Is There a Real Difference between $|V_{ub}|_{\text{incl}}$ vs. $|V_{ub}|_{\text{excl}}$ from Semi-leptonic $B$ Decays?

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Abstract

For a long time there have been active discussions about the sizable differences between $|V_{ub}|_{\text{incl}}$ & $|V_{ub}|_{\text{excl}}$. However, the real $|V_{ub}|_{\text{incl}}$ might be smaller than usually stated. The connection of the worlds of hadrons and quarks (& gluons) is subtle: we cannot ignore final states with a pair of $\bar{K}K$ mesons plus pions: $R(B \to l\nu\pi's) + R(B \to l\nu(\bar{K}K + \pi's)) \approx R([b\bar{q}] \to l\nu u(q_1q_2q_3q_4\ldots\bar{q})$ with $\bar{q} = \bar{u}, \bar{d}$ & $q_i = u, d, s$. It might show the limits or violations of duality close to thresholds. While inclusive FS cannot be measured, exclusive ones can be done soon by LHCb and later also by Belle II about $B^+ \to l^+\nu K^+K^- & B^0 \to l^+\nu K^+K^-\pi^+$. Present data have not given us the information about resonances in the region of 1 - 2 GeV that we need to understand the underlying dynamics in $\Delta S = 0$. Future data for exclusive $B \to l\nu\pi\pi$ might also narrow the gap between $|V_{ub}|_{\text{incl}}$ & $|V_{ub}|_{\text{excl}}$ in the opposite direction. I comment about $B^0_s \to l^+\nu X_u(\Delta S = -1)$.

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1 The ‘Problem’

There is a ‘long’ history about the differences between exclusive vs. inclusive semi-leptonic decays of B mesons. I focus on $B \rightarrow l\nu\pi/\rho$ vs. $B \rightarrow l\nu X_u$ leading to $|V_{ub}|_{\text{excl}} < |V_{ub}|_{\text{incl}}$, but only mention $B \rightarrow l\nu D^{(*)}$ vs. $B \rightarrow l\nu X_c$. Literature use different tools, which is enough for this exercise; for more details one can find it in PDG2014:

$$|V_{ub}|_{\text{incl}} \simeq (4.41 \pm 0.15^{+0.15}_{-0.17}) \cdot 10^{-3}$$  \hspace{1cm} (1)

$$|V_{ub}|_{\text{excl}} \simeq (3.28 \pm 0.29) \cdot 10^{-3}.$$  \hspace{1cm} (2)

It is important to see the details how one needs to produce the numbers [1]:

$$|V_{ub}|_{\text{incl}} \simeq 4.42(1 \pm 0.045_{\text{exp}} \pm 0.034_{\text{th}}) \cdot 10^{-3}$$  \hspace{1cm} (3)

$$|V_{ub}|_{\text{excl}} \simeq 3.23(1 \pm 0.05_{\text{exp}} \pm 0.08_{\text{th}}) \cdot 10^{-3}.$$  \hspace{1cm} (4)

It is not clear why one should average these two values. We realize that the situation is complex, as you can see in the literatures:

$$|V_{ub}|_{\text{excl}} \simeq (3.7 \pm 0.2) \cdot 10^{-3} \hspace{1cm} \text{MILC}$$  \hspace{1cm} (5)

$$|V_{ub}|_{\text{excl}} \simeq (3.3 \pm 0.3) \cdot 10^{-3} \hspace{1cm} \text{HFAG}$$  \hspace{1cm} (6)

MILC describes form factor of $\langle \pi | J_\mu | B \rangle$ from LQCD, while HFAG uses exclusive data. Furthermore one can probe the ‘golden triangle’ based on correlations leading to:

$$|V_{ub}| \simeq (3.45^{+0.23}_{-0.10}) \cdot 10^{-3} \hspace{1cm} \text{CKMfitter}$$  \hspace{1cm} (7)

Some will say these numbers show sizable differences between the values from one ‘camp’ on the higher side of 0.004 and those from the three ‘camps’ on the lower one. While the central numbers have hardly changed in ten years, the gaps have enhanced. Several authors of Refs.[1][2][3] have thought and worked on both sides of this ‘Problem’.

