Counting pions in the nucleon

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ABSTRACT

The number of pions in the nucleon is a theoretical concept and its value is model dependent. It is, however, a useful tool to understand the pionic contribution to several nucleon observables such as electric charge radii, magnetic moments, axial charge, pion-nucleon coupling constant, amplitudes for electroexcitation of baryon resonances and electroproduction of pions above such resonances.

KEYWORDS

Pion cloud, static properties of the nucleon, electroexcitation of \( \Delta \), electroproduction of pions, linear sigma model, chiral chromodielectric model, cloudy bag model

INTRODUCTION

Many nucleon properties can be rather successfully explained by assuming a cloud of pions surrounding the quark core. We shall discuss here what information on the pion cloud we can extract from \textit{static observables} of the nucleon as well as from \textit{dynamic observables} such as the amplitudes for electroexcitation of baryon resonances. We shall particularly emphasize the role of the number of pions \( (n_\pi) \) as a tool to explore the relative pion contribution to the observables and the model (in)dependence of the results. We shall try to distinguish two effects: the one in which the pionic contribution is proportional to the number of pions, and the other where it is proportional to the specific contribution of each pion which depends on its radial profile.

Next, it is an important question whether \( n_\pi \) is also a useful phenomenological concept counting pions as partons eagerly waiting to be kicked out in the process \( eN \rightarrow e'X\pi \) (Güttner, Chanfray, Pirner and Povh, 1984). The theoretical number of pions does not have to coincide with the number of partons in an impulse approximation, but it is more “natural” if it does. By “natural” we mean that such a model has a good chance to make realistic prediction without new ad hoc parameters. An “unnatural” or effective model, however, has to introduce effective charges and effective currents in order to relate the theoretical number of pions to the properties of the electroproduction process.

Finally, \( n_\pi \) is a useful mathematical-physical tool to determine the regime of the quark-pion coupling: we have shown (Čibej, Fiolhais, Golli and Rosina, 1992) that for \( n_\pi < 0.3 \) the solution is in the weak-coupling (perturbative) regime, while for \( n_\pi > 3 \) it is in the strong-coupling regime. In many models the best fit to experimental observables requires \( n_\pi \sim 1 \) so that none of the
extremes applies. This is an important warning to assess the quality of some approximate schemes which unjustly assume either perturbative regime or strong coupling regime. For example, the semiclassical or cranking projection of angular momentum and isospin in some formalisms is valid only in the strong coupling regime; in the CBM, the first order perturbation is justified only for \( R \geq 1 \text{ fm} \).

The definition of the number of pions is somewhat arbitrary. One possibility is to count the “free pions” \( \sum_{tklm} a_{tlm}^\dagger (k) a_{tlm} (k) \) defined by the creation and annihilation operators in the plane wave or spherical wave expansion of the pion field operator:

\[
\hat{n}_\pi (\mathbf{r}) = \sum_{k} \sqrt{\frac{2}{\pi}} j_1(kr) Y_{lm}(\hat{\mathbf{r}}) \frac{1}{\sqrt{2\omega_k}} [a_{tlm}(k) + (-1)^{l+m} a_{tl-m}^\dagger(\mathbf{k})]
\]

and to define observables as normal products with respect to the free vacuum \( |0\rangle \),

\[ a_{tlm}(k)|0\rangle = 0. \]

Here \( \omega_k^2 = k^2 + m^2 \) and \( \sum_k \) corresponds to \( \int k^2 dk \).

Instead of “free pions” we could have counted “distorted pions”

\[ \tilde{n}_\pi = \sum_i \tilde{a}_i^\dagger \tilde{a}_i \]

in a more general canonical basis defined by

\[ \tilde{a}_i^\dagger = \sum_{tklm} x_{i,tklm} a_{tlm}^\dagger (k) + y_{i,tklm} a_{tlm} (k). \]

The usefulness of such a Bogoliubov transformation has been explored very little. We found in a variational calculation of the projected hedgehog nucleon in a schematic model that the additional variation of \( x \) and \( y \) coefficients lowered the ground state energy at most by a few percent. It would be confusing if different authors were comparing the number of pions in different Bogoliubov basis; therefore we advocate to compare the number of free pions.

We should emphasize that a unitary transformation to “distorted pions” in which \( a^\dagger \) is only a superposition of \( a^\dagger \)s without \( as \ (y = 0) \) does not change the number of pions, therefore it is the same if we count “plane wave pions” or “spherical wave pions” or pions with any radial profile.

