Multiparticle production and perturbative QCD

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Abstract

The perturbative quantum chromodynamics (QCD) is quite successful in the description of main features of multiparticle production processes. Ten most appealing characteristics are described in this brief review talk and compared with QCD predictions. The general perturbative QCD approach is demonstrated and its problems are discussed. It is shown that the analytical calculations at the parton level with the low-momentum cut-off reproduce experimental data on the hadronic final state surprisingly accurately even though the perturbative expansion parameter is not very small. Moreover, the perturbative QCD has been able not only to describe the existing data but also to predict many bright qualitative phenomena.

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1 Introduction

Multiparticle production is the main process of very high energy particle interactions. Studying it, one hopes to get knowledge on validity of our general ideas about the structure of the matter at smallest distances, on new states of matter which could be created at these extreme conditions, on asymptotical properties of strong interactions, on confinement etc. One should understand these processes also to separate the signals for new physics from the conventional QCD background.

Theoretical interpretation of these processes evolved from statistical and hydrodynamical approaches to multiperipheral models and QCD. Many models have been elaborated and their computerized Monte Carlo versions are available for a detailed comparison with experimental data. In particular, A. Capella told us about the dual parton model proposed and developed in Orsay in the late 1970s by a group of theorists including J. Tran Thanh Van. Neither these phenomenological models nor QCD approach can evade a problem of transition from partons (quarks, gluons) to the observed particles. This stage is treated phenomenologically in any of them. It introduces additional parameters which can give us a hint to the confinement property, but they are sometimes hard to control.

The whole process is considered as a cascade of consecutive emissions of partons each of which produces observed hadrons in a jet-like manner. Jets from primary quarks were discovered in 1975 with
the angular distribution expected for spin 1/2 quarks. Gluons emitted by quarks at large transverse momenta can be described by perturbative QCD due to the asymptotical freedom property, and such processes are used to determine the value of the coupling strength. However, one can try to proceed to lower transverse momenta when many jets (and consequently many hadrons) are created. These processes are of main concern for this survey.

Perturbative QCD analysis can be directly applied to $e^+e^-$-data where the initial state is determined by quark jets or to jets separated from final states in $ep$, $p\bar{p}$ etc processes.

Usually, Monte Carlo models deal with matrix elements (actually, probabilities) of a process at the parton level plus hadronization stage. They properly account for the energy-momentum conservation laws. One gets all possible exclusive characteristics at a given energy but fails to learn their asymptotics. On the contrary, analytical QCD pretends to start with asymptotical values and proceed to lower energies accounting for conservation laws, higher order perturbative and simplified non-perturbative effects.

The perturbative evolution is terminated at some low scale $Q_0 \sim 1\text{GeV}$ for transverse momenta or virtualities of partons. Some observed variables, e.g., such as thrust, are insensitive to this “infrared” cut-off. For others, like inclusive distributions, the local parton-hadron duality (LPHD) is assumed which declares that the distributions at the parton level describe the hadron observables up to some constant factor. This concept originates from the preconfinement property of quarks and gluons to form colorless clusters. It works surprisingly well when applied for comparison with experiment.

Even more amazing look two other features of the perturbative QCD approach: probabilistic description and applicability to comparatively soft processes. At high energies, the quantum interference of different amplitudes for parton production results in angular (or, more precisely, transverse momentum) ordering of successive emissions of gluons which favors the probabilistic equations for these processes. The solutions of these equations obtained via the modified perturbative expansion seem to be applicable sometimes even for rather soft processes where the expansion parameter is not small enough and, moreover, it is multiplied by some large factors increasing with energy. Thus, it can be justified only because some subseries of the purely perturbative expansion ordered according to their high energy behavior are summed first and then the asymptotic series is cut off at the proper order. In this framework, the perturbative QCD has demonstrated its very high predictive power.

If asked to choose 10 most spectacular analytical predictions already confirmed by experiment I would mention:

1. the energy dependence of mean multiplicities,
2. oscillations of cumulant moments of multiplicity distributions as functions of their rank,
3. difference between quark and gluon jets,
4. the hump-backed plateau of inclusive rapidity distribution and energy dependence of its maxima,
5. difference between heavy- and light-quark jets,
6. color coherence in 3-jet events,
7. intermittency and fractality,
8. the energy behavior of higher moments of multiplicity distributions,
9. subjet multiplicities,
10. jet universality.

