Turbulent energy routes in viscoelastic wall turbulence

E. De Angelis, C.M. Casciola & R. Piva

1DIEM, II Faculty of Engineering, University of Bologna, Italy
2DIMA, University of Rome "La Sapienza", Italy
E-mail: e.deangelis@unibo.it

Abstract. A small amount of long chain polymers dissolved in an otherwise Newtonian flow is known to reduce dramatically drag in wall bounded flows. This corresponds to a drastic change in the mean velocity profile, where the slope of the log-law in the near wall region passes from 2.5 to 11.7, see Virk (1975). This phenomenon is the most reported of a thorough alteration of the dynamics of wall turbulence, but it is still lacking a satisfactory explanation. To advance in this direction, in the present contribution we will extend the scale by scale analysis of a viscoelastic channel flow already presented in our previous paper in Palma & Lopes (2007) discussing the alteration of the energy fluxes from production to Newtonian and non-Newtonian dissipation. The proposed framework, which had been used by Marati et al. (2004) and recently applied to larger Reynolds number Newtonian turbulence, Cimarelli et al. (2011), simultaneously describes the dynamics of turbulent fluctuations in the space of scales and in the physical space, and can be used to understand the alteration of the mechanisms of formation and sustainment of the turbulent fluctuations in the near-wall region.

1. Introduction

Long chain polymers added to an otherwise Newtonian liquid can deeply modify the structure of turbulence. Recent studies have in fact shown that the new rheology can lead to very interesting phenomena like the alteration of the heat transfer, Benzi et al. (2010), or the modification of the turbulent interface dynamics, Liberzon et al. (2009). In spite of these very interesting findings, which confirm a complex coupling between the polymers molecules and the dynamics of turbulent fluctuations, the most striking and robust effect still remains the reduction of friction drag in wall bounded flows.

Since the first observations by Toms in 1948 a large number of experimental investigations in the past and numerical analysis more recently have appeared. Such an intense activity can be justified certainly by the unicity of such phenomenon, a small amount of solute can produce a very large effect, but also by its complexity due to the interplay of wall turbulence and polymer dynamics. Actually the much investigated phenomenon of Drag Reduction (DR), see Virk (1975) for a thorough account, is only the most evident aspect of the effect of polymers in solution and it is always accompanied by a deep alteration of wall turbulence structure. A persistent observation is a large depletion of Reynolds stresses, as reported by Wei & Willmarth (1992), and Warholic et al. (1999), associated with an increase of polymer stresses as can be directly checked in simulations, see De Angelis et al. (2002) and Min et al. (2003) for some...
examples. The previous findings have been normally reported together with an increase in the streamwise component of the velocity fluctuations and a decrease in the wall normal direction. More in general an alteration of both length and time scales have been observed, in particular an increased streak spacing, a decrease of the characteristic burst frequency, see Luchik & Tiederman (1988), have been often reported. More recently thanks to numerical simulations, the phenomena described have been substantiated and the most robust observation has been a negative correlation between the turbulent fluctuations and fluctuating polymeric forces, see De Angelis et al. (2002) and Dubief et al. (2004), that can at least phenomenologically explain the weakening of the Reynolds stresses.

Altogether, the extensive analysis carried out have provided a large body of knowledge on the alteration of turbulence structure via flexible polymers. However, a fundamental explanation is still missing. In this scenario, the present paper addresses the dynamics of turbulent energy in wall turbulent flows in the framework of the generalized Kolmogorov equation already used to study the energy fluxes for a Newtonian turbulent channel flow by Marati et al. (2004). This approach, already extended to the polymeric case, allows for a description of the dynamics of turbulent fluctuations in the space of scales and in the physical space and can be used to understand how the drag reduction phenomenon, typically associated with large scales, can be influenced by a small scales interference operated by the polymer molecules. In particular, we will discuss how only a full comprehension of the energy routes starting from the buffer layer and toward the rest of the channel allows for an explanation of polymer drag reduction.

