Variation Law and Experimental Study on Multi-stage Critical Speed of an Aero-engine Power Turbine Rotor

Weijian Nie\textsuperscript{1,2}, Wangqun Deng\textsuperscript{1,2} and Yanong Chen\textsuperscript{1,2}

\textsuperscript{1} AECC Hunan Aviation Powerplant Research Institute, Zhuzhou 412002, China
\textsuperscript{2} Key Laboratory of Aero engine Vibration Technology, Aero Engine Corporation of China, Zhuzhou 412002, China

E-mail: nwjxj@126.com

Abstract. Finite element method was used to establish the calculation model of the dynamic characteristics of the simulated power turbine rotor, and the dynamic characteristics of the rotor were systematically calculated and analysed. The variation law of the first three stages critical speeds of the rotor with the supporting stiffness and the mass of each disk were revealed. The dynamic characteristics test of the simulated power turbine rotor in the full speed range was completed, and the calculation results and test results were compared and analysed. The results show that the calculation model reflects the dynamic characteristics of the rotor well, and the rationality of the rotor structure design, supporting layout and critical speed distribution were verified. The study provides a theoretical basis for the adjustment of critical speed based on the supporting stiffness and disk’s mass, it also provides technical reference for the real rotor structure design and test.

1. Introduction

Rotor is an important rotating part of turbine machineries such as motors, pumps, compressors, gas turbines and aero-engines. It is of great significance to study rotor dynamics. Many scholars like Zhou [1], He[2], Zhang[3], Tai[4], Du[5] and other researcher[6-9] have studied the dynamics of turbo machinery rotor and achieved notable results. Compared with the rotor of other turbo machinery, the rotor of aeroengine was characteristic of high speed, complex structure and bad working condition, there were great risks and difficulties in the dynamic research of the real rotor directly. Therefore, the dynamic analysis and research of the simulated rotor (connection structure, supporting layout, mass inertia, etc. were basically consistent with the real rotor) were usually carried out firstly. Deng [10,11], Liu [12] and Wu [13], conducted dynamic analysis and experimental research on the simulated rotor of the aeroengine, which provided a reference for the structure design and test of the real rotor. Critical speed design is the core content of the rotor dynamics design of small and medium-sized aeroengine, which plays an important role in the development of small and medium-sized aeroengine. In order to meeting the margin requirements of the rotor critical speed design, it can be achieved by selecting appropriate supporting stiffness, changing the mass distribution, optimizing the rotor structure and other methods. Deng [14,20], Nie [15,16], Hong [17], Bai [18], Mei [19] and other scholars have carried out a systematic study on the influencing factors of the critical speed of the aeroengine, providing a theoretical basis for the adjustment of the critical speed. Whether the structure design, supporting layout and critical speed distribution of the rotor are reasonable or not needs to be verified by experiments.
In this paper, dynamic analysis and experimental research were carried out on the flexible rotor with complex structure of aeroengine. The dynamic characteristic calculation model of the simulated power turbine rotor was established by finite element method. The dynamic characteristic of the rotor was systematically calculated and analysed. The laws of the first three critical speeds of the rotor change with supporting stiffness and each disk’s mass were revealed, and the dynamic characteristic in the full speed range was carried out. The rationality of rotor structure design and dynamic design was verified by experimental research, which laid a solid foundation for the dynamic research of real rotor.

2. Rotor Structure
The schematic diagram of the simulated power turbine rotor is shown in figure 1. The whole rotor was mainly composed of power turbine shaft, first stage simulated power turbine disk, second stage simulated power turbine disk and other parts. The two stages simulated power turbine disks were connected by end teeth, the first stage simulated power turbine disk and power turbine shaft were connected by splines for transmitting torque with cylindrical centering at both ends, and the second stage simulated power turbine disk and power turbine shaft were connected by circle cylinder interference fit. The length of the rotor’s power turbine shaft was nearly 1.5 meters, and the hollow structure was adopted. The mass center position, mass and moment of inertia of the simulated power turbine disks were in good agreement with the real disks. There were four bearings in the rotor, among which the No.1 bearing was ball bearing, the No.2, No.6.5 and No.7 bearings were all roller bearing. The simulated power turbine rotor was a slender, hollow structure flexible rotor with large aspect ratio.

