Twisted Quadrupole Topological Photonic Crystals

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Topological manipulation of waves is at the heart of cutting-edge metamaterial research. Quadrupole topological insulators were recently discovered in 2D flux-threading lattices that exhibit higher-order topological wave trapping at both the edges and corners. Photonic crystals (PhCs), lying at the boundary between continuous media and discrete lattices, however, are incompatible with the present quadrupole topological theory. Here, quadrupole topological PhCs triggered by a twisting degree-of-freedom are unveiled. Using a topologically trivial PhC as the motherboard, it is shown that twisting induces quadrupole topological PhCs without flux-threading. The twisting-induced crystalline symmetry enriches the Wannier polarizations and leads to the anomalous quadrupole topology. Versatile edge and corner phenomena are observed by controlling the twisting angles in a lateral heterostructure of 2D PhCs. This study paves the way toward topological twist photonics as well as the quadrupole topology in the quasi-continuum regime for phonons and polaritons.

1. Introduction

The discovery of higher-order topology[1–26] opens a new horizon in the study of topological phenomena. Higher-order topological insulators (HOTIs)[1–26] are intriguing topological phases where topological mechanisms manifest themselves in multiple dimensions, unveiling a paradigm beyond the bulk-edge correspondence. For instance, 2D quadrupole topological insulators (QTIs)[1,4] exhibit 1D gapped edge states and 0D corner states. The concept of QTI generalizes the conventional Bloch band topology to the Wannier band topology[1-4] that is described by the so-called “nested Wannier bands.” A $\pi$-flux lattice model for QTIs was proposed by Benalcazar et al.[1] that was later realized in experimental systems based on mechanical metamaterials,[10] microwave systems,[11] electric circuits,[12] and coupled optical ring resonators.[19] However, the $\pi$-flux lattice picture is incompatible with conventional subwavelength photonic crystals (PhCs) that lie at the boundary between continuous media and discrete-lattice systems. A straightforward generalization of the $\pi$-flux lattice to PhCs would fail since there is no mechanism for flux-threading. On the other hand, quadrupole topological PhCs, where light–matter interaction can be much enhanced by strong photon confinement on the edges and corners at subwavelength scales, are on demand for topological photonics in the nonlinear and quantum regimes.

Recently, twisting has been discovered as an invaluable approach toward exotic states of matter with nontrivial topology, strong correlation, or superconductivity in 2D van der Waals materials.[27–29] However, so far, there is no connection between HOTI and twisting. Moreover, the power of twisting has not been unleashed in photonics where twisting is, in fact, experimentally more accessible than in electronic systems.

In this work, we illustrate that twisting can be an efficient tool to bring about HOTI in photonics. Instead of the bilayer Moiré patterns, we use a lateral heterostructure of 2D photonic crystals with opposite twisting angles to realize photonic corner states emerging from HOTIs. Interestingly, the underlying HOTI is a photonic anomalous QTI (AQTI) emerging due to the twisting-induced crystalline symmetry. Specifically, exploiting a common square-lattice PhC with trivial topology as the motherboard, we show that a particular twisting deformation can induce the AQTI phase when the mirror symmetries are removed while the glide symmetries emerge. In AQTIs, the flux-threading mechanism is not needed, and the system is not based on tight-binding models but rather rely on the quadrupole topology due to the glide symmetries in the quasi-continuum regime where the photonic bands are induced by Bragg scatterings. Moreover, the twisting approach leads to PhCs with “anomalous quadrupole topology” that differs fundamentally from the conventional quadrupole topology as follows: First, the twisting-induced nonsymmorphic symmetry doubles the band representation, hence at least four bands are needed below the quadrupole topological band gap. Second, in the Wannier representation, four nondegenerate Wannier bands are required for a minimal description of the anomalous quadrupole topology. Third, due to the lack of mirror symmetry, the edge polarizations include a topological (quadrupolar) contribution and a trivial ($C_2$-symmetric) contribution.

