Doping and energy evolution of quasiparticle transport in cuprate superconductors with extended impurities

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Abstract. Within the nodal approximation of the quasiparticle excitations and scattering processes, we study the effect of the extended impurity scatterers on the quasiparticle transport of cuprate superconductors based on the framework of the kinetic energy driven superconducting mechanism. It is shown that there is a cusplike shape of the energy dependent microwave conductivity spectrum. In contrast with the dome shape of the doping dependent superconducting gap parameter, the minimum microwave conductivity occurs around the optimal doping, and then increases in both underdoped and overdoped regimes.

1. Introduction
Understanding the role of impurities in cuprate superconductors has taken many years of great effort [1]. This follows from a fact that the physical properties of cuprate superconductors in the superconducting (SC) state are extreme sensitivity to the impurity effect than the conventional superconductors due to the finite angular-momentum charge carrier Cooper pairing [2]. Experimentally, By virtue of systematic studies using the microwave conductivity measurements, some essential features of the evolution of the quasiparticle transport with energy have been established [3], where the low temperature experimental results show the existence of the very long-live excitation deep in the SC state, as evidenced by the sharp cusplike energy dependent microwave conductivity spectrum, while the width of the sharp peak is nearly temperature independent, and main behaviors of the microwave conductivity are governed by thermally excited quasiparticles being scattered by impurities or other defects. Theoretically, an agreement has emerged that the simple Bardeen-Cooper-Schrieffer (BCS) formalism with the d-wave SC gap function is useful in the phenomenological description of the quasiparticle transport [4, 5]. In this case, the microwave conductivity of cuprate superconductors has been phenomenologically discussed in zero temperature and energy by including the contributions of the vertex corrections [4]. Recently, these phenomenological discussions have been generalized to study the energy dependence of the quasiparticle transport [5]. However, to the best of our knowledge, the microwave conductivity of cuprate superconductors has not been treated starting from a microscopic SC theory, and no explicit calculations of the doping dependence of the microwave conductivity has been made so far. In this paper, we start from the kinetic energy driven SC mechanism [6], and study the effect of the extended impurity scatterers on...
the microwave conductivity of cuprate superconductors. We evaluate explicitly the microwave conductivity within the nodal approximation of the quasiparticle excitations and scattering processes, and qualitatively reproduced some main features of the microwave conductivity measurements on cuprate superconductors in the SC state [3].

In cuprate superconductors, the essential physics can be described by the $t$-$J$ model [7],

$$H = -t \sum_{i\sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{i+\hat{n}\sigma} + \mu \sum_{i\sigma} \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma} + J \sum_{i \neq j} \hat{S}_i \cdot \hat{S}_{i+\hat{n}},$$

(1)

where $\hat{n} = \pm \hat{x}, \pm \hat{y}$, $C_{i\sigma}$ ($C_{i\sigma}$) is the electron creation (annihilation) operator, $\hat{S}_i = (\hat{S}^x_i, \hat{S}^y_i, \hat{S}^z_i)$ is the spin operator, and $\mu$ is the chemical potential. This $t$-$J$ model (1) is subject to an important local constraint $\sum_{\sigma} C_{i\sigma}^\dagger C_{i\sigma} \leq 1$ to avoid the double occupancy [8]. The strong electron correlation in the $t$-$J$ model manifests itself by this local constraint [8], which can be treated properly in analytical calculations within the charge-spin separation (CSS) fermion-spin theory [9], where the constrained electron operators are decoupled as $C_{i\uparrow} = h_{i\uparrow}^1 S_{i\uparrow}$ and $C_{i\downarrow} = h_{i\downarrow}^1 S_{i\downarrow}^\dagger$, with the spinful fermion operator $h_{i\sigma} = e^{-i\Phi_{i\sigma}} h_i$ describes the charge degree of freedom together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (charge carrier), while the spin operator $S_i$ describes the spin degree of freedom (spin), then the $t$-$J$ model (1) can be rewritten in this CSS fermion-spin representation as,

$$H = t \sum_{i \neq j} (h_{i+\hat{n}\uparrow}^1 h_{i\downarrow}^1 S_{i+\hat{n}\uparrow}^\dagger S_{i\downarrow}^\dagger + h_{i+\hat{n}\downarrow}^1 h_{i\uparrow}^1 S_{i+\hat{n}\downarrow}^\dagger S_{i\uparrow}^\dagger) - \mu \sum_{i\sigma} h_{i\sigma}^1 h_{i\sigma} + J_{\text{eff}} \sum_{i \neq j} \hat{S}_i \cdot \hat{S}_{i+\hat{n}},$$

(2)

where $J_{\text{eff}} = (1 - \delta)^2 J$, and $\delta = \langle h_{i\sigma}^\dagger h_{i\sigma} \rangle = \langle h_{i\sigma}^1 h_i \rangle$ is the doping concentration.

