Quantized magneto-thermoelectric transport in low-dimensional junctions

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Abstract – Quantization of the magneto-thermoelectric transport is studied when an external d.c. magnetic field is applied to the C/N-knot formed as crossing between a narrow stripe of conducting atomic monolayer C on the one hand, and a metal stripe N on the other hand. The temperature gradient in C is created by injecting the non-equilibrium electrons, holes and phonons from the heater H thereby directing them toward the C/N-knot. A non-linear coupling between electron states of the C/N-knot counter-electrodes causes splitting of the heat flow into several fractions owing to the Lorentz force acting in the C/N-knot vicinity, thereby inducing the magneto-thermoelectric current in N, whereas the phonons pass and propagate along C further ahead. The heat flow along C generates a transversal electric current in N showing a series of maxima when dimensions of the Landau orbits and the C/N-knot match each other. It allows observing the interplay between the quantum Hall effect and the spatial quantization.

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Study of the magneto-thermoelectric transport in the low-dimensional conductors improves our knowledge about their nature and motivates further work [1,2]. Typical examples are the atomic monolayers (AM) such as graphene and transition metal dichalcogenides (TMDCs) [3]. The electron transport in AMs is governed, e.g., by applying the gate voltages. It allows changing the charge carrier concentration and the magnitude of electrical conductance in wide limits [4]. Alternatively, electric transport is controlled with the d.c. magnetic field. At low temperatures, it causes the quantum Hall effect which is manifested as quantization of the electrical conductance of the narrow stripe giving \( \sigma = I_{\text{ch}}/I_{\text{Hall}} = \nu e^2/h \), where \( \nu \) is the integer number. Furthermore, quantization of transverse electron motion occurs owing to spatial confinement of the electron states inside the narrow stripes. The above phenomena are utilized in the low-dimensional elements of electronic circuits, and also during the thermoelectric cooling and energy generation [5,6].

In this letter we focus on the other aspects of the problem concerning filtering, spatial separation, redirecting and conversion of the different components of the thermal and electric transport. Such phenomena take place when the d.c. magnetic field \( B = \{0,0,B_z\} \) acts on the crossing of a narrow AM stripe C with a metal stripe N. There are many monoatomic layer conductors called in this letter as C. One example is graphene [7], an extremely thin electric and thermal conductor [8], showing high carrier mobility [9], and surprising properties of the molecular barrier [10,11]. Other 2D materials are TMDCs [4], transition metal oxides like titania- and perovskite-based oxides [12,13], and graphene analogues such as boron nitride (BN) [14,15]. Atomic monolayers of TMDCs which display a wide range of interesting electronic, optical, mechanical, chemical and thermal properties, have a strong potential for a variety of applications. Currently, there is a strong interest in the monatomic materials with relatively small energy gap \( \Delta_l \simeq 1–100 \text{meV} \). The interest is motivated by a variety of potential applications in the nanoelectronics, e.g., THz sensors and spectral analyzers, lasers, high-speed logic, and superfast computers. Typically, the pristine monoatomic layer materials are either semiconducting (e.g., TMDCs such as MoX_2 and WX_2) or metallic (e.g., NbX_2 and TaX_2). The energy minigap \( \Delta_l \) in the excitation spectrum of a monatomic metal is induced using one of many approaches [16]. For instance, a minigap can arise due to the interaction-driven electronic order when a high magnetic field is applied; it also can result from the influence of substrates such as SiC or hexagonal boron nitride (BN), although the lattice mismatch might...
cause rather complex effects. The superlattice patterns on graphene is created using the electron beam-induced deposition of adatoms allowing to achieve a pattern periodicity as short as ~5 nm. The semiconducting and metallic superlattices routinely are used for manipulating the electronic structure of materials [3].

