\( \tau \) PHYSICS AT \( B \)-FACTORIES

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Today the \( B \)-factories BaBar and Belle have accumulated largest samples of \( \tau^+\tau^- \) events and are competing to be called \( \tau \)-factories. Among the problems to be tested and measurements to be done by BaBar and Belle are check of CP and CPT invariance in tau decays, measurement of strange and non-strange spectral functions, extraction of mass of strange quark and \(|V_{us}|\), searches for lepton flavor violation processes. In this paper, the latest results in tau physics by these two experiments and measurements to be done within next few years are reviewed.

I. INTRODUCTION

BaBar\cite{1} and Belle\cite{2} are the \( e^+e^- \) collider experiments running at \( \sqrt{s} \) equal to \( \Upsilon(4S) \) mass. In spite of their commonly used name \( B \)-factories they also provide largest and cleanest samples today for study of tau physics. Indeed, the cross-section of \( e^+e^- \rightarrow \tau^+\tau^- (\sigma_{\tau^+\tau^-}) \) at this energy is 0.89 nb, of the same order as \( B\bar{B} \) cross section at \( \Upsilon(4S) \) and just a bit smaller than \( \sigma_{\tau^+\tau^-}^{th} \approx 1.2 \) nb at \( \tau^+\tau^- \) production threshold\cite{3}.

The \( \tau^+\tau^- \) event produced at \( \Upsilon(4S) \) has very characteristic topology. The decay products of two taus are well separated in space, such that if space is split on two hemispheres with respect to the axis of the event thrust, the decay products of two taus are mostly contained within opposite hemispheres. From another side, the boost is not as large as at LEP experiments, and the tracks are well separated. As both BaBar and Belle are multipurpose spectrometers, the full particle identification of the event can be performed. Usually tau decays have one (1-prong) or three (3-prong) charged particles in the final state, therefore the multiplicity of the \( \tau^+\tau^- \) events is relatively small.

The typical backgrounds are radiative Bhabha and di-muon events which can be suppressed by vetoing leptons or high momentum tracks, and hadronic \( q\bar{q} \) events which are more isotropic and have in average more neutral particles.

Currently, BaBar has recorded about \( 6 \cdot 10^8 \) tau decays, and Belle has recorded about \( 10^9 \) tau decays with large part of the statistics used in the analyses presented below. In section II we review high precision measurements of tau mass and lifetime, which allow test of CPT invariance, and discuss perspectives of measuring CP-violation in tau decays.

Section III is concentrated on the description of the measurements of the hadronic tau decays. In the section IV the searches of lepton flavor violation in tau decays are described and the conclusions are drawn in section V. The future perspectives and limitations of different measurements are discussed throughout the paper.

II. STANDARD MODEL TESTS

The basic tasks of the \( \tau \)-factories are to measure mass \( m_\tau \) and time of life \( \tau_\tau \) of tau lepton. Belle has recently presented a preliminary measurement of \( m_\tau = (1776.71 \pm 0.25_{stat} \pm 0.62_{sys}) \) MeV\cite{5} using pseudo-mass technique pioneered by ARGUS\cite{6}. The sample of 253 fb\(^{-1}\) was used. Although the measurement is dominated by systematic uncertainties, it is largely due to the size of the control samples, and therefore is likely to be improved with increased statistics. The sample is also used to probe the difference between \( m_{\tau^+} \) and \( m_{\tau^-} \) which is found to be negligible, \( |m_{\tau^+} - m_{\tau^-}|/m_\tau < 5.0 \cdot 10^{-4} \) at 90\% confidence level (CL). This number is statistically limited as most systematic uncertainties are canceled in the ratio.

At the same time BaBar has concentrated on the tau lifetime measurement. The flight distance transverse to the beam \( \lambda_T \) is measured and corrected by polar angle of 3-prong system (\( \Theta_{3pr} \)) to calculate the total decay length \( \lambda = \lambda_T/\sin \Theta_{3pr} \). The dependence on azimuthal angle \( \phi (\lambda(\phi)) \) is fitted to minimize the systematic uncertainties due to alignment of the vertex detector. The preliminary result is \( \tau_\tau = 289.40 \pm 0.91_{stat} \pm 0.90_{sys} \) fs\cite{7}. It is in agreement with PDG value \( \tau_\tau = 290.6 \pm 1.1 \) fs\cite{7} and is the most precise measurement up to date. 80 fb\(^{-1}\) were used in the analysis. As in case of \( m_\tau \) measurement, the systematic uncertainty are partially limited by statistics of control samples and is likely to be improved with luminosity, although not more than by factor of two. The preliminary study
of $\tau_{\tau^+}$ and $\tau_{\tau^-}$ showed no difference $\tau_{\tau^-} - \tau_{\tau^+}/\tau_{\tau^-} + \tau_{\tau^+} = (0.12 \pm 0.32_{\text{stat}})^\%$, where the systematic uncertainty is to be estimated but likely to be small.

