The three-loop Yang–Mills condensate dark energy model and its cosmological constraints

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Abstract. This work is a comprehensive investigation of the Yang–Mills condensate (YMC) dark energy (DE) model, which is extended to include the three-loop quantum corrections. We study its cosmic evolution and the possibility of crossing the phantom divide \( w = -1 \), examine in detail the Hubble parameter \( H \), the deceleration parameter \( q \), the statefinder \((r, s)\) diagnostic and the \( w - w' \) diagnostic for the model without and with interaction, and compare our results with other DE models. Also, using the observational data for type Ia supernovae (SNIa), the shift parameter from the cosmic microwave background (CMB), and the baryon acoustic oscillation peak from large scale structures (LSS), we give the cosmological constraints on the three-loop YMC model. It is found that the model can solve the coincidence problem naturally, and its prediction of the aforementioned parameter is much closer to the \( \Lambda \)CDM (CDM: cold dark matter) model one than those from other dynamical DE models; the introduction of the matter–DE interaction will make the YMC model deviate from the \( \Lambda \)CDM model, and will give an equation of state crossing \(-1\). Moreover, it is also found that, for fitting the latest SNIa data alone, the \( \Lambda \)CDM model is slightly better than the three-loop YMC model; but in fitting the combination of SNIa, CMB and LSS data, the three-loop YMC model performs better than the \( \Lambda \)CDM model.

Keywords: dark energy theory, supernova type Ia

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1. Introduction

Observations on the type Ia supernovae [1], CMB anisotropies [2] and large scale structure [3] all indicate the existence of mysterious dark energy that is driving the current accelerating expansion of the Universe. In efforts to interpret the physics of dark energy, there have been a large number of models proposed. The simplest one is the ΛCDM model, which can fit the observations so far, but is plagued with the fine-tuning problem and the coincidence problem [4]. While the former problem exists for almost all the DE models, the latter one can be solved in the framework of dynamical DE models. Among these are quintessence [5], phantom [6], $k$-essence [7], quintom [8], tachyonic [9], holographic [10], and ‘agegraphic’ [11] models. There are also other interesting models based either on effective gravity [12] or on the Born–Infeld quantum condensate [13]. In our previous works [14] a dynamical model is proposed, in which the renormalization-group improved effective YMC serves as the dark energy; and in a recent work [15], this model was extended to the two-loop quantum corrections. Unlike the scalar models, our model is based on a vector-type quantum effective Yang–Mills (YM) field and does not suffer from the difficulties of scalar models mentioned in [16]. The effective YMC in our model is a coherent boson field system at low temperatures, and the DE is viewed as the ground state energy of this YMC field. As is known, a quantum system of bosons with a conservative ‘charge’ (such as the particle number, the electric charge, and the color in QCD) will experience Bose–Einstein condensation when the temperature is low enough or the charge density is high enough. This applies to many systems, such as Q-balls [17, 18], charged relativistic scalar bosons [19, 20], and the gluon condensate in the effective QCD models [21, 22]. And this is the physical origin of our Yang–Mills condensate model. From the viewpoint of quantum effective field theory at low temperatures, it would be beneficial if one could include high order quantum corrections, as much as possible. In this work we will extend the YMC DE model to the three-loop quantum corrections, focusing on its cosmic evolution and the issue of crossing $w = -1$.

As an important next step, one needs to confront DE models with observational data. In a recent work, by using the differential ages of passively evolving galaxies, Simon et al gave nine observational $H(z)$ data points in the range $0 \leq z \leq 1.8$ [23], which have
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been used to constrain various DE models [24, 25]. Besides, the deceleration parameter \(q(z)\), containing the second-order derivative of the scale factor \(a(t)\), is also important in confronting DE models [26–28]. Moreover, a new geometrical diagnosis pair \((r, s)\), called the statefinder diagnostic, has been introduced to distinguish DE models [29]. Since the pair \((r, s)\) contains the third-order derivative of \(a(t)\) and their values are expected to be available from the future SNAP observation [30], statefinder diagnosis has attracted a lot of attention [31–41]. Finally, another dynamical diagnostic, \(w - w'\), involving the EOS and its time derivative, is also extensively used in the literature [42–51]. In viewing these different diagnosis methods, we shall present a comprehensive analysis for \(H(z)\), \(q(z)\), \((r, s)\) and \(w - w'\), respectively, in the three-loop YMC model. As regards the issues listed above, it is more common to test various DE models by using the observational data for SNIa. In particular, in a recent work, Riess et al released the up-to-date 182 gold sample of SNIa [52], which has been used in [53–55]. In addition, as useful complements to data from SNIa, the shift parameter \(R\) from CMB observations [56, 57] and the baryon acoustic oscillation (BAO) peak parameter \(A\) from LSS [58] are also crucial for constraining various DE models. So in this work, we shall utilize a combination of SNIa, CMB, and LSS data to give the cosmological constraints on our model.

