Luminosity Evolution of Gamma-ray Pulsars

Kouichi Hirotani

Theoretical Institute for Advanced Research in Astrophysics (TIARA), Academia Sinica, Institute of Astronomy and Astrophysics (ASIAA), PO Box 23-141, Taipei, Taiwan

We investigate the electrodynamic structure of a pulsar outer-magnetospheric particle accelerator and the resultant gamma-ray emission. By considering the condition for the accelerator to be self-sustained, we derive how the trans-magnetic-field thickness of the accelerator evolves with the pulsar age. It is found that the thickness is small but increases steadily if the neutron-star envelope is contaminated by sufficient light elements. For such a light element envelope, the gamma-ray luminosity of the accelerator is kept approximately constant as a function of age in the initial ten thousand years, forming the lower bound of the observed distribution of the gamma-ray luminosity of rotation-powered pulsars. If the envelope consists of only heavy elements, on the other hand, the thickness is greater but increases less rapidly than what a light element envelope has. For such a heavy element envelope, the gamma-ray luminosity decreases relatively rapidly, forming the upper bound of the observed distribution. The gamma-ray luminosity of a general pulsar resides between these two extreme cases, reflecting the envelope composition and the magnetic inclination angle with respect to the rotation axis. The cutoff energy of the primary curvature emission is regulated below several GeV even for young pulsars, because the gap thickness, and hence the acceleration electric field is suppressed by the polarization of the produced pairs.

1. Introduction

The Large Area Telescope aboard Fermi Gamma-ray Space Telescope (Atwood et al. 2009) has proved remarkably successful at discovering rotation-powered pulsars emitting photons above 0.1 GeV. Thanks to its superb sensitivity, the number of gamma-ray pulsars has increased from six in Compton Gamma Ray Observatory era (Thompson 2004) to more than one hundred (Nolan 2012). Plotting their best estimate of the gamma-ray luminosity, $L_{\gamma}$, against the spin-down luminosity, $L_{\text{spin}} = 4\pi^{2}I \dot{P} P^{-3}$, Abdo et al. 2010 found the important relation that $L_{\gamma}$ is approximately proportional to $L_{\text{spin}}^{0.5}$ (with a large scatter), where $I$ refers to the neutron-star (NS) moment of inertia, $P$ the NS rotational period, and $\dot{P}$ its temporal derivative. However, it is unclear why this relationship arises, in spite of its potential importance to discriminate pulsar emission models such as the polar-cap model (Daugherty & Harding 1982, Dermer 1994, Harding et al. 1978), the outer-gap model (Chiang & Romani 1992, Hirotani 2008, Romani 1996, Takata et al. 2004, Wang et al. 2011, Zhang & Cheng 1997), the pair-starved polar-cap model (Venter et al. 2009) (see also Yuki & Shibata 2012 for the possible co-existence of such models), and the emission model from the wind zone (Aharonian et al. 2012, Bai & Spitkovski 2010a,b, Petri 2011).

Recent gamma-ray observations suggest that the pulsed gamma-rays are emitted from the higher altitudes of a pulsar magnetosphere. This is because the observed light curves (Abdo et al. 2010) favor fan-like emission geometry, which scan over a large fraction of the celestial sphere, and because the Crab pulsar shows pulsed photons near and above 100 GeV (Aleksić et al. 2011a,b, Aliu et al. 2011), which rules out an emission from the lower altitudes, where strong magnetic absorption takes place for $\gamma$-rays above 10 GeV. Consequently, higher-altitude emission models such as the outer-gap model (Cheng et al. 1986a,b), the high-altitude slot-gap model (Muslimov & Harding 2004), or the pair-starved polar-cap model (Venter et al. 2009), gathered attention. It is noteworthy that the outer-gap model is presently the only higher-altitude emission model that is solvable from the basic equations self-consistently (Hirotani 2011a). In the present paper, therefore, we focus on the outer-gap model and derive the observed relationship $L_{\gamma} \propto L_{\text{spin}}^{0.5}$ both analytically and numerically.

