Time Resolved Measurement of Electron Cloud Densities from Dispersion of Transverse Electric Pulses

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The measurement of electron cloud densities in particle accelerators using microwaves has proven to be an effective, non-invasive and inexpensive method. So far the experimental schemes have used continuous waves. This has either been in the form of travelling waves that are propagated, or standing waves that are trapped, in both cases within a segment of the accelerator chamber. The variation in the wave dispersion relation caused by the periodic creation and decay of the electron cloud leads to a phase modulation in the former case, and a frequency modulation in the latter. In general, these methods enable the measurement of a time averaged electron cloud density. In this paper we propose a time resolved measurement by using pulses propagated over a finite length of the accelerator chamber. The pulses are launched periodically, once after a bunch train has passed and then again half a revolution period later. This results in pulses alternating between a dispersion that is either affected by a cloud or not. The resulting spectrum of the signal can be related to the electron density sampled by the pulse that propagates through the cloud. By varying the delay of the launch of the pulse with respect to the train passage, one can map the cloud density at different points behind the train.

I. INTRODUCTION

A better understanding of electron clouds is of prime importance for the optimal performance of a number of present and future accelerators. One area of this study includes methods of measuring the density of the cloud. The idea of using the dispersion of microwaves caused by electron clouds, acting like a plasma was first introduced by F. Caspers [1, 2]. Since then a number of related studies have been conducted based on this principle. Overall, these studies can be divided into two categories (1) The use of travelling waves [3] and (2) the use of standing waves [4]. Both the methods involve generating a continuous wave, and they typically provide an electron cloud density estimate that is integrated over time. In the former case, the electron cloud produces phase modulation and then again half a revolution period later. This results in pulses alternating between a dispersion that is either affected by a cloud or not. The resulting spectrum of the signal can be related to the electron density sampled by the pulse that propagates through the cloud. By varying the delay of the launch of the pulse with respect to the train passage, one can map the cloud density at different points behind the train.

II. ALTERNATING PULSES WITH A STATIC PERTURBATION IN THE DISPERSION

Suppose we produce a pulse with shape \( f(t) \) that has a periodicity \( T_0 \). Then the signal produced by the series of pulses may be expressed as

\[
F(t) = \sum_{n=-\infty}^{\infty} f(t - nT_0)
\]

(1)

We may express \( F(t) \) such that

\[
F(t) = \sum_{n=-\infty}^{\infty} F_n e^{i\omega_0 t}
\]

(2)

where \( \omega_0 = 2\pi/T_0 \). We can then see that,

\[
F_n = \frac{1}{T_0} \int_{0}^{T_0} dt F(t)e^{-i\omega_0 t}
\]

(3)

If we substitute the above expression for \( F(t) \), we have

\[
F_n = \frac{1}{T_0} \int_{0}^{T_0} dt \sum_{m=-\infty}^{\infty} f(t - mT_0)e^{-i\omega_0 t}
\]

(4)
If we substitute \( t = t' + mT_0 \) and interchange the order of summation and integration, we have
\[
F_n = \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \int_{-mT_0}^{-(m-1)T_0} dt' f(t') e^{-in\omega_0 t'}
= \frac{1}{T_0} \int_{-\infty}^{\infty} dt' f(t') e^{-in\omega_0 t'} = \frac{1}{T_0} \hat{f}(n\omega_0)
\] (5)
where \( \hat{f}(\omega) \) is the Fourier transform of \( f(t) \) given by

\[
\hat{f}(\omega) = \int_{-\infty}^{\infty} dt f(t) e^{-i\omega t}
\]
and consequently, the inverse transform is given by

\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{f}(\omega) e^{i\omega t}
\]
(6)

