The generalized equation of motion for the orbital dynamics in the presence of current

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The orbital dynamics induced by the charge current is studied theoretically. The equation of motion for the isospin vector \( T^A \) in the SU(N) case is derived in the presence of the current, and \( N=2 \) corresponds to the \( e_g \) orbitals and \( N=3 \) to the \( t_{2g} \) orbitals. Especially, the anisotropy of the current driven orbital domain wall motion is discussed.

I. INTRODUCTION

The dynamics of the spins in the presence of the current is an issue of recent intensive interest\(^{1,2,3,4,5,6,7,8,9,10,11,12,13} \). Especially it has been predicted theoretically\(^{3,4,5,10,11,12,13} \) and experimentally confirmed that the magnetic wall domain (DW) motion is driven by the spin polarized current in the metallic ferromagnets\(^{3,4} \) and also magnetic semiconductors\(^2 \). The basic mechanism of this current driven dynamics is the spin torque due to the current, which is shown to be related to the Berry phase\(^2 \). In addition to the spin, there is another internal degree of freedom, i.e., orbital, in the strongly correlated electronic systems. Therefore it is natural and interesting to ask what is the current driven dynamics of the orbital, which we shall address in this paper. In the transition metal oxides, there often occurs the orbital ordering concomitant with the spin ordering. Especially it is related to the colossal magnetoresistance (CMR)\(^4 \) in manganese oxides. One can control this ordering by magnetic/electric field, and/or the light radiation. Even the current driven spin/orbital order melting has been observed. On the other hand, an orbital liquid phase has been proposed for the ferromagnetic metallic state of manganites\(^3,4,5,10,11,12,13 \). The effect of the current on this orbital liquid is also an interesting issue.

There are a few essential differences between spin and orbital. For the doubly degenerate \( e_g \) orbitals, the SU(2) pseudospin can be defined analogous to the spin. However, the rotational symmetry is usually broken in this pseudospin space since the electronic transfer integral depends on the pair of the orbitals before and after the hopping, and also the spacial anisotropy affects as the pseudo-magnetic field. For \( t_{2g} \) system, there are three degenerate orbitals, and hence we should define the SU(3) Gell-Mann matrices to represent its orbital state. There is also an anisotropy in this 8 dimensional order parameter due to the same reason as mentioned above.

In this paper, we derive the generic equation of motion of the SU(N) internal degrees of freedom in the presence of the orbital current. N=2 corresponds to the \( e_g \) orbitals and N=3 to the \( t_{2g} \) orbitals. Especially, the anisotropy of the DW dynamics is addressed. Surprisingly, there is no anisotropy for the \( e_g \) case while there is for the \( t_{2g} \) case. Based on this equation of motion, we study the DW dynamics between different orbital orderings. The effect of the current on the orbital liquid is also mentioned.

II. CPN−1 FORMALISM

In this section, we are only interested in the orbital dynamics. To be general, we consider a system with \( N \) electronic orbital degeneracy. The model that we investigate is very similar to the lattice CP\(^{N−1} \) sigma model with anisotropic coupling between the nearest-neighbor spinors. In contrary to the prediction in the band theory, the system is an insulator when it is \( 1/N \) filled, namely one electron per unit cell. Because of the strong on-site repulsive interaction, the double occupancy is not allowed. Therefore, it costs high energy for electrons to move, and the spin degrees of freedom is quenched to form some spin ordering. Due to the complicate spin, orbital, and charge interplay scheme, it is convenient to use the slave-fermion method in which we express the electron as \( d_{\sigma i} = h_i^{\dagger} \tilde{h}_{\gamma i}^{(s)} \), where \( h_i^{\dagger} \), \( \tilde{h}_{\gamma i}^{(s)} \), and \( z_{\sigma i} \) are baptized as holon, spinon, and pseudo spinon (for the orbital) respectively, and the index \( \gamma \) denotes the position\(^{15} \). Holes are introduced into the system, their mobility leans to frustrate the spin ordering and thus leads to a new phase which might possess finite conductivity. Therefore, we consider the following effective Lagrangian

