       | 29.33       | 26.36        | 0  | 6.00e+00 | 1.12e−01|
| ...        | 36.20       | 30.14       | 29.53       | 26.92        | 1  | 4.98e+00 | 1.74e−01|
| ...        | 35.16       | 30.36       | 29.77       | 27.40        | 2  | 6.44e+00 | 9.22e−02|
| ...        | 42.50       | 31.80       | 27.13       | 13.40        | 3  | 2.23e+02 | 4.30e−48|
| ...        | 31.62       | 30.06       | 30.13       | 30.06        | 4  | 2.41e+01 | 2.34e−05|
| ...        | 25.18       | 26.83       | 29.41       | 35.70        | 5  | 1.38e+02 | 9.56e−30|
| 503.70     | 22.59±1.22  | 22.45±1.22  | 18.79±0.90  | 18.02±1.10   | ...| ...      | ...|
| ...        | 24.48       | 19.80       | 19.34       | 17.36        | 0  | 7.83e+00 | 4.96e−02|
| ...        | 23.89       | 19.89       | 19.48       | 17.76        | 1  | 6.18e+00 | 1.03e−01|
| ...        | 23.18       | 20.05       | 19.67       | 18.11        | 2  | 5.04e+00 | 1.69e−01|
| ...        | 28.05       | 21.12       | 18.03       | 9.00         | 3  | 8.97e+01 | 2.53e−19|
| ...        | 20.84       | 19.86       | 19.92       | 19.90        | 4  | 1.11e+01 | 1.12e−02|
| Period (s) | $UV$ (mmag) | $B$ (mmag) | $R$ (mmag) | $NIR$ (mmag) | $l$ | $\chi^2$ | $Q$       |
|-----------|-------------|------------|------------|--------------|----|---------|---------|
| ...       | 16.52       | 17.65      | 19.39      | 23.61        | 5  | 6.68e+01| 2.09e-14|
| 528.70    | 7.65± 1.22  | 8.25± 1.22 | 6.79± 0.90 | 6.88± 1.10   | ...| ...     | ...     |
| ...       | 8.84        | 7.13       | 6.96       | 6.24         | 0  | 2.16e+00| 5.39e-01|
| ...       | 8.62        | 7.17       | 7.03       | 6.41         | 1  | 1.66e+00| 6.45e-01|
| ...       | 8.35        | 7.24       | 7.10       | 6.55         | 2  | 1.22e+00| 7.48e-01|
| ...       | 9.99        | 7.58       | 6.47       | 3.27         | 3  | 1.50e+01| 1.83e-03|
| ...       | 7.51        | 7.19       | 7.21       | 7.22         | 4  | 1.09e+00| 7.80e-01|
| ...       | 5.96        | 6.40       | 7.05       | 8.61         | 5  | 6.79e+00| 7.91e-02|
Table 6. Fits of predicted ULTRACAM amplitudes to those observed for the pulsation modes of HS 0039+4302 reported by J04

| Period (s) | $u'$ (mmag) | $g'$ (mmag) | $r'$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|-----------|-------------|-------------|-------------|-----|-----------|-----|
| 134.44    | 0.890±0.156 | 0.730±0.063 | 0.600±0.081 | ... | ...       | ... |
|           | 0.914       | 0.690       | 0.660       | 0   | 9.81e−01  | 6.12e−01 |
|           | 0.900       | 0.690       | 0.663       | 1   | 9.99e−01  | 6.07e−01 |
|           | 0.886       | 0.693       | 0.666       | 2   | 1.02e+00  | 6.02e−01 |
|           | 1.178       | 0.676       | 0.654       | 3   | 4.62e+00  | 9.95e−02 |
|           | 0.794       | 0.679       | 0.687       | 4   | 1.81e+00  | 4.04e−01 |
|           | 0.678       | 0.676       | 0.735       | 5   | 5.38e+00  | 6.80e−02 |
| 180.13    | 1.590±0.156 | 1.350±0.063 | 1.410±0.081 | ... | ...       | ... |
|           | 1.820       | 1.363       | 1.300       | 0   | 4.06e+00  | 1.31e−01 |
|           | 1.780       | 1.367       | 1.311       | 1   | 3.06e+00  | 2.16e−01 |
|           | 1.732       | 1.374       | 1.321       | 2   | 2.18e+00  | 3.36e−01 |
|           | 2.152       | 1.358       | 1.093       | 3   | 2.83e+01  | 7.12e−07 |
|           | 1.540       | 1.383       | 1.372       | 4   | 6.04e−01  | 7.39e−01 |
|           | 1.322       | 1.340       | 1.487       | 5   | 3.89e+00  | 1.43e−01 |
| 181.89    | 3.590±0.156 | 2.960±0.063 | 2.640±0.081 | ... | ...       | ... |
|           | 3.820       | 2.859       | 2.728       | 0   | 5.90e+00  | 5.23e−02 |
|           | 3.734       | 2.868       | 2.749       | 1   | 4.82e+00  | 8.98e−02 |
|           | 3.628       | 2.878       | 2.768       | 2   | 4.22e+00  | 1.21e−01 |
|           | 4.542       | 2.874       | 2.313       | 3   | 5.54e+01  | 9.15e−13 |
|           | 3.216       | 2.890       | 2.867       | 4   | 1.48e+01  | 6.10e−04 |
|           | 2.750       | 2.788       | 3.097       | 5   | 6.83e+01  | 1.51e−15 |
| 182.79    | 2.390±0.156 | 2.000±0.063 | 1.800±0.081 | ... | ...       | ... |
|           | 2.582       | 1.932       | 1.843       | 0   | 2.95e+00  | 2.28e−01 |
|           | 2.524       | 1.938       | 1.858       | 1   | 2.21e+00  | 3.31e−01 |
|           | 2.452       | 1.946       | 1.871       | 2   | 1.67e+00  | 4.35e−01 |
|           | 3.064       | 1.941       | 1.562       | 3   | 2.82e+01  | 7.66e−07 |
|           | 2.174       | 1.954       | 1.939       | 4   | 5.37e+00  | 6.82e−02 |
Table 6—Continued

