Dark–bright solitons in spinor polariton condensates under nonresonant pumping

Xingran Xu\(^1\), Lei Chen\(^4\), Zhidong Zhang\(^1,2\) and Zhaoxin Liang\(^3\)

\(^1\) Shenyang National Laboratory for Materials Science, Institute of Metal Research, Chinese Academy of Sciences, Shenyang, People’s Republic of China
\(^2\) School of Materials Science and Engineering, University of Science and Technology of China, Hefei, People’s Republic of China
\(^3\) Department of Physics, Zhejiang Normal University, Jinhua, 321004, People’s Republic of China
\(^4\) School of Physics and Electronic Science, Zunyi Normal University, Zunyi 563006, People’s Republic of China

E-mail: zhxliang@gmail.com

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Abstract
By adopting a mean-field Gross–Pitaevskii description of spinor polariton Bose–Einstein condensates under nonresonant pumping, we investigate the static and dynamical properties of dark–bright (DB) solitons. We derive the equation of motion for the center of mass of the DB solitons’ center analytically by using the Hamiltonian approach. The resulting equation captures how the combination of the open-dissipative character and the spin degrees of freedom of a polariton Bose–Einstein condensate affects the properties of DB solitons, i.e. DB solitons relax by blending with the background at a finite time. In this case, we also determine the lifetime of the DB solitons. Further numerical solutions of the modified dissipative two-component Gross–Pitaevskii equations are in excellent agreement with the analytical results. In the presence of the Langevin noise, we demonstrate that the DB solitons can still propagate for a long time, which is sufficient for their experimental observations within current facilities.

Keywords: dark–bright solitons, polariton Bose–Einstein condensate, mean-field method, stochastic GP equations

(Some figures may appear in colour only in the online journal)

1. Introduction
A soliton [1] is a solitary wave packet which can maintain its shape due to self-stabilization against dispersion through nonlinear interaction [2–6]. In the one-component nonlinear Schrödinger equation (NLSE), the dark [7, 8] (bright [6, 9]) soliton can exist provided the interaction is repulsive (attractive). By contrast, in the two-component (spinor) NLSE with repulsive interaction, i.e. the vector variant of the NLSE, a dark–bright (DB) soliton [10] can appear even though a bright soliton alone is forbidden; a density dip in the form of a dark soliton in one component plays the role of an effective potential well for a bright soliton created in the second component [10]. This remarkable phenomenon highlights the important role of the spin degree of freedom (two-component) of a system, which, when interplaying with dispersion and nonlinearity, can give rise to novel solitons with no analog in its one-component counterpart [11–15]. DB solitons have been extensively studied in a wide variety of physical systems. In particular, they have been experimentally observed in the context of nonlinear optics [16, 17] and recently in atomic condensates [18–21].

The recent realization of Bose–Einstein condensates (BECs) of polaritons [22–24] in quantum well semiconductor microcavities has opened up intriguing possibilities to explore the DB solitons beyond thermal equilibrium. As polaritons undergo rapid radiative decay, their population in the condensate is maintained by persistent optical pumping. Hence a polariton condensate is inherently in non-equilibrium with an open-dissipative character [23]. Its mean-field physics can be well captured by a Gross–Pitaevskii equation (GPE) with gain and loss [25, 26], where the nonlinearity results from the
strongly and repulsively interacting excitons [22]. Further, polaritons naturally possess peculiar spin properties [27]: due to the strong couplings between excitons and photons, the spin-up and spin-down projections of the total angular momentum of excitons along the growth axis of the structure directly come from the right- and left-circularly polarized photons absorbed or emitted by the cavity, respectively. Combinations of these exceptional properties promise polariton BECs as a novel platform for realization and investigation of various nonlinear phenomena, such as solitons.

In the one-component polariton condensates formed under a resonant pumping laser, a series of experiments have demonstrated the existence of the oblique dark solitons and vortices [28–30], or bright spatial and temporal solitons [31, 32]. In condensates created spontaneously under incoherently pumping, the formation and behavior of dark solitons have been investigated in [33–35] theoretically and observed in Ref. [36] experimentally. The existence of stable dark soliton trains has been reported in the non-resonantly driven spinor polariton BEC at one dimension [37]. Yet, the existence of DB solitons in non-equilibrium polariton BEC remains unexplored. It is the aim of the present work to investigate the interplay of the nonlinearity, dispersion and the spin degree of freedom of system on the DB solitons in the presence of incoherent driving and dissipation.

In this work, we study the physics of DB solitons in a spinor polariton BEC formed under nonresonant pumping by solving two-component dissipative GPEs with a combination of analytical and numerical approaches. Our goal is to explore the combined effects of the open-dissipative and spinor nature on the dynamics of DB solitons. To this end, we first use the Hamiltonian approach and analytically derive the evolution equations for the soliton parameters, i.e. the inverse width of the soliton. We compare this analytical result with the numerical solutions for the trajectory of DB solitons directly obtained from the GPE, and find a remarkable agreement between the two. Further, we have demonstrated that, while noise will eventually destroy the dissipative DB solitons, their lifetime remains sufficient for a feasible experimental observation. Our results open a route to observe stable DB solitons in non-equilibrium spinor polariton BEC within the current experimental capabilities.

The paper is organized as follows. In section 2, we introduce two-component dissipative GPEs coupled to the rate equation of a spin-unpolarized reservoir, which can well describe the static and dynamical properties of the polariton BECs under nonresonant pumping. In section 3, we present a brief review on the collective excitations in the uniform stationary state using Bogoliubov’s theory, and provide analytical results for the parameter regimes where the system is modulationally stable. In section 4, we use the Hamiltonian approach to derive the equation of motion for the center of mass of the DB solitons. We then compare the analytical results with the exact numerical ones. We moreover demonstrate how these dissipative solitons can be affected by Langevin noise by numerically solving the stochastic GPEs. In section 5, we conclude with a summary of our main results and final remarks.

2. Model system

Many studies have been done in the quantum gas field [38–40] where a spinor exciton–polariton BEC created under nonresonant pumping is considered [26, 27]. At the mean-field level, the condensate can be well described by a two-component time-dependent wave function [ψ1, ψ2] [41–43]. For the excitonic reservoir, we assume that the spin relaxation of the reservoir is sufficiently fast so that the reservoir on the relevant timescales can be modeled by a scalar density denoted by nR(t) [22, 27].

The dynamics of the spinor polariton BEC can be described by the following driven-dissipative two-component GPEs [22, 25, 26, 44–47]

\[
\frac{i\hbar}{\partial t} \psi_1 = \left[ -\frac{\hbar^2 \nabla^2}{2m} + g |\psi_1|^2 + g_{12} |\psi_2|^2 + g_R n_R \right] \psi_1 + \frac{\hbar}{2} (R n_R - \gamma_C) \psi_1 + i\hbar \frac{\partial \psi_1}{\partial t},
\]

(1)

\[
\frac{i\hbar}{\partial t} \psi_2 = \left[ -\frac{\hbar^2 \nabla^2}{2m} + g |\psi_2|^2 + g_{12} |\psi_1|^2 + g_R n_R \right] \psi_2 + \frac{\hbar}{2} (R n_R - \gamma_C) \psi_2 + i\hbar \frac{\partial \psi_2}{\partial t}.
\]

(2)

Here, m is the mass of polariton, g (g_{12}) is the interaction between polaritons with same (opposite) spins and gR is the interaction strength between reservoir and polariton. Further, the terms \(d\psi_i^2 = 2D_i dW_i \) (\(i = 1, 2\)) account for the fluctuations induced by white noise [48–50]. Here dW_i is a Gaussian random variable characterized by the correlation functions [48–50]

\[
\langle dW_i dW_j \rangle = \delta_{ij} dt, \quad \langle dW_i dW_j \rangle = 0,
\]

(3)

where i, j are indices for the component of spinor polariton BEC. Below we will ignore the noise term in equation (3) when calculating the stationary states, analyzing the modulation stability parameter regimes and deriving the equation of motion for the DB soliton’s width. We will add the noise terms afterwards, which serves to test the stability of DB solitons against fluctuations. Furthermore, in equations (1) and (2) we have ignored the effects of the transverse-electric and transverse-magnetic splitting [51, 52]. Note that going beyond the GPEs (1) and (2) to fully include the quantum and thermal fluctuations of the quantum field (e.g., Keldysh path-integral method [53–56]) is beyond the scope of this work.

