Non-reciprocal mu-near-zero mode in $\mathcal{PT}$-symmetric magnetic domains

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(Dated: December 19, 2014)

We find that a new type of non-reciprocal modes exist at an interface between two parity-time ($\mathcal{PT}$) symmetric magnetic domains (MDs) near the frequency of zero effective permeability. The new mode is non-propagating and purely magnetic when the two MDs are semi-infinite while it becomes propagating in the finite case. In particular, two pronounced nonreciprocal responses could be observed via the excitation of this mode: one-way optical tunneling for oblique incidence and unidirectional beam shift at normal incidence. When the two MDs system becomes finite in size, it is found that perfect-transmission mode could be achieved if $\mathcal{PT}$-symmetry is maintained. The unique properties of such an unusual mode are investigated by analytical modal calculation as well as numerical simulations. The results suggest a new approach to the design of compact optical isolator.

PACS numbers: 41.20.Jb, 78.20.Ls, 11.30.Er

I. INTRODUCTION

Over the past few decades there has been much activity on the non-reciprocity effect in optics.1–13. Non-reciprocal optical elements, such as optical isolators, have attracted great attention owing to its capability of allowing light to propagate only along a single direction, while strongly suppressing backward scattering. The traditional way for creating nonreciprocal devices relies on magneto-optic Faraday effect in the presence of an external magnetic field. However, the intrinsic weakness of Faraday effects based on available magneto-optical (MO) materials makes the Faraday rotator bulky and hinders miniaturization of such devices. Later, the photonic crystal (PC) made of MO materials2 was suggested to enhance the nonreciprocal response, and create compact and integrated isolators and circulators. Recently, Raghu and Haldane3,4 theoretically predicted one-way edge modes could be observed in MO photonic crystals, as optical counterparts to chiral edge states of electrons in the quantum Hall effect. These modes are confined to the region near the edge of the 2D PC, displaying one-way propagation characteristics. Subsequently, experimental realizations and observations of such electromagnetic one-way edge states in different magneto-optical photonic crystal (MPCs) were reported by several groups5–6. Nonreciprocal behavior has also been demonstrated by considering dynamic modulation in standard materials7–8, the use of opto-mechanical9,10, and opto-acoustic effects11 and optical nonlinearities12–13.

On the other hand, considerable efforts have been intensively devoted to a new class of artificial optical materials having balanced loss and gain - parity-time ($\mathcal{PT}$)-symmetric metamaterials14–34. Such $\mathcal{PT}$-symmetric systems have non-Hermitian Hamiltonians, exhibiting with entirely real eigenvalues as long as $\mathcal{PT}$ symmetry holds. Remarkably, the system may undergo an abrupt phase transition (spontaneous $\mathcal{PT}$ symmetry-breaking) at some non-Hermiticity threshold, beyond which some of the eigenvalues become complex. To date, several $\mathcal{PT}$-symmetric models have been demonstrated with some intriguing light propagation behaviors, including power oscillations21, double refraction20, unidirectional invisibility21,24, non-reciprocal light transmission25–28 and unattenuated surface modes29–31.

It turns out that $\mathcal{PT}$-symmetry has a strong linkage to perfect transmission states32. This type of spatial-temporal symmetry can be more general than the usual symmetry-related perfect transmission associated with mirror symmetry or inversion symmetry. Since such a $\mathcal{PT}$-symmetry-related perfect transmission is complementary to non-reciprocity, it is also useful for the design of optical isolator displaying one-way perfect transmission with no gain medium such as the case in this paper. In the present work, we consider a structure composed of two MDs with $\mathcal{PT}$ symmetry17,18, magnetized homogeneously in opposite directions, and find a new type of non-reciprocal mu-near-zero (MNZ) modes at the interface separating two MDs near the frequency of zero effective permeability. The broken $\mathcal{P}$ and $\mathcal{T}$ symmetries, induced here simultaneously by the geometry and the orientation of the external magnetic field, result in the asymmetrical dispersion relations of the interface mode, whereas the unbroken $\mathcal{PT}$ symmetry leads to the emergence of the perfect transmission mode32. Furthermore, two pronounced nonreciprocal behaviors are exhibited by application of such a MNZ mode for incident plane waves: one-way complete optical tunneling at oblique incidence and unidirectional beam shift at normal incidence. Calculations on nonreciprocal dispersion relations, reflection spectra and field patterns for such a

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A PT-symmetric system are employed to verify our conclusions.