In the following section a thorough description of the simplest dumbbell model for a dilute polymer solution, i.e. Oldroyd B, will be offered. After that, the alteration of wall turbulence caused by the introduction of polymers will discussed by the analysis of DNS data obtained via a spectral method. A detailed description of the small scale dynamics via an extension of the Kolmogorov equation to inhomogeneous flows for dilute polymer solutions will follow.

2. Viscoelastic wall turbulence

The parameter which couples the dynamics of the polymer chains and the turbulence is the ratio of two times scales, namely the Deborah (or Weissemberg) number $D_{e*} = \tau_p / \tau_*$. Here $\tau_* = \nu / u_*^2$ is the friction time scale and $\tau_p$ is the principal relaxation time of the chain, the estimated time needed to recover equilibrium after the external strain is removed. As an order of magnitude, in most of the experiments this ratio is of the order of one for dilute systems at the onset of drag reduction. Hence the internal dynamics of the chain is unlikely to get substantially coupled to the turbulence. This allows for the description of the system with a single internal degree of freedom, this model is called Oldroyd-B. The resulting dumbbell model consists of two mass-less beads, acted upon by friction in the relative motion with respect to the carrier fluid, connected through an elastic force of Brownian origin. Given the huge size of the chains, their diffusivity in the solvent is negligible. Each dumbbell is represented in terms of the vector separation of the two beads $R$. Averaging the force balance on the dumbbell leads to the evolution equation for the conformation tensor $\mathcal{R} = \langle R \otimes R \rangle$,

$$\frac{D \mathcal{R}}{Dt} = \mathcal{R} \cdot \nabla u + \nabla u^\top \cdot \mathcal{R} - \frac{1}{D_{e*}} (\mathcal{R} - \mathcal{I}) \cdot \nabla u$$

(1)

The model is completed with the momentum equation for an incompressible flow ($\nabla \cdot u = 0$) augmented with the extra-stress

$$\frac{Du}{Dt} = -\nabla p + \frac{1}{Re_*} \nabla^2 u + \nabla \cdot T_p$$

(2)

where

$$T_p = \eta_p/Re_* \langle (R \otimes R - \mathcal{I})/3 \rangle$$

(3)
Figure 1. Left: Mean velocity profiles in viscous variables. In the different cases the friction Reynolds is the same, \( Re_s = 300 \). The solid line gives the Newtonian data. The polymeric data are: \( De_s = 18 \) (triangles), \( De_s = 36 \) (squares), \( De_s = 72 \) (circles), \( De_s = 90 \) (diamonds). Right: Fluctuation intensities in viscous variables, streamwise \( u'_{\text{rms}} = \sqrt{(u'/u_s)^2} \) (filled symbols) and wall-normal \( v'_{\text{rms}} = \sqrt{(v'/u_s)^2} \) (empty symbols). The values of \( De_s \) are coded as in the left panel.

For the present DNS data, the dimensions of the integration domain are \( 2\pi h \times 2h \times 1.2\pi h \), where \( h \) is half the channel height. The simulations have been performed on a channel flow at a nominal Reynolds number of 10000, for all the cases, Newtonian and viscoelastic. The fluctuations are periodic in the streamwise, \( x \), and span-wise, \( z \), coordinates. The numerics consists of a Fourier \( \times \) Chebyshev \( \times \) Fourier spectral method, see De Angelis et al. (2002) for the details. The flow has been forced on average with the same pressure drop, so the resulting Reynolds number based in the friction velocity is the same and equal to \( Re_s = 300 \). The values for the polymers parameters are ranging from \( De_s = 0, 90 \). In this framework drag reduction amounts to an increased throughput \( Q \) and is measured by the fractional flow rate enhancement compared to Newtonian, \( S = (Q_p - Q_N)/Q_N \). As shown in the left panel of fig. 1, increasing the value of the relaxation parameter causes a shift upwards of the mean velocity profile and hence a monotonic increase of the parameter \( S \) up to a value of 65\% for \( De_s = 90 \). As commonly observed in wall turbulent flows, drag reduction is related with an increase of the streamwise fluctuations and a decrease of the wall normal ones, see the right panel of figure 1. Moreover, the depletion of the Reynolds stresses with drag reduction is apparent, left panel of figure 2. The production and dissipation terms of the budget of the turbulent kinetic energy, \( \epsilon \), is shown in figure 2, where each term is scaled with \( u_s^4/\nu \). For the viscoelastic fluid an additional terms due to the extra-stress has to be added to the usual Newtonian expression,

