Photon time delay resulting from vacuum fluctuations: 
the vacuum as a dielectric

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It is not possible to detect a vacuum fluctuation without a test particle interacting with the vacuum fluctuation in a measurable manner. In the quantum electrodynamics calculation presented here, a photon traveling through the vacuum is used as the test particle to determine properties of the vacuum resulting from vacuum fluctuations. In particular, this article (1) discusses the mathematical procedure for describing a lepton-antilepton bound state that is a vacuum fluctuation, (2) describes the photon interacting with and being incorporated into the bound state to form a quasi-stationary bound state, and (3) calculates the electromagnetic decay rate of the quasi-stationary bound state.

I. INTRODUCTION

According to perturbation theory in quantum electrodynamics, as a photon travels through the vacuum it can be viewed as splitting into a charged, virtual, particle-antiparticle pair that then annihilates, emitting a photon identical to the original. Such interactions contribute to the photon propagator but, even to infinite order, do not change the location of the pole of the propagator. Because the position of pole does not change, the speed of a photon does not change as a result of interacting with virtual particles. In this article the term “virtual particle” refers exclusively to a particle associated with a Green’s function or, equivalently, with a propagator that arises in perturbative calculations using quantum field theories.

In addition to virtual particles, there is a second concept associated with the vacuum, that of “vacuum fluctuations” (VFs). The term VFs refers exclusively to particles that appear spontaneously within the vacuum and are not associated with a Green’s function. VFs cannot be observed directly but, as will be demonstrated in the present article, can be observed indirectly by the effect of their interactions with photons moving through the vacuum. Using only quantum electrodynamics to describe a photon traveling through the vacuum, it is possible to show that the presence of certain types of VFs slows the progress of photons.

The result presented here can be viewed as being foreshadowed by Dimopoulos, Raby, and Wilczek who state, “The vacuum is a dielectric.”[1] Their statement is made in reference to the well-understood electromagnetic, weak, and color charge renormalization. Here, however, the possibility is examined that the statement, “The vacuum is a dielectric,” is more generally true. Specifically, the energy associated with a particular type of VF, a lepton-antilepton pair, is postulated to increase until the lepton and antilepton are on mass shell, so that they can be treated as external particles, and are bound into an atom in its ground state. Not only does the binding into atoms minimize the transient violation of conservation of energy allowed by what is commonly referred to as the time-energy uncertainty principle, it also provides atoms that can interact with photons. The term “dielectric” can then be used in the usual sense: a photon passing through a dielectric is slowed by its interactions with atoms in the dielectric, a concept familiar from discussions of a physical dielectric [2,4]. Because there is a nonzero, finite lifetime associated with this quasi-stationary state, there will be a delay in the progress of the photon through the vacuum, this delay being increased with each interaction of the photon with a lepton-antilepton VF. The effect of this interaction will appear in the propagator only as a change in the value of $c$, not as a change in the position of the pole. As required by conservation of energy, when the quasi-stationary state annihilates, the vacuum reabsorbs the released energy.

II. CALCULATION OF THE DECAY RATE OF PHOTON-EXCITED PARAPOSITRONIUM

Here the decay rate is calculated for electron-positron pairs that are VFs and appear as parapositronium, the positronium state with the lowest energy that also happens to have zero angular momentum. The formula for the decay rate immediately generalizes to yield decay rates for VFs of muon-antimuon and tau-antitau pairs that are bound into atoms in their ground states. These three decay rates play an essential role in determining the speed of photons as they propagate through the vacuum. The decay is kinematically forbidden for ordinary parapositronium but is allowed for parapositronium that is a VF since the latter parapositronium does not enter into overall energy conservation because it contributes no net
energy.
Labeling the initial (incident) and final (emitted) photons, respectively, by $\gamma_i$ and $\gamma_f$, to lowest order the two Feynman diagrams that contribute to the process $\gamma_i + \text{positronium} \rightarrow \gamma_f$ are shown in FIG. 1.

