A Samplable Multimodal Observation Model for Global Localization and Kidnapping

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**Abstract:** Global localization and kidnapping are two challenging problems in robot localization. The popular method, Monte Carlo Localization (MCL) addresses the problem by sampling uniformly over the state space, which is unfortunately inefficient when the environment is large. To better deal with the the problems, we present a proposal model, named Deep Multimodal Observation Model (DMOM). DMOM takes a map and a 2D laser scan as inputs and outputs a conditional multimodal probability distribution of the pose, making the samples more focusing on the regions with higher likelihood. With such samples, the convergence is expected to be much efficient. Considering that learning based Samplable Observation Model may fail to capture the true pose sometimes, we furthermore propose the ADAPTIVE MIXTURE MCL, which adaptively selects updating mode for each particle to tolerate this situation. Equipped with DMOM, ADAPTIVE MIXTURE MCL can achieve more accurate estimation, faster convergence and better scalability compared with previous methods in both synthetic and real scenes. Even in real environment with long-term changing, ADAPTIVE MIXTURE MCL is able to localize the robot using DMON trained only on simulated observations from a SLAM map, or even a blueprint map.

**Keywords:** Global Localization, Samplable Observation Model, Multimodal

1 Introduction

The ability to accurately localize a robot is the fundamental requirement for many robotics applications, including motion planning, decision making and control [1, 2, 3, 4]. In this paper, we focus on the global localization and kidnapping problem on 2D scenes with 2D laser observation. Given real-time motion information and 2D laser observation, the goal of global localization is to estimate the pose of the robot with respect to a map of the environment without any prior about the robot pose. In the kidnapping problem, where the robot is suddenly taken to some other place without being told, the algorithm should be able to detect this situation and recover from it.

Out of the conventional frameworks for robot localization, including [5, 6, 7, 8], Monte Carlo Localization (MCL) is arguably the most popular and efficient one [9] for the global localization and kidnapping problem. MCL uses a set of particles to represent the estimated probability distribution of the robot pose and iteratively deploys Bayes rule to update this set with the Motion Model and Observation Model. In [7, 10, 11], variants of MCL are proposed to boost its performance, among which MIXTURE MCL [7] achieves the best performance. The key to the success of MIXTURE MCL is DUAL MCL with a handcrafted-feature-based Samplable Observation Model, which can use handcrafted features of observation to provide a probability distribution over state space for sampling particle while traditional MCL is only able to sample from a uniform distribution. However, when working in an environment that is highly symmetrical or dynamic, MIXTURE MCL still faces the problem of high computation cost and unsatisfactory estimation accuracy of localization. The bottleneck lies in the inaccurate Samplable Observation Model. First of all,
the three handcrafted features in [7] are not able to extract enough meaningful information from the laser ranges, resulting in an imprecise probability distribution. Also, MixTure MCL requires physical data collection in the testing environment, which is sometimes not applicable in the real application and limits its generalization ability.

As deep learning techniques can learn a model in a data-driven mode and generalize across datasets, there comes a surge of deep learning models that can be used to construct Samplable Observation Models. They can be classified into the following two categories by how many probability peaks they provide.

1. **Unimodal**: Represented by [12, 13], a classifier or a regression network is trained to find the pose with the largest possibility to obtain the observation in a given map, which provides only a unimodal distribution. Intuitively, due to the fact that the robot can appear on only one pose at a timestamp, a perfect Samplable Observation Model should be unimodal. However, as the environment can be very symmetrical and dynamic (with people walking around or unknown obstacles), observation can be very similar and correspond to several robot poses. Under this kind of situation, unimodal distribution might easily lose track of the robot pose.

2. **Top N**: Embraced by [14, 15, 16], this kind of methods train deep models to extract features of the RGB or 3D point cloud information and features of templates in the database, with which they are able to propose top $N$ possible poses of the robot. There are two limitations in this category. First, it can only choose the top $N$ regions, where $N$ is a fixed handcrafted parameter. However, in real scenes, an observation might correspond to different numbers of poses and $N$ limits the flexibility of this model to approximate the true multimodal distribution. Secondly, the output of this kind of model is simply $N$ regions and additional effort should be taken to construct a probability distribution over the state space.

