ACOUSTIC SOURCE LOCALIZATION WITH THE ANGULAR SPECTRUM APPROACH IN CONTINUOUSLY STRATIFIED MEDIA

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July 13, 2020

The following article has been submitted to JASA Express Letters. After publication, it will be found here.

ABSTRACT

The angular spectrum approach (ASA)—a fast, frequency domain method for calculation of the acoustic field—enables passive source localization and modeling forward propagation in homogeneous media with high computational efficiency. Here we show that, if the medium is continuously stratified, a first-order analytical solution may be obtained for the field at arbitrary depth. Our simulations show that the stratified ASA solution enables accurate source localization as compared to the uncorrected ASA (error from $1.2 \pm 0.3$ to $0.49 \pm 0.3$ wavelengths) at scalings relevant to biomedical ($kL \sim 500$, where $L$ is the length of the measurement aperture), underwater ($kL \sim 800$), and atmospheric ($kL \sim 10$) acoustic applications. Overall the total computation was on the order milliseconds on standard hardware ($225 \pm 84$ ms, compared with $78 \pm 63$ ms for the homogeneous ASA formulation over all cases). Collectively, the results suggest the proposed ASA phase correction enables efficient and accurate method for source localization in continuously stratified environments.

1 Introduction

Generally, the angular spectrum approach (ASA)—a fast, frequency domain method for calculation of the acoustic field—enables passive source localization and modeling forward propagation in homogeneous media with high computational efficiency. Here we show that, if the medium is continuously stratified, a first-order analytical solution may be obtained for the field at arbitrary depth. Our simulations show that the stratified ASA solution enables accurate source localization as compared to the uncorrected ASA (error from $1.2 \pm 0.3$ to $0.49 \pm 0.3$ wavelengths) at scalings relevant to biomedical ($kL \sim 500$, where $L$ is the length of the measurement aperture), underwater ($kL \sim 800$), and atmospheric ($kL \sim 10$) acoustic applications. Overall the total computation was on the order milliseconds on standard hardware ($225 \pm 84$ ms, compared with $78 \pm 63$ ms for the homogeneous ASA formulation over all cases). Collectively, the results suggest the proposed ASA phase correction enables efficient and accurate method for source localization in continuously stratified environments.

Source localization from the beamformed maps is a problem of significant interest in biomedical, underwater, and aeroacoustic applications. However, in such applications the propagation environment typically varies as a function of the vertical direction. This variation not accounted for by the ASA, whose derivation assumes a uniform
speed of sound, and thus the heterogeneity of the medium will induce aberration in the image and result in errors in the source localization.

Herein we derive a first order analytical correction for the ASA and for the case of a stratified medium. Then, through simulated acoustic propagation, we demonstrate corrected sub-wavelength localization in stratified environments at scales relevant to biological, atmospheric, and underwater acoustics. Finally, we consider the point source localization errors, and the computational cost of the different algorithm permutations.

2 Methods

2.1 Angular Spectrum Approach for Stratified Media

The angular spectrum $P$ of a monochromatic ($\propto -i\omega t$) field $\tilde{p}$ defined by

$$P(k_x, k_y, z) = \mathcal{F}_k[\tilde{p}(x, y, z)] = \iint_{-\infty}^{\infty} \tilde{p}(x, y, z) e^{-i(k_xx + k_yy)} \, dx \, dy.$$

(1)

Taking the 2D Fourier transform to the homogeneous Helmholtz equation $(\nabla^2 + \omega^2/c_0^2)\tilde{p} = 0$ with a constant speed of sound $c_0$ gives an ordinary differential equation for $P$

$$\frac{d^2P}{d^2z} + k_x^2 P = 0,$$

(2)

where $k_x^2 = (\omega/c_0)^2 - k_y^2 - k_z^2$. Knowledge of the boundary condition $P_0$ at $z = 0$, and the assumption that there are no backward-travelling waves, then Eq. (2) has the solution

$$P = P_0 e^{ik_z z}.$$

(3)

The acoustic field in any plane may then be reconstructed by with Eq. (3) and evaluation of the inverse transform.

