LCR and AFD of the Products of Nakagami-m and Nakagami-m Squared Random Variables: Application to Wireless Communications Through Relays

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Abstract

The paper considers level crossing rate (LCR) and average fade duration (AFD) of the product of independent and identically distributed (i.i.d) Nakagami-m (NM) and double NM squared (also known as gamma-gamma) random variables (RVs). The derived statistics are then directly applied to the radio-frequency (RF) - free space optical (FSO), dual-hop (DH), amplify-and-forward (AF) relaying system over non turbulent-induced-fading channels (nTIFCs) and turbulent-induced-fading channels (TIFCs). The obtained results for LCR and AFD of DH-AF, RF-FSO system over TIFCs and nTIFCs are numerically evaluated and graphically presented for various system model parameters.

Keywords AFD · AF relaying · FSO · LCR · second order statistical (SOS) · FSO

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1 Introduction

The relaying wireless systems play an important role in wireless communications. The huge variety of technologies that includes relays are all radio-frequency (RF) relaying schemes, all free-space-optical (FSO) relaying schemes, mixed RF-FSO relaying schemes, millimeter-wave (mmW)-FSO relaying schemes [1–4]. In particular, amplify-and-forward (AF) relaying systems has been proposed for application in fronthaul/backhaul 5G systems [5], vehicle-to-everything (V2X) communications [6, 7], unmanned ariel vehicle (UAV) communications [8], device-to-device (D2D) communications [9], mobile-to-mobile (M2M) communications [10–13] and underwater optical-wireless (UOW) communications [14]. The FSO relaying enabled communications can ensure high capacity and wide bandwidth [15]. Moreover, FSO communications are (1) cost effective, (2) non sensitive to co-channel interference and (3) spectrum license free that makes this type of technology a promising choice for future wireless systems. The main factor of FSO system performance degradation is caused by atmospheric turbulence (also known as scintillation). The weather conditions and pointing errors can result in an additional FSO system performance deterioration. The references [16–18] investigates RF-FSO relay systems while FSO-FSO relay systems are explored in [19]. The secrecy performance evaluation of cooperative RF-FSO system is investigated in [20]. Moreover, the re-configurable intelligent surface (RIS) assisted RF-FSO communications link is investigated in [21] while the satellite-aerial-terrestrial link is considered in [22]. The above-mentioned works consider the first-order statistics (FOS) of mixed RF-FSO relay systems.

Besides the FOS, the second order statistics (SOS) such as average level crossing rate (LCR) and average fade duration (AFD) can more adequately characterize the time-variant fading channels. The LCR of gamma-gamma random variable (RV) is addressed in [23], while the LCR over Malaga turbulence-induced-fading channels (TIFCs) is addressed in [24]. The LCR expression of the ratio of two gamma-gamma RVs is derived in [25]. Moreover, the SOS over log-normal TIFCs of multi-hop FSO system with pointing errors are addressed in [26], while the SOS over gamma-gamma RVs of multi-hop FSO transmission system is considered in [27]. The SOS of V2V of mixed RF-FSO-RF over NM non TIFCs (nTIFCs) and gamma-gamma TIFCs are considered in [6]. The [28, 29] gives some experimental results for SOS of FSO systems. Moreover, SOS of satellite-UAV FSO communications where the TIFC is modeled with Log-normal distribution are presented in [30]. The Nakagami-m (NM) RV can be used to model nTIFC in RF environments [31–33], while gamma-gamma (double squared NM) RV is the most often used to model the fading for FSO communications in moderate to strong TIFC environments [34–36].

