Understanding \(Y(4274)\) and \(X(4320)\) in the \(J/\psi\phi\) invariant mass spectrum

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Abstract

In this work we study the structures near 4.3 GeV in the \(J/\psi\phi\) invariant mass spectra in \(B\) meson decay process \(B^+ \rightarrow J/\psi\phi K^+\) and two photon fusion process \(\gamma\gamma \rightarrow J/\psi\phi\). The \(Y(4274)\) as a \(D_sD_{s0}(2317)\) molecular is studied in the Bethe-Salpeter equation approach with quasipotential approximation. The absence of \(Y(4274)\) in \(\gamma\gamma \rightarrow J/\psi\phi\) channel can be well explained by the decay widths of \(Y(4274)\) decaying to \(\gamma\gamma\) and \(J/\psi\phi\). The distribution of mass difference released by CMS collaboration is reproduced by two resonances near 4.3 GeV, \(Y(4274)\) and \(X(4320)\). The different production mechanism suggests \(X(4320)\) observed in the \(B\) decay should be the missing \(3^3P_1\) charmonium state \(\chi''_{c1}\) and different from \(X(4350)\) observed in two photon fusion which can be assigned as \(\chi''_{c2}\).

Keywords: molecular state, Bethe-Salpeter equation, charmonium

1. Introduction

Very recently, CMS collaboration released their results about the \(J/\psi\phi\) spectrum in \(B^+ \rightarrow J/\psi\phi K^+\) \cite{1}. Two peaks at mass values of \(4148\pm2.0\) (stat)\(\pm4.6\) (syst) MeV and \(4316.7\pm3.0\) (stat)\(\pm7.3\) (syst) MeV, remarked as \(X(4320)\) here and hereafter, were reported. If we recall the previous experimental results as shown in Fig. \ref{jpsi_phi_spectrum} \(J/\psi\phi\) invariant mass spectrum near 4.3 GeV is confusing to some extent. In the energy region near 4.3 GeV, three structures, \(Y(4274)\), \(X(4320)\) and \(X(4350)\) were reported by CDF Collaboration, CMS collaboration and Belle collaboration, respectively. No evidence of \(Y(4274)\) was reported in the measurement by LHCb collaboration \cite{2}. However, As shown in Fig. \ref{jpsi_phi_spectrum} there exists an obvious bump structure near 4.3 GeV, which was also suggested by Yi in a recent review article \cite{8}.

To understand the \(J/\psi\phi\) invariant mass spectrum near 4.3 GeV, the following questions should be answered,
Figure 1: The invariant mass spectrum $m(J/\psi\phi)$ by CMS collaboration [1], LHCb collaboration [2], CDF collaboration [3, 4] and Belle collaboration [5]. The three full thick vertical lines from left to right (red, blue, green and brown) are for $Y(4140)$, $Y(4274)$, $X(4320)$ and $X(4350)$, respectively. The curves are the fitting results by the experimental collaborations. The two full thin vertical lines are the thresholds for $D^*D^{*-}$ and $D^*D_s^0(2317)$ [6]. The two dashed thin vertical lines are the masses of $\chi_{c1}{''}$ and $\chi_{c2}{''}$ predicted by GI potential [7].

- The structure $Y(4274)$ is found by CDF Collaboration in the $B$ decay process. Why is it not found in two photon fusion?
- If $Y(4274)$ exists, it should be explained why the $Y(4274)$ is not reported by CMS collaboration in the same channel $B^+ \rightarrow J/\psi\phi K^+$.
- Is the structure $X(4320)$ different from $Y(4274)$ considered the large mass difference? Why is it not found in CDF experiment?
- Are the structure $X(4320)$ found in the $B$ decay and the structure $X(4350)$ found in the two photon fusion the same?
Among all questions, we should first confirm the existence and the internal structure of $Y(4274)$. The first choice is putting $Y(4274)$ in the frame of the constituent quark model with structure $c\bar{c}$. By checking the Table I in Ref. [7] and considering $Y(4274)$ observed in the $J/\psi\phi$ invariant mass spectrum, we can conclude that $Y(4274)$ should be a $P$-wave state with the second radial excitation with the quantum number $I^G(J^{PC}) = 0^+(J^{++})$ with $J = 0, 1$ or 2. However, the calculation in $3P_0$ mode shows that the total widths of the second radial excitations of $\chi_{c0}$ and $\chi_{c1}$ are larger than the experimental observed width of $Y(4274)$ and the candidate of the second radial excitation of $\chi_{c2}$ has been assigned to $X(4350)$ [9]. Besides, because mass of $Y(4274)$ is far beyond the open charm threshold, it would be expected that the dominant decay channel of $Y(4274)$ is open charm pairs and the branching fraction in $J/\psi\phi$ channel is tiny [8]. Hence, the assignment of $Y(4274)$ as a charmonium is not preferred. Of course, at present we cannot fully exclude the $P$-wave charmonium explanation of $Y(4274)$, since the uncertainty of $3P_0$ model is not under control.

