Electromagnetic Origin for Planck Mass and Dark Energy

M. Arık\textsuperscript{1*}, N. Katırcı\textsuperscript{1,2†}

\textsuperscript{1} Department of Physics, Boğaziçi University, Bebek Istanbul, Turkey
\textsuperscript{2} Department of Physics, Doğuş University, Acıbadem Istanbul, Turkey

Abstract

The origin of dark energy remains to be one of the challenges of modern cosmology. We modify Jordan-Brans-Dicke theory using a vector field instead of a scalar field and theory becomes similar to a simple Einstein-aether theory. The time component of the vector field picks up a cosmological background value. Identifying the vector field to be the photon field, a small photon mass leads to late time inflation. The time dependent background electrical potential of the photon permeates the universe and explains the weakness of the gravitational interaction by coupling to curvature. This theory relates the smallness of the photon mass to the smallness of the Hubble parameter. The model predicted photon mass is far below observational constraints.

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\textsuperscript{*}e-mail:metin.arik@boun.edu.tr
\textsuperscript{†}e-mail: nkatirci@dogus.edu.tr
1 Introduction

The universe is expanding at an ever increasing rate according to observations of supernovae (SNe). This has been known since 1998 [1–4], for a review see [5]. Cosmologists have studied to explain the source of this behavior by modifying Einstein’s theory of general relativity or proposing new types of gravitational theories. Cosmological constant $\Lambda$ in $\Lambda$CDM, is currently the best candidate for the source of the accelerated expansion. It may arise from vacuum fluctuations, however there is a large (at least sixty orders of magnitude) discrepancy between the predicted energy density of the vacuum in particle physics and energy density of the cosmological constant from fitting of $\Lambda$CDM model to observations such as Supernova type 1A explosions, cosmic microwave background radiation (CMBR) and baryon acoustic oscillations (BAO) [4, 6, 7].

Einstein-aether theories have been recently revived with the purpose of fixing the divergences of quantum field theory (QFT) by breaking Lorentz invariance and putting a short distance - large energy cutoff for energy and momentum [8]. Minkowski spacetime is invariant under Lorentz transformations, which are true in all inertial frames. When gravity comes into play, Minkowski spacetime turns into Friedmann-Robertson-Walker spacetime, which models expansion of space and Lorentz invariance is broken, whereas spatial isotropy and homogeneity are preserved. Lorentz symmetry is locally a good symmetry of spacetime, however on large scales, it may be broken by the buildings of the universe such as matter or radiation. For a recent discussion see [9]. If the background value of the vector field chooses a preferred spatial direction, it contradicts the isotropy of the universe, stated by the cosmological principle. A vector field that has zero spatial components, but nonzero time component also breaks local Lorentz symmetry down to the rotation subgroup. Will and Nordtvedt investigated detectable effects of this phenomenon on the motion of the solar system relative to a preferential reference frame [10]. In [11] it was stated that perturbation spectra do not stringently depend on Lorentz invariance breaking model parameters, and studies on the compact stars or black holes (BH) [12] can be more effective to determine the constraints.

The cosmological motivation for the vector field is the exact knowledge about the presence of the vector field when compared to that of the scalar field. The possible relation between the photon mass and Hubble parameter comes to mind from the approximately same experimental values. In [13] the Hubble parameter in the early inflation era is calculated as $H_I \approx 10^{21} eV$ and it is stated that photon acquires a mass ($m_{\gamma,I} \approx 10^{21} eV$) of the same order of magnitude in that era. This predicts that the current phase of small accele-
tion causes a nonzero, but very small photon mass of the order of magnitude $H_0 = 10^{-33}eV$ [13].

The best current laboratory bound on the photon mass $10^{-14} eV$, derived from measurements of potential deviations from the Coulomb law [14], is far above the cosmological constraint on the photon mass. From the measurements of Earth’s magnetic field [15] and the Pioneer-10 measurements of Jupiter’s magnetic field [16], $m_\gamma$ is obtained approximately as $10^{-15}eV$. Whereas an upper limit of $10^{-27}eV$ has been determined using effects of photon mass to galactic magnetic fields [17]. The cosmological evolution of the electric potential has been considered in [18] and where it is stated that the value of the electric potential during the early universe is $10^{-3}eV$ and it evolves to $10^{27}eV$ in the present era [18].