![Figure 1. Structural Sketch of the Rotor.](image)

3. Finite Element Analysis Model

3.1. Concentrated Mass
In order to facilitating the model, parts of the body of the two stages simulated power turbine disks were simulated with the concentrated mass, which characteristics are shown in table 1.

| Parts                          | Mass (kg) | Polar Moment of Inertia (kg m²) | Diameter Moment of Inertia (kg m²) |
|-------------------------------|-----------|---------------------------------|-----------------------------------|
| Partial First Stage Simulated Power Turbine Disk | 10.399    | 89169                           | 177583                           |
| Partial Second Stage Simulated Power Turbine Disk | 14.337    | 137028                          | 272823                           |

3.2. Supporting Stiffness
The supporting stiffness of the rotor is shown in table 2.

| Supporting Number | No.1 | No.2 | No.6.5 | No.7 |
|-------------------|------|------|--------|------|
| Stiffness(E+7N/m) | 5    | 1.78 | 3.5    | 0.0154 |
3.3. Finite Element Model

During the pre-processing of the model, the structure of the rotor has been simplified, and some small local structures such as chamfering and threaded holes have been neglected. The finite element software PATRAN was used to establish the model based on beam element, and then the analysis software SAMCEF/ROTOR was used to improve the calculation model (mainly including the establishment of concentrated mass elements and bearing elements). The model has 708 beam elements, 717 nodes, 2 concentrated mass elements and 4 bearing elements. The finite element calculation model of the rotor established is shown in figure 2.

![Finite Element Model of The Rotor](image)

Figure 2. Finite Element Model of The Rotor.

4. Calculation Results of Dynamic Characteristics

4.1. Results of Critical Speeds and Mode Shapes

Under the condition of supporting stiffness in table 2, the critical speeds and mode shapes of the first three stages of the simulated power turbine rotor were calculated. The results of the first three stages critical speed and their margins are shown in table 3, and the first three stages mode shape are shown in figure 3.

| Stage | First | Second | Third |
|-------|-------|--------|-------|
| Critical Speed(rpm) | 4730 | 9250 | 26533 |
| Margin (%) | >35 | >18 | >50 |

![Mode Shapes](image)

(a) First Stage.  (b) Second Stage.  (c) Third Stage.

Figure 3. The First Three Stages Mode Shape of The Rotor.

It can be seen that the simulated power turbine rotor worked above the two stages critical speed, the second stage critical speed margin was slightly less than 20%, and the other two stages critical speed meet the design requirement. The first three stages mode shapes of the simulated power turbine rotor were all bending mode shapes, and the slender power turbine shaft was the main reason for the bending deformation of the rotor.

4.2. Law of Critical Speeds Change with Supporting Stiffness

Without changing the structure and mass distribution of the simulated power turbine rotor, the stiffness of the rotor in the table 2 was taken as the reference stiffness, and the law of the first three stages critical speeds of the rotor change with the supporting stiffness was obtained through calculation, which provides a theoretical basis for the adjustment of critical speeds based on the supporting stiffness.
4.2.1. Law of Critical Speeds Change with No.1 Supporting Stiffness. When the stiffness of No.2, No.6.5 and No.7 supporting stayed unchanged, when No.1 supporting stiffness varied from 2E+6N/m to 20E+7N/m, curve of the first three stages critical speeds of the simulated power turbine rotor change with No.1 supporting stiffness is shown in figure 4, and the change rates of the first three stages critical speed are shown in table 4. It can be seen that the first, second and third stage critical speeds of the simulated power turbine rotor increased by 8.36%, 11.65% and 81.97% respectively when the No.1 supporting stiffness increased from 2E+6N/m to 5E+7N/m. With the continuous increase of the No.1 supporting stiffness, the first and third critical speeds of the rotor almost no longer changed.

