Anomalous $U(1)$ Symmetry and Lepton Flavor Violation

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Abstract

We show that in a large class of models based on anomalous $U(1)$ symmetry which addresses the fermion mass hierarchy problem, leptonic flavor changing processes are induced that are in the experimentally interesting range. The flavor violation occurs through the renormalization group evolution of the soft SUSY breaking parameters between the string scale and the $U(1)_A$ breaking scale. We derive general expressions for the evolution of these parameters in the presence of higher dimensional operators. Several sources for the flavor violation are identified: flavor–dependent contributions to the soft masses from the $U(1)_A$ gaugino, scalar mass corrections proportional to the trace of $U(1)_A$ charge, non–proportional $A$–terms from vertex corrections, and the $U(1)_A D$–term. Quantitative estimates for the decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are presented in supergravity models which accommodate the relic abundance of neutralino dark matter.

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1 Introduction

The Standard Model, while highly successful in explaining all experimental data, does not provide an explanation for the observed hierarchy in the masses and mixings of quarks and leptons. Extended symmetries are often speculated to address this problem. Family-dependent $U(1)$ symmetry is a widely studied extension. An attractive scenario is the Froggatt–Nielsen scheme \[1\]. In this scenario all the Yukawa couplings are assumed to be of order one, but the ones which generate the light fermion masses arise only as nonrenormalizable operators suppressed by powers of a small parameter $\epsilon \equiv \langle S \rangle / M$, where $\langle S \rangle$ is the flavor symmetry breaking order parameter and $M$ is a more fundamental mass scale. With the flavor $U(1)$ charges of fermions differing only by order one, large hierarchy factors, such as $m_u/m_t \sim 10^{-6}$, are explained.

A natural origin for the flavor $U(1)$ symmetry is the anomalous $U(1)_A$ gauge symmetry of string theory \[2\]. The small expansion parameter $\epsilon$ arises in a natural way in anomalous $U(1)$ models through the Fayet–Iliopoulos term induced by the gravitational anomaly \[3\]. Such models have been extensively studied in the literature for understanding the fermion mass hierarchy puzzle \[4, 5\]. The purpose of this paper is to address flavor changing neutral currents in this class of models.

In the presence of low energy supersymmetry (SUSY), a family–dependent anomalous $U(1)_A$ symmetry will induce flavor changing processes, even when there is no flavor violation in the soft SUSY breaking parameters at the fundamental scale – taken to be the string scale. Such violations will be generated through the renormalization group evolution (RGE) of the SUSY breaking parameters between $M_{\text{string}}$ and the $U(1)_A$ breaking scale $\langle S \rangle$. We derive general expressions for the evolution of these parameters in the presence of higher dimensional operators. Our results can be applied to a wide class of Froggatt–Nielsen models. We have found several sources of flavor violation. In the momentum interval $\langle S \rangle \leq \mu \leq M_{\text{string}}$, the $U(1)_A$ gaugino is active and will contribute differently to the soft masses of different families. Because $\text{Tr} U(1)_A$ is not zero in anomalous $U(1)$ models, there are nonuniversal RGE contributions to the soft scalar masses arising from the $D$–term proportional to the respective flavor charges. Furthermore, the trilinear $A$–terms will receive vertex corrections from the $U(1)_A$ gaugino that are not proportional to the respective Yukawa couplings. In addition to these RGE effects, upon symmetry breaking, the $D$–term associated with the $U(1)_A$ will also induce nonuniversal masses for the sfermions \[6–8\].

In this paper we investigate the combined effects of nonuniversality for lepton flavor violating (LFV) decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in a class of anomalous $U(1)_A$ models. Quantitative predictions for the branching ratios are presented in two specific models of fermion mass hierarchy. We find that the branching ratio for $\mu \to e\gamma$ is around the current experimental limit, while $\tau \to \mu\gamma$ may be accessible in the future. In our analysis we also include the right–handed neutrino–induced LFV effects, which have been widely studied in the literature \[9–11\]. These effects turn out to be significant in some but not all cases that we study. In fact, within our framework, if leptogenesis is assumed to be the source of the observed cosmological baryon asymmetry \[12\], it turns out that the right–handed neutrino induced LFV effects are negligible. Related effects from SUSY GUT thresholds for LFV have been studied in Ref. \[13\]. The flavor violating effects are more prominent in the leptonic sector compared to the quark sector since quark flavor violation is diluted somewhat by the gluino focusing effects which arise during the evolution of the
SUSY breaking parameters below the $U(1)_A$ breaking scale. This is especially so when the SUSY parameter are chosen, as we do, such that the lightest neutralino is the dark matter with an acceptable cosmological abundance.

The structure of the paper is as follows. In Section 2 we introduce our models. In 2.1 we describe our models of fermion mass hierarchy, in 2.2 we present the details of anomalous $U(1)$ models. In 2.3 we present our fermion mass fits for two models, Model 1 and Model 2. In 2.4 we discuss baryogenesis through leptogenesis in these models. In Section 3 we derive generalized expressions for the evolution of the soft SUSY breaking parameters appropriate for a class of Froggatt–Nielsen models. Section 4 is devoted to the numerical analysis of the branching ratios for the processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$. In 4.1 we outline the qualitative features of flavor violation arising from various sources. 4.2 has our numerical fits. Figures 3 and 4 show our results for the branching ratios for $\mu \to e\gamma$ in the two models of fermion masses. In 4.3 we address the $D$-term splitting problem. Our conclusions are given in Section 5.

2 Fermion Mass Matrices and Anomalous $U(1)_A$ Flavor Symmetry

In this section we present two specific models of fermion mass hierarchy based on a flavor-dependent anomalous $U(1)_A$ symmetry using the Froggatt–Nielsen mechanism [1]. The $U(1)_A$ symmetry is broken by an MSSM singlet flavon filed $S$ slightly below the string scale ($M_{st}$). This provides the small expansion parameter $\epsilon = \langle S \rangle / M_{st}$ needed for explaining the fermion mass hierarchy. We also present explicit models of anomalous $U(1)_A$ symmetry.

2.1 Fermion Mass Hierarchy

A general form of the superpotential which can explain the fermion masses and mixing hierarchy through the Froggatt–Nielsen mechanism has the form

$$W = \frac{y_{ij}^u}{n_{ij}^u} Q_i u_j^c H_u \left( \frac{S}{M_{st}} \right)^{n_{ij}^u} + \frac{y_{ij}^d}{n_{ij}^d} Q_i d_j^c H_d \left( \frac{S}{M_{st}} \right)^{n_{ij}^d} + \frac{y_{ij}^e}{n_{ij}^e} L_i e_j^c H_d \left( \frac{S}{M_{st}} \right)^{n_{ij}^e} + \frac{y_{ij}^\nu}{n_{ij}^\nu} L_i \nu_j^c H_u \left( \frac{S}{M_{st}} \right)^{n_{ij}^\nu} + \mu H_u H_d ,$$

where $i, j = (1, 2, 3)$ are family indices, $n_{ij}^u, n_{ij}^d, n_{ij}^e, n_{ij}^\nu$ and $n_{ij}^\nu$ are positive integers fixed by the choice of $U(1)_A$ charge assignment. $y_{ij}^u$ etc. are Yukawa coupling coefficients which are all taken to be of order one. Here $M_R$ is the right-handed ($\nu^c_j$) neutrino mass scale. We use the standard notation for the MSSM fields.

The soft supersymmetry breaking terms which will induce LFV have the form given by

$$-L_{soft} = \left\{ \frac{a_{ij}^u}{n_{ij}^u} Q_i u_j^c H_u \left( \frac{S}{M_{st}} \right)^{n_{ij}^u} + \frac{a_{ij}^d}{n_{ij}^d} Q_i d_j^c H_d \left( \frac{S}{M_{st}} \right)^{n_{ij}^d} \right\}.$$
Here a tilde stands for the scalar components of the matter superfields, and \( \lambda_i \) and \( \lambda_F \) are the MSSM gauginos and the flavor \( U(1)_A \) gaugino. \( (M_{1/2}^i, M_{\lambda_F}) \) and \( ((\tilde{m}_s^2)_{ab}, \tilde{m}_g^2) \) are the gaugino and scalar soft masses respectively. \( \tilde{f}_a \) stands for the MSSM sfermions including the right–handed sneutrinos. Note that the generalized \textbf{A}–terms in Eq. \( (2) \) has the same structure as the corresponding superpotential terms of Eq. \( (1) \).

