A Fractional-Order Transitional Butterworth-Butterworth Filter and Its Experimental Validation

SHIBENDU MAHATA, NORBERT HERENC&SAR, DAVID KUBANEK, RAJIB KAR, DURBADAL MANDAL, AND İ. CEM GÖKNAR

1Department of Instrumentation and Electronics Engineering, Dr. B. C. Roy Engineering College, Durgapur, West Bengal 713206, India
2Department of Telecommunications, Faculty of Electrical Engineering and Communication, Brno University of Technology, 61600 Brno, Czechia
3Department of Electronics and Communication Engineering, National Institute of Technology Durgapur, Durgapur 713209, India
4Department of Electrical and Electronics Engineering, Işik University, 34980 Şile/Istanbul, Turkey

Corresponding author: Norbert Herencsár (herencsarn@ieee.org)

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ABSTRACT

This paper introduces the generalization of the classical Transitional Butterworth-Butterworth Filter (TBBF) to the Fractional-Order (FO) domain. Stable rational approximants of the FO-TBBF are optimally realized. Several design examples demonstrate the robustness and modeling efficacy of the proposed method. Practical circuit implementation using the current feedback operational amplifier employed as an active element is presented. Experimental results endorse good agreement ($R^2 = 0.999968$) with the theoretical magnitude-frequency characteristic.

INDEX TERMS

Analog filter approximation, analog signal processing, current feedback operational amplifier, fractional-order filter, transitional filter.

I. INTRODUCTION

The modeling techniques and realization of classical (integer-order) analog filters are well-established. To further improve the performance of such filters (e.g., reduction in passband error, sharper transition-band characteristic), the use of graphical methods [1] and optimal procedures [2]–[4] have been adopted.

Recently, the theoretical concept of fractional calculus, which deals with the generalization of the classical definitions of differentiation and integration, has been applied to achieve a more precise attenuation behavior of analog filters [5]. This is possible due to the generalization of the classical Laplacian operator $s$ to the Fractional-Order (FO) form $s^\alpha$, where $\alpha \in (0, 1)$, which causes additional degrees of freedom in system modeling. The impedance function containing the $s^\alpha$ operator may be realized using fractance devices or Constant Phase Elements (CPE) [6]. Due to the commercial unavailability of these devices, CPE emulators in the integrated form [7] or discrete-components-based [8] have been reported. The $s^\alpha$ operator forms the basic building block of the FO transfer functions, which can lead to generalizations of classical Butterworth filter [9], oscillators [10], and resonators [11]. Both active and passive elements have been employed to realize the FO impedances [12], [13]. Another popular method is to approximate the FO system using the integer-order transfer function [14]. The exact dynamics of a FO system can be theoretically achieved by a system of infinite integer order. For practical purposes, the characteristics of the FO filter need to be approximated using a finite-order rational approximant. An integer-order model of lower-order is desirable since it results in smaller hardware overhead. The rational approximation of $s^\alpha$ may be achieved using frequency-domain-based curve fitting [15], a weighted sum of first-order optimal high-pass filter sections [16], etc.

Transitional filters merge the frequency responses of various classical filters (e.g., Butterworth, Chebyshev, Bessel, Legendre, Thomson) to attain conciliation between the amplitude and group delay characteristics [17]. Transitional filters may be designed by combining different filter poles using the arithmetic or geometric interpolation, as exemplified by the transitional Legendre-Thomson filter [18], and the transitional ultraspherical-ultraspherical filter [19]. An alternative
design technique involves combining the classical filter polynomials [20]. The magnitude squared function of the classical Transitional Butterworth-Butterworth Filter (TBBF) is given by (1) [20]:

\[
|H(j\omega)|^2 = \frac{1}{1 + \varepsilon^2(\omega^{2n} + \omega^{2k})},
\]

where \( n \) and \( k \) are integers, \( 0 \leq k \leq n \); \( \varepsilon \) is the ripple constant; and \( \omega \) is the angular frequency in radians per second (rad/s). For \( n = k \), and rewriting the ripple constant as \( \varepsilon / \sqrt{2} \), the magnitude characteristics of the \( n \)th order Butterworth filter can be also obtained from (1). The response of the TBBF comprises the arithmetic interpolation between two classical Butterworth filters. It may be inferred from (1) that for \( n > k \), the dominating responses in the passband and stopband regions are due to the \( k \)th order and the \( n \)th order Butterworth filters, respectively. Hence, the passband and stopband responses of the TBBF can be nearly independently adjusted.