We consider that equations (1) and (2) are coupled to a (scalar) incoherent reservoir as mentioned earlier, which is described by a rate equation, i.e.

\[
\frac{\partial n_R}{\partial t} = P - \gamma R n_R - R (|\psi_1|^2 + |\psi_2|^2) n_R.
\]

(4)

Here, P is the rate of an off-resonant continuous-wave (cw) pumping, \(\gamma_R^{-1}\) describes the lifetime of reservoir polaritons and R is the stimulated scattering rate of reservoir polaritons into the spinor condensate.
The emphasis and value of this work are to take account of the polariton spin degree of freedom and look for the possibility of the existence of DB solitons in a spinor polariton BEC. We note that in an ultracold quantum gas context, one usually has almost equal values of the inter-interaction, g, and intra-interaction, $g_{12}$, coupling constant, which supports the existence of DB solitons naturally. In contrast, one has $g > 0$, $g_{12} < 0$, and $g \gg |g_{12}|$ [27, 57] in typical polariton systems. We remark that, recently, several experiments [58, 59] have demonstrated the possibility of tuning the interaction constants using biexcitonic Feshbach resonance in resonantly created polariton condensate and a single quantum background. Two-component polaritons have two steady states (see appendix B) as $P > P_{\text{th}}$ if we can adjust the interaction and we can verify this by ground state analysis [26] or pseudospin evolution [26, 61–63].

In our subsequent analytical treatment, we will consider the situation when $g > 0$ [27, 57] and $g_{12} > 0$ with $g = g_{12}$. We note that while in present experiments one typically has $g_{12} < 0$ with $|g_{12}| \ll g$, recent experimental progress in realizing tunable cross-spin interaction properties has opened up the prospect of realizing polariton BECs in regimes $g_{12} > 0$ and $g_{12}/g \geq 1$ [58, 59]. Under these conditions, in the steady state, the spinor polariton BEC has a total condensate density $n_{0} = (P - P_{\text{th}})/\gamma_{C}$, which is equally distributed in each component, while the reservoir density is given by $n_{R}^{0} = \gamma_{C}/R$.

For convenience, we will recast equations (1)–(4) into a dimensionless form (see appendix A). After rescaling $\psi_{i} \rightarrow \psi_{i}/\sqrt{m_{i}}$ and introducing $m_{R} = n_{R} - n_{R}^{0}$, we obtain

$$i\frac{\partial}{\partial t}\psi_{1} = \frac{1}{2}\nabla^{2}\psi_{1} + (|\psi_{1}|^{2} + |\psi_{2}|^{2}) - 1)\psi_{1} + \bar{g}_{R}m_{R}\psi_{1} + \frac{i}{2}\bar{R}m_{R}\psi_{1} + \frac{m_{R}}{\alpha_{1}}\psi_{1},$$

$$i\frac{\partial}{\partial t}\psi_{2} = \frac{1}{2}\nabla^{2}\psi_{2} + (|\psi_{1}|^{2} + |\psi_{2}|^{2}) - \bar{\mu})\psi_{2} + \bar{g}_{R}m_{R}\psi_{2} + \frac{i}{2}\bar{R}m_{R}\psi_{2} + \frac{\alpha_{2}}{m_{R}}\psi_{2},$$

$$\frac{\partial}{\partial t}m_{R} = \gamma_{C}(1 - |\psi_{1}|^{2} - |\psi_{2}|^{2}) - \gamma_{R}m_{R} - \bar{R}(1 - |\psi_{1}|^{2} - |\psi_{2}|^{2})m_{R},$$

$$3. \text{Stability of a homogeneous condensate}$$

Here we briefly revisit the known results of the linear collective excitations of a spinor polariton BEC under incoherent uniform pumping. We will determine analytically the regime of modulational instability (MI) [60] of a homogeneous background. Two-component polaritons have two steady states (see appendix B) as $P > P_{\text{th}}$, if we can adjust the interaction and we can verify this by ground state analysis [26] or pseudospin evolution [26, 61–63].

Following standard procedures of stability analysis, we linearize equations (5)–(7) around the steady-state solution by:

$$\psi_{1} = \psi_{1}^{0}e^{-i\omega_{1}t} + \sum_{k} [u_{1k}e^{i(kx - \omega_{1}t)} + v_{1k}^{*}e^{-i(kx + \omega_{1}t)}],$$

$$\psi_{2} = \psi_{2}^{0}e^{-i\omega_{2}t} + \sum_{k} [u_{2k}e^{i(kx - \omega_{1}t)} + v_{2k}^{*}e^{-i(kx + \omega_{1}t)}],$$

$$n_{R} = n_{R}^{0} + \sum_{k} [w_{k}e^{i(kx - \omega_{1}t)} + w_{k}^{*}e^{i(kx + \omega_{1}t)}],$$

and substitute equations (8)–(10) into equations (5)–(7) to get a $5 \times 5$ matrix about vector $(u_{1k}, v_{1k}, u_{2k}, v_{2k})$. The eigenvalue problem for the dispersion relation can be solved by diagonalization [26]; however, we just look at the spin-up polarization phase in this work

$$\left(\omega^{2} - k^{2}/4\right) \times \left[\omega^{3} + i(\gamma_{R} + R)\omega^{2} - (\omega_{R}^{2} + \bar{R}\gamma_{C})\omega - i\omega_{R}^{2}(\gamma_{R} + R) + i\gamma_{C}\bar{g}_{R}k^{2}\right] = 0,$$

with $\omega_{R}^{2} = k^{2}/4$.

We now determine the MI regime from equation (11). Specifically, there are five complex dispersion branches $\omega_{k} = \mathcal{M}(\omega_{k}(k)) + i\mathcal{J}(\omega_{k}(k)), \ (s = 1, 2, 3, 4, 5)$. The modulation instability occurs when $\mathcal{J}(\omega) > 0$ at certain momenta. In our case, MI is found to occur $\bar{g}_{R}\gamma_{C} > \gamma_{R} + \bar{R}$ in a regime $k \in [k_{1}, k_{2}]$ with $k_{1} = 0$ and $k_{2} = 2\sqrt{\bar{g}_{R}\gamma_{C}/(\gamma_{R} + \bar{R})} - 1$.

In the remaining part of our work, we will restrict our consideration in the fast reservoir limit $\gamma_{R} \gg \gamma_{C}$, where the modulation stability condition is naturally satisfied.

