Prediction of primary energy consumption using NDGM(1,1,k,c) model with Simpson formula

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Primary energy consumption; Grey forecasting model; Simpson formula; Non-homogeneous index sequence; Prediction accuracy.

**Abstract.** Energy consumption plays a key role in economic development for all countries. Keeping up with the future trend of energy consumption is essential for governments and energy companies. In this research, the primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam is systematically studied using different forecasting models. Based on the actual data derived from the year 2006 to 2016, a novel discrete grey forecasting model termed NDGM\textsubscript{3}(1,1,k,c) is proposed where the Simpson numerical integration formula is applied to construct the background value. The expressions of the present model are all derived and then, its unbiased property is proved. As demonstrated by the results, the NDGM\textsubscript{3}(1,1,k,c) model can achieve better prediction accuracy than other forecasting models, and it is quite applicable to predicting a sequence based on homogeneous/non-homogeneous exponential law.

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1. Introduction

Primary energy is an energy resource found in nature that has not been subjected to any human engineered conversion or transformation process. It is also called natural energy contained in oil, coal, natural gas, water energy, and modern renewable energy used to generate electricity. The BP Statistical Review of World Energy 2017 [1] demonstrated that global primary energy consumption expanded by 1% in 2016, following the growth of 0.9% in the year 2015 and 1% in the year 2014. This trend is measured with the 10-year average of 1.8% a year. Primary energy consumption in Saudi Arabia grew by 1.9% in 2016 corresponding to 266.5 million tonnes of oil equivalent (Mtoe), which is 2.0% of the total international primary energy consumption. In addition, the annual growth rate during the years 2005 to 2015 is 5.1%. For India's primary energy consumption, a growth is 5.4% in 2016 corresponding to 723.9 Mtoe, which is 5.5% of the total global consumption. From 2005 to 2015, primary energy consumption grew at an incredible average rate of 5.1%. In Philippines, it grew by 11.3% in 2016 corresponding to 42.1 Mtoe, which is 0.3% of the total global consumption. The annual growth rate from 2005 to 2015 is 3.6%. For Vietnam, the mentioned energy consumption growth was 1.5% in...
2016 corresponding to 64.8 Mtoe, which is 0.5% of the total global consumption. From the year 2005 to 2015, this consumption grew at an incredible average rate of 7.5%. Presently, energy markets are accommodated and the near-term strength may continuously ease. It is important for decision-makers and government departments to develop a better understanding and judgment of the energy resource plan scientifically and formulate appropriate energy plans.

Energy is and has been receiving remarkable attention over a long time because of its importance all over the world. A variety of methods and techniques have been advanced for energy forecast utilization such as cointegrated panel analysis [2], artificial neural network [3], time series analysis [4,5], coupling mathematical models [6,7], hybrid forecasting models [8,9], grey models [10–14], etc. Among those excellent methods/techniques, grey system theory that was presented by Deng [15] is a feasible and efficient prediction technique to analyze uncertain problems. In his work, the first-order linear model with single variable termed GM(1,1) model was discussed in detail. The main advantage of grey models is that they require a small number of samples to describe the system. Over the past three decades, the GM(1,1) model has significantly generalized with the following aspects: the univariate linear grey models [16,17], the univariate nonlinear grey models [18–20], and the multivariate grey models [21–23].

Recently, Cui et al. [24] studied the continuous non-homogeneous grey model named NGM(1,1,k) model where bk is grey action quantity. The yearly amount of concave soil in Xuyi of China and the CSI 300 index specimen data were used to illustrate the NGM(1,1,k) model and their optimized model was effective. However, Chen and Yu’s work [25] identified that the parameters of the NGM(1,1,k) model had a fatal flaw that badly affected the application value. Based on Cui’s work, a modified model named NGM(1,1,k,c) was proposed in which bk+c was grey action quantity. This model is truly feasible for simulation and forecasting of approximate non-homogeneous exponential sequence and can achieve outstanding prediction accuracy. Meanwhile, Xie et al. [26] developed an NDGM model where the background value was derived from the trapezoid formula and the initial point was optimized. The expression of this model was obtained and the prediction precision was found to be dependent on the pure non-homogeneous index sequence.

Encouraged by these works [24–26] and considering the non-homogeneous exponential sequence existing in the real world [27,28], the present study focuses on the non-homogeneous discrete NGM(1,1,k,c) model called NDGMs(1,1,k,c) where the background value is computed using the Simpson numerical integration formula. Its solutions, properties, and applications are derived in this paper. Meantime, we prove the new model is able to simulate a linear sequence and a homogeneous/non-homogeneous exponential sequence without error. Further, the primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam is calculated by grey models, Auto-Regressive Integrated Moving Average model (ARIMA), and Support Vector Machines (SVMs). It is noted that the NDGMs(1,1,k,c) model presents high accuracy in the primary energy consumption.

This paper is organized below. A brief introduction to the NGM(1,1,k) model is given in Section 2. Detailed discussions of the NDGMs(1,1,k,c) model are given in Section 3. Applications of the primary energy consumption are arranged in Section 4. The last section concludes the paper.

2. The existing NGM model

Next, a brief analysis of the continuous NGM model is conducted based on the Trapezoid formula.

2.1. Grey NGM(1,1,k) model

It is assumed that an original non-negative data sequence with n entries is $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$, where $x^{(0)}(k)$ stands for the value of the data at the time index $k$.

Let $X^{(1)} = (x^{(0)}(i), x^{(0)}(i), \ldots, x^{(0)}(i))$ be the first accumulated generating operation (1-AGO) sequence of $X^{(0)}$.

We denote $Z^{(1)}$ as the background value of the grey forecasting model where $Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n))$ with $z^{(1)}(k) = \frac{1}{2}x^{(1)}(k) + \frac{1}{2}x^{(1)}(k - 1), k = 2, 3, \ldots, n$.

From Ref. [24], the mathematical model of the NGM(1,1,k) is as follows:

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt,$$

which is a linear differential equation, $a$ is the developing coefficient, and $bt + c$ is the grey action quantity.

The values for the unknown parameters $a$ and $b$ of NGM(1,1,k) model are computed by the least squares estimation:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (\Lambda^T \Lambda)^{-1} \Lambda^T \eta,$$

where:

$$\Lambda = \begin{bmatrix} z^{(1)}(2) & -2 \\ z^{(1)}(3) & -3 \\ \vdots & \vdots \\ z^{(1)}(n) & -n \end{bmatrix}, \quad \eta = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}.$$

The time response function of the NGM(1,1,k) model is:
\[
\dot{z}^{(1)}(k) = \left( x^{(0)}(1) + \frac{b}{a^2} - \frac{b}{a} \right) e^{-a(k-1)} - \frac{b}{a^2} k - \frac{b}{a^2},
\]
\[
k = 2, 3, \cdots , n.
\]
(3)

The restored values of \( \dot{z}^{(0)}(k) \) are:
\[
\dot{z}^{(0)}(k) = \left( x^{(0)}(1) + \frac{b}{a^2} - \frac{b}{a} \right) \left( 1 - e^{-a(k-1)} \right) \left( \frac{b}{a} \right),
\]
\[
k = 1, 2, \cdots , n.
\]
(4)

2.2. Grey \( \text{NGM}(1,1,k,c) \) model

From Ref. [25], the mathematical form of the \( \text{NGM}(1,1,k,c) \) model is:
\[
\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt + c,
\]
(5)
where \( a \) is the developing coefficient and \( bt + c \) is the grey action quantity.

