Investigation of Dynamic Factor, Rail Stress and Track Foundation Modulus of Ballasted Railway Track for Different Bed Conditions Using Numerical Methods

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Abstract. The track stiffness is the primary function of roadbeds thickness and subgrade characteristics. For this purpose, numerical scale track finite element technique representing the ballasted track with multi layered substructure founded on subgrade was simulated. The track deflection, stress was abstracted in static and dynamic conditions. The track significant design parameters: Foundation modulus, rail fatigue strength, rail bending stress and stress on subgrade levels were evaluated by using improved current track design numerical methods and compared against field test results which were carried out on part of MG Double track high speed main line (1600 km). Mathematical equations were developed to correlate the variables; ballast thickness, settlement, track stiffness, rail bending stress and rail fatigue strength on varying subgrade soil modulus. Incorporation of this parametric study will improve and optimise the conventional track design and maintenance standard. A simple improved track design was introduced by using single track stiffness parameter from conventional plate bearing test (PBT) on Force Displacement (FD) conventional curve method. The improved method with deriving equivalent track stiffness from rail pad and track substructure tested C value are accurate and simple. The current test method to determine the track stiffness in live track condition is expensive and unsafe with operational requirements. This PBT is simple, cost saving on labour, safe and without applying live train load.

1. Introduction
Vertical track stiffness is a function of the modulus of elasticity of different layers and components in the railway track systems [1]. Track compendium. PMC Media House. The properties included are rail pad modulus, rail flexural bending and roadbed modulus which is joint performance of ballast, sub ballast and subgrade below as shown in Figure 1.

A study by Pita et al. [2] showed that the optimum study recommends criteria to consider the global stiffness must have more than 52 kN/mm with limited settlement of 28 mm and for soft 32 kN/mm with 46 mm. The maximum bending stress evaluated in UK Rail is 110 MPa. Recent study by [3] concluded that the optimum value of track stiffness is depend on respective line specification and universally accepted value not applicable. The aim of this paper is to evaluate the response on the track foundation modulus C values, rail bending stress, fatigue strength to the varying ballast thickness on changing subgrade soil material value for the local train projects. Therefore, it is necessary to compare the experimental methods with numerical static and dynamic analysis taking into account the elasto plastic properties and non-linear behaviour of track material and repetitive loading to the various speeds.
In the present study, the author studied the response of rail fatigue and bending stress and its influences on C value (function of track stiffness and deformation) on different bed (ballast thickness) conditions on changing subgrade soil modulus. The ballast thickness (100 – 500 mm), sub ballast 300mm were chosen. The subgrade soil modulus was chosen to the range (25 – 48 MPa). The study focused on (PBT) plate bearing test method and deriving the track stiffness and response on rail stress evaluation as per project requirement.

1.1. The (C) definitions and influence
The (C) is track stiffness and called in different name in different textbooks that are; Ballast modulus, Coefficient of Ballast, Track Foundation Modulus (C) [4]. It is denoted in letter (C). Here the author refers as (C). The (C) indicates at which value of (N/mm²) surface pressure the sleeper subsides by 1 mm. The unit is N/mm² or Kg/cm³.

2. Review methodology
This chapter briefs on how the review was carried out:
   i. Rail fatigue and bending stress evaluation from the Field Test result (Plate Bearing Test -(C) value determination test.
   ii. Evaluation of (C) Test and its response for varying ballast thickness on varying subgrade soil condition with numerical modelling.

3. Current design concept of ballasted track structure
The track is designed utilizing load bearing approach to sustain the static and dynamic loads which are transferred to the subgrade through sleeper ballast by ensuring that the strength of the components is not exceeding their capacity and to limit allowable rail bending stress, sleeper bending stress ballast pressure and subgrade pressure [5].
   Current approach on Winkler model (WM) and discrete support model (DSM) are Semi empirical. The dynamic response on the characteristic of the track components is not well understood to form the basis of rational design. It relies on observed dynamic response to an equivalent static response by making use of various load factors. It is understood that ballast and subgrade properties need to be examined and included.

