ABSTRACT

The aim of this paper is to introduce and study a new class of topological groups called \( P_0 \)-topological group. By using some nonstandard techniques, we investigated some properties of \( P_0 \)-monads in \( P_0 \)-topological group.

**Keywords:** nonstandard analysis, topological group.

1- Introduction:

In 1982, Mashhour A.S. *et al.* [6], defined a new version of nearly open sets which is significant notion to the field of general topology called preopen sets.

There are several important concepts in topology in which can be defined in terms of preopen sets.

In 2000 Dontchev J. *et al.* [3] introduced the concept of pre-\( \theta \)-open sets, in this work we use the notion of pre-\( \theta \)-open sets to define and study a new type of topological groups called \( P_0 \)-topological group, also we study some properties of \( P_0 \)-monads in \( P_0 \)-topological group. For this investigation, we need the following basic background in general topology and nonstandard analysis.

2- Basic Backgrounds in General Topology:

Throughout this work, \((X, \tau)\) or (simply X) denotes a standard topological space on which no separation axioms are assumed unless explicitly stated, we recall the following definitions, notational conventions and characterizations.
The closure (resp., interior) of a subset $A$ of a space $X$ is denoted by $\text{Cl}A$ (resp. $\text{Int}A$).

**Definition 2.1:** A subset $A$ of a space $X$ is said to be
- **preopen** set [6] if and only if $A \subseteq \text{Int}\text{Cl}A$.
- **$\beta$ -open** set [2] if and only if $A \subseteq \text{Cl}\text{Int}\text{Cl}A$.
- **preclosed** set[2] if and only if $X \setminus A$ is preopen set. Equivalently, $\text{Cl}\text{Int}A \subseteq A$.
- **$\beta$ -closed** set[2] if and only if $X \setminus A$ is $\beta$ -open. Equivalently, $\text{Int}\text{Cl}\text{Int}A \subseteq A$.
- **$\emptyset$ -open** set [6] if and only if $A \subseteq X$  is preopen set. Equivalently, $\text{Cl}\text{Int}A \subseteq A$.
- **pre-$\emptyset$ -open** set if and only if $x \in A$, there is an open subset $G$ of $X$ such that $x \in G \subseteq \text{Cl}G \subseteq A$.
- **sp-$\emptyset$ -open** set if and only if $x \in A$, there is a preopen subset $G$ of $X$ such that $x \in G \subseteq \text{pCl}G \subseteq A$.
- **$\beta$ -closed** if and only if $X \setminus A$ is $\emptyset$ -open set.
- **$\emptyset$ -closed** if and only if $X \setminus A$ is pre-$\emptyset$ -open set.
- **sp-$\emptyset$ -closed** if and only if $X \setminus A$ is sp-$\emptyset$ -open set.
- **$\emptyset$ -closed** if and only if $X \setminus A$ is $\emptyset$ -open set.

The intersection of all pre-closed (resp., $\beta$ -closed) sets containing $A$ is called **pre-closure** (resp., **$\beta$ -closure**) and denoted by $\text{pCl}A$ (resp., $\beta\text{Cl}A$).

The intersection of all $\emptyset$ -closed (resp., $\emptyset$-closed, sp-$\emptyset$ -closed, and $\emptyset$-closed) sets containing $A$ is called **$\emptyset$ -closure** (resp., $\text{pCl}^0A$, $\text{spCl}^0A$, $\text{pCl}^0A$) and denoted by $\text{Cl}^0A$ (resp., $\text{pCl}^0A$, $\text{spCl}^0A$, $\text{pCl}^0A$).

The family of all pre-open (resp., $\beta$ - open, $\emptyset$ -open, pre-$\emptyset$ -open, sp-$\emptyset$ -open, and $\emptyset$-open) sets of a space $X$ is denoted by $\text{PO}(X)$ (resp., $\beta\text{O}(X)$, $\emptyset\text{O}(X)$, $\text{PO}(X)$, $\text{SP}\text{O}(X)$, and $\text{PO}(X)$).

The family of all pre-closed (resp., $\beta$ - closed, $\emptyset$ - closed, pre-$\emptyset$ - closed, sp-$\emptyset$ - closed, and $\emptyset$-closed) sets of a space $X$ is denoted by $\text{PC}(X)$ (resp., $\beta\text{C}(X)$, $\emptyset\text{C}(X)$, $\text{PC}(X)$, $\text{SP}\text{C}(X)$, and $\text{PC}(X)$).

