INFLATIONARY COSMOLOGY AND OSCILLATING UNIVERSES IN LOOP QUANTUM COSMOLOGY

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We study oscillatory universes within the context of Loop Quantum Cosmology. We make a comparative study of flat and positively curved universes sourced by scalar fields with either positive or negative potentials. We investigate how oscillating universes can set the initial conditions for successful slow-roll inflation, while ensuring that the semi-classical bounds are satisfied. We observe rich oscillatory dynamics with negative potentials, although it is difficult to respect the semi-classical bounds in models of this type.

I. INTRODUCTION

The study of cyclic/oscillatory universes has a long history. One of the primary motivations for such a study was to circumvent the need for initial conditions in cosmology. This was, however, shown to be extremely difficult to achieve within the context of general relativity (GR), without encountering singularities.

Recent developments in String/M-theory inspired braneworld models have revived hopes for the possibility of constructing such universes (see e.g. Ref. [1]). One such attempt is the so called cyclic/ekpyrotic scenario (see Ref. [2] and references therein). Despite the attractiveness of this possibility, a successful (non-perturbative) treatment of bounces within the context of M-Theory is still lacking.

More recently such universes have been studied in the context of loop quantum gravity, which at present is the main background independent and non-perturbative candidate for a quantum theory of gravity (see for example Refs. [3] and [4]). To date, cosmological applications have focused on minisuperspace/midisuperspace settings with a finite number of degrees of freedom. This program is referred to as Loop Quantum Cosmology (LQC). In this context the evolution of the universe can be divided into 3 distinct phases: an initial high energy and high curvature quantum phase, described by a (or a set of) difference equation(s); an intermediate semi-classical phase, with continuous evolution equations modified due to non-perturbative quantization effects; and finally, a classical phase, where the usual continuous cosmological equations are recovered. It has recently been shown that such semi-classical effects can result in bouncing FRW universes which avoid singularities [5]. This has in turn led to the possibility of non-singular cyclic universes in LQC settings [6, 7].

In this paper we make a comparative study of recent results concerning both spatially flat [8], and positively curved oscillating universes [6], with both positive and negative potentials. We extend these results in a number of areas, in particular, in the case of models with negative potentials. This latter class of models is of direct relevance to the ekpyrotic/cyclic models considered recently.

II. EFFECTIVE FIELD EQUATIONS IN LOOP QUANTUM GRAVITY

The semi-classical phase in isotropic LQC refers to the regime where the scale factor lies in the range $a_i < a < a_*$, where $a_i \equiv \sqrt{\gamma} l_p$, $a_* \equiv \sqrt{\gamma j/3} l_p$, $\gamma = \ln 2/\sqrt{3} \pi \approx 0.13$ and $j$ is a quantization parameter which must take half integer values. Below the scale $a_i$, the discrete nature of space-time becomes important, whereas the standard classical cosmology is recovered above $a_*$. The parameter $j$ therefore sets the effective quantum gravity scale. The modified Friedmann equation is given by

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi l_p^3}{3} \left[ \frac{1}{2} \frac{\dot{\phi}^2}{D} + V(\phi) \right] - k a^{-2}, \quad (1)$$

where $k$ takes the values 0 or 1 for a flat or positively curved universe, respectively, and the quantum correction factor $D(q)$ is defined by

$$D(q) = \left( \frac{8}{77} \right)^6 q^{3/2} \left\{ \frac{7}{3} \left[ (q+1)^{11/4} - |q-1|^{11/4} \right] - 11q \left[ (q+1)^{7/4} - \text{sgn}(q-1) |q-1|^{7/4} \right] \right\}^6 \quad (2)$$
with \( q \equiv (a/a_*)^2 \). As the universe evolves through the semi-classical phase, this function varies as \( D \propto a^{15} \) for \( a \ll a_* \), has a global maximum at \( a \approx a_* \), and falls monotonically to \( D = 1 \) for \( a > a_* \).

