Scalar Cosmological Perturbations in M-theory with Higher Derivative Corrections

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Abstract

We investigate the inflationary expansion of the universe induced by higher curvature corrections in M-theory. The inflationary evolution of the geometry is discussed in ref. \cite{1}, thus we succeed to analyse metric perturbations around the background. Especially we focus on scalar perturbations and analyse linearized equations of motion for the scalar perturbations. By solving these equations explicitly, we evaluate the power spectrum of the curvature perturbation. Scalar spectrum index is estimated under some assumption, and we show that it becomes close to 1.
1 Introduction

Recent remarkable progress on astrophysical observations enables us to reveal the evolution of our universe\cite{2,3,4}. Particularly these results support the inflationary scenario, in which the universe exponentially expands before the Big Bang\cite{5,6,7,8}. There are a lot of models in which the inflation is caused by introducing a scalar field named inflaton field. The inflaton field slowly rolls down its potential and the vacuum geometry becomes de-Sitter like\cite{9}-\cite{13} (see also \cite{14}-\cite{18} and references there). Among a lot of inflationary models, superstring theory is a good candidate for the inflation scenario since it possesses many scalar fields after the compactification. Positions of D-branes are also described by scalar fields, and it is possible to identify one of these with the inflation field\cite{19}-\cite{24} (see also \cite{25} and references there).

Besides the inflaton field, there are a lot of models in which the inflation is realized by modifying the gravity theory\cite{5,26,27,28,29}. Especially, the predictions of the Starobinsky model\cite{5}, which contains curvature squared term in the action, are good agreement with the observations. In fact, it predicts scalar spectral index $n_s = 0.967$ and tensor to scalar ratio $r = 0.003$ when the number of e-folds is 60. Since the curvature squared term in the Starobinsky model is considered as the quantum effect of the gravity, it is natural to ask the origin of the quantum effect in more fundamental theory, such as the superstring theory or M-theory. Actually heterotic superstring theory contains Gauss-Bonnet term, and type II superstring theories or M-theory contain quartic terms of the Riemann tensor\cite{30}-\cite{35}. As examples, a study of the inflationary solutions in the heterotic superstring theory was done in ref. \cite{36}, and studies of the inflationary solutions in the M-theory were executed in refs. \cite{37,38,39,40,1}.

In ref. \cite{1}, we investigated the effect of leading curvature corrections in M-theory with respect to the homogeneous and isotropic geometry. There we found that such corrections induce exponentially rapid expansion at early universe. Furthermore the inflation naturally ends when the corrections are negligible compared to leading supergravity part. Since the higher derivative corrections are universal in the superstring theory or the M-theory, the above is a promising scenario to explain the origin of the inflation. Therefore it is important to evaluate scalar spectral index and tensor to scalar ratio in the presence of the higher curvature corrections. In this paper we propose a method to evaluate the scalar spectral index $n_s$ in the presence of higher derivative corrections. And we show that $n_s$ is close to 1, if the power spectrum is constant at the beginning of the universe.

The organization of this paper is as follows. In section 2 we briefly review the inflationary scenario discussed in ref. \cite{1}. In section 3 first we consider perturbations around the background metric, and examine their infinitesimal variations under the general coordinate
transformation. Next we derive linearized equations of motion for the perturbations. Then we concentrate on the scalar perturbations and reduce their equations of motion by removing auxiliary fields. In section 4, we derive the second order effective action with respect to the scalar perturbations. And we rederive the equations of motion for the scalar perturbations obtained in section 3. In section 5, we solve the equations of motion perturbatively and obtain explicit form of the curvature perturbation. Finally we evaluate the power spectrum of the curvature perturbation and calculate the scalar spectral index, which becomes close to 1. Supplementary equations are listed in appendix A.

2 Review of Inflationary Solution in M-theory

In this section, we briefly review the inflationary solution in M-theory\(^1\). The effective action for the M-theory consists of supergravity part and higher derivative corrections. Although the complete form of the higher derivative corrections is not known, we have control over leading curvature corrections. Thus we truncate the effective action up to the leading higher curvature terms, which is written as \[^{34, 35}\]

\[
S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11} x \, e(R + \Gamma Z),
\]

\[
Z \equiv 24(W_{abcd} W^{abcd} W_{efgh}^2 - 64W_{abcd} W^{ae} W^{bf} W^{cd} W^{eh}) + 2W_{abcd} W^{ae} W^{bf} W^{cd} W^{ef} + 16W_{abcd} W^{ae} W^{bf} W^{cd} W_{efgh}
\]

\[
- 16W_{abcd} W^{ae} W^{bf} W^{ef} W^{cd} - 64W_{abcd} W^{ae} W^{bf} W^{ef} W^{cd} - 16W_{abcd} W^{ae} W^{bf} W^{ef} W^{cd} - 16W_{abcd} W^{ae} W^{bf} W^{ef} W^{cd}
\]

where \(a, b, c, \cdots g, h\) are local Lorentz indices and \(W_{abcd}\) is Weyl tensor. There are two parameters, gravitational constant \(2\kappa_{11}^2\) and expansion coefficient \(\Gamma\), in the effective action. These are expressed in terms of 11 dimensional Planck length \(\ell_p\) as

\[
2\kappa_{11}^2 = (2\pi)^8 \ell_p^9, \quad \Gamma = \frac{\pi^2 \ell_p^6}{2^{22} 3^2}.
\]

By varying the effective action \(^{1}\), we obtain following equations of motion\(^{11}\).

\[
E_{ab} = R_{ab} - \frac{1}{2} \eta_{ab} R + \Gamma \left\{ - \frac{1}{2} \eta_{ab} Z + R_{cdea} Y^{cde} - 2D_{(c} D_{d)} Y^{cde} - 2D_{(c} D_{d)} Y^{cde} - \frac{1}{2} \eta_{ab} Z + R_{cdea} Y^{cde} \right\} = 0.
\]

Here \(D_a\) is a covariant derivative with respect to the local Lorentz index. The tensor \(Y_{abcd}\) in the above is defined as

\[
Y_{abcd} = X_{abcd} - \frac{1}{9} (\eta_{ac} X_{bd} - \eta_{bc} X_{ad} - \eta_{ad} X_{bc} + \eta_{bd} X_{ac}) + \frac{1}{90} (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}) X,
\]

\(^1\) Calculations in this paper are done by using Mathematica codes. See ref. \(^{42}\)
and the tensor $X_{abcd}$ is given by

$$X_{abcd} = \frac{1}{2} \left( X'_{[ab][cd]} + X'_{[cd][ab]} \right), \quad X_{ab} = X'_{acb}, \quad X = X^a_{\ a}, \quad (5)$$

$$X'_{abcd} = 96 \left( W_{abcd} W_{efgh} W_{efgh} - 16 W_{abce} W_{d fgh} W_{efgh} + 2 W_{abef} W_{cdgh} W_{efgh} \right. \right.$$

$$+ 16 W_{abce} W_{b fgh} W_{efgh} - 16 W_{abef} W_{c dgh} W_{efgh} - 16 W_{abef} W_{cdgh} W_{efgh} + 8 W_{abef} W_{cegh} W_{d fgh} \right)\right).$$

Note that $Y_{acb} = 0$.

Below we solve the equations of motion (3) up to the linear order of $\Gamma$. We assume that the 11 dimensional coordinates $X^\mu$ are divided into 4 dimensional spacetime $(t, x^i)$ and 7 internal directions $y^m$, where $i = 1, 2, 3$ and $m = 4, \cdots, 10$. The ansatz for the metric is given by

$$ds^2 = -dt^2 + a(t)^2 dx_i^2 + b(t)^2 dy_m^2, \quad (6)$$

$a(t)$ and $b(t)$ are scale factors of 3 dimensions and 7 internal ones, respectively. Now we define Hubble parameter $H(t) = \frac{a(t)}{a(t)}$ and similar one $G(t) = \frac{b(t)}{b(t)}$. Then the equation (3) is expressed in terms of $H(t)$ and $G(t)$, and the solution up to linear order of $\Gamma$ is given by

$$H(\tau) = \frac{H_1}{\tau} + \Gamma \frac{c_h H_1^7}{\tau^7}, \quad G(\tau) = -\frac{7 + \sqrt{21}}{14} \frac{H_1}{\tau} + \Gamma \frac{c_g H_1^7}{\tau^7}. \quad (7)$$