Numbers are not always give the best information we can get from the data. In my view there are three camps to describe final states (FS) with somewhat different tools: inclusive ones from $b \rightarrow ul\bar{\nu}$; MILC and HFAG describe similar landscapes with different tools; results from CKMfitter are based on correlations with other transitions.

We can learn from this situation with three options: (a) We do not understand how to deal with inclusive transitions. (b) It has been suggested that we might have underestimated the uncertainties in $B \rightarrow l\nu\pi$. One was discussed in Ref.[2] about disentangling observables in $B^- \rightarrow l^-\bar{\nu}\pi^+\pi^-$. It is not trivial to measure them, but using tools like HQET, SCET etc. give us more information, if they are within their limits. It was discussed also to probe $B^- \rightarrow l^-\bar{\nu}\pi^+\pi^-$ based on dispersion relations to get model-insensitive analyses [4]. (c) ND is hiding in the correlations & its uncertainties of CKMfitter.

Personally I find options (c) & (b) very interesting. Maybe I am biased; however, I see no reason, why option (a) is a real one.

I add a comment about a very interesting paper [5]. They are courageous to solve both the differences between $|V_{cb}|_{\text{excl}}$ vs. $|V_{cb}|_{\text{incl}}$ and $|V_{ub}|_{\text{excl}}$ vs. $|V_{ub}|_{\text{incl}}$. They found in
the ‘global’ landscape they considered there is hardly any chance to find ND without too much impact on the measured widths of $Z \rightarrow \bar{b}b$.

Instead I focus on $|V_{ub}|_{\text{excl}}$ vs. $|V_{ub}|_{\text{incl}}$ on a narrow landscape; we have to measure semi-leptonic decays of $B$ mesons with two & three light hadrons in the final states (FS). My main point is that ”duality” is more subtle. Thus I do not give up on option (c) yet.

We can use heavy quark parameters extracted from $B \rightarrow l\nu X_{c}$. It is not trivial, but we know how to do it. There is another challenge, namely to compare $B^{+} \rightarrow l^{+}\nu X_{u}$ vs. $B^{0} \rightarrow l^{+}\nu X_{u}$ [6]. It is not easy, but there is a prize: probing the impact of ‘weak annihilation’ (WA) give us novel lessons about non-perturbative QCD (and more). Furthermore $\Gamma(B \rightarrow l\nu X_{u})$ are under better theoretical control than $\Gamma(B \rightarrow l\nu X_{c})$ in the SM, since the expansion parameter is smaller ($\frac{\mu}{m_{b}}$ vs. $\frac{\mu}{m_{b}-m_{c}}$), and $O(\alpha_{S}^{2})$ corrections are known. In the real world there arise more challenges: to distinguish $b \rightarrow u$ from huge $b \rightarrow c$ backgrounds one applies cuts on variables like charged lepton energy $E_{l}$, hadronic mass $M_{X}$ and the lepton-pair invariance mass $q^{2}$ to ‘manageable’ proportions. We have learnt from detailed analyses of radiative $B \rightarrow \gamma X_{s}$ decays [7]. Furthermore one has to deal with other backgrounds that are not connected with the production of beauty hadrons.

BaBar and Belle have measured the energy of the charged lepton in the region of $\sim 2.0 - 2.6$ GeV. When one gets close to $\sim 2.0$ GeV from above, it is more difficult to measure that. Below one has to depend on the extrapolation down from the measured region; one expects a long tail of $|V_{ub}|$ amplitudes. The questions are: how much and where. The state-of-the-art theoretical tools had been applied to $M_{X}$ & $q^{2}$ in $B \rightarrow l\nu X_{u}$ & also in $B \rightarrow \gamma X_{s}$ and discussed in details [5]. Measuring the hadronic recoil spectrum up to a maximal values $M_{X}^{\text{max}}$ captures the lion share of the $b \rightarrow u$ rate if $M_{X}^{\text{max}}$ is below 1.7 GeV; yet it is still vulnerable to theoretical uncertainties in the low-$q^{2}$ region. One can think to apply combinations, and it has been done for $|V_{ub}|_{\text{incl}}$ in the world of quarks [6].

