Hadronic transitions in quarkonium

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Abstract. We report on a new approach to compute quarkonium hadronic transitions which does not rely on the standard twist expansion of the intermediate octet propagator. The latter cannot be justified for the energy release of hadronic transitions between heavy quarkonium states with different principal quantum numbers. Instead, we use that the octet state spectrum corresponds to the hybrid quarkonium one which can be obtained from nonrelativistic effective field theories (EFT) of QCD and lattice computations of the heavy-quark-antiquark static energies.

1. Introduction

Hadronic transitions in quarkonium have been studied since the late 70’s. The traditional approach has been to parametrize the transition amplitudes using chiral symmetry (current algebra in the early works and later on with EFT’s) and to input information on the bound state dynamics to partially or fully determine the free parameters left. To carry out the latter part of the program, it was noted that, in the context of the multipole expansion, hadronic transitions can be thought as a two step process. First, a gluon is emitted by the initial quarkonium states and the singlet quarkonium state becomes a color-octet. This octet state propagates for a brief period of time and emits a second gluon transforming into the final singlet quarkonium state. In a longer time scale, dictated by $\Lambda_{QCD}$, the two gluons hadronize into the final state pions. The multipole expansion allows for the derivation of selection rules and to relate transitions between different partial waves, however by itself does not allow for the full determination of the transition amplitude since it is still in terms of a nonlocal correlator involving the octet propagator and glue operators. The final step in the traditional approach, was proposed by Voloshin and consists in an operator product expansion of the octet propagator, which results in the amplitude being written in terms of local glue operators, for this reason this is often referred as the twist expansion of the transition amplitude. The hadronization of the local glue operators into the final pion states (at least the leading order term) can be determined making use of the scale \cite{1,2} and axial \cite{3,4} anomalies for the cases of two and one pions respectively. Nevertheless, the use of the twist expansion is only well justified when the energy of the final state pions is smaller than the typical binding energy of the quarkonium and therefore not valid for transitions between quarkonium states with different principal quantum number \cite{5}.

Our proposal is to study these transitions using an EFT approach, based on a hadronic version of weakly-coupled pNRQCD \cite{6,7}. The hadronic EFT is built in terms of the singlet (standard heavy quarkonium), hybrid and pion fields, organized according the $1/m_Q$ and multipole expansions as well as chiral symmetry. We also organize the computation according to the large $N_c$ expansion, which allows us to argue that $B/D$ mesons and possible tetraquarks contributions are, a priori, subleading. In this EFT the intermediate octet states in the hadronic transitions correspond to the hybrid quarkonium spectrum
and the amplitude can be written in terms of local glue operators which can be hadronized with the scale and axial anomalies leading to formally similar results as the ones using twist expansion. In Ref. [8] we have studied $Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi\pi$, $Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1S)\pi^0(\eta)$ and $Q\bar{Q}(2S) \rightarrow Q\bar{Q}(1P)\pi^0$ transitions. In the following we review the first case.

2. The Effective field theory

The EFT that describes the heavy-quark bound states on the gluonic static energies is discussed in Refs. [9, 10, 11, 12, 13]. The bound states are small energy fluctuations around the minima of the static energies, thus the binding energy $E_b \sim \sqrt{\Lambda_{QCD}^2/m} \ll \Lambda_{QCD}$, and therefore the relative distance scales as $r \sim 1/(m_Q\Lambda_{QCD}) \lessgtr 1/\Lambda_{QCD}$. It is interesting to study the matching in the short-distance regime $r \ll 1/\Lambda_{QCD}$. In this case the scale associated to the relative heavy-quark momentum $m v \gg \Lambda_{QCD}$ can be integrated out perturbatively leading to an EFT formally identical to weakly-coupled pNRQCD [6, 7].