**Definition 2.2[4]:** A topological space $(X, \tau)$ is called **p-regular** (resp., **p*-regular) if and only if for each $x \in X$ and each closed (resp., preclosed) set $F$ such that $x \notin F$, there exist two disjoint preopen sets $A$ and $B$ such that $x \in A$ and $F \subseteq B$.

**Definition 2.3 [2]:** A space $X$ is extremely disconnected if the closure of every open set is an open set.

**Theorem 2.4 [1]:** Any union of pre-$\emptyset$-open sets is a pre-$\emptyset$-open set.

**Theorem 2.5 [1]:** For any space $X$, the following statements are true:

i) Every pre-$\emptyset$-open set is preopen set.

ii) Every $\emptyset$-open set is pre-$\emptyset$-open set.

iii) Every pre-$\emptyset$-open set is $\emptyset$-open set.

**Theorem 2.6 [1]:**
i) If \( X \) is extremely disconnected, then \( \text{PO}(X) = \text{PO}(\delta_0(X)) \).

ii) If \( X \) is \( p_\rho \)-regular space, then \( \tau \subset \text{PO}(X) \).

iii) If \( X \) is \( p^* \)-regular space, then \( \text{PO}(X) = \text{PO}(\delta_0(X)) \).

**Proposition 2.7** [1]: Let \( X_1 \) and \( X_2 \) be two topological spaces and \( X = X_1 \times X_2 \) be the topological product, let \( A_i \in \text{PO}(X_i) \) for \( i = 1, 2 \) then \( A_1 \times A_2 \in \text{PO}(X) \).

**Definition 2.8:** A mapping \( f : (X, \tau) \to (Y, \rho) \) is said to be

i) \( \delta_0 \)-irresolute [4], if \( f^{-1}(G) \in \text{PO} \text{PO}(X, \tau) \), for each \( G \in \text{PO} \text{PO}(Y, \rho) \).

ii) \( \delta_0^{**} \)-continuous [4], if \( f^{-1}(G) \in \tau \) for each \( G \in \text{PO} \text{PO}(Y, \rho) \).

iii) completely preirresolute [2], if \( f^{-1}(G) \in \text{PO}(X, \tau) \), for each \( G \in \text{PO}(Y, \rho) \).

iv) faintly precontinuous [2], if \( f^{-1}(G) \in \text{PO}(X, \tau) \), for each \( G \in \text{PO}(Y, \rho) \).

v) strongly faintly precontinuous [2], if \( f^{-1}(G) \in \text{PO}(X, \tau) \), for each \( G \in \text{PO}(Y, \rho) \).

Note that to define a \( \delta_0 \)-topological group, we introduce the following new type of continuity in topological spaces called \( \text{PO} \)-irresolute function, some characterizations and relations are obtained for this definition.

**Definition 2.9:** A mapping \( f : (X, \tau) \to (Y, \rho) \) is said to be \( \text{PO} \)-irresolute at a point \( \in X \) if for each \( \text{PO} \)-open set \( V \) of \( Y \) containing \( f(x) \), there exists a \( \text{PO} \)-open set \( U \) of \( X \) such that \( f(U) \subseteq V \).

If \( f \) is \( \text{PO} \)-irresolute at every point \( x \in X \), then it is called \( \text{PO} \)-irresolute.

**Theorem 2.10:** For any mapping \( f : (X, \tau) \to (Y, \rho) \), the following statements are equivalent

i) \( f \) is \( \text{PO} \)-irresolute.

ii) The inverse image of every \( \text{PO} \)-open set in \( Y \) is \( \text{PO} \)-open set in \( X \).

iii) The inverse image of every \( \text{PO} \)-closed set in \( Y \) is \( \text{PO} \)-closed set in \( X \).

iv) \( f(p\text{Cl}_\rho(A)) \subseteq p\text{Cl}_\rho(f(A)) \), for each subset \( A \) of \( X \).

v) \( p\text{Cl}_\rho(f^{-1}(B)) \subseteq f^{-1}(p\text{Cl}_\rho(B)) \), for each subset \( B \) of \( Y \).

vi) \( f^{-1}(p\text{Int}_\rho(B)) \subseteq p\text{Int}_\rho(f^{-1}(B)) \), for each subset \( B \) of \( Y \).

