Numerical and experimental analysis of pressure fluctuations in the draft-tube of a Francis turbine using the swirl number

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Abstract. Pressure fluctuations in the draft-tubes of hydraulic turbines are the most dominant issues that may cause damages to a hydraulic power plant. When the frequency of the fluctuations coincides with one natural frequencies of the plant, resonance with high amplitude vibrations and pressure pulsations occurs, dramatically impacting the integrity of the plant. This research focuses on the part-load behaviour of a medium specific-speed Francis turbine (with specific speed 184 m.kW), designed by the authors in the framework of an industrial project. This paper combines the results of model tests with CFD simulation to investigate the part-load behaviour of the turbine. To do so, performance of the turbine is numerically simulated at one operating condition by solving the RANS equations via the commercial code ANSYS CFX 19.0. The k-ω SST turbulence model is used to predict the turbine performance. In addition, model tests are conducted in the range of 20% to 110% guide vane opening. To capture pressure fluctuations in the draft-tube, 12 static pressure sensors are installed in 4 different sections of the draft-tube. The no-swirl and other iso-swirl lines of the draft tube flow are first determined by using CFD results. Furthermore, pressure signals from model tests are analysed by performing cross spectral density analysis. Two transitions in the behaviour of the precessing vortex under part-load conditions occurring at given values of swirl number are observed. Finally, it is observed that linear correlations between the Strouhal number of the vortex rope frequency and the swirl number can be established within the 2nd and 3rd part-load regimes, independently of the values of the speed coefficient.

1. Introduction

In today’s world of ever-increasing energy costs, many industries are looking for more affordable alternative sources of energy rather than fossil fuels. In addition, air pollution in big cities, and, effect of greenhouse gases on global climate changes led the world to seek for clean and sustainable energy resources. Hence, it is observed that from mid 80s feasibility studies on using renewable energies came into existence and followed the progress path toward its universal utilization which we can notice nowadays. It stands to reason that harvesting renewable energies, e.g. wind, solar, and so forth, depends extremely on climate changes putting at risk the grid stability. So, utilizing hydro-power, which has
shown more flexible operating capabilities, attracts enormous amount of attention in recent years. Achieving this, however, requires a wide range of operating conditions, from low load to high load, in order for the generating units to adjust their output power.

In case of hydraulic machines, operating at off-design condition is usually accompanied with undesirable phenomena, such as cavitation, precessing vortex rope, pressure surge, and so forth [1]. A precessing vortex rope occurs in the draft tube of Francis turbines operating in part-load conditions due to the high residual swirl leaving the runner, leading to vortex breakdown. The precession of the vortex rope with a frequency between 0.2 and 0.4 times the runner frequency induces pressure fluctuations in the draft tube and the whole hydraulic circuit [2,3] that can be decomposed into two components at the precession frequency, namely, synchronous, and convective components [4,5]. While the convective component consists of a local fluctuation observed only in the draft tube, the synchronous pulsations propagate through the complete hydraulic circuit, and can cause resonance if the precession frequency coincides with one of the natural frequencies of the system [6-8]. Although much research has contributed to our understanding of this phenomenon in the past decades, the prediction of the dynamics of the precessing vortex rope and the induced pressure pulsations on the complete operating range of the machine remains challenging.

The current research is conducted under the framework of an industrial project to investigate the performance and the draft-tube flow behavior of a medium specific speed Francis turbine under part-load condition. The value of the swirl number in the draft tube is predicted by using results of CFD and model tests and the analytical expression developed by Favrel et al. [9]. Model tests are conducted over a wide range of operating points, more than 110, to insure fulfilling an extensive study of the turbine performance.

