Energy of partial $A$-body problem for $^9\Lambda\text{Be}$ and $^{10}\Lambda\Lambda\text{Be}$

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Abstract. The energy of the degenerate doublet $(3^+ / 2, 5^+ / 2)$ of $^9\Lambda\text{Be}$, treating it as a partial nine-body system in the $\Lambda\alpha\alpha$ cluster model, has been calculated in the variational Monte Carlo framework. A simplified treatment, with the central two-body Urbana type $\Lambda N$ and the three-body dispersive and two-pion exchange $\Lambda NN$ forces along with the central two- and three-body correlations, is found to be adequate in explaining the energy of observed $\gamma$-ray transition from the excited degenerate doublet to the ground state. This partial nine-body model of $^9\Lambda\text{Be}$ has been extended for the case of partial ten-body problem $^{10}\Lambda\Lambda\text{Be}$ in the $\Lambda\Lambda + \alpha\alpha$ model where nucleonic degrees of freedom of $\alpha$s are taken into consideration ignoring antisymmetrization between two $\alpha$s. The central two-body $\Lambda N$ and $\Lambda\Lambda$ and the three-body dispersive and two-pion exchange $\Lambda NN$ forces, constrained by the $\Lambda p$ scattering data and the observed ground state energies of $^3\Lambda\text{He}$ and $^5\Lambda\Lambda\text{He}$, are employed. The product type trial wavefunction predicts binding energy for the ground state considerably less than for the event reported by Danysz et al but however, consistent with the value deduced assuming a $\gamma$-ray of 3.04 MeV must have escaped undetected in the decay of the product $^9\Lambda\text{Be}^* \rightarrow ^9\Lambda\text{Be} + \gamma$ of the emulsion event $^{10}\Lambda\Lambda\text{Be} \rightarrow \pi^- + p + ^3\Lambda\text{He}^*$ and for the excited $2^+$ state closer to the value measured in the Demachi-Yanagi event. The results of the present work are consistent with the earlier three- and four-body cluster model approaches where $\alpha$s are assumed to be structureless entities.

1. Introduction

Recently, Shoeb [1] and coworkers [2, 3, 4] have analysed the ground state binding energies of $s$- and $p$-shell hypernuclei and excited states of $p$-shell hypernuclei in the $\alpha$ cluster model using the variational Monte Carlo (VMC) method. In these analyses [1, 2, 3], a phenomenological dispersive three-body $\Lambda\alpha\alpha$ force of Yukawa shape was proposed in analogy with the one suggested in explaining the spectra of $^{12}\text{C}$ using the three-body cluster $\alpha\alpha\alpha$ potential [5, 6]. The phenomenological dispersive three-body $\Lambda\alpha\alpha$ force [1, 2, 3] along with the appropriate $\Lambda\alpha$, $\alpha\alpha$ and $\Lambda\Lambda$ potentials explains the ground and excited state energies of $^9\Lambda\text{Be}$ and $^{10}\Lambda\Lambda\text{Be}$. The need to incorporate a repulsive dispersive three-body force has been pointed out not only in the $\alpha$ cluster model analysis [7] but investigated in detail in the earlier microscopic study [8]. In contrast to analyses [1, 2, 3, 4], Hiyama et al [9] have found the modified odd/even state $\Lambda N$ potential important in explaining the binding energy and excited state data of $S = -1$ and $-2$ hypernuclei. Although these potentials have been microscopically calculated but were tailored to adjust the $\Lambda$-cluster potentials so that ground-state or ground- and excited-state energies of hypernuclei containing clusters are reproduced thus rendering these as phenomenological ones. In the calculation of the properties of $^9\Lambda\text{Be}$ in the $\Lambda\alpha\alpha$ model, using the Faddeev method, Cravo et al [10] needs neither a dispersive force nor modified odd/even state $\Lambda N$ potential. In the last
few years, other cluster model analyses [11, 12] have also been performed for the binding energy of s-shell hypernuclei. Here it is noteworthy to point out that oldest calculation for the excited state of $^{9}_\Lambda$Be is by Ali et al [13].

