The Nambu-Jona Lasinio mechanism and electroweak symmetry breaking in the Standard Model

Edoardo Di Napoli

University of Rome "La Sapienza" P.le Aldo Moro, 5 Rome Italy

Abstract

In this paper I examine the breaking of internal symmetries from another point of view showing that is possible to reproduce the electroweak panorama of the traditional Standard Model in a exhaustive and self consistent way. The result is reached applying the main futures of the old Nambu-Jona Lasinio (NJL) mechanism to an electroweak invariant Lagrangian. In this context the use of functional formalism for composite operators naturally leads to a different dynamical approach. While the Higgs mechanism acts on the Lagrangian form, a NJL like model looks directly at the physics of the system showing the real dynamical content hidden in the Green functions of the theory.

1 Introduction

The aim of this model is to develop and analyze an alternative version of the traditional electroweak sector of the Standard Model (SM). In the latter the mechanism of mass "generation", for the gauge and matter field, is the direct consequence of the insertion in the Lagrangian of a gauge invariant scalar field with Yukawa coupling terms. Modifying suitably the potential parameters, the appearance of a vacuum expectation value introduces mass terms in the Lagrangian leading to a spontaneous symmetry breaking effect. A different approach reaches the same results through a process that doesn’t act on the Lagrangian form but directly on the Green functions. The starting point of the whole mechanism draws inspiration from the basic work of Nambu and Jona Lasinio (NJL) that gave origin to the homonymous model [1]. NJL, aware of the tight analogy with the BCS theory of superconductivity, applied Bogoliubov’s quasi particle method to the relativistic field theory. Through a simple four fermion chirally invariant interaction, they obtained a gap equation for an order parameter that is merely the dynamical fermion mass. Moreover
the model points out the presence of induced relativistic bound states as much as an unavoidable logarithmic dependence on the cut-off of the analytical results which, paradoxically, enriches the theory. In the first part of this work the fundamental ideas of this model are presented and then used with the combined support of the composite operators functional formalism for effective action and 1/N expansion techniques. I start by introducing a simplified four fermion (4-F) interaction term in a Lagrangian invariant under the $SU_L(2) \otimes U_Y(1)$ symmetry group. Using stationary condition on the effective action I find a Schwinger-Dyson (SD) equation for the top quark propagator in the form of a self consistent condition for its dynamical mass. After having fine tuned this condition, setting the system in the asymmetrical phase (breaking chiral invariance), I proceed to extract the correct Bethe-Salpeter equations. The latter show the presence of a series of bound states playing the role of Goldstone and Higgs bosons of the traditional SM. Going on, calculating the gauged SD equations, it is possible to ascertain the absorption of the pseudo-Goldstone modes by the electroweak gauge propagators. In this way mass terms are induced for $W^\pm$ and $Z^0$, while the photonic propagator remains mass free. Section 2 is a kind of toolbox in which a short explanation of the mathematical instruments, used in the rest of the paper, is given. In section 3 I introduce a 4-F interaction and analyze the SD equations for the fermion propagators. Section 4 is devolved to the relativistic bound states and to their mass spectrum. In section 5 I take in consideration the SD equation for the gauge electroweak propagators showing the mass inducing effect caused by the pseudo-Goldstone amplitudes. Considerations and conclusions follow in the last section.

2 Functional formalism toolbox

The first problem to be resolved is the possibility of using the same formalism to analyze order parameters (as the mass operator functions) and bound states amplitudes. Second, but not less important, is the quest for an equivalent approximation order for all the results obtained in the course of the work. The functional effective action for composite operators seemed to have all the properties we were looking for. First of all its stationarity under variation with respect to the second Legendre variable (on physical states) gives directly a SD equation for a local propagator that includes all the corrections coming from the interaction terms in the Lagrangian. Then, the second derivative respect this propagator represents the inverse bilocal connected function of the theory, that in simple words is the connected part of a four point particle scattering amplitude. The second requirement on bound states is then accomplished. It remains the approximation problem. It was evident that I should consider some kind of cutting process so to make possible the practical handling of
expressions in terms of the most relevant contributions. Moreover the order of approximation had to be the same for every explicit formula. In this case the use of functional formalism points out, as natural solution, the approximation in series of $1/N_c$ (where $N_c$ is number of colors) showing the high level of self consistency of this approach.

For simplicity let me introduce the functional formalism for composite operators in case of a scalar field $\phi(x)$. As we already know, the application of the Legendre transform to the usual generating functional $Z(J)$, that depends on the local source $J(x)$, gives the generator of the 1-particle irreducible graphs of the theory. If we formally define a generating functional that also depends on a bilocal source $K(x, y)$

$$Z(J, K) = \exp \left[ \frac{i}{\hbar} W(J, K) \right] = \int d\phi \ \exp \left\{ \frac{i}{\hbar} \left[ \int d^4x \ (\mathcal{L}(\phi) + \phi(x) J(x)) + \int d^4x d^4y \ \phi(x) K(x, y) \phi(y) \right] \right\}. \quad (1)$$

we can imagine to recover an analogous generator with a similar transform. I will call the latter second Legendre transform,

$$\Gamma(\phi_c, G) = W(J, K) - \int d^4x \ \phi_c(x) J(x) - \frac{1}{2} \int d^4x d^4y \ \phi_c(x) K(x, y) \times \phi_c(y) - \frac{1}{2} \hbar \int d^4x d^4y \ G(x, y) K(x, y) \quad (2)$$

with

$$\frac{\delta W(J, K)}{\delta J(x)} = \phi_c(x) = \langle 0 | \phi(x) | 0 \rangle$$

$$\frac{\delta W(J, K)}{\delta K(x, y)} = \frac{1}{2} \left[ \phi_c(x) \phi_c(y) + \hbar G(x, y) \right] = \langle 0 | \phi(x) \phi(y) | 0 \rangle. \quad (3)$$

