Gap nodes in the superconducting phase of the itinerant ferromagnet $UGe_2$.

I.A.Fomin,
P. L. Kapitza Institute for Physical Problems,
Kosygina 2, 117334 Moscow, Russia

Abstract

For the $UGe_2$ ferromagnetic superconductor the forms of the order parameter admitted by the crystal symmetry within the strong spin-orbit coupling scheme are written down. For each of the two possible phases existence of gap nodes required by symmetry is discussed and the nodes are found. Some consequences of presence of the nodes, which may be useful for experimental identification of the phases are discussed as well.

PACS numbers: 74.25.Dw 74.70.Tx 75.50.Cc

1 Introduction

The itinerant ferromagnet $UGe_2$ becomes superconducting in a pressure interval $11 < P_c < 16kbar$ at a temperature below $0.8K$ [1,2]. This temperature is much smaller then the Curie temperature $T_c$ for the same pressure. The estimated splitting of (quasi)spin-up and spin-down Fermi surfaces is $2 - 3$ orders of magnitude greater then the superconducting gap. This condition rules out the possibility of the spin-singlet Cooper pairing. At a triplet pairing the order parameter is a complex vector function $d(k)$. As a consequence of time reversal symmetry breaking superconducting phases of a ferromagnet generally speaking are nonunitary; i.e. $d$ and $d^*$ are not proportional to each
other, or \( \mathbf{d} \times \mathbf{d}^* \neq 0 \). Possible forms of the order parameters for the superconductive phases emerging continuously from a given normal phase can be classified if the group of symmetry of the normal phase is known \[3\]. An attempt of such classification for UGe\(_2\) has been made in a ref.\[4\]. It has been observed that the magnetic group of an orthorombic ferromagnetic crystal \( D_2(C_2) \) is isomorphic to \( D_2 \) and the basis functions of four different representations of the group \( D_2 \): \( B_1, B_2, B_3 \) were suggested as possible forms of the order parameter. It was shown later (ref. \[4\]) that not all of the suggested functions are independent. Because of the specific rules for the multiplication of antiunitary elements of magnetic groups the basis functions corresponding to the representations \( A \) and \( B_1 \) are equivalent i.e. are transformed according to one corepresentation of the magnetic group. Two other functions are also mutually equivalent. As a result there are only two essentially different types of symmetry of the superconducting order parameter.

For an experimental identification of a type of the order parameter, which is realized in UGe\(_2\) the existence and of gap nodes and their character for each of the phases are of importance. The aim of the present paper is to find the nodes. The following arguments apply as well to another orthorombic itinerant superconducting ferromagnet – URhGe \[6\].

2 Basis functions

Let us find first a general form of the functions \( \Psi_A \) and \( \Psi_B \), which are transformed under two different corepresentations \( A \) and \( B \) of magnetic group \( D_2(C_2) \). The group \( D_2(C_2) \) contains four operators. Two of them – the unity operator \( E \) and the rotation for an angle \( \pi \) around \( z \)-direction \( C_2^z \) are unitary. Two others – \( RC_2^x \) and \( RC_2^y \) contain time reversal \( R \) and are nonunitary. Corepresentations are formed by matrices \( G_1 \) and \( G_z \) corresponding to the unitary operators and \( F_x, F_y \) – to the nonunitary. In the present case corepresentations are one-dimensional and the matrices \( G_1, G_z, F_x, F_y \) are just complex numbers. According to the rules of multiplication of matrices forming corepresentations \[4\] they satisfy the following equations

\[
G_1^2 = G_1, \quad F_x \cdot F_x^* = G_1, \quad F_y \cdot F_y^* = G_1, \quad F_x \cdot F_y^* = G_z.
\]

