Learning complex inflectional paradigms through blended gradient inputs

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Abstract

Through Gradient Symbolic Computation (Smolensky and Goldrick, 2016), in which input forms can consist of gradient blends of more than one phonological realization, we propose a way of deriving surface forms in complex inflectional paradigms that dispenses with direct references to inflectional classes and relies solely on relatively simple blends of input expressions.

In languages whose inflectional systems have a highly complex paradigmatic structure, it becomes a challenge to explain how a speaker can produce a correct inflectional form when the number of possible forms is exceedingly large. As Ackerman and Malouf (2013, p. 429) (henceforth A&M) comment: “That speakers are able to do this is a truly puzzling accomplishment given the extraordinary variation and complexity attested in the morphological systems of the world.” When it becomes extremely difficult for a speaker to memorize every possible inflectional form for every lexeme in their language, it becomes attractive to posit some means by which they can produce a correct unmemorized form.

Three interacting types of paradigmatic complexity that put a burden on learning are addressed here: (a) differences in inflectional material across lexemes, which, descriptively, result in division of lexemes into inflectional classes; (b) syncretism, where the same inflectional material occurs in different paradigm cells; (c) independent subsystems of inflectional class behaviour, evident in Mazatec (see §3.) These complexities are all ways in which inflectional patterns depart from canonicity as defined in detail by Corbett (2009); Baerman and Corbett (2010); Stump (2016).

Through Gradient Symbolic Computation (Smolensky and Goldrick, 2016), a type of Harmonic Grammar, and with examples from Russian and Mazatec, we propose a system that enables a speaker to produce correct forms of complex paradigms through learnable input representations without indexing to inflectional classes.

The paper is organized as follows. §1 introduces GSC and how it can be applied to learning exponents of inflectional paradigms. §2 shows how Russian noun inflection can be acquired through this framework. §2.1 shows how GSC limits the kinds of inflectional patterning that are possible. §3 analyses complex paradigms in Mazatec, where inflectional classes vary in three cross-cutting dimensions. §4 discusses testing and comparison with other models. §5 summarizes how this framework can both explain departures from canonicity in inflectional paradigms and also constrain the degree of departure from the canonical.

1 In response to a reviewer’s question about the effects and degree of simplification of the paradigms of these two languages, we contend that (a) any linguistic analysis will have some degree of simplification, (b) the paradigms are no more simplified than those analysed by A&M and in fact include more paradigm positions than are represented in their table A6 (p. 457), and (c) nothing in the analysis was excluded because the model couldn’t handle it.
2016; Zimmermann, 2017b,a, forthcoming). In the examples in §2 from Russian noun inflection, the surface exponents that can represent gen-
itive singular among the descriptive inflectional classes, form a blend of input segments, which we shall call ‘inflectional input’: \{a, i\}. A lexe-
me (basic element of the lexicon) in descriptive class 1 whose genitive sg. is *vina*, ‘wine’ contributes an input we shall call ‘base input’ that consists of
what is traditionally thought of as a stem plus a blend of word-final segments \{a, u, o\} that mir-
rors segments that can occur in surface-final position. We propose that it derives as follows, with
numerical details shown later in table 4.

| Lexical base | Inflectional affix |
|--------------|--------------------|
| Input: \( \text{root} \) \( \text{v}in{\{a, u, o\}} \) GEN.SG. = \{a, i\} | Output: \( \text{root exponent} \) \( a \) affixal exponent |

Table 1: Gen.sg. of lexeme, descriptive class 1

In this framework, a discrete output form is chosen from possible candidates by the action of Faithfulness constraints that are familiar from Optimality Theory (Prince and Smolensky, 1993), but which, in GSC, have weighted values and evaluate gradient input activations. We assume a highly-weighted quantization constraint quanti-
fied by an equation in Cho et al. (2017) that disfavs blended outputs and gives a higher Har-
mony to discrete non-blended forms. The tableaux below thus ignore blended output candidates.

Surface material that some models view as de-
rivng solely from an affix input is derived here from two distinct input sources, thus blending two competing analyses: a constructivist approach that builds whole words out of morphemes and a word-
based approach that seeks relationships between whole word forms. The input representation of a lexe-
me that includes affixal material as an in-
trinsic part of the word is based upon a whole-
word model; an affix as a fully independent in-
put follows a morpheme-based model. A simi-
lar approach is taken by Smolensky and Goldrick (2016), treating French liaison as derived from both the end of a preceding word and the begin-
ing of a following word, thus combining two competing approaches to liaison in the literature.

