NN-EMD: Efficiently Training Neural Networks using Encrypted Multi-sourced Datasets

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Abstract

Training complex neural network models using third-party cloud-based infrastructure among multiple data sources becomes a promising approach in existing machine learning solutions. However, privacy concerns of large-scale data collection and recent regulations or acts have restricted the availability and use of privacy sensitive data in the third-party infrastructure. To address the privacy issue, a promising emerging approach is to train a neural network model over an encrypted dataset. Specifically, the model training process can be outsourced to a third party such as a cloud service that is backed by significant computing power, while the encrypted training data keeps the data confidential from the third party. Compared to training a traditional machine learning model over encrypted data, however, it is extremely challenging to train a deep neural network (DNN) model over encrypted data for two reasons: first, it requires large-scale computation over huge datasets; second, the existing solutions for computation over encrypted data, such as using homomorphic encryption, is inefficient. Further, for enhanced performance of a DNN model, we also need to use huge training datasets composed of data from multiple data sources that may not have pre-established trust relationships among each other. We propose a novel framework, NN-EMD, to train DNN over multiple encrypted datasets collected from multiple sources. Toward this, we propose a set of secure computation protocols using hybrid functional encryption schemes. We evaluate our framework for performance with regards to the training time and model accuracy on the MNIST datasets. We show that compared to other existing frameworks, our proposed NN-EMD framework can significantly reduce the training time, while providing comparable model accuracy and privacy guarantees as well as supporting multiple data sources. Furthermore, the depth and complexity of neural networks do not affect the training time despite introducing a privacy-preserving NN-EMD setting.

1 Introduction

Deep neural networks (DNN), also known as deep learning, have been increasingly used in many fields such as computer vision, natural language processing, and speech/audio recognition [37]. Such DNN-based solutions usually consist of two phases: the training phase and the inference phase. In the training phase, a well-designed neural network is provided as input a training dataset and an appropriate optimization algorithm to generate optimal parameters for the neural network; then, in the inference phase, the generated model (i.e., optimal parameters) is used for inference tasks, namely, predicting a label for an input sample.

One of the critically needed components in DNN-based applications is a powerful computing infrastructure with higher performance CPU and GPU, larger memory storage, etc., [27]. The volume of training data is another critical component. For instance, existing commercial Machine Learn-
Table 1: Comparison of representative privacy-preserving approaches in DEEP LEARNING

| Proposed Work     | Training | Prediction | Privacy △ | DS † | Underlying Technique                        |
|-------------------|----------|------------|-----------|------|---------------------------------------------|
| [51]              | ✓        | ✓          | ○         | -    | Federated Setting*                           |
| [1]               | ✓        | ✓          | ○         | -    | Differential Privacy                        |
| [42] (SecureML)   | ✓        | ✓          | ○         | ❌    | Customized SMC (Garbled Circuits)           |
| [48] (DeepSecure) | -        | ✓          | ○         | ❌    | General SMC (Garbled Circuits)              |
| [30], [18], [33]  | ✓        | ✓          | ○         | 🌠    | Homomorphic Encryption                      |
| [25] and [43]     | ✓        | ✓          | ○         | 🌠    | Homomorphic Encryption                      |
| [59] (CryptoNN)   | ✓        | ✓          | 🌠         | -    | Functional Encryption                       |
| NN-EMD (our work) | ✓        | ✓          | 🌠         | ✓    | Hybrid Functional Encryption                 |

△ The column denotes privacy guarantee degree such as mild approach ○ (e.g., differential privacy) and strong guarantee ⚫ (e.g., confidentiality level privacy guarantee).
† Multiple Data Source: the minus symbol indicates the approach supports multiple data sources to some extent.
* The model is trained in a federated manner where each data owner trains a partial model on their private data.

In Table 1, we summarize existing representative privacy-preserving approaches used by deep learning systems. Existing solutions such as a federated learning approach and those based on differential privacy cannot provide strong privacy guarantees because of inference attacks, as demonstrated in the literature [22], [52], [44]. The general secure multi-party computation (garbled circuits based) approaches, such as those proposed in [42], [48], have a limitation with regards to large volumes of encrypted data that need to be transferred during the execution of the associated secure protocols. Except for the recently proposed solutions such as CryptoNN [59] and [43], [25], most of these SMC approaches only address privacy issues in the inference phase rather than in the training phase; this is mainly due to the efficiency challenges related to both computation and communication.

Furthermore, none of the existing solutions consider the fact that training data may be coming from multiple data sources distributed horizontally or vertically. The training dataset may have different composition cases; it may include data from multiple data sources, where: (i) each data source provides a dataset that includes all the features; (ii) each one provides a dataset that has only a subset of the features; but, collectively these datasets cover the complete set of features; or (iii) it is a hybrid of (i) and (ii). Even though existing solutions such as CryptoNN [59] support case (i), cases (ii) and (iii) still pose a huge challenge when considering privacy-preserving training of a neural network model.
In this paper, we propose a framework to train a Neural Network over Encrypted Multi-sourced Datasets (NN-EMD). That is, the NN-EMD trains a neural network using a dataset that is composed of independently encrypted datasets from many different sources. Each data source may provide its encrypted data that may include a complete set of features or only a subset of features. The goal here is to provide a strong privacy guarantee, while training a DNN model more efficiently as compared to the most recently proposed solutions, namely, those in [59, 43]. The most related work CryptoNN [59] and our NN-EMD both use functional encryption (FE) to address the problem of computing over encrypted data. To tackle the challenge in the aforementioned three cases, in essence, NN-EMD proposes a new method (i.e., hybrid FE solution) instead of the simple FE solution as adopted in CryptoNN. The consequent implementation and experiments of NN-EMD are also different from CryptoNN, which will be discussed later. To the best of our knowledge, NN-EMD is the first efficient and more practical approach for training a DNN over a set of encrypted/private multi-sourced datasets.

Specifically, to tackle the challenges of secure computation between a server, and a client pool (data sources) contributing multiple datasets that are composed in various ways, we first propose two non-interactive secure computation protocols between a server and the client pool, namely, a secure two-party horizontally partitioned computation (S2PHC) and a secure two-party vertically partitioned computation (S2PVC) protocols. We construct these two protocols by using two types of functional encryption schemes. The NN-EMD uses these as the building blocks; in particular, S2PHC and S2PVC are used in each training iteration according to varying data composition cases.

We also implemented an NN-EMD system that can be deployed in a real cloud environment to support training neural networks using a set of encrypted datasets from multiple sources that are independently pre-processed locally. Each data source can independently encrypt its data samples with complete or partial features, or incomplete data samples with partial features. Based on the collected encrypted (incomplete) data samples, the remotely deployed NN-EMD system is able to train a global model.

We analyze the security and privacy properties of our proposed NN-EMD approach and show that it satisfies the security and privacy goals. We evaluate the performance of NN-EMD with regards to training time, local pre-processing time and model accuracy. The experimental results on the MNIST dataset show that our proposed NN-EMD approach achieves significant efficiency improvements by reducing training time by more than 90% compared to that of the best existing homomorphic encryption based approach while achieving the comparable model accuracy and privacy guarantees by adopting the functional encryption scheme with a built-in third-party authority to provide key service. Furthermore, the depth and complexity of neural networks do not affect the training time despite introducing a privacy-preserving NN-EMD setting.

**Organization.** In Section 2, we introduce the background and preliminaries. We propose NN-EMD in Section 3.1, the associated threat model in Section 3.2, the underlying secure computation approaches in Section 3.3, and the details of the framework in Section 3.4. The security and privacy analysis is presented in Section 4 and the evaluation is presented in Section 5. We discuss the related work in Section 6 and conclude the paper in Section 7.
In this paper, we focus on a DNN approach that uses a client-server architecture with two parties: (i) the **cloud service provider** (server) with powerful computational infrastructure that can be employed for training a DNN model; and (ii) the **client pool** (data sources) that have privacy-sensitive datasets and need to build a DNN model based on these training datasets without leaking private information.

Such privacy-preserving DNN needs novel secure computation protocols to support efficient computation and interactions between the client pool and the server, while offering strong privacy guarantees. Existing general secure multi-party computation (SMC) solutions (i.e., garbled circuits) have limitations because they need to perform several rounds of communication involving transmission of large volumes of intermediate data. Using these techniques for DNN is cost prohibitive because of the huge volumes of training data needed. Cryptography-based solutions (e.g., homomorphic encryption-based SMC) also has computational efficiency problem.

