The unitary aqueduct method: a new tool for the preliminary design of air chambers

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Abstract. Modern aqueducts supply pressurized water from the available sources to the demanding urban-industrial or agricultural centers. Air chambers, in which air is stored at the pipeline pressure, are widely used in aqueducts, to reduce and to control the effects of the hydraulic transients produced immediately after the accidental or programmed stoppage of the pumps of the system. Dimensionless graphs have been proposed for the preliminary design of the air chambers: Allievi (1937), Angus (1937), Evans and Crawford (1954), Parmakian (1955), to name the classics. These graphs, obtained for the elastic and for the rigid column models of water hammer, allow the calculation of the initial air volume in the chamber and of the maximum and minimum pressures resulting from the hydraulic transients. However, dimensionless equations have not been proposed for the preliminary design of the air chambers. The method of the Unitary Aqueduct, for the preliminary design of the air chambers, is proposed in this paper, presenting dimensionless equations with only two dimensionless parameters, in which all aqueducts are represented, and design tables, from which the complete dimensions of the air chambers can be defined. The findings of this method are compared with the dimensions of a number of existing relevant aqueducts in Mexico, with very good correspondence. The Unitary Aqueduct is defined as a 10 km long aqueduct, with a flow of 2 m$^3$/s, a water velocity of 2 m/s, and an absolute head of 110 m, typical of an aqueduct able to supply a city of 500 000 inhabitants. All possible aqueducts will be a multiple or a fraction of this aqueduct, and since a set of unitary air chambers for this unitary aqueduct can be completely pre-designed, for different relative minimum pressures, the design of an air chamber for a specific aqueduct will be a multiple or a fraction of one of the unitary air chambers of the prototype. Finally, the maximum possible upsurge is calculated and shown in a dimensionless figure. This maximum pressure can be compared with the maximum transient head desired, the difference between the two representing the head to be eliminated at the air chamber entrance or at the pipe connecting the returning flow with the air chamber. The method is also validated with the findings of a large number of experiments, carried out in an experimental installation.

Keywords: Air chambers, waterhammer, hydraulic transients, aqueducts

1. Introduction
We can consider the Russian professor Nikolai Joukowsky (1847 – 1921) and the Italian engineer Lorenzo Allievi (1856 – 1941) as the pioneers in the study of waterhammer. The first, known also for its contributions to aeronautics (he is considered the father of Soviet aeronautics), developed, in the Moscow of the last years of the XIX century, a series of theoretical analyses and experiments from which he could explain the waterhammer, in its causes and effects, such as the overpressures and downpressures, including the extreme case of water column separation. He obtained the expression defining the celerity of the pressure waves in a pipeline that is not rigid but elastic, subject to rapid changes in its boundary conditions, and conveying water considered compressible. He calculated also the maximum magnitude of the pressure change during waterhammer, known today as the “Joukowsky overpressure”, and was the first to propose air chambers to control the downpressures and the overpressures (Simin, 1904).
Lorenzo Allievi, whose professional life had been dedicated to managing industrial enterprises, decided to dedicate instead his time to the study of waterhammer, in the first years of the XX century, following a serious accident occurred in a hydroelectric installation, close to the city of Terni, in which he worked. The accident had been caused by the sudden interruption of flow in one of the main pipes of the installation. He proposed the first solution to the waterhammer equations, known as the “Allievi chain solution”, as well as the first dimensionless parameters to describe the waterhammer (Allievi, 1921).

At this moment, more than one hundred years after the first theoretical developments and many decades after the first manual and graphical calculation methods, precise numerical solutions and computer programs exist for the calculation of waterhammer, and a number of structures and mechanisms have been developed for its control, namely, the surge tanks, which can be simple or restricted at their base; the unidirectional tanks; and the compressed air chambers, all with their respective calculation programs and well defined boundary conditions (Wylie and Streeter, 1993).

Since the first studies of pressurized air chambers (Allievi, 1937; Angus, 1937; Evans and Crawford, 1954; Parmakian, 1955), dimensionless graphs have been proposed for the preliminary design of these chambers, calculating with them the initial air volume in the chamber and the minimum and maximum pressures, resulting from waterhammer, at the pump station and at the midlength of the pipeline. There are two types of graphs: those derived from elastic column models, which consider the compressibility of water and the elasticity of the pipeline, and those coming from rigid column models, which do not consider the compressibility of water nor the elasticity of the pipeline.

