Sound absorption performance of underwater anechoic coating in plane wave normal incidence condition

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Abstract. On account of the theory of non-uniform waveguide and the wave propagation theory in layered media, a complete theoretical model is established for predicting the sound absorption performance of multi-layer underwater composite anechoic coating. Furthermore, the correctness of the theoretical calculation is verified by the finite-element simulation results. Then, the velocity nephograms corresponding the three absorption peak frequencies of the overburden are analyzed to reveal the sound-absorbing mechanism.

1. Introduction
With the increasing demand for acoustic performance of modern underwater equipment, some measures related to vibration damping and noise reduction continue to emerge. Among them, the installation of the underwater anechoic coating on the outer surface of the equipment is an important move, which has the effects of reducing the acoustic target strength of structure itself, suppressing the structural sound radiation, and improving the sound insulation of internal noise. Therefore, it is necessary to study the sound absorption properties of the underwater anechoic overburden.

The research methods for the acoustic properties of sound-absorbing coatings vary depending on their structural forms. For example, Stepanishen and Strozeski [1] analyzed the reflection-transmission performance of a layered viscoelastic lining by transfer matrix method (TMM). Gao et al. [2] proposed a Generalized Multiscale Finite-Element Method (GMsFEM) to study the wave propagation characteristics in non-homogeneous coatings. Haberman et al. [3] and Bonfoh et al. [4] studied the material and acoustic properties of periodically scattering composite sound-absorbing structures containing spherical inclusions and ellipsoidal coated particles, respectively, through self-consistent theory and generalized self-consistent theory. Tao [5] established a complete two-dimensional theoretical model for predicting the acoustic performances of an overburden embracing periodic cylindrical or conical cavities. In addition, for anechoic coatings with complex cavity structures such as spherical, ellipsoidal, super-ellipsoid and non-spherical cavums, Ivansson [6] investigated the sound-deadening effect by finite element method (FEM). Furthermore, Zhong et al. [7] also adopted the FEM to explore the low-frequency sound-absorbing property of a viscoelastic overburden embodying periodic horizontal cavities with infinitely long eccentric cylinders.

Therefore, this paper proposes a multi-layer underwater composite anechoic coating, which is formed by embedding two heterogeneous linings containing periodic cavity structures in two homogeneous plates. Meanwhile, we introduce the theory of non-uniform waveguide and combine the wave propagation theory in layered media to investigate the sound absorption performance of the present coating.
2. Theoretical modeling

2.1. Structural elaboration

The underwater anechoic coating shown in Fig. 1 consists of four rubber layers, labeled from top to bottom as layers \( m \) with thicknesses \( h_m \) (\( m = 1,2,3,4 \)), respectively, wherein layers 1 and 4 are homogeneous linings, and layers 2 and 3 are heterogeneous overburdens containing periodic cavity structures. The coating divides the service water environment into two upper and lower semi-infinite spaces, namely the water and backing conditions in Fig. 1(a), marked as layers 0 and 5, respectively. Moreover, the two diameters, small upper diameter, and large lower diameter, of the truncated cone cavity in layer 2 are respectively \( r_1 \) and \( r_2 \), while the diameter of the cylindrical cavity in layer 3 is \( r_2 \), as illustrated in Fig. 1(b). In addition, the basic structure of the blue dotted square in Fig. 1(b) is a periodic unit having the length-width dimension of \( l_b \). Furthermore, a global Cartesian coordinate system is set up on the upper surface of the coating, where the \( x \)- and \( y \)-axes are along the array directions of periodic cavities, and the \( z \)-axis is perpendicular to the \( xy \)-plane downward. When a plane wave having a sound pressure amplitude of \( p_i \) impinges on the upper surface of the overburden normally, the reflected and transmitted waves with amplitudes \( p_r \) and \( p_t \) are derived respectively.