In experiments, we use 2D subwavelength PhCs made of Al$_2$O$_3$ cylinders to realize the photonic AQTIs via twisting the...
unit-cell structure. Using the electromagnetic near-field scanning methods, we directly measure and visualize the photonic wavefunctions of the bulk, edge and corner states. We reveal that the twisting angle can effectively control the photonic corner and edge states, leading to versatile topological boundary phenomena. Exploiting such controllability, we demonstrate that when the frequencies of the edge and corner states are tuned close, their mutual couplings enable excitation of corner states via the edge states. This finding unveils the twisted quadrupole topological PhCs with rich corner and edge phenomena and may inspire future developments of twisting photonicics in the up-rising field of topological photonics.\textsuperscript{[30–35]}

2. Results

2.1. System and Symmetry

The 2D square-lattice PhC has four identical cylinders made of Al\textsubscript{2}O\textsubscript{3} (radius 0.25 cm) in each unit-cell. The lattice constant is \(a = 2\) cm. The dielectric PhC is placed in a 2D cavity formed by metallic cladding above and below, where the transvers-magnetic harmonic modes (i.e., the modes with electric fields along the \(z\) direction) dominate the photonic bands at low frequencies. We start with a configuration where the four cylinders are located at the positions of \((\pm \frac{a}{2}, \pm \frac{a}{2})\) and the unit-cell has \(C\textsubscript{4v}\) point-group symmetry. By twisting the four cylinders along the dashed lines as illustrated in Figure 1a, the crystalline symmetry is reduced to the nonsymmetric group \(P\textsubscript{4g}\) that contains the two glide symmetries, \(G\textsubscript{x} = (x, y) \rightarrow (\frac{a}{2} - x, \frac{a}{2} + y)\) and \(G\textsubscript{y} = (x, y) \rightarrow (\frac{a}{2} + x, \frac{a}{2} - y)\); the four-fold rotation symmetry, \(C\textsubscript{4}\); and the inversion symmetry \(I = (x, y) \rightarrow (-x, -y)\). Note that the two glide symmetries do not commute with each other, \(G\textsubscript{x}G\textsubscript{y} \neq G\textsubscript{y}G\textsubscript{x}\). Here, the twist is not a pure rotation, but is designed to preserve the glide symmetries. When the dielectric cylinders overlap with each other, we combine the overlapping cylinders together.

2.2. Photonic Bands

We now point out the consequences of the glide symmetries on the photonic bands. First, the glide symmetries lead to double degeneracy of photonic bands at the Brillouin zone boundaries (i.e., the MX and MY lines; they are equivalent since the system has the \(C\textsubscript{4}\) rotation symmetry). This can be illustrated via the anti-unitary operators \(\Theta \equiv G\textsubscript{x}I\) (i.e., \(x, y\)). Here, \(I\) is the time-reversal operator that is explicitly the complex conjugation operator for the electromagnetic wavefunctions. One finds that \(\Theta\textsuperscript{x}\psi\textsubscript{a,k} = -\psi\textsubscript{-a,-k}\), if \(k\textsubscript{x} = \pi/a\), and \(\Theta\textsuperscript{y}\psi\textsubscript{a,k} = -\psi\textsubscript{-a,-k}\), if \(k\textsubscript{y} = \pi/a\), for any photonic Bloch wavefunction \(\psi\textsubscript{a,k}\) with \(a\) and \(k\) being the band index and the wavevector, respectively. Similar to the Kramers theorem, these algebraic properties give rise to the double degeneracy for all photonic bands at the MX and MY lines. In addition, the inversion operator, \(I\), and the anti-unitary operator, \(\Theta\), anti-commute at the X (Y) point. Therefore, the doubly degenerate bands at the X (Y) point always include an odd-parity band and an even-parity band (see Figure 1a; see Note S1, Supporting Information, for the proof). From the parity-inversion between the \(\Gamma\) and X (Y) points (see Figure 1a), we conclude that the Wannier dipole is quantized to \(\vec{P} = (\frac{1}{2}, \frac{1}{2})\) for the partial photonic band gap between the second and the third bands, whereas the Wannier dipole is quantized to \(\vec{P} = (0, 0)\) for the complete photonic band gap between the fourth and fifth bands. The vanishing dipole polarization of the complete photonic band gap provides a necessary condition for the emergence of the quadrupole topology.

