Research Article

Heterogeneous Cognitive Radio Sensor Networks for Smart Grid: Markov Analysis and Applications

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Cognitive radio sensor networks (CRSN) have the potential to vastly improve spectrum utilization among heterogeneous applications for smart grid. To date, there has been little queueing theoretic modeling conducted of such systems that provide a quantitative estimate of the benefits from CRSN. We propose a novel queuing model which incorporates service rate and functional heterogeneity on the servers and implement preemptive priority among the varying service classes. Initially, we present a continuous-time Markov chain for performance analysis of CRSN for two colocated cognitive systems with various priority classes and bandwidth requirements. Closed form results for spectrum utilization, blocking probability, and optimal traffic intensities are then derived for the scenario of two heterogeneous secondary systems. A channel packing scheme is then proposed to pack smaller bandwidth users into clusters of adjacent channels to alleviate blockage of users requiring larger bandwidth requirements. Based on the numerical results of benefits of our scheme, we propose a feasible application for smart grid.

1. Introduction

Several recent studies on wireless spectrum usage have highlighted (temporal) underutilization of licensed spectrum as a key malaise. To improve spectrum utilization, the Federal Communications Commission (FCC) has suggested cognitive radio (CR) [1] as a broad policy to encourage new (cognitive or secondary) users to utilize licensed bands, provided their transmissions do not cause harmful interference to the authorized (primary) users. In addition, both ITU and Chinese government [2, 3] are pushing CR techniques into real application. Clearly, a key component of the CR etiquette places the burden of sensing network status on the secondary users, so as to proactively avoid causing excessive interference to existing users.

By their very definition, CRSN are largely heterogeneous in nature; that is, the colocated primary and secondary users will use different radio technologies. As such, there is great need for developing models for coexistence of such heterogeneous networks, yet almost all existing CRSN models focus exclusively on homogeneous networks (systems of like nodes). Consider the IEEE 802.22 standard that proposes reuse of the 52–862 MHz spectrum, part of which (in both VHF and UHF bands) is allocated for licensed TV broadcast. Since most of electricity utilities are lacking wireless spectrum resources, CRSN technologies significantly attract them for the low cost in construction and convenience in mobility.

Our work is not scenario specific in the sense that the models derived herein consider a suite of future possibilities. For example, since over-the-air TV broadcast schedules are largely deterministic, these licensed spectra are potential candidates for opportunistic sharing among heterogeneous secondary user networks when the primary user is off-air, which is known. Potential secondary users like new ultra-wideband (UWB) condition maintenance devices in smart grid, using 500 MHz channels, can coexist in such VHF bands along with Walkie Talkie (that operates at 400–470 MHz) over 200 KHz channels. In addition, FCC has permitted unlicensed devices to operate in certain parts of the abovementioned spectrum previously licensed, notably the
military (267–322 MHz) and aeronautical radio navigation (322–328.6 MHz) bands [4]. Since the user occupancy in these underutilized licensed bands is stochastic, these are well-suited for coexistence of primary and secondary users. Our models encompass both these cases and concentrate on system modeling that reflects the bandwidth heterogeneity inherent in such scenarios.

In this paper, a queuing theoretic model is developed for CRSN in two disparate scenarios: (i) coexistence of one primary and one secondary user network and (ii) coexistence of two cognitive heterogeneous secondary networks. The complexity of our model is enhanced due to varying bandwidth requirements, also termed in our paper as bandwidth heterogeneity, from primary and secondary users. Next, class-based service is assumed in which arriving users are immediately served based on server availability, subject to preemptive priority assigned to primary users. We have assumed no buffering for simplicity in mathematical analysis, derived from the underlying concept of $M/M/m/m$ model. It needs to be pointed out here that $M/M/m/m$ model assumes a constant value for the total number of servers $m$, but it does not satisfy the property of bandwidth heterogeneity. For better modeling of CRSN, we have also considered functional heterogeneity (different servers serve primary and secondary users) and service rate heterogeneity (service rates for the various traffic classes, primary and secondary, are variable). In a nutshell, our research contributions are as follows:

(i) Queuing theoretic modeling of critical aspects of CRSN using bandwidth heterogeneity, functional heterogeneity, and service rate heterogeneity along with preemptive priority for primary users.

(ii) Channel packing: this novel scheme [5] encouraging packing of users with smaller bandwidth requirements to adjacent subchannels, so as to improve spectrum utilization.

(iii) Closed form derivations of blocking probability and spectrum utilization for two secondary systems and for a special case of coexisting primary and secondary systems.

(iv) Numerical results of blocking probability and spectrum utilization for different sensing schemes in heterogeneous networks.

(v) Feasible applications of this CRSN technique for smart grid.

The rest of this paper is organized as follows. Related works are introduced in Section 2. The following Section 3 describes the system model and the formulation of CRSN for heterogeneous systems. Channel packing scheme is then proposed in Section 4. After that, Sections 5–7 provide the continuous-time Markov chain (CT-MC) channel model for one primary system and one secondary system in different scenarios; the performance analysis is also given in terms of spectrum utilization and blocking probability. Numerical results of coexisting multiple primary and secondary systems are shown in Section 8 in terms of various sensing schemes. A feasible application for smart grid is then provided in Section 9. The paper concludes with Section 10.

