Spreading Information in Mobile Wireless Networks

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Abstract—Device-to-device (D2D) communication enables us to spread information in the local area without infrastructure support. In this letter, we focus on such information spreading in mobile wireless networks where all nodes move around. We derive the average number of nodes that have successfully received a given information packet as a function of the transmission range and the number of transmissions. Based on these results, we formulate a redundancy minimization problem under the maximum transmission range and delay constraints. By solving the problem, we provide an optimal rule for the transmission range of the source node.

Index Terms—Information spreading, mobility, redundancy minimization, mobile wireless network.

I. INTRODUCTION

Near field communication (NFC) technologies enable devices in close proximity to exchange mutual information without any support from infrastructure. Device-to-device (D2D) communication in 3GPP LTE (Long Term Evolution) also facilitates information exchange between adjacent devices. We call this information spreading throughout this letter. Such information spreading boosts various services, for example, mobile marketing and advertising in local areas [1], [2].

An accurate prediction about the number of nodes that have successfully received a given information packet as time goes is necessary for efficient information spreading. A classical research issue in computer science is to calculate the cover time that defines the expected number of transmissions (or hops) until all nodes in a given network receive a specific packet [3]. Applications of the cover time analysis include searching/querying, routing, membership services and group-based communications. However, the cover time analysis is limited to the wired or the static network.

In this letter, we focus on the information spreading in mobile wireless networks where all nodes move around. Node mobility improves the capacity of wireless networks [4]. Similarly, node mobility brings positive effects on the information spreading because moving nodes can deliver information anywhere by direct transmission. However, this may cause packet delay, which is an important parameter in the information spreading.

Another parameter is the number of redundant receptions [5] (i.e., waste of resources). If the maximum transmission range is not limited, the number of redundant receptions will be minimized when increasing the transmission range as large as the target number of nodes in a network can receive a given information packet at one transmission. In practice however, the transmission range is limited by either regulatory or application constraints, and multiple transmissions are needed to deliver the information packet to the target number of nodes. In this case, increasing the transmission range reduces the number of required transmissions but it may cause more redundant receptions at each transmission. This means that finding an optimal solution for minimizing the total number of redundant receptions is not trivial, in particular when the maximum transmission range is limited. Then, we have questions regarding information spreading in mobile wireless networks:

• How many transmissions are required for delivering a given information packet to a certain percentage of nodes in the network?

• What is the optimal transmission range for minimizing the total number of redundant receptions, while keeping the delay within a reasonable level?

In this letter, we derive the average number of nodes that have successfully received a given information packet as a function of the transmission range and the number of transmissions. With this, we formulate a redundancy minimization problem under the maximum transmission range and delay constraints. By solving the problem, we provide the optimal transmission range and transmission number.

II. REDUNDANCY MINIMIZATION PROBLEM IN INFORMATION SPREADING

Consider a wireless network composed of one mobile source node and other $N$ nodes that are able to move around the whole area $S$ (Figure 1). The moving source node tries to spread an information packet throughout the entire network without relaying. If the distance between source and destination nodes is shorter than the transmission range $R$,
the destination node can successfully receive the packet \(5\), \(6\). This is similar to the protocol model \(7\). Reflecting transmission power constraint and radio signal attenuation, we assume that the transmission range \(R\) is not greater than \(\bar{R}\). Let us define that a node is covered if the node receives the information packet from the source node at least once. The number of covered nodes by the \(k\)-th time slot is a random variable, denoted as \(\hat{N}_k\). Another random variable \(M_k\) denotes the number of nodes within the transmission range of the source node at the \(k\)-th time slot, out of which \(M_k\) denotes the number of nodes that have been already covered (before \(k\)-th time slot).

During the spreading process, redundant receptions may occur, which we need to minimize as formulated below:

\[
(P) \quad \min_{R, k} f(R, k), \quad \text{s.t.} \quad \frac{E[\hat{N}_k]}{N} \geq \gamma,
\]

\[0 \leq R \leq \bar{R}, \quad 1 \leq k \leq \hat{k}.
\]

The objective function \(f(R, k)\) denotes the number of redundant receptions. Note that the control parameters are \(R\) and \(k\), which means that we jointly determine how large the transmission range is set and how many times the information packet is repeatedly transmitted. The first constraint requires that the ratio of the covered nodes should be higher than or equal to a target value \(\gamma\). Particularly, all nodes should be covered when \(\gamma = 1\). The second constraint determines the maximum transmission range. The last constraint says that the number of required transmission slots (i.e., delay) should be less than \(k\) slots.

