Pseudo Nambu-Goldstone Dark Matter: Examples of Vanishing Direct Detection Cross Section

DIMITRIOS KARAMITROS∗
National Centre for Nuclear Research, ul. Hoża 69, 00-681 Warsaw, Poland

February 6, 2019

Abstract
We consider cases where the dark matter-nucleon interaction is naturally suppressed. We explicitly show that by extending the standard model scalar sector by a number of singlets, can lead to a vanishing direct detection cross section, if some softly broken symmetries are imposed in the dark sector. In particular, it is shown that if said symmetries are $SU(2)$ ($SU(N)$) and $U(1) \times S_N$, then the resulting pseudo-Nambu-Goldstone bosons can constitute the dark matter of the Universe, while naturally explaining the missing signal in nuclear recoil experiments.

1 Introduction
The current status of direct detection experiments reduces the allowed number of dark matter (DM) models with DM particle masses around the electroweak (EW) scale (typically $\mathcal{O}$(GeV) – $\mathcal{O}$(TeV)), as indicated by recent results from the XENON1T collaboration [1]. The main reason for this is the incompatibility of the experimental results with what one would expect from dimensional arguments (i.e. the so-called WIMP miracle [2]), indicating that a DM particle with mass around the EW scale, its interactions should have an EW strength. That is, if the dark matter particle has a mass accessible to direct detection experiments, why its interaction rate with nucleons is not within the detectable range?

There are various ways that the missing direct detection signal can be explained, e.g. via suppressed interaction with the nucleons due to a heavy (integrated-out) mediator [3, 4, 5, 6, 7, 8], the appearance of “blind spots” [9, 10, 11] or the smallness of the DM mass [12, 13, 14, 15, 16, 17, 18] which makes the DM particle inaccessible to such experiments. [1] Among the most appealing scenarios, however, direct detection experiments are unable to detect the WIMP due to symmetry arguments [25, 26, 27, 28, 29]. In such models there is a symmetry that is responsible for the suppression of the DM-nucleon cross-section, usually through the cancellation of the tree-level DM-nucleon interaction.

In the present work, we focus on the pseudo-Nambu-Goldstone boson (PNGB) DM scenario, which can explain the lack of direct detection signal. The general idea is that Nambu-Goldstone bosons (NGB), which result from a spontaneous breaking of a global symmetry, have derivative couplings with other particles, and so their interactions vanish at zero momentum. On the other hand, a PNGB (a DM cannot be NGB, since it should be massive) is a result of a spontaneously broken approximate global symmetry, which could induce new interactions resulting to a non-vanishing direct detection cross section. However, there are examples of a cancellation that allows the tree-level DM-nucleon interaction to vanish at the zero momentum transfer [27, 29], making models featuring such cancellation suitable DM candidates. Furthermore, since such models belong to the family of Higgs portal DM

∗email: dimitrios.karamitros@ncbj.gov.pl

However, this could change soon as the efforts for detection of light DM intensifies [19, 20, 21, 22, 23, 24].
(e.g. [30, 31, 32, 33, 34]), this expands the ongoing search for simple models that can explain the DM content of the Universe while respecting experimental constraints.

In our effort to identify PNGB models featuring the aforementioned cancellation, we extend the SM by a scalar field (singlet under the SM gauge symmetry) and doublet under a softly broken $SU(2)$ global symmetry. We also show that the PNGBs in this case remain stable due to the symmetry properties of the interaction terms. Furthermore, we show how these arguments apply to a softly broken $SU(N)$ global symmetry.

Then, we move to another case, where we add two scalar fields (again singlet under the SM), and we note that the cancellation of PNGB-nucleon interaction occurs assuming a permutation symmetry. However, in contrast to the minimal case [27], the PNGB is not naturally stable unless a dark CP-symmetry is imposed. We also show that this model can be generalized to an arbitrary number ($N$) of scalar fields, provided an $S_N$ symmetry assumption.

The outline of the paper is the following: in section 2, we discuss the DM content and the natural suppression of the DM-nucleon cross section in the $SU(2)$. At the end of this section, we also show how these results are generalized in the $SU(N)$ case. In sec. 3 we consider the $U(1) \times S_2$ case, and show how the cancellation of the direct detection cross section takes place, which we then generalize to $U(1) \times S_N$. Finally, in section 4 we summarize our results, and comment on possible future directions.

2 The $SU(2)$ case

In this section examine a dark sector with a softly broken $SU(2)$ symmetry, in order to determine if the cancellation takes place. Specifically, the SM is extended by a scalar ($\Phi$) which is a gauge singlet under the SM gauge group, and a doublet under a softly broken $SU(2)$. We show that indeed this model can provide us with naturally stable (multi-component) DM, which exhibits a cancellation of the DM-nucleon interaction. We also show that this holds for $SU(N)$ and $\Phi$ in the fundamental representation.

The potential and mass terms

The potential is comprised of two parts, the symmetric and the soft breaking ones. The symmetric part (global $SU(2)$ invariant) is

$$V_0 = -\frac{\mu^2}{2}|H|^2 + \frac{\lambda}{2}|H|^4 + \frac{\lambda_{\Phi}}{2}|\Phi|^4 + \lambda_{H\Phi}|H|^2|\Phi|^2,$$

while the softly breaking part of the potential can be written as

$$V_{\text{soft}} = \sum_{i=1}^{2} \sum_{j=1}^{2} \left[ m^2_{\Phi,ij} \Phi_i \Phi_j + \text{h.c.} \right] + m^2_{\Phi} \Phi_i \Phi_j^\dagger,$$

with $m^2_{\Phi,12} = m^2_{\Phi,21}$, $m^2_{\Phi,12} = (m^2_{\Phi,21})^*$, and $m^2_{\Phi} \in \mathbb{R}$. Also, note that the potential, $V = V_0 + V_{\text{soft}}$, becomes $SU(2)$-invariant if $m_{\Phi} = m_{\Phi,12} = 0$ and $m^2_{\Phi} = m^2_{\Phi,22}$. Assuming that both $H$ and $\Phi$ develop VEVs,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v \end{pmatrix}, \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi + is \\ \rho + i\chi + v\phi \end{pmatrix},$$

2 Similar models have been studied in great detail [35], however we focus on the cancellation of the DM-nucleon cross section and show explicitly that this takes place regardless of the form of the soft breaking terms.