**SOME POPULAR MODELS FOR THE PION CLOUD**

We shall analyze the results of three different models, defined by the corresponding Hamiltonians.

(i) Linear sigma model (LSM) (Birse, 1990; and references therein)

\[
H = \int d^3x \left[ \frac{1}{2} \left[ \tilde{P}^2_\pi + \nabla \tilde{\pi} \cdot \nabla \tilde{\pi} + P^2_\sigma + \nabla \sigma \cdot \nabla \sigma \right] + \bar{\psi} [-i \gamma \cdot \nabla + g(\sigma + i\gamma_5 \tilde{\tau} \cdot \tilde{\pi})] \psi + \frac{\lambda^2}{4} \left( \sigma^2 + \tilde{\pi}^2 - \nu^2 \right)^2 - 4 \pi m^2_\pi \sigma \right],
\]

(ii) Chiral chromodielectric model (CDM) (Birse, 1990)

\[
H = \int d^3x \left\{ \frac{1}{2} \left[ \tilde{P}^2_\pi + \nabla \tilde{\pi} \cdot \nabla \tilde{\pi} + P^2_\sigma + \nabla \sigma \cdot \nabla \sigma + P^2_\chi + \nabla \chi \cdot \nabla \chi \right] + \bar{\psi} [-i \gamma \cdot \nabla + \frac{g}{\chi} (\sigma + i\gamma_5 \tilde{\tau} \cdot \tilde{\pi})] \psi + \frac{\lambda^2}{4} \left( \sigma^2 + \tilde{\pi}^2 - \nu^2 \right)^2 - 4 \pi m^2_\pi \sigma + \frac{1}{2} M^2_\chi \chi^2 \right\},
\]
(iii) Cloudy bag model (CBM) (Thomas, 1983)

\[
H = \int d^3x \left[ \frac{1}{2} [\vec{P}_\pi^2 + \nabla \vec{\pi} \cdot \nabla \vec{\pi} + m_\pi^2 \vec{\pi}^2] + \frac{i}{2f_\pi} \bar{\Psi} \gamma_5 \vec{\pi} \cdot \vec{\pi} \delta(x - R) - i\bar{\Psi} \gamma_5 \nabla \Psi \Theta(R - x) \right].
\]

In all these expressions, \( \Psi \) stands for the quark field, \( \vec{\pi} \) and \( \sigma \) for the chiral meson fields, and \( \chi \) for the scalar-isoscalar chiral singlet field which (dynamically) generates confinement in the CDM. The model parameters are: the pion mass \( m_\pi = 0.14 \text{ GeV} \) and the pion decay constant \( f_\pi = 0.093 \text{ GeV} \) which appear in the three models (these parameters also enter in the mexican-hat potential for the chiral fields in LSM and CDM); the sigma mass \( m_\sigma = 1.2 \text{ GeV} \) which enters in the mexican-hat in LSM and CDM. In the LSM there remains just one (dimensionless) parameter \( g \). In the CDM there are two free parameters, the coupling constant \( g \) (with dimension of energy) and the \( \chi \) mass \( M_\chi \). However, in practice the results are sensitive mainly to one combination, \( G = \sqrt{gM_\chi} \). Finally, in the CBM the free parameter is the bag radius \( R \).

The purpose of comparing the three models is to generate situations with a wide range of \( n_\pi \) and to explore model dependence of our conclusions.

The first two models offer only a small range of \( n_\pi \). In the LSM, for small \( n_\pi \) \( (n_\pi \ll 1) \) the system of three quarks is not bound, and for a large one \( (n_\pi \gg 1) \) the vacuum gets unstable against baryon-antibaryon creation. Reasonable values of observables are obtained in the range \( 4.5 < g < 5.5 \) and \( n_\pi \sim 1 \). In the CDM the system of three quarks is always confined by the \( \chi \) field and therefore the sigma and the pion fields are much suppressed with respect to their values in the LSM. Correspondingly, \( n_\pi \) is smaller in comparison with its typical values in the LSM. Even for very high coupling constant \( G \), the selfconsistent field \( \chi \) is such that a strong effective coupling between quarks and pions is never achieved. Therefore the number of pions always remains small \( (n_\pi \sim 0.2) \). The physical range of the coupling constant is \( 0.175 \text{ GeV} < G < 0.205 \text{ GeV} \). The cloudy bag is a suitable model to offer a wide range of \( n_\pi \) since the quarks are confined by a bag and pions are not indispensable for binding. However, the number of pions is very sensitive to changes of \( R \) so that, by varying \( R \) in a wide range we can obtain \( n_\pi \) in a wide range too. In the presentation of our results, we consider unrealistic values of \( n_\pi \) just for the purpose of studying the \( n_\pi \)-dependence of the observables.

