The non-adiabatic pressure in general scalar field systems

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Abstract
We discuss the non-adiabatic or entropy perturbation, which controls the evolution of the curvature perturbation in the uniform density gauge, for a scalar field system minimally coupled to gravity with non-canonical action. We highlight the differences between the sound and the phase speed in these systems, and show that the non-adiabatic pressure perturbation vanishes in the single field case, resulting in the conservation of the curvature perturbation on large scales.

Key words: Scalar fields in cosmology, non-adiabatic pressure, sound speed

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1. Introduction

Providing a mechanism to generate the primordial spectrum of density fluctuations that later on in the history of the universe source the Cosmic Microwave Background anisotropies and the Large Scale Structure is one of the strongest points in favour of inflation [1, 2]. The calculation of this spectrum is a crucial tool in studying the early universe and inflation. Instead of following the evolution of the field perturbations directly it is easier to relate them to conserved quantities which are constant in time on super-horizon scales, such as the curvature perturbation on uniform density hypersurfaces, \( \zeta \). This allows us to side-step unresolved issues of inflationary cosmology such as the decay of the scalar field(s) driving inflation into the standard model particles, and directly relate the epoch when the primordial density spectrum is generated to the epoch of structure formation after the perturbations reenter the horizon.

Conserved quantities, such as \( \zeta \), will only be constant on super-horizon scales and in the very simplest cases such as slow-roll single field inflation, and will evolve in others such as multi-field or non slow-roll inflation. The evolution equation for the curvature perturbation \( \zeta \) follows directly from the divergence of the energy-momentum tensor [3], and can be written on large scales as

\[
\dot{\zeta} = -\frac{H}{(\rho_0 + P_0)} \delta P_{\text{nad}},
\]

where \( \delta P_{\text{nad}} \) is the non-adiabatic pressure perturbation which we shall study in detail below. Hence \( \zeta \) will be constant if \( \delta P_{\text{nad}} = 0 \). On the other hand, the non-adiabatic pressure can be used to generate or amplify the curvature perturbation, which is exploited in the curvaton mechanism [4].

There has been a lot of recent interest in scalar fields with non-canonical actions. Early work in the realm of the early universe included that of k-inflation [5, 6], and more current work includes that of the...
string theory motivated Dirac-Born-Infeld (DBI) inflation \[7, 8, 10, 11\]. Scalar fields with non-canonical actions have also recently been considered as dark energy candidates (see, e.g. Ref. \[12\]).

We study here if and under what conditions the curvature perturbation $\zeta$, remains constant on large, super-horizon scales, for a scalar field system derived from a general, non-canonical action. We focus on the classical aspects of the system, which allows us to treat the scalar fields as a non-standard perfect fluid. For aspects of quantisation and a more field-centred view see Ref. \[13\].

The Letter is organised as follows: in the next section we study thermodynamic relations between the pressure, energy density and entropy of the system. This is followed by a discussion regarding the adiabatic sound speed and phase speed of a system. In Section 3 we give the governing equations for the non-canonical scalar field system, both in the background and to first order in the perturbations and describe specific examples of adiabatic sound speed and phase speeds. In Section 4 we present conditions for the non-adiabatic pressure perturbation to vanish, and we conclude the Letter with a short discussion in Section 5.

Throughout this Letter we assume a spatially flat Friedmann-Robertson-Walker (FRW) background spacetime and use coordinate time, $t$. Derivatives with respect to coordinate time are denoted by a dot. Greek indices, $\mu, \nu, \lambda$, run from 0, ..., 3, while lower-case Latin indices, $i, j, k$, take the value 1, 2, or 3.

2. Thermodynamic relations

Considerable physical insight can be gained by studying the thermodynamic properties of a system. In order to keep the discussion as simple as possible, we focus here on a single fluid system. For an analysis including an arbitrary number of interacting fluids and canonical fields see e.g. Ref. \[14\].

