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Toward an efficient approximate analytical solution for 4-compartment COVID-19 fractional mathematical model

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ABSTRACT

With the recent trend in the spread of coronavirus disease 2019 (Covid-19), there is a need for an accurate approximate analytical solution from which several intrinsic features of COVID-19 dynamics can be extracted. This study proposes a time-fractional model for the SEIR COVID-19 mathematical model to predict the trend of COVID-19 epidemic in China. The efficient approximate analytical solution of multistage optimal homotopy asymptotic method (MOHAM) is used to solve the model for a closed-form series solution and mathematical representation of COVID-19 model which is indeed a field where MOHAM has not been applied. The equilibrium points and basic reproduction number (R0) are obtained and the local stability analysis is carried out on the model. The behaviour of the pandemic is studied based on the data obtained from the World Health Organization. We show on tables and graphs the performance, behaviour, and mathematical representation of the various fractional-order of the model. The study aimed to expand the application areas of fractional-order analysis. The results indicate that the infected class decreases gradually until 14 October 2021, and it will still decrease slightly if people are being vaccinated. Lastly, we carried out the implementation using Maple software 2021a.

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1. Introduction

The deadly disease called (Covid-19) is a family of infectious diseases believed to have originated from Wuhan (Chinese city). It is disclosed that the virus originated from bat, and the transmission might link to a seafood market (Huanan Seafood Wholesale Market) exposure [1–4]. The virus history can be dated back to 1965, when Tyrrell and Bynoe recognized a virus named B814 [5]. This B814 virus can be identified in human respiratory tract of an adult, resulting into the spread of the disease [6]. Some of the early research on this disease includes coronavirus antibody experiment [7], human coronavirus infections [8], and coronaviruses isolation with some epidemiological observations [9]. Based on that, several countries have stopped airplane for a while and have declared lock-down so that some preventive measures can take place to lessen great loss of human lives. At the same time, some countries are trying to restrict the unnecessary movement of people to decrease the number of cases in their countries. This Covid-19 outbreak had affected economics of so many countries, human society and currently affecting 223 countries and territories with confirmed cases...
of 242,688,319, confirmed deaths 4,932,928, recovered 221,523,414, new cases 101,867,2,102, and a total of 6,655,399,359 vaccines have been administered as of 20 October 2021 that includes Pfizer–BioNTech, AstraZeneca/Oxford, and Janssen (Johnson & Johnson’s) according to (WHO). The situation region includes America 61,284,892, Europe 51,007,204, South-East Asia 20,669,435, Eastern Mediterranean 8,918,834, Africa 3,285,051, and Western Pacific 2,373,140. In stage III trials, many COVID-19 vaccines have exhibit effectiveness as high as 95% in preventing symptomatic COVID-19 infections. As of April 2021, 13 vaccines are already approved by at least one national monitory authority for citizens use: two RNA vaccines (the Pfizer–BioNTech vaccine and the Moderna Vaccine), five conventional inactivated vaccines (BBIBP-CorV, CoronaVac, Covaxin, WIBP-CorV, and CovIVac), four viral vector vaccines (Sputnik V, the Oxford–AstraZeneca vaccine, Convidecia, and the Johnson & Johnson vaccine), and two protein subunit vaccines (EpiVacCorona and RBD-Dimer). In sum total, as of March 2021, 308 vaccines are in different phases of progress, with 73 in clinical research, including 24 in stage I trials, 33 in stage II trials, and 16 in stage III headway. According to WHO, the current update on covid-19 from China which is the epicentre of the disease, confirmed cases 125,620, death 5696, new cases 55, recovered 9,620 and 2,238,760,403 vaccines have been administered as of 14 October 2021 WHO [10,11].

A mathematical model is a representation of a system using mathematical view and language. In literature, this model play an important role in describing the behaviour of a system. The model can describe infectious disease control and forecast its spread. To grasp the spread of an infectious diseases COVID-19 [12], we used the perception of differentiation and integration to model the non-linear dynamic system of infectious disease in the term of FDEs to predict the spread within a specific population. The universal problem of the spread has enticed the interest of several authors of various areas, giving rise to different suggestions to investigate and predict the progress of the global pandemic. There exist several models in literature to describe infectious diseases, beginning from the normal SIR model to more additional compartment that includes the most recent studies; mathematical modelling and vaccination campaign of COVID-19 [13], vaccination effect of two infectious disease Sars–CoV-2 mathematical model [14] reformed SIR COVID-19 model for the epidemic in Italy [15], SIR COVID-19 model in several communities [16], SIR COVID-19 model with time-dependent [17], applications of the SIR model to COVID-19 [18], golden ratio model in the epidemic of COVID-19 [19], and a macroeconomic SIR model for COVID-19 [20]. The SEIR model which contains exposed compartment has been considered as an improvement of SIR for an exposed case. The latest studies include: dynamics of SEIR model of the COVID-19 [21], hybrid SEIR and Regression Model for COVID-19 spread in India [22], SEIR COVID 19 model of community outbreak [23], SEIR COVID-19 model for spread in Indonesia [24], SEIR COVID-19 model for social distancing [25], generalized SEIR COVID-19 model [26], a modified SEIR model for COVID-19 outbreak in Spain and Italy [27], modified SEIR and AI prediction of COVID-19 in China [28], another SEIR model that classifies infected class into two compartments (asymptomatic and symptomatic classes) [29], insights on SEIR Covid-19 pandemic model [30], time-delay stochastic COVID-19 epidemic model [31], and SEIR COVID-19 model for the analysis of tailored social distancing [32]. Many numerical methods have been used to solving SEIR model that includes, Newton’s iterative method for COVID-19 model [33], Euler method for Covid-19 model [34], homotopy perturbation method for COVID-19 model [35], Euler finite-difference scheme for Covid-19 model [36], fixed point approach and fractional Adams–Bashforth method for Covid-19 model [37], variational iteration method (VIM) [38] and differential transformation method (DTM) for Covid-19 model [39], predictor–corrector method for Covid-19 model [40], Adams–Bashforth–Moulton predictor–corrector scheme for Covid-19 model [41], analytical scheme of time-fractional for numerical computation [42], computational technique for a biological population fractional-order model [43], and analysis of fractional model of guava for biological pest control with memory-effect [44].

The recent development in fractional calculus has attracted authors and researchers to implement fractional derivative operators on Covid-19 model that includes; stability analysis of fractional-order SIR model [45], modelling and analysis of fractional derivatives COVID-19 epidemics real data from Pakistan [46], synchronization and numerical solutions of a fractional-order chaotic system [47], governmental action and individual reaction of COVID-19 fractional-order model [48], and analysis and dynamics of Covid-19 fractional order model [49], Atangana–Baleanu for SEIR [50], Caputo–Mittag type fractional derivative for SEIR [51–53], Caputo fractional derivative for SEIR [54,55], Caputo fuzzy fractional derivatives for SEIR [56], conformable derivative operator sense [57,58], non-integer order and application [59], conformable derivative operator for heat conduction equation [60], conformable fractional derivative for non-linear programming problem [61], and conformable derivative operator for gradient based dynamic system [62]. The goal of this study is to predict the transmission of COVID-19 through time-fractional SEIR model. Also, to transform the SEIR model using newly developed conformable derivative operator. The purpose of using conformable derivative operator includes to enlarge the stability region of the non-linear dynamical system of the model, to use conformable derivative operator with multistage optimal homotopy analysis method (MOHAM) approach for COVID-19 model for behaviour, performance, and mathematical representation, and to predict the spread of COVID-19 accurately. The main contributions in this are summarized as follows:

- The latest published articles about the coronavirus disease 2019 (Covid-19) have been reviewed and discussed.
- The transmission rates are modelled as a system of non-linear fractional mathematical models.
- The multistage optimal homotopy asymptotic method is suggested to find approximate analytical solutions for such equations.
- Another contribution here is to predict epidemic in China. The data for the confirmed cases in China have been used in the computational simulations, and some predictions are discussed.
From the computational simulations results, the number of infected people has decreased gradually until 14 October 2021, then it will be decreased slightly if people are being vaccinated.

