Different proof methods of Dalton's partial volume law equivalent to Dalton's partial pressure law

Ping Zhu¹,² and Yufa Lu¹,²

¹School of Science and Technology, Puer University, Puer 665000, Yunnan, China
²Open Key Laboratory of Mechanics of University in Yunnan Province, Puer 665000, Yunnan, China

*Email: zhuupp@163.com

Abstract. As we all know, Dalton's law of the partial pressure is equivalent to Dalton's law of the partial volume. On the other hand, through experiments, it can be proved that Dalton's law of the partial volume is correct, and Dalton's law of the partial pressure can be derived from Dalton's law of the partial volume, implying that both laws are equivalent to each other. Combined with the teaching of middle school physics and university basic physics, with different ideas and methods, this paper presents a variety of proof methods of Dalton's law of the partial volume to be equivalent to Dalton's law of the partial pressure. Different proof methods can improve the understanding and application of Dalton's law of the partial pressure and law of the partial volume. It is proved that the relevant content of Dalton's ideal gas partial pressure law is further supplemented and perfected, which plays a role throwing some light on this field.

As we all know, Dalton's law of the partial pressure is equivalent to Dalton's law of the partial volume. On the other hand, we can prove the correctness of Dalton's partial volume law by experiment. At the same time, Dalton's partial pressure law is derived from Dalton's partial volume law, so Dalton's partial volume law is yet equivalent to Dalton's partial pressure law. They are equivalent to each other. We have not seen the discussion that Dalton’s partial volume law is equivalent to Dalton’s partial pressure law in the relevant literature [1-6]. In this paper, with different ideas and methods, we give a variety of methods to prove that Dalton's partial volume law is equivalent to Dalton's partial pressure law. Different proof methods can further improve the understanding and application of Dalton's partial pressure law and Dalton's partial volume law.

There are N ideal gas mixtures in a container, where the volume of which is $V$, the temperature is $T$, and the pressure is $p$. Through experiments, we can prove that Dalton's law of partial volume holds.

The experiment shows that if in the container, we discharge all other gases, and leave only the i-th gas, in which we still maintain the temperature of the gas is $T$, and the pressure is $p$, and then the volume of the gas is $V_i$, then we have

$$V = \sum V_i,$$

and $pV_i = v_iRT$.

Then, the state equation for $N$ ideal gas mixtures is given by

$$pV = NRT.$$
\[ pV = \sum_i pV_i = (\sum_i v_i)RT, \] (2)

where \( v_i \) is the molar quantity of the i-th gas.

Assuming there is alone the i-th gas in the container, the temperature of which is \( T \), the volume is \( V_i \), and the pressure is \( p_i \), where \( p_i \) is called as the partial pressure of the i-th gas, then the pressure of \( N \) kinds of the mixed gases is satisfied by

\[ p = \sum p_i \] (3)

This result is called Dalton's partial pressure law. Dalton's law of the partial volume is equivalent to Dalton's partial pressure law, where, we will give the proof in different ways.

1. **State transformation method**

In Dalton's partial volume law, \( V_i \) is the volume of the i-th gas at the state of the pressure \( p \) and the temperature \( T \). Now, let's make it change isothermal to reach the state of the pressure to be \( p_i \) and the volume to be \( V_i \). Then we have

\[ pV_i = p_iV \]

\[ V_i = \frac{pV}{p} \] (5)

Substituting (5) into (1), we get

\[ V = \sum V_i = \sum \frac{pV}{p} = V\sum p_i, \] (6)

\[ \frac{\sum p_i}{p} = 1, \] (7)

\[ p = \sum p_i. \] (8)

2. **Molecular density method**

When the i-th ideal gas is alone at the partial volume state \( (p,V_i,T) \), its state equation may be written as

\[ p = n_i kT. \] (9)

When \( N \) kinds of gases are in the mixed equilibrium state \( (p,V,T) \), its state equation may be written as

\[ p = nkT, \] (10)

so

\[ n = n_i, \] (11)

where \( n \) is the molecular density of \( N \) kinds of the mixed ideal gases. On other hand, when the i-th ideal gas is alone at the partial pressure state \( (p_i,V,T) \), its state equation may be written as

\[ p_i = n_i kT, \] (12)

From the partial volume state and the partial pressure state, we can deduce easily \( n = \sum n_i \). Then
from (9) and (10) we have

\[ p = n_i kT = nkT = \left( \sum n_i' \right) kT = \sum (n_i' kT) = \sum p_i \]  

(13)

3. **Molar mass method**

When the i-th ideal gas is alone at the partial volume state \((p_i, V_i, T)\), its state equation may be written as

\[ p V_i = v_i kT \]  

