Problem of Statistical Model in Deep Inelastic Scattering Phenomenology

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Abstract:

Recent Deep Inelastic data leads to an up-down quark asymmetry of the nucleon sea. Explanations of the flavour asymmetry and the di-lepton production in proton-nucleus collisions call for a temperature $T \approx 100$ MeV in a statistical model. This $T$ may be conjectured as being due to the Fulling-Davies-Unruh effect. But it is not possible to fit the structure function itself.

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There is lot of excitement because of availability of very precise data on the structure functions of the proton and the neutron (deuteron) by the NMC collaboration[1]. They find that the structure functions for proton and neutron violate the Gottfried sum rule[2] implying that the sea is not flavour symmetric. There is more d anti-quark in proton sea than the u antiquark.

The Fermilab experiment E772 reported the measurements of the yields of di-muons in a 800 GeV protons colliding with isoscalar and neutron excess targets[3]. The asymmetry in $\bar{u}(x)$ and $\bar{d}(x)$ distribution will have its mark in the cross section. Any model describing the Gottfried defect should also explain this Drell-Yan process.

The process $P + A \rightarrow \mu^+\mu^- + X$ is dominated in low $X_F$ region by a quark $q$ of a particular flavour annihilating the antiquark $\bar{q}$ of same flavour. Hence the cross-section is sensitive to the distribution of the anti-quark in the target nucleus. In the E772 experiment the target nuclei were isoscalars $^2H$, C and neutron rich W. The ratio of cross-sections per nucleon $\sigma_A$ in proton collision with a nucleus A, to that $\sigma_{iso}$, with an isoscalar target, is given by

$$R_A \equiv \frac{\sigma_A}{\sigma_{iso}} \approx 1 + \frac{(N - Z) \bar{d}(x) - \bar{u}(x)}{A \bar{d}(x) + \bar{u}(x)}$$

(1)

The differential cross section is given by

$$m^3 \frac{d\sigma}{dm dX_F} = \frac{8\pi\alpha^2}{9} \frac{\tau}{\sqrt{X_F}^2 + 4\tau} \sum_i e_i^2 [q_i(x_1)\bar{q}_i(x_2) + (1 \leftrightarrow 2)]$$

(2)

where $\tau = m^2/s$, $m^2 = x_1 x_2 s$ is the di-muon invariant mass square. $x_1$ and $x_2$ are the momentum fraction for the $q$ and the $\bar{q}$, $X_F = x_1 - x_2$ the Feynman variable. $s = 2 \times M \times E$, where $M$ is the mass of the proton and $E$ is the beam energy.

We go back to NMC experiment. The difference in the structure function for proton and neutron when integrated gives rise to

$$S_G \equiv \int_{0.004}^{0.8} [F_2^p(x) - F_2^n(x)] \frac{dx}{x} = 0.227 \pm 0.007 \text{ (stat)} \pm 0.014 \text{ (sys)}$$

(3)
It is claimed [4] that when extrapolated to $x = 0 \text{ to } 1$, this integral may give values much less, about half of the $1/3$ expected from symmetric sea! The unpolarized structure functions of proton and neutron in the quark-parton model are respectively given by

$$ F_2^{e,p}(x) = \frac{4}{9}[u^p(x) + \bar{u}^p(x)] + \frac{1}{9}[d^p(x) + \bar{d}^p(x)], \quad (4) $$

$$ F_2^{e,n}(x) = \frac{4}{9}[u^n(x) + \bar{u}^n(x)] + \frac{1}{9}[d^n(x) + \bar{d}^n(x)]. \quad (5) $$

Assuming $u^p = d^n = u$ and $d^p = u^n = d$, the eqn.(3) leads to

$$ S_G = \frac{1}{3} + \frac{2}{3} \int_0^1 \left[ \bar{u}(x) - \bar{d}(x) \right] dx \quad (6) $$

so that

$$ I_G \equiv \int_0^1 [\bar{u}(x) - \bar{d}(x)] dx = -0.140 \pm 0.024 \quad (7) $$

The valence quark distributions in the proton is

$$ \int_0^1 u_v(x) dx \equiv \int_0^1 [u(x) - \bar{u}(x)] dx = 2, \quad (8) $$

and

$$ \int_0^1 d_v(x) dx \equiv \int_0^1 [d(x) - \bar{d}(x)] dx = 1. \quad (9) $$

It is evident from eqns. (1) and (4) that there is a consistency requirement for the distribution function.

In the ref. [3] the data for the Drell-Yan ratio have been compared with different model-calculations [5], [6] and [7]. Although all the models were consistent with NMC data only the last model was found to be within the experimental error bars for the Drell-Yan ratio. Later, Eichten, Hinchcliffe and Quigg [8] showed that their model is consistent
with NMC as well as E772 experiments. In this perspective let us investigate what the statistical model predicts.

In this model the partons are described as a gas inside the confining hadron at finite temperature. Although very speculative, the model has been studied by many workers in the field [9], [10].

How does one reconcile oneself with such a temperature? We have pointed out earlier that this temperature could possibly arise because of Fulling-Davies-Unruh effect (FDU) [11]: accelerating particles feel a hot vacuum [12]. The applicability of FDU to Dirac particles was first treated by Soffel, Müller and Greiner [13]. This should also be of importance to hadron physics, since light quarks encounter very rapid change of velocity at the confining boundary of hadrons. The velocity of quarks is nearly equal to that of light, even in the constituent quark model and at the border of the confining region the quark must turn back sharply, in order that the confinement paradigm, to which we subscribe, should be valid.