It has been suggested to measure semi-leptonic $B \rightarrow l\nu\pi\pi$ [4] to test the number of $|V_{ub}|_{\text{excl}}$ from $B \rightarrow l\nu\pi$ using dispersion relations. This class of tools has a long history to describe three-body FS with strong & electromagnetic forces (including chiral symmetry) like $\eta \rightarrow 3\pi$ or $\eta^{(f)} \rightarrow \pi\pi\gamma$.

The usually stated value of $|V_{ub}|_{\text{incl}}$ is based on ”duality” in HEP. In this paper I suggest that its real value might be smaller and thus narrows the gap between $|V_{ub}|_{\text{excl}}$ and $|V_{ub}|_{\text{incl}}$: in semi-leptonic decays of $B$ mesons might not enter the region where semi-local duality works with good accuracy. The reason? To be honest: good/bad luck due to threshold(s) in the region of 1 - 2 GeV with resonances with $\Delta S = 0$ decaying to $\bar{K}K$ plus $\pi$’s produce unusually impact. I cannot predict that, but miracles can happen – rarely. After working hard and with successes about semi-leptonic inclusive rates, I cannot give up easily; instead I suggest we have to follow the data in several ways, which I discuss below. There is another challenge, namely to discuss the different ‘cultures’ in Hadrodynamics & HEP and their definitions of the item ”duality”. Still fundamental dynamics are formulated in the world of quarks, not pions, kaons & $\eta^{(f)}$ and their resonances; therefore ”duality” means for connections between the worlds of quarks & hadrons.
2 Duality: subtle tools & their limits

Operator Product Expansion (OPE) combined with heavy quark expansion (HQE) have been applied to quantify non-perturbative effects in a number of important processes since the early 1980’s [9] including heavy flavor hadrons. These papers and others had mostly focused on inclusive $b \rightarrow c$ amplitudes for semi- & non-leptonic decays of beauty hadrons with final states (FS) of $D$ and resonances $D^*$, $D^{**}$ etc. and the perturbative domain. Later OPE & HQE had been applied also to $b \rightarrow u$ amplitudes with very refined tools like in Refs.[8] and others. Often the impact of WAmplitudes was ignored, but not in Ref.[8]. It might produce a difference in the endpoint of semi-leptonic $B^+$ vs. $B^0$ transitions; however present analyses show little impact of WA [10, 1].

It was explained in Ref.[11] in the beginning of this century why duality cannot be seen as additional assumption beyond quantum field theory. The landscape of quark-hadron duality can be subtle: in the regions of thresholds one has to apply semi-averaging (or ‘smearing’) duality in intelligent ways with a well-known example: we describe $e^+e^- \rightarrow H_c\bar{H}_c^+$ light flavor hadrons in the region of $\psi(3770)$: below we have two very narrow resonances $\psi(1S)$ & $\psi(2S)$). Pairs of open charm mesons appear just below $\psi(3770)$. Thus there are two gaps in the description of hidden & open charm hadrons: a small one between $\psi(2S)$ & just below $\psi(3770)$; a large one between $\psi(1S)$ & $\psi(2S)$. In the world of charm quarks with $m_c \sim 1.3 \text{ GeV}$ without gaps in the world of quarks. It tells us that we have to use semi-averaged duality over a region $\sim 1 - 1.5 \text{ GeV}$ in the world of quarks. Do we have enough space to describe well semi-leptonic decays of beauty mesons Still duality gives unitarity in subtle ways.