The functional so obtained is consequently the generator of the 2-particle irreducible Green functions\footnote{We call 2 point irreducible a function whose graph remains connected cutting any couple of internal lines.} with all the internal lines expressed in terms of the complete propagator $G(x, y)$. The word complete here is used because $G(x, y)$ automatically takes into account the corrections coming from the interaction terms in the Lagrangian. This functional together with its two stationary conditions

$$\frac{\delta \Gamma(\phi_c, G)}{\delta \phi_c(x)} = -J(x) - \int d^4y \ K(x, y) \ \phi_c(y) \quad (4)$$
\[
\frac{\delta \Gamma(\phi_c, G)}{\delta G(x, y)} = -\frac{1}{2} \hbar K(x, y).
\]

give the Jackiw, Cornwall and Tomboulis\cite{2} variational method for composite operators. On physical states, when all the sources are equal to zero, (4) become a system of equations from which is formally possible to extract the expressions for \(\phi_c\) and \(G\). In particular the latter equation, depending on self-energy, represents essentially the SD equation for \(G\). In order to extract an explicit expression for \(G\) we have to obtain a practical method that gives formulas to be handled with approximation procedures. One of the most known is the loop expansion\cite{2}\cite{4}. Referring to the usual scalar field action
\[
S(\phi) = \int d^4 x \, \mathcal{L}(\phi(x))
\]
and defining
\[
\mathcal{D}^{-1}(\phi; x, y) = \frac{\delta^2 S(\phi)}{\delta \phi(x) \delta \phi(y)} = iD^{-1}(x - y) + \frac{\delta^2 S_{\text{int}}(\phi)}{\delta \phi(x) \delta \phi(y)}
\]
with \(D^{-1}\) the inverse free propagator, it is possible to write the effective action as
\[
\Gamma(\phi_c, G) = S(\phi_c) + \frac{1}{2} i\hbar \text{Tr} \left[ \ln \left( D G^{-1} \right) + \mathcal{D}^{-1}(\phi_c) G - 1 \right] + \Gamma_2(\phi_c) G. \quad (5)
\]
An infinite number of terms are stored in \(\Gamma_2\) and its definition is possible through a slightly different version of the classic action where the field \(\phi\) is translated by the quantity \(\phi_c\). This defines a new interaction \(S_{\text{int}}(\phi; \phi_c)\) whose vertices explicitly depend on \(\phi_c\). With this premise \(\Gamma_2\) is made by the collection of all 2-particle irreducible vacuum graphs determined by a theory with the interaction \(S_{\text{int}}(\phi; \phi_c)\) and all the internal lines equal to \(G\). These graphs are easily classifiable in a loop series giving a coherent expression for (5). This argument is equally reproducible for gauge boson and fermion fields with the exceptions that for the latter \(\psi_c = \langle 0 | \psi | 0 \rangle\) is identically zero (with the consequence that \(\mathcal{D}^{-1}\) is equal to \(D^{-1}\)) and all the \(-\frac{1}{2}\) factors must be replaced with 1. Let us imagine, now, to set the first stationary condition such that \(\phi_c = 0\), the functional \(W(0, K) = \frac{1}{i} \ln Z(0, K)\) becomes the generator of the bilocal connected Green functions
\[
W(0, K) = \sum_{n=0}^\infty \frac{(i)^{n-1}}{n!} G_c^{(n)}(x_1 y_1; \ldots; x_n y_n) K(x_1; y_1) \ldots K(x_n; y_n) \quad (6)
\]
with
\[
G_c^{(n)}(x_1 y_1; \ldots; x_n y_n) = \frac{\delta^n W(K)}{\delta K(x_1; y_1) \ldots \delta K(x_n; y_n)}
\]
while \(\Gamma(0, G)\) is the sum of all 2-particle irreducible (2PI) vacuum graphs of
the same theory. If we substitute now the scalar field with a generic spinorial one, the last formula do not involve any ansatz on $\psi_c$, but are a natural consequences of the antisymmetric character of the field. In this scenario the combined effect of the two formula

$$\frac{\delta W(K)}{\delta K(x; y)} = -G(x; y) \quad \frac{\delta \Gamma(G)}{\delta G(x; y)} = K(x; y)$$

leads to the integro differential equation

$$\int \int d^4x \, d^4y \frac{\delta^2 \Gamma}{\delta G_{\alpha\beta}(x_1; y_1) \delta G_{\gamma\delta}(x; y)} \frac{\delta^2 W}{\delta K_{\delta\gamma}(x; y) \delta K_{\eta\theta}(x_2; y_2)} =$$

$$= - [\delta_{\alpha\eta} \delta_{\beta\theta} \delta(x_1 - x_2) \delta(y_1 - y_2)]$$

where the Greek indices are spinorial. This demonstrates that $\Gamma^{(2)}(x_1 y_1; x_2 y_2)$ is the inverse of the four point connected Green function $G^{(2)}_{c}(x_1 y_1; x_2 y_2)$. Using now the loop expansion to calculate $\Gamma^{(2)}$, we simply recover a compact expression for $G^{(2)}_{c}$ in momentum space

$$G^{(2)}_{c;\alpha\beta,\gamma\delta} (p; q; P) = G^{(2)}_{0;\alpha\beta,\gamma\delta} (p; q; P) + \frac{1}{(2\pi)^6} \int \int d^4q' \, d^4p' \, G^{(2)}_{0;\alpha\beta,\rho\tau} (p; q'; P) \times$$

$$\times K_{\rho\tau,\sigma\eta} (p'; q'; P) G^{(2)}_{c;\sigma\eta,\gamma\delta} (q'; q; P)$$

where

$$-i \frac{\delta^2 \Gamma_2}{\delta G \delta G} (p; q; P) = K (p; q; P)$$

$$G^{(2)}_{0;\alpha\beta,\gamma\delta} (p; q; P) = (2\pi)^4 \delta(p - q)G_{\alpha\delta}(p + \eta P)G_{\gamma\beta}(p - (1 - \eta)P).$$

This is exactly the Bethe-Salpeter (BS) equation cited at the beginning of the section. For cases in which its expression is relatively simple, this equation is the starting point from which to detect the presence of bound states. In the present one the expansion in series of $\frac{1}{N_c}$ of the kernel $K (p; q; P)$, permits to solve it in an elegant and self consistent way.

