In order to clear up the sensitivity of the nucleus–nucleus reaction cross sections $\sigma_R$ to the nuclear matter distributions in exotic halo nuclei, we have calculated the values of $\sigma_R$ for scattering of $^6$He, $^{11}$Li, and $^{19}$C nuclei on several nuclear targets at the energy of 0.8 GeV/nucleon. The calculations were performed in the "rigid target" approximation to the Glauber theory, different shapes of the nuclear density distributions in $^6$He, $^{11}$Li, and $^{19}$C being assumed.

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## 1. INTRODUCTION

Reaction cross sections $\sigma_R$ serve as one of the main sources of information on sizes of exotic halo nuclei. The root mean square (rms) radius of an exotic nucleus is determined by comparing the experimental reaction cross section for nucleus-nucleus scattering involving the nucleus of interest with theoretical model predictions. At the same time, the reaction cross sections depend not only on the rms nuclear matter radius, but also to some extent on the radial shape of the studied nucleus. Bush with coauthors [1] investigated the sensitivity of reaction cross sections to $^{11}$Li density distributions, and they concluded that $^{11}$Li-target cross sections at fixed rms matter radii retain a significant sensitivity to higher radial moments of the assumed $^{11}$Li density which is quite different for light and heavy targets. According to their consideration, reaction cross sections for nucleus–nucleus scattering are significantly more sensitive to the matter density at the nuclear periphery than those for proton–nucleus scattering. Therefore, measuring reaction cross sections for nucleus-nucleus scattering...
and nucleus-proton scattering one can determine the rms nuclear matter radius and obtain information on the radial shape of the studied nucleus. However, calculations of cross sections in [1] were performed using the Glauber theory in the optical limit approximation, which as is well known [2] significantly overestimates nucleus-nucleus reaction cross sections, especially for the case of halo nuclei. For this reason, it is not clear whether the conclusion of Bush et al. [1] that nucleus-nucleus cross sections are more sensitive to the nuclear periphery than nucleus-proton cross sections reflects the real physical case, or this conclusion was made due to the optical approximation to the Glauber theory used in their calculations. Bush et al. pointed out that it would be important to repeat similar calculations using a more accurate approach for calculations of the reaction cross sections.

In the present work, we investigate the sensitivity of reaction cross sections to the shape of the nuclear matter distribution by performing calculations of reaction cross sections for scattering of halo nuclei $^6\text{He}$, $^{11}\text{Li}$ and $^{19}\text{C}$ on different nuclear targets at the energy of 0.8 GeV/nucleon. The calculations of reaction cross sections were performed using the Glauber theory within the “rigid target” approximation [3], that is, at first the amplitude of scattering of one nucleon on the nuclear target was calculated, and then this amplitude was used in the calculations of the cross sections for scattering of an exotic nucleus consisting of several nucleons. As was shown in [2], the reaction cross sections for scattering of exotic nuclei on nuclear targets calculated within the rigid target approximation are very close to those calculated with the exact Glauber formula.

2. CROSS SECTION CALCULATIONS

The main formulas used in the present paper for calculations of the reaction cross sections $\sigma_R$ for nucleus–nucleus scattering are given below:

$$\sigma_R = \sigma_{tot} - \sigma_{el},$$

$$\sigma_{tot} = \frac{4\pi}{k} \text{Im}[F_{el}(0)], \quad \sigma_{el} = \frac{2\pi}{k^2} \int |F_{el}(q)|^2 q dq,$$

$$F_{el}(q) = \frac{ik}{2\pi} \int d^2b \exp (iqa) d^3r_1 d^3r_2 ... d^3r_A \cdot \rho(r_1, ..., r_A) \times$$
\[ \times \left\{ 1 - \prod_{j=1}^{A} [1 - \Gamma(b - s_j)] \right\}, \]

\[ \Gamma(b) = \int d^3r_1d^3r_2...d^3r_{A_T} \rho_T(r_1,...,r_{A_T}) \times \left\{ 1 - \prod_{m=1}^{A_T} [1 - \gamma(b - s_m)] \right\}. \quad (4) \]