\[
\begin{bmatrix}
    p_{1,m} \\
    v_{1,m}
\end{bmatrix} = \begin{bmatrix}
    \cos(k_m h_m) & ip_m c_m \sin(k_m h_m) \\
    i \sin(k_m h_m) & P_m c_m \cos(k_m h_m)
\end{bmatrix} \begin{bmatrix}
    p_{2,m} \\
    v_{2,m}
\end{bmatrix} = A^\mathrm{pre}_m \begin{bmatrix}
    p_{2,m} \\
    v_{2,m}
\end{bmatrix}, \tag{1}
\]

where subscript \( m \) represents the \( m \)-th layer in the entire overburden, while subscript 1 and 2 indicate the front and rear ends of the coating; superscript \( \mathrm{pre} \) denotes the pressure state; \( \rho, c, h, k, A \) are respectively the density, complex sound velocity, thickness, complex wavenumber, and transfer matrix of the homogeneous layer; \( i (i^2 = -1) \) is the imaginary unit.

Thus, the transfer matrix of a homogeneous layer in the force state is

![Figure 1](image-url). Schematic diagram of the present coating: (a) assembly drawing and (b) sectional view.

2.2. Matrix synthesis and absorption calculation

For a single homogeneous lining similar to layer 1 or 4, the relationship between sound pressure and particle velocity on the front and rear interfaces is [8]:
with
\[ A_m = S_e \otimes A_m^{\text{pre}}, \]  
\[ S_e = \begin{bmatrix} 1 & s_b \\ 1/s_b & 1 \end{bmatrix}, \]
where symbol \( \otimes \) indicates the Kronecker product between matrices, \( s_b = l_b^2 \) is the sectional area of one periodic element.

In layer 2 with periodic truncated cone cavities, the propagation of sound waves in the coating is similar to that in a highly viscous liquid with variable cross-section waveguides, so the wave equation is [9]
\[ \frac{\partial^2 \xi}{\partial z^2} + \frac{1}{s} \frac{\partial \xi}{\partial z} + k^2 \xi = 0, \]
where \( \xi \) and \( s \) are the particle displacement and cross-sectional area of the coating, respectively, and both are all function of \( z \). Therefore, the expression is a nonlinear equation, which can be transformed into a linear equation to obtain the analytic solution only when the medium cross-sectional area \( s(z) \) satisfies the following formula:
\[ (\sqrt{s})''/\sqrt{s} = \mu^2 \quad (\mu = \text{constant}). \]

Since the total pressure and particle velocity on the front and rear interfaces in the heterogeneous layer 2 are continuous, the cone-shaped waveguide \( \sqrt{s(z)} = az + b \) is used to approximate the truncated cone cavity, and the transfer relations of \( F_1, F_2, v_1, \) and \( v_2 \) can be expressed as
\[ \begin{bmatrix} F_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} b_{2,11} & b_{2,12} \\ b_{2,21} & b_{2,22} \end{bmatrix} \begin{bmatrix} F_2 \\ v_2 \end{bmatrix}, \]
with
\[ b_{2,11} = \sqrt{s_1/s_2} \cos (K_2 h_2) + \frac{1}{K_2 h_2} \left( \frac{s_2}{s_1} - 1 \right) \sin (K_2 h_2) \]
\[ b_{2,12} = \frac{i \rho_s c_2 \sqrt{s_1 s_2}}{k} \left( \frac{\cos (K_2 h_2)}{h_2} \right) \left( 2 - \frac{s_1}{s_2} - \frac{s_2}{s_1} \right) + K_2 \sin (K_2 h_2) \left[ 1 - \left( \frac{1}{K_2 h_2} \right)^2 \left( 1 - \frac{s_1}{s_2} \right) \right] \]
\[ b_{2,21} = \frac{i k_2}{K_2 \rho_s c_2 \sqrt{s_1 s_2}} \sin (K_2 h_2) \]
\[ b_{2,22} = \sqrt{s_2/s_1} \cos (K_2 h_2) - \frac{1}{K_2 h_2} \left( \frac{s_1}{s_2} - 1 \right) \sin (K_2 h_2) \]
where \( K_2 = \sqrt{k^2 - \mu^2} \); \( s_1 = l_1^2 - \pi r_1^2 \) and \( s_2 = l_2^2 - \pi r_2^2 \) are the upper and lower interface cross-sectional areas of layer, respectively.

