A Risk Budgeting Approach Improved by Genetic Algorithms

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Abstract. The risk budgeting approach has been applied to manage and monitor the portfolio risk of large and sophisticated institutional investors. How to decide proper parameters to obtain the optimal or near-optimal solutions for the risk budgeting approach is a problem. In this paper, a risk budgeting approach improved by genetic algorithms is proposed. Experiment results on real financial data demonstrate that, compared with some traditional methods, the proposed algorithm is capable of generating a set of parameters which can attain near-optimal return on investment with high probability and computational complexity of searching process also has been reduced.

Keywords: asset portfolio, risk budgeting, genetic algorithms.

1. Introduction

The risk budgeting (RB)\cite{1}\cite{2} is one of special cases of a more general class of risk-based asset allocation models. It has been applied to a universe of multi-assets classes to manage and monitor the portfolio risk of large and sophisticated institutional investors like pension funds. How to properly decide a sort of parameters which can obtain the optimal or near-optimal solutions for asset allocation models is a challenging problem. In practice, the parameters are chosen by experience or with the help of traversing in a limited parameters space. Without reliable experience, RB cannot be reused easily. On the other hand, if parameters are decided with the help of traversing strategy (TS), the application condition is restricted. TS has the problems of weak effectiveness.

In this paper, we propose a risk budgeting approach improved by genetic algorithms (RBIGAs). In the space expended by parameters, near-optimal solutions for risk budgeting models can be obtained. The possible problems of “stochastic” and “inefficient” in TS, which may be induced by traditional method in practice, can be avoided. Through analysis and experiments upon financial data, the performance is compared with some traditional methods. The results demonstrate that the proposed algorithm is capable of generating the set of parameters with better performance in most cases.

2. Background

2.1. Risk budgeting approach

Researchers prefer to define the portfolio according to weights and the risk budgets in relative value. RB\cite{3}\cite{4} is used to build diversified portfolios. In these portfolios, if one asset has a negative risk, the
risk is concentrated in the other assets [1]. There are $n$ assets in a portfolio $x = (x_1, x_2, \ldots, x_n)$. $x_i$ represents the weight of the $i$th asset and $\mathcal{R}(x_1, x_2, \ldots, x_n)$ represents the risk of portfolio. If $\mathcal{R}$ is convex and coherent, it can be decomposed as:

$$RC_i(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} x_i \frac{\partial \mathcal{R}(x_1, x_2, \ldots, x_n)}{\partial x_i}$$

(1)

A set of risk budgets is given as the risk value of above portfolio and RB can be defined as Eq. 2:

$$\begin{cases}
x_i \cdot (\sum x_i) = b_i \cdot (x^T \sum x) \\
b_i \geq 0 \\
x_i \geq 0 \\
\sum_{i=1}^{n} b_i = 1 \\
\sum_{i=1}^{n} x_i = 1
\end{cases}$$

(2)

Generally, the mathematical system in following equations can also be used to define RB as Eq. 3:

$$\begin{cases}
x_i \cdot (\sum x_i) = b_i \cdot (x^T \sum x) \\
b_i \geq 0 \\
x_i \geq 0 \\
\sum_{i=1}^{n} b_i = 1 \\
\sum_{i=1}^{n} x_i = 1
\end{cases}$$

(3)

RB need a set of parameters, such as how to provide the risk budgets $b$, how long to hold assets, which measure can be used to represent the risk before, how long to measure the risk before, which measure can be used to regulate portfolio, how long to regulate portfolio. In some case, the researcher cannot provide accurate the risk budgets and they hope that a proper risk budgets can be attained by automatic search.

2.2. Genetic algorithms
Based on principles of evolution, genetic algorithms [5][6] are used as the important optimizing technique in randomized search. Genetic algorithms can search in a complicated searching space to obtain near-optimal solutions for the optimization problem.