This paper is based on the idea that quantum electrodynamics (QED) may affect physics in long scales in a different manner, when compared to its small scale behavior. We realize a model which shows the relation between the photon mass and the Hubble parameter. However, our model is restricted to the present era, the mass parameter is a constant and thus cannot explain the reason for the huge difference between the Hubble parameters (the rate of expansion) at the primordial and the present era. We modify Jordan-Brans-Dicke theory (JBD) in a way similar to Einstein-aether theory (AE) using a Lagrange multiplier field to impose the condition that the Brans-Dicke (BD) field (the square of the Jordan scalar field) is equal to the norm of the electromagnetic vector potential. Different cosmological applications of Lagrange multiplier and their implications such as Lorentz symmetry breaking and power counting renormalizable gravitational theories have also been investigated in [19–26].

The outline of the paper is as follows. In section 2, we present the model and its cosmological solutions for otherwise empty space-time. We also consider a matter dominated and radiation dominated energy momentum tensors and show that it leads to the standard $\Lambda$CDM result. We discuss the physical photon mass predicted by our model. The last section encompasses our concluding remarks.

## 2 The Model

The proposed action is

$$S = \int d^4x \sqrt{-g} \left[ -\frac{\phi^2}{8\omega} R + \frac{1}{2} \nabla_{\alpha} \phi \nabla^{\alpha} \phi - \frac{\lambda^2}{2} \left( \phi^2 - A_\alpha A^\alpha \right) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} m^2 A_\alpha A^\alpha \right] + S_M$$  \hspace{1cm} (2.1)
where \( S_M \) is the matter action. \( \varphi \) is the Jordan field, \( \lambda \) is the Lagrange multiplier field which imposes \( \varphi^2 = A^2 \). Equations of motion, obtained from the variation with respect to metric, \( A_\mu, \varphi \) and \( \lambda \) are given as,

\[
\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^\mu\nu} = -\frac{\varphi^2}{8\omega} \nabla^\mu \nabla_\nu \varphi^2 - \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{4} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + \frac{\lambda^2}{2} A_\mu A_\nu + \frac{1}{4} g_{\mu\nu} \lambda^2 (\varphi^2 - g^{\alpha\beta} A_\alpha A_\beta) + \frac{1}{4} g_{\mu\nu} m^2 A_\alpha A_\alpha - \frac{1}{2} m^2 A_\mu A_\nu - \frac{1}{2} (F_{\mu\beta} F^\beta_\nu - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}) + \frac{T_{\mu\nu}}{2} = 0
\] (2.2)

\[
\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta A_\mu} = \nabla_\mu F^{\mu\nu} + \lambda^2 A^\mu - m^2 A^\mu = 0
\] (2.3)

\[
\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \varphi} = -\frac{\varphi}{4\omega} R - \Box \varphi - \lambda^2 \varphi = 0
\] (2.4)

\[
\frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \lambda} = \lambda (g^{\alpha\beta} A_\alpha A_\beta - \varphi^2) = 0.
\] (2.5)

We use a metric signature (+ − − −) for Friedmann-Robertson-Walker (FRW) metric with flat space-like sections. The non-zero components of the Ricci tensor and the Ricci scalar are given by \( R_{00} = -3\ddot{a}/a \), \( R_{\alpha\beta} = (\ddot{a} + 2\dot{a}^2) \delta_{\alpha\beta} \), \( \alpha, \beta = 1, 2, 3 \) and \( R = -6(\ddot{a}/a + \dot{a}^2/a^2) \), respectively. \( T_{\mu\nu} \) is the energy-stress tensor for a perfect fluid given by \( T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p) \). We consider a cosmological background value as \( A_\mu = (A_0(t), 0, 0, 0) \) which by (2.5) gives that the scalar field \( \varphi(t) \) equals \( A_0(t) \) for nonzero \( \lambda \). For \( \lambda = 0 \) and \( A_\mu = 0 \), standard Jordan-Brans-Dicke theory is obtained \[27\]. Using equation (2.3), \( \lambda^2 = m^2 \) and we obtain Eqs. (2.7)-(2.9). The fractional rate of change of the scale factor, Hubble parameter \( H \), and a constant fractional rate of change for the scalar (or time component of the vector field) field, \( B \) are defined as, respectively

\[
H = \frac{\dot{a}(t)}{a(t)}, \quad B = \frac{\dot{\varphi}(t)}{\varphi(t)} = \frac{\dot{A}_0(t)}{A_0(t)},
\] (2.6)

where \( a \) is the scale factor and field equations above are written in terms of \( H, B \) become

\[
6HB + 3H^2 - 2\omega B^2 - 2\omega m^2 = \frac{4\omega}{\varphi^2} \rho_M,
\] (2.7)
\[-2\dot{B} - B^2(4 + 2\omega) - 4HB - 2\dot{H} - 3H^2 + 2\omega m^2 = \frac{4\omega}{\varphi^2} p_M, \quad (2.8)\]

\[\dot{B} + B^2 + 3HB = \frac{1}{2\omega}(3\dot{H} + 6H^2 - 2\omega m^2), \quad (2.9)\]

