Hadronic axion, baryogenesis and dark matter

Bengt-Åke Lindholm

Center for Theoretical Physics
Seoul National University
Seoul 151-742
Korea

Abstract

We investigate the prospects of an extra global symmetry, added to the Standard Model in the context of baryogenesis. The $PQ$ symmetry is studied as an example. We show that the hadronic axion provides a protection of the B-asymmetry even if $B - L = 0$ and that there are no limits on the neutrino masses. It is also shown that there is a crucial difference between the two possible invisible axion models. A connection between the baryon asymmetry and dark matter is also pointed out.

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1 Introduction

It has been known for a long time that $B + L$ symmetry is anomalous and that $B - L$ symmetry is anomaly free in the Standard Model. However, the operator induced by the anomaly has negligible effect at zero temperature [1]. Some years ago the sphaleron solution to the classical $SU(2)$ gauge-Higgs system was found [2]. It is a saddle point configuration and is unstable. Soon after this [3], it was realized that the sphaleron mediates $B + L$ violating interactions at a substantial rate at high temperatures. Since then, this circumstance has been under intense investigation. Recent reviews on this topic are given in refs. [4]. Contrary to earlier beliefs it was shown in refs. [3] and [5] that a $B$ asymmetry created at GUT scale not necessarily is washed out by the sphalerons. Their conclusions were that the $B$ asymmetry is proportional to a $B - L$ asymmetry that could have been created at GUT scale. However, this immediately raises the question of $B - L$ violating operators. Operators of dimension five or higher coming from sources beyond the Standard Model are posing a threat to the $B - L$ asymmetry are they in equilibrium. These issues have been investigated in refs. [5], [6] and [7]. The lowest dimension operator that violates $B$ or $L$ is the following $\Delta L=2$ dimension-five operator:

$$m_\nu \frac{(l_L H)^2}{v^2}$$

(1)

$l_L$ and $H$ are the left-handed lepton doublet and Higgs doublet respectively. Below the electroweak symmetry breaking scale this operator induces a majorana mass for the neutrino. It also mediates different scattering processes all of which break $L$-conservation. The requirement that this operator be out of equilibrium, so that $B - L$ asymmetry is not washed out, puts limits on the neutrino mass and in ref. [5] the following limit was derived:

$$m_\nu < \frac{4 eV}{(T_{B-L}/10^{10} GeV)^{1/2}}$$

(2)

where $T_{B-L}$ is the temperature at which the $B - L$ asymmetry is generated. This was elaborated further in ref. [8] where decays of heavy right-handed neutrinos also were considered. In that case the limit is much stronger:

$$m_\nu < 10^{-3} eV$$

(3)

It has been pointed out in ref. [8] that there are ways to circumvent this limit. Above a certain temperature in the minimal supersymmetric Standard Model there are two more global symmetries that will protect a primordial $B - L$ asymmetry. These symmetries have $SU(2)$ and $SU(3)$ anomalies. The
constraint on the neutrino mass coming from (1) is in this case relaxed by four
orders of magnitude and reads:

\[ m_\nu < 10^{eV} \quad (4) \]

Investigations along the line of extra global symmetries have also been pursued
in ref. [9]. In that paper one more unbroken global symmetry with an \( SU(2) \)
anomaly was introduced. Several new fields as compared to the Standard
Model were also introduced. It will protect a primordial \( B \) asympmetry even if
\( B - L = 0 \). At the same time there is also the possibility to have an asymmetry
in the new charge that could be responsible for the Dark Matter.

## 2 An extra global symmetry

In this paper we examine the prospects of the \( PQ \) symmetry [10] in protecting
a primordial \( B \) asymmetry. This extra symmetry has no \( SU(2) \) anomaly which
is a difference compared to the previously studied extra global symmetries that
can protect a primordial \( B \) asymmetry. Our main interest is the hadronic axion
model [11]. The DFSZ-axion [12] cannot protect a primordial \( B \) asymmetry
as will be shown. To be more explicit, the model to be studied is the Standard
Model with \( N \) generations of quarks and leptons and one Higgs doublet. In
order to implement the hadronic axion we also introduce one extra \( SU(2) \)-
singlet quark with electric charge \( q \) and a complex \( SU(2) \)-singlet scalar.

