Research on Control Strategy of Three-Phase Inverter Based on Fractional Calculus

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Abstract: Three-level inverters has widely used in Medium pressure due to its advantages of low total harmonics, low voltage resistance of power switch tubes and low switching losses. In order to further improve the output voltage waveform quality and optimal control effect of the Three-level inverter, this paper introduces fractional calculus in the control method of the three-level inverter circuit. First, designed a control strategy for the fractional voltage outer loop. Secondly, the use of vector representation simplifies the parameter tuning of fractional-order controller, the optimization process of the parameters of the fractional order PI and integer-order PI is completed by utilizing the ITAE rules. At the same time, the Oustaloup approximation method realizes the fractional integral operator in the controller, thereby achieving the purpose of improving the output voltage of the system. For three-level inverter circuit model, comparative analysis of the control effects of the fractional-order PI and integer-order PI. Finally, simulation results show that fractional-order PI controllers have excellent flexibility, robustness.

1. Introduction
In a multilevel inverter topology [1], the three-level inverter has a simple topology and meets the performance requirements of medium and high voltage large-capacity applications. Therefore, the object of this study is the midpoint clamp Bit-type three-level inverter [2-3]. A large number of practical engineering experiences have fully verified that the traditional control strategy has obtained the best control effect. Because the design goal of the traditional controller is mainly an integer-order PID controller designed for the controlled object described by the integer-order PID mathematical model. The research on control theory of fractional calculus has attracted extensive attention of researchers [4]. Researchers also found that the actual system or nonlinear system described by the fractional calculus equation has a clearer physical meaning and more accurate physical characteristics; the parameter tuning method based on the fractional order PI\(^\frac{\alpha}{\beta}\) controller has three tuning parameters to make the system control effect better [5]. Therefore, the theoretical and application research of the fractional-order PI\(^\frac{\alpha}{\beta}\) urgently need to further deepened and perfected [6]. Literature [7] proposes to apply fuzzy control to the adaptive adjustment of fractional order parameters, and its control strategy is applied to the twelve-pulse rectifier circuit, and proves that the fractional-order controller PI\(^\frac{\alpha}{\beta}\) have better results in the process of nonlinear systems. Literature [8] applied fractional-order controllers in the power electronics field of single-phase voltage source inverters, designed the fractional order controller with ideal transfer function, and obtained Fractional integral operator by Oustaloup approximation method. Reference [9] applied the fractional-order controller to the current closed loop of the three-level inverter. Therefore, the application of fractional order PI\(^\frac{\alpha}{\beta}\) controllers on NPC three-phase three-level inverters has very
important research significance. This article focuses on three-level inverters, it uses vector notation to set fractional-order PI\(^\alpha\)D\(^\beta\) parameters and integer-order PI control parameters to obtain the optimal value of the fractional order PI\(^\alpha\)D\(^\beta\) control parameters. Finally, simulation research shows that the fractional-order PI\(^\alpha\)D\(^\beta\) control strategy has strong robustness and good dynamic performance.

2. Principle of three-level inverter

The three-level inverter is shown in Figure 1, where \(V_{in}\) is DC side input power, \(C_1, C_2\) is voltage balancing capacitor on the DC side, and they each bear 1/2 of the input voltage, point O is neutral point of inverter output; the main circuit topology of the three-level inverter is composed of 12 power switch tubes. Each bridge arm contains 4 power Switch tube and 2 midpoint clamp diodes; \(L\) is the filter inductance on the output AC side; \(C\) is the filter capacitor on the output AC side; \(r\) is the equivalent resistance of line impedance and filter inductance loss; \(R\) is the resistive load on the AC side; \(V_A, V_B,\) and \(V_C\) is AC the three-phase output phase voltage on the side.

