Assessment of wear dependence parameters in complex model of cutting tool wear

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Abstract. This paper addresses wear dependence of the generic efficient life period of cutting tools taken as an aggregate of the law of tool wear rate distribution and dependence of parameters of this law’s on the cutting mode, factoring in the random factor as exemplified by the complex model of wear. The complex model of wear takes into account the variance of cutting properties within one batch of tools, variance in machinability within one batch of workpieces, and the stochastic nature of the wear process itself. A technique of assessment of wear dependence parameters in a complex model of cutting tool wear is provided. The technique is supported by a numerical example.

1. Introduction
In order to solve the problem of assessing the efficient life period of cutting tools, modern mechanical engineering, which is commonly characterized by using a very broad range of cutting tools [1], uses traditionally wear dependencies that describe only the dependence of average efficient life of cutting tools on cutting modes. Such wear characteristics do not consider the stochastic nature of cutting tool wear, which depends on a large number of factors: cutting modes, cutting properties of tools, type of machining, hardness of machined articles, allowance amount, pre-existing strain-stress distribution, vibration, machine inaccuracies, etc [2–7].

This study considers wear dependence as the generic efficient life period of cutting tools taken as an aggregate of the law of tool wear rate distribution and dependence of law’s parameters on the cutting mode. In this case, the factors that must be taken into account are the variance of cutting properties within one batch of tools, variance in workpiece hardness and allowance, and the stochastic nature of the wear process. All of these factors lead to variance in the efficient life period of cutting tools; and even within one batch of cutting tools, their efficient life can vary broadly (15 – 35 %) [8].

Depending on the aforementioned factors, the following mathematical models of cutting tool wear and failures can be used [9]: a vibered wear model, a wear accumulation model, a complex model of wear, a fracture model, and a generalized model.

In the case of the complex model of wear, the following random factors are taken into account: variance of cutting properties within one batch of tools (tool bits), variance in machinability within one batch of workpieces (for example, related to variance in workpiece hardness and allowance), as well as the stochastic nature of the wear process itself.

2. Materials and methods
In the complex model, wear Y of a particular tool after operating life t as a random variable follows the normal distribution with the density:
\[
\varphi_t(y) = \frac{1}{\sqrt{2\pi} \sigma^2 t} \exp\left[-\frac{(y-u \cdot t)^2}{2\sigma^2 t}\right],
\]

where \( u \) is the average wear rate of a particular tool, \( \sigma^2 \) is the wear dispersion per operating life unit, \( t \) is the tool's operating life of installation that is measured, for example, in cutting time minutes or machined part units.

When a tool is changed, distribution parameters (1) \( u \) and \( \sigma \) are also changed. Like in cases of machining equipment failure [10], the complex model suggests that \( u \) as a random value has lognormal distribution with the density:

\[
\psi(u) = \frac{1}{\sqrt{2\pi} \delta \cdot u} \exp\left[-\frac{(\ln u - \ln \hat{U})^2}{2\delta^2}\right],
\]

where \( \hat{U} \) is geometric mean wear rate for all tool bits in the batch; \( \delta \) is standard deviation of wear rate logarithm \( u \). The mean wear rate for all tool bits in the batch (expected value) and the variance factor of this rate are:

\[
\bar{U} = \hat{U} \cdot \exp(\delta^2/2), \quad K_u = \sqrt{\exp(\delta^2) - 1}.
\]

Direct experimentation data about the correlation between \( u \) and \( \sigma \) are yet insufficient, but theoretical considerations allow one to assume that under first approximation, standard deviation of wear per unit of operating life and standard deviation of wear rate change proportionally to average wear rate \( u \), that is:

\[
\sigma = \nu \cdot u.
\]

Proportionality factor \( \nu \) is the factor of variance of wear per unit of operating life. It depends on the variance of properties of machined workpieces [9].