Suppose the generated pulse is allowed to travel through a dispersive medium, then the shape of the pulse would evolve as

\[
f(t, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{f}(\omega) e^{i(kx - \omega t)}
\]
(8)
where \( k = k(\omega) \) is a function of \( \omega \) and is given by the dispersion relationship of the medium. If the relationship between \( k \) and \( \omega \) is linear, we know that the shape of the pulse is preserved. We may express this propagated signal as

\[
F(t, x) = \sum_{n=-\infty}^{\infty} F_n(x) e^{i\omega_0 t}
\]
(9)
where \( F_n(0) = F_n \). It is easy to show that \( F_n(x) = F_0 e^{-ik(n\omega_0)x} \). Using Eq (8), we have

\[
F_n(x) = \frac{1}{T_0} \hat{f}(n\omega_0, x)
\]
(10)
Using Eq (8) and the definition of Fourier transform, this gives

\[
F_n(x) = \frac{1}{2\pi T_0} \int_{-\infty}^{\infty} dt d\omega \hat{f}(\omega) e^{-i(kx - \omega t)} e^{-i\omega t}
\]
(11)
Using the standard expression for the dirac-delta function \( \delta(\alpha - \beta) = (1/2\pi) \int_{-\infty}^{\infty} dp e^{ip(\alpha - \beta)} \) we get

\[
F_n(x) = \frac{1}{T_0} \int_{-\infty}^{\infty} d\omega \hat{f}(\omega) e^{-ik(n\omega_0)x} \delta(\omega - n\omega_0)
= \frac{1}{T_0} \hat{f}(n\omega_0) e^{-ik(n\omega_0)x} = F_n e^{-ik(n\omega_0)x}
\]
(12)

We now study the case where we generate a pulse between intervals of \( T_0/2 \). The pulse is allowed to propagate a distance \( x \) before being detected. We further assume that the dispersion relationship varies slightly between alternate pulses. Assume that at one instance of pulse propagation the wave number is \( k(\omega) \), and at the next it is \( k(\omega) + \Delta k(\omega) \), with this pattern repeating itself. We denote the modification of the pulse shape due to \( \Delta k(\omega) \) as \( f + \delta f \), and that of the corresponding amplitude of the \( n \)th harmonic as \( F_n + \delta F_n \). The signal is then given by

\[
F(t, x, \Delta k) = \sum_{n=-\infty}^{\infty} [f(t - nT_0, x) + \delta f(t - nT_0, x)]
\]
(13)
where \( \delta f(t, 0) = 0 \). This leads us to

\[
F(t, x, \Delta k) = \sum_{n=-\infty}^{\infty} \frac{F_2n(x) e^{2in\omega_0 t} + \sum_{n=-\infty}^{\infty} \delta F_n(x) e^{in\omega_0 t}}{
\sum_{n=-\infty}^{\infty} \delta F_n(x)}
\]
(14)
where we have in analogy to Eq (8),

\[
F_{2n}(x) = \frac{2}{T_0} \hat{f}(2n\omega_0, x)
\]
(15)
and

\[
\delta F_n(x) = \frac{1}{T_0} \delta \hat{f}(n\omega_0, x)
\]
(16)
Eq (14) may be rewritten as

\[
F(t, x, \Delta k) = \sum_{n=-\infty}^{\infty} \frac{F_2n(x) + \delta F_{2n}(x) e^{2in\omega_0 t}}{
\sum_{n=-\infty}^{\infty} \delta F_n(x)}
\]
(17)
Thus, at every even harmonic of \( \omega_0 \), we see a signal amplitude of \( F_n(x) + \delta F_n(x) \) and at every odd harmonic we see a signal amplitude of \( \delta F_n(x) \). We proceed to derive a relationship between the ratio of even and odd harmonic amplitudes, and the corresponding value of \( \Delta k \) at that harmonic. Analogous to Eq (8), we have

\[
f(x, t) + \delta f(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{f}(\omega) e^{-i[k(\omega)x + \Delta k(\omega)\omega_0 x - \omega t]}
\]
(18)
A Taylor expansion to first order in \( \Delta k(\omega)x \) gives

\[
f(x, t) + \delta f(x, t) \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{f}(\omega) e^{-i[k(\omega)x - \omega t]}
\times [1 + i\Delta k(\omega)x]
\]
(19)
This gives to first order

\[
\delta f(x, t) = \frac{-ix}{2\pi} \int_{-\infty}^{\infty} d\omega \Delta k(\omega) \hat{f}(\omega) e^{-i[k(\omega)x - \omega t]}
\]
(20)
Taking the Fourier transform of this, and following the same process as before of using the expression for the dirac-delta function as in Eq (12) we get