4.2.2. Law of Critical Speeds Change with No.2 Supporting Stiffness. When the stiffness of No.1, No.6.5 and No.7 supporting stayed unchanged, when No.2 supporting stiffness varied from 2E+6N/m to 20E+7N/m, curve of the first three stages critical speeds of the simulated power turbine rotor change with No.2 supporting stiffness is shown in figure 5, and the change rates of the first three stages critical speed are shown in table 5. It can be seen that the first, second and third stage critical speeds of the simulated power turbine rotor increased by 21.22%, 37.20% and 72.49% respectively when the No.2 supporting stiffness increased from 2E+6N/m to 5E+7N/m. With the continuous increase to 20E+7N/m of the No.2 supporting stiffness, the first three stages critical speed increased by 1.91%, 8.12% and 7.72% respectively.

| Ranges of No.1 Supporting Stiffness (E+7N/m) | 0.2-5 | 5-20 |
|---------------------------------------------|-------|------|
| Change Rate of First Stage Critical Speed (%) | 8.36  | 0.59 |
| Change Rate of Second Stage Critical Speed (%) | 11.65 | 0.80 |
| Change Rate of Third Stage Critical Speed (%) | 81.97 | 0.52 |

| Ranges of No.2 Supporting Stiffness (E+7N/m) | 0.2-5 | 5-20 |
|---------------------------------------------|-------|------|
| Change Rate of First Stage Critical Speed (%) | 21.22 | 1.91 |
| Change Rate of Second Stage Critical Speed (%) | 37.20 | 8.12 |
| Change Rate of Third Stage Critical Speed (%) | 72.49 | 7.72 |

4.2.3. Law of Critical Speeds Change with No.6.5 Supporting Stiffness. When the stiffness of No.1, No.2 and No.7 supporting stayed unchanged, when No.6.5 supporting stiffness varied from 2E+6N/m
to $2\times 10^7$ N/m, curve of the first three stages critical speeds of the simulated power turbine rotor change with No. 6.5 supporting stiffness is shown in figure 6, and the change rates of the first three stages critical speed are shown in table 6. When the No. 6.5 supporting stiffness increased from $2\times 10^6$ N/m to $5\times 10^7$ N/m, the first stage critical speeds of the rotor increased by 131.28%, with the continuous increase to $2\times 10^7$ N/m of the No. 6.5 supporting stiffness, the first stage critical speeds increased by 8.23%. The second stage critical speeds of the rotor increased by 35.40% when the No. 6.5 supporting stiffness increased from $2\times 10^6$ N/m to $2\times 10^7$ N/m. There was no substantial change in the third stage critical speed of the rotor when the No. 6.5 supporting stiffness increased from $2\times 10^6$ N/m to $2\times 10^7$ N/m.

Table 6. Change Rates of the First Three Stages Critical Speeds (No.6.5 Supporting Stiffness Changes).

| Ranges of No.6.5 Supporting Stiffness (E+7N/m) | 0.2-5 | 5-20 |
|---------------------------------------------|-------|------|
| Change Rate of First Stage Critical Speed (%) | 131.28 | 8.23 |
| Change Rate of Second Stage Critical Speed (%) | 20.62 | 14.78 |
| Change Rate of Third Stage Critical Speed (%) | 1.04 | 1.88 |

Table 7. Change Rates of the First Three Stages Critical Speeds (No.7 Supporting Stiffness Changes).

| Ranges of No.7 Supporting Stiffness/(E+7N/m) | 0.01-5 | 5-20 |
|---------------------------------------------|-------|------|
| Change Rate of First Stage Critical Speed (%) | 101.29 | 0 |
| Change Rate of Second Stage Critical Speed (%) | 57.19 | 20.05 |
| Change Rate of Third Stage Critical Speed (%) | 1.50 | 2.16 |