Here, \( \sigma_{\text{tot}} \) is the total cross section for nucleus-nucleus scattering, \( \sigma_{\text{el}} \) is the integrated cross section for elastic nucleus-nucleus scattering, \( k \) is the value of the wave vector of the incident exotic nucleus, \( F_{\text{el}}(q) \) is the amplitude of elastic nucleus-nucleus scattering, \( q \) is the momentum transfer, \( b \) is the impact parameter vector, \( \rho(r_1,...,r_A) \) and \( \rho_T(r_1,...,r_{A_T}) \) are the many-body densities correspondingly of the studied exotic nucleus and the target nucleus, \( r_1,...,r_A, r_1,...,r_{A_T} \) and \( s_1,...,s_A, s_1,...,s_{A_T} \) stand for the radius vectors of the nucleons in these nuclei and their transverse coordinates, \( A \) and \( A_T \) are the total numbers of nucleons in the exotic and target nuclei, \( \Gamma(b) \) is the profile function of scattering of one nucleon of the studied exotic nucleus on the target nucleus, and \( \gamma(b) \) is the profile function of the nucleon–nucleon interaction. Spin-independent isospin-averaged amplitude of the free nucleon–nucleon (NN) scattering was employed, the traditional high-energy parametrization of this amplitude and the corresponding profile function

\[ \gamma(b) = \frac{\sigma_{NN}(1 - i\epsilon_{NN})}{2} \frac{1}{2\pi\beta_{NN}} \exp\left( - \frac{b^2}{2\beta_{NN}} \right) \quad (5) \]

being taken with the same parameters as in [1]: the total cross section \( \sigma_{NN} = 41 \text{ mb} \) and the amplitude slope \( \beta_{NN} = 0 \text{ fm}^2 \). The ratio of the real to imaginary part \( \epsilon_{NN} \) of the \( NN \) scattering amplitude has very little influence on the calculated value of \( \sigma_R \). Within the optical limit Glauber model, this parameter has no influence on \( \sigma_R \) at all. In our calculations we set \( \epsilon_{NN} = 0 \).

3. DENSITY DISTRIBUTIONS

In the present calculations it was assumed that the \(^6\text{He}, ^{11}\text{Li}, \) and \(^{19}\text{C} \) nuclei consist of a nuclear core of four, nine, and 18 nucleons, correspondingly, the number of halo neutrons being correspondingly two, two, and one. The many-body densities in the projectile exotic nucleus and in the target nucleus were presented as products of
one-body densities:

\[
\rho(r_1, ..., r_A) = \prod_{j=1}^{A_c} \rho_c(r_j) \times \prod_{j=A_c+1}^{A} \rho_h(r_j),
\]

\[
\rho_T(r_1, ..., r_{A_T}) = \prod_{j=1}^{A_T} \rho_T(r_j).
\]

Here, \( \rho_c(r_j) \) and \( \rho_h(r_j) \) are the one-body densities of the core and halo of the exotic nucleus, correspondingly; \( \rho_T(r_j) \) is the one-body density of the target nucleus; \( A_c \) and \( A_h \) are the numbers of the core and halo nucleons \((A_c + A_h = A)\).