Suppose now that the sound speed has weak spatial dependence (i.e., that the sound speed changes over scales that are large compared with the wavelength), such that $c_0 \rightarrow c(r)$ in the Helmholtz equation. Then, with the definitions $\mu = c_0^2/c^2(r)$ and $\lambda = (1 - \mu)\omega^2/c_0^2$ for a constant reference sound speed $c_0$, it can be shown that the governing equation is

$$\frac{d^2P}{d^2z} + k_x^2 P = \Lambda * P.$$

(4)

Here $\Lambda = \mathcal{F}_k[\lambda]$, and $*$ represents the 2D convolution is over the wavenumber components $k_x$ and $k_z$. If $c$ varies only with the axial coordinate $z$, the convolution may be evaluated to give

$$\frac{d^2P}{d^2z} + k_x^2 P = \lambda P.$$

(5)

Assuming a WKB–type solution of the form

$$P = A(k_x, k_y, z) e^{ik_z z},$$

(6)

where $A$ is a complex amplitude. Substitution of Eq. (6) into Eq. (5) and evaluation of the derivatives yields

$$\frac{d^2A}{d^2z} + 2ik_z \frac{dA}{dz} - \lambda A = 0.$$

(7)

To first order (see below), the first term in Eq. (7) can be neglected to obtain a first-degree ODE for $A$, which may be integrated directly to give

$$A = A_0 \exp \left( \frac{1}{2ik_z} \int_0^z \lambda(z') \, dz' \right).$$

(8)

Application of the boundary condition (i.e., the Fourier transform of the measured field at $z = 0$) gives

$$A = P_0 \exp \left( \frac{1}{2ik_z} \int_0^z \lambda(z') \, dz' \right).$$

(9)
We have shown in a previous work that the angular spectrum at arbitrary \( z \) is then

\[
P = \left[ P_0 \exp \left( \frac{1}{2ikz} \int_0^z \lambda(z') \, dz' \right) \right] e^{ikz}.
\]

Neglecting the second-order term in Eq. (7), yields the solution given by Eq. (9). This assumption requires that \( |d^2A/dz^2| \) is small compared to both \( |(dA/dz)/k_z| \) and \( |\lambda| \) [recall that here \( \lambda \equiv k_0^2(1 - \mu) \) rather than the wavelength].

Evaluation of the derivatives in Eq. (9) gives this condition as

\[
\frac{d^2A}{dz^2} \sim \frac{d\lambda/dz}{2ikz} - \frac{\lambda^2}{4k_z^2} \Rightarrow \frac{d\lambda/dz}{2ikz} - \frac{\lambda}{4k_z^2} \ll 1
\]

Thus the first term then requires that \( |(d\mu/dz)/k_z(1 - \mu)| \ll 1 \), i.e., the sound speed must change slowly compared to a wavelength. The second term that dictates \( |(k_0^2/k_z^2)(1 - \mu)| \) is negligible, so for \( k_0 \sim k_z \) (paraxial approximation), this requirement is that \( \mu \sim 1 - 2c'/c_0 \sim 1 \), i.e., that the relative magnitude of the sound speed changes should be small.

### 2.1.1 Interpretation of Solution

Equation (10) represents an additional phase delay \( \phi \) to the homogeneous medium case given by

\[
\phi = \frac{k_0^2}{2k_z} \int_0^z 1 - \mu(z') \, dz'.
\]

Note that for a homogeneous medium, then \( \mu = 1 \), and the uniform case [Eq. (3)] is recovered. Equation (12) may be thought of as accumulation of phase shifts incurred as the wave travels through an infinitesimal width \( dz \), i.e.,

\[
\phi = \int \, d\phi \quad \Rightarrow \quad d\phi = \frac{k_0^2}{2k_z} (1 - \mu) \, dz.
\]

