Casimir forces: An exact approach for periodically deformed objects

T. Emig
Institut für Theoretische Physik, Universität zu Köln - 50923 Köln, Germany, and
Physics Department, Massachusetts Institute of Technology
Cambridge, MA 02139, USA

(received 7 November 2002; accepted in final form 14 March 2003)

PACS. 03.70.+k – Theory of quantized fields.
PACS. 12.20.-m – Quantum electrodynamics.
PACS. 42.50.Ct – Quantum description of interaction of light and matter; related experiments.

Abstract. – A novel approach for calculating Casimir forces between periodically deformed objects is developed. This approach allows, for the first time, a rigorous non-perturbative treatment of the Casimir effect for disconnected objects beyond Casimir’s original two-plate configuration. The approach takes into account the collective nature of fluctuation-induced forces, going beyond the commonly used proximity force theorem (PFT) which is a valid approximation for objects with a radius of surface curvature much larger than their distance. As an application of the method, we exactly calculate the Casimir force due to scalar field fluctuations between a flat and a rectangular corrugated plate. Maximal distinction from the PFT towards a much stronger force is found for small corrugation lengths.

Casimir forces between electrically neutral solids are a macroscopic consequence of material-dependent changes in the zero-point vacuum fluctuations of the electromagnetic field [1–3]. The Casimir effect for two parallel metallic plates has been referred to as one of the least intuitive consequences of quantum electrodynamics [4]. However, this effect can be as well regarded as a macroscopic manifestation of many-body van der Waals forces between the particles that form the plates. This point of view is supported by Schwinger’s equivalent approach which considers no vacuum fluctuations but only the fields generated by the fluctuating dipoles themselves [4,5]. In addition, fluctuation-induced forces can be observed in a plethora of other systems where quantum, thermal or disorder fluctuations are modified by external objects [6]. Examples of recent interest are as diverse as the thickening of helium films near the superfluid transition [7,8], forces between vortex matter in anisotropic superconductors [9], Casimir energy densities in cosmological models [10], and interactions between proteins on biological membranes [7].

Initiated by the first quantitative verification of the electrodynamic Casimir effect by Lamboreaux [11], high-precision experiments have motivated a resurgence in the field of Casimir force measurements in the last few years [12]. All experiments confirm the theory for Casimir’s original flat-plate geometry within a few percent of accuracy. Much less clear is the situation for non-trivial geometries. If the separation between the objects is large compared to characteristic wavelengths of the material, e.g., the plasma wavelength, the force becomes a universal function of the geometry with the energy scale set by $\hbar$. However, the entropic origin and collective nature of the Casimir force can cause this universal function to have a non-trivial and unexpected dependence on the shape of the interacting objects. There is little intuition
even as to whether the interaction is attractive or repulsive as demonstrated strikingly by the positivity of the Casimir energy of a conducting spherical shell [13]. In particular, the latter result raises the important question if repulsive forces can also emerge between disconnected objects, being of potential importance for the behavior of microelectromechanical systems due to the short-scale separations between their mobile parts [14]. Recently, in searching for non-trivial shape dependences, Mohideen et al. measured the force between corrugated surfaces with a corrugation length being larger than their distance [15,16] and obtained agreement [17] with a theory based on a pairwise summation of van der Waals forces. However, the pairwise summation is expected to break down at small corrugation lengths [18], motivating the development of our new approach.

Calculations of Casimir forces beyond the simple situation of two parallel flat plates are notoriously difficult due to the collective nature of the many-body interaction. To date, this simple geometry indeed appears to be the only case amongst disconnected macroscopic objects for which an exact result is available. This is due to the lack of rigorous, non-perturbative methods for calculating the force between deformed objects. The simplest and commonly used approximation is the proximity force theorem (PFT) which yields the force between deformed plates as the average of the flat-plate force over the local distance between the plates [19]. A conceptual different approximation is the pairwise summation of renormalized retarded van der Waals forces [3]. For ideal metal plates of the geometry studied here both approximations yield the same result [20]. However, Lifshitz’s theory for dielectric bodies (with flat surfaces) demonstrates that in general the interaction cannot be obtained from a pairwise summation [21]. The pairwise summation can even give a sign for the Casimir interaction which contradicts, e.g., for a conducting cubic shell, results from the zeta-function method [22] which itself is unreliable due to the artificial neglect of exterior fluctuations, and of volume and edge divergences. Thus it is fair to conclude that these approaches are to some extent uncontrolled. For corrugated metal plates, the failure of the PFT at a small corrugation length $\lambda$ has been shown by perturbation theory with respect to the deformation amplitude $a$ [18]. Another perturbative approach, based on a multiple-scattering expansion, has been applied only in the large separation limit [23]. However, perturbative approaches break down for $\lambda/a \lesssim 1$, a limit where maximal distinction from the PFT is expected [18]. Thus, in order to design experimental systems which will be able to probe the distinction of Casimir interactions from the PFT predictions and the pairwise additive interactions, it is highly desirable to gather information about the perturbatively non-accessible regime.