Any matching phonological material from the
two sources will combine through coalescence. When there are multiple descriptive inflectional classes, the exponent for a given inflectional com-
bination varies, depending not only on the lexeme, but on the stem when there is stem allomorphy (Stump, 2016). The current proposal for blended word-final input segments that mirror affixes, di-
rectly encodes the fact that lexeme/stem choice
will affect affix-choice. Non-concatenative and suppletive alternations can thus be represented as
well, as blends of different stem alternants occur-
ring underlyingly.

2 Learning Russian noun inflection

For ease of exposition, we first examine the relatively simple paradigms of case/number noun suf-
fixes in Russian, with data from A&M p. 460 given in table 2. For presentation purposes, we ignore here the paradigm defectiveness of some Russian nouns (Corbett, 2007).

The exponent that surfaces for a given stem and person/number combination is the one with the highest aggregate activation that surpasses a threshold determined by MAX and DEP con-
straints to be defined below. When two in-
stances of the same exponent occur in both base and inflectional input, they can coalesce together in
the output, with an aggregate input activa-

| CLASS | SINGULAR | 2 | 3 | 4 |
|-------|----------|---|---|---|
| NOM   | -o       | -∅ | -a | -∅ |
| ACC   | -o       | -∅ | -u | -∅ |
| GEN   | -a       | -a | -i | -i |
| DAT   | -u       | -u | -e | -i |
| LOC   | -e       | -e | -e | -i |
| INST  | -om      | -om | -oj | -ju |

Table 2: Russian noun paradigm


2A reviewer asks what advantages this model has over a “genuine connectionist model” such as
Goldsmith and O’Brien (2006); Kann and Schütze (2016); Malouf (2018). In this model, the knowledge it contains is
completely transparent, whereas in the last two models cited by the reviewer, it is not clear what kind of knowledge is
contained in those networks. (See §4 for further discussion.)

3But see Zimmermann (2017b,a, 2018) for analyses of other phenomena in which outputs are also gradient.
The following simple algorithm learned blended base and inflectional input values such that the optimal output candidate is the correct target of learning for all 48 forms. Following Faust and Smolensky (2017, page 2), not just an individual segment but also other structures can have an activity level.

- Initialize all activations at zero.
- Calculate the Harmony for each descriptive stem-class/number/case combination.
- If the wrong affix is predicted, decrease its two input activations if nonzero and increase the activations of the desired affix. Stepsize 0.02.
- Repeat until all 48 paradigm positions are correctly predicted.

Tables 6 and 7 show the learned input values after 72 iterations.

Nonzero inputs for an inflectional combination indicate its set of possible exponents. A co-occurring nonzero base input narrows down the choice of those exponents. For example, the locative singular affix is -e for all descriptive classes except class 4, where -i surfaces. The activation of 0.12 for -i for a class 4 base input allows it to surpass other competitors where it surfaces.

In summary, representing both base and inflectional inputs with a blend of partially activated exponents allows a speaker to encode all the information in a multi-class inflectional paradigm directly on the relevant input forms.

An affixless candidate incurs no MAX reward or DEP penalty and will be optimal when all other candidates have negative Harmony (table 5.)
### 2.1 Restricting possible paradigms

The Russian noun paradigm exhibits both syncretism within descriptive inflectional classes and variance of exponents for a given position across inflectional classes. Our model predicts that there are limits to such departures from canonicity. In the dative singular, the affix is $e$ in class 3 and $i$ in class 4. In the genitive singular, it is $i$ for both classes, making $i$ syncretic across those two feature combinations for class 4. Our model predicts the impossibility of the pattern in the right-hand half of table 8.

| Class | 1 | 2 | 3 | 4 |
|-------|---|---|---|---|
| a     | 0.1 | | 0.04 | |
| e     | | | 0.12 | |
| u     | 0.02 | | 0.06 | |
| om    | | 0.04 | | |
| ov    | | 0.02 | | |
| ax    | | | | |
| am    | | | | |
| am'i  | | | | |
| i     | 0.1 | | 0.12 | |
| ej    | | | | |

#### Table 6: Learned nonzero input values

| Cell | Real | Impossible |
|------|------|------------|
|      | Class 3 | Class 4 | Class 3 | Class 4 |
| Gen. Sg. | i     | i     | i     | i     |
| Dat. Sg.  | e     | i     | e     | i     |