4. Dynamics of dark–bright solitons

In the limit of a fast reservoir, in this section we derive the equation of motion for the center of mass of the DB soliton supported by equations (5)–(7) in the absence of noise. We find that it is more transparent to rewrite equations (5)–(6) in
the following form
\[ i \frac{\partial}{\partial t} \psi_1 + \frac{1}{2} \nabla^2 \psi_1 - (|\psi_1|^2 + |\psi_2|^2 - 1) \psi_1 = R_t, \tag{13} \]
\[ i \frac{\partial}{\partial t} \psi_2 + \frac{1}{2} \nabla^2 \psi_2 - (|\psi_1|^2 + |\psi_2|^2 - \mu) \psi_2 = R_s, \tag{14} \]
where we have introduced the notation
\[ R_{1,2} = \left( \gamma_R m_R + i \frac{1}{2} R m_R \right) \psi_{1,2}. \tag{15} \]
The dynamics of the perturbation \( m_R \) in equation (15) is governed by the rate equation
\[ \frac{\partial}{\partial t} m_R = \gamma_c (1 - |\psi_1|^2 - |\psi_2|^2) - \gamma_r m_R - R (|\psi_1|^2 + |\psi_2|^2) m_R. \tag{16} \]
Equations (13) and (14) can be viewed as two coupled NLSEs subjected to time-dependent perturbations provided by \( R_1 \) and \( R_2 \). (We note that the two equations obviously reduce to that in [34] if one of the components vanishes.)

As a first step, we consider \( R_{1,2} = 0 \), i.e. in the absence of the open-dissipative effects. Under the boundary conditions \( \psi_1 \to 1 \) and \( \psi_2 \to 0 \) as \( |x| \to \infty \), there exists an exact one-DB-soliton solution, which takes the form
\[ \psi_D = \cos \phi \tan \left[ D(x - x_0(t)) \right] + i \sin \phi, \tag{17} \]
\[ \psi_B = \eta \sec \left[ D(x - x_0(t)) \right] e^{ik[x - x_0(t)] + \mu(t)}. \tag{18} \]
Here \( \phi \) is the dark soliton’s phase angle, \( \cos \phi \) and \( \eta \) are amplitudes of the dark and bright solitons, \( D \) and \( x_0(t) \) are associated with the inverse width and the center position of the dark and bright solitons. Further, \( k = D \tan \phi \) is referred to as the wavenumber of the bright soliton. The amplitude of bright soliton \( \eta \) is connected to the number \( n_2 \) of atoms in the bright soliton through the following relation [34]
\[ \int_{-\infty}^{\infty} |\psi_B|^2 dx = \frac{2n_2^2}{D} = n_2. \tag{19} \]
Notice that above parameters of the DB solitons are not independent, related to each other by \( D^2 = \cos^2 \phi - \eta^2 \), \( x_0 = D \tan \phi \), and \( \theta(t) = (D^2 - k^2)t/2 + (\mu - 1)t \).

Next, we account for the open-dissipative effects as captured by \( R_{1,2} \neq 0 \). We will rely on the Hamiltonian approach [64] of the perturbation theory which allows for analytical treatment of the effect of \( R_{1,2} \) [see (15)] on the DB solitons. We note that, different from [34], the system considered here has spin degrees of freedom and is described by two coupled GP equations. At the heart of the Hamiltonian approach of quantum dynamics for DB solitons is the assumption that, in presence of perturbation, the parameters of the solitons become slow functions of time, but the functional form of the soliton remains unchanged, i.e. \( \phi \to \phi(t) \) and \( D \to D(t) \), while keeping \( D^2(t) = \cos^2 \phi(t) - \eta^2 \), \( x_0(t) = D(t) \tan \phi(t) \). As such, the time evolution of the parameters \( \phi(t) \) and \( D(t) \) can be obtained from the time evolution of the DB soliton’s energy [34], i.e.
\[ \frac{dE}{dt} = -2 \mathcal{R} \int \left( R_1 \frac{\partial}{\partial t} \psi_1^* + R_2 \frac{\partial}{\partial t} \psi_2^* \right) dx, \tag{20} \]
where \( E \) is the energy of the DB solitons given by
\[ E = \frac{1}{2} \int dx \left( |\nabla \psi_1|^2 + |\nabla \psi_2|^2 + (|\psi_1|^2 + |\psi_2|^2 - 1)^2 \right). \tag{21} \]
We remark that equations (20) and (21) can be regarded as the so-called adiabatic invariant approximation of the perturbation theory of solitons in [65, 66]. We also stress that our theory so far assumes a perturbative regime of reservoir excitations \( m_R(\vec{r}, t) \ll 1 \).

Substituting equations (17) and (18) into equation (21), we obtain the time-dependent energy of the DB soliton as follows
\[ E = \frac{4}{3} D^3(t) + \frac{n_2}{2} D^2(t) \sec^2 \phi(t). \tag{22} \]
The first (second) term in equation (22) corresponds to the dark (bright) soliton’s energy, respectively. For \( n_2 = 0 \), i.e. when the bright soliton vanishes, equation (22) reduces to the energy of a single dark soliton [see equation (30) in [34]]. However, when the dark soliton disappears, the bright soliton will vanish as well. To see this, note that the darkness of the dark soliton in DB solitons can be approximately measured by the velocity \( \sin \phi \) via the relation \( \eta^2_{\phi} = \sin^2 \phi \). Hence, when the dark soliton vanishes for \( n^0_{\phi \phi} \to 1 \) (i.e. when \( \phi = \pi/2 \)), the second term in equation (22) becomes divergent, meaning the bright soliton will be destroyed. This provides an intuitive understanding that DB solitons can exist in the sense that a density dip in the form of a dark soliton in one component creates a potential well for the second component which traps a bright soliton therein. It then follows from equation (22) that the time variation of the DB soliton’s energy is described by
\[ \frac{dE}{dt} = 4D^2 \dot{D} + n_2 D^2 \sin^2 \phi \dot{D} + n_2 D^2 \tan^2 \phi \dot{\phi}. \tag{23} \]
Next, calculations of the right-hand side of equation (20) require knowledge of the reservoir density \( m_R \). In the limit of a fast reservoir, we have
\[ \dot{m}_R = \frac{\gamma_c}{\gamma_r} (1 - |\psi_1|^2 - |\psi_2|^2). \tag{24} \]
Using equations (15) and (24), we write the right side of equation (20) as
\[ \Re \int \left( R_1 \frac{\partial}{\partial t} \psi_1^* + R_2 \frac{\partial}{\partial t} \psi_2^* \right) dx \]
\[ = \frac{2R}{3} \frac{\gamma_c}{\gamma_r} D^3 \sin^2 \phi + \frac{8R}{3} \frac{\gamma_c}{\gamma_r} \cos^2 \phi(D + 4D \tan \phi \dot{\phi}) \]
\[ - n_2 \frac{\gamma_c}{\gamma_r} D^2 \tan^2 \phi + n_2 \frac{8R}{6} \frac{\gamma_c}{\gamma_r} DD \dot{D}. \tag{25} \]
In equation (25), terms in the second (third) lines result from the coupling of dark (bright) solitons with the reservoir. When the bright soliton vanishes \( (n_2 = 0) \), the first term in the second line exactly recovers the corresponding result in [34]. Whereas, the second term in the second line comes from the reservoir-induced modification of the interaction of polaritons. This effect was ignored in [34].
Finally, substitution of equations (23) and (25) into equation (20) readily yields the equation of motion for the inverse width of soliton $D$, i.e.

$$
\left[1 - \frac{8g_2 \gamma_C}{2 \gamma_R} + \frac{n_2^2}{4D^2(2D + n_2)^2}\right]D = -\frac{1}{3} \frac{\dot{\gamma}_C}{\gamma_R} D(1 - D^2) + \frac{1}{3} \frac{\dot{\gamma}_C}{\gamma_R} n_2 D.
$$

Equation (26) for $n_2 = 0$ reduces to the well-known results in [34]. In this case, equation (26) can be simplified into

$$
\left(1 - \frac{8g_2 \gamma_C}{2 \gamma_R}\right)D = -\frac{1}{3} \frac{\dot{\gamma}_C}{\gamma_R} D(1 - D^2).
$$

In terms of $D = \sqrt{1 - v_t^2}$ as in [34], we have $\frac{dD}{dt} = \frac{1}{2\tau}(1 - v_t^2)v_t$ with $\tau = \frac{1}{2} \left(1 - \frac{8g_2 \gamma_C}{2 \gamma_R}\right)^{-\frac{1}{2}}$. Here, for vanishing $g_R$, our result can exactly recover the previous result, i.e. equation (33) in [34]. However, different from [34], our calculations have considered the interactions between the condensate and the reservoir characterized by $g_R$. The presence of such coupling will modify the effective local self-induced potential exhibited by the condensate and, therefore, changes the width and darkness of the solitonic state.