### 3 The breaking of chiral invariance

After the technical premises of the previous section we can enter the core of the model. Let us consider a local $SU_L(2) \otimes U_Y(1)$ gauge invariant Lagrangian divided in the following groups of terms: a pure gauge electroweak sector $\mathcal{L}_B$, a
pure fermionic sector $\mathcal{L}_F$ including all three families of quarks and leptons and a 4-F interaction $\mathcal{L}_{4-F}$ (note that we are not including any Higgs or Yukawa term). A sufficiently general 4-F interaction expression could be

$$\mathcal{L}_{4-F} = K_{\alpha,\beta}^{a,b} \left[ (\bar{\Psi}_L^a \psi_R^a) (\bar{\Psi}_R^\beta \Psi_L^\beta) \right]$$  \hspace{1cm} (8)

$\alpha, \beta$ and $a, b$ being respectively family and isospin indexes. In order to be gauge invariant, the possible indexes of this expression are constrained by the values of the weak hypercharge quantum numbers. This implies that both the pairs $\alpha, \beta$ and $a, b$ must be at the same time quark like or lepton like. Moreover $a$ and $b$ must always refer to the same isospin orientation. We need now to make a reasonable hypothesis, on a phenomenological basis, so as to restrict the number of possible interaction terms. A strong indication comes from the abnormal heaviness of the quark top. It could be a significative signal of its major role in comparison with the other lighter fermions suggesting to ignore all other interaction terms apart from those involving it directly. This doesn’t mean that all the other terms are canceled, but that, for the moment, we consider a simplified model with

$$\mathcal{L}_{4-F} = K_{3q,3q}^{t,t} \left[ (\bar{\Psi}_L^{3q} \iota_R^{3q}) (\bar{\Omega}_R^{3q} \Psi_L^{3q}) \right]$$  \hspace{1cm} (9)

where $b$ indicates quark bottom and $t$ quark top. Expression (9) is crucial for the determination of $\Gamma_2$ in the loop series expansion of the effective action

$$\Gamma (G_t, G_b) = iN_c \sum_{a=b,t} \int \int d^4x \, d^4y \, \text{Tr} \left[ \ln S_a^{-1}(x-y) G_a(y;x) + -S_a^{-1}(x-y) G_a(y;x) + 1 \right] + \Gamma_2 (G_t, G_b)$$  \hspace{1cm} (10)

where every $N_c$ comes from the sum over color for each quark loop considered and $S_a^{-1}$ is an inverse free propagator of the theory. The loop expansion of $\Gamma_2$ gives schematically

$$\Gamma_2 (G_t, G_b) = N_c F_{-1} (\text{Tr}G_t) + F_0 (\text{Tr}G_t + \text{Tr} (G_t; G_b) + \ldots) +$$

$$+ \frac{1}{N_c} F_1 (\text{Tr}G_t + \text{Tr} (G_t; G_b) + \ldots) + \ldots$$

with

$$F_{-1} = -G \int d^4x \left\{ \left[ \text{Tr} \left( \frac{1}{2} G_t(x;x) \right) \right]^2 + \left[ \text{Tr} \left( \frac{1}{2} i\gamma_5 G_t(x;x) \right) \right]^2 \right\}$$
\[ F_0 = G \int d^4x \left\{ \text{Tr} \left[ \left( \frac{1}{2} i \gamma_5 G_t(x; x) \right)^2 \right] + \text{Tr} \left[ \left( \frac{1}{2} G_t(x; x) \right)^2 \right] + \right. \\
+ \left. \text{Tr} \left[ \frac{1 - \gamma_5}{2} G_t(x; x) \left( \frac{1 + \gamma_5}{2} G_b(x; x) \right) \right] \right\} + \ldots \]

Taking into consideration the highest order in the \( \frac{1}{N_c} \) expansion we finally arrive to

\[ \Gamma (G_t, G_b) = i N_C \sum_{a=b,t} \int d^4x \int d^4y \ \text{Tr} \left[ \ln S^{-1}_a(x - y) G_a(y; x) \right. \]
\[ - S^{-1}_a(x - y) G_a(y; x) + 1 \] \[ - G \int d^4x \ N_C \left\{ \text{Tr} \left[ \frac{1}{2} G_t(x; x) \right]^2 \right\} + \]
\[ + \left[ \text{Tr} \left( \frac{1}{2} i \gamma_5 G_t(x; x) \right) \right]^2 \right\}. \quad (11) \]

As we can see this expression contains terms of the same order and so its solution will be completely self-consistent. But before starting to search for it, it is necessary to make some remarks. Let’s go back to the stationary conditions (4) and analyze their meanings on the mass shell

\[ \frac{\delta \Gamma(\phi_c, G)}{\delta \phi_c(x)} = 0 \]
\[ \frac{\delta \Gamma(\phi_c, G)}{\delta G(x, y)} = 0. \quad (12) \]