Using the above numbers averaged with PDG values and leptonic branching fractions\cite{14} of tau one can compare lepton charged current coupling constants:

\[
\frac{g_e}{g_\mu} = \sqrt{\frac{B(\tau \to e\nu\nu)}{B(\tau \to \mu\nu\nu)}} \left(1 + C_{\tau\mu}\right) = 0.9997 \pm 0.0024
\]

\[
\frac{g_\mu}{g_\tau} = \sqrt{\frac{\tau_\mu}{\tau_\tau}} \left(\sqrt{\frac{m_\tau}{m_\mu}}\right) \frac{1}{B(\tau \to e\nu\nu)} = 0.9980 \pm 0.0022,
\]

where $C_{\tau\mu} = -0.004$, $C_{\tau\mu} = -0.0313$ and $C_{\mu\tau} = -0.0044$ are radiative corrections. No significant deviation from SM is observed.

The subject which is still in to do list of both experiments is a search of CP-violation in tau decays. While no such CP-violation is expected in SM, other contribution, like e.g. charged Higgs exchange can result in non-negligible effect in angular and visible mass distributions of tau decay products due to interference of vector and scalar parts. CLEO has searched for such effect in the decays $\tau \to K_S^0\pi^0\nu\bar{\nu}$ and $\tau \to \pi^0\nu\bar{\nu}$ with 13.3 fb$^{-1}$. While no signal was found, the CLEO collaboration has put limits on imaginary part of charged Higgs coupling of $-0.172 < \text{Im}(\Lambda) < 0.067$ from $\tau^- \to K_S^0\pi^-\nu$ data assuming $K^* (1430)$ scalar contribution and $-0.046 < \text{Im}(\Lambda) < 0.022$ from $\tau \to \pi^0\nu\bar{\nu}$ for maximal scalar contribution. The limits are at 90% CL. The largest source of the uncertainty here is the size of the sample recorded. Given that $B$-factors have almost two orders of magnitude more data, it should be possible to improve CLEO result significantly. It is clear, however, that the understanding of the systematic uncertainty will require a careful work, in particular, study of possible charge asymmetry in the detector.

Of course, there are more SM tests to be performed, such as measurement of tau electric and anomalous magnetic dipole moments, Michael parameters, measurement of $\nu_\tau$ helicity. However, it is unlikely, that either BaBar or Belle will be able to improve previous measurements soon.

### III. STUDY OF HADRONIC TAU DECAYS

Due to simplicity of the SM tau decays involving $W^-$ exchange, it is possible to study the hadronization process in details. All hadronic tau decays are of interest, starting from the most common 1-prong $\tau \to \pi\pi^0\nu\bar{\nu}$ up to not yet observed 7-prong tau decay. The analysis of spectral function of $\pi\pi^0$ is to be used for comparing the measurement of anomalous magnetic moment of muon with SM prediction. The analysis of tau strange decays provides an information on mass of strange quark and $|V_{us}|$ element of CKM matrix. The 5-prong tau decays are studied with large statistics and not observed 7-prong decays are used to probe non-SM contributions.

#### A. Non-strange spectral function

The calculation of hadronic part of the anomalous magnetic moment of muon $a_\mu^{\text{had,LO}}$ includes integral of the cross-section $e^+e^- \to \text{hadrons}$ multiplied with QED kernel $K(s)$. The structure of $K(s)$ is such, that 75% of $a_\mu^{\text{had,LO}}$ is covered by two pion final state dominated by $\rho(770)$ resonance. Assuming isospin invariance, $\sigma(e^+e^- \to \pi^+\pi^-)$ can be estimated from the branching fraction $B(\tau^- \to \pi^-\pi^0\nu)$\cite{10}. Currently, the results based on tau data together with results of Muon $g$-2 experiment\cite{4} give $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (9.4 \pm 10.5) \cdot 10^{-10}$, while calculation based solely on $e^+e^-$ data is $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.2 \pm 9.2) \cdot 10^{-10}$\cite{10}. Belle has recently presented new preliminary study of $\tau \to \pi\pi^0\nu$ decay\cite{11}. The measured $\pi^-\pi^0$ invariant mass spectrum is corrected for the detector deficiency and distortions.
using the unfolding technique and then fitted with Gounaris-Sakurai function as shown on Fig. 1a. The distribution also exposes $\rho'''$ resonance, evident in this decay for the first time. The obtained $\pi\pi$ contribution to $a_\mu$ is $a_\mu^{\pi\pi} = (462.4 \pm 0.6_{\text{stat}} \pm 3.2_{\text{syst}} \pm 2.3_{\text{isospin}}) \times 10^{-10}$, which yields $a_\mu^{\exp} - a_\mu^{SM} = (11.0 \pm 10.5) \times 10^{-10}$, in good agreement with ALEPH and CLEO data. Although, only 72 fb$^{-1}$ were used in this analysis, the systematic uncertainties, such as on the track and $\pi^0$ reconstruction efficiency, dominate and it is unlikely to be improved with larger statistics.