Based on the CSS fermion-spin theory [9], we have developed a kinetic energy driven SC mechanism [6], where the charge carrier-spin interaction from the kinetic energy term in the $t$-$J$ model (2) induces the charge carrier pairing state with the d-wave symmetry by exchanging spin excitations, then the electron Cooper pairs originating from the charge carrier pairing state are due to the charge-spin recombination, and their condensation reveals the d-wave SC ground-state. In particular, this d-wave SC state is controlled by both SC gap function and quasiparticle coherence. Moreover, we have shown that this SC state is the conventional BCS like with the d-wave symmetry [10], so that the basic BCS formalism with the d-wave SC gap function is still valid in quantitatively reproducing all main low energy features of the electronic structure of cuprate superconductors, although the pairing mechanism is driven by the kinetic energy by exchanging spin excitations. Following our previous discussions [6, 10], the full charge carrier Green’s function in the SC state can be obtained in the Nambu representation as,

$$\tilde{g}(\mathbf{k}, \omega) = Z_{hF} \frac{1}{\omega^2 - E_{h\mathbf{k}}^2} \left( \frac{\omega + \xi_\mathbf{k}}{\Delta_{h\mathbf{k}}(\mathbf{k})} \frac{\Delta_{h\mathbf{k}}(\mathbf{k})}{\omega - \xi_\mathbf{k}} \right) = Z_{hF} \frac{\omega \tau_0 + \Delta_{h\mathbf{k}}(\mathbf{k}) \tau_1 + \xi_\mathbf{k} \tau_3}{\omega^2 - E_{h\mathbf{k}}^2},$$

(3)

where $\tau_0$ is the unit matrix, $\tau_1$ and $\tau_3$ are Pauli matrices, the renormalized charge carrier excitation spectrum $\xi_\mathbf{k} = Z_{hF} \xi_\mathbf{k}$, with the mean-field charge carrier excitation spectrum $\xi_\mathbf{k} = Z \chi \gamma_\mathbf{k} - \mu$, the spin correlation function $\chi = \langle S^z_i S^z_{i+\hat{n}} \rangle$, $\gamma_\mathbf{k} = (1/Z) \sum_{\vec{q}} e^{i\vec{q}\cdot\vec{r}}$, $Z$ is the number of the nearest neighbor sites, the renormalized charge carrier pair gap function $\Delta_{h\mathbf{k}}(\mathbf{k}) = Z_{hF} \Delta_{h}(\mathbf{k})$, where the effective charge carrier pair gap function $\Delta_{h}(\mathbf{k}) = \Delta_{h}(\mathbf{k})$ with $\gamma_\mathbf{k}^{(d)} = (\cos k_x - \cos k_y)/2$, and the charge carrier quasiparticle spectrum $E_{h\mathbf{k}} = \sqrt{\xi_\mathbf{k}^2 + |\Delta_{h\mathbf{k}}(\mathbf{k})|^2}$, while the charge carrier quasiparticle coherent weight $Z_{hF}$ and effective charge carrier gap parameters $\Delta_{h}$ are determined by the self-consistent calculation [6, 10].

In the CSS fermion-spin theory [9], the electron Green’s function is a convolution of the spin Green’s function and charge carrier Green’s function. Since the spins center around the $[\pi, \pi]$
point in the mean-field level [6, 10], then the main contributions for the spins comes from the \([\pi, \pi]\) point. In this case, the electron Green's function can be approximately reduced as the BCS formalism with the d-wave SC gap function [6],

\[
\tilde{G}(\mathbf{k}, \omega) = Z_F \frac{1}{\omega^2 - E_{\mathbf{k}}^2} \left( \frac{\omega + \xi_{\mathbf{k}}}{\Delta_{\mathbf{k}}(\mathbf{k})} \xi_{\mathbf{k}} \right) = Z_F \frac{\omega \tau_0 + \Delta_{\mathbf{k}}(\mathbf{k}) \tau_1 + \xi_{\mathbf{k}} \tau_3}{\omega^2 - E_{\mathbf{k}}^2}.
\] (4)

where the electron quasiparticle coherent weight 

\[
Z_F = \frac{Z_{hF}}{2}, \quad \tilde{\xi}_{\mathbf{k}} = Z_F \xi_{\mathbf{k}}, \quad \xi_{\mathbf{k}} = Z t \chi_{\gamma} + \mu,
\]

\[
\tilde{\Delta}_{\mathbf{k}}(\mathbf{k}) = \Delta_{h\mathbf{k}}(\mathbf{k})/2, \quad \text{and electron quasiparticle spectrum } E_{\mathbf{k}} \approx \sqrt{\xi_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}(\mathbf{k})|^2}.
\]

In the presence of impurities, the unperturbed electron Green's function (4) is dressed via the impurity scattering as 

\[
\tilde{\tilde{G}}(\mathbf{k}, \omega) = \tilde{G}(\mathbf{k}, \omega) \left[ 1 - \tilde{\Sigma}(\mathbf{k}, \omega) \right],
\]

with the self-energy 

\[
\tilde{\Sigma}(\mathbf{k}, \omega) = \sum_{\alpha} \Sigma_{\alpha}(\mathbf{k}, \omega) \tau_{\alpha}.
\]