The scalar field equation has the form

\[
\ddot{\phi} = -3H \left( 1 - \frac{1}{3} \frac{d \ln D}{d \ln a} \right) \dot{\phi} - DV',
\]

where a prime denotes differentiation with respect to the scalar field. Differentiating Eq. (1) and substituting from Eq. (3) then gives

\[
\dot{H} = \frac{-4\pi G \phi^2}{D} \left( 1 - \frac{1}{6} \frac{d \ln D}{d \ln a} \right) - \frac{k}{a^2}.
\]

We can immediately see two interesting features. First, the correction to the scalar field equation (3) causes the frictional/anti-frictional \( \dot{\phi} \) term to change sign when \( d \ln D/d \ln a \) passes through 3. Secondly, an expanding universe naturally undergoes a period of super-inflationary expansion (\( \dot{H} > 0 \)) \[9, 10\], when \( d \ln D/d \ln a > 6 \) (assuming the curvature term to be either zero or negligible).

An interesting consequence of these features is that both a collapsing positively curved universe \[5, 6\], and a flat universe sourced by a scalar field with a negative potential \[6, 7\], can undergo a non-singular bounce. We can most clearly see how the bounce arises by reformulating Eqs. (1) and (3) in the standard form of the Einstein field equations sourced by a perfect fluid \[2\]:

\[
H^2 = \frac{8\pi G}{3} \rho_{\text{eff}} - \frac{k}{a^2},
\]

\[
\dot{\rho}_{\text{eff}} = -3H (\rho_{\text{eff}} + p_{\text{eff}}),
\]

where

\[
\rho_{\text{eff}} = \frac{1}{2} \frac{\dot{\phi}^2}{D} + V,
\]

\[
p_{\text{eff}} = \frac{1}{2} \frac{\dot{\phi}^2}{D} \left( 1 - \frac{1}{3} \frac{d \ln D}{d \ln a} \right) - V,
\]

define the effective energy density and pressure of the fluid, respectively. The effective equation of state, \( w \equiv p_{\text{eff}}/\rho_{\text{eff}} \), is given by

\[
w = -1 + \frac{2\dot{\phi}^2}{\ddot{\phi}^2 + 2DV} \left( 1 - \frac{1}{6} \frac{d \ln D}{d \ln a} \right).
\]

Hence, the LQC corrections to the cosmic dynamics can be entirely parametrized in terms of the equation of state. When \( d \ln D/d \ln a > 6 \), the fluid represents ‘phantom’ matter (\( w < -1 \)) that violates the null energy condition (\( \rho_{\text{eff}} + p_{\text{eff}} \geq 0 \)), independent of the form of the potential. During a collapse the energy density starts to decrease when this condition is met, and in a positively curved universe, the density term in the Friedmann equation is eventually balanced by the growing curvature term, thereby leading to a bounce. In the case of a flat universe sourced by a scalar field with a negative potential, the decreasing energy density instantaneously vanishes as the positive kinetic energy component is balanced by the negative potential. In both cases, \( H \) tending to zero necessarily leads to a bounce because \( \dot{H} > 0 \).

In the following sections we shall study the question of initial conditions for inflation in both flat and positively curved universes, with both positive and negative potentials and we will investigate whether other interesting scenarios involving negative potentials can be developed.

### III. SCALAR FIELDS WITH POSITIVE POTENTIALS

In this section we consider a semi-classical universe sourced by a scalar field self-interacting through a positive potential. We are particularly interested in how the dynamics of the semi-classical region can set the initial conditions for slow-roll inflation if the field is situated at or near the minimum of its potential. Our discussion applies for an arbitrary potential apart from the weak assumptions that it has a global minimum at \( V_{\text{min}}(0) = 0 \) and is a positive-definite and monotonically varying function when \( \phi \neq 0 \) such that \( V'' > 0 \). We shall also assume that the field is initially located at the minimum of its potential. For definiteness, we focus in this section on a massive field with a quadratic potential, \( V(\phi) = m^2 \phi^2/2 \).
A. Flat case

