CONSTRaining UNIversal EXTRA DImENSIONS THROUGH B DECAYS

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We analyze the exclusive rare $B \to K^{(*)}\ell^+\ell^-$, $B \to K^{(*)}\nu\bar{\nu}$ and $B \to K^*\gamma$ decays in the Applequist-Cheng-Dobrescu model, an extension of the Standard Model in presence of universal extra dimensions. In the case of a single universal extra dimension, we study the dependence of several observables on the compactification parameter $1/R$, and discuss whether the hadronic uncertainty due to the form factors obscures or not such a dependence. We find that, using present data, it is possible in many cases to put a sensible lower bound to $1/R$, the most stringent one coming from $B \to K^*\gamma$.

Keywords: Rare B decays; Universal Extra Dimensions.

1. Introduction

Rare B decays induced by $b \to s$ transition play a peculiar role in searching for new Physics, being induced at loop level and hence suppressed in the Standard Model (SM). They can also be useful to constrain extra dimensional scenarios. This is the case of the Applequist-Cheng-Dobrescu (ACD) model in which universal extra dimensions are considered, which means that all the fields are allowed to propagate in all available dimensions. In the case of a single extra dimension compactified on a circle of radius R, Tevatron run I data allow to put the bound $1/R \geq 300$ GeV. To be more general, we analyze a broader range $1/R \geq 200$ GeV.

In Refs. 4, 5 the effective hamiltonian relative to inclusive $b \to s$ decays was computed within the ACD model. In this paper, we summarize the results obtained in Ref. 6 for exclusive $b \to s$-induced modes. In this case, the uncertainty in the form factors must be considered, since it can overshadow the sensitivity to the compactification parameter $1/R$. Indeed we find that computing the branching ratios of $B \to K^{(*)}\ell^+\ell^-$ and the forward-backward lepton asymmetry in $B \to K^*\ell^+\ell^-$ for a representative set of
form factors, a bound can be put. We also study the modes $B \to K^{(*)} \nu \bar{\nu}$, for which no signal has been observed, so far, and the $BR(B \to K^{*} \gamma)$ versus $1/R$, which allows to establish the most stringent bound on $1/R$.

2. The ACD model with a single UED

The ACD model\textsuperscript{3} consists in the minimal extension of the SM in $4 + \delta$ dimensions; we consider $\delta = 1$. The fifth dimension $x_5 = y$ is compactified to the orbifold $S^1/Z_2$, i.e. on a circle of radius $R$ and runs from 0 to $2\pi R$ with $y = 0, y = \pi R$ fixed points of the orbifold. Hence a field $F(x, y)$ (denoting the usual $3+1$ coordinates) would be a periodic function of $y$, and it could be expressed as $F(x, y) = \sum_{n=-\infty}^{+\infty} F_n(x) e^{in y/R}$. If $F$ is a massless boson field, the KK modes $F_n$ obey the equation $(\partial^\mu \partial_\mu + n^2/R^2) F_n(x) = 0$, $\mu = 0, 1, 2, 3$ so that, apart the zero mode, they behave in four dimensions as massive particles with $m_n^2 = (n/R)^2$. Under the parity transformation $P_5 : y \to -y$ fields having a correspondent in the 4-d SM should be even, so that their zero mode in the expansion is interpreted as the ordinary SM field. On the other hand, fields having no SM partner should be odd, so that they do not have zero modes.

Important features of the ACD model are: i) there is a single additional free parameter with respect to the SM, the compactification radius $R$; ii) conservation of KK parity, with the consequence that there is no tree-level contribution of KK modes in low energy processes (at scales $\mu \ll 1/R$) and no production of single KK excitation in ordinary particle interactions. A detailed description of this model is provided in Ref. 4.

