Analysis of vibration isolation of a cargo mounted on polyurethane shock absorbers

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Abstract. The mathematical model of vibration isolation system of cargo, mounted in the container on tunnel-type polyurethane shock absorbers, is suggested. This type of shock absorbers allows working modes with finite deformations and has increased damping properties. The model considers nonlinearities of elastic and inelastic characteristics of shock absorbers and asymmetry of load curve. Coefficients of vibration isolation, which correspond to different container’s accelerations, are determined by harmonic and direct linearization methods of ODE of motion.

1. Introduction
It is very promising to use tunnel-type polyurethane shock absorbers (shown in figure 1(a)) in shock and vibration isolating systems. Shock absorbers of such type can work under finite deformations and have high dissipative properties [1]. The “displacement – force” relationship of these shock absorbers in the range of working mode are nonlinear (figure 1(b)). Shock absorbers with characteristics of this type provide high energy capacity of the vibration protection system with limited forces transmitted to the protected object.

![Image](https://example.com/image1)

**Figure 1.** Tunnel-type polyurethane shock absorber (a) and its “displacement – load” curve (b).
Operability justification of such cargo protection systems requires the thorough theoretical and experimental research, which leads to the necessity to take into account the nonlinearities of tunnel-type shock absorbers. The purpose of this work is to develop a mathematical model of a system for cargo vibroisolation system on polyurethane shock absorbers, taking into account their nonlinear properties under harmonic oscillations with large amplitudes.

2. An analytical description of the elastic and inelastic properties of shock absorbers

We choose an analytical representation of the restoring force $f$ as a function of the shock absorber displacement $\delta$. Approximately $f$ can be represented as a sum of the elastic $f_e(\delta)$ and inelastic $f_i(\delta, \dot{\delta})$ components:

$$f(\delta, \dot{\delta}) = f_e(\delta) + f_i(\delta, \dot{\delta}).$$

In accordance with the type of load curves of the shock absorber (figure 1), we assume that the elastic part of the restoring force has the form

$$f_e(\delta) = c_1 \delta - c_2 \delta^2 + c_3 \delta^3,$$

where $c_1$, $c_2$, $c_3$ — stiffness constants to be determined from results of static compression tests of a single shock absorber. In this work, static tests were carried out on a *Galdabini Quasar 50* testing machine with small-sized models used in [3]. Definition of elastic stiffness constants (1) was realized for dispositions within range of $\delta \leq 12$ mm using the least squares method in MATLAB. The obtained stiffness constants are $c_1 = 215$ N/mm, $c_2 = 34.7$ N/mm² and $c_3 = 2.16$ N/mm³, they correspond to the value of the coefficient of determination $R^2 = 0.968$.

The inelastic part of the restoring force is characterized by hysteretic damping properties of the polyurethane. It is known that the area of elastomers hysteresis loop depends on the frequency of deformation cycles (in the low frequency range) and more on the amplitude of deformation. Following the suggestion of Ya.G. Panovko [4], we assume that for a harmonic deformation cycle, the shape of the hysteresis loop is elliptical, i.e.

$$f_i(\delta, \dot{\delta}) = B \delta_0'' \text{sign} \dot{\delta} \left(1 - \frac{\delta^2}{\delta_0^2}\right)^{1/2},$$

where $\delta_0$ — amplitude of shock absorber displacement, $B$, $\mu$ — damping constants. We evaluate them with results of experiments on cargo vibration isolation [3], preserving the equivalence of the physical and mathematical models with respect to the energy absorbing parameters. For the damping model (2), the energy dissipated during the period of harmonic oscillations with a circular frequency $\omega$ has the form

$$W = \int_0^{2\pi/\omega} f_i(\delta, \dot{\delta}) \dot{\delta} \, dt = \pi B \delta_0'' \omega^{\mu - 1}.$$

