PRE-IMAGES OF EXTREME POINTS OF THE NUMERICAL RANGE, AND APPLICATIONS

Ilya M. Spitkovsky
College of William and Mary, ims2@nyu.edu

Stephan Weis

Follow this and additional works at: https://scholarworks.wm.edu/aspubs

Recommended Citation
Spitkovsky, Ilya M. and Weis, Stephan, PRE-IMAGES OF EXTREME POINTS OF THE NUMERICAL RANGE, AND APPLICATIONS (2016).
10.7153/oam-10-58
PRE–IMAGES OF EXTREME POINTS OF THE NUMERICAL RANGE, AND APPLICATIONS

ILYA M. SPITKOVSKY AND STEPHAN WEIS

Abstract. We extend the pre-image representation of exposed points of the numerical range of a matrix to all extreme points. With that we characterize extreme points which are multiply generated, having at least two linearly independent pre-images, as the extreme points which are Hausdorff limits of flat boundary portions on numerical ranges of a sequence converging to the given matrix. These studies address the inverse numerical range map and the maximum-entropy inference map which are continuous functions on the numerical range except possibly at certain multiply generated extreme points. This work also allows us to describe closures of subsets of 3-by-3 matrices having the same shape of the numerical range.

Mathematics subject classification (2010): 47A12, 54C10, 62F30, 94A17.

Keywords and phrases: Numerical range.

REFERENCES

[1] Y. H. AU–YEUNG, Y. T. POON, A remark on the convexity and positive definiteness concerning Hermitian matrices, Southeast Asian Bull. Math. 3 (1979) 85–92.
[2] S. K. BERBERIAN, G. H. ORLAND, On the closure of the numerical range of an operator, Proceedings of the American Mathematical Society 18 (3) (1967) 499–503.
[3] T. BONNESEN, W. FENCHEL, Theory of Convex Bodies, BCS Associates, Moscow, Idaho USA, 1987.
[4] E. S. BROWN, I. M. SPITKOVSKY, On flat portions on the boundary of the numerical range, Linear Algebra and its Applications 390 (2004) 75–109.
[5] J. CHEN, Z. JI, C.-K. LI, Y.-T. POON, Y. SHEN, N. YU, B. ZENG, D. ZHOU, Discontinuity of maximum entropy inference and quantum phase transitions, New Journal of Physics 17 (8) (2015) 083019.
[6] M.-T. CHIEN, H. NAKAZATO, Joint numerical range and its generating hypersurface, Linear Algebra and its Applications 432 (1) (2010) 173–179.
[7] D. COREY, C. R. JOHNSON, R. KIRK, B. LINS, I. SPITKOVSKY, Continuity properties of vectors realizing points in the classical field of values, Linear and Multilinear Algebra 61 (2013) 1329–1338.
[8] I. Csiszár, F. Matuš, Closures of exponential families, The Annals of Probability 33 (2) (2005) 582–600.
[9] J. ELDRED, L. RODMAN, I. SPITKOVSKY, Numerical ranges of companion matrices: flat portions on the boundary, Linear and Multilinear Algebra 60 (2012) 1295–1311.
[10] G. FISCHER, Plane Algebraic Curves, Providence, Rhode Island: AMS, 2001.
[11] T. GALLAY, D. SERRE, Numerical measure of a complex matrix, Communications on Pure and Applied Mathematics 65 (3) (2012) 287–336.
[12] I. M. GELFAND, M. M. KAPRANOV, A. V. ZELEVINSKY, Discriminants, Resultants, and Multidimensional Determinants, Boston, MA: Birkhäuser Boston, 1994.
[13] B. GRÜNBBAUM, Convex Polytopes, 2nd Edition, New York: Springer, 2003.
[14] P. R. HALMOS, Ten problems in Hilbert space, Bull. Amer. Math. Soc. 76 (5) (1970) 887–933.
[15] F. HAUSDORFF, Der Wertvorrat einer Bilinearform, Math. Z. 3 (1) (1919) 314–316.
[16] J. W. HELTON, I. M. SPITKOVSKY, The possible shapes of numerical ranges, Operators and Matrices 6 (2012) 607–611.
[17] R. A. Horn, C. R. Johnson, *Topics in Matrix Analysis*, 10th printing, Cambridge Univ. Press, 1991.

[18] E. T. Jaynes, *Information theory and statistical mechanics*, Phys. Rev. **106** (1957), 620–630 and **108** (1957), 171–190.

[19] M. Joswig, B. Straub, *On the numerical range map*, Journal of the Australian Mathematical Society **65** (1998) 267–283.

[20] K. Kato, F. Furrer, M. Murao, *Information-theoretical analysis of topological entanglement entropy and multipartite correlations*, Physical Review A **93** (2016), 022317.

[21] D. S. Keeler, L. Rodman, I. M. Spitkovsky, *The numerical range of $3 \times 3$ matrices*, Linear Algebra and its Applications **252** (1997) 115–139.

[22] R. Kippenhahn, *Über den Wertevorrat einer Matrix*, Math. Nachr. **6** (1951) 193–228.

[23] T. Leake, B. Lins, I. M. Spitkovsky, *Pre-images of boundary points of the numerical range*, Operators and Matrices **8** (2014) 699–724.

[24] T. Leake, B. Lins, I. M. Spitkovsky, *Inverse continuity on the boundary of the numerical range*, Linear and Multilinear Algebra **62** (2014) 1335–1345.

[25] C.-K. Li, Y.-T. Poon, *Convexity of the joint numerical range*, SIAM J. Matrix Anal. A **21** (2) (2000) 668–678.

[26] P. X. Rault, T. Sendova, I. M. Spitkovsky, *3-by-3 matrices with elliptical numerical range revisited*, Electronic Journal of Linear Algebra **26** (2013) 158–167.

[27] F. Rellich, *Perturbation Theory of Eigenvalue Problems*, Research in the Field of Perturbation Theory and Linear Operators, Technical Report No. 1, Courant Institute of Mathematical Sciences, New York University, 1954.

[28] L. Rodman, I. M. Spitkovsky, *$3 \times 3$ matrices with a flat portion on the boundary of the numerical range*, Linear Algebra and its Applications **397** (2005) 193–207.

[29] L. Rodman, I. M. Spitkovsky, A. Szkoła, S. Weis, *Continuity of the maximum-entropy inference: Convex geometry and numerical ranges approach*, Journal of Mathematical Physics **57** (20-16), 015204.

[30] R. Schneider, *Convex Bodies: The Brunn-Minkowski Theory*, 2nd Edition, Cambridge University Press, 2014.

[31] O. Toeplitz, *Das algebraische Analogon zu einem Satze von Fejér*, Math. Z. **2** (1–2) (1918) 187–197.

[32] S. Weis, *Quantum convex support*, Linear Algebra and its Applications **435** (12) (2011) 3168–3188; correction (2012) ibid. 436 xvi.

[33] S. Weis, *A note on touching cones and faces*, Journal of Convex Analysis **19** (2012) 323–353.

[34] S. Weis, *Information topologies on non-commutative state spaces*, Journal of Convex Analysis **21** (2014) 339–399.

[35] S. Weis, *Continuity of the maximum-entropy inference*, Communications in Mathematical Physics **330** (3) (2014) 1263–1292.

[36] S. Weis, *Maximum-entropy inference and inverse continuity of the numerical range*, Reports on Mathematical Physics **77** (2) (2016), 251–263.

[37] S. Weis, A. Knauf, *Entropy distance: New quantum phenomena*, J. Math. Phys. **53** (10) (2012) 102206.