Neutrino Pair Bremsstrahlung in Neutron Star Crusts: a Reappraisal

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Abstract

We demonstrate that band-structure effects suppress bremsstrahlung of neutrino pairs by electrons in the crusts of neutron stars at temperatures of the order of $5 \times 10^9$ K and below. Taking this into account, together with the fact that recent work indicates that the masses of neutron star crusts are considerably smaller than previously estimated, we find neutrino pair bremsstrahlung to be much less important for the thermal evolution of neutron stars than earlier calculations suggested.
Observations of neutron star temperatures have potential for probing the interiors of neutron stars, and in recent years much effort has been devoted to making such measurements, especially with the Einstein Observatory and ROSAT. On the theoretical front there have also been developments regarding neutrino emission in matter in the cores of neutron stars, which is the main cooling mechanism early in the life of a neutron star. In this Letter we consider bremsstrahlung of neutrino pairs by electrons scattering from the Coulomb field of nuclei in the crust of a neutron star, \( e^- + Z \rightarrow e^- + Z + \nu + \bar{\nu} \). According to the usual picture of neutron star cooling, this process can play an important role, especially if neutrons are superfluid and/or protons superconducting\(^1\). In the latter case, neutrino emission in the core will be suppressed at temperatures below the transition temperatures, and neutrino pair bremsstrahlung in the crust can dominate. Even if nucleons in the interior are normal, the crust bremsstrahlung process has been estimated to be comparable in importance to the modified Urca process, which for the past quarter of a century has been regarded as the “standard” process.

The theory of the bremsstrahlung process in dense matter was developed by Festa and Ruderman\(^2\), and subsequently extended by other workers \(^3, 4, 5, 6\). The basic assumption common to these treatments is that the electron-ion interaction may be treated in first-order perturbation theory, and the conclusion is that, for a perfect lattice, the rate of emission of energy by neutrinos varies approximately as \( T^6 \). What we demonstrate in this Letter is that the process is suppressed exponentially at low temperatures. The physical reason for this is that repeated interactions of an electron with the lattice give rise to splittings between bands which can range up to 1 MeV. Consequently, at temperatures of order \( 10^9 K \sim 0.1 \text{ MeV} \), at which the neutrino bremsstrahlung process has been thought to be important, the rate of the process is much less than previously estimated. We begin by describing the microscopic calculations of the bremsstrahlung rate, and then, incorporating recent developments in the theory of matter in the crust of a neutron star, we explore implications for the thermal evolution of neutron stars.

To set the scene, let us examine the electron spectrum. In most of the crust of a neutron star, electrons move in a periodic lattice of nuclei. At the lower densities, the nuclei are essentially spherical, and the lattice bcc, but at densities approaching that at the inner boundary of the crust, nuclei may be rod-like or plate-like. For densities
above $\sim 10^6 \text{ g cm}^{-3}$ the electrons are relativistic. As we shall demonstrate, splittings between bands are generally small compared with electron energies, and therefore they may be estimated in the nearly-free-electron approximation. Since the Fermi energy is much greater than the electron rest mass, it is an excellent approximation to work in the extreme relativistic limit, in which the electron helicity is a good quantum number. This simplifies significantly the calculations, and the errors introduced are small, of order $(m_e c^2/\mu_e)^2$.

The crystal potential has most effect on states for which the free particle energies $\epsilon_p$ and $\epsilon_p - \hbar K$ are almost equal for some reciprocal lattice vector $K$. The energy eigenvalues for the upper and lower bands, denoted by $+$ and $-$ respectively, are given by

$$E^\pm(p) = \frac{\epsilon_p + \epsilon_{p-hK}}{2} \pm \sqrt{\left(\frac{\epsilon_p - \epsilon_{p-hK}}{2}\right)^2 + |V_K|^2}, \quad (1)$$

