NNLO HTLpt predictions for the curvature of the QCD phase transition line

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We present predictions for the second- and fourth-order curvature coefficients of the QCD phase transition line using the NNLO HTLpt-resummed thermodynamic potential. We present three cases corresponding to (i) \( \mu_s = \mu = \mu_B/3 \), (ii) \( \mu_s = 0, \mu = \mu_B/3 \) and (iii) \( S = 0, Q/B = 0.4, \mu = \mu_B/3 \).

In all three cases, we find excellent agreement with continuum extrapolated lattice QCD results for \( \kappa_2 \), given current statistical uncertainties. We also make HTLpt predictions for \( \kappa_4 \) in all three cases, finding again excellent agreement with lattice extractions of this coefficient where available.

I. INTRODUCTION

It is well established that, at low energy density, quarks and gluons are confined within hadrons via the long-range strong interactions present in Quantum Chromodynamics (QCD) and that there is a large chiral condensate. At high temperatures and low net baryon density, numerical lattice QCD calculations find that nuclear matter becomes deconfined and chiral symmetry is restored. The resulting phase diagram of QCD encodes the temperature and chemical potential dependence of these transitions, including the order of each phase transition. The nature of the QCD phase diagram, at finite temperature and baryon density, including the dependence of the phase transition temperature on the net baryon density, has been the subject of very active research in recent years [1–19].

Two regions of the QCD phase diagram are of particular interest, namely, (i) large net baryon density and vanishing temperature and (ii) large temperature and vanishing net baryon density. At large to moderate values of the net baryon density, it is expected that confined nuclear matter becomes deconfined via a first order phase transition. If one moves along the phase transition line from high to low values of the net baryon density, this first order phase transition line ends at the QCD critical point and, beyond this point, there is a smooth crossover at small to vanishing net baryon density. The large temperature and small net baryon density region of the QCD phase diagram can be accessed via various heavy-ion experiments. Currently, the Large Hadron Collider at CERN is focused on the study of deconfined matter at small net baryon densities, whereas the Beam-Energy-Scan program at the Relativistic Heavy Ion Collider in New York is currently focusing on the nature of the transition at larger net baryon density. This latter program will be complemented by future facilities at the Facility for Antiproton and Ion Research in Darmstadt, Germany and the Joint Institute Nuclear Research in Dubna, Russia.

In parallel to the worldwide experimental program, theorists have used various techniques to determine the curvature of the QCD phase transition line in the \( T-\mu_B \) plane, where \( \mu_B \) is the baryochemical potential associated with a given net baryon density. Various methods have been used to extract the curvature of the QCD transition line, including but not limited to direct numerical lattice QCD calculations [1–8], Dyson-Schwinger-Equation approaches [9–11], the Polyakov-loop improved Nambu-Jona-Lasinio model [12], Functional Renormalisation Group approaches [13], and phenomenological extractions of the freeze-out temperature using thermal models [14–19]. In this brief report we calculate the curvature of the QCD phase transition line using next-to-next-to leading-order (NNLO or three-loop) hard-thermal-loop perturbation theory (HTLpt).

For the purposes of this work, we will make use of the temperature and chemical potential dependence of the NNLO HTLpt pressure which was calculated analytically in Ref. [20]. This NNLO HTLpt calculation showed good agreement between the resummed perturbative calculations and continuum extrapolated lattice results for a wide array of lattice observables, including the pressure versus temperature and various susceptibilities. The success of HTLpt can be attributed to the fact that shifting the starting point for the finite-temperature/density loop expansion builds fundamental classical physics related to plasma screening and damping into the calculation, thereby resumming a large class of diagrams that are important in a high temperature/density QGP and curing all infrared problems in the chromoelectric sector [21–40].

Herein, we compare NNLO HTLpt calculations of the curvature of the QCD phase transition line with available lattice data. Typically, lattice studies are restricted to the region of small \( \mu_B \). In the case of the HTLpt predictions, however, we find that the typical quartic fit form used to extract the curvature coefficients works well for \( \mu_B/T \lesssim 1 \). For our main results, we will compare the curvature coefficients extracted in three cases that have been considered in the lattice literature, namely (1) \( \mu_s = \mu_l = \mu_B/3 \), (2) \( \mu_s = 0, \mu_l = \mu_B/3 \), and (3) \( S = 0, Q/B = 0.4, \mu_l = \mu_B/3 \), where \( \mu_s \) and \( \mu_l \) are the strange and light quark chemical potentials, respectively.

In the third case, which most closely mimics real-word conditions generated in relativistic heavy-ion collisions, the strange quark chemical potential is a function of \( T \) and \( \mu_B \), which guarantees that the net strangeness (\( S \)) is zero and that the charge to baryon number ratio (\( Q/B \))
is held fixed. We find that the existing analytic NNLO HTLpt result of Ref. [20] results in very good agreement between resummed perturbative QCD and the world’s lattice data for the curvature coefficients \( \kappa_2 \) and \( \kappa_4 \) in all three cases.

