Compressible impurity flow in the TJ-II stellarator

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Abstract

Fully ionized carbon impurity flow is studied in ion-root, neutral beam heated plasmas by means of charge exchange recombination spectroscopy (CXRS) in the TJ-II stellarator. Perpendicular flows are found to be in reasonable agreement with neoclassical calculations of the radial electric field. The parallel flow of the impurity is obtained at two locations of the same flux surface after subtraction of the calculated Pfirsch–Schluter parallel velocity. For the medium density plasmas studied, $\dot{n}_e \in (1.2–2.4) \times 10^{19}$ m$^{-3}$, the measured impurity flow is found to be inconsistent with a total incompressible flow, i.e. $\nabla \cdot u_i \neq 0$, thus implying a non-constant impurity density on those flux surfaces. The experimentally observed velocity deviations are compared with the parallel return flow calculated from a modelled impurity density redistribution driven by ion-impurity friction. Although the calculated return flow substantially modifies the incompressible velocity pattern, modifications at the locations of the CXRS measurements are generally smaller in magnitude and opposite in sign when compared to the experimentally observed deviations. Small inhomogeneities of the electrostatic potential in a surface are also shown to affect the impurity redistribution but do not provide a better understanding of the measurements.

Keywords: impurities in plasmas, plasma dynamics and flow, stellarators, optical measurements, impurity transport, plasma fluid equations, impurity density asymmetry

(Some figures may appear in colour only in the online journal)

1. Introduction

The flow of mass along flux surfaces in magnetically confined plasmas has come to be regarded as an important factor in determining plasma stability, radial transport and performance of these devices. The $E \times B$ flow pattern is of particular importance with regard to transport. Indeed, a sufficiently strong radial velocity shear is generally accepted as reducing turbulence and transport [1]. The flow patterns of the different species present in the plasma (main ions, electrons and impurity ions) deviate from the $E \times B$ flow and from each other through their different diamagnetic velocity components and parallel force balances. These diamagnetic and parallel flows give rise to currents that are a fundamental part of the stellarator MHD equilibrium in high-beta reactor-relevant plasmas. Parallel currents are generally split into Pfirsch–Schluter (PS) and parallel mass flows. The former arises in response to the compressibility of the perpendicular diamagnetic current and carries no net toroidal current, but can nevertheless cause a radial (Shafranov) shift of magnetic surfaces as the pressure gradient increases. The reduction of this current is a design requirement of modern stellarators because of its detrimental effect on high-beta stability and neoclassical (NC) transport [2]. On the other hand, the bootstrap current carries a net toroidal current\textsuperscript{4} and can potentially change the iota profile, which is of particular importance for island divertor configurations in stellarators and is taken advantage of in tokamak non-inductive scenarios. For these reasons experimental validation of first-principle theory-based models of plasma flows and currents is of considerable importance. In [4] measurements of fully ionized carbon impurity flow were undertaken using CXRS [5] in low density, Electron Cyclotron Resonance heated plasmas in the TJ-II stellarator. It was verified that the in-surface variation of the parallel impurity flow was consistent with an incompressible total flow tangent to flux surfaces. In addition, the measured perpendicular and parallel mass flows were compared with NC calculations of the radial electric field and ion parallel mass flow, finding a good agreement in those low-density plasmas.

In this work, CXRS measurements of C$^{6+}$ flows in neutral beam injection (NBI) heated, ion-root plasmas of the TJ-II stellarator are presented. As indicated in [4] significant and reproducible deviations in the measured impurity flow from

\textsuperscript{4} In the parallel mass flow we therefore include all the relevant parallel forces that determine the parallel flows of the different species, e.g. the NBI-driven currents. It is also noted that, to some extent, the split of the parallel currents is a matter of convention [3].
an incompressible pattern are observed as density increases, which points to a redistribution of impurity density within the flux surfaces. We present these flow measurements and compare the observed deviations with the parallel return flow from a modelled impurity density redistribution driven by ion-impurity friction [6]. Such a friction model was adapted to a general stellarator geometry with the bulk ions in the PS regime of collisionality in [7] and is extended here for main ions in the plateau regime, provided that the ion temperature gradient is small (a plausible assumption for the plasmas under consideration). The calculated return flow substantially modifies the incompressible velocity pattern, being comparable to the impurity parallel PS flow. However, it is shown that the calculated modifications at the CXRS measurement locations are small in comparison to the measured in-surface variations of impurity parallel flow and in the opposite direction for most cases. The inclusion of inertial and parallel electric field forces in the parallel momentum balance does not provide a better understanding of inertial and parallel electric field forces in the parallel return flow substantially modifies the incompressible velocity pattern, being comparable to the impurity parallel PS flow. However, it is shown that the calculated modifications at the CXRS measurement locations are small in comparison to the measured in-surface variations of impurity parallel flow and in the opposite direction for most cases. The inclusion of inertial and parallel electric field forces in the parallel momentum balance does not provide a better understanding of the experimental observations.

The fact that impurity density inhomogeneities can alter their radial transport [6] makes the understanding of these inhomogeneities particularly relevant. Also, from the data interpretation point of view, the parallel return flows associated with a density inhomogeneity might complicate the comparison of CXRS rotation measurements to standard NC theory, particularly in the presence of large main-ion gradients [8,9]. Observations of in–out flow variations have been reported in the CHS stellarator [10] and the C-Mod and ASDEX-U tokamaks [11, 12], which have been recently shown to be caused by a poloidal redistribution of the impurities with direct impurity density measurements [13,14]. In this work we restrict ourselves to the discussion of flow deviations from incompressibility as an indirect measure of the C\(^{6+}\) density inhomogeneity, since several instrumental uncertainties of the TJ-II CXRS system prevent the interpretation of signal intensities as relative density measurements. On the other hand, parallel and perpendicular impurity flows and temperatures are routinely provided by the CXRS system and have been shown to be in reasonable agreement with other diagnostics and/or NC theory predictions [4,15].

This paper is organized as follows: in section 2, the diagnostic set-up and geometry are presented together with the methodology used to relate the flow fields to the CXRS velocity measurements through the appropriate geometric quantities. In section 3 the impurity flow measurements and their compressible asymmetries are described. These asymmetries are compared with the results of an ion-impurity friction model in section 4, where modifications to the incompressible impurity flow pattern, caused by an in-surface impurity density variation, are detailed. In section 5 the validity of the friction model is examined and the impurity parallel force balance is extended to account for inhomogeneities of the electrostatic potential within a magnetic surface. Finally, conclusions are drawn in section 6.

