A NOVEL EMISSION SPECTRUM FROM A RELATIVISTIC ELECTRON MOVING IN A RANDOM MAGNETIC FIELD

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ABSTRACT

We numerically calculate the radiation spectrum from relativistic electrons moving in small-scale turbulent magnetic fields expected in high-energy astrophysical sources. Such a radiation spectrum is characterized by the strength parameter $a = \lambda_B e |B| / mc^2$, where $\lambda_B$ is the length scale of the turbulent field. When $a$ is much larger than the Lorentz factor of a radiating electron $\gamma$, synchrotron radiation is realized, while $a \ll 1$ corresponds to the so-called jitter radiation regime. Because for $1 < a < \gamma$ we cannot use either approximations, we should have recourse to the Lienard–Wiechert potential to evaluate the radiation spectrum, which is performed in this Letter. We generate random magnetic fields assuming Kolmogorov turbulence, inject monoenergetic electrons, solve the equation of motion, and calculate the radiation spectrum. We perform numerical calculations for several values of $a$ with $\gamma = 10$. We obtain various types of spectra ranging between jitter radiation and synchrotron radiation. For $a \sim 7$, the spectrum takes a novel shape which had not been noticed up to now. It is like a synchrotron spectrum in the middle energy region, but in the low frequency region it is a broken power law and in the high frequency region an extra power-law component appears beyond the synchrotron cutoff. We give a physical explanation of these features.

Key words: gamma-ray burst: general – magnetic fields – radiation mechanisms: general – turbulence

1. INTRODUCTION

A major part of non-thermal emission from high-energy astrophysical objects is almost always characterized by the radiation from relativistic electrons moving in magnetic fields. Usually it is interpreted in terms of the synchrotron radiation. However, the synchrotron approximation is not always valid, particularly when the magnetic fields are highly turbulent. Electrons suffer from random accelerations and do not trace a helical trajectory. In general, the radiation spectrum is characterized by the strength parameter

$$a = \lambda_B e |B| / mc^2,$$

where $\lambda_B$ is the typical scale of turbulent fields, $|B|$ is the mean value of the turbulent magnetic fields, $e$ is the elementary charge, $m$ is the mass of electron, and $c$ is the speed of light (Reville & Kirk 2010). When $a \gg \gamma$, where $\gamma$ is the Lorentz factor of radiating electron, the scale of turbulent fields is much larger than the Larmor radius $r_L = \gamma mc^2 / e |B|$, and electrons move in an approximately uniform field, so that the synchrotron approximation is valid. In contrast, when $a \ll 1$, $\lambda_B$ is much smaller than the scale $r_L / \gamma = \lambda_B / a$, which corresponds to the emission of the characteristic synchrotron frequency. In this regime, electrons move approximately straightly, and jitter approximation or the weak random field regime of diffusive synchrotron radiation (DSR) can be applied (Medvedev 2010; Fleishman & Urtiev 2010). For $1 \lesssim a \lesssim \gamma$, no simple approximation of the radiation spectrum has been known.

The standard model of gamma-ray bursts (GRBs) is based on the synchrotron radiation from accelerated electrons at the internal shocks. The observational spectra of prompt emission of a GRB can be well fitted by a broken power-law spectrum, which is called the Band function. Around one third of GRBs show a spectrum in the low-energy side harder than the synchrotron theory predicts. To explain this, other radiation mechanisms are needed. Medvedev examined relativistic collisionless shocks in relevance to internal shocks of GRBs and noticed the generation of small-scale turbulent magnetic fields near the shock front (Medvedev & Loeb 1999). Then he analytically calculated the radiation spectrum from electrons moving in small-scale turbulent magnetic fields, to make a harder spectrum than that of the synchrotron radiation (Medvedev 2000). However, he assumed that the strength parameter $a$ is much smaller than 1 and that turbulent field is of one-dimensional structure, which may be an oversimplification in general (Fleishman 2006). Medvedev also calculated three-dimensional structure assuming that the turbulent field is highly anisotropic (Medvedev 2006). He conclude that the harder spectrum is achieved in the “head on” case, and that in the “edge-on” case, the spectrum is softer than it is for synchrotron radiation. The spectral index depends on the angle $\theta$ between the particle velocity and shock normal, with the hard spectrum obtained when $\theta \lesssim 10^{\circ}$ (Medvedev et al. 2009).

