Design of modified 2-degree-of-freedom proportional–integral–derivative controller for unstable processes

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Abstract
Concerning first-order unstable processes with time delays that are typical in chemical processes, a modified 2-degree-of-freedom proportional–integral–derivative control method is put forward. The system presents a two-loop structure: inner loop and outer loop. The inner loop is in a classical feedback control structure with a proportional controller intended for implementing stable control of the unstable process; the outer loop is in a 2-degree-of-freedom structure with feedforward control of set points, where the system’s tracking response of set points is separated from its disturbance response. To be specific, the system has a feedforward controller that is designed based on the controlled object models and mainly used for regulating the system’s set point tracking characteristics; besides, it has a feedback controller that is designed on the ground of direct synthesis of disturbance suppression characteristics to improve the system’s disturbance rejection. To verify the effectiveness, the system is put into a theoretical analysis and simulated comparison with other methods. Simulation results show that the system has good set point tracking characteristics and disturbance suppression characteristics.

Keywords
Unstable processes with time delays, 2-degree-of-freedom, set point tracking, disturbance suppression

Introduction
Proportional–integral–derivative (PID) controller remains one of the most universal control methods used in current industrial production due to its advantages of a simple control structure, sound robustness, and reliability among others.1–3 However, it is challenging to make effective control in the traditional PID method for unstable processes with time delays. To cope with such unstable processes with time delays, De Paor and O’Malley4 proposed a Z-N-structure PID controller tuning method; Shafiei and Shenton5 raised a graphical method by PID controller using D-divide method; and some advanced methods are targeted at common unstable integral processes and first-order processes with time delays6,7 but such methods do not suffice regarding the control of system performance.

To design a controller for practical industrial control processes, in addition to stable control of the unstable processes, focus should also be given to optimization of system performance indicators, among which set point tracking characteristics and disturbance suppression characteristics dominate. Compared with the traditional PID control, the internal model control (IMC)8–10 works better in terms of control of processes with delays; furthermore, a modified IMC method is proposed in Yang et al.11 and Tan et al.12 to handle unstable processes with time delays, where the IMC is designed by Pade approximation. The IMC control structure is kept and the IMC sets parameters in a clear way; however, the IMC only has one adjustable parameter, thus it fails to access set point tracking characteristics and disturbance suppression characteristics at the same time, which is a performance compromise method.13 To guarantee stable operation and sustainable controlled performance of unstable processes with
time delays, the idea of 2-degree-of-freedom PID control is proposed and promoted\cite{14-16} and put into practice. In recent years, many scholars have put forward 2-degree-of-freedom-based modified control methods one after another. Limebeer et al.\cite{17} came up with the method of designing a robust 2-degree-of-freedom controller, where the system’s feedback controller and prediction filter are separately designed to make the system have a greater performance in design. Prempain and Bergeon\cite{18} raised a method of designing a 2-degree-of-freedom controller based on Youla parameterization,\cite{19} which suppresses internal disturbance and external disturbance in a multi-variable control system through H-infinity optimization\cite{20} to solve unstructured uncertainty problems. Based on the 1-degree-of-freedom-to-2-degree-of-freedom span, a double 2-degree-of-freedom control method\cite{21,22} is also put forward, where four independent controllers are designed for stabilizing the open-loop unstable or integral processes with delay, improving set point response tracking performance and enhancing the system’s disturbance suppression characteristics. In this method, the system’s performance can be well improved, but its 4-controller design structure is too complicated. In contrast, Rico and Camacho\cite{23,24} offer a modified 2-degree-of-freedom controller method based on Smith predictor structure, which has a much simpler controller structure. In this method, a Smith prediction filter and a preset controller are designed for predictive compensation of the processes delays\cite{25} and set point tracking response. This method can decouple and tune the system performance in spite of exceptional unstable processes but is dependent on the precision of controlled models in the system. Since the IMC structure bears a great similarity with Smith control, an optimal H2 PID controller\cite{26} is designed in line with the IMC principle for unstable processes with right half plane (RHP) zero poles and time delays, whose PID parameters\cite{27} are accessed by Maclaurin series approximation or stability controller parameters\cite{28} accessed in the Blaschke product method; moreover, the controller’s closed-loop control performance of unstable processes with time delays is enhanced in conjunction with rules for adjustment of maximum sensitivity tuning parameters. In Humaidi and Hasan,\cite{29} 2 two-degree-of-freedom control strategy is combined with sliding mode control. Meanwhile, to improve the accuracy of real-time recognition of the permanent magnet synchronous motor model and its control performance over servo drive systems, a fractional-order and generalized predictive control (GPC) 2-degree-of-freedom PI control method is presented in Qiao et al.,\cite{30} but it is hardly implemented in practice since fractional control\cite{31} and GPC\cite{32} involve too many setting parameters. In addition, many scholars combine intelligent optimization with PID control\cite{33-35} to effectively modify the performance of control systems. However, such control algorithms involve too much calculation and are hardly implemented in practical systems.