FIG. 1. (a) Photon $\gamma_i$ interacts with a positron that then annihilates with an electron, emitting photon $\gamma_f$. (b) Photon $\gamma_i$ interacts with an electron that then annihilates with a positron, emitting photon $\gamma_f$.

In the diagrams $p_-, p_+, k_i$, and $k_f$ are, respectively, the four-momenta of the electron, positron, initial photon, and final photon.

For ordinary positronium the process is kinematically forbidden. In the center-of-mass rest frame of positronium, $p_- + p_+ = 0$. Therefore, in this frame [3],

\begin{align}
  p_- &= (E_-, p_-) = (\sqrt{m_e^2 + p_-^2}, p_-), \\
  p_+ &= (E_+, p_+) = (\sqrt{m_e^2 + p_+^2}, p_+), \\
  &= (\sqrt{m_e^2 + p_-^2}, -p_-) = (E_-, -p_-), \\
  k_i &= (\omega_i, k_i) = (|k_i|, k_i), \\
  k_f &= (\omega_f, k_f) = (|k_f|, k_f).
\end{align}

Conservation of energy and momentum requires $p_+ + p_- + k_i = k_f$. Squaring both sides of the above equation yields

$$E_+^2 + E_-\omega_i = 0. \quad (2)$$

Eq. (2) cannot be satisfied for ordinary positronium since both terms on the left-hand side are positive. However, after a photon excites positronium that is a VF, a photon can be emitted, but only when the positronium vanishes into the vacuum because only then does $E_- \rightarrow 0$, allowing (2) to be satisfied.

When performing an electrodynamics calculation, if a factor $\exp(\pm(ip \cdot x))$ is associated with a particle that is a VF when it appears in its initial state, a factor $\exp(\mp(ip \cdot x))$ is associated with a particle that is a VF when it vanishes into the Grid. This just eliminates the contribution of the particle that is a VF to overall energy-momentum conservation. When progressing along an energy-momentum line in a Feynman diagram, the energy-momentum associated with a particle that is a VF is not further used after the particle vanishes. Since the electron and positron that are VFs are treated as external particles, they must be on-shell. Using the notation of [7], the S-matrix for the transition “photon + positronium that is a VF → photon” is

$$S_{fi} = \frac{e^2}{V^2} \sqrt{\frac{m_e}{E_+}} \sqrt{\frac{m_e}{E_-}} \frac{1}{\sqrt{2\omega_i}} \frac{1}{\sqrt{2\omega_f}} (2\pi)^4 \delta(k_i - k_f) \times$$

$$\bar{v}(p_+, s_+)[(-i\gamma_i^f)(-\vec{p}_+ - \vec{k}_i - m_e(-i\gamma_i^f) +$$

$$(-i\gamma_i^f)(\vec{p}_- + \vec{k}_i - m_e(-i\gamma_i^f))] u(p_-, s_-). \quad (3)$$

In (3) the fermion wave functions are normalized to unit probability in a box of volume $V$. The equality $1/(p \pm m_e) = (p \mp m_e)/(p^2 - m_e^2)$ is used to rewrite the above two propagators. Then the equation

$$g_h = -\bar{\psi} \psi + 2a \cdot b I, \quad (4)$$

where $a$ and $b$ are four-vectors and $I$ is the identity matrix, yields $\bar{\psi}^+ \psi_+ = -\bar{\psi}^+ \psi_+ + 2p_+ \cdot \epsilon_i I$ and $\bar{\psi}_- \psi_+ = -\bar{\psi}_- \psi_+ + 2p_- \cdot \epsilon_i I$. Also $\bar{v}(p_+ + s_+) \psi^+_+ = -\bar{v}(p_+ + s_+) m_e$ and $\bar{\psi}_- u(p_-, s_-) = m_e u(p_-, s_-)$, with the result that (3) can be rewritten as