These equations have two solutions, giving rise to two corepresentations. One of them (referred as \( A \)) has a form:

\[
G_1 = 1; \quad G_z = 1; \quad F_x = e^{2i\phi}; \quad F_y = e^{2i\phi}.
\]
The other – B a form:

$$G_1 = 1; G_z = -1; F_x = e^{2i\phi}; F_y = -e^{2i\phi},$$ (2)

where \(\phi\) is a real scalar. Let us write down \(\Psi_A\) in a form adopted for the strong spin-orbit coupling scheme \[4\]:

$$\Psi_A = x f_x(k) + y f_y(k) + z f_z(k),$$ (3)

where \(x, y, z\)- unit vectors directed along the symmetry axes \(b, c, a\). In the ferromagnetic phase \(a\) is an easy magnetization axis. All functions \(f_{x,y,z}(k)\) are odd with respect to \(k\), i.e. \(f_x(-k) = -f_x(k)\) etc.. \(\Psi_A\) under the action of operators \(E; C^*_2; R C^*_{2z}; R C^*_{2y}\) is multiplied by the numbers specified by the eq. (1). That imposes constraints on the functions \(f_x(k), f_y(k), f_z(k)\).

Since all operators in question are linear or antilinear the constraints are imposed separately on each function \(f_x, f_y, f_z\). For \(f_x(k)\) it is

$$f_x(-k_x, -k_y, k_z) = -f_x(k_x, k_y, k_z);$$
$$f_x^*(k_x, -k_y, -k_z) = e^{2i\phi} f_x(k_x, k_y, k_z);$$
$$f_x^*(-k_x, k_y, -k_z) = e^{2i\phi} f_x(k_x, k_y, k_z).$$

The constraints for \(f_y(k)\) are obtained by interchange of indices \(x\) and \(y\). The constraints for \(f_z(k)\) are:

$$f_z(-k_x, -k_y, k_z) = f_z(k_x, k_y, k_z);$$
$$f_z^*(k_x, -k_y, -k_z) = -e^{2i\phi} f_z(k_x, k_y, k_z);$$
$$f_z^*(-k_x, k_y, -k_z) = -e^{2i\phi} f_z(k_x, k_y, k_z).$$

The above conditions are satisfied by the following function

$$\Psi_A = e^{-i\phi_A}\{\hat{x}k_x(a_{11} + ik_xk_ya_{10}) + \hat{y}k_y(a_{22} + ik_xk_ya_{20}) + \hat{z}k_z(a_{33} + ik_xk_ya_{30})\},$$ (4)

where \(\phi_A, a_{11},...\) are real functions of \(k_x^2, k_y^2, k_z^2\). \(\Psi_A\) defined by eq. (4) differs from that given by eq.(2) of ref.\[4\] by the phase factor \(e^{-i\phi_A}\). When \(\phi_A = \pi/2, \Psi_A\) defined by eq. (4) can be cast in a form coinciding with \(\Psi_{B1}\) of ref.\[4\].

For the corepresentation B the constraints for \(f_x(k)\) are:

$$f_x(-k_x, -k_y, k_z) = -f_x(k_x, k_y, k_z),$$
$$f_x^*(k_x, -k_y, -k_z) = e^{2i\phi} f_x(k_x, k_y, k_z),$$
$$f_x^*(-k_x, k_y, -k_z) = e^{2i\phi} f_x(k_x, k_y, k_z),$$

\[\]
and for $f_z(k)$:

\[
\begin{align*}
    f_z(-k_x, -k_y, k_z) &= f_z(k_x, k_y, k_z) ; \\
    f_z^*(k_x, -k_y, -k_z) &= -e^{2i\phi} f_z(k_x, k_y, k_z) ; \\
    f_z^*(-k_x, k_y, -k_z) &= -e^{2i\phi} f_z(k_x, k_y, k_z).
\end{align*}
\]

A general form of the basis function for the corepresentation $B$ reads as:

\[
\Psi_B = e^{-i\phi_B} \{ \hat{x} k_z (b_{13} + ik_x k_y b_{10}) + \hat{y} k_z (i b_{23} + k_x k_y b_{20}) + \hat{z} k_x (b_{31} + ik_x k_y b_{30}) \}.
\]

With a suitable choice of the phase factor $e^{-i\phi_B}$ it can be transformed either in $\Psi_{B_{3}}$ or in $\Psi_{B_{4}}$ of ref. [4].