We assign flavor \( U(1)_A \) charges to the MSSM fields such that the observed fermion mass and mixing hierarchies are obtained with all Yukawa couplings being order one. As we will show explicitly, the expansion parameter \( \epsilon = \langle S \rangle / M_{\text{st}} \) is naturally of order 0.2 in anomalous \( U(1) \) models. We use the idea of “lopsided” mass matrices for generating large neutrino mixings \([14, 15]\), while maintaining small quark mixings.

We analyze two specific textures for the fermion mass matrices, Model 1 and Model 2. Model 2 is the texture advocated in Ref. \([5]\), and Model 1 is a slight variant of it. The textures for the two models are

\[
M_u \sim \langle H_u \rangle \begin{pmatrix} \epsilon^8 & \epsilon^{6-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^{6-\alpha} & \epsilon^4 & \epsilon^2 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \quad M_d \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^{4-\alpha} & \epsilon^{4-\alpha} \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix},
\]

\[
M_e \sim \langle H_d \rangle \epsilon^p \begin{pmatrix} \epsilon^{5-\alpha} & \epsilon^3 & \epsilon \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \\ \epsilon^{4-\alpha} & \epsilon^2 & 1 \end{pmatrix}, \quad M_{\nu_D} \sim \langle H_u \rangle \epsilon^8 \begin{pmatrix} \epsilon^2 & \epsilon & 1 \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},
\]

\[
M_{\nu_e} \sim M_R \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \quad \Rightarrow \quad M_{\nu}^{\text{light}} \sim \frac{\langle H_u \rangle^2}{M_R} \epsilon^{2s} \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}. \quad (3)
\]

Here \( M_u, M_d \) and \( M_e \) are the up–quark, down–quark, and the charged lepton mass matrices (written in the basis \( u M_u u^c \), etc.). \( M_{\nu_D} \) is the Dirac neutrino mass matrix, and \( M_{\nu_e} \) is the right–handed neutrino Majorana mass matrix. The light neutrino mass matrix \( M_{\nu}^{\text{light}} \) is derived from the seesaw mechanism. We have not exhibited order one coefficients in the matrix elements of Eq. \( (3) \). The expansion parameter is \( \epsilon \sim 0.2 \). The exponent \( p \) appearing in the overall factor \( \epsilon^p \) multiplying \( M_d \) and \( M_e \) is assumed to take values 0, 1 or 2 corresponding to large (\( \sim 20 \)), moderate (\( \sim 10 \)), and small (\( \sim 5 \)) values of \( \tan \beta \) (\( \equiv \langle H_u \rangle / \langle H_d \rangle \)) respectively.

The parameter \( \alpha \) is allowed to take two values, 0 and 1, corresponding to Model 1 \( (\alpha = 0) \) and Model 2 \( (\alpha = 1) \). The two models differ only in the masses and mixings of the first family. Both models give excellent fits to the fermion masses and mixings including neutrino oscillation parameters. Their predictions for LFV are however noticeably different, which we analyze in Section 4.
2.2 Anomalous $U(1)_A$ Models

The mass matrix textures of Eq. (3) can be obtained from an anomalous $U(1)$ gauge symmetry that is flavor dependent. In Table 1 we present a set of such $U(1)$ charges. These charges are integer multiples of the charge of the flavon field $S$ which is taken to be $-1$. Note that the texture alone does not fix the exponent $s$ appearing in $M_{
u_D}$ of Eq. (3).

| Field $Q_1, Q_2, Q_3$ | $U(1)_A$ Charge $4 - \alpha, 2, 0$ | Charge notation $q^Q_i$ |
|-------|-----------------|-------------------|
| $L_1, L_2, L_3$ | $1 + s, s, s$ | $q^L_i$ |
| $u^c_1, u^c_2, u^c_3$ | $4 - \alpha, 2, 0$ | $q^u_i$ |
| $d^c_1, d^c_2, d^c_3$ | $1 + p, p, p$ | $q^d_i$ |
| $e^c_1, e^c_2, e^c_3$ | $4 + p - s - \alpha, 2 + p - s, p - s$ | $q^e_i$ |
| $\nu^c_1, \nu^c_2, \nu^c_3$ | $1, 0, 0$ | $q^{\nu}_i$ |
| $H_u, H_d, S$ | $0, 0, -1$ | $(h, \bar{h}, q_s)$ |

Table 1: The flavor $U(1)_A$ charge assignments for the MSSM fields and the flavon field $S$ in the normalization of $q_s = -1$. Here $\alpha$ is 0 (1) for Model 1 (Model 2). In the third column we list the generic notation for the charges used in the RGE analysis.

We use the Green–Schwarz (GS) mechanism [2] for anomaly cancellation associated with $U(1)_A$ gauge symmetry. These conditions are given by

$$\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \frac{A_A}{k_A} = \frac{A_{\text{gravity}}}{12},$$  \hspace{1cm} (4)

where $A_i$ and $A_A$ are the coefficients $U(1)^2_Y \times U(1)_A$, $SU(2)_L^2 \times U(1)_A$, $SU(3)_C^2 \times U(1)_A$ and $U(1)^3_A$ gauge anomalies respectively. $k_i$ and $k_A$ are the Kac–Moody levels. $A_{\text{gravity}}$ is the mixed gravitational anomaly coefficient which is given by the trace of the $U(1)_A$ charges over all fields. With the non–Abelian levels $k_2 = k_3 = 1$, which is the simplest possibility, from Table 1 and Eq. (4) one finds

$$A_2 = \frac{19 - 3\alpha + 3s}{2},$$

$$A_3 = \frac{19 - 3\alpha + 3p}{2}.$$  \hspace{1cm} (5)

This implies that $p = s$. Furthermore,

$$A_1 = \frac{5}{6}(19 - 3\alpha + 3p),$$  \hspace{1cm} (6)

which fixes the level $k_1$ to be $5/3$.

With $p = s$ the charges given in Table 1 are compatible with $SU(5)$ unification. In string theory gauge coupling unification can occur without a simple covering group. The string unification condition is [16]

$$k_1 g_1^2 = k_2 g_2^2 = k_3 g_3^2 = k_A g_F^2.$$  \hspace{1cm} (7)
Our result $k_1 = 5/3$ is what is needed for consistency of the observed unification of gauge couplings in the MSSM. The discrepancy in the unification scale derived from low energy data versus perturbative string theory evaluation can be reconciled in the context of M–theory by making use of the radius of the eleventh dimension \cite{17}. We assume such a scenario.

From now on we shall assume the $SU(5)$ normalization for $g_1$. If one assumes that the field content of the model is just the one listed in Table 1, the gravitational anomaly $A_{gravity}$ would not satisfy the GS condition. One simple solution is the introduction of additional MSSM singlet (hidden sector) fields $X_k$. Then Eq. (4) leads to the following result:

$$A_{gravity} = 5 \left( 13 - 2\alpha + 3p \right) + \sum_k q^X_k = 6 \left( 19 - 3\alpha + 3p \right), \quad (8)$$

where $q^X_k$ are the $U(1)_A$ charges of the extra fields $X_k$. From this, one gets $\sum_k q^X_k = (49 - 8\alpha + 3p)$. We assume for simplicity that all the $X_k$ fields have the same flavor charge equal to 1. The number $n^X$ of $X_k$ fields is then fixed to be

$$n^X = 49 - 8\alpha + 3p. \quad (9)$$

We are now in a position to determine the level $k_A$ as well as the $U(1)_A$ gauge coupling $g_F$ at the unification scale. We renormalize the $U(1)_A$ charges by a factor $|q_s|$ so that the charge of the flavon field is now $-|q_s|$. $|q_s|$ is determined by demanding $g^2_F = g^2_2$ at the unification scale. Eq. (4) and the number $n^X$ in Eq. (9) then fix $|q_s|$ to be

$$|q_s| = \sqrt{\frac{19 - 3\alpha + 3p}{10(4 - \alpha)^3 + 5((1 + p)^3 + 2p^3) + n^X}}. \quad (10)$$

For $p = (0, 1, 2)$ one has $|q_s| = (0.165, 0.172, 0.166)$ for Model 1 ($\alpha = 0$) and $|q_s| = (0.225, 0.228, 0.203)$ for Model 2 ($\alpha = 1$).