In the next section, we describe node mobility and derive the average number \((E[\hat{N}_k])\) of covered nodes. With this, we solve the redundancy minimization problem \((P)\) and provide the optimal transmission range \((R^*)\) and transmission number \((k^*)\) in Section \(\text{V}\) (Proposition 4).

### III. Mobility Model: Homogeneous Condition

To describe node mobility, we define the homogeneous condition \(8\) as follows:

**Definition 1**: If \(E[M_k] = N \frac{\pi R^2}{S}\) and \(E[\frac{\hat{N}_k}{M_k}] = E[\frac{\hat{N}_{k-1}}{N}]\) for all \(k\), then node mobility is said to satisfy homogeneous condition.

To understand the homogeneous condition, let us regard covered nodes as molecules of a chemical solute. Then, the homogeneous condition resembles a homogenous solution where the solute concentrations in any location are the same owing to the high speed of molecular movement. Similarly, if all nodes move around the network with high speed such as the i.i.d. mobility model \(9\), then the network satisfies the homogeneous condition, which also applies to our focused network.

**Proposition 1**: If all nodes are randomly distributed in the whole area and move anywhere independently of their previous positions (i.e., the i.i.d. mobility model), then the network is under the homogeneous condition.

**Proof**: If all nodes have the i.i.d. mobility, they are uniformly distributed in the network at each time slot. This means a node is within the transmission range of the source node with the probability, \(\pi R^2 / S\). Therefore, \(M_k\) follows a binomial distribution \(Bi(N, \pi R^2 / S)\), and \(E[M_k] = N \pi R^2 / S\).

Moreover, in the i.i.d. mobility model, the distribution of \([M_k|M_{k-1}, \hat{N}_{k-1}]\) follows a binomial distribution \(Bi(M_k, \hat{N}_{k-1}/N)\). Therefore, using the total probability theorem, we calculate \(E[M_k/M_{k-1}]\) as follows:

\[
E \left[ \frac{M_k}{M_{k-1}} \right] = E_{\hat{N}_{k-1}} \left[ E_{M_k} \left[ M_k \left| M_{k-1}, \hat{N}_{k-1} \right. \right] \right] = E_{\hat{N}_{k-1}} \left[ \frac{1}{M_{k-1}} \left( \frac{\hat{N}_{k-1}}{N} \right) \right] = E_{\hat{N}_{k-1}} \left[ \frac{\hat{N}_{k-1}}{N} \right]
\]

Another mobility model that satisfies the homogeneous condition is the random way-point (RWP) with high speed. In the RWP mobility model \(10\), all nodes’ destinations, speeds and moving directions are chosen randomly and independently of other nodes. If the nodes have high speed that is enough to reach their own destinations in one time slot, the RWP mobility model is equivalent to the i.i.d. mobility model. Therefore, the RWP mobility model with sufficiently high speed satisfies the homogeneous condition, which is verified by our simulation results of Figure \(2\).

### IV. Number of Covered Nodes

In this section, we derive the average number \((E[\hat{N}_k])\) of covered nodes by the \(k\)-th time slot. We consider two transmission modes, broadcast and unicast. In the broadcast mode, all nodes within the transmission range receive the packet. On the other hand, in the unicast mode, the packet can be delivered to only one node that is either the nearest or the randomly selected one within the transmission range.

