Significance of tension for gravitating masses in Kaluza-Klein models

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In this report, we consider the six-dimensional Kaluza-Klein models with spherical compactification of the internal space. Here, we investigate the case of bare gravitating compact objects with the dustlike equation of state $\rho_0 = 0$ in the external (our) space and an arbitrary equation of state $\rho_1 = \Omega \epsilon$ in the internal space. These models satisfy the classical gravitational tests. However, we show that gravitating masses acquire effective relativistic pressure in the external space. Such pressure contradicts the observations of compact astrophysical objects (e.g., the Sun). The equality $\Omega = -1/2$ (i.e., tension) is the only possibility to preserve the dustlike equation of state in the external space. Therefore, tension plays a crucial role for the considered models.

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I. INTRODUCTION

The multidimensionality of spacetime is an essential property of the modern theories of unification such as superstrings, supergravity and M-theory, which have the most self-consistent formulation in spacetime with extra dimensions [1]. Obviously, these physical theories should be consistent with observations. For example, in the weak-field limit they must satisfy gravitational experiments such as the perihelion shift, the deflection of light, the time delay of radar echoes and parameterized post-Newtonian parameters. It is well known that general relativity in four-dimensional spacetime is in good agreement with these experiments [2]. Therefore, to investigate the similar correspondence for multidimensional theories, in our papers [3–5] we have considered popular Kaluza-Klein models with toroidal compactification of the internal space. We have shown that a matter source in the form of a dustlike compact gravitating object failed with the observations. Here, the dustlike equation of state $p = 0$ holds in all spatial dimensions. The obtained result was surprising to us because this approach is the most natural one for the ordinary astrophysical objects (such as our Sun) and it works well in general relativity [6]. It turned out that to satisfy the experimental data, the matter source must have negative pressure (i.e., tension) in the internal spaces [4, 5]. We have shown that latent solitons (in particular, the uniform black strings and black branes) satisfy the gravitational experiments at the same level of accuracy as general relativity. In general case, the variation of the total volume of the internal spaces generates the fifth force [11]. This is the main reason of the problem. However, in the case of the latent solitons, tension of the gravitating source eliminates the fifth force, resulting in agreement with the observations. Therefore, tension plays a crucial role here. For uniform black strings/branes with toroidal compactification, the equation of state in the internal spaces is $\rho_1 = -\epsilon/2$.

A dustlike (in all spatial dimensions) matter source was also considered for Kaluza-Klein models with spherical compactification of the internal space [12, 13]. In contrast to the case of toroidal compactification, this model can satisfy the gravitational experiments if the internal space is stabilized what happens for a positive six-dimensional cosmological constant [13, 14]. Here, the fifth force is replaced by Yukawa interaction which is short-range for large Yukawa masses. Therefore, at large three-dimensional distances, the effect of this interaction is negligibly small. Roughly speaking, the agreement with observations occurs asymptotically. Moreover, all models where a matter source has the dustlike equation of state $\rho_0 = 0$ in the external (our) space and an arbitrary equation of state $\rho_1 = \Omega \epsilon$ in the internal space satisfy asymptotically the gravitational experiments [14]. We have shown that all these models tend asymptotically to the exact black brane solution with spherical compactification [14]. For such exact black brane solution, the parameter of state $\Omega = -1/2$, in full analogy with black branes with toroidal compactification. Therefore, for any $\Omega$ (including the dustlike value $\Omega = 0$), considered models can satisfy the gravitational experiments. However, the analysis conducted in the present paper shows that in all models with $\Omega \neq -1/2$ a gravitating matter source acquires effective relativistic pressure in the external (our) space. Obviously, this is not acceptable for astrophysical objects such as our Sun. Therefore, in spite of the agreement (asymptotical) with the gravitational experiments, such models fail with the observations. Only in the case of the black brane with $\Omega = -1/2$, a matter source remains dustlike in the external space. Therefore, tension also plays a crucial role in models with spherical compactification.

In Sec. II, we demonstrate that a compact gravitating source in the considered models acquires effective relativistic pressure in the external space except the case of tension in the internal space. This is the only possibility.
to preserve the dustlike equation of state in the external space. The main results are briefly summarized in concluding Sec. III.

