Damped Oscillating Dark Energy: Ideal Fluid and Scalar-Tensor description

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(Dated: April 4, 2015)

In this paper, we study damped oscillating form of dark energy for explaining dynamics of universe. First of all, we consider universe is filled with an ideal fluid which has damped oscillating dark energy in terms of this case we calculate several physical quantities such as Hubble parameter, acceleration parameter, energy density, pressure and others for dark energy, dark energy-matter coupling and non-coupling cases. Secondly, we consider as universe is filled with scalar field instead of an ideal fluid we obtain these physical quantities in terms of scalar potential and kinetic term for the same cases in scalar-tensor formalism. Finally, we show that ideal fluid description and scalar-tensor description of dark energy give mathematically equivalent results for this EoS parameter, even if they haven’t same physical meaning.

I. INTRODUCTION

According to recent cosmological observations data, current universe is expanding with acceleration [11, 2] caused by dark energy. There is considered as, currently, that the universe entered through the inflationary phase at early epoch and there are big evidences that it is accelerating at present. Although several models are proposed for explaining cosmic acceleration, it has not known what is the origin and dynamics of universe as well as current value of the dark energy equation of state (EoS) parameter ($w$) yet. The dynamical behavior of Universe which is determined by its EoS parameter. If $w$ is equal to $-1$, it is namely as standard ΛCDM cosmology or cosmological constant model. On the contrary to this, observational data indicates that effective EoS parameter ($w_{eff}$) of dark energy lies in the interval: $-1.48 < w_{eff} < -0.72$ [3, 4]. Therefore, EoS parameter may be time varying such as quintessence scalar field model $w > -1$ and phantom scalar field model $w < -1$ [5, 11]. Recent cosmological data [12] accept the possibility of time-dependent $w$ which lies close to $-1$ currently. There are various possibilities are proposed to overcome general dark energy problem such as time-dependent EoS [13, 14], the inhomogeneous equation of state [5, 6, 13] and multiple-Lambda cosmology [16]. For understanding the dynamical properties of dark energy, several parametrization are suggested [12, 17, 18, 26, 29]. Ideal dark fluids and scalar field models may reproduce unification of late time acceleration with early time inflation. Cosmological consequences of dark energy with time-dependent EoS parameter are described. This EoS parameter may explain the transition from phantom era to non-phantom era (if the dark energy is non-phantom phase, currently), or transition from deceleration to acceleration. If the universe is filled with a time-dependent EoS, ideal fluid may present be in the acceleration epoch of quintessence or phantom phase. It is known as well that an ideal fluid satisfying a time-dependent EoS can lead to a phantom era or non-phantom era. In addition to this, dark energy may behave scalar field instead of an ideal fluid, and therefore using scalar-tensor formalism, behavior of dark energy can be explained. Moreover, phase transitions between deceleration and acceleration or non-phantom and phantom are shown scalar theory [30].

In this study, we consider, as an example, damped oscillating EoS parameter

$$w(t) = -1 + w_0 e^{-bt} \cos \omega t \tag{1}$$

$t$ is the cosmic time. $w_0$ and $b$ are positive and real parameters of time-dependent $w(t)$ function which leads to acceleration expansion or damped oscillating of the universe. Additionally, $\omega$ is a frequency. Here, $w_0, \omega > 0$. EoS parameter has been considered as $[31, 32]$ in which it may be consistent with observations result for some values of parameters. According to this EoS model, the additional term $w_0 e^{-bt} \cos \omega t$ in Eq.(1) corresponds to perturbation of dark energy. Using this EoS parameter, we will discuss some physical quantities such as energy density, pressure and others for dark energy, both dark energy-matter coupling and dark energy-matter non-coupling cases in terms of an ideal fluid description for this time-dependent EoS parameter. Additionally using scalar-tensor description of dark energy we will obtain these physical quantities in terms of scalar potential and kinetic term for the same cases. Finally, we will show that ideal fluid description and scalar-tensor description of dark energy give mathematically equivalent results for this EoS parameter. Moreover, the ideal fluid description transforms a fluid in a scalar-tensor theory taking into consideration of the same FRW scale factor and the process can be reversed. The organization of this paper is structured, respectively: In Sec.II, theoretical framework of the FRW equations is presented. In Sec.III, the same physical parameters and dynamics of universe for damped oscillating dark energy are computed and discussed. In Sec.IV, the analytical results in the scalar-tensor description are presented. Finally, conclusion is given in Sec.V.
II. THEORETICAL FRAMEWORK