**Proof:** Straightforward.

**Theorem 2.11:** If \( X \) is extremely disconnected space, then every \( \delta_0 \)-irresolute mapping is \( \text{PO} \)-irresolute.

**Proof:** Let \( f : (X, \tau) \to (Y, \rho) \) be a \( \delta_0 \)-irresolute mapping, and let \( G \in \text{PO} \text{PO}(Y) \). Then, by Theorem 2.5(iii) we have \( G \in \text{PO} \text{PO}(Y) \), since \( f \) is \( \delta_0 \)-irresolute function, then \( f^{-1}(G) \in \text{PO}(X) \).

Since \( X \) is extremely disconnected space, by Theorem 2.6(i), we get \( f^{-1}(G) \in \text{PO}(X) \).

Hence \( f \) is \( \text{PO} \)-irresolute.

**Theorem 2.12:** If \( X \) is \( p \)-regular space, then every \( \delta_0^{**} \)-continuous mapping is \( \text{PO} \)-irresolute mapping.

**Proof:** The proof is similar to Theorem 2.11.

**Theorem 2.13:** If \( X \) is \( p^* \)-regular space, then every completely preirresolute mapping is \( \text{PO} \)-irresolute mapping.

**Proof:** It follows directly from Theorem 2.6(iii) and their definitions.
Theorem 2.14: For any mapping \( f : (X, \tau) \to (Y, \rho) \), the following statements are true
i) Every \( p_{\theta} \)-irresolute is faintly precontinuous function.
ii) Every strongly faintly precontinuous is \( p_{\theta} \)-irresolute.

Proof: The proof is easy, and therefore is omitted.

Theorem 2.15: Let \( f : X \to Y \) and \( g : Y \to Z \) be two \( p_{\theta} \)-irresolute functions. Then the composition mapping \( g \circ f : X \to Z \) is \( p_{\theta} \)-irresolute.

Proof: The proof is obvious.

Theorem 2.16: If \( f_i : X_i \to Y_i \) are \( p_{\theta} \)-irresolute for \( i=1,2 \). Then, the mapping \( h : X_1 \times X_2 \to Y_1 \times Y_2 \) defined by \( h(x_1,x_2) = (f_1(x_1), f_2(x_2)) \) is also \( p_{\theta} \)-irresolute.

Proof: Let \( G_1 \times G_2 \subseteq Y_1 \times Y_2 \), where \( G_1 \) and \( G_2 \) are \( \theta \)-open sets in \( Y_1 \) and \( Y_2 \) respectively, by Proposition 2.7 we have \( G_1 \times G_2 \in P_{\theta} O(Y_1 \times Y_2) \).
Since \( f_1 \) and \( f_2 \) are \( p_{\theta} \)-irresolute mappings, then \( f_1^{-1}(G) \in P_{\theta} O(X_1) \) and \( f_2^{-1}(G) \in P_{\theta} O(X_2) \) and \( h^{-1}(G_1 \times G_2) = f_1^{-1}(G) \times f_2^{-1}(G) \in P_{\theta} O(X_1 \times X_2) \).
Which implies that \( h \) is \( p_{\theta} \)-irresolute.

Theorem 2.17: Let \( X, Y_1 \), and \( Y_2 \) be any topological spaces, and \( f_i : X \to Y_i \), for \( i=1,2 \) are mappings. If \( g : X \to Y_1 \times Y_2 \) defined by \( g(x) = (f_1(x), f_2(x)) \) is a \( p_{\theta} \)-irresolute then \( f_1 \) and \( f_2 \) are \( p_{\theta} \)-irresolute.

Proof: The proof is similar to Theorem 2.16.