The Fayet–Iliopoulos term for the anomalous $U(1)_A$, generated through the gravitational anomaly, is given by \cite{3}

$$\xi = \frac{g^2_{st}M^2_{st}}{192\pi^2}|q_s|A_{gravity}, \quad (11)$$

where $g_{st}$ is the unified gauge coupling at the string scale. By minimizing the potential

$$V = \frac{|q_s|^2g^2_F}{8} \left( \frac{\xi}{|q_s|} - |S|^2 + \sum_a q^f_a |\tilde{f}_a|^2 + \sum_k q^X_k |X_k|^2 \right)^2 \quad (12)$$

in the unbroken supersymmetric limit we obtain

$$\epsilon = \frac{\langle S \rangle}{M_{st}} = \sqrt{\frac{g^2_{st}A_{gravity}}{192\pi^2}}. \quad (13)$$

The numerical values of $\epsilon$ derived from Eq. (13) for different $p$ and $\alpha$ are listed in Table 2 by making use of Eq. (10). This is the small expansion parameter appearing in the mass matrices of Eq. (3). Here we took $g^2_{st}/4\pi \simeq 1/24$. 

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The mass of the flavor $U(1)_A$ gauge boson is found to be

$$M_F = \frac{|q_s| g_F \langle S \rangle}{\sqrt{2}}. \quad (14)$$

Between the string scale $M_{st}$ and $M_F$ the flavor gaugino contributes to flavor violating processes. This mass can now be determined:

$$M_F = \left( \frac{M_{st}}{88.3}, \frac{M_{st}}{84.6}, \frac{M_{st}}{86.3} \right) \text{ for } p = (0, 1, 2), \quad (15)$$

in the case of Model 1 and

$$M_F = \left( \frac{M_{st}}{68.7}, \frac{M_{st}}{66.8}, \frac{M_{st}}{72.9} \right) \text{ for } p = (0, 1, 2), \quad (16)$$

in the case of Model 2.

### 2.3 Fermion Mass Fits

Here we present numerical fits to the fermion masses and mixings for Model 1 and Model 2. These fits will be used in our quantitative analysis of lepton flavor violation.

As input at low energy we choose the following values for the running quark masses

$$m_u(1 \text{ GeV}) = 5.11 \text{ MeV}, \quad m_c(m_c) = 1.27 \text{ GeV}, \quad m_t(m_t) = 167 \text{ GeV},$$
$$m_d(1 \text{ GeV}) = 8.9 \text{ MeV}, \quad m_s(1 \text{ GeV}) = 130 \text{ MeV}, \quad m_b(m_b) = 4.25 \text{ GeV}. \quad (17)$$

The CKM mixing matrix elements are chosen to be $|V_{us}| = 0.222$, $|V_{ub}| = 0.0035$, $|V_{cb}| = 0.04$ and $\eta = 0.33$ (the Wolfenstein parameter of CP-violation). Using two–loop QED and QCD renormalization group equations we obtain these running parameters at the SUSY breaking scale, $M_{SUSY} = 500 \text{ GeV}$, with $\alpha_s(M_Z) = 0.118$, to be

$$r_f \equiv \frac{m_f(M_{SUSY})}{m_f(m_f)}, \quad (18)$$

where

$$(r_t, r_b, r_\tau, r_{u,c}, r_{d,s}, r_{e,\mu}) = (0.943, 0.605, 0.991, 0.395, 0.398, 0.989). \quad (19)$$

| $\epsilon$ | $p = 0$ | $p = 1$ | $p = 2$ |
|----------|----------|----------|----------|
| $\alpha = 0$ | 0.177 | 0.191 | 0.204 |
| $\alpha = 1$ | 0.163 | 0.177 | 0.191 |

Table 2: Numerical values for the small expansion parameter $\epsilon$ corresponding to different fermion mass hierarchy structure.
Using two–loop SUSY RGE evaluation above $M_{SUSY}$ we obtain the Yukawa couplings at the $U(1)_{A}$ breaking scale ($\sim 10^{15}$ GeV) to be

\[
(Y_u, Y_c, Y_t) = \left( 5.135 \times 10^{-6}, 1.426 \times 10^{-3}, 0.538 \right),
\]

\[
(Y_d, Y_s, Y_b) = \left( 3.459 \times 10^{-5}, 5.052 \times 10^{-4}, 2.768 \times 10^{-2} \right),
\]

\[
(Y_e, Y_\mu, Y_\tau) = \left( 1.024 \times 10^{-5}, 2.118 \times 10^{-3}, 3.572 \times 10^{-2} \right),
\]

\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = \left( 3.515 \times 10^{-4}, 8.419 \times 10^{-4}, 1.131 \times 10^{-2} \right),
\]

(20)

for $\tan \beta = 5$,

\[
(Y_u, Y_c, Y_t) = \left( 4.999 \times 10^{-6}, 1.389 \times 10^{-3}, 0.518 \right),
\]

\[
(Y_d, Y_s, Y_b) = \left( 6.844 \times 10^{-5}, 9.997 \times 10^{-4}, 5.470 \times 10^{-2} \right),
\]

\[
(Y_e, Y_\mu, Y_\tau) = \left( 2.027 \times 10^{-5}, 4.192 \times 10^{-3}, 7.094 \times 10^{-2} \right),
\]

\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = \left( 1.708 \times 10^{-3}, 4.105 \times 10^{-2}, 5.519 \times 10^{-2} \right),
\]

(21)

for $\tan \beta = 10$, and

\[
(Y_u, Y_c, Y_t) = \left( 4.996 \times 10^{-6}, 1.387 \times 10^{-3}, 0.518 \right),
\]

\[
(Y_d, Y_s, Y_b) = \left( 1.40 \times 10^{-4}, 2.045 \times 10^{-4}, 0.113 \right),
\]

\[
(Y_e, Y_\mu, Y_\tau) = \left( 4.132 \times 10^{-5}, 8.545 \times 10^{-3}, 0.147 \right),
\]

\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = \left( 8.551 \times 10^{-3}, 2.059 \times 10^{-2}, 0.278 \right),
\]

(22)

for $\tan \beta = 20$. $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$ and $|V_{ts}|$ are multiplicatively renormalized by an RGE factor of 0.9 in going from the low energy scale to the $U(1)_{A}$ breaking scale.

We have determined the Dirac neutrino Yukawa couplings as follows. First we note that the anomaly cancellation conditions in Eq. (11) implies $p = s$, which means that the Dirac neutrino Yukawa couplings are fixed to be of the same order as the charged lepton Yukawa couplings. Now, if one takes the right–handed Majorana neutrino mass matrix to be proportional to the transpose of the Dirac neutrino Yukawa coupling matrix for simplicity, $M_{\nu} = Y_\nu^T M_R e^p$, then the light neutrino mass matrix is given by

\[
M_{\nu}^{\text{light}} = Y_\nu M_{\nu}^{-1} Y_\nu^T v^2 \sin^2 \beta = Y_\nu \frac{v^2 \sin^2 \beta}{M_R e^p}.
\]

(23)

This simplified choice is certainly consistent with the fermion mass structures we have chosen in Eqs. (13). We adopt this choice in our analysis. $Y_\nu$ is determined from a fit to the light neutrino oscillation parameters with $M_R = 10^{14}$ GeV. This fit corresponds to $m_{\nu_e} = 2.7 \times 10^{-3}$ eV, $m_{\nu_\mu} = 6.4 \times 10^{-3}$ eV and $m_{\nu_\tau} = 8.6 \times 10^{-2}$ eV and the leptonic mixing matrix given by

\[
V_{MNS} = \begin{pmatrix}
0.848 & -0.526 & -0.0409 \\
0.349 & 0.619 & -0.72 \\
-0.4 & -0.59 & -0.7013
\end{pmatrix}.
\]

(24)
We also consider a scenario where the Dirac neutrino Yukawa couplings are maximized by choosing $M_R = 4 \times 10^{14}$ GeV. In this case we have

\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = \left(1.406 \times 10^{-3}, 3.368 \times 10^{-3}, 0.453 \times 10^{-2}\right) \text{ for } \tan \beta = 5,
\]
\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = \left(6.843 \times 10^{-3}, 1.645 \times 10^{-2}, 0.222\right) \text{ for } \tan \beta = 10,
\]
\[
(Y_{\nu_1}, Y_{\nu_2}, Y_{\nu_3}) = \left(3.514 \times 10^{-2}, 8.464 \times 10^{-2}, 1.237\right) \text{ for } \tan \beta = 20. \quad (25)
\]

We now present our fits to the observables of Eqs. (20)–(25) consistent with the texture of Eq. (3). This cannot be done uniquely since the right–handed rotation matrices are unknown from low energy data, so we make a specific choice. In our lepton flavor violation analysis we shall make use of this specific fit. One should bear in mind that there are uncertain coefficients of order one in the Yukawa matrices of our fit, which can lead to an order of magnitude uncertainty in the branching ratios for LFV processes.