In the traditional SM the Lagrangian depends on an explicit complex scalar doublet, invariant under global \( SU_L(2) \) transform. The effective action is then the first Legendre transform of the functional action \( Z(J) \), meanwhile the first stationary condition (12) (the only one in this case) determines univocally \( \phi_c \). It is just a positive value of \( \phi_c \), on the mass shell, that breaks the global \( SU_L(2) \) leading the electroweak breaking. In the present case the attention is, instead, focused on the second relation of (12). The main target is to find a solution for \( G_t \) that breaks chiral invariance spontaneously giving a dynamical value to the mass operator. As the global symmetry breaking in the Higgs mechanism, the chiral symmetry breaking is just a means trough which it is possible to reach the electroweak breaking. We can now impose on (11) the second of the (12), calculated for the quark top propagator, remembering that the breaking of a loop, in the derivation process, carries a multiplicative \( \frac{1}{N_c} \) term

\[ \frac{\delta \Gamma(G_t, G_b)}{\delta G_t(t; z)} = i G_t^{-1}(p) - i S_t^{-1}(p) - \frac{1}{2 (2\pi)^2} \int d^4k \ \text{Tr} \left[ G_t(k) \right] = 0. \quad (13) \]
A possible solution for $G_t$ is obtained with the help of the additional hypothesis of linearity à la Hartree-Fock. Writing $G_t^{-1}(p) = S_t^{-1}(p) - \Sigma_t(p)$, the (13) becomes an integro-differential equation for the mass operator $\Sigma_t(p)$

$$\Sigma_t(p) = \frac{iG}{2(2\pi)^4} \int d^4k \ Tr \left[ k^\alpha \gamma_\alpha + \Sigma_t(k) \right].$$

(14)

Because of momentum independence, $\Sigma_t(p)$ can now be written as $m_t$. This fact transforms the last equation in a self-consistent condition that will be frequently used in the following analysis of the bound states

$$1 = \frac{2iG}{(2\pi)^4} \int \frac{d^4k}{k^2 - m_t^2}.$$  

(15)

After regularization, (15) becomes an important expression linking together cut off, dynamical mass and 4-F coupling constant $G$

$$G^{-1} = \frac{\Lambda^2}{8\pi^2} \left[ 1 - \frac{m_t^2}{\Lambda^2} \ln \left( \frac{\Lambda^2}{m_t^2} \right) \right].$$

(16)

This is the first important result and needs some observations. First it contains two cut off dependent terms, one logarithmically and the other quadratically divergent. In some way it resembles the quadratic divergence of the Higgs mechanism where a fine tuning of the Yukawa and scalar quartic coupling constants were required. Second, for positive dangerous values of $m_t^2$ it implies that $\mathcal{K} = \frac{G}{\alpha} \geq 1$ giving to $G_c = \frac{8\pi^2}{\Lambda^2}$ the significance of a critical coupling constant, dividing the symmetrical from the asymmetrical phase\footnote{For what concerns $G_b$, its solution here is not considered for two main reasons: the dependence on powers of $\frac{1}{N_c}$ is of higher order respect that of $G_t$ and the experimental values of $m_t$ and $m_b$ suggest to work in the condition of maximal isospin violation.}. This means that even the (16) needs a fine tuning process in order to respect the hierarchy of scales $m_t \ll \Lambda$, together with the continuum limit $\lim_{\Lambda \to \infty} \mathcal{K} = 1$. From this point of view the spontaneous and dynamical symmetry breaking mechanisms resolve their divergence problems in a similar way with the only difference that from the point of view of renormalization group the continuum limit for $\mathcal{K}$ can be interpreted as an ultraviolet stable point

$$\lim_{\Lambda^2/m_t^2 \to \infty} \beta_\mathcal{K} = \frac{d\mathcal{K}}{d\ln \Lambda} = 0.$$
Vertex operators

The breaking of chiral invariance, just described, is an indispensable ingredient for the development of the entire model. In particular, as already stated, the use of the (15) in the (7) eliminates quadratic divergences from vertex amplitudes and makes possible the individuation of poles in their spectrum. Depending on the interaction form there are three kinds of vertices: a scalar, a pseudoscalar one and two mixed. In all the typologies the basic instrument of research is the BS equation. In fact the vertices operators can be written as

\[
\tilde{\Gamma}_S(x, y) = \frac{G}{2N_C} \langle 0 | T \left[ \bar{t}(0) t(0) t(x) \bar{t}(y) \right] | 0 \rangle_c = -G^{(2)}_{c_{\alpha\beta}, \gamma\delta}(xy; 00) \frac{G}{2N_C} \delta_{\gamma\delta}
\]

\[
\tilde{\Gamma}_{PS}(x, y) = \frac{G}{2N_C} \langle 0 | T \left[ \bar{t}(0) \gamma_5 t(0) t(x) \bar{t}(y) \right] | 0 \rangle_c = -G^{(2)}_{c_{\alpha\beta}, \gamma\delta}(xy; 00) \frac{G}{2N_C} \delta_{\gamma\delta}
\]

\[
\tilde{\Gamma}_{M+} = \frac{G}{4N_C} \langle 0 | T \left[ \bar{t}(1 + \gamma_5) tb \bar{t} \right] | 0 \rangle_c = -G^{(2)}_{c_{\alpha\beta}, \gamma\delta}(xy; 00) \frac{G}{4N_C} (1 + \gamma_5)_{\gamma\delta}
\]

\[
\tilde{\Gamma}_{M-} = \frac{G}{4N_C} \langle 0 | T \left[ \bar{t}(1 - \gamma_5) tb \bar{t} \right] | 0 \rangle_c = -G^{(2)}_{c_{\alpha\beta}, \gamma\delta}(xy; 00) \frac{G}{4N_C} (1 - \gamma_5)_{\gamma\delta}
\]

so that their Fourier transforms can be inserted in the (7). Again the principal dynamical informations that enter in the equations are collected in \( \Gamma_2 \) in the form of a second derivative kernel. The problem, here, resides in the fact that the higher order in powers of \( \frac{1}{N_C} \) is not easily given by the first graphs of the loop expansion of \( \Gamma_2 \). In fact its double derivative carries extra \( \frac{1}{N_C} \) factors that forced us to take into account an infinite series of graphs with the consequence of complicate combinatorial calculus. Moreover we should take great care of the spinorial indices that often are the keys of the right approximation. For reasons of convenience is preferable to treat the scalar and pseudoscalar vertices independently from the mixed ones. Thus we have for the kernel