In all the above-mentioned analyses, $\alpha$ clusters are treated as rigid entities devoid of structure and thus the effects of the role played by the dynamical correlations among the baryons are not explicitly manifested in the energy calculation. Therefore, such studies are deficient in accommodating the realities of the internal structure and seem, therefore, far from satisfactory. Earlier, Shoeb et al [14] have calculated the ground state energy of $^{9}_\Lambda$Be, treating it as a partial nine-body system in the $\Lambda + 2\alpha$ model. This work presupposes the existence of the $\alpha\alpha$ structure. The two-body $NN$ correlations within $\alpha\alpha$ were explicitly incorporated. However, the effect of $NN$ correlations, where each $\alpha$ contributes a nucleon, is simulated through $\alpha\alpha$ correlation. Thus, antisymmetrization between two $\alpha$s has been ignored. However, soft repulsive core in the $\alpha\alpha$ potential [6] simulates the effect of $NN$ antisymmetrization in the wavefunction. The three-body $\Lambda NN$ correlations were included in the trial wavefunction but $\Lambda N$ space-exchange correlations were ignored. From this study it was concluded that dispersive three-body $\Lambda NN$ force and space-exchange $\Lambda N$ potential can not be determined uniquely. The presence of one masks the determination of other. However, $B_\Lambda$ of $^{9}_\Lambda$Be is satisfactorily explained.

The energy of the ground and excited states of $^{10}_{\Lambda\Lambda}$Be has been extensively analysed in the $\Lambda\Lambda\alpha\alpha$ cluster model in [1, 2, 9, 15, 16] using a variety of methods: VMC framework, Gaussian-basis coupled-rearrangement channel method and Faddeev-Yakubovsky method. The input $\Lambda \Lambda$, $\Lambda \alpha$, and $\alpha\alpha$ potentials with soft repulsive core and reasonable shapes, constrained by the data relevant to the each interacting pair, have been used. The analyses [2, 9] predict the ground state $\Lambda\Lambda$ binding of $^{10}_{\Lambda\Lambda}$Be, $B_{\Lambda\Lambda}$ about 15% less than the currently accepted experimental [17] value 17.6 ± 0.4 MeV. Thus strengthening the speculation that in the measurement of $B_{\Lambda\Lambda}$, a $\gamma$-ray [18] of 3.04 MeV must have escaped undetected from the decay products of $^{10}_{\Lambda\Lambda}$Be in the emulsion. Further, the cluster model calculations of Shoeb [1] and Hiyama et al [9] for the energy of the 2$^+$ excited state of $^{10}_{\Lambda\Lambda}$Be incidentally agree with the $\Lambda\Lambda$ binding energy $B_{\Lambda\Lambda}$ = 12.33$^{+0.35}_{-0.21}$ MeV found in the E373 experiment, named as Demachi-Yanagi event [19]. Thus Demachi-Yanagi event, based on the calculation of $\Lambda\Lambda\alpha\alpha$ cluster model, is identified with the 2$^+$ excited state of $^{10}_{\Lambda\Lambda}$Be.

The satisfactory application of a partial nine-body problem [14] within the $\Lambda + 2\alpha$ model in explaining the $B_\Lambda$ of $^{9}_\Lambda$Be, though for a single set of potential parameters, motivated us to apply it to analyse the experimental [18] binding energy $B_\Lambda = 3.67$ MeV of the degenerate doublet ($3^+/2$, $5^+/2$) of $^{9}_\Lambda$Be. We have also extended the partial nine-body model for the case of $^{10}_{\Lambda\Lambda}$Be, treating it as a partial ten-body problem in the $\Lambda\Lambda + \alpha\alpha$ model and to investigate how far it succeeds in explaining the energy of the ground and 2$^+$ excited states. We may point out that antisymmetrization which has been ignored among the nucleons of two well separated $\alpha\alpha$ is being simulated through the soft repulsive core in the $\alpha\alpha$ potential [6, 20]. Following the spirit of the earlier work [14], for the simplicity of the calculation, we have chosen simple baryon-baryon potentials along with the corresponding simple correlation functions.