\[
-\langle u' v' \rangle \frac{dU}{dy} - \frac{d\langle q'^2 \rangle}{2dy} \frac{d\langle p' v' \rangle}{dy} + \frac{1}{Re_s} \frac{d^2\langle q'^2 \rangle}{2dy^2} - \epsilon_N - \pi_p + \frac{d\langle \mathbf{T}'_p \cdot \mathbf{u}' \rangle}{dy} = 0,
\]

where \( q'^2 \) is the squared modulus of the fluctuation velocity and \( \pi_p = \langle \mathbf{T}'_p \cdot \nabla \mathbf{u}' \rangle \), is discussed in the right panel of figure 2. It is splitted in two components, \( -\pi_p \) represents the energy drain in favour of the microstructure, \( \langle \mathbf{T}'_p \cdot \mathbf{u}' \rangle \) contributes to the spatial flux in the wall normal direction, \( \phi_s = \langle q'^2 \rangle/2 + \langle p' v' \rangle - \langle \mathbf{T}'_p \cdot \mathbf{u}' \rangle - d\langle q'^2 \rangle/(2Re_s dy) \). The energy available at a certain location, due to local production and divergence of the spatial flux, is partly dissipated by ordinary viscosity, \( \epsilon_N \), and partly moved to the polymers, \( \pi_p \).
3. The Kolmogorov’s equation generalized to viscoelastic wall-turbulence

Wall-bounded turbulence is characterized by several processes which maybe thought as belonging to two different classes: phenomena which occur in physical space and phenomena which take place in the space of scales. The most significant aspect of the former is the spatial flux of turbulent kinetic energy and of the latter is the energy transfer among scales due to the coupling between eddies of different size. As a consequence, a full understanding of these phenomena requires a detailed description of the processes occurring simultaneously in physical and scale space. For a planar channel or pipe flow, a simultaneous view of small and large scale dynamics can be achieved by a suitable generalization of the budget of $\langle \delta u^2 \rangle$ here extended to polymeric flows. In this case the scale energy is a function of the separation vector $r_i$ and of the mid-point $X_{ci} = 1/2(x_i' + x_i)$, allowing for the description of the scale-dependent energy processes in the presence of inhomogeneity

$$
2\nu \left( \nabla_r^2 + \frac{\partial^2}{8\partial Y_{c}^2} \right) \langle \delta q^2 \rangle - 4\epsilon_N + 4 \left( \nabla_r \cdot \langle T_{p}^{\prime \prime} \cdot \delta u' \rangle + \frac{\partial \langle Y_{c} \cdot \delta T_p^{\prime \prime} \cdot \delta u' \rangle}{\partial Y_{c}} - \pi_p^{*} \right),
$$

the asterisk denotes the half-sum of the quantity at the two points, $\hat{y}$ is the wall-normal unit vector, $u'$, $v'$ and $p'$ are streamwise, wall-normal velocity fluctuations and pressure fluctuation, respectively. This simplified form holds when the two points lay at the same wall-normal distance, see also Marati et al. (2004). Terms with $r$-derivates are related to the flux through scales, due to the nonlinear terms, the viscosity and to polymers respectively, while the ones with $Y$-derivates arise due to inhomogeneity, with a further term which is due to pressure. The term $2\langle \delta u \delta v' \rangle (dU/dY_{c})^{*}$ represents the production at scale $r$. It is easy to show that the large scale limit of (5) is equal to four times the balance of turbulent kinetic energy (4). Equation (5), as it is, offers a large amount of information and can provide a tool to assess the relative importance of the various processes occurring in viscoelastic turbulence at different scales and distances from the wall. To ease the interpretations, as already done in Marati et al. (2004), we will firstly define and discuss an $r$-average of the terms in (5) as

$$
Q_{c}(r, Y_{c}) = \frac{1}{r^2} \int_{-\frac{r}{2}}^{\frac{r}{2}} \int_{-\frac{r}{2}}^{\frac{r}{2}} q(r_x, 0, r_z|Y_{c}) \, dr_x \, dr_z,
$$

