Symmetries of the equations of two-dimensional shallow water over a rough bottom

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Abstract. The two-dimensional shallow water system over a rough bottom is considered. An overdetermined system of equations for finding the allowed symmetries is obtained. The consistency of this overdetermined system of equations is investigated. A general form of the solution of this system is obtained. The kernel of the symmetry operators is found. Cases where the kernel extensions of symmetry operators exist are presented. The corresponding classifying equations are given. The results of the group classification have indicated that the two-dimensional shallow water system above rough bottom cannot be linearized by point transformation in contrast to the one-dimensional shallow water system in cases of horizontal and inclined bottom profiles.

1. Introduction
In papers [1, 2], the one-dimensional shallow water system above rough bottom was considered and the group classification problem was solved.

In dimensionless variables, the two-dimensional shallow water system above rough bottom has the form [3]

\[ \begin{align*}
    u_t + uu_x + v u_y + \eta_x &= 0, \\
    v_t + uv_x + vv_y + \eta_y &= 0, \\
    \eta_t + [u(\eta + h)]_x + [v(\eta + h)]_y &= 0.
\end{align*} \tag{1} \]

Here \( z = -h(x, y) \), \( h(x, y) \geq 0 \) is the bottom profile, \( u = u(x, y, t) \), \( v = v(x, y, t) \) are the velocities components averaged on the horizontal depth, and \( \eta = \eta(x, y, t) \) is the deviation of the free surface.

In this paper, we solve the problem of group classification of the system (1) for all the bottom profiles.

2. Group classification
The task of the group classification is to find Lie symmetries admitted by the considered system of equations depending on the unknown function entering the system [4].

We seek symmetry operators of the system (1) as

\[ X = \xi_1(x, y, t, u, v, \eta) \frac{\partial}{\partial x} + \xi_2(x, y, t, u, v, \eta) \frac{\partial}{\partial y} + \xi_3(x, y, t, u, v, \eta) \frac{\partial}{\partial t} + \eta_1(x, y, t, u, v, \eta) \frac{\partial}{\partial u} + \eta_2(x, y, t, u, v, \eta) \frac{\partial}{\partial v} + \eta_3(x, y, t, u, v, \eta) \frac{\partial}{\partial \eta}. \]
Applying the criterion of invariance [4], we obtain an overdetermined linear homogeneous system of determining equations, which is not given here because of its bulkiness. Consistency testing of this determining system leads to the following result

\[ \xi^1 = -\dot{a}x - 2By + 2k(t), \quad \eta^1 = (\dot{a} + 2C)u - 2Bv - \ddot{a}x + 2\dot{k}, \]
\[ \xi^2 = -\dot{a}y + 2Bx + 2l(t), \quad \eta^2 = (\dot{a} + 2C)v + 2Bu - \ddot{a}y + 2\dot{l}, \]
\[ \xi^3 = -2a(t) - 2Ct, \quad \eta^3 = (2\dot{a} + 4C)\eta + \frac{x^2 + y^2}{2}\dddot{a} - 2\dddot{k}x - 2\dddot{l}y + \dot{f}. \]

For each \( h(x, y) \) functions \( a(t), k(t), l(t), f(t) \) and constants \( B, C \) are determined from the classifying equation

\[ (-\dddot{a}x - 2B\dddot{y} + 2k(t))h_x + (-\dddot{a}y + 2B\dddot{x} + 2l(t))h_y - (2\dddot{a} + 4C)\cdot h(x, y) = \]
\[ = -\frac{x^2 + y^2}{2} \cdot \dddot{a} + 2x \cdot \dddot{k} + 2y \cdot \dddot{l} - \dot{f}. \]

Next the results of the group classification will be shown.
1. Let \( h(x, y) \) be arbitrary function and
\[ X_1 = \frac{\partial}{\partial t}. \]

This operator appears for all bottom profiles.

