Minimal Glider-Gun in a 2D Cellular Automaton

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Abstract
To understand the underlying principles of self-organisation and computation in cellular automata, it would be helpful to find the simplest form of the essential ingredients, glider-guns and eaters, because then the dynamics would be easier to interpret. Such minimal components emerge spontaneously in the newly discovered Sayab-rule, a binary 2D cellular automaton with a Moore neighborhood and isotropic dynamics. The Sayab-rule has the smallest glider-gun reported to date, consisting of just four live cells at its minimal phases. We show that the Sayab-rule can implement complex dynamical interactions and the gates required for logical universality.

keywords: universality, cellular automata, glider-gun, logical gates.

1 Introduction
The study of 2D cellular automata (CA) with complex properties has progressed over time in a kind of regression from the complicated to the simple. Just to mention a few key moments in CA history, the original CA was von Neumann’s with 29 states designed to model self-reproduction, and by extension – universality[18]. Codd simplified von Neumann’s CA to 8 states[5], and Banks simplified it further to 3 and 4 states[2,1]. In modelling self-reproduction its also worth mentioning Langton’s “Loops”[12] with 8 states, which was simplified by Byl to 6 states[4]. These 2D CA all featured the 5-cell “von Neumann” neighborhood.

Another line of research was based on the larger 9×9 “Moore” neighborhood. Conway’s famous “Game-of-Life” binary CA[3,9] featured the first emerging gliders, and Gosper was able to devise “glider-guns” to fire a stream of gliders. Interactions involving glider-streams and “eaters” enabled the demonstration of universal computation. A few “Life-Like” CA featuring glider-guns were subsequently discovered that follow the Game-of-Life birth/survival paradigm[7].
Figure 1: Left: One of the Sayab-rule’s minimal glider-gun patterns, of 4 live cells. Right: the glider-gun GG1 in action shooting two diagonal glider streams with a frequency of 20 time-steps and glider spacing of 5 cells. Each glider stream is stopped by an eater. Because the system is isotropic, any orientation of the glider-gun is equally valid. Green dynamic trails are set to 10 time-steps.
Note: Green dynamic trails mark any change on a zero (white) cell within the last 10 time-steps, giving a glider a green trailing wake. 10 time-steps is the setting in all subsequent figures with green dynamic trails.

Figure 2: The Sayab-rule glider-gun attractor cycle[19] with a period of 20 time-steps composed of two phases, where opposite glider-gun patterns are flipped. The direction of time is clockwise. A small patch was isolated around a glider-gun by two close eaters. Left: A detail of a patch with a minimal glider-gun (green denotes change) alongside the same pattern on the attractor cycle.
More recently, CA that feature glider-guns, but not based on birth/survival, have been found, including Sapin’s R-Rule [16], and the authors’ X-Rule [10] and Precursor-Rule [11]. Glider-guns have also been discovered in CA with 6 and 7 cell neighborhoods on a hexagonal 2D geometry with 3 values [21, 22]. From this we can see that the architecture of CA that is demonstrably able to support emerging complex dynamics is becoming simpler — arguably a positive development since a minimal system becomes easier to interpret. This is important if the underlying principles of universal computation in CA are to be understood, and by extension the underlying principles of self-organisation in nature.

The essential ingredients for a recipe to create logical universality in CA are gliders, glider-guns, eaters, and the appropriate diversity of dynamical interactions between them including bouncing and destruction. Of these the glider-gun or “pulse generator”, a devise that ejects gliders periodically, is the most critical and elusive structure. To some extent glider-guns have been demonstrated in 1D [6], and to an lesser extent in 3D [23], but here we consider the more familiar and much more studied 2D space, which is also easier to represent and manipulate. Up to now, glider-guns in 2D CA comprise periodic structures that involve at least tens of cells in the on state in their minimum phase. Here we present a much smaller glider-gun which emerges spontaneously in the newly discovered Sayab-rule, named after the Mayan-Yucatec word for a spring (of running water).

The Sayab-rule is a binary 2D CA with a Moore neighborhood and isotropic dynamics. Though analogous to the game-of-Life and the recently discovered Precursor-rule, the Sayab-rule has the smallest glider-gun reported to date, consisting of just four live cells at its minimal phase, as well as eaters and other essential ingredients. We show that the Sayab-rule can implement a diversity of complex dynamical structures and the logical gates required for logical universality [1] and supports analogous complex structures from the Game-of-Life lexicon — still lives, eaters, oscillators and spaceships.

