Multijet Structure of High $E_T$ Hadronic Collisions

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Abstract

Multijet events at large transverse energy ($\sum E_T > 420$ GeV) and large multijet invariant mass ($m_{jets} > 600$ GeV) have been studied by the CDF Collaboration at the Fermilab Tevatron. The observed jet multiplicity distribution can be understood in a QCD inspired exponentiation model, in regions of phase space which require going beyond fixed order perturbation theory.
With the completion of run I at the Fermilab Tevatron, a sufficient amount of data has been collected to study rare hard scattering events, with jet transverse energies in the hundreds of GeV range. Such high $E_T$ jet events are interesting in their own right, as the discussion of a possible “excess” in the single jet $E_T$ spectrum above $E_T \approx 200$ GeV [1] has shown. Another aspect is the fact that events with jets in this $E_T$ range, while rare at the Tevatron, will be produced abundantly at the much higher energy of the CERN LHC, where they will constitute important backgrounds to searches for new heavy degrees of freedom. Thus, the study of high $E_T$ QCD events at the Tevatron allows one to develop strategies for new physics searches at the LHC.

The aspect to be considered below is the multiplicity of additional soft jets which arise via the emission of soft gluons in a hard scattering process. At the LHC, one application is the study of weak boson scattering. In events like $qQ \rightarrow qQWW$, which are mediated by $t$-channel exchange of electroweak bosons, medium $E_T$ jets in the central region are a rare occurrence. This is a consequence of color coherence between initial and final state radiation which leads to gluon emission mainly between the forward scattered quarks and the beam directions [2]. At the same time the modest transverse momentum of the scattered quarks severely limits the transverse momentum of emitted gluons [3]. Typical background events like $t\bar{t} \rightarrow bW^{+}\bar{b}W^{-}$ or $q\bar{q} \rightarrow W^{+}W^{-}$, on the other hand, show a large probability for QCD radiation in the central region, with gluon transverse momenta sufficient to produce visible jets [4]. As a result, a central jet veto is quite effective for background suppression [5,6].

In applications like these it is necessary to correctly model the angular distribution of emitted minijets and to reliably determine the hard scales which govern their transverse momenta. These features are automatically included by using full tree level QCD matrix elements. For an effective jet veto, however, one is interested in the phase space region where the probability of extra QCD radiation becomes of order unity, and this is exactly the region where fixed order perturbation theory ceases to be applicable. In Refs. [4,7] it was suggested to use a soft gluon exponentiation model to extend the perturbative calculation into this region of large minijet multiplicities. In the following we show that existing data on multijet events at the Tevatron [8,9] allow one to
test and refine this model for multiple minijet emission. In particular one can experimentally
determine “good” choices for the factorization and renormalization scales, information which can
then be used to more reliably predict minijet emission in other processes, at the Tevatron or at
the LHC.

Consider dijet production in $p\bar{p}$ collisions at the Tevatron. The next-to-leading order (NLO)
QCD corrections to the cross section for producing two or more jets, $\sigma_{2,\text{incl}}$, is available in the
form of a full NLO Monte Carlo program, JETRAD \[\text{[10]}\]. The same program also provides the
tree level cross section, $\sigma_3$, for three-jet production, within an arbitrary phase space region. In
the following we use the CDF cone algorithm to define jets \[\text{[11]}\], with a radius of $R_0 = 0.7$ in
pseudorapidity–azimuthal angle space; a cluster of partons of transverse energy $E_T > 20$ GeV
and pseudorapidity $|\eta| < 4.2$ is defined as a jet. (Variations of the $E_T$ threshold will be considered
later.) Following the analysis of the CDF Collaboration \[\text{[8]}\] we study two- and three-jet events
with total jet transverse energy,

$$\sum E_T > 420 \text{ GeV} \ ,$$

and invariant mass of the multijet system,

$$m_{\text{jets}} > 600 \text{ GeV} \ .$$

In addition the scattering angle \[\text{[12]}\], $\theta^*$, of the highest $E_T$ jet in the multijet center of mass frame
must satisfy

