Dynamic Response Measurement of Elastic Structures Through Smart Digital Image Correlation Method and Calculation

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Abstract. Based on the proper improvement and development of the basic principle of digital image correlation method, a 3D-DIC measurement system is built. Its measurement principle and application process can be divided into four steps: camera calibration, image acquisition, image matching and three-dimensional reconstruction. The measurement system can realize non-contact and full-field measurement and has the advantages of not being affected by the test environment, not imposing additional mass on the test model, and high signal-to-noise ratio of the measured data. Taking the elastic plate model as an example, the accuracy and reliability of the measurement system in the measurement of static and dynamic displacement response are verified by using the measurement results of laser displacement meter as a reference. In addition, the tensile strain of the projectile aluminum alloy tail model and the ground mode of the elastic plate are tested to further verify the practicability of the test system.

Keywords: Digital image correlation method, non-contact and full-field measurement, laser displacement meter, strain, modal.

1. Introduction
At present, there are more and more methods to measure the dynamic response of objects in structural dynamics test, such as acceleration sensor, velocity sensor, laser displacement meter, GPS (Global Positioning System), etc. Based on the characteristics of high sensitivity and high signal-to-noise ratio of accelerometer, non-contact measurement and high reliability of laser displacement meter, Takanori Uchiyama and Keita Shinohara used the results of accelerometer and laser displacement meter to identify MMGs (mechanomyograms) [1]. Asei Tezuka used laser displacement meter to measure the surface pressure of the object under low velocity flow [2]. Ivan Godler developed a rotating acceleration sensor, which enlarged the measuring range and applied it to the acceleration measurement of servo control system [3]. These applications have achieved good results, but because the accelerometer itself has a certain quality, and the measurement range of laser displacement meter is limited, these two methods will no longer be applicable in the long-distance measurement of the deformation of flexible objects. Ding Xiao-li uses a multi-antenna GPS system with all-weather, high efficiency and multi-function to monitor the deformation and movement of slopes and large structures
on the earth [4]. However, due to the limitation of GPS measurement accuracy, it is difficult to apply it to the measurement of small deformation of small objects.

The development of non-contact optical measurement methods based on digital image correlation method began in the 1980s. Initially, the two-dimensional digital image correlation method proposed by relevant scholars has been widely concerned and developed because of its advantages of simple optical path, convenient use and strong anti-interference ability [5]. However, due to the limitation of the method itself, the two-dimensional digital image correlation method is limited to measure the in-plane motion of the object surface or plane, and requires the camera optical axis to be perpendicular to the measured object surface. Nowadays, combined with computer vision technology, three-dimensional digital image correlation method (3D-DIC) emerges as the times require [6-8]. This method overcomes the limitations of two-dimensional method, can measure the shape and deformation of three-dimensional objects, and greatly expands the application of DIC method in engineering measurement [9-11]. With the improvement of digital camera resolution and image processing methods, 3D-DIC method has become an important part of experimental mechanics and has been widely used in optical measurement because of its advantages of non-contact, full-field measurement, high measurement accuracy, convenient use and high signal-to-noise ratio of collected data [12-15].

2. Basic principles

The general steps of dynamic response measurement by using 3D-DIC are camera calibration, image acquisition, image matching, three-dimensional reconstruction.

2.1. Camera calibration

Using the camera to take pictures of the calibration board at different positions, the internal and external parameters of the camera can be calibrated. Among them, the internal parameters mainly refer to the focal length and centre coordinates of the camera, while the external parameters mainly refer to the relative position relationship between the cameras, namely translation and rotation matrix. In this paper, Zhang Zhengyou’s calibration method is used. Figure 1 shows the calibration board, and the grid point is the calibration grid point.

![Calibration Board](image)

Figure 1. Calibration board

The three-dimensional points on the target plane are marked as \( M = [x, y, z]^T \), the two-dimensional points on the image plane are marked as \( m = [u, v]^T \), and the corresponding homogeneous coordinates are \( \tilde{M} = [x, y, z, 1]^T \) and \( \tilde{m} = [u, v, 1]^T \). Based on pinhole imaging model, the mapping relationship between spatial \( M \) and image point \( m \) is as follows:

\[
s\tilde{m} = A[R \ t] \tilde{M}
\]

In which, \( s \) is an arbitrary non-zero scale factor, rotation matrix \( R \) and translation vector \( t \) are the external parameter matrix of the camera, \( A \) is the internal parameter matrix of the camera, which can be written as follows:
\( A = \begin{pmatrix} a_x & r & u_0 \\ 0 & a_y & v_0 \\ 0 & 0 & 1 \end{pmatrix} \) (2)