Recently, several particle-in-cell (PIC) simulations of relativistic collisionless shocks have been performed to study the nature of turbulent magnetic fields that are generated near the shock front (e.g., Frederiksen et al. 2004; Kato 2005; Chang et al. 2008; Haugbolle 2010). The characteristic scale of the magnetic fields is the order of skin depth as predicted by the analysis of Weibel instability. Then, the wavelength of the turbulent magnetic field $\lambda_B$ is described by using a coefficient $\kappa$ as

$$\lambda_B = \kappa c / \omega_{pe} \Gamma_{int},$$

where $\omega_{pe}$ is the plasma frequency and $\Gamma_{int}$ is the relative Lorentz factor between colliding shells. The energy conversion fraction into the magnetic fields,

$$\epsilon_B = B^2 / 8\pi \Gamma_{int} n m_{pe} c^2,$$
is $10^{-3}$–0.1, where $B^2/8\pi$ is energy density of magnetic fields and $\Gamma_{\text{rms}} m_p c^2$ is the kinetic energy density of the shell. The Lorentz factor of electrons is similar to $\Gamma_{\text{int}}$, and that $\kappa$ is typically 10 from the result of PIC simulations, the strength parameter $a$ can be estimated as

$$a = \sqrt{2} k_B e_B^{1/2} \left( \frac{m_p}{m_e} \right) \sim O(10^{-2}).$$

(4)

Thus, the assumption $a \ll 1$ on which jitter radiation and DSR weak random field regime are based is not necessarily valid when we consider the radiation from the internal shock region of GRB.

Fleishman & Urtiev (2010) calculated the radiation spectrum for $a > 1$ using a statistical method, but their treatment of the “small-scale component” is somewhat arbitrary. They introduced the critical wavelength $\lambda_{\text{crit}}$ and called components with $\lambda \leq \lambda_{\text{crit}}$ the “small-scale component,” where $\lambda_{\text{crit}}$ obeys the inequality

$$r_g \ll \lambda_{\text{crit}} \ll r_g/\gamma$$

(Toptygin & Fleishman 1987). The inequality can be transformed to $1 \ll \lambda_{\text{crit}}(B)/mc^2 \ll \gamma$, so that the division through $\lambda_{\text{crit}}$ may be ambiguous when we calculate the radiation spectrum for $1 < a < \gamma$.

The synthetic spectra from PIC simulations were recently calculated (e.g., Hededal 2005; Sironi & Spitkovsky 2009; Frederiksen et al. 2010; Reville & Kirk 2010; Nishikawa et al. 2011). Although their magnetic fields are realistic and self-consistent, it is inevitable that the fields are described by discrete cells in PIC simulation. Reville & Kirk (2010) developed an alternative method of calculation of radiation spectra that uses the concept of photon formation length, which costs much less time than the first principle method utilizing the Lienard–Wiechert potential.

In this Letter, we rather use the first principle method to obtain a spectrum that is as exact as possible. We adopt the field description method developed by Giacalone & Jokipii (1999) and used by Reville & Kirk (2010). We assume isotropic turbulent magnetic fields which have broader power spectra $k_{\text{max}} = 100 \times k_{\text{min}}$ and calculate the radiation spectra in the regime of $1 < a < \gamma$. In Section 2, we describe calculation method and numerical results. In Section 3, we give a physical interpretation.

2. METHOD OF CALCULATION

Because we focus our attention on calculating the radiation spectrum, we assume a static field with the required properties of $a$, and neglect the back reaction of radiating electrons to the magnetic field. We accurately solve the electron trajectory in each time step and calculate the radiation spectrum.

