Heavy Quark Anti-Quark Free Energy and the Renormalized Polyakov Loop

O. Kaczmarek, F. Karsch, P. Petreczky and F. Zantow

Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

ABSTRACT

We calculate the colour averaged and colour singlet free energies of static quark anti-quark sources placed in a thermal gluonic heat bath. We discuss the renormalization of these free energies using the short distance properties of the zero temperature heavy quark potential. This leads to the definition of the renormalized Polyakov loop as an order parameter for the deconfinement phase transition of the SU(3) gauge theory which is well behaved in the continuum limit.
1 Introduction

Universal properties of the finite temperature phase transition in non-abelian $SU(N)$
gauge theories generally are discussed in terms of the Polyakov loop, which is an or-
der parameter for the confinement-deconfinement transition. In a somewhat loosely
defined terminology the Polyakov loop, $L$, is said to be related to the free energy of
a heavy, static quark put as a test charge into a thermal medium, $L \sim \exp(-F_q/T)$.
However, it is well known that the Polyakov loop calculated on the lattice is a priori
not properly renormalized. Taking the continuum limit at fixed temperature will
lead to a vanishing Polyakov loop expectation value even in the deconfined phase as
there are divergent self-energy contributions to the free energy. A proper renor-
malization of the Polyakov loop thus is needed in order to relate it to the heavy quark
free energy in the continuum limit which then can also be used to define a proper
order parameter that survives the continuum limit and can, for instance, be used to
construct effective actions for the confinement-deconfinement transition [1].

We suggest here that a proper renormalization of Polyakov loop can be ob-
tained through the renormalization of the finite temperature free energy of a static
quark anti-quark pair calculated at short distances. At short distances the quark
anti-quark pair interacts through the exchange of gluons, which may be calculated
perturbatively. More importantly we expect that this interaction is essentially tem-
perature independent for separations $r$ which are smaller than the average separation
between partons in the thermal medium. At short distances the finite temperature
heavy quark anti-quark free energy thus will be given by the zero temperature heavy
quark potential. This allows to remove divergent self-energy contributions in the free
energy, $F_{\bar{q}q}(r,T)$, through a matching of its short distance behaviour to that of the
heavy quark anti-quark potential, $V_{\bar{q}q}(r)$, at zero temperature. After having done
so also the large distance behaviour of $F_{\bar{q}q}(r,T)$ is fixed. As this, in turn, is re-
lated to the Polyakov loop expectation value, it allows to define a renormalized
Polyakov loop, i.e. as usual no additional divergences will show up in the heavy
quark free energy at finite temperature, once it has been renormalized properly at
zero temperature.

It should be obvious that the program outlined above equally well holds for the
heavy quark free energy calculated in QCD, i.e. in the presence of light dynamical
quarks. It will be of even greater importance in this case as $F_{\bar{q}q}(r,T)$ will be finite
for all temperatures and its large distance behaviour in the low temperature, chiral
symmetry broken phase, will reflect the temperature dependence of the string break-
ing energy. In fact, the conceptual approach we are going to present and analyze
here has already been anticipated in the presentation of heavy quark anti-quark free
energies given in Ref. [2].

This paper is organized as follows: In Section 2 we summarize basic definitions
for colour averaged and singlet free energies and outline our approach to renormalize the Polyakov loop expectation value. In Section 3 we present a detailed calculation of the heavy quark anti-quark free energy at short distances. These calculations are performed within the SU(3) gauge theory on lattices with temporal extent up to $N_{\tau} = 16$ so that small distances in units of the temperature can be resolved, $rT \sim 1/N_{\tau}$. We also point out the usefulness of the colour singlet free energy for the matching to the heavy quark potential at short distances and the determination of the asymptotic behaviour of free energies at large distances. In Section 4 we discuss the properties of the renormalized Polyakov loop. Section 5 contains our conclusions.

