An improved Hausdorff distance method for locating single phase to ground fault in neutral non-effectively grounded system

Wenhao Wu | Xinhui Zhang | Jun Zhang | Yu He | Wenyuan Bai

School of Electrical and Electronic Engineering, Shandong University of Technology, Zibo, China

Correspondence
School of Electrical and Electronic Engineering, Shandong University of Technology, Zibo 255000, Shandong, China.
Email: zhxh@sdut.edu.cn

Funding information
Science and Technology Project of State Grid Corporation, Grant/Award Number: SGHADK00PJJS1900078

Abstract
Neutral non-effectively grounded mode is widely used in medium-voltage distribution networks in China. When a single-phase grounding fault occurs in distribution networks, abundant transient signals will be generated. Here, a fault location method based on improved Hausdorff distance is proposed with transient zero-sequence current and transient zero-sequence voltage of bus. First, according to the impedance characteristic analysis of the zero-sequence network, the selected frequency band (SFB) is determined, and the transient signals in the SFB are extracted by the digital filter. Second, the projection components of transient zero-sequence currents at each monitoring point of faulty feeder are obtained by the principle of orthogonalization. Finally, the projection component values of each section are calculated by the improved Hausdorff distance, and the maximum value is selected. If the maximum value is much larger than the sum of the values of other sections excluding the maximum value, the section where the maximum value is located is judged as a faulty section, otherwise, the downstream section of the last monitoring point is determined as the faulty section. Matlab/Simulink simulation shows that the proposed method can locate accurately under different fault conditions.

1 | INTRODUCTION

The neutral non-effectively grounded system is still the dominant type of distribution network in China. More than 80% of the total faults are single-phase grounding (SPG) fault in distribution network. When an SPG fault occurs in the neutral non-effectively grounded system, the line voltage between the three phases remains symmetrical and the system can be allowed to operate continuously for 1–2 h. However, if the SPG fault cannot be isolated in time, the insulation between the lines will be broken down and the faulty range will be expanded [1, 2]. Therefore, it is a crucial problem in relay protection technology to locate the faulty section quickly in the neutral non-effectively grounded system.

More recently, there are many published studies in the field of small-current grounding fault location, which can be roughly divided into three directions: methods using steady-state quantities, methods using the transient quantities and active methods.

Methods using steady-state fault quantities [3–6] include comparison of the amplitude and phase of the zero-sequence current of each feeder method, zero-sequence reactive power direction method and fifth harmonic component method, which are susceptible to the influence of noise and Petersen coil and result in the failure of fault location.

Compared with the above methods, the methods of using transient fault characteristics [7–12] are not affected by the Petersen coil, and can locate the faulty section quickly and accurately. Aiming at the successful case of traveling wave ranging method used in transmission line in [7], this method is applied to the distribution line with longer lines and fewer branches in [8], however, considering the presence of cable-overhead hybrid, short distribution network lines and many branches, it is difficult to locate fault section. The analysis of the similarity of the transient zero-mode current waveform on the upstream and downstream of the fault point was used for fault location in [9]. Simulation and field tests have found that when the fault point is in some special positions, the waveform of transient currents between upstream and downstream is similar in this method, that is, there are certain blind areas in the algorithm. A new method of faulty section location is proposed in [10], which eliminates the location blind zone by the consistency of the polarity relationship between the power frequency and the
Transient current and solves the deficiencies of the method proposed in [9]. The method of using empirical mode decomposition to obtain the intrinsic mode function was studied in [11, 12], which leads to the failure of the method under severe fault conditions.

The active methods [13–16] include the S injection method and the method adjusting the compensation degree of the Petersen coil. Different from other methods, additional equipment is required, which increases the difficulty of field operation. Besides, these methods are difficult to locate faulty section when the fault resistance is relatively high and they are not suitable for intermittent arc grounding fault.

In addition, with the access of the cable line, the fault situation of actual distribution network is more complicated and the transient information generated are more abundant. Therefore, it is very important to study reliable locating methods for the distribution network of cable-overhead hybrid lines.

Here, based on the traditional algorithm, an improved Hausdorff distance faulty location algorithm is proposed. First, the transient zero-sequence voltage of the bus and the transient zero-sequence current of each line are taken as fault characteristic quantities, a digital filter is designed to extract the transient characteristic quantities of the selected frequency band (SFB). After that, the faulty feeder is determined by comparing the amplitude of transient zero-sequence current of each feeder. Second, based on the principle of orthogonalization, the transient zero-sequence current of each monitoring point of the faulty feeder in the SFB are projected to the transient zero-sequence voltage of the bus. Finally, the projection components are calculated by the improved Hausdorff distance algorithm and then the faulty section is located according to the set fault criterion.

2 TRANSIENT CURRENT CHARACTERISTICS OF NEUTRAL NON-EFFECTIVELY GROUNDED SYSTEM

According to the model simplification principle and simplification steps proposed in [17], the SPG fault of the resonant grounding system is simplified, and the simplified circuit diagram is shown in Figure 1. \(U_{1f}\) denotes the zero-sequence virtual voltage source of the fault point; \(L_p\) is expressed as three times equivalent inductance of Petersen coil; \(R\) denotes the sum of line-mode, zero-mode resistance of the upstream at the fault point of faulty feeder and three times of grounding resistance; \(L\) denotes the sum of line-mode and zero-mode inductance of the upstream at the fault point of the faulty feeder; \(C\) denotes the sum of distributed capacitances of all outgoing lines of the system to ground.

