Mathematical formulation

The vertical impermeable wall, the associated system of reference as well as the thermal and the concentration conditions are presented by figure 1: the x-y system of co-ordinates, the wall constant

1. Introduction
The double diffusion convection process received a great attention in the scientific community due to its applications. The research papers where this type of convection is triggered by a vertical impermeable wall embedded in a fluid environment saturated with a certain constituent [1-3] can be encountered in great number in the last decades.

The scale analysis combined with the finite differences method for solving the conservation equations are encountered [1-9] as methods for revealing the mass and/or heat driven convection process in the boundary layers that occur near the vertical wall.

This paper considers the case of a vertical impermeable wall embedded in a fluid saturated with a certain constituent when \( Pr > 1 \) and \( Le > 1 \). The temperature of the wall and the mass flux of the constituent at the wall are constant. At infinity, the temperature of the fluid is constant, while the concentration of the constituent varies linearly. Using the scale analysis of the governing equations, this work shows that, depending on the parameters set, a HDC or a HDC–MDC regime succession attains the equilibrium state along the wall. The finite differences method is used to solve the conservation equations for two particular parameter sets and, in this way, to illustrate the scale analysis results.

2. Mathematical formulation
The vertical impermeable wall, the associated system of reference as well as the thermal and the concentration conditions are presented by figure 1: the x-y system of co-ordinates, the wall constant

3. Results
The scale analysis results are illustrated for two parameter sets by solving the conservation equations using the finite differences method.

4. Conclusion
The results show that the natural convection near a vertical impermeable wall embedded in a fluid containing a certain constituent can lead to a HDC or a HDC–MDC regime succession depending on the parameters set. The finite differences method is used to illustrate the scale analysis results.

Acknowledgment
The author thanks the anonymous reviewers for their valuable comments and suggestions which improved the presentation of the results.

References
[1] Neagu, M. (2018) Double diffusion convection in a mass stratified fluid near a vertical impermeable wall. IOP Conf. Series: Materials Science and Engineering 444, 082020.
[2] Neagu, M. (2017) Double diffusion convection in a mass stratified fluid near a vertical impermeable wall. IOP Conf. Series: Materials Science and Engineering 347, 012047.
[3] Neagu, M. (2016) Double diffusion convection in a mass stratified fluid near a vertical impermeable wall. IOP Conf. Series: Materials Science and Engineering 270, 012045.

Natural convection near a vertical wall of constant mass flux and temperature situated in a mass stratified fluid

M Neagu

1Manufacturing Engineering Department, "Dunărea de Jos" University, Galați, Romania

E-mail: Maria.Neagu@ugal.ro

Abstract. This paper presents the analysis of the natural convection process that occurs near a vertical impermeable wall embedded in a fluid containing a certain constituent. The temperature of the wall is constant and the mass flux of the constituent is constant at the wall. At infinity, the fluid has a constant temperature and a linearly varying concentration of the constituent. The scale analysis of the governing equations reveals the succession of the heat (HDC) and/or mass (MDC) driven convection regime that attains the equilibrium state along the wall. Two possibilities are encountered: a heat driven convection (HDC) regime if \( Ra \cdot \gamma ^2 \cdot Sc \cdot Le ^{4/3} \cdot N^{-3} \geq 1 \) and a HDC–MDC regime succession if \( Ra \cdot \gamma ^2 \cdot Sc \cdot Le ^{4/3} \cdot N^{-3} < 1 \), where Ra is the Rayleigh number, Sc is the mass dimensionless stratification parameter, N is the buoyancy ratio, Le is the Lewis number, \( \gamma = 1 - \left( 1 + Pr ^{1/2} \right) ^{1/2} \) and Pr is the Prandtl number. These results are illustrated for two parameter sets by solving the conservation equations using the finite differences method.
temperature, $T_w$, the mass flux of a certain constituent at the wall, $m_w$, the environment temperature, $T_\infty$, and the linearly variable concentration of the constituent at infinity, $C_{\infty,x} = C_{\infty,0} + s_C \cdot x$, where $s_C$ is the stratification parameter. All the properties are constant except the fluid density that obeys the Boussinesq approximation.

