Some necessary and sufficient conditions for Hypercyclicity Criterion

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Abstract. We give necessary and sufficient conditions for an operator on a separable Hilbert space to satisfy the hypercyclicity criterion.

Keywords. Strong operator topology; Hilbert–Schmidt operators; Hypercyclicity Criterion.

1. Introduction

Suppose that $X$ is a separable topological vector space and $T$ is a continuous linear mapping on $X$. If $x \in X$, then the orbit of $x$ under $T$ is defined as $\text{Orb}(T,x) = \{x, Tx, T^2x, \ldots \}$. An operator $T$ is called hypercyclic if there is a vector $x$ such that $\text{Orb}(T,x)$ is dense in $X$ and in this case $x$ is called a hypercyclic vector for $T$ (see [14] for an exhaustive survey on hypercyclicity).

It is interesting that many continuous linear mappings can actually be hypercyclic. The first example of hypercyclicity appeared in the space of entire functions, by Birkhoff [3] in 1929. He showed the hypercyclicity of the translation operator, while MacLane [19] proved the hypercyclicity of the differentiation operator in 1952. Hypercyclicity on Banach spaces was discussed in 1969 by Rolewics [20], who showed that $\lambda B$ is hypercyclic whenever $B$ is the unilateral backward shift (on $\ell^p$ and $c_0$) and $|\lambda| > 1$.

A nice condition for hypercyclicity is the Hypercyclicity Criterion (Theorem 1.1 below), which was developed by Kitai [17] and independently by Gethner and Shapiro [12]. This criterion has been used to show that certain classes of composition operators [6], weighted shifts [21], adjoints of multiplication operators [7], and adjoints of subnormal and hyponormal operators [5], are hypercyclic. Hypercyclicity has also been established in various other settings by means of this criterion [14,68,12,13,16]. Salas [21] showed that every perturbation of the identity by a unilateral weighted backward shift with nonzero bounded weights is hypercyclic, and he also gave a characterization of the hypercyclic weighted shifts in terms of their weights. But, then Montes and Leon showed that these hypercyclic operators do satisfy the criterion as well (§2 of [17] and Proposition 4.3 of [13]). Bes and Peris proved that a continuous linear operator $T$ on a Frechet space satisfies the Hypercyclicity Criterion if and only if it is hereditarily hypercyclic. In particular they show that hypercyclic operators with either a dense generalized...
kernel or a dense set of periodic points must satisfy the criterion. Also, they provide a characterization of those weighted shifts $T$ that are hereditarily hypercyclic with respect to a given sequence $\{n_k\}$ of positive integers, as well as conditions under which $T$ and $\{T^{n_k}\}$ share the same set of hypercyclic vectors [2].

**Theorem 1.1 (The Hypercyclicity Criterion).** Suppose $X$ is a separable Banach space and $T$ is a continuous linear mapping on $X$. If there exists two dense subsets $Y$ and $Z$ in $X$ and a sequence $\{n_k\}$ such that:

1. $T^{n_k}y \to 0$ for every $y \in Y$,
2. there exists functions $S_{n_k}: Z \to X$ such that for every $z \in Z$, $S_{n_k}z \to 0$, and $T^{n_k}S_{n_k}z \to z$,

then $T$ is hypercyclic.

Note that the sequence $\{n_k\}$ in Theorem 1.1 need not be the entire sequence $\{n_k\} = \{k\}$ of positive integers. Salas [22] and Herrero [15] have shown that there are hypercyclic operators on Hilbert spaces that do not satisfy the Hypercyclicity Criterion for the entire sequence $\{k\}$, but so far no hypercyclic operator has been found that does not satisfy the Hypercyclicity Criterion in its general form. In this paper our work was stimulated by the well-known question: Does every hypercyclic operator satisfy the hypothesis of the Hypercyclicity Criterion? (see [2]).

