Small-$x$ evolution with $Q$ dependence and unitarity

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Abstract

We propose a modified Balitsky-Fadin-Kuraev-Lipatov equation from the viewpoint of the resummation technique, which contains an intrinsic dependence on momentum transfer $Q$, and satisfies the unitarity bound. The idea is to relax the strong rapidity ordering and to restrict phase space for real gluon emissions in the evaluation of the BFKL kernel. It is shown that the power-law rise of the gluon distribution function with the small Bjorken variable $x$ turns into a logarithmic rise, and that the predictions for the proton structure function $F_2(x, Q^2)$ are consistent with the HERA data.
1. Introduction

It is known that the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation \[1\] sums leading logarithms \(\ln(1/x)\), \(x\) being the Bjorken variable, which are produced from the reggeon ladder diagrams with the rung gluons obeying the strong rapidity ordering. This equation predicts a rise of the gluon distribution function and thus a rise the proton structure function \(F_2\) involved in deep inelastic scattering (DIS) at small \(x\), which have been confirmed by the recent HERA data \[2\]. However, some controversies remain unsolved.

Since the BFKL equation is independent of momentum transfer \(Q\), its predictions are insensitive to the variation of \(Q\). On the contrary, the experimental data for \(F_2(x, Q^2)\) exhibit a stronger \(Q\) dependence \[2\]: The rise at low \(x\) is slower for smaller \(Q\), corresponding to so-called soft pomeron exchanges. The rise obtained from the BFKL equation is always larger, which corresponds to hard pomeron exchanges. To explain the data, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation \[3\], which sums single logarithms \(\ln Q\) produced from strong transverse momentum ordering, is then employed. The relevant splitting function \(P_{gg}\) contains a term \(1/x\), which also gives a rise at small \(x\). An alternative solution \[4\] is the Ciafaloni-Catani-Fiorani-Marchesini (CCFM) equation, which embodies the DGLAP and BFKL equations \[5\]. For this unified evolution equation, the \(Q\) dependence comes from the summation of the ladder diagrams with strong angular ordering.

On the other hand, the increase of the gluon distribution function at \(x \to 0\) predicted by the BFKL equation is power-like, such that \(F_2\) and the DIS cross section \(\sigma_{\text{tot}}\) rise as a power of \(x\), i.e., as a power of \(s\) for \(x = Q^2/s\), \(s\) being the total energy. This behavior does not satisfy the Froissart bound \(\sigma_{\text{tot}} \leq \text{const.} \times \ln^2 s\), and violates unitarity. Hence, the BFKL equation can not be the final theory for small \(x\) physics. Though it has been expected that the inclusions of next-to-leading \(\ln(1/x)\) \[6\] and of higher-twist effects from the exchange of multiple pomerons \[7\] may soften the BFKL rise, the attempts have not yet led to a concrete conclusion.

Recently, we have proposed a modified BFKL equation from the viewpoint of the resummation technique \[8\], which involves an intrinsic \(Q\) dependence \[9\]. This \(Q\) dependence was introduced by cutting off the longitudinal component of the real gluon momentum at a scale of order \(Q\), when evaluating the BFKL kernel. The motivation is that it is unlikely for a real gluon to carry an infinite momentum. The gluon distribution function derived from
this $Q$-dependent evolution equation gives predictions of $F_2$, which are in good agreement with the data. Unfortunately, the predictions still rise as a power of $x$. In [10] we observed that relaxing the strong rapidity ordering for real gluon emissions results in a destructive correction, and the power-law rise of the gluon distribution function turns into a logarithmic rise. The unitarity bound is thus satisfied.

In this letter we shall combine the above two modifications to derive a new small-$x$ evolution equation, which depends on $Q$ and satisfies the unitarity bound at the same time. We demonstrate that the predictions for $F_2$ are consistent with the current data for $x > 10^{-4}$, and that the logarithmic rise of $F_2$ turns on as $x < 10^{-4}$.

2. The formalism

We refer the detailed derivation of the BFKL equation using the resummation technique to [11]. Here we summarize only the basic idea. The unintegrated gluon distribution function $F(x, k_T)$, describing the probability of a parton carrying a longitudinal momentum fraction $x$ and transverse momenta $k_T$, is defined by

$$F(x, k_T) = \frac{1}{p^+} \int \frac{dy^-}{2\pi} \int \frac{d^2 y_T}{4\pi} e^{-i(x p^+ y^- - k_T \cdot y_T)} \times \frac{1}{2} \sum_{\sigma} \langle p, \sigma | F^+_{\mu}(y^-, y_T) F^{\mu+}(0) | p, \sigma \rangle ,$$  

in the axial gauge $n \cdot A = 0$, $n$ being a gauge vector with $n^2 \neq 0$. The ket $|p, \sigma\rangle$ denotes the incoming proton with the light-like momentum $p^\mu = p^+ \delta^{\mu+}$ and spin $\sigma$. An average over color is understood. $F^+_{\mu}$ is the field tensor.