We examine the microscopic mechanism of controllable redirection and conversion of one type of transport to another taking place in such crossing (C/N-knot), where the electric and heat currents are separated from each other as shown in Fig. 1. Thus, the C/N-knot represents a crossing of the narrow atomic monolayer material (AM) stripe C with the metal stripe N where the thermal flow components, i.e., electrons, holes, and phonons, are rerouted in different directions as shown in Fig. 2(a). The quantum dot is created on the C/N-knot as follows. Owing to the difference between work functions of the two materials at the C/N-interfaces, the Schottky barriers are formed. Furthermore, there are also barriers (marked in Figs. 1(a), (b), (d), (e) as brown) created during the fabrication process due to presence of the atomic impurities localized at the interface, separating the open C-sections on the one hand, and the C′-section located underneath the metal stripe N on the other hand. Coupling of electron states of the C and N electrodes leads to several consequences: i) it modifies the Landau states, ii) induces transmission of charge carriers between C and N and also iii) shifts the energy profile by a finite value Δ0 serving as the bottom of the quantum dot. The heater H (shown in Fig. 1(a) as yellow) injects the non-equilibrium phonons (ph), electrons (e), and holes (h) into the C-stripe, leading to the temperature difference δT∗ = T∗ − TC∗ arising between the section C′′ beneath the heater on the left and C′ beneath the metal N on the right. The effective local temperature T∗ in C′′ much exceeds the steady state temperature TC in the C-ends. According to the Fourier law, the finite value of δT∗ ≠ 0 initializes the flow of heat Q = ΛΔT∗ along C between the sections C′′ and C′ as shown in Fig. 1. Here we introduced the thermal conductance Λ = Λph + Λe + Λh consisting of the phonon (Λph), electron (Λe), and hole (Λh) components [17]. The most interesting case is when the numbers of the excited electrons and holes are equal to each other and they, jointly with phonons, carry only the heat along C but not the electric current, i.e., Iq ≡ 0. This happens for the case of electron-hole symmetry of the excitation spectrum. A more detailed review of the corresponding C-materials is given in ref. [3]. As the counter-electrode N, one can use a normal metal stripe, e.g., Pd, Ni or Nb. In the

Fig. 1: (Color online) (a) The injection of heat into the C′′-section under the heater. (b) Electron tunneling from the C′-section into the stripe of metal N. (c) Main figure: splitting the flow of heat in the C/N-knot when a finite d.c. magnetic field B is applied perpendicular the C/N-knot plane. Pictures (d) and (e) represent the two different energy diagrams of the C/N-knot, which correspond to distinct trajectories of the electrons C/C′/C (green arrow) and C/C′/N (blue and red arrows).

Fig. 2: (Color online) (a) The C/N-knot model for computing of the transmission probabilities Tε,C/C′. (b) The transmission probabilities for an electron emerging from C into the C′-section along the x-direction, Tε (dotted black curve), turning to the right, Tε,right (solid blue curve), and turning to the left, Tε,left (dashed red curve) computed for geometry (a) by solving the non-linear boundary conditions. (c) The non-equilibrium electron distribution function fε(z) in the C-section. The brown curve is fε (z) which is altered only due to the phonon drag, whereas the green curve is fε (z) which is altered only due to the thermal injection of electron and holes (see parameters in the text). The inset shows the phonon-drag-induced non-equilibrium deviation of the distribution function δfε,h consisting of the phonon (ε = h) and holes (ε = h) in the C-section for the same parameters. (d) Driving factor of the transversal magnetoelectric current computed assuming that the quantum dot bottom Δ0 is set at Δ0 = 0, whereas the minigap is Δ = 1 (the units are explained in the text). The red dashed curve shows the influence of only the non-equilibrium thermal injection, whereas the solid blue curve shows the combined influence of the non-equilibrium thermal injection and phonon drag.
tron excitation spectrum of a narrow conducting stripe, transmission probabilities states of the C- and N-stripes by introducing the partial states/(eV \cdot \text{“bare” electron density of states (DOS) as a constants}

