Proton Spin, Sum Rules, QCD and Higher Twist

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Abstract

We examine the present status of the Bjorken sum rule in the light of recent data on the spin structure functions of the proton, neutron and deuteron obtained by the CERN and SLAC experimental groups. We also discuss the role of possible higher-twist contributions and higher-order PQCD corrections and comment on the extraction of the necessary parameters, $D$ and $F$, obtained from hyperon semi-leptonic decays.

Résumé

Nous examinons l’état actuel de la règle de somme de Bjorken à la lumière des données récentes sur les fonctions de structures du spin du proton, neutron et deutéron obtenues par des groupes expérimentaux au CERN et à SLAC. Nous discutons aussi le rôle des contributions éventuelles de twist plus élevé et des corrections de PQCD à ordre plus élevé et nous commentons l’extraction des paramètres nécessaires, $D$ et $F$, obtenus à partir des désintégrations semi-leptoniques des hypérons.

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1. Introduction

Polarization effects provide valuable insight into the dynamics of hadronic interactions and can be sensitive to bound-state and other non-perturbative physics. In particular, the Bjorken sum rule (BSR) [1] is a measurable quantity that may be used to test theoretical predictions based on the light-cone expansion in PQCD. The experimental precision now attainable is at the ten-percent level or better, while on the theoretical side, all relevant PQCD calculations have been carried out to at least two-loop order [2] (i.e., approximately one-percent level) and for the BSR itself to three loops [3].

In the quark-parton model the structure function $g_1(x,Q^2)$ [4] is related to polarized quark distributions, in a manner analogous to $F_1(x,Q^2)$:

$$g_1(x,Q^2) = \frac{1}{2} \sum_f e_f^2 \Delta q_f(x,Q^2),$$  \hfill (1)

where $q_f^\pm(x,Q^2)$ are the densities of quarks of flavour $f$ and positive or negative helicity with respect to the parent hadron.

Experimentally an asymmetry is measured and the polarized structure function is then extracted via

$$F_1(x,Q^2) = \frac{1}{2} \sum_f e_f^2 q_f(x,Q^2).$$  \hfill (2)

The polarized and unpolarized quark densities are defined in the following manner:

$$\Delta q_f(x,Q^2) = q_f^+(x,Q^2) - q_f^-(x,Q^2),$$  \hfill (3)

$$q_f(x,Q^2) = q_f^+(x,Q^2) + q_f^-(x,Q^2).$$  \hfill (4)

where $q_f^\pm(x,Q^2)$ are the densities of quarks of flavour $f$ and positive or negative helicity with respect to the parent hadron.

Experimentally an asymmetry is measured and the polarized structure function is then extracted via

$$g_1(x,Q^2) = \frac{A_1(x,Q^2) F_2(x,Q^2)}{2x(1 + R(x,Q^2))},$$  \hfill (5)

where $R_1(x,Q^2)$ is the ratio of longitudinal to transverse unpolarized structure functions and $A_1(x,Q^2)$ is the measured asymmetry.

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2. The Bjorken system of equations

The full SU(3) algebra of the baryon octet admits three independent quantities, which may be expressed in terms of the SU(3) axial-vector currents:

\[ A_3 = \bar{u} \gamma_5 u - \bar{d} \gamma_5 d, \]
\[ A_8 = \bar{u} \gamma_\gamma u + \bar{d} \gamma_5 d - 2 \bar{s} \gamma_5 s, \]
\[ A_0 = \bar{u} \gamma_\gamma u + \bar{d} \gamma_5 d + \bar{s} \gamma_5 s. \] (6)

And thus

\[ \langle p^\uparrow | A_i | p^\uparrow \rangle = g_i \quad (i = 3, 8, 0), \] (7)

The r.h.s. of (7) for \( i = 3, 8 \) corresponds to axial-vector couplings accessible in hyperon semi-leptonic decays \((g_3 = 1.2573 \pm 0.0028 \ [5] \text{ and } g_8 = 0.629 \pm 0.039 \ [6])\), but \( g_0 \), corresponding to the flavour-singlet axial-vector current, is unknown. Thus, a prediction for the proton integral alone is impossible. A further independent combination of \( u, d \) and \( s \) axial-current matrix elements is accessible via \( n-p \) elastic scattering \([7]\) and this would allow a prediction for single nucleon targets. However, the precision of such data is still very poor.