II. EFFECTIVE ENERGY DENSITY AND PRESSURE OF THE GRAVITATING MASS

As we pointed out in papers [12, 13], the matter source in the Kaluza-Klein models with spherical compactification should consist of two parts. First, it is the homogeneous perfect fluid which provides spherical compactification of the internal space. Second, it is the gravitating object, which is spherically symmetric and compact (i.e. pointlike) in the external space and uniformly smeared over the internal space. The total energy-momentum tensor is the sum of these parts with the following nonzero components:

\[ T^0_0 = \varepsilon + \varepsilon^1 + \rho(r_3)c^2, \]
\[ T^a_a = \varepsilon + \varepsilon^1, \quad a = 1, 2, 3, \]
\[ T^4_4 = T^5_5 = -\omega_1 \varepsilon - \omega_1 \varepsilon^1 - \Omega \rho(r_3)c^2, \]

where \( \varepsilon \) is the energy density of the homogeneous perfect fluid, \( \rho(r_3) \) is the rest mass density of a compact gravitating object and \( \varepsilon^1 \) is the excitation of the background matter energy density by this object. The background matter is fine-tuned with the radius \( a \) of the two-sphere: \( \varepsilon = [(1 + \omega_1)\kappa_6a^2]^{-1} \), and a free parameter \( \omega_1 \) defines the equation of state of this matter in the internal space. The model may also include a six-dimensional cosmological constant \( \Lambda_6 \), which is fine-tuned with the parameters of the model: \( \Lambda_6 = \omega_1 \varepsilon_6 \). This bare cosmological constant is absent if \( \omega_1 = 0 \). The gravitating compact object has the dustlike equation of state in the external (our) space \( \rho = 0 \) and an arbitrary equation of state \( \rho_3 = \Omega \rho(r_3)c^2 \) in the internal space. We also suppose that this object is uniformly smeared over the internal space: \( \rho(r_3) = \rho_3(r_3)/V_2 \) where \( V_2 = 4\pi a^2 \).

In the case of a pointlike mass in the external space \( \rho_3(r_3) = m\delta(r_3) \). It also worth noting that the six-dimensional and Newton’s gravitation constants are related as follows: \( \kappa_6/V_2 = \kappa_N = 8\pi G_N/c^4 \).

The metrics for the considered model in isotropic coordinates takes the form (see for details [12, 13])

\[ ds^2 = A^2dt^2 + Bdx^2 + Cdy^2 + Ddz^2 + E(d\xi^2 + \sin^2 \xi d\eta^2) \]

with \( A \approx 1 + A^1(r_3), B \approx -1 + B^1(r_3), C \approx 1 - C^1(r_3), D \approx -1 + D^1(r_3), E \approx -a^2 + E^1(r_3) \), where all metric perturbations \( A^1, B^1, C^1, D^1, E^1 \) are of the order \( O(1/c^2) \) and can be found with the help of the Einstein equations. They read

\[ A^1 = \frac{2\varphi_N}{c^2} + \frac{E^1}{a^2}, \]
\[ B^1 = C^1 = D^1 = \frac{2\varphi_N}{c^2} - \frac{E^1}{a^2}, \]
\[ E^1 = a^2\frac{\varphi_N}{c^2} (1 + 2\Omega)e^{-r_3/\lambda}, \quad \lambda \equiv a/\sqrt{\omega_1}, \]

where the Newton’s potential is \( \varphi_N = -G_N m/r_3 \). The solution (15) takes place for \( \omega_1 > 0 \). In the opposite case \( \omega_1 < 0 \), we get the nonphysical oscillating solution. If \( \Omega \neq -1/2 \), Eq. (14) demonstrates that conformal variations of the internal space volume generate the Yukawa interaction. The admixture of such interaction to \( A^1, B^1, C^1, D^1 \) is negligible at distances \( r_3 \gg \lambda \), and we achieve good agreement with the gravitational tests in this region.

The Einstein equations also lead to the following important relation: \( \varepsilon^1 = E^1/(\kappa_6a^4) \). Eq. (6) shows that this background perturbation is localized around the gravitating mass and falls off exponentially with the distance \( r_3 \) from the gravitating object. Therefore, the bare gravitating mass is covered by this ”coat”. For an external observer, this coated gravitating mass is characterized by the effective energy-momentum tensor with the following nonzero components:

\[ T_0^{0(\text{eff})} \approx \varepsilon^1 + \rho(r_3)c^2 \]
\[ = -\left(1 + 2\Omega\right)\frac{mc^2}{V_2^2r_3} \exp\left(-\frac{\sqrt{\omega_1}}{a}r_3\right) + \frac{1}{V_2}mc^2\delta(r_3), \]
\[ T_{\alpha}^{(\text{eff})} \approx \varepsilon^1 \]
\[ = -\left(1 + 2\Omega\right)\frac{mc^2}{V_2^2r_3} \exp\left(-\frac{\sqrt{\omega_1}}{a}r_3\right), \quad \alpha = 1, 2, 3, \]
\[ T_4^{(\text{eff})} = T_5^{(\text{eff})} \approx -\omega_1 \varepsilon^1 - \Omega \rho(r_3)c^2 \]
\[ = \left(1 + 2\Omega\right)\frac{\omega_1 mc^2}{V_2^2r_3} \exp\left(-\frac{\sqrt{\omega_1}}{a}r_3\right) - \frac{\Omega}{V_2}mc^2\delta(r_3). \]

