A Pure Mathematical Argument for Finiteness of Velocity of Material Objects

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Abstract. Purely by mathematical means a theorem of the finiteness of the velocity of material objects is proved without reference to the special theory of relativity. The Proof is valid under the condition that in material objects there are always present interaction signals (carrier particles) without rest mass that are propagated at a velocity greater than any achieved velocity of the particles and bodies with rest mass, moreover, the interaction signals (carrier particles) colliding with elements of these objects continuously initiate interactions and processes in all objects. The condition does not in itself contain any limitation of the velocity of material objects. Neither does it contradict it. The arguments presented in this paper can be tied to any inertial reference system by virtue of the formal equality of these systems.

1. Introduction

It is generally known that the velocities of material bodies cannot achieve the speed of light, \( c \). This is generally explained in the special theory of relativity as a consequence of the fact that it requires an infinitely great amount of energy to accelerate a body through the light velocity barrier [1-3].

However, this explanation only makes it possible to state the fact of the finiteness of the velocities of material objects without explaining why the light speed barrier exists. We would like here to go beyond this type of explanation and search for an underlying cause of the finiteness of the velocities of material objects and that of speed of light.

Hence, we shall give an answer to the question: Why are material bodies restricted to move at velocities smaller than the speed of light? What is the reason that a material body, no matter how and how long it is being accelerated, never achieves the speed \( c \)?

Also, why does the speed of light, being finite, act as if it is infinite in the sense that it is unattainable for material bodies?

One may wonder: Will the reason for the existence of the light barrier ever be found, or must one always be content with pointing to the special theory of relativity? Maybe there exists some deeper reason which has nothing to do with the special theory of relativity? That is the question we shall explore here.

2. A condition implying inconsistency of unlimited great velocity for material objects

Most people think that the finiteness of the velocity of material objects needs no explanation besides that given in the special theory of relativity. And not searching for any other explanation of course means that no other explanation will be found. Here we want to explore another possibility.
We will show that it is possible to postulate a condition concerning the constitution of matter, which makes the fact of the finiteness of the velocity of material objects become a logical necessity. In this way the limitation of the velocities of material objects will no longer contradict common sense. On the other hand according to this condition a boundless increase of the velocity of a material object would then appear utterly artificial and even impossible.

The condition can be stated as follows.

**Condition C1**: In massive material objects there are always present massless (without rest mass) interaction signals (carrier particles) that are propagated at a velocity \( V \) greater than any achieved velocity \( v \) of the massive (with rest mass) particles and bodies moreover the massless interaction signals (carrier particles) travelling the mean free path \( \lambda \) collide with elements of the material objects committing interactions and continuously initiating processes in all objects.

The condition **C1** does not in itself contain any limitation of the velocity of massive material particles and bodies. Neither does it contradict it.

Leaving out of account the limitation of velocity coming from the special theory of relativity we *a priori* admit the existence of unlimited velocities \( v \) of massive bodies and particles that may have any value. Nevertheless, the velocities \( v \) of the massive bodies are assumed to be less than the massless signal and carrier particles velocity \( V \), i.e. \( v < V \).

### 3. A proof of limitation of velocity of material bodies

We prove logically that according to the condition **C1** there is a limit to the allowable achieved velocity of massive objects, admitting that without the condition **C1** there is no such limiting velocity.

**Theorem**: If the condition **C1** is valid then the velocity \( V \), of massless interaction signals (carrier particles) at the distance \( L > \lambda \) is finite.

**Proof**: Assume that there is a point A located a distance \( L \) from a certain material object O at rest in a given reference frame. A massless signal moves to A where it is immediately reflected, and moves back to O. Can the signal move so fast that it returns instantaneously after having performed the to and fro travel?

Let us first discuss the meaning of the concept “instantaneously”. What does this concept mean?

In a metrological sense “instantaneously” means a vanishing time interval between emission and return of the signal. In a physical sense “instantaneously” means that no physical process or change of the object can occur between these events. The emission moment in time and the return moment must merge together in this instance.

We now ask whether it is possible for an arbitrarily fast moving signal to traverse from O to A and back to O while no changes occur to a object at rest O in which constantly \( n \) signals are present.

This is not possible if the condition **C1** of the theorem is valid.

Consider a situation with a material object at rest O and a massless signal travelling a distance \( 2L \) from object O to point A and back. While the massless particle outside the object travels the distance \( 2L \), then inside the material object O at rest \( n \) signals are committing a certain kind of interaction, and there occur \( 2nL/\lambda \) events in the object, where \( \lambda \) is the average free path of the signals committing this kind of interaction within the object, and it is assumed that \( \lambda < L \). Since there are interactions in the object, it will change in time, and therefore an arbitrarily accurate clock will show a non-vanishing time interval during the travel of the signal.

Since in accordance with condition **C1** the velocity, \( v \), of massive bodies and particles is less than the finite velocity \( V \) of massless signal, the velocity \( v \) is finite as well.