2.1. Entropy and non-adiabatic pressure

A general thermodynamic system is fully characterised by three variables, of which only two are independent. Here we choose the energy density $\rho$, and the entropy $S$, as independent variables, with the pressure $P$ being $P \equiv P(\rho, S)$. The pressure perturbation can then be expanded into a Taylor series as

$$\delta P = \frac{\partial P}{\partial S} \delta S + \frac{\partial P}{\partial \rho} \delta \rho.$$

(2)

This can be cast in the more familiar form

$$\delta P = \delta P_{\text{nad}} + c_s^2 \delta \rho,$$

(3)

by introducing the adiabatic sound speed

$$c_s^2 \equiv \left. \frac{\partial P}{\partial \rho} \right|_S,$$

(4)

and by identifying the non-adiabatic pressure perturbation as $\delta P_{\text{nad}} \equiv \left. \frac{\partial P}{\partial S} \right|_\rho \delta S$. Equations (2) and (3) provide an intuitive and convenient definition for the adiabatic and the entropy, or non-adiabatic, perturbations in our system: the single adiabatic degree of freedom is proportional to the energy density, all other degrees of freedom will contribute to the non-adiabatic perturbation. This definition can easily be extended to more than two degrees of freedom as we shall show below, and can also be extended to higher order in the perturbations, as we shall briefly demonstrate at the end of this section.

With the above definitions, we see from Eq. (3) that the adiabatic sound speed $c_s^2$ is of zeroth order in the perturbations, and since all background quantities depend only on time we get from Eq. (4)

$$c_s^2 = \frac{\dot{P}_0}{\dot{\rho}_0},$$

(5)

where subscript “0” denotes a background quantity. Together with the behaviour of $\delta P$ and $\delta \rho$ under a gauge-transformation $\tilde{t} = t - \delta t$, namely $\delta \rho = \delta \rho + \dot{\rho}_0 \delta t$ and $\delta P = \delta P + \dot{P}_0 \delta t$, see e.g. Ref. \[15\], only the
definition for the sound speed $c_s^2$ given in Eq. renders the non-adiabatic pressure perturbation $\delta P_{nad}$ gauge-invariant.

We now focus on a system with two degrees of freedom. In order to obtain the non-adiabatic pressure perturbation in terms of different variables, we simply change from $\rho$ and $P$ to the new variables denoted here $\varphi$ and $X$, which we shall identify in the following section with the amplitude of the scalar field and its kinetic term, respectively. The energy density and the pressure are then functions of $\varphi$ and $X$, that is $P = P(\varphi, X)$ and $\rho = \rho(\varphi, X)$, and we can therefore write

$$\delta P = \frac{\partial P}{\partial \varphi} \delta \varphi + \frac{\partial P}{\partial X} \delta X,$$

and

$$\delta \rho = \frac{\partial \rho}{\partial \varphi} \delta \varphi + \frac{\partial \rho}{\partial X} \delta X.$$

Substituting Eqs. (6) and (7) into Eq. (2) we obtain

$$\delta P_{nad} = \rho_{,\varphi} \left( \frac{P_{,\varphi}}{\rho_{,\varphi}} - c_s^2 \right) \delta \varphi + \rho_{,X} \left( \frac{P_{,X}}{\rho_{,X}} - c_s^2 \right) \delta X,$$

where, for example, $P_{,\varphi} \equiv \partial P/\partial \varphi$. Note that in Eq. (8) the terms in brackets are of zeroth order, and hence $\delta P_{nad}$ can be evaluated once $P(\varphi, X)$ has been specified using background quantities only, which we shall do below in Section 4.

Equation (8) can readily be extended to more than two degrees of freedom. For example, if the energy density and the pressure are functions of $N$ fields $\varphi_I$ and $X$, then

$$\delta P_{nad} = \sum_K \rho_{,\varphi_K} \left( \frac{P_{,\varphi_K}}{\rho_{,\varphi_K}} - c_s^2 \right) \delta \varphi_K + \rho_{,X} \left( \frac{P_{,X}}{\rho_{,X}} - c_s^2 \right) \delta X.$$

