In the baryogenesis via leptogenesis framework the baryonic asymmetry depends on lepton mass matrices. In a previous paper we used SO(10)-inspired mass matrices and we found few possibilities to obtain a sufficient level of asymmetry. In the present paper we use SU(5)-inspired mass matrices, which also allow to check the dependence of the baryonic asymmetry on Dirac neutrino masses. In particular, we find that the large mixing matter solution to the solar neutrino problem, which within SO(10) gives too small asymmetry, can now be favoured.
I. INTRODUCTION

The origin of the baryonic asymmetry of the universe is one of the fundamental questions in theoretical physics [1]. In this paper we consider the baryogenesis-via-leptogenesis mechanism [2,3], where the out-of-equilibrium decays of heavy Majorana neutrinos produce a leptonic asymmetry which is partially converted into a baryonic asymmetry by electroweak sphaleron processes [4]. A minimal framework to realize this mechanism is the standard model with heavy right-handed neutrinos, but the same mechanism works also within unified theories, such as the SU(5) model with heavy right-handed neutrinos, and the SO(10) model. In any case the light left-handed Majorana neutrinos are obtained through the seesaw mechanism [5].

The baryogenesis via leptogenesis has been discussed by several authors [6–10]. In a previous paper [11] we have calculated the baryonic asymmetry assuming SO(10)-inspired quark-lepton symmetry. In fact, we have used both the relations $M_e \sim M_d$ and $M_\nu \sim M_u$ for the Dirac mass matrices. A natural motivation to consider different lepton mass matrices, in the context of leptogenesis, is that the slightly favoured solution to the solar neutrino problem, namely the large mixing MSW, was not able to produce enough baryonic asymmetry. We notice here that the SU(5) model allows more freedom for mass matrices in that again $M_e$ is related to $M_d$, but $M_\nu$ is independent from $M_u$. Moreover, the right-handed neutrino mass matrix $M_R$ is generated as bare mass term, and not from a Yukawa coupling term with the Higgs field that breaks the unified or intermediate gauge symmetry as happens in SO(10). In particular, we will check the effect of several mass hierarchies for Dirac neutrino masses.

In the next section we collect the relevant formulas of the baryogenesis-via-leptogenesis mechanism. These depend on both the Dirac and the heavy Majorana neutrino mass matrices. In the seesaw mechanism such two matrices are related through the effective (light) Majorana neutrino mass matrix $M_L$. Therefore, in section III we give an outline on how to determine the effective neutrino mass matrix from neutrino oscillation data, and in section IV we comment about Dirac mass matrices in the SU(5) model. In section V the calculation of the baryonic asymmetry is carried out and in section VI we give our conclusions.
II. BARYOGENESIS VIA LEPTOGENESIS

A baryonic asymmetry can be generated from a leptonic asymmetry \[2\]. The baryonic asymmetry is defined as \[12\]

\[
Y_B = \frac{n_B - n_{\bar{B}}}{7.04n_{\gamma}},
\]

(1)

where \(n_B, n_{\bar{B}}, n_{\gamma}\) are number densities. Due to electroweak sphaleron effect, the baryonic asymmetry \(Y_B\) is related to the leptonic asymmetry \(Y_L\) by \[13\]

\[
Y_B = \frac{a}{a-1} Y_L
\]

(2)

where

\[
a = \frac{8N_f + 4N_H}{22N_f + 13N_H}.
\]

\(N_f\) is the number of families and \(N_H\) the number of Higgs doublets. For \(N_f = 3\) and \(N_H = 1\) or \(N_H = 2\), it is \(a \simeq 1/3\), so that \(Y_B \simeq -Y_L/2\).

The leptonic asymmetry can be written as \[3\]

\[
Y_L = d \frac{\epsilon_1}{g^*}
\]

(3)

where \(\epsilon_1\) is a CP-violating asymmetry associated with the decay of the lightest heavy neutrino, \(d\) is a dilution factor, and \(g^* = 106.75\) in the standard case or 228.75 in the supersymmetric case. For the standard case the asymmetry \(\epsilon_1\) is given by \[14,15\]

\[
\epsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}^2} \sum_{j=2,3} \text{Im}[(M_D^\dagger M_D)_{j1}]^2 f\left(\frac{M_j^2}{M_1^2}\right),
\]

(4)

where \(M_D\) is the Dirac neutrino mass matrix when \(M_\epsilon\) and \(M_R\) are diagonalized, \(M_i\) \((i = 1, 2, 3)\) are the three eigenvalues of \(M_R\), \(v = 175\) GeV is the VEV of the Higgs doublet, and

\[
f(x) = \sqrt{x}\left[1 - (1 + x)\ln\frac{1+x}{x} - \frac{1}{x-1}\right].
\]

