Torque ripple minimization for PMSM considering harmonic magnet flux phase

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Abstract. Due to the existence of cogging torque, magnet flux harmonics and inverter dead time, permanent magnet synchronous motor (PMSM) has torque ripple during operation, which limits its application in high-precision situations. Stator current optimization is an easy-to-apply and accurate torque ripple suppression approach. However, existing methods fail to consider the effect of magnet flux phase angle on optimal current calculation. In this paper, a torque ripple minimization method considering harmonic magnet flux phase is presented, which uses torque waveform of an electric angular period as the optimization object. By deducing the electromagnetic torque model considering harmonic magnet flux, the torque is expressed as a dense sequence. Based on this, the objective function including torque ripple and losses minimization is constructed and optimized by genetic algorithm (GA) to obtain the optimal harmonic current. Experiment results show that the proposed approach can calculate the optimal harmonic current accurately and suppress the torque ripple effectively.

1. Introduction

Permanent magnet synchronous motor (PMSM) is now widely used in aerospace, high-precision CNC machines, robots and other fields [1]-[2]. However, because of the cogging effect, the rotor magnetic pole structure distortion and nonlinear characteristics like dead time caused by inverter, the motor will produce larger torque ripple, which makes the motor system to produce vibration and noise. Therefore, how to suppress torque ripple effectively has become a significant research issue.

At present, the methods of suppressing torque ripple can be divided into two directions. The first approach is to optimize the motor structure such as winding type optimization and rotor magnetic circuit optimization [3]-[4]. The other is to optimize the stator current to offset the harmonic components in the torque. This method does not need to change the structure of the motor, and can be flexibly applied to different motor driving systems, so it has a broader application prospect.

In order to obtain optimal stator current, researchers have proposed a series of methods. In [5]-[6], iterative learning controller is designed. Literature [7] established a three-phase current geometric model based on the torque model, and designed a neural network algorithm to calculate the optimal stator current. In [8], the LaGrange multiplier method is used as the optimization algorithm to derive the optimal harmonic current analytical formula. However, these methods need to extract the amplitude of the harmonic torque through complex formula derivation, and often ignore the influence of harmonic magnet flux phase angle, which reduces the applicability of these methods.

In order to solve above problems, a non-ideal magnet flux PMSM torque ripple minimization method considering harmonic magnet flux phase angle is proposed in this paper. The waveform of torque is expressed as sequence. The RMS value of torque ripple in a period of electric angle are taken
as the optimization object, and the harmonic torque amplitude is not required to be derived. In addition, this method takes into account the problem of motor loss and establishes a multi-objective function, which can guarantee the torque ripple and minimize the loss simultaneously. Finally, the effectiveness of the proposed method is verified by experiments.

2. Torque ripple modeling of non-ideal rotor magnet flux PMSM

For facilitating the analysis of PMSM mathematical model, the rotor magnet flux is usually assumed as sinusoids in the air gap. But in practice, if the rotor magnet flux cannot achieve a perfectly ideal sinusoidal distribution, it will contain both fundamental and harmonic components. Also, the stator current has harmonic components. Thus, according to the magnetic co-energy model [9], the electromagnetic torque of PMSM can be expressed as

\[ t_e = T_0 + (t_{h1} + t_{h2} + t_{h3} + t_{cog}) \] (1)

with

\[ T_0 = K_p (\lambda_0 I_{q0} + L_{d0} I_{d0} I_{q0}) \] (2)

\[ t_{h1} = K_p [\lambda_{dih} I_{q0} + L_{d} (I_{q0} I_{dih} + I_{d0} I_{qih})] \] (3)

\[ t_{h2} = K_p [I_{q0} (\dot{\lambda}_{dih} + \frac{d\lambda_{dih}}{d\theta_e}) - I_{d0} (\dot{\lambda}_{qih} - \frac{d\lambda_{qih}}{d\theta_e})] \] (4)