Equation (26) with $n_2 \neq 0$ allows us to analyze the combined effects of the spinor and open-dissipative nature of polariton BEC on the dynamics of the DB soliton. Notice that the velocity of the dark soliton is defined as $v_t = D \tan \phi$. By virtue of the relation $v_t = \sqrt{(2D)} - n_2 D^2(2D + n_2)$, equation (26) can be written as

$$
m_{\text{eff}} \frac{d\psi_t}{dt} = F_{\text{eff}}(\psi_t).
$$

Here $m_{\text{eff}}$ plays the role of the effective mass of the DB solitons

$$
m_{\text{eff}} = 1 - \frac{8g_2 \gamma_C}{2 \gamma_R} + \frac{n_2^2}{4D^2(2D + n_2)^2},
$$

and $F_{\text{eff}}$ represents an effective force given by

$$
F_{\text{eff}} = \frac{1}{3} \frac{\dot{\gamma}_C}{\gamma_R} D \left[(1 - D^2) - \frac{n_2^2}{2D + n_2}\right] \times \frac{D[2(2D + n_2) - n_2^2] + n_2}{(2D - n_2)[2D(2D + n_2)](2D - n_2)}.
$$

Equation (27) allows us to interpret the dynamics of the DB soliton in terms of the motion of a classical particle of mass $m_{\text{eff}}$ subjected to an external force $F_{\text{eff}}$. Equation (28) trivially recovers the corresponding result in [34] when $n_2 = 0$. Since the dark and bright solitons in a DB soliton share the same velocity, we will focus below on analyzing the behavior of the dark one. An interpretation of equation (27) is straightforward following from equation (13) for the dark soliton wave function $\psi_1$, which can be rewritten as

$$
\frac{\partial \psi_1}{\partial t} + \frac{\Lambda}{2} \psi_1 + g_r(1 - |\psi|^2) \psi_1 - i\psi_1 = iP.
$$

Here $g_r = 1 - \frac{8g_2 \gamma_C}{2 \gamma_R}$ is the effective interaction constant, $V = (1 - g_r \gamma_C f/\gamma_R) |\psi|^2$ is the effective external potential induced by the bright soliton and $P = \frac{1}{2} \frac{\dot{\gamma}_C}{\gamma_R} (g_r(1 - |\psi|^2) |\psi|^2) \psi$. Based on equation (30), the equation of motion for the center of mass of the dark solitons in equation (27) can be explained as follows.

(i) The effective mass in equation (28) involves two corrections due to the coupling of polariton BEC to reservoir (see second and third terms). To understand the first modification, we note that according to equation (30), the interaction between the dark soliton and the reservoir, characterized by $g_R$, modifies the effective local self-induced potential exhibited by the condensate [see the term containing $g_r$ in equation (30)]. This effect changes both the width and darkness of the solitonic state and, therefore, the first correction. On the other hand, the coupling of the bright soliton and reservoir results in an effective external potential [see the term of $V$ in equation (30)], hence explaining the second correction to the effective mass in equation (28).

(ii) To understand the effective force in equation (29), we note that the stimulated scattering term proportional to $\dot{R}$ on the right side of equation (30) is responsible for the variation of the dark soliton’s velocity. The effective force in equation (29) contains two competitive parts: while the action of the first part leads to acceleration of the dark soliton, the second part—which is induced by the bright soliton—will slow down the motion of the dark soliton.

We now proceed to solve the dynamics of the DB solitons governed by equation (26). Searching for the equilibrium solution, we set $\dot{D} = 0$ in equation (26), i.e. $(1 - D^2) - \frac{n_2^2}{2D + n_2} = 0$, which yields $\phi_{eq} = 0$, $x_{eq} = 0$ and $D_{eq} = \frac{n_2}{\sqrt{4 + \sqrt{n_2^2/16 + 1}}}$. Next, we expand the solutions around the equilibrium values, i.e. $x_0(t) \rightarrow 0 + x_0(t)$, $\phi(t) \rightarrow 0 + \phi(t)$, $D(t) \rightarrow D_{eq} + D_1(t)$. We recall that the three parameters are not independent and are related to each other as described earlier. Therefore, as $\phi(t) \approx 0$ nears the equilibrium value, we have $D_1(t) = -\dot{D_0}\phi(t)$ with $\dot{D} = (2D_{eq} + n_2/2)^{-1}$. With these approximations, equation (26) can be readily calculated as

$$
\frac{d\phi(t)}{dt} = \frac{D_{eq} \dot{\gamma}_C (4D_{eq} + D_{eq} n_2^2 - 2n_2) \phi(t)}{4g_2 \gamma_C^2 (DD_{eq} + Dn_2 - 2) + 3g_r \gamma C^2 (8DD_{eq} + 2Dn_2 - Dn_2 n_2)}.
$$

The solution takes the form

$$
\phi(t) = \phi_0 e^{t/\tau},
$$

where $\phi_0$ is the initial phase angle of dark soliton. Here $\tau$ provides the characteristic timescale for the existence of solitons in the presence of dissipation. It is explicitly given by

$$
\tau = \frac{3 \gamma_C}{\dot{\gamma}_C g_r} \left( g_r, n_2 \right) = \frac{3}{\gamma_C \gamma_0} \left( \frac{P_n}{P_n} \right) \left( g_r, n_2 \right),
$$
with
\[ f(\bar{g}_R, n_2) = \frac{8DD_{eq} + 2\bar{D}n_2 - D_{eq}n_2}{2\bar{D}n_2 - 2n_2D_{eq}}. \]

Notice that equation (33) can be simplified into the expression \( \tau = 3\bar{g}_R/\bar{R}n_C^2 \), as shown in [34] if \( n_2 = 0 \) and the term proportional to \( \bar{g}_R \) is ignored. The knowledge of \( \phi \) then allows us to determine the velocity of the dark soliton via \( v_t = D \tan \phi \) and hence the darkness of the dark soliton through \( \eta^2_{\min} = v_t^2(\tau) \). It is immediately clear from equation (32) that the dark soliton speeds up exponentially in time with a rate \( \tau^{-1} \) until it disappears eventually.

