Transversely Anisotropic Damage Characteristics of Loaded Rock under Freeze-thaw Cycles

Xiangdong Liu, Huimei Zhang
Department of Mechanics, Xi’an University of Science and Technology, Xi’an, China
E-mail: 18192194331@163.com

Abstract. Flake metamorphic rocks and sedimentary rocks tend to show obvious transversely isotropic mechanical properties in deformation and strength because of the existence of weak structural surface or soft-hard interlayer structure. The rock in cold area is affected by load and freeze-thaw cycles, and the prevention and control of freeze-thaw disaster has become an urgent problem to be solved in the construction of cold area. Based on the strength of Rock Micro Element subjecting to Weibull random distribution and the macroscopic statistical damage model of rock, this paper used the transversely isotropic elastomer model to describe the damage evolution equation of freeze-thaw rocks in the whole process of the load deformation, and established the corresponding damage constitutive model. The results showed that this model more accurately describes the effect of freeze-thaw cycles on rock mechanical behavior, and had positive theoretical and practical significance for solving engineering problems in complex rock body.

1. Introduction
The freeze-thaw erosion of rock slope in cold area, the freezing expansion rupture of tunnel surrounding rock, the freezing expansion and thawing settlement of building foundation and so on are becoming more and more prominent, and groundwater and freeze-thaw cycles are the main factors causing rock weathering [1-2]. With the expansion of resource development, in the construction and operation of mines, railways, highways, tunnels and other civil works in cold areas, more and more involves the rocks under the environment of low temperature and alternating temperature changes, and the prevention and control of freeze-thaw disasters has become an urgent problem to be solved in engineering construction. The study of the physical and mechanical properties of loaded rock under the action of freeze-thaw cycles is of great significance to reveal the mechanism of freeze-thaw disaster.

Xu [3] studied the mechanical properties of Jiangxi Red Sandstone and Hubei shale under drying and saturation by uniaxial compression at low temperature and triaxial compression tests at different temperatures. Fahey [4], Prick [5] compared the effects of freeze-thaw cycles and dry and wet cycles on the strength of shale through experiments. Matsuoka [6] focused on the mechanism of freeze-thaw failure of rock and the relationship between rock strength and freeze-thaw times. Nicholson [7] carried out freeze-thaw cycles tests on 10 kinds of rocks to study the effect of lithology on the deterioration degree of rock freeze-thaw damage. Yang [8] performed CT scans on soft rock at different freezing temperatures, and evaluated the propagation law of freeze-thaw damage with CT number changes. Zhang [9,10] He carried out the freeze-thaw cycles and uniaxial compression test of sandstone and shale, and established the damage model of freeze-thaw rock in unidirectional stress state. Li [11] established the damage deterioration model of frozen and melted fractured rock, and discussed the damage deterioration mechanism of fractured rock mass under the coupling of freeze-thaw cycles and load based on the microscopic damage theory and macroscopic statistical damage model. Zhang [12]
studied the deterioration mechanism and mechanical characteristics of rock freeze-thaw damage through the freeze-thaw cycles test of different hydration environment. Chen [13] performed freeze-thaw cycles and single-axis compression test, established the damage expression of rock under different environmental conditions, and analyzed the effect of initial damage on rock freeze-thaw.

In order to reduce the complexity of the problem, the previous research in most cases the rock is regarded as isotropic body, the proposed failure criterion does not describe the influence of anisotropy on material damage well. Because the rock in the formation process has the characteristics of layer, chip and so on, or in a certain direction has a very developed joint system, which forms a certain degree of layered phenomenon. Therefore, there are different elastic constants along the direction of the layer and the direction perpendicular to the layer. If the modulus of elasticity is several times different, more than 10 times or even dozens of times. And all directions parallel to the layer, the so-called transverse, have the same elastic properties as the rock mass is called the transversely isotropic rock mass. Obviously, if the rock mass is still calculated as an isotropic body, it will not be realistic. Heng [14] confirmed that shale has obvious anisotropy characteristics, the elastic modulus is the largest in the direction parallel to the layer, the direction perpendicular to the layer is the smallest, and the increase of confining pressure makes its increasing rate decrease. Gao [15] carried out single-axis and triaxial tests on sand slab rocks in surrounding rock to study the influence of fine layers in sand slate on rock deformation and strength characteristics, and the results show that the anisotropic characteristics of rock mechanical properties are significant.

