Calibrating the Amati relation for Gamma Ray Bursts using measurements from Cosmic Chronometers

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Abstract

Gamma-Ray Bursts (GRBs) can be used as a tool to probe the universe at high redshift. In this regard, the Amati relation which correlates the isotropic equivalent radiant energy ($E_{iso}$) and the spectral peak energy in the GRB rest frame ($E_p$) allows us to use GRBs as distance indicators. However, the circularity issue that arises due to the lack of GRBs at low redshift has motivated several authors to come up with model-independent approaches to investigate this relation. For this same purpose, we use Hubble parameter measurements obtained from the differential age of the galaxies to circumvent the circularity problem. In this work, we apply a non-parametric approach namely Gaussian Process on the observational Hubble data (without assuming any cosmological model or parameters) to determine the luminosity distances needed to calculate $E_{iso}$. We find that the best fit values of the Amati relation parameters are in concordance with the earlier works.

1 Introduction

Accelerated expansion of the universe is fairly well established now [1, 2, 3, 4]. The simplest model consistent with the observations is the spatially flat ΛCDM model, according to which universe is mostly made of dark energy and dark matter with the baryonic matter being a small fraction. The nature of the dark sector is still unknown and hence it becomes important to analyse the model with different observational data. For the same reason, people have been trying to explore new probes in cosmology. GRB is one such tool that has the potential to explore the universe even at large redshift.

Gamma Ray Bursts (GRBs) are very high energy jets (of the order of tens of keV to GeV) and are believed to form from the collapse of massive spinning stars [5]. On the basis of the burst duration, GRBs are classified in two categories: Short GRBs and Long GRBs. The short GRBs usually last for less than 2 seconds while the long GRBs can last from 2 seconds to several minutes [6]. Though the exact mechanism behind these high energy explosions is not very clear, it is believed that the short and long GRBs form from the mergers of the binary neutron stars and core collapse of supernovae respectively [7, 8].

GRBs have been observed up to very high redshifts ($z \sim 9$). Furthermore, the emission is unaffected by the intervening dust. Due to these reasons, GRBs can be considered a good candidate to study the universe

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at high redshifts. Keeping this in mind, several energy-luminosity correlations for GRBs have been proposed in the literature. The earliest correlation discovered by Amati et al., is known as the “Amati Relation” [9, 10, 11, 12]. Other relations like the “Ghirlanda relation”, “Yonetoku relation” and “Liang-Zhang relation” were also proposed later that could make GRBs more suitable for standard candles [13, 14, 15, 16]. The Ghirlanda relation correlates the collimation-corrected energy of the bursts ($E_p$) and the source frame peak energy ($E_p$), i.e., $E_p \propto E_p^{0.7}$. Yonetoku relation is the correlation between the source frame peak energy ($E_p$) and the isotropic luminosity while according to the Liang-Zhang relation, the isotropic gamma-ray energy ($E_{\gamma}^{iso}$) depends on $E_p$ and the rest frame break time of the optical afterglow light curves ($t_b$). It is believed that the tightest out of all these is the Ghirlanda relation. Liang-Zhang correlation doesn’t have any theoretical explanation, it relies only on the phenomenological considerations unlike the Ghirlanda one. On the other hand, it has been found that the Yonetoku relation weakly depends on the redshift which may be responsible for dispersion in the correlation [17]. Though Ghirlanda is the tightest relation its use is limited. This is because, to use the Ghirlanda relation, observations of the jet break is important which is fairly difficult and hence the sample for testing this relation is very small. On the other hand, Swift and Fermi detector observations have confirmed the Amati relation for the long GRBs, making it a widely used relation.

The Amati relation correlates the isotropic equivalent radiant energy ($E_{iso}$) with the spectral peak energy in the GRB rest frame ($E_p$). Isotropic equivalent radiant energy is related to the luminosity distance, ($E_{iso} = 4\pi d_L^2 S(1+z)^{-1}$). Therefore, in order to calculate $E_{iso}$ from the observed GRB prompt fluence ($S$), we need to first estimate the luminosity distance. However, due to the sparsity of GRBs at low redshift, one has to assume a cosmological model. This means that the Amati relation is calibrated by assuming a cosmological model and a “Circularity Problem” is introduced if the same data is used to constrain the cosmological parameters. Apart from this, the Amati relation has also faced the extrinsic scatter problem. However with better data, the scatter in the data have reduced and the relation has become more realistic.