The manuscript is organized as follows: In Section 2, a brief study of some fractional derivative operators and some theorems are given and the solution procedure. In Section 3, we show the mathematical model. The basic idea of the proposed method is given in Section 4. The usefulness of our model, mathematical illustration, and discussion are given in Section 5. We end with Section 6 of conclusions.

2. A brief study of some fractional derivative operators

2.1. The Mittag-Leffler function

The Mittag-Leffler function has two definitions in the literature:

\[
E_\alpha (z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \alpha \in \mathbb{C}, \mathcal{R} (\alpha) > 0,
\]

and

\[
E_{\alpha, \beta} (z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \alpha, \beta \in \mathbb{C}, \mathcal{R} (\alpha) > 0, \mathcal{R} (\beta) > 0. \tag{2.1}
\]

This rule makes the Taylor series of the exponential function in the case \( \alpha = 1 \). Here, we observe only a few basic attributes of the Mittag-Leffler functions:

\[
\begin{align*}
E_{\alpha, \beta} (z) &= 2E_{\alpha, \beta+1} (z) + \frac{1}{\Gamma(\beta)}, \\
E_{\alpha, \beta} (z) &= \beta E_{\alpha, \beta+1} (z) + \alpha z \frac{d}{dz} E_{\alpha, \beta+1} (z), \\
\frac{d^m}{dz^m} \left[ z^{\beta-1} E_{\alpha, \beta}(z^\alpha) \right] &= z^{\beta-m-1} E_{\alpha, \beta-m}(z^\alpha), \mathcal{R} (\beta-m) > 0, m = 0, 1 \ldots n. \\
\frac{d}{dz} E_{\alpha, \beta} (z) &= E_{\alpha, \beta-1} (z) - \frac{\beta-1}{\alpha} E_{\alpha, \beta} (z).
\end{align*}
\]

More details and properties concerning these functions can be found [63].

2.2. The Riemann-Liouville and Caputo derivative

Two other related types of fractional derivatives, named Riemann–Liouville and Caputo derivative, respectively, are delimited by:

\[
D^\alpha_0 f (x) = J^{m-\alpha}_0 \left( \frac{d^m}{dx^m} f (x) \right), x > 0, m = \lceil \alpha \rceil, \\
D^\alpha_0 f (x) = \frac{d^m}{dx^m} \left( J^{m-\alpha}_0 (f (x)) \right), x > 0, m = \lceil \alpha \rceil. \tag{2.3}
\]

The integral operator is given as

\[
J^\alpha_0 f (x) = \frac{1}{\Gamma (\alpha)} \int_0^x \frac{f (s)}{(x-s)^{1-\alpha}} ds, x > 0. \tag{2.4}
\]

It is noted that the fractional derivative of a function depends on which definition is being used. Still, there exist also correspondences between the Caputo and Riemann–Liouville derivatives. A significant result connecting these two derivatives is presented with the following result:

**Theorem 2.1.** Let \( f \in L^1([0, \infty)) \cap C^m([0, \infty)) \) and \( m-1 < \alpha \leq m \) for giving \( m \in \mathbb{N} \). And so:

\[
D^\alpha_0 f (x) = D^\alpha_0 f (x) + \sum_{k=0}^{m-1} \frac{f^{(k)} (0^+)}{\Gamma (1+k-\alpha)} x^{k-\alpha}, x > 0. \tag{2.5}
\]

Where the notation \( f^{(k)} (0^+) = \lim_{x \to 0^+} f^{(k)} (x) \) is being used. As a result of this theorem, we find then: if \( f^{(k)} (0^+) = 0, k = 0, 1, \ldots, m-1 \), then \( D^\alpha_0 = D^\alpha_0 \) which connects the Caputo and Riemann–Liouville derivative for particular types of functions [64].
2.3. The chain rules

Unfortunately, the chain rule, \( (f \circ g)(x) \circ f'(g(x))g'(x) \) for ordinary derivatives, cannot be applied to fractional derivatives. In general,

\[
D^\alpha [f \circ g (x)] \neq f' (g(x)) D^\alpha g (x).
\]

Counterexamples can be found in Refs. [65]. To close this section: one way of understanding fractional derivatives is to view them as a non-linear interpolation of ordinary derivatives in terms of a convolution with a weakly-singular kernel. We refer to more detailed fractional derivatives [66].

2.4. The conformable derivative operator

The definition of conformable derivative operator preserves many properties of classical order derivatives [67].

**Definition 2.1.** Let \( f : [0, \infty) \to \mathbb{R} \) be a given function. The \( \alpha \)-order conformable derivative operator of \( f \) is given by

\[
T^\alpha (f) (t) = \lim_{\epsilon \to 0} \frac{f (t + \epsilon t^{1-\alpha}) - f (t)}{\epsilon}, \quad \forall \ t > 0 \text{ and } \alpha \in (0, 1].
\]

If \( f \) is \( \alpha \)-differentiable in some \( (0, a) \), \( a > 0 \), and \( \lim_{t \to 0^+} f^{(\alpha)}(t) \) exists, then define

\[
f^{(\alpha)} (0) = \lim_{t \to 0^+} f^{(\alpha)} (t).
\]

We sometimes write \( f^{(\alpha)}(t) \) for \( T^\alpha (f)(t) \), to denote the conformable fractional derivatives of \( f \) of order \( \alpha \). If the conformable fractional derivative of \( f \) of order \( \alpha \) exists, we simply say \( f \) is \( \alpha \)-differentiable. We should remark that \( T^\alpha \left( t^p \right) = pt^{p-\alpha} \). Further, our definition coincides with R-L and Caputo’s classical definitions of polynomials (up to a constant multiple). As a result of the above definition, we obtain the following valid theorem.

**Theorem 2.2.** If a function \( f : [0, \infty) \to \mathbb{R} \) is \( \alpha \)-differentiable at \( t_0 > 0 \), \( \alpha \in (0, 1] \), then \( f \) is continuous at \( t_0 \).

**Theorem 2.3.** Let \( \alpha \in (0, 1] \), and \( f, g \) be \( \alpha \)-differentiable at a point \( t > 0 \). Then

1. \( T^\alpha (af + bg) = aT^\alpha (f) + bT^\alpha (g) \), for all \( a, b \in \mathbb{R} \),
2. \( T^\alpha \left( t^p \right) = pt^{p-\alpha} \) for all \( p \in \mathbb{R} \),
3. \( T^\alpha \left( \lambda \right) = 0 \), for all constant functions \( f (t) = \lambda \),
4. \( T^\alpha \left( fg \right) = f T^\alpha \left( g \right) + g T^\alpha \left( f \right) \),
5. \( T^\alpha \left( \frac{1}{t} \right) = \frac{\frac{\alpha}{\alpha-1} \left( T^\alpha \left( \frac{1}{t} \right) \right)}{t^2} \),
6. If \( f \) is differentiable, then \( T^\alpha (f) (t) = t^{1-\alpha} \frac{df}{dt} (t) \).