This paper considers the SOS of the product of independent and identically distributed (i.i.d) NM RV and gamma-gamma RV (modeled as the product of two squared NM RVs). The derived statistical measures are directly applied to dual-hop (DH) AF relay RF-FSO communications over mixed nTIFC and TIFC. The derived integral expressions as well as the closed form approximations for LCR and AFD are numerically evaluated and graphically presented in relation to DH AF RF-FSO system model set of parameters. To the best of author’s knowledge there is no publications on the SOS of DH-AF mixed RF-FSO systems.
2 System Model

The cascaded, double and composite fading channels can be modeled as the product of RVs \([26, 27, 37–41]\). On the other hand, interference limited environments can be modeled as the ratio of RVs \([42–44]\). Thus, the products and the ratios of RVs play an important role in the performance analysis of wireless communication systems. It has been shown by \([45]\, Eq. (39)] that the signal envelope at the output of AF relaying over non turbulence-induced-fading channels (nTIFCs) for fixed gain relays can be modeled as the product of N Rayleigh RVs. The FSO multi-hop AF relaying over turbulence-induced-fading channels (TIFCs) are considered as a product of log-normal RVs in \([26]\, Eq. (13)]

Thus, we model the fading signal envelope from source to destination, \(z_{SD}\) of DH-AF relay (shown in Fig. 1) as the product of the NM RV, \(y_{NM,1}\) and gamma-gamma RV, \(y_{GG}\) (modeled as the product of two NM squared i.i.d RVs, \(y_{NM,2}^2\) and \(y_{NM,3}^2\), respectively) \([31]\, Eq. (2.55)]:

\[
\begin{align*}
z_{SD} &= y_{NM,1}^{Ihop} y_{GG}^{Ihop} = y_{NM,1}^{Ihop} y_{NM,2}^2 y_{NM,3}^2
\end{align*}
\]

where \(y_{NM,j}\) are NM RVs, representing the signal envelopes over nTIFC and TIFC from source to destination, whose probability density functions (PDFs) in terms of fading severity parameters and average powers, denoted as \(m_{NMi}\) and \(\omega_{NMi}\) respectively, are:

\[
p_{Y_{NM,j}}(y_{NM,j}) = \frac{2(m_{NM}/\omega_{NM})^{m_{NM}}}{\Gamma(m_{NM})} y_{NM,j}^{2m_{NM}-1} e^{-\frac{m_{NM} y_{NM,j}^2}{\omega_{NM}}} y_{NM,j}^2
\]

The PDFs parameters of the FSO NM squared RVs can be expressed as \([46]\, Eqs. (10–11)] and \([34]\, Eqs. (7–8)], respectively:

\[
\omega_{NM2} = \omega_{NM3} = 1
\]

\[
\alpha_{GG} = m_{NM2} = \left[ \exp \left( \frac{0.49\delta^2}{(1 + 0.18d^2 + 0.56\delta^{12/5})^{1/6}} \right) - 1 \right]^{-1}
\]

Fig. 1 Simplified scheme of RF-FSO dual-hop AF relaying
\[ \beta_{GG} = m_{NM3} = \left[ \exp \left( \frac{0.51 \delta^2 (1 + 0.69 \delta^{1/2})^{-5/6}}{(1 + 0.9d^2 + 0.62d^2 \delta^{1/2})^{5/6}} \right) - 1 \right]^{-1} \]  

where \( \alpha_{GG} \) and \( \beta_{GG} \) are FSO small-scale and large-scale atmospheric cell parameters, respectively, \( \delta^2 = 0.5C_n^2k_{FSO}^2L_{FSO}^{1/6} \) is the FSO Rytov variance and \( d = \sqrt{k_{FSO}D_{FSO}^2/4L} \) is the FSO wave number. Moreover, \( C_n^2 \) is FSO refractive index, \( k_{FSO} = 2\pi/\lambda \) is FSO wave-number \((\lambda \text{-wavelength})\), \( D_{FSO} \) is FSO receiver aperture diameter and \( L \) is FSO distance between source and destination.

### 3 Second Order Statistics (SOS)

The level crossing rate (LCR) of the product of NM and double squared NM RV for a given threshold \( z_{th,SD} \) can be expressed as \( \dot{z}_{SD} \) [39, Eq. (20)],

\[ N_{Z_{SD}}(z_{th,SD}) = \int_0^\infty \dot{z}_{SD} p_{Z_{SD}Z_{SD}}(z_{th,SD} \dot{z}_{SD}, d\dot{z}_{SD} \)  

which can be for independent RVs further expressed as in [45, Eq. (13)]:

\[
p_{Z_{SD}Z_{SD}Y_{NM,2}Y_{NM,3}}(z_{SD} \dot{z}_{SD} Y_{NM,2} Y_{NM,3}) = p_{Z_{SD}/Z_{SD}Y_{NM,2}Y_{NM,3}}(z_{SD} \dot{z}_{SD} Y_{NM,2} Y_{NM,3}) \\
\times p_{Z_{SD}/Y_{NM,2}Y_{NM,3}}(\dot{z}_{SD}/Y_{NM,2} Y_{NM,3}) p_{Y_{NM,2}/Y_{NM,3}}(Y_{NM,3}) p_{Y_{NM,3}/Y_{NM,3}}(Y_{NM,3})  
\]

where, \( p_{Z_{SD}/Y_{NM,2}Y_{NM,3}}(z_{SD} \dot{z}_{SD} Y_{NM,2} Y_{NM,3}) = \left| \frac{dy_{NM,1}}{dz_{SD}} \right| p_{Y_{NM,1}}(z_{SD}/Y_{NM,2} Y_{NM,3}) \).

The variance of \( \dot{Z}_{SD} \), denoted as \( \sigma_{Z_{SD}}^2 \), under assumption that \( \dot{Z}_{SD} \) is a zero mean Gaussian RV can be expressed through the variances of \( Y_{NM,1} \), \( Y_{NM,2} \) and \( Y_{NM,3} \), denoted as, respectively, \( \sigma_{Y_{NM,1}}^2 \), \( \sigma_{Y_{NM,2}}^2 \) and \( \sigma_{Y_{NM,3}}^2 \):

\[
\sigma_{Z_{SD}}^2 = \gamma_{NM,2}^4 \gamma_{NM,3}^4 \sigma_{Y_{NM,1}}^2 \left( 1 + \frac{4\sigma_{Z_{SD}}^2}{\gamma_{NM,2}^4 \gamma_{NM,3}^4} + \frac{4\sigma_{Z_{SD}}^2}{\gamma_{NM,2}^4 \gamma_{NM,3}^4} \right)  
\]

where derivation of the \( \sigma_{Z_{SD}}^2 \) is provided in the Appendix. After evaluating the integral [47, Eq. (16)],

\[
\int_0^\infty \dot{z}_{SD} p_{Z_{SD}/Z_{SD}Y_{NM,2}Y_{NM,3}}(z_{th,SD} \dot{z}_{SD} Y_{NM,2} Y_{NM,3}) d\dot{z}_{SD} = \frac{1}{\sqrt{2\pi}} \sigma_{Z_{SD}} \]

the \( N_{Z_{SD}}(z_{th,SD}) \) can be expressed after some manipulations as:

\[
N_{Z_{SD}}(z_{th,SD}) = \frac{8^{m_{NM1}/2} \alpha_{NN1}^m \sigma_{Y_{NM,1}}^2 \beta_{GG} \sigma_{Y_{NM,2}}^2 \gamma_{NN1}^{2m_{NM1}+1} \sqrt{2\pi \Gamma(m_{NM1}) \Gamma(\alpha_{GG}) \Gamma(\beta_{GG})}}{J_1}  
\]
where,
\[
J_1 = \int_0^\infty dy_{NM,2} \int_0^\infty dy_{NM,3} \times \sqrt{1 + \frac{4z_{th,SD}^2}{y_{NM,2}^6} \frac{\sigma_{y_{NM,1}}^2}{\sigma_{y_{NM,2}}^2} + \frac{4z_{th,SD}^2}{y_{NM,3}^6} \frac{\sigma_{y_{NM,1}}^2}{\sigma_{y_{NM,3}}^2}} \]

\[
\times e^{\frac{\alpha_{NM} y_{NM,2}^2 - \beta_{NM} y_{NM,3}^2}{2}} - (2\alpha_{GG} - 4m_{NM1} + 1) \ln(y_{NM,2}) \times e^{2(\beta_{GG} - 4m_{NM1} + 1) \ln(y_{NM,3})}
\]

