Generalized Neutrino Equations

Valeriy V. Dvoeglazov  
UAF, Universidad Autónoma de Zacatecas  
Apartado Postal 636, Zacatecas 98061 Zac., México  
E-mail: valeri@fisica.uaz.edu.mx  
URL: http://fisica.uaz.edu.mx/~valeri/

Abstract

I discuss generalized spin-1/2 equations for neutrinos. They have been obtained by means of the Gersten’s method for derivation of arbitrary-spin relativistic equations. Possible physical consequences are discussed.

1 Introduction

Gersten [1] proposed a method for derivations of massless equations of arbitrary-spin particles. In fact, his method is related to the van der Waerden-Sakurai [2] procedure for the derivation of the massive Dirac equation. I commented the derivation of Maxwell equations [1a] in [3]. Then, I showed that the method is rather ambigious because instead of free-space Maxwell equations one can obtain generalized $S = 1$ equations, which connect the antisymmetric tensor field with additional scalar fields. The problem of physical significance of additional scalar chi-fields should be solved, of course, by experiment.

In the present article I apply the van der Waerden-Sakurai-Gersten procedure to the spin-1/2 fields. As a result one obtains equations which generalize the well-known Weyl equations. However, these equations are known for a long time [4]. Raspini [5, 6, 7, 8, 9] analized them again in detail. I add some comments on physical contents of the generalized spin-1/2 equations.

\footnote{In fact, the $S = 1$ quantum equations.}
2 Derivation

I use the equation (4) of the Gersten paper [1a] for the two-component spinor field function:

\[(E^2 - c^2 \vec{p}^2) I^{(2)} \psi = \left[ EI^{(2)} - c \vec{p} \cdot \vec{\sigma} \right] \left[ EI^{(2)} + c \vec{p} \cdot \vec{\sigma} \right] \psi = 0 \quad (\text{eq.}(4) \ of \ [1a]).\]

(1)

Actually, this equation is the massless limit of the equation which has been presented (together with the corresponding method of derivation of the Dirac equation) in the Sakurai book [2]. In the latter case one should substitute \(m^2 c^4\) into the right-hand side of eq. (1). However, instead of equation (3.25) of [2] one can define the two-component ‘right’ field function

\[\phi_R = \frac{1}{m_1 c} \left( i h \frac{\partial}{\partial x_0} - i h \vec{\sigma} \cdot \nabla \right) \psi, \quad \phi_L = \psi\]

with an additional mass parameter \(m_1\). In such a way we come to the system of the first-order differential equations

\[\left( i h \frac{\partial}{\partial x_0} + i h \vec{\sigma} \cdot \nabla \right) \phi_R = \frac{m_2^2 c}{m_1} \phi_L,\]

(3)

\[\left( i h \frac{\partial}{\partial x_0} - i h \vec{\sigma} \cdot \nabla \right) \phi_L = m_1 c \phi_R.\]

(4)

It can be re-written in the 4-component form:

\[
\begin{pmatrix}
 i h (\partial / \partial x_0) & i h \vec{\sigma} \cdot \nabla \\
 -i h \vec{\sigma} \cdot \nabla & -i h (\partial / \partial x_0)
\end{pmatrix}
\begin{pmatrix}
 \psi_A \\
 \psi_B
\end{pmatrix}
= \frac{c}{2}
\begin{pmatrix}
 (m_2^2 / m_1 + m_1) & (m_2^2 / m_1 - m_1) \\
 (m_2^2 / m_1 - m_1) & (m_2^2 / m_1 + m_1)
\end{pmatrix}
\begin{pmatrix}
 \psi_A \\
 \psi_B
\end{pmatrix}
\]

(5)

for the function \(\Psi = \text{column}(\psi_A \ \psi_B) = \text{column}(\phi_R + \phi_L \ \phi_R - \phi_L)\). The equation (5) can be written in the covariant form.

\[
\left[ i \gamma^\mu \partial_\mu - \frac{m_2^2 c (1 - \gamma^5)}{m_1 \hbar} - \frac{m_1 c (1 + \gamma^5)}{\hbar} \right] \Psi = 0.\]

(6)

The standard representation of \(\gamma^\mu\) matrices has been used here.

If \(m_1 = m_2\) we can recover the standard Dirac equation. As noted in [4b] this procedure can be viewed as the simple change of the representation of \(\gamma^\mu\) matrices (unless \(m_2 \neq 0\)).
Furthermore, one can either repeat a similar procedure (the modified Sakurai procedure) starting from the massless equation (4) of [1a] or put $m_2 = 0$ in eq. (6). The massless equation is

$$
\left[ i\gamma^\mu \partial_\mu - \frac{m_1 c (1 + \gamma^5)}{\hbar} \right] \Psi = 0 .
$$

Then, we may have different physical consequences following from (7) with those which follow from the Weyl equation. The mathematical reason of such a possibility of different massless limits is that the corresponding change of representation of $\gamma^\mu$ matrices involves mass parameters $m_1$ and $m_2$ themselves. The corresponding transformation matrix may be non-existent (its elements tend to infinity in the certain limit).