The mass, momentum, energy and species concentration conservation equations:

\begin{align}
\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} &= 0 \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \beta_T T + g \beta_C C \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= \frac{\partial}{\partial x} \left( D \frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2} \right) + \frac{\partial}{\partial y} \left( \frac{\partial^2 C}{\partial y^2} \right)
\end{align}

are subjected to the following boundary conditions:

\begin{align}
  u = v = 0, & \quad T = T_w, \quad \frac{\partial C}{\partial y} = \frac{m_w}{D} = \Gamma_w \text{ at } y = 0 \\
  v = 0, & \quad T = T_\infty, \quad C = C_{\infty,x} \text{ as } y \to \infty \\
  v = 0, & \quad T = T_\infty, \quad C = C_{\infty,0} \text{ at } x = 0 \\
  \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 T}{\partial x^2} = \frac{\partial^2 C}{\partial x^2} = 0 \text{ at } x = h
\end{align}

where the following notations were used: $t$ – the time, $T$ – the temperature, $C$ – the concentration, $h$ – the upper limit of the computational domain, $u$ and $v$ – the horizontal and the vertical velocities, $p$ – the pressure, $\alpha$ – the thermal diffusivity, $D$ – the mass diffusivity and $\nu$ is the kinematic viscosity.
3. Scale analysis

This section is divided in three different parts that analyse:

- the transient state, section 3.1;
- the heat driven convection (HDC) regime, section 3.2;
- the mass driven convection (MDC) regime, section 3.3.

3.1. Scale analysis of the transient state

In the beginning, the equilibrium of the inertia and the horizontal diffusion of heat characterises the equation (4): \( \frac{\partial T}{\partial t} = \alpha \cdot \frac{\partial^2 T}{\partial y^2} \) or \( \frac{\Delta T}{t} = \alpha \cdot \frac{\Delta T}{\delta_t^2} \). The temperature difference registered across the thermal boundary layer is \( \Delta T \sim 1 \), while the temperature boundary layer thickness, in the transient state, is:

\[
\delta_T \sim \alpha^{1/2} \cdot t^{1/2}.
\]

Similarly, the equilibrium between the inertia and the horizontal diffusion of the species, in the equation (5), requires:

\[
\frac{\partial C}{\partial t} = D \cdot \frac{\partial^2 C}{\partial y^2} \quad \text{or} \quad \frac{\Delta C}{t} = D \cdot \frac{\Delta C}{\delta_C^2}
\]

and, consequently, the concentration boundary layer thickness, \( \delta_C \), is:

\[
\delta_C \sim D^{1/2} \cdot t^{1/2}.
\]

As \( \Delta C \sim \Gamma_w \cdot \delta_C \), the concentration difference across the boundary layer is: \( \Delta C \sim \Gamma_w \cdot D^{1/2} \cdot t^{1/2} \).

Two aspects are of great importance in this point of the analysis: the time when the HDC–MDC transition occurs and the time when the dominance of the \( v \cdot \frac{\partial C}{\partial x} \) or \( v \cdot s_C \) shifts in equation (5):

- The HDC and MDC regimes presence depends on the relative magnitude of the \( \beta_i \Delta T \) and the \( \beta_j \Delta C \) terms across the boundary layer. As \( \Delta T \sim 1 \) and \( \Delta C \sim \Gamma_w \cdot D^{1/2} \cdot t^{1/2} \), it is clear that, in the beginning, a heat driven convection regime is installed at each \( x \) co-ordinate of the wall. The heat \( \rightarrow \) mass driven convection transition is taking place or not depending on the magnitude of the equilibrium time for each \( x \) co-ordinate.

The HDC–MDC transition takes place when \( \beta_i \Delta T > \beta_j \Delta C \) or \( \beta_j > \beta_i \Gamma_w \cdot D^{1/2} \cdot t^{1/2} \). Defining the buoyancy ratio as \( N = \beta \cdot \Gamma_w \cdot L \cdot \beta_i \cdot t^{-1} \), the transition time becomes:

\[
t_{trz} \sim L^2 \cdot (N^2 \cdot D)^{1/4}.
\]

If the equilibrium time is greater than \( t_{trz} \), then, a mass driven convection regime replaces the heat driven convection regime that was initially evolving at that \( x \) co-ordinate.

- The second aspect of interest – the time when the dominance of \( v \cdot \frac{\partial C}{\partial x} \) or \( v \cdot s_C \) shifts in the equation (5) – is established considering the expression of \( \Delta C \) and it takes the following form:

\[
t_s \sim \frac{s_C \cdot x^2}{\Gamma_w \cdot D}.
\]

If the equilibrium time of the concentration field is smaller that \( t_s \), then the \( v \cdot s_C \) term is dominant in the left hand side of equation (5). Otherwise, the \( v \cdot \frac{\partial C}{\partial x} \) term becomes dominant.
Considering the time evolution of the convection regimes at each x co-ordinate of the wall, the scale analysis of the conservation equations begins with the HDC regime analysis (section 3.2) followed by the MDC regime analysis (section 3.3). In this later case, the \( \nu \cdot s_C \) dominant influence on the system will be analyzed first, followed by the influence due to the dominance of the \( \nu \cdot \frac{\partial C}{\partial x} \) term.