We introduce the following notation:

\[
Y_{ij}^{f} \equiv y_{ij}^{f} e^{n_{ij}^{f}}. \quad (26)
\]

In Model 1, a good fit to the Yukawa couplings matrices is found to be

\[
Y^u = y_{33}^u \begin{pmatrix} 3.91 \epsilon^8 & 0.226 \epsilon^6 & 0.375 \epsilon^4 \\ 0.226 \epsilon^6 & 1.91 \epsilon^4 & 0.499 \epsilon^2 \\ 0.375 \epsilon^4 & 0.499 \epsilon^2 & 1 \end{pmatrix},
\]

\[
Y^d = y_{33}^d \epsilon^p \begin{pmatrix} (1.56 + 0.115i) \epsilon^5 & (0.909 + 0.054i) \epsilon^4 & (0.658 + 0.131i) \epsilon^4 \\ -2.89 \epsilon^3 & 1.02 \epsilon^2 & 1.22 \epsilon^2 \\ (-0.878 + 0.88 \times 10^{-3}i) \epsilon & 0.412 + 0.11 \times 10^{-6}i & 1 + 0.73 \times 10^{-6}i \end{pmatrix},
\]

\[
Y^e = y_{33}^e \epsilon^p \begin{pmatrix} 1.89 \epsilon^5 & 1.57 \epsilon^3 & 0.812 \epsilon \\ 0.487 \epsilon^4 & 2.14 \epsilon^2 & 0.316 \\ 1.10 \epsilon^4 & 1.52 \epsilon^2 & 1 \end{pmatrix},
\]

\[
Y^\nu = y_{33}^\nu \epsilon^p \begin{pmatrix} 1.51 \epsilon^2 & -0.358 \epsilon & -0.438 \epsilon \\ -0.358 \epsilon & 0.339 & 0.485 \\ -0.438 \epsilon & 0.485 & 1 \end{pmatrix}. \quad (27)
\]

Here

\[
y_{33}^u = (0.539, 0.523, 0.519),
\]

\[
y_{33}^d = (0.650, 0.257, 0.106),
\]

\[
y_{33}^e = (0.840, 0.333, 0.139),
\]

\[
y_{33}^\nu = (0.225, 0.219, 0.221), \quad (28)
\]

for $(p = 2, 1, 0)$ which we shall associate with $\tan \beta = (5, 10, 20)$. Here we have taken $\epsilon = 0.2$. For simplicity we assumed the leptonic Yukawa couplings to be all real.
In Model 2 we have the following fit for the Yukawa coupling matrices:

\[
Y^u = y^u_{33} \epsilon^p \begin{pmatrix}
0.876 \epsilon^6 & 1.30 \epsilon^5 & 0.499 \epsilon^3 \\
1.30 \epsilon^5 & 2.59 \epsilon^4 & 0.993 \epsilon^2 \\
0.499 \epsilon^3 & 0.993 \epsilon^2 & 1
\end{pmatrix},
\]

\[
Y^d = y^d_{33} \epsilon^p \begin{pmatrix}
(3.01 + 0.13i) \epsilon^4 & (2.66 + 0.13i) \epsilon^3 & (1.21 + 0.13i) \epsilon^3 \\
1.79 \epsilon^3 & 2.26 \epsilon^2 & 1.42 \epsilon^2 \\
1.00 + 0.33 \times 10^{-3}i \epsilon & 0.987 + 0.582 \times 10^{-5}i & 1 + 0.21 \times 10^{-5}i
\end{pmatrix},
\]

\[
Y^e = y^e_{33} \epsilon^p \begin{pmatrix}
1.19 \epsilon^4 & 1.68 \epsilon^3 & 0.579 \epsilon \\
0.892 \epsilon^3 & 2.18 \epsilon^2 & 0.350 \\
1.36 \epsilon^3 & 1.45 \epsilon^2 & 1
\end{pmatrix},
\]

\[
Y^\nu = y^\nu_{33} \epsilon^p \begin{pmatrix}
1.53 \epsilon^2 & -0.329 \epsilon & -0.406 \epsilon \\
-0.329 \epsilon & 0.293 & 0.449 \\
-0.406 \epsilon & 0.449 & 1
\end{pmatrix}.
\]

(29)

Here

\[
y^u_{33} = (0.535, 0.52, 0.515),
\]

\[
y^d_{33} = (0.650, 0.257, 0.107),
\]

\[
y^e_{33} = (0.840, 0.333, 0.139),
\]

\[
y^\nu_{33} = (0.233, 0.228, 0.229),
\]

(30)

for three different values of \( p = (2, 1, 0) \) identified with \( \tan \beta = (5, 10, 20) \).

### 2.4 Leptogenesis Constraints

In this subsection we show that baryogenesis through leptogenesis [12] can occur naturally in our models of fermion mass hierarchy. This requirement does put significant restrictions on the parameter \( p \), which is related to the value of \( \tan \beta \). We find that an acceptable baryon asymmetry would require \( p = 2, 1 \), corresponding to small to moderate values of \( \tan \beta \). In this case, lepton flavor violation resulting from the right–handed neutrino Dirac Yukawa couplings tend to be suppressed.

From the neutrino Majorana mass matrix of Eq. (3), we find that the heavy Majorana masses of \( \nu^c_{2,3} \) denoted by \( M_{2,3} \) are of order \( M_R \), while that of \( \nu^c_1 \) (denoted by \( M_1 \)) is of order \( \epsilon^2 M_R \). Lepton asymmetry is generated in the out of equilibrium decay of \( \nu^c_1 \). The induced asymmetry parameter is given by

\[
\epsilon_1 = \frac{1}{8\pi v_u^2} \frac{1}{\left\langle M^\dagger_{\nu_D} M_{\nu_D} \right\rangle_{11}} \sum_{j=2,3} \text{Im} \left[ \left( M^\dagger_{\nu_D} M_{\nu_D} \right)_{1j} \right] f \left( \frac{M_j^2}{M_1^2} \right),
\]

(31)

where \( v_u \approx 174 \sin \beta \) GeV, and the function \( f(x) \approx -3/\sqrt{x} \). Let us focus on the asymmetry induced through the exchange of \( \nu^c_2 \). Denote the mass ratio \( M_1/M_2 = a \epsilon^2 \), where \( a \) is a coefficient of order one. We also have from Eq. (3) \( (M^\dagger_{\nu_D} M_{\nu_D})_{11} = b \epsilon^{2+2} v_u^2 \),
\( (M_{\nu_D}^\dagger M_{\nu_D})_{12} = c \epsilon^{2s+1} v_u^2 \), where \( b, c \) are also order one coefficients. \( \epsilon_1 \) can then be estimated to be

\[
\epsilon_1 \simeq \frac{3ac}{8\pi b} 2s+2 \sin \phi ,
\]

where \( \sin \phi \) is an order one phase parameter. The leptonic asymmetry parameter \( Y_L \) is obtained from \( \epsilon_1 \) via the relation \( Y_L = d\epsilon_1/g^* \), where \( d \) is the dilution factor and \( g^* \) is the number of degrees of freedom in thermal equilibrium (\( g^* \approx 200 \) in the supersymmetric Standard Model). The dilution factor \( d \) depends on the ratio \( k \) of the decay rate versus the Hubble expansion rate \[19]\:

\[
k = \frac{M_{Pl} \left( M_{\nu_D}^\dagger M_{\nu_D} \right)_{11}}{1.66\sqrt{g^*}(8\pi v_u^2)M_1} .
\]

