Hinge Moment Coefficient Prediction Tool and Control Force Analysis of Extra-300 Aerobatic Aircraft

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Abstract. This paper presents the development of tool that is applicable to predict hinge moment coefficients of subsonic aircraft based on Roskam’s method, including the validation and its application to predict hinge moment coefficient of an Extra-300. The hinge moment coefficients are used to predict the stick forces of the aircraft during several aerobatic maneuver i.e. inside loop, half cuban 8, split-s, and aileron roll. The maximum longitudinal stick force is 566.97 N occurs in inside loop while the maximum lateral stick force is 340.82 N occurs in aileron roll. Furthermore, validation hinge moment prediction method is performed using Cessna 172 data.

Keywords: hinge moment coefficient, prediction tool, control force, Extra-300, Microsoft Excel

1. Introduction

When an aircraft flies, the air passes through its control surfaces will change the load on them and generate force and moment around the hinge line. This moment has to be countered by pilot through cockpit control devices to deflect the corresponding control surfaces [1]. There are two kinds of aircraft control concept, reversible and irreversible [2].

In reversible system, the pilot controls are directly connected using combinations of pushrods, cables, and pulleys to the control surface. As the consequence, if pilot controls are deflected, the corresponding control surface is deflected and at the same time if the control surface is deflected, then the pilot controls will be deflected. In irreversible system, there are additional power augmentation system that support pilot to counter the hinge moment. The augmentation system consists usually of hydraulic system or pneumatic (electro-mechanical actuators).

Figure 1. Illustration of elevator linkage system [3]
Hinge moment usually expressed as coefficient of hinge moment \( (C_h) \). Generally, specific aircraft has unique hinge moment coefficient as its aerodynamic characteristic. Hinge moment analysis has three main following purposes:

1. To compute stick, wheel, and pedal cockpit control forces so they can be checked against airworthiness requirements.
2. To compute actuator force level so that hydraulic or electro-mechanical actuators can be properly sized [4].
3. To design proper artificial feel or control loading system.

Roskam’s method of hinge moment derivative is a method that is applicable to predict hinge moment coefficient of subsonic aircraft. Hinge moment coefficient is function of control surface geometry, aircraft specification, angle of attack, and surface deflection.

A hinge moment prediction tool is developed based on the Roskam’s method to calculate hinge moment coefficient. This tool is designed to calculate hinge moment coefficient in ailerons and elevators and these values will be validated using reference data of Cessna 172S. The advantages of this tool is the convenience to inspect the calculation steps and to debug if any problem occurs so it is suitable for both educational, design, or other analysis.

The tool is applied to Extra-300 aerobatic aircraft which operates in high load factor region during its operation. There is various level of aerobatic maneuvers known i.e. basic maneuver, basic combination maneuver, and basic sequence maneuver [5]. In this work, inside loop, half cuban 8, split-s, and aileron roll maneuver is selected. Longitudinal and lateral control forces are analyzed during those maneuvers. Control force is one kind of cueing aspect in aircraft. In complex, high consequence environments such as aviation, the capacity to acquire, integrate, and respond to task-related cues is critical [6]. Pull back stick that generate forward control force is considered as positive control force [7].

To conclude the introduction statements, the purpose of this work is to explain the development of a hinge moment prediction tool based on Roskam’s method. The tool is applied to analyze hinge moment coefficient of Extra-300. Furthermore, flight simulation is conducted and the control forces are analyzed during some aerobatic maneuvers.

2. Hinge Moment Prediction

Roskam’s method is used as basic theory for the developed tool. The following is the general equation of hinge moment coefficient \( (C_h) \)

\[
C_h = C_{ho} + C_{ha} \cdot \alpha + C_{h\delta} \cdot \delta + C_{h\delta t} \cdot \delta t
\]  

Hinge moment coefficient is separated into three derivatives i.e. angle of attack, control surface deflection, and trim tab deflection. Besides, it’s also contribution of \( C_{ho} \) which is function of airfoil feature. For Extra-300 in which the airfoils are symmetrical, the value of \( C_{ho} \) is 0.