The photonic band structure for the PhC with \(\theta = 22°\) is shown in Figure 1a that exhibits a large photonic band gap (\(\approx 30\%)\) between the fourth and fifth bands, ranging from 9.46 to 13.2 GHz, corresponding to a subwavelength regime (i.e., the structure features and the lattice constant are smaller than the wavelengths in free-space). Throughout this article, the permittivity of the dielectric cylinders is taken as \(\varepsilon = 6.2\) for the 2D simulation to represent the experimental measurements in the quasi-2D systems approximately (see Note S2, Supporting Information, for the detailed comparison between the 2D approximation and the 3D simulation).

2.3. Quadrupole Topology

We illustrate below that such a photonic band gap carries the anomalous quadrupole topology. The underlying physics of the AQTI phase does not rely on any tight-binding model, but can be directly characterized through the “nested Wannier bands” approach\textsuperscript{[11–14]} using the photonic wavefunctions from first-principle calculations (see Figure 1). The nontrivial quadrupole topology in the \(P\textsubscript{4g}\) PhCs originates from the symmetry-enforced quantization of the quadrupole polarization due to the glide symmetries (see Note S3, Supporting Information, for the proof).

According to Benalcazar et al.,\textsuperscript{[11–14]} the quadrupole topology requests the following key elements: First, the gapped Wannier bands and the vanishing dipole polarization, meaning that the Wannier centers are away from 0 and \(\frac{1}{2}\) and come in pair with both positive and negative values, that is, \((-\nu, \nu)\) with \(0 < \nu < \frac{1}{2}\). Second, the nontrivial, quantized quadrupole moment as manifested in the Wannier sector polarizations. The former is a necessary condition for the latter. In previous theories,\textsuperscript{[11–14]} the first condition is realized via the flux-threading mechanism that leads to the non-commutative mirror symmetries. Without the flux-threading, the commutative mirror symmetries lead to gapless Wannier bands and hence forbid the quadrupole topology. In our flux-free systems, the motherboard PhC has commutative mirror symmetries and gapless Wannier bands in all combinations. When the mirror symmetries are removed by twisting, the non-commutative glide symmetries give rise to the quadrupole topology.

The Wannier bands are calculated from the photonic Bloch wavefunctions using the Wilson-loop approach. The Wilson-loop operator along, for example, the \(y\) direction is defined as\textsuperscript{[14,51]}

\[
\tilde{W}_{y,k} = \exp \left[ \int \vec{A}(k) \cdot dk \right]
\]

where the subscript \(y\) and \(k\) specify, respectively, the direction and the starting point of the loop. \(\tilde{A}(k)\) is the matrix (non-Abelian) formulation of the photonic Berry connection with its matrix...
Figure 1. Photonic anomalous QTIa. a) Left: Unit-cell structure and Brillouin zone. Twisting is along the dashed lines with \( \theta \) denoting the (clockwise) twisting angle. Right: Photonic band structure for \( \theta = 22^\circ \pm \) denote the parity of the photonic bands at the \( \Gamma \) and \( X \) points (+/– for even/odd parity, respectively). The photonic band gap between the fourth and fifth bands exhibits the anomalous quadrupole topology. b) Wannier bands and their combinations. c) Nested Wannier bands (left) and the illustration of edge polarizations and quadrupole topology (right). The edge polarizations include the topological contribution with a quadrupolar geometry and the trivial contribution with a \( C_4 \)-symmetric geometry. d) Evolutions of the Wannier bands and their combinations with the twisting angle \( \theta \).

