Graded CTL Model Checking for Test Generation

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Abstract

Recently there has been a great attention from the scientific community towards the use of the model-checking technique as a tool for test generation in the simulation field. This paper aims to provide a useful mean to get more insights along these lines. By applying recent results in the field of graded temporal logics, we present a new efficient model-checking algorithm for Hierarchical Finite State Machines (HSM), a well established symbolism long and widely used for representing hierarchical models of discrete systems. Performing model-checking against specifications expressed using graded temporal logics has the peculiarity of returning more counterexamples within a unique run. We think that this can greatly improve the efficacy of automatically getting test cases. In particular we verify two different models of HSM against branching time temporal properties.

1. INTRODUCTION

The model-checking is a widely used technique to verify correctness of hardware and software systems. A model checker explores the state space of a model of a given system to determine whether a given specification is satisfied. Usually such specifications are expressed by means of formulas in a temporal logic, such as the Computational Temporal Logics CTL. A very useful feature to fix the possible errors in the model is that when the model checker detects that the specification is violated then it returns a counterexample. In the last years this feature has also been exploited in the simulation framework. In fact, it is nowadays a well-established fact that formal (both software and hardware) analysis is a valid complementary technique to simulation and testing (see e.g., [7]). On one side, the model checking approach, allows a full verification of system components to be free of errors, but its use is limited to small and medium sized models, due to the so-called state explosion problem. On the other hand the testing and simulation approaches are usually applied to larger systems: they check the presence of errors in the system behavior through the observation of a chosen set of controlled executions. Shortly, the efficacy of testing relies on the creation of test benches and that of model-checking on the ability of formally defining the properties to be verified, through temporal logic formulas. More explicitly, the complementarity of the two techniques lies in the fact that the counterexamples generated by a model-checker can be interpreted as test cases. A good choice of the test suite is the key for successful deductions of faults in simulation processes. It is now more than a decade that model-checking is used for this purpose, see [10, 15, 4, 2, 3, 11]. In this context, a high level abstraction of the System Under Test (SUT), is necessary. Such abstraction should be simple and easy to model check, but precise enough to serve as a basis for the generation of test cases. This approach can be usefully adopted also in the DEVS modeling and simulation framework, [16].

However not surprisingly, the most challenging problem is the performance and two issues are crucial: the choice of an efficient tool to generate the test suite and the choice of a suitable abstract model to check.

For the first issue, we propose the use of graded temporal logic specifications. In fact standard model-checking tools generate only one counterexample for each run and the check stage (of the model against a specification) is often expensive, in terms of time resources. We claim that it is highly desirable to get more meaningful counterexamples with a unique run of the model checker. For the second issue we propose the use of HSM as an abstract model of a DEVS modeling the SUT, which preserves the hierarchical structure while abstracting the continuous variables. Thus we focus on how to generate simulation scenarios for DEVS by providing a tool which automatically generates multiple counter-examples in an unique run, using hierarchical state machines as abstract model. The sequence of events of each counterexample will then be used to create a timed test trace for DEVS simulation. In Figure 1 a small example of our idea is shown (the states labeled Try1 and Try2 are states on a higher hierarchy level standing for the graph $M_1$). Suppose we want to check whether the (timed) model in the figure satisfies the specification (clearly false) stating that if a Fail occurs in the first attempt (Try1) of sending a message, then an Abort event is eventually reached. We can model-check an (untimed) over-approximation of the model (shown on the left) obtaining the error trace Start, Try1.(Send, Wait, Timeout, Fail), Try2.(Send, Wait, Ack), Success. This trace lets us concentrate on the portion of the model with a potential error and can guide the simulation process to detect the error in the timed model. Let us now briefly detail the two notions of graded logics and
HSM. In order to get more counterexamples in a unique run we use specifications expressed in graded-CTL, recently introduced in [9]. Graded-CTL strictly extends classical CTL with graded modalities: classical CTL can be used for reasoning about the temporal behavior of systems considering either all the possible futures or at least one possible future, while graded-CTL uses graded extensions on both existential and universal quantifiers. With graded-CTL formulas one can describe a constant number of future scenarios. For example, one can express that in $k$ different cases it is possible that a waiting process never obtains a requested resource, or that there are $k$ different ways for a system to reach a safe state from a given state.

The notion of finite state machine with a hierarchical structure has been used for many years for modeling discrete systems, since the introduction of Statecharts, [12], and is actually applied into many fields as a specification formalism. In particular, in the model-checking framework, one of the most considered models is the Hierarchical State Machine (HSM) (see e.g. [1]). A generalization of HSM is introduced in [13], as an exponentially more succinct model where also higher level states, called boxes, are labeled with atomic propositions. The intended meaning of such labeling is that when a box $b$ expands to a machine $M$, all the vertices of $M$ inherit the atomic propositions of $b$ (scope), such that different vertices expanding to $M$ can place $M$ into different scopes. Such model is called a hierarchical state machine with scope-dependent properties (Scope-dependent Hierarchical State Machine, shortly SHSM).

