Abstract. We study the spectra of charmonia, charmed mesons, singly and doubly charmed baryons using Lattice QCD, with 2+1 flavours of fermions. In the case of mesons, we include higher spin states, while for baryons, both positive and negative parity channels were investigated. By means of the variational method, we were able to extract a clean signal from the correlation functions and information about the excited states.

1. Introduction

In recent years the spectroscopy of hadrons containing charm quarks has received special attention. Concerning charmonium-like states, since 2002, new resonances have been found that are unlikely to be conventional $\bar{c}c$ states (see ref. [1] for a summary of quarkonium states). The triggering point was the discovery of the $X(3872)$ resonance by the Belle collaboration [2], later confirmed by BaBar, CDF and D0, whose mass lies close to the $D\bar{D}^*$ threshold. It has been widely studied and no consensus on its inner structure has been reached so far. After this observation, a plethora of puzzling chamonium-like states emerged from the experiments, the most interesting being the ones lying close to the open charm thresholds. These new states could correspond to loosely bound hadronic molecules, hybrid states or tetraquark states.

Concerning charmed baryons, there exist 17 experimentally well established singly charmed baryons [5], most of which have been found in the last decade, in the $e^+e^-$ colliders and at Fermilab. The charm quark is sufficiently massive for the states to be described as a combination of a heavy quark and a light di-quark and their properties can be understood in terms of Heavy Quark Effective Theory (HQET). Moreover, doubly-charmed baryons are also interesting, since they combine two scales of QCD: the size of the two heavy quark ($QQ$) system and $\Lambda^{-1}$ (where $\Lambda$ is a typical binding energy). Two possible pictures are shown in Figure 1. In the charmonium-like picture (left), the di-quark is formed by a heavy and a light quark. The resulting object will interact with the remaining heavy $Q$, as if it was a charmonium system. The radius of the $QQ$
system is much larger than $\Lambda^{-1}$. In the HQET picture (right), the $QQ$ diquark system binds itself into the $\bar{3}$ representation of SU(3). In that case the radius of the heavy-heavy diquark is smaller than $\Lambda^{-1}$.

![Figure 1](image1.png)

**Figure 1.** Structure of a doubly charmed baryon: Charmonium-like picture (left), HQET picture (right).

In the next few years, more results are expected to appear in currently running experiments, e.g. Belle, BES-III and LHC and the future PANDA experiment at the FAIR facility. In order to study these systems from a theoretical point of view, we need a non-perturbative approach to QCD. Lattice QCD achieves this through a Monte Carlo evaluation of the path integral after a discretisation of space time on a finite lattice with lattice spacing $a$. To keep systematic errors under control, we need $a^{-1}$ to be much larger than the relevant physical scales of the problem, the size of the box $L$ to be larger than the typical size of the hadrons and an extrapolation to the physical quark masses to be carried out (simulating physical masses is computationally expensive).

### 2. Methods and computational details

We have employed $N_f = 2+1$ gauge configurations generated using the tree level $O(a^2)$ Symanzik improved Wilson action for the gluonic degrees of freedom. The fermionic action uses non-perturbatively improved Wilson fermions with stout links in the derivative terms (SLiNC [6]). The quark masses were first tuned to the SU(3)$^f$-symmetric point where the flavour singlet mass average $m_q = (m_u + m_d + m_s)/3$ takes its physical value. Then, the different quark masses are varied while keeping the singlet quark mass fixed [7, 8]. At present, there is only one $\beta$ value available, $\beta = 5.5$, corresponding to $a \sim 0.0795$ fm.

| $\kappa_f$  | $\kappa_s$ | $a$ (fm) | # meas (meson) | # meas (baryon) | $M_\pi$ (MeV) |
|------------|------------|---------|----------------|----------------|---------------|
| 0.12090    | 0.12090    | 0.0795(3) | 941            | --             | 442           |
| 0.12104    | 0.12062    | 0.0795(3) | 450            | 450            | 348           |

