Bearing Fault Detection in Induction Motor-Gearbox Drivetrain

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Abstract. The main contribution in the hereby presented paper is to investigate the fault detection capability of a motor current signature analysis by expanding its scope to include the gearbox, and not only the induction motor. Detecting bearing faults outside the induction motor through the stator current analysis represents an interesting alternative to traditional vibration analysis. Bearing faults cause changes in the stator current spectrum that can be used for fault diagnosis purposes. A time-domain simulation of the drivetrain model is developed. The drivetrain system consists of a loaded single stage gearbox driven by a line-fed induction motor. Three typical bearing faults in the gearbox are addressed, i.e. defects in the outer raceway, the inner raceway, and the rolling element. The interaction with the fault is modelled by means of kinematical and mechanical relations. The fault region is modelled in order to achieve gradual loss and gain of contact. A bearing fault generates an additional torque component that varies at the specific bearing defect frequency. The presented dynamic electromagnetic dq-model of an induction motor is adjusted for diagnostic purpose and considers such torque variations. The bearing fault is detected as a phase modulation of the stator current sine wave at the expected bearing defect frequency.

Keywords: Motor current signature analysis; Condition monitoring; Gearbox; Bearing fault detection; Torque variations;

1. Introduction

This research work is motivated by the offshore oil & gas sector, within the framework of implementing condition-based maintenance (CBM) technology into offshore applications. Induction machines coupled with gearboxes are crucial parts of many applications in the offshore oil & gas industry, e.g. in drilling machines, winches, and pumps. Such drivetrains are subject to harsh conditions and its failures can be dangerous and/or lead to expensive downtime in the drilling or production process. Therefore there is a need for continuous condition monitoring of the drivetrain and thereby being able to predict failure in advance. Such early warning system will improve reliability and availability, in accordance with CBM standards, i.e. effective maintenance and replacement operations, safety, uptime productivity and low failure rate.

Gearbox bearing faults are one major cause for such drivetrain failures, typically initiated by insufficient lubrication, [1]. Several studies, [2], [3], [4], [5], and [6] report that the gearbox bearing is the main source for failures in a drivetrain. Motor current signature analysis (MCSA) is one of the condition monitoring techniques for detecting and predicting drivetrain faults, [7].
It has been shown recently by [8], that bearing faults inside the induction machine affect the stator current signature and that such modulation is detectable and analyzable.

Although MCSA is a frequently quoted model for bearing fault detection inside induction motors, [9], to the author’s knowledge, no literature refers to the capability of MCSA to detect bearing faults outside the induction motor. To detect bearing faults outside the induction motor through stator current analysis seems to be an interesting alternative to traditional vibration monitoring of characteristic bearing fault-related frequencies. Compared with vibration monitoring, the MCSA is cost-effective and sensorless by utilizing the electric drive system as a diagnostic instrumentation. This means that quantities, such as the stator current feedback, are already being measured for control and protection purpose.

This paper investigates the detection of typical rolling element bearing faults in a gearbox by monitoring the stator current in the driving induction machine. This paper follows and expands previous research work by [10], where a computational model of the drivetrain system consisting of a single stage gearbox coupled with an induction motor was developed with focus on the detection of the gearbox bearing with an outer ring fault, through analysis of the motor current signature. The previous drivetrain model is expanded by a new bearing model and a new motor model. The bearing model is expanded by two other frequent bearing faults, i.e. inner ring fault and rolling element fault. The dynamic electromagnetic model of the line-fed induction motor is modelled by means of $dq$-windings. The $dq$-transformation is commonly introduced for the control purpose. However, so far, the presented method has not been previously applied for the diagnostics of bearing faults outside the induction motor. This research work is also the theoretical background for an experimental verification.

2. System Description - Induction Motor-Gearbox Drivetrain

The investigated object is a model of an induction motor-gearbox drivetrain with a faulty bearing located in the gearbox, illustrated in Fig. 1.

![Figure 1. The induction motor-gearbox drivetrain model.](image)

The driving machine is a line-fed, three phase, low-voltage and squirrel-cage induction motor. The selected induction motor is described by the rated parameters, including the per-phase equivalent circuit data, in table 1.

The driven speed reduction gearbox is single stage with two spur gear wheels, two shafts and four rolling element bearings. The gearbox is considered as ideal, i.e. noise, disturbances and backlash are neglected. The selected gearbox is described by the parameters given in table 2.

Basically the rolling element bearing consists of the fixed outer ring, the rotating inner ring and a number of rolling elements. The main geometrical parameters of the selected bearing SKF – 6005 are stated in table 3. A part of the geometry and the components are shown in Fig. 2.
### Table 1. Rated parameters of the induction motor.

| $P$ | $V_{l-rms}$ | $I_{rated}$ | $Y/\Delta$ | $\eta$ | $\cos\varphi$ | $p$ | $n_{rated}$ |
|-----|-------------|-------------|-------------|--------|---------------|-----|------------|
| 2.4 kW | 460 V | 4A | Y | 88.5% | 80.0% | 4 | 1750 RPM |
| $f_{syn}$ | $R_s$ | $R_r$ | $X_{ls}$ | $X_{lr}$ | $X_m$ | $s_{rated}$ | $J_{im}$ |
| 60 Hz | 1.77 $\Omega$ | 1.34 $\Omega$ | 5.25 $\Omega$ | 4.57 $\Omega$ | 139.0 $\Omega$ | 1.72% | 0.0125 kgm$^2$ |

### Table 2. The parameters of the gearbox.

| Ratio $i$ | Module $m$ | Bull gear pitch radius $r_{p,bg}$ | Pressure angle $\psi$ | Mom. of inertia (motor side) $J_{gb}$ |
|-----------|------------|-----------------------------------|-----------------------|--------------------------------------|
| 5.57      | 3          | 175.5 mm                          | 20°                   | 0.0125 kgm$^2$                       |

### Table 3. The parameters of the selected rolling element bearing SKF – 6005.

| Number of rolling elements $n_{re}$ | Radius of rolling element $r_{re}$ | Outer radius of inner ring $r_{ir}$ |
|-------------------------------------|------------------------------------|-----------------------------------|
| 10                                  | 3.5 mm                             | 14.5 mm                           |
| Radius of cage centerline $r_{cage}$ | Inner radius of outer ring $r_{or}$ | Pitch angle of rolling elements $\gamma_p$ |
| 18 mm                               | 21.5 mm                            | 36°                               |

**Figure 2.** A part of the bearing; the geometry and the components.

**Figure 3.** Kinematic model of the rolling element bearing.

### 3. Model of the Rolling Element Bearing

The hereby presented 1-DOF model describes the rolling element bearing and addresses its three typical localized faults, such as defects in the outer ring, the inner ring and in the rolling elements. The model of the interaction between the bearing element and the fault is based on the kinematic and mechanical relations.

