Effects of Vertical Advection on Multimessenger Signatures of Black Hole Neutrino-dominated Accretion Flows in Compact Binary Coalescences

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Abstract

In the coalescence events of binary neutron star (NS) or a black hole (BH) and an NS, a BH hyperaccretion disk might be eventually formed. At very high mass accretion rates, MeV neutrinos will be emitted from this disk, which is called a neutrino-dominated accretion flow (NDAF). Neutrino annihilation in the space out of the disk is energetic enough to launch ultrarelativistic jets to power gamma-ray bursts. Moreover, vertical advection might exist in NDAFs, which can generate the magnetic buoyancy bubbles to release gamma-ray photons. In this paper, we visit the effects of the vertical advection in NDAFs on the disk structure and gamma-ray and neutrino luminosities for different accretion rates. Then we study the anisotropic emission of kilonovae and the following gravitational waves (GWs) driven by the gamma-ray photons and neutrinos from NDAFs. Comparing NDAFs without vertical advection, the neutrino luminosity and GW strains slightly decrease for the case with vertical advection, and the kilonovae will be brightened by the injected gamma-ray photons. The future joint multimessenger observations might distinguish whether the vertical advection exists in NDAFs or not after compact binary coalescences.

Unified Astronomy Thesaurus concepts: Accretion (14); Black holes (162); Neutrino astronomy (1100); Gravitational waves (678); Transient sources (1851); Gamma-ray bursts (629)

1. Introduction

Short-duration gamma-ray bursts (SGRBs) commonly occur in the scenario of compact binary coalescences, i.e., binary neutron star (NS) or a black hole (BH) and an NS. When the binary gets into gravitational fields of each other and spiral toward one another, it will radiate gravitational waves (GWs), e.g., Cutler & Flanagan 1994; Sathyaprakash & Schutz 2009; Baiotti & Rezzolla 2017. After coalescence events, optical/near-infrared emission from the radioactive decay of rapid neutron captured (r-process) elements are produced by the neutron-rich ejecta during the merger and postmerger phases (e.g., Lattimer & Schramm 1974, 1976; Symbalisty & Schramm 1982; Li & Paczynski 1998; Kasen et al. 2013; Just et al. 2015; Metzger 2019; Nakar 2020; Cowan et al. 2021). These transient events were named as “kilonovae” because their luminosity was approximately 1000 times brighter than typical novae and its emission timescale would last for several days or even longer (e.g., Metzger et al. 2010; Metzger 2017, 2019). Besides, the remnants are expected to emit large amounts of MeV neutrinos that is because the violent collision heats up the material which goes through rapid decompression from the debris of compact object (e.g., Sekiguchi et al. 2011; Kyutoku & Kashiwama 2018). Meanwhile, the newborn BH constantly deforms matters and thus produces ultrarelativistic jets from bipolar, which would be detected as SGRBs if they point toward the Earth (e.g., Paczynski 1986; Narayan et al. 1992; Popham et al. 1999; Liu et al. 2017a).

The successful observations to GW170817 by the advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO)/Virgo (Abbott et al. 2017a) and its accompanied GRB 170817A (e.g., Abbott et al. 2017b) has indicated the beginning of a new era of multimessenger astronomy; and about 10 hr after merger event, the electromagnetic counterpart, kilonova AT 2017gfo, was detected (e.g., Abbott et al. 2017b; Andreoni et al. 2017; Arcavi et al. 2017; Coulter et al. 2017), which was indubitably another landmark of multimessenger signals. Its light curve cannot be explained in the r-process depended kilonova model by using only one single set of parameters. Instead, the different mass, velocity, morphology, and opacity of the ejecta all need to be taken into consideration (e.g., Cowperthwaite et al. 2017; Perego et al. 2017; Tanaka et al. 2017; Villar et al. 2017; Kawaguchi et al. 2018; Wu et al. 2019). Following the observation of GW170817, IceCube, ANTARES, Super-Kamiokande (Abe et al. 2018), and the Pierre Auger Observatory (Albert et al. 2017) all attempted to detect the accompanying high-energy neutrinos, but unfortunately, none of them received significant neutrinos from GW170817. After that, on 2020 January 5, the LIGO/Virgo detected the first BH–NS merger event, GW200105, and 10 days later, the second BH–NS merger event, GW200115, was also discovered (Abbott et al. 2021). Both accretion and ejection of material from a BH produce electromagnetic radiation, which was not observed in either of these events (e.g., Zhu et al. 2021; Qi et al. 2022). It is possible that the NS did not disintegrate, but was swallowed intact by the BH and did not produce a noticeable electromagnetic counterpart.

A stellar-mass BH surrounded by a hyperaccretion disk will be formed in the center of compact binary coalescences or massive collapsars. For the very high accretion rates ($10^{-3} M_e$ s$^{-1} \lesssim \dot{M} \lesssim 10 M_e$ s$^{-1}$), such a disk has an extremely high density ($\rho \sim 10^{10}$–$10^{13}$ g cm$^{-3}$) and temperature...
(\(T \sim 10^{10} - 10^{11} \) K). In the inner region of the disk, neutrino cooling dominate to balance the viscous heating. This type of accretion disk is named the neutrino-dominated accretion flow (NDAF; for reviews, see Liu et al. 2017a; Zhang 2018). It was first proposed by Popham et al. (1999) and explains the energy supply of gamma-ray bursts (GRBs) in terms of neutrino annihilation in the space outside of the disk. Subsequently, the NDAF model was widely studied (e.g., Narayan et al. 2001; Di Matteo et al. 2002; Kohri & Mineshige 2002; Kohri et al. 2005; Lee et al. 2005; Gu et al. 2006; Chen & Beloborodov 2007; Kawanaka & Mineshige 2007; Liu et al. 2007; Lei et al. 2009; Zalamea & Beloborodov 2011; Kawanaka et al. 2013; Xue et al. 2013; Yi et al. 2019). Considering that BHs accrete gas constantly, this causes relativistic effects, and the GW emission leading to the brighten the kilonovae and also affect the emission of neutrinos and GWs triggered by NDAFs.