We give necessary and sufficient conditions in terms of open subsets for an operator on a separable Hilbert space to satisfy the Hypercyclicity Criterion. For this, see Theorem 2.6, Corollary 2.11 and Proposition 2.12. Also, in the proof of Theorem 2.6, we pay attention to hypercyclicity on the operator algebra $B(H)$ and the algebra of Hilbert–Schmidt operators, $B_2(H)$. Recall that if $\{e_i\}$ is an orthonormal basis for a separable Hilbert space $H$, $A \in B(H)$ and

$$\|A\|_2 = \left( \sum_{i=1}^{\infty} \|Ae_i\|^2 \right)^{1/2},$$

then $\|A\|_2$ is independent of the basis chosen and hence is well-defined. If $\|A\|_2 < \infty$, then $A$ is called a Hilbert–Schmidt operator and by this norm $B_2(H)$ is a Hilbert space. Indeed, $B_2(H)$ is a special case of the Schatten $p$-class of $H$ when $p = 2$. For more details about these classes of operators, see [10,23].

Chan [9] showed that hypercyclicity can occur on the operator algebra $B(H)$ with the strong operator topology (SOT-topology) that is not metrizable. For example, when $T$ satisfies the Hypercyclicity Criterion, then the left multiplication operator $L_T$ is SOT-hypercyclic on $B(H)$, that is, $L_T$ is hypercyclic on $B(H)$ with strong operator topology.

### 2. Main results

From now on we suppose that $H$ is a separable infinite-dimensional Hilbert space.

**DEFINITION 2.1.**

Let $L: B(H) \to B(H)$ be linear and bounded. We say that $L$ is SOT-hypercyclic if there exist some $T \in B(H)$ such that the set $\text{Orb}(L, T) = \{T, LT, L^2T, \ldots\}$ is dense in $B(H)$ in the strong operator topology. Also we say that $L: B_2(H) \to B_2(H)$ is $\|\cdot\|_2$-hypercyclic if there exists some $T \in B_2(H)$ such that $\text{Orb}(L, T)$ is dense in $B_2(H)$ with $\|\cdot\|_2$-topology.
DEFINITION 2.2.

For any operator $T \in B(H)$, define the left multiplication operator $L_T: B(H) \to B(H)$ by $L_T(S) = TS$ for every $S \in B(H)$.

Note that $B_2(H)$ is an ideal of $B(H)$ and hence $L_T: B_2(H) \to B_2(H)$ is also well-defined. We show that $B_2(H)$ and $B_2(H)$, respectively with the strong operator topology and $|||·|||_2$-topology, are separable. For this, see the following Lemma 2.3.

Suppose $\{e_i; i \geq 1\}$ is an orthonormal basis for a separable Hilbert space $H$ and $S(H)$ denotes the set of all finite rank operators $T$ such that there exists $N_T \in \mathbb{N}$, satisfying $Te_i = 0$ for $i \geq N_T$.

**Lemma 2.3.** Suppose $E = \{e_i; i \geq 1\}$ is a basis for a separable Hilbert space $H$, then $S(H)$ is SOT-dense in $B(H)$ and also $|||·|||_2$-dense in $B_2(H)$; moreover, $S(H)$ is separable.

**Proof.** Suppose that $A \in B_2(H)$ and $\varepsilon > 0$. Then there exist $N \in \mathbb{N}$ such that $\sum_{i=N+1}^{\infty} |||Ae_i|||^2 < \varepsilon^2$. Now define the finite rank operator $F$ by $F = A$ on $\{e_k; 1 \leq k \leq N\}$ and $F = 0$ on $\{e_k; 1 \leq k \leq N\}$. Then $|||A - F|||^2 = \sum_{i=N+1}^{\infty} |||Ae_i|||^2 < \varepsilon^2$ and so $S(H)$ is $|||·|||_2$-dense. Also, of [9] p. 234 implies that every $|||·|||_2$-dense subset of $B_2(H)$ is SOT-dense in $B(H)$, and so it follows that $S(H)$ is SOT-dense. Now the proof is complete.

The following result is the main tool that we used to show that an operator is hypercyclic. Versions of this result have appeared in the work of Godefroy and Shapiro ([13], Theorem 1.2) and Kitai ([17], Theorem 2.1).