3. Basic Backgrounds in Nonstandard analysis:

In this section, we use E. Nelson’s Nonstandard Analysis construction, based on a theory called internal set theory IST[7]. The axioms of IST is the axiom of Zermelo-Frankel with the axiom of choice (briefly ZFC) together with three axioms which are the transfer principle (T), the idealization principle (I), and the standardization principle (S), are stated by the following

Transfer principle
Let \( A(x, t_1, t_2, ..., t_n) \) be an internal formula with free variables \( x, t_1, t_2, ..., t_n \)
Only then
\[ \forall^{st} t_1, t_2, ..., t_n (\forall^{st} x A(x, t_1, t_2, ..., t_n)) \Rightarrow (\forall x A(x, t_1, t_2, ..., t_n)) \]

Example 3.1: Consider the following statement:
\[ \exists^{st} x \in R \forall^{st} y \in R, such that x, y = x \]
Applying transfer principle, we have \( \exists y \in R, \forall x \in R, such that x, y = x \)
Thus, we may assert that \( R \) has a unique multiplicative identity. Furthermore, recalling that we can identify \( R^{st} \) as a subset of \( R \), we can say that this is 1.

The primary use of the transfer principle is that if one wishes to prove a theorem about the standard universe, it suffices to prove an analogous theorem with standard parameters in the enlarged universe.

Idealization Principle (I)
Let \( B(x, y) \) be an internal formula with free variables \( x, y \) and with possibly other free variables then
\[ \forall^{st} z \exists x \forall y \in Z \wedge B(x, y) \Rightarrow \exists x \forall^{st} y B(x, y) \]

Standardization Principle (S)
Let \( F(Z) \) be a formula, internal or external with free variables \( z \) and with possibly other free variables. Then,
\[ \forall x, y \in X, \forall z \in Y \Rightarrow z \in X \land F(z). \]

Every set or element defined in a classical mathematics is called standard. Any set or formula which does not involve new predicates "standard, infinitesimal, limited, unlimited is called internal, otherwise it is called external

**Definition 3.2 [5]:** Let \((X, \tau)\) be a standard topological space. Then, the P\(\theta\)-monad at a standard point \(a \in X\) is defined as follows

\[ \text{P}\(\theta\)-monad} = \bigcap \{ \text{pClA} : A \in \mathcal{P}(X) \text{ and } a \in A \}, \]

and is denoted by \(\mu_{\text{P}\(\theta\)}(a)\).

**Theorem 3.3 [5]:** Let \((X, \tau)\) be a standard topological space, and let \(a \in X\) be any element. Then, there exists a standard preopen \(H\) such that \(\text{pClH} \subseteq \mu_{\text{P}\(\theta\)}(a)\).

**Theorem 3.4 [5]:** Let \(A\) be a standard subset of a standard space \(X\). Then, \(A\) is pre-\(\theta\)-open set if and only if \(\mu_{\text{P}\(\theta\)}(a) \subseteq A\) for each \(a \in A\).

**Proof:** Assume that \(A\) is pre-\(\theta\)-open set and let \(a \in A\) then there exists a standard pre-open \(G\) such that \(a \in G \subseteq \text{pClG} \subseteq A\).

Now \(\bigcap \{ \text{pClG} : G \in \mathcal{P}(a) \} \subseteq \text{pClG} \subseteq A\), by Definition 3.2, \(\mu_{\text{P}\(\theta\)}(a) \subseteq A\).

Conversely, suppose that \(\mu_{\text{P}\(\theta\)}(a) \subseteq A\) for each \(a \in A\), then by Theorem 3.3, there exists a standard preopen set \(G\) such that \(\text{pClG} \subseteq \mu_{\text{P}\(\theta\)}(a)\).

Thus \(a \in G \subseteq \text{pClG} \subseteq A\), for each standard \(a\).

Therefore, by transfer principle, we have \(a \in G \subseteq \text{pClG} \subseteq A\), for each \(a\).

Hence, \(A\) is pre-0-open set.

### 4. P\(\theta\)-Topological Groups

**Definition 4.1:** Let \(G\) be a standard group and \((G, \tau)\) be a standard topological space. Then, \((G, \tau)\) is said to be **p\(\theta\)-topological group**, if the mappings

\[ g: G \times G \to G, \text{ defined by } g(x, y) = xy \]

\[ h: G \to G, \text{ defined by } h(x) = x^{-1} \]

are p\(\theta\)-irresolute.

**Example 4.2:** Let \(G = \mathbb{Z}_2\) be a group of integer modulo 2, and \(\tau\) be an indiscrete space then the maps

\[ g: G \times G \to G, g(x, y) = xy \]

\[ h: G \to G, h(x) = x^{-1} \]

are p\(\theta\)-irresolute.