\[
K_{\alpha\beta, \gamma\delta} = -\frac{i}{2N_C} \frac{G}{\delta G_{t,\alpha\beta} \delta G_{t,\gamma\delta}} \left[ \delta_{\alpha\beta} \delta_{\gamma\delta} + (i\gamma_5)_{\alpha\beta} (i\gamma_5)_{\gamma\delta} \right] + \frac{iG}{2N_C} \left\{ \delta_{\alpha\beta} \delta_{\gamma\delta} \left[ 1 + \sum_{n=1}^{\infty} [F_S]^n \right] + (i\gamma_5)_{\alpha\beta} (i\gamma_5)_{\gamma\delta} \left[ 1 + \sum_{n=1}^{\infty} [F_{PS}]^n \right] \right\} + 0 \left( \frac{1}{N_C^2} \right)
\]

with
\[ F_S(P) = i \frac{G}{2(2\pi)^4} \int d^4q \ Tr \left[ G_t \left( q + \frac{P}{2} \right) G_t \left( q - \frac{P}{2} \right) \right] \] (18)

\[ F_{PS}(P) = -i \frac{G}{2(2\pi)^4} \int d^4q \ Tr \left[ \gamma_5 G_t \left( q + \frac{P}{2} \right) \gamma_5 G_t \left( q - \frac{P}{2} \right) \right] . \]

The next step, after the insertion of \( K_{\alpha\beta,\gamma\delta} \), consists in an infinite reiteration process of the BS equations. The spinorial indices play, in this case, a major role in selecting which reiterated terms must be kept or just ignored. The point of the question resides in the rising of spinorial traces of propagators, each carrying an \( N_c \) factor with it. Thus only the first term of \( K_{\alpha\beta,\gamma\delta} \) is kept producing a great simplification of the final expressions

\[ \Gamma_{S;\alpha\beta}(P) = -\frac{G}{2N_C} \delta_{\alpha\beta} \left\{ 1 + \sum_{n=1}^{\infty} [F_S(P)]^n \right\} \] (19)

\[ \Gamma_{PS;\alpha\beta}(P) = -\frac{G}{2N_C} (\gamma_5)_{\alpha\beta} \left\{ 1 + \sum_{n=1}^{\infty} [F_{PS}(P)]^n \right\} . \]

After regularization, it is always possible to locate a set of values of \( P^2 \) for which \( F(P) < 1 \), the last expressions becoming a geometrical series and so can be formally rewritten as

\[ \Gamma_{S;\alpha\beta}(P) = -\frac{G}{2N_C} \delta_{\alpha\beta} [1 - F_S(P)]^{-1} \] (20)

\[ \Gamma_{PS;\alpha\beta}(P) = -\frac{G}{2N_C} (\gamma_5)_{\alpha\beta} [1 - F_{PS}(P)]^{-1} . \]

Calculating (18) explicitly it is easy to express (20) suitably, showing they contain a term equal to the self-consistent equation (15), and arrive to the final expressions for the vertex functions

\[ \Gamma_S(P) = -i \frac{G}{4N_C} \frac{\left( 4m^2_t - P^2 \right)}{\left( 2\pi \right)^4} \int d^4k \ \frac{1}{\left[ (k + P)^2 - m^2_t \right] \left[ k^2 - m^2_t \right]} \]^{-1} \]

\[ \Gamma_{PS}(P) = -i\gamma_5 \frac{G}{4N_C} \frac{P^2}{\left( 2\pi \right)^4} \int d^4k \ \frac{1}{\left[ (k + P)^2 - m^2_t \right] \left[ k^2 - m^2_t \right]} \]^{-1} . (21)

A similar procedure takes to the evaluation of the mixed vertex functions. The differences, in this case, are a slight change of spinor indices in the BS equation and a more complicate kernel

\[ K_{\alpha\beta,\gamma\delta} [q; (p - k)] = -i \frac{G}{4N_C} (1 + \gamma_5)_{\delta\alpha} (1 - \gamma_5)_{\beta\gamma} \left\{ 1 + \sum_{n=1}^{\infty} [F_M(P)]^n \right\} . \]
leading to

\[ \Gamma_{M_{\pm};\alpha\beta}(P) = -\frac{G}{4N_C} (1 \pm \gamma_5)_{\alpha\beta} - i \frac{G}{4N_C} \frac{(1 \pm \gamma_5)_{\alpha\beta}}{(2\pi)^4} \left\{ 1 + \sum_{n=1}^{\infty} [F_M(P)]^n \right\} \times \]

\[ \times \int d^4q \, N_C \text{Tr} \left[ (1 \mp \gamma_5) G_t(q + P) \Gamma_{M_{\pm}}(P) G_b(q) \right] \]  \hspace{1cm} (22)

with

\[ F_M(P) = \frac{iG}{4(2\pi)^4} \int d^4l \, \text{Tr} \left[ (1 - \gamma_5) G_t(l + P)(1 + \gamma_5) G_b(l) \right]. \]

Unfortunately the reiteration of (22) doesn’t eliminate any term and produces a final expression that, only under the right decomposition, becomes

\[ \Gamma_{M_{\pm};\alpha\beta}(P) = -\frac{G}{4N_C} (1 \pm \gamma_5)_{\alpha\beta} \left[ \frac{F_M(P)^2}{1 - F_M(P)} \right] \]  \hspace{1cm} (23)

with

\[ 1 - F_M(P) = \frac{1}{2} - \frac{iG}{(2\pi)^4} \int \frac{d^4k \, (k^2 - P^2)}{(P + k)^2 (k^2 - m_t^2)} \xrightarrow{P^2 \to 0} 0 \]  \hspace{1cm} (24)

\[ F_M(P)^2 = \left\{ -\frac{1}{2} - \frac{iG}{(2\pi)^4} \int \frac{d^4k \, (k^2 - P^2)}{(P + k)^2 (k^2 - m_t^2)} \right\}^2 \xrightarrow{P^2 \to 0} 1. \]