2. Related Work

The inclusion of bandwidth heterogeneity (with respect to primary and secondary user channel access) marks a clear distinction from the existing literature in cognitive radio networks, where performance modeling concentrated exclusively on the homogeneous case. For example, Chou et al. [6] provided the basic $M/G/1$ model for the CR network performance analysis, used by Tang and Mark [7] and Zhu et al. [8] to estimate blocking probability and queueing delay for primary and secondary systems with the same bandwidth requirements, while most realistic CR scenarios involve subnetworks with different channel bandwidths. Xing et al. [9] developed a continuous-time Markov chain (CT-MC) model for two types of cognitive secondary systems, but without any primary systems. Zhu et al. [10] presented CT-MC models for one primary and one secondary system for simple cases, but they assume that the spectrum has to book a pool of channels for secondary users, which is not easy to implement in reality. Raspopovic and Thompson [11] evaluated channel access availability of narrow and wideband users, but functional heterogeneity and service rate heterogeneity were not considered.

Moreover, existing research in queuing theory has considered service rate and functional heterogeneity separately and not together. For example, in [12], Dharmaraja considered only service rate heterogeneity, that is, a system consisting of servers with varying service rates. The varying service rates are utilized in serving jobs from a single queue to minimize delay and increase system throughput. In [13], Lippolt et al. considered service rate heterogeneity and functional heterogeneity as separate cases for two classes of services. The authors claimed that compared to service rate heterogeneity, functional heterogeneity has a substantial impact on the system performance.

3. Modeling with Heterogeneous Primary and Secondary Networks

3.1. Heterogeneous Systems. In our system model shown in Figure 1, we consider opportunistic sharing of the licensed spectrum in heterogeneous networks. Channel allocation depends on the bandwidth requirements for heterogeneous systems and specific search schemes, which will be discussed in Section 4. In this paper, we assume that the available system bandwidth $B_{\text{overall}}$ with $M$ licensed channels is represented as

$$B_{\text{overall}} = MB_c,$$  \hspace{1cm} (1)

where $B_c$ is the bandwidth of a channel.

First, we consider the case of two colocated systems, one primary and the other secondary, and the channel bandwidth demand for the systems satisfies

$$B_p = KB_p,$$  \hspace{1cm} (2)

where $B_p$ and $B_s$ are the bandwidth requirements of primary and secondary users, respectively. Two scenarios of $K \geq 1$ and $0 < K < 1$ will be separately discussed. The former scenario implies that $K$ is the number of subchannels
| Channel 1 | Channel 2 | Channel 3 | ... | Channel M |
|-----------|-----------|-----------|-----|-----------|

Heterogeneous networks

$B_p = B_s$

Service rate heterogeneity

- PU server
  - $\mu_p$
- SU servers
  - $\mu_s$

Functional heterogeneity

- [K = 3]
  - $(A + 1, B)$
  - $\lambda_p (A + 1) \mu_p$
  - $\lambda_s (B + 1) \mu_s$

$b$: $0 < K < 1$

$K$: ratio of $B_p$ to $B_s$ ($K = B_p / B_s$)

$B_L = 2B_C$

Service rate heterogeneity

- SU (larger BW requirement) server
  - $\mu_L$
- SU (smaller BW requirement) servers
  - $\mu_s$

Functional heterogeneity

- [K = 1/3]
  - $(C + 1, D)$
  - $\lambda_L (C + 1) \mu_L$
  - $\lambda_S (D + 1) \mu_S$

$c$: $L = 1/2, k = 2$

$B_L / \lambda_L / \mu_L$: bandwidth/arrival rate/departure rate of secondary users with larger (smaller) bandwidth requirements

$K$: ratio of $B_L$ to $B_S$ ($K = B_L / B_S$)

$L$: ratio of $B_L$ to $B_C$ ($L = B_L / B_C$)

$B_{L,1}/\lambda_{L,1}/\mu_{L,1}$: bandwidth/arrival rate/departure rate of primary (secondary) users

**Figure 1**: Left side: bandwidth heterogeneity of spectrum access in heterogeneous networks with (a) one primary system and one secondary system, $K \geq 1$, (b) one primary system and one secondary system, $0 < K < 1$, and (c) two different secondary systems. Right side: Markovian model without boundary conditions.
each channel by assuming $K$ to be an integer. In this case, the bandwidth requirement of primary users is no less than that of secondary users. The latter scenario implies that each channel is composed of $1/K$ subchannels by assuming $1/K$ to be an integer, which means a secondary user occupies more spectrum resources compared with a primary user. Additionally, we assume that a system with larger bandwidth requirements will occupy a channel per access. Thus a primary (secondary) user occupies a channel (subchannel) per access in the $K \geq 1$ scenario and a subchannel (channel) per access in the $0 < K < 1$ scenario. Consider

$$
B_c = B_p, \quad K \geq 1,
$$
$$
B_c = B_s, \quad 0 < K < 1.
$$

(3)

Examples of both scenarios are demonstrated in Figures 1(a) and 1(b).

Further, we also consider the case of two different secondary systems. The channel bandwidth for the two secondary systems satisfies

$$
B_L = kB_S,
$$

(4)

where $k$ is a positive integer and the subscripts $L$ ($S$) indicate larger (smaller) bandwidths, respectively. Here, we assume that $B_c = LB_L$. Thus, we have

$$
B_{overall} = MB_c = MLB_L = MLkB_S,
$$

(5)

where we assume that $ML$ is a positive integer ($L \geq 1/M$). An example of this scenario is shown in Figure 1(c).