3. Decays $B \to K^{(*)} \ell^+ \ell^-$

In the Standard Model the effective $\Delta B = -1, \Delta S = 1$ Hamiltonian governing the transition $b \to s \ell^+ \ell^-$ is $H_W = 4 G_F V_{td} V_{ts}^{*} \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$. $G_F$ is the Fermi constant and $V_{ij}$ are elements of the Cabibbo-Kobayashi-Maskawa mixing matrix; we neglect terms proportional to $V_{ub} V_{us}^{*}$, $O_1$, $O_2$ are current-current operators, $O_3, ..., O_6$ QCD penguins, $O_7$, $O_8$ magnetic penguins, $O_9$, $O_{10}$ semileptonic electroweak penguins. We do not consider the contribution to $B \to K^{(*)} \ell^+ \ell^-$ with the lepton pair coming from $c \bar{c}$ resonances, mainly due to $O_1$, $O_2$. We also neglect QCD penguins whose coefficients are very small compared to the others. Therefore, in the case of the modes $B \to K^{(*)} \ell^+ \ell^-$, the relevant operators are: $O_7 = \frac{\epsilon}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$, $O_9 = \frac{\epsilon^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \ell^{\mu} \ell$, $O_{10} =$
\[
\frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_{L\alpha}) \bar{\ell}_\mu \gamma_5 \ell. \]
Their coefficients have been computed at NNLO in the Standard Model\textsuperscript{7} and at NLO for the ACD model\textsuperscript{4,5}: we use these results in our study. No new operators are found in ACD, while the coefficients are modified because particles not present in SM can contribute as intermediate states in loop diagrams. As a consequence, they are expressed in terms of functions \(F(x_t, 1/R)\), \(x_t = m_t^2/M_W^2\), generalizing the corresponding SM functions \(F_0(x_t)\) according to \(F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n)\), where \(x_n = m_n^2/M_W^2\) and \(m_n = n/R\textsuperscript{4,5}\). For large values of \(1/R\) the SM phenomenology should be recovered, since the new states, being more and more massive, decouple from the low-energy theory.

The exclusive \(B \to K^{(*)}\ell^+\ell^-\) modes involve the matrix elements of the operators in the effective hamiltonian between the \(B\) and \(K\) or \(K^*\) mesons, for which we use the standard parametrization in terms of form factors:

\[
\langle K(p')|\bar{s} \gamma_\mu b|B(p)\rangle = (p + p')_\mu F_1(q^2) + \frac{M_B^2 - M_K^2}{q^2} q_\mu \left( F_0(q^2) - F_1(q^2) \right);
\]

\[
\langle K(p')|\bar{s} i\sigma_{\mu\nu}q^\nu b|B(p)\rangle = \left[ (p + p')_\mu q^2 - (M_B^2 - M_K^2) q_\mu \right] \frac{F_T(q^2)}{M_B + M_K};
\]

\[
\langle K^*(p', \epsilon)|\bar{s} \gamma_\mu (1 - \gamma_5) b|B(p)\rangle = \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu\rho\sigma} p^\alpha p^\beta \cdot 2V(q^2) \frac{A_2(q^2)}{M_B + M_K^*} - \left( \epsilon^* \cdot q \right) \frac{2M_{K^*}}{q^2} \left( A_2(q^2) - A_0(q^2) \right) q_\mu ;
\]

\[
\langle K^*(p', \epsilon)|\bar{s} \sigma_{\mu\nu}q^\nu \frac{(1 + \gamma_5)}{2} b|B(p)\rangle = i\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu\rho\sigma} p^\alpha p^\beta \cdot 2T_1(q^2) + \left[ \epsilon^*_{\mu} \frac{M_B^2 - M_K^*}{2M_{K^*}} - \left( \epsilon^* \cdot q \right) (p + p')_\mu \right] T_2(q^2) + \left( \epsilon^* \cdot q \right) \left[ q_\mu - \frac{q^2}{M_B^2 - M_K^*} (p + p')_\mu \right] T_3(q^2),
\]

where \(q = p - p'\), \(A_3(q^2) = \frac{M_B + M_{K^*}}{2M_{K^*}} A_1(q^2) - \frac{M_B - M_{K^*}}{2M_{K^*}} A_2(q^2)\) with the conditions \(F_1(0) = F_2(0)\), \(A_3(0) = A_0(0)\), \(T_1(0) = T_2(0)\).

We use two sets of form factors: the first one (set A) obtained by three-point QCD sum rules based on the short-distance expansion\textsuperscript{8}; the second one (set B) obtained by light-cone QCD sum rules\textsuperscript{9}. For both sets we include in the numerical analysis the errors on the parameters.
In Fig. 1 we plot, for the two sets of form factors, the branching fractions relative to $B \to K(\ast)\ell^+\ell^-$ versus $1/R$ and compare them with the experimental data provided by BaBar$^{10,11}$ and Belle$^{12,13}$:

$$BR(B \to K\ell^+\ell^-) = (5.50 \pm 0.75 \pm 0.27 \pm 0.02) \times 10^{-7} \quad (Belle)$$
$$= (3.4 \pm 0.7 \pm 0.3) \times 10^{-7} \quad (BaBar)$$
$$BR(B \to K^*\ell^+\ell^-) = (16.5 \pm 2.3 \pm 0.9 \pm 0.4) \times 10^{-7} \quad (Belle)$$
$$= (7.8 \pm 1.9 \pm 1.2) \times 10^{-7} \quad (BaBar).$$

Set B excludes $1/R \leq 200$ GeV. Improved data will resolve the discrepancy between the experiments and increase the lower bound for $1/R$.

