Conformal theory of everything

F. F. Faria*
Centro de Ciências da Natureza,
Universidade Estadual do Piauí,
64002-150 Teresina, PI, Brazil

Abstract

By using conformal symmetry we unify the standard model of particle physics with gravity in a consistent quantum field theory which describes all the fundamental particles and forces of nature.

PACS numbers: 104.60.-m, 98.80.-k, 04.50.+h

* felfrafar@hotmail.com
1 Introduction

It is well known that the standard model (SM) of particle physics is consistent with the experiments performed so far on particle accelerators such as the large hadron collider (LHC). However, the theory presents some problems such as the hierarchy and the Landau pole problems. Several modifications of the SM at scales between the electroweak and Planck scales, such as GUT [1, 2, 3, 4] or SUSY [5, 6, 7, 8, 9, 10, 11, 12, 13], have been proposed to solve such problems. However, it is likely that there is no new physics beyond the SM all the way up to the Planck scale [14]. This leads us to suppose that the SM is a low energy limit of a fundamental theory defined at the Planck scale. Since gravitational effects are expected to be important around the Planck scale, it is natural to conjecture that this fundamental theory includes quantum gravity.

One of the most straightforward ways to extend and unify physical theories is to change their symmetries. A strong candidate to be incorporated in the unification of the SM with gravity is the (local) conformal symmetry, which perform a multiplicative rescaling of all fields according to

\[ \tilde{\Phi} = \Omega(x)^{-\Delta_\Phi} \Phi, \]  

where \( \Omega(x) \) is an arbitrary function of the spacetime coordinates, and \( \Delta_\Phi \) is the scaling dimension of the field \( \Phi \), whose values are \(-2\) for the metric field, \(0\) for gauge bosons, \(1\) for scalar fields and \(3/2\) for fermions.

The consideration of the conformal symmetry as one of the fundamental symmetries of physics is justified for several reasons. First of all, the classical SM action and the power spectrum of the cosmic fluctuations are approximately conformal invariant. Second, the conformal symmetry might play an important role in the solution of both the hierarchy and the Landau pole problems [15]. In addition, the black hole complementarity principle can be explained with the use of the conformal symmetry [16]. Last but not least, quantum field theories in curved spacetime (for a nice review, see [17]) may become asymptotically conformally invariant in the limit of a strong gravitational field [18].

The aim of this work is to consistently unify the SM with gravity through the use of conformal symmetry. In Section 2 we describe a consistent conformal theory of quantum gravity, called massive conformal gravity (MCG). In Section 3 we develop an extended SM conformally coupled with gravity. In Section 4 we combine MCG with the extended SM to form what we call
conformal theory of everything (CTOE). Finally, in Section 5 we present our conclusions.

2 Gravity

The conformally invariant theory of gravity that we consider here is MCG, whose action is given by

\[ S_{\text{MCG}} = \int d^4 x \sqrt{-g} \left[ \varphi^2 R + 6 \partial^\mu \varphi \partial_\mu \varphi - \frac{1}{2\alpha^2} C^2 - \eta E \right], \]  

(2)

where \( \varphi \) is a scalar field called dilaton, \( \alpha \) and \( \eta \) are dimensionless coupling constants,

\[ C^2 = C^{\alpha\beta\mu\nu} C_{\alpha\beta\mu\nu} = E + 2W \]  

(3)

is the Weyl tensor squared, \( E = R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 \) is the Euler density, \( W = R^{\mu\nu} R_{\mu\nu} - (1/3) R^2 \), \( R^{\alpha\mu\beta\nu} \) is the Riemann tensor, \( R_{\mu\nu} = R^{\alpha\mu\alpha\nu} \) is the Ricci tensor, and \( R = g^{\mu\nu} R_{\mu\nu} \) is the scalar curvature. Considering that \( \sqrt{-g} E \) is a total derivative, and integrating by parts, we can write (2) as

\[ S_{\text{MCG}} = \int d^4 x \sqrt{-g} \left[ \varphi^2 R - 6 \varphi \Box \varphi - \frac{1}{\alpha^2} W \right], \]  

(4)

where \( \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu \) is the generally covariant d’Alembertian.

The variation of (4) with respect to \( g^{\mu\nu} \) and \( \varphi \) gives the field equations

\[ \varphi^2 G_{\mu\nu} + 6 \partial_\mu \varphi \partial_\nu \varphi - 3 g_{\mu\nu} \partial^\rho \varphi \partial_\rho \varphi - g_{\mu\nu} \Box \varphi^2 - \nabla_\mu \nabla_\nu \varphi^2 - \alpha^{-2} B_{\mu\nu} = 0, \]  

(5)

\[ \varphi R - 6 \Box \varphi = 0, \]  

(6)

where

\[ B_{\mu\nu} = \nabla^\alpha \nabla^\beta C_{\alpha\mu\nu\beta} - \frac{1}{2} R^{\alpha\beta} C_{\alpha\mu\nu\beta} \]  

(7)

is the Bach tensor, and

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \]  

(8)

is the Einstein tensor.

