A Study of the Roper Resonance as a Hybrid State from $J/\psi$ Decays

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Abstract

The structure of the Roper resonance as a hybrid baryon is investigated through studying the transitional amplitudes in $J/\psi \rightarrow \bar{p}N^*, \bar{N}^*N^*$ decays. We begin with perturbative QCD to describe the dynamical process for the $J/\psi \rightarrow 3\bar{q} + 3q$ decay to the lowest order of $\alpha_s$, and by extending the modified quark creation model to the $J/\psi$ energy region to describe the $J/\psi \rightarrow 3\bar{q} + 3q + g$ process. The non-perturbative effects are incorporated by a simple quark model of baryons to evaluate the angular distribution parameters and decay widths for the processes $J/\psi \rightarrow \bar{p}N^*, \bar{N}^*N^*$. From fitting the decay width of $J/\psi \rightarrow \gamma p\bar{p}$ to the experimental data, we extract the quark-pair creation strength $g_1 = 15.40$ GeV. Our numerical results for $J/\psi \rightarrow \bar{p}N^*, \bar{N}^*N^*$ decays show that the branching ratios for these decays are quite different if the Roper resonance is assumed to be a common $3q$ state or a pure hybrid state. For testing its mixing properties, we present a scheme to construct the Roper wave function by mixing $|qqqg\rangle$ state with a normal $|qqq, 2s\rangle$ state. Under this picture, the ratios of the decay widths to that of the $J/\psi \rightarrow p\bar{p}$ decay are re-evaluated versus the mixing parameter. A test of the hybrid nature of the Roper resonance in $J/\psi$ decays is discussed.

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1 Introduction

The theoretical studies on the hadronic structure remain as long outstanding problems in particle physics. About four decades ago, the naive quark model was proposed to count for SU(3) symmetries in hadronic spectroscopies[1]. After that, much progresses have been made in many theoretical aspects, such as the predictions on the mass spectrum, decay rate and electromagnetic coupling properties for hadrons (for details, see recent review[2]). So far, much more phenomenological models based on the quark constituents have been developed with a large number of successes for further understanding the relations between quarks’ degrees of freedom and fundamental field of quantum chromodynamics (QCD) theory. Despite these successes the quark models acquired, they are still confronted with some challenges. One of the outstanding questions is the fact that, in addition to the conventional quark states, QCD theory also predicts the existence of hadrons with 'excited glue', which is called hybrid states...
historically. Therefore, searching for these new hadronic states is of great importance both to QCD theory and the common quark models. If no hybrid meson/baryon states are found, it would imply that our current understanding of QCD theory should be modified, and the dynamics within quark model would have to be changed. On the other hand, if an unconventional baryon state is discovered in experiment, it would be necessary to determine whether it is a hybrid state or a common quark state, or their mixture. However, the answer to this question seems to be more difficult from experimental aspects due to the fact that it depends on a comparison with theoretical predictions on the specific properties for the new states.

Therefore, the identification of hybrid states would require a close collaboration between experimental and theoretical experts[3]. As our knowledge about common hadronic structures is mainly acquired from their mass spectrum, hadronic spectroscopy will continue to be a key tool in this field. Various models and methods have been used to predict the spectrum of hybrid mesons/baryons, such as the bag model, QCD sum rule, the flux tube model, and so on. Though each model assumes a particular description of excited glue, fortunately they often reach similar conclusion regarding the quantum numbers and approximate masses of these states. For instance, the predictions on the light hybrid mesons are in good agreement with each other, with the so-called exotic number $J^{PC} = 1^{-+}$ and the mass about $1.5 \sim 2.0\text{GeV}$. Experimentally we now have some candidates for $J^{PC}$ exotic mesons, for example, $\pi_1(1400)$ with $1^{-+}$ seen in $\eta\pi[4]$, and $\pi_1(1600)$ seen in $\rho\pi, \eta\pi$ and $b\pi[5]$. In baryon sector, the Roper resonance $N^*(1440)$ has been suggested to be a potential candidate of hybrid baryons for a long time. In the pioneering work of the evaluation of the hybrid baryon mass, Barnes and Close once used the bag model to predict that the lightest hybrid baryon was an "extra" $\frac{1}{2}^{+} N^*_1(P_{11})$ state with energy at about 1.6 GeV[6]. The subsequent identification of this hybrid baryon with the Roper resonance was confirmed basically by Golowich, Haqq and Karl[7]. The estimation of its mass was also carried out by using the QCD sum rule, a general prediction came with a conclusion that the lightest $\frac{1}{2}^{+}$ hybrid baryon is near 2.1 GeV[8]. Recently, this value has been corrected by Kisslinger to be about 1.5 GeV, very suggestive of the the Roper resonance[9]. It seems that one might find the state with excited gluonic degrees of freedom by studying baryonic mass spectrum. Unfortunately to identify a hybrid baryon by the mass spectrum remains an unconquerable difficulty, mainly due to the lack of $J^P$ exotics (this situation is quite different from that in hybrid mesons). Hence, for their identification we have to predict other experimental observables characterized by their distinct properties as a signature of hybrid baryons. For example, strong decay amplitudes[10], photoproduction and electroproduction (EM) amplitudes and so on, can be served for this purpose[11]. Especially, the study of the $N^*$ spectroscopy from the $J/\psi$ hadronic decays at Beijing Electronic-Positron Collider (BEPC) may be an alternative tool to identify the nature of the Roper resonance[12]. One might expect hybrid baryons to have a larger production amplitudes from $J/\psi$ decays for hybrid baryons than the conventional $qqq$ baryons, because the three gluonic intermediate states produced in $J/\psi$ annihilations may have a large overlap with final hybrid baryons. Furthermore, this approach has an additional advantage that it is an isospin $I=1/2$ filter, so that no $\Delta$ (and hybrid $\Delta$) states are present to complicate the analysis.

Recently, BESII has finished data-taking for 58 million more $J/\psi$ events, which is about two order of magnitude more statistics than MARKII data, and one order of magnitude more statistics than BESI data. With such statistics, partial wave analysis of relevant channels is possible, the angular distributions and the decay widths of processes $J/\psi \rightarrow p N^*(1440)$ and $J/\psi \rightarrow \bar{N}^*(1440) N^*(1440)$ are expected to be available from partial wave analysis in the near future.

What properties are able to serve as a signature to distinguish the Roper resonance between a hybrid baryon and a conventional three quark baryon? As well-known, the spatial distribution of constituent components bound in the Roper resonance is described as the first radially excited state in the naive quark model, while as a ground state in the hybrid picture. These different structure pictures, together with their different spin-flavor structures,
may provide us an effective method to test the hybrid nature of the Roper resonance through studying transitional amplitude involved in a given $J/\psi$ decay process. In this paper we pick out some relevant processes, in which the $J/\psi$ particles decay into a baryon and an anti-baryon pair including Roper resonance to investigate the Roper structure as a hybrid state.

This paper is organized as follows: in the second section, we present our model of the $J/\psi$ decay into a hybrid baryon and an anti-baryon, and make a choice of wave functions for hybrid baryons. With this model, in section 3, we formulate the amplitudes of $J/\psi$ decays into $\bar{p}N^*$ and $\bar{N}^*N^*$ baryon pairs, and its corresponding decay widths and angular distributions for these two decay modes. In the fourth section, the determination of the quark pair creation strength from the process $J/\psi \rightarrow \gamma p\bar{p}$ is formulated. Then, we present our main numerical results on angular distributions and decay widths versus the mixing parameter for the two decay modes $J/\psi \rightarrow \bar{p}N^*, \bar{N}^*N^*$, where the structure of the Roper resonance in two different pictures are assumed. We also discuss our results in the last section. Some matrix elements of the transitional amplitude are appended at the end of paper.