3.1. Track structure design approach
The steps involved are: -
   i. The (C) value parameter derivation from PBT results.
   ii. Mean Rail bending stress (σmean)
   iii. Determination of dynamic factor (DF)
   iv. Calculation of Maximum Bending Stress in rail and Verification of bending stresses against fatigue strength using SMITH diagram.
3.1.1. (C) Derivation methods. The (C) is the track parameter and derivative of track stiffness. The common methods adopted in railway industry are classified into 3 Major groups, that are theoretical, theoretical-experimental, and experimental. Theoretical method is questionable on validity, experimental methods about their field test which leads to time, cost, speed restrictions and safety concern on live track. Hence not possible for all rail roads in all places. Considering the practicality, constructability, minimisation of train traffic, safety, cost, and labour, one of the simple experimental tests is (PBT) on two cycles.

3.1.2. The (C) value using Plate Bearing Test (PBT). The (C) value is derived from the settlement using the (PBT). The conventional (PBT) is carried out on the prepared roadbed surface (which is completed to lay the sleeper) and measured the total settlement by applying rail seat load in two cycles. The basic principle is the intensity of pressure (kPa) exerted by the bearing plate to the resultant deformation provides the total elasticity modulus of the roadbed and subgrade. The plate bearing test with two cycles confirming to the British (BS 1377) [6] and German (DIN 18134:2012-04) [7] standards. The PBT consists of applying static load in uniform increment on a circular plate that is placed on close contact (roadbed) and measuring the deflection for each increment in two cycles. During the first cycle at initial stage, the voids in ballast are closed similar to what happens in actual track. The behaviour on load deformation is linear as shown in Figure 2a. The second cycle represents the track simulation stage on accumulative load wherein the track ballast particles further densified and create a low-voids medium. The behaviour is non-linear due to accumulative load and properties of subgrade and roadbed material [8]. The idealised theoretical and experimental test load settlement behaviour of track simulation immediately after construction i.e., at cycle 1 and after tamping i.e., cycle 2 are shown in figure 2(a) and (b). From the FD developed graph, the slope (s) is obtained from applied load and residual settlement in second cycle. The C value is derived by dividing the area of the loading plate (A) from (s) value as shown in Equation (1).

![Figure 2](image_url)

**Figure 2.** (a) Idealised theoretical load-settlement graph [9], (b) Idealised field experimental test load-settlement graph.

The C value was derived from load deformation behaviour graph on second cycle.

\[
C = \frac{s}{A} \quad (1)
\]

For load evaluation main line passenger cum freight network operated by KTM trains (Locomotive and EMU) on metre gauge Class 26 Blue Tiger with maximum 20-ton axle load arranged in 1.850m front axle spacing (figure 3) was considered. The rail section UIC 54 kg/m, concrete sleeper on 600 mm spacing were considered. The maximum rail seat load transfers to the immediate sleeper ranging 0.43 to 0.65 of (p) [10] i.e., wheel load and transfers to 3 adjacent sleepers. The maximum rail seat load of 50 kN (0.5p) was considered.
Figure 3. DE AC 33 C- HP 3300- KTMB 26 class-Blue Tiger Locomotive: axle load 20 tonnes.

3.2. Development of C value - rail bending stress working equations

In the current practise, the rail design is based on the theory developed by Winkler and later by Zimmerman [4]. Based on recent study [11], negligible differences were evaluated between two track models, that are one on continuous support Winkler Model (WM) assuming elastic foundation and another is discrete supports models (DM) which comprises rail pad sleeper ballast and subgrade stiffness. The WM slightly increase in DM and negligibly decreases in rail deflection [12].

Conventional track consists basically of two parallel continuous beams, the rails, which are fixed at regular intervals onto sleepers’ support. The ballast bed rests on a formation. Winkler hypothesis applies to track support. At each point of support the compressive stress is proportional to the local compression. This relation is,

\[ \alpha = Cw \]  

(2)

Where,

\[ \alpha = \text{Local compressive stress on the support (N/m}^2\text{)} \]
\[ w = \text{local subsidence of the support (m)} \]
\[ C = \text{Foundation Modulus (N/m}^3\text{)} \]

In discreet support

\[ K_d = C A_{rs} \]
\[ K_d = \text{Sum F/Sum w} \]
\[ K = k d/ a \]
\[ a = \text{Spacing between centres of discrete support} \]

Zimmermann’s method is appropriate which considered the influence of adjacent wheel hogging effect on deflection, bending moment and characteristic length. The total Equivalent stiffness (\( k_{eq} \)) was derived with tested standard rail pad stiffness (\( k_p \)) with ballast subgrade stiffness (\( k_s \)).