3 There is a python module available [github.com/dkaramit/pseudo-Goldstone_DM] that can be used to obtain Feynman rules and LanHEP [36] input files for the $SU(N)$ case.
where, without loss of generality we have assumed that the lower component of $\Phi$ obtains a non-zero VEV. By minimizing the potential, we obtain the following relations

$$m_{\Phi}^2 = \sum_i m_{\Phi i}^2 \in \mathbb{R}$$

$$\mu_H^2 = \lambda_H v^2 + \lambda_H v^2$$

$$m_{\Phi 12} = \frac{-m_{\Phi 21}^2}{2}$$

$$\lambda_\Phi = -\frac{1}{v_\Phi^2} \left[ \lambda_H v^2 + 4m_{\Phi 22}^2 + 2m_{\Phi 22}^2 \right] , \quad (2.4)$$

The Lagrangian mass terms can be written as

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \left( G^T M_G^2 G + S^T M_S^2 S \right) , \quad (2.5)$$

where $G = (\chi, s, \phi)^T$ are the PNGBs and $S = (h, \rho)^T$. The mass matrices become

$$M_G^2 = \begin{pmatrix} -4m_{\Phi 22}^2 & 2\Re(m_{\Phi 12}^2) & -2\Re(m_{\Phi 11}^2) \\ 2\Re(m_{\Phi 12}^2) & -2m_{\Phi 22}^2 + m_{\Phi 11}^2 - m_{\Phi 11}^2 & -2\Re(m_{\Phi 11}^2) \\ -2\Re(m_{\Phi 12}^2) & -2\Re(m_{\Phi 11}^2) & -2m_{\Phi 22}^2 + m_{\Phi 11}^2 - m_{\Phi 11}^2 \end{pmatrix}$$

$$M_S^2 = \begin{pmatrix} \lambda_H v^2 & \lambda_H v v_\Phi & \lambda_H v v_\Phi \\ \lambda_H v v_\Phi & \lambda_H v v_\Phi & \lambda_H v v_\Phi \end{pmatrix} , \quad (2.6)$$

with $\lambda_\Phi$ given by eq. (2.4). It is also evident that, as expected, $M_G^2$ becomes a zero matrix (i.e. the pseudo-Nambu-Goldstone bosons become massless) in the limit of SU(2) invariance.

### Stability of PNGBs

It is straightforward to show that the PNGBs in the mass eigenstate basis are stable, by simply writing down the interaction terms. The reason for the stability comes from the fact that $SU(2)$ is explicitly broken by dimension-2 terms, and so the interaction terms are still $SU(2)$ invariant. After SSB, the interaction terms involving the PNGBs become $O(3)$ invariant. Therefore, the diagonalization of their mass matrix, does not induce any non-diagonal three-point interactions, e.g. only $\xi_i^2 h$ three-point interactions (with $\xi_i$ the PNGBs in the mass eigenstate basis). This, in turn, means that the Lagrangian, is $Z_2^{(\xi_1)} \times Z_2^{(\xi_2)} \times Z_2^{(\xi_3)}$ symmetric (i.e. each PNGB carries its own $Z_2$ parity) that not only forbids decays of the PNGBs, but also PNGB conversions as well. Thus, all pseudo-Nambu-Goldstone bosons are stable, resulting to a three-component DM content.

### The pseudo-Nambu-Goldstone–nucleon interaction

Since all PNGBs are stable, we need to calculate three amplitudes for the direct detection cross section. However, due to the $O(3)$ symmetry of the interaction terms, the amplitude for the $\xi_i$-nucleon elastic scattering ($\xi_i n \rightarrow \xi_i n$) is proportional to $G_i$-nucleon elastic scattering amplitude and it is independent of $i$. In general, the interaction of the three-point terms pertinent to this interaction can be written as

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^2 Y_{ij}^{(k)} G_i G_j S_k . \quad (2.7)$$

But due to the $O(3)$ symmetry, we expect that

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} \left( G_1^2 + G_2^2 + G_3^2 \right) \sum_{k=1}^2 Y^{(k)} S_k = -\frac{1}{2} \left( \xi_1^2 + \xi_2^2 + \xi_3^2 \right) \sum_{k=1}^2 Y^{(k)} S_k . \quad (2.8)$$
From the potential \( V_0 \) and the relations \( M^2 \), we obtain

\[
L_{\text{int}} = -\frac{1}{2} \left( \xi_1^2 + \xi_2^2 + \xi_3^2 \right) \left( \begin{array}{c} \lambda_H \Phi v \\ \frac{1}{v} \left( \lambda_H \Phi v^2 + 4m_{\Phi}^2 + 2m'_{\Phi} \right) \end{array} \right)^T \left( \begin{array}{c} h \\ \rho \end{array} \right).
\]

Since we are interested in the zero-momentum transfer limit, the propagator is proportional to the inverse of the mass-matrix \( M^2 \). Then the direct detection amplitude for all PNGBs (the Feynman diagram is shown in Fig. 1) becomes

\[
A_{DD} \sim \left( \begin{array}{c} -\lambda_H v \Phi v \\ \lambda_H \Phi v^2 + 4m_{\Phi}^2 + 2m'_{\Phi} \end{array} \right)^T \left( \begin{array}{c} \lambda_H \Phi v^2 + 4m_{\Phi}^2 + 2m'_{\Phi} \lambda_H \Phi v^2 \\ -\lambda_H v^2 \end{array} \right) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) = 0,
\]

which concludes the proof of the claim that the DM-nucleon cross section vanishes at tree-level and zero momentum transfer. However, this only indicates that the direct detection cross section is “naturally suppressed”. In practice, loop corrections need to be included as well, since these effects could allow for a possible direct detection signal \[27, 37, 38\].

