ON THE GRAVITATIONAL STABILITY AND MASS ESTIMATION OF STELLAR DISKS

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\textbf{Abstract.} We estimate the masses of disks of galaxies using the marginal gravitational stability criterion and compare them with the photometrical disk mass evaluations. The comparison reveals that the stellar disks of most of spiral galaxies we considered cannot be substantially overheated (at least within several radial scalelengths) and are therefore unlikely to have experienced a significant merging event in their history. However, for substantial part of S0-type galaxies a stellar velocity dispersion is well in excess of the gravitational stability threshold suggesting a major merger event in the past. For four low surface brightness galaxies we found that the disk masses corresponding to the marginal stability condition are significantly higher than it may be expected from their brightness. Either their disks are dynamically overheated, or they contain a large amount of non-luminous matter.

\textbf{Key words:} Galaxies: individual: M33, ESO 186-55, ESO 206-14, ESO 234-13, ESO 400-37: evolution, structure

1. INTRODUCTION
A dynamical evolution of disks is the matter of hot debates, and the analysis of their kinematical characteristics plays a key role in the study of their structure and history (see e.g. Zasov, Silchenko (2010) and references therein). By kinematical characteristics we mean a rotation velocity and velocity dispersion of old stars, making up the bulk of disk mass. In general, stellar velocity dispersion either can reflect the velocity of turbulent motion of gas which gave the birth to the disk stars, or can be the result of dynamical heating of disk caused by the internal or external reasons. The minimal velocity dispersion of old stars at given $r$ is constrained by disk local marginal stability condition. Numerical simulations of models of initially weakly unstable 3D disks show the rapid transition of a disk into the marginally stable state, after which the growth of velocity dispersion practically ceases (see e.g. Khoperskov et al. (2003)).

2. THE LOCAL CRITERION OF GRAVITATIONAL STABILITY
The constraints on the disk surface density distribution may be found from the radial velocity dispersion, using the condition of gravitational stability of disk. The critical value of radial velocity dispersion which makes a thin, collisionless isothermal disk stable against the gravitational perturbations is usually written as:

\[ c_{cr} = Q_c \cdot c_T \approx Q_c \cdot \frac{3.4 \cdot G \cdot \sigma_*}{\kappa}, \]
where $Q_c$ is the stability parameter, $c_T$ is the Toomre critical radial velocity dispersion, $\sigma_*$ is the disk surface density, $\kappa$ is the epicyclical frequency. A finite disk thickness makes disk more stable, while non-radial perturbations have the opposite effect. The stability condition taking into account both of these effects cannot be expressed in analytic form. As numerical simulations of 2D and 3D disks show, the parameter $Q_c$ for a wide range of $r$ lies in the interval $1.2 – 2.5$ (see e.g. Bottema (1993), Khoperskov et al. (2003)).

Radial velocity dispersion can be estimated from the observed line-of-sight velocity dispersion measured along the major axis: 

$$c_{\text{obs}}(r) = (c_z \cos^2(i) + c_\phi \sin^2(i) + c_r \sin^2(i))^{0.5},$$

where $c_{\text{obs}}(r)$ is the line-of-sight velocity dispersion, $c_z, c_\phi, c_r$ are the vertical, azimuthal and radial components, $i$ is the disk inclination. To separate the components of the velocity dispersion one can introduce two additional conditions: $c_r = 2\Omega \cdot c_\phi/\kappa$ (Lindblad formula for the epicyclical approximation) and $c_z = m \cdot c_r$. Both the analysis of the available measurements of the galactic disk velocity dispersion (Shapiro et al., 2003) and the results of numerical modeling (see e.g. Zasov et al., 2008) show that in most cases $m \approx 0.4 – 0.7$.

By accepting these approximations and assuming that the disks are marginally stable, one can put constraints on their surface densities $\sigma_*$. In general, when marginal stability condition does not hold, the resulting density and mass estimates can be treated as the upper limits.

3. THE RESULTS OF DISK MASS ESTIMATIONS

We used the marginal stability condition for galactic disks and the stellar velocity dispersion data found in the literature for spiral and S0 galaxies to place the upper limits of the disk local surface density $\sigma(2h)$ at the radial distance of about two radial scalelengths $r \approx 2h$, where the disk contribution to the observed velocity curve is maximal. Extrapolating these estimates, we constrained the total mass of the disks $M_d = 2\pi h^2 \sigma(2h) e^2$ and compare these estimates to those based on the photometry and color of stellar populations. We assumed $Q(2h) \approx 1.5$ (Khoperskov et al. (2003)) and the ratio $m = 0.5$. To compare the obtained disk masses with the photometric estimates we calculated disk mass-to-light ratios in $B$ band ($M/L_B$). For Sa—S0 galaxies the contribution of bulge was taken into account (for more details see Zasov et al. 2011).