\((u_0, v_0)\) is the coordinate of the main image points, \(a_x\) and \(a_y\) are the length factors of the \(u\)-axis and \(v\)-axis respectively, and \(r\) is the non-vertical factor of the \(u\)-axis and \(v\)-axis. Usually, it can be assumed that the target plane is on the \(x-y\) plane of the world coordinate system, which is \(z=0\). Record column \(i\) of rotation matrix \(R\) as \(r_i\). From Formula (1), we can get that:

\[
\begin{bmatrix} u \\ y \\ 1 \end{bmatrix} = A \begin{bmatrix} r_1 & r_2 & r_3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ 1 \end{bmatrix} = A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\]

(3)

In this case, \(M = [x, y, z]^T\), \(\tilde{M} = [x, y, z, 1]^T\), the relationship between point \(M\) on target plane and image point \(m\) is as follows:

\[ s\tilde{m} = H\tilde{M} \]

(4)

Where \(H = \lambda A [r_1, r_2, t]\) is a matrix of 3 x 3 and \(\lambda\) is a constant factor. Let \(H = [h_1, h_2, h_3]\), that is:

\[
\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} = \lambda A \begin{bmatrix} r_1 & r_2 & t \end{bmatrix}
\]

(5)

The translation vector \(t\) is the vector from the origin of the actual coordinate system to the centre of light of the camera and \(r_1\) and \(r_2\) are the direction vectors of the projection of the two coordinate axes in the image coordinate system in the world coordinate system. Because \(r_1\) is orthogonal to \(r_2\), so \(\det ([r_1, r_2, t]) \neq 0\) and because \(\det [A] \neq 0\), so \(\det [H] \neq 0\).

The calculation of \(H\) is the process of minimizing the residual between the actual image coordinate \(m_i\) and the image coordinate \(\tilde{m}\) calculated by formula (1). The objective function is:

\[
\min \sum_i \|m_i - \tilde{m}_i\|^2
\]

(6)

After calculating \(H\), two basic equations can be obtained according to the orthogonality of \(R\) and formulas (5):

\[
\begin{align*}
\begin{bmatrix} h_1^T A^{-T} & h_2^T A^{-T} & h_3^T A^{-T} \end{bmatrix} & = 0 \\
\begin{bmatrix} h_1^T A^{-T} \end{bmatrix} A^{-1} h_1 & = h_2^T A^{-T} A^{-1} h_2
\end{align*}
\]

(7)

Formula (5) is two basic constraints on camera internal parameters.

A quadric surface in space can be expressed as \(x^T B x = 0\), where \(\tilde{x} = (x, y, z, 1)^T\), \(B\) is a symmetric matrix of 4x4. Obviously, \(B\) multiplied by any number that is not zero can still represent the same quadric surface. A quadric surface on a plane can be expressed as \(\tilde{x}^T B \tilde{x} = 0\), where \(\tilde{x} = (x, y, 1)^T\), \(B\) is a symmetric matrix of 3x3. Similarly, \(B\) multiplied by any number that is not zero also represents the
same quadric surface. Therefore, \(A^T A^{-1}\) describes the projection of absolute quadratic curve on image plane. Let:

\[
B = A^T A^{-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}
\]

\[
= \begin{bmatrix} \frac{1}{\alpha_x^2} & -\frac{r}{\alpha_x^2 \alpha_y} & \frac{v_0 - r u_0 \alpha_y}{\alpha_x^2} \\ -\frac{r}{\alpha_x^2 \alpha_y} & \frac{r^2}{\alpha_y^2} + \frac{1}{\alpha_y^2} & \frac{r (v_0 - r u_0 \alpha_y)}{\alpha_x^2 \alpha_y} - \frac{v_0}{\alpha_y^2} \\ \frac{v_0 - r u_0 \alpha_y}{\alpha_x^2 \alpha_y} & \frac{r (v_0 - r u_0 \alpha_y)}{\alpha_x^2 \alpha_y} & \frac{(v_0 - r u_0 \alpha_y)^2}{\alpha_x^2 \alpha_y^2} + \frac{v_0^2}{\alpha_y^2} + 1 \end{bmatrix}
\]

Since \(B\) is a symmetric matrix, it can be expressed as the following six-dimensional vector:

\[
b = [B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33}]^T
\]

Let the vector of column \(i\) in \(H\) be \(h_i = [h_{i1}, h_{i2}, h_{i3}]^T\), so:

\[
h_i^T Bh_i = v_i^T b
\]