Table 8: Crossing diagonals

In the impossible paradigm, lines connecting the two instances of $i$ (coloured blue) and connecting the two instances of $e$ (coloured red) would cross. Such diagonal crossing can be shown to mathematically imply activation hierarchies that lead to a contradiction (table 9), where $\iota$ represents the activation of $i$ and $\epsilon$ of $e$ for a given (subscripted) feature value or descriptive inflectional class.

$$
\iota_{g.sg.} + \iota_3 > \iota_{g.sg.} + \iota_3 \quad (1)
$$

$$
\iota_{g.sg.} + \iota_4 > \iota_{g.sg.} + \iota_4 \quad (2)
$$

$$
\iota_{dat.sg.} + \iota_4 > \iota_{dat.sg.} + \iota_4 \quad (3)
$$

$$
\iota_{dat.sg.} + \iota_3 > \iota_{dat.sg.} + \iota_3 \quad (4)
$$

$$
(1) + (2) : \iota_3 + \iota_4 > \iota_3 + \iota_4 \quad (5)
$$

$$
(3) + (4) : \iota_4 + \iota_3 > \iota_4 + \iota_3 \quad (6)
$$

(6) contradicts (5)

Table 9: Contradictory inequalities

Thus, in spite of a common misconception that ‘you can do anything with numbers’, the proposed GSC model predicts that many conceivable paradigms are impossible.

### 3 Learning inputs in Mazatec

We now consider verb paradigms in Chihihtlán Mazatec, a language with a high degree of paradigmatic complexity. Jamieson (1982) shows that its verbal inflectional morphology has three separate dimensions in the affirmative forms of the neutral and incompletive aspects: (a) a stem formative (table 12), with 35 possible exponents, (b) a final vowel (table 19), with 11 possible exponents, and (c) a tone pattern (table 15), with 15 possible exponents. For example, in the neutral 1st person singular form in table 10, $ba^3\text{se}^1$, the stem is merely the segment $s$. The stem formative is $ba$, the final vowel is $\text{se}$ and the tone pattern is 3-1. Yet these $35 \times 11 \times 15 = 5775$ hypothetically
Possible combinations are actually reduced to 20 in Jamieson’s list of paradigms. Table 10 gives the paradigm for ba\(^3\)se\(^2\) ‘remember’ from A&M:450, taken from Jamieson (1982, p. 167). This lexeme is in stem formative descriptive class 11, tone pattern class B 3-1 and final vowel class 2.

Combinations of morphosyntactic features are represented by combinations of multiple exponents, but there is no apparent correspondence between individual feature values and individual exponents. The surface form for a verb expresses feature values for aspect, person and number, with a stem formative, a tone pattern and a suffix vowel.

Stems in many cases consist merely of a single segment. As A&M:447 observe, a stem is much more easily identified by “its membership in an inflectional class in each of three distinct crosscutting dimensions: tone pattern, final vowel, and stem formative.” In the proposed blended representations, a lexeme’s base input carries much of the information that predicts what exponents it occurs with, in this way identifying what can descriptively be called its stem. It also directly encodes what is descriptively its inflectional class in a way that is not possible in a model in which paradigm information and stem information are separate. Table 11 shows, with activation values omitted for now, an example basic input-output structure, which contains gradient blends of consonants in the input.

| Class | Stem formatives | Representations |
|-------|----------------|-----------------|
|       | Neu. Incomp.  | Neu. Incomp.   | blend | vowel |
| 1     | be             | k\(^e\)        | b, k\(^e\) | e     |
| 2     | ba             | k\(^a\)        | b, k\(^a\) | a     |
| 3     | bo             | sk, k\(^o\)    | b, sk, k\(^o\) | o     |
| 4     | bu             | cu             | b, sk, k\(^u\) | u     |
| 5     | hu             | k\(^u\)        | b, sk, k\(^u\) | u     |
| 6     | hi             | k\(^i\)        | b, sk, k\(^i\) | i     |
| 7     | ha             | k\(^h\)       | b, k\(^h\) | a     |
| 8     | ci             | n\(^i\)        | n, i     | i     |
| 9     | su             | s\(^u\)        | s, n     | u     |
| 10    | ku             | k\(^u\)        | b, k\(^u\) | u     |
| 11    | ba             | k\(^a\)        | b, k\(^a\) | a     |
| 12    | ka             | ca             | b, k\(^a\) | a     |
| 13    | ka             | k\(^a\)       | b, sk, k\(^a\) | a     |
| 14    | ha             | n\(^a\)        | b, k\(^h\), n | a     |
| 15    | bi             | k\(^i\)       | b, k\(^i\) | i     |
| 16    | bu             | s\(^u\)        | b, sk, k\(^u\) | u     |
| 17    | hi             | s\(^i\)        | h, s, c, s | i     |
| 18    | ba             | k\(^h\)       | b, k\(^h\), c | a     |

Table 12: Representations of stem formatives

Consider first the stem formatives, shown in table 12 (Data from Jamieson (1982)).

