Effects of surface roughness, MHD and couple stress on squeeze film characteristics between curved circular plates

Hanumagowda B N¹, Salma A² and Nagarajappa C S³
¹,²,³REVA University, Bangalore -560064, India.
Email: hanumagowda123@rediffmail.com, salma.alla@gmail.com

Abstract: The theoretical discussion is carried out for understanding the combined study of MHD, rough surface and couple-stress in the presence of applied magnetic field between two curved circular plates is present analysis. Modified Reynolds Equations accounting for rough surface using stochastic model of Christensen are mathematically formulated. The close form derivations for pressure, load-supporting capacity and response-film time are obtained. Our results shows that, there is an significant increase (decrease) for pressure, load-supporting capacity and squeeze film time due to the effect of azimuthal (radial) roughness parameter when compared to the Hanumagowda.et.al [14] and numerical data of load supporting capacity and response time are given in Table for engineering applications.

1. Introduction
In earlier times, the study of rough surface plays a vital role in the field of science, engineering and industrial applications. In bearings, surface texture is considered as roughness component. It is determined by the variation in the direction of the normal vector of a real surface from its ideal form. Surface is rough, if the variations are large and the surface is smooth, if they are small. In the field of mechanical, roughness is taken as a good predictor of the performance due to its irregularities in the surface which form nucleation sites for cracks or corrosion. Since the roughness variation is very large along with the mean separation for sliding surfaces but it appears normal to view the film thickness in a bearing as a stochastic process characterized by a number of statistical parameters. All bearing surfaces are rough to some extent, hence more interest is shown to study roughness using hydrodynamic lubrication theory recently. In the literature to discuss the effect of surface roughness on bearing system, many authors have proposed a tremendous work. First paper which discuss the effect of rough surface was by Ting [1]. Later, Burton [2] presented rough surface by using concept of Fourier series type approximation. The stochastic models for hydrodynamic lubrication of rough surface was developed by Christensen [3] and Elord [4] and their study derive an important results which is applicable to rough bearings was the Reynold’s equation in general form. Based on this result, Prakash and Tiwari [5] and Gururajan and Prakash [6] have done survey on different types of porous bearings with Newtonian fluid. The combined study of roughness and lubricant additives oil for distinct porous bearing systems is studied by Naduvinamani et al. [7] and from the result due to presence of lubricant additives there is an considerable influence on the characteristics of rough porous bearings.
In recent times, the study of magneto hydrodynamic (MHD) have given more importance by many authors in the field of lubrication bearings, since it prevents the unexpected difference of viscosity for
lubricant with temperature under sever operating conditions and due to magnetic effect in lubrication several authors have been attracted from various field particularly authors from the field of space and nuclear engineering. Work done by Syeda Tasneem Fathima et.al [8] shows that the combined study of rough surface in presence of MHD for circular plates results, increase in surface asperity and a large amount of load is released in the bearing and increases the response time of squeeze-film motion as compared to the smooth case. The combined study on MHD and surface roughness is carried out by many authors [9-11] and from their result it’s found that the surface roughness in the presence of magnetic field is significant on squeeze film characteristics of the bearings.

In the Tribology literature, so far the influence of transverse magnetic field between rough curved plates with couple stress squeeze film lubrication has not been discussed. Hence, the present paper aims to analysis the mixed discussion of rough surface and MHD over the couple stress squeeze film characteristics between curved plates and obtained numerical findings are compared with classical case studied by Hanumagowda.et.al [14].

2. Mathematical Formulation.

A pictorial display of the bearing system in which the lower plate is rough and separated by central thickness $h_0$ of fluid film in the presence of applied transverse magnetic field $B_0$ which is perpendicular to plates is shown in figure:1

![Figure-1: Bearing Geometry of Curved Circular plates.](image-url)
The fluid film thickness \( h(r) \) considered as

\[
h(r) = h_o \left\{ \exp(-\beta r^2) - \frac{1}{1 + \gamma r} + 1 \right\}; \quad 0 \leq r \leq a
\]

(1)

Where \( \beta \) and \( \gamma \) are upper and lower plate curvature parameters respectively.