2. Methodology

2.1. Test case

The test case is a real-size model of a medium specific-speed Francis turbine (Ns. 184 m.kW) designed in the framework of an industrial project between Waseda University, EAML engineering, and Meidensha companies. The turbine features 15 runner blades. The hydraulic performance of this turbine has been presented in a previous paper [10]. Table 1 presents the general specifications of the turbine at its design point, and Figure 1 shows the 3D model and the physical model of the investigated turbine.

| Effective head | 13.45 [m]   | Runner inlet diameter | 0.3587 [m] | Discharge | 0.36 [m³/s] | Number of runner vanes | 15          |
|----------------|-------------|-----------------------|------------|-----------|-------------|------------------------|-------------|
| Rotational speed | 730 [rpm]   | Number of guide vanes | 18 [-]     | Specific speed (Nₚ) | 184 [m.kW]  | Number of stay vanes  | 9 [-]      |

![Table 1. General specifications of the turbine at design point.](image)

**Figure 1.** Geometry of the turbine.
2.2. Model tests
The model tests were conducted in an open-loop hydraulic test rig located at EAML Engineering Co., Ltd. The main purpose of the model tests was to determine the performance of the turbine under no-cavitation conditions from no-load to full-load operating range. Model tests were performed in the range of 20% to 110% guide vane openings, for 10 nEDs.

Since the model tests were conducted in an open-loop test rig, it was not possible to set the cavitation number precisely. The tail water level was however high enough to avoid cavitation onset at most of the operating conditions. To measure pressure fluctuations in the draft-tube, 12 static pressure sensors were installed at 4 different sections, as shown in Figure 2.

![Figure 2. Pressure measurement sections in the draft-tube.](image)

2.3. Numerical setup
The performance of the turbine is numerically simulated at one operating point (87% guide vanes opening, \( n_{ED}^* = 1 \), \( Q_{ED}^* = 0.9838 \)) by solving the steady-state RANS equations (Reynolds-averaged Navier-Stokes). To obtain a more accurate prediction, numerical simulation is performed over the whole fluid domain, including the spiral casing, all stay vanes, guide vanes, runner vanes, and the draft-tube. Simulation condition is explained in the previous paper [10]. It was shown previously [10] that the results of CFD are in an acceptable agreement with experiments in term of efficiency.

2.4. Swirl number definition
One of the most common causes of pressure fluctuations in Francis turbine draft-tubes is the development of a precessing vortex core resulting from a vortex breakdown in the swirling flow in the draft tube. This phenomenon occurs specifically at part-load conditions with flow rates lower than at the Best Efficiency Point (BEP). It is observed that the dynamics of the vortex core mainly depends on the swirl intensity of the incoming flow, i.e. the flow leaving the runner in the case of hydraulic turbines [11,12]. The swirl number (S), as defined in equation 1, is a common dimensionless definition to characterize the swirl intensity in the draft tube of hydraulic machines [13].

\[
S = \frac{\int_0^R C_m C_u r^2 dr}{\int_0^R C_m \cdot r dr}
\]  

Where \( C_m \) and \( C_u \) are the meridional and tangential velocity components at runner outlet, respectively. Favrel et al. have obtained an analytical expression of the swirl number expressed as a function of the operating parameters of the machines, i.e. the discharge and speed factors, see Equation 2 [9]. By performing Laser Doppler Velocimetry (LDV) measurements along the diameter of one cross-
section in the draft-tube cone, they showed that their analytical expression is in fair agreement with the experimental data. The advantage of using Equation 2 is to simply link the speed and discharge factors to the swirl number.

\[
S = n_{ED} \frac{\pi^2}{8} \left( \frac{1}{Q_{ED}} - \frac{1}{Q_{0ED}} \right)
\]

where \( n_{ED} \) and \( Q_{ED} \) are speed, and discharge factors, respectively, as described below [14]:

\[
n_{ED} = \frac{nD}{\sqrt{E}} \quad Q_{ED} = \frac{Q}{D^2 \sqrt{E}}
\]

and, \( Q_{0ED} \) in Equation 2 represents the discharge factor corresponding to the no-swirl condition for a given \( n_{ED} \) value.