For a fixed parton momentum $k^+$, $F$ varies with $p^+$ implicitly through $x = k^+/p^+$, and we have $-x dF/dx = p^+ dF/dp^+$. That is, the differentiation of $F$ with respect to $x$ is related to the differentiation with respect to $p^+$. Because of the scale invariance in $n$ of the gluon propagator $-i N^{\mu\nu}(l)/l^2$ with

$$N^{\mu\nu} = g^{\mu\nu} - \frac{n^{\mu} l^{\nu} + n^{\nu} l^{\mu}}{n \cdot l} + n^2 \frac{l^{\mu} l^{\nu}}{(n \cdot l)^2} ,$$  

$F$ must depend on $p$ and $n$ via the ratio $(p \cdot n)^2/n^2$. Hence, there exists a
chain rule relating $p^+ d/dp^+$ to $d/dn$,

$$ p^+ \frac{d}{dp^+} F = -\frac{n^2}{v \cdot n} v_\beta \frac{d}{dn_\beta} F , \tag{3} $$

$v_\beta = \delta_{\beta +}$ being a vector along $p$. The operator $d/dn_\beta$ applies to a gluon propagator, giving

$$ \frac{d}{dn_\beta} N^{\nu\nu'} = -\frac{1}{n \cdot l} (l^\nu N^{\beta\nu'} + l^{\nu'} N^\nu_{\beta \beta}) . \tag{4} $$

The loop momentum $l^\nu$ $(l'^\nu)$ contracts with the vertex the differentiated gluon attaches, which is then replaced by a special vertex $\dot{v}_\beta = n^2 v_\beta / (v \cdot n n \cdot l)$. This special vertex can be read off from the combination of Eqs. (3) and (4).

Summing the diagrams with different differentiated gluons, and employing the Ward identity for the contraction of $l^\nu$ $(l'^\nu)$, those diagrams embedding the special vertices cancel by pairs, leaving the one in which the special vertex moves to the outer end of the parton line [8]. We then obtain the derivative,

$$ -x \frac{d}{dx} F(x, k_T) = 2 \bar{F}(x, k_T) , \tag{5} $$

where the new function $\bar{F}$ contains one special vertex [11]. The coefficient 2 comes from the equality of the new functions with the special vertex on either of the two parton lines.

Next we factorize the new function into the convolution of the subdiagram involving the special vertex with the original gluon distribution function. The resultant factorization formula will lead to the small-$x$ evolution equation. The kinematic approximation, i.e., the rapidity ordering of radiative gluons, is specified only at the stage of computing the subdiagram, which is then identified as the evolution kernel. Therefore, in our approach it is easier to handle the approximation applied to the evolution kernel, and to investigate its effect on the behavior of the gluon distribution function.

The only important region of the loop momentum $l$ flowing through the special vertex is soft, in which the subdiagram containing the special vertex can be factorized. Including the color factor, the factorization formula is written as [9, 10]

$$ \bar{F}(x, k_T) = \frac{iN_c g^2}{(2\pi)^4} \int \frac{d^4 l}{4\pi} N^{\nu\beta}(l) \frac{\dot{v}_\beta v_\nu}{v \cdot l} \left[ 2\pi i \delta(l^2) F(x + l^+ / p^+, |k_T + l_T|) \right] $$
\[ + \frac{\theta(k_T^2 - l_T^2)}{l_T^2} F(x, k_T) \].

The first term in the brackets corresponds to the real gluon emission, where \( F(x + l^+/p^+, |k_T + l_T|) \) implies that the parton coming out of the proton carries the momentum components \( xp^+ + l^+ \) and \( k_T + l_T \) in order to radiate a real gluon of momentum \( l \). The second term corresponds to the virtual gluon emission, where the \( \theta \) function sets the upper bound of \( l_T \) to \( k_T \) to ensure a soft momentum flow.

