Lepton anomalous magnetic moments – a theory update

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Standard Model contributions to the electron, muon, and tau lepton anomalous magnetic moments, \(a_l = (g_l - 2)/2\), are reviewed and updated. The fine structure constant is obtained from the electron \(g_e - 2\) and used to refine the QED contribution to the muon \(g_\mu - 2\). Recent advances in electroweak and hadronic effects on \(g_\mu - 2\) are summarized. Examples of “New Physics” probed by the \(a_\mu\) Brookhaven experiment E821 are outlined. The prediction for \(a_\tau\) is also given.

1. Introduction and summary

During the past few years, there has been growing interest in leptonic anomalous magnetic moments. On the experimental side, the extraordinary measurements of \(a_e \equiv (g_e - 2)/2\) at the University of Washington currently provide the best determination of the fine structure constant, \(\alpha\), when compared with theoretical predictions:

\[
\alpha^{-1} = 137.035 \, 999 \, 59(38)(13).
\]

Similarly, a new effort in progress at Brookhaven National Lab (Experiment E821) aims to improve the measurement of \(a_\mu\) by a factor of 20 or better. Although not competitive with \(a_e\) in precision, \(a_\mu\) is much more sensitive to electroweak loop effects as well as “New Physics” which give contributions \(\sim m_\tau^2\), i.e. \(4 \times 10^4\) enhancement in \(a_\mu\) relative to \(a_e\). Besides being able to observe the electroweak loop corrections to \(a_\mu\) predicted by the Standard Model, E821 is capable of detecting the presence of “New Physics.” For example, supersymmetry loop effects can potentially provide a large contribution to \(a_\mu\). If a significant deviation from Standard Model expectations is observed, supersymmetry is likely to provide the leading candidate explanation.

To exploit experimental progress requires detailed calculations of the Standard Model contributions to \(a_e\) and \(a_\mu\). QED computations at the 4 and 5 loop level, hadronic vacuum polarization and other loop effects, 1 and 2 loop electroweak effects, and “New Physics” contributions must be thoroughly scrutinized and refined. That effort challenges our theoretical tools and abilities. It provides an important synergism between theory and experiment.

| \(a_{\mu}\) (Theory) | \(a_{\mu}\) (Experiment) |
|----------------------|-------------------------|
| 0.001 159 652 153 5(240) | 0.001 159 652 188 4(43) |
| 0.001 159 652 187 9(43) | 0.001 159 652 187 9(43) |
| 0.001 165 915 96(67) | 0.001 165 923 50(730) |
| 0.001 176 9(4) | 0.004 35 |

In this paper we update some of the recent progress in theoretical calculations of \(a_e\) and \(a_\mu\) and thereby provide a status report (see Table 1). In the case of \(a_\mu\), we briefly describe a few examples of the “New Physics” sensitivity of E821 underway at Brookhaven. Comparison of the present experimental results with the Standard Model prediction gives the following 95% C.L. bound on “New Physics” contributions to...
the muon anomalous magnetic moment
\[-710 \times 10^{-11} < a_\mu^{\text{New Physics}} < 2210 \times 10^{-11}.\] (2)

For completeness, we also give an updated prediction for \(a_e\), even though an experimental measurement of that quantity is far from current capabilities.

2. Electron

To match the present experimental precision one has to include the following QED contributions to \(a_e = (g_e - 2) / 2\)

\[a_e^{\text{QED}} = \sum_{n=1}^{4} A_n \left( \frac{\alpha}{\pi} \right)^n + [B_2(e, \mu) + B_2(e, \tau)] \left( \frac{\alpha}{\pi} \right)^2 + B_3(e, \mu) \left( \frac{\alpha}{\pi} \right)^3 \] (3)

where \(B_n(l, l')\) describe the contributions of loops containing lepton \(l'\) to \(a_l\), while \(A_n\) contain pure QED contributions [11-14]:

\[A_1 = \frac{1}{2} \]
\[A_2 = \frac{3}{4} \zeta_3 - \frac{\pi^2}{2} \ln 2 + \frac{\pi^2}{12} + 197 + 63(3)\]
\[
\approx -0.3284789656
\]
\[A_3 = \frac{83}{72} \zeta_3 - \frac{215}{24} \zeta_3 - \frac{239}{2160} \pi^4 + \frac{139}{18} \zeta_3 - 2520 \ln (\frac{m_\mu}{m_e}) + \frac{25}{9} \pi^2 \ln 2 + \frac{17101}{810} \pi^2 + \frac{28259}{5184} \]
\[
\approx 1.811245
\]
\[A_4 = -1.5098(384) \] (4)

and

\[B_2(e, \mu) = 5.197 \times 10^{-7} \]
\[B_2(e, \tau) = 1.838 \times 10^{-9} \]
\[B_3(e, \mu) \approx -7.3739 \times 10^{-6}. \] (5)