Equation (33) shows that, as also pointed out in [34], the lifetime of soliton \( \tau \) is proportional to \( \gamma_R \) while inversely proportional to \( \bar{R} \) and \( \gamma_C \), as has been numerically verified. In figure 1 (see the plots in the left and middle columns), we numerically solve equations (13) and (14) for the condensate density distribution of the dark soliton \( |\psi_d(x, t)|^2 \) and bright soliton \( |\psi_b(x, t)|^2 \), taking \( \bar{g}_R = 2 \), \( \gamma_C = 3 \) and \( \phi_0 = 0.2 \), while varying parameters \( \gamma_R \) and \( \bar{R} \). We compare the numerical results for the darkness \( \eta^2_{\min} \) of the dark solitons with the analytical predictions from \( \eta^2_{\min} = v_t^2(\tau) \). A remarkable agreement between the two is found, as illustrated in the right column of figure 1. To be more specific, we compare figures 1(a3) and (b3), corresponding to the cases with \( \bar{R} = 0.5 \) and \( \bar{R} = 0.1 \) by taking \( \gamma_R = 30 \), respectively. Whereas, comparisons of figures 1(a3) and (c3), both taking \( \bar{R} = 0.5 \) while \( \gamma_R \) is different, demonstrate the effects of \( \gamma_R \). Obviously, the analytical results agree increasingly well with the numerical ones for larger \( \gamma_R \) and smaller \( \bar{R} \). In all plots, we note that the typical polariton relaxation time is about \( \gamma_C^{-1} = 10 \text{ ps} \) [22] and the timescaling variable expressed as \( \tau_0 = \gamma_C/\gamma_C \) with \( \gamma_C = 3 \) takes the physical value of 30 ps. The corresponding propagation time for the solitonic state shown in figure 1 reaches \( \tau = 3000 \text{ ps} \), which is much longer than the condensate and reservoir relaxation times.

In the above context of this section, we now compare our results with the dark soliton [34]. Finally, we add the Langevin noise term equation (3) into equations (13) and (14),

Figure 1. Dynamics of the one-dimensional DB solitons of spinor polariton BECs created under nonresonant pumping. Shown are the contour plots of the dark soliton of \( |\psi_d|\overline{|^2 \text{ (left column) } \right., \right. \) bright soliton of \( |\psi_b|\overline{|^2 \text{ (middle column) } \right., \right. \) and the dependence of the darkness \( \eta^2_{\min} \) by numerically solving equations (13)–(14) (right column, blue plus signs). The solid red lines in (c1)–(c3) are calculated using the analytical results in equation (26) in the main text. In all plots, we have chosen \( \bar{g}_R = 2 \), \( \gamma_C = 3 \), \( \phi_0 = 0.2 \). For other parameters: (a1)–(a3) \( \bar{R} = 0.5 \), \( \gamma_R = 30 \); (b1)–(b3) \( \bar{R} = 0.1 \); \( \gamma_R = 30 \); and (c1)–(c3) \( \bar{R} = 0.5 \), \( \gamma_R = 45 \).
which serves both as an initial seed for the condensate and to test the stability of the DB solitons against fluctuations. Obviously, the soliton’s lifetime becomes shorter and unstable. We compare the dynamics of the dark soliton with (the third column of figure 2) and without noise (the first column of figure 2). (The results for the bright soliton are compared in the second and fourth columns.) According to equation (26), we get the equation for the inverse width of the DB solitons:

\[
\bar{\gamma}_R \frac{\bar{\gamma}_C}{2} \left[ D^2 - \frac{2D}{2D + n_2^2} \right] \\
= \frac{1}{3} \bar{\gamma}_R \frac{\bar{\gamma}_C}{\bar{\gamma}_R} D \\
\left. \left( \frac{1}{4D^2(2D + n_2^2)} \right) \right|_{D}.
\]

However, the lifetime of DB solitons in polaritons is hard to solve as the bright soliton component cannot be ignored. As is illuminated in figure 2, the added noise makes it clear that the small ratio of \(\frac{\bar{\gamma}_R}{\bar{\gamma}_C}\) and large \(\bar{R}\) make solitons spread farther. It is reasonable for \(\bar{\gamma}_R\) to represent the reservoir loss rate, while \(\bar{R}\) means that the stimulated scattering rate for polaritons can be replenished from the reservoir. While noise in general leads to a faster decay of solitons, they remain stable until \(t = 20\), corresponding to the real time of \(t = 600\) ps. This means that DB solitons can still propagate for a sufficiently long time even in the presence of noise, hence making it possible to feasibly observe them in experiments.

5. Discussion and conclusion

As is known, a dark soliton in DB solitons is more dynamically stable than a dark soliton alone. In the equilibrium case, it has been established that the dark soliton stripes in conservative atomic condensates and optical fields are always unstable against transverse excitations that have wavelength greater than their extension \([67-69]\), leading to undulation and an eventual breakup of dark solitons into multivortex

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**Figure 2.** Effects of Langevin noise on the dynamics of DB solitons in a spinor polariton BEC under nonresonant pumping by numerically solving equations (5)–(7). As a comparison, the first and second columns depict the time evolution of DB solitons without Langevin noise, while the third and fourth columns correspond to the presence of Langevin noise [see equation (3)] with \(d\rho = 0.01dW\). In all plots, we take \(\bar{\gamma}_R = 2, \bar{\gamma}_C = 3, \phi = 0.2\). For other parameters, (a1)–(a4) \(\bar{R} = 1.5, \bar{\gamma}_R = 15\); (b1)–(b4) \(\bar{R} = 0.5, \bar{\gamma}_R = 15\); (c1)–(c4) \(\bar{R} = 0.5, \bar{\gamma}_R = 45\); (d1)–(d4) \(\bar{R} = 0.5, \bar{\gamma}_R = 45\).
patterns [70, 71]. However, the large DB solitons are expected to transcend this restriction, because their size can be much larger than their extension when the number of bright soliton becomes very large [18]. In the non-equilibrium case, a natural question arises as to how the dissipative nature affects the snake instability of DB solitons. Addressing this issue is beyond the scope of this work, and we will leave the snake instability of DB solitons for future investigations.

In summary, we have investigated the dynamics of dark–bright solitons appearing in spinor polaron Bose–Einstein condensates under nonresonant pumping. We have derived analytically the evolution equations for the soliton parameters. Within the framework of Hamiltonian approach, our analytical results capture the essential physics as to how the combined effects of the open-dissipative and spinor nature affects the dark–bright solitons by blending with the background at a finite time. We also solve the modified dissipative two-component GPEs in a numerically exact fashion. The numerical results find remarkable agreement with the analytically ones. We also demonstrate that these dissipative solitons can exist in a substantially long time, even in presence of noise, rendering them available for experimental observations.

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Appendix A. Dimensionless method

In this appendix, we will discuss how to get dimensionless equations (5)–(7) in detail. As we all know, mean-field theory is a powerful tool to study cold atom physics. Two coupled polaritons’ wave functions can be written as

\[
\begin{align*}
\frac{i}{\hbar} \frac{\partial}{\partial t} \psi_1 &= -\frac{\hbar^2}{2M_1} \nabla^2 \psi_1 + g |\psi_1|^2 \psi_1 + g_{12} |\psi_2|^2 \psi_1 \\
&+ g_{2R} \bar{n}_R \psi_1 + i\hbar \left( R \bar{n}_R - \gamma_C \right) \psi_1, \\
\frac{i}{\hbar} \frac{\partial}{\partial t} \psi_2 &= -\frac{\hbar^2}{2M_2} \nabla^2 \psi_2 + g |\psi_2|^2 \psi_2 + g_{12} |\psi_1|^2 \psi_2 \\
&+ g_{2R} \bar{n}_R \psi_2 + i\hbar \left( R \bar{n}_R - \gamma_C \right) \psi_2,
\end{align*}
\]

(A.1)

and the reservoir

\[
\frac{\partial}{\partial t} \bar{n}_R = P(r, t) - [\gamma_R + R(|\psi_1|^2 + |\psi_2|^2)] \bar{n}_R.
\]

(A.3)