In this letter we will introduce an avenue along which the shortcomings of the commonly used approximations can be bypassed for objects with uniaxial shape modulations. To illustrate our new approach, we consider the force between a rectangular corrugated and a flat plate, a geometry which cannot be treated by perturbation theory due to the edges in the surface profile. Before coming to this specific case, let us outline the general approach which is based on path-integral quantization. Although one can apply this formalism to the electromagnetic vector field (either by gauge field quantization [24] or by separation into scalar field modes [18]), we focus on the most commonly used scalar electrodynamics model [25] in favor of a concise introduction to the new approach. The fluctuations of the scalar field are governed by the Euclidean action $S[\Phi] = \frac{1}{2} \int d^D X (\nabla \Phi)^2$ after a Wick rotation to imaginary time. While this model describes for $D = 3$ also a superfluid with $\Phi$ the phase of the complex order parameter [6], electromagnetic vector field fluctuations ($D = 4$) in uniaxial geometries can be reduced to two decoupled scalar field systems, corresponding to transversal magnetic (TM) and electric (TE) modes, in analogy to normal modes in waveguides [18,26]. Both types of modes are described by the same action $S[\Phi]$ but are subject to different boundary conditions. Our scalar electrodynamics model is equivalent to TM waves which obey Dirichlet boundary
conditions, $\Phi|_\Omega = 0$, on perfectly conducting surfaces $\Omega$. We have checked that TE waves can be treated with our approach as well by imposing Neumann boundary conditions [27]. However, the computations become then more involved due to a spatially varying orientation in the boundary conditions whose treatment is beyond the scope of this letter. Instead, we will give a brief summary of our findings for TE modes at the end of this article. The considered geometry consists of a flat surface in the $xy$-plane at $z = H$ and a uniaxially corrugated surface given by $z = h(x)$ with periodic height profile $h(x + \lambda) = h(x)$ of wavelength $\lambda$. At zero temperature, the Casimir force per surface area $F = -\partial E/\partial H$ between the two objects corresponds to the change with the objects distance in the ground-state energy density $E$ of the field $\Phi$. Employing the path integral method to implement the boundary conditions at the surfaces [24, 28], the energy density is given by $E = -\hbar c \ln Z/AL$ with $L$ the Euclidean system size along imaginary time and

$$Z = Z_0^{-1} \int \mathcal{D}\Phi \prod_{\alpha=1}^2 \prod_{\chi} \delta[\Phi(X_{\chi})] e^{-S[\Phi]/\hbar},$$

with the partition function $Z_0$ of empty space with no objects, and $X_{\chi}$ ($\chi = 1, 2$) the positions on the two surfaces $\Omega_{\chi}$ in 4D Euclidean space. The product of delta-functions can be rewritten as a functional integral over an auxiliary field $\psi_\chi$ on each surface, $\prod_{\chi} \delta[\Phi(X_{\chi})] = \int \mathcal{D}\psi_\chi \exp[i \int_\Omega d\chi \psi_\chi \Phi]$. Next, the Gaussian integration over $\Phi$ is performed, resulting in $Z = \int \mathcal{D}\psi_\chi \exp[-S_{\text{eff}}]$ with the effective action $S_{\text{eff}} = \frac{1}{2} \int_{r} \int_{r'} \sum_{\alpha,\beta} \psi_\alpha(r) M_{\alpha\beta}(r, r') \psi_\beta(r')$ and $r = (ict, x, y) \equiv (X^0, X^1, X^2)$. With the surface parameterization $X_1(r) = [r, h(x)]$, $X_2(r) = [r, H]$, the matrix kernel of $S_{\text{eff}}$ reads

$$M(r, r') = \begin{pmatrix} G[r - r', h(x) - h(x')] & G[r - r', H - h(x')] \\ G[r - r', H - h(x')] & G[r - r', 0] \end{pmatrix},$$

where $G(r, z) = (r^2 + z^2)^{-1}/4\pi^2$ is the Green’s function of the 4D Laplacian operator [29]. The Casimir force per surface area $A$ between the objects is then given by

$$F = -\frac{\hbar c}{2AL} \partial H \ln \det M = -\frac{\hbar c}{2AL} \text{Tr} \left( M^{-1} \partial H M \right).$$

This expression is suited for a non-perturbative approach since the force must be finite, in contrast to the energy density $E$ which contains, without an explicit cutoff, infinite but $H$-independent contributions from the individual surfaces.