Collecting results from (21), (23) and (24) it is straight now to observe that the scalar vertex presents a pole for a transferred momentum \( P = 2m_t \), while the pseudoscalar and mixed ones have poles at \( P^2 = 0 \) corresponding to bosonic bound states on their mass shell. In particular the scalar channel represents a top-antitop state that could be compared to a sort of pseudo-Higgs boson because of its extremely massive characteristic. Moreover the absence of a negative mass squared bound state is a first indication of the stability of the asymmetrical phase. The natural conclusion is that the pseudoscalar and mixed resonances are a sort of pseudo-Goldstone bosons coming from the breaking of the chiral invariance. At this point the logic hope is that these latter amplitudes shall be absorbed in some ways in the gauge electroweak propagators in the form of mass terms. The analogy with the Higgs mechanism would be, then, complete.
5 Electroweak symmetry breaking

Having all the basic prerequisites in our hand it is now possible to approach the main target of this work. Because the SD equation coming from the stationary condition of the effective action is the main tool used in here, let us recall the loop expansion in terms of gauge propagators

\[ \Gamma(\mathcal{D}_{\mu\nu}, G_t, G_b) = \frac{i}{2} \sum_{i=W,A,Z^0} \left[ \int \int d^4x \, d^4y \, Tr \left\{ \ln \left[ D_{\mu\nu}^a(x; y) D_{\mu\nu}^{-1}(y; x) \right] + D_{\mu\nu}^{-1}(y; x) D_{\mu\nu}^a(x; y) - 1 \right\} + \Gamma_2(\mathcal{D}_{\mu\nu}, G_t, G_b) \right] \]  \tag{25}

We obviously didn’t mention terms dependent exclusively on fermion propagators because they would be eliminated by the stationary condition after the derivation respect \( \mathcal{D}_{\mu\nu}^a \). Again the richness of the theory is enclosed in \( \Gamma_2 \) and so depends on the form of interaction terms. In this case such terms are not only given by the 4-F interaction, but depends directly on the form of the neutral and charged currents \( J_Y^\mu = \frac{2}{3} (\mathcal{T}_\gamma^\mu t) - \frac{1}{3} (\mathcal{T}_\gamma^\mu b) - J_b^\mu \) and \( J^\mu = \mathcal{B} (1 + \gamma_5) \gamma^\mu t \) with its complex conjugate. Consequently the combinatorics of 2PI vacuum graphs becomes more complicate and necessitates an accurate evaluation that keeps an infinite number of terms of the same order in power of \( N_c \). Taking into account only the higher order terms, the final expression, in the case of charged weak propagators is

\[ \Gamma_2 \left( \mathcal{D}_W^\mu, G_t, G_b \right) = -\frac{g_2^2}{16} N_c \int d^4x \, d^4y \, \left\{ Tr \left[ G_t(y; x) \gamma^\nu (1 - \gamma_5) \times G_b(x; y) \right] \right\} + \]

\[ -\frac{ig_2^3 G}{64} N_c \left[ \sum_{n=1}^\infty F^{(n)}_W \left( \mathcal{D}_W^\mu, G_t, G_b \right) \right] \]  \tag{26}

with

\[ F^{(n)}_W = \left( \frac{iG}{d} \right)^{n-1} \int \cdots d^4x d^4z_1 \cdots d^4z_n \int d^4y \, y \left\{ Tr \left[ G_b(z_2; z_1) (1 - \gamma_5) G_t(z_1; z_2) (1 + \gamma_5) \times \gamma^\nu (1 - \gamma_5) \right] \right\} \]

\[ \times \cdots \times Tr \left[ G_b(z_n; z_{n-1}) (1 - \gamma_5) G_t(z_{n-1}; z_n) (1 + \gamma_5) \right] \times \]

\[ \times \left[ Tr \left[ G_t(y; z_n) (1 + \gamma_5) G_b(z_n; y) \gamma^\mu (1 - \gamma_5) \right] \mathcal{D}_W^{\mu \nu} (x; y) \right] \].

Applying the stationary condition \( \frac{\delta \Gamma(\mathcal{D}_W^\mu, G_t, G_b)}{\delta \mathcal{D}_W^{\mu \nu}} = 0 \) to the (25) one directly obtains the SD equation for the complete propagator \( \mathcal{D}_W^{\mu \nu} \) as a complicate
function of the free one $D_{\mu \nu}^W$ and the fermionic $G_t$, $G_b$, that demonstrate the complete absorption of the mixed vertex functions

$$D_{\mu \nu}^{-1} = D_{\mu \nu}^{-1} + \frac{iNCG_2^2}{8} \{ \text{Tr} [G_t \gamma^\nu (1 - \gamma_5) G_b (1 + \gamma_5) \gamma^\mu] + i R (F_M (P)) \times \text{Tr} [G_t \gamma^\nu (1 - \gamma_5) G_b \Gamma_{M_+}] \text{Tr} [\Gamma_{M_+} G_t G_b \gamma^\mu (1 - \gamma_5)] \}. \quad (27)$$