The organization of this paper is as follows. In section 2, we extend our previously proposed YMC DE model to the three-loop quantum corrections, present its prediction of cosmic evolution, and explore the possibility of crossing the phantom divide \(w = -1\). In section 3, we study the Hubble parameter \(H\), the deceleration parameter \(q\), the statefinder diagnostic \((r, s)\), and the \(w - w'\) diagnostic in the three-loop YMC model, and compare our results with those from other DE models. In section 4, the observational SNIa, CMB, and LSS data are employed to give the cosmological constraints on the three-loop YMC model. Section 5 is a short summary. In this paper, units with \(c = \hbar = 1\) are used.

2. The three-loop YMC model

In the renormalization-group improved effective YM field theory [59, 60], the running coupling constant up to the three-loop level [61, 62], should have the following form:

\[
g^2(F) = \frac{1}{b} \left[ \frac{1}{\tau} - \frac{\ln |\tau|}{\tau^2} + \eta^2 \frac{\ln^2 |\tau| - \ln |\tau| + C}{\tau^3} \right] + O \left( \frac{1}{\tau^3} \right),
\]

(1)

where \(\tau \equiv \ln |F/\epsilon \kappa^2|\), \(F \equiv -\frac{1}{2} F_{\mu \nu}^a F^{a \mu \nu} = E^2 - B^2\) plays the role of the order parameter of the YMC, and the parameter \(\kappa\) is the renormalization scale with the dimension of squared mass. For the gauge group \(SU(N)\) without fermions, \(b = 11N/3(4\pi)^2\), \(\eta \equiv 2\beta_1/\beta_0^2 \simeq 0.8\). Here \(C \equiv 8\beta_0\beta_2/\beta_1^2 - 1\), and the numerical coefficients \(\beta_0, \beta_1, \beta_2\) are given in [62]. It should be stressed that, although the one-loop and two-loop corrections are uniquely fixed, the three-loop correction to \(g^2\) is renormalization-scheme dependent, and so is the coefficient \(C\) [63]. Notice that the Lorentz invariance is preserved in the effective YM theory, because the Lagrangian is constructed out of combinations of \(F_{\mu \nu}^a F^{a \mu \nu}\) [21, 22, 64]. For simplicity, we only discuss the case of a pure ‘electric’ condensate with \(F = E^2\) (the case including a magnetic component was discussed in [65]). The effective Lagrangian, defined as \(L_{\text{eff}} = F/2g^2(F)\), is given by

\[
L_{\text{eff}} = \frac{1}{2} b \kappa e^y \left[ (y - 1) + \eta \ln |y - 1 + \delta| - \eta^2 \frac{\ln^2 |y - 1 + \delta| - \ln |y - 1 + \delta|}{y - 1 + \delta} \right],
\]

(2)

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where the variable \( y \equiv \tau + 1 = \ln |F/\kappa^2| \), and \( \delta \), a dimensionless parameter, represents the higher order corrections, including terms such as \( C/\tau^3 \) in equation (1). In the bracket of equation (2) the \( \eta \) term is the two-loop contribution, and the \( \eta^2 \) term is the three-loop one, which adds no new parameter other than the two-loop model one. Notice that the YM field introduced here is not the gluon field of QCD, nor the gauge boson field of the electroweak unification. From the effective Lagrangian in equation (2) follow the energy density and the pressure of the YMC DE:

\[
\rho_y = \frac{1}{2} b \kappa^2 e^y \left[ (y + 1) + \eta (Y_1 + 2Y_2) - \eta^2 (Y_3 - 2Y_4) \right],
\]

(3)

\[
p_y = \frac{1}{6} b \kappa^2 e^y \left[ (y - 3) + \eta (Y_1 - 2Y_2) - \eta^2 (Y_3 + 2Y_4) \right],
\]

(4)

where

\[
Y_1 \equiv \ln |y - 1 + \delta|, \quad Y_2 \equiv \frac{1}{y - 1 + \delta},
\]

(5)

\[
Y_3 \equiv (Y_1 - 1)Y_1Y_2, \quad Y_4 \equiv (Y_1 - 3)Y_1Y_2^2.
\]

(6)

As a consistency check, the well-known conformal trace anomaly [21,66]

\[
T^\mu_\mu = \rho_y - 3p_y = 2F \frac{d}{d\tau} \left[ \frac{1}{g^2(F)} \right]
\]

(7)

is satisfied up to the three-loop level. It is known that the trace anomaly occurs as a quantum effect of the vacuum and only violates the traceless condition of the stress tensor \( T_{\mu\nu} \) without violating the Lorentz invariance. Also the form of the stress tensor \( T_{\mu\nu} \) of YM fields is consistent with homogeneity and isotropy of the Universe. The EOS for the YMC is given by

\[
w = \frac{p_y}{\rho_y}.
\]

(8)

If one ignores the \( \eta^2 \) terms from equation (1) to equation (4), the two-loop model [15] is obtained, and if one further sets \( \eta = 0 \), the one-loop model [14] is recovered.