In the case of $B \to K^*\ell^+\ell^-$ the investigation of the forward-backward asymmetry $A_{fb}$ in the dilepton angular distribution may also reveal effects beyond the SM. In particular, in SM, due to the opposite sign of the coefficients $C_7$ and $C_9$, $A_{fb}$ has a zero the position of which is almost independent of the model for the form factors$^{14}$. Let $\theta_L$ be the angle between the $\ell^+$ direction and the $B$ direction in the rest frame of the lepton pair (we consider massless leptons). We define:

$$A_{fb}(q^2) = \frac{\int_0^1 dq^2 \int d\cos\theta_L dq^2 \int d\cos\theta_L - \int_{-1}^0 dq^2 \int d\cos\theta_L dq^2 \int d\cos\theta_L}{\int_0^1 dq^2 \int d\cos\theta_L dq^2 \int d\cos\theta_L + \int_{-1}^0 dq^2 \int d\cos\theta_L dq^2 \int d\cos\theta_L}. \quad (2)$$

We show in Fig. 2 the predictions for the SM, $1/R = 250$ GeV and $1/R = 200$ GeV. The zero of $A_{fb}$ is sensitive to the compactification parameter, so that its experimental determination would constrain $1/R$. At present, the analysis performed by Belle Collaboration indicates that the relative sign of $C_9$ and $C_7$ is negative, confirming that $A_{fb}$ should have a zero$^{15}$.

4. The decays $B \to K(\ast)\nu\bar{\nu}$

In the SM the effective Hamiltonian governing $b \to s\nu\bar{\nu}$ induced decays is

$$H_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2(\theta_W)} V_{ts}V_{tb}^* \eta_X X(x_t) \bar{b} \gamma^\mu (1 - \gamma_5)s \bar{\nu} \gamma_\mu (1 - \gamma_5)\nu \quad (3)$$

obtained from $Z^0$ penguin and box diagrams dominated by the intermediate top quark. In (3) $\theta_W$ is the Weinberg angle. We put to unity the QCD factor $\eta_X$. The function $X$ was computed in Refs. 16, 17, 18 in the SM and in the ACD model in Refs. 4, 5.

$B \to K(\ast)\nu\bar{\nu}$ decays have been studied within the SM$^{20,21}$, while in Ref. 6 the dependence of $BR(B \to K\nu\bar{\nu})$ and $BR(B \to K^*\nu\bar{\nu})$ on $1/R$
has been derived. However, only an experimental upper bound exists for 
\[ B \to K\nu\bar{\nu}; \quad BR(B^- \to K^-\nu\bar{\nu}) < 3.6 \times 10^{-5} \] (90% CL) \(^{22}\), 
\[ BR(B^- \to K^-\nu\bar{\nu}) < 5.2 \times 10^{-5} \] (90% CL)\(^{23}\), furthermore the 1/R dependence turns
out to be too mild for distinguishing values above 1/R \(\geq 200\) GeV.
5. The decay $B \rightarrow K^*\gamma$

The transition $b \rightarrow s\gamma$ is described by the operator $O_7$. The most recent measurements for the exclusive branching fractions are $24, 25$:

\[
BR(B^0 \rightarrow K^{*0}\gamma) = (4.01 \pm 0.21 \pm 0.17) \times 10^{-5} \ (Belle) \\
= (3.92 \pm 0.20 \pm 0.24) \times 10^{-5} \ (BaBar) \\
BR(B^- \rightarrow K^{*-}\gamma) = (4.25 \pm 0.31 \pm 0.24) \times 10^{-5} \ (Belle) \\
= (3.87 \pm 0.28 \pm 0.26) \times 10^{-5} \ (BaBar)
\]

In Fig. 3 the branching ratio computed in the ACD model is plotted versus $1/R$: the sensitivity to the this parameter is evident; a lower bound of $1/R \geq 250$ GeV can be put adopting set A, and a stronger bound of $1/R \geq 400$ GeV using set B, which is the most stringent bound that can be currently put on this parameter from the $B$ decay modes we have considered.

![Fig. 3. BR($B \rightarrow K^*\gamma$) versus $1/R$ using set A (left) and B (right) of form factors. The horizontal band corresponds to the experimental result.](image)

6. Conclusions and Perspectives

We have shown how the predictions for $B \rightarrow K^{(*)}\ell^+\ell^-$, $B \rightarrow K^{(*)}\nu\bar{\nu}$, $B \rightarrow K^*\gamma$ decays are modified within the ACD scenario. The constraints on $1/R$ are slightly model dependent, being different using different sets of form factors. Nevertheless, various distributions, together with the lepton forward-backward asymmetry in $B \rightarrow K^*\ell^+\ell^-$ are very promising in order to constrain $1/R$, the most stringent lower bound coming from $B \rightarrow K^*\gamma$. Improvements in the experimental data, expected in the near future, will allow to establish more stringent constraints for the compactification radius.
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