**Table 1.** Details of the configurations used so far in this study.

Let $\hat{O}_1$ and $\hat{O}_2$ be two interpolating operators overlapping with the state we are interested in. A correlation function can then be defined,

$$C(\hat{O}_1, \hat{O}_2, t) = \langle \hat{O}_2(0)|\hat{O}_1^\dagger(t)\rangle = \lim_{T \to \infty} \frac{1}{Z(T)} \text{Tr} \left[ e^{-(T-t)\hat{H}} \hat{O}_2 e^{-t\hat{H}} \hat{O}_1^\dagger \right] = \sum_n \langle 0|\hat{O}_2|n\rangle \langle n|\hat{O}_1|0\rangle e^{-E_n t}, \quad Z(T) = e^{-\hat{H} T},$$

with $T$ being the temporal extent of the lattice. Every correlation function contains a tower of states. Since we are primarily interested in the lower lying states, the contributions for higher excitations can represent a problem if they do not die out sufficiently fast. The variational method [9] allows us to disentangle different states. The idea is to choose a basis of operators $\hat{O}_i$
and construct a cross correlation matrix \([C(t)]_ij = C(\hat{O}_i, \hat{O}_j, t)\). One then solves the generalised eigenvalue problem (GEVP),
\[
C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)v^\alpha(t, t_0) = \lambda^\alpha(t, t_0)v^\alpha(t, t_0).
\]
(2)
It can be shown that the eigenvalues behave like,
\[
\lambda^\alpha(t, t_0) \propto e^{-(t-t_0)E_\alpha} \left[ 1 + O \left( e^{-\Delta E_\alpha(t-t_0)} \right) \right] \Delta E_\alpha = E_{\alpha'} - E_\alpha \text{ with } \alpha' > \alpha.
\]
(3)
For both mesons and baryons we use a basis of three operators. Each operator has the same Fock structure but different spatial extent. The latter is achieved by applying gauge invariant smearing on the quark fields.

3. Mesons with hidden and open charm
To compute the meson spectra, we used a subset of the operators given in Ref [10], which have the general form,
\[
O(x) = \bar{q}_1(x)\Gamma Dq_2(x),
\]
(4)
where the indices denote flavour, \(\Gamma\) is a combination of gamma matrices and \(D\) represents a covariant derivative operator, containing zero, one or two derivatives depending on the spatial angular momentum of the particle under consideration (we have explored states with \(L \leq 2\)). The results obtained for the spectra of the \(D_s\) and charmonium systems are shown in Figure 2, for the flavour symmetric ensemble (\(\kappa_f = \kappa_s\)). We could resolve the first excited state as well as the ground state for all operators. In most charmonium states and for two \(D_s\) operators, the second excited state was also extracted. We can see that the experimental spectra are qualitatively reproduced. Before making a more detailed comparison, results at different light quark masses and volumes are required. This work is in progress. The good signals obtained for radially excited and non-zero angular momentum states gives us confidence in our methods.

For the \(D_s\) system, we will study the mixing of the 0\(^+\) and 1\(^+\) states with the \(DK\) and \(DK^*\) molecules, respectively. Analogously, we will study molecule mixing for the charmonium-like 1\(^{++}\) state, lying close to the \((D^0)^*D^0\) threshold.

![Figure 2. \(D_s\) (left) and charmonium (right) spectra from \(N_f = 2 + 1\) configurations.](image)

4. Charmed baryons
4.1. Interpolating operators
Even though flavour symmetry is not an exact symmetry of nature, SU(2)\(_{\text{flavour}}\) is reasonably well respected. If we include the strange quark, since \(m_s - m_u < A_{QCD}\), we still observe
an approximate flavour symmetry (SU(3)_{flavour}) in the baryon spectrum. Pushing things further, when including the charm quark, flavour symmetry is not a good symmetry of the system. In principle, it is not clear that the interpolating operators falling into the irreducible representations of SU(4)_{flavour}, cf. Figure 3 have significant overlaps with the actual baryon states. However, empirically, we have found that this is indeed the case.