\[
\delta F_n(x) = \frac{1}{T_0} [ix\Delta k(n\omega_0) \hat{f}(n\omega_0) e^{-ik(n\omega_0)x}]
= \frac{1}{T_0} [ix\Delta k(n\omega_0) \hat{f}(n\omega_0, x)]
\]
(21)
Using the above equation and Eq (15), we obtain the ratio between an even and an odd harmonic amplitude to be

\[
R_{mn} = \frac{|\delta F_{2m-1}(x)|}{F_{2n}(x) + \delta F_{2n}(x)} = \frac{|\delta F_{2m-1}(x)|}{F_{2n}(x)} = x\delta k(2m-1,\omega_0) \left| \frac{f(2m-1,\omega_0, x)}{f(2m, \omega_0, x)} \right| \frac{1}{2}
\]

Thus, \( R_{mn} \) depends on the product of two terms. (1) The quantity \( x\delta k((2m-1)\omega_0) \) which is the phase shift induced by the variation in the dispersion relation for the wave at frequency \((2m-1)\omega_0\) after it propagates a distance \(x\). (2) The ratio between the peaks at the corresponding even and odd harmonics. Thus in order to determine \( \delta k((2m-1)\omega_0) \), two measurements need to be made which are, (1) of \( R_{mn} \) done by sending alternating pulses, perturbed and unperturbed, at every \( T_0/2 \), and (2) the ratio between the peaks at the corresponding even and odd harmonics of the unperturbed pulse done by propagating the same pulse at intervals of \( T_0 \).

### III. A TABLE-TOP EXPERIMENT REPLICATING THE PROCEDURE

This process may be replicated with the help of a pair of identical waveguides, some dielectric material that can fill up a part of one waveguide, a signal generator, a spectrum analyzer and a fast switch. We can generate a periodic pulse and let it propagate through one of the waveguides, or alternate between waveguides with and without a dielectric with the help of the fast switch. In this manner the two step experimental process described in the previous section may be carried out. The thickness of the dielectric may be altered, resulting in a change in value of \( R_{mn} \). The value of \( \delta k \) may be determined using the values of \( R_{mn} \) and the ratio of the peaks of even and odd harmonics as described in the previous section, and this may be compared with the expected value as predicted by the properties of the dielectric material. It is important for the electrodes that launch the wave to be well matched with the arriving signal. In addition, the propagation and the reception of the signal need to be efficient enough so that the odd harmonics are well above the noise floor. The use of amplifiers at the input and/or output may be useful in reducing the signal to noise ratio. Demonstrating this concept through such an experiment would help establish confidence in applying it to an operating storage ring. This table top experiment can be done easily, and interpretation of the results is straightforward as the dielectric properties remain the same for a given frequency through out their propagation. The procedure and analysis associated with performing such a measurement in an operating storage ring is more involved and is the topic of the next section.

### IV. GENERALIZATION TO A DYNAMIC PERTURBATION IN THE DISPERSION

In a storage ring, the beam creates a cloud density that varies with time and position, leading to a dispersion relation that is dynamic. Let us assume that the storage ring has a single train of bunches, and all the electron cloud created by the beam is cleared away, i.e, none of it is trapped. We propose launching a pulse at two instances during a revolution period. Once after the train has passed the launch point, with a certain delay \( \tau \), and half a revolution period later when the beam is half way around the ring, when presumably there is no electron cloud along the length of propagation of the pulse. The pulse is propagated a distance \( x \) before being detected, and the detector is shielded from the signal produced by the beam using a fast gate. We assume that the tail of the last bunch in the train passes the launch point at \( t = 0 \), and \( x = 0 \) corresponds to the launch point. Rather than an abrupt appearance and disappearance of the dielectric medium as in the table-top experiment we have a periodic buildup, and dissipation of the dielectric medium which here is the electron cloud. The perturbation in the wave number is now a function of \( \omega, t, x \) with periodicity \( T_0 \). We denote this as \( \Delta K \), with \( \delta k \) as the function that repeats itself at intervals of \( T_0 \). In this case, \( \delta k \) is a function of \( \omega, t, x \) and is a consequence of the spatial and temporal dependence of the cloud density. We may express the relationship between \( \Delta K \) and \( \delta k \) as

\[
\Delta K = \sum_{n=-\infty}^{\infty} \delta k(\omega, t - nT_0 - x/v_b)
\]