In the supersymmetric version \(v \rightarrow v \sin \beta\),

\[
f(x) = -\sqrt{x}\left[\ln\frac{1+x}{x} + \frac{2}{x-1}\right],
\]

and a factor 4 is included in \(\epsilon_1\), due to more decay channels. The formula (4) arises from the interference between the tree level and one loop decay amplitudes of the lightest heavy neutrino, and includes vertex \[2\] and self-energy \[16\] corrections.
The dilution factor takes into account the washout effect produced by inverse
decay and lepton number violating scattering. A good approximation for $d$ can be
inferred from refs. [17–19]:

$$d = (0.1 \ k)^{1/2} \exp[-(4/3)(0.1 \ k)^{1/4}] \quad (5)$$

for $k \gtrsim 10^6$,

$$d = 0.24/k(\ln k)^{3/5} \quad (6)$$

for $10 \lesssim k \lesssim 10^6$, and

$$d = 1/2k, \quad d = 1 \quad (7)$$

for $1 \lesssim k \lesssim 10$, $0 \lesssim k \lesssim 1$, respectively, where the parameter $k$ is

$$k = \frac{M_P}{1.7v^232\pi^3\sqrt{g^*}} \frac{(M_D^T M_D)_{11}}{M_1}, \quad (8)$$

and $M_P$ is the Planck mass. In the supersymmetric case the critical value $10^6$ for $k$
is lowered, but in our calculation $k$ remains always much smaller, so that only (6)
and (7) are really used.

In order to calculate the baryonic asymmetry by means of the foregoing formulas,
we have to determine $M_D$ and the heavy neutrino masses $M_i$ (and hence both
$M_\nu$ and $M_R$). This is the aim of the next two sections.

**III. NEUTRINO MASSES AND MIXINGS**

According to the seesaw mechanism, the light (left-handed) neutrino mass matrix
is given by the formula

$$M_L = -M_\nu M^{-1}_R M^T_\nu. \quad (9)$$

Of course, this formula can be inverted to give the matrix $M_R$ in terms of a theoretical $M_\nu$ and a phenomenological $M_L$,

$$M_R = -M^T_\nu M^{-1}_L M_\nu. \quad (10)$$
In this section we discuss about $M_L$ and in the next section about $M_\nu$. The effective matrix $M_L$ can be written as

$$M_L = U^\dagger D_L U^*, \quad (11)$$

where $D_L = \text{diag}(m_1, m_2, m_3)$ contains the light neutrino masses and $U$ is the lepton mixing matrix \[20\] in the basis with $M_e$ diagonal; $\nu_{\alpha L} = U_{\alpha i} \nu_{i L} \ (\alpha = e, \mu, \tau; i = 1, 2, 3)$. The mixing matrix $U$ can be parametrized as the CKM matrix times a diagonal phase matrix $D = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$ \[21\], so that it depends on three angles and three phases. From neutrino oscillation data we can infer the three angles \[22\] and, assuming hierarchy $m_1 \ll m_2 \ll m_3$, also $m_2$ and $m_3$. In particular, for atmospheric neutrinos we use the SuperKamiokande best fit \[23\]

$$\Delta m_a^2 = 3.5 \times 10^{-3}\text{eV}^2$$

$$\sin^2 2\theta_a = 1.0,$$

that is maximal mixing. For solar neutrinos we have three matter (MSW) solutions \[24\], namely the small mixing angle (SMA)

$$\Delta m_s^2 = 5.4 \times 10^{-6}\text{eV}^2$$

$$\sin^2 2\theta_s = 0.006,$$

the large mixing angle (LMA)

$$\Delta m_s^2 = 1.8 \times 10^{-5}\text{eV}^2$$

$$\sin^2 2\theta_s = 0.76,$$

the low-$\Delta m^2$ (LOW)

$$\Delta m_s^2 = 7.9 \times 10^{-8}\text{eV}^2$$

$$\sin^2 2\theta_s = 0.96,$$

and the vacuum oscillation (VO) solution

$$\Delta m_s^2 = 8.0 \times 10^{-11}\text{eV}^2$$

$$\sin^2 2\theta_s = 0.75.$$
The latest day-night and spectral data favour the LMA solution, but do not exclude the others \cite{25,26}. A further information on neutrino oscillations comes from the CHOOZ experiment \cite{27} which gives the bound $|U_{e3}| < 0.2$, while $U_{e2}$ and $U_{\mu3}$ are related to the above best fits for atmospheric and solar neutrinos. Moreover, if light neutrino masses are hierarchical, we have $m_3^2 \simeq \Delta m^2_{a}$, $m_2^2 \simeq \Delta m^2_{s}$, $m_1 \lesssim 10^{-1} m_2$. Therefore, we have three free phases, $\delta \equiv \arg(U_{e3})$, $\varphi_1$, $\varphi_2$, and two bounded positive parameters, $|U_{e3}|$, $m_1$. Choosing values for these five parameters leads to a complete determination of $M_L$.

