Dark Energy from ferromagnetic condensation of cosmic magninos

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It is proposed that an ultra-light fermionic species, dubbed cosmic magnino has condensed into a ferromagnetic state in the Universe. The extended structure of domain walls associated with this ferromagnetism accounts for the observed Dark Energy. In modification of the situation with an electron gas, it is proposed that the Stoner criterion is satisfied due to magnetic dipolar repulsion. The cosmological requirements then yield a lower bound on the magnetic moment of the cosmic magnino. The proposed magnetism is supposed to be associated with a new non-standard electromagnetism. If the magnino is also electrically charged under this electromagnetism, the corresponding oppositely charged heavier species would account partially or entirely for the Dark Matter in the Universe.

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INTRODUCTION

The discovery of the Dark Energy component\cite{1,2} of the cosmological energy density from direct observations, in concordance with the WMAP precision data\cite{3}, presents a new challenge to fundamental physics. The extremely small value of the mass scale associated with this energy density makes it unnatural as Cosmological Constant\cite{4}, and therefore demands an unusual mechanism for relating it to the known physics of elementary particles. On the other hand a new window to the very low mass physics has been opened up by the discovery of the low mass scale of neutrinos\cite{5}. Further, a variety of theoretically motivated ultra-light species are currently being sought experimentally\cite{6}. We may therefore exploit the presence of an ultra-light sector to explain the Dark Energy phenomenon autonomously at a low scale, without direct reference to its high scale connection with known physics.

The analysis of \cite{3} assumes the Λ-CDM model, with equation of state of the Dark Energy constrained to \( \rho/\rho_0 = w = -1 \). However alternative analyses (see for instance \cite{11}) show that dynamically evolving \( w \) is also consistent with data. It has been argued, early in \cite{13} that the equation of state obeyed by the observed contribution to the energy density could be well fitted by a network of frustrated domain walls\cite{9}. Further, a variety of theoretically motivated ultra-light species are currently being sought experimentally\cite{6}. We may therefore exploit the presence of an ultra-light sector to explain the Dark Energy phenomenon autonomously at a low scale, without direct reference to its high scale connection with known physics.

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The proposed model involves a new fermionic species responding to a hidden electromagnetism and whose condensed state can be characterised by the familiar Stoner criterion\cite{12} of ferromagnetism. A novel feature I propose is to assume the Stoner mechanism to arise from dipolar force between magnetic moments. The reason is that the fermion gas in the cosmological setting has to be extremely rarefied in which case the screened Coulomb potential which is exponentially cut off becomes ineffective, while magnetic force is not screened and obeys a power law. The proposed mechanism of condensation due to magnetic dipolar interaction between neutral fermions was first proposed in \cite{13}, which led to the calculations of \cite{14,15,16}. Stoner ansatz has received extensive theoretical attention in the past decade. In the context of a three dimensional gas, controlled verification of Stoner mechanism for neutral ultra-cold gas of spinless atoms is reported in \cite{17}, where a Feshbach resonance is used for tuning the repulsion. It would be interesting to also test the dipolar mechanism proposed here in a laboratory system.

We dub this species magnino, being an ultra-light fermionic species whose predominant property manifesting itself today is magnetism. As to the magnetic moment, there are two possibilities. One is that the magnetic moment is induced, and the other that it is intrinsic. In the latter case, the magnino is also electrically charged, and another species, equally and oppositely charged should be present for neutrality of the Universe. This gives rise to the further possibility that it is heavier, not participating in the condensation mechanism, but be a component of Dark Matter.

The proposed new species, plus one or more accompanying species such as the corresponding photons, would contribute to the tally of relativistic species present at the epoch of Big Bang Nucleosynthesis (BBN). This is justified by the analyses of new data\cite{3,18} which admit the presence of approximately one new effective degree of freedom. The proposed model can accommodate a Dark Matter candidate, in which case it would also provide...
an explanation for the “cosmic concordance”. An additional possibility in the mechanism is an explanation of the seeds required to produce the inter-galactic magnetic fields, as I discuss at the end. In the following I use the units \( \hbar = c = 1 \) such that \( \hbar c = 1 \approx 200 \text{MeV-fermi} \), and express all dimensionful quantities in the units of eV.

