I. INTRODUCTION

As one of the cornerstones of modern physics, the principle of invariance of the speed of light implies Lorentz symmetry. Experimentally, superluminal particles have not been found, which strongly indicates that Lorentz symmetry is well reserved in the lower energy region. Nevertheless, in the developments of theoretical physics and astrophysics, more and more models and theories require the breaking of Lorentz invariance (LI) in the higher energy region. For example, the beta decay theory of Fermi, and the Greisen-Zatsepin-Kuzmin cutoff in the study of high-energy cosmic rays. Furthermore, the most notorious difficulty in general relativity is its non-renormalizable because Einstein’s gravity satisfies LI. As a result, it is impossible to describe gravity systems at Plank scale by general relativity. The quantum cosmology and the details of the very early universe (for example, the details of inflation of very early universe), for the most part, are still unknown to us. In addition, some models of dark matter also require the violation of LI.

In order to construct a renormalizable gravity, Hořava proposed Hořava-Lifshitz (HL) gravity in 2009. In HL theory, the LI is broken, and the time derivative terms are not higher than second order, but the spatial derivatives contain up to 6th order terms and the renormalizability is achieved. However, the violation of LI only appears in the higher energy region, in the sense that the higher order spatial derivative terms are insignificant at the lower energy region, thus the HL theory is not in contradiction with today’s physical experiments. Since the advent of HL gravity, the theory is plagued by some difficulties, such as strong coupling, instability and ghost problems. So some new physical conditions, such as the detailed balance condition and new scalar fields $A$ and $\varphi$ with $U(1)$ symmetry, are introduced to handle the above difficulties. Our recent works have studied several versions of HL gravity which in principle avoid all the above difficulties, and it is shown that our model of HL passes the test of Post-Newtonian experiments. It’s worth noting that the work of indicated that the constant term of projectable HL theory might be a dark matter candidate.

Recently, it is shown that the HL theory becomes an effective Einstein-Æther theory at the lower energy region, and this conclusion may greatly simplify the study of the HL gravity. Einstein-Æther theory also violates the LI, where superluminal particles are allowed. Therefore, these modified gravitational theories face a question in black hole physics: Do black holes still exist in those theories without LI? According to the modified gravity without LI, the Killing horizon can not be used to define a black hole because superluminal particles inside of Killing event horizon could still travel through the surface, and therefore escape from inside the Killing horizon. So, does it mean that the singularity will be exposed? Fortunately, Einstein-Æther theory introduce a khronon scalar, $\phi$, which can play the role of time. In other words, $\phi$ can redefine physical time. It is further found out that Einstein-æther black holes have a universal horizon which can trap any particle with arbitrarily high velocity. In our recent work, we considered the khronon scalar $\phi$ and æther vector $u^\alpha$ (which is defined by $\phi$) as the test field in normal black hole spacetimes, and redefined time by $\phi$. We successfully prove that the universal horizon also exists for black holes of these gravitational theories.

It is understood that the investigation of Einstein-Æther black hole solutions are helpful to understand the

---

**Charged Einstein-Æther black holes in $n$-dimensional spacetime**

Kai Lin $^{a,b,c}$ and Fei-Hung Ho $^f$

$^a$ Institute for Advanced Physics & Mathematics, Zhejiang University of Technology, Hangzhou 310023, China

$^b$ Instituto de Física e Química, Universidade Federal de Itajubá, CEP 37500-903, Itajubá, Brazil

$^c$ Escola de Engenharia de Lorena, Universidade de São Paulo, CEP 12602-810, Lorena, SP, Brazil

(Dated: June 14, 2017)

In this work, we investigate the $n$-dimensional charged static black hole solutions in the Einstein-Æther theory. By taking the metric parameter $k$ to be 1, 0, and $-1$, we study the spherical, planar, and hyperbolic spacetimes respectively. Three choices of the cosmological constant, $\Lambda > 0$, $\Lambda = 0$ and $\Lambda < 0$, are investigated, which correspond to asymptotically de Sitter, flat and anti-de Sitter spacetimes. The obtained results demonstrate the existence of the universal horizon which traps any particle with arbitrarily large velocity. We analyze the horizon and the surface gravity of 4- and 5-dimensional black holes, and the relations between the above quantities and the electrical charge. It is shown that when the æther coefficient $c_{13}$ or the charge $Q$ increases, the outer Killing horizon shrinks and approaches the universal horizon. Furthermore, the surface gravity decreases and approaches zero in the limit $c_{13} \to \infty$ or $Q \to Q_c$, where $Q_c$ is the extreme charge.

