Quadrupole lenses scheme for RS FEL

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Abstract. The scheme with quadrupole lenses is presented for realization relativistic strophotron type Free electron laser. Equations of motion are solved and trajectories are found. It is shown, that movement of electrons in presented scheme is stable in both transverse directions.

1. Introduction

Usually the concept FEL is associated with undulator the magnetic field of which is periodical function of longitudinal coordinate z [1-11]. There exist other systems too allowing transverse oscillations of electrons, and consequently, there are other possibilities for creation FELs [3]. In one of such system movement of electrons was considered in plane parabolic potential, i.e. in the field whose potential do not depend on one of the transverse coordinates (for example, y) and on the longitudinal z and has square dependence on the other transverse coordinate x. For the systems of this type sometimes the term strophotron is used.

Single quadrupole lens, as it was well known, does not satisfy the above mentioned conditions: his potential square depends on both transverse coordinates x and y (near axis Oz), if dependence on x is potential through then the dependence on y is inverted through. So, in the direction Ox there is focusing and there are possible oscillations, and in the direction Oy movement is aperiodic and the beam decay occurs. This means, that for beam stability needed correction.

In the present article self consistent scheme is described of correction by use of additional quadrupole lenses providing electron beam stability and oscillation periodicity in both transverse coordinates.

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2. Quadrupole lenses scheme and equations of electron motion in strophotron static fields

A scheme is presented in Fig. 1. It’s base is long electrostatic or magnetic quadrupole lens focusing electrons in direction Ox and defocusing in direction Oy.

Let scalar potential $\phi(x, y)$ of long electrostatic quadrupole lens has a form $\phi = \frac{\phi_0}{d^2}(x^2 - y^2)$, where $\phi_0$ and $2d$ height and width of potential through in direction Ox. The equations of electron motion in this through have a form

$$\frac{dp_x}{dt} = -\frac{2e\phi_0}{d^2}x, \quad \frac{dp_y}{dt} = \frac{2e\phi_0}{d^2}y, \quad \frac{dp_z}{dt} = 0,$$

where $p_{x,y,z}$ are components of electron momentum along axes Ox, Oy, Oz.

Fig. 1. Basic and corrective lenses layout.
According to third Eq. of (1) the longitudinal momentum and energy are conserved  \( p_z = \text{const} \), \( \varepsilon_z = \left( p_z^2 + m^2 \right)^{1/2} = \text{const} \) (here the system is used where \( c=1 \)). Taking into account 
\[
\dot{p}_x = \dot{x} \left( \varepsilon_x^2 + p_x^2 + p_y^2 \right)^{1/2}, \quad \dot{p}_y = \dot{y} \left( \varepsilon_y^2 + p_x^2 + p_y^2 \right)^{1/2}
\]
the second and third Eqs. Give
\[
\ddot{x} + \Omega^2 x \left( 1 - \dot{x}^2 \right)^{3/2} = 0, \quad \Omega^2 = \frac{2e\phi_0}{d^2\varepsilon_z} \tag{2}
\]
\[
\ddot{y} - \Omega^2 x \left( 1 - \dot{y}^2 \right)^{3/2} = 0. \tag{3}
\]
At \( |x| \leq 1 \) Eq. (2) for \( x(t) \) turns to equation of harmonic oscillator having a solution
\[
x(t) = x_0 \cos \Omega t + \frac{\dot{x}_0}{\Omega} \sin \Omega t, \tag{4}
\]
where \( x_0 \), \( \dot{x}_0 \) initial coordinate and velocity along Ox axis, \( \alpha \equiv \sin \alpha = p_x / p \) electron entry angle in strophotron.

At \( |y| \leq 1 \) Eq. (3) gives exponential growing solution
\[
y(t) = y_0 \cosh \Omega t + \frac{\dot{y}_0}{\Omega} \sinh \Omega t, \tag{5}
\]
where now \( y_0 \), \( \dot{y}_0 \) initial coordinate and velocity along Oy axis.

