INTERMITTENCY AND CORRELATIONS
AT LEP AND AT HERA

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A review on recent investigations of local fluctuations and genuine correlations in
\(e^+e^-\) annihilations at LEP and in \(e^+p\) collisions at HERA is given.

1 Introduction

Local fluctuations and genuine correlations of hadrons in high energy inter-
actions provide us with important information about the multihadron pro-
duction mechanism. This information gives details which are beyond those
obtained from studies of single-particle distributions.

The peculiarity of hadrons to group when produced have been studied a
long time using correlation functions, while in the recent decade the interest
in correlations have been revived due to a new method of factorial moments
and due to the obtained intermittency phenomenon, i.e. self-similarity, or
fractality, of hadron production.\(^1\)

The fractal structure of particle distribution in \(e^+e^-\) collisions has been
realized many years ago due to a jet evolution picture (see e.g. \(^2\)). However,
even if the parton shower is expected to exhibit intermittency, this does not
 guarantee the effect to appear at hadron level. Recently, different approaches
have been proposed to describe the experimentally observed fractality by analyt-
cal QCD calculations.\(^3\)

In this talk, we review experimental studies of intermittency and correlations
at LEP \(^4\)\(^5\)\(^6\)\(^7\)\(^8\) and at HERA,\(^9\)\(^10\) which appeared recently and provide
us with further information, in addition to discussions of recent reviews.\(^1\)

From the studies, it is seen that, despite considerable success in understand-
ing different properties of the phenomenon, further investigations are needed.

2 Definitions

2.1 A Tool of Factorial Moments and Cumulants

In order to measure local dynamical fluctuations, a method of normalized
factorial moments, \(F_q\), is applied.\(^4\) The factorial moment of order \(q\) is defined

\(F_q = \left\{ \frac{\sum_{i=1}^{N} x_i^q}{N} \right\} \)
as a function of a phase-space region size $\delta$, 

$$F_q(\delta) = \langle n(n-1)\cdots(n-q+1)/\langle n\rangle^q. \quad (1)$$

Here, $n$ is the number of particles in the $\delta$-region, and the brackets $\langle \cdot \rangle$ denote averaging over events.

The normalised factorial moments allow us to extract dynamical fluctuations. For uncorrelated particle production, the moments are independent of $\delta$, $F_q \equiv 1$. Correlations between particles lead to an increase of factorial moments with decreasing $\delta$ (increasing number $M$ of $\delta$-regions), and if exhibiting a power-law the dependence is called intermittency.

To extract genuine correlations contributing to the fluctuations, one uses the technique of normalised factorial cumulant moments, cumulants. The cumulants, $K_q$, are constructed from the unnormalised factorial moments in a way that they vanish whenever particles in $q$-tuple are statistically independent. The normalised cumulants are defined as

$$K_q(\delta) = k_q/\langle n\rangle^q. \quad (2)$$

with the Mueller moments $k_q$,

$$k_1 = \langle n \rangle, \quad k_2 = \langle n(n-1) \rangle - \langle n \rangle^2, \quad k_3 = \langle n(n-1)(n-2) \rangle - 3\langle n(n-1) \rangle \langle n \rangle + 2\langle n \rangle^3, \quad \text{etc.} \quad (3)$$

Normalised cumulants share with the normalised factorial moments their property to measure the dynamical component of the underlying particle density.

From Eqs. (2) and (3), one finds the interrelations between normalised factorial moments and cumulants,

$$F_1 = K_1, \quad F_2 = K_2 + 1, \quad F_3 = K_3 + 3K_2 + 1, \quad \text{etc.}, \quad (4)$$

which provide us with the information whether the $p$-order genuine correlations are important in the $q$-particle dynamical fluctuation.

### 2.2 Analytical QCD Predictions

The QCD description of multihadron production is based on a partonic picture, i.e. on gluon cascades radiated off the initial parton. In order to describe the hadron production mechanism, one has to cut off the parton cascade at some scale $Q_0 \leq 1$ GeV, while the following non-perturbative hadronization is considered within the concept of Local Hadron Parton Duality (LPHD), which connects multihadron final states and partons.
The QCD calculations are given in angular phase space, i.e. in 1-dimensional, 1D, rings (or 2-dimensional, 2D, cones) around a jet axis with mean opening angle $\Theta$ (a direction $(\Theta, \Phi)$) and a half width (opening angle) $\delta \equiv \vartheta$.

For the normalised cumulants and factorial moments, the power-law,

$$K_q(\Theta, \vartheta) \text{ or } F_q(\Theta, \vartheta) \propto (\Theta/\vartheta)^{(q-1)(D-D_q)},$$

with fractal or Rényi dimensions $D_q$ is predicted. $D$ is a dimensional factor: $D = 1$ for ring regions and 2 for cones.

