Quiet SDS’ Josephson Junctions for Quantum Computing

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Unconventional superconductors exhibit an order parameter symmetry lower than the symmetry of the underlying crystal lattice. Recent phase sensitive experiments on YBa$_2$Cu$_3$O$_7$ single crystals have established the d-wave nature of the cuprate materials, thus identifying unambiguously the first unconventional superconductor [1]. The sign change in the order parameter can be exploited to construct a new type of s-wave–d-wave–s-wave Josephson junction exhibiting a degenerate ground state and a double-periodic current–phase characteristic. Here we discuss how to make use of these special junction characteristics in the construction of a quantum computer. Combining such junctions together with a usual s-wave link into a SQUID loop we obtain what we call a ‘quiet’ qubit — a solid state implementation of a quantum bit which remains optimally isolated from its environment.

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Quantum computers take advantage of the inherent parallelism of the quantum state propagation, allowing them to outperform classical computers in a qualitative manner. Although the concept of quantum computation has been introduced quite a while ago [2], widespread interest has developed only recently when specific algorithms exploiting the character of coherent state propagation have been proposed [3]. Here we deal with the device aspect of quantum computers, which is flourishing in the wake of the recent successes achieved on the algorithmic side. Two conflicting difficulties have to be faced by all hardware implementations of quantum computation: while the computer must be scalable and controllable, the device should be almost completely detached from the environment during operation in order to minimize phase decoherence. The most advanced propositions are based on trapped ions [4,5], photons in cavities [7], NMR spectroscopy of molecules [8], and various solid state implementations based on electrons trapped in quantum dots [9], the Coulomb blockade in superconducting junction arrays [10,11], or the flux dynamics in Superconducting Quantum Interference Devices (SQUIDs) [12]. Nanostructured solid state quantum gates offer the attractive feature of large scale integrability, once the limitations due to decoherence can be overcome [13].

Here we propose a new device concept for a (quantum) logic gate exploiting the unusual symmetry properties of unconventional superconductors. The basic idea is sketched in Fig. 1: connecting the positive (100) and negative (010) lobes of a d-wave superconductor with a s-wave material produces the famous π-loop with a current carrying ground state characteristic of d-wave symmetry [14]. Here we make use of an alternative geometry and match the s-wave superconductors (S) to the (110) boundaries of the d-wave (D) material. As a consequence, the usual Josephson coupling ∝ (1 − cos ϕ) vanishes due to symmetry reasons and we arrive at a bistable device, where the leading term in the coupling takes the form $E_d \cos 2\phi$ with minima at $\phi = \pm \pi/2$ (here, $\phi$ denotes the gauge invariant phase drop across the junction). In our design we need the minima at the positions $\phi = 0, \pi$ — the necessary shift is achieved by going over to an asymmetric SDS’ junction with a large DS’ coupling, see Fig. 1. The static DS’ junction shifts the minima of the active SD junction by the desired amount $\phi = \pm \pi/2$. A similar double-periodic junction has recently been realized by combining two d-wave superconductors oriented at a 45° angle [14].

The ground states of our SDS’ junction are degenerate and carry no current, while still being distinguishable from one another: e.g., connecting the junction to a large inductance loop, the π state is easily identified through the induced current. It is this double-periodicity and the associated degeneracy in the ground state of the SDS’ junction which we want to exploit here for quantum com-

![FIG. 1. Geometrical arrangements between s-wave and d-wave superconductors producing a π-loop (used in the phase sensitive experiment by Wollman et al. [1]) and a qubit, the basic building block of a quantum computer.](image-url)
putation: combining the SDS’ junction, a capacitor, and a conventional s-wave junction into a SDS’ SQUID loop, we construct a bistable element which satisfies all the requirements for a qubit, the basic building block of a quantum computer. Below we give a detailed account of the operational features of our device.

Consider a small-inductance ($L$) SQUID loop with $I_s L \ll \Phi_0$, where $I_s$ denotes the (Josephson) critical current of the loop and $\Phi_0 = h c / 2 e$ is the flux unit. Such a loop cannot trap magnetic flux ($\Phi = 0$) and the gauge invariant phase differences $\phi_1$ and $\phi_2$ across the two junctions are slaved to each other, as the uniqueness of the wave function requires that $\phi_1 - \phi_2 = 2 \pi \Phi / \Phi_0$, see [15]. Combining a SDS’ junction with a coupling energy $E_c$ and a conventional s-wave junction (coupling $E_s$) into a SDS’ SQUID loop, we obtain a potential energy

$$V(\phi) = E_c(1 - \cos 2\phi) + E_s(1 - \cos \phi),$$  \hspace{1cm} (1)

exhibiting two minima at $\phi = 0, \pi$, see Fig. 2. The switch $s$ allows us to manipulate their energy separation, choosing between minima which are either degenerate or separated by $2E_s$.