The Reynolds equation in the modified form to discuss the MHD couple stress squeeze film between curved circular plates was derived by Hanumagowda.et.al [14] and is,

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( rf(h,l,M_o) \frac{\partial p}{\partial r} \right) = \mu V
\]

(2)

Where,

\[
f(h,l,M_o) = \begin{cases}
\frac{h_o^3}{M_o^3} \left\{ \frac{2l}{A^2 - B^2} \left( \frac{B^2}{A} \tanh \frac{Ah}{2l} \right) \frac{A^2}{B} \tanh \left( \frac{Bh}{2l} \right) \right\} + h, & \text{for } M_o^3 l^2 / h_o^3 < 1 \\
\frac{h_o^3}{M_o^3} \left\{ \frac{h}{2 \sec h} \left( \frac{h}{2 \sqrt{2l}} \right) \left( \frac{h}{2 \sqrt{2l}} \right) \right\} + h, & \text{for } M_o^3 l^2 / h_o^3 = 1 \\
\frac{h_o^3}{M_o^3} \left\{ \frac{2lh_o}{M_o} \left( \frac{A_2 \cot \theta - B_2}{\cos B_2 h + \cosh A_2 h} \right) + h, & \text{for } M_o^3 l^2 / h_o^3 > 1
\end{cases}
\]

For stochastic model of the rough surface, the film thickness is having two parts and given by

\[
H_i = h_i + h_j(r, \theta, \xi)
\]

(3)

The probability distribution function is given \( f(h_i) \), where \( h_j \) is the stochastic film thickness.

The modified stochastic Reynolds equation is found by taking the stochastic average of (15) with respect to \( f(h_i) \)

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( rf(h,l,M_o) \frac{\partial p}{\partial r} \right) = \mu V
\]

(4)

Where

\[
E(*) = \int_{-\infty}^{\infty} f(h_i) dh_i
\]

(5)

According to Christensen [3], we have

\[
f(h_i) = \begin{cases}
\frac{35}{32c^3} (c^3 - h_i^3)^3, & -c < h_i < c \\
0, & \text{elsewhere}
\end{cases}
\]

(6)

According to Christensen [3] stochastic theory, rough surface in one dimension consists of two parts, namely radial roughness and azimuthal roughness configuration.

**Radial roughness configuration.**

In 1-D radial roughness configuration, the roughness configuration is in the form of ridges and valleys running in radial(\( r \)) direction and non-dimensional film thickness is given by
\[ H_i' = h_i + h_i(\theta, \xi) \]  

Thus modified-stochastic Reynold’s equation (4) is written as

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( rE(f(h,l,M_o)) \frac{\partial E(p)}{\partial r} \right) = \mu V \]  

**Azimuthal roughness configuration.**

In 1-D Azimuthal roughness configuration, the roughness configuration is in the form of long, narrow ridges and valleys running in \( \theta \)- direction and non-dimensional film thickness is given by

\[ H_i' = h_i + h_i(r, \xi) \]  

Thus modified-stochastic Reynold’s equation (4) is written as

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r}{E(1/f(h,l,M_o))} \frac{\partial E(p)}{\partial r} \right) = \mu V \]  

Combining equations (8) and (10), the resultant expression is written as

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( G(H_i,l,M_o,C)r \frac{\partial E(p)}{\partial r} \right) = \mu V \]  

Where,

\[ G(H_i,l,M_o,C) = \begin{cases} E(f(h,l,M_o)), & \text{Radial} \\ E(1/f(h,l,M_o)), & \text{Azimuthal} \end{cases} \]  

For Radial: \( E(f(h,l,M_o)) = \frac{35}{32c^3} \int_{-c}^{c} f(h,l,M_o)(c^2 - h_i^2)^3 dh_i \)  

For Azimuthal: \( E \left( \frac{1}{f(h,l,M_o)} \right) = \frac{35}{32c^3} \int_{-c}^{c} \frac{(c^2 - h_i^2)^3}{f(h,l,M_o)} dh_i \)  

Introducing the following dimensionless variables

\[ r^* = \frac{r}{a}, h^* = \frac{h}{h_0}, l^* = \frac{2l}{h_0}, K = \beta a^2, C = \gamma a, P^* = -\frac{h_0 E(p)}{\mu a^2 V}, C = \frac{c}{h_0} \]  