\footnote{This action is obtained from the action of Ref. \[19\] by the reparametrizations \( \beta \lambda^{-2}/2k_c \rightarrow 1 \) and \( \alpha/k_c \rightarrow 1/\alpha^2 \). Additionally, we included in (2) the Gauss-Bonnet term \( \int d^4 x \sqrt{-g} E \) in order to provide renormalizability.}
In addition to the conformally invariant field equations (5) and (6), MCG also has conformally invariant line element
\[ ds^2 = (\varphi/\varphi_c)^2 g_{\mu\nu} dx^\mu dx^\nu, \] (9)
and geodesic equation
\[ \frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} + \frac{1}{\varphi} \varphi \frac{\partial \varphi}{\partial x^\rho} \left( g^\lambda^\rho + \frac{dx^\lambda}{d\tau} \frac{dx^\rho}{d\tau} \right) = 0, \] (10)
where \( \varphi_c \) is a constant scalar field and
\[ \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda^\rho} \left( \partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu} \right) \] (11)
is the Levi-Civita connection. Although the theory has not yet been fully tested with classical tests\(^2\), only its quantum behavior, which will be addressed in more detail next, is relevant to this work.

### 2.1 Renormalizability

By using the flat background field expansions
\[ g_{\mu\nu} = \eta_{\mu\nu} + \alpha h_{\mu\nu}, \quad \varphi = \varphi_c + \sigma, \] (12)
and keeping only the terms of second order in the quantum fields \( h^{\mu\nu} \) and \( \sigma \), we find that (11) reduces to the linearized action
\[ \bar{S}_\text{MCG} = \int d^4x \left[ m^2 \bar{\mathcal{L}}_{\text{EH}} + 2m\sigma \bar{R} - 6\sigma \Box \sigma - \left( \bar{R}^{\mu\nu} \bar{R}_{\mu\nu} - \frac{1}{3} \bar{R}^2 \right) \right], \] (13)
where \( m = \alpha \varphi_c \) is the graviton mass,
\[ \bar{\mathcal{L}}_{\text{EH}} = -\frac{1}{4} \left( \partial^\mu h^{\nu\rho} \partial_{\rho} h_{\mu\nu} - 2\partial^\mu h^{\nu\rho} \partial_{\nu} h_{\rho\mu} + 2\partial^\mu h_{\mu\nu} \partial^\nu h - \partial^\mu h \partial_{\mu} \bar{R} \right) \] (14)
is the linearized Einstein-Hilbert Lagrangian density,
\[ \bar{R}_{\mu\nu} = \frac{1}{2} \left( \partial_\mu \partial^\nu h_{\rho\mu} + \partial_\nu \partial^\rho h_{\mu\rho} - \Box h_{\mu\nu} + \partial_\mu \partial^\rho h_{\rho\nu} \right) \] (15)
\( ^2\)It has been shown so far that MCG is consistent with solar system observations\([20]\), has no vDVZ discontinuity\([21]\), can reproduce the orbit of binaries by the emission of gravitational waves\([22]\) and describes the late universe without the cosmological constant problem\([23]\).
is the linearized Ricci tensor, and
\[
\bar{R} = \partial^\mu \partial^\nu h_{\mu\nu} - \Box h
\]  
(16)
is the linearized scalar curvature, with \( \Box = \eta^{\mu\nu} \partial_\mu \partial_\nu \) and \( h = \eta^{\mu\nu} h_{\mu\nu} \).

The linearized action (13) is invariant under the coordinate gauge transformation
\[
h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \chi_\nu + \partial_\nu \chi_\mu,
\]  
(17)
where \( \chi^\mu \) is an arbitrary spacetime dependent vector field, and under the conformal gauge transformations
\[
h_{\mu\nu} \to h_{\mu\nu} + \eta_{\mu\nu} \Lambda, \quad \sigma \to \sigma - \frac{1}{2} \Lambda,
\]  
(18)
where \( \Lambda \) is an arbitrary spacetime dependent scalar field. As usual, these gauge symmetries require the addition of gauge fixing and Faddeev–Popov ghost terms.

