HORIZON RUN 3: TOPOLOGY AS A STANDARD RULER

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ABSTRACT

We study the physically self-bound cold dark matter halo distribution, which we associate with the massive galaxies within Horizon Run 3, to estimate the accuracy of the determination of the cosmological distance scale measured by the topology analysis. We apply the routine “Contour 3D” to the 108 Mock Survey of $\pi$ steradians out to redshift $z = 0.6$, which effectively corresponds to the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) survey, and compare the topology with that of a Gaussian random phase field. We find that given three separate smoothing lengths $\lambda = 15, 21$, and $34 \, h^{-1}$ Mpc, the least $\chi^2$ fit genus per unit volume ($g$) yields a $1.7\%$ fractional uncertainty in smoothing length and angular diameter distance to $z = 0.6$. This is an improvement on former calibrations and presents an error estimate competitive with baryon acoustic oscillation scale techniques. We also present three-dimensional graphics of the Horizon Run 3 spherical mock survey to show a wealth of large-scale structures of the universe that are expected for surveys like BOSS.

Key words: cosmology: theory – distance scale – large-scale structure of universe – methods: numerical

1. INTRODUCTION

The most popular model for the generation of primordial density fluctuations is the inflationary scenario (Guth 1981; Linde 1982; Albrecht & Steinhardt 1982; Linde 1983). This model assumes primordial Gaussian random phase perturbations and it has been shown that such initial conditions produce a sponge-like topology on large scales (Gott et al. 1986, 1987). At such scales where the power spectrum has not been transformed by nonlinear growth, the topology of the structure in the early universe is well preserved, and small deviations from random phase predictions provide important information about primordial non-Gaussianity, biased galaxy formation, and nonlinear clustering (Matsubara 1994; Park et al. 1998, 2005b, 2012; Park & Gott 1991).

The genus statistic is central to these studies, and is now a well-tested quantitative measure (Gott et al. 1986, 1987, 1989; Hamilton et al. 1986; Vogele et al. 1994; Hikage et al. 2002, 2003; Choi et al. 2010; Park et al. 2005a, 2005b), having been applied to both the Sloan Digital Sky Survey (SDSS) Luminous Red Galaxy (LRG) sample (Gott et al. 2009; Strauss et al. 2002; Eisenstein et al. 2011), and the cosmic microwave background (CMB; Park et al. 1998). Its utility lies in the existence of the “genus curve,” an analytical expression for genus as a function of density, which allows the comparison of observed topology with that expected from a standard big bang inflationary model (Hamilton et al. 1986).

So far, fitting the Gaussian random phase (hereafter, GRP) genus curve to mock surveys in a ΛCDM cosmology has been remarkably successful. The genus has now been suggested as a cosmic standard ruler (Park & Kim 2010) and as a means for probing dark energy (Park & Kim 2010; Zunckel et al. 2011; Slepian et al. 2014). The baryon acoustic oscillation (BAO) feature, detectable in the power spectrum and galaxy two-point correlation function, is the established “standard ruler” (Anderson et al. 2012), with a reported fractional uncertainty in angular diameter distance to $z = 0.6$ of $1.1\%$ expected for the SLOAN survey when completed. Now, with the introduction of ever larger galaxy samples, such as the CMASS Data Release 10 sample of the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS), topology is becoming another attractive technique for probing the expansion of the universe and constraining the equation of state of dark energy. We apply the genus to 108 LRG mock surveys, derived from the Horizon Run 3 (HR3) $N$-body Simulations (Kim et al. 2011), in order to ascertain the statistical accuracy of said “topological distance measure.”