However, for the inhomogeneous layer 3 containing periodical cavities, which is a special case of layer 2, with \( s_1 = s_2 = l_p^2 - \pi r_2^2 \) and \( K_3 = k_3 \), so the transfer matrix of sound pressure and particle velocity on both end faces of layer 3 is
\[ B_3 = \begin{bmatrix} \cos (k_3 h_3) & i \rho_s c_2 s_2 \sin (k_3 h_3) \\ i \sin (k_3 h_3) & \rho_s c_2 s_2 \cos (k_3 h_3) \end{bmatrix}, \]
Moreover, the global transfer matrix of the present coating in the force state can be assembled as follows:
\[ T = A_1(h_1) \times B_2(h_2) \times B_3(h_3) \times A_4(h_4). \]  
\[ T = A_1(h_1) \times B_2(h_2) \times B_3(h_3) \times A_4(h_4). \]

Then, the reflection and absorption coefficients at the incident interface of the coating are:

\[ r = \frac{Z_{in} - Z_w}{Z_{in} + Z_w}, \]  
\[ r = \frac{Z_{in} - Z_w}{Z_{in} + Z_w}, \]

\[ \alpha = 1 - r \cdot r^*, \]  
\[ \alpha = 1 - r \cdot r^*, \]

with

\[ Z_{in} = \frac{T_1F_5 + T_4v_5}{s_b(T_{21}F_5 + T_{22}v_5)}, \]  
\[ Z_{in} = \frac{T_1F_5 + T_4v_5}{s_b(T_{21}F_5 + T_{22}v_5)}, \]

where superscript * denotes the conjugate of the reflection coefficient, \( Z_w = \rho_w c_w \) is the characteristic impedance of the aqueous medium in the 0-th layer.

3. Model validations and discussions

3.1. Simulation verification

The value of the present theoretical model is determined by both the calculation accuracy and computation rate, in which the model validity is an indispensable basis. For the underwater anechoic coating as shown in Fig. 1, the materials of layers 1 and 4 are selected as high-elastic rubber (HER), with densities \( \rho_1 = \rho_4 = 1300 \text{ kg/m}^3 \), Young’s moduli \( E_1 = E_4 = 4.96 \times 10^7(1+i) \text{ Pa} \), Poisson ratios \( \nu_1 = \nu_2 = 0.47 \), and thicknesses \( h_1 = 0.5h_4 = 15 \text{ mm} \). Moreover, the material properties of ordinary conventional rubber (OCR) in layers 2 and 3 are \( \rho_2 = \rho_3 = 1600 \text{ kg/m}^3 \), \( E_2 = E_3 = 2.98 \times 10^8(1+0.4i) \text{ Pa} \), \( \nu_1 = \nu_2 = 0.37 \), \( h_2 = 3h_3 = 30 \text{ mm} \), and \( r_1 = 0.5r_2 = 6 \text{ mm} \). The density and sound velocity of water in the incident medium are \( \rho_w = 1000 \text{ kg/m}^3 \) and \( c_w = 1500 \text{ m/s} \), respectively, while the backing condition is rigid.

Then, a fully coupled finite-element model of the present anechoic coating shown in Fig. 2 is established by COMSOL software in accordance with the following four steps: first, using the perfectly matched layer (PML) to simulate the semi-infinite water space. Second, employing the Floquet periodicity boundaries on the x- and y-direction planes in one cell to elaborate an infinite lining. Again, approximating the rigid backing by the fixed constraint boundary with vibration velocity of zero. Finally, treating the fluid-solid interfaces between fluids such as water or air and rubbers by the acoustic-structure boundaries.