Basic parameters of the C/N-knot are deduced by measuring the thermal and electric currents. Besides, parameters are determined from the first-principle numeric calculations. In particular, for 2H-NbSe$_2$ [19], using the effective electron mass $m^* = 0.6 m$ and lattice constants $a = 3.45$ Å, $c = 12.54$ Å, one evaluates the “bare” electron density of states (DOS) as $N(0) \approx 2.8$ states/(eV · unit cell) and the Fermi velocity as $v \approx 1.6 \times 10^6$ m/s. The electron minigap $\Delta_m$ arising in 2H-NbSe$_2$ due to the presence of a superlattice with a “long” period $L_{SL} \gg \max\{a,c\}$ is evaluated as follows. Let us consider a wide range of densities, expressed as a multiple of the base value $n_0 = 10^{13}$ cm$^{-2}$, under the conditions $k_F < k_c$ and $\varepsilon_F < \varepsilon_c$. For the latter number, we evaluate the Fermi wave vector as $k_F = \sqrt{\pi n_0} = 5.6 \times 10^6$ m$^{-1}$, and $p_F = h k_F = 3.7 \times 10^{-8}$ eVs/m. For the superlattice period $L = 50$ nm and the Fermi velocity $v = 1.6 \times 10^6$ m/s, $K = 2\pi/3a$, we find the minigaps $\Delta_m \approx |f_m| - 5.9 \times 10^{-2}$ eV spaced with $\Delta E_x = 6.6 \times 10^{-2}$ eV. Thus, both the minigap magnitude $\Delta_m$ and spacing $\Delta E_x$ are adjusted by changing the superlattice period and of the amplitude of the “slow” potential $V(x)$. Similar sub-bands arising in the electron excitation spectrum of a narrow conducting stripe, or when applying a d.c. magnetic field, can be studied with the method suggested in this letter.

We describe the non-linear coupling between electron states of the C- and N-stripes by introducing the partial transmission probabilities $T_{x,y}$. Let us derive the boundary conditions (BC) for the electron wave functions of the C/N-knot influenced by the external d.c. magnetic field as shown in fig. 2(a). First, we assume that after emerging from the open left C-section, an electron then enters the C/N-knot (see fig. 2(a)). For an electron passing through the C/N-knot, there are three possible further trajectories: a) an electron is deflected from the straight trajectory toward the right wing of N (blue arrow in fig. 2(a)); b) it traverses the C/N-knot toward the open C-section on the opposite side further ahead (green arrow in fig. 2(a)); c) an electron, instead to be deflected toward the right wing of N, “wrongly” deviates from C toward the left wing of N. In the same figure, the red arrow depicts the hole trajectory. We emphasize that only the $e$- and $h$-deviated trajectories (see the blue and red lines in fig. 1(c)) are contributing to the transverse electric current generated in N. The aforementioned processes effectively are represented as tunneling through the asymmetric C/C’/N-junction, the energy diagram of which is shown in fig. 1(e). The green arrow directed ahead as shown in fig. 1(c) depicts another type of trajectory. The latter process is represented as a tunneling via the symmetric C/C’/C-junction (see the energy diagram in fig. 1(d)). Such process gives no contribution to the electric current in N.

The transverse magneto-thermoelectric current $I_L$ is computed assuming that the non-equilibrium electrons and holes in C are excited by two mechanisms: i) due to the phonon drag of the electrons and holes along C, and ii) via the thermal injection of the non-equilibrium quasiparticles from the heat source H into C (see fig. 2(a)). More detailed description of the mechanisms i) and ii) is given in ref. [3]. Furthermore, we suppose that the C/N-interface transparency is high, thereby ensuring that the transmission probability $T_{ph}$ through the C/C’/N-knot is considerable for electrons (and holes) emerging from section C, then traversing the section C’, and being transmitted, e.g., in the left (or right) wing of N, provided $T_{ph} \ll 1$. The transmission probabilities $T_{ph}$ are determined by the work functions of C and N. Simultaneously, the transparency $T_{ph}^{CN}$ for phonons of the same C’/N-interface is very low, i.e., $T_{ph}^{CN} \ll 1$, though propagation of phonons inside C is ballistic. That happens because redirecting of the phonons from C to N is accompanied by a significant change of the phonon momentum $\Delta q = \mp q_N - q_C$, where $q_N$ and $q_C$ are the phonon momentum components along N and C, respectively [17]. Another reason why $T_{ph}^{CN} \ll 1$ is that the phonon spectra of C and N are very distinct thereby considerably reducing the phonon transmission through the C/N-interface. Therefore, for the sake of simplicity, we disregard the phonon transmission from C to N by setting $T_{ph}^{CN} \approx 0$. On the one hand, we assume that phonons pass the knot and propagate along C further ahead. On the other hand, we will see that the Lorentz force reroutes electrons and holes propagating between C and the perpendicular stripe of the metal N. As follows from further calculations, the redirection occurs because the Lorentz force pulls electrons to the right (blue arrow in fig. 1(c)) while simultaneously pushing the holes toward the left (red arrow in fig. 2(c)). In this way, the heat flow (yellow arrow in figs. 1(a), (c)) along C induces the electric current $I_L$ (magenta arrow) in N (brown).

