Correlations between the nuclear matter symmetry energy, its slope, and curvature.

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Abstract. By using point-coupling versions of finite range nuclear relativistic mean field models containing cubic and quartic self interactions in the scalar field $\sigma$, a nonrelativistic limit is achieved. This approach allows for an analytical expression for the symmetry energy ($J$) as a function of its slope ($L$) in a unified form, namely, $L = 3J + f(m^*, \rho_o, B_o, K_o)$, where the quantities $m^*$, $\rho_o$, $B_o$ and $K_o$ are bulk parameters at the nuclear matter saturation density $\rho_o$. This result establishes a linear correlation between $L$ and $J$ which is reinforced by exact relativistic calculations we have performed. An analogous analytical correlation can also be found for $J$, $L$ and the symmetry energy curvature ($K_{sym}$). Based on these results, we propose a graphic constraint in $L \times J$ plane which finite range models should satisfy.

1. Introduction
The investigation on correlations between observables is an important issue in physics since the knowledge of one observable may carry information about other. Several bulk parameter quantities help understanding the nuclear matter properties. One of them is the symmetry energy $S$, which can be expanded as a function of the nuclear density $\rho$ as $S(\rho) = J + Lx + \frac{1}{2}K_{sym}x^2 + \frac{1}{6}Q_{sym}x^3 + O(x^4)$, where $x = (\rho - \rho_o)/3\rho_o$ and $\rho_o$ is the nuclear matter saturation density. The coefficients of this expansion, namely, $J$, $L$, $K_{sym}$ and $Q_{sym}$ are, at the saturation energy, the symmetry energy at the saturation density, the slope, curvature, and third derivative (skewness) of $S$ respectively. In this work we use a non-relativistic limit from point-coupling versions of finite range nuclear relativistic mean field models for establish a correlations between $L$ and $J$.

2. The Model
The relativistic nonlinear point-coupling (NLPC) versions of the Boguta-Bodmer models are described by the following Lagrangian density

$$\mathcal{L}_{NLPC} = \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi - \frac{1}{2}G_\nu^2(\bar{\psi}\gamma^\mu \psi)^2 + \frac{1}{2}G_\nu^2(\bar{\psi}\psi)^2 + \frac{A}{3}(\bar{\psi}\psi)^3 + \frac{B}{4}(\bar{\psi}\psi)^4 - \frac{1}{2}G_\nu^2(\bar{\psi}\gamma^\mu \gamma^\nu \psi)^2,$$

(1)
that mimics the two, three and four body point-like interactions with the fermionic spinor field \( \psi \) associated to the nucleon of mass \( M \). In this equation, the last term was included in order to take into account the asymmetry of the system (different number of protons and neutrons). In the nonrelativistic limit of the NLPC model, and using the mean-field approach, the energy density functional at zero temperature for asymmetric nuclear matter is written as

\[
\varepsilon^{(NR)}(\rho, y) = (G^2_\nu - G^2_\pi)\rho^2 - A\rho^3 - B\rho^4 + G^2_{TV}\rho^2(2y - 1)^2 + \frac{3}{10M^*(\rho, y)}\lambda\rho^\frac{5}{3},
\]

where the effective mass is

\[
M^*(\rho, y) = \frac{M^2}{(M + G^2_\nu\rho + 2A\rho^2 + 3B\rho^3)H^2_5},
\]

with \( H^2_5 = 2\frac{4}{3}y^\frac{5}{3} + (1 - y)^{\frac{5}{3}} \), \( \lambda = (3\pi^2/2)^\frac{2}{3} \), and \( y = \rho_p/\rho \) being the proton fraction of the system. The proton density is \( \rho_p \). For a detailed derivation of Eq. (2) from Eq. (1) in the \( y = 1/2 \) case, we address the reader to Ref. [1].

The symmetry energy \( S \) and its slope can be performed analytically. For this purpose, we first use Eq. (2) to write \( S(\rho) \) and \( \rho_p \) are defined, respectively, by \( P^{(NR)}(\rho, y) = \rho^2 \frac{\partial S^{(NR)}(\rho)}{\partial \rho} \) and \( K^{(NR)}(\rho, y) = \frac{9}{2} \rho_p^{(NR)} \).

An advantage of this approach is to obtain simple analytical expressions for the equations of state (EOS) of the model, in comparison to those calculated in the exact finite range (FR) relativistic mean-field (RMF) models. Thus, the study of the correlation between the symmetry energy and its slope can be performed analytically. For this purpose, we first use Eq. (2) to write \( S(\rho) = \frac{1}{8} \left[ \frac{\partial^2 S^{(NR)}(\rho)}{\partial \rho^2} \right]_{\rho=\rho_p} \). Then, \( J = S(\rho_o) \) is given by

\[
J = \frac{\lambda\rho_o^\frac{5}{3}}{6M} + \left( G^2_s + 2A\rho + 3B\rho_o^2 \right) \frac{\lambda\rho_o^\frac{5}{3}}{6M^2} + G^2_{TV}\rho_o.
\]

The symmetry energy \( S(\rho) \) is used again in order to obtain \( L = 3\rho_o \left( \frac{\partial S}{\partial \rho} \right)_{\rho=\rho_p} \). The result is

\[
L = \frac{\lambda\rho_o^\frac{2}{3}}{3M} + \left( 5G^2_s + 16A\rho + 33B\rho_o^2 \right) \frac{\lambda\rho_o^\frac{2}{3}}{6M^2} + 3G^2_{TV}\rho_o.
\]

From Eq. (4) it is possible to determine the last coupling constant \( G^2_{TV} \), by imposing the model to present a particular value for \( J \).