Thus a good Samplable Observation Models should be able to provide an accurate multimodal probability distribution with adaptive number of probability peaks. Besides, generalization ability, low time consumption and scalability over scene sizes are also important requirements. Moreover, as training samples for Samplable Observation Models may not cover every possible situation, the predicted probability distribution can sometimes fail to capture the correct pose. As MixTURE MCL deploys traditional MCL with a probability of $p_{MCL}$ or DUAL MCL with a probability of $1 - p_{MCL}$ in every updating iteration, the particle set must lose track of the robot pose if MixTURE MCL barely use the DUAL MCL branch under the case above.

To address these challenges, we propose the Deep Multimodal Observation Model (DMOM) and ADAPTIVE MixTURE MCL. As shown in Figure 1, DMOM first aggregates features of the observation and a pre-built map without obstacles, and then computes their similarity, followed by a distribution decoder to lift the similarity matrix to a probability distribution over the state space with adaptive number of probability peaks. Different from the way in which MixTURE MCL combines the traditional MCL and DUAL MCL, ADAPTIVE MixTURE MCL first divides the whole particle set into highly trusted set and untrusted set and then deploys traditional MCL and DUAL MCL with DMOM respectively for these two parts. Experiments demonstrate that DMOM has the ability to generalize across different synthetic environments. In real environments, DMOM can be trained by only simulation laser scans in given SLAM-based or floorplan maps and applied to infer the probability distribution with real 2D laser scans. Detailed experiments in both synthetic and real environments demonstrate that ADAPTIVE MixTURE MCL outperforms MCL and MixTURE MCL in both effectiveness and efficiency, even when the Samplable Observation Models in MixTURE MCL is trained by data containing the testing sequences. Even in real, highly dynamic environments like a palace, ADAPTIVE MixTURE MCL is able to accurately localize the robot.

## 2 Problem Formulation

We use $s_t = (x_t, y_t, \theta_t)$ to denote the state (pose) of the robot at timestamp $t$, where $(x_t, y_t)$ is the position and $\theta_t$ indicates the orientation. $Bel(s_t)$ is the probability distribution of $s_t$. We use
Figure 1: The architecture of Deep Multimodal Observation Model.

$M_{env}$ to denote the occupancy matrix for the environment and $a_t$ and $o_t$ respectively for the motion and observation information (2D laser scan in our setting) at timestamp $t$. Note that $a_{t-1}$ is the control command executed during the time interval $[t-1, t]$. Then as shown in (1), the problem is to estimate $Bel(s_t)$ with given sequences of observation and motion information.

$$Bel(s_t) = p(s_t|o_{0:t}, a_{0:t-1}, M_{env})$$  \hfill (1)

Based on Recursive Bayes Filtering, Monte Carlo Localization [7] transforms (1) into (2) with an Markov assumption and Bayes Theorem [17] [18]. In (2), $\eta$ is the normalization term, $p(o_t|s_t, M_{env})$ stands for the Observation Model, $p(s_t|s_{t-1}, a_{t-1}, M_{env})$ describes the Motion Model and $Bel(s_{t-1})$ indicates the Prior (distribution at the last timestamp).

$$Bel(s_t) = \eta p(o_t|s_t, M_{env}) \int p(s_t|s_{t-1}, a_{t-1}, M_{env})Bel(s_{t-1})ds_{t-1}$$  \hfill (2)

To approximate $Bel(s_t)$, MCL [7] uses a set of particles $P_t = \{s^i_t, w^i_t\}$, where $s^i_t$ is the state of the $i$th particle and $w^i_t$ is the weight of it at timestamp $t$. Beginning with random sampling over the state space, MCL iteratively updates the particle set from the previous timestamp by the following steps:

1. **Sampling:** Sample $P_t$ from $P_{t-1}$ according to $w^i_{t-1}$ and update particle states in $P_t$ with $a_{t-1}$.
2. **Weighting:** Reweight particles in $P_t$ with Weighting Observation Model.

