Mechanics of inspection robot traveling over the electrical line with constant velocity

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Abstract. This article aims to the construction of mathematical models for a situation in which the electrical line stretched and supported by two isolators, one horizontal and the other vertical and robot inspector moves with a constant speed on it. The expansion theorem is used to construct the equation of motion of the system and has been solved by two methods: expansion method and Galerkin method. As a result of even constant speed of motion of robot inspector along the electrical line, vibrations have been observed. These vibrations can cause damage to the line or the robot structure as well. Then, the effect of the resistance force, wind force, has been studied. As shown in results, resistance force acts as a damper to the system and causes a decrease in the amplitude of vibrations. The practical results are useful for engineers to select proper parameters and speed for designing and constructing the robot inspectors.

1. Introduction

Nowadays electrical energy has an inseparable role in human life. Inspection and maintenance of electrical transmission lines are necessary and done regularly according to the standards of each country [1–2], since any problem during the transmission of the electrical energy lines to cities, industries, etc. may affect human life.

Usually, inspection and maintenance of electrical transmission lines are done by human forces. By this, human forces are located in a potential hazard environment. Therefore, to reduce this risk, different types of robot inspectors are designed and used in some regions [1–10]. Each of these robots is capable of doing different tasks. One of these robots shown in figure 1 [11].

Executive and mathematical modeling experience shows that even with a constant speed movement of the robot over the line, vibrations occur [3–10, 12–16]. These vibrations can cause damage as well to the electrical line as to the robot. Therefore, it is needed to conduct mathematical modeling for different situations that may robot faces [11–16].

This article, which is the extended work of [12–15], deals with the construction of mathematical models for a situation in which the electrical line stretched and supported by two isolators, one horizontal and the other vertical and robot inspector moves with a constant speed on it. It follows by studying the effect of wind force (resistance force).
2. The fluctuation of the stretched string with a moving point load
This section deals with the dynamics of the conductor with a moving load is considered (figure 2). We consider a case in which the electrical line stretched and supported by two isolators, one horizontal and the other vertical (figure 2a). The horizontal one acts as simple pin support and the other is a massless roller that can move vertically. As illustrated in figure 2b, here, we consider the conductor is a taut string while the robot inspector is considered as a moving point load.

![Figure 2. (a) Electrical line hangs between two supports, and (b) concentrated load moves over the stretched string.](image)

2.1. The expansion method
We start to construct the problem statement for the dynamic deflection of a string using the expansion method. The kinetic energy ($K$) and the potential energy ($U$) of the string are:
where $T$ is the string tension force, $\rho$ is the density, $l$ is the string length. Differentiation with relative to the coordinate $x$ and time $t$ is shown by prime and dot.

Utilizing the Lagrangian $L = K - U$, when a system is subject to generalized forces $q(x, t)$, one can introduce them in extension Hamilton’s principle through the virtual work expression $\delta A = q(x, t) \delta u$. Extension Hamilton’s principle can be written as $\delta \int_0^t (L + A) dt = 0$.

The equation of motion can be obtained as explained in [17]:

$$
\int_0^l \rho \ddot{u} \delta u + \int_0^l Tu' \delta u + \int_0^l (Tu'' - \rho \ddot{u}) \delta u \ dx dt + q(x, t) = 0.
$$

Equating the third term in equation (2) to zero leads to the dynamic equation for the string as:

$$
\rho \ddot{u} - Tu'' = q(x, t).
$$

Since the variation $\delta u$ at the initial and final times are zero, then the first term in equation (2) is always zero. Equating the third term in equation (2) to zero leads to the dynamic equation for the string as:

$$
\rho \ddot{u} - Tu'' = q(x, t).
$$

Setting the second term in equation (2) to zero gives us the boundary conditions of the problem, i.e.:

$$
x = 0: \quad u = 0, \quad \dot{u} = 0; \\
x = l: \quad Tu' = 0,
$$

Initial conditions are $t = 0$: $u = 0, \dot{u} = 0$.

The loading point (Constant $P$) moves with a constant speed of $v$. Therefore, the force concentrated at a point $x$ will be $q(x, t) = P \delta(x - vt)$ ($\delta$ is the Dirac delta function).

The equation of motion for the systems described above can be written in the general form

$$
\ddot{u} - c^2 \dddot{u} = p \delta(x - vt),
$$

$$
c = \sqrt{\frac{T}{\rho}}, \quad p = \frac{P}{\rho}.
$$

The solution to the problem (5) can be constructed by the method of eigenfunctions [18] knowing the eigenvalues and eigenfunctions are given by $\omega_n = \lambda_n c, \varphi_n = \sin \lambda_n x, \quad \lambda_n = \frac{n \pi}{l}$ as:

$$
u(x, t) = \sum_{n=1}^{\infty} u_n(t) \varphi_n(x),
$$

where the unknown modal coordinate is designated by $u_n(t)$.