In the case of magnetic lens with vector potential \( A \square Oz \), \( A_z = -\frac{A_0}{d^2} \left( x^2 - y^2 \right) \) (the corresponding magnetic field \( H_y = -\frac{2A_0}{d^2} y, H_x = \frac{2A_0}{d^2} x \) ) the equations of electron motion are
\[
\frac{dp_x}{dt} = -\frac{2eA_0}{d^2} x v_x, \quad \frac{dp_y}{dt} = \frac{2eA_0}{d^2} y v_y, \quad \frac{dp_z}{dt} = \frac{2eA_0}{d^2} \left( x \ddot{x} - y \ddot{y} \right). \tag{6}
\]
In this case total energy of electron is conserved \( \varepsilon = \left( p_x^2 + p_y^2 + p_z^2 + m^2 \right)^{1/2} = \text{const} \).

Equations (6) can be written as equations for \( x, y, z \), because of \( \mathbf{p} = \varepsilon \mathbf{v} \)
\[
\ddot{x} = v_x = -\frac{2eA_0}{d^2 \varepsilon} x \dot{x}, \quad \ddot{y} = v_y = \frac{2eA_0}{d^2 \varepsilon} y \dot{y}, \quad \ddot{z} = v_z = \frac{2eA_0}{d^2 \varepsilon} \left( x \ddot{x} - y \ddot{y} \right). \tag{7}
\]
The third equation of (6) is immediately integrated giving

\[
\dot{z} = \frac{eA_0}{\varepsilon d^2} \left( (x^2 - x_0^2) - (y^2 - y_0^2) \right) + v_0 \cos \alpha
\]

\[
= \frac{\Omega^2}{2} \left( (x^2 - x_0^2) - (y^2 - y_0^2) \right) + v_0 \cos \alpha, \quad \text{where} \quad \Omega = \left( \frac{2eA_0}{\varepsilon d^2} \right)^{1/2}.
\]  

(8)

Putting \(\dot{z}\) (8) into first and second equations of (7) we find nonlinear equations for \(x(t)\) and \(y(t)\)

\[
\ddot{x} + \Omega^2 x \left[ v_0 \cos \alpha + \frac{1}{2} \Omega^2 \left( x^2 - x_0^2 \right) - \frac{1}{2} \Omega^2 \left( y^2 - y_0^2 \right) \right] = 0
\]  

(9)
\[
\ddot{y} - \Omega^2 y \left[ v_0 \cos \alpha + \frac{1}{2} \Omega^2 \left( x^2 - x_0^2 \right) - \frac{1}{2} \Omega^2 \left( y^2 - y_0^2 \right) \right] = 0. \tag{10}
\]

Equations (9) and (10) differ from corresponding equations (2) and (3) for electric field. But as before in equations (9) and (10) small anharmonicity can be neglected if \(|\alpha| \ll 1\), \(|x_0| \Omega \ll 1\), \(|y_0| \Omega \ll 1\). In the harmonic approximation equations (9), (10) have the same solutions (4) and (5). Those equations show that there is a focusing in Ox direction and there are possible electron oscillations, and there is defocusing in direction Oy and occurs beam decay of the beam. This means that for beam stabilization its correction is needed.

The well known hard focusing with crossed quadrupole lenses is used for beam stabilization. Arrows in the figure indicate the positions of corrective lenses along the Oz axis of the system at a distance \(\pi / \Omega\) from each other. Indexes x and y show those transverse directions in which corresponding lens is focusing. The corrected lenses are assumed to be short, so that their length \(l\) is much less than focal length \(f\). This assumption allows us to not analyze in detail the electron motion in the corrective lenses, describing their action instantaneous change in a component of the electron velocity on \(\pm x_i f\), \(\mp y_i f\), where \(x_i\) and \(y_i\) values of the transverse coordinate at the intersection of electron i-th corrected lens.