The unknown parameters \( a, b, \) and \( c \) of the \( \text{NGM}(1,1,k,c) \) model are determined by the least squares estimation:
\[
\begin{pmatrix}
a \\
b \\
c
\end{pmatrix} = (A^T \Lambda)^{-1} A^T \eta,
\]
(6)
where:
\[
\Lambda = \begin{pmatrix}
\dot{z}^{(1)}(2) & -2 & -1 \\
\dot{z}^{(1)}(3) & -3 & -1 \\
\vdots & \vdots & \vdots \\
\dot{z}^{(1)}(n) & -n & -1
\end{pmatrix}, \quad \eta = \begin{pmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{pmatrix}.
\]

The time response function of the \( \text{NGM}(1,1,k,c) \) model is:
\[
\dot{z}^{(1)}(k) = \left( x^{(0)}(1) + \frac{c}{a} - \frac{b}{a} \right) e^{-a(k-1)} - \frac{b}{a^2} k - \frac{b}{a^2} + \frac{c}{a},
\]
\[
k = 2, 3, \cdots , n.
\]
(7)

The restored values of \( \dot{z}^{(0)}(k) \) are:
\[
\dot{z}^{(0)}(k) = \left( x^{(0)}(1) + \frac{c}{a} - \frac{b}{a} + \frac{b}{a^2} \right) \left( 1 - e^{-a(k-1)} \right) \left( \frac{b}{a} \right),
\]
\[
k = 2, 3, \cdots , n.
\]
(8)

3. The \( \text{NDGM}_5(1,1,k,c) \) model

3.1. Representation of the \( \text{NDGM}_5(1,1,k,c) \) model

In this subsection, we plan to derive the discrete \( \text{NDGM}_5(1,1,k,c) \) model from the Simpson numerical integration formula. Considering the integration of Eq. (5) at the interval \( [k-1, \ k + 1] \), it follows that:
\[
\int_{k-1}^{k+1} dx^{(1)}(t) + a \int_{k-1}^{k+1} x^{(1)}(t) dt = b \int_{k-1}^{k+1} t dt + c \int_{k-1}^{k+1} dt.
\]
(9)

From Eq. (9), we have:
\[
x^{(1)}(k+1) - x^{(1)}(k-1) + a \int_{k-1}^{k+1} x^{(1)}(t) dt = 2kb + 2c.
\]
(10)

Applying the Simpson numerical integration formula, we realize that:
\[
\int_{k-1}^{k+1} x^{(1)}(t) dt = \frac{1}{3} x^{(1)}(k-1) + \frac{4}{3} x^{(1)}(k) + \frac{1}{3} x^{(1)}(k+1).
\]
(11)

By substituting Eq. (11) into Eq. (10), it turns to be:
\[
(3 + a)x^{(1)}(k + 1) + 4ax^{(1)}(k) - (3 - a)x^{(1)}(k - 1) = 6kb + 6c.
\]
(12)

It follows from Eq. (12) that:
\[
x^{(1)}(k + 1) - w x^{(1)}(k)
\]
\[
= \frac{a - 3}{w(a + 3)} \left[ x^{(1)}(k) - w x^{(1)}(k - 1) \right] + \frac{6b}{a + 3} k + \frac{6c}{a + 3}.
\]
(13)

where \( w = \sqrt{a^2 + 2a + 2} \). Iterating Eq. (13) by itself, we obtain that:
\[
x^{(1)}(k + 1) - w x^{(1)}(k)
\]
\[
= \frac{a - 3}{w(a + 3)} \left\{ \frac{a - 3}{w(a + 3)} \left[ x^{(1)}(k - 1) - w x^{(1)}(k - 2) \right] \right\} + \frac{a - 3}{w(a + 3)} \left[ (k - 1) \frac{6b}{a + 3} + \frac{6c}{a + 3} \right]
\]
\[
+ k \frac{6b}{a + 3} + \frac{6c}{a + 3}
\]
\[
= \left( \frac{a - 3}{w(a + 3)} \right)^2 \left[ x^{(1)}(k - 1) - w x^{(1)}(k - 2) \right] + \frac{6b}{a + 3} \sum_{m=0}^{k-1} \left( \frac{a - 3}{w(a + 3)} \right)^m (k - m)
\]
\[
+ \frac{6c}{a + 3} \sum_{m=0}^{k-1} \left( \frac{a - 3}{w(a + 3)} \right)^m
\]
\[
= \left( \frac{a - 3}{w(a + 3)} \right)^k \left[ x^{(1)}(2) - w x^{(1)}(1) \right]
\]
\[ + \frac{6c}{a + 3} \sum_{m=0}^{k-2} \left( \frac{a - 3}{w(a + 3)} \right)^m \]

\[ + \frac{6b}{a + 3} \sum_{m=0}^{k-2} \left( \frac{a - 3}{w(a + 3)} \right)^m (k - m) \]

\[ = \alpha^{k-1} \left[ x^{(1)}(2) - wx^{(1)}(1) \right] \]

\[ + \frac{6b}{a + 3} \sum_{m=0}^{k-2} \alpha^m (k - m) + \frac{6c}{a + 3} \sum_{m=0}^{k-2} \alpha^m, \quad (14) \]

with \( \alpha = \frac{a-3}{w(a+3)} \). Note that:

\[ x^{(1)}(k + 1) - w^{k-1}x^{(1)}(2) \]

\[ = \sum_{i=0}^{k-2} w^i [x^{(1)}(k + 1 - i) - wx^{(1)}(k - i)] \]

\[ = \sum_{i=0}^{k-2} w^i \alpha^{k-1} \left[ x^{(1)}(2) - wx^{(1)}(1) \right] \]

\[ + \sum_{i=0}^{k-2} w^i \frac{6b}{a + 3} \sum_{m=0}^{k-2} \alpha^m (k - i - m) \]

\[ + \sum_{i=0}^{k-2} w^i \frac{6c}{a + 3} \sum_{m=0}^{k-2} \alpha^m \]

\[ = \alpha^{k-1} - \frac{(w\alpha^2)^{k-1}}{1 - w\alpha} \left[ x^{(1)}(2) - wx^{(1)}(1) \right] \]

\[ + \frac{6b}{a + 3} \sum_{i=0}^{k-2} \sum_{m=0}^{k-2} w^i \alpha^m (k - i - m) \]

\[ + \frac{6c}{a + 3} \sum_{i=0}^{k-2} \sum_{m=0}^{k-2} w^i \alpha^m. \quad (15) \]

The 1-AGO sequence \( \hat{X}^{(1)} \) of discrete NDGMs\( (1, 1, k, c) \) is:

\[ \hat{x}^{(1)}(k + 1) = w^{k-1}x^{(1)}(2) \]

\[ + \frac{\alpha^{k-1} - (w\alpha^2)^{k-1}}{1 - w\alpha} \left[ x^{(1)}(2) - wx^{(1)}(1) \right] \]

\[ + \frac{6b}{a + 3} \sum_{i=0}^{k-2} \sum_{m=0}^{k-2} w^i \alpha^m (k - i - m) \]

\[ + \frac{6c}{a + 3} \sum_{i=0}^{k-2} \sum_{m=0}^{k-2} w^i \alpha^m, \quad k = 1, 2, \cdots, n - 1. \quad (16) \]

The IAGO on \( \hat{X}^{(1)} \) is applied to obtain:

\[ \dot{x}^{(0)}(k + 1) = \hat{x}^{(1)}(k + 1) - \hat{x}^{(1)}(k), \]

\[ k = 1, 2, \cdots, n - 1. \quad (17) \]