In order to no part of the graviton propagator to fall off slower than \( p^{-4} \) for large momenta, we choose the gauge fixing terms
\[
\bar{S}_{GF1} = -\frac{1}{2\xi_1} \int d^4x \left( m^2 - \Box \right) \left( \partial^\mu h_{\mu\nu} - \frac{1}{2} \partial_\nu h \right)^2,
\]  
(19)\[
\bar{S}_{GF2} = \frac{1}{6\xi_2} \int d^4x \left( \bar{R} - 6\xi_2 m\sigma \right)^2,
\]  
(20)
and its corresponding Faddeev–Popov ghost terms
\[
\bar{S}_{FP1} = \int d^4x \bar{c}^\mu \left( \Box - m^2 \right) \Box c_\mu, \quad \bar{S}_{FP2} = 2 \int d^4x \bar{c} \left( \Box - \xi_2 m^2 \right) c,
\]  
(21)\( (22)\)
where \( \xi_1 \) and \( \xi_2 \) are coordinate and conformal gauge fixing parameters, \( (\bar{c}^\mu)c^\mu \) is a vector (anti-ghost)ghost field, and \( (\bar{c})c \) is a scalar (anti-ghost)ghost field.
Integrating by parts, and performing a long but straightforward calculation, we can write
\[ S = S_{\text{MCG}} + S_{\text{GF1}} + S_{\text{G2}} + S_{\text{FP1}} + S_{\text{FP2}} \]
in the form
\[
S = -\int d^4x \left\{ \frac{1}{4} h^{\mu\nu} \left[ \left( \square - m^2 \right) \square P_{\mu\nu,\alpha\beta}^{(2)} - \frac{2}{\xi_2} \left( \square - \xi^2 m^2 \right) \square P_{\mu\nu,\alpha\beta}^{(0-s)} \right.ight.
\]
\[
+ \left. \frac{1}{2\xi_1} \left( \square - m^2 \right) \square \left( 2P_{\mu\nu,\alpha\beta}^{(1)} + 3P_{\mu\nu,\alpha\beta}^{(0-s)} - \sqrt{3} \left( P_{\mu\nu,\alpha\beta}^{(0-sw)} - P_{\mu\nu,\alpha\beta}^{(0-ws)} \right) \right) \right. 
\]
\[
+ \left. P_{\mu\nu,\alpha\beta}^{(0-w)} \right] h^{\alpha\beta} + 6\sigma \left( \square - \xi^2 m^2 \right) \sigma - \tilde{c}^\mu \left( \square - m^2 \right) \tilde{c}_\mu
\]
\[-2\tilde{c} \left( \square - \xi^2 m^2 \right) c \right\},
\]
(23)
where
\[
P_{\mu\nu,\alpha\beta}^{(2)} = \frac{1}{2} \left( \theta_{\mu\alpha} \theta_{\nu\beta} + \theta_{\mu\beta} \theta_{\nu\alpha} \right) - \frac{1}{3} \theta_{\mu\nu} \theta_{\alpha\beta},
\]
(24)
\[
P_{\mu\nu,\alpha\beta}^{(1)} = \frac{1}{2} \left( \theta_{\mu\alpha} \omega_{\nu\beta} + \theta_{\mu\beta} \omega_{\nu\alpha} + \theta_{\nu\alpha} \omega_{\mu\beta} + \theta_{\nu\beta} \omega_{\mu\alpha} \right),
\]
(25)
\[
P_{\mu\nu,\alpha\beta}^{(0-s)} = \frac{1}{3} \theta_{\mu\nu} \theta_{\alpha\beta},
\]
(26)
\[
P_{\mu\nu,\alpha\beta}^{(0-w)} = \omega_{\mu\nu} \omega_{\alpha\beta},
\]
(27)
\[
P_{\mu\nu,\alpha\beta}^{(0-sw)} = \frac{1}{\sqrt{3}} \theta_{\mu\nu} \omega_{\alpha\beta},
\]
(28)
\[
P_{\mu\nu,\alpha\beta}^{(0-ws)} = \frac{1}{\sqrt{3}} \omega_{\mu\nu} \theta_{\alpha\beta},
\]
(29)
are the spin projectors, with \( \theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu} \) and \( \omega_{\mu\nu} = \partial_\mu \partial_\nu / \square^2 \) being the transverse and longitudinal projectors.

By inverting the kinetic matrix of \( h^{\mu\nu} \) shown in (23) and going over to momentum space, we obtain the MCG graviton propagator
\[
D_{\mu\nu,\alpha\beta} = -i \begin{cases} 
2P_{\mu\nu,\alpha\beta}^{(2)}(p) & + \xi_1 \left[ 2P_{\mu\nu,\alpha\beta}^{(1)}(p) + 4P_{\mu\nu,\alpha\beta}^{(0-w)}(p) \right] \\
p^2 \left( p^2 + m^2 \right) & + \xi_2 \left[ P_{\mu\nu,\alpha\beta}^{(0-s)}(p) + \sqrt{3} \left( P_{\mu\nu,\alpha\beta}^{(0-sw)}(p) + 3P_{\mu\nu,\alpha\beta}^{(0-ws)}(p) \right) + 3P_{\mu\nu,\alpha\beta}^{(0-w)}(p) \right] \\
p^2 \left( p^2 + \xi^2 m^2 \right) & 
\end{cases}
\]
(30)
The $p^{-4}$ behavior of all terms of (30) at high energies makes the theory power-counting renormalizable\cite{24, 25}. Explicitly, the one-loop MCG divergences are given by\cite{26}

$$
\Gamma_{\text{MCG}}^{(1)} = -\frac{1}{(4\pi)^2\varepsilon} \int d^4x \sqrt{-g} \left[ \frac{103}{45} E + \frac{797}{60} W - \frac{11}{72} \phi^{-4} (\phi^2 R - 6\phi\square\phi)^2 - \frac{13}{6} \alpha^2 (\phi^2 R - 6\phi\square\phi) + \frac{5}{2} \alpha^4 \phi^4 \right],
$$

(31)

where $\varepsilon$ is the dimensional regularization parameter.