The QCD expectations for $D_q$ are as follows (see 3 and refs. therein).

- In a fixed-coupling regime ($\alpha_s = \text{const.}(\vartheta)$) of the Double Log Approximation (DLA),

$$D_q \equiv D_q^{(c)} = \frac{q}{q+1}, \quad \gamma_0(q) = \sqrt{2N_c\alpha_s/\pi}, \quad Q = E\Theta, \quad (6)$$

for moderately small angular regions, $\vartheta \leq \Theta$.

- In a running-coupling regime of the DLA,

  (a) $D_q = D_q^{(c)} \left(1 + \frac{q^2 + 1}{4q^2} \cdot \varepsilon\right)$,  
  (b) $D_q = 2D_q^{(c)} \left(1 - \frac{\sqrt{1 - \varepsilon}}{\varepsilon}\right)$,  
  (c) $D_q = 2\gamma_0(q) \frac{q - w(q, \varepsilon)}{\varepsilon(q - 1)}$,  

$$w(q, \varepsilon) = q\sqrt{1 - \varepsilon} \left(1 - \frac{\ln(1 - \varepsilon)}{2q^2}\right). \quad (7)$$

- In the Modified Leading Log Approximation (MLLA), Eq. (7a) remains valid but $\gamma_0$ is replaced by an effective one, $\gamma_0^{\text{eff}}(Q) = \gamma_0(Q) + \gamma_0^{2}(Q) \cdot f(q, N_f, N_c)$.

Here, $E$ is the jet energy, $N_c$ and $N_f$ are the number of colors and flavors, respectively.

A scaling variable,

$$\varepsilon = \frac{\ln(\Theta/\vartheta)}{\ln(E\Theta/\Lambda)} \quad (8)$$

is utilised in the calculations. For the maximum phase space, $\vartheta = \Theta$, $\varepsilon = 0$.

The analytical predictions involve only one adjustable parameter, the QCD scale $\Lambda$, while a strong coupling $\alpha_s$ is based on first-order QCD relation,

$$\alpha_s = \frac{\pi\beta^2}{6} \frac{1}{\ln(Q/\Lambda)}, \quad \beta^2 = 12 \left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)^{-1}. \quad (9)$$

The QCD calculations are made at asymptotic energies, which corresponds to an infinite number of partons in an event. No energy-momentum conservation is taken into account in the calculations above.
3 Experimental Results

3.1 Spatial Fluctuations and Correlations

At LEP, L3 and OPAL have studied fluctuations and correlations in the $Z^0 \rightarrow e^+e^- \rightarrow \text{hadrons}$ process.

L3 measured fluctuations in rapidity and in the 4-momentum difference ($Q_{ij}^2 = -(p_i - p_j)^2$ of all $ij$-pairs) using factorial moments for the former variable and the bunching parameters for both variables. The measurements have shown the multifractal character of the local fluctuations, as it is expected from the QCD parton shower picture.

A large statistics of more than 4 million events has been used by OPAL to measure local fluctuations and genuine correlations in one-, two- and three-dimensional subspaces of rapidity, azimuthal angle, and transverse momentum (w.r.t. the sphericity axis). As L3, OPAL has observed multifractal behavior of factorial moments in one dimension, rapidity and azimuthal angle. Such an intermittency behaviour gets more pronounced with increasing dimension, which is ascribed to a jet-like structure of the events as predicted.

OPAL has measured factorial cumulants and found genuine correlations up to 5th order, being especially large in the rapidity vs. azimuthal angle subspace. Using Eqs. (4) reduced to some $p$th order, $p<q$ (e.g. $F^{(2)}_3 = 3K_2 + 1$), OPAL checked the importance of the genuine correlations in the dynamical fluctuations obtained. It was found that genuine correlations of high-order are needed to describe the intermittency effect, see Fig. 1.

![Figure 1. Decomposition of factorial moments $F_q$ into correlation contributions $F^{(p)}_q$ in the subspace of rapidity vs. azimuthal angle, measured by OPAL (see Eq. (4) and text).](image)

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In both Collaborations’ studies, Monte Carlo (MC) models which have been tuned to reproduce global event-shape distributions and single-particle inclusive spectra in $e^+e^-$ annihilations, were found to reproduce the trend, but not the magnitudes of the measured moments, especially for small $\delta$.