To the best of our knowledge, the approach proposed by Bost et al. in [14] is the first work that supports both training and predictive analysis over encrypted data. Their approach achieves this by integrating several crypto schemes, i.e., Quadratic Residuosity cryptosystem, Paillier cryptosystem, and homomorphic encryption, with secure protocols designed for them. However, their approach only supports limited types of basic ML models such as naïve bayes, decision trees and support vector machine, but not DNNs. Most recently, approaches proposed by Nandakumar et al. in [43], and Xu et al. in [59] are the only ones that support training neural networks over encrypted data; their approaches use homomorphic and functional encryption, respectively. Insight from these two approaches indicates that the crypto-based secure computing techniques are promising for the training phase of a DNN model. However, there are two key challenges toward achieving effective and efficient training of neural networks over encrypted datasets that we address in this paper: (i) **Efficiency of Training Process**: The existing secure computing protocols are not efficient, as mentioned above. For instance, with optimized approaches (e.g., multiple threads, training data distillation) in [43], training time for one mini-batch, with 60 samples, is around 40 minutes because the computation of each layer of neural networks is over ciphertext. This indicates that training time in the case of a larger volume of training data will be significantly higher. (ii) **Multiple Data Sources**: There is a lack of consideration of real complex datasets composed of horizontally and vertically partitioned datasets coming from multiple data sources. Meanwhile, the training techniques also provide strong confidentiality-level privacy guarantees.

### 2.2 Functional Encryption

In this paper, we use functional encryption to construct our secure computing protocols instead of homomorphic encryption that has been employed by most of the existing privacy-preserving machine learning approaches.

Generally, functional encryption (FE) belongs to a public-key encryption family [39][12], where the decrypting party can be issued a secret key, also known as a functionally derived key, to allow it to learn the result of a function over a ciphertext without leaking the corresponding plaintext. To construct functional encryption schemes for general functionality, most of the recently proposed approaches such as those in [26][13][57][23][17][38] only focus on the theoretical feasibility or the existence of functionalities, and not on the computational efficiency issues. These schemes rely on strong primitives such as indistinguishable obfuscation or multilinear maps that are prohibitively inefficient [35].

As most underlying computational operations of the training and inference phases of a DNN can be classified as matrix multiplications, or more precisely, vector inner-products, we employ **functional encryption for inner-product (FEIP)** scheme instead of the general functional encryption scheme that provides general functionality at the expense of inefficiency. To be specific, we adopt two kinds of FEIP schemes: **single-input FEIP** and **multi-input FEIP**. The security of both these schemes is based on the decisional Diffie-Hellman (DDH) assumption.

**Single-input FEIP.** We adopt the single-input FEIP (SI-FEIP) construction proposed in [2]. In SI-FEIP scheme, the supported function is described as $f_{SIIP}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{\eta} (x_i y_i)$ s.t. $|\mathbf{x}| = |\mathbf{y}| = \eta$, where $\mathbf{x}$ and $\mathbf{y}$ are two vectors of length $\eta$, from different parties. The SI-FEIP scheme $\mathcal{F}_S$ includes four algorithms: Setup, SKGenerate, Encrypt, Decrypt, and is defined as $\mathcal{F}_S = (\mathcal{F}_S.S, \mathcal{F}_S.K, \mathcal{F}_S.E, \mathcal{F}_S.D)$ in the rest of the paper.
Multi-input FEIP. We employ the multi-input FEIP (MI-FEIP) construction derived from the work proposed in [3]. In the MI-FEIP scheme, the support function is defined as:

$$f_{MIIP}(x_1, x_2, ..., x_n, y) = \sum_{i=1}^{n} \sum_{j=1}^{\eta_i} (x_{ij} y \sum_{k=1}^{i-1} \eta_k + j) \text{ s.t. } |x_i| = \eta_i, |y| = \sum_{i=1}^{n} \eta_i,$$

where $x_i$ and $y$ are vectors from different parties. Accordingly, the MI-FEIP scheme $F_M$ includes five algorithms: Setup, PKDistribute, SKGenerate, Encrypt, Decrypt, and is defined as $F_M = (F_M.S, F_M.PK, F_M.SK, F_M.E, F_M.D)$ in the rest of the paper.

Note that there exist three roles/entities in the SI-FEIP and MI-FEIP schemes: (i) an encryptor that employs encrypt algorithm to protect the sensitive data; (ii) a decryptor that uses decrypt algorithm to acquire function result; (iii) a third-party authority (TPA) that runs the setup algorithm to initialize the cryptosystem and then runs pkdistribute and skgenerate algorithms to provide key service for both encryptors and decryptors.

2.3 Neural Networks

Deep learning models are typically achieved by DNN in a hierarchical and non-linear architecture consisting of multiple layers. Each layer includes several neural units (a.k.a, neurons) to receive the data generated from its previous layer and outputs the processed data for its next layer. Such a structure allows higher-level, abstract features to be represented as lower-level features through non-linear function computation at each layer.

Usually, a DNN includes three types of layers: one input layer; one output layer; and several hidden layers. In particular, the raw data is encoded properly and fed into the input layer. Then, the features are abstracted and mapped from the raw data gradually from the first layer to the last layer via non-linear activation functions and iterative update (a.k.a, gradient descent optimization algorithm) until the convergence condition (e.g., the limited training time, the specified number of iterations, the expected training accuracy) is reached. Such mapping abstractions, also known as the learned model or neural weights, can be used to perform the inference/predictive tasks.

Suppose that a DNN includes $L$ layers. The computation of layer $l$ can be represented as $a_l = f_{act}(w_l \cdot a_{l-1})$, where $a_l$ is the output of layer $l$, $f_{act}$ is the activation function, $w_l$ is the weight matrix of layer $l$. The goal of training a DNN is to learn optimal neural weights $W$ based on a given training dataset $D = \{(x^{(1)}, y^{(1)}), ..., (x^{(n)}, y^{(n)})\}$ and a specified loss function $L$. Such a problem can be described as $\min_{W} E_D(W) = \min_{W} \sum_{i=1}^{n} L(y^{(i)}, a^{(i)}_l)$. As a non-linear optimization problem, common solutions include gradient descent and its variants. In our work, our proposed secure computation approaches support the computation over the matrix. Thus, we use the well-known mini-batch stochastic gradient descent (SGD) algorithm \([15, 40]\) as the optimization algorithm, where the secure computation is over a mini-batch for each iteration.
3 The Proposed NN-EMD Framework

3.1 Overview

In NN-EMD, we have the following three roles/entities: the client pool, the server, and a trusted third-party authority (TPA):

- A client pool of multiple data sources that collaboratively contribute to the final training dataset composed of horizontally and vertically partitioned data, or a hybrid mix of the two. Each data source still keeps its data confidential from the rest.
- A server responsible for training a DNN over a training dataset composed of multiple private datasets.
- A trusted TPA that initializes the underlying cryptosystems by setting secret credentials. Then, it distributes the associated public keys to data sources in the client pool and the server, and provides private key service to the server during the training phase. Note that the TPA cannot acquire/access the encrypted training data.

Figure 1 illustrates the essence of the NN-EMD framework. Before the model training, the server collects the meta information about the training datasets from the client pool and then launches the privacy-preserving entity resolution mechanism with each data source if the final training dataset is a vertical composition of datasets from sources in the client pool. We assume that for each data sample, there exists at least one data source having the label and only one data source’s labels are enrolled in the training phase. Meanwhile, both the client pool and the server acquire associated cryptographic keys from the TPA. Then, each data source in the client pool pre-processes its data as required by the framework and outsources the encrypted data to the server. The server starts to train the model by setting up the proper training hyperparameters, e.g., learning rate, number of iterations, and the total number of data sources, etc. For instance, suppose we have two data sources \(d_1\) and \(d_2\) with datasets \(X_{d_1}\) and \(X_{d_2}\), respectively. In the case of horizontally partitioned dataset, \(d_1\) and \(d_2\) first prepare two types of ciphertext, \(\text{Enc}(X_{d_1})\), \(\text{Enc}(X_{d_1}^T)\) and \(\text{Enc}(X_{d_2})\), \(\text{Enc}(X_{d_2}^T)\) using the secure2pc approach, respectively. With received ciphertext, the server can launch the training using our proposed training algorithm based on secure2pc approach. More detail will be presented in Section 3.4.

3.2 Threat Model and Assumptions

We assume that there exists a trusted TPA. This TPA is an independent third-party that is widely trusted by all the data sources in the client pool and the server. Note that it is also a common assumption in cryptosystems such as [12; 11; 28]. The role of a trusted TPA is similar to the role of a trusted certificate authority in existing public key infrastructures. In this paper, we consider the following threat model:

(i) Honest-but-curious Server; which is a common assumption in most of the existing approaches ([10; 59; 43]). Here, the server follows the instructions of a protocol or algorithm, but may try to learn private information by inspecting the collected encrypted dataset and decrypted functional results during the training phase.

