Efficient generation of NOON states on two microwave-photon resonators

Ming Hua, Ming-Jie Tao, and Fu-Guo Deng†

Department of Physics, Applied Optics Beijing Area Major Laboratory, Beijing Normal University, Beijing 100875, China

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We present an efficient scheme for the generation of NOON states of photons in circuit QED assisted by a superconducting charge qutrit. It is completed with two kinds of manipulations, that is, the resonant operation on the qutrit and the resonator, and the single-qubit operation on the qutrit, and they both are high-fidelity operations. Compared with the one by a superconducting transmon qutrit proposed by Su et al. (Sci. Rep. 4, 3898 (2014)), our scheme does not require to maintain the qutrit in the third excited state with a long time, which relaxes the difficulty of its implementation in experiment. Moreover, the level anharmonicity of a charge qutrit is larger and it is better for us to tune the different transitions of the charge qutrit resonant to the resonator, which makes our scheme faster than others.

Key words: entanglement production, NOON states, microwave-photon resonators, superconducting charge qutrit, circuit QED

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I. INTRODUCTION

Quantum information is an important branch of quantum physics. It includes mainly quantum communication and quantum computation [1,2]. By far, many interesting quantum systems have been presented for quantum information processing, such as nuclear magnetic resonance [3,4], quantum dots [8,12], diamond nitrogen vacancy (NV) centers [14,19], photonic systems [20,21], circuit quantum electrodynamics (QED) [25,33], and so on. Due to the good scalability [36] and convenient operation on superconducting qubits, circuit QED has attracted much attention in recent years.

Composed of the superconducting circuit and the superconducting 1D resonator, circuit QED [37] has some good characters for completing quantum information processing. The superconducting circuit can act as a qubit perfectly. The energy-level structure of the qubit can be divided into Ξ, Λ, V, and Δ types [38] which can not be found in atom systems. A relative long life time of a superconducting qubit has been realized to reach 0.1 ms [39]. The strong coupling strength between a superconducting qubit and a superconducting resonator [25] has been demonstrated in the experiment. All these characters make circuit QED as a good platform for the quantum computation based on superconducting qubits. In 2009, DiCarlo et al. demonstrated a two-qubit algorithms with a superconducting quantum processor [40]. In 2012, Reed et al. realized a three-qubit quantum error correction with superconducting circuits [41], and in the same year, Lucero et al. computed the prime factors with a Josephson phase qubit quantum processor [36] in which they integrated five superconducting resonators and four superconducting qubits in a quantum processor.

A superconducting resonator can act as a cavity and a quantum bus, which can be coupled to the distant qubits. The quality of the resonator can be reached to $10^6$ and even $10^{12}$ [42], that is, the superconducting resonators can also afford a powerful platform for quantum information processing. In 2007, Schuster et al. resolved the photon number states in a superconducting circuit [43]. In 2010, Johnson et al. realized a quantum nondemolition detection of single microwave photons in a circuit [44], and in the same year, Strauch et al. presented a method to synthesize an arbitrary quantum state of two superconducting resonators [45]. In 2012, Strauch proposed an all-resonant control of superconducting resonators with a drive field [46]. In 2013, they proposed a selective-resonance scheme to perform a fast quantum entangling operation for quantum logic gates on superconducting qubits [47], assisted by one or two superconducting resonators. By combination of the selective resonance and the tunable period relation between a wanted quantum Rabi oscillation and an unwanted one besides the positive influence from the non-computational third levels of the superconducting qubits, these universal quantum gates are significantly faster than previous proposals and do not require any kind of drive fields.