| Period (s) | $u'$ (mmag) | $g'$ (mmag) | $r'$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|-------------|-------------|-------------|-----|----------|-----|
| ...        | 1.860       | 1.886       | 2.096       | 5   | 2.81e+01 | 7.78e−07 |
| 192.58     | 5.730±0.156 | 4.350±0.063 | 4.050±0.081 | ... | ...      | ... |
| ...        | 5.764       | 4.309       | 4.108       | 0   | 9.90e−01 | 6.10e−01 |
| ...        | 5.624       | 4.320       | 4.140       | 1   | 1.93e+00 | 3.80e−01 |
| ...        | 5.454       | 4.337       | 4.170       | 2   | 5.38e+00 | 6.79e−02 |
| ...        | 6.776       | 4.351       | 3.501       | 3   | 9.09e+01 | 1.79e−20 |
| ...        | 4.824       | 4.352       | 4.321       | 4   | 4.49e+01 | 1.77e−10 |
| ...        | 4.122       | 4.191       | 4.671       | 5   | 1.71e+02 | 6.19e−38 |
| 234.11     | 6.110±0.156 | 4.930±0.063 | 4.620±0.081 | ... | ...      | ... |
| ...        | 6.536       | 4.843       | 4.607       | 0   | 9.38e+00 | 9.17e−03 |
| ...        | 6.328       | 4.862       | 4.658       | 1   | 3.32e+00 | 1.90e−01 |
| ...        | 6.072       | 4.888       | 4.704       | 2   | 1.59e+00 | 4.53e−01 |
| ...        | 7.290       | 4.953       | 3.993       | 3   | 1.17e+02 | 3.37e−26 |
| ...        | 5.336       | 4.904       | 4.890       | 4   | 3.59e+01 | 1.60e−08 |
| ...        | 4.554       | 4.699       | 5.315       | 5   | 1.87e+02 | 3.07e−41 |
Table 7. Fits of predicted $uvby$ amplitudes to those observed for the pulsation modes of BAL 090100001 reported by O05

| Period (s) | $u$ (mmag) | $v$ (mmag) | $b$ (mmag) | $y$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|-----------|-------------|-------------|-------------|-------------|-----|-----------|-----|
| 351.70    | 16.73± 0.08 | 17.43± 0.09 | 16.84± 0.09 | 15.76± 0.08 | ... | ...       | ... |
|           | ...         | 21.02       | 13.97       | 15.27       | 13.27 | 0         | 5.62e+03 |
|           | ...         | 20.24       | 14.63       | 15.63       | 14.07 | 1         | 3.51e+03 |
|           | ...         | 19.37       | 15.30       | 15.93       | 14.83 | 2         | 1.89e+03 |
|           | ...         | 20.87       | 14.83       | 15.03       | 13.15 | 3         | 4.99e+03 |
|           | ...         | 17.27       | 16.26       | 16.60       | 16.32 | 4         | 2.71e+02 |
|           | ...         | 14.49       | 16.21       | 16.29       | 18.70 | 5         | 2.35e+03 |
| 356.30    | 49.36± 0.11 | 41.34± 0.11 | 38.55± 0.10 | 38.03± 0.10 | ... | ...       | ... |
|           | ...         | 54.58       | 36.23       | 39.63       | 34.40 | 0         | 5.84e+03 |
|           | ...         | 52.05       | 37.63       | 40.20       | 36.19 | 1         | 2.34e+03 |
|           | ...         | 49.34       | 39.00       | 40.62       | 37.81 | 2         | 8.86e+02 |
|           | ...         | 54.02       | 38.46       | 38.99       | 34.10 | 3         | 4.04e+03 |
|           | ...         | 43.19       | 40.71       | 41.56       | 40.88 | 4         | 4.90e+03 |
|           | ...         | 35.77       | 40.04       | 40.20       | 46.21 | 5         | 2.24e+04 |
Table 8. Fits of predicted $UBVR$ amplitudes to those observed for the pulsation modes of BAL 090100001 reported by B05