3. Results and discussions

3.1. Prediction of iso-swirl lines

The no-swirl zone is usually identified during model tests by visualizing the draft tube flow: it is assumed that the conditions featuring no cavitation in the draft tube correspond to the no-swirl conditions. In the present test-case, model tests were performed in cavitation-free conditions and the no-swirl zone cannot therefore be identified. In this section, the no-swirl conditions and other iso-swirl lines are estimated based on the approximation of the swirl number (equation 2) and results of CFD. To do so, it is first necessary to find at least three no-swirl points on the hill chart. By using equation 1 along with the axial and tangential velocity profiles at the runner outlet obtained by CFD, the swirl number is estimated at \( S = -0.1324 \) for the simulated operating point. By using Equation 2, the value of the discharge factor in no-swirl conditions for \( n_{ED}^* = 1 \) can be estimated at \( Q_{0ED}^* = 0.917 \).

To determine two additional no-swirl points by using the procedure as described in the previous paragraph, it is necessary to find the axial and tangential velocity profiles (\( C_m \) and \( C_u \)) at the runner outlet for two additional operating points. In the current research however, the authors adopted a simple method to find two other no-swirl points. It is assumed that the \( C_m \) distribution remains constant along the same iso-swirl line for operating points close to the simulated one. Two points with \( C_m \) equal to ±3% of the value of the simulated point are considered as shown in Figure 3. The values of \( C_m \) presented in Figure 3 are normalized with the maximum value of \( C_m \) from simulation.

![Figure 3. C_m profiles at the runner outlet.](image)

![Figure 4. Schematic velocity triangles at the runner outlet.](image)
As shown in Figure 4, runner speed is modified for the additional points to maintain similar velocity triangle. Based on the velocity triangles at the runner outlet presented in Figure 4, the $C_u$ components can be computed for these two operating points by using Equation 3.

$$C_u = U - \frac{C_m}{\tan(\beta_2)}$$  \hspace{1cm} (3)

where $U$ and $\beta_2$ are circumferential velocity and blade angle at runner outlet, respectively.

By using the procedure used to determine the first no-swirl point, two additional no-swirl points are determined, as described in Table 2 and Figure 5.

| Point | $n_{ED}^*$ | $Q_{ED}^*$ | $S$ | $Q_{ED}^{D*}$ |
|-------|------------|------------|-----|---------------|
| [-3% $C_m$] | 0.9658 | 0.9541 | -0.1324 | 0.889 |
| [CFD $C_m$] | 1 | 0.9838 | -0.1324 | 0.917 |
| [+3% $C_m$] | 1.0256 | 1.0131 | -0.1324 | 0.944 |

Table 2. Zero-swirl points.

As shown in Figure 5, the no-swirl line is defined by linear interpolation of the three no-swirl points defined previously ($Q_{ED}^{D*} = 0.9023n_{ED}^* + 0.0159$). This linear relation is extrapolated to the complete range of $n_{ED}$ values. By using this relation along with Equation 2, it is possible to predict iso-swirl lines on the complete operating range.

3.2. Precessing vortex frequency

One common method to determine the frequency of the precessing vortex core is to perform cross spectral density analysis of two pressure signals measured in the same section of the draft-tube. The cross spectral density is the Fourier transform of the cross-correlation function, which is a similarity function to find the displacement of one signal relative to the other one [15]. Furthermore, the coherence between both signals at the vortex frequency is also computed. Figure 6 shows examples of cross spectral density and coherence for two signals at $n_{ED} = 1$ and $Q_{ED}^* = 0.53$. 

![Figure 5. Extrapolation of zero-swirl points into zero-swirl line.](image)
Figure 6. Examples of cross spectral density calculation at $n_{ED}^* = 1$ and $Q_{ED}^* = 0.53$.

In the following, the frequency of the precessing vortex core ($f_{pvc}$) is expressed as a Strouhal number as defined in Equation 4., representing the dimensionless frequency of vortex.

$$St = \frac{f_{pvc}D_2}{Q}$$

Figure 7 presents the Strouhal number plotted against the swirl number for $n_{ED}^*$ ranging from 0.6 to 1.65. Two transitions in the vortex frequency can be identified at given values of swirl number, $S$ equal to 0.4 and 1.7. Linear correlations between the Strouhal number of the vortex frequency and the swirl number are observed in the 2nd and 3rd regimes.

The linear correlations in regimes 2 and 3 are presented in Table 3. The interesting point to mention is that the slope of the correlation in regime 2 is almost as twice high as the slope in regime 3.

