Interacting particles at a metal-insulator transition

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We study the influence of many-particle interaction in a system which, in the single particle case, exhibits a metal-insulator transition induced by a finite amount of onsite potential fluctuations. Thereby, we consider the problem of interacting particles in the one-dimensional quasiperiodic Aubry-André chain. We employ the density-matrix renormalization scheme to investigate the finite particle density situation. In the case of incommensurate densities, the expected transition from the single-particle analysis is reproduced. Generally speaking, interaction does not alter the incommensurate transition. For commensurate densities, we map out the entire phase diagram and find that the transition into a metallic state occurs for attractive interactions and infinite small fluctuations — in contrast to the case of incommensurate densities. Our results for commensurate densities also show agreement with a recent analytic renormalization group approach.

I. INTRODUCTION

The metal-insulator transition (MIT) in disordered electronic systems has been the subject of intense research activities over the last two decades and still continues to attract much attention. For non-interacting electrons in disordered systems the scaling hypothesis can successfully predict many of the universal features of the MIT. However, the influence of many-particle interactions on the MIT is not equally well understood and recent investigations of an apparent MIT in two-dimensional systems even question the main assumptions of the scaling hypothesis. In the single-particle case, the one-dimensional quasiperiodic Aubry-André model is known rigorously to exhibit an MIT for all states in the spectrum as a function of the quasiperiodic potential strength $\mu$. The ground state wave function is extended for $\mu < 1$ and localized for $\mu > 1$. The system at $\mu_c = 1$ is critical: there the wave functions decrease algebraically, not exponentially as in the localized case. This behavior is contrary to the localization of non-interacting particles in a one-dimensional random potential, where the ground state wave function becomes localized for infinite small disorder strength. An ingenious theoretical approach to the interplay of interactions and disorder is based on the two-interacting-particles (TIP) problem in one-dimensional random potential. Furthermore, numerical results for spinless fermions in a random potential at finite particle density have given additional insight. In general, these investigations have shown that changes in the wave function interference due to many-particle interactions can lead to a rather large enhancement of the localization lengths in one and two dimensions. Recently, we examined the TIP problem in a quasiperiodic potential by means of the transfer-matrix and the decimation method together with a careful finite-size-scaling analysis. We found that it exhibits the MIT at $\mu_c = 1$ as in the single-particle case — independent of interaction.

As an independent extension of these low-density TIP results, Chaves and Satija have studied a model of nearest-neighbor interacting spinless fermions at finite particle density in the same quasiperiodic potential by means of Lanczos diagonalization for small systems up to chain length $M = 13$. They find evidence for a critical region in which the behavior of the charge stiffness is different from its behavior in the metallic and localized regimes. In order to reach much larger system sizes for interacting systems, we employ the numerical density-matrix renormalization group (DMRG) which has been shown to be very useful. In particular, the ground state properties of interacting many-particle systems in one dimension can be obtained very accurately. In the present paper, we shall study the quasiperiodic model of Ref. at various densities and interaction strengths by DMRG and compare our results to the TIP data.

For the interacting many-body system in a quasiperiodic potential we recover the transition at $\mu_c = 1$ independent of interaction, provided we consider densities like $\rho = 1/2$ which are incommensurate compared to the wave vector of the quasiperiodic potential — an irrational multiple of $\pi$. Thus, the low-density TIP case is comparable to finite but incommensurate densities. On the other hand, for commensurate densities, we find that the system can be completely localized even for $\mu < 1$, due to a Peierls resonance between electronic and quasiperiodic potential degrees of freedom. Whereas for repulsive interactions the ground state remains localized, we find a region of extended states for attractive interaction due to the interplay between interaction and quasiperiodic potential. The behavior may be described by a weak-coupling renormalization group (RG) treatment. Thus, the physics of the model is dominated...
by whether the density is commensurate or incommensurate and only then by interaction effects.

The paper is organized as follows. The finite-density many-body system is introduced in section II. Results for incommensurate and commensurate densities are presented in section III and IV, respectively. We summarize and conclude in section V.

