Segregation of granular binary mixtures by a ratchet mechanism

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We report on a segregation scheme for granular binary mixtures, where the segregation is performed by a ratchet mechanism realized by a vertically shaken asymmetric sawtooth-shaped base in a quasi-two-dimensional box. We have studied this system by computer simulations and found that most binary mixtures can be segregated using an appropriately chosen ratchet, even when the particles in the two components have the same size, and differ only in their normal restitution coefficient or friction coefficient. These results suggest that the components of otherwise non-segregating granular mixtures may be separated using our method.

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While segregation is often an undesired effect, sometimes separating the components of a granular mixture is the ultimate goal. Since ancient times sieves have been used by humans to separate small grains from bigger ones. But nature manages to separate different kinds of grains also without sieves. Many of the segregation processes in granular matter\textsuperscript{[11–13]} have recently been studied in great detail such as segregation according to particle size, shape or friction properties in a shaken box\textsuperscript{[4–6]}, in a rotating drum\textsuperscript{[7–10]} or when poured into a thin box\textsuperscript{[1–3]}. However, so far no method has been proposed for segregating particles which only differ in hardness.

The setup we use for segregation is as follows: particles of a binary mixture are falling into a two-dimensional box from above at a specified place. The base of the box having an asymmetric sawtooth profile is shaken harmonically in vertical direction. The transport properties of homogenous granular media in a similar setup were studied earlier\textsuperscript{[14]} inspired by recent progress in the theoretical understanding of molecular motors\textsuperscript{[15,16]}. In the corresponding models, known as thermal ratchets, fluctuation-driven transport phenomena can be interpreted in terms of overdamped Brownian particles moving through a periodic but asymmetric (typically sawtooth shaped) potential in the presence of nonequilibrium fluctuating forces (such as periodic driving or switching between potentials). When a particle jumps out of the box at either the right or the left boundary, it is removed and counted, and finally the segregation quality is determined. For details on the geometry see Fig. 1. The segregation quality is \( q = \left( 2 \max\{N_{1\rightarrow} + N_{2\rightarrow}, N_{1\leftarrow} + N_{2\leftarrow}\}\} / (N - 1) \right) \times 100\%\), where \( N_{i\rightarrow} \) (\( N_{i\leftarrow} \)) denotes the number of particles of the \( i \)th component (\( i = 1, 2 \)) leaving the box on the left (right) side, and \( N \) is the number of all particles in the mixture. Thus the quality of random segregation, when the particles go to the left or right side with equal probabilities, is 0\%. In this Letter we show results for binary mixtures in which the components contain equal number of particles, in other cases different quality definitions may also be appropriate. Unlike other segregation phenomena, in which segregation is due to the collective behaviour of the grains, here the interaction between the base and the individual particles is dominant, and the efficiency of segregation usually decreases with increasing number of particle-particle collisions. However, setting a sufficiently low load rate \( R \), the quality can be high, and still, “parallelizing” the procedure the separation capacity can be large (see bellow). In the simulation an event-driven algorithm\textsuperscript{[17]} is applied with a hard-sphere collision model\textsuperscript{[18]}, in which the sphere-shaped particles have five parameters: mass \( m \), radius \( r \), normal restitution coefficient \( e \), friction coefficient \( \mu \) and maximum tangential restitution coefficient \( \beta_0 \). The particles can rotate around the axis going through their center and perpendicular to the plane of the box, their moment of inertia is \( 2/5 m r^2 \) about their center. Since the mass of the particles does not play a role in collisions with the base but only in binary collisions, it is enough to specify that the particles have the same mass density, so the mass is cubically proportional to the radius.
If there are only few particles in the box at the same time, then, as a first approximation, the interaction between the particles can be neglected. Therefore, we investigated the motion of one particle in an infinitely wide box in detail, and found it to be chaotic in most cases. A similar, but simpler model was also reported to show chaotic behaviour [20]. Depending on the parameters, it is possible that the particle follows a periodic trajectory, travelling with velocity \( v = ub/c \), where \( b \) and \( c \) are integer numbers, meaning that in one period, which lasts for \( c \) vibration cycles, the particle jumps over \( b \) teeth. These periodic trajectories are not interesting for practical applications for the following reasons: (1) the transients are usually very long, and therefore may be more important than the asymptotic periodic trajectory for the segregation behaviour, (2) periodic trajectories are not robust against collisions with other particles and other sources of noise, and, last but not least, (3) for certain conditions two periodic trajectories with opposite directions can coexist for the same type of particles. In this case one cannot predict on which side the particle tends to leave the box. We explain below how these periodic trajectories can be avoided when searching for the ratchet parameters suitable for segregating a given binary mixture. In the chaotic regime, however, the time evolution of the particle’s horizontal position can be well described as drift–diffusion. The connection between chaotic motion and diffusion has been investigated extensively recently [21] [22]. A simple explanation for this drift–diffusion behavior can be that on time scales larger than the typical time it takes for the particle to jump to another sawtooth, the kicks of the base can be considered to be independent of each other. Furthermore, the asymmetry of the ratchet leads to an average velocity in the left or right direction. Consequently, the horizontal motion in the chaotic regime can be described statistically by two parameters: a drift velocity \( v \) and a diffusion coefficient \( D \). An example for the drift–diffusion motion can be seen in Fig. 4.