The rail mean (\( \alpha_{\text{mean}} \)) is derived using general equation (3) and (4) with developed moving three-wheel load influence envelope from the Zimmermann Analysis [10] (figure 4).

\[ M_{max} = \frac{Q L}{4} \]  

(3)

\[ L = \frac{\sqrt{4EI}}{k} \]  

(4)

Where,

\[ w = \text{Rail deflection (m)} \]
\[ Q = \text{wheel load (N)} \]
\[ EI = \text{Rail Bending stiffness, Nm}^2 \]
\[ L = \text{Characteristic length, (m)} \]
\[ k = \text{Foundation coefficient / Track modulus} \]
(N/m/m)
k = Support stiffness
kd = Spring constant of the support = C*A

The Dynamic Factor (DF) is applied to the static load to consider dynamic impact on stresses and deflection in the railroad bed and sub grades. as in equation (5).

\[ \alpha_{max} = DF \alpha_{mean} \]  

The Eisenmann’s DF (\( \phi \)) influences on three factors as in Equation (6)

\[ \phi = 1 + t \cdot \varepsilon \left(1 + \frac{(V-60)}{140}\right) \]  

Where,
t= multiplication factor of standard deviation 1 for 68.3 Probability occurrences, 2 for 95.4 and 3 for 99.7
\( \varepsilon \)= factor depending on track quality 0.1 for very good, 0.2 good and, 0.3 bad condition
s = Speed Factor, V= Train speed (kmph)

![Figure 4](image)

**Figure 4. Moving load bending moment and deflection profile-Zimmermann analysis [10].**

### 3.3. **Verification of bending stresses against fatigue bending strength using SMITH diagram envelop.**

From the Smith Diagram [4] the permitted limit of stress to the fatigue condition for UIC 54 kg UTS 900 was derived. The ultimate tensile strength is 900 N/mm². The upper limit should not exceed yield limit FY (580 N/mm²). The permissible range of stress fluctuation is 280 N/mm² (DK) \( \sigma_q \) (Live Load stress) while the constant background stress of 180 N/mm². For any mean stress along the line OA the safe stress range is AP and AQ. Apart from this additional constant background stress in the rail are due to temperature variation and residual stress while manufacturing process. These are derived with maximum thermal stress of 50 N/mm² as per project requirement and residual stress on manufacturing process of 130 N/mm². The developed fatigue bending strength smith diagram is shown in figure 5.
4. Field test results evaluation and rail bending stress derivation

The C value test on completed double track were carried out on Two sites. Three locations along Site 1 i.e Rawang- Ipoh –Padang Besar Track project (509 kM) and four locations along Site 2 i.e Gemas- Johore Bahru Track Project (200 kM). The test carried out on 14 m width 30m length along the track area over ballast. The plate of size range 450-600 mm dia. with 25 mm thick were used. The increment of loading in 2 cycles are 24 and 48 kN at Site One and increment loading of 150 kPa and 300 kPa in Site 2. The settlement was measured.

Table 1. The summary of C value, rail bending stress derived from plate bearing test.

| Subgrade Modulus N/mm² | Vertical Displacement (mm) | Ballast Thickness (mm) | Derived C Value kg/cm³ | Derived Mean rail Stress N/mm² |
|------------------------|---------------------------|------------------------|------------------------|-------------------------------|
| 48                     | -0.861                    | 100                    | 36.526                 | 52.734                        |
|                        | -0.843                    | 200                    | 37.308                 | 52.726                        |
|                        | -0.838                    | 250                    | 37.507                 | 53.137                        |
|                        | -1.057                    | 300                    | 29.765                 | 52.722                        |
|                        | -0.836                    | 350                    | 37.600                 | 52.726                        |
|                        | -0.838                    | 400                    | 37.509                 | 52.732                        |
|                        | -0.842                    | 450                    | 37.368                 | 52.739                        |
|                        | -0.846                    | 500                    | 37.181                 | 52.726                        |
| 35                     | -0.996                    | 100                    | 31.571                 | 55.060                        |
|                        | -0.960                    | 200                    | 32.763                 | 51.620                        |
|                        | -0.949                    | 250                    | 32.763                 | 52.000                        |
|                        | -1.147                    | 300                    | 33.149                 | 53.170                        |
|                        | -0.937                    | 350                    | 27.408                 | 51.000                        |
|                        | -0.936                    | 400                    | 33.149                 | 52.000                        |
|                        | -0.936                    | 450                    | 33.585                 | 53.000                        |
|                        | -0.939                    | 500                    | 33.606                 | 54.000                        |
| 25                     | -1.185                    | 100                    | 26.539                 | 56.139                        |
|                        | -1.117                    | 200                    | 28.162                 | 53.631                        |
4.1. Evaluation of plate bearing test
The field plate bearing tests (PBT) load-settlement graphs were developed. The $K_{eq}$ was derived from tested Rail Pad stiffness of 100 kN/mm in project requirement. The rail means bending stress $\sigma_{\text{mean}} = M/Z$ was evaluated. The summary of derived C value and Mean Rail stress $\sigma_{\text{mean}}$ are shown in table 2.
1-1: Site 1 in location 1
2-1: Site 2 in location 2