In the world of quarks one uses amplitudes $T([b\bar{q}] \rightarrow l^-\bar{\nu}uq)$ to describe inclusive transitions in $T(B \rightarrow l^-\bar{\nu}[\pi's + K K'\pi's])$. Of course, it is easy to add internal pairs of $u$, $d$ & $s$ (anti-)quarks to connect with the world of hadrons. As first guess one might suggest ratios of $\bar{u}u/\bar{d}d \sim 1$ and $\bar{s}s/\bar{u}u \sim 1/3 - 1/2$. It is naive to say that $\sim 14 - 20$ % of the FS comes from $(\bar{u}s)(\bar{s}q)$ and therefore for $K\bar{K}$ pions. Anyway, it enhances the averaged mass of $X_u(\Delta S = 0, -1)$. We have a better understanding of that due to mixing of $\langle 0 |\bar{u}u|0\rangle$, $\langle 0 |\bar{d}d|0\rangle$ between $\langle 0 |\bar{s}s|0\rangle$ with scalar resonances that are not OZI suppressed [13], but so far not quantitatively. Its impact could be smaller. However, the landscape is even much more complex.

Since I had worked with applying OPE & HQE (and trust them), I might be seen as biased; however, I know that experienced HEP people have thought about them and discussed without solving the ‘Problem’ [1]. It seems it could be a good time to look at unusual ways, although the probability to solve that is sizably less than 50 % as I admit.

1 PDG2014 gives us for the charm quark $m_c = (1.275 \pm 0.025) \text{ GeV}$.

2 As well known the definition of quark masses is subtle. We cannot use pole masses as explained in details [12]; instead we have to use scale depending value like for ‘kinetic’ quark masses.
3 Idea about a new road

Decays of $B \rightarrow l^+\nu + X_c$ give mostly described by a few hadrons in the FS, namely $D$, $D^*$ and narrow $D^{**}$ resonances. The situation is more ‘complex’ in $B \rightarrow l^+\nu + X_u$ decays with many hadrons in the FS (and large background). I ‘paint’ the landscape (when I refer to diagrams) to make the value of $|V_{ub}|_{incl}$ smaller. Of course, the ‘primary intermediate’ $u\bar{q}$ (with $\bar{q} = \bar{u}, \bar{d}$) produce ‘secondary’ $u\bar{q}, u\bar{q}, \ldots \bar{q} & \bar{q}$ ones with $u, d, s$ before they give FS $K\bar{K} + \pi's$ with non-zero probability due to (strong) final states interactions (FSI) (or re-scattering). The question is what does that mean quantitatively?

Adding a pair of $\bar{s}s$ to intermediate $u\bar{q}$ (or $u\bar{s}s\bar{q}, \bar{q} ...$) is one thing, but understanding the dynamics is quite another thing. Resonances happen and have impact; one cannot describe that with local operators in general. Close to thresholds we have to deal with semi-averaged duality; its impacts are less predictable. They depend on the features of the FSI, not just their existence\(^3\).

Data list in PDG2014 show $\text{BR}(B^{0,+} \rightarrow l^+\nu X_u(\Delta S = 0)) \sim 2 \cdot 10^{-3}$. However, they depend on extrapolations down from the measured region. Most people think that inclusive semi-leptonic decays of $B^{0,+}$ mesons consist basically of FS with only pions ($+ \eta'^i$). I talk about the combination of two items: (i) $\Delta S = 0$ resonances that produce a pair of strange mesons plus pions in the region of 1 - 2 GeV. (ii) Their impacts might be enhanced being close to thresholds. Those can go to $B^+ \rightarrow l^+\nu[K^++K^−/K^+K^−/K^−K^−/K^+/K^−\pi^+/K^−\pi^−/\pi^−/\pi^+/...]$ and $B^0 \rightarrow l^+\nu[K^−K^−\pi^−/K^−K^−\pi^+/\pi^−/\pi^+/...]$. I focus on FS that LHCb collab. can probe.