| Period (s) | $U$ (mmag) | $B$ (mmag) | $V$ (mmag) | $R$ (mmag) | $l$ (mmag) | $\chi^2$ | $Q$ |
|------------|-------------|-------------|-------------|-------------|------------|---------|-----|
| 356.19     | 75.23± 0.25 | 57.71± 0.18 | 53.34± 0.16 | 50.26± 0.17 | ...        | ...     | ... |
| ...        | 77.92       | 58.14       | 52.05       | 49.37       | 0          | 2.15e+02 | 2.68e−46 |
| ...        | 73.18       | 57.96       | 53.38       | 51.31       | 1          | 1.07e+02 | 3.84e−23 |
| ...        | 68.50       | 57.76       | 54.53       | 52.85       | 2          | 1.01e+03 | 2.91−219 |
| ...        | 77.85       | 59.92       | 52.99       | 46.11       | 3          | 8.61e+02 | 2.11−186 |
| ...        | 58.83       | 56.55       | 55.98       | 56.27       | 4          | 5.87e+03 | 0.00e+00 |
| ...        | 46.98       | 49.55       | 55.85       | 63.17       | 5          | 2.08e+04 | 0.00e+00 |
| 354.20     | 26.71± 0.25 | 21.92± 0.18 | 20.53± 0.16 | 19.92± 0.17 | ...        | ...     | ... |
| ...        | 29.52       | 22.03       | 19.73       | 18.71       | 0          | 2.02e+02 | 1.57e−43 |
| ...        | 27.77       | 22.00       | 20.26       | 19.47       | 1          | 2.81e+01 | 3.55e−06 |
| ...        | 26.04       | 21.94       | 20.72       | 20.08       | 2          | 9.53e+00 | 2.30e−02 |
| ...        | 29.50       | 22.69       | 20.06       | 17.46       | 3          | 3.61e+02 | 7.16e−78 |
| ...        | 22.43       | 21.55       | 21.33       | 21.44       | 4          | 4.03e+02 | 6.13e−87 |
| ...        | 17.98       | 18.96       | 21.37       | 24.16       | 5          | 2.14e+03 | 0.00e+00 |
| 354.01     | 15.82± 0.25 | 12.86± 0.18 | 11.62± 0.16 | 11.57± 0.17 | ...        | ...     | ... |
| ...        | 17.14       | 12.79       | 11.46       | 10.87       | 0          | 4.61e+01 | 5.27e−10 |
| ...        | 16.12       | 12.77       | 11.76       | 11.30       | 1          | 4.91e+00 | 1.78e−01 |
| ...        | 15.11       | 12.73       | 12.02       | 11.65       | 2          | 1.51e+01 | 1.71e−03 |
| ...        | 17.13       | 13.17       | 11.65       | 10.14       | 3          | 1.01e+02 | 7.47e−22 |
| ...        | 13.00       | 12.49       | 12.36       | 12.43       | 4          | 1.78e+02 | 1.91e−38 |
| ...        | 10.41       | 10.98       | 12.38       | 13.99       | 5          | 8.02e+02 | 1.76−173 |
| 353.81     | 5.86± 0.25  | 5.12± 0.18  | 4.71± 0.16  | 4.59± 0.17  | ...        | ...     | ... |
| ...        | 6.75        | 5.04        | 4.51        | 4.28        | 0          | 1.77e+01 | 4.99e−04 |
| ...        | 6.35        | 5.03        | 4.63        | 4.45        | 1          | 4.99e+00 | 1.73e−01 |
| ...        | 5.96        | 5.02        | 4.74        | 4.59        | 2          | 4.92−01  | 9.21e−01 |
| ...        | 6.74        | 5.18        | 4.58        | 3.99        | 3          | 2.57e+01 | 1.13e−05 |
| ...        | 5.13        | 4.93        | 4.88        | 4.91        | 4          | 1.42e+01 | 2.69e−03 |
Table 8—Continued

| Period (s) | $U$ (mmag) | $B$ (mmag) | $V$ (mmag) | $R$ (mmag) | $\chi^2$ | $Q$ |
|------------|------------|------------|------------|------------|---------|----|
| ...        | 4.12       | 4.35       | 4.90       | 5.54       | 5       | 9.93e+01 | 2.23e−21 |
| 350.39     | 2.23± 0.25 | 1.81± 0.18 | 2.08± 0.16 | 1.45± 0.17 | ...     | ...     |
| ...        | 2.53       | 1.89       | 1.69       | 1.61       | 0       | 8.33e+00 | 3.96e−02 |
| ...        | 2.38       | 1.89       | 1.74       | 1.67       | 1       | 6.81e+00 | 7.82e−02 |
| ...        | 2.23       | 1.88       | 1.78       | 1.72       | 2       | 6.31e+00 | 9.75e−02 |
| ...        | 2.54       | 1.95       | 1.72       | 1.50       | 3       | 7.17e+00 | 6.66e−02 |
| ...        | 1.93       | 1.85       | 1.83       | 1.84       | 4       | 9.20e+00 | 2.67e−02 |
| ...        | 1.54       | 1.62       | 1.83       | 2.07       | 5       | 2.43e+01 | 2.14e−05 |
| 350.18     | 2.01± 0.25 | 1.72± 0.18 | 1.31± 0.16 | 1.48± 0.17 | ...     | ...     |
| ...        | 2.15       | 1.60       | 1.44       | 1.36       | 0       | 1.82e+00 | 6.11e−01 |
| ...        | 2.02       | 1.60       | 1.47       | 1.41       | 1       | 1.63e+00 | 6.53e−01 |
| ...        | 1.89       | 1.59       | 1.50       | 1.46       | 2       | 2.21e+00 | 5.31e−01 |
| ...        | 2.15       | 1.65       | 1.46       | 1.27       | 3       | 2.85e+00 | 4.15e−01 |
| ...        | 1.62       | 1.56       | 1.54       | 1.55       | 4       | 5.48e+00 | 1.40e−01 |
| ...        | 1.30       | 1.37       | 1.54       | 1.74       | 5       | 1.64e+01 | 9.45e−04 |
| 349.83     | 1.94± 0.25 | 1.45± 0.18 | 1.58± 0.16 | 1.45± 0.17 | ...     | ...     |
| ...        | 2.13       | 1.59       | 1.43       | 1.35       | 0       | 2.45e+00 | 4.84e−01 |
| ...        | 2.00       | 1.59       | 1.46       | 1.41       | 1       | 1.26e+00 | 7.38e−01 |
| ...        | 1.88       | 1.58       | 1.49       | 1.45       | 2       | 8.91e+00 | 8.28e−01 |
| ...        | 2.13       | 1.64       | 1.45       | 1.26       | 3       | 3.61e+00 | 3.07e−01 |
| ...        | 1.62       | 1.56       | 1.54       | 1.55       | 4       | 2.37e+00 | 5.00e−01 |
| ...        | 1.30       | 1.38       | 1.55       | 1.75       | 5       | 9.80e+00 | 2.04e−02 |
| 264.82     | 5.94± 0.25 | 4.73± 0.18 | 4.16± 0.16 | 4.07± 0.17 | ...     | ...     |
| ...        | 6.09       | 4.64       | 4.21       | 4.01       | 0       | 8.73e−01 | 8.32e−01 |
| ...        | 5.84       | 4.62       | 4.27       | 4.10       | 1       | 1.00e+00 | 8.01e−01 |
| ...        | 5.59       | 4.62       | 4.34       | 4.19       | 2       | 4.15e+00 | 2.46e−01 |
| ...        | 6.45       | 4.73       | 4.17       | 3.64       | 3       | 1.04e+01 | 1.55e−02 |
Table 8—Continued

