Vortex pinning in two-gap superconductors

Jun Goryo, Tatsuro Saito, and Hiroshi Matsukawa
Department of Physics and Mathematics, Aoyamagakuin University, 5-10-1, Fuchinobe, Sagamihara, Kanagawa, 229-8558, Japan
E-mail: jungoryo@phys.aoyama.ac.jp

Abstract. We discuss vortices and their pinning in two-gap superconductors, in which two superconducting gaps are opened around two different Fermi surfaces. We found that there are two kinds of vortices with continuously valuable fractions of the unit flux quanta $\frac{hc}{2e}$. These two kinds of vortices are tightly bound and cannot be separated in the ground state, but can be when deconfinement occurs by the finite temperature effect as is pointed out (See, Jun Goryo, Shingo Soma, and Hiroshi Matsukawa, Europhys. Lett. 80 (2007) 17002). In this paper, we show that two kinds of vortices can be split by the pinning effect.

1. Introduction
It is well known that the pinning of vortex lattice in type II superconductors is closely related to the science of friction. The vortex lines are subject to the Lorentz force under external current and begin to flow perpendicular to the current and magnetic field when the Lorentz force exceeds the maximum pinning force. This behavior is quite analogous to the maximum static friction between two solid surfaces.

In this paper, we discuss novel features of vortices and their pinning in recently discovered two-gap superconductors, in which two superconducting gaps exist in two different seats of Fermi surface. One of the most famous example is MgB$_2$ [1]. Besides MgB$_2$ there are many candidates of two-gap superconductors [2]. Hence, it is important to investigate properties of two-gap superconductors.

2. Fractional vortices with continuously variable fractions of the unit flux quanta
In this section, we discuss the presence of fractional vortices in two-gap superconductors. The essence of the vortex in usual superconductors with single gap is $2\pi$-winding of a phase of the gap function around a vortex center. The supercurrent, which is proportional to the gradient of the phase, is then circulating around the vortex center and the magnetic flux is induced whose magnitude is topologically quantized as an integer times the magnetic flux quanta $\Phi_0 = \frac{hc}{2e}$. In two-gap superconductors, there are two gap functions $\Delta_{1,2} = |\Delta_{1,2}| \exp[i\theta_{1,2}]$ and two phases exist. The London equation for two-gap superconductors is

$$\lambda \nabla \times \nabla \times \mathbf{H} = -\mathbf{H} + \frac{\Phi_1}{2\pi} \nabla \times \nabla \theta_1 + \frac{\Phi_2}{2\pi} \nabla \times \nabla \theta_2,$$

where $\lambda$ is the London penetration depth and $\Phi_{1,2} = \frac{K_{1,2}|\Delta_{1,2}|^2}{K_1|\Delta_1|^2 + K_2|\Delta_2|^2}$ ($K_{1,2}$: the coefficients of the gradient terms of $\Delta_1$ and $\Delta_2$ in two-gap Ginzburg-Landau free energy [3]). This equation
tells us that when two phases are winding $2\pi$ around different points, i.e.,

$$\nabla \times \nabla \theta_{1,2}(r) = 2\pi \delta^2(r - r_{1,2})$$  \hspace{1cm} (2)

with $r_1 \neq r_2$, two kinds of supercurrents $j_{s1,2} = K_{1,2}|\Delta_{1,2}|^2 \nabla \theta_{1,2}$ are circulating around $r_1$ and $r_2$, and induce $\Phi_1$ and $\Phi_2$, respectively. Details of $\Phi_1$ and $\Phi_2$ are as follows: (i) $\Phi_1 + \Phi_2 = \Phi_0$, i.e., we have a usual vortex when $r_1 = r_2$. (ii) The factors before $\Phi_0$ in both fluxes are related to the amplitudes of two gap functions. In general, these depend on temperature and magnetic field. Therefore, we obtain a remarkable conclusion that $\Phi_1$ and $\Phi_2$ are arbitral fraction of $\Phi_0$ and can be changed continuously by varying temperature and magnetic field[4, 5, 6, 7].

We should argue the stability of this novel fractional flux state $r_1 \neq r_2$[7]. We note that there is a Josephson-type interaction

$$\gamma (\Delta_1^* \Delta_2 + h.c.) = |\Delta_1| |\Delta_2| \cos(\theta_1 - \theta_2)$$  \hspace{1cm} (3)

in the two-gap Ginzburg-Landau free energy[3]. This term favors to lock the relative phase at some constant values. For instance, if $\gamma > 0$, $\theta_1 - \theta_2 = 2\pi n$ ($n = 0, \pm 1, \pm 2...$) is favored. To agree with the phase-winding conditions Eq. (2), the relative phase becomes a multi-valued function. It is constant in the bulk region and jumps $\pm 2\pi$ at a "string" which is connecting two fractional vortices (See Fig. 1). The string has a tension $\tau$ and its creation energy is proportional to the string length $L_{st}$. On the other hand, by using the random walk argument, we can see that the string has a configuration entropy which is also proportional to $L_{st}$. Then, below a certain temperature $T_\tau$ corresponding to the energy scale of the string tension, i.e., $k_B T_\tau \sim \tau \xi$ ($\xi$: the coherence length of the superconducting order parameter, the cut off length in the short length scale in this system), the string tightly binds two fractional vortices and linear confinement occurs. Above $T_\tau$, the string looses its tension by the entropic effect. Then, deconfinement occurs and two fractional vortices are freely exist. In other words, fractional vortices are stabilized thermodynamically[7].