2.1. Setting

The isotropic turbulent field is generated by using the discrete Fourier transform description as developed in Giacalone & Jokipii (1999). It is described as a superposition of $N$ Fourier modes, each with a random phase, direction, and polarization:

$$B(x) = \sum_{n=1}^{N} A_n \exp[i(k_n \cdot x + \beta_n)] \hat{\xi}_n.$$  

(6)

Here, $A_n$, $\beta_n$, $k_n$, and $\hat{\xi}_n$ are the amplitude, phase, wave vector, and polarization vector for the $n$th mode, respectively.

The polarization vector is determined by a single angle $0 < \psi_n < 2\pi$:

$$\hat{\xi}_n = \cos \psi_n e'_x + i \sin \psi_n e'_y,$$  

(7)

where $e'_x$ and $e'_y$ are unit vectors, orthogonal to $e'_z = k_n/k_n$. The amplitude of each mode is

$$A_n = \sigma^2 G_n \left[ \sum_{n=1}^{N} G_n \right]^{-1},$$

(8)

where the variance $\sigma$ represents the amplitude of the turbulent field. We use the following form for the power spectrum

$$G_n = \frac{4\pi k_n^2 \Delta k_n}{1 + (k_n L_c)^2},$$

(9)

where $L_c$ is the correlation length of the field. Here, $\Delta k_n$ is chosen such that there is an equal spacing in logarithmic $k$–space, over the finite interval $k_{\text{min}} \leq k \leq k_{\text{max}}$, where $k_{\text{max}} = 100 \times k_{\text{min}}$ and $N = 100$, where $k_{\text{min}} = 2\pi/L_c$ and $\alpha = 11/3$. We have no reliable constraint for value of $\alpha$ from GRB observation, so we tentatively adopt the Kolmogorov turbulence $B^2(k) \propto k^{-5/3}$, where the power spectrum has a peak at $k_{\text{min}}$. Then we define the strength parameter using $\alpha$ and $k_{\text{min}}$ as

$$a \equiv \frac{2\pi e\sigma}{mc^2k_{\text{min}}}.$$  

(10)

We isotropically inject 32 monoenergetic electrons with $\gamma = 10$ in the prescribed magnetic fields, and solve the equation of motion

$$\gamma m_e \ddot{v} = -e \beta \times B$$

(11)

using the second-order Runge–Kutta method. We pursue the orbit of electrons up to the time $300 \times T_g$, where $T_g$ is the gyro time $T_g = 2\pi \gamma mc/e\sigma$. We calculate the radiation spectrum using acceleration $\dot{v} = \beta c$. The energy $dW$ emitted per unit solid angle $d\Omega$ (around the direction $n$) and per unit frequency $d\omega$ to the direction $n$ is computed as

$$\frac{dW}{d\omega d\Omega} = \frac{e^2}{4\pi c^2} \left[ \int_{-\infty}^{\infty} dt \frac{\mathbf{n} \times [(\mathbf{n} - \beta) \times \hat{B}]}{(1 - \beta \cdot \mathbf{n})^2} \right] \times \exp \left\{ i\omega \left( t' - \frac{\mathbf{n} \cdot \mathbf{r}(t')}{c} \right) \right\}^2,$$

(12)

where $\mathbf{r}(t')$ is the electron trajectory, $t'$ is retarded time (Jackson 1999).

2.2. Results

First, we show the radiation spectrum for $a = 3$ in Figure 1. The frequency is normalized by the fundamental frequency $\omega_0 = e\sigma/(\gamma mc)$, and the magnitude is arbitrarily scaled. The jagged line is the calculated spectrum, while the straight line drawn in the low frequency region is a line fitted to a power-law spectrum. The fitting is made in the range of 1–350$\omega_0$, and the spectral index turns out to be 0.44. The straight line drawn in the high frequency region shows a spectrum of $\propto \omega^{-5/3}$ expected for diffusive synchrotron radiation for reference (Toptygin & Fleishman 1987). The spectrum is well described by a broken power law, and the spectral index of the
above a power-law spectrum. The high frequency side above the peak indicates an excess spectrum of the low frequency side becomes a broken power-law spectrum. The peak frequency of this spectrum is located at around $10^3$. Low-energy side is harder than synchrotron theory predicts. The synchrotron radiation, while below the break the index is 0 in the middle frequency region, is different from that of $a = 3$ in both sides of the peak. The spectral shape changes from that of $a = 3$ in both sides of the peak.

