Conjecture on the Avoidance of the Big Crunch

Cheng-Yi Sun∗ and De-Hai Zhang†
Department of Physics,
The Graduate School of The Chinese Academy of Sciences,
Beijing 100049, P.R.China.

July 2, 2021

Abstract
In [1], KKLT give a mechanism to generate de Sitter vacua in string theory. And recently, the scenario, landscape, is suggested to explain the problem of the cosmological constant. In this scenario, the cosmological constant is a de Sitter vacuum. The vacuum is metastable and would decay into an anti-de Sitter vacuum finally. Then the catastrophe of the big crunch appears. In this paper by conjecturing the physics at the Planck scale, we modify the definition of the Hawking temperature. Hinted by this modification, we modify the Friedmann equation. We find that this avoid the singularity and gives a bouncing cosmological model.

PACS: 98.80.Cq, 98.80.Jk, 04.20.Gz

Key words: big crunch, Hawking temperature, Friedmann equation

1 Introduction

The data from the observation of the first year Wilkinson Microwave Anisotropy Probe (WMAP) [2, 3] and the observation of the SNe Ia [4] make

∗cysun@mails.gscas.ac.cn
†dhzhang@gscas.ac.cn
us almost be sure that the expansion of the universe is accelerating. The simplest explanation is that a small but non-zero cosmological constant, the de Sitter vacuum, dominates the present universe. Recent years, great efforts basing on the string theory have been paid to solve the cosmological constant problem and to construct a complete process of the cosmological evolution. In [1], a mechanism, KKLT mechanism, is given to get de Sitter vacua in string theory. And a scenario named Landscape has been suggested [5, 6]. In this scenario it is argued that string theory has a landscape of vacua. The supersymmetric (SUSY) sector of the landscape has the zero vacuum energy. The non-SUSY sector has the stochastic distribution of vacua energies around the zero vacuum energy, where some vacua are de Sitter vacua with positive vacuum energy and others are anti-de Sitter vacua with negative vacuum energy. One of the de Sitter vacua describes the present acceleration of our universe, and this vacuum is a metastable state. It would decay into another vacuum with a lower cosmological constant. Unfortunately, the detailed information of the landscape is absent. So the exact process of the decay is unknown. However, due to the exist of the anti-de Sitter vacua, we are sure that the final vacuum should be an anti-de Sitter vacuum. On the other hand, it is familiar to us that, in cosmology, the decay of dS vacua to AdS vacua would cause the big crunch inside the vacuum-decay bubble.

The catastrophe of the big crunch seems to be inevitable and discouraging. Even, this is one of the reasons that lead Banks to conjecture that the dS vacua are stable in sensible gravity theories [7]. However, this may be solved in another path. We know that the energy density of the universe increases as the contraction. It must have reached the Planck energy scale before the singularity is reached. On the other hand, the physical laws which is used to derive the appearance of the singularity is applicable only to the energy scale much lower than the Planck energy scale. So we think the singularity of the big crunch may be eliminated due to the new physics near the Planck scale. Unfortunately, the physics near the Planck scale is unclear up to now. But this can not block us to conjecture the physics at Planck scale.

In this paper, we first investigate the Hawking radiation of a black hole. We find that the concept of the Hawking temperature is questionable for a black hole with the Planck mass. By modifying the definition of the Hawking temperature we solve it. Then, analogously, we modify the Friedmann equation. This modification avoid the singularity of the big crunch by giving a bounce at the Planck scale.
2 the Hawking temperature of black holes at the Planck scale

In this section we investigate the Hawking radiation of a black hole. First, we consider a Schwarzschild black hole with mass \( m \). Hereafter, we take \( 8\pi G = M_{pl}^{-2} \). Then the radius of the horizon of the black hole is \( r_H = \frac{m}{4\pi M_{pl}} \) and the mass density of the black is \( \rho = \frac{3}{4\pi} m r_H^{-3} \). Hawking has shown (1974, 1975) that a distant observer will detect a thermal spectrum of particles coming from the black hole, at a temperature \( T = M_{pl}^2 m^{-1} \). This is just the well-known Hawking radiation. When the mass is at the scale of a star, the temperature is very low and the concept of the temperature makes sense. However, we know the mass of the black hole decreases as the radiation processing. When the mass approaches the Planck scale, the temperature approaches the Planck scale, too. This means the average energy of the radiation particles, \( \sim T \), would be about the Planck energy, which is nearly equal to the mass of the black hole. It implies that one radiation particle carries away nearly the whole energy of the black hole. What does this mean? In fact this is just the translation of the black hole and no radiation exists at this time. So we expect that the radiation temperature should disappear at the Planck scale. But the semiclassical definition of the Hawking temperature give a very high temperature. The reason, we think, is the backreaction effects of the emitted particles are neglected in the definition. However, from the analysis above, we know that the backreaction is not negligible when the mass of a black hole approaches the Planck scale.