2. THE GENUS STATISTIC

Gott et al. (1986) presented the genus as a reliable description of topology. Traditionally, the genus comes from the Gauss–Bonnet theorem, which states that the integral of Gaussian curvature $K = 1/(r_1 r_2)$ (where $r_1$ and $r_2$ are the principle radii) over a compact two-dimensional surface is given by

$$\int K \, dA = 4\pi (1 - G_b).$$

(1)

We use a slightly altered form of the Gauss–Bonnet genus, $G = G_b - 1$, so that it has a more intuitive meaning for cosmology:

$$G = \text{(No. of doughnut holes)} - \text{(No. of isolated regions)}.$$

(2)

See Park et al. (2013) for the relation to the Euler characteristic and the Betti numbers. With this definition, the genus of a sphere is $G = -1$, of a toroid is $G = 0$, of three isolated spheres is $G = -3$, and of a figure eight pretzel is $G = 1$ (two holes, one isolated body). Essentially, the genus is a measure of connectivity. A highly connected structure—such as a sponge—will have many holes, a single body, and therefore a large, positive genus. A sparse array of objects—a meatball topology (Soneira & Peebles 1978; Press & Schechter 1974)—will have many isolated regions, relatively few holes, and therefore a negative genus. An array of isolated voids will also produce a negative genus.
To calculate the genus, we smooth the physically self-bound (PSB) subhalo distribution of HR3 (Kim et al. 2011) with a Gaussian smoothing ball of radius \( \lambda \) (Equation (5)).

We picked the most massive physically bound subhalos to match the number density of LRG galaxies projected for the SLOAN III survey when completed. Because HR3 is a pure cold dark matter simulation, we make the simple assumption that the most luminous red galaxies will form at the centers of the most massive cold dark matter halos. In the simulation, we applied the one-to-one correspondence model (or subhalo-galaxy abundance matching; Kim et al. 2008) to identify LRG galaxies with subhalos. This model is simple to employ and has proved to be powerful in reproducing the observed galaxy distribution among various galaxy formation models available until now (Choi et al. 2010).

We then create isodensity contour surfaces of the smoothed density distribution, labeling them as \( v \), which is related to the volume fraction \( f \) on the high-density side of the contour by

\[
f = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-x^2/2} dx,
\]

where \( x \) is the density parameter. The value \( f = 50\% \) corresponds to the median volume fraction contour (\( v = 0 \)). For GRP initial conditions, the genus curve is

\[
g_{\text{GRP}}(v) = A(1 - v^2)e^{-v^2/2},
\]

where the amplitude \( A \) is the average value of the squared wave vector, \( k^2 \), in the smoothed power spectrum (Gott et al. 1986) or the slope of the two-point correlation function.

The shape of a genus curve, and its deviation from the random phase prediction, can be parameterized by several variables. First, there is the \( \chi^2 \) best-fit amplitude, which is measured by fitting the GRP curve (Equation (4)) to the observed curve. This provides information about the power spectrum and phase correlation of the density fluctuation. Second, there are three variables that characterize deviations from a GRF: \( \Delta v \), \( A_v \), and \( A_C \) (Park et al. 1992). \( \Delta v \) measures any shift in the central part of the genus curve. The GRP curve has \( \Delta v = 0 \). A negative value of \( \Delta v \) is referred to as a meatball shift, caused by a greater prominence of isolated high-density structures, pushing the genus curve to the left. \( A_v \) and \( A_C \) measure the relative number of voids and clusters with respect to GRP expectations.

3. THE N-BODY SIMULATIONS

Cosmological N-body simulations have played a crucial role in explaining the observed universe with theories or physical models, especially for the large-scale structures of the universe and the highly nonlinear galaxy formation therein. There have been many “big” cosmological simulations up to now for the study of galaxies (Millennium: Springel et al. 2005; Millennium-II: Boylan-Kolchin et al. 2009; Bolshoi: Klypin et al. 2011) and cosmology (Horizon-4π: Teyssier et al. 2009; Horizon Run: Kim et al. 2009; Horizon Run 2 and 3: Kim et al. 2011; Millennium-XXL: Angulo et al. 2012; DEUS-FUR: Alimi et al. 2012; MultiDark: Prada et al. 2012; MICE-GC: Fosalba et al. 2013; BQG6912: Harnois-Déraps et al. 2013; Europa Hubble Volume: Watson et al. 2014; Dark Sky Simulations: Skillman et al. 2014), where a somewhat arbitrary division is made on a simulation box size of \( L_{\text{box}} = 1000 h^{-1}\) Mpc. Every simulation made a boast of its successful applications to various fields of cosmology and galaxy formation. For example, the Horizon Runs 2 and 3 were specifically designed for the comparison of cosmic topology between the ΛCDM model and SDSS observations (Kim et al. 2014b; Choi et al. 2013).