2 Heavy quark anti-quark free energy

The free energy of a pair of static quark anti-quark sources in a thermal medium

$$\frac{F_{\bar{q}q}(r, T)}{T} = -\ln \left( \langle \overline{\text{Tr}} L_{\vec{x}} \overline{\text{Tr}} L_{\vec{y}} \rangle \right) + c(T) \quad , \quad rT = |\vec{x} - \vec{y}|/N_{\tau} ,$$

is represented in terms of the Polyakov loop, which on an Euclidean lattice of size $N_{a}^3 \times N_{\sigma}$ is defined as a product of gauge field variables $U_{x_0, \vec{x}}$,

$$L_{\vec{x}} = \prod_{x_0=1}^{N_{\tau}} U_{x_0, \vec{x}} . \quad (2)$$

In Eq. 1 and in what follows we use the notation $\overline{\text{Tr}} = \frac{1}{3} \text{Tr}$. Eq. 1 defines the colour averaged free energy up to an additive normalization constant, $c(T)$, which is related to the self-energy of the quark and anti-quark sources. The Polyakov loop expectation value, $\langle L \rangle \equiv \langle N_{a}^{-3} \sum_{\vec{x}} \overline{\text{Tr}} L_{\vec{x}} \rangle$, is closely related to the long (infinite) distance behaviour of the free energy,

$$F_{\infty}(T) = \lim_{r \to \infty} F_{\bar{q}q}(r, T) = -T \ln |\langle L \rangle|^2 + c(T) T . \quad (3)$$

It is obvious that we need to fix the normalization constant $c(T)$ in order to give a physical meaning to this free energy.

In the pure SU(3) gauge theory the expectation value of the Polyakov loop, provides an order parameter for the deconfinement phase transition; in the confined
phase exp(\(-F_\infty(T) / 2T\)) vanishes, while it stays non-zero in the deconfined phase. We would like to interpret \(F_\infty\) as the change in free energy due to the presence of two well separated quarks, each of which is screened by a cloud of gluons. In order to do so we have to clarify the normalization of \(F_{\bar{q}q}(r, T)\) which unambiguously fixes the \(T\)-dependent additive constant in Eq. 1. Before doing so it, however, is worthwhile to discuss in a bit more detail the properties of \(F_{\bar{q}q}(r, T)\).

In addition to the colour averaged potential defined through Eq. 1 we introduce the colour singlet potential [4],

\[
\frac{F_1(r, T)}{T} = -\ln \left( \langle \bar{T}rL_\bar{g}L_\bar{g}^\dagger \rangle \right) + c'(T) .
\]  

As it stands this correlation function is gauge dependent; for its evaluation we have to fix a gauge. However, it recently has been shown that a gauge independent definition of the singlet free energy can be given [5] and that this definition coincides with the definition given by Eq. 1 in gauges which have a positive transfer matrix representation on the lattice. Calculating \(F_1\) from the Polyakov loop correlation function given in Eq. 4 in, for instance, the Coulomb gauge thus yields the singlet potential.

The colour averaged as well as the singlet free energy will approach finite, non-zero values for all temperatures \(T > T_c\) (and diverge below \(T_c\)). In fact, if the separation between the quark anti-quark sources gets large the relative orientation of the charges in colour space will not influence the screening of the individual charges. We thus choose the relative normalization of \(F_1\) and \(F_{\bar{q}q}\) such that they are identical in the limit of large spatial separations. The colour averaged free energy defined in Eq. 1 may then be represented by a thermal average over free energies of a \(\bar{q}q\)-pair in singlet (\(F_1\)) and octet (\(F_8\)) representations, respectively [3, 4],

\[
\exp(-F_{\bar{q}q}(r, T)/T) = \frac{1}{9} \exp(-F_1(r, T)/T) + \frac{8}{9} \exp(-F_8(r, T)/T) .
\]  

At distances much shorter than the inverse temperature \((rT << 1)\) the dominant scale is set by \(r\), the running coupling will be controlled by this scale, \(g(r, T) \simeq g(r, T = 0) \equiv g(r)\), and will become small for \(r << 1/\Lambda_{QCD}\). In this limit the singlet and octet free energies are dominated by one-gluon exchange and are calculable within ordinary zero temperature perturbation theory,

\[
F_1(r, T = 0) = -8F_8(r, T = 0) + O(g^4) = -\frac{g^2(r)}{3\pi r} \left( 1 + O(g^2) \right) ,
\]  

where \(F_1(r, T = 0) \equiv V_{\bar{q}q}(r)\) is the static, zero temperature heavy quark potential. As the singlet potential is attractive and the octet potential is repulsive at short
distances it follows from Eq. 5 that in this limit the colour averaged free energy will be dominated by the singlet contribution. We then may deduce from Eq. 5 also the asymptotic short distance behaviour of $F_{\bar{q}q}$ and $F_1$,

$$\lim_{r \to 0} (F_{\bar{q}q}(r, T) - F_1(r, T)) = T \ln 9 \quad \text{for all } T.$$