### 3.2. Parameters estimation of the discrete NDGMs\( (1, 1, k, c) \) model

Based on the definition of 1-AGO, we obtain:

\[ x^{(1)}(k + 1) - x^{(1)}(k - 1) = \sum_{i=1}^{k} x^{(0)}(i) - \sum_{i=1}^{k} x^{(0)}(i) \]

\[ = x^{(0)}(k + 1) + x^{(0)}(k). \]

Upon employing the Simpson numerical integration formula, the background value of \( X^{(1)} \) is provided below:

\[ z^{(1)}(k) = \frac{1}{3} x^{(1)}(k - 1) + \frac{4}{3} x^{(1)}(k) + \frac{1}{3} x^{(1)}(k + 1), \]

\[ k = 2, 3, \ldots, n. \]

Substituting \( z^{(1)}(k) \) into Eq. (10), we have that:

\[ x^{(0)}(k) + x^{(0)}(k + 1) + az^{(1)}(k) = 2kb + 2c. \quad (18) \]

It follows from Eq. (18) that:

\[ \begin{aligned}
  x^{(0)}(2) + x^{(0)}(3) &= -az^{(1)}(2) + 4b + 2c, \\
  x^{(0)}(3) + x^{(0)}(4) &= -az^{(1)}(3) + 6b + 2c, \\
  & \vdots \\
  x^{(0)}(n - 1) + x^{(0)}(n) &= -az^{(1)}(n - 1) + 2(n - 1)b + 2c.
\end{aligned} \quad (19) \]

By applying the least squares estimation method, the model parameters \( \hat{\xi} = (a, b, c)^T \) of the NDGMs\( (1, 1, k, c) \) are:

\[ \hat{\xi} = (B^TB)^{-1}B^TY, \quad (20) \]

where:

\[ B = \begin{pmatrix}
  z^{(1)}(2) & -4 & -2 \\
  z^{(1)}(3) & -6 & -2 \\
  \vdots & \vdots & \vdots \\
  z^{(1)}(n - 1) & -2(n - 1) & -2
\end{pmatrix}, \]

\[ Y = \begin{pmatrix}
  x^{(0)}(3) + x^{(0)}(2) \\
  x^{(0)}(4) + x^{(0)}(3) \\
  \vdots \\
  x^{(0)}(n) + x^{(0)}(n - 1)
\end{pmatrix}. \]

Here, we give a short explanation to demonstrate that \( \hat{\xi} \) is the least squares estimation of the model. It is known that to determine the least squares estimation
of the NDGM$_S(1,1,k,c)$ model is to pursue an $\hat{\xi}$, thus making the subsequent equation minimum:

$$s(\xi) = \sum_{i=2}^{n-1} (x^{(i)}(i)+x^{(i+1)}(i+1)+a x^{(i)}(i)-2i b-2c)^2$$

$$= (Y - B\xi')^T (Y - B\xi) = ||Y - B\xi||^2.$$  \hspace{1cm} (21)

If $\hat{\xi}$ is the least squares estimation of the model, there must be $s(\hat{\xi}) \geq s(\xi)$ for any $\xi$. Let $\xi'$ be the solution of Eq. (20), that is, $\xi' = (B^T B)^{-1}B^T Y$.

For any values of $\xi$, we have:

$$s(\xi) = (Y - B\xi' + B\xi - B\xi')^T (Y - B\xi' + B\xi - B\xi)$$

$$= (Y - B\xi')^T (Y - B\xi') + (Y - B\xi')^T (B\xi - B\xi')$$

$$+ (B\xi - B\xi')^T (Y - B\xi') + (B\xi - B\xi')^T (B\xi - B\xi')$$

$$= s(\xi') + (Y^T B - \xi'^T B)B(\xi' - \xi)$$

$$+ (\xi' - \xi)^T (B^T Y - B^T B\xi')$$

$$+ (B\xi - B\xi')^T (B\xi' - B\xi) = s(\xi') + ||B(\xi' - \xi)||^2.$$ \hspace{1cm} (22)

This means that $\xi'$ is the least square estimation of the NDGM$_S(1,1,k,c)$ model. Furthermore, taking $\hat{\xi}$ into Eq. (22), we acquire that:

$$s(\hat{\xi}) = s(\xi') + ||B(\xi' - \xi)||^2.$$ \hspace{1cm} (23)

It follows from Eq. (23) that $s(\hat{\xi}) \geq s(\xi)$. As $\xi$ is the least square estimation of the model, we know that $s(\hat{\xi}) \leq s(\xi)$. Thus, $s(\hat{\xi}) = s(\xi)$ and $B(\xi' - \xi) = 0$. Then, $B^T B\xi' = B^T B\xi = B^T Y$ which leads to $\xi' = (B^T B)^{-1}B^T Y$.

### 3.3. Modeling evaluation criteria

To examine the forecasting correctness of the NDGM$_S(1,1,k,c)$ model, the Absolute Percentage Error (APE), the mean absolute simulation percentage error (MAPE$_{simu}$), the mean absolute prediction percentage error (MAPE$_{pred}$), and the overall mean absolute percentage error (MAPE$_{over}$) are applied. In general, the APE, MAPE$_{simu}$, MAPE$_{pred}$, and MAPE$_{over}$ are defined as follows:

$$\text{APE}(k) = \left| 1 - \frac{\hat{x}^{(i)}(k)}{x^{(i)}(k)} \right| \times 100\%, \quad k = 2, 3, \ldots, n.$$  \hspace{1cm} (24)

$$\text{MAPE}_{simu} = \frac{1}{n-1} \sum_{k=2}^{n} \left| 1 - \frac{\hat{x}^{(i)}(k)}{x^{(i)}(k)} \right| \times 100\%.$$ \hspace{1cm} (25)

$$\text{MAPE}_{pred} = \frac{1}{n-1} \sum_{k=2}^{n} \left| 1 - \frac{\hat{x}^{(i)}(k)}{x^{(i)}(k)} \right| \times 100\%.$$ \hspace{1cm} (26)

$$\text{MAPE}_{over} = \frac{1}{n-1} \sum_{k=2}^{n} \left| 1 - \frac{\hat{x}^{(i)}(k)}{x^{(i)}(k)} \right| \times 100\%.$$ \hspace{1cm} (27)

### 3.4. Unbiased property of the NDGM$_S(1,1,k,c)$ model

In this subsection, we prove that the NDGM$_S(1,1,k,c)$ model is unbiased to simulate a linear sequence and a homogeneous/non-homogeneous exponential sequence without inaccuracy.

#### 3.4.1. Simulate a linear sequence

Suppose that a linear sequence is $X^{(0)} = \{rk+\theta, k = 1, 2, \ldots, n\}$. Then, we obtain:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) = \frac{1}{2}(k+1)kr + k\theta.$$ \hspace{1cm} (28)

The 1-AGO of $X^{(0)}$ is stated by:

$$X^{(1)} = \left\{ r + \theta, 3r + 2\theta, 6r + 3\theta, \frac{(n+1)n}{2}r + n\theta \right\}.$$ \hspace{1cm} (29)

By using these expressions into the matrix $B$ and $Y$, it can be found that:

$$B = \begin{pmatrix} -19r/3 - 4\theta & 4 & 2 \\ -37r/3 - 6\theta & 6 & 2 \\ \vdots & \vdots & \vdots \\ -r(3n^2 - 3n + 1)/3 -(2n-2)r & 2(n-1) & 2 \end{pmatrix}.$$ \hspace{1cm} (30)

After some calculations, we acquire:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = (B^T B)^{-1}B^T Y = \begin{pmatrix} 0 \\ r/2 + \theta \end{pmatrix}.$$ \hspace{1cm} (30)

Then, we can easily get:

$$w = 1, \quad \alpha = -1.$$ \hspace{1cm} (30)