We now demand that the light neutrino mass \( m_{\nu_3} \), which can be written from the last expression of Eq. \[3\] as \( m_{\nu_3} = h c^{2s} v_u^2 / M_2 \), should equal 0.05 eV, to be consistent with the atmospheric neutrino oscillation data. This then determines \( k \) to be \( k \simeq |b/(ah)|(33.3) \). For this range of \( k \) (assuming that \( b/(ah) \) is not smaller than about 0.3) the dilution factor can be written approximately as

\[
dl \simeq \frac{0.3}{k[\ln(k)]^{0.6}}, \quad (10 \leq k \leq 10^6).
\]

Using \( Y_B \simeq -Y_L/2 \), we find

\[
Y_B = \frac{3.6 \times 10^{-6} x \epsilon^{2s}}{[\ln(33.3y)]^{0.6}} ,
\]

where \( x = (a^2 c^2 h \sin \phi)/b^2 \) and \( y = b/(ah) \) are both expected to be of order one. For the case of \( s = 2 \), we choose \( x = 0.05 \), \( y = 3 \) to obtain \( Y_B = 1 \times 10^{-10} \), which is near the central value of the observed baryon asymmetry. Note that this choice of \( x, y \) is quite natural and consistent with the flavor structure we have adopted. For the case of \( s = 1 \), we can fit \( Y_B = 1 \times 10^{-10} \) by choosing \( x = 1.7 \times 10^{-3}, y = 3 \). Considering that \( x \) is a nontrivial combination of order one parameters, this choice of \( x \) cannot be considered unnatural. On the other hand, for the case of \( s = 0 \), a good fit to \( Y_B \) requires, for example, \( x = 7 \times 10^{-5}, y = 3 \). Such a low value of \( x \) does not go well with the hierarchical structure dictated by the anomalous \( U(1)_{A} \) symmetry.

We conclude that baryogenesis via leptogenesis works in a simple and natural way with the mass matrix textures suggested in Eq. \( 3 \), but only for low to medium values of \( \tan \beta \), corresponding to \( s = p = 1, 2 \).

### 3 Generalized RGE Analysis of Soft SUSY Breaking Parameters

In this section we give a general RGE analysis of the soft SUSY breaking parameters including higher dimensional operators as shown in Eqs. \[1\] and \[2\]. This includes the effects of the flavor \( U(1)_A \) gaugino sector. Our analysis of this section should apply to a large class of Froggatt-Nielsen models.
The one–loop $\beta$–functions for the soft scalar masses of the sleptons are found to be

\[
\beta (\tilde{m}_L^2)_{ij} = \beta (\tilde{m}_L^2)_{ij}^{\text{MSSM}} + \frac{1}{16\pi^2} \left\{ \left( \tilde{m}_L^2 Y^{\nu\nu} Y^{\nu\nu} + Y^{\nu\nu} Y^{\nu\nu} \tilde{m}_L^2 \right)_{ij} + 2 \left( Y^{\nu\nu} \tilde{m}_v^2 Y^{\nu\nu} + \tilde{m}_{H_u}^2 Y^{\nu\nu} + A^{\nu\nu} A^{\nu\nu} \right)_{ij} + 2q_i^e g_F^2 \delta_{ij} \left( \sigma - 4q_i^e (M_{\chi_F})^2 \right) \right\},
\]

(36)

\[
\beta (\tilde{m}_e^2)_{ij} = \beta (\tilde{m}_e^2)_{ij}^{\text{MSSM}} + \frac{1}{16\pi^2} 2q_i^e g_F^2 \delta_{ij} \left( \sigma - 4q_i^e (M_{\chi_F})^2 \right),
\]

(37)

\[
\beta (\tilde{m}_\nu^2)_{ij} = \frac{1}{16\pi^2} \left\{ 2 \left( \tilde{m}_\nu^2 Y^{\nu\nu} Y^{\nu\nu} + Y^{\nu\nu} Y^{\nu\nu} \tilde{m}_\nu^2 \right)_{ij} + 4 \left( Y^{\nu\nu} \tilde{m}_\nu^2 Y^{\nu\nu} + \tilde{m}_{H_u}^2 Y^{\nu\nu} + A^{\nu\nu} A^{\nu\nu} \right)_{ij} + 2q_i^\nu g_F^2 \delta_{ij} \left( \sigma - 4q_i^\nu (M_{\chi_F})^2 \right) \right\}.
\]

(38)

Similarly the $\beta$–functions for the squark soft masses are given by

\[
\beta (\tilde{m}_Q^2)_{ij} = \beta (\tilde{m}_Q^2)_{ij}^{\text{MSSM}} + \frac{1}{16\pi^2} 2q_i^Q g_F^2 \delta_{ij} \left( \sigma - 4q_i^Q (M_{\chi_F})^2 \right),
\]

(39)

\[
\beta (\tilde{m}_u^2)_{ij} = \beta (\tilde{m}_u^2)_{ij}^{\text{MSSM}} + \frac{1}{16\pi^2} 2q_i^u g_F^2 \delta_{ij} \left( \sigma - 4q_i^u (M_{\chi_F})^2 \right),
\]

(40)

\[
\beta (\tilde{m}_d^2)_{ij} = \beta (\tilde{m}_d^2)_{ij}^{\text{MSSM}} + \frac{1}{16\pi^2} 2q_i^d g_F^2 \delta_{ij} \left( \sigma - 4q_i^d (M_{\chi_F})^2 \right).
\]

(41)

Here $\sigma$ is defined as

\[
\sigma = 3 \text{Tr} \left( 2q_i^Q \tilde{m}_Q^2 + q_i^u \tilde{m}_u^2 + q_i^d \tilde{m}_d^2 \right) + \text{Tr} \left( 2q_i^L \tilde{m}_L^2 + q_i^c \tilde{m}_e^2 + q_i^\nu \tilde{m}_\nu^2 \right) + q_s \tilde{m}_s^2 + \sum_k q_k X_k \tilde{m}_{X_k}^2,
\]

(42)

where $\tilde{m}_{X_k}$ is the soft mass of the extra particles $X_k$ and the trace is over family space. Here $\beta (\tilde{m}_L^2)_{ij}^{\text{MSSM}}$ stands for the MSSM $\beta$–function without the $\nu^c$ or the flavor $U(1)_A$ contributions [20].

The contributions proportional to $\sigma$ in Eqs. (36)–(41) arise from the diagram in Figure 1 (a) which has its origin from the $U(1)_A$ $D$–term. We call this the trace contributions. For a non–anomalous $U(1)$ gauge symmetry with universal scalar masses the trace term would vanish. However, for an anomalous $U(1)$ gauge symmetry, trace of the flavor charges is not zero, so this term will induce flavor non–universal masses. The diagram in Figure 1 (b) is the source of flavor non–universal contributions proportional to the gaugino mass $M_{\chi_F}$ in Eqs. (36)–(41).

Now we give the expressions for the one–loop contributions to the $\beta$–function of the SUSY breaking $A$–terms of Eq. (2). Let us introduce the following notation:

\[
A_{ij}^f \equiv a_{ij}^f e^{\nu_{ij}}.
\]

(43)
There are two types of contributions to the $\beta$–functions of $A^f_{ij}$: one from the gaugino and the other from the $A$–terms. The flavor gaugino contribution arises from diagrams such as the one in Figure 2. The $A$–term contribution to $\beta \left( A^f \right)$ cannot have the flavon field $S$ propagating in the loop, so that contribution is included in the MSSM piece.

The gaugino vertex contribution to $\beta \left( A^e_{ij} \right)$ (see Figure 2) is

$$\beta(a^e_{ij})^V = \frac{1}{4\pi^2} M_{\lambda_F} g_F^2 y_{ij}^e \left( q_i^L q_j^e + q_i^L h + q_j^e h + n_{ij} q_s (q_i^L + q_j^e + h) + \frac{1}{2} n_{ij}^e (n_{ij}^e - 1) q_s^2 \right).$$  (44)

Eq. (44) is obtained by summing all possible gaugino exchange diagrams.

