Study of the section optimization of gravity dam based on improved PSO

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Abstract. As the linearly-decreasing inertia weight of particle swarm optimization (PSO) can easily lead to local extremum, this paper proposes an improved PSO which changes the dynamic changing pattern of Inertia Weight w based on the properties of trigonometric function. The inertia weight value of improved PSO is large at the early stage and relatively small at the later stage, thus improving the global searching ability as well as the convergence performance of the algorithm. The Optimal Section Area A(X) of non-overflow gravity dam calculated through improved PSO is 5147.3 m² which is 6.93% lower than that calculated through SPSO (A(X)=5530.7 m²). To obtain the optimal solution, the calculation steps needed by improved PSO and SPSO respectively are 15 and 22. The convergence rate of improved PSO is 31.8% higher than that of SPSO. The calculation results of the two algorithms show that the improved PSO is able to obtain better optimization results in higher convergence rate and thus is suitable for the optimization design of large-scale hydraulic structures.

1. Introduction

Particle swarm optimization (PSO) is a swarm-intelligence-based evolutionary optimization algorithm proposed by Kennedy and Eberhart, inspired by artificial life research results [1]. Compared with other bionics algorithms, PSO has fewer control parameters, higher convergence rate, simpler operation, and better performance in solving practical problems [2].

In recent years, scholars both home and abroad have carried out intensive researches on the convergence performance of PSO as well as its applications in engineering, achieving considerable research findings. Chen et al. [3], through numerical experiment, studied the influence of Inertia Weight w on the convergence performance of PSO. Fu et al. [4] studied convergence and divergence performances of PSO in each sub-region through a series of numerical simulation experiments and analyzed the correlation between eigenvalue and algorithmic parameters. Pan et al. [5], through the difference model of PSO, studied the influences of Inertia Weight w and Acceleration Factor c on the premature convergence and divergence of PSO. Ahmad Nickabadi et al. [6] analyzed the influence of self-adaptively-changed Inertia Weight w on the global convergence performance of standard PSO algorithm through numerical experiments. The research results show that constant inertia weight of standard PSO algorithm can easily lead to the local optimal solution, which will affect the global convergence performance of the algorithm. Meanwhile, the real search process of PSO is nonlinear, and the particle trajectory changes dynamically [7]. Therefore, based on the findings of previous studies, this paper, according to the properties of trigonometric function, proposed an improved PSO algorithm which can dynamically adjust the inertia weight value. The proposed PSO algorithm is applied in the optimization design and computational analysis of non-overflow gravity dam section so
as to provide references for the optimization design of large-scale hydraulic structures.

2. Improved Particle Swarm Optimization

2.1. Standard Particle Swarm Optimization

In standard particle swarm optimization (SPSO), the optimal solution researched by the Particle X_{ik} is called self-historical optimal solution, expressed as phi-best. The optimal solution researched by all particles is called global historical optimal solution, expressed as pg-best. Then the velocity and position of particle can be updated through the equations below.

\[ v_{i}^{k+1} = w \cdot v_{i}^{k} + c_{1}r_{1}(p_{hi-best} - X_{ik}^{k}) + c_{2}r_{2}(p_{g-best} - X_{ik}^{k}) \]  
\[ X_{ik}^{k+1} = X_{ik}^{k} + v_{i}^{k+1} \]

Where \( X_{ik}^{k} \) refers to the position of the \( i \)-th particle at the \( k \)-th iteration, \( v_{i} \) refers to the velocity of the \( i \)-th particle at the \( k \)-th iteration, \( w \) refers to inertia weight, \( c_{1} \) and \( c_{2} \) are learning factors, \( r \) is restraint coefficient or convergence factor, and \( r_{1} \) and \( r_{2} \) are random numbers between \([0,1]\).

2.2. Nonlinear Decreasing Strategy of Inertia Weight

In SPSO, the particles tend to converge to local extremum earlier (known as premature convergence), especially in the linear decreasing strategy of inertia weight. In the search process of algorithm, the lack of diversity in the positions of particles will lead to premature convergence. Therefore, the Inertia Weight \( w \) can be adjusted dynamically.