2. Diagnostic set-up and data analysis

The CXRS process of interest in TJ-II involves electron capture from accelerated hydrogen by fully ionized carbon ions into a highly excited state of C\(^{5+}\), followed by spontaneous decay via photon emission, i.e. the C\(^{6+}\) line at 529.07 nm (\(\nu = 8 \rightarrow 7\)). For this, a compact diagnostic neutral beam injector (DNBI) provides a 5 ms long pulse of neutral hydrogen accelerated to 30 keV. Its 1/e-radius at focus is 21 mm. For light collection, commercial lenses focus the emitted light onto fibre optic bundles that transport it to the input of a transmission grating spectograph [16]. Correctly performed instrument wavelength calibration and optical alignment are essential to minimize the CXRS experimental uncertainties. For the first case, a neon pencil lamp is inserted between each light collection lens and corresponding fibre head between discharges to determine the wavelength dispersion at the focal plane for each fibre. In addition, corrections are made for fine-structure, Zeeman broadening and the so-called pseudo-velocities [17] before Doppler shifts and widths are determined. The uncertainties associated with the integration of geometrical quantities along sightlines are also accounted for. See [4] for a complete description. In this way, the uncertainty achieved in measured velocity is 1–2\(\times\)10\(^{-4}\).

The procedure for aligning the diagnostic is detailed in section 2.1 of [4]. The location of the flux surfaces is known accurately from the vacuum field. Note that the helical axis of TJ-II makes the Shafranov shift very small ((\(\leq 3\) mm for \(\beta = 1\%\), see e.g. [18])). This shift is smaller than the spot size of the fibres. Consequently, the inboard and outboard measurements are directly mapped to flux coordinates using the known magnetic field geometry, and in contrast to the tokamak experiments [11, 12], no additional relative shift between the inboard and outboard measurements is needed to align the carbon temperature profiles. Moreover, an additional check to confirm the goodness of the toroidal \(\rho\) mapping is made. For this, the spectrograph grating (set at 529 nm [16]) was exchanged for one centred at 656.2 nm. Then, by injecting the DNBI beam into the vacuum chamber with no magnetic fields, spectra with Doppler-shifted H\(_{\alpha}\) line emission from the beam were collected and analysed. Hence, by determining the Doppler shift of the beam H\(_{\alpha}\) for each sight line, and knowing the beam energy, the beam to sight line angles were obtained and the beam/sight line intersection points could be determined. These intersection points were compared with the values obtained using the illumination method described in [4] thereby confirming the uncertainties in the alignment of toroidal fibres, i.e. \(\pm 3\) mm. Note: the separation between toroidal sightlines, \(\geq 3\) cm, is fixed by the fibre bundle and focusing lens.

A schematic layout of the diagnostic sightlines is presented in figure 1, together with a poloidal cut of several magnetic surfaces of TJ-II. The plasma minor radius region spanned by nearly symmetric poloidal views is \(\rho \in (0.25, 0.85)\) in the magnetic configurations studied in this work. Here the normalized radius is defined as \(\rho = \sqrt{V/V_0}\), where \(V\) and \(V_0\) are the volumes enclosed by the surface of interest and the last closed magnetic surface, respectively. In the figure only the bottom poloidal array is presented for clarity (see [4] for details). On the other hand, the toroidal fibres cover both sides of the magnetic axis, from \(\rho \sim -0.75\) to \(\rho = 0.6\) at 10 locations (in figure 1 the toroidal sightlines, plotted as open circles, extend into/out of the page). The region in which both poloidal and toroidal measurements are taken is labelled
as outboard, while the zone where only toroidal measurements are made is labelled as inboard. The nomenclature $\rho > 0$ (outboard) and $\rho \leq 0$ (inboard) is also utilized to define these regions.

In the outboard region, poloidal and toroidal fibres view the same surfaces at $\rho \sim 0.2, 0.4$ and $0.6$, see figure 1. Therefore, the 2D-flow velocity is completely determined at these locations. The redundant inboard-toroidal measurements have been recently used to demonstrate that, in low-density TJ-II plasmas (with line-averaged electron densities $n_e \leq 10^{19}$ m$^{-3}$), impurity rotation is incompressible and follows the NC theory \[4\]. The general form of the impurity flows and the methodology used to assess their incompressibility is described next.

### 2.1. Spatial variation of the flow

From the radial force balance for a single species $s$ it follows that the perpendicular velocity is given by the $\mathbf{E} \times \mathbf{B}$ and diamagnetic flows \[19\],

$$u_{s\perp} = \frac{B}{B^2} \times \left( \nabla \Phi + \frac{1}{n_s q_s} \nabla p_s \right) + O(\delta_s^2 v_s),$$

(1)

where $n_s, q_s = eZ_s, p_s = n_s T_s$, and $v_s = \sqrt{2T_s/m_s}$ are the density, charge, pressure and thermal velocity of the species $s$, respectively, and $\delta_s = \rho_s/L$ is the gyro-radius normalized by the system characteristic length. In the following the subscript $i$ is used for main ions and $z$ for impurities with charge $Z$. The impurity diamagnetic term in equation (1) is usually neglected against the $\mathbf{E} \times \mathbf{B}$ flow, that is generally comparable to the main-ion diamagnetic term. Hereafter, this approximation is adopted for the C6+ CXRS measurements. The perpendicular impurity flow is thus

$$u_{s\perp} = E_s \frac{B}{B^2} \times \nabla \rho + E_{\phi} \frac{d}\rho = \frac{d\Phi}{d\rho},$$

(2)

with $\Phi$ the electric potential. Note that any parallel variation of $\Phi$ is neglected against the radial variation. For main ions the perpendicular diamagnetic flow is included in $E_i(\rho) = d\Phi/d\rho + (n_i e^{-1}) d\rho_i/d\rho$.

Given the form of the perpendicular $s$ flow, the parallel component is obtained from the (steady-state) $s$ number conservation, $\nabla \cdot (n_s u_s) = 0$. If density of $s$ is constant on flux surfaces then $\nabla \cdot u_s = 0$ and a local parallel flow (PS) must compensate for the compression of the perpendicular flows. The general expression of the parallel flow is then (see e.g. \[4, 20\])

$$u_{i\parallel} = (E_i(\rho) h + \Lambda_i(\rho)) B,$$

(3)

with the function $h(\rho, \theta, \phi)$ ($\theta, \phi$: poloidal and toroidal angles) satisfying

$$B \cdot \nabla h = \frac{2}{B^2} B \times \nabla \rho \cdot \nabla \left( \ln B \right).$$

The integration constant for $h$ is fixed by the condition $(hB^2) = 0$. With this choice the flux-constant $\Lambda_i(\rho)$ in (3) is given by $\Lambda_i = (u_{i\parallel} \cdot B)/(B^2)$, where $\langle \cdot \rangle$ denotes a flux-surface average. The first term on the right of equation (3), $u_{i\parallel}^\text{PS} = E_i h B$, is the well-known PS flow. In the following, $\Lambda_i B$ is referred as the parallel mass flow of the $s$ species, i.e. the parallel flow without the PS contribution.