DUAL MCL is a “reverse” version of MCL, which deploys three handcrafted features to train a kd-tree as a Samplable Observation Model. To start with, observation data in the same scene are collected and the features of these data are computed to form the Samplable Observation Model. Then DUAL MCL follows the steps below to update the particle set:

1. **Sampling:** Compute features of $o_t$ and sample $P_t$ with the Samplable Observation Model.
2. **Weighting:** Find related particles in $P_{t-1}$ for each particle in $P_t$ with $a_{t-1}$ and set the weights of particles $P_t$ the same as those of related particles.

The most important difference between DUAL MCL and traditional MCL lies in the way they sample the new particle set. Traditional MCL starts from uniform distribution across the state space while DUAL MCL is able to sample from a multimodal prior. However, the Samplable Observation Model might be unstable. Thus Mixture MCL combines both to achieve a relatively satisfying performance on global localization and kidnapping problems.
3 Deep Multimodal Observation Model

In this section, we propose our Deep Multimodal Observation Model (DMOM). As shown in Figure 1, the inputs of DMOM are two images, one for the environment and one for the observation image (here we transform the 2D laser ranges to a 2D image), represented by two matrices $M_{env} \in \mathbb{R}^{H_{env} \times W_{env}}$ and $M_{scan} \in \mathbb{R}^{H_{scan} \times W_{scan}}$. PM, the output of DMOM, is the grid approximation of the probability distribution in Section 3.2. Section 3.1 shows details in these four parts and the loss function. Then we explain why DMOM can approximate the multimodal probability distribution in Section 3.2.

3.1 Architecture

Observation Encoder. As described in (3), $F_{enc}^{scan}$ is first deployed to encode $M_{scan}$ to a feature map and then a Multi-Layer Perceptron (MLP) transforms this feature map to the feature representation $S \in \mathbb{R}^{K \times D}$. We can see that there are $K$ feature vectors in $S$, each of which is related to the feature at each discretized rotation angle. As illustrated in Figure 1, $F_{enc}^{scan}$ consists of several encoding blocks, which contains convolutional layers, batch normalization layers, non-linear activation layers and pooling layers.

$$S = MLP (F_{enc}^{scan} (M_{scan}))$$  

Environment Encoder. As (4) shows, $F_{enc}^{env}$ is similar to $F_{enc}^{scan}$, transforms the map of the environment $M_{env} \in \mathbb{R}^{H_{env} \times W_{env}}$ to its feature map $M \in \mathbb{R}^{H_{env} \times W_{env} \times K}$.

$$M = F_{enc}^{env} (M_{env})$$  

Similarity Computation. Cosine similarity is deployed to compute the similarity between $S$ and $M$, resulting in the similarity feature map $SIM \in \mathbb{R}^{H_{env} \times W_{env} \times K}$. As described in (5), $S_k \in \mathbb{R}^D$ is the $k^{th}$ feature vector in $S$, $M_{i,j} \in \mathbb{R}^D$ is the feature vector on pixel $(i,j)$ on $M$ and $\epsilon$ is a very small number to avoid dividing by zero. $SIM_{i,j,k}$ indicates the $(i,j,k)$ entry in $SIM$.

$$SIM_{i,j,k} = \frac{S_k^T M_{i,j}}{\max \left( \|S_k\|_2 \cdot \|M_{i,j}\|_2, \epsilon \right)}$$  

Distribution Decoder. In (6), $F_{dec}$ and a softmax activation $\sigma$ lift SIM to PM, which stands for the grid approximation of probability distribution over the state space. $F_{dec}$ consists of several decoding blocks, each of which contains convolutional layers, batch normalization layers, non-linear activation layers and an unpooling layer. The unpooling indexes are the same as the pooling indexes in the environment encoder.

$$PM = \sigma (F_{dec} (SIM))$$  

Loss Function. To help the network better capture the useful information for probability distribution generation, Kullback-Leibler divergence Loss [19] (KLD Loss) is deployed to guide the training process of the network. A unimodal probability distribution at the exact pose where the robot obtains the observation is computed as the ground truth, GT. Details about the generation of GT are shown in Appendix C. (7) illustrates how KLD Loss works, where $GT_{i,j,k}$ and $PM_{i,j,k}$ respectively indicate the $(i,j,k)$ entry in GT and PM.