| Model | \( g/G/R \) | \( n_\pi \) | \( \langle r_i^2 \rangle \) | \( \langle r_{ij}^2 \rangle \) | \( \mu_{\pi} \) | \( \mu_{N} \) | \( g_\Lambda \) | \( \frac{g}{2M_N} \) | \( g_{-NN} \) |
|-------|-------------|----------|----------------|----------------|----------|----------|--------|----------------|-----------|
| LSM   | 4.6 0.92    | 0.61     | −0.10          | 0.18           | 2.53     | 1.80     | 1.25   |                |           |
|       | 5.0 0.98    | 0.59     | −0.11          | 0.17           | 2.57     | 1.82     | 1.26   |                |           |
|       | 5.4 1.03    | 0.56     | −0.11          | 0.16           | 2.60     | 1.83     | 1.26   |                |           |
| CDM   | 0.19 0.20   | 0.98     | −0.06          | 0.28           | 1.94     | 1.37     | 0.94   |                |           |
|       | 0.20 0.22   | 0.91     | −0.06          | 0.26           | 1.88     | 1.38     | 0.94   |                |           |
|       | 1.3 0.12    | 0.95     | −0.05          | 0.39           | 2.28     | 1.06     | 0.78   |                |           |
| CBM   | 1.0 0.24    | 0.59     | −0.06          | 0.28           | 1.93     | 1.03     | 0.76   |                |           |
|       | 0.7 0.52    | 0.33     | −0.07          | 0.17           | 1.69     | 0.97     | 0.72   |                |           |
| Exp.  | – –         | 0.69     | −0.12          | 0.44           | 2.35     | 1.26     | 1.01   |                |           |

Table 1: Static quantities of the nucleon shown in dependence of the coupling constant \( g \) (in LSM), \( G \) [GeV] (in CDM) and bag radius \( R \) [fm] (in CBM) and of the number of pions. Charge radii are in units of fm\(^2\), magnetic moments in n.m.
We calculated nucleon properties in all three models using a projected hedgehog wavefunction (Cibej, Fiolhais, Golli and Rosina, 1992) for the three valence quarks and the meson (and chromodielectric) fields. The fields were treated quantum mechanically as coherent states. All radial profiles were determined variationally, with variation after projection, subject to virial constraints of the type $\langle \Psi | [H, P(r)] | \Psi \rangle = 0$ (Amoreira, Fiolhais, Golli and Rosina, 1995). Using such a constraint guarantees the proper asymptotic behaviour of the pion field as well as some other consistency relations. The calculated static properties are shown in Table 1.

All figures show the observables as a function of $n_\pi$. Such diagrams were not shown before and we believe that they are very instructive. In order to recognize the respective coupling strength in each model which corresponds to a given $n_\pi$, the “vocabulary” is given in Fig. 1.

Figure 1: Coupling constants $g$ [MeV] in CDM (with $M_\chi$ fixed to 1.4 GeV) and $g$ [dimensionless] in LSM (a) and bag radius $R$ [fm] in CBM (b) as a function of $n_\pi$.

Nucleon–Delta mass difference

In all three models the $\Delta - N$ mass splitting is mostly due to pions. For low $n_\pi$ the mass splitting increases with the number of pions because nucleon and $\Delta$ differ in the kinetic energy of pions but not in the interaction energy of pions with the quark source; with increasing coupling constant the number of pions $n_\pi$ grows and also the kinetic energy per pion grows (since all radial profiles shrink). For high $n_\pi$ the mass splitting decreases because both states belong to a rotational band and the moment of inertia increases with $n_\pi$. There is a peak between $n_\pi = 1$ and 10 (Fig. 2). It is favourable to have $n_\pi$ around 1 in order to avoid exceedingly large additional terms such as chromomagnetic repulsion.