The Horizon Runs, provided by the Korean Institute of Advanced Study (KIAS), provide some of the best raw material for calibrating a topological study of LRG surveys (Park et al. 2005b; Kim et al. 2011). These N-body simulations replicate the topology of the SDSS LRGs exquisitely (Gott et al. 2009; Eisenstein et al. 2001). We use the HR3 data set exclusively, which adopts a pressureless cold dark matter cosmology with a pure cosmological constant. The basic HR3 cosmological parameters were fixed by the WMAP5 data (Spergel et al. 2003; Komatsu et al. 2011; Hinshaw et al. 2013) and the initial linear power spectrum was calculated using the CAMB source code (Lewis & Bridle 2002). The entire simulation is a cube of 374 billion pixels, spanning a volume of \((10.815 h^{-1}\text{Gpc})^3\). The initial redshift was \( z = 27 \) and \( N_{\text{step}} = 600 \) discrete timesteps were taken.

3.1. Mock LRG Survey Construction

The selection of cold dark matter halo masses uses the friends-of-friends algorithm (Audit et al. 1998; Davis et al. 1985; Huchra & Geller 1982; Knebe et al. 2011) where the separation cut off distance is 20% of the mean particle separation. To improve cluster identification, HR3 searches for PSB subhalos that are gravitationally self bound and not tidally disruptable (Kim & Park 2006). This provides a substantial increase in the similarity between simulation and observational data (Kim et al. 2008; Choi et al. 2010), as these dark matter subhalos are sites for LRG formation.

To simulate the SDSS survey dimensions, HR3 places 27 observers evenly within its cubical simulation volume and allows each observer to see out to \( z < 0.7 \). This creates 27 independent, non-overlapping spherical regions. The comoving positions and velocities of all CDM particles are saved as they cross their past light cone and PSB subhalos are identified from this data. In preparation for the SDSS-III LRG catalog, it was assumed that a volume-limited sample would yield a constant number density of \( 3 \times 10^{-4}/(h^{-1}\text{Mpc})^3 \). In order to match this prediction, the minimum mass limit of the PSB subhalos was varied with redshift. Given these parameters, the physical properties of the HR3 mock surveys match very well with the most recent LRG surveys (Choi et al. 2010; Gott et al. 2008, 2009).

4. METHODS

4.1. Smoothing and Discretization

We smooth the 27 past light cone PSB subhalo distributions with a Gaussian smoothing ball,

\[
W(r) = \frac{1}{(2\pi)^{3/2}} e^{-\frac{r^2}{2\lambda^2}},
\]

smearing out on scales smaller than \( \lambda \). The mock survey data are placed into a three-dimensional pixel grid of density values, and we choose \( \lambda \) to always be greater than 2.5 pixel side lengths \( s \). For cold dark matter models, smoothing with a Gaussian recovers the topology of the initial density field.
provided that the smoothing length $\lambda$ is sufficiently greater than the correlation length $R_0$ and nonlinear effects are avoided.\footnote{\textit{R}_0 \text{ is approximately } 5h^{-1}\text{Mpc for LRG.}}

4.2. Conversion and Trimming

With this smoothed mock survey in hand, we convert from comoving spherical coordinates to redshift coordinates using a comoving line-of-sight distance formula (Hogg 1999). PSB subhalo peculiar velocities are converted into redshift distortions by

$$\Delta \xi = \frac{v_r - v_{pec}}{c} = \hat{r} \cdot v_{pec},$$

where $v_r$ is the radial velocity, $\hat{r}$ is the unit radial vector, and $v_{pec}$ is the Cartesian peculiar velocity of the subhalo. After redshift converting and correcting, we save PSB subhalo counts within a grid of dimensions $650^3$, with a cubical pixel volume of $s^3 = (6h^{-1}\text{Mpc})^3$. The entire grid spans a volume of $(1950h^{-1}\text{Mpc})^3$ (see Figure 1 for an example of the Spherical HR3 mock survey).