The direct calculation shows that extensions of the Lie algebra of symmetries can be presented only in the following forms of the bottom
2. \( h(x, y) = H(x) + a_0y^2 + b_0y \).
2.1. \( H(x) = c_0x^2 + d_0x + e_0 \). In this case unknown functions \( a(t), k(t), l(t), f(t) \) and constants \( B, C \) are found in the system of equations

\[ 8a_0\dot{a} + 8a_0C = \dddot{a}, \quad (a_0 - c_0)B = 0, \]
\[ 2b_0B - 4d_0C + 4c_0k(t) - 3d_0\dot{a} = 2\dddot{k}, \]
\[ -2d_0B - 4b_0C - 3b_0\dot{a} + 4a_0l(t) = 2\dddot{l}, \]
\[ -4c_0C - 2c_0\dot{a} + 2d_0k(t) + 2b_0l(t) = -\dot{f}, \]

that can be solved easily.
2.2. \( H(x) = c_0x^3 + d_0x^2 + e_0x + f_0 \), \( c_0 \neq 0 \). In this case unknown functions \( a(t), k(t), l(t), f(t) \) and constants \( B, C \) are found in the system of equations

\[ 8a_0\dot{a} + 8a_0C = \dddot{a}, \quad B = 0, \]
\[ -3\dot{a}b_0 + 4a_0l(t) - 4b_0C = 2\dddot{l}, \]
\[ 4C + 5\dot{a} = 0, \quad 8b_0(\dot{a} + C) - 12c_0k(t) = \dddot{a}, \]
\[ c_0(4C + 3\dot{a}) - 4d_0k(t) = -2\dddot{k}, \]
\[ f_0(4C + 2\dot{a}) - 2c_0k(t) - 2b_0l(t) = \dot{f}, \]

that can be solved easily.
2.3. $H(x) = e_0 \ln(c_0 x - d_0) + f_0 x^2 + g_0 x + h_0$, $c_0 \neq 0$, $e_0 \neq 0$. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

\[
\begin{align*}
8a_0 \ddot{a} + 8a_0 C = \ddot{a}, & \quad B = 0, \\
-3 \dot{a} b_0 + 4a_0 l(t) - 4b_0 C = 2 \ddot{l}, & \quad \ddot{a} = 8f_0(\dot{a} + C), \\
-4c_0 g_0 C + 4d_0 f_0 C - 3c_0 g_0 \dot{a} + 4d_0 f_0 \dot{a} + 4c_0 f_0 k(t) = \frac{d_0 \ddot{a}}{2} + 2c_0 \dddot{k}, \\
& -4c_0 h_0 C + 4d_0 g_0 C - c_0 e_0 \dot{a} - 2c_0 h_0 \dot{a} + 3d_0 g_0 \dot{a} + 2c_0 g_0 k(t) - \\
& \quad -4d_0 f_0 k(t) + 2b_0 c_0 l(t) = -2d_0 \dddot{k} - c_0 \dddot{f}, \\
4d_0 h_0 C + 2d_0 h_0 \dot{a} + 2c_0 e_0 k(t) - 2d_0 g_0 k(t) - 2b_0 d_0 l(t) = d_0 \ddot{f},
\end{align*}
\]

that can be solved easily.

2.4. $H(x) = e_0 (c_0 x - d_0) \ln(c_0 x - d_0) + f_0 x^2 + g_0 x + h_0$, $c_0 \neq 0$, $e_0 \neq 0$. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

\[
\begin{align*}
8a_0 \ddot{a} + 8a_0 C = \ddot{a}, & \quad B = 0, \\
-3 \dot{a} b_0 + 4a_0 l(t) - 4b_0 C = 2 \ddot{l}, \\
3 \dot{a} + 4 C = 0, & \quad c_0 k(t) + d_0 (\dot{a} + 2C) = 0, \\
8f_0 \dot{a} + 8f_0 C = \ddot{a}, \\
-3g_0 \dot{a} - 4g_0 C + 4f_0 k(t) = 2 \dddot{k}, \\
2b_0 (\dot{a} + 2C) - 2g_0 k(t) - 2b_0 l(t) = \ddot{f},
\end{align*}
\]

that can be solved easily.