3.1 Impact of $K\bar{K}$, $K\bar{K}\pi$ etc. resonances

Obviously LHCb cannot measure inclusive transitions. Maybe also Belle II cannot solve the challenge of ”$|V_{ub}|_{incl}$” > $|V_{ub}|_{excl}$ directly. I suggest that limits or violation of duality can be larger than expected due to combine two items: resonances produce sizable pairs of $K\bar{K}$ & $K\bar{K}\pi$ can be enhanced by being close to thresholds. Maybe just luck? Re-scattering \cite{14,15,16} is crucial for $u\bar{u} \rightarrow ss$, $sq\bar{s}$ etc., which are not described by local operators. It seems to me unlikely to measure inclusive decays $B \rightarrow l\nu K\bar{K} + \pi's$ due a huge background from $B \rightarrow l\nu X_c$ and other sources.

3.1.1 Exclusive semi-leptonic $B^{0,+}$ decays with $\Delta S = 0$

So far we have measured exclusive rates with a (pseudo-)single hadron in the FS including well-known resonances:

$$\text{BR}(B^+ \rightarrow l^+\nu\omega) = (1.19 \pm 0.09) \cdot 10^{-4} \quad , \quad \text{BR}(B^+ \rightarrow l^+\nu\rho^0) = (1.58 \pm 0.11) \cdot 10^{-4}$$

$$\text{BR}(B^+ \rightarrow l^+\nu\pi^0) = (7.80 \pm 0.27) \cdot 10^{-5} \quad , \quad \text{BR}(B^+ \rightarrow l^+\nu\eta') = (2.3 \pm 0.6) \cdot 10^{-5} \quad (8)$$

It was pointed out in Ref.\cite{17} that a more definite conclusion has not been found yet about the impact of gluonia contributions to $\eta'^i$ amplitudes. In my view that is not an

\(^3\)Actually the definitions of limits or violations of duality are fuzzy.
academic discussion. I will come back to that item below.\footnote{The landscape of $B^+ \to l^+ \nu p\bar{p}$, $B^0 \to l^+ \nu p\bar{p}$, $B^0 \to l^+ \nu p\bar{p}K^0$ is very interesting about the impact of FSI & non-perturbative QCD, but these data show no impact on measuring $|V_{ub}|_{\text{incl.}}$.}

\begin{align*}
\text{BR}(B^0 \to l^+ \nu \pi^-) &= (1.45 \pm 0.05) \cdot 10^{-4} \\
\text{BR}(B^0 \to l^+ \nu \rho^-) &= (2.94 \pm 0.21) \cdot 10^{-4}.
\end{align*}

Both $B^+$ & $B^0$ produce around 20% of the inclusive ones in measured exclusive ones. On the other hand multi-body FS give around 80% including both narrow & broad resonances.

Diagrams show with $\bar{q} = \bar{u}, \bar{d}$, added pairs of (anti-) $q_{i,j,k} = u, d, s$ in the FS and connect with hadrons:

\begin{align*}
[b\bar{q}] &\Rightarrow l\nu [u\bar{q}][q_i\bar{q}] \simeq B \to l\nu (\pi\pi/\bar{K}K) \\
[b\bar{q}] &\Rightarrow l\nu u\bar{q}i\bar{q}j\bar{q}k\bar{q} \simeq B \to l\nu (3\pi/\bar{K}K\pi) \\
[b\bar{q}] &\Rightarrow l\nu u\bar{q}i\bar{q}j\bar{q}k\bar{q}k\bar{q} \simeq B \to l\nu (4\pi/\bar{K}K2\pi)
\end{align*}

Some of my comments are based on hand-waving arguments now, but not all.

(a) In these quark diagrams I use "\(\Rightarrow\)" instead of the usual "\(\to\)" to emphasize the impact of FSI; in general those cannot be described with local operators.