The paper is organized as follows: In the next section, we briefly describe the Hamiltonians of $^{9}_\Lambda$Be and of hypernucleus $^{10}_{\Lambda\Lambda}$Be in the $\alpha$ cluster model along with the trial wavefunctions and the energy calculations. The expressions for the $\Lambda N$, $NN$, $\alpha\alpha$ and $\Lambda\Lambda$ and three-body $\Lambda NN$ potentials are given in our earlier work [21, 22]. In section 3, the results and discussion are presented. The conclusions of our study are given in the last section.
2. Hamiltonians of the p-shell hypernuclei in the α cluster model, Trial Wavefunctions and Energy Calculations

2.1. Hamiltonians of the p-shell hypernuclei

The 2+ state of $^{10}_{\Lambda\Lambda}$Be is assumed to be built on the first excited state $J_C = 2^+$ of the core nucleus $^8$Be. $J_C = 2^+$ is a coupled state of $L_C = 2$, and $S_C = 0$. The two Λ particles of $s_\Lambda = 1/2$ coupled to spin singlet function $\chi^0_0$ when combined to the $J_C = 2^+$ of $^8$Be core in $^{10}_{\Lambda\Lambda}$Be gives rise the $J = 2^+$ spin state. The Hamiltonian of the $A(=10)$ baryons system $^{10}_{\Lambda\Lambda}$Be in the $2\alpha + 2\Lambda$ is given as:

$$
H^{10}_H = \sum_{i=1}^{A-2} K_N(i) + \sum_{i<j}^{A-2} V_{NN}(r_{ij}) + \sum_{i<j}^{A-2} V_N(r_{ij}) + V^{(0)}_{\alpha\alpha}(r_{\alpha_1\alpha_2}) + \sum_{k=1}^{2} (K_{\Lambda_k} + \sum_{i=1}^{4} V_{\Lambda\Lambda}(r_{\Lambda_i}))
$$

$$
+ \sum_{i=5}^{A-2} V_{\Lambda\Lambda}(r_{\Lambda_i}) + \sum_{i<j}^{A-2} V_{NN}(r_{\Lambda_i\Lambda_j}) + V_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2}),
$$

where labels 1 to 8 specify the nucleons and symbols $\alpha_1$ and $\alpha_2$ are identified with the $\alpha$ particles. $V_{xy}$ denotes the potential for a pair of particles $xy$ ($=NN$, $\Lambda N$, $\Lambda \Lambda$, $\alpha\alpha$) and in the case of $\alpha\alpha$ pair, $V^{(0)}_{\alpha\alpha}$ is potential in the relative angular momentum $l$ ($=0$ for the ground and 2 for the excited state). The three-body potential $V_{\Lambda\Lambda}$ is the sum of the dispersive force $V^{D}_{NN}$ and the two-pion exchange (TPE) three-body force $V^{TPE}_{NN}$. The contribution of $(V^{D}_{NN})$ to the energy, from triads $\Lambda NN$, where each $\Lambda$ contributes a nucleon, is substantial as shown in [8, 14], neglecting it not only over binds the $^9\Lambda$Be but also $^{10}_{\Lambda\Lambda}$Be. The expressions for the $\Lambda N$, $NN$, $\alpha\alpha$ and $\Lambda\Lambda$ and for three-body $\Lambda NN$ potentials used here are given in [21, 22]. The suppression of $\Lambda$ index in equation (1) leads to the Hamiltonian of the partial nine-body problem $^3\Lambda$Be.