(ii) Curious and Colluding Data Sources: In the client pool, some of curious data sources may try to collude to infer any private information of other non-colluding data sources by inspecting their outsourced encrypted data.

3.3 Secure Computation Approaches

Here, we present our proposed privacy-preserving secure computation approach between a server and a client pool (data sources). To be specific, we propose two secure computation protocols, namely, secure two-party horizontally partitioned computation protocol (S2PHC, see Figure 3) and secure two-party vertically partitioned computation protocol (S2PVC, see Figure 4). Both the protocols are non-interactive, secure two-party computation protocols, where there is no interaction between the server and the client pool; i.e., they have only one-way communication between them.

The difference between the two secure computation protocols is mainly with regards to how the input datasets from client pool are composed. Suppose that there is a secure computation task such
as a matrix multiplication $X^{l\times m}W^{m\times l}$ between the client pool and the server, where the client pool has the matrix $X^{l\times m}$ that is composed of data from different data sources $\{d_k\}$ and the server has $W^{m\times l}$. As shown in Figure 2, $X^{l\times m}$ represents horizontal or vertical composition of data from multiple sources.

**Secure Two-party Horizontally Partitioned Computation Protocol.** We present the detailed description of the non-interactive S2PHC protocol in Figure 3. The protocol is built from the single-input functional encryption scheme. Here, we suppose that each data source in the client pool has the same column length related to $X$ as illustrated in Figure 2. It indicates that each data source owns complete features for each data sample, and those data samples constitute a training dataset. Note that the S2PHC protocol can be considered as an improvement of the secure matrix computation approach proposed in [59] where the possibility of multiple horizontal data sources had been mentioned, but no theoretical analysis and practical implementation were presented. Unlike in [59], we present specific practical construction in our protocol with the experimental evaluation in Section 5.

**Secure Two-party Vertically Partitioned Computation Protocol.** Figure 4 shows the details of the S2PVC protocol. Here, we assume that each data source from the client pool has the same row length with regards to $X$. It means that each data source that owns partial features can provide the same size of data samples to constitute a training batch, while those partial features can compose the complete features. The S2PVC protocol is constructed using the multi-input functional encryption scheme as the key underlying scheme.

### 3.4 NN-EMD Training

Here, we present the details of our proposed NN-EMD framework. As mentioned above, NN-EMD mainly includes two parties: the server and the client pool, and they use S2PHC and S2PVC protocols.

Suppose that there exists data sources $S_d=\{d_1,\ldots,d_m\}$, where each data source $d_k \in S_d$ has dataset $X_{d_k}$. The goal of the NN-EMD framework is to train a neural network model based on the dataset $X$ that is composed of $\{X_{d_1},\ldots,X_{d_m}\}$ without leaking $X$ to the server, and without disclosing $X_{d_i}$ to $d_j$ where $d_i, d_j \in S_d \land d_i \neq d_j$. Such an assumption is common in existing vertical machine learning related literature, and also indicates there are no overlapping features among those data sources except for the identity feature used for the privacy-preserving entity resolution.

Algorithm 1 illustrates how our proposed S2PHC and S2PVC protocols are integrated in the training process of a neural network model. First, we initialize S2PHC and S2PVC protocols with proper security parameter $\lambda$ and function parameters $(\eta, \vec{\eta}, n)$ as defined in Section 3.3. Then, the server acquires the basic meta-information of the training dataset from each source from the client pool, and decides several training hyperparameters such as proper mini-batch size $p_{\text{batch}}$ and dataset type $T_{d_k}$ shared with each data source (lines 1-3). Note that we define dataset types: $T_f$ and $T_p$ to indicate a dataset with a full or partial set of features corresponding horizontally and vertically partitioned datasets cases, respectively.
Secure Two-party Horizontally Partitioned Computation Protocol

**Initialization and Key Services**

⇒ TPA initializes the system as follows:

- initializes the single-input FEIP cryptosystem by generating a common public key and master private key, \( pk_{\text{SI-FEIP}} \) and \( msk_{\text{SI-FEIP}} \), \( F_S, S(1^\lambda, 1^\nu) \) by giving parameters \( \lambda \) and \( \eta \), where \( \lambda \) is the security parameter indicating the bit length of security credentials, while \( \eta \) denotes the maximum length of all possible input vectors of the inner-product function \( f_{\text{SIIP}} \) during the execution phase of the protocol.

- initializes a private authenticated channel with the server and the client pool, respectively.

- delivers the public key \( pk_{\text{com}} \) and the parameter \( \eta \) to both parties, namely, client pool and server.

⇒ TPA provides key services as follows:

- receives a functional private key request, and \( w \) from the server.

- checks the \( w \) to prevent potential inference attack by making sure \( |w| \leq \eta \) and non-zero elements of \( w \) is less than the threshold \( \tau \) using the weights filter module.

- executes private key generation algorithm to generate private key \( sk_{\text{SI-FEIP,w}} \) \( \leftarrow F_S.K(msk, w) \), and sends back the key via the private authenticated channel.

**Party: Client Pool** ⇒ all data sources in the client pool agree on an encoding precision \( \epsilon_{\text{client}} \). For each data source \( d_i \in \{d_1, \ldots, d_l\} \) in the client pool, each client in the pool executes the following steps:

- receives the public key \( pk_{\text{com}} \) and \( \eta \) from the TPA and verifies the validity of \( pk_{\text{com}} \).

- encodes elements in data from floating-point format \( X_{\text{fp}} \) into integer format \( X_{\text{int}} \) with encoding precision \( \epsilon_{\text{client}} \).

- counts the shape of the length of \( X_{\text{int}} \) \( \rightarrow (s_{d_i}.r,s_{d_i}.c) \), and checks \( s_{d_i}.c \leq \eta \).

- for each row \( x_i \) of \( X_{\text{int}} \), calls SI-FEIP encryption algorithm \( ct_{x_i,d_i} \leftarrow F_S.E(pk_{\text{com}}, x_i) \).

- if any above operations (assertion, verification, encoding, encryption) fails, abort.

- sends all ciphertexts \( \{ct_{x_1,d_1}, \ldots, ct_{x_1,d_l}\} \) and parameters \( \epsilon_{\text{client}}, (s_{d_1}.r,s_{d_1}.c) \) to the server.

**Party: Server** ⇒ the server executes the following steps:

- receives the public key \( pk_{\text{SI-FEIP,com}} \) and \( \eta \) from the TPA and verifies the validity of \( pk_{\text{SI-FEIP,com}} \).

- collects ciphertexts \( ct \leftarrow \{ct_{x_1,d_1}, \ldots, ct_{x_1,d_l}\} \) and parameters \( \epsilon_{\text{client}}, \{ (s_{d_1}.r,s_{d_1}.c) \} \) from the client party.

- sets up the encoding precision \( \epsilon_{\text{server}} \), and encodes each element in input weights from floating-point format \( W_{\text{fp}} \) into integer format \( W_{\text{int}} \).

- counts the shape of \( W_{\text{int}} \) \( \rightarrow (s_{\text{server}}.r,s_{\text{server}}.c) \), and checks \( \forall i, j, s_{\text{client}}.c \leftarrow s_{d_i}.c = s_{d_j}.c \) and \( s_{\text{server}}.c = s_{\text{client}}.c \land s_{\text{server}}.r \leq \eta \).

- for each column \( w_i \) of \( W_{\text{int}} \), sends a function private key request to the TPA, and collects the received private keys \( sk \leftarrow \{sk_{f_{\text{SIIP}},w_1}, \ldots, sk_{f_{\text{SIIP}},w_m}\} \) with verification.

- if all above operations (assertion, verification, encoding, encryption) fails, abort.

- initializes a matrix \( Z \) with shape \((|ct|, |sk|)\), and for each \( i \in \{1, \ldots, |ct|\} \) and \( j \in \{1, \ldots, |sk|\} \), calls decryption algorithm \( w_{i,j} \leftarrow F_S.D(pk_{\text{SI-FEIP,com}}, ct[i], sk[j], w_j) \).

- decodes each element in \( Z \) from integer format into floating-point format using \( \epsilon_{\text{server}} \) and \( \epsilon_{\text{client}} \).

![Figure 3: Detailed description of non-interactive secure two-party horizontally partitioned computation protocol. Note that arrows indicate assignment operation, while the equal sign is a comparison operation.](image)

According to different compositions of final training data \( X \), we propose three different training approaches: horizontally partitioned based training, vertically based partitioned training, and hybrid partitioned based training.

