Calculation of CP Violation in Non-leptonic Kaon Decay on the Lattice

J. Noaki for RBC Collaboration

RIKEN BNL Research Center, Bldg. 510A, Brookhaven National Laboratory, Upton NY, 11973

Abstract. We give a progress report of our lattice calculation of direct and indirect CP violation in kaon decays, parametrized as $\varepsilon'/\varepsilon$ and $B_K$, which require non-perturbative calculation of the matrix elements of the Standard Model effective Hamiltonian.

INTRODUCTION

In the investigations of the Standard Model, a very important issue is the theoretical treatment of $K \to \pi\pi$ decay to the accuracy such that comparison with the experimental results is possible. In particular, the ratio of direct and indirect CP violation, $\varepsilon'/\varepsilon$, has been determined experimentally in recent years and theoretical calculation is desired to test Kobayashi-Maskawa theory. In the theoretical calculation, numerical simulation of lattice QCD is the most systematic method to estimate the non-perturbative effect of QCD which is the main source of the error. Using the operator product expansion, the interaction in this decay is written as $H_W = \frac{G_F V_{ud}}{\sqrt{2}} \sum_i W_i(\mu) Q_i$, where the coefficients $W_i$ contain the effects of the energy scales higher than the matching point $\mu$ and can be obtained perturbatively. Non-perturbative QCD effects will appear in $K \to \pi\pi$ matrix elements of the local operators $\langle \pi\pi | Q_i | K \rangle$, which should be calculated on the lattice. A couple of years ago, CP-PACS and RBC Collaboration calculated all of the matrix elements using the domain-wall fermion formalism to realize the chiral symmetry required in this calculation and reported small and negative values of $\varepsilon'/\varepsilon$ in conflict with the experimental result. Another work using staggered fermion has obtained a larger negative value. In these works, however, there are several uncontrolled systematic errors such as 1) the effect of the small, but non-zero, chiral symmetry breaking, 2) the effect of finite lattice spacing, 3) the effect of the perturbative treatment of the charmed quark in the matrix elements, 4) quenching effect, and 5) $K \to \pi\pi$ matrix elements are obtained from $K \to \pi$ and $K \to 0$ (vacuum) by using lowest order chiral perturbation theory.

In order to examine all of these systematic errors except the fifth one, we are performing two types of numerical simulation with domain-wall fermion and the DBW2 gluonic action to improve the chiral symmetry on the lattice. “Numerical Simulation I” is the quenched calculation including directly the effect of the charm quark on the lattice. The degree of chiral symmetry breaking is decreased by a factor 1/10 compared with the previous work of RBC Collaboration. In addition, we are generating gauge configurations with $N_f = 2$ dynamical quarks in “Numerical Simulation II.” In the rest of this
article, we present the contents of these numerical simulations and report preliminary results of the matrix elements which numerically dominate $\varepsilon'/\varepsilon$ and kaon B-parameter $B_K$.

**NUMERICAL SIMULATION I**

We are generating gauge configurations on a relatively fine $24^3 \times 48$ lattice with the scale $a^{-1} = 2.86(9)$. The residual quark mass $m_{\text{res}}$ which measures the chiral symmetry breaking is as small as $\lesssim 0.3$ MeV. Since quark mass $m_f$ is introduced as a parameter of the boundary condition in the fifth dimension in domain-wall QCD, the localization of chiral modes on domain-walls in the fifth dimension tends to fail for a heavy quark mass $m_f$. However, our small lattice spacing made the value of $m_c a$ acceptable as a domain-wall fermion: $m_c a \approx 0.45$. We found that, around this value, the behavior of wave function in the fifth dimension is qualitatively same as the case of much smaller quark mass.

At the lowest order of chiral perturbation theory, $K \to \pi\pi$ matrix elements are in proportion to $K \to \pi$ matrix elements calculated on the lattice. For $i = 1 - 6, 9, 10$, these matrix elements are related as,

\begin{equation}
\left\langle \pi^+ \pi^- | Q_i^{(f)} | K^0 \right\rangle = \frac{m_K^2 - m_\pi^2}{f_\pi^2} \left\langle \frac{1}{m_{PS}} \pi^+ | Q_i^{(f)} | K^0 \right\rangle + O(p^2),
\end{equation}

in particular, for $\Delta I = 1/2$, or $I = 0$, subtraction of a lower dimension operator is needed:

\begin{equation}
\left\langle \pi^+ | Q_i^{(0)} | K^+ \right\rangle \right|_{\text{subt}} = \left\langle \pi^+ | Q_i^{(0)} - \alpha_i Q_{\text{sub}} | K^+ \right\rangle, \tag{2}
\end{equation}

\begin{equation}
Q_{\text{sub}} = (m_s + m_d) \bar{s}d - (m_s - m_d) \bar{s}\gamma_5 d, \quad \alpha_i = \left\langle 0 | Q_i^{(0)} | K^0 \right\rangle / \left\langle 0 | Q_{\text{sub}} | K^0 \right\rangle \tag{3}
\end{equation}