Concerning common first-order unstable processes with time delays in chemical production, a modified 2-degree-of-freedom PID control structure is put forward in this paper. This structure comprises three parts: an internal setting controller, a feedforward controller, and a feedback controller. Based on the controlled object models, the feedforward controller can make the entire system have sound set point tracking characteristics while the feedback controller is mainly designed by direct synthesis of disturbance suppression characteristics. In this way, the control system can stabilize unstable processes while acquiring good set point tracking and disturbance suppression. The paper is organized as follows. Section “Problem statement” gives a brief description to typical feedback control and traditional 2-degree-of-freedom control structures. In section “Modified 2-degree-of-freedom control,” the modified 2-degree-of-freedom PID control structure is designed on the basis of unstable processes. Section “Design of controllers” is concerned with the details of design analysis on internal stability controller, feedforward controller, and feedback controller. Section “Simulation” illustrates the superiority of the method through simulation and comparison. This paper ends with a conclusion in section “Conclusion.”

**Problem statement**

In practical industrial process control, many factors need to be simultaneously considered in terms of control performance to meet the expected control requirement. To this end, 2-degree-of-freedom control is an appropriate design method, where two independent controllers with independent parameters are designed to optimize the system’s disturbance suppression performance and set point tracking performance at the same time. It is different from 1-degree-of-freedom control methods that can only reach a compromise between the two performance indicators and hence fails to yield satisfactory control effects.

**Typical feedback control**

A typical feedback control structure is shown in Figure 1.

In Figure 1, \( r, d, n, \) and \( y \) represent the set point, disturbance input, noise input, and system output,
respectively; \( G_c(s) \) represents the controller while \( G_p(s) \) represents the controlled process.

Through Figure 1, the closed-loop transfer function between the system output and set point is

\[
G_{yr}(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(1)

A closed-loop transfer function between the system output and disturbance is

\[
G_{yd}(s) = \frac{G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(2)

A closed-loop transfer function between the system output and noise input is

\[
G_{yn}(s) = \frac{-G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(3)

Through an analysis on equations (1)–(3), any one closed-loop transfer function can be used to derive the other two functions. Therefore, this system is a 1-degree-of-freedom control system that cannot give a consideration to both set point tracking performance and disturbance suppression performance at the same time.

**Traditional 2-degree-of-freedom control**

Based on the typical feedback control, a 2-degree-of-freedom control structure is designed as shown in Figure 2.

In Figure 2, \( F(s) \) represents the set point filter.

Similarly, such closed-loop transfer functions as \( G_{yr}(s), G_{yd}(s), \) and \( G_{yn}(s) \) are derived from Figure 2

\[
G_{yr}(s) = \frac{F(s)G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(4)

\[
G_{yd}(s) = \frac{G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(5)

\[
G_{yn}(s) = \frac{-G_c(s)G_p(s)}{1 + G_c(s)G_p(s)}
\]  

(6)

Equations (4)–(6) show that the feedforward transfer function is only related to \( G_{yr}(s) \) and any two of \( G_{yd}(s), \) \( G_{yd}(s), \) and \( G_{yn}(s) \) can be independently adjusted. Besides, disturbance suppression characteristics are related to the controller \( G_c(s) \) only. Therefore, in the controller design and setting, sound disturbance suppression performance of the controller \( G_c(s) \) is the primary factor in consideration, followed by fulfillment of the expected set point tracking performance through the design of \( F(s) \).

**Modified 2-degree-of-freedom control**

For unstable processes with time delays, the control structure is shown in Figure 3. Here, \( P(s) \) represents the controlled process, \( C_1(s) \) represents the feedforward controller, \( C_2(s) \) represents the feedback controller, \( r \) represents the input of the control system, \( y \) represents the output of the control system, \( d \) represents disturbance signals, and \( C_3(s) \) represents the inner loop feedback controller. The entire dotted box shows how the generalized controlled process \( G_p(s) \) is derived from the controller \( C_3(s) \) acting on the controlled process \( P(s) \).

