Spectral density calculations in a heavy-light meson-meson system *

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A system of two static quarks, at fixed distances \( r \), and two light quarks is studied on an anisotropic lattice. Excitations by operators emphasizing quark or gluon degrees of freedom are examined. The maximum entropy method is applied in the spectral analysis. These simulations ultimately aim at learning about mechanisms of hadronic interaction.

1. INTRODUCTION

Features of hadronic interaction can in principle be gleaned from two-hadron energy spectra in finite boxes [1]. For two hadrons containing one heavy (static) quark each [2,3], the relative distance \( r \) is well defined, and the excitation spectra as functions of \( r \) should give insight into the physics of the strong interaction.

Refined analysis techniques able to deal with excitations are desirable. We have employed Bayesian inference [4] to extract energy spectra from correlation functions of a set of meson-meson operators. In lattice QCD Bayesian techniques had not been used until only very recently [5,6]. Experience with this analysis tool is thus somewhat limited. In this contribution we present selected results from Bayesian curve fitting with an entropic prior.

2. OPERATORS AND LATTICE

The following operators describe systems of two pseudo scalar mesons in the \( I = 2 \) channel

\[
\Phi_1(t) = \sum_{\vec{x}, \vec{y}} \delta_{\vec{x}, \vec{y}} - \vec{g} \gamma_5 q_A(\vec{x}t) \bar{q}_B(\vec{y}t) \\
\Phi_2(t) = \sum_{\vec{x}, \vec{y}} \delta_{\vec{x}, \vec{y}} - \vec{g} \gamma_5 q_B(\vec{x}t) \bar{q}_A(\vec{y}t) \\
\]

Heavy and light quark fields are denoted by \( Q \) and \( q \), respectively, and \( A, A', B, B' \) are color indices.

Meson fields in \( \Phi_1 \) are local. Those in \( \Phi_2 \) are spread out over a distance \( \vec{r} \) connected by link variable products \( U_P \) along straight spatial paths \( P \) between \( \vec{x} \) and \( \vec{y} \), within time slice \( t \).

Among the building blocks of the \( 2 \times 2 \) correlation matrix \( C_{ij}(t, t_0) = \langle \hat{\Phi}_j^\dagger(t) \hat{\Phi}_i(t_0) \rangle \) are light-quark propagators, which are computed from random \( Z_2 \)-source estimators living on time slice \( t_0 \) only, and heavy-quark propagators, which are treated in the static approximation. Gaussian smearing of the quark fields and APE fuzzing of the gauge field links is applied.

Simulations are done on a \( L^3 \times T = 10^3 \times 30 \) anisotropic lattice with a (bare) aspect ratio of \( a_s/a_t = 3 \). We use the tadpole improved gauge field action of [7] with \( \beta = 2.4 \). This corresponds to a spatial lattice constant of \( a_s \simeq 0.25 \text{fm} \), \( a_s^{-1} \simeq 800 \text{MeV} \). The (quenched) anisotropic Wilson fermion action is augmented with a clover term limited to spatial planes. Only spatial directions are improved with renormalization factors \( u_s = (\Box)^{1/4} \), while \( u_t = 1 \) in the time direction. The Wilson hopping parameter \( \kappa = 0.0679 \) leads to the mass ratio \( m_\pi/m_\rho \simeq 0.75 \).

3. SPECTRAL DENSITY

Consider an operator combination \( \Phi_\nu = v_1 \Phi_1 + v_2 \Phi_2 \) and the corresponding time correlation function \( C_\nu(t, t_0) = \langle \hat{\Phi}_\nu^\dagger(t) \hat{\Phi}_\nu(t_0) \rangle \). We will fit an \( \omega \)-discretization of the spectral model

\[
F(\rho_T|t, t_0) = \int_{-\infty}^{+\infty} d\omega \rho_T(\omega) \exp(-\omega(t - t_0)) \]

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where $\exp_T(\omega, t) = e^{-\omega t + \omega T \Theta(-\omega)}$. The spectral density $\rho_T(\omega)$ is a discrete sum of $\delta$-peaks

$$\rho_T(\omega) = \sum_{n \neq 0} \delta(\omega - \omega_n) |\langle n|\hat{\Phi}_v(t_0)|0\rangle|^2 c_n, \quad (4)$$

here $c_n = e^{-\omega_n T \Theta(-\omega_n)}$. The $\chi^2$-distance of the spectral model from the lattice data, involving $C_v(t_1, t_0) = F(\rho_T(t_1, t_0))$, is computed with the full covariance matrix between all time slices $t_1 \neq t_0$.