**Theorem 2.4.** Let \( 0 < \alpha \leq 1 \) and \( (f, g) \) be \( \alpha \)-differentiable at a point \( t > 0 \) if \( f \) is a differentiable function, then \( \frac{df}{dt} = t^{1-\alpha} \frac{df}{dt} \).

**Definition 2.2.** \( I^\alpha_a f(t) = I_{\alpha}^a \left( \frac{1}{1-\alpha} f \right) = \int_a^t \frac{f(\tau)}{1-\alpha} d\tau \), where the integral is the regular Riemann improper integral, and \( 0 < \alpha \leq 1 \).

**Theorem 2.5.** Let \( f \) be any continuous function in the domain of \( I^\alpha \), then \( T^\alpha I^\alpha_a f(t) = f(t) \forall t \geq a. \)

**Theorem 2.6.** Let \( f : (a, b) \to \mathbb{R} \) be differentiable and \( 0 < \alpha \leq 1 \). Then for all \( t > a \) we have \( I^\alpha_a T^\alpha f(t) = f(t) - f(a) \). The advantage of using fractional derivative operator and why we considered conformable fractional operator are given below.

(a) It gave a more useful knowledge of a non-linear real problem than traditional-order.
(b) It provided a new dimension and gave information in between two different integer-order.
(c) More also, as a non-local operator in sense, it considers the fact that the future state not only relies upon the present state but also upon all the history of the previous states.
(d) We considered using conformable derivative operator, because it has not been formulated with MOHAM approach to study the SEIR model of infectious disease for behaviour, performance, and mathematical representation.
(e) To help authors gain more knowledge of this powerful mathematical tool and advance its application.
(f) The conformable derivative operator is also employed to enlarge the stability region of the considered system.
(g) To utilize the simplicity and efficiency of the newly conformable derivative operator.
(h) Conformable derivative operator appeared in more than thousands plus articles and its gaining popularities till today.
2.5. The solution procedure

The solution procedure is given in detail as below.

1. We started by literature review on non-linear fractional mathematical model.
2. We propose an approximate analytical solution of multistage optimal homotopy asymptotic method (MOHAM).
3. The proposed method is used to construct a conformable non-linear fractional mathematical model.
4. We run the numerical simulations of the non-linear fractional mathematical model to study their system behaviour, performance, and mathematical representation.
5. We write Maple codes for the non-linear fractional mathematical models.
6. The study show that the proposed technique is efficient and reliable.

3. The formulation of SEIR COVID-19 model

We looked into the spread of COVID-19 in humans. The total population is represented by $N_h(\tau)$ which is splitted into four compartments; Susceptible, Exposed, Infected, and Recovery compartments represented by $S_h(\tau)$, $E_h(\tau)$, $I_h(\tau)$, and $R_h(\tau)$. The collaboration among these compartment has been shown in Fig. 1.

From the model above, the recruitment rate of susceptible populations is given by $\Lambda_h$ which denotes birth rate, $\eta_h$ denotes death rate of each compartment, $\lambda_h$ denotes interaction rate between susceptible and infected populations with the route $\frac{\lambda_h S_h I_h}{N}$, $\psi_h$ denotes the rate at which exposed completed their incubation period and entered into the infected class, $\gamma_h$ is the removal or recovery rate of infected population.

\[
\frac{dS_h(\tau)}{d\tau} = \Lambda_h - \eta_h S_h - \frac{\lambda_h S_h I_h}{N},
\]

\[
\frac{dE_h(\tau)}{d\tau} = \frac{\lambda_h S_h I_h}{N} - \eta_h E_h - \psi_h E_h - \psi_h E_h,
\]

\[
\frac{dI_h(\tau)}{d\tau} = \psi_h E_h - \eta_h I_h - \gamma_h I_h,
\]

\[
\frac{dR_h(\tau)}{d\tau} = \gamma_h I_h - \eta_h R_h.
\]
Corresponding non-negative initial conditions

\[ S(0) = S^* \geq 0, \quad E(0) = E^* \geq 0, \quad I(0) = I^* \geq 0, \quad \text{and} \quad R(0) = R^* \geq 0. \]

In order to capture the underlying features that are not captured by the integer order derivatives, we formulate the model with a fractional-order time derivative as

\[
\frac{d^\alpha}{dt^\alpha} S_h (\tau) = \Lambda_h - \eta_h S_h - \frac{\lambda_h S_h I_h}{N},
\]

\[
\frac{d^\alpha}{dt^\alpha} E_h (\tau) = \frac{\lambda_h S_h I_h}{N} - \eta_h E_h - \psi_h E_h,
\]

\[
\frac{d^\alpha}{dt^\alpha} I_h (\tau) = \psi_h E_h - \eta_h I_h - \gamma_h I_h,
\]

\[
\frac{d^\alpha}{dt^\alpha} R_h (\tau) = \gamma_h I_h - \eta_h R_h.
\]

and the corresponding non-negative conditions remain the same as

\[ S(0) = S^* \geq 0, \quad E(0) = E^* \geq 0, \quad I(0) = I^* \geq 0, \quad R(0) = R^* \geq 0, \quad \text{and} \quad 0.7 \leq \alpha \leq 1. \]

3.1. Equilibrium points, basic reproduction number, and local stability analysis

The possible fixed points of (3.5)–(3.8) model are included in this section. Two possible equilibrium points are calculated, i.e. Disease-free equilibrium (DFE) and endemic equilibrium (EE). Also, the basic reproduction number is obtained using the next generation approach and discouse the stable local analysis of these equilibrium points. The steady-state solution of the model is obtained from the equation:

\[
\frac{dS_h(\tau)}{d\tau} = \frac{dE_h(\tau)}{d\tau} = \frac{dI_h(\tau)}{d\tau} = \frac{dR_h(\tau)}{d\tau} = 0.
\]

Using (3.9) on (3.5)–(3.8) becomes:

\[
0 = \Lambda_h - \eta_h S_h - \frac{\lambda_h S_h I_h}{N},
\]

\[
0 = \frac{\lambda_h S_h I_h}{N} - \eta_h E_h - \psi_h E_h,
\]

\[
0 = \psi_h E_h - \eta_h I_h - \gamma_h I_h,
\]

\[
0 = \gamma_h I_h - \eta_h R_h.
\]

From a steady-state system (3.10)–(3.13), DFE is obtained by assuming \( E_h = I_h = R_h = 0 \) and is represented by

\[
\psi_{DFE} = (S^0, E^0, I^0, R^0) = \left( \frac{\Lambda_h}{\eta_h}, 0, 0, 0 \right).
\]