(b) I expect that LHCb can probe $B^+ \to l^+ \nu K^+K^-$ and $B^0 \to l^+ \nu K^+K^-\pi^-$ on the level of branching ratios of a few$\times10^{-4}$. It might narrow the gap between $|V_{ub}|_{\text{incl.}}$ and $|V_{ub}|_{\text{excl.}}$ claimed before. If nothing was found there, it is the end of my idea. However, if LHCb will find non-zero data about $B^+ \to l^+ \nu K^+K^-$ & $B^0 \to l^+ \nu K^+K^-\pi^-$, we have to think how much values we expect from limits or violations of duality. It is possible to predict $B \to l\nu K\bar{K}$ decays; however it would need large work analyzing such FS like based on dispersion relations (& hoping to apply chiral symmetry). I do not suggest to work on a project, unless new data show the roads to understand such exclusive FS.

(c) We cannot stop at $\pi, \rho, \omega$ & $\eta(\rho)$. We have to analyze $\pi\pi$ in general as pointed out $[2, 4]$; likewise to go beyond $\omega$. Measuring exclusive $B \to l^+ \nu [3\pi, 4\pi]$ would help our understanding of duality semi-quantitatively and its limits at least.

3.1.2 Future data about resonances with $\Delta S = 0$

Actually there are still different roads leading to the goal. The PDG2014 shows that there are many resonances in the region of 1 - 2 GeV that could contribute in this landscape. Actually there are four classes now:

- One basically produces only pions.
- The second one gives some $\bar{K}K$ pairs like $f_2(1270), f_1(1285), a_2(1320), f_0(1500)$.
- The third one mostly contributes the leading source of $\bar{K}K$ pairs like $\phi(1020), f_1(1420), \eta(1475)$.
There is a fourth one, where we know little about the landscape: \( a_1(1260), \eta(1405), a_0(1450), \eta_2(1645), f_0(1710), \pi(1800), f_2(1950) \). We need more data at low energies and probe with refined analyses.

It might show the connection of low energy collisions of strong forces with weak dynamics. More data and/or analyses can change the situation up or down about FS with a pair of \( \bar{K} \) & \( K \). Actually it might also give new information about the impact of gluonic contributions to \( \eta(0) \) amplitudes mean that \(|V_{ub}|_{\text{incl}}\) is smaller than claimed; i.e., the discussion of \( \eta(0) \) wave functions [17] enters a new stage here.

### 3.2 Another comment about diagrams

We have some experience about the complex landscapes of diagrams vs. local operators. We can describe transitions of \( \bar{q}q \to s \bar{s}, q = u, d \) with a local operator and used for \( s \to q\bar{s}q \) or \( q \to s\bar{s}q \). However the situation is very different for suppressed semi-leptonic \( B \) amplitudes. Diagrams can show (strong) re-scattering [14, 15, 16]. However the latter cannot be described with local operators, while the impact of re-scattering is crucial.

We can see other examples from non-leptonic transitions, namely to look at the data of \( B \to 2\pi, 3\pi \) & \( 4\pi \): \( \text{BR}(B^0 \to \pi^+\pi^-) = (5.12 \pm 0.19) \times 10^{-6}, \text{BR}(B^0 \to \rho^0\rho^0) = (2.0 \pm 0.5) \times 10^{-6}, \text{BR}(B^0 \to \rho^0\pi^\pm) = (2.30 \pm 0.23) \times 10^{-5}, \text{BR}(B^0 \to 2\pi^+2\pi^-) < 1.93 \times 10^{-5} \) and \( \text{BR}(B^+ \to \rho^0\pi^+) = (1.52 \pm 0.14) \times 10^{-5} \). It hardly suggests we can describe this landscape with local operators.

### 3.3 Exclusive semi-leptonic \( B_s^0 \) decays with \( \Delta S = -1 \)

So far we have hardly any information about \( \text{BR}(B_s^0 \to l^+\nu K^-\bar{K}^+) \) – actually even for predicted rates: \( X_u^{(s)}(\Delta S = -1) = \left[ K^-/K_S\pi^-/K^-\pi^+\pi^-/K^-K^+K^-/K^-K_SK^+\pi^-/... \right] \). Can one find \( B_s^0 \to l^+\nu K^-\bar{K}^+ \) due to re-scattering? Possible, however PDG2014 shows no sign for resonances in the region of 1 - 2 GeV leading to hadronic FS with \( K\bar{K}K \) (except \( \phi \)) and \( K\bar{K}K\pi \). Of course, we have to probe \( B_s^0 \to l^+\nu K^-\pi^+\pi^- \). It was pointed out to analyze \( B_s^0 \to l^+\nu K^-\pi^+\pi^- \) and compare with \( B \to K^*l^+l^- \) [18]. Furthermore we have to include also broad resonances.

