Two-object remote quantum control

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We consider the two-object remote quantum control for a special case in which all the object qubits are in a telecloning state. We propose a scheme which achieves the two-object remote quantum control by using two particular four-particle entangled states.

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Quantum entanglement exhibits nonlocal correlation between separate systems, which describes that the results of measurements on one system can not be specified independent of the parameters of the measurements on the other systems. The quantum entanglement in multipartite systems describes the quantum correlation between more than two parties, and is more complex and richer than the bipartite entanglement. For example, the bound entanglement (B) is a distinguished kind of entanglement in multipartite systems. The multipartite quantum entanglement has been extensively applied into quantum information processing and quantum computation, such as quantum teleporting (3), quantum remote information concentration (4) and multi-output programmable quantum processor (5). Here we apply the multipartite quantum entanglement into remote quantum control (6). Recently quantum remote control is deeply studied by S. F. Huelga (6,7), B. Reznik (8) and Chui-ping Yang et al. The previous works are based on one object or aimed to creating interaction between separate systems. However, it is important to remotely control several objects simultaneously and individually in quantum network. For simplicity, we consider two-object remote control.

An unknown state $|\chi\rangle = \alpha |0\rangle + \beta |1\rangle$ of a single qubit can be teleported to two spatially separated receivers by a telecloning process (3) simultaneously. After the telecloning process, the qubits held by the two receivers and an ancillary qubit are in a pure three-qubit state $|\psi\rangle_{ABC} = \alpha |\psi_0\rangle_{ABC} + \beta |\psi_1\rangle_{ABC}$, where $\alpha$ and $\beta$ are real numbers, and satisfy $\alpha^2 + \beta^2 = 1$. $|\psi_0\rangle_{ABC}$ and $|\psi_1\rangle_{ABC}$ are defined as:

$$|\psi_0\rangle_{ABC} = \sqrt{\frac{2}{3}} |0\rangle_A |0\rangle_B |0\rangle_C + \sqrt{\frac{1}{6}} (|1\rangle_A |0\rangle_B |1\rangle_C + |1\rangle_A |1\rangle_B |1\rangle_C),$$

$$|\psi_1\rangle_{ABC} = \sqrt{\frac{2}{3}} |1\rangle_A |1\rangle_B |1\rangle_C + \sqrt{\frac{1}{6}} (|0\rangle_A |0\rangle_B |1\rangle_C + |0\rangle_A |1\rangle_B |0\rangle_C),$$

where the qubits B and C are held by the two receivers, the qubit A is an ancilla. That is, the two receivers obtain an optimal copy of the state $|\chi\rangle$ with the fidelity of $\frac{2}{3}$, respectively. In terms of the density matrices, namely,

$$\rho_B = Tr_{A,C}(|\psi\rangle_{ABC}\langle\psi|) = \frac{5}{6} |\chi\rangle \langle \chi | + \frac{1}{6} |\chi^\perp\rangle \langle \chi^\perp |,$$

$$\rho_C = Tr_{A,B}(|\psi\rangle_{ABC}\langle\psi|) = \frac{5}{6} |\chi\rangle \langle \chi | + \frac{1}{6} |\chi^\perp\rangle \langle \chi^\perp |,$$

where $|\chi^\perp\rangle$ denotes a state orthogonal to the state $|\chi\rangle$. Now the problem present is that if the sender is required to remotely control the two optimal cloning state only by local operations and classical communication (LOCC), how do the sender, i.e. the controller to do it? Namely, the controller will remotely control two objects (the two optimal cloning state) by LOCC simultaneously. The task of the two-object remote control can be achieved by pre-sharing two particular four-particle entangled states, a four-party unlockable bound entangled state (3)

$$\rho_{DEFG}^{ub} = \frac{1}{4} \sum_{i=0}^{3} |\Phi^i\rangle_{DE} \langle \Phi^i | \otimes |\Phi^i\rangle_{FG} \langle \Phi^i |,$$

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where $|\Phi^i\rangle_{i=0,1,2,3}$ represent the four Bell states $|\Phi^{0,1}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Phi^{2,3}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$, and a four-particle free entangled state

$$|\varphi\rangle_{PA'B'C'} = \frac{1}{\sqrt{2}}(|0\rangle_P |\phi_0\rangle_{A'B'C'} + |1\rangle_P |\phi_1\rangle_{A'B'C'}),$$

where the states $|\phi_0\rangle_{A'B'C'}$ and $|\phi_1\rangle_{A'B'C'}$ are defined as in Eq. (1). The procedure is illuminated in the Fig. 1. Supposing that three spatially separated parties Alice, Bob and Charlie hold qubits A, B and C in the state $|\psi\rangle_{ABC}$, respectively. The controller sends in advance the qubits E and A' to Alice, the qubits F and B' to Bob, and the qubits G and C' to Charlie, and leaves the qubits D and P for himself. Then the controller begins the work of two-object remote control by LOCC.