We schematically depict the pulsed outer-magnetospheric accelerator (i.e., the outer gap) in figure 1 of Hirotani 2013. As the NS rotates, there appears the light cylinder, within which plasmas can co-rotate with the magnetosphere. The magnetic field lines that become tangential to the light cylinder at the light cylinder radius, $\varpi_{LC} = cP/2\pi$, are called the last-open magnetic field lines, where $c$ refers to the speed of light. Pairs are produced via photon-photon pair production mostly near the null-charge surface and quickly polarized by the magnetic-field-aligned electric field, $E_{\parallel}$, in the gap. In this paper, we assume that the rotation and magnetic axes reside in the same hemisphere to obtain $E_{\parallel} > 0$, which accelerates positrons ($e^{+}$'s) outwards while electrons ($e^{-}$'s) inwards. These ultra-relativistic particles have Lorentz factors, $\gamma \sim 10^{7.5}$, to emit photons efficiently by the curvature process.
2. Analytical examination of outer-gap luminosity

In this section, we analytically derive how the gamma-ray luminosity of an outer gap evolves with time. In the outer magnetosphere, only the dipole component remains in the magnetic field; thus, the inhomogeneous part of the Maxwell equation (i.e., the Poisson equation for the electro-static potential) gives the magnetic-field-aligned electric field \[ E_{||} \approx \frac{\mu}{2\bar{\omega} \rho_c^3} h_m^2, \] where \( \mu \) denotes the NS magnetic dipole moment, and \( h_m \) the trans-magnetic-field thickness of the gap. Since the Poisson equation is a second-order differential equation, \( E_{||} \) is proportional to \( h_m^2 \). Electrons (\( e^- \)'s) and positrons (\( e^+ \)'s) are created via photon-photon (and sometimes via magnetic) pair production, being subsequently polarized by \( E_{||} \) and accelerated in the opposite directions, to finally attain the terminal Lorentz factor \( \gamma = \left( \frac{3\beta_c^2}{2e} E_{||} \right)^{1/4}, \nbl\) where \( \beta_c \) refers to the radius of curvature of particle’s motion in the three-dimensional magnetosphere, \( e \) the charge on the positron. Photons are radiated by such ultra-relativistic \( e^\pm \)'s via curvature process with characteristic energy, \[ h\nu_c = \frac{3}{2} \hbar \gamma^3 \beta_c^2 \rho_c, \] where \( h \) denotes the Planck constant, \( \hbar \equiv h/2\pi \). Once \( h_m \) is obtained, we can readily compute the \( \gamma \)-ray luminosity of curvature radiation from an outer gap by \[ L_{\gamma} \approx 2.36(\nu F_{\nu})_{\text{peak}} \times 4\pi d^2 f_{\Omega} \approx 1.23 f_{\Omega} h_m^3 \frac{h^2 \Omega^4}{c^3}, \] where \( f_{\Omega} \), which has been conventionally assumed to be approximately unity, refers to the flux correction factor \[ \text{Romani, R. & Watters 2010}, \] and \( \Omega = 2\pi/P \) the rotation angular frequency of the NS. Here, it is assumed that the current flowing in the gap is comparable to the Goldreich-Julian value \[ \text{Goldreich, & Julian 1969}. \] The last factor, \( \mu^2 \Omega^4/c^3 \) is proportional to the spin-down luminosity, \( L_{\text{spin}} \). Therefore, the evolution law, \( L_{\gamma} \propto L_{\text{spin}}^{0.5} \), is crucially governed by the evolution of \( h_m \) as a function of the NS age, \( t \).

The evolution of \( h_m \) is essentially controlled by the photon-photon pair production in the pulsar magnetosphere. To analytically examine the pair production, we assume the static dipole magnetic field configuration for simplicity, and consider the plane on which both the rotational and magnetic axes reside. On this two-dimensional latitudinal plane, the last-open field line intersects the NS surface at magnetic co-latitude angle \( \theta_m^\ast \) (measured from the magnetic dipole axis) that satisfies \[ \frac{\sin^2 \theta_m^\ast}{r_\ast} = \frac{\sin^2(\theta_{LC} - \alpha)}{\bar{\omega}_{LC}/\sin \theta_{LC}}, \] where \( r_\ast \) denotes the NS radius, \( \theta_{LC} \) the angle (measured from the rotation axis) of the point where the last-open field line becomes tangential to the light cylinder, and \( \alpha \) the inclination angle of the dipole magnetic axis with respect to the rotation axis. A magnetic field line can be specified by the magnetic co-latitude (measured from the dipole axis), \( \theta_\ast \), where it intersects the stellar surface. A magnetic field line does not close within the light cylinder (i.e., open to large distances) if \( 0 < \theta_\ast < \theta_m^\ast \). Thus, the last-open field lines, \( \theta_\ast = \theta_m^\ast \), corresponds to the lower boundary, which forms a surface in a three-dimensional magnetosphere, of the outer gap.