Our contribution aims in providing also strong theoretical evidence of the soundness of our approach. In particular we study the problem of verifying whether an SHSM models a given graded-CTL formula. We first give an algorithm to solve the graded-CTL model-checking of an HSM, and then we extend it to model-check general SHSMs. We show that the problem has the same computational complexity as CTL model checking, and we show how to solve it both for HSM and SHSM, with an extra factor in the exponent which is logarithmic in the maximal grading constant occurring in the CTL formula. Let us stress that the experimental results for flat models reported in [9] shows that this extra factor does not have real effects in the running time of the algorithms (currently we are implementing also the algorithms presented here for hierarchical structures and the initial tests are very promising).

The rest of the paper is organized as follows: in Sections 2 and 3 we give basic definitions and known results of graded-CTL, and of SHSM, respectively; in Section 4 we give the algorithm to model-check SHSM against graded-CTL specifications. In Section 5 we give our conclusions.

2. GRADED CTL

In this section we first recall the definitions of CTL and then give that of graded-CTL, see [9]. The temporal logic CTL [5] is a branching-time logic in which each temporal operator, expressing properties about a possible future, has to be preceded either by an existential or by an universal path quantifier. So, in CTL one can express properties that have to be true either immediately after now ($X$), or each time from now ($G$), or from now until something happens ($\exists$), and it is possible to specify that each property must hold either in some possible futures ($E$) or in each possible future ($A$). Formally, given a finite set of atomic propositions $AP$, CTL is the set of formulas $\varphi$ defined as follows:

$$\varphi ::= p | \neg \psi_1 | \psi_1 \land \psi_2 | E \psi_1 | G \psi_1 | E \psi_1 U \psi_2$$

where $p \in AP$ is an atomic proposition and $\psi_1$ and $\psi_2$ are CTL formulas. The semantics of a CTL formula is defined with respect to a Kripke Structure $\mathcal{K}$ by means of the classical relation $\models$. As usual, a Kripke structure over a set of atomic propositions $AP$ is a tuple $\mathcal{K} = (S, S_0, R, L)$, where $S$ is a finite set of states, $S_0 \subseteq S$ is the initial state, $R \subseteq S \times S$ is a transition relation with the property that for each $s \in S$ there is $t \in S$ such that $(s, t) \in R$, and $L: S \rightarrow 2^{AP}$ is a labeling function. A path in $\mathcal{K}$ is denoted by the sequence of states
graded-CTL formulas. The semantics of graded-CTL is still defined as follows:

- \((\mathcal{K}, s) \models p \in AP\) iff \(p \in L(s)\);
- \((\mathcal{K}, s) \models \neg \psi_1\) iff \(\neg((\mathcal{K}, s) \models \psi_1)\) (in short, \((\mathcal{K}, s) \not\models \psi_1)\);
- \((\mathcal{K}, s) \models \psi_1 \land \psi_2\) iff \((\mathcal{K}, s) \models \psi_1\) and \((\mathcal{K}, s) \models \psi_2\);
- \((\mathcal{K}, s) \models \psi_1 \land \psi_2\) iff there exists \(s' \in S\) such that \((s, s') \in R\) and \((\mathcal{K}, s') \models \psi_1\) (the path \(s, s'\) is called an evidence of the formula \(\psi_1\));
- \((\mathcal{K}, s) \models \psi_1 \land \psi_2\) iff there exists an infinite path \(s\) starting from \(s\) (i.e., \(\pi(0) = s\)) such that for all \(j \geq 0\), \((\mathcal{K}, \pi(j)) \models \psi_1\) (the path \(\pi\) is called an evidence of the formula \(\psi_1\));
- \((\mathcal{K}, s) \models \psi_1 \land \psi_2\) iff there exists a finite path \(\pi\) with length \(|\pi| = r + 1\) starting from \(s\) such that \((\mathcal{K}, \pi[r]) \models \psi_2\) and, for all \(0 \leq j < r\), \((\mathcal{K}, \pi[j]) \models \psi_1\) (the path \(\pi\) is called an evidence of the formula \(\psi_1 \land \psi_2\)).

We say that a Kripke structure \(\mathcal{K} = (S, s_m, R, L)\) models a CTL formula \(\phi\) iff \((\mathcal{K}, s_m) \models \phi\). Note that we have expressed the syntax of CTL with one of the possible minimal sets of operators. Other temporal operators as well as the universal path quantifier \(A\), can be easily derived from those. **Graded-CTL** extends the classical CTL by adding graded modalities on the quantifier operators. Graded modalities specify in how many possible futures a given path property has to hold, and thus generalize CTL allowing to reason about more than a given number of possible distinct future behaviors. Let us first define the notion of *distinct*. Let \(\mathcal{K} = (S, s_m, R, L)\) be a Kripke structure. We say that two paths \(\pi_1\) and \(\pi_2\) on \(\mathcal{K}\) are *distinct* if there exists an index \(0 \leq i < \min\{|\pi_1|, |\pi_2|\}\) such that \(\pi_1[i] \neq \pi_2[i]\). Observe that from this definition if a path is the prefix of another path, then they are not distinct. The graded existential path quantifier \(E^{>k}\), requires the existence of \(k + 1\) pairwise distinct evidences of a path-formula. Given a set of atomic proposition \(AP\), the syntax of graded-CTL is defined as follows:

\[
\varphi := p | \neg \psi_1 | \psi_1 \land \psi_2 | E^{>k} x \psi_1 | E^{>k} \neg \psi_1 | E^{>k} \psi_1 \land \psi_2
\]

where \(p \in AP\), \(k\) is a non-negative integer and \(\psi_1\) and \(\psi_2\) are graded-CTL formulas. The semantics of graded-CTL is still defined with respect to a Kripke structure \(\mathcal{K} = (S, s_m, R, L)\) on the set of atomic propositions \(AP\). In particular, for formulas of the form \(\neg \psi_1\) and \(\psi_1 \land \psi_2\) the semantics is the same as in the classical CTL. For the remaining formulas, the semantics is defined as follows:

- \((\mathcal{K}, s) \models E^{>k} \theta\), with \(k \geq 0\) and either \(\theta = x \psi_1\) or \(\theta = \neg \psi_1\) or \(\theta = \psi_1 \land \psi_2\), iff there exist \(k + 1\) pairwise distinct evidences of \(\theta\) starting from \(s\).

It is easy to observe that classical CTL is a proper fragment of graded-CTL since the simple guarded formula \(E^{>1} x p\) cannot be expressed in CTL, whereas any CTL formula is also a graded-CTL formula (note that \(E^{>0} \theta\) is equivalent to \(E\theta\)). We can also consider the graded extension of the universal quantifier, \(A^{\leq k}\), with the meaning that all the paths starting from a node \(s\), but at most \(k\) pairwise distinct paths, are evidences of a given path-formula. The quantifier \(A^{\leq k}\) is the dual operator of \(E^{>k}\) and can obviously be re-written in terms of \(\neg E^{<k}\). However, while \(A^{\leq k} x \psi_1\) and \(A^{\leq k} \neg \psi_1\) can be easily re-written respectively as \(\neg E^{>k} x \neg \psi_1\) and \(\neg E^{>k} \neg \neg \psi_1\), the transformation of the formula \(A^{\leq k} \psi_1 \land \psi_2\) with \(k > 0\) in terms of \(\neg E^{<k}\) deserves more care (see [9] for a detailed treatment).

The **graded-CTL model-checking** is the problem of verifying whether a Kripke structure \(\mathcal{K}\) models a graded-CTL formula \(\phi\). The complexity of the graded-CTL model-checking problem is linear with respect to the size of the Kripke structure and to the size of the formula, (this latter being the number of the temporal and the boolean operators occurring in it). Let us remark that this complexity is independent from the integers \(k\) occurring in the formula.

### 3. SCOPE-DEPENDENT HIERARCHICAL STATE MACHINES

In this section we formally define the Scope-dependent Hierarchical State Machines and recall some known results. The Scope-dependent Hierarchical State Machines are defined as follows.

**Definition 1.** A Scope-dependent Hierarchical State Machine (SHSM) over \(AP\) is a tuple \(M = (M_1, M_2, \ldots, M_h)\), each \(M_i = (V_i, in_i, OUT_i, TRUE_i, expn, E_i)\) is called machine and consists of:

- a finite set of vertices \(V_i\), an initial vertex \(in_i \in V_i\) and a set of output vertices \(OUT_i \subseteq V_i\);
- a labeling function \(TRUE_i : V_i \rightarrow 2^{AP}\) that maps each vertex with a set of atomic propositions;
- an expansion function \(expn_i : V_i \rightarrow \{0, 1, \ldots, h\}\) such that \(expn_i(u) < i\), for each \(u \in V_i\), and \(expn_i(u) = 0\), for each \(u \in \{in_i\} \cup OUT_i\);
- a set of edges \(E_i\) where each edge is either a couple \((u, v), with u, v \in V_i\) and \(expn_i(u) = 0\), or a triple \((u, z, v), with u, v \in V_i, expn_i(u) = j, j > 0, and z \in OUT_j\).

In the rest of the paper we use \(h\) as the number of machines of an SHSM \(M\) and \(M_h\) is called top-level machine.
We assume that the sets of vertices $V_i$ are pairwise disjoint. The set of all vertices of $M$ is $V = \bigcup_{i=1}^{k} V_i$. The mappings $\expn : V \rightarrow \{0, 1, \ldots, h\}$ and $\true : V \rightarrow 2^A$ extend the mappings $\expn_i$ and $\true_i$, respectively. If $\expn(u) = j > 0$, the vertex $u$ expands to the machine $M_j$ and is called box. When $\expn(u) = 0$, $u$ is called a node. Let us define the closure $\expn^+ : V \rightarrow 2^{\{0, 1, \ldots, h\}}$, as: $h \in \expn^+(u)$ if either $h = \expn(u)$ or there exists $u' \in \expn(u)$ such that $h \in \expn^+(u')$.

We say that a vertex $u$ is an ancestor of $v$ and $v$ is a descendant from $u$ if $v \in V_h$, for $h \in \expn^+(u)$.

A vertex $v \in V_i$ is called a successor of $u \in V_i$ if there is an edge $(u, v) \in E_i$, and it is called a $z$-successor of $u$, for $z \in \OUT_{\expn(u)}$, if $((u, z), v) \in E_i$.

