MOND and asymptotic safe gravity

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Abstract: The modified Newtonian dynamics (MOND) paradigm is discussed in the context of asymptotic safe gravity. We estimate quantum correction to the logarithmic potential which is well known to account for the constancy of the circular velocity $v$ of the spiral galaxies. We determine plausible bounds on $v$.

1 Introduction

In the past several years an alternative but an attractive viable hypothesis to galactic dark matter has emerged out of the modification of Newtonian dynamics (MOND) (see [1,2] for reviews). As early as in 1983, Milgrom [3] recognized the inadequacy of Newton’s universal inverse square law of gravitation based on the observational signatures coming from the extra-galactic regimes. In a series of remarkable papers [4], Milgrom pointed out the need to deviate from the conventional wisdom and proposed to replace it by MOND taking into consideration a critical value $a_0$ of acceleration below which Newton’s law of gravitation could break down. The typical scales of the extragalactical acceleration are less than the estimate of $a_0 \approx 1.2 \times 10^{-10} \text{ms}^{-2}$, and indeed much smaller than those encountered in the solar system. One of the key achievements of MOND [5] is that it explained quite well the previously known Tully-Fisher empirical relation [6] between the intrinsic luminosity of a spiral galaxy and its asymptotic rotation velocity. Recently, a new gravitational theory has been advanced for modified Newtonian dynamics (MOND) [7] that matches with the observed universal galactic acceleration scale and the phenomenology following there from. It also suggests the possibility of undiscovered gravity that could explain characteristics prevalent in the cosmic microwave background. For a study on certain aspects of MOND in cosmic structures see [8].

The existence of radial acceleration relation of satellite galaxies within the standard model of cosmology, the $\Lambda$CDM, provides a direct evidence of MOND wherein the characteristic acceleration embodies $a_0$ [10]. However, the question of covariance in MOND [11] is...
intriguing. Of course it is not a strong necessity but one has to remember that MOND is a phenomenologically emergent theory with the issue of fundamentality of the constants like $c$, $a_0$ or $l_M$ (the MOND length) not clear. Secondly, the emergent laws of dynamics are also generically time nonlocal and this nonlocality may be in tune with the nonlocality in MOND. Thirdly, in MOND, the measure of an absolute acceleration is required.

Observational data show that the rotational velocities of different stars and gas in the outskirts of spiral galaxies flatten out to constant values as evidenced in the discovery of Rubin et al. [12] and receiving confirmation in subsequent analyses. This feature runs counter to Newtonian expectation that it should be inversely proportional to the square root of the distance $r$ of the star from the nucleus. Herein a tacit assumption is made that the contributing mass is all luminous, with the centre of the galaxy carrying the bulk luminosity. On the other hand, the MOND facilitates arriving at the result of constant rotational velocity by invoking weak equivalence principle - the inertial mass $m$ defined in Newton’s second law of motion, which gives the force as $ma$, $a$ being the acceleration, is identified with the mass $M$ which acts upon it and determines the magnitude of the gravitational force $\frac{GmM}{r}$, where $G$ is the Newtonian gravitational constant, the two masses $m$ and $M$ are situated at a distance $r$ apart. We thus see that $m$ cancels out and we are left with the result for the acceleration as $a \propto \frac{1}{r^2}$. Interpreting it to be the centripetal acceleration namely, $a = \frac{v^2}{r}$, one readily finds the rotational velocity $v$ to be constant, $v = (GMc_0)^\frac{1}{4}$. An implication of this is that since the luminosity $L$ is proportional to $M$, $v^4$ is proportional to $L$ [6,14]. Behind this result is the force driven by the Newtonian potential along with a small logarithmic correction (see, for example, [19–21])

$$V = V^{(N)} + V^{(\log)}$$

(1.1)

where

$$V^{(N)} = -\frac{GM}{r}$$

(1.2)

$$V^{(\log)} = \sqrt{GMa_0} \ln \left( \frac{r}{r_0} \right)$$

(1.3)

where we have set $m = 1$ and $r_0$ is a constant. The logarithmic potential yields a circular velocity which is asymptotically constant as just noted, agreeing with what is observed in the outside vicinity of spiral galaxies. In the context of baryonic Tully-Fisher law, $M$ stands for the total baryonic mass of the galaxy.