In this paper, we investigate the NDAF with vertical advection around a fast-rotating BH (as shown in Figure 1) and the effects of the vertical advection on the disk structure, kilonovae, and neutrino and GW emissions. The paper is structured as follows. In Section 2, we model the NDAFs with vertical advection, the anisotropic kilonovae induced by the ejecta and gamma-ray photons together, the spectra of electron antineutrinos from NDAFs by considering the general relativistic effects, and the GW emission leading to the anisotropic neutrinos from NDAFs. The main results are shown in Section 3. A brief summary is made in Section 4.

2. Model

2.1. NDAFs with Vertical Advection

Considering that BHs accrete gas constantly, this causes them to accumulate angular momentum, and in most GRB engine candidates the central BHs are rotating rapidly, so we adopt the relativistic correction factors (Riffert & Herold 1995) to describe the Kerr BHs, i.e.,

\[
A = 1 - \frac{2GM_{\text{BH}}}{c^2 r} + \left(\frac{GM_{\text{BH}} a_\ast}{c^2 r}\right)^2, \tag{1}
\]

\[
B = 1 - \frac{3GM_{\text{BH}}}{c^2 r} + 2a_\ast \left(\frac{GM_{\text{BH}}}{c^2 r}\right)^{3/2}, \tag{2}
\]

\[
C = 1 - 4a_\ast \left(\frac{GM_{\text{BH}}}{c^2 r}\right)^{3/2} + 3 \left(\frac{GM_{\text{BH}} a_\ast}{c^2 r}\right)^2, \tag{3}
\]

\[
D = \int_{r_m}^r \frac{\frac{x^2 c^4}{8G^2} - \frac{3x M_{\text{BH}} c^2}{4G} + a_\ast^2 M_{\text{BH}}^2 c^2}{x^2 c^4 - \frac{3x M_{\text{BH}} c^2}{G} + 2 \sqrt{a_\ast^2 M_{\text{BH}}^2 c^2}} \, dx, \tag{4}
\]

where \(r\) and \(r_m\) are the disk radius and the inner boundary of the disk, respectively. In our model, we consider a fast-rotating stellar-mass BH with \(M_{\text{BH}} = 3 M_\odot\) and the dimensionless spin parameter \(a_\ast = 0.9\).

The kinematic viscosity is written as

\[
\nu = \frac{c^2}{\Omega_k}, \tag{5}
\]

where \(\alpha = 0.1\) is the viscous parameter of the disk, \(c_s = \sqrt{P/\rho}\) is the isothermal sound speed with \(P\) being the pressure and \(\rho\) the density, and \(\Omega_k = (GM_{\text{BH}}/r)^{1/2}\) is the Keplerian angular velocity. In the Kerr metric, the basically dynamic equations of NDAFs are given as follows (e.g., Liu et al. 2010, 2017a):

\[
\dot{M} = -4 \pi r v_r \rho H, \tag{6}
\]

and

\[
\dot{M} \frac{D}{A} = 4 \pi v_r \rho H \frac{A}{\sqrt{BC}}, \tag{7}
\]

where \(H = \sqrt{Pr^3/\rho GM_{\text{BH}}} \sqrt{B/C}\) is the half-thickness of the disk and \(v_r\) is the radial velocity of the accreted gas. Here, two typical accretion rates \(\dot{M} = 0.1 M_\odot\) and \(1 M_\odot\) s\(^{-1}\) are considered.

The total pressure consists of four items: radiation pressure, gas pressure, electron degeneracy pressure, and neutrino pressure. Thus, the equation of state is expressed as (e.g., Liu et al. 2010):

\[
P = \frac{11}{12} a T^4 + \frac{\rho k T}{m_p} \left(1 + 3 X_{\text{nuc}}\right) \frac{4}{3} \left(\frac{\rho}{\mu_e}\right)^{4/3} + \frac{u_e}{3}, \tag{8}
\]

where \(a\) is the radiation constant, \(m_p\) is the proton rest mass, \(k\) is the Boltzmann constant, \(h\) is the Planck constant, \(\mu_e\) is the electron chemical potential, and \(X_{\text{nuc}} \approx 34.8 \rho_{10}^{-3/4} T_{11}^{9/8} \exp(-0.61/T_{11})\) is the mass fraction of free nucleons with \(T_{11} = T/(10^{11} \text{ K})\) and \(\rho_{10} = \rho/(10^{10} \text{ g cm}^{-3})\); (e.g., Gu et al. 2006; Liu et al. 2007). In addition, the neutrino energy density \(u_e\) is given by (e.g., Popham & Narayan 1995; Liu et al. 2007):

\[
u_e = (7/8)a T^4 \sum \frac{\tau_{\nu e}/2 + 1/\sqrt{3}}{\tau_{\nu e}/2 + 1/\sqrt{3} + 1/(3 \tau_{\nu e})}, \tag{9}
\]
where $\tau_{ij} = \tau_{a,i} + \tau_{n,j}$ is the sum of the total absorptive and scattering optical depths for each neutrino flavor ($\nu_e$, $\nu_{\mu}$, $\nu_{\tau}$); (e.g., Di Matteo et al. 2002; Gu et al. 2006; Liu et al. 2010).