Substituting these values into Eq. (16), we have:

$$\hat{x}^{(1)}(k+1) = (3r + 2\theta) + \frac{(-1)^{k-1} - 1}{2}(2r + \theta)$$

$$+ 2r \sum_{i=0}^{k-2} \sum_{m=0}^{i-2} (-1)^m(k-i-m)$$

$$+ (r + 2\theta) \sum_{i=0}^{k-2} \sum_{m=0}^{i-2} (-1)^m$$

$$= (3r + 2\theta) + \frac{(-1)^{k-1} - 1}{2}(2r + \theta)$$
\[ + 2r \left( \frac{k^2}{4} + \frac{3}{8}(-1)^k + \frac{k}{2} - \frac{3}{8} \right) \]
\[ + (r + 2\theta) \left( \frac{k}{2} - \frac{1}{4} + \frac{1}{4}(-1)^k \right) \]
\[ = \left( \frac{k^2}{2} r + kr + \frac{k}{2} r + r \right) + (k + 1)\theta \]
\[ + \left[ 2r + 3(-1)^k - 3r - \frac{1}{4} + \frac{r}{4}(-1)^k + (-1)^{k-1}r - r \right] \]
\[ + \left( \theta - \frac{1}{2}\theta + \frac{k}{2}(-1)^k + \frac{k}{2}(1 - \theta - \frac{1}{2}) \right) \]
\[ = \left( \frac{k + 2)(k + 1)}{2} + (k + 1)\theta \right) = \nu^{(1)}(k + 1). \] (31)

From Eq. (31), the proposed NDGM_{S(1,1,k,c)} model can simulate a linear sequence without errors.

3.4.2. Simulation of a homogeneous/non-homogeneous exponential sequence

Assume that a non-homogeneous exponential sequence is \( X^{(0)} = \{rq^k + \theta, k = 1, 2, \cdots, n\}. \) Then, we possess:

\[ \nu^{(1)}(k) = \sum_{i=1}^{k} \nu^{(0)}(i) = \frac{rq(1 - q^k)}{1 - q} + k\theta, \]
\[ k = 1, 2, \cdots, n. \] (32)

The 1-AGO of \( X^{(0)} \) is given by:

\[ X^{(1)} = \left\{ rq + \theta, \frac{rq(1 - q^2)}{1 - q} + 2\theta, \frac{rq(1 - q^3)}{1 - q} + 3\theta, \cdots, \frac{rq(1 - q^n)}{1 - q} + n\theta \right\}. \] (33)

Substituting these expressions into the matrices \( B \) and \( Y \), the equations shown in Box I is obtained. After some calculations, we obtain:

\[
\begin{pmatrix}
\alpha \\
\beta \\
\gamma
\end{pmatrix} = (B^T B)^{-1} B^T Y = \begin{pmatrix}
\frac{3 (1 - q^k)}{1 + q^2 + q^k} \\
\frac{3 \theta (1 - q^k)}{1 + q^2 + q^k} \\
\frac{3 q (1 + q^2 + q^k)}{1 + q^2 + q^k}
\end{pmatrix}.
\] (34)

Then, we can easily get:

\[ w = q, \quad \alpha = -\frac{q + 2}{2q + 1}. \]

Further, we have:

\[ \frac{6b}{a + 3} = \frac{3\theta(1 - q^2)}{2q + 1}, \]
\[ \frac{6c}{a + 3} = \theta(1 + 4q + q^2) + 3\theta(1 + q). \]

Substituting these values into Eq. (16), we have:

\[ \dot{x}^{(1)}(k + 1) = q^{k-1}(r_q + r_q^2 + 2\theta) \]
\[ + (r + 2\theta - q\theta) \sum_{i=0}^{k-2} q^i \alpha^{k-i-1} \]
\[ + \sum_{i=0}^{k-2} q^i \left( \frac{6b}{a + 3} + \frac{6c}{a + 3} \right) \sum_{m=0}^{k-i-2} \alpha^m \]
\[ - \sum_{i=0}^{k-2} q^i \left( \frac{6b}{a + 3} + \sum_{m=0}^{k-i-2} m\alpha^m \right) \]
\[ = q^{k-1}(r_q + r_q^2 + 2\theta) \]
\[ + (r + 2\theta - q\theta) \sum_{i=0}^{k-2} q^i \alpha^{k-i-1} \]
\[ + \sum_{i=0}^{k-2} q^i \left( \frac{3\theta(1 - q^2)}{2q + 1} \right) \sum_{m=0}^{k-i-2} \frac{(k - i - 1)\alpha^{k-i-1}}{1 - \alpha} \]
\[ + \sum_{i=0}^{k-2} q^i \left( \frac{\theta(1 + 4q + q^2) + 3\theta(1 + q)}{2q + 1} \right) \sum_{m=0}^{k-i-2} \frac{(k - i - 1)\alpha^{k-i-1}}{(1 - \alpha)^2} \]
\[ - \sum_{i=0}^{k-2} q^i \left( k - i - 1 \right) \alpha^{k-i-2} \left( 1 - \alpha \right). \]

Box I
Table 1. Results of the NGM(1,1,k,c) and NDGMs(1,1,k,c) models for a nonhomogeneous exponential sequence with \( r = 0.06 \), \( q = 2.25 \), and \( \theta = 3 \).

| \( k \) | Actual value | NGM(1,1,k,c) model \( \bar{x}^{(0)}(k) \) | APE(k)% | NDGMs(1,1,k,c) model \( \bar{x}^{(0)}(k) \) | APE(k)% |
|---|---|---|---|---|---|
| 1 | 3.1350 | 3.1350 | 0 | 3.1350 | 0 |
| 2 | 3.3038 | 5.0186 | 51.9055 | 3.3038 | 0 |
| 3 | 3.6834 | 7.3563 | 99.7130 | 3.6834 | 4.8346 \times 10^{-11} |
| 4 | 4.5377 | 12.4014 | 173.2942 | 4.5377 | 8.8666 \times 10^{-11} |
| 5 | 6.4599 | 23.2891 | 260.5184 | 6.4599 | 1.5553 \times 10^{-10} |
| 6 | 10.7848 | 46.7861 | \textbf{333.8160} | 10.7848 | \textbf{2.0291 \times 10^{-10}} |
| 7 | 20.5158 | 97.4950 | 375.2302 | 20.5158 | 2.3222 \times 10^{-10} |
| 8 | 42.4105 | 206.9302 | \textbf{387.9227} | 42.4105 | 2.3950 \times 10^{-10} |
| 9 | 91.6735 | 443.1029 | 383.3489 | 91.6735 | 2.3462 \times 10^{-10} |
| 10 | 202.5154 | 952.7886 | 370.4771 | 202.5154 | 2.2340 \times 10^{-10} |
| 11 | 451.9097 | 2052.7440 | 354.2377 | 451.9097 | 2.0963 \times 10^{-10} |
| 12 | 1013.0470 | 4436.5639 | 336.9556 | 1013.0467 | \textbf{1.9434 \times 10^{-10}} |

MAPE_{simu}(\%) = \frac{183.8494}{9.0090 \times 10^{-11}}

MAPE_{pred}(\%) = \frac{368.0270}{2.2228 \times 10^{-10}}

MAPE_{over}(\%) = \frac{284.3099}{1.6629 \times 10^{-10}}

\begin{align*}
= q^{k-1}(rq + q^2) + rq \sum_{i=0}^{k-2} q^i \alpha^{k-i-1} \\
+ \sum_{i=0}^{k-2} q^i (2q \alpha^{k-i-1} + \theta q^{k-1}) \\
+ \sum_{i=0}^{k-2} q^i \left\{ (2-q) \alpha^{k-i-1} + (1-q)(k-i)(1-\alpha^{k-i-1}) \right\} \\
= \frac{rq(1-q^{k+1})}{1-q} + \theta q^{k-1} + \sum_{i=0}^{k-2} q^i \left\{ (2-q) \alpha^{k-i-1} + (1-q)(k-i)(1-\alpha^{k-i-1}) \right\}
\end{align*}

From Eq. (35), the proposed NDGMs(1,1,k,c) model can attain an unbiased simulation of a homogeneous/non-homogeneous exponential sequence.