Our brief report is structured as follows. In Sec. II we present a brief overview of the HTLpt formalism. In Sec. III we present our NNLO HTLpt results for the second- and fourth-order curvatures and compare to existing lattice results for \( \kappa_2 \) and \( \kappa_4 \). We present our conclusions in Sec. IV.

II. HTLPT FORMALISM

The Minkowski-space QCD Lagrangian density with massless quarks is [27]

\[
\mathcal{L}_{\text{QCD}} = -\frac{1}{2} \text{Tr} [G_{\mu\nu} G^{\mu\nu}] + \sum_i \bar{\psi}_i [i\gamma^\mu D_\mu - \gamma_0 \mu_i] \psi_i \\
+ \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{ghost}} + \Delta \mathcal{L}_{\text{QCD}} ,
\]

where \( \Delta \mathcal{L}_{\text{QCD}} \) contains the necessary counterterms that cancel the vacuum ultraviolet divergences in perturbative calculations. The gluon field strength is \( G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \) and \( D_\mu = \partial_\mu - igA_\mu \) is the covariant derivative in the fundamental representation. Additionally, in the quark sector there is an explicit sum over the \( N_f \) quark flavors, whereas \( \mathcal{L}_{\text{ghost}} \) represents the ghost term that depends on the choice of the gauge-fixing term \( \mathcal{L}_{\text{gf}} \).

HTL perturbation theory (HTLpt) is a reorganization of bare perturbation theory for QCD at finite temperature and chemical potential(s). The HTLpt Lagrangian density is obtained as

\[
\mathcal{L} = (\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{HTL}})|_{g \to \sqrt{g}} + \Delta \mathcal{L}_{\text{HTL}} ,
\]

where \( \mathcal{L}_{\text{QCD}} \) is the QCD Lagrangian density (1) and \( \mathcal{L}_{\text{HTL}} \) represents the HTL improvement term which can be written as [25, 27, 41]

\[
\mathcal{L}_{\text{HTL}} = (1 - \delta) i m_q^2 \bar{\psi} \gamma^\mu \frac{y_\mu}{y \cdot D} \psi \\
- \frac{1}{2} (1 - \delta) m_q^2 \text{Tr} \left[ G_{\mu\alpha} \left( \frac{y^\alpha y_\beta}{(y \cdot D)^2} \right) y^{\mu\beta} \right] ,
\]

where the first term is due to the quark sector and the second term is due to the gluon and ghost sector. In the quark sector, \( D_\mu \) is the covariant derivative in fundamental representation whereas, in the gluon sector, \( D_\mu \) represents the covariant derivative in the adjoint representation. The four-vector \( y^\mu \equiv (1, \hat{y}) \) is a light-like four-vector which encodes the velocity of the hard quarks and gluons. The parameter \( \delta \) in Eq. (3) is the formal expansion parameter in HTLpt. To calculate the thermodynamic potential, we should truncate the relevant expression at some specific order in \( \delta \), which depends upon the loop-order we are interested in and in the end, one should set \( \delta = 1 \). Note that if one sets \( \delta = 1 \) in Eq. (3), one gets back QCD Lagrangian.

The quantities \( m_q \) and \( m_D \) can be identified with the thermal quark mass and Debye screening mass, respectively. In principle, these are unknown parameters and can be determined in HTLpt by a variational prescription or using effective field theory methods. As in the case of vacuum QCD, the HTLpt expansion produces ultraviolet divergences. In bare perturbation theory, the ultraviolet divergences can be regularized by the counterterm \( \Delta \mathcal{L}_{\text{QCD}} \). Although there is not yet a general proof, it has been shown through three-loop order that all ultraviolet divergences not removed by \( \Delta \mathcal{L}_{\text{QCD}} \) can be removed using simple mass and coupling constant renormalizations which can be collected in \( \Delta \mathcal{L}_{\text{HTL}} \) [20].

III. EXTRACTING THE CURVATURE OF THE QCD PHASE DIAGRAM

At small baryochemical potential, the chemical potential dependence of the crossover line can be expressed as

\[
\frac{T_\mu}{T_0^c} = 1 - \kappa_2 \left( \frac{\mu_B}{T_0^c} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T_0^c} \right)^4 ,
\]

where \( T_\mu \) is the chemical potential dependent crossover temperature and \( T_0^c \) is the crossover temperature at \( \mu_B = 0 \).