The energy equation is written as

$$Q_{\text{vis}}^+ = Q_{\text{adv}} + Q_{\nu}^- + Q_{\text{photo}}^- + Q_{\gamma}^-.$$  \hspace{1cm} (10)

The above equation illustrates the balance between heating because of viscous dissipation $Q_{\text{vis}}^+$ and cooling due to advection $Q_{\text{adv}}$, neutrino losses $Q_{\nu}^-$, photodisintegration $Q_{\text{photo}}^-$, and vertical advection $Q_{\gamma}^-$.

The heating rate is expressed as (e.g., Liu et al. 2010)

$$Q_{\text{vis}}^+ = \frac{3GM\dot{M}D}{8\pi r^3 B},$$  \hspace{1cm} (11)

and we take the advective cooling term in Liu et al. (2007), which is written as

$$Q_{\text{adv}} \simeq \nu_r \frac{H}{r} \left[ \frac{11}{3} a T^4 + \frac{3}{2} \frac{p T}{m_p} + \frac{1}{4} + \frac{4u_e}{3} \right].$$  \hspace{1cm} (12)

We adopt a bridging formula for calculating the neutrino transport, i.e.,

$$Q_{\nu}^- = \sum \frac{(7/8\sigma T^4)}{(3/4)(\tau_{\nu}/2 + 1/\sqrt{3} + 1/(3\tau_{\nu}^{\nu})))}. \hspace{1cm} (13)$$

Moreover, the rate of the cooling due to photodissociation is always ignored because it is much less than the neutrino cooling rate in the inner disk (e.g., Liu et al. 2007).

The vertical advection effect leads to magnetic buoyancy bubbles, which carry the gamma-ray photons and float them up from the equatorial plane of the disk. We introduce a new cooling term $Q_{\gamma}^-$ to describe this process. Figure 1 provides a schematic description of an NDAF with the vertical advection effect.

First, we consider that the averaged velocity of the vertical advection process, $V_z$, can be estimated by

$$V_z = \lambda c_s,$$  \hspace{1cm} (14)

where $\lambda$ is a dimensionless parameter, and we take $\lambda = 0.1$ by referring to the typical vertical velocity given by the numerical simulations in Jiang et al. (2014). Second, based on the mixing-length theory (Prandtl 1925), a turbulent eddy viscosity is related to the mixing length $\ell$ and the gradient of mean velocity, i.e., $\nu = \ell^2 \partial u / \partial z$. Thus, by considering Equations (5) and (14) we can roughly estimate the mixing length of the bubbles $\ell \sim H$, suggesting that these bubbles can reach the disk surface. Third, the optical depth of the bubbles is mainly determined by the vertical distribution of the disk density. In order to maintain dynamical equilibrium at the surface of bubbles, the bubbles should expand and keep a lower density within when they float up. Finally, they will mix with the material at the disk surface. The optical depth there is thin enough to release photons. Moreover, the process of the bubble expansion is nonadiabatic because photons can be produced at any height, and the matter is extremely dense and optically thick for them, so these photons tend to inject into the existing bubbles or generate new bubbles where the density is lower than that of NDAFs. In absence of detailed modeling of the disk vertical structure, we utilize physical quantities at the disk equatorial plane to characterize the cooling process of vertical advection in NDAFs for simplicity.

Based on the above analyses, we adopt the formula

$$Q_{\gamma}^- = V_z (u_{\text{ph}} + u_\nu + u_{\text{gas}}),$$  \hspace{1cm} (15)

where $u_{\text{ph}}$, $u_\nu$, and $u_{\text{gas}}$ are the energy density of photons, neutrinos, and gases, respectively (Yi et al. 2017). In our calculation, the third term $V_z u_{\text{gas}}$ is ignored because the amount of gas that escapes through magnetic buoyancy is tiny compared to the other terms. The vertical advection term describes the released photons and neutrinos due to the magnetic buoyancy, which can dominate over the normal diffusion process.

Although the roughly estimated mixing length of the bubbles approaches the half disk thickness, one can notice that not all of the bubbles from any radius of the disk can reach the disk surface in simulations (see Figures 9 and 14 of Jiang et al. 2014). Moreover, the yields of the photons will decrease once the place of production deviates from the equatorial plane of the disk. Thus, we consider that this description denotes the upper limit of the cooling rate due to vertical advection, and therefore the gamma-ray photons released from bubbles that serve as the new sources of energy that brighten up the kilonovae might be exaggerated to a certain extent.

### 2.2. Kilonovae

As first suggested by Lattimer & Schramm (1974), the material that is ejected from compact binary mergers is a favorable place for the generation of heavy elements through r-process nucleosynthesis. These extremely erratic heavy nuclei promptly decay to power the so-called kilonovae (e.g., Li & Paczyński 1998; Metzger et al. 2010; Kasen et al. 2013; Tanaka & Hotokezaka 2013; Liu et al. 2017a; Metzger 2017, 2019, and references therein). Studies of AT 2017gfo indicate that a single-component model of a kilonova is not sufficient to characterize the observations in multiple bandpasses, so two or more components are required (e.g., Cowperthwaite et al. 2017; Tanaka et al. 2017; Tanvir et al. 2017). It is widely accepted that kilonova emission covers multiple phases, from the early optical and ultraviolet band, which is known as the “blue” component (BC), to the later infrared band, which called “red” component (RC). The occurrence of this phenomenon can be attributed to the fact that the opacity $\kappa$ of the merger ejecta may not have a single value, but instead depend on the composition of the ejecta material resulting from the different proportions of lanthanides and actinides in it. When the ejected material undergoes higher rates of r-process nucleosynthesis, it will generate abundant lanthanides and actinides, and thus it can explain the RC because lanthanides can reach $\kappa \gtrsim 10$ cm$^2$ g$^{-1}$, much higher than substances composed of iron group elements (e.g., Barnes & Kasen 2013; Tanaka & Hotokezaka 2013); but if the ejecta experiences only partial r-process nucleosynthesis, it will not form the lanthanide elements, in which case the BC appears, which has a lower typical $\kappa \sim 0.1$ cm$^2$ g$^{-1}$ (e.g., Cowperthwaite et al. 2017).