To overcome the circularity issue in the Amati relation, various methods have been proposed in the literature. Some authors have used the GRB data with the $H(z)$ and BAO data to determine the cosmological and GRB correlation parameters simultaneously [18, 19]. However in another approach, several ancillary probes have been used to determine the luminosity distance in a model independent method. For example, authors have used Type Ia SNe data to calibrate the distance modulus of GRBs in [17, 20, 21, 22]. Observational Hubble data have been used in literature to calibrate the Amati relation and approximate the cosmic evolution through a Bezier parametric curve [18, 24, 25, 26]. In a recent work, the angular diameter distances from galaxy clusters have been used to calibrate the Amati relation at low redshifts [27]. In a similar fashion, without making use of any cosmological model, we calibrate the Amati relation at the low redshifts using the $H(z)$ data obtained from the differential age of galaxies. Our work is different from earlier work in the sense that we have not assumed any parametric form or any cosmological assumption to determine the luminosity distance from the $H(z)$ data.

The paper is organized as follows. We describe the data set and our methodology in Section 2 and Section 3 respectively. Results are discussed in Section 4. Finally in Section 5, we discuss our results and present some conclusions.

2 Data set

2.1 GRB data

We used GRB data having 220 data points (referred as A220) in the redshift range, $0.0331 \leq z \leq 8.20$. This data is consolidated in literature (Tables 7 and 8 of ref. [28]). In the data, corresponding to each sample source, the name of the GRB, its redshift, spectral peak energy in the rest frame ($E_p$), and measurement of the bolometric fluence ($S_{bol}$) calculated in the standard rest-frame energy band, i.e. $1-10^4$ keV along with $1\sigma$ confidence level are mentioned. A220 is the union of two samples, i.e A118 ($0.3399 \leq z \leq 8.2$) and A102 ($0.0331 \leq z \leq 6.32$). The A118 data that has 118 long GRBs is further composed of two subsamples, i.e. 93 GRBs and 25 GRBs, collected from [29] and [30] respectively. A102 consist of 102 long GRBs taken from [18, 29].
2.2 Observational Hubble Data

To estimate the luminosity distance of the GRBs, we use Hubble data. The Hubble parameter can be obtained by integrating the luminosity distance. Other than this \( H(z) \) can also be obtained (i) from the differential ages of passively evolving galaxies, also referred as ‘Cosmic Chronometers (CC)’ \([31, 32]\) (ii) measurements of the peaks of Baryons Acoustic Oscillations (BAO) \([3, 33, 34, 35, 36]\) and (iii) redshift drift \([37, 38]\).

In FLRW metric, \( H(z) \) can be expressed as

\[
H(z) = -\frac{1}{(1 + z)} \frac{dz}{dt}
\]  

(1)

It becomes essential to take proper care when using the differential age approach to estimate \( H(z) \). With the spectroscopy of extragalactic objects, redshift measurement is possible upto an accuracy of \( \delta z / z < 10^{-3} \) but measurement of \( dt \) is crucial. To estimate \( dt \) various methods like full spectrum fitting, absorption feature analysis and calibration of specific spectroscopic features are used \([39, 40, 41]\). One of the important issue while using this approach is the degeneracy between the age-metallicity and age-star formation history. To overcome this, passively evolving red galaxies are used. It is assumed that the star formation in such galaxies has been quenched and hence their spectra are dominated by the older stellar population \([31]\). The measurement of differential age minimizes the systematics that could be there if we measure the absolute age. Measurement of \( H(z) \) in this way does not rely on any cosmological model and is purely spectroscopic, making it a strong candidate to check the viability of any cosmological model and assumption. Recently, Mağan et. al (2018) compiled 31 datapoints measurements of \( H(z) \) using differential ages of passively evolving galaxies \([42]\). For our work, we have used 32 Hubble measurements that includes 31 from the Mağan et. al (2018) compilation and one additional data point compiled by Borghi et al. (2022) at \( z = 0.75 \) \([43]\). The redshift range of the data is \( 0.06 \leq z \leq 1.965 \).