Through Figure 3, the closed-loop transfer function between the system output and input is

\[
G_{yr}(s) = \frac{C_2(s)G_p(s)}{1 + C_2(s)G_p(s)} + \frac{C_1(s)G_p(s)}{1 + C_2(s)G_p(s)}
\]  

(7)

\[
G_{yd}(s) = \frac{G_p(s)[C_1(s) + C_3(s)]}{1 + C_2(s)G_p(s)}
\]  

(8)

A transfer function between the process output and disturbance is

Equations (7) and (8) indicate that the controller \( C_1(s) \) incurs no impact on the disturbance of the processes but \( C_2(s) \) bears a relation with the system’s set point tracking and disturbance suppression characteristics. If \( C_1(s)G_p(s) = 1 \) and there is no disturbance signal, the output of the generalized processes will be equal to the input of such processes, indicating that the system is an open-loop control system with good set point tracking characteristics.

**Design of controllers**

**Stability controller**

To stabilize the controlled processes, the design of a controller \( C_3(s) \) is needed in the first place.
The controlled process is defined as follows:

\[ G_p(s) = \frac{P(s)}{1 + C_s(s)P(s)} \]  

Here, \( C_s(s) \) serves for the stabilization of the process first. The proportional \((P)\) controller has a simple and implementable structure. \( C_s(s) \) is set to be the \( P \) controller valued by proportional gain \( K_3 \).

The controlled process \( P(s) \) is an unstable process with time delays: \( P(s) = (K/(Ts - 1))e^{-\theta s} \).

Then, the generalized controlled process is as follows:

\[ G_p(s) = \frac{P(s)}{1 + C_s(s)P(s)} = \frac{Ke^{-\theta s}}{Ts - 1 + K_3Ke^{-\theta s}} \]  

The time delay item in the denominator of equation (10) is expanded and approximated by Taylor series as follows:

\[ e^{-\theta s} \approx 1 - \theta s \]  

Furthermore, \( G_p(s) \) can be derived as follows:

\[ G_p(s) = \frac{Ke^{\theta s}}{(T - K_3K\theta)s + (K_3K - 1)} \]  

A characteristic equation is extracted from equation (12), where the value range of \( K_3 \) can be derived by Hurwitz stability criterion to stabilize the system

\[ \frac{1}{K} < K_3 < \frac{T}{K\theta} \]  

Within the range of equation (13), the value of \( K_3 \) is adjusted and selected to guarantee the system’s closed-loop response characteristics.

**Feedforward set point tracking controller**

The controller \( C_1(s) \) is designed to decompose \( G_p(s) \)

\[ G_p(s) = G_{p+}(s)G_{p-}(s) \]  

In equation (15), \( n \) represents the order of the controller that can be implemented. Since first-order processes are targeted, \( n \) is selected as 1 and \( l \) represents the to-be-set parameter.

**Feedback disturbance suppression controller**

Last but not least, a controller \( C_2(s) \) is designed.

Through equation (8), the form of \( C_2(s) \) is derived as follows:

\[ C_2(s) = \frac{G_p(s) - G_{yd}(s)}{G_p(s)G_{yd}(s)} \]  

According to equation (16), where the generalized controlled process or the generalized model is obtained after \( C_3(s) \) controls the unstable process, the controller \( C_2(s) \) can be designed so long as the closed-loop transfer function between the output and disturbance in the process is known.

It is considered to design the controller \( C_2(s) \) as a PID controller

\[ C_2(s) = K_p\left(1 + \frac{1}{Ts} + TDs\right) \]  

Equations (12) and (17) are substituted into equation (8) to derive

\[ G_{yd}(s) = \frac{G_p(s)}{1 + C_2(s)G_p(s)} = \frac{Ke^{-\theta s}}{1 + \frac{Ke^{-\theta s}}{(T - K_3K\theta)s + (K_3K - 1)K_p\left(1 + \frac{1}{Ts} + TDs\right)}} \]  

The time delay item of the denominator in equation (18) is approximated by Padé approximation as follows:

\[ e^{-\theta s} \approx \frac{1 - 0.5\theta s}{1 + 0.5\theta s} \]  

Equation (19) is simplified and substituted into equation (18) to derive

\[ G_{yd}(s) = \frac{\frac{1}{Ts}(1 + 0.5\theta s)e^{-\theta s}}{\left(\frac{T - K_3K\theta}{kk_\theta} - 0.5\theta T;TD\right)s^2 + \left(\frac{T - K_3K\theta}{kk_\theta} + 0.5\theta T;TD\right)s + \left(\frac{T - K_3K\theta}{kk_\theta} + T;TD - 0.5\theta T\right)s^2 + \left(\frac{T;TD - 0.5\theta T}{kk_\theta} + T;D - 0.5\theta \right)s + 1} \]  