![Figure 1. Fractional vortices and the string.](image)

3. Vortex pinning in two-gap superconductors
Let us consider the vortex pinning as another mechanism to appear the fractional flux state $r_1 \neq r_2$, since we can expect easily that if one of the fractional vortices is pinned easier than
the other, splitting of these two occurs when we apply the external force. We introduce a two-dimensional simple model. We provide two kinds of $15 \times 15$ vortices, each of which makes a square lattice with a lattice constant $a$. It corresponds to the lattice consists of vortex-1 or vortex-2. Initially, these lattices are completely overlapped. We pull these lattices weakly and make a pinning state. The equations of motion of each lattice cites, that is each vortex, are

$$\eta^{(a)} \frac{d r_{i,\mu}^{(a)}}{dt} = -k_{11} \left[ r_{i+\mu,\mu}^{(a)} + r_{i-\mu,\mu}^{(a)} - 2r_{i,\mu}^{(a)} \right] - k_{12} \left[ r_{i+\mu,\mu}^{(2)} + r_{i-\mu,\mu}^{(2)} - 2r_{i,\mu}^{(1)} \right] - \tau \frac{r_{i,\mu}^{(1)} - r_{i,\mu}^{(2)}}{|r_{i}^{(1)} - r_{i}^{(2)}|} + g_{p,\mu}^{(a)} + f_{ext,\mu}^{(a)},$$

$$\eta^{(2)} \frac{d r_{i,\mu}^{(2)}}{dt} = (1 \leftrightarrow 2 \text{ in r.h.s. of Eq.(4)}).$$

Here, $r_{i,\mu}^{(a)}$ ($a = 1, 2; \mu = x, y$) denotes the $\mu$-component of the displacement vector for vortex-$a$ on the $i$-th cite, the indices $i \pm \mu$ denote the nearest neighbor cites of the $i$-th cite along $\mu$-axis, $\eta^{(a)}$ ($a = 1, 2$) is viscosity, $k_{11}, k_{22}$ and $k_{12} = k_{21}$ are elastic constants, $g_{p,\mu}^{(a)}$ and $f_{ext,\mu}^{(a)}$ are a force from a pinning center and an external force acting to vortex-$a$, respectively. We denote the maximum force from a pinning center $f_{p,\mu}^{(a)} (= \max g_{p,\mu}^{(a)})$.

| Table 1. Parameters |
|---------------------|
| **Case (A)** | $\eta_1/\eta_2 = 1.0$, $k_{22}/k_{11} = 0.1$, $k_{12}/k_{22} = 0.3$, $f_{p2}/f_{p1} = 1.0$, $f_{p1} = 0.5k_{11}a$ |
| $f_{ex1y}/f_{ex1x} = 1.0$, $f_{ex2}/f_{ex1} = 1.0$ |
| **Case (B)** | $\eta_1/\eta_2 = 1.0$, $k_{22}/k_{11} = 0.1$, $k_{12}/k_{22} = 0.3$, $f_{p2}/f_{p1} = 2.5$, $f_{p1} = 0.5k_{11}a$ |
| $f_{ex1y}/f_{ex1x} = 1.0$, $f_{ex2}/f_{ex1} = 1.0$ |

Figure 2. A part of the pinning state in Case (A) with $f_{ex1} = 1.0k_{11}a$ and $\tau = 0.01k_{11}a$. A green dot and a red one denote Vortex-1 and -2, respectively.

By using parameters of Case (A) in Table 1, in which Lattice-1 is harder than Lattice-2, we numerically obtain a pinning state. We see splitting between vortex-1 and vortex-2 in Fig.
2. Dependence of the mean value of splitting distance \( < d > \) on the string tension \( \tau \) is shown in Fig. 3. As might have been expected, the distance decreases when we increase the string tension. For the parameters of Case (B) in Table 1, the external force for vortex-2 is weaker than that for vortex-1 and the maximum pinning force for vortex-2 is stronger than that for vortex-1. Therefore, Lattice-2 is easier to be pinned. The \( \tau \)-dependence of \( < d > \) in this case is shown in Fig. 4. The behavior is basically same with that in Case (A) but the splitting is larger.

We show the \( \tau \)-dependence of the total maximum pinning force \( F_p \) in Fig. 5 for Case (B). We see that \( F_p \) is almost independent of \( \tau \). The result in case (A) shows essentially same behavior. Such results would be obtained whenever we use the elastic model. A more realistic model including the plastic flow should be considered. Drastic changes in the \( \tau \)-dependence of
$F_p$ are expected in such a model. Finite temperature effect should be also included to discuss the splitting and the deconfinement simultaneously.

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