In the quantum case, the phase fluctuates as a consequence of the particle–phase duality [15]. The phase fluctuations are driven by the electrostatic energy required to move a Cooper pair across the junction and are described by the kinetic energy $T(\phi) = (\hbar / 2 e) C \dot{\phi}^2 / 2$, where $C$ denotes the loop capacitance. The dynamics of $\phi$ is manipulated by inserting a large switchable (switch $c$) capacitance $C_{\text{ext}}$ into the loop acting in parallel with the capacitances $C_d$ and $C_s$ of the d- and s-wave junctions. Note that the Lagrangean $L = T - V$ of our loop is formally equivalent to that of a particle with ‘mass’ $m \propto C$ moving in the potential $V(\phi)$.

With the switch settings $c$ on and $s$ off, see Fig. 2(a), the loop capacitance is large and the junction exhibits a doubly degenerate ground state which we characterize via the phase coordinate $\phi$, $|0\rangle$ and $|\pi\rangle$. Closing the switch $s$, see Fig. 2(b), the degeneracy is lifted and while $|0\rangle$ becomes the new ground state, the $|\pi\rangle$-state is shifted upwards by the energy $2E_s$ of the s-wave junction, the latter being frustrated when $\phi = \pi$. On the other hand, opening the switch $c$, see Fig. 2(c), completely isolates the d-wave junction and leads to the new ground and excited states $|\pm\rangle = (|0\rangle \pm |\pi\rangle) / \sqrt{2}$ separated by the tunneling gap $2\Delta_d$. The latter relates to the barrier $2E_d$ and the capacitance $C_d$ of the d-wave junction via $\Delta_d \propto E_d \exp(-2\sqrt{C_d E_d / \hbar})$. Closing the switch $c$, the capacitance is increased by $C_{\text{ext}}$ and the tunneling gap is exponentially suppressed. Using the above three settings, we perform all the necessary single qubit operations:

**Idle-state:** The switch settings $c$-on and $s$-off define the qubit’s idle-state. While the large capacitance $C_{\text{ext}}$ inhibits tunneling, the degeneracy of $|0\rangle$ and $|\pi\rangle$ guarantees a parallel time evolution of the two states. This idle-state is superior to other designs, where the two states of the qubit have different energies and one has to keep track of the relative phase accumulated between the basis states.

**Phase shifter:** Closing the switch $s$ separates the energies of the basis states $|0\rangle$ and $|\pi\rangle$ by an amount $2E_s$. Using a spinor notation for the two-level system, the relative time evolution of the two states is described by the Hamiltonian $\mathcal{H}_s = -E_s \sigma_z$, with $\sigma_z$ a Pauli matrix. Keeping the switch $s$ on during the time $t$, the time evolution of the two states is given by the unitary rotation $u_z(\varphi) = \exp(-i \sigma_z \varphi / 2)$ with $\varphi = -2E_s t / \hbar$.

**Amplitude shifter:** Assume we have prepared the loop in the ground state $|0\rangle$ and wish to produce a superposition by shifting some weight to the $|\pi\rangle$ state. Opening the switch $c$ in the loop, see Fig. 2(c), the time evolution generated by the Hamiltonian $\mathcal{H}_d = \Delta_d \sigma_x$ of the open loop induces the rotation $u_x(\vartheta) = \exp(-i \sigma_x \vartheta / 2)$ with $\vartheta = 2\Delta_d t / \hbar$. The system then oscillates back and forth between $|0\rangle$ and $|\pi\rangle$ with frequency $\omega = \Delta_d / \hbar$ and keeping the switch $c$ open for an appropriate time interval $t$ we obtain the desired shift in amplitude (note that the qubit remains isolated from the environment during these Rabi oscillations).