For a flat universe the anti-friction mechanism simply accelerates the scalar field up the potential $V$. A major problem with this mechanism is that during the anti-friction era the energy density of the field increases. On the other hand, the Hubble length must remain larger than the limiting value of the scale factor consistent with the semi-classical regime, i.e., $|H/a| < 1$. (This is roughly equivalent to the requirement that energy scales remain below the Planck scale and is referred to as the Hubble bound). Since this length decreases during the period of anti-friction, we need to determine how far the field can be moved before this condition is violated. To achieve 60 e-folds of inflation for a quadratic potential, $V = m^2 \phi^2/2$, the field must be displaced by at least $3l_{pl}^{-1}$ from the minimum of its potential at the onset of inflation. It was recently found that given an initial value $\phi_{init}$, the field could only be moved as far as $2.4l_{pl}^{-1}$ or less (for sufficiently large $j$) without violating the Hubble bound.

The behavior of the field can be understood by approximating the evolution into two epochs. In the first, between $\dot{\phi}_{init}$ and the end of the semi-classical regime $a_s$, the asymptotic form, $D_{approx} = (12/7)^6(a/a_s)^{15}$, is used. In the second with $a > a_s$, we assume $D = 1$. Here $a_s$ is defined as the value of the scale factor where $D_{approx}$ first reaches unity. The field is assumed to be massless up to the turning point in its evolution, which is estimated to occur when $m^2 \dot{\phi}_t^2/2 \approx \dot{\phi}_t^2$, where the subscript $t$ stands for “turning point”. Hence, neglecting the potential in the Friedmann equation (1), we have $(d\phi/d\ln a)^2 = 6D/8\pi l_{pl}^2$. Integrating for $\phi$, and using the approximation above for $D$, we then obtain

$$\phi_t = \phi_{init} + \left(\frac{6}{8\pi l_{pl}^2}\right)^{1/2} \left\{ \frac{2}{15} \left(\frac{12}{7}\right)^3 \left[ \left(\frac{a_s}{a_s}\right)^{15/2} - \left(\frac{a_{init}}{a_s}\right)^{15/2} \right] + \ln \left(\frac{a_t}{a_{init}}\right) \right\}. \quad (10)$$

The explicit dependence on $a_t$ can be removed by using the expression

$$\dot{\phi}_t = \dot{\phi}_{init} \left(\frac{a_s}{a_t}\right)^3 \left(\frac{a_{init}}{a}\right)^{12} \quad (11)$$

which itself arises from using the first integral

$$\dot{\phi} = \dot{\phi}_{init} \left(\frac{a_s}{a}\right)^3 \frac{D}{D_{init}}. \quad (12)$$

in both epochs, along with the form for $D_{approx}$. Finally, applying the condition for the turning point, we arrive at the estimate

$$\phi_t \exp \left[\sqrt{12\pi l_{pl}} (\phi_t - \phi_{init})\right] = \left(\frac{\sqrt{2}}{m} \frac{\dot{\phi}_{init}}{\dot{\phi}_t} \left(\frac{7}{12}\right)^{24/5} \exp \left[\frac{6}{15} \left(1 - (12/7)^3 (15/4)\right)\right] \right). \quad (13)$$

We note that for a given initial value $a_{init} = (a_{init}/a_s)^2$, the value of $\phi$ when inflation occurs is independent of $j$ and proportional to $\dot{\phi}_{init}$. This approximation was compared to numerical results $[3]$, and found to be in very good agreement. We emphasize that this approximation only works for initial conditions such that $a_{init} \ll a_s$ because the approximation for $D$ involves that assumption.