#### 3.1. Kinematical Model

For a fault free bearing the planetary motion of the rolling elements is assumed to be kinematically determined due to the rolling contact with both rings. In addition, the equidistance between rolling elements is assured by the function of the cage. The counterclockwise rotation of
the inner ring and the clockwise rotation of the rolling element are selected as positive directions. The contact angle and the clearance of the rolling elements are assumed to be zero, which justifies the kinematic model. The outer ring is fixed in the gearbox and the inner ring rotates with the input shaft. With the assumption of no slippage between the rings and the rolling elements and the theory for relative motion, the kinematic analysis is established. The variables used in the kinematic analysis of the bearing are illustrated in Fig. 3. The kinematic analysis is based on the bearing geometry and angular velocity of the inner ring, \( \omega_{ir} \). The velocity and the angular position of the rolling element, \( \omega_{re} \) and \( \varphi_{re} \), are given by respectively:

\[
\omega_{re} = \omega_{ir} \left( \frac{r_{ir}}{2r_{re}} \right)
\]

(1)

\[
\varphi_{re} = \varphi_{ir} \left( \frac{r_{ir}}{2r_{re}} \right) + \varphi_{re,0}
\]

(2)

, where \( \varphi_{re,0} \) is the initial rolling element position. The angular velocity and the position of the cage, \( \omega_{cage} \) and \( \varphi_{cage} \), are given by:

\[
\omega_{cage} = \omega_{ir} \left( \frac{r_{ir}}{2r_{cage}} \right)
\]

(3)

\[
\varphi_{cage} = \varphi_{ir} \left( \frac{r_{ir}}{2r_{cage}} \right) + \varphi_{cage,0}
\]

(4)

, where \( \varphi_{cage,0} \) is the initial cage position. The angular position of the \( j \)-th rolling element center, \( \varphi_{rec,j} \), is a function of the cage position, \( \varphi_{cage} \), Eq. 4, and the pitch angle between rolling elements, \( \gamma_p \):

\[
\varphi_{rec,j} = \varphi_{cage} + (j - 1) \gamma_p
\]

(5)

The angular position of the \( j \)-th rolling element surface point related to origin, \( \varphi_{re,j/o} \), is expressed as follows:

\[
\varphi_{re,j/o} = \varphi_{re,j/cc} + \varphi_{rec,j}
\]

(6)

, where

\[
\varphi_{re,j/cc} = \arcsin \left( \frac{r_{re}}{r_{re,j/o}} \sin(\varphi_{rec,j} + \varphi_{re}) \right)
\]

, and

\[
r_{re,j/o} = \sqrt{r^2_{cage} + r^2_{re} - 2r_{cage}r_{re} \cos(\varphi_{rec,j} + \varphi_{re})}
\]

3.2. Mechanical Model
In the static mechanical model of the fault-free rolling element bearing, the main interest is put on rolling elements in the lower part of the bearing and the distribution of the load onto the elements. The load on the elements is modelled to vary as illustrated in Fig. 4. The radial load on the bearing origins from the external torque on the gearbox. The gear mesh force:

\[
F_{gm} = F_{tg} \cos(\psi)
\]

, where \( F_{tg} \) represents the tangential force applied by the driving pinion gear, \( F_{tg} = \frac{\tau_{pg}}{r_{pg}} \).

In order to determine the nominal radial load carried by each rolling element, the rolling elements are thought of as a radially loaded springs with unit stiffness, \( k \), as depicted in Fig. 4. The compression occurs only on rolling elements in the load zone, i.e. the lower half of the bearing. Rolling elements located in the upper half are not expected to carry any load.

The opposite reaction force of each \( j \)-th compressed rolling element, \( F_{re,j} \), is to be calculated, and represents the nominal radial force to be carried. The static model of a loaded rolling element is illustrated in Fig. 5.
The radial deformation of the rolling elements is handled separately for $\Delta x$ and $\Delta y$ displacements:

- $\Delta x$: 
  \[ F_{x,\text{re},j},\Delta x = -k \Delta x \cos^2 \varphi_{\text{rec},j} \]
  \[ F_{y,\text{re},j},\Delta x = -k \Delta x \sin \varphi_{\text{rec},j} \cos \varphi_{\text{rec},j} \]
  \[ F_{r,\text{re},j},\Delta x = k \Delta x \cos \varphi_{\text{rec},j} \]

- $\Delta y$: 
  \[ F_{x,\text{re},j},\Delta y = -k \Delta y \sin \varphi_{\text{rec},j} \cos \varphi_{\text{rec},j} \]
  \[ F_{y,\text{re},j},\Delta y = -k \Delta y \sin^2 \varphi_{\text{rec},j} \]
  \[ F_{r,\text{re},j},\Delta y = k \Delta y \sin \varphi_{\text{rec},j} \]

The equilibrium equations are set up for the shaft in both the $x$- and $y$-directions.

\[
\Delta x \left( \sum - \cos^2 \varphi_{\text{rec},j} \right) + \Delta y \left( \sum - \sin \varphi_{\text{rec},j} \cos \varphi_{\text{rec},j} \right) = 0
\]

\[
\Delta x \left( \sum - \sin \varphi_{\text{rec},j} \cos \varphi_{\text{rec},j} \right) + \Delta y \left( \sum - \sin^2 \varphi_{\text{rec},j} \right) = \frac{F_{gm}}{k}
\]

The equilibrium equations are solved for the unknown deflections $\Delta x$ and $\Delta y$. The aim is not to determine the actual deflections but the deflections are used to compute the load distributed to each of the rolling elements in the lower half:

\[
F_{x,\text{re},j} = k \left[ \Delta x \left( - \cos^2 \varphi_{\text{rec},j} \right) + \Delta y \left( - \sin \varphi_{\text{rec},j} \cos \varphi_{\text{rec},j} \right) \right]
\]

\[
F_{y,\text{re},j} = k \left[ \Delta x \left( - \sin \varphi_{\text{rec},j} \cos \varphi_{\text{rec},j} \right) + \Delta y \left( - \sin^2 \varphi_{\text{rec},j} \right) \right]
\]

The nominal radial load of the $j$-th loaded rolling element is $F_{r,\text{re},j} = \sqrt{F_{x,\text{re},j}^2 + F_{y,\text{re},j}^2}$.

As an example the load for rolling element number one is shown in the Fig. 6. The load is seen to be symmetric around the $y$-axis of the bearing. The element only carries load when located in the lower half of the bearing, i.e. when the angular position of the center of the element is in the interval $180 - 360^\circ$. 

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**Figure 4.** Radially loaded rolling elements, illustrated as springs.

**Figure 5.** The static model of rolling element deformation.
3.3. Bearing Fault Model

In the presented bearing fault model, each fault type is considered as a localized pit with defined size, geometry and location. The bearing fault size is the fundamental parameter given by the fault width, \( w \), and the depth, \( d \). The fault width, \( w \), is assumed to be \( \frac{1}{10} \) of the rolling element radius, \( w = \frac{1}{10} r_{re} \), and the depth, \( d = \frac{w}{2} \), for all three considered bearing fault types. The bearing fault zone geometry is given by the angular width, \( \theta \), relative to the bearing origin. The fault location relative to the origin, is defined by the angular positions of fault center, \( \varphi_C \), the rolling-in fault edge A, \( \varphi_A \), and the rolling-out fault edge B, \( \varphi_B \). The geometric definition of rolling element fault is illustrated in Fig 7.

In service more faults may develop in the bearing. However it is unlikely that two faults develop at the same time and hence the first fault is expected to be discovered during the time when it is the only one. Therefore the model in this paper does not take multiple faults into account.

3.3.1. Interaction

The conditions of fault interaction are the same for all rolling elements, allowing the following theory presentation to be generalized for one rolling element. The model of the interaction between the bearing fault and another bearing element is based strictly on the geometric relations. The bearing elements are assumed to remain in contact, while passing the fault. The change in load to be carried by the other rolling elements, when a single rolling element is in the proximity of the fault, is not considered here.

When located in a fault the contact force on the rolling element depends on the angular position in the bearing and the position of the rolling element relative to the fault. The latter determines if the element is rolling into or out of the fault.

3.3.2. Contact Model - Adjusted Radial Force

[11] presented a contact model of rolling element and bearing fault as a step function with sudden loss and instant gain of contact. Such assumption leads to large undesirable impact forces. In the model in this paper, the nominal radial force of each rolling element, \( F_{re,j} \), is adjusted in order to model the interaction with the fault region and to reflect the path of the rolling element.