\[ L = i\hbar \sum_i \bar{z}_{\sigma i} \left( \gamma_{\sigma i} - \delta_{\sigma i} \right) \dot{z}_{\sigma i} + \sum_{\gamma j} \left( t_{\gamma j}^\beta \bar{h}_i h_j \bar{z}_{\alpha i} z_{\beta j} + \text{c.c.} \right) \]

where \( z \) is short for the spinor \( z^{(s)} \). The \( t_{\gamma j}^\alpha \) in Eq. (1) is the transfer integral which is in general anisotropic because of the symmetry of the orbitals. Therefore, the system does not have SU(N) symmetry in general. To introduce the current, we consider the following mean field:

\[ < \bar{h}_i h_j > = x e^{i\theta_{ij}} \]

where \( x \) denotes the doping concentration, and \( \theta_{ij} \) is the bond current with the relation \( \theta_{ij} = -\theta_{ji} \). Then, the
Lagrangian can be written as
\[ L = i\hbar \sum_i (1 - x) z_{\alpha i} \dot{z}_{\alpha i} + \sum_{<ij>} (t^a_{ij} x e^{i\theta_{ij}} z_{\alpha i} \dot{z}_{\beta j} + \text{c.c.}) \] (3)

Note that the constraint \( \sum_{\alpha} |z_{\alpha i}| = 1 \) is imposed, and the lowest energy state within this constraint is realized in the ground state. The most common state is the orbital ordered state, which is described as the Bose condensation of \( z \), i.e., \( <z_{\alpha i}> \neq 0 \). In the present language, it corresponds to the gauge symmetry breaking. On the other hand, when the quantum and/or thermal fluctuation is enhanced by frustration etc., the orbital could remain disordered, i.e., the orbital liquid state can be obtained. Then the Lagrangian Eq. (3) describes the liquid state without the gauge symmetry breaking. In this case, the gauge transformation
\[ z_{\alpha i} \to e^{-i\varphi_i} z_{\alpha i} \]
\[ \dot{z}_{\alpha i} \to e^{i\varphi_i} \dot{z}_{\alpha i} \]
(4)
is allowed. Given \( \tilde{r}_{ij} = \tilde{r}_{ij} \cdot \tilde{j} \), where \( \tilde{r}_{ij} = \tilde{r}_{i} - \tilde{r}_{j} \), the local gauge transformation in Eq. (4) with \( \varphi_i = \tilde{r}_{i} \cdot \tilde{j} \) corresponding to the simple shifts of the momentum from \( k \) to \( k + \tilde{j} \). Therefore, the presence of current does not affect the state significantly since the effect is canceled by the gauge transformation. On the other hand, if \( z_{\alpha i} \) represent an orbital ordering, the current couples to the first order derivative of \( z \) in the continuum limit. Define \( a_\mu = iz\partial_\mu z \), the second term in Eq. (3) can be written as \(-j^\mu a_\mu \) in the continuum limit. Namely, the current couples to the Berry’s phase connection induced by the electron hopping. Therefore, we expect some non-trivial effect similar to the spin case.