We now list the full one–loop $\beta$ function for each $A^f_{ij}$. This generalizes the results of Martin [21].

$$\beta(A^f_{ij}) = \beta(A^e_{ij})^{MSSM} + \frac{1}{16\pi^2} \left\{ A^e \left[ Y^{\nu^i \nu} - 2 \left( (q_i^L)^2 + (q_j^e)^2 + h^2 \right) g_F^2 \right] 
+ 2 Y^{\nu^i} A^f \right\}_{ij} + \frac{1}{4\pi^2} g_F^2 Z_{ij}^e Y_{ij} M_{\lambda_F},$$  (45)

$$\beta(A^e_{ij}) = \frac{1}{16\pi^2} \left( A^f \left[ 5 Y^{\nu^i \nu^j} + Y^{e^i} Y^e \right] + \text{Tr} \left( 3 Y^{\nu^i} Y^{\nu^j} + Y^{\nu^i} Y^{\nu^j} \right) \right)$$
From the charges listed in Table II we have

\[
Z^e = - \begin{pmatrix}
(11, 13, 16) & (4, 6, 9) & (1, 3, 6) \\
(10, 11, 13) & (3, 4, 6) & (0, 1, 3) \\
(10, 11, 13) & (3, 4, 6) & (0, 1, 3)
\end{pmatrix}
\]  

(50)

in Model 1 (\(\alpha = 0\)) and

\[
Z^e = - \begin{pmatrix}
(7, 9, 12) & (4, 6, 9) & (1, 3, 6) \\
(6, 7, 9) & (3, 4, 6) & (0, 1, 3) \\
(6, 7, 9) & (3, 4, 6) & (0, 1, 3)
\end{pmatrix}
\]  

(51)

in Model 2 (\(\alpha = 1\)) for the three different values of \(p = (0, 1, 2)\).

### 4 Lepton Flavor Violating Decays \(\mu \to e\gamma\) and \(\tau \to \mu\gamma\)

#### 4.1 Qualitative Analysis

In the Standard Model with small neutrino mass, lepton flavor violating processes are highly suppressed. On the other hand, in the presence of low energy supersymmetry, LFV effects can be quite significant. In particular, LFV induced by the right-handed
neutrino Yukawa couplings in the MSSM can lead to $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decay rates near the current experimental limits \[24, 11\].

Here we focus on flavor violation in leptonic processes. The slepton soft masses are more sensitive to the $U(1)_A$ gaugino corrections compared to those in the squark sector. This is because flavor violation in the squark sector is diluted by the gluino focusing effects. This is especially so when one considers the cosmological constraints on the lightest SUSY particle (LSP) mass. Demanding that the neutralino LSP constitutes an acceptable cold dark matter imposes the condition $m_0 \approx M_{1/2}/4.4$ in the context of supergravity models. This condition results from the coannihilation mechanism \[22, 11\] for diluting the dark matter density which requires $\tilde{\tau}_R$ mass to be about $5 - 15$ GeV above the LSP mass.

The approximate formulae for the sfermion soft masses in terms of the universal soft scalar mass $m_0$ and the common gaugino mass $M_{1/2}$ (for small to medium $\tan \beta$) are \[23\]

$$
\begin{align*}
\tilde{m}_L^2 & \approx m_0^2 + 0.52 M_{1/2}^2, \\
\tilde{m}_e^2 & \approx m_0^2 + 0.15 M_{1/2}^2, \\
\tilde{m}_d^2 & \approx m_0^2 + 6.5 M_{1/2}^2, \\
\tilde{m}_u^2 & \approx \tilde{m}_d^2 \approx m_0^2 + 6.1 M_{1/2}^2. \\
\end{align*}
$$

From these expressions with $m_0 \approx M_{1/2}/4.4$ we see that the gaugino focusing effects make the squark soft masses much less sensitive to any flavor violating contributions.

The right–handed neutrino induced LFV effects in our models depend on the overall factor $e^\beta$ in Eq. \[3\]. These processes will be suppressed for $p = 1, 2$ corresponding to low values of $\tan \beta$. As we noted in Sec. 2.3, $p = 2$ is preferred for leptogenesis, in which case the $\nu^c$ effects are small.

There are three different sources of LFV in our models: (i) RGE effects between $M_{st}$ and the $U(1)_A$ symmetry breaking scale $M_F$ induced by the $U(1)$ gaugino, (ii) RGE effects between $M_{st}$ and the right–handed neutrino mass scale $M_R$ induced by the neutrino Dirac Yukawa couplings, and (iii) the $U(1)_A D$–term. Here we discuss only the RGE effects (i) and (ii). We call them the flavor gaugino induced LFV and $\nu^c$–induced LFV. (The $D$–term contributions are not included in our numerical analysis, but are discussed in subsection 4.3.) We give approximate formulas for these LFV processes by integrating the relevant $\beta$–functions derived in Section 3.

We adopt the minimal supergravity scenario (mSUGRA) for supersymmetry breaking. We assume universality of scalar masses and proportionality of the $A$–terms and the respective Yukawa couplings at the string scale. Gaugino mass unification is also assumed.

The various flavor violating effects are summarized below:

1. (Right-handed neutrino contributions to the scalar soft masses arising from Eq. \[36\]) proportional to the Dirac neutrino Yukawa couplings:

$$
\delta \left( \tilde{m}_L^{\nu^c} \right)_{ij} \simeq - (Y^{\nu^c} Y^{\nu})_{ij} \left( 3 m_0^2 + A_0^2 \right) \frac{\ln (M_{st}/M_F)}{8 \pi^2}. 
$$

2. Trace correction from $D_A$–term in Eqs. \[36\] and \[37\] from Figure 1(a):

$$
\delta \left( \tilde{m}_F^A \right)_{ij} \simeq - q_i^L |q_s| g_F^2 \delta_{ij} \left( 3 m_0^2 \sum_{i=1,3} (n^{u}_i + n^{d}_i) + m_0^2 \sum_{i=1,3} (n^c_i + n^\nu_i) + n^X m_0^2 - \tilde{m}_s^2 \right) \frac{\ln (M_{st}/M_F)}{8 \pi^2}, 
$$
\[ \delta (\tilde{m}_e^2)^A_{ij} \simeq -q_i^e |q_s| g^2 \delta_{ij} \left( 3 m_0^2 \sum_{i=1,3} (n_{ii}^u + n_{ii}^d) + m_0^2 \sum_{i=1,3} \left( n_{ii}^e + n_{ii}^\nu + n^X m_0^2 - \tilde{m}_s^2 \right) \right) \ln \left( \frac{M_{st}/M_F}{8\pi^2} \right). \] (54)

(3) Gaugino mass correction from Figure 1(b):

\[ \delta (\tilde{m}_L^2)^G_{ij} \simeq (q_i^L g_F)^2 \delta_{ij} (M_\lambda F)^2 \ln \left( \frac{M_{st}/M_F}{2\pi^2} \right), \]

\[ \delta (\tilde{m}_e^2)^G_{ij} \simeq (q_i^e g_F)^2 \delta_{ij} (M_\lambda F)^2 \ln \left( \frac{M_{st}/M_F}{2\pi^2} \right). \] (55)

(4) Right–handed neutrino induced vertex correction to the $A^e$–terms (see Eq. (45)):

\[ \delta A^e_{ij} \simeq -3A_0 \left( Y^e Y'^\nu Y^\nu \right)_{ij} \ln \left( \frac{M_{st}/M_R}{16\pi^2} \right). \] (56)

(5) Flavor gaugino vertex correction to the $A^e$–terms arising from Figure 2 (see the last term of Eq. (45)):

\[ \delta A^e_{ij} \simeq -M_\lambda F g_F^2 Y^e Z^e_{ij} \ln \left( \frac{M_{st}/M_F}{4\pi^2} \right). \] (57)

In addition, we have flavor charge dependent wave function renormalization of the $A$–terms as given in Eq. (55). These are however not significant since they are diagonalized simultaneously with the corresponding Yukawa couplings. On the other hand, the vertex corrections to the $A$–terms given in Eqs. (56) and (57) will induce nonproportionality in going from $M_{st}$ to $M_F$.

The matrix elements $Z^e_{ij}$ in Eq. (54) are given in Eqs. (50) and (51) for different values of $p$. The elements in the $(1,2)$ block of $Z^e$ are rather different from each other, suggesting that the gaugino vertex contributions can be very important for the process $\mu \rightarrow e\gamma$. On the other hand, the elements in the second and the third rows are identical, hence, $A_{23}^e$ and $A_{33}^e$ run at the same rate as their corresponding Yukawa couplings do in the short momentum interval. Therefore, this vertex correction for the process $\tau \rightarrow \mu\gamma$ is always suppressed in models with the texture of Eq. (3).