While the \( j \)-th rolling element passes the fault, the smooth loss/gain of contact between rolling element and ring, is achieved by the adjusted radial force, \( F_{bf,j} \), and the undesirable impact pulses are avoided. The smooth variation is obtained in the model with the following equation:

\[
F_{bf,j} = F_{re,j} \left[ \frac{1}{2} \sin \left( \pi \left( \frac{\varphi_{re,j} - \varphi_C}{\Delta \theta} - \frac{\pi}{2} \right) \right) + \frac{1}{2} \right] \quad (7)
\]
The nominal radial force is adjusted for each bearing fault type individually, according to its fault center location, $\varphi_C$, and fault angular width, $\Delta \theta$, both related to the bearing origin.

3.3.3. Bearing Fault-related Forces and Torque Variations

Torque variation is a physical effect, when the bearing fault comes into contact with another bearing element. When rolling into the fault, the rolling element produces a torque on the inner ring in the direction of rotation of the inner ring. This contribution reduces the external load on the induction motor. Contrary a torque is produced on the inner ring in the direction opposite to the rotation when the rolling element rolls out of the fault. This contribution increases the load on the induction motor. Based on static equilibrium the relations between the radial and tangential forces $F_{bf}$ and $T_{bf}$ and the torque on the inner ring $\tau_{bf}$ are established.

3.3.4. Rolling Element Fault

Not many publications concern in detail with physical modelling of a localized geometrical fault in the rolling element. The reference [12] introduced a bearing model with a localized fault in a rolling element including the depth of the fault and the difference in curvature between the inner and outer ring of the bearing. In the hereby presented bearing fault model, the kinematic interaction between rolling element fault and rings is presented as point contact related to origin, similarly as in fault-free case. The bearing elements are assumed to remain in contact, while passing the fault. The geometric definition of the rolling element fault is illustrated in Fig. 7.

The half angular width of the rolling element fault, is defined as: $\Delta \theta = \arcsin \left( \frac{w}{r_{re}} \right)$.

Related to origin, the angular widths of the rolling element fault sensed by the outer and the inner ring are not the same, specifically: $\theta_{ir} > \theta_{or}$. This results from different curvature of the rings and is also a function of radius of the rolling element, $r_{re}$. The angular widths of the rolling element fault sensed by the individual rings, are illustrated on Fig. 7.

![Figure 7. Rolling element fault as sensed by the outer ring and the inner ring.](image-url)
The half angular width of the rolling element fault sensed by the outer ring is defined as $\Delta \theta_{or} = \arctan \left( \frac{w_2}{r_{cage} + a} \right)$, and sensed by the inner ring $\Delta \theta_{ir} = \arctan \left( \frac{w_2}{r_{cage} - a} \right)$. The distance $a = r_{re} \cos (\Delta \theta)$. The radial distance of the fault center: $r_C = r_{re} - d$.

The rolling element fault rotates at the rolling element spin speed, $\omega_{re}$, which is a kinematical function of the rotor shaft speed, $\omega_{ir}$, as stated in Eq. 1. This means that the rolling element fault shares rotating characteristics with the inner ring fault and defines a main difference from the fixed fault in the outer ring.

Related to the rolling element center, the angular position of the rolling element fault center, $\varphi_C$, is associated with the position of the rolling element, $\varphi_{re}$, stated in Eq. 2. The angular position is defined as $\varphi_C = \varphi_{re} + \varphi_{C,0}$, where $\varphi_{C,0}$ is the initial position of the rolling element fault center. The angular positions of rolling-in/out fault edges are defined as: $\varphi_A = \varphi_C + \Delta \theta$ and $\varphi_B = \varphi_C - \Delta \theta$. Related to origin, the angular positions of rolling element fault center, $\varphi_{C/o}$, and fault edges, $\varphi_{A/o}$, $\varphi_{B/o}$ are calculated similarly as the position of the $j$-th rolling element surface point related to origin, $\varphi_{re,j/o}$, see Eq. 6.

The loss/gain of contact occurs twice for each complete rotation of the loaded rolling element, due to the contact with both inner ring and outer ring. Different expressions are set up for the contact depending on if the fault on the rolling element is in contact with the outer ring or inner ring respectively and if the ring is rolling into or out of the fault. The derivation of contact conditions, sensed by both rings, is illustrated in Fig. 8, while rolling-in.

![Figure 8](image-url)
The adjusted radial force on loaded rolling element with fault is established by Eq. 7. However, since the angular widths of fault, related to origin, are different for sensing by each ring, the radial force is adjusted according to actual sensed ring: $F_{\text{ref-}a/o}$ and $F_{\text{ref-}i/o}$.

The free-body-diagrams of rolling element fault sensed by outer and inner ring, while rolling-in, are shown in Fig. 8. For the case of rolling element fault sensed by the outer ring, the rolling element fault-related angle $\alpha$ is defined for rolling in and out, respectively: $\alpha = \varphi_{\text{rec}} - \varphi_{A/o}$, and $\alpha = \varphi_{B/o} - \varphi_{\text{rec}}$. For the case of rolling element fault sensed by the inner ring: $\alpha = \varphi_{A/o} - \varphi_{\text{rec}}$, and $\alpha = \varphi_{B/o} - \varphi_{\text{rec}}$.

For the case of rolling element fault sensed by the outer ring, the tangential force, $T$, on the inner ring caused by tangential force, $\tau_{\alpha}$, and by assumption that the radial force $F_1$ is adjusted: $F_1 = F_{\text{ref-}a/o}$.

$$\sum M_A = T_1b - F_1a = 0 \rightarrow T_1 = F_{\text{ref-}a/o} \frac{a}{b}$$

The moment arm $b$ is defined for rolling in and out, respectively: $b = r_{A/o} \cos \alpha - r_{ir}$, and $b = r_{B/o} \cos \alpha - r_{ir}$. Similarly, the moment arm $a$: $a = r_{A/o} \sin \alpha$, and $a = r_{B/o} \sin \alpha$. The torque on the inner ring is established using static equilibrium with moment equation around point, $A$, and by assumption that the radial force $F_1$ is adjusted: $F_2 = F_{\text{ref-}i/o}$.

$$\sum M_P = T_1b - F_2a = 0 \rightarrow T_1 = F_{\text{ref-}i/o} \frac{a}{b}$$

The moment arm $b$ is defined for rolling in and out, respectively: $b = r_{or} \cos \alpha - r_{A/o} + r_{or} \frac{\sin^2 \alpha}{\cos \alpha}$, and $b = r_{or} \cos \alpha - r_{B/o} + r_{or} \frac{\sin^2 \alpha}{\cos \alpha}$. The moment arm $a$: $a = r_{or} \tan \alpha$. The torque on the inner ring caused by tangential force, $T_1$, is defined for rolling in and out, respectively: $\tau_{\text{ref-}i/o} = T_1r_{A/o}$, and $\tau_{\text{ref-}i/o} = -T_1r_{B/o}$.

The resultant torque on the inner ring caused by the rolling element fault, $\tau_{\text{ref}}$, is given:

$$\tau_{\text{ref}} = \tau_{\text{ref-}a/o} + \tau_{\text{ref-}i/o}$$

When the rolling element fault is not in contact with the rings, no tangential force is applied on the inner ring and therefore the resultant torque $\tau_{\text{ref}} = 0$ Nm.

