A survey on the techniques to find elementary feasible solutions for fuzzy transportation problems

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Abstract. To obtain an optimum solution for the fuzzy transportation problem, there is requisite to find an elementary feasible solution. In this paper, a survey is presented on the current methods to get feasible solutions for the fuzzy transportation problems. The methods are also showed with a mathematical example for fuzzy data and a comparative study is also presented.

Keywords: Fuzzy transportation problem, elementary feasible solution, Optimum solution, Fuzzy data

1. Introduction

The transportation problem (TP) is related to provide some things from the basis to the terminus with least aggregate cost so that supply and demand restrictions will be satisfied. The minimum whole cost is the main criteria of TP for an optimum solution. Cost of transportation becomes an essential module for many companies because they spend lot of money in it.

Hitchcock in (1941) [11] developed the TP initially, by taking all the parameters as the exact values. Different methods were given by Dantzig in (1951) [7] & by Charnes, et al. in (1953) [6]. Due to uncontrollable factors, situations in real life are not certain. After introduction of fuzzy numbers by Zadeh (1965) [28] and Zimmerman (1996) [12] the vagueness and uncertainty in transportation problem was considered. Hadi Basir zadeh (2011)[9] given a new approach to solve transportation problem with fuzzy concept. This type of problems are called, fuzzy transportation problems (FTP) follow the scientific way to find the optimum solution as shown in following flow chart in figure 1.1

![Flowchart for solution of fuzzy transportation problem](image-url)

**Figure 1.1.** Flow chart for solution of fuzzy transportation problem.
There are many diverse methods developed by the researchers to find the elementary feasible solution (EFS) for selection of appropriate source and terminus to minimize the shipping cost. The most applied methods to find an elementary feasible solution (EFS) of TP are- Least Cost Method and North West Corner Method by Hamdy A T (2007) [10], Vogel’s Approximation Method by Hamdy A T (2007) [10] and Pandian & Natarajan (2010) [20]. Column Minimum Method by Schrage L Lindo (1991) [25] and Row Minimum Method by Reinfield, N V and W R Vogel (1958) [24].

Presently, some new exploratory methods have been framed viz. - Ahmed M M et al. (2016) [2] proposed, allocation table method, to find an EFS for transportation problems. Abdallah A Hlayel et al. (2012) [1], in their paper proposed, the best candidate method, in which they obtained the solution which is closest to optimal solution with a least computation time for shipping problem. Ravi Kumar R. et al. (2018) [23] proposed Direct Sum Method & its helpfulness is matched with standard approaches. Md. Amirul Islam et al. (2012) [19], had given Extremum Difference Formula which is based on total opportunity cost. A new technique to minimize the transportation cost by using operations on a total opportunity cost table. The distribution indicators are considered by the difference of highest and lowest unit cost.

S. Narayanamoorthy & S. Kalyani (2015) [27], proposed an algorithm for finding the elementary feasible solution of a FTP in their paper. The first step is finding, fuzzy difference between the highest fuzzy cost and lowest fuzzy cost in each row and column i.e. fuzzy penalty cost. Rajshri Gupta et al. (2019) [22], proposed a technique Sum of Minimum Costs Method, for finding elementary feasible solution to the TP for crisp data as well as fuzzy data, which is easy to calculate and nearer to the optimal solution of the problem. Kirca and Satir (1990) [17], introduced a technique Total Opportunity Cost Matrix (TOCM) to get initially basic feasible solution (IBFS). TOCM converts, by adding row and column opportunity cost matrix, the leading matrix into the preliminary matrix. Mathirajan et al. (2004) [18] united TOC concept with Vogel’s Approximation Method. Islam et al. (2012) [14] combined TOC table with different two maximal penalties. TOCM-SUM was presented by Khan et al. (2015a) [15]. Khan et al. (2015b) [16] started the method with Total Opportunity Cost Matrix by counting the distribution indicator for each cell. Amaliah et al. (2019) [3] proposed a method called Total Opportunity Cost Matrix-Minimal Total. Azad and Hossain (2017) [4] suggested a method for IBFS by calculating mean row penalty & mean column penalty. Soomro et al. (2015) [26] established Modified Vogel’s Approximation Method. Das et al. (2014b) [8] suggested Advanced Vogel’s Approximation Method to find IBFS. Babu et al. (2013) [5] familiarized Lowest Allocation Method (LAM) which started from the least supply or demand. Hosseini (2017) [13] presented Total Differences Method 1 by considering row penalty.

2. Objective
The objective of this paper is to study & compare the existing methods to find elementary feasible solutions for fuzzy transportation problems.

3. Mathematical Model of Transportation Problem
A transportation problem is mathematically, a special linear programming problem in which the purpose is to minimize the cost of transportation by considering demand and supply limitations.