An HSM is an SHSM such that $\true(b) = \emptyset$, for any box $b$.

As an example of an SHSM $M$ see Figure 2 where $p_1, p_2, p_3$ are atomic propositions labeling nodes and boxes of $M$, $i_1$, and $z_1$ are respectively entry nodes and exit nodes for $i = 1, 2, 3,$ and $\expn(b'_j) = j - 1$ for $i = 0, 1$ and $j = 2, 3$.

**Semantics.** The semantics of an SHSM $M$ is given by a flat Kripke structure, denoted $M^F$.

A sequence of vertices $\alpha = u_1 \ldots u_m$, $1 \leq m$, is called a well-formed sequence if $u_{\ell+1} \in \expn(u_\ell)$, for $\ell = 1, \ldots, m - 1$. Moreover, $\alpha$ is also complete when $u_1 \in V_i$ and $u_m$ is a node.

A state of $M^F$ is $\langle \alpha \rangle$ where $\alpha$ is a complete well-formed sequence of $M$. Note that the length of a complete well-formed sequence is at most $h$, therefore the number of states of $M^F$ is at most exponential in the number of machines composing $M$. Transitions of $M^F$ are obtained by using as templates the edges of $M$. Figure 3 shows the Kripke structure which is equivalent to the SHSM of Figure 2. We formally define $M^F$ as follows. Given an SHSM $M = (M_1, M_2, \ldots, M_h)$, it is immediate to observe that the tuple $M_j = (M_1, M_2, \ldots, M_j)$, $1 \leq j \leq h$, is an SHSM as well. Clearly, $M_h = M$. In the following, we sketch how to compute recursively the flat Kripke structures $M^F_j$.

We start with $M^F_1$ which is obtained from machine $M_1$ by simply replacing each vertex $u$ with a state $\langle u \rangle$ labeled with $\true(u)$ (recall that by definition all vertices of $M_1$ are nodes). Thus, for each edge $\langle v, w \rangle \in E_1$ we add a transition $\langle \langle v \rangle, \langle w \rangle \rangle$ in $M^F_1$.

For $j > 1$, $M^F_j$ is obtained from $M_j$ by simply replacing each box $u$ of $M_j$ with a copy of the Kripke structure $M^F_{\expn(u)}$. More precisely, for each node $u \in V_j$, $\langle u \rangle$ is a state of $M^F_j$ which is labeled with $\true(u)$ and for each box $u \in V_j$ and state $\langle \alpha \rangle$ of $M^F_{\expn(u)}$, $\langle \langle u \alpha \rangle \rangle$ is a state of $M^F_j$ and is labeled with $\true(u) \cup \true(\langle \alpha \rangle)$. The transitions of $M^F_{\expn(u)}$ are all inherited in $M^F_j$, that is, there is a transition $\langle \langle u \alpha \rangle, \langle \beta \rangle \rangle$ in $M^F_j$ for each transition $\langle \alpha, \beta \rangle$ of $M^F_{\expn(u)}$. The remaining transitions of $M^F_j$ correspond to the edges of $M_j$:

- for each node $v \in V_j$ and edge $\langle u, v \rangle \in E_j$ (resp. $\langle u, z \rangle \in E_j$) there is a transition from $\langle u \rangle$ (resp. $\langle u \rangle$) to $\langle v \rangle$;

- for each box $v \in V_j$ and edge $\langle u, v \rangle \in E_j$ (resp. $\langle u, z \rangle \in E_j$) there is a transition from $\langle u \rangle$ (resp. $\langle u \rangle$) to $\langle v \rangle$.

A box $u$ expanding into $M_j$ is a placeholder for $M^F_j$ and determines a subgraph in $M^F$ isomorphic to $M^F_j$. This is emphasized in Figure 3 where we have enclosed in shades of the same shape and color the isomorphic subgraphs corresponding to a same graph $M^F_j$. Therefore, Figure 3 also illustrates the recursive definition of $M^F$.

If two distinct boxes $u_1$ and $u_2$ both expand into the same machine $M_j$, that is $\expn(u_1) = \expn(u_2) = h$, then the states of $M^F_j$ appear in $M^F$ in two different scopes, possibly labeled with different sets of atomic propositions: in one scope this set contains $\true(u_1)$ and in the other it contains $\true(u_2)$. The atomic propositions labeling boxes represent scope-properties. In fact, for a given box $u$, the set $\true(u)$
of atomic propositions is meant to hold true at \( u \) and at all its possible descendants.

**Succinctness.** Clearly, any hierarchical structure, either an HSM or an SHSM, is in general more succinct than a traditional Kripke structure. Scope properties make SHSM possibly even more succinct than HSM. In fact, two isomorphic Kripke structures can be represented in an SHSM by two different machines in an HSM. Let us recall two main results from [13] on the succinctness of these models, where a restricted SHSM \( M \) is a SHSM where for all vertices \( u,v \) such that \( u \) is an ancestor of \( v \) in \( M \) it holds that \( \text{true}(u) \cap \text{true}(v) = \emptyset \).

**Theorem 1 ([13]).** Restricted SHSMs can be exponentially more succinct than HSMS and finite state machines.

There is an exponential gap also between restricted SHSMs and SHSMs as shown in the following proposition.

