Rotons and their damping in elongated Bose-Einstein condensates.

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We discuss finite temperature damping of rotons in elongated Bose-condensed dipolar gases, which are in the Thomas-Fermi regime in the tightly confined directions. The presence of many branches of excitations which can participate in the damping process, is crucial for the Landau damping and results in significant increase of the damping rate. It is found, however, that even rotons with energies close to the roton gap may remain fairly stable in systems with the roton gap as small as $1\,\text{nK}$.

I. INTRODUCTION

The spectrum of elementary excitations is strongly influenced by the character of interparticle interactions and is a key concept for understanding the behavior of quantum many-body systems. For Bose-condensed systems with a short-range interparticle interaction the low-energy part of the spectrum represents phonons with a linear energy-momentum dependence. In some cases, the excitation spectrum has an energy minimum at rather large momenta with roton excitations around it, which is separated by a maximum (maxon excitations) from the low-energy phonon part. The roton-maxon excitation was first observed in liquid $^4$He, and intensive discussions during decades arrived at the conclusion that the presence of the roton is related to the tendency to form a crystalline order [1, 2]. The presence of rotons in the excitation spectrum of dipolar Bose-Einstein condensates was first predicted in Refs. [3, 4] and is considered as a precursor of the formation of a supersolid phase (for a review of the supersolid phase and its experimental manifestations see, for example, Ref. [5]). During last several years, supersolid phases were observed experimentally in systems of ultracold trapped bosonic magnetic atoms (Dy, Er) [6–8], as well as the presence of the roton excitations and their role in the formation of the supersolid state [9, 12]. In the systems of magnetic atoms, the formation of the supersolid state and the appearance of the roton excitations are attributed to the magnetic dipole-dipole interaction. The present theoretical description of the excitations is based on the numerical solutions of the three-dimensional Bogoliubov-de Gennes equations in a trapped geometry at zero temperature [11, 13], and is focused on the real part of their dispersion, without addressing the question of the excitation damping. The damping, however, strongly affects system response to external perturbations which are used to probe the system properties (see, for example, [14]). Therefore, studies of the excitation damping and its temperature dependence have not only theoretical interest, but also direct experimental relevance. These studies should also indicate how stable are rotonic excitations, in particular for building up roton-induced density correlations in non-equilibrium systems [15]. Another issue is the spatial roton confinement in trapped Bose-Einstein condensates [16].

Damping of rotons in quasi-1D dipolar Bose-condensed gases has been discussed in Refs. [17–19]. In this paper we investigate the damping of rotons in an elongated Bose-condensed polarized dipolar gas, which is in the Thomas-Fermi (TF) regime in the tightly confined directions [20, 21]. In this case, there is a large number of branches in excitations spectrum, many of which can contribute to the Landau damping, the leading damping mechanism at finite temperatures [22]. This may significantly increase the damping rate and make rotons unstable.

II. GROUND STATE CONDENSATE WAVE FUNCTION

We consider an elongated Bose-Einstein condensate of polarized dipolar particles (magnetic atoms or polar molecules). The motion in the $z$ direction is free, and in the $x,y$ directions it is harmonically confined with frequency $\omega$. We consider the case where the dipolar polarization is orthogonal to the $z$ axis, let say, is along the $x$ direction. The ground state condensate wave function $\Psi_0(r)$ obeys the Gross-Pitaevskii (GP) equation:
where $\rho^2 = x^2 + y^2$, $\mu$ is the chemical potential of the system, and

$$V(r) = g\delta(r) + V_d(r),$$

(2)

with $g$ being the coupling constant of the short-range (contact) interaction, and $V_d(r)$ the potential of the dipole-dipole interaction between two atoms. For the dipoles $d$ oriented in one and the same direction we have

$$V_d(r) = \frac{d^2r^2 - 3(d\mathbf{r})^2}{r^5}.$$  

(3)