As in [1], the matter density distributions in the core were described by Gaussian distributions

\[
\rho_c(r_j) = \frac{3}{2\pi R_c^2} \exp\left(-\frac{3r_j^2}{2R_c^2}\right),
\]

whereas the density distributions in the halo were described by a 1p-shell harmonic oscillator-type function

\[
\rho_h(r_j) = \frac{5}{3} \frac{5}{2} \frac{2\pi}{R_h^2} \left(\frac{r_j}{R_h}\right)^2 \exp\left(-\frac{5r_j^2}{2R_h^2}\right).
\]

Here, \( R_c \) and \( R_h \) are the root-mean-square (rms) radii of the core and halo matter density distributions. Calculations with the halo density distributions containing long density “tails” (to be discussed later) were also performed. Note that the rms radius \( R_m \) of the total matter density distribution is connected with the core and halo radii \( R_c \) and \( R_h \) as

\[
R_m = \left[\frac{(A_c R_c^2 + A_h R_h^2)}{A}\right]^{1/2}.
\]

We used the matter density distributions in the target nuclei \(^4\)He, \(^{12}\)C, \(^{28}\)Si, \(^{58}\)Ni, and \(^{208}\)Pb the same as those in [4].

As is known, no nucleon correlations are taken into account in cross-section calculations in the optical limit. When performing calculations of the cross sections in the rigid target approximation, we did not take the nucleon correlations into account either.

**4. RESULTS AND DISCUSSION**

At first, using the Glauber formulas in the optical limit approximation we repeated
the calculations of Bush et al. [1] of reaction cross sections $\sigma_R$ for scattering of $^{11}\text{Li}$ on protons and nuclear targets $^{12}\text{C}$ and $^{208}\text{Pb}$ at several fixed $^{11}\text{Li}$ rms matter radii $R_m$ with different ratios $R_h/R_c$. The calculated cross sections are shown in Fig. 1 versus the ratio $\epsilon = \langle r^4 \rangle / \langle r^2 \rangle^2$. The results of our calculations are practically the same as in [1]. (A small difference between the cross sections calculated in our study and in [1] is evidently due to slightly different target nuclear matter distributions used in the calculations.) In addition to the nuclear targets $^{12}\text{C}$ and $^{208}\text{Pb}$, considered by Bush et al., we also performed calculations for the $^4\text{He}$ target. It is seen that the dependence of $\sigma_R$ upon $\epsilon$ is essentially different for light and heavy targets. According to [1], these results indicate that reaction cross sections for heavy targets have higher sensitivity to valence nucleons of exotic nuclei than that for light targets.

Then, we repeated similar calculations using the Glauber theory within the rigid target approximation (Fig. 2). The cross sections calculated in the rigid target approximation are somewhat different from those calculated in the optical limit approximation. At the same time, for the given $R_m$ values, the variations of the cross sections as a function of $\epsilon$ are more or less similar in both cases. With $\epsilon$ increasing, the reaction cross sections $\sigma_R$ for light targets (protons and $^4\text{He}$) decrease, while for heavy targets ($^{208}\text{Pb}$) they increase. For $^{11}\text{Li}-^{12}\text{C}$ scattering, the cross section calculated in the optical limit approximation increases by a few percent with $\epsilon$ increasing, while the cross section calculated in the rigid target approximation is almost constant, the variation of $\sigma_R$ with $\epsilon$ being very small.

We also carried out calculations of the reaction cross sections $\sigma_R$ for scattering of halo nuclei $^6\text{He}$ and $^{19}\text{C}$ on protons and nuclear targets $^4\text{He}$, $^{12}\text{C}$ and $^{28}\text{Si}$. As compared to $^{11}\text{Li}$, the considered nuclei $^6\text{He}$ and $^{19}\text{C}$ have different relative amount of halo neutrons. The results of calculations (Figs. 3 and 4) show that here also with $\epsilon$ increasing the cross sections $\sigma_R$ decrease in the case of light target nuclei (protons and $^4\text{He}$), while they increase for a heavier nucleus $^{28}\text{Si}$. The performed calculations confirm the conclusions of Bush et al. that the cross sections $\sigma_R$ at fixed rms radii
Figure 1: Variation of reaction cross sections $\sigma_R$ with $\epsilon = \langle r^4 \rangle / \langle r^2 \rangle^2$ at fixed total rms matter radii $R_m$ of $^{11}\text{Li}$ equal to 2.9 fm (dashed lines), 3.0 fm (solid lines), and 3.1 fm (dotted lines). The cross sections are calculated in the optical limit approximation to the Glauber theory for scattering of $^{11}\text{Li}$ on hydrogen, $^4\text{He}$, $^{12}\text{C}$, and $^{208}\text{Pb}$ targets.