First, by exploiting the bound entangled state $\rho^{ab}_{DEF}$, the controller concentrates the information diluted in the state $|\psi\rangle_{ABC}$ back to the state $|\chi\rangle$ of the qubit D under the cooperations of the three parties Alice, Bob and Charlie $\sigma^1$. The three parties are asked to perform individually the Bell-state measurements (BSMs) on the pairs of the qubits in hand, i.e. the qubits A and E, the qubits B and F, and the qubits C and G. Each of them obtains one of the possible outputs $\{ |\Phi^i\rangle_{i=0,1,2,3}\}$ of the BSM, which is associated with a corresponding Pauli operator in the set $\{\sigma^i_{(i=0,1,2,3)}\}$, and communicates the result with the controller, respectively. The controller determines a Pauli operator $\sigma^i$ on his qubit D for retrieving the state $|\chi\rangle$ on the qubit D, according to the product of the three Pauli operators pertaining to each one of the three BSMs. That is, up a global phase factor, $\sigma^i$ is equal to $\sigma^i_{AE} \sigma^i_{BF} \sigma^i_{CG}$, where $l, j, k = 0, 1, 2, 3$ and the subscripts denote the BSMs on the corresponding pairs of qubits. Finally, the pure state $|\chi\rangle$ is recreated on the qubit D in the location of the sender (i.e. the controller).

The process can be analytically interpreted. We joint the state $|\psi\rangle_{ABC}$ with $\rho^{ab}_{DEF}$:

$$|\psi\rangle_{ABC} \otimes \rho^{ab}_{DEF} \otimes_{ABC} |\bar{\psi}\rangle = (|\alpha\rangle_{ABC} + |\beta\rangle_{ABC})\rho^{ab}_{DEF}(\alpha^*_{ABC} \langle \alpha | + \beta^*_{ABC} \langle \beta |)
\rightarrow \frac{1}{64} ((|I\rangle_{AE, BF, CG} \sigma^0_{0D} + |\beta^0\rangle_{1D}) (|\alpha^0\rangle_{0D} + |\beta^0\rangle_{1D}) (|\alpha^0\rangle_{0D} + |\beta^0\rangle_{1D}) (|\alpha^0\rangle_{0D} + |\beta^0\rangle_{1D}))$$
$$+(|II\rangle_{AE, BF, CG} \sigma^1_{0D} + |\beta^1\rangle_{1D}) (|\alpha^1\rangle_{0D} + |\beta^1\rangle_{1D}) (|\alpha^1\rangle_{0D} + |\beta^1\rangle_{1D}) (|\alpha^1\rangle_{0D} + |\beta^1\rangle_{1D})$$
$$+(|III\rangle_{AE, BF, CG} \sigma^2_{0D} + |\beta^2\rangle_{1D}) (|\alpha^2\rangle_{0D} + |\beta^2\rangle_{1D}) (|\alpha^2\rangle_{0D} + |\beta^2\rangle_{1D}) (|\alpha^2\rangle_{0D} + |\beta^2\rangle_{1D})$$
$$+(|IV\rangle_{AE, BF, CG} \sigma^3_{0D} + |\beta^3\rangle_{1D}) (|\alpha^3\rangle_{0D} + |\beta^3\rangle_{1D}) (|\alpha^3\rangle_{0D} + |\beta^3\rangle_{1D}) (|\alpha^3\rangle_{0D} + |\beta^3\rangle_{1D}).$$

where $|I\rangle\langle I|$ marks a set of the combining results of the three BSMs as

$$|I\rangle\langle I| = |\Phi^0\rangle_{AE} \langle \Phi^0 | + |\Phi^0\rangle_{BF} \langle \Phi^0 | + |\Phi^0\rangle_{CG} \langle \Phi^0 | + |\Phi^0\rangle_{AE} \langle \Phi^0 | + |\Phi^0\rangle_{BF} \langle \Phi^0 | + |\Phi^0\rangle_{CG} \langle \Phi^0 |$$
$$+ |\Phi^1\rangle_{AE} \langle \Phi^1 | + |\Phi^1\rangle_{BF} \langle \Phi^1 | + |\Phi^1\rangle_{CG} \langle \Phi^1 | + |\Phi^2\rangle_{AE} \langle \Phi^2 | + |\Phi^2\rangle_{BF} \langle \Phi^2 | + |\Phi^2\rangle_{CG} \langle \Phi^2 |$$
$$+ |\Phi^3\rangle_{AE} \langle \Phi^3 | + |\Phi^3\rangle_{BF} \langle \Phi^3 | + |\Phi^3\rangle_{CG} \langle \Phi^3 | + |\Phi^0\rangle_{AE} \langle \Phi^0 | + |\Phi^0\rangle_{BF} \langle \Phi^0 | + |\Phi^0\rangle_{CG} \langle \Phi^0 |$$
$$+ |\Phi^1\rangle_{AE} \langle \Phi^1 | + |\Phi^1\rangle_{BF} \langle \Phi^1 | + |\Phi^1\rangle_{CG} \langle \Phi^1 | + |\Phi^2\rangle_{AE} \langle \Phi^2 | + |\Phi^2\rangle_{BF} \langle \Phi^2 | + |\Phi^2\rangle_{CG} \langle \Phi^2 |$$
$$+ |\Phi^3\rangle_{AE} \langle \Phi^3 | + |\Phi^3\rangle_{BF} \langle \Phi^3 | + |\Phi^3\rangle_{CG} \langle \Phi^3 |.$$
measurement. Then he broadcasts the results to the three parties Alice, Bob and Charlie. The three parties rotate their respective qubits A’, B’ and C’ by a corresponding Pauli operator σ^i pertaining to the the result of the BSM. At last, the state of the three qubits A’, B’ and C’ are mapped to a state

\[ |\psi\rangle_{A'B'C'} = \alpha' |\phi_0\rangle_{A'B'C'} + \beta' e^{i\theta} |\phi_1\rangle_{A'B'C'} \]  

