Approximating Permutations with Neural Network Components for Travelling Photographer Problem

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Abstract—Most of current inference techniques rely upon Bayesian inference on Probabilistic Graphical Models of observations, and does prediction and classification on observations rather well. Event understanding of machines with observation inputs needs to deal with understanding of the relationship between sets of observations, and thus there is a crucial need to build models and come up with effective data structures to accumulate and organize relationships between observations. Given a set of states probabilistically-related with observations, this paper attempts to fit a permutation of states to a sequence of observation tokens (The Travelling Photographer Problem). We have devised a machine learning inspired architecture for randomized approximation of state permutation, facilitating parallelization of heuristic search of permutations. Our algorithm is able to solve The Travelling Photographer Problem with very small error. We demonstrate that by mimicking components of machine learning such as normalization, dropout, lambda layer with randomized algorithm, we are able to devise an architecture which solves TPP, a permutation NP-Hard problem. Other than TPP, we are also able to provide a 2-Local improvement heuristic for the Travelling Salesman Problem (TSP) with similar ideas.

Keywords—attention mechanism, simulation, approximation algorithm, randomized algorithm, permutations, problem-driven architecture

I. INTRODUCTION

Given a macro-level phenomenon, we often have models of how likely an observation is likely to occur under macro-level context. But when bombarded with a plethora of time-series observation trails, can we simulate phenomenon level simulated roll-out to reconcile the macro-level model with observation trails. This effort will provide context to observation trails and ease reasoning about often unorganized time-series observation trails. A macro-level phenomenon often doesn’t directly dictate the occurrence of any single observation, often only probabilistically correlated with each observation and can be well-modelled with Bernoulli distribution.

\[ \text{observation}_i | \text{state}_m \sim \text{Bernoulli}(p_i^{(m)}) \]

To solve this problem, we draw inspirations and ideas from successful machine learning models such as activation function, layer normalization and working in embedded space to build a permutation approximation architecture. Our permutation approximation algorithm has proven to be a success on synthetic data. The contribution of this paper can be summarized as follows:

- Devised an approximation and randomized architecture which incorporates randomness from Bernoulli simulation in a compact way
- Devised problem-driven architecture which is able to learn from the structure of the problem from model, giving prediction with 1 data point in 1 shot
- Devised an algorithm to solve the Bernoulli stochastic permutation problem at hand.
- Potentially for parallelizing heuristic search for permutations
- Devised a highly accurate data-specific dropout function

The rest of this paper is organized as follows. Section 2 is literature review, followed by definition of Travelling Photographer Problem in Section 3. An overview of our algorithm for Travelling Photographer is introduced in Section 4, followed by elaboration on pseudo state rollout in Section 5. In section 6, we elaborate upon simulated attention mechanism, and customized dropout layer for our architecture. In section 7, we explore Machine Learning Architecture for Approximation (MLAA) as a template, for real-valued version of Travelling Photographer Problem. We perform ablation studies on both synthetic data in section 8 and actual dataset in section 9. In section 10, we customize another MLAA to solve the Travelling Salesman Problem, demonstrating the generalizing capabilities of the architecture across problem types. Finally, we list the limitations of method in section 11 and the paper concludes in section 12.

II. RELATED WORK

This section will introduce related works in two areas, attention mechanism and Travelling Salesman Problem (TSP).

A. Attention Mechanism

Attention has taken the NLP world by storm with the seminal work "Attention is All You Need"[39]. There are many
extensions to the attention mechanisms evident in the number of variants of Transformers, such as Set Transformer[31], Longformer, InsertionTransformer etc. Attention can easily be expanded to solve problems other than NLP, it has also demonstrated capabilities in solving routing problems, image captioning by focusing its attention on some internal subsets of the data. Transformer’s self-attention mechanism is a weighted average importance score of its internal words with respect to a certain word. Attention performed much better on various machine translation task benchmarks, BLEU than traditional statistical machine translation. Since then, there had been many works studying the structure and kernel of Attention[36], claiming that the effectiveness of attention is due to its entangling of positional embedding and word embedding. However, recent work by Yu, Luo and team [44] studied the attention mechanism for computer vision task, and conclude that it is possible to replace attention mechanism with much simpler computational gadgets such as a pooling layer. This view is further supported by Microsoft’s DeBERTa’s [41] super-human performance on SuperGLUE, DeBERTa disentangles positional and word embedding.