2 Model description

The exclusive decays of the charmonium have been investigated by many authors within perturbative QCD[13]. The main dynamical mechanism is simply assumed that the $c\bar{c}$ quarks annihilate into a minimum number of gluons constrained by the charge and parity conservation. The decay of the $J/\psi$ particle into a baryon-antibaryon pair is currently assumed to proceed via two steps as illustrated by Fig.1(a). In the first step, the $c\bar{c}$ pair annihilates into three gluons, followed by the materialization of each gluon into a pair of quark-antiquarks. Since the $c\bar{c}$ quarks annihilate only if their mutual separation is less than about $1/m_c$ ($m_c$ is the mass of $c$ quark), which is smaller than the non-perturbative charmonium radius, and since the average energy of three virtual gluons is of the order 1-2 GeV which lies in the QCD perturbative region. Thus this step is usually carried out from perturbative QCD to the lowest order of $\alpha_s$. Then in the second step the three quarks on the one hand and the three antiquarks on the other hand combine to form a baryon and antibaryon, respectively. The nonperturbative dynamics is included by the quark wave functions inside the baryons from the quark model.

As in naive quark model, the quarks and the gluon bound in hybrid baryons are treated as constituent components with mass $m_q$ and $m_g$, respectively, and their spatial wave function are chosen to be harmonic-oscillator eigen wave functions in the center of mass (CM) system of the baryon. The construction of the hybrid wave function is completed by combining the quark and the gluon sector into a color-singlet state, which is indeed included in the representation of group $SU(3)_c \otimes SU(3)_c \otimes SU(3)_c \otimes SU(8)_c$. Let $I,J$ denote the quantum numbers of total isospin and angular momentum for $|qqqg\rangle$ state with $I_3$ and $J_3$, the z-projection components, then the wave function of the hybrid baryon denoted by $|Nq,I,J\rangle_{2I,2J}$ can be expressed as follows.
mass spectra and the photoproduction amplitudes. On the other hand, some theoretical studies have predicted the existence of a hybrid state at this mass region[6, 8, 9]. So the mixing of those two configurations should be considered. Li[11] once mixed the

\[|N, I_3j_3\rangle\]

with an equal ratio[11], i.e.

\[|qqq\rangle = \frac{1}{\sqrt{2}}(|N, I_3j_3\rangle + |N, I_3j_3\rangle) \times \text{symmetric spatial part}.\]  

Figure 1: Lowest-order diagrams for (a)\(J/\psi \rightarrow |\bar{q}q\rangle + |qqq\rangle\), (b)\(J/\psi \rightarrow |\bar{q}q\rangle + |qqq\rangle\), (c)\(J/\psi \rightarrow \gamma p\bar{p}\)

\[|Ng, I_3j_3\rangle_{2I, 2J} = \frac{1}{\sqrt{8}} \sum_{\alpha=1}^{8} \sum_{js} C_{j_3j_3-1j_3}^{I} |N, I_3j_3\rangle_{2I, 2J} |g, 1J3 - j3\rangle_{\alpha}.\]  

where \(|N, I_3j_3\rangle\) is the color-octet \(qqq\) part in the hybrid state, with quantum number of the total isospin \(I\)

\[|\bar{q}q\rangle \rightarrow |\bar{q}q\rangle + |qqq\rangle + |qqq\rangle + |qqq\rangle + |qqq\rangle.\]  

Especially, for \(I=1/2\), the wave functions for the color-octet \(qqq\) part[6] are

\[4s : |N, I_3j_3\rangle_{1,3}^{n} = \frac{1}{\sqrt{2}} (\phi^s \psi^s - \phi^s \psi^s) \chi^s,\]  

\[2s : |N, I_3j_3\rangle_{1,1}^{n} = \frac{1}{2} (|\phi^s \chi^s - \phi^s \chi^s\rangle \psi^s - (\phi^s \chi^s + \phi^s \chi^s) \psi^s).\]  

Here \(\phi, \chi, \psi\) denote the flavor, spin and color wave functions, respectively. \(s, \lambda, \rho\) denote the symmetry or mixed symmetry representations of the total permutation of particles (\(\lambda\) is symmetric and \(\rho\) antisymmetric under (23) interchange). The study of the electromagnetic transitional properties suggested that the wave function for the \(|qqq\rangle\) state should be chosen as a mixing of \(4s\) and \(2s\) with an equal ratio[11], i.e.

\[|qqq\rangle = \frac{1}{\sqrt{2}} (|Ng, I_3j_3\rangle_{1,3} + |Ng, I_3j_3\rangle_{1,1}) \times \text{symmetric spatial part}.\]  

In the constituent quark model, the wave function of the nucleon and the Roper resonance are assigned as a ground state and the first radial excitation of three constituent quarks in the harmonic oscillator potential, respectively. The nucleon wave function in the color-spin-flavor space reads,

\[|p\rangle \equiv |qqq, 1s\rangle = \frac{1}{\sqrt{2}} \psi^s (\chi^s \phi^s + \chi^s \phi^s) \times \text{symmetric spatial part},\]  

where \(\psi^s\) is the asymmetric color wave function, \(\chi^s(\chi^s)\) and \(\phi^s(\phi^s)\) are the spin and flavor wave functions for the \(qqq\) cluster with \(\rho\) (or \(\lambda\)-type) mixing symmetry, respectively.

The identification of the Roper resonance as a \(|qqq, 2s\rangle\) state meets some difficulties in the reproduction of the mass spectra and the photoproduction amplitudes. On the other hand, some theoretical studies have predicted the existence of a hybrid state at this mass region[6, 8, 9]. So the mixing of those two configurations should be considered. Li[11] once mixed the \(|qqq\rangle\) part with \(|qqq, 1s\rangle\) in the study of \(N^*(1440)\) in the photoproduction process, where the wave function of nucleon \(|p\rangle\) and the Roper resonance \(|N^*\rangle\) are assumed to be linear combination
The full decay width of the $J/\psi$ particle into three gluons has been investigated many years ago. Since the charmonium decay process involves two different physical scale: the binding energy $\epsilon$ of the bound state and the heavy $c$-quark mass $m_c(m_c \gg \epsilon)$, now it is widely accepted that the transitional amplitudes for charmonium decays can be expanded to order $O(p/m)$, where $p$ is the relative momentum of the $c\bar{c}$ quarks. For $J/\psi$ particles described as a $1s$ bound state, only the first order term contributes dominantly to the decay width. This approximation results in that the decay width is proportional to the value of the $J/\psi$ wave function at the origin $|\Phi(0)|$. To study the exclusive decays of $J/\psi$ into a baryon anti-baryon pair, we focus on the dynamical behavior of the gluon properties, the outgoing quarks and the information on the baryon structure. We feel that the non-perturbative properties of the bound $c\bar{c}$ quarks and their dynamical effects do not play important roles in the $J/\psi$ exclusive decays at least in the limit of the non-relativistic approximation, since in which the contribution from $c\bar{c}$ quarks are independent on their relative momenta. Hence the non-perturbative effects within this energy level, together with the contribution from the $c\bar{c}$ quarks and the decay constant of $J/\psi$ and so on are all parameterized into an overall constant $C_0$.

The gluons from the $c\bar{c}$ annihilation play a different role in our model. In the process $J/\psi \rightarrow 3g \rightarrow 3\bar{q} + 3\bar{q}$, each virtual gluon behaves with equal status due to the fact that each gluon finally creates a $q\bar{q}$ quark pair. While in the process $J/\psi \rightarrow 3g \rightarrow 3\bar{q} + 3q + g$, of all three gluons, there being one gluon does not create a $q\bar{q}$ quark pair and instead is bound inside the hybrid baryon in the final state. We consider this gluon to be a constituent one in the bound state.