Taking into account (1), (2) and (4), wear distribution \( Y \) after operating life \( t \) has density:

\[
f_t(y) = \int_0^\infty \psi(u) \cdot \varphi_t(y) du = \frac{1}{2\pi \delta \sqrt{\nu^2 t}} \int_0^\infty \frac{1}{u^2} \exp\left[-\frac{\ln^2(u/\hat{U})}{2\delta^2} - \frac{(y-u \cdot t)^2}{2(\nu \cdot u)^2 t}\right] du.
\]

This density is used to determine other practically important characteristics.

The reliability function or probability of failure-free operation during operating life \( t \) is:

\[
P(t) = \int_0^L f_t(y) dy.
\]

The average value of the efficient life period (expected value) is:

\[
\bar{T} = \int_0^\infty P(t) dt.
\]

Rough calculation should be performed with formula \( \bar{T} = L/\hat{U} \).

The variance factor of efficient life period is:

\[
K_T = \frac{1}{\bar{T}} \sqrt{\int_0^\infty [P(t) dt - \bar{T}^2].}
\]
3. Assessment of wear dependence parameters in a complex model of wear

Parameters of distribution (5) $\hat{U}$, $\delta$, and $\nu$ are assessed based on experience from the wear resistance experiment or using statistics obtained in production environment based on the results of processing protocols of specially organized observations.

Let us first address the case of assessing parameters under a fixed cutting mode, based on the "operating life - wear" statistics:

$$[t_{ij}, Y_{ij}], j = 1, ..., M_i; j = 1, ..., N].$$

Here $i$ is the number of a tool bit in the batch under consideration, $j$ is the number of wear measurement of $i$th tool bit, $M_i$ is the quantity of bit's wear measurements, $t_{ij}$ is operating life, and $Y_{ij}$ is the increment of wear during this operating life for the $i$th tool bit during $j$th wear measurement.

Let us assess parameters $\hat{U}$, $\delta$, and $\nu$ in two stages. First, let us assess parameters $u$ and $\sigma$ of density (1) for each tool bit in the sample. The maximum likelihood method produces the following assessment:

$$u_i = \frac{Y_i}{t_i}, \sigma_i^2 = \frac{1}{M_i} \left( \sum_{j=1}^{M_i} \frac{Y_{ij}^2}{t_i} - \frac{Y_i^2}{t_i} \right),$$

where $t_i = \sum_{j=1}^{M_i} t_{ij}$ is the total operating life of the $i$th tool bit and $Y_i = \sum_{i=1}^{M_i} Y_{ij}$ is the total wear of this tool bit. At the second stage, let us assess $\hat{U}$, $\delta$, and $\nu$ immediately. Let us assess parameters $\hat{U}$ and $\delta$ using the maximum likelihood method proceeding from lognormal distribution (2) and statistics $(u_i, i = 1, ..., N)$. For these parameters, one obtains the following assessment:

$$\hat{U} = \left( \prod_{i=1}^{N} u_i \right)^{1/N}, \quad \delta^2 = \frac{1}{N} \sum_{i=1}^{N} [\ln(u_i / \hat{U})]^2.$$

For parameter $\nu$, assessment is provided like that for $\hat{U}$ as a geometric mean of $\nu_i = \sigma_i / u_i$, that is:

$$\nu = \left( \prod_{i=1}^{N} \nu_i \right)^{1/N}.$$

To optimize cutting modes and prevent cutting tool failures, it is necessary to know not only the law of wear distribution like (5) or the reliability function (6), but also the dependence of parameters $\hat{U}$, $\delta$, and $\nu$ on the cutting mode. In practice, researchers often use power-law wear dependence of the following form:

$$T = C_T / [V^m S^x h^y (H_B/200)^z],$$

where $V$ is cutting speed, $S$ is feed, $h$ is cutting depth, $H_B$ is hardness of the processed workpiece, $T$ is efficient life period, $C_T$, $m$, $x$, $y$, and $z$ are experimental constants. A shortcoming of this dependence is that it does not take into account the random factors mentioned above. It also does not reflect the hump-shaped nature of the dependence of the efficient life period on cutting speed and possible correlation between $V$ and $S$ or $S$ and $h$. To fix these shortcomings, other dependencies of $T$ on $V$ were proposed, but they do not include the random factor.
Here, it is proposed to determine the dependence of parameters $\hat{U}$, $\delta$, and $\nu$ on cutting mode parameters in the following way. Let us determine parameter $\hat{U}$ as an exponential function of the polynomial of logarithms $V$, $S$, $h$, and $H_B$, that is:

$$
\hat{U}(b_0,\ldots,b_8) = \exp[b_0 + b_1 \ln V + b_2 \ln^2 V + b_3 \ln^3 V + b_4 \ln S + b_5 \ln h + b_6 \ln V \cdot \ln S + b_7 \ln S \cdot \ln h + b_8 \ln(H_B/200)].
$$

(14)

If one considers that $T = L/U$, then formula (14) generalizes formula (13) by adding the second and third powers $\ln V$ to address the hump-shaped nature of the dependence of $T$ on $V$, and allows one to factor in the correlation between $V$ and $S$, $S$ and $h$. Such representation of wear dependence allows one to address easily other possible correlations given there is sufficient statistics.

Parameter $\delta$ characterizes variance in tool bit cutting properties and depends on the cutting mode only through wear rate $U$. Parameter $\nu$ characterizes variance in workpiece machinability properties and also depends on cutting modes only through wear rate $U$.

In order to obtain the dependence of parameters $\hat{U}$, $\delta$, and $\nu$ on cutting modes, it is necessary to have statistics like (9), but under different cutting modes that are assigned, for example, according to the experiment planning theory. Let us consider a variant of statistics:

$$
[V_i, S_i, h_i, H_{Bi}, (t_{ij}, Y_{ij}, j = 1,\ldots,M_i), i = 1,\ldots,N].
$$

(15)

Here it is assumed that each of $N$ tool bits is tested under its own combination of cutting modes $V_i, S_i, h_i, H_{Bi}$. But for each tool bit, wear $Y_{ij}$ after operating life $t_{ij}$ is controlled $M_i$ times ($M_i > 1$).

As it has already been done for the fixed cutting mode, let us write statistics (15) as:

$$
[V_i, S_i, h_i, H_{Bi}, u_i, v_i = 1,\ldots,N],
$$

using the same formula (10). To assess $\delta$ and coefficients $b_0,\ldots,b_8$, let us use the same maximum likelihood method in application to density (2), but under $\hat{U}$, defined as in (14). In this case, the maximum likelihood function will take the following form:

$$
I(\delta, b_0,\ldots,b_8) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\delta u_i} \exp\left[-\frac{(\ln u_i - \ln \hat{U})^2}{2\delta^2}\right].
$$

Assessments of $\delta$ and $b_0,\ldots,b_8$ are located as a result of solving the system of 10 equations:

$$
\frac{\partial \ln(I)}{\partial b_0} = 0,\ldots, \frac{\partial \ln(I)}{\partial b_8} = 0, \quad \frac{\partial \ln(I)}{\partial \delta} = 0,
$$

(16)

which allows one to achieve the maximum of the likelihood function.

After differentiating and carrying out the necessary transformations to determine $b_0,\ldots,b_8$, one obtains the following system of 9 linear equations:

$$
\sum_{m=0}^{8} b_mB_{nm} = C_n, \quad n = 0,\ldots,8.
$$

(17)

Here $B_{nm}$ are statistical moments written as follows:

$$
B_{nm} = \frac{1}{N} \sum_{i=1}^{N} X_{ni} \cdot X_{mi},
$$

where
Free terms of system (17) are determined as:

\[ C_n = \frac{1}{N} \sum_{i=1}^{N} \ln u_i \cdot X_{ni}. \]

System (17) is solved using standard subroutines for solving linear equation systems. Parameter \( \delta \) is determined after solving system (17) using the following formula, obtained from last equation (16):

\[ \delta^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \ln u_i - \sum_{n=0}^{8} b_n X_{ni} \right)^2. \]  

(18)

Assessment (12) remains actual for parameter \( \nu \).

