Controlled generation of higher-order Poincaré sphere beams from a laser

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The angular momentum of light can be described by positions on a higher-order Poincaré sphere, where superpositions of spin and orbital angular momentum states give rise to laser beams that have many applications, from microscopy to materials processing. Many techniques exist to create such beams but none so far allow their creation at the source. Here we report on a new class of laser that is able to generate all states on the higher-order Poincaré sphere. We exploit geometric phase control inside a laser cavity to map polarization to orbital angular momentum, demonstrating that the orbital angular momentum degeneracy of a standard laser cavity may be broken, producing pure orbital angular momentum beams, and that generalized vector vortex beams may be created with high purity at the source. This work paves the way to new lasers for structured light based on intracavity geometric phase control.
allowing generalized vector vortex beams to be created. mode (of opposite helicity) for each of the two polarizations, controlled. We use a geometric phase element to create an OAM states within the cavity. But as the polarization is not degenerate polarization, or equivalently, particular modes to polarization. It is therefore possible to map particular phase changes to has not been used inside a laser cavity for mode control. The circulating light is circularly polarized. In such a cavity the fraction of light that leaks out. It follows that in region B the polarizing beam splitter (PBS) and quarter-wave plate (QWP) ensure that the polarization state in region A is always linearly polarized, with the orientation dependent on that of the PBS. Traditionally such cavities are used to output light from the PBS, with the orientation of the QWP acting as a control on the fraction of light that leaks out. It follows that in region B the circulating light is circularly polarized. In such a cavity the polarization at any position is controlled and repeated after every round trip. Our central idea is to exploit the SAM control as a proxy for OAM control, thereby realizing generalized modes on the HOP sphere. We employ geometric phase, which until now has not been used inside a laser cavity for mode control. The geometric phase change has a sign that is polarization-dependent. It is therefore possible to map particular phase changes to polarization, or equivalently, particular modes to polarization states within the cavity. But as the polarization is not degenerate and can easily be controlled, so by proxy the modes can be controlled. We use a geometric phase element to create an OAM mode (of opposite helicity) for each of the two polarizations, allowing generalized vector vortex beams to be created.

Our non-homogenous polarization optic, in the form of a $q$-plate, acts as a SAM–OAM converter inside the cavity. The $q$-plate ladders some incoming OAM state following the selection rules: $|\ell, L\rangle \rightarrow |\ell + 2q, R\rangle$ and $|\ell, R\rangle \rightarrow |\ell - 2q, L\rangle$, where $L$ and $R$ refer to left and right circularly polarized light and $q$ is the charge of the $q$-plate (see Supplementary Information). This concept is illustrated graphically in Fig. 2b. By modifying the standard cavity to that shown in Fig. 2c, OAM-carrying beams are created within the cavity. The doubling of the elements ensures that the spatial mode and polarization states are repeated after each complete round trip. The QWP and $q$-plate angles provide the two degrees of freedom necessary to traverse the entire HOP sphere. It can be shown (see Supplementary Information) that our repeating mode in the cavity can be described by

$$V_{ou} = \cos(\frac{\Theta}{2}) \exp(-i\frac{\Phi}{2}) |L_\ell\rangle + \sin(\frac{\Theta}{2}) \exp(i\frac{\Phi}{2}) |R_\ell\rangle \quad (1)$$

**Results**

**Concept.** In contrast to the complexity and challenges of producing OAM beams and vector vortex beams from lasers, the control of polarization or SAM inside laser cavities is a well-established technique. Consider a standard solid-state laser cavity in a Fabry–Pérot configuration, as shown in Fig. 2a. The inclusion of a polarizing beam splitter (PBS) and quarter-wave plate (QWP) ensures that the polarization state in region A is always linearly polarized, with the orientation dependent on that of the PBS. Traditionally such cavities are used to output light from the PBS, with the orientation of the QWP acting as a control on the fraction of light that leaks out. It follows that in region B the circulating light is circularly polarized. In such a cavity the polarization at any position is controlled and repeated after every round trip. Our central idea is to exploit the SAM control as a proxy for OAM control, thereby realizing generalized modes on the HOP sphere. We employ geometric phase, which until now has not been used inside a laser cavity for mode control. The geometric phase change has a sign that is polarization-dependent. It is therefore possible to map particular phase changes to polarization, or equivalently, particular modes to polarization states within the cavity. But as the polarization is not degenerate and can easily be controlled, so by proxy the modes can be controlled. We use a geometric phase element to create an OAM mode (of opposite helicity) for each of the two polarizations, allowing generalized vector vortex beams to be created.