Optimization techniques were employed to approximate the characteristics of the FO Butterworth Filter (FOBF) [21], [22]. However, to the best of the authors’ knowledge, no literature exists on the FO modeling of TBBFs. This paper introduces the definition of FO-TBBF characteristic by removing the restrictions of integer values for \( n \) and \( k \) imposed in (1). Optimal rational approximations are proposed, which can meet the theoretical magnitude-frequency behavior of the FO-TBBF. Design stability is ensured by representing the denominator polynomial of the proposed model as a cascade of first-order and second-order terms comprising positive coefficients. Thus, inequality constraints are avoided to meet the s-domain stability criteria. Table 1 compares the advantages and limitations of the proposed method with those of the FOBF [9], [21], [22], and TBBF [20] design techniques. Several design cases are considered to evaluate the performance of the proposed technique. Current Feedback Operational Amplifier (CFOA) [23] based hardware circuit implementation of the proposed FO-TBBF approximant is demonstrated. Simulation and experimental results confirm excellent agreement with the ideal magnitude characteristics.

In the rest of the paper, the proposed technique is presented in Section II. MATLAB simulations are carried out to highlight the modeling efficiency in Section III. Section IV presents the circuit implementation and measurement results, while conclusions are drawn in Section V.

### Table 1. Comparison with the existing FOBF and TBBF design techniques.

| Reference | Design | Advantages | Limitations |
|-----------|--------|------------|-------------|
| [9]       | FOBF   | Analytically derived expressions for model coefficients | Cannot model TBBF characteristics; Non-optimal technique |
| [20]      | TBBF   | Based on interpolation of classical Butterworth filters | Cannot model fractional-step characteristics of TBBFs |
| [21], [22]| FOBF   | Circuit realization does not require CFOE or CFOE emulators | Cannot model TBBF characteristics; Stability conditions based on both bound and nonlinear inequality constraints |
| Present Work | FO-TBBF | Optimal TBBF models exhibiting fractional-step behavior; Guaranteed stability using lower bound constraints only | Iterative method |

### II. DESIGN METHODOLOGY

#### A. DEFINITION

The theoretical squared-magnitude function for the FO-TBBF is proposed according to (2):

\[
|B(j\omega)|^2 = \frac{1}{1 + \varepsilon^2(\omega^{2(n+\alpha)} + \omega^{2(n+\beta)})},
\]

where \( n_1 \) and \( n_2 \) are integer numbers, \( \alpha, \beta \in [0, 1] \), and \( (n_1 + \alpha) \geq (n_2 + \beta) \). For \( \alpha = \beta = 0 \) and 1, the TBBF can be treated as a special case of the FO-TBBF. Note that (2) may yield the definition of a FOBF when \( (n_1 + \alpha) = (n_2 + \beta) \). The proposed definition also allows the exponents of \( \omega \) in (2) to attain any value between 0 and 2, which is not possible using the classical TBBF.

#### B. PROPOSED TECHNIQUE

The proposed FO-TBBF approximant \( G(s) \) is modeled according to (3):

\[
G(s) = \begin{cases} 
  \frac{k(s^2 + z_1s + z_2)}{(s + p_1)(s + p_2)...(s + p_{n_1+3})}, & \text{if } n_1 \text{ is odd;} \\
  \prod_{i=1}^{(n_1+3)/2} (s^2 + p_is + p_i'), & \text{if } n_1 \text{ is even.}
\end{cases}
\]

The resulting integer order of the approximant \( G(s) \), as defined by (3), is determined as \( n_1 + 3 \). The approximation of the magnitude-frequency response of the normalized FO-TBBF is formulated as an optimization problem by minimizing the cost function \( f \), as proposed in (4):

\[
f = \sum_{i=1}^{L} [20 \log_{10} |B(j\omega_i)| - 20 \log_{10} |G(j\omega_i, X)|]^2.
\]