Figure 2. Left panel: Reynolds shear stress, $-T_{R}^{\prime \prime} = \langle u'v' \rangle / u_{*}^{2}$. The values of $De_{*}$ are coded as in fig. 1. Right panel: Fluctuation intensities in viscous variables, streamwise $u_{rnm}^{+} = \sqrt{(u'/u_{*})^{2}}$ (filled symbols) and wall-normal $v_{rnm}^{+} = \sqrt{(v'/u_{*})^{2}}$ (empty symbols). Symbols as in fig. 1.
where \( r = (r_x, 0, r_z) \) and \( r_x = r_y \). This approach allows to evaluate the effective role of the different process at a given scale \( r \), averaging out the directional information carried by equation (5).

Applying this average operator and casting together the terms in the round parenthesis, the equation (5) reads, in a compact form,

\[
T_r(r, Y_c) + \Pi_e(r, Y_c) = D_e(r, Y_c) + E(r, Y_c) + G_e(r, Y_c) + E_p(r, Y_c). \tag{7}
\]

In analogy with a homogeneous and isotropic case, equation (7) can be interpreted as the sum of transfer through the scales, \( T_r \) and effective production balanced by the flux toward the polymers and by the dissipative contributions due both to the solvent and the polymers.

In order to exploit in a different way the structure of equation (5), it can be rewritten in a conservative form,

\[
\nabla \cdot \Phi(r, Y_c) = \xi(r, Y_c), \tag{8}
\]

where \( \nabla \cdot \) is the divergence in the \((r_x, r_y, r_z, Y_c)\)-space of the energy flux \( \Phi = (\Phi_{r_x}, \Phi_{r_y}, \Phi_{r_z}, \Phi_c) \) and \( \xi = 2(\delta u \delta v) \left( \frac{dU}{dy} \right)^* - 4\epsilon_0^* - 4\pi_p^* \) is the energy source/sink as it reads when the additional dissipation due to the solvent is taken into account. It is worth pointing out that for the polymeric case the components of the flux \( \Phi \) retain a viscoelastic component which depends on the scale and on the distance from the wall. The form (8) highlights that the energy flux vector field, composed by a flux \( \Phi_r = (\Phi_{r_x}, \Phi_{r_y}, \Phi_{r_z}) \) through different scales and a flux \( \Phi_c \) through different wall-distances, is driven by the scalar field \( \xi \). In homogeneous conditions, this term is always negative, \( \xi_{\text{hom}}(r) \leq 0 \), consistently with the classical notion of the Richardson cascade, even in viscoelastic turbulence, see De Angelis et al. (2005). The energy transfer is initialized at the largest scales where production equals dissipation, \( \xi_{\text{hom}}(\infty) = 0 \), and then scale-energy moves toward the energy sink ranges at small scales. In wall-turbulence, as already discussed in Cimarelli et al. (2011), the source term might reach positive values, \( \xi(r_x, r_y, r_z, Y_c) > 0 \), meaning that production exceeds dissipation at certain scales and wall-distances. This is a distinguishing feature of actual inhomogeneous flows which is modulated by the presence of the polymers as will be discussed in Sec 4.2.
4. Results