If we assume the Roper resonance is a pure hybrid state, i.e. without mixing with $|qqg\rangle$ component in its wave functions, there only being one precess as shown in fig.1(b) should be considered. In general, the Roper resonance is assumed to be a mixture of the $|qqq\rangle$ and $|qqg\rangle$ components. In this case of mixing picture, to study the decay of $J/\psi$ into the Roper, it is essential to calculate the basic transitional amplitudes of both decay modes given in fig.1.(a) and (b). The basic amplitudes for these two processes can be expressed as,

$$
<\bar{N}N|T_1C_1|J/\psi^{(A)}\rangle_{g_{s},s_{t}} = \Psi_{s_{t}}(q',s')C_1T_1|J/\psi^{(A)}\rangle
$$
\[
\begin{align*}
&= \sum_{s_i,s_i'} \left( \prod_{i=1}^{3} d^{2}q_i \, d^{2}q_i' \right) < \Psi_N(q',s') \Psi_N(q,s)|q_i,s_i,q_i',s_i', i = 1..3 > \\
&\times < q_i,s_i,q_i',s_i', i = 1..3|C_1T_2|J/\psi^{(A)}>, \\
&\text{for fig.1.(a), and} \\
&\equiv < \Psi_h(q',s') \Psi_N(q,s)|C_2T_2|J/\psi^{(A)} > \\
&= \sum_{s_i,s_i'} \left( \prod_{i=1}^{4} d^{2}q_i \, d^{2}q_i' \right) < \Psi_h(q',s') \Psi_N(q,s)|q_i,s_i,q_j,s_j', i = 1..4, j = 1..3 > \\
&\times < q_i,s_j,q_j',s_i', i = 1..4, j = 1..3|C_2T_2|J/\psi^{(A)}>,
\end{align*}
\]

for fig.1.(b). Here \(N\) and \(N_h\) stand for the \(qqq\) and \(qqqg\) states, respectively. \(\Psi_N(q',s')\) and \(\Psi_h(q',s')\) are the flavor-spin-spatial wave functions for \(qqq\) and \(qqqg\) cluster, respectively.

The hard scattering part in above equations can be written explicitly by the standard Feynman rules, i.e.

\[
< q_i,s_i,q_i',s_i'|C_1T_2|J/\psi^{(A)} > = C_0 < C_1 > (ig)^{-A}\epsilon_{j/\psi}^{(A)\lambda} \frac{g_{\mu\lambda}g_{\nu\rho} + g_{\mu\rho}g_{\nu\lambda} + g_{\nu\lambda}g_{\mu\rho}}{(q_i + q_i')^2(q_2 + q_2')^2(q_3 + q_3')^2} \\
\times \hat{u}(q_i',s_i')\gamma^{\mu}v(q_i,s_i)\hat{u}(q_2',s_2')\gamma^{\nu}v(q_2,s_2)\hat{u}(q_3',s_3')\gamma^{\nu}v(q_3,s_3),
\]

and

\[
< q_i,s_i,q_j,s_j'|C_2T_2|J/\psi^{(A)} > = C_0 < C_2 > (ig)^{5}g_{\nu}\epsilon_{j/\psi}^{(A)\lambda} \epsilon_{j/\psi}^{(A)\mu} \frac{g_{\mu\lambda}g_{\nu\rho} + g_{\mu\rho}g_{\nu\lambda} + g_{\nu\lambda}g_{\mu\rho}}{(q_i + q_i')^2(q_2 + q_2')^2(q_3 + q_3')^2} \\
\times \hat{u}(q_i',s_i')\gamma^{\nu}v(q_i,s_i)\hat{u}(q_2',s_2')\gamma^{\nu}v(q_2,s_2)\hat{u}(q_3',s_3')v(q_3,s_3),
\]

where \(\epsilon_{j/\psi}^{(A)\lambda}, \epsilon_{j/\psi}^{(A)\mu}\) are the polarization vectors for \(J/\psi\) and the gluon with helicity values \(\Lambda\) and \(\lambda_i\), respectively. We perform the calculation in the \(J/\psi\) rest system. For \(J/\psi\) produced in \(e^{+}e^{-}\) annihilation, its helicity is limited to be \(\Lambda = \pm 1\). \(q_i(q_i')\) and \(s_i(s_i')\) are the four vector momentum and spin z projection of the anti-quarks (quarks), respectively. \(\langle C_1 \rangle\) and \(\langle C_2 \rangle\) are color factors corresponding to the decay mode as depicted in fig.1.(a) and (b), and \(g = \sqrt{4\pi\alpha_s}\) is a strong coupling constant. \(g_i\) is an effective strength of the created quark pairs in the \(^3P_0\) quark model which will be determined from the decay mode \(J/\psi \rightarrow \gamma p\bar{p}\). Other constants and contributions from charmonium bound properties, which are assumed to be independent on a given process, are all put into a single overall constant \(C_0\). \(v(q_i,s_i)\) and \(v(q_i,s_i)\) are assumed to be free Dirac spinors for quarks and anti-quarks, respectively. They are explicitly given by

\[
u(q_i',s_i') = \sqrt{\frac{m_i + E_i'}{2E_i'}} \left( \frac{1}{\sqrt{m_i + E_i'}} \chi_{s_i'} \right) \chi_{s_i}, \quad v(q_i,s_i) = \sqrt{\frac{m_i + E_i}{2E_i}} \left( \frac{\sqrt{m_i + E_i}}{1} \right) \chi_{s_i},
\]

where \(E_i \equiv q_i^0\) is the energy of a quark.

The evaluation of the color factor \(< C_1 >\) and \(< C_2 >\) for these decay modes is straightforward in terms of the SU(3) structure constant \(d_{abc}\) and the SU(3) color matrices \(T_{abc}\), they are

\[
< C_1 > = \frac{1}{24\sqrt{3}}d_{abc}\epsilon^{ijk}T^{a}_{il}T^{b}_{jm}T^{c}_{kn}\epsilon^{lmn} = \frac{5}{18\sqrt{3}},
\]

\[
< C_2 > = \frac{1}{48}\epsilon_{ijk}T^{b}_{il}T^{c}_{jm}\delta_{kl}\psi^{\nu}(lmn) = \frac{5i}{144},
\]

where \(T^{(a)}(a = 1.8)\) are Gellmann matrices, and \(\psi^{\nu}(lmn)\) is the color-octet wave function for three quarks in the \(|qqqg\) state with antisymmetry by exchanging quark (2) and (3), while the \(\lambda\)-type wave function \(\psi^{\lambda}(lmn)\), which
symmetric properties of Gellmann matrices. It is worthy to note that the ratio $\langle C_2 \rangle/\langle C_1 \rangle \approx 0.2$ from Eqs. (12) and (13), which means that the decay of the $J/\psi$ particle into the hybrid state is a color suppressed process.

