Development of mathematical models of micromechanical devices

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Abstract. The article is devoted to the temperature error of micromechanical accelerometers, as well as the development of structural and technological solutions to improve the accuracy parameters of micromechanical devices, primarily the stabilization of output parameters. To reduce the error, a number of methodologies and design and technological solutions were developed that helped to improve accuracy, i.e. reduce temperature error. Analytical methods for estimating errors, methods for analyzing the effect of temperature on a measuring circuit and identifying elements that lead to a distortion of the measuring function of conversion were considered. Different types of errors were considered. Used statistical methods. Calibration issues considered. The proposed techniques can significantly improve the accuracy of determining the error model coefficients, as a result of which a more accurate accelerometer calibration is obtained. The use of the algorithmic component has significantly reduced the effect of temperature on the output signals.

1. Introduction
As can be seen from studies [1-5] the issues of reducing the temperature error of micromechanical devices are relevant at the present time. Approximately 50% of industrial processes are related to temperature measurement and control. Converters are often used in industry they operate in accordance with thermoelectric principles. To improve the reliability of measurements, it is necessary to calibrate these converters, preferably with low cost. The separation of integrally formed (complex) technologies, which include the technology of micromechanical system, is a characteristic feature of the global information technology development. Not only in our country but also abroad there is a steady growth of interest in the integrated sensors development which is associated with the possibility of effective solving a number of monitoring and control tasks. The current designs of integrated acceleration sensors do not meet modern requirements due to the high level of manufacturing complexity, as well as the temporary instability of metrological characteristics and low resource.

The accuracy of the model calibration coefficients cannot be higher than their instability. It is the instability of the sensing transducer (ST) error model coefficients which principally characterize the accuracy of the inertia mass (IM) movement parameters.

In practice, the development of an error model for sensors and a method for determining its coefficients are closely related and constitute a single task.

Temperature, vibration and cross acceleration are main reasons concerned errors of the MEMS-accelerometer measurement. Temperature change of the environment leads to a change of the dielectric
constant ε, the gap between the pendulum plate and the covers.

2. Materials and methods
The error model of the accelerometers is similar but distinguished by some additional components, taking into account the design features and principle of operation. It normally includes the following values: zero shift, scale coefficient, angles of non-orthogonality.

The sensing transducer error model is becoming more complicated with an increase in the level of the required accuracy. Depending on the required accuracy, the quantity and range of physical quantities taken as source data, the available test-bench equipment and the required measurement range of the output values, the error models can differ significantly in their detail, include the variability of the above parameters, their dependence on external influences. In addition, if the calibration of the ST is performed as part of the IM, then it also defined the parameters for recalculation of accelerometer readings to one point and correlation of the ST data moments to one timeline [6].

The analysis of mass-produced micromechanical devices shows that the trend towards a decrease in their weight and size characteristics is ahead of the trend towards a decrease in power consumption. High installation densities and power consumption increase the thermal load on inertial sensors, which leads to an increase in scale factor instability and temperature drift of the output signals.

Temperature change of the environment is one of the main reasons causing measurement errors of the micromechanical accelerometer (MMA). Additional zero shift due to temperature variation:

\[ \Delta W_f = \kappa_f \cdot \Delta T = \kappa_f \cdot T \cdot t, \]

where \( \kappa_f \) – the thermal drift of the zeroes of accelerometers; \( \Delta T \) – the temperature change during the test, \( T \) – the rate of temperature change; \( t \) – is the time of the test.

The most important MMA parameters are the range of measured accelerations, sensitivity, usually expressed as the ratio between the signal in volts and acceleration, nonlinearity in percent of the full scale, noise, fluctuations (shift) of zero temperature and sensitivity. Due to these characteristics, they have been applied in various fields: military and civil aviation; automotive industry; aerospace equipment; robotics; oil and gas industry; sport; medicine. In some cases, the natural frequency is the natural frequency of the sensor or the resonant frequency, which determines the range of the operating frequency of the sensor [7]. In most applications, the temperature range and the maximum allowable overloads (characteristics related to the operating conditions of the sensors) are important. The determining parameters affect the accuracy of the acceleration determination equal to zero, sensitivity changes (mainly temperature) and sensor noise, which limits the device resolution threshold.

Analytical methods for error detecting and evaluating are based on a functional analysis of the measurement methodology. The use of methods for error detecting and evaluating usually precedes the hypothesis of the error presence from a particular source, including instrumental errors, methodological errors, errors due to conditions other than normal, subjective errors.

Analytical methods are more often used to calculate instrumental and methodological errors, as well as errors due to discrepancy under normal measurement conditions. Special models are created for calculations.