Let us assume that the gap upper boundary coincides with the magnetic field lines that are specified by \( \theta_\ast = (1 - h_m/2)\theta_m^\ast \). Numerical examinations show that \( h_m \), indeed, changes as a function of the distance along the field line and the magnetic azimuthal angle (measured counter-clockwise around the dipole axis). Nevertheless, except for young pulsars like the Crab pulsar, an assumption of a spatially constant \( h_m \) gives a relatively good estimate. Thus, for an analytical purpose, we adopt a constant \( h_m \) in this analytical examination. In this case, we can specify the middle-latitude field line by the magnetic co-latitude \( \theta_\ast = (1 - h_m/2)\theta_m^\ast \). Screening of \( E_{||} \) due to the polarization of the produced pairs, takes place mostly in the lower altitudes. It is, therefore, appropriate to evaluate the screening of \( E_{||} \) around the point \( (r_0, \theta_0) \) where the null-charge surface intersects the middle-latitude field line.

An inwardly migrating electron (or an outwardly migrating positron) emits photons inwards (or outwards), which propagate the typical distance \( l_1 \) (or \( l_2 \)) before escaping from the gap. Denoting the cross section of the inward (or outward) horizontal line from the point \( (r_0, \theta_0) \) and the upper boundary as \( (r_1, \theta_1) \) (or as \( (r_2, \theta_2) \)), and noting \( r_0 \cos \theta_0 = r_1 \cos \theta_1 = r_2 \cos \theta_2 \), we obtain \[ l_1 = r_0 \cos \theta_0 (\tan \theta_0 - \tan \theta_1), \] \[ l_2 = r_0 \cos \theta_0 (\tan \theta_2 - \tan \theta_0). \] Along the upper-boundary field line, we obtain \[ \frac{\sin^2(\theta_1 - \alpha)}{r_1} = \frac{\sin^2(\theta_2 - \alpha)}{r_2} = \frac{\sin^2[(1 - h_m)\theta_m^\ast]}{r_\ast}. \]
whereas along the middle-latitude field line, we obtain

\[ \frac{\sin^2(\theta_0 - \alpha)}{r_0} = \frac{\sin^2[(1 - h_{m}/2)\theta_{m}^{\text{max}}]}{r_s}. \]  

Combining these two equations, and noting \( \theta_{m}^{\text{max}} \ll 1 \), we find that \( \theta_1 (\approx \theta_0) \) and \( \theta_2 (\gg \theta_0) \) can be given by the solution \( \theta \) that satisfies

\[ \cos \theta \sin^2(\theta - \alpha) = \left( \frac{1 - h_{m}}{1 - h_{m}/2} \right)^2 \cos \theta_0 \sin^2(\theta_0 - \alpha), \]  

where \( \theta_0 \) is given by

\[ \tan \theta_0 = \frac{1}{2} \left( 3 \tan \alpha + \sqrt{9 \tan^2 \alpha + 8} \right). \]  

Thus, if we specify \( \alpha \), we can solve \( \theta = \theta_1 \) and \( \theta = \theta_2 \) as a function of \( h_m \) by equation (10). Substituting these \( \theta_1 \) and \( \theta_2 \) into equations (5) and (7), we obtain \( l_1 \) and \( l_2 \), where \( r_0 \) depends on \( P \).