Similarly, EE of the model (3.5)–(3.8) is obtained from the system (3.10)–(3.14) and is given by: \( \psi_{EE} = (S^*, E^*, I^*, R^*) \), where

\[
S^* = \frac{N (\eta_h^2 + \eta_h \gamma_h + \eta_h \psi_h + \gamma_h \psi_h)}{\lambda_h \psi_h},
\]

\[
E^* = \frac{\Lambda_h \lambda_h \psi_h - N \eta_h^3 - N \eta_h^2 \gamma_h - N \eta_h \gamma_h \psi_h - N \eta_h \gamma_h \psi_h}{\psi_h (\eta_h + \gamma_h \lambda_h)};
\]

\[
I^* = \frac{\Lambda_h \lambda_h \psi_h - N \eta_h^3 - N \eta_h^2 \gamma_h - N \eta_h \gamma_h \psi_h - N \eta_h \gamma_h \psi_h}{\lambda_h (\eta_h^2 + \eta_h \gamma_h + \eta_h \psi_h + \gamma_h \psi_h)};
\]

\[
R^* = \frac{\gamma_h (\Lambda_h \lambda_h \psi_h - N \eta_h^3 - N \eta_h^2 \gamma_h - N \eta_h \gamma_h \psi_h - N \eta_h \gamma_h \psi_h)}{\eta_h \lambda_h (\eta_h^2 + \eta_h \gamma_h + \eta_h \psi_h + \gamma_h \psi_h)}.
\]

The basic reproduction number \( R_0 \) is calculated using the next-generation approach [68]. The F and V matrices at DFE \( \psi_0 \) are given as follows:

\[
F = \begin{bmatrix}
0 & \lambda_h \\
0 & 0
\end{bmatrix},
\]

\[
V = \begin{bmatrix}
\eta_h + \psi_h & 0_h \\
\psi_h & \eta_h + \gamma_h
\end{bmatrix}.
\]
Using the spectral radius $\rho(FV^{-1})$, the required reproduction number $R_0$ is calculated as

$$R_0 = \frac{\lambda_h \psi_h}{(\eta_h + \psi_h) + (\eta_h + \gamma_h)}.$$  

(3.21)

**Theorem 3.1.** The DFE $\Psi_0$ of the system (3.5)–(3.8) is locally asymptotically stable if $R_0 < 1$.

**Proof.** The Jacobian matrix of system (3.5)–(3.8) at DFE is given by:

$$J_{\psi_0} = \begin{bmatrix} -\eta_h & 0 & -\lambda_h & 0 \\ 0 & -\eta_h - \psi_h & \lambda & 0 \\ 0 & \psi_h & -\eta_h - \gamma_h & 0 \\ 0 & 0 & \gamma_h & -\eta_h \end{bmatrix}.$$  

(3.22)

Suppose $\lambda$ denote the eigenvalues of the Jacobian matrix $J_{\psi_0}$. Here, the two eigenvalues of the above matrix are negative, i.e., $-\eta_h$ (twice). The following characteristic equation can obtain the remaining required eigenvalues:

$$\lambda^2 + \kappa_1 \lambda + \kappa_2 = 0,$$  

(3.23)

Where

$$\kappa_1 = \gamma_h + 2\eta_h + \psi_h,$$

$$\kappa_2 = \eta_h \gamma + \eta_h^2 + \gamma_h \psi_h - \lambda_h \psi_h,$$

$$\Rightarrow \kappa_2 = (\eta_h + \gamma_h)(\eta_h + \psi_h).$$  

(3.24)

From (3.21), it can be observed that $\kappa_1 > 0$. Likewise, from (3.22), it can be seen that $\kappa_1 > 0$ when $R_0 < 1$. So, all the coefficients of the characteristic equation are non-negative. In addition, using Routh–Hurwitz criteria [69], the eigenvalues of the above characteristics equations are negative. Thus, all the eigenvalues of the Jacobian matrix (3.19) are negative for $R_0 < 1$. Hence the disease-free equilibrium (3.5)–(3.8) is locally asymptotically stable when $R_0 < 1$.

**4. Multistage optimal homotopy asymptotic method**

We started by looking into basic concept of the optimal homotopy asymptotic method (OHAM) where $T^\omega$ is the CFD, $L_k$ is a linear operator, $N_k$ is a non-linear operator, $t$ is an independent variable, $x_k(t)$ is an unknown function, $\phi$ is the problem domain, and $g_k(t)$ is a known function, we have [70–73]. $T^\omega \xi_k(t) + L_k(\xi_k(t)) + N_k(\xi_k(t)) = g_k(t)$ $t \in [0, 1]$ $k = 1, 2 \ldots m$, with initial conditions

$$\xi_k(0) = a_i.$$  

(4.1)

According to OHAM, we formulate a homotopy map $H_k(\phi(t, p)) : \phi \times [0, 1] \rightarrow \phi$ which satisfies (3.5)–(3.8) can be constructed using OHAM as

$$1 - \epsilon[T^\omega(N_k(t, \epsilon))] = H_k(\epsilon)[T^\omega N_k(t, \epsilon) + N_k N_k(t, \epsilon) + L_k N_k(t, \epsilon) + g_k(t)].$$  

(4.2)

where embedding parameter $(\epsilon)$ is $0 \leq \epsilon \leq 1$, auxiliary function $H_k(\epsilon)$ $\forall \epsilon \neq 0$, unknown function (3.5)–(3.8) and $H(0) = 0$. When $\epsilon = 0$ and $\epsilon = 1$, it holds that $N_k(t, 0) = \psi_{k,0}(t)$ and $N_k(t, 1) = \psi_k(t)$ respectively. Thus as $\epsilon$ moves from 0 to 1, the solution $N_k(t, \epsilon)$ approaches from $\psi_{k,0}(t)$ to $\psi_k(t)$, where initial guess $\psi_{k,0}(t)$ satisfies the linear operator generated from (4.2) for $\epsilon = 0$ as

$$T^\omega(\psi_{k,0}(t)) = 0, \psi_{k,0}(0) = 0.$$  

(4.3)

The $H_k(\epsilon)$ is given as

$$H_k(\epsilon) = \sum_{j=1}^{n} \epsilon^j C_j,$$  

(4.4)

where $C_j$ can be known later. We get an approximate solution by expanding $N_k(t, \epsilon, C_j)$ in Taylor’s series in terms of $\epsilon$,

$$N_k(t, \epsilon, C_j) = \psi_{k,0}(t) + \sum_{j=1}^{n} \psi_{i,k}(t, C_j) \epsilon^j = 1, 2, \ldots, n,$$  

(4.5)

using above in (4.2) with collections of the coefficient like the power of $\epsilon$ gives the governing equations $\psi_{i,0}(t)$ in a linear form in (4.3). Then 1st problems are given as

$$T^\omega(\psi_{k,1}(t)) + g_k(t) = C_{1} N_{0}(\psi_{k,0}(t)), \psi_{k,1}(0) = 0,$$  

(4.6)
the general governing equations for \( \psi_{k,i}(t) \) are

\[
\mathcal{T}^{\alpha} \left( \psi_{k,i}(t) \right) - \mathcal{T}^{\alpha} \left( \psi_{k,i-1}(t) \right) = C_i N_{k,0} \left( \psi_{k,0}(t) \right) + \sum_{m=1}^{i-1} C_{j,m} \left[ \mathcal{T}^{\alpha} \left( \psi_{k,i-m}(t) \right) + N_{k,i-m} \left( \psi_{k,i-1}(t) \right) \right],
\]

\[
\psi_{k,0}(b) = 0 \quad i = 2, 3, \ldots, m,
\]

where \( N_{k,m}(\psi_0(t), \psi_1(t), \ldots, \psi_m(t)) \) is the coefficient of \( \varepsilon^m \), produced by expanding \( N_k(\psi_k(t, \varepsilon, C_j)) \) in series relating to \( \varepsilon \)

\[
N_k(\psi_0(t), \psi_1(t), \ldots, \psi_m(t)) = N_{k,0}(\psi_0(t)) + \sum_{m=1}^{\infty} N_{k,m}(\psi_0, \psi_1, \ldots, \psi_m)\varepsilon^m.
\]

