Model-checking Driven Black-box Testing Algorithms for Systems with Unspecified Components

[Extended Abstract]

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1. INTRODUCTION

Component-based software development is a systematic engineering method to build software systems from prefabricated software components that are previously developed by the same organization, provided by third-party software vendors, or even purchased as commercial-off-the-shelf (COTS) products. Though this development method has gained great popularity in recent years, it has also posed serious challenges to the quality assurance issue of component-based software since externally obtained components could be a new source of system failures. The issue is of vital importance to safety-critical and mission-critical systems. For instance, in June 1996, during the maiden voyage of the Ariane 5 launch vehicle, the launcher veered off course and exploded less than one minute after taking off. The report of the Inquiry Board indicates that the disaster resulted from insufficiently tested software reused from the Ariane 4. The developers had reused certain Ariane 4 software component in the Ariane 5 without substantially testing it in the new system, having assumed that there were no significant differences in these portions of the two systems.

Most of the current work addresses the issue from the viewpoint of component developers: how to ensure the quality of components before they are released. However, this view is obviously insufficient: an extensively tested component (by the vendor) may still not perform as expected in a specific deployment environment, since the systems where a component could be deployed may be quite different and diverse and they may not be tried out by its vendor.

So, we look at this issue from system developers’ point of view: (*) how to ensure that a component functions correctly in the host system where the component is deployed.

In practice, testing is almost the most natural resort to resolve this issue. When integrating a component into a system, system developers may have three options for testing: (1) trust the component provider’s claim that the component has undergone thorough testing and then go ahead to use it; (2) extensively retest the component alone; (3) hook the component with the system and conduct integration testing. Unfortunately, all of the three options have some serious limitations. Obviously, for systems requiring high reliability, the first option is totally out of the question. The second option may suffer from the following fact. Software components are generally built with multiple sets of functionality, and indiscriminately testing all the functionality of a software component is not only expensive but sometimes also infeasible, considering the potentially huge state space of the component interface. Additionally, it is usually difficult to know when the testing over the component is adequate. The third option is not always applicable. This is because, in many applications, software components could be applied...
for dynamic upgrading or extending a running system that is costly or not supposed to shut down for retesting at all. Even without all the above limitations, purely testing-based strategies are still not sufficient to establish the solid confidence for a reliable component required by mission-critical or safety-critical systems, where formal methods like model-checking are highly desirable. However, one fundamental obstacle for using a formal method to address the issue of (*) is that design details or source code of an externally obtained software component is generally not fully available to the developers of its host system. Thus, existing formal verification techniques (like model-checking) are not directly applicable.

Clearly, this problem plagues both component-based software systems and some hardware systems with a modularized design. Generally, we call such systems as systems with unspecified components (in fact, in most cases, the components are partially specified to which our approach still applies).

In this paper, we present a new approach, called model-checking driven black-box testing, which combines model-checking techniques and black-box testing techniques to deal with this problem. The idea is simple yet novel: with respect to some temporal requirement about the unspecified component from the rest of the system, this condition guarantees that the system satisfies the requirement iff the condition is satisfied by the unspecified component, which can be checked by adequate black-box testing over the unspecified component with test-cases generated automatically from the condition.

We provide algorithms for both LTL and CTL model-checking driven black-box testing. In the algorithms, the condition mentioned earlier is represented as communication graphs and witness driven black-box testing. In the algorithms, the condition guarantees that the system satisfies the requirement if the condition is satisfied by the unspecified component, which can be checked by adequate black-box testing over the unspecified component with test-cases generated automatically from the condition.

The advantages of our approach are obvious: a stronger confidence about the reliability of the system can be established through both formal verification and adequate functional testing; system developers can customize the testing with respect to some specific system properties; intermediate model-checking results (the communication and witness graphs) can be reused to avoid (repetitive) integration testing when the component is updated, if only the new component’s interface remains the same; our algorithms are both sound and complete.

Though we do not have an exact complexity analysis result, our preliminary studies show that, in the liveness testing algorithm for LTL, the maximal length of test-cases run on the component is bounded by $O(n \cdot m^2)$. For CTL, the length is bounded by $O(k \cdot n \cdot m^2)$. In here, $k$ is the number of CTL operators in the formula to be verified, $n$ is the state number in the host system, and $m$ is the state number in the component.

The advantages of our approach are obvious: a stronger confidence about the reliability of the system can be established through both formal verification and adequate functional testing; system developers can customize the testing with respect to some specific system properties; intermediate model-checking results (the communication and witness graphs) can be reused to avoid (repetitive) integration testing when the component is updated, if only the new component’s interface remains the same; our algorithms are both sound and complete; most of all, the whole process can be carried out in an automatic way.

The rest of this paper is organized as follows. Section 2 provides some background on temporal logics LTL and CTL along with our model of systems containing unspecified components. The main body of the paper consists of Section 3 and Section 4 which propose algorithms for LTL and CTL model-checking driven black-box testing, respectively, over the system model. Section 5 illustrates the algorithms through an example. Section 6 lists some of the related work. Section 7 concludes the paper with some further issues to be resolved in the future.

Details on some algorithms are omitted in this extended abstract. At http://www.eecs.wsu.edu/~gxie a full version of this paper is available.

2. PRELIMINARIES

2.1 The System Model

In this paper, we consider systems with only one unspecified component (the algorithms generalize to systems with multiple unspecified components). Such a system is denoted by $Sys = (M, X)$, where $M$ is the host system and $X$ is the unspecified component. Both $M$ and $X$ are finite-state transition systems communicating synchronously with each other via a finite set of input and output symbols.

Formally, the unspecified component $X$ is defined as a deterministic Mealy machine whose internal structure is unknown (but an implementation of $X$ is available for testing). We write $X$ as a triple $(\Sigma, \nabla, m)$, where $\Sigma$ is the set of $X$’s input symbols, $\nabla$ is the set of $X$’s output symbols, and $m$ is an upper bound for the number of states in $X$ (as a convention in black-box testing, the $m$ is given). Assume that $X$ has an initial state $s_{init}$. A run of $X$ is a sequence of symbols alternately in $\Sigma$ and $\nabla$: $a_0\beta_0\alpha_1\beta_1\ldots$, such that, starting from the initial state $s_{init}$, $X$ outputs exactly the sequence $\beta_0\beta_1\ldots$ when it is given the sequence $\alpha_0\alpha_1\ldots$ as input. In this case, we say that the input sequence is accepted by $X$.

The host system $M$ is defined as a 5-tuple $(S, \Gamma, R_{env}, R_{comm}, I)$ where

- $S$ is a finite set of states;
- $\Gamma$ is a finite set of events;
- $R_{env} \subseteq S \times \Gamma \times S$ defines a set of environment transitions, where $(s, \alpha, s') \in R_{env}$ means that $M$ moves from state $s$ to state $s'$ upon receiving an event (symbol) $\alpha \in \Gamma$ from the outside environment;
- $R_{comm} \subseteq S \times \Sigma \times \nabla \times S$ defines a set of communication transitions, where $(s, \alpha, \beta, s') \in R_{comm}$ means that $M$ moves from state $s$ to state $s'$ when $X$ outputs a symbol $\beta \in \nabla$ after $M$ sends $X$ an input symbol $\alpha \in \Sigma$; and,
- $I \subseteq S$ is $M$’s initial states.

Without loss of generality, we further assume that, there is only one transition between any two states in $M$ (but $M$, in general, could still be nondeterministic).

An execution path of the system $Sys = (M, X)$ can be represented as a (potentially infinite) sequence $\tau$ of states and symbols, $s_0\alpha_0\beta_0\gamma_1\ldots$, where each $s_i \in S$, each $c_i$ is either a symbol in $\Gamma$ or a pair $\alpha_i\beta_i$ (called a communication) with $\alpha_i \in \Sigma$ and $\beta_i \in \nabla$. Additionally, $\tau$ satisfies the following requirements:

- $s_0$ is an initial state of $M$, i.e., $s_0 \in I$;
- for each $c_i \in \Gamma$, $(s_i, c_i, s_{i+1})$ is an environment transition of $M$;
- for each $c_i = \alpha_i\beta_i$, $(s_i, \alpha_i, \beta_i, s_{i+1})$ is a communication transition of $M$.

The communication trace of $\tau$, denoted by $T_\tau$, is the sequence obtained from $\tau$ by retaining only symbols in $\Sigma$ and $\nabla$ (i.e., the result of projecting $\tau$ onto $\Sigma$ and $\nabla$). For any given state $s \in S$, $\tau$...
we say that the system $Sys$ can reach $s$ iff $Sys$ has an execution path $\tau$ on which $s$ appears and $\tau X$ (if not empty) is also a run of $X$.

