Two-loop splitting in double parton distributions

Jonathan Gaunt (CERN)
[arXiv:1812.xxxxx], …
In collaboration with Markus Diehl, Peter Plößl, Andreas Schäfer

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In double parton scattering (DPS), two partons from a proton could have arisen as a result of one parton perturbatively splitting into (at least) two: $1 \to 2$ mechanism.

This will be dominant contribution at small (perturbative) transverse separation between partons, $y$.

All-order form of double parton distribution $F$ at small $y$ is:

$$F_{a_1 a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \sum_{a_0} \int \frac{dz}{z^2} V_{a_1 a_2, a_0} \left( \frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log \frac{\mu^2 y^2}{b_0^2} \right) f_{a_0}(z, \mu)$$

Overall $1/y^2$ dependence

This $1/y^2$ dependence causes a power divergence when naïve formulation of DPS cross section is used: $\int d^2y F(y)F(y)$. Related to leaking of DPS into SPS region.
Recap of DPS framework of Diehl, Gaunt, Schönwald (DGS)

Use double parton distributions (DPDs) in $y$ space, insert cut-off into $y$ integration:

$$\sigma_{\text{DPS}} = \int d^2 y \Phi^2(\nu y) F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$

Cuts off integral for $y \lesssim 1/\nu$, regulates power divergence

Use subtraction term in sum of SPS and DPS to avoid double counting:

$$\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$$

$\nu$ dependence cancelled order by order

DPS cross section with both DPDs replaced by fixed order splitting expression

$F$ must reduce to perturbative expression at small $y$. When modelling $F$ we used a sum of two terms:

$$F = F_{\text{split}} + F_{\text{non-split}}$$

$$F_{\text{perturb.}}(y^*) e^{-y^2 \Lambda^2} \quad \text{with} \quad 1/y^2 = 1/y^2 + 1/y_{\text{max}}^2$$

$$f(x_1; \mu_0) f(x_2; \mu_0) \Lambda^2 e^{-y^2 \Lambda^2} / \pi$$
NLO corrections to DPS

Some key advantages of the DGS framework
• Can be formulated at all orders, with corrections that can be practicably computed. Opens the way for NLO calculations of DPS!
• Makes maximal use of existing SPS quantities.

What perturbative ingredients do we need for NLO DPS cross sections?
• NLO corrections to partonic cross sections: already known for many processes from SPS calculations ✓
• NLO ‘usual’ splitting functions - needed for evolution of $F(y)$: already known since the 80s ✓
• NLO corrections to the splitting (i.e. NLO $V$): not yet known ×

$F_{a_1a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \sum_{a_0} \int_{x_1 + x_2}^{1} \frac{dz}{z^2} V_{a_1a_2, a_0} \left( \frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log \frac{\mu^2 y^2}{b_0^2} \right) f_{a_0}(z, \mu)$

In this talk: computation of $V$ at NLO.
One can also consider $\Delta$-space DPDs, where all divergences regularised using $\text{dimreg} + \overline{\text{MS}}$, and compute matching onto PDFs:

$$F_{a_1a_2}(x_1, x_2, \Delta, \mu) = \sum_{a_0} \int_{x_1 + x_2}^1 \frac{dz}{z^2} W_{a_1a_2,a_0} \left( \frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log \frac{\mu^2}{\Delta^2} \right) f_{a_0}(z, \mu)$$

Evolution of $\Delta$-space DPDs involves an inhomogeneous $1 \to 2$ splitting term:

$$\frac{d}{d \ln \mu^2} F_{a_1a_2}(x_1, x_2, \Delta, \mu) = \sum_{a_0} \int_{x_1 + x_2}^1 \frac{dz}{z^2} P_{a_1a_2,a_0} \left( \frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu) \right) f_{a_0}(z, \mu) + \{\text{homogeneous terms}\},$$

We compute also $W$ and $P$ at NLO.
For our purpose: $W$s are needed to link our $y$-space DPDs to $\overline{\text{MS}}$ $\Delta$-space DPDs, latter of which satisfy momentum and number sum rules at $\Delta = 0$. Allows us to check to what extent our models for $F(y)$ satisfy the sum rules, and construct improved models.

[Gaunt, Stirling JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, arXiv:1811.00289]

$$F_{\Phi}^{a_1a_2}(x_1, x_2; \Delta = 0; \mu, \nu) = \int d^2y \Phi(y) F_{\Phi}^{a_1a_2}(x_1, x_2, y; \mu)$$

$$F_{\overline{\text{MS}}}^{a_1a_2}(x_1, x_2, \Delta = 0; \mu) = F_{\Phi}^{a_1a_2}(x_1, x_2, \Delta = 0; \mu, \nu) + F_{\text{match}}^{a_1a_2}(x_1, x_2; \mu, \nu)$$

Satisfy momentum and number sum rules (see talk by Peter tomorrow)

In this talk, I’ll focus on computation of matching coefficients and splitting functions for colour-singlet, unpolarised DPDs, for all parton channels. These will be made available shortly in [arXiv:1812.xxxxx].