Since it was required that \( c'/c_0 \) is small, \( \mu(z) = (1 + c'/c_0)^{-2} \) can be expanded so that Eq. (13) becomes

\[
d\phi \approx \frac{k_0^2}{2k_z} \left[ 1 - \left( 1 - 2\frac{c'}{c_0} \right) \right] \, dz
\]

\[
\approx \frac{k_0^2}{2k_z} \left( 2\frac{c'}{c_0} \right) \, dz = \left( \frac{c'}{c_0}k_0 \right) \left( \frac{k_0}{k_z} \right) \, dz.
\]

The term \( (c'/c_0)k_0 \) has the form of an effective wavenumber, accounting for the dilation of contraction of the wavelength due to the difference in sound speed from \( c_0 \). The second term \( (k_0/k_z) \, dz \) is the distance between the two infinitesimally separated planes for a plane wave traveling with propagating wavenumber \( k_z \). The extra phase then has a the familiar form \( \phi \sim k_{eff}d \).

### 2.1.2 Comparison of Results

We have shown in a previous work[13] that a numerical marching scheme may be applied to obtain an approximate solution to Eq. (4) in the case of general heterogeneity

\[
P^{n+1} \approx P^n e^{ikz\Delta z} + \frac{e^{ikz\Delta z}}{2ikz} (P^n * \Lambda) \times \Delta z.
\]

To compare with the analytical result in the case of a stratified medium, Eq. (10) can be re-written with an expansion of the exponential term (see below) to give

\[
P \approx P_0 e^{ikz} \left[ 1 + \frac{1}{2ikz} \int_0^z \lambda(z') \, dz' + \ldots \right].
\]

Retention of first order terms and approximation of the integral as a left Riemann sum gives

\[
P^{n+1} \approx P^n e^{ikz\Delta z} + \frac{e^{ikz\Delta z}}{2ikz} P^n \lambda(z) \Delta z + \mathcal{O} \left[ (\Delta z)^2 \right].
\]
Figure 1: Correction of aberration with phase correction. (a) Emissions from an acoustic source are in a stratified medium are recorded with a receiver array. (b) Use of conventional ASA beamforming results in aberration and error in source localization. (c) Use of phase corrections $\phi$ computed from the sound speed field $c(z)$ reduces the error.

In the stratified medium case, $\Lambda \ast P = \lambda P$, so that Eq. (15) agrees with Eq. (17) to $O[(\Delta z)^2]$. This is expected as the first-order solution of Eq. (7) was used.

Use of the truncated expansion in Eq. (16) requires that

$$\frac{1}{4} \left( \frac{k_0}{k_z} \right)^2 \left[ k_0 \int_0^z (1 - \mu) \, dz \right]^2 \ll 1.$$ (18)

In the far field (e.g., for PAM), the paraxial approximation dictates that first term is of order 1. Equation (18) is true then if $\mu \equiv (c_0/c)^2 \approx 1$, i.e., for relatively weak heterogeneity, which was assumed for use of the first-order solution of Eq. (7).

### 2.2 Simulations and Numerical Implementation

To determine the improvement in source localization, acoustic sources were simulated in k-Wave. For computational efficiency, simulations were performed in 2D, and thus in the simulations and reconstructions, $y = k_y = 0$. The PAM reconstruction routines were written in MATLAB and computation times are reported for a standard desktop computer (Intel Core i7, four cores at 2.8 GHz and 16 GB memory); no parallel or graphical processing techniques were employed. Sources were simulated as Gaussian pulses with 5% bandwidth. The resulting pressure was measured by a virtual linear array, the RF data were then beamformed to compute $P(x, z)$ with Eq. (3) or Eq. (10), for the uncorrected and corrected cases respectively, and the intensity field was then calculated from

$$I(x, z) = \sum_\omega \| F_k^{-1} [P(k_x, z)] \|^2.$$ (19)