Chen et al, proposed in Literature [7] that, by slowing the decline rate of inertia weight, the global optimization performance can be improved at the early stage of the algorithm and the local optimization performance can be improved at the later stage. The decreasing strategy of the Inertia Weight Value \( w \) includes a downward parabola which is shown in equation (3) and an upward parabola which is shown in equation (4).

\[ w_{1} = (w_{start} - w_{end})(t/t_{max})^2 + w_{start} \]  
\[ w_{2} = (w_{start} - w_{end})(t/t_{max})^2 + (w_{end} - w_{start})(2t/t_{max}) + w_{start} \]

During the initial stage of the algorithm, the Inertia Weight Value \( w \) should be set as large as possible so that the particles can be searched in the whole search space, improving the diversity of particle positions. Once the algorithm starts to search in the neighborhood of the global extremum, the Inertia Weight Value \( w \) should be reduced rapidly and kept at a small value at the later stage of the search, and then the swarm is able to converge to global extremum with strong local search capacity. According to the abovementioned analyses, an improved PSO algorithm which can dynamically adjust the Inertia Weight Value \( w \) is proposed based on trigonometric function.

The changing formula of inertia weight is:

\[ w_{3} = [0.65 + 0.25 \cos(\pi \cdot k/k_{max})] \cdot [a \cdot \sin(2\pi \cdot k/k_{max}) + 1] \]

Where \( k_{max} \) refers to the maximum number of iterations, \( w_{3} \) refers to the \( k+1 \) inertia weight values obtained through \( k \) iterations. Adjustment factor \( a \) is set at 0.02 in this paper after a series of numerical experiments. Therefore, the velocity of particle can be updated through the following equations:

\[ v_{i}^{k+1} = w_{3} \cdot v_{i}^{k} + c_{1}r_{1}(p_{hi-best} - X_{ik}^{k}) + c_{2}r_{2}(p_{g-best} - X_{ik}^{k}) \]

The classical test function Griewank function is adopted in this paper to test the optimization performance of Algorithms \( w_{1}-PSO \), \( w_{2}-PSO \) and \( w_{3}-PSO \). The global minimum of the function is 0, the number of particles in the algorithms is 40, \( c_{1}=c_{2}=2 \), and the stop condition for optimization is the iteration precision reaches \( 10^{-10} \) or the number of iterations exceeds 1000. Fig.1 shows the variation of experimental average optimal solution with the number of iterations. It can be known from Fig.1 that: the optimization performance of \( w_{2}-PSO \) is the worst; the global extremum of Algorithm \( w_{3}-PSO \) is closer to the theoretical value and the early convergence rate of Algorithm \( w_{3}-PSO \) is higher. Therefore, the dynamic adjustment strategy of Inertia Weight Value \( w \) proposed on the basis of
trigonometric function is better.

Figure 1. Optimization iterative curve of Griewank function

2.3. Constraint-Handling Technique
In unconstrained optimization, PSO algorithm is able to converge to global extremum with bounded particles. While in constrained optimization, the diversity of particle swarm will decrease during the convergence, thus affecting the global convergence performance of the algorithm. Therefore, constraint-handling technique is the key technology for improving the efficiency of algorithm in practical constrained optimization problems.

To solve the abovementioned problem, the penalty function which converts constrained nonlinear extremum problem into a series of unconstrained extremum problems is defined:

\[ \psi(X, \gamma) = F(X) + \gamma \sum_{j=1}^{m} g_j(X) \]  

(7)

Where \( \gamma \) is penalty factor (a monotonically-increasing positive coefficient sequence: \( 0 \leq \gamma \leq \infty \)) and \( g_j(X) \) is the parenthesis operator whose meaning is as follows:

\[ g_j(X) = \begin{cases} g_j(X) & g_j(X) > 0 \\ 0 & g_j(X) \leq 0 \end{cases} \]  

(8)

r is a non-negative constant (r=2).

3. Instance Calculation Based on Improved PSO

3.1. Optimization Design of Gravity Dam Section
In the concrete gravity dam, the dam height is 132.0m, the design water level is 129.5m, and the downstream water level is 17 m. The buoyant unit weight of the sediment is 5 kN/m³. The Shear Friction Coefficient \( f' \) between the foundation surface and foundation rock is 0.82. The Cohesion Force \( c' = 610 \text{kPa} \). The distance between the drainage hole and the upstream face is 12 m.