**Cosmological setting**: A gas of slowly moving thin domain walls has an average stress tensor given by [19]

\[
\langle T^\mu_\nu \rangle = \frac{\eta^3}{3L} \text{diag}(3, -2, -2, -2)
\]

where \( \eta \) is the mass scale associated with the surface tension of the walls, and \( L \) is the average separation between the walls. Thus the wall gas (WG) satisfies the equation of state \( p_{\text{WG}} = (-2/3)\rho_{\text{WG}} \). A Universe containing non-relativistic matter, and domain wall structure formed on scales much smaller than the Hubble scale, is described by Friedmann-Robertson-Walker scale factor \( R(t) \) obeying

\[
\left( \frac{1}{R} \frac{dR}{dt} \right)^2 = \frac{8\pi}{3} G \left( \frac{R_0^3 P_{nm0}}{R^3} + \frac{\rho_{\text{WG}} R_0}{R} \right)
\]

where the subscript 0 refers to the present epoch. Let \( t_1 \) be the time when the the energy density of the walls gas equals the energy density in non-relativistic matter. Using the value of density fraction of matter \( \Omega_m \approx 0.3 \) and that of Dark Energy \( \Omega_{\Lambda} \approx 0.7 \) at the present epoch, gives \( (R_1/R_0)^2 = 3/7 \). Photon temperature at this epoch is \( T_1 = 4.18 K = 5.0 \times 10^{-4} \text{eV} \). The current contribution of Dark energy to the total energy density has the value \( \rho_{\text{DE}} \approx (3 \times 10^{-3} \text{eV})^4 \).

We shall assume the magnino (M) to be a very light species, of mass \( m_M \leq T_1 \), with a conserved number so that the abundance of the species relative to photons remains constant during the epochs under consideration. The number density of the species can be parameterised as \( N_M(t_1) = 3.56 \times 120 \text{Ycm}^{-3} \approx 3.2 \times 10^{-12} \text{Y(eV)}^3 \), where \( \text{Y} \) is an unknown factor.

**Ferromagnetism**: According to the Stoner ansatz [12] spontaneous ferromagnetism is a consequence of a shift in single particle energies, proportional to the difference between the spin up \( (N_\uparrow) \) and the spin down \( (N_\downarrow) \) populations. A parameter \( I \) is introduced to incorporate this, the single-particle energy spectrum being

\[
E_{\uparrow, \downarrow}(k) = E(k) - I \frac{N_\uparrow - N_\downarrow}{N}
\]

Using this it is shown [20, 21] that the ferromagnetic susceptibility is

\[
\chi = \frac{\chi_F}{1 - \frac{I}{\chi_F}}
\]

where \( \beta \) is a factor of order unity depending upon the geometry of the Fermi surface; for the spherical case having value \( \frac{4}{3} \). The criterion for spontaneous magnetization is \( \chi < 0 \). A sufficient condition for the gas to be spontaneously magnetised at zero temperature is the Stoner criterion,

\[
I > \frac{E_F}{\beta}
\]

The origin of such a large energy shift is supposed to be a repulsive interaction among the fermions, which makes it favourable for them to enter the state of aligned spins, which in turn due to Pauli exclusion principle ensures large enough a separation among them so as to reduce the repulsive energy. An estimate of the size of this “exchange hole” [22, 21] is given by the density deficit of same spin fermions in the vicinity of a given fermion, \( \Delta n_M = -0.86 n_M \). Let the two-particle long range interaction energy be \( \gamma^2 \) which is repulsive. This energy reduction should be proportional to \( \Delta n_M \). For the Stoner parameter \( I \) therefore stipulate the relation

\[
I = \gamma^2 \frac{\Delta n_M}{n_M}
\]

I now make the assumption that for the fermions under consideration, this coupling arises from magnetic dipole-dipole interaction, which is dominated by a repulsive contribution in an appropriate ferromagnetic state. The resulting increase in single particle energy can be estimated as

\[
\gamma^2 = \kappa_{JM} \frac{\mu^2_M |\Delta n_M|}{n_M}
\]

The favorable ferromagnetic state, the JM ansatz [12] turns out to be spheroidal, ensuring dominance of repulsion, and the related parameter \( \kappa_{JM} \) is a factor of order unity. Note that the interaction energy between non-relativistic dipoles goes as inverse third power of interparticle separation and hence consistent with scaling as \( |\Delta n_M| \).