The convergence of series solution (4.8) relies on \( C^j_i \). If it is convergent at \( \varepsilon = 1 \) gives solution to (4.1) as

\[
\psi_k(t, C_j) = \psi_k(0) + \sum_{k=1}^{m} \psi_{k,k}(t, C_j), j = 1, 2, \ldots, n,
\]

using (4.9) in (4.1), we have an expression for the residual error as

\[
R_k(t, C_j) = \mathcal{T}^{\alpha}(\psi_k(t, C_j)) + L_k(\psi_k(t, C_j)) + N_k(\psi_k(t, C_j) - g_k(t)).
\]

If

\[
R_k(t, C_j) = 0,
\]

then \( \psi_k(t, C_j) \) is the exact solution. Usually, such a case does not occur. We adopt optimization technique of Galerkin method to find the optimal values \( C^j_i \) as given below

\[
\epsilon_k = \frac{\partial \psi_k(t, C_j)}{\partial C_j} = 0 \quad k = 1, 2, \ldots, m,
\]

minimize the functional

\[
\Delta_k(C_j) = \int_0^b \epsilon_k \times R_k(t, C_j) dt.
\]

Where the values of \( a \) and \( b \) depend on the given problem. With these known \( C^j_i \), the approximate analytical solution (4.1) is well known.

If the interval of the time variable is long, then the OHAM fails to reach accurate solutions. MOHAM overcome this shortcoming by partitioning the time interval \([t_0, T]\), into \( N \) subintervals \([t_0, t_1], \ldots, [t_{N-1}, t_N]\) where \( t_N = T \) and the OHAM will be utilized over each subinterval. The endpoint in each sub-interval denotes an initial approximation to the solution over the following interval. The procedure will continue until we obtain a pre-assigned time \( (T) \). Utilization of MOHAM is relative to OHAM, with some minor changes from \( C_i \) to \( C_{ij} \) respectively. Also, initial approximation in \([t_{y-1}, t_y]\), \( y = 0, 1, \ldots, N-1 \), will be considered as

\[
u_{0,j}(t) = \alpha_j, \quad j = 1..N.
\]

In addition, the deformation equation in each subinterval will change to the following

\[
(1 - p)[L_j(u_j(t, p)) - u_{0,j}(t)] = H_j(P, t)[L_j(u_j(t, P)) + g(t) + N_j(u_j(t, P))].
\]

\( H(p, t) \) will be generalized as follows,

\[
H_j(P, t) = (C_{1,j} + C_{2,j}t + C_{3,j}t^2 + \cdots)P, \quad j = 1, \ldots, N.
\]

For \( i = 1, 2, \ldots, m \), and \( j = 1, 2, \ldots, N \), we have

\[
u_j(t, C_{ij}) = u_{0,j}(t) + \sum_{k=1}^{m} u_{k,j}(t, C_{ij}).
\]

\[
R_j(t, C_{ij}) = \mathcal{T}^{\alpha}(u_j(t, C_{ij})) + L_j(u_j(t, C_{ij})) + N_j(u_j(t, C_{ij})) - g_j(t),
\]

\[
J_j(C_{ij}) = \int_{t_y}^{t_{y+h}} R_j^2(s, C_{ij}) ds = 0, 1, \ldots, N - 1.
\]

The length of the subinterval \([t_y, t_{y+1}]\) is \( h \), and the number of subintervals is \( N = \lfloor T/N \rfloor \). Now we consider the derivatives (4.19) for \( C_{ij} \) to zero. We define \( \alpha_j = u_j(t_y) \) in each subinterval \([t_y, t_{y+1}]\). Therefore, the convergence control
parameters can be determined from the solution of the following system of equations.
\[
\frac{\partial f_j}{\partial c_{1,j}} = \frac{\partial f_j}{\partial c_{2,j}} = \cdots = \frac{\partial f_j}{\partial c_{m,j}} = 0.
\] (4.20)

We calculate the approximate analytical solutions on each subinterval as follows.
\[
u(t) = \begin{cases} 
u_1(t), & t_0 \leq t < t_1, \\ 
u_2(t), & t_1 \leq t < t_2, \\ \cdots \\ 
u_{N}(t), & t_{N-1} \leq t \leq T.
\end{cases}
\] (4.21)

We calculate the correctness of MOHAM by
(1) Error norm \(L_2\)
\[
L_2 = \|\psi^\text{exact} - \psi_N\| \approx \sqrt{\frac{b - a}{N} \sum_{k=0}^{N} |\psi_k^\text{exact} - (\psi_N)_k|^2},
\] (4.22)
(2) Error norm \(L_\infty\)
\[
L_\infty = \|\psi^\text{exact} - \psi_N\|_\infty \approx \max |\psi_k^\text{exact} - (\psi_N)_k|.
\] (4.23)

5. Mathematical illustration

The proposed method provides an approximate analytical solution in the form of infinite series [74,75]
\[
\nu_1[F_1(t, \nu)] = T^\nu F_1(t, \nu),
\]
\[
\nu_2[F_2(t, \nu)] = T^\nu F_2(t, \nu),
\]
\[
\nu_3[F_3(t, \nu)] = T^\nu F_3(t, \nu),
\]
\[
\nu_4[F_4(t, \nu)] = T^\nu F_4(t, \nu),
\]
(5.1)

\[
\nu_1[F_1(t, \nu)] = T^\nu F_1(t, \nu) - A_h + \eta_h F_1(t, \nu) + \frac{\lambda_h F_1(t, \nu) F_3(t, \nu)}{N},
\]
\[
\nu_2[F_2(t, \nu)] = T^\nu F_2(t, \nu) - \frac{\lambda_h F_1(t, \nu) F_3(t, \nu)}{N} + \eta_h F_2(t, \nu) + \psi_h F_2(t, \nu),
\]
\[
\nu_3[F_3(t, \nu)] = T^\nu F_3(t, \nu) - \psi_h F_2(t, \nu) + \eta_h F_3(t, \nu) + \gamma_h F_3(t, \nu),
\]
\[
\nu_4[F_4(t, \nu)] = T^\nu F_4(t, \nu) - \gamma_h F_3(t, \nu) - \eta_h F_4(t, \nu),
\]
(5.2)

using (4.2)
\[
(1 - \nu) T^\nu F_1(t, \nu) = H_h(\nu) \left[ T^\nu F_1(t, \nu) - A_h + \eta_h F_1(t, \nu) + \frac{\lambda_h F_1(t, \nu) F_3(t, \nu)}{N} \right],
\]
\[
(1 - \nu) T^\nu F_2(t, \nu) = H_h(\nu) [T^\nu F_2(t, \nu) - \frac{\lambda_h F_1(t, \nu) F_3(t, \nu)}{N} + \psi_h F_2(t, \nu)] + \eta_h F_3(t, \nu),
\]
\[
(1 - \nu) T^\nu F_3(t, \nu) = H_h(\nu) [T^\nu F_3(t, \nu) - \psi_h F_2(t, \nu) + \eta_h F_3(t, \nu) + \gamma_h F_3(t, \nu)],
\]
\[
(1 - \nu) T^\nu F_4(t, \nu) = H_h(\nu) [T^\nu F_4(t, \nu) - \gamma_h F_3(t, \nu) - \eta_h F_4(t, \nu)].
\] (5.3)