PACS numbers: 04.50.Kd, 04.20.Jb, 04.70.Dy
properties of HL theory and other modified gravity. In this work, we study the static solutions of $n$ dimensional Einstein-Æther gravity coupling with Maxwell field and discuss the universal horizon and surface gravity of those black holes. In section II, we introduce the action of Einstein-Æther-Maxwell theory and discuss the spherical, planar and hyperbolic solutions in asymptotically flat and (anti-)de Sitter spacetimes. In section III, we investigate the relations between universal horizon, Killing horizon, surface gravity and electrical charge in asymptotically flat spacetime. The conclusions are given in Section IV.

II. CHARGED STATIC SOLUTIONS OF EINSTEIN-ÆTHER THEORY

According to the Einstein-Æther theory, the properties of gravitational field are determined by the spacetime metric $g_{\mu\nu}$ and æther vector $u^\alpha$. In $n$-dimensional spacetime, the general action of Einstein-Æther-Maxwell theory reads [9,11,16],

$$ S = \int d^n x \sqrt{-g} \left( R - 2\Lambda + \mathcal{L}_m - \alpha F_{\mu\nu} F^{\mu\nu} \right) (2.1) $$

and

$$ \mathcal{L}_m = c_1 (\nabla_\alpha u_\beta) (\nabla^\alpha u^\beta) + c_2 (\nabla^\gamma u_\gamma)^2 
+ c_3 (\nabla^\gamma u_\alpha) (\nabla^\alpha u^\gamma) - c_4 u^\alpha u^\beta (\nabla_\alpha u_\mu) (\nabla_\beta u^\mu) 
+ \lambda (u_\beta u^\beta + 1), \quad (2.2) $$

where $c_1, c_2, c_3$ and $c_4$ are the coupling constants in æther Lagrangian $\mathcal{L}_m$, and $\lambda$ is Lagrange multiplier, so that $u_\alpha$ satisfies the condition $u_\beta u^\beta + 1 = 0$. In fact, $u_\mu$ can be defined by the kchronon scalar field $\phi$ [13]:

$$ u_\mu = \frac{\partial_\mu \phi}{\sqrt{-g_{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi}} \quad (2.3) $$

where $\phi$ is a timelike scalar field, so this scalar can be used to redefine physical time. In [12], we have shown that the speed of the kchronon scalar is given by

$$ c_\phi^2 = \frac{c_{123}}{c_{14}}, \quad (2.4) $$

where $c_{14} \equiv c_1 + c_4$, $c_{123} \equiv c_1 + c_2 + c_3$, and it implies that the most important cases are $c_{123} = 0$ ($c_{\phi}^2 = 0$) and $c_{14} = 0$ ($c_{\phi}^2 \to \infty$).

According to above action, one may obtain the gravitational field equation, æther field equation and Maxwell equation, found in [10,11].

In order to investigate the charged static black hole solutions, we consider the ansatz of the solutions given by

$$ ds^2 = -F(r) dt^2 + 2 dr dv + r^2 d\Sigma_{n-2}^2, $$

$$ u^\alpha = \delta^\alpha_0 u^r (r) + \delta^\alpha_i V(r), $$

$$ \zeta^\alpha = \delta^\alpha_0, $$

$$ A_\alpha = \delta^\alpha_0 A_0 (r) \quad (2.5) $$

where $\zeta^\alpha$ is the Killing vector, and we can use the condition of $u^\alpha$ to obtain $u^\nu = (V + \sqrt{G})/F$ with $G = F + V^2$. The definition of $d\Sigma_{n-2}^2$ is given by

$$ d\Sigma_{n-2}^2 = \begin{cases} d\theta^2 + \sin^2 \theta d\Omega_{n-3}^2 & k = 1 \\
 dx_i dx^i & k = 0 \\
 d\theta^2 + \sinh^2 \theta d\Omega_{n-3}^2 & k = -1 \end{cases} $$

$k = 1, 0, -1$ corresponds sphere, planar and hyperboloid spacetimes respectively, and $d\Sigma_{n-2}^2$ is the spherical metric.