The proposed representation for each base input is simply a blend of all its possible initial consonants. The vowels stay the same across a class. This consonant surfaces depends on how this blend interacts through coalescence of identical consonants with an inflectional input blend of consonant features.

Tables 13 and 14 show the results of the same kind of algorithm discussed on page 3 for learning input activations of inflectional person-number combinations and base inputs respectively.

It took 22 iterations to learn activations that derive the correct stem formative consonants for each of 4 person-number groups in 18 classes. Abstracting away for now from tone and final vowel

Tone and final vowel omitted here

| Tone and final vowel omitted here | Lexical base | Stem formative |
|----------------------------------|--------------|----------------|
| Input:                           | {b, k\(^w\), c} | 3rd.p.def.incomp. |
| Output:                          | k\(^b\), k\(^h\), k\(^a\), s | stem formative, root exponent |

Table 11: Input-output for ‘remember’ 3rd.p.def.inc.

Some further complexities in Jamieson’s data are omitted here for simplicity of exposition but can be explained by our analysis. These include suffixing of verbs, verb compounding, verbs with no stem formative\(^5\) and stem suppletion.\(^6\)

Among the 35 possible stem formatives, 11 possible final vowels and 15 possible tone patterns, a given lexeme’s input form only needs a blend of a small subset of each of these exponents.

3.1 Learning inputs for stem formatives

Consider first the stem formatives, shown in table 12 (Data from Jamieson (1982)).

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\(^5\)If a stem formative is absent from the base input, an anchoring constraint will prevent an inflection-based stem-formative input from surfacing.

\(^6\)Stem suppletion can be accounted for by an input that includes a blend of different stem allomorphs each linked to different inflectional features.

\(^7\)Following Golston and Kehrein (1998), we take what Jamieson transcribes as complex onsets and nuclei to be simple segments with secondary articulations.
vowels, for kwa\textsuperscript{se} \textsuperscript{2}‘remember’ (class 11), 3rd p. def. incompl., the activations for the input given above in table 11 are as follows.

Base input: (0.1-b, 0.3-k\textsuperscript{w}, 0.3-\cacute)
as

Inflectional input:
(0.21-sk, 0.21-k\textsuperscript{wh}, 0.12-s, 0.12-k\textsuperscript{w})

For each candidate, an inflectional input can coalesce with a base input. The segment with the highest combined activation surfaces, namely k\textsuperscript{w}, with an underlying activation of 0.3+0.12 = 0.42. We assume the stem consonant and the vowel of the stem formative to both have full 1.0 underlying activations; thus no Harmony is gained by having a gradient inflectional stem formative consonant incorrectly coalesce with the stem consonant nor for a gradient inflectional final vowel to incorrectly coalesce with the stem formative vowel.

### 3.2 Learning inputs for tone patterns

Tone patterns, which fall into 11 descriptive classes, were also learned by an algorithm that separately learned patterns for the first and second syllables.

Table 15 shows tone patterns listed by class and person-number-aspect combination. (Data from Jamieson (1982).)

There is some phonological predictability for the tones of the second syllable, where 1 is the highest tone and 4 the lowest:

- The initial tone on the second syllable can be no lower than the final tone of the first syllable.
- If the initial tone of the second syllable is 3 or 4 and the same as the final tone of the first, there is a further rise of two tone levels:
  - e.g. 4 can be followed by 42 but not simple 4;
  - 3 can be followed by 31 but not 3.
  - 2 can only be followed by 2 or 24.
- 1 can only be followed by 1.
- We can take the level 4 tone that occurs last in
### Table 15: Tone patterns

the 1st exclusive to be a separate tonal input for 1st-exclusive.

Tables 16 and 17 show the results of learning algorithms for first and second syllable tones. Ten iterations were required for syllable 1 and 22 iterations for syllable 2, with a stepsize of 0.05 for syllable 1 and 0.1 for syllable 2. Activations were initialized at 0.2 for each base-input and tonal person-number input for each tone pattern that actually occurs. We assume there exists a Harmonic Grammar analysis of the above constraints, in which they have higher weight than Faithfulness, which will otherwise derive the pattern with the highest aggregate activation.