**PROPOSITION 2.4.**

If $T$ is a continuous operator on a separable Banach space $X$, then $T$ is hypercyclic if and only if for any two non-void open sets $U$ and $V$ in $X$, $T^nU \cap V \neq \emptyset$ for some positive integer $n$.

Godefroy and Shapiro ([13], Corollary 1.3) also gave a sufficient condition for hypercyclicity that is a direct consequence of Proposition 2.4.

**COROLLARY 2.5.**

An operator $T$ on a separable Banach space $X$ is hypercyclic if for each pair $U, V$ of non-void open subsets of $X$, and each neighborhood $W$ of zero in $X$, there are infinitely many positive integers $n$ such that both $T^nU \cap W$ and $T^nW \cap V$ are non-empty.

**Remarks.**

(i) In Proposition 2.4, the condition $T^nU \cap V \neq \emptyset$ is equivalent to the condition $U \cap T^{-n}V \neq \emptyset$.

(ii) If an operator $T$ is hypercyclic, then it automatically has a dense set of hypercyclic vectors. For, if a vector $x$ is hypercyclic for $T$, then so is $T^nx$ for any positive integer $n$. Thus the condition ‘$T^nU \cap V \neq \emptyset$ for some positive integer $n$’, in Proposition 2.4, can be replaced by the condition ‘$T^nU \cap V \neq \emptyset$ for infinitely many positive integers $n$’.

(iii) Equivalent to the hypothesis of Corollary 2.5 is the apparently weaker requirement that the sets $T^nU \cap W$ and $T^nW \cap V$ be non-empty for a single $n$. 
The following theorem shows that the converse of the above corollary is equivalent to the Hypercyclicity Criterion. Remember that for vectors \( g, h \) in \( H \) the operator \( g \otimes h \) denotes a rank one operator and is defined by \( (g \otimes h)(f) = \langle f, h \rangle g \).

**Theorem 2.6.** For any operator \( T \in B(H) \), the following are equivalent:

(i) \( T \) satisfies the hypothesis of the Hypercyclicity Criterion.

(ii) For each pair \( U, V \) of non-void open subsets of \( H \), and each neighborhood \( W \) of zero, \( T^nU \cap W \neq \emptyset \) and \( T^nW \cap V \neq \emptyset \) for some integer \( n \).

**Proof.** It is easy to see that (i) implies (ii) (for details see Corollary 1.4 in [7]). For the converse, assume that \( T \) satisfies property (ii). First we show that for each pair \( U', V' \) of non-void \( \| \cdot \|_2 \)-open subsets of \( B_2(H) \) there is an integer \( n \geq 1 \) such that \( U' \cap L^2 \delta V' \neq \emptyset \). For this, fix an orthonormal basis \( E = \{e_i; i \geq 1\} \) for \( H \). By using Lemma 2.3 there exist finite rank operators \( A \) and \( B \) such that \( A \in S(H) \cap U' \) and \( B \in S(H) \cap V' \), whence for a certain integer \( n \geq 1 \) we have \( A(e_i) = B(e_i) = 0 \) for \( i > N \). But for some \( \varepsilon > 0 \) we have

\[
\{D \in S(H): \|D - A\|_2 < \varepsilon\} \subseteq S(H) \cap U',
\]

and

\[
\{D \in S(H): \|D - B\|_2 < \varepsilon\} \subseteq S(H) \cap V'.
\]

Now consider the following open sets:

\[
U_i = \left\{h \in H: \|h - Ae_i\| < \frac{\varepsilon}{\sqrt{N}}\right\},
\]

\[
V_i = \left\{h \in H: \|h - Be_i\| < \frac{\varepsilon}{\sqrt{N}}\right\}
\]

for \( i = 1, 2, \ldots, N \). Note that Corollary 2.5 or remark (iii) implies that \( T \) is hypercyclic. Now by using Proposition 2.4 repeatedly (indeed by remark (ii)), it follows that there exist integers \( 0 = n_0 < n_1 \leq n_2 \leq \cdots \leq n_{N-1} \) and \( 0 = m_0 < m_1 \leq m_2 \leq \cdots \leq m_{N-1} \) such that

\[
U = U_1 \cap T^{-m_1}U_2 \cap T^{-m_2}U_3 \cap \cdots \cap T^{-m_{N-1}}U_N \neq \emptyset
\]