The components involving Dirac spinors in Eqs.(9,10) can be expressed more explicitly as

$$\epsilon_{J/\psi}^+ = (0, \mp \frac{1}{\sqrt{2}}, -i \frac{1}{\sqrt{2}}, 0),$$

$$\bar{u}(q', s_i) \gamma^0 v(q, s_i) = \left\langle s_i' \begin{pmatrix} \vec{\sigma} \cdot \vec{q}' \cr m_i + E_i \end{pmatrix} + \begin{pmatrix} \vec{q}' \cdot \vec{\sigma} \cr m_i + E_i \end{pmatrix} s_i \right\rangle,$$

$$\bar{u}(q', s_i) \gamma^5 v(q, s_i) = \left\langle s_i' \begin{pmatrix} \vec{\sigma} \cdot \vec{q}' \cr m_i + E_i \end{pmatrix} - \begin{pmatrix} \vec{q}' \cdot \vec{\sigma} \cr m_i + E_i \end{pmatrix} s_i \right\rangle,$$

$$\bar{u}(q', s_i) \bar{\gamma}^\mu v(q, s_i) = \left\langle s_i' \begin{pmatrix} \vec{\sigma} \cdot \vec{q}' \cr m_i + E_i \end{pmatrix} + \frac{(\vec{\sigma} \cdot \vec{q}') \vec{\sigma} \cdot \vec{q}'}{m_i + E_i} s_i \right\rangle,$$

where $E_i$ and $m_i$ are the energy and the mass of the quark.

From these amplitudes, one may obtain elements of the transitional amplitudes $M_{s_i s_i'}^{(A)}$ for processes $J/\psi \rightarrow \bar{p}p, \bar{p}N^*, N^*N^*$ by projecting them on the wave functions of the nucleon or the Roper resonance (see Appendix B). In our calculation, we ignore the amplitude for $J/\psi \rightarrow \bar{q}qg$ + $|\bar{q}qg\rangle$ because we argue that the contribution of matrix elements for this process is trivial due to the color factor and an additionally suppressed factor $e^{-\frac{\vec{q}^2}{2m^2}}$ in the spatial wave function for the $|\bar{q}qg\rangle$ cluster. Then the differential decay width for $J/\psi \rightarrow BB(B : \text{baryon})$ can be expressed as,

$$\frac{d\Gamma(J/\psi^{(A)} \rightarrow BB)}{dOmega} = \frac{1}{32\pi^2} \left[ |M^{(A)}_{\frac{1}{2},\frac{1}{2}}|^2 + |M^{(A)}_{\frac{1}{2},-\frac{1}{2}}|^2 + |M^{(A)}_{-\frac{1}{2},\frac{1}{2}}|^2 + |M^{(A)}_{-\frac{1}{2},-\frac{1}{2}}|^2 \right] \left| \vec{q} \right|^2 \frac{1}{M_{J/\psi}^2},$$

where $M_{J/\psi}$ is the mass for $J/\psi$, and the $\vec{q}$ is the momentum of outgoing baryons.

From the symmetry consideration, the differential decay width of $J/\psi \rightarrow BB$ can be expressed in a more general form,

$$\frac{d\Gamma(J/\psi^{(A)} \rightarrow BB)}{dOmega} = N_{BB}(1 + \alpha_B \cos^2 \theta),$$

where $N_{BB}$ is a constant directly related to the experimental total branching ratio of $J/\psi \rightarrow BB$, and can be used to fix the overall constant $C_0$, and $\theta$ is the angle between the positron beam direction and the direction of the outgoing baryon. $\alpha_B$ characterizes the angular distribution of $J/\psi \rightarrow BB$, which can be extracted from the experiments.

4 Strength of quark-pair creation in $J/\psi$ decays

The quark-pair creation model was first introduced by Micu[15] in 1969 in a study of meson decays, which suggested that the strong decay of mesons proceeds through a simple quark-antiquark ($q\bar{q}$) pair creation from the vacuum, with a quantum number $J^{PC} = 0^{++}$. With a $q\bar{q}$ pair creation operator and a dimensionless constants $\gamma$, the widths of the two-body hadron decays can be evaluated easily from definite transitional amplitudes. Many authors have developed Micu’s original suggestion and applied it extensively to a number of baryon and meson decays with considerable successes. In this model, the constant $\gamma$ is a parameter and is obtained from the fitting to the light meson or baryon decay widths.

For the OZI rule allowed decays of the $J/\psi$ particle, the $q\bar{q}$ pair might be created from either a gluon annihilation or the QCD vacuum. However, in the energy region of the $J/\psi$ decay the strength of the $q\bar{q}$ pair creation from the QCD vacuum may be different from that in the light meson or baryon decays. Therefore, we adopt the modified quark-pair creation model to extract the strength of the quark-pair creation by evaluating the decay width of the
The radiative decay $J/\psi \rightarrow \gamma p\bar{p}$ is assumed to proceed via two steps as illustrated by fig.1(c). In the first step, the $c\bar{c}$ pair annihilates with the emission of a photon and two gluons. In the second step, each one of the gluons creates a quark-antiquark pair and another quark-antiquark pair is created from the vacuum, which can be described by quark creation model with the quark pair creation strength $g_I$. By a complicated final state interaction, a proton and an antiproton are formed. This complicated hadronization process is accomplished by the bound state quark wave functions of the nucleon, which is constructed in the constituent quark model.

In our quark pair creation model, the quark-antiquark pair with any color and flavor can be created anywhere from the QCD vacuum with equal strength. But only those pairs whose color-flavor wave function and space wave function overlap with that of baryons in the final state can make a contribution to the decay width. Following the usual procedure, the Hamiltonian for the quark pair creation can be defined in the modified $^3P_0$ model[16] in terms of quark and antiquark creation operators $b^+$ and $d^+$,

$$H_I = \sum_{i,j,o,\alpha,s} \int d^3k d^3k' g_I [\bar{u}(k',s')v(k,\bar{s})] b_{\alpha,i}(\vec{k}',s') d_{\alpha,j}(\vec{k},s) \delta^3(\vec{k} - \vec{k}') \delta_{\alpha\beta} \hat{C}_I,$$

where $\alpha(\beta)$ and $i(j)$ are the flavor and color indexes of the created quarks (anti-quarks) , and $u(k',s')$ and $v(k,s)$ are free Dirac spinors for quarks and antiquarks, respectively. They are normalized as $u^+(k,s)u(k',s') = \delta_{k,k'} \delta_{\alpha\beta}$. $\hat{C}_I = \delta_{ij}$ is the color operator for $q\bar{q}$ and $g_I$ is the strength of the decay interaction, which will be extracted from fitting the width $\Gamma(J/\psi \rightarrow \gamma p\bar{p})$ to the experimental data. In the non-relativistic limit, $g_I$ can be related to $\gamma$, the strength of the conventional $^3P_0$ model, by $g_I = 2\pi\gamma[16]$.

We first define an operator for $J/\psi \rightarrow \gamma + 2q + 2\bar{q}$ transitions, i.e.

$$\hat{O}(J/\psi \rightarrow \gamma,qq,\bar{q}\bar{q}) = C_0 \hat{C}(i\epsilon)(\imath g)\frac{\alpha(\lambda)\epsilon^{\mu(\lambda)}\epsilon^{\mu(\lambda)}}{(q_1 + q_1')^2(q_2 + q_2')^2} \cdot \bar{u}(q_1',s_1') \gamma^\mu v(q_1,s_1)\bar{u}(q_2',s_2') \gamma^\mu v(q_2,s_2),$$

where $q_i(i = 1,2,3)$ and $q'_i(i = 1,2,3)$ are the four-momentum vectors for anti-quarks and quarks, respectively. $C_0$ is an overall constant which can be determined from fitting the decay width of the process $J/\psi \rightarrow p\bar{p}$. $\hat{C} = \frac{1}{2\pi} \delta_{\alpha\beta} T_{\lambda \mu}^{\alpha \beta} r_{\mu \nu}^\lambda$ is the color operator in terms of SU(3) color matrices $T$, and $g$ is a strong coupling constant. $\epsilon^{\lambda(\Lambda)}\lambda$ is a photon polarization vector with helicity $\lambda = \pm 1$. $\epsilon^{\Lambda(\Lambda)}_{J/\psi}$ is the $J/\psi$ polarization vector.