But one has to be careful about the non-uniform acceleration. It is known that a uniformly accelerated detector in the Minkowski vacuum feels a thermal bath characterized by a $T$, proportional to its proper acceleration. For non-uniform acceleration, or for example a sudden deceleration felt by a lepton during a DIS process, - it is not at all clear that a simple thermal bath picture is adequate. Even for uniform acceleration there are problems of divergence in the excitation rate for finite-time detectors [14]; only recently it has been shown that no divergence appears provided the detectors are turned on and off continuously as in a more realistic picture for modeling physical detectors [15].

An effective temperature in a hadron may affect the structure functions and in particular may affect the difference effect in neutrons and protons. We shall, in particular, follow the work of Mac and Ugaz [10]. In their language, one could not “ascribe to the effective temperature, for example, any deep physical meaning (or permanent physical reality) concerning such a complicated bound system as the nucleon”. In our point of view, the presence of some kind of an average temperature is natural, once one admits
that the fast moving quarks have large average accelerations $a$.

We looked at Gottfried sum rule, which addresses the difference in the structure functions of the proton and the neutron and simultaneously at the related problem of the distribution function in Drell-Yan process. The parameters of the statistical model are the temperature $T$, chemical potentials for the $u$ and $d$ and the radius of the nucleon. We intend to fit these parameters to get the phenomenology right.

We cannot hope to fit the proton structure function itself, since we are using a model where quarks are bound, and as pointed out by Reya [16] all bound state approaches to DIS have problem since the scale of bound state problem is $100\text{ MeV}$. This is well known at large $x$, close to 1 where bound state structure functions do not go to zero. In a recent paper Donnachie and Landshoff [18] analyzed the problem from the phenomenological point of view. They point out that the variation of the structure function $\nu W_2$ with $Q^2$ at small $Q^2$ cannot be described by perturbative QCD: it is unsafe to use any perturbative evolution equation until $Q^2$ is at least so large that $\nu W_2$ has fully recovered from its need to vanish at $Q^2 = 0$. The latest NMC data on the structure function at small $x$ [17] is contrary to all earlier expectation and theoretical fits! It is found to increase at small $x$. This is incorporated in the ref. [18] along with the real photon data, which makes the fit very attractive. It may be that some bound state model can meet this phenomenological model halfway, when it has recovered from its boundedness. It is missing in our model.

As mentioned above, a defect of the model of Mac and Ugaz or any bound state model for partons is that for $x = 1$, the structure functions do not go to zero. It appears the model tries to ameliorate this problem by choosing a radius which is very large, about 2 fm. This is unsatisfactory, but the point of current interest is the antiparticle distributions and this is only substantial for small $x$. So we expect the model that we have adopted reproduces the essential physics of small $x$ DIS.

We hope that since the Gottfried sum rule refers to the difference between the proton and the neutron, the large $x$ part cancels out. The structure functions show asymmetry in spin and isospin only at small $x$. This was pointed out in the papers by Carlitz and
We start with the mean number of quarks with two polarizations and momentum within $p$ to $p + dp$:

$$q_i(p) = \frac{6V}{(2\pi)^3} \left[ 1 + \exp \left( \frac{\epsilon - \mu_i}{T} \right) \right]^{-1},$$

where $i$ is the flavour label, $\mu_i$ is the chemical potential for the respective quark, $V$ is the volume and $\epsilon$ the corresponding energy. The quark distribution in the infinite momentum frame is given by Mac and Ugaz [10] to be

$$q_i(x) = \frac{6V}{(2\pi)^2 M^2 T_x ln} \left[ 1 + \exp \left( \frac{\mu_i - Mx/2}{T} \right) \right],$$

where now $q_i(x)dx$ is the probability of finding a quark carrying the momentum fraction between $x$ and $x + dx$ of the total nucleon momentum.

Fixing $M = 938 \text{ MeV}$ we find the parameter set $T = 103 \text{ MeV}$, $\mu_u = 148 \text{ MeV}$, $\mu_d = 83.4 \text{ MeV}$ and $R = 1.28 \text{ fm}$ giving eqns. (8, 9) as

$$S_G = 0.22$$
$$I_G = 0.14$$

They compare very well with the experimental numbers (eqns. 3 and 7).

In Figs. 1 and 2 we plot the Drell-Yan ratio and the differential cross section (eqns. 1-2). The fit seems to be good. Regarding $F_2$ itself the situation is hopeless. Our $F_2$ has a peak of about 0.7 - 0.8 and then goes down, whereas the recent data show a flat structure. In [10] they have given formulae for using four particle gluon graphs with an extra parameter $K$, which measures the gluon coupling. The value of all quantities including $F_2$, now involves a tedious integral over the gluon variable $y$. But performing extensive time-consuming searches we found that we cannot improve the fits even with the extra available parameter $K$. We should also mention that the lower temperature given in refs. [9-10] is inadequate to explain Gottfried defect and Drell-Yan processes. So we conclude that a simple statistical model can give a rough description of the antiparticle
cloud, but a better model is certainly necessary to understand all aspects of deep inelastic scattering.

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**Figure caption**

Fig. 1 : Drell-Yan Ratio $R_A$ (eqn. 1) vs. $x$. Experimental data are from [3].

Fig. 2 : Drell-Yan differential cross-section (eqn. 2) vs $X_F$. Experimental data are from [3].