In a previous work \[4\], the above expressions for an incompressible impurity flow pattern were shown to agree well with C6+ CXRS measurements made in low-density plasmas. The so-obtained $E_i(\rho)$ and $\Lambda_i(\rho)$ were found to be in agreement with NC calculations of the ambipolar radial electric field and main-ion parallel mass flow, $\langle u_{i\parallel} \cdot B \rangle/(B^2)$, respectively. For the higher density plasmas under study in this work a systematic deviation of the parallel flows from the form given by equation (3) is observed. The deviation is interpreted to be caused by variations of impurity density $n_z$ within the flux surfaces, in which case the reduction of the number conservation condition to the incompressibility of total impurity flows no longer holds. This situation is treated by allowing the function $\Lambda_i$ to have angular dependencies. To quantify the deviations it is convenient to define an impurity parallel return flow as $\Lambda(\rho, \theta, \phi) = \Lambda_i(\rho, \theta, \phi) - \Lambda_i(\rho)$, which is associated with parallel gradients of the impurity density. With this particular choice, impurity flows are written as the sum of an incompressible flow,

$$u_{i\parallel} = E_i \frac{B \times \nabla \rho}{B^2} + (\Lambda_i + E_i h) B,$$

(4)

plus the return flow, $\Lambda B$, which compensates for the impurity density redistribution, i.e.

$$u_{i\parallel} = u_{i\parallel}^\text{PS} + \Lambda(\rho, \theta, \phi) B.$$

(5)

Note that this velocity field is the same as that given by equations (2) and (3), but with $\Lambda_i(\rho, \theta, \phi) = \Lambda_i(\rho, \theta, \phi) + \Lambda_i(\rho)$. The case $\Lambda = 0$ reduces to an incompressible flow pattern with the $z$ impurities dragged by the ion parallel flow, i.e. $\Lambda_i = \Lambda_i(\rho)$. 

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**Figure 1.** Schematic diagram of CXRS diagnostic sightlines with a poloidal cut of several magnetic surfaces of TJ-II. The inboard and outboard regions of measurement are highlighted. The magnetic surfaces in which poloidal and toroidal outboard measurements are coincident, $\rho \sim 0.2, 0.4$ and $0.6$, are coloured in red, blue and black, respectively.
2.2. Data analysis

The data analysis employed here, which is an adaptation of that of [4], accounts for the possible compressible variations of the parallel impurity flow discussed previously. The method makes use of the three independent flow measurements performed at the same flux surface (two of them at the same point on a surface, refer to figure 1) to obtain independent measurements of the impurity parallel mass flow \( \Lambda_z \) at two locations of the same flux surface. The logic can be summarized as follows: first, the intersecting poloidal and toroidal outboard sightlines are used to extract the local parallel and perpendicular flows. The perpendicular flow provides a direct estimate of \( E_z(\rho) \), which is then used to subtract the PS component from the parallel velocity to get a measurement of \( \Lambda_z \) in the outboard region, i.e. \( \Lambda_z^{\text{out}} \). Next, the obtained value of \( E_z(\rho) \) is used to calculate the projections of the perpendicular and parallel PS flows on the inboard-toroidal measurement at the same flux surface. After subtraction of these projections a second value of the impurity parallel mass flow, \( \Lambda_z^{\text{in}} \), is obtained in the inboard region. To make the above description more explicit a dimensionless vector \( f \) is defined as

\[
f = -\frac{\langle B \rangle}{|\nabla \rho|} \left( \frac{B \times \nabla \rho}{B^2} + \hbar B \right),
\]

and the average radial electric field and perpendicular velocity as \( E_z(\rho) \equiv -\langle |\nabla \rho| \rangle \Phi / d\rho \) and \( U_z(\rho) \equiv E_z / (\hbar B) \), respectively. The impurity flow given by equation (5) is then

\[
u_z = fU_z + \Lambda_z \beta_z B.
\]

The perpendicular flow constant \( U_z(\rho) \) is calculated from the intersecting poloidal and toroidal sightlines at three radial positions in the outboard region \( \rho \sim 0.2, 0.4 \) and 0.6, see figure 1. These values of \( U_z \) are used together with the condition \( U_z(0) = 0 \) to interpolate \( U_z \) at the radial locations of all CXRS toroidal measurement with \( |\rho| \lesssim 0.6 \). Next, the quantity \( \Lambda_z(\rho, \theta, \phi) \) is obtained from toroidal velocity measurements as

\[
\Lambda_z = \frac{U_z - fU_z}{\beta_z}.
\]

Here, the sub-index \( t \) indicates the projection of a vector \( (u_z, f \) and \( B) \) in the toroidal viewing direction, \( e_z \). Figure 3 shows several examples of these profiles, that are discussed in the next section. Finally, the differences in the parallel mass flow (divided by the local magnetic field strength) are

\[
\Delta \Lambda_z = \Delta \left( \frac{U_z}{\beta_z} \right) - U_z \Delta \left( \frac{\hbar}{\beta_z} \right),
\]

where \( \Delta(X) \equiv X^{\text{in}} - X^{\text{out}} \). Note that the differences in the impurity return flow defmed in equation (5), i.e. \( \Delta \Lambda_z = \Delta \Lambda \). Therefore, if flows are incompressible, \( \Lambda_z \) is a flux function proportional to the \( U_z(\rho) \) defined in [4]. The in–out differences in \( \Lambda_z \) (see figure 3 of section 3) can then be compared with the symmetry in \( U_z \) profiles found in low-density density plasmas (figures 7 and 10 of that reference).

In this work, two close magnetic configurations are considered: 100,\( ^{44} \)\_64 and 100,\( ^{40} \)\_63. Here, the nomenclature reflects currents in the central, helical and vertical coils, respectively. For both configurations, on-axis magnetic field is about 1 T. The vacuum rotational transform, \( \tau \), covers the range \( 1.55 \lesssim \tau \lesssim 1.65 \) and \( 1.509 \lesssim \tau \lesssim 1.608 \), and the volumes are \( 1.098 \text{ m}^3 \) and \( 1.043 \text{ m}^3 \), respectively. These two configurations have been studied in [21, 22] from the NC point of view. For similar plasma profiles and momentum input, no qualitative differences are predicted in the flows within the surface. The plasmas presented here are heated by one of the two tangential NBIs (\( NBI \leq 100 \text{ ms} \)), either in the direction of the magnetic field (co-injection), or in the opposite direction (counter-injection). The line-averaged densities scanned in this paper cover the range \( \bar{n_e} \in (1.2-2.4) \times 10^{19} \text{ m}^{-3} \).