3.2 Angular Fluctuations and Correlations

L3, DELPHI, and ZEUS have measured fluctuations and correlations in angular phase space. While the thrust or sphericity axes are chosen as a jet axis in LEP experiments, the special Breit frame is considered in the HERA experiment, to separate the hadronic final state from the radiation and to mimic a single $e^+e^-$ reaction hemisphere.

Comparing the measurements to the first-order analytical predictions, Eqs. (6), (7), and (9), it has been found that the DLA and MLLA qualitatively describe the general features of the data (Figs. 2, 3). The measured moments rise approximately linearly for large angles $\vartheta$ (small $\varepsilon$) as expected from the parton shower multifractality (cf. Eq. (5)), while the levelling off at smaller angles is believed to be due to the running effect of the coupling $\alpha_s$. The 2D moments rise much more steeply than the 1D ones. The factorial moments are found to increase as the energy increases, see Fig. 3. Note that the $\vartheta$- and D-dependences are analogous, respectively, to the multifractality of a parton shower and to the jet structure, obtained in spatial analyses, as discussed above.

Figure 2: L3 factorial moments $F_q(z \equiv \varepsilon)/F(0)$ at $\Lambda=0.16$ GeV (left) and $\Lambda=0.04$ GeV (right) compared to the QCD calculations according to DLA Eqs. (6), (7) and MLLA ($\gamma_0 \to \gamma^{\text{eff}}_0$).
On the quantitative level, one finds some deviations between the QCD predictions and the data. The analytical calculations are very sensitive to the QCD parameters, $\Lambda$ and $N_f$, and are not able to describe simultaneously the factorial moments at all orders and at different dimensions. A better agreement between the factorial moments calculated and the data is obtained for $\Lambda=0.04$ GeV than that is found at the expected larger value, $\Lambda=0.16$ GeV, Figs. 2 and 3. However, at small $\Lambda$, the theory overestimates the data for large $\varepsilon$. The measured cumulants are far from the predictions, and the reduction of the $\Lambda$-value was not found to sensibly improve the situation, see Fig. 4.

Likely reasons for the failure of the QCD calculations seem to be their asymptotic character and lack of energy-momentum conservation, as it was mentioned in Sec. 2.2. However, DELPHI has analysed some high-energy events (Fig. 3) and no improvement at small $\varepsilon$-values was found. Inclusion of energy-conservation terms was observed to lead to even larger discrepancies.

ZEUS has compared their results on 2-particle angular correlation functions with the DELPHI analysis to check energy-dependence of the correlations, see Fig. 5. According to DELPHI, there is a steeper rise of the 2-particle correlation functions at $\sqrt{s}=183$ GeV than at $\sqrt{s}=91$ GeV. No such dependence is confirmed by Fig. 5, although ZEUS data are taken at lower energy than those of DELPHI. All the possible checks (of experimental and calculation procedures) kept the results unchanged. It would be interesting to carry out an analysis to understand whether the expected universality of
The cumulants and factorial moments are normalized by $C_{n}/(n/0)$ and $F_{n}/(n/0)$ for easy comparison of the measured shapes with the analytical predictions. There is generally good agreement between the Monte Carlo simulation (open circles) and the corrected data (full circles).

Figure 1: The cumulants of orders $n=2$ and $n=3$ in one-dimensional rings around jet cones normalized by $C_{n}/(n/0)$ and compared with the predictions of ref. [4], eq. [8] (solid lines) with $n_f = 5$ for $a) / Q = 0.15 \text{ GeV}$, $b) / Q = 0.04 \text{ GeV}$. The statistical errors are shown by the error bars, the systematic errors by the shaded regions.

/* A clear disagreement is observed: the predictions lie well below the data and differ in shape (Fig. 1a). Using a lower value of $\Lambda$ (i.e. $\Lambda = 0.04 \text{ GeV}$ instead of $0.15 \text{ GeV}$) does not help, as can be seen in Fig. 1b (neither does a smaller value of $n_f$, not shown here).*/

Figure 2: DELPHI cumulants (circles), calculated via Eq. (4), are compared to the QCD Eq. (7c) calculations (lines). The statistical errors (error bars) are shown along with the systematic ones (shaded areas).

Figure 3: The factorial moments of orders $2$, $3$, $4$, and $5$ normalized by $F_{n}/(n/0)$, together with the predictions of refs. [4/6], in one- and two-dimensional angular intervals (i.e. rings and side cones) for various numerical values of $\Lambda$ and $n_f$./* The correlations in one-dimensional rings around jets, expressed by factorial moments, are not described well by the theoretical predictions using the QCD parameters $\Lambda = 0.15 \text{ GeV}$ and $n_f = 5$ (Fig. 3a). The predictions lie below the data for not too large $\Lambda$, differing also in shape./* Choosing $n_f = 3$ (Fig. 3b) instead of $n_f = 5$ as in Fig. 3a reduces the discrepancies.*/

Figure 4: DELPHI cumulants (circles), calculated via Eq. (4), are compared to the QCD Eq. (7c) calculations (lines). The statistical and total errors are shown by the inner and outer bars, respectively.