When an SPG fault occurs in a neutral non-effectively grounded system, the transient current flowing through the fault point consists of transient capacitor current and transient inductor current, namely \(i_f = i_C + i_L\), calculated as follows:

\[
i_f = (I_{cm} - I_{bm}) \cos (\omega t + \varphi)
+ I_{cm} \left( \frac{\omega f}{\omega} \sin \varphi \sin \omega t - \cos \varphi \cos \omega f / \omega t \right) e^{-\frac{t}{\tau_c}}
+ I_{bm} \cos \varphi e^{-\frac{t}{\tau_l}},
\]

where \(I_{cm}, I_{bm}\) are steady-state capacitance current amplitude and inductor current amplitude flowing through the fault point when the SPG fault occurs; \(\tau_c, \tau_l\) are time constants; \(\omega = 1 / \sqrt{LC}\) is resonant angular frequency of inductance capacitor circuit. The first term in formula (1) is the difference between the steady-state capacitive current and the inductor current, that is the steady-state component. Due to the overcompensation effect of the Petersen coil, the amplitude and polarity of the steady-state component will be greatly affected, which will further affect the SPG fault detection in the resonant grounding system. Therefore, the transient component, \(i_{ts}\), will be extracted as follows:

\[
i_{ts} = I_{cm} \left( \frac{\omega f}{\omega} \sin \varphi \sin \omega t - \cos \varphi \cos \omega f / \omega t \right) e^{-\frac{t}{\tau_c}}
+ I_{bm} \cos \varphi e^{-\frac{t}{\tau_l}}.
\]

In the equivalent zero-sequence network shown in Figure 2, \(C_{01}, C_{02}, C_{0G}\) denote the ground capacitance of line 1, line 2 and generator end, respectively; \(C_{0a}, C_{0b}, C_{0d}, C_{0e}\) denote the capacitance to ground of the section ab, bf, fd, de of line 5, respectively; \(f\) is the fault point; \(U_{1f}\) denotes the zero-mode virtual voltage source at the fault point, which is numerically equal to the transient zero-mode voltage at the fault point. When a SPG fault occurs, the polarity of transient zero-sequence current flowing through the faulty feeder is contrary to that of the non-faulty feeder, and the amplitude of the former is larger than that of the latter. The length of upstream line at the fault point is much longer than that of downstream line, so the amplitude of
The neutral point ungrounded system is taken as an example. When a SPG fault occurs at point \( f \) in Figure 2, the transient zero-sequence current flowing through monitoring point \( b \), \( i_{0b} \), is equal to the sum of \( i_{0a} \) and the capacitance current to ground of section \( ab \). Similarly, the transient zero-sequence current flowing through the monitoring point \( d \), \( i_{0d} \), is equal to the sum of \( i_{0e} \) and the capacitance current to ground of section \( de \); The transient zero-sequence current flowing through the monitoring point \( a \), \( i_{0a} \), is equal to the sum of the capacitance current to ground of other non-faulty feeders.

Due to the short length of section \( ab \), the capacitance current to ground of the section \( ab \) can be neglected compared with the sum of the zero-sequence capacitance current of the non-faulty feeder. Therefore, the transient zero-sequence current flowing through monitoring points \( a \) and \( b \) is approximately equal, namely, \( i_{0a} \approx i_{0b} \).

3 | ANALYSIS OF IMPEDANCE PHASE FREQUENCY CHARACTERISTIC

3.1 | Analysis of line impedance

In the zero-sequence network of neutral non-effectively grounded system, in the condition of terminal open circuit, the input impedance, \( Z_{oc} \), of the line is

\[
Z_{oc}(\omega) = Z_s \coth(\gamma l)
\]

\[
= \sqrt{\frac{R_0 + j\omega L_0}{j\omega C_0}} \coth\left(\frac{l\sqrt{\omega R_0 C_0 - \omega^2 L_0 C_0}}{\omega C_0}\right),
\]

(3)

where \( Z_s = \sqrt{\frac{R_0 + j\omega L_0}{j\omega C_0}} \) is the wave impedance of the line; \( \gamma = \sqrt{\omega R_0 C_0 - \omega^2 L_0 C_0} \) is the propagation coefficient; \( l \) represents the line length.

The amplitude–frequency and phase–frequency characteristics of the input impedance are shown in Figure 3. It can be seen from Figure 3 that the input impedance shows capacitive and inductive alternatively with the increase of the frequency. When the input impedance changes from capacitive to inductive, the amplitude of the impedance reaches a minimum and the series resonance occurs in this line. The line impedance is capacitive in the frequency range of \( 0 < \omega < \omega_k \), and when in the frequency range of \( \omega > \omega_k \), it will show inductive and capacitive alternatively, where

\[
\omega_k = \frac{\pi}{2l\sqrt{L_0 C_0}}.
\]

(4)

In formula (4), \( \omega_k \) denotes the minimum resonant frequency of the series resonance of the line.

3.2 | Determination of the SFB

From the above, the series resonance or the parallel resonance will occur in the line with the increase of frequency, and generate transient resonance current, and the amplitude of transient current is affected by the excitation source. As a virtual power source of the excitation source, the frequency is increased from the power frequency, and the amplitude of transient current is
monotonically decreasing. Therefore, the amplitude of transient current generated by minimum resonant frequency of the series resonance or the parallel resonance will be the largest. From frequency 0 to the frequency band of the minimum value of the first resonant frequency of all non-faulty feeders is defined as the SFB [18]. The frequency range of the main transient components is usually greater than 200 Hz [19], therefore, the SFB is selected as $400\pi < \omega < \omega_k$. In the SFB, all non-faulty feeder and faulty feeder can be equivalent to a capacitor. Most of transient currents are distributed to all non-faulty feeders through faulty feeders (the amplitude of transient currents of non-faulty feeder is inversely proportional to their equivalent capacitive reactance). That is, the polarity of the transient zero-sequence current of the faulty feeder is contrary to the non-faulty feeder, and the amplitude of the former is larger than that of the latter. Similarly, the polarity of transient zero-sequence current of the upstream at the fault point is opposite to that of downstream, and the amplitude of the former is much larger than the latter.

