Analysis on Existence of New Stable Attitude of Spatial Flexible Damping Beam

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Abstract. In my previous works, the attitude stability of the spatial flexible damping beam was investigated employing the structure-preserving method and a new stable attitude was found when the non-sphere perturbation effects were considered. But the new stable attitude state is lack of theoretical explanation. In this paper, the existence of the new stable attitude of the spatial flexible damping beam with the non-sphere perturbation will be discussed referring to the coupling dynamic model describing the couple motion between the orbit motion, the attitude evolution and the transverse vibration of the spatial flexible damping beam. The conclusions obtained in this paper illustrate the existence of the new stable attitude of the spatial flexible damping beam and verifies the numerical results presented in my previous job.

1. Introduction

With the development of the space science, the structure of the spacecraft becomes more and more complex. To realize the assumed mission, all spacecraft must work in the assumed attitude. What is the assumed attitude for a certain spacecraft? This problem is determined by the possible stable attitude of the certain spacecraft. The stable attitude discussed in this paper means the stable on-orbit attitude without any attitude control input. Strictly, in the stale attitude, the spacecraft has minimum mechanical energy. Usually, the on-orbit attitude state of the spacecraft is designed as close as possible to one of the stable attitude states of which to minimize the needed attitude control energy. Thus, the stable attitude states of the spacecraft provide the target of the control strategy designed [1-3], which implies that seeking the stable attitude states of the spacecraft is the precondition of the design of the control strategy. But, for the complexity of the space environment and the structural form of the spacecraft, the stable attitude state is difficult to be determined.

In many cases, the slender spacecraft, such as the Solar Power Satellite Via Arbitrarily Large Phased Array concept shown in Fig. 1, can be simplified as a flexible beam and the stable attitude state can be determined with this simplified model approximately. It has been proved that, the attitude state with $\alpha=0$ is stable for the flexible beam on orbit. But, if this stable attitude state is taken as the attitude control target, many space missions cannot be achieved. For example, this stable attitude state of the Solar Power Satellite System (SPSS) [4] implies that the axis of the SPSS points to the Sun and the collection of the solar energy cannot be performed in this state. In another word, adjusting the
SPSS departure this stable attitude state forcibly to make it can receive the solar energy will cost vast energy. In this case, the economic efficiency of the SPSS should be assessed again.

Figure 1. The Solar Power Satellite Via Arbitrarily Large Phased Array concept

Considering the effects of the gravity gradient and the non-sphere perturbation, a new stable attitude state $\alpha = \pi/2$ [5] was found employing the structure-preserving method [6-11] when the initial attitude angle is close to $\pi/2$ and the initial attitude angle speed is small enough. If this stable attitude state truly exists, the stable axis of the SPSS can be adjusted to deviate the Sun and the efficiency of the collecting solar energy can be improved. Based on this background, the evidences of the existence of the new stable attitude state will be seek in detail in this paper.

2. Dynamic model of the spatial flexible damping beam
As the previous job [5], a homogeneous spatial flexible damping beam, a typical slender component [12], moving around the Earth in the XOY plane is considered in this paper, see Fig. 2.

Figure 2. Dynamic model of spatial flexible beam

The plane motion and the transverse vibration of the beam can be formulated by the state vector $[r, \alpha, \theta, u]^T$, in which, the orbit radius $r = r(t) = |Oo|$ and the orbit true anomaly of the center of mass $\theta = \theta(t)$ determine the plane motion of the beam, the attitude angle $\alpha = \alpha(t)$ describes the attitude of the
beam and the transverse displacement $u = u(x,t)$ in the local coordinate system $u_0$, $v$ denotes the transverse vibration induced by the plane motion of the beam. The Hamiltonian function of the beam can be formulated as \cite{[3]},

$$
H = T + U
$$

$$
= \frac{\rho l}{2} \left[ \dot{r}^2 + (r \dot{\theta})^2 \right] + \frac{1}{2} \frac{\rho l^3}{12} (\dot{\theta} + \ddot{\alpha})^2 + \frac{\rho l}{2} \int_{x/2}^{l/2} \left[ \ddot{u}^2 + u^2 (\dot{\theta} + \alpha)^2 \right] dx
$$

$$
- \frac{\mu pl^3}{24 r^4} (1 - 3 \cos^2 \alpha) - \frac{\mu J R^2 \rho l}{2 r^4} - \frac{3 \mu J_2^2 R^4}{r^4} + \frac{EI l}{2} \int_{x/2}^{l/2} \ddot{\alpha}^2 u dx
$$

where $T$ is the total kinetic energy of the spatial flexible beam, including the translational energy, the rotational kinetic energy as well as the transverse vibrational kinetic energy. $U$ is the total potential energy of the spatial flexible beam, the variation of which is determined by the gravity gradient, the non-sphere perturbation and the transverse vibration of the beam.

Then, the coupling dynamic model for the spatial flexible damping beam can be obtained based on the Hamilton variational principle,

\begin{equation}
\begin{aligned}
- \rho l \dddot{r} + \rho l r \ddot{\theta} - \mu pl/r^2 + \mu pl^3 (1 - 3 \cos^2 \alpha)/8r^4 - 3 \mu J_2 R^4 \rho l/2r^4 - 9 \mu J_2 R^4 \rho l/r^4 &= 0 \\
\rho l^3 \ddot{\theta} + 2 \rho rl \dot{r} \ddot{\theta} + \rho l^3 (\dot{\theta} + \alpha)/12 + \rho \int_{x/2}^{l/2} [2u \ddot{u} + u^2 (\ddot{\theta} + \alpha)] dx &= 0 \\
\rho l^3 (\dot{\theta} + \alpha)/12 + \mu pl^3 \cos \alpha \sin \alpha /4 r^3 + \rho \int_{x/2}^{l/2} [2u \ddot{u} + u^2 (\ddot{\theta} + \alpha)] dx &= 0 \\
\rho \dddot{\alpha} + c \dddot{u} - \rho u (\ddot{\theta} + \alpha)^2 + EI \dddot{\alpha} &= 0
\end{aligned}
\end{equation}

where, $\rho$ and $l$ represent the linear density and the length of the beam respectively, $\mu$ is the gravitation constant of the Earth, $J_2$ and $J_2$ denote the coefficient of the Earth zonal harmonic term and the Earth tesserl harmonic term respectively, $R$ is the average equatorial radius of the Earth, $EI$ is the flexural stiffness of the beam.