(23)

The term containing \( v_b \), the velocity of the beam causes the medium to follow the beam. Let us define \( t' = t - x/v_b \). Then following the steps analogous to Eq (1-5), we have

\[
\Delta K(\omega, t') = \sum_{n=-\infty}^{\infty} K_n(\omega)e^{in\omega_0t'}
\]

(24)

where

\[
K_n(\omega) = \frac{1}{T_0} \delta k(\omega, n\omega_0)
\]

(25)

We may then rewrite \( \Delta K \) as

\[
\Delta K(\omega, t, x) = \sum_{n=-\infty}^{\infty} K_n(\omega)e^{in\omega_0(t-x/v_b)}
\]

(26)
In this section, we require that the pulse shape is symmetric about a mid point. We denote the pulse delay $\tau$ as the time period between $t = 0$ and time of generation of the mid point of the pulse. The signal from this delayed pulse launched between intervals of $T_0/2$ may be expressed as

$$F(t, \tau) = \sum_{n=-\infty}^{\infty} f(t - \frac{nT_0}{2} - \tau)$$

(27)

as before, this gives

$$F(t, \tau) = \sum_{n=-\infty}^{\infty} F_{2n} e^{i2n\omega_0(t - \tau)}$$

(28)

with $F_{2n} = (2/T_0) \tilde{f}(2n\omega_0)$. The propagated pulse shape, with the delay $\tau$ taken into account would be

$$f(t, x, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \tilde{f}(\omega)e^{-i[k(\omega)x - \omega(t - \tau)]}$$

(29)

further, the signal produced by the propagated pulses will be given by

$$F(t, x, \tau) = \sum_{n=-\infty}^{\infty} F_{2n} e^{-i[k(2n\omega_0)x - 2n\omega_0(t - \tau)]}$$

(30)

The perturbation of the pulse shape may be obtained by replacing $\delta k$ by $\Delta K$ into Eq (20), and using Eq (26). We then have

$$\delta f(x, t, \tau) = -\frac{ix}{2\pi} \int_{-\infty}^{\infty} d\omega \sum_{n=-\infty}^{\infty} K_n(\omega)e^{i\omega_0(x - t - \tau)}$$

$$\times \tilde{f}(\omega)e^{-i[k(\omega)x - \omega t + \omega \tau]}$$

(31)

taking a Fourier transform of this gives

$$\tilde{\delta f}(\omega, t, \tau) = -\frac{ix}{2\pi} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\omega' \tilde{f}(\omega)e^{-i[k(\omega)x - \omega t + \omega' \tau]}$$

$$\times e^{-i\omega_0 x/v_b}e^{-i[k(\omega + \omega')x + \omega' \tau]}$$

(32)

Intercalating the order of summation and integration, and integrating over $t$ by using the usual expression for the dirac-delta function, that was used in Eq (12), we can reduce this to

$$\delta f(\omega, t, \tau) = -ix \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega' K_n(\omega')\tilde{f}(\omega')$$

$$\times e^{-i\omega_0 x/v_b}e^{-i[k(\omega + \omega')x + \omega' \tau]}\delta(n\omega + \omega')$$

(33)

From Eq (10), we know that the $n^{th}$ harmonic amplitude of the signal produced by $\delta f$ is $(1/T_0)\delta f(n\omega_0, x, \tau)$. Thus, by performing the above integration we obtain

$$\delta F_n(x, \tau) = -\frac{ix}{T_0} \sum_{m=-\infty}^{\infty} K_m([m - n]\omega_0)e^{-i\omega_0 x/v_b}$$

$$\times e^{-i[k([m - n]\omega_0)x + ([m - n]\omega_0 + \omega_0)\tau]}\tilde{f}([m - n]\omega_0)$$

(34)

By substituting $(m - n) = N$, this may be further simplified to

$$\delta F_n(x, \tau) = -\frac{ix}{T_0} e^{-i\omega_0 x/v_b} \sum_{N=-\infty}^{\infty} K_{N+n}(N\omega_0)$$

$$\times e^{-i[k(N\omega_0)x + (\tau + x/v_b)N\omega_0]}\tilde{f}(N\omega_0)$$

(35)