5. Numerical analysis
The numerical model of plate bearing test similar to experimental test on track within tank of size (1.5m x 1.5m x 1.2 m) was simulated. The track substructure (ballast sub-ballast and subgrade) was simulated like field test using ABAQUS programme. The static and dynamic repeated load to the frequency range (1.5-31 Hz) were applied and evaluated. The load frequency of a train is derived as $f=V/L$, where $V$ is the speed (10 to 200 kmph) and $L$ is the distance between axles. The front axle distance of 1.80m for 20t axle load locomotive was assumed. The response on overall deformation and stress distribution were evaluated to determine the C value on different bed (ballast thickness) conditions on changing subgrade soil modulus [13].

### Table 2. Material properties used in numerical analysis.

| Item          | Density ($t/mm^3$) | Elastic Modulus (N/mm²) | Poisson | Friction | $\phi$ | Cohesion (Cu) (N/mm²) | Thickness (mm) |
|--------------|--------------------|-------------------------|---------|----------|-------|-----------------------|----------------|
| Steel Plate  | 7E-9               | 20000                   | 0.35    | -        | -     | -                     | 450 dia 25     |
| Ballast      | 2E-9               | 127                     | 0.35    | 45       | 15    | 0.001                 | Varies (100-500) |
| Sub ballast  | 1.9E-9             | 127                     | 0.35    | 45       | 15    | 0.001                 | 300            |
| Sub-grade    | 1.8E-9             | 25-48 Varies            | 0.35    | -        | -     | -                     | 600            |

Figure 6. Test set up on prepared track ballast - plate bearing test (field photograph).
5.1. Analysis - determination of C value

The numerical model consists of three layers (substructure and subgrade) the top layer is 100-500 mm thick ballast (varying), middle layer is 300 mm thick sub ballast and bottom layer is 600 mm thick subgrade. The subgrade base is fixed support as founded on rigid base. The model is shown Figure 7a. The parameters of materials assumed are shown in table 3. The aim of static analysis is to evaluate the total vertical settlement, static stress on track bed layers for different roadbeds thickness on varying subgrade conditions. From the deformation value the (c) value and rail stress were derived.

Two cycles loadings applied incrementally similar to the field plate bearing test. In first cycle 25 kN and 50 kN in second cycle. In second stage the dynamic analysis were carried out. The aim of dynamic analysis is to evaluate the dynamic factor (DF) i.e., the ratio of dynamic stress and static stress on subgrade.

The vertical settlement and von mises stress distribution are shown in figure 7 (a), (b), (c) and (d).

![FEM modelling](image)

**Figure 7.** (a) FEM simulated model, (b) Displacement at ballast level in E45 subgrade maximum 1.05mm, (c) Load displacement on two cycles for E 47 sub grade modulus, (d) Dynamic stress on subgrade.

| Soil Subgrade | Subgrade Modulus (MPa) | Settlement From Ballast Thickness | C Value From Ballast Thickness | Rail Bending Stress From Ballast Thickness |
|---------------|------------------------|----------------------------------|-------------------------------|-------------------------------------------|
| High          | 47                     | Y= 0.0002x-1.0268                | Y=0.005x+29.444               | Y=-0.0015x*58.714                        |
| Medium        | 35                     | Y = 6e-5x-0.8902                 | Y=0.005x+29.444               | Y=-0.0009x*57.716                        |
| Low           | 25                     | Y = 0.0003x-1.2163               | Y=0.005x+29.444               | Y=0.0004x+56.928                         |

Table 3. Mathematical equation and dynamic factor.
5.2. Results - C value rail bending stress dynamic factor derivation
From the results the linear relationship was observed. The mathematical expressions were obtained for various subgrade with different sub ballast thickness on C value and rail bending stress. The speed factor(s) was derived on mathematical equation, 0.0155X+0.646, X is speed in kmph. The DF Factor of 2.9 for maximum speed of 160 kmph was derived (figure 8(a)).