In [11], the exact solutions and their thermodynamics for $n = 3$ case was studied, so in what follows we will focus on $n \geq 4$ case. First of all, it is easy to find the solution of the Maxwell equations,

$$ A_0 (r) = A_c - \frac{Q r^{3-n}}{n-3}, \quad (2.6) $$

where $A_c$ and $Q$ are constant of integration in Maxwell equations. However, it is difficult to obtain the general solutions of the gravitational equation and æther equation, so we will discuss the case with $c_{14} = 0$ and $c_{123} = 0$ case respectively.

When $c_{14} = 0$ ($c_{\phi}^2 \to \infty$), we find the solutions are given by

$$ F(r) = F_a (r) = k - \frac{2 M_a}{r^{n-3}} + \frac{\bar{Q}^2}{2(n-2)(n-1)} - \frac{2 \Lambda - \rho_a B^2 r^2}{r^{2n-4}} - \frac{4 c_{13} B^2}{r^{2n-4}} $$

$$ V(r) = V_a (r) = 2 Br \left( \frac{r^{1-n}}{r^{1-n}} - r^{1-n} \right) \quad (2.7) $$

where

$$ \bar{Q}^2 = \frac{2 \alpha Q^2}{(n-3)(n-2)} $$

$$ c_{13} = c_1 + c_3 $$

$$ \rho_a = \frac{(n-1)(n-2)c_{13} - (n-1)c_{123}}{2(n-2)} \quad (2.8) $$

and $r_z, M_a$ and $B$ are constants. It is worth mentioning that $V(r_z) = 0$, so $u^\alpha \propto \zeta^\alpha$ at $r = r_z$. It is interesting to point out that when $n = 4, k = 1$, the above results can be reduced to the formula (5.1) in [10].

On the other hand, when $c_{123} = 0$ ($c_{\phi}^2 = 0$), it is convenient to calculate the solutions of $F(r)$ and $G(r)$, and $V(r)$ is determined by $G = F + V^2$. Therefore we obtain

1 There is a slight difference in notations from those in [9,11], the signs before the coefficients $c_i$, $(i = 1, 2, 3, 4)$ are inverted in the present work.
\[ F(r) = F_b(r) = k + \beta - \frac{2M_b}{r^{n-3}} + \frac{\bar{Q}^2 + \rho_b}{r^{2n-6}} - \frac{2\kappa^2}{(n-2)(n-1)(1+c_{13})} \]

\[ G(r) = G_b(r) = \frac{C_G}{r^{2n}} \left( r_{UH}^3 r^n - r_{UH}^n r^3 \right)^2 \]  

(2.9)

where

\[ \bar{Q}^2 = \frac{2\alpha Q^2}{(n-3)(n-2)(c_{13} + 1)} \]

\[ \rho_b = \frac{C_G(n-2)c_{13} - (n-3)c_{14}}{(n-2)(1+c_{13})r_{UH}^2} \]

\[ C_G = \frac{\beta + (\beta + k)c_{13}}{c_{13} r_{UH}^6} \]  

(2.10)

and \( r_{UH}, M_b \) and \( \beta \) are constants. It is noted that when \( n = 4, k = 1, \beta = 0 \), the above results reduce to the formula (5.1) in [10]. According to the physical content of the universal horizon, we find that \( r_{UH} \) is no other than the position of the universal horizon. What’s more, refs. [12,13] have shown that the universal horizon should be between outer Killing horizon and inner Killing horizon in charged spacetimes, and this condition requests that universal horizon must coincide with the Killing horizon at extreme black hole case, namely

\[ F(r_{UH}) = F'(r_{UH}) = 0. \]  

(2.11)

Therefore, we have the relation

\[ M_b = (k + \beta) r_{UH}^{n-3} - \frac{2\bar{Q}_{UH}^{n-1}}{(n-3)(n-1)(c_{13} + 1)}. \]  

(2.12)

In the next section, we will investigate horizons of the above spacetimes.