Using the classical background field equation (6) in (31)\footnote{The propagator \cite{20} reduces to the propagator considered in Refs. \cite{24, 25} after the imposition of the Feynman gauge $\xi_1 = \xi_2 = 1$.}, and considering the relation (3), we obtain the on-shell divergent part of the one-loop MCG effective action

$$
\Gamma_{\text{MCG}}^{(1)\text{on-shell}} = -\frac{1}{(4\pi)^2\varepsilon} \int d^4x \sqrt{-g} \left[ \frac{797}{120} C^2 - \frac{1567}{360} E + \frac{5}{2} \alpha^4 \phi^4 \right].
$$

(32)

Since the first two terms of (32) are of the same type presented in the original action (2), they are renormalizable. In order to renormalize the last term of (32), we must add to (2) a quartic self-interacting term of the dilaton field $\lambda \int \sqrt{-g} \phi^4$. However, the inclusion of this term makes the flat metric no longer a solution of the field equations, which invalidates the $S$-matrix formulation. Fortunately, this problem is solved if we consider the renormalized value of $\lambda$ equal zero so that the self-interacting term is present in the renormalized action only to cancel out divergent terms like the last one of (32). In this way, the theory is one-loop renormalizable.

Inserting the on-shell MCG effective action up to one-loop into the trace of the energy-momentum tensor

$$
T = g^{\mu\nu} T_{\mu\nu} = g^{\mu\nu} \frac{2}{\sqrt{-g}} \frac{\delta \Gamma_{\text{eff}}}{\delta g^{\mu\nu}},
$$

(33)

it can be shown that MCG has the conformal (trace) anomaly

$$
T = \frac{1}{(4\pi)^2} \left[ \frac{797}{120} C^2 - \frac{1567}{360} E + \frac{5}{2} \alpha^4 \phi^4 \right] \neq 0,
$$

(34)

\footnote{This result corresponds to the one derived in Ref. \cite{24} with $\lambda = 0$.}

\footnote{In order to \cite{6} remain valid even in the presence of matter, which will be necessary for the renormalizability of the full theory, we will assume that the dilaton field does not couple with matter in Section 3.}
which breaks the conformal symmetry of the theory at the one-loop level. A possible consequence of this symmetry breaking is the emergence of non-renormalizable \[ \int \sqrt{-g} R^2, \] Einstein-Hilbert and cosmological constant divergent terms beyond the one-loop level. However, by performing the background conformal transformations

\[ \tilde{g}_{\mu\nu} = \left( \varphi / \varphi_c \right)^2 g_{\mu\nu}, \quad \tilde{\varphi} = \varphi_c, \quad (35) \]

we can turn these divergent terms into the same types as the last three terms in (31). Thus, despite having a conformal anomaly, MCG is completely renormalizable.

### 2.2 Unitarity

Writing the propagator (30) in the form

\[ D_{\mu\nu,\alpha\beta} = -i \frac{2}{m^2} \left[ \frac{1}{p^2} - \frac{1}{p^2 + m^2} \right] D_{\mu\nu,\alpha\beta}^{(2)}(p) + \text{gauge terms}, \quad (36) \]

we can see that the renormalizability of the theory is achieved at the cost of the emergence of a massive ghost pole at \( p^2 = -m^2 \) with negative residue, in addition to the usual massless graviton pole at \( p^2 = 0 \) with positive residue. In most cases, the presence of a ghost violates the unitarity of the theory. However, the fact that the mass of the MCG ghost is above the normal threshold of the massless graviton production makes it unstable. In this case, the ordinary perturbation theory breaks down and we must use a modified perturbation series in which the bare propagator \( D(p^2) \) is replaced by the dressed propagator \[ D(p^2) = \left[ D^{-1}(p^2) - \Pi(p^2) \right]^{-1}, \quad (37) \]

where \( \Pi(p^2) \) is the sum of all one-particle irreducible (1PI) self-energy parts.

We can find the dressed MCG graviton propagator by coupling \( N \) fermionic fields to the action (13), carrying out a \( 1/N \) expansion, and using the Cauchy’s integral theorem. The result of such calculation can be write in the spectral form \[ D_{\mu\nu,\alpha\beta} = -i \frac{2}{m^2} \left[ \frac{1}{p^2} + \frac{\mathcal{R}}{p^2 - M^2} + \frac{\mathcal{R}^*}{p^2 - M^*} + \frac{1}{2\pi} \int_C \frac{\rho(a)}{p^2 - a} da \right] D_{\mu\nu,\alpha\beta}^{(2)}(p) + \text{gauge terms}, \quad (38) \]
where $M$ and $M^*$ are the positions of a complex-conjugate pole pair, $\mathcal{R}$ and $\mathcal{R}^*$ are the corresponding residues of the complex poles, $\rho(a)$ is a spectral function of the continuum states and $C$ is an appropriate path in the complex plane.

In the place of the unstable massive ghost pole found in the bare propagator (36), the dressed propagator (38) has a pair of complex-conjugate poles in the physical riemannian energy sheet. By using the Becchi-Rouet-Stora-Tyutin (BRST) method [29, 30, 31, 32], and the Ward-Takahashi identities [33, 34], we can find the Nielsen identities [35] for the position of the complex massive pole $M^6$

\[
\frac{\partial M^2}{\partial \xi_1} = 0,
\]

\[
\frac{\partial M^2}{\partial \xi_2} + C(\langle \sigma \rangle, \xi_2) \frac{\partial M^2}{\partial \langle \sigma \rangle} \neq 0,
\]

where $C(\langle \sigma \rangle, \xi_2)$ is determined order by order in the loop expansion of the theory and $\langle \sigma \rangle$ is the vacuum expectation value (VEV) of $\sigma$.