In the case when $X$ is fully specified, the system can be regarded as an I/O automaton [26].

2.2 Model-checking

Model-checking [9, 10, 26, 20] is an automatic technique for verifying a finite-state system against some temporal specification. The system is usually represented by a Kripke structure $K = (S, R, L)$ over a set of atomic propositions $AP$, where

- $S$ is a finite set of states;
- $R \subseteq S \times S$ is the (total) transition relation;
- $L : S \to 2^{AP}$ is a function that labels each state with the set of atomic propositions that are true in the state.

The temporal specification can be expressed in, among others, a branching-time temporal logic (CTL) or a linear-time temporal logic (LTL). Both CTL and LTL formulas are composed of path quantifiers $A$ and $E$, which denote “for all paths” and “there exists a path”, respectively, and temporal operators $X$, $F$, $U$ and $G$, which stands for “next state”, “eventually”, “until”, and “always”, respectively.

More specifically, CTL formulas are defined as follows:

- Constants $true$ and $false$, and every atomic proposition in $AP$ are CTL formulas;
- If $f_1$ and $f_2$ are CTL formulas, then so are $\neg f_1$, $f_1 \land f_2$, $f_1 \lor f_2$, $\Box f_1$, $\Diamond f_1$, $EF f_1$, $AF f_1$, $E[f_1 \land f_2]$, $A[f_1 \lor f_2]$, $EG f_1$, $AG f_1$.

Due to duality, any CTL formula can be expressed in terms of $\neg$, $\lor$, $EX$, $EU$ and $EG$. A CTL model-checking problem, formulated as

\[ K, s \models f \]

is to check whether the CTL formula $f$ is true at a state $s$. For example, $AF f$ is true at state $s$ if $f$ will be eventually true on all paths from $s$; $E[f \lor g]$ is true at state $s$ if there exists a path from $s$ on which $f$ is true at each step until $g$ becomes true.

LTL formulas, on the other hand, are all in the form of $A f$ where $f$ is a path formula defined as follows:

- Constants $true$ and $false$, and every atomic proposition in $AP$ are path formulas;
- If $f_1$ and $f_2$ are path formulas, then so are $\neg f_1$, $f_1 \land f_2$, $f_1 \lor f_2$, $X f_1$, $F f_1$, $[f_1 \land f_2]$, $G f_1$.

An LTL model-checking problem, formulated as

\[ K, s \models Af \]

is to check whether the path formula $f$ is true on all paths from a state $s$. For example, $AGF f$ is true at $s$ if on all paths from $s$, after a future point $f$ will be always true; $AGF f$ is true at $s$ if on all paths from $s$, $f$ will be true infinitely often.

More detailed background in model-checking and temporal logics can be found in the textbook [11]. The system $Sys = (M, X)$ defined earlier can be understood as a Kripke structure (with a given labeling function and atomic propositions over states in $M$). Since $X$ is an unspecified component, in the rest of the paper, we mainly focus on how to solve the LTL/CTL model-checking problems on the $Sys$ through black-box testing on $X$.

2.3 Black-box Testing

Black-box testing (also called functional testing) is a technique to test a system without knowing its internal structure. The system is regarded as a “black-box” in the sense that its behaviour can only be determined by observing (i.e., testing) its input/output sequences. As a common assumption in black-box testing, the unspecified component $X$ (treated as a black-box) has a special input symbol $reset$ which always makes $X$ return to its initial state regardless of its current state. We use $Experiment(X, reset\pi)$ to denote the output sequence obtained from the input sequence $\pi$, when $X$ runs from the initial state (caused by the $reset$). After running this $Experiment$, suppose that we continue to run $X$ by providing an input symbol $\alpha$ following the sequence $\pi$. Corresponding to this $\alpha$, we may obtain an output symbol $\beta$ from $X$. We use $Experiment(X, \alpha)$ to denote $\beta$. Notice that this latter $Experiment$ is a shorthand for “the last output symbol in $Experiment(X, reset\alpha)$".

Studies have shown that if only an upper bound for the number of states in the system and the system’s inputs set are known, then its (equivalent) internal structure can be fully recovered through black-box testing. Clearly, a naive algorithm to solve the LTL/CTL model-checking problem over the $Sys$ is to first recover the full structure of the component $X$ through testing, and then to solve the classic model-checking problem over the fully specified system composed from $M$ and the recovered $X$. Notice that, in the naive algorithm, when we perform black-box testing over $X$, the selected test-cases have nothing to do with the host system $M$. Therefore, it is desirable to find more sophisticated algorithms such as the ones discussed in this paper, that only select “useful” test-cases wrt the $M$ as well as the temporal specification of $M$ that needs to be checked.

3. LTL MODEL-CHECKING DRIVEN BLACK-BOX TESTING

In this section, we introduce algorithms for LTL model-checking driven black-box testing for the system $Sys = (M, X)$ defined earlier. We first show how to solve a liveness analysis problem. Then, we discuss the general LTL model-checking problem.

3.1 Liveness Analysis

The liveness analysis problem (also called the infinite-often problem) is to check: starting from some initial state $s_0 \in I$, whether the system $Sys$ can reach a given state $s_f$ for infinitely many times.

When $M$ has no communications with the unspecified component $X$, solving the problem is equivalent to finding a path $\rho$ that runs from $s_0$ to $s_f$ and a loop $C$ that passes $s_f$. However, as far as communications are involved, the problem gets more complicated. The existence of the path $\rho$ does not ensure that the system can indeed reach $s_f$ from $s_0$ (e.g., communications with $X$ may never allow the system to take the necessary transitions to reach $s_f$). Moreover, the existence of the loop $C$ does not guarantee that the system can run along $C$ forever either (e.g., after running along $C$ for three rounds, the system may be forced to leave $C$ by the communications with $X$).

We approach this infinite-often problem in three steps. First, we look at whether a definite answer to the problem is possible. If we can find a path from $s_0$ to $s_f$ and a loop from $s_f$ to $s_f$ that involve only environment transitions, then the original problem (i.e., the infinite-often problem) is definitely true. If such a path and a loop, no matter what transitions they may involve, do not exist at all, then the original problem is definitely false. If no definite answer is possible, we construct a directed graph $G$ and use it to generate
test-cases for the unspecified component $X$. The graph $G$, called a communication graph, is a subgraph of $M$, represents all paths and loops in $M$ that could witness the truth of the problem (i.e., paths that run from $s_0$ to $s_f$ and loops that pass $s_f$). The graph $G$ is defined as a pair $(N, E)$, where $N$ is a set of nodes and $E$ is a set of edges. Each edge of $G$ is annotated either by a pair $\alpha\beta$ that denotes a communication of $M$ with $X$, or has no annotation. We construct $G$ as follows.

- Add one node to $G$ for each state in $M$ that is involved in some path between $s_0$ and $s_f$ or in a loop that passes $s_f$.
- Add one edge between two nodes in $N$ if $M$ has a transition between two states corresponding to the two nodes respectively. If the transition involves a communication with $X$, then annotate the edge with the communication symbols.

It is easy to see that the liveness analysis problem is true if and only if the truth is witnessed by a path in $G$. Therefore, the last step is to check whether $G$ has a path along which the system can reach $s_f$ from $s_0$ first and then reach $s_f$ for infinitely many times. More details of this step are addressed in the next subsection.

See appendix B.1 for details on the above operations.

### 3.2 Liveness Testing

To check whether the constructed communication graph $G$ has a path that witnesses the truth of the original problem, the straightforward way is to try out all paths in $G$ and then check, whether along some path, the system can reach $s_f$ from $s_0$ first and then reach $s_f$ for infinitely many times. The check is done by testing $X$ with the communication trace of the path to see whether it is a run of $X$. However, one difficulty is that $G$ may contain loops, and certainly we can only test $X$ with a finite communication trace. Fortunately, the following observations are straightforward:

- To check whether the system can reach $s_f$ from $s_0$, we only need to consider paths with length less than $mn_{n_1}$ where $n_1$ is the maximal number of communications on all simple paths (i.e., no loops on the path) between $s_0$ and $s_f$ in $G$, and $m$ is an upper bound for the number of states in the unspecified component $X$;
- To check whether the system can reach from $s_f$ to $s_f$ for infinitely many times, we only need to make sure that the system can reach $s_f$ for $m - 1$ times, and between $s_f$ and $s_f$, the system goes through a path no longer than $n_2$ that is the maximal number of communications on all simple loops (i.e., no nested loops along the loop) in $G$ that pass $s_f$.