$$|\cos \theta^*| < \frac{2}{3} \ .$$

Within these cuts $\sigma_{2,\text{incl}}$ depends weakly on the minimal jet transverse energy since the two-jet
inclusive events are largely defined by the two hardest jets, which must have transverse energies
$E_{T1}, E_{T2} \approx \sum E_T/2 > 210$ GeV. The three-jet cross section,

\[1\] Here and in the following, the “cross section for n-jet inclusive events”, $\sigma_{n,\text{incl}} = \sum_{k \geq n} \sigma_{k,\text{jets}}$, 
directly corresponds to the rate of such events. No jet multiplicity factor is included in its
definition.
\[
\sigma_3(E_{T,\text{min}}) = \int_{E_{T,\text{min}}}^{\infty} dE_T^3 \frac{d\sigma_3}{dE_T^3},
\]

on the other hand, is a steeply falling function of the transverse energy threshold, \(E_{T,\text{min}}\). For sufficiently low threshold one will eventually reach a region with \(\sigma_3(E_{T,\text{min}}) > \sigma_{2,\text{incl}}\); clearly, the interpretation of \(\sigma_3\) as the cross section for either three jet inclusive or three-jet exclusive events is not tenable in this region. The unphysical relation \(\sigma_3 > \sigma_{2,\text{incl}}\) is a sign that fixed order perturbation theory is breaking down and that multiple gluon emission needs to be resummed.

For small \(E_{T,\text{min}}\), soft gluon emission from the hard dijet production process will dominate, and, analogous to soft photon emission, one may show that this soft-gluon radiation approximately exponentiates when the soft gluons go unobserved \([13]\). Here we consider a phenomenological model which assumes that the analogy to multiple soft photon emission can be taken further, namely, that the probability \(P_n\) for observing \(n\) soft jets beside the two hard jets of the basic hard scattering event is given by a Poisson distribution,

\[
P_n(\bar{n}) = \frac{\bar{n}^n}{n!} e^{-\bar{n}},
\]

with

\[
\bar{n} = \bar{n}(E_{T,\text{min}}) = \frac{1}{\sigma_{2,\text{incl}}} \int_{E_{T,\text{min}}}^{\infty} dE_T^3 \frac{d\sigma_3}{dE_T^3}.
\]

We will call this model the “exponentiation model” in the following \([14]\). The exponentiation model has a number of appealing features:

1. By construction the cross section for two jet inclusive events is given by \(\sigma_{2,\text{incl}}\).

2. \(\sigma_{2,\text{incl}} P_0\) gives the correct Sudakov suppressed rate for two jet exclusive events \([14]\); this is not surprising since \(P_0\) is the probability for not seeing any additional jets, and here soft gluon exponentiation can be proved.

3. For sufficiently large \(E_{T,\text{min}}\), when \(\sigma_3(E_{T,\text{min}}) \ll \sigma_{2,\text{incl}}\), the three jet rate, \(i.e.\) the cross section for events with one soft jet, is given by

\[
\sigma_{2,\text{incl}} P_1 \approx \sigma_{2,\text{incl}} \bar{n} = \sigma_3(E_{T,\text{min}}),
\]

\[4\]
and thus reproduces the perturbative result at $O(\alpha_s^3)$. From a perturbative point of view
\[\sigma_{2,\text{incl}} P_1 = \sigma_3(E_{T,\text{min}})(1 + O(\alpha_s)),\]
thus, $\sigma_{2,\text{incl}} P_1$ has the same level of accuracy as the tree level calculation of $\sigma_3(E_{T,\text{min}})$.

4. The full angular and transverse momentum information contained in the tree level result is retained in the estimated multijet emission probabilities.