Here $\tau$ is dimensionless time coordinate given by

$$\tau = \frac{(-1 + \sqrt{21}) H_1 t + 2}{2} \quad (8)$$

and numerical coefficients $c_h$ and $c_g$ are expressed as

$$c_h = \frac{13824 \left( 477087 - 97732 \sqrt{21} \right)}{8575} \sim 47111, \quad (9)$$

$$c_g = \frac{-41472 \left( 532196 - 110451 \sqrt{21} \right)}{60025} \sim -17996.$$

It is easy to integrate the eq. (7), and log $a$ and log $b$ are solved as

$$\log a = \log a_E + \frac{1 + \sqrt{21}}{10} \log \tau - \frac{1 + \sqrt{21}}{60} c_h \Gamma H_1^6 \frac{1}{\tau^6},$$

$$\log b = \log b_E - \frac{3\sqrt{21} - 7}{70} \log \tau - \frac{1 + \sqrt{21}}{60} c_g \Gamma H_1^6 \frac{1}{\tau^6}. \quad (10)$$

From this we see that $a(\tau)$ is rapidly expanding and $b(\tau)$ is rapidly deflating during $1 \leq \tau \leq 2$. $a_E$ or $b_E$ are integral constants and correspond to scale factors just after the inflation or the deflation. After $\tau = 2$, the higher derivative corrections are suppressed and the scale factors behave like

$$a_0 = a_E \frac{1 + \sqrt{21}}{10} \tau, \quad b_0 = b_E \frac{3\sqrt{21} - 7}{70} \tau. \quad (11)$$
The behavior of $a_0$ is similar to radiation dominated era.

The motivation for the inflation is to resolve the horizon problem. This requires that the particle horizon $\int \frac{dt}{a(t)}$ during the inflationary era is almost equal to that after the radiation dominated era. The particle horizon during the inflationary era is given by

$$\sqrt{\frac{21}{10} + 1} \int_1^2 \frac{d\tau}{a(\tau)} = \sqrt{\frac{21}{10} + 1} \int_1^2 d\tau \tau^{-\frac{1+\sqrt{21}}{10}} e^{\frac{1+\sqrt{21}}{60} c_h \Gamma H_1^6 \tau}.$$  \hspace{1cm} (12)

On the other hand, if we simply apply the eq. (11) for the scale factor after $\tau = 2$, the particle horizon during this era is evaluated like

$$\sqrt{\frac{21}{10} + 1} a_0(\tau_0) = \sqrt{\frac{21}{10} + 1} a_0(\tau_0) \sim \sqrt{\frac{21}{10} + 1} a_0(\tau_0)$$

where $\tau_0$ is the value at current time $t_0$. Now we define the e-folding number as $N_e = \log \frac{a(t_0)}{a(t)}$.

This means that $\tau_0 = 2e^{\frac{\sqrt{21}}{2}N_e}$. By equating the eq. (12) with the eq. (13), we obtain

$$\int_1^2 d\tau \tau^{-\frac{1+\sqrt{21}}{10}} e^{\frac{1+\sqrt{21}}{60} c_h \Gamma H_1^6 \tau} \sim \frac{9 + \frac{21}{60} e^{\sqrt{21}/2} N_e}{6}.$$  \hspace{1cm} (14)

This gives a relation between $\Gamma H_1^6$ and $N_e$, and we obtain $\Gamma H_1^6 \sim 0.014$ for $N_e = 69$, for example.

### 3 Scalar Perturbations around the Background Geometry

Perturbations around homogeneous and isotropic universe are important directions to sort out inflationary models via observations. In this section, first we consider general metric perturbations around the background metric (6), and examine infinitesimal variations of perturbations under general coordinate transformation. Next we derive linearized equations of motion for the perturbations. Finally we focus on scalar perturbations and simplify their equations of motion by removing auxiliary fields. Main results are given by eqs. (43) and (46).

#### 3.1 Metric Perturbations and General Coordinate Transformation

Let us consider perturbations around the background geometry (6). We choose the metric with perturbations as follows.

$$ds^2 = -(1 + 2\alpha(t))dt^2 - 2a(t)\beta_i dt dx^i + a(t)^2 (\delta_{ij} + h_{ij})dx^i dx^j$$

$$- 2b(t)\beta_m dt dy^m + b(t)^2 (\delta_{mn} + h_{mn}) dy^m dy^n + 2a(t)b(t) h_{im} dx^i dy^m,$$  \hspace{1cm} (15)

where $i, j = 1, 2, 3$ and $m, n = 4, \cdots, 10$. $\alpha(X)$, $\beta_i(X)$ and $h_{ij}(X)$ are perturbations for the 4 dimensional spacetime, and $h_{mn}(X)$ is that of the internal space. $\beta_m(X)$ and $h_{im}(X)$

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^2Note that the definition of the e-folding number is different from that in ref. [1].
are off-diagonal perturbations between 4 dimensional spacetime and the internal space. As usual, we decompose vectors and tensors as

\[ \beta_{i} = \hat{\beta}_{i} + \partial_{i}\beta, \quad h_{ij} = \hat{h}_{ij} + 2\partial_{i}\hat{\gamma}_{ij} + 2\partial_{j}\hat{\gamma} + 2\psi\delta_{ij}, \quad \beta_{m} = \hat{\beta}_{m} + \partial_{i}\beta_{i}, \quad h_{mn} = \hat{h}_{mn} + 2\partial_{i}\hat{\gamma}_{mn} + 2\partial_{m}\hat{\gamma} + 2\psi\delta_{mn}, \]

\[ h_{im} = \hat{h}_{im} + \partial_{i}\lambda_{m} + \partial_{m}\lambda_{i} + 2\partial_{i}\partial_{m}\lambda. \]

Here hatted vectors are divergenceless, and hatted tensors are divergenceless and traceless. Note that \( \hat{\gamma} \) are off-diagonal perturbations between 4 dimensional spacetime and the internal space. As usual, we decompose vectors and tensors as

\[ Y \]

\[ \delta Y \]

\[ \delta X \]

\[ \delta X = \frac{1}{2}(\delta X_{\alpha\beta} + \delta X_{\beta\alpha}), \]

\[ \delta X_{\alpha\beta} = 96 \delta W_{\alpha\beta\gamma\delta} \frac{\partial}{\partial y_{\gamma\delta}} W_{\alpha\beta\gamma\delta} + 2W_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} - 16\delta W_{\alpha\beta\gamma\delta} \delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}, \]

\[ -\delta \delta W_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} - 16\delta W_{\alpha\beta\gamma\delta} \delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}, \]

\[ + 2\delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}. \]

\[ \delta X_{\alpha\beta} = \frac{1}{2}(\delta X'_{\alpha\beta} + \delta X'_{\beta\alpha}), \]

\[ \delta X'_{\alpha\beta} = 96 \delta W_{\alpha\beta\gamma\delta} \frac{\partial}{\partial y_{\gamma\delta}} W_{\alpha\beta\gamma\delta} + 2W_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} - 16\delta W_{\alpha\beta\gamma\delta} \delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}, \]

\[ -\delta \delta W_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} - 16\delta W_{\alpha\beta\gamma\delta} \delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}, \]

\[ + 2\delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}. \]

3.2 Equations of Motion for Metric Perturbations

Let us derive linearized equations of motion for the metric perturbations. This is simply done by varying the eq. (8). First of all, variation of \( Y_{abcd} \) is evaluated as

\[ \delta Y_{abcd} = \delta X_{abcd} - \frac{1}{9}(\eta_{ab} \delta X_{bd} - \eta_{bd} \delta X_{ab} - \eta_{ad} \delta X_{bc} + \eta_{bd} \delta X_{ac}) + \frac{1}{90}(\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}) \delta X, \quad (19) \]

and variation of \( X_{abcd} \) is given by

\[ \delta X_{abcd} = \frac{1}{2}(\delta X'_{\alpha\beta} + \delta X'_{\beta\alpha}), \]

\[ \delta X'_{\alpha\beta} = 96 \delta W_{\alpha\beta\gamma\delta} \frac{\partial}{\partial y_{\gamma\delta}} W_{\alpha\beta\gamma\delta} + 2W_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} - 16\delta W_{\alpha\beta\gamma\delta} \delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}, \]

\[ -\delta \delta W_{\alpha\beta\gamma\delta} W_{\alpha\beta\gamma\delta} - 16\delta W_{\alpha\beta\gamma\delta} \delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}, \]

\[ + 2\delta W_{\alpha\beta\gamma\delta} W_{\gamma\delta}. \]
Second, variation of $D_cD_dY^c_{ab\,d}$ is calculated as

$$\delta(D_cD_dY^c_{ab\,d}) = \delta e_{\mu}^a D_{\mu} D_d Y^c_{ab\,d} + \delta \omega_c^e D_d Y^e_{ab\,d} - \delta \omega_{ce(a} D_d Y^{ce)}_{b\,d}$$

$$- \delta \omega_{ce(a} D_d Y^{ce)}_{b\,d} e_d + D_c \delta (D_d Y^c_{ab\,d})$$

$$= \delta e_{\mu}^a D_{\mu} D_d Y^c_{ab\,d} + \delta \omega_c^e D_d Y^e_{ab\,d} - \delta \omega_{ce(a} D_d Y^{ce)}_{b\,d}$$

$$- \delta \omega_{ce(a} D_d Y^{ce)}_{b\,d} e_d + D_c \delta (\delta e_{\mu}^a D_{\mu} Y^c_{(ab\,d)} - \delta \omega_d^c e_d Y^c_{(ab\,d)}),$$

where $\delta \omega_a^c \equiv e_{\mu}^a \delta \omega_a^c$.