In the model (2), the first equation mainly controls the evolution of the orbit radius. The second equation mainly controls the evolution of the orbit true anomaly of the center of mass. The third equation mainly controls the evolution of the beam. The last equation mainly controls the transverse vibration of the flexible beam. Meanwhile, the model (2) is an infinite-dimensional coupling dynamic system, which determines that the explicit analytical solution of which cannot be obtained and the numerical results of which cannot be proved by the analytical solution directly. Thus, the numerical results should be verified by other approaches.

3. Explanation of the existence of the new stable attitude state

There is only one stable attitude state for the spatial flexible damping beam when only the gravity gradient is considered, while the new stable attitude state appears when both the gravity gradient and the non-sphere perturbation of the Earth are considered. The theoretical evidence will be proposed in this section in detail to support the numerical results presented in our previous job.

Comparing the number of the stability of the attitude state of the spatial flexible beam, it can be speculate that the appearance of the new stable attitude state of the spatial beam results from the consideration of the non-sphere perturbation, which implies that the existence of the new stable attitude state may be explained by the competition between the gravity gradient and the non-sphere perturbation in the evolution of the attitude.

Let we review the first equation of the control model (2),

$$
- \rho l \dddot{r} + \rho l r \ddot{\theta} - \mu pl/r^2 + \mu pl^3 (1 - 3 \cos^2 \alpha)/8r^4 - 3 \mu J_2 R^4 \rho l/2r^4 - 9 \mu J_2 R^4 \rho l/r^4 = 0
$$

\begin{equation}
\end{equation}
which mainly controls the evolution of the orbit radius and contains both the gravity gradient and the non-sphere perturbation.

When \( \alpha_0 \rightarrow \pi/2 \), \( \cos \alpha \rightarrow 0 \), the effect of the gravity gradient formulated in Eq. (3) can be simplified as the following form approximately,

\[
\mu \rho l^3 (1 - 3 \cos^2 \alpha) / 8r^4 \approx \mu \rho l^3 / 8r^4
\]

which implies that the gravity gradient is determined by the orbit radius and is almost independent on the value of the attitude angle. Form Eq. (4), it can be found that: on the one hand, the evolution of the orbit radius is almost independent of the initial value of the attitude angle. On the other hand, the evolution of the orbit radius is almost independent of the real-time value of the attitude angle.

In this case, both \( \mu \rho l^3 / 8r^4 \), \( -3 \mu J_2 R_x^2 \rho l / 2r^4 \) and \( -9 \mu J_2 R_x^2 \rho l / r^4 \) are only dependent on the orbit radius and are in direct proportion to \( 1/r^4 \). The difference is that, \( \mu \rho l^3 / 8r^4 \) is plus while \( -9 \mu J_2 R_x^2 \rho l / r^4 \) and \( -3 \mu J_2 R_x^2 \rho l / 2r^4 \) are minus, which implies that the considering of the non-sphere perturbation will decrease the effects of the gravity gradient on the dynamic behaviors of the spatial flexible beam theoretically. For the existence of the gravity gradient, the gravity of the end of the beam close to the earth is larger than that of the end of the beam far from to the earth, the difference between them results in the gravity gradient moment acting on the spatial beam.

On the geostationary earth orbit (GEO), the values of these three terms are comparable, which implies that, the negative Earth tesseral harmonic term weakens the effect of the gravity gradient. When the effect of the gravity gradient is weakened, the accompanying gravity gradient moment that make the axis of the spatial flexible damping beam point to the Earth is also weakened. While the existence of the non-sphere perturbation will result in the transverse vibration of the spatial flexible beam, which reduces the stability of the attitude state \( \alpha = 0 \) and enhances the stability of the attitude state \( \alpha = \pi/2 \) of the beam. With the negligible gravity gradient moment, the attitude angle needn’t to be converging to zero and the new stable attitude state is permitted.

In the previous numerical experiments \(^{[7]}\), it has been proved that, the gravity gradient term increases the vibration time of the beam and break the attitude stability of the beam when \( \alpha \rightarrow \pi/2 \). The above factors are just weakened by the Earth tesseral harmonic term, that is, when \( \alpha \rightarrow \pi/2 \), the vibration of the beam will decrease monotonously and the attitude angle of the spatial flexible damping beam will tend to \( \pi/2 \) steadily if the attitude angle speed is small enough. The above analysis implies that the new stable attitude state exists without question.

4. Conclusions
To prove the numerical results presented in the previous work, the dynamic model of the spatial flexible beam is investigated in detail in this paper. From the analytical results obtained in the section 3, it can be concluded that: ① Considering the non-sphere perturbation will reduce the effects of the gravity gradient on the attitude evolution. ② When \( \alpha_0 \rightarrow \pi/2 \), the effects of the non-sphere perturbation and those of the gravity gradient on the stability of the attitude tend to counteract, which will results in the new stable attitude angle found in our previous work. The above conclusions will help us to comprehend the existence of the new stable attitude angle \( \alpha = \pi/2 \) theoretically.

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