We now assume that $\Psi_0^2$ depends only on $\rho$, i.e. is symmetric in the $x,y$ plane and $z$-independent. In the Thomas-Fermi regime the kinetic energy of the condensate is omitted, and one expects that $\Psi_0^2$ has the shape of inverted parabola. Then, using the identity

$$\frac{r^2 - 3r^2}{r^5} = -\nabla_i \frac{1}{r} - \frac{4\pi}{3} \delta(r),$$

(4)

we obtain

$$\int V(r-r')\Psi_0^2(r')d^3r' \equiv \hat{V}\Psi_0^2(r) = g(1+\eta/2)\Psi_0^2(\rho),$$

(5)

where

$$\eta = \frac{4\pi d^2}{3g}.$$  

(6)

Equation [1] then takes the form

$$\frac{m\omega^2 \rho^2}{2} \psi_0(\rho) + g(1+\eta/2)\Psi_0^2(\rho) = \mu \Psi_0(\rho),$$

(7)

and, hence, the condensate wavefunction is given by

$$\Psi_0^2(\rho) = n_0 \left(1 - \frac{\rho^2}{R^2}\right); \rho \leq R,$$

(8)

where

$$\Psi_0^2 = 0; \rho > R,$$

(9)

with $n_{1D}$ being the one-dimensional density (the number of particles divided by the length of the system in the $z$ direction). The radius of the condensate in the $x,y$ plane is given by

$$R^2 = \frac{2\mu}{m\omega^2}.$$  

(10)

The chemical potential and density are related to each other as

$$\mu = (g + g_d/2) \frac{2}{\pi R^2} n_{1D} = n_0 g (1 + \eta/2),$$

(11)

with

$$g_d = \frac{4}{3^5}\eta g,$$

(12)

and the validity of the TF regime requires the chemical potential (interaction between particles) to be much large $r$ than the level spacing between the trap levels,

$$\frac{\mu}{\hbar \omega} \gg 1.$$  

(13)

III. ELEMENTARY EXCITATIONS. GENERAL RELATIONS AND LOW-MOMENTUM LIMIT.

Representing the field operator of the non-condensed part of the system as $\Psi'(r,t) = \sum_{\nu} [u_\nu(r)b_\nu - v_\nu(r)b_\nu^\dagger]$, where the index $\nu$ labels eigenstates of the excitations, we have the Bogoliubov-de Gennes equations for the functions $u$ and $v$:

$$-\frac{\hbar^2}{2m} \nabla^2 u(r) + \frac{m\omega^2 \rho^2}{2} u(r) - \mu u(r) + [\hat{V}\Psi_0^2(\rho)] u(r) + [\hat{V}\Psi_0(\rho)u(r)] \Psi_0(\rho) - [\hat{V}\Psi_0(\rho)v(r)] \Psi_0(\rho) = Eu(r),$$

(14)

$$-\frac{\hbar^2}{2m} \nabla^2 v(r) + \frac{m\omega^2 \rho^2}{2} v(r) - \mu v(r) + [\hat{V}\Psi_0^2(\rho)] v(r) + [\hat{V}\Psi_0(\rho)v(r)] \Psi_0(\rho) - [\hat{V}\Psi_0(\rho)u(r)] \Psi_0(\rho) = -Ev(r),$$

(15)

Turning to the functions $u \pm v = f_\pm$ and representing

$$f_\pm(r) = f_\pm(\rho)e^{ikz},$$

(16)
where $k$ is the momentum of the motion along the $z$ axis, using the GP equation (1) we transform Eqs. (14) and (15) to

$$
\frac{\hbar^2}{2m} \left( -\nabla^2_r + k^2 + \frac{\nabla^2 r \Psi_0}{\Psi_0} \right) f_+ = E f_-,
$$

(17)

$$
\frac{\hbar^2}{2m} \left( -\nabla^2_r + k^2 + \frac{\nabla^2 r \Psi_0}{\Psi_0} \right) f_- + 2[V \Psi_0 f_-(r)] \Psi_0 = E f_+.
$$

(18)

