Effects of hydraulic uncertainty on slope stability in an embankment dam

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Abstract. It is well-recognized that hydraulic properties are highly variable and heterogeneous suffering from great uncertainty. In this study, the effects of uncertainty of saturated hydraulic conductivity, k_s, on pore water pressure head, displacement, and factor of safety of an embankment dam are studied by Monte Carlo simulation based on 500 random fields. The uncertainty of displacement and pressure head will increase as the uncertainty of k_s increases. The standard deviation of the factor of safety will increase with the increase of the uncertainty of k_s, whereas the mean value is almost unchanged. As the coefficient of variation of k_s increases, the factor of safety gradually tends to a normal distribution.

1. Introduction

It is well-recognized that hydraulic properties are highly variable and heterogeneous suffering from great uncertainty [1]. This hydraulic spatial variability may affect preferential flow paths and pore water pressure, and finally change the deformation and stability of slopes [2, 3]. Slope stability analysis is an essential procedure for predicting the safety of soil embankment due to various loading and environmental conditions. Therefore, it is necessary to study the effects of hydraulic uncertainties on seepage, deformation, and stability of embankment dams.

Several scholars have studied the influence of spatial variability of hydraulic parameters on embankment dams of unsaturated soil. Gui et al employed a probabilistic approach to investigate the effects of seepage through an embankment, and the hydraulic conductivity was modeled as a stationary random field [4]. Cho studied the effect of spatial variability of the saturated hydraulic conductivity on the seepage flow in a two-layered embankment [5]. Tan et al assessed the effects of spatial variability of hydraulic parameters on seepage behavior in an earth dam combining Monte Carlo simulation and random field theory [6]. Calamak & Yanmaz investigated the effects of uncertainty in hydraulic conductivity and van Genuchten parameters on transient seepage through the embankment by stochastic analyses [7]. Yang et al proposed an efficient probabilistic back estimation method for characterization of spatial variability by integration of the Karhunen-Loève expansion method, the Polynomial Chaos Expansion method, and the Markov Chain Monte Carlo method [8]. Liu et al investigated the steady seepage through an embankment on a soil foundation with non-stationary random fields [9]. However, most of the studies generally do not involve slope stability analyses or use simple methods such as...
simplified Bishop method [9]. The influence of hydraulic uncertainty on slope stability is poorly understood.

In this study, the numerical model of an embankment dam is firstly established by Darcy’s Law and Mohr-Coulomb model. Random field theory is adopted to account for the spatial variability of hydraulic parameters. The strength reduction method is developed to compute the factor of safety (FOS) of the slope. The effects of uncertainty of hydraulic conductivity, \( k_s \) on pore water pressure head, displacement, and FOS are studied by Monte Carlo simulation based on 500 random fields of \( k_s \).

2. Methods

2.1. Random field

In this study, \( k_s \) is viewed as a spatial variable to represent hydraulic uncertainty. \( k_s \) can be modeled by random field theory following a log-normal distribution. The mean value \( \mu_{\ln k_s} \) and the standard deviation \( \sigma_{\ln k_s} \) to simulate log-normal distribution can be calculated as:

\[
\sigma_{\ln k_s}^2 = \ln(1 + \frac{\sigma_{k_s}^2}{\mu_{k_s}^2})
\]

\[
\mu_{\ln k_s} = \ln \mu_{k_s} - \frac{1}{2} \sigma_{k_s}^2
\]

where \( \mu_{k_s} \) and \( \sigma_{k_s} \) are the mean value and the standard deviation of \( k_s \), respectively. In this study, the mean value of \( k_s \) is assumed to be \( 5 \times 10^{-5} \) m/s (silty soil).