According to equation (20), the controller \( C_2(s) \) is designed by direct synthesis of disturbance suppression given in Vilanova et al.\(^1\) The closed-loop transfer function of the expected disturbance suppression performance is \( Q_{yd}(s) \), which is in the following form

\[ Q_{yd}(s) = \frac{\frac{1}{Ts}(1 + 0.5\theta s)e^{-\theta s}}{(\zeta s + 1)^2} \]  

In formula (21), \( \zeta \) represents the to-be-adjusted parameter.
Through a combination of formulas (20) and (21), the PID controller parameter of the controller \( C_2(s) \) is solved as follows where the coefficients before the same power are equal

\[
T_I = \frac{(3\xi + 0.5\theta)(1.5\theta - K_3K\theta^2 - 0.5\theta^2) - (\xi^3 + 1.5\theta\xi^2)(K_3K - 1)}{[1.5T - K_3K\theta - 0.5\theta + 0.25(K_3K - 1)\theta]\theta}
\]

(22)

The value of \( T_I \) is worked out then the other two related parameters can be solved as follows

\[
K_P = \frac{(K_3K - 1)T_I}{K(3\xi + 0.5\theta - T_I)}
\]

(23)

\[
T_D = \frac{0.5\theta(T - K_3K\theta)\xi + 0.5\theta - T_I + \xi^3}{0.5\theta T_I}
\]

(24)

**Simulation**

In Liu et al., \(^{36}\) a first-order unstable process with time delay is considered

\[
P(s) = \frac{e^{-0.4s}}{s - 1}
\]

Here, \( T, K, \) and \( \theta \) are 1, 1, and 0.4, respectively. In the controller design methods set out in section “Design of controllers,” the following can be derived through equation (13): \( 1 < K_3 < 2.5 \), wherein \( K_3 \) is equal to 2. Therefore, equation (12) is presented as follows

\[
G_P(s) = \frac{e^{-0.4s}}{0.2s + 1}
\]

From equations (14) and (15), the form of the controller \( C_1(s) \) is derived as follows

\[
C_1(s) = \frac{0.2s + 1}{\lambda s + 1}
\]

(22)

Through equations (22)–(24), respectively, as \( T_I = 0.58, K_P = 1.12, \) and \( T_D = 0.41. \) For a better evaluation on the performance, a disturbance is added at \( t = 25 \) s. Then, the method is compared with the method proposed by Liu et al. \(^{36}\) (where, \( \lambda_c = \lambda_f = 0.4, K_P = 2.90, T_I = 0.72, T_D = 0.47 \)). The responses are shown in Figure 4, where they have given nearly the same set point tracking; however, the proposed method can yield very good control effects in set point tracking and disturbance suppression at the same time without any over-adjustment.

Note that practical processes may not match the models; it is supposed that the maximum mismatch degree of parameters is ±20% to produce three sets of mismatch parameters randomly using the Monte Carlo method:

Set 1: Mismatch process parameters: \( K = 1.1904, T = 0.9223, \theta = 0.4638; \)

Set 2: Mismatch process parameters: \( K = 0.8324, T = 0.8583, \theta = 0.4611; \)

Set 3: Mismatch process parameters: \( K = 1.0325, T = 0.9283, \theta = 0.4106. \)

Figures 5–7 show the responses under model/plant mismatch cases. Through a comparison between the proposed method in this paper and that of the method proposed by Liu et al., \(^{36}\) it is found that in the event of model/plant mismatch, the method in this paper can
ensure that the closed-loop system is stable with good disturbance suppression.

Conclusion
A modified 2-degree-of-freedom PID control structure is proposed in this paper for first-order unstable processes with time delays that are common in chemical engineering. This structure has three parts: an internal setting controller, a feedforward controller, and a feedback controller. Based on the feedforward controller designed, the entire system has sound set point tracking performance. The feedback controller is mainly designed by direct synthesis of disturbance suppression characteristics. This design method can ensure system stability while obtaining good set point tracking and disturbance suppression. It is characterized by a simple control structure and convenience in controller parameter setting.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was partially supported by the Science and Technology Research Project of Zhejiang (2018R52029) and the Hong Kong Research Grants Council (16207717).

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