of halo nuclei retain a significant sensitivity to higher radial moments of the nuclear density which is different for light and heavy targets. This means that information about the rms radius can only be meaningfully extracted from the measured cross section if some form is assumed for the radial density distribution of the studied nucleus. At the same time, analyzing reaction cross sections measured for several nuclear targets of different size one can more precisely assess the rms nuclear matter radius and also get information on the radial shape of the studied nucleus. (Note that in the case of heavy targets Coulomb dissociation of the exotic nuclei should
Equation (9) with $R_h > R_c$ can describe an extended nucleon distribution of valence nucleons. However, this distribution at large distances from the nuclear center decreases with the radius $r$ increasing faster than it is predicted by theory. According to [5, 6], in addition to the main halo component which can be described to a first approximation by (9), the halo in exotic nuclei with low binding energy contains also a long density “tail”, which decreases exponentially with the radius $r$ increasing (Figs. 5, 6). Though such a density tail contains a small amount of matter (of the order of 1 %), it can produce a noticeable effect on the total rms nuclear matter radius. Due to smallness of the density tails, it can be expected that the sensitivity of the reaction cross sections $\sigma_R$ to the density tails is relatively
Figure 3: Variation of reaction cross sections $\sigma_R$ with $\epsilon$ at fixed total rms matter radii $R_m$ of $^6\text{He}$ equal to 2.4 fm (dashed lines), 2.5 fm (solid lines), and 2.6 fm (dotted lines). The cross sections are calculated in the rigid target approximation for scattering of $^6\text{He}$ on hydrogen, $^4\text{He}$, $^{12}\text{C}$, and $^{28}\text{Si}$ targets.

In the present study we investigate the sensitivity of reaction cross sections to the density tails by performing calculations of $\sigma_R$ using model nuclear density distributions with a tail and without it.

We performed calculations of $\sigma_R$ for scattering of $^6\text{He}$ and $^{11}\text{Li}$ on protons and several nuclear targets. The nuclear density distributions in $^6\text{He}$ and $^{11}\text{Li}$ used in the calculations are shown in Figs. 5 and 6. The solid curves in these figures show the density distributions without a tail with the following parameters: $R_c = 1.95$ fm, $R_h = 2.88$ fm, $R_m = 2.30$ fm for $^6\text{He}$, and $R_c = 2.30$ fm, $R_h = 5.86$ fm, $R_m = 3.37$ fm for $^{11}\text{Li}$ (see [7, 8]). The dashed curves show the same distributions including...
Figure 4: Variation of reaction cross sections $\sigma_R$ with $\epsilon$ at fixed total rms matter radii $R_m$ of $^{19}$C equal to 3.0 fm (dashed lines), 3.1 fm (solid lines), and 3.2 fm (dotted lines). The cross sections are calculated in the rigid target approximation for scattering of $^{19}$C on hydrogen, $^4$He, $^{12}$C, and $^{28}$Si targets.

The tails corresponding to the theoretical density distributions FC of [5] and P2 of [6]. Inclusion of these tails increases the rms matter radii by 0.20 fm and 0.26 fm, correspondingly in $^6$He and $^{11}$Li.

Table 1 presents the relative increase $\Delta \sigma_R / \sigma_R$ of the calculated reaction cross sections when the density tails are taken into account. For comparison, we also show the relative increase $\Delta \sigma'_R / \sigma_R$ of the cross sections calculated without density tails, but with the increased halo radii $R_h$ so that the total rms matter radii $R_m$ are larger by 0.20 fm and 0.26 fm, correspondingly in $^6$He and $^{11}$Li.