3 Methodology

3.1 Gaussian Process

In this work, we need a model-independent estimate of luminosity distance. For this, we first estimate the comoving distance from the Hubble parameter measurements in the redshift range \( 0 < z < 2 \). We apply Gaussian Process (GP) to obtain a continuous smooth curve of \( H(z) \) and after integrating the \( H(z) \) values we get the corresponding comoving distances (\( dC \)). GP is a well known hyper-parametric regression method which aims to reconstruct the shapes of physical functions from data without assuming a parametrized form of the function \([44]\). GP has been extensively used and because of its flexibility and simplicity, it is very useful for functional reconstructions. For example, from a set of measurements, we have \( H(z) \) values and the uncertainty, i.e. \( H(z) \pm \sigma_H \), where the value of \( H(z) \) follows a Gaussian distribution at every point of \( z_i \). Suppose, at an unknown point \( z' \), we want to estimate the value of the function. For this, we have a covariance or kernel function \( k(z, z') \) which indicates that the value of function at \( z \) is not independent of its value at \( z' \) but instead the values are correlated by the kernel function.

Gaussian Process is a non-parametric technique because it only depend on the choice of the covariance function not on the model parameters or any functional form. The covariance function often solely depends on the separation between the \( |z - z'| \)-points. In this analysis, we consider the Squared Exponential or Gaussian kernel function \([41, 45]\) since this function has the characteristic of being infinitely differentiable, which is important for reconstructing a derivative of a function. The Squared Exponential kernel function is

\[
k(z, z') = \sigma_f^2 \exp \left( -\frac{(z - z')^2}{2l^2} \right)
\]  

(2)
where $\sigma_f$ and $\ell$ are the GP hyperparameters which basically regulate the correlation-strength of the function value and the length scale of the correlation in $z$ respectively. Using the observed data, one can estimate the value of $\sigma_f$ and $\ell$ parameters by minimizing a log marginal likelihood function. For maximization, we use flat priors for the $\sigma_f$ and $\ell$ parameters of kernel function.

Once we get the reconstructed $H(z)$ in the required redshift range, $0 < z < 2$, we use the Simpson $\frac{3}{8}$ method for numerical integration of Eq. [3] to obtain the continuous values of $d_C$ in the same redshift range.

$$d_C(z) = \int_0^z \frac{c dz'}{H(z')}$$

where, $c$ is speed of light.

Finally, to obtain luminosity distance we use $d_L = (1 + z)d_C$. The corresponding uncertainties are obtained by propagating the error obtained in $d_C$ using Gaussian Process as shown in Fig. [1]. The $d_L$ obtained by this method is then used with $S_{\text{bolo}}$ given in the GRB data to calculate $E_{\text{iso}}$. It is important to note that only 118 data points lying in the redshift range $0 < z < 2$ of A220 data are considered for this purpose as the Hubble data we have used and hence the luminosity data exists only up to this redshift.

3.2 Parameter Estimation

For this analysis, we use a linear regression relation using the logarithms of $E_{\text{iso}}$ and $E_P$. This relation is generally referred to as the Amati relation which is basically a correlation between isotropic equivalent energy ($E_{\text{iso}}$) and spectrum peak energy in the comoving frame ($E_P$). The Amati relation can be parametrized as

$$\log \left[ \frac{E_P}{1 \text{ keV}} \right] = m \log \left[ \frac{E_{\text{iso}}}{1 \text{ erg}} \right] + c$$

where,

$$E_{\text{iso}} = \frac{4\pi d_L^2 S_{\text{bolo}}}{(1 + z)}$$

Here $S_{\text{bolo}}$ is the bolometric fluence and $d_L$ is luminosity distance which we estimate from the comoving distance as $d_L = d_C(1 + z)$. The factor $(1 + z)$ accounts for the cosmological time dilation effect. The spectrum peak energy, i.e., $E_P$ in the observer frame is given by

$$E_P = E_{P, \text{obs}} (1 + z)$$

Defining,

$$y \equiv \log \left[ \frac{E_P}{1 \text{ keV}} \right], \quad x \equiv \log \left[ \frac{E_{\text{iso}}}{1 \text{ erg}} \right]$$