(1)

and

\[
V = V_1 \cap T^{-m_1}V_2 \cap T^{-m_2}V_3 \cap \cdots \cap T^{-m_{N-1}}V_N \neq \emptyset.
\]

(2)

Put \( W = \{h: \|h\| < \delta\} \) where

\[
\delta = \min \left\{\frac{\varepsilon}{2\sqrt{N}\|T\|^{n_i-1}}, \frac{\varepsilon}{2\sqrt{N}\|T\|^{m_i-1}}; i = 1, 2, \ldots, N\right\}.
\]

(3)

Since \( T \) satisfies the hypothesis (ii) of Theorem 2.6, then there exists some \( x \in W \) and \( y \in U \) such that \( T^n x \in V \) and \( T^n y \in W \) for some integer \( n \). The relations (1) and (2) imply that

\[
\|T^{n_i-1}y - Ae_i\| < \frac{\varepsilon}{2\sqrt{N}}, \quad \|T^n(T^{m_i-1}x) - Be_i\| < \frac{\varepsilon}{2\sqrt{N}}
\]

(4)

for \( i = 1, 2, \ldots, N \). Now define \( S_1 = \sum_{i=1}^N T^{n_i-1}y \otimes e_i \) and \( S_2 = \sum_{i=1}^N T^{m_i-1}x \otimes e_i \). Let \( S = S_1 + S_2 \). Then \( S \) is a Hilbert–Schmidt operator, because it has finite rank. Note that by
(3) \( \|T^{m_i-1}x\| \leq \|T\|^{m_i-1}\|x\| < \delta \|T\|^{m_i-1} < \frac{\epsilon}{2\sqrt{N}} \). Now by using (4) we get the following inequalities:
\[
\|S - A\|_2 \leq \|S_1 - A\|_2 + \|S_2\|_2 \\
= \left\{ \sum_{i=1}^{N} \|S_ie_i - Ae_i\|^2 \right\}^{1/2} + \left\{ \sum_{i=1}^{N} \|S_ie_i\|^2 \right\}^{1/2} \\
= \left\{ \sum_{i=1}^{N} \|T^{m_i-1}y - Ae_i\|^2 \right\}^{1/2} + \left\{ \sum_{i=1}^{N} \|T^{m_i}x\|^2 \right\}^{1/2} < \epsilon.
\]
Hence \( S \in U' \). Also note that since \( T^n y \in W \), by (3) we get \( \|T^{m_i-1}(T^n y)\| \leq \|T\|^{m_i-1}\delta < \frac{\epsilon}{2\sqrt{N}} \), and thus we have
\[
\|L_T^n S - B\|_2 \leq \|L_T^n S_2 - B\|_2 + \|L_T^n S_1\|_2 \\
= \left\{ \sum_{i=1}^{N} \|T^n S_2 e_i - Be_i\|^2 \right\}^{1/2} + \left\{ \sum_{i=1}^{N} \|T^n S_1 e_i\|^2 \right\}^{1/2} \\
= \left\{ \sum_{i=1}^{N} \|T^n(T^{m_i-1}x) - Be_i\|^2 \right\}^{1/2} + \left\{ \sum_{i=1}^{N} \|T^{m_i-1}(T^n y)\|^2 \right\}^{1/2} < \epsilon.
\]
So \( L_T^n S \in V' \). Now it follows that \( U' \cap L_T^n V' \neq \phi \) and so by Proposition 2.4, \( L_T \) is \( \|\|_2 \)-hypercyclic. This also implies that \( \bigoplus_{n=1}^{\infty} T: \bigoplus_{n=1}^{\infty} H \rightarrow \bigoplus_{n=1}^{\infty} H \) is hypercyclic, because the left multiplication operator \( L_T: B_2(H) \rightarrow B_2(H) \) is unitary equivalent to the operator \( \bigoplus_{n=1}^{\infty} T: \bigoplus_{n=1}^{\infty} H \rightarrow \bigoplus_{n=1}^{\infty} H \) (see [11], p. 6). Now Theorem 2.3 in [2] implies that \( T \) satisfies the Hypercyclicity Criterion, and so the proof is now complete.