Here we consider the most spectacular case of the electron-hole symmetry in C assuming that the energy gap $\Delta$ is relatively small. The magnitude of the transversal electric current generated in N is found according the Landauer-Büttiker formula

$$I_L = \frac{4e}{h} \int_0^{\infty} d\varepsilon M_{\min}(\varepsilon) \left( T_{ph}^{\varepsilon} - T_{ph}^{-\varepsilon} \right) \times (f_C(\varepsilon - V_H) - f_N(\varepsilon)), \quad (1)$$

where $M_{\min}(\varepsilon)$ is the minimum number of quantum channels per energy interval, $T_{ph}^{\varepsilon}$ is the energy-dependent transmission probability through the C/C’/N-knot for an
electron emerging from section C, traversing the section C', and getting into the left (or right) wing of N, V_{th} is the thermal Hall voltage induced by the d.c. field across C as explained in ref. [3]. When the excitation spectrum in C is non-symmetric, and one type of charge carriers (either electrons or holes) prevails, in addition to the heat flow, there is also a finite thermoelectric current, also flowing along C.

The transmission probabilities $T_{y}(\varepsilon)$ through the C/N-knot, entering eq. (1) are computed using the model depicted in fig. 1(c) and fig. 2(a). As a trial electron wave function $\Psi_{C}(x, y)$, we use a combination of two-dimensional waves formed inside the rectangular box situated on the C-section underneath the metal N as shown in fig. 2(a). The box represents the rectangular 2D quantum dot whose bottom is at the energy value $\varepsilon = \Delta_{0}$. For the sake of simplicity, we disregard the particle’s chirality [7]. The value of $\Delta_{0}$ is set owing to coupling between the atomic orbitals of C and N. We assume that an electron with the momentum $p = (k, q)$ enters the C/N-knot on the left side from C at $x = 0$ and $0 < y < W$. The electron leaves the C/N-knot toward N either on the right ($y = 0$, $0 < x < L$), on the left ($y = W$, $0 < x < L$), or propagates toward C further ahead ($x = L$, $0 < y < W$) where $L$ is the metal stripe width. While traversing the C/N-knot, the electrons acquire a transversal momentum component $q = \pm (v_{F}/c) B_{x} y + t^{2} g/v$ where the former term is due to the magnetic field whereas the last term is caused by the C/N-coupling whose energy is $t^{2} g$. Here $t$ is the matrix element of tunneling between C and N, $g$ is the electron density of states in the metal stripe N, $v$ is the Fermi velocity in C. The confinement effect inside the C-stripe is modelled by introducing the spreading of parabolic potential in the $y$-direction $U(y) = m \omega_{c}^{2} y^{2}/2$, where $m$ is the electron mass and $\omega_{c}$ characterizes the steepness of the confinement potential. Then the dispersion law of an electron inside C is

$$E_{n,k} = E_{s} + \hbar \omega_{c} \left( n + \frac{1}{2} \right) + \frac{\hbar^{2} k^{2}}{2m} \frac{\omega_{c}^{2}}{\omega_{c}^{2}},$$

(2)

where $n$ is a non-negative integer, $\omega_{c} = eB/m_{e}$ is the cyclotron frequency, and $\omega_{c0} = \sqrt{\omega_{c}^{2} + \omega_{0}^{2}}$. From eq. (2), it follows that both the confinement of electron states and the d.c. magnetic field cause minibands in the electron excitation spectrum of the C-stripe. For the rectangular geometry of the C/N-knot shown in fig. 2(a), the trial electron wave function takes the form

$$\Psi_{C}(x, y) = (\alpha_{x} e^{ik_{x}x} + \beta_{x} e^{-ik_{x}x})(\alpha_{y} u_{n}(\zeta_{x}) + \beta_{y} u_{n}(\zeta_{-})).$$