According to (39) and (40), the position of the complex massive pole $M$ is independent of $\xi_1$ but depends on $\xi_2$. Similarly, we can show that the same is valid for the position of the complex massive pole $M^*$. This means that we can move the positions of the complex massive poles around by varying $\xi_2$, which causes the excitations represented by the complex poles do not contribute to the gauge-invariant absorptive part of the $S$-matrix. Thus, the $S$-matrix connects only asymptotic states with positive norm, leading to the unitarity of the theory.

2.3 Symmetry breaking

In order to verify the possibility for spontaneous symmetry breaking in the MCG universe, we need to analyze the effective potential of the dilaton field,

\[\text{The presence of the extra } \Box \text{ in (19) does not affect the results of Ref. 36.}\]
which is given by [37]

\[ V_{\text{eff}}^{(1)} = \sqrt{-g} \langle \varphi \rangle^2 R + \frac{\sqrt{-g}}{2(4\pi)^2} \left\{ \left( \log \frac{\langle \varphi \rangle^2}{\mu^2} + \frac{3}{2} \right) \left( \frac{10}{9} C^2 - \frac{985}{72} W \right) \right. \\
\left. + \frac{1}{2} \langle \varphi \rangle^4 \left[ 5\alpha^4 \left( \log \frac{\langle \varphi \rangle^2}{\mu^2} - \frac{25}{6} \right) + \frac{144}{\mu^4} \left( \frac{10}{9} C^2 - \frac{985}{72} W \right) \right] \\
- \frac{24}{\mu^2} \langle \varphi \rangle^2 \left( \frac{10}{9} C^2 - \frac{985}{72} W \right) \right\}, \tag{41} \]

where \( \langle \varphi \rangle \) is the VEV of the dilaton field and \( \mu \) is a renormalization mass scale.

Considering that the metric of the MCG universe is given by [23]

\[ ds^2 = -c^2 dt^2 + a(t)^2 \left( \frac{dr^2}{1 + r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \tag{42} \]

where \( a(t) = \sqrt{bt + t^2} \) and \( b \) is a positive constant, we find that

\[ R = C = 0, \quad W < 0. \tag{43} \]

In this case, the effective potential (41) has a minimum \( \langle \varphi \rangle = \varphi_0 \) away from the origin, which means that quantum corrections of the dilaton field spontaneously breaks the conformal symmetry via the Coleman-Weinberg mechanism [38].

After the spontaneous breaking of the conformal symmetry, the MCG action (2) becomes

\[ S_{\text{MCG}} = \int d^4 x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2\alpha^2} \frac{C^2}{E} - \eta E \right], \tag{44} \]

where

\[ M_P^2 = 2\varphi_0^2 \tag{45} \]

is the reduced Planck mass. It follows from (44) that the conformal symmetry of the theory is spontaneously broken near the Planck scale.

---

7It is worth noting that due to the MCG energy continuity equation

\[ \frac{d}{dt} \left[ (c^2 \rho + p) a^4 \right] = 0, \]

the metric (42) is valid in all epochs of the MCG universe.
3 Matter

Here, we follow the GR nomenclature and call “matter” all the fields of nature that are source of gravity, with the exception of the gravitational field itself. As in the case of the gravitational action, the actions of the matter fields should be conformally invariant in the model considered here. In addition, we consider that such actions couple with gravity through the interaction of the matter fields with the metric field only and not with the dilation field, as stated early in Subsection 2.1. The description of the matter actions conformally coupled with gravity follows below.

3.1 Yang-Mills

The Yang-Mills (YM) theory [39] describes the dynamics of 12 gauge fields (spin-1 bosons), namely, 1 photon $A_\mu$, which mediates the electromagnetic interaction, 3 weak bosons $W_\mu^\pm$ and $Z_\mu^0$, which mediate the charged and neutral current weak interactions, and 8 gluons $G_\mu^a$ ($a = 1, 2, \ldots, 8$), which mediate the strong interactions. Since the standard YM action in flat spacetime is already conformally invariant, its conformal coupling with gravity is performed only through the minimal coupling $\eta_{\mu\nu} \rightarrow g_{\mu\nu}$ and $d^4x \rightarrow d^4x \sqrt{-g}$.

The YM action conformally coupled with gravity is given by

$$S_{\text{CYM}} = -\frac{1}{4} \int d^4x \sqrt{-g} \left[ g^{\mu\alpha} g^{\nu\beta} (B_{\mu\nu} B_{\alpha\beta} + W^i_{\mu\nu} W^i_{\alpha\beta} + G^a_{\mu\nu} G^a_{\alpha\beta}) \right], \quad (46)$$

where

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (47)$$

is the field strength of the $B_\mu$ gauge field that corresponds to the phase transformations $U(1)_Y$ symmetry group,

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - \eta g \epsilon^{ijk} W^j_\mu W^k_\nu \quad (48)$$

is the field strength of the $W^i_\mu$ ($i = 1, 2, 3$) gauge fields that correspond to the chiral $SU(2)_L$ symmetry group, and

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - \eta s g_s f^{abc} C^b_\mu G^c_\nu \quad (49)$$

is the field strength of the $G^a_\mu$ gauge fields that correspond to the color $SU(3)_C$ symmetry group.
In (48) and (49), we have that \( \eta = \pm 1 \), \( g \) is the coupling constant associated with the \( SU(2)_L \) group, \( \epsilon^{ijk} \) is the Levi-Civita symbol, \( \eta_s = \pm 1 \), \( g_s \) is the coupling constant associated with the \( SU(3)_C \) group, and \( f^{abc} \) are the structure constants of the \( SU(3)_C \) group, satisfying \( [T^a, T^b] = if^{abc}T^c \), with \( T^a \) being the generators of the \( SU(3)_C \) group, which can be represented by the Gell-Mann matrices \( \lambda^a \) according to \( T^a = (1/2)\lambda^a \). Note that the generic signs in the \( \eta \) parameters are necessary to specify the different notations found in literature.