The material from a compact binary merger could be ejected from the whole solid angle, similar to a thick crust of spheres with a central engine in the middle, but this shell geometry is not necessarily of a uniform distribution; that is, the ejecta need not be isotropic. In fact, the dynamic ejecta produced by a BH–NS merger is primarily from along the equatorial plane, and
looks exhibits a crescent shape (e.g., Kyutoku et al. 2013, 2015; Kawaguchi et al. 2016; Zhu et al. 2020; Qi et al. 2022). The ejected mass varies from $10^{-3}$ to $10^{-2} M_\odot$ and has a relativistic velocity that accelerates with ejecta diffusion from 0.1 to 0.4 c. Particularly, in the equatorial direction where most of the ejection happens, neutron-rich ejecta can produce large amounts of lanthanides. This part of the ejecta is primarily produced via the tidal tail, which has a high opacity and thus reddens the kilonovae, i.e., the RC. By contrast, in the polar direction where the ejecta might be injected by the neutrino-driven disk winds or outflows, the electron abundance is high, and therefore the $r$-process is restricted and elements with atomic mass number $A \geq 130$ cannot be readily synthesized. The kilonovae in this direction will look more “blue”, i.e., the BC. Based on the above considerations, the differences in the geometry and opacity of the ejecta in each angle make it necessary to use an anisotropic kilonova emission model.

In addition to the radioactive decay of the ejecta, there are a few other potentially important energy outputs, such as the neutrino radiation from the accretion disk, which is expected to heat the surroundings and create the disk winds (e.g., McLaughlin & Surman 2005; Metzger et al. 2008; Dessart et al. 2009), and winds from magnetized NSs (e.g., Yu et al. 2013; Metzger & Piro 2014; Yu et al. 2018; Ren et al. 2019). The interaction between relativistic jets and ejecta can also cause shock heating (e.g., Bucciantini et al. 2012; Gottlieb et al. 2018; Piro & Kollmeier 2018). In addition, a large number of studies have taken the accretion disk out flows into account (e.g., Just et al. 2015; Wu et al. 2016; Perego et al. 2017; Siegel & Metzger 2017; Song et al. 2018; Qi et al. 2022), which is another important source of mass and energy.

In our model, with the presence of the vertical advection mechanism, the gamma-ray photons that have escaped from the magnetic buoyancy bubbles on the disk carry enormous energy that will trigger brighter kilonovae, which could have a further impact on observations. As we mentioned before, after the coalescence event, the formation of a fast-rotating central BH largely affects the gravitational field in its vicinity, including the direction of photon emission from the disk. These photons will move along the geodesic of the Kerr metric; that is, this will intensify their anisotropic emission. In order to calculate these photon trajectories, we use a ray-tracing method (Fanton et al. 1997) and apply it also to the calculation of neutrinos (see Section 2.3). This radiant energy is supposedly injected into the nearest material (normal incidence is assumed), which is the first shell layer of the ejecta. As shown in Figure 2, we divide it into $N(\geq 1)$ layers and use the subscript to denote the mass layers $i = 1, 2, \ldots, N$, where $i = 1$ and $N$ represent the innermost and outermost layers, respectively. Each one has a different expansion speed $v_i$: the first layer has the lowest speed $v_1 = v_{\text{min}}$ and the outermost layer has the maximum $v_N = v_{\text{max}}$, based on a simplified radiation transfer model. We set the values of $v$ from 0.1 to 0.3 c. (e.g., Kasen & Bildsten 2010; Metzger 2019; Qi et al. 2022).

At a time $t$, $R_i(t) = v_i t$ represents the location of the $i$th layer, so the maximum and minimum position of the ejecta can be easily calculated by $R_{\text{max}}(t) = v_{\text{max}} t$ and $R_{\text{min}}(t) = v_{\text{min}} t$. The density distribution is taken as (e.g., Nagakura et al. 2014)

$$\rho_{\text{ej},i} = \frac{(3 - \delta)m_{\text{ej}}}{\Delta \Omega_k R_{\text{max}}^3} \left[ 1 - \left( \frac{R_{\text{min}}}{R_{\text{max}}} \right)^{3-\delta} \right]^{-1} \left( \frac{R}{R_{\text{max}}} \right)^{-\delta},$$

where $\delta$ is the power-law index of the density distribution and is between 1 and 3. The ejecta is supposed to be symmetric along the equatorial axis; because of its crescent-shaped distribution, we manually partition $k$ ($k = 1, 2, \ldots, 10$) discrete clumps in equivalent solid angles $\Delta \Omega_k$ from the polar axis to the equatorial plane, with an increasing mass for each block as shown in Figure 2. The overall mass of the ejecta should be around several times $0.01 M_\odot$ (Arnett 1980). Here, we set the total mass of the ejecta, $M_{\text{ej}} = 0.04 M_\odot$, and the mass distribution of the ejecta meets $\sim \theta^2$.