The Bjorken sum rule \([1]\), with PQCD radiative corrections, then reads

\[ \Gamma_1^{p-n} = \int_0^1 dx \, g_1^{p-n}(x, Q^2) \]
\[ = \frac{1}{3} g_3 \left[ 1 - \alpha_s / \pi - c_2(\alpha_s / \pi)^2 - \ldots \right], \]

(9)

where the Wilson coefficients, \( c_n \), are known up to \( n = 3 \). Note that in what follows the higher-order corrections will be suppressed for simplicity, but it should always be borne in mind that all expressions receive QCD corrections known to at least second order.

3. The \( D \) and \( F \) parameters

Central then to the theoretical analysis are the values of the SU(3) constants, \( D \) and \( F \), parametrizing the axial couplings, \( g_i \), involved in hyperon semi-leptonic decays \([6]\). There has, over the past few years, been considerable discussion on the validity of the standard approach to their extraction from data \([8-11]\). The problem is essentially that of how to account reliably for SU(3) breaking.

A systematic approach to the problem was presented in ref. \([8]\), where the so-called “recoil” correction was taken into account, together with possible differences in the strange and \( u, d \) sea-quark wave-functions. Later, also taking advantage of the more precise data available, it was shown that the latter correction is strongly disfavoured by the data and that the former alone provides a very satisfactory account of SU(3) breaking \([6]\). The principle results of this analysis have since been confirmed by various authors \([9]\).

Recently two new approaches have been proposed: the first is based on a phenomenological parametrization of SU(3) breaking in the ratio \( F/D \) on which we shall comment shortly \([10]\), the second has been discussed at this meeting and the reader is referred to the paper appearing in these proceedings \([11]\).

In \([10]\) it was noticed that \( F/D \), as inferred from the three axial coupling constants extracted from angular/spin correlations alone, apparently obeys a simple linear law in an SU(3)-breaking mass parameter defined by

\[ \delta = \frac{m_i + m_f + m_p - m_n}{m_i + m_f + m_p + m_n}, \]

(10)

where \( m_{i,f} \) are the initial- and final-state baryon masses and \( m_{p,n} \) the reference proton and neutron masses.

Solely on this basis the authors propose extrapolating their fit to \( \delta = 0 \), for which they obtain

\[ F/D = 0.40 \pm 0.07, \]

(11)

where the rather large error directly reflects the extrapolation procedure adopted. The interest in such a value is that it would allow the Ellis-Jaffe sum rule to be saturated without recourse to strange-quark polarization. Note, however, that it could not, of course, simultaneously explain any discrepancy with the Bjorken (or neutron) sum rule.

There are two strong criticisms to be levelled at this approach: one is of a theoretical nature and the other more experimental. The theoretical difficulty has to do with the fundamental nature of the \( D \) and \( F \) parameters themselves; these are universal constants describing the symmetric and antisymmetric parts of the SU(3) couplings and appear with different Clebsch-Gordan coefficients in the various decay matrix elements. The hidden implication of such an approach is that, due to SU(3) breaking, \( D \) and \( F \) are miraculously renormalized in just such a way as to preserve the particular combination, \( F + D = g_3 \). Note that precisely the same combination also governs \( \Xi \rightarrow \Sigma^0 \nu \), which is thus predicted to remain unrenormalized despite the large value of the breaking parameter, \( \delta \), for this decay.

