Numerical study on dynamic buckling of composite cylindrical shell with one edge simply supported and opposite edge clamped

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Abstract. Considering stress wave effect, axial inertia and rotational inertia are calculated, and dynamic buckling control equation of cylinder shell is obtained by Hamilton principle based on the Donnell thin shell theory. The dimensionless of the equation is solved by differential method, which discuss about effects of different initial imperfection, different impact mass, different impact speed and different lay-up angle on the buckling of cylindrical composite shell. The result shows significant effects of stress wave and axial inertia on dynamic buckling.

1. Introduction

As the most commonly used basic components in engineering structures, composite cylindrical shells are widely used in large machinery, aerospace, weaponry and other fields due to their high strength and high stiffness. The dynamic buckling of cylindrical shell is subjected to actual axial impact in engineering occurred frequently, so the dynamic buckling of composite cylinder shell has attracted the attention and research of researchers. A.V Lopatin¹ obtained the analytical solution of the buckling problem of composite cylindrical shell subjected to hydrostatic pressure by Fourier decomposition and the Galerkin method. Buckling behavior of axisymmetric cylindrical shell subjected to uniform axial load is studied based on the first-order shear deformation theory. The analytical expression of stress is obtained and the effects of the geometric properties on buckling stress are investigated by F.M Nasrekani². Y.L Pi³ use an energy method to investigate the nonlinear elastic dynamic in-plane buckling of a pinned–fixed shallow circular arch under a central concentrated load which is applied suddenly. The principle of conservation of energy is used to establish the criterion for dynamic buckling of the arch, and analytical solution for the dynamic buckling load is derived. Bisagni⁴ studied composite cylinder shell about static and dynamic buckling tests under axial compression. H.J Gu⁵ studied dynamic buckling behaviour of cylindrical shell by dropping a hammer experiment. The results show that the number of the edges of a folded part increases with increase of the ratio of diameter to thickness, and the energy absorption is proportional to diameter of the cylindrical shell. F Taheribeheooz⁶ employed experimental and numerical procedures to investigate effects of initial geometric imperfection on the buckling behavior of the perfect and perforated composite cylinders.

In summary, most studies do not take into account stress wave effect and axial inertia. After considering the axial inertia, the problem which is rigid mass impacting the dynamic buckling of
composite cylindrical shell cannot obtain an analytical expression. Therefore, difference method and finite element method become the most important methods to solve this problem.

2. The control equations of dynamic buckling
As shown in Figure 1, composite cylindrical shell is simple at the left edge, clamped at right edge. \( L \) stands for the length of cylinder shell, \( R \) denotes radius, \( h \) is thickness and \( k \) is number of lay-up. Select cylindrical coordinate system \( x, y, z \) and its corresponding displacement is \( u, v, w \). The left end is subjected to impacting of a rigid mass. The stress wave propagates along the \( x \) direction, and internal forces in each section of cylindrical shell are shown in Figure 2, where \( l_{cr} \) is the critical length, \( t_{cr} \) is critical time, and \( c \) is stress wave’s velocity.

![Figure 1 Composite cylindrical shell impacting by a rigid quality block.](image1)

![Figure 2 Transmitting of stress wave diagram.](image2)

According to the theory of composite plates and shells, ignoring effects of shear, the expressions of internal force and internal moment of composite thin-walled cylindrical shell is as follows[7]:

\[
\begin{bmatrix}
N_x \\
N_\theta \\
N_{x\theta} \\
M_x \\
M_\theta \\
M_{x\theta}
\end{bmatrix}
= 
\begin{bmatrix}
A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & e_x \\
A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & e_\theta \\
A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} & e_{\theta\theta} \\
B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & \kappa_x \\
B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & \kappa_\theta \\
B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} & \kappa_{\theta\theta}
\end{bmatrix}
\begin{bmatrix}
v_0 \\
x \\
u \\
v \\
w \\
z
\end{bmatrix}
\]  
\( \text{(1)} \)

Where \( A_{ij} \) stand for tensile stiffness, \( B_{ij} \) denotes Coupling stiffness, \( D_{ij} \) is Bending stiffness, they all defined by the following formulas[7]:

\[
A_{ij} = \sum_{k=1}^{n} (Q_{ij})_k (z_k - z_{k-1}) \quad B_{ij} = \frac{1}{2} \sum_{k=1}^{n} (Q_{ij})_k (z_k^2 - z_{k-1}^2) \quad D_{ij} = \frac{1}{3} \sum_{k=1}^{n} (Q_{ij}) (z_k^3 - z_{k-1}^3)  
\]  
\( \text{(2)} \)

Where \( Q_{ij} \) is engineering elastic constant.