![Figure 3](image)

**Figure 3.** SU(4) multiplets of baryons containing u, d, s, c quarks. a) Totally symmetric 20-plet with the SU(3) decuplet in the lowest level. b) 20-plet with the SU(3) octet in the lowest level. c) Totally antisymmetric quadruplet (¯4). The last one can only occur for spatial angular momentum with odd values. Figure from [5].

Alternatively, following HQET, we can use operators that describe a heavy baryon as a light di-quark in the presence of a heavy quark. They are listed in Table 2. Comparing results from the two sets of operators can help us understand the inner structure of the baryons.

| Singly charmed | J^P = \frac{1}{2}^+ | J^P = \frac{3}{2}^+ |
|---------------|-----------------|-----------------|
| (S) (I) s_q^a | (qq)Q | O | Name | Name |
| (0) (0) (0) | (ud)c | O_\mu = \epsilon_{abc}(u^{\sigma T}C\gamma_\mu d^c)^c | \Lambda_c | \Sigma_c |
| (0) (1) (1)* | (uu)c | O_\mu = \epsilon_{abc}(u^{\sigma T}C\gamma_\mu u^b)^c | \Sigma_c | \Sigma_c^* |
| (−1) (\frac{1}{2}) (0)* | (us)c | O_\mu = \epsilon_{abc}(u^{\sigma T}C\gamma_\mu s^b)^c | \Xi_c | \Xi_c^* |
| (−1) (\frac{1}{2}) (1)* | (us)c | O_\mu = \epsilon_{abc}(u^{\sigma T}C\gamma_\mu s^b)^c | \Xi_c | \Xi_c^* |
| (−2) (0) (1)* | (ss)c | O_\mu = \epsilon_{abc}(s^{\sigma T}C\gamma_\mu b^c)^c | \Omega_c | \Omega_c^* |

| Doubly charmed | J^P = \frac{1}{2}^+ | J^P = \frac{3}{2}^+ |
|---------------|-----------------|-----------------|
| (S) (I) s_q^a | (QQ)q | O | Name | Name |
| (0) (0) (1)* | (cc)u | O_\mu = \epsilon_{abc}(c^{\sigma T}C\gamma_\mu c^b)^u^c | \Xi_{cc} | \Xi_{cc}^* |
| (−1) (\frac{1}{2}) (1)* | (cc)s | O_\mu = \epsilon_{abc}(c^{\sigma T}C\gamma_\mu c^b)^s^b | \Omega_{cc} | \Omega_{cc}^* |

**Table 2.** Summary of quantum numbers of heavy baryons and interpolating operators following the HQET approach. Some of these operators were suggested in [11] for lattice calculations.

**4.2. Results**

The singly and doubly charmed baryon spectra for the κ_l = 0.12104, κ_s = 0.12062 ensemble are shown in Figure 4. On the left hand side, we present our preliminary singly (above) and doubly (below) charmed baryon spectra for the two bases of operators chosen, including positive and negative parity channels. As was the case for meson spectroscopy, the simulations are still at an early stage. In our results, we can see that the mass differences between baryons containing u, d quarks and the ones with s quarks are smaller than the experimental values. This is to be expected as we are far from the physical point in terms of the masses of the quarks: the singlet quark m_q is tuned to the physical value which means that the u, d and s quark masses are respectively heavier and lighter than their physical values. On the right hand side, we can see a summary of lattice results for the singly and doubly charmed spectra, with different systematics [12, 13, 14, 15, 16, 17]. We can see that, overall, lattice results agree with experiment.
Figure 4. Singly charmed (top) and doubly charmed (bottom) low lying spectrum. On the left hand size, results from the SLiNC configurations are shown ($M_\pi = 348\,\text{MeV}$). Errors are statistical only. On the right hand size, a summary of lattice results is presented.

5. Acknowledgements

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6. References

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