II. THE NEAREST-NEIGHBOR HAMILTONIAN AT FINITE DENSITY AND THE DMRG

Let us consider \( N \) interacting spinless fermions on a ring of circumference \( M \) in the Aubry-André potential \( \tilde{V} \) such that

\[
H = -t \sum_{m} \left( c_{m+1}^{\dagger} c_{m} + \text{h. c.} \right) + V \sum_{m} n_{m+1} n_{m} + 2\mu \sum_{m} n_{m} \cos(\alpha m + \beta).
\]

The operators \( c_{m}^{\dagger}, c_{m}, \) and \( n_{m} \), respectively, denote as usual Fermi creation, annihilation and number operators; \( 2\mu \cos(\alpha m + \beta) \) is the quasiperiodic potential of strength \( \mu \) with \( \alpha/2\pi \) being an irrational number. \( \beta \) is an arbitrary phase shift and we choose \( \alpha/2\pi = (\sqrt{5} - 1)/2 \), i.e., the inverse of the golden mean. In addition, we set \( t = 1 \). The particle density is \( \rho = N/M \). The model has been studied extensively in two limits, namely, independent particles, \( V = 0 \), and the clean case, \( \mu = 0 \). The first case corresponds of course to the single-particle problem discussed briefly in section I. In the second case, the model can be mapped onto the anisotropic Heisenberg (XXZ) model for which a closed Bethe-ansatz solution exists. It shows three distinct phases at zero temperature. For half filling \( (\rho = 1/2) \) and strong repulsive interaction \( (V > 2) \) the system is a charge-density-wave insulator. For weak and intermediate interaction strength and away from half filling it is a metal and can be described as a Luttinger liquid with linear energy dispersion and gapless excitations. The metal, at least, is separated for all fillings by a first-order transition at \( V = -2 \) from an insulator where the fermions form clusters (phase-separated state), corresponding to the ferromagnetic state of the spin model.

In a previous analysis of the interacting quasiperiodic model, a large enhancement of the Drude weight \( D \) (or Kohn stiffness) and the superconducting fluctuations in the ground state was found near the first-order transition at \( V \approx -2 \) using exact Lanczos diagonalization. However, the system sizes attainable by the diagonalization were restricted to \( M \leq 13 \). Using the DMRG, it is possible to extend the tractable system lengths to about \( M \approx 100 - 200 \). We use the finite lattice algorithm for non-reflection-symmetric models as described in Ref. 35. In our simulations we perform five lattice sweeps and keep 300 (small systems) to 500 (larger systems, \( M = 89 \) and 144) states per block. As our observable of the phase transition we choose the phase sensitivity

\[
M\Delta E = M(-1)^{N}[E(0) - E(\pi)]
\]

of the ground state, which is connected to \( D \) in the clean case (Luttinger phase) and equal to \( \pi^{2}D \) for non-interacting fermions. Here \( E(\Phi) \) measures the reaction of the system due to a twist in the boundary condition, \( c_{M+1} = \exp(i\Phi)c_{1} \). The prefactor \((-1)^{N}\) cancels the odd-even effects. In addition, it is believed that the phase sensitivity matches the character of the wave function. It does not depend on system size if the wave function is extended, e.g., for \( \mu = 0 \), and it decreases exponentially for large systems if the wave function is localized. As argued in Ref. 35, this can be transferred to the critical case. Therefore, we suppose that the phase sensitivity decreases algebraically with increasing system size, if the wave function is critical. Thus, we have to compare at the very least three different chain lengths in order to characterize the length dependence of the wave function.

III. INCOMMENSURATE PARTICLE DENSITIES

In order to systematically study finite-size effects, we should in principle compute \( M\Delta E \) for fixed particle density \( \rho \) and increasing \( M \). However, with fixed \( \alpha/2\pi = (\sqrt{5} - 1)/2 \), the quasi-periodic potential is incompatible with periodic boundary conditions and the results for \( M\Delta E \) depend on the choice of \( \beta \). Averaging over many potentials with different \( \beta \) can in principle be used in order to obtain \( \beta \)-independent results. However, we have found that the statistical fluctuations for densities like \( \rho = 1/2, 1/3, 1/4 \) at fixed \( \alpha/2\pi = (\sqrt{5} - 1)/2 \) are rather large such that we could not detect any significant \( M \) dependence of the phase sensitivity.