Element written as \( A_{nm}^j(k) = i(E_n(k)|\partial_{k_j}|E_m(k)) \) where \( |E_n(k)\rangle \) is the periodic part of the photonic Bloch wavefunction for the electric field along the \( z \) direction. The ket–bra symbols and the inner product for the photonic wavefunctions are defined in Note S4, Supporting Information. The first four photonic bands below the topological band gap are numerated by \( n,m = 1,2,3,4 \). \( \mathcal{T}_e \) represents the path-ordering operator along a closed loop in the Brillouin zone. Here, the Wilson-loop path has fixed \( k_z \), but with \( k_y \) traversing the whole region of \( [0, \frac{2\pi}{a}] \).

The Wannier bands are obtained by diagonalizing the Wilson-loop operator, \( \hat{W}_{\gamma^j} \gamma_j^{\kappa} | \gamma_j^{\kappa} \rangle = e^{2\pi i j/4} | \gamma_j^{\kappa} \rangle \) for \( j = 1,2,3,4 \). The \( j \)th Wannier band is explicitly the dependence of the Wannier center \( \gamma_j(k_x) \) on the wavevector \( k_x \) (see Figure 1b). The eigenvectors \( | \omega_j \rangle \) are used to construct the Wannier band bases, \( | \omega_j^{\kappa} \rangle = \frac{1}{4} \sum_{n=1}^4 | \gamma_j^{\kappa} \rangle | E_n(k) \rangle \) with \( | \gamma_j^{\kappa} \rangle \) denoting the \( n \)th element of the eigenvector. The Wannier band bases can be regarded as the Wannier “wavefunctions” from which the topology of the Wannier bands can be defined. For instance, the polarizations of the gapped, non-degenerate Wannier bands are given by \( p_{\gamma_j^{\kappa},\gamma_j^{\kappa}}(k_x) = \frac{1}{2\pi} \int A_{\gamma_j^{\kappa},\gamma_j^{\kappa}}(k)dk_x \) where \( A_{\gamma_j^{\kappa}}(k) = (i\omega_j^{\kappa}(k)|\partial_{k_x}|\omega_j^{\kappa}(k)) \) is the Berry connection for the \( j \)th Wannier band. The Wannier band polarizations, \( p_{\gamma_j^{\kappa},\gamma_j^{\kappa}}(k_x) \), which characterize the topological properties of the Wannier bands, are termed as the “nested Wannier bands” (see Note S4, Supporting Information, for calculation details).
Figure 2. Bulk, edge, and corner states modulated by the twisting angle. a) Nested Wannier bands for AQTIα and AQTIβ as functions of the twisting angle. b) Schematic illustration of the edge and corner states for a lateral heterostructure where the central region is the AQTIα and the outside region is the AQTIβ. In between them is the region with the PEC boundaries. The arrows represent the edge states and their topological polarizations, while the red and blue dots represent the corner states. c) The lateral heterostructure without the PEC boundaries. The boundary between the two PhCs is denoted by the black dashed lines. Orange double arrows represent interactions between the edge states from the two sides of the boundary. d) Evolution of the bulk, edge (including odd- and even-parity edge modes), and corner states with the twisting angle $\theta$. The bulk spectrum is obtained from the unit-cell calculation, while the edge and corner spectra are from the ribbon- and box-like (as shown in (c)) supercell calculations, respectively. e) The experimental setup that realizes (c). f) The detailed structure of the PhCs at the corner. Upper panel: top-down view; lower panel: bird-view photograph.