**Horizontal Partitioning Based Training**. This approach deals with the case where each data source’s dataset has a full set of features needed in the training. That is, \( X \) is horizontally composed of \( \{X_{d_1}, \ldots, X_{d_l}\} \). In this case, each data source first divides its local dataset into several mini-batches according to the received batch parameter. Then, for each mini-batch, the data source executes S2PHC protocol twice with input mini-batch \( X_{d_k,batch} \) and its transpose \( X_{d_k,batch}^\top \), re-
Secure Two-party Vertically Partitioned Computation Protocol

Initialization and Key Services
⇒ TPA initializes the system as follows:
- initializes the multi-input FEIP crypto schemes by generating a common public key, master public key and private key $pk_{MI-FEIP}, mpk_{MI-FEIP}, msk_{MI-FEIP} \leftarrow F_M.S(\lambda, \bar{\eta}, n)$ by giving parameters $\lambda$ and $\{\bar{\eta}, n\}$, where $\{\bar{\eta}, n\}$ indicates the allowed $n$ maximum number of data sources where each data source has maximum input length represented as $\bar{\eta}$, during the computation execution of $f_{MIP}$.
- assigns a identity $id_{d_k}$ for each registered data source $d_k$ in the client pool.
- initializes a private authenticated channel with the server and the data sources, respectively.
- delivers $d_k$-associated $pk_{MI-FEIP}, id_{d_k} \leftarrow F_M.PK(mpk_{MI-FEIP}, msk_{MI-FEIP}, id_{d_k}), \eta_{id_{d_k}} \leftarrow \bar{\eta}$ and the common public key $pk_{MI-FEIP,com}$ to each data source $id_{d_k}$, respectively.
- delivers the common public key $pk_{MI-FEIP,com}, \bar{\eta}, n$ to the server.
⇒ TPA provides key services:
- receives the request $w$ from the server.
- checks $w$ to prevent potential inference attack by checking that non-zero elements of $w$ is less than the threshold $\tau$ using weights filter module.
- generates private key $sk_{MI-FEIP, w} \leftarrow F_M.SK(mpk_{MI-FEIP}, msk_{MI-FEIP}, w)$, and sends back the key via the private authenticated channel.

Party: Client Pool ⇒ all data sources agree on an encoding precision $\epsilon_{client}$. For each data source $d_k \in \{d_1, ..., d_n\}$ in the client pool, each client executes the following steps:
- receives the public key $pk_{MI-FEIP,com}, pk_{MI-FEIP, id_{d_k}}$ and $\eta_{id_{d_k}}$ from the TPA and verifies the validity of $pk_{MI-FEIP,com}$ and $pk_{MI-FEIP, id_{d_k}}$.
- encodes elements in data from floating-point $X_{fp}$ into integer $X_{int}$ with encoding precision $\epsilon_{client}$.
- counts the shape of the length of $X_{int} \rightarrow (s_{d_k}, r, s_{d_k}, c)$, and checks $s_{d_k}, c \leq \eta_{id_{d_k}}$.
- for each row $\mathbf{x}_i$ of $X_{int}$, calls MI-FEIP encryption $ct_{\mathbf{x}_i, d_k} \leftarrow F_M.E(pk_{com, \mathbf{x}_i})$.
- if any above operations (assertion, verification, encoding, encryption) fails, abort.
- sends all ciphertexts $\{ct_{\mathbf{x}_1, d_k}, ..., ct_{\mathbf{x}_i, d_k}\}$ and parameters $\epsilon_{client}, (s_{d_k}, r, s_{d_k}, c)$ to the server.

Party: Server ⇒ the server executes the following steps:
- receives the public key $pk_{MI-FEIP,com}, \bar{\eta}, n$ from the TPA and verifies the validity of $pk_{MI-FEIP,com}$.
- collects $ct \leftarrow \{ct_{\mathbf{x}_1, d_1}, ..., ct_{\mathbf{x}_i, d_n}\}$ and parameters $\epsilon_{client}, (s_{d_k}, r, s_{d_k}, c)$ from client pool.
- sets up the encoding precision $\epsilon_{server}$ and encodes each element in input weights from floating-point number $W_{fp}$ into integer number $W_{int}$.
- counts the shape of $W_{int} \rightarrow (s_{server}, r, s_{server}, c)$, and checks $\forall i, j, s_{d_k}, r = s_{d_k}, r$ and $s_{server}, r = \sum s_{d_k}, c \wedge s_{server}, r \leq \sum \bar{\eta} \wedge |ct| < n$.
- for each column $\mathbf{w}_i$ of $W_{int}$, sends a function private key request to the TPA, and collects the received keys $sk \leftarrow \{sk_{MIP, \mathbf{w}_1}, ..., sk_{MIP, \mathbf{w}_m}\}$ with verification.
- if all above operations (assertion, verification, encoding, encryption) fails, abort.
- re-organizes $ct \rightarrow ct'$ by aggregating by $ct$ index.
- initializes a matrix $Z$ with $|ct'|$ rows and $|sk|$ columns, and for each $i \in \{1, ..., |ct'\}$ and $j \in \{1, ..., |sk|\}$, and calls decryption algorithm $wu_{i,j} \leftarrow F_M.D(pk_{com}, ct[i], sk[j], \mathbf{u}_j)$.
- decodes each element in $Z$ from integer format into float point format using $\epsilon_{server}$ and $\epsilon_{client}$.

Figure 4: Detailed description of non-interactive secure two-party vertically partitioned computation protocol.

spectively. The generated ciphertexts $S_{ct_{d_k}}$ and $S_{ct_{d_k}}$ are used in feed-forward computation and backpropagation computation in the training phase, respectively (lines 6-11).

On the server side, weights are randomly initialized for the model (line 22). For each mini-batch iteration, $S2PHC$ protocol is executed with $S_{ct_{d_k}}$ to support the secure computation that occurs between the input layer and the first hidden layer (line 25). As the output is in plaintext, the normal feed-forward operations can be continued as in a normal neural network training phase (line 27). In the back-propagation phase, the normal gradient computation can be done first from the last layer
Algorithm 1: NN-EMD Training Algorithm

Input: secure parameter $\lambda$, functionality parameters $(\eta, \bar{\eta}, n)$, data sources $S_d = \{d_k\}$, each data source $d_k$ has dataset $X_{d_k}$.
Output: trained model $W$

1. initialize S2PHC protocol by setting $(\lambda, \eta)$;
2. initialize S2PVC protocol by setting $(\lambda, \bar{\eta}, n)$;
3. $\text{ph}_{d_k} := \text{exchange meta-information of } \{X_{d_k}\}$;
4. party pre-process$(X_{d_k}, \text{ph}_{d_k}, p; X_{d_k})$

foreach $d_k \in S_{d_k}$ do
if $T_{d_k} = T_+$ then
foreach mini batch $X_{d_k, \text{batch}} \in X_{d_k}$ do
   $S_{ct_{d_k}} \leftarrow \text{S2PHC}(d_k, X_{d_k, \text{batch}})$;
   $S_{ct_{d_k}} \leftarrow \text{S2PHC}(d_k, X_{d_k, \text{batch}})$;
else
   start entity resolution with shuffle;
   foreach mini batch $X_{d_k, \text{batch}} \in X_{d_k}$ do
      $S_{ct_{d_k}} \leftarrow \text{S2PVC}(d_k, X_{d_k, \text{batch}})$;
      $S_{ct_{d_k}} \leftarrow \text{S2PHC}(d_k, X_{d_k, \text{batch}})$;
   sends $S_{ct_{d_k}}, X_{d_k}$, $T_{d_k}$ and $Y$ if $d_k$ has the label;
party server $\text{training}(S_{ct_{d_k}}, X_{d_k, \text{batch}}, T_{d_k}, Y)$
5. $W \leftarrow \text{initialize model weights}$;
6. foreach iteration do
   foreach mini batch $X_{d_k, \text{batch}} \in S_{ct_{d_k}}, X_{d_k} \in S_{ct_{d_k}}$ do
      if $T_{d_k} = T_+$ then
         $A_1 \leftarrow \text{S2PVC}(d_k, X_{d_k, \text{batch}})$;
      else
         $A_1 \leftarrow \text{S2PHC}(d_k, X_{d_k, \text{batch}})$;
      feed-forward$(A_1, W)$;
      $\nabla_{A_1, \ldots, A} \leftarrow \text{gradient compute}(Y, W, A)$;
      $A_1 \leftarrow \text{S2PHC}(d_k, X_{d_k, \text{batch}})$;
5. $W \leftarrow W - \alpha \nabla$;
return $W$

(line 28). When it comes to the first layer, the server executes the S2PHC protocol with different ciphertext, namely, $S_{ct_{d_k}}$ (line 29). Finally, the weights are updated using the learning rate and current gradients (line 30) defined in Section 2.5.