4. Illustration of the method of estimating model parameters

Let us illustrate the described method of assessing parameters of wear dependence using a numerical example.

The authors proceed from statistics like (15), but without varying workpiece hardness. An example of such statistics is shown in Table 1. This statistics is for illustrative purposes.

| No. | \( V \), m/min | \( S \), mm/rev | \( h \), mm | \( t_1 \), pcs | \( Y_1 \), mm | \( t_2 \), pcs | \( Y_2 \), mm |
|-----|----------------|----------------|-----------|--------------|-------------|--------------|-------------|
| 1   | 50             | 0.1            | 0.2       | 50           | 0.044       | 70           | 0.066       |
| 2   | 70             | 0.1            | 0.2       | 50           | 0.018       | 70           | 0.025       |
| 3   | 100            | 0.1            | 0.2       | 50           | 0.025       | 70           | 0.034       |
| 4   | 50             | 0.3            | 0.2       | 17           | 0.591       | 23           | 0.734       |
| 5   | 70             | 0.3            | 0.2       | 20           | 0.632       | 28           | 0.838       |
| 6   | 100            | 0.3            | 0.2       | 11           | 0.639       | 15           | 0.778       |
| 7   | 50             | 0.3            | 0.5       | 2            | 0.114       | 3            | 0.374       |
| 8   | 70             | 0.3            | 0.5       | 3            | 0.782       | 4            | 1.01        |
| 9   | 100            | 0.3            | 0.5       | 2            | 0.533       | 3            | 0.932       |
| 10  | 50             | 0.1            | 0.5       | 50           | 0.132       | 70           | 0.184       |
| 11  | 70             | 0.1            | 0.5       | 50           | 0.141       | 70           | 0.193       |
| 12  | 100            | 0.1            | 0.5       | 50           | 0.173       | 70           | 0.235       |
| 13  | 50             | 0.1            | 0.2       | 50           | 0.044       | 70           | 0.068       |
| 14  | 70             | 0.1            | 0.2       | 50           | 0.028       | 70           | 0.037       |
| 15  | 100            | 0.1            | 0.2       | 50           | 0.026       | 70           | 0.038       |
| 16  | 50             | 0.3            | 0.2       | 25           | 0.67        | 34           | 0.922       |
| 17  | 70             | 0.3            | 0.2       | 17           | 0.541       | 24           | 0.916       |
| 18  | 100            | 0.3            | 0.2       | 11           | 0.516       | 16           | 0.802       |
| 19  | 50             | 0.3            | 0.5       | 4            | 0.215       | 5            | 0.334       |
| 20  | 70             | 0.3            | 0.5       | 4            | 0.981       | 5            | 1.333       |
| 21  | 100            | 0.3            | 0.5       | 2            | 0.487       | 3            | 0.57        |
| 22  | 50             | 0.1            | 0.5       | 50           | 0.217       | 70           | 0.309       |
| 23  | 70             | 0.1            | 0.5       | 50           | 0.138       | 70           | 0.194       |
| 24  | 100            | 0.1            | 9.5       | 50           | 0.197       | 70           | 0.291       |
| 25  | 50             | 0.1            | 0.2       | 50           | 0.03        | 70           | 0.042       |
| 26  | 70             | 0.1            | 0.3       | 50           | 0.027       | 70           | 0.037       |
The first column of the Table contains numbers of tests. Each of the 36 tests was carried out with a new tool bit from a single-consignment batch. The cutting mode, i.e. cutting speed \( V \), feed \( S \), and cutting depth \( h \) were assigned proceeding from a complete factorial experiment plan when the cutting speed varied in three levels (50, 70, 100), the feed varied in two levels (0.1, 0.3), and the cutting depth also varied in two levels (0.2, 0.5). Furthermore, every mode variant was tested three times, i.e. using three tool bits. The tool bit operating life was measured in pieces of machined parts. Wear control for each tool bit was performed twice after the operating life of 50 and 70 pcs. Control was carried out under a lower operating life as in Table 1 so that wear did not exceed maximum rate \( L = 0.8 \) mm. Workpiece hardness in this example was not controlled and did not vary.