The \( J_1 \) can be approximated by [45], Eq. I.3:
\[
\int_0^\infty dy_{NM,2} \int_0^\infty g(y_{NM,2}, y_{NM,3}) e^{-\gamma f(y_{NM,2}, y_{NM,3})} dy_{NM,3} \approx \frac{2\pi g(y_{NM,2}(0), y_{NM,3}(0))}{\gamma} \left[ e^{-\gamma f(y_{NM,2}(0), y_{NM,3}(0))} \right] \]

\[
(12)
\]

where, \( y_{NM,2}(0) \) and \( y_{NM,3}(0) \) are positive and real values obtained from the defined set of equations,
\[
\frac{\partial f(y_{NM,2}(0), y_{NM,3}(0))}{\partial y_{NM,2}(0)} = 0, \quad \frac{\partial f(y_{NM,2}(0), y_{NM,3}(0))}{\partial y_{NM,3}(0)} = 0
\]

\[
(13)
\]

and where \( detB \) is determinant of 2 \times 2 matrix \( B \),
\[
B = \begin{bmatrix}
\frac{\partial^2 f(y_{NM,2}(0), y_{NM,3}(0))}{\partial y_{NM,2}(0) \partial y_{NM,3}(0)} & \frac{\partial^2 f(y_{NM,2}(0), y_{NM,3}(0))}{\partial y_{NM,2}(0) \partial y_{NM,1}(0)} \\
\frac{\partial^2 f(y_{NM,2}(0), y_{NM,3}(0))}{\partial y_{NM,3}(0) \partial y_{NM,1}(0)} & \frac{\partial^2 f(y_{NM,2}(0), y_{NM,3}(0))}{\partial y_{NM,3}(0) \partial y_{NM,1}(0)}
\end{bmatrix}
\]

\[
(14)
\]

The LCR can be approximated for the following choice of the parameter \( \gamma \) and the functions \( g(y_{NM,2}, y_{NM,3}) \) and \( f(y_{NM,1}, y_{NM,2}) \), respectively: \( \gamma = 1 \),
\[
g(y_{NM,2}, y_{NM,3}) = \sqrt{1 + \frac{4z_{th,SD}^2}{y_{NM,2}^6} \frac{\sigma_{y_{NM,1}}^2}{\sigma_{y_{NM,2}}^2} + \frac{4z_{th,SD}^2}{y_{NM,3}^6} \frac{\sigma_{y_{NM,1}}^2}{\sigma_{y_{NM,3}}^2}}
\]

\[
(15)
\]

\[
f(y_{NM,1}, y_{NM,2}) = \frac{m_{NM1}}{\omega_{NM1}} \frac{z_{th,SD}}{y_{NM,2}^4 y_{NM,3}^4} + \alpha_{GG} y_{NM,2}^2 + \beta_{GG} y_{NM,3}^2
\]

\[
- (2\alpha_{GG} - 4m_{NM1} + 1) \ln(y_{NM,2}) - (2\beta_{GG} - 4m_{NM1} + 1) \ln(y_{NM,3})
\]

Finally, the closed form approximation of \( N_{ZSD} (z_{th,SD}) \) becomes:
\( N_{zd}(z_{th,SD}) \approx \frac{16\pi (m_{NM1}/\omega_{NM1})^{m_{NM1}} a_{GG}^G \beta_{GG}^G \sigma_{Y_{NM3}}^2 z_{th,SD}^{2m_{NM1} - 1}}{\sqrt{2\pi \Gamma(m_{NM1})^{\Gamma(a_{GG}^G)\Gamma(\beta_{GG}^G)}}} \times \sqrt{\frac{1 + \frac{4z_{th,SD}^2}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)}}{\frac{\sigma_{Y_{NM1}}^2}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)}} + \frac{4z_{th,SD}^2}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)}} \times \frac{\sigma_{Z_{SD}}^2}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)}} \times \sqrt{\det B} \times e^{-\frac{m_{NM1}}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)}} + a_{GG}^G y_{NM3}^2 (0) + \beta_{GG}^G y_{NM3}^2 (0) \times e^{(2a_{GG}^G - 4m_{NM1} + 1) \ln (y_{NM3} (0)) + (2\beta_{GG}^G - 4m_{NM1} + 1) \ln (y_{NM3} (0))} \right) \) (17)

