Gauge Invariant Smearing and the Extraction of Excited State Masses Using Wilson Fermions at $\beta = 6.2$

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We present an investigation of gauge invariant smearing for Wilson fermions on a $24^3 \times 48$ lattice at $\beta = 6.2$. We demonstrate a smearing algorithm that allows a substantial improvement in the determination of the baryon spectrum obtained using propagators smeared at both source and sink, at only a small computational cost. We investigate the matrix of correlators constructed from local and smeared operators, and are able to expose excited states of both the mesons and baryons.

1. Introduction

In this talk we present the results of an investigation into gauge invariant smearing on a $24^3 \times 48$ lattice at $\beta = 6.2$. We begin by comparing effective masses obtained from local correlators with those from Wuppertal or “scalar wave function” smeared correlators. We introduce a variant of the iterative Wuppertal scheme which enables us to achieve the results of scalar wave function smearing but at a much lower cost in computer time. Using this scheme, we perform detailed measurements of the hadron spectrum using local and smeared sources and sinks. Finally, we investigate the $2 \times 2$ matrices of correlators formed from propagators computed with these four source-sink combinations, and by diagonalising these matrices are able to isolate the first excited states.

2. Gauge Invariant Smearing

2.1. Wuppertal Smearing

Wuppertal smearing uses a scalar propagator to smear a delta function source at the origin. The smeared source is given by the solution of the three dimensional, gauge invariant, Klein-Gordon equation. For our initial investigation of the dependence of the effective masses on the smearing radius, quark propagators were computed on an ensemble of 7 configurations using Wuppertal smeared sources at $\kappa_S = 0.180$ and 0.184, corresponding to $r \simeq 2$ and $r \simeq 4$ respectively.

The effective mass of the nucleon obtained using LL and LS propagators on the same ensemble of configurations is shown in figure 1 (we adopt the convention of LS representing a propagator smeared at the source and local at the sink). The lightest state of the nucleon is isolated nearer the origin with increasing smearing radius.

The Wuppertal smearing algorithm creates sig-
significant computational overheads when used to smear the propagator at the sink; the smearing algorithm must be implemented on every time slice, not just the source time slice, and for every spin component. Indeed, for $\kappa_S = 0.184$, the computational effort required to smear at the sink is comparable with that required to compute the propagator, and larger values of the smearing radius are prohibitively expensive.

2.2. Jacobi Smearing

An alternative smeared source, $J(x)$, can be obtained by solving the three dimensional Klein-Gordon equation as a power series in $\kappa_S$, stopping at some finite power $N$.

$$J(x) = \sum_{n=0}^{N} \kappa_S^n \Delta^n \delta_{x,0}$$

(1)

where

$$\Delta_{x,x'} = \sum_{\mu=1}^{3} U_{\mu}(x) \delta_{x',x+\mu} + U_{\mu}^\dagger(x - \mu) \delta_{x',x-\mu}$$

This can be achieved easily using Jacobi iteration. When $\kappa_S$ is smaller than some critical value, the power series converges and one recovers the scalar wave function. For sufficiently large $\kappa_S$, the series diverges but provides an acceptable smeared source for suitable choices of $N$. A similar iterative scheme has been used by the Wuppertal group\[4\].

We found that for a radius of approximately 4, smearing with the Jacobi iteration was a factor of ten faster. The remainder of this paper will, therefore, concentrate on results obtained using $\kappa_S = 0.250$, $N = 50$, corresponding to $r \simeq 4$, on the same 18 configurations which were used in\[4\].

3. Hadron Masses

We present in table 1 the masses of the pseudoscalar, vector, nucleon and $\Delta$ using the LL, LS, SL and SS propagators, together with the corresponding time ranges used for the fits. All fits are performed using the full covariance matrix, with the errors extracted using a bootstrap analysis\[5\]. The improvement in the determination of the baryon masses through the use of smeared sources and sinks is substantial. We expect that the study of a larger sample of configurations would lead to a still more marked improvement through the ability to perform a fit to the full covariance matrix for the SS correlators over a larger fitting range than that quoted in table 1. The lack of improvement in the determination of the meson masses suggests that mesons are already well exposed using local sources and sinks.

A recent analysis\[6\] of the hadron spectrum obtained using both wall sources and Wuppertal sources suggests that there is a systematic difference in the baryon masses in the two cases, and that this difference is particularly noticeable in the determination of the $\Delta$ mass. To test for such a discrepancy in our data, we perform a bootstrap analysis of the $\Delta$ mass differences for the various combinations of sources and sinks, using the time ranges of table 1. For the LS and SL correlators, we find

$$m_{\Delta}^{LS} - m_{\Delta}^{SL} = 0.026 ^{+31}_{-20}. \quad (2)$$

This discrepancy is a $1\sigma$ effect. However, since the expectation values of the LS and SL correlators should be identical, we attribute the discrepancy to limited statistics.