Diagrams “$(M/L_B) \alpha – (B – V)_0$” for stellar disks are shown in Fig. 1. The straight line reproduces model relation obtained by Bell, de Jong (2001) for stellar systems with different present-day star formation rate (SFR) using the bottom-light Salpeter initial mass function (IMF). In Fig. 1a the entire sample of galaxies is shown; pair members are marked by asterisks. The same diagram as in Fig. 1a, but after the exclusion of S0/a—S0 galaxies is shown in Fig. 1b. Though the scatter of points in these diagrams is large ($\sim 0.3$ dex), it is compatible with the errors of individual mass estimates. Therefore there is a general agreement between $(M/L_B)\alpha$ estimates based on marginal stability condition and those based on the stellar population modeling. It is remarkable that most of the galaxies, which significantly deviate from model relation, have a red color $(B – V)_0 > 0.7$. At least half of these galaxies are above the straight line, that is most of them have disks with a significant dynamical overheating. Note, that there are both paired and field galaxies among the latter: it seems that only a strong gravitation perturbation can be a cause of the disk dynamical overheating.

If to consider the disky galaxies as systems with marginally stable disks, one
can plot the relation $M_d - V_c$ (baryonic Tully-Fisher relation), connecting the most important parameter of stellar disk (a mass) with the parameter determined mostly by massive halo (a rotation velocity). This diagram is shown in Fig. 2. The relation obtained by McGaugh (2005) for a large sample of spiral galaxies, where disk masses were estimated from their luminosities and colors, is also shown by straight line. A comparison reveals the absence of systematical difference between the two methods of mass estimation.

A more detailed analysis was performed for the nearby galaxy M33. For this galaxy we applied the modified marginal gravitational stability criterion taking into account the influence of gas on the disk stability (see Saburova & Zasov 2012). We used the rotation curve of Corbelli (2003) and the radial profile of line-of-sight velocity dispersion of the disk planetary nebulae obtained by Ciardullo et al. (2004). Fig. 3 demonstrates the radial profiles of local $M/L_K$, estimated for marginally stable disk. The K-band surface brightness was taken from Regan, Vogel (1994). The dotted line denotes $M/L_K$ profile taken from the photometric model of Bell, de Jong (2001) for the observed color indices: $(B - V)$ for the inner
part (Guidoni et al. 1981) and \((H - K)\) for the outer part (Regan, Vogel 1994) of the disk. A thin dotted line in Fig. 3a includes the correction for internal dust extinction (Verley et al. 2009).

Figure 3: Radial profiles of the mass-to-light ratio \(M/L_K\) of M33 assuming that the stellar disk is marginally stable and \(m = c_z/c_r = 0.4\) (solid line). The dashed lines correspond to the profiles based on the color indices and stellar population models by Bell, de Jong, 2001: (a) for the inner part of the galaxy and \((B - V)\) profile and (b) for the outer disk and \((H - K)\) profile.

From Fig. 3 it follows that the surface density of the disk of M33, corresponding to the marginal gravitational stability, is in a good agreement with the photometry-based estimates — with the exception of the most distant point \((r > 7 \text{ kpc})\), where the dynamical overheating is quite possible. It gives evidence that the disk of M33 within several radial scalelengths have not experienced a significant dynamical heating or minor merging events during its evolution.

The situation may be different for giant low surface brightness (LSB) galaxies. We applied the gravitational stability criterion (1) to four LSB-galaxies, for which the distribution of the velocity dispersion and rotation curves of ionized gas and stars, parallel with the photometrical profiles in R-band, were given by Pizzella et al., 2008 (for more details see Saburova 2011). The obtained surface density profiles were used to estimate \((M/L_R)_d\) for different galactocentric distances (see Fig. 4). The filled and open symbols in Fig. 4 correspond to the estimates based on the gas and stellar circular velocities. The dashed lines denote \((M/L_R)_d\) ratios, predicted by stellar population models of Bell, de Jong, 2001 for the metallicity \(Z = 0.008\). From Fig. 4 it follows that \((M/L_R)_d\) ratios which satisfy the marginal stability criterion are several times higher than those predicted by stellar population synthesis models. It can indicate either a strong dynamical overheating of LSB disks, or the presence of a large amount of dark (or faintly luminous) matter in disks. The latter case means, that the disks may contain the unusually large amount of low massive stars (a heavy bottom IMF, see Lee et al. 2004), or a significant fraction of non-stellar dark matter — either in baryonic (e.g. cold molecular gas, see Pfenniger, Combes, 1994) or non-baryonic form.

4. CONCLUSION

Here we show on concrete examples for different types of galaxies that the comparison of local velocity dispersion of stellar disk with the minimal value needed
Figure 4: Radial profiles of $(M/L_R)d$ ratios for LSB-galaxies, obtained by applying the local gravitational stability criterion. Open and filled circles correspond to the data based on the gas and stellar circular velocities. The dashed lines denote $(M/L_R)d$ ratios, predicted by stellar population models of Bell de Jong 2001.

for its gravitationally stability enables to put the constraints on its density and dynamical evolution.

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