Figure 2: $\Delta - N$ mass difference (in GeV) in CDM, LSM and CBM as a function of $n_\pi$. 

4
Charge radii

The charge radius squared of the proton is a weighted mean between $\langle r_q^2 \rangle$ of the quark distribution and $\langle r_\pi^2 \rangle$ of the pionic distribution. One might expect that $\langle r^2 \rangle$ increases with pion number since the pionic distribution is broader. One is, however, surprised to find that $\langle r^2 \rangle$ decreases with $n_\pi$ rather than increases (Fig. 3a). The explanation is twofold. First, the pionic distribution is not much broader. One intuitively thinks of the extension of the Yukawa field which is of the order of $1/m_\pi \sim 1.4$ fm. The charge distribution of pions, however, is much narrower than the field. This can be seen from the relation between the pion field $\Phi$ and the “pion wavefunction” $\phi$ in momentum space

$$\tilde{\Phi}(k) \sim (\tilde{\phi}^*(k) + \tilde{\phi}(k))/\sqrt{2\omega_k}, \quad \omega_k \approx k.$$  

Since $\tilde{\phi}(k) \sim \sqrt{k} \tilde{\Phi}(k)$ is enhanced at large momenta, the charge distribution in coordinate space is enhanced at smaller $r$. For example, for typical model parameters in the LSM ($g = 4.8$) we get for the pion profile a radius squared 0.87 fm$^2$, not drastically larger than for the quark profile (0.48 fm$^2$).

On the other hand, all radial profiles (of quarks and pions) shrink with increasing $n_\pi$ (increasing interaction strength $g$) which overcompensates the increase due to the higher weight of pions.

The charge radius squared of the neutron (lower curves in Fig. 3a) remains small and negative for all values of $n_\pi$. The reason is similar as before, the negative charge distribution of pions is not much broader than the positive charge distribution of the quark core so that they almost cancel, consistently with the experimental value.

Magnetic moments

While the isovector magnetic moment where quarks and pions contribute is well described the isoscalar magnetic moment which lacks a pionic contribution is twice too small (Fig. 3b). This is a common feature of all models with quarks and pions (contrary to models with quarks only which automatically give the correct ratio 5:1). One should not conclude from this that the “best” number of pions is zero but rather that some additional degree of freedom is missing on which the isoscalar magnetic moment is sensitive.

![Figure 3: Proton and neutron charge square radii [fm$^2$] (a) and isoscalar and isovector magnetic moments [n.m.] (b) as a function of $n_\pi$ for CDM, LSM and CBM.](image-url)
Axial charge

In the LSM and the CDM, the axial charge appears as a cross term between the pion and sigma fields. It is so large in the LSM (Fig. 4a) because of the strong sigma field along with a relative large \( n_\pi \). Therefore one cannot blame only \( n_\pi \) but rather the strong sigma field. In the CBM the pions do not contribute directly to this observable.

Pion-nucleon coupling constant

In the expression (Čibej, Fiolhais, Golli and M. Rosina, 1992)

\[
\frac{m_\pi}{2M_N} g_{\pi NN} = \frac{m_\pi^3}{2\sqrt{3}} \sqrt{2\pi\omega} \int_0^{\infty} r^3 \xi_0(r) dr \langle (a_{00} + a_{00}^\dagger) \rangle
\]

the expectation value of \((a + a^\dagger)\) “counts pions” and increases approximately as \( \sqrt{n_\pi} \), while the radial integral decreases (Fig. 4b). Because of these effects it is difficult to deduce the “experimental” value of \( n_\pi \) from this observable. We can, however, better understand the stability of the result with respect to the coupling strength. In the CBM the \( g_{\pi NN} \) comes out too small since we are using the physical value for \( f_\pi \). In the perturbative calculations of the CBM \( f_\pi \) was a fitting parameter adjusted to reproduce the experimental value of \( g_{\pi NN} \) (Théberge, Miller and Thomas, 1982).