Recently, the generation of the NOON state [48] on two resonators attracted much more attention. In 2010, Strauch, Jacobs, and Simmonds [45] proposed a scheme for completing the generation of the NOON state on two resonators without using the third non-computational excited energy level. The superconducting qubit was operated with a selective rotation by using a drive field whose amplitude should much smaller than the photon-number-dependent Stark shifts on the qubit. That is, the operation time of the qubit should be extended a little longer. In 2010, Merkel and Wilhelm [49] proposed a
theoretic scheme for generating NOON states on two resonators by using two superconducting qubits and three superconducting resonators. In 2011, Wang et al. demonstrated Merkel-Wilhelm scheme in experiment. In Ref. [46], a novel method was proposed to generate the NOON state on two resonators by using a complicated classical microwave pulse and an all-resonant manipulation. It can get a very high-fidelity NOON state within a much shorter time without using the non-computational excited energy level. In 2013, Su et al. [51] proposed an interesting scheme for the generation of the NOON state on two resonators with the resonant operation between the transmon qubit and the superconducting resonator, assisted by the single-qubit rotation. The scheme can be completed with $2N$ steps, and in the first $N$ steps, the qubit should be maintained in the third-excited state corresponding to the case that the photon number in each resonator is zero.

In this paper, we proposed a scheme to produce the NOON state on two resonators in a quantum processor composed of two tunable superconducting resonators coupled to a tunable Ξ-type three-energy-level superconducting qutrit. Our scheme requires two kinds of quantum operations. One is the resonant operation on the superconducting qutrit and the resonators. The other is the single-qubit manipulation which can be completed by applying a drive field on the qutrit. Our scheme can be used to produce the NOON state on two resonators effectively, in a simple and fast way, compared with Merkel-Wilhelm scheme. Moreover, it does not require us to remain the qutrit in the third-excited state all the time, which relaxes largely the requirements of its implementation in experiment, compared with the previous work in Ref. [51].

II. GENERATION OF THE NOON STATE ON TWO MICROWAVE-PHOTON SUPERCONDUCTING RESONATORS

Let us consider a quantum system composed of two superconducting resonators coupled to a superconducting qutrit, shown in Fig. 1(a). The energy-level structure of the qutrit is the Ξ type, which can be found in a superconducting charge qubit, shown in Fig. 1(b). In order to construct the NOON state on two resonators $r_1$ and $r_2$, we exploit the lowest three energy levels of the qutrit, denoted by $|g\rangle_q$, $|e\rangle_q$, and $|a\rangle_q$ with the energy $E_g < E_e < E_a$. The Hamiltonian of the system composed of the two resonators and the qutrit is (under the rotating-wave approximation, and we choose $\hbar = 1$ below)

$$H = \sum_{l=g,e,a} \sum_{i=1,2} E_l |l\rangle_q \langle l| + \sum_{i=1,2} \left[ \omega^{r_i} a_i^+ a_i + g^{g,e}_i (a_i^+ \sigma_{g,e}^+) + a_i^+ \sigma_{g,e} (a_i^+ \sigma_{g,e}^+ + a_i^+ \sigma_{g,e}^+) \right].$$

Here, $\omega^{r_i}$ and $a_i^+$ are the transition frequency and the creation operator of the resonator $r_i$, respectively. $\sigma_{g,e}^+$ are the creation operators of the two transitions $|g\rangle_q \rightarrow |e\rangle_q$ and $|e\rangle_q \rightarrow |a\rangle_q$ of the qutrit, respectively. $g^{g,e}_i$ is the coupling strength between the resonator $r_i$ and the qutrit in the two transitions $|g\rangle_q$ and $|e\rangle_q$, and $g^{e,a}_i$ is the coupling strengths between the resonator $r_i$ and the qutrit in the two transitions $|e\rangle_q$ and $|a\rangle_q$.

In order to turn on or off the interaction between the resonators and the qutrit, on one hand, one can tune the transition frequency of the qutrit by using the external magnetic flux, or turn the transition frequency of the resonator to make them resonate or largely detune with each other. On the other hand, one can tune the coupling strength between the qutrit and the resonator. It worth noticing that a tunable resonator [52] and a tunable coupling qubit [53, 54] have been demonstrated in experiment.

The principle of our scheme for generating the NOON state on two microwave-photon resonator qudits. There are two resonators coupled to a superconducting qutrit. The transition frequencies of the qutrit and the resonators are tunable. (b) The structure for the energy levels of a charge qutrit, $r_1$ and $r_2$ are the two microwave-photon resonators. $g^{g,e}_i$ is the coupling strength between the resonator $r_i$ and the superconducting qutrit in the transition between the states $|g\rangle_q$ and $|e\rangle_q$ ($|e\rangle_q$ and $|a\rangle_q$).