Combining the above results, we see that the variation of the eq. (3) is evaluated as

$$\delta E_{ab} = \delta R_{ab} - \frac{1}{2} \eta_{ab} \delta R + \Gamma \left\{ \frac{1}{2} \eta_{ab} \delta R_{cedf} Y^{cedf} + \delta R_{ceda} Y^{cede} + R^{cede} a \delta Y_{cedb} \right\}$$

$$- 2 \delta e_{\mu}^a D_{\mu} D_d Y^c_{ab\,d} - \delta \omega_c^e D_d Y^e_{ab\,d} + 2 \delta \omega_{ce(a} D_d Y^{ce)}_{b\,d}$$

$$+ 2 \omega_{ce(a} D_d Y^{ce)}_{b\,d} e_d - D_c \delta (\delta e_{\mu}^a D_{\mu} Y^c_{(ab\,d)} - \delta \omega_d^c e_d Y^c_{(ab\,d)}).$$

(22)

### 3.3 Equations of Motion for Scalar Perturbations

In this subsection, we restrict the metric perturbations to the scalar perturbations. So the metric is chosen as

$$ds^2 = -(1 + 2\alpha)dt^2 - 2a \partial_i \beta dt dx^i + a^2 (\delta_{ij} + 2 \partial_i \partial_j \gamma + 2 \psi \delta_{ij}) dx^i dx^j$$

$$- 2b \partial_m \tilde{\beta} dt dy^m + b^2 (\delta_{mn} + 2 \partial_m \partial_n \tilde{\gamma} + 2 \tilde{\psi} \delta_{mn}) dy^m dy^n + 4ab \partial_i \partial_m \lambda dx^i dy^m.$$

(23)

If we choose some gauge, the above metric is equivalent to the vielbein of the form

$$e_{\mu}^a + \delta e_{\mu}^a$$

$$= \begin{pmatrix} 1 + \alpha & 0 & 0 & 0 & 0 & \cdots & 0 \\ -\partial_1 \beta & a(1 + \partial_1^2 \gamma + \psi) & 0 & 0 & 0 & \cdots & 0 \\ -\partial_2 \beta & 2a \partial_1 \partial_1 \gamma & a(1 + \partial_1^2 \gamma + \psi) & 0 & 0 & \cdots & 0 \\ -\partial_3 \beta & 2a \partial_2 \partial_1 \gamma & 2a \partial_3 \partial_1 \gamma & a(1 + \partial_1^2 \gamma + \psi) & 0 & \cdots & 0 \\ -\partial_4 \beta & 2a \partial_3 \partial_2 \lambda & 2a \partial_4 \partial_2 \lambda & 2a \partial_3 \partial_3 \lambda & b(1 + \partial_2^2 \tilde{\gamma} + \tilde{\psi}) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ -\partial_{10} \bar{\beta} & 2a \partial_{10} \partial_1 \lambda & 2a \partial_{10} \partial_2 \lambda & 2a \partial_{10} \partial_3 \lambda & 2a \partial_{10} \partial_4 \lambda & \cdots & b(1 + \partial_{10}^2 \tilde{\gamma} + \tilde{\psi}) \\ \end{pmatrix},$$

up to the linear order of the perturbations. Here $e_{\mu}^a$ is the background vielbein and $\delta e_{\mu}^a$ linearly depends on the scalar perturbations.

Now we define following quantities.

$$\chi = a(\beta + a \gamma), \quad \bar{\chi} = b(\tilde{\beta} + b \tilde{\gamma}),$$

$$\Psi = H^{-1} \psi, \quad \bar{\Psi} = G^{-1} \tilde{\psi}, \quad \sigma = ab \lambda - \frac{a^2}{2} \gamma - \frac{b^2}{2} \tilde{\gamma}.$$
Consulting the eq. (17), we see that $\alpha, \chi, \Psi, \bar{\Psi}$ are invariant under $\xi$ and $\xi$ transformations, and $\sigma$ is invariant under the general coordinate transformation. By inserting scalar perturbations (24) into eq. (22), and expanding all perturbations by Fourier modes, such as Mathematica codes, and portions of results are written as

$$\Psi(t, x, y) = \int d^3k d^3l \{ \Psi(t, k, l)e^{ikx+ilmym} + \Psi(t, k, l)^*e^{-ik\bar{x}+ilmym} \},$$

we obtain following 8 linearized equations with respect to $\bar{\Psi} = \{ \alpha(t, k, l), \chi(t, k, l), \Psi(t, k, l), 
\bar{\chi}(t, l), \bar{\Psi}(t, k, l), \sigma(t, k, l) \}$.

$$E_1 \equiv \delta E_{00} = 0, \quad E_2 \equiv \frac{a}{k_a} \delta E_{0a} = 0, \quad E_3 \equiv \frac{a^2}{k_a k_b} \delta E_{ab} = 0, \quad E_4 \equiv \delta E_{aa} - \frac{k_a}{k_b} \delta E_{ab} = 0,$$

$$E_5 \equiv \frac{b}{l_m} \delta E_{om} = 0, \quad E_6 \equiv \frac{b^2}{l_m l_n} \delta E_{mn} = 0, \quad E_7 \equiv \delta E_{mn} - \frac{l_m}{l_n} \delta E_{mn} = 0, \quad E_8 \equiv \frac{ab}{k_a l_m} \delta E_{am} = 0.$$

Here the indices are not contracted. In order to evaluate the above equations we employed Mathematica codes, and portions of results are written as

$$E_1 = \frac{k^2}{a^2} \left\{ -(7G + 2H)\chi + 2H\Psi + 7G\bar{\Psi} \right\} + \frac{l^2}{b^2} \left\{ -3(2G + H)\bar{\chi} + 3H\Psi + 6G\bar{\Psi} \right\}$$

$$+ \frac{k^2l^2}{a^2b^2}2\sigma - 6(7G^2 + 7GH + H^2)\alpha + 3\dot{H}(7G + 2H)\Psi + 3H(7G + 2H)\bar{\Psi}$$

$$+ 21\dot{G}(2G + H)\bar{\Psi} + 21G(2G + H)\dot{\Psi} + \Gamma \tilde{S}_1(\bar{\Psi}, H, G),$$

$$E_2 = \frac{l^2}{b^2} \left\{ -\chi + \frac{\chi}{2} + 2H\sigma - \dot{\sigma} \right\} + (7G + 2H)\alpha - 2\dot{H}\Psi - 2H\bar{\Psi}$$

$$- 7(G^2 - GH + \dot{G})\bar{\Psi} - 7G\bar{\Psi} + \Gamma \tilde{S}_2(\bar{\Psi}, H, G),$$

$$E_3 = - \frac{2l^2\sigma}{b^2} - \alpha + (7G + H)\chi + \bar{\chi} - H\Psi - 7G\bar{\Psi} + \Gamma \tilde{S}_3(\bar{\Psi}, H, G),$$

$$E_4 = \frac{k^2}{a^2} \left\{ -\alpha + (7G + H)\chi + \bar{\chi} - H\Psi - 7G\bar{\Psi} \right\}$$

$$+ \frac{l^2}{b^2} \left\{ -\alpha + 2(3G + H)\bar{\chi} + \bar{\chi} - 2H\Psi - 6G\bar{\Psi} \right\} - \frac{k^2l^2}{a^2b^2}2\sigma$$

$$+ 2(28G^2 + 14GH + 3H^2 + 7\dot{G} + 2\dot{H})\alpha + (7G + 2H)\dot{\alpha}$$

$$- 2(7GH + \dot{H} + 3H\dot{H})\Psi - 2(7GH + 3H^2 + 2\dot{H})\bar{\Psi} - 2H\bar{\Psi}$$