The dimensionless graphs, in which all problems of a certain type are solved, are useful to visualize a certain problem and to understand its relevant variables. The dimensionless graphs can be used as well to define the first design possibilities of a project, to study different alternatives, and to choose those that will be part of the final studies in which precise numerical models and eventually physical models will be used for the definitive design (Autrique and Rodal, 2016b).

For this paper, a large number of laboratory experiments are made, in an installation described below, in which an air chamber is installed at the beginning of a pipeline, immediately downstream of a butterfly valve, whose rapid closure produces an extreme waterhammer. The experimental findings are plotted in dimensionless graphs with parameters corresponding to elastic and to rigid column models. As the closure of the valve is practically instantaneous, the inertial effects of the pumps of the system are out of consideration. Under the light of the experiments, the different dimensionless graphs proposed in the literature are analyzed.

A dimensionless equation is derived from the mass oscillation equation, the continuity equation, and the Boyle-Mariotte’s Law for gases. This dimensionless equation relates the main variables in an aqueduct with an air chamber, and is used as the basis of the Unitary Aqueduct Method proposed in this paper for the calculation of air chambers. This method allows the rapid definition, for a particular aqueduct, of one or more preliminary designs of air chambers, having selected previously one or more values of the relative minimum pressure desired. The air chambers of several existing Mexican aqueducts are recalculated with this method.

2. Experimental installation
The experimental installation is shown in Figure 1. It consists in a steel pipe with an outside diameter (OD) of 100 mm, thickness of 1.5 mm (D/t = 67), 282 m long, coupled in series with 2 m of transparent polyvinyl chloride (PVC) pipe, with two hydropneumatic tanks at the beginning and at the end of the pipeline, which guarantees constant heads. A different configuration was also used, in which 112 m of high density polyethylene (HDPE), with D/t = 7, replaced the steel pipe.

Figure 1. Experimental installation layout, with steel pipes in grey and transparent PVC pipes in blue.
At the upstream end of the pipeline, an air chamber in transparent PVC was installed, 300 mm in diameter and 3 m in height. The details of the communication between the air chamber and the steel pipe are shown in Figure 2. All the connection elements between the chamber and the main pipe are also in transparent PVC. A programmable butterfly valve at the upstream end, with a closing time of 0.2 s, allows extreme transients. The main pipe is fed by one or two 11 kw centrifugal pumps, which can be installed in series or in parallel, being able to regulate pressures, flows and water velocities. Transient pressures are measured 500 psi transducers with a frequency of 500 Hz. The non-return (“check”) valve was kept open to simplify the chamber performance (Figure 1). The air chamber does not have a differential restriction at its base, the exit and entry head loss coefficients being similar. The experimental installation is located at the Policonductos plant in San Luis Potosí, México, and was designed with the collaboration of the IIUNAM: Instituto de Ingeniería (Engineering Research Institute) of the National Autonomous University of Mexico (UNAM).

A second installation, similar to the one previously described, is located at the Hydromechanics Laboratory of the IIUNAM, with 104 m of 100 mm PVC pipe, and an acrylic transparent chamber, 127 mm in diameter and 1.3 m in height. The experiments were developed in the frame of an industry-university collaboration. Both physical models are available for future research projects with universities or research institutes.

3. Development of experiments
A total of 45 experiments were made: 25 with a 112 m long HDPE pipe, pressure wave celerity of 500 m/s, absolute pressures from 23 to 58 mwc (meters of water column, expressed as meters (m) from now on), water velocities from 1.3 to 3.1 m/s; 16 experiments with a 290 m long steel pipe, pressure wave celerity of 1120 m/s, initial absolute pressures from 30 to 41 m, water velocities from 1.3 to 2.0 m/s; and 4 experiments with a 104 m PVC pipe, 344 m/s wave celerity, initial absolute pressure of 13 m, and water velocity of 1.0 m/s.

Table 1 shows a sample of the experimental results, for HDPE pipe. With the values of $H_0$, $V_o$, $a$, $A$, $L$, and $C_o$ for each experiment, $H_{min}$ is measured. The dimensionless parameters $\alpha$, $\beta_o$, and $\eta = H_{min}/H_0$ are calculated from $H_{min}$ or from the waterhammer magnitude $M$. All the relevant variables and dimensionless parameters are defined at the end of this paper.