Imposing the condition $E_d \gg E_s$, $\Delta_d$ on the coupling...
energies, we make sure that the two states $|0\rangle$ and $|\pi\rangle$ are well defined while simultaneously involving only the low energy states $|0\rangle$ and $|\pi\rangle$ of the system. Furthermore, all times involved should be smaller than the decoherence time $\tau_{\text{dec}}$, requiring $E_s, \Delta_d \gg \hbar/\tau_{\text{dec}}$.

The present setup differs significantly from the conventional (large inductance) SQUID loop design, where the low-lying states are distinguished via the different amount of trapped flux and their manipulation involves external magnetic fields $H$ or biasing currents $I$. SQUID loops of this type are being used in the design of classical Josephson junction computers and have been proposed for the realization of quantum computers, too, see [12]. However, this setup suffers from the generic problem that the flux moving between the loops leads to a magnetic field mediated long-ranged interaction between the individual loops and further produces an unwanted coupling to the environment. By contrast, our device remains decoupled from the environment, the operating states do not involve currents, and switching between states can be triggered with a minimal contact to the external world — we therefore call our qubit implementation a ‘quiet’ one.

Next, we discuss how to perform two-qubit operations within an array of SDS’ SQUID loops. A two-qubit state is a coherent superposition of single qubit states and can be expressed in the basis $\{|xy\rangle\}$, where $x, y \in \{0, \pi\}$ denote the phases on the $d$-wave junctions of the first ($x$) and second ($y$) qubit, respectively. Unitary operations acting on these states are represented as $4 \times 4$ unitary matrices. Single-qubit operations $u$ acting on the second qubit take the block-matrix form

$$U_2 = \begin{pmatrix} u & 0 \\ 0 & u \end{pmatrix},$$

(2)

and a similar block form selecting odd and even rows and columns defines the single-qubit operations on the first qubit. As all logic operations on two qubits can be constructed from combinations of single-qubit operations and the Controlled-NOT gate it is sufficient to define the operational realization of the latter. The Controlled NOT gate performs the following action on two qubits: with the first (controller) qubit in state $|x\rangle$ and the second (target qubit) in state $|y\rangle$ the operation shall leave the target qubit unchanged if $x = 0$, while flipping it between 0 and $\pi$ when $x = \pi$, in matrix notation

$$U_{\text{CNOT}} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \sigma_x \end{pmatrix}.$$ (3)

The above Controlled NOT operation can easily be constructed from the ‘phase shifter’: Connecting two individual qubits in their idle-state over a $s$-wave junction into a SQUID loop, the states $|00\rangle$ and $|\pi\pi\rangle$ become separated from the states $|0\pi\rangle$ and $|\pi0\rangle$ by the energy $2E_s$, of the $s$-wave junction. Keeping the two qubits connected during the time $t$ introduces a phase shift $\chi = -2E_s t/\hbar$ between the two pairs of states,

$$U_{ps}(\chi) = \begin{pmatrix} u_x(\chi) & 0 \\ 0 & u_x(-\chi) \end{pmatrix},$$ (4)

The Controlled NOT gate then can be constructed from the phase-shifter via the following sequence of single- and two-qubit operations (see [4] for a similar realization of the CNOT gate),

$$U_{\text{CNOT}} = \exp(-i\pi/4)U_{2y}(\pi/2)U_{1x}(-\pi/2)U_{2x}(-\pi/2),$$

(5)

where the single qubit operations $U_{pm}(\theta)$ rotate the qubit $i$ by an angle $\theta$ around the axis $\mu(U_{pm}(\theta) = \exp(-i\sigma_p\theta/2)$ acting on $i$) while leaving the other qubit unchanged.

A key element in our design are the switches and a valid suggestion is the single electron transistor discussed in the literature [17]. Here we propose a quiet switch design optimally adapted to our SDS’ qubits. The basic idea derives from frustrating the junctions in a SQUID loop resulting in a ‘phase blockade’: Combining a SDS’ junction with energy $E_d$, a $\pi$-junction with $E_s \ll E_d$, and a $s$-wave junction with $E_s = E_\pi$ into a (small inductance) SQUID loop, see Fig. 2(d), we obtain the following switching behavior: The phase $\phi = 0$ on the SDS’ junction frustrates the remaining junctions and the loop’s energy-phase relation is a constant, $E_{\text{sw}}(\phi = \phi_s = \pi) = 0$. A voltage pulse coming down the signal lines and switching the SDS’ junction into the $|\pi\rangle$ state changes the phase relation between the $\pi$- and the $s$-wave junctions and closes the switch: the energy $E_{\text{sw}}(\phi = \pi) = 2E_\pi(1 - \cos \phi_s)$ produces the current-phase relation $I = (2e/\hbar)\partial\phi/E_{\text{sw}}$. The appropriate voltage pulses can be generated by driving an external SDS’ SQUID loop unstable.