$$- 7(2\dot{G}H + \dot{G} + 8G\dot{\bar{\chi}})\bar{\Psi} - 14(4G^2 + GH + \dot{G})\bar{\Psi} - 7G\bar{\Psi} + \Gamma \tilde{S}_4(\bar{\Psi}, H, G),$$

$$E_5 = \frac{k^2}{a^2} \left\{ \chi - \frac{\chi}{2} + 2G\sigma - \dot{\sigma} \right\} + (2G + H)\alpha + 3(GH - H^2 - \dot{H})\Psi - 3H\bar{\Psi}$$

$$- 6\dot{G}\bar{\Psi} - 6G\bar{\Psi} + \Gamma \tilde{S}_5(\bar{\Psi}, H, G),$$

$$E_6 = - \frac{k^2}{a^2}2\sigma - \alpha + (5G + 3H)\chi + \bar{\chi} - 3H\Psi - 5G\bar{\Psi} + \Gamma \tilde{S}_6(\bar{\Psi}, H, G),$$

$$E_7 = \frac{b}{l_m} \delta E_{om} = 0, \quad E_8 \equiv \frac{ab}{k_a l_m} \delta E_{am} = 0.$$
Again the explicit forms of \( S \) order, and 1 does one which is linear order of \( \Gamma \). As will be clear soon, it is useful to define following gauge invariant quantity.

\[ E_7 = \frac{k^2}{a^2} \left\{ -\alpha + (6G + 2H)\chi + \dot{\chi} - 2H\Psi - 6G\bar{\Psi} \right\} \\
+ \frac{l^2}{b^2} \left\{ -\alpha + (5G + 3H)\bar{\chi} + \dot{\bar{\chi}} - 3H\Psi - 5G\bar{\Psi} \right\} - \frac{k^2l^2}{a^2b^2} 2\sigma \\
+ 6(7G^2 + 6GH + 2H^2 + 2\dot{G} + \dot{H})\alpha + 3(2G + H)\dot{\alpha} \\
- 3(6G\dot{H} + \ddot{H} + 4H\dot{H})\Psi - 6(3GH + 2H^2 + \dot{H})\bar{\Psi} - 3H\bar{\Psi} \\
- 6(3\dot{G}H + \dot{G} + 7G\dot{G})\bar{\Psi} - 6(7G^2 + 3GH + 2\dot{G})\bar{\Psi} - 6G\ddot{\Psi} + \Gamma \bar{\Psi}\bar{\Theta}_7(\bar{\Theta}, H, G) \]

(34) 

\[ E_8 = -\alpha + \frac{1}{2}(5G + 3H)\chi + \frac{1}{2} \ddot{\chi} + \frac{1}{2}(7G + H)\bar{\chi} + \frac{1}{2} 3\dot{\chi} - 2H\Psi - 6G\bar{\Psi} \]

- 2(7G^2 + GH + \dot{G})\sigma + (5G + H)\dot{\sigma} + \Gamma \bar{\Psi}\bar{\Theta}_8(\bar{\Theta}, H, G),

(35)

where \( k^2 = k_i k^i \) and \( l^2 = l_m l^m \). Note that one of these equations is redundant because of \( D^a E_{ab} = 0 \), so we have 7 independent equations. The explicit forms of \( \tilde{S}_u(u = 1, \cdots, 8) \) can be found in ref. [42].

Now we simply set \( l_m = 0 \), because \( \frac{l^2}{b^2} \) becomes very large after the rapid expansion and such massive modes will decouple. Anyway the full analyses including \( l_i \neq 0 \) modes will be discussed elsewhere. When \( l_i = 0 \), the above 7 independent equations reduce to following 4 equations, which are linear on \( \bar{\Theta} = \{ \alpha, \chi, \Psi, \bar{\Psi} \} \).

\[ E_1 = \frac{k^2}{a^2} \left\{ - (7G + 2H)\chi + 2H\Psi + 7G\bar{\Psi} \right\} \\
- 6(7G^2 + 7GH + H^2)\alpha + 3\ddot{H}(7G + 2H)\Psi + 3H(7G + 2H)\bar{\Psi} \\
+ 21\dot{G}(2G + H)\dot{\Psi} + 21G(2G + H)\dot{\bar{\Psi}} + \Gamma S_1(\bar{\Theta}, H, G) = 0, \]

(36)

\[ E_2 = (7G + 2H)\alpha - 2H\Psi - 2H\bar{\Psi} - 7(G^2 - GH + \dot{G})\bar{\Psi} - 7G\ddot{\Psi} + \Gamma S_2(\bar{\Theta}, H, G) = 0, \]

(37)

\[ E_3 = -\alpha + (7G + H)\chi + \dot{\chi} - H\Psi - 7G\bar{\Psi} + \Gamma S_3(\bar{\Theta}, H, G) = 0, \]

(38)

\[ E_7 = \frac{k^2}{a^2} \left\{ -\alpha + (6G + 2H)\chi + \dot{\chi} - 2H\Psi - 6G\bar{\Psi} \right\} \\
+ 6(7G^2 + 6GH + 2H^2 + \dot{G} + \dot{H})\alpha + 3(2G + H)\dot{\alpha} \\
- 3(6G\dot{H} + \ddot{H} + 4H\dot{H})\Psi - 6(3GH + 2H^2 + \dot{H})\bar{\Psi} - 3H\bar{\Psi} \\
- 6(3\dot{G}H + \dot{G} + 7G\dot{G})\bar{\Psi} - 6(7G^2 + 3GH + 2\dot{G})\bar{\Psi} - 6G\ddot{\Psi} + \Gamma S_7(\bar{\Theta}, H, G) = 0. \]

Again the explicit forms of \( S_u(u = 1, 2, 3, 7) \) can be found in ref. [42].

Let us solve above equations up to the linear order of \( \Gamma \) expansion, so we expand the perturbation \( \bar{\Theta} \) as \( \bar{\Theta} = \bar{\Theta}_0 + \Gamma \bar{\Theta}_1 \). Here the subscript 0 represents the quantity at the leading order, and 1 does one which is linear order of \( \Gamma \). As will be clear soon, it is useful to define following gauge invariant quantity.

\[ P \equiv \Psi - \bar{\Psi} = H^{-1}\Psi - G^{-1}\bar{\Psi}. \]

(40)

Below we will show that equations of motion for scalar perturbations can be collected into single differential equation with respect to \( P \), after eliminating auxiliary fields \( \alpha \) and \( \chi \).
First let us consider the equations of motion at the leading order of $\Gamma$ expansion. From eqs. (36) and (37), $\alpha_0$ and $\chi_0$ are solved as

$$\alpha_0 = -\frac{9 + \sqrt{21}}{3} H_0 P_0 + \dot{\Psi}_0 + \frac{\sqrt{21}}{3} \dot{P}_0,$$

$$\chi_0 = \frac{a^2}{k^2} \left\{ - \frac{3(\sqrt{21} - 1)}{2} H_0^2 P_0 + 3 H_0 \dot{P}_0 \right\} + \Psi_0 + \frac{\sqrt{21}}{3} P_0. \tag{42}$$

Here $H_0$ and $G_0$ are leading parts of $H$ and $G$, respectively, and we used $G_0 = \frac{7 + \sqrt{21}}{14} H_0$ and $\dot{H}_0 = \frac{1 - \sqrt{21}}{2} H_0^2$. By inserting the above into the eq. (38) with $\Gamma = 0$, we obtain

$$0 = \ddot{P}_0 - \frac{\sqrt{21} - 1}{2} \dot{H}_0 \dot{P}_0 + \left( \frac{k^2}{a^2} \right) \frac{\sqrt{21} - 11}{2} H_0^2 P_0. \tag{43}$$

Note that the eq. (39) is automatically satisfied. So we only need to solve the eq. (43) at the leading order of $\Gamma$.