**Theorem 2 ([13]).** SHSMs can be exponentially more succinct than restricted SHSMs.

Observe that HSMS, restricted SHSMs and SHSMs can all be translated to equivalent finite state machines with a single exponential blow-up. Thus, the two succinctness results do not add up to each other, in the sense that it is not true that SHSMs can be double exponentially more succinct than HSMS.

### 4. Model Checking Problem

The **CTL model-checking** is the problem of verifying whether a Kripke structure \( K \) models a CTL formula. For an SHSM \( M \), the **CTL model-checking** is the problem of verifying whether the flat structure \( M^F \) models a CTL formula. It is known that the CTL model-checking problem can be solved in linear time in the size of both the formula and the machine, see [5], while it is exponential for both HSM and SHSM. More precisely, the following theorem holds.

**Theorem 3 ([11],[13]).** The CTL model-checking of an SHSM \( M \) for a formula \( \varphi \) can be solved in \( O(|M| \cdot 2^{\theta(d)} \cdot |AP_{\varphi}|) \) time, where \( d \) is the maximum number of exit nodes of \( M \) and \( AP_{\varphi} \) is the set of atomic proposition occurring in \( \varphi \). Moreover, if \( M \) is an HSM, then it can be solved in \( O(|M| \cdot 2^{\theta(d)}) \) time.

In this section we extend the result to model-checking a hierarchical structure against a graded-CTL formula. We first show an algorithm for graded-CTL model-checking of an HSM, and then we extend it to model-check SHSMs.

The aim of the algorithm is to determine, for each node \( u \) in a machine \( M_j \) of \( M \) and each subformula \( \psi \) of \( \varphi \), whether \( u \) satisfies \( \psi \) or not. Anyway, the concept of satisfiability may be ambiguous, since whether \( u \) satisfies \( \psi \) or not may depend on the possible different sequences of boxes which expand in \( M_j \). Thus, the algorithm transforms \( M \) in such a way that either for every box sequence \( b_1, \ldots, b_m \) it holds that \( (M^F, \langle b_1, \ldots, b_m u \rangle) \models \psi \) (and in this case we say that \( u \) satisfies \( \psi \)), or for every \( b_1, \ldots, b_m \) it holds that \( (M^F, \langle b_1, \ldots, b_m u \rangle) \not\models \psi \). This transformation determines multiple copies of each \( M_j \), for \( j \leq h \) (clearly, since there are no nodes expanding in the top-level machine \( M_h \), there is not such ambiguity for a \( u \in M_h \)).

The algorithm considers the subformulas \( \psi \) of \( \varphi \), starting from the innermost subformulas, and, for each node \( u \) in \( M \) sets \( u.\psi = \text{TRUE} \) if \( u \) satisfies \( \psi \), modifying possibly the hierarchical structure. If \( \psi \) is an atomic proposition or it is either \( \neg \theta \) or \( \theta_1 \wedge \theta_2 \), the algorithm is trivial. For subformulas with temporal operators and grade 0, then the algorithm behaves exactly as in [1] for the CTL model-checking. We now show how it behaves for subformulas of the form \( \psi = E^k \theta \), with \( k > 0 \) and \( \theta \in \{ x \theta_1, \neg \theta_1, \theta_1 \cap \theta_2 \} \). By inductive hypothesis, we assume that the algorithm has already set \( u.\theta_i = \text{TRUE} \) if
u satisfies $\theta_i$, for $i = 1, 2$.

The algorithm for $\psi = E^{\geq k}x \theta_1$ is rather simple. It starts from the nodes of $M_1$, setting $u, \psi = \text{TRUE}$ if $u$ satisfies $\psi$, and then inductively considers all the machines. Let $u$ be a node of $M_j$. If $u \notin OUT_j$, then it satisfies $\psi$ if there are at least $k + 1$ successors in $M_j$ satisfying $\theta_1$. For an output node $z \in OUT_j$, whether $z$ satisfies $\psi$ depends also on the successors of a box expanding in $M_j$. Multiple copies of $M_j$ are then created, denoted $M_j^g$, where $g : OUT_j \rightarrow \{0, \ldots, k + 1\}$, which correspond to the different contexts in which $M_j$ occurs. The nodes of $M_j^g$ are $u^g$, for a node $u$ of $M_j$, and the boxes are $b^g$, for a box $b$ of $M_j$. The idea is that $g(z)$ is the number of $z$-successors, satisfying $\theta_1$, of a box expanding in $M_j$ (recall that the edges outgoing from a box $b$ are of the type $((b, z), v)$, and we call such $v$ a $z$-successor of $b$). Thus, the algorithm sets $z^g, \psi = \text{TRUE}$ if the sum of $g(z)$ and the number of successors in $M_j$ satisfying $\theta_1$ is greater than $k$. Moreover, for each box $b$, the algorithm calculates the number of $z$-successors of $b$ satisfying $\theta_1$. The new HSM is then obtained by defining the new expansion of $b$ in $M_j$: $b$ expands in the copy $M_j^g$ such that $g(z)$ is the number of $z$-successors of $b$ satisfying $\theta_1$.

Consider now formulas of the type $\psi = \text{expn}_b \theta_1$ and let us call $\psi^l = E^{> k}G \theta_1$.