Using identity (4) we obtain

$$
\int V(r) e^{ikz} dz = -\frac{3g\eta}{2} \frac{\partial^2}{\partial x^2} K_0(k\rho) + g(1-\eta)\delta(\rho).
$$

(19)

When acting with operator $[19]$ on the function that depends only on $\rho$ it is useful to explicitly differentiate in Eq. (19) and make an average over the azimuthal angle, at least for small and large $k$. This gives

$$
\int V(r)e^{ikz}dz = g(1+\frac{\eta}{2})\delta(\rho) - \frac{3g\eta}{2\pi} k_0(k\rho) \equiv A(\rho),
$$

(20)

with $K_0$ being the modified Bessel function of the second kind.

In the TF regime we omit the first and third terms in the round brackets in the l.h.s of Eq. (18). We then express $f_-$ through $f_+$ from Eq. (17) and substitute it into Eq. (18). This yields

$$
\frac{\hbar^4k^2}{4m^2} \left( k^2 - \nabla^2_r + \frac{\nabla^2 r \Psi_0}{\Psi_0} \right) f_+ + 2 \frac{\hbar^2}{2m} \int d^2r' A(\rho' - \rho') \Psi_0(\rho') \left( k^2 - \nabla^2_r + \frac{\nabla^2 r \Psi_0}{\Psi_0} \right) f_+(\rho') \Psi_0(\rho) = E^2 f_+(\rho).
$$

(21)

In the low momentum limit, $kR \ll 1$, we omit the first term of Eq. (21) and angular momentum dependent terms in the expression (20) for $A(\rho - \rho')$. Representing $f_+ = W(\rho) \sqrt{1-\rho^2/R^2}$ for excitations with zero orbital momentum (of the motion around the $z$ axis) we find

$$
(1-\rho^2)(\tilde{k}^2 - \nabla^2_r)W(\tilde{\rho}) + 2\rho \frac{dW(\tilde{\rho})}{d\tilde{\rho}} \tilde{k} = 2\epsilon^2 W(\tilde{\rho}),
$$

(22)

where we turned to dimensionless momenta, energy, and coordinates: $\tilde{k} = kR$, $\epsilon = E/\hbar \omega$, and $\tilde{\rho} = \rho/R$. In terms of the variable $s = \tilde{\rho}^2$ equation (22) becomes

$$
(1-s) s^2 W + (1 - 2s) \frac{dW}{ds} + \left[ \frac{\epsilon^2}{2} - \frac{k^2}{4} + \frac{s k^2}{4} \right] W = 0.
$$

(23)

Omitting the term $\tilde{k}^2s/4$ in Eq. (22) is nothing else than the hypergeometric equation. The solution which is regular at the origin and finite at $s \to 1$ ($\rho \to R$) reads

$$
W = CF(-j, j+1, 1, s), \quad j = 0, 1, 2, ...,
$$

(24)

where $j$ is a non-negative integer, and $C$ is the normalization constant. The related energy spectrum is given by $\epsilon^2 = \left[ \tilde{k}^2/2 + 2j(j+1) \right]$. The functions $W_j$ are normalized by the condition

$$
\int W_j(\tilde{\rho}) W_i(\tilde{\rho}) 2\pi \tilde{\rho} d\tilde{\rho} = \delta_{ji}.
$$

(25)

From Eq. (18) we obtain

$$
2\frac{\mu}{E} (1-\tilde{\rho}^2) f_- \approx f_+.
$$

(26)

For the lowest branch of the spectrum ($j=0$) the excitation energy has a linear dependence on $k$;

$$
E_0 = \frac{\hbar \omega R}{2} k.
$$

(33)
IV. ROTON PART OF THE SPECTRUM

In the opposite limit, $kR \gg 1$, we keep the term $(\hbar^2 k^2/2m)^2$ in Eq. (21). In this limiting case the main contribution to the integral over $d^2 \rho'$ in equation (21) comes from distances $\rho'$ very close to $\rho$, and this equation takes the form (for zero orbital momentum)

$$\left(\frac{\hbar^2 k^2}{2m}\right)^2 f_+ (\rho) + 2g(1 - \eta) \frac{\hbar^2}{2m} \psi_0 \left( k^2 - \nabla^2 + \frac{\nabla^2 \psi_0 (\rho)}{\psi_0 (\rho)} \right) f_+ (\rho) \psi_0 (\rho) - 6g \eta \frac{\hbar^2}{2m} \psi_0 (\rho) \frac{d^2 f_+ \psi_0}{d \rho^2} = E^2 f_+ (\rho).$$