Inside a baryon there are strong interactions among three constituent quarks. To form a baryon from the three quarks is a non-perturbative process. Following the usual approach, we account for all strong interaction effects by the bound-state quark wave-functions in the final (anti-)baryon state. The amplitude for $J/\psi \rightarrow \gamma p\bar{p}$ transition can be written as,

$$\mu^{(A)}(s_z,s_{z}',p_1,p_2,p_3) = \langle \psi_p(s_z,p_1,q_1,q_2,q_3)\psi_p(s_{z}',p_2,q_1',q_2',q_3') | \hat{O}H_I | \psi^{(A)}_{J/\psi}(P) \rangle,$$

where $s_z$ and $s_{z}'$ are the spin z-projections for the anti-baryon and baryon, respectively. $\psi_p(s_z,p_1,q_1,q_2,q_3)$ and $\psi_p(s_{z}',p_2,q_1',q_2',q_3')$ are the spin-flavor-spatial wave functions of the baryon and anti-baryon,respectively. In general , the structure of spin-flavor wave functions of baryons can be constructed in the constituent quark model. The spin and flavor wave functions of the proton and antiproton are similar, i.e.

$$\psi^p_{SF} = \psi^{\bar{p}}_{SF} = \frac{1}{\sqrt{2}} (x^\rho \phi^\rho + x^\lambda \phi^\lambda),$$

where $\chi^\rho$ and $\chi^\lambda$ are the spin-$\frac{1}{2}$ wave functions of the quark pair with mixed-symmetry (see Eqs.(31,32) in Appendix A).

The spatial wave functions for the proton or antiproton are chosen as simple harmonic-oscillator eigenfunctions in their center-of-mass(c.m.) system. However, our calculation is carried out in the $J/\psi$ rest system, where the
function from the c.m. system of the baryon to the laboratory system. Here we only perform the Lorentz boosts for the spatial wave function and ignore the Melosh rotations of the quark spinors. This approximation will be discussed in the next section.

The decay widths for the process $J/\psi \rightarrow \gamma p\bar{p}$ can be expressed as:

$$\Gamma = \frac{(2\pi)^3}{2M} \sum_{A} \sum_{s_z,s_{z'},p_1,p_2,p_3} |\mu(A, s_z, s_{z'}, p_1, p_2, p_3)|^2 \prod_{i=1}^{3} \frac{d^3p_i}{(2\pi)^3 2E_i} \delta^3(P - p_1 - p_2 - p_3),$$

(21)

where $P$ and $M$ are the four momentum vector and the mass of the $J/\psi$ particle, respectively. In the $J/\psi$ c.m. system, $\vec{P} = 0$. $E_1, E_2$ and $E_3$ are the energies of the anti-baryon, baryon and photon, respectively.

The photon in the process $J/\psi \rightarrow \gamma p\bar{p}$ may also come from the bremsstrahlung of the outgoing proton or anti-proton following the strong decay $J/\psi \rightarrow p\bar{p}$. However, this bremsstrahlung mainly contributes to the low-energy photons[17]. On the other hand, the existing measurements for the radiative decays are performed with the condition $m(p\bar{p}) < 2.79 GeV/c$[18], which implies that photons with energies smaller than 130 MeV are cut off. In our calculation we ignore the contribution from the bremsstrahlung in the final state to the radiative decay width and perform the calculation with the experimental condition to evaluate the quark-pair creation strength. We use the branching fraction $Br(J/\psi \rightarrow \gamma p\bar{p}) = 3.8 \times 10^{-4}$[19], the light quark mass $m_q = 0.22 GeV$ and the harmonic constant 0.08 GeV$^2$ to extract the strength $g_I = 15.40 GeV$. We also change the harmonic oscillator parameter from 0.08 GeV$^2$ to 0.16 GeV$^2$ and find that the strength $g_I$ is not sensitive to the choice of this parameter.

This value is much larger than that from fitting to the light meson decay widths by Ackleh et al.[16]. From their calculation, we get $g_I = 7.3 GeV$. The difference may indicate that the strength $g_I$ is energy-dependent. Besides, in Ackleh’s calculation, the non-relativistic limit $E_q = m_q$ is used and the Lorentz contraction for the wave functions of outgoing mesons is ignored. While in our evaluation, the relativistic transformation on the baryonic radial wave functions between two different systems are carried out.

5 Numerical results and discussions

Although the decay of the $J/\psi$ into three gluons and the creation of a quark-antiquark pair from one gluon may be treated by perturbative QCD, the formation of a baryon or hybrid baryon is certainly beyond the asymptotic regime. We account for this effect by explicitly including the bound state wave function for the three-quark cluster or quark-gluon cluster. As in the naive quark model, we treat the quarks and gluon in the baryon as constituent components with masses $m_q$ and $m_g$, respectively. We assume that the momentum distribution of the bound components in the cluster $|qqq\rangle$ and $|qqqg\rangle$ can be described by wave functions $\phi_{nl}(p_p,p_\lambda)$ and $\phi_{h}(p_a,p_b,p_c)$, respectively. Since the spin-independent potential is used in the constituent quark model, the spatial wave functions of the constituents can be chosen as simple harmonic oscillator eigenfunctions in their center-of-mass (CM) system [see Eqs.(42,43)].

In the CM system of the $J/\psi$, the two baryons are moving in opposite directions with highly relativistic speeds. One has to transfer the quark wave function from the CM system of the baryon to the laboratory system. The two baryons’ radial wave functions in the laboratory system are related to the wave functions in their CM system by the Lorentz transformation [20, 21]. In principle the quark spin wave functions have to be changed by the Melosh rotations of the quark spinors simultaneously. For simplicity, this relativistic transformation can be simplified by neglecting Melosh rotation effects in the framework of the naive quark model as in the studies of the resonance electroproduction[22]. Since the non-relativistic descriptions for constituent quarks are employed in our calculation, the momentum dependence of the spin wave function can be ignored. On the other hand, for the $J/\psi$ decay process we can take the $z$ axis in the direction of the moving baryons. So the third component of the quark spin remains the
we found that this treatment can be compensated by taking the parameters describing the baryonic properties as effective values from fitting to experimental data. Therefore, in our present calculations, we only perform the Lorentz boosts for the spatial wave function as follows,

$$\phi_p(p^\mu, q^\nu) = \left| \frac{\partial(p^\mu, q^\nu)}{\partial(p^\mu, q^\nu)} \right|^{1/2} \phi(p^\mu, q^\nu),$$

and

$$\phi_h(p^\mu, q^\nu) = \left| \frac{\partial(p^\mu, q^\nu)}{\partial(p^\mu, q^\nu)} \right|^{1/2} \phi(p^\mu, q^\nu).$$

In most quark model calculations, the value of $\alpha$ has been chosen in the range of 0.06 $\sim$ 0.22GeV$^2$, which corresponds to the nucleon radius in the range of 0.52 $\sim$ 0.98fm. The range of $\alpha$ in our model has been determined elsewhere by fitting the angular distribution of $J/\psi \to p\bar{p}[23]$. We take the mass of the light quark $m_q = 0.22$GeV and $\alpha = 0.08$GeV$^2$, which corresponds to the central value of the angular distribution parameter $\alpha_{pp} = 0.62$ for $J/\psi \to p\bar{p}$. The assignment of a mass to the constituent gluon is model-dependent. For instance, the lattice QCD calculations predict the mass of the constituent gluon $m_g = 0.5 \sim 0.6$ GeV. But from the phenomenological theory, the mass of gluons are commonly taken in the range $m_g = 0.3 \sim 0.7$GeV at the level of $J/\psi$ decays[24, 26]. In our model, if we assume that the Roper is composed of one constituent gluon and three light quarks in 1s state, we approximately take the constituent gluon mass as $m_g \sim M_{N^*} - M_p \approx 0.5$GeV. In our present calculation, the gluon is regarded as a relativistic constituent particle, its mass can also be adjusted to a lower value $m_g = 0.37$GeV as taken in Ref.[25]. The coupling constant is chosen to be $\alpha_s = 0.28[14]$, and the strength of the quark pair creation, $g_1$, is determined from the calculation of the decay width $\Gamma(J/\psi \to \gamma p\bar{p})$ in the previous part.