The ratio between even and odd harmonics as given in Eq (22) will now be far more complex. Some simplification would occur because we have required $f(t)$ to be symmetric about $t = 0$. As a result of this $\delta f(\omega)$ is purely real. This would lead to $|\tilde{f}((2m - 1)\omega_0, x, \tau)| = \tilde{f}((2m - 1)\omega_0)$ as we know that variation in $x$ and $\tau$ only leads to a phase shift. Consequently,

$$R_{mn} \approx \frac{|\delta F_{2m-1}(x, \tau)|}{|\tilde{F}_{2n}(x, \tau)|} = x \sum_{N=-\infty}^{\infty} K_{N+2m-1}(N\omega_0) \times$$

$$e^{-i[k(N\omega_0)x + (\tau + x/v_b)N\omega_0]}\tilde{f}(N\omega_0) \left|\frac{2\tilde{f}(2n\omega_0)}{2f(2n\omega_0)}\right|$$

(36)

Typically, it is only the absolute values of the harmonics of $F$ that are measurable. Using the fact that $\delta f(\omega)$ is purely real, the quantity $\tilde{f}(N\omega_0)/\tilde{f}(2n\omega_0)$ may be measured by propagating pulses at every $T_0$ when the beam is half way around the ring, and all the cloud has presumably cleared away. This would give values of $\tilde{f}(n\omega_0)$ for all $n$, even and odd. In general, the components of $K$ will have real and imaginary parts since $\delta k$ is need not be symmetric about any value of $t$. Thus the Fourier transform of $\delta k$ will be complex, which implies from Eq (25) that the components of $\delta k$ will also be complex.

The unknown quantities in Eq (36) are the components of $K$. To be able to determine them up to a sufficiently high order, we need to solve a large enough set of simultaneous equations. One can generate a sufficient number of such equations by varying $m$, or by varying $\tau$ in steps, and inserting the values into Eq (36), which will need to be truncated at a large enough value in $N$. These equations are nonlinear because the components of $K$ have real and imaginary parts. The set of equations can be solved using a standard numerical scheme such as the Newton-Raphson method. This will require initial guess values which will undergo an iterative process till they converge to a set of values that satisfy the equations. The guess values may be obtained from some idea of a typical cloud build up pattern. The cloud decay time itself can be readily estimated by scanning over a range of $\tau$. When $\tau$ is large enough so that the electron cloud has decayed away, we get the signal produced by pairs of identical pulses, where the perturbed signal seen at the odd harmonics are absent. Knowing the components of $K$ will give us the periodic variation of the perturbation in the dispersion, which is $\delta k$. From this one can determine the time variation of the cloud density if we know the relationship between $\delta k$ and the electron density.

The dispersion relation for a waveguide filled with a dielectric medium is well known. For an electron cloud in a
region free of external magnetic fields, the dispersion relationships have been worked out in Refs [3], [4]. It is also shown that the relationship between $\delta k$ and the electron density is linear for frequencies that are not too close to the waveguide cutoff frequencies. In addition, the linear relationship is valid for electron densities corresponding to plasma frequencies that are small compared to the wave frequency and the cutoff frequency. These approximations are reasonable for typical conditions of electron cloud generation in accelerators.

V. SUMMARY

This paper proposes a method to perform time resolved measurements of electron cloud density using transverse electric pulses. Given that this is a far more sensitive measurement compared to earlier methods that used continuous waves, it offers various challenges. Since the duration of the pulse is much smaller than its periodicity, the average power in the transmission can be low. One needs to determine if the loss during the transmission is significant, and if it depends on the frequency. A frequency dependence in the rate of attenuation in the transmitted power would distort the measurement. A significant advantage over the measurements using continuous waves is that the signal produced by the particle beam can be completely shielded. The strong signal from the beam increases the level of the noise floor, and its absence would contribute toward improving the signal to noise ratio. It needs to be determined from experimental tests if such a measurement requires specialized instrumentation instead of beam position monitors (BPM) that have typically be in use. Additionally, a method of suppressing a possible reflection of the pulse after its transmission over the required distance might be necessary. The method and the concept presented in this paper are novel and will prove to be a powerful diagnostic tool for electron clouds in particle accelerators, if the challenges mentioned above could be overcome.

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