These components define the effective energy density and pressure of the coated gravitating mass. For example, from Eq. (9) we conclude that this mass acquires relativistic pressure \( p^{(\text{eff})} = -T_\alpha^{(\text{eff})} \) in the external space. To demonstrate it more clearly, we can replace the rapidly decreasing exponential function by the delta function:

\[ \frac{1}{r_3} \exp\left(-\frac{\sqrt{\omega_1}}{a}r_3\right) \rightarrow \int \frac{1}{r_3} \exp\left(-\frac{\sqrt{\omega_1}}{a}r_3\right) dV' \times \delta(r_3) \]
\[ = \frac{V_2}{\omega_1^3} \delta(r_3). \]

Then, Eqs. (7), (8) read

\[ T_0^{0(\text{eff})} \rightarrow -\left(1 + 2\Omega\right)\frac{mc^2}{V_2^2\omega_1}\delta(r_3) + \frac{1}{V_2}mc^2\delta(r_3) \]
\[ = \rho(r_3)c^2 \left(1 - \frac{1 + 2\Omega}{2\omega_1}\right), \]
\[ T_{\alpha}^{(\text{eff})} \rightarrow -\left(1 + 2\Omega\right)\frac{mc^2}{V_2\omega_1}\delta(r_3) \]
\[ = -\rho(r_3)c^2 \frac{1 + 2\Omega}{2\omega_1}, \quad \alpha = 1, 2, 3, \]
\[ T_4^{(\text{eff})} \rightarrow \left(1 + 2\Omega\right)\frac{mc^2}{V_2^2}\delta(r_3) - \frac{\Omega}{V_2}mc^2\delta(r_3) \]
\[ = \rho(r_3)c^2 \frac{1}{2}. \]
These equations give us the effective energy density and pressure of the coated gravitating mass. We see that the effective energy density $\varepsilon^{(\text{eff})} = T_0^{0(\text{eff})}$ and effective pressure in the external (our) space $p_0^{(\text{eff})} = -T_0^{0(\text{eff})}$ depend on parameter $\Omega$, which defines the equation of state of the bare gravitating mass in the internal space. On the other hand, the effective pressure in the internal space $p_4^{(\text{eff})} = -T_4^{4(\text{eff})}$ does not depend on $\Omega$ and is negative. From Eq. (12), we clearly see that the coated gravitational mass acquires relativistic pressure in the external (our) space. Obviously, it is not the case for compact astrophysical objects, such as our Sun. Usually, they have nonrelativistic velocities in the three-dimensional space, and their pressure is much less than the energy density. Moreover, in the weak-field limit the pressure for these objects is taken in the dustlike form $p_0 = 0$. It can be easily seen that the equality $\Omega = -1/2$ is the only possibility to achieve $p_0^{(\text{eff})} = 0$ for our model. It means that the bare gravitating mass should have tension with equation of state $\bar{p}_0 = -\bar{\varepsilon}/2$ in the internal space. Then, the effective and bare energy densities coincide with each other and the gravitating mass remains pressureless in our space. In the internal space the gravitating mass still has tension with the parameter of state $-1/2$. Therefore, tension plays a crucial role for models with spherical compactification.

III. CONCLUSION

In this report, we have considered the six-dimensional Kaluza-Klein models with spherical compactification of the internal space. We have investigated the case when a compact gravitating mass satisfies asymptotically the gravitational experiments. This bare mass has the dust-like equation of state $p_0 = 0$ in the external (our) space and an arbitrary equation of state $\bar{p}_1 = \Omega \bar{\varepsilon}$ in the internal space. The bare mass disturbs the background matter which provides spherical compactification of the internal space. This perturbation is localized around the bare mass. As a result, the mass is covered by this coat. We have shown that the coated gravitating mass acquires the effective relativistic pressure in the external space. Such pressure contradicts the observations of the compact astrophysical objects. These objects usually have negligible pressure. The equality $\Omega = -1/2$ is the only possibility to preserve the dust-like equation of state in the external space. Therefore, to be in agreement with observations, bare gravitating masses in the models with spherical compactification should have tension with $\Omega = -1/2$ in the internal space.

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