In our model the Universe is filled with three major kinds of energy components: the dark energy represented by the YMC, the matter (baryons and dark matter), and the radiation (consisting of CMB and other massless particles). The overall cosmic expansion is determined by the Friedmann equation

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} (\rho_y + \rho_m + \rho_r),
\]

(9)

where \( \rho_m \) is the energy density of matter, and \( \rho_r \) is that of radiation. The dynamical evolutions of the three components are given by

\[
\dot{\rho}_y + 3\frac{\dot{a}}{a}(\rho_y + p_y) = -\Gamma \rho_y,
\]

(10)

\[
\dot{\rho}_m + 3\frac{\dot{a}}{a} \rho_m = \Gamma \rho_y,
\]

(11)

\[
\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 0,
\]

(12)
where \( p_r \) is the radiation pressure, \( \Gamma \) is the rate of decay of the YMC into matter, a parameter of the model. If \( \Gamma = 0 \), the YMC does not couple to matter; if \( \Gamma > 0 \), the interaction term \( \Gamma \rho_y \) in equations (10) and (11) represents the rate of energy transfer from the YMC to matter. The sum of equations (10)–(12) guarantees that the total energy of the three components is still conserved. Replacing the old variables \( t, \rho_m, \rho_r, \) and \( \rho_y \) with new variables \( N \equiv \ln a(t), x \equiv \rho_m/(2\pi^2 b n^2), \) \( r \equiv \rho_r/(2\pi^2 b n^2), \) and \( y, \) and, making use of the Friedmann equation at \( z = 0 \), the set of equations (10)–(12) take the following form:

\[
\frac{dy}{dN} = -\frac{4[y + \eta(Y_1 + Y_2) - \eta^2(Y_3 - Y_4)] + (\Gamma_0/H_0 \zeta)[(y + 1) + \eta(Y_1 + 2Y_2) - \eta^2(Y_3 - 2Y_4)]}{(y + 2) + \eta(Y_1 + 3Y_2 - 2Y_4) - \eta^2(Y_3 - 3Y_4 + 4(Y_1 - 4)Y_1 Y_2)},
\]

(13)

\[
\frac{dx}{dN} = \frac{\Gamma_0}{H_0 \zeta} e^{\eta y}[(y + 1) + \eta(Y_1 + 2Y_2) - \eta^2(Y_3 - 2Y_4)] - 3x,
\]

(14)

\[
\frac{dr}{dN} = -4r,
\]

(15)

where

\[
\zeta = \sqrt{e^{\eta y}[(y + 1) + \eta(Y_1 + 2Y_2) - \eta^2(Y_3 - 2Y_4)] + x + r},
\]

(16)

\( \zeta_0 = \zeta(z = 0) \), and \( H_0 = H(z = 0) \). Once the parameters \( \Gamma \) and \( \delta \), as well as the initial conditions, are specified, the solution of this set of equations follows immediately. As our calculation will show, for the YMC to be a sensible model of dynamical DE, the order of magnitude of the decay rate \( \Gamma \) should be less than or at most of the order of the expansion rate, i.e., \( \Gamma \leq H_0 \). To be specific, we take the decay rate \( \Gamma = 0.31H_0 \) and the parameter \( \delta = 4 \). The initial conditions for equations (13)–(15) are chosen at a very high redshift \( z_i = 10^{10} \) during the big bang nucleosynthesis (BBN) era. To ensure radiation–matter equality occurring at a redshift \( z = 3454 \) [2], the initial radiation and matter are taken as

\[
x_i = 1.22 \times 10^{20}, \quad r_i = 3.52 \times 10^{35}.
\]

(17)

Besides, to ensure that the BBN occurs as usual [67], the initial YMC fraction should be \( \sim 10\% \) or less [15]; for concreteness we take the upper limit

\[
y_i \leq 74, \quad \text{i.e., } \frac{\rho_y}{\rho_r} \leq 3 \times 10^{-2}.
\]

(18)

In figure 1, we plot the dynamical evolutions of \( \rho_y, \rho_m, \) and \( \rho_r \) in the three-loop YMC model without and with interaction. For a whole range of initial \( y_i = (1, 74) \), which corresponds to the initial energy fraction of the YMC \( \rho_y/\rho_r \simeq (2.5 \times 10^{-35}, 3 \times 10^{-2}) \), ranging over \( \sim 33 \) orders in magnitude, the YMC always has a desired tracking solution, i.e., during the radiation era the YMC follows the radiation as \( y \propto r \propto a(t)^{-4} \), then during the matter era it follows the matter approximately as \( y \propto \rho_m \propto a(t)^{-3} \). Rather later, around \( z \simeq 0.5 \), it becomes dominant, and then it levels off and becomes asymptotically a constant for \( z \leq 0 \). The existence of a tracking solution can be analytically proved, also. For any DE model, the energy density and the pressure of
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Figure 1. Dynamical evolution of $\rho_y$, $\rho_m$, and $\rho_r$ in the three-loop YMC model without and with interaction.