A state with both hidden charm and hidden strange should be easy to be produced in the $J/\psi\phi$ channel and the threshold of two charm-strange mesons $D_s\bar{D}_s(2317)$ is 4.287 GeV. In Ref. [10], a systemic study of $D_s\bar{D}_s(2317)$ in one-boson-exchange (OBE) model have been done. It has been suggested in the literatures with one-boson-exchange (OBE) model and QCD sum rule that the structure $Y(4274)$ can be assigned as a $S$-wave $D_s\bar{D}_s(2317)$ molecular state with bound energy about 10 MeV with a internal structure $(|D_s^+D_s^0\rangle + |D_s^-D_s^0\rangle)/\sqrt{2}$, which has quantum number $I^G(J^{PC}) = 0^+(0^{-+})$ [11, 12, 13]. In the previous works [11, 12], the mass of $Y(4274)$ have been reproduced in a non-relativistic OBE model by solving Schrödinger equation. The three body decay of $Y(4274)$ is also discussed in Ref [12]. However, there does not exist a theoretical study about the decays of $Y(4274)$ into $J/\psi\phi$ and $\gamma\gamma$ where it is observed.

The molecular state is a loose bound state of two hadrons, so the Bethe-Salpeter (BS) equation is an appropriate tool to deal with the molecular state. In Refs. [14, 15], the $K\bar{K}$ and $Z_b(10610)$ have been studied in the BS equation approach with quasipotential approximation. Moreover, we have studied the $Y(4274)$ and its three body decay in the BS equation approach with non-relativistic approximation [12]. And this method is successfully applied to the $D^*_b(2400)N$ system and found $\Sigma_c(3250)$ reported by BarBar collaboration recently can be explained as a $D_s^0(2400)N$ molecular state [16]. To explore the internal structure of $Y(4274)$, in this work we will study $Y(4274)$ as $D_s^0D_{s0}(2317)$ molecular state in BS equation approach and discuss its decay pattern to explain the absence of $Y(4274)$ in the two photon fusion.

The coincidence of the mass of $\chi_{c2}''$ predicted by Godfrey-Isgure (GI) potential [7] and the $X(4320)$ reported by CMS collaboration as shown in Fig. 1 suggests that the $X(4320)$ may be a good candidate of the missing $3P_1$ charmonium $\chi_{c2}''$ in the constituent quark model. Besides, the large mass difference does not support that the structures $Y(4274)$ and $Y(4320)$ are a same resonance. Hence, the bump structure near 4.3 GeV reported by CMS collaboration and CDF collaboration may be composed of two structures. In fact, in Fig. 1 one can find a hint of double-peak feature in $J/\psi\phi$ invariant mass spectra by CMS and CDF collaborations (The peaks in CDF data are close to the ones in CMS data with a translation about 10 MeV). To verify this assumption we will fit the distribution of the mass difference,
that is, the $J/\psi\phi$ invariant mass spectrum, released by CMS collaboration recently with two resonances $Y(4274)$ and $X(4320)$. It will be also helpful to understand the absence of $X(4350)$ in the $B$ decay process $B^+ \to J/\psi\phi K^+$.

This work is organized as follows. In the next section we will study the bound state of $D_s D_{s0}(2317)$ system through solving the BS equation. The two body decay pattern of $Y(4274)$ will be calculated with the wave function obtained. In the section 3 the invariant mass spectrum is analyzed. In the last section, a summary and discussion will be given.

2. $Y(4274)$ as $D_s D_{s0}(2317)$ molecular state

In this section we will study whether the $D_s D_{s0}(2317)$ system can generate a bound state which decay pattern consistent with the experimental observation of $Y(4274)$ by solving the BS equation. Due to the complication of direct solution of the BS equation, we will apply two popular forms of 3-dimension reduction, Blankenbecler-Sugar-Logunov-Tavkhelidz (BSLT) and Gross formalisms [17,18,19,20] to find the solution of the BS equation. With the wave functions obtained, the decay widths of $Y(4274)$ in $\gamma\gamma$ and $J/\psi\phi$ channels can be calculated.

2.1. The bound energy of $D_s D_{s0}(2317)$ system

The 3-dimension BS equation for the normalized wave function $|\phi\rangle$ can be written as

$$ (W - E_1 - E_2)|\phi\rangle = \int \frac{d^3k}{(2\pi)^3} \tilde{\mathfrak{F}} V \phi, \quad \text{with} \quad \tilde{\mathfrak{F}}_G = 1, \quad \tilde{\mathfrak{F}}_B = \frac{\sqrt{4(E_1+E_2)(E_1+E_2')}}{E_1+E_2+W}, \quad (1) $$

where $W, E_1 = \sqrt{k^2+m_1^2}, E_2 = \sqrt{k^2+m_2^2}$, are the energies of $D_s D_{s0}(2317)$ system, $D_s$ and $D_{s0}(2317)$, respectively. The reduced potential $V = i\sqrt{2\gamma^2 E_1 E_2 E_1' E_2'}$, which is reduced to the usual one-boson-exchange model after non-relativization [11,12]. The $\tilde{\mathfrak{F}}_G$ and $\tilde{\mathfrak{F}}_B$ are for Gross and BSLT formalisms, respectively. The explicit derivation can be found in [Appendix A].

