ELECTROWEAK PROCESSES OF THE DEUTERON IN EFFECTIVE FIELD THEORY *

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We review our recent calculations of electroweak processes involving the deuteron, based on pionless effective field theory with dibaryon fields. These calculations are concerned with neutron-neutron fusion and \( np \rightarrow d\gamma \) at BBN energies.

1. Introduction

The study of electroweak processes plays an important role in few-body physics. Effective field theory (EFT) provides a systematic way of calculating the transition amplitudes for those processes. It can also establish, through the symmetry of QCD, useful relations between the amplitudes for weak- and strong-interaction processes. Some of the important processes, e.g., neutron \( \beta \)-decay\(^1\) and the electroweak processes involving the deuteron, have been studied in the framework of EFT\(^2\). In this talk, we review two recent studies on neutron-neutron fusion\(^3\) and \( np \rightarrow d\gamma \) for big-bang nucleosynthesis (BBN)\(^4\); these studies employ pionless EFT with dibaryon fields (dEFT)\(^5\).\(^6\) As regards \( nn \)-fusion, we pay particular attention to the consequences of uncertainties in the existing experimental data on the neutron-neutron scattering length and effective range. As for the \( np \rightarrow d\gamma \) cross section at BBN energies, a Markov Chain Monte Carlo (MCMC) is adapted to analyze the relevant experimental data and determine the low energy constants (LECs) in dEFT.

2. Neutron-Neutron Fusion, \( nn \rightarrow de^-\bar{\nu}_e \)

Ultra-high-intensity neutron-beam facilities are currently under construction at, e.g., the Oak Ridge National Laboratory and J-PARC and are expected to bring great progress in high-precession experiments concerning the fundamental properties of the neutron. Besides these experiments that focus on the properties of a single neutron, one might consider processes that involve the interaction of two free neutrons, which allow the model-independent determination of the neutron-neutron scattering length and effective range, \( a_0^{nn} \) and \( r_0^{nn} \). In this talk, we first

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\(^2\)We refer to it as “dibaryon EFT” (dEFT) in this talk.
consider the $nn$-fusion process for neutrons of very low energies such as the ultra- 
cold neutrons and thermal neutrons. It is worth noting that, for very low en-
ergy neutrons, the maximum energy $E_{\text{e}}^{\text{max}}$ of the outgoing electrons from $nn$-fusion
is $E_{\text{e}}^{\text{max}} \simeq B + \delta_N \simeq 3.52$ MeV, where $B$ is the deuteron binding energy and
$\delta_N = m_n - m_p$. The value of $E_{\text{e}}^{\text{max}}$ is significantly larger than the maximum energy
of electrons from neutron $\beta$-decay, $E_{\text{e},\beta-\text{decay}}^{\text{max}} \simeq \delta_N \simeq 1.29$ MeV, and thus the $nn$-
fusion electrons with energies larger than $\delta_N$ are in principle distinguishable from
the main background electrons of neutron $\beta$-decay.

Diagrams for the $nn$-fusion process up to next-to leading order (NLO) are shown
in Fig. 1, from which the cross section is calculated $^3$. We also include the Fermi
function and $\alpha$-order radiative corrections pertaining to the one-body interaction
$^1$ to ensure accuracy better than 1 % in the cross section. The two low-energy constants
(LECs), $e_R^V$ and $l_{1A}$, appear in our calculation. Using the formula for neutron $\beta$-
decay $^1$ and the recent values of $G_F$, $V_{ud}$, $g_A$, and the neutron lifetime $\tau$ in the
literatures, we deduce $\frac{\alpha^2}{2\pi} e_R^V = (2.01 \pm 0.40) \times 10^{-2}$. The LEC, $l_{1A}$, which also
contributes to other processes, e.g., $pp$-fusion and $\nu$-$d$ reactions, can in principle
be fixed from the tritium $\beta$-decay data. However, there has been no attempt to
include the weak current into the three-nucleon system in dEFT. So we make use
of the result from the pionful EFT $^6$, and obtain $l_{1A} = -0.33 \pm 0.03$. Hence the
uncertainties due to the errors in these LECs and higher order terms should be less
than 1%. The prime uncertainty in the cross section comes from $a_0^{nn}$ and $r_0^{nn}$, $^7$
\begin{equation}
    a_0^{nn} = -18.5 \pm 0.4 \text{[fm]}, \quad r_0^{nn} = 2.80 \pm 0.11 \text{[fm]}.
\end{equation}