The magnetic moment introduced above could either be intrinsic or induced,

\[
\mu_M = \frac{g_M e_X h}{2m_M}
\]

where \( e_X \) is the unit of charge of the new electromagnetic, and \( m_M \) is the mass of the magnino. For charged magnino, \( g_M \) at tree level has the Dirac value 2. If the magnino is to be electrically neutral, the factor \( g_M \) has to arise from radiative corrections or from compositeness. For example, for neutrinos the radiatively induced magnetic moment is expected to be small [23], or \( \mu_M/\mu_B < 10^{-15} \) as derived in [24] under certain reasonable assumptions. In a more general setting, \( g_M \) can be order unity as in the case of the neutron. Thus the Stoner criterion [25] becomes

\[
\alpha_X n_M \left( \frac{g_M}{m_M} \right)^2 > \frac{4}{3} \left\{ \left( 3\pi^2 n_M \right)^{2/3} + m_M^2 \right\}^{1/2} - m_M
\]
where $\alpha_X = e^2$ is the fine structure constant, and we have assumed $|\Delta n_M| \approx n_M$, and $\kappa = 1$ for simplicity. In the non-relativistic approximation when this mechanism is assumed to operate, this requirement can be simplified to $m_M < \frac{1}{4} \alpha_X \eta_{M}^{1/3}$. The critical temperature for such a phase transition is estimated to be $T_c = I/4[21]$. In the present case, this value becomes $T_c \approx m_M/\alpha_X^2$. This is far too high, since it allows creation of magnino pairs and condensation cannot be stable. We assume in the following that there is an epoch $t_2$ preceding $t_1$ when this phase transition is accomplished, with the requirement that the temperature $T_{h2}$ of the hidden photons at $t_2$ satisfies $T_{h2} < m_M$.

**Domain walls:** The energy density of the condensed state can be separated into a sum of two contributions, one from the configuration space degrees of freedom and the other due to the spin degrees of freedom. The latter may be modelled by a Landau-Ginzberg Lagrangian for one from the configuration space degrees of freedom and $\sigma^2$. Here $\sigma$ determines the magnitude of the magnetization per unit volume, estimated to be $\mu_M |\Delta n_M| [21]$ up to a factor of order unity. From standard solitonic calculation the domain walls have a width $w \sim (\sqrt{2} \sigma)^{-1}$ and energy per unit area $\eta^3 \sim \sqrt{2} \sigma^3$. These domain walls are not expected to be topologically stable. This is because the vacuum manifold which is a 2 dimensional sphere allows for the wall to develop holes, though classically suppressed by an energy barrier. The rate of decay of the walls is thus governed by tunneling processes which can be much slower than the age of the Universe. According to the JM ansatz for the ferromagnetic state introduced in [17] the fermi surface of the condensate is spheroidal, leading also to ferro nematic order in configuration space.

As for the momentum space degrees of freedom, they continue to behave like a degenerate quantum gas; however the formation of the domains breaks their almost scale invariant behaviour. Let us assume the average size of the domains to be characterised by a length scale $l$. Equivalently, there is one wall passing through a cubic volume of size $l^3$ on the average. The energy density trapped in such a wall is $\eta^3/w$. The scale $l$ of this microstructure is expected to be many orders of magnitude smaller than the scale of galactic clusters, $l_{gc}$, and certainly the Hubble scale. In the following we assume that the momentum degrees of freedom which are $O(l_{gc}^{-1})$ and larger, and certainly those that are $l^{-1}$ and larger cease to respond to the cosmic expansion. Only the degrees of freedom of very low momentum, smaller than $l_{gc}^{-1}$, continue to redshift as $R^{-1}$. The corresponding separation of the number densities may be denoted $n_{M,>}$ for the large momentum, which would be most of the number density contribution, and a small fraction $n_{M,<}$ for the low momentum modes.

This separation of scales makes it clear how the microscopic relation $\sigma \propto n_M$, could be consistent with the fact that the coarse grained wall gas energy density obeys the equation of state $p_{WG} = -(2/3)\rho_{WG}$. Once the walls have formed, $\sigma$ is essentially determined by $n_{M,>}$ and remains a constant. I assume therefore that the density $\rho_{WG}$ is set by the initial value $\sigma^2(t_2)$ at the epoch $t_2$ of emergence of wall gas, and subsequently scales as $1/R$ as required by the covariant conservation equation $d(p_{WG} R^3) + p_{WG} d R^3 = 0$. With this caveat, if the wall gas is to comprise the entire Dark Energy component of the Universe, we get the relation