Let $n = \max(n_1, n_2)$. The following procedure $TestLiveness$ uses a bounded and nested depth-first search to traverse the graph $G$ while testing $X$. It first tests whether the system can reach $s_f$ from $s_0$ along a path with length less than $mn$, then it tests whether the system can further reach $s_f$ to $s_f$ for $m - 1$ more times. The algorithm maintains a sequence of input symbols that has been successfully accepted by $X$, an integer variable $level$ that records how many communications have been gone through without reaching $s_f$, and an integer variable $count$ that indicates how many times $s_f$ has been reached. At each step, it chooses one candidate from the set of all possible input symbols at a node, and feeds the input sequence concatenated with the candidate input symbol to $X$. If the candidate input symbol and the output symbol (corresponding to the candidate input symbol) of $X$ match the annotation of an edge originating from the node, the procedure moves forward to try the destination node of the edge with $level$ increased by 1. If there is no match, then the procedure tries other candidates. But before trying any other candidate, we need to bring $X$ to its initial state by sending it the special input symbol $reset$. The procedure returns $false$ when all candidates are tried without a match, or when more than $mn$ communications have been gone through without reaching $s_f$. After $s_f$ is reached, the procedure increases $count$ by 1 and resets $level$ to 0. The procedure returns $true$ when it has already encountered $s_f$ for $m$ times.

#### Procedure $TestLiveness(X, \pi, s_0, s_f, level, count)$

**If level \(> mn\)** Then
Return false;
Else If $s_0 = s_f$ Then
If $count \geq m$ Then
Return true;
Else
$count := count + 1$; $level := 0$;
For each $(s_0, s') \in E$ Do
Experiment$(X, \pi, s_0, s_f, level, count)$
If $TestLiveness(X, \pi, s', s_f, level, count)$ Then
Return true;
Inputs := $(\alpha(s_0, \alpha\beta, s') \in E)$;
For each $\alpha \in Inputs$ Do
Experiment$(X, \pi, s_0, s_f, level, count)$
If $\exists s' : (s_0, \alpha\beta, s') \in E$ Then
If $TestLiveness(X, \pi, s', s_f, level + 1, count)$ Then Return true;
Return false.

In summary, our liveness testing algorithm to solve the liveness analysis problem has two steps: (1) build the communication graph $G$; (2) return the truth of

$TestLiveness(X, reset, s_0, s_f, level = 0, count = 0)$.

### 3.3 LTL Model-Checking Driven Testing

Recall that the LTL model-checking problem is, for a Kripke structure $K = (S, R, L)$ with a state $s \in S$ and a path formula $f$, to determine if $K, s \models Af$. Notice that $K, s \models Af$ if and only if $K, s \models \neg E \neg f$. Therefore it is sufficient to only consider formulas in the form $E f$. The standard LTL model-checking algorithm first constructs a tableau $T$ for the path formula $f$. $T$ is also a Kripke structure and includes every path that satisfies $f$. Then the algorithm composes $T$ with $K$ and obtains another Kripke structure $P$ which includes exactly the set of paths that are in both $T$ and $K$. Thus, a state in $K$ satisfies $E f$ if and only if it is the start of a path (in the composition $P$) that satisfies $f$.

Define $sat(f)$ to be the set of states in $T$ that satisfy $f$ and use the convention that $(s, s') \in sat(f)$ if and only if $s' \in sat(f)$. The LTL model-checking problem can be summarized by the following theorem:

**Theorem 1.** $K, s \models E f$ if and only if there is a state $s'$ in $T$ such that $(s, s') \in sat(f)$ and $P, (s, s') \models EG \true$ under fairness constraints $\{sat(\exists(g U h) v h) \mid g U h \text{ occurs in } f\}$.

Note that the standard LTL model-checking algorithm still applies to the system $Sys = (M, X)$, although it contains an unspecified component $X$. To see this, the construction of the tableau $T$ from $f$ and the definition of $sat$ are not affected by the unspecified component $X$. The composition of $Sys$ and $T$ is a new system $Sys' = (P, X)$ where $P$ is the composition of $M$ and $T$. Then one can show

**Corollary 1.** $\langle M, X \rangle, s \models E f$ if and only if there is a state $s'$ in $T$ such that $(s, s') \in sat(f)$ and $\langle P, X \rangle, (s, s') \models EG \true$
under fairness constraints \( \{ \text{sat}(\neg(g \ U \ h) \lor h) \mid g \ U \ h \text{ occurs in } f \} \).

Obviously, checking whether there is a state \( s' \) in \( T \) such that \( (s, s') \in \text{sat}(f) \) is trivial. To check whether \( (P, X), (s, s') \models EG \text{ true} \) under the fairness constraints is equivalent to checking whether there is computation in \( (P, X) \) that starts from \( (s, s') \) and on which the fairness constraints are true infinitely often. One can show that this is equivalent to the liveness analysis problem we studied in the previous subsection, and thus, the LTL model-checking problem can be solved by extending our algorithms for the liveness analysis problem. Moreover, the algorithms are both complete and sound.

4. CTL MODEL-CHECKING DRIVEN BLACK-BOX TESTING

In this section, we introduce algorithms for CTL model-checking driven black-box testing for the system \( Sys = (M, X) \).

4.1 Ideas

Recall that the CTL model-checking problem is, for a Kripke structure \( K = (S, R, L) \), a state \( s_0 \in S \), and a CTL formula \( f \), to check whether \( K, s_0 \models f \). The standard algorithm \( \text{ProcessCTL} \) for this problem operates by searching the structure and, during the search, labeling each state \( s \) with the set of subformulas of \( f \) that are true at \( s \). Initially, labels of \( s \) are just \( L(s) \). Then, the algorithm goes through a series of stages during the \( i \)-th stage, subformulas with the \((i - 1)\)-nested CTL operators are processed. When a subformula is processed, it is added to the labels for each state where the subformula is true. When all the stages are completed, the algorithm returns \text{true} \) when \( s_0 \) is labeled with \( f \), or \text{false} \) otherwise.

However, if a system is not completely specified, the standard algorithm does not work. This is because, in the system \( Sys = (M, X) \), transitions of \( M \) may depend on communications with the unspecified component \( X \). In this section, we adapt the standard CTL model-checking algorithm \( \text{ProcessCTL} \) to handle the system \( Sys \) (i.e., to check whether \( (M, X), s_0 \models f \) holds where \( s_0 \) is an initial state in \( M \) and \( f \) is a CTL formula over \( M \)).

The new algorithm follows a structure similar to the standard one. It also goes through a series of stages to search \( M \)'s state space and label each state during the search. However, during a stage, processing the subformulas is rather involved, since the truth of a subformula \( h \) at a state \( s \) can not be simply decided (it may depend on communications). Similar to the algorithm for the liveness analysis problem, our ideas here are to construct a graph representing all the paths that witness the truth of \( h \) at \( s \). But, the new algorithm is far more complicated than the liveness testing algorithm for LTL, since the truth of a CTL formula is usually witnessed by a tree instead of a single path. In the new algorithm, processing each subformula \( h \) is sketched as follows.

When \( h \) takes the form of \( EX \ g \), \( E[g_1 \ U \ g_2] \), or \( EG \ g \), we construct a graph that represents exactly all the paths that witness the truth of \( h \) at some state. We call such a graph the subformula’s witness graph (WG), written as \( [h] \). We also call \( [h] \) an EX graph, an EU graph, or an EG graph if \( h \) takes the form of \( EX \ g \), \( E[g_1 \ U \ g_2] \), or \( EG \ g \), respectively.

Let \( k \) be the total number of CTL operators in \( f \). In the algorithm, we construct \( k \) WGs, and for each WG, we assign it with a unique ID number that ranges between 2 and \( k + 1 \). (The ID number 1 is reserved for constant \text{true} \.) Let \( T \) be the mapping from the WGs to their IDs; i.e., \( T([h]) \) denotes the ID number of \( h \)'s witness graph, and \( T^{-1}(i) \) denotes the witness graph with \( i \) as its ID number, \( 1 \leq i \leq k + 1 \). We label a state \( s \) with ID number 1 if \( h \) is true at \( s \) and the truth does not depend on communications between \( M \) and \( X \). Otherwise, we label \( s \) with \( 2 \leq i \leq k + 1 \) if \( h \) could be true at \( s \) and the truth would be witnessed only by some paths which start from \( s \) in \( T^{-1}(i) \) and, on which, communications are involved.

When \( h \) takes the form of a Boolean combination of subformulas using \( \land \) and \( \lor \), the truth of \( h \) at state \( s \) is also a logic combination of the truths of the component subformulas at the same state. To this end, we label the state with an ID expression \( \psi \) defined as follows:

- \( ID := 1 \ | 2 \ | \ldots \ | k + 1 \); 
- \( \psi := ID \ | \neg \psi \ | \psi \lor \psi \).