The other objection is against an arbitrarily selective use of data; the decay-rate data are completely ignored and these both increase the overall precision and shift the final answers, while also highlighting the inability of this approach to globally describe the data well. Note that the decay-rate data are both more numerous and more varied in their \( D-F \) dependence than the angular/spin correlation data alone. Let me then stress that a simple – physically motivated – recoil correction provides good agreement between all present data and that the low value of \( F/D \) proposed in ref. \([10]\) would appear highly improbable \([12]\).
4. The Ellis-Jaffe Sum Rule

Arguments may be made for setting the strange-quark axial matrix element to zero [13]: the strange quarks in the proton are very few and are concentrated below $x_B \approx 0.1$, where all correlations with the parent nucleon are expected to be very weak. Thus, the matrix elements in eq. (7) for $i = 8$ and 0 should be approximately equal, leaving only two independent quantities and allowing predictions for the proton and neutron separately:

$$\Gamma_1^{(n)} = (-) \frac{1}{12} g_3 + \frac{5}{36} g_8 + \frac{1}{2} \langle p^\uparrow | \bar{s} \gamma_3 \gamma_5 s | p^\uparrow \rangle, \quad (12)$$

where the last term is then assumed negligible. Conversely, given the value of $\Gamma_1$, the value of either the strange-quark or singlet axial-vector matrix elements may be extracted from these equations. There is no space here to discuss in detail the strange-quark spin problem; the interested reader is referred to [14,15], where a bound on the non-diffractive component and thus on the strange-quark polarization was derived, references to critiques of these papers may be found therein. In short, this analysis leads to the following bound: $|\Delta s| \leq 0.02$. In what follows, we shall not in face place great emphasis on the bound, but recall it here lest this physically intuitive result be forgotten.

5. A Comparison of Theory and Experiment

We now compare the experimental results with theoretical predictions based on the framework outlined above. In performing the calculations we have used the very precise value of $\Lambda_{QCD}^{(4)}$ recently extracted in a three-loop analysis of scaling violations in deep-inelastic scattering (DIS) [16], which is thus most suited to our purposes. Such an analysis also allows an examination of the possible improvement to be attained on increasing the order of the perturbative corrections included. It should always be stressed that, for consistency, all quantities must be evaluated at the same loop order and that, in particular, it is meaningless to insert a two-loop $\alpha_s$ into a three-loop expression and vice versa.

**EMC** [17] $\Gamma_1^\uparrow(11) = 0.128 \pm 0.010 \pm 0.015$

**SMC** [18] $\Gamma_1^\uparrow(10) = 0.136 \pm 0.011 \pm 0.011$

**E143** [19] $\Gamma_1^\uparrow(3) = 0.127 \pm 0.004 \pm 0.010$

**SMC** [20] $\Gamma_1^\uparrow(5) = 0.023 \pm 0.020 \pm 0.015$

**E143** [21] $\Gamma_1^\uparrow(3) = 0.049 \pm 0.004 \pm 0.003$

**E142** [22] $\Gamma_1^\uparrow(2) = -0.022 \pm 0.006 \pm 0.009 \quad (13)$

where the number in parenthesis refers to the mean value of $Q^2$ in GeV$^2$ and, where necessary, nuclear corrections have already been included [20,21]. The short-fall in the proton integral with respect to the Ellis-Jaffe prediction (taking $\Delta s = 0$) is evident. This observation led to coining the phrase *Spin Crisis*. A similar observation may be made for the E143 deuteron integral. In contrast, the neutron sum rule appears well satisfied by the E142 data. Thus, in terms of the strange-quark contribution, both the EMC and SMC data imply $\Delta s \approx -0.15$ while that of E142 leads to $\Delta s \approx -0.04$.

A measure of the discrepancy between the data and theory may be obtained by extracting, experiment-by-experiment, the singlet axial-vector matrix elements: the results are