Next let us investigate linear order of $\Gamma$ expansion. Again, from eqs. (36) and (37), auxiliary fields $\alpha$ and $\chi$ are solved up to linear order of $\Gamma$ as

$$\alpha = \frac{2 \dot{H} \dot{\Psi} + 2 \dot{H} \dot{\Psi} + 7(\dot{G} - G H + G^2) \dot{\Psi} + 7 G \ddot{\Psi}}{7 G + 2 H} + \Gamma \left[ \frac{1536(14229047 + 734623 \sqrt{21})}{8575} H_0^7 \dot{\Psi}_0 + \frac{3072(828991 \sqrt{21} - 1479901)}{8575} H_0^7 \dot{\Psi}_0 \right]$$

$$+ \Gamma \left[ \frac{1536(-2965613 - 1496367 \sqrt{21})}{8575} H_0^6 \dot{P}_0 + \frac{54681(136 + 9 \sqrt{21})}{245} H_0^5 \dot{P}_0 - \frac{6144(1771 + 1014 \sqrt{21})}{1225} H_0^4 \dot{P}_0 \right]$$

$$- \frac{k^2}{a^2} \left( \frac{1536(1677 \sqrt{21} - 152147)}{1225} H_0^5 P_0 + \frac{3072(497 + 1348 \sqrt{21})}{1225} H_0^4 P_0 \right), \tag{44}$$

$$\chi = \frac{a^2}{k^2} \frac{63 G^2 \dot{H} \dot{\Psi} - 21 G(3 H \dot{G} - 12 G H^2 + 14 G^3 - 2 H^3) \dot{\Psi} + 63 G^2 H \ddot{P}}{(7 G + 2 H)^2} + \frac{2 H \dot{P} + 7 G \ddot{\Psi}}{7 G + 2 H}$$

$$+ \Gamma \left[ \frac{a^2}{k^2} \left\{ \frac{3072(34956697 - 13586977 \sqrt{21})}{8575} H_0^7 \dot{\Psi}_0 - \frac{3072(26812583 + 1603297 \sqrt{21})}{8575} H_0^7 \dot{\Psi}_0 \right\} - \frac{3072(25037012 - 2136942 \sqrt{21})}{8575} H_0^7 \dot{P}_0 - \frac{3072(628026 - 719166 \sqrt{21})}{8575} H_0^6 \dot{P}_0 \right]$$

$$- \frac{3072(34463 + 217 \sqrt{21})}{1225} H_0^5 \dot{P}_0 + \frac{3072(5902 \sqrt{21} - 573447)}{8575} H_0^6 P_0$$

$$+ \frac{4608(861 + 855 \sqrt{21})}{245} H_0^5 P_0 - \frac{6144(1771 + 1014 \sqrt{21})}{1225} H_0^4 P_0 - \frac{k^2}{a^2} \frac{3072(497 + 1348 \sqrt{21})}{1225} H_0^4 P_0 \right]. \tag{45}$$

By inserting the above into (38), of course we obtain the eq. (43) at the leading order, and

$$0 = \ddot{P}_1 - \frac{\sqrt{21} - 1}{2} \dot{H}_0 \dot{P}_1 + \left( \frac{k^2}{a^2} \right) \frac{\sqrt{21} - 11}{2} H_0^2 P_1$$

$$+ \frac{1536(69692339 \sqrt{21} - 70593438)}{8575} H_0^8 P_0 + \frac{768(36412229 \sqrt{21} - 124991079)}{1715} H_0^7 \dot{P}_0$$

$$+ \frac{768(5604373 \sqrt{21} - 3606833)}{1715} H_0^6 \dot{P}_0 + \frac{12288(6383 \sqrt{21} - 17688)}{245} H_0^5 \dot{P}_0$$

$$+ \frac{3072(2261 \sqrt{21} - 23271)}{1225} H_0^4 \dot{P}_0 + \frac{k^2}{a^0} \left\{ \left( \frac{768(3567079 \sqrt{21} - 29260239)}{8575} H_0^6 - 2 a_1 \right) P_0 \right\}$$

$$+ \frac{3072(55331 \sqrt{21} - 265416)}{1225} H_0^5 P_0 + \frac{1536(9479 \sqrt{21} - 66369)}{1225} H_0^4 P_0$$

$$+ \frac{k^4}{a_0} \frac{6144(1633 \sqrt{21} - 9288)}{1225} H_0^4 P_0. \tag{46}$$
at the linear order of $\Gamma$. Here $\bar{a}_1$, which comes from $\frac{k^2}{a^2}P_0$, is defined as

$$
\bar{a}_1 = -\frac{1 + \sqrt{21}}{60}c_h H_0^6 = -\frac{1 + \sqrt{21}}{60}c_h H_0^6.
$$

In summary, we have derived the eq. (43) and the eq. (46) for the scalar perturbation $P$. We will solve these equations up to the linear order of $\Gamma$ in section 5.

4 Effective Action for Scalar Perturbations

In the previous section, we derived equations of motion for scalar perturbations, which are expressed by the eq. (43) and the eq. (46). In this section, we consider effective action which is second order with respect to the scalar perturbations. We will reproduce the equations of motion (43) and eq. (46) from this effective action.

First let us substitute the metric (15) into the action (1), and expand it up to the second order with respect to the scalar perturbations. By setting $l_m = 0$, the result is written as

$$
S^{(2)}_{pt} = S^{(2,0)}_{pt} + \Gamma S^{(2,1)}_{pt}
$$

$$
= \frac{1}{2\kappa_0^2} \int d^3x d^2y a^6 \left[ -42 G^2 \dot{\psi}^2 - 6H^2 \dot{\psi}^2 - 42 G H \dot{\psi} \dot{\bar{\psi}} + 42 G \dot{\bar{\psi}}^2 \left( 2G^3 + 18 G^2 H + 6G H^2 + 3G \dot{H} + 13 \dot{G} G + 3 \ddot{G} H + \ddot{G} \dot{H} \right) + 42 G \dot{\psi} \dot{\bar{\psi}} \left( 21 G^2 H + 18 G H^2 + 6 G \dot{H} + 6H + 6 \dot{G} H + 7 H \dot{H} \right) + 6 H^2 \ddot{\psi} \left( 28 G^2 H + 14 G H^2 + 7 \dot{G} H + 7 \dot{G} \dot{H} + 3 H^3 + 5 \ddot{H} H + \ddot{H} \dot{H} \right) + 6 \alpha \dot{\psi} \left( 21 G^2 H + 21 G H^2 + 7 \dot{G} H + 3 H^3 + 2 \ddot{H} H \right) + 6 H \alpha \dot{\psi} \left( 7 G + 2 H \right) \right] + \Gamma \mathcal{L}^{(2,1)}_{pt}(\Upsilon, H, G) \right],
$$

where $S^{(2,0)}_{pt}$ represents the leading order part of $\Gamma$ expansion in the second order terms with respect to the scalar perturbations. Similarly $S^{(2,1)}_{pt}$ or $\mathcal{L}^{(2,1)}_{pt}(\Upsilon, H, G)$ does linear order part of $\Gamma$. The explicit forms of $\mathcal{L}^{(2,1)}_{pt}$ and other complicated equations in this section can be found in ref. [42]. By varying the above action, we obtain following equations of motion for scalar perturbations.

$$
E_\alpha = \frac{k^2}{a^2} \left( 7G \ddot{\psi} - (7G + 2H) \chi + 2H \dot{\psi} \right) + 3 \ddot{\psi} \left( 21 G^2 H + 21 G H^2 + 7G \dot{H} + 3 H^3 + 2 \ddot{H} H \right) + 3 H \dot{\psi} \left( 7G + 2H \right) + 21 \ddot{\psi} \left( 7 G^3 + 7 G^2 H + G H^2 + \dot{G} H + 2 \dot{G} G \right) + 21 G \dot{\psi} \left( 2G + H \right) + \Gamma \mathcal{S}_\alpha(\Upsilon, H, G) = 0,
$$

\[49\]
\[ E_\chi = -\alpha(7G + 2H) + 2\dot{H}\Psi + 2H\ddot{\Psi} + 7\Psi(G^2 - \dot{G}H + \dot{G}) + 7G\ddot{\Psi} + \Gamma S_\chi(\mathcal{Y}, H, G) = 0, \] (50)

\[ E_\Psi = \frac{k^2}{a^2} \left\{ 14G\Psi + 2\alpha - 2\chi(7G + H) + 2H\Psi - 2\dot{\chi} \right\} - 3(7G + 2H)\dot{\alpha} + 6\Psi(28G^2H + 14GH^2 + 7\dot{G}H + 7G\dot{\Psi} + 3H^3 + \dot{H} + 5\dot{H}H) + 21\dot{\Psi}(28G^3 + 14G^2H + 3GH^2 + 2G\dot{H} + 2\dot{G}H + \ddot{G} + 15\dot{G}G) \] (51)

\[ + 6\Psi(7GH + 3H^2 + 2\dot{H}) + 42\dot{\Psi}(4G^2 + GH + \dot{G}) + 6\dot{H}\Psi + 21G\ddot{\Psi} + \Gamma S_\Psi(\mathcal{Y}, H, G) = 0, \] (52)

where \( S_I (I = \alpha, \chi, \Psi, \bar{\Psi}) \) represents linear terms with respect to \( \Gamma \). At least at the leading order, it is straightforward to show that these are equivalent to the equations of motion \( \{45\} \). For example, \( E_\Psi \) is expressed by a combination of \( E_2, \dot{E}_2 \) and \( \frac{k^2}{a^2} E_3 \). Thus the effective action \( \{48\} \) is consistent with the results in the previous section.