Figure 1b shows that there are four Wannier bands that are non-degenerate and gapped, distributing symmetrically in the positive and negative regions. Unlike the conventional QTIs with two Wannier bands, the four Wannier bands in AQTIs enable rich combinations. We find that only the “1 + 3” and “2 + 4” Wannier sectors yield gapped, composite Wannier bands that are necessary for the emergence of quadrupole topology. We denote the Wannier sector “1 + 3” (“2 + 4”) as I (II). Since the Wannier band is negative (positive) for the Wannier sector I (II), it is adiabatically connected with the edge states at the lower (upper) edge.[1,4] The Wannier band polarization $P^{I}_{x,y} = P^{II}_{x,y}$ then gives the topological polarization of the lower (upper) edge due to the bulk. Such analysis accounts for the topological edge polarizations of the edge band gap, which are used to describe the quadrupole polarization and the induced topological corner states.[1,4]

Remarkably, our calculation shows that $P^{0}_{x,y} = -P^{II}_{x,y} = \frac{1}{2}$. Such a quantization is due to the glide symmetries, as proved in Note S3, Supporting Information. Since the $x$ and $y$ directions are equivalent here, one finds that $P^{0}_{x,y} = -P^{II}_{x,y} = \frac{1}{2}$. The quadrupole polarization is then quantized as $q_{xy} = 2P^{II}_{x,y} = \frac{1}{2}$ (Figure 1c). Because mirror symmetry is absent in P4g crystals, the polarization of a physical edge contains both the topological contribution and the trivial contribution. Importantly, the trivial edge polarizations, due to their $C_4$-symmetric configurations, do not contribute to the corner charge or the bulk-corner correspondence[1,4] (see Figure 1c; see the analysis in Note S5, Supporting Information). We find that for the case with a negative twisting angle, the signs of the edge polarizations are flipped (see Figure 1c for schematics and Figure 2a for the topological edge polarization). Nevertheless, the quadrupole polarization remains nontrivial, $q_{xy} = \frac{1}{2}$. We denote the topological phase with positive $\theta$ as AQTIα, whereas the topological phase with negative $\theta$ as AQTIβ (see Figure 1c,d).

The evolution of the Wannier bands with the twisting angle $\theta$ is shown in Figure 1d. The Wannier sectors “1 + 3” and “2 + 4”
Figure 3. Controlling the edge and corner states by tuning the geometry. a) Evolution of the odd-parity edge states at \( k_x = 0 \) with the twisting angle \( \theta \). The green dashed line represents the boundary between the two PhCs with opposite twisting angles. Similar variation of the even-parity edge states is presented in the Figure S10, Supporting Information. The edge states merge into the bulk in both small and large \( \theta \) limit. b) Evolution of the lower-frequency corner states with the twisting angle \( \theta \). The variation of the higher-frequency corner states is presented in the Figure S7, Supporting Information. Note that the corner states merge into the edge in both the small and large \( \theta \) limit. For the small \( \theta \) limit, the corner state evolves into the odd-parity edge state, whereas in the large \( \theta \) limit, it evolves into the even-parity edge state.

(i.e., sectors I and II) become gapless in the limits: \( \theta \to 45^\circ \) and \( \theta \to 0 \) where the mirror symmetries are recovered. These properties agree with the observations made by Benalcazar et al.\(^4\) that mirror-symmetric systems without flux-threading cannot support nontrivial quadrupole topology. In addition, the photonic band gap closes in the limit of \( \theta \to 45^\circ \). The twisting angle, thus, controls the Wannier band gap and the photonic band gap simultaneously.

Within the \( P4g \) space group, we did not find any PhC with gapped Wannier bands but vanishing quadrupole polarization. Nevertheless, we can exploit the two PhCs with opposite twisting angles to construct a supercell with the edge and corner states. The scenario is illustrated in Figure 2b,c. If a perfect-electric-conductor (PEC) boundary is used to separate AQTI\(\alpha\) and AQTI\(\beta\) as illustrated in Figure 2b, then the topological edge (corner) states emerge at the edges (corners) for both AQTI\(\alpha\) and AQTI\(\beta\) (see Note S6, Supporting Information, for the photonic spectra and wavefunctions of the edge and corner states with PEC boundaries). Note that in Figure 2b, the outer boundaries of the AQTI\(\alpha\) have the same edge polarization configurations as the inner boundaries of the AQTI\(\beta\), which is in accordance with the analysis in Figure 1. However, the use of PEC is incompatible with our experimental system based on the transverse-magnetic harmonic modes and thus the PEC boundaries should be avoided.