For $\mu \rightarrow e\gamma$ we find that the most dominant effect is from the flavor gaugino contributions to the soft masses. This is due to the following reason. It is proportional to the flavor charge squared and to the flavor gaugino mass squared (recall that we have $m_0 \simeq M_{1/2}/4.4$, both of which are large. On the other hand, the trace contributions to the soft masses depend linearly on the flavor charges and are proportional to $m_0^2$, which make them relatively small although the trace of the $U(1)_A$ charges itself is large. The right–handed neutrino contributions are significant only for $p = 0$. For other values of $p$ the $\nu^c$–contributions to the branching ratio for $l_i \rightarrow l_j \gamma$ is suppressed by $e^{4p}$.

We find that the gaugino contribution to the $\tau \rightarrow \mu\gamma$ decay rate is always suppressed since $\tau_L$ and $\mu_L$ have the same flavor charges and since the $\tau_R^c - \mu_R$ mixing angle is of order $\epsilon^2$. The only significant effect to this process is from the right–handed neutrino effects when $p = 0$. 


4.2 Numerical Results

In this section we present our numerical results for the LFV processes $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. We adopt the mSUGRA scenario for the SUSY breaking parameters. At the string scale, taken to be $M_{st} = 10^{17}$ GeV, we assume a universal scalar mass $m_0$ and a common gaugino soft masses $M_{1/2}$. The unified gauge coupling at $2 \times 10^{16}$ GeV is taken to be $\alpha_G \simeq 1/24$. We assume the $U(1)_A$ gauge coupling $g_F$ to be equal to $g_2$ at the string scale. We evolve the soft SUSY breaking parameters from $M_{st}$ to the $U(1)_A$ gaugino mass $M_F \simeq M_{st}/80$ (see Eqs. (15) and (16)). We use the numerical values of the Yukawa couplings given in Eqs. (27)-(30) for this evolution.

As explained previously, we take $m_0 = M_{1/2}/4.4$ so that the relic abundance of neutralino dark matter can be reproduced correctly. With this choice we always find the neutralino to be the LSP with the $\tilde{\tau}_R$ mass higher than the LSP mass by $5 - 15$ GeV.

We impose radiative electroweak symmetry breaking condition. The SUSY higgs mass parameter $\mu$ is chosen to be positive which is favored by $b \rightarrow s\gamma$.

We take $M_{1/2}$ to vary in the range 250 GeV to 1 TeV. The lower value satisfies the lightest higgs boson mass limit. We present the results for three different values of $\tan \beta = (5, 10, 20)$. The corresponding values of the exponent $p$ are taken to be $p = (2, 1, 0)$. The results are presented for two different values of $A_0 = (0, 300)$ GeV. When $\tan \beta = 20$, the lower limit on $M_{1/2}$ is around 300 GeV, or else the radiative electroweak symmetry breaking would fail. The branching ratios are plotted against universal gaugino mass $M_{1/2}$ in Figures 3–14.

In Figure 3, we plot the branching ratio for the process $\mu \rightarrow e\gamma$ including all the LFV effects described earlier for Model 1 as a function of $M_{1/2}$ for different values of $\tan \beta$ and $A_0$. We see that the branching ratio is in the experimentally interesting range for most of the parameter space. In this Figure for $\tan \beta = 20$ we also show the branching ratio when the Dirac neutrino Yukawa coupling effects are maximized (denoted by “large $Y^\nu$”). The horizontal line corresponds to the current experimental limit.

In Figure 4 the combined effect for $\mu \rightarrow e\gamma$ is plotted for Model 2.

In Figure 5 we plot $B(\mu \rightarrow e\gamma)$ induced solely by the right–handed neutrino Yukawa couplings. This result is identical for Models 1 and 2 since neutrino textures are the same for the two models. In Figure 6 we plot the branching ratio induced by the right–handed neutrino effects and the flavor gaugino effects for Model 1. Figure 7 has the same plot for Model 2. In Figure 8 (9) we plot $B(\mu \rightarrow e\gamma)$ induced by the trace term and the right–handed neutrino for Model 1 (2). Figure 10 (11) is a plot of the branching ratio including the effects of $A$–terms and $\nu^c$ for Model 1 (2). Figures 12 and 13 are the branching ratios for $\tau \rightarrow \mu\gamma$ including all LFV effects for Model 1 and Model 2. Figure 14, which is valid for both Models 1 and 2, has the branching ratios for $\tau \rightarrow \mu\gamma$ induced only by the $\nu^c$ Yukawa coupling effects.

From these figures we see that the decay $\mu \rightarrow e\gamma$ is within the reach of forthcoming experiments. Discovery of $\tau \rightarrow \mu\gamma$ decay will strongly hint, within our framework, an origin related to the right–handed neutrino Yukawa couplings.
Figure 3: Branching ratio for the process $\mu \rightarrow e\gamma$ including all corrections for Model 1. The solid line corresponds to $A_0 = 300$ GeV and the dashed line corresponds to $A_0 = 0$ GeV. For $\tan\beta = 20$ we give two sets of curves, the upper one corresponds to the maximal value of the neutrino Yukawa coupling $Y^\nu$. Here and in other plots, the straight horizontal line corresponds to the current experimental limit $B(\mu \rightarrow e\gamma)_{exp} < 1.2 \times 10^{-11}$ [24].
Figure 4: Branching ratio for the process $\mu \to e\gamma$ including all corrections for Model 2.

Figure 5: Branching ratio for the process $\mu \to e\gamma$ induced by only the right–handed neutrino Yukawa coupling effects. This result holds for both Models 1 and 2.
Figure 6: Branching ratio for the process $\mu \rightarrow e\gamma$ induced by the gaugino corrections (plus $\nu^e$ effects) for Model 1.

Figure 7: Branching ratio for the process $\mu \rightarrow e\gamma$ induced by the gaugino corrections (plus $\nu^e$ effects) for Model 2.
Figure 8: Branching ratio for the process $\mu \to e\gamma$ induced by the trace correction (plus $\nu^c$ effects) for Model 1.

Figure 9: Branching ratio for the process $\mu \to e\gamma$ induced by the trace correction (plus $\nu^c$ effects) for Model 2.
Figure 10: Branching ratio for the process $\mu \to e\gamma$ from the vertex corrections (plus $\nu^c$ effects) for Model 1.

Figure 11: Branching ratio for the process $\mu \to e\gamma$ from the vertex corrections (plus $\nu^c$ effects) for Model 2.
Figure 12: Branching ratio for the process $\tau \rightarrow \mu \gamma$ including all the effects for Model 1.

Figure 13: Branching ratio for the process $\tau \rightarrow \mu \gamma$ including all the effects for Model 2.
Figure 14: Branching ratio for the process $\tau \to \mu \gamma$ induced by only the right–handed neutrino Yukawa coupling effects for Models 1 and 2.

4.3 The D–term Splitting Problem

Any model based on a gauged flavor symmetry has a potential $D$–term splitting problem, which could give rise to large FCNC processes even with a universal choice of soft scalar masses. Here we quantify this problem in the class of anomalous $U(1)$ models. We point out that this problem is not as serious as it might naively appear, if the hierarchy $m_0 = M_{1/2}/4.4$ needed for an acceptable relic abundance of neutralino dark matter is assumed.