$$LOSS = \sum_{i=1}^{H_{env}} \sum_{j=1}^{W_{env}} \sum_{k=1}^{K} \left[ GT_{i,j,k} \log \frac{GT_{i,j,k}}{PM_{i,j,k}} \right]$$  

3.2 Multi-Modal Effect in DMOM

Why can the unimodal ground truth guide the network to generate a multimodal probability distribution? Assuming that there exists a set of similar observations $\{M_{scan}^l | l = 1, 2, ..., m\}$ obtained
at a set of different states (poses) \( \{ s^l | l = 1, 2, ..., m \} \) in the training set, all the \( S \) generated by observation encoder are extremely similar while environment encoder should obtain the same feature map \( M \), resulting in a set of highly similar probability map \( \{ \text{PM}^l | l = 1, 2, ..., m \} \). The sum of the KLD Losses of these \( m \) samples is minimized only when the output \( \text{PM} \) is an even-distributed multimodal distribution on these \( m \) poses.

We here prove the case in which \( \{ \text{GT}^l | l = 1, 2, ..., m \} \) are one-hot (1 at the ground truth pose and 0s at other entries) and \( M_{\text{scan}} \) are the same. Thus all the \( \text{PM}^l \) are the same, denoted as \( \text{PM} \). The sum of losses is described in (8), where \( \text{PM}_{s^l} \) indicates value on the entry \( s^l \) of \( \text{PM} \).

\[
\text{Loss} = \sum_{l=1}^{m} \left\{ \sum_{i=1}^{H_{\text{enc}}} \sum_{j=1}^{W_{\text{enc}}} \sum_{k=1}^{K} \text{GT}^l_{i,j,k} \log \frac{\text{GT}^l_{i,j,k}}{\text{PM}^l_{i,j,k}} \right\}
\]

\[
= \sum_{l=1}^{m} -\log \text{PM}_{s^l}
\]

Thus the problem turns to:

\[
\text{minimize}_{\text{PM}} \quad \sum_{l=1}^{m} -\log \text{PM}_{s^l}
\]

subject to \( \sum_{s} \text{PM}_s = 1 \) (9)

After applying Lagrange multiplier to it, the problem turns to minimize the term below:

\[
\text{minimize}_{\text{PM}, \lambda} \quad L = \sum_{l=1}^{m} -\log \text{PM}_{s^l} + \lambda \left( 1 - \sum_{s} \text{PM}_s \right)
\]

(10)

Let the derivative of \( L \) with respect to every entry in \( \text{PM} \) and \( \lambda \) be 0, we get:

\[
\frac{1}{\lambda} = \text{PM}_{s^1} = \text{PM}_{s^2} = \ldots = \text{PM}_{s^m}
\]

(11)

Also with \( \sum_{s} \text{PM}_s = 1 \), we can get that only when \( \text{PM}_{s^1} = \text{PM}_{s^2} = \ldots = \text{PM}_{s^m} = \frac{1}{m} \) can the sum of these \( m \) KLD Losses get minimized. As training samples can cover part of the state space, it is obvious that the KLD Loss is able to guide the network to approximate the ground truth multimodal distribution.

4 Adaptive Mixture MCL

As training data is not able to cover the whole state space, \( \text{PM} \) sometimes fails to approximate the accurate probability distribution. To compensate this problem, we propose a novel filter, Adaptive Mixture MCL, which deploys \( \text{PM} \) as a Samplable Observation Model, to tackle with the global localization as well as kidnapping problem.