(14)

From Dalton’s law of the partial volume of gas (1), we have

\[ V = \sum V_i = \sum \frac{v_i R T}{p} \]  

(15)

When the i-the gas exists alone at the partial pressure state \((p_i, V, T)\), its state equation may be written as

\[ p_i V = v_i R T \]  

(16)

Substituting (16) into (15), we obtain after manipulations

\[ V = \sum V_i = \sum \frac{p V}{p} \]  

(17)

\[ p = \sum p_i \]  

(18)

4. **Parameter configuration method**

From (2), we obtain

\[ \frac{p V}{V_i} = \frac{\left( \sum v_i \right)}{v_i} R T \]  

(19)

\[ \frac{V}{V_i} = \frac{\left( \sum v_i \right)}{p V_i} R T \]  

(20)

The dimension of \( p V / \left( \sum v_i \right) \) possesses the dimension of the pressure \( p \). Defining

\[ p_i = \frac{p v_i}{\left( \sum v_i \right)} \]  

(21)

we have

\[ \sum p_i = \frac{\sum p V_i}{\left( \sum v_i \right)} = p \]  

(22)

\[ V \sum p_i = p V = \left( \sum v_i \right) R T \]  

(23)

The result is
\[ p = \sum p_i. \]  \hspace{1cm} (24)

5. **Molecular average translational kinetic energy method**

The i-th ideal gas is alone at the partial volume state \((p_i, V_i, T)\), at the partial pressure state \((p_i, V, T)\), and N ideal gases mixed is at the state \((p, V, T)\), the temperature of which are T. So their molecular average translational kinetic energy is the same.

At the partial volume state \((p_i, V_i, T)\), the molecular average translational kinetic energy is \(\varepsilon_i\), the molecular density is \(n_i\), and the pressure is given by

\[ p = \frac{2}{3} n_i \varepsilon_i. \] \hspace{1cm} (25)

At the partial pressure state \((p_i, V, T)\), the molecular average translational kinetic energy is \(\varepsilon_i'\), the molecular density is \(n_i'\), and the pressure is given by

\[ p_i = \frac{2}{3} n_i' \varepsilon_i'. \] \hspace{1cm} (26)

At the mixed state \((p, V, T)\), the molecular average translational kinetic energy is \(\varepsilon\), the molecular density is \(n\), and the pressure is given by

\[ p = \frac{2}{3} n \varepsilon. \] \hspace{1cm} (27)

From (24) and (26), we may get

\[ n_i = n. \] \hspace{1cm} (28)

Since \(n = \sum n_i\), and \(\varepsilon = \varepsilon' = \varepsilon\), then we have

\[ p = \frac{2}{3} n \varepsilon = \sum \frac{2}{3} n_i \varepsilon_i = \sum \frac{2}{3} n_i \varepsilon_i = \sum p_i, \] \hspace{1cm} (29)

\[ p = \sum p_i. \] \hspace{1cm} (30)

In the above work, starting from the correctness of Dalton's law of partial volume of gas proved by experiments, we prove that Dalton's law of the partial volume is equivalent to Dalton's law of the partial pressure through many different methods. Therefore, both laws are equivalent to each other. Different proof methods can further improve the understanding and application of Dalton's partial pressure law and Dalton's partial volume law. The proof further complements and perfects the relevant contents of Dalton's ideal gas partial pressure law, which can make some contribution to the future development in this field.

**References**

[1] F. Enrico. Thermodynamics. Dover Publications, New York, 1956.
[2] S. J. Blundell and K. M. Blundell. Concepts in Thermal Physics, Oxford University Press. 2nd, New York, 2009.
[3] J. G. Yan Wylen and R. E. Sonntag. Fundamentals of Thermodynamics. 2nd ed. Wiley, New
Jersey, Printed in the United States of America, 2012.

[4] Y. H. Qin. General physics course: thermotics.3th ed. Higher education press, Beijing, 2011(In Chinese).

[5] C. Li, L. Y. Zhang, and S. W. Qian. General physics course: thermotics.3th ed. Higher education press, Beijing, 2015(In Chinese).

[6] Z. C. Wang. Thermodynamic statistical physics. 3th ed. Higher education press, Beijing, 2008(In Chinese).

[7] K. H. Zhao and W. Y. Luo. New concept physics course: thermotics.2nd ed. Higher education press, Beijing, 2005(In Chinese).

[8] K. Wark, Jr. and E. R. Donald. Thermodynamics. 6th ed. Tsinghua university press, Beijing, 2006 (In Chinese).