The spatially-flat homogeneous and isotropic FRW universe, the metric is

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2$$

Then the FRW equations are given by

$$H^2 = \frac{\kappa^2}{3} \rho, \quad \dot{H} = -\frac{\kappa^2}{2} (p + \rho)$$

where $\kappa^2 = 8\pi G$ is the gravitational constant and $H \equiv \frac{\dot{a}}{a}$ is Hubble rate. Here, $a(t)$ is the scale factor of the three-dimensional flat, $\rho$ is energy density and $p$ is pressure.

The evolution of the energy density of the fluid is determined by solving the continuity equation

$$\dot{\rho} + 3H (p + \rho) = 0$$

where $(\cdot)$ indicates the time derivative. The relation between $\rho$ and $p$ is given as

$$p = w\rho$$

where $w$ is EoS parameter of FRW universe which is constant and equal to exactly $-1$ for FRW framework. However, EoS parameter $w$ may be chosen depending on time $t$. Especially, in order to explain time evolution of the universe for different time epoch, it is supposed that $w$ changes with time $[3] [6] [13] [23]$. In this case time dependent EoS is given by

$$p = w(t) \rho$$

Taking into account Eq.s (3)’s first equation, (4) and (6), we obtain energy conservation

$$\dot{\rho} + \frac{\kappa}{\sqrt{3}} (1 + w(t)) \frac{\rho}{2} = 0.$$  

Integrating of Eq.(7) we can obtain time-dependent energy density is

$$\rho = \frac{4}{3\kappa^2 (\int dt (1 + w(t)))^2}.$$  

If Eq.(8) is put into in Eq.(3)’s first equation, time dependent Hubble parameter can be presented as

$$H = h(t) \equiv \frac{2}{3 \int dt (1 + w(t))}.$$  

However, when $w$ is chosen as a constant, the standard expression of Hubble is recovered

$$H = \frac{2}{3(1 + w) (t - t_s)}.$$  

The dynamics of universe depends on time and EoS parameter $w$. For instance, the cosmological time $t$ should be restricted to be $t_s$ and if $\int dt (1 + w(t)) = 0$ which occurs at $t = t_s$, $H$ diverges which behavior of corresponds to the Big Rip singularity. On the other hand, in the case of $w < -1$, Hubble parameter takes $H > 0$ which corresponds to phantom era and $w > -1$ corresponds to quintessence where the Hubble takes $H < 0$.

III. IDEAL FLUID DESCRIPTION

A. Dark Energy Case

In this section we consider the universe filled with dark energy where $w(t)$ is time-dependent, damped oscillating function. This may explain the effective value of $w$ is being currently roughly to $-1$. Using this time-dependent function defined in Eq.(1), $w(t) = -1 + w_0 e^{-bt} \cos \omega t$, we will obtain some physical quantities and give numerical solutions. The time dependence behavior of Eq.(1) is plotted in Fig.(1) for $w_0 = b = \omega = 1$. It can be seen from figure that the EoS is $w(t) = 0$ at $t = 0$ which is maximum value of the EoS and it is damped oscillating with time and reach to $-1$ at large time. The effective value of the EoS parameter may be adjusted roughly equal to $-1$ currently, which corresponds to the observational bounds at present. In Fig.(1), $w$ changes with time this interval: $-1 < w < 0$ which has been shown in these previous studies $[33] [35]$, as it can be seen our EoS model is fitting with these works. Although so many EoS models are consistent with current cosmological observations best, time evolution of the dark energy is still debate, therefore, alternative EoS models can be proposed $[33] [35]$.