The observation that the horizontal motion can be well approximated by drift–diffusion enables us to predict segregation-related quantities [22]. First of all, the probability that a particle jumps out of the box through the right (\( n_+ \)) or the left (\( n_- \)) boundary is: 
\[
 n_+(u, L, \alpha) = \frac{1-e^{-u/L}}{1-e^{-u/L}} \quad \text{and} \quad n_-(u, L, \alpha) = 1 - n_+(u, L, \alpha),
\]
where we introduce the notation \( u = v/D \), since the probabilities depend on the drift velocity and the diffusion coefficient only through this combination. The asymptotic behaviour of these probabilities in \( L \) is exponential with characteristic length \( \alpha^{-1}|u|^{-1} \), which means that if the drift velocity is, say, positive and the box width is large enough, most or all of the particles arrive at the right end. The approximate explanation for this is the following: the displacement of the average particle position increases linearly in time, while the width of the probability distribution is proportional only to the square root of time (although this is exactly true only if the box width is infinite, for large box widths it is still a good approximation). Therefore, on length scales larger than \( |u|^{-1} \), the drift dominates over diffusion, so that most likely the particle leaves the system at the end towards which it drifted.

As a consequence, for the segregation of a binary mixture the theory suggests that if the drift velocities of the particles of the two components (\( v_1 \) and \( v_2 \)) have opposite directions, then one can obtain an arbitrarily good segregation quality by choosing the box wide enough. For a fixed box width, the best segregation quality is given by 
\[
 q_{\text{opt}} = [n_-(u_1, L, \alpha_{\text{opt}}) + n_-(u_2, L, \alpha_{\text{opt}}) - 1] \times 100\%,
\]
where \( u_1 < 0 < u_2 \), and \( \alpha_{\text{opt}} = 1 - \frac{|u_1|e^{x_2x_1} - 1}{|u_2|e^{x_1x_2} - 1} \) gives the optimal place of loading to obtain the best possible quality. The asymptotic behaviour of \( q_{\text{opt}} \) as a function of \( L \) shows that it exponentially approaches 100% with characteristic length \( (|u_1| + |u_2|)|u_1u_2|^{-1} \).

Figure 3 shows an example for segregation of particles differing only in friction coefficient. In this example the particle parameters are: \( r_1 = r_2 = 1.5 \text{ mm}, e_1 = e_2 = 0.4, \mu_1 = 0.1, \mu_2 = 0.3, \beta_{11} = \beta_{02} = 0.4, \) the ratchet parameters are: \( w = 8 \text{ mm}, h = 6 \text{ mm}, a = 0.4, \theta = -0.08, A = 2 \text{ mm}, f = 20 \text{ s}^{-1}, \) and the segregation parameters are: \( \alpha = 0.51, H = 1 \text{ cm}, R = 0.5 \text{ s}^{-1}. \) The corresponding diffusion parameters are \( v_1 = -1.58 \text{ cm s}^{-1}, D_1 = 1.36 \text{ cm}^2 \text{s}^{-1}, v_2 = 1.86 \text{ cm s}^{-1} \) and \( D_2 = 1.83 \text{ cm}^2 \text{s}^{-1}, \)
FIG. 3. Segregation of particles differing only in friction coefficient. The segregation quality (solid line) rapidly grows to 100% with increasing box width, the dashed line shows the theoretical prediction. At small box widths the deviation from the theoretical prediction (dashed line) is due to that (1) the drift-diffusion approximation is valid on length scales larger than the sawtooth width, and (2) the starting state (position and velocity) of a particle is untypical to the drift-diffusion motion. Inset: The drift velocity and diffusion coefficient (measured in an infinitely wide box) as functions of the friction coefficient $\mu$. The drift velocity changes its sign twice, at $\mu = 0.14$ and at $\mu = 0.4$, and is only coincidentally zero at $\mu = 0$.