The potential with light meson exchanges can be obtained with the Lagrangian from the heavy quark field theory [21],

$$ \mathcal{L} = \frac{i}{\hbar} \int \left( P_{\mu}^{a} \partial_{\mu} p_{b} + P_{ba}^{a} \partial_{\mu} P_{\mu}^{a} - \frac{\beta\gamma V}{\sqrt{2}} P_{ba}^{a} \partial_{\mu} p_{b} p_{a}^{\mu} + \frac{i\beta' g V}{\sqrt{2}} P_{ba}^{a} \partial_{\mu} p_{b} + \right) \quad (2) $$

where the coupling constants $\hbar = -0.56 \pm 0.28, \beta\beta' = 0.90, g_V = m_{\rho}/m_{\pi} = 5.8$ with $m_{\pi} = 132$ MeV [11, 12, 21, 22]. Since $\beta$ and $\beta'$ are not well determined in the literature. Different values $\beta\beta' = 0.9\eta\beta\beta'$ with $\eta\beta\beta' = 1.2$ will be considered in this work.

The annihilation operations $P$, $P_{a}^{\mu}$, $P_{b}^{\mu}$, and $P_{ba}^{a}$ satisfy the normalization relations

$$ \langle 0 | P | Q \bar{q}(0^-) \rangle = 1, \quad \langle 0 | P_{a}^{\mu} | Q \bar{q}(0^-) \rangle = \epsilon_{a}, \quad \langle 0 | P_{b}^{\mu} | Q \bar{q}(0^+) \rangle = 1, \quad \langle 0 | P_{ba}^{a} | Q \bar{q}(1^+) \rangle = \epsilon_{a}. \quad (3) $$

In Eq. (2), the octet pseudoscalar and nonet vector meson matrices read as

$$ P = \begin{pmatrix} \pi^0 \sqrt{2} + \eta / \sqrt{6} \\ \pi^- \sqrt{2} + \eta / \sqrt{6} \\ \eta / \sqrt{6} \end{pmatrix}, \quad V = \begin{pmatrix} \rho^0 \sqrt{2} + \omega / \sqrt{6} \\ \rho^- \sqrt{2} + \omega / \sqrt{6} \\ -\omega / \sqrt{6} \end{pmatrix}. \quad (4) $$

4
The interaction mechanism for the $D_sD_{s0}(2317)$ is shown in Fig. 2. Only pseudoscalar light meson $\eta$ and vector light meson $\phi$ exchange are possible because the vertices of charm-strange meson and light meson with $u/d$ quark are suppressed by OZI rule.

The explicit form of the kernel $V = V_\phi + 2/3V_\eta$ can be written as

$$
V_\phi = -i \frac{\beta \beta' g_V^2}{2} (p'_i + p'_f) \frac{-q''\eta + q''\phi/m^2_\phi}{q^2 - m^2_\phi} (p''_i + p''_f),
$$

$$
V_\eta = -i \frac{h^2}{f_\pi^2} (p_i + p_f) \cdot \frac{q}{q^2 - m^2_\eta} (p'_i + p'_f) \cdot q,
$$

where $p_i,f$ and $p'_i,f$ are the momenta of the particle 1 and 2 in the initial (final) state and $q = p_f - p_i$. $m_\phi$ and $m_\eta$ are the masses of exchanged mesons $\phi$ and $\eta$. A monopole form factor $F(q^2) = (\Lambda^2 - m^2)/(\Lambda^2 - q^2)$ is introduced to compensate the off-shell effect of the changed light meson.

With the potential (5), the 3-dimension BS equation can be solved numerically with the recursion method. The explicit can be found in Appendix B. Since the parameters $\beta \beta'$ and $h$ is not well determined in the literature. We will present the results with different values of parameters. The obtained bound energies $E = W - m_{D_s} - m_{D_{s0}}$ are shown in Fig. 3.

In the left part of Fig. 3, the results by the non-relativistic (NR) one-boson exchange potential in Refs. [11, 12] are given. We find the results obtained with the numerical method in this work is same to the original model [11, 12], which can be seen as a verification of our numerical method. In NR model the contribution from $\eta$ exchange is negligible for reasonable cut-off $\Lambda$ (the cut-off $\Lambda$ should be in the region $1\sim 5$ GeV). So only the parameter $h$ is adjusted to find a cut off $\Lambda$, with which the bound energy is 13 MeV as suggested by experiment, and a value $h = -1.6$ is adopted in Refs. [11, 12].

However, the $h$ is better determined with value $h = 0.56 \pm 0.28$ obtained by the sum rule [21] than $\beta \beta'$. Moreover, as shown in the left part of Fig. 3 the contributions from $\eta$ exchange become more important in BS approach, which is relativistic, than in NR model.

![Figure 2: The diagram for the interaction of $D_sD_{s0}(2317)$ system by exchanging $\eta$ meson (a) and $\phi$ meson (b).](image-url)
Figure 3: The bound energy for $D_sD_{s0}$ system with different cutoff $\Lambda$ and coupling constants. Here $\beta\beta' = 0.9\eta_{\beta\beta'}$. NR, G and B stand for non-relativistic model, BS equation with Gross formalism and BS equation with BSLT formalism.