We are now in a position to carry out numerical calculations of the electron
spectrum and the cross section. Since the $nn$-fusion cross section obeys the $1/v$
law, where $v$ is the relative velocity between the two neutrons, we may concentrate
on a particular value of the incident neutron energy. We consider here a head-
on collision of two ultra-cold neutrons (UCN) ($v_{\text{UCN}} \simeq 5$ m/sec), and thus $v =
2v_{\text{UCN}} \sim 10$ m/sec. In Fig. 2, we plot the calculated electron spectrum, $d\sigma/dE_e$,
as a function of $E_e$. As mentioned, the electrons with $E_e > \delta_N = 1.29$ MeV are
in principle distinguishable from the electrons coming out of neutron $\beta$-decay. The
total cross section $\sigma$ is calculated to be
\begin{equation}
    \sigma = (38.6 \pm 1.5) \times 10^{-40} \text{[cm$^2$]}.
\end{equation}
Figure 2. Spectrum of the electrons from neutron-neutron fusion, $nn \to d\nu$.

We find that the significant uncertainty ($\sim 4\%$) in the cross section comes solely from the current experimental errors of $a_0^{nn}$ and $r_0^{nn}$. Since the cross section obtained here is very small, the experimental observation of this reaction does not seem to belong to the near future.

3. $np \to d\gamma$ at the BBN energies

Primordial nucleosynthesis processes take place between $1$ and $10^2$ seconds after the big bang at temperatures ranging from $T \simeq 1$ MeV to 70 keV. Predictions of primordial light element abundances, D, $^3$He, $^4$He, and $^7$Li, and the comparison of them with observations are a crucial test of the standard big bang cosmology. The uncertainties in these predictions are dominated by the nuclear physics input for the reaction cross sections. Reaction databases are continuously updated$^8$, with more attention now paid to the error budget. The cross section of the $np \to d\gamma$ process at the BBN energies has been thoroughly studied by using pionless EFT up to $N^3$LO by Chen and Savage$^9$, and up to $N^4$LO by Rupak$^{10}$. In this part of talk, we present an estimation of the cross section employing a new method, i.e., a combination of dEFT up to NLO and an MCMC analysis with the aid of the relevant experimental data. We find that this method leads to a result comparable with that obtained by Rupak, and we discuss that the estimated $np \to d\gamma$ cross section at the BBN energies is reliable to within 1%.

Figure 3. Diagrams for the $np \to d\gamma$ process up to NLO in dEFT.
Diagrams for the $np \rightarrow d\gamma$ process up to NLO in dEFT are shown in Fig. 3. From these diagrams we calculate the amplitudes for the $S(1S_0$ and $3S_1)$- and $P$-waves of the initial two-nucleon. We note that since the $3S_1$ amplitude is highly suppressed due to the orthogonality of the scattering and bound $3S_1$ states, we neglect it in our calculations. Using these amplitudes, we can easily calculate the cross section for $np \rightarrow d\gamma$.

Five parameters, $a_0$, $r_0$, $\gamma$, $\rho_d$, and $l_1$, appear in the amplitudes. We determine the values of the four parameters, $a_0$, $r_0$, $\rho_d$, and $l_1$, by the MCMC analysis of the relevant low energy experimental data; the total cross section of the $np$ scattering at the energies $\leq 5$ MeV (2124 data) from the NN-OnLine web page, the $np \rightarrow d\gamma$ cross section from Suzuki et al.,¹¹ and Nagai et al.,¹² including two thermal capture data,¹³ the $d\gamma \rightarrow np$ cross section from Hara et al.,¹⁴ and Moreh et al.,¹⁵, and the photon analyzing power from Tornow et al.,¹⁶ and Schreiber et al.,¹⁷. Meanwhile, we constrain $\gamma$ from the accurate value of $B$. In Table 1 we give our estimates of the parameters obtained from the present MCMC analysis along with the values obtained in the previous method ("Prev. Method"). We find small differences ($\leq 2\%$) between the values of the parameters for the two cases; we will come back to this later.