B. Travelling Salesman Problem Solver

This section will discuss approximation solvers for the travelling salesman problem. In 1960s, the Christofides algorithm is an approximation algorithm which solves TSP with a worst case approximation factor of 1.5. Christofides algorithm was originally thought of as a placeholder soon to be replaced by more sophisticated algorithm, however, it remained to be the best worst-case bound approximation algorithm for TSP until today. From heuristic search side, the long-standing of heuristic search solvers like LKH-solver remains to be better performing than recent efforts to solve TSP with machine learning models despite much recent efforts. LKH solver is a pair improvement strategy which improves upon a particular TSP solution. Recent efforts to use NLP machine learning tools to solve TSP has attempted to cast TSP encoding and decoding as a translation problem, or as a general heuristic learning problem. There are works that has attempted to train a TSP solver with reinforcement learning methods, whereas [30] trains a soft-actor critic network to learn improvement heuristic for TSP solutions instead of a search heuristic. Both [33], which combined reinforcement learning strategy selector with LKH, and [6] which pre-trains on small-cases of TSP, have successfully solved various hard-cases of TSP. In the general machine learning for NP-hard problem scheme, powerful NP-hard solvers like ReduMIS for Maximum Independent Set problem and Z3 for has been paralleled by a recent work which does general tree search under the guidance of trained neural network. Chen’s work [17] uses a 2-improvement local search strategy on a reduced graph, a combination of search and reinforcement learning techniques.

The Travelling Salesman Problem remains to be a central problem in graph learning community that is open to this day.

III. PROBLEM FORMULATION

Given a set of states which is probabilistically related to observations (model), and a string which collates observations from state traversal (observation trail), we attempt to find the permutation of state which has most likely generated the observation trail.

A. The Travelling Photographer Problem

Given the following paragraph, and an observation trail and some states of agent (Bob), the goal is to identify a sequence of states which has most likely generated the observation trail.

Suppose there is a photographer who is travelling between different cities, one city per day. At different cities, he takes pictures of different scenes at will randomly. Sometimes different cities have exactly the same scenes. Now that the photographer is back at home, he forgot all about his travelling schedule, he now wants to figure out his travelling schedule (permutation of cities). The photos have date it was taken printed below, he wish to use that to find the order of which he visited the cities. The problem can be summarized into identifying a state permutation to a series of noisy observations.

Travelling Photographer Problem can be summarized as the following. City can also be interpreted as states. Given a model of cities. City A:

$$\text{obs}_1 \sim \text{Ber}(0.1); \text{obs}_2 \sim \text{Ber}(0.2)$$

City B:

$$\text{obs}_2 \sim \text{Ber}(0.9); \text{obs}_3 \sim \text{Ber}(0.3)$$

City C:

$$\text{obs}_1 \sim \text{Ber}(0.8); \text{obs}_2 \sim \text{Ber}(0.5); \text{obs}_2 \sim \text{Ber}(0.7)$$

Find a permutation of \{A, B, C\} which has most likely led to observation sequence \{1; 2; 2\&4\}.

IV. HEURISTIC SEARCH FOR PERMUTATION

The algorithm is a heuristic search over the space of permutations on a user-defined embedded space for observations. The scoring of each permutation is in essence a Hammersley-Clifford theorem inspired numerical compression and activation of a state-rollout. We attempt to embed the features of each observation episode via a numerical embedding function as input into neural approximation architecture, and then perform a ranking on the approximated values. The process is divided mostly into two major portions, embedding and activation of input state permutation with respect to transition matrix and embedding and activation of observation with respect to input state permutation. We need to interpret both how likely is a certain observation
sequence given input state permutation and how likely is a certain input state permutation is given the transition matrix. To the ends stipulated above, we have customized activation function which is similar to tanh function for both input state permutation and observation interpretation.