Buckling control equations of composite cylindrical shell are derived by Hamilton principle[8].

\[
\frac{\partial N_x}{\partial r} + \frac{1}{R} \frac{\partial N_{\theta}}{\partial \theta} - I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^3 w_0}{\partial \theta \partial t^2} = 0
\]
\[
\frac{\partial N_{\theta}}{\partial r} + \frac{1}{R} \frac{\partial N_x}{\partial \theta} - I_0 \frac{\partial^2 u_0}{\partial \theta^2} + I_1 \frac{\partial^3 w_0}{\partial r \partial t^2} = 0
\]
\[
\frac{\partial^2 M_x}{\partial r^2} + \frac{1}{R} \frac{\partial^2 M_\theta}{\partial \theta^2} + 2 \frac{\partial^3 M_{\theta}}{\partial r \partial \theta \partial t} - N_\theta \frac{\partial}{\partial r} \left[ N \frac{\partial u_0}{\partial r} \right] = I_0 \frac{\partial^2 v_0}{\partial \theta^2} + I_1 \frac{\partial^4 u_0}{\partial r \partial \theta \partial t^2} + I_2 \frac{\partial^4 w_0}{\partial r^2 \partial t^2} - I_3 \frac{\partial^4 v_0}{\partial \theta^2 \partial t^2}
\]  
\( \text{(3)} \)

For a special orthotropic symmetric laminated shell and a normal symmetric orthogonally laid laminated shell, there are \( A_{16}=A_{26}=D_{16}=D_{26}=B_{ij}=0 \). It is assumed that internal forces of cylindrical shell are uniformly distribution along circumferential direction.

\[
N = e_x A_{11} + A_{12} \frac{w_0}{R}  
\]  
\( \text{(4)} \)
The dynamic buckling control equations of a rigid mass impacting composite cylindrical shell are as follows:

$$A_{11}u_{xx} + \frac{1}{R^2} A_{66} u_{yy} = \rho hu_u + \frac{1}{R} A_{12} w_x - \frac{1}{4} \rho h^2 w_{xx}$$

$$D_1 \dddot{w}_{xxxx} + \frac{2}{R^2} (D_{12} + 2D_{66}) \dddot{w}_{xxyy} + \frac{1}{R^2} A_{22} \dddot{w} + A_1 u_{xx} w_x + A_{11} \dddot{w}_{xx} u_x$$

$$- \frac{1}{R} A_{12} w_x w_y - \frac{1}{R} A_{22} \dddot{w}_{yy} = \frac{1}{12} \rho h^3 w_{xxxx} + \frac{1}{12} \rho h^3 w_{yyyy} + \rho h w_u - \frac{1}{4} \rho h^2 u_y$$

(5)

Take the following dimensionless quantities:

$$c_r, c_l, c_1, c_2, c_3, c_4, \rho_c, M = \frac{l}{l_{cr}} A_y, c_x = A_{22}/A_{11}, c_3 = A_{22}/A_{11}, c_4 = R/l_{cr}$$

(6)