B. Positively curved case - massless scalar field

Before considering the positively curved case with a positive potential, it is illuminating to consider the dynamics of such a universe sourced by a massless scalar field. This was discussed in detail in Ref. $[3]$. We can, however, easily explain the form that the dynamics will take by the following simple argument. We know from the above discussion that a collapsing positively curved universe will always bounce into a new expanding phase no matter what form the potential takes, and consequently, this bounce will still occur even for $V = 0$. We also know that an expanding, positively-curved, classical cosmology sourced by a massless scalar field will rapidly undergo a re-collapse since the energy density scales as $\rho \propto a^{-6}$, and therefore redshifts much more quickly than the curvature term in Eq. (4). After the bounce, the universe expands and once it has entered the classical phase a re-collapse will inevitably occur. The process then repeats itself indefinitely.

Since the effective equation of state $[4]$ is independent of the field’s kinetic energy when $V = 0$, we see that identical cycles must occur. Furthermore, Eq. (12) implies that the field’s kinetic energy never vanishes during a given cycle since the scale factor always remains finite. The value of the field therefore increases or decreases monotonically with time.
C. Positively curved case — massive scalar field

To understand how the cyclic dynamics is modified by the presence of a potential, we recall that the equation of state can change from cycle to cycle depending on the field’s position on the potential. Assuming that the potential is initially unimportant, the dynamics is qualitatively similar to that described in section 3.2. This implies that the kinetic energy of the field never vanishes and a re-collapse always occurs. However, since the field moves monotonically away from the minimum, the potential becomes progressively more important with each completed cycle and it must therefore become dynamically significant after a finite number of cycles have been completed. In general, a re-collapse occurs if the matter redshifts more rapidly than the curvature term. This requires the strong energy condition to hold, \( w > -1/3 \), which in turn implies \( V < \dot{\phi}^2 \). As the field moves up the potential, \( w \) is pushed towards \(-1\) and it becomes progressively harder to satisfy the strong energy condition. Eventually, therefore, a cycle is reached where \( V > \dot{\phi}^2 \) and the condition \( w > -1/3 \) is violated. This leads to an epoch of slow-roll inflationary expansion, with the field slowing down, reaching a point of maximum displacement and rolling back down the potential. For some initial conditions and choices of parameters, this may occur during the first cycle, in which case the scenario discussed for flat models, we saw that the maximum value attained by the field as it moves up the potential is independent of \( \dot{\phi} \) given an initial ratio \( a_{init}/a_* \) and \( \dot{\phi}_{init} \). For positively curved models, on the other hand, the field moves further up the potential before the onset of slow-roll inflation for smaller values of \( j \). This behavior can be quantitatively understood by assuming that the total energy at the point of collapse is conserved through the cycles and that the potential is dynamically insignificant during the first cycle. The massless case, therefore, gives us a good estimate of the total energy, with the evolution of this energy being given by Eq. (12). Now, assuming the universe to be classical at the point of collapse, the potential becomes progressively more important with each completed cycle and it must be understood by assuming that the total energy at the point of collapse is conserved through the cycles and that the conservation of energy is a good approximation. This requires the strong energy condition to be violated. This leads to an epoch of slow-roll inflationary expansion, with the field slowing down, reaching a point of maximum displacement and rolling back down the potential. For some initial conditions and choices of parameters, this may occur during the first cycle, in which case the scenario discussed for the flat model is recovered.

Thus far, we have discussed the cyclic dynamics for a given value of the quantization parameter, \( j \). In spatially flat models, we saw that the maximum value attained by the field as it moves up the potential is independent of \( j \) given an initial ratio \( a_{init}/a_* \) and \( \dot{\phi}_{init} \). For positively curved models, on the other hand, the field moves further up the potential before the onset of slow-roll inflation for smaller values of \( j \). This behavior can be quantitatively understood by assuming that the total energy at the point of collapse is conserved through the cycles and that the potential is dynamically insignificant during the first cycle. The massless case, therefore, gives us a good estimate of the total energy, with the evolution of this energy being given by Eq. (12). Now, assuming the universe to be classical at \( \phi_i \), using the condition \( V_i = \dot{\phi}_i^2 \) to estimate the point at which inflation starts, and the relation \( 8\pi G \rho_t/3 \approx 1/a_i^2 \) to eliminate \( a_i \), one obtains

\[
\dot{\phi}_i^2 = \frac{1}{\dot{\phi}_{init}} m^2 \left( \frac{8\pi G \rho_t}{2} \right)^{-3/2} \frac{D_{init}}{q_{init}^{3/2} a_i^3}.
\]  