Let \( \Psi \) denote the set of all ID expressions. For each subformula \( h \), we construct a labeling (partial) function \( L_h : S \rightarrow \Psi \) to record the ID expression labeled to each state during the processing of the subformula \( h \), and the labeling function is returned when the subformula is processed.

The detailed procedure, called \( \text{ProcessCTL} \), for processing subformulas will be given in Section 4.2. After all subformulas are processed, a labeling function \( L_f \) for the outer-most subformula (i.e., \( f \) itself) is returned. The algorithm returns \text{true} \) when \( s \) is labeled with 1 by \( L_f \). It returns \text{false} \) when \( s \) is not labeled at all. In other cases, a testing procedure over \( X \) is applied to check whether the ID expression labeled in \( L_f(s) \) could be evaluated true. The procedure, called \( \text{TestWG} \), will be given in Section 4.3. In summary, the algorithm (to solve the CTL model-checking problem \( (M, X), s_0 \models f \)) is sketched as follows:

**Procedure** \( \text{CheckCTL}(M, X, s_0, f) \)

\[
L_f := \text{ProcessCTL}(M,f) \\
\text{If } s_0 \text{ is labeled by } L_f \text{ Then} \\
\text{If } L_f(s_0) = 1 \text{ Then} \text{ Return } \text{true}; \\
\text{Else} \text{ Return } \text{false};
\]

**Else** \( \text{Return } \text{TestWG}(X, \text{reset}, s_0, L_f(s_0)); \)

**Else** (i.e., \( s_0 \) is not labeled at all) \text{Return } \text{false}.

4.2 Processing a CTL formula

Processing a CTL formula \( h \) is implemented through a recursive procedure \( \text{ProcessCTL} \). Recall that any CTL formula can be expressed in terms of \( \land, \lor, \neg, EX, EU \), and \( EG \). Thus, at each intermediate step of the procedure, depending on whether the formula \( h \) is atomic or takes one of the following forms: \( g_1 \lor g_2 \), \( \neg g \), \( EX \ g \), \( EG \ g \), \( E[g_1 \ U \ g_2] \), or \( EG \ g \), the procedure has only six cases to consider and when it finishes, a labeling function \( L_h \) is returned for formula \( h \).

**Procedure** \( \text{ProcessCTL}(M, h) \)

**Case**

- \( h \) is atomic: Let \( L_h \) label every state with 1 whenever \( h \) is true on the state;
- \( h = g_1 \lor g_2 \):
  \[
  L_{g_1} := \text{ProcessCTL}(M,g_1); \\
  L_{g_2} := \text{ProcessCTL}(M,g_2); \\
  L_h := \text{HandleUnion}(L_{g_1}, L_{g_2});
  \]
- \( h = \neg g \):
  \[
  L_g := \text{ProcessCTL}(M,g); \\
  L_h := \text{HandleNegation}(M,L_g);
  \]
- \( h = EX \ g \):
  \[
  L_g := \text{ProcessCTL}(M,g);
  \]
Let $L_h := \text{HandleEX}(M, L_g)$;

$h = E [g_1 \cup g_2];$

$L_{g_1} := \text{ProcessCTL}(M, g_1);$

$L_{g_2} := \text{ProcessCTL}(M, g_2);$

$L_h := \text{HandleEU}(M, L_{g_1}, L_{g_2});$

$h = EG \, g;$

$L_g := \text{ProcessCTL}(M, g);$

$L_h := \text{HandleEG}(M, L_g);$ 

\textbf{Return} $L_h.\) 

In the above procedure, when $h = g_1 \lor g_2$, we first process $g_1$ and $g_2$ respectively by calling \text{ProcessCTL}, then construct a labeling function $L_h$ for $h$ by merging (i.e., \text{HandleUnion}, see Appendix B.2 for details)) $g_1$ and $g_2$’s labeling functions $L_{g_1}$ and $L_{g_2}$ as follows:

- For each state $s$ that is in both $L_{g_1}$’s domain and $L_{g_2}$’s domain, let $L_h$ label $s$ with 1 if either $L_{g_1}$ or $L_{g_2}$ labels $s$ with 1 and label $s$ with ID expression $L_{g_1}(s) \lor L_{g_2}(s)$ otherwise;

- For each state $s$ that is in $L_{g_1}$’s domain (resp. $L_{g_2}$’s domain) but not in $L_{g_2}$’s domain (resp. $L_{g_1}$’s domain), let $L_h$ label $s$ with $L_{g_1}(s)$ (resp. $L_{g_2}(s)$).

When $h = \neg g$, we first process $g$ by calling \text{ProcessCTL}, then construct a labeling function $L_h$ for $h$ by “negating” (i.e., \text{HandleNegation}, see Appendix B.3 for details)) $g$’s labeling function $L_g$ as follows:

- For every state $s$ that is not in the domain of $L_g$, let $L_h$ label $s$ with 1;

- For each state $s$ that is in the domain of $L_g$ but not labeled with 1 by $L_g$, let $L_h$ label $s$ with ID expression $\neg L_g(s)$.

The remaining three cases (i.e., for $EX$, $EU$, and $EG$) in the above procedure are more complicated and are handled in the following three subsections respectively.

### 4.2.1 Handling EX

When $h = EX \, g$, $g$ is processed first by \text{ProcessCTL}. Then, the procedure \text{HandleEX} is called with $g$’s labeling function $L_g$ to construct a labeling function $L_h$ and create a witness graph for $h$ (we assume that, whenever a witness graph is created, the current value of a global variable $id$, which initially is 2, is assigned as the ID number of the graph, and $id$ is incremented by 1 after it is assigned to the graph).

The labeling function $L_h$ is constructed as follows. For each state $s$ that has a successor $s'$ in the domain of $L_g$, if $s$ can reach $s'$ through an environment transition and $s'$ is labeled with 1 by $L_g$ then let $L_h$ also label $s$ with 1, otherwise let $L_h$ label $s$ with the current value of the global variable $id$.

The witness graph for $h = EX \, g$, called an $EX$ graph, is created as a triple:

$$[h] := (N, E, L_g),$$

where $N$ is a set of nodes and $E$ is a set of annotated edges. It is created as follows:

- Add one node to $N$ for each state that is in the domain of $L_g$.

- Add one node to $N$ for each state that has a successor in the domain of $L_g$.

- Add one edge between two nodes in $N$ to $E$ when $M$ has a transition between two states corresponding to the two nodes respectively; if the transition involves a communication with $X$ then annotate the edge with the communication symbols.

When \text{HandleEX} finishes, it increases the global variable $id$ by 1 (since one new witness graph has been created).

See Appendix B.4 for details.

### 4.2.2 Handling EU

The case when $h = E [g_1 \cup g_2]$ is more complicated. We first process $g_1$ and $g_2$ respectively by calling \text{ProcessCTL}, then call procedure \text{HandleEU} with $g_1$ and $g_2$’s labeling functions $L_{g_1}$ and $L_{g_2}$, to construct a labeling function $L_h$ and create a witness graph for $h$.

We construct the labeling function $L_h$ recursively. First, let $L_h$ label each state $s$ in the domain of $L_{g_2}$ with $L_{g_2}(s)$. Then, for state $s$ that has a successor $s'$ in the domain of $L_h$, if both $s$ and $s'$ is labeled with 1 by $L_{g_1}$ and $L_h$ respectively and $s$ can reach $s'$ through an environment transition then let $L_h$ also label $s$ with 1, otherwise let $L_h$ label $s$ with the current value of the global variable $id$. Notice that, in the second step, if a state $s$ can be labeled with both 1 and the current value of $id$, let $L_h$ label $s$ with 1. Thus, we can ensure that the constructed $L_h$ is indeed a function.

The witness graph for $h$, called an $EU$ graph, is created as a 4-tuple:

$$[h] := (N, E, L_{g_1}, L_{g_2}),$$

where $N$ is a set of nodes and $E$ is a set of edges. $N$ is constructed by adding one node for each state that is in the domain of $L_h$, while $E$ is constructed in the same way as that of \text{HandleEX}. When \text{HandleEU} finishes, it increases the global variable $id$ by 1.

See Appendix B.5 for details.

### 4.2.3 Handling EG

To handle formula $h = EG \, g$, we first process $g$ by calling \text{ProcessCTL}, then call procedure \text{HandleEG} with $g$’s labeling function $L_g$ to construct a labeling function $L_h$ and create a witness graph for $h$.

The labeling function $L_h$ is constructed as follows. For each state $s$ that can reach a loop $C$ through a path $p$ such that every state (including $s$) on $p$ and $C$ is in the domain of $L_g$, if every state (including $s$) on $p$ and $C$ is labeled with 1 by $L_g$ and no communications are involved on the path and the loop, then let $L_h$ also label $s$ with 1, otherwise let $L_h$ label $s$ with the current value of the global variable $id$.