(7)

From Eqs. 6 and 7 it follows that $F_{\bar{q}q}$ as well as $F_1$ will show Coulomb-like behaviour at short distances and up to a normalization constant will approach the zero temperature heavy quark potential. We note, that this power-like short distance behaviour of the free energies is quite different from the behaviour of the colour averaged free energy at large distances ($rT >> 1$) where the dominant scale for the running coupling is set by the temperature ($g(r, T) \equiv g(T)$) and high temperature perturbation theory is used to show that the leading order contribution to $F_{\bar{q}q}$ is given by two-gluon exchange [3, 6]. In general the statement is that

$$\frac{\Delta F_{\bar{q}q}(r, T)}{T} \sim \left(\frac{\Delta F_1(r, T)}{T}\right)^2 \quad \text{for} \quad rT >> 1,$$

(8)

where $\Delta F_i \equiv F_i(r, T) - F_\infty$ with $i = 1, \bar{q}q$. In a somewhat loose notation it thus often is argued that $F_{\bar{q}q}/T \sim (F_1/T)^2$ [3, 6] at large distances. In the spirit of our previous discussion this statement, however, has to be formulated a bit more carefully as $F_\infty$ in general will not be zero. When fixing the overall normalization of the free energies at short distances one no longer has the freedom to assume that they approach zero at large distances. In this limit the colour averaged as well as the singlet free energy will approach a finite value, $F_\infty(T)$. Using this asymptotic value for the heavy quark anti-quark free energy we now can define the renormalized Polyakov loop,

$$L^{\text{ren}}(T) = \exp(-F_\infty(T)/2T).$$

(9)

It is obvious that $L^{\text{ren}}(T)$ is zero below $T_c$ on a lattice with infinite spatial extent, as $F_\infty$ will be infinite in the confined phase. Moreover, as the renormalization performed at short distances only involves zero temperature physics it does not influence the non-analytic structure of $F_\infty$ which will show up in the vicinity of $T_c$, the renormalized Polyakov loop. $L^{\text{ren}}$ thus will also reproduce all the universal properties extracted in the past from unrenormalized Polyakov loops [7].

3 Numerical results for heavy quark free energies

We have calculated the colour singlet and colour averaged free energy of a heavy quark anti-quark pair in a $SU(3)$ gauge theory on lattices of size $N_\sigma^3 \times N_\tau$ with
Figure 1: The difference of the colour averaged and singlet heavy quark free energies calculated at two temperatures above $T_c$ calculated on lattices of size $32^3 \times N_\tau$. The free energies have been assumed to coincide in the limit of infinite separation between the quark and anti-quark sources.

$N_\sigma = 32$ and $N_\tau = 4, 8$ and 16. For our numerical calculations we used a tree-level improved gauge action which includes the standard Wilson plaquette term and planar six link loops. This action has previously been used extensively for the calculation of thermodynamic quantities \cite{footnote}. These calculations also provided detailed information on the non-perturbative $\beta$-function determined through the string tension $\sigma a^2$. We use this information as well as the relation $T_c/\sqrt{\sigma} = 0.635$ \cite{footnote} to fix the temperature scale. In order to get access also to the singlet potential we have fixed the Coulomb gauge.

Most of our calculations have been performed in the deconfined phase of the SU(3) gauge theory. The colour averaged and singlet free energies, calculated according to the definitions given in Eqs. 1 and 4, approach a constant value at infinite distance. As argued above the free energy should not depend on the relative orientation of the quarks in colour space and we thus have fixed the relative normalization of the free energies such that they coincide in this limit. We then can analyze the spatial dependence of the difference $(F_{\bar{q}q}(r,T) - F_1(r,T))/T$. For large distances this difference now vanishes by construction and we should find in the short distance limit, $rT \to 0$, the relation given in Eq. 7. As shown in Figure 1 the difference indeed approaches the value $\ln 9$ as expected. This also shows that the colour averaged potential at short distances indeed is dominated by the singlet contribution. For $T \gtrsim T_c$ and distances $rT \lesssim 0.1$ we find that the difference deviates by less than
Figure 2: The shifted colour averaged \( (F_{av} \equiv F_{qq} - T \ln 9) \) and singlet \( (F_1) \) heavy quark anti-quark free energies calculated at \( T = 1.5 T_c \) on lattices with temporal extent \( N_\tau = 8 \) and 16. The free energies and the spatial separation \( r \) have been expressed in units of the square root of the string tension. The free energies have been matched to the zero temperature heavy quark potential (solid line) at the shortest distance available. The arrow points at the value \( (F_\infty - T \ln 9)/\sqrt{\sigma} \), with \( F_\infty \) determined from the colour singlet potential.