With a non-zero contribution from $\eta$ exchange, the solution can be found in BS approach if we adopt $h = 0.56 \pm 0.28$ obtained by the sum rule$^{[21]}$ as shown in the right part of Fig. 3. In the NR model, no solution with bound energy about 10 MeV can be found with such value of $h$. If adopting a larger $\beta\beta' = 1.8$ the solution with bound energy about 10 MeV can be found with a cut-off $\Lambda$ about 2.5 GeV. As shown in Fig. 3 with all parameters considered here, a loose bound state can be found when solving BS equation. Hence, a molecular state $Y(4274)$ can be generated from $D_sD_{s0}(2317)$ system.

From Fig. 3 the results from Gross formalism, BSLT formalism and NR model are close to each other for the small bound energy. The solution with bound energy near zero appears with the almost same cut off $\Lambda$. In other word, if we focus on the molecular state with very small bound energy, the three models give a very similar results. If a deep bound molecular state is considered, more complicated formalism should be adopted.
2.2. The decay pattern of $Y(4274)$

Now we have obtained the wave function, which contain the information of vertex, as well as the bound energy. We will estimate the decay widths of $Y(4274)$ to $J/\psi \phi$ and $\gamma \gamma$ with the wave function. Assuming $Y(4274)$ is a molecular state, the dominant decay mechanism is exchanging charm/light quarks in the two constituent mesons as shown in Fig. 4.

\[
\begin{align*}
Y(4274) &\rightarrow J/\psi \phi \\
Y(4274) &\rightarrow \gamma \gamma
\end{align*}
\]

Figure 4: Decay mechanism of $Y(4274)$ to $J/\psi \phi$ and $\gamma \gamma$. The left diagram is in the quark level and right diagrams are in hadron level.

The decay mechanism in quark level can be described by a hadronic loop, and the amplitudes can be written as

\[
\mathcal{M} = \Gamma G A = \Gamma (g + \Delta G) A \approx \Gamma g A \equiv \int \frac{d^3k}{(2\pi)^4} \psi(|\vec{k}|) A(|\vec{k}|),
\]

(6)

where the $\Gamma$, $G$ and $A$ are the vertex for the $Y(4274)$ decaying to $D_s$ and $D_{s0}(2317)$, propagators of $D_s$ and $D_{s0}(2317)$, and the amplitudes for $D_s$ and $D_{s0}(2317)$ to $J/\psi \phi$ and $\gamma \gamma$ by exchanging $D_s^*$. Here the term with $\Delta G$ is omitted as usual.

The Lagrangian used for $A_{J/\psi \phi}$ are [21, 23],

\[
\begin{align*}
\mathcal{L} &= -2g_3\sqrt{m_Dm_{D'}}\psi \cdot P_a^* P_a - i2g_3\sqrt{m_Dm_{D'(s)}}/m_{D'(s)} \\
&\quad \cdot \varepsilon_{\beta\mu\nu\tau} \partial^\beta \psi^\mu (P_a^\dagger \gamma^\tau P_a + P_a^* \gamma^\mu \partial^\tau P_a) \\
&\quad - i\sqrt{2}\lambda \gamma^\nu \varepsilon_{\lambda\alpha\beta\mu} \partial^\lambda \psi^\mu (P_a^* \gamma^\alpha \partial^\nu P_a + P_a^* \gamma^\nu \partial^\alpha P_a) \\
&\quad - \sqrt{2}c_v \sqrt{m_{D_0}m_{D'}} (P_0^* P_{b0}^* + P_a^* P_0^* P_{b0}) V_{aba} \\
&\quad \pm \sqrt{2}c_v (P_0^* \gamma^\alpha P_{b0}^* - P_a^* \gamma^\nu \partial^\alpha P_{b0}) (\partial_\beta V_{\alpha\beta} - \partial_\alpha V_{\beta\alpha})_{b0},
\end{align*}
\]

(7)
where $g_3 = \sqrt{m_\psi/f_\psi}$ and $g_2 = \sqrt{m_\psi/(2m_D f_\psi)}$ with $f_{J/\psi} = 405 \pm 14$ MeV, $\lambda = 0.56$ GeV$^{-1}$, $\zeta = 0.727$ and $\bar{\omega} = 0.364$ [21, 23]. For the two photon decay,

$$
\mathcal{L} = g_{D_s D_s \gamma^*} D_s A^{\mu \nu} F_{\mu \nu} + g_{D_s D_s \gamma^*} D_s \epsilon_{\mu \nu \alpha \beta} A^{\mu \nu} F_{\alpha \beta}.
$$

(8)

The coupling constant can be obtained by the decay widths. The radiative decay of $D_{s0}(2317)$ to $D_s^*$ have been calculated theoretically in literature, here we adopt a typical value $\Gamma_{D_{s0}D_s^* \gamma^*} = 1$ keV [24]. For the decay of $D_{s0}^*$ to $D_s$, we choose the upper limit of the values in PDG as $1.9 \times 94.2\%$ MeV [6]. The decay widths with the variation of the cut-off $\Lambda$ are shown in Fig. 5.