Focal lengths of all corrected lenses (except first) we accept to be

\[
f = \frac{1}{2\Omega} = \frac{f'}{2} = \frac{f_0}{\pi}, \tag{11}
\]

where \(f' = 2f\) is focal length of first lens located at the entrance \((z=0)\), and \(f_0\) is focal length of basic quadrupole lens \(f_0 = \pi / 2\Omega \ll L\). This choice provides the electron motion periodicity on both transverse coordinates (see Fig.2.). Electron velocity in direction Oy, \(\dot{y}\) changes his direction(sign) during every correction (Fig.2. a,b), electron velocity in direction Ox, \(\dot{x}\) at the intersection of corrective lenses abruptly changed to \(\pm 2x_0 \Omega\), and \(\pm x_0 \Omega\) at intersection first lens (see Fig.2.). These abrupt changes in speed \(\dot{x}\) are small in comparison with its initial value \(\dot{x} \approx \alpha\) if the condition holds

\[
\Omega \Delta x_0 \ll \alpha \tag{12}
\]
where \( \Delta x_0 \) is transverse size of the beam in the Ox direction corresponding to the electrons fly in the vicinity of the Oz axis and giving an effective contribution to the resonant radiation and equal \( \frac{4}{\Omega \sqrt{\pi L}} \), where L is system length, \( \lambda \) - radiation wavelength.

If the necessity of correction and focusing of the beam in the Oy direction is evident, the need for focus adjustment in the Ox direction is associated with the requirements of periodicity of the oscillations and the absence of accumulated change of velocity \( \dot{x} \).

Under condition (12) jumps of velocity \( \dot{x} \) are small and do not essentially affect the nature of the oscillations \( x(t) \), Fourier expansion of which has a form

\[
x(t) = x_0 \cos \Omega t + \frac{\alpha}{\Omega} \sin \Omega t - \frac{2x_0}{\pi} \left( 1 - 2 \sum_{k=1}^{\infty} \frac{1}{4k^2-1} \cos 2k\Omega t \right).
\]  
(13)

Under the selected installation scheme of corrective lenses movement in a second transverse coordinate y is also periodic with the same period \( 2\pi / \Omega \) if \( \dot{y}_0 = 0 \). Condition \( \dot{y}_0 \neq 0 \) violates periodicity. During one period \( 2\pi / \Omega \), y increases on \( y / \Omega \). The deviation from the strict periodic motion in y means that for \( \dot{y}_0 \neq 0 \) is a certain degree of instability. This instability is small under condition

\[
\frac{L \dot{y}_0}{\lambda \Omega} = \frac{L \dot{y}_0}{2\pi} d_{ey}
\]  
(14)

where L is the system length, \( \lambda = 2\pi / \Omega \). The condition (14) means that the increase of y independence on \( \dot{y}_0 \neq 0 \) on the whole length L should be less than transverse dimension of the beam in the y direction \( d_{ey} \).

With values \( L = 300 \text{cm} \), \( d_{ey} = 1 \text{cm} \) the condition (14) gives

\[
\dot{y}_0 \leq \frac{2\pi d_{ey}}{L} = 2 \times 10^{-2}
\]  
(15)

The condition of used approximation has a form

\[ e^\varepsilon d_{ey} < D_y, \quad 1/\Omega, \]  
(16)

where \( D_y \) is a lens aperture in the Oy direction.
3. Conclusion

From the above it can be seen, that the movement of the electron beam in the system of quadrupole lenses of the type described has a two-dimensional character, and in each of them many harmonics of transverse oscillation frequencies are essential. The problems of investigation spontaneous and stimulated radiation in such a system on a higher harmonics of resonant frequency are very complicated. Therefore below the simplified mathematical model will be considered – model of plane parabolic through. It will be shown that in this simplified model, there are a number of physical factors that were not previously taken into account, but are essential for the correct evaluation of the real possibilities of amplification in FEL, based on macroscopic channeling effect in the quadrupole lenses.

Note finally the stability of the motion of electrons in the system with a quadrupole lenses. A small deviation of the focal distance $\delta f \ll f$ does not lead to instability in the movement in the $x$ direction. In the direction $y$ it leads to increase of $y$ coordinate, so that $\delta y_0 / y_0 \ll N \frac{\delta f}{f}$, where $y_0$ is coordinate $y$ at the entrance in system, $\delta y_0$ is increase on the system length, $N$- number of periods, and $f$ is lens focal length. The condition of small deviation along $y$ is $\delta y_0 / y_0 < 1$, which is equivalent to the requirement $\frac{\delta f}{f} < \frac{1}{N}$.

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