(6)

with respect to the state \( |\chi\rangle_D \). The process also can be simply expressed in a formulair way,

\[ |\chi_D\rangle \otimes |\varphi\rangle_{PA'B'C'} = (\alpha' |0\rangle_D + \beta' e^{i\theta} |1\rangle_D) \otimes \frac{1}{\sqrt{2}} (|0\rangle_P |\phi_0\rangle_{A'B'C'} + |1\rangle_P |\phi_1\rangle_{A'B'C'}) \]

\[ = \frac{1}{2} (|\Phi^0\rangle_{DP} (\alpha' |\phi_0\rangle_{A'B'C'} + \beta' e^{i\theta} |\phi_1\rangle_{A'B'C'}) + |\Phi^1\rangle_{DP} (\alpha' |\phi_0\rangle_{A'B'C'} - \beta' e^{i\theta} |\phi_1\rangle_{A'B'C'}) + |\Phi^2\rangle_{DP} (\alpha' |\phi_1\rangle_{A'B'C'} + \beta' e^{i\theta} |\phi_0\rangle_{A'B'C'}) + |\Phi^3\rangle_{DP} (\alpha' |\phi_1\rangle_{A'B'C'} - \beta' e^{i\theta} |\phi_0\rangle_{A'B'C'})). \]

It is shown that the state \( |\psi\rangle_{A'B'C'} = \alpha' |\phi_0\rangle_{A'B'C'} + \beta' e^{i\theta} |\phi_1\rangle_{A'B'C'} \) can be retrived by performing corresponding Pauli operators \( \sigma^i \) on each of the three qubits A’, B’ and C’ as analyzed in [3].

From the state \( |\psi\rangle_{A'B'C'} \) after tracing out ancillary qubit A’ and the other qubit the density matrices of the qubits B’ and C’ are given as.

\[ \rho_{B'} = Tr_{A',C'} \left( |\psi\rangle \langle \psi'\rangle \right) = \frac{5}{6} |\chi\rangle \langle \chi' | + \frac{1}{6} |\chi'\rangle \langle \chi' |. \]

\[ \rho_{C'} = Tr_{A',B'} \left( |\psi\rangle \langle \psi'\rangle \right) = \frac{5}{6} |\chi\rangle \langle \chi' | + \frac{1}{6} |\chi'\rangle \langle \chi' |. \]

(7)

By checking the Eqs. (2) and (7), it is found that \( \rho_{B'} \) and \( \rho_{C'} \) are the completely transformed versions of \( \rho_B \) and \( \rho_C \),

\[ \rho_{B'} = Tr_{A',C'} \left( |\psi\rangle \langle \psi'\rangle \right) = UT_{RA,C} (|\psi\rangle \langle \psi|) U^\dagger = U \rho_B U^\dagger, \]

\[ \rho_{C'} = Tr_{A',B'} \left( |\psi\rangle \langle \psi'\rangle \right) = UT_{RA,B} (|\psi\rangle \langle \psi|) U^\dagger = U \rho_C U^\dagger. \]

Consequently, The controller achieves the task of two-object remote control only by LOCC and pre-shared entanglement.

As stated in [8], there is no distillable pairwise entanglement in the unlockable bound entangled state \( \rho^{ub}_{DEFG} \) if no joint operations are allowed for qubits in different locations. So the entanglement in the bound entangled state alone cannot be enough for faithfully transmitting of quantum information by LOCC. It is the entanglement existing in the state \( |\psi\rangle_{ABC} \) that assists the bound entangled state for transmitting the quantum information. The entanglement in the state \( |\psi\rangle_{ABC} \) is crucial for the remote information concentration back to the controller. Therefore the ancillary qubit A plays important role in the two-object remote control so that it cannot be discarded.

As a result, our scheme only uses two particular four-party entangled state to achieve deterministically complete quantum control on two remote objects. The two object qubits and an ancillary qubit are in an entangled state.

Our scheme is easily generalized to the case of more than two objects by using more complex multipartite entangled states. Of course, the entanglement in the whole initial state of the objects and ancillas is required. The multi-object remote quantum control is important in future quantum network. And the scheme can act as a multi-receiver quantum key distribution if the quantum key is encoded in quantum state as operation information. The receivers can retrieve the key by comparing the initial sending state with the final receiving state. Since our scheme involves complicated multipartite entangled state and many qubits, whether there is more effective and simpler scheme of multi-objects remote quantum control is still an open question.

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Fig. 1 The sketch for two-object remote quantum control