In the more conventional approaches, the zero-point energy is calculated in terms of the allowed frequencies $\omega_n$ of the space between the objects, using $E = \frac{1}{2} \sum_n \hbar \omega_n$ and by applying substraction schemes in order to obtain a regularized energy which in turn yields the force. An advantage of our approach is that the frequencies need not be calculated explicitly but the force is directly obtained in terms of the free-space Green’s function without any necessity for regularization schemes. Of course, the functional inverse of the matrix kernel $M(r, r')$ in eq. (3) is in general difficult to evaluate. In the following, we will demonstrate how this problem can be tackled for uniaxial geometries. Due to the symmetry of the geometry, it is useful to consider the matrix kernel $M(p, q)$ in momentum space. The periodicity of the surface profile along the $X^1$-direction and uniformity in the $X^0X^2$-plane impose the general form

$$M(p, q) = \sum_{m=-\infty}^{\infty} N_m(p_\perp, p_1) (2\pi)^3 \delta(p_\perp + q_\perp) \delta(p_1 + q_1 + 2\pi m/\lambda)$$

on the kernel. This Fourier decomposition defines the matrices $N_m(p_\perp, p_1)$ with $p_\perp = (p_0, p_2)$ and $p_\perp = |p_\perp|$. Thus the matrix $M(p, q)$ has non-vanishing entries only along the diagonal
and its periodically shifted analogues. An important observation is that matrices of this structure can be transformed to block-diagonal form by applying row and column permutations. Physically, the existence of such a transformation is due to the non-mixing property of the kernel $M(p, q)$ for Fourier modes whose moments differ by non-integer multiples of $2\pi/\lambda$. For the time being, let us assume that the objects have the extension $W$ along the $X^1$-direction, leading to a discrete set of momenta $p_1$. Then the transformed matrix $M(p, q)$ consists of (infinite dimensional) block matrices $M_j$ along the diagonal with $j = 1, \ldots, N \equiv W/\lambda - 1$. Since this matrix form can always be realized by an even number of permutations, we get

$$\det M = \prod_{j=1}^N \det M_j.$$  

For discrete momenta $p_1$, the matrices $M_j$ can be parametrized by integer indices $k$, $l = -\infty, \ldots, \infty$. The elements $M_{j, kl}$ can be related to the matrices $N_m$ in eq. (4) by looking at the permutations which transform $M(p, q)$ to block-diagonal form. A convenient method is to arrange the matrices $N_m$ in the plane spanned by the discrete set of momenta $p_1$ and $q_1$ so that the structure of eq. (4) is reflected by non-zero entries on bands which are parallel to the line $p_1 + q_1 = 0$. Next we superimpose the $p_1 q_1$-plane with a square grid of lattice constant $2\pi/\lambda$ so that each unit cell contains $N^2$ discrete momenta. The $N_m$ which are localized on the grid are shifted by the permutations into one particular block $M_j$. By moving the grid successively by one site along the line $p_1 + q_1 = 0$, one obtains $N$ non-equivalent superpositions of the grid and the $p_1 q_1$-plane, leading to $N$ different block matrices $M_j$ which are then given by

$$M_{j, kl}(p_\perp, q_\perp) = (2\pi)^2 \delta(p_\perp + q_\perp) B_{kl}(p_\perp, 2\pi j/W)$$

with the matrix $B_{kl}$ given by

$$B_{kl}(p_\perp, p_1) = N_{k-l}(p_\perp, p_1 + 2\pi l/\lambda).$$

Moreover, from eq. (5) we obtain easily $\partial_H (\ln \det M) = \sum_{j=1}^N \text{Tr}(M_j^{-1} \partial_H M_j)$. Thus the separation-dependent part of the ground-state energy $E$ is given by the sum of the individual energies of the decoupled subsystems which are described by the matrices $M_j$. Next, it is useful to define the function