2.5. $H(x) = e_0 (c_0 x - d_0)^2 \ln(c_0 x - d_0) + f_0 x^2 + g_0 x + h_0$, $c_0 \neq 0$, $e_0 \neq 0$. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

\[
\begin{align*}
8a_0 \ddot{a} + 8a_0 C = \ddot{a}, & \quad B = 0, \\
-3 \dot{a} b_0 + 4a_0 l(t) - 4b_0 C = 2 \ddot{l}, \\
\dot{a} + C = 0, & \quad 2k(t)c_0 + d_0 C = 0, \\
8f_0 \dot{a} + 8f_0 C = \ddot{a}, \\
-4g_0 C - 3g_0 \dot{a} + 4f_0 k(t) = 2 \dddot{k}, \\
2b_0 (\dot{a} + 2C) - 2g_0 k(t) - 2b_0 l(t) = \ddot{f},
\end{align*}
\]

that can be solved easily.

2.6. $H(x) = e_0 (c_0 x - d_0)^3 + f_0 x^2 + g_0 x + h_0$, $c_0 \neq 0$, $e_0 \neq 0$, $i_0 \neq 0, 1, 2, 3$. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

\[
\begin{align*}
8a_0 \ddot{a} + 8a_0 C = \ddot{a}, & \quad B = 0, \\
-3 \dot{a} b_0 + 4a_0 l(t) - 4b_0 C = 2 \ddot{l}, & \quad \ddot{a} = 8f_0(\dot{a} + C), \\
-4c_0 g_0 C + 4d_0 f_0 C + 4c_0 f_0 k(t) - 3c_0 g_0 \dot{a} + 4d_0 f_0 \dot{a} - \frac{d_0 \ddot{a}}{2} - 2c_0 \dddot{k} = 0, \\
-4c_0 h_0 C + 4d_0 g_0 C + 2c_0 g_0 k(t) - 4d_0 f_0 k(t) + 2b_0 c_0 l(t) - 2c_0 h_0 \dot{a} + \\
& \quad + 3d_0 g_0 \dot{a} + 2d_0 \dddot{k} + c_0 \dddot{f} = 0, \\
\dot{a}(2 + i_0) + 4 C = 0, & \quad c_0 i_0 k(t) + 2d_0 C + d_0 \dot{a} = 0, \\
4d_0 h_0 C - 2d_0 g_0 k(t) - 2b_0 d_0 l(t) + 2d_0 h_0 \dot{a} = d_0 \ddot{f},
\end{align*}
\]
that can be solved easily.

2.7. Let $H(x)$ be arbitrary function, excluding cases 2.1–2.6. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

$$8a_0\dot{a} + 8a_0C = \ddot{a}, \quad B = 0,$$
$$2a_0l(t) = \ddot{l}, \quad \dot{a} = 0,$$
$$k(t) = 0, \quad C = 0,$$
$$2b_0l(t) = -\dot{f},$$

that can be solved easily.

3. $h(x, y) = H(y - a_0x) + b_0x^2 + c_0xy + d_0x$, except case 2. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

$$(1 + a_0^2)\dddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 4B(a_0^2c_0 - c_0 + 2a_0b_0),$$
$$2B(1 + a_0^2)\dot{H} - (4c_0\dot{a} + 4c_0C + 4B(b_0 + a_0c_0) - a_0\ddot{a})z =$$
$$= -2(2b_0 + a_0c_0)k(t) - 2c_0l(t) + 2\ddot{k} + 2a_0\ddot{t} + d_0(2a_0B + 3\dot{a} + 4C),$$
$$(2a_0B - \dot{a})z - 2a_0k(t) + 2l(t)H' - (2\dot{a} + 4C)H(z) =$$
$$= \left(-\frac{1}{2}\dddot{a} + 2c_0B\right)z^2 + (2\dddot{l} - 2c_0k(t) + 2d_0B)z - 2d_0k(t) - \dot{f},$$

where $z = y - a_0x$.