It is interesting to note that we can also repeat this procedure for the definition (or for even more general definitions);

$$
\phi_L = \frac{1}{m_3 c} (i\hbar \frac{\partial}{\partial x_0} + i\hbar \sigma \cdot \nabla ) \psi, \quad \phi_R = \psi .
$$

This is due to the fact that the parity properties of the two-component spinor are undefined in the two-component equation. The resulting equation is

$$
\left[ i\gamma^\mu \partial_\mu - \frac{m_3 c (1 + \gamma^5)}{\hbar} \right] \tilde{\Psi} = 0 ,
$$

which gives us yet another equation in the massless limit ($m_4 \to 0$):

$$
\left[ i\gamma^\mu \partial_\mu - \frac{m_3 c (1 - \gamma^5)}{\hbar} \right] \tilde{\Psi} = 0 ,
$$

The above procedure can be generalized to any Lorentz group representations, i.e., to any spins. In some sense the equations (7,10) are analogous to the $S = 1$ equations [3, (4-7,10-13)], which also contain additional parameters.

\(^2\)It is necessary to stress that the term ‘massless’ is used in the sense that $p_\mu p^\mu = 0$.

\(^3\)Remember that the Weyl equation is obtained as $m \to 0$ limit of the usual Dirac equation.
3 Physical Interpretations and Conclusions

Is the physical content of the generalized $S = 1/2$ massless equations the same as that of the Weyl equation? Our answer is ‘No’. The excellent discussion can be found in [4a,b]. First of all, the theory does not have chiral invariance. Those authors call the additional parameters as measures of the degree of chirality. Apart of this, Tokuoka introduced the concept of the gauge transformations (not to confuse with phase transformations) for the 4-spinor fields. He also found some strange properties of the anti-commutation relations (see §3 in [4a] and cf. [11b]). And finally, the equation (7) describes four states, two of which answer for the positive energy $E = |\mathbf{p}|$, and two others answer for the negative energy $E = -|\mathbf{p}|$.

I just want to add the following to the discussion. The operator of the chiral-helicity $\hat{\eta} = (\alpha \cdot \hat{\mathbf{p}})$ (in the spinorial representation) used in [4b] (and re-discovered in [11a]) does not commute, e.g., with the Hamiltonian of the equation (7)

$$[\mathcal{H}, \alpha \cdot \hat{\mathbf{p}}] = 2\frac{m_1 c}{\hbar} \left(1 - \gamma^5\right) \left(\gamma \cdot \hat{\mathbf{p}}\right). \quad (11)$$

For the eigenstates of the chiral-helicity the system of corresponding equations can be read (\(\eta = \uparrow, \downarrow\))

$$i\gamma^\mu \partial_\mu \Psi_\eta - \frac{m_1 c}{\hbar} \frac{1 + \gamma^5}{2} \Psi_{-\eta} = 0. \quad (12)$$

The conjugated eigenstates of the Hamiltonian $|\Psi_\uparrow + \Psi_\downarrow\rangle$ and $|\Psi_\uparrow - \Psi_\downarrow\rangle$ are connected, in fact, by $\gamma^5$ transformation $\Psi \rightarrow \gamma^5 \Psi \sim (\alpha \cdot \hat{\mathbf{p}}) \Psi$ (or $m_1 \rightarrow -m_1$). However, the $\gamma^5$ transformation is related to the $PT$ ($t \rightarrow -t$ only) transformation [4b], which, in its turn, can be interpreted as $E \rightarrow -E$, if one accepts the Stueckelberg idea about antiparticles. We associate $|\Psi_\uparrow + \Psi_\downarrow\rangle$ with the positive-energy eigenvalue of the Hamiltonian $E = |\mathbf{p}|$ and $|\Psi_\uparrow - \Psi_\downarrow\rangle$, with the negative-energy eigenvalue of the Hamiltonian $(E = -|\mathbf{p}|)$. Thus, the free chiral-helicity massless eigenstates may oscillate one to another with the frequency $\omega = E/\hbar$ (as the massive chiral-helicity eigenstates, see [10a] for details). Moreover, a special kind of interaction which is not symmetric with respect to the chiral-helicity states (for instance, if the left chiral-helicity eigenstates interact with the matter only) may induce changes in the oscillation frequency, like in the Wolfenstein (MSW) formalism.

\footnote{Do not confuse with the Dirac Hamiltonian.}
The question is: how can these frameworks be connected with the Ryder method of derivation of relativistic wave equations, and with the subsequent analysis of problems of the choice of normalization and of the choice of phase factors in the papers [10, 11, 12]? However, the conclusion may be similar to that which was achieved before: the dynamical properties of the massless particles (e. g., neutrinos and photons) may differ from those defined by the well-known Weyl and Maxwell equations.