Maps were formed over a frequency range $\omega$ of three frequency bins, centered at the source frequency. The reconstructed source location $(x_r, z_r)$ was taken to be the position of peak intensity of $I$, and the error defined relative to the known source position $(x_{true}, z_{true})$ from the simulation

$$\epsilon = \sqrt{\epsilon_x^2 + \epsilon_z^2} = \sqrt{(x_r - x_{true})^2 + (z_r - z_{true})^2}.$$ (20)

Because $P$ is proportional to $\exp i k_z z$, back propagation incurs multiplication by $\exp -i k_z z$. Evanescent components of the angular spectrum, for which $k_z$ is pure imaginary, will then grow exponentially. Therefore, all measured
angular spectra $P_0$ were windowed with a Tukey window with cosine fraction $R = 0.25$ to taper these components. Additionally, all initial spectra were zero padded such that their computational extent was four times larger than their physical extent. Sound speed fields $c(r)$ were padded with their edge values replicated to match the grid size of the padded $P_0$, and reconstructions were performed at depth steps $\Delta z = f_0/6c_0$ (i.e., 1/6 of the reference wavelength).

For the atmospheric simulations, 1 Hz sources ($N = 209$) were simulated over altitudes from 0–10 km and beamformed with a 6 km array. The medium density and sound speeds were defined from the 1972 US Standard Atmosphere and the altitude- and frequency-dependent attenuation were taken from Ref. [19]. For underwater simulations, 1 kHz sources ($N = 70$) were performed for depths to 300 m and beamformed with a 190 m array. The sound speed profile was taken from Ref. [20] at 0.5$^\circ$N and 100.5$^\circ$W, with attenuation defined from an empirical frequency-dependent model. In the biomedical-scale applications, a mean sound speed of 1540 m/s was augmented by a 25% Gaussian profile with variance of 30 mm. A mean density of 1043 kg/m$^3$ and attenuation of 0.54 dB/cm/MHz were defined for the entire medium, and $N = 99$ sources were simulated at 1 MHz. In all cases, a perfectly matched layer of at least two wavelengths was used at all boundaries (with the number of points chosen to improve the simulation efficiency).

3 Results

3.1 Biomedical Range

Figure 2 demonstrates the improvement in source localization due to the phase correction for 1 MHz sources. Without the phase correction, the error was 2.05 ± 1.00 mm, while with the phase correction it was 0.97 ± 0.20 mm. The error in both the corrected and uncorrected case was principally in the axial direction ($\mid \epsilon_x, \text{avg}/\epsilon_z, \text{avg} \mid = 34.6$ and 35.6 for the corrected and uncorrected cases, respectively) and is plotted in Fig. 2(b). To understand the effect of the beamforming aperture, we beamformed and localized using three sub-apertures. For the $L = 50$ mm aperture, some localizations in the corrected case have larger errors, however for larger apertures (75 mm and 100 mm) which cover the transverse extent of the sources, the corrected localization accuracy was within approximately one half wavelength [Fig. 2(c), orange]. In the uncorrected case, (purple) the error was largest for depths near 25 mm and 75 mm. These are the extrema of $dc/dz$, and thus where discounting the medium variation is most egregious.

Finally, absolute the localization accuracy did not depend strongly on the wavelength [Fig. 2(d)]. The absolute localization error was similar for all frequencies (mean 0.91 mm, 0.97 mm, and 0.88 mm for 0.5 MHz, 1.0 MHz, and 1.5 MHz, respectively), as was the improvement relative to the uncorrected case at that frequency (mean improvement 55%, 57%, and 53%). The short wavelength criterion [Eq. (11)] was roughly met in all cases; for the longest wavelength (500 kHz), $|d^2A/dz^2| \sim 0.4$.