5. The probabilities $P_n$ all remain finite at small $E_{T,\text{min}}$, with $0 < P_n < 1$. This renders the exponentiation model superior to the use of e.g. $\sigma_3/(\sigma_{2,\text{incl}} - \sigma_3)$ or $\sigma_3/\sigma_{2,\text{incl}}$ as an estimate for the ratio of three-jet to two-jet exclusive or inclusive rates.

6. Only the hard scattering cross section (here $\sigma_{2,\text{incl}}$) and the cross section for emission of one additional soft parton (here $\sigma_3$) need to be known perturbatively. Thus, the model can easily be applied to more complicated processes [4,7].

For the exponentiation model to be useful, it is necessary that minijet multiplicities in hard scattering events at least approximately follow a Poisson distribution. Recently the CDF collaboration has published results on multijet production within the cuts of Eqs. (1–3). Out of a total of $N_{\text{tot}} = 1874$ events, $N_0 = 345$ have exactly 2 jets with $E_T > 20$ GeV, $N_1 = 612$ have 3 such jets, and the number of events with $n \geq 2$ minijets, in addition to the hard dijet system, are $N_2 = 554$, $N_3 = 250$, $N_4 = 88$, $N_5 = 21$, and $N_6 = 4$ [8]. For a Poisson distribution the average multiplicity of minijets in the CDF event sample, $\langle n_{\text{jets}} - 2 \rangle = 1.57 \pm 0.03$ (where the error is statistical only), and the values for $\bar{n}(N_n/N_{\text{tot}})$ extracted from the fraction of events with a fixed number of minijets, via the relation
\[f_n = \frac{N_n}{N_{\text{tot}}} = P_n(\bar{n}),\] (8)
should all agree. The results of these different extractions of $\bar{n}$ are compared in Fig. 1. A Poisson ansatz for the jet multiplicity distribution works surprisingly well. For larger minijet multiplicities ($n \geq 4$) the observed rates fall more and more below a Poisson distribution. This can be understood qualitatively, however, as an effect of the reduced phase space (in $\eta, \phi$) which
FIG. 1. Poisson parameter $\bar{n}$ extracted from the fraction of events with $n$ “minijets” in the CDF multijet sample [8], according to Eq. (8). Errors are statistical only. A Poisson distribution of the minijet multiplicity would yield the dashed line at $\bar{n} = 1.57$, the mean observed minijet multiplicity. No solution to Eq. (8) exists for $n = 2$ since the observed fraction, $f_2 = 0.296 \pm 0.011$, is larger than the maximal Poisson probability for two events, $P_2(\bar{n} = 2) = 0.271$. The dashed line gives $P_2(\bar{n} = 1.57) = 0.257$ which is $3.5\sigma$ below the measured $f_2$.

is available for additional minijets due to the finite cone size of jets. Also, one should note that a systematic error of 15% only on all $N_n$, (in addition to the statistical errors shown in Fig. [1]) would suffice to render a Poisson fit perfectly acceptable.

Since the CDF data are well described by a Poisson distribution of minijet multiplicities we may now compare the measured average multiplicity, $\bar{n} = 1.57$ for $E_T,\text{min} = 20 \text{ GeV}$, with the perturbative QCD prediction given in Eq. (3). For this purpose we have calculated the total multijet rate, within the cuts of Eqs. (1–3) and 10% Gaussian smearing of jet momenta [8] to simulate detector effects. Using the JETRAD program [10] we find $\sigma_{2,\text{incl}} = 33 \text{ pb}$. Here we have used CTEQ 4HJ parton distribution functions [15] and the factorization and the renormalization
scale have been set to $\mu_F = \mu_R = \sum E_T / 4$, with $\alpha_s$ evaluated at two-loop order and $\alpha_s(M_Z) = 0.116$ as required by the evolution of the parton distribution functions.

