**Abstract**

Large fermion mass effect on the Yukawa process $Z \rightarrow b\bar{b} \rightarrow b\bar{b}h(A)$ in the entire range of the neutral Higgs boson masses is found. It is particularly important for light Higgs bosons, which are still not excluded experimentally in a general two-Higgs-doublet model.

The light Higgs bosons in the context of the standard model (SM) and its minimal supersymmetric extension (MSSM) have been ruled out from zero up to a value that depends on the model assumed. In the framework of the SM the dominant Higgs boson production process is the Bjorken process

$$Z \rightarrow H_{SM}Z^* \rightarrow H_{SM}f\bar{f}$$

and the mass limit $m_{H_{SM}} \geq 65.1$ GeV has been established based on the results of 4 collaborations collecting data at LEP. In extensions of the SM, like MSSM or general two-Higgs-doublet model (2HDM), the Higgs sector is much richer: there are two CP-even neutral Higgs bosons denoted by $h$ and $H$ (we assume that $m_h \leq m_H$), a CP-odd neutral $A$, and a pair of charged scalars $H^\pm$. The CP-even Higgs bosons can be produced via the process (1) however with lower rates because they share the couplings to $Z$ boson, i.e.

$$\Gamma(Z \rightarrow hZ^*) = \Gamma_{SM}(Z \rightarrow H_{SM}Z^*) \sin^2(\beta - \alpha)$$

$$\Gamma(Z \rightarrow HZ^*) = \Gamma_{SM}(Z \rightarrow H_{SM}Z^*) \cos^2(\beta - \alpha)$$

where $\alpha$ and $\beta$ are mixing angles in the neutral and charged Higgs sectors, respectively, with $\tan \beta$ given by the ratio of the vacuum expectation values of the Higgs doublets, $\tan \beta = v_2/v_1$.  

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In both MSSM and 2HDM models there is another production process, namely the Higgs pair production, which is complementary to (2) and (3) in the sense that

\[ \Gamma(Z \rightarrow hA) = 0.5 \Gamma(Z \rightarrow \nu\bar{\nu}) \cos^2(\beta - \alpha) \lambda^p \]

\[ \Gamma(Z \rightarrow HA) = 0.5 \Gamma(Z \rightarrow \nu\bar{\nu}) \sin^2(\beta - \alpha) \lambda^p \]

where \( \lambda = (1 - \kappa_h - \kappa_A)^2 - 4\kappa_h\kappa_A \), with \( \kappa_i = m_i^2/m_Z^2 \) and \( p = 3/2 \).

In the MSSM the parameters of the Higgs sector are constrained and only two of them are independent, for example \( \tan \beta \) and \( m_A \). This remains true also after taking radiative corrections, which are important for a heavy top quark, although the relations among them are modified and depend on \( m_t \) and some other parameters of the supersymmetric sector. The allowed domain for \( m_h \) and \( m_A \) is restricted theoretically and for any given \( m_h \) and \( m_A \) the values of \( \sin^2(\beta - \alpha) \) and \( \cos^2(\beta - \alpha) \) are restricted to vary in a certain range. In the MSSM the heavier Higgs boson \( H \) cannot be produced at LEP (because \( m_H \geq m_Z \)) and combining the negative search results from processes (2) and (4) the Delphi Collaboration\(^2\) recently set the mass limits \( M_h \geq 44 \) GeV for any \( \tan \beta \) and \( M_A \geq 27 \) GeV for \( \tan \beta \geq 1 \) assuming the mass of the top quark \( m_t = 170 \) GeV and degeneracy of the top-squarks with \( m_{sq} = 1 \) TeV. There is, however, no lower limit on \( M_A \) when \( M_h \geq 60 \) GeV.

On the other hand in a general 2HDM the masses and mixing angles in the Higgs sector are unrelated and unconstrained theoretically and therefore much weaker bounds can be established. From the process (2) one can derive experimentally an upper limit \( \sin^2(\beta - \alpha) < 0.1 \) for \( m_h < 50 \) GeV\(^3\). Therefore one can imagine a scenario in which \( \sin^2(\beta - \alpha) \sim 0 \). In addition, if one assumes the process (4) to be forbidden kinematically (i.e. \( M_A + m_h > M_Z \)), then either very light \( h \) or very light \( A \) cannot be ruled out on the basis of negative searches via processes (2) and (4).

For large \( \tan \beta \), however, there is still another important process in the 2HDM\(^4\) namely the bremsstrahlung of the Higgs boson from the fermion line in the final state, which we will call a Yukawa process (Fig.1.)

\[ Z \rightarrow f\bar{f} \rightarrow f\bar{f}h \text{ or } f\bar{f}A \]

It can produce a substantial number of Higgs bosons for \( f = b, \tau \) because of strong enhancement of the Higgs couplings to down-type fermions by a factor \( \sim \tan \beta \). The Higgs boson then would predominantly decay to the heaviest possible fermion pair. Note that it is a single Higgs boson production process with different topology than in the Bjorken process. Such events have not been fully analysed experimentally yet, although the possible importance of such a process has been pointed out in the literature long time ago\(^5\), \(^6\), \(^7\), \(^8\).