Let \(S_a, S_b, S_c\) be switching function of the bridge arm in the NPC three-level inverter shown in equation (1).

\[
S_{abc} = \begin{cases} 
1 & \text{Switch on upper bridge arm} \\
0 & \text{Bridge arm middle switch tube conduction} \\
-1 & \text{Lower bridge arm switch on} 
\end{cases} \tag{1}
\]

Therefore, output voltage is:

\[
\begin{bmatrix} 
U_A \\
U_B \\
U_C 
\end{bmatrix} = \begin{bmatrix} 
S_a \\
S_b \\
S_c 
\end{bmatrix} \times \frac{1}{2} V_{in} \tag{2}
\]

According to the topology of the three-level inverter circuit shown in Figure 1, the Kirchhoff current and voltage laws can be obtained respectively:

\[
\begin{align*}
C \frac{dV_A}{dt} &= I_a - i_a \\
C \frac{dV_B}{dt} &= I_b - i_b \\
C \frac{dV_C}{dt} &= I_c - i_c 
\end{align*} \tag{3}
\]
Where: $I_a$, $I_b$, and $I_c$ are Filtered inductor current value at the output side of the inverter; $i_a$, $i_b$, and $i_c$ are load current values; $V_A$, $V_B$, $V_C$ is filter capacitor voltage value; $U_A$, $U_B$, and $U_C$ are output voltage value of the inverter. The Clarke transformation "equivalent" $dq$ coordinate transformation matrix is:

$$M_{abc/\alpha\beta} = \frac{2}{3} \begin{bmatrix} 1 & -\sqrt{3}/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$

(5)

Substitute equation (5) into equation (3) and equation (4), we know:

$$\begin{align*}
& \frac{dI_a}{dt} + rl_a = U_A - V_a \\
& \frac{dI_b}{dt} + rl_b = U_B - V_b \\
& \frac{dI_c}{dt} + rl_c = U_C - V_c
\end{align*}$$

(4)

After Park transformation, there is no coupling relationship between the state variables in the mathematical model, but the output of the transformation is still AC, which is not convenient for the design of the controller. Therefore, the two-phase static $\alpha\beta$ coordinate coefficient model needs to be changed to a system through Park transformation. The "equivalent" $dq$ coordinate transformation matrix of the Park transformation is:

$$M_{\alpha\beta/dq} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

(7)

Where $\theta$ is the angle the reference voltage vector has turned. Take equation (7) into equation (6), we can see:

$$\begin{align*}
& \frac{dV_d}{dt} = I_d - i_q + \omega CV_q \\
& \frac{dV_q}{dt} = I_q - i_q - \omega CV_d \\
& \frac{dI_d}{dt} = U_d - V_d - rl_d + \omega LI_q \\
& \frac{dI_q}{dt} = U_q - V_q - rl_q - \omega LI_d
\end{align*}$$

(8)

After Park conversion, the AC output from the three-level inverter is converted into DC in the two-phase rotating $dq$ coordinate. It can be seen from equation (8) that two-phase rotation the mathematical model in the coordinate system is coupled, so when the controller is added, it needs to be decoupled.
3. Fractional Calculus

Grünewald and Letnikov from the original concept of differential quotient limit definition differentiation, the definition of G-L fractional calculus is obtained:

\[ D_t^{-\alpha}f(t) = \lim_{h \to 0^+} \sum_{j=0}^{[\alpha]} \frac{\Gamma(\alpha+j)}{j!\Gamma(\alpha)} f(t-jh) \]  

(\ref{eq9})

This paper mainly focuses on the design of fractional-order control of the voltage outer loop.

\[ G_{ov}(s) = \frac{k_{ov}s + k_{vi}}{C_s^2(4T_s^2s + 1)} \]  

(\ref{eq10})

Equation (\ref{eq10}) contains an integer-order PI controller. When using vector representation is used, the controlled object of the voltage outer loop needs to be used. Therefore, the equation (\ref{eq10}) is taken out of the controlled object, and its change function is:

\[ G_i(s) = \frac{1}{C_s(4T_s^2s + 1)} \]  

(\ref{eq11})

In equation (\ref{eq11}), C is the voltage equalizing capacitance on the DC side, and T_s is the sampling period. The frequency characteristic of the controlled object is as follows:

\[ A(\omega) = \frac{1}{C_s\sqrt{(4T_s\omega)^2 + 1}} \]

\[ \text{Arg}[G_i(j\omega)] = -\arctan(4T_s\omega) - 90^\circ \]  