Time traces of a representative plasma discharge, #32577, in the 100,\( ^{44} \)\_64 configuration are shown in figure 2. The evolution of the reference discharge (#32576) used to remove background \( C^{6+} \) passive contribution. From top to bottom, time traces of: line-averaged electron density, \( \bar{n_e} \); plasma current (reversed), \( -I_p \); radiation monitors: bolometer (solid line) and \( C^{6+} \) (dashed); \( H_e \) signal. The DNBI injection is shown in grey.

3. Experimental results

Figure 2. Time evolution of two similar plasma discharges. In red, shot#32577 with a DNBI pulse and in blue, shot#32576 without DNBI, used to remove the \( C^{6+} \) passive contribution. From top to bottom, time traces of: line-averaged electron density, \( \bar{n_e} \); plasma current (reversed), \( -I_p \); radiation monitors: bolometer (solid line) and \( C^{6+} \) (dashed); \( H_e \) signal. The DNBI injection is shown in grey.
flows were shown to be incompressible in [4], and is included here as a reference. The measured $\Lambda_{\perp}$ profile departs from the incompressible expectation in the co-NBI discharge #32306 and the counter-NBI discharges #31100, #32577 and #32580. The reproducibility of the $\Lambda_{\perp}$ profile for the counter-NBI discharges, that are similar in terms of $n_c$, $T_e$ and $T_i$ profiles, but otherwise distant in time and impurity content, reinforces the reproducibility of the observed flow deviations. The general tendency observed in the experimental database, with few exceptions, is that the inboard parallel flow is more positive than the outboard one, and thus, the in–out differences in the parallel mass flow, $\Delta \Lambda_{\parallel}$ from equation (9), are always positive. This observation is nearly independent on the magnetic configuration and the direction of injection of the heating NBIs.

As indicated in section 2, toroidal and poloidal view lines of the CXRS system overlap at three locations ($\rho \approx 0.2, 0.4$ and 0.6) on the outboard side of the DNBI path. This enables unambiguous determination of the perpendicular and parallel flow components at those locations, relying only on the assumption of a small radial flow component compared to perpendicular and parallel flows. The perpendicular impurity flow component is expected to be dominated by the $E \times B$ flow because of the $1/Z$ factor of the diamagnetic flow. Figure 4 shows the comparison of this experimental approximation to the radial electric field with the NC expectations, calculated as in [22]. The database shown here is comprised of 12 discharges. Data taken in the 100,44,64 and 100,40,63 magnetic configurations are presented as circles and squares, respectively. Note that the impurity diamagnetic term is not included in computing the measured $E_r$ (see text).

### Figure 3

Left: profiles of the electron density ($n_e$, in gray) and temperature ($T_e$, in red), together with carbon temperature profiles ($T_i$, in blue). Right: measured profile of $\Lambda_{\perp}$, in blue, and the incompressible expected values extrapolated from the outboard measurements, in grey. The calculated PS contribution is displayed in red. The discharge #28263, in which flows were demonstrated to be incompressible [4], is included here as a reference. The discharges #31100, #32577 and #32580, heated with one NBI in counter-injection (consistent with $\Lambda_{\perp} < 0$) are presented to highlight the reproducibility of the discharge from incompressibility observed in $\Lambda_{\perp}$. Discharges #31100 and #32306 were performed in the configuration 100,40,63.

### Figure 4

Comparison of the experimentally measured radial electric fields for the outboard region, $E_{\text{out}}$, with the corresponding NC values for several TJ-II discharges. Dashed lines correspond to the NC value (diagonal) and the region of confidence $E_{\text{out}} = E_{\text{NC}} \pm 1 \, \text{kV m}^{-1}$ (upper and lower diagonals), respectively. Here, circles and squares represent data from the 100,44,64 and 100,40,63 magnetic configurations, respectively. Red points indicate NBI in counter-$B$ direction (consistent with $\Lambda_{\perp} < 0$) while open points indicate co-injection. Note that the impurity diamagnetic term is not included in computing the measured $E_r$ (see text).
signals while ignoring the calibration deficiencies mentioned in section 2) typically results in absolute values \( \leq 1 \text{ kV m}^{-1} \) at \( \rho = 0.6 \), with little or no impact for more internal regions, as expected from the \( 1/Z \) dependence. This estimation is consistent with the main-ion diamagnetic velocities calculated from experimental data; \( |v_{\text{diam},i}| \leq 4 \text{ km s}^{-1} \) for \( |\rho| \leq 0.6 \).

Despite this uncertainty in the estimated radial electric field, we note that the radial electric field does not enter any of the expressions for the impurity flows alone (sections 2.1 and 4), but rather in combination with the diamagnetic component as the total perpendicular flow. Such a velocity component is provided by the overlapping CXRS velocity measurements through geometric factors only and is not subjected to such uncertainties. In the following, the measured parallel mass flow deviations in the region \( |\rho| \in (0.2, 0.6) \) are studied in light of an impurity density redistribution model.

4. Friction-driven impurity density redistribution

Impurity temperature and parallel mass flow are generally taken to be a proxy of those of the main ions, as the impurity fluid is typically strongly collisionally coupled to the main-ion fluid. For similar temperatures (\( T_i \approx T_f \)) and large mass difference \( (m_z > m_i) \), impurity \( z \) and ion \( i \) collisionalities relate to each other through \( v_{z,i} = (m_i/m_z)^{1/2} (q_i/q_z)^{1/3} v_{\text{th},i} \) with \( v_{\text{th},i} = v_{\text{th}} R/(v_{\text{th}}) \) (see e.g. [24]). The pre-factor is \( \sim 10 \) for a hydrogen plasma and \( C_i^z \) impurity which, for the plateau ions characteristic of TJ-II [22], places the impurity under study in the PS collisional regime. This collisional character of medium to high-\( Z \) impurities can cause their density variations within a surface to be comparable with the mean value on that surface [6], and thus, the impurity parallel mass flow to differ from that of the main ions. In order to study the measured parallel mass flow deviations from an incompressible pattern, the continuity equation

\[
B \cdot \nabla \left( \frac{n_z R_{z,l}}{B} \right) = -E_z B \times \nabla \cdot \nabla \left( \frac{n_z}{B^2} \right),
\]

and the impurity parallel force balance

\[
T_z \nabla n_z = R_{z,l},
\]

need to be solved for the unknown functions \( n_z \) and \( \Lambda_z \). Here, \( R_{z,l} \) is the parallel friction on the impurities. The inclusion of other forces in (11) (the impurity inertia and the parallel electric field) is described and evaluated in section 5, while the impurity parallel viscosity is neglected against the parallel impurity pressure gradient, \( \nabla \cdot p_z \) [6]. As also shown in [6], the strong ion-impurity energy equilibration keeps the impurity temperature close to the ion one and thus \( T_z \approx T_i(\rho) \).