![Figure 1](image-url)  

**FIG. 1.** This figure reflects the time dependence behavior of Eq.(1) is presented in this work for $w_0 = b = \omega = 1$. 

Hubble parameter and its time derivative for EoS in Eq.(1) are respectively given as

$$H(t) = \frac{2}{3 w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t)}$$

and

$$\dot{H} = \frac{2}{3 \left( w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t) \right)^2}$$

where $w_1$ is an integration constant. The Hubble parameter oscillates damping on account of exponential term. Time dependence behavior of Hubble parameter and its time derivative are respectively plotted in Fig.(2) and (3) for constant parameters $w_0 = w_1 = b = \omega = 1$. As it can be seen from Fig.(2) while the value of Hubble parameter is maximum at $t = 0$. It is damped oscillating with
time and reaches to constant value at large time. On the contrary to this, the time derivative of Hubble parameter shows different behavior from Hubble parameter because the time derivative of Hubble parameter takes negative values and its value increases with time up to reach approximately zero. Although Hubble parameter is a constant in FRW model, in this study, it changes with time for time-dependent EoS parameter. Moreover, the universe lives in phantom phase where $H > 0$ and in non-phantom phase $H < 0$. In this case, in Eq.(12), if $w_0 \cos \omega t < 0$, the universe is in phantom phase but if $w_0 \cos \omega t > 0$, the universe is in non-phantom phase. Depending on the select of parameters in the EoS of the dark energy, $H(t)$ can correspond to either phantom or non-phantom universe. Both phantom and non-phantom phases the universe expands with acceleration.

The Universe enters through different epoch, it may be given by the acceleration parameter

$$\frac{\ddot{a}}{a} = H^2 + \dot{H}.$$  \hspace{2cm} (13)

For our EoS model, acceleration parameter can be found

$$\frac{\ddot{a}}{a} = \frac{4}{9} \left[ w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t) \right]^2 \left(1 - \frac{3}{2} w_0 e^{-bt} \cos \omega t \right).$$  \hspace{2cm} (14)

This acceleration parameter is plotted versus time in Fig.(4) for constant parameters $w_0 = w_1 = b = \omega = 1$. As it can be seen from Fig.(4), acceleration parameter for constant values of parameters fastly maximum value in sort time range and later it approximately to a constant value at large time. It may be considered that the sort time regime where rapidly increasing of acceleration parameter corresponds to inflation era of the universe. The large time behavior of the acceleration parameter in Fig.(4) also shows that universe reaches to constant acceleration parameter. Additionally, the derivative of scale factor $\ddot{a} > 0$, the universe expands.

Dark energy density (15) satisfies conservation laws (4) and (7). The time-dependence behavior of dark energy density is given in Fig.(5). As it can be seen from this figure, energy density has maximum value at $t = 0$. Moreover, dark energy density is also damped oscillating with time and it reaches to constant value at large time. The behavior of the energy density is consequences of EoS parameter. The behavior of the energy density can be interpreted that universe is expanding, as unexpectedly, or dark energy may transform from one of kind to another form due to interactions energy-energy or energy-matter so on. Moreover, we note that $\rho$ has a physical meaning. So, $\rho > 0$, Universe is in phantom phase and the energy density grows. On the contrary to this, in the non-phantom phase, $\rho < 0$ and energy density decreases. Additionally, in the phantom phase, the entropy

\[ \text{FIG. 3. Time derivative of Hubble parameter for EoS in Eq. (1).} \]

\[ \text{FIG. 4. Time dependence behavior of acceleration parameter for EoS in Eq. (1).} \]
FIG. 5. This figure reflects the time dependence behavior of energy density for EoS work. In this figure the parameters are chosen as $w_0 = w_1 = \kappa = b = \omega = 1$. It may become negative, as it has been shown \[22\]. For our EoS model, $\dot{\rho}$ is less than zero, therefore, this case is consistent with non-phantom phase of Universe.