**Generalization to \( SU(N) \)**

It is straightforward to generalize the above result in the case where \( \Phi \) is in the fundamental representation of a softly broken \( SU(N) \) global symmetry, since the form of \( V_0 \) is the same as in eq. (2.1), with the soft breaking terms being

\[
V_{\text{soft}} = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ m_{\Phi}^2 \Phi_i \Phi_j + \text{h.c.} + m'_{\Phi} \Phi_i^* \Phi_j \right],
\]

where in analogy to \( SU(2) \) \( m_{\Phi} \) and \( m'_{\Phi} \). Assuming that the \( N \)th component of \( \Phi \) develops a VEV, one can show that the minimization of the potential requires

\[
m_{\Phi}^2 \in \mathcal{R}
\]

\[
\mu_H^2 = \lambda_H v^2 + \lambda_H v^2
\]

\[
m_{\Phi}^2 \in \mathcal{R}
\]

\[
\lambda_{\Phi} = -\frac{1}{v^2} \left[ \lambda_H v^2 + 4m_{\Phi}^2 + 2m'_{\Phi} \right],
\]

This results to \( 2N - 1 \) PNGBs, \( \chi, \phi_i \) and \( s_i \) with \( i = 1, 2, \ldots, N - 1 \), where, similarly to the \( SU(2) \) case, all pseudo-Nambu-Goldstone bosons are stable particles. The Lagrangian pertinent to the pseudo-Nambu-Goldstone boson–nucleon interaction takes the familiar form
3 The $U(1) \times S_2$ case

In this section, we examine another case, which we denote as $U(1) \times S_2$. In this case, the SM is extended by two scalars ($S_{1,2}$) charged only under a softly broken global $U(1)$. For the desired cancellation to occur, we impose a permutation symmetry on $S_{1,2}$. As we will see, this symmetry provides a sufficient condition for the vanishing of the PGB-nucleon cross section.

3.1 The cancellation mechanism for this model

The Potential

In the case of two scalars, each transforming as $S_i \rightarrow e^{-i\alpha}S_i$, the $U(1) \times S_2$ symmetric potential, assuming that all parameters are real numbers (we shall call this assumption dark CP–invariance), is

$$V_0 = -\frac{\mu_0^2}{2} |H|^2 + \frac{\lambda_H}{2} |H|^4 + \lambda_{HS1} |H|^2 \left( |S_1|^2 + |S_2|^2 \right) + \lambda_{HS2} |H|^2 \left( S_1 S_2^\dagger + \text{h.c.} \right)$$

$$- \frac{\mu_S^2}{2} (|S_1|^2 + |S_2|^2) - \frac{\mu_{S2}^2}{2} \left( S_1 S_2^\dagger + \text{h.c.} \right) + \frac{\lambda_S}{2} \left( |S_1|^4 + |S_2|^4 \right) + \frac{\lambda_{S2}}{2} \left( (S_1 S_2^\dagger)^2 + \text{h.c.} \right)$$

while the $S_2$-symmetric soft breaking potential is written as

$$V_{\text{soft}} = -\frac{\mu_{S2}^2}{2} \left( S_1^2 + S_2^2 + \text{h.c.} \right) - \mu_{S2}^2 \left( S_1 S_2 + \text{h.c.} \right) .$$

with the total potential given by $V = V_0 + V_{\text{soft}}$. In order to find the minimization conditions, we expand the fields around their VEVs

$$S_{1,2} = \frac{1}{\sqrt{2}} (v_S + s_{1,2} + i \chi_{1,2}) ,$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} ,$$

$$V = \frac{1}{2} \sum_{i=1}^{2N-1} \xi_i^2 \left( \frac{1}{v} (\lambda_H \Phi v^2 + 4m_{\Phi_2}^2 + 2m_{\Phi_2}^2) \right)^T \left( \frac{h}{\rho} \right) ,$$

Since the mass matrix $M_2^2$ is independent of $N$ (i.e. it is always given by eq. (2.6)), the amplitude for the process $\xi_i N \rightarrow \xi_i N$ at tree-level and zero momentum transfer, vanishes as in the $SU(2)$ case.

One should keep in mind that the cancellation takes place only if $\Phi$ is in the fundamental representation of $SU(N)$. It is not clear if $A_{\text{DD}}$ would cancel if another (irreducible) representation of $\Phi$ was assumed, as there are additional interactions, corresponding to all the possible contractions of the $SU(N)$ indices. For example, for $N = 2$ and $\Phi$ in the adjoint representation, there is an interaction term of the form

$$V_{\text{int}} \sim |H|^2 \sum_{i,j,k,l} \epsilon^{ij} \epsilon^{kl} \Phi_{ij} \Phi_{kl},$$

which can potentially change the mixing between the particles in a non-trivial way. Since the number of such interactions increases greatly with the dimension of each representation of $SU(N)$, it becomes hard to generalize. Thus, we postpone such analysis for the future.⁴

---

⁴ However, if the imposed symmetry is $SU(N) \times U(1)$, such interactions are not allowed, which means the mechanism under consideration holds for other representations as well.
where this particular choice of \( \langle S_{1,2} \rangle \), ensures that the potential remains symmetric under simultaneous permutations of \((s, \chi)_1 \leftrightarrow (s, \chi)_2\). Due to the permutation symmetry, there are only two independent stationary point conditions, which read

\[
\begin{align*}
\mu_H^2 &= \lambda_H v^2 + 2v_S^2 (\lambda_{HS_1} + \lambda_{HS_2}) , \\
\mu_{S_1}^2 &= \lambda v^2 (\lambda_{HS_1} + \lambda_{HS_2}) - \left[ \mu_{S_2}^2 + 2 \left( \mu_{S_1}^2 + \mu_{S_2}^2 \right) \right] + v_S^2 (\lambda_{S_1} + \lambda_{S_2} + \lambda_{S} + 4c) .
\end{align*}
\]  

Spectrum of the CP–odd scalars

In order to calculate the direct detection amplitude, we first need to identify the PNGB. This can be done by diagonalizing the mass matrix of the CP–odd fields to its eigenvalues. Once the eigenvalues are found, one of them should vanish in the limit where the \( U(1) \) is restored, which should correspond to the PNGB. From eq. (3.5), we obtain the mass matrix for the \( \chi's \)

\[
M_\chi^2 = \begin{pmatrix}
-\frac{v^2 \lambda_{HS_2}}{2} & -\frac{v_S^2 (\lambda_{S_2} + c)}{2} \pm \frac{\mu_{S_1}^2 + \mu_{S_2}^2 + \mu_{S_2}^2}{2} & v_S^2 (\lambda_{S_2} + c) + \mu_{S_1}^2 - \frac{\mu_{S_2}^2}{2} \\
-\frac{v_S^2 (\lambda_{S_2} + c)}{2} & -\frac{v^2 \lambda_{HS_2}}{2} & -v^2 (\lambda_{S_2} + c) + 2 \mu_{S_1}^2 + \mu_{S_2}^2 + \frac{\mu_{S_2}^2}{2}
\end{pmatrix},
\]

from which we find the eigenvalues

\[
\begin{align*}
m_{\xi_1}^2 &= 2(\mu_{S_1}^2 + \mu_{S_2}^2) \\
m_{\xi_2}^2 &= 2\mu_{S_1}^2 + \mu_{S_2}^2 - v^2 \lambda_{HS_2} - 2v_S^2 (\lambda_{S_2} + c) .
\end{align*}
\]  