In the SFB, LC series resonance will inevitably occur in the system, the resonance frequency of this resonance process is the smallest, which concentrates most of the energy of the transient signals. Therefore, the fault information in the SFB is selected as the fault characteristic quantity.

### 3.3 Extraction of transient characteristic quantities

Based on the above, the transient zero-sequence voltage of the bus and the transient components of the zero-sequence current of each monitoring point will be extracted as the fault characteristic quantity. The finite impulse response (FIR) is utilized to extract transient component.

The unit impulse response of FIR filter is a finite-length sequence, its unit impulse response $b(n)$ length is set as $L$ and its system function $H(\zeta)$ and difference function $\gamma(n)$ are

$$H(\zeta) = \sum_{n=0}^{L-1} b(n) \zeta^{-n},$$

$$\gamma(n) = \sum_{m=0}^{L-1} b(m) \times (n - m). \quad (6)$$

The performance of the designed digital filter can be improved by choosing an appropriate window function. The energy in the main lobe of the spectral function will be increased by adjusting the window function, which will decrease the passband and stopband fluctuations and increase the stopband attenuation. Based on the above description and the comparison of the advantages and disadvantages of each window function, the Kaiser window is chosen to extract transient signals [20, 21].

The Kaiser window is a single parameter window function group and it is used in digital signal processing, its optimization criterion is that for limited signal energy, it is required to determine a signal waveform with a limited time width, which maximizes the energy within the bandwidth. In other words, the energy in the frequency band of the Kaiser window is mainly concentrated in the main lobe, which has the best side lobe suppression performance. Its definition is shown in formula (7).

$$\nu(n) = \begin{cases} I_0 \left( \frac{\pi \alpha}{\sqrt{1 - (\frac{2n}{N-1})^2}} \right) & 0 \leq n \leq N-1, \\ 0 & \text{otherwise} \end{cases}, \quad (7)$$

where $N$ is the length of the sequence; $I_0$ is the first kind of modified Bessel function of zero order; $\alpha$ is any non-negative real number, it is used to adjust the shape of the Kaiser window. In the frequency domain, the width of the main lobe and the size of the side lobe can be selected.

As shown in Figure 4a,b, it can be concluded that the transient zero-sequence component shows an exponential decay trend. The FIR digital filter is used to extract fault information in the SFB in order to suppress the interference of power frequency and other low-frequency components.

![FIGURE 3 The amplitude–frequency and phase–frequency characteristic of the input impedance](image-url)
FIGURE 4  Current and voltage after an SPG.  
(a) Transient zero-sequence current of fault point.  
(b) Transient zero-sequence voltage of the bus

4  |  FAULT LOCATION PRINCIPLE  
BASED ON IMPROVED HAUSDORFF  
DISTANCE

4.1  |  Orthogonal processing of fault  
characteristic quantities

In linear algebra, if a set of vectors on the inner product space  
can form a subspace, it is called a base of this subspace. Gram–Schmidt  
orthogonalization provides a method, which can get  
an orthogonal basis of the subspace from a basis in this subspace  
and then get the corresponding standard orthogonal  
basis.

Suppose \( \alpha_1, \alpha_2, \ldots, \alpha_m \) \((m \leq n)\) is a linearly independent  
vector group in \( \mathbb{R}^n \), if

\[
\begin{align*}
\beta_1 &= \alpha_1, \\
\beta_2 &= \alpha_2 - \frac{\langle \alpha_2, \beta_1 \rangle}{\langle \beta_1, \beta_1 \rangle} \beta_1, \\
\vdots \\
\beta_m &= \alpha_m - \frac{\langle \alpha_m, \beta_1 \rangle}{\langle \beta_1, \beta_1 \rangle} \beta_1 - \frac{\langle \alpha_m, \beta_2 \rangle}{\langle \beta_2, \beta_2 \rangle} \beta_2 - \cdots - \frac{\langle \alpha_m, \beta_{m-1} \rangle}{\langle \beta_{m-1}, \beta_{m-1} \rangle} \beta_{m-1}.
\end{align*}
\]  
(8)

Then, a set of orthogonal vectors, \( \beta_1, \beta_2, \ldots, \beta_m \), is calculated by  
formula (8). Let

\[
e_i = \frac{\beta_i}{|\beta_i|} \ (i = 1, 2, \ldots, m). 
\]  
(9)

According to formula (9), a standard orthogonal vector set,  
\( e_1, e_2, \ldots, e_m \), is obtained. As shown in Figure 5, \( \varphi \) is the angle  
between vector \( \dot{U} \) and vector \( \dot{I}; i_p \) is the projected scalar of the
vector \( \mathbf{I} \) on the vector \( \mathbf{U} \); \( \mathbf{I}_p \) is the projection vector of the vector \( \mathbf{I} \) on the vector \( \mathbf{U} \).

As can be seen from Figure 5:

\[
\begin{align*}
  i_p &= |I| \cos \varphi, \\
  \mathbf{I}_p &= \frac{\mathbf{U}}{|U|} i_p
\end{align*}
\]

(10)

According to formula (10), the projection component of the vector \( \mathbf{I} \) on the vector \( \mathbf{U} \) can be obtained as follows:

\[
\mathbf{I}_p = \frac{\mathbf{U}}{|U|^2} \langle \mathbf{U} \cdot \mathbf{I} \rangle,
\]

(11)

where \( \langle \mathbf{U} \cdot \mathbf{I} \rangle \) is the inner product between vector \( \mathbf{U} \) and vector \( \mathbf{I} \). According to formula (11), the current projection of \( \mathbf{I}_p \) in the SFB can be calculated.