and the corresponding states are

$$\Psi_{p,\sigma}^+(r) = u_p \epsilon^{\hat{p}r} u_\sigma(p) + v_p \epsilon^{\hat{(p-hK)}r} u_\sigma(p-hK)$$
$$\Psi_{p,\sigma}^-(r) = v_p \epsilon^{\hat{p}r} u_\sigma(p) - u_p \epsilon^{\hat{(p-hK)}r} u_\sigma(p-hK), \quad (2)$$

where $u_\sigma$ is a (four-component) spinor of helicity $\sigma$, and the “coherence factors” are given by

$$u_p^2 = \frac{1}{2} \left(1 + \frac{\xi_p}{\mathcal{E}_p}\right), \quad v_p^2 = \frac{1}{2} \left(1 - \frac{\xi_p}{\mathcal{E}_p}\right), \quad \text{and} \quad u_p v_p = \frac{V_K}{2 \mathcal{E}_p}, \quad (3)$$

with $\xi_p = (\epsilon_p - \epsilon_{p-hK})/2$ and $\mathcal{E}_p = \sqrt{\epsilon_p^2 + |V_K|^2}$.

These results are essentially the same as for the non-relativistic case, except that the free-particle dispersion relation is that for massless particles, and the matrix element of the electron-lattice interaction is modified. The splitting between bands due to the periodic potential is $2|V_K|$, where

$$V_K = -v_\perp \frac{4\pi e \rho_K}{K^2}. \quad (4)$$
In this equation $v_\perp = \sqrt{1 - (K/(2k_F))^2}$, with $k_F \simeq \mu_e/(\hbar c)$ being the Fermi wavenumber. The quantity $\rho_k$ is the Fourier transform of the total charge density, $\rho_k = en_p F(k)/\varepsilon(k)$, and $n_p$ is the proton density. Here $F(k)$ is the form factor, which has contributions from the shape of the nuclear charge distribution as well as from thermal vibrations (the Debye-Waller factor), and $\varepsilon = 1 + k_{FT}^2/k^2$ is the static dielectric function, where $k_{FT} = \sqrt{4\alpha/\pi} k_F$ is the Fermi-Thomas screening wavenumber, with $\alpha = e^2/(\hbar c)$. The factor $v_\perp$ is essentially the overlap between helicity states familiar in calculations of scattering of relativistic electrons. For point-like nuclei, with atomic number $Z$, and for the smallest reciprocal lattice vector in a bcc lattice, the splitting is $\simeq 0.018(Z/60)^{2/3} \mu_e$. In the inner crust of neutron stars, electron Fermi energies range up to $\sim 75$ MeV, and $Z$ can be as large as $\sim 60[7]$ so splittings can be 1 MeV or more.

Let us now estimate the rate of energy emission. The basic process is shown in Fig.1(a). Here an electron moving in the lattice potential emits a neutrino-antineutrino pair. It follows directly from Fermi’s golden rule that the rate of energy emission in neutrino pairs, per unit volume, is given by

$$\dot{E} = \frac{2\pi}{\hbar} \sum \delta(E_f - E_i) f_1(1 - f_2)|H_{fi}|^2(E_\nu + E_\bar{\nu}) . \tag{5}$$

Here $f$ is the Fermi distribution function, and the factor $f_1(1 - f_2)$ ensures that the initial electron state, 1, is occupied, and the final state, 2, vacant. We assume that neutrinos can escape freely from matter, and therefore there are no blocking factors for neutrinos. The sum is over momenta and helicities of incoming and outgoing particles.