I have selected some of the most impressive results. More complete list with a detailed survey can be found in the books \[1,2\] and in more recent review papers \[3,4,5,6\].

2 QCD equations

First, let me describe main theoretical tools used for prediction of all these features. The most general approach starts from the equation for the generating functional. The generating functional contains
Let us write them down explicitly for the generating functions now.

\[ G(\{u\}, \kappa_0) = \sum_n \int d^3k_1...d^3k_n u(k_1)...u(k_n) P_n(k_1, ..., k_n; \kappa_0), \]  

where \( P_n(k_1, ..., k_n; \kappa_0) \) is the probability density for exclusive production of particles with momenta \( k_1, ..., k_n \) at the initial virtuality (energy) \( \kappa_0 \), and \( u(k) \) is an auxiliary function. For \( u(k) = \text{const} \), one gets the generating function of the multiplicity distribution \( P_n(\kappa_0) \). The variations of \( G(\{u\}) \) over \( u(k) \) (or differentials for constant \( u \)) provide any inclusive distributions and correlations of arbitrary order, i.e. complete information about the process. The general structure of the equation for the generating functional describing the jet evolution for a single species partons can be written symbolically as

\[ G' \sim \int \alpha_S K[G \otimes G - G]. \]  

It shows that the evolution of \( G \) indicated by its variation (derivative) \( G' \) is determined by the cascade process of the production of two partons by a highly virtual time-like parton (the term \( G \otimes G \)) and by the escape of a single parton \( (G) \) from a given phase space region. The weights are determined by the coupling strength \( \alpha_S \) and the splitting function \( K \) defined by the interaction Lagrangian. The integral runs over all internal variables, and the symbol \( \otimes \) shows that the two partons share the momentum of their parent. This is a non-linear integrodifferential probabilistic equation with shifted arguments in the \( G \otimes G \) term under the integral sign.

For quark and gluon jets, one writes down the system of two coupled equations. Their solutions give all characteristics of quark and gluon jets and allow for the comparison with experiment to be done. Let us write them down explicitly for the generating functions now.

\[ G'_G = \int_0^1 dx K_G^G(x) \gamma_0^2 [G_G(y + \ln x)G_G(y + \ln(1 - x)) - G_G(y)] + n_f \int_0^1 dx K_G^F(x) \gamma_0^2 [G_F(y + \ln x)G_F(y + \ln(1 - x)) - G_F(y)], \]  

\[ G'_F = \int_0^1 dx K_F^G(x) \gamma_0^2 [G_G(y + \ln x)G_F(y + \ln(1 - x)) - G_F(y)], \]  

where \( G'(y) = dG/dy, y = \ln(p\Theta/Q_0) = \ln(2Q/Q_0), p \) is an initial momentum, \( \Theta \) is the angle of the divergence of the jet (jet opening angle), assumed here to be small, \( Q \) is the jet virtuality, \( Q_0 = \text{const} \), \( n_f \) is the number of active flavours,

\[ \gamma_0^2 = \frac{2N_c\alpha_S}{\pi}, \]  

the labels \( G \) and \( F \) correspond to gluons and quarks, and the kernels of the equations are

\[ K_G^G(x) = \frac{1}{x} - (1 - x)[2 - x(1 - x)], \]  

\[ K_G^F(x) = \frac{1}{4N_c} [x^2 + (1 - x)^2], \]  

\[ K_F^G(x) = \frac{C_F}{N_c} \left[ \frac{1}{x} - 1 + \frac{x}{2} \right], \]  

where \( N_c = 3 \) is the number of colours, and \( C_F = (N_c^2 - 1)/2N_c = 4/3 \) in QCD. The asymmetric form \( (\) of the three-gluon vertex can be used due to symmetry properties of the whole expression. The variable \( u \) has been omitted in the generating functions.