4.2.4. Law of Critical Speeds Change with No.7 Supporting Stiffness. When the stiffness of No. 1, No.2 and No.6.5 supporting stayed unchanged, when No.7 supporting stiffness varied from $1\times 10^5$ N/m to $2\times 10^7$ N/m, curve of the first three stages critical speed of the simulated power turbine rotor changes with No.7 supporting stiffness is shown in figure 7, and the change rates of the first three stages critical speed are shown in table 7. When the No.7 supporting stiffness increased from $1\times 10^5$ N/m to $5\times 10^7$ N/m, the first stage critical speeds of the rotor increased by 101.29%, with the continuous increase to $2\times 10^7$ N/m of the No.7 supporting stiffness, the first stage critical speed almost no longer changed. The second stage critical speeds of the rotor increased by 77.24% when the No.7 supporting stiffness increased from $1\times 10^5$ N/m to $2\times 10^7$ N/m. There was no substantial change in the third stage critical speed of the rotor when the No.7 supporting stiffness increased from $1\times 10^5$ N/m to $2\times 10^7$ N/m.

4.3. Law of Critical Speeds Change with Simulated Power Turbine Disks
Without changing the supporting stiffness and the disks’ structure, it was revealed that the first three stages critical speed of the rotor varied with the disks’ mass, which only depends on the density of the disks. The supporting stiffness for calculation is shown in table 2, and the disk material and their properties are shown in table 8.

**Table 8. The Disks’ Properties.**

| Material       | Density (kg m\(^{-3}\)) | Modulus of Elasticity (GPa) | Poisson’s Ratio |
|----------------|--------------------------|----------------------------|-----------------|
| Magnesium Alloy| 1780                     | 40.2                       | 0.34            |
| Aluminium Alloy| 2800                     | 71                         | 0.31            |
| Titanium Alloy | 4440                     | 109                        | 0.34            |
| Stainless Steel| 7800                     | 196                        | 0.3             |
| Superalloy     | 8010                     | 205                        | 0.29            |
| Superalloy     | 8400                     | 191                        | 0.3             |

4.3.1. Law of Critical Speeds Change with Mass of The First Stage Simulated Power Turbine Disk.

When the density of first stage simulated power turbine disk varied from 1780 kg m\(^{-3}\) to 8400 kg m\(^{-3}\), the curve of the first three stages critical speeds of the rotor change with density of the first stage simulated power turbine disk is shown in figure 8, and the change rates of the first three stages critical speeds are shown in table 9. When the density of the first stage simulated disk increased from 1780 kg m\(^{-3}\) to 8400 kg m\(^{-3}\), the change rates of the first three stages critical speeds of the simulated power turbine rotor were all less than 5%, indicating that the mass of the first stage disk has little effect on the first three stages critical speeds of the simulated power turbine rotor.

**Figure 8. Curve of Critical Speeds Change with Density of The First Stage Disk.**

4.3.2. Law of Critical Speeds Change with Mass of The Second Stage Simulated Power Turbine Disk.

When the density of second stage simulated power turbine disk varied from 1780 kg m\(^{-3}\) to 8400 kg m\(^{-3}\), the curve of the first three stages critical speeds of the rotor change with density of the second stage simulated power turbine disk is shown in figure 9, and the change rates of the first three stages critical speeds are shown in table 10. When the density of the second stage simulated disk increased from 1780 kg m\(^{-3}\) to 8400 kg m\(^{-3}\), the change rates of the first two stages critical speed of the rotor were reduced 30.67% and 8.57% respectively, and there was no substantial change in the third critical speed. It shown that the mass of the second stage simulated disk has a certain influence on the first two critical speeds of the rotor, but has no substantial influence on the third critical speed.