\(B_2(e, l)\) describe loops with the lepton \(l\) inserted in the Schwinger diagram; they are calculated using

\[B_2(e, l) = \frac{1}{3} \int_{4m_l^2}^{\infty} ds \sqrt{s - 4m_l^2} \frac{s + 2m_l^2}{s^2} \]

\[\times \int_0^1 \frac{x^2(1-x)}{x^2 + (1-x)m_e^2} dx \] (6)

\(B_3(e, \mu)\) is a sum of three groups of diagrams [10-12]:

\[B_3(e, \mu) = A^{(4,2)}(m_\mu/m_e) + B^{(2,4)}(m_\mu/m_e) + B_3^{\gamma}(e, \mu). \] (7)

They describe, respectively, diagrams with either two muon loops or photon corrections within a single muon loop; and the remaining diagrams with a single muon loop (they contain either a photon not attached to the muon loop or electron loop). Finally, \(B_3^{\gamma}(e, \mu)\) is a contribution of light-by-light scattering with a muon loop [12]. Their numerical values are

\[A^{(4,2)}(m_\mu/m_e) + B^{(2,4)}(m_\mu/m_e) \approx \]

\[\left( \frac{m_e}{m_\mu} \right)^2 \left[ \frac{23}{135} \ln \left( \frac{m_\mu}{m_e} \right) - \frac{2}{45} \pi^2 + \frac{10117}{24300} \right] \]

\[+ \left( \frac{m_\mu}{m_e} \right)^4 \left[ \frac{19}{2520} \ln^2 \left( \frac{m_\mu}{m_e} \right) - \frac{14233}{132300} \ln \left( \frac{m_\mu}{m_e} \right) + \frac{49}{768} \zeta_3 - \frac{11}{495} \pi^2 \right] \]

\[+ \frac{2976691}{296352000} \approx -0.000021768 \] (8)

and

\[B_3^{\gamma}(e, \mu) \approx \left( \frac{m_e}{m_\mu} \right)^2 \left[ \frac{3}{2} \zeta_3 - \frac{19}{16} \right] \]

\[+ \left( \frac{m_\mu}{m_e} \right)^4 \left[ -\frac{161}{810} \ln^2 \left( \frac{m_\mu}{m_e} \right) - \frac{16189}{48600} \ln \left( \frac{m_\mu}{m_e} \right) + \frac{13}{18} \zeta_3 - \frac{161}{9720} \pi^2 \right] \]

\[- \frac{831931}{972000} \approx 0.0000143945 \] (9)

The hadronic contributions arise from vacuum polarization insertion in the Schwinger diagram [13], in the two-loop QED diagrams [14], and from the hadronic light-by-light diagram, estimated as \(a_e^{\text{had}}(\text{light-by-light}) \approx m_e^2 / m_\mu^2\); the result is

\[a_e^{\text{had}} = 1.63(3) \times 10^{-12}. \] (10)

The electroweak contribution up to two loops is [15]

\[a_e^{\text{EW}} = 0.030 \times 10^{-12}. \] (11)
In total, the current Standard Model prediction for $a_e$ is given by

$$a_e = 0.5 \frac{\alpha}{\pi} - 0.328478444 \left( \frac{\alpha}{\pi} \right)^2 + 1.181234 \left( \frac{\alpha}{\pi} \right)^3 - 1.5098 \left( \frac{\alpha}{\pi} \right)^4 + 1.66 \times 10^{-12}$$

where the last term consists of the electroweak and hadronic contributions, whose dependence on $\alpha$ can be neglected for the purpose of $\alpha$ determination at the present level of accuracy. The QED part of the expansion appears to converge very well, with alternating signs and no considerable growth of the coefficients. We can use the average of the latest experimental results for $a_e$. 