At a solid angle $\Delta \Omega_k$, the emission of the ejecta is correlated with the thermal energy $E_i$, for which the evolution can be described as (e.g., Ren et al. 2019)

$$\frac{dE_i}{dt} = (1 - e^{-\Delta \Omega_k})e^{\gamma_i - \tau_{\text{th}}}L_{\text{ph}} + m_i q_i \tau_{\text{th}} - \frac{E_i}{R_i} \frac{dR_i}{dt} - L_i,$$

for $i = 1, 2, \ldots, N$,

$$\Delta \Omega_k = \frac{dE_i}{dt} = (1 - e^{-\Delta \Omega_k})e^{\gamma_i - \tau_{\text{th}}}L_{\text{ph}} + m_i q_i \tau_{\text{th}} - \frac{E_i}{R_i} \frac{dR_i}{dt} - L_i.$$
The radioactivity power per unit mass derived from elaborate nucleosynthesis calculations (e.g., Korobkin et al. 2012) is

\[ \dot{q}_i = 4 \times 10^{18} \left[ \frac{1}{2} - \frac{1}{\pi} \arctan \left( \frac{t - t_0}{\sigma} \right) \right]^{1.3} \text{erg s}^{-1} \text{g}^{-1}, \]

where \( \sigma = 0.11 \text{ s}, t_0 = 1.3 \text{ s}, \) and the thermalization efficiency can be written as (e.g., Barnes et al. 2016)

\[ \eta_{\text{th}} = 0.36 \exp(-0.56t_{\text{day}}) + \frac{\ln(1 + 0.34\sqrt{0.74}t_{\text{day}})}{0.34t_{\text{day}}}, \]

with \( t_{\text{day}} = t/1 \text{ day} \). This equation is satisfied in the case of \( m_{\text{ej}} = 0.01M_\odot \) and \( v = 0.1c \), with random magnetic fields. In order to ensure that our calculations are compatible with this equation, as mentioned above, we divided the mass into 10 clumps, where the mass of four blocks near the equator is in the range of 5 \( \times 10^{-3} - 0.01 \text{ M}_\odot \).

\( L_i \) is the observed radiation luminosity of the \( i \)th layer, which can be calculated by (e.g., Yu et al. 2018; Ren et al. 2019)

\[ L_i = \frac{E_i}{\max(t_{\text{dl}}, t_{\text{d},i})}, \]

where the light-crossing time \( t_{\text{dl}} = R_i/c \) gives the time limit, and \( t_{\text{d},i} \) represents the radiation diffusion timescale during which the thermal heat can escape from the entire ejecta

\[ t_{\text{d},i} = \frac{3K_\ell}{\Delta\Omega_i R_i c} \sum_{n=1}^{N-1} m_n. \]

The form of \( \max(t_{\text{dl}}, t_{\text{d},i}) \) guarantees the causality, especially in the optically thin layer, which is close to the outermost shell.

By summarizing the contributions of each layer and all \( \Delta\Omega_i \), the total bolometric luminosity of the merger ejecta can be obtained by

\[ L_{\text{bol}} = \sum_i L_i. \]

Assuming that the spectrum is always that of a blackbody and is beaming from the photosphere \( R_{\text{ph}} \), the effective temperature of the kilonova emission can be calculated as follows

\[ T_{\text{eff}} = \left( \frac{L_{\text{bol}}}{\sigma_{\text{SB}} \Delta\Omega_i R_i^2 R_{\text{ph}}^{-4}} \right)^{1/4}, \]

where \( \sigma_{\text{SB}} \) is the Stephan–Boltzmann constant. The photosphere radius \( R_{\text{ph}} \) is determined by setting \( \tau_{\text{ph}} = \int_{R_{\text{ph}}}^{R_{\text{max}}} K(\rho, R) dR = 1 \) for the case \( \tau_{\text{tot}} > 1 \). However, if the total optical depth of the ejecta \( \tau_{\text{tot}} \leq 1 \), the radius of the photosphere \( R_{\text{ph}} \) is taken as the minimum radius \( R_{\text{min}} \) (e.g., Yu et al. 2018; Ren et al. 2019; Qi et al. 2022). The polar character of this ejection allows the substance to increase its electron potential of the emission location. The total observed spectrum in the MeV range after the \( r \)-process are close to the gamma-ray photon energies released from the accretion disk (e.g., Metzger 2019). Thus, we use an identical set of opacities for our calculation.

The flux density of the kilonova emission from a solid angle with photon frequency \( \nu \) can be given by

\[ dF(\nu, t_{\text{obs}}) = \frac{2\pi\hbar\nu^3}{c^2} \frac{1}{1 - \exp(h\nu/k_B T_{\text{eff}})} \frac{R_{\text{ph}}^2 d\Omega}{4\pi D_L^2}, \]

where \( D_L \) is the luminosity distance.

Finally, we consider the isochronous surface. Tracing the travel of light, if a photon is emitted at time \( t \), it will be observed at time (e.g., Qi et al. 2022)

\[ t_{\text{obs}} = t + \frac{[R_{\text{ph}}(\theta_{\text{obs}}) - R_{\text{ph}}(\theta)] \cos \Delta\theta}{c}. \]

If the observer is located at \( \varphi = 0^\circ \), the range of longitudes visible to the observer in a given \( \theta \) is \( [-\varphi(\theta), \varphi(\theta)] \). According to the half-day arc equation,

\[ \varphi(\theta) = \arccos(-\cot \theta \cot \theta_{\text{obs}}). \]

Here, we take \( D_L = 40 \text{ Mpc} \) as the distance from the source, and the AB magnitude \( M_\nu \) can be calculated from the flux density as \( M_\nu = -2.5 \log_{10}(F_\nu/3631 \text{Jy}). \)

