Undulator-Based and Crystal-Based Gamma Radiation Sources for Positron Generation

A Potylitsyn\textsuperscript{a,b}

\textsuperscript{a}National Research Tomsk Polytechnic University, 634050, Tomsk, Russia
\textsuperscript{b}National Research Nuclear University “MEPhI”, 115409, Moscow, Russia

E-mail: potylitsyn@tpu.ru

Abstract. Oriented Si or diamond crystals with thicknesses about 10-20 mm can be considered as an effective “solid-state undulator” if electron energy will be higher than 10 GeV. The scheme where such a crystal (photon radiator) placed upstream of the amorphous positron convertor with thickness less than 1 radiation length (hybrid scheme) may provide the intensity of produced and accelerated positrons comparable with an intensity of the initial electron beam. In this case the deposited energy in both parts of the hybrid scheme positron source may be much less than the damage threshold.

1. Positron sources for future colliders, such as, for instance, the International Linear Collider (ILC) [1], have to provide a high-energy positron beam intensity which is at least, equal to an intensity of the counter propagated electron beam. Currently there are two approaches to construct a positron source with the required parameters. The first one is the conventional source based on an electromagnetic cascade-shower process whereby a heavy-metal target is irradiated by high energy electrons from the linear accelerator [2]. The positron yield is naturally proportional to the incident electron intensity, but when the intensity exceeds a certain limit, the target may be destroyed due to the excessive heat load.

The second approach is rather new and based on a long undulator (~100 m) where an electron beam with an energy of $E_e > 100$ GeV is generating an undulator radiation (UR) beam with mean photon energy $\bar{\omega} \sim 10$ MeV. For such a long undulator the photon flux is enough to create a few positrons per each initial electron in a thin amorphous target ($\sim 0.4X_0$, $X_0$ is radiation length) [3, 4]. In this case the heat power deposited in the target and, consequently, the temperature stress remains at an acceptable level.

An oriented crystal may be considered as some kind of a solid-state undulator with extremely low period. In principle, the appropriate crystal may be used as a “photon radiator” instead of an undulator with huge length [5]. The oriented crystal target (photon radiator) can be separated from an amorphous positron convertor (so-called hybrid scheme). Experiments with electron energy $E_e \sim 1$ GeV [6, 7] demonstrated the possibility to obtain intense radiation in oriented crystal targets with thickness $t \sim (0.1 \div 0.2)X_0$ $\gg L_d$ ($L_d$ stands for the dechanneling length [8]) to produce photon beam with spectrum enriched by “soft” photons.

In the work [7] authors demonstrated an applicability of a hybrid scheme using 4 mm tungsten crystal and 4mm amorphous converts for electrons with $E_e \leq 10$ GeV. Authors of the papers [9] showed that, electrons with an energy $E_e \sim 6$ to 10 GeV interacting with oriented thick tungsten crystals may produce positrons with characteristics appropriated for acceptance in acceleration.
2. Let’s compare the main characteristics of emitted spectra for ordinary bremsstrahlung (BS), coherent bremsstrahlung (CBS), and channeling radiation (ChR). Radiation losses for BS may be estimated as

$$\Delta E_{\text{BS}} \approx \frac{t}{X_0} E_0, \ t \ll X_0. \ (1)$$

The mean photon energy in the BS spectrum is:

$$\bar{\omega}_{\text{BS}} \approx \frac{\Delta E_{\text{BS}}}{N_{\text{BS}}}, \ (2)$$

where $N_{\text{BS}}$ is the mean number of photons emitted by an electron [10]:

$$N_{\text{BS}} \approx \frac{t}{X_0} \int_{\omega_0}^{E_0/dE} \frac{d\omega}{\omega} = \frac{t}{X_0} \ln \frac{mc^2}{\omega_p} \approx 10 \frac{t}{X_0}. \ (3)$$

Here $\gamma$ is the Lorentz-factor, $\omega_p$ is the plasmon energy of target material ($\omega_p \approx 30$ eV for Al and Si).