Next, we here provide a numerical experiment to illustrate the accuracy of the NGM(1,1,k,c) and NDGMs(1,1,k,c) models to simulate and predict the non-homogeneous index sequence. Let \( x^{(0)}(k) = rq^k + \theta, k = 1, 2, \ldots, 12, r > 0 \). For ease of referencing, the following notation is defined:

\[ \varepsilon = |\hat{a} - a| + |\hat{b} - b|, \]

where \( \hat{a} \) and \( \hat{b} \) are approximated parameters of NGM(1,1,k,c) and NDGMs(1,1,k,c) models. In addition, parameters \( a \) and \( b \) are determined using Eq. (34). Table 1 gives results for \( r = 0.06, q = 2.25 \), and \( \theta = 3 \). It can be seen in Table 1 that the maximum APEs for simulation of NGM(1,1,k,c) and NDGMs(1,1,k,c) are 333.8160% and 1.7146 \times 10^{-11}% and those for prediction are 387.9227% and 2.0291 \times 10^{-10}%.
284.3099\%; those for the NDGM\(_5(1,1,k,c)\) are 9.9990 \times 10^{-11}\% \), 2.2228 \times 10^{-10}\% \), and 1.6629 \times 10^{-10}\%, respectively. Clearly, the APEs of the NDGM\(_5(1,1,k,c)\) model are caused by the round-off error of computer, while the APEs of the NGM\((1,1,k,c)\) model are caused by its inconsistency.

Further, we select the parameter \(q\) given at the interval [0.1,5.0] by the step 0.01 and the parameters \(r\) and \(\theta\) randomly generated at the interval [1,15] and [1,5] by the discrete uniform distribution, respectively. Computational results are depicted in Figure 1. According to Figure 1, the maximum \(\varepsilon\) is only 1.8867 \times 10^{-9}, which is obviously a truncation error occurring in the computer program process.

### 4. Applications

In this part, the NDGM\(_5(1,1,k,c)\) model is utilized to predict the primary energy consumption in Saudi Arabia, India, Philippines, and Vietnam. Outcomes are compared to those of discrete DGM\((1,1)\) model, non-homogeneous NGM\((1,1,k)\) model, NGM\((1,1,k,c)\) model, ARIMA, and SVMs.

The raw data of the primary energy consumption belonging to Saudi Arabia, India, Philippines, and Vietnam are announced from the BP Statistical Review of World Energy 2017. These observations are divided into two categories: The observations from 2006 to 2013 utilized to construct different prediction models and the observations from 2014 to 2016 used to verify and differentiate the forecasting results. Raw observation of the primary energy consumption are given in Table 2.

#### 4.1. The primary energy consumption in Saudi Arabia

We first take the NDGM\(_5(1,1,k,c)\) model as an example to explain how to build and calculate the simulation and prediction values. From Table 2, the values of \(X^{(0)}\), \(X^{(1)}\), and \(Z^{(1)}\) of the Saudi Arabia are given below:

\[
X^{(0)} = (164.5, 171.4, 186.9, 196.5, 216.1, 222.2, 235.7, 237.4,)
\]

\[
X^{(1)} = (164.5, 335.9, 522.8, 719.3, 933.4, 1157.6, 1393.3, 1630.7,)
\]

\[
Z^{(1)} = (676.9667, 1048.8, 1445.1333, 1872.8333, 2319.7, 2787.1667,)
\]

It follows from Subsection 3.2 that:

\[
B = \begin{pmatrix}
-676.9667 & 4 & 2 \\
-1048.8 & 6 & 2 \\
-1445.1333 & 8 & 2 \\
-1872.8333 & 10 & 2 \\
-2319.7 & 12 & 2 \\
-2787.1667 & 14 & 2
\end{pmatrix}
\]

\[
Y = \begin{pmatrix}
358.3 \\
383.4 \\
412.6 \\
436.3 \\
457.9 \\
473.1
\end{pmatrix}
\]

The system parameters can be further resolved to:

\[
\begin{pmatrix}
a \\
b \\
c
\end{pmatrix} = (B^T B)^{-1} B^T Y = \begin{pmatrix}
0.1262 \\
38.4235 \\
144.1316
\end{pmatrix}
\]

Moreover, the expression of the NDGM\(_5(1,1,k,c)\) model is:

\[
\hat{x}^{(1)}(k + 1) = (0.8815)^{k-1} \hat{x}^{(1)}(2) + \frac{(-1.0429)^{k-1} - (0.9587)^{k-1}}{1.9193} \times (\hat{x}^{(1)}(2) - 0.8815 \hat{x}^{(1)}(1))
\]

### Table 2. Raw data of the primary energy consumption.

| Year | Saudi Arabia | India | Philippines | Vietnam |
|------|--------------|-------|-------------|---------|
| 2006 | 164.5        | 414.0 | 27.6        | 28.1    |
| 2007 | 171.4        | 470.2 | 26.7        | 30.6    |
| 2008 | 186.9        | 475.7 | 27.6        | 38.2    |
| 2009 | 196.5        | 513.2 | 28.0        | 39.3    |
| 2010 | 216.1        | 537.1 | 28.8        | 44.3    |
| 2011 | 222.2        | 568.7 | 29.5        | 50.3    |
| 2012 | 235.7        | 611.6 | 30.5        | 52.5    |
| 2013 | 237.4        | 621.5 | 32.5        | 54.8    |
| 2014 | 252.1        | 663.6 | 34.4        | 59.8    |
| 2015 | 260.8        | 685.1 | 37.7        | 63.7    |
| 2016 | 266.5        | 723.9 | 42.1        | 64.8    |
\[ + 73.7425 \sum_{i=0}^{k-2} \sum_{m=0}^{k-i-2} (0.8815)^i (-1.0429)^m (k-i-m) \]
\[ + 276.6278 \sum_{i=0}^{k-2} \sum_{m=0}^{k-i-2} (0.8815)^i (-1.0429)^m, \]
\[ k = 1, 2, \ldots, 10. \quad (37) \]

Finally, the values of \( \hat{x}^{(0)}(k) \) are obtained through Eqs. (17) and (37).

Similarly, the expressions of DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), and ARIMA models are provided below:

The DGM(1,1) model:
\[ \hat{x}^{(1)}(k + 1) = 1.0554^k x^{(1)}(1) \]
\[ \quad - \frac{169.0427}{0.0554} (1 - 0.10554^k). \quad (38) \]

The NGM(1,1,k) model:
\[ \hat{x}^{(1)}(k + 1) = (x^{(1)}(1) + 94.0293)e^{-0.7000k} \]
\[ + 229.0920k - 94.0293. \quad (39) \]

The NGM(1,1,k,c) model:
\[ \hat{x}^{(1)}(k + 1) = (x^{(1)}(1) + 842.0723)e^{-0.1472k} \]
\[ + 288.7606k - 1130.8389. \quad (40) \]

The ARIMA model:

\[ (1 - 0.9661B)(1 - B)x^{(0)}(k) = (1 - 0.5342B)x_k. \quad (41) \]

where \( B \) and \( x_k \) are the lag operator and error terms, respectively.