3.1. $H(z) = e_0z^3 + f_0z^2 + g_0z + h_0$. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

$$(1 + a_0^2)\dddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 4B(a_0^2c_0 - c_0 + 2a_0b_0),$$
$$4B(1 + a_0^2)c_0 - (4c_0\dot{a} + 4c_0C + 4B(b_0 + a_0c_0) - a_0\ddot{a}) = 0,$$
$$2B(1 + a_0^2)f_0 = -2(2b_0 + a_0c_0)k(t) - 2c_0l(t) + 2\ddot{k} + 2a_0\ddot{t} + d_0(2a_0B + 3\dot{a} + 4C),$$
$$2c_0(2a_0B - \dot{a}) - (2\dot{a} + 4C)e_0 = \left(-\frac{1}{2}\dddot{a} + 2c_0B\right),$$
$$(2a_0B - \dot{a})f_0 + 2c_0(-2a_0k(t) + 2l(t)) - f_0(2\dot{a} + 4C) = (2\dddot{l} - 2c_0k(t) + 2d_0B),$$
$$(-2a_0k(t) + 2l(t))f_0 - (2\dot{a} + 4C)g_0 = -2d_0k(t) - \dot{f},$$

that can be solved easily.

3.2. $H(z) = e_0z^3 + f_0z^2 + g_0z + h_0$, $e_0 \neq 0$. In this case unknown functions $a(t), k(t), l(t), f(t)$ and constants $B, C$ are found in the system of equations

$$(1 + a_0^2)\dddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 4B(a_0^2c_0 - c_0 + 2a_0b_0),$$
$$B = 0, \quad 4c_0\dot{a} + 4c_0C - a_0\ddot{a} = 0, \quad 5\dot{a} + 4C = 0,$$
$$-2(2b_0 + a_0c_0)k(t) - 2c_0l(t) + 2\ddot{k} + 2a_0\ddot{t} + d_0(3\dot{a} + 4C) = 0,$$
$$- 6a_0c_0k(t) - 4f_0C - 4f_0\dot{a} + 6c_0l(t) = -\dddot{a},$$
$$-4a_0f_0k(t) - 4g_0C - 3g_0\dot{a} + 4f_0l(t) = 2\dddot{l} - 2c_0k(t),$$
$$-2a_0g_0k(t) - 4h_0C - 2h_0\dot{a} + 2g_0l(t) = -2d_0k(t) - \dot{f},$$

that can be solved easily.
3.3. \( H(z) = g_0 \ln(e_0 z - f_0) + h_0 z^2 + i_0 z + j_0, e_0 \neq 0, g_0 \neq 0. \) In this case unknown functions \( a(t), k(t), l(t), f(t) \) and constants \( B, C \) are found in the system of equations

\[
(1 + a_0^2) \ddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 0, \quad \ddot{a} + 2C = 0,
\]
\[
B = 0, \quad 4c_0 \ddot{a} + 4c_0 C - a_0 \dddot{a} = 0, \quad h_0 C = 0,
\]
\[-2(2b_0 + a_0c_0)k(t) - 2c_0 l(t) + 2 \ddot{k} + 2a_0 \dddot{l} + d_0(3 \ddot{a} + 4C) = 0,
\]
\[-4a_0e_0h_0k(t) - 4e_0i_0C + 4f_0h_0C - 3e_0i_0 \ddot{a} + 4f_0h_0 \ddot{a} + 4e_0h_0l(t) = \frac{f_0 \dddot{a}}{2} + e_0(2 \dddot{a} - 2c_0 k(t)),
\]
\[-2a_0e_0i_0k(t) + 4a_0f_0h_0k(t) - 4e_0j_0C + 4f_0i_0C - e_0g_0 \ddot{a} - 2e_0j_0 \ddot{a} + 3f_0i_0 \ddot{a} + 2e_0i_0l(t) =
\]
\[= 4f_0h_0l(t) - f_0(2 \dddot{a} - 2c_0 k(t)) + e_0(-2d_0k(t) - \dot{j})],
\]
\[-2a_0e_0g_0k(t) + 2a_0f_0i_0k(t) + 4f_0j_0C + 2f_0j_0 \ddot{a} + 2e_0g_0l(t) - 2f_0i_0l(t) = -f_0(-2d_0k(t) - \dot{j}),
\]
that can be solved easily.