Table 3. Linear correlations between precessing vortex frequency and swirl intensity.

| Regime     | Correlation      | Slope   |
|------------|------------------|---------|
| Two & three| $St.1 = 0.304S + 0.2954$ | 0.304   |
| Three      | $St.2 = 0.1485S + 0.1951$ | 0.1485  |
Three part-load regimes are shown on the hill chart in Figure 8, where the transitions occur at $S = 0.4$ and $S = 1.7$.

![Figure 8. Part-load regimes.](image)

### 4. Conclusions

This paper is devoted to study the part-load behaviour of a medium specific-speed Francis turbine. The main objective of this paper is to first predict the swirl intensity by using results of CFD and to correlate the vortex frequency with the swirl number over the complete part-load range. Three operating points located on the no-swirl line are first determined by using CFD results. Then, by performing cross spectral analysis of model tests’ results, the frequency of the precessing vortex is calculated for all tested part-load operating conditions. Two transitions in the behavior of the precessing vortex frequency while changing the swirl intensity are observed, defining three distinct part-load regimes. Finally, two linear correlations between the precessing vortex frequency and the swirl number are established for regimes 2 and 3. It is of great interest to investigate the physics behind the two transitions in future works.

### References

[1] Rheingans W 1940 Power swings in hydroelectric power plants *Transactions of the ASME* 62 171-184

[2] Arpe J, Nicolet C, and Avellan F 2009 Experimental evidence of hydroacoustic pressure waves in a Francis turbine elbow draft tube for low discharge conditions *Journal of Fluid Engineering* 131(8)

[3] Lowys P, Andre F, Ferreira da Silva A, Duarte F, and Payre P 2014 Hydro plant operating range extension transverse approach for increasing turbine flexibility *Proceedings of Hydrovision* USA

[4] Nishi M, Matsunaga S, and Kubota T 1984 Surging characteristics of conical and elbow-type draft tubes *Proceedings of the 12th IAHR Symposium on Hydraulic Machinery and System* Scotland

[5] Duparchy A, Guillozet J, De Colombel T, and Bornard L 2014 Spatial harmonic decomposition as a tool for unsteady flow phenomena analysis *IOP Conference Series: Earth and Environmental Science* 22

[6] Dörfler P 1982 System dynamics of the Francis turbine half load surge *Proceedings of the 11th IAHR Symposium on Operating Problem of Pump Stations and Powerplants* Netherlands

[7] Fritsch A, and Maria D 1988 Dynamic behaviour of a partial load Francis water turbine:
model/prototype comparison *La Houille Blanche* 3(3-4)

[8] Favrel A, Landry C, Müller A, Yamamoto K, and Avellan F 2014 Hydro-acoustic resonance behaviour in presence of a precessing vortex rope: Observation of a lock-in phenomenon at part-load Francis turbine operation *IOP Conference Series: Earth and Environmental Science* 22

[9] Favrel A, Pereira J G, Landry C, Müller A, Nicolet C, and Avellan F 2017 New insight in Francis turbine cavitation vortex rope: role of the runner outlet flow swirl number *Journal of Hydraulic Research*

[10] Khozaei M H, Yamaguchi N, Miyagawa K 2019 Numerical analysis of hydraulic loss in a medium specific-speed Francis turbine *The 2nd IAHR-Asia Symposium on Hydraulic Machinery and Systems* Korea

[11] Cassidy J J, and Falvey H T 1970 Observations of unsteady flow arising after vortex breakdown *Journal of Fluid Mechanics* 41(4)

[12] Cala C E, Fernandes E C, Heitor M V, and Shtork S I 2006 Coherent structures in unsteady swirling jet flow *Experiments in Fluids* 40(2)

[13] Gupta A K, Lilley D G, and Syred N 1984 *Swirl Flows* (Tunbridge Wells, Kent: Abacus Press) vol 1

[14] IEC standards 1999 60193: Hydraulic turbines, storage pumps and pump-turbines – model acceptance tests (2nd ed.) Geneva: International Commision

[15] Stoica P, and Moses R 2005 *Spectral Analysis of Signals* (Upper Saddle River, New Jersey: Prentice Hall)