In Adaptive Mixture MCL, each particle has three elements \( (s^i, w^i_{\text{norm}}, w^i) \), where the first one indicates the pose of this particle, \( w^i \) is the original observation weight from the Weighting Observation Model and \( w^i_{\text{norm}} \) is the normalized weight. In every updating iteration, we first evaluate the degree we trust a particle by computing \( \frac{w^i}{w_{\text{perfect}}} \), where \( w_{\text{perfect}} \) is the weight of “perfect observation” (the same as we can obtain in the given map of the environment). According to the values computed in the previous step, we divide the whole set into \( \mathcal{H} \) and \( \mathcal{L} \). To endure noise in the observation, we introduce a parameter \( w_{\text{cut}} \in (0, 1) \) and if \( \frac{w^i}{w_{\text{perfect}}} > w_{\text{cut}} \), we directly add this particle to \( \mathcal{H} \). If not, this particle is added to \( \mathcal{H} \) with a probability of \( \frac{w^i}{w_{\text{perfect}}} \) or \( \mathcal{L} \) with a probability of \( 1 - \frac{w^i}{w_{\text{perfect}}} \). Finally we deploy normal MCL update for \( \mathcal{H} \) and Dual MCL update with \( \text{PM} \) for \( \mathcal{L} \). An algorithm of Adaptive Mixture MCL is shown in Appendix A.
5 Experiments and Results

In this section, we conduct extensive experiments in both synthetic and real scenes to investigate the following three questions: 1) How well does DMOM approximate the multimodal probability distribution? 2) How well does ADAPTIVE MIXTURE MCL solve the global localization and kidnapping problem? 3) What kind of generalization ability does ADAPTIVE MIXTURE MCL have?

5.1 Experiments Setup

Environments. For synthetic environments, we manually create 20 maps. We collect training data in 18 of them and testing sequences in the other 2 maps. To train DMOM, we add random obstacles in the environments and simulate laser to get the observation. What we feed into DMOM are the original map without obstacles and the observation. For the testing sequential data, we put unknown obstacles or remove some parts of the environment along the path where the robot moves.

For real environments, we conduct experiments on two benchmark real-world dataset: the Royal Alcazar of Seville dataset (UPO) [20] and the Rawseeds indoor dataset collected in the Universit di Milano-Bicocca (Bicocca) [21] [22]. For the Bicocca dataset, we divide the whole scene into two smaller parts for experiments. To train DMOM, we add random obstacles to the map given by these datasets (SLAM-based or floorplan) and collect training data using simulating laser. With these synthetic data, we train DMOM, after which it is applied to real observation in the same scene.

Settings in Baselines and Adaptive Mixture MCL. There are two types of baselines in our experiments: traditional MCL with random sampling and MIXTURE MCL [7]. As the Weighting Observation Model in these two baselines influences their performance a lot, we try several models introduced in [23] [24]. Also we try our best to tune the parameters in these two baseline for better performance than the original parameter set in [23] [24], leading to the final baselines. Note here that the Sampleable Observation Models in MIXTURE MCL introduced in [7] is not robust and we use data containing testing sequences (except for the exact pose at a given timestamp) to train it, which to some extend is “cheating” because an algorithm should not have any prior about the testing sequences. We set the particle number to 500 for these two methods and ADAPTIVE MIXTURE MCL. The random sampling rate of traditional MCL is set to 0.2. The network structure in DMOM remains the same in different experiments to demonstrate the scalability of it. \( w_{cut} \) is set to 0.6. And results for parameter sensitivity experiments on \( w_{cut} \) are shown in Appendix B.

Evaluation Metric. For each testing sequence and method, we run experiments for 100 times. Then the error of the estimated position \((x_{est}, y_{est}, \theta_{est})\) is computed at each updating iteration as follow: 

\[
E_{pos} = \sqrt{(x_{est} - x_{gt})^2 + (y_{est} - y_{gt})^2} \quad \text{and} \quad E_{rot} = |\theta_{est} - \theta_{gt}|. 
\]

Thus at each updating iteration on each testing sequence, we have 100 error values for each method.

To show the converging process of each method, we compute the mean value as well as 95% confidence interval of the 100 error values \(E_{pos}\) at each updating step and plot the changing estimation error of all methods on the same chart to compare their performance. To evaluate how accurate the algorithms converge, we do two “box plots’’ respectively for \(E_{pos}\) in global localization and kidnapping problems. To evaluate how fast and stable an algorithm converges, we first set a condition for converging to the accurate pose: If \(E_{pos}\) is lower than 1 meter and the error in orientation \(E_{rot}\) is lower than 10 degrees for 10 consecutive updating steps, we consider that the algorithm converges to the accurate pose. If an algorithm converges, \(STEPS\) is computed as the number of steps from the beginning of the problem to the first step in these 10. Finally we deploy histogram to show the comparison on converging speeds and rates.