Therefore, considering the failure mechanism of transversely anisotropic rock is of positive theoretical and practical significance to solve the engineering problems in complex rock mass. In this paper, a transversely isotropic elastomer model was proposed to describe the damage evolution equation of loaded rocks under freeze-thaw cycles in the whole deformation process, and the corresponding damage constitutive model was established, which provided a theoretical basis for the design, construction and safe operation of rock mass engineering in cold area.

2. Isotropic Loaded Rock Damage Constitutive Model under Freeze-thaw Cycles

From the point of view of damage, the microscopic cracks and pores randomly distributed inside the rock can be regarded as a kind of damage field, which is affected by freeze-thaw cycles and load, and the microscopic defects continue to expand, showing the deterioration of the macroscopic mechanical properties of the rock, and when the change of the microscopic structure reaches a certain degree, the material is destroyed. Under the action of load, rock grains produce slip and staggered, and the behavior of material progressive damage caused by the continuous accumulation of micro-defects can be expressed by the damage parameter $D$. Based on the inhomogeneity of rock microstructure [16], the distribution of mechanical properties of the internal elements of the material is probabilistic, and the statistical distribution characteristics of rock strength are expressed by Weibull function distribution, and according to stress-strain relation [17], the damage evolution equation of the loaded rock can be

$$D = \int_0^\varepsilon \varphi(x) \, dx = 1 - e^{-\left(\frac{\varphi(x)}{\varphi(x)}\right)}$$

where $\varphi(\varepsilon)$ is the damage rate of primitive bodies of loaded rocks, $\varepsilon$ is the strain, $\varepsilon_r$ is the maximum strain, $m = \frac{1}{\ln}\left(\frac{E}{\sigma_r}\right)$ is the rock uniformity coefficient, $\sigma_r$ is the maximum stress.

In the literature [9], the concepts of freeze-thaw damage, load damage and total damage were put forward, and freeze-thaw and load had different mechanical mechanisms to promote the initiation and expansion of cracks in rocks, thus showing different damage characteristics. After the rock undergoes the freeze-thaw cycles, it is equivalent to the first stage loading, then the loaded rock under the freeze-thaw can be equivalent to two-stage loading. According to the principle of strain equivalence after popularization in the literature [18], the reference damage state of rock is treated as the first damage state, and the state after freeze-thaw damage is used as the second damage state, and the damage variable is defined as
where, \( A_0 \) and \( A_n \) are effective bearing area of reference damage state and freeze-thaw damage state respectively, \( D_n \) is the freeze-thaw damage variable, the subscript \( n \) is a freeze-thaw cycles coefficient. It is assumed that the rock material is isotropic, that is, the damage variable is isotropic.

According to the concept of macroscopic image damage mechanics, the response of rock macroscopic physical properties can represent the degree of deterioration within the material. The elastic modulus of the material is more convenient to be analyzed and measured during the freeze-thaw cycles, and the rock freeze-thaw damage variable can be expressed by Eq (2) as

\[
D_n = 1 - \frac{E_n}{E_0} \tag{3}
\]

where \( E_0 \) and \( E_n \) are the isotropic elastic modulus of initial damage state and freeze-thaw damage state respectively.

The total damage variable of loaded rock under freeze-thaw cycles is

\[
D_m = D + D_n - DD_n \tag{4}
\]

where \( D_m \) is the total damage variables of loaded rocks under freeze-thaw cycles.