### 4.2 Principle of improved Hausdorff distance algorithm

The Hausdorff distance algorithm can be applied to image processing. It is a fuzzy distance measure to measure the similarity between two finite point sets. In image matching and recognition, the image is regarded as a point set, and the matching degree of the image is expressed by the similarity of the two point sets [22–24]. Suppose there are two finite point sets as \( \mathcal{A} = \{a_1, a_2, \ldots, a_n\} \), \( \mathcal{B} = \{b_1, b_2, \ldots, b_n\} \), then the Hausdorff distance can be used to measure the distance between these two point sets, namely

\[
H(\mathcal{A}, \mathcal{B}) = \max \{b(\mathcal{A}, \mathcal{B}), b(\mathcal{B}, \mathcal{A})\},
\]

(12)

where \( b(\mathcal{A}, \mathcal{B}) \) is the one-way Hausdorff distance from point set \( \mathcal{A} \) to point set \( \mathcal{B} \). \( b(\mathcal{B}, \mathcal{A}) \) is the one-way Hausdorff distance from point set \( \mathcal{B} \) to point set \( \mathcal{A} \).

\[
b(\mathcal{A}, \mathcal{B}) = \max_{a \in \mathcal{A}} \left( \min_{b \in \mathcal{B}} \| a - b \| \right),
\]

(13)

\[
b(\mathcal{B}, \mathcal{A}) = \max_{b \in \mathcal{B}} \left( \min_{a \in \mathcal{A}} \| b - a \| \right),
\]

(14)

\( \| a - b \|, \| b - a \| \) represent the Euclidean distance between point set \( \mathcal{A} \) and point set \( \mathcal{B} \).

The calculation results of the traditional Hausdorff distance will be greatly disturbed by isolated points or occlusions in the image, which will affect the accuracy of image matching. In order to improve the accuracy of the calculation results, the Hausdorff distance algorithm is improved by using the average value instead of the maximum value when calculating \( b(\mathcal{A}, \mathcal{B}) \) and \( b(\mathcal{B}, \mathcal{A}) \) there.

The improved Hausdorff distance from point set \( \mathcal{A} \) to point set \( \mathcal{B} \) is

\[
b_{\text{mod}}(\mathcal{A}, \mathcal{B}) = \frac{1}{|\mathcal{A}|} \sum_{a \in \mathcal{A}} \left( \min_{b \in \mathcal{B}} \| a - b \| \right).
\]

(15)

The improved Hausdorff distance from point set \( \mathcal{B} \) to point set \( \mathcal{A} \) is

\[
b_{\text{mod}}(\mathcal{B}, \mathcal{A}) = \frac{1}{|\mathcal{B}|} \sum_{b \in \mathcal{B}} \left( \min_{a \in \mathcal{A}} \| b - a \| \right).
\]

(16)

According to formulas (12), (15), and (16), the two-way improved Hausdorff distance between point set \( \mathcal{A} \) and point set \( \mathcal{B} \) is calculated as follows:

\[
H_{\text{mod}}(\mathcal{A}, \mathcal{B}) = \max \{b_{\text{mod}}(\mathcal{A}, \mathcal{B}), b_{\text{mod}}(\mathcal{B}, \mathcal{A})\}.
\]

(17)

In formula (17), \( H_{\text{mod}}(\mathcal{A}, \mathcal{B}) \) reflects the degree of matching between point set \( \mathcal{A} \) and point set \( \mathcal{B} \). The smaller the value of \( H_{\text{mod}}(\mathcal{A}, \mathcal{B}) \) is, the higher the similarity between point set \( \mathcal{A} \) and point set \( \mathcal{B} \). The three paths in Figure 6 are taken as an example to compare the calculated results of the traditional and improved Hausdorff distance.

In Figure 6, the nodes through paths X, Y and Z are (1–4, 4, 9, 10), (6–10) and (11, 16–20, 15), respectively. Node 11 through path Z is taken as an example to compare the value of \( H(Z, X) \) and \( H_{\text{mod}}(Z, X) \). Among all the nodes through path X, the closest to node 11 is node 1, and its distance is 2 p.u., that is, \( \min \| x_{(1)} - x \| = 2 \) p.u.. Similarly, it can be proved that \( \min \| x_{(10)} - x \| = 3 \) p.u., \( \min \| x_{(17)} - x \| = 3 \) p.u., \( \min \| x_{(18)} - x \| = 3 \) p.u., \( \min \| x_{(20)} - x \| = 2 \) p.u., \( \min \| x_{(15)} - x \| = 1 \) p.u., according to formulas (13) and (15), it can be obtained that \( b(Z, X) = 3 \) p.u., \( b_{\text{mod}}(Z, X) = 3+3+3+2+2+1/7 = 2.285 \) p.u., then, according to formulas (14) and (16), it can be obtained that \( b(X, Z) = 4 \) p.u., \( b_{\text{mod}}(X, Z) = 2.5 \) p.u. Finally, according to formulas (12) and (17), it can be seen that \( H(Z, X) = 4 \) p.u., \( H_{\text{mod}}(Z, X) = 2.5 \) p.u. The improved Hausdorff distance value of 2.5 p.u. is smaller than the traditional Hausdorff distance value of 4 p.u., which indicates that the calculation result of
the improved Hausdorff distance algorithm is more accurate, and the matching degree of paths Z and X is higher. Therefore, compared with the traditional algorithm, using the improved algorithm to measure the similarity of the zero-sequence current of each feeder is better. The same method as above is used to compare the similarity of path X and Y, and path Y and Z, the conclusions are the same.