$$S_{fi} = -i \frac{e^2}{V^2} \sqrt{\frac{m_e}{E_+}} \sqrt{\frac{m_e}{E_-}} \frac{1}{\sqrt{2\omega_i}} \frac{1}{\sqrt{2\omega_f}} (2\pi)^4 \delta(k_i - k_f) \times$$

$$\bar{v}(p_+, s_+) \left[ \frac{(2\epsilon_i \cdot p_+ + \epsilon_i \cdot \vec{k}_i)\gamma_i^f}{(p_+ + k_i)^2 - m_e^2} + \frac{\epsilon_i \cdot (2p_- \cdot \epsilon_i + \vec{k}_i \cdot \gamma_i^f)}{(p_- + k_i)^2 - m_e^2} \right] u(p_-, s_-). \quad (5)$$
To obtain the decay rate to lowest order, in $S_{fi}$, the respective velocities $v_-$ and $v_+$ of the electron and positron can be neglected. Thus

\begin{align}
E_- &\to m_e, \quad E_+ \to m_e, \quad (6a) \\
p_\pm &\to (m_e, 0).
\end{align}

The respective polarization vectors $\epsilon_i$ and $\epsilon_f$ of the initial and final photons are chosen to be space-like: $\epsilon_i = (0, \epsilon_i), \quad \epsilon_f = (0, \epsilon_f)$ where

\begin{align}
k_i \cdot \epsilon_i &= -k_i \cdot \epsilon_i = 0, \\
k_f \cdot \epsilon_f &= -k_f \cdot \epsilon_f = 0.
\end{align}

Using $|S_i|^2$ becomes

\begin{align}
S_i &= i \epsilon_i^2 \frac{1}{\sqrt{2 \omega_i}} \frac{1}{\sqrt{2 \omega_f}} \frac{1}{2m_e \omega_i} (2\pi)^4 \delta^4(k_i - k_f) \\
&\quad \delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-).
\end{align}

\begin{align}
S_i &= -i \epsilon_i^2 \frac{1}{\sqrt{2 \omega_i}} \frac{1}{\sqrt{2 \omega_f}} \frac{1}{2m_e \omega_i} (2\pi)^4 \delta^4(k_i - k_f) \\
&\quad \delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-).
\end{align}

Then,

\begin{align}
|S_i|^2 &= \frac{\epsilon_i^2}{V^2} \frac{1}{\sqrt{2 \omega_i}} \frac{1}{\sqrt{2 \omega_f}} \frac{1}{4m_e^2 \omega_i} V T \langle 2\pi \rangle^4 \delta^4(k_i - k_f) \\
&\quad \delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-) \\
&\quad \delta(p_-, s_-)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_+, s_+).
\end{align}

As mentioned previously, $\hat{k}_i$ anti-commutes with both $\epsilon_i$ and $\epsilon_f$, with the result that $|S_i|^2$ becomes

\begin{align}
|S_i|^2 &= \frac{\epsilon_i^2}{V^2} \frac{1}{\sqrt{2 \omega_i}} \frac{1}{\sqrt{2 \omega_f}} \frac{1}{4m_e^2 \omega_i} V T \langle 2\pi \rangle^4 \delta^4(k_i - k_f) \\
&\quad \delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-) \\
&\quad \delta(p_-, s_-)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_+, s_+).
\end{align}

The average cross section is now calculated for photon-excited positronium that is a VF to annihilate and emit a photon: $|S_i|^2$ is divided by $V T$ to form a rate per unit volume, divided by the electron-positron flux $|v_+ - v_-|/V$, and divided by the number of target particles per unit volume $1/V$. Averaging over the spins and positron ($\frac{1}{2} \sum_{s_f}$), summing over the polarizations of the final photon ($\frac{1}{2} \sum_{s_f}$), averaging over the polarizations of the initial photon ($\frac{1}{2} \sum_{s_i}$), summing over the number of states of the final photon in the momentum interval $d^3k_f$, averaging over the number of states of the initial photon in the momentum interval $d^3k_i$, an expression for the cross section $\sigma$ is obtained:

\begin{align}
\sigma &= \frac{\epsilon_i^2}{4 \frac{1}{V^2} \frac{1}{\sqrt{2 \omega_i}} \frac{1}{\sqrt{2 \omega_f}} \frac{1}{4m_e^2 \omega_i} V T \langle 2\pi \rangle^4 \delta^4(k_i - k_f) \\
&\quad \delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-) \\
&\quad \delta(p_-, s_-)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_+, s_+) \\
&\quad \delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-) \\
&\quad \delta(p_-, s_-)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_+, s_+).
\end{align}