Algorithm 1: Permutation ML-APPROX

Input: Model $model$, State Transition $T$,
Observation $obs$

Output: Best Fit State Permutation $best$

1. $fn =$ DROPOUT FUNCTION $(model, T, obs)$;
2. relevant_perms = permutations (states, dropout = $fn$);
3. foreach perm in relevant_perms do
4. score = SEQUENCE SCORER $(obs, perm, T)$;
5. if score > max_score then
6. best = perm;
7. max_score = score;
8. end
9. end
10. return best

Our architecture incorporates common machine learning layers such as attention linear layer, normalization layer, activation function, lambda layer with multiplication operations. We designed such that optimization space is a monotonically increasing space where if a state is more likely to have produced a set of observation, that particular episode is given a higher score than another state which is less likely to have led to the same set of observation. The algorithm is such that instead of finding a reward from actually rolling out a state’s Bernoulli observations, we try to draw numerical values from a distribution parameterized by the Bernoulli probabilities to replace a roll-out. We score each state’s roll-out with the product of cliques which is positively correlated with the likelihood of a state producing observations at hand. Since the number of observations per episode is not constant, we need to normalize the products of each episode with respect to the number of observations.

A. Attention Mechanism Simulation

We attempt to simulate attention mechanism which tries to learn balance between state roll-out and likelihood of state permutation sequence. The input layer takes both observation to state mapping for each episode, and the probability of length-3 sub-sequences of input state permutation. The algorithm need to also simulate the self-attention mechanism which input state permutation performs on itself.

V. PSEUDO STATE ROLLOUT

This paper attempts to score a pseudo state rollout by repeatedly sampling from a known model, activate the probability that an observation occur with a custom activation function, and return the product of all the activation values as a heuristic for likelihood of state generating the set of observation. The correctness and positive correlation is proven by Hammersley-Clifford Theorem.

Theorem 1 (Hammersley-Clifford Theorem): Assuming satisfaction of weak positivity condition. Let $U$ denote the set of all random variables under consideration, and let $\Theta, \phi_1, \phi_2, ..., \phi_n \subseteq U$ and $\psi_1, \psi_2, ..., \psi_m \subseteq U$ denote
arbitrary sets of variables. If

\[ Pr(U) = f(\Theta) \prod_{i=1}^{n} g_i(\phi_i) = \prod_{j=1}^{m} h_j(\psi_j) \]

for functions \( f, g_1, g_2, \ldots, g_n \) and \( h_1, h_2, \ldots, h_m \), then there exist functions \( h'_1, h'_2, \ldots, h'_m \) and \( g'_1, g'_2, \ldots, g'_n \) such that

\[ Pr(U) = (\prod_{j=1}^{m} h'_j(\Theta \cap \psi_j))(\prod_{i=1}^{n} g'_i(\phi_i)) \]

In other words, \( \prod_{i=1}^{m} h_j(\psi_j) \) provides a template for further factorization of \( f(\Theta) \).

**Definition 1 (Weak Positivity Condition):** Functionally compatible densities \( \pi_1, \ldots, \pi_n \) satisfy the weak positivity condition on a set \( A \subset S \), if there exists a point \( x' \in A \) and a permutation \( (r_1, \ldots, r_n) \) on \( \{1, \ldots, n\} \) such that for almost all points \( x \in A \) and all \( j = 1, \ldots, n \),

\[ \pi_{r_j}(x'_{r_j} | x_{r_1}, \ldots, x_{r_{j-1}}, x_{r_{j+1}}, \ldots, x_{r_n}) > 0 \]

Pseudo State Rollout is an approximation of the reward function from a state roll-out, where reward function rewards occurrences of observation. To give a score which is positively correlated with the likelihood that the given state has produced the observations at hand. The state roll-out can be generated and be embedded with the parameters. The product of all the possible n-cliques of the edge is positively correlated with the factorized probability that a set of observation is associated with a particular state. Factorized clique compressed a state roll-out into a score, it performs the function of an embedding layer. The functions of layer normalization is being simulated with constant multiplications in Pseudo State Rollout.

**Algorithm 3: Pseudo State Rollout**

**Input:** Observation Set at episode t **obs**, State **state**, Model **model**, Clique Size **size**

**Output:** Activation Value

1. initialize **value** as 1;
2. foreach **ob** in **obs** do
   3. **activations** = ROLLOUT
      4. **ACTIVATION**(obs, state, model);
      5. **combis** = combinations(activations, size);
      6. **constant** = \( e^{-\text{sum}(\text{combis})} \);
      7. foreach clique in combis do
         8. **values** = multiply(clique) * **constant**;
      9. end
   10. **score** = \( \sqrt{\text{value} \cdot \text{len}(\text{obs})} \);
11. return **score**

**A. Activation Functions**

The rollout activation function score is proportional to the likelihood that an observation is observed under a particular state. The activation value is thresholded from below with minimum threshold value like a ReLU. The activation function for simulated self-attention mechanism for state permutations is a minimum, instead of maximum, but also possess threshold from below feature.

