CMB spectroscopy at third order in cosmological perturbations

Atsuhsa Ota (DAMTP) with Nicola Barltro (Padva U)
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Linear CMB was successful

Density fluctuations are
• Almost scale invariant
• Adiabatic
• Gaussian
Small scales are not observed!
Neither is non-Gaussianity (Bispectrum)
Small scales are not observed!

Neither is non-Gaussianity (Bispectrum)
We know very little on $k > 1 \text{ Mpc}^{-1}$
How can we observe small scales?
CMB spectral distortions
COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE

Theory and observation agree

Intensity, 10^-4 ergs/cm² sr sec cm⁻¹

Waves / centimeter

http://lambda.gsfc.nasa.gov/product/cobe/cobe_image_table.cfm
Spectral distortions

Deviations from the isotropic Planck distributions.
Deviations from Planck

Spectral distortions

Deviations from the isotropic Planck distributions.

Compton y

Chemical potential \( \mu \)

200

400

600

800

1000

1200

GHz

www

GSPN1MBODL

http://lambda.gsfc.nasa.gov/product/cobe/cobe_image_table.cfm
Spectral distortions

Deviations from the isotropic Planck distributions.
Spectral distortions and energy release in the early Universe

\[
\mu = 1.4 \frac{\Delta \rho}{\rho} \quad | \quad 5 \times 10^4 < z < 2 \times 10^6 \\
y = \frac{1}{4} \frac{\Delta \rho}{\rho} \quad | \quad z < 5 \times 10^4
\]

[Sunyaev+1970, Daly1991, Hu+1994]
• Acoustic damping
• Annihilation of darkmatter
• PBH evapoation
• etc...
\[
\frac{\Delta \rho}{\rho} \bigg|_{\text{Silk}} = 6 \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle \sim \langle \zeta^2 \rangle
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CMB at second order!

Short mode \times Short mode

→ homogenous mode

Khatri Sunyaev (2013)
Summary at second order

• COBE FIRAS constraints on $0.1 < k \text{ Mpc} < 10^4$

\[ \mu, y < 10^{-5} \iff \mathcal{P}_\zeta < 10^{-5} \]

• **Complementary constraints** on density perturbations.

• Future measurements: $\mu < 10^{-8}, y < 10^{-9}$
CMB at third order
→ Bispectrum?
Boltzmann equation

\[
\frac{\partial f_\gamma}{\partial t} + \frac{dx}{dt} \frac{\partial f_\gamma}{\partial x} + \frac{dp_\gamma}{dt} \frac{\partial f_\gamma}{\partial p_\gamma} = \int_{p'_\gamma, p_e, p'_e} \delta^{(3)}(p_{\text{tot.}}) \sum_{\text{spins}} |M|^2 \times \left[ f'_\gamma f'_e (1 + f_\gamma) (1 - f_e) - f_\gamma f_e (1 + f'_\gamma) (1 - f'_e) \right]
\]

Expand this up to **third order** in perturbations...
$n^{th}$ order Boltzmann equations

If $T_e/m_e \ll 1$, then derivative operators in the Compton scattering collision terms are $(p\partial/p)^n$

[AO 2016]
3rd order Boltzmann equations

Up to third order we only have

\[(p\partial/\partial p)^1, (p\partial/\partial p)^2, (p\partial/\partial p)^3\]

- Ansatz can be written as some derivatives of Planck dist.
- No need to follow full phase space evolution.
Solution at third order

Sum of $\delta T/T$, $y$, and $\kappa$ (ignore $\mu$ for simplicity)
Equation to solve

- Simple even at third order!

\[
\frac{d\langle\langle \kappa \rangle\rangle}{d\eta} = -2\langle\langle yA \rangle\rangle
\]

First order temperature collision term:

\[
\dot{\tau}^{-1} A = \frac{\Theta_{00}}{\sqrt{4\pi}} - \Theta + V + \frac{1}{10} \sum_{m=-2}^{2} Y_{2m} \Theta_{2m},
\]

[See textbook by Dodelson]
Results

• For scale invariant perturbations (local):

\[ \kappa \approx 10^{-18} f_{NL}^{loc} \left( \frac{y}{4.0 \times 10^{-9}} \right) \]

• \( \kappa < 10^{-9} \) in Future space mission.

• Too small (?)
Results

• Normalization of nonlinear parameter:

\[ f_{\text{NL}} \sim \frac{\langle \zeta^3 \rangle}{\langle \zeta^2 \rangle^2} \]

• No one knows the power on small scales

\[ 10^{-18} f_{\text{NL}}^{\text{loc.}} \left( \frac{y}{4.0 \times 10^{-9}} \right) \rightarrow \langle \zeta^2 \rangle^2 f_{\text{NL}}^{\text{loc.}}. \]
Forecast in $P_\zeta$-$f_{\text{NL}}$ plane

\[ f_{\text{NL}}^{\text{loc.}} \langle \zeta^2 \rangle^2 < 10^{-10} \]
Summary

• Third order distortion arises from non-Gaussianity.

• It is small for scale invariant perturbations.

• But, no one knows both bispectrum and powerspectrum on small scales.

• non-Gaussianity can be constrained for bigger powerspectrum.
Future works

• Are cluster contaminations (relativistic SZ effects) crucial?

• What about equilateral NG?
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\[ k_1 = k_2 = k_3 = 1 \text{ to } 100 \text{ Mpc}^{-1} \]