Lepton Flavour Violating $Z$ Decays in the MSSM

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The possibility to observe lepton flavour violating $Z$ decays in the GigaZ option of DESY’s TESLA project consistently with present bounds from other processes is analyzed in the context of the minimal supersymmetric standard model. In particular, constraints on the slepton mass matrices from radiative lepton decays are updated and taken into account. Their correlation to the present measurement of the muon anomalous dipole moment is briefly discussed.

1. LEPTON FLAVOUR VIOLATION

1.1. Motivation

Lepton flavour number is perturbatively conserved in the Standard Model (SM) but a tiny lepton flavour violation (LFV) is expected in charged sectors when including light massive neutrinos ($\nu$SM) compatible with the observed neutrino oscillations, e.g. $\text{BR}(\ell_j \to \ell I \gamma) \lesssim 10^{-48}$ and $\text{BR}(Z \to \ell I \ell J) \lesssim 10^{-54}$ ($\ell I \neq \ell J$) [1]. Such effects are far beyond the reach of present or future experiments. Therefore, the observation of charged LFV would be a clear signature of physics beyond the SM.

Supersymmetric (SUSY) models introduce mixings in the sneutrino and the charged slepton sectors which could imply flavour–changing processes at rates accessible to upcoming experiments. Following a recent work [2], I report on the TESLA GigaZ [3] potential to observe $Z \to \ell I \ell J$ in SUSY models with unbroken $R$–parity, where the slepton mass matrices constitute a natural source of LFV. At GigaZ the present limits on LFV $Z$ decays from LEP can be improved by a factor of a hundred to a thousand. An appropriate parameterization of the slepton matrices is used. No assumption on the origin of SUSY breaking is made. The relevant LFV parameters are allowed to vary consistently with the present lower limits on SUSY masses and the currently most constraining LFV processes: $\mu \to e\gamma$, $\tau \to e\gamma, \mu\gamma$. Others have smaller ratios in SUSY, e.g. $\text{BR}(\ell J \to 3\ell I) \approx \alpha_{\text{em}} \text{BR}(\ell J \to \ell I \gamma)$ or $\text{R}(\mu Ti \to eTi) \approx 5 \times 10^{-3} \text{BR}(\mu \to e\gamma)$, giving in both cases weaker bounds, according to present experiments.

For a study of $Z \to d I d J$ in SUSY and two–Higgs–doublet models see [4].

1.2. Properties of the effective vertex

In order to compare $Z \to \ell I \ell J$ and $\ell J \to \ell I \gamma$ let us first examine the Lorentz structure of the effective $V\ell I \ell J$ vertex, since it already reveals some distinctive features.

The most general vertex $V\ell I \ell J$ coupling on–shell fermions (leptons) to a vector boson ($V = \gamma, Z$) can be parameterized in terms of four form factors: $F_V, F_A, F_M$ and $F_E$. Unlike the vector and axial–vector (the first two), the magnetic and electric dipole form factors are chirality flipping and therefore they are proportional to a fermion mass, either external or internal (due to virtual fermions running in loops).

For an on–shell (massless) photon $F_V^\gamma = 0$, and, in addition, if $m_{\ell I} \neq m_{\ell J}$ then $F_V^\gamma = 0$. As a consequence, the flavour–changing process $\ell J \to \ell I \gamma$ is determined by (chirally–flipping) dipole transitions only. In contrast, all form factors and chiralities contribute to the decay of a $Z$ boson.

The anomalous magnetic dipole moment is related to $F_M^\gamma$ for identical external leptons.
2. LFV IN SUPERSYMMETRY

2.1. SUSY contributions

The genuine supersymmetric contributions to one loop to the LFV processes \( Z \to \ell_I \ell_J \) and \( \ell_J \to \ell_I \gamma \) are summarized in Fig. 1. There is no contribution at tree level. Note that chargino–sneutrino diagrams in (A) and neutralino–charged leptons in (B) do not couple to the photon. Diagrams of type (C) are not relevant to the photon processes either, since they do not give dipole contributions.

It is remarkable that only the sum of all chargino–sneutrino diagrams on one side and neutralino–charged sleptons on the other are ultraviolet finite and, at the same time, exhibit the decoupling of heavy SUSY particles running in the loops. This helps as a crosscheck for the calculation.