![Figure 5: The decay widths for $Y(4274) \rightarrow J/\psi\phi/\gamma\gamma$ and bound energy with $h = 0.84$ and $\eta_{\mu \nu} = 1$. The upper subfigure is for the decay widths of $Y(4274)$. The lower subfigure is the bound energy obtained with the same parameter.](image)

An interesting phenomena can be found that the decay width increase with the increases of bound energy $|E|$, which is different from the case of three body decay [12, 16] where the decay width decreases. For a molecular state, which is a bound state of consistent hadrons, a large bound energy (which also means small radius of the molecular state as many calculation suggested, for example Ref. [16]) will be helpful to the occurrence of the exchanging of quarks in the constituents, which is the main mechanism of the two body
decay. For the three body decay, the main decay mechanism is the collapse of the molecular state in sequential three-body decay, a looser bound system is beneficial to the collapse of the constituents. Hence our results in the two and three body decays of $Y(4274)$ reflect the internal structure of the molecular state.

3. The $J/\psi \phi$ invariant mass spectrum

As shown in the previous section, a bound state can be generated from the $D_s D_{s0}(2317)$ system. The theoretical results about $J/\psi \phi$ and $\gamma \gamma$ decays of this state are consistent with the experimental observed structure $Y(4274)$. However, In the CMS experiment [1], $Y(4274)$ was not reported while a structure $X(4320)$, which is about 50 MeV higher than $Y(4274)$, was reported. In this section, we will analysis the invariant mass spectrum released by CMS collaboration to answer the questions proposed in introduction.

Now we make an analysis about the decays of $B^+$ to $X/Y$s. The leading order of the weak decay can be described as a four-quark local interaction by the effective Hamiltonian [23, 26] as shown in Fig. 6.

$$H = \frac{G_F}{\sqrt{2}} V_{cb} V_{c\bar{s}}^* \left[ (C_2 + \frac{C_3}{3}) O_2 + \cdots \right],$$

(9)

where $O_2 = (\bar{c}\Gamma_\mu c)(\bar{s}\Gamma^\mu b)$ with $\Gamma_\mu = \gamma_\mu (1 - \gamma_5)$.

![Figure 6: Weak decay mechanism of $B^+ \rightarrow K^+ X/Y$.](image)

The decay amplitude can be factorized by splitting the matrix into two pieces,

$$\mathcal{A} = iF \langle B(p)|J^W_\mu|K(p')\rangle \langle X(q)|J^{\mu(cc)}|0\rangle,$$

(10)

with $F = \frac{G_F}{\sqrt{2}} V_{cb} V_{c\bar{s}}^* (C_2 + \frac{C_1}{3})$. The $B$ to $K$ part can be described as,

$$\langle B(p)|J^W_\mu|K(p')\rangle = f_+(q^2)P_\mu + f_-(q^2)q_\mu.$$

(11)

The explicit form and the parameterizations for $f_{\pm}(q^2)$ can be found in Ref. [27].
The matrix $\langle X|J|0\rangle$ for pseudoscalar and axial vector meson can be parameterized with decay constant as

\begin{equation}
\langle X_0^-(q)|J^{\mu(\mu)}|0\rangle = -if_P q^\mu,
\end{equation}

\begin{equation}
\langle X_1^+(q)|J^{\mu(\mu)}|0\rangle = f_A m_A \epsilon_\mu(q),
\end{equation}

where $f_A$, $f_P$, $m_A$ and $\epsilon_\mu$ are the decay constants for pseudoscalar and axial vector meson, mass of axial vector meson, and polarized vector of the axial vector meson, and $q$ are the momentum of the meson.

The $X(4350)$ observed in two photon fusion have been suggested as a $3^3P_2$ charmonium state $\chi''_{c2}$ [9], which vanishes in the $B$ decay in the factorization approximation. Such suppression is obvious in the production of $1^3P_1$ states $\chi_{cJ}$ in $B \to \chi_{cJ} K^+$ channel [6]. The observation of $X(4320)$ in $B$ decay suggested it should not be $X(4350)$ found in the two photon fusion. The P-wave charmonium $\chi'_{c1}$ should have a mass close to $\chi''_{c2}$, and $X(4320)$ observed by CMS collaboration have a mass close to the value 4317 MeV predicted in constituent quark model [7] as shown in Fig 1. Moreover as an axial vector meson it can not be produced in the two photon fusion as the Belle experiment suggested while the production of $\chi''_{c1}$ should be considerable in the $B$ decay. Hence it is reasonable to assign $X(4320)$ as the missing $\chi''_{c1}$. The absence of $Y(4140)$ in the two photon fusion indicts it should not be a molecular state [8, 25], which is also disfavored by the large bound energy. Here, we adopt the assignment of $Y(4140)$ as an tetraquark $cc\bar{s}s$ with $J^P = 1^{++}$, which can explain the absence in two photon fusion and its mass and decay pattern [28].

Hence, in the $J/\psi\phi$ invariant mass spectrum from $B$ decay there exist three resonances, $Y(4140)$ with $1^{++}$, $X(4320)$ with $1^{++}$ and $Y(4274)$ with $0^{-+}$. The Lagrangians for the sequential decays of the pseudoscalar and axial vector mesons $X/Y$ to the $J/\psi\phi$ can be written as

\begin{equation}
\mathcal{L} = \frac{g_P}{M_X} i\epsilon_{\mu\nu\alpha\beta} \psi^{\mu\nu} \phi_{\alpha\beta} X + g_A i\epsilon_{\mu\nu\alpha\beta} \phi^{\mu} X^{\nu} \tilde{\psi}^{\alpha} \tilde{\phi}^{\beta},
\end{equation}

where $\phi^{\alpha\beta} = \partial^\alpha \phi^\beta - \partial^\beta \phi^\alpha$, and $\tilde{\phi}^{\beta} = (g^{\beta\rho} - k^\beta k^\rho/m^2_X) \phi_\rho$ to keep the gauge invariance. $\psi^{\mu\nu}$ and $\tilde{\psi}^{\alpha}$ for $J/\psi$ is analogous. $g_P$ and $g_A$ are the coupling constants for pseudoscalar and axial vector mesons, respectively.