It is also possible to express the effective action in terms of \( P_0 \) and \( P_1 \). First by eliminating auxiliary fields \( \alpha_0 \) and \( \chi_0 \) by using the eq. \( \{41\} \) and the eq. \( \{42\} \), the leading order action \( S_{pt}^{(2,0)} \) is written as

\[ S_{pt}^{(2,0)} = \frac{1}{2k_1^2} \int dt d^3x d^7 y 6a_0^3b_0^7H_0^2 \left\{ - \left( \frac{k^2}{a_0^2} - \frac{\sqrt{21} - 11}{2} H_0^2 \right) P_0^2 + \dot{P}_0^2 \right\}. \] (53)

It is easy to confirm that the eq. \( \{40\} \) can be derived from this action.

Next let us express \( S_{pt}^{(2)} \) in terms of \( P_0 \) and \( P_1 \). By eliminating auxiliary fields \( \alpha \) and \( \chi \) by inserting the eq. \( \{41\} \) and the eq. \( \{45\} \) into the action \( \{48\} \), we obtain

\[ S_{pt}^{(2)} = \frac{1}{2k_1^2} \int dt d^3x d^7 y 6a_0^3b_0^7H_0^2 \left\{ - \left( \frac{k^2}{a_0^2} - \frac{\sqrt{21} - 11}{2} H_0^2 \right) P_0^2 + \dot{P}_0^2 \right\} + \Gamma \left\{ - 2\dot{P}_0 P_1 + (\sqrt{21} - 11)H_0 \dot{P}_0 P_1 + (\sqrt{21} - 11)H_0^2 P_0 P_1 \\
- \frac{3072(2261\sqrt{21} - 23271)}{1225}H_1^4 \dot{P}_0^2 + (3\bar{a}_1 + 7\bar{b}_1) + \frac{768(6043049\sqrt{21} - 44300079)}{8575}H_0^6 \dot{P}_0^2 \\
+ (\sqrt{21} - 11)(3\bar{a}_1 + 7\bar{b}_1) + \frac{768(16193341\sqrt{21} - 29555691)}{1225}H_0^6 P_0^2 \\
+ \frac{k^2}{a_0^2} \left( - 2\bar{P}_1 P_0 - (\bar{a}_1 + 7\bar{b}_1) + \frac{768(1807903\sqrt{21} - 20672673)H_0^6}{8575} \right) P_0^2 \\
+ \frac{1536(9479\sqrt{21} - 66369)}{1225}H_1^2 \dot{P}_0^2 - \frac{k^4}{a_0^4} \frac{6144(1633\sqrt{21} - 9288)}{1225} H_0^4 P_0^2 \right\}. \] (54)
where \( \bar{b}_1 \) is defined as
\[
\bar{b}_1 = -\frac{1 + \sqrt{21}}{60} c_g \frac{H^6}{\tau^6} = -\frac{1 + \sqrt{21}}{60} c_g H_{0}^6.
\] (55)

Note that \( \dot{\bar{a}}_1 \) and \( \dot{\bar{b}}_1 \) are written as
\[
\dot{\bar{a}}_1 = H_1 = c_h \frac{H^7}{\tau^7} = c_h H_{0}^7,
\]
\[
\dot{\bar{b}}_1 = G_1 = c_g \frac{H^7}{\tau^7} = c_g H_{0}^7,
\] (56)
respectively. Then it is possible to check that this action consistently reproduces the equations of motion (43) and (46).

5 Analyses of Scalar Perturbations

In this section, first we solve the eq. (43), and then do the eq. (46). From these, it is possible to obtain the explicit form of the curvature perturbation \( \psi \) and estimate its power spectrum.

5.1 Solutions of \( P_0 \) and \( P_1 \)

In order to solve the eq. (43), we introduce new time coordinate \( \eta \) instead of \( t \), which is defined by
\[
dt = \frac{1 + \sqrt{21}}{10H_1} d\tau = a_0 d\eta.
\]
Note that \( a_0 = a_E \frac{1 + \sqrt{21}}{10} \) is leading part of the scale factor \( a \). Therefore \( \eta \) is considered to be a conformal time after the inflationary expansion. Then \( \eta \) is expressed in terms of \( \tau \) like
\[
\eta = \frac{1 + \sqrt{21}}{10H_1} \int \frac{d\tau}{a_0} = \frac{3 + \sqrt{21}}{6a_E H_1} \tau^{\frac{9 - \sqrt{21}}{10}}.
\] (57)

By solving inversely, \( \tau \) is expressed in terms of \( \eta \) as
\[
\tau = \left( \frac{\sqrt{21} - 3}{2} a_E H_1 \eta \right)^{\frac{9 + \sqrt{21}}{6}},
\] (58)
and \( a_0 \) is given by
\[
a_0 = a_E \left( \frac{\sqrt{21} - 3}{2} a_E H_1 \eta \right)^{\frac{9 + \sqrt{21}}{6}}.
\] (59)

Remind that the inflationary expansion is realized during \( 1 \leq \tau \leq 2 \). Now \( \tau = 1 \) corresponds to \( a_E H_1 \eta = \frac{3 + \sqrt{21}}{6} \sim 1.26 \), and \( \tau = 2 \) does to \( a_E H_1 \eta = \frac{3 + \sqrt{21}}{6} \frac{9 - \sqrt{21}}{10} \sim 1.72 \).

Hubble parameter \( H_0 \) with respect to the time coordinate \( \eta \) is evaluated as
\[
H_0 = \frac{a'_0}{a_0} = a_0 H_0 = \frac{3 + \sqrt{21}}{6} \frac{1}{\eta},
\] (60)
where \( ' = \frac{d}{d\eta} \). And derivatives of \( P_0 \) with respect to \( t \) are replaced with
\[
\dot{P}_0 = \frac{1}{a_0} P_0', \quad \ddot{P}_0 = \frac{1}{a_0^2} (P_0'' - H_0 P_0').
\] (61)
Then by multiplying $a_0^2$ to the eq. (43), it becomes

$$0 = P''_0 - \frac{\sqrt{21}}{2} H_0 P'_0 + \left( k^2 - \frac{\sqrt{21}}{2} H_0^2 \right) P_0$$

$$= a_0^2 \left[ U''_0 + \left( k^2 + \frac{1}{4\eta^2} \right) U_0 \right],$$

(62)

where $P_0$ and $U_0$ are related as

$$P_0 = a_0 \sqrt{\frac{\sqrt{21} + 1}{4\eta^2}} U_0.$$  

(63)

Now the eq. (62) can be solved as

$$U_0 = c_1 \sqrt{k\eta} J_0(k\eta) + c_2 \sqrt{k\eta} Y_0(k\eta).$$  

(64)

$J_0$ and $Y_0$ are Bessel functions of the first and second kind, respectively. $c_1$ and $c_2$ are integral constants and both have mass dimension $-1$. In order to fix the ratio of $c_2/c_1$, we demand that $U_0$ behaves like $e^{-ik\eta}$ as $\eta$ goes to the infinity. This is reasonable if we assume that the perturbations, such as the eq. (26), are canonically expressed in terms of Fourier modes as $\eta$ goes to the infinity. Since $\sqrt{x} J_0(x) \sim \sqrt{\frac{2}{\pi}} \cos(x - \frac{\pi}{4})$ and $\sqrt{x} Y_0(x) \sim \sqrt{\frac{2}{\pi}} \sin(x - \frac{\pi}{4})$ as $x \to \infty$, we choose $c_2/c_1$ as

$$\frac{c_2}{c_1} = -i,$$

(65)

and $U_0$ is given by

$$U_0 = c_1 \sqrt{k\eta} H_0^{(2)}(k\eta),$$

(66)

where $H_0^{(2)}$ is Hankel function of the second kind.