Figure 1. Radiation spectrum for $a = 3$, $\alpha = 11/3$, and $\gamma = 10$. The straight line in the low frequency region shows a power-law spectrum with an index 0.44. The straight line in the high frequency region is $dW/d\omega \propto \omega^{-5/3}$ for reference. Power-law index of the low frequency spectrum is harder than the synchrotron theory predicts.

Figure 2. Radiation spectrum for $a = 5$, $\alpha = 11/3$, and $\gamma = 10$. Two straight lines drawn in low frequency region are fitted one to a power-law spectrum in the range of $0.5 \times 10^0 \omega_g$ and $10^{-1} \omega_g$, respectively. The power-law index is 0.58 and 0.33. The latter corresponds to the synchrotron radiation. The straight line in the high frequency region is $dW/d\omega \propto \omega^{-5/3}$, and we see a broad hump in the peak region, which is identified with the synchrotron spectrum with an exponential cutoff.

low-energy side is harder than synchrotron theory predicts. The peak frequency of this spectrum is located at around $10^3 \omega_g$. This frequency corresponds to approximately the typical frequency of synchrotron radiation $\omega_{\text{syn}} = 3\gamma^2e\alpha/2mc \sim 10^3 \omega_g$, for $\gamma = 10$.

Figure 2 shows the spectrum for $a = 5$. The spectral shape changes from that of $a = 3$ in both sides of the peak. The spectrum of the low frequency side becomes a broken power law with a break around $10 \omega_g$, above which the spectrum is fitted by a power law with an index of 0.33, as expected for synchrotron radiation, while below the break the index is 0.58. The high frequency side above the peak indicates an excess above a power-law spectrum $dW/d\omega \propto \omega^{-5/3}$. It looks like an exponential cutoff. The whole spectrum is described by a superposition of a synchrotron spectrum and a DSR broken power-law spectrum. This spectral shape is totally novel and is different from the one by described Fleishman & Urtiev (2010). He reported that the spectrum is described by broken power law in the same range of $a$ as this work ($1 < a < \gamma$, $a \sim 10^2$, and $\gamma \sim 10^5$) (Fleishman & Urtiev 2010).

To confirm our inference, we calculate the case of $a = 7$, for three different values of $\alpha$, i.e., $\alpha = 14/3$, $11/3$, and $8/3$, and the results are shown in Figure 3. The curved black line is a theoretical curve of synchrotron radiation, and three straight black lines are $dW/d\omega \propto \omega^{-2/3}$ and $\omega^{-3/3}$ expected for DSR theory for reference. The green line corresponding to $\alpha = 14/3$ reveals a clear exponential cutoff, and reveals a DSR component in only the highest frequency region. The power-law index of this component in the highest frequency region coincides with the expected value $-\alpha + 2 = -8/3$. The red and blue lines correspond to $\alpha = 11/3$ and $8/3$, respectively. They indicate a common feature to the green one.

3. INTERPRETATION

We give a physical explanation of the spectra obtained in the previous section. At first, we interpret the broken power-law spectrum for $a = 3$ by using DSR theory. Next, we consider a physical interpretation of the complex shape of radiation spectrum for $a = 5$ and $a = 7$. Finally, we compare our work with previous studies.