It is apparent that \( m_{\xi_1}^2 \) vanishes in the limit \( \mu_{S,1,2} \to 0 \), thus the particle corresponding to this mass can be identified as the would-be Nambu-Goldstone boson of the \( U(1) \), i.e. the PNGB of this model. The eigenstates corresponding to these masses are

\[
\xi_1 = \frac{1}{\sqrt{2}}(\chi_1 + \chi_2) , \quad \xi_2 = \frac{1}{\sqrt{2}}(\chi_1 - \chi_2) .
\]  

It is worth noting that the PNGB \( (\xi_1) \) is symmetric under \( \chi_1 \leftrightarrow \chi_2 \). This property of the PNGB, will be proven helpful especially in the \( N \)-particle generalization of this model, since it will allow us to calculate the desired direct detection amplitude easily. The imposed dark CP–invariance can potentially keep both of the states \( \xi_{1,2} \) stable, since there are only interactions involving even numbers of CP–odd particles, e.g. there is no \( \xi_1 h^2 \) interaction term while the vertex \( \xi_1^2 h \) exists. However, since we are interested in the scenario where the DM particle is a PNGB, we need to impose an extra hierarchy condition, so that \( \xi_1 \) will be stable while \( \xi_2 \) will be able to decay. This condition is \( m_{\xi_1} < m_{\xi_2} \), with their difference \( (m_{\xi_2} - m_{\xi_1}) \) at least larger than the mass of the lightest CP–even particle (e.g. \( m_{\xi_2} - m_{\xi_1} > m_H \approx 125 \text{GeV} \) if the Higgs boson is the lightest one). This is not too restrictive, and it does not affect the vanishing of the PNGB-nucleon cross section, but it must be pointed out for the sake of completeness.

The direct detection amplitude

The calculation of the quark-\( \xi_1 \) scattering amplitude is a relatively straightforward task. We just need to calculate the corresponding Feynman diagram (fig. [1]). In fact, since we are interested in the zero momentum transfer limit, the ingredients that we need in order to show that the direct detection cross section vanishes, are the inverse of the mass matrix of the CP–even scalars and the three-point interaction of a pair of PNGBs with them (i.e. vertices of the form \( \xi_1^2 h \) and \( \xi_1^2 s_{1,2} \)). The mass terms for the CP–even scalars can be written in a compact form as

\[
\mathcal{L}_{\text{hs}} = -\frac{1}{2} \Phi^T M_\Phi^2 \Phi ,
\]  

6
with $\Phi = (h, s_1, s_2)^T$ and

$$M_\Phi^2 = \begin{pmatrix} v^2\lambda_H & v_{VS}(\lambda_{HS_1} + \lambda_{HS_2}) & v_{VS}(\lambda_{HS_1} + \lambda_{HS_2}) \\ v_{VS}(\lambda_{HS_1} + \lambda_{HS_2}) & 2v^2(c+\lambda_{s_1})+\mu_2^2+2\mu_3^2-\lambda_{HS_2}v^2 & 2v^2(3c+\lambda_{s_2}+\lambda_{s_1}'+1)v\lambda_{HS_2}^\prime-\mu_2^2-2\mu_3^2 \\ v_{VS}(\lambda_{HS_1} + \lambda_{HS_2}) & 2v^2(3c+\lambda_{s_2}+\lambda_{s_1}'+1)v\lambda_{HS_2}^\prime-\mu_2^2-2\mu_3^2 & 2v^2(c+\lambda_{s_1})+\mu_2^2+2\mu_3^2-\lambda_{HS_2}v^2 \end{pmatrix}$$

Observing that only $h$ couples to SM fermions, we only need the following few terms of the inverse of $M_\Phi^2$

$$[M_\Phi^2]^{-1}_{11} \sim v_S^2 \left( \lambda_{HS_1} + \lambda_{HS_2} + \lambda_{HS_1}' + 4c \right)$$
$$[M_\Phi^2]^{-1}_{12} \sim -v_{VS}(\lambda_{HS_1} + \lambda_{HS_2}).$$

With the interaction term of the Lagrangian terms responsible for the $\xi_1$-nucleon elastic scattering being $^3$.

$$\mathcal{L}_{\text{int}} = -\frac{1}{8} \xi_1^2 \begin{pmatrix} v_S \left( \lambda_{HS_1} + \lambda_{HS_2} + \lambda_{HS_1}' + 4c \right) \\ v_S \left( \lambda_{HS_1} + \lambda_{HS_2} + \lambda_{HS_1}' + 4c \right) \end{pmatrix}^T \begin{pmatrix} h \\ s_1 \\ s_2 \end{pmatrix},$$

we can show that the amplitude for the $\xi_1$-nucleon elastic scattering vanishes. That is

$$A_{DD} \sim \begin{pmatrix} 2v_S \left( \lambda_{HS_1} + \lambda_{HS_2} \right) \\ v_S \left( \lambda_{HS_1} + \lambda_{HS_2} + \lambda_{HS_1}' + 4c \right) \end{pmatrix}^T \begin{pmatrix} v_S^2 \left( \lambda_{HS_1} + \lambda_{HS_2} + \lambda_{HS_1}' + 4c \right) \\ -v_{VS}(\lambda_{HS_1} + \lambda_{HS_2}) \end{pmatrix} = 0.$$

3.2 Generalization to $U(1) \times S_N$

As we saw in sec. 3.1, the cancellation mechanism holds when the model consists of two scalars under the assumption that the potential is symmetric under permutations of these scalars. This symmetry fixes the PNGB-$s_{1,2}$ interactions and the relevant components of $M_\Phi^2$ in such way that $A_{DD}$ vanishes. However, there is no guarantee that this also happens if we add more scalars, since more interaction terms are allowed. In this section, we investigate whether $A_{DD}$ vanishes in a model consisting of an arbitrary number of scalars. We denote this model as $U(1) \times S_N$, and it is a direct generalization of $U(1) \times S_2$ with $N$ number of scalars.