Where

\[
v_{ij} = \left[h_{i1} h_{j1} + h_{i2} h_{j2} + h_{i3} h_{j3}, h_{i1} h_{j1} + h_{i2} h_{j2} + h_{i3} h_{j3}, h_{i1} h_{j1} + h_{i2} h_{j2} + h_{i3} h_{j3}, h_{i1} h_{j1} + h_{i2} h_{j2} + h_{i3} h_{j3}\right]
\]

Therefore, formula (7) can be written into two homogeneous equations with \(b\) as an unknown number:

\[
\begin{bmatrix} v_{12}^T \\
(v_{11} - v_{22})^T \end{bmatrix} b = 0
\]

Suppose \(n\) images are taken on the target plane and \(n\) equations are superposed to obtain, where \(V\) is a matrix of \(2N \times 6\).

\[
Vb = 0
\]

After solving \(b\), Cholesky matrix decomposition algorithm can be used to find \(A^{-1}\), and then the inverse can be obtained \(A\). At this point, the external parameters of cameras can be obtained by formula (5), that is:

\[
\begin{align*}
r_1 &= \lambda A^{-1} h_1 \\
r_2 &= \lambda A^{-1} h_2 \\
r_3 &= r_1 \times r_2 \\
t &= \lambda \left(A^{-1} h_3\right)
\end{align*}
\]

Where \(\lambda = 1 \parallel A^{-1} h_1\| = 1 \parallel A^{-1} h_2\|\).

Usually, the lens of the camera is distorted. Therefore, it is necessary to take the obtained parameters as initial values and search through optimization algorithm to calculate the exact values of the required parameters. In practical application, two cameras are calibrated simultaneously by using calibration target, and the parameters of two cameras are obtained respectively.

2.2. Image acquisition

Two digital cameras are used to focus the measured object from different angles, and the surface digital images of the measured object before and after deformation are obtained. In order to make the image have matching features, spray paint is usually used to process the surface of the object to make
the image have random texture distribution. As shown in the figure 2, it is an example of speckle texture.

![Figure 2. Speckle texture example](image)

The three-dimensional digital image correlation method can also measure the three-dimensional deformation of the curved surface, so the measured surface can be a curved surface, and at the same time, it allows the normal (off-plane) direction of movement/deformation. However, it is generally necessary to ensure that the motion/deformation of the measured object is within the range of the camera's field of view and depth of field. Therefore, in practical experiments, different lenses or even cameras should be selected according to the motion/deformation of the measured object.

2.3. Image matching

For two digital images, the area to be measured is determined by a specified method, and then the grid points are divided evenly in the area to be measured, which is the point to be measured. Finally, the corresponding position of the point to be measured in the target image is calculated according to the image matching algorithm. In this paper, sub-pixel matching method is used for image matching [6].

The target points and its adjacent image sub-regions are used to match and the reference sub-regions are selected with the target point as the center in the reference image. The corresponding sub-regions in the target image are found by the sub-region matching method. The central position of the target sub-regions is the corresponding position of the target point in the target image. Zero-mean Normalized Sum Squared Difference (ZNSSD) Correlation Function is selected as the correlation function.

\[
C_{ZNSD} = \frac{1}{M} \sum_{m=0}^{M} \sum_{n=0}^{M} \left( \frac{\sum_{m=0}^{M} \sum_{n=0}^{M} (f(x,y)-\bar{f})(g(x,y)-\bar{g})}{\sum_{m=0}^{M} \sum_{n=0}^{M} (g(x,y)-\bar{g})^2} \right)^2
\]  

(10)

In practical applications, considering that the measured object may undergo large deformation or even rotation, it is necessary to introduce a shape function \( W(x,y;p) \) to correspond the pixels in the reference sub-region with those in the target sub-region:

\[
\begin{bmatrix} x' \\ y' \end{bmatrix} = W(x,y;p)
\]  

(11)

Usually, the first-order shape function can allow translation or rotation of the target subarea, even shear and expansion deformation, and can accurately represent the change of the shape of the target subarea.
\[
W_I(x, y; p_I) = \begin{bmatrix}
u_x + I & u_y & u
v_y + I & v & l
\end{bmatrix}
\begin{bmatrix}
u
v
l
\end{bmatrix}
\]

\[
p_I = (u, u_x, u_y, v, v_x, v_y)^T
\]

Based on the first-order shape function and ZNSSD correlation function, the Gauss-Newton (GN) method is used to iteratively calculate the optimal matching between the target sub-region and the reference sub-region [16].

2.4. Three-dimensional reconstruction

Using the internal and external parameters of the camera obtained by camera calibration, the matching results obtained by the two cameras are reconstructed to obtain the three-dimensional coordinates of the monitoring points.