III. THE UNIVERSAL HORIZON AND SURFACE GRAVITY IN ASYMPOTICALLY FLAT SPACETIME

As explained before, it is understood that \( \phi \) is a time-like scalar, and therefore it can be used to redefine the physical time. The universal horizon traps any particle with arbitrarily high velocity [9,15], in the sense that the geodesics can not reach outside of universal horizon. It implies that black hole still exists in modified gravity with violation of Lorentz symmetry, and the Cosmic Censorship Hypothesis is still valid. In this section, we will study the universal horizon and compare it with the Killing horizon.

According to the definition of universal horizon \( r_{UH} \), which satisfies

\[ u_\alpha \xi^\alpha |_{r=r_{UH}} = 0 \]  

(3.1)

The ansatz of static spacetime gives

\[ G'(r_{UH}) = G(r_{UH}) = 0. \]  

(3.2)

and the surface gravity of universal horizon is [12,13]

\[ \kappa_{UH} \equiv \frac{u_\alpha}{2} \nabla_\alpha \left( u_\lambda \xi^\lambda \right) \bigg|_{r=r_{UH}} = \frac{V(r)}{2\sqrt{2}} \sqrt{G''(r)} \bigg|_{r=r_{UH}} \]  

(3.3)

On the other hand, the Killing horizon \( r_{KH} \) satisfies

\[ u_\alpha \xi^\alpha |_{r=r_{KH}} = 0, \]

\[ F(r_{KH}) = 0. \]  

(3.4)

and the surface gravity at Killing horizon is

\[ \kappa_{KH} \equiv \frac{F'(r)}{2} \bigg|_{r=r_{KH}}. \]  

(3.5)

For the \( c_{14} = 0 \) case, we can reduce 5 constants \( (r_0, r_{UH}, r_{KH}, Q \) and \( \Lambda \) to 4 independent parameters \( (r_U \equiv r_{UH}/r_0, r_k \equiv r_{KH}/r_0, Q^{1/(n-3)}/r_0 \) and \( \Lambda r_0^2) \) where \( r_0 = (2M_0)^{1/(n-3)} \), and \( r_0r_{KH} \) and \( r_0r_{UH} \) are the functions of the above new parameters.

Similarly, for the \( c_{123} = 0 \) case, we also find that the 4 constants \( (r_{UH}, r_{KH}, Q \) and \( \Lambda \) can be reduced to 3 independent parameters \( (r_{KH}/r_{UH}, Q^{1/(n-3)}/r_{UH} \) and \( \Lambda r_0^2) \), and \( r_{UH}\kappa_{KH} \) and \( r_{UH}\kappa_{UH} \) are the functions of above new parameters.

In what follows, we will study asymptotically flat spherical spacetime with \( k = 1 \) and \( \Lambda = 0 \). The cases for asymptotically (Anti-)de Sitter spherical spacetime and for asymptotically Anti-de Sitter planar spacetime are given in the Appendix.

For a black hole, the most important surface is the horizon, but the definition of horizon can be challenged in the gravitational theory without LI. As discussed above, recent works [17] show the universal horizon can be used to redefine black hole even if the gravity allows the superluminal particles. Therefore, we will apply the above conditions to investigate the universal horizon, Killing horizon and surface gravity.

In asymptotically flat spacetime, the observer is at infinite \( r \to \infty \). The physics in our world requires \( u^r \) to satisfy the condition \( u^r |_{r \to \infty} \propto \xi^\alpha |_{r \to \infty} \) in static spacetime it means

\[ V(r) |_{r \to \infty} = 0 \]  

(3.6)

so we choose \( r_2 \to \infty \) for the \( c_{14} = 0 \) case and \( \beta = 0 \) for the \( c_{123} = 0 \) case respectively.