#### A. Unicast Mode

Let \(\hat{X}_k^U\) denotes the number of nodes covered at the \(k\)-th time slot, where the superscript \(U\) represents the unicast mode. At the \(k\)-th time slot, \(\hat{X}_k^U = 1\) if a randomly selected node (or the nearest node to the source) has not been covered yet. Otherwise, \(\hat{X}_k^U = 0\). By definition, we get the following equations:

\[
\hat{N}_k^U = k \sum_{i=1}^{\hat{X}_k^U} \hat{N}_{k-1}^U, \quad N_0^U = 0.
\]

Note that \(\hat{X}_k^U\) follows a Bernoulli distribution \(B(1 - M_k^U / M_k^U)\). Using this, we derive \(E[\hat{N}_k^U]\) in the following proposition:
 Proposition 2: In the unicast mode, the average number \( E[\hat{N}_k^U] \) of covered nodes by the \( k \)-th time slot is
\[
E[\hat{N}_k^U] = N \left[ 1 - \left( 1 - \frac{1}{N} \right)^k \right].
\]

Proof: From \( \hat{X}_k^U \sim B(1 - \hat{M}_k^U / \hat{M}_k^U) \), we get a recurrence relation and solve it as follows:
\[
E[X_k^U] = E \left[ 1 - \frac{\hat{M}_k^U}{\hat{M}_k^U} \right] \leq E \left[ 1 - \frac{\hat{N}_k^{U-1}}{N} \right],
\]
\[
= 1 - \frac{1}{N} \left( 1 - \frac{1}{N} \right)^{k-1}, \quad (2)
\]
where \((a)\) and \((b)\) follow from the homogeneous condition and Equation \((1)\), respectively. By solving the recurrence relation, we achieve \((c)\). With Equations \((1)\) and \((2)\), we have
\[
E[\hat{N}_k^U] = \sum_{i=1}^{k} E[X_i^U] = N \left[ 1 - \left( 1 - \frac{1}{N} \right)^k \right]. \quad (3)
\]

B. Broadcast Mode

Let \( \hat{X}_k^B \) denotes the number of nodes covered at the \( k \)-th time slot, where the superscript \( B \) represents the broadcast mode. In this mode, all the uncovered nodes within the transmission range are covered simultaneously. By definition, we get the following equations:
\[
\hat{N}_k^B = \sum_{i=1}^{k} X_i^B, \quad \hat{N}_0^B = 0.
\]

Then, we derive \( E[\hat{N}_k^B] \) as follows:

 Proposition 3: In the broadcast mode, the average number \( E[\hat{N}_k^B] \) of covered nodes by the \( k \)-th time slot is
\[
E[\hat{N}_k^B] = N \left[ 1 - \left( 1 - \frac{\pi R^2}{S} \right)^k \right].
\]

Proof: By definition, \( X_i^B \) is equal to \( M_k^B - \hat{M}_k^B \). Then, we get the recurrence relation for \( E[X_i^B] \) and solve it as follows:
\[
E[X_k^B] = E \left[ M_k^B - \hat{M}_k^B \right] = E \left[ M_k^B \left( 1 - \frac{\hat{M}_k^B}{M_k^B} \right) \right]
\]
\[
\leq E[M_k^B] E \left[ 1 - \frac{\hat{M}_k^B}{M_k^B} \right] = N \frac{\pi R^2}{S} E \left[ 1 - \frac{\hat{N}_k^{B-1}}{N} \right]
\]
\[
= N \frac{\pi R^2}{S} \left( 1 - \frac{1}{N} \right)^{k-1} \left( 1 - \frac{\pi R^2}{S} \right)^k. \quad (5)
\]
where \((a)\) and \((b)\) follow from independence between the number of nodes and the covered ratio, and the homogeneous condition, respectively. By solving the recurrence relation, we achieve \((c)\). With Equations \((4)\) and \((5)\), we have \( E[\hat{N}_k^B] \):
\[
E[\hat{N}_k^B] = \sum_{i=1}^{k} E[X_i^B] = N \left[ 1 - \left( 1 - \frac{\pi R^2}{S} \right)^k \right].
\]

From Propositions 2 and 3, we observe that the broadcast mode is reduced to the unicast mode by taking \( R = \sqrt{S / \pi N} \) where there is one node on average in the transmission area. Thus, we will consider the broadcast mode in redundancy minimization problem (Section V).