Using $|S_i|^2$, $\frac{1}{\sqrt{2 \omega_i}} \frac{1}{\sqrt{2 \omega_f}} \frac{1}{4m_e^2 \omega_i} V T \langle 2\pi \rangle^4 \delta^4(k_i - k_f)$, $\delta(p_+, s_+)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_-, s_-)$, $\delta(p_-, s_-)(\epsilon_i \epsilon_f - \epsilon_f \epsilon_i) \hat{k}_i \hat{k}_f u(p_+, s_+)$, the unit polarization vectors for the initial photon $\epsilon_i^a$ and $\epsilon_i^b$ are chosen in the x- and y-direction, respectively. Because the delta function in $|S_i|^2$ imposes the condition $k_f = k_i$, the unit polarization vectors for the final photon $\epsilon_f^a$ and $\epsilon_f^b$ can also be chosen in the x- and y-direction, respectively. The sum over polarizations in $|S_i|^2$ is now
easily performed:

\[
\sum_{\epsilon_f} \sum_{\epsilon_i} 1 = 4, \tag{18a}
\]

\[
\sum_{\epsilon_f} \sum_{\epsilon_i} (\epsilon_i \cdot \epsilon_f)^2 = \left(\epsilon_i^a \cdot \epsilon_f^a \right)^2 + \left(\epsilon_i^b \cdot \epsilon_f^b \right)^2 + \left(\epsilon_i^c \cdot \epsilon_f^c \right)^2 = (-1)^2 + 0 + 0 + (-1)^2 = 2. \tag{18b}
\]

Using \[14\] and the identity \[8\],

\[
\delta(\omega^2 - a^2) = (1/2a)[\delta(\omega - a) + \delta(\omega + a)], \quad a > 0, \tag{19}
\]

it is straightforward to show that

\[
\int_{-\infty}^{\infty} \frac{d^3k}{2\omega_i} = \int_{-\infty}^{\infty} d^4k_i \delta(k_i^2)\theta(k_{i0}). \tag{20}
\]

The theta function \(\theta(k_{i0}) = 0\) if \(k_{i0} < 0\) and \(\theta(k_{i0}) = 1\) if \(k_{i0} > 0\). With the aid of \[20\], the second line in \[17\] can be rewritten as

\[
\int_{-\infty}^{\infty} \frac{d^3k}{2\omega_f} \int_{-\infty}^{\infty} \frac{d^3k}{2\omega_i} \delta^4(k_i - k_f) = \int_{-\infty}^{\infty} \frac{d^3k_f}{2\omega_f} \delta(k_f^2)\theta(k_{f0}). \tag{21}
\]

Factoring \(k_i^2 = k_{i0}^2 - |k_f|^2\) in the above \(\delta\)-function, using \[19\], rewriting \(d^3k_f\) as \(d\Omega_f/|k_f|^3 d|k_f|\), performing the angular integration over \(d\Omega_f\), which yields a factor of 4\(
\pi\), and then integrating over \(|k_f|\),

\[
\int_{-\infty}^{\infty} \frac{d^3k_f}{2\omega_f} \int_{-\infty}^{\infty} \frac{d^3k}{2\omega_i} \delta^4(k_i - k_f) = \pi. \tag{22}
\]