On the other hand, pressure can be obtained from second FRW equation

$$p = -\frac{1}{\kappa^2}(2\dot{H} + 3H^2) .$$

(16)

Using Eqs (11) and (12), pressure of the dark energy for damped oscillating EoS can be obtained as

$$p(t) = \frac{4(\omega^2 + b^2)^2}{3\kappa^2[w_1 + w_0 e^{-bt}(\omega \sin \omega t - b \cos \omega t)]^2(e^{-bt}w_0 \cos \omega t - 1)} .$$

(17)

B. Coupling Case of Dark Energy and Matter

So far we discuss some quantities for only dark energy which is defined in Eq.(1). On the contrary to this, in a more realistic situation, universe is filled with matter and dark energy. Moreover it is generally supposed that the matter interacts with dark energy. In a such case, the total energy density is given by $\rho_{tot} = \rho + \rho_m$ where $\rho_m$ is the density of matter in the universe. Here, we can write energy conservation law for matter and dark energy are separately. This case is defined by respectively

$$\rho_m + 3H(\rho_m + p_m) = 0$$

(18)

and

$$\dot{\rho} + 3H(\rho + p) = 0 .$$

(19)

From Eq.(18), we obtain energy density of matter

$$\rho_m = \rho_0 a^{-3(1+w_m)}$$

(20)

$\rho_0$ is a constant and $w_m$ is EoS parameter for matter. Therefore, in the presence matter dark energy interaction, the dark energy $\rho$ is described as

$$\rho = \frac{3}{\kappa^2}H^2 - \rho_0 a^{-3(1+w_m)} .$$

(21)

In addition to this, the total pressure for matter and dark energy interact is given by $p_{tot} = p + p_m$ where $p$ indicates dark energy pressure and $p_m$ corresponds to matter pressure which is defined by

$$p_m = w_m \rho_m .$$

(22)

From the second FRW equation, dark energy pressure in the presence of dark energy and matter coupling can be
written as

\[ p = -\frac{1}{\kappa^2} (2\dot{H} + 3H^2) - w_m \rho_0 a^{-3(1 + w_m)} . \]  

We now consider the case that universe is filled with damped oscillating dark energy and the matter as well as they are interacting with each other. In the presence matter and damped oscillating dark energy interaction, the dark energy density \( \rho \) is described as

\[ \rho = \frac{4}{3\kappa^2} \left[ \frac{[\omega^2 + b^2]^2}{w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t)^2} - \rho_0 \alpha^{-3(1 + w_m)} \right] . \]  

Additionally, the total pressure for matter and damped oscillating dark energy combination can be obtained as

\[ p = -\frac{4}{3\kappa^2} \left[ \frac{[\omega^2 + b^2]^2}{w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t)^2} (1 - e^{-bt} w_0 \cos \omega t) \right] - w_m \rho_0 \alpha^{-3(1 + w_m)} . \]  

Now we can define a new EoS parameter for the coupling dark energy and matter can be written as

\[ w(t) = -1 - \frac{2\dot{H}}{H^2} + (1 + w_m) \rho_0 \alpha^{-3(1 + w_m)} \]  

The denominator of Eq. (26) is always positive. Moreover, we note that all physical quantities can be redefined for a suitable scale factor \( a \).