| Period (s) | $U$ (mmag) | $B$ (mmag) | $V$ (mmag) | $R$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|-------------|-------------|-------------|-------------|-----|----------|-----|
| ...        | 4.83        | 4.54        | 4.46        | 4.46        | 4   | 2.96e+01 | 1.70e−06 |
| ...        | 3.85        | 4.05        | 4.47        | 4.99        | 5   | 1.17e+02 | 4.05e−25 |
| 264.08     | 2.58± 0.25  | 1.83± 0.18  | 1.51± 0.16  | 1.60± 0.17  | ... | ...      | ...       |
| ...        | 2.40        | 1.82        | 1.65        | 1.57        | 0   | 1.37e+00 | 7.12e−01 |
| ...        | 2.29        | 1.82        | 1.68        | 1.61        | 1   | 2.40e+00 | 4.95e−01 |
| ...        | 2.19        | 1.81        | 1.70        | 1.64        | 2   | 3.92e+00 | 2.71e−01 |
| ...        | 2.54        | 1.86        | 1.64        | 1.43        | 3   | 1.69e+00 | 6.39e−01 |
| ...        | 1.89        | 1.77        | 1.74        | 1.74        | 4   | 1.06e+01 | 1.43e−02 |
| ...        | 1.50        | 1.58        | 1.74        | 1.94        | 5   | 2.67e+01 | 6.83e−06 |
Table 9. Fits of predicted ULTRACAM amplitudes to those observed for the pulsation modes of PG 0014+067 reported by J05