We can rewrite Eq. [4] as

$$y = mx + c$$

And the associated uncertainties with $y$ and $x$ are given as

$$\sigma_y = \frac{1}{\ln(10)} \left( \frac{\sigma_{E_P}}{E_P} \right), \quad \sigma_x = \frac{1}{\ln(10)} \left( \frac{\sigma_{E_{\text{iso}}}}{E_{\text{iso}}} \right)$$
In the Amati relation mentioned above, we have two parameters namely the slope \((m)\) and the intercept \((c)\). These parameters can be estimated by directly fitting Eq. [8] with the observed GRBs data. In this analysis, these parameters are determined by maximizing the likelihood \((\mathcal{L})\) defined as

\[
-2\ln \mathcal{L} = \sum_i \ln 2\pi \sigma_i^2 + \sum_i \frac{[y_i - (mx_i + c)]^2}{\sigma_i^2}
\]  

(10)

where \(\sigma_i^2 = \sigma_y^2 + m^2 \sigma_x^2 + \sigma_s^2\) and \(\sigma_s\) denotes the intrinsic scatter which specifies the tightness of the Amati relation.

4 Results

Our aim is to constrain the Amati relation parameters \((m, c)\) and the intrinsic scatter \((\sigma_s)\) by maximising the likelihood. For this we require \(E_P\) and \(E_{\text{iso}}\), where \(E_P\) can be obtained directly from the GRB data. But to estimate \(E_{\text{iso}}\), we need to determine luminosity distance \((d_L)\) corresponding to the redshift of GRBs. We used a non-parametric method (Gaussian Process) to reconstruct luminosity distance from the observational Hubble data. The reconstructed \(d_L\) vs \(z\) curve is shown in Fig. [1]. The red dashed curve represents the reconstructed \(d_L\) line while the dark and light grey colors are 1\(\sigma\) and 2\(\sigma\) confidence regions respectively. Black line shows variation of the luminosity distance with redshift for a flat \(\Lambda\)CDM model with \(\Omega_{m0} = 0.3\).

![Figure 1: Red dashed line indicates the reconstructed \(d_L\) versus \(z\) curve and the black line is for a flat \(\Lambda\)CDM model with \(\Omega_{m0} = 0.3\). Bands in dark and light grey color show the 1\(\sigma\) and 2\(\sigma\) confidence regions respectively. The luminosity distance curve for a flat \(\Lambda\)CDM model lies well within the 68\% confidence region of the reconstructed \(d_L\) estimated from Hubble parameter measurements.](image-url)
The best fit values of $m$, $c$ and $\sigma_s$ with 68% confidence level obtained using GRBs dataset.

As stated earlier, the reconstructed $d_L$ values are used to estimate the $E_{\text{iso}}$ which are further used in Eq. [10] along with the spectral peak energy to constrain the Amati relation parameters and the intrinsic scatter. The 68%, 95% and 99% contours of $m$, $c$ and $\sigma_s$ are displayed in Fig. [2]. The contours have been produced using the emcee package in Python.

| Parameters | Galaxy cluster [27] | $H(z)$, our results |
|------------|---------------------|---------------------|
| $m$        | $0.44^{+0.07}_{-0.09}$ | $0.499^{+0.026}_{-0.026}$ |
| $c$        | $-20.10^{+4.62}_{-3.84}$ | $-23.545^{+1.373}_{-1.375}$ |
| $\sigma_s$ | $0.45^{+0.091}_{-0.066}$ | $0.317^{+0.016}_{-0.015}$ |

Table 1: The best fit values of $m$, $c$ and $\sigma_s$ with 68% confidence level obtained using GRBs dataset.

Figure 2: 68%, 95% and 99% contours of the Amati relation parameter, i.e. $m$, $c$ and $\sigma_s$ for the subset of A220 GRB data upto $z < 2$.

5 Conclusions and Discussions

In order to alleviate the circularity problem, it has been proposed that the Type Ia SNe data available in redshift range as GRBs can be used to calibrate luminosity correlations of GRBs [21, 46]. It is assumed that the objects at same redshift should have the same luminosity distance in any cosmology hence the luminosity distance of the Type Ia SNe can be assigned to the GRBs existing at the same redshift or various model independent methods can also be used to obtain $d_L$ vs $z$ plot. But the lack of Type Ia SNe beyond $z \sim 2$ makes it hard to calibrate the correlation at high redshift. One can only calibrate the relation for low redshift GRBs and extend it to high redshift. In the process of doing so, one has to make an assumption that the
GRBs correlation is not evolving with redshift. As most of the GRBs are available at the high redshift, it becomes crucial to test this hypothesis. For this reason, cosmologists have been trying to test this relation with various data sets and methods.