We will calculate the \( B \) and \( L \) number in equilibrium in the early Universe. We
assign a chemical potential to each of the \( N \) generations of quarks and leptons,
to the \( SU(3) \times SU(2) \times U(1) \) gauge bosons, to the complex Higgs doublet, to
one complex scalar singlet and to one \( SU(2) \)-singlet heavy quark. Here we
will use the notation of ref. [11] and the chemical potentials are assigned as
follows: \( \mu_W \) for \( W^- \), \( \mu_0 \) for \( \phi^0 \), \( \mu_- \) for \( \phi^- \), \( \mu_{uL} \) for all the left handed up-quark
fields, \( \mu_{uR} \) for all the right handed up-quark fields, \( \mu_{dL} \) for all the left handed
down-quark fields, \( \mu_{dR} \) for all the right handed down-quark fields. \( \mu_{iL} \) for all
the left handed charged leptons fields, \( \mu_{iR} \) for all the right handed charged
lepton fields. \( \mu_i \) for all the left handed neutrino fields, \( i = 1 \) to \( N \). \( \mu_\sigma \) for the
complex singlet scalar field and \( \mu_{QL}, \mu_{QR} \) for the left and right handed singlet
heavy quarks. Rapid interactions in the early Universe enforce the following
equilibrium relations among the chemical potentials:

\[
\begin{align*}
\mu_W &= \mu_- + \mu_0, \quad W^- \leftrightarrow \phi^0 + \phi^- \\
\mu_{dL} &= \mu_{uL} + \mu_W, \quad W^- \leftrightarrow \bar{u}_L + d_L \\
\mu_{iL} &= \mu_i + \mu_W, \quad W^- \leftrightarrow \bar{\nu}_{iL} + e_{iL}
\end{align*}
\]
\[ \begin{align*}
\mu_{uR} & = \mu_0 + \mu_{uL}, \quad \phi^0 \leftrightarrow \bar{u}_L + u_R \\
\mu_{dR} & = -\mu_0 + \mu_{dL}, \quad \phi^0 \leftrightarrow \bar{d}_R + d_L \\
\mu_{iR} & = -\mu_0 + \mu_{iL}, \quad \phi^0 \leftrightarrow \bar{e}_{iR} + e_{iL} \\
\mu_{QR} & = \mu_{QL} + \mu_\sigma, \quad \sigma \leftrightarrow \bar{Q}_L + Q_R
\end{align*} \tag{5} \]

At temperatures high above the weak scale we can assume that all the chemical potentials for the particles in the same \( SU(3) \times SU(2) \times U(1) \) multiplet are the same and hence all the corresponding gauge bosons have vanishing chemical potential. Moreover, due to cabbibo mixing the chemical potentials for the different quark flavours are the same, whereas the chemical potentials for the leptons are in general different.

There are \( 10 + 3N \) chemical potentials and \( 5 + 2N \) relations. This gives us \( 5 + N \) independent chemical potentials chosen to be \( \mu_{uL}, \mu_0, \mu_W, \mu_{QL}, \mu_{QR} \) and \( \mu_i \) which corresponds to the conserved charges: \( Q, T_3, PQ, B, B_Q \) and \( L_i \). \( B_Q \) is the fermion number carried by the extra \( SU(2) \)-singlet quark. The \( SU(2) \) anomaly induces the following operator:

\[ (q_L q_L l_L)^{N_g} \tag{6} \]

where \( q_L \) and \( l_L \) are the left-handed doublets. \( N_g \) is the number of generations. This will give us the following relation between the chemical potentials:

\[ N(\mu_{uL} + 2\mu_{dL}) + \sum \mu_i = 0 \tag{7} \]

The number of chemical potentials is reduced by one and \( B \) and \( L_i \) are no longer separately conserved but instead we have \( B - \sum L_i \) conserved\(^2\). The \( PQ \)-symmetry has an \( SU(3) \) anomaly which induces the following operator:

\[ (q_L q_L (u_R)^c (d_R)^c)^{N_g} (Q_L (Q_R)^c) \tag{8} \]

c is charge conjugation. Whether this operator gives rise to fast enough \( PQ \)-breaking processes at finite temperature is still an open question. It is of course tempting to assume the existence of \( QCD \)-sphalerons which would mediate transitions in a similar way as the \( SU(2) \)-sphalerons mediate \( B + L \) violation in the weak sector. It seems established that the rate of \( B + L \) violating processes in the symmetric phase of the electroweak sector are non-zero \([13]\). In ref. \([14]\) it is assumed that the \( SU(2) \) sector, in the symmetric phase, in all essentials are the same as QCD. For this reason the rate for the processes induced by \((8)\) should also be non-zero. However, there are reasons to be a little bit careful here since the sphaleron is a solution of \( SU(2) \)-gauge-Higgs

\(^2\)Strictly speaking there are \( N \) conserved charges: \( \frac{1}{2N}B - L_i \)
system in the broken phase. Although there are indications that the Higgs field decouples in the symmetric phase [13] of the SU(2) sector there are still uncertainties about the role of the Higgses in the symmetric phase [16]. We will in this paper assume that the operator in (8) is out of equilibrium.