(3)

where we introduced the auxiliary functions $u_{n}(\zeta) = \exp(-\zeta^{2}/2) H_{n}(\zeta)$, where $H_{n}(\zeta)$ is the Hermite polynomial, $\zeta_{\pm} = (y, \pm k)$, and $q_{y} = \pm \lambda_{c} k$. Here $q_{y} = \sqrt{m \omega_{c}^{2}/\hbar}$, $\lambda_{c} = (\omega_{c}/\omega_{c0})^{2} x_{0}$ and $\lambda = \sqrt{\hbar/eB}$ is the magnetic length. Furthermore, at $B = 0$, $\pi/q_{y} = W^{*}$ is the effective width of C-stripe. For the magnetic flux density $B = 16 T$ one obtains $\lambda = \sqrt{\hbar/eB} \sim 6.4 \text{nm}$, $\hbar \omega_{c} = 2 \text{meV}$, $\hbar \omega_{0} = 5 \text{meV}$, $\hbar \omega_{c0} \sim 5.3 \text{meV}$, and the effective width of the stripe is $W^{*} = \pi/q_{y} \sim 75 \text{nm}$, $U_{0} = \Delta_{0} \sim 5.5 \text{meV}$, where $\Delta_{0} = E_{s} + \hbar \omega_{c0}/2$ is the bottom energy.

Outside the C/N-knot region, the trial wave functions are taken in the form of plane waves. For an electron which is propagating along the C-stripe, at $x < 0$ and $y = 0$ we use

$$\Psi_{C}(x, y = 0) = r_{x} e^{-i k_{x} x} + e^{i k_{x} x},$$

(4)

whereas for an electron inside the N-stripe $x = 0$ we use

$$\Psi_{N}(x = 0, y) = r_{y}^{R(L)} e^{i q_{y} (y + W/2)},$$

(5)

where the upper (lower) signs correspond to $y > W$ ($y < -W$). In the above formulas (4), (5) we disregarded the dependence on $y$, which enters into the coefficients $r_{x}$ and $r_{y}^{R(L)}$. The transmission probabilities are defined, e.g., as $T_{y}(\varepsilon) = |t_{y}(\varepsilon)|^{2}$. The energy dependence of $k$ is then obtained in the form $k(\varepsilon) = (\omega_{c0}/\omega_{y})^{2}(2m\Delta_{y}/\hbar)(\varepsilon - \Delta_{y})/\Delta_{y}$, where $\Delta_{y} = E_{s} + \hbar \omega_{c0}(n + 1/2)$. The boundary conditions (BC) yield the system of 8 non-linear transcendental equations for the 8 coefficients entering the trial wave function (3). Technically, non-linear coupling between the electron states in the C- and N-strips originates from the products like $\alpha_{x} \alpha_{y}$, $\alpha_{x} \beta_{y}$, $\beta_{x} \alpha_{y}$, which are present in the boundary conditions. The non-linear BC equations are solved numerically. The obtained solutions allow to compute the transmission probabilities $T_{y}(\varepsilon)$ which enter into eq. (1). Our simple model assumes that the propagation of particles within the C/N-knot is ballistic. The model also can be extended to include the roughness of substrate and interfaces as described, e.g., in ref. [20].

The electric current $I_{L}$ is induced in the N-stripe owing to the thermal injection [2] and the phonon drag [18]. Thereby, $I_{L}$ depends on the non-equilibrium distribution functions of the electrons $f_{c}$ and phonons $N_{C}$ inside the C/N-knot. The functions $f_{c}$ and $N_{C}$ are found as solutions of the kinetic equations

$$\dot{f}_{C} = \mathcal{L}_{th} + \mathcal{L}_{ep},$$

(6)

$$\dot{N}_{C} = \mathcal{P}_{th} + \mathcal{P}_{pe},$$

(7)

where $\mathcal{L}_{th}$ and $\mathcal{P}_{th}$ describe, respectively, the thermal injection of the non-equilibrium electrons, holes, and phonons from the heat source H into the C" section as shown in fig. 1(a), $\mathcal{L}_{ep}$ is the electron-phonon collision integral, whereas $\mathcal{P}_{pe}$ is the phonon-electron collision integral. In eqs. (6), (7) we have disregarded other processes having lesser importance.