### 4.3 Principle of fault location

Assuming that there are m sections in the faulty feeder of the neutral non-effectively grounded system, for the transient zero-sequence currents $i_{0i}$, $i_{0j}$ of adjacent sections $i, j$, the improved Hausdorff distance between two adjacent sections can be obtained from formula (17):

$$H_{\text{mod}}(i, j) = \max \left(b_{\text{mod}} \left(i_{0i}, i_{0j}\right), b_{\text{mod}} \left(i_{0j}, i_{0i}\right)\right), \quad (18)$$

$H_{\text{mod}}(i, j)$ is defined as the degree of mismatch of the transient zero-sequence currents of adjacent sections $i, j$. The larger the value of $H_{\text{mod}}(i, j)$ is, the more obvious the difference between the transient zero-sequence current waveforms.

When a SPG fault occurs in the distribution network, the amplitude of the transient zero-sequence current of the upstream at the fault point is much larger than that of downstream and the polarities of the two are opposite. Therefore, the amplitude difference of projected component of the monitoring points on both sides of the fault point is high, the waveform similarity is low and the corresponding $H_{\text{mod}}(i, j)$ vary greatly. However, the amplitude difference of projected component of each monitoring point of the upstream at the fault point is low, the waveform similarity is high and the corresponding $H_{\text{mod}}(i, j)$ vary slightly. For each monitoring point of the downstream at the fault point, a similar conclusion can be concluded. When an SPG fault occurs at the end of the line, the waveforms of the projected components of the adjacent monitoring points of the faulty feeder are highly similar and the corresponding $H_{\text{mod}}(i, j)$ are not much different. $H_{\text{max}}$ is defined as the maximum value of the improved Hausdorff of the projection components of each section at the faulty feeder. $\sum H_{\text{mod}}$ is defined as the sum of the improved Hausdorff distance values of the projection components of each section at the faulty feeder excluding the $H_{\text{max}}$. According to the relationship between the two to judge the faulty section, the fault criterion is

$$H_{\text{max}} > \sum H_{\text{mod}}, \quad (19)$$

If the value $H_{\text{mod}}(i, j)$ of each section of the faulty feeder satisfies formula (19), it is judged as the faulty section, otherwise determined as that the downstream section of the last monitoring point is the faulty section.

### 4.4 Fault location flowchart

For the distribution network with overhead-cable hybrid outgoing lines, the flow chart of faulty section location based on the above principles is shown in Figure 7. The specific fault location steps are as follows:

1. The zero-sequence voltage of the bus of the system, $U_0$, is monitored. When $U_0$ is greater than $kU_N$ ($U_N$ is defined as the bus rated voltage, and the value of $k$ is generally set as 0.15), the system is identified as a faulty system.
2. The zero-sequence current to each line is obtained by zero-sequence current mutual inductor. The method of comparing the amplitude of transient zero-sequence current is used to determine the feeder where the fault is located, and then searches for the faulty section by section along the feeder.
3. The zero-sequence voltage of the bus and the zero-sequence current of each monitoring point of the faulty feeder in the first cycle are sampled after a SPG fault, and the digital filter is designed to extract the transient zero-sequence voltage of the bus and the transient zero-sequence current at each monitoring point of the faulty feeder in the SFB.
4. Formula (11) is used to obtain the projection components of each monitoring point of the faulty feeder, and formula (18) is used to calculate the improved Hausdorff distance value of adjacent monitoring points with the projected current.
5. According to the obtained improved Hausdorff distance value, select the maximum value $H_{\text{max}}$, if $H_{\text{max}}$ is much larger than $\sum H_{\text{mod}}$, the section with the maximum value $H_{\text{max}}$ is judged as the faulty section, otherwise, the downstream section of the last monitoring point is the faulty section.

### 5 SIMULATION VERIFICATION

#### 5.1 Simulation modeling

Matlab/Simulink software is used to build a 10 kV cable-overhead hybrid neutral non-effectively grounded system model as shown in Figure 8. The model contains three overhead and one cable mixed feeders: feeders $L_1, L_2, L_3$, one cable feeder$L_4$, one overhead feeder$L_5$, one cable feeder$L_6$. The line parameters are shown in Table 1.

For the resonant grounding network, the system is set to be overcompensated and the compensation degree of the Petersen coil is set to 110%, $\psi$ is detune degree, which indicates the compensation of the Petersen coil, $\psi = I_C - I_f/I_C$. The inductance value of Petersen coil is calculated to be 0.414 H, and the active power loss of the Petersen coil is about 2.5–5% of the inductive reactive power loss, in general, it is taken into 4% and the resistance value of the Petersen coil is calculated to be 5.2 $\Omega$. The sampling frequency is set as 10 kHz.
5.2 Simulation analysis under different fault conditions

In order to verify the effectiveness of the proposed method under different fault conditions, a great deal of simulations were carried out by changing the grounding resistance, the fault inception angle, and the position of the fault point in the neutral non-effectively grounded system. Some simulation results are shown in Tables 2–4.

By analysing data in Table 2–4, it is found that with the increase of the grounding resistance, the fault transient characteristic quantity is weakened, and the improved Hausdorff distance value of the faulty section also decreases, but the maximum value $H_{\text{max}}$ of the faulty section is still far larger than...
TABLE 2  Improved Hausdorff distance value of each section under different fault inception angles

| Grounding resistance (Ω) | Fault inception angle (rad) | Improved Hausdorff distance of sections 1–5 | Location result |
|--------------------------|----------------------------|---------------------------------------------|-----------------|
|                          |                            | Section 1 | Section 2 | Section 3 | Section 4 | Section 5 |                      |
| 0                        | 0                          | 0.535     | 75.156    | 0.504     | 0.560     | 0.474     | Section 2             |
| 0                        | π/6                        | 2.961     | 385.189   | 2.987     | 2.698     | 2.374     | Section 2             |
| 0                        | π/4                        | 4.5177    | 554.855   | 3.574     | 4.063     | 3.481     | Section 2             |
| 0                        | π/3                        | 5.3991    | 671.918   | 4.059     | 4.730     | 4.239     | Section 2             |
| 0                        | π/2                        | 6.643     | 780.9240  | 5.158     | 5.802     | 5.026     | Section 2             |

the sum of the distance values in the remaining section $\sum H_{\text{rest}}$. After changing the position of the fault point and the fault inception angle, the faulty section can still be accurately located according to the fault criterion. Therefore, the fault location results are not affected by the fault inception angle, the value of the grounding resistance and the position of the fault point, which verifies the effectiveness of the fault location method based on improved Hausdorff distance.