Equation (11) becomes

\[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( G^*(H_i^*,l^*,M_o,C)r^* \frac{\partial P^*}{\partial r^*} \right) = -1 \]  

Where,

\[ G^*(H_i^*,l^*,M_o,C) = \begin{cases} E(f(h^*,l^*,M_o)), & \text{for Radial} \\ E(1/f(h^*,l^*,M_o)), & \text{for Azimuthal} \end{cases} \]


\[ G'(H', l', M_0, C) = \begin{cases} \frac{1}{M_0^2} \left\{ \frac{l'}{(A' - B')^2} \left( \frac{B'}{A'} \tan h' - \frac{A'}{B'} \tan h' + h' \right) \right\}, & \text{for } M_0^2 l'^2 < 1 \\ \frac{h'}{2} \sec h' \left( \frac{h'}{2l'} \right) \left( \frac{3l'}{2l'} \right) \tan h' \left( \frac{h'}{2l'} \right) + h', & \text{for } M_0^2 l'^2 = 1 \\ \frac{1}{M_0^2} \left\{ \frac{l'}{M_0 (\cos B'_* h' + \cosh A'_* h')} + h' \right\}, & \text{for } M_0^2 l'^2 > 1 \end{cases} \]

The corresponding boundary condition for the fluid pressure is given by

\[ \frac{\partial P^*}{\partial r^*} = 0 \quad \text{at } r^*=0 \quad \text{and} \quad P^* = 0 \quad \text{at } r^*=a. \]  

(16)

Using boundary condition (16), integrating equation (15), and the expression for the dimensionless film pressure is given by

\[ P^* = \frac{-1}{2} \int_{r^*}^r G'(H', l', M_0, C) dr^*. \]  

(17)

The mean load supporting capacity is obtained by integrating the pressure field over the film region and relation is,

\[ W^* = \frac{E(w)h_0^2}{\pi \mu a^4 V} = -\frac{1}{2} \int_0^r \int_{r^*}^r G'(H', l', M_0, C) r^* dr^* \]  

(18)

The response film time for the film thickness is,

\[ T^* = \frac{E(w)h_0^2}{\pi \mu a^4} = -\frac{1}{2} \int_0^r \int_{r^*}^r G'(H', l', M_0, C) r^* dr^* dh_0^* \]  

(19)

Where \( h^*_0 = h_0 \left\{ \exp(-\beta r^*) - \frac{1}{1+\gamma r} + 1 \right\}, \quad h^*_0 = h_0, \quad h^*_0 = h_0 \)

3. Results and Discussions.

In order to analysis the squeeze film characteristics on the rough circular curved plates with non-Newtonian fluid, the numerical calculations are carried out for various non-dimensional parameters namely roughness parameter \( C \), Hartmann number \( M_0 \), couple stress parameter \( l^* \), lower (\( \gamma \)) and upper (\( \beta \)) curvature parameters for couple stress fluids on the basis of Stokes theory [12] and the Christensen’s stochastic theory [3] for rough surfaces.

For the discussion of squeeze film behaviour the numerical values of \( C = 0.0-0.4, \quad M_0 = 0.4, \quad l^* = 0.0-0.4, \quad \gamma = 0.8 \) and \( \beta = 0.8 \) are chosen. Results so obtained are parallel verified with classical case (\( C = 0 \)), Newtonian case (\( l^* = 0 \)) and Non–magnetic case (\( M_0 = 0 \)).