The amplitude $\mathcal{M}_{X/Y}$ for $X$ can be obtained from Eqs. (12, 13) and the propagator of $X/Y$ which involves Breit-Wigner mass $m$ and width $\Gamma$. With a constant background $\mathcal{L}_{bk} = C g^{\mu\nu}$, the square of the amplitude can be written as,

\begin{equation}
|\mathcal{M}|^2 = 4C^2 + |\mathcal{M}_{Y(4140)}|^2 + |\mathcal{M}_{Y(4274)}|^2 + |\mathcal{M}_{X(4320)}|^2 + 2Re(e^{i\phi}\mathcal{M}_{Y(4140)}^{\mu\nu}\mathcal{M}_{Y(4274)}^{\nu\mu}).
\end{equation}

One can find only one interference term are left.

With the square of the amplitude, the $J/\psi\phi$ invariant mass spectrum for process $B^+ \to K^+ J/\psi\phi$ can be calculation. The parameters are the strength constants $N$, Breit-Wigner masses $m$ and widths $\Gamma$ of $Y(4140)$, $Y(4274)$ and $X(4320)$. Here strength constant $N$ involve the coupling constants of corresponding resonance and a general normalization.
Now, we determine the parameters by fitting the distribution of mass difference \( m(\mu^+\mu^- - K^+K^-) - m(\mu^+\mu^-) \), this is, the \( J/\psi\phi \) invariant mass spectrum, released by CMS collaboration by binned maximum likelihood \(^6\) with three resonances. It is done through minimizing

\[-2 \ln L = 2 \sum_{i}^{N} [n_i - \nu_i + \ln \frac{n_i}{\nu_i}] \]

where \( n_i \) and \( \nu_i \) are the experimental and theoretical values in \( i^{th} \) bin using the MINUIT code. The results are shown in Fig. 7.

![Figure 7: Distribution of the mass difference \( m(\mu^+\mu^- - K^+K^-) - m(\mu^+\mu^-) \) and Dalitz plot for \( B^+ \to J/\psi\phi K^+ \). The histogram (black) is for the fitting results in the current work. The long dashed (green), short dashed (blue), dotted (magenta) and dash dotted (cyan) lines are for the contributions from \( X(4140) \), \( X(4274) \), \( X(4320) \) and background. The full circle (blue) and full line (red) are for the experimental data and the fitting results of CMS Collaboration \(^1\).](image)

Different from the two Breit-Wigners fitting in the experimental article \(^1\), here we use three Breit-Wigners. One can found the CMS data is well fitted with three Breit-Wigners especially below \( m(\mu^+\mu^- - K^+K^-) - m(\mu^+\mu^-) = 1.3 \) GeV. The first structure around 1.05 GeV is about 0.15 GeV below the second structure around 1.2 GeV, and dominant with the contribution of resonance \( Y(4140) \). Hence, the fitting result in the current work is close to that in Ref. \(^1\). For the second structure, only one Breit-Wigner are used in the fitting of the CMS collaboration, and a resonance with \( m = 4316.7 \) MeV was reported. In the current
work, this structure is fitted with two resonances, $Y(4274)$ and $X(4320)$, which results in an excellent reproduction of the experimental distribution of the mass difference around 1.2 GeV. The Dalitz plot for $B^+ \to J/\psi\phi K^+$ is also presented for reference, which is obtained by FOWL program in CERNLIB.

The explicit about the fitted parameters is shown in Table 1. The Berit-Wigner masses of $Y(4140)$, $Y(4274)$ and $X(4320)$ obtained in the current fitting are 4147.8 MeV, 4269.4 MeV and 4320.8 MeV, which are close to the values suggested in the original experimental articles as shown in Fig. 1[1, 3, 5]. The Breit-Wigner width of $Y(4274)$ is 9.4 MeV, which is much smaller than the value of PDF collaboration, $32.3^{+21.9}_{-15.3}(\text{stat})\pm7.6(\text{syst})$ MeV. It is easy to understand because two resonances $Y(4274)$ and $X(4320)$ are considered in the current work. The total decay width of $X(4320)$ is consistent to the constituent quark model and $^3P_0$ model predictions with a assignment of $\chi''_{c1}$[7, 9].

|       | $Y(4140)$  | $Y(4274)$  | $X(4320)$  |
|-------|------------|------------|------------|
| $N$   | 0.139±0.007| 0.069±0.006| 0.094±0.007|
| $m$   | 4147.8±2.6 | 4269.4±1.5 | 4320.8±1.6 |
| $\Gamma$ | 29.6±2.3   | 9.4±1.4    | 25.0±1.7   |
| $\text{Sig}$ | 13.3[11.7] $\sigma$ | 4.7[5.2] $\sigma$ | 6.2[8.4] $\sigma$ |

Table 1: The fitted strength constant $N$, Breit-Wigner masses $m$ and widths $\Gamma$ for the three resonances. The best fitted phase angle $\phi = 0.23 \pm 0.09$ and the background constant $C = 1.98 \pm 0.04$. The last line is the significance $-2\Delta \ln L$ by taking off the corresponding resonance in the full model or adding a resonance into background (with bracket).