**Algorithm 4: Rollout Activation Function**

**Input:** Observation Set at episode t **obs**, State **state**, Model **model**

**Output:** Activation Value

1. initialize **draws** and **thresh** values, and **activations** as empty array;
2. foreach **ob** in **obs** do
   3. if **ob** in **model**(state).key() then
      4. **bp** = **model**(state)[**ob**];
      5. \( \triangleright \) Bernoulli Probability of Observation Occurring Under State:
      6. **distribution** = Gaussian(\( \frac{\text{bp}}{\sqrt{\text{bp}}} \));
      7. **samples** = sample(\( \text{distribution} \), **draws**);
      8. **val** = max(\( \text{thresh} \), max(\( \text{samples} \)));
      9. **activations**.append(**val**);
   10. end
11. end
12. return **activations**

Our activation function incorporates randomness in simulation by activating values drawn from distribution. The usage of activation functions, min, max, median is also intuitive for users, one can direct whether one wants to consider the worst-case, average-case or the best-case in a simulated state roll-out. Our architecture has customized activation function for different input sections of the architecture.

**VI. Attention for Transitions and State Rollouts**

We take both activated state permutation and observation graph as input, and simulate embedding both of it, before feeding both streams of data into a common multiplicative layer. There are two types of attention, self-attention and encoder-decoder attention.

\[ \text{Attention} = \text{Softmax}(\frac{Q \cdot K}{\text{Scale}}) \cdot V^T \]

- \( Q = \) Encoder Layer; \( K = \) Decoder Layer; \( V = \) Learnt Vector
- Self-Attention: \( Q = K = V = \) Source
- Encoder-Decoder Attention: \( K, V = \) Source; \( Q = \) Target
We experimented with different types of encoding and decoding combinations. Proportional to the richness of representation of the mechanism in Natural Language Processing (NLP), the prediction performance also increases accordingly. In FIRST ORDER ATTENTION, we simulated self-attention for input state permutation and performed self-attention on state roll-out keys \( \{ \text{state}[i], \text{obs}[i] \} \) as a whole. Decoder is a multiplicative function which attempts to strike a balance between the two components.

### Algorithm 5: FIRST ORDER ATTENTION

**Input:** Transitions \( \text{transitions} \), State Rollouts \( \text{rollout} \)

**Output:** Heuristic Score

1. initialize \( \text{draws} \) and \( \text{thresh} \) values ;
2. episodes = length(rollout) ;
3. transition\_product, rollout\_product = 1, 1 ;
4. for \( t \) in range(episodes - 2) do
   5. activated = TRANSITION ACTIVATION \((\text{transition}[t] \ast \text{transition}[t + 1])\) ;
   6. transition\_product\_* = activated \* e\(^{-\text{thresh} / \text{draws}}\) ;
5. end
6. transition\_products = \text{transitions}[-1] ;
7. foreach state in rollout do
   8. rollout\_products = rollout ;
5. end
10. score = \( \sqrt{\text{transition\_product} \ast \text{rollout\_product}} \) ;
11. return score

In the second order attention, we process state-observation roll-out and likelihood of state sequence in tandem with each other, drawing inspirations from Bidirectional Encoder Transformer (BERT) models. We score the likelihood of two time-steps of observation being matched to a two time-step state sequence. The likelihood of the matching the entire sequence is positively correlated with the likelihood of each of the two time-step matching in the sequence. We performed self-attention on both input sequences and tuple \( \{ \text{obs}[t], \text{prob(state}[t], \text{state}[t + 1], \text{obs}[t + 1] \} \) as encoder input to the attention block.

The above simulation of attention has proven to be success in producing good decoder score. However, the architecture can be confused when there are too many states to a small number of positions. This leads us to try to improve the architecture with a denoising filter in the next section.

