Development of Unscented Kalman Filter Algorithm for stock price estimation

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Abstract. Stock market is established in order to bring together the stock sellers and buyers. Securities often traded in the stock market are shares. Shares are securities as proof of participation or ownership of a person or legal entity in a company. In choosing a safe and appropriate investment in stocks, investors need a way to assess the price of the shares to be purchased or the ability of the stock to provide dividends in the future, so as to optimize profits. The correct way to analyze the risk for investors in investing is to estimate the stock price. The purpose of this paper is to analyze the comparison of share price estimates using the Unscented Kalman Filter (UKF) and Unscented Kalman Filter Square Root (UKF-SR) methods. The simulation results show that both methods have a significantly high accuracy of less than 2%. We conclude that the two methods can be used to estimate the stock prices.

1. Introduction
In the development of the business world, companies are very dependent on investment. Investment contributes to the development of a business, because with investment, it is expected to get more profits. One of them is financial investment, whereas financial investment is only a proof of ownership of the company but does not have a direct contribution to the production of the company, such as stocks, bonds, and other securities. Shares are securities as proof of a person's participation or ownership in a company. In daily stock trading activities, stock prices experience fluctuations either increases or decreases [1].

The right way of analysis will reduce the investment risk for investors, that is, an ability to estimate stock prices. One method for estimating increases and decreases in stock prices is by making estimates. The reason is that very often a problem can be solved using previous information or previous data related to the problem. Kalman filter is a method of estimating state variables from a discrete linear dynamic system that minimizes covariance estimation errors. Some of the developments of the Kalman Filter (KF) method are Ensemble Kalman Filter (EnKF), Ensemble Kalman Filter Square Root (EnKF-SR), Fuzzy Kalman Filter (FKF), Unscented Kalman Filter (UKF), and Unscented Kalman Filter Square Root (UKF-SR). EnKF is the development of KF by generating a number of good ensembles, that is, 100, 200, or 300 ensembles [2,3]. EnKF-SR is the development of EnKF by adding a square root scheme at the correction stage [4,5]. And, FKF is a combination of KF and fuzzy methods [6]. UKF is a development of the KF method with unscented transformation, while UKF-SR is the development of UKF with the addition of a square root scheme. This paper
focuses on the application of the UKF and UKF-SR method to estimate stock prices for high and low prizes, which can be used as basis of consideration by investors.

2. **Unscented Kalman Filter (UKF)**

Algorithm of Unscented Kalman Filter is written as follows [7]:

- **Initiation at** \( k = 0 \):
  \[
  \hat{x}_0 = E[x_0] \\
  P_x^0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \\
  P_p^0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] = \begin{bmatrix}
  P_x & 0 & 0 \\
  0 & P_y & 0 \\
  0 & 0 & P_n
  \end{bmatrix}
  \] (1)

  For \( k = 1, 2, 3, \ldots, \infty \):

1) **Count sigma point**

\[
X_{k-1}^a = \begin{bmatrix}
  \hat{x}_{k-1}^a \\
  \hat{x}_{k-1}^a + \gamma \sqrt{P_{k-1}} \\
  \hat{x}_{k-1}^a - \gamma \sqrt{P_{k-1}}
\end{bmatrix}
\]

Where:

\[
\gamma = \sqrt{L + \lambda} \\
\lambda = \alpha^2(L + \kappa) - L
\] (2)

2) **Time-update (prediction stage)**

\[
\hat{x}_{k}^- = \sum_{i=0}^{2L} W_i^{(m)} X_{i,k|k-1} \\
P_{x_k}^- = \sum_{i=0}^{2L} W_i^{(c)} (X_{i,k|k-1} - \hat{x}_{k}^-)(X_{i,k|k-1} - \hat{x}_{k}^-)^T \\
Z_{k|k-1} = H(X_{k|k-1}^-, X_{k|k-1}^n) \\
\hat{z}_{k}^- = \sum_{i=0}^{2L} W_i^{(m)} Z_{i,k|k-1}
\] (3)

3) **Measurement update (correction stage):**

\[
P_{z_k, x_k} = \sum_{i=0}^{2L} W_i^{(c)} (Z_{i,k|k-1} - \hat{z}_{k}^-)(Z_{i,k|k-1} - \hat{z}_{k}^-)^T \\
P_{x_k, z_k} = \sum_{i=0}^{2L} W_i^{(c)} (X_{i,k|k-1} - \hat{x}_{k}^-)(X_{i,k|k-1} - \hat{x}_{k}^-)^T \\
K_k = P_{z_k, x_k} P_{x_k, z_k}^{-1} \\
\hat{x}_k = \hat{x}_{k}^- + K_k (z_k - \hat{z}_{k}^-) \\
P_{x_k} = P_{x_k}^- - K_k P_{x_k, z_k} K_k^T
\] (4)

3. **Square Root Matrix**

The Unscented Kalman Filter Square Root (UKF-SR) algorithm is a development of the UKF algorithm, where there is a Singular Value Decomposition (SVD) and a square root matrix. SVD is a matrix in the form of a diagonal multiplication containing singular values, with a matrix containing corresponding singular vectors. Singular value decomposition is a technique that has been used widely to decompose matrices into several matrix components [8].