To extend Eq. (2) to higher order we simply do not truncate the Taylor expansion at the linear order, that is

$$\delta P = \frac{\partial P}{\partial S} \delta S + \frac{\partial P}{\partial \rho} \delta \rho + \frac{1}{2} \left[ \frac{\partial^2 P}{\partial S^2} \delta S^2 + \frac{\partial^2 P}{\partial \rho \partial S} \delta \rho \delta S + \frac{\partial^2 P}{\partial \rho^2} \delta \rho^2 \right] + \ldots. $$

The entropy, or non-adiabatic pressure perturbation at second order, for example, is then found from Eq. (11), and expanding $\delta P = \delta P_1 + \frac{1}{2} \delta P_2$, and similarly $\delta \rho$, we get

$$\delta P_{2nad} = \delta P_2 - c_s^2 \delta \rho_1 - \frac{\partial c_s^2}{\partial \rho} \delta \rho_1^2.$$

After this higher order excursion, we return to linear theory below.

2.2. Sound speed and phase speed

Many oscillating systems can be described by a wave equation, that is a second order evolution equation of the form

$$\frac{1}{c_{ph}^2} \ddot{\phi} - \nabla^2 \phi + F(\phi, \dot{\phi}) = 0,$$

where $\phi$ is, for example, the velocity potential or the scalar field amplitude, $F(\phi, \dot{\phi})$ is the damping term, and $c_{ph}^2$ is the phase speed, i.e. the speed with which perturbations travel through the system.

There is some confusion in the literature on the meaning of adiabatic sound speed and phase speed. Although this seems not to affect the results of previous works, and the adiabatic sound speed is often
simply used as a convenient shorthand for $\dot{\rho}_0/\rho_0$, as defined in Eq. (5), it is often confusingly used to denote the phase speed. We take the opportunity to clarify these issues here.

The adiabatic sound speed defined above describes the response of the pressure to a change in density at constant entropy, and is the speed with which pressure perturbations travels through a classical fluid. A collection of scalar fields can also be described as fluid, but the analogy is not exact. Whereas in the fluid case phase speed $c^2_{\text{ph}}$ and adiabatic sound speed $c^2_s$ are equal, this is not true in the scalar field case and the speed with which perturbations travel is given only by $c^2_{\text{ph}}$. We shall illustrate the difference in $c^2_{\text{ph}}$ and $c^2_s$ for a concrete example in Section 3.3 below.

3. Governing equations

We now derive the governing equations for a scalar field system with general non-canonical action. Consider a general Lagrangian for a single scalar field $\phi$,

$$L = f(X, \phi),$$  \hspace{1cm} (13)

where $X = -\frac{1}{2}g^{\mu\nu}\phi,_{\mu}\phi,_{\nu}$.

The energy-momentum tensor is defined as \[ T_{\mu\nu} = -2\frac{\partial L}{\partial g^{\mu\nu}} + g_{\mu\nu}L, \] (14)
and substituting the Lagrangian Eq. (13) into Eq. (14) we get

$$T^\mu_\nu = f,X g^{\mu\lambda}\phi,_{\lambda}\phi,_{\nu} + \delta^\mu_\nu f.$$  \hspace{1cm} (15)

The energy-momentum tensor for a perfect fluid is

$$T^\mu_\nu = (\rho + P)u^\mu u_\nu + P\delta^\mu_\nu,$$  \hspace{1cm} (16)

where $P$ and $\rho$ are the pressure and energy density, respectively, and $u^\mu$ is the four-velocity, subject to the constraint $u^\mu u_\mu = -1$.

We consider here only scalar perturbations in a flat FRW background with line element

$$ds^2 = -(1 + 2A)dt^2 + 2aB, dx^i dt + a^2[(1 - 2\psi)\delta_{ij} + 2E,_{ij}] dx^i dx^j,$$  \hspace{1cm} (17)

where $a = a(t)$ is the scale factor, $A$ is the lapse function, $B$ and $E$ make up the shear ($\sigma = a^2\dot{E} - aB$), and $\psi$ is the dimensionless curvature perturbation. The four-velocity is then given by

$$u^\mu = \left[(1 - A), \frac{1}{a}v^i\right],$$  \hspace{1cm} (18)

where $v$ is the scalar velocity potential of the fluid.