Substituting \[18\] and \[22\] into \[17\] yields the formula for the cross section for the annihilation into a photon of photon-excited positronium that is a VF,

\[
\sigma = \frac{2\pi\alpha^2}{m_e^2} \frac{1}{|\mathbf{v}_+ - \mathbf{v}_-|}. \tag{23}
\]

From the formula for the cross section, a formula for the decay rate is readily obtained. The logic is the same as that used to calculate the decay rate for positronium decaying into two photons \[4\]: para- positronium, orthopositronium, and a photon have respective charge conjugation parities of +1, -1, and -1. Thus photon-excited parapositronium has charge conjugation parity of -1 while photon-excited orthopositronium has charge conjugation parity of +1. Since electromagnetic interactions are invariant under charge conjugation, photon-excited parapositronium, but not photon-excited orthopositronium, can decay into a single photon.

In obtaining \[23\] the electron and positron spins were averaged over all four spins, resulting in the sum being divided by four. But the annihilating state is parapositronium, the singlet state. Orthopositronium, the triplet state, does not contribute. Since only one of the four spin states contributes to the cross section, the formula for the cross section should not have been divided by four, it should have been divided by the number one. Thus the formula for \(\sigma\) in \[23\] should be multiplied by a factor of four to obtain the cross section, abbreviated \(\sigma_{p-\text{Ps}}\), for the annihilation into a photon of photon-excited parapositronium that is a VF,

\[
\sigma_{p-\text{Ps}} = \frac{8\pi\alpha^2}{m_e^2} \frac{1}{|\mathbf{v}_+ - \mathbf{v}_-|}. \tag{24}
\]

For the annihilation of photon-excited parapositronium that is a VF into a photon, the electromagnetic decay rate \(\Gamma_{p-\text{Ps}}\) is calculated using the mechanism for the annihilation of ordinary parapositronium \[9\]. The Schrödinger wave function \(\psi(x)\) for parapositronium is just the ground-state hydrogen atom wave function with the reduced mass of hydrogen, which is approximately \(m_e\), replaced by \(m_e/2\), the reduced mass of parapositronium:

\[
\psi(x) = \frac{1}{\sqrt{\pi}} \left(\frac{\alpha m_e}{2}\right)^{3/2} e^{-\alpha m_e r/2}. \tag{25}
\]

In the above formula \(x\) is the magnitude of the vector \(x = x_e - x_p\), where \(x_e\) and \(x_p\) are, respectively, the positions of the electron and the positron.

The decay rate \(\Gamma_{p-\text{Ps}}\) is the product of \(\sigma_{p-\text{Ps}}\) and the flux of a parapositronium atom, which is the relative velocity of approach of the electron and positron in parapositronium multiplied by \(|\psi(0)|^2\), the probability density that the electron and positron collide and annihilate.

\[
\Gamma_{p-\text{Ps}} = \sigma_{p-\text{Ps}} |\mathbf{v}_+ - \mathbf{v}_-| |\psi(0)|^2,
\]

\[
= \frac{8\pi\alpha^2}{m_e^2} \frac{1}{|\mathbf{v}_+ - \mathbf{v}_-|} \frac{1}{6} \left(\frac{\alpha m_e}{2}\right)^3,
\]

\[
= \alpha^6 m_e. \tag{26}
\]

The above decay rate is twice that of ordinary parapositronium into two photons \[4\].

From \[26\] it immediately follows that the corresponding decay rates for muon-antimuon and tau-antitau bound, ground states that are VFs are obtained by replacing the electron mass with the muon or tau mass, respectively. As photons travel through the vacuum, these three decay rates describe how photons interact with lepton-antilepton pairs that are VFs.

### III. CONCLUSION

When photons interact with virtual particles as described by quantum electrodynamics, the interactions do not affect the speed of the photons. In contrast, when photons interact with lepton-antilepton pairs that are VFs, the speed of photons is decreased similarly to the way that their speed is decreased in a dielectric consisting of ordinary matter. In an article to be submitted,
the decay rates calculated here are used to determine how these interactions primarily determine the speed of photons in the vacuum.

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