$$\Delta S = 0.17 \pm 0.17 \quad \text{EMC proton}$$

$$= 0.25 \pm 0.15 \quad \text{SMC proton}$$

$$= 0.24 \pm 0.09 \quad \text{E143 proton}$$

$$= 0.09 \pm 0.25 \quad \text{SMC deuteron}$$

$$= 0.32 \pm 0.05 \quad \text{E143 deuteron} \quad (14)$$

$$= 0.46 \pm 0.10 \quad \text{E142 neutron}$$

$$= 0.31 \pm 0.05 \quad \text{global deuteron}$$

$$= 0.23 \pm 0.07 \quad \text{global proton},$$

where $\Delta S$ is the invariant sum of quark polarizations as in eq. (7) with $i = 0$, *i.e.*, evaluated for $Q^2 = \infty$. Comparison of the last three lines of (14) reveals the nature of the problem: unless a very large PQCD (or otherwise) correction is invoked the proton data imply a significantly smaller value of $\Delta q$ than do those for the neutron. We remark in passing that the SLAC deuteron data is perfectly in line with the mean of the proton and neutron data; thus, providing reassurance as to the validity of the theoretical nuclear corrections introduced and attesting the overall consistency of the experimental picture.

Alternatively, the strange-quark spin contribution may be fit [23]; taking the SLAC proton and neutron data and performing completely consistent fits at one- and three-loop order, we obtain respectively $\chi^2 = 3.7$, 3.8 and 3.2 for one degree of freedom. Using the Particle Data Group [5] preferred value of $\Lambda_{QCD}^{(4)} = 260^{+56}_{-46}$ in a two-loop fit (for consistency with the extraction of $\Lambda_{QCD}$), the situation is marginally improved to give $\chi^2 = 2.8$.

6. Higher-Twist Contributions

Given the low $Q^2$ of the SLAC data, it is natural to worry about the possibility of higher-twist “contamination.” Two approaches to this problem are either to theoretically estimate the size of such effects (e.g., using a bag model [24] or QCD sum rules [25,26]) or to deduce limits from the well-documented higher-twist behaviour of unpolarized data [27].
higher-twist contributions found is too small to have any real impact, even on the SLAC neutron data (by an odd quirk, the higher-twist contribution to $g_1^{n}$ is typically much smaller even than that in the case of $g_1^{p}$).

Furthermore, note that while the inclusion of large higher-twist contributions can in principle restore agreement between predictions and data as far as the sum-rule integrals at fixed $Q^2$ are concerned, the data seem to prefer only mild (logarithmic) scaling violations. A similar situation has already been noted in the case of the Gross-Llewellyn Smith sum rule [28], where the size of corrections (perturbative or not) required by the sum rule is larger than, and incompatible with, that deduced from the $Q^2$ variation [29].

We should also mention an approach based on the known limiting behaviour for $Q^2 \rightarrow 0$ [30,31]. The resulting effects are found to be rather large and even in conflict with the observed $Q^2$ dependence. Moreover, this analysis depends crucially on an assumed smooth interpolation through the low-$Q^2$ resonance region.

7. A Possible Explanation and Consequences

It is interesting to ask what happens if the normalization condition on the Wilson coefficients is relaxed, i.e., if PQCD is ignored and current algebra is used only to fix ratios of matrix elements [27]. In this case, adopting our strange-quark bound to effectively set $\Delta s = 0$, any one data set may be used to fix the overall normalization. The SLAC proton data, for example, then lead to the following “prediction” for the neutron:

$$0.002 \leq \Gamma_n^s \leq -0.026,$$

in rather good agreement with the SLAC neutron data.

Alternatively, the quark spins may be deduced from the proton and neutron data (with absolutely no assumption on the strange-quark spin) and the following relation is then obtained:

$$\Gamma_n^s = -\frac{1}{3} \Gamma_p^s + \frac{2}{3} \Delta s,$$

which leads to a strange-quark polarization of

$$\Delta s = -0.03 \pm 0.03,$$

a non-trivial result perfectly compatible with the bound. The total spin of the quarks is then found to be

$$\Delta \Sigma = 0.33 \pm 0.03.$$

Thus, a satisfactory and self-consistent picture may be rendered, provided we are willing to admit non-perturbative effects in the overall normalization of the relevant operator matrix elements. That such contributions might be important should not be too surprising given that the “real” intermediate quark and gluon states, on which the PQCD Wilson-coefficient calculations are based, clearly do not correspond to the physical hadronic states of the real world.

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