As to how MOND brings about a modification of Newtonian inertia was also enquired by Milgrom [22]. This was to introduce a deformed force $F$ by making a simple ansatz

$$F = ma\mu \left( \frac{a}{a_0} \right)$$

(1.4)
where $a$ is the Newtonian acceleration, $a_0$ is its threshold value, and $\mu(a/a_0)$ is some kind of an interpolating function which is designed to obey the typicality of Newtonian force in the absence of the deformation parameter $a_0$, and also to meet the observed mass discrepancy in galactic systems. Motion of stars around the centre of their galaxies, acting as a kind of nucleus, reveals circular motion but possessed with very small acceleration i.e. $a << a_0$ (deep MOND regime). Milgrom conjectured that in such a situation the $\mu$-function could be approximated by $\mu = \frac{a}{a_0}$ resulting in the force taking the form

$$F = m \left( \frac{a^2}{a_0} \right)$$

There has been a wide range of models that have tried to assess other variants of the $\mu$-function that retain the limiting behaviour [23]. The choice of the $\mu$-function can be adjusted according to the merits of the problem as several recent studies have shown (see [1] for a discussion on this aspect). Also, as noted by Milgrom [5] in a recent review article, the choice of an interpolating function can vary with situation like for instance, there could be one for circular orbits and another for linear constant-acceleration trajectories.

In this paper we will concentrate on how the theory of asymptotic safe gravity (ASG) [24] influences MOND. ASG was first conjectured by Weinberg [25] who gave a renormalizability condition to shun the theory from ultraviolet divergences and thus rendering safe the physical quantities from them, by admitting nontrivial non-Gaussian fixed points. It tries to provide a consistent picture of the quantum theory of the gravitational field and its different manifestations [26–31]. The scale dependence of the couplings is special in the ultraviolet limit when all their dimensionless combinations remain finite. Our purpose will be to derive the quantum-corrected logarithmic potential in the ASG scenario and estimate the bounds for the spiral galaxy velocity.

2 MOND and asymptotic safe gravity

The idea of MOND as a regime of quantum gravity for length scales exceeding that of the horizon has been advocated by Smolin [32]. The dynamics of classical theory is known to be scale symmetric. But in the case of quantum field theory in which the usual feature of the scale-dependence of the couplings exists, small fluctuations can appear that break the classical symmetry [33,34]. This implies that a theory developed at one scale may not remain consistent as the scale is tuned to smaller distance values where divergences in the couplings could show up. One of the ways to circumvent it is through the restoration of scale symmetry and have an extension of the theory to short distance scales (asymptotic freedom) [33].

In the context of ASG, the running gravitational constant is characterized by [35]

$$G(r) = \frac{G_N}{1 + \eta G \left( \frac{1}{r^2} + \frac{GM}{r^3} \right) \hbar}$$

where $G_N$ is the Newtonian gravitational constant, $\eta$ and $\gamma$ are numerical constants, and $c = k_B = 1$ has been set.. Note that both $\frac{1}{r^2}$ and $\frac{1}{r^3}$ terms in the denominator influence $G(r)$. Impact of ASG in different branches of cosmology has been widely pursued. For instance, while ASG inevitably modifies Schwarzschild metric [35] thereby bringing changes in the results of black hole thermodynamics [36], it influences [37] the features of Planck stars [38].

In a recent article, Scardigli and Lambiase [37] attempted a Taylor expansion of $G(r)$ to write for large $r$
\[ G^{(r)} = G_N \left[ 1 - \left( \frac{\eta \hbar}{r^2} G_N + \frac{\eta \gamma_M G_N^2}{r^3} G_N^2 \right) + O \left( \frac{\eta^2 \hbar^2}{r^4} G_N^2 \right) \right] \] (2.2)

Subsequently, they sought quantum correction to the Newtonian potential \( V^{(N)} \) through replacing the gravitational constant \( G \) by the above estimate of \( G^{(r)} \)

\[ V^{(N)} \rightarrow - \frac{G^{(r)} M}{r} \] (2.3)

The quantum correction in (3.3) is evidenced by the appearance of \( \hbar \) term in \( G^{(r)} \), with no classical correction coming from post-Newtonian approximation. Comparing with the standard leading quantum corrected expectation derived from the low energy limit of the scattering amplitude which reads up to the order of \( \hbar \) \[39,40\]

\[ V^{(q)} = - \frac{G_N M}{r} \left[ 1 + \frac{41}{10 \pi} \frac{G_N \hbar}{r^2} + ... \right] \] (2.4)

they determined the \( \eta \)-value to be \( \eta = - \frac{41}{10 \pi} \).