DE can be written as

$$\rho_y = E_k + V(y), \quad p_y = E_k - V(y),$$

where $E_k$ denotes the kinetic energy, and $V(y)$ denotes the potential energy. For our model, one can easily obtain $V$ from equations (3) and (4). Following [68], we introduce a key function

$$\Delta \equiv \frac{V''V}{(V')^2},$$

whose properties determine whether tracking solutions exist. The prime means the derivative with respect to $y$. Steinhardt et al [68] had demonstrated that the tracking solutions exist if: (1) $\Delta$ is nearly constant, i.e., $|\Delta^{-1}(d(\Delta - 1)/Hdt)| \approx |\Delta'/(\Delta V'/V)| \ll 1$; and (2) $\Delta > 1$ for $w < w_B$ or $\Delta < 1$ for $w_B < w < (1/2)(1 + w_B)$. (Here $w_B$ denotes the EOS of the background component.) Our calculation shows that for the range $y > 1$ in the YMC model, $|\Delta'/(\Delta V'/V)| \ll 1$ is always satisfied; besides, $w < w_B$ and $\Delta > 1$ also hold true in our model. Therefore, there is a tracking solution existing in our model.

For a smaller initial value $\rho_{yi}$, $\rho_y(t)$ still tracks $\rho_r(t)$, but for a shorter period, and then approaches the same constant. When the initial value $y_i$ is sufficiently small, the YMC dark energy is effectively similar to the cosmological constant $\Lambda$. For the non-interacting case, the matter component maintains $\rho_m \propto a(t)^{-3}$ and always decays with time $t$. For the case of YMC decaying into matter, the matter density $\rho_m$ deviates from $\propto a(t)^{-3}$ around $z \sim 0$ and finally becomes a constant. For the decay rate $\Gamma = 0.31H_0$, the dynamical equation for $(x, y)$ at $t \to \infty$ has a fixed point $(x_f, y_f) = (0.052, -0.887)$, regardless of initial conditions. Let we study the stability of this fixed point analytically. On the basis
of equations (14) and (13), one can obtain
\[
\frac{dx}{dN} = f(x, y), \quad \frac{dy}{dN} = g(x, y),
\]
where \(f(x, y)\) and \(g(x, y)\) are the terms on the right-hand side of equations (14) and (13), respectively. By a standard procedure, expanding \(x = x_i + \varepsilon\) and \(y = y_i + \eta\) (\(\varepsilon\) and \(\eta\) are small perturbations around the fixed point), and keeping up to the first order of small perturbations, equation (21) is reduced to
\[
\frac{d}{dN} \begin{pmatrix} \varepsilon \\ \eta \end{pmatrix} = M \begin{pmatrix} \varepsilon \\ \eta \end{pmatrix},
\]
where \(M\) is a \(2 \times 2\) matrix, whose elements are \(M_{11} = \partial f(x, y_i)/\partial x\), \(M_{12} = \partial f(x, y_i)/\partial y\), \(M_{21} = \partial g(x, y_i)/\partial x\), and \(M_{22} = \partial g(x, y_i)/\partial y\), respectively. The general solution for the linear perturbations is of the form
\[
\varepsilon = C_1 e^{\mu_1 N} + C_2 e^{\mu_2 N}, \quad \eta = C_3 e^{\mu_1 N} + C_4 e^{\mu_2 N},
\]
where \(C_1, C_2, C_3,\) and \(C_4\) are constants, \(\mu_1\) and \(\mu_2\) are the eigenvalues of matrix \(M\). As long as \(\mu_1\) and \(\mu_2\) are both negative, the fixed point \((x_i, y_i)\) is stable, and the solution is an attractor. By calculating, we find the matrix
\[
M = \begin{pmatrix} -3.16068 & 0.18234 \\ 0.20061 & -2.55983 \end{pmatrix},
\]
and its two eigenvalues are \(\mu_1 = -2.50411\) and \(\mu_2 = -3.21640\), both negative. Thus the fixed point of this model is stable against perturbation, and the solution is an attractor. Therefore, the big rip would not happen in our model. For an initial YMC DE ranging over \(\sim 33\) orders in magnitude, and for both cases without and with interaction, the fractional densities \(\Omega_{\gamma 0} = 0.73\) and \(\Omega_{m0} = 0.27\) are always achieved at \(z = 0\). In addition, from the theoretical point of view [68], for any DE model with an attractor-like behavior, as long as \(w \leq w_B\) is satisfied, the coincidence problem can be naturally solved. Both of these conditions are satisfied in our model. Therefore, the coincidence problem is naturally solved in the three-loop YMC model.