Vertical Partitioning Based Training. This approach is for the case where each data source's dataset has a subset of features, however, these partial features collected from all the sources form the complete set of features; i.e., $X$ is vertically composed of $\{X_{d_1}, \ldots, X_{d_m}\}$. Note that we assume that each $X_{d_k}$ has an identity column so that the privacy-preserving entity resolution mechanism can be executed; there are no overlapping features that will be used in the training. In this case, each data source starts with a privacy-preserving entity resolution mechanism with the server that plays the role of a coordinator, similar to those in other approaches such as in [50, 32]. Here, each data source sends the encoded identical features to the server for entity matching. Then, the server generates a proper permutation for each data source to re-order its local data. As a result, a data source does not know which entity in its dataset has been enrolled in the training; and the server still cannot learn the training dataset. As entity resolution is not the core contribution in our framework, we refer the reader to [32] for more details.

Here, each data source generates $S_{ct_{d_k}}$ by executing the S2PVC with input $X_{d_k, \text{batch}}$, while generating $S_{ct_{d_k}}$ by executing S2PHC with input $X_{d_k, \text{batch}}$ (lines 14-17). The server acquires the output of the first hidden layer by executing the S2PVC protocol with corresponding $S_{ct_{d_k}}$ (line 25).

Hybrid Partitioning Based Training. Our NN-EMD framework can also be naturally applied to the hybrid case where $X$ is composed of the data from multiple data sources using a mix of horizontal and vertical composition. Algorithm 1 is for processing the hybrid training case by integrating the horizontally partitioned based training approach with the vertically partitioned based training approach.

Comparison with Existing Solutions. Here we briefly compare our NN-EMD framework with CryptoNN [59] and the one in [43]. CryptoNN is actually a special instance of our NN-EMD frame-
work in the horizontal partitioning based training setting. Unlike those in [59] [43], NN-EMD does not protect the label information in the training dataset. Actually, the encrypted label information in CryptoNN framework can be easily inferred, while the design of encrypting label in [43] is required by the adoption of underlying homomorphic encryption. We argue that NN-EMD satisfies the privacy requirements even though the label is exposed to the server; we analyze this in Section 4. In [43], all the outputs of each layer are still in ciphertext form. The output of the first hidden layer in NN-EMD is in plaintext; because of which the training time does not increase as in [43].

Note that we do not present the inference phase of the neural network model since the inference can be viewed as one iteration of feed-forward computation in the training phase, as shown in Algorithm

4 Security and Privacy Analysis

4.1 Security of Underlying Cryptosystems

S2PHC and S2PVC protocols are critical components of NN-EMD framework that provides the basis for privacy guarantees. As presented in Section 2.2, we add protocols to deliver the public keys and private keys generated by the TPA on the originally proposed constructions of single-input and multi-input functional encryption schemes that we adopt for our proposed scheme.

For the formal proof of security of adopted functional encryption schemes we refer the readers to [2][3]. In our adoption of these schemes, the added public key distribution and private key delivery methods are managed by the TPA. This, however, does not affect the ordinal encryption and decryption constructions as compared to the originally proposed schemes. With regards to the public-key setting in our framework with multiple data sources, each data source has its respective public key pk_{SI-FEIP} and they all have a common public key pk_{MI-FEIP}. Here, we analyze the possible security concern that a colluding data source monitors or inspects the encrypted outsourced datasets from other data sources/clients. Intuitively, such settings could enable the colluding data sources in the client pool to infer the target encrypted data by iteratively encrypting its candidate data and then checking the ciphertext with target encrypted data as all sources share a common public key pk_{SI-FEIP}. However, such an inference is prevented by the ciphertext indistinguishability property implied in the adopted functional encryption scheme [2][3]. For instance, for same input data x, with the same public key pk_{SI-FEIP}, the encrypted ciphertexts $c_1 = E_{pk_{SI-FEIP}}(x), c_2 = E_{pk_{SI-FEIP}}(x), ..., c_n = E_{pk_{SI-FEIP}}(x)$ are indistinguishable. That ciphertext indistinguishability is guaranteed by the IND-CPA security of SI-FEIP [2]. Thus, there is still a non-negligible advantage for the attackers by increasing the number of colluding data sources to brute-force the encrypted data from the non-colluding data source [2]. As a result, our framework can resist such a brute-force attack by the colluding data sources.

As mentioned earlier, the labels in our framework are not protected. We argue that such a design does not disclose the private information of the training data. Essentially, in the binary classification task, the label is encoded into meaningless value such as using {1,-1} to represent positive and negative labels rather than using a meaningful/concrete label such as “this x-ray image represents cancer”. The server can only learn group information of the encrypted data such as the information that $E_{FE}(X_{y=1})$ belongs to label $y = 1$, but the server cannot learn $X_{y=1}$, as it is protected by the cryptosystems, and what $y = 1$ means. The server is also not able to launch the enrollment inference attack where the curious server tries to infer whether a target data is enrolled in the training or not, because the training data is encrypted via functional encryption. In particular, the adopted FE schemes have the IND-CPA security guarantee, where the ciphertexts $c_i = E_{pk_{SI-FEIP}}(x), c_j = E_{pk_{SI-FEIP}}(x)$ of the same data $x$ is indistinguishable [2]. Let us suppose the target of enrollment inference attack is $x_{target}$. The data source encrypt $x_{target}$ to $c_{target} = E_{pk_{SI-FEIP}}(x_{target})$. Even though the server has the original data $x_{target}$, it is not able to infer whether $x_{target}$ is in the training dataset nor not, because the generated ciphertext of $c_{server} = E_{pk_{SI-FEIP}}(x_{target})$ by the server is indistinguishable from the ciphertext $c_{target}$.

4.2 Privacy Analysis

NN-EMD also ensures the privacy of the output of the secure computation protocols. Here, we present two types of inference attacks launched by the honest-but-curious server.
Inference Type I. Our proposed $S2PHC$ and $S2PVC$ protocols adopt the functional encryption as the underlying cryptosystems. For both functions $f_{SIP}(x, w)$ and $f_{MIP}(\langle x_1, ..., x_n \rangle, w)$ as described in Section 2.2, the server is able to acquire the decryption results (i.e., the output of the first layer in NN), and the weights of the first layer (i.e., $w$). The security of functional encryption scheme can ensure that the server cannot break/infer the input $x$ or $\langle x_1, ..., x_n \rangle$. However, an inference attack may be possible by iteratively employing FE on a specific $x$. Consider the iterative training such that the curious server may be able to collect enough polynomial equations for a specific training sample. For instance, suppose we have one training data sample $x$. For each iteration $i$ in the training phase, the server is able to acquire $f_i = \langle x, w_i \rangle$, where $f_i$ and $w_i$ are available or visible to the server. Obviously, with enough pairs of $(w_i, f_i)$, the server is able to solve the linear equation system $\{f_i = \langle x, w_i \rangle\}$ and acquire $x$. Formally, suppose that the sample $x$ has $n_{\text{feature}}$ features, i.e., $x = (x_1, x_2, ..., n_{\text{feature}})$, and each sample is used once in one training epoch. Let the total number of training epoch be $n_{\text{epoch}}$, and the number of periodical shuffle operations is $n_{\text{shuffle}}$. We have the following Lemma:

**Lemma 1.** $NN-EMD$ is able to prevent Inference Type I, if $\frac{n_{\text{epoch}}}{n_{\text{shuffle}}} < n_{\text{feature}}$

**Proof.** Suppose that the curious server has advantage $\epsilon$ to infer $x$, which indicates it has $\epsilon$ advantage to solve the system of linear equation problems $\{f_i = \langle x, w_i \rangle\}$ with determined solution. According to theorem of PSSLS in linear algebra [8], the curious server has the advantage $\epsilon$ to collect $n_e$ linear equations for the specific sample $x$, where $n_e \geq n_{\text{feature}}$.

However, in $NN-EMD$, the server has non-negligible advantage to distinguish the ciphertext of $x$ among all encrypted training samples as proved in [2][3]. After encrypted sample shuffle by the data source, the server also has non-negligible advantage to learn the position of $x$ in the training set. Thus, the server only has the advantage to collect $n_e = \frac{n_{\text{epoch}}}{n_{\text{shuffle}}} \cdot n_{\text{feature}}$ linear equations. Here, $\frac{n_{\text{epoch}}}{n_{\text{shuffle}}} < n_{\text{feature}}$ in $NN-EMD$ is subject to the requirement of PSSLS theorem, namely, $n_e < n_{\text{feature}}$. As a result, the curious server has no advantage to infer $x$.