In the relativistic limit the number densities are related to the chemical potentials as [17]:

$$\frac{n_+ - n_-}{s} = \frac{15g}{4\pi^2 g_s T} \mu \text{ for fermions}$$

$$\frac{n_+ - n_-}{s} = \frac{15g}{2\pi^2 g_s T} \mu \text{ for bosons}$$

(9)

$n_+$ and $n_-$ are the number densities for particles and anti-particles respectively. $g$ and $g_s$ are respectively the number of internal and total degrees of freedom. $s$ is the entropy density. We now have eqs. (5), (7), (9) together with $\mu_W = 0$ and the constraint of electric charge neutrality of the Universe. From this we can easily write down the different number densities as:

$$B = \frac{15g}{4\pi^2 g_s T} 4N\mu u_L$$

$$L = \frac{15g}{4\pi^2 g_s T} (-\frac{14N^2 + 9N}{2N + 1} \mu u_L + \frac{Nq}{2(2N + 1)} (\mu QL + \mu QR))$$

$$B_Q = \frac{15g}{4\pi^2 g_s T} (\mu QL + \mu QR)$$

$$PQ = \frac{15g}{4\pi^2 g_s T} (2\mu_\sigma + \frac{1}{2} (\mu QL - \mu QR))$$

(10)

As can be seen, the simple proportionality between $B$ and $B - L$ is lost here. There are three conserved charges: $B - L$, $PQ$, and $B_Q$. We can afford to break or initially put to zero two of them and still have one chemical potential that is non-zero and proportional to the $B$ number. This can be done in various ways. $B_Q$ is conserved in the low energy sector, $PQ$ is spontaneously broken at a scale $F_\alpha$ and $B - L$ can be violated by higher dimension operators from beyond the standard model.

2.1 $B_Q \neq 0$

Below the $PQ$ breaking scale $\mu_\sigma$ is equal to zero. From eq.(10) we then get $\mu QL = \mu QR$ which gives

$$B = \frac{15g}{4\pi^2 g_s T} 4N\mu u_L$$

3We will in this paper neglect mass effects.
\[
L = \frac{15g}{4\pi^2g_sT}(-\frac{14N^2 + 9N}{2N + 1}\mu_{uL} + \frac{Nq}{2N + 1}\mu_{QL})
\]

\[
B_Q = \frac{15g}{4\pi^2g_sT}2\mu_{QL}
\]  \hfill (11)

If the operator in (1) is in equilibrium there will be the extra condition \(\mu_0 = -\mu_i\) which will change \(L\) and \(B_Q\) to:

\[
L = -\frac{15g}{4\pi^2g_sT}12N\mu_{uL}
\]

\[
B_Q = -\frac{15g}{4\pi^2g_sT}2q(10N + 3)\mu_{uL}
\]  \hfill (12)

As can be seen, in this case there is still one chemical potential, corresponding to the \(B_Q\) asymmetry. All the other charges can now be expressed in terms of \(B_Q\).

\[
B = -\frac{2Nq}{10N + 3}B_Q
\]

\[
L = \frac{6Nq}{10N + 3}B_Q
\]

\[
B - L = -\frac{8Nq}{10N + 3}B_Q
\]

\[
B + L = \frac{4Nq}{10N + 3}B_Q
\]  \hfill (13)

We do not need to worry about \(L\) violating operators and of course there is no limit on neutrino masses. \(B_Q\) protects both \(B\) and \(B - L\) from becoming zero. The exotic quark may make up part of the dark matter and in this model there is a connection between \(B_Q\) and \(B\) which depend on the charge of the exotic quark.

### 2.2 \(B_Q = 0\)

From eq.(10) we can see that \(B_Q = 0\) gives \(\mu_{QL} = -\mu_{QR}\). For \(B\) and \(L\), now only the terms proportional to \(\mu_{uL}\) in eq.(10) will survive. In this case we arrive at the same expressions as derived in [3]:

\[
B = \frac{8N + 4}{22N + 13}(B - L)
\]

\[
L = -\frac{14N + 9}{22N + 13}(B - L)
\]

\[
B + L = -\frac{6N + 5}{22N + 13}(B - L)
\]  \hfill (14)
This will be the case regardless of what happens with the $PQ$ symmetry. With $B_Q = 0$ it reduces to: $PQ \propto -\mu_{QL}$. This chemical potential will not show up in the expressions for $B$, $L$, $Q$ or the sphaleron condition. The derivation of eq. (14) will therefore go through unaffected of whether $PQ$ is zero or not or whether the operator in (8) is in equilibrium or not. In this case the limit on neutrino masses from eq.(3) will be valid. From this we can see that the DFSZ-axion is unable to protect a primordial $B$ asymmetry since the extra fields in that model will not show up in the expressions for $B$, $L$, $Q$ or the sphaleron condition.

3 Conclusions

We have looked into effects of having extra global symmetries in the Standard Model in the context of baryogenesis. As an example we have taken the $PQ$ symmetry. If $B_Q \neq 0$ than the hadronic axion will provide a protection for a primordial $B$ asymmetry in much the same way as $B - L$ asymmetry can protect a primordial $B$ asymmetry. At the same time all limits on the neutrino masses are abolished. In this model there is also a connection between dark matter abundance and $B$ asymmetry. A difference between the two models of $PQ$ symmetry has been shown. It turns out that the hadronic axion but not the DFSZ axion protects the $B$ asymmetry.

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