[We are also computing the polarised + colour interference channels].
We initially compute bare $\Delta$-space DPDs at $\mathcal{O}(\alpha_s^2)$ in a partonic state $a_o$: $F_B^{(1)}(\Delta)$

Fourier transform in $2 - 2\epsilon$ dimensions

Renormalise, extract matching coefficient

$W^{(1)}(\Delta)$ (from $\epsilon^0$ part of $F_B^{(1)}$)

$P^{(1)}$ (from $\epsilon^{-1}$ part of $F_B^{(1)}$)

Renormalise, extract matching coefficient

$V^{(1)}(y)$ (from $\epsilon^{-1}$ part of $F_B^{(1)}$)

Dimensional analysis $\Rightarrow$ $F_B^{(n)}$ depends on $\Delta$ like this

$\int \frac{d^{2-2\epsilon} \Delta}{(2\pi)^{2-2\epsilon}} e^{-i\Delta y} \left( \frac{\mu}{\Lambda} \right)^{2\epsilon n} = \frac{\Gamma(1-\epsilon)}{(\pi y^2)^{1-\epsilon}} \left( \frac{y\mu}{b_0} \right)^{2\epsilon n} n^e T_{\epsilon,n}$

$T_{\epsilon,n} = 1 + \zeta_2 n\epsilon^2 + \ldots$
In light-cone gauge, graphs to compute:

+ virtuals
Compute graph expressions (FORM, FeynCalc).
Integrate over minus components using contours.

\[ D_1 = \frac{(k_1 + \Delta)^2}{x_1} + \frac{(k_2 - \Delta)^2}{x_2} + \frac{(k_1 + k_2)^2}{x_3} \]
\[ D_3 = (k_1 + \Delta)^2 \]
\[ D_4 = k_2^2 \]
\[ D_5 = (k_1 + k_2)^2 \]

\[ I_1(a_1, a_2, a_3, a_4) = \int \frac{d^{d-2}k_1 d^{d-2}k_2}{\prod_{i=1,4} D_i^{\alpha_i}} \]
\[ I_2(a_1, a_2, a_3, a_4, a_5) = \int \frac{d^{d-2}k_1 d^{d-2}k_2}{\prod_{i=1,4} D_i^{\alpha_i} \prod_{i=4,5} D_i^{\beta_i}} \]

Integration-by-parts reduction to master integrals (LiteRed)

[Lee, J. Phys. Conf. Ser. 523 (2014)]

Construct differential equations in \( x_1 \) and solve (Fuchsia)

\[ \begin{bmatrix}
\frac{\partial I_1(1,1,0,0)}{\partial x_1} \\
\frac{\partial I_1(0,1,1,0)}{\partial x_1} \\
\frac{\partial I_1(1,1,1,0)}{\partial x_1} \\
\frac{\partial I_1(1,0,1,1)}{\partial x_1} \\
\frac{\partial I_1(2,1,1,1)}{\partial x_1}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
\bullet & 0 & 0 & 0 & 0 \\
\bullet & \bullet & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet & \bullet
\end{bmatrix}
\]

Results for bare graphs!

Computation of \( x_3 \to 0 \) limit of master integrals using method of regions (boundary conditions)

\[ I_1(0, 1, 1, 0) \to x^{3-2\epsilon} x_3^{-\epsilon} (x_1 x_2)^{\epsilon} \frac{\Gamma[-\epsilon]}{\sin[2\pi \epsilon] \Gamma[1 - 3\epsilon]} \]
Cross-checks

- Full computation of bare graphs done using light-cone and covariant Feynman gauge ✓
- Master integrals satisfy differential equation in $x_2$ ✓
- Master integrals all checked numerically at 10 random points using FIESTA ✓
- Individual graphs have poles in $\epsilon$ up to $\epsilon^{-3}$, as well as rapidity divergences. $\epsilon^{-3}$ pole + rapidity divergences cancel after summing over graphs, $\epsilon^{-2}$ pole is as predicted by renormalisation group equation ✓
- Splitting functions $P_{a_1a_2,a_0}^{(1)}$ satisfy constraints related to number and momentum sum rules:

$$
\int_{0}^{1-x_1} dx_2 \left[ P_{a_1q,a_0}(x_1,x_2) - P_{a_1\bar{q},a_0}(x_1,x_2) \right] = \left( \delta_{a_1\bar{q}} - \delta_{a_1q} - \delta_{a_0\bar{q}} + \delta_{a_0q} \right) P_{a_1a_0}(x_1),
$$

$$
\sum_{a_2} \int_{0}^{1-x_1} dx_2 x_2 P_{a_1a_2,a_0}(x_1,x_2) = (1-x_1) P_{a_1a_0}(x_1) \quad ✓
$$

[Gaunt, Stirling JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, arXiv:1811.00289]
Small $x_1, x_2$ limit

Interesting processes/regions for studying DPS typically involve small $x$ values (higher density of partons $\rightarrow$ greater chance of DPS, plus smaller $Q$ such that power suppression is reduced).

