A mathematical model of switched-reluctance motor drive for electrical spindle in synthetic yarn machine

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Abstract. The article deals with the modeling of switched-reluctance drive motor for electrical spindle in synthetic yarn machine. We propose to use the backward differentiation formula to solve a system of algebraic-differential equations that governs electromagnetic processes in switched-reluctance electrical machine. The considered approach allows us to avoid the use of numerical differentiation of current and turning angle dependence of the phase flux linkage. As a result, the mathematical model does not significantly depends on basic data and has good convergence. This article shows the switched-reluctance electrical machine modeling. The use of method proves the correctness of the chosen approach.

1. Introduction

The production of composite materials of synthetic yarn is one of the most important areas of modern technologies. Synthetic yarns properties largely determine the characteristics of composite materials made of them. Therefor the improvement of synthetic yarn quality is the crucial task. One of the way to solve this problem is the spinning machines automation. The tension uniformity during winding has a significant effect on the yarn quality. We must use an adjustable electric drive to achieve the required quality of winding.

The switched-reluctance electrical machine is the most appropriate as an electrical spindle drive in synthetic yarn spinning machines. Its design simplicity and precise torque and speed control are its main advantages. The switched-reluctance electrical machine consists on electromechanical converter and power converter. Principles and methods of control have a decisive influence on the characteristics of such a machine. To realize all the benefits of switched-reluctance electrical machine, we need to optimize the active layer configuration of the electromechanical converter, laws, algorithms and parameters of control system. For this purpose, it is necessary to use the mathematical model of switched-reluctance electrical machine. This model must allow analysis of electromagnetic and electromechanical processes in an electromechanical converter and a power converter, determination of the parameters and characteristics of switched-reluctance electrical machine with various algorithms and control parameters.

The determination of current pulses parameters and control system testing at the stage of designing are the most difficult tasks of electric drives creation. This article describe the model that allows...
solving these tasks and takes into account the design and control system particularities of the switched-reluctance electrical machine.

2. Problem statement.
The use of models based on field theory allows us to receive the most adequate information about electromagnetic processes in the electric machine under consideration. The disadvantages of such models is a calculation complexity. Therefore, in most cases, the mathematical model uses the approaches based on the circuit theory [1-5]. The equation below describes the phase processes:

\[ \frac{d\Psi_f}{dt} = U_f - i_f R_f, \]  

where \( \Psi_f, i_f, U_f \) are flux linkage, phase current and applied voltage; \( R_f \) is phase resistance.

The flux linkage depends on phase current and the stator and rotor relative position:

\[ \Psi_f = f(i_f, \gamma_r). \]  

The joint solution of equations 1 and 2 models switched-reluctance electrical machine. The switched-reluctance electrical machine has a significant magnetic saturation [5]. The saturation is especially manifested when the rotor and phase are in a consistent position and when the stator and rotor teeth axis match. The characteristics phase flux linkage versus phase current is non-linear.

Equations (1) and (2) are the system of nonlinear algebraic-differential equations, which is difficult to solve directly. Therefore, the flux linkage is the product of phase current and phase inductance [1, 5].

Then, the phase flux linkage derivative is:

\[ \frac{d\Psi_f}{dt} = \frac{\partial\Psi(i_f, \gamma)}{\partial i} \cdot \frac{di_f}{dt} + i_f \omega \cdot \frac{\partial\Psi(i_f, \gamma)}{\partial \gamma}, \]  

where \( \omega = \frac{dy}{dt} \) is rotor rotational speed.

This change allows us to go to the solution of the phase current differential equation.

The expression (3) consist of derivatives of phase flux linkage versus current and rotor rotation angle. Dependencies (2) are currently determined using field theory methods. The phase weber-ampere characteristics are calculated at different rotor positions. The interpolation methods is used to determine the flux linkage at intermediate values of current and rotor position. When differentiating the resulting dependencies, significant errors may appear.

Another factor cumbering the switched-reluctance electrical machine modeling using expression (3) is the fact that a voltage in the form of rectangular pulses with a steep edge is applied to the phase. The edge has breaks of the first kind. It renders difficult to apply multistep methods for solving differential equations.