5.2 Visualization Results on the Output of DMOM

In Figure 2, it is clear that DMOM is able to take the observation and map without obstacles as inputs and generate different numbers of probability peaks including the ground truth, which demonstrates DMOM as an acceptable Sampleable Observation Model.
Figure 2: Visualization results on the output of DMOM. As shown in (a)(c)(e), observation with obstacles around is indicated by blue lines. Red squares in (a)(c)(e) indicate the ground truth position. The input of DMOM are observation in (a)(c)(e) and a “clean” map. Green clusters in (b)(d)(f) show the predicted probability peaks. Blue and purple lines in (b)(d)(f) show the observation on the predicted probability peaks. The size of the scene is 43 meters in both width and height.

5.3 Ablation Study

We compare four methods in a testing synthetic environment: MCL, MIXTURE MCL, Mixture MCL with DMOM as Samplable Observation Model and ADAPTIVE MIXTURE MCL. The results for global localization and kidnapping with these four methods are shown in Figure 3. It can be found that MIXTURE MCL and Mixture MCL with DMOM achieve similar performance while the Samplable Observation Model in the former one is “cheating”. This demonstrates that DMOM is able to generalize across synthetic environments and provide an accurate probability distribution over the state space for sampling. Also, it is obviously observed that Adaptive Mixture MCL converges faster and achieves more accurate estimation than Mixture MCL with DMOM, demonstrating the effectiveness of Adaptive Mixture MCL.

Figure 3: Results on the ablation study experiment.

Figure 4: Results on synthetic environments. (a)(b) respectively show the results of $E_{pos}$ in global localization and kidnapping problem. (c)(d) are histograms for converging steps among 600 experiments in global localization and kidnapping problem.

5.4 Results on Localization and Generalization

Train in some synthetic maps and test in “unseen” synthetic maps. Six sequences of synthetic data are collected in two synthetic maps to evaluate the performance of the three methods, leading to 600 experiments for each method. For each sequence of testing data, we first start from global localization and then kidnap the robot for one time. As shown in Figure 4, it can be found that ADAPTIVE MIXTURE MCL achieves significantly better estimation results than the other two. Also
**Adaptive Mixture MCL** achieves the highest converging rate and requires least steps to converge. These demonstrate that DMOM is able to generalize across different synthetic environments and aid **Adaptive Mixture MCL** to accurately localize the robot in "unseen" synthetic scenes.

![Map and trajectories](image1.png)

(a) Map and trajectories

![Converging Results](image2.png)

(b) Converging Results

Figure 5: (a) shows the map of UPO dataset and multiple trajectories they collect. We random select three testing clips among them. (b) shows the localization results.

**Train with simulation observation on SLAM-based or floorplan maps and test on real observation.** As shown in Figure 5 and 6, it can be found that under most cases in real scenes, **Adaptive Mixture MCL** can achieve a more accurate estimation even than the **Mixture MCL** with a "cheating" Samplable Observation Model. These results indicate that DMOM can learn from synthetic laser range data in the given map of a real environment (SLAM-based or floorplan) and generalize to real observation in the same environment, which helps **Adaptive Mixture MCL** to attain accurate estimation of the robot pose in real environments. We also notice that the results of the first experiment on the Bicocca dataset are not satisfying enough. We think the problem lies in that the real scene in this experiment may be very different from that of the floorplan.

![Experiment in part 1 of Bicocca dataset](image3.png)

(a) Experiment in part 1 of Bicocca dataset.

![Experiment in part 2 of Bicocca dataset](image4.png)

(b) Experiment in part 2 of Bicocca dataset.