The outcomes of the primary energy consumption in Saudi Arabia are tabulated in Table 3 and Figure 2. The errors are listed in Table 4 and Figure 3.

From Table 3 and Figure 2, we can notice that DGM (1,1), NGM(1,1,k,c), NDGM(1,1,k,c), ARIMA (1,1,1), and SVMs models successfully catch the tendency of the primary energy consumption in Saudi Arabia. The numerical outcomes by the NDGM(1,1,k,c) model are usually closer to the raw data than the outcomes of the other models.

| Year | Data | DGM(1,1) | NGM(1,1,k) | NGM(1,1,k,c) | NDGM(1,1,k,c) | ARIMA(1,1,1) | SVMs |
|------|------|----------|------------|-------------|--------------|--------------|------|
| 2006 | 164.5| 164.5    | 164.5      | 164.5       | 164.5        | 164.5        | 189.2396 |
| 2007 | 171.4| 177.1631 | 97.7947    | 150.9900    | 171.4        | 164.5        | 183.4264 |
| 2008 | 186.9| 186.9856 | 164.4757   | 169.0332    | 185.2058     | 174.3795     | 186.9510 |
| 2009 | 196.5| 197.3527 | 197.2919   | 186.1150    | 201.4061     | 195.1848     | 198.4368 |
| 2010 | 216.1| 208.2945 | 213.4420   | 200.1677    | 211.4817     | 205.0715     | 211.9135 |
| 2011 | 222.2| 219.8430 | 221.3901   | 212.2967    | 224.7475     | 229.1427     | 222.2509 |
| 2012 | 235.7| 232.0318 | 225.3016   | 222.7653    | 231.8629     | 231.8020     | 229.6540 |
| 2013 | 237.4| 244.8963 | 227.2266   | 231.8007    | 242.9132     | 246.6592     | 237.4510 |
| 2014 | 252.1| 258.4741 | 228.1740   | 239.5992    | 247.6754     | 243.9880     | 246.0452 |
| 2015 | 260.8| 272.8047 | 228.6402   | 246.3301    | 257.0601     | 261.9677     | 250.5585 |
| 2016 | 266.5| 287.9290 | 228.8697   | 252.1306    | 259.9228     | 259.8285     | 246.5711 |
Table 4. Errors of the primary energy consumption for Saudi Arabia by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMs(1,1,k,c), ARIMA(1,1,1), and SVMs models.

| Year | DGM(1,1) | NGM(1,1,k) | NGM(1,1,k,c) | NDGMs(1,1,k,c) | ARIMA(1,1,1) | SVMs |
|------|----------|------------|-------------|---------------|--------------|------|
| 2006 | 0        | 0          | 0           | 0             | 15.0333      |
| 2007 | 3.3624   | 42.9436    | 11.9200     | 0             | 4.0257       | 7.0166|
| 2008 | 0.0158   | 11.9980    | 9.1315      | 0.9065        | 6.6990       | 0.0273|
| 2009 | 0.4339   | 0.4030     | 5.2850      | 2.4967        | 0.6093       | 0.9856|
| 2010 | 3.6120   | 1.2299     | 7.3726      | 2.1371        | 5.1334       | 1.9373|
| 2011 | 1.0608   | 0.3645     | 4.4569      | 1.1465        | 3.1245       | 0.0229|
| 2012 | 1.5563   | 4.4117     | 5.4878      | 1.6258        | 1.6538       | 2.5651|
| 2013 | 3.1577   | 4.2853     | 2.3866      | 2.2323        | 3.9003       | 0.0215|
| 2014 | 2.5284   | 9.4907     | 4.987       | 1.7551        | 3.2174       | 2.4017|
| 2015 | 4.0300   | 12.3312    | 5.5843      | 1.4340        | 0.1477       | 3.9270|
| 2016 | 8.0112   | 14.1202    | 3.3885      | 2.4860        | 1.2490       | 7.4780|

MAPE_{simu} = 1.8898 \quad MAPE_{pred} = 5.0576 \quad MAPE_{over} = 2.8402

As shown in Table 4, MAPE_{pred} and MAPE_{over} for the NDGMs(1,1,k,c) model are 1.8857% and 1.6292%; those for the DGM(1,1) model are 5.0576% and 2.8402%; those for the NGM(1,1,k) model are 11.9807% and 10.1578%; those for the NGM(1,1,k,c) model are 5.2985% and 6.1908%; those for the ARIMA model are 1.6380% and 3.0090%; and those for the SVM model are 4.6022% and 2.6383%, respectively.

It can be concluded that the new model exceeds other models in this application.

4.2. The primary energy consumption of India

This subsection studies the performance of DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMs(1,1,k,c), ARIMA, and SVMs models in predicting the primary energy consumption in India. Computation results and raw data are shown in Tables 5 and 6 and Figures 4 and 5.

It can be seen in Table 6 that MAPE_{simu}, MAPE_{pred}, and MAPE_{over} for the DGM(1,1) are 1.0430%, 2.5142%, and 1.4844%; those for the NGM(1,1,k) model are 9.9016%, 14.6999%, and 11.3411%; those for the NGM(1,1,k,c) model are 3.8010%, 3.8784%, and 3.8242%; those for the NDGMs(1,1,k,c) model are 0.7936%, 0.7412%, and 0.7778%; those for the ARIMA model are 3.0268%, 2.9249%, and 2.9962%; and those for the SVMs are 1.4642%, 5.1290%, and 2.5633%, respectively.

According to Tables 5 and 6 as well as Figures 4 and 5, the predicted values from the NDGMs(1,1,k,c) model are closer to raw samples than other prediction models. The computation results illustrate that the
Table 5. Computational results of the primary energy consumption for India by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMs(1,1,k,c), ARIMA(1,1,1), and SVMs models.

| Year | DGM(1,1) | NGM(1,1,k) | NGM(1,1,k,c) | NDGMs(1,1,k,c) | ARIMA(1,1,1) | SVMs   |
|------|----------|------------|--------------|----------------|---------------|--------|
| 2006 | 414      | 414        | 414          | 414            | 414           | 482.4519 |
| 2007 | 450.2    | 454.3107   | 253.8983     | 425.7961       | 450.2         | 471.0947 |
| 2008 | 475.7    | 480.1950   | 422.8578     | 458.4002       | 475.8445      | 485.5684 |
| 2009 | 513.2    | 507.5540   | 506.8931     | 490.1910       | 512.0522      | 498.6237 |
| 2010 | 537.1    | 536.4717   | 548.6897     | 521.1506       | 537.7097      | 541.6501 |
| 2011 | 568.7    | 567.6371   | 569.4781     | 551.3010       | 573.9289      | 559.3171 |
| 2012 | 611.6    | 599.3139   | 579.8176     | 580.6631       | 599.5994      | 593.5762 |
| 2013 | 621.5    | 633.4914   | 584.9602     | 609.2577       | 635.8302      | 617.6630 |
| 2014 | 663.6    | 669.5845   | 587.5180     | 637.1048       | 661.5137      | 633.6752 |
| 2015 | 685.1    | 707.7339   | 588.7901     | 664.2239       | 697.7562      | 697.1583 |
| 2016 | 723.9    | 748.0560   | 589.4228     | 690.6341       | 723.4526      | 705.7656 |

Table 6. Errors of the primary energy consumption for India by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMs(1,1,k,c), ARIMA(1,1,1), and SVMs models.