Next let us solve the differential equation for $P_1$. We replace derivatives of $P_0$ with respect to $t$ by using the eq. (61) and

$$\ddot{P}_0 = \frac{1}{a_0^2} \left\{ P'''_0 - 3 H_0 P''_0 + \frac{1 + \sqrt{21}}{2} H_0^2 P'_0 \right\},$$

$$\ddot{P}_0 = \frac{1}{a_0^2} \left\{ P'''_0 - 6 H_0 P''_0 + (5 + 2\sqrt{21}) H_0^2 P''_0 - \frac{21 + \sqrt{21}}{2} H_0^3 P'_0 \right\}.$$

(67)

Then by multiplying $a_0^2$ to the eq. (60), it becomes

$$P''_1 = \frac{\sqrt{21} + 1}{2} H_0 P'_1 + \left( k^2 - \frac{\sqrt{21}}{2} H_0^2 \right) P_1$$

$$= \frac{1}{a_0^2} \left[ P''_0 - \frac{1536(49692383\sqrt{21} - 70534338)}{5875 H_0^8 P_0} + \frac{1536(15053494\sqrt{21} - 40585737)}{1715 H_0^7 P'_0} \right.$$  

$$+ \frac{6912(1814321\sqrt{21} - 16802761)}{5875 H_0^6 P''_0} + \frac{6144(57047\sqrt{21} - 1070677)}{1225 H_0^5 P''_0} + \frac{3072(2261\sqrt{21} - 23271)}{1225 H_0^4 P''_0}$$

$$+ \frac{768(3567079\sqrt{21} - 29260239)}{8575 H_0^3 P'''_0} + \frac{64 (161183\sqrt{21} - 464463)}{1225 H_0^2 P'''_0} + \frac{2163(9479\sqrt{21} - 66369)}{1225 H_0^4 P''_0} \right\}$$

$$+ k^2 \left\{ \frac{768(3567079\sqrt{21} - 29260239)}{8575 H_0^3 P'''_0} + \frac{64 (161183\sqrt{21} - 464463)}{1225 H_0^2 P'''_0} + \frac{2163(9479\sqrt{21} - 66369)}{1225 H_0^4 P''_0} \right\} + k^4 \frac{6144(1633\sqrt{21} - 9288)}{1225 H_0^4 P_0} = 0.$$  

(68)
In the above, we used
\[ H'_0 = \frac{3 - \sqrt{21}}{2} H_0^2, \quad H''_0 = 3(5 - \sqrt{21}) H_0^3, \] (69)
and neglected higher order terms on \( \Gamma \). The remaining eq. (39) is automatically satisfied. If we rescale \( P_1 \) as
\[ P_1 = a_0^{\frac{3}{4} - 1} U_1, \] (70)
we obtain
\[
0 = U''_1 + \left( k^2 - \frac{3(\sqrt{21} - 5)}{8} H_0^2 \right) U_1 + \frac{1}{a_0^6} \left[ - \frac{3456(64904378 \sqrt{21} - 301020693)}{8575} H_0^8 U_0 + \frac{13824(676051 \sqrt{21} - 2068971)}{1715} H_0^7 U_0' + \frac{2304(4137503 \sqrt{21} - 32998023)}{8575} H_0^6 U_0'' + 36864(7757 \sqrt{21} - 15827) H_0^5 U_0''' + \frac{1536(452 \sqrt{21} - 46542)}{1225} H_0^4 U_0'''' + \frac{12}{a_0^6} 6 a_0^6 a_1 U_0 + \frac{9216(22123 \sqrt{21} - 66353)}{1225} H_0^3 U_0'' + \frac{1536(9479 \sqrt{21} - 66399)}{1225} H_0^2 U_0'''' \right] + k^2 \left( \frac{602616417 + 131422261}{300125} \right) \left( \eta \right) \left( c_1 J_1(\eta) + c_2 Y_1(\eta) \right) + k^2 \left( \frac{602616417 + 131422261}{300125} \right) \left( \eta \right) \left( c_1 J_0(\eta) + c_2 Y_0(\eta) \right).
\] (71)

In the last line, we substituted the eq. (60) and the eq. (64). Particular solution of the above is given by
\[ U_1 = - \frac{288(20727 - 4523 \sqrt{21})}{300125} \frac{H_0^2 \sqrt{K_0}}{ \sqrt{21} - 3 } \left( c_1 U_{111} - c_2 \left( \frac{41 + 9 \sqrt{21}}{10} \right) \sqrt{K_0} U_{12} \right). \] (72)

Since the explicit expressions of \( U_{11} \) and \( U_{12} \) are quite long, we put them in the appendix [A].

The ratio of \( \frac{c_2}{c_1} \) should be fixed by \( \frac{c_2}{c_1} = -i \) as explained around the eq. (59). Thus we have solved the eqs. (43) and (40), and \( P = P_0 + \Gamma P_1 \) is given by
\[
P_0 = c_1 a_0^{\frac{3}{4} - 1} \sqrt{K_0} H_0^{(2)}(\eta), \quad P_1 = \frac{288(20727 - 4523 \sqrt{21})}{300125} c_1 a_0^{\frac{3}{4} - 1} \frac{H_1^2 \sqrt{K_0}}{ \sqrt{21} - 3 } \left( c_1 U_{11} + c_2 \left( \frac{41 + 9 \sqrt{21}}{10} \right) \sqrt{K_0} U_{12} \right). \] (73)

5.2 Curvature perturbation

Let us investigate the curvature perturbation \( \psi \). If we choose \( \tilde{\psi} = 0 \) gauge, the curvature perturbation is expressed as \( \psi = H P \). Up to the linear order of \( \Gamma \) expansion, \( \psi \) is written as
\[ \psi(\eta, x) = \psi_0 + \Gamma \psi_1 = H_0 P_0 + \Gamma (H_1 P_0 + H_0 P_1), \] (74)
where $\eta$ is related to $\tau$ via the eq. (57). Now we expand $\psi(\eta, x)$ as
\[
\psi(\eta, x) = \int d^3k \left\{ \psi(\eta, k)e^{ik\cdot x} + \psi(\eta, k)^* e^{-ik\cdot x} \right\}.
\] (75)

Then Fourier component of $\psi_0$ is evaluated as
\[
\psi_0(\eta, k) = \tilde{c}_1 H_0^{(2)}(k\eta), \quad \tilde{c}_1 = c_1 a_E^{\frac{\sqrt{27}+1}{4}} H_1 \left( \frac{\sqrt{21} + 3}{6} \frac{k}{a_E H_1} \right)^{\frac{1}{2}}.
\] (76)

If we take $k\eta \to \infty$, $\psi_0(\eta, k)$ approaches to $\tilde{c}_1 \sqrt{\frac{2}{\pi k\eta}} e^{i\frac{\pi}{4}} e^{-ik\eta}$.

Next Fourier component of $\psi_1$ is calculated as
\[
\psi_1(\eta, k) = H_0(c_2 H_0^6 P_0 + P_1)
\]
\[
= c_1 a_0^{\frac{\sqrt{27}+1}{4}} H_0 \sqrt{k\eta} \left\{ c_2 H_0^{(2)}(k\eta) - \frac{288(20727-4523\sqrt{21})}{300125} \left( U_{11} + i \left( -\frac{41+9\sqrt{21}}{10} \right) \sqrt{\eta} U_{12} \right) \right\}
\]
\[
= \frac{\tilde{c}_1}{\tau^6} \left\{ c_2 H_0^{(2)}(k\eta) - \frac{288(20727-4523\sqrt{21})}{300125} \left( U_{11} + i \left( -\frac{41+9\sqrt{21}}{10} \right) \sqrt{\eta} U_{12} \right) \right\}.
\] (77)

Here we used $\tau^6 = (\frac{\sqrt{27}+3}{2} a_E H_1 \eta)^{9+9\sqrt{21}}$. Note that $\psi_1$ decreases faster than $\psi_0$ as $\tau$ goes to the infinity.