The non-linear coupling between the electron states of counter-electrodes in the C/N-knot causes the leakage of the electrons and holes from the C-stripe to N as illustrated in fig. 2. The corresponding numeric results are presented in figs. 2(b)–(d). In fig. 2(b) we plot the transmission probabilities for an electron incoming from
C into the C'-section along the \( \hat{z} \)-direction, \( T_x \) (dotted black curve), turning to the right, \( T_y \) (solid blue curve), and turning to the left, \( T_y' \) (dashed red curve) obtained as solutions of the non-linear boundary conditions. The probabilities of electron transmission \( T_y \) and \( T_x \) contain series of peaks related to the geometrical resonances arising when the dimensions of the Landau orbit with index \( n = 3 \) and the C/N-knot dimensions coincide. Notice the buildup of the transmission probabilities at energies \( 0 < \varepsilon < \Delta_0 \), where \( \Delta_0 \) is the bottom energy of the C/N-knot. The buildup arises when the effective width \( W^* = \pi/q_0 \) of the C-stripe is finite. In other words, the maxima occur when the position of the Landau level \( n = 3 \) matches with the localized energy levels originating from the spatial quantization. Thus, stronger transversal deviation of electrons and holes traversing through the C/N-knot occurs at certain values of the electron energy \( \varepsilon \) and magnetic field \( B_z \). To illustrate the difference between the two mechanisms of creation of the non-equilibrium excitations (i.e., the phonon drag and thermal injection) in C, we set either \( \delta f_{\text{pd}} \equiv 0 \) while holding \( \delta f_{\text{th}} \neq 0 \) or, otherwise, \( \delta f_{\text{th}} \equiv 0 \) while \( \delta f_{\text{pd}} \neq 0 \). The calculation parameters depend on properties of the C- and N-materials. Furthermore, certain parameters, like the magnetic length \( \lambda \) and the cyclotron frequency \( \omega_c \), also depend on whether the electron excitations are the chiral fermions or not. Therefore, it is convenient to operate with material-independent dimensionless units. We use the following parameters: \( \tau_{\text{ep}} = 10^{-12} \text{s}, \sigma_0 = 0.2, N_0 = 1, \Delta = 1, T_C = 0.1 \) is the temperature in the C'-section, \( s = 0.02 \) is the sound velocity, \( v = 1 \). The dimensionless length units correspond to \( v/2\Delta_0 \), whereas the wave vectors are in units of \( 2\Delta_0/v \) where both \( \Delta_0 \) and \( v \) are material dependent. For the sake of convenience, in fig. 2(b) and below, the parameters with energy and temperature dimensions are expressed in units of the semiconducting gap \( \Delta \) in C (we set \( h = 1 \)). Parameters with the wave number dimensions (like \( k, q_0, q_c, etc. \)) are expressed in units of \( \Delta/v \), whereas the length parameters (like \( L, W, \lambda, \lambda_{\text{el}}, etc. \)) are expressed in units of \( v/\Delta \). Then \( \Delta = 2h\nu/W = 0.4 \text{eV}, \Delta/\nu = 7.5 \times 10^8 \text{m}^{-1} \) and \( v/\Delta = 1.3 \text{nm} \), where we also take \( v = 8.1 \times 10^5 \text{m/s} \) and \( s = 2.1 \times 10^4 \text{m/s} \). In fig. 2(c), the brown curve represents the distribution function of electrons in C, \( f_C(\varepsilon) \), altered owing to only the phonon drag, whereas the green curve is \( f_C(\varepsilon) \) altered only by thermal injection of the electron and holes (see parameters in text). The inset in fig. 2(c) shows the phonon-drag-induced non-equilibrium deviation of the distribution function \( \delta f_{\text{el},h} \) of electrons (blue curve) and holes (red curve) in the C-section for the same parameters as listed above. The driving factor \( \delta f_C = f_{C} - f_{S} = \delta f_{\text{pd}} + \delta f_{\text{th}} \) for the transversal magneto-thermoelectric current \( I_L \) at \( U_0 = 0 \) is plotted in fig. 2(d). The red dashed curve corresponds to the influence of the non-equilibrium thermal injection only (for the moment we set \( \delta f_{\text{pd}} = 0 \)) while the blue solid curve shows a combined influence of both the non-equilibrium thermal injection and the phonon drag.