The absorption coefficient during oscillations of a system with effective stiffness $c_{eff}$ at a resonant frequency $\nu_0$ with a maximum value of the vibration isolation coefficient $\beta_{max} = \max(a_{out})/a_{in}$ can be estimated as

$$\psi = \frac{2W}{c_{eff} \delta_{max}^2} = \frac{2\pi B \delta_{max}^{\mu - 1}}{c_{eff}} \approx \frac{2\pi B}{c_{eff}} \left(\frac{\beta_{max} a_{in}}{4\pi^2 \nu_0^2}\right)^{\mu - 1},$$

where the resonant amplitude of the shock absorber displacement $\delta_{max}$ is approximately determined through the acceleration $a_{out}$ of the vibrating stand table and cargo acceleration $a_{in}$. Since the properties of the physical model [3] were determined only under the resonance mode ($a_{in} = 0.5g$, $\nu_0 = 11.1$ Hz,
c_{eff} = 195 \text{ N/mm}, \quad \beta_{\text{max}} = 6.2, \quad \psi = 0.63), \text{ within the conditions of limited experimental data the selection of parameters } B \text{ and } \mu \text{ has to be done with some additional assumptions. Let us consider } \mu = 3 \text{ and obtain } B = 0.5 \text{ N/mm}^3 \text{ by the known absorption coefficient (3). Further calculations were performed with these values of damping parameters.}

3. Dynamics of cargo with longitudinal (vertical) installation

The system (figure 2(a)) is in the field of gravity, \( y \) — cargo displacement, \( z \) — container displacement. The lower shock absorber is preloaded by the weight of the cargo. The upper shock absorber is brought into contact with the cargo, but is not deformed and does not create an additional reaction. The peculiarity of this nonlinear oscillatory system is that the initial preload can significantly change its stiffness. Figure 2(b) shows the shift of the load curves of the shock absorbers (dashed curves) in the presence of the initial preload of the lower shock absorber.

![Figure 2](image)

Figure 2. The system of isolation of cargo (a) and “displacement – load” curves (b) of the lower (1) and upper (2) shock absorbers (dashed curves) and the curves of the restoring force without taking into account the weight of the cargo (3) and taking it into account (4).

The load curve of the system as a whole becomes asymmetric when the shock absorber displacement \( \delta = y - z \) is measured from the position of static equilibrium. The curve region \( \delta > 0 \) corresponds to the compression of the lower shock absorber, region \( -\delta_a < \delta < 0 \) corresponds to the collaboration of two shock absorbers. On the region \( \delta < -\delta_a \) there is a loss of contact of the cargo with the lower shock absorber.

Accepting the assumption that there is no influence of static preload on the amount of energy dissipated during oscillations and assuming that the process is close to harmonic, we will use a linearized expression of the internal friction force \( f_i(\delta) = b_{eq} \dot{\delta} \) with a viscosity coefficient determined by the harmonic linearization method [2]. Assuming \( \delta = \delta_0 \cos \omega t \), we find \( b_{eq} \) for model (2) by the formula

\[ b_{eq} = \frac{1}{\pi \omega \delta_0^2} \int_{0}^{2\pi} f_i(\delta_0, \cos \xi, -\omega \delta_0 \sin \xi) \sin \xi \, d\xi = \frac{B \delta_0^{\mu-1}}{\omega}. \]

Then the equation of cargo motion will take the form

\[ m\ddot{\delta} + \frac{B \delta_0^{\mu-1}}{\omega} \dot{\delta} + f(\delta) = -m\ddot{z}, \]
where \( f(\delta) = H(\delta_u^s + \delta)f_1(\delta_u^s + \delta) - H(-\delta)f_2(-\delta) - mg \) — restoring force of the system (curve 4 on figure 2), \( H(\delta) \) — Heaviside step function, \( f_1(\delta) = c_1\delta - c_2\delta^2 + c_3\delta^3 \) — reaction of lower shock absorber, \( f_2(-\delta) = -(c_1\delta + c_2\delta^2 + c_3\delta^3) \) — reaction of upper shock absorber.