Let us note that these equations can be exactly solved \( [7] \) if the coupling strength is assumed fixed, i.e. independent of \( y \). For the running coupling strength, the Taylor series expansion can be used \( [8] \) to get the perturbative expansion of physically measurable quantities.
A typical feature of any field theory with a dimensionless coupling constant (quantum chromodynamics in particular) is the presence of the singular terms at $x \to 0$ in the kernels of the equations. They imply the uneven sharing of energy between newly created jets and play an important role in jet evolution giving rise to its more intensive development compared with the equal proportions (nonsingular) case.

Even though the system of equations (3), (4) is physically appealing, it is not absolutely exact; i.e. it is not derived from first principles of quantum chromodynamics. One immediately notices this since, for example, there is no four-gluon interaction term which is contained in the lagrangian of QCD. Such a term does not contribute a singularity to the kernels and its omission is justified in the lowest orders. Nevertheless, the modified series of the perturbation theory (with three-parton vertices) is well reproduced by such equations up to the terms including two-loop and three-loop corrections. As shown in Ref. [2], the neglected terms would contribute at the level of the product of, at least, five generating functions. Physical interpretation of the corresponding graphs would lead to treatment of the ‘colour polarizability’ of jets. Sometimes, the effective infrared-safe coupling constant is used as a substitute for the phenomenological parameter $Q_0$. It must be universal for different processes and tend to a constant limit at low virtualities. However, we will use the more traditional approach.

There are some problems with the definition of the evolution parameter, with preasymptotic corrections etc. For example, the lower and upper limits of integration over $x$ in Eqns (3), (4) change in the preasymptotical region. It is imposed by the restriction on the transverse momentum which is given by

$$k_t = x(1 - x)p\Theta' > Q_0/2.$$  \hspace{1cm} (9)

This condition originates from the requirement that the formation time of a gluon ($t_{form} \sim k/k_t^2$) should be less than its hadronisation time ($t_{had} \sim kR^2 \sim k/Q_0^2$) for the perturbative QCD to be applicable. It leads to the requirement that the arguments of the generating functions in Eqns (3), (4) should be positive. Therefore, we must integrate in Eqns (3), (4) over $x$ from $\exp(-y)$ to $1 - \exp(-y)$. However these limits tend to 0 and 1 at high energies. That is why it seems reasonable to learn more about the solutions of the equations (3), (4) in the asymptotical region, and then take the neglected terms into account as corrections to these solutions.

Moreover, it is of physics importance. With limits equal to $\exp(-y)$ and $1 - \exp(-y)$, the partonic cascade terminates at the perturbative level $Q_0/2$ as is seen from the arguments of the generating functions in the integrals. With limits equal to 0 and 1, one extends the cascade into the non-perturbative region with low virtualities $Q_1 \approx xp\Theta'/2$ and $Q_2 \approx (1 - x)p\Theta'/2$ less than $Q_0/2$. This region contributes terms of the order $\exp(-y)$, power-suppressed in energy. It is not clear whether the equations and LPHD hypothesis are valid down to some $Q_0$ only or the non-perturbative region can be included as well.

Some approximations are used to solve these equations. The asymptotical results are obtained in the so-called double-logarithmic (DLA) or leading order (LO) approximation when the terms $(\alpha_s \ln^2 s)^n$ are summed. Here $s$ is the cms energy squared. The emitted gluons are assumed so soft that the energy-momentum conservation is neglected. The corrections accounting for conservation laws in the $G \otimes G$ term and in limits of the integration as well as the higher order terms in the weight $\alpha_s K$ (in particular, the non-singular terms of the kernels $K$) appear in the next-to-leading (NLO or MLLA - modified leading logarithm approximation) and higher (2NLO, ...) orders. Formally, these equations have been proven only for the next-to-leading (NLO) order of the perturbative QCD. However, one can try to consider them as kinetic equations in higher orders and/or generalize them including the abovementioned effects in a more rigorous way than it is usually implied.