To get LFV two conditions have to be fulfilled: sleptons must mix and their spectrum must be non-degenerate.

2.2. Slepton mass matrices

Since SUSY is broken, fermion and scalar mass matrices will be diagonalized by different rotations in flavour space. This supplies new sources of LFV.

We assume that the mixing takes place only between two generations (\( IJ \)), neglecting the slepton mixing with a third family. Furthermore it is natural to assume no alignment between fermion and scalar fields, which implies a mixing angle of order one. Only the slepton mass splitting is then left as a free LFV parameter. The different contributions are separated in LL, RR or LR, according to the entries of the slepton mass matrix involved the splitting (only LL for sneutrinos). The relevant entries of the (symmetric) mass matrices can be written as

\[
M^2_{\tilde{e}} = \tilde{m}^2 \begin{pmatrix}
1 & \delta^1_{IJ}
0 & 1
\end{pmatrix},
\]

(1)

\[
M^2_{\tilde{\ell}} = \tilde{m}^2 \begin{pmatrix}
1 & \delta^1_{IJ}
\delta^1_{LR} & \delta^1_{JJ}
\delta^1_{RL} & \delta^1_{JR}
\delta^1_{RJ} & \delta^1_{RR}
\end{pmatrix},
\]

(2)

where only one \( \delta^{IJ} \) is taken different from zero in each case. Note that \( \delta^1_{LR} \) and \( \delta^1_{JR} \) are flavour conserving. There are two free parameters: the mass eigenvalues \( \tilde{m}_1^2 \) and \( \tilde{m}_2^2 \) or else the mass scale \( \tilde{m}^2 = m_1 \tilde{m}_2 \) and the mass splitting \( \delta = (\tilde{m}_2^2 - \tilde{m}_1^2)/(2\tilde{m}^2) \):

\[
\tilde{m}_1^2 = \tilde{m}^2(\sqrt{1+\delta^2} \mp \delta).
\]

The splitting is responsible for any flavour-changing process: \( \delta = 0 \) corresponds to the flavour-conserving case, \( \delta \ll 1 \) can be treated as a non-diagonal mass insertion, and \( \delta \to \infty \) gives \( \tilde{m}_2^2 \to \infty \) (a decoupled second family). The last case implies a maximum flavour-changing rate.

2.3. Calculation

We have obtained analytical expressions for the amplitudes of the processes under study in terms of generic couplings and one-loop tensor integrals [2]. The complete set of Feynman rules, including full chargino and neutralino mixings, have been implemented to get the numerical results.
Table 1
Approximate lower bounds on SUSY mass parameters based on \[5\].

| Component       | Lower Bound            |
|-----------------|------------------------|
| sleptons ($\tilde{m}_1$) | $m_\rho > 45 \text{ GeV}$ |
|                 | $m_{\tilde{\ell}_{L,R}} > 90 \text{ GeV}$ |
| charginos       | $m_{\chi^+_i} > 75 \text{ GeV}$, if $m_\rho > m_{\chi^+_i}$ |
|                 | $m_{\chi^+_i} > 45 \text{ GeV}$, otherwise |
| neutralinos     | $m_{\tilde{\chi}^0_i} > 35 \text{ GeV}$ |

have taken into account the direct constraints on SUSY masses in Table 1. Note that, for negligible scalar trilinears $m_\rho^2 = m_{\ell_L}^2 + M_Z^2 c_W^2 \cos 2\beta$, and the bounds on $m_\rho$ and $m_{\ell_L}$ are correlated. For instance: $m_\rho > 65 \text{ (40) GeV for } \tan\beta = 2 \text{ (50)}$.

There are five input parameters in the SUSY sector to be scanned consistently with above bounds: $M_2$, $\mu$ and $\tan\beta$ (controlling the spectra and couplings of charginos and neutralinos), $\tilde{m}_1$ (relevant lightest scalar mass) and $\delta$ (LFV parameter).

It is instructive to examine the diagrams in terms of gauginos, current eigenstates and mass insertions (the amplitudes are approximately proportional to $\delta$ when this is small). The dominating diagrams are depicted in Fig. 2. The lepton chirality is specified. All the diagrams contributing to $\ell_j \to \ell_i \gamma$ are proportional to $\tan\beta$ times an external fermion mass (because they pick a Yukawa coupling), except for the last one that is proportional to a gaugino mass. Due to the weaker experimental bounds on sneutrino masses, the dominant contributions to $Z \to \ell_i \ell_j$ come from the diagrams mediated by charginos and sneutrinos.