10\% from the asymptotic value, \( \ln 9 \). We also note that the shortest distance that can be resolved on a lattice with finite temporal extent \( N_\tau \) is \( rT = 1/N_\tau \). In order to analyze the short distance behaviour of the potential for \( rT \equiv 0.1 \) it thus was mandatory to use the large temporal lattices which we have used here for the first time to study the finite temperature heavy quark free energies.

As discussed in the previous section we expect that at short distances the colour averaged heavy quark anti-quark free energy is dominated by the singlet contribution and, moreover, approaches the zero temperature heavy quark potential, \( V_{qq}(r) \). In order to compare our finite temperature calculations of the free energies with \( V_{qq}(r) \) we first have to specify the latter. Using lattices with small lattice spacing \( V_{qq}(r) \) has recently been calculated for the SU(3) gauge theory for distances larger than 0.05fm and the results have been extrapolated to the continuum limit \[ \bar{r} \]. For distances larger than a scale \( r_0 \), which is defined through the slope of the heavy quark potential \[ \bar{r} \],

\[
r_0^2 \left( \frac{dV_{qq}(r)}{dr} \right)_{r=r_0} = 1.65 \quad ,
\]

(10)
it is known that $V_{qq}$ is well described by the simple linear confining potential corrected by a Coulomb-like term arising from string fluctuations, i.e. $V_{qq}(r) = \sigma r - \pi/12r$ for $r > r_0$ with

$$\sigma r_0^2 = 1.65 - \pi/12 .$$

For distances smaller than $r_0$ we use a polynomial interpolation of the lattice data of Ref. [9] normalized such that the resulting potential smoothly joins the confinement potential for $r > r_0$, i.e. we fix the free constant in the lattice results such that $r_0 V_{qq}(r = r_0) = 1.65 - \pi/6$. In some cases we also need the zero temperature potential at distances smaller than $0.1r_0 \approx 0.05\text{fm}$. Here we use the perturbative 3-loop calculation of the potential in so-called $qq$ scheme [11] which agrees well with lattice calculations up to distances $0.25r_0$. For the comparison of free energies with the zero temperature potential we give them in units of $\sqrt{\sigma}$ which is straightforward since $r_0$ and $\sqrt{\sigma}$ are related through Eq. 11. This zero temperature potential is shown in Figure 2 as a solid line.

For the matching of free energies to $V_{qq}$ we also use calculations of $F_1(r, T)$ and $F_{qq}(r, T)$ at off-axis separations on the lattice. At short distances these calculations show violations of rotational symmetry, also known from other studies of short distance properties on the lattice. As the matching is done at short distances we thus should try to adjust for these lattice artefacts. Following Ref. [9] we replace $F_{1,qq}(r)$ by $F_{1,qq}(r_I)$ where $r_I^{-1}(r) = 4\pi \int_{-\pi}^{\pi} \frac{d\vec{k}}{(2\pi)^3} \exp(i\vec{k} \cdot \vec{r}) D_{00}^{(0)}(k)$, with $D_{\mu\nu}^{(0)}$ denoting the tree level lattice gluon propagator evaluated in [12]. We thus replace the lattice separation $r$ by the separation $r_I$ which corrects for the violation of the rotational symmetry in the Coulomb potential calculated on the lattice.

A comparison of the colour averaged free energy ($F_{qq}/\sqrt{\sigma}$), the singlet free energy ($F_1/\sqrt{\sigma}$) and the heavy quark potential ($V_{qq}/\sqrt{\sigma}$) is given in Figure 3. For clarity we show in this figure the shifted colour averaged free energy, $F_{av}(r, T) = F_{qq}(r, T) - T \ln 9$, which should coincide with the heavy quark potential at short distances. From Figure 3 it is evident that indeed the shifted colour averaged free energy and the singlet free energy approach the zero temperature heavy quark potential at short distances. The good agreement of $F_{av}/\sqrt{\sigma}$ and $F_1/\sqrt{\sigma}$ at short distances, furthermore, supports our assumption that the singlet and colour averaged ($F_{qq}/\sqrt{\sigma}$ rather than $F_{av}/\sqrt{\sigma}$) free energies approach the same constant at large distances. We also note that the color singlet potential calculated for $N_T = 8$ and $N_T = 16$ agree well with each other indicating that residual cutoff effects are small. Similar results hold for all temperatures examined by us.
Figure 3: The colour singlet heavy quark anti-quark free energy ($F_1$) in units of $\sqrt{\sigma}$ versus $r\sqrt{\sigma}$ calculated on lattices of size $32^3 \times 8$ for several values of the temperature above $T_c$. The corresponding gauge couplings are given in Table 1. The free energies have been matched to the zero temperature heavy quark potential (solid line) at the shortest distance available.