3.2. Markov Chain Modeling of CRSN. Detailed Markovian models will be presented in Sections 5-6, and we just show basic examples of Markov chain models in Figure 1. Since primary users have prioritized access to the spectrum compared to secondary users, they are unaware of secondary users’ occupation of the spectrum. Thus, the arrival rate of primary users into the network is independent of the presence of secondary users. The primary users’ arrivals follow Poisson distribution with rate $\lambda_p$, and their service time is exponentially distributed with rate $\mu_p$. Similarly, secondary users are also Poisson with rate $\lambda_s$ and exponential service rate $\mu_s$. A secondary user is forced to immediately release a channel due to the arrival of any primary user and instantaneously transition into other available spectrum resources. Spectrum sharing among the queued primary and secondary users is interpreted via Markovian description, shown in Figure 1. The possible state transitions include the following: (i) with rate $\lambda_p$, a new primary user $(A + 1)$ from the priority queue occupies a channel; (ii) a free subchannel is occupied by the next incoming secondary user $(B + 1)$ with rate $\lambda_s$; (iii) transition occurs from states $(A+1, B)$ or $(A, B+1)$ to the state $(A, B)$ with service rate $(A+1)\mu_s$ or $(B+1)\mu_s$, due to departure of primary or secondary users, respectively.

For the scenario of two different secondary systems, we assume that users with larger bandwidth requirements arrive with Poisson process of rate $\lambda_L$ and their service time is exponentially distributed with rate $\mu_L$. On the other hand, users with smaller bandwidth requirements follow Poisson arrival and exponential departure with rates $\lambda_S$ and $\mu_S$. Similar to the scenario with one primary and one secondary system, the transitions in the basic Markov chain among states $((C, D), (C + 1, D)$, and $(C, D + 1))$ are illustrated in Figure 1.

3.3. Heterogeneous Systems: Formulation. Multiple radio systems are presumed to operate in the band of interest in the presence of a master node that serves as the access controller for arbitration among heterogeneous systems. The rules of spectrum occupancy for the different types of users are defined below. Note that a primary user can only be blocked by other primary users but a secondary user can be blocked by other secondary and primary users.

Rule 1. Arrival of a primary user wishing to access a channel currently occupied by secondary user(s) will cause the secondary user(s) to immediately vacate those occupied spectral resources.

Rule 2. The secondary user that vacates a channel (or subchannel) due to the arrival of a primary user will occupy other available spectral resources in a very short transition time $T_s$. 

Rule 3. An arrival of a secondary user wishing to access a channel (or subchannel) currently occupied by another secondary/primary user will cause blockage of the new secondary user.

4. Channel Packing Scheme

As can be understood from Section 3, efficient channel sensing impacts the overall performance in heterogeneous CRSN, generally speaking. While the choice of (optimal) sensing schemes in CRSN is a new direction of research [14–18], none of the cited work actually integrates the role of channel sensing into the analysis of heterogeneous system performance. To alleviate the problem encountered in serial search (SS) or random search (RS) sensing in heterogeneous networks, that is, unnecessary secondary user blocking, we introduce a novel noncooperative sensing scheme called channel packing scheme (CPS).

CPS comprises two steps: in the first step, an incoming user with smaller bandwidth requirement identifies a channel that includes subchannels already occupied by other users of the same type. At the next step, the first available subchannel in sequence is allocated for this new primary (secondary) user. We assume that each channel is composed of $r$ subchannels. The scheme is described as below:

1. Serially search and identify a channel with some subchannels already occupied by users of the same type.
2. If such a channel exists, use serial search to occupy an available subchannel inside it.
3. Otherwise, repeat step (1).
(4) If such a channel is not found over the whole spectrum, serially search for any available subchannel from the first channel of the band.

(5) If a user of the same type finishes the occupation and leaves a spectrum hole, the last user in sequence moves to this spectrum hole.

In CPS, step (3) occurs when the targeted channel is fully occupied either by primary or by secondary users. In this case, all subchannels inside this targeted channel are all unavailable, and thus the incoming users will serially search for the next channel which contains available subchannels. The number of trials to repeat step (1) depends on occupancy of users and successful detection probability. For example, we assume $N$ existing users with larger bandwidth requirement are identically independently distributed among these $M$ channels, which means that there are $M - N$ available channels. Then the average number of repeating step (1) will be $(M - N)/M$ if we assume perfect sensing [19].

CPS avoids unnecessary blocking from traditional non-cooperative sensing schemes, because users with smaller bandwidth requirements will be packed in clusters by step (1). Thus an incoming user with larger bandwidth requirement will not be blocked if the remaining available spectrum resource is sufficient. The primary reason we use serial instead of random search in CPS is that SS can pack users with smaller bandwidth requirement can be accommodated. With ideal channel packing, the largest number of primary users $h$ that can operate is

$$h = M - \left\lceil \frac{n}{r} \right\rceil,$$

where $\left\lceil x \right\rceil$ denotes the smallest integer that is no less than $x$ and $h$ gives the upper bound for $(h + 1)r + n > Mr$.

5. One Primary System and One Secondary System, $K \geq 1$

We first analyze the performance of two colocated systems, one primary and the other secondary, in both single-channel ($M = 1$) scenario and multichannel ($M > 1$) scenario where $K \geq 1$. The secondary users can explore the allocated bandwidth for spectrum holes to fill. On the other hand, primary users can preemptively force existing secondary users to evacuate. As is well known, conventional non-cooperative sensing schemes such as RS or SS have the same performance (in terms of mean time to detect a spectrum hole) in this scenario [20]. Accordingly, we focus on the Markovian analysis below and not on the specific sensing mode.