We just consider continuous-wave and spatially uniform pumping, \(P(r, t) = P_0 = \text{const} \) and the system has a threshold pump \(P_{th} = \gamma_C / R \). If \( P < P_{th} \), there are no condensates for \( n_1 = n_2 = 0 \). If \( P > P_{th} \), condensates appear, and the condensate densities and reservoir density are:

\[
n_1 + n_2 = (P_0 - P_{th}) / \gamma_C, \quad \bar{n}_R = \gamma_C / R.
\]

(A.4)

Here, we just consider \( M_1 = M_2 = M \) and both sides are divided by \( g n_i \):

\[
\begin{align*}
\frac{\hbar}{\partial (g n_i) \bar{n}_R} &= \frac{\hbar P}{g n_1^2} - \left[ \frac{\hbar R}{g n_1^2} + \frac{\hbar R}{g n_2^2} (|\psi_1|^2 + |\psi_2|^2) \right] \bar{n}_R \\
\frac{\hbar}{\partial (g n_i) \bar{n}_R} &= \bar{n}_R.
\end{align*}
\]

(A.7)

To simplify the problem, we just discuss the condition \( n_1 = n_2 = n/2 \), so the above equations can be written in a dimensionless form by using scaling unit of healing length: \( \bar{r} = r / (\sqrt{m} a) \) with \( c_r = a \sqrt{m} \) is sound velocity and \( t = t / \sqrt{m} a / c_r \) is timescaling. Then we will use transformation \( \bar{\psi}_1 = \psi_1 / \sqrt{\bar{n}_R} \) and define new parameters: \( \bar{g}_{12} = g_{12} / g, \bar{g}_R = g_{2R} / g, \bar{R} = R / \sqrt{m} a / c_r, \bar{\gamma}_C = \gamma_C / g n_1^2, \bar{n}_R = n_R / n_1, \bar{P} = \frac{\hbar P}{g n_1^2}, \) and \( \bar{\gamma}_R = R / g n_1^2 \) to get the dimensionless form

\[
\begin{align*}
\frac{i}{\hbar} \frac{\partial}{\partial \bar{t}} \bar{\psi}_1 &= -\frac{\hbar^2}{2M_1} \nabla^2 \bar{\psi}_1 + (|\bar{\psi}_1|^2 + g_{12} |\bar{\psi}_2|^2) \bar{\psi}_1 \\
&+ \bar{\gamma}_R \bar{n}_R \bar{\psi}_1 + i\hbar \left( \bar{R} \bar{n}_R - \bar{\gamma}_C \right) \bar{\psi}_1, \\
\bar{\psi}_1 &= \psi_1(0, t), \\
\end{align*}
\]

(A.8)

\[
\begin{align*}
\frac{i}{\hbar} \frac{\partial}{\partial \bar{t}} \bar{\psi}_2 &= -\frac{\hbar^2}{2M_2} \nabla^2 \bar{\psi}_2 + (g_{12} |\bar{\psi}_1|^2 + |\bar{\psi}_2|^2) \bar{\psi}_2 \\
&+ \bar{\gamma}_R \bar{n}_R \bar{\psi}_2 + i\hbar \left( \bar{R} \bar{n}_R - \bar{\gamma}_C \right) \bar{\psi}_2, \\
\end{align*}
\]

(A.9)

\[
\frac{\partial}{\partial \bar{t}} \bar{n}_R = \bar{P} - (\bar{\gamma}_R + \bar{R} (|\bar{\psi}_1|^2 + |\bar{\psi}_2|^2)) \bar{n}_R.
\]

(A.10)

The aim of this work is to study the propagation and stability of nonlinear waves; for this, the perturbation of condensate wave function and the reservoir density can written as

\[
\begin{align*}
\bar{\psi}_1 &= \bar{\psi}_1^0 (t) \phi_1 (r, t), \\
\bar{n}_R &= \bar{n}_R^0 + \phi_1 (r, t)
\end{align*}
\]

(A.11)

(A.12)

with \( \bar{\psi}_1^0 (t) = \exp(-i \omega_0 t), \omega_0 = 1 + \bar{g}_R \bar{n}_R^0 \) and \( \bar{n}_R^0 = \bar{\gamma}_C / \bar{R} \).

Here, \( \bar{n}_R^0 \) and \( \bar{\psi}_1^0 \) are steady states and \( \phi_1 (r, t) \) and \( \phi (r, t) \) are perturbations. Next, we will discuss how these perturbations
influence equations. Condensate densities can be described as
\[
\frac{\partial}{\partial t} \psi_i(t) \phi_i(r, t) = -i \left[ \frac{\psi_i(t)}{2} \nabla^2 \phi_i(r, t) + ((\phi_i(r, t))^2 + \delta_{12} |\phi_{3-}(r, t)|^2) \psi_i(t) \phi_i(r, t) + \frac{\delta_{31}}{\delta_{31}} \nabla^2 \phi_i(t) \phi_i(r, t) + \frac{i}{2} (\bar{R} \delta_{31} + m_R(r, t) - \gamma_c) \psi_i(t) \phi_i(r, t) \right]
\] (A.13)

\[
\frac{\partial}{\partial t} \phi_i(r, t) = -\frac{1}{2} \nabla^2 \phi_i(r, t) + ((\phi_i(r, t))^2 + \delta_{12} |\phi_{3-}(r, t)|^2 - 1) \phi_i(r, t) + \frac{\delta_{31}}{\delta_{31}} \nabla^2 \phi_i(t) \phi_i(r, t) + i \frac{m_R(r, t)}{2} \phi_i(r, t)
\] (A.14)

and reservoir density
\[
\frac{\partial}{\partial t} n_R = \tilde{P} - [\gamma_R + \bar{R}(|\tilde{\psi}|^2 + |\tilde{\phi}|^2)] n_R
\] (A.16)

\[
\frac{\partial}{\partial t} m_R(r, t) = \tilde{P} - \gamma_R n_R - \bar{R}(|\tilde{\psi}|^2 + |\tilde{\phi}|^2) n_R - \gamma_R m_R(r, t) - \bar{R}(|\tilde{\psi}|^2 + |\tilde{\phi}|^2) m_R(r, t)
\] (A.17)

\[
= \tilde{P} - \gamma_R \frac{\gamma_c}{R} - \bar{R}(|\tilde{\psi}|^2 + |\tilde{\phi}|^2) \frac{\gamma_c}{R}
- \gamma_R m_R(r, t) - \bar{R}(|\tilde{\psi}|^2 + |\tilde{\phi}|^2) m_R(r, t)
\] (A.18)

for a threshold \( \tilde{P}_0 = \frac{\gamma_c}{R} \), and so the total density of the condensates \( n = \frac{\tilde{P} - \tilde{P}_0}{\tilde{P}} = \frac{\gamma_c}{R} (|\tilde{\psi}|^2 + |\tilde{\phi}|^2) \), the reservoir can be written as
\[
\frac{\partial}{\partial t} m_R(r, t) = \gamma_R (1 - |\tilde{\psi}|^2 + |\tilde{\phi}|^2)
- \gamma_R m_R(r, t) - \bar{R}(|\tilde{\psi}|^2 + |\tilde{\phi}|^2) m_R(r, t).
\] (A.20)