Inference Type II. The curious server could also launch another type of inference attack by specifying “malicious” $w$ to acquire the functional private key. For instance, by specifying $w = (1, 0, ..., 0)$, the decryption result of $\langle x, w \rangle$ will disclose the first element $x_1$ of $x$. To prevent such an attack, we have introduced inference weights filter into the TPA. Specifically, the filter module will check the vector $w = (1, 0, ..., 0)$ to ensure that the number of non-zero elements is greater than a threshold $\tau$, basically, $\tau \geq 2$. As a result, it is impossible to launch the above inference attack.

5 Evaluation

We evaluate the following aspects of $NN-EMD$:

(i) To present the efficiency advantage of training time of our $NN-EMD$ framework, we compare its training time with that of only those closely related solutions proposed in [43][59]. We also explore the impact of network architecture and the number of network layers in the training time in our $NN-EMD$ framework.

(ii) With respect to the trained model accuracy, we compare our $NN-EMD$ framework in a horizontal partitioning based training setting and a vertically partitioned based training setting with a baseline model, namely, a normal neural network without any privacy-preserving settings.

(iii) As the underlying cryptosystems only work on the integer field, while the training of neural networks model works on the floating-point number field, we try to evaluate the impact of the precision on the model performance after the numeric encoding/decoding.

Note that the impact of data distribution such as non-iid and imbalanced data is beyond the scope of $NN-EMD$ because our framework only provides secure computation features into existing the DNN model rather than modifying the intrinsic properties of the underlying DNN model such as the network architecture.
5.1 Experimental Setup

To benchmark the performance of the NN-EMD framework, we train a model of a neural network with the same topology as the one used in [43] on the publicly available MNIST dataset of handwritten digits [36] that includes 60000 training samples and 10000 test samples. In our evaluation, each sample (28 × 28 image) in the MNIST dataset is mapped to a vector with a length of 784. Besides, we also explore the framework performance on different neural network architectures and different numbers of network layers. Essentially, we run the experiments for 5 data sources forming the client pool. Each data source is randomly assigned 60000/5 = 12000 data samples from the MNIST dataset for the horizontal partitioning based training, while in the vertical partitioning based training, each data source is assigned 60000 data samples but only around 784/5 ≈ 157 features for each sample. Note that such a vertical setting over the MNIST data is only for illustration purposes. We use comparable settings when evaluating the impact of the number of data sources on the model performance. In all the experiments we utilize the same model hyperparameters of a neural network model such as learning rate, l2 regularization parameter, etc.

Implementation Consideration. We have implemented the NN-EMD framework based on the NumPy library to use the high-level mathematical functions in Python programming language. The underlying cryptosystems, namely, the functional encryption schemes, are also implemented in Python based on the gmpy2 library, which is a C-coded Python extension module that supports multiple-precision arithmetic and relies on the GNU multiple precision arithmetic (GMP) library.

In contrast to the implementation of functional encryption in [59], we incorporate the acceleration techniques used in [58] in the proposed work. By tracking the time cost of each decryption step in the functional encryption scheme, we find that the most inefficient computing step is the final step that computes the discrete logarithm of a small integer. To be specific, it involves computing $f$ in $h = g^f$, where $h$, and $g$ are big integers while $f$ is a small integer. To accelerate such discrete logarithm computations, we employ a bounded-table-lookup method by initially setting up a hash table to store pair $(g, f)$ with a specified public key parameter $g$ and a positive bound $f_b$ where $-f_b \leq f \leq f_b$. The size of the hash table depends on the allowed encoding precision on encryption over the floating-point numbers. Then, the final discrete logarithm computation is a table lookup operation with complexity $O(1)$, which is better compared to traditional baby-step giant-step algorithm that has complexity $O(n^2)$.

Environment Setup. All the experiments have been performed on two test platforms: Test Platform I (TP I) that is a local Macbook Pro with 2.3GHz Intel Core i9 8-Core CPU and 32GB RAM, and Test Platform II (TP II) that is a remote cloud service, i.e., AWS m5d.8xlarge instance with 2.5GHz Intel Xeon 8124M 32 vCPUs and 128GB RAM. For the evaluations of model performance, where the client pool and the server are put on the same platform, we repeat the experiments in both the test platforms. To simulate real scenarios, we use TP I as the client pool and TP II as the server.

5.2 Experimental Results

5.2.1 Comparison with Contracted Frameworks

| Proposed work | Network architecture | CPU † | Threads | Mem | Training time |
|---------------|----------------------|-------|---------|-----|---------------|
| CryptoNN [59] | 784 → 128 → 32 → 10 | Type 2: 8-Core | 30 | 16GB | 2 days |
| CryptoNN [59] | 784 → 128 → 32 → 10 | Type 2: 8-Core | 8 | 16GB | 94m |
| NN-EMD (HPT) | 784 → 128 → 32 → 10 | Type 2: 8-Core | 1 | 32GB | 49.83s |
| NN-EMD (VPT) | 784 → 128 → 32 → 10 | Type 2: 8-Core | 1 | 32GB | 31.71s |
| NN-EMD (HPT) | 784 → 128 → 32 → 10 | Type 3: 32 vCPUs | 1 | 128GB | 55.63s |
| NN-EMD (VPT) | 784 → 128 → 32 → 10 | Type 3: 32 vCPUs | 1 | 128GB | 33.67s |

† Type 1: 2.3GHz Intel Xeon E5-2698v3 16-Core; Type 2: 2.3GHz Intel Core i7 8-Core; Type 3: 2.3GHz Intel Core i9 8-Core;
As shown in Table 2, we compare the training time of our NN-EMD framework with the approaches proposed in [43, 59]. Note that as the codes and experimental platforms for work in [43] are not publicly available, we report the experimental results reported in [43] directly. We also include the test environment reported in their papers. In our evaluation, we use comparable experimental platforms used in [43, 59], and train the model on the same MNIST dataset with the same neural network architecture.

We evaluate the NN-EMD framework both in horizontal partitioning based training (HPT) and vertical partitioning based training (VPT) settings. Our experimental results show that the training time of one mini-batch including 60 samples in our NN-EMD only needs 49.83 seconds and 55.63 seconds in TP I and TP II environments, respectively. Compared to the existing best result (i.e., 40 minutes) as reported in [43] where each training sample is extracted from 28 × 28 to 8 × 8 to reduce the input size and the multithreaded parallelism technique is employed in the training phase, our proposed NN-EMD reduces the training time by approximately 97.7%.

In contrast to the approach in [43], where computation at each layer of neural networks is over encrypted data, in essence the computation over the encrypted data only occurs at the first layer in NN-EMD. That is the main reason NN-EMD can significantly reduce the training time. Actually, there is a trade-off between the privacy and efficiency. As the focus of NN-EMD is the proposed secure computation approaches, only the input layer is protected in our illustration and evaluation, while the rest of the parameters are still plaintext to the server. Thus, to some extent, low-level features (e.g., the output of the second layer) may reveal partial private information. It is possible to protect more layers of neural networks by integrating SplitNN technique [54] into our framework, and hence to some extent, it may sacrifice training time; such integration is beyond the scope of this paper.

5.2.2 Impact of NN Architectures and Number of Layers

As reported in Table 2, the training time of existing solutions such as the framework proposed in [43] increases significantly as the network architecture changes. To evaluate the impact of network architectures on the training time in our NN-EMD framework, we train neural network models with different architectures on the MNIST dataset with the same number of data sources. As presented in Table 3, the training time for our proposed approach is only impacted by the number of nodes in the first hidden layer. When the network architecture of the rest of the layers changes, the training time does not change compared to the normal neural networks without a privacy-preserving setting.

For further verification of such a claim, we conducted additional experiments with a large number of hidden layers. As shown in Figure 5c, we measure the training time of one mini-batch in our NN-EMD framework in different training settings (i.e., HPT and VPT) and vary the number of hidden layers from 1 to 30 where each layer includes 64 neural nodes. As can be seen, the training time does not change drastically like in existing solutions as the number of hidden layers increases.
Figure 5: Comparison of model accuracy and training time of one mini-batch as number of hidden layers increase. Note that the network architecture used for model accuracy comparison in Normal-NN and NN-EMD is $784 \rightarrow 512 \rightarrow 256 \rightarrow 128 \rightarrow 64 \rightarrow 32 \rightarrow 10$. Each hidden layer of network architecture in right figure includes 64 neural nodes and the results are generated on the TP II platform.