5.1. Markovian Analysis for Single-Channel ($M = 1$) Scenario

We first note that since primary users will not be impacted by the presence of secondary users, the behavior of primary system can be modeled straightforwardly as $M/M/1/1$ queue [21] with mean arrival rate $\lambda_p$ and corresponding service rate $\mu_p$. A Markovian description of the heterogeneous system is based on state $(i, j)$ that denotes $i$ primary and $j$ secondary users in the system. Based on the CT-MC diagram shown in Figure 2, the steady-state equations are then given by

$$\mu_p P_{1,0} = \lambda_p \sum_{i=0}^{K} P_{0,i},$$

$$\sum_{i=0}^{K} P_{1,i} = 1.$$
In this scenario, the average spectrum utilization of primary and secondary systems is

\[
\overline{SU}_p = P_{1,0} = \frac{\rho_p}{1 + \rho_p},
\]

\[
\overline{SU}_s = \frac{1}{K} \sum_{i=0}^K iP_{0,i}.
\]  \hfill (9)

A primary user coming into the spectrum is blocked only if there is another primary user occupying the channel. On the other hand, a secondary user is blocked if the channel is already occupied either by a primary user or by \(K\) secondary users. Thus, the blocking probability of the primary and secondary users is

\[
P^b_p = \frac{\rho_p}{1 + \rho_p}, \hfill (10)
\]

\[
P^b_s = \frac{\rho_p}{1 + \rho_p} + P_{0,K}.
\]

For the special case of \(K = 1\), we can derive the steady-state probabilities as

\[
P_{1,0} = \frac{\rho_p}{1 + \rho_p},
\]

\[
P_{0,0} = \frac{\rho_p \gamma + \gamma}{(1 + \rho_p)(\rho_p + \gamma + \rho_s \gamma)}, \hfill (11)
\]

\[
P_{0,1} = \frac{\rho_s \gamma}{(1 + \rho_p)(\rho_p + \gamma + \rho_s \gamma)},
\]

where \(\rho_s = \lambda_s / \mu_s\) is the traffic intensity of secondary users and \(\gamma = \mu_s / \mu_p\) is the ratio of the service rates of secondary and primary users. The closed-form blocking probability and average spectrum utilization of the secondary system are

\[
\overline{SU}_s = P_{0,1} = \frac{\rho_s \gamma}{(1 + \rho_p)(\rho_p + \gamma + \rho_s \gamma)},
\]

\[
P_s^b = P_{1,0} + P_{0,1} = \frac{1}{1 + \rho_p} \left( \frac{\rho_s \gamma}{\rho_p + \gamma + \rho_s \gamma} + \rho_p \right).
\]  \hfill (12)

5.2. Case Study: \(M = 1, K = 2\). Solving the steady-state equations (7) by applying \(M = 1, K = 2\), we derive the steady-state probabilities for this scenario as

\[
P_{1,0} = \frac{\rho_p}{1 + \rho_p},
\]

\[
P_{0,0} = \frac{(\rho_p^2 + 3\rho_p \gamma + \rho_p \rho_s \gamma + 2\gamma^2)}{D},
\]

\[
P_{0,1} = \frac{\rho_s \gamma (\rho_p + 2\gamma)}{D},
\]

\[
D = \left(1 + \rho_p\right) \cdot \left(\rho_p^2 + 3\rho_p \gamma + 2\rho_p \rho_s \gamma + 2\gamma^2 + 2\rho_s \gamma^2 + \rho_s^2 \gamma^2\right).
\]  \hfill (13)

The spectrum utilization and blocking probability for secondary users in this single-channel scenario are given by

\[
\overline{SU}_s = \frac{P_{0,1} + P_{0,2}}{2} = \frac{(\rho_s \gamma / (\rho_p + \gamma) + \rho_s \gamma^2)}{D},
\]

\[
P_s^b = P_{1,0} + P_{0,2} = \frac{\rho_p}{1 + \rho_p} + \frac{\rho_s \gamma^2}{D}.
\]  \hfill (14)

From (14), if \(\rho_p \gg \rho_s\), we have \(P_s^b \approx \rho_p / (1 + \rho_p)\). This implies that \(\rho_p\), that is, the traffic intensity of primary users, is the dominant factor in deciding the blocking probability of secondary users. Conversely, for \(\rho_p \ll \rho_s\), the spectrum utilization and blocking probability for secondary users are expressed as

\[
\overline{SU}_s = \frac{\rho_s^2 + 2\rho_s}{\rho_s^2 + 2\rho_s + 2},
\]

\[
P_s^b = \frac{\rho_p}{\rho_s^2 + 2\rho_s + 2}.
\]  \hfill (15)

Since higher traffic intensity of primary users leads to higher blocking probability for secondary users, the upper bound and the lower bound of blocking probability for secondary users are \(\rho_p / (1 + \rho_p)\) and \(\rho_s^2 / (\rho_s^2 + 2\rho_s + 2)\), respectively.