This paper is organized as follows. In Sec. II, the exact analytical modal description is employed to investigate the non-reciprocal MNZ mode in the PT-symmetric system we proposed. Sec. III shows the numerical results of reflection spectra and field patterns for the finite-size PT-symmetric system. Finally, the conclusions are given in Sec. IV.

II. ANALYTICAL MODAL DESCRIPTION OF NON-RECIPROCAL MU-NEAR-ZERO MODE

We start with two semi-infinite MDs constructed by MO media oppositely magnetized in the Voigt geometry as shown in Fig. 1(a). Under the external static magnetic field along ±z, the two semi-infinite MDs are, characterized respectively by identical permittivities $\epsilon_0$ and magnetic permeability tensors $\mu_{(x>0)}$ and $\mu_{(x<0)}$,

$$
\mu_{(x>0)} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_1 \end{pmatrix}, \quad \mu_{(x<0)} = \begin{pmatrix} \mu'_1 & i\mu'_2 & 0 \\ -i\mu'_2 & \mu'_1 & 0 \\ 0 & 0 & \mu'_1 \end{pmatrix}.
$$

We take the following parameters for MDs $\mathbb{R}$, i.e., $\mu_1 = \mu'_1 = 1 + i\omega_m\omega_h/(\omega^2_h - \omega^2)$, $\mu_2 = -i\mu'_2 = -i\omega_m\omega/(\omega^2_h - \omega^2)$, where $\omega_h = \gamma H_0$ is the precession frequency, $\gamma$ is the gyromagnetic ratio, $H_0$ is the applied magnetic field on the two MDs, $\omega_m = c\kappa_m = 4\pi\gamma M_s$, and $c$ denotes the speed of light in vacuum, $4\pi M_s$ is the saturation magnetization. The parameters are chosen to fulfill PT symmetry $\mu_{(x>0)} = \mu^*_{(x<0)}$, which will lead to perfect transmission modes. It should be noted that only transverse electric (TE) polarization (i.e., electric field along the z direction) is considered, and the $e^{-\omega t}$ time-dependent convention for harmonic field is used in this work.

To form guided waves at the interface between two MDs, the field should decay exponentially away from the interface, and can be written as follows: $E(x>0) = (0,0,A)e^{-\gamma x + ik_y y}$ and $E(x<0) = (0,0,B)e^{\beta x + ik_y y}$.

Here $A$ and $B$ are the amplitudes of the corresponding electric field components in two MDs. $\gamma$ and $\beta$ denote positive decay parameters, displaying the relations with the parallel component of wave vector $k_y$: $k^2_y - \gamma^2 = k^2_y - \beta^2 = \epsilon_m\mu_{eff}\omega^2/c^2$ in two homogenous gyromagnetic materials, with identical effective permeability defined as $\mu_{eff} = (\mu^2_1 - \mu^2_2)/\mu_1$. By solving the Maxwell’s equations, we have magnetic fields components $H = (H_x, H_y, 0)e^{-\gamma x + ik_y y}$ for the $x > 0$ space satisfying the following relations,

$$
(\mu^2_1 - \mu^2_2)H_x(x>0) = \frac{A}{\omega}(\mu_1 k_y - \mu_2 \gamma),
$$

$$
(\mu^2_1 - \mu^2_2)H_y(x>0) = -\frac{A}{\omega}(\mu_1 \gamma - \mu_2 k_y),
$$

By replacing $A, \gamma$ and $\mu_2$ by $B, -\beta$ and $-\mu_2$, respectively, we could obtain the corresponding equations of magnetic field for the space $x < 0$.