### 5.1 Roper as a pure hybrid baryon or conventional baryon

At first, we consider the Roper resonance as a pure hybrid state, namely a $qqqg$ bound state. With selected parameters varying within our parameter window, we get the angular distribution parameters $\alpha_+ = 0.42 \sim 0.57$ and $\alpha_{++} = (-0.1) - (-0.9)$ for the decay modes $J/\psi \to \bar{p}N^*$, $\bar{N}^*N^*$, respectively.

Although the dynamical behavior of gluons and created quarks are important to the decay mode, we find that the structure of the bound cluster play a dominant role in the evaluation of contributions of amplitudes to those decay processes. Varying the mass of a constituent gluon from 0.37GeV up to 0.50GeV, we find that the amplitude for the hybrid decay modes is much smaller than that for the normal decay mode, and one obtains

$$\langle <qqq, 1s; qqq, 1s|T_2C_2|J/\psi(\Lambda) > |_{s_{s'}}, < \rangle < <qqq, 1s; qqq, 1s|T_1C_1|J/\psi(\Lambda) > |_{s_{s'}}, < \rangle < <qqq, 2s; qqq, 1s|T_2C_2|J/\psi(\Lambda) > |_{s_{s'}}, < \rangle < <qqq, 1s; qqq, 1s|T_1C_1|J/\psi(\Lambda) > |_{s_{s'}}, < \rangle <$$

$$< 0.15;$$

$$< 0.04.$$  

This implies that if the Roper is a pure $qqqg$ hybrid, the decay widths for the process $J/\psi \to \bar{p}N^*$, $\bar{N}^*N^*$ are almost less than 2% and 0.2% of that for the process $J/\psi \to p\bar{p}$, respectively. Where does this suppression come from? A great part is from the depressed color factor due to the different dynamical behaviors of the quarks and gluons in these two decay processes. One could imagine the fact that if a gluon to combine with a color-octet three-quark cluster to form a color singlet object, not all colored gluons are allowed. This situation is well understood from the calculation of the color factor $<C_2>$, which is almost the fifth of the color factor $<C_1>$. This indicates that $J/\psi$ decays into a hybrid baryon is a color depressed process. While in the normal decay of the $J/\psi$ particle, the gluons and the constituent quarks appear with an equal status due to the fact that each of the three gluons decays into a quark and an anti-quark pair and then three quarks (anti-quarks) form a color singlet baryon (anti-baryon). The color factor for this process is much larger. In other words, the decay mode for a hybrid baryonic production is color
depressed process. Other source is coming from the different behaviors of the constituent quarks and gluons bound in the baryon. We simply assume that the proton is a pure three-quark system and the $N^*$ is a pure $qqqg$ system. To calculate the decay amplitude one has to project the basic amplitude onto the bound state wave functions of three quarks (and the gluon). The overlap of the basic amplitude with the wave functions of three quarks is much larger than that with the wave functions of three quarks and a gluon. By adjusting the gluon mass from $m_g = 0.37$ GeV up to $0.50$GeV, and using the harmonic oscillator parameter $\alpha = 0.08$GeV$^2$, we finally obtain the ratios of the $J/\psi \rightarrow pN^*$ and $J/\psi \rightarrow N^*N^*$ decay widths to the width of the process $J/\psi \rightarrow p\bar{p}$.

\[
\frac{\Gamma(J/\psi^{(\Lambda)} \rightarrow pN^*)}{\Gamma(J/\psi^{(\Lambda)} \rightarrow \bar{p} \bar{p})} \bigg|_{\delta=0} = \begin{cases} 
1.4 \times 10^{-2} & (m_g = 0.50 \text{GeV}); \\
1.8 \times 10^{-3} & (m_g = 0.37 \text{GeV}); 
\end{cases} 
\]

\[
\frac{\Gamma(J/\psi^{(\Lambda)} \rightarrow N^*N^*)}{\Gamma(J/\psi^{(\Lambda)} \rightarrow \bar{p} \bar{p})} \bigg|_{\delta=0} = \begin{cases} 
1.2 \times 10^{-4} & (m_g = 0.50 \text{GeV}); \\
2.3 \times 10^{-4} & (m_g = 0.37 \text{GeV}); 
\end{cases} 
\]

We also change the harmonic oscillator parameter of nucleons $\alpha$ from $0.06$GeV$^2$ to $0.21$GeV$^2$. The results are plotted in Fig.2.

Another extreme situation is that the Roper resonance is a pure common 3q state, namely $\delta = 1$ in Eq.(6), which is assigned as the first radial excitation state $|qqq,2s\rangle$. In this case, we deal only with one decay mode, $J/\psi \rightarrow |qqq\rangle + |\bar{q}\bar{q}\rangle$ as illustrated in fig.1(a). With the same parameters for the quark masses and the harmonic oscillator the numerical results are

\[
\frac{\Gamma(J/\psi^{(\Lambda)} \rightarrow pN^*)}{\Gamma(J/\psi^{(\Lambda)} \rightarrow \bar{p} \bar{p})} \bigg|_{\delta=1} = 2.0 \sim 4.5, 
\]

\[
\frac{\Gamma(J/\psi^{(\Lambda)} \rightarrow N^*N^*)}{\Gamma(J/\psi^{(\Lambda)} \rightarrow \bar{p} \bar{p})} \bigg|_{\delta=1} = 3.2 \sim 22.0, 
\]

and their angular distribution parameters are $\alpha_\star = 0.22 \sim 0.70$ and $\alpha_{\star\star} = 0.06 \sim 0.08$, respectively. This indicates that the the $J/\psi \rightarrow pN^*$ decay width is much larger if the $N^*$ is a normal three-quark excitation than that if the $N^*$ is a hybrid.

### 5.2 Roper in the hybrid picture mixing with the $|qqq,2s\rangle$ state

If the Roper resonance is taken as a mixture of a normal $|qqq,2s\rangle$ state with a pure hybrid state, the decay widths for $J/\psi \rightarrow pN^*, \bar{N}^* N^*$ decay modes can be evaluated numerically from Eqs.(39)~(41). The predicted ratios of decay widths in logarithmic scale are plotted in Fig.2 in terms of the mixing parameter $\delta$. The dashed, solid and dotted curves correspond to the different choice of the harmonic oscillator parameters $\alpha = 0.06$GeV$^2, 0.08$GeV$^2$ and $0.21$GeV$^2$, respectively. The light quark masses are taken as $m_q = 0.22$GeV. For each type of curves in small $\delta$ region, the upper and lower curves correspond the different choice of the gluonic mass $m_g = 0.37$ or $0.50$GeV as illustrated in the figure caption. It is clear to see from these results that the ratios grow rapidly with the increase of the mixing parameter in small $\delta$ region. While in larger $\delta$ region, the ratios almost remain the same as the variation of the gluonic mass from $0.37$GeV up to $0.50$GeV. This behavior is easily understood from calculations of the contributions of the hybrid decay mode to the transition amplitudes. Since the $J/\psi$ decays into a hybrid baryon are color-depressed processes, the normal decay modes contribute to the transition amplitudes dominantly. Hence comparing the calculation of the ratios in $J/\psi$ decays with the measurements might provide a new approach to probe the structure of the Roper resonance.
Figure 2: The ratios of decay widths are plotted as a function of the mixing parameter $\delta$. The $N^*(1440)$ is assumed as a mixture of the $|qqq, 2s\rangle$ and $|qqqg\rangle$ states with mixing parameter $\delta$. The dashed, solid and dotted curves correspond to the choices of the harmonic oscillator parameter $\alpha = 0.06\text{GeV}^2, 0.08\text{GeV}^2$ and $0.21\text{GeV}^2$, respectively. The values for light quark masses are taken as $m_q = 0.22\text{GeV}$.