PROPOSITION 2.7.
If \( T \in B(H) \), then the following are equivalent:

(i) \( T \) satisfies the hypothesis of the Hypercyclicity Criterion.
(ii) \( T \) is hypercyclic and for each non-void open subset \( U \) and each neighborhood \( W \) of zero, \( T^n U \cap W \neq \phi \) and \( T^{-n} U \cap W \neq \phi \) for some integer \( n \).

Proof. By Theorem 2.6 it suffices to show that (ii) implies (i). So let (ii) hold. By Theorem 2.6, it suffices to show that (ii) in Theorem 2.6 holds. Since \( T \) is hypercyclic, by Proposition 2.4, \( U \cap T^{-m} V \neq \phi \) for some positive integer \( m \). Let \( G \) be a neighborhood of zero that is contained in \( W \cap T^{-m} W \). By condition (ii), there exists some positive integer \( n \) such that \( T^{-n} G \cap (U \cap T^{-m} V) \neq \phi \) and \( G \cap T^{-n}(U \cap T^{-m} V) \neq \phi \). But \( T^{-n} G \cap (U \cap T^{-m} V) \) is a subset of \( T^{-m} W \cap U \), hence \( T^{-m} W \cap U \neq \phi \). Also \( G \cap T^{-n}(U \cap T^{-m} V) \) is a subset of \( T^{-m} W \cap T^{-n}(T^{-m} V) = T^{-m}(W \cap T^{-n} V) \) which implies that \( T^{-n} V \cap W \neq \phi \). Thus, hypothesis (ii) of Theorem 2.6 holds and so the proof is complete.
Remark 2.8. We say that the sequence \( \{ T_n \}_{n=1}^{\infty} \) of bounded linear operators on a Hilbert space \( H \) is hypercyclic provided that there exists some \( x \in H \) such that the collection of images \( \{ T_n x : n = 1, 2, \ldots \} \) is dense in \( H \). Note that Theorem 1.1, Proposition 2.4 and Corollary 2.5 can be extended to the case where hypercyclicity of \( T \) is replaced by hypercyclicity for the sequence \( \{ T_n \}_{n=1}^{\infty} \) of bounded linear operators that have dense range. In particular we say that \( \{ T_n \}_{n=1}^{\infty} \) satisfies the hypothesis of the Hypercyclicity Criterion if in the hypothesis of Theorem 1.1, we use \( T_{nk} \) instead of \( T^n \). It also implies that if the sequence \( \{ T_n \}_{n=1}^{\infty} \) satisfies the hypothesis of the Hypercyclicity Criterion, then \( \{ T_n \}_{n=1}^{\infty} \) is hypercyclic (see Theorem 1.2, Corollaries 1.3 and 1.5 in [13]).

It is not difficult to see that Theorem 2.6 and Proposition 2.7 work for the sequence \( \{ T_n \}_{n=1}^{\infty} \) of bounded linear operators provided that \( T_n T_m = T_m T_n \) for each pair \( m, n \) of positive integers. Hence we can deduce the following corollary.