In order to extract a phase transition temperature in HTLpt one can extract the temperature at which the NNLO HTLpt resummed pressure goes to zero. Below
this temperature, $T_{P=0}^\mu$, the QGP phase is unstable. In practice, this provides a lower limit on the phase transition temperature since the crossover to a hadron gas occurs before the deconfined QGP phase pressure goes to zero. To obtain the HTLpt estimate for the pressure, we use the NNLO HTLpt expression for the pressure contained in Eq. (4.7) of Ref. [20]. For the Debye mass, thermal quark masses, and one-loop running coupling constant, we use the same prescriptions as Ref. [20]. The final NNLO HTLpt result depends on the temperature $T$, the individual quark chemical potentials $\mu_i$, and the scaled renormalization scales for the quark and gluon sectors, $\Lambda_q$ and $\Lambda_g$. The central values for these two scales are taken to $\Lambda^0_q = 2\pi T$ and $\Lambda^0_g = 2\pi \sqrt{T^2 + \mu^2/\pi^2}$. To estimate the uncertainty associated with the renormalization scale choice, we vary the renormalization scales by a factor of two around the central values, i.e. $\Lambda_q = c\Lambda^0_q$ and $\Lambda_g = c\Lambda^0_g$, with $c \in [1/2, 2]$. The resulting NNLO HTLpt pressure can be seen in Fig. 1 of Ref. [20]. For the central values of the two scales and $\mu_i = 0$, one finds that the pressure vanishes at $T_{P=0}^\mu \approx 148.4$ MeV. The bands shown in Fig. 1 of Ref. [20] indicate the variation of the pressure under $c \in [1/2, 2]$.

For each choice of $c$, one can numerically solve for $T_{P=0}^\mu$ and then fit $\kappa_2$ and $\kappa_4$ using a polynomial fit to the form given in Eq. (4). In practice, we evaluated 100 points in the range $0 \leq \mu_B \leq 100$ MeV. We do this in three distinct physical cases corresponding to

(i) $\mu_s = \mu_l = \mu_B/3$,
(ii) $\mu_s = 0, \mu_l = \mu_B/3$,
(iii) $S = 0, Q/B = 0.4, \mu_l = \mu_B/3$.

The last case corresponds to the physical case with fixed charge to baryon number and zero net strangeness. In this case, $\mu_s$ is a non-trivial function of $\mu_B$ and $T$. Herein, we use lattice data provided by the authors of Ref. [4] for case (3). Our results in these three cases are presented in Figs. 1 and 2. Our results are compared to lattice QCD results obtained using different methods. Refs. [1, 2, 4] used the imaginary chemical potential method and Refs. [3, 5] used the Taylor expansion method to measure $\kappa_2$. Refs. [5] and [4] also reported measurements for $\kappa_4$. These are all collected in Figs. 1 and 2 and Tables I and II. As these Figures and Tables demonstrate, we find excellent agreement between the NNLO HTLpt results and lattice data.

IV. SUMMARY

In this brief report, we presented NNLO HTLpt predictions for the second- and fourth-order curvatures of the QCD phase transition line and compared our results with available lattice QCD measurements of these coefficients. Figs. 1 and 2, together with Tables I and II, demonstrate that NNLO HTLpt is consistent with existing lattice calculations of $\kappa_2$ and $\kappa_4$ in all three cases considered in the lattice QCD literature. In the case of $\kappa_4$, lattice measurements only exist in one of the three cases considered. We made predictions for the other two cases, which can be checked with future lattice calculations.

V. ACKNOWLEDGEMENTS

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TABLE I. Comparison of NNLO HTLpt predictions for $\kappa_2$ with lattice results.

| Case | Lattice $\kappa_2$ | HTLpt $\kappa_2$ |
|------|-------------------|-----------------|
| $\mu_s = \mu_l = \mu_B/3$ | 0.020(4) [1] | 0.0204$^{+0.0006}_{-0.0012}$ |
| $\mu_s = 0, \mu_l = \mu_B/3$ | 0.0135(20) [2] | 0.0145(25) [3] |
| $S = 0, Q/B = 0.4, \mu_l = \mu_B/3$ | 0.0153(18) [4] | 0.0124(5) [5] |
| $\mu_s = 0, \mu_l = 0$ | 0.000032(67) [4] | 0.0000(4) [5] |
| $\mu_l = \mu_B/3$ | 0.000253$^{+0.000012}_{-0.000006}$ |

TABLE II. Comparison of NNLO HTLpt predictions for $\kappa_4$ with lattice results.

| Case | Lattice $\kappa_4$ | HTLpt $\kappa_4$ |
|------|-------------------|-----------------|
| $\mu_s = \mu_l = \mu_B/3$ | $-\mu_s$ | $-\mu_s$ |
| $\mu_s = 0, \mu_l = \mu_B/3$ | $0.00058^{+0.00015}_{-0.00015}$ |
| $\mu_l = \mu_B/3$ | $-\mu_l$ | $-\mu_l$ |
| $S = 0, Q/B = 0.4, \mu_l = \mu_B/3$ | $0.00032(67)$ [4] | $0.0000(4)$ [5] |

FIG. 2. Filled circles are lattice calculations of $\kappa_4$ from Refs. [4, 5], from top to bottom, respectively. The color coding etc. for the symbols is the same as in Fig. 1.
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