Figure 1: (a) Schematic diagram for generating the NOON state on two microwave-photon-resonator qudits. There are two resonators coupled to a superconducting qutrit, and the transition frequencies of the qutrit and the resonators are tunable. (b) The structure for the energy levels of a charge qutrit, $r_1$ and $r_2$ are the two microwave-photon resonators. $g^{g,e}_i$ is the coupling strength between the resonator $r_i$ and the superconducting qutrit in the transition between the states $|g\rangle_q$ and $|e\rangle_q$ ($|e\rangle_q$ and $|a\rangle_q$).

Here the subscripts 1 and 2 represent the two resonators $r_1$ and $r_2$, respectively. That is, the qutrit is in the state $|a\rangle_q$ ($|e\rangle_q$), and the resonators are in the state $|0\rangle_{1,2}$. Here and below, $|n\rangle_i$ is the Fock state of the resonator $r_i$, which means there are $n$ microwave photons in the resonator $r_i$ ($i = 1, 2$). To generate the NOON state
The state of the system becomes
\[ |\phi\rangle_{\text{NOON}} = \frac{1}{\sqrt{2}}(|N\rangle_1 |0\rangle_2 + |0\rangle_1 |M\rangle_2) \] (3)
on \(r_1\) and \(r_2\) \((N = M\) is a special situation of the NOON state), our scheme needs \(N + M\) steps. The first \(N\) steps are described as follows.

Step 1: By making both \(r_1\) and \(r_2\) detune largely with the qutrit, one can use a drive field with the frequency equivalent to the transition frequency \(\omega_{e,a}\) of the qutrit to pump the state of the qutrit from \(|e\rangle_q\) to \(|a\rangle_q\). The amplitude of the drive field is chosen with a proper value for avoiding to pump the state from \(|g\rangle_q\) to \(|e\rangle_q\). Here \(\omega_{e,a} \equiv E_a - E_e\). After the operation time \(\Omega_{e,a} t = \pi\) \((\Omega_{e,a}\) is the proper amplitude of the drive field for pumping the qutrit from \(|e\rangle_q\) to \(|a\rangle_q\)), the state of the system evolves into
\[ \frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 - i |a\rangle_q |0\rangle_1 |0\rangle_2) \] (4)

Subsequently, one can tune the transition frequencies of the qutrit and the two resonators to make \(r_1\) resonate with the qutrit in the transition \(|e\rangle_q \leftrightarrow |a\rangle_q\). If the coupling strength between \(r_1\) and the qutrit is tuned with a proper value before the resonance, one can neglect the interaction between \(r_1\) and the qutrit in the transition \(|g\rangle_q \leftrightarrow |e\rangle_q\). Meanwhile, \(r_2\) and the qutrit detune largely with each other. After the interaction time \(g_{t,a}^2 t = \pi\), the state of the system becomes
\[ \frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 - |e\rangle_q |1\rangle_1 |0\rangle_2) \] (5)

Step \(j\) \((j = 2, 3, ..., N)\): By repeating the operation of the step 1 for \(N - 1\) times and maintaining \(r_2\) detuning largely with \(r_1\) and the qutrit all the time, the state of the system is changed to be
\[ \frac{1}{\sqrt{2}}(|g\rangle_q |0\rangle_1 |0\rangle_2 + (-1)^N |e\rangle_q |N\rangle_1 |0\rangle_2) \] (6)
The whole operation time is
\[ t = t_d + t_r \] (7)
Here, \(t_d = \sum_N \frac{N\pi}{\omega_{g,c}}\) is the rotated-operation time of the qutrit and \(t_r = \sum_N \frac{\pi}{2g_d \sqrt{N}}\) is the resonated-operation time between the qutrit and the \(r_1\).

The details of the first \(N\) steps have been described above. The next \(M\) steps are described as follows.