Subject to:

\[
\begin{align*}
  z_i > 0, & (i = 1, 2); \\
  p_i, p_i' > 0, & (i = 1, 2, \ldots, \frac{n_1+3}{2}) \text{ if } n_1 \text{ is odd;} \\
  p_0 > 0, p_i, p_i' > 0, & (i = 1, 2, \ldots, \frac{n_1+2}{2}) \text{ if } n_1 \text{ is even.}
\end{align*}
\]

where \( L \) denotes the total number of frequency points logarithmically distributed in the interval \( \omega \in [\omega_{\min}, \omega_{\max}] \) rad/s; and \( X \) represents the vector of decision variables. For odd values of \( n_1, X = [k z_1 z_2 p_1 p_1' p_2 p_2' \cdots p_{(n_1+3)/2} p_{(n_1+3)/2}']; \)
TABLE 2. Optimal design variables vector (X) and the coefficient of determination (R²) for the proposed FO-TBBFs.

| No. | n₁ | n₂ | α  | β   | X          | R²     |
|-----|----|----|----|-----|------------|--------|
| 1   | 0  | 0  | 0.8| 0.5 | 3.4577    | 20.3781 |
| 2   | 1  | 0  | 0.6| 0.8 | 10.7612   | 40.2385 |
| 3   | 1  | 1  | 0.9| 0.2 | 2.2825    | 35.6773 |
| 4   | 2  | 0  | 0.4| 0.7 | 30.9803   | 30.5457 |
| 5   | 1  | 0.5| 0.5| 1   | 18.6865   | 41.7971 |
| 6   | 2  | 0.8| 0.1| 1   | 3.8256    | 44.7456 |
| 7   | 3  | 0.1| 0.3| 1   | 206.5309  | 14.8654 |
| 8   | 3  | 0.7| 0.6| 1   | 6.4343    | 45.9104 |
| 9   | 2  | 0.5| 0.2| 1   | 19.0725   | 43.1617 |
| 10  | 3  | 0.9| 0.4| 1   | 2.3133    | 47.3617 |
| 11  | 4  | 0.2| 0.4| 1   | 96.0684   | 17.5756 |
| 12  | 4  | 0.6| 0.3| 1   | 11.2103   | 49.1477 |
| 13  | 4  | 0.8| 0.9| 1   | 3.9069    | 54.3350 |
| 14  | 4  | 0.7| 0.7| 1   | 6.6053    | 53.0182 |
| 15  | 4  | 0.9| 0.1| 1   | 2.3428    | 57.3502 |

C. ALGORITHM IMPLEMENTATION

Algorithm 1 presents the pseudocode to implement the proposed optimization routine for a single trial run. In order to guarantee the generation of a stable rational approximant, the lower bound (Lb) for all decision variables (except \( L_b \)) for all decision variables is set as \( 10^{-2} \); in the case of \( k, Lb \) may be fixed as 0. A large value of \( Ub \) needs to be avoided since it may result in a large dispersion of the decision variables. A wide variation in the coefficients of the FO-TBBF transfer function will lead to larger spreading (ranging from a few ohms to several mega-ohms) in the values of passive components, which is undesirable for the practical implementation. To attain the passive components values within practical limits, \( Ub \) may be chosen as 1000. The initial point is randomly varied between \( Lb \) and \( (Lb + c) \), where \( c \in \mathbb{Z}^+ \). A single trial run of the optimization algorithm generates an \( iter \) number of solutions; the best solution (\( X_{best} \)) is the one that attains the smallest value of the error fitness function (\( f_{min} \)). Table 3 presents the minimum (min), maximum (max), mean, and standard deviation (SD) indices of \( f_{min} \) for all considered design cases based on 30 runs. Out of the 15 examples, cases yield the same fitness values for min, max, and mean indices. This implies that the same solution quality is obtained irrespective of the number of independent trial runs of the optimization technique. The excellent robustness of the proposed technique is further highlighted by the small value of the SD index.