3.4. \( H(z) = g_0(e_0 z - f_0)^2 \ln(e_0 z - f_0) + h_0 z^2 + i_0 z + j_0, e_0 \neq 0, g_0 \neq 0. \) In this case unknown functions \( a(t), k(t), l(t), f(t) \) and constants \( B, C \) are found in the system of equations

\[
(1 + a_0^2) \ddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 0, \quad 3 \dot{a} + 4C = 0,
\]
\[
B = 0, \quad 4c_0 \ddot{a} + 4c_0 C - a_0 \dddot{a} = 0, \quad h_0 \dot{a} = 0,
\]
\[-2(2b_0 + a_0c_0)k(t) - 2c_0 l(t) + 2 \ddot{k} + 2a_0 \dddot{l} + d_0(3 \ddot{a} + 4C) = 0,
\]
\[-2a_0e_0g_0k(t) + 4f_0g_0C + 2f_0g_0 \ddot{a} + 2e_0g_0l(t) = 0,
\]
\[-e_0g_0 \ddot{a} - 4a_0h_0k(t) - 4i_0C - 3i_0 \ddot{a} + 4h_0l(t) = 2 \dddot{a} - 2c_0k(t),
\]
\[-2a_0e_0g_0k(t) - 2a_0i_0k(t) + 2e_0g_0l(t) - 4j_0C - 2j_0 \ddot{a} + 2i_0l(t) = -2d_0k(t) - \dot{j},
\]
that can be solved easily.

3.5. \( H(z) = g_0(e_0 z - f_0)^2 \ln(e_0 z - f_0) + h_0 z^2 + i_0 z + j_0, e_0 \neq 0, g_0 \neq 0. \) In this case unknown functions \( a(t), k(t), l(t), f(t) \) and constants \( B, C \) are found in the system of equations

\[
(1 + a_0^2) \ddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 4B(a_0^2c_0 - c_0 + 2a_0b_0),
\]
\[
B = 0, \quad 4c_0 \ddot{a} + 4c_0 C - a_0 \dddot{a} = 0, \quad \dot{a} + C = 0,
\]
\[-2(2b_0 + a_0c_0)k(t) - 2c_0 l(t) + 2 \ddot{k} + 2a_0 \dddot{l} + d_0(3 \ddot{a} + 4C) = 0,
\]
\[-2a_0e_0k(t) + f_0 C + 2c_0 l(t) = 0, \quad \ddot{a} = 0,
\]
\[4a_0e_0f_0g_0k(t) - 4f_0^2 g_0 C - 2f_0^2 g_0 \ddot{a} - 4e_0f_0g_0l(t) = 0,
\]
\[-2a_0e_0g_0k(t) + e_0f_0h_0 \ddot{a} + 2e_0^2 g_0l(t) - 4a_0h_0k(t) - 4i_0C - 3i_0 \ddot{a} + 4h_0l(t) = 2 \dddot{a} - 2c_0k(t),
\]
\[2a_0e_0f_0g_0k(t) - 2e_0f_0g_0l(t) - 2a_0i_0k(t) - 4j_0C - 2j_0 \ddot{a} + 2i_0l(t) = -2d_0k(t) - \dot{j},
\]
that can be solved easily.

3.6. \( H(z) = g_0(e_0 z - f_0)^k_0 + h_0 z^2 + i_0 z + j_0, e_0 \neq 0, g_0 \neq 0, k_0 \neq 0, 1, 2, 3. \) In this case unknown
functions \(a(t), k(t), l(t), f(t)\) and constants \(B, C\) are found in the system of equations

\[
(1 + a_0^2)\ddot{a} - 8(b_0 + a_0c_0)(\dot{a} + C) = 4B(a_0^2c_0 - c_0 + 2a_0b_0),
\]

\[
B = 0, \quad 4c_0\dot{a} + 4c_0C - a_0\ddot{a} = 0,
\]

\[
- 2(2b_0 + a_0c_0)k(t) - 2c_0l(t) + 2\ddot{k} + 2a_0\dddot{t} + d_0(3\dot{a} + 4C) = 0,
\]

\[
\dot{a}(2 + k_0) + 4C = 0, \quad \ddot{a} = 8h_0(\dot{a} + C),
\]

\[
- 2a_0e_0g_0k(t) + 2e_0g_0k_0l(t) + 4f_0g_0C + 2f_0g_0\dot{a} = 0,
\]

\[
- 4a_0e_0h_0k(t) - 4e_0i_0C + 4f_0h_0C - 3e_0i_0\dot{a} + 4f_0h_0\dot{a} + 4e_0h_0l(t) = \frac{f_0\ddot{a}}{2} + e_0(2\dddot{t} - 2c_0k(t)),
\]