The ratchet segregating a certain binary mixture most efficiently is searched in the following way: for both components the diffusion parameters of the one particle motion ($v_1, D_1$ and $v_2, D_2$) are measured using many different ratchets. Then we select those ratchets for which the drift velocities have opposite directions (it may happen that no such ratchet is found). Setting the box width to a reasonable value, for each of the selected ratchets the best segregation quality is predicted, and the ratchet with the highest segregation quality is chosen for segregating the mixture. However, it is possible that with this ratchet one or both of the particles have periodic trajectories, which is undesired for segregation. We describe here one possible solution to this problem: any kind of noise can destroy periodic trajectories. For example, in the computer simulation the angle of the relative velocity after a particle–particle collision is changed by an amount uniformly chosen from an interval $[-\delta \phi, \delta \phi]$. We found that a rebounding angle noise $\delta \phi \approx 0.05$ (measured in radian) is enough to destroy the periodic trajectories, drastically changing the drift velocity. Hence, the diffusion parameters for both particles and each ratchet are also measured with $\delta \phi = 0.05$ and 0.1. Then, for selecting a ratchet it is not enough that the drift velocities have opposite direction when $\delta \phi = 0$, but their values should not change too much when the noise level is increased to 0.05 and 0.1 (we allowed a maximum relative change of 50% and 70%, respectively). The best ratchet is chosen from this restricted set by a rule which takes into account that (1) the quality should be high also in the noisy case, (2) $|v_1|$ and $|v_2|$ should be as large as possible to allow a high load rate, (3) $\alpha_{opt}$ should not change much in the presence of noise, otherwise the segregation quality may decrease much due to interaction between particles. This choice of noise is not only good for avoiding periodic trajectories, but also may serve as a first approximation for taking into account the deviation of the particles’ shape from a sphere. Therefore, the ratchets selected by this method are robust to some extent against deviations in shape and presumably in other parameters as well.

We checked the practicability of this procedure by finding appropriate ratchets to segregate $27^3 = 351$ different binary mixtures composed from 27 particle types, measuring the diffusion parameters for each type by simulating 3000 seconds of particle trajectories for 3125 ratchets. The parameters which were varied are listed in Table I, the maximum tangential restitution coefficient and the vibration amplitude were fixed $\beta_0 = 0.4$ and $\beta = 2$ mm, respectively. Then we performed a segregation simulation for each mixture with the selected ratchet (for three mixtures no proper ratchet was found in this set). The results show that in most cases good segregation quality can be achieved even if the rebounding angle noise is relatively high or the particles within the components are not uniform (see Fig. 3).

If no ratchet is found, even in an extended set, which segregates a certain mixture (i.e., the drift velocities always have the same sign), then there is an alternative procedure which we describe here only briefly. Unless the particles in the components are exactly the same, there exists a ratchet for which $u_1$ and $u_2$ are different. If, e.g., $0 < u_1 < u_2$, then choosing an appropriate loading place near the left end, more particles from component 1 will be collected on the left side than from component 2. Then the particles collected on the right side are reloaded again and again. For a given box width and number of reloads one can determine the optimal load place to obtain the best possible segregation quality, which tends

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$r$ & $\epsilon$ & $\mu$ & $w$ & $h$ & $a$ & $f$ & $\theta$ \\
[mm] & [mm] & [mm] & $[s^{-1}]$ & [rad] \\
\hline
1 & 0.4 & 0.1 & 6 & 6 & 0 & 16 & -0.16 \\
1.5 & 0.6 & 0.3 & 8 & 8 & 0.1 & 18 & -0.08 \\
2 & 0.8 & 0.5 & 10 & 10 & 0.2 & 20 & 0.0 \\
12 & 12 & 0.3 & 22 & 0.08 \\
14 & 14 & 0.4 & 24 & 0.16 \\
\hline
\end{tabular}
\caption{The parameter values used to produce $3^3 = 27$ particle types and $5^5 = 3125$ ratchet types.}
\end{table}
FIG. 4. Segregation quality for 351 binary mixtures. In all cases the segregation parameters are $L = 30\,\text{cm}$, $H = 3\,\text{cm}$ and $R = 0.25\,\text{s}^{-1}$. a, The particles are uniform within the components, and the rebounding angle noise is $\delta \phi = 0$. The number of mixtures in the last column (quality is better than 95%) is 219, and the quality is better than 70% for 320 mixtures. b, The same as in a but the rebounding angle noise is $\delta \phi = 0.3$. The quality is better than 70% for 270 mixtures. c, The same as in a but the particles are not uniform within the components, all of the parameters ($r$, $e$, $\mu$ and $\beta_0$) are varied by maximum 10%. The quality is better than 70% for 263 mixtures. d, The rebounding angle noise is $\delta \phi = 0.3$ and the particle parameters are varied by maximum 10%. The quality is better than 70% for 220 mixtures.

to 100% with increasing number of reloads (results not shown here).