As shown in the figure 3, it is assumed that \( O - xyz \) of the left camera is just at the origin of the world coordinate system without rotation. Its image coordinate system is \( O_l - X_lY_l \) and its effective focal length is \( f_l \). The right camera coordinate system is \( O - x_ry_rz_r \), the image coordinate system is \( or - X_Y \) and the effective focal length is \( f_r \).

**Figure 3.** Three-dimensional reconstruction of spatial points

Firstly, for the right camera, the relationship between the image coordinate system and the world coordinate system is as follows:

\[
s_r [X_r] = \alpha \begin{bmatrix}
0 & u_y & 0
0 & v_x & 0
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
r_7 & r_5 & r_3 & t_z
r_2 & r_5 & r_6 & t_z
r_2 & r_5 & r_6 & t_z
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_w
Y_w
Z_w
1
\end{bmatrix}
\]

\[
S_r = \begin{bmatrix}
\alpha r_7 + u_y r_7
\alpha r_2 + v_x r_2
\alpha r_5 + u_y r_5
\alpha r_3 + u_y t_z
\alpha r_7 + u_y r_7
\alpha r_2 + v_x r_2
\alpha r_5 + u_y r_5
\alpha r_3 + u_y t_z
\end{bmatrix}
\begin{bmatrix}
X_w
Y_w
Z_w
1
\end{bmatrix}
\]

Simplify the formula and eliminate \( S_r \) can obtain:
\[
\begin{pmatrix}
-\alpha_x & 0 & X_x - u_x \\
0 & -\alpha_y & Y_y - v_y
\end{pmatrix}
\begin{bmatrix}
t_x \\
t_y \\
t_z
\end{bmatrix}
= \begin{pmatrix}
\alpha_x r_x + (u_x - X_x) r_x & \alpha_y r_y + (u_y - X_y) r_y & \alpha_z r_z + (u_z - X_z) r_z \\
\alpha_x r_x + (v_x - Y_x) r_x & \alpha_y r_y + (v_y - Y_y) r_y & \alpha_z r_z + (v_z - Y_z) r_z
\end{pmatrix}
\begin{bmatrix}
X_w \\
Y_w \\
Z_w
\end{bmatrix}
\] (14)

Then, for the left camera, because it is located at the origin of the world coordinate system and does not rotate, \( R \) is a unit array and \( t \) are a zero array. \( u_{ij} = v_{ij} = 0 \). The relationship between the image coordinate system and the world coordinate system is as follows:

\[
\begin{bmatrix}
x_l \\
y_l \\
1
\end{bmatrix} = \begin{pmatrix}
\alpha_x & 0 & 0 & 0 & 1 \\
0 & \alpha_y & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{bmatrix}
X_w \\
Y_w \\
Z_w
\end{bmatrix}
\] (15)

Simplify the formula and eliminate \( S_i \), we can obtain:

\[
Z_w \begin{bmatrix}
x_l \\
y_l
\end{bmatrix} = \begin{pmatrix}
\alpha_x \\
\alpha_y
\end{pmatrix} \begin{bmatrix}
X_w \\
Y_w
\end{bmatrix}
\] (16)

Combining Formula 14 and Formula 16, we can find that:

\[
\begin{align*}
X_w &= Z_w X_l / \alpha_x, \\
Y_w &= Z_w Y_l / \alpha_y
\end{align*}
\]

\[
Z_w = \begin{pmatrix}
(X_x - u_x) r_x - \alpha_x r_x & (Y_y - v_y) r_y - \alpha_y r_y \\
\alpha_x r_x + (u_x - X_x) r_x & \alpha_y r_y + (v_y - Y_y) r_y
\end{pmatrix} / \alpha_x / \alpha_y
\] (17)

Where \( X_w \), \( Y_w \), \( Z_w \) are the three-dimensional coordinates of the monitoring points in the world coordinate system.

3. Application of measurement verification

3.1. Dynamic displacement response measurement of spring steel sheet

The figure shows the calibration board image collected by a camera and the object image collected by a dual camera at a certain time.
Before dynamic displacement measurement, the static results of only normal translation and bending-torsion deformation of the measured object are measured by using 3D-DIC system, and compared with the results of laser displacement meter, as shown in Fig. 6. It can be seen from the figure that the 3D-DIC system has a high accuracy in static deformation measurement. The reason why the measurement errors of 3D-DIC and laser displacement meter are larger when the bending and torsion deformation is larger is that the measurement points of laser displacement meter are larger when the bending and torsion deformation is larger.