After a long and accurate regularization [3] that doesn’t break the gauge invariance one recover an expression that makes evident how the absorption process is a concrete "mass generation" mechanism giving body to the electroweak symmetry breaking

$$\frac{1}{g_2^2} D_{\mu \nu}^{-1} = i \left( \frac{P^\mu P^\nu}{P^2} - g^{\mu \nu} \right) \left( \frac{1}{g_2^2 (P^2)} P^2 - \tilde{h}^2 (P^2) \right) \quad (28)$$

with

$$\frac{1}{g_2^2 (P^2)} = \frac{1}{g_2^2} + \frac{N_C}{48 \pi^2} \left[ \ln \frac{\Lambda^2}{m_t^2} + P^2 \int_{m_t^2}^\infty \frac{d^4k}{k^2 (k^2 - P^2)} \left( 1 - \frac{m_t^2}{k^2} \right)^2 \left( 1 + \frac{2m_t^2}{k^2} \right) \right]$$

$$\tilde{h}^2 (P^2) = \frac{N_C m_t^2}{32 \pi^2} \left[ \ln \frac{\Lambda^2}{m_t^2} + P^2 \int_{m_t^2}^\infty \frac{d^4k}{k^2 (k^2 - P^2)} \left( 1 - \frac{m_t^2}{k^2} \right) \right].$$

The (28) has, evidently, a pole in correspondence of $M_W^2 = P^2 = g_2^2 (P^2) \tilde{h}^2 (P^2)$ that is just the gauge mass term searched.

A similar procedure, even if a bit more lengthy, applies for the neutral gauge propagators. After the additional transform that rotate fields from $W_3 \mu, B_\mu$ to $Z^0_\mu, A_\mu$ changing the interaction terms, the relevant difference is contained in the explicit expressions of the $\Gamma_2$ expansion

$$\Gamma_2 = \Gamma_2^A + \Gamma_2^{Z^0} \quad (29)$$

where the photonic part is

$$\Gamma_2^A = -\frac{2}{9} e^2 N_C \left\{ \int d^4x \int d^4y \left\{ \text{Tr} [G_t (y; x) \gamma^\mu G_t (x; y) \gamma^\nu] D_{\mu \nu}^A (x; y) \right\} + \frac{i e^2 G}{9 N_C} \left\{ \sum_{n=1}^\infty F_{A,S}^{(n)} \right\} + \frac{i e^2 G}{9 N_C} \left\{ \sum_{n=1}^\infty F_{A,PS}^{(n)} \right\} \right\} \quad (30)$$

with
meanwhile the $Z^0$ part has the following form

$$
\Gamma_2^{Z^0} = -\frac{g_2^2 N_C}{32 \cos^2 \theta_W} \int \int d^4 x d^4 y \left\{ \text{Tr} \left[ G_t (x; z_1) \gamma^\mu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] G_t (z_1; x) \right\} \times
\times \left\{ \text{Tr} \left[ G_t (z_2; z_1) G_t (z_1; z_2) \right] \times \text{Tr} \left[ G_t (z_3; z_2) \right] \times \ldots \times \text{Tr} \left[ G_t (z_n; z_{n-1}) G_t (z_{n-1}; z_n) \right] \times \text{Tr} \left[ G_t (y; z_n) G_t (z_n; y) \gamma^\nu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \right\}
$$

with

$$
F_{Z^0,S}^{(n)} = \left( \frac{iG}{2} \right)^{n-1} \int \cdots \int d^4 x d^4 z_1 \ldots d^4 z_n d^4 y \left\{ \text{Tr} \left[ G_t (x; z_1) \gamma^\mu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] G_t (z_1; x) \right\} \times
\times \left\{ \text{Tr} \left[ G_t (z_2; z_1) G_t (z_1; z_2) \right] \times \text{Tr} \left[ G_t (z_3; z_2) \right] \times \ldots \times \text{Tr} \left[ G_t (z_n; z_{n-1}) G_t (z_{n-1}; z_n) \right] \times \text{Tr} \left[ G_t (y; z_n) G_t (z_n; y) \gamma^\nu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \right\}
$$

$$
F_{Z^0,PS}^{(n)} = \left( \frac{iG}{2} \right)^{n-1} \int \cdots \int d^4 x d^4 z_1 \ldots d^4 z_n d^4 y \left\{ \text{Tr} \left[ G_t (x; z_1) \gamma^\mu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] G_t (z_1; x) \right\} \times
\times \left\{ \text{Tr} \left[ G_t (z_2; z_1) G_t (z_1; z_2) \right] \times \text{Tr} \left[ G_t (z_3; z_2) \right] \times \ldots \times \text{Tr} \left[ G_t (z_n; z_{n-1}) G_t (z_{n-1}; z_n) \right] \times \text{Tr} \left[ G_t (y; z_n) G_t (z_n; y) \gamma^\nu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \right\}.
$$

Again considering the stationary conditions for the neutral gauge boson propagators lead us to
\[ D_{\mu\nu}^{-1}(P) = D_{\mu\nu}^{-1}(P) + \frac{4iNCe^2}{9} \left\{ \text{Tr} \left[ G_\gamma \gamma^\mu G_\gamma \gamma^\nu \right] + ight. \\
- i\Gamma_s \text{Tr} \left[ G_\gamma \gamma^\mu G_\gamma \gamma^\nu \right] \text{Tr} \left[ G_\gamma G_\gamma \right] + \\
\left. - i\Gamma_{PS} \text{Tr} \left[ G_\gamma_5 G_\gamma \gamma^\mu \right] \text{Tr} \left[ G_\gamma_5 G_\gamma \gamma^\nu \right] \right\} \] (32)

and

\[ D_{\mu\nu}^{Z_0}^{-1}(P) = D_{\mu\nu}^{Z_0}^{-1}(P) + \frac{iNCg_2^2}{16 \cos^2 \theta_W} \left\{ \text{Tr} \left[ G_\gamma \gamma^\mu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \times \\
\times G_\gamma \gamma^\nu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right\} + \\
- i\Gamma_s \text{Tr} \left[ G_\gamma G_\gamma \gamma^\mu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \times \\
\times \text{Tr} \left[ G_\gamma G_\gamma \gamma^\nu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] + \\
- i\Gamma_{PS} \text{Tr} \left[ G_\gamma_5 G_\gamma \gamma^\mu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \times \\
\times \text{Tr} \left[ G_\gamma_5 G_\gamma \gamma^\nu \left( \left( 1 - \frac{8}{3} \sin^2 \theta_W \right) - \gamma_5 \right) \right] \right\} (33) \]