$$g(p_\perp, p_1) = \text{tr} \left( B^{-1}(p_\perp, p_1) \partial_H B(p_\perp, p_1) \right),$$

where the lower-case symbol $\text{tr}$ denotes the partial trace with respect to the discrete indices $k$, $l$, cf. eq. (7), at fixed $p_\perp, p_1$. It follows from the structure of the matrix $B_{kl}$ that $g(p_\perp, p_1 + 2\pi n/\lambda) = g(p_\perp, p_1)$. In addition, choosing the height profile to be symmetric, $h(-x) = h(x)$, imposes the condition $N_m(p_\perp, -p_1) = N_{-m}(p_\perp, p_1)$ on the matrices $N_m$ which in turn leads after row and column permutations of the matrix $B_{kl}$ to $g(p_\perp, -p_1) = g(p_\perp, p_1)$. Taking at this point the thermodynamic limit $W, N \to \infty$ transforms the sum over the energies of the $N$ subsystems into an integral. Thus we obtain, after taking the total trace over all degrees of freedom, the final result for the Casimir force density,

$$F = -\frac{\hbar c}{4\pi^2} \int_0^\infty dp_\perp p_\perp \int_0^{\pi/\lambda} dp_1 g(p_\perp, p_1),$$

where we have used the symmetries of $g(p_\perp, p_1)$. For a given Fourier decomposition of the kernel $M$ into the matrices $N_m$, this formula together with eqs. (7), (8) yields the exact result for the force between the objects. This result has the advantage of being particularly suited for a numerical analysis as will be shown below.

In the following, we apply the general formula in eq. (9) to obtain a non-perturbative result for the force between a flat and a rectangular corrugated surface. We choose the height profile
on a basic interval as \( h(x) = a \) for \( |x| < \lambda/4 \) and \( h(x) = -a \) for \( \lambda/4 < |x| < \lambda/2 \) so that \( H \) is the mean distance between the surfaces, see fig. 1.

The matrices \( N_m(p_\perp,p_1) \) can be calculated by Fourier transformation of \( M(r,r') \) in eq. (2). Using the fact that \( h(x) \) is piecewise constant and can be written as the series
\[
h(x) = 4a/\pi \sum_{m=1}^{\infty} \frac{(-1)^{m-1}}{2m-1} \times \cos((2m-1)2\pi x/\lambda),
\]
we obtain, after some manipulations,
\[
N_0 = \begin{pmatrix}
\frac{1}{2p} + e^{-2ap} - 1 + \Phi_0(p) & e^{-pH} \cosh(ap) \\
\frac{e^{-pH}}{2p} \cosh(ap) & \frac{1}{2p}
\end{pmatrix},
\]
\[
N_m = c_m \begin{pmatrix}
0 & e^{-pH} \sinh(ap) \\
e^{-p_m H}/2p_m \sinh(a\tilde{p}_m) & 0
\end{pmatrix},
\tag{10}
\]
for odd \( m \neq 0 \), and
\[
N_m = \begin{pmatrix}
\Phi_m(p) & 0 \\
0 & 0
\end{pmatrix},
\tag{11}
\]
for even \( m \neq 0 \) with \( \tilde{p}_m = \sqrt{p_\perp^2 + (p_1 + 2\pi m/\lambda)^2} \), \( c_m = 2(-1)^{(\lfloor m \rfloor - 1)/2}/\pi |m| \) and
\[
\Phi_m(p) = \frac{(-1)^{m/2}}{\pi^2} \sum_{n=-\infty}^{\infty} \frac{f[p_\perp,p_1 + (2n-1)2\pi/\lambda]}{(2n-1)(m-2n+1)},
\]
where \( f(p_\perp,p_1) = (\exp[-2ap] - 1)/p \). In order to calculate the function \( g(p_\perp,p_1) \), we truncate the matrix \( B_{kl} \) symmetrically around \((k,l) = (0,0)\) at order \( M \) so that \( k, l = -(M-1)/2, \ldots, (M-1)/2 \). This defines via eq. (8) a series of approximations \( g_M(p_\perp,p_1) \) which converges to \( g(p_\perp,p_1) \) for \( M \to \infty \).

Before we use the general form of the decomposition, let us consider two instructive limiting cases. In the limit \( \lambda/a \to 0 \), one would expect that \( g_M(p_\perp,p_1) \) converges very rapidly since the contributions from matrices \( N_m \) decrease with \( m \), cf. eq. (4). To test this guess, we make use of the fact that the matrices \( N_m(p_\perp,p_1 + 2\pi l/\lambda) \) simplify considerably for \( \lambda/a \to 0 \), so that the series \( g_M \) can be explicitly obtained order by order from the truncated matrix \( B_{kl} \).