### 3 Comparison with experiment

Let us turn directly to the comparison of results obtained with available experimental data. The main bulk of the data is provided by $e^+e^-$-processes at $Z^0$ energy.
3.1 The energy dependence of mean multiplicity

The equations for the average multiplicities in jets are obtained from the system of equations (3), (4) by expanding the generating functions in $u - 1$ and keeping the terms with $q=0$ and 1 with account of the definition

$$
\left. \frac{dG}{du} \right|_{u=1} = \sum n P_n = \langle n \rangle.
$$

They read

$$
\langle n_G(y) \rangle' = \int dx \gamma_0^2[K_G^G(x)(\langle n_G(y + \ln x) \rangle + \langle n_G(y + \ln(1-x)) \rangle - \langle n_G(y) \rangle)
+ n_f K^F_G(x)(\langle n_F(y + \ln x) \rangle + \langle n_F(y + \ln(1-x)) \rangle - \langle n_G(y) \rangle)],
\langle n_F(y) \rangle' = \int dx \gamma_0^2 K^G_F(x)(\langle n_G(y + \ln x) \rangle + \langle n_F(y + \ln(1-x)) \rangle - \langle n_F(y) \rangle).
$$

Herefrom one can learn about the energy evolution of the ratio of multiplicities $r$ and of the QCD anomalous dimension $\gamma$ (the slope of the logarithm of average multiplicity in a gluon jet) defined as

$$
r = \frac{\langle n_G \rangle}{\langle n_F \rangle}, \quad \gamma = \frac{\langle n_G \rangle'}{\langle n_G \rangle} = (\ln \langle n_G \rangle)' = \langle n_G \rangle' - \langle n_G \rangle.
$$

They have been represented by the perturbative expansion at large $y$ as

$$
\gamma = \gamma_0(1 - a_1 \gamma_0 - a_2 \gamma_0^2 - a_3 \gamma_0^3) + O(\gamma_0^5),
$$

$$
r = r_0(1 - r_1 \gamma_0 - r_2 \gamma_0^2 - r_3 \gamma_0^3) + O(\gamma_0^4).
$$

Using the Taylor series expansion of $\langle n \rangle$ at large $y$ in Eqns (11), (12) with (14), (15) one gets the coefficients $a_i, r_i$.

One of the most spectacular predictions of QCD states that in the leading order approximation, where $\gamma = \gamma_0$, average multiplicities should increase with energy [10, 11, 12] like $\exp[c\sqrt{\ln s}]$, i.e., in between the power-like and logarithmic dependences predicted by hydrodynamical and multiperipheral models. Next-to-leading order results account for the term with $a_1$ in Eqn. (14), (13, 14, 15) and contribute the logarithmically decreasing factor to this behavior whereas the higher order terms do not practically change this dependence [10, 17]. The fitted parameters in the final expression are an overall constant normalization factor which is defined by confinement and a scale parameter $Q_0$. The $e^+e^-$-data are well fitted by such an expression as seen in Fig. 1. Let us note here that the expansion parameter $\gamma$ is rather large at present energies being about 0.4 - 0.5.

3.2 Oscillations of cumulant moments

The shape of the multiplicity distribution can be described by its higher moments related to the width, the skewness, the kurtosis etc. The $q$-th derivative of the generating function corresponds to the factorial moment $F_q$, and the derivative of its logarithm defines the so-called cumulant moment $K_q$. The latter ones describe the genuine (irreducible) correlations in the system (it reminds the connected Feynman graphs).

$$
F_q = \frac{\sum_n P_n n(n - 1) \cdots (n - q + 1)}{(\sum_n P_n n)^q} = \left. \frac{1}{\langle n \rangle^q} \cdot \frac{d^q G(z)}{du^q} \right|_{u=1},
$$

$$
K_q = \left. \frac{1}{\langle n \rangle^q} \cdot \frac{d^q \ln G(z)}{du^q} \right|_{u=1},
$$

where

$$
\langle n \rangle = \sum_{n=0}^{\infty} P_n n
$$
is the average multiplicity. These moments are not independent. They are connected by definite relations that can easily be derived from their definitions in terms of the generating function:

$$F_q = \sum_{m=0}^{q-1} C_{q-1}^m K_{q-m} F_m,$$

(19)

which are nothing other than the relations between the derivatives of a function and of its logarithm at the point where the function itself equals 1. Here

$$C_{q-1}^m = \frac{(q-1)!}{m!(q-m-1)!} = \frac{\Gamma(q)}{\Gamma(m+1)\Gamma(q-m)} = \frac{1}{mB(q,m)},$$