For $i = 7, 8$, we have the simpler relation

\begin{equation}
\left\langle \pi^+ \pi^- | Q_i^{(f)} | K^0 \right\rangle = -\frac{1}{\sqrt{2} f} \left\langle \pi^+ | Q_i^{(f)} | K^0 \right\rangle + O(p^2). \tag{4}
\end{equation}

In particular, $Q_6^{(0)}$ and $Q_8^{(2)}$ have the largest contribution to $\varepsilon'/\varepsilon$, numerically. In FIG.11 results of $K \to \pi$ matrix elements of these operators are plotted. In particular, the left panel, which is the example with $m_c a = 0.40$, shows that there is a severe cancellation in the subtraction in (2) for $Q_6^{(0)}$. Since the slope of the subtracted matrix elements have the error of $\sim 200\%$, we cannot quote a result of $K \to \pi\pi$ matrix element with the current statistics. And its dependence on $m_c a$ is not visible, so far.

Lattice value of kaon B parameter which is defined by

\begin{equation}
B_K = \frac{\left\langle K | Q_{\Delta S=2} | K \right\rangle}{8/3 \left\langle K | A_\mu | 0 \right\rangle \left\langle 0 | A_\mu | K \right\rangle}, \tag{5}
\end{equation}
FIGURE 1. $\mathcal{K} \to \pi$ matrix element of $Q_{6}^{(0)}$ (left) and $Q_{6}^{(2)}$ (right) as a function of $m_f a$ from Numerical Simulation I. In the left panel, data for matrix elements before (circle) and after (diamond) the subtraction and the subtraction term $-\alpha_6 \langle \pi | Q_{\text{sub}} | K \rangle$ (square) are plotted from 50 statistics. Linear extrapolation was used for all of plots.

FIGURE 2. Lattice value of $B_K$ as a function of $m_f a$ from 77 configurations. The fit function used is $B_K = \xi_0 [1 + C m_f a \ln (m_f a)] + \xi_1 m_f a$ with $C$ taken from analytic result [11]. The physical result for $B_K$ can be obtained at $m_f = m_s / 2$ (the filled symbol). To obtain the physical value, we are now calculating the $Z$ factor for $B_K$ by non-perturbative renormalization proposed in [12] and the preliminary result is roughly consistent with the previous works [13, 4].

NUMERICAL SIMULATION II

Since the dynamical simulation demands much more resources than a quenched one, dynamical domain-wall QCD has been explored by only our collaboration [10], so far.
In this calculation, we generated three kinds of gauge configuration on a $16^3 \times 32$ lattice with the mass of the sea quark ($u$ or $d$ quark) being $m_{\text{sea}}a = 0.02, 0.03$ and 0.04. For each series of configurations, $K \to \pi$ and $K \to 0$ matrix elements are calculated in the same way as Simulation I with the five valence quark masses $m_{\text{val}} = 0.01–0.05$, and basic parameters $a^{-1} \approx 1.8$ GeV and $m_{\text{res}} \approx 3$ MeV are obtained. FIG. 3 shows the same matrix elements as in FIG. 1 as an example of the case of $m_{\text{sea}} = 0.03$. Although the signal seems to be reasonable, we need much more statistics to take correct chiral limit $m_{\text{sea}} = m_{\text{val}} \to 0$ using three data points with $m_{\text{sea}}a = m_{\text{val}} = 0.02, 0.03$ and 0.04. $B_K$ at the physical point such that $m_{\text{sea}} = m_{u/d}$ and $m_s \sim 120$ MeV will be also obtained after a careful treatment of our data and non-perturbative renormalization which is now under calculation.

We thank RIKEN, BNL and the U.S. DOE for providing the facilities essential for the completion of this work.

REFERENCES

1. Alavi-Harati, A. et al., Phys. Rev. Lett. 83, 22, 1999; Fanti, A. et al., Phys. Lett. B465, 335, 1999.
2. For a review, see Buchalla, G., Buras, A. J., Leutenbacher, M E., Rev. Mod. Phys 68, 1125, 1995.
3. CP-PACS Collaboration, Noaki, J. et al., Phys. Rev. D68, 014501, 2003.
4. RBC Collaboration, Blum, T. et al., hep-lat/0110075.
5. Kaplan, D., Phys. Lett. B288, 342, 1992.
6. Shamir, Y., Nucl. Phys. B406, 90, 1993; Furman, V. and Shamir, Y., Nucl. Phys. B439, 54, 1995.
7. Pekurovsky, D. and Kilcup, G., Phys. Rev. D64, 074502, 2001.
8. Bernard, C., Draper, T., Soni, A., Politzer, H. D. and Wise, M. B., Phys. Rev. D32, 2343, 1985.
9. RBC Collaboration, Aoki, Y. et al., hep-lat/0211023.
10. RBC Collaboration, Izubuchi, T. et al., hep-lat/0210011.
11. Sharpe, S., Phys. Rev. D46, 3146, 1992.
12. Martinelli, G. et al., Nucl. Phys. B445, 81, 1995.
13. CP-PACS Collaboration, Ali Khan, A. et al., Phys. Rev. D64, 114506, 2001.
14. Golterman, M. and Leung, K-C., Phys. Rev. D57, 5703, 1998.