![Figure 2](image-url)  
**Figure 2.** Acoustic-structure fully coupled finite-element model of the present coating: (a) multiphysics geometry model and (b) mesh model.
Therefore, based on the two methods of theoretical calculation and simulation analysis, the sound absorption coefficients of the underwater anechoic overburden are compared as illustrated in Fig. 3. Obviously, the trends of the two curves are consistent, and they all fluctuate with increasing frequency and eventually become stable. In addition, the two highly coincident curves rise rapidly to 0.6, which just shows that the present coating’s design is reasonable and the developed theoretical model has higher accuracy.

![Graph showing sound absorption coefficients](image)

**Figure 3.** Comparison between the theoretical analysis and simulation result of the present coating.

### 3.2. Sound-absorbing mechanism exploration

It can be seen from Fig. 3 that the two curves of theoretical calculation and simulation result have three peak frequencies in the research frequency range of 10Hz ~ 10 kHz. In order to explore the energy dissipation mechanisms for the absorbing peaks in the sound absorption curve of the present overburden, the vibration velocity of the coating in the whole research frequency range is simulated and analyzed. The velocity nephograms corresponding to the three peak frequencies are displayed in Fig. 4.

![Velocity nephograms at three peak frequencies](image)

**Figure 4.** Velocity nephograms of the present anechoic coating at three peak frequencies: (a) 464.16 Hz, (b) 3274.55 Hz and (c) 7564.63 Hz.

From this diagram, we can know that the mechanisms for generating the respective sound absorption peaks are different. For the first peak frequency in the low-frequency band, the sound-absorbing peak at 464.16 Hz is mainly attributable to the axial piston-like resonance energy dissipation generated by the layer 1 coupling the truncated cone cavities in layer 2, as shown in Fig. 4(a), the velocity magnitudes of layers 1 and 2 are dominant. As can be seen from Fig. 4(b), the vibration velocity magnitudes of the heterogeneous layers 2 and 3 are significantly concentrated and superior to other layers. Therefore, the two sound-absorbing modes, the radial drum-like resonance energy consumption of the periodic cavity structures in layers 2 and 3, and the waveform conversion in layer 2, synergistically cause the second sound absorption crests at the frequency 3274.55 Hz in the
mid-frequency band. However, for the velocity contour at 7564.63 Hz in the high-frequency range depicted in Fig. 4(c), a number of distinct velocity stratifications are formed within the anechoic coating, while the interface between layer 1 and the aqueous medium is the most intense. This is because the viscous energy dissipation in the anechoic coating plays a major role, so that the sound absorption coefficient of the entire structure is stable in the high-frequency region.

4. Conclusions
This paper concerns with the sound absorption performance of an underwater composite anechoic coating. It is composed of four stratified rubber structures, in which the upper and lower layers are homogeneous linings, while the center two layers are heterogeneous overlays, each containing periodically truncated cone cavums and cylindrical cavities. On account of the theory of non-uniform waveguide and the wave propagation theory in layered media, a complete theoretical model is established for analyzing the acoustic properties of the present coating.

Furthermore, the homogeneous layers 1 and 4 are selected to be high-elastic rubber, and the ordinary conventional rubber is used in place of the non-homogeneous layers 2 and 3, thereby completing the following two aspects of research work. One is to verify the correctness of the developed theoretical model by finite element simulation method. The other is to study and analyze the velocity nephograms corresponding to the three peak frequencies to explore the absorbing mechanisms of the sound absorption crests. Thus, the following two main conclusions are drawn: first, the present theoretical model has higher accuracy. Second, the sound-absorbing mechanisms for the three absorption peaks of the underwater anechoic coating are different, respectively, by the axial piston-like resonance energy dissipation, radial drum-like resonance energy consumption coupled waveform conversion, viscous energy dissipation.

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