At this point, we rewrite the coupling constants of the model, namely, \( G^2_s, G^2_\nu, A, B \), in terms of the bulk parameters \( m^*, \rho_o, B_o, \) and \( K_o \). Therefore, it is possible to write \( L \) explicitly as \( L = L(m^*, \rho_o, B_o, K_o) \). By doing so, and subtracting \( 3J \) from \( L \), we finally find a clear correlation between \( J \) and \( L \) in the following form

\[
L = 3J + f(m^*, \rho_o, B_o, K_o),
\]

where the function

\[
f(m^*, \rho_o, B_o, K_o) = \frac{5E_p^{(0)}}{(3M^2 - 19E_p^{(0)}M + 18E_p^{(0)})} \times \left\{ \frac{2M}{9m^*} (3M - 14E_p^{(0)}) - M (M + K_o/9) + E_p^{(0)} (5M + 6B_o) \right\}
\]
Figure 1. Effect of $\Delta f$ in the $L - J$ correlation of Eq. (6) for (a) $0.50 \leq m^* \leq 0.80$, and (b) $250 \leq K_o \leq 315$ MeV.

exhibits a dependence with the inverse of the effective mass, with $E^o_F = 3\lambda \rho^2_o/10M$.

Usually, in nuclear mean-field models, the binding energy and the saturation density are well established close around the values of $B_o = 16$ MeV and $\rho_o = 0.15$ fm$^{-3}$. The same assumption does not apply to the incompressibility and effective mass. Thus, from Eq. (7), it is straightforward to check that for a fixed value of $m^*$, the variation in $f$ will be given by

$$ (\Delta f)_{K_o} = -\frac{5M E^o_F}{9(3M^2 - 19E^o_F M + 18E^o_F^2)} \Delta K_o. $$

(8)

For the range of $250 \leq K_o \leq 315$ MeV, recently proposed in Ref. [2], one can verify that $|\langle(\Delta f)_{K_o}\rangle| = 0.32$ MeV. On the other hand, by choosing two different models presenting the same incompressibility $K_o$ but with two different effective masses $m^*_1$ and $m^*_2$, the $f$ variation can be inferred by

$$ (\Delta f)_{m^*} = -\frac{5M E^o_F(3M - 14E^o_F)}{9(3M^2 - 19E^o_F M + 18E^o_F^2)} \frac{\Delta m^*}{m^*_1 m^*_2}, $$

(9)

where $\Delta m^* = m^*_2 - m^*_1$. For a typical range of $0.50 \leq m^* \leq 0.80$, presented by FR-RMF models, one has $|\langle(\Delta f)_{m^*}\rangle| = 18$ MeV.

Fig. 1 shows how such variations affect the correlation given in Eq. (6). From this figure we can conclude that different models presenting the same effective mass, will exhibit points in a $L \times J$ graph situated very close to a same line, since in this case the variation of the linear coefficient in Eq. (6) is very small compared to that one of the case in which $K_o$ is fixed. This leads us to draw the conclusion that in the NR limit of the NLPC models described by Eq. (1), the linear correlation between $J$ and $L$ in Eq. (6) is achieved for the more distinct models under the condition that their effective masses are equal.
3. PREDICTIONS ON FR-RMF MODELS

3.1. Symmetry energy slope

Now, we pose the question whether the NR correlation obtained in Eq. (6), and the results showed in Fig. 1 with the subsequently conclusions, still remain valid for exact FR models. The answer is given by the study we have done for a set of representative FR models, whose results are displayed in Fig. 2.

Confident on the correlation between $L$ and $J$, found in the NR approximation and confirmed for the relativistic calculations, we have constructed a region of possible $L$ values as a function of $J$, and according the FRS constraint [3], that FR-RMF Boguta-Bodmer models must satisfy in order to give values for the finite nuclei spin-orbit splitting compatible with well established experimental values. In Fig. 3, we present our prediction for the lowest and highest values for $L$ in comparison with other values found in the literature, by taking into account the region in a range of $25 \leq J \leq 35$ MeV for the symmetry energy. Notice that our limits for $L$ are comparable with other models.

3.2. Symmetry energy curvature

In the NR framework it is also possible to find an analytical expression for $K_{\text{sym}} = 9\rho_0^2 \left( \frac{\partial^2 S}{\partial \rho^2} \right)_{\rho=\rho_0}$.

It reads

$$K_{\text{sym}} = \frac{5E_v^o}{(3M^2 - 19E_v^o M + 18E_v^o M^2)} \left\{ \frac{2M}{3m^*} (5M - 18E_v^o) 
+ M(18B_0 - 4M + 18E_v^o) - \frac{K_0}{9} (19M - 18E_v^o) \right\}$$

(10)
Figure 3. Comparison between the limits of $L$ obtained in this work and those from Dong et al., Carbone et al., Liu et al., Tsang et al., Warda et al., Danielewicz and Lee, Shetty et al., Chen et al., Möller et al., and Chen; See reference [4].

Note than the symmetry energy curvature $K_{\text{sym}}$ depends on $m^*$, scaling as $1/m^*$, and is linearly correlated with $K_o$ in the NR approach, see Eq. (10). Such dependences are not negligible. Using the Eqs. (6) and (10), we can rewrite $K_{\text{sym}}$ as function $L$ (or $J$) and establish the correlation.

4. Conclusion
In this work, by studying the nonrelativistic limit of the NLPC models, we have seen that the symmetry energy slope $L$ is linearly correlated with $J$. This correlation depends explicitly on the effective nucleon mass $m^*$ and the incompressibility $K_o$. However, the $K_o$ dependence of $L$ has been verified to be negligible. In the same way, we have also shown that it is possible to establish correlation between the symmetry energy curvature $K_{\text{sym}}$ as function $L$ (or $J$). However, a more detailed study regarding this correlation is still underway.

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