The removal of the PEC boundaries leads to the coupling between the edge states from the outer boundaries of the AQTI\(\alpha\) and those from the inner boundaries of the AQTI\(\beta\) (see Figure 2c). With such couplings, the hybridized edge states form the symmetric and anti-symmetric (i.e., even-parity and odd-parity) edge modes (see Note S7, Supporting Information, for their wavefunctions), because the edge boundaries are mirror symmetric, if AQTI\(\alpha\) and AQTI\(\beta\) have opposite twisting angles. In such a configuration, the evolution of the bulk, edge, and corner spectra with the twisting angle \( \theta \) for the setup in Figure 2c is presented in Figure 2d. The corresponding experimental system is illustrated in Figure 2e,f. As elaborated below, the AQTI\(\alpha\) and AQTI\(\beta\) form a lateral heterostructures with tunable and versatile properties of the edges and corners. For instance, the coupling between the edge states is reflected by the splitting between the symmetric and anti-symmetric modes, which can be controlled by the twisting angle \( \theta \), as shown in Figure 2d. With \( \theta \) approaching 0, as the two PhCs are tuning into the same geometry, the edge states merge into the bulk (as shown in Figure 3a; Figure S10, Supporting Information). In the other limit, with \( \theta \) approaching 45°, the close of the bulk band gap also leads to delocalization and annihilation of the edge states. In this process, the edge band gap closes before the bulk band gap closing (see Figure 2d). Later, these edge states merge into the bulk states (see Figure 3a; Note S8, Supporting Information).

The corner states also experience strong modulation when the twisting angle is tuned. Figure 3b shows the controllability of the lower-frequency corner states by the twisting angle (the study of the higher-frequency corner states is presented in the Figure S7, Supporting Information). With \( \theta \) going from 27° to 5°, the corner states gradually merge into the lower-branch (i.e., odd-parity) edge states and then into the bulk states. In the other direction, with increasing \( \theta \), the corner states traverse the edge band gap and gradually merge into the upper-branch (i.e., even-parity) edge states, as \( \theta \to 35^\circ \). Such a behavior of traversing the
edge band gap is a sign of topological corner states. In contrast, the higher-frequency corner states do not have such a property. We also notice from simulations that the lower-frequency corner states are more robust than the higher-frequency corner states (see Note S11, Supporting Information). These findings indicate that twisting can be used as an efficient tool to transfer photonic states from 0D corner states to 1D edge states or 2D bulk states, in reconfigurable PhCs.

3. Experimental Section

The experimental setup used to verify the physics elaborated above is shown in Figure 2e where the box-like PhC structure realizes the schematic illustration in Figure 2c. The whole structure contains 20 × 20 unit-cells (including 10 × 10 unit-cells for AQTIα in the center and other 300 unit-cells for the AQTIβ at the outside). Electromagnetic waves were excited in the AQTIα region and absorbed by the absorber outside the structure. The detection for the bulk, edge, and corner modes was realized by three probes located in the bulk region of AQTIα, on the edge, and at the corner, separately. They are termed as the bulk-probe, edge-probe, and corner-probe, respectively.