The AFD can be then computed as [31, Eq. (2.106)], [48, Eq. (2.9)]:

\[
AFD(z_{th,SD}) = \frac{F_{zd}(z_{th,SD})}{N_{zd}(z_{th,SD})} \tag{18}
\]

where \( F_{zd}(z_{th,SD}) \) is the cumulative distribution function (CDF) for the specified threshold \( z_{th,SD} \). The \( F_{zd}(z_{SD}) \) can be expressed as [31, Eq. (2.107)], [48, Eq. (2.4)]:

\[
F_{zd}(z_{SD}) = \int_0^{z_{SD}} p_{zd}(s) ds \tag{19}
\]

where \( p_{zd}(z_{SD}) \) is the probability density function (PDF) at the output of DH-AF RF-FSO proposed model and can be further expressed as:

\[
p_{zd}(z_{SD}) = \frac{4(a_{GG}^G)^{m_{NM1}} \gamma_{NM3}^2 \gamma_{NM2}^2}{\Gamma(m_{NM1})^{\Gamma(a_{GG}^G)\Gamma(\beta_{GG}^G)}} \times \left( \frac{z_{SD}^2}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)} \right)^{m_{NM1}} (m_{NM1} - 1)! \times \frac{\Gamma(a_{GG}^G)\Gamma(\beta_{GG}^G)}{4(a_{GG}^G)^{a_{GG}^G} \gamma_{NM2}^2 \gamma_{NM3}^2} \left( \frac{\omega_{NM1}}{m_{NM1}} \right)^{m_{NM1}} \sum_{k=0}^{m_{NM1}} \frac{(\frac{z_{SD}^2}{\gamma_{NM3}^2 (0)y_{NM3}^2 (0)})^k}{k!} J_2 \tag{20}
\]

where \( J_2 \) in (21) is given:

\[
J_2 = \int_0^{\infty} dy_{NM2} \int_0^{\infty} \frac{y_{NM2}^{2a_{GG}^G - 4k - 1}}{y_{NM3}^{2\beta_{GG}^G - 4k - 1}} \times e^{-\frac{m_{NM1} \gamma_{NM3}^2 (0)}{\gamma_{NM2}^2 (0)y_{NM3}^2 (0)}} \times e^{-a_{GG}^G y_{NM2}^2 (0) + \beta_{GG}^G y_{NM3}^2 (0)} \times dy_{NM3} \tag{21}
\]
The closed form approximation of $F_{\text{sd}}(z_{\text{sd}})$ can be calculated with a help of (12)-(14) for the following choices of parameter $\gamma$ and functions $g(Y_{NM2},Y_{NM3})$ and $f(Y_{NM2},Y_{NM3})$, respectively: $\gamma = 1$, $g(Y_{NM2},Y_{NM3}) = 1$

$$f(Y_{NM2},Y_{NM3}) = \frac{m_{NM1}}{\omega_{NM1} y_{NM2}^4 y_{NM3}^4} + \alpha_{GG} y_{NM2}^2 + \beta_{GG} y_{NM3}^2$$

$$- (2\alpha_{GG} - 4k - 1) \ln(y_{NM2}) - (2\beta_{GG} - 4k - 1) \ln(y_{NM3})$$

(23)

Finally, the closed form $F_{Z_{\text{sd}}}(z_{\text{th},SD})$ is:

$$F_{Z_{\text{sd}}}(z_{\text{th},SD}) \approx \frac{4(\alpha_{GG})^y_{GG}(\beta_{GG})^\beta_{GG}(\frac{m_{NM1}}{\omega_{NM1}})^{m_{NM1}}(m_{NM1} - 1)!}{\Gamma(m_{NM1})\Gamma(\alpha_{GG})\Gamma(\beta_{GG})} \times \left( \frac{\omega_{NM1}}{4(\alpha_{GG})^y_{GG}(\beta_{GG})^\beta_{GG}m_{NM1}} \right)^{m_{NM1}} - \sum_{k=0}^{m_{NM1} - 1} \frac{(\frac{m_{NM1}^2}{\omega_{NM1}})^k}{k!} J_3$$