The rolling element fault-related torque variations appear periodically with respect to the spin frequency of the rolling element, $\omega_{\text{rev}}$, stated in Eq. 1, over rings in the load zone. The diagram in Fig. 9a) shows the torque on the inner ring due to the faulty rolling element relative to the rotation of the inner ring, $\varphi_{ir}$.

**Figure 9.** a) The rolling element fault-related torque variations, $\tau_{\text{ref}}$, applied on the inner ring. b) Rolling element located within the load zone.
Since the radial load distribution effect in the bearing is considered, the rolling element fault-related torque varies periodically in amplitude, according to the position of the rolling element fault, within the load zone. The sequence in Fig. 9b) is for the interval when the rolling element is located in the lower half of the bearing. When the rolling element is located in the upper half the torque $\tau_{ref}$ on the inner ring is zero. The maximum value of $\tau_{ref}$ appears when the rolling element and the ring is in contact at the bottom of the bearing.

3.3.5. Outer Ring Fault

The outer ring fault model has been previously introduced in [10]. Fig. 10 shows the resultant torque variation characteristic applied on the inner ring: a) by consecutive rolling elements; b) by rolling element number 1, in the outer ring fault zone, relative to the inner ring position $\phi_{ir}$.

The outer ring fault-related torque variations, $\tau_{orf}$, appear periodically with respect to the passing frequency of the rolling elements, i.e. cage speed, $\omega_{cage}$, which is a kinematical function of the rotor shaft speed, $\omega_{ir}$, as stated in Eq. 3.

Since the outer ring fault is located at the maximum of the distributed load, i.e. where the defect is most likely to occur, the outer ring fault-related torque variations, $\tau_{orf}$, have constant amplitudes.

![Figure 10. The outer ring fault-related torque variations, $\tau_{orf}$, applied on the inner ring by: a) consecutive rolling elements; b) rolling element number 1.](image)

3.3.6. Inner Ring Fault

The inner ring fault is modelled in the same way as the fault in the outer ring. The resultant inner ring fault-related torque variations, $\tau_{irf}$, appear periodically with respect to the passing frequency of the inner ring fault over rolling elements in the load zone, i.e. the inner ring speed, $\omega_{ir}$. Fig. 11 shows the torque variation applied on the inner ring: a) by consecutive rolling elements in the inner ring fault zone; b) by three consecutive rolling elements, according to inner ring position, $\phi_{ir}$.

![Figure 11. The inner ring fault-related torque variations, $\tau_{irf}$, applied on the inner ring by: a) consecutive rolling elements; b) three consecutive rolling elements](image)
The radial load distribution effect in the bearing is considered, while the inner ring fault is constantly moving in and out of the load zone as the ring rotates with the rotor shaft. Therefore, the inner ring fault-related torque varies in amplitude, according to the position of the inner ring fault and rolling element, within the load zone. The maximum torque appears when the rolling element hits the inner ring fault in the middle of the load zone.

All presented graphs of bearing fault-related torques characteristic correspond to double impulse phenomenon, previously introduced in the literature, in the case of entry into and exit from a localized fault. The model of the different types of faults in the bearing are implemented into the time domain simulation of the drive train in section 5. This is done to test the capabilities of the stator current analysis to detect the bearing fault.

4. Dynamic Model of the Induction Motor
The hereby presented dynamic electromagnetic model considers the above described bearing fault-related torque variations. One of the methods used to study the influence of the rotor displacement on the stator current is based on the air-gap field calculation by means of magnetomotive force (MMF). The reference [8] as a first applied MMF approach to the analysis of bearing faults in induction motors.

A different approach is taken here. The theoretical development of this model is based on the air-gap field calculation by means of dq-windings transformation, described by [13]. Usually, the main reason for the dq-analysis of an induction machine is for the control purpose, such as vector control. Here, the main purpose for application of dq-theory is rather for the diagnostics of bearing faults outside the motor, where the dq-approach has not been applied yet.

4.1. The dq-Windings Transformation
The transformation of the input stator a-b-c voltages, $\vec{v}_s(t)$, into equivalent stator dq-voltages, $\vec{v}_{sdq}(t)$, is established by means of a direct dq-transformation matrix: $\vec{v}_{sdq}(t) = [T_s(t)] \vec{v}_s(t)$.

$$\begin{bmatrix} v_{sd}(t) \\ v_{sq}(t) \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix} \cos \theta_{da}(t) & \cos \left(\theta_{da}(t) - \frac{2}{3} \pi\right) & \cos \left(\theta_{da}(t) - \frac{4}{3} \pi\right) \\ -\sin \theta_{da}(t) & -\sin \left(\theta_{da}(t) - \frac{2}{3} \pi\right) & -\sin \left(\theta_{da}(t) - \frac{4}{3} \pi\right) \end{bmatrix} \begin{bmatrix} i_{a}(t) \\ i_{b}(t) \\ i_{c}(t) \end{bmatrix}$$  

(8)

In order to obtain the output stator a-b-c currents, $\vec{i}_s(t)$, in terms of dq-currents, $\vec{i}_{sdq}(t)$, the reverse dq-transformation is made by means of the inverse matrix: $\vec{i}_s(t) = \left[\mathbf{T}_s(t)\right]^{-1} \vec{i}_{sdq}(t)$.

$$\begin{bmatrix} i_{a}(t) \\ i_{b}(t) \\ i_{c}(t) \end{bmatrix} = \frac{\sqrt{2}}{3} \begin{bmatrix} \cos \theta_{da}(t) & -\sin \theta_{da}(t) \\ \cos \left(\theta_{da}(t) + \frac{4}{3} \pi\right) & -\sin \left(\theta_{da}(t) + \frac{4}{3} \pi\right) \\ \cos \left(\theta_{da}(t) + \frac{2}{3} \pi\right) & -\sin \left(\theta_{da}(t) + \frac{2}{3} \pi\right) \end{bmatrix} \begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \end{bmatrix}$$  

(9)

$\theta_{da}(t)$ is the angle of the dq-winding set in the stator reference frame rotating at the speed $\omega_d(t)$.

4.2. The dq-Flux Linkages
The dq-flux linkages, $\vec{\lambda}_{dq}(t)$, are derived by means of voltage equations for the dq-windings, i.e. application of Kirchhoff’s voltage law and the hypothesis of imposed voltages, by the time derivative of the corresponding flux linkage. The time derivative of the stator dq-flux linkages stated in vector form are:

$$\frac{d}{dt} \vec{\lambda}_{sdq}(t) = \vec{v}_{sdq}(t) - R_s \vec{i}_{sdq}(t) - \omega_d(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\lambda}_{sdq}(t)$$  

(10)
The matrix $[M_{rot}]$ has the role of rotating the space vector $\vec{X}_{rdq}(t)$ by $\frac{\pi}{2}$. The stator flux linkage equations show their dependence on $\theta_{da}(t)$, by the speed term $\omega_d(t) = \frac{d}{dt}\theta_{da}(t)$.