Hinge moment derivative due to angle of attack can be determined using this equation

\[
C_{ha} = \frac{A \cdot \cos \Lambda c}{A + 2 \cdot \cos \Lambda c} \cdot C_{haM} + \Delta C_{ha}
\]

The term \( C_{haM} \) is the contribution of 2D hinge moment derivative which is function of Mach number and balance ratio factor as shown in the equation below. Balance ratio represents the ratio between forward and backward part of control surface in respect to the hinge line.

\[
C_{haM} = \frac{(C_{ha})_{balance}}{(1-M^2)^{\frac{1}{2}}}
\]

\( C_{haM} \) can be obtained through some steps and the first is examining trailing edge (TE) angles by checking whether following equation (4) is satisfied. TE angle notation explained in the Figure 4

\[
\tan\left(\frac{\phi_{te}}{2}\right) = \tan\left(\frac{\phi_{te}}{2}\right) = \tan\left(\frac{\phi_{te}}{2}\right) = \frac{c}{e}
\]
The second step is calculating $C_h'$ using equation (5) and the value of $\frac{C_h'}{(C_h)_{theory}}$ and $(C_h)_{theory}$ can be obtained from Figure 2.

$$C_h' = \frac{C_h'}{(C_h)_{theory}} \cdot (C_h)_{theory}$$  \hspace{1cm} (5)

**Figure 2.** trailing edge definition

**Figure 3.** (a) Graph $(C_h)_{theory}$ vs $c_f/c$, (b) Graph $\frac{C_h'}{(C_h)_{theory}}$ vs $c_f/c$, (c) $\frac{c_l}{(C_l)_{theory}}$ vs $\tan(\frac{\Phi_{te}}{2})$
If equation (4) is satisfied, then \( C_{h\alpha}'' = C_{h\alpha}' \), but if it is not, \( C_{h\alpha}'' \) can be calculated using equation (6)

\[
C_{h\alpha}'' = C_{h\alpha}' + 2. (C_{l\alpha})_{\text{theory}} \cdot \left[ 1 - \frac{C_{l\alpha}}{(C_{l\alpha})_{\text{theory}}} \right] \cdot \left[ \tan \frac{\phi_{\text{te}}}{2} - \frac{f}{c} \right]
\]  
(6)

Then the next step is calculating \((C_{h\alpha})_{\text{balance}}\) using following equation

\[
(C_{h\alpha})_{\text{balance}} = C_{h\alpha}'' \cdot \frac{(C_{h\alpha})_{\text{balance}}}{C_{h\alpha}}
\]  
(7)

As the feature of aircraft’s control surface, balance ratio equation is stated below

\[
BR = \sqrt{\left( \frac{c_{\text{bf}}}{c_{\text{cf}}} \right)^2 - \left( \frac{t}{2c_f} \right)^2}
\]  
(8)

Then, balance ratio information is applied to Figure 3(a) to obtain the value of \( \frac{(C_{h\alpha})_{\text{balance}}}{C_{h\alpha}} \).

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**Figure 4.** (a) Balance ratio graph, (b) \( \frac{\Delta C_{h\alpha}}{C_{l\alpha} B_2 K_a \cos \Lambda_c} \) graph

After completing previous steps, the value of \( C_{h\alpha M} \) can be calculated from equation (2). The next step is examining \( \Delta C_{h\alpha} \). The equation is stated below

\[
\Delta C_{h\alpha} = \left[ \frac{\Delta C_{h\alpha}}{C_{l\alpha} B_2 K_a \cos \Lambda_c} \right] \cdot C_{l\alpha} \cdot B_2 \cdot K_a \cdot \cos \Lambda_c
\]  
(9)

The term \( \frac{\Delta C_{h\alpha}}{C_{l\alpha} B_2 K_a \cos \Lambda_c} \) can be evaluated from figure 3(b) while the values of \( C_{l\alpha} \) and \( \cos \Lambda_c \) are the property of airfoil and wing or HTP plane respectively. \( K_a \) can be calculated using the equation and graph 4(a) below:

\[
K_a = \frac{K_a_0 (1-\eta_a)-K_{ad} (1-\eta_a)}{\eta_0-\eta_1}
\]  
(10)

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**Figure 5.** (a) Graph \( K_a \) vs \( \eta \), (b) Graph \( B_2 \) vs \( c_{f/c} \)
The variable $\eta_i$ and $\eta_o$ refer to inboard and outboard span of control surface as fraction of the main surface semi span respectively. The value of B2 can be obtained from the graph 4(b). Return to equation (1), all variables have been known hence value of $C_{h\alpha}$ can be calculated by this equation

$$C_{h\alpha} = \frac{Acos\Lambda_c}{A+2cos\Lambda_c} \cdot C_{h\alpha M} + \Delta C_{h\alpha}$$