Using eq. (8), we get

\[
g_M(p_\perp,p_1) = \begin{cases}
-\frac{2p(1 + e^{-2ap})}{1 + e^{-2ap} - 2e^{2(H-a)p}}, & \text{for } M = 1, \\
-\frac{2p}{1 - e^{2p(H-a)}}, & \text{for } M \geq 3,
\end{cases}
\tag{12}
\]
for positive \( a \). This result indeed shows a rather rapid convergence; the series \( g_M \) is even invariant for \( M \geq 3 \) with increasing dimension of the matrix \( B_{kl} \). Using eq. (9), we get from eq. (12) the force density for \( a > 0 \),

\[
F_0 = -\frac{\pi^2}{480} \frac{hc}{(H-a)^4}.
\tag{13}
\]
Physically, this result appears to be quite natural since the most contributing modes of the fluctuating field cannot probe the valleys of the corrugated surface in the limit $\lambda/a \to 0$. Thus, the surfaces experience a force which is given by the force between two flat surfaces at reduced distance $H - |a|$ corresponding for $a > 0$ to eq. (13) [18]. In the following we call this limit the reduced-distance (RD) regime. For small $a/H$, the correction $\sim \hbar c|a|/H^5$ to the flat-surface result is non-analytic in $a$, and thus cannot be obtained in perturbation theory.

Next, we consider the limit $\lambda/H \to \infty$. In the limit of small surface curvature, the PFT is justified [19]. For our geometry, this yields

$$F_{\infty} = -\frac{\pi^2}{480} \frac{\hbar c}{2} \left( \frac{1}{(H + a)^4} + \frac{1}{(H - a)^4} \right)$$

since 50% of the local surface’s distances are $H + a$, $H - a$, each.

In order to see how these two limits fit into the complete theory as described by eq. (9), one has to resort to a numerical analysis. The recipe for this analysis is straightforward: At fixed order $M$, the truncated matrix $B_{kl}$ is calculated from eqs. (11), then its inverse is constructed to obtain the $M$-th order approximation $g_M$ to the function $g$. Finally, numerical integration in eq. (9) yields a corresponding series of force densities which can be extrapolated to the final result for $M \to \infty$. The results are summarized in fig. 2. For fixed $\lambda/a$, the relative increase $\delta = F/F_{\text{flat}} - 1$ of the force compared to the force between two flat surfaces is shown in fig. 2a. At small $\lambda/a$, the exact result $F_0$ of eq. (13) is recovered. For larger $\lambda/a$, there is a crossover between two scaling regimes: For $H \gg \lambda$ the RD scaling $\delta \sim H^{-1}$ remains valid with an amplitude which decreases with increasing $\lambda/a$. For $H \ll \lambda$ the PFT regime is entered with $\delta \sim H^{-2}$. As a consequence, the force is always attractive with $F_0$ as upper and $F_{\infty}$ as lower bound. Figure 2b shows the crossover between the PFT and the RD regimes for fixed separations $H/a$. Independent of $H/a$, the crossover appears at $\lambda/a \approx 10$. The two limits are approached as $(F_0 - F)/F_{\text{flat}} \sim \lambda/a$ for $\lambda \to 0$ and $(F - F_{\infty})/F_{\text{flat}} \sim a/\lambda$ for $\lambda \to \infty$, cf. the inset of fig. 2b.

Finally, we note that we have applied the same approach to Neumann boundary conditions, yielding the force $F_{\text{TE}}$ due to TE modes [27] and thus the full electrodynamic Casimir force as $F_{\text{TM}} + F_{\text{TE}}$. We find the same qualitative behavior as for TM modes with the two scaling regimes of fig. 2a and $F_{\infty} \leq F_{\text{TE}} \leq F_0$, where the lower and upper bound is approached for $\lambda/a \to \infty$ and $\lambda/a \to 0$, respectively. A good measure for the quantitative difference between

![Image](image-url)
the two modes is the ratio $\kappa = F_{TM}/F_{TE}$ of the corresponding forces which is always close to one. Since the bounds on both forces are equal, we find the limit $\kappa \to 1$ both for $\lambda/a \to 0$ and $\lambda/a \to \infty$. At intermediate $\lambda/a$ we obtain $\kappa > 1$ for $\lambda/a \lesssim 5$, $\kappa < 1$ for $\lambda/a \gtrsim 10$, while for $5 \lesssim \lambda/a \lesssim 10$ both situations are possible, depending on $H/a$. Generally, for $H/a \to 1$ and $H/a \to \infty$ we observe again $\kappa \to 1$ which is consistent with the flat plate result $\kappa = 1$.