If \( h_m \ll 1 \), we can expand the left-hand side of equation (10) around \( \theta = \theta_0 \), where \( \theta = \theta_1 \) for inward (or \( \theta = \theta_2 \) for outward) \( \gamma \)-rays to find \( \theta_2 - \theta_0 = \theta_0 - \theta_1 \sim \sqrt{h_m} \). That is, the leading terms in the expansion vanish and we obtain \( l_1 = l_2 \) from the next-order terms, which are quadratic to \( \theta - \theta_0 \). Assuming \( L_X \propto t^{-\beta} \), where \( \beta \approx 0.48 \) is appropriate for \( t < 10^5 \) years for a light-element-envelope NS and for \( t < 10^6 \) years for a heavy-element NS, we find \( h_m \propto P^{5/6} m^{-1/6} \mu^{1/2} \), and hence \( L_\gamma \propto P^{-3/2} m^{3/2} \mu^{3/2} \). Since the dipole radiation formula gives \( P \propto \mu^{1/2} \), we obtain \( L_\gamma \propto m^{3/2} \mu^{3/2} \propto t^{-0.03} \). Thus, when the gap is very thin, which is expected for a light-element-envelope NS, \( L_\gamma \) little evolves with the pulsar age, \( t \).

However, if \( h_m > 0.2 \), say, the rapidly expanding magnetic flux tube gives asymmetric solution, \( l_2 > l_1 \). That is, the third and higher order terms in the expansion contribute significantly compared to the quadratic terms. Thus, we must solve equation (10) for \( \theta = \theta_1 \) or \( \theta_2 \) without assuming \( |\theta - \theta_0| \ll 1 \), in general.

Let us now consider the condition for a gap to be self-sustained. A single ingoing \( e^- \) or an outgoing \( e^+ \) emits

\[ (N_\gamma)_1 = eE_\parallel l_1/(\hbar c) \]  

or

\[ (N_\gamma)_2 = eE_\parallel l_2/(\hbar c) \]  

photons while running the typical distance \( l_1 \) or \( l_2 \), respectively. Such photons materialize as pairs with probability

\[ \tau_1 = l_1 F_1 \sigma_1 / c \]  

or

\[ \tau_2 = l_2 F_2 \sigma_2 / c, \]  

where \( F_1 \) and \( F_2 \) denote the X-ray flux inside and outside of \( (r_0, \theta_0) \), respectively; \( \sigma_1 \) and \( \sigma_1 \) are the pair-production cross section for inward and outward photons, respectively. Thus, a single \( e^- \) or \( e^+ \) cascades into

\[ (N_\gamma)_1 \tau_1 = \frac{eE_\parallel}{\hbar c} F_1 l_1^2 \sigma_1 \]  

pairs or into

\[ (N_\gamma)_2 \tau_2 = \frac{eE_\parallel}{\hbar c} F_2 l_2^2 \sigma_2 \]  

pairs within the gap. That is, a single inward-migrating \( e^- \) cascades into pairs with multiplicity \( (N_\gamma)_1 \tau_1 \). Such produced pairs are polarized by \( E_\parallel \).

Each returning, outward-migrating \( e^+ \) cascades into pairs with multiplicity \( (N_\gamma)_2 \tau_2 \) in outer magnetosphere. As a result, a single inward \( e^- \) cascades eventually into \( (N_\gamma)_1 \tau_1 \cdot (N_\gamma)_2 \tau_2 \) inward \( e^- \)'s, which should become unity for the gap to be self-sustained. Therefore, in a stationary gap, the gap thickness \( h_m \) is automatically regulated so that the gap closure condition,

\[ (N_\gamma)_1 \tau_1 \cdot (N_\gamma)_2 \tau_2 = 1, \]  

may be satisfied.

Approximately speaking, a single, inward-migrating \( e^- \) emits \( (N_\gamma)_1 \sim 10^5 \) curvature photons, a portion of which head-on collide the surface X-ray photons to materialize as pairs with probability \( \tau_1 \sim 10^{-3} \) within the gap. Thus, a single \( e^- \) cascades into \( (N_\gamma)_1 \tau_1 \sim 10 \) pairs in the gap. Each produced \( e^+ \) return outwards to emit \( (N_\gamma)_2 \sim 10^5 \) photons, which materialize as pairs with probability \( \tau_2 \sim 10^{-6} \) by tail-on colliding with the surface X-rays. In another word, \( (N_\gamma)_1 \tau_1 \sim 10 \) holds regardless of the nature of the pair production process (e.g., either photon-photon or magnetic process [Takata et al. 2010] in the lower altitudes, because it is determined by the pair-production efficiency in the outer magnetosphere \( (N_\gamma)_2 \tau_2 \sim 0.1 \), which is always due to photon-photon pair production.