3.1.1 The constraint conditions for the optimization design of dam section
- Upstream dam slope: \( 0 \leq n \leq 0.4 \)
- Downstream dam slope: \( 0.5 \leq m \leq 1.0 \)
- Ratio of the height of upstream folding point to dam height: \( 0 \leq h_1 / H \leq 1 \)
- Ratio of the height of downstream folding point to dam height: \( 0 \leq h_2 / H \leq 1 \)
- Ratio of width of dam bottom to dam height: \( 0.6 \leq L / H \leq 1.0 \)
- Stability constraints: \( \sum P \leq f' \sum W + c' A \)
- Dam heel stress: \( \sigma_x = \left( \sum W / B + 6 \sum W / B' \right) \geq 0 \)
- Dam toe stress: \( \sigma_x = \left( \sum W / B - 6 \sum W / B' \right) \leq [\sigma_{a,1}] \)
Stress of upstream folding point: \( \sigma_{\text{uc}} = \left( \sum W_i / B_i + 6 \sum W_i / B_i ^2 \right) \geq 0 \).

3.1.2 Optimization Model. The basic section of gravity dam is shown in Fig.3. The Design Variable \( X \) are the Height \( x_1 \) between the upstream folding point and the design water level, the Horizontal Distance \( x_2 \) from the dam heel to the upstream edge of dam crest, and the Horizontal Distance \( x_3 \) from the dam toe to the upstream edge of dam crest. The objective function is the non-overflow Section Area \( A(X) \) of the gravity dam.

The optimization model is:

Find the Design Variable \( X = \{ x_1, x_2, x_3 \} ^T \) which meets \( A(X) \rightarrow \text{ymin} \) and the Constraint Conditions mentioned above.

The partial coefficient limit-state design method is used in the section optimization of gravity dam. The anti-sliding stability of dam foundation and the compressive strength of dam toe are calculated under ultimate limit state. The dam heel stress is worked out under serviceability limit state.

3.2 Optimization Results and Analysis

The section optimization design of gravity dam section is worked out through the calculation program of PSO algorithm improved by MATLAB. The results of the program are as follows: \( x_1 = 92.1235 \text{m}; x_2 = 14.7506 \text{m}; x_3 = 64.8523 \text{m} \).

Table 1. The optimization design obtained through improved PSO, GA and SPSO

| Calculation scheme | Design parameters of gravity dam section (m) | Stress (MPa) | Objective function \( A(X) \) (m²) |
|-------------------|---------------------------------------------|-------------|-----------------------------------|
|                   | \( x_1 \) | \( x_2 \) | \( x_3 \) | Dam toe | Dam heel | |
| GA                | 82.4675  | 16.2596  | 74.2265  | 2.3452  | 0.3149   | 5861.0 |
| SPSO              | 87.2265  | 15.8456  | 69.8568  | 2.8236  | 0.3652   | 5530.7 |
| Improved PSO      | 92.1235  | 14.7506  | 64.8523  | 3.1890  | 0.8956   | 5147.3 |

To verify the feasibility of the improved PSO algorithm in the optimized design of structure and the rationality of its optimization results, the results of the enetic algorithm (GA) and the standard particle swarm optimization (SPSO) are compared in Table 1. It can be known from Table 1 that the optimal result of improved PSO is 12.2% lower than that of GA and 6.93% lower than that of SPSO, which is much more economical than the other two. To obtain the optimal solution, the calculation steps needed by improved PSO, GA, and SPSO respectively are 15, 26, and 22. The convergence rate of improved PSO is 42.3% higher than that of GA and 31.8% higher than that of SPSO. Compared with SPSO, the improved PSO proposed in this paper reaches the critical value in the upstream dam slope, downstream dam slope, and ratio of dam bottom width to dam height. Besides, the compressive stress...
of dam toe reduces while the stress reserve increases. Therefore, the calculation result of improved PSO is closer to the global optimal solution.

4. Conclusions
The Inertia Weight w of PSO algorithm is an important control parameter which is able to control the ability of the algorithm in searching for the potential feasible solution. As the linearly-decreasing inertia weight of PSO can easily lead to local optimal solution, this paper proposes a nonlinear decreasing strategy in which the inertia weight dynamically changes with time so as to improve the convergence performance of the algorithm. The proposed improved PSO algorithm is used in the shape optimization design of gravity dam and its optimization results are compared with those of GA and SPSO. The comparison results show that improved PSO is superior to GA and SPSO in the optimized design of hydraulic structure. The improved PSO has the characteristics by few control parameters, fast convergence, simple operation, and no need to determine the search direction by function derivatives. It has strong adaptability in design variables, objective functions, and constraint functions and has broad application prospects in the optimization design of large-scale hydraulic structures.

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