The correlation length is a parameter to measure the distance over which points of soil samples are significantly correlated in a domain. If the distance of two points is greater than the correlation length, the soil samples at these two points are largely uncorrelated. In probabilistic analysis, the effects of correlation length need to be considered by random field theory, which provides a convenient measure of relevance of soil properties in space [10]. It is reported that the correlation length of \( \ln k_s \) varies from less than 1 m to around 10 m [1]. Therefore, the horizontal and vertical correlation lengths (\( l_x \) and \( l_z \)) of the natural logarithm of \( k_s \), \( \ln k_s \) are assumed to be 10 m and 2 m, respectively. The exponential covariance function \( C(x) \) is used to simulate the spatial variability of \( k_s \):

\[
C(x) = \sigma_{\ln k_s}^2 \exp\left\{-\frac{(x_1 - x_2)^2}{l_x^2} + \frac{(z_1 - z_2)^2}{l_z^2}\right\}
\]

where \( x = [(x_1, z_1), (x_2, z_2)] \) represent the coordinates of the two points in the domain.

2.2. Slope stability analysis by strength reduction

Three steps are used to analyze the slope stability. First, the pore pressure in the soil is modeled by Darcy’s law. In the unsaturated zone, the hydraulic conductivity is the function of soil suction (Figure 1(a)). Second, the Mohr-Coulomb yield function is used to simulate the soil deformation. The angle of internal friction \( \Phi \) is given as:

\[
\Phi = \tan^{-1}\left(\tan\phi_u\right) (p < 0) + \tan^{-1}\left(\tan\phi_i\right) (p \geq 0)
\]

where \( \phi_u \) and \( \phi_i \) are angles of internal friction for unsaturated and saturated soils, and \( p \) is the pore pressure given by Darcy’s law.

To compute the factor of safety (FOS), the strength reduction method is developed in the third step. The Mohr-Coulomb material parameters are functions of FOS [11]:

\[
c' = \frac{c}{FOS}
\]

\[
\Phi' = \tan^{-1}\left(\frac{\tan\phi}{FOS}\right) (p < 0) + \tan^{-1}\left(\frac{\tan\phi}{FOS}\right) (p \geq 0)
\]
where $c$ is the cohesion of soil; $c'$ and $\Phi'$ are the cohesion and angle of internal friction used in Mohr-Coulomb yield function. Decreasing the material parameters results in the reduction of the shear strength of the soil, which eventually leads to the collapse of the embankment. At this moment, the value of FOS is the safety factor of the embankment. In this study, $c$, $\phi_u$, and $\phi_s$ are assumed to be 25 kPa, 30°, and 20°, respectively [12]. The Young's modulus, Poisson's ratio, soil density, and porosity are $5 \times 10^5$ kPa, 0.4, 2000 kg/m$^3$, 0.3, respectively.

### 2.3. Simulation of embankment dam

The numerical model of an embankment dam is simulated by COMSOL Multiphysics. The geometry of the embankment is shown in Figure 1(b), the height of the embankment is 12 m and the width is 55 m. The water level is 10 m and the possible seepage height is 4 m. Therefore, the left and right sides are constant head boundaries and other boundaries are zero-flux boundaries. In the next step to simulate soil deformation, the hydraulic pressure on the left side of the saturated zone of the embankment is used as boundary load combined effect of gravity. No external stresses are applied in regions of unsaturated soil since pores are considered interconnected and at constant atmospheric pressure.

![Figure 1](image.png)
**Figure 1.** Numerical model of an embankment dam (steady-state): (a) Permeability function; (b) Initial and boundary conditions.

### 3. Results and discussions

#### 3.1. Deterministic results

The deterministic results are shown in Figure 2. Figure 2(a) displays the displacement of the embankment. The maximum displacement is over 0.34 m located on the side of the reservoir level. The pressure head in the embankment dam is shown in Figure 2(b). In the saturated zone, the pressure head value varies from top to bottom is 0 m to 10 m. In the unsaturated zone, the maximum matric suction head is around 3 m (negative pressure head).

![Figure 2](image.png)
**Figure 2.** Deterministic results: (a) Total displacement; (b) Pressure head.
The analysis of slope stability by the strength reduction method is shown in Figure 3. The maximum displacement versus FOS is shown in Figure 3(a). The FOS increases from 1.00 to 1.92 with a step of 0.01. The maximum displacement increases significantly at FOS = 1.86 indicating the onset of the collapse of the slope. The plastic strain of FOS = 1.86 is shown in Figure 3(b). The slip surface is well reflected in the figure and the arrows show the direction of the landslide based on the displacement.