| Period (s) | $u'$ (mmag) | $g'$ (mmag) | $r'$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|-----------|-------------|-------------|-------------|----|----------|----|
| 100.29    | 0.900±0.200 | 0.690±0.070 | 0.600±0.100 | ... | ... | ... |
| ...       | 0.864       | 0.674       | 0.648       | 0   | 3.20e−01 | 8.52e−01 |
| ...       | 0.854       | 0.674       | 0.650       | 1   | 3.55e−01 | 8.37e−01 |
| ...       | 0.842       | 0.675       | 0.650       | 2   | 3.87e−01 | 8.24e−01 |
| ...       | 1.200       | 0.662       | 0.697       | 3   | 3.47e+00 | 1.77e−01 |
| ...       | 0.766       | 0.676       | 0.667       | 4   | 9.35e−01 | 6.27e−01 |
| ...       | 0.676       | 0.661       | 0.710       | 5   | 2.63e+00 | 2.68e−01 |
| 139.14    | 0.800±0.200 | 0.710±0.080 | 0.800±0.100 | ... | ... | ... |
| ...       | 0.952       | 0.736       | 0.707       | 0   | 1.56e+00 | 4.59e−01 |
| ...       | 0.934       | 0.738       | 0.711       | 1   | 1.36e+00 | 5.05e−01 |
| ...       | 0.912       | 0.741       | 0.715       | 2   | 1.19e+00 | 5.51e−01 |
| ...       | 1.162       | 0.734       | 0.565       | 3   | 8.89e+00 | 1.18e−02 |
| ...       | 0.822       | 0.744       | 0.738       | 4   | 5.77e−01 | 7.49e−01 |
| ...       | 0.728       | 0.725       | 0.794       | 5   | 1.69e−01 | 9.19e−01 |
| 140.98    | 1.200±0.300 | 1.100±0.100 | 1.000±0.100 | ... | ... | ... |
| ...       | 1.370       | 1.060       | 1.017       | 0   | 5.13e−01 | 7.74e−01 |
| ...       | 1.342       | 1.061       | 1.022       | 1   | 4.25e−01 | 8.08e−01 |
| ...       | 1.306       | 1.062       | 1.024       | 2   | 3.29e−01 | 8.48e−01 |
| ...       | 1.754       | 1.113       | 0.857       | 3   | 5.47e+00 | 6.50e−02 |
| ...       | 1.166       | 1.056       | 1.048       | 4   | 4.34e−01 | 8.05e−01 |
| ...       | 1.010       | 1.007       | 1.103       | 5   | 2.33e+00 | 3.12e−01 |
| 141.01    | 5.900±0.300 | 5.030±0.090 | 4.600±0.100 | ... | ... | ... |
| ...       | 6.336       | 4.900       | 4.702       | 0   | 5.24e+00 | 7.28e−02 |
| ...       | 6.202       | 4.902       | 4.721       | 1   | 4.51e+00 | 1.05e−01 |
| ...       | 6.038       | 4.909       | 4.735       | 2   | 3.83e+00 | 1.47e−01 |
| ...       | 8.078       | 5.127       | 3.948       | 3   | 9.63e+01 | 1.20e−21 |
| ...       | 5.390       | 4.883       | 4.845       | 4   | 1.16e+01 | 3.10e−03 |
| Period (s) | $u'$ (mmag) | $g'$ (mmag) | $r'$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|-----------|-------------|-------------|-------------|-----|----------|-----|
| ...       | 4.690       | 4.675       | 5.122       | 5   | 5.91e+01 | 1.50e−13 |
| 141.06    | 4.500±0.200 | 3.200±0.070 | 3.040±0.090 | ... | ...     |     |
| ...       | 4.166       | 3.222       | 3.092       | 0   | 3.22e+00 | 2.00e−01 |
| ...       | 4.082       | 3.226       | 3.108       | 1   | 5.07e+00 | 7.91e−02 |
| ...       | 3.976       | 3.233       | 3.118       | 2   | 7.83e+00 | 2.00e−02 |
| ...       | 5.196       | 3.298       | 2.540       | 3   | 4.49e+01 | 1.74e−10 |
| ...       | 3.564       | 3.229       | 3.204       | 4   | 2.54e+01 | 3.07e−06 |
| ...       | 3.128       | 3.118       | 3.416       | 5   | 6.59e+01 | 4.90e−15 |
| 146.50    | 4.000±0.200 | 3.030±0.060 | 2.680±0.080 | ... | ...     |     |
| ...       | 3.834       | 2.963       | 2.842       | 0   | 6.05e+00 | 4.85e−02 |
| ...       | 3.750       | 2.964       | 2.855       | 1   | 7.53e+00 | 2.32e−02 |
| ...       | 3.646       | 2.969       | 2.863       | 2   | 9.43e+00 | 8.97e−03 |
| ...       | 4.766       | 3.060       | 2.361       | 3   | 3.08e+01 | 2.04e−07 |
| ...       | 3.258       | 2.959       | 2.938       | 4   | 2.55e+01 | 2.86e−06 |
| ...       | 2.858       | 2.854       | 3.133       | 5   | 7.32e+01 | 1.25e−16 |
| 150.47    | 0.900±0.200 | 0.690±0.070 | 0.620±0.090 | ... | ...     |     |
| ...       | 0.876       | 0.677       | 0.649       | 0   | 1.54e−01 | 9.26e−01 |
| ...       | 0.858       | 0.678       | 0.653       | 1   | 2.08e−01 | 9.01e−01 |
| ...       | 0.834       | 0.680       | 0.656       | 2   | 2.87e−01 | 8.66e−01 |
| ...       | 1.076       | 0.696       | 0.538       | 3   | 1.62e+00 | 4.45e−01 |
| ...       | 0.746       | 0.679       | 0.674       | 4   | 9.79e−01 | 6.13e−01 |
| ...       | 0.654       | 0.654       | 0.719       | 5   | 2.98e+00 | 2.25e−01 |
| 150.78    | 0.800±0.100 | 0.570±0.070 | 0.600±0.100 | ... | ...     |     |
| ...       | 0.774       | 0.598       | 0.573       | 0   | 2.97e−01 | 8.62e−01 |
| ...       | 0.762       | 0.602       | 0.580       | 1   | 3.99e−01 | 8.19e−01 |
| ...       | 0.746       | 0.608       | 0.587       | 2   | 6.06e−01 | 7.39e−01 |
Table 9—Continued