(11)

This method is applicable for both ailerons in wings and elevator in horizontal tail. The value of angle of attack in wing is not equal to in tail. There is a correction due to the presence of downwash is computed using the equation below

$$\frac{d\alpha_H}{d\alpha} = \frac{[A^2(1-M^2)+4]^{0.5}-2}{[A^2(1-M^2)-4]^{0.5}+2}$$

(12)

The next step is calculating hinge moment derivative due to control surface deflection. The general formula of hinge moment derivative due to control surface deflection is stated below

$$C_{h\delta} = \cos\Lambda_c \cdot \cos\Lambda_h \cdot [C_{h\delta M} + a\delta \cdot C_{h\alpha M} \cdot \frac{2cos\Lambda_c}{A+2cos\Lambda_c} + \Delta C_{h\delta}]$$

(13)

From the previous steps, the value of $C_{h\alpha M}$ is already known. The value of $\alpha\delta$ can be obtained from the following graph

![Figure 6. Graph $\alpha\delta$ vs surface deflection](image)

Equivalent to the hinge moment derivative due to angle of attack, the deflection factor is also started from 2D hinge moment derivative formulated as

$$C_{h\delta M} = \frac{(C_{h\delta})_{balance}}{(1-M^2)^{1/2}}$$

(14)

After examining the trailing edge angles which has been done before in angle of attack discussion, the second step is calculating $C'_{h\delta}$ using this equation :

$$C'_{h\delta} = \frac{c_{h\delta}}{(C_{h\delta})_{theory}} \cdot (C_{h\delta})_{theory}$$

(15)
The value of $\frac{\delta}{\delta \text{delta}}$ and $(C_{h\delta})_{\text{theory}}$ can be obtained from Figure 6.

Figure 7. (a) Graph $\frac{c_{h\delta}}{c_{h\delta \text{delta}}}$ vs cf/c, (b) Graph $(C_{h\delta})_{\text{theory}}$ vs cf/c, (c) Graph $\frac{c_{l\delta}}{c_{l\delta \text{theory}}}$ vs $\tan(\frac{\phi_{t\epsilon}}{2})$, (d) Graph $(C_{l\delta})_{\text{theory}}$ vs cf/c

If equation (4) is satisfied then $C_{h\delta}'' = C_{h\delta}'$, but if not $C_{h\delta}''$ can be calculated by using equation (16)

$$C_{h\delta}'' = C_{h\delta}' + 2 \cdot (C_{l\delta})_{\text{theory}} \cdot [1 - \frac{c_{l\delta}}{(C_{l\delta})_{\text{theory}}} \cdot [\tan(\frac{\phi_{t\epsilon}}{2}) - \frac{1}{2}]]$$

$(C_{l\delta})_{\text{theory}}$ can be obtained from graph 6(d). The next step is calculating $(C_{h\delta})_{\text{balance}}$ using following equation

$$(C_{h\delta})_{\text{balance}} = C_{h\delta}'' \cdot \frac{(C_{h\delta})_{\text{balance}}}{C_{h\delta}}$$

Next, balance ratio equation is stated below

$$BR = \sqrt{\left(\frac{2}{c_f} \right)^2 - \left(\frac{1}{2c_f} \right)^2}$$
Then, balance ratio information is applied to this graph to obtain the value of $\frac{(C_{h\delta})_{balance}''}{C_{h\delta}''}$.

Figure 8. Balance ratio graph

The next step is examining the value of $\Delta C_{h\delta}$ in which the steps are similar to the calculation of $\Delta C_{ha}$. Return to the general equation:

$$C_{h\delta} = \cos\Lambda_c \cos\Lambda_{ht}; \frac{2\cos\Lambda_c}{\Lambda_c^2 + 2\cos\Lambda_c} + \Delta C_{h\delta}$$ (20)

All the variables can be completely obtained from the steps explained before. Therefore, the value of hinge moment coefficient derivative as the effect of control surface deflection can be calculated. The method of calculation the effect of tab is similar to the surface deflection calculation.