It is important to emphasize that our results depend on contributions with opposite signs that often cancel when varying a parameter. For example, one would expect that the branching ratios are optimized for light slepton masses. However, we observe frequently the opposite effect. The rates can increase by raising the mass of the sleptons up to values of 500 GeV, and only at masses above $1-2$ TeV the asymptotic regime is reached [2].

Figure 2. Dominating diagrams contributing to (a) $Z \to \mu e$ and (b) $\mu \to e\gamma$, showing the approximate linear dependence on the $\delta$ insertions (crosses), the fermion mass insertions (big dots) and the Yukawa couplings (open circles).

### 3. SUSY Predictions for $Z \to \ell_i \ell_j$

Let us first consider the process $Z \to \ell_i \ell_j$ uncorrelated from other LFV processes. For SUSY masses above the current limits it is possible to have $Z \to \mu e; \tau e; \tau \mu$ at the reach of GigaZ. The maximum rate is obtained when the second slepton, $\tilde{\ell}_1$, is very heavy ($\delta^{\ell_1} \to \infty$).

The largest contribution comes from virtual sneutrino–chargino diagrams (all other contributions are at least one order of magnitude smaller). It gives $\text{BR}(Z \to \ell_1 \ell_j)$ from $2.5 \times 10^{-8}$ for $\tan\beta = 2$ to $7.5 \times 10^{-8}$ for $\tan\beta = 50$, practically independent of the lepton masses. The variation is due to the mild dependence of chargino and sneutrino masses on $\tan\beta$. We find that a branching ratio larger than $2 \times 10^{-9}$ ($2 \times 10^{-8}$) can be obtained with sneutrino masses of up to 305 GeV (85 GeV) and chargino masses of up to 270 GeV (105 GeV). These branching ratios are well below LEP limits of $\mathcal{O}(10^{-6})$ but range within GigaZ sensitivities.

However, most of these values of $\text{BR}(Z \to \ell_1 \ell_j)$
are correlated with an experimentally excluded rate of $\ell_J \rightarrow \ell_I \gamma$. In particular, after scanning for all the parameters in the model we find that $\text{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ implies BR($Z \rightarrow \mu e$) < $1.5 \times 10^{-10}$, below GigaZ reach.

A more promising result is obtained for the processes involving the $\tau$ lepton. It turns out (see next Section) that the bounds from $\tau \rightarrow e\gamma; \mu\gamma$ can be avoided while still keeping $Z \rightarrow \tau e; \tau \mu$ at the reach of the best GigaZ projection (see Fig. 3). In particular, for large $\delta_{LL}^{\ell \ell I}$ and a light sneutrino we get BR($Z \rightarrow \tau e$) $\approx$ BR($Z \rightarrow \tau \mu$) $\approx 1.6 \times 10^{-8}$ for BR($\tau \rightarrow e\gamma$) $\approx$ BR($\tau \rightarrow \mu\gamma$) $\approx 3.5 \times 10^{-8}$ (two orders of magnitude below current limits). We obtain events at the reach of GigaZ with lightest sneutrino masses from 55 to 215 GeV, lightest chargino from 75 to 100 GeV, and tan $\beta$ up to 7. They all need large and negative values of the Higgsino mass parameter $\mu$. The contributions due to charged slepton mixing saturate the experimental bounds to $\tau \rightarrow e\gamma; \mu\gamma$ giving at most an effect one order of magnitude below the reach of GigaZ in $Z \rightarrow \tau e; \tau \mu$.

4. BOUNDS FROM $\ell_J \rightarrow \ell_I \gamma$

The bounds on the mass insertions $\delta^{\ell \ell I}$ establish how severe is the flavour problem in the lepton sector of the MSSM, since they provide the most stringent constraints on the slepton LFV mass terms available today. We have updated the early works by Masiero and collaborators [6] to include more recent experiments and, in particular, the complete calculation in the MSSM: only photino–mediated diagrams had been considered in [6], but they are typically subdominant as pointed out already by Ref. [7]. Moreover, the frequent cancellations of different contributions demand a careful treatment.