For the indicated reasons, the modeling time significantly increases when using such a software as MatLAB Simulink and in some cases, the solving becomes impossible due to problems of numerical methods convergence [1]. Therefore, when switched-reluctance electrical machine modeling, we must not use the numerical determination of phase linkage derivatives versus current and relative position of the stator and rotor.

3. Switched-reluctance electrical machine mathematical model.
A switched-reluctance electrical machine consist on the mechanical converter which is synchronous reluctance machine with concentrated tooth windings on the stator and tooth rotor without winding. Unipolar current pulses provide power supply of the switched-reluctance electrical machine. Phases are not galvanically connected to each other. The current pulses are formed by individual, autonomous half-bridge voltage inverters, that are connected to the common DC link.

Each phase module of a power converter consists of two power keys (S1 and S2, S3 and S4, S5 and S6) and two diodes (D1 and D2, D3 and D4, D5 and D6). Figure 1 shows the structural-functional diagram of the switched-reluctance electrical drive for electric spindle.
When mathematical modeling of the switched-reluctance electrical drive we accept the following assumptions:

- phases in switched-reluctance electrical machine are not magnetically coupled, we consider independently all processes in each of them;
- power semiconductor devices in power converter present key elements with great resistance in off state, and with low resistance in on-state; state transition is prompt;
- constant voltage source power a switched-reluctance electrical machine, in the DC link we take into account the E.M.F., active resistance, inductance and capacitance;
- design model topology does not change during calculations.

Figure 2 shows the design model of the switched-reluctance electrical drive for electric spindle.

The equation system (1) and (2), that describe electromagnetic processes in phase of a switched-reluctance electrical machine, can be solved by Backward Differentiation Formula (BDF). In this case, the solution of the algebraic-differential equations system comes down to the consistent solution of an algebraic equations system. When using the first order BDF, phase flux linkage is as follows:

\[
\frac{d\Psi_f}{dt} = \frac{\Psi_{f_{k+1}} - \Psi_{f_k}}{\Delta t},
\]

where \(\Psi_{f_{k+1}}, \Psi_{f_k}\) is the stator phase flux linkage at time moment \(t_{k+1}\) and \(t_k\); \(\Delta t\) is interval of time.

Then the equations (1) and (2) can be as follows:

\[
\begin{align*}
\frac{1}{\Delta t} \Psi_{f_{k+1}} &= U_{f_{k+1}} - R_f i_{f_{k+1}} + \frac{1}{\Delta t} \Psi_{f_k}; \\
\Psi_{f_{k+1}} &= L_f i_{f_{k+1}},
\end{align*}
\]

where \(U_{f_{k+1}}, i_{f_{k+1}}\) are the applied voltage and phase current at time moment \(t_{k+1}\); \(R_f\) is the phase active resistance;

\[L_f = \frac{\Psi_f(l_f)}{i_f}\]

is the phase static inductance at current rotor position.

Provided that the switched-reluctance electrical machine magnetic system is saturated, the equation system (5) is non-linear, since the value of static inductance depends on the phase current. We used the iterative method with the forecast – correction method to solve the equation system in each modeling stage. At the beginning of the iterative process, we use the static inductance value that was received in previous modeling stage. Then goes the forecast that means the equation system (5) solving. We use the obtained current value to specify the inductance value. Then we use the specified inductance value to correct and resolve the equation system (5). The correction stage can be performed several times to achieve the required accuracy.

The power semiconductor devices of power converter define the switched-reluctance electrical machine phase voltage. It can be defined as follows:
where \( c_f \) is phase switching function and \( U_{dc} \) is capacitor voltage in the DC link.

The control of switched-reluctance electrical machine is carried out by the formation in phases of unipolar current pulses. After the set current becomes nonzero, both phase power keys are in on-state, the positive voltage is applied to the phase and the switching function takes on value “1”. After the measured phase current reaches the set value, one of the power keys is at the off state and the phase is shorted through the other power key in on-stage and a diode, phase current decreases. The switching function takes the value "0". After the current reducing to the lower permissible value, both power keys are in on-stage and the cycle repeats. When the current becomes zero, the power keys are in on-stage, and the phase gives the accumulated energy through the diodes to the capacitance in the DC link. The switching function has a value of “0”.