Table 1. Experimental results, HDPE, SLP, 1711.

| No | $Q_o$ | $H_0$ | $V_o$ | $\Delta h_j$ | $M$ | $\Delta h_f$ | $\Delta h_f/ho$ | $y_o$ | $C_o$ | $H_{min}$ | $\alpha$ | $\beta_o$ | $\eta$ |
|----|------|------|------|-------------|----|-------------|--------------|------|-----|---------|---------|---------|-------|
| 1  | 10.0 | 42.0 | 2.0  | 107.5       | 2.56| 6.21        | 0.18         | 0.20 | 0.015 | 24.0    | 6.9     | 2.7     | 0.57  |
| 2  | 10.0 | 42.0 | 2.0  | 107.5       | 2.56| 6.21        | 0.18         | 0.40 | 0.029 | 28.5    | 13.8    | 5.4     | 0.68  |
| 3  | 10.0 | 42.0 | 2.0  | 107.5       | 2.56| 6.21        | 0.18         | 0.60 | 0.044 | 31.5    | 20.7    | 8.1     | 0.75  |
| 4  | 10.0 | 42.0 | 2.0  | 107.5       | 2.56| 6.21        | 0.18         | 1.00 | 0.073 | 33.0    | 34.5    | 13.5    | 0.79  |
| 5  | 9.2  | 23.0 | 1.8  | 98.9        | 4.30| 5.26        | 0.35         | 0.20 | 0.015 | 13.0    | 7.5     | 1.7     | 0.57  |
4. Results

4.1 Dimensionless parameters and dimensionless graphs.

4.1.1 Elastic column model.

For this model, the dimensionless parameter will be called $\alpha$, defined as $\alpha = \frac{(Co/AL)}{a/Vo}$, in which the initial absolute head $Ho$ is not explicit. The dimensionless parameter $\alpha$ is the ratio between the potential energy of the compressed air and the energy stored in the pipeline, as a product of the compressibility of the water and the elasticity of the pipe. In the form that it is presented, can be seen as the product of a volume ratio and a velocity ratio. This parameter can be plotted versus the relative downsurge, or versus the absolute minimum relative pressure. The best known dimensionless graphs are those of Parmakian (1955), which are equivalent to those of Evans and Crawford (1954), and those of Graze and Horlacher (1982).

These dimensionless graphs must include as one of their parameters the magnitude $M$ of the waterhammer (which is the parameter $2\rho$ proposed by Allievi). Separate graphs exist to consider different values of the relative head losses, although the influence of these losses is minor for the effects of the first drawdown, which will define the starting point of the envelope of minimum pressures.

The experimental results are shown in Figure 4, in which we can appreciate clearly the grouping of the experimental points with the same values of $M$, which are compared with the theoretical curves, which are drawn with discontinuous lines.

![Figure 4](image_url) - Experimental results. Elastic column parameters, logarithmic horizontal scale.

![Figure 5](image_url) - Experimental results. Rigid column parameters, logarithmic horizontal scale.

4.1.2 Rigid column model.

For this model, the dimensionless parameter will be called $\beta_0$, defined as $\beta_0 = \frac{(Co/AL)}{g Ho/Vo^2}$, used by Stephenson (2002), although his notation is different. This parameter, which contains explicitly the initial absolute head $Ho$, was proposed originally by Allievi, as $\sigma = \frac{(Co/AL)}{2g Ho/Vo^2}$. The dimensionless parameter $\beta_0$ represents the ratio between the potential energy of the compressed air in the chamber and the kinetic energy contained in the pipeline. In the form that it is presented, it can be seen as the product of a volume ratio and an energy ratio, between potential and kinetic energies. If we divide the “elastic” parameter $\alpha$ by the “rigid”
parameter $\beta_0$, we obtain $M$, the magnitude of the waterhammer: $\alpha / \beta_0 = M$. If we write $\beta_0 = \alpha / M$, we see that the $M$ parameter can be eliminated in a dimensionless graph. In agreement with the rigid column hypotheses, friction is considered negligible. Experimental results are shown in Figure 5. The experimental points have the tendency to group around the theoretical curve for $\beta_0$, as the parameter $M$ does not need to be represented in the dimensionless graph.