In fig. 3 we show the obtained transmission probabilities \( T_x(\varepsilon) \), \( T_y(\varepsilon) \) and the transversal electric current \( I_L \) vs. magnetic field \( B_z \) and temperature deviation \( \delta T^* = T^* - T_C \). The effective electron temperature \( T^* \), as a characteristic of the non-equilibrium effect, is introduced as \( T^* = (\Omega_{\text{CN}}/k_B)^{\varepsilon_{\text{el}}}, \) where \( \Omega_{\text{CN}} = hW \) is the C/N-knot volume, \( h \) and \( W \) are, respectively, the thickness and width of C, and \( L \) is the width of N. The energy of non-equilibrium electrons per unit volume \( \varepsilon_{\text{el}} \) is given by eq. (12) of ref. [3].

Maxima of the electric current component \( I_L^{(3)} \) in fig. 3 arise from the matching of the Landau orbit \( (n = 3) \) with dimensions of the C/N-knot. In other words, they constitute an interplay between the Landau and localized levels arising in the C/N-knot. Figure 3 suggests that the basic C/N-knot parameters can be extracted straight from the magneto-thermoelectric characteristics. In fig. 3, we use \( L = 0.6, W = 0.67, \) and \( h = 10^{-3} \), all expressed in the dimensionless units. The bottom energy \( \Delta_0 \) of the C/N-knot and its confinement steepness wave number \( q_0 \) are, respectively, \( \Delta_0 = 0.5 \) and \( q_0 = 0.5 \). When the C-stripe is made of, e.g., NbSe\(_2\), the dimensionless units listed for fig. 3

Fig. 3: (Color online) (a) Longitudinal transmission probability through the C/N-knot vs. the electron energy \( \varepsilon \) (in units of \( 2\Delta_0 \)) and the cyclotron wave number \( q_c = \sqrt{\epsilon B_z/h} \) (in units of \( 2\Delta_{0}/v \)) computed assuming \( \Delta = 0 \) and \( 2\Delta_0 = 1 \). (b) The transversal transmission probability for \( 2\Delta_0 = 1 \) and \( \Delta = 0.4 \). The dashed lines indicate resonances \( \delta q_{0,1} \) related to the Landau and spatial quantization. Notice also features related to the bottom energy \( \Delta_0 \), the energy gap \( \Delta \), and the cyclotron wave number \( q_c \). (c) The transversal magneto-thermoelectric current \( I_L \) (in units of \( 2\Delta_{0}/(\epsilon/h) \)) generated in N by the heat flow along C as a function of the cyclotron wave number \( q_c \) for different temperature deviations \( \delta T^* \) in C (see text). (d) Corresponding contour plot of the steady state \( I_L \) for the same parameters of the C/N-knot as before.
correspond to the following parameters dependent on the physical properties of the material. The d.c. magnetic flux density (magnetic induction) is $B = 16 \text{T}$, $\lambda = 6.4 \text{nm}$, the cyclotron energy $\hbar \omega_c = 1.85 \text{meV}$, the confinement steepness energy $\hbar \omega_c = 5 \text{meV}$, $\hbar \omega_c \approx 5.3 \text{meV}$, the effective C-stripe width $W^* = \pi / q_0 = 80 \text{nm}$, $v/2\Delta_0 = 130 \text{nm}$, $2\Delta_0 / v = 7.6 \times 10^6 \text{m}^{-1}$, $\Delta_0 = E_a + \hbar \omega_c / 2 = 0.5 \text{meV}$. Figure 3(a) shows the longitudinal transmission probability $T_x (\epsilon)$ through the C/N-knot vs. the electron energy $\epsilon$ and the cyclotron wave number $q_c = \sqrt{eB_c / \hbar}$ setting the energy gap $\Delta = 0$ and the bottom energy $2\Delta_0 = 1$. In fig. 3(b) we present the transversal transmission probability $\overline{T}_y (\epsilon)$ through the C/N-knot vs. $\epsilon$ (in units of $2\Delta_0$) and $q_c$ in units of $2\Delta_0 / v$, assuming that the energy gap is finite, $\Delta = 0.4$. In fig. 3(c) we plot the transversal magneto-thermoelectric current $I_\perp (3)$ (in units of $2\Delta_0 / \sqrt{v}$) by setting $\Delta = 0$ (again) induced along N vs. the cyclotron wave number $q_c$ (in units of $2\Delta_0 / v$) for different effective temperature deviations $\delta T^*$ = 0.08, 0.8, 1.4, and 2 (in units of $2\Delta_0 / k_B$) for curves 1–4, respectively. As an illustration, in fig. 3(d), we also show the corresponding contour plot of $I_\perp (3)$ for the same C/N-knot parameters as in the former fig. 3(c). From figs. 3(c), (d) one can see that $I_\perp (3)$ is roughly proportional to the temperature deviation $\delta T^*$ in C.