### 3.2. Scale analysis of the heat driven convection (HDC) regime

For \( Pr > 1 \), \( Le \geq 1 \), the scientific literature [5] indicates an amplitude of the vertical velocity:

\[
\nu_T \approx \frac{g \beta_s \alpha}{\nu} \gamma^2
\]

where \( \gamma = 1 - \left(1 + Pr^{1/2}\right)^{-1} \), the velocity boundary layer thickness \( \delta_v = \left(Pr\right)^{1/2} \delta_T \), \( Pr = \frac{\nu}{\alpha} \). In the HDC regime, the equilibrium is reached when the heat flux diffused in the \( Y \) direction equals the heat flux convected in the \( X \) direction: \( \nu_T \frac{\partial T}{\partial x} - \alpha \frac{\partial^2 T}{\partial y^2} \) or \( \nu_T \cdot \frac{\Delta T}{\delta_T} - \alpha \frac{\Delta T}{\delta_T} \). Replacing the equations (14) and (10), the equilibrium time, the temperature boundary layer thickness and the maximum vertical velocity are:

\[
\left(t_{ech,T}\right)_T \sim \left(\frac{xv}{g \beta_s \alpha}\right)^{1/2} = \frac{1}{\alpha} \left(\frac{xL^3}{Ra}\right)^{1/2} = \frac{L^2}{\alpha} \left(\frac{X}{Ra \cdot \gamma^2}\right)^{1/2}
\]

\[
\left(\delta_{ech,T}\right)_T \sim L \cdot \left(\frac{X}{Ra \cdot \gamma^2}\right)^{1/4}
\]

\[
\nu_T \approx \frac{\alpha}{L} \left(X \cdot Ra \cdot \gamma^2\right)^{1/2} \quad \text{or} \quad V_T \sim \left(X \cdot Ra \cdot \gamma^2\right)^{1/2}
\]

where \( X = \frac{x}{L} \), \( Ra = \frac{g \beta_s L^3}{\alpha \nu} \), \( V = \frac{vL}{\alpha} \).

**Scale analysis of the concentration field in the HDC regime.** For the \( Pr > 1 \), \( Le \geq 1 \) case, the order of magnitude of the velocity in the concentration field \( \left(v_C\right)_T = \nu_T \frac{\partial C}{\partial x} \). Depending on the relative magnitude of the \( \frac{\partial C}{\partial x} \) or the \( s_C \) term, two situations are encountered:

a) If \( \nu \cdot \frac{\partial C}{\partial x} \) is greater than \( \nu \cdot s_C \), in the equation (5), then the scale analysis requires:

\[
\nu_T \frac{\partial C}{\partial x} \sim D \frac{\partial^2 C}{\partial y^2}.
\]

Using the equations (11), (16) and (17), the equilibrium time of the concentration field is:

\[
\left(t_{ech,C}\right)_T \sim L^2 \left(\frac{X}{Ra \cdot \gamma^2}\right)^{1/2} \quad \text{Le}^{1/3}
\]

while the concentration boundary layer thickness is
\[
(\delta_{ech,C})_T \sim L \left( \frac{X}{Ra \cdot \gamma^2} \right)^{1/4} \frac{I}{Le^{1/3}} = \frac{(\delta_{ech,T})_T}{Le^{1/3}}.
\]

(20)

The equilibrium time, \((t_{ech,C})_T\), is bigger than the transition time, \(t_{trz}\), if:

\[
X > X_{trz,C} = Ra \cdot \gamma^2 \cdot Le^{4/3} \cdot N^{-4}.
\]

(21)

b) If the \(v \cdot s_C\) term is greater than the \(v \cdot \frac{\partial C}{\partial x}\) term, then the scale analysis reveals that:

\[
v_T \frac{\delta_{C}}{\delta_T} \cdot s_C \sim D \left( \frac{\partial^2 C}{\partial y^2} \right).
\]

(22)

Using the equations (11), (16) and (17), the equilibrium time becomes:

\[
(t_{ech,Sc})_T \sim \frac{L^2}{\alpha} \frac{I}{S_C} \left( X \cdot Ra \cdot \gamma^6 \right)^{-1/4}
\]

(23)

while the concentration boundary layer thickness is:

\[
(\delta_{ech,Sc})_T \sim D^{1/2} \frac{1}{\alpha} \left( X \cdot Ra \cdot \gamma^6 \right)^{-1/8} (Le \cdot S_C)^{-1/2}.
\]

(24)

In the equations (23) and (24), the Lewis number \(Le = \frac{\alpha}{D}\) and the dimensionless stratification parameter \(S_C = \frac{S_C}{\Gamma_w}\). Further, the equilibrium time \((t_{ech,Sc})_T\) will be compared to \(t_{trz}\) and \(t_T\):

b1) the equilibrium time \((t_{ech,Sc})_T\) is smaller than the transition time, \(t_{trz}\), if:

\[
X > N^8 \left( Ra^3 \cdot \gamma^6 \cdot S_C^4 \cdot Le^4 \right)^{1/4} = X_{trz,Sc};
\]