Thus, the total damage evolution equation is obtained

\[
D_m = 1 - \frac{E_n}{E_0} e^{-\left(\frac{1}{m_{(\varepsilon)}}\right)^\nu} \tag{5}
\]

The damage constitutive relationship of the isotropic loaded rocks under different freeze-thaw cycles is

\[
\sigma = E_0 (1 - D_m) \varepsilon = E_n e^{-\left(\frac{1}{m_{(\varepsilon)}}\right)^\nu} \varepsilon \tag{6}
\]

3. Establishment of Damage Constitutive Model of Transversely Anisotropic Loaded Rock under Freeze-thaw Cycles

3.1. Generalized Hooker's Law of Transversely Isotropic Body

For a particular orthotropic body, which is characterized by the same elastic properties in all directions parallel to a plane (that is, the so-called transverse), and the orthotropic body is called the transversely isotropic body. Many layered rocks and flake rocks can be regarded as transversely isotropic bodies. For this type of rock, the direction parallel to the layer is transverse, and the direction perpendicular to the layer is longitudinal. The \( z \) axis is now located in the longitudinal direction of one of the elastic principal direction, and the \( x \) axis and the \( y \) axis are located horizontally, as shown in Figure. 1. Because the elastic properties of the coordinate plane \( xoy \) are the same in any direction, that is, the three axes in the diagram are all in the elastic principal axes. The elastic properties of the \( x \) direction and the \( y \) direction are the same.

Generalized Hooker's law of the transversely isotropic body is as follows,
where $E$ and $E'$ are the elastic modulus of the isotropic plane and its vertical direction, respectively, $G = E/[2(1 + \mu)]$ is the shear modulus of the isotropic plane, $G'$ is the shear modulus on the plane orthogonal to the isotropic plane, $\mu$ is the Poisson’s ratio which determines the vertical shrinkage with the tensile direction in the plane when the isotropic plane is stretched; $\mu'$ is the Poisson’s ratio of the shrinkage in the isotropic plane is determined when stretching vertically in the direction of the isotropic surface; Here are a total of 5 independent elastic constants $E$, $E'$, $G'$, $\mu$, $\mu'$, and $G = E/[2(1 + \mu)]$ is independent elastic constant.

\[ \begin{bmatrix}
    \sigma_x \\
    \sigma_y \\
    \sigma_z \\
    \tau_{yz} \\
    \tau_{xz} \\
    \tau_{xy}
\end{bmatrix} = \begin{bmatrix}
    \frac{1}{E} & -\mu/E & -\mu'/E' & 0 & 0 & 0 \\
    -\mu/E & \frac{1}{E} & -\mu'/E' & 0 & 0 & 0 \\
    -\mu'/E' & -\mu'/E' & \frac{1}{E'} & 0 & 0 & 0 \\
    0 & 0 & 0 & 1/G & 0 & 0 \\
    0 & 0 & 0 & 0 & 1/G' & 0 \\
    0 & 0 & 0 & 0 & 0 & 1/G' \\
\end{bmatrix} \begin{bmatrix}
    \varepsilon_x \\
    \varepsilon_y \\
    \varepsilon_z \\
    \gamma_{yz} \\
    \gamma_{xz} \\
    \gamma_{xy}
\end{bmatrix} \quad (7)
\]

\[ \Delta = \begin{bmatrix}
    1 & -\mu & -\mu' \\
    -\mu & 1 & -\mu' \\
    -\mu' & -\mu' & 1
\end{bmatrix} \]

The stress-strain relationship that can be expressed by flexibility

\[ \begin{bmatrix}
    \sigma_x \\
    \sigma_y \\
    \sigma_z \\
    \tau_{yz} \\
    \tau_{xz} \\
    \tau_{xy}
\end{bmatrix} = \begin{bmatrix}
    \frac{1 - \mu}{EE'} & \frac{\mu + \mu'}{EE'} & \frac{\mu' + \mu'}{EE'} & 0 & 0 & 0 \\
    \frac{\mu + \mu'}{EE'} & \frac{1 - \mu}{EE'} & \frac{\mu' + \mu'}{EE'} & 0 & 0 & 0 \\
    \frac{\mu' + \mu'}{EE'} & \frac{\mu' + \mu'}{EE'} & \frac{1 - \mu}{EE'} & 0 & 0 & 0 \\
    G_{yz} & 0 & 0 & G_{yz} & 0 & 0 \\
    G_{xz} & 0 & 0 & 0 & G_{xz} & 0 \\
    G_{xy} & 0 & 0 & 0 & 0 & G_{xy}
\end{bmatrix} \begin{bmatrix}
    \varepsilon_x \\
    \varepsilon_y \\
    \varepsilon_z \\
    \gamma_{yz} \\
    \gamma_{xz} \\
    \gamma_{xy}
\end{bmatrix} \quad (8)
\]
The stress and strain are converted from the elastic principal coordinate system to any coordinate system, and there are