$\rho_M$ and $p_M$ are the energy density and pressure of matter. Note that, the electromagnetic field

\[F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \quad (2.10)\]

does not contribute to the Eqs. (2.7)-(2.9) since for $A_0$ being only function of time and $A_i = 0$, it becomes zero. Thus Eqs. (2.7)-(2.9) are similar to the equations obtained from a massive Brans-Dicke Model without any electromagnetic field [28, 29].

### 2.1 Vacuum Solution

The vacuum solution ($\rho = p = 0$) yields a solution giving a constant Hubble parameter ($H$) and the rate of change of the scalar or vector field ($B$) as

\[B = \frac{H}{2(\omega + 1)}, \quad H^2 = \frac{4\omega(1 + \omega)^2}{(2\omega + 3)(3\omega + 4)} m^2. \quad (2.11)\]

Since the limit on the BD parameter is $\omega > 10^4$ [30–33], $m$ should be less than $10^{-35}$ eV which is far below the observational constraints mentioned above. We identify this solution as the dark energy solution for $t \to \infty$. Considering only the leading terms in $\omega$, $H$ and $B$ for the dark energy era become

\[H_\infty = \sqrt{\frac{2\omega}{3} m}, \quad B_\infty = \frac{H_\infty}{2\omega}. \quad (2.12)\]

Now, we consider that the relation

\[B = \frac{1}{2\omega} H \quad (2.13)\]

which is obtained from equation (2.11) by neglecting higher order terms in $\frac{1}{\omega}$ and place it into equation (2.9). Again neglecting higher order terms in $\frac{1}{\omega}$, we obtain

\[H = H_\infty \coth\left(\frac{3H_\infty t}{2}\right), \quad a = a_1 \sinh^2\left(\frac{3H_\infty t}{2}\right) \quad (2.14)\]

which gives $p = 0$ in equation (2.8) and

\[H^2 = H_\infty^2\left(1 + \left(\frac{a_1}{a}\right)^3\right). \quad (2.15)\]
Thus this is natural that the matter dominated era is followed by the dark energy era in this model. This means the solution theoretically knows how should a matter dominated era be experienced on the timeline to the late time inflationary (dark energy dominated) era.

Denoting derivatives with respect to $a$ by prime, one can integrate

$$\frac{\varphi'}{\varphi} = \frac{B}{aH}. \tag{2.16}$$

Using the relation between $H$ and $B$ in equation (2.11) yields

$$|\varphi^2| = |\varphi_0^2| \left[ \frac{a}{a_0} \right]^{\frac{1}{1+\omega}} \tag{2.17}$$

Since $\omega > 10^4 \varphi$ is nearly constant, thus the electric potential of the vector field permeates all the universe and its value is found as approximately $10^{30}$ eV from

$$\frac{1}{16\pi G_N} = \frac{M_p^2}{2} = \frac{A_0^2}{8\omega} = \frac{|\varphi_0^2|}{8\omega} \tag{2.18}$$

similar to the result of [18] where $A_0$ has magnitude as $10^{27}$ eV for the late dark energy era. The mass of the photon behaves as dark energy and the coupling of the vector field to gravitation is interpreted as the Planck Mass. The difference of this model is the non-minimal coupling of gravitation to the scalar field which is identified with the norm of the vector field. This interpretation provides to relate the smallness of the Hubble parameter that expands the universe to the smallness of the photon mass.

Current measurements show that the present content of matter density in the universe is 25% and the dark energy content is 75% and asymptotically universe will contain only dark energy. Our model predicts the same results as the standard model of cosmology and interprets the dark energy as electromagnetic dark energy.