The witness graph for $h$, called an $EG$ graph, is created as a triple:

$$[h] := (N, E, L_g),$$

where $N$ is a set of nodes and $E$ is a set of annotated edges. The graph is constructed in the same way as that of \text{HandleEU}. When \text{HandleEG} finishes, it also increases the global variable $id$ by 1.

See Appendix B.6 for details.

### 4.3 Testing a Witness Graph

As mentioned in Section 4.1, the procedure for CTL model-checking driven black-box testing, \text{CheckCTL}, consists of two parts. The first part, which was discussed in Section 4.2, includes \text{ProcessCTL} that processes CTL formulas and creates witness graphs. The second part is to evaluate the created witness graphs through testing $X$. We will elaborate on this second part in this section.

In processing the CTL formula $f$, a witness graph is constructed for each CTL operator in $f$ and a labeling function is constructed for each subformula of $f$. As seen from the algorithm \text{CheckCTL} (at the end of Section 4.1), the algorithm either gives a definite “yes” or “no” answer to the CTL model-checking problem, i.e.,
\((M,X), s_0 \models f\), or it reduces the problem to checking whether the ID expression \(\psi\) labeled to \(s_0\) can be evaluated true at the state. The evaluation procedure is carried out by the following recursive procedure \(\text{TestWG}\), after an input sequence \(\pi\) has been accepted by the unspecified component \(X\).

**Procedure** \(\text{TestWG}(X, \pi, s_0, \psi)\)

**Case**

\[
\psi = \psi_1 \lor \psi_2;
\]

If \(\text{TestWG}(X, \pi, s_0, \psi_1)\) Then

Return \(true\);

Else

Return \(\neg\text{TestWG}(X, \pi, s_0, \psi_2)\);

\[
\psi = \neg\psi_1;
\]

Return \(\neg\text{TestWG}(X, \pi, s_0, \psi_1)\)

\[
\psi = 1;
\]

Return \(true\);

\[
\psi = i \text{ with } 2 \leq i \leq k + 1;
\]

When \(\tilde{T}^{-1}(i)\) is an \(EX\) graph

Return \(\text{TestEX}(X, \pi, s_0, \tilde{T}^{-1}(i))\);

When \(\tilde{T}^{-1}(i)\) is an \(EU\) graph

Return \(\text{TestEU}(X, \pi, s_0, \tilde{T}^{-1}(i), level = 0)\);

When \(\tilde{T}^{-1}(i)\) is an \(EG\) graph

Return \(\text{TestEG}(X, \pi, s_0, \tilde{T}^{-1}(i))\).

In \(\text{TestWG}\), the first three cases are straightforward, which are consistent with the intended meaning of ID expressions. The cases \(\text{TestEX}, \text{TestEU}, \text{TestEG}\) for evaluating \(EX, EU, EG\) graphs are discussed in the following three subsections.

4.3.1 TestEX

The case for checking whether an \(EX\) graph \(G = (N, E, L_0)\) can be evaluated true at a state \(s_0\) is simple. We just test whether the system \(M\) can reach from \(s_0\) to another state \(s' \in \text{dom}(L_0)\) through a transition in \(G\) such that the ID expression \(L_0(s')\) can be evaluated true at \(s'\).

See Appendix B.7 for details.

4.3.2 TestEU

To check whether an \(EU\) graph \(G = (N, E, L_{01}, L_{02})\) can be evaluated true at a state \(s_0\), we need to traverse all paths \(p\) in \(G\) with length less than \(mn\) and test the unspecified component \(X\) to see whether the system can reach some state \(s' \in \text{dom}(L_{02})\) through one of those paths. In here, \(m\) is an upper bound for the number of states in the unspecified component \(X\) and \(n\) is the maximal number of communications on all simple paths between \(s_0\) and \(s'\).

In the meantime, we should also check whether \(L_{02}(s')\) can be evaluated true at \(s'\) and whether \(L_{01}(s_1)\) can be evaluated true at \(s_1\) for each \(s_1\) on \(p\) (excluding \(s'\)) by calling \(\text{TestWG}\).

See Appendix B.8 for details.

4.3.3 TestEG

For the case to check whether an \(EG\) graph \(G = (N, E, L_0)\) can be evaluated true at a state \(s_0\), we need to find an infinite path in \(G\) along which the system can run forever.

The following procedure \(\text{TestEG}\) first decomposes \(G\) into a set of SCCs. Then, for each state \(s_f\) in the SCCs, it calls another procedure \(\text{SubTestEG}\) to test whether the system can reach \(s_f\) from \(s_0\) along a path not longer than \(mn\), as well as whether the system can further reach \(s_f\) from \(s_f\) for \(m - 1\) times. The basic idea of \(\text{SubTestEG}\) (see Appendix B.9 for details) is similar to that of the TestLiveness algorithm in Section 4.3 except that we need also check whether \(L_0(s_i)\) can be evaluated true at \(s_i\) for each state \(s_i\) that has been reached so far by calling \(\text{TestEG}\). Here, \(m\) is the same as before while \(n\) is the maximal number of communications on all simple paths between \(s_0\) and \(s_f\).

**Procedure** \(\text{TestEG}(X, \pi, s_0, G = (N, E, L_0))\)

\[
\text{SCC} := \{C|C \text{ is a nontrivial SCC of } G\};
\]

\[
T := \bigcup_{C \in \text{SCC}} \{s | s \in C\};
\]

For each \(s \in T\) Do

Experiment(\(X, \text{reset}\pi\));

If \(\text{SubTestEG}(X, \pi, s_0, s, G, level = 0, count = 0)\); Return \(true\);

Return \(false\).

In summary, to solve the CTL model-checking problem

\((M,X), s_0 \models f\)

our algorithm CheckCTL in Section 4.1 either gives a definite yes/no answer or gives a sufficient and necessary condition in the form of ID expressions and witness graphs. The condition is evaluated through black-box testing over the unspecified component \(X\). The evaluation process will terminate with a yes/no answer to the model-checking problem. One can show that our algorithm is both complete and sound.

5. EXAMPLES

In this section, to better understand our algorithms, we look at some examples.

![Diagram](image1)

**Figure 1**

Consider a system \(\text{Sys} = (M, X)\) where \(M\) keeps receiving messages from the outside environment and then transmits the message through the unspecified component \(X\). The only event symbol in \(M\) is \(\text{msg}\), while \(X\) has two input symbols \(\text{send}\) and \(\text{ack}\), and two output symbols \(\text{yes}\) and \(\text{no}\). The transition graph of \(M\) is depicted in Figure 1 where we use a suffix ? to denote events from the outside environment (e.g., \(\text{msg}\)), and use an infix / to denote communications of \(M\) with \(X\) (e.g., \(\text{send/yes}\)).

Assume that we want to solve the following LTL model-checking problem

\((M, X), s_0 \models \text{EG} s_2\)

i.e., starting from the initial state \(s_0\), the system can reach state \(s_1\) infinitely often. Applying our liveness analysis algorithms, we can obtain the (minimized) communication graph in Figure 2.

![Diagram](image2)

**Figure 2**

From this graph and our liveness testing algorithms, the system satisfies the liveness property if the communication trace

\(\text{send yes}(\text{send yes ack yes})^{m-1}\)

\(\text{1}\)The transition graphs in the figures in this section are not made total for the sake of readability.
is a run of $X$, where $m$ is an upper bound for number of states in $X$. Now, we slightly modified the transition graph of $M$ into Figure 3 such that when a send fails, the system shall return to the initial state.

**Figure 3**

For this modified system, its (minimized) communication graph with respect to the liveness property would be as shown in Figure 4.

**Figure 4**

From Figure 4 and the liveness testing algorithms, the system satisfies the liveness property iff there exist $0 \leq k_1, k_2 \leq 2m$ such that the communication trace

$$(send\ no)^{k_1}send\ yes((send\ yes\ ack\ yes)(send\ no)^{k_2})^{m-1}$$

is a run of $X$.

Still consider the system in Figure 3, but we want to solve a CTL model-checking problem $(M, X), s_0 = AFs_3$; i.e., along all paths from $s_0$, the system can reach state $s_3$ eventually. The problem is equivalent to

$$(M, X), s_0 = \neg EG\neg s_2.$$ Applying our CTL algorithms to formula $h = EG\neg s_2$, we construct an $EG$ witness graph $G = (N, E, L_{true})$ whose ID number is 2 and a labeling function $L_h$, where $L_{true}$ labels all three states $s_0, s_1, s_2$ with ID expression 1 (as defined in Section 4.3) which stands for $true$, and $L_h$ labels all three states $s_0, s_1, s_2$ with 2. The graph $G$ is depicted in Figure 5. From this graph as well as $L_h$, the algorithms conclude that the model-checking problem is true iff the communication trace $(send\ no)^{m-1}$ is not a run of $X$.