5. Deduction of the dimensionless equations

Considering three equations: 1, Boyle-Mariotte’s Law, which assumes a polytropic exponent $n = 1$:

$$
Co Ho = (Co + Cw) H\text{min}
$$

(1)

2, the continuity equation:

$$
\frac{dQ}{dt} = AV
$$

(2)

and 3, the dynamic equation of mass oscillations, for zero friction:

$$
h + \frac{L}{g} \frac{dV}{dt} = 0
$$

(3)

If we integrate this last equation, substitute values from the first two equations, and assume that the decelerating head in the period of time considered is in average one half of the drawdown head $h_{\text{min}}$, we obtain the equation (Stephenson, 2002):

$$
\frac{Co \ g \ Ho}{AL \ Vo^2} = \frac{Ho}{h_{\text{min}}} \left(\frac{Ho}{h_{\text{min}}} - 1\right)
$$

(4)

Further algebra and the introduction of the dimensionless variables $\eta = H\text{min}/Ho$, representing the minimum relative pressure, and $\beta_0 = (Co/AL) \ g \ Ho/Vo^2$, representing the dimensionless initial pressurized air volume, lead to the following dimensionless and exact solution, from which the initial air volume $Co$ can be calculated for a desired minimum pressure:

$$
\beta_0 = \eta / (1 - \eta)^2
$$

(5)

Using again Boyle-Mariotte’s Law, we obtain the equation for $C = Co + Cw$, the total air volume of the chamber:

$$
\beta = 1 / (1 - \eta)^2
$$

(6)

where $\beta = (Co/AL) \ g \ Ho/Vo^2$. With these equations, we can calculate the initial air volume and the total volume of the air chamber, required to obtain the desired minimum relative pressure at the starting point of the pipeline, which we call in this paper $\eta$. This minimum pressure will be the starting point of the envelope of desired minimum pressures. The equations for $\beta_0$ and $\beta$ are drawn in Figure 5. As can be seen clearly in Figure 5, the experiments reported and the existing Mexican aqueducts are very well grouped around the $\beta_0$ curve.

In Table 2, the characteristics of several Mexican aqueducts protected with air chambers are shown (Carmona et al, 2002).

| Table 2. Selected Mexican aqueducts with air chambers. |
|---|---|---|---|---|---|---|---|---|
| | Papagayo II | Los Cabos | Ciudad Victoria | Chapala-Guadalajara | Cuchillo-Monterrey | Chetumal | Sta Rosa-León |
| $L$ (m) | 1358 | 2465 | 13055 | 25720 | 9567 | 8112 | 8400 |
| $D$ (m) | 1.52 | 0.76 | 0.88 | 2.10 | 2.13 | 0.61 | 0.91 |
| $h_{\text{atm}}$ (m) | 10.3 | 10.3 | 10.1 | 8.6 | 9.9 | 10.3 | 8.3 |
| $A$ ($m^2$) | 1.824 | 0.456 | 0.608 | 3.464 | 3.563 | 0.292 | 0.656 |
| $Vo$ (m/s) | 1.096 | 0.987 | 1.645 | 2.165 | 1.684 | 1.164 | 1.520 |
| $a$ (m/s) | 1000 | 906 | 1000 | 1000 | 1000 | 580 | 1000 |
| $Qo$ ($m^3/s$) | 2.0 | 0.45 | 1.0 | 7.5 | 6.0 | 0.34 | 1.0 |
| $H\text{min}$ (m) | 16.0 | 34.9 | 45.8 | 24.9 | 22.2 | 19.0 | 46.0 |
| $Ho$ (m) | 66.5 | 69.8 | 102.8 | 124.3 | 115.7 | 65.4 | 96.0 |
6. Method of the Unitary Aqueduct and recommendations for preliminary design

With the aim of calculating rapidly the order of magnitude or the approximate size of an air chamber for a specific aqueduct, we propose the definition of a “Unitary Aqueduct”, supplied with a “unitary air chamber”, of which any other aqueduct and its respective chamber would be “multiples”.