We show the relations between the horizon and the surface gravity in Figs.1,2,3 and 4.

In Fig.1, three curves above \( r_U \) are the outer Killing horizons \( r_b \) and, the others are the inner Killing horizons \( r_i \). So that \( r_b, r_i \) and the universal horizon satisfy the relation \( r_i \leq r_{UH} \leq r_b \) and they coincide at extreme black hole case. Two Killing horizons approach the universal horizon as \( c_{13} \) increases, and it implies that universal
of extreme case, one has $F > Q$. It signifies that the Third Law of black hole thermodynamics might be absolutely stationary, so it is essential to investigate rotational black hole solutions, and we plan to carry out such study in the near future.

Similarly, for the $c_{123} = 0$ case with fixed $c_{14}$, from Fig.[3] we find that outer and inner horizon approach the universal horizon as $c_{13}$ increases, and it implies that the three horizons will coincide as $c_{13} \to \infty$.

The Fig.4 also show similar behaviour with the surface gravity of $c_{14} = 0$ case: the larger $c_{13}$ leads to smaller $k_{KH}$ and $k_{UH}$, and it is indicated that surface gravities vanish as $c_{13} \to \infty$.

IV. CONCLUSION

In this work, we investigated the black holes solutions of $n$-dimensional Einstein-Æther-Maxwell theory, and then analyzed the behaviour of the killing and the universal horizons. The present work generalized the studies of [9–11], which correspond to specific solutions of $\alpha$-dimensional Einstein-Æther-Maxwell theory. Our study reveals that universal horizon exists in the higher dimensional black hole spacetimes.

Moreover, it is interesting to find that, in the modified gravitational theories, black hole still has extreme case, which is defined by Eq. (2.11). And the universal horizon coincides with the killing horizon at such extreme condition so that the surface gravities $k_{UH}$ and $k_{KH}$ vanish. Mathematically, one has $F(r_{UH}) = G(r_{UH}) = 0$ at extreme case, and it requires $V(r_{UH}) = 0$, so $k_{UH} = V(r_{UH})\sqrt{G''(r_{UH})}/(2\sqrt{2}) = 0$. This result implies that the black hole’s temperature $T = k/2\pi$ equals 0 at such extreme case. If one can prove that “It is not possible to form a black hole with vanishing Temperature”, (or “vanishing surface gravity is not possible to achieve”), it signifies that the Third Law of black hole thermodynamics is true for the case of black holes with universal horizon. When the electrical charge $Q$ is larger than $Q_e$ of extreme case, one has $F > 0$, and $G = F + V^2 > 0$, so that the universal horizon vanishes, which would result in a Naked singularity. It is prohibited by the Cosmic Censor Conjecture.[23].

As a matter of fact, few real black holes in our universe might be absolutely stationary, so it is essential to investigate rotational black hole solutions, and we plan to carry out such study in the near future.

Acknowledgments

We are thankful for Prof. Anzhong Wang, Prof. Chikun Ding and Prof. Wei-Liang Qian for valuable discussions and insightful comments. This work is supported in part by Brazilian funding agencies CAPES, CNPq and FAPESP, and Chinese funding agency NNSFC under Contracts No. 11573022 and No. 11375279.

Appendix: Asymptotically (Anti-)de Sitter spherical spacetime and Anti-de Sitter planar spacetime

A. Asymptotically de Sitter spherical spacetime

$k = 1$ and $\Lambda > 0$

In asymptotically de Sitter spacetimes, the position $r \to \infty$ is a singularity, and nobody can stay at this point, so the condition $u^\alpha|r_{\to \infty} \propto \zeta^\alpha|r_{\to \infty}$ is not necessary. It means that we don’t need the conditions $r_z \to \infty$ for the $c_{14} = 0$ case and $\beta = 0$ for the $c_{123} = 0$ case.

In charged de Sitter spacetimes, black holes have 3 Killing horizon: outer Killing horizon $r_p$, inner Killing horizon $r_i$ and cosmological Killing horizon $r_C$, and the relation between the Killing horizon and the universal horizon is given by $r_i < r_{UH} < r_p < r_C$.