The integration over \( l^- \) gives

\[
\bar{\alpha}_s \int \frac{d^2 l_T}{\pi} \left[ \int_{0}^{\infty} \frac{2l^+ n^2}{(2n - l^2 + n^2 l_T^2)^2} d l^+ \right] F(x + l^+/p^+, |k_T + l_T|) - \frac{\theta(k_T^2 - l_T^2)}{l_T^2} F(x, k_T),
\]

with \( \bar{\alpha}_s = N_c \alpha_s / \pi, N_c = 3 \) being the number of colors. We shall show that Eq. (7) reduces to the conventional BFKL equation, the modified BFKL equation with the \( Q \) dependence, the modified BFKL equation with unitarity, and the new evolution equation with both the modifications by removing the applied approximations gradually.

To derive the conventional BFKL equation, we simply assume the strong rapidity ordering, namely, approximate \( F(x + l^+/p^+) \) by its dominant value \( F(x) \). Performing the integration over \( l^+ \) to infinity, and substituting \( \bar{\alpha}_s \) into Eq. (5), we arrive at

\[
\frac{dF(x, k_T)}{d \ln(1/x)} = \bar{\alpha}_s \int \frac{d^2 l_T}{\pi l_T^2} \left[ F(x, |k_T + l_T|) - \theta(k_T^2 - l_T^2) F(x, k_T) \right],
\]

which is the BFKL equation adopted in [4].

However, the vanishing of \( F(x + l^+/p^+) \) at large momentum fraction constrains \( l^+ \) to go to infinity. To render the approximation \( F(x + l^+/p^+) \approx F(x) \) more reasonable, we truncate \( l^+ \) at the scale \( Q/\sqrt{2} \) for the real gluon emission in Eq. (7), and obtain the \( Q \)-dependent BFKL equation,

\[
\frac{dF(x, k_T)}{d \ln(1/x)} = \bar{\alpha}_s \int \frac{d^2 l_T}{\pi l_T^2} \left[ F(x, |k_T + l_T|) - \theta(k_T^2 - l_T^2) F(x, k_T) \right] \]

\[ - \bar{\alpha}_s \int \frac{d^2 l_T}{\pi} \frac{F(x, |k_T + l_T|)}{l_T^2 + Q^2}, \]

with \( \bar{\alpha}_s = N_c \alpha_s / \pi, N_c = 3 \) being the number of colors.
for the choice $n = (1, 1, 0)$. The extra term compared to the conventional BFKL equation comes from the upper bound of $l^+$. It is trivial to find that Eq. (9) approaches Eq. (8) as $Q \to \infty$. This correction, being negative, moderates the BFKL rise at low $Q$. Equation (9) has been solved in [1], and its predictions of the structure function $F_2$ match the data for various $x$ and $Q^2$.

As stated in the Introduction, the conventional BFKL equation gives a power-law rise for the gluon distribution function at $x \to 0$, which violates unitarity. It has been pointed out that the assumption of the strong rapidity ordering is the cause for the unitarity violation [1]. For most values of $l^+$, $F(x + l^+/p^+)$ is much smaller than $F(x)$. Hence, replacing the former by the latter in the whole integration range of $l^+$ overestimates the real gluon contribution, which is responsible for the rise. To derive an evolution equation with unitarity, we employ Eq. (7) directly without applying the approximation. Reexpress $F(x + l^+/p^+, |k_T + l_T|)$ as

$$F(x, |k_T + l_T|) + \left[ F(x + l^+/p^+, |k_T + l_T|) - F(x, |k_T + l_T|) \right], \quad (10)$$

in Eq. (7), where the first term, combined with the virtual gluon contribution, leads to the conventional BFKL equation, and the terms in the brackets are the correction from relaxing the strong rapidity ordering. We then derive

$$\frac{dF(x, k_T)}{d \ln(1/x)} = \bar{\alpha}_s \int \frac{d^2l_T}{2l^2_T} \left[ F(x, |k_T + l_T|) - \theta(k^2_T - l^2_T)F(x, k_T) \right]$$
$$+ \bar{\alpha}_s \int \frac{d^2l_T}{\pi} \int_0^\infty d l^+ \frac{4l^+}{(2l^+)^2 + l^2_T}$$
$$\times \left[ F(x + l^+/p^+, |k_T + l_T|) - F(x, |k_T + l_T|) \right]. \quad (11)$$

The above equation has been studied in [10], and the BFKL power-law rise was found to be moderated into a logarithmic rise at small $x$.