The results of calculation using the method described are summarized in Table 2.

| Variant | \( U(V,S,h) \) | \( b_0 \) | \( b_1 \) | \( b_2 \) | \( b_3 \) | \( b_4 \) | \( b_5 \) | \( b_6 \) | \( b_7 \) | \( \delta \) |
|---------|----------------|---------|---------|---------|---------|---------|---------|---------|---------|-------|
| 1       | 2.468          | 0.367   | –       | –       | 3.667   | 1.869   | –       | –       | 0.2918  |
| 2       | 11.078         | -3.69   | 0.477   | –       | 3.667   | 1.869   | –       | –       | 0.2905  |
| 3       | 9.123          | -2.308  | 0.150   | 0.026   | 3.667   | 1.869   | –       | –       | 0.2905  |
| 4       | -3.361         | 2.777   | -0.608  | 0.085   | -0.86   | 1.869   | 1.064   | –       | 0.2388  |
| 5       | -3.386         | 2.584   | -0.562  | 0.0814  | -1.03   | 1.610   | 1.064   | -0.148  | 0.2359  |

This Table summarizes the results of assessment of parameters \( b_0,\ldots,b_7,\delta \) for 5 variants of dependence \( U(V,S,h) \). The first variant takes into account only first powers \( \ln V \), \( \ln S \), and \( \ln h \); the second one also factors in second power \( \ln V \); the third one includes third power \( \ln V \). The fourth variant adds a component with product \( \ln V \cdot \ln S \), and the fifth one also adds a component with \( \ln S \cdot \ln h \). Value \( U \) was calculated according to formula (14) under \( b_8 = 0 \) because hardness did not vary during the tests.

Parameter \( v = 0.244 \) is for all five variants. Variant 5 has the maximum likelihood because it has minimum value \( \delta \), i.e. standard deviation of wear rate logarithm \( U \) calculated using formula (18).

This way, the assessment allowed one to obtain wear dependency in the form of the law of wear distribution (5) with parameters \( \delta = 0.2359 \), \( v = 0.244 \), and:

\[
\hat{U} = \exp[b_0 + b_1 \ln V + b_2 \ln^2 V + b_3 \ln^3 V + b_4 \ln S + b_5 \ln h + b_6 \ln V \cdot \ln S + b_7 \ln V \cdot \ln S \cdot \ln h],
\]

where

\[
b_0 = 0.244, \quad b_1 = 0.0273, \quad b_2 = 0.0396, \quad b_3 = 0.0342, \quad b_4 = -1.03, \quad b_5 = 1.61, \quad b_6 = 1.064, \quad b_7 = -0.148.
\]

The graphs shown in Figure 1 are calculated according to formula (7) taking into account (5), (6), (19), and (20).
Figure 1. Graphs of dependence of average wear resistance $\bar{T}$ on cutting speed under different feeds and cutting depths:

1 – S=0.1 mm; h=0.5 mm; 2 – S=0.1 mm; h=0.5 mm; 3 – S=0.15 mm; h=0.35 mm.

As can be seen in the figure, the obtained wear resistance reflects the hump-shaped nature of the dependence of the efficient life period on the cutting speed.

5. Conclusion
The addressed method of assessing parameters of wear resistance can be also used in other wear resistance experiment plans and with other values of tool operating life, such as cutting time, cutting path, etc.

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