2.2. Trial Wavefunctions

The variational wavefunction for $^{10}_{\Lambda\Lambda}$Be in the state $(J, J_z)$ as usual is constructed from the product of central two-body correlation functions $f_{xy}$ (for $xy = \alpha\alpha$, relative angular momentum state $l = 0$ or 2), three-body correlations $f_{NN\Lambda}$ and the $l$s coupled function $(y_{lm}(\Omega_{\alpha_1\alpha_2}) \otimes \chi^0_0)_{JJ_z}$, a appropriate combination of $\chi^0_0$, spin singlet function of the two $\Lambda$ particles and $y_{lm}(\Omega_{\alpha_1\alpha_2})$, spherical harmonic for the motion of two $\alpha$ in relative angular momentum state $l$:

$$
\Psi^{(10)}_H(J, J_z) = \prod_{k=1}^{2} \left[ \prod_{i=1}^{A-2} f_{NN}(r_{\Lambda_i}) \right] \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j}) \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j}) \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j})
$$

$$
\times \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j}) \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j}) \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j}) \prod_{i<j}^{A-2} f_{NN}(r_{\Lambda_i\Lambda_j})
$$

$$
\times f^{(0)}_{\alpha\alpha}(r_{\alpha_1\alpha_2}) f_{\Lambda\Lambda}(r_{\Lambda_1\Lambda_2})(y_{lm}(\Omega_{\alpha_1\alpha_2}) \otimes \chi^0_0)_{JJ_z}.
$$

The central two-body spin-independent correlation functions $f_{\alpha\alpha}$, $f_{NN}$, $f^{(0)}_{\alpha\alpha}(r)$ and $f_{\Lambda\Lambda}$, similar to earlier analyses [1, 2, 14, 15] are obtained from the procedure developed by the Urbana group and three-body correlations $f_{NN\Lambda}$ have the analytical forms as used in our earlier work [3]. The $f_{NN\Lambda}$ correlations arising from the triads $\Lambda NN$, where a participating nucleon comes from each $\alpha$, have been ignored as these make a negligibly small contributions owing to large $\alpha\alpha$ separation. The wavefunction $\Psi^{(10)}_H$ depends on a total of seventeen variational parameters $\kappa_{\Lambda\Lambda}$, $\kappa_{NN}$, $\kappa_{\alpha\alpha}$, $R_{\Lambda\Lambda}$, $s_{\Lambda\Lambda}$, $s_{NN}$, $a_{\alpha\alpha}$, $a_{\alpha\alpha}$, $a_{\alpha\alpha}$, $a_{\alpha\alpha}$, $a_{\alpha\alpha}$, $a_{\alpha\alpha}$ and $R_{\Lambda\Lambda}$ for ground state of $^{10}_{\Lambda\Lambda}$Be and exactly the same number of variational parameters for the excited state. The trial wavefunction for $^3\Lambda$Be involving 13 variational parameters, is obtained from wavefunction of $^{10}_{\Lambda\Lambda}$Be after suppressing the $\Lambda$ index and restricting baryon number to 9.
2.3. Energy calculations

The energy \(-B_{\Lambda\Lambda}(J, J_z)\) for a hypernucleus of baryon number \(A\) is the difference of energy of hypernucleus in the state \(\Psi_H^{(A)}(J, J_z)\) and of the nuclear core in the state \(\Psi_{C}^{(A-2)}(J_C, M_C)\) and is written as:

\[
-B_{\Lambda\Lambda}(J, J_z) = \frac{\langle \Psi_H^{(A)}(J, J_z)|H_H^{(A)}|\Psi_H^{(A)}(J, J_z)\rangle}{\langle \Psi_H^{(A)}(J, J_z)|\Psi_H^{(A)}(J, J_z)\rangle} - \frac{\langle \Psi_C^{(A-2)}(J_C, M_C)|H_C^{A-2}|\Psi_C^{(A-2)}(J_C, M_C)\rangle}{\langle \Psi_C^{(A-2)}(J_C, M_C)|\Psi_C^{(A-2)}(J_C, M_C)\rangle}.
\] (3)