**Corollary 2.9.**
Suppose that \( \{ T_n \}_{n=1}^{\infty} \) is a sequence of bounded linear operators on a Hilbert space \( H \) such that \( T_n T_m = T_m T_n \) for each pair \( m, n \) of positive integers and have dense range. Then the following are equivalent:

(i) \( \{ T_n \}_{n=1}^{\infty} \) satisfies the hypothesis of the Hypercyclicity Criterion.
(ii) For each pair \( U, V \) of non-void open subsets of \( H \), and each neighborhood \( W \) of zero, \( T_n U \cap W \neq \emptyset \) and \( T_n W \cap V \neq \emptyset \) for some integer \( n \).
(iii) \( \{ T_n \}_{n=1}^{\infty} \) is hypercyclic and for each non-void open subset \( U \) and each neighborhood \( W \) of zero, \( T_n U \cap W \neq \emptyset \) and \( T_n^{-1} U \cap W \neq \emptyset \) for some integer \( n \).

The following definition is introduced in [2].

**Definition 2.10.**
Suppose that \( T \in B(H) \) and \( \{ n_k \} \) is a sequence of positive integers. We say that \( T \) is hereditarily hypercyclic with respect to \( \{ n_k \} \) if for any subsequence \( \{ n_{k_m} \} \) of \( \{ n_k \} \), the sequence \( \{ T^{n_{k_m}} \} \) is hypercyclic.

Now we summarize all necessary and sufficient conditions for the Hypercyclicity Criterion in the following corollary.

**Corollary 2.11.**
For any operator \( T \in B(H) \), the following are equivalent:

(i) \( T \) satisfies the hypothesis of the Hypercyclicity Criterion.
(ii) \( T \) is hereditarily hypercyclic with respect to a subsequence \( \{ n_k \} \) of positive integers.
(iii) \( \bigoplus_{k=1}^{\infty} T \) is hypercyclic on \( \bigoplus_{k=1}^{\infty} H \).
(iv) The left multiplication operator \( L_T : B_2(H) \to B_2(H) \) is \( \| \cdot \|_2 \)-hypercyclic.
(v) For each pair \( U, V \) of non-void open subsets of \( X \), and each neighborhood \( W \) of zero, \( T^n U \cap W \neq \emptyset \) and \( T^n W \cap V \neq \emptyset \) for some integer \( n \).

**Proof.** The proof is an immediate consequence of Theorem 2.6, Proposition 2.7 and Theorem 2.3 in [2].
The following proposition represents some relation between hypercyclicity and the
Hypercyclicity Criterion.

**PROPOSITION 2.12.**

For any operator $T \in B(H)$ the following are equivalent:

(i) $T$ satisfies the hypothesis of the Hypercyclicity Criterion.

(ii) There exists a dense subset $Y$ in $X$ and a sequence $\{n_k\}$ such that $\{T^{n_k}\}$ is hypercyclic and $T^{n_k}y \to 0$ for every $y \in Y$.

(iii) There exists a sequence $\{n_k\}$ such that for each pair $U, V$ of non-void open subsets of $H$, there is $N \geq 1$ such that $T^{n_k}U \cap V \neq \emptyset$ for any $k \geq N$.

**Proof.**

(i) $\to$ (ii): It follows from condition (ii) of Corollary 2.11.

(ii) $\to$ (i): Let $T_k = T^{n_k}$, $U$ be any non-void open set and also let $W$ be any open neighborhood of zero. Then by Remark 2.8, $\{T_k\}_k$ is hypercyclic and so there is some sequence $\{m_k\}$ of positive integers such that $T_{m_k}W \cap U \neq \emptyset$ for every $k \geq 1$. Now if $y \in U \cap Y$, then $T_{m_k}y = T^{m_k}y \to 0$ which yields $T_{m_k}U \cap W \neq \emptyset$. It holds condition (iii) of Corollary 2.11, hence $\{T_k\}$ satisfies the hypothesis of the Hypercyclicity Criterion and so $\{T^{n_k}\}$ and consequently $T$ satisfy the Hypercyclicity Criterion.

(iii) $\to$ (i): It suffices to show that condition (iii) implies condition (v) of Corollary 2.11. For this let $U, V$ be a pair of non-void open subsets of $H$ and $W$ be any neighborhood of zero. Then for some integer $N$, we have

$$T^{n_k}U \cap W \neq \emptyset; \quad T^{n_k}W \cap V \neq \emptyset$$

for any $k > N$. Thus indeed condition (v) of Corollary 2.11 is consistent.