The proposed Unitary Aqueduct has the following characteristics: flow, 2 m$^3$/s; pipe inside diameter, 1.13 m; pipe transversal area, $A$, 1.0 m$^2$; water velocity, $V_0$, 2.0 m/s; absolute pumping initial head, $H_0$, 110 m; length, $L$, 10 km. This unitary aqueduct has a characteristic value $K_u$, defined as $K_u = ALV_0^2 / H_0$, equal to 364.

This aqueduct, operating 20 hours per day, can supply the potable water requirements of a medium size city of 500,000 inhabitants, with a provision of 288 l/person/day.

For this Unitary Aqueduct, the initial air volume in the chamber and the total chamber volume can be calculated, for a range of minimum relative pressures $\eta$ between 0.2 and 0.7, which represent a reasonable range of minimum pressure envelopes, as shown in Figure 6. There will be one unitary chamber for every value of $\eta$.

If we define the air chamber diameter as 4 times the pipe diameter, we obtain a chamber diameter of 4.50 m, which is in the limit of transportability by road.

Table 3 shows the calculated dimensions of the unitary air chambers, in which each minimum relative pressure has its corresponding unitary air chamber, defined by its initial air volume and by its total volume. From the definitions of $K_u$ and $\beta_o$, we obtain the expression $C_o = K_u \beta_o / g$, and for the proposed characteristics of the Unitary Aqueduct, we obtain $C_o = 37 \beta_o$ for the initial air volume and $C = 37 \beta$ for the total chamber volume, calculating $\beta_o$ and $\beta$ with equations (5) and (6). A safety factor of 1.25 has been considered for the total design height of the chamber ($h_{design}$), and, to calculate its weight, a $D/t$ ratio of 150, which is safe against vacuum collapse (Autrique and Rodal, 2016a). The estimated weights of the air chambers, fabricated in steel, are also shown, to envisage its transportation and its erection.

Figure 6 shows three examples, corresponding to three different topographical profiles, to three minimum pressure envelopes, and to three values of the relative minimum pressure $\eta$, that is, 0.3, 0.5 and 0.7. The three air chambers and their air and water volumes are shown in scale in the figure. The topographical Profile 1 requires the minimum pressure Envelope 1, with $\eta = 0.7$. The corresponding air chamber and its initial air volume are very large. The chamber corresponding to Profile 2 and to Envelope 2, with $\eta = 0.5$, shows a...
reasonable size and balanced initial air and water volumes. Profile 3 and Envelope 3, with $\eta = 0.3$, require a very small chamber, with a small initial air volume.

From Table 3, from Figure 6, and from Boyle-Marriott’s Law (equation 1), which shows that the value of $\eta$ represents as well the fraction of the initial air volume relative to the total volume of the air chamber, that is, $\eta = Co/C$, we can conclude that, from the point of view of the dimensions of the chamber and its effectiveness, values of $\eta$ between 0.4 and 0.6 are advisable, as they control adequately the depressions and have an adequate weight and size. Of particular interest are the chambers with $\eta = 0.5$, for which the initial air volume is one half of the total chamber volume, meaning that the air and water volumes are exactly the same during normal operation conditions and at the beginning of a transient. For values of $\eta$ below 0.4, in which large downsurges can be allowed, the chambers are relatively small, the initial air volume will occupy only a small fraction of them, water occupying most of the chamber total volume. On the contrary, for values of $\eta$ larger than 0.6, for which the topography allows only limited downsurges, the chambers become very large, and the initial air volumes are larger than the initial water volumes.

| $\eta$  | $\beta_o$ | $\beta$ | $Co$  | $C$  | $h_{air}$ | $h_{total}$ | $h_{design}$ | $C_{design}$ | $W_{cham}$ |
|--------|-----------|---------|-------|------|-----------|-------------|--------------|-------------|-----------|
| 0.20   | 0.31      | 1.56    | 11.4  | 57.0 | 0.72      | 3.60        | 4.50         | 72          | 25        |
| 0.25   | 0.44      | 1.78    | 16.2  | 64.8 | 1.02      | 4.08        | 5.10         | 81          | 27        |
| 0.30   | 0.61      | 2.04    | 22.2  | 74.0 | 1.40      | 4.67        | 5.84         | 93          | 29        |
| 0.35   | 0.83      | 2.37    | 30.2  | 86.3 | 1.90      | 5.43        | 6.79         | 108         | 32        |
| 0.40   | 1.11      | 2.78    | 40.4  | 101.0| 2.54      | 6.35        | 7.94         | 126         | 36        |
| 0.45   | 1.49      | 3.31    | 54.2  | 120.5| 3.41      | 7.58        | 9.48         | 151         | 41        |
| 0.50   | 2.00      | 4.00    | 72.7  | 145.4| 4.57      | 9.14        | 11.43        | 182         | 48        |
| 0.55   | 2.72      | 4.94    | 98.9  | 179.8| 6.22      | 11.31       | 14.14        | 225         | 57        |
| 0.60   | 3.75      | 6.25    | 136.4 | 227.3| 8.58      | 14.30       | 17.88        | 284         | 69        |
| 0.65   | 5.31      | 8.16    | 193.1 | 297.1| 12.14     | 18.68       | 23.35        | 371         | 87        |
| 0.70   | 7.78      | 11.11   | 282.9 | 404.1| 18.42     | 26.31       | 32.89        | 523         | 119       |