\[ t_{h3} = K_p (\lambda_{dih} i_{qih} - \lambda_{qih} i_{dih} + L_{d} i_{dih} i_{qih} + i_{dih} \frac{d\lambda_{dih}}{d\theta_e} + i_{qih} \frac{d\lambda_{qih}}{d\theta_e}) \] (5)

where \( t_e \) is the total torque of PMSM; \( K_p = \frac{3p}{2} \) and \( p \) is the number of pole pairs; \( \lambda_0 \) is the fundamental component of the \( d \)-axis magnet flux; \( \lambda_{dih} \) and \( \lambda_{qih} \) are the harmonic components of the \( dq \)-axis magnet flux; \( I_{d0} \) and \( I_{q0} \) are the fundamental components of the \( dq \)-axis current; \( i_{dih} \) and \( i_{qih} \) are the harmonic components of the \( dq \)-axis current; \( \theta_e \) is the electrical rotor position; \( t_{cog} \) is the cogging torque; \( L_d = L_d - L_q \) and \( L_d \) and \( L_q \) are the \( dq \)-axis inductance.

In (1)-(5), \( t_{h1} \) represents the harmonic torque caused only by harmonic currents, \( t_{h2} \) represents the harmonic torque caused only by harmonic magnet flux, \( t_{h3} \) represents the harmonic torque caused by the interaction of harmonic current and harmonic magnet flux, and \( t_{cog} \) represents the cogging torque. For a designed PMSM, its magnet flux and cogging torque cannot be changed. However, \( t_{h1} \) and \( t_{h3} \) can be controlled by adjusting the stator harmonic currents. Therefore, torque ripple minimization is to find the optimal stator harmonic current which can make the controllable part of harmonic torque cancel out the uncontrollable part, so as to minimize the total harmonic torque.

3. Sequence representation of torque

General methods of establishing objective function is to substitute the specific expansion formula of harmonic flux, current and cogging torque into the torque formula to extract the amplitude of torque and minimize it. However, the derivation process is complex and the expression form of the results is lengthy. Moreover, the influence of \( t_{h3} \) term and harmonic magnet flux phase are often ignored. As shown in [8], in order to facilitate the extraction of the amplitude expression of harmonic torque, harmonic magnet flux phase are set as 0, which is different from the actual situation.

In order to solve the above problems, the waveform of harmonic magnet flux and current can be expressed in sufficiently dense sequences. In a period of electric angle, the variation range of \( \theta_e \) is divided into \( N \) equal parts, so that each angle has a corresponding magnet flux and current value. The harmonic magnet flux and harmonic current can be expressed as \( N \)-dimensional sequences (6)-(8)
\( \lambda_{dth} = \sum_k \lambda_{dq} \cos(k\theta - \varphi_{dth}) = \sum_k \{a_1^k, a_2^k, a_3^k, \ldots, a_m^k, \ldots, a_N^k\} \) (6) \\
\( \lambda_{qth} = \sum_k \lambda_{dq} \sin(k\theta - \varphi_{qth}) = \sum_k \{b_1^k, b_2^k, b_3^k, \ldots, b_m^k, \ldots, b_N^k\} \) \\
\( i_{dth} = \sum_k I_{dth} \cos(k\theta - \varphi_{dth}) = \sum_k \{c_1^k, c_2^k, c_3^k, \ldots, c_m^k, \ldots, c_N^k\} \) (7) \\
\( i_{qth} = \sum_k I_{qth} \cos(k\theta - \varphi_{qth}) = \sum_k \{d_1^k, d_2^k, d_3^k, \ldots, d_m^k, \ldots, d_N^k\} \) \\
\( t_{cog} = \sum_k T_{ck} \sin(k\theta + \phi_{cog}) = \sum_k \{e_1^k, e_2^k, e_3^k, \ldots, e_m^k, \ldots, e_N^k\} \) (8)