where
\[
F_1(t, \nu) = S_0(t) + \sum_{j \leq 1} S_{1,j}(t, \zeta_j) \nu^j,
\]
\[
F_2(t, \nu) = E_0(t) + \sum_{j \leq 1} E_{1,j}(t, \zeta_j) \nu^j,
\]
\[
F_3(t, \nu) = I_0(t) + \sum_{j \leq 1} I_{1,j}(t, \zeta_j) \nu^j,
\] (5.4)
\[ F_4(t, \mathcal{P}) = \mathcal{R}_0(t) + \sum_{j \leq 1} \mathcal{R}_{1,j}(t, \mathcal{C}) \mathcal{P}^j. \]

and

\[ H_k(\mathcal{P}) = \mathcal{P}\mathcal{C}_1 + \mathcal{P}^2\mathcal{C}_2 + \mathcal{P}^3\mathcal{C}_3 + \cdots k = 1, 2 \ldots m. \]

substitute \( F_1(t, \mathcal{P}), F_2(t, \mathcal{P}), F_3(t, \mathcal{P}), F_4(t, \mathcal{P}) \) and \( H_k(\mathcal{P}) \) into (5.3), and equating the coefficient of like powers of \( \mathcal{P} \), gives linear FDEs as,

\[ \begin{align*}
\mathcal{P}^0: & \quad T^\alpha S_0(t) = 0, \\
\alpha \mathcal{P}^0: & \quad T^\alpha \mathcal{E}_0(t) = 0, \\
\mathcal{P}^0: & \quad T^\alpha \mathcal{I}_0(t) = 0, \\
\mathcal{P}^0: & \quad T^\alpha \mathcal{R}_0(t) = 0.
\end{align*} \]  

(5.5)

\[ \begin{align*}
\mathcal{P}^1: & \quad T^\alpha S_1(t) = 1.315726475 \times 10^{-9} \mathcal{S}_0(t) \mathcal{I}_0(t) \mathcal{C}_1 + T^\alpha \mathcal{S}_0(t) \mathcal{C}_1 \\
& \quad - 0.8C_1 - T^\alpha \mathcal{S}_0(t) - 0.01483679525 \mathcal{S}_0(t) \mathcal{C}_1 = 0,
\end{align*} \]  

(5.6)

\[ \begin{align*}
\mathcal{P}^1: & \quad T^\alpha \mathcal{E}_1(t) = 1.315726475 \times 10^{-9} \mathcal{S}_0(t) \mathcal{I}_0(t) \mathcal{C}_1 - T^\alpha \mathcal{E}_0(t) \mathcal{C}_1 \\
& \quad + 0.1391367952 \mathcal{E}_1(t) \mathcal{C}_1 + T^\alpha \mathcal{S}_0(t) = 0,
\end{align*} \]

\[ \begin{align*}
\mathcal{P}^1: & \quad T^\alpha \mathcal{I}_1(t) = T^\alpha \mathcal{I}_0(t) \mathcal{C}_1 - T^\alpha \mathcal{I}_0(t) + 0.1243 \mathcal{E}_0(t) \mathcal{C}_1, \\
& \quad - 0.1120567952 \mathcal{I}_0 \mathcal{C}_1 = 0,
\end{align*} \]

(5.6)

\[ \begin{align*}
\mathcal{P}^1: & \quad T^\alpha \mathcal{R}_1(t) = T^\alpha \mathcal{R}_0(t) \mathcal{C}_1 - T^\alpha \mathcal{R}_0(t) + 0.09722 \mathcal{I}_0(t) \mathcal{C}_1, \\
& \quad - 0.01483679525 \mathcal{R}_0 \mathcal{C}_1 = 0,
\end{align*} \]

\[ \begin{align*}
\mathcal{P}^2: & \quad T^\alpha S_2(t) = T^\alpha \mathcal{S}_0(t) \mathcal{C}_2 + T^\alpha \mathcal{S}_1(t) \mathcal{C}_1 + 1.315726475 \times 10^{-9} \mathcal{S}_0(t) \mathcal{I}_0(t) \mathcal{C}_2, \\
& \quad + 1.315726475 \times 10^{-9} \mathcal{S}_0(t) \mathcal{I}_1(t) \mathcal{C}_1, \\
& \quad + 1.315726475 \times 10^{-9} \mathcal{S}_1(t) \mathcal{I}_0(t) \mathcal{C}_1 + 0.8C_2 + T^\alpha \mathcal{S}_1(t), \\
& \quad + 0.1483679525 \mathcal{S}_1(t) \mathcal{C}_1 + 0.1483679525 \mathcal{S}_0(t) \mathcal{C}_2 = 0.
\end{align*} \]

(5.7)

Using the operator Definitions 2.1–2.2 and Theorems 2.2–2.6 on the above equations with initial conditions gives

\[ \begin{align*}
S_0(t) &= 1, \\
S_0(t) &= 0.5, \\
I_0(t) &= 0.5, \\
R_0(t) &= 0,
\end{align*} \]

(5.8)

\[ \begin{align*}
S_1(t, C_1) &= 7.851632034 t^{1/4} C_1 + 1, \\
E_1(t, C_1) &= 0.6956839628 t^{1/4} C_1 + 0.5, \\
I_1(t, C_1) &= 0.4990679520 t^{1/4} C_1 - 0.499067952 C_1 + 1, \\
R_1(t, C_1) &= 0.8238320475 t^{1/4} C_1 - 0.8238320475 C_1 + 1,
\end{align*} \]

(5.9)