**3. The new equation**

The $Q$-dependent BFKL equation (9), though phenomenologically successful, still gives a power-law rise of the gluon distribution function. Hinted by Eq. (11), we attempt to combine the above two modifications by switching
the upper bound of \( l^+ \) from \( \infty \) to \( Q/\sqrt{2} \) in Eq. (7), and obtain

\[
\frac{dF(x, k_T)}{d \ln(1/x)} = \bar{\alpha}_s \int \frac{d^2l_T}{\pi} \left[ \int_0^1 dy \frac{2yQ^2}{(y^2Q^2 + l_T^2)^2} F(x + y\sqrt{x}, |k_T + l_T|) \right.
\]

\[
- \frac{\theta(k_T^2 - l_T^2)}{l_T^2} F(x, k_T) \right],
\] (12)

where the variable change \( l^+ = y\sqrt{x}p^+ \) has been adopted, and the upper bound of \( y \) is determined by the kinematic relation \( Q \approx \sqrt{2xp} \). Equation (12) is the new small-\( x \) evolution equation we shall investigate in more details below.

In order to simplify the analysis, the \( \theta \) function for the virtual gluon emission is replaced by \( \theta(Q_0^2 - l_T^2) \) [9], where the parameter \( Q_0 \) can be determined from data fitting. This simplification is reasonable, because the virtual gluon contribution only plays the role of a soft regulator for the real gluon emission, and setting the cutoff of \( l_T \) to \( Q_0 \) serves the same purpose. We then Fourier transform Eq. (12) into the \( b \) space conjugate to \( k_T \) with Eq. (10) inserted, deriving

\[
\frac{d\tilde{F}(x, b)}{d \ln(1/x)} = S(b, Q) \tilde{F}(x, b) + 2\bar{\alpha}_s(1/b)Qb \int_0^1 dy K_1(yQb)[\tilde{F}(x + y\sqrt{x}, b) - \tilde{F}(x, b)],
\]

(13)

where

\[
S(b, Q) = -2\bar{\alpha}_s(1/b) [\ln(Q_0b) + \gamma - \ln 2 + K_0(Qb)]
\]

(14)

comes from the combination of the first term in Eq. (10) and the virtual gluon emission term. Note that \( S \) is the evolution kernel for Eq. (9) in \( b \) space. \( K_0 \) and \( K_1 \) are the Bessel functions, and \( \gamma \) the Euler constant. The argument of \( \alpha_s \) has been set to the natural scale \( 1/b \).

An initial condition \( \tilde{F}(x_0, b) = \tilde{F}^{(0)}(x_0, b) \) must be assumed when solving Eq. (13), \( x_0 \) being the initial momentum fraction. For instance, a “flat” gluon distribution function [4]

\[
\tilde{F}^{(0)}(x, b) = 3N_g(1 - x)^5 \exp(-Q_0^2b^2/4),
\]

(15)
for $x \geq x_0$, $N_g$ being a normalization constant, has been proposed. Therefore, the initial function $\tilde{F}(0)(x + y\sqrt{x}, b)$ should be substituted for $\tilde{F}(x + y\sqrt{x}, b)$ in Eq. (13) as $x + y\sqrt{x} > x_0$.

Before solving Eq. (13), we extract the behavior of $\tilde{F}$ analytically. Inserting a guess $\tilde{F} \propto x^{-\lambda}$ into Eq. (13), $\lambda$ being a parameter, we obtain

$$\lambda = S + 2\bar{\alpha}_s Q b \int_0^1 dy K_1(y Q b) \left[ \left( \frac{x + y\sqrt{x}}{x} \right)^{-\lambda} - 1 \right].$$

(16)

It can be numerically verified that a solution of $\lambda$, $0 < \lambda < S$, exists for $x < x_0$. That is, $\tilde{F}$ increases as a power of $x$, consistent with the results from the conventional and $Q$-dependent BFKL equations. While the correction term, i.e., the second term on the right-hand side of Eq. (16), diverges as $x \to 0$, and no solution of $\lambda$ is allowed, implying that $\tilde{F}$ can not maintain the power-law rise at extremely small $x$. We then substitute another guess $\tilde{F}|_{x \to 0} \propto \ln(1/x)$ with a milder rise into Eq. (13). In this case the correction term, increasing as $\ln^{1.2}(1/x)$, only slightly dominates over the first term $S\tilde{F} \propto \ln(1/x)$ (the derivative term $-xd\tilde{F}/dx \propto 1$ is negligible), and Eq. (13) holds approximately. At last, we assume $\tilde{F}|_{x \to 0} \propto \text{const.}$ as a test. It is easy to find that the first term becomes dominant, and the correction term and the derivative term vanish, i.e., no const. $\neq 0$ exists. These observations indicate that $\tilde{F}$ should increase as $\ln(1/x)$ at most when $x$ approaches zero.

