Microscopic calculations of elastic scattering between light nuclei based on a realistic nuclear interaction

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Abstract. The elastic phase shifts for the $\alpha + \alpha$ and $\alpha + ^3\text{He}$ collisions are calculated in a cluster approach by the Generator Coordinate Method coupled with the Microscopic $R$-matrix Method. Two interactions are derived from the realistic Argonne potentials AV8' and AV18 with the Unitary Correlation Operator Method. With a specific adjustment of correlations on the $\alpha + \alpha$ collision, the phase shifts for the $\alpha + \alpha$ and $\alpha + ^3\text{He}$ collisions agree rather well with experimental data.

1. Introduction

The aim of this work is to give a microscopic description of elastic collisions based on a realistic interaction while keeping rather simple basis wave functions. We choose to limit our approach to light colliding nuclei and we illustrate it on the $\alpha + \alpha$ and $\alpha + ^3\text{He}$ collisions.

For a long time the elastic scattering between light nuclei has been studied in the framework of the Generator Coordinate Method (GCM) [1]. In this model, the system made up of the colliding nuclei is described by a superposition of antisymmetrized states of two clusters separated by a variable distance. Each cluster is described by a Slater determinant of harmonic-oscillator states. After elimination of a Gaussian center-of-mass (c.m.) factor, the wave function of the system becomes invariant under translation (if the oscillator parameters of the two clusters are equal) and can be projected on angular momentum and parity [2]. While it has the required symmetries, the wave function does not have the correct asymptotic behavior of a scattering wave function. This problem is solved by the Microscopic $R$-matrix Method (MRM) [3, 4].

With the GCM, a large number of elastic collisions are described with a good precision [5, 6]. However, such results are obtained by using phenomenological interactions and fitting one or two parameters for each collision or each partial wave.

To obtain a more physical description of collisions and to approach an ab initio calculation, the phenomenological interaction has to be replaced by a realistic interaction reproducing nucleon-nucleon phase shifts and properties of the deuteron, such as the Argonne [7, 8] interactions. However, the study of elastic scattering with a realistic interaction is much more difficult and only a few collisions have been studied up to now [9, 10, 11].

The GCM basis is not suitable for realistic interactions. Thus, we adapt the wave function by applying a unitary operator or equivalently we adapt the interaction by applying the

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corresponding unitary transform, following the Unitary Correlation Operator Method (UCOM) [12, 13]. This method was used with success in nuclear structure calculations based on a realistic interaction [14] and very recently for reactions [11].

In this paper, we apply this correlation method to elastic scattering. To be specific, we study microscopically with the GCM coupled with the MRM, the $\alpha + ^3\text{He}$ and $\alpha + \alpha$ collisions with correlated interactions coming from the Argonne potentials AV8' [7] and AV18 [8].

2. Generator Coordinate Method

The microscopic approach is based on the Schrödinger equation for $A$ nucleons

$$ H\Psi(1, 2, \ldots, A) = E\Psi(1, 2, \ldots, A), $$

with

$$ H = \sum_{i=1}^{A} t_i - T_{cm} + \sum_{i<j}^{A} [v_{ij}^N + v_{ij}^C], $$

where $t_i$ is the kinetic energy of nucleon $i$, $T_{cm}$ is the c.m. kinetic energy, $v_{ij}^N$ is the nuclear interaction (the AV8' or AV18 Argonne potential) and $v_{ij}^C$ is the Coulomb potential.

The nuclei are described by Slater determinants of $0s$ states of harmonic oscillator with oscillator parameter $b$, noted $\phi_{S_1}$ and $\phi_{S_2}$. $S_1$ and $S_2$ designate the spins of nuclei 1 and 2.

The collision wave function $\Psi$ is expanded into the GCM basis [1]. The GCM basis functions are projected Slater determinants given by

$$ \psi_{JM}^{I_S}(R) = A \left[ \left( \phi_{S_1} \otimes \phi_{S_2} \right)^S \otimes Y_l(\rho_R) \right]_M^J \Gamma_l(\rho, R), $$

where $J$ is the total angular momentum, $M$ is its projection, $l$ is the orbital angular momentum of the relative motion of nuclei and $S$ is the total spin. $A$ is the antisymmetrization projector. $\rho = (\rho, \Omega_\rho)$ is the relative coordinate between the nuclei. $R$ is called the generator coordinate. The $\Gamma_l$ function is given by

$$ \Gamma_l(\rho, R) = \left( \frac{\mu}{\pi b^2} \right)^{3/4} e^{-\mu(\rho^2 + R^2)/2b^2} i_l \left( \frac{\mu\rho R}{b^2} \right), $$

where $i_l$ is a spherical Hankel function and $\mu$ is the reduced mass.

The phase shifts calculations can be performed systematically and fast with the GCM which makes this method very efficient. However, the GCM basis is not able to describe properly the colliding nuclei for realistic interactions. Indeed, a Gaussian wave function is not suitable for the repulsive core and the tensor component of the realistic nucleon-nucleon potential has no effect on the energy with this type of cluster. These problems are tackled with the UCOM. Moreover, the GCM wave function has not the correct asymptotic behavior of a scattering function. It is corrected by the MRM.