$\rightarrow$ Interesting to study matching coefficients and splitting functions in limits of small $x_i$. For example, small $x_1, x_2$ limit of $P_{gg,g}^{(1)}(x_1, x_2)$:

$$P_{gg,g}^{(1)}(x_1, x_2) \rightarrow \frac{C_A^2 \left( (1 - 6u + 6u^2) + \left( 8 - \frac{2}{u} - 4u + 4u^2 \right) \log[1 - u] + \{u \leftrightarrow 1 - u\} \right)}{x^2}$$

Same $1/x^2$ behaviour for other splitting functions, and $V$ kernels

$$V^{(1)}(x_1, x_2) \sim 1/x^2 \Rightarrow F(x_1, x_2, y) \sim \alpha_s^{n+2} \log^{n+1}(x)/x$$

(for NLO splitting)

i.e. NLL in small $x$ logarithms!

$$[V^{(1)}(x_1, x_2) \sim \log(x)/x^2 \Rightarrow F(x_1, x_2, y) \sim \alpha_s^{n+2} \log^{n+2}(x)/x, \text{ i.e. LL}]$$

Similar of usual splitting functions, where $P^{(1)}(x) \sim 1/x$ and not $\log(x)/x$. 

\hspace{1cm}
Comparison to other results in the literature

Various $1 \to 2$ splitting functions have been computed in the literature. Are they the same as our $P^{(1)}_{a_1a_2,a_0}$ functions?

Fracture functions

$$\frac{\partial M_{i,h/P}^r(\xi, \zeta, M^2)}{\partial \log M^2} = \frac{\alpha_s(M^2)}{2\pi} \left[ \int_{\xi}^{1} \frac{du}{u} \left[ P_{i\leftarrow j}^{(0)}(u) + \frac{\alpha_s(M^2)}{2\pi} P_{i\leftarrow j}^{(1)}(u) \right] \right] M_{j,h/P}^r \left( \frac{\xi}{u}, \zeta, M^2 \right)$$

$$+ \frac{\alpha_s(M^2)}{2\pi} \frac{1}{\xi} \int_{\xi}^{\xi+\zeta} \frac{du}{u} \int_{\xi}^{\frac{1-u}{u}} \frac{dv}{v} \left[ \tilde{P}_{ki\leftarrow j}^{(0)}(u,v) + \frac{\alpha_s(M^2)}{2\pi} P_{ki\leftarrow j}^{(1)}(u,v) \right] f_{j|h/P}^r \left( \frac{\xi}{u}, M^2 \right) D_{h/k}^r \left( \frac{\zeta}{v}, M^2 \right)$$

Some of these NLO functions computed in Daleo, Sassot [Nucl. Phys. B673 (2003) 357-384] + Garcia Canal [Nucl. Phys. B662 (2003) 334-358]

Suggested by Ceccopieri [Phys. Lett. B697 (2011) 482-487] that after a simple transformation, these functions are equal to DPS $P^{(1)}_{a_1a_2,a_0}$

We find that this is not the case – in fact we observe that above $P^{(1)}$'s are not symmetric under $x_1 \leftrightarrow x_2$
Comparison to other results in the literature

**Di-hadron fragmentation functions**

\[ D_{a_1 a_2, i}(x_1, x_2, Q^2) = \sum_{b_1, b_2, j} \int Y_0^Y dy \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} \frac{1}{z_1 + z_2} \]

\[ \times D_{a_1 b_1}(x_1/z_1, y) D_{a_2 b_2}(x_2/z_2, y) \]

\[ \times \hat{P}_{j\rightarrow b_1 b_2}(z_1/(z_1 + z_2)) D_{ji}(z_1 + z_2, Y - y) , \]

[Konishi, Ukawa, Veneziano, Nucl.Phys. B157 (1979) 45-107]

NLO ‘two-body decay probabilities’ computed by
Kalinowski, Konishi, Scharbach, Taylor, Nucl. Phys. B181 (1981) 253-276, Gunion, Kalinowski, Szymanowski, Phys. Rev. D32 (1985) 2303-2321

Generally different from our \( P_{a_1 a_2 a_0}^{(1)} \) - an exception is the non-singlet contribution to \( P_{qq,q}^{(1)} \). Likely because this is a very simple process: one Feynman diagram, and no subdivergences.
• NLO matching of DPDs onto PDFs, and NLO $1 \to 2$ splitting functions, computed in unpolarised colour-singlet case ✓

• Corresponding matching coefficients to come for polarised + colour-nonsinglet channels.

• Then numerics!
  • Look at effect of NLO corrections on DPD $y$-profiles, parton luminosities, cross sections, etc.
  • Investigate perturbative convergence of DPS cross sections
  • Look for observables where we might be able to detect differences between LO and NLO predictions.