5.3. Comparison of field and numerical values (settlement, C value, rail fatigue and bending stress)
The (C) value was derived from the PBT load-deformation data. The ballast thickness of 300 mm thick was constructed in both sites. Hence the comparison between FEM and Field is focused on 300 mm ballast thickness only.

Table 4. The summary of settlement, C value, rail stress, fatigue stress derived from site and FEM.

| Location/Model | Settlement (mm) | C Value (kg/cm³) | Mean Rail Stress (MPa) | Dynamic Stress N/mm² | Total Stress (MPa) (Live+Temp+Residual) | Free Stress (MPa) | Fatigue Limit stress Utilization % |
|----------------|----------------|-----------------|------------------------|-----------------------|------------------------------------------|------------------|---------------------------------|
| 1-1            | 0.46           | 65.43           | 53.99                  | 2.9                   | 156                                      | 336              | 124                            | 73 |
| 1-2            | 0.71           | 42.39           | 55.82                  | 2.9                   | 161                                      | 341              | 119                            | 74 |
| 1-3            | 0.35           | 86.00           | 53.06                  | 2.9                   | 153                                      | 333              | 127                            | 72 |
| 2-1            | 1.37           | 21.90           | 59.49                  | 2.9                   | 172                                      | 352              | 108                            | 77 |
| 2-2            | 2.83           | 10.60           | 64.72                  | 2.9                   | 187                                      | 367              | 93                             | 80 |
| 2-3            | 1.20           | 25.00           | 58.67                  | 2.9                   | 170                                      | 350              | 110                            | 76 |
| 2-4            | 1.14           | 26.32           | 58.36                  | 2.9                   | 169                                      | 349              | 111                            | 76 |
| FEM (47 MPa)   |                |                 |                        |                       |                                          | 347              | 113                            | 76 |
| Subgrade       | 1.06           | 28.30           | 57.93                  | 2.9                   | 167                                      |                  |                                 |    |
6. Evaluation and discussions
The displacement decreases with increase of subgrade modulus. It implies that soil subgrade modulus is the dominant factor influencing on rail deflection vertical stress on subgrade, ballast, sub ballast levels. The (track stiffness) C increases with decrease in rail bending stress.

Increase in ballast has two significant effects firstly increase the thickness secondly the distance increases from sleeper base. Hence the stress spreading resulting the reduction on deviatory stress [14] on subgrade. The mathematical linear equations were evaluated relating the variables ballast thickness, C value, rail stress on varying subgrade based on deformations results.

In FEM average settlement of 1.30mm and static stress at subgrade 44 kPa were evaluated. These values are closure to 4mm settlement and 40 kPa stress estimated by Harry Tan. It implies FEM is reliable and realistic.

From the Dynamic Analysis, the maximum dynamic stress was evaluated 190 kPa at the subgrade at the resonance frequency of 15 Hz. The first resonance frequency occurred at 15 hZ. (Refer Figure 8b). It is close to the few literatures evaluated railway frequency value of 16 hZ [15, 16], 15-25 hZ [16].

On comparison of the maximum settlement, Site 1 (0.7mm) is less than Site 2 (2.8mm) Relatively the C value higher, resulting the rail experiences less bending stress. It implies that Site 1 is much stiff than Site 2. Consequently, the Fatigue limit Derived from smith Diagram (Refer Figure 8b). Site 2 reaches close to the limit 280 MPa when comparing to Site 1. It implies for 50 MPa temperature and 130 MPa residual stress, the allowable live load Stress is 280 MPa. The maximum utilization on Site 1 (161MPa-74% of 280MPa), Site 2, (187 MPa - 80 % of 280 MPa). The low stiffness in Site 2 resulted in 93 MPa (20 % of 280 MPa), remaining free strength to reach the limit of 280 MPa. It shows that Site 2 has the tendency to experience more fatigue stress resulting deterioration rate shall be higher than Site 1. Consequently, rail life significantly shall reduce and, maintenance cost shall increase.