2.3. MeV Neutrinos

In the NDAF model, the cooling process of neutrinos occurs in large quantities, and mainly the inner region of the disk (e.g., Liu et al. 2016; Wei et al. 2019; Song et al. 2020; Qi et al. 2022). As with gamma-ray photons, the general relativity effects from the central BH also affect the formation of the neutrino spectra. So, we use the same approach, i.e., the ray-tracing method (e.g., Fanton et al. 1997; Liu et al. 2016). Numerically, for every pixel of our observed image, the position of the emission source in the accretion disk can be traced back. For simplicity, we assume that the neutrinos are emitted isotropically at each radius from the equatorial plane, i.e., \( \Delta\Omega = \pi/2 \), and that the disk is in Keplerian rotation, and we ignore the shading effect caused by the thickness of the disk; therefore, the trajectory of these emitted neutrinos should satisfy the geodesic equation (Carter 1968), i.e.,

\[ \pm \int_{R_{\text{em}}}^{\infty} \frac{dR}{\sqrt{f(R)}} = \pm \int_{t_{\text{obs}}}^{t_{\text{em}}} \frac{dt}{\sqrt{f(t)}}. \]

The energy shift of neutrinos can be calculated by considering the corresponding velocity and the gravitational potential of the emission location. The total observed spectrum is obtained by integrating all the pixels. Specifically, the total observed flux can be expressed as

\[ F_{\text{obs}} = \int_{\text{image}} g^3 I_{\nu \text{obs}} d\Omega_{\text{obs}}, \]

\[ \Delta\Omega = \pi/2, \text{ and for equator direction, } \Delta\Omega = \pi. \]
where $E_{\text{em}}$ and $E_{\text{obs}}$ are the neutrino emission energy from the local disk and the observed neutrino energy, respectively. $g \equiv E_{\text{obs}} / E_{\text{em}}$ is the energy shift factor, and $\Omega_{\text{obs}}$ is the solid angle of the disk image toward the observer.

The local emissivity $I_{\text{em}}$ can be obtained from the cooling rate of electron antineutrinos $Q_{\nu e}$, i.e.,
\[
I_{\text{em}} = \frac{Q_{\nu e}}{\int F_{\text{em}} dE_{\text{em}}},
\]
where $F_{\text{em}} = E_{\text{em}}^2 / [\exp(E_{\text{em}}/kT - 1)]$ is the unnormalized Fermi–Dirac spectrum (e.g., Liu et al. 2016; Wei et al. 2019).

Hence, the luminosity distribution can be calculated as follows
\[
L_\nu = 4\pi D_L^2 F_{\nu \text{em}}.
\]

2.4. GWs

GW radiation induced by anisotropic neutrino emission was first investigated by Epstein (1978), and this approach has been applied to core-collapse supernovae (e.g., Burrows & Hayes 1996; Kotake et al. 2006, 2007) and NDAFs (e.g., Suwa & Murase 2009; Kotake et al. 2012; Liu et al. 2017b; Song et al. 2020; Wei & Liu 2020). Here, we adopt the method to calculate the GWs from NDAFs with vertical advection in compact binary merger scenarios.

For long-term neutrino emission, the GW amplitude will converge to a nonzero value $h_\infty$, and it is subject to the observation angle $\theta_{\text{obs}}$, which is derived as follows (Suwa & Murase 2009)
\[
h_\infty(\theta_{\text{obs}}) = \frac{2G(1 + 2 \cos \theta_{\text{obs}})}{3 c^4 D_L} \tan \frac{\theta_{\text{obs}} \mathcal{T}}{2},
\]
where $\mathcal{L}_\nu = 2\pi \int_0^\infty \int_v^\infty Q_{\nu e} R \, dR \, dt / \mathcal{T}$ is the mean neutrino luminosity above or below the disk, and $\mathcal{T}$ is the activity duration of the GRB central engine. If the GRB is considered as a single burst triggered by the NDAF, the time evolution of the neutrino luminosity $L_\nu(t)$ can then be described as
\[
L_\nu(t) = \mathcal{L}_\nu \Theta(t) \Theta(\mathcal{T} - t),
\]
where $\Theta$ is the Heaviside step function. For the possibly more realistic case of multiple pulses, it can be expressed as
\[
L_\nu(t) = \sum_{i = 1}^{N'} \mathcal{L}_\nu \Theta(t - \frac{N' t}{N'}) \Theta(\frac{N' t}{N'} \mathcal{T} + \delta t - t),
\]
where $N'$ is the number of bursts and $\delta t$ is the duration of one burst. $N' \delta t$ should be shorter than total duration $\mathcal{T}$, except in the case of a single pulse, $N' = 1$.

After the Fourier inversion, the form of $L_\nu(t)$ can be given as
\[
L_\nu(t) = \int_{-\infty}^{\infty} \tilde{L}_\nu(f) e^{-2\pi i f t} df,
\]
where $f$ is the frequency.

Considering an axisymmetric source, the only nonvanishing component of the GW amplitude of the NDAF is (see more details in Müller & Janka 1997)
\[
h_\nu(t) = \frac{2G}{3 D_L c^4} \int_{-\infty}^{t - D_L/c} L_\nu(t') dt'.
\]

Therefore, the local energy flux of GWs can be expressed as (e.g., Suwa & Murase 2009; Wei & Liu 2020)
\[
\frac{dE_{\text{GW}}}{D_L^2 \Omega dt} = \frac{c^3}{16\pi G} \left( \frac{d}{dt} h_\nu(t) \right)^2.
\]

The total GW energy can be written as
\[
E_{\text{GW}} = \frac{\beta G}{9 c^5} \int_{-\infty}^{\infty} L_\nu(t)^2 dt,
\]
with $\beta \sim 0.47039$. After deriving the previous equations, we can readily obtain the GW energy spectrum for the NDAF model, which is
\[
\frac{dE_{\text{GW}}(f)}{df} = \frac{2\beta G}{9 c^5} |\tilde{L}_\nu(f)|^2.
\]

To discuss the detectability of GWs, we write down the characteristic GW strain as
\[
h_\nu(f) = \sqrt{\frac{2}{\pi} \frac{G}{c^3} \frac{dE_{\text{GW}}(f)}{df}}.
\]

Finally, the signal-to-noise ratios ($S/N$) obtained through matched filtering from the GW experiments can also be derived; the optimal $S/N$ is
\[
(S/N)^2 = \int_{-\infty}^{\infty} \frac{h_\nu^2(f) \, df}{h_n^2(f) \, f},
\]
where $h_\nu(f) = [S_h(f)]^{1/2}$ is the noise amplitude, in which $S_h(f)$ is the spectral density of the strain noise.