In figure 2, we plot the evolution of the EOS \(w(z)\) in three-loop YMC model without and with interaction. At the early stage of the Universe (high energies limit), \(w(z)\) approaches that for radiation, i.e., \(w \rightarrow 1/3\). With the expansion of the Universe and the decreasing of the energy scale, \(w\) smoothly decreases, and the YMC component transits from radiation to matter, and to DE. For the non-interacting case, \(w\) does not cross but only asymptotically approaches to \(-1\). For the interaction case, \(w\) can smoothly cross \(w = -1\), as indicated by the preliminary observational data [69]–[71]. Adopting the initial \(y_i = 74\), the EOS of the YMC will cross \(-1\) around \(z \simeq 0.6\). For a smaller \(y_i\) the crossing occurs earlier. For example, taking \(y_i = 72\) in our model will make \(w\) crossing \(-1\) occur around \(z \simeq 1.5\), as suggested in [25]. Moreover, treating \(\Gamma\) as a parameter, we find that a constant interaction \(\Gamma\) corresponds to a constant present EOS \(w_0\), and a larger \(\Gamma\) yields a smaller \(w_0\). For instance, \(\Gamma = 0.31 H_0 \rightarrow w_0 = -1.05; \Gamma = 0.67 H_0 \rightarrow w_0 = -1.15;\) and \(\Gamma = 0.82 H_0 \rightarrow w_0 = -1.21\).
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![Figure 2](image)

**Figure 2.** Evolution of the EOS $w$ in the three-loop YMC model without and with interaction.

In comparison with the case for the two-loop YMC model, the transition to the DE-dominant era occurs at $z \simeq 0.5$ in the three-loop model, later than $z \simeq 0.6$ predicted by the two-loop model [15]. Besides, the three-loop model yields an EOS $w_0 = -1.06$, which is closer to the observational constraints on $w_0$ [69] than that from the two-loop model [15]. In addition, the three-loop model can also predict a larger age of the Universe than the two-loop model [72]. Notice that the YMC is subdominant during the early stages, so the nucleosynthesis and the recombination occur as in the standard big bang cosmology. Besides, since the DE becomes dominant at a very late era, the matter era is also long enough for structure formation. It should be mentioned that the scale $\kappa$ can be fixed by requiring that $\rho_y$ in equation (3) be equal to the dark energy density $\sim 0.73 \rho_c$, where $\rho_c$ is the critical density, yielding $\kappa^{1/2} \simeq 7.5 h_0^{1/2} \times 10^{-3}$ eV ($h_0$ is the Hubble parameter). At the moment we do not have an answer to the question of why $\kappa$ is so small, so the fine-tuning problem is still present in our model. As has been shown [14], in the case of the YMC decaying into matter and radiation, both $\rho_m$ and $\rho_y$ will asymptotically approach to their respective constant values, i.e., the future of the universe is a steady state, quite similar to that of the steady state model [73]. Therefore, in a sense, our model bridges between the big bang and the steady state model.

### 3. The diagnosis with $H, q, r-s,$ and $w-w'$ in the model

In this section the decay rate is taken to be $\Gamma = 0.31 H_0$ as before. First, let us discuss the Hubble parameter $H \equiv \dot{a}/a$. The expansion of the Universe is determined by the
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Figure 3. Comparison of the observed $H(z)$, as square dots, with the predictions from the ΛCDM model [23] and the coupled three-loop YMC model.

The Friedmann equations

\[ H^2 = \frac{8\pi G}{3} \rho, \]  
\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p), \]

where the total energy density $\rho = \rho_y + \rho_m + \rho_r$, and the total pressure $p = p_y + p_r$. The dynamical evolutions of $\rho_y$, $\rho_m$, and $\rho_r$ have already been given in the previous section, so the Hubble parameter $H$ can be easily obtained. In figure 3 we compare the observed expansion rate $H(z)$ [23] with that predicted by the ΛCDM model [23, 25] and by the coupled three-loop YMC model. The area surrounded by two dashed lines shows the 68% confidence interval [2]. It is seen that the coupled YMC model is quite close to the ΛCDM model in the range $z \leq 1$, and the two models approximately agree with the observations. The observed dip of $H(z)$ around $z \sim 1.5$ [23] is a difficulty for both models, but $H(z)$ in our model is slightly lower than that in ΛCDM model and is closer to the dip. For the non-interacting case in the three-loop YMC model, $H(z)$ is closer to the ΛCDM model. Since its evolution trajectory almost overlaps with that of the ΛCDM model, we do not plot its curve here.

Next, we turn to the deceleration parameter $q(z)$, which is given by

\[ q \equiv -\frac{\ddot{a}}{aH^2} = \frac{1}{2} (1 + 3\Omega_y w + \Omega_r), \]

where $\Omega_y = \rho_y/\rho$ and $\Omega_r = \rho_r/\rho$. In deriving this expression, the equations (8), (25), and (26) are used. In figure 4 we plot $q(z)$ in the three-loop YMC model without and
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Figure 4. Evolution of deceleration parameter $q(z)$ in the three-loop YMC model without and with interaction.

with interaction. In both cases, starting from a positive value during the matter era, $q(z)$ decreases with the expansion of the Universe, turns negative around $z \sim 1$, and approaches an asymptotic value $q = -1$ in the future. The current value is $q_0 = -0.572$ for the non-interacting case, denoted by a square dot; and is $q_0 = -0.656$ for the interacting case, denoted by a round dot. In comparison, the $\Lambda$CDM model with $\Omega_\Lambda = 0.73$ has $q_0 = -0.595$, denoted by a star symbol. So the non-interacting YMC model is closer to the $\Lambda$CDM model than the coupled YMC one.