(a) The ratio $\Gamma(J/\psi \to \bar{p}N^*)/\Gamma(J/\psi \to pp)$. For each type of curves, the upper one corresponds to the mass of the constituent gluons $m_g = 0.50\text{GeV}$ and the lower one is related to the choice $m_g = 0.37\text{GeV}$. (b) The ratio $\Gamma(J/\psi \to \bar{N}^*N^*)/\Gamma(J/\psi \to pp)$. For each type of curves, the upper solid curve, dash curve and lower dotted curve correspond to the mass of the constituent gluons $m_g = 0.37\text{GeV}$ and the other curves are related to the choice $m_g = 0.50\text{GeV}$. 
6 Conclusion

We have studied the decay widths and angular distributions of the processes $J/\psi \rightarrow \bar{p}N^*$, $\bar{N}^*N^*$ in constituent quark model. The strong decay of the $J/\psi$ is assume by the emission of three gluons. The transitional amplitude of dynamical process for $J/\psi \rightarrow |qqq⟩ + |\bar{q}\bar{q}\bar{q}⟩$ is numerically evaluated from perturbative QCD approach by assuming that each gluon creates a quark-antiquark pair. While in the process $J/\psi \rightarrow |\bar{q}\bar{q}\bar{q}⟩ + |qqqg⟩$, we suppose that two pairs of quark-antiquarks are created from the annihilation of two gluons and a light quark pair, $u\bar{u}$ or $d\bar{d}$ is created from the QCD vacuum with strength $g_I$. Then the three quarks (anti-quarks) or three quarks and a gluon are respectively combine together to form an ordinary baryon or a hybrid state by some final-state interactions. The determination of the strength $g_I$ is also carried out with the modified quark creation model by fitting the decay width for the process $J/\psi \rightarrow \gamma p\bar{p}$ to the experimental value, and the numerical result gives the strength $g_I = 15.40$ GeV by using the same experimental cut-off condition on the restriction of the outgoing photon energy. Then the transitional amplitudes and angular distribution parameters for the processes $J/\psi \rightarrow \bar{p}N^*$, $\bar{N}^*N^*$ are calculated numerically. If the Roper resonance is assigned as a pure hybrid state, our numerical results show that the ratio $\Gamma(J/\psi(\Lambda) \rightarrow \bar{p}N^*)/\Gamma(J/\psi(\Lambda) \rightarrow \bar{pp}) < 2\%$, and $\Gamma(J/\psi(\Lambda) \rightarrow \bar{N}^*N^*)/\Gamma(J/\psi(\Lambda) \rightarrow \bar{pp}) < 0.2\%$, and their angular distribution parameters are $\alpha_\ast = 0.42 \sim 0.57$ and $\alpha_{\ast\ast} = (-0.1) - (-0.9)$, respectively. However, when the Roper resonance is assumed to be a common $2S$ state, the results are quite different, with $\Gamma(J/\psi(\Lambda) \rightarrow \bar{p}N^*)/\Gamma(J/\psi(\Lambda) \rightarrow \bar{pp}) = 2.0 \sim 4.5$, and $\Gamma(J/\psi(\Lambda) \rightarrow \bar{N}^*N^*)/\Gamma(J/\psi(\Lambda) \rightarrow \bar{pp}) = 3.2 \sim 22.0$, and with the angular distribution parameter $\alpha_\ast = 0.22 \sim 0.70$, $\alpha_{\ast\ast} = 0.06 \sim 0.08$. This implies that, not only the dynamics of three gluons and created quarks, but also the structure of the final cluster state, i.e. $|qqq⟩$ or $|qqqg⟩$, play important roles in the evaluation of the amplitudes in these decay processes. So it is suggestive that an accurate measurement of the decay widths and angular distributions of these channels may provide us a novel tool to probe the structure of the Roper resonance. In our present calculation we also consider the mixing of the normal Roper state with the hybrid state. Under this picture, we calculate the decay widths for these decays versus the mixing parameter $\delta$.

In conclusion, we find that the contributions from the Roper structure to the transitional amplitudes play an important role in evaluating the decay widths and angular distribution parameters for processes $J/\psi \rightarrow \bar{p}N^*$ or $\bar{N}^*N^*$. Hence, the study of the transitional amplitudes for these decay modes may provide us a new laboratory to justify the nature of the Roper resonance. If an accurate experimental data on decay widths and angular distribution parameters for these processes are available in the future, one may discriminate the hybrid Roper baryons from conventional baryons.

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APPENDIX

A  The construction of the hybrid state wave function

Let I, J denotes the quantum number of total isospin and momentum for \(|qqg\) state with \(I_3\) and \(J_3\), the z-projection component, and the wave function of hybrid state denoted by \(|Ng, I_3J_3\rangle\) can be constructed according Eq.(1) by combining the gluonic part and the three quarks part’s flavor-spin-color wave function into a color singlet state. Here, we use the dummy index sum as \(\frac{1}{V_8}\sum_{a=1,8}\). Especially, for \(I=J=1/2\), we give explicitly their form in spin-flavor-color space as follows:

\[
|Ng, \frac{1}{2}1\rangle_{13} = \frac{1}{\sqrt{2}}|1\rangle - 1\frac{1}{\sqrt{2}}(\phi^\lambda \psi^\nu_\alpha - \phi^\nu \psi^\lambda_\alpha)\chi_{3/2}^s
\]

\[\text{if } x = y, \text{we must replace}\]

\[
|Ng, \frac{1}{2}1\rangle_{13} = \frac{1}{\sqrt{2}}|1\rangle - 1\frac{1}{\sqrt{2}}(\phi^\lambda \psi^\nu_\alpha - \phi^\nu \psi^\lambda_\alpha)\chi_{1/2}^s
\]

\[
|Ng, \frac{1}{2}1\rangle_{13} = \sqrt{\frac{2}{3}}|1\rangle \frac{1}{\sqrt{2}}[(\phi^\lambda \psi^\nu_\alpha - \phi^\nu \psi^\lambda_\alpha)\chi_{3/2}^s - (\phi^\nu \psi^\lambda_\alpha - \phi^\lambda \psi^\nu_\alpha)\chi_{1/2}^s]
\]

\[
|Ng, \frac{1}{2}1\rangle_{13} = \sqrt{\frac{2}{3}}|1\rangle \frac{1}{\sqrt{2}}[(\phi^\lambda \psi^\nu_\alpha - \phi^\nu \psi^\lambda_\alpha)\chi_{1/2}^s - (\phi^\nu \psi^\lambda_\alpha - \phi^\lambda \psi^\nu_\alpha)\chi_{3/2}^s]
\]

\[
|Ng, \frac{1}{2}1\rangle_{13} = \sqrt{\frac{2}{3}}|1\rangle \frac{1}{\sqrt{2}}[(\phi^\lambda \psi^\nu_\alpha - \phi^\nu \psi^\lambda_\alpha)\chi_{3/2}^s - (\phi^\nu \psi^\lambda_\alpha - \phi^\lambda \psi^\nu_\alpha)\chi_{1/2}^s]
\]