Table 4: Time cost for different data source # in client-server setting in NN-EMD

| Training Sources # | Pre-Process: Client (TP I) | Training: Remote AWS (TP II) |
|--------------------|-----------------------------|-----------------------------|
| HPT Setting 5      | 1.2769s                     | 56.02s                      |
| HPT Setting 7      | 1.0579s                     | 55.95s                      |
| HPT Setting 9      | 1.1195s                     | 55.86s                      |
| HPT Setting 11     | 1.1246s                     | 55.87s                      |
| HPT Setting 13     | 1.1463s                     | 56.04s                      |
| HPT Setting 15     | 1.1260s                     | 56.13s                      |
| VPT Setting 5      | 0.3716s                     | 33.75s                      |
| VPT Setting 7      | 0.3363s                     | 32.48s                      |
| VPT Setting 9      | 0.2372s                     | 31.93s                      |
| VPT Setting 11     | 0.2214s                     | 31.50s                      |
| VPT Setting 13     | 0.1867s                     | 31.24s                      |
| VPT Setting 15     | 0.1711s                     | 31.17s                      |

† The neural networks architecture used in this experiment is $784 \rightarrow 128 \rightarrow 32 \rightarrow 10$. Note that the cost time reported here is for only one mini-batch that includes 60 samples.

5.2.3 Evaluation of Accuracy

Except for the performance with respect to the training time, we compare our framework with a baseline neural network framework (Normal-NN) that has the same network architecture but without any settings of privacy-preserving approaches. As shown in Figure 5a, our proposed NN-EMD framework can achieve comparable model accuracy compared to a normal neural network both in HPT and VPT. Further, the results in Figure 5b shows that the precision setting does not have an effect on the model accuracy.

5.2.4 Deployment in Client-Server Scenario

To evaluate the impact of the number of data sources on the training time, we have deployed our end-to-end NN-EMD system in a client-server scenario. In this experiment, our local machine (TP I) plays the role of client pool with varying number of data sources to pre-process the encrypted training datasets, while the remote AWS instance (TP II) plays the role of the server to train the neural network model based on these encrypted data samples.

As shown in Table 4, we present the training time for both the client pool and the server. All reported times for the server side is based on one mini-batch, while the time reported for the client pool is for one mini-batch per data source. In the case of the horizontal partitioning based training, the training time of NN-EMD framework does not change drastically like existing solutions as the number of data sources increases. In the case of the vertical partitioning based training, each data source pre-processes the same number of data samples. As the total number of features is fixed, the number
of features from each data source decreases as the number of data sources increase, and hence the pre-processing time decreases, while the training time still does not change drastically.

6 Related Work

Secure Computation. Secure computation, also known as secure multi-party computation (SMC) or multi-party computation (MPC), has shown its promise in supporting computational tasks for applications that process privacy-sensitive data since the first formal secure two-party computation (2PC) was proposed by Yao et al. [60]. Even though the secure computation has been explored for around 40 years, the practical deployment of secure computation solutions is still a challenge, especially, in the era of big data.

Generally, there are two research directions towards achieving the goal of secure computation, namely, constructing general-purpose SMC, or proposing special-purpose SMC. To construct general purpose SMC, existing solutions can be of two categories: (i) protocols such as those proposed in [31, 56, 55, 19] that are usually built on the garbled circuits [9] and oblivious transfer techniques [6], and (ii) protocols such as those proposed in [41, 21, 7] that are based on the homomorphic cryptosystems [24] (e.g., fully homomorphic encryption). However, these two kinds of SMC solutions have limitations with regards to either the large volumes of ciphertexts that need to be transferred or the inefficiency of computation involved (i.e., unacceptable computational time). To provide more applicable secure computation solutions, several special-purpose SMC approaches such as those proposed in [16, 34, 5] have been proposed to address special computational tasks such as additive functions, as the general computational tasks are not required in most of the application scenarios. In this paper, our proposed secure computation protocols are also essentially special-purpose SMCs that only address the computation of inner-products and matrix multiplications.

Computable Ciphertext. As we discussed above, a branch of secure computation approaches is based on homomorphic encryption schemes. Homomorphic encryption is a form of cryptosystems that allows computation over ciphertexts, where the processed result is still in ciphertext, but the decryption of that ciphertext matches the result of the computation performed on the corresponding plaintexts. In general, fully homomorphic encryption [39] is able to achieve a general purpose SMC, while partially homomorphic encryption such as additive homomorphic encryption [46, 20] can construct additive secure computation protocols. Existing homomorphic encryption schemes are still not efficient enough, especially, when applying to the large-scale computational tasks. On the other hand, the recently proposed functional encryption approach [39, 12] shows another promising direction to achieve the task of computation over a ciphertext. For instance, to construct functional encryption schemes for general functionality, most of the recently proposed approaches such in [26, 13, 57, 23, 17, 38] focus on the theoretical feasibility or functionality existence. Unlike homomorphic encryption schemes, functional encryption schemes rely on a trusted third party authority to provide private key service. In this paper, we have adopted the functional encryption schemes that are only applicable to inner-products [2, 4].

Privacy-preserving Machine Learning. The goal of privacy-preserving machine learning is to protect the privacy of training data while still generating a well-trained model. To achieve that goal, most of the popular techniques such as differential privacy [1], federated learning [51], general-purpose SMC protocols [42, 48] and computable cryptosystems [59, 43, 25, 30, 33] have been applied in different machine learning models. Except for the approaches proposed in [59, 43], all existing cryptosystem based privacy-preserving machine learning approaches either focus on the simple traditional machine learning models such as a linear regression model [45] and a logistic regression model [4] or only work on the inference phase of the neural network model [25, 30, 33]. It is still a huge challenge to train a DNN model over encrypted data, especially, on encrypted training dataset composed of data from multiple sources.

7 Conclusion

Training neural network models over encrypted data show significant promise towards addressing strong privacy requirements of both the users and regulations, while taking full advantage of an existing ML platform as a service infrastructure. However, there is a lack of efficient and practical privacy-preserving solutions for training a neural network based ML system over privacy-sensitive
datasets. We have proposed NN-EMD, a novel neural network framework that supports training a neural network model on a dataset where the data is composed, both horizontally and vertically, of encrypted datasets from multiple data sources. Our evaluation shows that NN-EMD can reduce the training time by 97% while still providing the same model accuracy and strong privacy guarantee as compared to most of the recent comparable approaches. Furthermore, the depth and complexity of neural networks do not affect the training time despite introducing a privacy-preserving NN-EMD setting. Future work includes applying the NN-EMD framework in a complex edge computing environment.

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A Difference between Homomorphic Encryption and Functional Encryption

Homomorphic Encryption (HE) is a form of cryptosystem with an additional evaluation capability for computing over ciphertexts without access to the private secret key, in which the result of operations over the ciphertexts, when decrypted, match the result of operations as if they have been performed on the original plaintext. Some typical types of HE are partially homomorphic, somewhat homomorphic, leveled fully homomorphic, and fully homomorphic encryption according to the capability of performing different classes of computations. Unlike traditional encryption scheme that includes three main algorithms: key generation (Gen), encryption (Enc), and decryption (Dec), an HE scheme also has an extra evaluation (Eval) algorithm. Formally, a HE scheme $\varepsilon_{\text{HE}}$ includes the above four algorithms such that

$$\varepsilon_{\text{HE}}.\text{Enc}_{\text{pk}}(f, \varepsilon_{\text{HE}}.\text{Enc}_{\text{pk}}(m_1), ..., \varepsilon_{\text{HE}}.\text{Enc}_{\text{pk}}(m_n))) = f(m_1, ..., m_n),$$

where $\{m_1, ..., m_n\}$ are the message to be protected, $pk$ and $sk$ are the key pairs generated by the key generation algorithm. Regarding recent emerging deep neural networks model, the CryptoNets\cite{25} tries to apply neural networks to encrypted data by employing a leveled homomorphic encryption scheme to the training data, which allows adding and multiplying encrypted messages but requires that one knows in advance the complexity of the arithmetic circuit. Besides, the work in \cite{43} uses the open-source FHE toolkit HElib for neural network training in a stochastic gradient descent training method.

Functional Encryption (FE) is another form of cryptosystem that also supports the computation over the ciphertext. Typically, the FE $\varepsilon_{\text{FE}}$ includes four algorithms: setup, key generation, encryption and decryption algorithms such that

$$\varepsilon_{\text{FE}}.\text{Enc}_{\text{pk}}(m_1), ..., \varepsilon_{\text{FE}}.\text{Enc}_{\text{pk}}(m_n))) = f(m_1, ..., m_n),$$

where the setup algorithm creates a public key $pk$ and a master secret key $msk$, and key generation algorithm uses $msk$ to generate a new functional private key $sk_f$ associate with the functionality $f$. Those two algorithms usually are run by the a trusted third-party authority.