![Figure 3. Slope stability by strength reduction method: (a) Factor of safety (FOS); (b) Plastic strain and landslide.](image)

### 3.2. Effects of $k_s$ uncertainty

The coefficient of variation (COV) and standard deviation (SD) are two typical indices to measure the uncertainty of variables. It is generally recognized that $k_s$ is highly heterogeneous and its COV can vary from 2% ~ 160% [1]. Therefore, in this study, the COVs are chosen to be 5%, 40%, and 80% for parametric study. SD is used to illustrate the uncertainty of model responses.

![Figure 4. Effect of COV of $k_s$ on uncertainty of displacement: (a) COV = 5%; (b) COV = 40%; (c) COV = 80%.](images)

Figure 4 shows the effect of COV of $k_s$ on the uncertainty of displacement. As the COV increases, the SD of displacement increases significantly. When the COV of $k_s$ is 80%, the maximum SD of displacement is approximately 0.05 mm, which is around 16% of the deterministic result. In contrast, when the COV is 5%, the maximum SD is only around 0.003 mm. It is indicated that the variation of $k_s$ has a significant influence on the uncertainty of displacement. The uncertainty of displacement will increase as the uncertainty of $k_s$ increases and the uncertainty around the water table is significant.

Figure 5 shows the effect of COV of $k_s$ on the uncertainty of pressure head. The SD of pressure head increases rapidly as the COV increases. When the COV of $k_s$ is 80%, the maximum SD of pressure head is around 0.8 m. It is also proved that the variation of $k_s$ has a significant influence on pressure head and the uncertainty of pressure head will increase as the uncertainty of $k_s$ increases. In addition, it can be seen that the uncertainty of pore pressure is mainly distributed on the right side of the embankment. This may be because the right side is less affected by the boundary conditions.

![Figure 5](image)

**Figure 5.** Effect of COV of $k_s$ on uncertainty of pressure head: (a) COV = 5%; (b) COV = 40%; (c) COV = 80%.

![Figure 6](image)

**Figure 6.** Effect of COV of $k_s$ on distribution of FOS: (a) COV = 5%; (b) COV = 40%; (c) COV = 80%.
Figure 6 shows the effect of COV of $k_s$ on the distribution of FOS. As the COV grows from 5% to 80%, the distributions of FOS become dispersed, which could be illustrated by the SD value of FOS. It ranges from 0.0068 to 0.083. However, the mean value of FOS is around 1.86 constantly. In addition, as the COV increases, the distribution of FOS could be better fitted by a normal distribution. To conclude, the uncertainty of FOS will increase with the increase of the uncertainty of $k_s$, whereas the mean value of FOS is almost unchanged. Besides, as COV of $k_s$ increases, FOS gradually tends to a normal distribution.

4. Conclusions
In this study, random field theory is adopted to account for the spatial variability of hydraulic parameters. The effects of uncertainty of saturated hydraulic conductivity, $k_s$ on pore water pressure head, displacement, and FOS are studied by Monte Carlo simulation based on 500 random fields of $k_s$. However, this study only uses the pore pressure of the seepage field as the boundary condition of the stress field to realize the interaction of water and soil, and it is not a fluid-solid coupling in the true sense. The fluid-solid coupling problem of embankment stability analysis needs further study. Major conclusions were summarized below:

1. The uncertainty of displacement and pressure head will increase as the uncertainty of $k_s$ increases. When the COV of $k_s$ is 80%, the maximum SD of displacement is approximate 0.05 mm, and of pressure head is around 0.8 m. It is suggested that the design of embankment needs to be more conservative when the uncertainty of $k_s$ is significant.

2. The uncertainty of FOS will increase with the increase of the uncertainty of $k_s$, whereas the mean value of FOS is almost unchanged. As COV of $k_s$ increases, FOS gradually tends to a normal distribution.

Therefore, the distribution of FOS can be determined by low-order statistics to be understood by engineers when the variation of $k_s$ is high.

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