In most cases that the condition $\mu_{eff} \neq 0$ is fulfilled, the magnetic field could be then easily obtained from Eq. (2). Further, according to the boundary condition that the tangential field components should be continuous across the interface, we could have an usual solution for an interface mode, shown with black solid lines in Fig. 1(c) as well as in Ref. [15]. However, if we take into account the possibility of $\mu_{eff} = 0$ (here $\mu_1 = \mu_2$) at $\omega_0 = \omega_h + \omega_m$ in this specified case, there is an extra solution of another interface mode for this PT-symmetric system,

$$
A = B = 0, \quad \gamma = \beta = -k_y, \quad (3)
$$

called here as the $\mu_{eff} = 0$ mode. It is interestingly found that the purely magnetic field with no electric field components emerges for this mode, and simultaneously the two orthogonal components of magnetic field has unique relations,

$$
H_x(x>0) = -H_x(x<0) = -iH_y, \quad (4)
$$

indicating the certain phase difference between $H_x$ and $H_y$ with $\pi/2$ in the left domains region and $-\pi/2$ at the right. Moreover, in order to guarantee the positive decay rate ($\beta > 0, \gamma > 0$), the parallel component of wave vector $k_y$ should remain negative, which leads to the emergence of a nonreciprocal $\mu_{eff} = 0$ mode shown in Fig. 1(d).
by the red line in Fig. 1(c). Here, we use parameters provided in a previous experimental study \cite{3}, i.e. $\epsilon_m = 15.26$, $H_0 = 800$ Oe, and $4\pi M_s = 1884$ G.

However, the nonreciprocal $\mu_{eff} = 0$ modes between two semi-infinite domains form a flat band and thus they are non-propagating, which makes the modes difficult to be excited. To improve its optical response, we alter the infinite systems by the finite-size bilayer MDs still with $\mathcal{PT}$ symmetry [shown in Fig. 1(b)], and here assumed with identical thickness $a$, embedded in an uniform surrounding medium. Based on the transfer matrix approach \cite{16}, the radiative modes for such a bilayer system outside the light line for surrounding mediums could be well solved. Two kinds of mode solutions could be analytically separated as

$$\sin(k_x a) / k_x = 0 \quad (5)$$

for reciprocal (symmetrical) modes and

$$\frac{1}{k_{x0}} \left[ \cos(k_x a)(k_y^2 - k_{x0}^2) - \frac{\mu_{eff}}{\mu_0} \right] - \frac{\omega^2}{c^2} (\epsilon_m \mu_0 - \epsilon_0 \mu_{eff})$$

$$+ \frac{k_y k_0}{k_x \mu_0} \sin(k_x a)(k_y^2 \frac{\mu_0}{\mu_1} + \frac{\mu_{eff}}{\mu_1} - \frac{\omega^2}{c^2} (\epsilon_m \mu_0 + \epsilon_0 \mu_{eff}))$$

$$= 0 \quad (6)$$

for non-reciprocal (asymmetrical) ones. Here, $\epsilon_0$ and $\mu_0$ are the permittivity and permeability for surrounding medium, and the wave-vector components normal to the interface in background and magnetic materials are taken as $k_{x0} = \sqrt{\epsilon_0 \mu_0 \omega^2 / c^2 - k_y^2}$, and $k_x = \sqrt{\epsilon_m \mu_{eff} \omega^2 / c^2 - k_y^2}$, respectively. The reciprocal propagating modes in Eq. (5) for such bilayer MD systems are identical to those in a single slab layer of MD, simultaneously independent of surrounding mediums. It should be emphasized that the linear term of $k_y$ in Eq. (6) breaks the spectral reciprocity (i.e., the left-right symmetry of the dispersion relation), leading to strong non-reciprocal behaviors. Furthermore, in the limit of $a \rightarrow \infty$, there is always a solution at $\omega_0$ identical with Eq. (6) for the infinite system in Fig. 1(a).