(25)

b2) the possibility to have \((t_{ech,Sc})_T < t_T\) is restricted to the domain defined bellow:

\[
X > \left( Ra \cdot \gamma^2 \cdot Le^{4/3} \cdot S_C^4 \right)^{1/4} = X_{S,T}.
\]

(26)

Figure 2. The heat and mass driven natural convection regimes sequence. (a) \(Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} \geq 1\); (b) \(Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} < 1\).
Two distinct situations appear:

1. If the equation (27) is true, then \( X_{trc,sc} < X_{s,t} < X_{trc,c} \) and a HDC regime attains the equilibrium state along the wall (figure 2 (a)).

\[
Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} \geq 1
\]  
(27)

2. On the contrary, if \( Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} < 1 \), then: \( X_{trc,c} < X_{s,c} < X_{trc,sc} \); a HDC_c region is encountered on the \([0, X_{trc,c}] \) domain, followed by a MDC_c-MDC_{sc} regime on the \([X_{trc,c}, X_{trc,sc}] \) domain. Theoretically, the HDC_{sc} regime regains the \([X_{trc,sc}, \infty] \) domain, but, the results presented at the end of section 3.3 show that the concentration field continue to be mass driven beyond \( X_{trc,sc} \) (see figure 2 (b)).

### 3.3. Scale analysis of the mass driven convection (MDC) regime

Using an analysis similar to the HDC regime, the vertical velocity scale in the MDC regime is:

\[
v^c \sim g\beta_c \Gamma_v D^{3/2} t^{1/2} v^{-1/2}
\]  
(28)

where \( \lambda = 1 - (1 + Sch^{1/2})^{-1} \); here, Sch is the Smith number, \( Sch = v \cdot D^{-1} \). The velocity boundary layer thickness is \( \delta_v = (Sch)^{1/2} \delta_c \). The MDC_{sc} regime (the \( v \cdot s_c \) term is dominant) and MDC_{c} regime (the \( v \cdot \partial C / \partial x \) term is dominant) will be treated separately.

#### 3.3.1. MDC_{sc} regime

The equilibrium between the horizontal diffusion and the vertical convection of the mass requires: \( v_c \cdot s_c \sim D \left( \frac{\partial ^2 C}{\partial y^2} \right) \) or \( v_c \cdot s_c \sim D \left( \frac{\Gamma_v}{D^{1/2} t^{1/2}} \right) \). Replacing \( v_c \) from equation (28), the equilibrium time and the boundary layer thickness are:

\[
\left( t_{ech,sc} \right) \sim \frac{L^2}{\alpha} \left( \frac{Le}{N \cdot Ra \cdot \lambda^2 \cdot S_C} \right)^{1/2}
\]  
(29)

\[
\left( \delta_{ech,sc} \right) \sim \frac{L (Ra \cdot \lambda^2 \cdot N \cdot S_C \cdot Le)}{t^{1/4}}
\]  
(30)

At equilibrium, the vertical velocity is:

\[
v_c \sim \frac{D \cdot L \left( Ra \cdot \lambda^2 \cdot N \cdot Le \cdot S_C^{-3} \right)^{1/4}}{t^{1/4}} \quad \text{or} \quad v_c \sim \left( Ra \cdot \lambda^2 \cdot N \cdot S_C^{-3} \cdot Le^{-3} \right)^{1/4}
\]  
(31)

The inequality \( \left( t_{ech,sc} \right) \leq t_s \) or

\[
X = x \cdot L^{-1} > \left( Ra \cdot \lambda^2 \cdot N \cdot S_C^{-5} \cdot Le \right)^{1/4} = X_{s,c}
\]  
(32)

defines the X co-ordinate that separates the MDC_{c} and the MDC_{sc} regimes in the figure 2(b).

#### Scale analysis of the temperature field in the MDC_{sc} regime

The scale analysis requires: \( v_c \cdot \frac{\partial T}{\partial x} \sim \alpha \cdot \frac{\partial ^2 T}{\partial y^2} \) or \( v_c \cdot \frac{\Delta T}{\Delta x} \sim \alpha \cdot \frac{\Delta T}{\Delta y} \). Using the equation (10) and the equation (31), the equilibrium time is \( \left( t_{ech,T} \right) \sim X \cdot v_c^{-1} \), while the temperature boundary layer thickness is:
\[
\left(\delta_{\text{ech,T}}\right)_{\text{Sc}} \sim L \cdot X \cdot \text{Re}^{1/2} \cdot \left(\text{Ra} \cdot \chi^2 \cdot N \cdot \text{Le} \cdot S_C^2\right)^{1/8}. 
\] (33)