As is customary in the context of quasiperiodic systems, the value of \( \alpha/2\pi \) may be approximated by the ratio of successive Fibonacci numbers — \( F_{n} = F_{n-2} + F_{n-1} = 0, 1, 2, 3, 5, 8, 13, \ldots \). In this way, choosing \( \alpha/(2\pi) = F_{n-1}/F_{n} \) and \( M = F_{n} \), we can retain the periodicity of the quasiperiodic potential on the ring. Averaging over different \( \beta \) values is then no longer necessary since the computed ground-state energy of the finite-density Hamiltonian does not depend on \( \beta \).

For a convenient comparison with results obtained for the disordered systems, we would now want to study the phase sensitivity at various densities, say, \( \rho = 1/2 \). However, we then have to use \( M = 34, 144, 610, \ldots \). For third filling, we would be restricted to \( M = 21, 144, 987 \), and so on. In principle, such large system sizes \( M \gg 100 \) can be treated within the DMRG, but the
necessary accuracy ($\approx 10^{-6}$) is very hard to obtain for periodic boundary conditions. In addition, if the phase sensitivity decreases with system size, there is no possibility to decide whether the decrease is algebraic or exponential because the obtained value of $M\Delta E$ for such large system sizes is already zero within the computational accuracy.

For example, let us comment on $\rho = 1/2$. First, we check the occurrence of an MIT near $\mu = 1$ at fixed $M$. Calculating the phase sensitivity at fixed $M$ for different values of $\mu$ and $V$, we find that there is a clear transition near $\mu_c = 1$, as seen in Fig. 1. Whereas for small $\mu$ the phase sensitivity for non-interacting particles is larger than for interacting — as it is in the clean case —, interaction slightly enhances the phase sensitivity in the extended regime for intermediate $0.5 \lesssim \mu \lesssim 1$. Near $\mu = 1$, it seems that the MIT is not shifted appreciably within the accuracy of our calculation. As shown in Fig. 1, fitting the data results in a power-law behavior close to $\mu = 1$ with slightly larger $\mu_c = 1.025$ for attractive interaction and slightly smaller $\mu_c = 0.95$ for repulsive. Also, close to the transition, the phase sensitivity can be described by $M\Delta E \propto (\mu_c - \mu)^\nu$ with $\nu \approx 1$. This is similar to the TIP situation where $\nu = 1$ was found for the behavior of the finite-size-scaled localization lengths $F_{\rho \pi}$.

In the non-interacting case, we can also study the length dependence of the phase sensitivity by a standard diagonalization routine. We investigate the system sizes $M = 34, 144$, and $610$. $M = 8$ is excluded because $\alpha/(2\pi) = 5/8 = 0.625$ differs too much from the true value $(\sqrt{5} - 1)/2 \approx 0.618$. As shown in Fig. 2, the phase sensitivity at $V = 0$ is different in the localized, critical and extended regimes. Thus, for a system of free fermions at incommensurate density, we reproduce the expected transition at $\mu_c = 1$ in agreement with Refs. [18, 19]. Similar plots can be made for attractive and repulsive interactions at $\rho = 1/2$ as also shown in Fig. 4. Unfortunately only the two system sizes $M = 34$ and $144$ are available due to numerical instabilities for the interacting system. Therefore, further conclusions appear rather speculative. Nevertheless, we conjecture that the MIT at $\mu_c \approx 1$ for incommensurate densities is only weakly influenced by attractive and repulsive interaction in agreement with the results obtained in the TIP case [18, 19].