$$a_e^{\exp} = 1159652188.4(4.3) \times 10^{-12},$$

$$a_e^{\exp} = 1159652187.9(4.3) \times 10^{-12},$$

(13)

to deduce a value of $\alpha$:

$$\alpha^{-1} = 137.035 \text{ 999 59(38)(13)}$$

(14)

where the first error comes from the uncertainty $\Delta a_e^{\exp} = 4.3 \times 10^{-12}$ (dominated by systematic cavity effects), and the second from the theoretical uncertainty in $A_4$. The next most precise method of determining $\alpha$ is based on the Quantum Hall Effect (for a review see [17] and references therein), where one finds

$$\alpha^{-1}(qH) = 137.036 \text{ 003 70(270)}.$$  

(15)

We have used that value of $\alpha$ in obtaining the prediction for $a_e^{\text{theory}}$ in Table II. Although there is currently a small discrepancy (1.5$\sigma$) between $\alpha^{-1}(e)$ and $\alpha^{-1}(qH)$, the overall level of agreement is truly impressive and represents a triumph for QED. However, the good agreement is not a particularly severe constraint on “New Physics.” If one assumes that such effects contribute $\Delta a_e^{\text{New Physics}} \sim m_e^2/\Lambda^2$, where $\Lambda$ is the scale of “New Physics,” then a comparison of $\alpha^{-1}(e)$ and $\alpha^{-1}(qH)$ is currently sensitive to $\Lambda \lesssim 100$ GeV. To probe the much more interesting $\Lambda \sim O(\text{TeV})$ region would require an order of magnitude improvement in $a_e$ (possibly feasible; see [18] for a discussion), an improved calculation of $A_4$, and a better independent direct measurement of $\alpha^{-1}$.

The last requirement could be met by improving $\alpha^{-1}(qH)$ by 2 orders of magnitude or perhaps more likely combining the already precisely measured Rydberg constant with a better $m_e$ determination.

3. Muon

3.1. QED contribution

Because of the presence of virtual electron loops, higher order QED contributions to $a_\mu$ are enhanced in comparison with $a_e$. At present we need 5 terms of the expansion in $\alpha$:

$$a_\mu^{\text{QED}} = \sum_{n=1}^{5} C_n \left( \frac{\alpha}{\pi} \right)^n$$

(16)

with

$$C_1 = A_1 = 0.5,$$

$$C_2 = A_2 + a_1(m_e/m_\mu) + a_2(m_\tau/m_\mu) = 0.765 857 388(44),$$

$$C_3 = A_3 + C_3^{\gamma\gamma}(e) + C_3^{\gamma\gamma}(\tau) + C_3^{\text{vac. pol.}}(e) + C_3^{\text{vac. pol.}}(\tau) = 24.050 \text{ 509(2)},$$

$$C_4 = A_4 + 127.55(41) = 126.04(41),$$

$$C_5 = 930(170).$$

(17)

where $a_{1,2}$ are given in [19,20]. Taking $m_\mu/m_e = 206.768273(24)$ and $m_\tau = 1777.05(26)$ MeV, $m_e = 0.51099907(15)$ MeV we find

$$a_1(m_e/m_\mu) = 1.094258294(37),$$

$$a_2(m_\tau/m_\mu) = 0.000078059(23).$$

(18)

For the evaluation of $C_2$ the errors in $a_{1,2}$ have been added in quadrature.

In $C_3$ we have contributions from light-by-light scattering diagrams with $e$ and $\tau$ loops, and vacuum polarization diagrams with either $e$, or $\tau$, or both types of loops. For light-by-light we use the formulas of [12]. For vacuum polarization we use [1], with exception of the mixed $e-\tau$ diagram which we evaluate numerically using the kernel from [14]. With updated values of $m_\mu/m_e$ and $m_\tau$ we find

$$C_3^{\gamma\gamma}(e) = 20.9479246(7)$$
\[
C_3^\gamma(\tau) = 0.0021428(7), \\
C_3^{\text{vac. pol.}}(e) = 1.9204551(2), \\
C_3^{\text{vac. pol.}}(\tau) = -0.0017822(4), \\
C_3^{\text{vac. pol.}}(e, \tau) = 0.0005276(2) \quad (19)
\]

Even adding all errors in \( C_3 \) linearly, we end up with the uncertainty 20 times smaller than the previous update \([17]\). This is because of the availability of the analytical result for \( A_3 \) \([8]\), whose error previously dominated.