We split quantities into a time-dependent background and a time- and space-dependent perturbation, and get e.g. for the scalar field $\phi = \phi_0(t) + \delta\phi(t, x^i)$. The energy density and pressure of the scalar field are then, in the background

$$\rho_0 = 2f,X X_0 - f, \quad P_0 = f,$$  \hspace{1cm} (19)

where $X_0 = \frac{1}{2}\dot{\phi}_0^2$, and to first order in the perturbations

$$\delta\rho = (f,X + 2X_0 f,X) \delta X + (2X_0 f,_{X}\phi - f,_{\phi}) \delta\phi,$$

$$\delta P = f,X \delta X + f,_{\phi} \delta\phi,$$  \hspace{1cm} (20)

\[ ^4 \text{More intuitive is the introduction of the adiabatic sound speed using the compressibility, } \beta \equiv \frac{\dot{\rho}}{P} \bigg|_T = \frac{1}{\rho c_s^2}, \text{ which describes the change in density due to a change in pressure while keeping the entropy constant.} \]
where we have defined $\delta X = \dot{\varphi}_0 \dot{\varphi} - A \dot{\varphi}_0^2$. It is convenient to work with the covariant velocity perturbation, $V = a(v + B)$, which in terms of the scalar field is given from $T^0_0$ component of the energy-momentum tensor as

$$V = -\frac{\delta \varphi}{\dot{\varphi}_0}.$$  

(21)

3.1. Background

The conservation of energy-momentum is $T^\mu_{\nu,\mu} = 0$, which in the background reduces to

$$\dot{\rho}_0 = -3H(\rho_0 + P_0).$$  

(22)

In terms of the scalar field, this becomes the evolution equation

$$\dot{\varphi}_0 \left( f,_{XX} \varphi_0^2 + f,_{X} \dot{\varphi}_0^2 - f,_{\varphi} + 3Hf,_{X} \varphi_0 \right) = 0.$$  

(23)

The Friedmann constraint equation is given by

$$H^2 = \frac{8\pi G}{3} \rho_0,$$  

(24)

where $H \equiv \dot{a}/a$ is the Hubble parameter.

3.2. First order

Conservation of energy-momentum to linear order gives the perturbed energy conservation equation

$$\delta \dot{\rho} = -3H(\delta \rho + \delta P) + (\rho_0 + P_0) \left[ 3\dot{\psi} - \frac{\nabla^2}{a^2} (\sigma + V) \right].$$  

(25)

This can be readily rewritten in terms of the curvature perturbation on uniform density hypersurfaces $\zeta = -\dot{\psi} - H\delta \rho / \dot{\rho}$, and working in the uniform density gauge, where $\delta \rho \equiv 0$ and hence $\delta P = \delta P_{nad}$ and $\zeta \equiv -\dot{\psi}$, we get

$$\dot{\zeta} = \frac{-H}{(\rho_0 + P_0)} \delta P_{nad} - \frac{\nabla^2}{3a^2}(\sigma + V).$$  

(26)

Equation (26) reduces on large scales, where gradient terms can be neglected, to Eq. (1) and shows that if the pressure perturbation is adiabatic, then the curvature perturbation is conserved. See Ref. [3] for a more detailed discussion.

In the following we will also need the constraint equation (see e.g. Ref. [14]),

$$\delta \rho - 3H(\rho_0 + P_0) V + \frac{H}{4\pi G a^2} k^2 \sigma = 0,$$  

(27)

where we have replaced spatial Laplacians with the wave-numbers of their respective eigenvalues, that is $\nabla^2 \rightarrow -k^2$, and have chosen the flat gauge, where $\psi = 0 = E$. On large scales, where gradient terms can be neglected, we can rewrite Eq. (27) in terms of the scalar field, using Eqs. (20) and (21), as

$$(f,_{X} \varphi_0^2 f,_{X} \dot{\varphi}_0^2 - f,_{\varphi} + 3Hf,_{X} \varphi_0) \delta \varphi = 0,$$  

(28)

giving a convenient relation between $\delta X$ and $\delta \varphi$. 