So let’s consider the backreaction effects. We know the nonzero Hawking temperature produce the Hawking radiation, with the radiation energy density \( \rho_R \sim T^4 \). According to the three equations given in the last paragraph, we know \( T^2 \sim M_{pl}^{-2} \rho \). So we get \( \rho_R \sim M_{pl}^{-4} \rho^2 \). Naturally, this radiation would reduce the energy density of the black hole. Then considering this effect, we expect the temperature should be \( T^2 \sim M_{pl}^{-2} (\rho - \rho_R) = M_{pl}^{-2} (\rho - M_{pl}^{-4} \rho^2) \). Now, as a toy model, we conjecture naively that the temperature of the black hole should be modified as

\[
T^2 = \frac{\rho}{48\pi^2 M_{pl}^2} (1 - \frac{\rho}{M_{pl}^4})
\]

(1)

When the mass is much larger than the Planck mass, the density is much smaller than the Planck scale. Then the second term in the bracket on the
right-hand side is negligible and the modified equation is equivalent to the ordinary definition approximately. But the difference is remarkable as the mass approaches the Planck scale. It is obvious that Eq.(1) gives $T = 0$ at $\rho = M_{pl}^4$. So the difficulty in the last paragraph does not exist. In addition, this expression implies the temperature has the maximum, $T_m \sim \frac{1}{4} M_{pl}$. This is consistent with the concept of the Hagedorn temperature in string theory.

3 the evolution of the universe at the Planck scale

From the investigation in the black hole above, we know the character of the physics changes remarkably near the Planck scale. We know, a de Sitter universe with a cosmological constant, $\Lambda$, is similar to a black hole. It has also a temperature, $T \sim H$. The Hubble parameter, $H$, is governed by the Friedmann equation

$$H^2 = \frac{\dot{a}}{a} = \frac{1}{3M_{pl}^2} \Lambda,$$

(2)

where $a$ denotes the scale factor in the FRW metric and ”dot” denotes derivative with respect to the physical time. Hinted by Eq.(1), we conjecture that Eq.(2) should also be modified as

$$H^2 = \frac{1}{3M_{pl}^2} \Lambda (1 - \frac{\Lambda}{M_{pl}^4}).$$

(3)

Extending this modification to a general FRW universe directly, we get the modified Friedmann equation

$$H^2 = \frac{1}{3M_{pl}^2} \rho (1 - \frac{\rho}{M_{pl}^4}).$$

(4)

Obviously the second term in the bracket on the right-hand side is negligible when the energy density is much lower than the Planck scale. Then this modified Friedmann equation becomes the standard Friedmann equation. But, at the high energy density, the second term play a key role. It is obvious that this modified Friedmann equation avoid the singularity of the big crunch by giving a bounce of the universe at $\rho = M_{pl}^4$. 
Now let’s analyze the evolution of the universe in detail as $\rho$ approaching $M_{pl}^4$ in the big crunch. We assume the equation of state of the dominant component of the universe is $p = w\rho$. Here, we emphasize that we only consider the component with $w > 1$. We define the scale factor $a(t) = a(t_p) + x$, where $x$ is an infinitesimal parameter and $t_p$ denotes the moment of the bounce, $\rho = M_{pl}^4$. Then, as $\rho$ approaching $M_{pl}$, we get

$$\rho = M_{pl}^4[1 - \frac{3(1 + w)x}{a(t_p)}], \quad H^2 = \frac{x^2}{a(t_p)^2}$$

Substituting these into Eq.(4), we get

$$\frac{dx}{dt} = -\sqrt{M_{pl}^2(1 + w)a(t_p)x}.$$  \hspace{1cm} (6)

Solving this equation, we get

$$x = \frac{M_{pl}^2(1 + w)a(t_p)}{4}(t - t_p)^2.$$  \hspace{1cm} (7)

Now we are sure that the bounce at $t_p$ is smooth according to Eq.(4). So the catastrophe of the big crunch is eliminated. After the bounce, the universe begins to expand again. Naturally, we may take the bounce as a big bang after which the new evolution of the universe begins.

The similar modification has been shown in another view, braneworld scenarios [8]. The difference is that in braneworld cosmology this is taken as the effect of the extradimensions. Here we suppose that this is due to the effect of the physics near the Planck scale.

4 Summary

In this paper, in order to solve the disaster of the big crunch, we conjecture the physics at the Planck scale. We first investigate the concept of the Hawking temperature of the black hole when the mass of the black hole decreases to the Planck mass. We find some paradox appears. We try to solve it by modifying the definition of the temperature. Analogously, by modifying the Friedmann equation, we avoid the singularity of the big crunch. Of course, our modification is just a toy model. We do not think we have obtained the
law of physics at the Planck scale. We just take it as an interesting attempt. Finally, we emphasize that many concepts at lower energy scale can not be used directly to describe the physics at the Planck scale.

References

[1] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter Vacua in string theory,” Phys. Rev. D 68, 046005 (2003) [arXiv:hep-th/0301240].

[2] C. L. Bennett et al., “First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results,” Astrophys. J. Suppl. 148 (2003) 1.

[3] D. N. Spergel et al., “First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters,” Astrophys. J. Suppl. 148 (2003) 175.

[4] A. G. Riess et al., Astrophys. J. 607 (2004) 665-687.

[5] L. Susskind, “The Anthropic Landscape of string theory,” [arXiv:hep-th/0302219]

[6] N. Arkani-Hamed, S. Dimopoulos and S. Kachru, “Predictive Landscapes and New Physics at a Tev,” [arXiv:hep-th/0501082]

[7] T. Banks, “Heretics of the False Vacuum: Gravitational Effects On and Of Vacuum Decay2,” [arXiv:hep-th/0211160]

[8] Y. Shtanov and V. Sahni, “Bouncing Braneworld,” Phys. Lett. B 557(2003)1-6, [arXiv:gr-qc/0208047]