From (1), (2), (3) one may obtain

$$\bar{\omega}_{\text{BS}} \approx 0.1E_0. \ (4)$$

The mean energy for ChR process may be estimated from the following expression (see [8]):

$$u = \frac{\bar{\omega}_{\text{CBS}}}{E_0 - \bar{\omega}_{\text{CBS}}} \approx 2\gamma^2 \left( \frac{V_0}{mc^2} \right) \left( \frac{1}{2} \frac{V_0}{mc^2} + \frac{\gamma \theta}{a} \right), \ (5)$$

where $V_0$ is the axis or plane potential, $a_s$ is a screening radius, $mc^2 = 0.511$ MeV, $\lambda_a = 3.86 \times 10^{-3}$ m. For the <111> axis of Si target and $E_0 \sim 1$ GeV $\bar{\omega}_{\text{CBS}} \sim 15$ MeV $< \bar{\omega}_{\text{BS}}$. For the coherent bremsstrahlung the same value is:

$$u = \frac{\bar{\omega}_{\text{CBS}}}{E_0 - \bar{\omega}_{\text{CBS}}} - \frac{\pi \gamma \theta}{a}, \ (6)$$

where $\theta$ is the orientation angle, $a$ is the interplanar distance. For the axial orientation of a thick monocrystalline target a rough estimation of the angle $\theta$ can be obtained as:

$$\theta \sim \tilde{\theta}_{\text{BS}} = \frac{21}{E_0} \sqrt{\frac{1}{2} \frac{t}{X_0}}, \ E_0 \ \text{in MeV}. \ (7)$$

For instance, for Si, $t = 10$ mm and $E_0 \sim 1$ GeV $\bar{\omega}_{\text{CBS}} \sim 0.5E_0 \sim 50$ MeV, and

$$\bar{\omega}_{\text{CBS}} < \bar{\omega}_{\text{BS}} < \bar{\omega}_{\text{in}}. \ (7)$$

The radiation losses for an axial orientation of the crystal can be considered as consisting of two parts

$$\Delta E = \Delta E_{\text{BS}} + \Delta E_{\text{cr}},$$

where the first term is connected with a radiation from an amorphous target with the same thickness, but the second one is determined by coherent radiation processes (channeling radiation together with coherent bremsstrahlung). Ratio between these parts was measured for electron energy $E_0 = 900$ MeV [11]. Ratio $R = \Delta E_{\text{cr}}/\Delta E_{\text{BS}}$ was measured for electron energy $E_0 = 900$ MeV for a thick diamond target ($t = 10$ mm, <100>, $R_{\text{Di}} = 2.5$) and for a thick silicon target ($t = 10$ mm, <111>, $R = 1.8$) [11]. The thicknesses chosen for these crystal targets are close enough to the optimal thickness as estimated and proposed in the work [12]. Having in mind the relation (7) one may obtain:
\begin{equation}
N_{cr} \approx \frac{\Delta E_{cr}}{\sigma_{cr}} \gg N_{BS} \tag{8}
\end{equation}

because of \( \Delta E_{cr} > \Delta E_{BS} \), \( \bar{\sigma}_{cr} < \bar{\sigma}_{BS} \).

**Figure 1.** Radiation losses spectra for channeled electrons with \( E_0 = 10 \text{ GeV} \) in Si crystal with thickness 0.8 mm (a) and 3.0 mm (b). Solid lines are the simulation results [13].

Authors of the experiment [13] that was carried out with electron energy \( E_0 = 10 \text{ GeV} \) had measured the energy losses for \(<111>\) axial orientation of Si crystals with thickness \( t = 0.8 \) and 3.0 mm (figure 1). From Monte-Carlo simulation they have obtained the mean photon number (photon multiplicity)

\[ \bar{N} = \bar{N}_{BS} + \bar{N}_{cr} \]

for each case:

\[ \bar{N}(0.8\text{mm}) = 1.8\text{ph/e}^{-}, \quad \bar{N}(3.0\text{mm}) = 5.4\text{ph/e}^{-}. \]

The measurements and simulations were carried out for the threshold photon energy \( \omega_t = 20 \text{ MeV} \). The contribution to the multiplicity from bremsstrahlung is negligible (see eq. (4)):

\[ N_{BS}(0.8) \approx 0.9 \cdot 10^{-3} \text{ph/e}^{-}, \quad N_{BS}(3.0) \approx 3.2 \cdot 10^{-3} \text{ph/e}^{-}. \]