Now, we study the statefinder diagnostic defined by [29]

$$r \equiv \frac{\ddot{a}}{aH^2}, \quad s \equiv \frac{r - 1}{3(q - 1/2)},$$

Taking time derivative of equation (26) and making use of equations (10)–(12), one obtains

$$r = 1 + \frac{9}{2} \Omega_y w(1 + w) - \frac{3}{2} \Omega_y w' + 2 \Omega_\tau + \frac{3 \Gamma}{2H} \Omega_y w,$$

$$s = \frac{3 \Omega_y w(1 + w) - \Omega_y w' + (4/3) \Omega_\tau + (\Gamma/H) \Omega_y w}{3 \Omega_y w + \Omega_\tau},$$

where

$$w' \equiv \frac{dw}{dN} = \frac{dw}{dy} \frac{dy}{dN}$$

can be calculated by taking the variable $y$ derivative of equation (8). The expressions for $r$ and $s$ in equations (29) and (30) hold actually for a generic DE model. Since different cosmological models exhibit qualitatively different trajectories of evolution in the $r$–$s$
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Figure 5. Evolution trajectories of the statefinder in the $r$–$s$ plane for the three-loop YMC model without and with interaction.

In figure 5 we plot the evolution trajectories of the statefinder in the $r$–$s$ plane for the three-loop YMC model without and with interaction, starting from the redshift $z = 3$. The arrows along the curves indicate the direction of evolution. The overall profiles of statefinder diagnosis predicted by these two models are quite similar. As the Universe expands, $s$ increases to a maximum and $r$ decreases to a minimum; after that, the trajectories turn a corner and approach a final fixed point $(r, s) = (1, 0)$. This fixed point is the same as that predicted by the ΛCDM model [29], denoted by a star symbol on the plot. The current value of the statefinder is $(r, s) = (0.972, 8.660 \times 10^{-3})$ for the non-interacting case, and $(r, s) = (0.912, 2.528 \times 10^{-2})$ for the interacting case. On the basis of these calculated values, one sees that the non-interacting three-loop YMC model is closer to the ΛCDM model; and the introduction of interaction between matter and DE causes a deviation from the ΛCDM model. We also find that a larger interaction $\Gamma$ yields a larger deviation. For instance, $\Gamma = 0.67H_0$ yields $(r, s) = (0.851, 3.898 \times 10^{-2})$ at $z = 0$, which is further away from the value of ΛCDM. In comparison, the quintessence model [30] gives the current values $(r, s) = (0.4, 0.3)$, the Chaplygin gas model [31] gives $(r, s) = (1.95, -0.3)$, and the agegraphic model [40] with parameter $n = 2.0$ give $(r, s) \simeq (-0.2, 0.5)$. All the current values of $(r, s)$ predicted by these three kinds of DE models are far away from the $(1, 0)$ given by the ΛCDM model. The holographic model [41] without interaction gives $(r, s) \simeq (0.94, 0.01)$; when an interaction is included, it gives $(r, s) \simeq (0.75, 0.09)$, deviating away considerably from the ΛCDM model again. Therefore, our model is much closer to ΛCDM than other dynamical DE models.
Finally, we investigate the $w - w'$ diagnostic, defined in the equations (8) and (31). In figure 6 we plot the evolution trajectories of $w - w'$ for the three-loop YMC model without and with interaction, starting from the redshift $z = 3$. The arrows along the curves denote the direction of evolution. For both models, with the expansion of the Universe, $w$ decreases and $w'$ increases, and the $w - w'$ diagnostic approaches a fixed point asymptotically. For the non-interacting case, the current value is $(w, w') = (-0.982, -2.778 \times 10^{-2})$, and the fixed point is $(-1, 0)$, which is same as that of the ΛCDM model, denoted by a star symbol on the plot. Therefore, the non-coupling YMC model cannot cross the phantom divide $w = -1$. For the interacting case, the situation is different: $\Gamma = 0.31H_0$ yields a current value $(w, w') = (-1.063, -5.430 \times 10^{-2})$, and approaches a fixed dot $(-1.12, 0)$ asymptotically. Therefore, the interaction between matter and DE causes a deviation from the ΛCDM, and gives an equation of state (EOF) crossing the phantom divide $w = -1$.