The dimensionless form of the control equations are calculated by bringing formula (6) into formula (5):

$$- \tilde{u}_{xx} + c_4 c_l \tilde{u}_{yy} = \tilde{u}_u + c_4 \tilde{w}_x - \frac{1}{4} \tilde{w}_{xx}$$

$$\dddot{w}_{xxxx} + (2c_1 + 4c_2) c_4 \dddot{w}_{xxyy} + c_3 c_4 \dddot{w}_{yyyy} + c_1 c_4 \dddot{u} + c_3 c_4 \dddot{w} + \dddot{u}_{xx} \dddot{w}_x + \dddot{w}_{xx} \dddot{u}_x - c_4 c_4 \dddot{w}_x \dddot{w}_x$$

$$- c_4 c_4 \dddot{w}_{xx} \dddot{w}_x = \frac{1}{12} \dddot{w}_{xxxx} + \frac{1}{12} \dddot{w}_{yyyy} - \dddot{w}_u - \frac{1}{4} \dddot{u}_y$$

(7)

The dynamic buckling control equations of composite cylindrical shell satisfy the following initial conditions and boundary conditions:

$$\tilde{u}(0, \theta) = \tilde{u}((l_{cr}, \theta) = \tilde{u}_{xx}(0, \theta) = \tilde{u}_{xx}(l_{cr}, \theta) = 0$$

$$\dddot{w}(0, \theta) = \dddot{w}(l_{cr}, \theta) = \dddot{w}_{xx}(0, \theta) = \dddot{w}_{xx}(l_{cr}, \theta) = 0$$

(8)

The axial initial displacement is calculated by formula (9).

$$\dddot{w}_0 = \frac{M v_0}{\rho c l_{cr}} e^{(-\frac{\rho c x}{M})} + \frac{M v_0}{\rho c l_{cr}}$$

(9)

Cite w-direction initial displacement as formula (10).

$$\dddot{w}_0 = c_0 \sin(x) \sin(m \theta) - \frac{n}{n+1} \sin(n \theta)$$

(10)

Where $M$ is impact mass, $v_0$ denotes impact velocity, $A$ stand for cross-sectional area, $\rho$ is material density, $c_0$ is a parameter, and $n$ is transverse mode number, $m$ is circumferential mode number.

3. Difference format for control equations

Bring central differential and backward differential formats into control equations, the difference format of the control equations are obtained as follows.

$$\dddot{u}(i, j, t + 1) = \Delta t^2 [\dddot{u}_{xx}(i, j, t) + c_4 c_4 \dddot{u}_{yy}(i, j, t) - c_1 c_4 \dddot{u}_x(i, j, t) + c_2 c_4 \dddot{u}_y(i, j, t)] + 2\dddot{u}(i, j, t)$$

$$- \dddot{u}(i, j, t - 1)$$

$$\dddot{w}(i, j, t + 1) = -\Delta t^2 [\dddot{w}_{xxxx}(i, j, t) + (2c_1 + 4c_2) c_4 \dddot{w}_{xxyy}(i, j, t) + c_3 c_4 \dddot{w}_{yyyy}(i, j, t) + c_1 c_4 \dddot{w}_x(i, j, t) + c_3 c_4 \dddot{w}_y(i, j, t) + \dddot{w}_{xx}(i, j, t) \dddot{w}_x(i, j, t) + \dddot{w}_{yy}(i, j, t) \dddot{w}_y(i, j, t) - c_4 c_4 \dddot{w}_x(i, j, t) \dddot{w}_x(i, j, t) - \frac{1}{12} \dddot{w}_{xxxx}(i, j, t) + \frac{1}{12} \dddot{w}_{yyyy}(i, j, t) + \frac{1}{4} \dddot{u}_{xx}(i, j, t) + \frac{1}{4} \dddot{u}_{yy}(i, j, t)] + 2\dddot{w}(i, j, t)$$

$$- \dddot{w}(i, j, t - 1)$$

(11)

Where $i, j$ corresponds to $x$ and $y$ spatial points of the cylindrical shell, $t$ indicates time points. $\Delta x, \Delta y$ is a dimensionless space displacement step and $\Delta t$ stand for a dimensionless time step.
4. Numerical Simulation

In this paper, dynamic buckling of carbon/epoxy cylindrical shell is discussed and material parameters are as shown in Table 1 [8]. The influence of different initial imperfections, impact mass and lay-up angle on dynamic buckling of composite cylindrical shell is discussed by programming formula (11) with Matlab. Take displacement step $\Delta x = 0.0237$, $\Delta y = 0.0189$, and time step $\Delta t = 0.025$. Then, obtained result is converted to the actual displacement by $w = w \times l_{cr}$.