We use the discretization $\Delta \omega = 0.04$, $\omega_k = \Delta \omega k$, $k = -50 \ldots + 125$, and $\rho_k = \Delta \omega \rho_T(\omega_k)$.

Assuming minimal information about $\rho$ the Bayesian prior probability is $\propto e^{-\alpha S}$, where $\alpha$ is a parameter and

$$S = \sum_{k}(\rho_k - m_k - \rho_k \ln \frac{\rho_k}{m_k}) \quad (5)$$

is the entropy relative to a default model $m$.

Minimizing the functional

$$W[\rho] = \frac{\chi^2}{2} - \alpha S \quad (6)$$

then yields the most likely spectral density distribution $\rho$. Unlike in [3], our implementation is based on

$$Z_W = \int [d\rho] e^{-\beta_W W[\rho]} \quad (7)$$

using simulated annealing, or cooling, $\beta_W^{-1} \to 0$. Local Metropolis updates $\rho_k \to x \rho_k$ where $x$ is randomly drawn from the p.d.f. $p_2(x) = xe^{-x}$ are employed.

4. RESULTS

Analyzing the correlation function of a single local heavy-light meson operator, see Fig. 3, reveals two distinct spectral peaks for $\omega > 0$. The dominant narrow peak corresponds to the wide log-linear stretch of the correlation function. The smaller and wider peak originates with data on a few early time slices, as closer examination shows. This attests to the sensitivity of the method. The nature of this excited state is not clear.

The examples of spectral densities shown in Fig. 3 correspond to the two-meson operators [4] and [4], respectively, both at relative distance $r = 4$. The nonlocal operator data are noisy to an extent that precludes the use of standard plateau methods. Bayesian inference still yields a spectral peak, though broad. Empirically, the length of a log-linear stretch (plateau) and the width of the corresponding peak appear to be inversely related. The spectral densities in Fig. 3 stem from averaging over four annealing starts. The spikes tend to smooth out for a large number of starts.

There are three main issues with the current approach: Dependence of the results on the entropy weight parameter $\alpha$, on the default model $m$, and on the start configuration for annealing.

The minimum of the functional $W[\rho]$ is unique. Different annealing start configurations $\rho$ will however move only into the vicinity of the minimum. (Information about the shallowness of $W$ at the minimum may thus be inferred.) It turns out that integrated quantities like the peak volume and the peak energy

$$Z_n = \sum_{k \in \Delta_n} \rho_k = |\langle n|\hat{\Phi}_v(t_0)|0\rangle|^2 \quad (8)$$

$$E_n = Z_n^{-1} \sum_{k \in \Delta_n} \rho_k \omega_k, \quad (9)$$

where $\Delta_n$ is the domain of peak $n$, are insensitive to annealing starts, whereas the micro structure (fringes) of the peaks can vary considerably. In
this context isolated spikes, like in Fig. 3, are common with fine discretization, but have almost no effect on $Z_n$ and $E_n$. Those quantities are also extremely stable with respect to changes of $m$ and $\alpha$. Typical ranges of stability are a remarkable $m \approx 10^{-12} \ldots 10^0$ and $\alpha \approx 10^{-5} \ldots 10^1$.

Figure 2. Two-meson correlators and spectral densities at $r = 4$. The top and bottom pair of figures correspond to $\Phi_1$ and $\Phi_2$, respectively.

We have used 628 gauge configurations. The corresponding statistical errors on $Z_n, E_n$ are comparable to fluctuations due to different annealing starts. Errors bars in Fig. 3 reflect annealing start variances.

Preliminary results for the $r$ dependence of selected quantities are shown in Fig. 3. The $Z_2/Z_1$ ratio points at an enhancement of $\Phi_2$-excitations (gluon degrees of freedom) over $\Phi_1$-excitations (quark d.o.f.) as the relative distance is decreased. Those two mechanisms of interaction are repulsive (in $I = 2$), but move in opposite directions with $r$. They possibly compete at small $r < 1$ in the chiral limit [7]. The adiabatic potential $V_{\text{ad}}(r) = \text{Min}(E_1, E_2)$ is repulsive for the present set of lattice parameters.

5. CONCLUSION

Spectroscopic analysis by way of Bayesian inference is a very powerful method to treat excitations of hadronic systems. The excitations of a heavy-light meson-meson give insight into aspects of strong interaction physics.

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