Similarly the time derivative of the rotor $dq$-flux linkages stated in vector form are:

$$\frac{d}{dt}\vec{X}_{rdq}(t) = \vec{v}_{rdq}(t) - R_r \vec{i}_{rdq}(t) - \omega_{dA}(t) [M_{rot}] \vec{X}_{rdq}(t)$$  \hspace{1cm} (11)

The rotor flux linkage equations show their dependence on the position of the $dq$-winding set in the rotor reference frame, $\theta_{dA}(t)$, by the speed term $\omega_{dA}(t) = \frac{d}{dt}\theta_{dA}(t)$. $\omega_{dA}(t)$ is the instantaneous speed of the $dq$-winding set in the air gap with respect to the rotor $A$-axis:

$$\omega_{dA}(t) = \omega_d(t) - \omega_m(t)$$  \hspace{1cm} (12)

$\omega_m(t)$ is the equivalent rotor winding speed, and is related to the mechanical speed of the rotor shaft $\omega_{mech}(t)$, by the pole-pair:

$$\omega_m(t) = \frac{p}{2} \omega_{mech}(t)$$  \hspace{1cm} (13)

By $\omega_m(t) = \omega_d(t) - \omega_{dA}(t)$ and $\omega_m(t) = \frac{d}{dt}\theta_m(t)$ the stator and rotor $dq$-flux linkages $\vec{X}_{sdq}(t)$ and $\vec{X}_{rdq}(t)$ depend on the equivalent rotor winding angle $\theta_m(t)$, i.e. depend on the mechanical position of the rotor $\theta_{mech}(t)$.

4.3. The $dq$-Currents

The main function of the hereby presented induction motor model is to express output $dq$-currents, $\vec{i}_{dq}(t)$, in terms of $dq$-flux linkages, $\vec{X}_{dq}(t)$, representing the state variables, while the stator $dq$-voltages, $\vec{v}_{sdq}(t)$, are inputs. The $dq$-currents are expressed, in terms of flux linkages, by inversion of the inductance matrix: $\vec{i}_{dq}(t) = [M_{L}]^{-1} \vec{X}_{dq}(t)$.

$$\begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \\ i_{rd}(t) \\ i_{rq}(t) \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}^{-1} \begin{bmatrix} \lambda_{sd}(t) \\ \lambda_{sq}(t) \\ \lambda_{rd}(t) \\ \lambda_{rq}(t) \end{bmatrix}$$  \hspace{1cm} (14)

The inductance matrix $[M_{L}]$, including leakage inductances, is defined by the mutual magnetizing inductance $L_m = \frac{X_m}{\omega_{sym}}$, the stator total self inductance $L_s = L_{ls} + L_m$, and the rotor total self inductance $L_r = L_{lr} + L_m$, where the stator leakage inductance $L_{ls} = \frac{X_{ls}}{\omega_{sym}}$, and the rotor leakage inductance $L_{lr} = \frac{X_{lr}}{\omega_{sym}}$.

The resultant net electromagnetic torque on the rotor is expressed in terms of the mutual magnetizing inductance, $L_m$, and $dq$-currents, $\vec{i}_{dq}(t)$:

$$\tau_{em}(t) = \frac{p}{2} L_m (i_{sq}(t)i_{rd}(t) - i_{sd}(t)i_{rq}(t))$$  \hspace{1cm} (15)

5. Time-domain Simulation of Induction Motor-Gearbox

The time-domain simulation of the induction motor-gearbox drivetrain has been conducted in the simulation software SimulationX. The program includes relevant signal blocks used in this paper, such as function blocks, signal generators and integrators. By default the program conducts the routines for time integration of the governing equations for the drivetrain system.
5.1. Assumptions
The drivetrain is assumed to be operating in apparent steady-state, i.e. the electromagnetic torque, \( \tau_{\text{em}}(t) \), is assumed to be equal to the constant load torque component: \( \tau_{\text{em}}(t) = \tau_{\text{gb}} \).
However, the dq-model considers the bearing fault-related additional torque load, \( \tau_{\text{bf}}(t) \), varying at the bearing defect frequency, \( f_{\text{bf}} \).

The assumptions for the simulation of the dq-model are following. At \( t = 0 \), the initial angle of the \( d \)-axis is \( \theta_{\text{da}}(0) = 0 \), i.e. \( d \)-axis is aligned to the stator phase-\( a \) axis. The dq-reference frame is rotating synchronously with \( \omega_{d} = \omega_{\text{syn}} \). The speed of the \( d \)-axis, \( \omega_{d} \), relative to rotor winding speed \( \omega_{m}(t) \), is considered as a slip speed \( \omega_{dA}(t) = s(t)\omega_{\text{syn}} \). The airgap is uniform and defined by constant permeance \( \Lambda \) and constant length \( l_{g} \). Rotating stator fields produced by additional bearing fault-related stator current components are neglected.

5.2. Initial Conditions
In order to carry out the time-domain simulation the initial values of the state variables the dq-flux linkages, \( \hat{\lambda}_{dq}(0) \), are calculated in terms of initial dq-currents, \( \hat{i}_{dq}(0) \). Initial dq-currents, \( \hat{i}_{dq}(0) \), are obtained by application of phasor analysis of per-phase equivalent circuit and space vectors under an initial balanced sinusoidal steady-state condition.

**Phasor Analysis:** For a given stator voltage \( \hat{V}_{a} \), the current phasors \( \hat{I}_{a} \) and \( \hat{I}_{A} (= -\hat{I}_{rA}) \), are defined by means of a per-phase steady-state equivalent circuit of the induction motor including leakage inductances as illustrated in Fig. 12. Assuming that the stator phase-\( a \) voltage has a positive peak at \( t = 0 \), \( \hat{V}_{a} = \hat{V}_{a} \angle 0^\circ \), the peak is \( \hat{V}_{a} = \sqrt{3}V_{ll,\text{rms}} \). Hence, for a given line-to-line stator voltage rms, \( V_{ll,\text{rms}} \): \( \hat{V}_{a} = \sqrt{3}V_{ll,\text{rms}} \angle 0^\circ \).

![Figure 12. Per-phase equivalent circuit in steady-state including leakage inductances.](image)

The rotor branch impedance is \( Z_{\text{rotor}} = jX_{lr} + \frac{R_{r}}{s} \). The stator branch impedance is \( Z_{\text{stator}} = R_{s} + jX_{ls} \). The magnetizing impedance is \( Z_{m} = jX_{m} \). Then the resultant equivalent circuit impedance is computed as:

\[
Z_{eq} = Z_{\text{stator}} + \frac{Z_{m}Z_{\text{rotor}}}{Z_{m} + Z_{rotor}} = (R_{s} + jX_{ls}) + \left(\frac{jX_{m}(jX_{lr} + \frac{R_{r}}{s})}{R_{s} + j(X_{m} + X_{lr})}\right)
\]

Thereby, the phase-\( a \) stator current phasor is \( \hat{I}_{a} = \frac{\hat{V}_{a}}{Z_{eq}} \). Recalling Kirchhoff’s voltage law, the voltage across the magnetizing branch is \( \hat{E}_{ma} = \hat{V}_{a} - Z_{\text{stator}} \hat{I}_{a} \). The rotor branch current phasor is defined as \( \hat{I}_{A} = -\hat{I}_{ra} = -\frac{\hat{E}_{ma}}{Z_{\text{rotor}}} \).

**Initial Space Vectors:** Stator voltage \( \hat{V}_{a}(0) = \frac{3}{2}\hat{V}_{a} \), with peak \( \hat{V}_{s}(0) = \frac{3}{2}\hat{V}_{a} \). Stator current \( \hat{i}_{s}(0) = \frac{3}{2}\hat{I}_{a} \), with peak \( \hat{I}_{s}(0) = \frac{3}{2}\hat{I}_{a} \). Rotor branch current \( \hat{i}_{r}(0) = \frac{3}{2}\hat{I}_{A} \), with peak \( \hat{I}_{r}(0) = \frac{3}{2}\hat{I}_{A} \).