4. Cosmological constraints from SNIa, CMB, and LSS

In the following, by using the maximum likelihood method, we will perform a best-fit analysis on our three-loop YMC model with the latest observational SNIa, CMB and LSS data. First, we derive the constraints on the model from SNIa. Recently, the up-to-date gold sample of 182 SNIa data was compiled by Riess et al [52]. It provides the apparent magnitude $m(z)$ of supernovae, which is related to the luminosity distance $d_L(z)$.
of supernovae through
\[ m(z) = M + 5 \log d_L(z) + 25, \]  
(32)
where \( M \) is the absolute magnitude, that can generally be considered to be the same for SNIa. In a flat universe the luminosity distance satisfies
\[ d_L = H_0^{-1}(1 + z) \int_0^z \frac{dz'}{E(z')}, \]  
(33)
where \( E(z) \equiv H(z)/H_0 \) and the Hubble scale \( H_0^{-1} = 2997.9 \, h^{-1} \, \text{Mpc} \). The data points of the latest 182 SNIa gold data set compiled in [52] are given in terms of the distance modulus
\[ \mu_{\text{obs}}(z_i) \equiv m_{\text{obs}}(z_i) - M. \]  
(34)
On the other hand, the theoretical distance modulus is defined as
\[ \mu_{\text{th}}(z_i) \equiv m_{\text{th}}(z_i) - M = 5 \log_{10} d_L(z_i) + 25. \]  
(35)
The theoretical model parameters are determined by minimizing
\[ \chi^2_{\text{SN}}(\mathbf{p}) = \sum_{i=1}^{182} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}, \]  
(36)
where \( \sigma \) is the corresponding 1\( \sigma \) error, and \( \mathbf{p} \) denotes the model parameters. In this work, we will determine the best-fit values of corresponding model parameters (including the present fractional matter density \( \Omega_{m0} \), the Hubble constant \( h \), and the decay rate \( \Gamma \)) in the three-loop YMC model.

Next, we also consider the constraints from CMB [2] and LSS [3] observations. For the CMB data, we use the CMB shift parameter \( R \), which is perhaps the most model-independent parameter that can be extracted from CMB data. The CMB shift parameter is given by [56]
\[ R \equiv \Omega_{m0}^{1/2} \int_0^{z_{\text{rec}}} \frac{dz'}{E(z')}, \]  
(37)
where the redshift of recombination \( z_{\text{rec}} = 1090 \), which is given by WMAP5 [74]. The shift parameter \( R \) relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at \( z_{\text{rec}} \) and the angular scale of the first acoustic peak in the CMB power spectrum of temperature fluctuations [56,57]. The measured value of \( R \) has been updated to \( R_{\text{obs}} = 1.710 \pm 0.019 \) from WMAP5 [74]. On the other hand, for the LSS data, we use the parameter \( A \) from the measurement of the BAO peak in the distribution of SDSS luminous red galaxies, which is given by [58]
\[ A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[ \frac{1}{z_b} \int_0^{z_b} \frac{dz'}{E(z')} \right]^{2/3}, \]  
(38)
where \( z_b = 0.35 \). The SDSS BAO measurement [58] gives \( A_{\text{obs}} = 0.469 (n_s/0.98)^{-0.35} \pm 0.017 \). Here the scalar spectral index \( n_s \) is taken to be 0.96 from the five-year WMAP data [74]. Since both parameters, \( R \) and \( A \), are independent of the Hubble constant \( H_0 \) and can be easily obtained from CMB and LSS observations, they provide robust
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Figure 7. The $\chi^2_{\text{SN}}$ and the corresponding likelihood $\mathcal{L}$ of the three-loop YMC, where the best-fit parameter $h = 0.626$ is adopted. These results are obtained from the latest 182 SNIa gold data set.

constraints on DE models as useful complements to the SNIa data. We perform a joint analysis of the latest 182 SNIa gold data set, the shift parameter $R$ from the CMB and the BAO peak measurement $A$ from LSS to constrain the three-loop YMC model. The total $\chi^2$ is given by

$$ \chi^2 = \chi^2_{\text{SN}} + \chi^2_{\text{CMB}} + \chi^2_{\text{LSS}}, $$

where $\chi^2_{\text{SN}}$ is given by equation (36), and the latter two terms are defined as

$$ \chi^2_{\text{CMB}} = \frac{(R - R_{\text{obs}})^2}{\sigma^2_R}, $$

and

$$ \chi^2_{\text{LSS}} = \frac{(A - A_{\text{obs}})^2}{\sigma^2_A}. $$

The corresponding 1σ errors are $\sigma_R = 0.03$ and $\sigma_A = 0.017$, respectively. As usual, assuming that the measurement errors are Gaussian, the likelihood function is

$$ \mathcal{L} \propto e^{-\chi^2/2}. $$

The model parameters that yield a minimal $\chi^2$ and a maximal $\mathcal{L}$ will be favored by the observations. The results of our analysis are presented in the following.

First for the non-interacting three-loop YMC model, utilizing the latest 182 SNIa gold data set alone, we plot the $\chi^2_{\text{SN}}$ and the corresponding likelihood $\mathcal{L}$ of the three-loop YMC.
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Figure 8. The $\chi^2$ and the corresponding likelihood $L$ of the three-loop YMC, where the best-fit parameter $h = 0.634$ is adopted. These results are obtained from the combined SNIa, CMB and LSS data.