We show the relation between horizon and surface gravity with $r_z = 3r_0, 10r_0, \infty$ and $\beta = 0, 1, 10$ in Figs. [8].

B. Asymptotically Anti-de Sitter spherical spacetime

$k = 1$ and $\Lambda < 0$

Anti-de Sitter spacetime implies negative $\Lambda$, and this spacetime plays an important role in the research of modern theoretical physics, owing to the correspondence between Anti-de Sitter and conformal field theory (AdS/CFT correspondence). Therefore, it is meaningful to study the Anti-de Sitter black hole in Einstein-Æther theory.

In asymptotically Anti-de Sitter spherical spacetimes, because of the same reason for de Sitter case, we don’t need the conditions $r_z \to \infty$ at $c_{14} = 0$ case and $\beta = 0$ for the $c_{123} = 0$ case. We show the relation between horizon and surface gravity with $r_z = 3r_0, 10r_0, \infty$ and $\beta = 0, 1, 10$ in Figs. [11].

The results of de Sitter black hole and anti-de Sitter black hole show that the properties of at (anti-)de Sitter spacetimes are very similar to those of asymmetrically flat case. However, it is interesting that the electrical charge values of the extreme case are different as $\beta \neq 0$.

C. Asymptotically Anti-de Sitter planar spacetime

$k = 0$ and $\Lambda < 0$

Another well-known black hole solution is the Anti-de Sitter planar case, and it is very convenient to apply this spacetime to investigate holographical superconductor,
which is an application of AdS/CFT Correspondence\[13, 19–22\]. In this section, we investigate the planar black hole of Einstein-Æther theory.

In asymptotically Anti-de Sitter Planar spacetime, we show the relation between the horizon and surface gravity with $r_z = 3r_0, 10r_0, \infty$ and $\beta = 0, 1, 10$ in Figs.13-16.