Against this background, it is interesting to look for the quantum correction to the MOND form of the logarithmic potential which is consistent with the empirical baryonic Tully-Fisher law. We therefore propose the following modification to \( V^{(\log)} \) in an ASG

\[ V^{(ASG)} \rightarrow \sqrt{G_N M a_0} \ln \left( \frac{r}{r_0} \right) \left[ 1 + \frac{41}{10 \pi} \left( \frac{G_N}{r^2} + \frac{\gamma_M G_N^2}{r^3} \right) \right] \] (2.5)

The combination of (3.3) and (3.5) make up the departure from the MOND expectation in the background of ASG.

Keeping only order of \( G \) terms, we find a modified force for the above extended potential

\[ F^{(ASG)} \rightarrow - \sqrt{G_N M a_0} \left[ \frac{G_N \hbar}{r^3} \frac{41}{10 \pi} \left( 1 - 2 \ln \left( \frac{r}{r_0} \right) \right) + \frac{1}{r} \right] \] (2.6)

When compared with the centripetal force, the quantum-corrected square of the asymptotic velocity of the receding stars for a cut-off non-zero galactic radius \( (r_0) \) turns out to be

\[ v^{ASG} \rightarrow (G_N M a_0)^{\frac{1}{2}} \left( 1 + \frac{41}{10 \pi} \frac{G_N \hbar}{4 r_0^2} \right) > 0 \] (2.7)

This is somewhat higher than the Milgrom’s estimate of \( \sqrt{G_N M a_0} \) in the outskirts of spiral galaxies. However, we get back the classical result by turning off the quantum correction \( (h = 0) \). Interestingly, at very small \( r_0 \)-values, the quantum correction becomes pronounced. It is to be remarked that the velocity of the spiral galaxy would satisfy the constraint inequality

\[ (M G_N a_0)^{\frac{1}{2}} \leq v_0 \leq (M G_N a_0)^{\frac{1}{2}} \left[ 1 + \frac{41}{10 \pi} \frac{G_N \hbar}{4 r_0^2} \right] \] (2.8)

Few comments are in order. It needs to be pointed out that the running gravitational constant considered above is with respect to the Newtonian gravitational constant \( G_N \). Implication of MOND has already been studied for the Newtonian gravitational law. On the other hand, Milgrom’s works on the effects of MOND in the realm of Newtonian mechanics has opened up two clear distinctions: one for very low scales of acceleration
called the deep MOND regime and the other for large accelerations which coincide with what Newton’s second law dictates. This change of behaviour of the force of inertia with respect to the acceleration scale prompted us to look for the impact of a running gravitational constant not only for the Newton’s gravitation potential but also for its logarithmic tail. With constant velocity resulting from the logarithmic potential, it is of interest to enquire how such a potential is modified in the ASG theory. With the inclusion of \(O(h)\) effect, as we estimated in (2.8), a constraint inequality restricts the velocity of the rotation curve.

3 Summary

Signatures coming from the extragalactic regime, reveal a breakdown from the standard picture of Newton’s universal law of gravitation mainly at low scales of acceleration. Indeed, a clear-cut verification of some of the features of Newtonian dynamics for galaxies and galaxy clusters, and also to a larger measure, the general theory of relativity to any desired accuracy, is still awaited at low accelerations. In this background, we extended MOND to include the effect of the running Newtonian gravitational constant. By keeping terms of the \(O(G_N)\), our purpose was to determine \(O(h)\) contribution to the logarithmic potential called \(V^{(ASG)}\) as a correction to the already known ASG-modified Newtonian potential. Subsequently, we obtained the corresponding force and in consequence the quantum correction to the rotational velocity of a spiral galaxy.

