Form Factors of the Nucleon Axial Current

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Abstract

A symmetry-preserving Poincaré-covariant quark+diquark Faddeev equation treatment of the nucleon is used to deliver parameter-free predictions for the nucleon’s axial and induced pseudoscalar form factors, $G_A$ and $G_P$, respectively. The result for $G_A$ can reliably be represented by a dipole form factor characterised by an axial charge $g_A = G_A(0) = 1.25(3)$ and a mass-scale $M_A = 1.23(3) m_N$, where $m_N$ is the nucleon mass; and regarding $G_P$, the induced pseudoscalar charge $g_A^p$ = 8.80(23), the ratio $g_A^p/g_A$ = 7.04(22), and the pion pole dominance Ansatz is found to provide a reliable estimate of the directly computed result. The ratio of flavour-separated quark axial charges is also calculated: $g_A^d/g_A^u$ = −0.16(2). This value expresses a marked suppression of the size of the $d$-quark component relative to that found in nonrelativistic quark models and owes to the presence of strong diquark correlations in the nucleon Faddeev wave function – both scalar and axial-vector, with the scalar diquark being dominant. The predicted form for $G_A$ should provide a sound foundation for analyses of the neutrino-nucleus and antineutrino-nucleus cross-sections that are relevant to modern accelerator neutrino experiments.

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1. Introduction

In a step beyond the Standard Model, it is now known that neutrinos have mass\cite{1,2}; and it may be that this mass is not generated by couplings to the Higgs boson\cite{3}. The mass splittings between different neutrino species and the angles that describe mixing between them have been measured with precision\cite{4, Sec. 14}. Yet, there are unresolved problems, inter alia: what are the masses of the individual neutrinos; are neutrinos their own antiparticles; and have neutrinos played a pivotal role in Universe evolution? With these questions bearing upon fundamental mysteries in Nature, high profile experiments are underway, being constructed or planned worldwide\cite{4, Sec. 14}.

The analysis and reliable interpretation of modern neutrino experiments relies on sound theoretical knowledge of neutrino/antineutrino-nucleus ($\nu$/$\bar{\nu}$-A) interactions\cite{5–7}. An important element in such calculations is the nucleon axial form factor, $G_A(Q^2)$, whose value at $Q^2 = 0$ is the nucleon’s nonsinglet axial charge, $g_A(0) = 1.2756(13)$\cite{4}, which determines the rate of neutron-to-proton $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}$. Significantly, at the structural level, $g_A$ measures the difference between the light-front number-density of quarks with helicity parallel to that of the nucleon and the density of quarks with helicity antiparallel\cite{8}.

$G_A(Q^2)$ has long been a subject of interest. It was extracted from $\nu p$ and $\nu$-deuteron, $d$, scattering experiments performed over thirty years ago\cite{9–12}, yielding results that are consistent with dipole behaviour characterised by a mass-scale $M_A \approx 1.1 m_N$, where $m_N$ is the nucleon mass. This value was also obtained in a more recent $p(e, e'\pi^-)n$ experiment\cite{13} and in a new analysis of the world’s data on $\nu d$ scattering, albeit with a larger uncertainty than estimated in the original analyses\cite{14}. On the other hand, modern experiments using $\nu$ scattering on an array of heavy targets (water, iron, mineral oil, Kevlar, and carbon) yield results covering the range $1.1 \leq M_A/m_N \leq 1.60$\cite{15–19}. Important issues here are the reliability of the model used to describe the nuclear target and differences between the models used by the collaborations.

There have been many model analyses of the nucleon’s axial current, e.g. Refs.\cite{20–23} and citations thereof. Today, numerical simulations of lattice-regularised quantum chromodynamics (QCD) are also being deployed to determine $G_A(Q^2)$. The contemporary status of such analyses is summarised in Ref.\cite{24}: the results correspond to dipole masses in the range $1.1 \leq M_A/m_N \leq 1.7$.

Evidently, considering both experiment and theory, $M_A$ is not known to better than 50%. It has been argued\cite{6} that if the precision can be increased to 10% or better, then $G_A$ will become a subdominant source of error in the determination of neutrino properties in $\nu$-oscillation experiments. This is good motivation for a new analysis of the nucleon’s axial current form factors.

Continuum Schwinger function methods\cite{25–30} have been...
where we have assumed isospin symmetry, \( {\mathbf{A}}\) are implicitly expressed in the formulation.

2. Nucleon Axial Current

The nucleon’s isovector axial-vector current is

\[
J_{\lambda}^\mu(K, Q) = \bar{u}(P_f) i \tau^\lambda \mathbf{\Lambda}_{\mathbf{p}}(K, Q) u(P_i)
\]

\[
= \bar{u}(P_f) \gamma_5 \frac{1}{2} i \tau^\lambda \left[ \gamma_\mu G_A(Q^2) + i \frac{Q_\mu}{2m_N} G_P(Q^2) \right] u(P_i),
\]

where we have assumed isospin symmetry, \( \{\tau^i | i = 1, 2, 3\} \) are Pauli matrices, \( K = (P_f + P_i)/2, Q = (P_f - P_i), P_i^2 = -m_N^2 = P_f^2, G_A(Q^2) \) is the axial form factor, and \( G_P(Q^2) \) is the induced pseudoscalar form factor. The kindred pseudoscalar current is

\[
J_5^\mu(K, Q) = \bar{u}(P_f) \gamma_5 \frac{1}{2} \tau^\lambda \Lambda_5(K, Q) u(P_i)
\]

\[
= \bar{u}(P_f) i \gamma_5 \frac{1}{2} \tau^\lambda G_5(Q^2) u(P_i).
\]