(i) $\to$ (iii): Note that by condition (ii) of Corollary 2.11, $T$ is hereditarily hypercyclic with respect to a sequence $\{n_k\}$ of positive integers. Now suppose that (iii) does not hold. So there exist some pair $U, V$ of non-void open sets such that $T^{n_k}U \cap V = \emptyset$ for some subsequence $\{n_{k_m}\}$ of $\{n_k\}$. But $\{T^{n_{k_m}}\}$ is hypercyclic and so it is a contradiction. Hence for every pair $U, V$ of non-void open sets, there is $N \geq 1$ such that $T^{n_k}U \cap V \neq \emptyset$ for any $k \geq N$. The proof is now complete.

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**References**

[1] Aron R and Bes J, Hypercyclic differentiation operators, Function Spaces, Contemporary Mathematics, *Am. Math. Soc.* (Providence, RI) (1999) vol. 232, pp. 39–46

[2] Bes J and Peris A, Hereditarily hypercyclic operators, *J. Func. Anal.* 167(1) (1999) 94–112.

[3] Birkhoff G, Demonstration dun theoreme sur les fonctions entieres, *C. R. Acad. Sci. Paris* 189 (1929) 473–475
Bonet J and Peris A, Hypercyclic operators on non-normable Frechet spaces, J. Funct. Anal. 159 (1998) 587–595

Bourdon P S, Orbits of hyponormal operators, Mich. Math. J. 44 (1997) 345–353

Bourdon P S and Shapiro J H, Cyclic phenomena for composition operators, Memoirs of the Amer. Math. Soc., Am. Math. Soc. (Providence, RI) (1997) vol. 125

Bourdon P S and Shapiro J H, Hypercyclic operators that commute with the Bergman backward shift, Trans. Am. Math. Soc. 352(11) (2000) 5293–5316

Chan K C and Shapiro J H, The cyclic behaviour of translation operators on Hilbert spaces of entire functions, Indiana Univ. Math. J. 40 (1991) 1421–1449

Chan K C, Hypercyclicity of the operator algebra for a separable Hilbert space, J. Operator Theory 42 (1999) 231–244

Conway J B, The theory of subnormal operators, Mathematical Surveys and Monographs, American Mathematical Society, 1991

Curto R, Spectral theory of elementary operators, in: Elementary operators and Applications (ed.) Martin Mathiea (World Scientific) (1992)

Gethner R M and Shapiro J H, Universal vectors for operators on spaces of holomorphic functions, Proc. Am. Math. Soc. 100 (1987) 281–288

Godefroy G and Shapiro J H, Operators with dense invariant cyclic manifolds, J. Funct. Anal. 98 (1991) 229–269

Grosse-Erdmann K-G, Universal families and hypercyclic operators, Bull. Am. Math. Soc. 36 (1999) 345–381

Herrero D A, Limits of hypercyclic and supercyclic operators, J. Funct. Anal. 99 (1991) 179–190

Herzog G and Schomoeger C, On operators $T$ such that $f(T)$ is hypercyclic, Studia Math. 108 (1994) 209–216

Kitai C, Invariant closed sets for linear operators (Dissertation, Univ. of Toronto) (1982)

Leon-Saavedra F and Montes-Rodriguez A, Linear structure of hypercyclic vectors, J. Funct. Anal. 148 (1997) 524–545

MacLane G R, Sequences of derivatives and normal families, J. D. Analyse Math. 2 (1952) 72–87

Rolewicz S, On orbits of elements, Studia Math. 32 (1969) 17–22

Salas H N, Hypercyclic weighted shifts, Trans. Am. Math. Soc. 347 (1995) 993–1004

Salas H, A hypercyclic operator whose adjoint is also hypercyclic, Proc. Am. Math. Soc. 112 (1991) 765–770

K Zhu, Operator theory in function spaces (New York: Marcel Dekker, Inc.) (1990)