To derive the equation of motion for the orbital moments, we use the SU(N) formalism. Introducing the SU(N) structure factors
\[ \{\lambda^A, \lambda^B\} = i f_{ABC} \lambda^C \]
\[ \{\lambda^A, \lambda^B\} = d_{ABC} \lambda^C + g_{A\beta} \delta^{AB} \]
(5)
(6)
where \( \lambda_{A\beta} \) are the general SU(N) Gell-Mann matrices, \( [\ ] \) are the commutators, and \( \{ \} \) are the anti-commutators. The \( f_{ABC} \) is a totally-anti-symmetric tensor, and \( d_{ABC} \) is a totally-symmetric tensor. Let us express \( t^a_{ij} \) in this basis
\[ t_{ij} = t^0_{ij} 1 + t^A_{ij} \lambda^A \]
(7)
We will only consider the nearest-neighbor hopping: \( t_{ij} = t_{<ij>} \). In the rest of the paper, \( t^A_{i=\pm k} \) will be written as \( t^A_k \), so does \( \theta_{ij} \). Define the CPN-1 superspin vector as
\[ T^A(i) = z_{\alpha i} \lambda^A_{\alpha i} \]
(8)
The equation of motion of \( T^A(i) \) given by Eq. (9) can be obtained as
\[ T^A(i) = \frac{xa}{(1 - x)\hbar} \big[ -2 \cos \theta_k f_{ABC} t^B_k T^C(i) \]
\[ - 2 \sin \theta_k t^0_k \Delta_k T^A(i) - \sin \theta_k t^B_k d_{ABC} \Delta_k T^C(i) \]
\[ + f_{ABC} t^B_k \sin \theta_k \Delta_C(i) \big] \]
(9)
where the dummy \( k \) is summed over \( x, y, \) and \( z \) direction, \( a \) is the lattice constant, and the orbital current \( \tilde{j}_C \) is given by
\[ \tilde{j}_C(i) = i(\Delta \dot{z}_{\alpha i} \lambda^C_{\alpha i} - \dot{z}_{\alpha i} \lambda^C_{\alpha i} \Delta z_{\alpha i}) \]
(10)
which is the second order in \( \theta \). Up to the first order in \( \theta \), the first term in the right hand side of the Eq. (9) is zero provided that
\[ t^A_k + t^B_k + t^C_k = 0 \]
(11)
which is true for most of the systems that we are interested in. Consequently, the dominant terms in Eq. (9) will be the second and the third ones, which can be simplified as
\[ T^A(i) = - \frac{xa}{(1 - x)\hbar} \big[ 2 \sin \theta_k t^B_k \Delta_k T^C(i) \]
\[ + \sin \theta_k t^B_k d_{ABC} \Delta_k T^C(i) \big] \]
(12)
which is one of the main results in this paper. Using Eq. (12), we will discuss the orbital DW motion in the \( e_g \) and the \( t_{2g} \) systems.

Here some remarks are in order on the mean field approximation Eq. (4) for the Lagrangian Eq. (11). First, it is noted that the generalized Landau-Lifshitz equation obtained by Bazaliy et al. \( \dot{a} \) can be reproduced in the present mean field treatment when applied to the spin problem. As is known, however, there are two mechanisms of current-induced domain wall motion in ferromagnets. One is the transfer of the spin torque and the other is the momentum transfer. The latter is due to the backward scattering of the electrons by the domain wall. Our present mean field treatment and that in Bazaliy’s paper take the former spin torque effect correctly, while the latter momentum transfer effect is dropped, since the scattering of electrons is not taken into account. However, the latter effect is usually small because the width of the domain wall is thicker than the lattice constant, and we can safely neglect it.

III. \( N=2, e_g \) SYSTEM

First, we consider the application on the (La,Sr)MnO (113 or 214) system. Without losing the generality, we show the result on the 113 system. The other one can be obtained in a similar way.

In LaMnO\(_3\), the valence of Mn ion is Mn\(^{3+}\) with the electronic configuration \((t_2g)^3(e_g)^1\). By doping with Sr, one hole is introduced to Mn\(^{3+}\) and make it to be Mn\(^{4+}\). The transfer integral between Mn ions depends on the Mn \( 3d \) and O \( 2p \) orbitals. After integrating over the oxygen \( p \) orbitals, the effective hopping between the Mn \( d \) orbitals can be obtained. If we denote the up state as \( d_{3z^2-r^2} \), the electron-phonon coupling is calculated as
\[ V_{ep} \]

It is noted that the generalized Landau-Lifshitz equation obtained by Bazaliy et al. \( \dot{a} \) can be reproduced in the present mean field treatment when applied to the spin problem. As is known, however, there are two mechanisms of current-induced domain wall motion in ferromagnets. One is the transfer of the spin torque and the other is the momentum transfer. The latter is due to the backward scattering of the electrons by the domain wall. Our present mean field treatment and that in Bazaliy’s paper take the former spin torque effect correctly, while the latter momentum transfer effect is dropped, since the scattering of electrons is not taken into account. However, the latter effect is usually small because the width of the domain wall is thicker than the lattice constant, and we can safely neglect it.
and the down state as $d_{x^2-y^2}$, $t_{ij}$ have the following form