3. Results

3.1. Disk Structure

The calculation results show that the vertical advection process plays an important role in shaping the physical properties of the disk. By solving Equations (1)–(15) above for NDAFs, we present the radial profiles of the disk density and temperature for the cases with and without vertical advection in Figure 3. Two typical accretion rates $M = 0.1$ and $1 M_\odot \, \text{s}^{-1}$ are indicated in the graphs by red and blue lines, respectively. It can be noted that when the accretion rate varies by different orders of magnitude, there are considerable differences in properties of the disk. For a given accretion rate and disk radius, the disk with vertical advection will have a higher density and lower temperature compared with the other.

The physical explanation is that, through the vertical advection process, a large quantity of trapped photons are radiated from the originally optically thick hyperaccretion disk due to the magnetic buoyancy. The energy is carried out by these gamma-ray photons, resulting in a lower temperature and lower radiation pressure, but a higher disk density. It is worth noting that such significant gamma-ray photons should be detectable, or they may inject into the electromagnetic counterpart in binary compact object merger events, contributing to the luminosity of kilonovae.
3.2. Kilonovae

The anisotropic emission that results from the different rates of energy injection of gamma-ray photons at different angles is the main focus in the present work, but first we discuss the case where there is no energy injection, i.e., a kilonova triggered by the radioactive decay of ejecta alone.

We show the viewing-angle-dependent uJKB-band light curves of a pure ejecta-driven kilonova in Figure 4 at a distance of 40 Mpc. The black, red, blue, and green lines indicate the observation angles $\theta_{\text{obs}} = 0^\circ$, $30^\circ$, $45^\circ$, and $90^\circ$, respectively. The K, J, r, and u bands are represented by the solid, dashed, dotted, and dashed–dotted lines, respectively. As we can see, the kilonova luminosity has typical magnitudes of no more than $\sim 19$ mag, and each band decreases correspondingly with the decrease in observation angle $\theta_{\text{obs}}$. That is because the mass and velocity of the ejecta change with the angle distribution. In fact, the biggest effect on the anisotropy is due to the difference in opacity at $\theta_{\text{obs}}$ as a boundary, i.e., the red and blue components of the kilonova, because the differences in the angle-dependent opacity amplify the contribution to the anisotropy. The peak is caused by the BC produced by low-opacity ejecta near the polar axis.

Next, we compare in Figure 5 the four cases of the presence of energy injection, i.e., kilonovae only driven by gamma-ray photons, and by ejecta radioactive decay and gamma-ray photons together at an accretion rate of 0.1 or 1 $M_\odot$ s$^{-1}$, respectively. One can find that a distinct common feature is that at the very early phase of the light curve there is a sharp bulge covering the original ejecta curve, where the luminosity increases dramatically and peaks at $\sim 1$ day. With an accretion rate of 0.1 $M_\odot$ s$^{-1}$, the peak magnitudes increase to over $18$ mag in the ultraviolet band and over $19$ mag in the infrared band. However, at an accretion rate of 1 $M_\odot$ s$^{-1}$, these values increase to 16 mag and 18 mag, respectively. After that, the luminosity curve decays rapidly and then goes into a smooth declining phase at $\sim 4$ days. Actually, comparing between panels (a) and (c) or (b) and (d) in Figure 5, the gamma-ray photons contribute throughout the whole light curve. It is worth noting that at higher accretion rates, the peak becomes more apparent. That is because the appearance of the peak structure in the light curves is related to the high value of the injected energy of gamma-ray photons that are released from the accretion disk. In the early stage, the optical depth is high, and the gamma-ray photons are mainly absorbed by the inner region of the ejecta, which means the radiative cooling of the ejecta occurs mainly in the outer region. That is, the early energy injection is mainly deposited in the inner layer. As the optical depth $\tau_{\text{tot}}$ decreases, these deposited energies will be released and the value of the energy injection is associated with the size of the bulge. By comparing Figures 5(a) and 5(c) with an accretion rate of 0.1 $M_\odot$ s$^{-1}$, we find the consistency of the early curves. The luminosity driven by the gamma-ray photons fades rapidly after $\sim 3$ days. Instead, in the later phases, the radioactive decay emission produced by the ejecta emerges and dominates the emission. The transition thus causes an “ankle” at $\sim 3$ days, as seen in the light curves, with later stages decaying more slowly than the earlier stages. This timescale corresponds to the diffusion timescale of the ejecta. After the transient injection, the energy needs to pass through layers of ejecta, from the innermost layer to the outermost layer, a process that is essentially determined by the mass and velocity of the ejecta. If we compare the case of an accretion rate of 1 $M_\odot$ s$^{-1}$, the energy of the radioactive decay is less important, the luminosity of the kilonova is increased considerably, and the whole process is practically dominated by the injected energy. The observed radiation would be mainly from the
gamma-ray photons. The trajectory of a photon changes due to the gravitational force of the BH, so the magnitude of the injected energy varies with the angle, which still shows anisotropy.