**Theorem 4.3:** Let \(G\) be a standard group having a standard topology \(\tau\). Then \((G, \tau)\) is a standard p\(\theta\)-topological group, if and only if the mapping

\[ f: G \times G \to G, \quad f(x, y) = xy^{-1} \]

is p\(\theta\)-irresolute.

**Proof:** The proof is obvious.

**Theorem 4.4:** \((G, \tau)\) is a standard p\(\theta\)-topological group, if and only if the following conditions are satisfied

i) For every standard \(x, y \in G\) and each standard pre-0-open set \(H\) containing \(x, y\), there exist a standard pre-0-open sets \(U\) and \(V\) of \(x\) and \(y\) respectively such that \(U, V \subseteq H\).

ii) For every standard \(x \in G\) and each standard pre-0-open set \(V\) containing \(x^{-1}\), there exists a standard pre-0-open sets \(U\) of \(x\) such that \(U^{-1} \subseteq V\).

Where \(U^{-1} = \{ x^{-1} : x \in U \}\), \(U, V = \{ x, y : x \in U \text{ and } y \in V \}\).

**Proof:** Let \((G, \tau)\) be a standard p\(\theta\)-topological group,
and let $H$ be a standard pre-$\theta$-open set containing $x, y$, since the mapping $f: G \times G \to G$, defined by $f(x, y) = xy$, is p$\theta$-irresolute, then $f^{-1}(H) = \{(x, y) \in G \times G; f(x, y) \in H\} = \{(x, y) \in G \times G; x \cdot y \in H\}$ is pre-$\theta$-open subset of $G \times G$.

Thus, there exist pre-$\theta$-open sets $U$ and $V$ of $x$ and $y$, respectively in $G$ such that $f^{-1}(H) = U \times V$

Now, $U \cdot V = \{x, y; x \in U \text{ and } y \in V\} = \{x, y; (x, y) \in U \times V\}$

$= \{x, y; (x, y) \in f^{-1}(H)\} = \{x, y; f(x, y) \in H\}$

Let $V$ be a pre-$\theta$-open set containing $x^{-1}$, since the mapping $h: G \to G$, defined by $h(x) = x^{-1}$ is p$\theta$-irresolute function, then $h^{-1}(V)$ is pre-$\theta$-open set.

Therefore, there exists a pre-$\theta$-open set $U$ of $x$ such that $h^{-1}(V) = U$.

Now $U^{-1} = \{x^{-1}; x \in U\}$

$= \{x^{-1}; x \in h^{-1}(V)\}$

$= \{x^{-1}, h(x) \in V\}$

$= \{x^{-1}, x^{-1} \in V\} \subseteq V$.

The converse part is obvious.

**Definition 4.5:** A mapping $f: (G, \tau) \to (G^*, \tau^*)$ is called p$\theta$-homeomorphism if

i) $f$ is bijective.

ii) $f$ is p$\theta$-irresolute.

iii) $f^{-1}$ is p$\theta$-irresolute.

**Theorem 4.6:** Let $(G, \tau)$ be a standard p$\theta$-topological group, then the following mappings are p$\theta$-homeomorphism.

i) $r_\alpha: (G, \tau) \to (G, \tau)$, defined by $r_\alpha(x) = xa$.

ii) $l_\alpha: (G, \tau) \to (G, \tau)$, defined by $l_\alpha(x) = ax$.

iii) $f: (G, \tau) \to (G, \tau)$, defined by $f(x) = x^{-1}$.

iv) $g: (G, \tau) \to (G, \tau)$, defined by $g(x) = axa^{-1}$.

are p$\theta$-homeomorphism, for a fixed $\alpha \in G$.

**Proof:** As a sample we proof (i)

It is clear that $r_\alpha: (G, \tau) \to (G, \tau)$, defined by $r_\alpha(x) = xa$ is bijective mapping.

Let $H$ be a pre-$\theta$-open set containing $x \cdot a$. Since $(G, \tau)$ is a p$\theta$-topological group, then by Theorem 4.4, there exists a pre-$\theta$-open sets $U$ and $V$ of $x$ and $a$ respectively such that $U \cdot V \subseteq H$.