5.3  Gaussian white noise interference

The influence of noise interference is also taken into consideration. In the neutral non-effectively grounded system, different fault inception angles and grounding resistances are set and Gaussian white noise interference is added to the resonant grounding system. Table 5 shows the comparison of fault location results with no noise added and white Gaussian noise with a signal-to-noise ratio (SNR) of 60 dB added to the system.

It can be seen from Table 5 that when a metallic grounding occurs in a neutral non-effectively grounded system, the results of fault location are not affected by the white noise interference.

TABLE 3  Improved Hausdorff distance value of each section under different grounding resistances

| Fault inception angle (rad) | Grounding resistance (Ω) | Improved Hausdorff distance of sections 1–5 | Location result |
|-----------------------------|--------------------------|---------------------------------------------|-----------------|
|                             |                          | Section 1 | Section 2 | Section 3 | Section 4 | Section 5 |                      |
| 0                           | 0                        | 0.535     | 75.156    | 0.504     | 0.560     | 0.476     | Section 2             |
| 0                           | 10                       | 0.254     | 30.086    | 0.199     | 0.228     | 0.193     | Section 2             |
| 0                           | 100                      | 0.040     | 4.5358    | 0.027     | 0.027     | 0.026     | Section 2             |
| 0                           | 500                      | 0.009     | 1.002     | 0.006     | 0.006     | 0.005     | Section 2             |
| 0                           | 1000                     | 0.005     | 0.510     | 0.003     | 0.003     | 0.003     | Section 2             |

5.3  Gaussian white noise interference

The influence of noise interference is also taken into consideration. In the neutral non-effectively grounded system, different fault inception angles and grounding resistances are set and Gaussian white noise interference is added to the resonant grounding system. Table 5 shows the comparison of fault location results with no noise added and white Gaussian noise with a signal-to-noise ratio (SNR) of 60 dB added to the system.

It can be seen from Table 5 that when a metallic grounding occurs in a neutral non-effectively grounded system, the results of fault location are not affected by the white noise interference.

TABLE 4  Improved Hausdorff distance value of each section when metallic grounding

| Fault inception angle (rad) | Faulty section | Improved Hausdorff distance of sections 1–5 | Location result |
|-----------------------------|----------------|---------------------------------------------|-----------------|
|                             |                | Section 1 | Section 2 | Section 3 | Section 4 | Section 5 |                      |
| 0                           | Section 1      | 65.940    | 0.357    | 0.333    | 0.408    | 0.422     | Section 1             |
| 0                           | Section 2      | 0.535     | 75.156   | 0.504    | 0.560    | 0.476     | Section 2             |
| 0                           | Section 3      | 0.448     | 0.542    | 63.853   | 0.514    | 0.439     | Section 3             |
| 0                           | Section 4      | 0.317     | 0.400    | 0.283    | 50.288   | 0.374     | Section 4             |
| 0                           | Section 5      | 0.250     | 0.297    | 0.227    | 0.194    | 41.039    | Section 5             |
| 0                           | End section    | 0.183     | 0.229    | 0.171    | 0.145    | 0.155     | End section           |
TABLE 5  Fault location results under the influence of Gaussian white noise

| SNR (dB) | Faulty section | Grounding resistance (Ω) | Fault inception angle (rad) | Improved Hausdorff distance of each section | Location result |
|----------|----------------|--------------------------|----------------------------|-------------------------------------------|----------------|
| 0        | Section 2      | 0                        | 0                          | 0.535, 75.156, 0.504, 0.560, 0.476        | Section 2      |
| 0        | Section 2      | 10                       | π/6                        | 1.403, 155.748, 1.804, 1.230, 1.156        | Section 2      |
| 0        | Section 2      | 100                      | π/4                        | 0.260, 31.405, 0.219, 0.253, 0.226         | Section 2      |
| 0        | Section 2      | 500                      | π/3                        | 0.071, 7.759, 0.052, 0.054, 0.050          | Section 2      |
| 0        | Section 2      | 1000                     | π/2                        | 0.043, 4.507, 0.033, 0.035, 0.031          | Section 2      |
| 60       | Section 2      | 0                        | 0                          | 0.540, 75.157, 0.503, 0.559, 0.475         | Section 2      |
| 60       | Section 2      | 10                       | π/6                        | 1.402, 155.748, 1.084, 1.230, 1.156        | Section 2      |
| 60       | Section 2      | 100                      | π/4                        | 0.260, 31.404, 0.219, 0.253, 0.226         | Section 2      |
| 60       | Section 2      | 500                      | π/3                        | 0.071, 7.759, 0.052, 0.054, 0.050          | Section 2      |
| 60       | Section 2      | 1000                     | π/2                        | 0.043, 4.507, 0.033, 0.035, 0.031          | Section 2      |

After adjusting the grounding resistance and the fault inception angle, the faulty section can still be accurately located under the influence of Gaussian white noise. It can be concluded that there is little influence on the proposed location method under strong noise interference.

5.4 | The superiority of the proposed locating method

The traditional Hausdorff distance algorithm is used to locate the faulty section, and the parameters required are consistent with the original text. Table 6 shows the comparison and analysis of the simulation results of faulty section location using traditional and improved Hausdorff distance algorithms when an SPG fault occurs in a neutral non-effectively grounded system with the fault inception angle is 0°.