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References

[1] B. Famaey and S. S. McGaugh, Living Rev. Relativity, 15 10 (2012).
[2] D. V. Bugg, Mond - a review, arXiv:1405.1695.
[3] M. Milgrom, Astro Phys. J. 270 365 (1983).
[4] M. Milgrom, Astro Phys. J. 270, 371, (1983); ibid. 270 384 (1983).
[5] M. Milgrom, Can. J. Phys. 93 107 (2015).
[6] R. B. Tully and J. R. Fisher, Astron. Astrophys. 54 661 (1977).
[7] C. Skordis and T. Zlosnik, Phys. Rev. Lett. 127 161302 (2021).
[8] C. Sivaram, K. Arun, and L. Rebecca, J. Astron. Astrophys. 41 4 (2020).
[9] P. M. Chesler and A. Loeb, Phys. Rev. Lett. 119, 031102 (2017).
[10] E. Garaldi, E. Romano-Diaz, C. Porciani, and M. S. Pawlowski, Phys. Rev. Lett. 120 261301 (2018).
[11] M. Milgrom, Phys. Rev, D 100 084039 (2019).
[12] V. C. Rubin, N. Thonnard, and W. K. Ford, Astro. J. 225 L107 (1978); V. C. Rubin, W. K. Ford, and N. Thonnard, ibid. 238 471 (1980).
[13] M. V. Berry, Principles of cosmology and gravitation, IOP Publishing Ltd. (1989).
[14] D. V. Bugg, Can. J. Phys., 91 668 (2013).
[15] E. Verlinde, JHEP 1104:029 (2011).
[16] B. Bagchi and A. Fring, Int. J. of Mod. Phys. B 33 1950018 (2019).
[17] E. Verlinde, Sci. Post Physics 2(3), 016 (2017).
[18] S. S. McGaugh, F. Lelli and J. M. Schombert, Phys. Rev. Lett. 117, 201101 (2016).
[19] J. Bekenstein and M. Milgrom, AstroPhys. J., 286 7 (1984).
[20] J. C. Fabris and J. Pereira Campos, Gen. Rel. Grav. 41 93 (2009).
[21] S. Das and S. Sur, Dark matter or strong gravity?, arXiv:2205.07153.
[22] M. Milgrom, XXV International school of theoretical physics ”Particles and Astrophysics - Standard Models and Beyond” - Ustroń, Poland (20010.
[23] R. Bonetto, Phys. Lett. A 382 2403 (2018).
[24] A. Bonanno et al, Front. Phys., 8, 269, (2020).
[25] S. Weinberg, Understanding the Fundamental Constituents of Matter, edited by A. Zichichi (Plenum Press, New York, 1978); General Relativity, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979)
[26] F. Saueressig, N. Alkofer, G. D’Odorico and F. Vidotto, PoS FFP14 174 (2016).
[27] J. M. Pawlowski and D. Stock, Phys. Rev. D 98 106008 (2018).
[28] A. Platania, Eur. Phys. J. C 79 470 (2019).
[29] L. Bosma, B. Knorr and F. Saueressig, Phys. Rev. Lett. 123 101301 (2019).
[30] A. Ishibashi, N. Ohta and D. Yamaguchi, Phys. Rev. D 104 066016 (2021).
[31] M. Nilton, J. Furtado, G. Alencar and R. R. Landim, Generalized Ellis-Bronnikov wormholes in asymptotically safe gravity, arXiv:2203.08860
[32] L. Smolin, Phys. Rev. D 96 083523 (2017).
[33] A. Eichhorn, Front. Astron. Space Sci. 5, 47 (2019)
[34] C. Wetterich, Quantum scale symmetry, arXiv:1901.04741.
[35] A. Bonanno and M. Reuter, Phys. Rev. D 62 043008 (2000); ibid. D 65 043508 (2002).
[36] R. Mandal and S. Gangopadhyay, Black hole thermodynamics in asymptotically safe gravity, Planck stars from asymptotic safe gravity, arXiv:2204.11616
[37] F. Scardigli and G. Lambiase, Planck stars from asymptotic safe gravity, arXiv:2205.07088
[38] C. Rovelli and F. Vidotto, Int. J. Mod. Phys. D 23 1442026 (2014)
[39] N. E. J. Bjerrum-Bohr, J. F. Donoghue and B. R. Holstein, Phys. Rev. D 67, 084033 (2003); J. F. Donoghue, Phys. Rev. D 50 3874 (1994).
[40] J. F. Donoghue, Front. Phys. 8 56 (2020).

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