### 2.2 An Alternative Approach to the Matter Dominated Solution

When matter is put into Eqs. (2.7) and (2.8) as $\rho_M = \rho_0 \left( \frac{a}{a_0} \right)^{-3}$ and $p_M = 0$, the relation in Eq.(2.11) between $H$ and $F$ is preserved. Eq.(2.9) is also satisfied and the Hubble parameter is derived as

$$H^2 = \frac{4\omega(1+\omega)^2}{(3\omega + 4)(2\omega + 3)} \left[ m^2 + \frac{2\rho_0}{|\varphi_0|^2} \left( \frac{a}{a_0} \right)^{\alpha} \right] \tag{2.19}$$

where $\alpha = - \left( 3 + \frac{1}{1+\omega} \right)$. As mentioned above, although the matter is not put into Eqs. (2.7) and (2.8), Hubble parameter in the $\omega \to \infty$ limit for vacuum case demonstrates the same behaviour as in Equation 2.15.
2.3 Photon Mass

The Proca Lagrangian density for a massive photon is given by

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\gamma}^2 A_\mu A^\mu \]  

(2.20)

where \( m_{\gamma} \) is the photon mass. To extract \( m_{\gamma} \) from Eq.(2.1), one needs to substitute \( \phi^2 = A_\alpha A^\alpha \) and then look at the coefficients of all the terms \( A_i A^i \) to find the mass of the physical photon. Using the relation between \( H \) and \( m \),

\[ m_{\gamma}^2 = -\frac{R}{4\omega} - m^2 = \frac{\omega(6\omega + 7)}{(2\omega + 3)(3\omega + 4)}m^2 \]  

(2.21)

this equation predicts a photon mass on the order of \( 10^{-35} \)eV for the vacuum solution.

Note that in our Lagrangian flat space limit of the photon mass is given by \( m_{\gamma}^2 = -m^2 \). Without this feature, our model would not work. Since during the dark energy era \( R = \text{const.} \), the term \( A_\mu A^\mu R \) also behaves as the photon mass so that \( m_{\gamma}^2 \) is positive in equation (2.21). This is reminiscent of the Higgs model where the mass term has the wrong sign, which is corrected by the \( \phi^4 \) interaction via spontaneous symmetry breaking.

We show that in our model there are two contributions to the photon mass. The first is related to the scalar curvature of the universe whereas the other comes from the bare photon mass in the action. In the late universe, the two contributions are comparable and partly cancel each other.

2.4 Radiation Dominated Solution

When we neglect \( \omega m^2 \) terms then \( a \propto \sqrt{t}, \phi = \text{const.} \) is a solution for radiation dominated era. To investigate the corrections to this solution, we expand \( H \) and \( B \) as power series at \( t = 0 \). We obtain the solution as

\[ H = \frac{1}{2t} + H_\infty^2 t + ..., \quad B = \frac{2m^2}{5} t + ... \]  

(2.22)

which gives \( a \propto \sqrt{t e^{H_\infty^2 t^2}} \) for equation of state \( p = \frac{2}{3} \). Note that the term \( H_\infty^2 t^2 \) in the exponent is reminiscent of inflation since it indicates a tendency for the decelerating universe \( a \propto \sqrt{t} \) to accelerate. However this term is insufficient to be interpreted as early inflation since the present lifetime of the universe of the order of \( H_\infty^{-1} \) and during the era of radiation dominated \( H_\infty^2 t^2 \) is negligible small. We can obtain standard early inflation only by putting \( p = -\rho \).
3 Conclusion

Cosmological background value of the time component of the photon field, $A_\mu$, may explain the late time expansion of the universe. Electric potential of the photon field causes constant Hubble parameter for the vacuum case and universe is considered as asymptotically deSitter space. This result is consistent with seven year WMAP data. This data has been analyzed and the dark energy "equation of state" parameter is $-1.10 \pm 0.14$, consistent with the cosmological constant (or equation of state parameter $-1$) [34]. The role of the cosmological constant in our model is played by the photon mass. Another feature of our model is that the Planck mass is interpreted as the scalar potential of the photon field that fills the universe. The huge amount of homogeneous electric potential may explain the weakness of the gravitational force. The smallness of the photon mass is related to the smallness of the present Hubble parameter. The difference between the Hubble parameter value in the primordial inflation era and present era has not yet been explained in this model. A possible mechanism for the change of the photon mass may be used to obtain the vastly different inflationary evolutions for both eras. The essence of this model is the embedded timelike vector field to Jordan-Brans-Dicke theory with the help of Lagrange multiplier field.

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