The process (4) has been analysed in the literature analytically\(^9\) for a pseudoscalar and numerically\(^5\), \(^6\) for a scalar Higgs bosons. The analytical treatment can however be done in the limit of vanishing fermion masses in which the the formulas for \( h \) and \( A \) production are the same (after appropriate change of the Higgs couplings and masses).

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\(^2\)In the MSSM the process (4) is never competitive with the sum (2)+(4) for \( M_h \leq 50 \) GeV.

\(^3\)For example, for \( M_h = 10 \) GeV, \( \tan \beta = 20 \) one expects about 3000 \( bbh \) events in \( 10^7 \) \( Z \) decays.
In this short note we would like to point out that the proper treatment of fermion masses is quite important not only at the edge of the available phase space when $2m_f + m_{h,A} \to m_Z$ but in the entire range of the Higgs boson mass.

The analytical expressions in the lowest order for the differential decay distributions $d\Gamma/dx_1 dx_2$ (with $x_1 = 2E_b/M_Z$ and $x_2 = 2E_{\bar{b}}/M_Z$) for both $Z \to b\bar{b}h$ and $Z \to b\bar{b}A$ can be inferred from eqs. (15) and (17) in ref.[7]. In the case of $\alpha = \beta$ (which we consider here) both distributions scale as $m_b^2 \tan^2 \beta$ due to the Higgs coupling $\sim m_b \tan \beta$ to the down-type fermion line. In addition, the fermion mass $m_b$ enters (i) the phase space integration limits when the contribution to the total decay rate is calculated, and (ii) the matrix element. If one neglects $m_b$ in the matrix elements the distributions for $Z \to b\bar{b}h$ and $Z \to b\bar{b}A$ are equal, and neglecting in addition $m_b$ in (i) the integral over $x_1$ and $x_2$ can be done in an elegant way[4]. The resulting decay widths as a function of the corresponding Higgs masses are shown in Figs. 2 and 3 (dotted lines). However, we find that although $m_b/M_Z \ll 1$ the fermion mass $m_b$ plays an important role in the entire range of Higgs boson mass. In Figs. 2 and 3 we show the effect of retaining the fermion mass (with $m_b = 5$ GeV) in the integration limits only and neglecting it in the matrix element (dashed lines), and keeping full $m_b$ dependence (solid lines).

In the case of $Z \to b\bar{b}A$ (Fig.2) we see that keeping $m_b \neq 0$ is very important – it cures the fake IR behaviour of the massless approximation and results in lowering the predicted decay rate by at least 15% for all $M_A$. Comparing the dashed and solid lines in Fig.2 we notice that once the kinematical limits are properly taken into account in (i) one can neglect $m_b$ in the matrix element (ii).

On the other hand for $Z \to b\bar{b}h$ (Fig.3) keeping $m_b \neq 0$ in (i) and (ii) leads to an enhancement of the scalar $h$ production in $Z$ decays up to $m_b = 50$ GeV (solid line). Also, contrary to the pseudoscalar case, one cannot neglect $m_b$ in the matrix element because it would lead to wrong IR behaviour (dashed line).

Similar mass effects also appear in the process (8) with $f = \tau$ lepton in the final state. The results presented here have been obtained for $\tan \beta = 20$. For other values of $\tan \beta$ the results can simply be obtained by rescaling.

The observed fermion mass effects are phenomenologically important as they modify significantly the predicted decay rate of the $Z$ boson as compared to the massless approximation. They have to be taken into account when performing the detailed search for a light Higgs boson or in deriving the experimental limits on the parameters of the two-Higgs-doublet model[8].

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4 $\Gamma = 1$ MeV corresponds to 4000 events per $10^7$ Z decays
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Figure Captions

Fig.1. The Feynman diagrams for the neutral Higgs boson production ($h$ or $A$) via the Yukawa process.

Fig.2. The decay width $\Gamma(Z \to b\bar{b}A)$ as a function of the pseudoscalar Higgs boson mass in the 2HDM for $\tan\beta = 20$. The solid line represents the results with full $m_b$ dependence, dashed lines - with $m_b$ neglected in the matrix element, and dotted lines - with $m_b$ neglected in the matrix element and integration limits.

Fig.3. The decay width $\Gamma(Z \to b\bar{b}h)$ as a function of the scalar Higgs boson mass in the 2HDM for $\tan\beta = 20$. Tla labeling of lines is the same as in Fig.3.
Fig. 1a
Fig. 2a

M_A (GeV)

Decay width (MeV)

\(\tan(\beta) = 20.0\)

mb = 5.0

ME with mb = 0

mb = 0
Decay width (MeV) vs. $M_A$ (GeV) for $\tan(\beta) = 20.0$.

- Solid line: $mb = 5.0$
- Dashed line: ME with $mb = 0$
- Dotted line: $mb = 0$

Fig. 2b
Fig. 3a

Decay width (MeV) vs. $M_h$ (GeV) with $\tan(\beta) = 20.0$.

- $mb = 5.0$
- ME with $mb = 0$
- $mb = 0$

Fig. 3a
Fig. 3b

$\tan(\beta) = 20.0$

MB with $m_b = 5.0$

$M_B$ with $m_b = 0$

$mb = 0$