The quiet device concept proposed above heavily relies on the double periodicity of the SD junction. As the second harmonic is strongly suppressed in a SID tunnel junction, a more feasible suggestion for the realization of a $\cos 2\phi$ junction is the SND sandwich, where the superconductors are separated by a thin metallic layer $N$. For a clean metallic layer, the coupling energies for the $n$-th harmonic are large and of order $E_n \sim k_F^2 A \hbar v_F / d$, producing the well known saw-tooth shape in the current-phase relation [18] (here, $v_F$ denotes the Fermi velocity in the $N$ layer while $d$ and $A$ are its width and area). In reality, it seems difficult to deposit a clean metallic film on top of a $d$-wave superconductor and we have to account for the reduction in the coupling $E_s$ due the finite scattering length $l$ in the metal layer. Using quasi-classical techniques to describe a dirty $\text{SN}_D$ junction, we obtain a second harmonic coupling energy $E_d \sim k_F^2 A (\hbar v_F / d)(l/d)^3 \sim (R_Q/R)(l/d)E_{\pi}$, where $l$ denotes the scattering length in the normal metal, $R_Q = \hbar/e^2$ is the quantum resistance, and $E_{\pi} \sim (\hbar v_F / d)(l/d)$ is the Thouless energy.
The second important device parameter is the tunneling gap $\Delta_d$, which depends quite sensitively on the coupling to the environment. The usual reduction in the tunneling probability produced by the environment is reduced if the system is effectively gapped at low energies [21]. This is the case for our SNDN'S' junction where the low-energy quasi-particle excitations in the metal are gapped over the Thouless energy $E_T$ [2]. The dynamics of the junction is only affected by the presence of virtual processes involving energies larger than $E_T$, leading to a renormalized capacitance $C_{ren} \sim h/R E_T$ (cf. [20]) and resulting in the reduced tunneling gap $\Delta_d \propto E_d \exp[-\nu(R_d/R)\sqrt{l/d}]$, with $\nu$ of order unity. Consistency requires that the tunneling process is ‘massive’ and hence slow, $h/\tau < E_T$. With a tunneling time $\tau \sim S/E_d$ ($S \sim h(R_d/R)\sqrt{l/d}$ = tunneling action) we find that the constraint $h/\tau E_T \sim \sqrt{l/d} < 1$ is satisfied. The condition $\Delta_d \ll E_d$ requires the tunneling gap $\Delta_d$ to be small, but large enough in order to allow for reasonable switching times, requiring $(R_d/R)\sqrt{l/d}$ to be of order 10. With typical device dimensions $d \sim 1000 \text{ Å}$, $l \sim 10 \text{ Å}$, and $R/R_d \sim (d/l)/(1/Ak^2) \sim 1/100$, this condition can be realized. Finally, the operating temperature $T$ is limited by the constraint $S/h > E_d/T$, guaranteeing that our device operates in the quantum regime, and the requirement $T < E_T$ that thermal quasi-particle excitations be absent. The first condition takes the form $T < h/\tau \sim \sqrt{l/d} E_T$ and is the more stringent one. Using the above parameters and a typical value $v_T \sim 10^8 \text{ cm/s}$, we obtain a Thouless energy $E_T \sim 1 \text{ K}$ and hence $T \ll 0.1 \text{ K}$.

In conclusion, we have discussed a novel device concept for logic gates in superconducting computers. The SDS’ SQUID loop realizes a number of attractive features which are potentially relevant both in classical Josephson computers based on RSFQ logics as well as in superconducting quantum computers. The most obvious advantage over previous designs is the quietness of the device: The SDS’ SQUID loop is a naturally bistable device and does not involve external bias currents or magnetic fields. Second, the basic states of the loop do not involve currents or trapped flux, hence long-range interactions between various elements of the computer are eliminated. Third, the qubits do not accumulate phase differences during idle time. And forth, all operations can be carried out via simple switching processes.

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