$$\rho_{WG} \approx \left( \frac{g_{M}^2 \alpha_X}{m^2} \right) (n_{M,>}(t_2) - w/l \frac{R^2}{R_0} \approx \rho_{DE}$$

where we have replaced $w/l$ by its average value. The ratio $R_{2}/R_{1} \approx R_{2}/R_{0}$ requires the details of the ferromagnetic phase transition. If we assume the emergence of wall gas to have been much earlier than when the equality $\rho_{WG} \approx \rho_{w}$ was reached, then using $g_{M}^2 \approx O(1)$, $\alpha_X \approx 10^{-2}$, and $(w/l) \lesssim 10$, we get a bound $m_M/\gamma \lesssim 10^{-8} eV$.

**Other dark components:** If $g_{M}$ arises from generic radiative corrections, it is proportional to $m_M$ and then the above scenario does not work. The possibility of the magnino being a neutral composite like the neutron remains open. A more appealing possibility is that it is charged, however the large intrinsic magnetic moment could not be of standard electromagnetism without being detected so far in some of the astrophysical phenomena such as the cooling rates of supernovae and red giants. Thus the corresponding electromagnetism must be new. Then an oppositely charged partner must also be present in the Universe for neutrality. The results derived so far remain unaffected if this partner is a heavier species, not participating in the ferromagnetism, in other words, if the new unobserved sector is also asymmetric under charge conjugation like the observed one. Let us designate this oppositely charged partner $Y$, and assume it to have equal and opposite charge, and therefore the same abundance as, the magnino.

The third new component, the photons of the new electromagnetic force should also be present, with an entropy density in a ratio $\gamma_{s}$ to the entropy density of the standard photons. Since $T_{h2} < m_M$ at an epoch when usual photons have a temperature $T \gtrsim 10^{-4} eV$, $\gamma_{s} \ll 1$. From the observational bounds on the new effective relativistic degrees of freedom, it is necessary that $2 \gamma + \gamma_{s} < 1$. If we assume $Y$ to be a massive non-relativistic species at present epoch, we can now obtain a bound on its mass. Using the standard values of relative density fractions of baryonic and non-baryonic matter and the baryon to photon ratio, we get $m_Y < (3.22/\gamma) eV$ in order for $Y$ not to overclose the Universe. Since $\gamma < \frac{1}{2}$, $Y$ turns out to be non-relativistic at cosmic temperatures $< 1 eV$, i.e., after most of the primordial neutral Hydrogen has come into existence. The mass range suggested seems too
light to act as Dark Matter to assist structure formation, however there is no model independent lower bound on the Dark Matter candidate.\cite{30,31}, and the possibility of Dark Matter from hidden world can be analysed along the lines of \cite{32}. Note that depending on the temperature $T_2$ to be deduced from details of the wall condensation mechanism, $T$ can be considerably smaller than unity, and $m_Y$ can then be sufficiently large for it to be acceptable Cold Dark Matter.

Another interesting outcome of the magnetic nature of this condensation mechanism is that it may explain the existing inter-galactic magnetic fields. For this it is necessary that the two electromagnetism mix through kinetic terms. Then over a large number of domains encompassing the scale of galactic clusters, the fluctuations from the average value zero of the net magnetic field may be large enough, that after mixing with standard electromagnetism it gives rise to the seeds required for generating the intergalactic magnetic fields.\cite{33,34}. These effects are the subject of ongoing work.

A characteristic prediction of this mechanism is degradation of Dark Energy. The form of the Stoner requirement Eq. \ref{eq:sto} means that as the Universe expands, the left hand side of the inequality diminishes faster than the right hand side, and eventually the inequality cannot be satisfied. Thus sometime before temperature $T \to 0$, the ferromagnetic state melts away, so does the wall gas masquerading as Dark energy, and the Universe returns to being matter dominated.

In conclusion I have introduced the cosmic magnino whose mass scale is extremely small, perhaps to be matched only by that of the lightest neutrino. Such light Dirac mass\cite{35} as also the extra $U(1)$ gauge symmetry can be obtained in the context of grand unification. Compositness is also a possibility for the magnino, however ensuring a very small mass and a large induced dipole moment may be difficult to arrange in this case. The easier alternative is for it to also be charged under an abelian gauge force. In this case the particle is further accompanied by a heavier oppositely charged partner $Y$, which can potentially be Cold Dark Matter.

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