In terms of the \( A \) function defined in (5) and \( n \equiv n_z/(n_i) \) these two equations are written as

\[
B \cdot \nabla (n \Lambda) = -u_{\perp 0} \cdot \nabla n,
\]

\[
A \cdot \nabla \cdot n = \gamma_i B^2 (A_{\rho} + B_{\theta} - \Lambda) \quad \text{(12b)},
\]

whose solubility condition is \( \langle A B^2 \rangle = B_i(\rho)(B^2) \), see the appendix. The compressible pattern (5) has been used to express the continuity equation (10) in its form (12a).

In addition, a flux-constant friction coefficient \( \gamma_i(\rho) \) and thermodynamic forces \( A_i(\rho) \) and \( B_i(\rho) \) have been defined in (12b) as

\[
\gamma_i \equiv \frac{m_i Z^2}{T_i \tau_{i,i}},
\]

\[
A_i \equiv \frac{T_i d \ln n_i}{e} \left( \frac{1}{2c} \frac{dT_i}{d\rho} \right),
\]

\[
B_i \equiv \frac{3}{5} \left( \frac{q_i}{B} \right)
\]

with \( \tau_{i,i} = 3 (2\pi)^{3/2} e_{0}^{-1/2} m_i^{1/2} T_i^{3/2} / (n_i e^4 \ln \Lambda) \) the ion self-collision time and \( q_i \) the ion heat flow. In deriving expression (12b) trace impurities are considered, \( \sum n_i Z^2 < n_i \), and so the parallel friction on the impurities is approximated by that exerted by main ions, i.e. \( R_{i,i} \approx R_{z,l} = -R_{z,i} \). The ion-impurity collision operator is modelled with a Lorentz pitch-angle scattering operator plus a term guaranteeing momentum conservation [6]. Finally, no assumption is made on bulk ion’s collisionality since its distribution function is expanded by Legendre and Laguerre polynomials, as is customary in the so-called moment approach to NC transport [20, 24]. Here, the so-called 13 M approximation [25] is adopted, i.e. contributions from \( j > 1 \) Laguerre components are neglected, see the appendix. We note that in the axisymmetric tokamak case, the impurity continuity equation (10) yields an algebraic relationship between the parallel impurity flow and the impurity density (see e.g. [26]), whereas such a simplification does not occur in general stellarator geometry [7].

The ion-impurity parallel friction is studied first in the next subsection. The effect of a parallel electric field and impurity inertial forces are considered in section 5. We anticipate here that the parallel momentum balance is dominated by the friction force and that the general behaviour of the solutions is to display \( \Delta \Lambda_z < 0 \), in contrast with the measured in-out variation. This can be heuristically understood by noting that the differences in the PS flow, \( A_i h B \) in equation (12b), drive the impurity density redistribution. As a consequence of the in-surface density variation, an impurity return flow \( \Lambda B \) is established (equation (12a)) which must act to reduce the overall impurity friction so that the density redistribution is not further amplified. Since the term \( A_i h B \) on the rhs of equation (12b) is more negative at the inboard side, the return flow \( \Lambda \) tends to behave similarly.

4.1. Calculation of the friction-driven impurity redistribution

The two coupled equations (12a) and (12b) can be recast as a second order partial differential equation for the unknown function \( n(\rho, \theta, \phi) \) (see appendix). The radial coordinate is a parameter in those equations which involve angular derivatives only. The required inputs are the main-ion parameters (temperature \( T_i \), density \( n_i \), parallel mass flow \( \Lambda_i \) and flux-surface averaged parallel heat flow \( q_i \cdot B \)) together with the impurity perpendicular flow. The CXRS and Thomson scattering systems provide measurements of these parameters, except for the ion parallel mass and heat flows. The latter is calculated with DKES [27], complemented with momentum correction techniques [28]. On the other hand, the measured \( \Lambda_z \) in the outboard region is used as a first guess for the ion parallel mass flow to solve the differential equations, \( \Lambda^{(0)}_z = \Lambda^{\text{Out}}_z \), since the external input of momentum is not...
included in the DKES $\Lambda_i$ calculations [21]. A new guess for the main-ion parallel flow is then obtained upon subtraction of the calculated impurity-ion flow difference, $\Lambda_i^{(0)}$ in our notation, i.e. $\Lambda_i^{(0)} = \Lambda_i^{\text{SM}} - \Lambda_i^{(0)}$. Note that at every step momentum conservation is imposed, i.e. $\langle \Lambda B^2 \rangle = B_i(\rho)(B_i^0)$ from equation (12b). The iteration of this process leads to a solution for the impurity flow that matches the outboard CXRS measurement, $\Lambda_i^{\text{out}} = \Lambda_i^{\text{SM}} + \Lambda_i^{(0)} = \Lambda_i^{\text{SM}}$. In practice only one iteration is necessary because the impurity return flow $\Lambda$ is not very sensitive to the ion parallel flow $A_i$ and the outward measurement locations happen to be close to a stagnation point of the calculated impurity return flow, so that $\Lambda_i = \Lambda_i^{\text{SM}}$ is already a good guess.

An example of the solution is shown in figure 5 for discharge #32577 presented in figures 2 and 3 and for the $\rho = 0.6$ magnetic surface. These results correspond to fully ionized carbon C6$^+$ impurity that is used for the CXRS measurements. From top to bottom the relative variations of magnetic field strength $B$ and impurity density, the corresponding impurity return flow and the parallel friction drive $A_i h B$ are plotted. The inboard/outboard toroidal measurement positions are shown as an open circle and solid square, respectively. As a reference the same quantities obtained for the low-density ECRH heated discharge #25801 are shown. The slightly hollow density profiles in typical ECRH discharges in TJ-II results in a small and negative thermodynamic force $A_i$, see equation (13a). Correspondingly, both the relative impurity density variations and return flow are small. Plasma profiles and CXRS flow measurements for this discharge can be found in [4]. In particular we recall that the measured impurity flows were shown to be nearly incompressible for this discharge.