3.2 Underwater Range

For the simulations with data from measured underwater sound speed profile, the magnitude of the $c'/c_0$ was small—approximately 2% [Fig. 3(a)]. Accordingly, the source localization in the uncorrected case was reasonable, with errors on the order of a wavelength (1.38 ± 0.12 m compared with the wavelength of 1.5 m) over all depths [Fig. 3(b)]. The correction successfully reduced the error by approximately 5-fold, to 0.26 ± 0.08 m. The axial component comprised most of the total error in the uncorrected case ($\mid \epsilon_x, \text{avg}/\epsilon_z, \text{avg} \mid = 9.2$), however in the corrected case, the axial and transverse errors were comparable ($\mid \epsilon_x, \text{avg}/\epsilon_z, \text{avg} \mid = 1.5$)

3.3 Atmospheric Range

In the atmospheric case, the sound speed decreased approximately linearly over the altitudes considered [up to 9 km, Fig. 4(a)]. Unlike the other cases considered here, the simulation medium also had significant density variations [$\rho'/\rho_0 \sim 0.5$, Fig. 4(b)]. Over the range of positions considered, the mean source localization error was reduced from 474.4 ± 395.6 m in the uncorrected case to 219.4 ± 226.3 m with the correction, compared with a wavelength of 343 m [Fig. 4(c)].

3.4 Computational Efficiency

The time required for evaluation of Eq. [19] depends on the number of frequency bins, signal duration, and the size of the computational grid. For the cases described here, reconstruction times were on the order of tens of microseconds for the uncorrected case (three frequency bins) and on the order of 100 ms when the correction was used. In all cases, this reduced to tens to a few hundred nanoseconds per pixel in the reconstructed image.
4 Discussion

In this paper, we have presented an analytical phase correction for ASA beamforming in a stratified medium. Under the assumptions that the sound speed changes slowly compared to a wavelength and that the magnitude of the change is small, the first order solution to the governing equation may be obtained. Beamforming of passively collected data with the correction mitigates aberration caused by the constant sound speed assumption of the canonical ASA (Fig. 1). Through simulations of point source radiation at biomedical (Fig. 2), underwater (Fig. 3), and atmospheric (Fig. 4) scalings, the error was reduced by 62.6%, averaged across all cases. Collectively, the results indicate that the method has value for passive source localization at several realistic environments where such information is of considerable interest.

While the computational cost was approximately three-fold increase in computational time (mean 147 ms), this is still orders of magnitude more efficient than time-domain methods that can handle such corrections natively. The computation time could be reduced further by reducing the number of receivers (provided the spacing between remains smaller than a half wavelength i.e., the spatial Nyquist frequency), reducing the signal duration or sampling rate $f_s$ (provided that the full source signal is captured and $f_s \geq 2f_{\text{max}}$, i.e., the Nyquist limit), or by use of parallel...
computations to reconstruct the maps at each frequency. Thus the localization method extends the real-time capabilities of the ASA.

The method has a few limitations. First, the derivations required that the changes in sound speed occur over long scales (i.e., $|\frac{dc}{dz}/c|\) is large compared to the wavelength) and with relatively small magnitudes (i.e., $|c/c_0|\sim 1$). Thus discrete impedance changes (e.g., layers) violate these assumptions (however such cases may be treated with a layer-by-layer approach\cite{2})). Second, only forward propagation is included, such that reflections (e.g., from the ground or water surface) are not accounted for.

5 Conclusion

Here, we derived an analytical, first-order correction to the angular spectrum approach for media with stratified material properties. This method extends the ability of the inherently method to localize sources at subwavelength accuracy with little additional computational burden.

Acknowledgements

Work supported by NIH Grant R00EB016971 (NIBIB) and NSF Grant 1830577 (LEAP HI).

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Figure 4: Atmospheric Scaling (a) Arrangement of sources (black circles) relative to the virtual sensor array and stratified sound speed (grayscale and plot at right; note $z$-direction is upward in this figure). (b) Vertically-dependent medium density used in the simulations. (c) Total radial error for the corrected (left) and uncorrected (center) cases. At right are shown the mean (line) and standard deviation (shaded region) of the errors at each depth $z$ averaged over all transverse positions $x$ for the corrected (orange) and uncorrected (purple) cases with the indicated aperture. For reference the wavelength of the 1 Hz signal is indicated at the dotted line.

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