The Potential for $N$ Scalars

In the case of $N$ scalar fields, each transforming as $S_i \to e^{-i\theta}S_i$ (similarly to sec. 3.1), the $U(1) \times S_N$ symmetric potential, assuming again dark CP–invariance, can be written as

$$V_0 = -\frac{\mu_2^2}{2} |H|^2 + \frac{\lambda_2^2}{2} |H|^4 - \sum_{i,j} \frac{\mu_{ij}^2}{2} S_i S_j^\dagger + \sum_{i,j} \frac{\lambda_{ij}^2}{2} (S_i S_j^\dagger)^2 + \sum_{i,j} \frac{\lambda_{ij}^2}{2} |S_i|^2 |S_j|^2$$

$$+ \sum_{i,j,k} c_{ijk} |S_i S_j S_k^\dagger|^2 + \sum_{i,j,k} c_{ijk}' (S_i S_j S_k^\dagger)^2 + \text{h.c.} + \sum_{i,j,k,l} d_{ijkl} S_i S_j S_k S_l^\dagger$$

$$+ \sum_{i,j} \lambda_{HS_1} |H|^2 S_i S_j^\dagger,$$

$^3$Note that since $\xi_1$ is an $S_2$ symmetric state, a pair of $\xi_1$ interacts in the same way with both $s_1$ and $s_2.$
where all the sums run over all scalars. This potential has some redundant terms, so we can set some of them to zero:

\[
\begin{align*}
\lambda_{Sii}^2 &= 0 \\
c_{iik} &= 0 \\
c'_{iij} &= c'_{ijj} = 0 \\
d_{ijjk} &= d_{ijjk} = d_{ijkk} = 0.
\end{align*}
\] (3.14)

Furthermore, the permutation symmetry, dictates:

\[
\begin{align*}
\mu_{Sij}^2 &= \begin{cases}
\mu_{S1}^2 & (i = j) \\
\mu_{S2}^2 & (i \neq j)
\end{cases} \\
\lambda_{Sij} &= \begin{cases}
\lambda_{S1} & (i = j) \\
\lambda_{S2} & (i \neq j)
\end{cases} \\
\lambda_{HSij} &= \begin{cases}
\lambda_{HS1} & (i = j) \\
\lambda_{HS2} & (i \neq j)
\end{cases} \\
\lambda'_{Sij} &= \lambda'_S (i \neq j)
\end{align*}
\] (3.15)

\[
\begin{align*}
c_{iji} &= c_{jjk} = c_1 & (i \neq j) \\
c_{ijk} &= c_2 & (i \neq j \neq k) \\
c'_{ijk} &= c'_i & (i \neq j \neq k) \\
d_{ijkl} &= d & (i \neq j \neq k \neq l).
\end{align*}
\]

As previously, we assume soft breaking of $U(1)$. That is, we add the following terms in the potential

\[
V_{\text{soft}} = -\sum_{i,j} \frac{\mu_{Sij}^2}{2} S_i S_j + \text{h.c.},
\] (3.16)

where, due to the $S_N$ symmetry, we have

\[
\mu_{Sij}^2 = \begin{cases}
\mu_{S1}^2 & (i = j) \\
\mu_{S2}^2 & (i \neq j)
\end{cases}
\] (3.17)

So, from eqs. (3.14), (3.15), and (3.17), the total potential becomes

\[
\begin{align*}
V &= -\frac{\mu_{H}^2}{2} |H|^2 + \frac{\lambda_{H}^2}{2} |H|^4 + \lambda_{HSi} \sum_i |H|^2 |S_i|^2 + \lambda_{HS2} \sum_{i \neq j} |H|^2 S_i S_j^\dagger \\
&\quad -\frac{\mu_{S1}^2}{2} \sum_i |S_i|^2 + \frac{\mu_{S2}^2}{2} \sum_{i \neq j} S_i S_j^\dagger + \frac{\lambda_{S1}}{2} \sum_i |S_i|^4 + \frac{\lambda_{S2}}{2} \sum_{i \neq j} (S_i S_j^\dagger)^2 \\
&\quad + c_1 \sum_{i \neq j} (S_i S_j S_j^\dagger S_i S_i^\dagger + S_i S_j^\dagger S_i^\dagger S_j^\dagger) + c_2 \sum_{i \neq j} S_i S_j^\dagger S_k |S_k|^2 \\
&\quad + \lambda'_S \sum_{j \neq i} |S_i|^2 |S_j|^2 + c'_i \sum_{i \neq j \neq k} (S_i S_j S_k^\dagger + \text{h.c.}) + d \sum_{i \neq j \neq k \neq l} S_i S_j S_k^\dagger S_l^\dagger \\
&\quad - \frac{\mu_{S1}^2}{2} \sum_i (S_i^2 + \text{h.c.}) - \mu_{S2}^2 \sum_i (S_i S_j + \text{h.c.}).
\end{align*}
\] (3.18)

At this point, it becomes clear that the $S_N$ symmetry helps keeping the number of new free parameters relatively small. This keeps the model as simple as possible, considering the potential large number of particles.

\footnote{There are 10, 12, and 13 free parameters for $N = 2$, $N = 3$, and $N \geq 4$, respectively.}
Similar to the previous, the scalars acquire VEVs

\[ S_i = \frac{1}{\sqrt{2}} (v_S + s_i + i \chi_i) , \]
\[ H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} , \]  

(3.19)

where, again, we have assumed that the potential remains symmetric under \((s, \chi)_i \leftrightarrow (s, \chi)_j\) after SSB. From eqs. (3.18) and (3.19), we observe that there are only two independent stationary point conditions, due to the \(S_N\) symmetry, (similar to 3.1), which are

\[
\begin{align*}
\mu_H^2 &= \lambda_H v^2 + N v_S^2 [\lambda_H S_1 + (N-1) \lambda_H S_2] , \\
\mu_{S_1}^2 &= v_S^2 [\lambda_{H S_1} + (N-1) \lambda_{H S_2}] - \left[2 \mu_{S_1}^2 + (N-1)(\mu_{S_2}^2 + 2\mu_{S_2}'^2)\right] \\
&\quad + v_S^2 \left[\lambda_{S_1} + (N-1)(\lambda_{S_2} + \lambda_S + 4c_1) + 2(N-1)(N-2)(c_2 + 2c')\right] \\
&\quad + 2(N-1)(N-2)(N-3)d , 
\end{align*}
\]  

(3.20)

These conditions further reduce the number of new parameters by one, i.e. the maximum number of new parameters introduced is 12 for \(N \geq 4\) (for \(N = 2\) and 3 these are 9 and 11, respectively).

