An Improvement for Quantum Tunneling Radiation of Fermions in a Stationary Kerr-Newman Black Hole Spacetime

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ABSTRACT: By introducing a specific etheric-like vector in the Dirac equation with Lorentz Invariance Violation (LIV) in the curved spacetime, an improved method for quantum tunneling radiation of fermions is proposed. As an example, we apply this new method to a charged axisymmetric Kerr-Newman black hole. Firstly, considering LIV theory, we derive a modified dynamical equation of fermion with spin 1/2 in the Kerr-Newman black hole spacetime. Then we solve the equation and find the increase or decrease of black hole’s Hawking temperature and entropy are related to constants $a$ and $c$ of the Dirac equation with LIV in the curved spacetime. As $c$ is positive, the new Hawking temperature is about $\sqrt{\frac{1+2a+2cr^2}{1+2a}}$ times higher than that without modification, but the entropy will decrease. We also make a brief discussion for the case of high spin fermions.
1 Introduction

Black hole is a mysterious cosmic body with extremely intense gravity. With the detection of LIGO and Virgo, more and more activities of black holes are found, so current theoretical researches for black holes become more and more significant. Black holes can be divided into three types: static black hole, stationary black hole and dynamic black hole.

Hawking firstly proved that black holes have thermal radiation in theory by studying the quantum effect near the horizon of black holes \[1, 2\]. Hawking radiation effectively links gravitational theory, quantum theory and thermodynamic statistics physics, and inspired other researchers to study the thermodynamical evolution of black holes \[3–6\]. Parikh et al. pointed that the behavior of Hawking radiation can be regarded as a quantum tunneling \[7, 8\]. In their assumption, the event horizon of black hole is a potential barrier, virtual particles yielded inside the horizon have a certain probability to escape from this barrier and be converted to real particles radiating out from the black hole. Refs.\[9–17\] adopted the quantum tunneling method to investigate Hawking radiation for different types of black holes. Srinivasan et al. derived the Hamilton-Jacobi equation in curved spacetime from a scalar field equation \[11, 12\]. Kerner and Mann et al. studied the tunneling radiation of Dirac particles using a semi-classical theory \[18–20\]. Lin and Yang proposed a new method to study the quantum tunneling radiation of fermions \[21–24\]. Their method can also be used to study the quantum tunneling radiation of bosons. The results obtained in Refs.\[21–24\] show that the Hamilton-Jacobi equation in curved spacetime is a basic equation of particle dynamic, which reflects the inherent consistency between Lorentz symmetry theory and the Hamilton-Jacobi equation. Recently, we considered a light dispersion relationship derived from string theory to research the modified quantum tunneling rates for spherical symmetry and axisymmetric black holes\[25, 26\].

General relativity is a gravitational theory that cannot be renormalized, so several modified gravitational theories have been proposed. Since the Lorentz Invariance Violation (LIV) may exist at high energy cases, various gravity models based on LIV have been
proposed [27–29]. In principle, LIV theory can solve the problem of renormalization of gravitational theory. In addition, some studies on LIV suggest that the dark matter may be just one of the effects of LIV theoretical models [30]. In string theory, electrodynamics and non-abel theory, LIV has attracted extensive attention [31–33]. In the recent years, Dirac equation with LIV term in the flat spacetime has been studied by introducing etheric-like field terms [34, 35]. In this theory, Lorentz symmetry disappears due to the existence of the ether-like field. Therefore, some special properties which are inconsistent with Lorentz symmetry theory will emerge at the high energy case. This is an interesting topic which needs to be explored in depth. On the other hand, the dynamics of fermions with LIV in the curved spacetime is also an attractive subject worthy further study, which will influence the correction to quantum tunneling radiation of black hole. At present, the quantum tunneling radiation of Dirac particles with etheric-like field terms has been investigated in spherically symmetric black holes [36]. We investigate the influence of different etheric-like vector $u^\alpha$ on the solution of the modified Hamilton-Jacobi equation by using both the semi-classical approximation and beyond the semi-classical approximation [37].