From the data in Table 6, it can be found that when fault characteristic information required are consistent with proposed method, when the ground resistance is 0, both the traditional and the improved Hausdorff distance can accurately locate the faulty section. However, as the grounding resistance increases, the method based on the traditional Hausdorff distance can cause misjudgment. Simulation results show that the method using the improved Hausdorff distance is not affected by the grounding resistance, which demonstrate that the proposed method is more reliable than the method using the traditional Hausdorff distance.

In order to reflect the advantage of the method proposed here, the simulation results of the existing location methods and the method proposed in this paper under different fault conditions are shown in Tables 7 and 8.

It can be seen from Table 7 that according to their respective fault criterion, the four fault location methods can accurately locate the faulty section under element fault conditions. As shown in Table 8, the fault characteristics are greatly affected by severe fault conditions, the four fault location methods are affected differently. The specific analysis is as follows:

1. The polarity of the energy weight coefficient of the EMD-based fault location method is positive in both section 2 and section 4, which will cause misjudgement.
2. According to the fault location method based on the correlation coefficient, the threshold value of the fault...
TABLE 7  Fault location results using different location methods (the grounding resistance is 10Ω and the fault inception angle is 45°)

| Fault location method     | Fault criterion                                                                 | Section number | Location result |
|---------------------------|---------------------------------------------------------------------------------|----------------|-----------------|
| Improved Hausdorff distance | $H_{max} > \sum H_{rest}$                                                        | 1  2  3  4  5 | Section 2       |
| Correlation coefficient method | Compare the degree of correlation coefficient with the threshold                   | 1  0.111  1  1  1  1 | Section 2       |
| Grey Relational degree    | Compare the degree of grey correlation with the threshold                          | 0.908 0.514 0.819 0.843 0.78 | Section 2       |
| Empirical mode decomposition | The positive and negative of the difference between adjacent energy weight coefficient | Negative  Positive Negative Negative Negative | Section 2       |

TABLE 8  Fault location results using different location methods (the grounding resistance is 500Ω and the fault inception angle is 90°)

| Fault location method     | Fault criterion                                                                 | Section number | Location result |
|---------------------------|---------------------------------------------------------------------------------|----------------|-----------------|
| Improved Hausdorff distance | $H_{max} > \sum H_{rest}$                                                        | 0.045 4.543 0.035 0.036 0.032 | Section 2       |
| Correlation coefficient method | Compare the degree of correlation coefficient with the threshold                   | 1  0.819 1 1 1 1 | Section 2       |
| Grey Relational degree    | Compare the degree of grey correlation with the threshold                          | 0.646 0.523 0.633 0.761 0.729 | Section 2       |
| Empirical mode decomposition | The positive and negative of the difference between adjacent energy weight coefficient | Negative  Positive Negative Positive Negative | No result       |

criterion is set in the range of 0.5–0.8, and the value of section 2 exceeds 0.8, therefore, different thresholds need to be set according to different fault conditions in the judgment process, which will increase the complexity of fault point judgment.

3. Based on the location method using grey correlation degree, the threshold in the fault criterion is set to 0.6, and the value of section 2, 0.523, is less than 0.6, which can locate the faulty section. However, when approaching the threshold, the value of the non-faulty section, 0.633, may result in misjudgement.

4. According to the fault criterion of the method proposed in this paper, the maximum value $H_{max}$ is equal to 227.071, which is much larger than the sum of the remaining values $\sum H_{rest}$, that is 7.228, under element fault conditions. The same conclusion, $H_{max} > \sum H_{rest}$, can be obtained under severe fault conditions.

Therefore, the method proposed there can accurately locate the faulty section regardless of the element fault conditions or the severe fault conditions. On the basis of the numerical comparison results, it can be seen that the proposed method is more reliable.

5.5  Analysis of the recorded field data

The recorded field data of the actual SPG fault event are used to evaluate whether the proposed method can identify the faulty section. The recorded field data of two actual SPG faults are collected by fault location device. The two faults occur at the same feeder and field data of four monitoring points of the faulty feeder are recorded. Both of the SPG faults occur between the first monitoring point and the second monitoring point, namely section 1. The two recorded waveforms collected by the fault location device are shown in Figures 9 and 10.

The method proposed in this paper is used to calculate the two sets of data recorded by the fault location device, and the results are shown in Table 9. Taking the results of the first set of recorded data as an example, the improved Hausdorff distance value of section 1 is 0.393, and the sum of improved Hausdorff distance values of sections 2 and 3 is 0.0753. Therefore, according to fault criterion, the maximum value $H_{max}$, 0.393, is much larger than the sum of the values $\sum H_{rest}$, 0.0753, and section 1 is selected as faulty section. The same conclusion can be obtained based on the second set of data, which comply with judgment of fault location device.
FIGURE 9 The first waveform recorded by fault location device. (a) Waveform of the zero-sequence voltage. (b) Waveform of the zero-sequence current of each monitoring point

TABLE 9 Location results of the field recorded data

| Improved Hausdorff distance of each section | Fault criterion ($H_{\text{max}} > \sum H_{\text{rest}}$) | Location result |
|-------------------------------------------|-----------------------------------------------|-----------------|
| 1  2  3                                   | 0.393 > 0.0753                                | Section 1       |
| 2.088 0.068 0.004                         | 2.088 > 0.072                                 | Section 1       |

6 | DISCUSSION

In view of the difficulty in locating the overhead-cable hybrid outgoing line of the distribution network, a fault location method based on transient zero-sequence current and transient zero-sequence voltage of bus information is proposed. The fault information of each section in the first cycle after the SPG fault is sampled. The improved Hausdorff distance algorithm is used to calculate the projection of each section to determine faulty section. The simulation results are not affected by the location of the fault point, the ground resistance, the fault inception angle and the noise.