One can then in turn integrate out the $\Lambda_{QCD}$ modes and match weakly-coupled pNRQCD to the EFT for hybrids [9, 11]. However, for application into quarkonium hadronic transitions it is convenient not to integrate out the $\Lambda_{QCD}$ modes since that is the scale of the energy gap between standard and hybrid quarkonium. In this case we work in a projection of weakly-coupled pNRQCD to the hybrid sector. The hadronic version of the pNRQCD Lagrangian reads

$$L_{pNRQCD}^{\text{had}} = \int d^3R d^3r \left[ S^t \left( i \partial_t - V_t(r) + \frac{\nabla_r^2}{m_Q} \right) S + \sum_{\lambda\lambda'} \Psi_r^t \left\{ (i \partial_t - (V_0^0 + \Lambda_0))\delta_{\lambda\lambda'} + \hat{r} \frac{\nabla_r \hat{r}}{m_Q} \hat{r} \right\} \Psi_r \right]$$

$$+ F_0^2 \left( \langle uu \rangle + \langle \chi_+ \rangle \right) + \left( r \cdot \hat{r} S^t \Psi + \text{h.c.} \right) \left( r^{(r1-r)} + t_d^{(r1-r)} F_0^2 \langle uu \rangle + t_d^{(r1-r)} F_0^2 \langle uu \rangle + t_m^{(r1-r)} F_0^2 \langle \chi_+ \rangle \right).$$

(1)

The fields $S$ and $\Psi_r$ should be understood as depending on $t$, $r$ and $R$ and correspond to the standard and hybrid quarkonium. The hybrid quarkonia we consider are the states with $k_{PC} = 1^-$. The unitary matrix $u = \exp(i \pi \lambda/(2F))$ contains the pion fields, which depend on $t$ and $R$, $F$ is the pion decay constant and

$$u_{\mu} = i \left( u^t (\partial_{\mu} - ir_{\mu}) u - u (\partial_{\mu} - il_{\mu}) u^t \right),$$

$$\chi_{\pm} = u^t \chi u^t \pm u \chi^t u,$$

(2)

where $\chi = 2B \text{diag}(m, m, m)$, with $B$ being related to the vacuum quark condensate. In the isospin limit, the pion mass is $m_\pi^2 = 2B m$. $\langle A \rangle$ stands for the trace of $A$ in the isospin index and the trace over spin indices is left implicit.

The matching conditions from the weakly-coupled pNRQCD to the hadronic EFT are

$$gO^0(t, r, R)E^0(t, R) \equiv Z_E^{1/2}(\Lambda_{QCD}) \Psi^0(t, r, R) + \ldots$$

(4)

where $O^0$ is the color-octet heavy quark-antiquark field. $Z_E$ is a normalization factor. The form of the potentials in the short distance can be generically described as follows: a sum of a perturbative part, which is typically nonanalytic in $r$ corresponding to the weakly-coupled pNRQCD potentials, and a nonperturbative part, which is a series in powers of $r$. The coefficients of the latter only depend on $\Lambda_{QCD}$ and can be expressed in terms of gluonic and light-quark correlators.

The singlet-hybrid mixing term in the hadronic theory is given by

$$Z_E^{1/2}(0)\Psi^0(t, r, R) \Psi^0(0, r, R) |0\rangle_{\text{amp.}} = i r \cdot \hat{r} \frac{Z_E^{1/2}}{2} r^{(r1-r)},$$

(5)

which according to the matching condition in Eq. (4) is matched to the following correlator of pNRQCD in term of quarks and gluons:
\begin{align*}
g(0)|\hat{\phi}_\Lambda^1(E^{\text{out}}(t, R'))O^\text{out}(t, r', R')S^\text{out}(0, r, R)|0\rangle_{\text{amp.}} &= i\sqrt{\frac{TF}{N_c}}\hat{\phi}_\Lambda^1 r^j(0)|gE^{\text{out}}(R)gE^{\text{out}}(R)|0\rangle, \\
\end{align*}
where \(\text{amp.}\) signals that only amputated contributions are considered (overall \(\delta(r'-r)\) are also factored out). The normalization of the \(\Psi_\Lambda\) field implies

\begin{equation}
\langle 0|gE^{\text{out}}(R)gE^{\text{out}}(R)|0\rangle = Z_E \delta^{ij} + \ldots
\end{equation}

Putting Eqs. (5)-(7) together we arrive at

\begin{equation}
t^{(r^{1-})} = \sqrt{\frac{TF}{N_c}Z_E}.
\end{equation}

The operators with an even number of pions in Eq. (1) are matched in a similar way. In the hadronic EFT

\begin{align*}
Z_{E}^{1/2} \int d^{4}x_{+}\cdots e^{ip_{-}x_{-}}\langle 0|\pi^{+}(x_{+})\pi^{-}(x_{-})S(t, r, R)\Psi_{\Lambda}^i(0, r, R)|0\rangle_{\text{amp.}} &= i4Z_{E}^{1/2}\hat{v} \cdot \hat{\lambda} \left(-i l_{d0}^{(r^{1-})} p_{+}^0 p_{-}^0 + i l_{di}^{(r^{1-})} p_{+}^1 p_{-} + i l_{m}^{(r^{1-})} m_{\pi}^2 \right),
\end{align*}