5
3.3. Speeds

The scalar field system we are considering here can be described by “borrowing” terminology from fluid dynamics, though the analogy is not exact. If we are interested in the speed with which perturbations travel through the system, we have to calculate the phase speed, which can be read off from the perturbed Klein-Gordon equation governing the evolution of the scalar field. This is just a damped wave equation, like Eq. (12), and so by comparing the coefficients of the second order temporal, and second order spatial derivatives, we obtain the phase speed.

For the general non-canonical Lagrangian, (13), the evolution equation for the first order scalar field perturbation is found by substituting Eq. (20) into (25), and is given here already in the form of Eq. (12) as

\[ \frac{1}{c_{ph}^2} \ddot{\delta \varphi} + \left( \frac{3H}{c_{ph}} + C_1 \right) \dot{\delta \varphi} + \left[ \frac{k^2}{a^2} + C_2 \right] \delta \varphi = 0 , \]

(29)

where the coefficients \( C_1 \) and \( C_2 \), both functions of \( \varphi \) and \( X \), are given in Eq. (37) in the Appendix. We can therefore read off the phase speed as

\[ c_{ph}^2 = \frac{f_{,X}}{f_{,X} + 2X_0 f_{,XX}} . \]

(30)

Using Eq. (19), this reduces to

\[ c_{ph}^2 = \frac{P_{0,XX}}{\rho_{0,X}} . \]

(31)

The adiabatic sound speed for the Lagrangian (13) is given by

\[ c_s^2 = \frac{f_{,X} \dot{\varphi}_0 + f_{,\varphi}}{f_{,X} \dot{\varphi}_0 - f_{,\varphi} + f_{,XX} \dot{\varphi}_0^2 + f_{,X \varphi} \dot{\varphi}_0^2} . \]

(32)

The Lagrangian for a canonical scalar field is obtained by setting \( f(X, \varphi) = X - U(\varphi) \). In this case the adiabatic sound speed reduces to

\[ c_s^2 = 1 + \frac{2U_{,\varphi}}{3H \dot{\varphi}_0} , \]

(33)

and becomes \( c_s^2 = -1 \) in slow-roll. The phase speed for a canonical scalar field is \( c_{ph}^2 = 1 \).

4. Non-adiabatic pressure for a non-canonical scalar field

We now return to the question of under what conditions the non-adiabatic pressure in a general scalar field system vanishes. The non-adiabatic pressure \( \delta P_{nad} \) for the system (13) is found by substituting the expressions for the energy and the pressure, Eq. (19), into Eq. (8),

\[ \delta P_{nad} = f_{,\varphi} (1 + c_s^2) - 2c_s^2 X_0 f_{,X \varphi} \delta \varphi + f_{,X} (1 - c_s^2) - 2c_s^2 X_0 f_{,XX} \delta X , \]

(34)

where \( c_s^2 \) was given in Eq. (32).

Using the constraint equation (28) and the evolution equation for the background field, Eq. (23), we obtain a simple relationship between \( \delta X \) and \( \delta \varphi \), namely,

\[ \delta X = \dot{\varphi}_0 \delta \varphi . \]

(35)

Substituting this into the expression for the non-adiabatic pressure perturbation, and using \( c_s^2 \), we obtain,

\[ \delta P_{nad} = 0 . \]

(36)
Therefore, the non-adiabatic pressure perturbation $\delta P_{\text{nad}}$ for the general action \[13\] vanishes in the large scale limit without the need to impose any further conditions, just as in the canonical case \[21\]. This is in agreement with Ref. \[13\].

This is at first glance a surprising result, since one might assume that the system has two degrees of freedom, $\delta \varphi$ and $\delta X$, and that one would need to impose slow-roll to “switch off” a degree of freedom in order to obtain zero non-adiabatic pressure.

Yet, in the super-horizon limit $k \to 0$, such a condition is not necessary since the constraint equation \[28\] eliminates one degree of freedom.

For multiple scalar fields, however, this equation does not hold, and hence $\delta P_{\text{nad}}$ is, in general, non-zero.