**Spectrum of the CP–odd scalars**

As in sec. 3.1 our next step is to find which mass eigenstate corresponds to the PNGB. To do so, we first have to find the mass matrix \((M_{\chi}^2)\) for the CP–odd scalars. Since the CP–odd and CP–even scalars do not mix (due to the dark CP–invariance), their mass terms are symmetric under permutations of the \(\chi\)'s. As a result, there are only two different entries in the mass matrix for \(\chi\)'s, the diagonal, \((M_{\chi}^2)_{ii}\), and the off diagonal, \((M_{\chi}^2)_{ij}\), ones. After some algebra, one can show that

\[
[M_{\chi}^2]_{ii} = -v^2(N-1)\frac{\lambda_{H S_2}}{2} + 2\mu_{S_1}^2 + \frac{1}{2}(N-1)(2\mu_{S_2}^2 + \mu_S^2) \\
- v_S^2 \left[(N-1)(\lambda_{S_2} + c_1) + \frac{1}{2}(N-1)(N-2)(c_2 + 6c')\right] \\
+ (N-1)(N-2)(N-3) , \\
[M_{\chi}^2]_{ij} = \frac{2 \left(\mu_{S_1}^2 + (N-1)\mu_{S_2}'^2\right) - [M_{\chi}^2]_{ii}}{(N-1)} \quad \text{for } i \neq j .
\]  

(3.21)

The eigenvalues of this matrix are

\[
m_{\xi_1}^2 = [M_{\chi}^2]_{jj} + (N-1) [M_{\chi}^2]_{ii} = 2 \left(\mu_{S_1}^2 + (N-1)\mu_{S_2}'^2\right) \\
m_{\xi_i}^2 = [M_{\chi}^2]_{ii} - [M_{\chi}^2]_{ij} \quad \text{for } i = 2,3,\ldots,N .
\]  

(3.22, 3.23)

The first \((m_{\xi_1}^2)\) corresponds to the particle \(\xi_1\), which is the PNGB \((m_{\xi_1}^2 \to 0\) as \(\mu_{S_2}'^2 \to 0)\), while the other particles \((\xi_2,3,\ldots,N)\) are degenerate with mass \(m_{\xi_2} = m_{\xi_3} = \cdots = m_{\xi_N}\). As it turns out (in analogy to sec. 3.1), the PNGB is the \(S_N\)-symmetric state

\[
\xi_1 = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \chi_i ,
\]  

(3.24)

where the others (not relevant to our discussion) can be found from orthonormality conditions. We also note again that some hierarchy conditions should be imposed in order for the PNGB to be the DM particle.
The Cancellation of the Direct Detection Cross Section

Again the ingredients that we need in order to show that the direct detection cross section vanishes, are the inverse of the mass matrix for the real part of the scalars and the interaction of a pair of pseudo-Nambu-Goldstone particles with them (i.e. $\xi_1 - h, s_1$).

As usual the mass terms for the CP–even scalars can be written in a compact form as

\[
L_{hs} = -\frac{1}{2} \Phi^T M^2_{\Phi} \Phi ,
\]

with $\Phi = (h \ s_1 \ s_2 \ldots)^T$ and

\[
[M^2_{\Phi}]_{ii} = v^2 \lambda_H ,
\]

\[
[M^2_{\Phi}]_{1i} = (M^2_{hs})_{1i} = v v_S [\lambda_{HS1} + (N - 1)\lambda_{HS2}] 
\text{for } i > 1 ,
\]

\[
[M^2_{\Phi}]_{ii} = -v^2 (N - 1)\frac{\lambda_{HS2}}{2} + \frac{N - 1}{2} (\mu_{S2} + \mu'_{S2}) + v^2 S [\lambda_{S1} + (N - 1)\lambda_{S2} + 4c_1 + 2(N - 2)(c_2 + 2c'_2) - (N - 1)(N - 2)(N - 3)d] 
\text{for } i > 1 ,
\]

\[
[M^2_{\Phi}]_{ij} = v^2 \frac{\lambda_{HS2}}{2} - \frac{1}{2} (\mu_{S2} + \mu'_{S2}) + v^2 S \left[ (\lambda_{S2} + \lambda'_{S2} + 3c_1) + \frac{5}{2} (N - 2)(c_2 + 2c'_2) + 3(N - 2)(N - 3)d \right] 
\text{for } i, j > 1 .
\]

The interaction term of the Lagrangian which is responsible for the $\xi_1 - N$ elastic scattering is

\[
L_{\text{int}} = -\frac{1}{4N} \xi_1^2 \left( Y_{\xi h}, Y_{\xi s}, Y_{\xi s}, \ldots \right) \left( \begin{array}{c} h \\ s_1 \\ s_2 \\ \vdots \end{array} \right) ,
\]

with

\[
Y_{\xi h} = v N [\lambda_{HS1} + (N - 1)\lambda_{HS2}] 
\]

\[
Y_{\xi s} = v S \left\{ \lambda_{S1} + (N - 1) \left[ \lambda_{S2} + \lambda'_{S2} + 4c_1 + 2(N - 2) (c_2 + 2c'_2 + (N - 3)d) \right] \right\} .
\]

Again, the propagator (i.e. the inverse of the $s - h$ mass matrix) should be multiplied by a column vector $\sim \delta_{ii}$ (since only $h$ interacts with SM fermions), so the elements of the inverse of $M^2_{hs}$ relevant to the DM-nucleon interaction are

\[
[M^2_{\Phi}]^{-1}_{ii} \sim [M^2_{\Phi}]_{22} + (N - 1)[M^2_{\Phi}]_{23} .
\]

\[
[M^2_{\Phi}]^{-1}_{ii} \sim -[M^2_{\Phi}]_{12}.
\]