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**Figure 1** HOP sphere representation of vector vortex beams. a. Local polarization vector states at various positions on the sphere. b. The intensity of the outputs are consistent beams with a central null. These beams are differentiated by the transmitted intensity from a linear polarizer oriented in the vertical, as depicted by the double-ended arrows. Expressions are provided for the states at the poles and for the special points on the equator with radial and azimuthal polarization.

**Figure 2** Laser concept. a. A standard Fabry–Pérot configuration controls the laser polarization using a PBS and a QWP. b. A $q$-plate is used to map the polarization to helically phased beams, with the handedness of the output depending on the incident state of the circular polarization. c. Experimental concept of the active selection of pure OAM LG$_{0\ell}$ by the intracavity coupling of SAM to OAM. The coupling is achieved by selecting a pure SAM state by transmitting light that is linearly polarized in the horizontal through a QWP rotated at angle $\gamma$ and consequently out-coupled through the FM. The two rotation angles may be varied accordingly to map out the HOP sphere. The inset illustrates the various polarization states operating in the cavity with their associated vectors.
A pure scalar mode of left handedness is observed at $\beta = 45^\circ$ after the wave-plate if the wave-plate axis is at $45^\circ$. The horizontally polarized Gaussian beam is converted this left circularly polarized beam into an OAM beam of charge $\ell = 1$ with right circular polarization. Reflection off the mirror inverts the entire state in both SAM and OAM, whereas the two remaining elements, oriented at opposite angles to the first two, reverse the process to create a vertically polarized Gaussian beam incident on mirror $R_2$. When this beam is propagated backwards through the cavity the modes invert again and return to mirror $R_1$ to the starting mode. The consequence is that the handedness of the light, as well as its vector nature, is completely defined by $\beta$ and $\gamma$. For example, if the QWP is rotated to produce linearly polarized light before the wave-plate, then superpositions of left- and right-handed light with opposite OAM charges is produced—our general vector beams.

The resonator concept as illustrated in Fig. 2c necessitates the use of a pair of $q$-plates and a pair of QWPs with a polarization-insensitive $45^\circ$ mirror (FM) positioned between the $q$-plates. This cavity may be equivalently constructed by resorting to a V-shaped cavity (see Methods) where the two arms are separated to a few degrees with a planar mirror positioned at the apex of the V allowing for an off-axis design where only a single $q$-plate and QWP are required; this allowed the polarization optics inside the cavity to be halved and thus reduced the losses to approximately that of the conventional cavity. It also ensured that the spatial mode and polarization states repeated after one round trip after taking into account the reflections inside the cavity. The losses introduced by the addition of the $q$-plate were negligible in comparison with the losses due to the output.

$$|L_i\rangle = e^{-i(\ell \phi)}|L_i\rangle, \quad |R_i\rangle = e^{i(\ell \phi)}|R_i\rangle,$$

with $|L_i\rangle$ and $|R_i\rangle$ representing uniform left circular and right circular polarization states, respectively, $\Theta = \pi/2 + 2\beta$ and $\Phi = 2\gamma - 2\beta$ where $\beta$ and $\gamma$ are the rotation angles of the QWP and $q$-plate, respectively. This is precisely the description of a point on the HOP sphere with coordinates $\Theta$ and $\Phi$, where the poles on the sphere represent the basis states $|R_i\rangle$ and $|L_i\rangle$. In other words, any HOP sphere beam can be realized from the laser. Examples of special cases are given in Table 1.

Heuristically the cavity can be understood by following the evolution of a Gaussian mode of linear polarization propagating in region A away from mirror $R_1$. The horizontally polarized Gaussian beam is converted into a left circularly polarized Gaussian beam after the wave-plate if the wave-plate axis is at $45^\circ$. The $q$-plate converts this left circularly polarized beam into an OAM beam of charge $\ell = 1$ with right circular polarization. Reflection off the mirror inverts the entire state in both SAM and OAM, whereas the two remaining elements, oriented at opposite angles to the first two, reverse the process to create a vertically polarized Gaussian beam incident on mirror $R_2$. When this beam is propagated backwards through the cavity the modes invert again and return to mirror $R_1$ to the starting mode. The consequence is that the handedness of the light, as well as its vector nature, is completely defined by $\beta$ and $\gamma$. For example, if the QWP is rotated to produce linearly polarized light before the wave-plate, then superpositions of left- and right-handed light with opposite OAM charges is produced—our general vector beams.