Recalling the properties of eigenfunctions

$$
\int_0^l \varphi_n \varphi_k dx = \delta_{kn} \equiv \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases} \Rightarrow u_n = \int_0^l u \varphi_n dx,
$$

and multiplying both sides of equation (5) by $\varphi_n$ and integrating, we obtain ordinary differential equations (ODE) for $u_n$:
\[ \ddot{u}_n + \omega_n^2 u_n = f_n(t), \]
\[ f_n(t) = \frac{\int_0^l \varphi_n(x) q(x,t) \, dx}{\int_0^l \varphi_n^2(x) \, dx} = \frac{1}{l} p \, \sin(\lambda_n vt). \]  

(8)

One can write the solution of equation (8) as:
\[ u_n(t) = C_{1n} \cos(\omega_n t) + C_{2n} \sin(\omega_n t) + \frac{2pl}{n^2 \pi^2 (c^2 - v^2)} \sin(\lambda_n vt), \]  

(9)

where \( C_{ij} \) and \( C_{ji} \) are arbitrary integration constants. Therefore, the solution for the string is obtained substituting equation (9) in equation (6) as:
\[ u(x,t) = \sum_{n=1}^{\infty} \left[ C_{1n} \cos(\omega_n t) + C_{2n} \sin(\omega_n t) + \frac{2pl}{n^2 \pi^2 (c^2 - v^2)} \sin(\lambda_n vt) \right] \sin(\lambda_n x). \]  

(10)

Under zero initial conditions, one can obtain \( C_{1n} = 0 \) and \( C_{2n} = -\frac{2plv}{\pi^2 (c^2 - v^2)} \).

Substituting integration constants in equation (10), we obtain the solution of problem (5):
\[ u(x,t) = \frac{2pl}{\pi^2 (c^2 - v^2)} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[ \sin(\lambda_n vt) - \frac{v}{c} \sin(\lambda_n ct) \right] \sin(\lambda_n x). \]  

(11)

The deflection of the conductor solving equation (11) is shown in figure 3. Parameters for calculations are: \( T=10 \) kN, \( P=1 \) kN, \( v=2 \) m/s, \( \rho=5 \) kg/m, \( l=200 \) m. One can see from figure 3 that even the robot moves with a constant speed over the line, oscillations appear in the vertical plane.

![Figure 3. Deflection of the electrical line with a moving load on it.](image)

2.2. Galerkin method

In this section, we construct an approximate solution of equation (3) in the matrix form:
\[ u(x,t) = \sum_{n=1}^{\infty} u_n(t) \phi_n(x) = U^T(t) \Phi(x), \]  

(12)
where $\Phi(x)$ is known as comparison functions which are orthogonal and orthonormal.

The approximate solution (12) will also satisfy all the boundary conditions, but in general, will not satisfy (3) identically since there remains an error defined by

$$e(x,t) = \ddot{U}^T \Phi - c^2 U^T \Phi^*, \quad (13)$$

which one can force it to have a zero projection on the chosen functions $\Phi(x)$.

$$\int_0^l e(x,t) \varphi_s \, dx = 0. \quad (14)$$

Substituting the expression of the error from equation (13) in equation (14), and rewriting, we have:

$$\begin{align*}
M\ddot{U} + KU &= f, \\
M &= \int_0^l \Phi \Phi^T \, dx, \\
K &= c^2 \int_0^l \Phi \Phi^* \, dx, \\
f &= \rho \Phi. 
\end{align*} \quad (15)$$

The equation (15) can be transited to a discrete model of a wire with a moving load to an ODE system (linear, with constant coefficients). System (15) is represented in the form of first-order equations:

$$\begin{align*}
\dot{\mathbf{U}} &= \mathbf{Z} \\
\dot{\mathbf{Z}} &= \mathbf{M}^{-1} (\mathbf{f} - \mathbf{KU}).
\end{align*} \quad (16)$$

The result is shown in figure 4. Parameters are the same as the previous section.

![Figure 4. Deflection of the conductor presented by two methods.](image-url)
3. Effect of wind force on the vibration of a stretched string with a moving load on it

In this section, we consider the resistance force from the wind on the electrical line. To take into account the distributed forces of external resistance in the string equation (3), the new form will be obtained [17–20]:

$$\rho \ddot{u} - Tu' + bu = q(x,t)$$  \hspace{1cm} (17)

To solve the equation (17) using the Galerkin method, the following change can be made in the equation (15):

$$M\ddot{\Phi} + K\Phi = f_{\text{new}},$$

$$M = \int_0^1 \Phi \Phi^T \, dx,$$

$$K = c^2 \int_0^1 \Phi \Phi_{,T}^T \, dx,$$

$$f_{\text{new}} = p\Phi - b\ddot{\Phi}.$$  \hspace{1cm} (18)

One can represent the system (19) in the form of first-order equations to solve:

$$\begin{cases}
\dot{U} = Z \\
\dot{Z} = M^{-1}(f - Ku - b\ddot{U})
\end{cases}$$  \hspace{1cm} (19)

The results are shown in figure 5. Graphs, shown in figure 5a, have parabolic shape since the quasistatic deflection (deflection obtained by setting all time-functions in equation (3) zero) is also included. To see a better difference in the effect of wind force, we omitted this deflection from the results and show just the difference $\Delta$ in figure 5b. Parameters are the same as the previous section including $b=0.5 \text{ Ns/m}$.