As in sec. 3.1, the Feynman diagram for the elastic PNGB-quark scattering is given fig. 1 with an amplitude proportional to

\[
A_{DD} \sim (Y_{\xi h}, Y_{\xi s}, Y_{\xi s}, \ldots) \left( \begin{array}{c} [M^2_{\Phi}]^{-1}_{11} \\ [M^2_{\Phi}]^{-1}_{11} \\ [M^2_{\Phi}]^{-1}_{11} \\ \vdots \end{array} \right) ,
\]

which, from eqs. (3.26), (3.27), and (3.30), can be shown that vanishes.
| $N$ | #phases |
|-----|---------|
| 1   | 1       |
| 2   | 3       |
| $\geq 3$ | $3 + \frac{3}{2}N(N - 1)$ |

Table 1: Number of phases for various values of $N$

A Note on the dark CP–invariance

In ref. [27] it was argued that the $U(1)$ case is invariant under $S \rightarrow S^\dagger$, because there is one phase which can be absorbed by $S$. This natural symmetry of the model guarantees that the imaginary part of $S$ (the CP–odd scalar) always interact in pairs and as a result it is stable. However, when the scalar sector consists of a larger number of particles, it is not possible to absorb all phases to the scalars, as shown in Table 1. Therefore, in order to guarantee the stability of the DM particle $\xi_1$, we have to assume that all parameters are real on top of the $S_N$ symmetry.

4 Conclusion and future direction

Inspired by an Abelian model which introduced a natural mechanism for the vanishing of the direct detection cross section, we have expanded the discussion on the explanation of the smallness of the DM direct detection cross section.

The first case under study (sec. 2) was a softly broken $SU(2)$ global symmetry. In this, we assumed that there is a doublet scalar (singlet under the SM gauge symmetry), which acquires a VEV. We showed that the resulting pseudo-Nambu-Goldstone bosons are all DM candidates, due to a remaining discrete symmetry that keeps them stable. We also showed that the DM–neucleon interaction vanishes. Then, we argued that this case can be generalized in a straightforward fashion to an $SU(N)$ symmetry, leading to the same result, i.e. vanishing of the DM–neucleon interaction.

Then in sec. 3.2 we examined the $U(1) \times S_N$ global symmetry, with $U(1)$ being softly broken, where we extended the scalar sector by adding $N$ scalars, charged only under a global $U(1)$. Assuming a dark CP–invariance, we calculated the form of the mass matrices and three-point interactions relevant to the pseudo-Nambu-Goldstone–nucleon interaction, which turned out to vanish.

A parameter space analysis of some simple cases (e.g. $U(1) \times S_2$ or $SU(2)$), will help us identify potential discovery channels at the LHC and astrophysical observations [39, 29]. Also, a calculation of 1-loop corrections will give us with precision the direct detection cross section, which can further be used to probe (or even exclude) the models discussed in this work. In addition, since the cases at hand should be treated as low-energy limits of complete models, an interesting direction would be to determine possible completions. These, can induce (parametrically or energetically suppressed [27, 29]) DM-nucleon interactions at the tree-level as well as decays of the PNGBs, allowing for a rich phenomenology, and connection of the DM problem with other open issues in particle physics (e.g. lepton number violation and neutrino masses [40]). Furthermore, there are some cases that we did not consider (i.e. the general irrep of the $SU(N)$ case), a study of other simple considerations (e.g. $SU(2)$-triplet) can be insightful, and help us identify similar classes of models. However, since we were only interested in furthering the discussion on the suppression of the DM-nucleon interaction, with a focus on simple realizations, we postpone these for a later project.

Acknowledgments

DK is supported in part by the National Science Council (NCN) research grant No. 2015- 18-A-ST2-00748. The author would like to thank Christian Gross, Alexandros Karam, Oleg Lebedev, and Kyriakos Tamvakis, for their involvement at the early stages of this project.
References

[1] XENON Collaboration, E. Aprile et al., Dark Matter Search Results from a One Ton-Year Exposure of XENON1T, Phys. Rev. Lett. 121 (2018), no. 11 111302, [arXiv:1805.12562].

[2] G. Bertone and D. Hooper, History of dark matter, Rev. Mod. Phys. 90 (2018), no. 4 045002, arXiv:1605.04909.

[3] H. Mebane, N. Greiner, C. Zhang, and S. Willenbrock, Constraints on Electroweak Effective Operators at One Loop, Phys. Rev. D88 (2013), no. 1 015028, arXiv:1306.3380.

[4] M. A. Fedderke, J.-Y. Chen, E. W. Kolb, and L.-T. Wang, The Fermionic Dark Matter Higgs Portal: an effective field theory approach, JHEP 08 (2014) 122, arXiv:1404.2283.

[5] J. Hisano, D. Kobayashi, N. Mori, and E. Senaha, Effective Interaction of Electroweak-Interacting Dark Matter with Higgs Boson and Its Phenomenology, Phys. Lett. B742 (2015) 80–85, arXiv:1410.3569.

[6] S. Matsumoto, S. Mukhopadhyay, and Y.-L. S. Tsai, Effective Theory of WIMP Dark Matter supplemented by Simplified Models: Singlet-like Majorana fermion case, Phys. Rev. D94 (2016), no. 6 065034, arXiv:1604.02230.

[7] A. Dedes, D. Karamitros, and V. C. Spanos, Effective Theory for Electroweak Doublet Dark Matter, Phys. Rev. D94 (2016), no. 9 095008, arXiv:1607.05040.

[8] J. Yepes, Top partners tackling vector dark matter, arXiv:1811.06059.

[9] C. Cheung, L. J. Hall, D. Pinner, and J. T. Ruderman, Prospects and Blind Spots for Neutralino Dark Matter, JHEP 05 (2013) 100, arXiv:1211.4873.

[10] S. Banerjee, S. Matsumoto, K. Mukaida, and Y.-L. S. Tsai, WIMP Dark Matter in a Well-Tempered Regime: A case study on Singlet-Doublets Fermionic WIMP, JHEP 11 (2016) 070, arXiv:1603.07387.

[11] T. Han, H. Liu, S. Mukhopadhyay, and X. Wang, Dark Matter Blind Spots at One-Loop, arXiv:1810.04679.

[12] L. Heurtier and D. Teresi, Dark matter and observable lepton flavor violation, Phys. Rev. D94 (2016), no. 12 125022, arXiv:1607.01798.

[13] A. Dedes, D. Karamitros, and A. Pilaftsis, Radiative Light Dark Matter, Phys. Rev. D95 (2017), no. 11 115037, arXiv:1704.01497.

[14] S. Knapen, T. Lin, and K. M. Zurek, Light Dark Matter: Models and Constraints, Phys. Rev. D96 (2017), no. 11 115021, arXiv:1709.07882.