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**Table 1 | QWP and $q$-plate settings with corresponding output states.**

| $\beta / \gamma$ | $\gamma = 0$ | $\gamma = \pi/2$ |
|------------------|--------------|------------------|
| $-\pi/4$         | $e^{-i(\ell \phi)}|L_i\rangle + |R_i\rangle$ | $e^{-i(\ell \phi)}|L_i\rangle + 0|R_i\rangle$ |
| $0$              | $(1/\sqrt{2})(|L_i\rangle + |R_i\rangle)$       | $(1/\sqrt{2})(-|L_i\rangle + |R_i\rangle)$ |
| $\pi/4$          | $0|L_i\rangle + e^{-i(\ell \phi)}|R_i\rangle$  | $0|L_i\rangle + e^{i(\ell \phi)}|R_i\rangle$ |

Specialized output states are realized by rotating the $q$-plate through angles $\beta$ and $\gamma$, respectively. These include a pure OAM state of $-\ell (-\ell = \pi/2, \gamma)$, pure OAM state of $+\ell (\pi/2, \gamma)$, radial polarization $(0,\gamma)$ and azimuthal polarization $(\ell, 0)$. The cavity was operated at $\ell = 0.5$ and $\gamma = 0^\circ$ for these tests.

**Figure 3 | Mode purity of OAM beams.**

(a) Recorded output of the laser as a function of $\beta$ with the insets showing the left- and right-circularly polarized components. A pure scalar mode of left handedness is observed at $\beta = -45^\circ$ and of right handedness at $\beta = 45^\circ$, with superpositions of SAM states in between (for example $\beta = 0^\circ$). (b) Experimental measured (data points) and theoretical prediction (curves) for the evolution of the relative weightings of the $|L_i\rangle$ and $|R_i\rangle$ states making up the final field as a function of $\beta$. (c) Modal analysis of the laser output ($\beta = \pm 45^\circ$) confirms pure $\text{LG}_{-1,1}$ and $\text{LG}_{0,1}$ modes, respectively, with their corresponding measurement channels (right). $m$, azimuthal index. (d) Results of radial and azimuthal modal decomposition show that $\approx 98\%$ of the power is contained in the desired mode ($\ell = -1$ and $p = 0$). The cavity was operated at $q = 1/2$ and $\gamma = 0^\circ$ for these tests.
The angle of rotation, \( \beta \), on rotation. Parallel to the orientation of the linear polarizer (indicated by double-ended arrows) on rotation.

Figure 4 | Measured HOP sphere beams. a, The output of the laser \( (\gamma = 0, \beta = 0) \) is radially polarized (this is confirmed in b). A lobe structure is observed after transmission through a linear polarizer. The lobed structure is parallel to the orientation of the linear polarizer (Fig. 4d). With \( \gamma = 90^\circ \) an azimuthally polarized beam is observed, and confirmed in d, d. The lobe structure now rotates out of phase with the polarizer. These results are also confirmed by Stokes polarimetry (see Supplementary Information). e, Experimental measured beams as represented on a HOP sphere, together with their state expression and an example of the transmission through a polarizer. The values in parentheses represent the angles \( \beta \) and \( \gamma \) used to create these beams.

Generation of HOP sphere beams. We initially set \( \gamma = 0 \) and varied the angle of rotation, \( \beta \), of the QWP. The output beam was an annular shaped beam (see Fig. 3a) independent of \( \beta \), as expected from theory. The state of the output is given by \( \psi_{\text{out}} = \alpha_1 |L_\ell \rangle + \alpha_2 |R_\ell \rangle \) where \( \alpha_1 \) and \( \alpha_2 \) are the relative weights of the states on the poles. A measurement of the polarization state (evident from the insets in Fig. 3a) confirms that the mode evolves from a left circularly polarized beam (\( \beta = -45^\circ \)) to a right circularly polarized beam (\( \beta = 45^\circ \)). To determine the accuracy in the variation of the polarization as shown in Fig. 3a, we measured the intensity of the relative weights of the left and right components of the transmitted light. These components describe the states on the poles of the HOP sphere and thus \( \alpha_1 = \cos(\theta/2)e^{i\Phi/2} \) and \( \alpha_2 = \sin(\theta/2)e^{i\Phi/2} \). The measured intensities of the respective components compare well with the numerical determination for \( \beta \) varied from \(-45^\circ \) to \(45^\circ\) as illustrated in Fig. 3b.