### B. Strange spectral function

Analysis of strange tau decays allows to extract mass of strange quark $m_s$ and $|V_{us}|$ element of CKM matrix via moments of the strange spectral function (SSF):

$$ R_{kl}^{\tau} = \int_0^{M_{\tau}^2} ds \left( 1 - \frac{s}{m_{\tau}^2} \right)^k \frac{s}{m_{\tau}^2} \frac{B(\tau \to X^{(S=1)}\nu)}{B(\tau \to e\nu)} \frac{dN_{X^{(S=1)}\nu}}{ds}. $$

$R_{kl}^{\tau}$ are calculable within operator product expansion framework with phenomenological hadronic parametrization [12].

The $R_{1(0,0)}$ moment is most sensitive to the $|V_{us}|$, while its dependence on $m_s$ is small and can be neglected [12]. This allows to extract $|V_{us}| = 0.2208 \pm 0.0033_{\text{exp}} \pm 0.0009_{\text{syst}}$ from results of OPAL [13] assuming $m_s(2\text{GeV}) = 95 \text{ MeV}$. The value is already very competitive with the estimate from $K \to \pi e\nu$ decays $|V_{us}| = 0.2200 \pm 0.0026$ [14] and unlike in $K \to \pi e\nu$ case the theoretical uncertainty is significantly smaller than experimental. Higher order moments are more sensitive to the $m_s$ and one extracts $m_s(2\text{GeV}) = (81 \pm 22) \text{ MeV}$ from the same data. The authors of [12] anticipate simultaneous extraction of $m_s$ and $|V_{us}|$ from the data in future.

The OPAL result is largely statistics limited with total of 162 thousands of identified tau events. While there is only a preliminary result from BaBar on $B(\tau \to K\pi^0\nu) = (4.38 \pm 0.04_{\text{stat}} \pm 0.22_{\text{syst}}) \times 10^{-3}$ available, one can expect a significant improvement of knowledge of SSF from $B$-factories. The statistical uncertainties is very small, and systematic uncertainties are largely correlated for different $\tau \to X^{(S=1)}\nu$ exclusive channels, and the measurement is expected to be few times more precises than current PDG value of $B(\tau \to X^{(S=1)}\nu) = (29.1 \pm 0.8) \times 10^{-3}$ [14].

### C. 5- and 7-prong tau decays

With such big sample at hand, one can look into underlying structure of rare tau decays. BaBar has recently published a study of 5-prong decays $\tau^- \to 3h^-2h^+\nu$ [15]. The branching fraction is $(8.56 \pm 0.05 \pm 0.42) \times 10^{-4}$ in
agreement with previous measurements\textsuperscript{[14]}. However, the invariant mass of five hadrons is different from the phase-space distribution assumed before (see Fig. 2). The contribution of $\rho$ meson is evident in the mass of two pion, and $f_1$ resonance is observed in the four pion mass distribution, $B(\tau^- \to f_1 h^- \nu_\tau) = (3.9 \pm 0.7 \pm 0.5) \cdot 10^{-4}$\textsuperscript{[15]}. BaBar has also searched for 7-prong tau decays. If observed, they would signal of non-SM contribution, as SM predicts $B(\tau^- \to 4\pi^- 3\pi^+ \nu) < 10^{-9}$. No signal is found in either exclusive or inclusive 7-prong tau decays and the obtained upper limits are

\begin{align}
B(\tau^- \to 4\pi^- 3\pi^+(\pi^0)\nu_\tau) &< 3.0 \cdot 10^{-7} \\
B(\tau^- \to 4\pi^- 3\pi^+ \nu_\tau) &< 4.3 \cdot 10^{-7} \\
B(\tau^- \to 4\pi^- 3\pi^+ \pi^0 \nu_\tau) &< 2.5 \cdot 10^{-7}
\end{align}

at 90\% CL\textsuperscript{[16]}. 232 fb$^{-1}$ is used in both analyzes.