**Initial dq-Currents, \( \hat{i}_{dq}(0) \), and Stator dq-voltages, \( \hat{v}_{sdq}(0) \):**

\[
\begin{align*}
\hat{i}_{sd}(0) &= \sqrt{\frac{2}{3}}\hat{I}_{r}(0) \cos(\theta_{s}(0) - \theta_{da}(0)) \\
\hat{i}_{rd}(0) &= \sqrt{\frac{2}{3}}\hat{I}_{r}(0) \cos(\theta_{r}(0) - \theta_{da}(0)) \\
\hat{i}_{sq}(0) &= \sqrt{\frac{2}{3}}\hat{I}_{s}(0) \sin(\theta_{s}(0) - \theta_{da}(0)) \\
\hat{i}_{rq}(0) &= \sqrt{\frac{2}{3}}\hat{I}_{r}(0) \sin(\theta_{r}(0) - \theta_{da}(0))
\end{align*}
\]
The rotor is short circuited, i.e. the rotor dq-voltages, \( \vec{v}_{rdq} \), are null:
\[
v_{sdq}(0) = \sqrt{2} V_s(0) \cos(\theta_{sa}(0) - \theta_{da}(0)) \quad v_{sq}(0) = \sqrt{2} V_s(0) \sin(\theta_{sa}(0) - \theta_{da}(0))
\]

The initial dq-flux linkages are calculated in terms of the initial dq-currents \( \vec{\lambda}_{dq}(0) = [\mathbf{M}_{L}] \vec{\lambda}_{dq}(0) \). The initial dq-currents, \( \vec{\lambda}_{dq}(0) \), further allow to obtain the initial steady-state electromagnetic torque, \( \tau_{em}(0) \), by Eq. 15.

5.3. The Simulation Model of the Induction Motor-Gearbox Drivetrain

The overall schematic simulation block diagram of the dynamic model of the induction motor-gearbox drivetrain, is shown in Fig. 13, including references to the governing equations and initial conditions for the integrator. This drivetrain model has been developed by means of the \( dq \)-theory, and the model of a gearbox with a bearing fault. The model is additionally adjusted for diagnostic purpose, by introducing:

- Extraction of the mechanical rotor position, \( \theta_{mech}(t) \), as input to the bearing fault model.
- Varying torque load, \( \tau_{load}(t) \), caused by the bearing fault in the gearbox.
- Monitoring of the electromagnetic torque, \( \tau_{em}(t) \), for torque spectrum analysis.
- Monitoring of the stator phase-a current, \( i_a(t) \), for current signature analysis.

![Figure 13](image_url)

**Figure 13.** Simulation model of the induction motor-gearbox drivetrain, in terms of \( dq \).
5.3.1. Model of Gearbox with Bearing Fault

The bearing fault model is primarily described by the angular position of the inner ring, \(\phi_{ir}(t)\). Due to the assumption of a rigid coupling between the induction motor and the gearbox, the input shaft of the gearbox (inner ring) turns with the same angular velocity as the output shaft of the driving induction motor \(\omega_{ir}(t) = \omega_{mech}(t)\) and thereby \(\phi_{ir}(t) = \theta_{mech}(t)\). Hence, the mechanical rotor position of the induction motor, \(\theta_{mech}(t)\), represents the input for the bearing fault model.

Bearing faults in the gearbox generate an additional varying torque on the inner ring, and thereby the total load torque, \(\tau_{load}(t)\), from the gearbox to the rotor shaft of the induction motor is varying. The constant torque component is represented by \(\tau_{gb}' = 5.4\) Nm, corresponding to the external load applied on the load side of the gearbox, and referred to the motor side. The varying torque component is represented by an additional bearing fault-related torque, \(\tau_{bf}(t)\), varying at the bearing defect frequency, \(f_{bf}\), corresponding to the overall steady state operation.

The total torque load on the rotor shaft is a function of the mechanical rotor position \(\theta_{mech}(t)\), i.e. is a function of time:

\[
\tau_{load}(t) = \tau_{gb}' + \tau_{bf}(t)
\] (16)

The total torque load from the gearbox, \(\tau_{load}(t)\), including the bearing fault-related torque variations is further implemented into the electrodynamic \(dq\)-model of the induction motor, in order to detect the bearing fault effect on the stator current signature.

5.4. Electrodynamics

In this section the governing equations for the dynamics of the induction machine is set up. The acceleration/deceleration of the rotor shaft is determined by the time-varying net torque, \(\tau_{net}(t)\), acting on the equivalent inertia, \(J_{eq}\), of the motor and the load. The net torque is the difference of the net electromagnetic torque, \(\tau_{em}(t)\), produced by the induction motor and the load torque at the motor side, \(\tau_{load}(t)\). The equation of motion of the rotor in terms of the actual mechanical speed of the rotor shaft \(\omega_{mech}(t)\):

\[
\frac{d}{dt}\omega_{mech}(t) = \frac{\tau_{net}(t)}{J_{eq}} = \frac{\tau_{em}(t) - \tau_{load}(t)}{J_{eq}}
\] (17)

The actual mechanical speed of the rotor shaft is obtained through integration of the angular acceleration, where \(\tau\) is a time variable of integration:

\[
\omega_{mech}(t) = \frac{1}{J_{eq}} \int_{t_0}^{t} (\tau_{em}(\tau) - \tau_{load}(\tau)) d\tau
\]

5.4.1. Bearing Fault Effect on the Stator Current

The hereby presented method to study the influence of the rotor displacement on the stator current caused by a bearing fault outside the motor is based on variations of the rotor \(dq\)-flux linkages, \(\vec{\lambda}_{rdq}(t)\), caused by varying slip speed \(\omega_{DA}(t)\).

The equation of motion in Eq. 17 and the assumption that the drivetrain is operating in apparent steady-state, i.e. \(\tau_{em}(t) = \tau_{gb}'\), leads to the effect of the bearing fault-related torque variations on the mechanical speed of rotor shaft, \(\omega_{mech}(t)\), as follows:

\[
\omega_{mech}(t) = -\frac{1}{J_{eq}} \int_{t_0}^{t} \tau_{bf}(\tau) d\tau + \omega_{mech,0}
\] (18)

The integration constant is the initial mechanical speed of the rotor \(\omega_{mech,0}\). Therefore, the mechanical speed \(\omega_{mech}(t)\) consists of a constant component \(\omega_{mech,0}\) and a bearing fault-related varying component.
The mechanical rotor position, $\theta_{\text{mech}}(t)$, is the integral of Eq. 18:

$$\theta_{\text{mech}}(t) = \int_{t_0}^{t} \omega_{\text{mech}}(\tau)d\tau = -\frac{1}{J_{\text{eq}}} \int_{t_0}^{t} \tau_f(\tau)d\tau + \omega_{\text{mech},0}t + \theta_{\text{mech},0}$$

(19)

The integration constant is the initial mechanical position of the rotor, $\theta_{\text{mech},0}$. For the case of a healthy drivetrain running in steady state, the mechanical position of the rotor is $\theta_{\text{mech}}(t) = \omega_{\text{mech},0}t + \theta_{\text{mech},0}$. However, in the case of a gearbox bearing fault torque variations are present which causes a non constant angular velocity.

Recalling the Faraday’s law of induction, the variations of the mechanical rotor speed, $\omega_{\text{mech}}(t)$, have an influence on the induced electromotive forces in the rotor $dq$-windings $\bar{\epsilon}_{rdq}(t)$. Thus, the rotor speed variations have an effect on the time derivative of rotor $dq$-flux linkages, $\frac{d}{dt}\bar{\lambda}_{rdq}(t)$. As stated in Eq. 11, the rotor $dq$-flux linkages, $\bar{\lambda}_{rdq}(t)$, depends on the slip angle, $\theta_{\text{A}}(t)$, by the slip speed term: $\omega_{\text{A}}(t) = \frac{d}{dt}\theta_{\text{A}}(t)$.