(\ref{eq13})

The phase angle conditions and amplitude conditions are shown in the following formula:

\[ \text{Arg}\left[G_f(j\omega)\right]_{\omega=\omega_m} = \text{Arg}\left[G_{pv}(j\omega)G_i(j\omega)\right]_{\omega=\omega_m} = -\pi + \varphi_m \]  

(\ref{eq14})

\[ |G_f(j\omega)|_{\omega=\omega_m} = |G_{pv}(j\omega)G_i(j\omega)|_{\omega=\omega_m} = 1 \]  

(\ref{eq15})

According to the above formula, the vector G_{pv}(s) expression obtained is:

\[ G_{pv}(j\omega) = \frac{1}{G_i(j\omega)} \varphi_m - \text{Arg}[G_i(j\omega)] - \pi = k_p + \frac{k_i}{j\omega} \]  

(\ref{eq16})

According to formula (\ref{eq13}), (\ref{eq14}), (\ref{eq15}), (\ref{eq16}), we know:

\[ k_p + \frac{k_i}{\omega^2} \cos\left(-\frac{\pi}{2}\lambda\right) = C_s\sqrt{(4T_s^2\omega)^2 + 1}\cos\left[\varphi_m - \pi - \text{Arg}[G_i(j\omega)]\right] \]  

(\ref{eq17})

\[ \frac{k_i}{\omega}\sin\left(-\frac{\pi}{2}\lambda\right) = C_s\sqrt{(4T_s^2\omega)^2 + 1}\sin\left[\varphi_m - \pi - \text{Arg}[G_i(j\omega)]\right] \]  

(\ref{eq18})

According to formula (\ref{eq17}) and formula (\ref{eq18}), we can find k_p, k_i, \lambda:

(1) First determine \varphi_m the phase angle margin of the voltage outer loop, the phase angle margin \varphi_m set in this paper is 55\degree.

(2) Secondly determine the cut-off frequency of the voltage outer loop, which is set as 350 rad/s in this article;

(3) Set the order range of the fractional order integration link to and set the step size of the order to 0.01;

(4) Within each step, equations (\ref{eq17}) and (\ref{eq18}) calculate the corresponding value for each order, and substitute the corresponding value in the closed-loop step response of the voltage outer loop, and then calculate the time. Multiplying absolute error integral criterion, according to the controller designed by ITAE, the transient response vibration is small and the followability is strong;

(5) According to the minimum ITAE value, select the integral order \lambda, k_p, k_i of the corresponding fractional-order PI^\lambda controller. Figure 2 shows the curve of ITAE with the order of fractional order \lambda.
When \( \lambda = 0.86 \), corresponding to \( k_p = 0.0122 \), \( k_i = 1.3412 \), the transfer function of the fractional-order PI\(^{\lambda}\) controller is:

\[
G_{\text{PI}^{\lambda}}(s) = k_p + k_i/s^\lambda = 0.0122 + 1.3412/s^{0.86}
\]

(19)

When \( \lambda = 0.95 \), corresponding to \( k_p = 0.0135 \), \( k_i = 2.2244 \), then the calculated transfer function is:

\[
G_{\text{PI}^{\lambda}}(s) = k_p + k_i/s^\lambda = 0.0135 + 2.2244/s^{0.95}
\]

(20)

When the fractional-order \( \lambda \) is 1, according to formula (17) and formula (18), and \( k_p = 0.0141 \), \( k_i = 2.9722 \) can be obtained, so the integer order controller of the voltage outer loop is:

\[
G_{\text{PI}}(s) = k_p + k_i/s = 0.0141 + 2.9722/s
\]

(21)

According to formula (19), formula (20) and formula (21), the closed-loop unit step response of the voltage outer loop can be drawn, as the picture 3 shows.