In general, \( (N_\gamma)_1, \tau_1, (N_\gamma)_2, \tau_2 \) are expressed in terms of \( h_m, P, \mu, T, \) and \( \alpha \), where \( T \) denotes the NS surface temperature. Note that we can solve \( P = 2\pi/\Omega \) as a function of the NS age, \( t \), from the spin-down law. Thus, specifying \( \alpha \) and the cooling curve, \( T = T(t) \), we can solve \( h_m \) as a function of \( t \) from the gap closure condition, \( (N_\gamma)_1 \tau_1 (N_\gamma)_2 \tau_2 = 1 \). Note also that the spin-down law readily gives the spin-down luminosity, \( L_{\text{spin}} \propto PP^{-3} \), as a function of \( t \), once \( P = P(t, \alpha) \) is solved. On these grounds, \( L_\gamma \) can be related to \( L_{\text{spin}} \) with an intermediate parameter \( t \), if we specify the cooling curve and the spin-down law.

Substituting equations (16) and (17) into (18), we obtain

\[ \frac{eE_\parallel \sqrt{F_1 \sigma_1 F_2 \sigma_2}}{\hbar c} l_1 l_2 = 1, \]
\[ F_i \sigma_i = \pi(1 - \mu_i) \left( \frac{r_i}{r_0} \right)^2 \int_{\nu_{th, i}}^{\infty} \frac{B_\nu(T)}{\hbar \nu} \sigma_\nu(\nu, \nu_\gamma, \mu_i) \]  
(20)

with \( i = 1, 2 \); \( \nu_\gamma \) denotes the \( \gamma \)-ray frequency, and \( B_\nu(T) \) the Planck function. We have to integrate over the soft photon frequency \( \nu \) above the threshold energy

\[ h \nu_{th, i} = \frac{2(m_e c^2)^2}{(1 - \mu_i)h \nu_\gamma}, \]  
(21)

where \( m_e c^2 \) refers to the rest-mass energy of the electron. The cosine of the collision angle \( \mu_i \) becomes \( 1 - \mu_1 = 1 - \sin \theta_0 \) for outward (or \( 1 - \mu_2 = 1 + \sin \theta_0 \) for inward) \( \gamma \)-rays. That is, collisions take place head-on (or tail-on) for inward (or outward) \( \gamma \)-rays. The total cross section is given by

\[ \sigma_\nu = \frac{3}{16} \sigma_T(1 - v^2) \left[ (3 - v^4) \ln \frac{1 + v}{1 - v} - 2v(2 - v^2) \right], \]  
(22)

where \( \sigma_T \) denotes the Thomson cross section and

\[ v \equiv \sqrt{1 - \frac{2}{1 - \mu_i}} \frac{(m_e c^2)^2}{h \nu h \nu_\gamma}. \]  
(23)

Pair production takes place when the \( \gamma \)-rays collide with the surface X-rays in the Wien regime, that is, \( h \nu \gg kT \). An accurate evaluation of \( \sigma_2 \) requires a careful treatment of the collision geometry, because the threshold energy, \( h \nu_{th, 2} \), strongly depends on the tiny collision angles. In the numerical method (next section), the pair-production absorption coefficient is explicitly computed at each point in the three-dimensional pulsar magnetosphere. However, in this section, for analytical purpose, we simply adopt the empirical relation,

\[ \sqrt{F_1 \sigma_1 F_2 \sigma_2} = \epsilon \sqrt{1 - \mu_1} \sigma_T F_X, \]  
(24)

where \( \epsilon = 0.004, 0.01, \) and 0.038 for \( \alpha = 45^\circ, 60^\circ, \) and \( 75^\circ, \) respectively; \( 1 - \mu_1 \approx 2 \). The X-ray flux is evaluated at \((r_0, \theta_0)\) such that

\[ F_X = \frac{L_X}{2 \pi k T} \frac{1}{4 \pi r_0^2}, \]  
(25)

where \( L_X \) refers to the luminosity of photon radiation from the the cooling NS surface. For a smaller \( \alpha \), the point \((r_3, \theta_0)\) is located in the higher altitudes, where the magnetic field lines begin to collimate along the rotation axis, deviating from the static dipole configuration. Thus, the collision angles near the light cylinder, and hence \( \sigma_2 \) decreases with decreasing \( \alpha \). The explicit value of \( \epsilon \) can be computed only numerically, solving the photon specific intensity from infrared to \( \gamma \)-ray energies in the three-dimensional pulsar magnetosphere.