The algorithm first determines which nodes of the HSM $M$ satisfy the CTL formula $\psi^l$. At the end of this step $M$ is modified in such a way that each node $u$ either satisfies $\psi^l$ or satisfies $\neg \psi^l$. In doing that, the size of $M$ may double (cf. $\Pi$). Call $S$ the set of the nodes satisfying $\psi^l$.

The algorithm determines, for each node $u \in S$, whether $u$ satisfies $\psi$ using the following idea. Let a sink-cycle be a cycle containing only nodes with out-degree 1.

**Claim 1.** Consider the graph induced by the states of $M^F$ where $\psi^l$ holds. Then, given a state $s$, $(M^F, s) \models \psi^l$ iff in this graph either there is a non-sink-cycle reachable from $s$, or there are $k + 1$ pairwise distinct finite paths connecting $s$ to sink-cycles.

The algorithm checks the property of the claim analyzing all the machines $M_j$ of $M$ starting from the bottom-level machine $M_1$, which contains no boxes. For each machine $M_j$, it performs a preliminary step to determine the set of non-sink-cycles $NSC_j \subseteq S$ of nodes $u \in V_j$ such that a non sink-cycle is reachable in $M^F_j$ from $\langle u \rangle$, through nodes of $S$.

Then, in a successive step, the algorithm detects the other nodes satisfying $\psi$. In particular for any detected node $u \in V_j$ and for any sequence $\alpha$ of boxes (below we show how to remove this dependency from $\alpha$) the following situation can occur:

- there is a non-sink cycle reachable in $M^F$ from a state $\langle \alpha u \rangle$ including only nodes in $S$;
- $k + 1$ paths start in $M^F$ from $\langle \alpha u \rangle$, each going through nodes belonging to $S$, and ending into sink-cycles.

Observe that, if the non-sink cycle is in $M^F_j$, but it is not in $M^F_j$, then $u \notin NSC_j$ and thus the former case has not been detected by the algorithm in the previous preliminary step.

In order to get that the above properties do not depend on the choice of $\alpha$, also in this case multiple copies of each $M_j$ are created, each for a different context in which $M_j$ occurs. Each copy is denoted $M^g_j$ where $g : OUT_j \rightarrow \{0, \ldots, k + 1\}$ is a mapping such that if $z$ does not satisfies $\psi^l$ then $g(z) = 0$. Its nodes and boxes are obtained by renaming nodes and boxes of $M_j$, as in the previous case.

Let us now give some details on how the above steps are realized.

The set $NSC_j$, for $j \in \{1, \ldots, h\}$, is computed by visiting a graph $M^F_j$, with the nodes in $V_j \cap S$. If $j \neq 1$, then $M^F_j$ contains also the boxes $b$ of $M_j$, such that $in_{expn}(b) \in S$, and new vertices $((b, z), v)$, for $z \in OUT_{expn}(b) \cap S$ (recall that there are no boxes in $M_1$). The edges of $M_j$ connecting the boxes and the nodes above are edges also of this graph, moreover, there is an edge from $b$ to $(b, z)$ if there is a path from $in_{expn}(b)$ to $z$ in $M_{expn}(b)$, constituted of all vertices not belonging to $NSC_{expn}(b)$.

The algorithm proceeds inductively, starting from $M_1$. When $M_j$ is considered, for $j > 1$, we assume that the sets $NSC_j$ have already been determined, for all $j' < j$, and that, for each $z \in OUT_j$, it has also been checked whether there is a path from $in_{f'}$ to $z$, constituted of all vertices not belonging to $NSC_{f'}$ (observe that this property is used to define the edges in $M^F_j$). Moreover, we assume that, if there is such a path, it has also been checked whether there are vertices in the path with out-degree greater than 1 and whether $z$ has an out-going edge within $M^F_j$. The result of this test is useful to detect the non-sink cycles and thus to determine the set $NSC_j$. In fact, if either a node $z \in OUT_{expn}(b)$ has an out-going edge or there is a vertex with out-degree at least 2 in the path from $in_{expn}(b)$ to $z$, then a cycle going through $(b, z)$ in $M^F_j$ determines a non-sink cycle on the corresponding flat machine.

Once the set $NSC_j$ has been computed, the algorithm sets $u, \psi = \text{TRUE}$ for all $u \in NSC_j$ and then it performs the successive step considering only the remaining nodes.

For each $j$ and each mapping $g : OUT_j \rightarrow \{0, \ldots, k + 1\}$, a dag $G^g_j$ is constructed with the nodes $u \in V_j \cap S$ such that $u \notin NSC_j$, the boxes $b$ and the new vertices $((b, z), v)$, for $z \in OUT_{expn}(b)$, such that both $in_{expn}(b)$ and $z$ satisfy $\psi^l$ and do not belong to $NSC_{expn}(b)$, and with the exception that the sink cycles are substituted by a single vertex. The edges in $G^g_j$ are those of $M_j$.