3.2 Higgs

The Higgs field was introduced in the SM for the mass terms of the \( W \) and \( Z \) gauge bosons, and fermions, to become \( SU(2)_L \times U(1)_Y \) invariant [40,41]. However, the explicit mass term of the Higgs boson breaks the conformal symmetry of the theory. This problem can be fixed if we consider that the Higgs mass is generated by the symmetry breaking of an additional scalar field.

The Higgs action conformally coupled with gravity that we consider here is

\[
S_{CH} = -\int d^4x \sqrt{-g} \left[ g^{\mu\nu} (D_\mu H)^\dagger D_\nu H + \frac{1}{2} g^{\mu\nu} \partial_\mu S \partial_\nu S + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] - \int d^4x V(H, S, \phi),
\]

(50)

where

\[
H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}
\]

(51)

is the Higgs doublet field with 2 complex scalar fields \( H^+ \) and \( H^0 \), \( S \) and \( \phi \) are real scalar singlet fields [4].

\[
D_\mu H = \left( \partial_\mu + i \eta g T^i W^i_\mu + i \eta' g' Y B_\mu \right) H
\]

(52)

is the \( SU(2)_L \times U(1)_Y \) gauge covariant derivative for the Higgs field, and

\[
V(H, S, \phi) = \sqrt{-g} \left\{ \frac{1}{6} (H^\dagger H) R + \frac{1}{12} S^2 R + \frac{1}{12} \phi^2 R + \frac{\lambda_H}{4} (H^\dagger H)^2 + \frac{\lambda_S}{4} S^4 \right. \\
\left. + \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_{HS}}{2} S^2 (H^\dagger H) + \frac{\lambda_{H\phi}}{2} \phi^2 (H^\dagger H) + \frac{\lambda_{S\phi}}{2} S^2 \phi^2 \right\}
\]

(53)

\(^8\)The addition of two scalars instead of just one will be justified in Section 4.
is the scalar potential, with $\lambda_H, \lambda_S, \lambda_{\phi}, \lambda_{HS}, \lambda_{H\phi}$ and $\lambda_{S\phi}$ being dimensionless coupling constants.

We have in (52) that $T^i$ are the generators of the $SU(2)_L$ group, satisfying $[T^i, T^j] = i\epsilon^{ijk}T^k$, $\eta' = \pm 1$, $g'$ is the coupling constant associated with the $U(1)_Y$ group, $\eta_Y = \pm 1$, and the hypercharge $Y$ is the generator of the $U(1)_Y$ group, which is related to the electron charge $Q$ and the $T^3$ generator of the $SU(2)_L$ group by $Q = T^3 + \eta_Y Y$. We can represent the generators of the $SU(2)_L$ group in the form $T^i = (i/2)\sigma^i$, where $\sigma^i$ are the Pauli matrices.

### 3.3 Fermions

The fermions are composed by 6 leptons $e_M$ and $\nu_M$, and 6 quarks $u^C_M$ and $d^C_M$, where $M = 1, 2, 3$ is the generation index such that $e_M$ is the electron, the muon or the tau, $\nu_M$ is the corresponding neutrino, $u^C_M$ is either the up, charm or top quark, and $d^C_M$ is the down, strange or bottom quark, with $C = 1, 2, 3$ corresponding to the three types of $SU(3)$ color $R, G, B$.

By conformally coupling the SM fermion action with gravity, we find

$$S_{CF} = -\int d^4x \sqrt{-g} \left( \sum_{\psi_L} i\bar{\psi}_L \slashed{D}_L \psi_L + \sum_{\psi_R} i\bar{\psi}_R \slashed{D}_R \psi_R \right), \quad (54)$$

where summation over the generation and color indices is implied, $\bar{\psi} = \psi^\dagger \gamma^0$ is the adjoint fermion field,

$$\slashed{D}_L \psi_L = e^a_\mu \gamma^a \left( \partial_\mu + i\eta g T^i W^i_\mu + i\eta' g' \eta_Y Y B_\mu - i\frac{4}{\omega_{\mu ab}} [\gamma^a, \gamma^b] \right) \psi_L \quad (55)$$
is the full gauge covariant derivative for the left-handed fermions

$$\psi_L = L, Q = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad (56)$$

and

$$\slashed{D}_R \psi_R = e^a_\mu \gamma^a \left( \partial_\mu + i\eta' g' \eta_Y Y B_\mu - i\frac{4}{\omega_{\mu ab}} [\gamma^a, \gamma^b] \right) \psi_R \quad (57)$$
is the full gauge covariant derivative for the right-handed fermions

$$\psi_R = \nu_R, e_R, u_R, d_R. \quad (58)$$

In (55) and (57), we have that $e^a_\mu$ are the tetrad fields, $\gamma^a$ are the Dirac matrices, which satisfy the anticommutation relation $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$, and $\omega_{\mu ab} = e^a_\mu e^b_\nu \Gamma^\lambda_{\mu \nu} - e^a_\nu \partial_\mu e^b_\lambda$ is the spin connection.
3.4 Yukawa

