Ultrasoft fermionic modes at high temperature

Yoshimasa Hidaka
Mathematical Physics Laboratory, RIKEN

Collaboration with Daisuke Satow and Teiji Kunihiro (Kyoto University)

Based on arXiv:1105.0423 [hep-ph]
and 1111.5015 [hep-ph], Nucl. Phys. A876, 93 (2012)
QCD Phase Diagram

This talk
Collective modes at high T

Massless fermion-boson system (Yukawa, QED, QCD)

Low energy excitations are collective.

In bosonic sector, hydro mode exists as zero modes.

(g: coupling constant)
ex) Hydro mode: Density fluctuation

Minami and Kunihiro (2009)

Sound mode

Thermal diffusion mode

\[ S_{nn}(k, \omega) = \langle (\delta n(k, t=0))^2 \rangle \]

\[ \omega \text{ [1/fm]} \]

\[ \nu_{TKO} = \text{Landau} \quad \text{Minimal} \]
Fermionic ultrasoft-modes at high $T$?

$\rho(\omega, p = 0)$

Normal mode

Antiplasmino

HTL effective theory works

$\omega \sim gT$

$\gamma \sim g^2 T$

cf: fermionic ultrasoft mode was suggested:
massive boson case: M. Kitazawa, T. Kunihiro and Y. Nemoto, PTP 117, 103 (2007),
QCD: V. V. Lebedev and A. V. Smilga, Annals Phys. 202, 229 (1990).
e.g., Yukawa model

\[ T \sim m \] boson mass

Kitazawa, Kunihiro and Nemoto ('06)
Does chiral symmetry imply the existence of zero modes?

Yes, in the vacuum.

Questionable at finite T.
Perturbation theory at high $T$

Naive perturbation does not work at ultrasoft momentum region

Bare propagators

**Fermion:** \( D_R(k) = \frac{k}{k^2 + i\epsilon k^0} \)

**Boson:** \( G_A(k) = \frac{1}{k^2 - i\epsilon} \)

(Scalar, photon, gluon,..)

One-loop analysis

\[
\approx g^2 \int \frac{d^4 k}{(2\pi)^4} \ k(n_F(k) + n_B(k))G_A(k)D_R(k + p)
\]

\[
\approx g^2 \int \frac{d^4 k}{(2\pi)^4} \ k(n_F(k) + n_B(k)) \frac{1}{2p \cdot k} (2\pi) \delta(k^2)
\]

Diverges as $p \to 0$. Need improvement.
Dressed perturbation theory

Dressed propagators

\[ D_R(k) = \frac{k}{k^2 - m_f^2 + 2ik^0\gamma_f} \]
\[ G_A(k) = \frac{1}{k^2 - m_b^2 - 2ik^0\gamma_b} \]

\( m_f^2, m_b^2 : \) thermal masses \hspace{1cm} \( \gamma_f, \gamma_b : \) damping rates

\[ g^2 \int \frac{d^4k}{(2\pi)^4} k(n_F(k) + n_B(k)) \frac{1}{\delta m^2 + 2(p \cdot k + ik^0\gamma)} \]

Fermion soft mode in QCD, Lebedev, Smilga ('90)

Finite, improved!

where \( \delta m^2 = m_b^2 - m_f^2 \hspace{1cm} \gamma = \gamma_b + \gamma_f \)
One loop results (QED)

Propagator

$$D_R \simeq -\frac{Z}{2} \left( \frac{\gamma^0 - \hat{p} \cdot \gamma}{p^0 + v|\mathbf{p}| + i\gamma} + \frac{\gamma^0 + \hat{p} \cdot \gamma}{p^0 - v|\mathbf{p}| + i\gamma} \right).$$

Pole \hspace{1cm} \omega = \pm \frac{1}{3} p + i\gamma \hspace{1cm} \text{Residue} \hspace{1cm} Z = \frac{e^2}{16\pi^2} \left( \frac{8\delta m^2}{e^2T^2} \right)^2

where \hspace{1cm} \delta m^2 = \frac{e^2}{12} \hspace{1cm} \gamma \sim \frac{e^2}{4\pi} \ln \frac{1}{e}$
But this is not the end of the story.

Infinite numbers of higher order diagrams can contribute to the leading order.
A similar situation: transport coefficients

Resummation of ladder diagrams is necessary

Boltzmann equation
Jeon ('94)

Resummation of ladder diagrams is necessary
Higher loop diagrams

All ladder diagram contributes to the leading order.
Resummation

Self-consistent equation

Self-energy in the leading order
Special case: Yukawa model

Yukawa model is a special case.

Ladder diagram is suppressed, tree vertex function is leading.

The ‘wave function’ in the ladder diagram
\[ \sim m_f^2 \sim g^2 \], so it is higher order correction.
The velocity is $1/3$, the residue is of order $g^2$.

| Yukawa model         | $\gamma \sim g^4 T$ | $C = 2/9$ |
|----------------------|----------------------|-----------|
| QED                  | $\gamma \sim g^2 T$ | $C = 1/9$ |
| QCD                  | $\gamma \sim g^2 T$ | $C = 4(N_f + 5)/3$ |
Ward-Takahashi identity

\[ k^\mu \Gamma_\mu(p, k) = \phi + k^\mu - \Sigma^R(p + k) - \phi + \Sigma^R(p). \]

Ward-Takahashi identity is satisfied in the leading order.
Origin of ultrasoft modes

Supersymmetry?

In a supersymmetric model:
Phonino as Goldstino due to the symmetry breaking of the SUSY at finite T.

Girardello, Grisaru, Salomonson ('81); Boyanovsky ('84); Aoyama, Boyanovsky ('84); Gudmundsdottir, Salomonson ('87). Lebedev, Smilga ('89).

For QCD:
At g=0, supersymmetry can be assigned to quarks and gluons, which is explicitly broken by the interaction.

Lebedev and Smilga ('90)
Origin of ultrasoft modes

Chiral symmetry  no explicit mass

Time reversal

In vacuum
\[ D^{-1}(\omega, 0) = -D^{-1}(-\omega, 0) \quad \rightarrow \quad \text{pole at } \omega = 0 \]

In medium
\[ \text{Re}D^{-1}(\omega, 0) = -\text{Re}D^{-1}(-\omega, 0) \]
\[ \text{Im}D^{-1}(\omega, 0) = \text{Im}D^{-1}(-\omega, 0) \]

If \( \text{Re}D^{-1} \) is continuous at \( \omega=0 \),
and if \( \text{Im}D^{-1} \) is small, pole at
\[ \text{Re}D^{-1}(0, 0) = 0 \]
\[ \omega = -i\gamma \]
Summary

Ultrasoft fermionic mode

| Pole         | Residue            |
|--------------|--------------------|
| $\omega = \pm \frac{1}{3} p + i\gamma$ | $Z = \frac{g^2}{16\pi^2} c$ |

Ward-Takahashi identity: OK
Outlook

Kinetic theory for the ultrasoft fermionic modes.
(in preparation Satow and YH)

Observables sensitive to the ultrasoft fermionic modes.