**Figure 9. Curve of Critical Speeds Change with Density of The Second Stage Disk.**
Table 9. Change Rates of the First Three Stages Critical Speeds (Density of The First Stage Disk Changes).

| The First-stage Disk’s Density (kg m⁻³) | 1780-8400 |
|----------------------------------------|-----------|
| Change Rate of First Stage Critical Speed (%) | 3.25 |
| Change Rate of Second Stage Critical Speed (%) | -0.89 |
| Change Rate of Third Stage Critical Speed (%) | 4.89 |

Table 10. Change Rates of the First Three Stages Critical Speeds (Density of The Second Stage Disk Changes).

| The Second-stage Disk’s Density (kg m⁻³) | 1780-8400 |
|----------------------------------------|-----------|
| Change Rate of First Stage Critical Speed (%) | -30.67 |
| Change Rate of Second Stage Critical Speed (%) | -8.57 |
| Change Rate of Third Stage Critical Speed (%) | 1.85 |

5. Dynamic Characteristics Test and Result Analysis

Dynamic characteristics test of the rotor was carried out on the high-speed rotating rig. In figure 1, a displacement sensor D₁ was arranged in the vertical direction of position A, a displacement sensor D₂ was arranged in the vertical direction of position C, and two displacement sensors D₃ and D₄ were respectively arranged in the vertical and horizontal directions of position B. The rotor deflection at positions A, B and C were measured respectively.

The curves of rotor deflection change with relative speed the rotor in the full speed range measured by the four sensors are shown in figure 10. In figure 10, the relative speed is defined as: relative speed = (actual speed / rated working speed) × 100%.

![Figure 10. Curves of Rotor Deflection Change with Relive Speed Measured by Four Sensors.](image)

The analysis of the critical speed calculation results and the experimental results were obtained, as shown in table 11. In table 11, calculation error was defined as: Calculation Error = ((Test Result - Calculation Result)/Test Result) × 100%.
### Table 11. Analysis of Calculation Result and Test Result of Critical Speeds.

| Calculation Result(rpm) | First Stage | Second Stage |
|-------------------------|-------------|--------------|
| Test Result(rpm)        | 4730        | 9250         |
| Error (%)               | Not Obvious | 8456~8638    |
| Margin of Test Result (%)| —           | 7.08~9.38    |

The calculation error of the second critical speed was 7.08-9.38%, less than 10%. The difference between rotor calculation model and actual structure is the main cause of error, and the measurement error is another source of the error. In fact, if the actual error of oil film stiffness, the measurement error of supporting stiffness and the complex structure of the rotor were considered, the calculation results still have good consistency with the test results. The rotor has obvious resonance peak at the critical speed, indicating that the rotor has been bending change, it is also consistent with the calculated vibration modes. From the test results, the margin of the second critical speed of the rotor was 23.71~25.31%, greater than 20%, which meets the critical speed margin requirement [21].

### 6. Conclusion

In this paper, the finite element calculation model of the dynamic characteristics of the slender simulated power turbine rotor was established, and the dynamic characteristics of the rotor were systematically calculated and analysed. The laws of the first three critical speeds of the rotor change with the supporting stiffness and the mass of each disk were revealed, which provides the theoretical basis for the adjustment of the critical speeds of the rotor, and the dynamic characteristics test verification of the rotor in the full speed range was completed. The main conclusions are as follows.

1. No.1 supporting stiffness has obvious influence on the third critical speed, No.2 supporting stiffness has obvious influence on the first three critical speeds, No.6.5 supporting stiffness has obvious influence on the first and second critical speeds, and No.7 supporting stiffness has obvious influence on the first and second critical speeds.

2. In a certain range of supporting stiffness, the first three critical speeds of the simulated rotor can be adjusted by adjusting the supporting stiffness.

3. The mass of the first stage simulated power turbine disk has no substantial influence on the first three critical speeds of the rotor. The mass of the second stage simulated power turbine disk has a significant influence on the first stage critical speed of the rotor, a certain influence on the second stage critical speed, and no substantial influence on the third stage critical speed.

4. Mode shapes of the first three stages mode shapes of the simulated power turbine rotor are all flexural mode shapes. The test results verified the rationality of the rotor structure design, supporting layout and critical speed distribution.

5. The calculated error of the finite element model is less than 10%, which is in good agreement with the experimental results.

### References

[1] Zhou C Y, Xue Z H, Liu X F and Wen J X, 2018, The rotor dynamic calculation analysis of pump rotor system, *Chinese Journal of Turbomachinery*, 60( S1), pp.39–48.