(14)

This demonstrates that for the quadratic potential, with a given initial \( a_{init}/a_* \), the value of \( V \) when inflation occurs is inversely proportional to \( a_i^2 \) and hence inversely proportional to \( j^{3/2} \). It also shows an inverse dependence on \( \dot{\phi}_{init} \). It is worth pointing out that in deriving Eq. (14), unlike in the flat case, we have not used any approximation to the general form of the correction factor \( D(q) \), and consequently this expression is still a good estimate of \( \phi_i \) when \( a_{init} > a_* \).

As in the flat model, it is important to ensure that the Hubble bound is not violated. However, due to the inverse dependence of \( \phi_i \) on \( \dot{\phi}_{init} \), this proves to be a much weaker constraint in the positively curved models. The field is moved up the potential in a series of small ‘kicks’ of anti-friction, and can easily be moved sufficiently to drive 60 e-folds of slow-roll inflation in the case of a quadratic potential.

In Fig. 1 we illustrate the dependence of the maximum value attained by the field before turnaround on the value of the quantization parameter \( j \) and the ratio \( a_{init}/a_* \) for a given initial value of the kinetic energy \( \dot{\phi}_{init} \). The horizontal lines represent the estimated value of \( \phi_i \) extracted from Eq. (14) and the lines with negative slope represent the estimate (14). Circles and triangles correspond to the actual values obtained by numerically integrating the equations of motion. When many oscillations occur before the onset of inflation, Eq. (14) yields a good estimate of \( \phi_i \). However, if the field is moved far enough in the first period of anti-friction for the strong energy condition to be violated, no oscillations occur and the situation is equivalent to the flat model. In this case, Eq. (14) gives a good estimate of \( \phi_i \). The points where the lines cross in Fig. 1 show the transitions between the two regimes.

The numerical results in Fig. 1 show a stepping behavior, with a range of \( j \) leading to roughly the same value of \( \phi_i \), before there is a drop to the next value. This becomes more pronounced at higher \( j \). This occurs because our approximation relies on two estimates: firstly, that \( V \equiv \phi^2 \) at the onset of inflation, and secondly, that \( 8\pi G \rho_t/3 \approx 1/a_i^2 \) when the field reaches its maximal value. Clearly, the two approximations cannot be satisfied simultaneously, since the first implies a violation of the strong energy condition, whereas the second represents a turning point in the expansion. In effect, since we assume that the potential becomes dynamically significant only during the last cycle, we are carrying over the second condition from the previous cycle. Since \( 8\pi G \rho_t/3 > 1/a_i^2 \) for the cycle in which inflation occurs, the second condition underestimates the amount of energy present and consequently underestimates the change in the value of the field before it turns around. A range of values of \( j \) will give rise to the same number of cycles: for the lowest value of \( j \) in a given range the condition \( 8\pi G \rho_t/3 \approx 1/a_i^2 \) is a good approximation, since a re-collapse is only just avoided, but for subsequent values the underestimate becomes progressively more severe. This gives rise to the steps which become more evident towards larger \( j \). This can be understood by recalling from Eq. (14) that \( \phi_i \propto j^{-3/4} \), which implies \( \Delta j \propto j^{7/4} \Delta \phi \), thus showing that the steps in \( j \) are wider for larger \( j \).
IV. SCALAR FIELDS WITH NEGATIVE POTENTIALS

In the spatially flat setting, we have seen that a bounce can occur when the universe is sourced by a scalar field self-interacting through a negative potential. It is important to investigate whether interesting cosmological scenarios can be developed using negative potentials. For example, a negative exponential potential plays a key role in the cyclic/ekpyrotic scenario [2]. Alternatively, bouncing cosmologies in which the maximal size of the universe increases with each successive bounce could offer a possible resolution of the flatness and horizon problems of standard, big bang cosmology (see Ref. [12] and references therein).