The Mathematical Model of Transportation Problem (MMTP) to minimize the total cost of transportation from ‘m’ sources of supply (i = 1, 2, 3, 4…m) to ‘n’ destinations (j = 1, 2, 3, 4 …n) is stated as follows:

Mathematically, the TP is stated as

Minimize (Total cost) \( F(x) = \sum \sum c_{ij} x_{ij} \) \hspace{1cm} (3.1)

Subject to: \( G(x) = A \) \hspace{1cm} (3.2)
\( H(x) = B \)
\[ x_{ij} \geq 0, \quad \text{for all } i \text{ and } j \]

Where,
- \( A \) = Total Number of the goods available \((a_i)\) at the origins
- \( B \) = Total Number of the goods needed \((b_j)\) at destinations
- \( c_{ij} \) = Shipping cost of one unit of a goods from origin \( i \) to destination \( j \).

4. Mathematical Model of Fuzzy Transportation Problem

Practically, the constraints like supply and demand in linear programming are not certain; the data is ambiguous. In this situation, it is necessary to use fuzzy linear programming (FLP). FLP problems are first converted into equivalent crisp linear problems, then it can be solved by standard methods.

Mathematically, the FTP is

Minimize

\[ z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \] \hspace{1cm} (4.1)

Subject to the constraints

\[ \sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{\alpha}_i \quad \text{for } i = 1,2,3,4,\ldots,m \]

\[ \sum_{i=1}^{m} \tilde{x}_{ij} \geq \tilde{\varepsilon}_j \quad \text{for } j = 1,2,3,4,\ldots,n \] \hspace{1cm} (4.2)

\[ \tilde{x}_{ij} \geq 0 \quad \text{for } i = 1,2,3,4,\ldots,m \text{ and } j = 1,2,3,4,\ldots,n \]

Where,
- \( \tilde{\alpha}_i \) = Fuzzy renders at a source \( i \)
- \( \tilde{\varepsilon}_j \) = Fuzzy requirements at a terminal \( j \)
- \( \tilde{c}_{ij} \) = Unit cost of transportation from source \( i \) to terminal \( j \)
- \( \tilde{x}_{ij} \) = Number of units transported from source \( i \) to terminal \( j \)

**Figure 4.1.** Transportation problem network.
5. Trapezoidal Fuzzy Number

A = (a, b, c, d) is said to be a trapezoidal fuzzy number if it has trapezoidal membership function denoted by μ_A (x). It is a function named “trapmf” and is given by equation 5.1.

\[
\mu_A(x) = \begin{cases} 
0, & x < a \\
\frac{x - a}{b - a}, & a \leq x \leq b \\
1, & b < x < c \\
\frac{d - x}{d - c}, & c \leq x \leq d \\
0, & x > d 
\end{cases}
\]  

(5.1)

Graphical representation of Trapezoidal Membership Function is shown in figure 5.1.

![Trapezoidal Membership Function](image)

**Figure 5.1.** Trapezoidal membership function.

6. Ranking of Fuzzy Numbers

Various features can be extracted for ordering of fuzzy quantities. There are different methods of fuzzy ranking methods to extracts a specific feature from fuzzy sets based on that feature. For the same fuzzy sets different ranking methods resulted into different ranking order.

In this paper FTP is changed into a crisp problem of transportation, using new approach of fuzzy ranking method by Rajshri Gupta et al. (2018) [21].

If \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \in R \) & \( A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \) is trapezoidal fuzzy number, then the defuzzified value or the crisp value of A is given as

\[
R(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \frac{2\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4}{8}
\]  

(6.1)

7. Numerical Example by Different Methods

In this section a numerical example having the fuzzy trapezoidal numbers, was solved for finding the elementary feasible solution by applying eight current methods. The results were compared to find suitable method.

7.1 Example

Consider the following balanced FTP, where \( S_1, S_2, S_3 \) has taken as three sources & \( D_1, D_2, D_3, D_4 \) as four destinations supply and demand are taken as trapezoidal fuzzy numbers as given in Table 7.1.
Table 7.1. Transportation problem with trapezoidal numbers.

|     | D1       | D2       | D3       | D4       | Demand   |
|-----|----------|----------|----------|----------|----------|
| S1  | (1,2,3,4)| (1,3,6,8)| (-1,0,1,2)| (3,5,6,8)| (0,2,4,6)|
| S2  | (4,8,12,16)| (6,7,11,12)| (2,4,6,8)| (1,3,5,7)| (2,5,9,13)|
| S3  | (1,5,9,13)| (0,4,8,12)| (0,6,8,14)| (4,7,9,12)| (2,4,6,7)|
| Supply | (1,3,5,7) | (0,2,4,6) | (1,3,5,7) | (1,3,5,7) |           |

By using definition, the problem is balanced, as explained below:

Sum of supply = \((1,3,5,7) + (0,2,4,6) + (1,3,5,7) + (1,3,5,7)\)  
= \((3, 11, 19, 27)\)

Sum of demand = \((0, 2, 4, 6) + (2, 5, 9, 13) + (2, 4, 6, 7)\)  
= \((4, 11, 19, 26)\)

\[R(3, 11, 19, 27) = \frac{2 \times 3 + 11 + 19 + 2 \times 27}{8} = 11.25\]

\[R(4, 11, 19, 26) = \frac{2 \times 4 + 11 + 19 + 2 \times 26}{8} = 11.25\]

Sum of supply = Sum of demand

By using ranking method by Rajshri Gupta et al. (2018) [21] the FTP can be written in crisp form as shown in Table 7.2.

Table 7.2. Transportation problem in crisp data

|     | D1 | D2 | D3 | D4 | Demand |
|-----|----|----|----|----|--------|
| S1  | 1.9| 3.4| 0.4| 4.1| 2.3    |
| S2  | 7.5| 6.8| 3.8| 3  | 5.5    |
| S3  | 5.3| 4.5| 5.3| 6  | 3.5    |
| Supply | 3  | 2.3| 3  | 3  | 11.3   |

By applying North West Corner Method the elementary feasible solution of the above problem is \((-130, 27, 189, 416)\) and we the crisp value of the fuzzy transportation problem is 98.5.

By Least cost method the elementary solution of the fuzzy transportation problem is \((-193,-1,175,456)\) and the crisp value of the FTP is 87.5.

The elementary feasible solution of the fuzzy transportation problem is \((-105, 15,149,361)\) and crisp value is 84.5, by Vogel’s Approximation Method and Column Minima Method, whereas by Row Minima Method the feasible solution is \((-51, 35, 155, 287)\) with crisp value 82.8.

By applying Direct Sum Method & Extremum Difference Method to the FTP the fuzzy solution and crisp solution are \((-113, 12,148,360)\) and 80.5 respectively.

Using sum of minimum cost method the elementary feasible solution of above problem is \((-113, 12,148,360)\) and crisp value is 80.5.
8. Results and Discussions
The elementary feasible solution of FTP by different methods are compared as shown in Table 8.1.

Table 8.1. Elementary feasible solutions of fuzzy transportation problem.

| S.N. | Methods                        | EFS Fuzzy Solution | EFS Crisp Solution |
|------|--------------------------------|--------------------|--------------------|
| 1    | North West Corner Method (NWCM) | (-130,27,189,416)  | 98.5               |
| 2    | Least Cost Method (LCM)         | (-193,-1,175,456)  | 87.5               |
| 3    | Vogel’s Approximation Method (VAM) | (-105,15,149,361)  | 84.5               |
| 4    | Column Minima Method (CMM)      | (-105,15,149,361)  | 84.5               |
| 5    | Row Minima Method (RMM)         | (-51,35,155,287)   | 82.8               |
| 6    | Direct Sum Method (DSM)         | (-113, 12,148,360) | 80.5               |
| 7    | Extremum Difference Method (EDM) | (-113, 12,148,360) | 80.5               |
| 8    | Sum of Minimum Costs Method (SMCM) | (-113, 12,148,360) | 80.5               |

Graphical representation for the comparison of elementary feasible solutions by all eight methods is shown in figure 8.1.

![Comparison: Example EFS Crisp Solution](image)

Figure 8.1. Comparison EFS crisp solutions.

From the Table-8.1 and figure 8.1, it is observed that the solution by Direct Sum Method, Extremum Difference Method and Sum of Minimum Costs Method give a least feasible solution than other current methods with less iterations. Hence these methods can be used to solve any type of fuzzy transportation problem.
9. Conclusion
In today’s world, competition is very high to transport goods to the clients economically. Transportation model plays an important role to determine the best ways for customer’s satisfaction. Profit of any firm is related to minimization of the transportation cost where in number of uncertainties occurs & hence the reliable way out, of the transportation problem with suitable technique is difficult.

This paper is providing a survey on the techniques for finding an elementary feasible solution of fuzzy transportation problems and also studies the approaches used to solve such problems. A comparative study is given of eight methods applied to solve a fuzzy transportation problem.

10. Future Scope
For the considered example, elementary feasible solution obtained by North West Corner Method, Least Cost Method, Vogel’s Approximation Method, Column Minima Method, and Row Minima Method is not optimal, whereas the elementary feasible solution obtained by Direct Sum Method, Extremum Difference Method and Sum of Minimum Costs Method, is an optimal solution. The elementary feasible solution attained by North West Corner Method is far from optimal for the transportation problems.

This paper has given the comparative study between the eight existing methods to find elementary feasible solutions for FTP and it may be helpful for further research on the topic.

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