**Figure 5**

Now we modify the system in Figure 1 into a more complicated one shown in Figure 6. For this system, we want to check

$$(M, X), s_0 = \neg E[\neg s_2U s_3],$$
i.e., starting from the initial state $s_0$, the system should never reach state $s_3$ earlier than it reaches $s_2$. Applying our CTL algorithms to formula

$$h = E[\neg s_2U s_3],$$
we obtain an $EU$ witness graph $G = (N, E, L_1, L_2)$ whose ID number is 2 and a labeling function $L_h$, where $L_1$ labels all four states $s_0, s_1, s_2,$ and $s_4$ with 1, $L_2$ just labels $s_3$ with 1, and $L_h$ labels states $s_0, s_1, s_2$, and $s_4$ with 2, and labels $s_3$ with 1. The graph $G$ is depicted in Figure 7. From this graph as well as $L_h$, the algorithms conclude that the model-checking problem is true iff none of communication traces in the form of $send\ no(ack\ yes\ send\ no)^*$ and with length less than $3m$ is a run of $X$.

**Figure 6**

**Figure 7**

For the same system, we could consider more complicated temporal properties as follows:

- $(M, X) \models AG(s_2 \rightarrow AF s_3)$; i.e., starting from the initial state $s_0$, whenever the system reaches $s_2$, it would eventually reach $s_3$.

- $(M, X), s_0 = AG(s_2 \rightarrow AXA[\neg s_2U s_3])$; i.e., starting from the initial state $s_0$, whenever it reaches state $s_2$, the system should never reach $s_2$ again until it reaches $s_3$.

We do not include the witness graphs and labeling functions for these two cases in this extended abstract. Nevertheless, it can be concluded that the two problems are true iff no communication traces with two consecutive symbol pairs $(send\ yes)$ can be runs of $X$.

See Appendix C.1 and Appendix C.2 for details about the above two examples.

6. RELATED WORK

The quality assurance problem for component-based software has attracted lots of attention in the software engineering community, as witnessed by recent publications in conferences like ICSE and FSE. However, most of the work is based on the traditional testing techniques and considers the problem from the viewpoint of component developers; i.e., how to ensure the quality of components before they are released.

Voas [37, 38] proposed a component certification strategy with the establishment of independent certification laboratories performing extensive testing of components and then publishing the results. Technically, this approach would not provide much improvement for solving the problem, since independent certification laboratories can not ensure the sufficiency of their testing either, and a testing-based technique alone is not enough to a reliable software component. Some researchers [4] [28] suggested an approach to
augment a component with additional information to increase the customer’s understanding and analyzing capability of the component behavior. A related approach \([22]\) is to automatically extract a finite-state machine model from the interface of a software component, which is delivered along with the component. This approach can provide some convenience for customers to test the component, but again, how much a customer should test is still a big problem. To address the issue of testing adequacy, Rosenblum defined in \([22]\) a conceptual basis for testing component-based software, by introducing two notions of \(C\)-adequate-for-\(P\) and \(C\)-adequate-for-\(M\) (with respect to certain adequacy criteria) for adequate unit testing of a component and adequate integration testing for a component-based system, respectively. But this is still a purely testing-based strategy. In practice, how to establish the adequacy criteria is an unclear issue.

Recently, Bertolino et al. \([8]\) recognized the importance of testing a software component in its deployment environment. They developed a framework that supports functional testing of a software component with respect to customer’s specification, which also provides a simple way to encode with a component the developer’s test suites which can be re-executed by the customer. Yet their approach requires the customer to have a complete specification about the component to be incorporated into a system, which is not always possible. McCamant and Ernst \([27]\) considered the issue of predicting the safety of dynamic component upgrade, which is part of the problem we consider. But their approach is completely different since they try to generate some abstract operational expectation about the new component through observing a system’s run-time behavior with the old component.

In the formal verification area, there has been a long history of research on verification of systems with modular structure. \([31]\) A key idea \([23]\) in modular verification is the assume-guarantee paradigm: a module should guarantee to have the desired behavior once the environment with which the module is interacting has the assumed behavior. There have been a variety of implementations for this idea (see, e.g. \([2]\)). However, the assume-guarantee idea does not immediately fit with our problem setup since it requires that users must have clear assumptions about a module’s environment.

In the past decade, there has also been some research on combining model-checking and testing techniques for system verification, which can be classified into a broader class of techniques called specification-based testing. But most of the work only utilizes model-checkers’ ability of generating counter-examples from a system’s specification to produce test cases against an implementation \([8]\) \([19]\) \([15]\) \([5]\) \([6]\) \([3]\).

Peled et al. \([30]\) \([17]\) \([29]\) studied the issue of checking a black-box against a temporal property (called black-box checking). But their focus is on how to efficiently establish an abstract model of the black-box through black-box testing, and their approach requires a clearly-defined property (LTL formula) about the black-box, which is not always possible in component-based systems. Kupferman and Vardi \([22]\) investigated module checking by considering the problem of checking an open finite-state system under all possible environments. Module checking is different from the problem in \((*)\) mentioned at the beginning of the paper in the sense that a component understood as an environment in \([22]\) is a specific one. Fisler et al. \([14]\) \([24]\) proposed an idea of deducing a model-checking condition for extension features from the base feature, which is adopted to study model-checking feature-oriented software designs. Their approach relies totally on model-checking techniques; their algorithms have false negatives and do not handle LTL formulas.

7. DISCUSSIONS

In this paper, we present algorithms for LTL and CTL model-checking driven black-box testing. The algorithms create communication graphs and witness graphs, on which a bounded and nested depth-first search procedure is employed to run black-box testing over the unspecified component. Our algorithms are both sound and complete. Though we do not have an exact complexity analysis result, our preliminary studies show that, in the liveness testing algorithm for LTL, the maximal length of test-cases fed into the unspecified component \(X\) is bounded by \(O(n \cdot m^2)\). For CTL, the length is bounded by \(O(k \cdot n \cdot m^2)\). In here, \(k\) is the number of CTL operators in the formula to be verified, \(n\) is the state number in the host system, and \(m\) is the state number in the component.

The next natural step is to implement the algorithms and see how well they work in practice. In the implementation, there are further issues to be addressed.

7.1 Practical Efficiency

Similar to the traditional black-box testing algorithms to check conformance between Mealy machines, the theoretical (worst-case) complexities are high in order to achieve complete coverage. However, worst-cases do not always occur in a practical system. In particular, we need to identify scenarios that our algorithms can be made more efficient. For instance, using existing ideas of abstraction \([12]\), we might obtain a smaller but equivalent model of the host system before running the algorithms. We might also, using additional partial information about the component, to derive a smaller state number for the component and to find ways to expedite the model-checking process. Notice that the number is actually the state number for a minimal automaton that has the same set input/output sequences as the component. Additionally, in the implementation, we also need a database to record the test results that have been performed so far (so repeated testing can be avoided). Algorithms are needed to make use of the test results to aggressively trim the communication/witness graphs such that less test-cases are performed but the complete coverage is still achieved. Also, we will study algorithms to minimize communication/witness graphs such that duplicate test-cases are avoided. Lastly, it is also desirable to modify our algorithms such that the communication/witness graphs are generated with the process of generating test-cases and performing black-box testing over the unspecified component \(X\). In this way, a dynamic algorithm could be designed to trim the graphs on-the-fly.

7.2 Coverage Metrics

Sometimes, a complete coverage will not be achieved when running the algorithms on a specific application system. In this case, a coverage metric is needed to tell how much the test-cases that have run so far cover. The metric will give a user some confidence on the partial model-checking results. Furthermore, such a metric would be useful in designing conservative algorithms to debug/verify the temporal specifications that sacrifice the complete coverage but still bring the user reasonable confidence.

7.3 More Complex System Models

The algorithms can be generalized to systems containing multiple unspecified components. Additionally, we will also consider cases when these components interacts between each other, as well as cases when the host system communicates with the components asynchronously. Obviously, when the unspecified component (as well as the host system) has an infinite-state space, both the traditional model-checking techniques and black-box techniques are not applicable. One issue with infinite-state systems is that, the in-
ternal structure of a general infinite-state system cannot be learned through the testing method. Another issue is that model-checking a general infinite-state system is an undecidable problem. It is desirable to consider some restricted classes of infinite-state systems (such as real-time systems modeled as timed automata) where our algorithms generalize. This is interesting, since through the study we may provide an algorithm for model-checking driven black-box testing for a real-time system that contains an (untimed) unspecified component. Since the algorithm will generate testcases for the component, real-time integration testing over the composed system is avoided.