Here we look at the general dynamics of a cosmic bounce with a negative potential. During the collapsing phase, once the scale factor falls sufficiently for the condition $\frac{d \ln D}{d \ln a} > 3$ to be met, the kinetic energy of the scalar field decreases due to the friction in Eq. (3). This is eventually balanced by the negative potential energy and leads to a bounce. Since the balance is achieved before the kinetic energy reaches zero, the field continues to move in the same direction during the bounce. This is similar to the dynamics of a positively curved model with a positive potential. After the bounce, a super-inflationary era ensues, and the standard scenario is recovered once the scale factor exceeds $a_*$. The field subsequently decelerates, and in the case where the potential is negative-definite, a re-collapse is initiated when the kinetic energy is again balanced by the potential. The qualitative behavior discussed above is then repeated indefinitely with the field always moving in the same direction. The behavior, however, is not symmetric since it responds to the region of the potential where the field is positioned. Consider, for example, the case of an exponential potential, $V \propto -\exp(-\alpha \phi)$: if the field is moving up the potential (towards more positive values), then the maximum size attained by the universe increases with each cycle whereas the minimal size decreases. This occurs because the universe has progressively more time to evolve before the kinetic energy is canceled by the decreasing negative potential term, and as the potential asymptotes to zero, the maximal/minimal size of the universe stabilizes. Conversely, if the field is moving towards more negative values, the reverse is true.

One question that naturally arises is whether this mechanism for realizing a non-singular bounce may be employed within the context of the cyclic/ekpyrotic scenario? This question has been asked previously [7], and the discussion given here is in agreement with the previous study. There are three major issues that need to be addressed. Firstly, in the cyclic/ekpyrotic scenario, the field is supposed to change direction at the bounce, but as we have discussed, this is difficult to achieve at the semi-classical level with the mechanism under discussion, although this problem might be alleviated by introducing a steep positive section to the potential. (This would cause the field to slow down and then reverse its direction of motion).

The second problem is more severe and is directly related to Hubble bound $H a_* < 1$ discussed above. During a classical collapse, the kinetic energy of the field increases due to the anti-friction in the field equation (3). The kinetic energy can start to fall only when the condition $d \ln D/d \ln a > 3$ is attained. Consistency at the semi-classical level requires that the Hubble bound must not be violated before this point is reached. (This problem of Planck scale
energies has been raised previously. In the case of a negative exponential potential, there exists a scaling solution in which the kinetic and potential energies of the field vary in proportion to one another. In principle, therefore, the total energy density may remain below the Planck scale. For the cyclic/ekpyrotic scenario, however, the potential has an exponential form only over a finite region or parameter space, and once the field evolves through this region, the cosmic dynamics moves away from the scaling solution. This implies that the kinetic energy is no longer balanced by the negative potential, and the total energy rapidly exceeds that of the Planck scale.