The photonic wavefunction of the bulk, edge, and corner states was measured by near-field scanning of the electromagnetic field (see Note S9, Supporting Information). The results are presented in Figure 4 for a part of the lateral heterostructure with θ = 22°. The photograph of the PhC structure is shown in Figure 4a. The boundary between the two types of PhCs is illustrated by the dashed lines, while the corner is denoted by the red dot. From low frequency to high frequency, five typical photonic wavefunctions are shown: the bulk state at 9.50 GHz (Figure 3b), the edge states at 10.18 and 12.30 GHz (Figure 3c, f), and the corner states at 10.27 and 12.22 GHz (Figure 3d, e). The real-part of the corner wavefunctions from both the simulation and the experiments is shown in Note S10, Supporting Information. The robustness of the corner states against disorder is studied in Note S11, Supporting Information. Here, it is shown that the corner states in all these studies, the measured electric field distributions agree well with the numerical simulation. These results directly visualize the corner and edge states, and confirm the physics picture elaborated in the previous section.

Then the scattering coefficient (S21) spectrum of the bulk-probe, edge-probe, and corner-probe was studied, which gave effectively the local density of states in the bulk, edge, and corner regions. The pump-probe spectroscopies for various twisting angles θ are presented in Note S12, Supporting Information. The frequency range for the bulk, edge, and corner modes was extracted from the S21 spectra. The results are shown in Figure 5a for six different twisting angles θ from 0° to 27°, which agree
Figure 5. Measuring the corner states and their evolution with the twisting angle. a) The spectral regions for the bulk, edge, and corner states for various twisting angles. b) Measurement of the electric field profiles along three directions (illustrated by the red, green, and blue lines in the inset) for the corner state (resonant frequency 10.27 GHz) at $\theta = 22^\circ$. c–f) Evolution of the wavefunction of the corner states with the twisting angle. g) Pump-probe spectroscopy for the setup with the source at the edge and the detector close to the corner (see the inset) for $\theta = 22^\circ$. h) The measured electric field profile for two corner states at resonant frequencies of 10.27 and 12.26 GHz, respectively.

quite well with the simulation in Figure 2d. Larger twisting angles were avoided because of the overlap of the Al$_2$O$_3$ cylinders at the unit-cell boundaries. To confirm the fully localized nature of the corner states, the electric field profile of the corner state around one of the corners was measured. Figure 5b presents the electric field profiles measured along three lines (the red, blue, and green lines as indicated in the inset) for $\theta = 22^\circ$ at the corner mode frequency of 10.27 GHz. The results show that the photonic wavefunction is well-localized at the corner and decays rapidly in all three directions, which confirm the observation of corner states as photonic bound states.

The evolution of the lower-frequency corner state with the twisting angle $\theta$ is studied in details in Figure 5c–f with $\theta$ varying from $22^\circ$ to $6^\circ$. These figures clearly show that the corner state gradually evolves from strongly localized to weakly localized. The corner state eventually merges into the bulk band and becomes a bulk state in the limit of $\theta \to 0$.

In the existing studies on HOTIs, the corner states are spectrally well-separated from the edge and bulk states. Such isolation makes it hard to utilize them for functional devices. Here, it is shown that, thanks to the tunable nature of the corner states in the PhCs in this study, the frequency of the corner states can be tuned to be close to the edge states and thus enable their mutual coupling in finite-sized systems. In experiments, the setup with the twisting angle $\theta = \pm 22^\circ$ was chosen where the corner and edge states have frequencies of 10.27 and 10.18 GHz, respectively. The coupling between the edge and corner states was revealed using the pump-probe measurement schematically illustrated in the inset of Figure 5g. The source is placed on the edge at the left side of the corner, while the detector is placed near the corner. The pump-probe spectroscopy in Figure 5g shows that there are two corner modes within the bulk band gap (indicated by the two red arrows): one at 10.27 GHz, the other at 12.26 GHz. The corner mode at 10.27 GHz is very close to the edge band (indicated by the blue arrow). The hybridization between the edge and corner modes is manifested directly in the measured electric field profiles in Figure 5h that are obtained by scanning the electric field at the two frequencies, 10.27 and 12.26 GHz. The electric field profile measured at 12.26 GHz indicates a clear feature of evanescent wave excitation of a spectrally isolated, strongly localized corner mode. In contrast, the electric field profile measured at 10.27 GHz indicates visible hybridization and coupling between the corner mode and the edge modes. Such coupled edge-corner system may serve as coupled waveguide-cavity systems in photonic chips.