In Table 2 the theoretical estimates of the $np \rightarrow d\gamma$ cross section at BBN energies are given as a function of the initial two-nucleon energy $E$ in the center of mass (CM) frame. The column labeled "dEFT(MCMC)" gives our preliminary results for the mean values and standard deviations obtained in MCMC. Table 2 also shows the results of four other methods: "dEFT(Prev. Meth.)" based on the parameter set "Prev. Method" in Table 1, pionless EFT up to $N^4$LO by Rupak, a high-precision potential model calculation including the meson-exchange current by Nakamura, and an R-matrix analysis by Hale. Good agreement is found among the different approaches except that the results of "dEFT(Prev. Meth.)" at $E = 0.5$ and 1 MeV and those of Hale exhibit some deviations, which are $\sim 0.6\%$ in the former and go up to 4.5 $\%$ in the latter. The $\sim 0.6\%$ difference at $E = 0.5$ and 1 MeV between "dEFT(MCMC)" and "dEFT(Prev. Meth.)" is significant compared to the small $\sim 0.3\%$ statistical errors obtained here. This difference can be accounted for by higher order terms that are not included in the amplitudes of dEFT up to NLO. By including the higher order terms associated with the $P$-wave scattering

| Table 1. Values of parameters |
|-----------------------------|
| Parameter  | MCMC       | Prev. Method |
| $a_0$      | $-23.7426 \pm 0.0081$ | $-23.749 \pm 0.008$ |
| $r_0$      | $2.783 \pm 0.043$    | $2.81 \pm 0.05$    |
| $\rho_d$   | $1.7460 \pm 0.0072$  | $1.760 \pm 0.005$  |
| $l_1$      | $0.798 \pm 0.029$    | $0.782 \pm 0.022$  |

bThe values of the effective ranges, $a_0$, $r_0$, and $\rho_d$, are taken from Ref.¹⁸, and the value of $l_1$ is obtained from the averaged value of the two thermal capture rates¹³.
Table 2. Theoretical estimates of the $np \rightarrow d\gamma$ cross sections at the BBN energies. $E$ is the initial two-nucleon energy in CM frame. See the text for more details.

| $E$ (MeV) | dEFT (MCMC) | dEFT (Prev. Meth.) | Rupak | Nakamura | Hale |
|-----------|-------------|-------------------|-------|----------|------|
| $1.265 \times 10^{-4}$ | 333.8(4) | 333.7(15) | 334.2(0) | 335.0 | 332.6(7) |
| $5 \times 10^{-4}$ | 1.667(2) | 1.666(8) | 1.668(0) | 1.674 | 1.661(7) |
| $1 \times 10^{-3}$ | 1.171(1) | 1.171(5) | 1.172(0) | 1.176 | 1.167(2) |
| $5 \times 10^{-3}$ | 0.4979(6) | 0.4976(21) | 0.4982(0) | 0.4999 | 0.4953(11) |
| $1 \times 10^{-2}$ | 0.3321(4) | 0.3319(14) | 0.3324(0) | 0.3335 | 0.3298(9) |
| $5 \times 10^{-2}$ | 0.1079(1) | 0.1079(4) | 0.1079(0) | 0.1084 | 0.1052(9) |
| 0.100 | 0.0634(7) | 0.0634(2) | 0.0635(2) | 0.06366 | 0.0605(10) |
| 0.500 | 0.03413(8) | 0.0343(1) | 0.0341(2) | 0.03416 | 0.0338(8) |
| 1.000 | 0.03502(10) | 0.0352(2) | 0.0349(3) | 0.03495 | 0.0365(8) |

volumes\(^9\), we can reproduce the “dEFT(MCMC)” results at $E = 0.5$ and 1 MeV in “dEFT(Prev. Meth.)”. This implies that the values fitted by MCMC mimic the roles of the higher order terms. Since our results “dEFT(MCMC)” agree quite well with those of Rupak and Nakamura’s calculations, and since in the N\(^4\)LO pionless EFT calculation by Rupak, various corrections due to the higher order terms have been studied, we infer that the estimated $np \rightarrow d\gamma$ cross section at the BBN energies should be reliable within 1% accuracy. A dEFT calculation provides a systematic perturbation scheme and a simple model-independent expression for the amplitudes in terms of a finite number of LECs. As demonstrated above, the combination of a dEFT calculation and an MCMC analysis of available experimental data would be a useful method to deduce reliable cross sections for other few-body processes.

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