At this point, can be concluded that an aircraft has its unique hinge moment coefficient. The coefficient depends on aircraft geometry, angle of attack, and control surface deflection and proportional to angle of attack and surface deflection. The more advance step is calculating control force of an aircraft during its operation or maneuvers. Control force is function of gearing ratio ($G$), hinge moment coefficient ($C_{he}$), HTP efficiency ($\eta_{htp}$) which is the dynamic pressure ratio between HTP and wing, air density ($\rho$), airspeed ($v$), surface area ($S$), and surface mean chord ($c$) as shown in equation (21)

$$F_e = \frac{1}{2} \cdot G \cdot C_{he} \cdot \eta_{htp} \cdot \rho \cdot v^2 \cdot S_e \cdot c_e$$ (21)

Those steps above have been implemented in the hinge moment prediction tool based on MS Excel. User needs to input aircraft specification including general data, wing and aileron data, and HTP and elevator data. The output of the tool is hinge moment derivative due to angle of attack ($C_{h\alpha}$) and control surface deflection ($C_{h\delta}$).

The hinge moment calculation tool has some features as shown in the Figure 8, 9, and 10.

1. Main sheet as the input and output center of the tool.
2. Airfoil analyzer to visualize the airfoil and calculate its angle especially for trailing edge.
3. Graph sheets, contains graph data and interpolation feature.
Figure 9. Main sheet of the prediction tool

Figure 10. Airfoil sheets as the trailing edge analyzer
3. Result and Discussion

The following table shows the calculation result and the reference data for Cessna 172S

|          | Cessna 172 | reference | calculation | error |
|----------|------------|-----------|-------------|-------|
| aileron  | Cha        | -0.05     | -0.078      | 55.38 |
|          | Chd        | -0.1      | -0.115      | 14.88 |
| elevator | Cha        | -0.295    | -0.283      | 4.04  |
|          | Chd        | -0.59     | -0.566      | 3.99  |

Table 1. Comparison between calculation result and reference for Cessna 172S

There are slight differences between reference data and calculation result of elevator for both angle of attack (4.04%) and control surface deflection (3.99%) derivatives. However, the errors are quite bigger in aileron with 55.38% for angle of attack and 14.88% for surface deflection. But these conditions happen because the coefficients based on reference data are relatively small. Furthermore, there are no contribution of angle of attack in aileron control force because the effect is canceling each other between right and left aileron.

Extra-300 aerobatic aircraft is the object of this research. Table below shows hinge moment coefficient of Extra-300.

| Aileron | Cha     | -0.10818 |
|         | Chd     | -0.41194 |
| Elevator| Cha     | -0.15284 |
|         | Chd     | -0.56721 |

Table 2. Hinge moment coefficient of Extra-300
Aircraft model of Extra-300 has been simulated using XPlane-10 doing normal flight and maneuvers. Each maneuvering flight consist of several maneuver executions. In this part, the most critical control force condition for each maneuver are presented. The maximum stick force in inside loop is 566.97 N while the most critical forces for half cuban 8 and split-s are 561 N and 480.05 N respectively as shown in the following graphs

![Figure 12. Stick force in inside loop maneuver](image)

![Figure 13. Stick force in half cuban 8 maneuver](image)

![Figure 14. Stick force in split-s maneuver](image)

Furthermore, since aileron roll involves aileron only, the critical force happen in aileron or called wheel force. The maximum wheel force is 340.82 N as shown in the figure below

![Figure 15. Wheel force in aileron roll maneuver](image)

Control force is a function several parameters including airspeed and control surface deflection. Force is proportional to airspeed hence higher airspeed results in higher force. Besides, surface deflection is proportional to hinge moment coefficient which is also proportional to force. The following split-s graph shows the relation between airspeed and force.
The following inside loop maneuver presents the relation between surface deflection and force.

**Figure 16. Relation between stick force and airspeed**

Besides, there is correlation between airspeed or deflection to maneuver shape i.e. loop diameter. Elevator deflection and entry airspeed are inversely proportional to loop diameter. Bigger deflection will generate more extreme inside loop with smaller diameter and greater load factor. Extra-300 can afford until 10 gravities [8].