[15] L. Darme, S. Rao, and L. Roszkowski, Light dark Higgs boson in minimal sub-GeV dark matter scenarios, JHEP 03 (2018) 084, arXiv:1710.08430.

[16] M. Dutra, M. Lindner, S. Profumo, F. S. Queiroz, W. Rodejohann, and C. Siqueira, MeV Dark Matter Complementarity and the Dark Photon Portal, JCAP 1803 (2018) 037, arXiv:1801.05447.

[17] P. Foldenauer, Let there be Light Dark Matter: The gauged $U(1)_{L_\mu-L_\tau}$ case, arXiv:1808.03647.

[18] S. Matsumoto, Y.-L. S. Tsai, and P.-Y. Tseng, Light Fermionic WIMP Dark Matter with Light Scalar Mediator, arXiv:1811.03292.
A. Dedes, I. Giomataris, K. Suxho, and J. D. Vergados, *Searching for Secluded Dark Matter via Direct Detection of Recoiling Nuclei as well as Low Energy Electrons*, Nucl. Phys. B826 (2010) 148–173, [arXiv:0907.0758](https://arxiv.org/abs/0907.0758).

R. Essig, T. Volansky, and T.-T. Yu, *New Constraints and Prospects for sub-GeV Dark Matter Scattering off Electrons in Xenon*, Phys. Rev. D96 (2017), no. 4 043017, [arXiv:1703.00910](https://arxiv.org/abs/1703.00910).

J. A. Evans, *Detecting Hidden Particles with MATHUSLA*, Phys. Rev. D97 (2018), no. 5 055046, [arXiv:1708.08503](https://arxiv.org/abs/1708.08503).

DarkSide Collaboration, P. Agnes et al., *Constraints on Sub-GeV Dark-Matter–Electron Scattering from the DarkSide-50 Experimemt*, Phys. Rev. Lett. 121 (2018), no 11 111303, [arXiv:1802.06998](https://arxiv.org/abs/1802.06998).

LDMX Collaboration, T. Åkesson et al., *Light Dark Matter eXperiment (LDMX)*, [arXiv:1808.05219](https://arxiv.org/abs/1808.05219).

FASER Collaboration, A. Ariga et al., *FASER: ForwArd Search ExpeRiment at the LHC*, [arXiv:1901.04468](https://arxiv.org/abs/1901.04468).

A. Dedes and D. Karamitros, *Doublet-Triplet Fermionic Dark Matter*, Phys. Rev. D89 (2014), no. 11 115002, [arXiv:1403.7744](https://arxiv.org/abs/1403.7744).

G. Arcadi, C. Gross, O. Lebedev, Y. Mambrini, S. Pokorski, and T. Toma, *Multicomponent Dark Matter from Gauge Symmetry*, JHEP 12 (2016) 081, [arXiv:1611.00365](https://arxiv.org/abs/1611.00365).

C. Gross, O. Lebedev, and T. Toma, *Cancellation Mechanism for Dark-Matter–Nucleon Interaction*, Phys. Rev. Lett. 119 (2017), no. 19 191801, [arXiv:1708.02253](https://arxiv.org/abs/1708.02253).

R. Balkin, M. Ruhdorfer, E. Salvioni, and A. Weiler, *Dark matter shifts away from direct detection*, JCAP 1811 (2018), no. 11 050, [arXiv:1809.09106](https://arxiv.org/abs/1809.09106).

T. Alanne, M. Heikinheimo, V. Keus, N. Koivunen, and K. Tuominen, *Direct and indirect probes of Goldstone dark matter*, [arXiv:1812.05996](https://arxiv.org/abs/1812.05996).

L. Lopez-Honorez, T. Schwetz, and J. Zupan, *Higgs portal, fermionic dark matter, and a Standard Model like Higgs at 125 GeV*, Phys. Lett. B716 (2012) 179–185, [arXiv:1203.2064](https://arxiv.org/abs/1203.2064).

A. Freitas, S. Westhoff, and J. Zupan, *Integrating in the Higgs Portal to Fermion Dark Matter*, JHEP 09 (2015) 015, [arXiv:1506.04149](https://arxiv.org/abs/1506.04149).

A. Karam and K. Tamvakis, *Dark matter and neutrino masses from a scale-invariant multi-Higgs portal*, Phys. Rev. D92 (2015), no. 7 075010, [arXiv:1508.03031](https://arxiv.org/abs/1508.03031).

L. Lopez Honorez, M. H. G. Tytgat, P. Tziveloglou, and B. Zaldivar, *On Minimal Dark Matter coupled to the Higgs*, JHEP 04 (2018) 011, [arXiv:1711.08619](https://arxiv.org/abs/1711.08619).

A. Filimonova and S. Westhoff, *Long live the Higgs portal!, [arXiv:1812.04628](https://arxiv.org/abs/1812.04628).

S. Bhattacharya, B. Melić, and J. Wudka, *Pionic Dark Matter*, JHEP 02 (2014) 115, [arXiv:1307.2647](https://arxiv.org/abs/1307.2647).

A. Semenov, *LanHEP — A package for automatic generation of Feynman rules from the Lagrangian. Version 3.2*, Comput. Phys. Commun. 201 (2016) 167–170, [arXiv:1412.5016](https://arxiv.org/abs/1412.5016).

D. Azevedo, M. Duch, B. Grzadkowski, D. Huang, M. Iglicki, and R. Santos, *One-loop contribution to dark-matter-nucleon scattering in the pseudo-scaler dark matter model*, JHEP 01 (2019) 138, [arXiv:1810.06105](https://arxiv.org/abs/1810.06105).
[38] K. Ishiwata and T. Toma, *Probing pseudo Nambu-Goldstone boson dark matter at loop level*, *JHEP* **12** (2018) 089, [arXiv:1810.08139](https://arxiv.org/abs/1810.08139).

[39] K. Huitu, N. Koivunen, O. Lebedev, S. Mondal, and T. Toma, *Probing pseudo-Goldstone dark matter at the LHC*, [arXiv:1812.05952](https://arxiv.org/abs/1812.05952).

[40] F. S. Queiroz and K. Sinha, *The Poker Face of the Majoron Dark Matter Model: LUX to keV Line*, *Phys. Lett.* B**735** (2014) 69–74, [arXiv:1404.1400](https://arxiv.org/abs/1404.1400).