In this paper, the quantum tunneling radiation of fermions is modified in the axisymmetric charged Kerr-Newman black hole by considering a specific ether-like field vector term. Our paper is organized as follows: In Sec.2, considering LIV theory, the dynamical equation of fermions with spin 1/2 is derived for Kerr-Newman black hole. In Sec.3, we solve this dynamical equation and obtain the corrected physical quantities such as Hawking temperature and tunneling rate of the black hole. We make some discussions in Sec.4.

## 2 Lorentz Invariance Violation Theory and Dirac-Hamilton-Jacobi Equation

In the Ref. [35], Nascimento et al. researched the particle’s action which includes LIV in the flat spacetime. Transferring the normal derivative to covariant derivative, and extending the commutation relation of gamma matrices $\bar{\gamma}^{\mu}$ and $\bar{\gamma}^{\nu}$ in the flat spacetime to that in the curved spacetime, we obtain Dirac equation of fermion, spin of which is 1/2, with LIV in the curved spacetime as:

$$\left\{ \gamma^{\mu} D_{\mu} [1 + \frac{\alpha}{m^2} (\gamma^{\mu} D_{\mu})^2] + \frac{b}{h} \gamma^5 + c h (u^\alpha D_\alpha)^2 - \frac{m}{h} \right\} \Psi = 0, \quad (2.1)$$

where $m$ is the mass of fermion, $a$, $b$, and $c$ is small constants. Gamma matrices $\gamma^\mu$ satisfy the condition:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} I, \quad (2.2)$$

$$\gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0, \quad (2.3)$$

where $g^{\mu\nu}$ is the inverse metric tensor, and $I$ is the unit matrix. In the flat spacetime, Eq.(2.2) reduces to $\bar{\gamma}^{\mu} \bar{\gamma}^{\nu} + \bar{\gamma}^{\nu} \bar{\gamma}^{\mu} = 2 \delta^{\mu\nu} I$, and Eq.(2.3) changes to $\bar{\gamma}^5 \bar{\gamma}^{\mu} + \bar{\gamma}^{\mu} \bar{\gamma}^5 = 0$. In Eq.(2.1)

$$D_{\mu} = \partial_{\mu} + \frac{i}{2} \Gamma^{\alpha\beta}_{\mu} \pi_{\alpha\beta} - \frac{i}{h} q A_{\mu}, \quad (2.4)$$
where \( q \) is the charge of fermion, and \( A_\mu \) is the electromagnetic potential of black hole. The second term at the right side of Eq.(2.4) is spin connection which is a very small term in the dynamical equation and thus can be ignored. \( u^\alpha \) is an etheric-like vector, which satisfies:

\[
u^\alpha u_\alpha = \text{const.} \tag{2.5}\]

In order to solve the Dirac equation for fermions with spin 1/2, we assume its wave function is

\[
\Psi = \psi_A e^{i\pi S} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\pi S}, \tag{2.6}
\]

where \( A \) and \( B \) are matrix elements in the column matrix, \( S \) is the Hamilton principal function. Substituting Eq.(2.4) and Eq.(2.6) into Eq.(2.1), we get

\[
\{i\gamma^\mu(\partial_\mu S - qA_\mu)[1 - \frac{q}{m^2}\gamma^\alpha\gamma^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta)]
\]

\[-cu^\alpha u^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta) + br^5 - m}\} \Psi = 0. \tag{2.7}

Using Eq.(2.2), we have

\[
\gamma^\alpha\gamma^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta) = g^{\alpha\beta}(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta). \tag{2.8}
\]