3.1 Non dimensional pressure.

In figure-2 displays the variation of Non-dimensional pressure \( P^* \) along the axial coordinate \( r^* \) for distinct values of \( C \) with the parametric values \( M_0 = 3, \quad l^* = 0.3, \quad \gamma = 0.8 \) and \( \beta = 0.8 \) for both the rough surface
configuration and found that at $C=0$ both the radial and azimuthal roughness configuration coincides. Further, increase in $P^*$ is more proclaimed with azimuthal roughness as seen in radial roughness configurations.
In Fig-3 and Fig-4, represents non-dimensional pressure $P^\ast$ along the axial coordinate $r^\ast$ for distinct values of Magnetic field $M_0$ and couple stress parameter $l^\ast$ along fixed values of $C = 0.3, \gamma=0.8$ and $\beta=0.8$ for both roughness configurations, it is found that the magnetic field and couple stress parameter enhances the pressure rise in the fluid film region as compared to the classical case.
3.2 Load supporting capacity.
In Fig-5, we plot dimensionless load supporting capacity $W^*$ versus upper plate curvature parameter $\beta$ for distinct values of $C$ with $M_0=3$, $l'=0.3$ and $\gamma=0.8$ for both the roughness configurations and found that both the roughness configurations coincides at $C = 0$. Further, for larger values of $C$, dimensionless load supporting capacity $W^*$ increases significantly for azimuthal roughness and decreases for radial roughness.

Fig-6 and Fig-7 depicts variation of non-dimensional load supporting capacity $W^*$ versus $\beta$ for distinct values of $M_0$ and $l'$ for constant values of $C = 0.3$ and $\gamma=0.8$ for both the roughness configurations and from the figures it is found that $W^*$ increases for increasing values $M_0$ and $l'$, as compared to classical case (i.e. $M_0=0$ and $l'=0$).

In Fig-8, the non-dimensional load supporting capacity $W^*$ is plotted along with lower plate curvature parameter $\gamma$ for various values of $C$ with $M_0=3$, $l'=0.3$ and $\beta=0.8$ for radial and azimuthal roughness configurations. As seen from the figure that the non-dimensional load supporting capacity $W^*$ is significant with greater values of roughness parameter and the effect is more prominent for azimuthal than those for radial roughness pattern. The dimensionless load $W^*$ along with $\gamma$ for various values of $M_0$ and $l'$ are plotted in Fig-9 and Fig-10, for radial and azimuthal patterns with fixed values of $C = 0.3$ and $\beta=0.8$. A significant increase is seen in $W^*$ for larger values of $M_0$ and $l'$ as compared to the smooth bearing surface but decreases with increasing values of lower plate curvature parameter.
Figure 7: Plot of $W^*$ along $\beta$ for distinct values of $l^*$, with $M_0=3$, $C=0.3$, $\gamma=0.8$.

Figure 8: Plot of $W^*$ along $\gamma$ for distinct values of $C$ fixed $M_0=3$, $l^*=0.3$, $\beta=0.8$. 
Figure 9: Plot of $W^*$ along $\gamma$ for distinct values of $M_0$ with $C=0.3, l^* = 0.3, \beta = 0.8$.

Figure 10: Plot of $W^*$ along $\gamma$ for distinct values of $l^*$, with $C=0.3, M_0=3, \beta = 0.8$.

3.3 Squeeze film time.

The dimensionless response film time $T_1^*$ along $h_1^*$ is plotted in Figure-11 for various values of $C$, with fixed parametric values $M_0=3, l^* = 0.3, \beta = 0.8$ and $\gamma=0.8$ for both radial and azimuthal configurations.
Figure 11: Plot of $T^*$ with $h_1^*$ for distinct values of $C$ with $l^* = 0.3, M_0^* = 3, \beta = 0.8, \gamma = 0.8$.

Figure 12: Plot of $T^*$ along $h_1^*$ for distinct values of $M_0^*$ with $l^* = 0.3, C = 0.1, \beta = 0.8, \gamma = 0.8$.

and seen that $T^*$ declined for raising values of $h_1^*$. Further it is found that effect of azimuthal (radial) roughness configurations is to incline (decline) $T^*$ for greater values of $C$ as compared to $C = 0$. Fig-12 and Fig-13, depicts the various results of $T^*$ along with $h_1^*$ for distinct values of $M_0$ and $l^*$ for both the
roughness configurations. As magnetic parameter and couple stress increases the squeeze film time is also increased when compared to $M_0=0$ and $l'=0$.