| Year | DGM(1,1) | NGM(1,1,k) | NGM(1,1,k,c) | NDGMs(1,1,k,c) | ARIMA(1,1,1) | SVMs    |
|------|----------|------------|--------------|----------------|---------------|---------|
| 2006 | 0.0931   | 43.9032    | 5.4296       | 0              | 8.4049        | 4.6412  |
| 2007 | 0.9449   | 11.1083    | 3.6367       | 0.0304         | 0.4214        | 2.0745  |
| 2008 | 1.1002   | 1.2289     | 4.4834       | 0.2377         | 2.8403        | 0.3032  |
| 2009 | 0.1169   | 2.1578     | 2.9695       | 0.1135         | 1.4057        | 0.7519  |
| 2010 | 0.2924   | 0.3368     | 3.0594       | 0.9195         | 1.6499        | 0.0272  |
| 2011 | 2.0039   | 5.1966     | 5.0583       | 1.9622         | 2.6200        | 2.6993  |
| 2012 | 1.9294   | 5.8793     | 1.9658       | 2.3058         | 4.2097        | 0.0250  |
| 2013 | 0.9018   | 11.4650    | 3.9926       | 0.3144         | 4.5095        | 2.2821  |
| 2014 | 3.3037   | 14.0578    | 3.0472       | 1.8473         | 1.7601        | 3.2010  |
| 2015 | 3.3370   | 18.5768    | 4.5954       | 0.0618         | 2.5051        | 9.9006  |
| 2016 | MAPE_{train} | 1.0430   | 9.9016       | 3.8010         | 0.7936        | 3.0268  | 1.4642  |
|      | MAPE_{pred} | 2.5412    | 14.6990      | 3.8784         | 0.7412        | 2.9219  | 5.1280  |
|      | MAPE_{test} | 1.4844    | 11.3411      | 3.8242         | 0.7778        | 2.9962  | 2.5633  |

NDGMs(1,1,k,c) model outperforms the DGM(1,1), GNM(1,1,k), NGM(1,1,k,c), ARIMA(1,1,1), and SVMs models; in addition, the NGM(1,1,k) has the worst performance.

4.3. The primary energy consumption of the Philippines

The simulation and forecasting results of the primary energy consumption of the Philippines are tabulated in Table 7 and Figure 6, while the errors are tabulated in Table 8 and Figure 7.

From Table 7 and Figure 6, we can notice that DGM(1,1), NGM(1,1,k), NDGMs(1,1,k,c), ARIMA(2,1,2), and SVMs models successfully catch the tendency of the primary energy consumption in the Philippines. The numerical results of the NDGMs(1,1,k,c) model are closer to the raw data than the results of the other models.

As shown in Table 8, MAPE_{train}, MAPE_{pred}, and MAPE_{test} for the DGM(1,1) model are 1.1396%, 10.6679%, and 3.9851%; those for the NGM(1,1,k) model are 9.6444%, 21.0239%, and 13.0582%; those for the NGM(1,1,k,c) model are 30.9143%, 76.0195%, and 44.4459%; those for the NDGMs(1,1,k,c) model are 0.8213%, 3.2315%, and 1.5443%; those for the ARIMA model are 3.0352%, 3.2165%, and 3.0896%; and those for the SVMs model are 0.3125%, 4.7765%, and 1.6517%, respectively. Obviously, according to
Table 8 and Figure 7, the proposed model outperforms the other models in the case.

4.4. The primary energy consumption in Vietnam

This subsection studies the performance of DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMS(1,1,k,c), ARIMA, and SVMs models in predicting the primary energy consumption in Vietnam. Computation results and raw data are shown in Tables 9 and 10 as well as Figures 8 and 9.

As can be seen in Table 10, the MAPE_{simu}, MAPE_{pred}, and MAPE_{error} for the DGM(1,1) are 3.7971%, 8.4716%, and 5.1995%; those for the NGM(1,1,k) model are 10.3366%, 14.0015%, and 11.4631%; those for the NGM(1,1,k,c) model are 7.4440%, 6.7516%, and 7.2963%; those for the NDGMS(1,1,k,c) model are 2.1315%, 3.1512%, and 2.4374%; those for the ARIMA(1,1,2) model are 10.6703%, 9.0352%, and 10.1798%; and those of the SVMs are 4.0866%, 3.6375%, and 3.9519%, respectively. Based on Tables 9 and 10 as well as Figures 8 and 9, the predicted values by the NDGMS(1,1,k,c) model are much closer to the raw data than the other models. The computation results illustrate that the NDGMS(1,1,k,c) model exceeds the DGM(1,1), GN(1,1,k), NGM(1,1,k,c), ARIMA(1,1,1), and SVM models; besides, the NGM(1,1,k) has the bad performance.

4.5. Discussions and suggestions

The primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam is systematically discussed in this paper by using the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMS(1,1,k,c), ARIMA, and SVM models and based on the actual data from 2006.
Table 7. Computational results of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM_{s}(1,1,k,c), ARIMA(2,1,2), and SVMs models.

| Year | Data | DGM (1,1) | NGM (1,1,k) | NGM (1,1,k,c) | NDGM_{s} (1,1,k,c) | ARIMA (2,1,2) | SVMs |
|------|------|-----------|-------------|--------------|--------------------|---------------|------|
| 2006 | 25.6 | 25.6      | 25.6        | 25.6         | 25.6               | 25.6         | 26.5284 |
| 2007 | 26.7 | 26.4819   | 14.9267     | 30.7052      | 26.7               | 25.6         | 26.7165 |
| 2008 | 27.6 | 27.3055   | 25.0760     | 32.2316      | 27.7639            | 25.9164      | 27.5835 |
| 2009 | 28.0 | 28.1546   | 28.3283     | 34.2071      | 27.7091            | 26.6613      | 28.2206 |
| 2010 | 28.8 | 29.0302   | 29.3705     | 36.7638      | 28.9468            | 27.6595      | 28.7835 |
| 2011 | 29.5 | 29.9329   | 29.7045     | 40.0728      | 29.3329            | 28.9488      | 29.4929 |
| 2012 | 30.5 | 30.8638   | 29.8116     | 41.3552      | 30.9366            | 30.3619      | 30.5465 |
| 2013 | 32.5 | 31.8236   | 29.8459     | 49.8976      | 31.9773            | 32.5095      | 32.1285 |
| 2014 | 34.4 | 32.8133   | 29.8568     | 57.0707      | 34.2522            | 35.3030      | 34.4616 |
| 2015 | 37.7 | 33.8337   | 29.8604     | 66.3512      | 36.3111            | 38.8936      | 36.6577 |
| 2016 | 42.1 | 34.8870   | 29.8615     | 78.3689      | 39.7505            | 43.7219      | 37.2514 |

Table 8. Errors of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM_{s}(1,1,k,c), ARIMA(2,1,2), and SVMs models.