We shall focus on temperatures low enough that the neutrino momentum is small compared with any reciprocal lattice vector. Under these circumstances it is easy to see that the important processes will be ones involving electrons lying close to the Fermi surface, and with crystal momenta having a component close to $K/2$ in the direction of some reciprocal lattice vector $K$. The electron spectra for two values of $p^\perp$, the component of $p$ perpendicular to $K$, are shown in Fig.2, as a function of $p^\parallel$, the component of $p \pm K/2$ parallel to $K$. If the initial and final electron states are in the same band, the bremsstrahlung process is kinematically forbidden, as we now show. On the one hand, the electron (group) velocity $\nabla_p E$ cannot exceed $c$, and therefore
the energy difference, $E_1 - E_2$, between electron states is less than $cq$, where $q$ is the total momentum of the neutrino pair. On the other hand, for the neutrino pair, the energy difference must exceed $cq$. Consequently it is impossible simultaneously to conserve energy and momentum. For states in different bands there will generally be a finite energy difference even for small momentum transfers, and so the process is kinematically allowed.

We now examine the process in which an electron in the upper band makes a transition to the lower one. In evaluating matrix elements of the weak interaction Lagrangian,

$$\mathcal{L} = -\sqrt{2} G \bar{\Psi}_\nu \gamma^\alpha P_L \Psi \bar{\Psi}(C_L \gamma^\alpha P_L + C_R \gamma^\alpha P_R) \Psi ,$$  

one must use Bloch electron states Eq.(2), rather than plane waves. In Eq.(6), $G$ is the Fermi coupling constant, $P_{L,R} = (1 \mp \gamma^5)/2$, and in terms of the weak mixing angle $\theta_W$, $C_L = 1 + 2 \sin^2 \theta_W$ and $C_R = 2 \sin^2 \theta_W$ for the emission of electron neutrinos. For the emission of muon and $\tau$ neutrinos, the corresponding couplings are $C_L' = -1 + 2 \sin^2 \theta_W$ and $C_R' = 2 \sin^2 \theta_W$. For emission of electron neutrinos one finds

$$\dot{E} = \frac{G^2}{24 \pi^6 \hbar^6 c} \frac{C_A^2 + C_V^2}{2} \int \frac{d^3 q}{\omega} \sqrt{u_1 v_2 v_1 u_2} \frac{1}{e^{\beta \omega} - 1} \theta(\omega - c|q|) ,$$  

where $C_V = (C_L + C_R)/2$, $C_A = (C_L - C_R)/2$, $v_\parallel = K/(2k_F)$, $\beta = 1/k_B T$, $k_B$ is the Boltzmann constant and $\omega = E_1 - E_2$. The total emission rate from all types of neutrinos is obtained by replacing $C_V^2 + C_A^2$ by $C_A^2 + C_V^2 + 2(1-C_A)^2 + (1-C_V)^2$. The result (7) is valid for arbitrary values of $T/|V_K|$ and to lowest order in $k_B T/\mu_e$. We remark that processes in which an electron initially in the “lower” (−) band makes a transition to a state in the “upper” (+) band is kinematically forbidden, even though the energy of the initial state can be higher than that of the final state.

Simple analytical results may be obtained in limiting cases, and we first consider temperatures small compared with $V_K$, where our results differ dramatically from earlier ones. In the low-temperature limit, one may expand the integrand in Eq.(7) in powers of $\xi/V_K$, and one finds
The exponential dependence reflects the fact that the minimum energy of the neutrino pair is $2|V_K|/(1 + v_\perp)$. This is easily seen by observing that the energy of the neutrino and antineutrino is given by $\omega = E_1 - E_2 \geq 2|V_K| + q_\perp v_\perp$, and that, in addition, the four-momentum of the neutrino pair must be time-like, $\omega \geq q \geq c|q_\perp|$.