4.1. ($r, Y$)-results
The generalized form of scale-energy budget, described in the previous paragraph and able to
discriminate between the different kinds of energy fluxes which occur either in physical and scale
space will be shown as an instrumental tool for the evaluation of the interaction of polymers with
the near-wall environment. In this framework the respective role of the nonlinear transfer and
the flux to the polymers can be in principle evaluated in different regions of the boundary layer.
Namely, to write equation (7), an effective production has been defined as the sum between
the turbulent transport and real production $\Pi(r, Y_c) = \Pi(r, Y_c) + T_c(r, Y_c) - P(r, Y_c)$ and
analogously also the viscous and polymeric contributions have been expressed as the sum of a
transport term, $D_e(r, Y_c)$ and $G_e(r, Y_c)$, grouping together the $r$-fluxes and the $Y_c$-fluxes, and
a local term, local dissipation, $E(r, Y_c)$, and local transfer to the microstructure, $E_P(r, Y_c)$,
respectively. In other words the energy available at given position $Y_c$ and scale $r$ is partly
produced locally by the production term $\Pi$ and partly arrives or leaves thanks to the spatial
transfer, the remaining part is either dissipated or transferred to the microstructure. The
following discussion will focus on the role of the polymeric terms in the budget for the channel
flow. To begin with we analyze the scale by scale budget (7) at fixed distances from the wall,
starting in the putative logarithmic region of the channel. The behavior of the fluctuations
resembles the one observed in the homogeneous and isotropic case De Angelis et al. (2005), at
large scale the dominant term is the effective production, then the energy is transferred through
the scales until it reaches a point where the transfer to the polymers becomes larger, thus the
dynamics seem affected by the polymers only at the very small scales. More instructive is the
analysis of the budget in the buffer layer. In this region, the larger scales are dominated by
production effects and the smaller scales are controlled by the polymers. However, around the
cross-over, energy production and polymer transfer interact directly, a clear indication of the
effect of polymers on the structure of turbulence in this region and another way of looking at
the fact that in this region the coherent structures of velocity, are correlated with the structures
of polymeric force, see De Angelis et al. (2002) and Dimitropoulos et al. (2005).

A physically oriented discussion is related to the role of the buffer layer as the engine of wall
turbulence and how this fact can interact with polymers. As already emerges from the standard
approach in terms of single-point statistics, the buffer can be described as the region where

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Nonlinear transport term $-T_r$ plotted as a function of the scale and of the distance
from the wall, overall behavior is shown in the main plot and a blow-up of the smaller scales
and distances from the wall in the inset. Left: Newtonian case, right: $De_* = 54$. Red is positive
and blue is negative.}
\end{figure}
Figure 5. Left panel: Contributions of the polymers to the turbulent kinetic energy balance, symbols as in fig. 1, \(-\pi_p\) (heavier symbols) and \(d\langle T'_p \cdot u'\rangle/dy\) (lighter symbols). Right: Polymer transport term \(G_e\) plotted as a function of the scale and of the distance from the wall. Red is positive and blue is negative.

production exceeds dissipation, thereby implying that turbulent kinetic energy is irradiated towards the remaining parts of the flow. An aspect of the buffer layer, which emerges in the present framework and has been already discussed in Marati et al. (2004), is the turbulent transport across scales \(-T_r\), which changes its sign from small to large scales. This behavior may be interpreted in the sense of a classical cascade of energy occurring in the small scales which turns into a reverse cascade at large separations. Those intermediate range of scales of the buffer layer, directly related to the dynamics of the coherent structures which are required to built up Reynolds stresses, are singled out as the effective engine of turbulent fluctuations in wall-bounded flows. A tampering with this range of scales, due to the polymers, can be easily identified as the origin of the strong alteration of the energy containing scales observed in drag reducing flows.