Next we measured the OAM state by a modal decomposition with a phase-only spatial light modulator (see Supplementary Information). We find that at \( \beta = -45^\circ \) the mode is a pure \( \ell = -1 \) helicity, whereas a pure \( \ell = 1 \) for \( \beta = 45^\circ \). This is illustrated graphically in Fig. 3c, together with the raw data for three of the modal decomposition channels, where a central peak indicates the present mode and a central null indicates the absence of that mode. The intracavity aperture ensures that the radial index of the mode is \( p = 0 \), and this too is confirmed by modal decomposition (see Fig. 3d). This approach presents a means to actively select the handedness of pure LG\(_{0,\pm 1}\) (Laguerre-Gaussian of radial order \( p \) and azimuthal order \( \ell \)) modes depending only on \( \beta \) and \( q \) of the q-plane. Experimental techniques produce beams with many radial modes (with \(<80\% \) of the power in the \( p = 0 \) mode for \( \ell = 1 \)) following a hypergeometric function (see Supplementary Information).

The energy content in the \( p = 0 \) mode drops dramatically with increasing \( \ell \), to as little as about 1% for \( \ell = 10 \). In other words, it is not possible to simultaneously maximize purity and modal power by external mode creation techniques. Here we demonstrate the selection of LG\(_{0,\pm 1}\) modes of high mode purity (>98%, \( p = 0 \))—a crucial requirement for certain applications, for example, in stimulated emission depletion (STED) microscopy.

Modes represented on the equator of a HOP sphere consist of a mixture of SAM and OAM states as determined by equation (1). The combination of SAM states is achieved by setting \( \beta \) to zero such that a pure linear state is incident on the q-plane resulting in a superposition output (as in the inset of Fig. 3a for \( \beta = 0^\circ \)). Consequently this also leads to a superposition of OAM and SAM states at the output, as given in Table 1. The non-separability of the polarization and spatial content of the mode means that on passing through a linear polarizer, the annular shaped output splits into two lobes that rotate with a rotation in the polarizer. With the laser operating under the conditions of \( \beta = 0^\circ \) and \( \gamma = 0^\circ \), we obtain an annular beam (Fig. 4a) that leads to a rotatable lobed beam (Fig. 4b) after transmission through a linear polarizer. The two-lobed structure is oriented parallel to the orientation of the linear polarizer (illustrated as double-ended arrows) thus presenting a pure radialy polarized vectorial vortex beam. With \( \gamma \) rotated by \( 90^\circ \) we select an annular beam (Fig. 4c) that is of pure azimuthal polarization, which is hallmarkled by the two-lobed structure being perpendicular to the orientation of the linear polarizer (Fig. 4d). We confirm the non-homogenous polarization map by Stokes polarimeter (on three output states, namely azimuthal, right circular (north pole) and radial, and by modal decomposition on each polarization component (see Supplementary Information). The remarkable capability of selectively exciting these vectorial vortex beams means that not only are CV vortex beams achievable but so are arbitrary vector states by controlling the input polarization state on the q-plane by selecting the correct value of \( \beta \). Proper control of \( \beta \) and \( \gamma \) allows for the entire HOP sphere to be mapped and to aid consistency in comparison to Fig. 1b, the states between the radial and azimuthal polarizations are accordingly determined as illustrated in Fig. 4e. The annular outputs are transmitted through a linear polarizer oriented vertically (depicted by the double-ended arrows) and are in excellent agreement with the anticipated intensities. Note that to create such modes external to the cavity would require several passes off an spatial light modulator (very lossy) or mixing beams with interferometry (difficult to align). Here, high mode purity is achieved across a range of modes by the rotation of just two optics.

This technique is not limited to LG\(_{0,\pm 1}\) modes, in fact a q-plane with a higher \( q \) value may be equivalently realized. We demonstrate this by replacing the q-plane of \( q = 1/2 \) with \( q = 5 \) thus allowing for the selection of LG\(_{0,\pm 10}\) modes without changing the physical properties of the cavity. The outputs for the cavity operating under \( \beta = -45^\circ, 45^\circ \) and \( 0^\circ \) with \( \gamma = 0 \) are illustrated in Fig. 5a and show well-defined annular beams. With the cavity operating at \( \beta = 0^\circ \), the annular output leads to a rotatable lobed beam after a linear polarizer as shown in Fig. 5b resulting in a radially polarized
output. Again, to infer the OAM of the mode, we execute an azimuthal inner product with the digitally encoded transmission function $e^{im\phi}$ for $m$ varying from $-12$ to $+12$ in unit steps and we identify an on-axis signal for $m = 10$ and $m = -10$ with zero elsewhere corresponding to operation for $\beta = 45^\circ$ and $-45^\circ$, respectively, as presented in Fig. 5c with some example measurement channels.