The paper is organised into the following further sections, (2) the Sayab-Rule definition, (3) the Sayab-Rule’s gliders-guns, eaters, collisions, and other complex structures, (4) logical universality by logical gates, and (5) the concluding remarks.

## 2 The Sayab-Rule definition

The Sayab-Rule is found in the ordered region of the input-entropy scatter-plot [20] close to the Precursor Rule [11], and from the same sample and short-list [10] [11]. The input-entropy criteria in this sample followed “Life-Like” constraints (but not birth/survival logic) to the extent that the rules are binary, isotropic, with a Moore neighborhood, and with the $\lambda$ parameter [13], the den-
sity of 1s in the look-up table, similar to the Game-of-Life where \( \lambda = 0.273 \). Isotropic mapping — the same output for any neighborhood rotation, reflection or vertical flip — reduces the full rule-table (figure 3) with \( 2^9 = 512 \) neighborhood outputs to just 102 effective outputs, from which just 29 “symmetry classes” map to 1 (figure 4).

Figure 3: Top The Sayab rule-table based on to all 512 neighborhoods, and Below expanded to show each neighborhood pattern. 131 black neighborhoods map to 1, 381 blue neighborhoods map to 0. Because the rule is isotropic, only 102 symmetry classes are significant, as described in figure 4.

3 Glider-guns, eaters and collisions

From the game-of-Life lexicon, we borrow the various names for characteristic patterns or objects, including glider-guns, gliders, eaters, still-lives, oscillators, and space-ships. A glider is a periodic mobile pattern that recovers its shape but at a displaced position, making it move at a given velocity, sometimes referred to as a mobile particle. A glider is usually identified as moving on the diagonal, whereas an orthogonal “glider” is called a space-ship. A glider-gun is a periodic pattern in a fixed location that sends, shoots, or sheds, gliders into space at regular intervals.

In the Sayab-rule, the spontaneous emergence of its basic glider-gun, as well as isolated gliders, is highly probable from a sufficiently large random initial state because the four glider patterns are very simple and likely to occur or emerge by chance – likewise, the smallest glider-gun patterns. Simple still-lives and oscillators (which may act as eaters which destroy gliders but remain active)
Figure 4: The Sayab-rule’s 29 isotropic neighborhood symmetry classes that map to 1 (the remaining 73 symmetry classes map to 0, making 102 in total). Each class is identified by the smallest decimal equivalent of the class, where the $3 \times 3$ pattern is taken as a string in the order $876543210$ — for example, the pattern is the string $001110111$ representing the symmetry class 119. The class numbers are colored depending on the value of the central cell to distinguish birth (blue) from survival (red), but no clear “Life-like” birth/survival logic is discernible.

are also likely to occur or emerge from random patterns. The basic glider-gun is also probable in subsequent evolution because it can result from the collision of two gliders, or a glider and an oscillator, though the glider-gun can also be destroyed by incoming gliders and other interactions.

Figure 5: The 4 phases of the Sayab-rule glider Ga, moving NE with speed $c/4$, where $c$ is the “speed of light”, in this case, for a Moore neighborhood, $c$ equals one cell per time-step, diagonally or orthogonally.

Figure 6: Examples of still-lives.
Figure 7: Sayab rule oscillators with the periods indicated.

Figure 8: A typical evolution emerging after 108 time-steps from a 50x50 30% density random zone. Two stable glider-guns have emerged, together with other gliders, still-lives and oscillators.

Figure 9: The glider-gun core for 10 successive time-steps — in the next next 10 time-steps the same reversed patterns are repeated, to make the period 20 attractor cycle (figure 2). The pattern sequence is from left to right. Any of these patterns are the seeds of a glider-gun, with the smallest, 4 live cells, being the most probable to occur in a random pattern.
As can be seen in its attractor (figure 2), the Sayab-rule’s basic glider-gun GG1 (figure 1) has a core that varies between just 4 and 11 live cells during its cycle of twenty time-steps, which is composed of two equivalent phases of 10 time-steps. After 10 time-steps the core patterns are reversed. In figures 2 and 9 the core and its twin 45° glider streams face towards the North, but the glider-gun can be oriented to face in any of 4 directions. The glider-gun shoots gliders at 20 time-steps intervals with a speed is $c/4$, and a glider takes 20 time-steps to traverse 5 (diagonal) cells, which is also the spacing of gliders in a glider stream. This spacing can be doubled (without limit) by combining the basic glider-guns into compound glider-guns (figures 16 and 17).