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APPENDIX

A. DEFINITIONS

B. ALGORITHMS

B.1 Liveness Analysis

Procedure CheckIO(⟨M, X⟩, s₀, s₁)

N := Φ; E := Φ;
If ⟨s₀, s₁⟩ ∈ Rₑv ∧ ⟨s₁, s₁⟩ ∈ Rₑv Then
  Return “Yes”;
Else if ⟨s₀, s₁⟩ ∉ Rₑv ∧ ⟨s₁, s₁⟩ ∉ Rₑv Then
  Return “No”;
End if
N := {⟨s₀, s₁⟩ ∈ Rₑv ∨ Rₑv := TransitiveClosure(Rₑv);
Rₑv := TransitiveClosure(Rₑv);
Integer id := 1;
End procedure

B.2 Union of Labeling Functions

Procedure Union(L₁, L₂)
L := Φ;
For each s ∈ dom(L₁) ∪ dom(L₂) Do
  If s ∈ dom(L₁) ∩ dom(L₂) Then
    If L₁(s) = 1 ∧ L₂(s) = 1 Then
      L := L ∪ {(s, 1)};
    Else
      L := L ∪ {(s, L₁(s) ∨ L₂(s))};
    End if
  Else if s ∈ dom(L₁) Then
    L := L ∪ {(s, L₁(s))};
  Else
    L := L ∪ {(s, L₂(s))};
  End if
End for
Return L;
End procedure

B.3 Negation of a Labeling Function

Procedure Negation(M, L₁)
L := Φ;
For each s ∈ S Do
  If s ∉ dom(L₁) Then
    L := L ∪ {(s, 1)};
  Else if f(s) ≠ 1 Then
    L := L ∪ {(s, ¬L₁(s))};
  End if
End if
Return L;
End procedure

B.4 Checking an EX Subformula

Procedure HandleEX(M, L₁)
N := dom(L₁); L := Φ;
For each t ∈ dom(L₁) Do
  For each s : Rₑv(s, t) Do
    N := N ∪ {s}
    If L₁(t) = 1 ∧ Rₑv(s, t) Then
      If s ∉ dom(L) Then
        L := L ∪ {(s, 1)};
      Else if L(s) ≠ 1 Then
        L := L |ₚ ;
      End if
    Else if s ∉ dom(L) Then
      L := L ∪ {(s, id)};
    End for
  End for
End procedure

B.5 Checking an EU Subformula

Procedure HandleEU(M, L₁, L₂)
L := L₂;
T₃ := dom(L₁); T₂ := dom(L);
While T₂ ≠ Φ Do
  Choose t ∈ T₂; T₂ := T₂ \ {t};
  For each s ∈ T₂ ∧ Rₑv(s, t) Do
    If L₁(s) = 1 ∧ L(t) = 1 ∧ Rₑv(s, t) Then
      If s ∉ dom(L) Then
        T₂ := T₂ ∪ {s}; L := L ∪ {(s, 1)};
      Else if L(s) ≠ 1 Then
        T₂ := T₂ ∪ {s}; L := L |ₚ ;
      End if
    Else if s ∉ dom(L) Then
      T₂ := T₂ ∪ {s}; L := L ∪ {(s, id)};
    End if
  End for
End while
N := dom(L);
E := {⟨s, s’⟩|s, s’ ∈ N ∧ ∃a : (s, a, s’ ∈ Rₑv
∪ {⟨s, a⟩|s, s’ ∈ N : (s, a, s’ ∈ Rₑv
|∃a : (s, a, s’ ∈ Rₑv
∪ {⟨s, a⟩|s, s’ ∈ N : (s, a, s’ ∈ Rₑv
Associate id with G = ⟨N, E, L₁⟩; id := id + 1;
Return L;
End procedure

B.6 Checking an EG Subformula

Procedure HandleEG(⟨X, π, s₀, G = ⟨N, E, L₀⟩)
SＣCₑv := {C} C is a nontrivial SCC of M and C contains 
no communication transitions ;
SＣCₑv := {C} C is a nontrivial SCC of M and C contains 
some communication transitions ;
L := {⟨(s, 1)|∃C ∈ SCCₑv : s ∈ C}
∪{(s, id)|∃C ∈ SCCₑv : s ∈ C}
T := dom(L);
While $T \neq \emptyset$ Do
Choose $t \in T$; $T := T \setminus \{t\}$
For each $s \in \text{dom}(L_1) \land R^t(s, t)$ Do
If $L(t) = 1 \land L_1(s) = 1 \land R^s_{env}(s, t)$ Then
If $s \notin \text{dom}(L)$ Then
$T := T \cup \{s\}; \quad L := L \cup \{(s, 1)\}$;
Else if $L(s) \neq 1$ Then
$T := T \cup \{s\}; \quad L := L|_{s-1}$;
End if
Else if $s \notin \text{dom}(L)$ Then
$T := T \cup \{s\}; \quad L := L \cup \{(s, id)\}$;
End if
End for
End While

Inputs $N := \text{dom}(L); \quad E := \{(s, s') | s, s' \in N \land \exists a : (s, a, s') \in R_{env}\} \cup \{(s, \alpha \beta, s') | s, s' \in N \land (s, \alpha, \beta, s') \in R_{comm}\}$; Associate id with $G = (N, E, L_1); \quad id := id + 1$;
Return $L$;
End procedure

B.7 Testing an EX Graph

The algorithm for testing an $EX$ graph is simple. It first checks whether $L_1(s')$ can be evaluated true at any state $s'$ such that the system can reach $s'$ from $s_0$ and reach a destination state. It returns true if it is the case. Otherwise, it chooses one candidate from the set of all possible input symbols from $s_0$, and checks the sequence $\pi$ concatenated with the input symbol to $X$. If the output symbol of $X$ and the input symbol matches the annotation of an edge originating from the node, it moves forward to try the destination node of the edge with $level$ increased by 1. If there is no match, then it tries other candidates. But before trying any other candidate, it brings $X$ to its initial state by sending it the special input symbol, reset. The algorithm returns false when all candidates are tried without a match.

Procedure TestEX($X, \pi, s_0, G = (N, E, L_1, level)$)
For each $(s_0, s') \in E : s' \in \text{dom}(L_1)$ Do
Experiment($X, reset\pi$);
If TestWG($X, \pi, s', L_1(s')$) Then
Return true;
End if
End for

Inputs := $\{\alpha | (s_0, \alpha \beta, s') \in E\}$;
For each $\alpha \in \text{Inputs}$ Do
Experiment($X, reset\pi$);
$\beta := \text{Experiment}(X, \alpha)$;
If $\exists s' : (s_0, \alpha \beta, s') \in E$ Then
If TestWG($X, \pi \alpha, s', L_1(s')$) Then
Return true;
End if
End if
End if
End each:
Return false;
End procedure

B.8 Testing an EU Graph

The procedure TestEU keeps a sequence of input symbols $\pi$ that has been successfully accepted by $X$ and an integer $level$ that records how many communications have been gone through without reaching a destination state. And the algorithm works as follows. At first, it checks whether it has gone through more than $mn$ communications without success, it returns false if it is the case. Then, it checks whether it has reached a destination state (i.e., $s_0 \in \text{dom}(L_2)$). If it is the case, it returns true when $L_2(s_0)$ can be evaluated true $s_0$. Next, it checks whether $L_1(s)$ can be evaluated true at $s_0$, it returns false if it is not the case. After that, it checks whether $L_1(s')$ can be evaluated true at any state $s'$ such that the system can reach $s'$ from $s_0$ through an environment transition. It returns true if it is the case. Otherwise, it chooses one candidate from the set of all possible input symbols from $s_0$, and feeds the sequence $\pi$ concatenated with the input symbol to $X$. If the output symbol of $X$ and the input symbol matches the annotation of an edge originating from the node, it moves forward to try the destination node of the edge with $level$ increased by 1. If there is no match, then it tries other candidates. But before trying any other candidate, it brings $X$ to its initial state by sending it the special input symbol, reset. The algorithm returns false when all candidates are tried without a match.