(34)

Representing $f_+ (\rho) = (1 - \rho^2/R^2) W$, in terms of dimensionless variables $s$, $\epsilon$ equation (34) reads

$$s(1 - s) W'' + (1 - 3s) W' + \left[ \frac{\epsilon^2 - \epsilon_j^2 (\tilde{k})}{3\eta} (1 + \eta/2) - \frac{(\eta - 1) \tilde{k}^2}{6\eta} s \right] W = 0,$$

(35)

where

$$\epsilon_j^2 (\tilde{k}) = \left( \frac{\hbar \omega}{4\mu} \right)^2 \tilde{k}^4 - \frac{\eta - 1}{2} \tilde{k}^2 + \frac{3\eta}{1 + \eta/2}.$$  

(36)

Omitting the term $-(\eta - 1) \tilde{k}^2 s W$, equation (35) becomes a hypergeometric equation. The solution regular at the origin and finite for $s \to 1$ is

$$W_j = \tilde{C} F(-j, j + 2, 1, s),$$

(37)

where $j$ is a non-negative integer, and $\tilde{C}$ is the normalization constant. The related energy spectrum is given by $\epsilon_j^2 = \epsilon_j^2 (q) + \frac{3\eta}{1 + \eta/2} j (j + 2)$. The functions $W$ are normalized by the condition

$$\int_0^1 W_{j_1} W_{j_2} \pi (1 - s) ds = \delta_{j_1 j_2}.$$  

(38)

From equations (17) and (18) in the limit of $kR \gg 1$ we have

$$f_+ \approx \frac{4\mu \epsilon}{\hbar \omega k^2} f_-,$$

(39)

and, hence,

$$f_+ = \sqrt{\frac{4\mu \epsilon}{\hbar \omega k^2}} \sqrt{1 - \tilde{\rho}^2 W},$$

(40)

$$f_- = \sqrt{\frac{\hbar \omega k^2}{4\mu \epsilon}} \sqrt{1 - \tilde{\rho}^2 W}.$$  

(41)

The omitted term $-(\eta - 1) \tilde{k}^2 s W_j$ we take into account perturbatively. The related calculations are given in the Appendix. Adding the first order correction to $\epsilon_j^2$ we obtain

$$\epsilon_j^2 = \epsilon_j^2 (\tilde{k}) + \frac{\eta - 1}{2} \tilde{k}^2 x_j + \frac{3\eta}{1 + \eta/2} j (j + 2),$$

(42)

V. DAMPING OF ROTONS

At finite temperatures, the leading damping mechanism for the roton excitation is the Landau damping. In particular, a roton with energy $\epsilon_0 (k)$ interacts with a thermal low-momentum ($|p| \ll k$) sound type excitation $\epsilon_j (p)$. Both get annihilated, and a roton excitation with a higher energy and momentum $\epsilon_l (k + p)$ is created, where $j, l$ are excitation branch numbers $j, l = 0, 1, 2, ...$. We calculate the damping rate for the lowest rotonic excitation which has momentum $k$. The damping rate is given by the Fermi golden rule

$$\frac{1}{\tau} = \sum_{j,l} \frac{1}{\tau_{jl}};$$

(44)
\[
\frac{1}{\tau_{jl}} = \frac{2\pi}{\hbar} \int_{-\infty}^{\infty} \frac{dp}{2\pi} \left|(k + p, l)|H_{\text{int}}|k, \{p, j\}\rangle|^2 - |(k, \{p, j\})|H_{\text{int}}|k + p, l\rangle|^2 \right| \delta(\epsilon_{k+p,l} - \epsilon_{k,0} - \epsilon_{p,j}). \tag{45}
\]