\section*{IV. SEARCHES FOR LEPTON FLAVOR VIOLATION}

One of the most interesting question in tau physics now is there a sizable lepton flavor violation (LFV) or not. Given the observation of neutrino oscillation by experiment\textsuperscript{[17]}, one expects charged LFV even in Standard Model extended with massive neutrinos. However, the expected branching fractions are negligible and far beyond reach of the current experiments. From other side, many other extensions of SM, e.g. supersymmetry, predict LFV on the level of $10^{-10} - 10^{-7}$\textsuperscript{[18]}, which can be probed with currently accumulated statistics. Analysis of different channels is important. While $\tau \to \mu \gamma$ is expected to be the largest tau LFV decay in most models, $\tau \to 3\ell$ can expose supersymmetric Higgs contribution\textsuperscript{[19]}, and $\tau^- \to \ell^+ h^- h^-$ violates not only lepton flavor, but also lepton number. If LFV process would be observed in an experiment, the combined analysis of different channels will allow to understand underlying mechanism and to differentiate between the models.

Both BaBar and Belle are very active in searches of lepton flavor violation. Unfortunately, no signal is found in any channel and upper limits are set on the level of $10^{-7}$ (see table\textsuperscript{[1]}. Both experiments plan to increase their samples by factor of 2 by 2008.

The obtained limits can already be used to restrict parameter space of the models. The Fig\textsuperscript{[3,21]} shows the exclusion plot for mSUGRA with right handed neutrinos as function of gaugino ($m_{1/2}$) and scalar ($m_0$) masses at grand unification scale $m_{\text{GUT}} = 5 \cdot 10^{15}$ GeV for $\tan \beta = 50$. The latest measurements of neutrino mixing matrix and masses\textsuperscript{[22]} are used for Yukawa couplings. The mass of right handed neutrinos is set to $M_{\nu_R} = 5 \cdot 10^{14}$ and normal hierarchy is assumed for left handed neutrinos. Everything but a green area is excluded by theory or cold
TABLE I: 90% CL upper limits on LFV tau decays obtained by B-factories\cite{20}. Numbers given in \(10^{-7}\) units. Second column for each experiment shows integrated luminosity used in analysis.

| Channel | BaBar UL | L | Belle UL | L |
|---------|----------|---|-----------|---|
| \(\tau^{-} \rightarrow \mu^{-} \gamma\) | 0.7 | 232 fb\(^{-1}\) | 3.1 | 86 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow e^{-} \gamma\) | 1.1 | 232 fb\(^{-1}\) | 3.9 | 87 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow e^{-} e^{+} e^{-}\) | 2.0 | 91 fb\(^{-1}\) | 3.5 | 87 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \mu^{-} \mu^{+} \mu^{-}\) | 1.9 | 91 fb\(^{-1}\) | 2.0 | 87 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \ell^{-} \ell^{\pm} \ell^{\mp}\) | (1-3) | 91 fb\(^{-1}\) | (2-4) | 87 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \ell^{-} h^{\pm} h^{-}\) | (1-3) | 221 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \ell^{-} h^{+} h^{-}\) | (0.7-5) | 221 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \ell^{-} \pi^{0}, \eta, \eta'\) | 2-10 | 154 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \Lambda \pi\) | 0.7 | 154 fb\(^{-1}\) |
| \(\tau^{-} \rightarrow \bar{\Lambda} \pi\) | 1.4 | 154 fb\(^{-1}\) |

FIG. 3: a) Exclusion plot of mSUGRA with right handed neutrinos as function of gaugino \((m_{1/2})\) and scalar \((m_{0})\) masses for \(\tan\beta = 50\). The green area is allowed by cold dark matter searches and blue curves show area excluded by \(\tau^{-} \rightarrow \mu^{-} \gamma\) as described in the text. b) Model independent upper limits of off-diagonal element of slepton mixing matrix \(M_{L23}^{2}/M_{L22}^{2}\) as function of \(m_{0}\) from \(B(\tau^{-} \rightarrow \mu^{-} \gamma)\) as described in the text.

V. CONCLUSIONS

Current B-factories have a large and interesting program to study tau physics. The accumulated statistics reaches \(10^{9}\) tau decays which allows very precise measurements and searches of very rare or forbidden tau decays. Among the most important measurements are mass and tau lifetime (systematics limited) and tests of CPT/CP violation (statistics limited). The study of the spectral function of \(\tau^{-} \rightarrow \pi^{0} \nu\) decay would help to clarify the comparison of measurement of anomalous magnetic moment of muon with its SM prediction, while strange spectral functions are to
be used for $m_s$ and $|V_{us}|$ estimates.

Among very interesting studies are searches of lepton flavor violation tau decays. While BaBar and Belle can be lucky to discover LFV and this way to probe the physics beyond Standard Model, one would need statistics of super-$B$ factory to be able to measure mixing of sleptons accurately.

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