The last factor, \( l_1 l_2 \), in the left-hand side of equation (19) is given by

\[ l_1 l_2 = r_0^2 \cos^2 \theta_0 (\tan \theta_0 - \tan \theta_1) (\tan \theta_2 - \tan \theta_0) \]  
(26)

Thus, equation (19) gives

\[ \epsilon E_\gamma \frac{L_X/e}{c \nu_\gamma 2.82 k T} \epsilon \sqrt{1 - \mu_1} \sigma_T \times \cos^2 \theta_0 (\tan \theta_0 - \tan \theta_1) (\tan \theta_2 - \tan \theta_0) \]  
(27)

where the \( r_0 \) dependence vanishes. Substituting equations (1), (2), (3) into (27), we can solve \( h_m \) as a function of \( L_X/k T, P, \) and \( \mu \).

To describe the evolution of \( P = P(t) = 2\pi / \Omega(t) \), we adopt in this paper

\[ - I \Omega \dot{\Omega} = C \frac{h_2^2 \Omega^4}{e^2}, \]  
(28)

where \( C = (2/3) \sin^2 \alpha \) for a magnetic dipole braking, while \( C = 1 + \sin^2 \alpha \) for a force-free braking [Spitkovski 2006]. Assuming a magnetic dipole braking, we obtain

\[ P = 39.2 ms \mu_{30} I_{45}^{1/2} (t/10^3 \text{years})^{1/2}, \]  
(29)

where \( \mu_{30} \equiv \mu/(10^{30} \text{G cm}^3) \) and \( I_{45} \equiv I/(10^{45} \text{g cm}^2) \). Thus, if we specify a cooling scenario, \( T = T(t) \), equation (27) gives \( h_m \) as a function of \( t \). Note that the \( \alpha \) dependence of the spin-down law is not essential for the present purpose; thus, \( C = 2/3 \) is simply adopted. Once \( h_m = h_m(t) \) is obtained, equation (1) readily gives \( L_X \) as a function of \( t \), and hence of \( L_{\text{spin}} \). It is worth noting that the heated polar-cap emission is relatively weak compared to the cooling NS emission, except for millisecond or middle-aged pulsars.

We adopt the minimal cooling scenario [Page et al. 2004]. Within the minimal cooling scenario, the cooling history of a NS substantially depends on the composition of the envelope. We adopt the cooling curves given in [Page et al. 2004] and consider the two extreme cases: light element and heavy element envelopes.

We present the solved \( h_m \) for a light and a heavy element envelope in figure 2 in [Hirotani 2013]. It follows that the gap becomes thinner for a light element case than for the heavy element cases. This is because the more luminous photon field of a light element envelope leads to a copious pair production, which prevents the gap to expand in the trans-field direction. As a result, the predicted \( L_\gamma \) becomes less luminous for a light element envelope than a heavy one.

In figure 4 we present the analytical results of \( L_\gamma \), versus \( L_{\text{spin}} \) as the dotted (or dashed) curve for a light (or a heavy) element envelope. As the pulsar spins down, \( L_\gamma \) evolves leftwards. It is interesting to note that \( L_\gamma \) little evolves for a light element envelope, as explained in [Hirotani 2013].
3. Numerical examination of outer-gap electrodynamics

Let us develop the analytical examination and look deeper into a self-consistent solution by a numerical method. To this end, we adopt the modern outer-gap model [Hirota, 2011] and solve the set of Maxwell and Boltzmann equations self-consistently and compute $E_\parallel$, distribution functions of $e^\pm$, and the photon specific intensity at each point in the three-dimensional pulsar magnetosphere. We consider not only the whole-surface, cooling NS emission but also the heated polar-cap emission as the photon source of photon-photon pair production in the numerical analysis. The former emission component is given as a function of the pulsar age from the minimum cooling scenario, in the same manner as in the analytical examination, while the latter emission component is solved consistently with the energy flux of the $e^-$'s falling on to the pulsar polar-cap surface.