The variational parameters entering in the wavefunction are varied to optimize the energy using the standard optimizing routine. The parameters \(\kappa_{\Lambda N}, \kappa_{NN}, \kappa_{\alpha\alpha}\) related to the separation energy of the pair \(xy\) are those on which the energy depends sensitively. The estimates for the energy were made for 100,000 points. The two terms in equation (3) were separately calculated. The second term in equation (3) is \(-31.20(-30.34)\) MeV for \(^4\)He and \(-62.3(-60.58)\) MeV for \(^8\)Be core for Malfliet-Tjon [23] (Volkov) [24] \(NN\) potential. Similarly, expression for the energy for partial nine-body problem \(^9\)Be is obtained.

3. Results and Discussion

3.1. Ground State and Degenerate Doublet \((3^+/2, 5^+/2)\) of \(^9\)Be

The experimental data for \(B_A\) of the systems \(^5\)\(^\Lambda\)He and \(^9\)\(^\Lambda\)Be have been taken from [18, 25]. Prior to the analysis of the energy of the degenerate doublet \((3^+/2, 5^+/2)\) of \(^9\)\(^\Lambda\)Be we need to recalculate the ground state energies of \(^5\)\(^\Lambda\)He and \(^9\)\(^\Lambda\)Be. The parameters of the two-body \(\Lambda N\) force are \(V = 6.15, 6.10\) and \(6.20\) MeV and \(\epsilon = 0.25\) and the strength \(W_d\) of dispersive force is adjusted for the sets of combinations of \(C_p\) and \(\tilde{c}\) to reproduce \(B_A\) of \(^5\)\(^\Lambda\)He for a chosen \(NN\) potential. The results for ground states of \(^5\)\(^\Lambda\)He and \(^9\)\(^\Lambda\)Be and for degenerate doublet \((3^+/2, 5^+/2)\) of \(^9\)\(^\Lambda\)Be are listed in our earlier work [21, 26]. Here we will only discuss the final results, listed in table 1, for the difference between excited- and ground-state energies for all the combinations of the three-body \(\Lambda NN\) potentials for Malfliet-Tjon (MT) and Volkov \(NN\) potentials.

We note that the difference \(\Delta E_{\Lambda}(=E^{(9)}_{\Lambda}(^6\Lambda\)Be)\) between excited and ground states energies for the strength parameter \(V = 6.15, 6.10\) and \(6.20\) MeV for MT and Volkov \(NN\) potential, is in good agreement with the observed emitted \(\gamma\)-ray [18] provided small spin-orbit splitting is ignored.

The Volkov \(NN\) potential induces weaker correlations compared to MT \(NN\) potential. However, the results for \(\Delta E_{\Lambda}\) listed in the fifth column of table 1 are not very different from those obtained for the MT potential. Thus both the central \(NN\) potentials with soft repulsive core give equally good fit to the energy spacing between the two states of \(^9\)\(^\Lambda\)Be.