We then apply an angular mask, splitting the 27 perfectly spherical mock surveys into four quadrants, each of which has $\pi$ steradians and radius $z = 0.6$, to approximate the area of sky coverage and depth in the SLOAN III survey. With these $4 \times 27 = 108$ smoothed mock surveys in hand, we calculate the genus using a polygonal approximation scheme developed by Weinberg (1988) and Hamilton et al. (1986) called “Contour 3D,” which adds up angle deficits at pixel vertices.

5. USING TOPOLOGY AS A STANDARD RULER

An application of quantitative topology to the SDSS LRG sample—other than testing the Gaussianity of the initial density fluctuations—is measuring cosmological parameters, such as those governing the expansion history of the universe. This can be done by measuring the genus statistic within a fixed volume at different redshifts. In the instance of $N$-body simulations, we know the correct cosmological model, and therefore the correct transformation $r \rightarrow z$. We smooth the density field with a known smoothing length $\lambda$ and then measures the median density genus within a volume $V$. This yields $g = G/V$, genus per unit volume, which we can use to indirectly measure any physical volume by counting structures. In order to more explicitly state the smoothing length dependence, the dimensionless quantity $g\lambda^3$ is often used, which is simply the genus per cubic smoothing length. This quantity can be analytically calculated from a full set of cosmological parameters and a linear power spectrum. Such a function $g\lambda^3(\lambda)$ has been examined closely for the WMAP3 and WMAP5 parameters (see Figure 1 of Park & Kim 2010 and Figure 1 of Zunckel et al. 2011; Kim et al. 2014a; see Figure 2).

In practice, we do not know the true cosmological model. Park & Kim (2010) have illustrated the effects of adopting incorrect values for $\Omega_m$ and $w$, but for completeness let us outline the consequences of applying incorrect cosmological parameters to a survey sample. If we underestimate the expansion rate of the universe—$H_0$, $w$, and $\Omega_m$—then our conversion from redshift to comoving space will put celestial objects too far from the Earth. This causes an overestimation of the survey volume (see Figure 2 of Park & Kim 2010). For a homogeneous and isotropic survey, the genus is linearly proportional to the volume and therefore an overestimation of $V$ will drive the genus at a certain smoothing length up ($G(\lambda) \uparrow$). At the same time, however, we have also adopted a comoving smoothing length $\lambda$ that is larger than intended. This will change the actual scale of study and erase all structure beneath the scale by convolution, decreasing the genus ($G(\lambda) \downarrow$). Luckily, the net effect is detectable since the amplitude $G$ of the genus curve effectively measures the slope of the power spectrum at the scale $\lambda$, which is not scale invariant (Park & Kim 2010; Zunckel et al. 2011).

Our procedure for measuring the angular diameter comoving distance to $z = 0.6$ is straightforward. We assume a $\Lambda$CDM flat cosmological model, $\Omega_m$, $h$, and $\Omega_\Lambda$ come from CMB fits with $l > 210$, which are insensitive to $w_\Lambda$ because dark energy has a negligible influence at recombination. These values are used to construct the power spectrum, and from that $g\lambda^3(\lambda)$ (see Figure 2). Now we measure $g\lambda^3$ and obtain a value; we look to our analytical plot (see Figure 2) and find the true value of
However, finite sample size, even at this level, introduces error density field of the entire N-body simulation in comoving space. Structures inside this volume is proportionate in error, then it will make no difference, as that will only distort shapes and structures inside this volume;\(^{8}\) and shot noise.