Very instructive is also the inspection of the single terms of (7) in the \((r - Y_c)\)-plane since provides at first glance the information on which is the range of scales and at which distance from the wall where a certain contribution has a prominent role. For example, in figure 4 \(T_r\) is shown as a function both of the scale \(r^+\) and the distance from the wall \(y^+\) both for the Newtonian flow and the case \(De_s = 54\). The inspection of the Newtonian plot confirms the existence of a well defined region in the buffer layer which is characterised by a reverse energy cascade. Whereas, it is possible to observe for the polymer case the depletion of the numerical values, sign of a weakening of the inertial cascade at the expenses of the energy flux toward the polymers. For both plot the positive values of \(-T_r\) for all the scales in the logarithmic region indicates the classical cascade of energy from the large to the small scales. Regarding figure 4, it is worth mentioning that this negative area increase with De, while the actual values are decreasing associated with the observation that at \(r \to \infty\) the values are locked to the single point statistics. An interesting feature emerges from the polymer transport term. In the right panel of figure 3 \(G_e\) is shown as a function both of the scale \(r^+\) and the distance from the wall \(y^+\) for the case \(De_s = 36\). In the left panel of same figure the polymer contribution to the kinetic energy budget is reported for reference. It is possible to observe that the maximum in the polymer transport term is located in small scales of the high buffer layer and in the logarithmic one, while a feeding to the large scales is observed in the buffer layer in analogy with the nonlinear transfer.
4.2. The source term
As already discussed in section 3, equation (5) can also be written in divergence form thus allowing, in principle, for an estimation of the energy fluxes in an augmented space of scales and distance from the wall \((r_x, r_y, r_z, Y_c)\). It will be shown in a different contribution about Newtonian turbulence Cimarelli et al. (2011) that the fluxes in this space corroborate the scenario of a reverse energy cascade from small to large scales. Here, instead, we will focus the discussion on the modified source term \(\xi = 2 \langle \delta u \delta v \rangle (dU/dy)^* - 4\epsilon^* N - 4\pi^* p\). As for the Newtonian counterpart, when \(\pi^*_p = 0\), the region of the space where \(\xi\) is positive, i.e. where production exceeds dissipation, represents the part of the augmented space where an excess energy can feed the fluctuations. It is known, even from a single point scenario, that the region of this excess energy is located in the buffer layer, however in the present framework, we are also able to identify at which scales. In particular we will show here the behavior of the source term as a function of \((r_z, Y_c)\) at various Deborah numbers.

As shown in the first plot of figure 6 which is a cut of the augmented space at \(r_x = 0\) and \(r_y = 0\) for the Newtonian case, the energy source region and, therefore, the peak of energy production, take place inside the spectrum of scales. The energy is not introduced at the top of the spectrum as the classical paradigm of turbulence leads to believe, i.e. but in the middle. It is worth noting that, beside the expected extremum in the buffer layer which is related to the near wall cycle of wall turbulence, an outer but much weaker peak appears at larger distances from the wall. The overall effect of the polymers at increasing Deborah number is an increase of the scale at which the maximum occurs in the buffer layer and, quite interestingly the disappearance

![Figure 6. Cross section of the energy source isolines in the \(r_z = 0\) and \(r_x = 0\) plane, for the Newtonian, \(De = 36, 54, 72\) respectively.](image-url)
of the outer peak. Both these ingredients are actually consistent with an apparent decrease of an effective Reynolds number.

5. Final remarks

In the present work we have presented a scale by scale analysis of a viscoelastic channel flow. The alteration of the energy fluxes from production to Newtonian and non-Newtonian dissipation has been sketched. The proposed framework, which had been used by Marati et al. (2004), offers the possibility to describe simultaneously the dynamics of turbulent fluctuations in the space of scales and in the physical space both for Newtonian and viscoelastic turbulence and can be used to understand the alteration of the mechanisms of formation and sustainment of the turbulent fluctuations in the near-wall region.

By using this investigation tool, we have highlighted a scenario that on its own can explain drag reduction. The buffer layer, which is the engine of wall turbulence, is characterized by a reverse cascade i.e. energy is transferred from the small to the large scales and in this region polymers are found to drain energy from the small scales of turbulent fluctuations. In these terms it is possible to argue that the coupling between the characteristics of viscoelastic turbulence and the inverse routes of energy, from small to large scales, in the near wall region is the key to explain why in bounded turbulence the introduction of polymers, which typically act at small scales, can produce a large scale phenomenon such as drag reduction.

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