The SM Yukawa action describes the interaction of the fermions with the Higgs field. These interactions are responsible for generating the masses of the fermions, with the exception of the neutrinos, when quantum corrections of the Higgs field spontaneously breaks the $SU(2)_L \times U(1)_Y$ (electroweak) symmetry. Including an extra interaction of the Higgs field with the neutrinos, the latter acquire similar masses to the other particles in the SM instead of the small masses found in nature. One way to solve this problem is through the seesaw mechanism \[42\], which assumes the SM interactions of the fermions with the Higgs field and introduces a Majorana mass term for the right-handed neutrinos. However, the Majorana mass term breaks the conformal symmetry of the theory. Therefore, in order to make the theory conformally invariant, we can consider that the neutrino masses are generated by the symmetry breaking of one of the extra scalars, from which we choose $S$.

The resulting conformally invariant Yukawa action coupled with gravity reads

$$S_{CY} = - \int d^4x \sqrt{-g} \left( \bar{L} Y^e \mathcal{H} e_R + \bar{Q} Y^u \tilde{H} u_R + \bar{Q} Y^d \mathcal{H} d_R 
+ \bar{L} Y^\nu \nu_R + \nu_R^T Y^M \mathcal{C} S \nu_R + \mathrm{h.c.} \right),$$

where summation over the generation and color indices is assumed, $Y^e$, $Y^u$ and $Y^d$ are the usual Yukawa matrices of the SM, $Y^\nu$ is a complex matrix that mediates the coupling of the SM fields to the right-handed neutrinos, $Y^M$ is a real diagonal matrix that describes the interactions of the right-handed neutrinos with the scalon and $\tilde{H} = i\sigma_2 \mathcal{H}^\dagger$.

4 Full theory

Finally, putting together all the pieces of the previous sections, we arrive at the CTOE action

$$S_{CTOE} = S_{MCG} + S_{CYM} + S_{CH} + S_{CF} + S_{CY},$$

where $S_{MCG}$, $S_{CYM}$, $S_{CH}$, $S_{CF}$ and $S_{CY}$ are given by \[2\], \[40\], \[50\], \[54\] and \[59\], respectively. Note that we not explicitly consider possible gauge-fixing terms as well as the compensating Faddeev-Popov ghost terms in \[60\].
It can be shown that the contribution of the matter fields to the on-shell divergences of the one-loop CTOE effective action are of the same types presented in (60) [45], which means that the full theory is one-loop renormalizable. The non-renormalizable divergences that may arise beyond the one-loop level due to the conformal anomaly produced by the contribution of the matter fields can be eliminated by performing the conformal transformations (35) and using the field equation (6), which makes CTOE completely renormalizable. Additionally, since there are no ghosts in the matter part of (60), the theory is also unitary.

The investigation of the spontaneous symmetry breaking in the CTOE matter part requires the minimization of the one-loop matter effective potential of the theory, which is a quite cumbersome thing to do due to the existence of three interacting scalar fields coupled with gravity in (53). However, in flat spacetime, the potential (53) reduces to

$$V(h, S, \phi) = \frac{\lambda_h}{4} h^4 + \frac{\lambda_S}{4} S^4 + \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_{hS}}{2} h^2 S^2 + \frac{\lambda_{h\phi}}{2} h^2 \phi^2 + \frac{\lambda_{S\phi}}{2} S^2 \phi^2,$$

(61)

where $h^2 = H^\dagger H$. Using the Gildener-Weinberg (GW) analytical approximation method [43], it was shown in Ref. [44] that the potential (61) leads to the correct phenomenology at low energies, while keeping the system stable and free of Landau poles up to the Planck scale, if we assume that $\phi$ does not develop a VEV and $S$ is a pseudo-Goldstone boson (PGB) that acquires a finite VEV during the electroweak symmetry breaking.

After the spontaneous breaking of the electroweak symmetry, the gauge bosons and fermions, with the exception of the right-handed neutrinos, acquire masses as in the standard model. In addition, the interactions of the Higgs field and the right-handed neutrinos with $S$, allow that the observed masses of these fields be generated after the spontaneous breaking of the electroweak symmetry. Since all scales of CTOE are generated by spontaneous breaking of symmetry, the theory is free of the hierarchy problem.

5 Final remarks

We have constructed here a renormalizable and unitary theory of everything by conformally coupling an extended SM with gravity. In the gravitational

---

9The effects of the top quark Yukawa coupling on the stability of the renormalization group (RG) running still need to be studied.
part of the theory there is a scalar field $\varphi$ called dilation whose quantum corrections spontaneously breaks the conformal symmetry near the Planck scale. In addition, the electroweak symmetry is spontaneously broken by quantum corrections of an extra scalar field $S$ introduced in the matter part of the theory, which also contains a second extra scalar $\phi$ with zero vacuum expectation value.