and the corresponding correlator in pNRQCD reads

\begin{equation}
\langle \pi^{+}(p_{+})\pi^{-}(p_{-})|S(t, r, R)\hat{v}_{\Lambda} \cdot E^{\text{out}}(0, R)O^{\text{out}}(0, r, R)|0\rangle_{\text{amp.}} = ig\sqrt{\frac{TF}{N_c}}\hat{v} \cdot \hat{\lambda} \langle \pi^{+}(p_{+})\pi^{-}(p_{-})|E(R) \cdot E(R)|0\rangle \\
= \frac{i}{3} \sqrt{\frac{TF}{N_c}}\hat{v} \cdot \hat{\lambda} \left(2 - \frac{9}{2} \kappa \right) p_{+}^0 p_{-}^0 - \left(2 + \frac{3}{2} \kappa \right) p_{+}^1 p_{-} + 3m_{\pi}^2
\end{equation}

where in the last step we hadronize the gluonic matrix element using the anomaly relation of the energy-momentum tensor of QCD \([1, 4, 2]\) which determines the matrix element up to the free parameter \(\kappa\).

Comparing Eq. (9) and Eq. (10) we obtain

\begin{align*}
l_{d0}^{(r^{1-})} &= \frac{2\pi^2}{3\beta_0} \sqrt{\frac{TF}{N_cZ_E}} \left(2 - \frac{9}{2} \kappa \right),
\end{align*}

\begin{align*}
l_{di}^{(r^{1-})} &= \frac{-2\pi^2}{3\beta_0} \sqrt{\frac{TF}{N_cZ_E}} \left(2 + \frac{3}{2} \kappa \right),
\end{align*}

\begin{align*}
l_{m}^{(r^{1-})} &= \frac{-2\pi^2}{\beta_0} \sqrt{\frac{TF}{N_cZ_E}}.
\end{align*}

For comparison purposes let us write the most general transition amplitude for two pion transitions according to chiral and heavy quark spin symmetry up to \(O(p^3)\)

\begin{equation}
\mathcal{A}_{\Lambda} = -a_1 p_{+}^0 p_{-}^0 + a_2 p_{+} \cdot p_{-} - a_3 m_{\pi}^2,
\end{equation}

Using the hadronic pNRQCD Lagrangian of Eq. (1) the free parameters of Eq. (12) take the form

\begin{align*}
a_1 &= \frac{-8\pi^2 TF}{3\beta_0 N_c}\beta_{r,Q}^{(12)} \left(2 - \frac{9}{2} \kappa \right),
\end{align*}

\begin{align*}
a_2 &= \frac{-8\pi^2 TF}{3\beta_0 N_c}\beta_{r,Q}^{(12)} \left(2 + \frac{3}{2} \kappa \right),
\end{align*}

\begin{align*}
a_3 &= \frac{-8\pi^2 TF}{\beta_0 N_c}\beta_{r,Q}^{(12)},
\end{align*}

where

\begin{align*}
\beta_{r,Q}^{(12)} &= \beta_{r,Q}^{(12)}
\end{align*}
with $\beta_{r,Q}^{(12)}$ the sum over the intermediate $k^{PC} = 1^{--}$ hybrid states

$$\beta_{r,Q}^{(\alpha n)} = \sum_m \langle S_r | \hat{r}_L \cdot r | \Psi_m \rangle \left( \frac{1}{m_n - m_m} + \frac{1}{m_{n'} - m_m} \right) \langle \Psi_m | \hat{r}_L \cdot r | S_n \rangle. \quad (14)$$

It is remarkable that the normalization factors $Z_E$ cancels out, which allows us to completely evaluate the transition amplitude except for the parameter $\kappa$.

We can now compare our results with experimental data. First let us look at the normalized decay width spectrum with respect to the dipion invariant mass. This normalized distribution is independent of the unknown normalization of the experimental spectrum making it a convenient observable to compare with, moreover the theoretical expression of the normalized decay width spectrum is independent of $\beta_{r,Q}^{(\alpha n)}$ and only depends on $\kappa$. Therefore, we can obtain the value of $\kappa$ by fitting the normalized decay width spectrum. The results are shown in Fig. 1.