4. Matrix Correlators

By studying the matrix of correlators formed using the local and smeared operators, $O_L$ and $O_S$ respectively, we can attempt to extract masses for the first excited state. We define the
$2 \times 2$ matrix, $C(t)$, of timeslice correlators by

$$C(t) = \begin{pmatrix} c_{LL}(t) & \omega c_{LS}(t) \\ \omega c_{SL}(t) & \omega^2 c_{SS}(t) \end{pmatrix}$$

with

$$c_{ij}(t) = \sum_{\vec{x}} \langle 0|O_i(\vec{x}, t)O_j^\dagger(\vec{0}, 0)|0\rangle.$$  

(3)

The factor $\omega$ arises from the arbitrary normalisation of the smearing function. Inserting a complete set of states in equation (3), we obtain

$$c_{ij}(t) = \sum_{n=0}^{\infty} \langle 0|O_i(\vec{0}, 0)|n\rangle\langle n|O_j^\dagger(\vec{0}, 0)|0\rangle e^{-E_n t} \frac{1}{2E_n}.$$  

(4)

We will now discuss two methods for studying the matrix $C(t)$. 

**Diagonalisation of the Transfer Matrix**

Consider the eigenvalue equation

$$C(t)u = \lambda(t, t_0)C(t_0)u.$$  

(5)

for fixed $t_0$. If the system comprises only two independent states, then the eigenvalues of eq. [3] are

$$\lambda_+(t, t_0) = e^{-(t-t_0)E_0}$$

$$\lambda_-(t, t_0) = e^{-(t-t_0)E_1}.$$  

(6)

The two states are separated exactly, and the coefficients in eq. [3] grow exponentially with $t_0$. In general, where there are more than two states, for sufficiently large $t$ we expect two states to be dominant. The coefficients of the contributions of the higher states to eq. [3] do not exhibit exponential growth with $t_0$ [8]. Hence ideally we wish to study $\lambda_-(t, t_0)$ for $t_0$ as large as possible. However, the increase in the noise in the data far from the source generally requires that we choose $t_0$ close to the origin.

**Diagonalisation of $C(t)$**

We can compute the eigenvalues, $\chi_+(t, \omega)$ and $\chi_-(t, \omega)$, of $C(t)$ directly, and at large times, $t$, obtain

$$\chi_+(t, \omega) = f_+(\omega)e^{-E_0 t}\{1 + g_+(\omega)O(e^{-\Delta E_1 t})\}$$

$$\chi_-(t, \omega) = f_-(\omega)e^{-E_1 t}\{1 + g_-(\omega)O(e^{-\Delta E_1 t})\},$$  

(7)

where

$$\Delta E_1 = E_1 - E_0$$

$$\Delta \tilde{E} = \min[E_1 - E_0, E_2 - E_1].$$  

(8)

As in the previous method, the eigenvalues $\chi_+(t, \omega)$ and $\chi_-(t, \omega)$ are dominated by the ground state and first excited state respectively. However, even when there are only two states, corrections to the leading behaviour of the eigenvalues remain. The coefficients of these corrections depend on the arbitrary parameter $\omega$ and, since we diagonalise $C(t)$ at each timeslice, these coefficients are not positive definite. We will exploit the dependence on $\omega$ to seek cancellations between contributions from higher states, and hence extend the plateau region in the effective mass closer to the source.

Such an approach has clear dangers. In particular, the effective masses can approach their asymptotic values either from above or from below as we vary $\omega$. One restriction does temper the uncontrolled nature of this method. We observe that for sufficiently large times the excited state contributions to eq. [3] will be negligible, and thus the effective masses derived from $\chi_+(t, \omega)$ and $\chi_-(t, \omega)$ must be insensitive to $\omega$.

**4.1. Results**

In figure 2, we show the effective mass derived from $\chi_-(t, \omega)$ for the first excited state of the nucleon at several values of $\omega$. We observe there is a region in $t$ for which the effective masses coincide for all $\omega$. As noted above this can be taken as a signal that the first excited state has been isolated. The plateau region in the effective mass appears to extend closer to the source as $\omega$ decreases. We attribute this to cancellations between contributions of the higher excited states to $\chi_-(t, \omega)$.

In figure 3 we show the effective mass of the first excited state of the nucleon derived from $\lambda_-(t, t_0)$ with $t_0 = 1$, and derived from $\chi_-(t, \omega)$ at the optimal value of $\omega$. The two methods yield consistent plateau, and using Method 2 we obtain

$$\tilde{m}_N(4 - \delta) = 0.98 \pm \frac{2}{4},$$  

(9)

where the fitting range is shown in brackets.
Figure 2. The variation with $\omega$ of the effective mass derived from $\chi^-(t, \omega)$ for the first excited state of the nucleon. All values of $\omega$ are quoted relative to $\omega_0$, where $\omega_0$ is defined as $\sqrt{c_{LL}c_{SS}}$ at $t = 12$.

Figure 3. The effective mass of the first excited state of the nucleon derived from $\lambda^-(t, t_0)$, and $\chi^-(t, \omega)$.

Comparing the computed ratio of the mass of the first excited state and that of the ground state for the nucleon with its physical value for light quarks, we find it is appreciably larger. We find $m_N/m_N \approx 2.0$, whereas $m_{N(1440)}/m_N = 1.5$. If this is disappointing, it should be noted that a mass of $O(a^{-1})$ is likely to be subject to considerable uncertainties arising from the non-zero lattice spacing. However, an unexpectedly large value for the mass of the nucleon excited state has been observed in other simulations [7].

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