3.3.2. MDC_C regime. The equilibrium between the horizontal diffusion and the vertical convection requires \( v_C \cdot \frac{\delta C}{\delta x} \sim D \left( \frac{\partial^2 C}{\partial y^2} \right) \) or \( v_C \cdot \frac{1}{x} \sim \frac{D}{\delta C} \). Replacing the equations (11) and (27), the equilibrium time and the concentration boundary layer thickness become:

\[
\left(\delta_{\text{ech,c}}\right)_C \sim \frac{1}{D} \left( \frac{x \cdot L^4}{\text{Ra} \cdot \chi^2 \cdot N \cdot \text{Le}} \right)^{2/5} = \frac{L^2}{D} \left( \frac{X}{\text{Ra} \cdot \chi^2 \cdot N \cdot \text{Le}} \right)^{2/5} 
\] (34)

\[
\left(\delta_{\text{ech,c}}\right)_C \sim L \cdot X^{1/5} \left(\text{Ra} \cdot \chi^2 \cdot N \cdot \text{Le}\right)^{1/5}. 
\] (35)

The maximum velocity becomes:

\[
v_C \sim \frac{\alpha}{L} \left( \text{Ra} \cdot \chi^2 \cdot N^3 \cdot Y \cdot \text{Le}^{-3}\right)^{1/5} \text{ or } V_C \sim \left(\text{Ra} \cdot \chi^2 \cdot N^3 \cdot X^3 \cdot \text{Le}^{-3}\right)^{1/5}. 
\] (36)

Scale analysis of the temperature field in the MDC_C regime. The scale analysis requires: \(\left(\delta_{\text{ech,T}}\right)_C \sim x \cdot v_C^{-1}\) and, consequently, the temperature boundary layer thickness scale is:

\[
\left(\delta_{\text{ech,T}}\right)_C \sim L \left( X \cdot \text{Le}^{5/2} \cdot \text{Ra} \cdot \chi^2 \cdot N^{-1}\right)^{1/5}. 
\] (37)

A more detailed analysis [1] shows that the borders between the HDC and the MDC regions are not sharp; there are adjacent regions of two special types:

- \((v_C)_T < v_c\) (in a HDC region, the concentration field is driven by \(v_C\)) which defines the regions: \(X > X_a = \frac{\text{Ra}}{N^4} \cdot \frac{\chi^2 \cdot Y}{\chi}\) in a HDC_C regime and \(X > X_b = \left(\text{Ra} \cdot \gamma^2 N^4 S_C^8\right)^{1/7}\) in a HDC_Sc regime;

- \(v_C < v_T\) (in a MDC region, the temperature field is driven by \(v_T\)) which defines the following regions: \(X < X_c = \frac{\text{Ra} \cdot \chi^2 \cdot Y}{N^4 \chi^2}\) in a MDC_C regime and \(X < X_d = \frac{\text{Le} \cdot \gamma^2}{S_C N \chi^2}\) in a MDC_Sc regime.

This detailed analysis goes beyond the aim of this study but it is essential because of one result: \(X_b < X_{\text{Sc,Sc}}\) if \(\text{Ra} \cdot \gamma^2 \cdot S_C \cdot \text{Le}^{4/3} / N^3 < 1\) which states that beyond \(X_{\text{Sc,Sc}}\) the concentration field continues to be driven by \(v_C\). Here, we expect to encounter values of the wall concentration corresponding to the MDC regime.

The validity of the scale analysis imposes the following conditions:

- if \(\text{Ra} \cdot \gamma^2 \cdot S_C \cdot \text{Le}^{4/3} / N^3 \geq 1\), the condition \(\left(\delta_{\text{ech,T}}\right)_T \ll X\) requires \(X > \left(\text{Ra} \cdot \gamma^2\right)^{1/3}\), a condition that defines a diffusion region [6].

- if \(\text{Ra} \cdot \gamma^2 \cdot S_C \cdot \text{Le}^{4/3} / N^3 < 1\), the following conditions are required: \(\left(\delta_{\text{ech,T}}\right)_T \ll x\) for \(X = X_{\text{Sc,Sc}}\); \(\left(\delta_{\text{ech,C}}\right)_C \ll x\) and \(\left(\delta_{\text{ech,T}}\right)_C \ll x\) for \(X = X_{\text{Sc,C}}\); \(\left(\delta_{\text{ech,T}}\right)_{\text{Sc}} \ll x\). This set of conditions requires: \(S_C \ll 1\), \(S_C \text{Le}^{1/2} \ll 1\) and \(\max(1, S_C^2 \text{Le}^{4/3} \cdot N^{-4} \chi^{-2}) \ll \text{Ra}^2 \cdot \text{Le}^{-3}\).
4. Numerical modeling