### 3 Nodes

At a triplet Cooper pairing gaps in the spectra of one-particle excitations are determined [8] by the eigenvalues of the matrix

\[ (\Delta_k \Delta_k^*)_{\alpha\beta} = d(k) \cdot d^*(k) \delta_{\alpha\beta} + i[d(k) \times d^*(k)] \sigma_{\alpha\beta}. \] (6)

For nonunitary phases this matrix has two different eigenvalues. Each of them gives a square of a gap for one projection of spin. In terms of the real and imaginary parts of $d(k)$, i.e. $d(k) = d_1(k) + id_2(k)$ the eigenvalues of $(\Delta_k \Delta_k^*)_{\alpha\beta}$ read as $|\Delta_{1,2}|^2 = |d_1(k)|^2 + |d_2(k)|^2 \pm 2|d(k) \times d^*(k)|$. The gap turns to zero for both projections of spin when $d_1(k) = 0$ and $d_2(k) = 0$; and only for one of the two projections when $|d_1(k)| = |d_2(k)|$ and $|d_1(k)| \perp |d_2(k)|$. Direct check shows that for a general form of unknown functions $a_{11}, a_{10}..., b_{13}, b_{10}...$ in the expressions (4) and (5) for both types of the order parameter $\Psi_A$ and $\Psi_B$ there are no nodes in the gaps. It means that the nodes are not required by symmetry. We have not used yet the fact that the splitting of spin-up and spin-down Fermi surfaces in UGe$_2$ and in URhGe is large.

The splitting of Fermi-surfaces suppresses the pairing amplitude for quasi-particles with different spin projections. For a singlet Cooper pairing superconductivity is completely destroyed at a splitting $2f > \sqrt{2}\Delta_0$, where $\Delta_0$ is a gap at zero temperature and zero splitting [9].
The formal reason for suppression of the pairing is the smearing of the singularity of the scattering amplitude of two quasiparticles with the opposite momenta and opposite spin projections at the polarization. When the polarization is absent the scattering amplitude in a second order on interaction has a singular contribution of a form: \( \ln \left( \frac{\omega_D}{\Delta_0} \right) \). At the polarization the splitting of two Fermi surfaces \( 2I \) occurs and the singular part turns into \( \ln \left( \frac{\omega_D}{I} \right) \). When \( I \gg \Delta_0 \) this contribution can be included in a regular part of the scattering amplitude. Transition from \( \Delta_{\uparrow\downarrow} \neq 0 \) to \( \Delta_{\uparrow\downarrow} = 0 \) must take place at \( I \sim \Delta_0 \sim T_s \). Both in UGe\(_2\) and in URhGe the condition \( I \gg T_s \) is well satisfied, so it is safe to assume that \( \Delta_{\uparrow\downarrow} = 0 \) for these compounds. This is equivalent to the condition \( d_z(k) = 0 \). With that constraint two types of the order parameter acquire the form:

\[
\Psi_A = e^{-i\phi_A} \{ \hat{x} k_x (a_{11} + ik_x k_y a_{10}) + \hat{y} k_y (a_{22} + ik_x k_y a_{20}) \}, \quad (7)
\]

\[
\Psi_B = e^{-i\phi_B} \{ \hat{x} k_x (b_{13} + ik_x k_y b_{10}) + \hat{y} k_y (ib_{23} + k_x k_y b_{20}) \}. \quad (8)
\]

\( \Psi_A \) together with the gaps on both Fermi surfaces turns to zero at the points \( k_x = 0, k_y = 0 \). These are symmetry nodes. To see it consider \( \Psi_A(0, 0, k_z) \) and apply to this function operation \( \hat{z} \): At the strength of eq. (1) \( \frac{\hat{z}}{2} \Psi_A(0, 0, k_z) = \Psi_A(0, 0, k_z) \). On the other hand from the definition of \( \hat{z} \):

\[
\frac{\hat{z}}{2} \Psi_A(0, 0, k_z) = -x f_x(0, 0, k_z) - y f_y(0, 0, k_z) = -\Psi_A(0, 0, k_z) \quad (9)
\]

Comparing two results we arrive at \( \Psi_A(0, 0, k_z) = 0 \).

In a similar way eq. (8) indicates that \( \Psi_B \) turns to zero on a line \( k_z = 0 \). These nodes are also required by symmetry since

\[
\frac{\hat{z}}{2} \Psi_B(k_x, k_y, 0) = -x f_x(-k_x, -k_y, 0) - y f_y(-k_x, -k_y, 0) = \Psi_B(k_x, k_y, 0) \quad (9)
\]

On the other hand from eq. (2) \( \frac{\hat{z}}{2} \Psi_B(k_x, k_y, 0) = -\Psi_B(k_x, k_y, 0) \), i.e. \( \Psi_B(k_x, k_y, 0) = 0 \).