It is attractive that the \( \tilde{P} \) pump term in above equations vanishes, so we only consider the perturbation of reservoir density \( m_R(r, t) \) as \( \delta n_R \) and \( \delta P_0 \) are canceled out. Finally, we use \( \psi_1 \) and \( m_R \) to mark the condensates’ wave functions and reservoir density and add white noise to our system. By using the above methods, the final two-component exciton–polariton dimensionless equations can be obtained in the main text
\[
i \frac{\partial}{\partial t} \psi_1 = -\frac{1}{2} \nabla^2 \psi_1 + ((|\psi_1|^2 + |\psi_2|^2 - 1) \psi_1
+ g_R m_R \psi_1 + i \frac{\bar{R} m_R \psi_1 + i \frac{d\psi_1^o}{dt}}{dt}.
\] (A.21)

\[
i \frac{\partial}{\partial t} \psi_2 = -\frac{1}{2} \nabla^2 \psi_2 + ((|\psi_2|^2 + |\psi_2|^2 - 1) \psi_2
+ g_R m_R \psi_2 + i \frac{\bar{R} m_R \psi_2 + i \frac{d\psi_2^o}{dt}}{dt}.
\] (A.22)

\[
\frac{\partial}{\partial t} m_R = \gamma_c (1 - |\psi_2|^2 - |\psi_2|^2) - \gamma_R m_R
- \bar{R}(|\psi_1|^2 + |\psi_2|^2) m_R.
\] (A.23)

Appendix B. Steady state and simulation of the pseudospin vector

The steady state of two-component exciton–polaritons has been studied by [26] and the spin problem is also widely researched [26, 61–63]. The pseudospin vector can be defined by \( \vec{S} = \frac{1}{2} (\Psi^\dagger \cdot \sigma \cdot \Psi) \), with \( \sigma \) the standard Pauli matrices and \( \Psi \) the complex spinor wave function, i.e.
\[
i \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + [g + g_{12}] S + (g - g_{12}) S \sigma \sigma] \Psi
\] (B.1)

\[
\frac{\partial}{\partial t} n_R = P - (\gamma_R + 2 R S) n_R.
\] (B.2)

We are interested in spatially homogeneous steady-state solutions, so we consider an ansatz \( \Psi(r, t) = e^{-i \Omega t} (\psi_1^0, \psi_2^0)^T = e^{-i \Omega t} (\sqrt{S} + S \sigma \cdot \vec{S} \cdot \sigma \cdot \vec{S})^T \) and \( n_R(r, t) = n_0^R \). Substituting this ansatz into equations (B.1) and (B.2) gives
\[
\dot{S}_x = -(\gamma_c - R n_R) S_x - 2d g S y S_y,
\] (B.3)

\[
\dot{S}_y = -(\gamma_c - R n_R) S_y + 2d g S x S_x,
\] (B.4)

\[
\dot{S}_z = -(\gamma_c - R n_R) S_z - \Omega S_z,
\] (B.5)

\[
\dot{S} = -(\gamma_c - R n_R) S,
\] (B.6)

\[
\dot{n}_R = P - (\gamma_R + 2 RS) n_R.
\] (B.7)

with \( d g = g - g_{12} \) and \( \Omega \) is Rabi frequency. Setting the left side of the above equations to zero, we can get the steady state of the pseudospins. For two-component BECs, there are two steady states, \( n_1 = n_2 \) and \( n_1 = n_2 \), and these states can be described by \( S_z \). Before presenting numerical calculations, we first briefly provide analytical stationary solutions.

• Spin-unpolarized phase: \( S_z = 0, S_x = 0 \) and \( S_z = S = n_0/2 \), which exists under the condition \( g_{12} < g < 2 \Omega/n_0 \) with \( n_0 = (P - P_0)/(2 \gamma_c) \).
Figure B1. Numerical calculations using equations (B.3)–(B.7) in different phases. (a) Occupations of two-component BECs with different interaction strength. (b) Steady states of pseudospins in the spin-unpolarized phase. (c) Steady states of pseudospins in the spin-polarized phase. Parameters used: $P/E_0 = 20$, $\Omega/E_0 = 0.1$, $\gamma_r/E_0 = 15$, $\gamma_c/E_0 = 1$ and the interaction between different components for spin-unpolarized phase: $g_{12}/g = 0.3$, and spin-polarized phase: $g_{12}/g = 1.5$. In all plots, the length, energy and time are scaled in units of $a = \sqrt{\hbar/m_\text{c}c}$, $E_0 = \hbar c/2$ and $\tau_c = \hbar/E_0$ with $a$ being the experimental length scale (e.g. in GaAs $a = 2\ \mu\text{m}$, $E_0 = 0.66\ \text{meV}$ and $\tau_c = 3\ \text{ps}$).

- Spin-polarized phase: $S_z = 0$, $S_y = 0$ and $S_x = \Omega/(g_{12} - g)$, which exists under the condition $g_{12} > g + 2\Omega/n_0$. Moreover, we obtain $S_z = \pm (n_0/2)\sqrt{1 - [2\Omega/(g - g_{12})n_0]^2}$, the sign being chosen randomly upon Bose condensation.

In figure B1(a), we provide numerical calculations using equations (B.3)–(B.7) for the occupation of the spin-up and spin-down components of the wave function $|\psi_1|^2 = S + S_z$ and $|\psi_2|^2 = S - S_z$, when $g_{12} < g + 2\Omega/n_0$ (solid lines) and when $g_{12} > g + 2\Omega/n_0$ (dotted lines). Parameters are provided in the figure caption. Moreover, we consider that the condensate is initialized with a small asymmetry in spin-up and spin-down occupations. The numerical results shown in figure B1 agree with our analytical results [26]; however, in this paper we just pay attention to the spin-unpolarized phase to study the dark–bright solitons in polaritons.

References

[1] Kevrekidis P G, Frantzeskakis D J, Frantzeskakis D J and Carrterero-González R 2015 The Defocusing Nonlinear Schrödinger Equation (Philadelphia, PA: SIAM)
[2] Kivshar Y S and Malomed B A 1989 Rev. Mod. Phys. 61 763–915
[3] Kartashov Y V, Malomed B A and Torner L 2011 Rev. Mod. Phys. 83 247–305
[4] Frantzeskakis D J 2010 J. Phys. A: Math. Theor. 43 213001
[5] Kevrekidis P and Frantzeskakis D 2016 Rev. in Phys. 1 140–53
[6] Liang Z X, Zhang Z D and Liu W M 2006 Mod. Phys. Lett. A 21 383
[7] Kivshar Y S and Luther-Davies B 1998 Phys. Rep. 298 81–197
[8] Burger S, Bongs K, Dettmer S, Ertmer W, Sengstock K, Sanpera A, Shlyapnikov G V and Lewenstein M 1999 Phys. Rev. Lett. 83 5198–201
[9] Liang Z X, Zhang Z D and Liu W M 2005 Phys. Rev. Lett. 94 050402
[10] Busch T and Anglin J R 2001 Phys. Rev. Lett. 87 010401
[11] Achilleos V, Kevrekidis P G, Rothsos V M and Frantzeskakis D J 2011 Phys. Rev. A 84 053626
[12] Yan D, Chang J J, Hamner C, Kevrekidis P G, Engels P, Achilleos V, Frantzeskakis D J, Carretero-González R and Schmelcher P 2011 Phys. Rev. A 84 053630
[13] Yan D, Tsitoura F, Kevrekidis P G and Frantzeskakis D J 2015 Phys. Rev. A 91 032619
[14] Katsimiga G C, Stockhofe J, Kevrekidis P G and Schmelcher P 2017 Phys. Rev. A 95 013621
[15] Wang W and Kevrekidis P G 2017 Phys. Rev. E 95 032201
[16] Chen Z, Segev M, Coskun T H, Christodoulides D N, Kivshar Y S and Afanasjev V V 1996 Opt. Lett. 21 1821
[17] Ostrovskaya E A, Kivshar Y S, Chen Z and Segev M 1999 Opt. Lett. 24 327
[18] Becker C, Stellmer S, Soltan-Panahi P, Dörchers S, Baumert M, Richter E M, Kronjäger J, Bongs K and Sengstock K 2008 Nat. Phys. 4 496 EP—article
[19] Hamner C, Chang J J, Engels P and Hoffer M A 2011 Phys. Rev. Lett. 106 065302
[20] Middelkamp S, Chang J, Hamner C, Carretero-González R, Kevrekidis P, Achilleos V, Frantzeskakis D., Schmelcher P and Engels P 2011 Phys. Rev. Lett. A 375 642–6
[21] Bersano T M, Gokhroo V, Khamcheh M A, D’Ambroise J, Frantzeskakis D J, Engels P and Kevrekidis P G 2018 Phys. Rev. Lett. 120 063620
[22] Deng H, Haug H and Yamamoto Y 2010 Rev. Mod. Phys. 82 1489–537
[23] Carusotto I and Ciuti C 2013 Rev. Mod. Phys. 85 299–366
[24] Byrnes T, Kim N Y and Yamamoto Y 2014 Nat. Phys. 10 803–13
[25] Wouters M and Carusotto I 2007 Phys. Rev. Lett. 99 140402
[26] Xu X, Hu Y, Zhang Z and Liang Z 2017 Phys. Rev. B 96 144511
[27] Shelykh I A, Kavokin A V, Rubo Y G, Liew T C H and Segev M 1999 Phys. Rev. A 59 032201
[28] Amo A, Pigeon S, Sanvitto D, Sala V G, Hivet R, Ciuti C and Bramati A 2011 Science 332 1167–70
[29] Grosso G, Nardin G, Morier-Genoud F, Léger Y and Deveaud-Plédran B 2011 Phys. Rev. Lett. 107 245301
[30] Grosso G, Nardin G, Morier-Genoud F, Léger Y and Deveaud-Plédran B 2012 Phys. Rev. B 86 020509