(20)

are the binomial coefficients, and $\Gamma$ and $B$ denote the gamma- and beta-functions, correspondingly. Thus there are only numerical coefficients in the recurrence relations (19) and the iterative solution (well-suited for computer calculation) reproduces all cumulants if the factorial moments have been given, and vice versa. In that sense, cumulants and factorial moments are equally suitable. The relations for the low ranks are

$$F_1 = K_1 = 1,$$

$$F_2 = K_2 + 1,$$

$$F_3 = K_3 + 3K_2 + 1.$$  

(21)

Solving the Eqns. (3), (4), one gets quite naturally the predictions [8, 7, 18] for the behavior of the ratio $H_q = K_q/F_q$. At asymptotically high energies, this ratio is predicted to behave as $q^{-2}$. However, the asymptotics is very far from our realm. At present energies, according to QCD, this ratio should reveal the minimum at $q \approx 5$ and subsequent oscillations. This astonishing qualitative prediction has been confirmed in experiment (for the first time in Ref. [19]) as in $e^+e^-$ (see Fig. 2 from [20]) as in hadronic processes. The predicted negative minimum of $H_q$ is clearly observed. It can correspond to the replacement of attractive forces (clustering) by repulsion (between clusters) in systems with different number of particles. The minimum position slowly changes with energy and with size of the phase space window. Also, for instanton induced processes it has been found at $q \approx 2$.

The quantitative analytical estimates are not enough accurate because, first of all, the expansion parameter becomes equal to the product $q\gamma$ which is close to one or even exceeds it for all $q > 1$. Therefore the perturbative approach is, strictly speaking, inapplicable to this problem. However, some tricks can be used to improve it. At the same time, the numerical computer solution [21] reproduces oscillations quite well. These new laws differ from all previously attempted distributions of the probability theory.

### 3.3 Difference between quark and gluon jets

The system of two equations for quark and gluon jets predicts that asymptotically the energy dependence of mean multiplicities in them should be identical. However, normalization differs, and gluon jets are more "active" so that the ratio $r = \langle n_G \rangle/\langle n_F \rangle$ of average multiplicities in gluon and quark jets should tend at high energies [22] to the ratio of Casimir operators $C_A/C_F = 9/4$. Once again, this prediction shows how far are we now from the true asymptotics because in experiment this ratio is about 1.5 at $Z^0$ energy and even smaller at lower energies. The higher order terms [23, 13, 17] (calculated now up to 3NLO) improve the agreement and approach the experimental value with an accuracy about 15% (see Fig. 3). The higher order terms change slightly also the energy behavior of quark jets compared to gluon jets as observed in experiment. However, the simultaneous fit of quark and gluon jets with the same set of fitted parameters is still not very accurate as is seen from the shaded area in Fig. 1. This failure is again due to insufficiently precise description of the ratio $r$.

Even better agreement has been achieved when the equations are modified to account for phase space limitations imposed by energy-momentum conservation [24]. However, some problems arise for
higher moments of the multiplicity distribution in such an analytical approach \[25\]. The exact computer solution of the equations \[26\] has led to better agreement on the ratio \(r\) at \(Z^0\) but about 20% discrepancy is still left at lower energies of \(\Upsilon\).

The widths of the multiplicity distributions differ in quark and gluon jets, the former being somewhat wider. Qualitatively, QCD describes this tendency but quantitative estimates are rather uncertain yet as is discussed in the subsection 3.8.