Combining Eq.(2.8) and Eq.(2.7), they yields

\[
i\gamma^\mu(\partial_\mu S - qA_\mu)\Psi = [1 - \frac{q}{m^2}\gamma^\alpha\gamma^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta)]^{-1}
\]

\[-cu^\alpha u^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta) + br^5 + m}\} \Psi
\]

\[= [1 + \frac{q}{m^2}\gamma^\alpha\gamma^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta) + O(a^2)]
\]

\[-cu^\alpha u^\beta(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta) + br^5 + m}\} \Psi
\]

\[= [1 + \frac{5}{m^2} + \frac{m}{m^2}g^{\alpha\beta}(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta)
\]

\[-\frac{b}{m^5}m\Psi. \tag{2.9}\]

Since \( b < m \), so \( \frac{b}{m}r^5 \) is very small. Multiplying \( i\gamma^\nu(\partial_\nu S - qA_\nu) \) at both sides of Eq.(2.9), we get

\[
-i\gamma^\nu(\partial_\nu S - qA_\nu)(\partial_\nu S - qA_\nu)\Psi = m^2 + 2cmu^\alpha u^\beta + ag^{\alpha\beta}
\]

\[(\partial_\alpha S - qA_\alpha)(\partial_\beta S - qA_\beta)\Psi + O, \tag{2.10}\]

i. e.,

\[
[g^{\mu\nu}(\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu)
\]

\[+ 2cmu^\mu u^\nu + ag^{\mu\nu})(\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu) + m^2]\} \Psi = 0. \tag{2.11}\]

This is a matrix equation. The condition for this matrix equation to have nontrivial solutions is that the value of determinant of the wave function’s coefficient matrix is zero, i. e.,

\[
(g^{\mu\nu} + 2cmu^\mu u^\nu + 2ag^{\mu\nu})(\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu) + m^2 = 0, \tag{2.12}\]

\[-3-\]
or

\[ (g^{\mu\nu} + \frac{2cmu^\mu u^\nu}{1 + 2a})(\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu) + m^2/(1 + 2a) = 0. \]  

(2.13)

Considering \( a \) is a very small constant and adopting Taylor expansion for the last term in the Eq.(2.13), we get

\[ (g^{\mu\nu} + \frac{2cmu^\mu u^\nu}{1 + 2a})(\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu) + m^2/(1 + 2a) = 0. \]  

(2.14)

From Eq.(2.1) to Eq.(2.14), we get a new dynamical equation for Dirac particles. Comparing with the normal Hamilton-Jacobi equation, one can find that, as \( 2cmu^\mu u^\nu = a = 0 \), Eq.(2.14) returns to the normal Hamilton-Jacobi equation. We call this deformed equation as Dirac-Hamilton-Jacobi equation. The choice of the Hamilton principal function \( S \) depends on the selected coordinate and line element. In a stationary spacetime, it can be generally expressed as \( S = S(t, r, \theta, \phi) \).

3 Correction to Tunneling radiation of Fermions in the Kerr-Newman Spacetime

In the Boyer-Lindquist coordinate, the line element of Kerr-Newman black hole is written as [38]

\[ ds^2 = \rho^2 \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \frac{\sin^2 \theta}{\rho^2} \left[ (r^2 + a^2_k)n d\varphi - a_k n dt \right]^2 - \frac{\Delta}{\rho^2} \left[ dt - a_k n \sin^2 \theta d\varphi \right]^2, \]  

(3.1)

where

\[ \Delta = r^2 + a^2_k - 2Mr + Q^2, \]

\[ \rho^2 = r^2 + a^2_k \cos^2 \theta, \]  

(3.2)

and inverse metric tensors

\[ g^{tt} = \frac{1}{\rho^2} (a^2_k \sin^2 \theta - \Delta), \]

\[ g^{rr} = \frac{\Delta}{\rho^2}, \]

\[ g^{\theta\theta} = \rho^2, \]

\[ g^{\varphi\varphi} = \frac{\sin^2 \theta}{\rho^2} \left[ (r^2 + a^2_k) - \Delta a^2_k \sin^2 \theta \right], \]

\[ g^{t\varphi} = \frac{a_k n \sin^2 \theta}{\rho^2} (Q^2 - 2Mr), \]  