When in a matrix \( A \in \mathbb{R}^{m \times k} \), there is a matrix of ortogonal \( U = [u_1, \ldots, u_m] \in \mathbb{R}^{m \times k} \), and \( V = [v_1, \ldots, v_m] \in \mathbb{R}^{k \times k} \), maka:
\[ A = U \Sigma V^T \]

(5)

With a matrix \( \Sigma \in \mathbb{R}^{m \times k} \) of which the diagonal entry is \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p \geq 0 \), \( p = \min[m,k] \) and the other entry is zero. The value of \( \sigma_i \geq 0, \ i = 1,2,\ldots, p \) called singular value of \( A \) [8].

The root root matrix is the square root of the positive definite matrix \( A \), that is,

\[ A^{1/2} = \sum_{i=1}^{\infty} \sqrt{\lambda_i} e_i e_i^T = U \Lambda^{1/2} U^T \]

(6)

where \( \Lambda^{1/2} \) is diagonal matrix with its digiagonal element \( \sqrt{\lambda_i} \) with

\[ \Lambda = \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_k \end{bmatrix} \]

and \( \lambda_i > 0 \). Variable \( \lambda_1, \lambda_2, \ldots, \lambda_k \) is the eigen values of \( A \).

4. Computational Result

This simulation made by applying the UKF and UKF-SR algorithms to the stock functions obtained from Mathematical software simulation showed the stock data for high, low and close price as in Table 1. The simulation results were evaluated and compared to the established stock functions, and the stock functions for high, low and close prices in equation (7) - (9) are as follows:

\[
\begin{align*}
 f_{\text{high}}(x) &= 15.35x^2 - 557.089x + 8235.77 \\
 f'_{\text{high}}(x) &= 30.70x - 557.089 \\
 f_{\text{low}}(x) &= 56.71x^2 - 892.25x + 3915.56 \\
 f'_{\text{low}}(x) &= 113.42x - 892.25 \\
 f_{\text{close}}(x) &= 48.45x^2 - 633.489x + 7982.1 \\
 f'_{\text{close}}(x) &= 96.9x - 633.489
\end{align*}
\]

(7)

The system requires discretation, so the stock functions model in equation (7) - (9) must be discreted by using the finite difference method. Equation (19) and (21), If \( f_{\text{high}} \) functional of high price stock and \( f_{\text{low}} \) functional of high price stock and \( f_{\text{close}} \) functional of close price stock

\[
\begin{align*}
 f_{\text{high}} &= f_{\text{high}}^k, f_{\text{low}} &= f_{\text{low}}^k, f_{\text{close}} &= f_{\text{close}}^k
\end{align*}
\]

(10)

The change of state variables in respect to the time is approximated by forward scheme of finite difference. Thus we will get

\[
\begin{align*}
 f_{\text{high}} &= \frac{df_{\text{high}}}{dt} \approx \frac{f_{\text{high}}^{k+1} - f_{\text{high}}^k}{\Delta t} \\
 f_{\text{low}} &= \frac{df_{\text{low}}}{dt} \approx \frac{f_{\text{low}}^{k+1} - f_{\text{low}}^k}{\Delta t} \\
 f_{\text{close}} &= \frac{df_{\text{close}}}{dt} \approx \frac{f_{\text{close}}^{k+1} - f_{\text{close}}^k}{\Delta t}
\end{align*}
\]

(11)

(12)

(13)

From equation (7) - (9) the modified the stock functions model in (11)-(13) are obtained as follows.

\[
\begin{bmatrix}
 f_{\text{high}}^{k+1} \\
 f_{\text{low}}^{k+1} \\
 f_{\text{close}}^{k+1}
\end{bmatrix} =
\begin{bmatrix}
 30.70x_k - 557.089 \Delta t \\
 113.42x_k - 892.25 \Delta t \\
 96.9x_k - 633.489 \Delta t
\end{bmatrix}
\]