3.2. Ground and Excited \(2^+\) States of \(^{10}\)\(^\Lambda\)Be

Before we proceed to discuss the results for the calculation of the energy of the system \(^{10}\)\(^\Lambda\)Be, we need to constrain some of the potential parameters from a fit to the ground state energies of \(^5\)\(^\Lambda\)He and \(^6\)\(^\Lambda\)He. The results for ground state energies of \(^5\)\(^\Lambda\)He and \(^6\)\(^\Lambda\)He have been discussed in our earlier work [21, 22, 26]. From the previous subsection, we have found that the results for the energy spacing of the excited degenerate doublet \((3^+/2, 5^+/2)\) from the ground state of \(^9\)\(^\Lambda\)Be for the MT and Volkov \(NN\) potentials are essentially the same. Therefore, we have carried out calculations for the energy of \(^{10}\)\(^\Lambda\)Be using the MT \(NN\) potential alone. We have also noticed from last subsection that the energy difference of the ground and excited states of \(^9\)\(^\Lambda\)Be for the strength parameter \(V = 6.15, 6.10\) and \(6.20\) MeV is not very different, therefore, we have restricted the calculations of the energy of \(^{10}\)\(^\Lambda\)Be for \(V = 6.15\) MeV alone. For the parameters \(V = 6.15\) MeV and \(\epsilon = 0.25\) of the two-body \(\Lambda N\) Urbana-type force, the strength \(W_d\) of dispersive force is adjusted for the sets of combinations of \(C_p\) and \(\tilde{c}\) from a fit to \(B_A\) of \(^5\)\(^\Lambda\)He for the \(NN\)
Table 1. The energy spacing $\Delta E_{\Lambda}$ (in MeV) between excited and ground states of $^{9}_{\Lambda}$Be for the MT and Volkov $NN$ potentials. All the potential strengths are in MeV and $\hat{c}$ in fm$^{-2}$. Experimental $\Delta E_{\Lambda} = 3.04$ MeV.

| $\bar{V}$ | $W_d$ | $C_P(\hat{c})$ | Theoretical $\Delta E_{\Lambda}$ |
|-----------|-------|----------------|-------------------------------|
|           |       |                | Malfliet-Tjon [23] | Volkov [24] |
| 6.15      | 0.012 | 0 (0)          | 3.18                         | 3.17       |
|           | 0.009 | 1 (1)          | 3.24                         | 3.19       |
|           | 0.010 | 1 (2)          | 3.00                         | 3.11       |
|           | 0.0115| 1 (3)          | 3.10                         | 3.19       |
|           | 0.006 | 2 (1)          | 3.06                         | 3.21       |
|           | 0.009 | 2 (2)          | 2.99                         | 3.16       |
|           | 0.016 | 2 (3)          | 3.08                         | 3.17       |
| 6.10      | 0.008 | 0 (0)          | 3.25                         | 3.10       |
|           | 0.005 | 1 (1)          | 2.98                         | 3.09       |
|           | 0.006 | 1 (2)          | 3.22                         | 3.13       |
|           | 0.0073| 1 (3)          | 3.27                         | 3.08       |
|           | 0.002 | 2 (1)          | 3.27                         | 3.00       |
|           | 0.005 | 2 (2)          | 3.23                         | 3.16       |
|           | 0.0115| 2 (3)          | 2.96                         | 3.12       |
| 6.20      | 0.017 | 0 (0)          | 3.10                         | 3.16       |
|           | 0.013 | 1 (1)          | 3.23                         | 3.24       |
|           | 0.013 | 1 (2)          | 3.08                         | 3.19       |
|           | 0.0153| 1 (3)          | 3.06                         | 3.11       |
|           | 0.010 | 2 (1)          | 2.99                         | 3.21       |
|           | 0.013 | 2 (2)          | 3.16                         | 3.23       |
|           | 0.020 | 2 (3)          | 3.17                         | 3.18       |

potential, as described in detail in [21, 26]. The sets of $\Lambda N$ and $\Lambda NN$ potential parameters, so obtained, along with $\Lambda \Lambda$ potential are used to analyse the $^{6}_{\Lambda}He$. The parameters sets that give a good account of the binding energies of $^{6}_{\Lambda}He$ and $^{8}_{\Lambda}He$, are employed to carry out the energy calculations for the ground and excited states of $^{10}_{\Lambda\Lambda}Be$. The experimental binding energy data for the systems $^{6}_{\Lambda\Lambda}He$ and $^{10}_{\Lambda\Lambda}Be$ are taken from [17, 19, 27].

Here, we may remark that for the TPE $\Lambda NN$ force, for the ease of computation, the calculations were restricted for two extreme values of cut-off radii $\hat{c} = 1$ and 3 fm$^{-2}$ as $\hat{c} = 2$ fm$^{-2}$ is expected not to give results different from the other two chosen values, such a situation has been observed in the last subsection for $^{9}_{\Lambda}Be$.