Figure 3. Top: genus per cubic smoothing length \(g\lambda^3\) for the WMAP5 parameters, with \(\chi^2\) best-fit data points and 1σ error bars. Bottom: the ensemble averaged genus curves \(G(v)\) for \(\lambda = 15, 21,\) and \(34\ h^{-1}\)Mpc.

\(\lambda\), which we will call \(\lambda_{\text{true}}\). If this is 1% smaller than the initial value of \(\lambda\) that we used, then it means that the comoving distance out to \(z = 0.6\) is also 1% smaller than previously thought. In this way, we can measure the comoving distance out to \(z = 0.6\). Also, with this as one data point, we can fit a cosmological model, leaving \(w_{\Lambda}\) as a parameter.

If the initial cosmological model is slightly wrong (i.e., \(w_{\Lambda}\) may not be exactly \(-1\), or may vary with time;\(^{9}\) Slepian et al. 2014), then this is inconsequential because we are just measuring the topology—counting the total number of structures inside \(z = 0.6\). If the radial comoving distance inside this volume is proportionate in error, then it will make no difference, as that will only distort shapes and structures slightly without altering their count (see Zunckel et al. 2011). An rms cosmic variance in the total genus \(\sigma_g\) out to \(z = 0.6\) in a survey sample will cause a fractional rms error of \(\sigma_g / g\) in \(g\lambda^3\), and given the slope of the curve, \((g\lambda^3)'\) at the applied \(\lambda\), this will introduce an rms error in \(\lambda\) (and therefore in the comoving distance at \(z = 0.6\)) of

\[
\frac{(g\lambda^3)'}{\lambda} \sigma_{\lambda} = \frac{\sigma_g}{g}.
\]

6. DISCUSSION OF UNCERTAINTIES

We examine the statistical variance of genus per unit volume \(g\) in the HR3 mock surveys, which is far from an “ideal” measurement.

An “ideal” measurement of \(g\) would be examining the initial density field of the entire N-body simulation in comoving space. However, finite sample size, even at this level, introduces error because of no power at large scales (or larger than the simulation box). The next best measurement of \(g\) would be to examine the final conditions of the entire N-body simulation in comoving space, which erases a portion of the cosmic variance associated with small survey size, but is subject to the effects of nonlinear gravitational infall and galaxy formation bias.

Observation of \(g\) in comoving space has obvious advantages to observation in redshift space, since we have complete knowledge of all PSB subhalo positions and velocities; knowledge that is destroyed by the transformation \(r \rightarrow z\). The redshift space distortion (RSD) of the genus curve has been studied by Choi et al. (2010) and has been shown that the redshift correction for peculiar velocity presents the worst source of error for the \(\chi^2\) best-fit amplitude of the genus curve. The application of peculiar velocity redshift corrections is in essence a smoothing routine of its own, in that real-space structures are radially smeared due to “fingers of god” effects. This effectively raises the observed smoothing parameter \(\lambda\) and yields a lower \(\chi^2\) best-fit genus amplitude \(g\).

Unavoidable sources of error, regardless of size or comoving versus redshift space, stem from the following: finite pixel size;\(^{7}\) boundary effects associated with a high surface area to volume ratio, where the structure is smoothed outside of a survey region;\(^{8}\) and shot noise.

An SDSS measurement of \(g\) uses a finite, redshift space sample where the aforementioned sources of error apply, and our predictions are meaningless unless we can quantitatively describe systematic effects and the constraints that observational data imposes.

Kim et al. (2014a) have completed an exhaustive study of the four main sources of systematic error in genus topology—finite pixel size, RSD, nonlinear evolution, and shot noise—using the Horizon Run 2 simulation. They find that the systematic drop in genus amplitude, \(g\), associated with finite pixel size can be reduced to 1% if we choose \(s\) to be at least three times smaller than the chosen smoothing length: \(\lambda > 3s\). Kim et al. (2014a) also find that RSD creates a systematic drop in \(g\), which depends on the growth factor and the degree of galaxy bias; however, the systematic effect is less than 2% along the entire genus curve \(g(v)\) at smoothing scales of \(15, 21,\) and \(34\ h^{-1}\)Mpc for the Horizon Run 2 Halo density field. Nonlinear gravitational

\(7\) Similar to peculiar velocity, finite pixel size applies an unintended smoothing scale to the data, systematically lowering the genus amplitude \(g\).