IV. COMMENSURATE PARTICLE DENSITIES

For the reasons outlined in the last section, we shall now turn our attention to the behavior at the commensurate densities $\rho_i \approx \lim_{n \to \infty} F_{n-i}/F_n \approx 0.618, 0.382, 0.236$, and $0.146$ corresponding to $i = 1, \ldots, 4$ in the following, where $\alpha = 2\pi \rho_1$.

A. The Peierls-like metal-insulator-transition

Let us first comment on the densities $\rho_1$ and $\rho_2$ and note that the case of $\rho_1$ is identical to $\rho_2$ due to the particle-hole symmetry in Eq. (1). Furthermore, the electronic Fermi wave-vector is $k_F = \pi \rho_1$ and therefore we have the resonance condition $\alpha = 2k_F$ for the wave-vector $\alpha$ of the quasiperiodic potential. Consequently, we expect critical behavior as found for the periodic potential at the Peierls transition [18, 19], i.e., a transition from insulating to metallic phase at $V = -\sqrt{2}$ and infinite small potential strength. Peierls [23] considered about 50 years ago the behavior of one-dimensional tight-binding electrons due to a periodic lattice distortion and found the same resonance condition. We therefore call the transition observed in the present quasiperiodic system a Peierls-like transition. The critical behavior for fermions in a periodic potential is equivalent to the critical behavior of the periodic hopping model. There the site-dependent hopping amplitude is given by $t_m = 1 - \delta \cos q m$. In this so-called Su-Schrieffer-Heeger model [39] the MIT occurs at $V = -\sqrt{2}$ and $\delta \to 0$, if filling factor $\rho$ and wave vector $q$ of the periodic hopping are commensurate, i.e., $\rho = q/(2\pi)$ or $q = 2k_F$. This happens for $\rho = 1/2$ and dimerization, one-third filling and trimerization, and so on [38]. For increasing $\delta$, $V_{\text{MIT}}$ decreases. The quasiperiodic model is expected to show this transition, at densities $\rho_1$ and $\rho_2$, because $\alpha = 2k_F$ in these cases. We have checked numerically that the Peierls-like transition is indeed observed in these cases.

Even for the other commensurate densities $\rho_3, \rho_4, \ldots,$ where the resonance condition is not strictly fulfilled, the critical behavior is still expected to be similar to the Peierls-case as shown in Ref. [13]. In our case, if $\mu$ is larger than a certain minimal $\mu_{\text{min}}$ value, we find that the phase diagrams for $\rho_3$ and $\rho_4$ are dominated by localized states as shown in Fig. 4. This localized regime extends further on to larger interactions, $-1.4 \lesssim V < \infty$, for $\mu > 0$. However, there also exists a sizeable region of extended states starting roughly at $V = -1.4$ for small $\mu$ and extending until $V \approx -2.5$ for larger $\mu$. As an example how well the transition point is defined numerically consider Fig. 3, where we plot the phase sensitivity versus interaction strength. The localization for $V > -1.7$ — where $M\Delta E$ decreases with system size — is caused by the quasiperiodic potential and for $V < -2$ by the clustering of the fermions in the phase-separated state. In between, the quasiperiodic potential is irrelevant and the ground-state wave-function is extended as for $\mu = 0$ — $M\Delta E$ increases with system size.

In the inset of Fig. 3 we show the transition at $\mu = 0.4$ in detail. A similar behavior close to the transition has also been found in data obtained recently for the TIP case [18, 19], albeit for varying $\mu$ and constant interaction and not for varying $V$ with constant $\mu$. We may nevertheless
perform a finite-size-scaling analysis. A simple ansatz assuming a power-law divergence of the correlation length \( \xi \sim |V - V_c|^{-\nu} \) at the transition yields reasonable scaling results as shown in Fig. 4. Taking into account non-linearities in the finite-size-scaling ansatz as in Ref. [13], we can determine \( V_c \) and \( \nu \) for various values of \( \mu \) as shown in Fig. 5. Our results indicate that \( \nu \) rises from \( \approx 1 \) at \( \mu = 0 \) to values close at \( 2 \) at \( \mu = 0.5 \) and then drops back to the expected \( \nu = 1 \) at the Aubry-André transition at \( \mu_c \). We emphasize that our data, due to the limited system sizes available, are not sufficient to distinguish between the power-law behavior of a second order phase transition and the expected Kosterlitz-Thouless behavior [41] at a Peierls-like transition.