| Period (s) | $u'$ (mmag) | $g'$ (mmag) | $r'$ (mmag) | $l$ | $\chi^2$ | $Q$ |
|------------|-------------|-------------|-------------|-----|----------|-----|
| ...        | 0.880       | 0.570       | 0.440       | 3   | 3.20e+00 | 2.02e−01 |
| ...        | 0.684       | 0.622       | 0.618       | 4   | 1.94e+00 | 3.80e−01 |
| ...        | 0.618       | 0.618       | 0.679       | 5   | 4.41e+00 | 1.10e−01 |
| 154.94     | 0.700±0.200 | 0.670±0.060 | 0.780±0.080 | ... | ...      | ... |
| ...        | 0.910       | 0.703       | 0.674       | 0   | 3.16e+00 | 2.05e−01 |
| ...        | 0.890       | 0.704       | 0.678       | 1   | 2.86e+00 | 2.39e−01 |
| ...        | 0.864       | 0.705       | 0.680       | 2   | 2.58e+00 | 2.76e−01 |
| ...        | 1.096       | 0.714       | 0.552       | 3   | 1.26e+01 | 1.87e−03 |
| ...        | 0.776       | 0.707       | 0.702       | 4   | 1.47e+00 | 4.81e−01 |
| ...        | 0.686       | 0.687       | 0.756       | 5   | 1.75e−01 | 9.16e−01 |
| 168.80     | 0.600±0.200 | 0.410±0.070 | 0.400±0.100 | ... | ...      | ... |
| ...        | 0.542       | 0.418       | 0.401       | 0   | 9.74e−02 | 9.52e−01 |
| ...        | 0.530       | 0.419       | 0.403       | 1   | 1.41e−01 | 9.32e−01 |
| ...        | 0.514       | 0.420       | 0.405       | 2   | 2.10e−01 | 9.00e−01 |
| ...        | 0.644       | 0.427       | 0.331       | 3   | 5.85e−01 | 7.46e−01 |
| ...        | 0.460       | 0.421       | 0.418       | 4   | 5.47e−01 | 7.61e−01 |
| ...        | 0.406       | 0.408       | 0.450       | 5   | 1.19e+00 | 5.52e−01 |
Table 10. Summary of partial mode identification in pulsating sdB stars

| Object                  | Period (s) | Possible $l$ identification (0–5) | Mode discrimination |
|-------------------------|------------|-----------------------------------|---------------------|
| KPD 2109+4401 (K98)    | 182.42     | 0,1,2                             | partial             |
| ...                     | 184.72     | 0,1,2,4                           | partial             |
| ...                     | 184.75     | 2,4                               | partial             |
| ...                     | 191.85     | 1,2,4                             | partial             |
| ...                     | 196.31     | 0,1,2                             | partial             |
| ...                     | 196.69     | 0,1,2,3,4,5                       | no                  |
| ...                     | 198.19     | 2,4                               | partial             |
| PG 1618+563B (S00)     | 139.30     | 0,1,2,3,4,5                       | no                  |
| ...                     | 143.90     | 0,1,2,3,4,5                       | no                  |
| PG 1605+072 (F03)      | 440.51     | 0,1,2,3,4,5                       | no                  |
| ...                     | 475.82     | 0,1,2,4                           | partial             |
| ...                     | 481.75     | 0,1,2                             | partial             |
| ...                     | 503.70     | 0,1,2,4                           | partial             |
| ...                     | 528.70     | 0,1,2,3,4,5                       | no                  |
| HS 0039+4302 (J04)     | 134.44     | 0,1,2,3,4,5                       | no                  |
| ...                     | 180.13     | 0,1,2,4,5                         | partial             |
| ...                     | 181.89     | 0,1,2                             | partial             |
| ...                     | 192.58     | 0,1,2                             | partial             |
| ...                     | 234.11     | 0,1,2                             | partial             |
| BAL 090100001 (O05)   | 351.70     | no acceptable fit                 | no                  |
| ...                     | 356.30     | no acceptable fit                 | no                  |
| BAL 090100001 (B05)   | 264.08     | 0,1,2,3,4                         | partial             |
| ...                     | 264.82     | 0,1,2,3                           | partial             |
| ...                     | 349.83     | 0,1,2,3,4,5                       | no                  |
| Object | Period (s) | Possible \( l \) identification (0–5) | Mode discrimination |
|--------|------------|-----------------------------------|---------------------|
| ...    | 350.18     | 0,1,2,3,4                         | partial             |
| ...    | 350.39     | 0,1,2,3,4                         | partial             |
| ...    | 353.81     | 1,2,4                             | partial             |
| ...    | 354.01     | 1,2                               | partial             |
| ...    | 354.20     | 1,2                               | partial             |
| ...    | 356.19     | 0,1                               | partial             |
| PG 0014+067 (J05) | 100.29  | 0,1,2,3,4,5                       | no                  |
| ...    | 139.14     | 0,1,2,3,4,5                       | no                  |
| ...    | 140.98     | 0,1,2,3,4,5                       | no                  |
| ...    | 141.01     | 0,1,2,4                           | partial             |
| ...    | 141.06     | 0,1,2                             | partial             |
| ...    | 146.50     | 0,1,2                             | partial             |
| ...    | 150.47     | 0,1,2,3,4,5                       | no                  |
| ...    | 150.78     | 0,1,2,3,4,5                       | no                  |
| ...    | 154.94     | 0,1,2,3,4,5                       | no                  |
| ...    | 168.80     | 0,1,2,3,4,5                       | no                  |
REFERENCES

Aerts, C., Jeffery, C.S., Fontaine, G., Dhillon, V.S., Marsh, T.R., & Groot, P. 2006, MNRAS, in press

Baran, A., Pigulski, A., Koziel, D., Ogloza, W., Silvotti, R., & Zola, S. 2005, MNRAS, 360, 737 (B05)

Brassard, P., Fontaine, G., Billères, M., Charpinet, S., Liebert, J., & Saffer, R.A. 2001, ApJ, 563, 1013