From the expression $Co = \beta_o AL Vo^2 / (g Ho)$, an aqueduct double in length will require a double initial air volume, or two unitary air chambers. If the flow is increased and the water velocity is kept, the pipe diameter and area will be increased and the the initial air requirements will grow proportionally with the flow. Larger heads will require lower initial air volumes, and lower heads will require larger initial air volumes.

In summary, any particular aqueduct will be a “multiple” of the Unitary Aqueduct, and the air chambers will be “multiples” of the unitary chambers. The diameter of the unitary chambers will be constant, 4.50 m. If the aqueduct under study is a fraction of the unitary aqueduct, the air chambers will reduce its height proportionally. If the aqueduct under study is more than one time the unitary aqueduct, the height or the number of air chambers will increase proportionally.

The value of $K = AL Vo^2 / Ho$ will define the “multiple” or fraction value of the aqueduct under study relative to the Unitary Aqueduct, or relative to its $Ku$ value, being $Ku = 364$. In this way, for example, the aqueduct of Ciudad Victoria, in Mexico (Table 2), with a value of $K = 208$ (Table 4, line 3), is an aqueduct that is $K/Ku = 208/364 = 0.57$ times the unitary aqueduct, and will require 0.57 times the initial air volume of the unitary aqueduct. As $\eta = 0.45$, according to line 6 of Table 3, the initial air volume in the chamber, $Co$, will be $Co = 0.57 * 54.2 = 31 m^3$ (the real value being 39 m$^3$), and the total volume of the chamber, $C$, will be $C = 0.57 * 120.5 = 69 m^3$ (the real value being 80 m$^3$).

| Aqueduct      | $\eta$ | $K$  | $K/Ku$ | $Co$     | $Co$ calculated | $Co$ real | $C$     | $C$ real |
|---------------|--------|------|--------|----------|----------------|-----------|--------|---------|
| Papagayo II   | 0.24   | 45   | 0.12   | 15.3     | 63.8          | 1.8       | 7.7    | 4.0     |
| Los Cabos     | 0.50   | 16   | 0.04   | 72.7     | 145.4         | 3.2       | 6.4    | 7.4     |
Table 4 shows the air chamber calculations for the Mexican aqueducts of Table 2, using the unitary aqueduct method. No security factors have been used. As in the previous example, good correspondence exists between the calculated dimensions of the chambers and the real chambers.

7. Calculation of maximum pressures

Having defined the size of the air chamber according to the allowable downsurge, an estimation of the maximum transient pressures is now required. We have seen that the pipeline friction can be considered negligible during the first drawdown. When the flow reverses in the pipeline and returns to the chamber, the pipeline friction must be considered, as it will reduce the maximum overpressure.

To estimate the maximum possible pressures caused by flow reversal, calculations were made with the TRANS computer program of the IIUNAM, which uses the elastic model equations and the method of characteristics (Wiley and Streeter, 1993), to calculate the maximum pressures for a wide range of cases, based in the Unitary Aqueduct, with variations in water velocity, initial head, and total length, considering friction losses but no chamber entrance losses. In Figure 7, the calculations are shown as colored dots, and three curves have been adjusted: the top curve, for “zero” friction \( f = 0.0001 \), the middle curve, for a hydraulically smooth pipe \( f = 0.01, \) or \( n = 0.009, \) for the unitary aqueduct diameter), and the lower curve, for a rough pipe with “maximum” friction \( f = 0.02, \) or \( n = 0.013, \) for the unitary aqueduct diameter).