C. Non-Coupling Case of Dark Energy and Matter

On the other hand, in the case of non-coupling between dark energy and matter, the pressure and energy density can be defined separately. However, EoS parameter cannot be defined separately. Therefore, EoS parameter can be described as

\[ p = w \rho + \frac{2}{\kappa^2} H g(t) \]  

where \( w = -1 \) for FRW and \( g(t) \) is an arbitrary function of time \( t \). Thus, EoS for dark energy fluid for non-coupling case is written as

\[ p = -\rho + \frac{4(1 + w(t))}{3\kappa^2 \int dt (1 + w(t))} - (1 + w_m) \rho_0 \exp \left\{ \frac{-2(1 + w_m)}{\int dt (1 + w(t))} \right\} . \]  

Finally using EoS in Eq.(1), Eq.(28) can be arranged as

\[ p = -\rho + \frac{4}{3\kappa^2} \frac{w_0 e^{-bt} \cos \omega t (\omega^2 + b^2)}{(w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t)^2)} - (1 + w_m) \rho_0 e^{-3(1 + w_m)} \frac{2(\omega^2 + b^2)}{2(w_1 + w_0 e^{-bt} (\omega \sin \omega t - b \cos \omega t)^2)} . \]  

Eq.(29) also satisfies energy conservation law (4).

IV. SCALAR-TENSOR DESCRIPTION

A. Scalar Field Case

In this part, we will obtain the damped oscillating dark energy with time-dependent, explicit EoS in an equivalent, scalar-tensor formalism following method in Refs. [39]-[41].

Let us start with following action

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \Omega(\phi) \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \right\} \]  

where \( \Omega(\phi) \) is the kinetic term and \( V(\phi) \) is the scalar potential, they are functions of the scalar field \( \phi \) which only depends on the time coordinate \( t \). The function \( \Omega(\phi) \) may be chosen +1 (−1) corresponds to the phantom phase (non-phantom phase). In this case, kinetic term relates to the sign of \( \Omega(\phi) \). Therefore, the sign of kinetic term is negative, phantom phase emerges so \( \dot{H} > 0 \) but the sign of kinetic term is positive, non-phantom phase
appears so $\dot{H} < 0$.

In the scalar-tensor formalism, energy density $\rho$ and $p$ are described as in terms of scalar field $\phi$ are respectively given by

$$\rho = \frac{1}{2} \Omega(\phi) \dot{\phi}^2 + V(\phi) \tag{31}$$

and

$$p = \frac{1}{2} \Omega(\phi) \dot{\phi}^2 - V(\phi) \tag{32}.$$  

Combining (3) with (31) and (32) we can find kinetic function and scalar potential as

$$\Omega(\phi) \dot{\phi}^2 = -\frac{2}{\kappa^2} \dot{H} \tag{33}$$

and

$$V(\phi) = \frac{1}{\kappa^2} (3H^2 + \dot{H}) \tag{34}$$

where $H$ is the Hubble parameter. In case of $\phi = t$ and $H = f(t)$, Eq.s (33) and (34) can be presented as

$$\Omega(\phi) = -\frac{2}{\kappa^2} f'(\phi) \tag{35}$$

$$V(\phi) = \frac{1}{\kappa^2} \left( 3 f(\phi)^2 + f'(\phi) \right) \tag{36}$$

where $f(\phi)$ is described as

$$f(\phi) = \frac{2}{3} \frac{\omega^2 + b^2}{w_1 + w_0 e^{-b\phi (\omega \sin \omega \phi - b \cos \omega \phi)}}. \tag{37}$$

These conditions in (35) and (36) satisfy scalar-field equation

$$0 = \Omega(\phi) \ddot{\phi} + \frac{1}{2} \Omega'(\phi) \dot{\phi}^2 + 3H \Omega(\phi) \dot{\phi} + V'(\phi) \tag{38}$$

where $dot$ indicates time derivation and $prime$ denotes derivation according to the scalar field. We note that the scalar field $\phi$ can be redefined as $\phi \rightarrow F(\phi)$ where $F(\phi)$ is the arbitrary function. Moreover the scalar field $\phi$ can be used as time coordinate such as $\phi = t$. Having in mind these details and using Eqs. (35) and (36), we obtain energy density and pressure in the form, respectively