IV. MASS MATRICES

In the present paper we adopt SU(5)-inspired quark and lepton mass matrices. In the SU(5) model, without loss of generality, the matrix $M_u$ can be taken diagonal (see for example \cite{28}). Moreover, the charged lepton mass matrix $M_e$ is related to the down quark matrix $M_d$. In particular, in the minimal model, where only the Higgs multiplet 5 contributes to Dirac masses, we have $M_e = M_d^T$, while contributions from the Higgs multiplet 45 give $(M_e)_{ij} = -3(M_d)_{ji}$ \cite{29,30}. A general approximate form for $M_d$ can be obtained from ref. \cite{31},

$$M_d = \begin{pmatrix}
0 & \sqrt{m_d m_s} & 0 \\
\sqrt{m_d m_s} & -im_s & m_s \\
0 & m_b/\sqrt{5} & 2m_b/\sqrt{5}
\end{pmatrix}.$$  \hspace{1cm} (12)

It is symmetric in the 1-2 sector and leads to the famous relation $V_{us} \simeq \sqrt{m_d/m_s} \simeq \lambda = 0.22$. Therefore, a suitable form for $M_e$ is

$$M_e = \frac{m_\tau}{m_b} \begin{pmatrix}
0 & \sqrt{m_d m_s} & 0 \\
\sqrt{m_d m_s} & 3im_s & m_b/\sqrt{5} \\
0 & m_s & 2m_b/\sqrt{5}
\end{pmatrix}.$$  \hspace{1cm} (13)

The latter matrix has been obtained by taking the transpose of $M_d$, and including a $-3$ factor in entry 2-2 in order to get good relations between charged lepton and down quark masses \cite{29}. The factor $m_\tau/m_b$ is an approximate factor which takes in
account the dependence of quark masses from the energy scale. At the unification scale \( m_b = m_{\tau} \), so that \( M_d \sim M_e \). At a lower scale \( M_d \sim (m_b/m_\tau)M_e \), see ref. [32].

It remains to consider the Dirac neutrino mass matrix \( M_\nu \). We have calculated the baryonic asymmetry using three hierarchies for a diagonal form of \( M_\nu \),

\[
M_\nu = \frac{m_\tau}{m_b} \text{diag}(\lambda^8, \lambda^4, 1) m_t \tag{14}
\]

\[
M_\nu = \frac{m_\tau}{m_b} \text{diag}(\lambda^4, \lambda^2, 1) m_b \tag{15}
\]

\[
M_\nu = \frac{m_\tau}{m_b} \text{diag}(\lambda^2, \lambda, 1) m_b, \tag{16}
\]

to be called 84t, 42b and 21b, respectively, where the first hierarchy is similar to the up quark case and the second to the down quark (or charged lepton) case. The scale of the heavy neutrino mass matrix \( M_R \) is given by \( M_R \sim m_t^2/m_1 \) or \( m_b^2/m_2 \) for the SMA solution, and by \( M_R \sim m_t^2/m_1 \) or \( m_b^2/m_1 \) for the LMA, VO, LOW solutions, in agreement with ref. [33].

V. THE BARYONIC ASYMMETRY

As done in ref. [11], we have random extracted the free neutrino parameters in order to determine the baryonic asymmetry according to the formulas included in the previous sections (4000 points, about 2000 giving a positive asymmetry). In figs. 1-4 we plot \( Y_B \) versus \(|U_{e3}|\) for the four different solar neutrino solutions and the three hierarchies 84t, 42b, 21b. The favoured range for the baryonic asymmetry, required for a successful description of nucleosynthesis, is \( Y_B = (1.7 \div 8.9) \times 10^{-11} \) [34], so that one can look at the region of \( Y_B \) between \( 10^{-11} \) and \( 10^{-10} \). It is clear that only the LMA solution with hierarchy 21b is fully reliable for leptogenesis. The SMA and LOW solutions with hierarchy 42b and the VO solution with hierarchy 84t are acceptable for \(|U_{e3}|\) tuned around the value 0.025. The SMA solution with hierarchy 21b is acceptable for very small values of \(|U_{e3}|\). The figures refer to the nonsupersymmetric case. However, the SU(5) model is consistent with the phenomenology only in its supersymmetric version [35]. In such a case the calculated
baryonic asymmetry is increased by a factor nearly 6 [11], so that the SMA and LOW solutions with hierarchy 84t become marginally acceptable. Also the LOW solution with hierarchy 21b becomes acceptable.

VI. CONCLUSION

The baryonic asymmetry $Y_B$ has been calculated using a random extraction for neutrino parameters and assuming SU(5)-inspired mass matrices. The results depend on both the solar neutrino solution and the Dirac neutrino mass hierarchy. The hierarchy 84t is marginally reliable for leptogenesis for the SMA, VO and LOW solutions. The hierarchy 42b is marginally acceptable for SMA and LOW solutions. In these cases $|U_{e3}|$ has to be tuned around a value. The hierarchy 21b is acceptable for SMA and LOW solutions and fully reliable for the LMA solution.

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FIG. 1. The baryonic asymmetry $Y_B$ vs. $|U_{e3}|$ for SMA and hierarchies 84t, 42b, 21b
FIG. 2. The baryonic asymmetry $Y_B$ vs. $|U_{e3}|$ for LMA and hierarchies 84t, 42b, 21b
FIG. 3. The baryonic asymmetry $Y_B$ vs. $|U_{e3}|$ for VO and hierarchies 84t, 42b, 21b
FIG. 4. The baryonic asymmetry $Y_B$ vs. $|U_{e3}|$ for LOW and hierarchies 84t, 42b, 21b