\[ t_x = t_0 \left( \frac{1}{4} \sigma^x - \frac{\sqrt{3}}{4} \sigma^z \right) = t_0 \left( \frac{1}{2} \frac{1}{2} - \frac{\sqrt{3}}{4} \sigma^x - \frac{1}{4} \sigma^z \right) \]

\[ t_y = t_0 \left( \frac{1}{4} \sigma^y + \frac{\sqrt{3}}{4} \sigma^z \right) = t_0 \left( \frac{1}{2} \frac{1}{2} + \frac{\sqrt{3}}{4} \sigma^x - \frac{1}{4} \sigma^z \right) \]

\[ t_z = t_0 \left( 1 0 0 \right) = t_0 \left( \frac{1}{2} + \frac{1}{2} \sigma^z \right) \]

(13)

where $\sigma^i$ are the Pauli matrices. For $N=2$, the pseudospin moment has the $O(3)$ symmetry given by $T^A = \bar{z} \sigma^A \bar{z}$. The consequent equations of motion is given by

\[ \left( \frac{\partial}{\partial t} + \frac{x}{1-x} \hbar a t_0 \bar{\theta} \cdot \bar{\Delta} \right) T^A(i) = 0 \]

(14)

where $\bar{\theta}$ is $(\theta_x, \theta_y, \theta_z)$, which can be related to the orbital current as $\bar{v}_o = -\frac{1}{\hbar} \sum a t_0 \bar{\theta}$. Taking the continuum limit, Eq. (14) becomes

\[ \left( \frac{\partial}{\partial t} - \bar{v}_o \cdot \bar{\Delta} \right) T^A(\bar{r}) = 0 \]

(15)

which suggests the solution to be the form $T^i(\bar{r} + \bar{v}_o t)$. The result is similar to spin case. While the spin domain wall moves opposite to the spin current, in our case, the orbital domain wall also moves opposite to the orbital current.

We can estimate the order of magnitude of the critical current to drive the orbital DW. The lattice constant $a$ is about 3Å. The transfer integral constant $t_0$ is around $2eV$ in the LSMO system estimated from the photoemission measurement. If we set $v$ around 1m/s, the critical current can be estimated as $e/a^2 \sim 6 \times 10^9$ A/m², which is roughly the same as the order of magnitude of that to drive the spin domain wall.

It should be noted that the current only couples to the first order derivative of $z$. The double exchange term which is given by the second order derivative is not shown in the equation of motion. However, the double exchange term plays a role to stabilize the DW configuration before the current is switched on.

There are two orbitals degenerate to $d_{x^2-y^2}$, which are $d_{y^2-z^2}$ and $d_{z^2-x^2}$. Similarly, $d_{3y^2-r^2}$ and $d_{3x^2-r^2}$ are degenerate to $d_{x^2-y^2}$. In the Manganese system, $d_{3z^2-r^2}$ and $d_{x^2-y^2}$ may have different energy due to slight structural distortion in the unit cell. Therefore, in most cases, domain walls are the type of those which separate two degenerate domains. Let’s consider the orbital domain wall separating $3y^2-r^2$ and $3x^2-r^2$. In Fig 1, the domain wall sits at $y=0$, and suppose the current is along the positive $x$ direction. $3y^2-r^2$ and $3x^2-r^2$ orbitals are described by the spinors $(-1/2, -\sqrt{3}/2)$ and $(-1/2, \sqrt{3}/2)$ respectively. The configuration given in Fig.1 is described by the spinor field $(\cos \theta(x), \sin \theta(x))$ where $\theta(x) = 2\pi / (3/2 \cot^{-1} e^{-x/w} + 1)$, where $w$ is the width of the domain wall.

Moreover, due to the special properties of SU(2) algebra, the equation of motion is isotropic regardless how anisotropic the transfer integral is. Therefore, the motion of orbital domain wall is undistorted as in the spin case.

![Fig. 1](Color online) An example of the domain structure in the $e_g$ system. The domain boundary lies on the $y$-axis.