\[
- 2a_0e_0i_0k(t) + 4a_0f_0h_0k(t) - 4e_0j_0C + 4f_0i_0C - 2e_0j_0\dot{a} + 3f_0i_0\dot{a} + 2e_0i_0l(t) - 4f_0h_0l(t) =
\]

\[
= -f_0(2\dddot{t} - 2c_0k(t)) + e_0(-2d_0k(t) - \dddot{f}),
\]

\[
2a_0f_0i_0k(t) + 4f_0j_0C + 2f_0j_0\dot{a} - 2f_0i_0l(t) = f_0(2d_0k(t) + \dddot{f}),
\]

that can be solved easily.

3.7. Let \(H(z)\) be arbitrary function, excluding cases 3.1–3.6. In this case unknown functions \(a(t), k(t), l(t), f(t)\) and constants \(B, C\) are found in the system of equations

\[
B = 0, \quad \dot{a} = 0, \quad C = 0,
\]

\[
- 2(2b_0 + a_0c_0)k(t) - 2c_0l(t) + 2\ddot{k} + 2a_0\dddot{t} + d_0(3\dot{a} + 4C) = 0,
\]

\[
l(t) = a_0k(t), \quad 2\dddot{t} - 2c_0k(t) = 0, \quad -2d_0k(t) - \dddot{f} = 0,
\]

that can be solved easily.

4. \(h(x, y) = \frac{g_0}{(x - a_0)^2} e^{\frac{h_0}{2} \arctan \frac{y}{x^2 + 1}} + c_0(x^2 + y^2) + d_0x + e_0y + f_0, \quad g_0 \neq 0, \quad z = (y - b_0)/(x - a_0)\).

In this case unknown functions \(a(t), k(t), l(t), f(t)\) and constants \(B, C\) are found in the system of equations

\[
\dddot{a} = 8c_0(\dot{a} + C), \quad Bb_0 = 2C,
\]

\[
\dddot{l} - 2c_0l(t) = -2c_0C - d_0B - \frac{3}{2}e_0\dot{a},
\]

\[
\dddot{k} - 2c_0k(t) = e_0B - 2d_0C - \frac{3}{2}d_0\dot{a},
\]

\[
\dddot{f} = 2f_0\dot{a} - 2d_0k(t) - 2c_0l(t) + 4f_0C,
\]

\[
- 2b_0h_0B - a_0h_0\dot{a} + 4a_0B - 2b_0\dddot{t} + 2h_0k(t) + 4l(t) = 0,
\]

\[
b_0h_0\dot{a} - 4b_0B - 4a_0C - 2a_0\dot{a} - 2h_0l(t) + 4k(t) = 0,
\]

that can be solved easily.

5. Let \(h(x, y)\) be a solution of the equation \((A_1x + A_2y + A_3)h_x + (A_1y - A_2x + A_4)h_y + A_5h = A_6(x^2 + y^2) + A_7x + A_8y + A_9\) for some constants \(A_1, ..., A_9\), among which there is a distinct from zero constant, excluding cases 1–4. In this case unknown functions \(a(t), k(t), l(t), f(t)\) and constants \(B, C\) are found in the system of equations

\[
(-\dot{a}x - 2By + 2k(t))h_x + (-\dot{a}y + 2Bx + 2l(t))h_y - (2\dot{a} + 4C) \cdot h(x, y) =
\]

\[
= -\frac{x^2 + y^2}{2} \cdot \dddot{a} + 2x \cdot \dddot{k} + 2y \cdot \dddot{l} + \dddot{f}
\]

Remark. Analysis of group classification results shows the impossibility of linearizing of the two-dimensional shallow water system above rough bottom with a point transformation.
3. Conclusion
Main results of the paper are the classification of bottom profiles where the kernel extensions of symmetry operators exist. Corresponding classifying equations for this profiles are derived. The distinctive property of a two-dimensional shallow-water system over a rough bottom is that it cannot be linearized by a point transformation, unlike a one-dimensional shallow-water system, when the bottom profiles are horizontal and inclined.

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