Therefore $r_\alpha(U) \subseteq H$.

Hence $r_\alpha$ is p$\theta$-irresolute.

Now let $y = r_\alpha(x)$, then $y = xa, x = ya^{-1}$

Which implies that $r_\alpha^{-1}(y) = r_\alpha^{-1}(x) = xa^{-1}$

By similar way one can prove that $r_\alpha^{-1}$ is p$\theta$-irresolute.

**Theorem 4.7:** Let $(G, \tau)$ be a p$\theta$-topological group, and let $U$ and $V$ be a subset of $G, g \in G$, then
i) If $V$ is a pre-$0$-open set, then $Vg, gV, gV g^{-1}$ and $V^{-1}$ are pre-$0$-open sets.

ii) If $U$ is a pre-$0$-closed set, then $Ug, gU, gU g^{-1}$ and $V^{-1}$ are pre-$0$-closed sets.

iii) If $V$ is a pre-$0$-open set and $A$ is any subset of $G$, then $VA$ and $AV$ are pre-$0$-open sets.

**Proof:** (i) and (ii) follow directly from Theorem 4.6.

(iii) Since, $VA = \bigcup_{\alpha \in A} V\alpha$ by part (i), $V\alpha$ is a pre-$0$-open set, by Theorem(2.4), we have $VA$ is pre-$0$-open set.

By similar way, we can prove that $AV$ is pre-$0$-open set.

**Theorem 4.8:**

A non-trivial standard p0-topological group has no fixed point property.

**Proof:** Let $(G, \tau)$ be a standard p0-topological group, for any $a \in G$ such that $a \neq e$, where $e$ is the identity element in $G$.

The mapping $r_a: (G, \tau) \to (G, \tau)$, defined by $r_a(x) = xa$ is p0-irresolute function

Suppose that $r_a(x) = x$, then $xa = x$, since $x \in G$ and $G$ is a group.

Therefore, $a = e$, which is contradiction

**5. Some properties of p0-monads in p0-topological groups:**

In this section, we give some properties of p0-monads in p0-topological groups, by using nonstandard techniques.

**Theorem 5.1:** Let $a$ and $b$ be any two standard points in p0-topological group $(G, \tau)$, then $\mu_{p0}(a), \mu_{p0}(b) \subseteq \mu_{p0}(a, b)$.

**Proof:** Let $x \in \mu_{p0}(a)$ and $y \in \mu_{p0}(b)$. We have to show that for any pre-$0$-open set $W$ of $a, b$ in $G$, $x, y \in W$. By Theorem 3.4 there exists two standard pre-$0$-open sets $U$ and $V$ of $a$ and $b$ respectively, then $x \in U$ and $y \in V$.

Since $(G, \tau)$ is a p0-topological group, by Theorem 4.4, for any standard pre-$0$-open set $W$ containing $a, b$, we have $U, V \subseteq W$, therefore $x, y \in W$.

Hence $\mu_{p0}(a), \mu_{p0}(b) \subseteq \mu_{p0}(a, b)$.

**Theorem 5.2:** Let $a$ be a standard points in p0-topological group $(G, \tau)$, then $\mu_{p0}(a^{-1}) = (\mu_{p0}(a))^{-1}$.

**Proof:** Let $V$ be a pre-$0$-open set containing $a^{-1}$. Since, $(G, \tau)$ is a standard p0-topological group, by Theorem 4.4 there exists a standard pre-$0$-open sets $U$ of $x$ such that $a^{-1} \subseteq U^{-1} \subseteq V$.

Then by Theorem 3.2 $\mu_{p0}(a^{-1}) \subseteq V, \mu_{p0}(a) \subseteq U$, and $U^{-1} \subseteq V$.

Therefore, $(\mu_{p0}(a))^{-1} \subseteq U^{-1} \subseteq V$, as $V$ was an arbitrary standard pre-$0$-open set, since $P0(X) \subseteq PO(X)$, and since $V \subseteq pClV$, we have $(\mu_{p0}(a))^{-1} \subseteq \bigcap \{pClV; V \in PO(X, a^{-1})\} = \mu_{p0}(a^{-1})$.

If we replace $a$ by $a^{-1}$, we get

$\mu_{p0}(a^{-1}) \subseteq (\mu_{p0}(a))^{-1}$.