### IV. $N=3$, $t_{2g}$ System

Let us consider the $t_{2g}$ systems, for example, in the Vanadate or Titanate systems. The $t_{2g}$ orbitals contain three orbits $d_{xy}$, $d_{yz}$, and $d_{zx}$. The hopping integral $t_{ij}$ between the $T^{i+}$ sites or the $V^{i+}$ ones is given as

\[ t_x = t_0 \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{array} \right) = t_0 \left( \frac{1}{2} \frac{1}{2} + \frac{\sqrt{3}}{2} \lambda^3 - \frac{1}{2 \sqrt{3}} \lambda^8 \right) \]

\[ t_y = t_0 \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right) = t_0 \left( \frac{1}{2} \frac{1}{2} + \frac{1}{2} \lambda^3 \right) \]

\[ t_z = t_0 \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right) = t_0 \left( \frac{1}{2} \frac{1}{2} - \frac{1}{2 \sqrt{3}} \lambda^3 - \frac{1}{2 \sqrt{3}} \lambda^8 \right) \]

(16)

where $\lambda^A$ are SU(3) Gell-Mann matrices with the normalization condition $Tr(\lambda^A \lambda^B) = 2 \delta^{AB}$. The super-spin $T^A$ given by $\bar{z} \lambda^A \bar{z}$ is normalized because $\bar{z} = 1$ and $\sum A \lambda^A_\alpha \lambda^A_\beta = 2 \delta_{\alpha \beta} \delta_{\beta} - \bar{\delta}_{\alpha \beta} \delta_{\beta}$. The equation of motion in this case will be anisotropic because $d_{ABC}$ is non-trivial in the SU(3) case. It is inspiring to work on an example to see how it goes. Let’s consider the $d_{xy} - d_{yz}$ orbital DW shown in the Fig.2. Because of the orbital symmetry, such DW can only be stabilized along the $y-$ direction. Similarly, the $d_{xy} - d_{zx}$ DW can only be stabilized along the $x-$ direction and so on. Therefore, the effect of current is anisotropic. In the
absence of current, the domain wall is stabilized to be

\[ z_\alpha(r') = \left( \begin{array}{c} \sin \tan^{-1} e^{-y_i/w} \\ \cos \tan^{-1} e^{-y_i/w} \\ 0 \end{array} \right) \]  

(17)

It can be easily seen that it takes no effect if the current is applied along \( x \) or \( z \) direction. The superspin component is given by

\[ T^A(r') = \left( \begin{array}{ccc} \sech(y_i/w) & 0 & 0 \\ 0 & \tanh(y_i/w) & 0 \\ 0 & 0 & 1 \end{array} \right) \]  

(18)

When the current is applied along the \( y \)– direction, Eq. (12) for each \( T_k \) are decoupled. For \( A = 1, 2, 3 \), they are given by

\[ \left( \frac{\partial}{\partial t} + \frac{2xT_0}{(1-x)\hbar} \theta_y \frac{\partial}{\partial y} \right) T^{1,2,3}(r') = 0 \]  

(19)

For \( A = 4, 8 \), they are given by

\[ \left( \frac{\partial}{\partial t} + \frac{xT_0}{(1-x)\hbar} \theta_y \frac{\partial}{\partial y} \right) T^{4,8}(r') = 0 \]  

(20)

At a glance, we obtain 2 characteristic drift velocity of the domain wall. It is not the case, because \( T^3(r') \) is zero and \( T^8(r') \) is constant. Only Eq. (19) determines the motion of the DW. Furthermore, the 8–dimensional super-spin space reduces to be 2–dimensional as summarized in Fig. 3. \( T^1 \) moment grows in the domain wall while \( T^3 \) moment distinguishes two domains. In the presence of current along \( y \)–direction, the wall velocity is \( |v| = \frac{2xT_0}{(1-x)\hbar} |\theta_y| \). Other type of domain structures can be analyzed in a similar way. As a result, even though the order parameter in the \( t_{2g} \) systems forms an 8-dimensional super-spin space, we can always reduce it to be 2-dimensional because of the anisotropic nature of the system. Furthermore, the DW moves without any distortion just like the isotropic case.

**V. DISCUSSION AND CONCLUSIONS**

In this paper, we formulated the orbital dynamics when the spin degree of freedom is quenched. We used SU(N) super-spin \( T^A \) to describe the orbital states and obtained the general equation of motion for it. We also showed some examples for the SU(2) and SU(3) cases corresponding \( e_y \) and \( t_{2g} \) systems, respectively. In the SU(2) case, the orbital dynamics is very similar to the spin case: undistorted and isotropic. In SU(3) case, the DW structure is anisotropic because of the orbital symmetry. In addition, the effective super-spin space is 2-dimensional, and the domain wall motion is also undistorted.