To begin with, we review the spectral feature of DSR based on the non-perturbative approach for $a < 1$ (Fleishman 2006). The typical spectrum takes a following form: $dW/d\omega \propto \omega^{-\alpha/3}$ in the low frequency region, $\propto \omega^0$ in the middle frequency region, and $\propto \omega^{8/3}$ in the high frequency region. The low frequency part and the middle frequency part are separated at $\omega_{\text{LM}} \sim a\omega_{\text{syn}}$. This spectral break corresponds to the break of the straight orbit approximation due to the effect of multiple scattering (Fleishman 2006). On the other hand, the middle frequency part and the high frequency part are separated at the typical frequency of jitter radiation $\omega_{\text{jit}}$, which is estimated by using the method of virtual quanta as $\omega_{\text{jit}} \sim \gamma^2 k_{\text{min}} c \sim a^{-1} \omega_{\text{syn}}$ (Medvedev 2000; Rybicki & Lightman 1979). Then, for $a \sim 1$, $\omega_{\text{LM}} \sim \omega_{\text{jit}}$ is achieved and the middle region may vanish. The spectrum for $a \sim 1$ becomes a broken power law with only one break, which is located at roughly the synchrotron
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Figure 4. Conceptual diagram of the radiation spectrum for $5 \leq a < \gamma$.

frequency $\omega_{\text{syn}}$. The power-law index of low frequency side is $\sim 0.5$ (which is harder than synchrotron radiation), and that of high frequency side is $-\alpha + 2 = -5/3$. Thus, the spectral feature for $a = 3$ can be explained by an extrapolation of the DSR non-perturbative approach for $a < 1$, if we consider that the middle frequency region is not conspicuous. Although our spectral index 0.44 slightly differs from 0.5 for DSR, this index is still harder than the synchrotron theory. The situation $a \sim 1$ can be achieved at the internal shock region of GRB, so that this may be responsible for harder spectral index than synchrotron observed for some GRBs.

Next, we interpret the spectral features for $a = 5$ and 7. The conceptual diagram of these spectra for $5 \leq a < \gamma$ is depicted in Figure 4, and a schematic picture of an electron trajectory is depicted in Figure 5. We explain the appearance of another break in the low frequency range seen at around $10\omega_{\text{syn}}$ in Figure 2. On the scale smaller than $\lambda_{\text{B}}$, the electron motion may be approximated by a helical orbit, while it is regarded as a randomly fluctuating trajectory when seen on scales larger than $\lambda_{\text{B}}$. Therefore, for the former scale, we can apply the synchrotron approximation to the emitted radiation. The beaming cone corresponding to the frequency $\omega$ is given by

$$\theta_{\text{cone}} = \frac{1}{\gamma} \left( \frac{3\omega_{\text{syn}}}{\omega} \right)^{1/3}$$

(Jackson 1999). The deflection angle $\theta_0$ of the electron orbit during a time $\lambda_{\text{B}}/c$ is estimated to be $\theta_0 = a/\gamma$ from the condition

$$\frac{\gamma \lambda_{\text{B}}}{a} \theta_0 = \lambda_{\text{B}},$$

as seen in Figure 5. Thus, the synchrotron theory is applicable only for $\theta_{\text{cone}} < \theta_0$, so that the break frequency is determined by $\theta_0 = \theta_{\text{cone}}$, and we obtain

$$\omega_{\text{br}} \sim a^{-3} \omega_{\text{syn}}.$$  

This break frequency is the same as that obtained by Medvedev (2010). We understand that as $a$ increases, the break frequency becomes lower, and when $a$ is comparable to $\gamma$, $\omega_{\text{br}}$ coincides with the fundamental frequency $e\sigma/\gamma mc$.