$$\{\sigma^{'}\} = [D^'] \{\varepsilon^{'}\}$$ \hspace{1cm} (9)

where $$[D^'] = [L][D][L]^T$$

$$[L] = \begin{bmatrix}
    l_1^2 & l_2^2 & l_3^2 & 2l_1l_2 & 2l_1l_3 & 2l_2l_3 \\
    m_1^2 & m_2^2 & m_3^2 & 2m_1m_2 & 2m_1m_3 & 2m_2m_3 \\
    n_1^2 & n_2^2 & n_3^2 & 2n_1n_2 & 2n_1n_3 & 2n_2n_3 \\
    l_1m_1 & l_2m_2 & l_3m_3 & l_1m_1 + l_2m_2 & l_1m_1 + l_3m_3 & l_2m_2 + l_3m_3 \\
    m_1n_1 & m_2n_2 & m_3n_3 & m_1n_1 + m_2n_2 & m_1n_1 + m_3n_3 & m_2n_2 + m_3n_3 \\
    n_1n_1 & n_2n_2 & n_3n_3 & n_1n_1 + n_2n_2 & n_1n_1 + n_3n_3 & n_2n_2 + n_3n_3 \\
\end{bmatrix}$$

where $$l_i$$, $$m_i$$, $$n_i$$ are the cosine of the angle between the elastic principal axes and any coordinate system, $$i = 1, 2, 3$$.

3.2. Establishment of the Damage Constitutive Model of Transversely Anisotropic Freeze-thaw Loaded Rock

Due to the complexity in the inelastic principal direction, only two independent damage variables are considered for freeze-thaw damage variables in the inelastic principal direction, assuming that the rock damage does not affect each other in the direction of the layer and the perpendicular to the layer. The main damage variables $$D_{n,1}$$, $$D_{n,3}$$ of transversely anisotropic are set, which are the damage variables in the horizontal direction and in the vertical direction of the material respectively, which are defined as

$$D_{n,i} = \frac{A_{n,i} - A_{0,i}}{A_{0,i}} \hspace{1cm} (i = 1, 3)$$ \hspace{1cm} (10)

where $$A_{0,i}$$ and $$A_{n,i}$$ are the cross-sectional areas in the horizontal direction and in the vertical direction respectively without damage, $$A_{0,i}$$ is the apparent cross-sectional area corresponding to the damage.

According to the concept of macroscopic phenomenological damage mechanics, the response of rock macroscopic physical properties can represent the degree of deterioration within the material. The elastic modulus of the material is more convenient to be analyzed and measured during the freeze-thaw cycles, and the rock freeze-thaw damage variable can be expressed by Eq (10) as

$$D_{n,i} = 1 - \frac{E_{n,i}}{E_{0,i}} \hspace{1cm} (i = 1, 3, 6)$$ \hspace{1cm} (11)

where $$E_{0,i}$$ and $$E_{n,i}$$ are the elastic modulus of the isotropic plane and in the vertical direction in the initial damage state respectively, $$E_{0,6}$$ is the shear modulus of the initial damage state in the plane perpendicular to the isotropic plane, $$E_{n,1}$$ and $$E_{n,3}$$ are the elastic modulus of the isotropic plane and in the vertical direction in the state of the freeze-thaw damage respectively. $$E_{n,6}$$ is the shear modulus of the initial damage state in the plane perpendicular to the isotropic plane in the freeze-thaw damage state, while the shear modulus of the isotropic plane is not independent.