The energies of ground and excited 2$^+$ states of $^{10}_{\Lambda\Lambda}Be$ are discussed in our earlier work [22, 26]. Here we will only quote the final results, listed in table 2, for the difference between excited- and ground-state energies for all the combinations of the three-body $\Lambda NN$ potentials for MT $NN$ potential.

The difference $\Delta E_{\Lambda\Lambda} = |E^{(10)}_{\Lambda\Lambda}(Be^*) - E^{(10)}_{\Lambda\Lambda}(Be)|$, between excited and ground states energy for the combinations of the potential parameters sets under consideration, is listed in the column three of table 2. We note from the table that its value 3.1—3.3 MeV is consistent with the cluster model calculation of Shoeb [1] but about 12% larger than the one found by Hiyama et al [9]. However, theoretically calculated $\Delta E_{\Lambda\Lambda}$ lies in between the two extreme experimental values 2.17 ± 0.4 MeV and 5.27 ± 0.4 MeV but closer to the lower limit.
Table 2. The energy spacing $\Delta E_{\Lambda\Lambda}$ (in MeV) along with the statistical error between $2^+$ excited and ground states of $^{10}_{\Lambda\Lambda}$Be. Other quantities are the same as in the preceding table. Experimental $\Delta E_{\Lambda\Lambda} = 5.27 \pm 0.4$ MeV from Danysz et al [17] (2.17 $\pm 0.4$ MeV deduced from the assumption of a missing $\gamma$-ray).

| $W_d$ | $C_p(\hat{c})$ | Theoretical $\Delta E_{\Lambda\Lambda}$ |
|-------|---------------|-------------------------------------|
| 0.012 | 0 (0)         | 3.24 $\pm$ 0.08                     |
| 0.009 | 1 (1)         | 3.11 $\pm$ 0.08                     |
| 0.015 | 1 (3)         | 3.24 $\pm$ 0.15                     |
| 0.006 | 2 (1)         | 3.32 $\pm$ 0.11                     |
| 0.016 | 2 (3)         | 3.13 $\pm$ 0.17                     |

4. Conclusions

We conclude that we have applied, to our knowledge, for the first time, the variational Monte Carlo method to analyse the degenerate doublet ($3^+/2, 5^+/2$) of $^9\Lambda$Be, treating it as a partial nine-body system in the $\Lambda\alpha\alpha$ cluster model and to analyse the energy of the ground and excited $2^+$ states of $^{10}_{\Lambda\Lambda}$Be, treating it as a partial ten-body system in the $\Lambda\Lambda\alpha\alpha$ cluster model using the simple $NN$ and $\Lambda N$ potentials and corresponding correlation functions. The use of Urbana $\Lambda N$ potential consistent with the $\Lambda p$-scattering data along with the dispersive $\Lambda NN$ or dispersive plus two-pion exchange $\Lambda NN$ forces not only explains $B_{\Lambda}$ of $^5\Lambda$He and $^9\Lambda$Be but also explains the emission of $\gamma$-ray of energy 3.04 MeV from the degenerate doublet to the ground state of $^9\Lambda$Be, ignoring very small spin-orbit force. The $\Lambda\Lambda$ potential constrained by $B_{\Lambda\Lambda}$ of $^6\Lambda\Lambda$He from NAGARA event gives energy of $2^+$ excited state of $^{10}_{\Lambda\Lambda}$Be close to the Demachi-Yanagi event thus confirming the spin and parity assignment $2^+$ of the event suggested from the $\alpha$ cluster model analyses. However, the ground state energy does not agree with the currently accepted experimental value. The partial $A$-body model gives results for $^9\Lambda$Be and $^{10}_{\Lambda\Lambda}$Be consistent with the earlier $\alpha$ cluster model approach. Therefore, our microscopic calculations strongly suggest that a fresh measurement be made for the ground state energy of $^{10}_{\Lambda\Lambda}$Be.

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