### A. Simulated Dropout Layer

Like in knowledge graph problems, we would like to capture how central is a particular state given an observation trail. We distribute the shard of probability of each of the observations to the states related, and propose that transitions are likely to take place between two states with larger portion of probability shard sum. Centralness of a state is defined as the sum of probability shard of all transitions.
that is connected to it, and is expressed by the matrix \( state, capacity, C \).

We have customized a dropout function for each input observation trail, the objective is to include the most central state for each of the observation sequence. Not only does the simulated dropout improves prediction accuracy, it also effectively speeds up computation, and allow for effective parallelization of the architecture. To achieve this aim, we again borrow ideas from the best practices from machine learning.

**Algorithm 7: Dropout Function**

**Input:** Model \( model \), State Transition \( T \), Observations \( obs \)

**Output:** Dropout Function \( dropout \)

1. initialize \( must\_traverse = [] \);
2. \( state\_capacity, C = \text{TRANSITION CAPACITY} (model, obs) \);
3. \( M = [C, CTC\_CT - CT] \);
4. foreach capacity in \( M \) do
   5. \( edge\_set = \text{MAXIMUM SPANNING EDGES} (C) \);
   6. \( state\_weights = \text{WEIGHT SUM} (edge\_set) \);
   7. \( ranking = \text{sort}(state\_weights, \text{decreasing} = True) \);
   8. \( must\_traverse.append(ranking.pop()) \);
5. \( dropout = \lambda s: \text{return all}(must\_traverse) \) in \( s \);
6. return \( dropout \)

Instead of a random dropout rate, we have an inclusion function which works very well empirically.

**VII. Architecture as Template**

Combining features stated in previous sections, here is a compact way of defining the overall algorithm with custom-made activation functions for each section of the algorithm. Users can possibly modify the behavior of the architecture when performing approximation with varied models and datasets.

**A. Real-Valued Observations**

In this subsection, we consider real-valued input data in observation trail. The model records the observation under state as a normal distribution or other continuous value distribution functions. The machine learning architecture in Permutation ML-Approx serves as a template for solving the problem, and can be adapted accordingly for approximating observation trails which include real-valued observations with customized activation functions which works with real-valued observations.

We input sampled values \( L_1 \) values from the distribution into our architecture to get a score that is proportional to these \( L_1 \) values, reflecting the likelihood that each state has led to these observations.

**Algorithm 8: ML Architecture for Approximation (MLAA)**

**Input:** Model \( model \), Rollout Activation Function \( rollout \), Transition Activation Function \( transition \), Dropout Activation Function \( dropout \), Attention Type \( attention \)

**Output:** Architecture

1. \( dropout\_fn = \text{DROPOUT FUNCTION} (dropout) \);
2. \( pseudo\_rollout = \text{PSEUDO STATE ROLLOUT} (rollout) \);
3. \( attention\_block = \text{ATTENTION}(attention, transition) \);
4. \( architecture = \text{PERMUTATION ML-APPROX} (dropout\_fn, pseudo\_rollout, attention\_block) \);
5. return \( architecture \)

**Algorithm 9: Real-Valued Permutation ML-Approx**

**Output:** function

1. \( function = \text{MLAA}(\text{REAL-VALUED ROLLOUT ACTIVATION}, \text{TRANSITION ACTIVATION FUNCTION}, \text{REAL-VALUED DROPOUT ACTIVATION FUNCTION}, \text{FIRST ORDER ATTENTION}) \);
2. return \( function \)

We have performed experiments on synthetic dataset, and the results are satisfying. By performing ablation studies on several hyperparameters of the neural approximation structure, we have found that this architecture is rather robust for sequence to sequence prediction.

**VIII. Ablation Studies**

**A. Experimental Settings**

The experiment configuration consists of 9 states, each states is probabilistically associated with at most 6 observations. Observation trails are generated by random walks on the transition matrix for number of episode time steps.

The procedure and architecture above has been able to blend well numerically and consistently produce reasonably good predictions. We can see from the plot below that both first and second order compilers improvements improve as the number of episodes increase.
The performance is less satisfactory for less number of episodes because there are more possibilities for each state, whereas for larger number of episodes every state must be bound to a certain position in the output permutation and therefore will return a sequence for which each state is being put to a position of maximum score. We also see that the performance is consistent with the size of clique and number of transitions in each granularity.