Next, we discuss on the high frequency radiation, which results from the electron trajectory on scales smaller than $\lambda_{\text{B}}$. The synchrotron theory applies between $\lambda_{\text{B}}/a = r_{\text{g}}/\gamma$ and $\lambda_{\text{B}}$. However, we should notice that electron motion suffers from acceleration by magnetic turbulence on scales smaller than $\lambda_{\text{B}}/a$. The trajectory down to the smallest scale of $2\pi/k_{\text{max}}$ is jittering, which is attributed to higher wavenumber modes as seen in the zoom up of Figure 5. If the field in this regime is relatively weak, i.e., $\alpha$ is relatively large (Figure 3, green line: $\alpha = 14/3$), the trajectory on the scale smaller than $\lambda_{\text{B}}/a$
does not deviate much from a helical orbit. In this case, the radiation spectrum reveals an exponential cutoff, and a power-law component appears only in the highest frequency region. On the contrary, if the smaller scale field is relatively strong, as in the case of α = 8/3 depicted in the blue line in Figure 3, the power-law component becomes predominant in the high frequency region, and the synchrotron exponential cutoff is smeared out. The intersection frequency of curved black line and straight black lines at around 10^3 ω_{br} in Figure 3 corresponds to ω_{br} as seen in Figure 4. Since the intersection frequency is determined by a, the frequency where the power-law component appears over the synchrotron cutoff is dependent on α. The excess from the theoretical curve in the middle frequency region in Figure 3 may be explained by the consideration of two effects. One is the contribution of hidden DSR component and the other is a range of synchrotron peak frequency caused by a fluctuation of magnetic field intensity. Fleishman reported that the spectrum for 1 < a < γ and 3 < a < 4 becomes a broken power law (Fleishman & Urtiev 2010). Medvedev asserted that the high frequency region of the spectrum reveals an exponential cutoff for 1 < a < γ (Medvedev 2010). Our result indicates that an exponential cutoff plus an extra power-law component appears, which is different from Fleishman’s remark and from Medvedev’s remark on the high frequency region. On the other hand, similar spectra to ours have been reported in Fleishman (2005) and Reville & Kirk (2010) when a uniform field is added to turbulent field. Because the high-energy power-law component arises from a turbulent spectrum over the wavenumber space, this component does not exist when the small-scale field is excited only in a narrow range of wavenumber space. Since the energy cascade of turbulent magnetic fields should exist at least to some degree, we regard the higher wavenumber modes as existing naturally. It depends on the set of parameters of σ and k_{max} whether this high-energy power-law component can be seen or not. If 2πσa/mc²k_{max} > 1, this component will not be seen. If the magnetic turbulence is excited by Weibel instability at the relativistic shocks, it is not possible for k_{max} to be much larger than k_{min}, because the wavelength of injection (λ_{B} = 2π/k_{min}) is only a few ten times the skin depth at most. Therefore, the component will not be seen for a ≫ 1 while for a ∼ O(1), this power-law component will be seen.

As for the frequency region lower than the break frequency ω_{br} = a^{-3}ω_{br}^2, Medvedev remarked that the spectrum is similar to small-angle jitter radiation (Medvedev 2010). However, it remains open whether it is so for 1 < a < γ, because the assumption that the straight orbit approximation of a radiating particle is broken. To predict the exact radiation spectrum of the frequency region lower than the break frequency, it is necessary to pursue the particle orbit to follow the long-term diffusion, which is a formidable task.

4. SUMMARY

We calculate the radiation spectrum from relativistic electrons moving in the small-scale turbulent magnetic fields by using the first principle calculation utilizing the Lienard–Wiechert potential. We concentrate our calculation on a range of the strength parameter of 1 < a < γ. We confirm that the spectrum for a ∼ 3 is a broken power law with an index of low-energy side ~0.5, and that some GRBs with a low-energy spectral index harder than synchrotron theory predicts may be explained. Furthermore, we find that the spectrum for a ∼ 7 takes a novel shape described by a superposition of a broken power-law spectrum and a synchrotron one. In particular, an extra power-law component appears beyond the synchrotron cutoff in the high frequency region, reflecting the magnetic field fluctuation spectrum. This is in contrast with previous works (Fleishman & Urtiev 2010; Medvedev 2010; our spectra for a = 5 and a = 7 are different from both of them). We have given a physical reason for this spectral feature. This novel spectral shape may be seen in various other scenarios in astrophysics. For example, the spectrum of 3C273 jet at the knot region may be due to this feature (Uchiyama et al. 2006).

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