The landscape of suppressed semi-leptonic \( B_s^0 \) decays is simpler than in \( B^{0,+} \) ones. Therefore one can compare the numbers \(|V_{ub}|_{\text{incl}}\) vs. \(|V_{ub}|_{\text{excl}}\) from \( B_s^0 \) decays with \(|V_{ub}|_{\text{incl}}\) vs. \(|V_{ub}|_{\text{excl}}\) ones.

### 3.4 Short comment about \( \Lambda_b^0 \to l^-\bar{\nu}[p...] \)

It is unlike that Belle II will go after beauty baryons. However, LHCb has measured \( \text{BR}(\Lambda_b^0 \to \mu^-\bar{\nu}p) = (3.9 \pm 0.8) \times 10^{-4} \) [19]. It is the first semi-leptonic suppressed decays of heavy flavor baryons in general. That is quite an achievement itself.

On the other hand I am not sure about the value given \(|V_{ub}| = (3.27 \pm 0.15 \pm 0.17 \pm 0.06) \times 10^{-3} \) is close to real value. It depends very much what LQCD gives us quantitatively.
It has to be tested by measuring $\Lambda^0_b \rightarrow l^- \bar{\nu} p \pi^+ \pi^-$, $l^- \bar{\nu} p K^+ K^-$, but we should not just trust simulations.

### 3.5 About the future of dispersion relations

If transitions of $B^- \rightarrow l^- \bar{\nu} \pi^+ \pi^-/K^+ K^-$ and/or $\bar{B}^0 \rightarrow l^- \bar{\nu} \pi^+ \pi^- \pi^+/K^+ K^- \pi^+$ were found, we make a strong case to use refined dispersion relations to predict semi-quantitatively \[13, 20, 4\]. As said before, it makes much more work and time, but it would be us a prize, namely to change a idea into an understand not only non-perturbative QCD, but solve a problem about the connection of the CKM matrix for flavor dynamics.

The authors of Ref.\[4\] made reasonable assumptions. However in my view they are not truly model-independent now. For examples: (a) They assumed that $\eta^{(t)}$ are describe by $\bar{q}_i q_i$, but not with two (constitute) gluon states; it was pointed out in Ref.\[17\] that we need more discussions & more data. (b) Can one ignore re-scattering $\pi \pi \rightarrow \bar{K} K$ on the very limited parts of the FS? It is an item about duality. It was pointed out in very recent review Ref.\[1\] in details about $|V_{ub}|_{incl}$. Of course, Ref.\[4\] was submitted before the review "The Physics of the B Factories"; however, the very active discussions have been going on about very subtle items for a long time.

It seems to me that the meaning of ‘duality’ is somewhat different in the world of hadrodynamics. For example, one can look at diagrams like $\bar{s}s \rightarrow f_0(980) \rightarrow \pi \pi$; one can try to describe them with ”effective operators”, but only in a small region and depend on chiral symmetry, not in general. Or one can discuss spectroscopies of $D^{(*)}D^{(*)}$ or $B^{(*)}B^{(*)}$ close to thresholds \[21\]. It is not clear to me, why one can use bare poles of heavy flavor mesons, when one goes for accuracy; it is the opposite for the world of quarks. Furthermore the situation is much more subtle, when one discuss weak transitions. Or one can look at the diagrams in Fig. 2 of Ref.\[4\]; hadrons are shown there, but not (anti-)quarks.

Even so, it shows the bridge between Hadrodynamics and HEP, but "a lot of water has still passing under the bridge"; i.e., it needs much more work, but also connections between Hadrodynamics and HEP.