The stream function formulation of the velocity field: \( \frac{\partial \Psi}{\partial X} = U \) and \( \frac{\partial \Psi}{\partial Y} = V \), and the vorticity definition: \( \zeta = \frac{\partial V}{\partial Y} - \frac{\partial U}{\partial X} \), define a new dimensionless form of the governing equations as follows:

\[
\zeta = \left( \frac{\partial^2 \Psi}{\partial Y^2} + \frac{\partial^2 \Psi}{\partial X^2} \right) \\
\frac{1}{Pr} \left( \frac{\partial \zeta}{\partial X} + U \frac{\partial \zeta}{\partial Y} + V \frac{\partial \zeta}{\partial X} \right) = \left( \frac{\partial^2 \zeta}{\partial Y^2} + \frac{\partial^2 \zeta}{\partial X^2} \right) + Ra \left( \frac{\partial \theta}{\partial Y} + N \frac{\partial \varphi}{\partial Y} \right) \\
\frac{\partial \theta}{\partial X} + U \frac{\partial \theta}{\partial Y} + V \frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial X^2} \\
\frac{\partial \varphi}{\partial X} + U \frac{\partial \varphi}{\partial Y} + V \frac{\partial \varphi}{\partial X} + V \frac{\partial \varphi}{\partial X} = \frac{\partial^2 \varphi}{\partial Y^2} + \frac{\partial^2 \varphi}{\partial X^2} \tag{39}
\]

where \( \theta = \frac{T - T_{\infty}}{T_w - T_{\infty}} \), \( \varphi = \frac{C - C_{\infty}}{C_w - C_{\infty}} \), \( U = \frac{uL}{\alpha} \) and \( \tau = \frac{\alpha L}{v} \). The boundary conditions become:

\[
\Psi = 0, \quad \frac{\partial \varphi}{\partial Y} = 0, \quad \frac{\partial \varphi}{\partial X} = 0 \quad \text{at} \quad Y = 0 \tag{42}
\]

\[
\frac{\partial \Psi}{\partial Y} = 0, \quad \zeta = 0, \quad \theta = 0 \quad \text{as} \quad Y = L \tag{43}
\]

\[
\Psi = 0, \quad \zeta = 0, \quad \theta = 0 \quad \text{as} \quad X = 0 \tag{44}
\]

\[
\frac{\partial^2 \Psi}{\partial X^2} = \frac{\partial^2 \Psi}{\partial Y^2} = \frac{\partial^2 \theta}{\partial X^2} = \frac{\partial^2 \varphi}{\partial X^2} = 0 \quad \text{at} \quad X = H \tag{45}
\]

The governing equations, \(38\)–\(41\), subjected to the boundary conditions, equations \(42\)–\(45\), were solved using the finite difference method, the higher order hybrid scheme \(10\) using a software created by the author. The computational domain was considered as being rectangular and it was discretized using a uniform grid. The dimensions of the domain and the number of points were tested for each particular case and they were established calculating, at each run, the dimensionless heat transfer coefficient (the Nusselt number) and the dimensionless mass transfer coefficient (the Sherwood number).

Second order finite-difference approximations were used for the diffusion terms and third order finite-difference approximations were used for the convection terms.

The implementation of the boundary conditions \(42\)–\(45\) required the following:

- at the left boundary of equation \(41\), a first order backward finite-difference representation with an exterior point was used for the implementation of the boundary condition \(42\);
- a second order central finite-difference approximation with an exterior point was considered at the right boundary of equation \(38\) for the implementation of the boundary condition \(43\);
- a second order finite-difference approximation with an exterior point was used at the upper boundary of equations \(40\) and \(41\) for the implementation of the boundary condition \(45\);
- second order backward finite-difference approximations were used at the upper boundary at the equation \(38\) for the implementation of the equation \(45\).
The system of equation (38)–(41) were solved using an iterative process: at each time step, equation (38) was solved iteratively till the relative error of \( \Psi \), at each point of the grid, became less than \( 10^{-6} \). The iterative process stopped when the relative errors of \( \theta \), \( \phi \) and \( \zeta \), at each grid point, became less than \( 10^{-3} \). An increase of this precision to \( 10^{-6} \) did not bring any significant change to the variables of interest.

5. Results and discussions

The results obtained during the scale analysis of the conservation equations are illustrated here for two particular parameter sets corresponding to the two possibilities presented graphically by figure 2. In each case, the computational domain has the dimensions of \( 0.7 \times 10.0 \) and the discretization grid has a number of \( 71 \times 501 \) uniformly distributed nodes. The iterative process used a time step of \( 5.0 \times 10^{-6} \).

The first example considers the following parameters: \( Ra = 10000 \), \( N = 10 \), \( Le = 1 \), \( Pr = 7.0 \) (the environment is water) and \( S_C = 0.03 \). Figure 3 presents the temperature, the concentration, the stream function and \( \frac{\partial \phi}{\partial x} \cdot S_C^{-1} \) fields for this case. Here, \( Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} = 0.157 < 1 \) and we expect a HDCMC—MDCC—MDCS regime succession.