5.3. Multichannel \((M > 1)\) Scenario. With the same set-up as before, the state transition diagram for the case of \(M\) - channel, \(K\) subchannels in each channel, is illustrated in Figure 3. Note that if a new primary user arrives and evicts \(K\) secondary users, state \((M - 2, 2K - 1)\) does not transition to \((M - 1, K - 1)\) as expected, because the remaining one subchannel \((MK - (M - 1)K - (K - 1) = 1)\) will be immediately occupied by one of the evicted secondary users. Thus state \((M - 2, 2K - 1)\) enters state \((M - 1, K)\) upon an arrival of a new primary user. This holds for other transitions as well, that is, from state \((M - i, q)\) to \((M - i + 1, (i - 1)K)\), where \(i\) is an integer \((i \in [1, M] \text{ and } q \in [(i - 1)K + 1, iK])\). Since primary users are not impacted by secondary users, channel access of the primary system is treated as a multiserver queue. Let \(P(m)\) denote the steady-state probability of \(m\) primary users in the band. Therefore, the steady-state equations are given by

\[
\lambda_p P(m - 1) = m \mu_p P(m) \quad m \in [1, M],
\]

\[
\sum_{j=0}^M P(j) = 1.
\]  \hfill (16)
Solving, we get

\[ P(m) = \frac{\rho_p^m / m!}{\sum_{i=0}^{M} \rho_p^i / i!}. \]  

(17)

The spectrum utilization and blocking probability are thus given by

\[ SU_p = \sum_{m=0}^{M} m P(m) = \frac{\sum_{j=0}^{M} \rho_p^j / (j! M)}{\sum_{i=0}^{M} \rho_p^i / i!}, \]  

(18)

\[ P_b^p = P(M) = \frac{\rho_p^M / M!}{\sum_{i=0}^{M} \rho_p^i / i!}. \]

We assume that the processing rate is proportional to the serving bandwidth, implying that \( \gamma = \mu_s / \mu_p = 1/K \). We define the overall spectrum utilization as the sum of spectrum utilizations of both primary and secondary systems. Numerical results for different values of \( M \) are shown in Figure 4. For the same \( \rho_p \) and \( \rho_s \), higher values of \( M \) result in

\[ SU = SU_p + SU_s \]

\[ P_b = P_b^p + P_b^s \]

We assume that the processing rate is proportional to the serving bandwidth, implying that \( \gamma = \mu_s / \mu_p = 1/K \). We define the overall spectrum utilization as the sum of spectrum utilizations of both primary and secondary systems. Numerical results for different values of \( M \) are shown in Figure 4. For the same \( \rho_p \) and \( \rho_s \), higher values of \( M \) result in
lower spectrum utilization, as a result of more vacant serving channels. Moreover, higher $\rho_p$ and $\rho_s$ lead to higher spectrum utilization.

6. One Primary System and One Secondary System, $0 < K < 1$

In this scenario, the whole band can support at most $M/K$ primary users at the same time. Similar to the derivations in Section 5, the steady-state probability for $m$ primary users in the band is $P(m) = (\rho_p^m / m!) \sum_{i=0}^{M/K} \rho_s^i / i!$. Thus the spectrum utilization and blocking probability of the primary system are

$$\bar{SU}_p = \sum_{m=0}^{M/K} \frac{m}{M/K} P(m) = \frac{\sum_{i=0}^{M/K} \rho_p^i / i!}{\rho_p^m / m!},$$

$$p^b_p = P(M/K) = \frac{\rho_p^{M/K} / (M/K)!}{\sum_{i=0}^{M/K} \rho_p^i / i!}.$$

6.1. Markovian Analysis for Single-Channel ($M = 1$) Scenario

Similarly as we draw the CT-MC diagram in Section 5, the steady-state equations are given by

$$P_{0,1} (\lambda_p + \mu_s) = P_{0,0} \lambda_s,$$

$$P_{0,0} (\lambda_p + \lambda_s) = P_{i,0} \mu_p + P_{i,1} \mu_s,$$

$$P_{1,0} \mu_p = P_{0,0} \lambda_p + P_{1,1} \lambda_s,$$

$$P_{0,0} \lambda_p = P_{i+1,0} (i+1) \mu_p, \quad i \in \left[1, \frac{1}{K} - 1\right].$$

Solving these equations for the steady-state probabilities, we have

$$P_{j,0} = \frac{\rho_p^j / j!}{\sum_{i=0}^{M/K} \rho_p^i / i!}, \quad j \in \left[1, \frac{1}{K}\right],$$

$$P_{0,0} = \frac{(\rho_p + \rho_s \gamma) / (\rho_p + \rho_s \gamma + \gamma)}{\sum_{i=0}^{M/K} \rho_p^i / i!},$$

$$P_{0,1} = \frac{\gamma / (\rho_p + \rho_s \gamma + \gamma)}{\sum_{i=0}^{M/K} \rho_p^i / i!}.$$

For $\bar{SU}_s = P_{0,1}$ and $p^b_s = 1 - P_{0,0}$ we have the following:

$$\bar{SU}_s = \frac{\gamma / (\rho_p + \rho_s \gamma + \gamma)}{\sum_{i=0}^{M/K} \rho_p^i / i!},$$

$$p^b_s = 1 - \frac{(\rho_p + \rho_s \gamma) / (\rho_p + \rho_s \gamma + \gamma)}{\sum_{i=0}^{M/K} \rho_p^i / i!}.$$

6.2. Multichannel ($M > 1$) Scenario

Although different sensing schemes do not impact system performance for $K > 1$ scenario, they DO impact the multichannel $0 < K < 1$ scenario. The reason is that secondary users with larger bandwidth requirements may be blocked by primary users with smaller bandwidth requirements due to preemptive priority of the latter, even if there are enough spectral resources. This unnecessary blockage can be attributed to the characteristics of the sensing scheme employed that causes users with smaller bandwidth to occupy subchannels that are scattered over the entire spectrum.