For a collapsing universe sourced by a massless scalar field, it is possible to estimate when the bounce must occur in order for the Hubble bound to remain satisfied throughout the evolution. In particular, we can derive a lower limit on the value of the ambiguity parameter, $j$. If the collapse starts well into the classical regime, it follows that $H_{\text{init}} = 0$, $D = 1$, and that the initial kinetic energy of the field is $\dot{\phi}^2_{\text{init}} = 6/8\pi^2 a_{\text{init}}^2$. The subsequent evolution of the field is determined from the relation $\dot{\phi}^2 = \dot{\phi}^2_{\text{init}}(a_{\text{init}}/a)^6$ and, after substituting this expression into the Friedmann equation and imposing the consistency condition $H a_i < 1$, we require that

$$j > \frac{3}{\gamma} \left( \frac{a_{\text{init}}}{a_i} \right)^4,$$

where $a_i$ has been employed as the point at which the energy density becomes maximized. Clearly, if the size of the universe is large compared with $a_i$ at the start of the collapse, $j$ must be very large indeed if a bounce is to occur within the validity of the semi-classical regime. Although a realistic collapse will involve a massive (or more generally a self-interacting) scalar field, the above estimate highlights the difficulties that arise: even if the field equations generically lead to a bouncing cosmology, the bounce may occur in a regime where the equations are no longer valid.

Finally, a third question that arises in developing a consistent bounce is also related to the size of the universe. For a consistent, semi-classical treatment, it is important that the bounce does not occur after the universe has collapsed beyond the minimal scale consistent with the semi-classical region, $a_i$, since the semi-classical Friedmann equations are no longer valid below this scale. In particular, this is relevant to any oscillating scenario involving a negative exponential potential in a flat universe where the maximal size of the universe increases with each successive cycle: as the value of the field increases with each cycle, the value of the scale factor at the bounce decreases and it is possible that it may fall below the critical value $a_i$.

Before concluding, it is also worth considering the case where the potential has a global minimum $V_{\text{min}}$, such that $V_{\text{min}} < 0$, with only a region around the minimum that is negative. One example of this class is a potential of the form $V(\phi) = C + m^2 \phi^2/2$ for some negative constant $C$. In positively-curved universes, the mechanism described in section 3.2 enables the field to work its way out of the negative region and continue up the potential until the cycles are broken. In Fig. 2 we illustrate the evolution of the scalar field, the scale factor and Hubble ratio for a quadratic potential $V = C + m^2 \phi^2 / 2$ in a positively-curved universe. The figure shows the stepwise evolution of the field up the potential followed by a short period of inflation between $\ln(mt) = [1, 1.5]$. As the field rolls down the potential, it once again reaches the negative section of the potential and this eventually causes the universe to re-collapse. During the collapse the Hubble parameter grows and is peaked around $a \approx a_\ast$. Below $a_\ast$, the potential prevents the scale factor from becoming singular and the field resumes the same stepwise behavior, but now in the opposite direction. This results in a further period of inflation. For spatially flat universes, the picture is similar although the oscillations come to an end as soon as the field reaches a positive region of the potential. Unfortunately, in the flat case the resulting amount of inflation is typically less than 60-efolds, even for the quadratic potential. (More inflation is possible but at the expense of violating the Hubble bound). Furthermore, for such a potential in a flat universe, there is the possibility that the potential becomes positive during the collapsing phase, in which case the universe can undergo no further bounces and ultimately collapses into a big crunch.

V. CONCLUDING REMARKS

We have summarized and extended recent results concerning the effects of loop quantum gravity corrections to the cosmological evolution equations. In particular, we have made a comparative study of the ability of oscillatory universes to set the initial conditions for successful slow-roll inflation. We have considered both flat and positively curved models sourced by a scalar field with either a positive or negative self-interaction potential. Negative potentials are not expected to arise naturally in string/M-theory motivated models such as the recently proposed ekpyrotic/cyclic scenario. We find that the requirement for self-consistency in the semi-classical regime leads to strong constraints that any realistic model must satisfy.
FIG. 2: Time evolution of $\phi$, $\ln(a)$ and $H$ for a positively-curved universe sourced by a quadratic potential $V(\phi) = C + m^2 \phi^2 / 2$, where $m = 10^{-6} l_{Pl}$, $j = 10$, $a_{init}/a_* = 0.5$, $H_{init} = 0$ and $C = -10^{-11} l_{Pl}^{-4}$. In the figure, the axes are labeled in Planck units.

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