In the paper [14] an approach was proposed to estimate the photon multiplicity from the measured distribution of the energy losses (energy straggling). Neglecting the bremsstrahlung losses, the photon multiplicity and the mean photon energy may be estimated in the following manner:

\[ \bar{N}_{cr} \approx \frac{\bar{Q}^2}{\sigma^2}, \quad \bar{\sigma}_{cr} = \frac{\Delta E_{cr}}{\bar{N}_{cr}} \tag{9} \]

where \( \bar{Q} \) is the mean value of the radiation losses calculated from energy losses distribution, \( \sigma \) is the distribution variance, \( \Delta E_{cr} \) is the total energy losses for axial orientation. Fitting the experimental results [13] by a smooth curve one may obtain for \( t = 3 \text{ mm} \) (see figure 1).

\[ \bar{Q} \approx 2.3 \text{ GeV}, \quad \sigma \approx 1.1 \text{ GeV} \] and, consequently:

\[ \bar{N}_{cr} \approx 4\text{ph/e}^{-} \text{ and } \bar{\sigma}_{cr} = 0.2\text{GeV} < \bar{\sigma}_{BS} = 1 \text{ GeV}. \]

The multiplicity estimated from this model agrees reasonably with the Monte-Carlo simulation. The model of photon multiplicity developed in [8] gives for the considered case (\( E_0 = 10 \text{ GeV}, \ t = 3 \text{ mm}, \ Si <111> \)) (see figure 17.3 there):
\[ \bar{N}_{\text{tot}} \approx 24 \text{ ph} / e^- . \]

It should be noted that the estimation of photon multiplicity accordingly to the model [8] gives much higher a photon number emitted by each electron due to low threshold energy.

**Figure 2.** a) Radiation losses spectra for channeled electrons with \( E_0 = 4.5 \) GeV in diamond crystal with thickness 10 mm [15]; b) Fit by a gaussian with \( Q = 1660 \) MeV, \( \sigma = 536 \) MeV, \( \Delta E = 1300 \) MeV.

Using the same approach let’s estimate the photon multiplicity in the experiment [15]. This experiment was performed at the electron beam with energy \( E_0 = 4.5 \) GeV for axial <111> orientation of the natural diamond target with thickness \( t = 10 \) mm (see figure 2a). Figure 2b shows the fit by a Gaussian (\( Q = 1660 \) MeV, \( \sigma = 536 \) MeV, \( \Delta E_{\alpha} = 1300 \) MeV), which gives the following result:

\[ \bar{N}_{\alpha} \approx 10 \text{ ph} / e^- , \alpha_{\alpha} \approx 130 \text{ MeV}. \] (10)

It allows to think that the photon multiplicity may reach the value \( \bar{N}_{\alpha} \sim 20 \) if one uses an electron beam with an energy \( E_0 \sim 10 \) GeV and a diamond target with thickness \( t \approx 20 \) mm. Data from Dubna’s experiment [16] show that the radiation losses of 10 GeV electron moving along <111> axis in the 10-mm silicon (~ 0.11 rad. length) reach 30 %. According to [8] losses from a diamond of the same radiation thickness (~ 13 mm) increases in \( a_{\alpha}/a_0 = 1.5 \) times. Due to this reason the axis potential (and, subsequently, radiation intensity) is much higher for a diamond crystal. The usage of diamond with thickness \( t \sim 20 \) mm leads to radiation increasing also. As a result one may expect that each electron will emit about 50% of the initial energy (\( \Delta E \approx 5 \) GeV).

The well known dependence of the multiple scattering angle on the energy and on the target thickness makes possible to estimate the mean photon energy for the considered case using formula (6):

\[ \bar{\sigma}_{\alpha}(E_2, t_2) \approx \sqrt{\frac{1}{t_1}} \bar{\sigma}_{\alpha}(E_1, t_1). \] (11)

For the considered case \( E_2 = 10 \) GeV, \( t_2 = 20 \) mm, \( E_1 = 4.5 \) GeV, \( t_1 = 10 \) mm, \( \bar{\sigma}_{\alpha}(E_1, t_1) \approx 130 \) MeV one can obtain \( \bar{\sigma}_{\alpha}(E_2, t_2) \approx 180 \) MeV and \( \bar{N}_{\alpha} \sim 14 \text{ ph} / e^- \). This photon flux may be used to produce positrons in a thin amorphous target in a complete analogy with an undulator radiation beam.