Algorithm 1 The Proposed Algorithm Pseudocode

Inputs: \( n_1, n_2, \alpha, \beta \)

Outputs: \( X_{best}, f_{min} \)

1 begin
2 set \( \omega_{min}, \omega_{max}, L, iter, c, Lb, Ub \)
3 for \( i = 1 \) to \( iter \) do
4 \( X_0(\text{Initial point of X}) \in \text{rand}(Lb, Lb + c) \)
5 \( B(s) \leftarrow k s^2 + k z s + k z^2 \)
6 if \( (n_1 == \text{Odd}) \) then
7 \( A(s) \leftarrow 1 \)
8 for \( j = 1 \) to \( (n_1 + 3)/2 \) do
9 \( A(s) \leftarrow A(s) \times (s^2 + p j s + p j^2) \)
10 else
11 \( A(s) \leftarrow (s + p_0) \)
12 for \( j = 1 \) to \( (n_1 + 2)/2 \) do
13 \( A(s) \leftarrow A(s) \times (s^2 + p j s + p j^2) \)
14 minimize \( (4) \) and store \( f_i \)
15 store \( X_i \)
16 \( f_{min} \leftarrow \text{min}(f_i) \)
17 \( X_{best} \leftarrow X_i \) corresponding to \( f_{min} \)
18 end

III. SIMULATION RESULTS

The MATLAB based optimization routine uses the solver fincon (algorithm: ‘active-set’) with the following arguments: \( \text{MaxFunEvals} = 50000; \text{MaxIter} = 5000; \text{ToI Func} = 1E-10; \) and \( \text{ToIF X} = 1E-10 \). The optimal values of the decision variables for 15 design examples, with \( [\omega_{min}, \omega_{max}] = [10^{-2}, 10^2] \) rad/s, \( \epsilon^2 = 0.5, L = 50, \text{iter} = 100, c = 10, \) and \( Ub = 1000 \), are presented in Table 2. To quantify the effectiveness of the modeling accuracy, the coefficient of determination (R²) index (evaluated for L magnitude-frequency data sample points) is also shown in Table 2. A higher value of R² (in ideal case 1) indicates a better fitting of the proposed model to the theoretical one. Except for no. 1 and 4 \( [(n_1, n_2, \alpha, \beta) = (0, 0, 0.8, 0.5) \) and \( (2, 0, 0.4, 0.7)] \), all other designs achieve \( R^2 > 0.9999 \), which highlights a good agreement in the magnitude responses of the optimal model with the ideal FO-TBBF. The proposed method can also attain the same solution quality for other values of \( c \), such as 100 and 1000. The magnitude plot of the proposed model for no. 1 \( [(n_1, n_2, \alpha, \beta) = (0, 0, 0.8, 0.5)] \) attains agreement with the
TABLE 3. Statistical indices to evaluate the average performance of the fitness value based on 30 runs.

| No. | n1 | n2 | α   | β   | min   | max   | mean | SD  |
|-----|----|----|-----|-----|-------|-------|------|-----|
| 1   | 0  | 0  | 0.8 | 0.5 | 1.0377| 1.0377| 1.0377| 2.78E-10 |
| 2   | 1  | 0  | 0.6 | 0.8 | 0.7588| 0.7794| 0.7770| 1.80E-3  |
| 3   | 1  | 1  | 0.9 | 0.2 | 0.5599| 0.5599| 0.5599| 2.51E-9  |
| 4   | 2  | 0  | 0.4 | 0.7 | 0.2139| 0.2139| 0.2139| 1.35E-9  |
| 5   | 2  | 1  | 0.5 | 0.5 | 0.2139| 0.2139| 0.2139| 1.35E-9  |
| 6   | 2  | 2  | 0.8 | 0.1 | 0.0427| 0.0427| 0.0427| 2.07E-9  |
| 7   | 3  | 0  | 0.1 | 0.3 | 0.0191| 0.0191| 0.0191| 1.14E-9  |
| 8   | 3  | 1  | 0.7 | 0.6 | 0.0478| 0.0492| 0.0483| 6.51E-4  |
| 9   | 3  | 2  | 0.5 | 0.2 | 0.0399| 0.0399| 0.0399| 1.74E-9  |
| 10  | 3  | 3  | 0.9 | 0.4 | 0.0132| 0.0132| 0.0132| 3.34E-9  |
| 11  | 4  | 0  | 0.2 | 0.4 | 0.0285| 0.0285| 0.0285| 2.16E-9  |
| 12  | 4  | 1  | 0.6 | 0.3 | 0.0246| 0.0246| 0.0246| 2.90E-9  |
| 13  | 4  | 2  | 0.8 | 0.9 | 0.0140| 0.0140| 0.0140| 6.44E-9  |
| 14  | 4  | 3  | 0.7 | 0.7 | 0.0184| 0.0184| 0.0184| 4.50E-9  |
| 15  | 4  | 4  | 0.9 | 0.1 | 0.0046| 0.0046| 0.0046| 1.61E-8  |