Ablation on Repetitions

With repetition and majority vote on state, the architecture is able to find unique state permutation to each observation trail, solving the problem at hand with very low error rate. The increase in accuracy from repetition comes from Law of Large Number.

IX. USE CASE

Given an actual dataset for which a model is not readily available, we need to first construct a model from the dataset itself or otherwise. On Kaggle’s Africa Economic Crisis Data, we have defined states with functions on subsets of columns, and found state permutations for economic history. Algorithms described in this paper have been able to capture contexts in each case’s dataset. For a series of economic observations, ML-A PPROX has been able to interpret economic time-series data with user-defined states using the case’s historical dataset as background context. Results from the architecture has provided a flexible and diversified way of interpreting time-series data.

Given the same series of figures, different conditions have been returned for different cases. The decoding is generally slightly more optimistic for cases with strong economic history and more pessimistic for cases with weak economic history.

X. TRAVELLING SALESMAN PROBLEM HEURISTIC SCORER

In this section, we will appropriate MLAA for the Travelling Salesman Problem. We will present a heuristic which attempts to greedily balance between edge cost minimization and every node’s potential as a local minimizer. While TSP APPROX cannot beat traditional heuristic solver like LKH solver, we think the scoring heuristic presented below is rather novel, and can potentially speed up a TSP heuristic solver because it helps with filtering. Given two permutations, we score both permutations, the one with the larger score is more likely to be part of an optimal solution. Our heuristic can be useful if the permutation space is too large and only subset of states is visible to the algorithm.

Algorithm 10: NODE ACTIVATION

Input: Cost Matrix cost, Permutation perm
Output: node_activation
1 initialize node_activation, prod, clique_act = 1, [], 1;
2 foreach i in perm do
3     edge_cost = \sum_{i,j \in \text{perm}} wij;
4     denom = \sum_{j=\Omega} wij - edge_cost;
5     prod.append(edge_cost/denom);
6 end
7 combis = permutations(prod, 3);
8 k = len(combis)²;
9 foreach combi in combis do
10     p1, p2, p3 = combi;
11     clique_act* = p1 * p2 * p3 * e^{k};
12 end
13 return \sqrt{\text{clique_act}}

We simulate attention mechanism with multiplication operations between node activation outputs and edge activation outputs. Node activation is the indication of how much of the local minimizer potential of each node in the permutation has been exploited by this permutation. We select all cliques
(triangles) and get a product of those to get a score that is proportional to the degree to which each node’s minimizing potential in the permutation is being exploited.

Algorithm 11: LOCAL MINIMA

Input: Cost Matrix cost, States states, Length length

Output: local minima permutation, score

1. \(perms = \text{permutations}(states, length)\);
2. \(\text{foreach perm in perms do}\)
3. \(\text{edge\_activation} = \text{EDGE ACTIVATION}(cost, perm)\);
4. \(\text{node\_activation} = \text{NODE ACTIVATION}(cost, perm)\);
5. \(\text{score} = \text{edge\_activation} \times \text{node\_activation}\);
6. \(\text{if score} \geq \text{best\_score} \text{ then}\)
7. \(\text{best} = \text{perm}\);
8. \(\text{best\_score} = \text{score}\);
9. \(\text{end}\)
10. \(\text{return best, best\_score}\)

Local Minima is simply a wrapper for simulated attention mechanism between edge and node activations. For a set of states, it picks a permutation of states which is the local minima permutation of a particular length, and returns that. We approximate the global optima by concatenating 3-4 local optimas in smaller node subsets, this can speed up computation drastically. Randomness comes from random distribution of states into smaller set of nodes, by repeating the experiments, we are evaluating new partitions of states and determining whether the best solution it produce be a good fit for TSP solution. The greater the score, the more recommendable is the current portion of the solution.

Ablation of TSP Approx

We appropriated MLAA to solve TSP, we obtained performance as follows. We can see that our heuristic is able to figure out an optimal solution very early on.

A. Simulated Attention as Improvement Heuristic

Our simulated attention for TSP can also be appropriated as an improvement heuristic, to give very satisfactory results for TSP20, TSP100 problems, often with error rate of less than 10%, returning the optimal solution at times. TSP solution from our methods can compare to modern heuristic solvers for TSP for TSP20, TSP100. The LKH heuristic is a type of 3-local heuristic, whereas ours is a 2-local heuristic. Our heuristic might possess huge potential at parallelization at large scale. We leave the empirical evaluation of this method on larger cases of TSPs as future work.