(3.3)

and inverse metric tensors

\[ g^{tt} = \frac{1}{\rho^2} \left( a^2_k \sin^2 \theta - \frac{r^2 + a^2_k}{\Delta} \right), \]

\[ g^{rr} = \frac{\Delta}{\rho^2}, \]

\[ g^{\theta\theta} = \frac{1}{\rho^2}, \]

\[ g^{\varphi\varphi} = \frac{1}{\rho^2} \left( \frac{1}{\sin^2 \theta} - \frac{a^2_k}{\Delta} \right), \]

\[ g^{t\varphi} = \frac{1}{\rho^2} \left( \frac{Q^2 - 2Mr}{\Delta} \right). \]  

(3.4)
According to null super-surface equation

$$g^{\mu\nu} \frac{\partial f}{\partial x^\mu} \frac{\partial f}{\partial x^\nu} = 0,$$

(3.5)

the event horizon of black hole \( r_H \) satisfies following equation

$$\Delta|_{r=r_H} = r_H^2 - 2Mr_H + a_k^2 + Q^2 = 0.$$  

(3.6)

The electromagnetic potential of the Kerr-Newman black hole

\[
A_{\mu} = (A_t, 0, 0, A_\varphi),
\]

\[
A_t = -\frac{Qr}{\rho^2},
\]

\[
A_\varphi = \frac{ka_n Q r \sin^2 \theta}{\rho^2}.
\]

(3.7)

Substituting Eq. (3.3) and (3.4) into Eq. (2.14), we get the dynamical equation of spin 1/2 fermions with mass \( m \) and charge \( q \) in the curved spacetime,

\[
g^{tt}(\partial_t S - q A_t)^2 + 2g^{r\varphi}(\partial_r S - q A_t)(\partial_{\varphi} S - q A_\varphi) + g^{\varphi\varphi}(\partial_\varphi S - q A_\varphi)^2
\]

\[
+ g^{\theta\varphi}(\partial_\theta S - q A_\theta)(\partial_{\varphi} S - q A_\varphi) + \rho^2 (1 - 2a)^2 + \rho^2 m^2 (1 - 2a) \cos^2 \theta + \frac{2mc}{1 + 2a} (u^t u^t (\frac{\partial S}{\partial t} - q A_t) + u^t u^\varphi (\frac{\partial S}{\partial \varphi} - q A_\varphi) + u^\theta u^\varphi (\frac{\partial S}{\partial \theta} - q A_\varphi) + 2u^t (\partial_t S - q A_t) (\partial_t S - q A_\varphi) + 2u^\varphi (\partial_\varphi S - q A_\varphi) + 2u^\theta (\partial_\theta S - q A_\theta)).
\]

(3.8)

After substituting the components of \( g^{\mu\nu} \) into Eq. (3.8), multiplying \( \rho^2 \) on both sides of the equation and merging the similar terms, the dynamical equation are as follows:

\[
\Delta (\frac{\partial S}{\partial t}) - \frac{1}{\Delta}[(r^2 + a_k^2)(\frac{\partial S}{\partial t}) + a_k (\frac{\partial S}{\partial \varphi}) + e Q r]^2 + r^2 m^2 (1 - 2a) + (\frac{\partial S}{\partial t})^2
\]

\[
+ (\sin \theta \frac{\partial S}{\partial \varphi})^2 + a_k \sin \theta (\frac{\partial S}{\partial \varphi})^2 + \rho^2 m^2 (1 - 2a) \cos^2 \theta + \frac{2mc}{1 + 2a} (u^t u^t (\frac{\partial S}{\partial t} - q A_t) + u^t u^\varphi (\frac{\partial S}{\partial \varphi} - q A_\varphi) + u^\theta u^\varphi (\frac{\partial S}{\partial \theta} - q A_\varphi) + 2u^t (\partial_t S - q A_t) (\partial_t S - q A_\varphi) + 2u^\varphi (\partial_\varphi S - q A_\varphi) + 2u^\theta (\partial_\theta S - q A_\theta)).
\]