To present the importance of each resonance in fitting, we also give the significance $-2\Delta \ln L$ of the binned maximum likelihood. Here $L$ and $L'$ are for the full model and changed model, respectively. By turning off the corresponding resonance from the full model, we find the significances of $Y(4274)$ and $X(4320)$ are 4.7 $\sigma$ and 6.2 $\sigma$ by turning of the the corresponding resonance in the full model or 5.2 $\sigma$ and 8.4 $\sigma$ by adding a resonance into background. Hence both $Y(4274)$ and $X(4320)$ are important to fit the distribution of mass difference.

4. Summary and discussion

In this work, we study the mass and the decay pattern of the $Y(4274)$ as a molecular state $D_sD_{s0}(2317)$ in the BS equation approach with quasipotential. The solution with reasonable parameters is found in the $D_sD_{s0}(2317)$ interaction by exchanging the light meson $\eta$ and $\phi$. The absence of the $Y(4274)$ in the $\gamma\gamma \to J/\psi\phi$ process can be explained by the decay widths of $Y(4272)$ in $\gamma\gamma$ and $J/\psi\phi$ channels, which are calculated with the wave function obtained in the solution of the BS equation.
The assignment of $X(4320)$ observed in $B$ decay and $X(4350)$ observed in two photon fusion as $P$-wave charmoniums $\chi''_1$ ($3^3P_1$) and $\chi''_2$ ($3^3P_2$) is consistent to existing experiment observations and the theoretical predictions about mass and decay width. Since the factorization approximation suggests suppression of $\chi''_c$ compared with $\chi''_1$ in $B^+ \to J/\psi\phi K^+$, the assignment of $X(4320)$ as $\chi''_1$ excludes the possibility to assign $Y(4274)$ as $\chi''_c$ combined with its considerable contributions in $B$ decay. It is also supported by the small width of $Y(4274)$, which conflict with the prediction in the $3P_0$ model [9].

Based on the calculation and analysis in this work we can reach following conclusions.

- The bump structure near 4.3 GeV in the $J/\psi\phi$ invariant mass spectrum in the $B^+ \to J/\psi\phi K^+$ is from two resonances, $Y(4274)$ as a $D_s D_{s0}(2317)$ molecular state and $X(4320)$ which can be assigned as $\chi''_1$.

- The absence of $Y(4274)$ in the two photon fusion can be explained by the decay pattern of $Y(4274)$.

- The structure $X(4350)$ found in two photon fusion can be assigned as $\chi''_2$, which contribution should be suppressed in $B^+ \to J/\psi\phi K^+$.

The more precise data from forthcoming BelleII and SuperB combined with explicit theoretical studies of $X/Y$ production in $B$ decay may provide clearer picture of double-peak feature and the internal structure of these resonances.

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Appendix A. The 3-dimension reduction of BS equation

The BS equation for the amplitudes $T$ can be written as [29, 30, 31]

$$ T = V + VGT, \quad (A.1) $$

where the propagator for the two particles $G = G_1 G_2 = i/(l_1^2 - m_1^2) \cdot i/(l_2^2 - m_2^2)$ and $V$ is the interaction kernel. For a bound state the amplitudes can be written as [29, 30, 31]

$$ T = \frac{|\Gamma\rangle i\langle\Gamma|}{P^2 - M^2}, \quad (A.2) $$

where $|\Gamma\rangle$ is the vertex of bound state and two constituent particles, $P$ and $M$ are the momentum and mass of the bound state.

Hence, the BS equation for the vertex can be written as

$$ |\Gamma\rangle = V G |\Gamma\rangle, \quad (A.3) $$
The BS equation for the vertex $|\Gamma\rangle$ can be rewritten as following form

$$|\Gamma\rangle = U g |\Gamma\rangle, \quad U = V + V\Delta G U,$$

(A.5)

Here $\Delta G = G - g$, $g$ and $U$ are called the quasipotential two-body propagator and the quasipotential kernel. As usual, we assume term with $\Delta G$ is small and can be neglected [17, 18, 20, 19, 30].

To satisfy the unitary condition, the propagator $g$ should satisfy the relation

$$g - g^\dagger = 2\pi i\delta((\eta_1(s)P + k)^2 - m^2)\delta((\eta_2(s)P - k)^2 - m^2),$$

(A.6)

where $\eta_1(s) + \eta_2(s) = 1$ with $s = P^2$. With $\epsilon_{1,2}(s) = (s + m_{1,2}^2 - m_2^2)/2\sqrt{s}$, we can define $\eta_{1,2} = \epsilon_{1,2}/(\epsilon_1 + \epsilon_2)$. Now we have many choice to write the propagator. The most popular form is [32, 33]

$$g = 2\pi \int \frac{ds'}{s' - s + i\epsilon} h(s', s) \delta([\eta'_1(s')P' + k]^2 - m_1^2) \delta([\eta'_2(s')P' - k]^2 - m_2^2)$$

(A.7)

with $P' = \sqrt{s'/s}P$.