where \( \lambda_{dth} \) and \( \lambda_{qth} \) are the amplitude of the \( dq \)-axis \( k \)th harmonic magnet flux; \( \varphi_{dth} \) and \( \varphi_{qth} \) are the phase angle of the \( dq \)-axis \( k \)th harmonic magnet flux; \( I_{dth} \) and \( I_{qth} \) are the amplitude and phase angle of the \( dq \)-axis \( k \)th harmonic current; \( T_{ck} \) and \( \phi_{cog} \) are the amplitude and phase angle of the \( dq \)-axis \( k \)th cogging torque; \( a_m^k \) is the value of the \( d \)-axis \( k \)th harmonic magnet flux at the angle \( m^{*}2/N \); \( b_m^k \) is the value of the \( q \)-axis \( k \)th harmonic magnet flux at the angle \( m^{*}2/N \); \( c_m^k \) is the value of the \( d \)-axis \( k \)th harmonic current at the angle \( m^{*}2/N \); \( d_m^k \) is the \( k \)th cogging torque at the angle \( m^{*}2/N \).

In order to express harmonic torque as sequences, the derivatived of the \( dq \)-axis harmonic magnet flux are needed. Therefore, we can get (9) by calculating the derivative of equation (6)

\[
\begin{align*}
\frac{d\lambda_{dth}}{d\theta} &= \sum_k \{f_1^k, f_2^k, f_3^k, \ldots, f_m^k, \ldots, f_N^k\} \\
\frac{d\lambda_{qth}}{d\theta} &= \sum_k \{g_1^k, g_2^k, g_3^k, \ldots, g_m^k, \ldots, g_N^k\}
\end{align*}
\] (9)

where \( f_m^k \) is the value of the derivative of the \( d \)-axis \( k \)th harmonic magnet flux at the angle \( m^{*}2/N \); \( g_m^k \) is the value of the derivative of the \( q \)-axis \( k \)th harmonic magnet flux at the angle \( m^{*}2/N \).

In a PMSM, the harmonic torque of different or ders can be suppressed separately by the use of corresponding harmonic currents. Therefore, taking the \( k \)th harmonic as an example, substituting (6)-(9) into (3)-(5) can obtain the \( k \)th harmonic torque sequence as

\[
t_k^t = \{h_1^k, h_2^k, h_3^k, \ldots, h_m^k, \ldots, h_N^k\}
\] (10)

with

\[
h_m^k = K_p [\lambda_q d_m^k + L_A (I_q 0 c_m^k + I_d 0 d_m^k)] + K_p [I_q 0 (a_m^k + g_m^k) - I_d 0 (b_m^k - f_m^k)] + K_p (a_m^k d_m^k - b_m^k c_m^k + L_A c_m^k d_m^k + c_m^k f_m^k + d_m^k g_m^k) + e_m^k
\] (11)

where \( t_k^t \) is the \( k \)th harmonic torque; \( h_m^k \) is the value of the \( k \)th harmonic torque at the angle \( m^{*}2\pi/N \).

From (10), the harmonic torque is obtained directly from the sequences of magnet flux and current, which not only avoids the complicated formula derivation process, but also considers the influence of magnet flux phase angle on the torque, so it has higher accuracy and applicability.

4. Optimal current design based on GA

In PMSM torque ripple minimization, the goal of current optimization is to find appropriate harmonic current \( i_{dth} \) and \( i_{qth} \) to minimize the RMS value of torque ripple. Meanwhile, the harmonic current will also cause additional losses which is related to the sum of squares of harmonic current amplitudes [10]. Thus we can build objective function as
\[ F(x) = k_1 \left( \frac{1}{N} \sum_{n=1}^{N} h_n^k \right)^{1/2} + k_2 (I_{d_k}^2 + I_{q_k}^2) \]  

(12)

where \( k_1 \) and \( k_2 \) are weight coefficient which influence the proportion of each objective in the function; \( x = \{ I_{d_1}, I_{q_1}, \phi_{d_1}, \phi_{q_1} \} \) is the solution of the function.