L. Amati et al. proposed a novel technique to determine $d_L$ from the $H(z)$ data in a model independent way. They approximated the Hubble function corresponding to the OHD data points using a Bezier parametric curve obtained from a linear combination of Bernstein basis polynomials [18]. This approach was further used in literature along with the enlarged Hubble data generated using various machine learning tools and BAO data to investigate the correlation [47, 48]. In a recent work, a model independent approach has been used to constrain the Amati relation parameters and the intrinsic scatter [27]. The authors used a non-parametric technique on the Galaxy cluster data to obtain $d_A$ to the GRBs redshift which is further used to determine $d_L$ assuming CDDR to be valid. Recently, N. Liang et al. (2022) calibrated the Amati relation of GRB using Gaussian Process with the Type Ia SNe data. They obtain GRB Hubble diagram with the A219 and A118 samples of GRB and used it to test $\Lambda CDM$ and $\omega CDM$ models [23].

In this work, we use a non-parametric technique (Gaussian Process) to obtain the $d_L$ as a function of $z$ from the Hubble data having 32 data points of Hubble parameter obtained from the differential age of the galaxies in the redshift range $0.0 < z < 2.0$. We used this luminosity distance and the bolometric fluence $s_{bol}$, given in the GRB data, to calculate the bolometric isotropic equivalent radiant energy of GRBs ($E_{iso}$) up to $z \leq 2$. Further, the spectral peak energy ($E_p$) of the GRBs along with the $E_{iso}$ is used to put constraints on the Amati relation parameters and the intrinsic scatter. We believe that our work is advantageous over the earlier works for the following reasons:

- We use $H(z)$ data obtained from the differential age of galaxies that does not rely on any cosmological assumption other than homogeneity and isotropy. In earlier work, a functional form of $H(z)$ (Bezier Polynomial) was used to obtain $d_L$ from $H(z)$ data. This method was applied to calibrate the Amati relation. However, in our work, we obtained $d_L$ using a non-parametric technique Gaussian Process.

- Recently N. Liang et al. used GP with Type Ia SNe data till redshift $z < 1.4$ [23] and G. Govindaraj et al. with Galaxy cluster data up to $z < 0.9$ [27], to calibrate the Amati relation. In the latter, authors have to assume that the Cosmic Distance Duality Relation (CDDR) is valid to get the luminosity distance from the $d_A$ obtained from Galaxy cluster data. It is important to note that we do not make any such cosmological assumption in our work making our data as well as work purely model independent. Also our data is in redshift range $0 \leq z \leq 2$ making it possible to test the validity of correlation up to comparatively higher redshift which is very crucial as most of the GRBs exist at high redshift. Another point to note is that by using this method, the errors in the parameters are smaller than the errors obtained by using the Galaxy Cluster data method as can be seen from Table 1.

Fig. [1] shows the variation of $d_L$ with redshift obtained on application of the Gaussian Process on the $H(z)$ data. This plot provides us the $d_L$ values at $z$ corresponding to the GRBs data up to $z \sim 2$. The $1\sigma$, $2\sigma$ and $3\sigma$ contours of the Amati relation parameters and intrinsic scatter along with their best fit values are shown in Fig. [2]. We found that our results are in good concordance with the recent work in which authors have used galaxy cluster data to constrain the Amati relation parameters (See Table 1).

The aforementioned evolutionary effects in GRB correlations is still under debate and several groups have been investigating the relation with different technique and data [49, 50, 51, 52, 53]. Lin et al. reported reasonable evidence of evolution with redshift for four relations and Wang et al. also found the same result for Amati relation [54, 55]. However some authors claimed that there is no redshift evolution in the Amati relation [28, 56, 57]. As a result, it becomes important to further examine the GRB relation in order to consider them as standard candles. For that reason, the Space-based multiband astronomical Variable Objects Monitor (SVOM), a Sino-French mission which is expected to be launched in the middle of the 2023 and is intended to study Gamma-Ray Burst (GRB) has great importance [58]. Another space mission, Transient High Energy Sky and Early universe Surveyor (THESEUS) is planned to be launched in 2032 [59]. It aims to exploit the high redshift Gamma Ray Bursts in order to explore the early universe. In the light...
of these next generation missions, it is expected that we would have data that could provide better insight on the use of GRBs as standard Candles.

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