It can be observed that the larger maximum relative heads are attained for small values of \( \eta \), and therefore for relative small chambers, as their smaller air volume can be easily compressed to produce a high pressure. In the same way, the maximum relative heads decrease for larger values of \( \eta \), and therefore for larger chambers, as their air volume is larger and more difficult to compress with the same static head. Curves shown in Figure 7 can be used to estimate the maximum relative heads, for initial absolute heads equal or larger than 100 m (for lower initial heads, the maximum relative heads will be increased, especially for \( \eta \) values lower than 0.5, due to the relative high value of the Joukowsky overpressure). For example, for \( \eta = 0.5 \), assuming “zero” friction, a maximum pressure of 2.2 times the initial head could be attained, when the flow returns and enters the chamber, but realistically, for the “smooth pipe” condition, the maximum pressure will be 1.5 times the initial head, for the “smooth pipe” condition. If we consider the examples of Figure 6, Envelope 3, with a minimum relative pressure \( \eta = 0.3 \), will require a very small air chamber. However, from the “smooth pipe” curve of Figure 7, an overpressure of 2.1 times the initial head will turn out from this small chamber. In this case, although more expensive, a larger air chamber could be selected, with \( \eta = 0.5 \), for example, producing an overpressure of only 1.5 times the initial head, with its corresponding savings in the pipeline cost.

![Figure 7. Maximum relative transient pressures (\( \chi \)) for different pipe friction conditions.](image-url)
The maximum relative pressures shown in Figure 7 can be considered adequate for the pressure rating of a certain pipeline material and \(D/t\) ratio, or they can be higher than the maximum desired or acceptable pressures, as represented in a preliminary maximum design pressure envelope. In this case, the pressure in excess of the desired pressure must be eliminated as a head loss at the entrance of the air chamber, through an adequate orifice plate design.

In Figures 8 and 9 the maximum relative pressures obtained from the dimensionless graphs of references [9] and [7] have been plotted out, for zero head losses (Figure 8), and for the estimated maximum possible head losses (0.7 times the initial head \(Ho\), including pipeline friction and air chamber entrance losses), Figure 9, having as background the adjusted curves of Figure 7. These previously published calculations confirm the decrease of the maximum relative pressures for increasing values of \(\eta\), and consequently, for increasing relative sizes of the chambers. The top curve, frictionless, not physically possible, is a reference, and the two lower curves, for smooth and rough pipes, cover a wide range of pipeline ages and materials. However, as the pipes show their lower friction coefficient when installed, we can conservatively select the curve corresponding to smooth pipes as the basis for design, as no lower friction coefficients are physically possible.

As an example, if we consider again the Ciudad Victoria aqueduct, for which \(\eta = 0.45\), the maximum pressure, for a new and hydraulically smooth pipe, will be 1.65 times the initial head (Figure 7). If we assume that the desired maximum pressure is 1.25 times the initial head \(Ho\), we must eliminate, at the entrance of the air chamber, 1.65 times minus 1.25 times = 0.4 times the initial head (which is 102.8 m, from Table 2, line 9), that is, 0.4 * 102.8 m = 41.1 m. This large and very local head loss should be eliminated through a series of orifice plates in the return pipe connection to the air chamber, whose detailed design is out of the scope of this paper.

8. Conclusions

The method of the unitary aqueduct, which includes a dimensionless equation that is an exact solution to the problem of the initial air volume in the pressurized air chamber required to obtain a desired minimum pressure at the beginning of the pipeline, was presented as a tool for the rapid and reasonably precise preliminary design of compressed air chambers. The method requires only the definition of the desired minimum dimensionless pressure at the pump station; from this value, the initial air volume and the total volume of the chamber can be calculated. This method is applicable to all aqueducts. It is shown that any particular aqueduct can be considered a multiple of the Unitary Aqueduct, as this last is defined, and that the possible air chambers for this particular aqueduct are also multiples of a set of pre-dimensioned air chambers of the unitary aqueduct, in terms of the desired minimum dimensionless pressure at the pipeline. The air chambers of several large Mexican aqueducts are calculated with the Unitary Aqueduct Method, finding reasonable correspondence between the calculated dimensions and the real dimensions of the chambers.