To estimate the MSSM prediction we combine low and high values of the relevant parameters: tan $\beta = 2; 50$, $\tilde{m}_1 = 100; 500$ GeV, and the gaugino and higgsino mass parameters $M_2 = 150; 500$ GeV and $\mu = \pm 150; \pm 500$ GeV. A summary of the results [2] follows.

The bounds on the first two families are very restrictive. For $\delta_{LL}^{\ell \ell I}$, $\delta_{LR}^{\ell \ell I}$ and $\delta_{RR}^{\ell \ell I}$ they range from $\mathcal{O}(10^{-3})$ to $\mathcal{O}(10^{-5})$ and are stronger for high tan $\beta$. This demands a very high degeneracy between the selectron and the smuon. For $\delta_{LR}^{12}$ the limits are of $\mathcal{O}(10^{-6})$, practically independent of tan $\beta$. This implies just that the scalar trilinears, usually assumed proportional to Yukawa couplings, are small.

The experimental bounds on the mass insertions involving the third family are much weaker. In particular, for small tan $\beta$ we find no bounds on any $\delta^{13}$ (except for $\delta_{LR}^{13}$). For large tan $\beta$ the bounds are (depending on the values of the SUSY–breaking masses) $\delta_{LL}^{13} = 0.03$ to 1.3; $\delta_{LR}^{13} = 0.14$ to $\infty$; and $\delta_{RR}^{13} = 0.11$ to $\infty$. For the $LR$ mass insertions we find $\delta_{LR}^{13} = 0.05$ to $\infty$, independent of tan $\beta$.

5. RELATION TO $(g_\mu - 2)$

A $g_\mu - 2$ correction would be generated by the diagrams in Fig. 2b if no mass insertions $\delta_{LL}^{\ell \ell I}$, $\delta_{LR}^{\ell \ell I}$, $\delta_{RR}^{\ell \ell I}$ are included and $\delta_{LR}^{13}$ is replaced by $\delta_{LR}^{22}$. In this sense, $g_\mu - 2$ is a normalization of the branching ratio BR($\ell_J \rightarrow \ell_I \gamma$) for processes changing the muon flavour.

The new data from the Brookhaven muon $g - 2$ experiment [8] confirms the previous measurement with twice the precision. Since the (revised) SM prediction [9] was already off by more than one standard deviation, the present ‘discrepancy’ has increased and is now approximately $\delta a_\mu = a_\mu^{\exp} - a_\mu^{\SM} = (24 \pm 10) \times 10^{-10}$. It is nevertheless affected by significant theoretical uncertainties, since different groups disagree in their estimate of the hadronic contributions.

Trying to extract conclusions of possible new physics is therefore rather speculative but, taking the discrepancy seriously, it seems to indicate that the muon dipole moment may need non–standard contributions of positive sign. Blaming it on SUSY, we obtain, in agreement with [10], positive or negative contributions correlated with the sign of the Higgsino mass parameter $\mu$ and similar in size to the weak corrections. Extra assumptions have to be made in order to constrain LFV processes from $(g_\mu - 2)$ but, in any case, the favourite region for $Z \rightarrow \tau \mu$ at GigaZ, requiring a negative $\mu$ parameter (Fig. 3), seems disfavoured.
Figure 3. \( \text{BR}(Z \rightarrow \tau \mu) \) and \( \text{BR}(\tau \rightarrow \mu \gamma) \) as a function of the lightest sneutrino mass (\( \tilde{m}_1 \)) with the other one decoupled (\( \delta_{LL}^{23} \rightarrow \infty \)), in several SUSY scenarios at the reach of the best GigaZ projection (\( \kappa = 2 \)).

6. CONCLUSIONS

SUSY models introduce LFV corrections which are proportional to slepton mass squared differences. We have shown that the non-observation of \( \mu \rightarrow e \gamma \) precludes the observation of \( Z \rightarrow \mu e \) at GigaZ and implies at least a one permil degeneracy between the lightest slepton families (maybe justifiable by the weakness of their Yukawa couplings). In contrast, the current bounds on \( \tau \rightarrow e \gamma; \mu \gamma \) introduce only weak constraints (no flavour problem for the third lepton family) and there is a (small) window for the observation of \( Z \rightarrow \tau \mu \) at the best projection of GigaZ.

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