where \(\phi, \chi, \psi\) are respectively responsible for the flavor, spin, and color wave function of the three quark parts in \(|qqqg\) cluster. The index \(s, \lambda, \rho\) denote total permutation symmetry or mixed representation. The mixed symmetric states are define as:

\[
\rho \text{ type: } |xyz|^\rho = \frac{1}{2}(|xyz\rangle - |xzy\rangle - |yzx\rangle - |yxz\rangle), \quad \text{(31)}
\]

\[
\lambda \text{ type: } |xyz|^\lambda = \frac{1}{2\sqrt{3}}(|xyz\rangle + |xzy\rangle + |yzx\rangle + |yxz\rangle - 2|xzy\rangle - 2|yxz\rangle), \quad \text{(32)}
\]

if \(x=y\), we must replace \(|xyz\rangle + |yxz\rangle\) with \(\sqrt{2}|xxz\rangle\) in the above equation. For example, the mixed symmetric spin wave functions read:

\[
\chi_{1/2, 1/2}^\nu = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle), \quad \text{(33)}
\]

\[
\chi_{1/2, 1/2}^\lambda = \frac{1}{\sqrt{6}}(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\rangle - 2|\uparrow\uparrow\uparrow\rangle), \text{etc.} \quad \text{(34)}
\]

B  The matrix elements of the amplitude for \(J/\psi \rightarrow |qq\rangle + |qqqg\rangle\)

Since the hybrid wave function is chosen as a mixture of the \(4^8\) state with the \(2^8\) state with equal ratio, e.g.
\[ J/\psi \to |\bar{q}q\rangle + |qqg\rangle. \] Note that \( < \psi^a | C_2 | \psi^\lambda > = 0 \), where \( \psi^a \) is the asymmetric color wave function of baryon, so we only calculate the part \( < \psi^a | C_2 | \psi^\nu > \).

\[
< N|C_2 T|Ng, \frac{1}{2} \frac{1}{2} >_{13} = \frac{1}{2\sqrt{3}} \left( 1 - 1 >^a (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}} \right) - \frac{1}{\sqrt{12}} (0 >^a (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}}) + \frac{1}{\sqrt{24}} (1 >^a (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}}) \]  

(35)

\[
< N|C_2 T|Ng, \frac{1}{2} \frac{1}{2} >_{11} = \frac{1}{2\sqrt{3}} \left( 1 - 1 >^a [(\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{1}{2}} - (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}}] \right) - \frac{1}{\sqrt{6}} (0 >^a [(\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{1}{2}} - (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}}]) \]  

(36)

\[
< N|C_2 T|Ng, \frac{1}{2} - \frac{1}{2} >_{11} = - \frac{1}{2\sqrt{3}} \left( 1 - 1 >^a [(\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{1}{2}} - (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}}] \right) + \frac{1}{\sqrt{6}} (0 >^a [(\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{1}{2}} - (\psi^a|C|\psi^\nu)^\lambda |T|\chi^\nu_{\frac{3}{2}}]) \]  

(37)

From those elements, one obtains the transitional amplitudes \( M^{(A)} \) for processes \( J/\psi \to p\bar{p}N^+, \bar{N}^*N^* \) by projecting them onto the wave functions of the proton or the Roper resonance. We ignore the trivial contribution from the process \( J/\psi \to |qqg\rangle + |\bar{q}q\rangle \) to the transitional amplitude. If one assumes that Roper is the mixture of \( |qqq, 2s > \) state with the hybrid state \( |qqqg > \), one gets

\[
M^{(A)}_{s_2, s'_{2}} (J/\psi^{(A)} \to p\bar{p}) = < \bar{q}q\bar{q}, 1s; qqq, 1s|T_1 C_1 | J/\psi^{(A)} >, \]  

(39)

\[
M^{(A)}_{s_2, s'_{2}} (J/\psi^{(A)} \to p\bar{N}^+) = \delta < \bar{q}q\bar{q}, 1s; qqq, 2s|T_1 C_1 | J/\psi^{(A)} > + \sqrt{1 - \delta^2} < \bar{q}q\bar{q}, 1s; qqq|T_2 C_2 | J/\psi^{(A)} >, \]  

(40)

\[
M^{(A)}_{s_2, s'_{2}} (J/\psi^{(A)} \to \bar{N}^*N^*) = \delta < \bar{q}q\bar{q}, 2s; qqq, 2s|T_1 C_1 | J/\psi^{(A)} > + \sqrt{1 - \delta^2} < \bar{q}q\bar{q}, 2s; qqq|T_2 C_2 | J/\psi^{(A)} >. \]  

(41)

In the decay mode \( J/\psi \to \bar{N}^*N^* \), we only consider the process illustrated in Fig.1(b), where the normal Roper state is treated as a \( |qqq, 2s > \) state, while the hybrid state is assumed to be a mixture of a \( |qqq, 2s > \) state and a \( |qqqg, 1s > \) state.

C     The radial wave function for \(|qqq\rangle\) and \(|qqqg\rangle\) clusters

The radial wave functions of protons or hybrid baryons are chosen as simple harmonic oscillator eigenfunctions in their center-of-mass (CM) system, i.e.

\[
\phi_{1s}(k^2_\rho, \mathbf{k}_\lambda) = \frac{1}{(\pi\alpha)^{\frac{3}{2}} e^{-\frac{k^2_\rho + K^2_\lambda}{2\alpha}}}, \]  

(42)

for \(|qqq, 1s\rangle\) cluster, and

\[
\phi_{2s}(k^2_\rho, \mathbf{k}_\lambda) = \sqrt{3} \frac{1}{(\pi\alpha)^{\frac{3}{2}}(1 - \frac{1}{3\alpha}(k^2_\rho + K^2_\lambda)) e^{-\frac{k^2_\rho + K^2_\lambda}{2\alpha}}}, \]  

(43)
for $|qqq, 2s\rangle$ cluster, where $\alpha = (3k m_q)^{1/2}$, $\vec{k}_\rho$, $\vec{k}_\lambda$ are relative coordinates defined as

$$
\vec{k}_\rho = \frac{1}{\sqrt{6}} (\vec{k}_1 + \vec{k}_2 - 2 \vec{k}_3), \quad (44)
$$

$$
\vec{k}_\lambda = \frac{1}{\sqrt{2}} (\vec{k}_1 - \vec{k}_2). \quad (45)
$$

Let $p_1, p_2, p_3$ be momenta for three quarks and $p_4$ for a gluon in the cluster $qqqg$ as depicted in Fig.3, and we define three relative coordinates as

$$
\vec{p}_a = \frac{1}{\sqrt{2}} (\vec{p}_1 - \vec{p}_2), \quad (46)
$$

$$
\vec{p}_b = \frac{1}{\sqrt{6}} (\vec{p}_1 + \vec{p}_2 - 2 \vec{p}_3), \quad (47)
$$

$$
\vec{p}_c = \frac{1}{\sqrt{12}} (\vec{p}_1 + \vec{p}_2 + \vec{p}_3 - 3 \vec{p}_4). \quad (48)
$$

One has a spatial wave function for $qqqg$ cluster in momentum space.

$$
\phi_b(p_a, p_b, p_c) = \frac{1}{(\pi \alpha)^{3/2}} e^{-\frac{(\vec{p}_a^2 + \vec{p}_b^2)}{2\alpha}} \frac{1}{(\pi \beta)^{3/4}} e^{-\frac{\vec{p}_c^2}{2\beta}}, \quad (49)
$$

where $\beta = (3k \tilde{M}_g)^{1/2}$ is harmonic-oscillator parameter ($\tilde{M}_g = \frac{2m_q m_g}{m_q + m_g}$).

Figure 3: The definition of the relative momentum for four bodies system in CM system
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