***

We would like to thank R. Golestanian, A. Hanke, M. Kardar and B. Rosenow for useful discussions. This work was supported by the Deutsche Forschungsgemeinschaft through the Emmy-Noether grant No. EM70/2-1 and by the NSF grant No. DMR-01-18213 at MIT.

REFERENCES

[1] Casimir H. B. G., Proc. K. Ned. Akad. Wet., 51 (1948) 793.
[2] Milonni P. W., The Quantum Vacuum (Academic, San Diego) 1994; Mostepanenko V. M. and Trunov N. N., The Casimir Effect and its Applications (Clarendon, Oxford) 1997.
[3] Bordag M., Mohideen U. and Mostepanenko V. M., Phys. Rep., 353 (2001) 1.
[4] Schwinger J., DeRaad L. L. Jr. and Milton K. A., Ann. Phys., 115 (1978) 1.
[5] Milton K. A., The Casimir Effect: Physical Manifestations of Zero-Point Energy (World Scientific) 2001.
[6] Kardar M. and Golestanian R., Rev. Mod. Phys., 71 (1999) 1233.
[7] Israelachvili J., Intermolecular and Surface Forces (Academic Press, San Diego) 1992.
[8] Garcia R. and Chan M. H. W., Phys. Rev. Lett., 88 (2002) 086101.
[9] B¨uchler H. P., Katzgraber H. G. and Blatter G., Physica C, 332 (2000) 402; Mukherji S. and Nattermann T., Phys. Rev. Lett., 79 (1997) 139.
[10] Bytsenko A. A., Cognola G., Vanzo L. and Zerbini S., Phys. Rep., 266 (1996) 1.
[11] Lamoreaux S. K., Phys. Rev. Lett., 78 (1997) 5.
[12] Mohideen U. and Roy A., Phys. Rev. Lett., 81 (1998) 4549; Chan H. B. et al., Science, 291 (2001) 1941; Phys. Rev. Lett., 87 (2001) 211801; Bressi G. et al., Phys. Rev. Lett., 88 (2002) 041804.
[13] Boyer T., Phys. Rev. A, 9 (1974) 2078.
[14] Serry F. M., Walliser D. and Maclay G. J., J. Microelectromech. Syst., 4 (1995) 193.
[15] Roy A. and Mohideen U., Phys. Rev. Lett., 82 (1999) 4380.
[16] Chen F., Mohideen U., Klimchitskaya G. L. and Mostepanenko V. M., Phys. Rev. Lett., 88 (2002) 101801.
[17] Klimchitskaya G. L., Zanette S. I. and Caride O. A., Phys. Rev. A, 63 (2001) 014101.
[18] Emig T., Hanke A., Golestanian R. and Kardar M., Phys. Rev. Lett., 87 (2001) 260402.
[19] Derjaguin B., Kolloid Z., 69 (1934) 155.
[20] Emig T., Hanke A., Golestanian R. and Kardar M., Phys. Rev. A, 67 (2003) 022114.
[21] Lifshitz E. M., Sov. Phys. JETP, 2 (1956) 73.
[22] Barton G., J. Phys. A, 34 (2001) 4083.
[23] Balian R. and Duplantier B., Ann. Phys. (N.Y.), 112 (1978) 165.
[24] Golestanian R. and Kardar M., Phys. Rev. Lett., 78 (1997) 3421; Phys. Rev. A, 58 (1998) 1713.
[25] Moore G. T., J. Math. Phys., 11 (1970) 2679; Dodonov V. V., Phys. Lett. A, 207 (1995) 126.
[26] The TE/TM modes are here defined relative to the symmetry ($y$) axis, cf. [18].
[27] Emig T., in preparation.
[28] Li H. and Kardar M., Phys. Rev. Lett., 67 (1991) 3275; Phys. Rev. A, 46 (1992) 6490.
[29] Here we have ignored metric functions which in general multiply $M$ but drop out in the force; we are indebted to Hanke A. for calling attention to this fact in the context of correlation functions, cf. Hanke A. and Kardar M., Phys. Rev. E, 65 (2002) 046121.