### 4 Summary

The difference between $|V_{ub}|_{excl}$ vs. $|V_{ub}|_{incl}$ values seems to be sizable after many discussions. I suggest that real value of $|V_{ub}|_{incl}$ might be smaller and makes the gap smaller with $|V_{ub}|_{excl}$. My main points are:

- The landscape of SM suppressed semi-leptonic $B \rightarrow l \nu X_u$ is more complex than expected before and more subtle due to the limits or violations of duality close to thresholds.
- There is not a true prediction. This idea can be found to be incorrect, or its impact is tiny – or it has a chance to put us on the right roads. If so, we cannot ignore
FS with $KK$, $\bar{K}K + \pi$’s in general, although their rates are smaller than with $\pi$’s. One can describe this landscape with somewhat different words: Re-scattering/FSI is important due to non-perturbative QCD.

- I am not saying that these inclusive ones can be measured now or ‘soon’. However in the future LHCb and later Belle II can test this idea by probe exclusive one, namely $B^+ \to l^+ \nu K^+ K^-$ & $B^0 \to l^+ \nu K^+ K^- \pi^-$ and maybe $B^0_s \to l^+ \nu K^- K^+ K^-$. Those enhance the averaged mass of $X_u(\Delta S = 0)$, have impact of $q^2$ and $\Gamma(B^{0,+} \to l\nu X_u(\Delta S = 0)), \frac{d}{dE}\Gamma(B^{0,+} \to l\nu X_u) \leftrightarrow$ low orders of moments. We can do it also for $X_u^{(s)}(\Delta S = -1)$ in different landscapes.

- There are very good reasons to probe light resonances of $\Delta S = 0$ with sizable impact of pairs of $\bar{K}K$ in the region of 1 - 2 GeV with more data and more refined analyses. It might narrow the gap between $|V_{ub}|_{\text{excl}}$ vs. $|V_{ub}|_{\text{incl}}$.

- Exclusive $B^{0,+} \to l\nu 3 \pi/4 \pi/\pi\eta^{(s)}$ have to be probed as much as possible, although the impact of thresholds is less.

- The future situation is simpler for $B^0 \to l^+ \nu X_u^{(s)}(\Delta S = -1)$. I see no reason why $B^0_s \to l^+ \nu K^- K^+ K^-$ give sizable impact on our understanding of fundamental dynamics. I think, we might hardly see a difference in measured values of $|V_{ub}|_{\text{incl}}$ vs. $|V_{ub}|_{\text{excl}}$ in suppressed semi-leptonic $B^0_s$ decays.

- Model-insensitive analyses are the important second step, but not the final one. Experience tells us before that the ‘best’ fitted analyses often do not give the best understanding of the underlying dynamics. The situations is probably different for $\bar{b}b$, $\bar{c}c$ and in particular $\bar{s}s$ states close to thresholds. The landscape of ”hadron-quark duality” is much more complex, namely the connection or not of local vs. non-local operators. Often ”effective operators” are used; however often their impact are not clear (at best) beyond diagrams. It was mentioned in the ‘Preface’ of the Memorial Book for Kolya Uraltsev [3] and discussed in some contributions there.

- We have refined theoretical tools (like dispersion relations) to apply to solve the ”Problem” in the difference of $|V_{ub}|_{\text{incl}}$ vs. $|V_{ub}|_{\text{excl}}$. However, it takes a lot of work to do that one way or another. It would not be fair to suggest a large project about inclusive transitions, unless we have data from exclusive ones showing that we are on the correct ‘road’. Furthermore we have to measure $B^+ \to l^+ \nu K^+ K^-$ and $B^0 \to l^+ \nu K^+ K^- \pi^-$; likewise for $B^0_s \to l^+ \nu K^- K^+ K^-$ and $B^0_s \to l^+ \nu K^- K^+ K^-$ and compare the results.

At least we get novel lessons about non-perturbative QCD – and possibly beyond. These suggestions fall first on the shoulders of my experimental colleagues. It is not unusual how theorists work.
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