One may think that the method presented here is not efficient for segregating real granular mixtures, since only a few grains can be present in the box at the same time, allowing only a low load rate. However, it is very easy to “parallelize” the procedure: as the box is essentially two-dimensional, many boxes can be placed onto a shaking machine, and possibly the walls between the boxes can be omitted. A rough estimation shows that the capacity of such a machine would be comparable to that of the machines in use today in the industry for cleaning, e.g., cereal grains (in which case the capacity is in the order of 1 ton per hour). We are planning to carry out experiments to check if mixtures containing particles of the same size differing only in normal restitution coefficient or friction coefficient can be segregated by this method.

We presented a computer simulation study of a method for segregating a binary granular mixture. The segregation is performed by a ratchet mechanism, and in contrast to other segregation schemes, here not the collective behavior of the particles is dominant but the interaction between the base and the individual particles. We found that good segregation quality can be achieved even if the particles of the two components differ only in friction coefficient or hardness.

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[1] H. M. Jaeger, and S. R. Nagel, Science 255, 1523-1531 (1992)
[2] A. Mehta (ed.), Granular Matter: An Interdisciplinary Approach (Springer, New York, 1994)
[3] H. J. Herrmann, J.-P. Hovi, and S. Luding (eds), Physics of Dry Granular Media (Kluwer, Dordrecht, 1998)
[4] A. Rosato, K. J. Strandburg, F. Prinz, and R. H. Swendsen, Phys. Rev. Lett. 58, 1038 (1987)
[5] J. B. Knight, H. M. Jaeger, and S. R Nagel, Phys. Rev. Lett. 70, 3728 (1993)
[6] W. Cooke, S. Warr, J. M. Huntley, and R. C. Ball, Phys. Rev. E 53, 2812 (1996)
[7] O. Zik, D. Levine, S. G. Lipson, S. Shtrikman, and J. Stavans, Phys. Rev. Lett. 73, 644 (1994)
[8] G. Baumann, I. M. Janosi, and D. E. Wolf, Phys. Rev. E 51, 1879 (1995)
[9] P.-Y. Lai, L.-C. Jia, and C. K. Chan, Phys. Rev. Lett. 79, 4994 (1997)
[10] D. Ktitarev and D. E. Wolf, Granular Matter 1, 141 (1998)
[11] P. Meakin, Physica A 163, 733 (1990)
[12] H. A. Makse, S. Havlin, P. R. King, and H. E. Stanley, Nature 386, 379 (1997)
[13] J. Baxter, U. Tuzun, D. Heyes, I. Hayati, and P. Fredlund, Nature 391, 136 (1998)
[14] Z. Farkas, P. Tegzes, A. Vukics, and T. Vicsek, Phys. Rev. E 60, 7022 (1999)
[15] F. Jülicher, A. Ajdari, and J. Prost, Rev. Mod. Phys. 69, 1260 (1997)
[16] R. D. Astumian, Science 276 917 (1997)
[17] B. D. Lubachevsky, J. Comput. Phys. 94, 255 (1991)
[18] O. R. Walton in Particulate Two-Phase Flow (ed Roco, O. M.) (Butterworth–Heinemann, Boston, 1992)
[19] G. L. Baker and J. P. Gollub, Chaotic dynamics: an introduction (Cambridge University Press, 1990)
[20] J. Duran, Europhys. Lett. 17, 679 (1992)
[21] P. Gaspard and G. Nicolis, Phys. Rev. Lett. 65, 1693 (1990)
[22] P. Gaspard et al., Nature 394, 865 (1998)
[23] C. P. Dettmann and E. G. D. Cohen, [nlin.CD/0001062]
[24] H. Risken, The Fokker–Planck Equation (Springer, Berlin, 1989)
[25] Z. Farkas and T. Fülöp, [cond-mat/0010325], T033104.