Step 1’: By making both \(r_1\) and \(r_2\) detune largely with the qutrit, one can apply a drive field with the frequency equivalent to the transition frequency \(\omega_{g,c}\) of the qutrit to rotate the states of the qutrit with \(|g\rangle_q \leftrightarrow |e\rangle_q\). By choosing the proper amplitude of the drive field, one can avoid to flip the qutrit with \(|e\rangle_q \leftrightarrow |a\rangle_q\). After the operation time \(\Omega_{g,c} t = \pi\), the state of the system evolves from Eq.(4) to
\[ \frac{1}{\sqrt{2}}(-i |e\rangle_q |0\rangle_1 |0\rangle_2 - (-1)^N i |g\rangle_q |N\rangle_1 |0\rangle_2) \] (8)

Applying a drive field with the frequency equivalent to the transition frequency \(\omega_{e,a}\) of the qutrit, one can pump the state of the qutrit from \(|e\rangle_q\) to \(|a\rangle_q\). The amplitude of the drive field is chosen with a proper value for avoiding to pump the state from \(|g\rangle_q\) to \(|e\rangle_q\). After the operation time \(\Omega_{e,a} t = \pi\), the state of the system evolves into
\[ \frac{1}{\sqrt{2}}(-|a\rangle_q |0\rangle_1 |0\rangle_2 - (-1)^N i |g\rangle_q |N\rangle_1 |0\rangle_2) \] (9)

Subsequently, one can tune the transition frequencies of the qutrit and the two resonators to make \(r_2\) resonate with the qutrit in the transition \(|e\rangle_q \leftrightarrow |a\rangle_q\). If the coupling strength between \(r_1\) and the qutrit is tuned with a proper value before the resonance, one can neglect the interaction between \(r_2\) and the qutrit in the transition \(|g\rangle_q \leftrightarrow |e\rangle_q\). Meanwhile, \(r_1\) and the qutrit detune largely with each other. After the interaction time \(g_{t,a}^2 t = \pi\), the state of the system becomes
\[ \frac{1}{\sqrt{2}}(|e\rangle_q |0\rangle_1 |1\rangle_2 - (-1)^N i |g\rangle_q |N\rangle_1 |0\rangle_2) \] (10)

Step \(j’\) \((j’ = 2, 3, ..., M - 1)\): By repeating the operation of the step 1’ for \(M - 2\) times, and maintaining \(r_1\) detuning largely with the qutrit all the time, the state of the system is changed to be
\[ \frac{1}{\sqrt{2}}((-1)^{M-1} i |e\rangle_q |0\rangle_1 |M-1\rangle_2 - (-1)^N i |g\rangle_q |N\rangle_1 |0\rangle_2) \] (11)

The final step: Applying a single-qubit operation to complete the rotations of the states \((-1)^{M-1} i |e\rangle_q \rightarrow i |e\rangle_q\) and \((-1)^N i |g\rangle_q \rightarrow |g\rangle_q\), the state shown in Eq.(11) is changed to be
\[ \frac{1}{\sqrt{2}}(|N\rangle_1 |0\rangle_2 + |0\rangle_1 |M\rangle_2) \otimes |g\rangle_q \] (12)

By resonating \(r_2\) and the qutrit in the transition \(|g\rangle_q \leftrightarrow |e\rangle_q\) and making \(r_1\) detune largely with the qutrit, the state shown in Eq.(12) is changed to be
\[ t’ = t_d’ + t_r’ \] (14)
Here, we have generated the NOON state on two microwave-photon resonators efficiently. The operation time of the second \(M\) steps is
\[ t_d’ = \sum_M \frac{M\pi}{\omega_{g,c}} \] is the rotation-operation time of the qutrit and \(t_r’ = \sum_M \frac{\pi}{2g_d’ \sqrt{M}}\) is the resonance-operation time between the qutrit and \(r_2\). In which, we neglect the operation time of the single-qubit operation in the final step for generating the NOON state with large number of the \(N\) and \(M\).
III. DISCUSSION AND SUMMARY