![Figure 4: Axial coupling constant (a) and the \( \pi NN \) coupling constant (b) as a function of \( n_\pi \) for CDM, LSM and CBM.](image)

Electric and magnetic polarizabilities

The electric polarizability (Golli and Sraka, 1993) comes twice too large in the LSM \((23 \cdot 10^{-4} \text{ fm}^3)\), particularly due to the large seagull term which possibly overcounts the pionic contribution. The situation is opposite for the magnetic polarizability: the relatively large \( n_\pi \) around one which we use is needed in order to compensate the large paramagnetic term due to the virtual \( \Delta \) excitation. This virtual excitation cannot be avoided or diminished (it contributes \( 20 \cdot 10^{-3} \text{ fm}^3 \)), therefore the diamagnetic effect of pions is necessary to approach the observed very small value (we get a pionic contribution \(-10 \cdot 10^{-4} \text{ fm}^3\)). A smaller number of pions which would improve the electric polarizability would spoil the magnetic one. We argue that it is in the electric polarizability that some effect is still missing and that a large \( n_\pi \) (which is needed for the magnetic polarizability) is not ruled out.
ELECTROEXCITATION OF BARYON RESONANCES

The amplitudes for electroproduction of low lying baryon resonances can also yield sensible information on the pion content of the nucleon and its excited states. In particular, relatively large quadrupole $E_2$ (or $E_{1+}$) and $C_2$ ($S_{1+}$) amplitudes in the vicinity of the $\Delta(1232)$ resonance can be almost entirely attributed to the cloud of p-wave pions (Foilhais, Golli and Širca, 1995). In quark models without the pion cloud, an unrealistically strong admixture of d-state quarks would have to be introduced in order to explain the experimental values.

In photoproduction, because of relatively low photon momentum which, in the centre-of-mass frame, is given by $k = k_W = (M_\Delta^2 - M_N^2) / 2M_\Delta = 258$ MeV, the quadrupole photon only “sees” the tail of the pion cloud in the region above 1 fm whose behaviour is determined by the Yukawa form and the $\pi N$ coupling constant. If this mechanism is correct we can expect that all those models containing the pion cloud which reproduce $g_{\pi NN}$ are also able to reproduce well the quadrupole amplitudes. The number of pions is therefore not important here. In order to be able to investigate the pion cloud in the interior of the baryon we have to consider electroproduction. Since such a process involves virtual photons with nonzero $Q^2$, $-Q^2 = \omega^2 - k^2$, it is possible to “sit” on the resonance while changing the photon momentum, $k^2 = [(M_\Delta^2 + M_N^2 + Q^2) / 2M_\Delta]^2 - M_N^2$. Because of kinematics, the $E_2$ amplitude is very small and difficult to measure precisely; on the other hand, the scalar quadrupole amplitude $C_2$ is clearly nonzero in electroproduction and may be an important indication of the presence of a strong pion cloud in the interior of the nucleon and the $\Delta$. In Fig. 5 we plot the $C_2$ amplitude calculated as a function of $n_\pi$ in the three models for photoproduction ($Q^2 = 0$) and for electroproduction at $Q^2 = 1$ GeV$^2$. We see, as anticipated, that for the former its value only weakly depends on the number of photons. For $Q^2 = 1$ GeV$^2$, however, it strongly depends on $n_\pi$; the experimental value $C_2 = -11.8 \pm 2.3 \cdot 10^{-3}$ GeV$^{-1/2}$ (Alder, 1972; Albrecht, 1971) suggests a rather strong pion cloud. (The experimental value at $Q^2 = 0$ is not measured directly. Using current conservation and the Siegert theorem it can be related to the value of $E_2$, $E_2 = -4.4 \pm 1.2 \cdot 10^{-3}$ GeV$^{-1/2}$ (Particle Data Group, 1994).)

![Figure 5: Electroexcitation electric quadrupole amplitude $C_2$ in units of $10^{-3}$ GeV$^{-1/2}$ as a function of $n_\pi$ for CBM, LSM and CBM at two values of photon virtuality: $Q^2 = 0$ GeV (thin lines) and $Q^2 = 1$ GeV (thick lines). Dotted lines denote the experimental uncertainty for $C_2$ at $Q^2 = 1$ GeV.](image-url)
ELECTROPRODUCTION OF PIONS

It is interesting to connect the number of “theoretical pions” to the number of pion-like partons kicked out in the reactions $e + p \to e' + n + \pi^+$ or $e + p \to e' + X + \pi^+$. Agreement would confirm the parton picture of pions in the nucleon (in an impulse approximation).