Next we consider the limit of temperatures high compared with $|V_K|$. The coherence factors may be expanded in powers of $V_K/\xi$ and the energy emission rate is

$$\dot{E}_\gamma = \frac{4\pi G^2}{567\hbar^3 c^6} \frac{C_A^2 + C_V^2}{2} \mu_e(k_B T)^6 \sum_K \frac{v_\parallel}{v_\perp^2} (1 - \frac{v_\perp^2}{v_\parallel^2}) \log \frac{1}{v_\perp^2} |V_K|^2 , k_B T \gg |V_K| \ . \ (9)$$

This result is consistent with what was found in earlier calculations \[3, 6\] in which the electron-lattice interaction was treated perturbatively. (See Fig.1(b).) To exhibit clearly scaling properties, we neglect all but the smallest reciprocal lattice vectors, put form factors equal to unity, and neglect the term in parentheses in Eq.(9). We then find an emissivity per unit mass from all species of neutrinos of $0.23 x Z T_8^6$ erg g$^{-1}$ s$^{-1}$ for spherical nuclei. Here $x = n_p/n$ is the proton fraction, where $n$ is the density of nucleons. Our results show that the neutrino emissivity due to the component of the lattice potential with wave-vector $K$ is reduced at low temperatures by a factor $\sim |V_K/k_B T|^2 \exp(-2|V_K|/[k_B T(1 + v_\perp)])$ compared with the high-temperature expression.

In the crusts of neutron stars many reciprocal lattice vectors contribute to neutrino emission, because the number of reciprocal lattice vectors for which $K < 2k_F$ is $\sim 4Z$. At the highest temperatures neutrino emission will be dominated by the smallest reciprocal lattice vectors, since they have the largest periodic lattice potential. However their contributions will be the first to be suppressed by band structure effects as the temperature is lowered, and consequently the most important reciprocal lattice vectors for neutrino emission will increase with decreasing temperature. To give a sense of this effect we have constructed a simple interpolation formula for the neutrino emission which, for each reciprocal lattice vector, agrees with the exact results in the high- and low-temperature limits. This is the sum of Eq.(8) with
an additional factor $\exp(-2|V_k|/|k_BT(1 + v_\perp)|)$ in the summand, and Eq.(8). In Fig.3 we show the energy loss rate per unit volume calculated from the interpolation formula, divided by the high-temperature rate, Eq.(9). The conditions assumed are those at the highest density at which nuclei in the crust are approximately spherical according to the calculations of Ref.[7], $\mu_e = 78$ MeV and $Z = 62$, and form factors are taken to be unity. This shows that neutrino emission is suppressed by a factor of 10 or more for temperatures less than about $10^9$ K. Including form factors reduces the total luminosity by more than a factor two, but for $T > 10^9$ K, the ratio of the interpolation formula result to $\dot{E}_\nu$ changes by no more than 30% when form factors are introduced.

We now explore the consequences of the recent discovery that in a significant fraction of the crustal matter of a neutron star nuclei are likely to be rod-like (spaghetti) or plate-like (lasagna), and not spherical[7, 8]. If one neglects spatial variations of the cross section of the rods or the thickness of the plates, the only reciprocal lattice vectors that provide scattering in the case of rods will be those that lie in a plane, while for lasagna the reciprocal lattice vectors must lie on a line. To the extent that neutrino emission is dominated by the lowest reciprocal lattice vectors, this fact implies that neutrino emission for spherical nuclei, spaghetti and lasagna would be in the ratio 6:3:1, reflecting the number of reciprocal lattice vectors for which the form factors do not vanish. Other effects influencing the emission are the dependence on nuclear shape of the magnitudes of reciprocal lattice vectors, especially the lowest ones, and of form factors.

Up to now we have assumed that the crystal is perfect, but in reality there may be impurities and/or lattice imperfections such as dislocations or grain boundaries. These will give rise to bremsstrahlung at low temperatures that will not be suppressed by the band-structure effects that we have considered above. Should they be present, their contribution, which varies as $T^6$, will dominate the loss of energy by neutrino bremsstrahlung at sufficiently low temperatures. For the case of impurity scattering in matter with spherical nuclei, the energy loss by pair bremsstrahlung depends on the impurity concentration, $x_i$, and the mean square deviation of the atomic number of the impurities from that of the host lattice, $\langle(\Delta Z)^2\rangle$, and it may be crudely estimated to be $\sim x_i \langle(\Delta Z/Z)^2\rangle$ times the bremsstrahlung rate for a perfect lattice evaluated neglecting band-structure effects, Eq.(8). If one uses the estimates of $x_i$ and $\langle(\Delta Z)^2\rangle$
from Ref.[4], one finds $x_i\langle(\Delta Z/Z)^2\rangle$ to be less than $10^{-5}$, and therefore impurities are unlikely to be important except at extremely low temperatures, where all other processes vary more rapidly with temperature than $T^6$.