We plot in Fig. 1(d) the corresponding radiative electromagnetic modes within the light cone for surrounding media with the refractive index $n = 4$. Each magnetic layer has equal thickness $a = 0.008m$. The reciprocal and non-reciprocal modes are shown by blue and red lines, respectively. It is found that the original flat and non-propagating $\mu_{eff} = 0$ mode interacts with the propagating modes in bilayer MDs, and extends to the bulk band for magnetic materials, thereby becoming dispersive. So we achieve a non-reciprocal mu-near-zero ($\mu_{eff} \approx 0$) radiative mode for thin films of MD structures, and expect to see the non-reciprocal optical response for the dispersive mode near the frequency $\omega_0$ corresponding to $\mu_{eff} = 0$, with direct illumination of external plane waves.

![Image](image.png)

**FIG. 2**: (color online) (a)(b) The reflection spectra for finite-size $\mathcal{PT}$-symmetric MDs shown in Fig. 1(b). (a) Contour plot - reflection as a function of $\omega$ and $k_y$, (b) 2D line plot - reflection as a function of $k_y$ with a frequency $\omega = 1.425\omega_m$ close to $\omega_0$, just along the horizontal dotted line in (a). Gray and dashed lines in (a) are the same as those in Fig. 1(d).

**III. NUMERICAL RESULTS ON FINITE-SIZE PT SYMMETRIC MAGNETIC DOMAINS**

To support our findings, we investigated the transport properties through finite-size $\mathcal{PT}$-symmetric MDs, with numerical calculations on reflection spectra shown in Fig. 2. Apparently, it is seen that reflection dips shown dark blue colors in Fig. 2(a) are in excellent agreement with those radiated modes in Fig. 1(d), and the dispersive and non-reciprocal $\mu_{eff} \approx 0$ mode could be well excited under external plane waves, also clearly shown in Fig. 2(b) with a particular example of the frequency $\omega = 1.425\omega_m$ close to $\omega_0$. In contrast to the usual $\mu_{eff} \neq 0$ interface mode indicated here with a very narrow dip in Fig. 2(b), the coupled $\mu_{eff} \approx 0$ mode shows strong non-reciprocal response, not sensitive to the incident angle of plane waves.

Further, 2D finite-element simulations using COMSOL Multiphysics were carried out to verify the electromagnetic non-reciprocal response of waves impinging on our proposed finite-size $\mathcal{PT}$-symmetric systems. Figure 3 depicts the spatial field distribution with a frequency of $\omega = 1.425\omega_m$ at oblique incidence. Counter-propagating plane waves are incident from surrounding mediums upon either side of the bilayer MD structures. For the case of the forward incidence shown in Fig. 3(a), full transmission could be obtained due to the excitation of $\mu_{eff} \approx 0$ mode on the interface. Interestingly, it is found that there exists a purely magnetic field with no electric field along the interface. To see more clearly, we zoom in and get a close-up view of the magnetic field $H = (H_z, H_y)$ in the two domains as shown in Fig. 3(b)(c), with black arrows representing the vector patterns of magnetic field. The fixed phase difference between $H_z$ and $H_y$ could be observed, such as $\pi/2$ in the left domain region and $-\pi/2$ at the right. These results are identical with the derivation of Eq. (4) for the infinite system. In contrast, for
back incidence in Fig. 3(d), such excitation of $\mu_{\text{eff}} \approx 0$ mode is almost completely suppressed, resulting in low transmission through the structure. Therefore a non-reciprocal optical response is attained with one-way tunneling for incident oblique waves through thin films of $\mathcal{PT}$-symmetric bilayer MD structure.

At normal incidence shown in Fig. 4, another interesting phenomenon of non-reciprocal beam shift could be seen by application of the $\mu_{\text{eff}} \approx 0$ mode through such a finite $\mathcal{PT}$-symmetric structure. In Fig. 4(a)(b) at a frequency of $\omega_A = 1.476\omega_m$ [corresponding to point A shown in Fig. 2(a)], both incoming Gaussian waves, including from left or right, undergo an upward lateral-shift perpendicular to the propagation direction after passing through the bilayer MDs. Meanwhile, by looking inside the magnetic domains at both of incidence cases, the direction of power flow indicated by black arrows always changes by an upswept angle with respect to the power flow of the incoming waves. The beam shift and non-reciprocal behavior can also be understood by the excitation of $\mu_{\text{eff}} \approx 0$ mode at point A, with an upswept-angle direction of wave group velocity $v_g$, evaluated as $v_g = \nabla_k \omega(k)$ from the dispersion relation of Fig. 1(d). For comparison, at another resonant frequency of $\omega_B = 1.634\omega_m$ [corresponding to point B in Fig. 2(a)], the incoming waves go straightforward with reciprocal response shown in Fig. 4(c)(d), because the reciprocal propagating mode is excited with the group velocity at point B keeping along the horizontal direction.