Instrumental errors include all measuring errors of instruments and auxiliary devices: instrument errors, adjustment errors, errors based on devices for linear-angular measurements, wires for connecting electrical measuring instruments, etc. Analytical calculations for measuring accuracy are carried out to assess their theoretical errors and permissible technological errors in the manufacture, that is an essential part of the design.

Errors due to nonadherence to normal measuring conditions are caused by any physical value that goes beyond the normalized range influences measured object and measuring instruments. Temperature, electromagnetic and other fields, atmospheric pressure, excessive humidity, vibrations and many other factors can lead to a distortion of the measurement value and / or measuring information about it.

To estimate the error of “conditions”, in general, one should take into account values influencing
measuring instruments and measured objects. To calculate the effect of the influencing value ψ on the measurement result, you need to know the function f(ψ) of measured physical value change and/or the signal of the measuring instrument when the argument changes (the influence value ψ) and the argument ψ. For example, the change in linear dimension (diameter or height of the measured part) under temperature other than normal is usually associated with the so-called "core model" and calculated using the elementary dependence:

\[ Δl = α(t_i - t_{20}) \],

where \( Δl \) – is the increment of length (positive or negative); \( α \) – is the temperature coefficient of linear expansion; \( t_i \) – is the temperature at measurement; \( t_{20} \) – is the nominal value of the normal temperature during measurement.

To assess the temperature influence on measuring instruments, it is necessary to analyze the temperature influence on the measuring circuit, identify those influenced elements which will distort the function of the measurement conversion, and define the distortion nature. It is often unproductive, because it requires a number of assumptions to build an analytical model of a complex measuring instrument, and it is not always possible to ensure their sufficient rigidity. An experimental estimation of an error is more often used.

Methodical errors arise as the results of theoretical assumptions and simplifications taken to measure or process, as well as the discrepancy of the real object to the assumed model.

Subjective errors may include result calculating errors and errors occurring during measuring instrument or measured object manipulations. To estimate the calculating errors from analog devices, it is possible to construct a geometric model of the error formation due to parallax, as well as a model for rounding or interpolating the fractional unit. In this case, a rigorous analytical assessment is impossible, therefore, the interpolation error is estimated by experimental methods or taken from information sources [8].

The revealing and error evaluation level depends on received information and can vary depending on name or relation scale assessment. The examples of qualitative assessments on the name scale can be a statement about the presence of an error due to certain reasons, a conclusion about the nature of the error ("systematic constant error of the object length when its temperature differs from normal" or "progressive error with a monotonic change of the object temperature").

The noise level of the accelerometer is a noise threshold uncorrelated with external influences in the form of a minimum sensor output that is distinguishable from background noise. Accelerometer noise negatively affects the minimum permissible angle of longitudinal and transverse inclinations and significantly affects speed and position calculating.

Accelerometers, like gyros, suffer from displacement drifts, non-leveling errors, temperature drifts and accelerations, non-linearity, and sensitivity drift. The most important characteristics of accelerometers for their comparative analysis are displacement and its drifts, displacement instability, and also noise. The sensitivity drift, nonlinearity coefficient and other parameters can also be taken into account.

Any accelerometer displacement without double integration acceleration causes a speed error proportional to integration time and a calculated position error squared in time. Uncontrolled zero shift causes an acceleration vector shift related to its true direction, and this applies not only to linear acceleration sensors, but also to gravitational, which must be subtracted from the overall output of the accelerometer. In inertial navigation systems, the drift of the accelerometer makes a significant contribution to the error in speed and position calculating [9]. During orientation measurement, the most significant errors are angular errors when calculating the slopes of the longitudinal and transverse directions.

Sensor displacement instability is a random displacement variation calculated as averaged over a specific time interval. This parameter is calculated using the Allan method for a stationary sensor. As the averaging time increases, the output noise decreases, the slope reaches its minimum point, and then
increases again. Accelerometer displacement instability in most specifications is determined by manufacturers as the best performance achieved under laboratory conditions (at 20 °C and no mechanical effects). The bias stability in real conditions represents the maximum drift of the residual bias error after external factors compensation - temperature, strikes, vibration, and aging [10].