First of all, the parameters in the solution space are encoded as chromosomes. A population is formed by a group of chromosomes. At the beginning of search, a random population is produced to represent different points in the searching space. Secondly, based on survival of the fittest, some strings in different chromosomes are selected. Cross-over operators and mutation operators are applied on these strings to yield a new generation of chromosomes. The process formed by selection, crossover and mutation is repeated for a certain number or meet a termination condition. Finally, the resulting chromosomes are decoded to produce solutions which is near-optimal for the optimization problem. The ordinary procedure of genetic algorithms can be found in [7]. In the area of asset allocation, searching for a set of appropriate parameters is an significant task. Therefore, the application of genetic algorithms for asset allocation appears to be appropriate.

3. A risk budgeting approach improved by genetic algorithms
Greedy search algorithms ensure that the searching process can be finished in limited times. However, some defects such as slow convergence rate and getting into local minimum are pointed out. Different from greedy search algorithms, GAs can avoid trapping into the local optimum and achieve the near-optimal (even optimal at sometimes) solution [8][9]. The parameters in GAs are not discussed in
this paper and they are tuned according to [10]. The procedure of the proposed algorithm (RBIGAs) is described as below:

Population P represents the set of chromosomes. Each chromosome indicates a possible selecting set of parameters. And in the Fig. 1, a case in point is consisted by the information of the allocation percentage of each asset, how long to measure the risk before and how long to regulate portfolio.

Assuming the risk weight of bond is chosen from 5% to 25%, the binary number “00000–11111” can cover all possible values. The period of measuring risk before is chosen in the range of [6, 12 and 36 months], the binary number “00–11” can cover the possible values. Similarly, the period of regulating portfolio is chosen in the range of [1, 3 and 6 month(s)], the binary number “00–11” can cover the possible values. In the chromosome of Figure 1, the characters from 1st to 5th bit, “10010”, represent that the risk weight of bond is 18%. The characters from 6th to 7th, “01”, represent that the period of measuring risk before is 6 months and those from 8th to 9th represent that the period of regulating portfolio is 6 months. Population size is set as 200, and maximal generation times is set as 50.

![Figure 1. The structure of chromosomes.](image1)

The fitness of every chromosome is the return of the asset portfolio controlled by some set of parameters.

\[
\text{fitness}(C) = J(X_c) - \text{penalty}(X_c)
\]  

\[
\text{penalty}(X_c) = \omega, \quad X_c \in D
\]

\(X_c\) is the set of parameters, \(J(X_c)\) is the return of the asset portfolio. Eq. 5 is the penalty function and it can ensure parameters is chosen in the reasonable range. For example, the period of measuring risk before is chosen in the range of [6 months, 12 months and 36 months]. “01”, “10”, “11” represent 6 months, 12 months and 36 months respectively, and “00” is out of reasonable range. The penalty coefficient is set as 1.

All chromosomes are ranked based on their fitness values. In order to ensure that the fitter chromosomes can survive at a higher probability than others, we adopt rank-based roulette-wheel selection scheme.

Generate the new population by using the standard crossover and mutation operators on selected chromosomes in Fig. 2. The probability of crossover is set as 0.6. The probability of mutation is set as 0.1.

![Figure 2. The procedure of crossover and mutation.](image2)
Judge whether one of the termination conditions is met. One is maximal generation time (10 times) and the other is the return of the asset portfolio has continuously increased for 5 times with an increasing scope less than 5% of the previous chromosome. Decode the best chromosome to get the resulting set of parameters and run the risk budgeting model with the resulting set of parameters.

4. Experiment

4.1. Real financial data
In this paper, we present the experiment of the portfolio based on two kind of assets, stocks and bonds in Chinese market. The CSI 300 index is used to represent stocks data and tracks the Shanghai and Shenzhen markets. The China Bond New Composite Index is used to represent bonds data and tracks the overall market of domestic bonds. Without considering the liquid requirement, the period of holding asset portfolio is 3 years, from Jan. 2015 to Dec. 2017.

4.2. Performance of the proposed algorithm (RBIGAs)
The optimal solution can be obtained by TS, we use this optimal solution to evaluate the performance of other algorithms in the return of portfolio. In the experiment, there are 189 sets of parameters in the solution space and only 18 resulting sets are listed briefly in Table 1. It appears that if the period of measuring risk before and that of regulating portfolio are short, the return of portfolio generally is higher. However, in some case, there are some outliers which are against the rule, just as some points in Fig. 3. One is which risk weight of bond is 8% and the period of regulating portfolio is 1 month, the other is which risk weight of bond is 12% and the period of regulating portfolio is 3 month. These outliers may led to the problem of the local minimum.

