THE FACTORIZABLE AMPLITUDE IN $B^0 \to \pi^+\pi^-$

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Using the measured spectrum shape for $B \to \pi\ell\nu$, the rate for $B^+ \to \pi^+\pi^0$, information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$, and theoretical inputs from factorization and lattice gauge theory, we obtain an improved estimate of the “tree” contribution to $B^0 \to \pi^+\pi^-$. We find the branching ratio $B(B^0 \to \pi^+\pi^-)|_{\text{tree}} = (5.25^{+1.67}_{-0.50}) \times 10^{-6}$, to be compared with the experimental value $B(B^0 \to \pi^+\pi^-) = (4.55 \pm 0.44) \times 10^{-6}$. The fit implies $|V_{ub}| = (3.62 \pm 0.34) \times 10^{-3}$. Implications for tree-penguin interference in $B^0 \to \pi^+\pi^-$ and for other charmless $B$ decays are discussed.

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I. INTRODUCTION

The semileptonic process $B \to \pi\ell\nu$ involves a form factor $F_+(q^2)$ related for $q^2 = m_\pi^2$ to the factorized color-favored “tree” contribution in $B^0 \to \pi^+\pi^-$ [1, 2, 3]. In previous work [4] we obtained an estimate of this contribution implying a branching ratio $B(B^0 \to \pi^+\pi^-)|_{\text{tree}} = (7.3 \pm 3.2) \times 10^{-6}$. A measurement of the spectrum $d\Gamma(B \to \pi\ell\nu)/dq^2$ has now been presented by the CLEO Collaboration [5] working at the Cornell Electron Storage Ring. Further results are expected from the BaBar and Belle Collaborations at asymmetric $e^+e^-$ colliders. Using the CLEO measurement and other inputs, we find in the present paper an improved estimate implying $B(B^0 \to \pi^+\pi^-)|_{\text{tree}} = (5.25^{+1.67}_{-0.50}) \times 10^{-6}$, to be compared with the observed branching ratio $B(B^0 \to \pi^+\pi^-) = (4.55 \pm 0.44) \times 10^{-6}$. This result has a number of implications for tree-penguin interference in $B^0 \to \pi^+\pi^-$ and for other charmless $B$ decays.

We review theoretical inputs, including constraints from factorization and lattice gauge theory calculations, in Sec. II, while data are discussed in Sec. III. We perform a global fit to these inputs in Sec. IV. The consequences of this fit are discussed for $B^0 \to \pi^+\pi^-$ and other charmless $B$ decays in Sec. V. We conclude in Sec. VI.

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II. THEORETICAL INPUTS

The $B \to \pi$ matrix element is parametrized by two independent form factors:

$$\langle \pi(p)|\bar{u}\gamma_{\mu}b|B(p+q)\rangle = \left(2p+q - q\frac{m_B^2 - m_{\pi}^2}{q^2}\right)\mu F_+(q^2) + q\mu \frac{m_B^2 - m_{\pi}^2}{q^2}F_0(q^2),$$

(1)

For massless leptons (assumed here), only $F_+(q^2)$ contributes to the differential decay rate

$$\frac{d\Gamma}{dq^2}(B^0 \to \pi^-\ell^+\nu_\ell) = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|\bar{\beta}_\pi|^3|F_+(q^2)|^2,$$

(2)

where $V_{ub}$ is the relevant CKM matrix element. In the factorization hypothesis, one replaces the lepton pair with a pion, giving what we term the “tree” contribution $T$ (in the notation of [6]) to the nonleptonic decay $B^0 \to \pi^+\pi^-$. In the limit of small $m_{\pi}$, the two processes are related by

$$\Gamma_{\text{tree}}(B^0 \to \pi^+\pi^-) = 6\pi^2f^2_{\pi}|V_{ud}|^2|a_1|^2 \frac{d\Gamma(B^0 \to \pi^-\ell^+\nu_\ell)}{dq^2} \bigg|_{q^2=m_{\pi}^2},$$

(3)

where $|a_1|$ is a QCD correction which we shall take equal to 1. The majority of QCD effects are expected to be associated with the form factors $F_{+0}(q^2)$ and thus are taken into account by the factorization ansatz.