![Figure 2. Change curve of ITAE with order \( \lambda \)](image)

Table 1. Performance indexes of different controllers

| controller     | ITAE    | ISE      | IAE      |
|----------------|---------|----------|----------|
| Integer order  | \(4.833 \times 10^{-4}\) | \(1.755 \times 10^{-3}\) | \(4.499 \times 10^{-3}\) |
| 0.86 order     | \(4.612 \times 10^{-4}\) | \(1.725 \times 10^{-3}\) | \(4.254 \times 10^{-3}\) |
| 0.95 order     | \(4.672 \times 10^{-4}\) | \(1.732 \times 10^{-3}\) | \(4.348 \times 10^{-3}\) |

According to Figure 3, the solid line represents the closed-loop step response of the fractional-order controller, the broken line represents the closed-loop step response of the integer-order PI controller. From the Table 1, the fractional-order controller has excellent control effect.
Figure 4. Sudden step response under the action of two controllers

From Figure 4 that red dotted line represents the sudden step response adjusted by the integer order controller; the solid blue line represents the sudden step response adjusted by the fractional-order PI\(^\lambda\). However, the rise times of the two controllers are the same, and the maximum adjustment time of the sudden step response.

Figure 5. Sudden reduction step response under the action of two controllers

Figure 5 shows the sudden step response curve compared fractional order PI\(^\lambda\) controller to integer-order controller. In Figure 5, the dashed red line represents the sudden step response adjusted by an integer order controller; the solid blue line represents the sudden step response adjusted by a fractional order controller. From Figure 5 that the two controllers also have good followability in the suddenly reduced step input signal, and the adjustment performance of the fractional order PI\(^\lambda\) controller is better than the integer-order PI controller. Therefore, the fractional-order PI\(^\lambda\) controller has better dynamic performance.

4. Simulation Research of NPC Three-phase Three-level Inverter
The simulation parameters of three-level inverter are set as follows:
Table 2. Simulation parameters of three-level inverter

| parameter | value  |
|-----------|--------|
| U         | 50V    |
| L         | 1mH    |
| C         | 4700uF |
| R         | 5Ω     |
| f         | 5kHz   |

In order to further study the control effect of fractional order PI\(^{\lambda}\) controller and integer order PI controller in three-level inverter system. In the simulation of three-level inverter, load simulation experiment and load reduction simulation experiment were carried out.

![Load output response curve of voltage outer loop in \(dq\) coordinate system](image)

Figure 6. Load output response curve of voltage outer loop in \(dq\) coordinate system

The \(dq\) is the load output response curve of the voltage outer loop in the coordinate system. Among them, red represents the response curve of integer order controller adjustment, and blue represents the response curve of fractional order PI\(^{\lambda}\) controller adjustment. The NPC three-level inverter simulation system suddenly increases the load at 1s. the fractional-order PI\(^{\lambda}\) controller has the same soaring speed, and the maximum overshoot under the action of the fractional-order PI\(^{\lambda}\) controller.

![Response curve of load shedding output of voltage outer loop in \(dq\) coordinate system](image)

Figure 7. Response curve of load shedding output of voltage outer loop in \(dq\) coordinate system

The \(dq\) is the load output response curve of the voltage outer loop in the coordinate system. Among them, red represents the response curve of integer order controller adjustment, and blue represents the

| parameter | value  |
|-----------|--------|
| U         | 50V    |
| L         | 1mH    |
| C         | 4700uF |
| R         | 5Ω     |
| f         | 5kHz   |
response curve of fractional order PI\(^\lambda\) controller adjustment. Three-level inverter simulation system suddenly reduces the load at 1s.

5. Summary
This article takes the three-phase diode clamped three-level inverter as the research object. First, the working principle of three-level inverter topology is studied, and a three-phase static coordinate coefficient model is established. Through further analysis of the mathematical model and getting the control strategy of the fractional voltage outer loop. Secondly, the vector representation method is used to simplify the setting calculation of the fractional-order PI\(^\lambda\). Meanwhile, the optimal parameters of the fractional order PI\(^\lambda\) controller are designed by using the ITAE criterion. To further realize the control method of the fractional order PI\(^\lambda\) controller, this paper adopts Oustaloup rationalized approximation method to achieve the fractional-order PI\(^\lambda\) controller. Finally, through simulation results verify the feasibility of the fractional-order PI\(^\lambda\) controller.

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