It is seen that the sensitivity of the reaction cross sections $\sigma_R$ for nucleus-proton
scattering to the density tails is rather poor, especially when the tail is long but contains small amount of nuclear matter, as in the case of $^{11}$Li. At the same time, the sensitivity of $\sigma_R$ to the density tails is more sizable in the case of nucleus-nucleus scattering for middle-weight nuclear targets ($^{28}$Si, $^{58}$Ni). In all the cases, the values of $\Delta\sigma_R / \sigma_R$ are smaller than those of $\Delta\sigma'_R / \sigma_R$. This means that taking the density tails into account in the analyses of the reaction cross sections $\sigma_R$ is important in order to deduce accurate values of the rms matter radii of the studied nuclei.
Figure 6: The nuclear matter density distribution in $^{11}\text{Li}$ (applied in the cross-section calculations) without a density tail (solid curve, $R_c = 2.30$ fm, $R_h = 5.86$ fm, $R_m = 3.37$ fm) and with a density tail (dashed curve, see the text).

Table 1. Relative increases of the calculated cross sections ($\Delta\sigma_R / \sigma_R$) when the density tails (shown in Figs. 5 and 6) are taken into account and ($\Delta\sigma'_R / \sigma_R$) when the halo radii are increased (see the text).

| Interacting nuclei | $\Delta\sigma_R / \sigma_R$ | $\Delta\sigma'_R / \sigma_R$ |
|-------------------|-----------------------------|-----------------------------|
| $^6\text{He} + p$ | 1.8 %                       | 3.9 %                       |
| $^6\text{He} + ^4\text{He}$ | 2.9 %                   | 4.8 %                       |
| $^6\text{He} + ^{12}\text{C}$ | 3.6 %                 | 4.6 %                       |
| $^6\text{He} + ^{28}\text{Si}$ | 3.6 %               | 4.2 %                       |
| $^6\text{He} + ^{58}\text{Ni}$ | 3.5 %                | 4.0 %                       |
| $^{11}\text{Li} + p$ | 0.4 %                     | 1.4 %                       |
| $^{11}\text{Li} + ^4\text{He}$ | 1.1 %                 | 3.1 %                       |
| $^{11}\text{Li} + ^{12}\text{C}$ | 2.2 %               | 5.2 %                       |
| $^{11}\text{Li} + ^{28}\text{Si}$ | 3.3 %              | 6.1 %                       |
| $^{11}\text{Li} + ^{58}\text{Ni}$ | 4.1 %              | 6.5 %                       |
5. CONCLUSION

We have performed calculations of the reaction cross sections $\sigma_R$ for nucleus-nucleus and nucleus-proton scattering assuming different model density distributions of the considered halo nuclei. The calculations were performed within the rigid target approximation to the Glauber theory, which is more accurate than the optical limit approximation used in previous calculations by Bush et al. [1]. Though the results of our calculations differ quantitatively from [1], they are in qualitative agreement with the conclusion of Bush et al. that the reaction cross sections for nucleus-nucleus scattering are more sensitive to the matter density at the nuclear periphery than those for nucleus-proton scattering. For all the considered cases, the calculated reaction cross sections $\sigma_R$ for nucleus-nucleus scattering depend not only on the rms matter radius of the studied exotic nucleus but also on the shape of its matter distribution. However, for the case of the $^{12}$C target this dependence is rather weak, the value of $\sigma_R$ depending basically on the rms nuclear matter radius. Analyzing the reaction cross sections for nucleus-proton and nucleus-nucleus scattering for light and middle-weight target nuclei it is possible to determine the rms matter radius of the exotic nucleus and to obtain information on the shape of its matter distribution. Our considerations also have shown that to deduce accurately the nuclear matter radius from the measured reaction cross sections it is important to take into account long matter density tails of the exotic nuclei at the nuclear far periphery.
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