The $D_A$–term contribution to the scalar potential including soft SUSY breaking mass for the flavon field $S$ is given by

$$V = \bar{m}_s^2 |S|^2 + \frac{|q_s|^2 g_F^2}{8} \left( \frac{\xi}{|q_s|} - |S|^2 + \sum_i q_i |\tilde{f}_i|^2 \right)^2$$

(58)

where $q_s$ and $q_i$ are the $U(1)_A$ flavor charges of the $S$ field and the MSSM fields $\tilde{f}_i$. Minimizing the potential one finds the mass splitting among the MSSM sfermions to be

$$\tilde{m}_{\tilde{f}_i}^2 - \tilde{m}_{\tilde{f}_j}^2 = \frac{q_i - q_j}{|q_s|} \bar{m}_s^2.$$  

(59)

If a universal scalar mass is assumed even for $\bar{m}_s^2$, this splitting could be unacceptably large. However, if we choose $m_0 = M_{1/2}/4.4$, this may be in an acceptable range. The low energy values for the slepton soft masses including the $D_A$–term corrections to Eq. (32) are

$$\left(\bar{m}_{\tilde{L}}^2\right)_{ii} = \frac{|q_s|^2}{|q_s|^2} \bar{m}_s^2 + m_0^2 + 0.52 M_{1/2}^2 \simeq \frac{|q_s|^2}{|q_s|^2} \bar{m}_s^2 + 11.1 m_0^2.$$
\begin{equation}
\left( \tilde{m}_e^2 \right)_{ii} = \frac{g_i^e}{|q_s|} \tilde{m}_s^2 + m_0^2 + 0.15 M_{1/2}^2 \simeq \frac{g_i^e}{|q_s|} \tilde{m}_s^2 + 3.9 m_0^2. \tag{60}
\end{equation}

Since the $D$–terms only contribute to the diagonal slepton mass splittings, any flavor violation arising from it must involve fermionic mixing angles. We find

\begin{align*}
\left( \delta_{RR} \right)_{12}^e & \simeq \frac{\delta q_{12}^e Y_{21}^e \tilde{m}_s^2}{Y_{22}^e (\tilde{m}_s^2)^2} , \tag{61} \\
\left( \delta_{LL} \right)_{12}^e & \simeq \frac{\delta q_{12}^L Y_{12}^e \tilde{m}_s^2}{Y_{22}^e (\tilde{m}_s^2)^2} , \tag{62}
\end{align*}

where \( \left( \delta_{RR} \right)_{12}^e \) is the $\tilde{e}_R$–$\tilde{\mu}_R$ mixing parameter in the supersymmetric basis. Here \( \delta q_{12}^e = (q_1^e - q_2^e)/|q_s| = 2 - \alpha \) and \( \delta q_{12}^L = (q_1^L - q_2^L)/|q_s| = 1 \), with \( \alpha = 0 \) (1) for Model 1 (Model 2). For $\mu \to e\gamma$ this gives

\begin{align*}
\left( \delta_{RR} \right)_{12}^e & \simeq C_R \frac{(2 - \alpha) \epsilon} {1 + 1.55 \left( \tilde{m}_s/m_0 \right)^2} , \tag{63} \\
\left( \delta_{LL} \right)_{12}^e & \simeq C_L \frac{(\tilde{m}_s/m_0)^2} {1 + 0.1 \left( \tilde{m}_s/m_0 \right)^2} , \tag{64}
\end{align*}

where

\begin{align*}
C_R = \frac{\epsilon Y_{21}^e}{3.9 Y_{22}^e} & \simeq \left( \frac{1}{86.9} , \frac{1}{46.9} \right) , \\
C_L = \frac{\epsilon Y_{12}^e}{11.1 Y_{22}^e} & \simeq \left( \frac{1}{75.6} , \frac{1}{72} \right) \text{ for (Model 1, Model 2)} . \tag{65}
\end{align*}

From the experimental bound on $\mu \to e\gamma$ one approximately has \( \frac{\tilde{m}_s/m_0}{\tilde{m}_s/m_0} \)

\begin{equation}
\left( \delta_{LL} \right)_{12}^e \big|_{\text{exp}} \simeq \left( \delta_{RR} \right)_{12}^e \big|_{\text{exp}} < 1.2 \times 10^{-2} \frac{1}{\tan \beta} \left( \frac{M_{\text{SUSY}}}{500 \text{ GeV}} \right)^2 . \tag{66}
\end{equation}

For $\tan \beta = 5$ and for $M_{\text{SUSY}} \simeq M_{1/2}$ (which is a reasonable choice) we find

\begin{equation}
\left( \delta_{LL,RR} \right)_{12}^e \big|_{\text{exp}} < 0.6 \times 10^{-3} \left( 1.0 \times 10^{-2} \right) \text{ for } M_{1/2} = 250 \text{ (1000) GeV} . \tag{67}
\end{equation}

This gives the following constraint on $\tilde{m}_s/m_0$:

\begin{equation}
\frac{\tilde{m}_s}{m_0} < 0.2 \text{ (0.9) for } M_{1/2} = 250 \text{ (1000) GeV} \tag{68}
\end{equation}

for Model 1 and

\begin{equation}
\frac{\tilde{m}_s}{m_0} < 0.17 \text{ (0.7) for } M_{1/2} = 250 \text{ (1000) GeV} \tag{69}
\end{equation}

for Model 2. For low values of $M_{1/2}$ this gives a significant constraint on $\tilde{m}_s$, while for large values of $M_{1/2}$ universality of all scalar masses may be maintained.
We wish to note that the $m_s^2$ appearing in Eq. (59) and in the subsequent discussions is the soft mass–squared of the scalar flavon field $S$ evaluated at the $U(1)_A$ breaking scale $M_F$. Since there are singlet fields $X_k$ in the theory needed for anomaly cancellation which are active between the $U(1)_A$ scale and $M_{st}$ (Cf. Eq. (9)), the $S$ field can have renormalizable Yukawa couplings of the type $Y_k X_k \bar{X}_k S$ where $\bar{X}_k$ are fields neutral under $U(1)_A$. We have examined the evolution of $m_s^2$ between $M_F$ and $M_{st}$ in the presence of such Yukawa couplings and found that even with the limits on $m_s^2$ given in Eqs. (68) and (69), $m_s^2 = m_0^2$ can be attained at the string scale for a range of Yukawa couplings $Y_k$.

The RGE running of $m_s^2$ between the string scale and the $U(1)_A$ symmetry breaking scale contains the same trace term and the same gaugino contributions that we considered for the MSSM sfermions in Eqs. (54) and (55) (see also terms proportional to $\sigma$ in Eqs. (36)-(41))

$$
\delta m_s^2 \simeq -qs |qs| g_F^2 \left( 3 m_0^2 \sum_{i=1,3} (n_{u_{ii}} + n_{d_{ii}}^d) \right)
+ m_0^2 \sum_{i=1,3} \left( n_{e_{ii}}^e + n_{\nu_{ii}}^\nu \right) + n^x m_0^2 - m_s^2 \right) \frac{\ln (M_{st}/M_F)}{8\pi^2}
+ (qs g_F)^2 (M_{\chi_F})^2 \frac{\ln (M_{st}/M_F)}{2\pi^2}.
$$

Interestingly, this contribution from the $D$-term evolution cancels the “trace contributions” in the sfermion masses (the terms proportional to $\sigma$ in Eqs. (36)-(41))

5 Conclusion

In this paper we have studied lepton flavor violation induced by a flavor–dependent anomalous $U(1)$ gauge symmetry of string origin in a class of models which addresses the fermion mass hierarchy problem via the Froggatt–Nielsen mechanism. We have derived a general set of renormalization group equations for the evolution of soft SUSY breaking parameters in the presence of higher dimensional operators. These results should be applicable to a large class of fermion mass models. We have shown that the $U(1)_A$ sector induces significant flavor violation in the SUSY breaking parameters during the RGE evolution from the string scale to the flavor symmetry breaking scale, even though this momentum range is very short. We have identified several sources of flavor violation: the $U(1)_A$ gaugino contribution to the scalar masses which is flavor dependent, a contribution proportional to the trace of $U(1)_A$ charge which is also flavor dependent, non–proportional $A$–terms arising from the $U(1)_A$ gaugino vertex correction diagrams, and the $U(1)_A D$–term. In addition, there are flavor violating effects in the charged lepton sector arising from the right–handed neutrino Yukawa couplings, which have also been included in our numerical analysis. The resulting flavor violation in the leptonic decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are found to be in the experimentally interesting range.

Adopting the minimal supergravity scenario for SUSY breaking, and choosing parameters such that the needed relic abundance of neutralino dark matter is realized, we have presented results for the branching ratios $B(\mu \rightarrow e\gamma)$ and $B(\tau \rightarrow \mu\gamma)$ in two specific models of fermion masses. Figures 3 and 4 are our main results for the two models for $B(\mu \rightarrow e\gamma)$, while Figures 13 and 14 are our results for $B(\tau \rightarrow \mu\gamma)$. The former should be
accessible to forthcoming experiments, while the latter is also in the observable range. Although we focused on two specific fermion mass textures these effects should be significant in a large class of models.

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