The time derivative of the rotor $dq$-flux linkages, $\frac{d}{dt}\bar{\lambda}_{rdq}(t)$ (rotor is short-circuited, $\bar{i}_{rdq} = 0$) is defined as:

$$\frac{d}{dt}\bar{\lambda}_{rdq}(t) = -R_r\bar{i}_{rdq}(t) - \omega_{\text{dA}}(t)[M_{\text{rot}}] \bar{\lambda}_{rdq}(t)$$

(20)

The mechanical speed of the rotor, $\omega_{\text{mech}}(t)$, is computed in terms of the slip speed, $\omega_{\text{dA}}(t)$, by Eq. 12 and Eq. 13:

$$\omega_{\text{mech}}(t) = \frac{2}{p}(\omega_d - \omega_{\text{dA}}(t))$$

(21)

The effect of the bearing fault-related torque variations, $\tau_{bf}(t)$, on the slip speed, $\omega_{\text{dA}}(t)$, by substituting Eq. 21 into Eq. 18:

$$\omega_{\text{dA}}(t) = \omega_d - \frac{p}{2} \omega_{\text{mech},0} + \frac{p}{2J_{\text{eq}}} \int_{t_0}^{t} \tau_f(\tau)d\tau$$

(22)

Therefore, the slip speed, $\omega_{\text{dA}}(t)$, consists of a constant component, $(\omega_d - \frac{p}{2} \omega_{\text{mech},0})$, and a bearing fault-related varying component, $(\frac{p}{2J_{\text{eq}}} \int_{t_0}^{t} \tau_f(\tau)d\tau)$.

Hence, the variations of the rotor $dq$-flux linkages, $\bar{\lambda}_{rdq}(t)$, caused by varying slip speed, $\omega_{\text{dA}}(t)$, are given by substitution of Eq. 22 into Eq. 20:

$$\frac{d}{dt}\bar{\lambda}_{rdq}(t) = -R_r\bar{i}_{rdq}(t) - \left(\omega_d - \frac{p}{2} \omega_{\text{mech},0}\right)[M_{\text{rot}}] \bar{\lambda}_{rdq}(t)$$

(21)

The bearing fault-related torque variations, $\tau_{bf}(t)$, at the bearing defect frequency, $f_{bf}$, are subsequently reflected in the stator $dq$-currents, $\bar{i}_{sdq}(t)$, by Eq. 14, followed by the reverse $dq$-transformation, Eq. 9, in order to obtain the stator a-b-c phase currents, $\bar{i}_s(t)$.

6. Simulation Results and Discussion
In this section the simulation results are discussed for each bearing fault type. At first, the torque spectrum of the loaded induction motor, with a bearing fault in the driven gearbox, is presented. The variations of electromagnetic torque are investigated in order to approve the bearing fault model and its torque variation effect.

Then, the stator current is analyzed in order to show that the bearing fault in the driven gearbox has an influence on the stator current in the driving induction motor. Thereby the bearing fault detection in the driven gearbox by analyzing the stator current spectrum is justified.
6.1. Electromagnetic Torque Variations - Torque spectrum

For all three fault types the influence arises from the impact between the edge of the fault and the opposite healthy component. Such localized defects generate vibrations, which are estimated at the specific bearing defect frequencies, $f_{bf}$, i.e. frequencies at which the rolling element passes the outer ring fault, the inner ring fault and the frequency of rolling element fault hitting both rings. Hence, the bearing defect frequency, $f_{bf}$, is specific for each defect location, i.e. for each bearing element: the outer ring, $f_{orf}$, the inner ring, $f_{irf}$, and the rolling element, $f_{ref}$.

The bearing defect frequency, $f_{bf}$, is determined by the bearing geometry ($r_{cage}$, $r_{re}$), number of rolling elements ($n_{re}$), and the mechanical frequency of the inner ring ($f_{ir}$). The frequency of the inner ring, corresponds to the rotor shaft speed by $f_{ir} = \frac{n}{60}$. The rated rotor shaft speed in this case is $n = 1769$ RPM.

The hereby presented torque spectrum of the loaded induction motor, under bearing fault in the driven gearbox, has been made by FFT-analysis with Hanning window. The dominant frequency component of the electromagnetic torque variation, $\tau_{em}(t)$, appears at the precalculated specific bearing defect frequency, $f_{bf}$. The match of those frequencies indicates the presence of a fault in the bearing.

As mentioned in subsection 3.1, the contact angle between the rolling element and the rings is assumed to be zero. However, a nonzero and unknown contact angle may lead to an uncertainty regarding the tracking of the expected defect frequency $f_{bf}$. A nonzero contact angle changes the effective radius of the inner ring, outer ring, and rolling element in the kinematic relations in subsection 3.1. This change alters the expected defect frequency so it is to be found in an interval. This is not investigated further in this paper.

6.1.1. Outer Ring Fault

The torque spectrum of the loaded motor with an outer ring fault in the gearbox bearing is shown in Fig. 14a). The dominant peak in the torque spectrum, $f_{orf}$, and its integer harmonics, $k \cdot f_{orf}$, as illustrated in Fig. 14b), simply corresponds to the defect frequency of the outer ring, given by:

$$f_{orf} = f_{ir} \left( \frac{n_{re}}{2} \right) \left( 1 - \frac{r_{re}}{r_{cage}} \right)$$

(23)

![Figure 14.](image)

**Figure 14.** a) Torque spectrum of the loaded motor, under outer ring fault in the gearbox bearing. b) The dominant peak and its harmonics.

6.1.2. Inner Ring Fault

The torque spectrum of the loaded motor, with an inner ring fault in the gearbox bearing, is shown in Fig. 15a). The dominant peak in the torque spectrum, $f_{irf}$, and its integer harmonics, $k \cdot f_{irf}$, corresponds to the defect frequency of inner ring, given by:

$$f_{irf} = f_{ir} \left( \frac{n_{re}}{2} \right) \left( 1 + \frac{r_{re}}{r_{cage}} \right)$$

(24)
Figure 15. a) Torque spectrum of the loaded motor, under inner ring fault in the gearbox bearing. b) The dominant peak and its harmonic including sidebands.

From the detailed torque spectrum shown in Fig. 15b), one can see the sideband peaks distributed around the dominant peak of the inner ring defect frequency, \( f_{irf} \). Sidebands indicate a classic amplitude modulation corresponding with the periodic amplitude variations of the inner ring fault-related torque.

Here, the inner ring defect frequency, \( f_{irf} \), represents the carrier (higher frequency), which is modulated by the modulating frequency (lower frequency) of the inner ring rotation, \( f_{ir} \), also stated as rotor shaft 1X running speed. The resultant amplitude-modulated torque signal generates the sidebands around the carrier, \( f_{irf} \). The sideband spacing is equal to the modulating frequency, i.e. the inner ring frequency, \( f_{ir} \), or 1X rotor shaft speed. Harmonics of a modulated frequency, \( k \cdot f_{irf} \), reproduce sideband peaks centered around them, as well.

6.1.3. Rolling Element Fault

The torque spectrum of the loaded motor with a rolling element fault in the gearbox bearing is shown in Fig. 16a). The dominant peak in the torque spectrum, \( f_{ref} \), and its harmonics, corresponds to the defect frequency of the rolling element, given by:

\[
f_{ref} = f_{ir} \left( r_{cage} \right) \left( 1 \left( \frac{r_e}{r_{cage}} \right)^2 \right)
\]

(25)

Figure 16. a) Torque spectrum of the loaded motor with a rolling element fault in the gearbox bearing. b) The dominant peak and its harmonic and subharmonics including sidebands.