We track the following scale analysis results:

- \( X_{MC,C} = 0.52 \), the co-ordinate that marks the HDC-MDC transition, is the co-ordinate where the wall concentration becomes smaller than \( 1/N = 0.1 \). Figure 3(b) reveals it as being \( X = 0.42 \);

![Figure 3](image_url)

**Figure 3.** The dimensionless temperature (a), concentration (b), stream function (c) and \( \frac{\partial \phi}{\partial x} \cdot S_C^{-1} \) (d) fields for \( Ra = 10000 \), \( N = 10 \), \( Le = 1 \), \( Pr = 7.0 \) and \( S_C = 0.03 \).
Figure 4. (a) $\theta$, (b) $\phi$ and (c) $V$ variations as a function of $Y$ and the scaled (d) $\theta$, (e) $\phi$ and (f) $V$ plots for the abscissas: 0.2, 0.4 and 0.6; $Ra=10000$, $N=10$, $Le=1$, $Pr=7.0$ and $S_C=0.03$.

Figure 5. (a) $\theta$, (b) $\phi$ and (c) $V$ variations as a function of $Y$ and the scaled (d) $\theta$, (e) $\phi$ and (f) $V$ plots for the abscissas: 0.6, 0.8, 1.0 and 1.2; $Ra=10000$, $N=10$, $Le=1$, $Pr=7.0$ and $S_C=0.03$. 
Figure 6. (a) $\theta$, (b) $\phi$ and (c) $V$ variations as a function of $Y$ and the scaled (d) $\theta$, (e) $\phi$ and (f) $V$ plots for the abscissas: 3.0, 4.0 and 5.0; $Ra = 10000$, $N = 10$, $Le = 1$, $Pr = 7.0$ and $S_C = 0.03$.

— The X co-ordinate where the $VS_C$ term becomes dominant in equation (41) is $X_{S,C} = 5.28$. Figure 3 (d) reveals a value of 1.37. This difference is due to the fact that the region where $\frac{\partial \phi}{\partial X}$ is dominant does not occupy the entire boundary layer.

— the boundary layer properties derived in section 3 for the regions HDC$_C$—MDC$_C$—MDC$_{S_C}$:
  ■ the HDC$_C$ region properties are verified analyzing the sections of three X co-ordinate: 0.2, 0.3 and 0.4. Figure 4 (a), (b) and (c) are presenting the $Y$ variation of temperature, concentration and vertical velocity, respectively, while the figures 4 (d), (e) and (f) are presenting the scaled versions of these plots. The scale analysis results of section 3.2 are verified through the collapse of the scaled graphs.
  ■ the MDC$_C$ region is analysed in four sections: 0.6, 0.8, 1.0 and 1.2. Similarly, Figure 5 presents the un-scaled and scaled forms of the temperature, concentration and vertical velocity plots verifying the validity of the results presented by section 3.3.2.
  ■ The MDC$_{S_C}$ region is analysed using the section done through the X co-ordinate: 3.0, 4.0 and 5.0. Similarly, figure 6 verifies the validity of the results derived in section 3.3.1.

The second example uses the parameters: $Ra = 2000$, $N = 1$, $Le = 1$, $Pr = 7.0$ (the environment is water) and $S_C = 0.05$. Figure 7 presents the temperature, concentration, stream function and $\left(\frac{\partial \phi}{\partial x}\right) S_C^{-1}$ fields for this parameters set. In this case $Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} = 52.66 > 1$ and we encounter a heat driven convection (HDC) process along the entire wall. Figure 7 (b) verifies that the concentration along the wall do not exceeds the value of $1/N = 1.0$ and, consequently, a HDC$_C$—HDC$_{S_C}$ regimes succession is attaining the equilibrium state.
Equation (26) shows that the dominance of $S_C$ over $\frac{\partial \phi}{\partial X}$ should occur at $X > X_{S,T}$, while Figure 7 (d) reveals that this transition occurs at an X co-ordinate of 1.63. This difference can be explained by noticing that, in Figure 7 (d), the region where $\frac{\partial \phi}{\partial X}$ is dominant does not occupy the entire boundary layer thickness.

Further, the scale analysis results are verified as follows:
- the HDC regime is analyzed using three values of the X co-ordinate: 0.8, 1.2 and 1.6, while the HDCC region is analyzed considering the following X co-ordinates: 4.0, 5.0 and 6.0;
- Figure 8 and figure 9 present the temperature, the concentration and the vertical velocity as well as the scaled plots for each co-ordinate. The collapse of all the scaled plots verifies the validity of the scale analysis of the HDC regime;
- a special aspect is encountered in the HDC region. Here the X co-ordinate $X_h = 0.01 < X_{S,T}$ and, even if we register a HDC region, the concentration field continues to be driven by $V_C$. Figure 9 (e) exemplifies this aspect by using equation (30) instead of equation (24) and obtaining, only in this way, the right order of magnitude of the values in the scaled plot.