We have described the process of our scheme for generating the NOON state on two superconducting resonators which are coupled to a Ξ-type-energy-level structure superconducting charge qutrit. It includes two kinds of quantum operations. The first one is the resonant operation on the qutrit and the resonators. The second one is the single-qubit operation on the qutrit. They are the high-fidelity, high-efficiency, and simple quantum operations in experiment in circuit QED systems. The whole operation time of our scheme for generating the NOON state $|\phi\rangle_{\text{noon}}$ is

$$T = \sum_{j=1}^{N} \left( \frac{j\pi}{2g_{i}^{e,a}\sqrt{j}} \right) + \sum_{j=1}^{M} \left( \frac{j'\pi}{\Omega_{e,a}^{j'}} \right).$$

In the calculation for the operation time in our scheme, we neglect the time for changing the transition frequencies of the superconducting qutrit and the superconducting resonator, and the operation time of the single-qubit operation in the final step. Compared with the one in Ref. [47], our scheme for generating the NOON state on superconducting resonators is much faster as it is composed of the resonant controls. Compared with the one in Refs. [49, 50], both the number of the resonators and that of the qutrits required in our scheme are much smaller as there are three superconducting resonators and two superconducting qutrits in the scheme in Refs. [49, 50], but only two superconducting resonators and a superconducting qutrit used in our scheme. Moreover, the single-qubit operation required in our scheme can be achieved with the simple classical drive field, and it is simpler than the one used in Ref. [46] as the amplitude of the drive field should be designed with a complex type and it is difficult to be realized in experiment in the latter. In Ref. [51], a similar method is used to generate the NOON state on two resonators. In their work, the transmon qutrit should be maintained in the first $N$ steps in the third excited state when there is no microwave photons in each resonators. It worth noticing that the higher excited states lead to a lower fidelity operation [46]. Luckily, our scheme does not require us to maintain the qutrit in its third excited state all the time, which relaxes the requirements of its implementation in experiment, compared with the one in Ref. [51]. Compared with a transmon qutrit, the level anharmonicity of a charge qutrit is larger and it is better for us to tune the different transitions of the charge qutrit resonant to the resonator.

In summary, we have proposed an efficient scheme to generate the NOON states on two superconducting resonators, assisted by a superconducting qutrit. It requires some high-fidelity quantum operations, that is, the resonant operation on the qutrit and the resonator and the single-qubit operation on the qutrit. Our scheme is a fast and simple one. Moreover, it does not require to maintain the qutrit in the third excited state with a long time, which relaxes the requirements of its implementation in experiment.

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[1] Nielsen M A, Chuang I L. Quantum Computation and Quantum Information (Cambridge University, Cambridge, 2000)
[2] Ren B C, Deng F G. Hyperentanglement purification and concentration assisted by diamond NV centers inside photonic crystal cavities. Laser Phys. Lett, 2013, 10: 115201
[3] Ren B C, Deng F G. Hyper-parallel photonic quantum computation with coupled quantum dots. Sci Rep, 2014, 4: 4623
[4] Wang Y H, Song H S. Preparation of multi-atom specially entangled W-class state and splitting quantum information. Chin Sci Bull, 2009, 54: 2599-2605
[5] Long G L, Xiao L. Parallel quantum computing in a single ensemble quantum computer. Phys Rev A, 2004, 69: 052303
[6] Long G L, Xiao L. Experimental realization of a fetching algorithm in a 7-qubit NMR spin Liouville space computer. J Chem Phys 2003, 119: 8473-8481
[7] Peng G R, Xu G F, Long G L. Experimental realization of nonadiabatic holonomic quantum computation. Phys Rev Lett, 2013, 110: 190501
[8] Hu C Y, Young A, O'Brien J L, et al. Giant optical Faraday rotation induced by a single-electron spin in a quantum dot: Applications to entangling remote spins via a single photon. Phys Rev B, 2008, 78: 085307
[9] Bonato C, Haupt F, Omrarsingh S S R, et al. CNOT and Bell-state analysis in the weak-coupling cavity QED regime. Phys Rev Lett, 2010, 104: 160503
[10] Wei H R, Deng F G. Universal quantum gates for hybrid systems assisted by quantum dots inside doublesided optical microcavities. Phys Rev A, 2013, 87: 022305
[11] Wang T J, Song S Y, Long G L. Quantum repeater based on spatial entanglement of photons and quantum-dot spins in optical microcavities. Phys Rev A, 2012, 85: 062311
[12] Wang C. Efficient entanglement concentration for partially entangled electrons using a quantum-dot and microcavity coupled system. Phys Rev A, 2012, 86: 012323
[13] Sheng Y B, Zhou L, Wang L, et al. Efficient entanglement concentration for quantum dot and optical microcavities systems. Quant Inform Proc, 2013, 12: 1885-1895
[14] Togan E, Chu Y, Trifonov A S, et al. Quantum entanglement between an optical photon and a solid-state spin qubit. Nature (London), 2010, 466: 730-734