giving expressions in which vertex functions appear naturally. Explicit calculations bring us to

\[ \frac{1}{g_2^2} D_{\mu\nu}^{Z_0}^{-1}(P) = i \left( \frac{P^\mu P^\nu}{P^2} - g^{\mu\nu} \right) \left( \frac{1}{g_2^2(P^2)} P^2 - \tilde{f}^2(P^2) \right) \] (34)

with

\[ \frac{1}{g_2^2(P^2)} = \frac{1}{g_2^2} + \frac{N_C}{96\pi^2 \cos^2 \theta_W} \left( \left( \frac{1}{3} \sin^2 \theta_W \right)^2 + 1 \right) \left[ \ln \frac{\Lambda^2}{m_t^2} + \\
+ P^2 \int_{4m_t^2}^{\infty} \frac{d^4k}{k^2(k^2 - P^2)} \left( 1 - \frac{4m_t^2}{k^2} \right) \left( 1 + 2 \frac{m_t^2}{k^2} \right) \right] \]

\[ \tilde{f}^2(P^2) = \frac{N_C m_t^2}{32\pi^2 \cos^2 \theta_W} \left[ \ln \frac{\Lambda^2}{m_t^2} + \frac{P^2}{2} \int_{4m_t^2}^{\infty} \frac{d^4k}{k^2(k^2 - P^2)} \left( 1 - \frac{4m_t^2}{k^2} \right)^{-\frac{1}{2}} \right] \]

giving to the Z\(^0\) field a mass term \( M_{Z_0}^2 = P^2 = \tilde{g}_2^2(P^2) \tilde{f}^2(P^2) \). Meanwhile for the photon propagator one obtains naturally no mass correction but only a coupling constant renormalization.

\[ \frac{1}{e^2} D_{\mu\nu}^{-1}(P) = i \left( P^\mu P^\nu - g^{\mu\nu} P^2 \right) \left( \frac{1}{e^2(P^2)} \right) \] (35)
with

\[
\frac{1}{e^2 (P^2)} = \frac{1}{e^2} + \frac{2N_C}{27\pi^2} \left[ \ln \frac{\Lambda^2}{m_i^2} + P^2 \int_{4m_i^2}^{\infty} \frac{d^4 k}{k^2 (k^2 - P^2)} \left( 1 - \frac{4m_i^2}{k^2} \right)^{\frac{3}{2}} \right].
\]

As we have already noted, in (27), (32) and (33) the inverse gauge propagators are corrected by means of terms that involve vertex functions. In a Feynman graph representation these terms contain internal lines corresponding to the resonance channels found in the section 4. In other words corrections to the gauge propagators imply inevitably the exchange of bound state particles. Moreover calculation details show \cite{3} that the only vertices that contribute to the mass generation are those relative to the pseudo-Goldstone modes, while the scalar amplitude remains outside the process: a strong hint towards the identification of latter’s resonance with a composite Higgs boson.

6 General considerations and conclusions

This work certainly doesn’t constitute the only attempt to build a model that try to justify the largeness of the quark top mass and use it as a breakthrough in the electroweak symmetry breaking. An entire class of so called ”Top Mode Standard Model” (TMSM) were written in the years that divide the last two decades\cite{5}. A second phase for TMSM arrived after a period of five years when a series of works tried to improve the mechanism giving to it the label ”renormalizable”\cite{6}\cite{7}\cite{8}. Despite their results and conclusions on the gauged NJL model and its renormalization, I would like to emphasize some aspects of the approach exposed here. Forgetting for the moment the divergencies included in the self-consistent equation (16), I show the importance of the chiral symmetry breaking as a means through which obtain a gauge symmetry breaking. The appearance of corrections depending on bound states amplitudes, in the SD equations for the inverse gauge propagators, makes the dynamical ”absorption process” a real alternative model to the static Higgs mechanism. This result is not conclusive obviously: the expressions obtained in section 5 gives in terms of \(\frac{1}{N_c}\) power series the following behavior

\[
D_{\mu\nu} = 0 (1) + \left( \frac{1}{N_c} \right) + \ldots.
\]

This implies that in the SD equation for \(G_t\) the neglect of boson contributions to \(\Gamma_2\) wasn’t the correct procedure. At least we should take in consideration the gauge propagator contribution to graphs in planar approximation which, in fact, carries the same order (in \(\frac{1}{N_c}\)) of the 4-F interaction. This was not done
for two main reasons: the preponderance of 4-F induced terms due to the over-
critical value of $\mathcal{G}$ respect to $g_1$ and $g_2$. Second, because the 4-F hypothesis was
analyzed in a low-energy regime (the SD without gauge contributions could be
considered the zero approximation for $m_t$ in a $P^2$ power series expansion) the
"generation" of gauge masses would be then justified as a tree-level approx-
imation. In this sense the next logical step would be the inclusion of gauge
corrections to all the equations of the model. The program is compatible with
fermionic SD and BS equations, but becomes really complex for gauge SD
equations because of the combinatorics of the graphs which clearly complicate
the localization of the "generated" mass terms.

Second important remark concerns the supposed predictability this model can
give of the SM mass spectrum. In section 3, in fact, I ignored some interaction
terms in order to simplify the analysis of the successive sections. Even if not
mentioned here I have studied [3] the consequences, on the fermion mass sector,
of the insertion of additional 4-F interactions in the Lagrangian. In analogy
with the work of Hasenfratz et Al. [9], I found that on this side gauged NJL
models don’t give any additional previsional information compared with the
traditional Higgs mechanism. This should not divert the attention from the
fundamental dynamical content that could help to enlighten several aspects of
existing problems. The NJL model is still a fruitful argument of analysis and
will be again a starring of future researches.

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