\(8\) For instance, the complicated boundaries of the SDSS; in particular, the three thin stripes along the southern Galactic cap, which are ignored altogether during genus analysis.
evolution for smoothing lengths $\lambda = 15, 22$, and $34 \ h^{-1} \ Mpc$ presents a systematic effect similar to that of RSD.

The primary source of systematic error in $g$ is due to shot noise, and Kim et al. (2014a) show clearly that the chosen smoothing length $\lambda$ should be at least as large as the mean galaxy separation, $\bar{n}_{1/3}$. If this inequality is satisfied, then we can expect $\pm 10\%$ error along the genus curve for halo density fields (see Figure 11, Kim et al. 2014a). Because of its size, this effect must be very accurately estimated, and Kim et al. (2014a) suggest varying the luminosity or mass-cut to make subsamples of galaxies—as compared to random selection from parent catalogs. This is exactly what two former studies have done, Gott et al. (2009) and Parihar et al. (2013), the latter of which catalogs this is exactly what two former studies have done, Gott et al. (2009) and Parihar et al. (2013), the latter of which reported 2.0% and 5.5% fractional uncertainties in genus per cubic smoothing length—for $\lambda = 21, 34 \ h^{-1} \ Mpc$—using the CMASS Data Release (DR10) sample of the SDSS-III BOSS.

The absence of a $15 \ h^{-1} \ Mpc$ smoothing length in Parihar et al. (2013) is due to this shot noise. When studying the topology of the large-scale structures, where galaxy number density varies with redshift, we must choose between a large redshift range—lots of comoving volume and cosmic structure, little variance in power, small boundary effects—and a high mean galaxy separation, $\bar{n}_{1/3}$. In the case of CMASS DR10, the galaxy number density peaks at around $z \sim 0.5$ and drops sharply around that point. If we were to smooth at a scale of $\lambda = 15 \ h^{-1} \ Mpc$, then we would have to study a very small redshift range of about $z \sim 0.5$, where the galaxy number density was sufficiently high and crippling shot noise could be avoided. Balancing between these two perils of small survey volume and shot noise, Parihar et al. (2013) chose a redshift range of $0.452 < z < 0.625$, where $\bar{n}_{1/3} \approx 20 \ h^{-1} \ Mpc$ at minimum.

As the SDSS continues, we will gain all-important survey volume and, perhaps, be able to push down the scale of study toward $15 \ h^{-1} \ Mpc$, where tight constraints on $g$ lie. We forecast this improvement, and use HR3’s enormous comoving volume to construct 108 “genus experiments” for the three smoothing lengths $\lambda = 15, 21$, and $34 \ h^{-1} \ Mpc$. Based on our results (see Table 1), we believe that the fractional uncertainty in smoothing length can be reduced to an order of 1.7%.

7. RESULTS

We measured the genus per cubic smoothing length for $\lambda = 15, 21$, and $34 \ h^{-1} \ Mpc$, studying the random and systematic error over 108 HR3 mock surveys. For $\lambda = 15 \ h^{-1} \ Mpc$, the fractional uncertainty in genus per cubic smoothing length was less than 1%, which translates to a fractional uncertainty in smoothing length—and angular diameter distance—of approximately 2.1% (Table 1).

Treating the variance at $\lambda = 15, 21$, and $34 \ h^{-1} \ Mpc$ as statistically independent—since HR3 adopts a random phase model and the smoothing volumes are significantly different—we add the three smoothing length rms errors in quadrature,

$$\frac{1}{\sigma_{eq}^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \frac{1}{\sigma_3^2},$$

yielding a 1.69% fractional uncertainty in smoothing length and an angular diameter distance out to $z = 0.6$. Combining only the 21 and $34 \ h^{-1} \ Mpc$ samples, we obtain a 2.97% fractional uncertainty in smoothing length.