The results of the calculations of impurity density redistribution for discharge #32577 and for several magnetic surfaces, $\rho \leq 0.8$, are plotted in figure 6 for the toroidal section of the CXRS measurements, $\phi = 75.5^\circ$. Again, the inboard and outboard toroidal measurement positions are shown as open circles and filled squares, respectively. The magnetic surfaces in which the inboard/outboard comparison is made, namely $\rho \sim 0.2, 0.4$ and 0.6, are also shown as dashed lines. The first two graphs of the figure are the normalized impurity density redistribution, $\Delta n = n_i(z)/\langle n_i \rangle - 1$, and impurity return flow, $A_i h B$. The PS impurity flow, $u_{PS} \sim E_i h B$, is presented at the bottom of the figure.

Some general comments on the solution can be made in light of the simulation results shown in figures 5 and 6. For the plasma profiles of the database used in this work, $\bar{n}_e \in (1.2–2.4) \times 10^{19}$ m$^{-3}$, carbon impurities tend to accumulate in the interior region of the bean-shaped plasma poloidal cross section (which is close the region of maximum magnetic field strength in TJ-II due to the proximity of the central coil). The resulting return flow is comparable in size to the PS impurity flow. Its angular dependence also shows a dominant $\cos \theta$ component. The difference in sign between the PS and return parallel flows is in line with the overall tendency heuristically described at the beginning of this section: the return flow $\Delta \lambda$ tends to compensate the $A_i h B$ friction drive in equation (12b), for in these ion-root plasmas $A_i$ and $d\Phi/d\rho$ are of similar magnitude and different signs so that $u_{PS} \approx (d\Phi/d\rho) h B \sim -A_i h B$. Consequently, the differences in the calculated impurity parallel return flow at the locations of the CXRS measurements, $\Delta \lambda_z \equiv \Delta \lambda$ from (5), are found to be negative for the ion-root plasma discharges in our CXRS database.
4.2. Comparison with experiment

Figure 7 shows the radial profiles of the friction-driven simulated (black) and experimental (blue) differences in the impurity parallel mass flow, $\Delta \Lambda_z$, for the discharges presented in figure 3. The calculated compressible modifications to the impurity flow become small at the CXRS measurement locations and do not account for the observed differences. A comparison of the experimental and theoretical values of $\Delta \Lambda_z$ is presented in figure 8 for the same database as in figure 4. Error bars in figures 7 and 8 come from the spread of the calculated velocities in the measurement volumes. As discussed in section 4.1, the parallel friction term in equation (12b) calculated from experimental profiles appears capable of producing a measurable impurity density asymmetry and parallel return flow, even for the internal positions considered in this work (the region of maximum gradient is typically located at $\rho \sim 0.7$–0.8 in TJ-II plasmas). Values of $\Delta \Lambda_z^{\text{tho}} \sim -2$ km s$^{-1}$ T$^{-1}$, or larger, are found in the simulation, while the experimental differences can easily reach $\Delta \Lambda_z^{\text{exp}} \sim 6$ km s$^{-1}$ T$^{-1}$. The overall tendency of the calculated return flows to be more negative in the inboard side is also clear from figure 8. Note that at the inboard positions the impurity return flow varies sharply, see figure 5, which translates into large error bars.

From this comparison it is concluded that, whereas the impurity-ion parallel friction (in its model form in equation (12b)) is capable of causing impurity density asymmetries and return flows of the order of magnitude of the observed in–out flow differences, the calculated return flows do not agree with the observed in-surface variation of the impurity parallel mass flow $\Lambda_z$, at the locations of the CXRS measurements for most cases. In the following section some of the assumptions made in the model (12b) are examined, and the parallel force balance (11) is extended to account for the impurity inertia and the effect of a parallel electric field.

5. Discussion on the validity and extensions of the model

Previous impurity parallel friction models for stellarators [7] consider main ions in the PS regime. This regime is not strictly applicable to the plasmas presented here ($n_i \in (0.5–3) \times 10^{19}$ m$^{-3}$, $T_i \in 100–200$ eV) since main ions are in the plateau...
regime $\tilde{v}_{ii} \sim 10^{-1}$–$10^{0}$ [22]. As indicated in section 4 and explained in the appendix, no assumption is made in this work on bulk ion collisionality, although the main-ion distribution function is truncated in the Laguerre expansion ($j \leq 1$, see the appendix) as in the 13 M approximation [24, 25].

In order to quantify the impact of this approximation let us consider main ions in the PS regime, as in [7]. In this regime of collisionality ($q_i B_i = 0$ (hence $B_i = 0$ in (13c)) and $A_{i,PS} = \left( T_e / e \right) \times (d \ln n_i / d \rho)$. Now, if the $j > 1$ truncation is applied to the exact collisional result, the thermodynamic force $A_i$ in (12b) results

$$A_{i,PS, \text{truncated}} = A_{i,PS} - \frac{1}{2e} \frac{d T_e}{d \rho},$$

which equals the general result $A_i$ in (13c). The resultant impurity redistribution and return flow as obtained from the exact and truncated collisional results are displayed in the left and right columns of figure 9, respectively. As observed, the impurity density in-surface variation reaches values up to $\Delta n \sim \pm 20\%$ in the exact collisional result while $\Delta n \sim \pm 13\%$ is found when truncating the main-ion distribution function. The simulated return velocity, $\Delta B$, is similarly affected by the truncation (values of $\pm 6$ and $\pm 4 \text{ km s}^{-1}$ are obtained in the exact and truncated friction models, respectively). The comparison in figure 9, and the proximity of TJ-II main-ion collisionalities to the PS regime, indicate that the inclusion of higher order Legendre components [25] in the modelled friction (12b) is unlikely to change the tendencies in the simulated impurity redistribution and return flow presented in section 4.

On the other hand, the generalization of the parallel friction presented in (12b) allows us to directly use the measured main-ion parameters (since no assumption is made on collisionality) and to include the effect of a non-zero parallel heat flow, thus extending previous friction models in stellarators [7].

Besides the above discussion on the generalization of the ion-impurity parallel friction, the impurity parallel force balance (11) can be extended to account for inertial and electrostatic parallel forces as

$$m_i n_i b \cdot (u_z \cdot \nabla u_z) + n_i Z_e e \nabla \Phi + T_z \nabla n_z = R_{iz},$$

where the first term is the impurity parallel inertia and the second one is the parallel electric field. The former can be approximated by

$$m_i n_i b \cdot (u_z \cdot \nabla u_z) \approx m_i n_i A_i^2 B \cdot \nabla B,$$

since the local PS and return parallel flows are expected to be smaller than the main-ion parallel mass flows in internal regions $|\rho| \leq 0.4$ of TJ-II NBI-heated plasmas. This same approximation leads to the centrifugal outboard accumulation of high-Z impurities in tokamaks [29]. In order to examine the impact of the inertia on the impurity density redistribution, let us consider parallel mass flows $\sim 20 \text{ km s}^{-1}$ are comparable to thermal velocities $\leq 50 \text{ km s}^{-1}$ (i.e. $\gamma_i \leq 0.4$), the large aspect ratio of TJ-II ($b^2 - 1 \sim a/R \sim 0.1$ makes $\Delta n \approx 2\%$ for all the plasma minor radius (here, $a \sim 0.2 \text{ m}$ and $R = 1.5 \text{ m}$ are the minor and major plasma radius). Such an estimation has been confirmed numerically. Therefore, the impurity inertia is neglected henceforth.