Which implies that $\mu_{p0}(a^{-1}) = (\mu_{p0}(a))^{-1}$.

**Theorem 5.3:**

Let $a$ and $b$ be any two standard points in standard p0-topological group $(G, \tau)$, then $\mu_{p0}(a), \mu_{p0}(b) = \mu_{p0}(a, b)$.
Proof: The proof is similar to Theorem 5.1.

**Theorem 5.4:**
If $U$ is a standard pre-0-open subset of $G$, then $U.a$ is also standard p0-open subset of $G$

**Proof:** Let $U$ be a standard pre-0-open subset of $G$, and let $b \in U$, then by Theorem 3.3, $\mu_p(b) \subseteq U^*$. by transfer principle we have $\mu_p(b) \subseteq U$.

Now, let $c \in U, a$, then $c = da$, for some $d \in U$

$\mu_p(d).a \subseteq \mu_p(d), \mu_p(a)$ by Theorem 5.3, we have

$\mu_p(d).a \subseteq \mu_p(d.a) = \mu_p(c) ....(1)$

If $e \in \mu_p(c)$, then $f = e \alpha^{-1}$ such that

$f \in \mu_p(e)\mu_p(\alpha^{-1}) = \mu_p(e, \alpha^{-1}) = \mu_p(d)$.

Thus, $e = e, \alpha^{-1}.a = f . a \in \mu_p(d).a$,

that is $\mu_p(c) \subseteq \mu_p(d).a$ .........(2)

from (1) and (2), we have $\mu_p(c) = \mu_p(d).a$

Hence, $\mu_p(c) \subseteq U.a$, by Theorem 3.4 we obtain that $U.a$ is pre-0-open set.

**Theorem 5.5:** Let $(G, \tau)$ be a standard p0-topological group, then $U$ is a standard pre-0-open subset of $G$, if and only if $U.a$ is also standard p0-open subset of $G$.

**Proof:** From Theorem 5.4, we have if $U$ is a standard pre-0-open subset of $G$, then $U.a$ is also standard pre-0-open subset of $G$.

It is enough to show that if $U.a$ is a standard pre-0-open subset of $G$, then $U$ is also standard pre-0-open subset of $G$, for any standard element $a$ in $G$, since $G$ is a group and $a \in G$, then $a^{-1} \in G$, by Theorem 5.4 $(U.a).a^{-1} = U$ is also pre-0-open set.

**Theorem 5.6:** $\mu_p(e)$ is a subgroup of a group $G$, where $e$ is the identity element in $G$.

**Proof:** It is clear that $e \in \mu_p(e)$, let $a, b \in \mu_p(e)$.

Then, $a, b \in \mu_p(e)$, by Theorem 5.1, we have $a, b \in \mu_p(e)$

Let $\in \mu_p(e)$, then $a^{-1} \in (\mu_p(e))^{-1}$, by Theorem 5.2, $a^{-1} \in \mu_p(e)$

Hence, $\mu_p(e)$ is a subgroup of $G$. 
REFERENCES

[1] Ahmed N.K., Hussein S.A. (2007), On $\Phi$-open sets in topological spaces, J. of Kirkuk Univ. Vol(2), No.(3) (125-132).

[2] Dontchev J. (2000), Survey on preopen sets. The proceedings of the Yatsushiro Topological Conference 1-18.

[3] Dontchev J., Ganster M., and Noiri T. (2000), On p-closed Spaces, Internat. J. Math. & Math. Sci., 24(3), 203-212.

[4] Hussein S.A. (2003), Application of $\phi$-open sets in topological spaces. M.Sc. Thesis, College of Education, Univ. of Salahaddin.

[5] Ismail T.H., Hamad I.O., and Hussein S.A., $\Psi$-monads in General Topology, submitted to publication.

[6] Mashhour A.S., Abd El-Monsef and El-Deeb S.N. (1982), On precontinuous and weak precontinuous mapping, Proc. Math. and Phys. Soc. Egypt 51, 47-53.

[7] Nelson, E. (1977); Internal Set Theory: A New Approach to Nonstandard Analysis, Bull. Amer. Math. Soc., Vol. 83, No. 6, pp. 1165-1198.