\[1\] J. Bhattacharyya, Ph.D. thesis, University of New Hampshire, Durham, NH, 2013.
\[2\] S. M. Carroll and E. A. Lim, Phys. Rev. D 70, 123525 (2004).
\[3\] S. Mukohyama, Class. Quant. Grav. 27:223101 (2010).
\[4\] P. Horava, Phys. Rev. D 79, 084008 (2009).
\[5\] P. Horava and C. M. Melby-Thompson, Phys. Rev. D 82, 064027 (2010).
\[6\] A. Borzou, K. Lin, and A. Wang, J. Cosmol. Astropart. Phys. 02 (2012) 025; A. Borzou, K. Lin, and A. Wang, J. Cosmol. Astropart. Phys. 05 (2011) 006; K. Lin, A. Wang, Q. Wu, and T. Zhu, Phys. Rev. D 84, 044051 (2011); T. Zhu, Q. Wu, A. Wang, and F.-W. Shu, Phys. Rev. D 84, 101502(R) (2011).
\[7\] K. Lin, S. Mukohyama, A. Wang, and T. Zhu, Phys. Rev. D 89, 084022 (2014); K. Lin, S. Mukohyama, and A. Wang, Phys. Rev. D 86, 104024 (2012); K. Lin and A. Wang, Phys. Rev. D 87, 084041 (2013).
\[8\] D. Blas, O. Pujolas, and S. Sibiryakov, Phys. Lett. B 688, 350 (2010); J. High Energy Phys. 04 (2011) 018.
\[9\] P. Berghlund, J. Bhattacharyya and D. Mattingly, Phys. Rev. D 85, 124019 (2012).
\[10\] C. Ding, A. Wang and X. Wang, Phys. Rev. D 92, 084055 (2015).
\[11\] C. Ding, C. Liu, A. Wang and J. Jing, Phys. Rev. D 94, 124034 (2016).
\[12\] K. Lin, O. Goldoni, M.F. da Silva and A. Wang, Phys. Rev. D 91, 024047 (2015).
\[13\] K. Lin, E. Abdalla, R.-G. Cai, and A. Wang, Int. J. Mod. Phys. D 23, 1443004 (2014).
\[14\] K. Lin, V. H. Satheeshkumar and A. Wang, Phys. Rev. D 93, 124025 (2016).
\[15\] K. Lin, F.-W. Shu, A. Wang, and Q. Wu, Phys. Rev. D 91, 044003 (2015).
\[16\] J. D. Bekenstein, Phys. Rev. D 70, 083509 (2004), Erratum: Phys. Rev. D 71, 069901 (2005); T. G. Zlosnik, P. G. Ferreira, and G. D. Starkman, Phys. Rev. D 74, 044037 (2006); T. G. Zlosnik, P. G. Ferreira, and G. D. Starkman, Phys. Rev. D 75, 044017 (2007); L. Blanchet and S. Marsat, Phys. Rev. D 84, 044056 (2011); M. Bonetti and E. Barausse, Phys. Rev. D 91, 084053 (2015); A. B. Balakin and H. Dehnen, Phys. Lett. B 681, 113 (2009); J. Latta and G. Leon, arXiv:1606.08586; J. Bhattacharyya and D. Mattingly, Int. J. Mod. Phys. D 23, 1443005 (2014); D. Garfinkle, T. Jacobson, Phys. Rev. Lett. 107, 191102 (2011).
\[17\] D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011); E. Barausse, T. Jacobson, and T. P. Sotiropou, Phys. Rev. D 83, 124043 (2011); J. Bhattacharyya, A. Coates, M. Colombo, and T. P. Sotiropou, arXiv:1512.04899; J. Bhattacharyya, M. Colombo, and T. P. Sotiropou, arXiv:1509.01558; M. Tian, X. Wang, M. F. da Silva, and A. Wang, arXiv:1501.04134; P. Horava, A. Mohd, C. M. Melby-Thompson, and P. Shawhan, Gen. Relativ. Gravit. 46, 1720 (2014); T. Sotiropou, I. Vega, and D. Vernieri, Phys. Rev. D 90, 044046 (2014); C. Elbing and Y. Oz, J. High Energy Phys. 11 (2014) 067; M. Saravani, N. Afshordi, and R. B. Mann, Phys. Rev. D 89, 084029 (2014); A. Mohd, arXiv:1309.0907; B. Cropp, S. Liberati, and M. Visser, Class. Quant. Grav. 30, 125001 (2013).
\[18\] R. M. Wald, General Relativity (The University of Chicago Press, Chicago, 1984); D. Blas, O. Pujolas, and S. Sibiryakov, Phys. Lett. B 688, 350 (2010); J. High Energy Phys. 04 (2011) 018; T. Jacobson and Mattingly, Phys. Rev. D 64, 024028 (2001); T. Jacobson, Proc. Sci., QG-PH2007 (2007) 020; T. Jacobson, Phys. Rev. D 81, 101502(R) (2010); A. Wang, On “No-go theorem for slowly rotating black holes in Hořava-Lifshitz gravity”, arXiv:1212.1040.
\[19\] S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, Phys. Rev. Lett. 101, 031601 (2008); J. High Energy Phys. 12, 015 (2008).
\[20\] K. Lin, E. Abdalla, A. Wang, Int. J. Mod. Phys. D 24, 1550038 (2015).
\[21\] K. Lin, J. de Oliveira, E. Abdalla, Phys. Rev. D 90, 124071 (2014).
\[22\] K. Lin, E. Abdalla, Eur. Phys. J. C 74, 3144 (2014).
\[23\] R. Wald, General Relativity, 299-308 (1984).
FIG. 1: The relation between $r_U \equiv r_{UH}/r_0$, $r_k \equiv r_{KH}/r_0$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically flat spherical spacetimes with $c_{14} = 0$, where we have chosen $\alpha = 1$. The blue curves are for the case of $c_{13} = 0$, black for $c_{13} = 2$ and, orange for $c_{13} = 10$.

FIG. 2: The relation between $r_{0KH}$, $r_{0UH}$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically flat spherical spacetimes with $c_{14} = 0$, where we have chosen $\alpha = 1$.