ACKNOWLEDGEMENTS

This work was supported by the Science and Technology Project of State Grid Corporation (grant number SGHADK00PJJJS1900078).

REFERENCES

1. Xiao, Y., et al.: Fault protection method of single-phase break for distribution network considering the influence of neutral grounding modes. Prot. Control Modern Power Systems, 5(1), 10 (2020)
2. Lin, X. et al.: Novel faulty feeder identification scheme in the neutral ineffectively grounded distribution networks. In: 12th IET International Conference on Developments in Power System Protection (DPSP 2014), Copenhagen, Denmark, pp. 1–6 (2014)
3. Yuan, S., Zhao, J., Song, Y.: “Analysis and comparison of several fault line selective methods in small current grounding power system,” 2008 China International Conference on Electricity Distribution, pp. 1–7 (2008)
4. Wang, Z., et al.: Faulty line selection and location based on zero-sequence amplitude increment ratio. In: 2020 5th Asia Conference on Power and Electrical Engineering (ACPEE), Chengdu, China, pp. 1477–1481 (2020)
5. Zhang, Z.X., et al.: The summarization of fault line selection of small current grounding system. Appl. Mech. Mater., 494-495, 1775–1778 (2014)
6. Gao, S., Lin, Z.: Study on section location of medium voltage distribution network based on fifth harmonic current fault component. In: 2018 IEEE 3rd Advanced Information Technology, Electronic and Automation Control Conference (IAEAC), Chongqing, China, pp. 1326–1331 (2018)
7. Lei, A., et al.: A novel current travelling wave based single-ended fault location method for locating single-phase-to-ground fault of transmission line. In: 2015 50th International Universities Power Engineering Conference (UPEC), Stoke on Trent, UK, pp. 1–6 (2015).
8. Zhao, J., et al.: Travelling wave fault location for distribution line based on improved morphological gradient algorithm. In: 2016 International Symposium on Computer, Consumer and Control (IS3C), Xi’an, China, pp. 156–159 (2016)
9. Ma, S.C., et al.: Transient current correlation based single phase earth fault location for distribution automation. In: 45th International Universities Power Engineering Conference UPEC2010, Cardiff, UK, pp. 1–4 (2010)
10. Xue, Y., et al.: Small current grounding fault transient analysis and a new method for segment location. Autom. Electr. Power Syst. 38(23), 101–107 (2014) (in Chinese)
11. Shu, H.-C., Li, B.: Research on fault line detecting based on empirical mode decomposition (EMD) in resonant grounded systems. In: 2009
12. Zhao, J., et al.: A fault section location method for small current grounding system based on HHT. In: 2018 China International Conference on Electricity Distribution (CICED), Tianjin, China, pp. 1769–1773 (2018)
13. Zhang, Y., Zhang, H.: Faulty line selection for single-phase earthed fault based on injected active component. In: 2011 International Conference on Electrical and Control Engineering, Yichang, pp. 5270–5273 (2011)
14. Li, J., Liang, J.: 10kV straight line fault location based on signal injection method. In: 2011 IEEE Power Engineering and Automation Conference, Wuhan, China, pp. 512–515 (2011)
15. Shen, Z., Qin, L.: Study on automatic arc suppression coil compensation technology and fault line selection and location based on fixed-frequency signal injection method. In: 2011 2nd International Conference on Artificial Intelligence, Management Science and Electronic Commerce (AIM-SEC), Deng Feng, China, pp. 4354–4357 (2011)
16. Wang, P., et al.: Fault location in resonant grounded network by adaptive control of neutral-to-earth complex impedance. IEEE Trans. Power Delivery, 33(2), 689–698 (2018)
17. Xue, Y., et al.: The transient equivalent circuit and transient analysis of the small current grounding fault of the neutral point through the arc suppression coil grounding system. Proc. Chin. Soc. Electr. Eng. 35(22), 5703–5714 (2015)
18. Yong-Duan, X., et al.: LC resonance mechanism analysis of fault transient for single phase earth fault in non-solidly earthed network. In: 2013 IEEE Power and Energy Society General Meeting, Vancouver, BC, Canada, pp. 1–5 (2013)
19. Xue, Y., et al.: Earth fault protection using transient signals in non-solid earthed network. In: Proceedings of the International Conference on Power System Technology, Kunming, China, pp. 1763–1767 (2002)
20. Wang, X., et al.: Location of single phase to ground faults in distribution networks based on synchronous transients energy analysis. IEEE Trans. Smart Grid, 11(1), 774–785 (2020)
21. Sulistyantoingsih, et al.: Performance comparison of Blackman, Bartlett, Hanning, and Kaiser window for radar digital signal processing. In: 2019 4th International Conference on Information Technology, Information Systems and Electrical Engineering (ICITISEE), Yogyakarta, Indonesia, pp. 391–394 (2019)
22. Dang-Nguyen, C., Do-Hong, T.: Robust line Hausdorff distance for face recognition. In: 2019 International Symposium on Electrical and Electronics Engineering (ISEE), Ho Chi Minh City, Vietnam, pp. 103–107 (2019)
23. Taha, A.A., Hanbury, A.: An efficient algorithm for calculating the exact Hausdorff distance, in IEEE Trans. Pattern Anal. Machine Int., 37(11), 2153–2163 (2015)
24. Jianwen, Z., et al.: Fault section location method using wide area centralized structure based on modified Hausdorff distance. In: 2017 IEEE Conference on Energy Internet and Energy System Integration (EI2), Beijing, pp. 1–6 (2017)

How to cite this article: Wu W., et al.: An improved Hausdorff distance method for locating single phase to ground fault in neutral non-effectively grounded system. IET Gener. Transm. Distrib. 2021;1–13. https://doi.org/10.1049/gtd2.12212