Breger, M., Handler, G., Garrido, R., Audard, N., Beichbuchner, F., Zima, W., Paparo, M., Zhi-Ping, L., Shi-Yang, J., Zong-li, L., Ai-ying, Z., Pikall, H., Stankov, A., Guzik, J.A., Sperl, M., Krzesinski, J., Ogloza, W., Pajdosz, G., Zola, S., Serkowitsch, E., Reegen, P., Rumpf, T., & Schmalwieser, A. 1997, A&A, 324, 566

Charpinet, S., Fontaine, G., & Brassard, P. 2003, in NATO ASIB Proc. 105: White Dwarfs, 69

Charpinet, S., Fontaine, G., & Brassard, P. 2001, PASP, 113, 785

Charpinet, S., Fontaine, G., Brassard, P., Billères, M., Green, E.M., & Chayer, P. 2005b, A&A, 443, 251

Charpinet, S., Fontaine, G., Brassard, P., Billères, M., Green, E.M., & Chayer, P. 2005c, in 14th European Workshop on White Dwarfs, ASP Conf. Series 334, ed. D. Koester & S. Moehler (San Francisco: ASP), 619

Charpinet, S., Fontaine, G., Brassard, P., Chayer, P., Rogers, F.J., Iglesias, C.A., & Dorman, B. 1997, ApJ, 483, L123

Charpinet, S., Fontaine, G., Brassard, P., Green, E.M., & Chayer, P. 2005a, A&A, 437, 575

Cugier, H., Dziembowski, W., & Pamyatnykh, A.A. 1994, A&A, 291, 143

Dupret, M.-A., De Ridder, J., De Cat, P., Aerts, C., Scuflaire, R., Noels, A., & Thoul, A. 2003, A&A, 398, 677

Falter, S., Heber, U., Dreizler, S., Schuch, S.L., Cordes, O., & Edelmann, H. 2003, A&A, 401, 289 (F03)

Fontaine, G., Brassard, P., Bergeron, P., & Wesemael, F. 1996, ApJ, 469, 320

Garrido, R., García-Lobo, E., & Rodríguez, E. 1990, A&A, 234, 262

Heber, U., Reid, I.N., & Werner, K. 1999, A&A, 348, 25

Heynderickx, D., Waelkens, C., & Smeyers, P. 1994, A&A, 105, 447

Jeffery, C.S., Aerts, C., Dhillon, V., Marsh, T., & Gänsicke, B.T. 2005, MNRAS, 362, 66 (J05)
Jeffery, C.S., Dhillon, V., Marsh, T., & Ramachandran, B. 2004, MNRAS, 352, 699 (J04)
Koen, C., 1998 MNRAS, 300, 567 (K98)
Montgomery, M.H., & O’Donoghue, D. 1999, in Delta Scuti Star Newsletter, No.13
Oreiro, R., Pérez Hernández, F., Ulla, A., Garrido, R., Østensen, R., & MacDonald, J. 2005, A&A, 438, 257 (O05)
Oreiro, R., Ulla, A., Pérez Hernández, F., Østensen, R., Rodríguez-López, C., & MacDonald, J. 2004, A&A, 418, 243
Press, W.H., Flannery, B.P., Teukolsky, S.A., & Vetterling, W.T. 1986, Numerical Recipes (Cambridge: Cambridge University Press)
Ramachandran, B., Jeffery, C.S., & Townsend, R.H.D. 2004, A&A, 428, 209
Randall, S.K., Fontaine, G., Brassard, P., & Bergeron, P. 2005, ApJS, 161, 456
Randall, S.K., Fontaine, G., Green, E.M., Brassard, P., Kilkenny, D., Crause, L., Terndrup, D.M., Daane, A., Kiss, L.L., Jacob, A.P., Bedding, T.R., For, B.-Q., Quirion, P.-O., & Chayer, P. 2006a, ApJ, in press
Randall, S.K., Fontaine, G., Green, E.M., Brassard, P., Terndrup, D.M., Brown, N., Fontaine, M., Zacharias, P., & Chayer, P. 2006b, ApJ, in press
Robinson, E.L., Mailloux, T.M., Zhang, E., Koester, D., Stiening, R.F., Bless, R.C., Percival, J.W., Taylor, M.J., van Citters, G.W. 1995, ApJ, 438, 908
Silvotti, R., Solheim, J.-E., Gonzalez Perez, J.M., Heber, U., Dreizler, S., Edelmann, H., Østensen, R., & Kotak, R. 2000, A&A, 359, 1068 (S00)
Watson, R.D. 1988, Ap&SS, 140, 255

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FIGURE CAPTIONS

Fig. 1 — Fits to the $U$, $B$, $V$, and $R$ pulsational amplitudes observed for the 182.42 s mode of KPD 2109+4401 by K98. The predicted amplitude-wavelength behaviors of modes with $l = 0$ to $l = 5$ have been fit to the observed values using a least-squares procedure. The curves with $l = 0$, $l = 1$, and $l = 2$ provide acceptable fits according to the quality-of-fit $Q$ quantity (see text). In contrast, the curves with $l = 3$, $l = 4$, and $l = 5$ do not provide viable fits and, consequently, these possible $l$ identifications must be rejected for that mode.

Fig. 2 — Similar to Fig. 1, but for the mode with a period of 196.31 s. Here the acceptable fits are again for values of the degree index $l = 0$, $l = 1$, and $l = 2$.