A two-syllable base input has a tone linked to the mora of each syllable. The inflectional input has a linearly ordered sequence of two tones on the tonal tier.\(^8\) A strongly-weighted anchoring constraint will require the tones on the inflectional input to line up with the tones on the base input when they coalesce.

Consider the second person singular incompletive of ‘remember’, \(\text{ca}^3\text{se}^2\). This is tonal class B-3-1 (8-13) with inputs shown in table 18. When the two input sources coalesce on each syllable, tone level 4 has the highest activation for syllable 1 of 0.25 + 0.1 = 0.35 and tone level 2 on syllable 2 of 0.4 + 0.0 = 0.4, both the correct tones.

### 3.3 Learning inputs for final vowels

Table 19, with data from Jamieson (1982) shows the 10 descriptive classes of final vowels.

\(^8\)In the 1st person exclusive, a level 4 tone will occur further to the right of other tones.

![Table](image)

### First syllable tones

![Table](image)

### Second syllable tones

![Table](image)

### Person/number

![Table](image)

The algorithm learns input values, shown in table 20, for ±hi, ±lo, ±bk and nas features, with coalescence of values from the two input sources. A negative value in the table represents a positive value for the negative binary feature: e.g. −0.1
hi means 0.1 –hi. We assume that ∀i both +fi and –fi do not occur in the same source. Table 21 shows input vowel features for ea1xe2 ‘remember’, final-vowel class 2 in the 2nd sg. incomp. (W0 denotes base input.)

We adopt a second, strongly-weighted quantization constraint from Cho et al. (2017) that requires the sum of activations from a binary feature group such as {+hi, –hi} to be 1. Thus –hi competes with +hi and –hi surfaces because its higher aggregate activation results in greater Harmony.

### 4 Testing and comparison with other models

To test how a learner could predict an unseen inflectional form for a given stem, we ran cross-validation on Mazatec stem formatives, tone patterns and final vowels, with training set items (70% of the total stemclass×inflection combinations) picked randomly from a Zipf-Mandelbrot distribution. On ten runs for each, the average test accuracy was 83% for stem formatives, 87% on tone 1, 92% on tone 2 and 89% on the final vowel. As a baseline, we simultaneously tested prediction of unseen forms based on frequency of occurring stem formatives, for which the accuracy was 9.5%. The relative success of the model in cross-validation is due to the syncretism that occurs both within paradigms and across classes. The following tableau shows how the correct stem formative kw is predicted for a never-encountered 3rd-definite-incomplete form of a class 7 stem ‘weave’ aña in the holdout set, using activation values that were learned for encountered forms.

The strength of this model is that it directly encodes knowledge of the tendency of an exponent to occur for a given stem and for a given morphosyntactic combination. For example, a Mazatec class 1 stem with a stem-formative blend (0.2 b, 0.3-kw) encodes the relative tendencies of these two exponents to occur with this stem, with zero-valued exponents having no inclination to occur. In Kann and Schütze (2016) and Malouf

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**Table 21: Values for final-vowel class 2, 2nd-sg.**

[-hi, –lo, –bk, 0 nas] = e, the correct final vowel, is the output after quantization.

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**Table 20: Input activations for vowel features**

We adopt a second, strongly-weighted quantization constraint from Cho et al. (2017) that requires the sum of activations from a binary feature group such as {+hi, –hi} to be 1. Thus –hi competes with +hi and –hi surfaces because its higher aggregate activation results in greater Harmony.

| Feature | 2nd-sg. | W0 | Winner |
|---------|---------|----|--------|
| hi      | 0.1     | -0.2 | –hi    |
| lo      | -0.3    | -0.1 | –lo    |
| bk      | -0.2    | -0.1 | –bk    |
| nas     | 0       | nas |

---

**Table 18: Aggregate activations**

| Class | 3def | 1s | 1in | 2s | 1ex | 2p |
|-------|------|----|-----|----|-----|----|
| 1     | i α  | e  | i   | i  | u   |    |
| 2     | e α  | e  | e   | i  | u   |    |
| 3     | α α  | e  | e   | i  | u   |    |
| 4     | u u  | u  | i   | i  | u   |    |
| 5     | o o  | o  | e   | i  | u   |    |
| 6     | a a  | a  | e   | i  | u   |    |
| 7     | i e  | e  | i   | i  | u   |    |
| 8     | e e  | e  | i   | i  | u   |    |
| 9     | u u  | u  | i   | i  | u   |    |
| 10    | a a  | a  | i   | i  | u   |    |