Thus, for repulsive and weak attractive interactions — including the non-interacting case —, we find that the ground state for \( \mu > \mu_{\text{min}} \) is localized. This is in agreement with previous studies for disordered and periodically disturbed systems [12 24 41]. We emphasize that an increase of the localization lengths as predicted by the arguments for TIP 3 is most likely too small 13 to be detected by the present accuracy. For more strongly attractive interactions at \( V \approx -1.4 \), the situation is more interesting. For all densities \( \rho_i \) and \( \mu > \mu_{\text{min}} \), the system shows Peierls-like behavior, i.e., a transition from insulating to metallic phase at \( V \approx -\sqrt{2} \). Changing the density, the transition occurs for decreasing particle density at increasing \( \mu \). This observation is confirmed by an investigation of the energy spectrum reported in Ref. 12. In the non-interacting case it contains no gap at \( \mu = 0 \) but \( M - 1 \) gaps at \( \mu_c = 1 \) for the given \( \alpha \) 12. As seen in Ref. 12 most of the gaps open successively for increasing \( \mu \). Especially, the first gap at \( k = \alpha/2 \) opens for \( \mu \to 0 \). Tuning the filling factor, the Fermi points fall into the additional gaps in the spectrum, leading to insulating behavior.

\[ M \Delta E \sim e^{-M/\xi} \] with \( \xi^{-1} \sim \mu^{2/3(2-K)} \),

where it is assumed that the localization length \( \xi \) is the only relevant length scale. We emphasize that this scaling is only reasonable in the localized regime. It is clearly found in the numerical data for, e.g., \( \rho_3 \) and \( V = -0.6 \) as shown in Fig. 4a). Near the phase transition, it is numerically difficult to distinguish between localized (exponential decay with system size) and critical (algebraic decay with system size) phases. For small system sizes and small \( \mu \), the decrease in the phase sensitivity is always algebraic, according to the RG equation. The exponential decrease with \( \xi \) sets in far from the transition points as shown in Fig. 4a). There, the exponential scaling is found for \( \mu \geq 0.5 \). We note, that despite the straight line was obtained by a fit only to the data for \( \mu = 0.7 \) and \( \mu = 0.8 \) the data for \( \mu = 0.5 \), and \( \mu = 0.6 \) fall likewise on this line. To show the deviations from this straight line for small \( \mu \) clearly, which is the most important feature in this plot, we do not show the data for \( \mu = 0.8 \). We call this intermediate region, where no universal algebraic or exponential decrease is found, transition region. A finite-size scaling corresponding to the previous section near the transition with \( \xi \sim |\mu - \mu_c|^{\nu} \) does not lead to conclusive results.

C. Enhancement near the first-order transition

As already mentioned in 20, the phase sensitivity shows a large enhancement around \( V = -2 \). We find that the first order phase transition at \( V = -2 \) remains unaffected for small \( \mu < 0.2 \). However, for larger \( \mu \) values, the metallic phase extends towards more negative \( V \) values. For \( \mu > 0.8 \) at \( V_p \approx -2.2 \) the phase sensitivity shows an unexpected sharp maximum in the delocalized regime, as shown in Fig. 4A. A similar maximum is also seen for \( \mu > 1 \) and small system sizes (\( M = 34, 55 \)), but the system is insulating. The exponential scaling in the localized region as shown in Fig. 4 does not change for \( V > -2 \) and \( V < V_p < -2 \) in the case of large \( \mu \). Therefore, we conclude that this regime belongs to the localized regime of the quasiperiodic potential. On the other hand, no scaling behavior is found for \( V < -2 \) and small \( \mu \) lower left corner in the phase diagrams, Fig. 4. Here, the localization is due to the formation of particle clusters in the phase-separated state. In the crossover regime between these two localized phases a metallic state is recovered. The maximum in the insulating regime is a rest of this crossover. At intermediate and strong \( \mu \) and \( V < V_p < -2 \) the phase sensitivity drops very rapidly to zero within the numerical accuracy. This behavior is comparable to the behavior at \( V = -2 \) and small \( \mu \) as seen in Fig. 4 at the first order transition. Thus, it seems that the first order transition is moved to smaller \( V \) with increasing \( \mu \). The
subtle interplay between the clustering and the quasiperiodic potential seems to delocalize the ground-state wave-function in a small parameter region. However, it is numerically difficult to follow the border between these two completely different localized phases for stronger $\mu$ and $-2 < V < -3$ in our approach.