(3.9)

To solve the above equation, we must choose special \( u^t, u^\varphi, u^\theta \) and \( u^\varphi \) which must satisfy Eq. (2.5). According to Eqs. (3.1)-(3.4) and the metric tensor of Kerr-Newman black hole, we choose the following \( u^\alpha \):

\[
u^t = \frac{k_3}{(\alpha_{kn}^2 \sin^2 \theta - \Delta)^{1/2}},
\]

\[
u^r = \frac{k_4}{\Delta^{1/2}},
\]

\[
u^\theta = \frac{k_5}{\rho},
\]

\[
u^\varphi = \frac{k_6}{\sin \theta \alpha_{kn}(Q^2 - 2Mr)}.
\]

(3.10)
where \( k_t, k_r, k_\theta, k_\phi \) are constants, which satisfies

\[
u^\alpha u_\alpha = k_\alpha^2,
\]

where \( \alpha \) denotes \( t, r, \theta, \phi \). Substituting Eq.(3.10) and Eq.(3.6) into Eq.(3.9), and considering the limit at the event horizon of black hole, we can get the dynamical equation of spin 1/2 fermions at the event horizon of black hole, that is,

\[
\Delta^2 |_{r \rightarrow r_H} \left( 1 + \frac{2cmk_r^2}{1 + 2a} \right) \left( \frac{\partial S}{\partial r} \right)^2 |_{r \rightarrow r_H} - \left[ (r_H^2 + a^2) \frac{\partial S}{\partial t} + a_k \frac{\partial S}{\partial \varphi} + qQr_H \right]^2 = 0. \tag{3.12}
\]

Because the quantum tunneling radiation of a black hole is a property of the radial direction of the black hole, we care the radial component of the Hamilton principal function. From Eq.(3.12),

\[
\frac{\partial S}{\partial r} |_{r \rightarrow r_H} = \pm \frac{(r_H^2 + a^2)\sqrt{1 + 2a}}{\Delta |_{r \rightarrow r_H} \sqrt{1 + 2a + 2cmk_r^2}} \left( \frac{\partial S}{\partial t} + \frac{a_k \frac{\partial S}{\partial \varphi} + qQr_H}{r_H^2 + a^2} \right). \tag{3.13}
\]

According to Eq.(3.1), we set

\[
S = -\omega t + R(r) + \Theta(\theta) + j\varphi, \tag{3.14}
\]

where \( \omega \) is particle energy, \( j \) is a constant which describes the \( \varphi \) component of general momentum. Eq.(3.13) can be reducible to

\[
\frac{\partial S}{\partial r} |_{r \rightarrow r_H} = \frac{dR}{dr} |_{r \rightarrow r_H} = \pm \frac{(r_H^2 + a^2)\sqrt{1 + 2a}}{\Delta |_{r \rightarrow r_H} \sqrt{1 + 2a + 2cmk_r^2}} (\omega - \omega_0), \tag{3.15}
\]

where

\[
\omega_0 = \frac{qQr_H + a_k j}{r_H^2 + a^2}, \tag{3.16}
\]

where \( \omega \) is the particle energy, \( \omega_0 \) is the chemical potential, which means the minimal energy of emission particles. Integrating the above equation from the inner side to the outer side of \( r_H \) with the residue theorem, we obtain

\[
S_\pm = R_\pm = \pm \int dr \frac{(r_H^2 + a^2)\sqrt{1 + 2a}}{\Delta |_{r \rightarrow r_H} \sqrt{1 + 2a + 2cmk_r^2}} (\omega - \omega_0),
\]