With 108 samples in hand, our fractional “uncertainty of the uncertainty” is $1/\sqrt{2(1-N)} = 6.8\%$. It is notable that the systematic effect for the $21 \ h^{-1} \ Mpc$ sample was very small, $-0.84\%$, and that the $\chi^2$ best-fit genus amplitudes modeled the $g\lambda^3$ curve extraordinarily well (see Figure 3).

Park & Kim (2010) have pointed out that for a BOSS-like survey at redshift $z \sim 0.5$, the fractional error in $w$ is approximately 9.5 times the fractional error in genus per unit volume: $\Delta_w = 9.5\Delta_g$. This clearly shows that if we measure $g$ with tight accuracy—as we have done in this paper, with an “ideal” quadrant survey of $\pi$ steradians, finding $\Delta_g = 0.919\%$, 1.245%, and 2.166% for $\lambda = 15, 21$, and

![Figure 4](http://ccpp.nyu.edu/~speare/LSS_topology.html)
$34 h^{-1} \text{ Mpc}$, respectively—we can constrain $w$ with an rms error significantly less than 10%, which is competitive with BAO scale techniques. ($\Delta_{w} \approx 7\%$ if we add errors in quadrature and use the approximation cited above.) Parihar et al. (2013) have reported 2.0% and 5.5% fractional uncertainties in genus per cubic smoothing length, which, when added in quadrature, yield a fractional uncertainty of $\Delta_{w} \sim 18\%$.

A possible extension of this work is to more accurately model the SDSS survey with 108 less “ideal” masks, or to measure the number of $\lambda_{7}$ could yield useful information about the evolution of error with scale.

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APPENDIX

3D GRAPHICS OF THE HORIZON RUN 3

We provide two figures (Figures 4 and 5) for the three-dimensional view of simulated cosmic structures. Online versions of them can be found at http://ccpp.nyu.edu/~speare/LSS_topology.html.