Procedure TestEU($X, \pi, s_0, G = (N, E, L_1, L_2, level)$)
If $level > mn$ Then$^2$
Return false;
Else if $s_0 \in \text{dom}(L_2)$ Then
If TestWG($X, \pi, s_0, L_2(s_0)$) Then
Return true;
End if
Else if not TestWG($X, \pi, s_0, L_1(s_0)$) Then
Return false;
End if
For each $(s, \alpha \beta, s') \in E$ Do
Experiment($X, reset\pi$);
If TestEU($X, \pi \alpha, s', G, level$) Then
Return true;
End if
End for
Return false;
End each:
End if
End if
End for:

B.9 Subroutine for Testing an EG Graph

The procedure SubTestEG keeps a sequence of input symbols that has been successfully accepted by $X$, an integer $level$ that records how many communications have been gone through without reaching $s_f$, and an integer $count$ that indicates how many times $s_f$ has been reached. It first checks whether it has gone through more than $mn$ communications without reaching $s_f$, it returns false if it is the case. Then, it checks whether it has reached the given state $s_f$. If it is the case, it returns true when it has already reached $s_f$ for $m$ times, it increases $count$ by 1 and resets $level$ to 0 otherwise. The next, it tests whether $L_1(s_0)$ can be evaluated true at $s_0$, and it returns false if it is not the case. After that it checks whether $L_1(s')$ can be evaluated true at any state $s'$ such that the system can reach $s'$ from $s_0$ through an environment transition. It returns true if it is the case. Otherwise, it chooses one candidate from the set of all possible input symbols from $s_0$, and feeds the sequence $\pi$ concatenated with the input symbol to $X$.

$^2$Here, $n$ always denotes the maximal number of communications on any simple paths in $G$. 
If there is no match, it tries other candidates. But before trying any other candidate, it brings X to its initial state by sending it the special input symbol reset. The algorithm returns false when all candidates are tried without a match.

**Procedure**

\[
\text{SubTestEG}(X, \pi, s_0, s_f, G) = \langle N, E, L_1, \text{level}, \text{count} \rangle
\]

- If \text{level} > \text{mn} Then 2
  - Return false;
- Else if \text{s} = \text{sf} Then
  - If \text{count} \geq \text{m} Then
    - Return true;
  - Else
    - \text{count} := \text{count} + 1; \text{level} := 0;
  - End if
- Else if not \text{TestWG}(X, \pi, s_0, L_1(s_0)) Then
  - Return false;
- End if

For \exists (s_0, s') \in E Do

- \text{Experiment}(X, \text{reset} \pi);
- If \text{SubTestEG}(X, \pi, s', s_f, G, \text{level}, \text{count}) Then
  - Return true;
- End if

End if

**Inputs** := \{ \alpha |(s_0, \alpha \beta, s') \in E \};

For each \alpha \in \text{Inputs} Do

- \text{Experiment}(X, \text{reset} \pi);
- \beta := \text{Experiment}(X, \alpha);
- If \exists s' : (s_0, \alpha \beta, s') \in E Then
  - If \text{SubTestEG}(X, \pi \alpha, s', s_f, G, \text{level} + 1, \text{count}) Then
    - Return true;
  - End if
- End if

End for

End if

End procedure

**C. EXAMPLES**

**C.1 Check** \((M, X) \models AG(s_2 \rightarrow AF s_3)\)

To check whether \((M, X) \models AG(s_2 \rightarrow AF s_3)\), is equivalent to checking whether

\((M, X) \models \neg E[true U(s_2 \land EG \neg s_3)]\).

We describe how the formula

\[ f = E[true U(s_2 \land TG \neg s_3)] \]

is processed by HandleCTL from bottom to up as follows.

1. the atomic subformula s_2 is processed by HandleCTL, and
   a labeling function \(L_1 = \{(s_2, 1)\}\) is returned;
2. the atomic subformula s_3 is processed, and a labeling function \(L_2 = \{(s_3, 1)\}\) is returned;
3. to process \neg s_3, HandleNegation is called with \(L_2\) to return a labeling function \(L_3 = \{(s_0, 1), (s_1, 1), (s_2, 1), (s_4, 1)\}\);
4. to process EG\neg s_3, HandleEG is called with \(L_3\) to construct an EG graph \(G_1 = \langle N, E, L_2\rangle\) with id 2 (see Figure 8) and return a labeling function \(L_4 = \{(s_0, 2), (s_1, 2), (s_2, 2)\}\);
5. to process \(s_2 \land EG \neg s_3\), HandleNegation and HandleUnion are called with \(L_1\) and \(L_4\) to return a labeling function \(L_5 = \{(s_2, 2)\}\);
6. to process \(E[true U(s_2 \land TG \neg s_3)]\), HandleEU is called with \(L_5\) to construct an EU graph \(G_2 = \langle N, E, L_3\rangle\) with id 3 (see Figure 9) and return a labeling function \(L_6 = \{(s_0, 3), (s_1, 3), (s_2, 3), (s_3, 3), (s_4, 3)\}\).

Since \(s_0\) is labeled by \(L_f\) with an ID expression 3 instead of 1 (i.e., true), we need to test whether the ID expression 3 can be evaluated true at \(s_0\) by calling TestWG with \(s_0\) and \(G_2\). It's easy to see that, essentially TestWG would be testing whether some communication trace (with bounded length) with two consecutive symbol pairs (send yes) is a run of \(X\). It returns false if such trace exists, or vice versa.

**C.2 Check** \((M, X), s_0 \models AG(s_2 \rightarrow AXA[\neg s_2 U s_3])\)

To check whether \((M, X), s_0 \models AG(s_2 \rightarrow AXA[\neg s_2 U s_3])\), is equivalent to checking whether

\((M, X) \models \neg E[true U(s_2 \land EX(E[\neg s_3 U(s_2 \land \neg s_3)] \lor EG \neg s_3)])\).

We describe how the formula

\[ f = E[true U(s_2 \land EX(E[\neg s_3 U(s_2 \land \neg s_3)] \lor EG \neg s_3))] \]

is processed by HandleCTL from bottom to up as follows.

1. the atomic subformula s_2 is processed by HandleCTL, and
   a labeling function \(L_1 = \{(s_2, 1)\}\) is returned;
2. the atomic subformula s_3 is processed, and a labeling function \(L_2 = \{(s_3, 1)\}\) is returned;
3. to process \neg s_3, HandleNegation is called with \(L_2\) to return a labeling function \(L_3 = \{(s_0, 1), (s_1, 1), (s_2, 1), (s_4, 1)\}\);
4. to process \(s_2 \land \neg s_3\), HandleNegation and HandleUnion are called with \(L_1\) and \(L_4\) to return a labeling function \(L_4 = \{(s_2, 1)\}\);
5. to process \(E[\neg s_3 U(s_2 \land \neg s_3)]\), HandleEU is called with \(L_3\) and \(L_4\) to construct an EU graph \(G_1 = \langle N, E, L_3\rangle\) with id 2 (see Figure 10) and return a labeling function \(L_5 = \{(s_0, 2), (s_1, 2), (s_2, 1)\}\).
6. to process $EG \neg s_3$, HandleEG is called with $L_3$ to construct an $EG$ graph $G_2 = \langle N, E, L_3 \rangle$ with id 3 (see Figure 11) and return a labeling function $L_6 = \{(s_0, 3), (s_1, 3), (s_2, 3)\};$

7. to process $E[\neg s_3 U (s_2 \land \neg s_3)] \lor EG \neg s_3$, HandleUnion is called with $L_5$ and $L_6$ to return a labeling function $L_7 = \{(s_0, 2 \lor 3), (s_1, 2 \lor 3), (s_2, 1)\};$

8. to process $EX(E[\neg s_3 U (s_2 \land \neg s_3)] \lor EG \neg s_3)$, HandleEX is called with $L_7$ to construct an $EX$ graph $G_3 = \langle N, E, L_7 \rangle$ with id 4 (see Figure 12) and return a labeling function $L_8 = \{(s_0, 4), (s_1, 1), (s_2, 4), (s_3, 4)\};$

9. to process $s_2 \land EX(E[\neg s_3 U (s_2 \land \neg s_3)] \lor EG \neg s_3)$, HandleNegation and HandleUnion are called with $L_1$ and $L_8$ to return a labeling function $L_9 = \{(s_2, 4)\};$

10. to process $E[true U (s_2 \land EX(E[\neg s_3 U (s_2 \land \neg s_3)] \lor EG \neg s_3))]$, HandleEU is called with $L_9$ to construct an $EU$ graph $G_4 = \langle N, E, L_9 \rangle$ with id 5 (see Figure 13) and return a labeling function $L_f = \{(s_0, 5), (s_1, 5), (s_2, 5), (s_3, 5), (s_4, 5)\}.$

Since $s_0$ is labeled by $L_f$ with an ID expression 5 instead of 1 (i.e., true), we need to test whether the ID expression 5 can be evaluated true at $s_0$ by calling TestWG with $s_0$ and $G_4$. It’s easy to see that, essentially TestWG would be testing whether some communication trace (with bounded length) with two consecutive symbol pairs (send yes) is a run of $X$. It returns false if such trace exists, or vice versa.