Longitudinal component of the Poynting vector of tightly focused cylindrical vector beam

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Abstract. We numerically investigated the tight focusing of cylindrical vector beam with wavelength of 532 nm by a diffractive lens with a numerical aperture NA = 0.95. The simulation was carried out using the Richards-Wolf formulae. It was shown that the focusing of cylindrical vector beam by a wide-aperture lens can produce intensity distribution with a negative component of the longitudinal component of the Poynting vector in the center of the focal spot.

1. Introduction

Recent years show an increase of the interest of researchers in the tight focusing of cylindrical vector beams (CVB) [1]. In investigation of the tight focusing, the attention of researchers is focused on achieving the minimum focal spot size. Since calculation of the intensity requires a calculation of the electric field only, the behavior of the magnetic field is not usually studied. However knowledge of the magnetic field distribution is necessary for investigation of the behavior of the Poynting vector.

According to [2], the force acting on the particle can be calculated as

\[ F = \frac{k}{2} \text{Im} \alpha \text{Re}(E \times H^*), \]  

(1)

where \( k \) is the wavenumber, \( \alpha \) is the polarizability of the particle, \( E \) and \( H \) are the electric and magnetic fields, respectively. From the Eq. (1) it follows that the longitudinal component of the force is proportional to the longitudinal component of the Poynting vector. If a beam contains negative values of the longitudinal component of the Poynting vector, then a particle illuminated by this beam moves toward the light source.

The existence of negative values of the longitudinal component of the Poynting vector is known in optics for a long time [3]. However, the absolute value of these negative area was small, therefore this effect was not studied in detail. Recently [4] it was discovered that it is possible to form a focal spot with the absolute values of the negative part of the Poynting vector comparable with positive values.

In this paper we numerically investigated the tight focusing of cylindrical vector beam with wavelength of 532 nm by a diffractive lens with a numerical aperture NA = 0.95. The simulation was carried out using the Richards-Wolf formulae. It was shown that the focusing of CVB by a wide-aperture lens can produce intensity distribution with a negative component of the longitudinal component of the Poynting vector in the center of the focal spot.
2. Numerical Simulation

2.1. Richards-Wolf formula

Our numerical simulation was performed using the Richards-Wolf formula [3]:

\[
U(\rho, \psi, z) = \frac{i}{k} \int_0^{2\pi} \int_0^\infty B(\theta, \phi) T(\theta) P(\theta, \phi) \exp \left( ik \left[ \rho \sin \theta (\cos \phi - \psi) + z \cos \theta \right] \right) \sin \theta \, d\rho \, d\phi
\]  

where \( U(\rho,\psi,z) \) is the electric \( E \) or magnetic field \( H \) in focal area, \( B(\theta, \phi) \) is the electrical or magnetic field of the input beam (\( \theta \) is the polar angle and \( \phi \) is the azimuthal angle), \( T(\theta) \) is apodization function, \( f \) is the focal length, \( k = 2\pi/\lambda \) is the wavenumber (\( \lambda = 532 \) nm), and \( P(\theta, \phi) \) is the polarization matrix:

\[
P(\theta, \phi) = \begin{bmatrix} 1 + \cos^2 \phi (\cos \theta - 1) & \sin \phi \cos \phi (\cos \theta - 1) \\ \sin \phi \cos \phi (\cos \theta - 1) & 1 + \sin^2 \phi (\cos \theta - 1) \end{bmatrix} \begin{bmatrix} a(0, \phi) + \sin \phi \cos \phi (\cos \theta - 1) b(0, \phi) \\ -\sin \phi \cos \phi (\cos \theta - 1) a(0, \phi) - \sin \phi \cos \phi (\cos \theta - 1) b(0, \phi) \end{bmatrix}
\]  

where \( a(\theta, \phi) \) and \( b(\theta, \phi) \) are polarization functions for the \( x \)- and \( y \)-components of the input beam. For the simulated field polarization functions were equal to

\[
E = \begin{bmatrix} a(0, \phi) \\ b(0, \phi) \end{bmatrix} = \begin{bmatrix} -\sin(2\phi) \\ \cos(2\phi) \end{bmatrix}
\]  

From (1) it follows that the investigated polarization differs from azimuthally-polarized light in that the azimuthal angle changes to doubled azimuthal angle. Fig. 1 shows direction of the polarization of the incident field.

![Direction of polarization of the input field.](image)

**Figure 1.** Direction of polarization of the input field.

In the simulation, we assume that a Fresnel zone plate \( T(\theta) = \cos^{3/2}(\theta), \) NA = 0.95) is illuminated by a plane wave \( B(0, \phi) = 1 \). Intensity in the focal spot was calculated as \( I = I_x + I_y + I_z = |E_x|^2 + |E_y|^2 + |E_z|^2 \). The on-axis projection of the Poynting vector was calculated as

\[
S_z = \frac{1}{2} \text{Re} \left[ \mathbf{E} \times \mathbf{H}^* \right]_z = \frac{1}{2} \text{Re} \left( E_z H_y^* - E_y H_z^* \right)
\]  

2.2. Results of the simulation

Fig 2-4 show results of the simulation. Fig. 2 shows intensity distribution in the focal spot.
Figure 2. Intensity distribution in focal spot (XY plane).

From Fig. 1 it can be seen that the focal spot has an asymmetrical shape. Tight focusing leads to energy redistribution between the components of intensity (Fig. 3) and to the asymmetry of the focal spot.

Figure 3. Intensity components $I_x$ (a), $I_y$ (b), and $I_z$ (c) in focal spot (XY plane).

Fig 4 shows z-axis projection of the Poynting vector in focal spot (Fig. 3a) and along the z-axis (Fig. 3b).
From Fig. 4 it can be seen that the central part of the focal spot contains negative values $S_z$. It should be noted that focal spot does not contain transverse components of Poynting vector $S_x, S_y$.

The results obtained by Richards-Wolf formulae were numerically reconfirmed using the FDTD-method implemented in Fullwave software [5].

3. Conclusions
We numerically investigated the tight focusing of cylindrical vector beam with wavelength of 532 nm by a diffractive lens with a numerical aperture NA = 0.95. The simulation was carried out using the Richards-Wolf formulae. It was shown that the focusing of CVB by a wide-aperture lens can produce intensity distribution with a negative component of the longitudinal component of the Poynting vector in the center of the focal spot.

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References
[1] Qiwen Zhan, "Cylindrical vector beams: from mathematical concepts to applications," Adv. Opt. Photon. 1, 1-57 (2009)
[2] Sukhov S and Dogariu A 2010 Opt. Lett. 35 3847.
[3] Richards B and Wolf E 1959 Proc. R. Soc. A Math. Phys. Eng. Sci. 253 358.
[4] Kotlyar V V and Nalimov A G 2017 Computer Optics 41 645.
[5] https://www.synopsys.com/optical-solutions/rssoft/passive-device-fullwave.html