REFERENCES

Albrecht, H., & Steinhardt, P. J. 1982, PhRvL, 48, 1220
Alimi, J.-M., Bouillot, V., Razera, Y., et al. 2012, arXiv:1206.2838
Anderson, L., Aubourg, E., Bailey, S., et al. 2012, MNRAS, 427, 3435
Angulo, R. E., Springel, V., White, S. D. M., et al. 2012, MNRAS, 426, 2046
Audit, E., Teyssier, R., & Alimi, J. 1998, A&A, 333, 779
Boylan-Kolchin, M., Springel, V., White, S. D. M., Jenkins, A., & Lemson, G. 2009, MNRAS, 398, 1150
Choi, Y.-Y., Kim, J., Rossi, G., Kim, S. S., & Lee, J.-E. 2013, ApJS, 209, 13
Choi, Y.-Y., Park, C., Kim, J., et al. 2010, ApJS, 190, 181
Davis, M., Efstathiou, G., Frenk, C., & White, S. D. M. 1985, ApJ, 292, 371
Eisenstein, D. J., Annis, J., Gunn, J. E., et al. 2001, AJ, 122, 2267
Eisenstein, D. J., Weinberg, D. H., Agol, E., et al. 2011, AJ, 142, 72
Fosalba, P., Crocce, M., Gaztañaga, E., & Castander, F. J. 2013, arXiv:1312.1707
Gott, J. R., Choi, Y.-Y., Park, C., & Kim, J. 2009, ApJL, 695, L45
Gott, J. R., Hambrick, D. C., Vogeley, M. S., et al. 2008, ApJ, 675, 16
Gott, J. R., Melott, A. L., & Dickinson, M. 1986, ApJ, 306, 341
Gott, J. R., Miller, J., Thuan, T. X., et al. 1989, ApJ, 340, 625
Gott, J. R., Weinberg, D. N., & Melott, A. L. 1987, ApJ, 319, 1
Guth, A. H. 1981, PhRvD, 23, 347
Hamilton, A. J. S., Gott, J. R., & Weinberg, D. W. 1986, ApJ, 308, L1
Harnois-DéRaps, J., Pen, U.-L., Iliev, I. T., et al. 2013, MNRAS, 436, 540
Hikage, C., Schmalzing, J., Buchert, T., et al. 2003, PASJ, 55, 911
Hikage, C., Suto, Y., Kayo, I., et al. 2002, PASJ, 54, 707 (The SDSS Collaboration)
Hinshaw, G., Larson, D., Agol, E., et al. 2013, ApJ, 208, 19
Hogg, D. W. 1999, arXiv:astro-ph/9905116
Huchra, J. P., & Geller, M. J. 1982, ApJ, 257, 1
Kim, J., & Park, C. 2006, ApJ, 659, 600
Kim, J., Park, C., & Choi, Y.-Y. 2008, ApJ, 683, 123
Kim, J., Park, C., Gott, J. R., & Dubinski, J. 2009, ApJL, 701, 1547
Kim, J., Park, C., Ross, G., Lee, S. M., & Gott, J. R. 2011, JKAS, 44, 217
Kim, Y.-R., Choi, Y. Y., Kim, S. S, et al. 2014, ApJS, 212, 22
Kim, Y.-R., Choi, Y.-Y., Kim, S. S., et al. 2014b, ApJS, 212, 22
Klypin, V., Trujillo-Gomez, S., & Primack, J. 2011, ApJ, 740, 102
Knebe, A., Knollmann, S. R., Muldrew, S. I., et al. 2011, MNRAS, 415, 2293
Komatsu, E., Smith, K. M., Dunkley, J., et al. 2011, ApJS, 192, 18
Lewis, A., & Bridle, S. 2002, PhRvD, 66, 103511
Linde, A. D. 1982, PhLB, 108, 389
Linde, A. D. 1983, PhLB, 129, 177
Matsubara, T. 1994, ApJL, 434, L43
Parihar, P., Vogeley, M., Gott, J. R., et al. 2014, ApJ, 796, 86
Park, C., Choi, Y.-Y., Kim, J., et al. 2012, ApJ, 759, L7
Park, C., Choi, Y.-Y., Vogeley, M. S., et al. 2005a, ApJ, 633, 11 (The SDSS Collaboration)
Park, C., Colley, W. N., Gott, J. R., et al. 1998, ApJ, 506, 473
Park, C., & Gott, J. R. 1991, ApJ, 378, 457
Park, C., Gott, J. R., & da Costa, L. N. 1992, ApJL, 392, 177
Park, C., Kim, J., & Gott, J. R. 2005b, ApJ, 633, 1
Park, C., & Kim, Y.-R. 2010, ApJL, 715, L185
Park, C., Pranav, P., Chingangbam, P., et al. 2013, JKAS, 46, 78
Park, C., Choi, Y.-Y., Vogeley, M. S., et al. 2005a, ApJ, 633, 11 (The SDSS Collaboration)
Park, C., Colley, W. N., Gott, J. R., et al. 1998, ApJ, 506, 473
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Skillman, S. W., Warren, M. S., Turk, M. J., et al. 2014, arXiv:1407.2600
Slepian, Z., Gott, J. R., & Zinn, J. 2014, MNRAS, 438, 1948
Soneira, R. M., & Peebles, P. J. E. 1978, ApJ, 83, 845
Springel, V. 2005, Natur, 435, 629
Stark, M. A., Weinberg, D. H., Lupton, R. H., et al. 2002, AJ, 124, 1810
Teyssier, R., Fires, S., Prunet, S., et al. 2009, A&A, 497, 335
Vogeley, M. S., Park, C., Geller, M. J., Huchra, J. P., & Gott, J. R. 1994, ApJ, 420, 525
Watson, W. A., Iliev, I. T., Diego, J. M., et al. 2014, MNRAS, 437, 3776
Weinberg, D. H. 1988, PASP, 100, 1373
Zunckel, C., Gott, J. R., & Lunnan, R. 2011, MNRAS, 412, 1401