It has been shown that all but the scalar component of the self-energy can be neglected or absorbed into \(\tilde{\Delta}_{\mathbf{k}}(\mathbf{k})\) [4, 5]. Based on the phenomenological d-wave BCS-type electron Green's function [4], the energy dependence of the microwave conductivity of cuprate superconductors has been fitted [5], where the electron self-energies \(\Sigma_0(\mathbf{k}, \omega)\) and \(\Sigma_3(\mathbf{k}, \omega)\) have been treated within the T-matrix approximation. Following their discussions [4, 5], we can obtain the microwave conductivity of cuprate superconductors in the present case as [11],

\[
\sigma(\omega, T) = -\frac{\text{Im} \Pi(\omega, T)}{\omega},
\] (5)

where \(\Pi(\omega, T)\) is the electron current-current correlation function in the SC state, and have been obtained in Ref. [11].

**Figure 1.** The microwave conductivity as a function of energy with \(T = 2\)K (solid line), \(T = 4\)K (dashed line), \(T = 8\)K (dash-dotted line), and \(T = 10\)K (dotted line) at \(\rho = 0.000014\) for \(t/J = 2.5\), \(V_1 = 58J\), \(V_2 = 49.32J\), and \(V_3 = 40.6J\) in \(\delta = 0.15\). Inset: the corresponding experimental result taken from Ref. [3].

**Figure 2.** The microwave conductivity as a function of doping with \(T = 2\)K and \(\omega \approx 1.81\)GHz at \(\rho = 0.000014\) for \(t/J = 2.5\), \(V_1 = 58J\), \(V_2 = 49.32J\), and \(V_3 = 40.6J\). The dashed line is the corresponding result of the superconducting gap parameter.

Now we can discuss the effect of the extended impurity scatterers on the quasiparticle transport in cuprate superconductors. In cuprate superconductors, although the values of \(J\) and
$t$ is believed to vary somewhat from compound to compound, however, as a qualitative discussion, the commonly used parameters in this paper are chosen as $t/J = 2.5$, with an reasonably estimmative value of $J \sim 1000 K$. We have performed a calculation for $\sigma(\omega, T)$ in Eq. (5), and the results of $\sigma(\omega, T)$ as a function of energy with temperature $T = 0.002 J = 2 K$ (solid line), $T = 0.004 J = 4 K$ (dashed line), $T = 0.008 J = 8 K$ (dash-dotted line), and $T = 0.01 J = 10 K$ (dotted line) under the slightly strong impurity scattering potential with $V_1 = 58 J$, $V_2 = 49.32 J$, and $V_3 = 40.6 J$ at the impurity concentration $\rho = 0.000014$ for the doping concentration $\delta = 0.15$ are plotted in Fig. 1 in comparison with the corresponding experimental results of cuprate superconductors [3] (inset). Obviously, the energy evolution of the microwave conductivity [3] is qualitatively reproduced. In particular, a low temperature cupshike shape of the microwave conductivity is obtained in the presence of the impurity scattering. At the low energy regime ($\omega < 0.0002 J$), this low temperature microwave conductivity $\sigma(\omega, T)$ rises rapidly from the universal zero-temperature limit to a much larger microwave conductivity. However, this low temperature microwave conductivity $\sigma(\omega, T)$ becomes smaller and varies from weakly energy dependence at the intermediate energy regime (0.0002 $J < \omega < 0.0008 J$), to the almost energy independence at the high energy regime ($\omega > 0.0008 J$).

For a better understanding of the physical properties of the microwave conductivity $\sigma(\omega, T)$ in cuprate superconductors, we have studied the doping evolution of the microwave conductivity, and the results of $\sigma(\omega, T)$ as a function of doping with $T = 0.002 J = 2 K$ and $\omega = 0.000087 J \approx 1.81$ GHz for $V_1 = 58 J$, $V_2 = 49.32 J$, and $V_3 = 40.6 J$ at $\rho = 0.000014$ are plotted in Fig. 2 (solid line). For comparison, the corresponding result of the SC gap parameter is also shown in the same figure (dashed line). Our result shows that in contrast to the dome shape of the doping dependent SC gap parameter [6, 10], $\sigma(\omega, T)$ decreases with increasing doping in the underdoped regime, and reaches a minimum in the optimal doping, then increases in the overdoped regime. This doping dependent behavior of the low energy microwave conductivity $\sigma(\omega, T)$ at low temperatures is also qualitatively consistent with the universal microwave conductivity limit [1] $\sigma \propto 1/\Delta$ at low energy as temperature $T \rightarrow 0$, if this SC gap parameter $\Delta$ in the phenomenological BCS formalism [1] has the similar dome shape doping dependence.

In conclusion we have shown very clearly in this paper that if the effect of the extended impurity scatterers is taken into account in the framework of the kinetic energy driven superconductivity, the microwave conductivity of the $t$-$J$ model calculated based on the nodal approximation of the quasiparticle excitations and scattering processes per se can correctly reproduce some main features found in the microwave conductivity measurements on cuprate superconductor in the SC state [3]. The theory also predicts a V-shaped doping dependent microwave conductivity, which should be verified by further experiments.

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