### 4. Time interval in physical clocks

As noted above, during the travel of a signal from the object O to point A and back \( 2Ln/\lambda \) events happen in a object in which constantly \( n \) signals committing a certain kind of interaction are present. Hence, the number of events of this kind \( \Delta n \) during this travel is
\[ \Delta \eta = 2 \ln \lambda. \]  

Each physical object including massive particles can be considered as a kind of imagined physical clocks, internally “measuring” time, but not giving visible to us readings. We can choose the number of events of the given kind induced by the signals in the object as invisible to us readings of physical clock. It should be noted that hypothetical imagined physical clocks of this type have a shortcoming in that their readings depend upon the size of that object. If \( m \) is the mass of the object (body or massive particle) that is considered as a physical clock then the “reading” of “clocks” with different masses would be different, because the greater the mass of the “clock”, the more events occur between two given events. In order to have a “clock” with a mass independent rate, we imagine the clock so that its rate of time is proportional to the ratio between the number of events of the given kind in a certain body or massive particle and the mass of this object. Hence \( \Delta t \sim \eta / m \eta_m \), where \( \eta_m \) is the number of events of the given kind per mass unit in the object defining a physical clock.

As far as measurements of time and motion are concerned, the set of physical (material) bodies that are at rest in a certain system of reference is assumed to characterize the fundamental kinematical properties of this system. Hence, the magnitude \( \eta_m \) can be called the ‘system time of given kind’. Unlike \( \eta \), the system time flows at the same rate in all bodies that are at rest in certain system. Since the number of events is dimensionless, the dimension of system time \( \eta_m \) is \([M^{-1}]\).

If the signal density is denoted by \( n_m = n/m \), then (1) can be rewritten as

\[ \Delta \eta_m = 2 \ln n_m / \lambda. \]  

This is the time interval for the travel of the signal from the object, \( O \), to the point, \( A \), and back to \( O \) expressed by the number of events of the given kind per unit mass. Hence the physical velocity of the signal is

\[ \Lambda_m = 2L / \Delta \eta_m = \lambda / n_m. \]  

We can divide the number of events in an object not by its mass, but by its total energy. Then instead of (2) we obtain

\[ \Delta \eta_E = 2 \ln n_E / \lambda \]  

and instead of (3):

\[ \Lambda_E = 2L / \Delta \eta_E = \lambda / n_E. \]  

The dimension of the time, \( \eta_m \), expressed in terms of mass density of signals and the dimension of velocity, \( \Lambda_m \), expressed in terms of the length and mass density of signals respectively are \([M^{-1}]\) and \([LM]\). The dimensions of the time, \( \eta_E \), and velocity, \( \Lambda_E \), expressed in terms of energy density of signals respectively are \([E^{-1}]\) and \([LE]\). We avoid here the use of the expended dimension \([L^2MT^{-1}]\) of energy in terms of length, mass and time, since the unit of time in this work is not initially equal to a second and is presented in an unusual form.

Let us consider only photons as carriers of interaction and a certain amount of photon gas as an object. By event we mean one complete oscillation of the electromagnetic wave of a photon. The
energy of photon gas containing \( n \) photons is equal to \( nh\nu \). The energy density, \( n_E \), of photons (the number of photons divided by the photon gas energy) is \( \frac{1}{h\nu} \), that is

\[
n_E = \frac{1}{h\nu}.
\]

It follows from (5) and (6) that the photon velocity \( \Lambda_E \) expressed by the number of events per unit energy equals \( h\nu\lambda \) there is

\[
\Lambda_E = h\nu\lambda.
\]

The number of oscillations of the electromagnetic field per unit energy of the photon gas per second is equal to \( n_E \) or taking into account (6) it equals \( \frac{1}{h} \). This value is the conversion factor from the time we entered to the usual time in seconds. To obtain the velocity \( V \) in the units of the SI system that we are familiar with, we must multiply the velocity, \( \Lambda_E \), by the coefficient \( \frac{1}{h} \). In this case we obtain the value of the speed \( V \) equal to \( \nu\lambda \).

5. Conclusion
We have introduced a condition of finiteness of the velocity of massless particles (interaction signals). This condition corresponds to the fundamental properties of material bodies and particles essential for understanding why an infinitely great velocity is unattainable for material bodies. The condition says that in all bodies there are massless interaction signals moving with a velocity greater than any velocity of material bodies and continuously causing changes in bodies.

We initially examined the conditions under which the velocity of material bodies has a limited nature based on the example of the model (the simulation of the special theory of relativity) described in references [3-6]. We later departed from this model and made the transition to the examination of the question independent of it. However, the considerations set forth in the aforementioned works in particular impelled us toward the logical solution of the problem of the finiteness of velocity and made it possible to answer the questions arising during the resolution of this problem.

The main result of this paper is that to make clear that we do not need the special theory of relativity in order to explain why material bodies cannot move arbitrarily fast. This is a consequence of the constitution of matter which contains massless interaction signals with a velocity which it is logically impossible to surpass by the massive particles and bodies. The limitation of the speed of massive particles is a consequence of interactions occurring continuously in all types of matter by means of massless particles. The arguments presented in this paper can be tied to any inertial reference system by virtue of the formal equality of these systems.

The results of our work may be useful for clarifying the possibility and courses of macroscopic violation of special relativity described in the papers of Nimtz [7-8]. They may be useful when discussing the subject of superluminal velocities, which recently again captured the attention of specialists in the wake of questionable publications [9] concerning experimental observations of these velocities.

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