Other contributions to charmless strangeness-preserving $B$ decays which we shall consider include color-suppressed tree ($C$) and penguin ($P$) amplitudes. The corresponding strangness-changing amplitudes are denoted by primes. We neglect smaller amplitudes which involve spectator quarks. For these and other details, see, e.g., Ref. [6]. The amplitude for $B^0 \to \pi^+\pi^-$ is then

$$A(B^0 \to \pi^+\pi^-) = -(T + P) = -|T|e^{i\gamma} - |P|e^{-i\beta}e^{i\delta},$$

(4)

where we have introduced phases of CKM elements, assuming the phase of the $\bar{b} \to \bar{d}$ penguin to be dominated by the top quark, and $\delta$ denotes a relative strong phase. A question which has been of interest for some time [4, 7, 8] is whether the small branching ratio for $B^0 \to \pi^+\pi^-$ reflects the effect of destructive tree-penguin interference. If so, by combining this information with CP-violating asymmetries in $B^0 \to \pi^+\pi^-$, one can learn a good deal about both weak (i.e., CKM) and strong phases [9].

We use notation in which the square of an amplitude directly gives a $B^0$ branching ratio in units of $10^{-6}$. The observed branching ratio $\mathcal{B}(B^0 \to \pi^+\pi^-) = (4.55 \pm 0.44) \times 10^{-6}$ (see Sec. III) then corresponds to $|T + P| = 2.13 \pm 0.10$ in our units. In previous work [4] we found $|T| = 2.7 \pm 0.6$, too large an error to display any possible tree-penguin interference.

Another amplitude which will be of use to us is $A(B^+ \to \pi^+\pi^0) = -(T + C)/\sqrt{2}$. The color-suppressed amplitude $C$ is expected to have small phase and magnitude relative to $T$ [10]. We shall use only the conservative range $0.08 < |C/T| < 0.37$. This will provide a useful bound on $T$ based on $\mathcal{B}(B^+ \to \pi^+\pi^0)$. 

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Lattice gauge theories predict not only the shape, but also the normalization, of the $B \to \pi$ form factors at large $q^2$ or small pion recoil momentum in the $B$ rest frame \cite{12,13,14,15}. These predictions turn out to be very helpful in constraining parameters on the basis of the $q^2$ spectrum in $B \to \pi l \nu$. However, they do not address the key question of the form factor behavior at small $q^2$ or large pion recoil.

The CKM parameter $V_{ub}$ is another key input whose determination is for the moment still subject to theoretical uncertainties. Good understanding of the $B \to \pi$ form factor would reduce these uncertainties. Independently of $B \to \pi l \nu$ (or the more complex process $B \to p l \nu$), however, various inclusive methods have been employed to extract $V_{ub}$ from semileptonic $b \to u$ decays, including the study of leptons with energy exceeding the endpoint for $b \to c l \nu$, the rejection of events with recoil mass above charm threshold, and the use of the photon energy distribution in $b \to s \gamma$ to measure the “Fermi distribution” of $b$ quarks inside a $B$ meson. These are summarized in a subsection of the Review of Particle Physics \cite{16}.

The form factor $F_+(q^2)$ is expected to have a $B^*$ pole, as well as possible higher-lying poles in $q^2$. In Ref. \cite{4} we approximated it with a dipole form proposed by Becirevic and Kaidalov \cite{17} on the basis of lattice gauge theory calculations:

$$F_+(q^2) = \frac{c_B(1-\alpha_B)}{(1-q^2/m_{B^*}^2)(1-\alpha_B q^2/m_{B^*}^2)}.$$  

(5)

A value of $\alpha_B$ between 0 and 1 would correspond to a pole lying above $m_{B^*}^2$. However, we were unable to achieve a good fit to the CLEO $B \to \pi l \nu$ spectrum with this form. (The $\chi^2$ of the fit is more than 3 for one degree of freedom.) A generalization of the above form factor is to multiply it by $(1+a q^2/m_{B^*}^2)$, where $a$ is an additional parameter. The resulting form factor is equivalent to an explicit dipole:

$$F_+(q^2) = \frac{R_1}{1-q^2/m_{B^*}^2} + \frac{R_2}{1-\alpha_B q^2/m_{B^*}^2}.$$  

(6)

However, we were unable to achieve any fit for any physical $\alpha_B$ to the numerical inputs in Sec. III which represented any improvement over the single pole with this form. We thus choose instead the two-parameter form

$$F_+(q^2) = \frac{F(0)}{1-q^2/m_{B^*}^2}(1+a q^2/m_{B^*}^2).$$  

(7)

### III. NUMERICAL INPUTS

We summarize some information on $B \to \pi \pi$ branching ratios \cite{18,19,20} in Table I. The central value of the $\pi^+\pi^0$ branching ratio exceeds that of $\pi^+\pi^-$ despite the fact that the coefficient of its dominant $T$ term is divided by $\sqrt{2}$ (see Sec. II). To extract an amplitude for comparison with $B^0$ decays we must first divide all $B^+$ branching ratios by the $B^+/B^0$ lifetime ratio \cite{21} $\tau_+/\tau_0 = 1.073 \pm 0.014$.