Even though the analogy to the spin case can be made, one must be careful about some crucial differences between the spin and orbital degrees of freedom. We have estimated the critical current to drive the domain wall assuming the uniform current flow in the metallic system, but most of the orbital ordered state is insulating. This is the most severe restriction when the present theory is applied to the real systems. An example of the metallic orbital ordered state is \( A \)-type antiferromagnetic state with \( x^2 − y^2 \) orbital ordering in NdSrMnO\(_2\). However it is insulating along the \( c \)-direction, and there is no degeneracy of the orbitals once the lattice distortion is stabilized. The ferromagnetic metallic state in LSMO is orbital disordered. According to the quantum orbital liquid picture, there is no remarkable current effect on the orbitals as explained in the Introduction. On the other
hand, when the classical fluctuation of the orbital plays
the dominant role for the orbital disordering, the short
range orbital order can be regarded as the distribution of
the domain walls, which shows the translational motion
due to the current as discussed in this paper. Note how-
ever that the current does not induce any anisotropy in
the orbital pseudospin space in the $e_g$ case.

Orbital degrees of freedom is not preserved in the vac-
uum or the usual metals, where the correlation effect is
not important, in sharp contrast to the spin. There-
fore the orbital quantum number cannot be transmitted
along the long distance and the pseudospin valve phe-
nomenon is unlikely in the orbital case.

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APPENDIX A: $e_g$ ORBITALS

There are two class of $e_g$ orbitals, i.e., one is $3x^2-r^2$,
$3y^2-r^2, 3z^2-r^2$, and the other is $x^2-y^2, y^2-z^2$,
and $z^2-x^2$. They can be described by a two-component
spinor. If we use $(1,0)$ to describe $3z^2-r^2$ and $(0,1)$ for
$x^2-y^2$, the other degenerate orbitals are given by

$$
\begin{align*}
3y^2-r^2 : & \left( -1/2, -\sqrt{3}/2 \right) \\
3x^2-r^2 : & \left( -1/2, \sqrt{3}/2 \right) \\
y^2-z^2 : & \left( -1, -1 \right) \\
z^2-x^2 : & \left( \sqrt{3}, -1/2 \right)
\end{align*}
$$

(A1)

Their pseudospin moments given by the transformation
$T^A = \bar{z}^{\lambda}_{\lambda} r$ are shown as the following

$$
\begin{align*}
\bar{T}_{3z^2-r^2}^1 = & \left( -\sqrt{3}/2, 0, -1/2 \right) \\
\bar{T}_{3y^2-r^2}^1 = & \left( \sqrt{3}/2, 0, -1/2 \right) \\
\bar{T}_{3z^2-r^2}^2 = & \left( 0, 0, 1 \right) \\
\bar{T}_{x^2-y^2}^1 = & \left( 0, 0, -1 \right) \\
\bar{T}_{y^2-z^2}^2 = & \left( \sqrt{3}/2, 0, 1/2 \right) \\
\bar{T}_{x^2-y^2}^2 = & \left( -\sqrt{3}/2, 0, -1/2 \right)
\end{align*}
$$

(A2)

APPENDIX B: GELL-MANN MATRICES

The Gell-Mann matrices used in the text are given ex-
plicitly in the following:

$$
\begin{align*}
\lambda_1 = & \left( \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right), \\
\lambda_2 = & \frac{1}{2} \left( \begin{array}{ccc} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{array} \right), \\
\lambda_3 = & \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right), \\
\lambda_4 = & \left( \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} \right), \\
\lambda_5 = & \left( \begin{array}{ccc} 0 & 0 & -i \\ 0 & i & 0 \\ -i & 0 & 0 \end{array} \right), \\
\lambda_6 = & \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right), \\
\lambda_7 = & \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{array} \right), \\
\lambda_8 = & \frac{1}{\sqrt{3}} \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{array} \right).
\end{align*}
$$

(B1)