Assuming that the isotropic plane is not affected by the damage of its vertical direction, the total damage variables of loaded rock under freeze-thaw cycles are deduced based on the use of the extended strain equivalence hypothesis in the literature [18].

$$D_{m,i} = D + D_{n,i} - D_D \hspace{1cm} (i = 1, 3, 6)$$ \hspace{1cm} (12)
where $D_{i,j}$ $(i = 1, 3)$ is the total damage variable of the isotropic plane of the loaded rock and its vertical direction under the action of freeze-thaw. $D_{i,j}$ $(i = 6)$ is the total shear damage variable in a plane perpendicular to the isotropic plane.

Therefore, the damage constitutive relationship of the loaded orthotropic rock under different freeze-thaw cycles is obtained by substituting Eq. (6) into Eq. (8)

$$
\left( \begin{array}{c}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xz} \\
\tau_{yz} \\
\tau_{xy}
\end{array} \right) =
\left( \begin{array}{ccc}
1 - \mu' \mu & \mu' + \mu' & \mu + \mu' \\
\mu' + \mu' & 1 - \mu' \mu & \mu' + \mu' \\
\mu' + \mu' & \mu' + \mu' & 1 - \mu \mu'
\end{array} \right) \left( \begin{array}{c}
E_{n,1} \\
E_{n,3} \\
E_{n,6}
\end{array} \right) -
\left( \begin{array}{c}
\frac{1}{E_{n,1}} \\
\frac{1}{E_{n,3}} \\
\frac{1}{E_{n,6}}
\end{array} \right) \left( \begin{array}{c}
\epsilon_x \\
\epsilon_y \\
\epsilon_z
\end{array} \right)
$$

(13)

where

$$
\Delta = \frac{1}{E_{n,3}E_{n,1}} \left( \begin{array}{ccc}
1 & -\mu & -\mu' \\
-\mu & 1 & -\mu' \\
-\mu' & -\mu' & 1
\end{array} \right)
$$

For the stress-strain relationship in the direction of the inelastic principal axes has a more complex form, the specific expression can be derived from the Eq. (9), which is abbreviated here. It can be seen that in the direction of inelastic principal axes, even transversely anisotropic rocks show general anisotropic properties, and there is a coupling effect between tangent strain and positive stress, as well as shear stress and line strain.

4. Examples

The paper cited the test data in the literature [10]. Water drilling method was adopted in rock sample processing. Drill the core along the direction perpendicular to the bedding plane, after initial grinding and fine grinding of the end face of the cutting specimen, and the cylindrical standard specimen with a diameter of 50mm and a height of 100mm was processed. All rock samples were placed in 105°C oven to dry for 48 hours, 3 pieces of dry rock samples were used for compressive strength test. The rest saturated rock samples were placed in the rapid freeze-thaw testing machine for freeze-thaw cycles test under the open system. Observe and record the freeze-thaw deterioration process of rock samples, and measure the quality of rock samples after they go through the set number of freeze-thaw cycles, and the compressive characteristics of the rock samples after 5, 10, 20, 40, 60, 100 freeze-thaw cycles were tested. The test results of the mechanical properties of shale under different water-bearing states and different freeze-thaw cycles are shown in Table. 1.
Table 1. Mechanical parameters of red sandstone under confining pressure

| Moisture condition | Freeze-thaw cycles n | Ultimate stress \( \sigma_u \)/MPa | Ultimate strain \( \varepsilon_u \)/\% | Elastic modulus \( E \)/GPa |
|--------------------|----------------------|----------------------------------|-------------------------------|--------------------------|
| dry                | 0                    | 52.075                           | 0.80                          | 9.859                    |
|                    | 5                    | 40.280                           | 0.68                          | 8.732                    |
|                    | 10                   | 34.275                           | 0.71                          | 6.628                    |
| saturated          | 20                   | 32.116                           | 0.87                          | 5.711                    |
|                    | 40                   | 30.855                           | 1.08                          | 5.169                    |
|                    | 60                   | 30.325                           | 1.28                          | 4.803                    |
|                    | 100                  | 30.153                           | 1.75                          | 4.715                    |

Since only the parameters perpendicular to the direction of the bedding plane are measured in the experiment, the parameters in the bedding plane can be determined according to the experimental results of the transversely anisotropic parameters of shale in the literature [14], and then the relationship between the elastic constants can be used to determine other material parameters by analogy.