6. Conclusions
This paper establishes a new understanding of the natural convection process that takes place near a vertical wall of constant temperature embedded in a fluid saturated with a certain constituent. The constituent mass flux is constant at the wall, while its concentration varies linearly at infinity. Far from the wall, the fluid temperature is constant. Referring to the $Le = 1$, $Pr > 1$ case, this work uses the scale analysis to establish the natural convection regimes that occur along the wall:
Figure 8. (a) $\theta$, (b) $\phi$ and (c) $V$ variations as a function of $Y$ and the scaled (d) $\theta$, (e) $\phi$ and (f) $V$ plots for the abscissas: 0.8, 1.2 and 1.6; $Ra = 2000$, $N = 1$, $Le = 1$, $Pr = 7.0$ and $S_C = 0.05$.

Figure 9. (a) $\theta$, (b) $\phi$ and (c) $V$ variations as a function of $Y$ and the scaled (d) $\theta$, (e) $\phi$ and (f) $V$ plots for the abscissas: 4.0, 5.0 and 6.0; $Ra = 2000$, $N = 1$, $Le = 1$, $Pr = 7.0$ and $S_C = 0.05$. 
- a heat driven convection (HDC) regime if $Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} \geq 1$;
- a heat–mass driven convection (HDC−MDC) regimes succession if $Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} < 1$.

These scale analysis results are illustrated, by solving the conservation equations using the finite differences method, for two particular parameter sets:

- $Ra = 10000 \ , \ N = 10 \ , \ Le = 1 \ , \ Pr = 7$ and $S_C = 0.03$ for $Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} < 1$ case;
- $Ra = 2000 \ , \ N = 1 \ , \ Le = 1 \ , \ Pr = 7.0$ and $S_C = 0.05$ for $Ra \cdot \gamma^2 \cdot S_C \cdot Le^{4/3} \cdot N^{-3} \geq 1$ case.

For each parameter set, the graphical representation of the temperature, concentration, stream function and $\frac{\partial \phi}{\partial x} \cdot S_C^{-1}$ fields are used to emphasize the following aspects:

- the HDC–MDC transition or the absence of this transition as the scale analysis results demonstrate;
- the point on the wall where $S_C$ becomes greater than $\frac{\partial \phi}{\partial X}$ in the equation (41) as well as the scale analysis results for this point.

Each natural convection regime is further analyzed through the temperature, the concentration and the vertical velocity plots and scaled plots in different sections along the wall. The scaled plots illustrate graphically the results of the scale analysis regarding the temperature, the concentration and the velocity boundary layer thicknesses as well as the vertical velocity order of magnitude.

These results show clearly that, for the particular case analyzed here, the natural convection regime type can be pre-determined using the results of this paper.

References
[1] Bennacer R and Gobin D 1996 Cooperating thermosolutal convection in enclosures– I. Scale analysis and mass transfer *International Journal of Heat and Mass Transfer* 39 pp 2671–2681
[2] Mongruel A, Cloitre M and Allain C 1996 Scaling of boundary–layer flows driven by double–diffusive convection *International Journal of Heat and Mass Transfer* 39 pp 3899–3910
[3] Bejan A 1995 *Convection Heat Transfer* (New York: Wiley)
[4] Bednarz T P, Lin W, Patterson J C, Lei C and Armfield S W 2009 Scaling for unsteady thermomagnetic convection boundary layer of paramagnetic fluids of Pr>1 in micro–gravity conditions *International Journal of Heat and Mass Transfer* 30 pp 1157–1170
[5] Patterson J C, Lei C, Armfield S W and Lin W 2009 Scaling of unsteady natural convection boundary layers with a non–instantaneous initiation *International Journal of Thermal Sciences* 48 pp 1843–1852
[6] Armfield S W, Patterson J C and Lin W 2007 Scaling investigation of the natural convection boundary layer on an evenly heated plate *International Journal of heat and Mass Transfer* 50 pp 1592–1602
[7] Saha S C, Patterson J C and Lei C 2010 Natural convection boundary–layer adjacent to an inclined flat plate subject to sudden and ramp heating *International Journal of Thermal Sciences* 49 pp 1600-1612
[8] Saha S C and Khan M M K 2012 An improved boundary layer scaling with ramp heating on a sloping plate *International Journal of Heat and Mass Transfer* 55 pp 2268–2284
[9] Saha S C, Brown R J and Gu Y T 2012 Scaling for the Prandtl number of the natural convection boundary layer of an inclined flat plate under uniform surface heat flux *International Journal of Heat and Mass Transfer* 55 pp 2394–2401
[10] Tannehill J C, Anderson D A and Pletcher R H 1997 *Computational Fluid Mechanics and Heat Transfer* (Washington: Taylor&Francis)