Table 1. Performance of RBIGAs.

|   | Fonds:Stocks | Measuring Risk Before (month) | Regulating Portfolio (month) | Annualized Return | Annualized Volatility |
|---|--------------|-------------------------------|-------------------------------|-------------------|-----------------------|
| 1 | 5:95         | 6                             | 1                             | 7.00%             | 5.46%                 |
| 2 | 25:75        | 6                             | 1                             | 4.61%             | 2.77%                 |
| 3 | 5:95         | 12                            | 1                             | 5.97%             | 6.01%                 |
| 4 | 25:75        | 12                            | 1                             | 4.13%             | 3.00%                 |
| 5 | 5:95         | 36                            | 1                             | 3.87%             | 5.80%                 |
| 6 | 25:75        | 36                            | 1                             | 3.19%             | 2.91%                 |
| 7 | 5:95         | 6                             | 3                             | 6.84%             | 5.84%                 |
| 8 | 25:75        | 6                             | 3                             | 4.53%             | 2.95%                 |
| 9 | 5:95         | 12                            | 3                             | 5.62%             | 6.17%                 |
| 10| 25:75        | 12                            | 3                             | 3.97%             | 3.11%                 |
| 11| 5:95         | 36                            | 3                             | 3.95%             | 5.82%                 |
| 12| 25:75        | 36                            | 3                             | 3.22%             | 2.94%                 |
| 13| 5:95         | 6                             | 6                             | 5.87%             | 6.70%                 |
| 14| 25:75        | 6                             | 6                             | 4.00%             | 3.38%                 |
| 15| 5:95         | 12                            | 6                             | 4.74%             | 6.79%                 |
| 16| 25:75        | 12                            | 6                             | 3.49%             | 3.47%                 |
| 17| 5:95         | 36                            | 6                             | 3.56%             | 6.32%                 |
| 18| 25:75        | 36                            | 6                             | 3.01%             | 3.21%                 |
The performance of RBIGAs is compared with that of TS and the gradient descent strategy (GDS)\cite{11} in Table 2, which is a typical greedy search algorithm. It is certain that TS can reach the optimal result at one time. The experiments times of other algorithms is set as 50. The starting points of GDS or RBIGAs are decided randomly, so the results may be different in each time.

Table 2. Hit ratio of different methods.

| Method | Experiments Times | Not Hit Times | Hit Ratio |
|--------|-------------------|---------------|-----------|
| TS     | 1                 | 0             | 100%      |
| GDS    | 50                | 7             | 86%       |
| RBIGAs | 50                | 4             | 92%       |

According results in Table 2, RBIGAs hits this optimal set at 46 times in all experiments and GDS hits this optimal set at 43 times. The experiments show that the resulting sets attained by GDS is more easily trapped in outliers than RBIGAs. In Table 3, searching times of three methods, including average times, maximum times and minimum times, are listed.

Table 3. Searching times of different methods.

| Method | Average Times | Minimum Times | Maximum Times | Standard Deviation |
|--------|---------------|---------------|---------------|--------------------|
| TS     | 279           | 279           | 279           | 0                  |
| GDS    | 18            | 9             | 26            | 5.9409             |
| RBIGAs | 101.7         | 90(5th)       | 109(6th)      | 5.7166             |

Because the traversing strategy must search for all sets in the solution space, so the searching time is equal to the number of all sets. Searching times of other two methods are less than the traversing strategy. In experiments, RBIGAs converged after searching 88 sets (5th generation) at least and 106 sets (6th generation) at most. GDS converged after searching for 9 sets at least and 26 sets at most. The searching time of GDS is less than other methods but it is not steady enough. According to the standard deviation, the stability of RBIGAs is better than GDS.

5. Conclusion
In this paper, we propose a risk budgeting improved by genetic algorithms (RBIGAs) in dealing with asset portfolio. Experiment results on real financial data demonstrate that, compared with some other methods, the proposed algorithm is capable of generating a set of parameters which can attain near-optimal return on investment with high probability and computational complexity of searching process also has been reduced.
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