The numerical method is described in [Hirota, 2013] in detail. We solve the set of partial and ordinary differential equations under the boundary conditions that $e^\pm$'s or $\gamma$-rays do not penetrate into the gap from outside. By this method, we can solve the acceleration electric field $E_\parallel$, particle distribution functions $n_{\pm}$, and the photon specific intensity $I_\nu$ (from $\nu = 0.005$ eV to 50 TeV), at each position in the three-dimensional magnetosphere of arbitrary rotation-powered pulsars, if we specify $P$, $\mu$, $\alpha$, and $kT$. We adopt the minimum cooling scenario in the same manner as in §2.

In figure 1 we plot the result of $L_\gamma$ as a function of $L_{\text{spin}}$ as the dash-dotted (or solid) curve for a light (or a heavy) element envelope, where $\mu_{\text{avg}} = 3.2$ is adopted in the same manner as in the analytical examination. It follows that these numerical solutions are consistent with the analytical ones, and that $L_\gamma$ decreases slowly until $10^4\text{.5}$ years. The physical reason why $L_\gamma$ increases with decreasing $L_{\text{spin}}$ at $t > 10^4$ years for a light element envelope, is the same as described at the end of §4. A realistic NS will have an envelope composition between the two extreme cases, light and heavy elements. Thus, the actual $L_\gamma$'s will distribute between the red solid (or dashed) and the blue dash-dotted (or dotted) curves. However, after $L_\gamma$ approaches $L_{\text{spin}}$ (thin dashed straight line; see Wang & Hirota, 2011 for the death line argument), the outer gap survives only along the limited magnetic field lines in the trailing side of the rotating magnetosphere because of a less efficient pair production; as a result, $L_\gamma$ rapidly decreases with decreasing $L_{\text{spin}}$. For a smaller $\alpha$, even for a light element envelope, $L_\gamma$ monotonically decreases as the dash-dot-dot-dot curve shows, because the gap is located in the higher altitudes, and because the less efficient pair production there prevents the produced electric current to increase with decreasing age around $t \sim 10^{4.5}$ years.

4. Discussion

To sum up, a light element envelope approximately corresponds to the lower bound of the (observationally inferred) gamma-ray luminosity of rotation-powered pulsars, whereas a heavy element one to the upper bound. The scatter of the intrinsic gamma-ray luminosity is physically determined by the magnetic inclination angle, $\alpha$, and the envelope composition. The cutoff energy of the primary curvature emission is kept below several GeV even for young pulsars, because the gap trans-field thickness, and hence the acceleration electric field, is suppressed by the polarization of the produced pairs in the lower altitudes.

To convert the observed $\gamma$-ray flux into luminosity, $L_\gamma$, one has conventionally assumed $f_{\Omega} = 1$. For example, the error bars of the observational data points in figure 1 do not contain any uncertainties incurred by $f_{\Omega}$. Nevertheless, if $\alpha$ and $\zeta$ can be constrained, we can estimate $L_\gamma$ more accurately, by applying the present quantitative outer-gap calculations. It is noteworthy that $L_\gamma$'s given in figure 1 little depend on the NS magnetic moment, $\mu$. This is particularly true for a light element case, which has $h_{\text{avg}} \ll 1$, by the reason described after equation (11). What is more, with an additional determination of $d$ (e.g., by parallax observations), we can infer the composition of individual NS envelopes, by using the constrained flux correction factor, $f_{\Omega}$ (fig. 6 in Hirota, 2013). We hope to address such a question as the determination of $\alpha$ and $\zeta$, and hence $f_{\Omega}$, for individual pulsars, by making an ‘atlas’ of the pulse profiles and phase-resolved spectra that are solved from the basic equations in a wide parameter space of $P$, $\mu$, $T$, $\alpha$, and $\zeta$, and by comparing the atlas with the observations.

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Figure 1: Evolution of the outer-gap luminosity as a function of the neutron-star spin-down luminosity. Analytical results are plotted as dotted and dashed curves, while numerical ones as dash-dotted, solid, and dash-dot-dot-dot ones. A light or a heavy element envelope is assumed, as indicated in the box. For comparison, the case of $\alpha = 45^\circ$ is also depicted. The green filled circles designates the normal gamma-ray pulsars, while the blue filled squares do those detected by the gamma-ray blind search technique.

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