7. Conclusions
In this paper, the response of static and dynamic analysis of ballasted track structure for different railroad bed condition on changing subgrade was evaluated by using numerical methods on taking in to account the elasto-plastic properties of subgrade soil and ballast, to consider the non-linear behaviour and to evaluate the plastic deformation due to train repetitive loading. The track stiffness comparison between two sites were evaluated and implies that Site 1 much stiff than Site 2 when comparing settlement, C value and rail stress. The response of rail bending and fatigue strength in different ballast thickness and different subgrade conditions were evaluated. The impact of rail bending stress was influenced by deformation and shows linear variation on Track foundation modulus. Based on reserve capacity of rail fatigue strength, the rail life, deterioration rate could be assessed. A simple improved track design was introduced by using single track stiffness parameter from conventional plate bearing test (PBT) on Force Displacement (FD) conventional curve method. The improved method with deriving equivalent track stiffness from rail pad and track substructure tested C value are accurate and simple. The current test method to determine the track stiffness in live track condition is expensive and unsafe with operational requirements. This PBT is simple, cost saving on labour, safe and without applying live train load.

Mathematical equations were developed to correlate the variables, ballast thickness settlement, track stiffness, rail fatigue strength on varying subgrade soil modulus. Incorporation of this equations and DF will improve the accuracy and reliability of the conventional track design methods and maintenance standards. Besides, it could assist to explore and assess on the quality control and quality assurance on functionality of the track system in the field.
8. References

[1] Lichtberger B 2005 Track Compendium: Formation, Permanent Way, Maintenance. Economics, 1. (Hamburg: Eurailpress Tetzlaff-Hestra GmbH & Co)
[2] Pita A L, Teixeira P F & Robusté F 2004 High speed and track deterioration: the role of vertical stiffness of the track, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 218 31-40
[3] Puzavac L, Popović Z and Lazarević L 2012 Influence of track stiffness on track behaviour under vertical load, Promet-Traffic&Transportation 24 405-412
[4] Esveld C 2001 Modern railway track (Vol. 385) (Zaltbommel: MRT-Production.
[5] Burrow M P N, Bowness D and Ghataora G S 2007 A comparison of railway track foundation design methods, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 221 1-12.
[6] Standard B 1999 BS 1377-7, Methods of test for Soils for civil engineering purposes-Part 7: Shear strength tests (total stress) (London: UK: British Standard Institute)
[7] DIN D 2012 18134: Soil–testing procedures and testing equipment–plate load test (Berlin, Germany).
[8] Sadeghi J M 2008 Experimental evaluation of accuracy of current practices in analysis and design of railway track sleepers, Canadian Journal of Civil Engineering 35 881-893
[9] Berggren E 2009 Railway track stiffness: dynamic measurements and evaluation for efficient maintenance PhD Thesis (Stockholm: KTH)
[10] Sadeghi J M 2005 Investigation on the accuracy of the current practices in analysis of railway track concrete sleepers, International Journal of Civil Engineering 3 31-45
[11] Sadeghi J & Barati P 2010 Evaluation of conventional methods in Analysis and Design of Railway Track System, International Journal of Civil Engineering 8 44-56
[12] Kouroussis G, Gazetas G, Anastasopoulos I, Conti C and Verlinden O J S D 2011 Discrete modelling of vertical track–soil coupling for vehicle–track dynamics, Soil Dynamics and Earthquake Engineering 31 1711-1723
[13] Mulugeta M 2015 Dynamic Simulation of T-Track: Under Moving Loads Advances in Intelligent Systems and Computing 334 147-161
[14] Sayeed M 2016 Design of ballasted railway track foundations using numerical modelling with special reference to high-speed trains PhD Thesis (Western Australia: Curtin University)
[15] Li Z and Wu T X 2008 Modelling and analysis of force transmission in floating-slab track for railways, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 222 45-57
[16] Chen R Zhao X Wang Z Jiang H and Bian X 2013 Experimental study on dynamic load magnification factor for ballastless track-subgrade of high-speed railway, Journal of Rock Mechanics and Geotechnical Engineering 5 306-311