For \( C_4 \) we use the difference between the muon and electron coefficients found in \([22]\), and the latest \( A_4 \) value; with errors added in quadrature we get

\[
C_4 = 127.55(41) + A_4 = 126.04(41). \quad (20)
\]

The first partial evaluation of \( C_5 \) was performed in \([23]\), where one-loop vacuum polarization insertions in the lowest order light-by-light diagram were computed, giving a contribution of about 570 to \( C_5 \). Those are the diagrams with the maximal power of \( \ln(m_\mu/m_e) \). However, these are not the only strongly enhanced diagrams. As had been shown in \([23]\) the light-by-light diagrams, in which the electron loop is connected with \( 2n + 1 \) photons to the muon loop, contains \( \pi^{2n} \ln(m_\mu/m_e) \). The numerical coefficient of such term in \( C_5 \) has been calculated in \([23]\). Some other diagrams, including two-loop vacuum polarization insertions, were estimated in \([24]\); we adopt the value of \( C_5 \) from this paper.

Some diagrams contributing to \( C_5 \) have been evaluated analytically in \([25,28]\). Other estimates of \( C_5 \) have also been made, using various methods including renormalization group \([27,29]\) and Padé approximation \([30]\). A class of contributions has even been computed to all orders \([31]\).

We now understand the ratio of the growth of the \( C_i \) coefficients describing the QED contribution to the muon \( g - 2 \). In the terms calculated so far, the characteristic increase of the coefficients is of the order of 10 for each power of \( \alpha/\pi \). The primary reason of the growth is an extra factor of \( \ln(m_\mu/m_e) \approx 5.3 \) due to electron loops inserted in the photon propagators, further enhanced by combinatorial factors (this also explains why the higher order terms have the same sign as the first one; the perturbative series tries to compensate for the fact that we are using a too small scale, and therefore too small value of \( \alpha \)). Another source of increase is due to the factor of \( \pi^2 \approx 9.9 \), which accompanies each two additional photon rungs added to the light-by-light diagrams. This reasoning justifies the truncation of the perturbative series at the fifth term. Our final estimate of the QED contribution to \( a_\mu \) is

\[
a_\mu^{\text{QED}} = 116 584 705.6(2.9) \times 10^{-11}. \quad (21)
\]

The error has been estimated by linearly adding roughly equal contributions from the uncertainty in \( \alpha \) and \( C_{4,5} \), and a small number estimating the higher order terms in the QED series for \( a_\mu^{\text{QED}} \).

### 3.2. Hadronic contributions

The bulk of hadronic contributions comes from vacuum polarization insertion in the Schwinger diagram and can be calculated using experimental data on \( e^+e^- \rightarrow \text{hadrons or hadronic } \tau \text{ decays, and dispersion relations (see } [32,33] \text{ where further references can be found). The most recent evaluation } [36] \text{ gives}

\[
a_\mu^{\text{had}}(4\text{th order}) = 6924(62) \times 10^{-11} \quad (22)
\]

That study relies in part on theoretical assumptions which are subject to some debate. For example, a preliminary update of the analysis \([33]\) arrives at a larger error estimate of \( 119 \times 10^{-11} \).

In addition, one has to include the vacuum polarization insertion in all two-loop QED diagrams \( [35,14] \),

\[
a_\mu^{\text{had}}(6\text{th order}) = -100(6) \times 10^{-11}, \quad (23)
\]

and hadronic light-by-light diagram \([30]\) for which we take an average \([35]\) of the values given in \([37,38] \),

\[
a_\mu^{\text{had}}(\text{light-by-light}) = -85(25) \times 10^{-11}. \quad (24)
\]

The total hadronic contribution is

\[
a_\mu^{\text{had}} = 6739(67) \times 10^{-11}. \quad (25)
\]

Its error dominates the present theoretical prediction of \( a_\mu \).

It would be very valuable to have at least an independent estimate of the hadronic light-by-light
contributions from lattice QCD. Such a study might be undertaken in the near future \textsuperscript{39}. Also, ongoing studies of $e^+e^- \rightarrow \text{hadrons}$ and hadronic tau decays could further reduce the theoretical uncertainty. A goal of $\pm 40 \times 10^{-11}$ or smaller is well matched to the prospectus of experiment E821 at Brookhaven which aims for that level of experimental accuracy.