Our estimate of $|T|$ based on $B^+ \to \pi^+\pi^0$ then proceeds as follows. After correcting for the lifetime ratio, we find $|T + C| = 3.13 \pm 0.24$. With \cite{11} $|T + C| = |T|(1.23 \pm 0.15)$
we then obtain $|T| = 2.55 \pm 0.37$. This is consistent with our previous determination \[4\] but with smaller errors. (The estimate $|T| = 3.0 \pm 0.3$ of Ref. \[22\] uses too restrictive a value of $|C/T|$ in our opinion.) We seek further information from the $B \to \pi l \nu$ spectrum shape and other sources. This value would imply $\mathcal{B}(B^0 \to \pi^+ \pi^-)|_{\text{tree}} = (6.5 \pm 1.9) \times 10^{-6}$, only 1$\sigma$ above the experimental branching ratio.

The CLEO Collaboration \[3\] measured $d\mathcal{B}(B \to \pi l \nu)/dq^2$ in three $q^2$ bins, each about 8 GeV/$c^2$ wide. The results are not very sensitive to the choice of form factor and we quote them for the form factor \[23\] which appears to fit the data best:

$$
\int dq^2 \frac{d\mathcal{B}}{dq^2}(B^0 \to \pi^- l^+ \nu_l) = \begin{cases} 
(0.431 \pm 0.106) \times 10^{-4} & (0 \leq q^2 \leq 8 \text{ GeV}^2) \\
(0.651 \pm 0.105) \times 10^{-4} & (8 \leq q^2 \leq 16 \text{ GeV}^2) \\
(0.245 \pm 0.094) \times 10^{-4} & (16 \text{ GeV}^2 \leq q^2)
\end{cases}
$$

Only statistical errors (dominant) are shown. These sum to a branching ratio of $\mathcal{B}(B^0 \to \pi^- l^+ \nu_l) = (1.33 \pm 0.18 \pm 0.11 \pm 0.01 \pm 0.07) \times 10^{-4}$, where the errors are statistical, experimental systematic, pion form factor uncertainty, and $\rho$ form factor uncertainty.

Lattice calculations of the form factor $F_+(q^2)$ have been presented in the past few years by the UKQCD \[12\], APE \[13\], Fermilab \[14\] and JLQCD \[15\] Collaborations. Numerical values of $F_+(q^2)$ are computed in the range $13.6 \text{ GeV}^2 \leq q^2 \leq 23.4 \text{ GeV}^2$. Although small variations are present among the four different calculations, all results are consistent with each other within errors. We will include them all in our fits.

For $|V_{ub}|$ we combine determinations presented in Ref. \[10\] in the following manner. All numbers will be quoted in units of $10^{-3}$. The inclusive LEP average is $4.09 \pm 0.37 \pm 0.44 \pm 0.34$ while the inclusive CLEO value is $4.12 \pm 0.34 \pm 0.44 \pm 0.33$, where the errors are statistical, experimental systematic, $b \to c$ uncertainty, and $b \to u$ uncertainty. In addition there are theoretical uncertainties estimated to range up to 15%. Combining the two inclusive numbers before folding in the theoretical uncertainties, and treating the last two errors as common, we obtain $4.11 \pm 0.61$. We shall use this in our fits. We do not include some preliminary results presented by CLEO \[24\] and Belle \[25\].

An earlier CLEO exclusive determination of $|V_{ub}|$ utilizes both $\pi l \nu$ and $\rho l \nu$ decays \[16\]. Its result, which we do not use in the present fit, amounts to an average of 3.25 with experimental and theoretical errors comparable to those in the inclusive determinations. Averaging with the inclusive value noted above, we should expect a global fit to give $10^3 |V_{ub}| \simeq 3.68 \pm 0.43$ with an additional 15% theoretical error, or approximately a 20% error overall. We shall see that a modest improvement upon this error is possible, while the central value does not change much.