| Parameter | Value |
|-----------|-------|
| $E_1$ (GPa) | 139 |
| $E_2 = E_3$ (GPa) | 9.4 |
| $G_{23}$ (GPa) | 2.98 |
| $G_{12} = G_{13}$ (GPa) | 4.5 |
| $\nu_{23}$ | 0.33 |
| $\nu_{12} = \nu_{13}$ | 0.3095 |
| $\rho$ (kg/m$^3$) | 1538 |
| $L$ (m) | 1 |

| Diameter (m) | 0.4 |
| Thickness (m) | 0.006 |
| Lay-up Angle(°) | 0/90/0/90/0 |
| Layers | 5 |

Figure 3 indicates that dynamic buckling mode diagrams of a composite cylindrical shell with different initial imperfections when impact mass, impact speed and the lay-up angle are constant. $l_{cr}$ in Figure (a) is 0.22m and 0.42m, and Figure (b) is same as Figure (a).

![Figure 3](a) Different initial imperfections of buckling mode on cylindrical shell.

Figure 3(a) shows that with stress wave propagation, $l_{cr}$ increase gradually, buckling will continue to grow. The number of buckling modes is increased, higher-order mode is excited, and peaks and troughs values are increased, and first half of the buckling wavelength decreased gradually. It also indicates that stress wave effect has a significant effect on dynamic buckling of the cylindrical shell.

As the number of lateral modes increased, initial imperfections have changes. As shown in Figure (b), great changes have taken place in buckling mode diagram. Compared (b) with (a), peaks and troughs values have drastically increased, and first half wavelength reduced substantially. The results show the initial imperfections have a prominent effect on the dynamic buckling of composite cylindrical shell.

Figure 4 expresses that dynamic buckling modes diagram of different impact mass when initial imperfections, impact speed and lay-up angle are constant.
Figure 4 shows that buckling continues to grow, the number of buckling modes is increased, and then higher-order mode is excited with $M$ increases. What’s more, peaks and troughs values are increased with the increased of $M$, and first half of the buckling wavelength decreased gradually. The changes of impact mass obtained from equation (9) leads to the changes of axial inertia, which has a notable effect on the dynamic buckling of composite cylindrical shell.

Figure 5 remarks that the buckling mode of different lay-up angles when initial imperfections, impact speed and impact mass are constant.

The results of figure 5 show that the peaks and troughs values are decreased with increase of the lay-up angles. The resistance buckling deformation of cylindrical shell subjected to axial impact load becoming better and better. And the resistance buckling deformation of cylindrical shell is best when the orthogonal symmetrical ply is used.

Figure 6 means that the buckling mode of different $\nu_0$ when the initial imperfections, impact mass and lay-up angle are constant.
Figure 6 shows that as impact speed increased, buckling continues to grow, the number of buckling modes is increased, and higher-order mode is excited. When $v_0$ increased, the peaks and troughs values gradually increased, and first half wavelength gradually decrease. They indicate that dynamic buckling of the shell has a significant effect. It also shows that impact speed has a definite effect on dynamic buckling of the cylindrical shell. The critical length decreased with increase of $v_0$, indicating that stress wave effect has a significant effect on dynamic buckling of the cylindrical shell.

5. Conclusions
(1) Considering stress wave effect, axial inertia and rotational inertia are calculated, and dynamic buckling control equation of cylindrical shell is obtained by Hamilton principle based on Donnell thin shell theory. The control equations are dimensionless which is solved by central difference and backward difference.

(2) Using Matlab to program differential equation of the control equations discuss initial imperfections, impact mass, lay-up angle and impact speed on dynamic buckling. The result shows impact quality, lay-up angle and impact velocity have a dramatic effect on dynamic buckling. It also reflects the significant effect of stress wave and axial inertia on composite dynamic buckling of composite cylindrical shell.

Acknowledgments
Thanks to the support from the National Natural Science Foundation of China (11372209).

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