XI. LIMITATIONS AND FUTURE WORK

Method discussed in this paper requires careful balancing of weighing factors with normalization constants, the outcome is sensitive to changes in normalization constants. Future works should address methods to robustify the simulated attention approach, to come up with an architecture which allows for both transparency and doesn’t rely heavily on normalization for correct results. It will be a huge improvements if the weights of different components in the input can be automatically explored with randomized optimizers like Stochastic Gradient Descent instead of relying on hardcoded architecture parameters. Future work can also choose to simulate skip connections and prompting.

On top of that, we would also like to expand our algorithms for larger cases of Travelling Photographer Problem and Travelling Salesman Problem with effective parallel implementations.

XII. CONCLUSION

In conclusion, currently machine learning tools still have difficulty to solve probabilistic permutation problems very well, this is evident for Travelling Photographer Problem. We theorized that it is due to the lack of structure for machine learning to converge or regress to. Thus, in order to provide better structure, we attempt to use an architecture inspired by machine learning to perform approximation. We are able to utilize simulated attention mechanism to effectively solve permutation problems, by combining attention to both edge and nodes independently yet simultaneously, we are able to further push the boundaries of randomized algorithms to solve probabilistic permutation (NP-Hard problems) with heuristics that take advantage of both repetition and randomness. Such implementations are not only more intuitive which help us understand the numerical landscape of attention mechanism better, it is also a heuristic for paralleling and randomizing search for permutations. For both Travelling Photographer Problem and Travelling Salesman Problem, our architecture is able to produce satisfactory results for miniature test cases, which serves as a
proof of concept for the feasibility of our machine learning inspired architecture for approximation to solve NP-Hard problems.

REFERENCES

[1] Akari Asai, Kazuma Hashimoto, Hannaneh Hajishirzi, Richard Socher, and Caiming Xiong. Learning to retrieve reasoning paths over wikipedia graph for question answering, 2019.

[2] Xavier Bresson and Thomas Laurent. The transformer network for the traveling salesman problem, 2021.

[3] Jane Chandlee, Rémi Eyraud, Jeffrey Heinz, Adam Jardine, and Jonathan Rawski. Learning with partially ordered representations. CoRR, abs/1906.07886, 2019.

[4] Uthsav Chitra and Benjamin J. Raphael. Random walks on hypergraphs with edge-dependent vertex weights. CoRR, abs/1905.08287, 2019.

[5] Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. BERT: pre-training of deep bidirectional transformers for language understanding. CoRR, abs/1810.04805, 2018.

[6] Zhang-Hua Fu, Kai-Bin Qiu, and Hongyuan Zha. Generalize a small pre-trained model to arbitrarily large tsp instances. Proceedings of the AAAI Conference on Artificial Intelligence, 35(8):7474–7482, May 2021.

[7] Carla P. Gomes, Willem-Jan van Hoeve, and Lucian Leahu. The power of semidefinite programming relaxations for max-sat. Integration of AI and Or Techniques in Constraint Programming, pages 104–118, 2006.

[8] et al. Griffith, M. Civic is a community knowledgebase for expert crowdsourcing the clinical interpretation of variants in cancer., 2017.

[9] Daniel Hsu, Sham M. Kakade, and Tong Zhang. A spectral algorithm for learning hidden markov models, 2012.

[10] Kejun Huang, Xiao Fu, and Nicholas D. Sidiropoulos. Learning hidden markov models from pairwise co-occurrences with application to topic modeling, 2018.

[11] Nathan Kallus. Recursive partitioning for personalization using observational data. volume 70 of Proceedings of Machine Learning Research, pages 1789–1798, International Convention Centre, Sydney, Australia, 06–11 Aug 2017. PMLR.

[12] Jin-Hwa Kim, Jaehyun Jun, and Byoung-Tak Zhang. Bilinear attention networks, 2018.

[13] Wouter Kool, Herke van Hoof, and Max Welling. Attention, learn to solve routing problems!, 2018.

[14] Zhenzhong Lan, Mingda Chen, Sebastian Goodman, Kevin Gimpel, Piyush Sharma, and Radu Soricut. Albert: A lite bert for self-supervised learning of language representations, 2019.