4 The renormalized Polyakov loop

The analysis of the heavy quark anti-quark free energies presented in the previous section shows that both can be renormalized by matching their short distance behaviour to that of the zero temperature heavy quark potential. In this way all divergent self-energy contributions are removed. Moreover, we have noted that the asymptotic large distance behaviour of $F_1(r, T)$ coincides with that of $F_{\bar{q}q}(r, T)$. The former thus equally well determines the asymptotic value of the free energy, $F_\infty(T)$. As it is obvious from Figure 2 that it is much easier to perform the matching to the zero temperature potential at short distances using the colour singlet free energy $F_1(r, T)$ we have used this free energy rather than $F_{\bar{q}q}(r, T)$ to determine the asymptotic value of the free energies, $F_\infty$. In particular, this allows to perform the matching to $V_{\bar{q}q}$ already with finite temperature free energies calculated on lattices with temporal extent $N_\tau = 4$. Results for the colour singlet free energy calculated at several values of the temperature above $T_c$ are shown in Figure 3. From these renormalized free energies we have determined $F_\infty(T)$ at different values of the temperature. Actually, in order to avoid any fits, we use the value of $F_1(r, T)$ at maximal on-axis separation of the quark anti-quark sources on a lattice with spatial extent $N_\sigma$, i.e. we define $F_\infty(T) \equiv F_1(N_\sigma/2, T)$. Systematic errors on the free energies have been estimated by matching them to $V_{\bar{q}q}$ at distance $rT = 1/N_\tau$ as well as
Table 1: Change in free energy due to the presence of a heavy quark anti-quark pair in a thermal gluonic heat bath. The table gives results for $F_\infty(T)/T$ obtained in calculations on a $32^3 \times 8$ lattice and the screening radius defined through Eq. 12.

Errors on $F_\infty(T)/T$ include an estimate of the systematic errors (see text). The temperatures corresponding to the gauge couplings $\beta$ are given in the first two columns.

| $T/T_c$ | $\beta$ | $F_\infty(T)/T$ | $r_{\text{screen}}T$ |
|---------|---------|-----------------|---------------------|
| 1.03    | 4.5592  | 1.42(4)         | 0.59                |
| 1.20    | 4.6605  | 0.93(4)         | 0.58                |
| 1.50    | 4.8393  | 0.53(4)         | 0.62                |
| 3.00    | 5.4261  | -0.03(3)        | 0.80                |
| 6.00    | 6.0434  | -0.18(1)        | 0.98                |

As can be seen in Figure 3 the singlet free energy rapidly changes from the Coulomb like short distance behaviour to a constant value. This, of course, reflects the exponential screening of the two static charges. As this change is so rapid, we may use $F_\infty(T)/T$ to define a screening radius $r_{\text{screen}}$, which characterizes the onset of screening, through the relation

$$\frac{V_{qq}(r_{\text{screen}})}{T} = \frac{F_\infty(T)}{T}, \quad (12)$$

The values for $r_{\text{screen}}/T$ determined in this way are also given in Table 1. As can be seen $r_{\text{screen}} \simeq 0.4$ fm in the vicinity of $T_c \simeq 270$MeV and drops to 0.2 fm for $T = 3T_c$. We do expect that asymptotically the screening radius will drop like the inverse Debye mass, \textit{i.e.} $r_{\text{screen}} \sim 1/g(T)T$.