Absorption of thermally excited phonons on electrons may be accompanied by neutrino bremsstrahlung. Flowers[3] has estimated the rate of energy emission by this process for matter with spherical nuclei and finds that it varies as $T^{11}$ at temperatures low compared with the Debye temperature (the ion plasma frequency), and that at the Debye temperature it is about 0.1 times the bremsstrahlung rate estimated neglecting suppression by the band structure effects considered in this Letter. At the melting temperature bremsstrahlung from phonons is found to be comparable to that from the static lattice, a conclusion confirmed by Itoh et al.[6] at densities less than $10^{12}$ g cm$^{-3}$. We conclude that at low temperatures the phonon process will dominate bremsstrahlung from the static lattice, but to make quantitative comparisons it is necessary to recalculate the rate of the phonon process with allowance for recent developments in the understanding of matter at sub-nuclear densities, including the non-spherical nuclear shapes already referred to.

We now assess the importance of neutrino bremsstrahlung from the static lattice for the thermal evolution of a neutron star. Consider the case when the result for the high-temperature limit is applicable. Our estimates of emissivities for matter with non-spherical nuclei indicate that these do not exceed those for matter with spherical nuclei, and therefore we take our estimate of the emissivity of matter at the highest density at which spherical nuclei exist as an upper bound on the emissivity of matter in the crust. The total luminosity of the crust is thus less than $\sim 1.1 \times 10^{33} T_8^6 M_{cr}/M_\odot$ erg s$^{-1}$, where $M_{cr}$ is the mass of the crust. According to the calculations of Friman and Maxwell[10], the luminosity due to the modified Urca process is $5.3 \times 10^{31} (n_0/n_{core})^{1/3} T_8^8 M/M_\odot$ erg s$^{-1}$ if neutron superfluidity and proton superconductivity are neglected. Here $n_{core}$ is the average baryon density in the core, and $n_0$ is the baryon density of nuclear matter at saturation. For a typical neutron star with mass $M = 1.4M_\odot$ and radius 10 km, and for crustal masses taken from Ref.[7], the modified Urca rates and the neutrino bremsstrahlung rate are comparable only at temperatures of order $5 \times 10^7$ K, almost one order of magnitude smaller than indicated by earlier calculations[4]. The effects of band structure and form factors will minimize the importance of the bremsstrahlung process still further, but to
determine whether it can ever be important, more detailed calculations of thermal evolution that allow for nucleon superfluidity and other effects are required.

To summarize, neutrino-pair bremsstrahlung from electrons in neutron star crusts is much less important than suggested by earlier estimates. One reason for this is that the basic process is suppressed by band structure effects and a second is that the amount of matter in the crust of a neutron star is considerably less than previously estimated.

This work was supported in part by NSF grant NSF PHY91-00283 and NASA grant NAGW-1583. We are grateful to D. G. Ravenhall and D. G. Yakovlev for helpful discussions.
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Figure Captions

FIG.1. a) The basic bremsstrahlung process. The double line is the propagator for a band electron. b) The process in first-order perturbation theory. The cross denotes an electron-lattice interaction, and the propagators are free ones.

FIG.2. Electron energy $E$ as a function of $p^\parallel$ for two values of $p^\perp$. The arrow shows a possible transition.

FIG.3. Energy emission rate according to the interpolation formula described in the text compared with the high temperature limit, $\dot{E}_\gamma$, Eq.(9), as a function of temperature.