If we study specific models, for example the DBI inflation models of Refs. \[10, 11\], and loosen the super-horizon limit $k \to 0$ by instead studying small but non-zero wavenumbers $k$, we can calculate the possible small deviation from zero of the non-adiabatic pressure or entropy perturbation. This is an additional observational constraint which can be imposed on the models, and which is already strongly constrained by the data \[2\]. Since these issues are outside the aim of this work, we will discuss them in detail in a forthcoming article \[22\].

5. Discussion and conclusions

We have studied whether the curvature perturbation in a system with a scalar field minimally coupled to gravity with non-canonical action is constant on super-horizon scales. Deriving the adiabatic and the non-adiabatic or entropy perturbation we found that imposing the strict super-horizon limit $k \to 0$ is sufficient to guarantee conservation of $\zeta$ and hence of the primordial power spectrum, just as in the canonical case.

The non-adiabatic pressure, or entropy, perturbation is in general non-zero, if there is more than one degree of freedom in the system, as is the case in multi-field systems. In this case the curvature perturbation evolves, even on large scales, and it is necessary to solve the evolution equations of the scalar fields to construct the non-adiabatic pressure. However, it is then more efficient to construct $\zeta$ directly using its definition and substituting in the energy densities in terms of the fields, instead of first constructing $\delta P_{\text{nad}}$ and then solving the evolution equation for $\zeta$, Eq. \[26\], see \[21, 13\].

We have also highlighted the difference between the phase speed and the adiabatic sound speed of a system in Section 3.3. Although these are the same in classical fluid systems, they are different in the scalar field models studied here, with only the phase speed describing the speed with which perturbations travel. We emphasise that only the definition for the adiabatic sound speed, Eq. \[5\], enters the gauge-transformation of the pressure perturbation, as pointed out in Section 2.1. Similarly, in the adiabatic case when $\delta P_{\text{nad}} = 0$, Eq. \[8\] reduces to $\delta P = c_s^2 \delta \rho$. Again, this convenient relation between the pressure and the energy density perturbation is only correct when using the adiabatic sound speed, as defined in Eq. \[3\].

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\[2\] We thank Sébastien Renaux-Petel for bringing this to our attention.
Appendix

The coefficients $C_1$ and $C_2$ in the perturbed evolution equation for the scalar field, (29), are given by

$$C_1 = \frac{c_{ph}}{f_X} \left[ f_{,\varphi} - 3H f_{,X\varphi} \right] \left[ 3f_{,XX\varphi} + \varphi_0^3 f_{,XXX} \right] + \frac{1}{f_X} \left[ \varphi_0 f_{,X\varphi} + \varphi_0^3 f_{,XXX} \right],$$

$$C_2 = \frac{c_{ph}^2}{f_X^2} \left[ f_{,\varphi} - 3H f_{,X\varphi} \right] \left[ f_{,X\varphi} + \varphi_0^2 f_{,XX\varphi} - \frac{4\pi G \varphi_0}{H c_{ph}} \left( 5\varphi_0^2 f_{,XX} + \varphi_0^4 f_{,XXX} + 2f_{,X} \right) \right]$$

$$- \frac{4\pi G \varphi_0}{H c_{ph}} \left[ 3H \varphi_0 f_{,X} + f_{,\varphi} - \frac{f_X \varphi_0}{H} (3H^2 + 2H) + c_{ph}^2 (\varphi_0^4 f_{,XXX} + f_{,\varphi}) \right]$$

$$+ \frac{1}{f_X} \left[ \varphi_0^2 f_{,XX\varphi} - f_{,\varphi\varphi} + 3H \varphi_0 f_{,X\varphi} \right]$$

(37)

where we have made use of the following expressions relating metric perturbations to perturbations in the scalar field derived from Ref. [14],

$$A = \frac{4\pi G}{H} \varphi_0 f_{,X} \delta \varphi,$$

(38)

$$\dot{A} = \frac{4\pi G}{H^2} \left[ H f_{,\varphi} - 4\pi G \varphi_0^3 f_{,XX} - \varphi_0 f_{,X} (3H^2 + 2H) \right] \delta \varphi + \frac{4\pi G}{H} \varphi_0 f_{,X} \delta \varphi,$$

(39)

and Eq. (27).

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8