The Hamiltonian \(H_{\text{int}}\) responsible for the damping represents the interaction between excitations and is given by

\[
H_{\text{int}} = \int d^3r \ d^3r' \left[ \Psi_0(r)\Psi^\dagger(r')V(r - r')\Psi_1(r) + \Psi^\dagger(r)\Psi^\dagger(r')V(r - r')\Psi_0(r) \right]. \tag{46}
\]

We use the non-condensed operator \(\Psi'\) in terms of the functions \(f^\pm\): \(\Psi'(r, t) = \sum_k [f^+_k + f^+_k \hat{b}_k - (f^+_k - f^-_k \hat{b}_k)/2, \; \text{where} \; \nu = \{k, j\}, \) and the functions \(f^\pm\) are given by Eqs. (28), (29), (40), and (41).

Substituting expression (46) into (45) and integrating over the momentum \(p\) and coordinates, we obtain for the damping rate of a roton with momentum \(k\) the expression

\[
\frac{1}{\tau_{jl}} = \frac{n_0 g^2}{4\hbar R^2} \left| E_{k+p,l} - E_{p,j} \right| Z_{jl}, \tag{47}
\]

where \(Z_{jl} = \left\{G_1(k)\bar{f}^+_k \bar{f}^-_{k+p,l} + G_2(k)\bar{f}^-_k \bar{f}^+_{k+p,l} + G_3(p)\bar{f}^+_p \bar{f}^-_{k+p,j} \bar{f}^+_{k+p,l} + \bar{f}^-_p \bar{f}^-_{k+p,j} \bar{f}^+_{k+p,l} \right\}

- G_1(k + p)\bar{f}^+_p \bar{f}^-_{k+p,j} \bar{f}^+_k \bar{f}^-_{k+p,l} + G_2(k + p)\bar{f}^-_p \bar{f}^-_{k+p,j} \bar{f}^+_k \bar{f}^-_{k+p,l} \right\}^2,
\]

where \(N_k = 1/(e^{E_{k}/T} - 1)\) are excitation occupation numbers, \(E_{k,l} = dE_{k,l}/dk\), and the momentum \(p\) is found from the energy conservation law:

\[
\epsilon_{k+p,l} = \epsilon_{k,0} + \epsilon_{p,j}. \tag{48}
\]

The functions \(\bar{f}^\pm\) are coordinate independent multipliers in Eqs. (28), (29), (40), and (41):

\[
\bar{f}^\pm_{p,j} = \left[ \frac{2\mu}{\hbar \omega \epsilon_{p,j}} \right]^{1/2} \; \bar{f}^\pm_{k,l} = \left[ \frac{4\mu \epsilon_{k,l}}{\hbar \omega k^2} \right]^{1/2}.
\]

For the functions \(G_i\) we obtain by the use of Eqs. (20) - (21) the following expressions:

\[
G_1(k) = \frac{\left( \epsilon_{k,l}^2 - \frac{\hbar \omega}{4\pi} \right)^2 k^4 (2 + \eta)}{k^2} \times \frac{\int_0^1 \left( 1 - s \right) F_1 F_2 ds}{\sqrt{\int F_1^2 \, ds} \sqrt{\int \left( 1 - s \right) F_2^2 \, ds}}, \tag{49}
\]

\[
G_2(k) = \frac{\left( \epsilon_{k,l}^2 - \frac{\hbar \omega}{4\pi} \right)^2 k^4 (2 + \eta)}{k^2} \times \frac{\int_0^1 F_1 F_2 ds}{\sqrt{\int \left( 1 - s \right) F_2^2 \, ds} \sqrt{\int \left( 1 - s \right) F_2^2 \, ds}}. \tag{50}
\]

VI. CONCLUSIONS

In this paper, we have calculated the finite temperature damping rate for rotons in an elongated Bose-condensed...
gas of polarized dipolar particles, which is in the Thomas-Fermi regime in the tightly confined directions. The specific of this case is the presence of a large number of excitation branches which can contribute to the damping process. We found out that this leads to a significant increase of the damping rate. Nevertheless, our calculations show that, even in this regime, rotons in the systems with the roton energy gap of the order of 1 nK are sufficiently long living to be observed as well-defined peaks in the excitation spectrum and contributions in response functions.