In order to determine the maximum possible pressures caused by the returning flow, and the means to limit them to a desired value, a method that considers conservatively a hydraulically smooth pipeline with minimum friction head losses is proposed for its approximate calculation. The difference between this maximum value and the maximum desired overpressure in the pipeline, will determine the head loss required at the entrance of the air chamber.
Findings from an experimental installation were presented in dimensionless graphs corresponding to elastic and to rigid column models, showing that, for a preliminary design, the rigid column method can represent adequately waterhammer in aqueducts protected with air chambers.

Although the dimensionless equations deduced and the Unitary Aqueduct Method presented are precise and useful for preliminary calculations of the air chamber dimensions, and allow the comparison of different design options and the approximation to a final solution, the definitive design must be obtained using adequate computer programs in which the waterhammer differential equations are solved with the method of characteristics and the air chamber boundary condition is formulated properly.

**Notation**

- \( A \): Transversal area of pipeline, \( \text{m}^2 \)
- \( A_c \): Transversal area of air chamber, \( \text{m}^2 \)
- \( a \): Celerity of pressure waves, \( \text{m/s} \)
- \( D \): Internal diameter of pipeline, \( \text{m} \)
- \( C_0 \): Initial air volume in air chamber, \( \text{m}^3 \)
- \( C_w \): Water volume in air chamber, \( \text{m}^3 \)
- \( C \): Total volume of air chamber, \( \text{m}^3 \)
- \( C_{dis} \): Design volume of air chamber, with a 1.25 safety factor, \( \text{m}^3 \)
- \( e \): Pipeline or air chamber thickness
- \( f \): Darcy-Weisbach friction coefficient
- \( g \): Acceleration of gravity, \( \text{m/s}^2 \)
- \( h_0 \): Initial absolute pressure at the upstream valve, \( \text{mwc} \)
- \( H_{max} \): Maximum absolute transient pressure, \( \text{mwc} \)
- \( h_{min} \): Minimum absolute transient pressure, \( \text{mwc} \)
- \( h_{atm} \): Local atmospheric pressure, \( \text{mwc} \)
- \( h_{air} \): Initial height of air in chamber, equal to \( C_0 / A_c \), \( \text{m} \)
- \( h_{o} \): Initial pressure at the upstream valve, \( \text{mwc} \)
- \( h_f \): Friction losses in pipeline, \( \text{mwc} \)
- \( h_{tot} \): Total height of air chamber, equal to \( C / A_c \), \( \text{m} \)
- \( h_{design} \): Design height of air chamber, with a 1.25 safety factor, \( \text{m} \)
- \( h_{min} \): Minimum head, measured as drawdown below \( h_0 \), \( \text{m} \)
- \( M \): Magnitude of hydraulic transient, \( \Delta h_f / h_0 \)
- \( K \): Characteristic parameter of an aqueduct under study, equal to \( ALVo^2/H_0 \)
- \( K_u \): Characteristic parameter of the unitary aqueduct
- \( L \): Length of the pipeline, \( \text{m} \)
- \( n \): Manning-Gauckler-Strickler friction coefficient
- \( y_0 \): Inicial air height in experimental air chamber, \( \text{m} \)
- \( \alpha \): Dimensionless parameter in elastic column model, equal to \( (C_0 AL) a/V_0 \)
- \( \beta_0 \): Dimensionless parameter for the initial air volume, rigid column model, equal to \( (C_0 AL) g h_0 V_0^2 \)
- \( \beta \): Dimensionless parameter for the total chamber volume, rigid column model, equal to \( (C_0 AL) g h_0 V_0^2 \)
- \( 2\rho \): Allievi dimensionless parameter, equal to \( M \)
- \( \sigma \): Allievi dimensionless parameter, equal to \( 2\beta_0 \)
- \( \eta \): Minimum relative transient pressure, equal to \( H_{min}/H_0 \)
- \( V_0 \): Velocity of water in pipeline, \( \text{m/s} \)
- \( \chi \): Maximum relative transient pressure, equal to \( H_{max}/H_0 \)
- \( \Delta h_f \): Joukowsky overpressure, equal to \( a V_0 / g \), \( \text{m} \)
- \( W_{cham} \): Total estimated weight of a unitary air chamber, Ton

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