The algorithm labels the vertices of $G^g_j$, starting from the leaves, as follows:

- $z \in OUT_j$ is labeled by $g(z)$,
• if \( x \) in \( G_f^g \) is not a box and has successors \( x_1, \ldots, x_s \), labeled by \( l_1, \ldots, l_k \), then \( x \) is labeled by \( l = \max\{l_1 + \cdots + l_k + 1\}\):

• for a box \( b \), such that \( \text{expn}(b) = f' \), let \( g' \) be the mapping such that \( g'(z) = r \) if \((b, z)\) is labeled by \( r \), for \( z \in \text{OUT}_f \).

If \( \text{in}_f \) has been labeled by \( i \) in the dag \( G_f^g \) then \( b \) is labeled \( i \) as well (observe that the labeling of \( \text{in}_f \) in \( G_f^g \) has already been determined, since \( f' < f \)).

As said above, new machines \( M_f^g \) have been constructed as copies of \( M_f \), by renaming its nodes and boxes. Now, for each \( u \in V_f \), the algorithm sets \( u^g.\psi = \text{TRUE} \) if \( u \) is labeled by \( k + 1 \) in \( G_f^g \).

Finally, the expansion mapping for \( M_f^g \) is defined as follows: if \( \text{expn}_f(b) = f' \) then \( b^F \) now expands into \( M_f^g \), where \( g' \) is such that \( g'(z) = r \) for \( z \in \text{OUT}_{f'} \) which has been labeled by \( r \) in \( G_f^g \).

Finally, for the case of a subformula \( \psi = E^k \theta_1 \Box \theta_2 \), for \( k > 0 \), the algorithm behaves in a similar way. It first determines the nodes of \( M_f \) which satisfy \( E^k \theta_1 \Box \theta_2 \) and then it determines, for each node \( u \in S \), whether \( u \) satisfies \( \psi \), with an approach suggested by the following claim.

**Claim 2.** Consider the graph induced by the states of \( M_f^g \) where \( E^k \theta_1 \Box \theta_2 \) holds, and by deleting the edges outgoing from states where \( \theta_1 \) does not hold. Then, given a state \( s \), \((M_f^g, s) = \psi \) iff in this graph either there is a non-sink-cycle reachable from \( s \), or there are \( k + 1 \) pairwise distinct finite paths connecting \( s \) to states where \( \theta_2 \) holds.

Thus, the main difference with respect to the steps described above, is in the definition of the graphs \( M_f^g \) and \( G_f^g \), since they now do not have edges outgoing from states where \( \theta_1 \) does not hold, in accordance to the Claim 2. We will omit further details.

Now we can state the first main result, where \( |\phi| \) is the number of the boolean and temporal operators in \( \phi \), \( d \) is the maximum number of exit nodes of \( M_f \) and \( k - 2 \) is the maximal constant occurring in a graded modalities of \( \phi \).

**Theorem 4.** The graded-CTL model-checking of an HSM \( M \) can be solved in \( O(|M| \cdot 2^{\log |\phi|}) \).

**Proof.** The algorithm sketched above considers the subformulas \( \psi \) of \( \phi \), and, for each node \( u \) in \( M_f \), sets \( u.\psi = \text{TRUE} \) if \( u \) satisfies \( \psi \). For \( \psi = E^k \theta \), with \( k > 0 \), and \( \theta = x \theta_1 \), the correctness of the algorithm is rather immediate, while if either \( \theta = \neg \theta_1 \) or \( \theta = \theta_1 \Box \theta_2 \), the correctness of the algorithm mainly relies on the given claims. For sake of brevity, we omit here the proof of the claims.

The crucial point is to prove that the algorithm detects all the nodes \( u \) in a machine \( M_f \) such that a non-sink cycle is reached from \( \langle b_1 \ldots b_m u \rangle \) along a path including only nodes satisfying \( E^0 \theta \). Let \( u \) be a node in \( M_f \). If there is a non-sink cycle reachable from \( \langle u \rangle \) in \( M_f^g \), including only nodes in the set \( S \) of nodes satisfying \( E^0 \theta \), then \( u \in \text{NSC}_f \) and the algorithm sets \( u.\psi = \text{TRUE} \). Now suppose that there are boxes \( b_1, \ldots, b_m \) and that a non-sink cycle is reachable from \( \langle b_1, \ldots, b_m u \rangle \) in \( M_f^g \) (again including only nodes in \( S \)) and suppose also that no non-sink cycles are reachable from \( \langle b_r, \ldots, b_m u \rangle \), for \( r > 1 \). This implies that there is \( z_i \in \text{OUT}_{\text{expn}(b_i)} \) and a non-sink cycle reachable from \( \langle b_1 z_1 \rangle \) in \( M_f^g \), and there are \( z_1, \ldots, z_m \) such that, for \( i = 1, \ldots, m \),

- \( z_i \in \text{OUT}_{\text{expn}(b_i)} \)
- \( \langle z_m \rangle \) is reachable from \( \langle u \rangle \) in \( M_f^g \)
- \( \langle z_i \rangle \) is reachable from \( \langle b_{i+1}, z_{i+1} \rangle \) in \( M_f^g_{\text{expn}(b_{i+1})} \)

In this case the algorithm sets \( \langle b_1, z_1 \rangle \in \text{NSC}_f \). Moreover, in the new HSM each \( b_i \) will expand in a copy \( M_{\text{expn}(b_i)} \) of \( M_{\text{expn}(b_i)} \), where \( g_i \) is such that \( g_i(z_i) = k + 1 \). And thus, called \( u^g \) the copy of \( u \) in \( M_f^g \), the algorithm sets \( u^g.\psi = \text{TRUE} \).