In the torque spectrum related to the rolling element fault a similar case of amplitude modulation is obvious. In this case, the amplitude modulation corresponds with periodic amplitude variations of the rolling element fault-related torque, while the rolling element hits rings in different parts of the load zone, as the rotor shaft turns. Here, the rolling element defect
frequency, $f_{\text{ref}}$, acts as the carrier (higher frequency), and the frequency of the cage, $f_{\text{cage}}$, represents the modulating frequency (lower frequency). By using Eq. 3, the frequency of the cage is found as $f_{\text{cage}} = \frac{\omega_{\text{cage}}}{2\pi}$.

The resultant amplitude-modulated torque signal generates the sidebands around the carrier, $f_{\text{ref}}$, with spacing of the modulating frequency, $f_{\text{cage}}$, as illustrated in Fig. 16b). Harmonics of a modulated frequency, $f_{\text{ref}}$, specifically the integer harmonics, $k \cdot f_{\text{ref}}$, and the half order subharmonics, $\frac{1}{2} \cdot f_{\text{orf}}$, reproduce sideband peaks centered around them, as well.

6.2. Stator Current Variations - Current spectrum

In this section the current spectra are being analyzed in order to find and show the relevant frequencies in relation to the bearing faults. The bearing fault-related torque variations, $\tau_{bf}(t)$, at the specific bearing defect frequency, $f_{bf}$, lead to phase modulation of the stator current sine wave, $i_a(t)$, i.e. the stator current is phase-modulated by the presence of a bearing fault. In the phase-modulated current, each bearing fault is expressed as a set of sidebands around the carrier, i.e. fundamental stator synchronous supply frequency, $f_{\text{syn}}$. The sideband spacing corresponds to the modulating frequency, i.e. specific bearing defect frequency, $f_{bf}$.

The bearing fault-related (torque variations) sidebands are defined as an additional frequency components, $f_{\tau v}$, in the stator current spectrum, as follows:

$$f_{\tau v} = |f_{\text{syn}} \pm k f_{bf}|,$$

where $k=1,2,3,...$

The hereby presented current spectrum of the loaded induction motor with a bearing fault in the driven gearbox, has been made by FFT-analysis with Hanning window. Hanning window is used in order to narrow smeared frequencies. Phase modulation generally exhibits a lot more sidebands in the spectrum compared to amplitude modulation. Hence, the amplitude of the power spectral density (PSD) is expressed in dB, in order to highlight these sidebands of interest. The amplitude PSD expressed in dB allows to display small amplitudes in the presence of large amplitudes. In this case, small amplitudes refer to the sidebands of the modulating frequency, i.e. bearing fault-related frequency, $f_{bf}$, and large amplitudes refer to the carrier of the modulated frequency, i.e. fundamental synchronous supply frequency, $f_{\text{syn}}$.

The sideband frequency components of the stator current variation, $i_a(t)$, appears at the precalculated sideband frequencies of interest, $f_{\tau v}$, related to the bearing fault and its torque variations. The match of those sideband frequency components indicates the presence of a fault in the gearbox bearing. This implies that the bearing fault detection in the driven gearbox, outside the motor, could be accomplished by tracking precalculated bearing fault-related sidebands in the stator current spectrum of the driving induction motor.

6.2.1. Outer Ring Fault

The current spectrum of the loaded motor with an outer ring fault in the gearbox bearing is shown in Fig. 17a). The sidebands around the dominant carrier peak, $f_{\text{syn}}$, correspond to the phase modulation of the stator current, $i_a(t)$, by modulating the frequency of the outer ring defect, $f_{orf}$, given in Eq. 23.

The sidebands of interest, related to the outer ring fault and its torque variations are found as:

$$f_{\tau v,orf} = |f_{\text{syn}} \pm k f_{orf}|,$$

where $k=1,2,3,...$

The outer ring fault-related sidebands, $f_{\tau v,orf}$, have spacing equal to the modulating frequency, i.e the outer ring defect frequency, $f_{orf}$, as illustrated in Fig. 17b).
6.2.2. Inner Ring Fault

The current spectrum of the loaded motor with an inner ring fault in the gearbox bearing is shown in Fig. 18a). From the detailed current spectrum shown in Fig. 18b), one can see the sideband peaks distributed around the dominant carrier peak of the supply frequency, \( f_{\text{syn}} \). The sidebands of interest related to the inner ring fault and its torque variations are defined as:

\[
f_{\tau v, \text{irf}} = |f_{\text{syn}} \pm k f_{\text{irf}}|, \text{ where } k=1,2,3,...
\]

The spacing of the inner ring fault-related sideband, \( f_{\tau v, \text{irf}} \), is equal to the modulating frequency, i.e. the inner ring defect frequency, \( f_{\text{irf}} \), given in Eq. 24. The inner ring fault-related sidebands, \( f_{\tau v, \text{irf}} \), also have their own sidebands corresponding to the periodic amplitude variations of the inner ring fault-related torque, i.e. the inner ring frequency, \( f_{\text{ir}} \).

6.2.3. Rolling Element Fault

The current spectrum of the loaded motor with a rolling element fault in the gearbox bearing is shown in Fig. 19a). From the detailed current spectrum shown in Fig. 19b), the sidebands around the carrier, \( f_{\text{syn}} \), are related to the rolling element fault and its torque variations and are defined as:

\[
f_{\tau v, \text{ref}} = |f_{\text{syn}} \pm k f_{\text{ref}}|, \text{ where } k=1,2,3,...
\]

The rolling element fault-related sidebands, \( f_{\tau v, \text{ref}} \), have spacing equal to the modulating frequency, i.e the rolling element defect frequency \( f_{\text{ref}} \), given in Eq. 25. The rolling element fault-related sidebands, \( f_{\tau v, \text{ref}} \), also have their own sidebands corresponding to the periodic amplitude variations of the rolling element fault-related torque, i.e. the frequency of the cage, \( f_{\text{cage}} \).

Figure 17. a) Spectrum of the stator phase-a current, \( i_a(t) \), of the loaded motor with an outer ring fault in the gearbox bearing. b) The synchronous frequency peak and its sidebands.

Figure 18. a) Spectrum of the stator phase-a current, \( i_a(t) \), of the loaded motor with an inner ring fault in the gearbox bearing. b) The synchronous frequency peak and its sidebands.

Figure 19. a) Spectrum of the stator phase-a current, \( i_a(t) \), of the loaded motor with a rolling element fault in the gearbox bearing. b) The synchronous frequency peak and its sidebands.
Figure 19. a) Spectrum of the stator phase-\(a\) current, \(i_a(t)\), of the loaded motor with a rolling element fault in the gearbox bearing. b) The synchronous frequency peak and its sidebands.

7. Conclusion

The main contribution in the hereby presented paper is the expansion of the bearing fault detection capability of motor current signature analysis to include bearings located outside the driving motor. In the presented case the bearings have been located in a driven gearbox. A time-domain simulation of the drivetrain model was developed by involving a new bearing fault model and a dynamic model of the induction motor. Three typical bearing faults in the gearbox have been investigated. The induction motor was modelled by means of \(dq\)-transformation, which has not been previously applied for the diagnostics of bearing faults outside the induction motor. As expected, the simulation results show an influence from the bearing faults onto the stator current in the induction motor. The pattern of the influence can be used for diagnostics purposes. The bearing fault detection outside the motor is accomplished by tracking expected bearing fault-related sidebands in the stator current spectrum of the driving induction motor. This research work also represents the theoretical background for an experimental verification. The experimental work will also show if the current variations are detectable in a real set up.

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