FIG. 3: The relation between $r_{KH}/r_{UH}$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically flat spherical spacetimes with $c_{123} = 0$, where we have chosen $\alpha = 1$ and $c_{14} = 0.3$. The blue curves are for the case of $c_{13} = 1$, black for $c_{13} = 3$ and, orange for $c_{13} = 10$.

FIG. 4: The relation between $r_{UH}\kappa_{KH}$, $r_{UH}\kappa_{UH}$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically flat spherical spacetimes with $c_{123} = 0$, where we have chosen $\alpha = 1$ and $c_{14} = 0.3$.
FIG. 5: The relation between $r_{KH}/r_{UH}$, $r_{C}/r_0$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically de Sitter spherical spacetimes with $c_{14} = 0$, where we have chosen $\Lambda = 0.1/r_0^2$, $\alpha = 1$ and $c_{123} = 3$. 
FIG. 6: The relation between $r_0\kappa_{KH}$, $r_0\kappa_{UH}$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically de Sitter spherical spacetimes with $c_{14} = 0$, where we have chosen $\Lambda = 0.1/r_0^2$, $\alpha = 1$ and $c_{123} = 3$
FIG. 7: The relation between $r_{KH}/r_{UH}$, $r_{C}/r_{UH}$ and $Q^{1/(n-3)}/r_{UH}$ in 4 and 5 dimensional asymptotically de Sitter spherical spacetimes with $c_{123} = 0$, where we have chosen $\Lambda = 0.1/r_0^2$, $\alpha = 1$ and $c_{14} = 0.3$. 
FIG. 8: The relation between $r_{UH} \kappa_{KH}$, $r_{UH} \kappa_{UH}$ and $Q^{1/(n-3)}/r_{UH}$ in 4 and 5 dimensional asymptotically de Sitter spherical spacetimes with $c_{123} = 0$, where we have chosen $\Lambda = 0.1/r_0^2$, $\alpha = 1$ and $c_{14} = 0.3$.

FIG. 9: The relation between $r_U \equiv r_{UH}/r_0$, $r_K \equiv r_{KH}/r_0$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically Anti-de Sitter spherical spacetimes with $c_{14} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{123} = 3$. 
FIG. 10: The relation between $r_{0KH}$, $r_{0UH}$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically Anti-de Sitter spherical spacetimes with $c_{14} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{123} = 3$.

FIG. 11: The relation between $r_{KH}/r_{UH}$ and $Q^{1/(n-3)}/r_{UH}$ in 4 and 5 dimensional asymptotically Anti-de Sitter spherical spacetimes with $c_{123} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{14} = 0.3$.
FIG. 12: The relation between $r_{UH}/r_{KH}$ and $Q^{1/(n-3)}/r_{UH}$ in 4 and 5 dimensional asymptotically Anti-de Sitter spherical spacetimes with $c_{123} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{14} = 0.3$

FIG. 13: The relation between $r_U \equiv r_{UH}/r_0$, $r_k \equiv r_{KH}/r_0$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically Anti-de Sitter planar spacetimes with $c_{14} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{123} = 3$
FIG. 14: The relation between $r_0 \kappa_{KH}$, $r_0 \kappa_{UH}$ and $Q^{1/(n-3)}/r_0$ in 4 and 5 dimensional asymptotically Anti-de Sitter planar spacetimes with $c_{14} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{123} = 3$

FIG. 15: The relation between $r_{KH}/r_{UH}$ and $Q^{1/(n-3)}/r_{UH}$ in 4 and 5 dimensional asymptotically Anti-de Sitter planar spacetimes with $c_{123} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{14} = 0.3$
FIG. 16: The relation between $r_{UH \kappa H}$, $r_{UH \kappa UH}$ and $Q^{1/(n-3)}/r_{UH}$ in 4 and 5 dimensional asymptotically Anti-de Sitter planar spacetimes with $c_{123} = 0$, where we have chosen $\Lambda = -0.1/r_0^2$, $\alpha = 1$ and $c_{14} = 0.3$