Fig. 3 — Similar to Fig. 1, but for the mode with a period of 198.19 s. Here the only acceptable fit from a formal point of view is the one corresponding to $l = 4$.

Fig. 4 — Fits to the $U$, $B$, and $V$ pulsational amplitudes observed for the 139.3 s mode of PG 1618+563B by S00. No mode discrimination is possible here.

Fig. 5 — Fits to the BUSCA ($UV$, $B$, $R$, and $NIR$) pulsational amplitudes observed for the 475.82 s mode of PG 1605+072 by F03. Partial mode discrimination is possible here, with the values $l = 3$ and $l = 5$ excluded.

Fig. 6 — Similar to Fig. 5, but for the mode with a period of 481.75 s. Here the acceptable fits are for values of the degree index $l = 0$, $l = 1$, and $l = 2$.

Fig. 7 — Similar to Fig. 5, but for the mode with a period of 503.70 s. Partial mode discrimination is possible, with the values $l = 3$ and $l = 5$ excluded.

Fig. 8 — Fits to the ULTRACAM ($u'$, $g'$, and $r'$) pulsational amplitudes observed for the 181.89 s mode of HS 0039+4302 by J04. The acceptable values of $l$ for that mode are $l = 0$, $l = 1$, and $l = 2$.

Fig. 9 — Similar to Fig. 8, but for the mode with a period of 192.58 s. Here the acceptable fits are for values of the degree index $l = 0$, $l = 1$, and $l = 2$.

Fig. 10 — Similar to Fig. 8, but for the mode with a period of 243.11 s. The acceptable fits are for values of the degree index $l = 0$, $l = 1$, and $l = 2$.

Fig. 11 — Fits to the $u$, $v$, $b$ and $y$ pulsational amplitudes observed for the 356.3 s mode of BAL 090100001 by O05. No mode discrimination is possible here.

Fig. 12 — Fits to the $U$, $B$, $V$, and $R$ pulsational amplitudes observed for the 356.19 s mode of BAL 090100001 by B05. There is no acceptable formal fit, but it is clear that the mode must have a degree index $l = 0$ or $l = 1$. 
Fig. 13 — Similar to Fig. 12, but for the mode with a period of 354.20 s. Formally, the only acceptable ($Q > 0.001$) fit is that with $l = 2$, although the solution with $l = 1$ should not be discarded since the data points fall in between those two model curves.

Fig. 14 — Similar to Fig. 12, but for the mode with a period of 354.01 s. The acceptable fits are for values of the degree index $l = 1$, and $l = 2$.

Fig. 15 — Similar to Fig. 12, but for the mode with a period of 353.81 s. The acceptable fits are for values of the degree index $l = 1$, $l = 2$, and $l = 4$.

Fig. 16 — Fits to the ULTRACAM ($u'$, $g'$, and $r'$) pulsational amplitudes observed for the 141.01 s mode of PG 0014+067 by J05. The acceptable values of $l$ for that mode are $l = 0$, $l = 1$, $l = 2$, and $l = 4$.

Fig. 17 — Similar to Fig. 16, but for the mode with a period of 141.06 s. Here the acceptable fits are for values of the degree index $l = 0$, $l = 1$, and $l = 2$.

Fig. 18 — Similar to Fig. 16, but for the mode with a period of 146.50 s. The acceptable fits are for values of the degree index $l = 0$, $l = 1$, and $l = 2$. 
Figure 1
Figure 2

Amplitude (mmag) vs. Wavelength (Å) for the 196.31 s mode with different $l$ values.
Figure 3
Figure 4

139.3 s mode

Amplitude (mmag)

Wavelength (Å)

$\ell = 5$

$\ell = 4$

$\ell = 1$

$\ell = 3$
Figure 5

Amplitude (mmag)

Wavelength (Å)

475.82 s mode

l = 0
l = 2
l = 3
l = 4
l = 5
Figure 7

The figure shows a graph of amplitude (mmag) versus wavelength (Å) for different $\ell$ modes. The $503.70$ s mode is highlighted with vertical bars indicating the amplitude at specific wavelengths for $\ell = 1, 2, 3, 4, 5$. The graph demonstrates the variation of amplitude across different wavelengths for each mode.
Figure 8

181.89 s mode

Amplitude (mmag)

Wavelength (Å)
Figure 9

Amplitude (mmag)

Wavelength (Å)

192.58 s mode

\( l = 0 \)
\( l = 2 \)
\( l = 3 \)
\( l = 4 \)
\( l = 5 \)
Figure 10

234.11 s mode

Amplitude (mmag)

Wavelength (Å)
Figure 11

356.3 s mode

Amplitude (mmag)

Wavelength (Å)
Figure 12

356.19 s mode

Amplitude (mmag)

Wavelength (Å)

$l = 5$

$l = 4$

$l = 2$

$l = 1$

$l = 0$

$l = 3$
Figure 13
Figure 14

354.01 s mode

Wavelength (Å)

Amplitude (mmag)
Figure 15
Figure 16
Figure 17

Amplitude (mmag) vs. Wavelength (Å) for the 141.06 s mode.
Figure 18

The figure shows a graph of amplitude (mmag) versus wavelength (Å) for the 146.50 s mode. The graph includes several lines labeled with different $l$ values: $l = 0$, $l = 2$, $l = 3$, $l = 5$, and $l = 1$. The lines represent different modes of oscillation.