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Appendix A: Perturbative corrections

Equation (35) can be written in the form \((H_0 + \tilde{V})W = \tilde{\epsilon}W\), with the perturbation

\[
\tilde{V} = -\frac{(\eta - 1)k^2}{6\eta} s, \tag{A1}
\]

and \(\tilde{\epsilon} = -\frac{\epsilon^2 - \epsilon_j(0)^2}{\eta}(1 + \eta/2)\). Substituting \(W = \sum c_n W_n^0\) into Eq. (35), multiplying it by \((1 - s)W_i\) and integrating over \(ds\) we obtain

\[
(\tilde{\epsilon} - \epsilon_j^0)c_j = \sum_l V_{jl}c_l, \tag{A2}
\]

where \(\epsilon_j^0 = j(j + 2),\) and \(V_{jl} = \int (1 - s)W_jVW_i \, ds\).

The first and second order corrections to the eigenvalues are equal to:

\[
\epsilon_j^{(1)} = \int \left( -\frac{(\eta - 1)k^2}{6\eta} s \right) (1 - s)|W_j|^2 \, ds, \tag{A3}
\]

and

\[
\epsilon_j^{(2)} = \sum_l \frac{|V_{jl}|^2}{\epsilon_j^{(0)} - \epsilon_l^{(0)}}. \tag{A4}
\]

For the lowest branch of the spectrum \((j = 0)\) we clearly have a roton minimum for \(\eta > 1\). The roton momentum \((\text{minimum of} \, \epsilon_0(\tilde{k}))\) is located at

\[
\tilde{k}_0 = \frac{2\mu}{\hbar\omega} \sqrt{\frac{\eta - 1}{1 + \eta/2}}. \tag{A5}
\]

Our approach assumes that \(\tilde{k}_0 = k_0R \gg 1\), which leads to the inequality

\[
\frac{4\mu^2}{(\hbar\omega)^2} \frac{\eta - 1}{1 + \eta/2} \gg 1. \tag{A6}
\]

The energy at \(\tilde{k} = \tilde{k}_0\) (roton gap) is

\[
\epsilon_0^2(\tilde{k}_0) = -\left(\frac{\mu}{\hbar\omega}\right)^2 \frac{(\eta - 1)^2}{(1 + \eta/2)^2} + \frac{3\eta}{1 + \eta/2}. \tag{A7}
\]

It should be larger than zero in order to avoid the instability and we have a restriction:

\[
\frac{\mu^2}{(\hbar\omega)^2} \frac{(\eta - 1)^2}{(1 + \eta/2)\eta} < 3. \tag{A8}
\]

Since \(\mu \gg \hbar\omega\), equation (A8) is compatible with inequality (A6). The applicability criterion of the perturbation theory

\[
|V_{jl}| \ll |\epsilon_j^0 - \epsilon_l^0| \tag{A9}
\]
takes the form
\[
\frac{(\eta - 1)\tilde{k}^2}{6\eta} \int_0^1 s(1 - s) W_j W_l \, ds \ll |j(j + 2) - l(l + 2)|,
\]
(A10)
and is fulfilled for momenta not far from the roton minimum. For example, for \(j = 0, l = 1\) Eq. (A10) gives the condition
\[
\frac{\mu^2}{(\hbar\omega)^2} \frac{(n-1)^2}{(1+\eta/2)^n} < 20 \quad \text{at} \quad \eta \sim 1, \quad \tilde{k} \sim \tilde{k}_0,
\]
which is satisfied due to inequality (A8).

Thus, we confine ourselves to the first order energy corrections, and using Eq. (A3) we obtain Eq. (42) for the spectrum.

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