![Figure 1. Plot of the normalized differential decay width spectrum. The dots are the experimental data for $\psi(2S) \to J/\Psi \pi^+\pi^-$ [15] and $\Upsilon(2S) \to \Upsilon(1S) \pi^+\pi^-$ [16] in blue and yellow respectively. In the same color scheme the continuous lines are the fits of the theoretical expression obtained from the amplitude computed with the hadronic pNRQCD Lagrangian. The variable $x$ is defined as $x = \frac{m_{ex} - 2m_\pi}{m_{2S} - m_{1S} - 2m_\pi}$.](image)

The values for $\kappa$ obtained from the fits are as follows

$$\kappa_c = 0.277 \pm 0.015, \quad \kappa_b = 0.229 \pm 0.016, \quad \kappa_{\text{joint}}$$(15)

We take the value of the joint fit to both sets of experimental data, $\kappa_{\text{joint}}$, in our numerical evaluations of the total widths. To estimate the uncertainty associated to subleading effects we take the difference of the value in Eq. (15), obtained with phase space factors computed with relativistic kinematics, with the value obtained using nonrelativistic kinematics. We then combine them in quadrature with the statistical error quoted in Eq. (15) and give $\kappa = 0.247(20)$.

We compute $\beta_{r,Q}^{(12)}$ with the $k = 1^{--}$ and $m^3S_1$ hybrid intermediate states $m = 1, \ldots, 4$ (we observe that the effect of introducing 3 or 4 hybrid states is comparatively small compared with other uncertainties). The hybrid masses and wave functions are obtained using the techniques developed in Ref. [9]. We obtain the following values for the decay widths

$$\Gamma_{\psi(2S) \to J/\Psi \pi^+\pi^-} = 46.2(10.3)/(+15.7)/(-13.9)\kappa(\pm21.2)_{s,p} \text{ keV}, \quad \Gamma_{\psi}^{\text{exp}} = 102.1(2.9) \text{ keV},$$

$$\Gamma_{\Upsilon(2S) \to \Upsilon(1S) \pi^+\pi^-} = 3.08(-0.58)/(+0.81)/(-0.22)\kappa(\pm1.49)_{s,p} \text{ keV}, \quad \Gamma_{\Upsilon}^{\text{exp}} = 5.71(48) \text{ keV}. \quad (16)$$
The uncertainties are labeled according to the source, with $\Lambda$ the lowest lying glue lump mass, and s.p. the different parametrization for the singlet and hybrid static potentials. Our numbers differ from the experimental ones by about a factor 2. One should keep in mind however that our estimates suffer from large uncertainties. We find a significant dependence on variations of the wave function of the hybrid states is somewhat smaller. These error estimates are of the right magnitude, though not large enough, to completely account for the difference with experiment. One should keep in mind however that, besides those errors already estimated, one error that has not been incorporated in this analysis is due to the uncertainties associated to the hadronization of the local operator such as $O(\alpha_s)$ corrections to the beta function.

It is interesting to consider the ratio of decay widths, where the uncertainties associated to the gluon hadronization cancel out, as well as a reduction on the impact of the other uncertainty sources

$$R_{bc,\pi\pi} = R(\frac{\Upsilon(2S) \rightarrow \Upsilon(1S) \pi^+ \pi^-}{\psi(2S) \rightarrow J/\psi \pi^+ \pi^-}) = 6.65((-0.30)_{-0.38})_{+0.02} \times 0.02, \quad (18)$$

$$R_{bc,\pi\pi}^{\exp} = 5.59(0.50) \times 10^{-2}.$$ 

This observable can be considered a rough measure of $\beta_{bc}^{(21)}/\beta_{ss}^{(21)}$. The agreement with experiment is remarkable: the discrepancies are below 20%, and could be accounted for by the quoted errors.

3. Conclusions
We have reviewed a new approach to study quarkonium hadronic transitions that does not rely on the twist expansion, which is not justified for transitions between states with different principal quantum number. This approach leads to a good description to the $2S$ to $1S$ two-pion transitions decay width spectrum in a similar way as the standard twist expansion-based analysis. The predictions for the absolute of the total widths turn out to be about a factor of 2 smaller than the experimental values but roughly consistent when the large uncertainties are considered. For the ratio of the decay widths, where several uncertainties cancel out, a better agreement of theory and experiment is achieved which indicates that the overall computational scheme is sound.

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