### 3.4 The hump-backed plateau

Dealing with inclusive distributions, one should solve the equations for the generating functional. It has been done up to NLO approximation. As predicted by QCD, the momentum (rapidity \(y\)) spectra of particles inside jets should have the shape of the hump-backed plateau \[10, 11, 12\]. This striking prediction of the perturbative QCD differs from the previously popular flat plateau advocated by Feynman. It has been found in experiment (Fig. 4). The depletion between the two humps is due to angular ordering and color coherence in QCD. The humps are of the approximately Gaussian shape near their maxima if the variable

\[
\xi = \ln \frac{1}{x}; \quad x = \frac{p}{E_j}
\]

is used. Here \(p\) is the particle momentum, \(E_j\) is the jet energy. This prediction was first obtained in the LO QCD, and more accurate expressions were derived in NLO \[27\]. Moments of the distributions up to the fourth rank have been calculated. The drop of the spectrum towards small momenta becomes more noticeable in this variable. The comparison with experimental data at different energies has revealed good agreement both on the shape of the spectrum (see Fig. 5 for \(e^+e^-\) from \[28\]) and on the energy dependence of its peak position (see Fig. 6 for \(e^+e^-, ep, p\bar{p}\) from \[29\]) and width.

### 3.5 Difference between heavy- and light-quark jets

Another spectacular prediction of QCD is the difference between the spectra and multiplicities in jets initiated by heavy and light quarks. Qualitatively, it corresponds to the difference in bremsstrahlung by muons and electrons where the photon emission at small angles is strongly suppressed for muons because of the large mass in the muon propagator. Therefore, the intensity of the radiation is lower in the ratio of masses squared. The coherence of soft gluons also plays an important role in QCD. For heavy quarks the accompanying radiation of gluons should be stronger depleted in the forward direction (dead-cone or ring-like emission). It was predicted \[30, 31\] that it should result in the energy-independent difference of companion mean multiplicities for heavy- and light-quark jets of equal energy. The naive model of energy rescaling \[32, 33, 34\] predicts the decreasing difference. The experimental data (see Fig. 7 from \[35\]) support this QCD conclusion.

### 3.6 Color coherence in 3-jet events

When three or more partons are involved in a hard interaction, one should take into account color-coherence effects. Several of them have been observed. In particular, the multiplicity can not be represented simply as a sum of flows from independent partons. QCD predicts that the particle flows should be enlarged in the directions of emission of partons and suppressed in between them. Especially interesting is the prediction that this suppression is stronger between \(q\bar{q}\)-pair than between \(gq\) and \(g\bar{q}\) in \(e^+e^- \rightarrow q\bar{q}g\) event if all angles between partons are large (the "string" \[36\] or "drag" \[37\] effect). All these predictions have been confirmed by experiment (see Fig. 8 from \[38\]). In \(q\bar{q}g\) events the particle population values in the \(qg\) valleys are found larger than in the \(q\bar{q}\) valley by a factor 2.23±0.37 compared to the theoretical prediction of 2.4. Moreover, QCD predicts that this shape is energy-independent up to an overall normalization factor.
Let us note that for the process \( e^+ e^- \rightarrow q\bar{q}\gamma \) the emission of additional photons would be suppressed both in the direction of a primary photon and in the opposite one. In the case of an emitted gluon, we observe the string (drag) effect of enlarged multiplicity in its direction and stronger suppression in the opposite one. This suppression is described by the ratio of the corresponding multiplicities in the \( q\bar{q} \) region

\[
R_\gamma = \frac{N_{q\bar{q}}(qg)}{N_{q\bar{q}}(q\gamma)}
\]  

which is found to be equal 0.58±0.06 in experiment whereas the theoretical prediction is 0.61.

The color coherence reveals itself as inside jets as in inter-jet regions. It should suppress both the total multiplicity of \( q\bar{q}g \) events and the particle yield in the transverse to the \( q\bar{q}g \) plane for decreasing opening angle between the low-energy jets. When hard gluon becomes softer, color coherence determines, e.g., the azimuthal correlations of two gluons in \( q\bar{q}gg \) system. In particular, back-to-back configuration (\( \varphi \sim 180^0 \)) is suppressed by a factor \( \sim 0.785 \) in experiment, 0.8 in HERWIG Monte Carlo and 0.93 in analytical pQCD. In conclusion, color coherence determines topological dependence of jet properties.