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**Table 19: Classes of final vowels**

| Class | hi | lo | bk | nas |
|-------|----|----|----|-----|
| 1     | 0.2| -0.1|    |    |
| 2     | -0.2| -0.1|    |    |
| 3     | -0.2| 0.2| -0.1|    |
| 4     | 0.4| -0.2| 0.1|    |
| 5     | -0.2| 0.1|    |    |
| 6     | 0.2| -0.2| 0.1|    |
| 7     | 0.2| -0.2| 0.2|    |
| 8     | -0.1| -0.1| 0.2|    |
| 9     | 0.4| -0.2| 0.1|    |
| 10    | 0.1|    | 0.1|    |
Table 22: Tableau for ‘weave’ class 7 3rd-def-inc.

| 3RD-DEF-INC(0.1·a, 0.15·sk, 0.09·kw, 0.12·s, 0.12·kw) | Harmony |
|---------------------------------------------|---------|
| + (0.20·hb, 0.20·kwh) aPy- 'weave' |         |
| sk-a'?y|-0.75 | -0.85 | -0.10 |
| kwh-a'?y| 1.45  | -0.71 | 0.74  |
| s-a'?y | 0.6  | -0.88 | -0.28 |
| kwh-a'?y| 0.6  | -0.88 | -0.28 |
| b-a'?y | 1.0  | -0.80 | -0.20 |

(2018), there is no indication of what kind of knowledge is represented by a given node or connection. Goldsmith and O’Brien (2006) is more similar to the present model but it is not clear how weights from input to hidden-layer nodes translate into feature-values on a symbolic level. And its winner-take-all mechanism with no threshold to surpass means that zero exponents in a paradigm must be represented as such, rather than resulting from any exponent failing to surpass a threshold. This results in awkward representations when a form is expressed by multiple zero affixes. The advantage of the GSC model, (see Smolensky et al. (2014, p. 1103)) is that it combines a subsymbolic neural level with a symbolic level, i.e. ‘microlevel representations and algorithms’ with symbolic grammatical theory, with an interface between the two. The subsymbolic level provides a platform for optimization and allows gradient activations. When output activations are quantized to discrete values they percolate to the symbolic level which contains symbolic descriptions that are familiar from symbol-based linguistic theory.

5 Summary

Inflectional patterns in Russian and Mazatec depart from canonicity in a number of ways, as was outlined on page 1. If these systems were completely canonical, each morphosyntactic combination of feature values would have a unique, lexeme-invariant exponent. In our model, in Russian, the variance of an exponent across inflectional classes is expressed directly by a blend of segments in an underlying form, e.g. (a, i) for genitive singular. Syncretism across inflectional feature combinations, such as multiple instances of -i in several classes is expressed by its occurrence as a base input, so that it can coalesce with a matching inflectional input for several feature combinations. If a different exponent occurred in each cell and there were no inflectional class divisions, the URs could simply be a single non-blended exponent for each inflectional input, with no inflectional formatives on base inputs. So the representations proposed here directly encode and capture the kinds of departures from canonicity that occur.

In Mazatec, blended inputs capture not just deviation from paradigm canonicity but patterns of predictability within its paradigm structure. Out of 12 possible stem-formative combinations that could occur in 12 paradigm positions, Jamieson only lists 18 classes. Blended input representations, in which no more than four phonemes occur for a base input or inflectional input, limit how many stem formatives occur for a given lexeme and along with the occurrence of the same blended inflectional inputs in multiple person-number combinations, derive the syncretism that occurs across those paradigm positions.

This approach can also derive subtractive morphology, suppletion, umlaut and moraic augmentation. For example, if an inflectional form is subtractive relative to a base, the subtractable part can have partial or negative-valued underlying activation in the UR and express morphosyntactic features that affect its surfacing. Umlaut (Trommer, 2017) can be derived from a blend of different vowel features where alternation occurs.

Given the vast number of ways that languages can deviate from canonical inflection (Corbett, 2009, 2007; Baerman and Corbett, 2010; Stump, 2016), we hope with future research to explore in the GSC framework how it is possible to learn complex paradigmatic patterns in other languages.

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\[\text{In terms of correspondence, this model employs the converse of what Pulleyblank (2008) proposes for his account of reduplication in which a single input has multiple outputs. Here, pairs of identical input features or segments can each coalesce on a single output feature or segment.}\]
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