The obtained results suggest that the electrons (e) and holes (h), having opposite electric charges and propagating in C under the influence of the Lorentz force (see fig. 1(b)) are diverted toward N in opposite directions. Thus, a finite field $B \neq 0$ splits the heat flow into several components, thereby separating the electrons and holes from each other as shown in fig. 1(a) and causing a finite electric current $I_e = I_h = I \neq 0$ along N. The magnitude of the electric current induced in N is proportional to the heat flow in C and also depends on the d.c. magnetic field $B$. The non-linear tunneling coupling between the electron states of the C- and N-stripes modifies the Landau states in the C/N-knot thereby introducing a difference between the magneto-thermoelectric transport in the C/N-knot and isolated C-stripe.

Our results illustrate that the study of magneto-thermoelectric phenomena enables a better understanding of the microscopic transport mechanisms in the atomic monolayer materials and junctions, thereby opening many additional opportunities in a variety of researches and applications. By measuring resonances of $I_\perp$, one establishes key parameters of the AM junction and examines the interplay between the Landau and spatial quantizations in the conditions of the thermal quantum Hall effect. This extends the capabilities to determine the parameters of low-dimensional junctions on the nanoscale. In principle, the filtering of the heat flow in the C/N-knot subjected to the d.c. magnetic field can be used, e.g., to probe the electron-electron interaction and to potentially reveal other spectroscopic features like, e.g., the parameters of the Luttinger liquid or exciton binding energy in narrow AM stripes. Furthermore, the phenomenon can be utilized for the thermoelectric cooling and generation of electricity [2,5].

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REFERENCES

[1] Kane C. L. and Fisher M. P. A., Phys. Rev. B, 55 (1997) 15832.
[2] Shafranjuk S. E., Eur. Phys. J. B, 87 (2014) 99.
[3] See appendix in Shafranjuk S. E., arXiv:1411.3996v1 [cond-mat.mes-hall],
[4] Wang Q. H., Kalantar-Zadeh K., Kis A., Coleman J. N. and Strano M. S., Nat. Nanotechnol., 7 (2012) 699.
[5] Rowe D. M. (Editor), Thermoelectric Handbook (Chemical Rubber Company, Boca Raton, Fl., 1995).
[6] Shafranjuk S. E., EPL, 87 (2009) 57007.
[7] Geim A. K., Science, 324 (2009) 1530.
[8] Balandin A. A. et al., Nano Lett., 8 (2008) 902.
[9] Geim A. K., Science, 324 (2009) 1530.
[10] Balandin A. A. et al., Nano Lett., 8 (2008) 902.
[11] Bunch J. S. et al., Nano Lett., 8 (2008) 2458.
[12] Nair R. R., Wu H. A., Jayaram P. N., Grigorieva I. V. and Geim A. K., Science, 335 (2012) 442.
[13] Osada M. and Sasaki T., Adv. Mater., 24 (2012) 210.
[14] Ayari A., Comas E., Ogundadege O. and Fuhrer M. S., J. Appl. Phys., 101 (2007) 014507.
[15] Dean C. R., Young A. F., Merec I., Lee C., Wang L., Sorgenfrei S., Watanabe K., Taniguchi T., Kim P., Shepard K. L. and Hone J., Nat. Nanotechnol., 5 (2010) 722.
[16] Pacile D., Meyer J. C., Ghiri C. O. and Zettl A., Appl. Phys. Lett., 92 (2008) 133107.
[17] Lado J. L., Phys. Rev. B, 88 (2013) 035448.
[18] Persson A. I., Koh Y. K., Cahill D. G., Samuelson L. and Linke H., Nano Lett., 9 (2009) 4484.
[19] Baily M., Phys. Rev., 157 (1967) 480.
[20] Valli T., Fedorov A. V., Johnson P. D., Glans P.-A., McGuinness C., Smith K. E., Andrei E. Y. and Berger H., Phys. Rev. Lett., 92 (2004) 086401.
[21] Xia F., Perebeinos V., Lin Y.-M., Wu Y. and Avouris Ph., Nat. Nanotechnol., 6 (2011) 179.