Considering the harmonic excitation of the system \( z = z_0\cos(\omega t + \varphi) \), we will find steady-state oscillations \( \delta = \Delta + \delta_0 \cos \omega t \) by the method of direct linearization of the restoring force function [5]. First, we determine the shift \( \Delta \) from the system’s potential energy equality for amplitude positions

\[
\int_{\Delta = \delta_0}^{\Delta + \delta_0} f(\delta) d\delta = 0. \tag{4}
\]

Nonlinear equation in \( \Delta \) (4) is solved with predetermined values of the shock absorber amplitude. The dependence of the shift, referred to the static preload displacement of the shock absorber \( \delta_0 \), on the amplitude divided by \( \delta_0 \) is shown in figure 3. A positive sign of \( \Delta \) at small amplitudes indicates the movement of the center of oscillations towards the flat region of the system “displacement – load” curve (figure 2, curve 4).

Further, using the direct linearization method with a weight function, we find the resonant frequency of the system

\[
\omega^2_0(\delta_0) = \frac{5}{2m\delta_0^3} \int_{-\delta_0}^{\delta_0} f(\delta - \Delta)\delta^2 d\delta, \tag{5}
\]

calculating the integral (5) by the Gaussian quadrature formulae. With a known backbone curve according to the equation of frequency response (FR) characteristic

\[
\omega^2 = \omega^2_0(\delta_0) \pm \left[ \frac{a_{in}}{\delta_0} \right]^2 - \left( \frac{B\delta_m^{\mu - 1}}{m} \right)^2, \]

frequencies \( \omega \) corresponding to the given values of \( \delta_0 \) are determined. Then the cargo coefficient of vibration isolation

\[
\beta = \frac{a_{out}}{a_{in}} = \left[ \frac{\omega^2_0(\delta_0) + \left( B\delta_m^{\mu - 1}/m \right)^2}{\left( \omega^2_0(\delta_0) - \omega^2 \right)^2 + \left( B\delta_m^{\mu - 1}/m \right)^2} \right]^{1/2}
\]

is calculated.

A rational calculation procedure is completely described in the monograph [2]. Without delving into the details of this numerical algorithm, we present the results in the form of curves of the cargo

\[
\Delta/\delta_{st}
\]

\[
\begin{align*}
&\Delta/\delta_{st} = \begin{cases} 0.2, & 1 \leq \delta_0/\delta_{st} \leq 2, \\
&0, & 2 < \delta_0/\delta_{st} < 3, \\
&-0.2, & 3 \leq \delta_0/\delta_{st} \leq 5. \\
&0, & \delta_0/\delta_{st} > 5,
\end{cases}
\end{align*}
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.png}
\caption{The dependence of oscillation shift on the amplitude of oscillations.}
\end{figure}
vibroisolation coefficient for various values of container acceleration amplitude 0.5g; 0.75g; 1.0g; 1.25g (figure 4). Note that the calculations were performed with load factors \( a_m / g \leq 1.25 \) implemented on a Data Physics Signal Force V400 vibrating stand with a cargo weight of 40 kg for the purpose of further experimental verification.

Figure 4. Cargo vibroisolation coefficients as a functions of frequency which correspond to different amplitudes of container’s accelerations: 1 – 0.5g; 2 – 0.75g; 3 – 1.0g; 4 – 1.25g.

4. Conclusions
The proposed mathematical model of the vibroisolation system takes into account the nonlinearity of the restoring forces of tunnel-type polyurethane shock absorbers and the asymmetry of load curve. Model parameters are identified according to the results of static and dynamic tests. Cargo vibration isolation coefficients under harmonic excitation with different amplitudes of the container acceleration are calculated. The developed model can be used in practical calculations of vibration isolation systems of objects.

References
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