[15] Neumann P, Kolesov R, Naydenov B, et al. Quantum register based on coupled electron spins in a room-temperature solid. Nature Phys, 2010, 6: 249-253

[16] Wei H R, Deng F G. Compact quantum gates on electron-spin qubits assisted by diamond nitrogen-vacancy centers inside cavities. Phys Rev A, 2013, 88: 042323

[17] Chen Q, Yang W L, Feng M, et al. Entangling separate nitrogen-vacancy centers in a scalable fashion via coupling to microtoroidal resonators. Phys Rev A, 2011, 83: 054305

[18] Zheng A S, Li J H, Yu R, et al. Generation of Greenberger-Horne-Zeilinger state of distant diamond nitrogen-vacancy centers via nanocavity input-output process. Opt Express, 2012, 20: 16902

[19] Yang W L, Yin Z Q, Xu Z Y, et al. One-step implementation of multiqubit conditional phase gate with nitrogen-vacancy centers coupled to a high-Q silica microsphere cavity. Appl Phys Lett, 2010, 96: 241113

[20] Knill E, Laflamme R, Milburn G J. A scheme for efficient quantum computation with linear optics. Nature (London), 2001, 409: 46-52

[21] Nemoto K, Munro W J. Nearly deterministic linear optical controlled-not gate. Phys Rev Lett., 2004, 93: 250502

[22] Lin Q, Li J. Quantum control gates with weak cross-Kerr nonlinearity. Phys Rev A, 2009, 79: 022301

[23] Beenakker C W J, DiVincenzo D P, Emary C, et al. Charge detection enables free-electron quantum computation. Phys Rev Lett, 2004, 93: 092501

[24] Ren B C, Wei H R, Deng F G. Deterministic photonic spatial-polarization hyper-controlled-not gate assisted by quantum dot inside one-side optical microcavity. Laser Phys Lett, 2013, 10: 095202

[25] Wallraff A, Schuster D I, Blais A, et al. Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics. Nature (London), 2004, 431: 162-67

[26] Liu Y X, You J Q, Wei L F, et al. Optical selection rules and phase-dependent adiabatic state control in a superconducting quantum circuit. Phys Rev Lett, 2005, 95: 087001

[27] Wang Y D, Xue F, Song Z, et al. Detection mechanism for quantum phase transition in superconducting qubit array. Phys Rev B, 2007, 76: 174519

[28] Cao Y, Huo W Y, Ai Q, et al. Theory of degenerate three-wave mixing using circuit QED in solid-state circuits. Phys Rev A, 2011, 84: 053846

[29] Huo W Y, Long G L. Entanglement and squeezing in solid-state circuits. New J Phys, 2008, 10: 013026

[30] Gao G L, Cai G C, Huang S S, et al. 1→N quantum controlled phase gate realized in a circuit QED system. Sci China-Phys Mech Astron, 2012, 55: 1422-1426

[31] Wang H, Sun H C, Zhang J, et al. Transparency and amplification in a hybrid system of the mechanical resonator and circuit QED. Sci China-Phys Mech Astron, 2012, 55: 2264-2272

[32] Qian Y, Zhang Y Q, Xu J B. Amplifying stationary quantum discord and entanglement between a superconducting qubit and a data bus by time-dependent electromagnetic field. Chin Sci Bulletin, 2012, 57: 1637-1642