In the Sayab rule, there are many possible outcomes resulting from collisions between two (or more) gliders, and between gliders and still-lives or oscillators. These have been examined experimentally but not exhaustively. The outcomes depend on the precise timing and points of impact, and can result in the destruction, survival, or modification of the various colliding objects. For the purposes of this paper we highlight some significant collision outcomes.

Eaters that are able to stop a stream of gliders, are a necessary component in the computation machinery. They can be derived from still-lives or oscillators (figure 10). The glider-gun itself can be the outcome of a collision between a glider and an oscillator (figures 11), or between two gliders (figure 12).

![Figure 10: Collisions between a glider and an eater, (a) derived from a still-life, and (b) from an oscillator.](image1)

![Figure 11: (a) three different collisions between a glider with an oscillator create a glider-gun (b) shown after 43 time-steps.](image2)
A particular but not infrequent collision situation can arise between a stream of gliders and an oscillator which results in a retrograde stable pattern moving backwards, a sort of footprint. This eventually destroys the originating glider-gun as illustrated in figure 13.

Figure 13: Glider-gun stream (a) collides with an oscillator resulting in a retrograde stable pattern (b) moving backwards that eventually destroys the glider-gun (c).

A small slow moving space-ship (an orthogonal glider) can result from a collision between a glider and an oscillator, as shown in figure 14. The spaceship that emerges has a frequency of 12 and speed of $c/12$, so it takes 12 time-steps to advance one cell. Larger space-ships with various frequencies are shown in figure 15.

Figure 14: (a) a glider collides with an oscillator creating a slow moving space-ship (b) shown after 25 time-steps. The 12 phases of the space-ship are shown.

Figure 15: Six large space-ships moving North with speed $c/2$. Periods, from left to right, are 2, 2, 2, 4, 4, 4.
Figure 16: (a) two pairs of gliders, each pair colliding at 90°, form a pre-image of GG2. (b) the compound glider-gun GG2 shown after 138 time-steps, shoots gliders with a frequency of 40 time-steps and glider spacing is 10 cells.

Figure 17: The compound glider-gun GG4 shoots gliders with a frequency of 80 time-steps and glider spacing is 20 cells.

A compound glider-gun (GG2) can be built from two interlocking GG1 glider-guns. GG2 shoots two glider streams in opposite directions with a frequency of 40 time-steps and a glider spacing is 10 cells (twice GG1). The dynamics depend on glider streams colliding at 90° resulting in the destruction of one glider-stream, and alternate gliders in the other glider-stream. Collisions leave behind a sacrificial “eater” which destroys one of the next pair of incoming gliders.

Two GG2 glider-guns can be combined into a larger compound glider-gun (GG4, figure 17) where analogous collisions result in doubling the GG2 frequency and spacing, so the GG4 glider-stream has a frequency of 80 time-steps and spacing of 20 cells. This doubling of glider-stream frequency and spacing with greater compound glider-guns can be continued without limit.
4 Logical Universality and Logical Gates

Post’s Functional Completeness Theorem\textsuperscript{[14, 8]} established that it is possible to make a disjunctive (or conjunctive) normal form formula using the logical gates NOT, AND and OR. Conway applies this as his 3rd condition for a cellular automata to be universal in the full sense. The three conditions, applied to the game-of-Life\textsuperscript{[3]}, state that the system must be capable of the following:

1. Data storage or memory.
2. Data transmission requiring wires and an internal clock.
3. Data processing requiring a universal set of logic gates NOT, AND, and OR, to satisfy negation, conjunction and disjunction.

This section is confined to demonstrating the logical gates, so Conway’s condition 3, for universality in the logical sense. To demonstrate universality in Conway’s full sense\textsuperscript{2} it would be necessary to also prove conditions 1 and 2.