$$\rho = \frac{3}{\kappa^2} f(\phi)^2 \tag{39}$$

and

$$p = -\frac{3}{\kappa^2} f(\phi)^2 - \frac{2}{\kappa^2} f'(\phi) \tag{40}.$$  

For the action (30), when $\phi = f^{-1}(\kappa \sqrt{\frac{\rho}{3}})$, we can obtain inhomogeneous EoS for dark energy in the form

$$p = -\rho - \frac{2}{\kappa^2} f'(f^{-1}(\kappa \sqrt{\frac{\rho}{3}})) \tag{41}.$$  

On the other hand, in the scalar-tensor formalism, using definitions (11) and (12), the kinetic term $\Omega(\phi)$ and the scalar potential $V(\phi)$ for damped oscillating EoS (1) can be written as respectively

$$\Omega(\phi) = \frac{4(\omega^2 + b^2)^2 w_0 e^{-b\phi (\omega \sin \omega \phi - b \cos \omega \phi)}}{3\kappa^2 [w_1 + w_0 e^{-b\phi (\omega \sin \omega \phi - b \cos \omega \phi)}]^2} \tag{42}$$

and

$$V(\phi) = \frac{(\omega^2 + b^2)^2 [4 - 2w_0 e^{-b\phi (\omega \sin \omega \phi - b \cos \omega \phi)}]}{3\kappa^2 [w_1 + w_0 e^{-b\phi (\omega \sin \omega \phi - b \cos \omega \phi)}]^2} \tag{43}.$$  

If we put Eq.s (42) and (43) into Eq.s (31) and (32), we get energy density and pressure relations

$$\rho = \frac{4(\omega^2 + b^2)^2}{3\kappa^2 [w_1 + w_0 e^{-b\phi (\omega \sin \omega t - b \cos \omega t)}]^2} \tag{45}$$

and

$$p(t) = \frac{4(\omega^2 + b^2)^2}{3\kappa^2 [w_1 + w_0 e^{-b\phi (\omega \sin \omega t - b \cos \omega t)}]^2} (e^{-bt} w_0 \cos \omega t - 1) \tag{47}.$$  

which confirms that EoS and scalar-tensor analysis are equivalent.

**B. Coupling Case of Scalar Field and Matter**

Let us consider a universe filled with scalar field $\phi$ and matter. Matter EoS is $p_m = w_m \rho_m$ ($w_m$ is constant) and scalar field $\phi$ depends on time $t$ for coupling case. In this case, action is given by

$$S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} \Omega(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) + \zeta_m \right] \tag{44}$$

where $\zeta_m$ is the matter Lagrangian density. For metric (2), corresponding FRW equations are given by

$$H^2 = \frac{\kappa^2}{3} (\rho_m + \rho_\phi) \tag{45}.$$
and
\[ \dot{H} = -\frac{\kappa^2}{2}(\rho_m + p_m + \rho_\phi + p_\phi) \]  \hspace{1cm} (46)
where \( \rho_\phi \) and \( p_\phi \) are energy density and pressure for scalar field \( \phi \), on the other hand, \( \rho_m \) and \( p_m \) are energy density and pressure for matter. Combining (45) and (46) with (33) and (34), we can obtain kinetic function and scalar potential term for coupling case between scalar field and matter, respectively.

These results clearly confirm that EoS and scalar-tensor analysis are mathematically equivalent.