Besides being at energies far beyond the reach of current particle accelerators, the dilaton field has no degrees of freedom and thus it is not detectable. On the other hand, the preferable energy of $S$ is of the order of a few GeV, which leads to additional Higgs decays that can be tested in future LHC runs. Since $\phi$, whose energy must lie between 300 GeV and 370 GeV, does not have a vacuum expectation value, it may be a good candidate for dark matter, which we intend to study further in future works.

Acknowledgments

The author is grateful to I. L. Shapiro and A. D. Pereira for very helpful discussions. In addition, the author would like to thank S. D. Odintsov, E. Elizalde, A. Salvio and A. Strumia for useful comments.

References

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974);

[2] H. Georgi, Particles and Fields, Proceedings of the APS Div. of Particles and Fields, ed. C. Carlson, p. 575 (1975).

[3] H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975).

[4] F. Gursey, P. Ramond and P. Sikivie, Phys. Lett. B 60, 177 (1976).

[5] H. Miyazawa, Prog. Theor. Phys. 36, 1266 (1966).

[6] Yu. A. Golfand and E.P. Likhtman, Sov. Phys. JETP Lett. 13, 323 (1971).

[7] J. L. Gervais and B. Sakita, Nucl. Phys. B 34, 632 (1971).

[8] D. V. Volkov and V. P. Akulov, Phys. Lett. B 46, 109 (1973).
[9] J. Wess and B. Zumino, Phys. Lett. B 49, 52 (1974).
[10] A. Salam and J. A. Strathdee, Nucl. Phys. B 76, 477 (1974).
[11] P. Fayet, Phys. Lett. B 78, 417 (1978).
[12] H. P. Nilles, Phys. Reports 110, 1 (1984).
[13] H. E. Haber and G.L. Kane, Phys. Reports 117, 75 (1987).
[14] C. D. Froggatt and H. B. Nielsen, Phys. Lett. B 368, 96 (1995).
[15] A. Gorsky, A. Mironov, A. Morozov and T. N. Tomaras, J. Exp. Theor. Phys. 120, 344 (2015), Zh. Eksp. Teor. Fiz. 147, 399 (2015).
[16] G. ’t Hooft, Subnucl. Ser. 47, 251 (2011).
[17] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, Effective action in quantum gravity (IOP Publishing, Bristol and Philadelphia, 1992).
[18] I. L. Buchbinder and S. D. Odintsov, Sov. J. Nucl. Phys. 40, 848 (1984), Yad. Fiz. 40, 1338 (1984); I. L. Buchbinder and S. D. Odintsov, Lett. Nuovo Cim. 42, 379 (1985); S. D. Odintsov, Fortsch. Phys. 39, 621 (1991);
[19] F. F. Faria, Adv. High Energy Phys. 2014, 520259 (2014).
[20] F. F. Faria, arXiv:1604.02210.
[21] F. F. Faria, Adv. High Energy Phys. 2019, 7013012 (2019).
[22] F. F. Faria, Eur. Phys. J. C 80, 645 (2020).
[23] F. F. Faria, Mod. Phys. Lett. A. 36, 2150115 (2021).
[24] F. F. Faria, Eur. Phys. J. C 76, 188 (2016).
[25] F. F. Faria, Eur. Phys. J. C 77, 11 (2017).
[26] E. S. Fradkin and A. A. Tseytlin, Nucl. Phys. B 201, 469 (1982).
[27] I. Antoniadis and E.T. Tomboulis, Phys. Rev. D 33, 2756 (1986).
[28] E. T. Tomboulis, Phys. Lett. B 70, 361 (1977).
[29] C. Becchi, A. Rouet and R. Stora, Phys. Lett. B 52, 344 (1974).
[30] C. Becchi, A. Rouet and R. Stora, Commun. Math. Phys. 42, 127 (1975).
[31] C. Becchi, A. Rouet and R. Stora, Ann. Phys.(N.Y.) 98, 287 (1976).
[32] I. V. Tyutin, Lebedev Institute Preprint, Report No: FIAN-39 (1975) (unpublished).
[33] J. C. Ward, Phys. Rev. 78, 182. (1950).
[34] Y. Takahashi, Il Nuovo Cimento. 6, 371 (1957).
[35] N. K. Nielsen, Nuclear Physics B 101, 173 (1975).
[36] F. F. Faria, Eur. Phys. J. C 78, 277 (2018).
[37] N. Matsuo, Gen. Relativ. Gravit. 22, 561 (1990).
[38] S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[39] C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
[40] R. Brout and F. Englert, Phys. Rev. Lett. 13, 321 (1964).
[41] P.W. Higgs, Phys. Lett. 12, 132 (1964).
[42] M. Gell-Mann, P. Ramond and R. Slansky, Supergravity (North Holland, Amsterdam, 1979).
[43] E. Gildener and S. Weinberg, Phys. Rev. D 13, 3333 (1976).
[44] A. J. Helmoldt, P. Humbert, M. Lindner and J. Smirnov, JHEP 2017, 113 (2017).
[45] I. L. Buchbinder and I. L. Shapiro, Yad. Fiz. 44, 1033 (1986).