FIGURE 1. MATLAB-simulated magnitude-frequency response comparison plots of the proposed FO-TBBFs. Note that the numbers within the parenthesis represent \((n_1, n_2, \alpha, \beta)\).

Theoretical behavior, as shown in Figure 1 (top). This figure also demonstrates that the roll-off characteristics for the FO-TBBF can extend below the one obtained for the lowest order of the classical TBBF \((n = k = 1)\). As another test case, Figure 1 (bottom) shows the magnitude response for the proposed design no. 5 \([(n_1, n_2, \alpha, \beta) = (2, 1, 0.5, 0.5)]\). A close match with the ideal characteristics \((R^2 = 0.999996)\) may be noted. The fractional stepping for the proposed FO-TBBF is highlighted in the same figure by presenting the magnitude plots of the TBBFs for \((n = 2, k = 1)\) and \((n = 3, k = 2)\).

Figure 2 (top) illustrates the smaller group delay of the proposed model no. 1 compared to the classical TBBF \((n = k = 1)\) reported in [20]. Group delay comparisons of the proposed approximant for no. 5 with the classical TBBFs cited in [20] for \((n = 2, k = 1)\) and \((n = 3, k = 2)\) are shown in Figure 2 (bottom). Results reveal that the group delay behavior of the optimal model lies in-between the responses of the classical filters.

To further highlight the FO modeling behavior of the proposed designs, the magnitude (top) and group delay (bottom) plots of the FO-TBBF for model no. 9 \([(n_1, n_2, \alpha, \beta) = (3, 2, 0.5, 0.2)]\) are compared with the classical TBBFs [20] for \((n = 3, k = 2)\) and \((n = 4, k = 3)\), as shown in Figure 3. Both these plots confirm that the proposed FO-TBBF can achieve the frequency responses which may not be yielded using the classical TBBFs.

The effectiveness of the proposed models in attaining a smaller group delay in the passband as compared to the FOBF is also demonstrated. For this purpose, the magnitude (top) and group delay (bottom) responses of the 1.6th-order FOBF reported in [22] are compared with the proposed FO-TBBF model no. 2 \([(n_1, n_2, \alpha, \beta) = (1, 0, 0.6, 0.8)]\), as presented in Figure 4. It may be noted that since the stopband attenuation characteristic for the FO-TBBF is dominated by the \((n_1 + \alpha)th\) order Butterworth filter, hence, the magnitude roll-off rate for the proposed FO-TBBF is similar to that of the FOBF of order 1.6. The magnitudes of the FO-TBBF at the frequencies of 10 rad/s and 100 rad/s are −29.06 dB and −61.08 dB, respectively. Therefore, the roll-off rate for the FO-TBBF is −32.02 decibel/decade (dB/dec), which is close to the theoretical value of −32.0 dB/dec obtained for...
The circuit realization of the FO-TBBF approximants is promising the stopband behavior. The complete circuit can be demonstrated using the CFOA employed in a follow-the-leader feedback topology [24]. The circuit comprises 6 CFOAs, 14 resistors, and 5 capacitors. The value of P is incremented from 1 to 4.