Using the values for $F_\infty(T)$ we can now construct the renormalized Polyakov loop defined in Eq. 3. Results are shown in Figure 4. The renormalized Polyakov loop is an order parameter for the deconfinement phase transition, which is well defined also in the continuum limit. Its magnitude is related to the free energy of a heavy quark placed in a thermal gluonic heat bath. It is obvious from Table 1 that $F_\infty/T$ becomes negative at large temperatures and consequently $L^{\text{ren}}$ becomes larger than unity. We note, however, that $L^{\text{ren}}$ is fixed only up to a multiplicative renormalization which results from fixing an arbitrary additive constant in the zero temperature heavy quark potential.
Figure 4: The renormalized Polyakov loop expectation value defined in Eq. 9 determined from the asymptotic behaviour of colour singlet free energies on lattices of size $32^3 \times N_T$.

5 Outlook

In this paper we have defined the renormalized Polyakov loop by matching the free energy of a static quark anti-quark pair at short distances to the zero temperature heavy quark potential. We have shown that the renormalized Polyakov loop can be determined from the large distance behaviour of the colour averaged as well as the colour singlet free energy of the $\bar{q}q$-pair. The approach has been used here to study the heavy quark free energy of the SU(3) gauge theory. It, however, generalizes without any difficulties to the case of QCD.

In the temperature regime analyzed by us, $T/T_c \leq 6$, the renormalized Polyakov loop is a monotonically rising function. It becomes larger than unity for $T/T_c \simeq 2.5$. In the future it will be interesting to analyze the asymptotic behaviour of the Polyakov loop at even larger temperatures and determine its infinite temperature limit. In this limit $L_{\text{ren}}$ is expected to approach a constant. Figure 4 suggests that this constant is close to unity. A more detailed analysis of the large temperature behaviour of $L_{\text{ren}}$ would also allow to make contact with perturbative calculations, which suggest that the asymptotic value may be approached from above [13].

Finally we note that our normalization of the heavy quark anti-quark free energies at short distances also opens the possibility for a new look at the heavy quark potential at finite temperature. Using the thermodynamic relations between entropy, energy and free energy, $S = -\partial F/\partial T$, $U = -T^2 \partial (F/T)/\partial T$, it is evident
from Figure 3 that $S_{q\bar{q}}(r, T)$ vanishes at short distances while it clearly is positive at large distances. It therefore will add a positive contribution to the total energy,

$$U_{q\bar{q}}(r, T) = F_{q\bar{q}}(r, T) + TS_{q\bar{q}}(r, T),$$

which becomes larger with increasing separation $r$. This shows that the change in energy due to the presence of a heavy quark anti-quark pair in a thermal bath is quite complex. In particular, its $r$-dependence is not only given by $F_{q\bar{q}}(r, T)$. This indicates that it may be misleading to use $F_1(r, T)$ in potential models for the study of heavy quark bound states.

Acknowledgements

We thank Edwin Laermann for very helpful discussions. This work has been supported by the DFG under grant FOR 339/1-2. It was in part based on the MILC collaboration’s public lattice gauge code. See [http://physics.utah.edu/~detar/milc.html](http://physics.utah.edu/~detar/milc.html)

References

[1] R. D. Pisarski, Phys. Rev. D 62 (2000) 111501; A. Dumitru and R. D. Pisarski, Phys. Lett. B 525 (2002) 95.

[2] F. Karsch, E. Laermann and A. Peikert, Nucl. Phys. B 605 (2001) 579.

[3] L.D. McLerran and B. Svetitsky, Phys. Lett. B98 (1981) 195 and Phys. Rev. D24 (1981) 450.

[4] S. Nadkarni, Phys. Rev. D 34 (1986) 3904

[5] O. Philipsen, Phys. Lett. B 535 (2002) 138.

[6] S. Nadkarni, Phys. Rev. D 33 (1986) 3738

[7] see e.g. J. Engels and T. Scheideler, Nucl. Phys. B 539 (1999) 557 and references therein.

[8] B. Beinlich, F. Karsch, E. Laermann and A. Peikert, Eur. Phys. J. C 6 (1999) 133.

[9] S. Necco and R. Sommer, Nucl. Phys. B 622 (2002) 328

[10] R. Sommer, Nucl. Phys. B 411 (1994) 839

[11] S. Necco and R. Sommer, Phys.Lett. B 523 (2001) 135

[12] P. Weisz, Nucl. Phys. B 212 (1983) 1; P. Weisz and R. Wohlert, Nucl. Phys. B 236 (1984) 397 and Erratum ibid. 247 (1984) 544

[13] for a recent discussion and further references see: R. D. Pisarski, ”Notes on the deconfining phase transition”, [hep-ph/0203271](http://arxiv.org/abs/hep-ph/0203271).