### 4 Discussion

The above argument shows that two possible superconducting phases of UGe\(_2\) differ in a character and position of the gap nodes. For the A-type phase
(eq.(7)) these are isolated nodes at points of intersection of the Fermi surfaces with the direction of easy magnetization axis. For the B-type phase (eq.(8)) these are lines of nodes on equators of the Fermi surfaces which are perpendicular to the above mentioned axis. The nodes are giving rise to power-law dependencies of thermodynamic quantities on temperature at $T \ll T_s$. The exponents depend on a character of the nodes. Investigation of the power-law dependencies of thermodynamic quantities is a standard tool for identification of unconventional superconducting phases [10]. Let us point out some specifics of the expected low temperature properties of UGe$_2$, stemming from its magnetic polarization. 1) The values of gaps are generally speaking different for different spin projections. At a large splitting of two Fermi surfaces one can expect that this difference can be large as well, for example $\Delta_\downarrow \ll \Delta_\uparrow$. Then in a temperature interval $\Delta_\downarrow \ll T < T_s$ the smaller gap practically does not influence temperature dependencies of the thermodynamic quantities and the contribution of spin-down quasiparticles to thermodynamics will be practically that of the normal phase. 2) Magnetic field, induced by spontaneous magnetization $H_M = 4\pi M$ in UGe$_2$ is $H_M \sim 1$. This is much greater then estimated $H_{C1}$ for that compound. It means that UGe$_2$ is in a mixed state (or in the spontaneous flux phase [11]). Combination of the vortices with the line of nodes oriented perpendicular to the axes of vortices according to ref. [12] gives rise to a finite density of states on a Fermi level, which in its turn renders a linear temperature dependence of specific heat at low temperatures with a coefficient proportional to a square root of the field: $c_s \sim c_n \sqrt{H_M/H_{c2}}$. For fields much smaller then $H_{c2}$ this contribution to the specific heat is more important then the contribution of the bound electron states in the cores of the vortices. According to the present analysis a contribution of the discussed type is expected in the B-type phase, but not in the A-type. One can conclude that the expected difference in the low temperature properties of A and B-type phases can be used for identification of superconducting phases realized in UGe$_2$ and in URhGe.

Part of this work was completed in CEA Grenoble. I am grateful to J.Flouquet for hospitality in this research center and for stimulating discussions, to Universite Joseph Fourier for the financial support of my stay in Grenoble, to V.P.Mineev and A.Huxley for valuable comments and discussions. This work was also supported by RFFS ander grant 010216714.
References

[1] S.S.Saxena *et al.*, Nature **406**, 587 (2000).

[2] A.Huxley *et al.*, Phys. Rev. B **63**, 144519 (2001).

[3] G.E.Volovik and L.P.Gor’kov, Zh. Exp. Teor. Fiz. **88**, 1412 (1985) [Sov. Phys. JETP **61**, 843 (1985)].

[4] I.A.Fomin, Pis’ma v ZhETPh, **74**, 116 (2001) [JETP Letters, **74**, 111 (2001)].

[5] V.P.Mineev, Superconducting states in ferromagnetic metals., arXiv:cond-mat/0204263.

[6] Dai Aoki *et al.*, Nature **413**, 613 (2001).

[7] E.P.Wigner, Group Theory, Academic Press, 1959, ch. 26.

[8] D.Vollhardt, P.Woelfle, The Superfluid Phases of Helium 3, Taylor and Francis, 1990, p. 71.

[9] A.A.Abrikosov, *Fundamentals of the Theory of Metals* (Nauka, Moscow, 1987; North-Holland, Amsterdam, 1988, Ch.21)

[10] V.P.Mineev and K.V.Samokhin, *Introduction to Unconventional Superconductivity*, Gordon and Breach Science Publishers, 1999.

[11] E.Sonin, cond-mat/0202193.

[12] G.E.Volovik, Pis’ma v ZhETPh, **58**, 457 (1993).