Figure 5 also shows the comparison between the light curve of AT2017gfo and our kilonova model with $\text{M} = 1 \, \text{M}_\odot \, \text{s}^{-1}$. One can notice that in the early phase, the model light curve is higher than the data, which is probably due to the overestimation of the efficiency of the transient injected energy or the production of the vertical advection cooling rates.

3.3. Neutrino Emission

The magnetic buoyancy produced by the vertical advection effect also leads to the escape of MeV neutrinos. We forecast the expected neutrino anisotropic radiation from the NDAF model. The neutrino energy is normally in the range of $\sim 1$–100 MeV, with a peak ranging from 10 to 20 MeV. Figure 6 shows the electron antineutrino spectra as a function of viewing angle of NDAFs with (solid lines) or without (dashed lines) vertical advection. The red, green, blue, and magenta lines correspond to $\theta_{\text{obs}} = 0^\circ$, $30^\circ$, $45^\circ$, and $90^\circ$, respectively. The energy of neutrinos is greater at higher accretion rates. The contribution of vertical advection in NDAFs to neutrino yields is very limited. In the absence of vertical advection, the peak of the neutrino emission is slightly shifted toward greater energy. In the high-energy range of the spectra, the neutrinos in the vertical advection model have a lower luminosity. This is due to the fact that neutrino emission is very sensitive to temperature, and similar to the results shown in Figure 3, the disk temperature is lower in the case with vertical advection, which leads to a lower reaction rate that generates neutrinos, even though a fraction of neutrinos are released by the bubbles. As a result, the neutrino energy is lower than in the case without vertical advection. Note that the optical depth of neutrinos is much smaller than that of the photons: the bubble actually cannot effectively confine many neutrinos inside. Therefore, the neutrinos are affected by the vertical advection effect, but only slightly compared to the photons. In addition, it is clear that the low-energy ($\lesssim 10$ MeV) neutrinos have a more anisotropic luminosity, and the amplitudes of the spectral lines decrease with increasing observation angles.

Figure 5. Viewing-angle-dependent $urJK$-band light curves of kilonovae at a distance of 40 Mpc for the cases of kilonovae driven by gamma-ray photons only and by ejecta radioactive decay and gamma-ray photons together at an accretion rate of $0.1$ or $1 \, \text{M}_\odot \, \text{s}^{-1}$. The black, red, blue, and green lines indicate the observation angles $\theta_{\text{obs}} = 0^\circ$, $30^\circ$, $45^\circ$, and $90^\circ$, respectively. The solid, dashed, dotted, and dashed–dotted lines represent the $K$, $J$, $r$, and $u$ bands, respectively. Panel (d): comparison between the $urJK$-band light curves for AT2017gfo and our kilonova model. The data are taken from Villar et al. (2017). The red, blue, magenta, and black dots represent the $K$, $J$, $r$, and $u$ bands, respectively.
3.4. GWs

Figure 7 shows the strains of GWs from NDAFs with or without vertical advection at a distance of 10 kpc for cases of SGRBs with a single pulse ($T \approx 1$ s) and multiple pulses ($T \approx 1$ s, $N' = 100$, and $\delta t = 0.001$ s) for the accretion rates $0.1$ and $1 M_\odot$ s$^{-1}$ (corresponding panels (a) and (b)). The black solid lines are the sensitivity curves (the noise amplitudes $h_n$) of aLIGO, ET, CE, LISA, Taiji, TianQin, DECIGO/BBO, and ultimate-DECIGO detectors. Comparing the cases with different accretion rates, when the accretion rate is higher, the more anisotropic neutrinos are produced on the accretion disk and the GW strain increases by several orders of magnitude. However, for the cases with identical accretion rates, the GW strain increases slightly without the vertical advection effect.

The typical GW frequencies are noted to be in the range of 1–1000 Hz, which is determined by the variability of the GRB. The GW waveforms caused by a single pulse and multiple pulses are quite different, which also results from the difference in the variability of the neutrino emission. It can be seen that the GWs (or memory) induced by the NDAF with vertical advection can be detected by ET, CE, DECIGO/BBO, and ultimate-DECIGO. This is essential to constrain the nature of the central engine, as well as the neutrino radiation. Since the GW strains depend entirely on the neutrino luminosity and the very limited contribution of vertical advection to neutrino yields (also see Figure 6), there is a tiny difference between the strains of GWs from NDAFs with and without vertical advection.

4. Summary

In the scenario of compact binary coalescences, a BH hyperaccretion disk is formed and is thought to be an NDAF because of the extreme physical conditions triggering the neutrino radiation. In particular, during the coalescences of a BH–NS or NS–NS, it is possible to detect multimessenger signal emission, such as SGRBs, kilonovae, neutrinos, and GWs, for which a theoretical prediction can be made and gives us a more comprehensive understanding of the source. In this paper, we model the NDAFs with an additional vertical
advection process and introduce the relativistic correction factors to correct the effect of the BH spin on the disk structure. We find that the vertical advection process slightly increases the disk density while decreasing the temperature. We present and analyze multiband ($uvwK$) kilonovae light curves derived from our model. We notice that the different masses, velocities, and opacity distributions of the ejecta are responsible for the anisotropy of the kilonovae, but the opacity causes the most significant effect. In addition, the gamma-ray photons escaping from the NDAF with vertical advection radiate anisotropically.

The GW signal from NDAFs with and without vertical advection at a distance of 10 kpc can be detected by CE, ET, ultimate-DECIGO, and DECIGO detectors. Although there are slight differences between neutrino spectra and GW shapes of NDAFs with and without vertical advection, the future joint multimessenger observations might distinguish them for sources in the local universe.

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**Abbreviations**

BBO. Brillouin scattering off backscattered photons.

**Abbreviation**

BBO Brillouin scattering off backscattered photons.
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