Acknowledgments. I greatly appreciate old discussions with Prof. A. Raspini and useful information from Prof. A. F. Pashkov. This work has been partly supported by the ESDEPED.

References

[1] Gersten A, Maxwell Equations as the One-Photon Quantum Equation, 1999 Found. Phys. Lett. 12 291; Quantum Equations for Massless Particles of Any Spin, 2000 ibid. 13 185.

[2] Sakurai J J 1967 Advanced Quantum Mechanics (Addison-Wesley), §3.2 (see also footnote on p. 91 and Dvoeglazov V, Interacción 'Oscilador' de Partículas Relativistas, 2000 Investigación Científica 2 5).

[3] Dvoeglazov V, Generalized Maxwell Equations from the Einstein Postulate, 2000 J. Phys A: Math. Gen. 33 5011.

[4] Tokuoka Z, A Proposal of Neutrino Equation, 1967 Prog. Theor. Phys. 37 603; Sen Gupta N D, The Wave Equation for Zero Rest-Mass Particles, 1967 Nucl. Phys. B4 147; Santhanam T S and Chandrasekaran P S, Clifford Algebra and Massless Particles, 1969 Prog. Theor. Phys. 41 264; Fushchich V I, On the P- and T- Non-Invariant Two-Component Equation for the Neutrino, 1970 Nucl. Phys. B21 321; On the Three Types of Relativistic Equations for Particles with Nonzero Mass, 1972 Lett. Nuovo Cim. 4 344; Fushchich V I and Grishchenko A, On the CP-noninvariant Equations for the Particle with Zero Mass and Spins = 1/2, 1970 Lett. Nuovo Cim. 4 927; Simon M T, On Relativistic Wave Equations for Massless Particles of Spin 1/2, 1971 Lett. Nuovo Cim. 2 616; Santhanam T S and Tekumalla A R, Relativistic Wave Equations Describing the Neutrino, 1972 Lett. Nuovo Cim. 3 190.
[5] Raspini A, *The Physical Neutrino Current From a Duality Transformation*, 1994 Int. J. Theor. Phys. **33** 1503.

[6] Raspini A, *Dirac Equation with Two Mass Parameters*, 1996 *Fizika B* **5** 159.

[7] Raspini A, *Modified Massless Dirac Equation*, 1997 *Fizika B* **6** 123.

[8] Raspini A, *Could Neutrino Flavours be Superpositions of Massive, Massless and Tachyonic States?*, 1998 *Fizika B* **7** 83.

[9] Raspini A, 1999 *A Review of Some Alternative Descriptions of Neutrino*. In *Photon and Poincaré Group*. Ed. V. Dvoeglazov. Series Contemporary Fundamental Physics (Commack, NY: Nova Science), p. 181-188.

[10] Dvoeglazov V, *Neutral Particles in Light of the Majorana-Ahluwalia Ideas*, 1995 *Int. J. Theor. Phys.* **34** 2467; *The Second-Order Equation from the (1/2, 0) ⊕ (0, 1/2) Representation of the Poincare Group*, 1998 *ibid* **37** 1909; *Extra Dirac Equations*, 1996 *Nuovo Cim.* **B111** 483; *Lagrangian for the Majorana-Ahluwalia Construct*, 1995 *ibid*. **A108** 1467; *Self/Anti-self Charge Conjugate States for j = 1/2 and j = 1*, 1997 *Hadronic J.* **20** 435; *Significance of the Spinorial Basis in Relativistic Quantum Mechanics*, 1997 *Fizika B* **6** 111; *Majorana–like Models in the Physics of Neutral Particles*, 1997 *Adv. Appl. Clifford Algebras* **7**(C) 303; *Unusual Interactions of the (1/2, 0) ⊕ (0, 1/2) Particles*, 1999 *ibid* **9** 231; *Gauge Transformations for Self/Anti-Self Charge Conjugate States*, 1998 *Acta Physica Polon.* **B29** 619; *Addendum to “The Relativistic Covariance of the B-Cyclic Relations*, 2000 *Found. Phys. Lett.* **13** 387.

[11] Ahluwalia D V., *Theory of Neutral Particles: McLennan-Case Construct for Neutrino, Its Generalization, and a Fundamentally New Wave Equation*, 1996 *Int. J. Mod. Phys.* **A11** 1855; *Nonlocality and Gravity-induced CP Violation*, 1998 *Mod. Phys. Lett.* **A13** 3123.

[12] Dvoeglazov V V, *Normalization and the m → 0 limit of the Proca theory*, 2000 *Czech. J. Phys.* **50** 225; *Longitudinal Nature of Antisymmetric Tensor Field After Quantization and Importance of the Normalization*. In *Photon: Old Problems in Light of New Ideas*. Ed. V. V. Dvoeglazov, pp. 319-337. Series Contemporary Fundamental Physics (Huntington, NY: Nova Science Publishers).