[33] Zhang S L, Zhang G F, Wang Y L, et al. A novel superconducting quantum interference device for biomagnetic measurements. Chin Sci Bulletin, 2013, 58: 2917-2919

[34] Li H J, Wang Y W, Wei L F, et al. Experimental demonstrations of high-Q superconducting coplanar waveguide resonators. Chin Sci Bulletin, 2013, 58: 2413-2417

[35] Yu L B, Tong N H, Xue Z Y, et al. Simulation of the spin-boson model with superconducting phase qubit coupled to a transmission line. Sci China-Phys Mech Astron, 2012, 55: 1557-1561

[36] Lucero E, Barends R, Chen Y, et al. Computing prime factors with a Josephson phase qubit quantum processor. Nature Phys, 2012, 8: 719-723

[37] Blais A, Huang R S, Wallraff A, et al. Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation. Phys Rev A, 2004, 69: 062320

[38] You J Q, Nori F. Atomic physics and quantum optics using superconducting circuits. Nature (London), 2011, 474: 589-597

[39] Rigetti C, Gambetta J M, Poletto S, et al. Superconducting qubit in a waveguide cavity with a coherence time approaching 0.1 ms. Phys. Rev. B, 2012, 86: 100506(R)

[40] DiCarlo L, Chow J M, Gambetta J M, et al. Demonstration of two-qubit algorithms with a superconducting quantum processor. Nature (London), 2009, 460: 240-244

[41] Reed M D, DiCarlo L, Nigg S E, et al. Realization of three-qubit quantum error correction with superconducting circuits. Nature (London), 2013, 497: 455-459

[42] Reagor M, Paik H, Catelani G, et al. Reaching 10 ms single photon lifetimes for superconducting aluminum cavities. Appl Phys Lett, 2013, 102: 192604

[43] Schuster D I, Houck A A, Schreier J A, et al. Resolving photon number states in a superconducting circuit. Nature (London), 2007, 445: 515-518

[44] Johnson B R, Reed M D, Houck A A, et al. Quantum non-demolition detection of single microwave photons in a circuit. Nature Phys, 2010, 6: 663-667

[45] Strauch F W, Jacobs K, Simmonds R W, et al. Arbitrary control of entanglement between two superconducting resonators. Phys Rev Lett, 2010, 105: 050501

[46] Strauch F W. All-resonant control of superconducting resonators. Phys Rev Lett, 2012, 109: 210501

[47] Hu M, Deng F G. Selective-resonance-based quantum entangling operation on qubits in circuit QED. arXiv:1310.0102

[48] Zhou L, Sheng Y B, Cheng W W, Gong I Y, Zhao S M. Efficient entanglement concentration for arbitrary less-entangled NOON states. Quantum Inf Process, 2013, 12: 1307-1320

[49] Merkel S T, Wilhelm F K. Generation and detection of NOON states in superconducting circuits. New J Phys, 2010, 12: 093036

[50] Wang H, Mariantoni M, Bialczak R C, et al. Deterministic entanglement of photons in two superconducting microwave resonators. Phys Rev Lett, 2011, 106: 060401

[51] Su Q P, Yang C P, Zheng S B. Fast and simple scheme for generating NOON states of photons in circuit QED. Sci Rep, 2014, 4: 3898

[52] Sandberg M, Wilson C M, Persson F, et al. Tuning the field in a microwave resonator faster than the photon lifetime. Appl Phys Lett, 2008, 92: 203501

[53] Allman M S, Altmare F, Whittaker J D, et al. rf-SQUID-mediated coherent tunable coupling between a superconducting phase qubit and a lumped-element resonator. Phys Rev Lett, 2010, 104: 177004
[54] Bialczak R C, Ansmann M, Hofheinz M, et al. ‘Fast tunable coupler for superconducting qubits. Phys Rev Lett, 2011, 106: 060501
[55] Haack G, Helmer F, Mariantoni M, et al. Resonant quantum gates in circuit quantum electrodynamics. Phys. Rev. B, 2010, 82: 024514.