It is found that the best-fit model parameters are $\Omega_{m0} = 0.331$ and $h = 0.626$, giving a minimal $\chi^2_{SN} = 158.858$. Besides, for a 68% confidence level [75], the ranges of parameters are determined as $\Omega_{m0} = 0.331^{+0.020}_{-0.021}$, $\Omega_y0 = 0.669^{+0.021}_{-0.020}$ and $h = 0.626 \pm 0.004$. In comparison, we also fit the $\Lambda$CDM model to the same SNIa data and find that the minimal $\chi^2_{SN,\Lambda} = 158.750$ for the best-fit parameter $\Omega_{m0} = 0.344$ and $h = 0.626$. So fitting to the 182 SNIa gold data set alone, the $\Lambda$CDM model is slightly better than the three-loop YMC model.

Moreover, utilizing the combination of SNIa, CMB and LSS data, we plot the $\chi^2$ and the corresponding likelihood $L$ of the three-loop YMC in figure 8. It is found that the best-fit model parameters are $\Omega_{m0} = 0.289$ and $h = 0.634$, giving a minimal $\chi^2_{min} = 160.317$. Besides, for a 68% confidence level [75], the ranges of parameters are determined as $\Omega_{m0} = 0.289^{+0.014}_{-0.013}$, $\Omega_y0 = 0.711^{+0.014}_{-0.013}$ and $h = 0.634 \pm 0.004$. As a comparison, we also fit the $\Lambda$CDM model to the same combined data. It is found that the minimal $\chi^2_{min,\Lambda} = 162.460$ for the best-fit parameter $\Omega_{m0} = 0.283$ and $h = 0.638$. Therefore, on fitting to the combination of SNIa, CMB and LSS data, the non-interacting three-loop YMC model is better than $\Lambda$CDM. This makes the three-loop YMC model more attractive.

Finally, we turn to the coupled YMC model and constrain the decay rate $\Gamma$ as the last model parameter. In table 1, we list the minimal $\chi^2_{SN, min}$ and $\chi^2_{min}$ for the three-loop YMC model with various values of $\Gamma$. For the latest SNIa data alone, the non-interacting YMC model is only slightly better than the coupled YMC. However, for the combination of SNIa, CMB and LSS data, the non-interacting YMC model is much more favored than the coupled YMC. This fact can be explained as follows. It is well known that most
current observations, including the SNIa, CMB and LSS ones, all favor the ΛCDM model. Therefore, to fit these observations well, a DE model should not deviate too far away from the ΛCDM model. Since the introduction of interaction between matter and DE will cause a deviation from the ΛCDM model, and a larger interaction Γ yields a larger deviation, the coupled YMC model with a large Γ will not be favored by the observations. That is to say, to have consistency with the current observations, any DE–matter interaction (if it exists) should be very small.

5. Summary

In this work, we extend our previously proposed YMC DE model to the three-loop quantum corrections. This model can solve the coincidence problem naturally, and it can give, in the interaction form, an EOF crossing the phantom divide \( w = -1 \). Next, we study the Hubble parameter \( H \), the deceleration parameter \( q \), the statefinder diagnostic \((r, s)\), and the \( w - w' \) diagnostic for the three-loop YMC model for the cases without and with interaction, and compare our results with those from other DE models. It is found that the three-loop YMC model is much closer to the ΛCDM model than other dynamical DE models; and the introduction of the matter–DE interaction will make the YMC model deviate from the ΛCDM model. Finally, by using the observational SNIa data, the shift parameter from the CMB, and the BAO peak from LSS, we give the cosmological constraints on the three-loop YMC model. Utilizing the latest SNIa data alone, the best-fit model parameters of the three-loop YMC are \( \Omega_{m0} = 0.331^{+0.020}_{-0.021}, \Omega_{y0} = 0.669^{+0.020}_{-0.021} \) and \( h = 0.626 \pm 0.004 \) (with \( 1\sigma \) uncertainty); and combining SNIa, CMB and LSS data, the best-fit model parameters are \( \Omega_{m0} = 0.289^{+0.014}_{-0.013}, \Omega_{y0} = 0.711^{+0.014}_{-0.013} \) and \( h = 0.634 \pm 0.004 \) (with \( 1\sigma \) uncertainty). To fit the latest SNIa data alone, the ΛCDM model is slightly better than the three-loop YMC model; but in fitting the combination of SNIa, CMB and LSS data, the three-loop YMC model performs better than the ΛCDM model. This makes the three-loop YMC model more attractive. In addition, the maximum likelihood analysis also shows that the interaction between matter and DE should be small.

There are also other observations that would be helpful for constraining the DE models, such as the Chandra x-ray observation [76], the lookback time data [77], the gamma-ray bursts [78] and so on. In addition, it is also very interesting to constrain the YMC model by using the global fitting to the full CMB and LSS data via Markov chain Monte Carlo analysis. These issues deserve further investigation in the future.

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