It should be noted that the $\mathcal{PT}$ symmetry in our system is actually not a necessary condition to achieve the spectral non-reciprocity. Nevertheless, the $\mathcal{PT}$ symmetry can help achieving perfect transmission mode in one direction as depicted in Fig. 5(a). For a non-$\mathcal{PT}$-symmetric structure with different applied magnetic field on the two magnetic domains, it is seen that transmission through the entire system would be partly suppressed, and the $\mu_{\text{eff}} \approx 0$ mode shift slightly.

Finally, owing to the possible difficulty in implementation in practice of our proposed finite-size $\mathcal{PT}$-symmetric structures, with two adjoined, but inversely magnetized MDs, we consider another structure by separating these two MDs with a little displacement, as illustrated in Fig. 5(b). Note that the $\mu_{\text{eff}} \approx 0$ mode shifts to the lower frequency shown in Fig. 5(c), due to the variation of the effective index of the structures.

FIG. 3: (color online) Electric-field distribution at $\omega = 1.425\omega_m$ under front illumination with $k_y = -4k_m$ (a), and back illumination with $k_y = 4k_m$ (d). Magnetic-field patterns in (a) of $H_x$ (b), and $H_y$ (c) in the regions filled with the two gyromagnetic materials at a zoom-in view. Black arrows in (b)(c) show vector patterns of the magnetic field $\mathbf{H} = (H_x, H_y)$. The big arrows shown in (a) and (d) guide us to see the wave propagation.

FIG. 4: (color online) Electric field distribution under the (a) front illumination and (b) back illumination of an incident gaussian wave normal to interface with a frequency of $\omega_A = 1.476\omega_m$. (c)(d) are similar to (a)(b), but for another case with a frequency $\omega_B = 1.634\omega_m$. These two particular cases are marked in Fig. 2(a) with point A and B, respectively. The vector patterns of power flow in our system are also illustrated with black arrows in (a)-(d). To see clearly, the power flow of incident waves is also shown by means of big arrows.

FIG. 5: (color online) (a) The reflection spectra at a specified incident angle with $k_y = -4k_m$ for a non-$\mathcal{PT}$-symmetric bilayer domain structure, with different applied magnetic field $H_{\text{cor}}$ on the right domain ($0 < x < a$). Here the applied field on the left domain ($-a < x < 0$) is fixed with $H_{\text{cor}} = 800$ Oe, and other parameters are the same with those in Fig. 1(d). (b) Schematic diagram of two bilayer MDs, similar to Fig. 1(b), but separating them with a little displacement of horizontal distance $d$. (c) The reflection spectra for the structure in (b), with $a = 0.004m$, and $d = 0.002m$. Other parameters and lines are identical with those in Fig. 2(a).
IV. CONCLUSION

In summary, we demonstrate a new type of non-reciprocal mu-near-zero radiative mode in the $\mathcal{PT}$-symmetric bilayer MDs, magnetized by opposite directions. Such an unusual mode occurs close to the frequency when the effective permeability for MDs approaches to zero, and could be well excited when the infinite system shrinks to a finite one. In particular, we see two pronounced non-reciprocal behavior for incident waves: one-way complete optical tunneling for oblique incident waves and unidirectional beam shift for normal incidence. Our theoretical results may provide a new way for designing compact isolators.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation of China under Grant No. 11204036, the Hong Kong Research Grant Council through the Area of Excellence Scheme (grant no. AoE/P-02/12), and the Hong Kong Polytechnic University through grant no. G-UA95.

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