Results for jets in \( ep \) and \( pp \) processes also favor theoretical expectations. Some proposals have been promoted for a special two-scale analysis of 3-jet events when the restriction on the transverse momentum of a gluon jet is imposed \[39, 40\]. They are under experimental study now.

3.7 Intermittency and fractality

The self-similar parton cascade leads to special multiparton correlations. Its structure with "jets inside jets inside jets..." provoked the analogy with turbulence and the ideas of intermittency \[41\]. Such a structure should result in the fractal distribution in the available phase space \[42\]. The fractal behavior would display the linear dependence of logarithms of factorial moments on the logarithmic size of phase space windows. The moments are larger in smaller windows, i.e. the fluctuations increase in smaller bins in a power-like manner (see the review paper \[43\]).

In QCD, the power dependence appears for a fixed coupling regime \[4\]. The running property of the coupling strength in QCD flattens \[41, 44, 46\] this dependence at smaller bins, i.e. the multifractal behavior takes over there. The slopes for different ranks \( q \) are related to the Renyi dimensions. Both the linear increase at comparatively large but decreasing bins and its flattening for small bins have been observed in experiment (see Fig. 9 from \[47\]). However, only qualitative agreement with analytical predictions can be claimed here. The higher order calculations are rather complicated and mostly the results of LO with some NLO corrections are yet available. In experiment, different cuts have been used which hamper the direct comparison. However, Monte Carlo models where these cuts can be done agree with experiment better. The role of partonic and hadronization stages in this regime is still debatable.

3.8 The energy behavior of higher moments of multiplicity distributions

The factorial moments increase both with their rank and with energy increasing. From the mentioned above behavior of \( H_q \)-moments one easily guesses that the the cumulant moments behave in a similar but somewhat different manner. The experimental results for 41.8 GeV gluon jets \( F_2^G = 1.023 \) and for 45.6 GeV \( uds \) quark jets, \( F_2^F = 1.082 \) are much smaller than the asymptotical predictions, viz. 1.33 and 1.75, respectively. The NLO terms improve the description of the data compared to leading order results. If one accepts the effective value of \( \alpha_s \) averaged over all the energies of the partons during the jet evolution to be \( \alpha_s \approx 0.2 \), one obtains the NLO values \( F_2^G \approx 1.039 \) and \( F_2^F \approx 1.068 \) at these energies which are quite close to experimental results. In this sense the NLO prediction can be said to describe the widths of the gluon and quark jet multiplicity distributions at \( Z^0 \) energy to within 10% accuracy.

Unfortunately, the 2NLO and 3NLO terms worsen the agreement with data compared to NLO (but not compared to LO) results \[48\]. The same is true for higher order moments. It raises the general
problem of the convergence of the perturbative expansion in view of the large expansion parameter \( q \gamma \) mentioned above. The attempts to account for conservation laws more accurately by the modified evolution equations for high moments \([25]\) have not led to the success yet. It is remarkable that the computer solution of the QCD equations \([21]\) provides a near-perfect description of the higher moments as well. This suggests that the failure of the analytic approach at higher orders is mainly a technical issue related to a treatment of soft gluons.

### 3.9 Subjet multiplicities

A single quark-antiquark pair is initially created in \( e^+e^- \)-annihilation. With very low angular resolution one observes two jets. A three-jet structure can be observed when a gluon with large transverse momentum is emitted by the quark or antiquark. However such a process is suppressed by an additional factor \( \alpha_S \), which is small for large transferred momenta. It can be calculated perturbatively. At relatively low transferred momenta, the jet evolves to angular ordered subjets (“jets inside jets...”). Different algorithms have been proposed to resolve subjets. By increasing the resolution, more and more subjets are observed. For very high resolution, the final hadrons are resolved. The resolution criteria are chosen to provide infrared safe results.