![Graphs showing deflection](image)

**Figure 5.** (a) General and (b) dynamic deflection of the conductor when resistance force considered.

4. Conclusion

The dynamical behavior of the electrical transmission lines robot inspector while moving over the line with a constant speed has been studied. To construct the equation of motion of the system by two methods: the expansion method and Galerkin method have been used. As a result of mathematical modeling, it is shown that motion of the robot even with a constant speed along the line vibration
appears in the vertical plane. Then, the effect of the resistance force, wind force, has been investigated. The practical results are useful for engineers to select proper parameters and speed for designing and constructing the robot inspectors.

References
[1] Aggarwal R K, Johns A T, Jayasinghe J A S B and Su W 2000 An overview of the condition monitoring of overhead lines Elect. Pow. Sys. Res. 53 15–22
[2] Dong G, Chen X and Wang B 2012 Inspecting transmission lines with an unmanned fixed-wing aircraft Appl. Rob. for the Pow. Ind. (CARPI) 173–174 DOI: 10.1109/CARPI.2012.6473355
[3] Toussaint K, Pouliot N and Montambault S 2009 Transmission line maintenance robots capable of crossing obstacles: state-of-the-art review and challenges ahead J. of Field Rob. 26 477–99
[4] Sawada J, Kusumoto K, Munakata T, Maikawa Y and Ishikawa Y A 1991 Mobile robot for inspection of power transmission lines IEEE Trans. on Power Del. 6 309–15
[5] Higuchi M, Maeda Y, Tsutani S and Hagihara S 1991 Development of a Mobile Inspection Robot for Power Transmission Lines J. of Robotic Soc. 9 57–63
[6] Tsujimura T and Morimitsu T 1997 Dynamics of mobile legs suspended from wire Rob. and Aut. Sys. 20 85–98
[7] Montambault S and Pouliot N 2007 Design and validation of a mobile robot for power line inspection and maintenance 6th Int. Conf. on Field and Ser. Rob. (FSR) 1–10
[8] Barbosa C F and Nallin F E 2014 Corrosion detection robot for energized power lines Appl. Rob. for the Pow. Ind. (CARPI) 1–6 DOI: 10.1109/CARPI.2014.7030059
[9] Alhassan A B, Zhang X, Shen H, Jian G, Xu H and Hamza K 2019 Investigation of Aerodynamic Stability of a Lightweight Dual-Arm Power Transmission Line Inspection Robot under the Influence of Wind Math. Prob. in Eng. 1–16
[10] Miller R, Abbasi F, and Mohammadpour J 2017 Power line robotic device for overhead line inspection and maintenance Ind. Rob. 44 75–84 DOI: 10.1108/IR-06-2016-0165
[11] Bahrami M R 2016 A novel design of electrical transmission line inspection machine Lect. Notes in Mech. Eng.: Adv. in Mech. Eng. 67–73
[12] Bahrami M R 2018 Mechanics of diagnostic machine on electrical transmission lines conductors MATEC Web of conferences 224 02021 DOI: 10.1051/matecconf/201822402021
[13] Eliseev V V and Bahrami M R 2016 Strength suspension of inspector robot on the electrical transmission line wire Bulletin of engineering (Moscow) 6 19–22
[14] Bahrami M R and Abed S A 2019 Mechanics of robot inspector on electrical transmission lines conductors: performance analysis of dynamic vibration absorber Vibroengineering PROCEdia 25 60–64 DOI:10.21595/vp.2019.20807
[15] Bahrami M R and Abed S A 2020 Mechanical challenges of electrical transmission lines inspection robot IOP Conf. Ser.: Mater. Sci. Eng. 709 022099
[16] Bahrami M R 2020 Mechanics of robot inspector on electrical transmission lines conductors with unequal heights supports Vibroengineering PROCEdia 30 20–25
[17] Hagedorn P and DasGupta A 2007 Vibrations and waves in continuous mechanical systems (Chichester: Wiley)
[18] Eliseev V V 2006 Mechanics of deformation of the rigid body. (St. Petersburg: Polytech Uni Pub)
[19] Tikhonov A N and Samarsky A A 1979 Equations of mathematical physics. (Moscow: Science)
[20] Oden J T and Reddy J N 2012 Variational methods in theoretical mechanics. (Springer Science and Business Media)