| Mode       | BaBar \[18\] | Belle \[19\] | CLEO \[20\] | Average |
|------------|--------------|--------------|-------------|---------|
| $\pi^+ \pi^-$ | $4.7 \pm 0.6 \pm 0.2$ | $4.4 \pm 0.6 \pm 0.3$ | $4.5^{+1.4+0.3}_{-1.2-0.4}$ | $4.55 \pm 0.44$ |
| $\pi^+ \pi^0$ | $5.5 \pm 1.0 \pm 0.6$ | $5.3 \pm 1.3 \pm 0.5$ | $4.6^{+1.8+0.6}_{-1.6-0.7}$ | $5.27 \pm 0.79$ |

Table I: Branching ratios for some charmless two-body $B$ decays, in units of $10^{-6}$. 
IV. GLOBAL FIT

We perform an overall three-parameter $\chi^2$ fit to the above-mentioned $B \to \pi l \nu$ branching fractions in the three $q^2$ bins, the averaged inclusive $|V_{ub}|$, and 26 lattice data points on $F_+(q^2)$. We neglect the small correlations among the three branching fractions. The quality of the fit is fairly good, with $\chi^2 = 8.7$ for 27 degrees of freedom. The $\chi^2$'s contributed by specific sources are summarized in Table II. More than 50% of the $\chi^2$ comes from the Fermilab lattice points, which appear to be of a somewhat different pattern from the other three lattice determinations.

The results of the fit are

$$a = 1.14^{+0.72}_{-0.42} \ ,$$
$$F(0) = 0.23 \pm 0.04 \ ,$$
$$|V_{ub}| = (3.62 \pm 0.34) \times 10^{-3} \ ,$$
$$\mathcal{B}(B^0 \to \pi^+ \pi^-)_{\text{tree}} = (5.25 \pm 1.67) \times 10^{-6} \ ,$$
$$|T| = 2.29 \pm 0.36 \ .$$

A theoretical error of $\simeq 15\%$ must be added to $|V_{ub}|$. The value of $|T|$ overlaps that ($|T| = 2.55 \pm 0.37$) obtained in Sec. III from $B^+ \to \pi^+ \pi^0$, but the $\pi^+ \pi^0$ value indicates that $\mathcal{B}(B^0 \to \pi^+ \pi^-)_{\text{tree}}$ is no smaller than $4.75 \times 10^{-6}$. Hence we shall truncate our parameter space at this lower limit, and quote

$$\mathcal{B}(B^0 \to \pi^+ \pi^-)_{\text{tree}} = (5.25^{+1.67}_{-0.50}) \times 10^{-6} \ ,$$
$$|T| = 2.29^{+0.36}_{-0.11} \ .$$

The ranges of parameters contributing to the global fit are illustrated in Fig. I where we show the points corresponding to minimum $\chi^2 = 8.7$ and the ellipses corresponding to $\Delta \chi^2 = 1$. The various projections are helpful in visualizing the full range of parameter variation. In particular, the value of $\mathcal{B}(B^0 \to \pi^+ \pi^-)_{\text{tree}}$ can vary substantially as a result of the uncertainty in $F(0)$, which is still not well constrained by the data. However, its $1\sigma$ upper limit of $6.92 \times 10^{-6}$ is well below that implied by the previous estimate of Ref. [4].
Figure 1: Left column: projections of error ellipsoid for global fit on the plane of two parameters for central values of the third. Right column: ellipses involving the tree amplitude corresponding to the variations shown in the left column. Note that $|T|^2 = B(B^0 \to \pi^+\pi^-)|_{\text{tree}}$ in units of $10^{-6}$. 
In Fig. 2 we show our best fit to the CLEO data [5] for the $B^0 \rightarrow \pi^- l^+ \nu_l$ spectrum (in three $q^2$ bins). The data favor a rather lower value of $F(0)$ than in our previous discussion [4], accounting for the lower magnitude of the tree amplitude in the present treatment. In Fig. 3 we show the comparison of the lattice data points with our best-fit form factor $F_+(q^2)$. As a consequence of the internal variations within the lattice results, a $\chi^2$ of about 5.5 (contributed by the lattice data) should be common for all fits; see Table II. Therefore, since the $B \rightarrow \pi l \nu$ spectrum is the second largest $\chi^2$ source, a significantly better overall fit can be achieved only if the measured $B \rightarrow \pi l \nu$ branching ratios in the three $q^2$ bins are fitted better. This will require the addition of a fourth parameter to affect the shape of $dB(B \rightarrow \pi l \nu)/dq^2$ so that it is suppressed at both low and high $q^2$ ends and enhanced in the middle while relatively unchanged in the region $13.6 \text{ GeV}^2 \leq q^2 \leq 23.4 \text{ GeV}^2$ where lattice data exist; see Fig. 2. Consequently smaller tree amplitudes are implied and we regard them as disfavored by the lower limit as obtained earlier.