### 3.3. Electroweak contributions

At the one loop level, the Standard Model predicts \textsuperscript{40–44}

$$a^\text{EW}_\mu (\text{1 loop}) = \frac{5}{3} \frac{G_\mu m^2}{\sqrt{2}\pi^2}$$

$$\times \left[ 1 + \frac{1}{5} (1 - 4 s^2_W)^2 + \mathcal{O} \left( \frac{m^2}{M^2} \right) \right]$$

$$\approx 195 \times 10^{-11}$$ (26)

where $G_\mu = 1.16639(1) \times 10^{-5}$ GeV$^{-2}$, $M = M_W$ or $M_{\text{Higgs}}$, and the weak mixing angle $\sin^2 \theta_W \equiv s^2_W = 1 - M^2_{Z}/M^2_{W} = 0.224$. We can safely neglect the $\mathcal{O} (m^2_{\mu}/M^2)$ terms in (26).

Two-loop corrections \textsuperscript{45–47}

$$a^\text{EW}_\mu (\text{2-loop}) = -44(4) \times 10^{-11},$$ (27)

decrease the electroweak contribution by about 23%, bringing it to

$$a^\text{EW}_\mu = 151(4) \times 10^{-11}.$$ (28)

This decrease is mainly due to the large logarithmic terms $\sim \ln(M^2_{Z}/m^2_{\mu})$. These leading logs have recently been resummed to all orders in $\alpha$ \textsuperscript{48}.

They found that the leading logs in order $\mathcal{O}(\alpha^2)$ increase the EW effects by a very small amount, $a^\text{EW}_\mu (\text{3-loop, LL}) = 0.5 \times 10^{-11}$ which is within the error.

The complete Standard Model prediction for $a_\mu$ is

$$a^\text{SM}_\mu = a^\text{QED}_\mu + a^\text{had}_\mu + a^\text{EW}_\mu$$

$$= 116 591 596(67) \times 10^{-11}.$$ (29)

Currently, the combined CERN \textsuperscript{2} and BNL \textsuperscript{3} measurements give

$$a^\text{exp}_\mu = 116 592 350(730) \times 10^{-11}.$$ (30)

It is expected that the ongoing run at BNL will reduce the uncertainty by a factor of three and in the long term $\pm 40 \times 10^{-11}$ is achievable.

Comparing experiment and theory, one finds

$$a^\text{exp}_\mu - a^\text{SM}_\mu \approx (750 \pm 733) \times 10^{-11}$$ (31)

which implies good consistency; but leaves open the possibility of “New Physics” contributions to $a_\mu$ as large as $\sim \mathcal{O}(10^{-8})$.

We subsequently consider several examples of “New Physics” effects on $a_\mu$. To estimate the sensitivity of ongoing measurements and theory, we assume $\pm 100 \times 10^{-11}$ combined precision is attainable. However, if the theoretical error can be further reduced and the experiment proceeds as planned, a reduction in that uncertainty by 2–3 appears feasible.

### 3.4. “New Physics” and Electroweak Radiative Corrections

“New Physics,” beyond Standard Model expectations, will in general give rise to additional $a_\mu$ contributions which we collectively call $a^\text{New Physics}_\mu$. Before discussing specific examples, we consider the electroweak radiative corrections to such “New Physics” effects.

Most “New Physics” effects contribute directly to the dimension 5 magnetic dipole operator. In that case, they are subject to the same EW suppression factor as the $W$ loop contribution to $a^\text{EW}_\mu$. From the calculation in Ref. \textsuperscript{15}, one finds a leading log suppression factor

$$1 - \frac{4\alpha}{\pi} \ln \frac{M}{m_\mu}$$ (32)

where $M$ is the characteristic “New Physics” scale. For $M \sim 100$ GeV, that factor corresponds to about a 6.4% reduction.

### 3.5. Supersymmetry

The supersymmetric contributions to $a_\mu$ stem from smuon–neutralino and sneutrino-chargino loops \textsuperscript{49,50} (see Fig. 1). They can be significant if the supersymmetric particles are not too massive and if $\tan \beta \equiv v_2/v_1$ is large. Indeed, the one loop effect is given in the large $\tan \beta$ limit by

$$a^\text{SU3}_\mu \approx \frac{\alpha}{8\pi^2} \frac{m^2}{m_\mu^2} \tan \beta$$ (33)

\textsuperscript{15}
where $\tilde{m}$ represents a typical SUSY loop mass. Including the EW suppression factor in Eq. (32) then implies

$$a^\text{SUSY}_\mu \approx 140 \times 10^{-11} \left( \frac{100 \text{ GeV}}{\tilde{m}} \right)^2 \tan \beta.$$  

(34)

For large $\tan \beta \approx 40$ scenarios, even an experimental sensitivity of $\pm 100 \times 10^{-11}$ probes $\tilde{m}$ at the 750 GeV level and becomes competitive with direct high energy collider searches.