Figure 6: Results on Bicocca dataset

### 6 Conclusion

In this paper, we propose a samplable deep multimodal observation model, DMOM, as well as **Adaptive Mixture MCL**, which is a novel localization filter for global localization and kidnapping problems. Our extensive experiments demonstrate that DMOM is able to generalize across different synthetic environments. Moreover it can generalize to real environments when trained on only simulated observations from a SLAM-based, or even blueprint maps.
Acknowledgments

If a paper is accepted, the final camera-ready version will (and probably should) include acknowledgments. All acknowledgments go at the end of the paper, including thanks to reviewers who gave useful comments, to colleagues who contributed to the ideas, and to funding agencies and corporate sponsors that provided financial support.

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Appendices

A Adaptive Mixture MCL

Algorithm 1 shows how Adaptive Mixture MCL works. First, the particles in $P_{t-1}$ are divided into two sets: $H$ and $L$ by how well we trust them (line 6 to 16). $w^i$ indicates the original weight (from Weighting Observation Model) of the $i^{th}$ particle and $w_{\text{perfect}}$ is the weight (from Weighting Observation Model) of a “perfect scan” (that is in each direction, the range from laser is the same as what we can observe in the given map). Then as illustrated in line 18 to 29, we deploy traditional MCL on $H$ and Dual MCL on $L$ with PM, which is the output of DMOM.

Algorithm 1: Adaptive Mixture MCL

Input: $P_{t-1}, o_t, a_{t-1}, w_{\text{perfect}}, M_{\text{env}}, w_{\text{cut}}$
Output: $P_t$

begin

PM ← DMOM($o_t, M_{\text{env}}$)

$P_t ← \emptyset$

/* Divide $P_{t-1}$ into $H$ and $L$, respectively for highly trusted set of particles and untrusted set of particles. */

$H ← \emptyset$

$L ← \emptyset$

for $(s^i, w^i_{\text{norm}}, w^i) \in P_{t-1}$ do

if $\frac{w^i}{w_{\text{perfect}}} > w_{\text{cut}}$ then

$H ← H \cup \{(s^i, w^i_{\text{norm}}, w^i)\}$

else

ξ ← random sampling from $[0, 1]$

if $\xi > \frac{w^i}{w_{\text{perfect}}}$ then

$H ← H \cup \{(s^i, w^i_{\text{norm}}, w^i)\}$

else

$L ← L \cup \{(s^i, w^i_{\text{norm}}, w^i)\}$

end

end

/* Normal MCL update for $H$ */

for $i = 1$ to $\|H\|$ do

sample $h$ from $H$ according to $w^i_{\text{norm}}, ..., w_{\text{norm}}$

sample $h' \sim p(h'|a_{t-1}, h)$

$w_{h'} = p(o_t|h')$

$P_t ← P_t \cup (h', w_{h'})$

end

/* Dual MCL update with PM for $L$ */

for $i = 1$ to $\|L\|$ do

sample $l'$ according to PM

sample $l' \sim p(l'|a_{t-1}, l)$

$w_{l'} = w_l$

$P_t ← P_t \cup (l', w_{l'})$

end

$w_{\text{norm}} ← \text{normalize} \; w \in P_t$

return $P_t$

end
B Parameter Sensitivity Experiment

We conduct parameter sensitivity experiments on two important parameters in ADAPTIVE MIXTURE MCL: $w_{cut}$ and the number of particles. Results are shown in Figure 7. When we look at (a) in Figure 7, it can be found that the performance of ADAPTIVE MIXTURE MCL is very stable when number of particles varies and ADAPTIVE MIXTURE MCL can converge to the accurate pose even with only 200 particles. This can significantly decrease the computation cost, especially when deployed in large scenes where traditional MCL requires large amount of particles to converge to the accurate pose.

As shown in (b) in Figure 7, when $w_{cut}$ varies, ADAPTIVE MIXTURE MCL achieves good estimation accuracy. When $w_{cut}$ is lower, ADAPTIVE MIXTURE MCL converges slower because it takes more time for the algorithm to recognize the wrong particles. When $w_{cut}$ is higher than 0.8, the estimation accuracy gets “turbulent”. The reason for this phenomenon lies in that with high $w_{cut}$, the algorithm lacks the ability to endure flaws in the observation and when PM fails to capture the correct pose, many new particles will be sampled at the wrong poses, resulting in higher estimation error and the “turbulent” performance.