Figure 5: ZEUS 2-particle correlation function, compared to the running $\alpha_s$ QCD calculations (solid line), to results of DELPHI, and to predictions for 30.6 GeV quark and gluon jets (dashed and dotted lines, respectively). The statistical and total errors are shown by the inner and outer bars, respectively.

Figure 6: ZEUS factorial moments $F_q(p_T^{cut})$, compared to MC models. The statistical and total errors are shown by the inner and outer bars, respectively.
the 2-particle inclusive density is violated. Note that QCD calculations for a gluon jet describe the data better than those of a quark jet, as shown in Fig. 5. This could be connected with the approximation used (DLA instead of, e.g., MLLA) in calculating the ratio of the mean multiplicity in gluon and quark jets. Nevertheless, it is seen that both predictions disagree with the data for small $\varepsilon$ values (large $\vartheta$).

A better agreement between the data and the calculations is found when one compares the measurements with MC simulations. At the $Z^0$ peak, as well as at high energies, the simulations reproduce the data well. This is presumably due to the fact that MC models take into account the energy-momentum conservation and are tuned to the data global variables. On the other hand, there is still a difference in the choice of the cut-off parameter $Q_0$, which "terminates" the parton cascade: $Q_0$ is about 0.3-0.6 GeV in the MC models used, while due to the LPHD it is expected to be $\approx 0.25$ GeV. However, although parton level MC studies by L3 indicate some disagreement with the LPHD assumption, the DELPHI investigation tells us that even a possible violation of LPHD seems unlikely to be a reason for the discrepancies between hadron and parton levels.

### 3.3 Fluctuations, Correlations and QCD Coherence

Recently, ZEUS has studied correlations in momentum-restricted regions in view of recent QCD+LPHD calculations. The normalised factorial moments of the multiplicity distributions are theoretically expected to behave as

$$F_q(p_{\text{cut}}) \simeq 1 + \frac{q(q-1)}{6} \frac{\ln(p_{\text{cut}}/Q_0)}{\ln(E/Q_0)}, \quad F_q(p_{\text{cut}}) \simeq \text{const} > 1, \quad (10)$$

when particles are restricted (cylindrically) in either the transverse momentum $p_t < p_{\text{cut}}$ or (spherically) in absolute momentum $|p| < p_{\text{cut}}$.

The predictions of Eq. (10) are the followings. There are correlations between partons because the factorial moments exceed unity. The correlations vanish for small $p_{\text{cut}}$, $F_q \to 1$ as $p_{\text{cut}} \to Q_0$, due to angular ordering of the partons in the jet (QCD coherence). This leads to the Poissonian (independent) emission. However, soft gluons with spherically limited momenta ($|p| < p_{\text{cut}}$) obey the non-Poissonian distribution even for small $p_{\text{cut}}$.

Fig. 6 of ZEUS shows a first evident disagreement between the LPHD hypothesis and the measurements. The data factorial moments represent a strongly increasing function of $p_t$ at $p_t < 1$ GeV which disagrees with what the theory predicts, Eq. (10). A similar behaviour is found for the factorial
moments in $p^{\text{cut}}$, while the errors are too large to conclude. MC models show good agreement with the data.

It would be interesting to find out whether this observation in $e^+p$ collisions agrees with that in $e^+e^-$ annihilation, where one does not need to transform to a specific frame, a possible reason of the above-mentioned disagreement in 2-particle correlation functions.

4 Conclusions

A review on recent results from the investigations of intermittency and genuine correlations by DELPHI, L3 and OPAL in $e^+e^-$ and by ZEUS in $e^+p$ collisions is given. The tool of factorial multiplicity and cumulant moments has been applied. The studies show an existence of strong correlations between produced hadrons. The analytical QCD calculations qualitatively describe the scaling behaviour of the measured moments, while on the quantitative level some discrepancies are found, especially for genuine correlations. Monte Carlo models reasonably well reproduce the trend in the data, although some disagreement in magnitudes is seen. ZEUS observed a violation of LPHD predictions for momentum-limited factorial moments. The universality between $e^+e^-$ and $e^+p$ collisions is analysed. The observations reviewed give clear evidence for the need of further efforts on studying the subject.

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