(iv) The passive components are selected from the E-24 standard industrial series for the resistors and the E-12 series for the capacitors. The following resistor values are set as: 

- $R_{F1} = 10$ kΩ, $R_{F2} = 10$ kΩ,
- $R_{F3} = 1$ kΩ, $R_{F4} = 10$ kΩ,
- $R_{G1} = 100$ kΩ, $R_{G2} = 10$ kΩ, $R_{G3} = 1$ kΩ, $R_{G4} = 10$ kΩ, and $R_{P1} = 270$ kΩ.

The values of the other R-C components are derived as follows: 

- $C_1 = 0.12$ nF, $C_2 = 10$ nF, $C_3 = 4.7$ nF, $C_4 = 15$ nF, $C_5 = 27$ nF, $R_2 = 7.5$ kΩ, and $R_3 = 750$ Ω.

The circuit for the proposed FO-TBBF no. 5 was assembled on a breadboard using the above listed R-C component values. The supply voltage for Analog Devices AD844AN amplifiers was provided by the Agilent E3630A power supply. The frequency responses of the FO-TBBF were measured by the OMICRON Lab Bode 100 network analyzer. 401 logarithmically spaced frequency points in the range 10 Hz to 100 kHz were considered. The level of the testing harmonic signal was set to 10 dBm (0.7071 V RMS). The receiver bandwidth of the analyzer was fixed at 10 Hz to obtain precise results.

The THRU calibration of the analyzer was performed before the measurement to eliminate the influence of the measurement setup. After connecting the proposed FO-TBBF circuit to the analyzer, the frequency responses were measured and displayed by the connected computer with the Bode Analyzer Suite software. The photograph of the hardware setup is presented in Figure 6.
The magnitude-frequency response measurements of the proposed FO-TBBF are compared with the ideal and simulated ones in Figure 7 (top). The practical filter demonstrates excellent agreement with the ideal characteristic up to nearly 70 kHz. The magnitude of the approximate at \( f_c = 1 \) kHz for measurement (–3.100 dB) demonstrates conformity with the ideal (–3.010 dB) and MATLAB simulations (–3.029 dB). \( R^2 \) of 0.999968 is achieved for the measured magnitude response data compared to the theoretical one. Figure 7 also depicts the experimental results for the phase (middle) and group delay frequency responses (bottom) of the FO-TBBF. Comparisons with the simulated plots highlight excellent matching of the phase plot for nearly 3 decades and the group delay for the entire design bandwidth.

V. CONCLUSION

Optimal and robust modeling of several frequency characteristics for the FO-TBBF is introduced. The generalization of the classical TBBF results in more precise control of the magnitude, phase, and group delay behaviors. The efficient modeling performance of the proposed technique is validated through numerical simulations and experiments made on CFOA-based circuit implementation.

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SHIBENDU MAHATA received the M.Tech. degree in instrumentation and electronics engineering from Jadavpur University, India, in 2010. He was a Field Operations and Panel Engineer with Reliance Industries Ltd., and a Research Assistant with the Department of Cybernetics, CSIR-CMERI. He has authored above 40 articles published in SCI-E peer-reviewed journals, conference proceedings, and book chapters. His research interests include optimal modeling of fractional-order analog and digital filters. He was awarded with the University Medal for his M.Tech. degree from Jadavpur University. He was a recipient of the 2019 Premium Award for Best Paper from IET Signal Processing.