### 3.6. Origin of Muon Mass

In models where the muon mass is generated by quantum loops, similar loop effects will also give additional contributions to $a_\mu$ [53]. Under very general assumptions, the induced $\delta a_\mu$ is given by

$$\delta a_\mu = C \frac{m^2_\mu}{\Lambda^2}, \quad C \sim \mathcal{O}(1),$$  

(35)

where $\Lambda$ is the scale of “New Physics” responsible for generating $m_\mu$. Examples of such mechanisms include: extended technicolor, multi-Higgs models, compositeness, etc. For $a_\mu^\text{New Physics}$ sensitivity of $\pm 100 \times 10^{-11}$ and $C \sim \mathcal{O}(1)$, $\Lambda \gtrsim 3$ TeV is probed.

### 3.7. Anomalous $W$ couplings

We generalize the $\gamma WW$ coupling such that the $W$ boson magnetic dipole moment is given by

$$\mu_W = \frac{e}{2m_W} (1 + \kappa + \lambda)$$  

(36)

and electric quadrupole moment by

$$Q_W = -\frac{e}{2m_W} (\kappa - \lambda)$$  

(37)

where $\kappa = 1$ and $\lambda = 0$ in the Standard Model, i.e. $g_W = 2$. For non-standard couplings, one obtains the additional one loop contribution to $a_\mu$ given by [54–57]

$$a_\mu (\kappa, \lambda) \approx \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \left[ (\kappa - 1) \ln \left( \frac{\Lambda^2}{m_W^2} \right) - \frac{1}{3} \lambda \right].$$  

(38)

where $\Lambda$ is the high momentum cutoff required to give a finite result. It presumably corresponds to the onset of “New Physics” such as the $W$ compositeness scale, or new strong dynamics. Electroweak effects reduce that contribution by roughly the suppression in Eq. (32). Probing $a_\mu$ at the $\pm 100 \times 10^{-11}$ level provides a sensitivity to $|\kappa - 1|$ of about $\pm 0.1$ (for $\Lambda \sim 1$ TeV) (see also [58]). Generally, one might expect $\kappa \sim (m_W/\Lambda)^2$ in theories with underlying strong dynamics at scale $\Lambda$.

The ongoing experimental effort to improve the accuracy of $a_\mu$ measurement will improve the constraints on the “New Physics” scenarios we mentioned above, as well as on other theories and phenomena, such as a general two-Higgs doublet model [59], leptoquarks [60], or four fermion contact interactions [61] (see also [62]).

### 4. Tau

The anomalous magnetic moment of the tau, $a_\tau$, is predicted to be

$$a_\tau = a_\tau^\text{QED} + a_\tau^\text{had} + a_\tau^\text{EW}$$  

(39)

where [63]

$$a_\tau^\text{QED} = 1.1732 \times 10^{-3},$$  

(40)

$$a_\tau^\text{had} = 3.2(4) \times 10^{-6},$$  

(41)

$$a_\tau^\text{EW} = 4.7 \times 10^{-7}.$$  

(42)

We have updated those contributions to incorporate $m_\tau = 1.77705(26)$ GeV, a theoretical estimate of $a_\tau^\text{had}$ from perturbative QCD [64] which is averaged with the result given quoted in [63], and two loop EW effects [15] which suppress $a_\tau^\text{EW}$ by about 15%. In total, one finds

$$a_\tau = 1.1769(4) \times 10^{-3}.\quad (43)$$

Experiments are currently not sensitive enough to measure $a_\tau$. They can, however, indirectly bound
an anomalously large \( a_\tau \) due to “New Physics.” For example, \( e^+e^- \to \tau^+\tau^- \) cross-section measurements imply

\[
|a_\tau| < 0.02. \tag{44}
\]

Also, at this meeting L. Taylor \cite{4} reported on analysis of \( e^+e^- \to \tau^+\tau^- \) by L3 and OPAL which gave a similar constraint (see Table \ref{tab:1}). If “New Physics” effects in \( a_\tau \) are of the form \( m_\tau^2/\Lambda^2 \), then one would need to extend such constraints to \( |a_\tau| < 0.0003 \) to probe the interesting regime \( \Lambda \gtrsim 100 \text{ GeV} \).

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