To verify our analysis, we conducted simulations, where we set the whole area \( S = 100 \times 100 \text{ m}^2 \), the number of nodes \( N = 100 \) and the transmission range \( R = 20 \text{ m} \). In the simulations, all nodes move according to the RWP mobility model with a maximum speed of 20 m/slot.

V. OPTIMAL RULE FOR REDUNDANCY MINIMIZATION

In this section, we solve the redundancy minimization problem \((P)\). Using the fact that \( \hat{M}_k^B \) is equal to the number of redundant receptions caused by the transmission of the source node at the \( k \)-th time slot, we derive the average number of redundant receptions by the \( k \)-th time slot in the following
We can calculate a sufficient condition for the minimum transmission range in the following proposition. Using Equations (6) and (7), we can rewrite (P) as follows:

\[
\begin{align*}
(P') & \quad \min_{R,k} \frac{\pi N}{S} k R^2 - N \left( 1 - \left(1 - \frac{\pi R^2}{S}\right)^k \right), \\
\text{s.t.} & \quad 1 - \left(1 - \frac{\pi R^2}{S}\right)^k \geq \gamma, \\
& \quad 0 \leq R \leq \bar{R}, \\
& \quad 1 \leq k \leq \bar{k}.
\end{align*}
\]

We jointly optimize \( R \) and \( k \) for \((P')\) and the result is described in the following proposition.

**Proposition 4** In the redundancy minimization problem, the optimal transmission range \((R^*)\) and the number of required transmission slots \((k^*)\) are

\[
R^* = \sqrt{\frac{S}{\pi} \left(1 - (1 - \gamma)^{1/k^*}\right)}, \quad k^* = \left\lceil \frac{\log(1 - \gamma)}{\log\left(1 - \frac{\pi R^2}{S}\right)} \right\rceil,
\]

where the symbol \([x]\) denotes the smallest integer that is larger than or equal to \(x\).

**Proof:** In \((P')\), the inequality of the first constraint should be equality because there is no reason to cover more nodes than the target. Therefore, we get the following equation:

\[
1 - \left(1 - \frac{\pi R^2}{S}\right)^k = \gamma \quad \Rightarrow \quad R^2 = \frac{S}{\pi} \left(1 - (1 - \gamma)^{1/k^*}\right). \quad (8)
\]

Moreover, the second term of the objective function in \((P')\) becomes \(N\gamma\), which is independent of the control variables \(R\) and \(k\). Then, we have only to minimize the first term \(\pi N k R^2 / S\), which can be transformed into \(Nk (1 - (1 - \gamma)^{1/k})\) by Equation \((8)\). Note that \(Nk (1 - (1 - \gamma)^{1/k})\) is an increasing function of \(k\) for \(0 < \gamma \leq 1\). Therefore, \(k^*\) should be the smallest integer that satisfies the second constraint in \((P')\). Using this and Equation \((8)\), we can calculate \(k^*\) and \(R^*\).

Proposition 4 provides an optimal rule for minimizing redundant receptions in information spreading, which is to transmit an information packet \(k^*\) times using the optimal transmission range \(R^*\). For example, if the maximum transmission range is sufficiently large or is not limited, then the optimal rule is to increase the transmission range as large as the target number of nodes can be covered at one transmission (i.e., \(k^* = 1\)). This coincides with our intuition. An interesting observation is that even though the maximum transmission range is limited, \(R^*\) is the largest one within the maximum transmission range \(R\), which makes corresponding \(k^*\) be the smallest integer.

**VI. Conclusions**

In this letter, we derived the average number of covered nodes as a function of the transmission range and the number of transmissions, and verified it by means of simulations. Using the result, we solved the redundancy minimization problem and provided the optimal transmission range and transmission number.

Our results can be employed in a variety of settings. One example is the wireless power transfer, an emerging battery charging technology without plugs or wires. In \([11]\), the authors pointed out that the network capacity decreases if the available transmission energy of each node is limited, and showed that the capacity can be improved by employing wireless charging vehicles. For the wireless power transfer, the source node and the information in this letter can be regarded as a wireless charging vehicle and the energy to deliver, respectively. In this case, the transmission range \(R\) and the number of transmissions \(k\) correspond to the charging range and the number of charging, respectively.

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