NORBERT HERENCSAR (Senior Member, IEEE) received the Ph.D. degree from Brno University of Technology (BUT), Czech Republic, in 2010. From 2013 to 2014, he was a Visiting Researcher with Boğaziçi University, Turkey, and Doğuş University, Turkey, for five months. In 2015, he has been an Associate Professor with the Department of Telecommunications, BUT. Since 2015, he has been collaborating on numerous research projects supported by Czech Science Foundation. From 2016 to 2021, he was the Science Communications Manager and an MC Member of the COST Action CA15225 “Fractional-Order Systems-Analysis, Synthesis and Their Importance for Future Design.” He has authored 104 articles published in SCI-E peer-reviewed journals and 122 papers in conference proceedings. His research interests include analog electronics, bi阻imance modeling, energy storage elements, fractional-order circuits and systems, impedance spectroscopy, instrumentation and measurement, sensory processing circuits and systems, and VLSI analog integrated circuits. Since 2013, he has been an Organizing or a TPC Member of AFRICON, ELECO, ICMTC, ICUMT, IWSIP, SETCAS, MWSCAS, and ICECS conferences. He is a Senior Member of IACST and IRED, and a member of IAENG, ACEEE, and RS. From 2008 to 2016 and from 2017 to 2020, he was an Organizing Committee Member and the General Co-Chair of the International Conference on Telecommunications and Signal Processing (TSP). From 2015, he has been serving for the IEEE Czechoslovakia Section Executive Committee as an SP/CAS/COM Joint Chapter Chair. Since 2021, he has been the General Chair of TSP. Since 2011, he has been contributing as a Guest Co-Editor to several special journal issues in AEU—International Journal of Electronics and Communications, Applied Sciences, Radioengineering, Telecommunication Systems, and Sensors. Since 2014, he has been serving as an Associate Editor for IEEE Access, IEICE Electronics Express (ELEX), and Journal of Circuits, Systems and Computers; an Editorial Board Member for the Elektronika ir Elektrotechnika and Radioengineering; and a Topics Board Member for the Nanomaterials. In July 2021, he was appointed as the Editor-in-Chief of the Engineering Section of Fractal and Fractional. He is ranked among World’s Top 2% Scientists reported by Stanford University.

DAVID KUBANEK received the M.S. degree in electronics and communication and the Ph.D. degree in informatics from Brno University of Technology (BUT), Czech Republic, in 2002 and 2006, respectively. Since 2006, he has been an Assistant Professor with the Department of Telecommunications, BUT. He has authored 24 articles published in SCI-E peer-reviewed journals and about 35 papers in conference proceedings. He has participated on numerous research projects supported by Czech Science Foundation. His research interests include design and analysis of analog electronic circuits, devices and elements, frequency filters, oscillators, impedance converters, non-linear circuits, and fractional-order circuits and systems. Since 2019, he has been serving as an Editorial Board Member for Fractal and Fractional Journal.

RAJIB KAR (Senior Member, IEEE) received the B.E. degree in electronics and communication engineering from Regional Engineering College, Durgapur, West Bengal, India, in 2001, and the M.Tech. and Ph.D. degrees from the National Institute of Technology, Durgapur, West Bengal, India, in 2008 and 2011, respectively. He is currently attached with the National Institute of Technology, Durgapur, as an Associate Professor with the Department of Electronics and Communication Engineering. He has published more than 130 research articles in international journals. He has guided ten Ph.D. students in his domain of research. His research interests include VLSI circuit optimization and signal processing via evolutionary computing techniques. He was awarded Visvesaraya YFPR by Meity, GOI, in 2016.

DURBADAL MANDAL (Member, IEEE) received the B.E., M.Tech., and Ph.D. degrees from the National Institute of Technology, Durgapur, West Bengal, India. He is currently attached with the National Institute of Technology, Durgapur, as an Associate Professor with the Department of Electronics and Communication Engineering. He has published more than 300 research papers in international journals and conferences. He has produced 13 Ph.D. students to date. His research interests include array antenna design and digital filter optimization via evolutionary optimization techniques.

İ. CEM GÖKNAR (Life Fellow, IEEE) was born in Istanbul, Turkey. He received the B.Sc. and M.Sc. degrees from Istanbul Technical University and the Ph.D. degree from Michigan State University, in 1969. He was a Visiting Professor at the University of California at Berkeley, the University of Illinois at Urbana-Champaign, the University of Waterloo, ON, Canada, and Technical University of Denmark, Lyngby. He gave lectures at the University of Sannio, Italy; the University of Lisbon, Portugal; and Brno University of Technology, Czech Republic. He was the Director of the Science and Technology Institute, İşık University, where he is currently a Professor. He has been on the European Circuit Society Council, in 1995, which he chaired from 2009 to 2011. He received NATO’s Senior Scientist Grant (1974) and Minma-James-Heinemann-Stiftung Award (1980). He was the IEEE-CAS Chapter Chair, Turkey, which received the IEEE CAS-Tr Chapter of the Year Award, in 2014.

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