The optimal harmonic current search process based on GA is shown in figure 1. At first we have to initialize the population and the total population of each generation remains unchanged. Then set maximum number of iterations MAXGEN, the crossover probability \( P_c \), mutation probability \( P_m \), the weight coefficients \( k_1 \) and \( k_2 \). The algorithm automatically generates the 0th generation, calculates their \( F(x) \) and stores the best solution \( x \) among them. Then, operations of selection, crossover and mutation are carried out in turn to generate new generation, and their \( F(x) \) are calculated to update the best solution \( x \). Repeat these until reached the maximum number of iterations, and output the stored best solution.

![Optimal harmonic current searching flow chart based on genetic algorithm](image)

**Figure 1. Optimal harmonic current searching flow chart based on genetic algorithm**

5. **Design of optimal harmonic current control system**

The PMSM optimal harmonic current control system is established, and its schematic diagram is shown in figure 2.

![Schematic diagram of optimal harmonic current control system](image)

**Figure 2. Schematic diagram of optimal harmonic current control system**

Figure 2 adopts the multi-synchronous rotating PI controller. It controls fundamental and harmonic currents respectively. Dc components of actual and reference currents in harmonic coordinate are achieved by using harmonic coordinate transformations and low-pass filters. Then, PI controllers are used on different coordinate to make actual currents track reference currents and generate harmonic voltage signal which is applied to inverter control to suppress torque ripple.

6. **Experimental results and discussions**

In this paper, the proposed torque ripple minimization method is experimentally verified. The platform includes the test PMSM, photoelectric encoder, torque sensor, inverter, load motor and servo...
controller. Wherein, the test motor is an interior PMSM (IPMSM), and the specific parameters are shown in Table 1.

| Parameter                      | value |
|--------------------------------|-------|
| Rated power $P_N$/kW           | 6     |
| Rated speed $n_S/(r/min)$      | 1500  |
| Rated torque $T_N$/Nm          | 40    |
| Winding resistance $R$/Ω       | 0.557 |
| Magnet flux $\Psi_f$/Wb        | 0.9876|
| Number of pole pairs $p$       | 2     |

In this experiment, the dominant component of harmonic magnet flux in the test PMSM is 18th harmonic. Therefore, in order to explain the effectiveness of the proposed torque ripple suppression method, this paper mainly aims at suppressing the 18th torque ripple. Load torque is 40 Nm, and the results before and after adding the suppression algorithm were compared.

![Figure 3. dq-axis currents before suppression](image)

![Figure 4. dq-axis currents after suppression](image)

![Figure 5. Torque before suppression](image)

![Figure 6. Torque after suppression](image)

![Figure 7. Currents before suppression](image)

![Figure 8. Currents after suppression](image)
It can be seen that after the suppression strategy is applied, the peak and peak value of the torque ripple is reduced from 4.83 Nm to 1.66 Nm, which is 65.63% lower. Due to the injection of extra harmonic current, current waveform after suppression shows a larger ripple. The above experimental results show that the torque ripple suppression method based on GA can effectively reduce the torque ripple by injecting additional harmonic current to generate additional harmonic torque, which is offset with the existing harmonic torque.

7. Conclusion
In this paper, a PMSM torque ripple suppression method considering harmonic magnet flux phase angle is designed. Through theoretical analysis and experimental verification, the following conclusions can be drawn. Firstly, harmonic magnet flux, current and torque are represented by sequences, which can reflect the influence of magnet flux phase angle on harmonic torque. Thus this method can reflect the torque change accurately and avoid the complicated formula derivation. Secondly, GA calculates the corresponding optimal harmonic current according to different working conditions, and generates the optimal harmonic current lookup table through off-line calculation, which makes the method suitable for different working conditions. At last, experiment results show that this method reduces torque ripple by about 67%, which has a great torque ripple suppression effect.

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