Subsequently, the dynamic response of the measured model is measured by using 3D-DIC system and laser displacement meter, and the displacement response of the same point is monitored at the same time. As shown in Fig. 7, the dynamic displacement response of the same monitoring point obtained by the two measuring methods is presented. It can be seen that compared with the measurement results of laser displacement meter, the 3D-DIC system has higher measurement accuracy and high signal-to-noise ratio.
Finally, in order to verify the full-field measurement characteristics of the 3D-DIC system, multiple monitoring points are selected for image matching at one time, as shown in Fig.8. Then three-dimensional reconstruction is performed to obtain displacement responses of multiple points at the same time, as shown in Fig.9.

3.2. Ground modal measurement of spring steel sheet

The spring steel sheet is selected for ground modal test, the displacement response of the whole field is obtained by using the 3D-DIC measurement system, and the modal information of the sheet is obtained by using the STD time domain modal parameter identification method. The spring steel sheet is shown in Fig.8. Its parameters are 100 ×150 ×0.2 mm and sweep angle are 30º. The root of the model is fixed in the test. Similar impulse excitation is applied to the model by sampling knock method, and the impulse response data of the whole field of the model are collected. Fig.10 is the impulse response curve and its frequency response function curve of some monitoring points on the surface of the model. From the frequency response function curve, it can be seen that the experimental modal frequencies are in good agreement with the simulation results.

As can be seen from the above figure, the first two modes of the model are better stimulated by the knocking method in this ground modal test. Therefore, for each monitoring point in the field, the first two modal arrays are obtained by using STD time domain modal parameter identification method, and the results of normalization of the maximum mode arrays are shown in Fig.11. The results are basically similar to Nastran modal analysis results, which proves that the measurement system combined with time domain modal parameter identification method, can conduct ground modal test, obtain modal frequency and modal array more accurately, and obtain modal damping ratio when necessary.
3.3. Strain measurement of aluminum alloy tensile

The measurement system can be applied to the strain measurement of supercavitating projectile. The stress and strain of the stabilized supercavitating projectile with tail are complex when it sails across the medium, and the tail is usually made of aluminum alloy.

Aluminium alloy 2A12 specimens were selected for tensile test, and full-field strain measurement was carried out by using 3D-DIC measuring system. The sample model of projectile tail is shown in Figure 12. One of the trapezoidal fins is taken, and the thickness of the sample is 1 mm, the length of the gauge section is 50 mm and the width is 10 mm. Before the test, the speckles are sprayed as uniformly and randomly as possible on the surface of the specimen, as shown in Fig .13

In-plane strain measurement is carried out with a single camera. At this time, the camera calibration only needs to calibrate the internal parameters of a single camera. By collecting the deformation image during the tensile process of the specimen, the full-field strain distribution of the specimen and its change with time can be obtained by image matching. Fig.14 is the strain field distribution cloud obtained after image matching. From Fig.14, it can be seen that the stress/strain of the specimen is large near the middle part and there is stress concentration. According to the test results, the specimen begins to shrink in the middle part of the tensile process and finally breaks from the middle part. At the same time, the strain curves of any three monitoring points on the specimen with time can be extracted from the analysis results, as shown in Fig .15. Therefore, the 3D-DIC measurement system can be used to measure the plane strain/stress field, and can extract the strain curve of the region of interest or the point of interest with time conveniently and quickly, and has high measurement accuracy.
4. Conclusions

Based on digital image correlation method, a displacement/strain measurement system - 3D-DIC measurement system is built. The basic principle and application process of the measurement system can be divided into four steps: camera calibration, image acquisition, image matching and three-dimensional reconstruction. Camera calibration is to obtain the internal and external parameters of two cameras. Image acquisition is to capture the vibration image of the model with cameras at different locations, and image matching is to obtain the coordinates of the monitoring point in the camera at different times, three-dimensional reconstruction is to transform the result of image matching into the physical displacement of each detection point changing with time by using the principle of keyhole imaging.

In order to verify the accuracy and reliability of 3D-DIC measurement system, this system is firstly applied to static deformation measurement, single-point dynamic measurement and multi-point dynamic measurement of flat plate model, and the measured displacement response results are compared with those of laser displacement meter to verify the practicability of the measurement system in displacement measurement. Secondly, the system is applied to the ground modal test of the spring steel sheet, and the first two modal frequencies and modal modes of the model are obtained more accurately. Finally, the system is applied to aluminium alloy test. The strain distribution nephogram of the specimen and the strain curve of the monitoring point are obtained by measuring the tensile strain of the specimen. The results are in agreement with the theoretical analysis results. In conclusion, it is proved that the 3D-DIC measurement system has high accuracy and reliability in static and dynamic displacement and strain measurement and meets the measurement requirements of the test.

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