Since the 3-jet cross section $\sigma_3(E_{T,\text{min}})$ is evaluated at tree level only, a variation of scales leads to substantial uncertainties here. We have analyzed four different choices. (i) $\mu_R = \mu_F = \sum E_T / 4$, which is the same choice as for the 2-jet inclusive rate. Such a large factorization scale is not entirely physical, however. Collinear initial state radiation with transverse energies between $E_{T,\text{min}}$ and the hard scale, $\sum E_T / 4$, is generated explicitly in terms of the third jet. When choosing a large factorization scale, this emission is considered twice, in terms of the third jet and via the evolution of the parton distribution functions. A factorization scale which is tied to the transverse energy of the third parton avoids such double counting. Similarly, a small renormalization scale may better match the small parton virtualities which appear in the emission of soft gluons. This motivates the following set of scales: (ii) $\mu_R = \mu_F = E_{T,3}$, where $E_{T,3}$ is the transverse energy of the softest of the three partons, and, finally, $\mu_F = \xi E_{T,3}$ with an $\alpha_s^3$ factor in the calculation of $\sigma_3$ which is given by

$$\alpha_s^3 = \prod_{i=1}^{3} \alpha_s(\xi E_{T,i}),$$

i.e. the transverse energy of each final state parton is taken as the relevant scale for its production. Here we have used overall scale factors (iii) $\xi = 1$ and (iv) $\xi = 1/2$.

Results for these four scale choices are shown in Fig. 2. For $E_{T,\text{min}} = 20$ GeV, estimates for the mean minijet multiplicity $\tilde{n}(E_{T,\text{min}}) = \sigma_3(E_{T,\text{min}})/\sigma_{2,\text{incl}}$ vary between 0.8 and 2.2 and thus bracket the CDF value of 1.57. From Fig. 2 it is clear, however, that a scale choice tied to the transverse energy of the soft jets is preferred by the data. The single CDF data point is not sufficient, of course, to optimize the scale choice: an analysis of the observed minijet multiplicity as a function of the minimal jet transverse energy would be needed.

We conclude that the exponentiation model provides a good description of existing data on minijet multiplicities in hard scattering events at the Tevatron. For a further quantitative comparison, the $E_{T,\text{min}}$ dependence of the multijet rates would be most useful. The freedom in choosing the renormalization and factorization scales for the tree level 3-jet cross section should
FIG. 2. Ratio of the tree level 3-jet cross section to the NLO cross section for 2-jet inclusive events within the CDF acceptance cuts [8]. The cross section ratio $\bar{n} = \sigma_3(E_{T,\text{min}})/\sigma_2,\text{incl}$, with $\sigma_2,\text{incl} = 33$ pb, is shown as a function of the transverse energy threshold, $E_{T,\text{min}}$, of the third jet. Results are given for four different scale choices in $\sigma_3$: $\mu_R = \mu_F = \sum E_T/4$ (solid line), $\mu_R = \mu_F = E_{T,3}$ (dashed line), and $\mu_F = \xi E_{T,3}$, $\alpha_s^3 = \prod_{i=1}^2 \alpha_s(\xi E_{T,i})$ with a scale factor $\xi = 1$ (dash-dotted line) and $\xi = 1/2$ (dotted line). The CDF value for the average minijet multiplicity, $\bar{n} = 1.57$, is given by the diamond.

allow for a reasonable parameterization of the data, as a function of $E_{T,\text{min}}$ and the parameters of the hard event, provided the approximate Poisson multiplicity distribution of jets remains valid beyond the phase space region probed in Ref. [8]. Finally we note that a comparison with tree level results for 4 or 5 jet cross sections [16], while complementary, will be subject to very large theoretical uncertainties. Just like in the exponentiation model, there is a strong scale dependence of the tree level $n$-jet cross sections ($\sigma_n,\text{jets} \sim \alpha_s^n(\mu_R)$). In addition one needs to use the tree level cross sections at the edge of the validity range of fixed order perturbative QCD and similar to the case of the 3-jet cross section discussed before, calculated $n$-jet rates will
become unphysically large for sufficiently low $E_{T,\text{min}}$. The exponentiation model addresses this last problem and should, thus, provide a valuable alternative in analyzing multijet emission in hard scattering events.

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