Applying CPS, the CT-MC diagram for this multichannel scenario is shown in Figure 5. Note that state $(i/K, M - i)$ denotes the full occupancy of the whole band. Any arrival of a secondary user will be blocked at this state but any arrival of a primary user will terminate the transmission of secondary users occupying the channel. Thus the arrival rate from state $(i/K, M - i)$ to $(i+1/K, M - i - 1)$ is $\lambda_p$, but state $(1+i/K, M - i - 1)$ can go to back to $(i/K, M - i)$. The steady-state equations are given by

$$P_{i,0} (\lambda_p + \lambda_s) = P_{i,0} \mu_p + P_{i,1} \mu_s,$$

$$P_{i,0} (\lambda_p + \lambda_s + i \mu_p) = P_{i-1,0} \lambda_p + P_{i+1,0} (i+1) \mu_p + P_{i+1,0} (i+1) \mu_p,$$

$$P_{i,0} (\lambda_p + i \mu_p) = P_{i-1,0} \lambda_p + P_{i+1,0} (i+1) \mu_p,$$

$$P_{M/K,0} \mu_p = P_{M/K-1,0} \lambda_p,$$

$$P_{0,j} (\lambda_p + \lambda_s + j \mu_s) = P_{0,j-1} \lambda_s + P_{0,j} \mu_p + P_{0,j+1} (j+1) \mu_s,$$

$$P_{0,M} (\lambda_p + M \mu_s) = P_{0,M-1} \lambda_s,$$

$$P_{i,j} (\lambda_p + \lambda_s + i \mu_p + j \mu_s) = P_{i-1,j} \lambda_p + P_{i+1,j} (i+1) \mu_p + P_{i+1,j} (i+1) \mu_p + P_{i+1,j+1} (j+1) \mu_s,$$

$$j \in \left[1, M - 1\right], \quad i \in \left[1, \frac{M-j-1}{K}\right].$$
where $\Phi$ is the set of possible states in this scenario. We express the overall spectrum utilization and blocking probability of secondary users as

$$SU_{\text{overall}} = \sum_{(i,j)\in \Phi} iK + j \frac{P_{i,j}}{M},$$

$$P_{b_s} = P_{0,M} + \sum_{j=0}^{M-1} \frac{(M-j)/K}{j} \sum_{i=(M-j-1)/K+1}^{i} P_{i,j}.\tag{24}$$

6.3. Comparison with Other Sensing Schemes. Because it is difficult to describe the CT-MC diagram for different sensing schemes such as RS or SS for the general multichannel scenario, we consider a specific case of $M=2$, $K=1/2$ to illustrate the above point. By assuming $\rho_p = 1.2$ and $\gamma = \mu_s/\mu_p = 1/K = 2$, we present numerical results in Figure 6. It can be seen that SS performs better than RS. This is because SS can pack some of primary users with smaller bandwidth into "clusters," thereby alleviating blockage to other secondary users with larger bandwidth requirements.

We continue with this case to investigate the performance of CPS vis-à-vis the conventional sensing schemes. Figure 6 shows that CPS has a much lower blocking probability than both SS and RS. Moreover, CPS has higher spectrum utilization than both conventional schemes. When $\rho_s = 2$, CPS has almost 10% and 20% gains in spectrum utilization compared to SS and RS. In this case, the blocking probability of CPS is just 40% of SS and RS. The more crowded the band (higher $\rho_s$), the more the unnecessary blockage of secondary users and the higher the gains from CPS.
7. Two Secondary Systems

The primary user occupancy is stochastic in major sections of the spectrum under consideration (52–862 MHz) in the IEEE 802.22 standard. Study of spectrum sharing in those bands between the primary and opportunistic secondary users is challenging (studied in Sections 5–6), since FCC has imposed transmission power limits to avoid interference on adjacent primary users. On the contrary, for over-the-air television broadcasting, the user occupancy is deterministic and scheduled over known periods of time. In such bands, the spectral resources are available for secondary systems over known time durations when the primary user is absent. In this section, we focus our attention on multiple heterogeneous secondary systems using the known idle durations in those deterministic television bands.

As before, we investigate the Markovian performance of two heterogeneous secondary systems in an $M$-channel band, where $M$ is the number of secondary systems using the known idle durations in those deterministic television bands.

We introduce a new parameter for spectrum utilization as a function of the blocking probability, termed as average effective spectrum utilization or EFU. It is measured as

$$\text{EFU} = \text{SU}_{\text{overall}} \times (1 - P_b^{\text{overall}}),$$

where the overall spectrum utilization $\text{SU}_{\text{overall}}$ is the sum of the spectrum utilizations of heterogeneous secondary systems and the overall blocking probability $P_b^{\text{overall}}$ is defined as the probability that ANY secondary user is denied. Thus, $P_b^{\text{overall}} = (P_b^S \lambda_S + P_b^L \lambda_L)/(\lambda_S + \lambda_L)$ and $\text{SU}_{\text{overall}} = \text{SU}_S + \text{SU}_L$.

It is not apropos to describe the average effective spectrum utilization or overall blocking probability in Sections 5 and 6, since there are different priorities among primary and secondary users. However, all heterogeneous secondary users hold the same priority in this section, leading to the introduction of both overall blocking probability and average effective spectrum utilization.