![Parameter Sensitivity Experiment on particle number](image1)

(a) Particle number

![Parameter Sensitivity Experiment on $w_{cut}$](image2)

(b) $w_{cut}$

Figure 7: Results on parameter sensitivity experiments

C GT Generation

To generate $\mathbf{GT} \in \mathbb{R}^{H_{env} \times W_{env} \times K}$ for a given pose $(x, y, \theta)$, we begin with an all-zero matrix for $\mathbf{GT}$. Then a GaussianBlur is deployed on a one-hot ground truth matrix of $(x, y)$, resulting in $\mathbf{GT}' \in \mathbb{R}^{H_{env} \times W_{env}}$. As $\theta$ can usually be different from the angle values in the discretized angle...
space, we find the nearest two values ($\theta_1$ and $\theta_2$) to $\theta$ and then a linear interpolation is deployed on these two values as described in (12). $I_{\theta_1}$ is the index for $\theta_1$ and $\text{GT}_{:\cdot;I_{\theta_1}}$ indicate all the entries where the third indicator equals to $I_{\theta_1}$.

$$\text{GT}_{:\cdot;I_{\theta_1}} = \left| \frac{\theta_1 - \theta}{\theta_1 - \theta_2} \right|$$

$$\text{GT}_{:\cdot;I_{\theta_2}} = 1 - \text{GT}_{:\cdot;I_{\theta_1}}$$  \hspace{1cm} (12)

Finally, we aggregate both the position and orientation ground truth as described in (13) and then a normalization is deployed on $\text{GT}$, leading to the final ground truth.

$$\text{GT}_{i,j,k} = \text{GT}_{i,j,k} \cdot \text{GT}_i'$$  \hspace{1cm} (13)

### D More Experiments in synthetic environments.

To demonstrate the robustness of ADAPTIVE MIXTURE MCL, we conduct more experiments in synthetic environments. In the two synthetic environments where we collect testing sequences, we generate 20 random testing sequences, each of which contains one kidnapping. Figure 8 illustrates 6 testing sequences among these 20, where red lines are two trajectories before and after kidnapping.

![Figure 8: Trajectories in synthetic environments.](image)

600 experiments are conducted on these 20 sequences. Results for position error and converging steps are respectively shown in Figure 9 and 10. It can be found that ADAPTIVE MIXTURE MCL achieves much more accurate position estimation results as compared to MIXTURE MCL and MCL. Also ADAPTIVE MIXTURE MCL converges the fastest and attains the highest converging rate.

![Figure 9: The results for position error in global localization and kidnapping problem among 600 experiments.](image)

### E Bayes Filtering

Here we make a brief summary for the probability framework introduced in Monte Carlo Localization [7], the heart of which is the Recursive Bayes Filtering. Bayes filter naturally introduces an
Applying the Markov assumption again, we can make the first term inside the integration simple in the pose of the robot. Thus, we have the following equation for what we have so far about the probability distribution described in (17).

Also, the third term in (15) can be expanded by integrating over the state at time stamp $t - 1$, as described in (17).

Thus, we have the following equation for what we have so far about the probability distribution of the pose of the robot.

Applying the Markov assumption again, we can make the first term inside the integration simple in (19).

(20) summarizes the recursive Bayes filter for computing the possibility distribution $Bel(s_t)$ of the robot pose, where $\eta$ is the normalization term, $p(o_t | s_t)$ stands for the Observation Model, $p(s_t | s_{t-1}, a_{t-1})$ describes the Motion Model and $Bel(s_{t-1})$ stands for the Prior (distribution at
the last time stamp).

\[
Bel(s_t) = \eta p(o_t|s_t, M_{env}) \int p(s_t|s_{t-1}, a_{t-1}, M_{env}) p(s_{t-1}|a_{0:t-1}, o_{0:t-1}, M_{env}) ds_{t-1}
\]

\[
= \eta p(o_t|s_t, M_{env}) \int p(s_t|s_{t-1}, a_{t-1}, M_{env}) Bel(s_{t-1}) ds_{t-1}
\]  

(20)