![Figure 13: Plot of $T^*$ along $h_1^*$ for distinct values of $l^*$ with $M_0=3, C=0.1, \beta=0.8, \gamma=0.8$.](image)

| $M_0$ | Hanumagowda et.al | Present analysis |
|-------|-------------------|-----------------|
|       | $l^*=0.2$ | $l^*=0.4$ | $l^*=0.2$ | $l^*=0.4$ | Radial | Azimuthal | Radial | Azimuthal |
| 0     | 0.9007 | 1.1636 | 0.9007 | 1.1636 | 0.8865 | 4.9565 | 1.1403 | 8.4800 |
| 2     | 1.2103 | 1.4758 | 1.2103 | 1.4758 | 1.2012 | 5.4719 | 1.4599 | 8.9980 |
| 4     | 2.1312 | 2.4089 | 2.1312 | 2.4089 | 2.1281 | 7.0153 | 2.4036 | 10.5508 |
| 6     | 3.6444 | 3.9526 | 3.6444 | 3.9526 | 3.6439 | 9.5778 | 3.9536 | 13.1352 |

| $W^*$ | 0 | 2 | 4 | 6 |
|-------|---|---|---|---|
| 0.000 | 4.6513 | 4.9637 | 5.9000 | 7.4574 |
| 2.000 | 8.8156 | 9.1292 | 10.069 | 11.6358 |
| 4.000 | 8.823 | 9.137 | 10.077 | 11.644 |
| 6.000 | 8.569 | 8.529 | 6.8757 | 8.4556 |
| 8.000 | 7.8446 | 8.2090 | 9.2810 | 11.017 |
| 10.000| 11.384 | 11.702 | 12.655 | 14.244 |

| $T^*$ | 0 | 2 | 4 | 6 |
|-------|---|---|---|---|
| 0.000 | 4.6545 | 4.967 | 5.9034 | 7.4610 |
| 2.000 | 8.823 | 9.137 | 10.077 | 11.644 |
| 4.000 | 5.6089 | 5.9259 | 6.8757 | 8.4556 |
| 6.000 | 7.8446 | 8.2090 | 9.2810 | 11.017 |
| 8.000 | 11.384 | 11.702 | 12.655 | 14.244 |

4. Concluding remarks.
A combined studied on MHD and surface roughness for conducting couple stress squeeze film behaviour between curved circular plates is analysed using Christensen stochastic theory for rough surface[3]. The following conclusions are drawn based on theoretical and numerical results obtain in the present study.
It is found that for $C = 0$, the results so obtained for both roughness configurations for one-dimensional curved circular plates can be reduced to smooth plates discussed by Hanumagowda et al. [14].

- The effect of Magnetic parameter and couple stress increases the non-dimensional pressure when considered to Non-magnetic case and Newtonian case for both roughness patterns.
- The dimensionless load carrying capacity is increased when considered to smooth case due to the effect of applied Magnetic field and couple stress.
- From the results obtained, effect of magnetic field and couple stress increases the response time when considered to the smooth case for both roughness patterns and the values are shown in Table 1.

5. References.
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6. Nomenclature.

\[B_0\] Transverse magnetic field

\[M_0\] Hartmann number \((= B_0 h_2 (\sigma/\mu)^{1/2})\)

\[p\] Pressure in the film region

\[l\] Couple stress parameter \((\eta/\mu_0)^{1/2}\)

\[l^*\] Non-dimensional couple stress parameter \((2l/h_0)\)

\[h^*\] Non-dimensional film thickness

\[u, w\] Velocity components in \(r\) and \(z\) directions

\[r, z\] Radial and Axial coordinates

\[W\] Load carrying capacity

\[W^*\] Non-dimensional load carrying capacity

\[T\] Squeeze film time

\[T^*\] Non-dimensional squeeze film time

\[c\] Maximum asperity deviation from the nominal film height

\[C\] Dimensionless roughness parameter \((c/h^3)\)

\[E\] Expectancy operator defined by Eq. (12)

\[H_i\] Film thickness \((h_i + h_s)\)

\[H_i^*\] Non-dimensional film thickness

Greek Symbols

\[\beta\] Upper plate curvature parameter

\[\gamma\] Lower plate curvature parameter

\[\eta\] Material constant responsible for couple stresses

\[\mu\] Lubricant viscosity

\[\sigma\] Electrical conductivity

\[\sigma\] Standard deviation \((c/3)\)