\[ \begin{align*}
S_2(t, C_1, C_2) &= 0.5824653329 t^{1/4} C_1^2 - 7.851632041 t^{1/4} C_1^2, \\
& \quad + 0.09722 \mathcal{I}_0(t) \mathcal{C}_1 - 0.01483679525 \mathcal{R}_0 \mathcal{C}_2, \\
& \quad - 0.1120567952 \mathcal{I}_0 \mathcal{C}_1 = 0.
\end{align*} \]
efficiency of the proposed method with other previous approximation analytical methods. The fractional-order results are locally asymptotically stable when $R_0$ and $WHO$ started from 4 January 2020 to 14 October 2021. From the stability analysis, we affirmed that the given model is globally asymptotically stable. The simulations have been obtained by using the MOHAM technique. The MOHAM technique was used to numerically simulate the qualitative behaviours, performance, and mathematical representation of the fractional-order models. The fractional derivative operator is based on conformable fractional derivative sense. We conduct numerical simulations to equate the proposed model results with the real data obtained from various reports from Worldometer and WHO. From the stability analysis, we affirmed that the given model is locally asymptotically stable when $R_0 < 1$. It really signifies that by taking $R_0 < 1$ the total population tends to DFE, and the disease will die out from the population. The comparisons with other approximation methods are shown in Figs. 2–5 for the value of $\alpha = 1$ with MOHAM. The effects of fractional-order $\alpha \in [0.7, 0.8, 0.9, 1]$ are examined while the model parameters are kept fixed as demonstrated in Figs. 6–9. In order to illustrate the effect of the fractional-order parameter $\alpha$ on the susceptible, exposed, infection, and recovered, we considered different values of it, that is, $\alpha \in [0.7, 0.8, 0.9, 1]$. Table 1 is the estimated and fitted values for different parameters. Tables 2–5 show the mathematical values and efficiency of the proposed method with other previous approximation analytical methods. The fractional-order results
in Tables 6–9 show the spreads realistic trajectory behaviour, and performance of the fractional-order of the model. The results show that the accurate series solution continually relies on the optimal values of the control-convergence parameters $C_k$ as described in (4.19)–(4.20). It is observed that as we move along the domain, we get consistent accuracy. From Tables 2–5 and 6–9, it is evident that with the increase in the order of approximation, the accuracy increases. Thus, the series solutions for COVID-19 model converge. We observed that the susceptible group decreases with time while that of the recovered group gradually increases due to the inclusion of the vaccinated susceptible group. The population of infected group decreases in the period of the epidemic. A decrease is noticed in the infected class by decreasing the values of conformable derivative operator parameter $\alpha$. Also, we observe that the reproduction number varies with the order of the fractional derivative $\alpha$. Bottom up, the use of the fractional derivative operator in the mathematical model for
Fig. 4. For the value of $\alpha = 1$ (MOHAM, OHAM, LHAM, and HAM) at infected.

Fig. 5. For the value of $\alpha = 1$ (MOHAM, OHAM, LHAM, and HAM) at Recovered.
Fig. 6. Susceptible cases with different $\alpha$.

Fig. 7. Exposed cases with different $\alpha$. 
COVID-19 model brought new patterns for the epidemics, unseen in the integer-order model. In addition, we also show the effects of fractional-order COVID-19 model on the stability of the equilibrium points, these numerical simulations are very important from biological point of view.
Table 1
Estimated and fitted values for different parameters China [76].

| Parameter | Description          | Value        | Source   |
|-----------|----------------------|--------------|----------|
| $\Lambda_h$ | Birth Rate       | 3,270,186.2462 | Estimated |
| $\eta_h$   | Natural Mortality Rate | 1.673343   | Fitted   |
| $\lambda_h$ | Contact Rate    | 0.29         | Fitted   |
| $\psi_h$   | Incubation Period | 0.1243       | Fitted   |
| $\gamma_h$ | Recovery Rate    | 0.09722      | Fitted   |

Table 2
The number of Susceptible (T) individuals case $\alpha = 1$.

| Time (s) | MOHAM | OHAM | LHAM | HAM | Abs Error |
|----------|-------|------|------|-----|-----------|
|          |       |      |      |     | [MOHAM-OHAM]       |
| 0        | 1.00000000 | 1.00000000 | 1.00000000 | 1.00000000 | 0.00000000 |
| 1        | 0.626018651 | 0.631179948 | 0.633760597 | 0.638921894 | 0.005161297 |
| 2        | 0.26700155 | 0.270001075 | 0.273551535 | 0.280652456 | 0.00710092 |
| 3        | 0.137588212 | 0.145984696 | 0.150182133 | 0.158577008 | 0.008394875 |
| 4        | 0.094107481 | 0.103813707 | 0.108689938 | 0.118373047 | 0.01243344 |
| 5        | 0.079396206 | 0.090467658 | 0.096033848 | 0.107048362 | 0.01671515 |
| 6        | 0.0763660401 | 0.093738948 | 0.10186638 | 0.11088413 | 0.013697492 |
| 7        | 0.079297693 | 0.09414778 | 0.101572823 | 0.11642291 | 0.014850087 |
| 8        | 0.082492196 | 0.09835027 | 0.106279307 | 0.12213738 | 0.015858074 |
| 9        | 0.08564134 | 0.102356489 | 0.11074063 | 0.12742912 | 0.016715149 |

Table 3
The number of Exposed (T) individuals case $\alpha = 1$.

| Time (s) | MOHAM | OHAM | LHAM | HAM | Abs Error |
|----------|-------|------|------|-----|-----------|
|          |       |      |      |     | [MOHAM-OHAM]       |
| 0        | 0.362456319 | 0.3698748 | 0.381288183 | 0.393053753 | 0.007418481 |
| 1        | 0.287847647 | 0.296179045 | 0.309130456 | 0.32264821 | 0.008331398 |
| 2        | 0.195375521 | 0.201792867 | 0.21186264 | 0.222337541 | 0.006417346 |
| 3        | 0.125637553 | 0.130055558 | 0.13697556 | 0.144263763 | 0.00448005 |
| 4        | 0.079293301 | 0.082221876 | 0.086818785 | 0.091672701 | 0.002928575 |
| 5        | 0.049752777 | 0.051699848 | 0.054684843 | 0.057875766 | 0.001917071 |
| 6        | 0.031194647 | 0.032445899 | 0.034417508 | 0.036508924 | 0.001251252 |
| 7        | 0.019585053 | 0.020402388 | 0.021692789 | 0.023064805 | 0.000817335 |
| 8        | 0.012321955 | 0.012856963 | 0.013703345 | 0.014605444 | 0.000535008 |
| 9        | 0.007769772 | 0.008120774 | 0.008677231 | 0.009271817 | 0.000351002 |
| 10       | 0.362456319 | 0.3698748 | 0.381288183 | 0.393053753 | 0.007418481 |

Table 4
The number of Infected (T) individuals case $\alpha = 1$.

| Time (s) | MOHAM | OHAM | LHAM | HAM | Abs Error |
|----------|-------|------|------|-----|-----------|
|          |       |      |      |     | [MOHAM-OHAM]       |
| 0        | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 1        | 0.73145273 | 0.738900226 | 0.754023475 | 0.769456256 | 0.007447496 |
| 2        | 0.662434043 | 0.679523332 | 0.69140168 | 0.711412926 | 0.009518288 |
| 3        | 0.501368108 | 0.509535604 | 0.526271919 | 0.543557959 | 0.008167496 |
| 4        | 0.356747927 | 0.362966197 | 0.375729788 | 0.388942207 | 0.00621827 |
| 5        | 0.248688984 | 0.253015789 | 0.262360533 | 0.27205041 | 0.00456806 |
| 6        | 0.171903385 | 0.175139687 | 0.181341443 | 0.188944416 | 0.003280581 |
| 7        | 0.11831934 | 0.12119743 | 0.12605288 | 0.131109001 | 0.00235891 |
| 8        | 0.082271057 | 0.083970203 | 0.087447497 | 0.09125035 | 0.001699146 |
| 9        | 0.057095428 | 0.058321773 | 0.060854052 | 0.06349628 | 0.001226346 |
| 10       | 0.039725433 | 0.040612826 | 0.042447522 | 0.044365102 | 0.000887393 |
Table 5
The number of Recovered (T) individuals case $\alpha = 1$.