We propose that the basic existential ingredients for constructing logical gates, and thus logical universality, are as follows:

1. A glider-gun or “pulse generator”, that sends a stream of gilders\textsuperscript{3} into space (figures 1 and 2).
2. An eater, based on a still-life or oscillator, that destroys an incoming glider and survives the collision, so can stop a glider stream (figure 10).
3. Complete self-destruction when two gilders collide at an angle. Any debris must quickly dissipate, and the gap between gilders must be sufficient so as not to interfere with the next glider collision (figure 18).

These ingredients exist in Sayab-rule dynamics, where collision outcomes depend on the precise timing and point of impact. Interacting GG1 glider-gun streams with glider/gap sequences with the correct spacing and phases representing a “string” of data, we present examples of the logical gates NOT, AND and OR, in figures 19, 20 and 21. Gaps in a string are indicated by grey circles, and dynamic trails of 10 time-steps are included. Any input strings can be substituted for those shown. Eaters are positioned to eventually stop gliders.

Figure 18: Two giders colliding at 90° self-destruct. 5 consecutive time-steps are shown. This is a key collision in making logical gates. Head-on collisions also self destruct, but are not as useful in this context.

\textsuperscript{2}Alternatively, full universality could be proved in terms of the Turing Machine, as was done by Randall\textsuperscript{[15]}.

\textsuperscript{3}Giders are not listed separately because they are implicit in the glider-gun.
Figure 19: An example of the NOT gate: \((-1, 1 \rightarrow 0 \text{ and } 0 \rightarrow 1)\) or inverter, which transforms a stream of data to its compliment, represented by gliders and gaps. The 5-bit input string A (11001) moving SE interacts with a GG1 glider-stream moving NE, resulting in NOT-A (00110) moving NE, shown after 94 time-steps.

Figure 20: An example of the AND gate \((1 \land 1 \rightarrow 1, \text{ else } \rightarrow 0)\) making a conjunction between two streams of data, represented by gliders and gaps. The 5-bit input strings A (11001) and B (10101) both moving SE interact with a GG1 glider-stream moving NE, resulting in A-AND-B (10001) moving SE shown after 174 time-steps. The dynamics making this AND gate first makes an intermediate NOT-A string 00110 (as in figure 19) which then interacts with input string B to simultaneously produce both the A-AND-B string moving SE described above, and also the A-NOR-B string 00010 moving NE.
Figure 21: An example of the OR gate \((1 \lor 1 \rightarrow 1, \text{else} \rightarrow 0)\) which makes a disjunction between two stream of data represented by two streams of gliders and gaps. The 5-bit input strings \(A\) (11001) and \(B\) (10101) both moving SE interact with two GG1 glider-streams, the lower GG1 shooting NE, and subsequently with an upper GG1 shooting SE, finally resulting in the A-OR-B string (11101) moving SE shown after 232 time-steps. The dynamics first makes an intermediate NOT-A string 00110 (as in figure 19), which then interacts with string \(B\) to simultaneously produce both the AND string (10001, which appears in the figure) and an intermediate A-NOR-B string 00010 — this is inverted by the upper glider-gun stream to make NOT(A-NOR-B) which is the same as the A-OR-B string (11101).

5 Concluding remarks

The Sayab-rule’s glider-gun is the smallest reported to date in 2D CA, consisting of just four live cells at its minimal phases. From this glider-gun and other artefacts it is possible to build the logical gates NOT, AND and OR required for logical universality, which are constructed by collision dynamics depending on precise timing and points of impact. Furthermore, the fact that the glider-gun can result from a collision between two gliders, or between a glider and a simple oscillator, opens up possibilities for making complex dynamical structures.

Three basic existential ingredients are proposed for constructing logical gates,
to summarise: a glider-gun, an eater, and self-destruction when two gliders collide at an angle. Rules with these ingredients are certainly elusive; in previous work [20, 10] we described how they can nevertheless be found. These methods and the frequency of such rules in rule-space requires further research. The rules occur as families of genetically related rules — this aspect in itself requires investigation — for example, variants of the Sayab-rule make up a family with related behaviour.

Finally, the minimal size of the Sayab-rule’s glider-gun is significant because it should make it easier to interpret its dynamical machinery, employing De Bruijn diagrams and other mathematical and computational tools. Such further research holds the promise of understanding how glider-guns and related artefacts can exist, and so reveal the underlying principles of self-organisation in CA, and by extension in nature itself.

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