7.1. Markovian Analysis for $ML = 1$. We start from the simplest scenario, where the whole band can support at most one secondary user with larger bandwidth. Let state $(L, S)$ denote the numbers of secondary users with larger and smaller bandwidth requirements. The steady-state probability equations can be formulated as

$$\frac{\rho_s}{i} P_{0,j-1} = P_{0,j}, \quad i \in [1, k],$$

$$\rho_L P_{0,0} = P_{1,0},$$

$$P_{1,0} + \sum_{j=0}^{k} P_{0,j} = 1.$$

Solving these equations, we have

$$P_{0,j} = \frac{\rho_S^j / j!}{(\sum_{i=0}^{k} (\rho_S^i/j! + \rho_L))},$$

$$P_{1,0} = \frac{\rho_L}{(\sum_{i=0}^{k} (\rho_S^i/j! + \rho_L))}.$$

The overall spectrum utilization and blocking probabilities are then derived as

$$\text{SU}_{\text{overall}} = P_{1,0} + \sum_{j=0}^{k} \frac{j}{k} P_{0,j}$$

$$= \frac{(\rho_L + \sum_{j=1}^{k} (\rho_S^j/(j-1)!))}{(\sum_{i=0}^{k} (\rho_S^i/j! + \rho_L))},$$

$$P_b^S = P_{1,0} + P_{0,k} = \frac{(\rho_L + \rho_S^k/k!)}{(\sum_{i=0}^{k} (\rho_S^i/j! + \rho_L))},$$

$$P_b^L = 1 - P_{0,0} = 1 - \frac{1}{(\sum_{i=0}^{k} (\rho_S^i/j! + \rho_L))}. $$

\[\text{SU}_{\text{overall}} = \text{SU}_{L,S} + \text{SU}_{L,S} \]

\[P_b^S = P_{1,0} + P_{0,k} \]

\[P_b^L = 1 - P_{0,0} \]
The overall blocking probability is given by

\[ P_{\text{overall}}^b = \frac{\rho_L + \tau \sum_{j=0}^{k} \left( \rho_S^j / j! \right) + (1 - \tau) \rho_S^k / k!}{\sum_{j=0}^{k} \left( \rho_S^j / j! \right) + \rho_L}, \]  

(31)

where \( \tau = \lambda_L / (\lambda_S + \lambda_L) \). We assume that the processing rate is proportional to the serving bandwidth, which means \( \mu_L = k \mu_S \). Therefore we have \( \tau = \rho_L / (\rho_L + \rho_S / k) \), and the overall blocking probability is given by

\[ P_{\text{overall}}^b = 1 - \frac{\rho_L + (\rho_S / k) \sum_{i=0}^{k-1} \left( \rho_S^i / i! \right)}{\left( \rho_L + \sum_{i=0}^{k} \left( \rho_S^i / i! \right) \right) \left( \rho_L + \rho_S / k \right)}. \]  

(32)

Now the blocking probability for the secondary system with larger bandwidth requirement satisfies

\[ \rho_L = \frac{1}{1 - P_L^b} - \frac{k \rho_S}{\prod_{i=0}^{k} \left( \rho_L + \rho_S / i! \right)}. \]  

(33)

Substituting (33) in (28), we have

\[ \overline{SU}_{\text{overall}} = 1 - \left( 1 - P_L^b \right) \sum_{i=0}^{k} \frac{\rho_S^i (k - i)}{k}. \]  

(34)

Equation (34) is a concave function in terms of \( \rho_S \), treating \( P_L^b \) as a constant. In other words, we seek to explore the optimization of (34) as a function of the system parameters with smaller bandwidth requirements. Local maxima of \( \overline{SU}_{\text{overall}} \) obtained by taking the 1st-order partial derivative lead to saddle points that are out of the achievable region of \( \rho_S \). Because \( P_L^b \) is fixed, it follows from (34) that maximizing \( \overline{SU}_{\text{overall}} \) is equivalent to minimizing \( f(k, \rho_S) = \sum_{i=0}^{k} \left( \rho_S^i / i! \right) (k - i) / k \). For any given \( k \), \( f(k, \rho_S) \) is a monotonic function of \( \rho_S \geq 0 \). Therefore, the optimal values for \( \rho_S \), \( \rho_L \), and \( \overline{SU}_{\text{overall}} \) are

\[ (\rho_S)_{\text{optimal}} = 0, \]  

\[ (\rho_L)_{\text{optimal}} = P_L^b, \]  

\[ (\overline{SU}_{\text{overall}})_{\text{max}} = P_L^b. \]  

(35)

7.2. General ML > 1 Scenario. The generalized case to formulate the heterogeneous networks of two different secondary systems is that \( ML > 1, k \geq 1 \). The CTMC diagram of the generalized case is based on the channel accessing scheme. Figure 7 shows the CTMC diagram for CPS. From the graph we can formulate the steady-state probability equations as below:

\[ \begin{array}{ccccccc}
\rho_S P_{0,0} = P_{0,1} & \rho_S^2 P_{0,1} = P_{0,2} & \cdots & \rho_S^{ML} P_{0,ML-1} = P_{0,MLk} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_S P_{ML-1,0} = P_{ML-1,1} & \cdots & \rho_S^{ML} P_{ML-1,ML-1} = P_{ML-1,ML-0} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_L P_{0,0} = P_{1,0} & \rho_L^2 P_{1,0} = P_{2,0} & \cdots & \rho_L^{ML} P_{ML-1,0} = P_{ML,0} \\
\sum_{i=0}^{ML} \sum_{j=0}^{ML-1} P_{i,j} = 1.
\end{array} \]  