D. Phase diagram

In Fig. 3, we summarize our results by showing two phase diagrams of model (I) for varying quasiperiodic potential strength $\mu$ and nearest-neighbor interaction $V$. They were obtained by studying the system size behavior of $M \Delta E$ up to $M = 144$ for $\rho_3 = 0.236$ and $\rho_4 = 0.146$. The localized parts of the phase diagrams for weak attractive and repulsive interactions are not shown, because they have no additional structure. For $\rho_4$, the intermediate phase is difficult to detect and is not separately marked in Fig. 3. There we can only distinguish whether the phase sensitivity decreases or not. As clearly seen, $\mu_{\min} = 0$ for $\rho_3$ and $\mu_{\min} = 0.1$ for $\rho_4$. In summary, the delocalized phase becomes larger for weak $\mu$ and smaller for strong $\mu$ with decreasing density. First, $\mu_{\min}(\rho_4) > \mu_{\min}(\rho_3)$ and second, the crossover region beyond $V = -2$ becomes smaller and extends only to $\mu = 0.9$.

V. CONCLUSIONS

In this work, we have studied the influence of interactions on an metal insulator transition in one dimension using the DMRG method. Thereby, we have compared the interplay of disorder and interactions for the quantum system (I) in a quasiperiodic potential at incommensurate and commensurate densities. Our results suggest that the delocalization found previously for low density TIP in the localized phase cannot simply be extrapolated to the finite-density situation. In the case of incommensurate densities with nearest-neighbor (repulsive or attractive) interaction the metal insulator transition is found at $\mu_c \approx 1$. The critical exponent, $\nu \approx 1$, is in agreement with the results of transfer-matrix-method calculations and finite-size scaling [13]. This indicates that the influence of interaction at this particular type of metal insulator transition is not strong enough to change the universality class of the Aubry-André model.

For commensurate densities $\rho_3$ and $\rho_4$, we have deduced the phase diagrams of system for varying quasiperiodic potential strength $\mu$ and interaction $V$. The numerically accessible filling factors are commensurate with the quasiperiodic potential and yield a Peierls-like behavior of the system with a metal insulator transition at attractive interaction and small quasiperiodic potential strength. Thus, although we find a strong enhancement of the phase sensitivity at the first order transition, the physics of the model at commensurate densities is dominated by the Peierls resonance condition which becomes irrelevant only for strong attractive interaction. A remainder of the single-particle localization is still found in this case — the region of the phase diagram, where the ground-state wave-function is extended, extends at most to $\mu = 1$. In addition, we have found a change in the exponent of the localization length from $\nu \approx 2$ to $\nu \approx 1$ at $\mu = 1$. In summary, we have seen that the single-particle localization in the Aubry-André model is not influenced by interaction. At commensurate densities, other effects such as a Peierls-like commensurability become important and dominate the transport properties.