On the other hand, the term containing the electrostatic potential variation in the flux surface, $\Phi = \Phi - \langle \Phi \rangle$, in equation (14) is considered. This portion of the full electrostatic potential, $\Phi$, results from imposing quasi-neutrality among the non-equilibrium density pieces of the coexistent species. Furthermore the calculation, carried out with the particle in cell code EUTERPE [30], considers adiabatic electron response and trace impurities. Under these approximations the resulting map of $\Phi$ mirrors that of the main-ion density. As an example, $\Phi$ is shown in figure 10 for the discharge #32577.

The parallel momentum balance in its form (15), together with particle conservation (12a), can be transformed into a second order partial differential equation for the unknown $n$, as in section (4.1). Figure 11 displays a mapping in the CXRS poloidal plane of measurement of the simulation results for discharge #32577 and for several magnetic surfaces, $\rho \leq 0.8$, after considering (left) only friction and (right) friction plus the $\nabla \Phi$ forces in its model form (15). As observed, the impurity redistribution and return flow patterns are affected by the inhomogeneity of the potential only at external radial locations $\rho > 0.7$. Nevertheless, the tendency
to display $\Delta A < 0$ remains unaltered, thus contradicting the experimental observations.

Finally, a possibly important omission of the impurity redistribution model used in this work could be the assumption of trace impurities. For the plasmas considered here values of $Z_{\text{eff}} \sim 1.2–1.6$ are obtained from soft x-ray emission, which would give rise to impurity strengths of $n_i Z^2/n_i \sim 0.2–0.6$. In such a case both the inhomogeneity of the electrostatic potential [26] and the collision operator used to model the parallel friction on the impurities [14] would change, thus modifying the impurity redistribution within a magnetic surface. The inclusion of these effects is out of the scope of this paper and is left to future work.

6. Conclusions

In this work fully ionized carbon impurity flows in ion-root, NBI heated, TJ-II plasmas are studied by means of charge exchange recombination spectroscopy. Perpendicular flows are found to be in reasonable agreement with neoclassical calculations of the radial electric field. The parallel flow of the impurity is obtained at two locations on the same flux surface and the calculated Pfirsch–Schlüter parallel velocity is subtracted. The remaining component of the flow is systematically observed to vary on each flux surface, pointing to a breakdown of impurity flow incompressibility in the medium density plasmas studied. The experimentally observed velocity deviations are compared with the parallel return flow calculated from a modelled impurity density redistribution driven by ion-impurity friction. Such a model is extended to account for impurity inertia and inhomogeneities in the electrostatic potential. The simulation results show that the parallel impurity force balance is dominated by parallel friction for the plasmas considered here, and demonstrate that the calculated return flow substantially modifies the incompressible velocity pattern. However, these modifications become small at the CXRS measurement locations and do not explain the in-surface variations of impurity parallel flow. The experimental validation of theoretical models of impurity density redistribution within a flux surface is of considerable importance as it provides an indirect validation of the model predictions for impurity radial transport.

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Appendix. Impurity density redistribution in a stellarator

In this appendix the parallel momentum balance (15) is derived and the method used to solved this equation consistently with particle number conservation is detailed. The impurity parallel momentum equation is taken to be

$$T_i \nabla n_i + n_i Z \nu \nabla \Phi = R_{\parallel},$$  \hspace{1cm} (A1)

where $R_{\parallel}$ is the parallel friction on the impurities and $\Phi$ the electrostatic potential. As demonstrated in [6] the impurity temperature is equilibrated with the bulk ion temperature and is therefore constant on the flux surface. As it is also shown in [6], impurity parallel inertia and viscosity can be neglected in (A1) if $\delta_i/(Z \nu_i) \ll 1$. For the plasmas considered in this work ($n_i \in (0.5–5) \times 10^{19} \text{m}^{-3}$, $T_i \in 100–200 \text{eV}$) typical values of the normalized ion gyro-radius are $\delta_i \sim (5–10) \times 10^{-3}$. Bulk ions in NBI-heated TJ-II plasmas are in the plateau regime, $\nu_i \sim 10^{-3}–10^{0}$, although close to the PS regime of collisionality [22]. Then, for fully ionized carbon...
impurity ions \((Z = 6), \delta i/ (Zv_{||}) \sim 10^{-3} - 10^{-2}\). Hence, the assumptions made in [6] to derive equation (A1) are applicable in this work. Furthermore, the expected variations of \(C_{\theta}\) density within the surface are \(\delta_i/ (n_{i}) \sim \delta i, Z^2 \sim 0.1 - 0.5\), thus justifying the present study.

In the trace impurity limit, \(\sum n_{Z} < n_{i}\), the parallel friction on the impurities may be approximated by [26]

\[
R_{i\parallel} \approx R_{i\perp} = - \frac{v_{i}}{m_{i}} C_{i\perp} \{ f_{i1} \},
\]  
(A2)

with \(f_{i1}\) the first order derivative of the bulk ion distribution function from a Maxwellian. The ion-impurity collision operator consists of a Lorentz operator plus a term guaranteeing momentum conservation [24]

\[
C_{i\perp} \{ f_{i1} \} = v_{i} \mathcal{L} \{ f_{i1} \} + \frac{m_{i} v_{i} u_{\parallel i} \eta_{\parallel i}}{T_{i}} f_{i0}.
\]  
(A3)

\[
\mathcal{L} = \frac{1}{2} \frac{\partial}{\partial \xi} \left( 1 - \xi^2 \right) \frac{\partial}{\partial \eta_{\parallel i}}.
\]  
(A4)

Here, \(v_{i} = 3 \pi^{1/2}/(4 \tau_{i} x_{i}^{3})\), \(\tau_{i} = \tau_{i}/(n_{i} Z^2)\) is the ion-impurity collision time, \(x_{i} = v_{i}/v_{i} = \sqrt{2 T_{i}/m_{i}}\) the ion thermal speed, \(\xi = v_{i}/v\) the pitch-angle and \(f_{i0} = n_{i}/(x_{i}^{3/2} \pi^{1/2}) \exp(-x_{i}^{2})\) is a flux-function Maxwellian. Since the collision operator is self-adjoint and \(\mathcal{L} [v_{i}] = -v_{i}\), the term in the parallel friction force arising from the Lorenz operator is written as

\[
- \int d^{3}v_{i} v_{i} \mathcal{L} \{ f_{i1} \} = \frac{3 \pi^{1/2}}{4 \tau_{i}} m_{i} v_{i} \int d^{3}x \frac{x_{i}^{2}}{x_{i}^{2}} f_{i1}.
\]  
(A5)

Let us consider now the expansion of \( f_{i1}(x, v, \hat{\xi}, \xi) \) in Legendre polynomials \( P_{l}(\hat{\xi}) \) [\(P_{1} = 1, P_{2} = \xi, \xi\)] [20].