[15] Juho Lee, Yoonho Lee, Jungtaek Kim, Adam R. Kosiorek, Seungjin Choi, and Yee Whye Teh. Set transformer. CoRR, abs/1810.00825, 2018.

[16] Hao Li, Zheng Xu, Gavin Taylor, Christoph Studer, and Tom Goldstein. Visualizing the loss landscape of neural nets. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett, editors, Advances in Neural Information Processing Systems 31, pages 6389–6399. Curran Associates, Inc., 2018.

[17] Zhuwen Li, Qifeng Chen, and Vladlen Koltun. Combinatorial optimization with graph convolutional networks and guided tree search, 2018.

[18] S. Lin and B. W. Kernighan. An effective heuristic algorithm for the traveling-salesman problem, operations research. Operations Research, 1973.

[19] Chenxi Liu, Liang-Chieh Chen, Florian Schroff, Hartwig Adam, Wei Hua, Alan Yuille, and Li Fei-Fei. Auto-deeplab: Hierarchical neural architecture search for semantic image segmentation, 2019.

[20] Jiasen Lu, Dhruv Batra, Devi Parikh, and Stefan Lee. Vilbert: Pretraining task-agnostic visiolinguistic representations for vision-and-language tasks, 2019.

[21] Brian McFee and Gert Lanckriet. Partial order embedding with multiple kernels. In Proceedings of the 26th Annual International Conference on Machine Learning, ICML ’09, page 721–728, New York, NY, USA, 2009. Association for Computing Machinery.

[22] Joseph Mellor, Jack Turner, Amos Storkey, and Elliot J. Crowley. Neural architecture search without training, 2020.

[23] Niv Nayman, Asaf Noy, Tal Ridnik, Itamar Friedman, Rong Jin, and Lihi Zelnik-Manor. Xnas: Neural architecture search with expert advice, 2019.

[24] Byambasuren Odmaa, Yang Yunfei, Sui Zhifang, Dai Dumai, Chang Baobao, Li Sujian, and Zan Hongying. Preliminary study on the construction of chinese medical knowledge graph. Journal of Chinese Information Processing, 33(10):1, 2019.

[25] Jeong Joon Park, Peter Florence, Julian Straub, Richard Newcombe, and Steven Lovegrove. Deepdfo: Learning continuous signed distance functions for shape representation, 2019.

[26] Giorgio Roffo. Ranking to learn and learning to rank: On the role of ranking in pattern recognition applications. CoRR, abs/1706.05933, 2017.

[27] Daniel Selsam, Matthew Lamm, Benedikt Bünz, Percy Liang, Leonardo de Moura, and David L. Dill. Learning a SAT solver from single-bit supervision. In 7th International Conference on Learning Representations, ICLR 2019, New Orleans, LA, USA, May 6-9, 2019. OpenReview.net, 2019.

[28] Mitchell Stern, William Chan, Jamie Kiros, and Jakob Uszkoreit. Insertion transformer: Flexible sequence generation via insertion operations. CoRR, abs/1902.03249, 2019.
[29] Yao-Hung Hubert Tsai, Shaojie Bai, Makoto Yamada, Louis-Philippe Morency, and Ruslan Salakhutdinov. Transformer dissection: A unified understanding of transformer’s attention via the lens of kernel, 2019.

[30] Yaoxin Wu, Wen Song, Zhiguang Cao, Jie Zhang, and Andrew Lim. Learning improvement heuristics for solving routing problems, 2020.

[31] Emre Yolcu and Barnabas Poczos. Learning local search heuristics for boolean satisfiability. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. Fox, and R. Garnett, editors, Advances in Neural Information Processing Systems 32, pages 7992–8003. Curran Associates, Inc., 2019.

[32] Jing Zhang and Xindong Wu. Multi-label inference for crowdsourcing. In Proceedings of the 24th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, KDD ’18, page 2738–2747, New York, NY, USA, 2018. Association for Computing Machinery.

[33] Jiongzheng Zheng, Kun He, Jianrong Zhou, Yan Jin, and Chu-Min Li. Combining reinforcement learning with lin-kernighan-helsgaun algorithm for the traveling salesman problem. Proceedings of the AAAI Conference on Artificial Intelligence, 35(14):12445–12452, May 2021.