3. The positron yield for an initial electron with energy \( E_0 \) interacting with a rather thin amorphous target (\( t \leq 1 \) rad. length) can be calculated using a simple analytical model [17]. In this model, an
electron crossing the layer with thickness \( t/3 \) emits bremsstrahlung photon which creates \( e^-e^+ \) pair at the next layer with thickness \( t/3 \) and, at last, the created positron moves through the remaining layer \( (t/3) \) changing its outgoing angle due to multiple scattering. The positron spectrum (neglecting ionization and radiation losses) can be written as

\[
\frac{dN_+}{d\varepsilon_+} = 0.07 \left( \frac{t}{X}\right)^2 \ln \left( \frac{1}{\lambda} \right) - 0.5 \left[ \frac{1}{\varepsilon_+} - \frac{1}{E_0} \right] \left( \frac{1}{\text{MeV}} \right),
\]

(12)

where \( \varepsilon_+ \) is the positron energy. This formula is valid for the case \( E_0 > \frac{2}{\lambda} \text{me}^2, \varepsilon_+ > \frac{2\text{me}^2}{\lambda}, \lambda = \frac{z^{1/3}}{111} \) is the screening parameter.

Accuracy of the expression (12) can be estimated upon comparison with the Monte Carlo simulations [17] (see figure 3).

This model can be developed for an initial photon beam. For the sake of simplicity let us consider a “flat” photon spectrum with a photon multiplicity \( N_{\text{ph}} \) (“triangle” intensity spectrum closed to UR and CBS ones):

\[
\frac{dN_{\text{ph}}}{d\omega} = \begin{cases} 
\frac{N_{\text{ph}}}{\omega_{\text{max}}}, & \omega \leq \omega_{\text{max}}, \\
0, & \omega > \omega_{\text{max}}.
\end{cases}
\]

(13)

Radiation losses may be calculated easily from (13):

\[
\Delta E = \int_{0}^{\omega_{\text{max}}} \omega \frac{dN_{\text{ph}}}{d\omega} d\omega = N_{\text{ph}} \frac{\omega_{\text{max}}}{2} = N_{\text{ph}} \bar{\varepsilon}, \quad \omega_{\text{max}} = 2\bar{\varepsilon}.
\]

The positron spectrum produced by photons with the energy \( \omega \) in the converter with thickness \( t \) may be approximated by the expression [17]:

\[
\frac{dN_+(\omega)}{d\varepsilon_+} = 0.14 \frac{t}{L_{\text{rad}}} \frac{\ln \frac{1}{\lambda} - 0.5}{\omega}, \quad \omega > \frac{2}{\lambda} \text{me}^2.
\]

(14)

After convolution of this spectrum with photon one (13) it is possible to obtain the analytical expression for the output positron spectrum:

\[
\frac{dN_+}{d\varepsilon_+} = 0.14 \frac{t}{X} \frac{N_{\text{ph}}}{\omega_{\text{max}}} \ln \left( \frac{\omega_{\text{max}}}{\varepsilon_+} \right) \left[ \frac{1}{2} \ln \left( \frac{\varepsilon_+ \omega_{\text{max}}}{\text{me}^2} \right) + 1.2 \right].
\]

(15)

Flottmann’s calculations [3] for UR with \( \omega_{\text{max}} = 21 \text{ MeV}, N_{\text{ph}} = 2\text{ph}/e^- \) give the result \( \frac{dN_+}{d\varepsilon_+} = 0.020 \) for \( \varepsilon_+ = 10 \text{ MeV} \) and \( t = 1 \text{ rad. length} \). Using (15) one obtain the value \( \frac{dN_+}{d\varepsilon_+} = 0.021 \) for the same parameters.

Figure 4 shows spectra of positrons generated in the 1 rad. length amorphous converter by a photon flux with \( N_{\text{ph}} = 20 \) created by an initial electron with energy \( E_0 = 10 \text{ GeV} \) in a diamond target (upper curve) and in a silicon target (lower curve). The thicknesses of the targets was 20 mm for both cases. The positron yield for an energy interval 5 MeV \( \leq \varepsilon_+ \leq 25 \text{ MeV} \) reaches \( \Delta N_+ \sim 3.1 \) per each initial electron integrating with diamond crystal. When taking into account the positron beam divergence one may expect that about 30 % of the yield can be accepted for acceleration.
In summary, it may be noted the considered scheme may provide an efficiency for the positron source as high as 1 e+/e- at least. In the work [18] authors considered a conventional positron source for ILC based on a long undulator. Author of the work [19] proposed to design a positron source for the JLAB facility and showed that utility of the 2 mm (0.6 X_0) tungsten target can provide a continuous positron current up to 3 μA generated by the 10 mA electron beam with an energy 126 MeV. The hybrid scheme looks as an attractive variant for such a positron source.

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