Thanks to the orthogonality properties of the \(P_{l}\) polynomials only the \(l = 1\) component of \(f_{i1}\) contributes to equation (A5). Such component is associated with the parallel particle and heat flows \(u_{\parallel i}\) and \(q_{i\parallel}\) respectively and is expanded by Laguerre (Sonine) polynomials \(L_{j}^{(3)}(x_{i}^{2})\) [\(L_{0}^{(3)} = 1, L_{1}^{(3)} = -x_{i}^{2} + 5/2, L_{2}^{(3)} = x_{i}^{2} - 2 - 7 x_{i}^{2} + 15/8\), etc] as \([20, 24]\)

\[
\begin{align*}
\hat{f}_{i1}^{(1)} & = \frac{2}{v_{i}} \frac{x_{i}^{2}}{\xi} \left[ u_{\parallel i} - \frac{L_{2}^{(3)}}{f_{i1}^{(1,1), j} \eta_{\parallel i}} \right] f_{i0} + \hat{f}_{i1}^{(1,1), j} \eta_{\parallel i}
\end{align*}
\]  
(A6)

Here, \(\hat{f}_{i1}^{(1,1), j} \eta_{\parallel i}\) denotes the sum of the \(j\)th Laguerre polynomial components with \(j \geq 2\). The inclusion of \(j > 1\) terms \([25]\) is out of the scope of this paper and thus \(\hat{f}_{i1}^{(1,1), j} \eta_{\parallel i}\) is taken in equation (A6), as it is customary in the moments approach to NC transport \([24]\) (the so-called 13 M approximation). See the comments in section 5 regarding the effect of this truncation. With this assumption the parallel friction on the impurities reads

\[
R_{i\parallel} \approx \frac{m_{i} n_{i}}{\tau_{i}} \left[ u_{\parallel i} - \frac{3 q_{i\parallel}}{5 p_{i}} \right],
\]  
(A7)

with \(u_{\parallel i}\) the impurity-parallel flow.

\[\text{Note: if the exact result for main-ion distribution function in the PS regime is used} \] [7]

\[
f_{i1}^{dnl} = \frac{2}{v_{i}} \left[ u_{\parallel i} - \frac{L_{2}^{(3)}}{f_{i1}^{(1,1), j} \eta_{\parallel i}} \right] f_{i0} + \hat{f}_{i1}^{(1,1), j} \eta_{\parallel i}
\]

the pre-factor \(-3/5\) accompanying the parallel heat flow in (A7) must be replaced by \(-2/5\).

For each particle species, \(\nabla \cdot \mathbf{q}_{\parallel} \approx 0\). Then the bulk ion parallel heat flow is

\[
q_{i\parallel} = \frac{5 p_{i}}{2e} \frac{\partial T_{i}}{\partial p_{\parallel}} h B + \mathbf{q}_{i\parallel} \cdot \mathbf{B} \approx B/\langle B^{2} \rangle.
\]  
(A8)

with the function \(h\) defined in section 2.1. Using the general expression for a compressible impurity heat flow, equation (5), and equation (A8), the parallel friction on the impurities is finally recast as

\[
R_{i\parallel} = \frac{m_{i} n_{i0}}{\tau_{i}} B \left[ \left(E_{i} - E_{c} - \frac{3 p_{i}}{2e} \frac{\partial T_{i}}{\partial p_{\parallel}} \right) h - \Lambda - \frac{3}{5} \left( \mathbf{q}_{i\parallel} \cdot \mathbf{B} \right) \right]
\approx p_{i} \gamma_{f} B (A_{i} h + B_{i} - \Lambda),
\]  
(A9)

with the flux constants \(\gamma_{f}(\rho), A_{i}(\rho)\) and \(B_{i}(\rho)\) given by equations (13). In the last step, the impurity diamagnetic term has been neglected against the main-ion one. With these assumptions (i.e. trace impurities, \(\sum n_{Z} < n_{i}\) and 13 M approximation, \(f_{i1}^{(1,1), j} \eta_{\parallel i}\)) the impurity parallel momentum balance (A1) results

\[
\mathbf{B} \cdot \nabla n_{i} = \gamma_{f} n_{i} B^{2} (A_{i} h + B_{i} - \Lambda) - n_{i} \frac{e Z}{T_{e}} \mathbf{B} \cdot \nabla \Phi,
\]  
(A10)

hence recovering equation (14) and the simplified form (12b), when the inhomogeneity of the potential, \(\Phi = \Phi - \Phi_{i}\), is neglected. In stellarator geometry, equation (A10) and the continuity equation (10) form a coupled system \([7]\) of partial differential equations (PDEs). This set of equations can be expressed as a parabolic PDE in the variable \(n = n_{i}/(n_{Z})\)

\[
\mathbf{B} \cdot \nabla (\mathbf{B} \cdot \nabla n) - \mathbf{g} \cdot \nabla \mathbf{n} - \mathbf{g} B^{2} u_{\parallel i} \cdot \nabla n = 0.
\]  
(A11)

where

\[
g(\rho, \theta, \phi) = \mathbf{B} \cdot \nabla \ln B_{\parallel} = \frac{e Z}{T_{e}} \mathbf{B} \cdot \nabla \Phi
\]  
(A12)

\[
f(\rho, \theta, \phi) = \gamma_{f} A_{i} \mathbf{B} \cdot \nabla \rho \cdot \nabla \ln B_{\parallel} + \epsilon Z \mathbf{B} \cdot \nabla \Phi.
\]  
(A13)

Equation (A11) is converted to an algebraic system of equations by applying finite differences to the variable \(n\). The angular periodicity of the TJ-II \((T_{\phi} = 2\pi, T_{\theta} = \pi/2)\) and the condition \((n) = 1\) are imposed. The parallel return flow \(A_{\parallel}\) is obtained from equation (12a). The system of PDEs has also been solved by Fourier expanding the variables in Boozer coordinates, showing consistency with the finite differences scheme.

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