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[1] B. Kramer and A. MacKinnon, Rep. Prog. Phys. 56, 1469 (1993).
[2] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).
[3] D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. 66, 261 (1994).
[4] S. V. Kravchenko et al., Phys. Rev. Lett. 77, 4938 (1996).
[5] D. Belitz and T. R. Kirkpatrick, Phys. Rev. B 58, 8214 (1998).
[6] V. M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, Pis’ma ZhETF 65, 887 (1997), [JETP Lett. 65 (1997)], cond-mat/9707052.
[7] S. Aubry and G. André, Ann. Israel Phys. Soc. 3, 133 (1980).
[8] D. L. Shepelyansky, Phys. Rev. Lett. 73, 2607 (1994).
[9] D. L. Shepelyansky, in Correlated fermions and transport in mesoscopic systems, edited by T. Martin, G. Montambaux, and J. T. T. Ván (Editions Frontieres, Proc. XXXI Moriond Workshop, Gif-sur-Yvette, 1996), p. 201.
[10] R. A. Römer and M. Schreiber, Phys. Rev. Lett. 78, 515 (1997).
[11] K. Frahm, A. Müller-Groeling, J. L. Pichard, and D. Weinnmann, Europhys. Lett. 31, 169 (1995).
[12] P. Schmitteckert et al., Phys. Rev. Lett. 80, 560 (1998).
FIG. 1. Phase sensitivity versus $\mu$ at fixed system size $M = 34$ for $V = -1.5$ (squares), 0 (circles), 1.5 (diamonds) at (incommensurate) half filling. The lines are fits corresponding $M\Delta E \sim |\mu - \mu_c|$. 

FIG. 2. Phase sensitivity versus system size for $V = -1$ (squares), 0 (circles), 1 (diamonds) at (incommensurate) half filling. The three cases $\mu = 0.9$ (solid lines), $\mu = 1$ (dashed lines), and $\mu = 1.1$ (dotted lines) are compared.
FIG. 3. Phase diagrams of the system described by Eq. (1) in terms of quasiperiodic potential strength $\mu$ and interaction $V$ for $\rho = \rho_3$ (left side) and $\rho = \rho_4$ (right side). An extended ground state wave function is marked by $\circ$, a localized by $\blacklozenge$. In addition, the transition regime (see text) is marked with a shaded $\blacklozenge$ in the left figure. The solid lines indicate the first-order transition at $V = -2$ and $\mu = 0$.

FIG. 4. Phase sensitivity versus interaction for three different system sizes, density $\rho_3$ and $\mu = 0.4$. The lines for $M = 34$ and 89 are guides to the eye, highlighting the different finite-size-scaling behaviors in the various regimes. The dotted lines indicate the transition between the localized and the extended wave functions. Inset: Additional data close to the transition and $M = 144 (\bigtriangleup)$ included. The lines denote the curves constructed by non-linear finite-size scaling.

FIG. 5. Scaling function (solid line) and scaled data points in the interval $V \in [-1.8, -1.4]$ close to the transition for $\mu = 0.4$ and system sizes $M = 34, 55, 89, 144$. Inset: Scaling parameter $\xi$ as a function of $V$.

FIG. 6. Estimates of $V_c$ ($\bigcirc$) and $\nu$ ($\blacklozenge$) obtained for $\rho_3$ by non-linear finite-size scaling at various $\mu$. The open symbols denote various fit functions and initial parameters, whereas the closed symbols indicate the best (according to $\chi^2$-statistics) of all these fit for a given value of $\mu$. Error bars mark the errors resulting from the Levenberg-Marquardt fitting method.

FIG. 7. (a) Phase sensitivity versus scaled system size $M\mu^{2/(2-K)}$, where $M = 34, 55, and 89$, for $V = -0.6$, filling $\rho = \rho_3$, and various potential strengths. The line indicates a plot of Eq. (4) with $K = \pi/2 \arccos(0.3) \approx 1.24$. (b) Phase sensitivity versus scaled system size, $M = 34, 55, and 89$, for $V = -1.5$ and $\rho_3$. The straight line is a fit to the data for $\mu = 0.7$ and 0.8.
FIG. 8. Phase sensitivity versus interaction at $\rho_3$ for different $\mu$ at size $M = 34$ (small grey symbols) and $M = 55$ (large open symbols). We compare two system sizes to indicate the regimes with the localized and extended ground state wave function. The lines connect the data points for different $\mu$ and $M$ values.