Finally the power spectrum of the Fourier mode up to linear order of $\Gamma$ is expressed as
\[
\mathcal{P}(\eta, k) = \left( \frac{k}{a_E H_1} \right)^3 |\psi(\eta, k)|^2
\]
\[
= \left| \tilde{c}_1 \right|^2 \left( \frac{k}{a_E H_1} \right)^3 \left| H_0^{(2)}(\eta, k) + \frac{\Gamma H_1^6}{\tau^6} \left\{ c_2 H_0^{(2)}(\eta, k) - \frac{288(20727-4523\sqrt{21})}{300125} \left( U_{11}(k\eta) + i \left( -\frac{41+9\sqrt{21}}{10} \right) \sqrt{\eta} U_{12}(k\eta) \right) \right\} \right|^2.
\] (78)

Now we assume that the power spectrum was some constant $A$ at the beginning of the universe, $\tau = 1$ or $\eta = \frac{3+\sqrt{21}}{66 a_E H_1}$. Then $\mathcal{P}(\frac{3+\sqrt{21}}{66 a_E H_1}, k) = A$, and $k$ dependence of $|\tilde{c}_1|$ is given by
\[
|\tilde{c}_1|^2 = A \left( \frac{a_E H_1}{k} \right)^3 \left| H_0^{(2)} \left( \frac{3+\sqrt{21}}{66 a_E H_1}, k \right) + \frac{\Gamma H_1^6}{\tau^6} \left\{ c_2 H_0^{(2)} \left( \frac{3+\sqrt{21}}{66 a_E H_1}, k \right) - \frac{288(20727-4523\sqrt{21})}{300125} \left( U_{11} \left( \frac{3+\sqrt{21}}{66 a_E H_1} k \right) + i \left( -\frac{41+9\sqrt{21}}{10} \right) \sqrt{\eta} U_{12} \left( \frac{3+\sqrt{21}}{66 a_E H_1} k \right) \right) \right\} \right|^2.
\] (79)

So far the analyses in this paper is reliable up to the linear order of $\Gamma$. Basically we don’t have any control over higher order terms, because we have poor knowledge about coefficients of those terms. At best we know that coefficient of those terms behave like $(\Gamma H_1^6)^n$, and expect that the $n = 1$ term in this paper is dominant during the inflationary era. Then the behavior of the power spectrum (78) with $\log \frac{k}{a_E H_1} = -30$ and $\Gamma H_1^6 = 0.014$ is plotted as in fig. 1. The shape of the plot does not change so much even if the wave number ranges $-40 < \log \frac{k}{a_E H_1} < -10$. The power spectrum is monotonically decreasing, but its tilt becomes slightly mild after the inflationary era. The behavior of the power spectrum at the horizon crossing $k = a H$ with $\Gamma H_1^6 = 0.014$ is shown in fig. 2. If we fit the data between $-36 < \log \frac{k}{a_E H_1} < -20$ in fig. 2, it is possible to draw a line $\log \frac{\mathcal{P}}{A} = -0.062 \log \frac{k}{a_E H_1} - 3.0$, and the spectral index is estimated as $n_s = 0.94$. This is close to the value of current observation, and we should investigate more seriously the era after the inflation.
6 Conclusion and Discussion

In this paper we investigated the inflationary scenario in M-theory. In addition to the ordinary supergravity part, the effective action of the M-theory contains higher curvature terms, which are expressed by products of 4 Weyl tensors. In the early universe, $H$ in the eq. (7) is relatively large and nonzero components of the Weyl tensor also become large. So the higher curvature terms become important and those induce the inflationary expansion. After sufficient expansion, $H$ becomes small and the Weyl tensor does small. Then the higher curvature terms are negligible and inflation naturally ends.

The main purpose of this paper is to explore the scalar perturbations in the above inflationary scenario. Actually, we considered the metric perturbations around the homogeneous and isotropic background, and derived the linearized equations of motion for the scalar per-
turbations. Originally there are 4 equations which linearly depend on $\alpha$, $\chi$, $\Psi$ and $\bar{\Psi}$, but after eliminating auxiliary fields $\alpha$ and $\chi$, we obtain only one equation for $P = \Psi - \bar{\Psi}$. The equation is expanded with respect to the parameter $\Gamma$ up to the linear order, and the results are given by the eqs. (43) and (46).

We also constructed the effective action for the scalar perturbations, and confirmed that the eqs. (43) and (46) can be reproduced from the effective action for $P$. Then we solved $P$ up to the linear order of $\Gamma$, as shown in the eq. (73), and obtained the power spectrum of the curvature perturbation $\psi$. As an initial condition, we assumed that the power spectrum of the curvature perturbation was some constant. Then the power spectrum is plotted against the time evolution in the fig. 1. The power spectrum is monotonically decreasing, but its tilt becomes mild after the inflationary era. We also plotted the power spectrum against the wave number in the fig. 2. The figure shows that the scalar spectral index becomes $n_s = 0.94$, if we fit the data for the wide range of the wave numbers. This is close to the value of current observation, and we should investigate more seriously the era after the inflation.

In the above analyses, we neglected the wave number $l_m$ of the internal space. So the next task is to include it and investigate the effect to the power spectrum. Of course, the tensor to scalar ratio should be evaluated in the above inflationary scenario. As a future work, it is interesting apply the method developed here to more complicated internal geometry, such as $G_2$ manifold [43]. It is also interesting to apply the analyses of this paper to the heterotic superstring theory with nontrivial internal space, which contains $R^2$ corrections [44], and reveal several problems in string cosmology [45].

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A Supplementary Notes

The explicit form of $U_{11}$ in the eq. (72) is given by

$$U_{11}(k\eta)$$

$$= \sqrt{\pi} J_0(k\eta) \left\{ 7350(13881 + 3029\sqrt{21})k^4\eta^4 G_{3,5}^{2,2}(k\eta, \frac{1}{2}, \frac{7+\sqrt{21}}{2}, \frac{7-\sqrt{21}}{2}, 0, 0, 0, 0, 0, 0, 0) 
- 1960(1712457 + 373688\sqrt{21})k^3\eta^3 G_{3,5}^{2,2}(k\eta, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{9+\sqrt{21}}{2}, 0) 
- (37841511589 + 8257680071\sqrt{21})k^2\eta^2 G_{3,5}^{2,2}(k\eta, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{9+\sqrt{21}}{2}, 0, 0, 0, 0, 0, 0) 
+ 78(1705246067 + 372115053\sqrt{21})k\eta G_{3,5}^{2,2}(k\eta, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0) \right\}$$

(80)

$$+ 2\pi Y_0(k\eta) \left\{ 7350(1449 + 316\sqrt{21})k^4\eta^4 F_3 \left( \frac{1}{2}, -\frac{5+\sqrt{21}}{2}; 1, 1, -\frac{3+\sqrt{21}}{2}; -k^2\eta^2 \right) 
- 245(714837 + 155983\sqrt{21})k^4\eta^4 F_3 \left( \frac{3}{2}, -\frac{5+\sqrt{21}}{2}; 2, 2, -\frac{3+\sqrt{21}}{2}; -k^2\eta^2 \right) 
- (3267117844 + 71937461\sqrt{21})k^2\eta^2 F_3 \left( \frac{1}{2}, -\frac{7+\sqrt{21}}{2}; 1, 1, -\frac{5+\sqrt{21}}{2}; -k^2\eta^2 \right) 
+ 39(147225227 + 32127118\sqrt{21})k^2\eta^2 F_3 \left( \frac{3}{2}, -\frac{7+\sqrt{21}}{2}; 2, 2, -\frac{5+\sqrt{21}}{2}; -k^2\eta^2 \right) 
- 6(1371162219 + 299210621\sqrt{21})k F_3 \left( \frac{1}{2}, -\frac{9+\sqrt{21}}{2}; 1, 1, -\frac{7+\sqrt{21}}{2}; -k^2\eta^2 \right) \right\}.$$
\[-312(35061208441 + 7650982944\sqrt{21})k\eta G_{3,5} \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 10 + \sqrt{21} \\ -1/2, -1/2, 1/2, 0, -1/2 \end{array} \right. \]

\[+ 72(127640510767 + 27853443103\sqrt{21})G_{3,5} \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 11 + \sqrt{21} \\ 1/2, \frac{1}{2}, 1/2, 0, -1/2 \end{array} \right. \}

\[+ Y_0(k\eta) \left\{ 7350(570801 + 124559\sqrt{21}) \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 7 + \sqrt{21} \\ 0, -1/2, -1, 0 \end{array} \right. \right. \]

\[-980(140837769 + 30733321\sqrt{21})k \eta G_{2,3} \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 6 + \sqrt{21} \\ -1/2, -1/2, 0, -1/2 \end{array} \right. \]

\[-2(778050877142 + 169784621803\sqrt{21})k^2 \eta^2 G_{2,3} \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 9 + \sqrt{21} \\ 0, -1/2, -1/2, 0 \end{array} \right. \]

\[+ 156(35061208441 + 7650982944\sqrt{21})k\eta G_{2,3} \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 8 + \sqrt{21} \\ 0, -1/2, -1/2, 0 \end{array} \right. \]

\[-36(127640510767 + 27853443103\sqrt{21})G_{2,3} \left( k\eta, \frac{1}{2} \right) \left| \begin{array}{c} 7 + \sqrt{21} \\ 0, -1/2, -1/2, 0 \end{array} \right. \}

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