(14)
Table 1. Stock data on for high, low, close price

| Month | High  | Low   | Close |
|-------|-------|-------|-------|
| 1     | 4000  | 3150  | 3930  |
| 2     | 5075  | 3950  | 4290  |
| 3     | 4410  | 4030  | 4400  |
| 4     | 5500  | 4250  | 5275  |
| 5     | 6700  | 5275  | 6100  |
| 6     | 6050  | 6150  | 5930  |
| 7     | 5075  | 3950  | 4290  |
| 8     | 4410  | 4030  | 4400  |
| 9     | 5500  | 4250  | 5275  |
| 10    | 6800  | 5475  | 6200  |
| 11    | 6700  | 5950  | 6150  |
| 12    | 6695  | 6025  | 6525  |
| 13    | 6550  | 5830  | 5950  |
| 14    | 6770  | 5850  | 6750  |
| 15    | 7100  | 6200  | 7000  |
| 16    | 5700  | 4500  | 4900  |
| 17    | 4870  | 4475  | 4530  |
| 18    | 4600  | 3870  | 3950  |
| 19    | 4245  | 3760  | 3760  |
| 20    | 4050  | 3725  | 3800  |
| 21    | 7225  | 6875  | 6925  |
| 22    | 6950  | 6555  | 6625  |
| 23    | 6720  | 6150  | 6300  |
| 24    | 6485  | 5600  | 5850  |
| 25    | 6250  | 5700  | 5725  |

After the function is obtained, then it is simulated with the Matlab software. In this paper a simulation is carried out by applying the UKF and UKF-SR algorithm in stock functions for high, low and close prices. The simulation results are evaluated by comparing the real conditions in the field with the results of the UKF and UKF-SR estimation. This simulation uses and 150 iterations for low and high prices, while for close prices uses 50 iterations. In figure 1 is a comparison of the results of estimates between UKF and UKF-SR at high price stock, figure 2 is the simulation results of the UKF and UKF-SR methods at low price stock. Figure 3 is the simulation results of the UKF and UKF-SR methods at close price stock.

Figure 1. Estimation of high stock price using UKF and UKF-SR method with 150 and 200 iteration
Figure 2. Estimation of low stock price using UKF and UKF-SR method with 150 and 200 iteration.

Figure 3. Estimation of close stock price using UKF and UKF-SR method with 150 and 200 iteration.

Figure 1 shows that the results of estimated high price stock have good accuracy with an error of less than 2%, whereas those of the estimated high price stock by the UKF method with RMSE of 0.2731 have higher accuracy than those by the UKF-SR with RMSE of 1.5179. Figure 2 shows that the results of high price stock have quite good accuracy with an error of less than 2%, whereas those of the estimated low price stock by the UKF method with RMSE of 0.26737 show better accuracy than those by the UKF-SR with RMSE of 2.1947. Likewise, in Figure 3 the results of the estimated close price stock have a good level of accuracy with an error of less than 2%.

Table 2. Comparison of the values of RMSE by the UKF and UKF-SR method by 100, 150 and 200 iteration.

| Mode       | 100 iteration | 150 iteration | 200 iteration |
|------------|---------------|---------------|---------------|
| High       | 0.2731        | 0.26737       | 0.25855       |
| Low        | 0.28145       | 0.27019       | 0.25177       |
| Close      | 0.27693       | 0.25515       | 0.24098       |
| Simulation | 4.67 s        | 5.85 s        | 7.21 s        |

In Table 2, it appears that the UKF method is more accurate than UKF-SR. UKF and UKF-SR Besides, the UKF method has faster computation time than UKF-SR with 100, 150 and 200 iterations. In general as seen in the Table 2, the results of the three simulations were highly accurate. The first simulation by generate 200 iteration with error of high stock price 0.25855 or accuracy of 99.2%.
The second simulation by generate 200 iteration with error of low stock price 0.25177 or accuracy of 99.1%, and the third simulation by generate 200 iteration with error of close stock price 0.24098 or accuracy of 98.8%. Overall, the UKF and UKF-SR methods can be used as a method to estimate stock prices for high, low and close prices with very good accuracy. The simulation results show that both methods have a significantly high accuracy of less than 2%. We conclude that the two methods can be used to estimate the stock prices.

5. Conclusion
Based on the results of the simulation analysis, the UKF method is more accurate than UKF-SR. The results of estimated high, low and close price stock using UF and UKF-SR have good accuracy with an error of less than 2%. Overall, the UKF and UKF-SR methods can be used as a method to estimate stock prices for high, low and close prices with very good accuracy.

Open problem. How to implemented Fuzzy Kalman Filter (FKF) and H-infinity for estimation of stock price.

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