V. HOW KNOWING THE TREE AMPLITUDE HELPS

The ratio $R_{\pi\pi}$ of the observed $B^0 \rightarrow \pi^+ \pi^-$ branching ratio to its value in the presence
of the tree amplitude alone helps to establish the relative magnitude and strong and weak phase of the penguin amplitude in this process [9]. On the basis of the previous determination of the tree amplitude [4] and the present world average for $B(B^0 \to \pi^+\pi^-)$ we quoted [26] $R_{\pi\pi} = 0.62 \pm 0.28$, which indicated that tree-penguin interference was not required but, if present in the rate, would be destructive. The new information on $|T|$ allows us to refine this estimate to obtain $R_{\pi\pi} = 0.87^{+0.11}_{-0.28}$, a value still consistent with both possibilities.

The ratio $|P/T|$ of penguin to tree amplitudes quoted in Ref. [26] was $|P/T| = 0.28 \pm 0.06$. This ratio is useful in interpreting CP-violating asymmetries in the decay $B^0 \to \pi^+\pi^-$ (see, e.g., [9]). With the new world average [26] $B(B^+ \to K^0\pi^+) = (19.6 \pm 1.4) \times 10^{-6}$ and the prescription [9] $|P/P'| = (f_\pi/f_K)\lambda/(1 - \lambda^2/2)$ we find for $f_\pi = 130.7$ MeV, $f_K = 159.8$ MeV, and [27] $\lambda = 0.224$ the values $|P'| = 4.28 \pm 0.16$, $|P| = 0.80 \pm 0.03$, and $|P/T| = 0.35^{+0.02}_{-0.06}$. (Here the prime denotes a $|\Delta S| = 1$ amplitude.) The “penguin pollution” thus is slightly greater than estimated previously. Corrections to the CKM phase $\alpha$ obtained from the asymmetry parameter $S_{\pi\pi}$ and the direct asymmetry

Figure 3: Comparison of lattice data points with our best-fit form factor $F_+(q^2)$. Lattice data are from UKQCD (squares), APE (stars), Fermilab (circles) and JLQCD (diamonds).
The tree/penguin ratio in $B^0 \rightarrow K^+\pi^-$ is also affected. By a similar analysis we found $r = |T'/P'| = 0.173 \pm 0.039$ in Ref. [20]; the new value is $0.151^{+0.024}_{-0.000}$. A bound on the CKM phase $\gamma$ quoted in Ref. [20] relied on the lower limit of $r$, which is slightly raised, so the bound is strengthened slightly. Since it was only at the 1$\sigma$ level, we do not present it here.

A further implication of the improved upper bound on $T$ is a lower bound on $C$. Given the 1$\sigma$ bound $|T + C| \geq 2.89$ based on the $B^+ \rightarrow \pi^+\pi^0$ branching ratio (see Sec. III) and the 1$\sigma$ upper bound $|T| \leq 2.65$ based on the present analysis, we conclude that if $C$ and $T$ have a small relative phase [10], then $\text{Re}(C/T) \approx 0.1$.

VI. CONCLUSIONS

The measurement of the $B \rightarrow \pi\ell\nu$ spectrum by the CLEO Collaboration [5] has provided valuable information allowing us to improve the determination of the “tree” contribution to $B^0 \rightarrow \pi^+\pi^-$. Combining this information with inclusive determinations of the CKM matrix element $|V_{ub}|$ and lattice gauge theory calculations of the $B \rightarrow \pi$ form factor $F_+(q^2)$, we have found $B(B^0 \rightarrow \pi^+\pi^-)|_{\text{tree}} = (5.25^{+1.67}_{-0.50}) \times 10^{-6}$, not significantly greater than the experimental value $B(B^0 \rightarrow \pi^+\pi^-) = (4.55 \pm 0.44) \times 10^{-6}$. The fit implies $|V_{ub}| = (3.62 \pm 0.34) \times 10^{-3}$, with an additional theoretical error of 15%. The relative strength of the penguin amplitude in this process, gauged using flavor SU(3) from the rate for $B^+ \rightarrow K^0\pi^+$, is slightly larger than estimated previously, amounting to $(35^{+2}_{-6})\%$ in amplitude. However, the need for strong destructive interference between this amplitude and the tree contribution is somewhat diminished in comparison with earlier estimates.

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