Similarly, the algorithm detects all the nodes \( u \) in \( M_f \), such that \( k + 1 \) paths start from \( \langle b_1 \ldots b_m u \rangle \) ending in sink cycles including only nodes in \( S \). To state the complexity of the algorithm, observe that, while processing a subformula \( \psi = E^k \theta \), with \( k > 0 \) and \( \theta \in \{ \neg \theta_1, \theta_1 \Box \theta_2 \} \), the algorithm creates several copies of each machine \( M_f \), denoted \( M_f^g \) where \( g : \text{OUT}_f \to \{0 \ldots, k + 1\} \). Thus the size of the current HSM grows for a factor not exceeding \( k^d \), where \( d \) is the maximum number of exit nodes of \( M_f \) and \( k - 2 \) is the maximal constant occurring in a graded modalities of \( \phi \). Since, for each operator in \( \phi \), the time spent by the algorithm is linear in the size of the current HSM, than the overall running time is \( O(|M| \cdot k^d) \).

Let us remark that, although the multiple copies created by the given algorithm can be seen as a step towards the flattening of the input HSM, the resulting structure is in general much smaller than the corresponding flat Kripke structure. To solve the graded-CTL model-checking for SHSM we show now how to reduce it to the model-checking problem for HSM. Let \( M = (M_1, M_2, \ldots, M_h) \) be an SHSM and let \( \phi \) be a graded-CTL formula. Let \( AP_\phi \) be the set of atomic propositions that occur in \( \phi \). The first step of our algorithm consists of constructing an HSM \( M_\phi^g \) such that \( M_\phi^g \) is isomorphic to \( M_f^g \). Let \( \text{index} : \{1, \ldots, h\} \times 2^{AP_\phi} \to \{1, \ldots, 2^{2^{AP_\phi}}\} \) be a bijection such that \( \text{index}(i, P) < \text{index}(j, P') \) whenever \( i < j \). Clearly, \( \text{index} \) maps \( (i, P) \) into a strictly increasing sequence of consecutive positive integers starting from \( 1 \). For a machine \( M_i = (V_i, \text{in}_i, \text{OUT}_i, \text{TRUE}_i, \text{expn}_i, E_i) \), \( 1 \leq i \leq k \) and \( P \subseteq AP_\phi \), define \( M_P^g \) as the machine \( (V_P^g, \text{in}_P^g, \text{OUT}_P^g, \text{TRUE}_P^g, \text{expn}_P^g, E_P^g) \) where:

- \( V_i^P = \{u^P \mid u \in V_i\} \), and \( \text{OUT}_i^P = \{u^P \mid u \in \text{OUT}_i\} \);
Let $h^j$ (from Theorem 4, we have the following second main result. $M$ can be solved in $O(\expn(u)^i, P \cup \text{TRUE}_j(u))$, otherwise;

- $\expn_j(u) = 0$ if $u$ is a node and $\expn_j(u) = \text{index}(\expn_j(u), P \cup \text{TRUE}_j(u))$, otherwise;

- $E_i^P = \{(u^P, v^P) \mid (u, v) \in E_i \} \cup \{(u^P, z^P, \text{TRUE}_j(u), v^P) \mid (u, z, v) \in E_i \}$.

Let $h' = h^{2 \expn}$. We define $M_{h'}$ be the tuple of machines $(M'_1, \ldots, M'_p)$ such that for $j = 1, \ldots, h'$, $M'_j = M_i^P$ where $j = \text{index}(i, P)$. From the definition of $M^p$ it is simple to verify that $M_{h'}$ is an HSM and $|M_{h'}|$ is $O(|M| 2^{2 \expn})$. Moreover, $M^F$ and $M_{h'}^F$ coincide, up to a renaming of the states. Thus, from Theorem 4 we have the following second main result.

**Theorem 5.** The graded CTL model checking of an SHSM can be solved in $O(|M| 2^{d \expn})$ time.

## 5. CONCLUSIONS

In this paper we have proposed the use of graded-CTL specifications to model-check hierarchical state machines. We think that the added power in the specification formalism can be fruitfully exploited in the simulation and testing community to get more meaningful test benches to perform simulation of more and more complex systems. We have given algorithms for checking classical HSMs and so-called SHSMs. Let us observe that the alternative approach of model-checking the fully expanded flat structure has in general a worse performance because of the exponential gap between an HSM and its corresponding flat structure. In fact the gain in size of the hierarchical model, is in practice much greater than the extra exponential factor paid, which depends on the size of the formula for the specification, usually quite small. One last consideration is that we have considered only sequential hierarchical finite state machines (as an abstraction of the DEVS model). It is a standard approach, when model checking concurrent systems, to first sequentialize the model of the SUT (possibly on-the-fly) and then check it with model checking algorithms for sequential models. Moreover, the cost of considering parallel and communicating machines would lead to a double exponential blow-up, the so-called state explosion problem.

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