| Year | DGM (1,1) | NGM (1,1,k) | NGM (1,1,k,c) | NDGM_{s} (1,1,k,c) | ARIMA (2,1,2) | SVMs |
|------|-----------|-------------|--------------|--------------------|---------------|------|
| 2006 | 0.8168    | 44.0947     | 15.0007      | 0                  | 4.1199        | 0.0618 |
| 2007 | 1.0672    | 9.1451      | 16.7812      | 0.5037             | 6.1000        | 0.0598 |
| 2009 | 0.5521    | 1.1736      | 22.1682      | 1.0389             | 4.7812        | 0.7879 |
| 2010 | 0.7901    | 1.9811      | 27.6522      | 0.5096             | 3.9601        | 0.0572 |
| 2011 | 1.4676    | 0.6033      | 35.8309      | 0.5666             | 1.8684        | 0.0230 |
| 2012 | 1.1928    | 2.5722      | 45.4270      | 1.4316             | 0.2031        | 0.0540 |
| 2013 | 2.0812    | 8.1666      | 53.5312      | 1.6084             | 0.2337        | 1.1430 |
| 2014 | 4.6126    | 13.2009     | 65.9032      | 0.4296             | 2.6309        | 0.0480 |
| 2015 | 10.2555   | 20.7948     | 76.0057      | 3.6840             | 3.1662        | 2.7647 |
| 2016 | 17.1357   | 29.0701     | 86.1494      | 5.5009             | 3.8525        | 11.569 |
| MAPE_{error} | 1.1396 | 9.6444 | 30.9143 | 0.8213 | 3.0352 | 0.3125 |
| MAPE_{pred}    | 10.0679 | 21.0229 | 76.0195 | 3.2315 | 3.2165 | 4.7765 |
| MAPE_{error}   | 3.9981  | 13.0582 | 44.4459 | 1.5443 | 3.0896 | 1.6517 |

Figure 7. Error values of the primary energy consumption for Philippines by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGM_{s}(1,1,k,c), ARIMA(1,1,1), and SVMs models.
Table 9. Computational results of the primary energy consumption for Vietnam by DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMS(1,1,k,c), ARIMA(1,1,2), and SVMs models.

| Year | Data | DGM(1,1) | NGM(1,1,k) | NGM(1,1,k,c) | NDGMS(1,1,k,c) | ARIMA(1,1,2) | SVMs |
|------|------|----------|------------|-------------|---------------|--------------|------|
| 2006 | 28.1 | 28.1     | 28.1       | 28.1        | 28.1          | 28.1         | 38.7583 |
| 2007 | 30.6 | 33.4598  | 18.7478    | 26.9497     | 30.6          | 28.1         | 37.5079 |
| 2008 | 38.2 | 36.5603  | 30.8373    | 32.5806     | 37.8519       | 33.2028      | 38.1755 |
| 2009 | 39.3 | 39.9482  | 38.9456    | 37.6267     | 39.3694       | 43.6051      | 40.3060 |
| 2010 | 44.3 | 43.6500  | 44.3837    | 42.1488     | 46.2959       | 35.4189      | 44.3244 |
| 2011 | 50.3 | 47.6948  | 48.0309    | 46.2012     | 47.3175       | 53.7805      | 49.0752 |
| 2012 | 52.5 | 52.1145  | 50.4770    | 49.8328     | 53.9123       | 47.6229      | 52.4755 |
| 2013 | 51.8 | 56.9437  | 52.1176    | 53.0872     | 54.4497       | 58.2137      | 55.2775 |
| 2014 | 59.8 | 62.2204  | 53.2180    | 56.0036     | 60.7831       | 52.2579      | 58.9776 |
| 2015 | 63.7 | 67.9860  | 53.9559    | 58.6171     | 60.8705       | 68.3572      | 61.6825 |
| 2016 | 64.8 | 74.2860  | 54.4509    | 60.9503     | 66.9822       | 60.1439      | 60.6723 |

Table 10. Errors of the primary energy consumption for Vietnam by the DGM(1,1), NGM(1,1,k), NGM(1,1,k,c), NDGMS(1,1,k,c), ARIMA(1,1,2), and SVMs models.

| Year | DGM(1,1) | NGM(1,1,k) | NGM(1,1,k,c) | NDGMS(1,1,k,c) | ARIMA(1,1,2) | SVMs |
|------|----------|------------|-------------|---------------|--------------|------|
| 2006 | 0        | 0          | 0           | 0             | 0            | 0    | 37.9298 |
| 2007 | 9.3457   | 38.7327    | 11.9291     | 0             | 8.1699       | 22.5747 |
| 2008 | 4.2923   | 19.2340    | 14.7104     | 0.9112        | 13.0817      | 0.0640 |
| 2009 | 1.6941   | 0.9018     | 4.2577      | 0.2452        | 10.9546      | 2.5078 |
| 2010 | 1.4673   | 0.1889     | 4.8560      | 4.5054        | 20.0475      | 0.0552 |
| 2011 | 5.1792   | 4.5111     | 8.1487      | 5.9294        | 6.9195       | 2.4350 |
| 2012 | 0.7343   | 3.8532     | 5.0804      | 2.6901        | 9.2897       | 0.4666 |
| 2013 | 3.9118   | 4.8948     | 3.1256      | 0.6393        | 6.2294       | 0.8714 |
| 2014 | 4.0474   | 11.0068    | 6.3485      | 1.6440        | 12.6123      | 1.3752 |
| 2015 | 6.7285   | 15.2568    | 7.7974      | 4.4419        | 7.3111       | 3.1673 |
| 2016 | 14.6388  | 15.9709    | 5.9271      | 3.3676        | 7.1823       | 6.3700 |
| MAPE_{intra} | 3.7971 | 10.3366    | 7.4440      | **21315**     | 10.6703      | 4.0866 |
| MAPE_{pred}   | 8.4716 | 14.0915    | 6.7516      | **31512**     | 9.0352       | 3.6375 |
| MAPE_{over}   | 5.1995 | 11.4631    | 7.2363      | **24374**     | 10.1798      | 3.9519 |

to 2016. The computational results show that the NDGMS(1,1,k,c) model outperforms the other prediction models in primary energy consumption.

The *BP Statistical Review of World Energy* states that the energy mix inches towards cleaner, lower carbon fuels determined by the environment needs and the technological progress. This result points out that the growth of worldwide primary energy consumption remained low in 2016. This growth is below average in all states except Europe (Saudi Arabia) & Eurasia (India, Philippines, Vietnam). As known, the fossil energy can produce harmful gases that pollute the environment and lead to ecological problems. Moreover, the government will play its role in meeting the dual challenge of supplying the energy for the nation’s needs to grow and prosper and reducing carbon emissions. The mentioned entity should reduce the traditional energy consumption and greatly increase clean energy consumption in the future. We hope that our computational results can provide a guidance for the government to formulate and adjust energy policies.
5. Conclusions

This research study investigated the discrete NDGM$(1,1,k,c)$ model with Simpson formula. Mathematical analysis was carried out to determine the properties of the proposed model. Further, the primary energy consumption for Saudi Arabia, India, Philippines, and Vietnam was carried out to verify the performance of our model with the DGM $(1,1)$, NGM$(1,1,k)$, NGM$(1,1,k,c)$, Auto-Regressive Integrated Moving Average (ARIMA), and the Support Vector Machines (SVMs) models. The results showed that the new NDGM$(1,1,k,c)$ model had high potential in the primary energy consumption with higher accuracy than the other models.

It needs to be pointed out that the GM$(1,1)$, DGM$(1,1)$, and their generalized models are homogeneous exponential models. However, it is difficult to meet data sequences with the significant growth of homogeneous exponent in real situations. This result illustrates that the homogeneous exponent models are inapplicable. According to the analysis of the NDGM$(1,1,k,c)$ model, it is known that the new model can be used as either a homogeneous exponent model or an non-homogenous model, which has a wide range of applications in the real world. Moreover, the proposed model is suitable for simulation and prediction data sequences with only a few samples (not less than four). However, the time series analysis and the computational intelligence technology require a large amount of data. It is sometimes impossible to get as many as observed samples in the real world.

In the future, the new NDGM$(1,1,k,c)$ model can be used for data forecasting such as nuclear energy consumption, the production of shale gas, etc. Further, the method for the NDGM$(1,1,k,c)$ model can be applied to analyze other grey models such as GM$(1,n)$ or Verhulst models.

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