A pioneering analysis of the pion distribution function in the nucleon $G_{\pi^*/p}(x)$ was performed by Güttner, Chanfray, Pirner and Povh, 1984, assuming

$$\frac{d^2\sigma_L(ep \to e'X\pi)}{dx dQ^2} = G_{\pi^*/p}(x) \frac{d\sigma_{el}(e\pi^* \to e'\pi)}{dQ^2}.$$ 

In principle, the pion distribution function $G$ is obtained as a ratio between the measured pion electroproduction cross section and the calculated cross section for the elastic scattering of electron on pion. Integrating $G$, one obtains the “experimental number of pions” $n^{\exp}_\pi$. The authors derived the probability 3% of the $\pi^+n$ configuration in the proton and estimate further 1.5% for the $\pi^0p$ and 3% for the $\pi\Delta$ configurations. This amounts at most to $n^{\exp}_\pi = 0.08$, favouring models with small number of pions.

This estimate is, however, inconclusive since the DESY experiment $e + p \to e' + n + \pi^+$ was not presented with full kinematics. The cross sections were given as a function of the squared momentum transfer onto the target nucleon, $t = (p_{\gamma} - p_{\pi})^2$ rather than as a function of the relevant Bjorken variable $x$. Therefore, $d^2\sigma(x, Q^2)$ was approximately reconstructed using statistical assumptions. Also, the proton and $\Delta$ final states were not measured in this reaction and the corresponding contributions were only estimated. Nevertheless, this analysis gave a strong stimulation for future research. A repeated experiment and new experiments would be very important.

A new proposal for the inclusive $e + p \to e' + X + \pi^+$ experiment has been presented and a theoretical estimate $n_\pi = 0.16$ given if all final states $X$, also beyond nucleon and $\Delta$, are taken (Pirner and Povh, 1993).

In order to plan well the future experiments, different model predictions have to be confronted so that one can anticipate the probable range of $n_\pi$. Recently, Baumgärtner, Pirner, Königsmann and Povh, 1995, proposed a very interesting model in which most pions act as private partons of each constituent quark. They could be probed at low $x$ and very high energy of electrons (at HERA). Since the model has been presented by Pirner at this School (Pirner, 1995) we shall only sketch the main idea.

If one assumes that the constituent quark consists of a bare quark plus a bare quark with a pion cloud, the angular momentum and charge gets distributed between the bare quark and the pion in a particular manner. This can simultaneously improve the Gottfried sum rule, the integrated polarized structure function of the proton and neutron, the quark contribution to nucleon helicity and the axial charge. The estimated pion admixture is 36% per constituent quark (around one pion per nucleon).

Such large $n_\pi$ resembles our results in the LSM. Of course, we do not distinguish whether pions are correlated with individual quarks or with the nucleon as a whole. We use a “mean source approximation” for the pions. Because the models are different the comparison is inconclusive, but it is suggestive.
CONCLUSION

The contribution of the pion cloud to nucleon observables can be conveniently analyzed as a product of the number of pions $n_\pi$ times the specific contribution per pion. While $n_\pi$ increases with increasing coupling strength of pions to the quark core, the specific contribution may increase, decrease or remain constant. Therefore we can classify several typical cases.

(i) The electric charge radius of the proton, the isovector magnetic moment, the axial coupling constant $g_A$ and the $\pi NN$ coupling constant show indifference or a slow decrease with $n_\pi$. The reason is that the specific contribution per pion decreases with $n_\pi$ since all radial profiles shrink. Therefore these observables are not suitable for “counting pions”.

(ii) The neutron electric charge radius squared contains a delicate cancellation and cannot suggest a reliable value of $n_\pi$.

(iii) The diamagnetic contribution to the magnetic polarizability as well as the electroexcitation of the $\Delta$ resonance show a strong dependence on $n_\pi$. They show that the contribution of pions is essential and suggest $n_\pi \sim 1$. Both theoretical and experimental analysis are still in a preliminary stage but they strongly encourage further theoretical and experimental studies of these quantities which are so promising for pion counting.

(iv) The electric polarizability cannot come out well with same parameters as the magnetic polarizability. The isoscalar magnetic moment does not contain a pion contribution and is not consistent with the isovector magnetic moment. The pion contribution to the axial coupling constant depends on the product of the pion and sigma fields and not on the pion field alone. Such inconsistent or ambiguous cases are also not suitable to determine $n_\pi$; other degrees of freedom are required.

Different models predict a wide range of values of $n_\pi$. It remains a challenge to relate this theoretical concept to the number of pions that are probed in electron scattering on the proton.

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