ORCID iDs
Xingran Xu @https://orcid.org/0000-0002-4418-7407
[31] Sich M, Krizhanovskii D N, Skolnick M S, Gorbach A V, Hartley R, Skryabin D V, Cerda-Méndez E A, Biermann K, Hey R and Santos P V 2011 Nat. Photonics 6 50 EP—article

[32] Ostrovskaya E A, Abdullaev J, Fraser M D, Desyatnikov A S and Kivshar Y S 2013 Phys. Rev. Lett. 110 170407

[33] Xue Y and Matuszewski M 2014 Phys. Rev. Lett. 112 216401

[34] Smirnov L A, Smirnova D A, Ostrovskaya E A and Kivshar Y S 2014 Phys. Rev. B 89 235310

[35] Xue Y, Jiang Y, Wang G, Wang R, Feng S and Matuszewski M 2018 Opt. Express 26 6267

[36] Ma X, Egorov O A and Schumacher S 2017 Phys. Rev. Lett. 118 157401

[37] Pinsker F and Flayac H 2014 Phys. Rev. Lett. 112 140405

[38] Bloch I, Dalibard J and Zwerger W 2008 Rev. Mod. Phys. 80 885–964

[39] Giorgini S, Pitaevskii L P and Stringari S 2008 Rev. Mod. Phys. 80 1215–74

[40] Buluta I and Nori F 2009 Science 326 108–11

[41] Lahaye T, Menotti C, Santos L, Lewenstein M and Pfau T 2009 Rep. Prog. Phys. 72 126401

[42] Kawaguchi Y and Ueda M 2012 Phys. Rep. 520 253–381 spinor Bose–Einstein condensates

[43] Carretero-González R, Frantzeskakis D J and Kevrekidis P G 2008 Nonlinearity 21 R139

[44] Borgh M O, Keeling J and Berloff N G 2010 Phys. Rev. B 81 235302

[45] Liew T C H, Egorov O A, Matuszewski M, Kyriienko O, Ma X and Ostrovskaya E A 2015 Phys. Rev. B 91 085413

[46] Li W, Chen L, Chen Z, Hu Y, Zhang Z and Liang Z 2015 Phys. Rev. A 91 023629

[47] Askitopoulos A, Kalini K, Liew T C H, Cilibrizzi P, Hatzopoulos Z, Savvidis P G, Berloff N G and Lagoudakis P G 2016 Phys. Rev. B 93 205307

[48] Wouters M, Carusotto I and Ciuti C 2008 Phys. Rev. B 77 115340

[49] Wouters M and Savona V 2009 Phys. Rev. B 79 165302

[50] Winkler K et al 2016 Phys. Rev. B 93 121303

[51] Shelykh I A, Kavokin K V, Kavokin A V, Malpuech G, Bigenwald P, Deng H, Weihs G and Yamamoto Y 2004 Phys. Rev. B 70 035320

[52] Shelykh I A, Rubio Y G, Malpuech G, Solnyshkov D D and Kavokin A 2006 Phys. Rev. Lett. 97 066402

[53] Elizatratov A A and Lozovik Y E 2016 Phys. Rev. B 93 104530

[54] Dunnett K and Zymańska M H 2016 Phys. Rev. B 93 195306

[55] Buchhold M, Everest B, Marcuzzi M, Lesanovsky I and Diehl S 2017 Phys. Rev. B 95 014308

[56] Altman E, Sieberer L M, Chen L, Diehl S and Toner J 2015 Phys. Rev. X 5 011017

[57] Ciuti C, Savona V, Piermarocchi C, Quattropani A and Schwendimann P 1998 Phys. Rev. B 58 7926–33

[58] Takemura N, Trebaol S, Wouters M, Portella-Oberli M T and Deveaud B 2014 Nat. Phys. 10 500–4

[59] Takemura N, Anderson M D, Navadeh-Toupehi M, Oberli D Y, Portella-Oberli M T and Deveaud B 2017 Phys. Rev. B 95 205303

[60] Kevrekidis P G and Frantzeskakis D J 2004 Mod. Phys. Lett. B 18 173

[61] Ohadi H, Dreisamm A, Rubio Y G, Pinsker F, del Valle-Inclan Redondo Y, Tsintzos S I, Hatzopoulos Z, Savvidis P G and Baumberg J J 2015 Phys. Rev. X 5 031002

[62] Ohadi H et al 2017 Phys. Rev. Lett. 119 067401

[63] Sigurdsson H, Ramsay A J, Ohadi H, Rubio Y G, Liew T C H, Baumberg J J and Shelykh I A 2017 Phys. Rev. B 96 155403

[64] Kivshar Y S and Yang X 1994 Phys. Rev. E 49 1657–70

[65] Konotop V V and Pitaevskii L 2004 Phys. Rev. Lett. 93 240403

[66] Kevrekidis P G, Wang W, Carretero-González R and Frantzeskakis D J 2017 Phys. Rev. Lett. 118 244101

[67] Kivshar Y S and Pelinovsky D E 2000 Phys. Rep. 331 117–95

[68] Kuznetsov E A and Turitsyn S K 1988 Sov. Phys. JETP 67 1583

[69] Ma M, Carretero-González R, Kevrekidis P G, Frantzeskakis D J and Malomed B A 2010 Phys. Rev. A 82 023621

[70] Tikhonenko V, Christou J, Luther-Davies B and Kivshar Y S 1996 Opt. Lett. 21 1129–31

[71] Anderson B P, Haljan P C, Regal C A, Feder D L, Collins L A, Clark C W and Cornell E A 2001 Phys. Rev. Lett. 86 2926–9