The theoretical curve of the constitutive model is calculated, and compared with the experimental curve, because the experiment only records the stress-strain data perpendicular to the direction of the laminar surface, only the calculation of the same direction is given here, which is in good agreement with the experimental results, as shown in Figure 2.

![Figure 2. Verification of damage constitutive model for rock](image)

5. Conclusions

Based on the safety evaluation of geotechnical engineering in cold area and the existing achievements, the changes of shale mechanical properties under the action of water and freeze-thaw cycles were studied in this paper. Considering the stochastic characteristics of rock defects, on the basis of the macroscopic statistical damage model of rock, aiming at the characteristics of transverse anisotropy of fractured rock during freeze-thaw cycles, this paper selected the common shale in engineering, studied the damage evolution process in geological environment, and established the transversely anisotropic damage constitutive model, the main conclusions were as follows:

1. Considering the formation process of natural rocks, the damage evolution equation and damage extended constitutive relationship of transversely anisotropic rocks are established, the non-uniformity of rock microscopic structure is fully taken into account, and the damage evolution and its influence...
on the macroscopic mechanical behavior of materials were described on the basis of the characteristics of rock element intensity obeying Weibull random distribution. It is revealed that the damage of rock in all directions affects each other.

(2) For rocks with obvious layered structure, the constitutive model of freeze-thaw damage based on isotropic rock does not reflect the damage evolution process well. The freeze-thaw damage model of rock transversely anisotropy established in this paper can more realistically reflect its occurrence environment, considering the objective fact that it has different material constants along the direction of the layer and in the direction perpendicular to the layer. The constitutive model of the transversely anisotropic loaded rock damage of the freeze-thaw cycles established in this paper can well reflect the change relationship of the stress and strain of shale under different freeze-thaw cycles, and more accurately describe the effect of freeze-thaw cycles on the mechanical behavior of rock.

6. References

[1] Guodong C 2001 Adv. Earthsci. 16 293-299.
[2] Ning L, Guodong C and Dingyi X 2001 Chinese J. Geotech. Eng. 23 268-272.
[3] Guangmiao X and Quansheng L 2005 Chinese J. Rock Mech. Eng. 24 3076-3082.
[4] Fahey B D 1983 Earth Surface Proc. 8 535-545.
[5] Prick A 1995 Catena 25 7-20.
[6] Matsuoka N 1990 Cold Regions Sci. Tech. 17 253-270.
[7] Nicholson H, Dawnt P and Nicholson F 2000 Earth Surface Proc. 25 1295-1308.
[8] Gengshe Y, Quansheng Z and Yibin P 2004 Chinese J. Geotech. Eng. 26 838-842.
[9] Huimei Z and Gengshe Y. 2013 J. China Coal Soc. 38 1756-1762.
[10] Huimei Z and Gengshe Y. 2014 J. Wuhan Uni. Technol. 36 95-99.
[11] LI Xinping, LU Yani and WENG Yangjun 2013 Chinese J Rock Mech. Eng. 32 2307-2315.
[12] Jizhou Z, Linchang M and Zhenfeng Y 2008 Chinese J Rock Mech. Eng. 27 1688-1694.
[13] Youliang C, Mingxing D, Mingliang Liu, Dandan W and Peng W 2013 Chinese Qua. Mech. 34 74-80.
[14] Shuai H, Chunhe Y, Baoping Z, Yintong G, Lei W and Yuanlong W 2015 Rock and Soil Mechanics 36 609-616.
[15] Chunyu G, Jin X, Zhonghong L and Jianhui D 2011 Rock and Soil Mech. 32 1360-1364.
[16] Tianhong Y, Xiaoli T, Bin Y, Yongbin Z, Lianchong L, Chunan T and Tham LG 2005 ACTA Mech. Sol. Sin. 26 333-337.
[17] Jianguo N and Zhiwu Z 2007 Chinese J. Theo. Appl. Mech. 39 70-76.
[18] Quansheng Z, Gengshe Y and Jianxi R 2003 Chinese J Rock Mech. Eng.: 22 30-34.