For further analysis, the maneuvering flight data are recorded and tabulated as follow:

**Table 3. Flight data during inside loop maneuver**

| throttle | entry airspeed | entry altitude | max altitude | loop diameter | max elevator deflection | max stick force |
|----------|----------------|----------------|--------------|---------------|------------------------|----------------|
| no       | m/s            | m             | m           | m             | m                      | N              |
| 1        | 87.5           | 565.2         | 884.1       | 318.9         | -11.28                 | 316.49         |
| 2        | 90.71          | 663.06        | 1074.51     | 411.45        | -8.43                  | 297.72         |
| 3        | 87.4           | 853.3         | 1197.63     | 344.33        | -10.63                 | 301.64         |
| 4        | 89.43          | 894.17        | 1152.84     | 258.67        | -14.91                 | 332.64         |
| 5        | 93.57          | 799.75        | 973.04      | 173.29        | -23.72                 | 566.97         |
| 6        | 86.62          | 796.31        | 1097.06     | 300.75        | -11.16                 | 315.73         |
| 7        | 90.04          | 873.39        | 1026.88     | 151.49        | -20.46                 | 504.07         |

**Figure 17. Relation between stick force and elevator deflection**

**Table 4. Flight data during half cuban 8 maneuver**

| throttle | entry airspeed | entry altitude | max altitude | max altitude change | max elevator deflection | max stick force | max wheel force |
|----------|----------------|----------------|--------------|---------------------|------------------------|----------------|----------------|
| no       | m/s            | m             | m           | m                   | m                      | N              | N              |
| 1        | 92.41          | 702.57        | 1101.09     | 447.51              | -8.65                  | -16.24         | 195.28         | -93.52         |
| 2        | 81.68          | 602.74        | 756.96      | 339.12              | -10.14                 | -23.94         | 130.09         | -234.18        |
| 3        | 89.37          | 732           | 1199.52     | 467.2               | -22.22                 | -31.4          | 302.8          | -348.7         |
| 4        | 89.21          | 737.81        | 564.95      | 192.86              | -25                    | -18.47         | 484.32         | -533.64        |
| 5        | 94.38          | 574.51        | 472.42      | 392.8               | -22.64                 | -10.93         | 561            | -146.52        |
Table 5. Flight data during split-s maneuver

| Throttle | Entry Airstream | Entry Altitude | Max Altitude | Curve Diameter | Max Elevator Deflection | Max Aileron Deflection | Max Stick Force | Max Wheel Force |
|----------|-----------------|----------------|--------------|---------------|------------------------|------------------------|----------------|----------------|
| %        | m/s             | m              | deg          | m             | deg                    | deg                    | N              | N              |
| 100%     | 88.21           | 370.39         | 227.62       | 327.25        | -11.76                 | -5.3                   | 446.95         | -194.95        |
| 75%      | 79.23           | 369.18         | 179.66       | 410.8         | -6.8                   | -3.3                   | 387.81         | -34.37         |
| 50%      | 61.71           | 355.31         | 194.44       | 360.87        | -10.85                 | 6.03                   | 430.31         | 190.82         |
| 37.5%    | 69.42           | 418.06         | 177.18       | 311.47        | -13.1                  | -4.49                  | 490.05         | -235.73        |
| 15%      | 83.97           | 402.99         | 45.91        | 357.08        | -9.93                  | 12.28                  | 423.13         | 202.88         |

Table 6. Flight data during aileron roll maneuver

| Throttle | Entry Airstream | Entry Altitude | Max Aileron Deflection | Max Wheel Force |
|----------|-----------------|----------------|------------------------|----------------|
| %        | m/s             | m              | deg                    | N              |
| 100%     | 78.13           | 461.9          | -5.9                   | -203.72        |
| 75%      | 78.84           | 450.89         | 6.39                   | 196.68         |
| 50%      | 76.07           | 488.06         | -6.93                  | -202.97        |
| 37.5%    | 79.29           | 465.67         | 8.13                   | 243.37         |
| 15%      | 80.1            | 433.69         | -10.62                 | -318.32        |
| 10%      | 79.27           | 418.22         | 14.36                  | 340.82         |
| 5%       | 80.99           | 407.92         | -7.76                  | -251.9         |
| 1%       | 79.65           | 448.52         | 8.76                   | 250.04         |
| 0%       | 81.45           | 409.93         | -7.09                  | -250.78        |

4. Conclusion

This research includes development, validation, and application of hinge moment prediction tool. The tool can be used to calculate coefficient of hinge moment of a subsonic aircraft. It has been validated using comparison data of Cessna-172.

Application on Extra-300 aerobatic aircraft generates hinge moment coefficient derivatives due to angle of attack of -0.108 for aileron and -0.152 for elevator. Besides hinge moment coefficient derivatives due to control surface deflection are -0.411 for aileron and -0.567 for elevator. The maximum stick force occurs in inside loop maneuver up to 566.97 N, while the maximum wheel force is 340.82 N in aileron roll. Control force is proportional to airspeed and control surface deflection.

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