**Discussion**

We have outlined the concept for a new class of laser that utilizes geometric phase control to realize arbitrary HOP sphere beams. We have demonstrated the concept in an otherwise conventional solid-state laser cavity and shown the controlled generation of such beams, including the special cases of OAM modes of high purity (>98%) as well as azimuthally and radially polarized light. These beams have found many applications, for example, in materials processing (tighter focusing and cleaner material edges), optical communication (vector mode multiplexing), mimicking quantum processes by virtue of their non-separable nature and in microscopy with structured light. The advantages of creating the desired modes at the source include significantly higher mode purity, direct compensation of losses by gain extraction and the flexibility of a fully integrated solution to produce such beams (table-top solutions will be invaluable in the applications of such beams).

In particular, as the first example of intracavity mode selection by the Pancharatnam–Berry phase, we believe this report will generate interest in this approach to designing integrated custom lasers. Such systems need not be based on Fabry–Perot cavities: for example, ring resonators, wavelength control in birefringent gain media and more compact solutions in the form of fibre lasers. As the mode at the back reflector of our cavity is a linearly polarized Gaussian beam, the standard approach to mode locking could be incorporated into the cavity with little difficulty by ensuring that the Kerr medium is at the Gaussian end of the cavity, thus providing a route towards spatial and temporal mode control. An interesting avenue to pursue would be the combination of geometric and dynamic phase control inside the cavity. For example, the radial mode ($p = 0$) here is fixed by mapping to a Gaussian function, selected by an aperture near the back reflector of the cavity. The aperture and back reflector could be replaced with a spatial light modulator, akin to the digital laser, to dynamically select a desired profile (for example, a higher-radial-order Laguerre–Gaussian mode).

We also point out that this concept extends to arbitrary mode selection beyond OAM and HOP sphere beams. As the concept outlined here does not require the mode to be the same everywhere in the cavity, in principle any mode can be selected at the two ends. For example, the left and right circularly polarized states can be mapped...
to pattern A and pattern B (which at the moment are two OAM modes of opposite handedness). By judicious choice of these spatial modes, together with the vector nature of the resulting beams, virtually any polarization pattern can be created directly at the source. This allows complete control over the vector nature of the modes at the output of the laser. These ideas can be extended to cascaded geometric phase elements, which can be switched on and off to create superpositions of these structured light fields. The switching and high mode purity may offer advantages in applications such as STED microscopy, laser materials processing and optical metrology with vector beams.

**Methods**

Methods and any associated references are available in the online version of the paper.

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Methods
Laser design. The V-shaped cavity was realized in a diode-pumped solid-state laser where a 0.5 at% Nd-doped YAG rod (4 × 50 mm rod) was side-pumped with a total input average pump power of ∼600 W operating at 805 nm. The end mirrors were both concave high reflectors with curvatures $R_1 = 400\text{mm}$ and $R_2 = 500\text{mm}$, respectively, with a planar mirror of 90% reflectivity positioned at the apex of the V. The separation distance between the two concave mirrors was 900 mm and the angle at which the two arms were separated by the plane mirror was on the order of 5°. The $q$-plate ($q = 1/2$) was designed to operate most efficiently when positioned on-axis and it was thus positioned sufficiently adjacent to the plane mirror. The QWP (multiorder operating at 1,064 nm) was required to transmit both arms and was thus positioned to incorporate its clear aperture of ∼12 mm. A lens of focal length $f = 400\text{mm}$ was inserted into the cavity to aid stability and to facilitate the clear aperture restriction imposed by the QWP. Finally a PBS was preferred for the selection of linear polarization in the horizontal. A further practical consideration was required to be met: with the present pump arrangement, multimode operation was favoured, which allows the existence of higher-order azimuthal and radial modes; we thus inserted a circular aperture with variable diameter such that the field incident on the $q$-plate was $LG_{00}$ in shape, that is, only radial indices of $p = 0$ were allowed. The forward-propagating wave in our configuration was considered as the propagation from $R_1$ to $R_2$ with the back propagating wave acting in reverse. These two waves impinged on the planar mirror thus presenting two outputs; we measured the output of the forward wave, as described by equation (1).