[2] He C H, Xue Z H and Liu X F, 2019, The rotor dynamic analysis of centrifugal pump rotor system, *Chinese Journal of Turbomachinery*, 61( Z1), pp.65–70.

[3] Zhang Y, Zhang Y C and Li R M, 2017, Rotor-dynamics system analysis of centrifugal blowers with high speed and efficiency, *Chinese Journal of Turbomachinery*, 59(3), pp.40–43.

[4] Tai X Y, Li Y, Yang S H and Zhang Y, 2019, Analysis of rotor dynamics characteristic for marine floating centrifugal compressor, *Chinese Journal of Turbomachinery*, 61(1), pp.63–70.
[5] Du G W, Wang Y Z and Yang G J, 2017, Rotor dynamic research of a 165kW helium blower with AMBs, Chinese Journal of Turbomachinery, 59(4), pp.63–70.

[6] Li Z H, Ren H and Luo Z Y, 2013, Study on dynamic behaviors of a certain type aero-engine rotor system and based on co-simulation technology, Machinery Design & Manufacture, (12), pp.174–176,180.

[7] Deng W Q, Guo F Y and Gao D P, 2016, Dynamic characteristics calculation for a high speed flexible rotor of an aeroengine, Journal of Vibration and Shock, 25(5), pp.130–133.

[8] Miao H, Wang K M and Ai S M, 2011, Finite element analysis on critical speeds of a dual-rotor system, Journal of Shenyang Aerospace University, 28(5), pp.27–31.

[9] Mo Y Y, Li Q T and Zhang B, 2011, Dynamic characteristic calculation of a engine's double rotors system, Machinery Design & Manufacture, (7), pp.117–119.

[10] Deng W Q, Tang G abd Shu S R, 2010, Dynamic characteristics research on simulated low pressure rotor of contra rotating engine, Aeroengine, 36(6), pp.6–8.

[11] Deng W Q, Fan P P and Xu Y L, 2017, Dynamic characteristic study of a simulated low-pressure rotor of a turboprop engine, Aeronautical Science & Technology, 28(8), pp.72–77.

[12] Liu W K, Deng W Q and Lu B, 2019, Research on the dynamic of a cantilever power turbine rotor with flexible transition section, Gas Turbine Experiment and Research, 32(2), pp.34–41.

[13] Wu G F, Chen G Z and Tu M P, 2006, Analysis and experimental study of the high speed flexible rotor dynamic behaviors, Journal of Aerospace Power, 26(3), pp.563–568.

[14] Deng W Q, Nie W J and He P, 2015, Change laws of critical speeds with supporting stiffness of a high-speed flexible rotor, Noise and Vibration Control, 35(3), pp.98–101.

[15] Nie W J, Deng W Q and He P, 2015, Influence of different draw-bars on dynamic characteristics of a high speed flexible rotor, Noise and Vibration Control, 35(3), pp.135–139.

[16] Nie W J, Deng W Q and Xu Y Y, 2015, Laws analysis on critical speeds change with supporting stiffness and disk mass of a high-speed flexible rotor, Gas Turbine Experiment and Research, 28(3), pp.19–24.

[17] Hong J, Wang H and Xiao D W, 2008, Effects of dynamic stiffness of rotor bearing on rotor dynamic characteristics, Aeroengine, 34(1), pp.23–27.

[18] Bai Z X, Wu W L and Liu H Z, 2012, The effect of the stiffness of a rotors supporting system on its dynamic characteristics, Machine Design and Research, 28(4), pp.18–21.

[19] Mei Q and Ou Y X, 2004, Effects of supporting distribution on dynamic characteristics of a rotor with double bearing, Journal of Vibration Engineering, 17(S), pp.156–158.

[20] Deng W Q, Wang Y and Nie W J, 2016, Influence analysis of supporting stiffness and supporting axial location on critical speeds of a low-pressure rotor of a counter rotating engine, Aeroengine, 42(3), pp.7–11.

[21] Fu C G, 2000, Rotor Dynamics and Whole Engine Vibration vol.19, (Beijing: Aviation Industry Press).