(36)

In addition, the equations above also satisfy

\[ \rho_L^i P_{i,j} = P_{i,j}, \quad i, j \in \Omega, \]  

(37)

where \( \Omega \) is denoted as the set of available steady states. Solving these equations we can derive the steady-state probability for generalized state \( P_{e,f} \) as

\[ P_{e,f} = \frac{\rho_S^e \rho_L^f (e! f!)}{\sum_{i=0}^{ML} \sum_{j=0}^{ML-1} (\rho_S^e \rho_L^f / i! f!)}. \]  

(38)

The blocking probability and average effective spectrum utilization are then given by
7.3. Comparison with Other Sensing Schemes. In order to show the advantage of CPS over other conventional sensing schemes, we analyze the performance of RS and SS in the multichannel scenario for heterogeneous secondary networks. Because the CTMC diagrams of RS and SS are very complex for the large values of $M_L$ and $k$, we consider the simplest case of $M_L = 2$, $k = 2$ to explore their performance by giving numerical results.

Figures 8-9 demonstrate the performance of three sensing schemes. It can be noticed that SS has lower blocking probability and higher average effective spectrum utilization than RS, although the gain is not significant. The lower probabilities of these two states result in less unnecessary blockage of secondary users with larger bandwidth requirements, hence leading to better performance. On the other hand, these two states will not happen in CPS. Therefore CPS has around 10% gain in blocking probability and 15% gain in average effective spectrum utilization over RS and SS.

8. Numerical Results of Multiple Heterogeneous Networks in Multiband Scenario

The generalized heterogeneous network can support multiple types of primary systems and secondary systems simultaneously operating in the same band. However, it is very difficult to describe the CT-MC diagram in a multidimensional plane and obtain the closed form analytical derivations; we then evaluate the performance of conventional RS and our CPS scheme in numerical results for a three-system scenario via
Matlab. Here we assume that the primary system and the secondary system with larger bandwidth requirements will occupy a channel per access. The other secondary system with smaller bandwidth requirements will occupy a subchannel per access. We assume no buffers for queueing and $M = 5$, $L = 1$, and $k = 2$. We set the parameters as $\mu_p = \mu_L = 2\mu_S$, where we have a utilization factor $\xi_S = \lambda_S/Mk\mu_S = 0.6$. Based on the observation that primary users will not be impacted by secondary users, we tune the utilization factors of the primary system and the secondary system with smaller bandwidth requirements ($\xi_p = \lambda_p/M\mu_p$, $\xi_L = \lambda_L/M\mu_L$) to investigate the overall spectrum utilizations of secondary systems under RS and CPS. Note that the spectrum utilization of the primary system is constant for a given value of $\xi_p$.

When the band is not fully occupied, any incoming cognitive secondary user with smaller bandwidth requirement will find an available subchannel in a very short time even though the first try fails, no matter via CPS or RS. If some secondary users with smaller bandwidth requirements are forced to terminate by primary users, they can also locate the available subchannels in a very short transition time. Thus the gain of CPS mainly comes from the benefit on the secondary system with larger bandwidth requirements. It can be noticed from Figure 10 that applying CPS leads to 10%–20% gain in spectrum utilization of secondary systems, especially for $\xi_p < 1$.

9. Feasible Applications for Smart Grid

In view of smart grid, the communication infrastructure is divided into three parts: wide area network (WAN), neighbor area network (NAN), and home area network (HAN) [22]. Multiple applications with heterogeneous communication methods will be applied in data transmission among power plants, substations, and users for controlling and monitoring. For example, some of sensors are using UWB technologies for condition monitoring while other sensors, such as remote terminal units (RTU), are using Zigbee techniques for controlling. Meanwhile, the technicians are likely to apply Walkie Talkie or FM radios which are operating at the same band at the same place. Figure 11 shows the scenario of such a situation where heterogeneous CRSN methods are applied in smart grid. In this scenario, intelligent electronic devices (IED) are connected to the control center by fiber wired communications at the transmission substations while...
wireless methods are required at the distribution substations and for home users due to the cost of constructions.

Because most of the electricity utilities are lacking wireless spectral resources, the mobile sensors applied heterogeneous CR technologies are then working at NAN and HAN to provide data transmission, where the QoS is not as high as data communications at the transmission substation side. For example, monitoring sensors can be seen as the primary users because of the security for the core data of utilities. On the other hand, smart meters can be assumed as secondary users since the data shared between home consumers and electricity utilities required low level of QoS. The different techniques and interfaces used by monitoring sensors and smart meters lead to different serving bandwidth in the same communication environment, which perfectly satisfies our heterogeneous model. We apply our CRSN approach on these IEDs so that they are able to perform efficient channel sensing in the smart grid scenario with all kinds of various equipment of distinct serving bandwidth.

10. Conclusions

CT-MC models of CRSN in heterogeneous networks for smart grid have been presented in this paper. By targeting cases of colocated primary and secondary systems with different bandwidth requirements, the analysis in the single-channel scenario provides the closed form derivations of the blocking probability and spectrum utilization for some special cases. In addition, a noncooperative channel packing scheme is proposed to alleviate the unnecessary blockage. Numerical results show that our CPS scheme can significantly decrease the blocking probability and increase the spectrum utilization, compared with traditional sensing schemes such as random search and serial search. A feasible application of heterogeneous CRSN for smart grid is then provided.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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