(24)

where,

$$J_3 \approx \sqrt{\det B} \times e^{-\frac{2\pi}{me}}$$

$$\times e^{(2\alpha_{GG} - 4k - 1) \ln(y_{NM2}(0)) + (2\beta_{GG} - 4k - 1) \ln(y_{NM3}(0))}$$

(25)

4 Numerical Results

The second order statistics (SOS) are provided through LCR and AFD statistical measures. The variance of NM RV in (8) is expressed as $\sigma^2_{Y_{NM1}} = \pi^2 f_m^2 m_{NM1}^2$, where $f_m$ is the maximum Doppler frequency. Moreover, the considered SOS are evaluated for $\omega_{NM1} = 1$. The variance of NM squared RVs, $\sigma^2_{Y_{NM2}}$ and $\sigma^2_{Y_{NM3}}$ in (8) are modeled as a zero mean Gaussian RV and expressed as, $\sigma^2_{Y_{NM1}} = \sigma^2_{Y_{NM2}} = f_0 \pi^2 \sigma^2_{GG} |Y_{GG}|$ [24], Eq. (13)]. Further, $\langle Y_{GG} \rangle = 1$ for double NM squared RV [46], Eqs. (10–11), the $\sigma^2_{GG}$ is given by [46], Eq. (15)] and $f_0 = \frac{1}{\pi n \sqrt{2}}$ is the FSO quasi frequency, where $t_0 = \sqrt{\frac{2L}{ut}}$ is turbulence FSO correlation time, $\lambda$ is the FSO wavelength, $L$ is the FSO distance and $ut$ is the speed of the wind directed towards the FSO part of the system [24].

Figure 2 shows that the closed form $N_{Z_{\text{sd}}}(z_{\text{th},SD})$ provided in (10) and presented for various RF-FSO sets of parameters ($f_m, \lambda$), constant $ut = 1m/s$ and $L=200m$ and under different turbulence-induced-fading channel (TIFC) and non TIFC (nTIFC) conditions, $(\alpha_{GG} = 3, \beta_{GG} = 3, m_{NM1} = 3), (\alpha_{GG} = 2, \beta_{GG} = 2, m_{NM1} = 2)$ and $(\alpha_{GG} = 1, \beta_{GG} = 1, m_{NM1} = 1)$ fits well with exact integral form solution provided in (17). Moreover, in the case when output signal experience increase in nTIFC and TIFC severity values, $N_{Z_{\text{sd}}}(z_{\text{th},SD})$ decreases, as
expected. The impact of the maximal Doppler frequency for \( f_m = 90 \text{Hz} \) and \( f_m = 120 \text{Hz} \) and FSO optical windows for \( (\lambda = 850 \text{nm}, \text{and} \lambda = 1550 \text{nm}) \) on \( N_{ZSD}(z_{th,SD}) \) is investigated. By increasing the FSO wavelength, the curves slightly decrease. Contrary, by increasing observed \( f_m \), the graphs slightly increase. Furthermore, it can be observed that \( nTIFC \) and \( TIFC \) parameters have stronger impact on \( N_{ZSD}(z_{th,SD}) \) than \( f_m \) and \( \lambda \) which means that RF and FSO channel severity conditions have much stronger impact on \( N_{ZSD}(z_{th,SD}) \). It can be further observed that for the \( z_{th,SD} \) values around 0 dB, \( N_{ZSD}(z_{th,SD}) \) reaches higher values due to the probability of increased signal envelope shifts from above to below \( z_{th,SD} \) threshold level and vice versa. Interestingly, around \( z_{th,SD} = 0 \), the \( N_{ZSD}(z_{th,SD}) \) is mainly independent of TIFC and \( nTIFC \) conditions. On the other hand for smaller and higher \( z_{th,SD} \) dB values, the \( N_{ZSD}(z_{th,SD}) \) takes lower values since the probability that signal envelope is either above or below \( z_{th,SD} \) dB threshold.