4. Conclusion and Outlook

In this work, we unveil twisting as a new degree-of-freedom toward higher-order topology in 2D dielectric, subwavelength PhCs. Here, the nonsymmorphic symmetry induced by twisting leads to anomalous quadrupole topology for photons. The intriguing properties of the photonic anomalous QTIs are revealed using a lateral heterostructure comprising two PhCs with opposite twisting angles. The photonic wavefunctions of the edge and
corner states are directly visualized using the near-field scanning methods. Consistent theory and experiments show that the photonic and Wannier band gaps of the PhCs can be controlled effectively by the twisting degree-of-freedom. Consequently, rich edge and corner phenomena are observed when the twisting angle is tuned, demonstrating efficient photonic states transfer among 0D corner states, 1D edge states, and 2D bulk states via twisting. With both simulation and experiments, we demonstrate that lateral heterostructures with different twisting angles can yield rich topological phenomena, and thus opens a new route toward topological photonics. Our study opens a new pathway toward topological photonics. It should be noted that at the final stage of this work, we became aware of recent works on QTIs in magnetized systems.[8,9,10]

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

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[40] M. G. Silveirinha, Phys. Rev. B 2015, 92, 125153.
[41] M. G. Silveirinha, Phys. Rev. B 2016, 93, 075110.
[42] X. J. Cheng, C. Jouvaud, X. Ni, S. H. Mousavi, A. Z. Genack, A. B. Khanikaev, Nat. Mater. 2016, 15, 542.
[43] F. Gao, Z. Gao, X. H. Shi, Z. J. Yang, X. Lin, H. Y. Xu, J. D. Jonanopoulos, M. Soljacic, H. S. Chen, L. Lu, Y. D. Chong, B. L. Zhang, Nat. Commun. 2016, 7, 11619.
[44] L. J. Maczewsky, J. M. Zeuner, S. Nolte, A. Szameit, Nat. Commun. 2017, 8, 13756.
[45] S. Banik, A. Karasahin, C. Flower, T. Cai, H. Miyake, W. DeGottardi, M. Hafezi, E. Waks, Science 2018, 359, 666.
[46] Y. T. Yang, Y. F. Xu, T. Xu, H. X. Wang, J. H. Jiang, X. Hu, Z. H. Hang, Phys. Rev. Lett. 2018, 120, 217401.
[47] F. F. Li, H. X. Wang, Z. Xiong, Q. Lou, P. Chen, R. X. Wu, Y. Poo, J. H. Jiang, S. John, Nat. Commun. 2018, 9, 2462.
[48] H. Jia, R. Zhang, W. Gao, Q. Guo, B. Yang, J. Hu, Y. Bi, Y. Xiang, C. Liu, S. Zhang, Science 2019, 363, 148.
[49] Y. Yang, Z. Gao, H. Xue, L. Zhang, M. He, Z. Yang, R. Singh, Y. Chong, B. Zhang, H. Chen, Nature 2019, 565, 622.
[50] T. Ozawa, H. M. Price, A. Amo, N. Goldman, M. Hafezi, L. Lu, M. C. Rechtsman, D. Schuster, J. Simon, O. Zilberberg, L. Carusotto, Rev. Mod. Phys. 2019, 91, 015006.
[51] H. X. Wang, G. Y Guo, J. H. Jiang, New J. Phys. 2019, 21, 093029.
[52] B. J. Wieder, Z. Wang, J. Cano, X. Dai, L. M. Schoop, B. Bradlyn, B. A. Bernevig, arXiv:1908.00016 2019.
[53] L. He, Z. Addison, E. J. Mele, B. Zhen, arXiv:1911.03980 2019.