In particular, one can predict the asymptotical ratio of subjet multiplicities in 3- and 2-jet events if one neglects soft gluon coherence:

\[
\frac{n_{3j}^{sj}}{n_{2j}^{sj}} = \frac{2C_F + C_A}{2C_F} = \frac{17}{8}. \tag{24}
\]

Actually, the coherence reduces this value to be below 1.5 in experiment for all acceptable resolution parameters. Theoretical predictions \([15]\) agree only qualitatively with experimental findings \([49, 50]\). Subjet multiplicities have also been studied in separated quark and gluon jets. The analytical results \([51]\) are seen (Fig. 10 from \([52]\)) to represent the data fairly well for large values of the subjet resolution scale \( y_0 \).

### 3.10 Jet universality

According to QCD, jets produced in processes initiated by different colliding particles should be universal and depend only on their own parent (gluon, light or heavy quark). This prediction has been confirmed by many experiments. In this talk, e.g., it is mentioned in subsections 3.2, 3.6, 3.7 and demonstrated in Fig. 6.

### 4 Conclusions and outlook

A list of successful analytical QCD predictions can be made longer. Quantum chromodynamics has already predicted spectacular qualitative features of soft processes. Quantitatively, analytical results show that higher order (NLO) terms always tend to improve the agreement with experiment compared to asymptotical (LO) predictions. The accuracy achieved is often better than 20% or even 10% that is surprising by itself considering rather large expansion parameter of the perturbative approach. Moreover, some characteristics are very sensitive to ever higher order terms and should be carefully studied. The astonishing success of the computer solutions and Monte Carlo schemes demonstrates the importance of the treatment of the boundary between the perturbative and non-perturbative regions which is approximately taken into account in the analytical approach via the cut-off parameter \( Q_0 \) and the limits of integration over the parton splitting variables.

A new era of multiparticle production studies opens with the advent of RHIC, LHC, TESLA. We are coming closer to the asymptotic region even though the predicted dependences are very slow.
Nevertheless, some results differ for various analytical approaches and Monte Carlo schemes at these energies. It will allow to distinguish between them.

The mean multiplicities will increase drastically. Now, in Au-Au collisions at 130 GeV per nucleon at RHIC the mean charged multiplicity exceeds 4000. It implies that the event-by-event analysis of various patterns formed by particles in the available phase space becomes meaningful. The results can be compared to exclusive probabilistic Monte Carlo schemes. The qualitative QCD predictions indicate the tendencies towards the asymptotical region. It would allow to analyze small color-suppressed effects, properties of minijets or clusters (with attraction-repulsion transition), other collective effects like elliptic flow (and even higher Fourier expansion terms), ring-like events (the probable signature of ”Cherenkov gluons”) etc. The event-by-event analysis of experimental exclusive data can be quantified locally if one uses wavelets (sometimes called ”mathematical microscope”) for pattern recognition in individual events [53]. We hope to confront QCD predictions with new findings as well as to separate new physics signals from conventional QCD background.

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Fig. 1. The mean charged particle multiplicity of $e^+e^-$ annihilation versus energy. Different fits are described in the Figure.

Fig. 2. Measured ratio of cumulant to factorial moments $H_q$ reveals oscillations as a function of the rank $q$, for the charged particle multiplicity distribution in $e^+e^-$ hadronic $Z^0$ decays.

Fig. 3. The multiplicity ratio of unbiased gluon and quark jets in comparison to QCD analytic predictions and HERWIG Monte Carlo.

Fig. 4. The inclusive rapidity distribution in $e^+e^-$ annihilation has a hump-backed plateau shape.
Fig. 5. The peak in $\xi$ is described by the distorted Gaussian fit to all energies using moments within NLO accuracy.

Fig. 6. Peak position of the inclusive distribution plotted against the di-jet scale in comparison with NLO prediction (central line). LO prediction is shown by lower line, and expectation from cascade without coherence by upper line.

Fig. 7. The difference between the mean charged multiplicities of bottom- and light-quark events. The perturbative QCD expectations and predictions based on energy rescaling are shown.

Fig. 8. Charged particle flow in 3-jet events in comparison to analytic prediction.
Fig. 9. The normalized factorial moments of different ranks as functions of the scaling variable. Different analytic approximations are compared to the data and show general trends but not a satisfactory agreement with data.

Fig. 10. The subjet multiplicities of separated gluon (a) and quark (b) jets in comparison to analytic results and to Monte Carlo predictions.