| Time (s) | MOHAM | OHAM | LHAM | HAM | Abs Error |
|----------|-------|------|------|-----|-----------|
| 0        | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
| 1        | 0.18097668 | 0.18394380 | 0.18526252 | 0.18789959 | -0.00296712 |
| 2        | 0.30372634 | 0.30885071 | 0.31088507 | 0.31529044 | -0.00456039 |
| 3        | 0.38271363 | 0.39003215 | 0.39179843 | 0.39738885 | -0.00629252 |
| 4        | 0.43179971 | 0.43896244 | 0.44217506 | 0.44855986 | -0.00718293 |
| 5        | 0.46160419 | 0.46936818 | 0.47284439 | 0.47976215 | -0.00778193 |
| 6        | 0.47934058 | 0.48752306 | 0.49116083 | 0.49843508 | -0.00818353 |
| 7        | 0.48966173 | 0.49811444 | 0.50187120 | 0.50938472 | -0.00845271 |
| 8        | 0.49549832 | 0.50413455 | 0.50796840 | 0.51564230 | -0.00863314 |
| 9        | 0.49866556 | 0.50741967 | 0.51131032 | 0.51909176 | -0.00875408 |
| 10       | 0.50027509 | 0.50911024 | 0.51303696 | 0.52089046 | -0.00883519 |

Table 6
The number of each individual case (MOHAM, $\alpha = 1$).

| Time (t) | Susceptible (t) | Exposed (t) | Infected (t) | Recovered (t) |
|----------|-----------------|-------------|--------------|---------------|
| 0        | 1.50000000      | 0.30000000  | 0.50000000   | 0.00000000    |
| 1        | 0.62601869      | 0.28784764  | 0.66243404   | 0.18097668    |
| 2        | 0.26290016      | 0.19537552  | 0.50136811   | 0.30372635    |
| 3        | 0.13758982      | 0.19537552  | 0.50136811   | 0.30372635    |
| 4        | 0.09410749      | 0.15637755  | 0.35674791   | 0.43179971    |
| 5        | 0.07936216      | 0.09792930  | 0.24846898   | 0.46160419    |
| 6        | 0.05761572      | 0.04975277  | 0.17190336   | 0.47934016    |
| 7        | 0.03640387      | 0.03194644  | 0.11831933   | 0.48661747    |
| 8        | 0.02927689      | 0.01958505  | 0.08227105   | 0.49549832    |
| 9        | 0.01249213      | 0.01231951  | 0.05709542   | 0.49866556    |
| 10       | 0.00856314      | 0.00776977  | 0.03972547   | 0.50027508    |

Table 7
The number of each individual case (MOHAM, $\alpha = 0.9$).

| Time (t) | Susceptible (t) | Exposed (t) | Infected (t) | Recovered (t) |
|----------|-----------------|-------------|--------------|---------------|
| 0        | 1.50000000      | 0.30000000  | 0.50000000   | 0.00000000    |
| 1        | 0.65892194      | 0.32753696  | 0.66908393   | 0.16440268    |
| 2        | 0.28065247      | 0.24957694  | 0.5743603    | 0.2761928     |
| 3        | 0.158577        | 0.16622328  | 0.4265583    | 0.34777367    |
| 4        | 0.11837306      | 0.10569933  | 0.30003232   | 0.39198945    |
| 5        | 0.10770485      | 0.06642473  | 0.20726109   | 0.41873096    |
| 6        | 0.10681993      | 0.04118301  | 0.14229353   | 0.43387606    |
| 7        | 0.11088412      | 0.02562603  | 0.09761902   | 0.44270224    |
| 8        | 0.1164229       | 0.01596406  | 0.06706035   | 0.44753643    |
| 9        | 0.12213738      | 0.00968289  | 0.04616442   | 0.45003175    |
| 10       | 0.12742922      | 0.00622964  | 0.03185099   | 0.4519087     |

Table 8
The number of each individual case (MOHAM, $\alpha = 0.8$).

| Time (t) | Susceptible (t) | Exposed (t) | Infected (t) | Recovered (t) |
|----------|-----------------|-------------|--------------|---------------|
| 0        | 1.50000000      | 0.30000000  | 0.50000000   | 0.00000000    |
| 1        | 0.65698648      | 0.29598178  | 0.59730419   | 0.14800868    |
| 2        | 0.30505097      | 0.2163945  | 0.49799639   | 0.24865024    |
| 3        | 0.18795907      | 0.14142088  | 0.36291096   | 0.31283338    |
| 4        | 0.15234485      | 0.08892523  | 0.25250325   | 0.35198936    |
| 5        | 0.14582493      | 0.05517314  | 0.17287841   | 0.37513772    |
| 6        | 0.15030163      | 0.03408935  | 0.11778389   | 0.38841195    |
| 7        | 0.15828534      | 0.02105148  | 0.08019287   | 0.39574274    |
| 8        | 0.16838021      | 0.01301254  | 0.05466187   | 0.39957454    |
| 9        | 0.17764063      | 0.00805548  | 0.03722617   | 0.40139753    |
| 10       | 0.18593224      | 0.00499479  | 0.02553744   | 0.40210665    |
Table 9
The number of each individuals case (MOHAM, α = 0.7).

| Time(t) | Susceptible(t) | Exposed(t) | Infected(t) | Recovered(t) |
|---------|----------------|------------|-------------|--------------|
| 0       | 1.50000000     | 0.30000000 | 0.50000000  | 0.00000000   |
| 1       | 0.67763167     | 0.26746664 | 0.5397594   | 0.13152469   |
| 2       | 0.33390937     | 0.18762383 | 0.43178541  | 0.22112569   |
| 3       | 0.22153856     | 0.12031927 | 0.30876053  | 0.27789309   |
| 4       | 0.19116976     | 0.07481312 | 0.2124319   | 0.31208419   |
| 5       | 0.19011074     | 0.04602282 | 0.14421451  | 0.33190449   |
| 6       | 0.19999501     | 0.02821756 | 0.09745956  | 0.34294785   |
| 7       | 0.22153856     | 0.01203193 | 0.06587749  | 0.35276412   |
| 8       | 0.22779855     | 0.00651325 | 0.03018002  | 0.35161265   |
| 9       | 0.24107293     | 0.00400471 | 0.02047536  | 0.35302243   |

6. Conclusions

This article presents the trend of COVID-19 in China using a SEIR compartmental fractional-time derivative model for prediction. The SEIR model shows the ability to predict the COVID-19 spread in China. The enactment of lockdown by the Chinese Government has proven to be efficient in reducing the $R_0$ subsequently to the number of active cases. It is noted that all these could not have been possible if the government received poor response from the public, which would hinder the positive trend current trajectory. In this study, we used fractional-time derivative SEIR model for prediction of COVID-19 in China. Two equilibrium points DFE and EEP are obtained for the model. The basic reproduction number $R_0$ is obtained using next generation approach. We generalized the SEIR model by applying the recently developed conformable derivative operator. Also, an approximate analytical method MOHAM is used to solve the proposed model. This procedure is effective and has a distinct advantage over the previous approximation analytical methods. The second-order approximation solutions were used unlike other approximate analytical methods with five-ten approximations solution. Lastly, we obtained numerical simulation and mathematical values as shown on graphs and tables. The MOHAM technique is reliable, dependable, and efficient for finding an approximate analytical solution of SEIR COVID-19 model and prediction. In future, researchers and authors can include the quarantine or vaccination or both the compartment to the model and show the effect of these compartments on the spread of this infectious disease. The MOHAM approach can also be extended to solving other non-linear fractional-order biological models including dengue fever, predator–prey, and tumour immune models. This infectious disease in not yet controlled in China as well as worldwide.

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