The equations (5) can be presented in the form of matrices

\[
\frac{\text{d}}{\text{d}t} \begin{bmatrix} \Psi_{f k+1} \\ i_{f k+1} \end{bmatrix} = \begin{bmatrix} R_f & -L \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Psi_f \\ i_f \end{bmatrix} + \begin{bmatrix} c_f U_{dck+1} \\ 0 \end{bmatrix},
\]

Equations (6) are the algebraic-linear equations system.

The following equations describe processes in a DC link:

\[
\begin{align*}
C_{dc} \frac{\text{d}U_{dc}}{\text{d}t} &= i_{dc} - \sum_{m=1}^{3} c_f m l_f m; \\
L_{dc} \frac{\text{d}i_{dc}}{\text{d}t} &= E_{dc} - R_{dc} i_{dc} - U_{dc}.
\end{align*}
\]

After using the BDF, formulas and transformations to the matrix form of the equation (7) are as follows:

\[
\begin{bmatrix}
\frac{\text{d}C_{dc}}{\text{d}t} & -1 \\
1 & R_{dc} + \frac{i_{dc}}{\text{d}t} 
\end{bmatrix} \begin{bmatrix} U_{dck+1} \\ l_{dck+1} \end{bmatrix} = \begin{bmatrix}
\frac{\text{d}C_{dc}}{\text{d}t} & 0 \\
0 & \frac{\text{d}i_{dc}}{\text{d}t} 
\end{bmatrix} \begin{bmatrix} U_{dck} \\ i_{dck} \end{bmatrix} + \begin{bmatrix} \sum_{m=1}^{3} c_f m l_f m k \\ E_{dc} \end{bmatrix},
\]

Equations (6), completed with the equations (8) describe the electromagnetic processes in the switched-reluctance electrical machine

4. Electric spindle motor drive model

We designed the one of the variant of the spinning machine electric spindle motor. Figure 3 shows the magnetic system drawing.

There is a three-phase switched-reluctance electrical machine with two pairs of poles. In has 12 teeth on the stator and 8 teeth on the rotor. The stator outer diameter is 100 mm, the rotor inner diameter is 30 mm, the clearance diameter is 60 mm, the air gap is 0.2 mm, the active core length is 100 mm. Each winding has 20 turns.

We determined the phase flux linkage versus the current at different rotor position angles. For this purpose, we calculated the distribution of the magnetic field in the magnetic system using the FEMM software package [6]. Figure 4 shows the magnetic field distribution in the computational space. Figure 5 shows the phase flux linkage and electromagnetic moment versus the stator phase current at different rotor position angles.
Figure 4. The magnetic field distribution in the computational space of the switched-reluctance electrical machine.

We simulated processes in the switched-reluctance electrical machine. Fig. 6 and 7 shows the results of simulation, the dependence of the phase current, current in the DC link, the switched-reluctance electrical machine electromagnetic moment created by one phase and the total electromagnetic moment.

Figure 5. The phase flux linkage (a) and electromagnetic moment (b) versus current at different rotor position angles.

Figure 6. The simulation results: phase current and current in the DC link

The analysis of the obtained results shows that the proposed modeling approach provide a good convergence and stability of the model. With a selected simulation step of 100 μs we needed only 2 or 3 μs in each stage of modeling to achieve the required accuracy. The described model can be used in such modeling system as MatLAB Simulink.
Figure 7. The simulation results: Electromagnetic moment

5. Conclusions.
The spinning machines automation is one of main ways of improving the synthetic yarn quality. The application of controlled individual electric drive of spinning machine spindles regulates winding. For this purpose, the use of switched-reluctance electrical machine is efficient. At the design stage, the use of mathematical model can help to optimize the design and determine the control system settings. The use of BDF allows us to reduce the impact of errors when determining the phase flux linkage versus phase current and rotor position. We defined that the use of first order BDF formulas is the most appropriate for this task. The model received by using such an approach has a good convergence and a high operation speed.

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