The stability of the transmitting coefficient of the device to the accelerometer largely depends on the magnitude and stability of the steepness of the torque sensor (TS), the steepness torque sensor, which, in turn, depend on the magnitude of the induction and coercivity of the magnet and its temperature:

\[
B_{\text{rt}} = B_{\text{r}0} \left[ 1 - \frac{\alpha(B_{\text{r}0}) \cdot (t_1 - 20)}{100} \right],
\]

\[
H_{\text{rt}} = H_{\text{r}0} \left[ 1 - \frac{\beta(H_{\text{r}0}) \cdot (t_1 - 20)}{100} \right],
\]

(3)

where \( B_{\text{rt}} \) – the residual induction of the permanent magnet; \( H_{\text{rt}} \) – the coercive force of the permanent magnet; \( B_{\text{r}0} \) and \( H_{\text{r}0} \) – are the induction and coercive force at 20°C; \( \alpha(B_{\text{r}0}) \) – the coefficient of working loss of residual induction, the value of which is equal to 0.013% / °C; \( \beta(H_{\text{r}0}) \) – the coefficient of working losses of the coercive force, the value of which is equal to 0.03% / °C.

It is known that the steepness of the TS is determined by the formula:

\[
K_{TS} = \frac{M_{\text{cm}}}{I_{\text{max}}},
\]

(4)

where \( M_{\text{cm}} \) – moment of control; \( I_{\text{max}} \) – maximum control current.

The analytical expression of the steepness of the TS is represented by the formula:

\[
K_{TS} = \frac{B_{l} \cdot D_{av_{w}} \cdot \lambda_{\alpha} \cdot l_{\text{ef}}}{2 \cdot b_{m} \cdot \frac{\pi \cdot D_{av_{eff}}}{h_{m}}} \cdot \lambda_{\alpha} + \frac{B_{l} \cdot H_{r} \cdot 2}{h_{m}} \cdot D_{av_{gap}},
\]

(5)

where \( B_{l} \) – the value of residual induction; \( D_{av_{w}} \) – the average diameter of the magnet; \( b_{m} \) – the length of the magnet; \( \alpha_{\text{w}} \) – the conductivity of the working gap; \( \alpha_{\text{total}} \) – the total scattering conductivity; \( l_{\text{ef}} \) – the length of the effective part of the magnet winding; \( h_{m} \) – the length of the magnet winding; \( H_{r} \) – the value of the coercive force; \( D_{av_{w}} \) – the average diameter of the working gap; \( w \) – the number of turns of the coil; \( D_{av_{gap}} \) – the average diameter of the winding.

The approximate expression of the slope of the TS at a temperature \( t \), \( K_{TS}(t) \), is represented by the formula:

\[
K_{TS}(t) = K_{TS}(t_{1}) - \frac{K_{TS}(t_{2}) - K_{TS}(t_{1})}{t_{2} - t_{1}} \cdot (t - t_{1}),
\]

(6)

\( K_{TS}(t_{1}) \) – the slope of the TS at a temperature of \( 1 \); \( K_{TS}(t_{2}) \) – the slope of the TS at a temperature of \( 2 \).

Determination of the coefficient of compensation for the temperature error of the TS steepness at temperature \( t \), \( K_{TS}(t) \) should be done by measuring its steepness at temperatures \( t_{1}, t_{2} \) with subsequent recalculation of the TS steepness using the formula:

\[
K_{TS}(t) = K_{TS}(t_{1}) + \frac{K_{TS}(t_{1}) - K_{TS}(t_{2})}{t_{2} - t_{1}} \cdot (t - t_{1}),
\]

(7)
$K_{TS}(t_1)$ – the slope of the TS at a temperature of 1; $K_{TS}(t_2)$ – TS steepness at a temperature of 2.

Accelerometer transmission coefficient error in the temperature range from minus 40°C to + 130°C after algorithmic temperature error compensation by the above method does not exceed 0.004%.

3. **Conclusion**

To sum up, we performed the analysis of commercially available accelerometers, which revealed their dominant errors, as well as the constructive-technological reasons for their formation.

The issues of calibration of the accelerometer error model coefficients are of particular importance, since the requirements for the accuracy of determining their scale factor and the orientation angles of the measuring axes are significantly higher than for the cardan systems. In addition, accelerometers should measure apparent acceleration and angular velocity at one point and refer to the same point in time of movement of a material point. In the course of this work, a calibration procedure was developed, which provides an increase in the bench calibration accuracy of the measuring module to achieve residual equivalent drifts.

The algorithmic compensation can significantly reduce the temperature effect on the sensing transducer output signals.

Studies have shown that the proposed calibration of IM by the navigation solution gives a significant gain in the development of navigation parameters compared with the traditional for dynamic effects.

The question analysis showed that nowadays dominant errors for this device are the value and instability of the zero signal and the spatial position of the sensitivity axis. The methods in this work can significantly improve the accuracy of IM error model coefficients determining. As a result, it is possible to estimate residual calibration errors of accelerometers.

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