3. Unitary Correlation Operator Method

The aim of the UCOM [12, 13] is to insert the correlation with a unitary operator $C$ in the uncorrelated GCM states $\Psi$

$$ \tilde{\Psi} = C\Psi. $$

The tilde symbol will be systematically used to represent a correlated object. $C$ is the product of a radial unitary correlator $C_r$ [12] and a tensor unitary correlator $C_\Omega$ [13]

$$ C = C_\Omega C_r. $$
The radial correlator modifies the wave function to take the repulsive core of the nuclear potential into account and the tensor correlator adds a D-wave component to the clusters.

In an equivalent way, the UCOM can be used to create an effective realistic interaction since

$$\langle \tilde{\Psi} | H | \tilde{\Psi} \rangle = \langle \Psi | C^\dagger H C | \Psi \rangle.$$  \hspace{1cm} (7)

The correlated Hamiltonian $C^\dagger H C$ has the same eigenvalues as $H$ and is phase equivalent to $H$. Moreover, $C^\dagger H C$ holds the symmetries of reflection, rotation and time reversal as $H$, because $C$ holds them too. $C^\dagger H C$ has the advantage to have a softer repulsive core and to be more suitable for the GCM basis. But $C^\dagger H C$ has the drawback to contain three- and more-body terms even if $H$ is limited to two-body terms

$$C^\dagger H C = \tilde{H}^{[1]} + \tilde{H}^{[2]} + \tilde{H}^{[3]} + \ldots$$ \hspace{1cm} (8)

To keep the calculations relatively easy, we neglect the three- and more-body terms.

4. Microscopic $R$-matrix Method

The principle of the MRM [3, 4] is to divide the configuration space into two regions at the channel radius $a$. In the internal region ($\rho < a$), the wave function is expanded as a sum of $N$ correlated GCM basis functions

$$\tilde{\Psi}_{\text{int}} = \sum_n f_n^{JSM} \tilde{\psi}_{JM} \left( \rho_n \right),$$ \hspace{1cm} (9)

where $f_n^{JSM}$ are generator parameters and $\rho_n$ are selected discrete values of the generator coordinate $R$.

In the external region ($\rho > a$), the antisymmetrization is neglected and the interaction between the two clusters is considered purely Coulombic

$$\tilde{\Psi}_{\text{ext}} = \phi_{\text{c.m.}} \left[ \tilde{\phi}_{S_1} \otimes \tilde{\phi}_{S_2} \right]^S \otimes Y_J (\Omega_{\rho}) \left[ \cos \delta_{IJ} F_I (\eta, k \rho) + \sin \delta_{IJ} G_I (\eta, k \rho) \right],$$ \hspace{1cm} (10)

where $\delta_{IJ}$ is the $IJ$-wave phase shift, $\eta$ is the Sommerfeld parameter, $k$ is the wavenumber and $F_I$ and $G_I$ are the regular and irregular Coulomb functions. The phase shifts are calculated by matching the internal and external pieces of the wave function at the channel radius.

5. Elastic phase shifts

The radial and tensor correlators are adjusted on the $\alpha + \alpha$ phase shifts. The calculated phase shifts are compared in Fig. 1 with experimental data [15].

The adjustment is made for AV8'. So, we detail the results for this case. The results for AV18 are only slightly different. The calculated $0^+$ phase shifts reproduce the experimental data, including the energy of the resonance at 92 keV with a precision of about 1 keV. The $2^+$ phase shifts are still in good agreement with the experimental data while the $4^+$ phase shifts are smaller than experiment. The too high $4^+$ resonance energy is probably explained by the excessive simplicity of our wave function. The space of basis functions is too limited to reproduce the rotational constant of the $\alpha + \alpha$ rotational band.

By keeping the same correlators, thus without specific adjustment, we calculate the $\alpha + ^3\text{He}$ phase shifts. These are compared in Fig. 2 with experimental data [16]. The calculated phase shifts reproduce the general behavior of the experimental data. The precision is very good for the $1/2^-$ and $3/2^-$ waves and slightly less good for the $1/2^+$ wave.

For this nuclear system, the comparison with experiment can be extended to the $^7\text{Be}$ bound states. For the AV8’ potential, the bound-state energies are $E_{3/2^-} = -1.348$ MeV.
Figure 1. Elastic phase shifts for the $\alpha + \alpha$ collision. The phase shifts are calculated for the interactions derived from AV8' (-----) and AV18 (- - - -). Experimental data [0$^+$($\Pi$), 2$^+$($\Omega$) and 4$^+$($\Delta$)] from [15].

Figure 2. Elastic phase shifts for the $\alpha + ^3$He collision. The phase shifts are calculated for the interactions derived from AV8' (-----) and AV18 (- - - -). Experimental data [1/2$^+$($\Omega$), 1/2$^-$($\Omega$) and 3/2$^-$($\Delta$)] from [16]. The 3/2$^-$ phase shifts are shifted by $-90^\circ$ for clarity.

and $E_{1/2^{-}} = -0.764$ MeV with respect to the threshold energy. They are slightly higher than the experimental values $E_{3/2^{-}} = -1.587$ MeV and $E_{1/2^{-}} = -1.156$ MeV. These results display an improvement of the GCM model by using a realistic interaction. The Minnesota potential with its parameters adjusted to the $\alpha + ^3$He phase shifts leads to strongly overbound states [6].

There exists no ab initio study of the $\alpha + ^3$He collision. However, a calculation of the elastic phase shifts based on a UCOM potential derived from the AV18 potential has recently been performed [11]. In spite of a much larger basis of wave functions, the phase shifts do not agree significantly better with experiment than ours and the bound-state energies are only slightly improved.

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