The choice of $h(s', s)$ is random to some extent. Here we adopt two widely used formalisms, BSLT and Gross formalisms [17, 18, 20]. For BSLT formalisms, we choose $h(s' - s) = 1$ and $\eta'(s') = \eta(s')$. For Gross formalism, $h(s', s) = (\sqrt{s'} + \sqrt{s})/\sqrt{s'}$ and $\eta'_1(s') = \eta_1(s)\sqrt{s'/s}$ and $\eta'_2(s') = 1 - \eta_1(s)\sqrt{s'/s}$.

The quasipotential propagators in the Gross (G) and BSLT (B) formalisms written down in the center of mass frame where $P = (W, \bar{0})$ are [17, 34]

$$g = \frac{-2\pi i}{2E_1E_2(E_1 + E_2 - W)} f_{GB} = \delta(\epsilon_1 + k^0 - E_1), \quad f_B = \frac{(E_1 + E_2)}{E_1 + E_2 + W}.\quad (A.8)$$
It is easy to find that in Gross formalism the particle 1 is set on-shell and in BSLT formalism \(k^0 = 0\). Due to the existence of the delta function, the BS equation in 4-dimension will be reduced into a 3-dimension equation.

The normalization of the vertex with quasipotential approximation in the center of mass frame has the following form,

\[
1 = \int \frac{d^4p}{(2\pi)^4} \Gamma \frac{\partial i g}{\partial W^2} \Gamma. \tag{A.9}
\]

The normalized wave functions of bound state can be introduced as \(|\phi\rangle = N|\psi\rangle\) with

\[
N_B = \frac{\sqrt{E_1 E_2}}{\sqrt{(2\pi)^3 2(E_1 + E_2)}}, \quad N_G = \frac{\sqrt{2E_1^2 E_2^2}}{\sqrt{(2\pi)^3 2W}}, \tag{A.10}
\]

and the wave function in 3-dimension normalized to \(\int d^3|\phi|^2 = 1\).

**Appendix B. Numerical solution of the 3-dimension BS equation**

The 3-dimension BS equation Eq. 1 can be rewritten as

\[
W \phi(\vec{k}) = \int \frac{d^3\vec{k}'}{(2\pi)^3} [3(W, \vec{k}, \vec{k}')V(W, \vec{k}, \vec{k}') + (E_1(\vec{k}')) + E_2(\vec{k}'))\delta(\vec{k}' - \vec{k})] \phi(\vec{k}'),
\]

\[\equiv \int \frac{d^3\vec{k}'}{(2\pi)^3} K(W, \vec{k}, \vec{k}') \phi(\vec{k}'), \tag{B.1}\]

Here the wave function \(\psi(\vec{k})\) is radially symmetric and the angular part are integrated out as refs. [14, 15]. After defining the kernel \(K(W, \vec{k}, \vec{k}')\) after integration as \(A(W, |\vec{k}|, |\vec{k}'|) = \int \frac{d^3\vec{k}'}{(2\pi)^3} K(W, \vec{k}, \vec{k}')\). We reach a integral equation with form

\[
W \psi(|\vec{k}|) = \int d|\vec{k}'| A(W, |\vec{k}|, |\vec{k}'|) \psi(|\vec{k}'|). \tag{B.2}\]

To solve the integral equation, we discrete the \(|\vec{k}|\) and \(|\vec{k}'|\) to \(|\vec{k}|_i\) and \(|\vec{k}'|_j\) by the Gauss quadrature, then the above equation transfer to a matrix equation

\[
W \psi_i = \sum_j A_{ij}(W) \omega_j \psi_j \equiv \sum_j \tilde{A}_{ij}(W) \psi_j, \tag{B.3}\]

which can be written as a compact form

\[
W \psi = \tilde{A}(W) \psi. \tag{B.4}\]

The integral equation involving a nonlinear dependence on the total energy \(W\) of the system reduce to a nonlinear spectral problem. Here we adopt the recursion method in
It proceeds by forming a sequence of approximations to $W$ and $\psi$ using the recursion relation:

$$W_n^{(l)}\psi = \tilde{A}(W_n^{(l-1)})\psi, \quad n = 1, 2, \ldots, s, \ldots$$  \hspace{1cm} (B.5)

where the upper index is the iteration number, and the lower index is the eigenvalue number. At the first iteration step, an input approximation of the sought eigenvalue is substituted into the kernel. In the problem here, we choose the $W$ with zero bound energy. Then $n$ eigenvalues can be obtained by the code of DGEEV function in NAG Fortran Library. If we are interesting in the $s^{th}$ eigenvalues. The eigenvalue with the fixed number $s$ is substituted in the integral equation kernel on each iterative loop. Then the linear spectral problem is solved again. The stopping criterion $|W_n^{(l)} - W_n^{(l-1)}| < \epsilon$ is tested on each iteration. The value $\epsilon$ is chosen to satisfy the required precision. As soon as stopping criterion is fulfilled, the iterative process is terminated. The eigenvalue $W_n^{(l)}$ and eigenfunction $\psi_n^{(l)}$ obtained on the last iteration are returned as the solution of the problem.

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