The \( AFD(z_{th,SD}) \) statistics is provided in Fig. 3. It can be noticed that curves fitting between closed form approximate expression and exact integral analytical expression for observed system RF-FSO DH-AF parameters is achieved, especially for higher \( z_{th,SD} \) dB values. By increasing \( nTIFC \) and \( TIFC \) RF-FSO severities, the \( AFD(z_{th,SD}) \) increases for lower thresholds values while the \( AFD(z_{th,SD}) \) decreases for higher thresholds values. Furthermore, the effect of various RF maximal Doppler frequencies \( (f_m = 90 \text{Hz} \) and \( f_m = 120 \text{Hz} \) and various FSO wavelengths \( (\lambda = 850 \text{nm}, \text{and} \lambda = 1550 \text{nm}) \) on \( AFD(z_{th,SD}) \) is also provided. It can be noticed that the impact of TIFC severities on the \( AFD(z_{th,SD}) \) for the observed severity parameters is stronger than observed \( f_m \) or \( \lambda \).

![Fig. 2 Comparison of exact and approximated results for LCR for different system model parameters under different TIFC and nTIFC conditions](image-url)
The paper considers second order statistics of the product of NM and double squared NM RVs. The provided statistical measures can be used in RF-FSO dual-hop (DH) amplify-and-forward (AF) relaying systems over turbulence-induced-fading channels (TIFCs) and non TIFCs (nTIFCs). Namely, we provide closed form approximative expressions as well as integral form exact expressions for LCR and AFD. The numerical examples show that approximations fit well with exact expressions, especially in higher signal envelope dB output regime. The system performance improvement can be reached for the RF-FSO system by designing the system with higher TIFC and nTIFC severity parameters. Moreover, the observed TIFC and nTIFC severities of the considered RF-FSO DH AF relaying system have stronger impact on \( N_{Z_{SD}}(z_{th,SD}) \) and \( AFD(z_{th,SD}) \) than observed \( f_m \) and \( \lambda \). Our future works are going to extend the proposed model to include the impact of co-channel interference in RF part of the system and pointing errors in FSO part of the system.

**Appendix A**

The variance of \( \dot{Z}_{SD} \) denoted as \( \sigma^2_{Z_{SD}} \) and given by (8) is obtained under the assumption that \( Z_{SD} \) is a zero-mean Gaussian RV [45]. Based on (1), the first derivative of \( Z_{SD} \) can be written as:

\[
\dot{Z}_{SD} = y^2_{NM,2} y^2_{NM,3} \dot{y}_{NM,1} + 2 y_{NM,1} y_{NM,2} y^2_{NM,3} \dot{y}_{NM,2} + 2 y_{NM,1} y^2_{NM,2} y_{NM,3} \dot{y}_{NM,3}
\] (A1)
where $\dot{Y}_{NM,1}$, $\dot{Y}_{NM,2}$ and $\dot{Y}_{NM,3}$ are the first derivatives of $Y_{NM,1}$, $Y_{NM,2}$ and $Y_{NM,3}$, respectively. Since the linear transformation of the Gaussian RVs is a zero mean Gaussian RV, the variance of $\sigma^2_{Z_{SD}}$ can be expressed through the variances of $\dot{Y}_{NM,1}$, $\dot{Y}_{NM,2}$ and $\dot{Y}_{NM,3}$ denoted as $\sigma^2_{Y_{NM,1}}$, $\sigma^2_{Y_{NM,2}}$ and $\sigma^2_{Y_{NM,3}}$, respectively:

$$
\sigma^2_{Z_{SD}} = y^4_{NM,2} y^4_{NM,4} \sigma^2_{Y_{NM,1}} + 4 y^2_{NM,1} y^2_{NM,2} y^2_{NM,3} \sigma^2_{Y_{NM,2}} + 4 y^2_{NM,1} y^2_{NM,2} y^2_{NM,3} \sigma^2_{Y_{NM,3}}
$$

(A2)

After using substitution $y_{NM,1} = \frac{Z_{SD}}{y^2_{SM,2} y^2_{NM,3}}$ and some algebra $\sigma^2_{Z_{SD}}$ is obtained as given by (8).

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**Declarations**

**Conflicts of interest/Competing interests** The authors declare that they have no conflict of interest or competing interests.

**Code availability** Not applicable.

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