\[
= \pm i \frac{\pi}{2} \frac{(r_H^2 + a^2)\sqrt{1 + 2a}}{(r_H - M)\sqrt{1 + 2a + 2cmk_r^2}} (\omega - \omega_0), \tag{3.17}
\]

where subscript + and - denote outgoing and incoming module, respectively. According to the tunneling theory of black hole, the tunneling rate is

\[
\Gamma = \exp[-2(\text{Im}S_+ - \text{Im}S_-)] = \exp(-\frac{\omega - \omega_0}{T_H'}), \tag{3.18}
\]

where \( T_H' \) is Hawking temperature after modification.
\[ T'_H = \frac{(r_H - M)(1 + 2a + 2c m k^2)}{2\pi (r_H^2 + a^2 kn)}^{1/2} \]
\[ = T_{KNh}(1 + 2a + 2c m k^2)^{1/2}/\sqrt{1 + 2a}, \quad (3.19) \]

where \( T_{KNh} \) is the Hawking temperature without LIV modification,
\[ T_{KNh} = \frac{(r_H - M)}{2\pi (r_H^2 + a^2 kn)}. \quad (3.20) \]

4 Summery and Discussions

In this paper, we have modified the dynamical equation of Dirac particles in the curved spacetime, considering LIV theory. Comparing Eq.(1) and Eq.(2.14), we simplify the complicate derivation process. By solving Eq.(2.14), the new Hamilton principal function \( S \) of Dirac particle is obtained. Since the quantum tunneling rate and Hawking temperature at the horizon of black hole depend on the imaginary part of \( S \), so the new quantities related to Hawking radiation are obtained naturally. These results are valuable for further study of LIV theory and quantum gravitational theory. Because LIV theory modifies the Hawking temperature at the event horizon of Kerr-Newman black hole, it will lead to the correction of black hole entropy. According to the first law of thermodynamics of black hole,
\[ dM = T dS + V dJ + U dQ, \quad (4.1) \]

where \( V \) and \( U \) are the rotation potential and electric potential of black hole respectively. From Eq.(3.19) and Eq.(3.20), the entropy with LIV theory correction in the Kerr-Newman black hole is
\[ S_H = \int dS_H = \int \frac{dM - V dJ - U dQ}{T'_H} \]
\[ = \int \frac{\sqrt{1 + 2a}}{\sqrt{1 + 2a + 2c m k^2}} dS_{KNh}, \quad (4.2) \]

where \( S_{KNh} \) denotes the entropy without modification,
\[ dS_{KNh} = \frac{dM - V dJ - U dQ}{T_{KNh}}. \quad (4.3) \]

It can be seen from Eq.(3.19) and Eq.(4.2) that the increase or decrease of black hole’s Hawking temperature and entropy are related to the value of constants \( a \) and \( c \) in Eq.(2.1). As \( c \) is positive, the new Hawking temperature is about \( \frac{\sqrt{1 + 2a + 2c m k^2}}{\sqrt{1 + 2a}} \) times higher than that without modification, but the entropy will decrease. Above results are from semi-classical approximation and based on spin 3/2 fermions. Beyond semi-classical approximation can refer our recent paper [37]. For spin 3/2 fermions, the wave function in Eq.(2.1) should be substituted by
\[ \Psi = \begin{pmatrix} A_V \\ B_V \end{pmatrix} e^{iS}, \quad (4.4) \]
where $A_V = [A \ B]^T$, $B_V = [A \ B][24]$. For arbitrary spin fermions, $\Psi$ in Eq.(2.1) should be substituted by $\Psi_{\alpha_1...\alpha_k}$, where the value of $\alpha_k$ corresponds to different spin. The larger $\alpha_k$, the higher the spin is. Furthermore, the above conclusions are also applicable to the other spherically symmetric and axisymmetric charged black holes. We will do further research at this aspect in the future.

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