C. Non-Coupling Case of Scalar Field and Matter

Now, let us consider non-coupling between dark energy and matter as a final case. In case of non-coupling,

\[ \Omega(\phi)^2 = -\frac{2}{\kappa^2} \dot{H} - (\rho_m + p_m) \]  \hspace{1cm} (47)

\[ V(\phi) = \frac{1}{\kappa^2}(3H^2 + \dot{H}) - \frac{\rho_m - p_m}{2}. \]  \hspace{1cm} (48)

By using Eqs. (11) and (12), kinetic function and scalar potential for coupling case to the damped oscillating dark energy are rewritten as respectively

\[ \Omega(\phi) = \frac{4(\omega^2 + b^2)^2w_0e^{-b\phi}\cos\omega\phi}{3\kappa^2[w_1 + w_0e^{-b\phi}(\omega\sin\omega\phi - b\cos\omega\phi)]^2} - (\rho_m + p_m) \]  \hspace{1cm} (49)

\[ V(\phi) = \frac{(\omega^2 + b^2)^2[4 - 2w_0e^{-b\phi}\cos\omega\phi]}{3\kappa^2[w_1 + w_0e^{-b\phi}(\omega\sin\omega\phi - b\cos\omega\phi)]^2} - \frac{(\rho_m - p_m)}{2}. \]  \hspace{1cm} (50)

Finally, if we put Eqs. (49) and (50) into Eqs. (31) and (32), we get energy density and pressure relations

\[ \rho = \frac{4}{3\kappa^2 \left[w_1 + w_0e^{-b\phi}(\omega\sin\omega\phi - b\cos\omega\phi)\right]^2} - \rho_0\alpha^{-3(1+w_m)} \]  \hspace{1cm} (24)

and

\[ p = -\frac{4}{3\kappa^2 \left[w_1 + w_0e^{-b\phi}(\omega\sin\omega\phi - b\cos\omega\phi)\right]^2} (1 - e^{-b\phi}w_0\cos\omega\phi) - w_m\rho_0\alpha^{-3(1+w_m)}. \]  \hspace{1cm} (25)

These results clearly confirm that EoS and scalar-tensor analysis are mathematically equivalent.
where \( F(\phi) = \int d\phi f(\phi) \) and \( F_0 \) is an integration constant. Using the relation between scale factor and arbitrary scalar function \( F(t) \)

\[
a(t) = a_0 e^{F(t)}, \quad a_0 = \left( \frac{\rho_{m0}}{F_0} \right)^{1/3(1+w_m)}.
\]

(55)

For this cases, inhomogeneous EoS for non-coupling case can be obtained as

\[
p = -\rho + 4 \frac{w_0 e^{-bt} \cos(\omega t) + b^2)}{3n^2 (w_1 + w_0 e^{-bt})} \left( \omega \sin \omega t - b \cos \omega t \right) \left( 1 + w_m \right) \rho m_0 e^{-3(1+w_m) \frac{2(\omega t + b^2)}{3(\omega t + b^2)}} - \frac{1}{3(1+w_m)} \frac{2(\omega t + b^2)}{3(\omega t + b^2)} \frac{2(\omega t + b^2)}{3(\omega t + b^2)}
\]

which also confirms that ideal fluid description and scalar-tensor description are mathematically equivalent for damped oscillating dark energy.

V. CONCLUSION

In this study, we discuss damped oscillating form of dark energy and computing some physical expression for dark energy, dark energy-matter coupling and dark energy-matter non-coupling cases both ideal fluid and scalar-tensor description of dark energy. As a result, we obtain that these descriptions are mathematically equivalent. We consider that damped oscillating EoS parameter may be used to dynamics of Universe similar to other EoS parameters.

ACKNOWLEDGMENTS

Authors would like to thank Istanbul University for financial support (Grant No. 48081). This work has been completed at Istanbul University, Graduate School of Natural and Applied Sciences and is the subject of the forthcoming M.Sc. Thesis of Nilay Bostan.
Rev. D 77, 106005 (2008).
[38] I. Brevik, O. Gorbunova, Gen. Relativ. Gravit. 37, 2039 (2005).
[39] I. Brevik, Int. Journ. of Mod. Phys. D 15, 767 (2006).

[40] J. Ren, X. Meng, Phys. Lett. B 633, 1 (2006).
[41] J. Ren, X. Meng, Phys. Lett. B 636, 5 (2006).