As presented above, the main similarity between the FE and HE support the computation over the ciphertext. In a high-level respective, the main difference between the functional encryption and the homomorphic encryption is that given an arbitrary function $f(\cdot)$, the homomorphic encryption allows to compute an encrypted result of $f(x)$ from an encrypted $x$, whereas the functional encryption allows to compute a plaintext result of $f(x)$ from an encrypted $x$. Intuitively, the function computation party in the HE scheme (i.e. the evaluation party) can only contribute its computation resource to obtain the encrypted function result, but cannot learn the function result unless it has the secret key, while the function computation party in the FE scheme (i.e., usually, the decryption party) can obtain the function result with the issued functional private key. Besides, expect for several most recently proposed decentralized FE schemes, the classic FE schemes are relied on a trusted third-party authority to provide key service such as issuing a functional private key associated to a specific functionality.

Unlike the HE-based secure computation techniques that have been widely adopted as a candidate solution for the secure computation for privacy-preserving machine learning (PPML), recently proposed FE-based PPML solutions such as in \cite{58, 59, 49} also show its promise in efficiency and practicality. The proposal in \cite{49} proposes a practical framework to perform partially encrypted and privacy-preserving predictions which combines adversarial training and functional encryption. The work in \cite{59} initialize a CryptoNN framework that supports training a neural network model over encrypted data by using the FE to construct the secure computation mechanism. In addition, the proposal in \cite{58} focuses on the privacy-preserving federated learning (PPFL) by utilizing the FE to construct the secure aggregation protocol to protect each participant’s input in the PPFL.

B Adopted Functional Encryption Schemes in Detail

Here we present the underlying adopted functional encryption schemes in our NN-EMD framework.
B.1 Single-Input Functional Encryption for Inner-Product

We adopt the single-input functional encryption for inner-product (SI-FEIP) proposed in [2]. In the SI-FEIP scheme, the supported function is described as

\[
    f_{\text{SIIP}}(x, y) = \sum_{i=1}^{\eta} (x_i y_i) \quad \text{s.t. } |x| = |y| = \eta,
\]

where \(x\) and \(y\) are two vectors of length \(\eta\), from different parties. The SI-FEIP scheme \(\mathcal{F}_S\) includes four algorithms: Setup, SKGenerate, Encrypt, Decrypt. Here, each algorithm is constructed as follows.

- Setup(\(1^\lambda, \eta\)): This algorithm generates a master private key and common public key pair \((pk_{\text{com}}, msk)\) based on a given security parameter \(\lambda\) and vector length parameter \(\eta\). Specifically, on the inputs of security parameters \(\lambda\) and \(\eta\), the algorithm first generates two samples as follows:

\[
    (G, p, g) \leftarrow \text{GroupGen}(1^\lambda)
\]

\[
    s = (s_1, ..., s_\eta) \leftarrow \mathbb{Z}_p^\eta
\]

and then sets \(pk_{\text{com}}\) and \(msk\) as follows:

\[
    pk_{\text{com}} = (g, h_i = g^{s_i})_{i \in [1,...,\eta]}
\]

\[
    msk = s
\]

It returns the pair \((pk_{\text{com}}, msk)\).

- SKGenerate(\(msk, y\)): This algorithm takes the master private key \(msk\) and one vector \(y\) as input, and generates a functionally derived key \(sk_{\text{fup}} = \langle y, s \rangle\) as output.

- Encrypt(\(pk_{\text{com}}, x\)): This algorithm outputs ciphertext \(ct\) of vector \(x\) using the public key \(pk_{\text{com}}\). Specifically, the algorithm first chooses a random \(r \leftarrow \mathbb{Z}_p\) and computes

\[
    ct_0 = g^r.
\]

For each \(i \in [1,...,\eta]\), it computes

\[
    ct_i = h_i^r \cdot g^{s_i}.
\]

Then the algorithm outputs the ciphertext \(ct = \{ct_0, \{ct_i\}_{i \in [1,...,\eta]}\}\).

- Decrypt(\(pk_{\text{com}}, ct, sk_{\text{fup}}, y\)): This algorithm takes the ciphertext \(ct\), the public key \(pk_{\text{com}}\) and functional key \(sk_{\text{fup}}\) for the vector \(y\) as input, and returns the inner-product \(f_{\text{SIIP}}(x, y)\). Specifically, the algorithm firstly compute the discrete logarithm in basis \(g\) as follows

\[
    g^{(x,y)} = \prod_{i \in [1,...,\eta]} ct_i^{s_i} / ct_0^{s_i}.
\]

Then, \(f_{\text{SIIP}}(x, y) = \langle x, y \rangle\) could be recovered.

B.2 Multi-Input Functional Encryption for Inner-Product

We employ the multi-input functional encryption for inner-product (MI-FEIP) construction derived from the work proposed in [3]. In the MI-FEIP scheme, the support function is defined as

\[
    f_{\text{MIIP}}(x_1, x_2, ..., x_n, y) = \sum_{i=1}^{n} \eta_i \sum_{j=1}^{\eta_i} (x_{i j} y_{\sum_{k=1}^{i-1} \eta_k + j}) \quad \text{s.t. } |x_i| = \eta_i, |y| = \sum_{i=1}^{n} \eta_i,
\]

where \(x_i\) and \(y\) are vectors from different parties. Accordingly, the MI-FEIP scheme \(\mathcal{F}_M\) includes five algorithms: Setup, PKDistribute, SKGenerate, Encrypt, Decrypt Below, we present the construction of each algorithm:

- Setup(\(1^\lambda, \eta, n\)): It generates a master private key and public key pair \((pk_{\text{com}}, mpk, msk)\) given security parameter \(\lambda\) and functional parameters \(\eta\) and \(n\), where \(n\) is the maximum number of input parties while \(\eta\) is a vector where each element represents the maximum input length vector.
of the corresponding party, and hence $|\eta| = n$. Specifically, the algorithm first generates secure parameters as

$$G = (G, p, g) \leftarrow s \text{GroupGen}(1^\lambda),$$

Then, it generates several samples as

$$a = (1, a)^T, a \leftarrow s \mathbb{Z}_p$$

$$W_i \leftarrow s \mathbb{Z}_p^{n_i \times 2}, i \in [1, \ldots, n]$$

$$u_i \leftarrow s \mathbb{Z}_p^{n_i}, i \in [1, \ldots, n]$$

Then, it generates the keys as

$$pk = (G, p, g)$$

$$mpk = (G, g^a, g^W)$$

$$msk = (W, (u_i)_{i \in [1, \ldots, n]})$$

• $PKDistribute(pk, msk, id_i)$: It delivers the public key $pk_i$ for party $id_i$ given the master public/private keys. Specifically, it looks up the existing keys via $id_i$ and returns the public key as

$$pk_i = (G, g^a, (Wa)_i, u_i).$$

• $SKGenerate(pk, msk, y)$: It takes the master public/private keys and vector $y$ as inputs, and generates a function derived key $sk_{f_{\text{MIP}}}$ as output. Specifically, the algorithm first partitions $y$ into $(y_1 | y_2 | \ldots | y_n)$, where $|y_i|$ is equal to $\eta_i$. Then it generates the function derived key as follows:

$$sk_{f, y} = (\{d_i^T \leftarrow y_i^T W_i\}, z \leftarrow \sum y_i^T u_i).$$

• $Encrypt(pk, x_i)$: It outputs ciphertext $ct_i$ of vector $x_i$ using the public key $pk_i$. Specifically, the algorithm first generates a random nonce $r_i \leftarrow_R \mathbb{Z}_p$, and then computes the ciphertext as follows:

$$ct_i = (t_i \leftarrow g^{a^r_i}, e_i \leftarrow g^{x_i^T u_i} (Wa)^{r_i}).$$

• $Decrypt(pk, Sct, sk_{f_{\text{MIP}}}, y)$: It takes the ciphertext set $Sct$, the public key $pk$ and functional key $sk_{f_{\text{MIP}}}$ as input, and returns the inner-product $f_{\text{MIP}}(\{x_i\}, y)$. Specifically, the algorithm first calculates as follows:

$$C = \prod_{i \in [1, \ldots, n]} (\frac{|y_i^T c_i|}{|d_i^T t_i|})^z,$$

and then recovers the function result as

$$f((x_1, x_2, \ldots, x_n), y) = \log_g (C).$$