In conclusion, the new evolution equation predicts a rapid power-like rise of $\tilde{F}$ for $x < x_0$ and a milder logarithmic rise at $x \to 0$.

The structure function $F_2$ is written, in terms of the $k_T$-factorization theorem [12], as

$$F_2(x, Q^2) = \int_x^1 \frac{d\xi}{\xi} \int_0^{p_e} \frac{d^2 k_T}{\pi} H(x/\xi, k_T, Q) F(\xi, k_T),$$

(17)

$p_e$ being the upper bound of $k_T$ which will be specified later. The hard scattering subamplitude $H$ denotes the contribution from the quark box diagrams, where both the incoming photon and gluon are off shell by $-Q^2$ and $-k_T^2$, respectively. $H$ has been computed in [9], and its expression is given by

$$H(z, k_T, Q) = \sum_q e_q^2 \frac{\alpha_s}{2\pi} \frac{z}{2} \left[ z^2 + (1 - z)^2 - 2z(1 - 2z) \frac{k_T^2}{Q^2} + 2z^2 \frac{k_T^4}{Q^4} \right].$$
\begin{equation}
\frac{1}{\sqrt{1 - 4z^2 k_T^2 / Q^2}} \ln \left( \frac{1 + \sqrt{1 - 4z^2 k_T^2 / Q^2}}{1 - \sqrt{1 - 4z^2 k_T^2 / Q^2}} - 2 \right),
\end{equation}

with $e_q$ the electric charge of the quark $q$. To require a meaningful $H$, the upper bound of $k_T$ in Eq. (17) is set to

\begin{equation}
p_c = \min\left( Q, \frac{\xi}{2x} Q \right).
\end{equation}

We choose $x_0 = 0.1$ and $Q_0 = 0.4 \text{ GeV}$, and evaluate Eq. (17) for $Q^2 = 15 \text{ GeV}^2$. Fitting the results to the corresponding data [2], the normalization constant is determined to be $N_g = 1.207$. $N_g$ for other $Q^2$ are then extracted by requiring that the gluon density, defined by

\begin{equation}
xg(x, Q^2) = \int_0^Q \frac{d^2 k_T}{\pi} F(x, k_T),
\end{equation}

has a fixed normalization $\int_0^1 xgdx$. $F_2$ for $Q^2 = 8.5, 12, \text{ and } 20 \text{ GeV}^2$ are computed, and the results along with the data [3] are displayed in Fig. 1. For comparison, we also present the results from the conventional and $Q$-dependent BFKL equations [4]. It is found that the shape of the curves from the conventional BFKL equation is almost independent of $Q$, and thus the match with the data is not very satisfactory. The curves from the $Q$-dependent BFKL equation, exhibiting smaller slopes for lower $Q$, match the data. However, they increase rapidly at $x \to 0$, and violate the unitarity bound. The predictions from the new evolution equation also agree with the data well. The curves have a steeper rise at a larger $Q$, which is the consequence of the cutoff at the scale of order $Q$. Because of $F(\xi, k_T) \leq \ln(1/\xi)$ at $\xi \to 0$, $F_2$ rises as $\ln^2(1/x)$ at most as indicated by the $\xi$ integration in Eq. (17), and thus satisfies the unitarity bound. It is observed that the ascent of the curves does not speed up as obtained from the conventional and $Q$-dependent BFKL equations at $x < 10^{-4}$. This is the consequence of relaxing the strong rapidity ordering.

\section{Conclusion}

In this letter we have combined the modifications to the conventional BFKL equation obtained in our previous studies, which are based on the
resummation formalism. The resultant small-$x$ evolution equation possesses the intrinsic $Q$ dependence and satisfies the unitarity requirement. The predictions of the structure function $F_2$ are in agreement with the HERA data for various $x$ and $Q^2$. We have also observed that the power-law rise turns into a logarithmic rise as $x < 10^{-4}$.

We emphasize that the cutoff of order $Q$ for the longitudinal momentum in the real gluon emission is phenomenologically motivated in this work. In the derivation of a new unified evolution equation for both intermediate and small $x$, we shall demonstrate that the scale $Q$ can be introduced in a rigorous way. This subject will be published elsewhere [13].

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**Figure Caption**

**FIG. 1.** The dependence of $F_2$ on $x$ for $Q^2 = 8.5, 12, 15,$ and $20$ GeV$^2$ from the conventional BFKL equation (dotted lines), from the $Q$-dependent BFKL equation, and from the new evolution equation (solid lines).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9806211v1