This pair of currents is related by an axial-vector Ward-Green-Takahashi identity:

\[
Q_\mu J_{\lambda}^\mu(K, Q) + 2im_q J_5^\mu(K, Q) = 0,
\]

where \( m_q \) is the current-quark mass; and this entails

\[
2m_N G_A(Q^2) - \frac{Q^2}{2m_N} G_P(Q^2) = 2m_q G_5(Q^2).
\]

Eq. (4) imposes tight constraints on any analysis of these form factors.

Faddeev Amplitude — The first step in the calculation of \( G_{A,P} \) is computation of the nucleon’s mass and Poincaré-covariant bound-state amplitude from the quark-diquark Faddeev equation introduced in Refs. [43–45] and depicted in Fig. 1. There is a large body of evidence supporting the presence of diquark correlations within baryons, based on experiment, phenomenology, and theory [42]. Importantly, the diquarks in Fig. 1 are nonpointlike and fully dynamical: they appear in a Faddeev kernel – shaded domain – which requires their continual breakup and reformation, and participate in interactions with all probes as allowed by their quantum numbers.

In solving the Faddeev equation, we use the following diquark masses (in GeV):

\[
m_{[ud]} = 0.80, \ m_{[uu]} = m_{[dd]} = 0.89;
\]

and light-quarks characterised by a Euclidean constituent mass \( m_{g_{udd}} = 0.33 \) GeV. The associated propagators and additional details concerning the Faddeev kernel are presented in Ref. [35, App. 1, App. 2]. Importantly, e.g., the light-quark mass function, illustrated in Ref. [37, Fig. 6], is in fair agreement with that obtained in modern gap equation studies [46–48].

These inputs are sufficient to obtain the nucleon Faddeev amplitude, \( \Psi \), and mass \( m_N = 1.18 \) GeV. This mass value is intentionally large because Fig. 1 describes the nucleon’s dressed-quark core. To obtain the complete nucleon, resonant contributions should be included in the Faddeev kernel. Such “meson cloud” effects generate the physical nucleon, whose mass is roughly 0.2 GeV lower than that of the core [49–51]. Their impact on nucleon structure can be incorporated using sophisticated dynamical coupled-channels models [36, 52], but that is beyond the scope of contemporary Faddeev equation analyses. Instead, we express all form factors in terms of \( x = Q^2/m_N^2 \), a procedure that has proved efficacious in developing sound comparisons with nucleon and nucleon-resonance electromagnetic observables [35–41]. In addition, we report a model uncertainty obtained by independently varying the diquark masses by \( \pm 5\% \), thereby changing \( m_N \) by \( \pm 3\% \).

Current Elements — The next step requires construction of the nucleon weak interaction current. The minimal form consistent with the axial-vector Ward-Green-Takahashi identities at the quark, diquark, and nucleon levels can be derived following the procedures in Refs. [53–55]. It is depicted in Fig. 2 and explained in Ref. [56]. Herein, we recapitulate the salient details.

The vertex describing interactions between a weak-boson and dressed-quark in Fig. 2 – diagrams (1) and (4) is [54]:

\[
\Gamma_i^{\lambda}(k_+, k_-, \ldots) = \gamma_5 \frac{\tau^\lambda}{2} \left[ \Gamma_i^{\lambda}(k_+, k_-, \ldots) + \frac{2Q_\mu}{Q^2 + m_N^2} \Sigma_\mu(k_+^2, k_-^2) \right],
\]

\[
\Gamma_i^{\lambda}(k_+, k_-, \ldots) = \gamma_\mu \Sigma_\mu(k_+^2, k_-^2) + 2k_\mu \gamma \cdot kA_\mu(k_+^2, k_-^2),
\]

where \( Q \) is the incoming momentum of the weak boson, \( k_+ = k \pm Q/2, m_\pi \approx 0.14 \) GeV is the pion mass, \( \Sigma_\mu(k_+^2, k_-^2) = (F(k_+^2 + F(k_-^2))/2, \Delta_\mu(k_+^2, k_-^2) = (F(k_+^2) - F(k_-^2))/2k_+^2, \ F = A, B \). Since the diquark correlations are nonpointlike and fully interacting, symmetry preservation requires that the nucleon electromagnetic current include seagull interactions, diagrams (5) and
Takahashi identity, Eq. (3): single line, also isoscalar, associated with diagrams in Fig. 3. Given that the scalar diquark correlation amplitude is and (3) in Fig. 2. These couplings are expressed by the sum of the weak-boson coupling to diquarks, indicated by diagrams (2) and (3). For the nucleon electromagnetic current, with the Faddeev kernel and weak current; but since we have closely followed the successful kindred treatment of electromagnetic processes, we judge those assumptions to be sound. Naturally, assumptions have been made in formulating the Faddeev amplitude and the weak current are completely determined, with construction of the latter being constrained by the axial-vector Ward-Green-Takahashi identity.

On the other hand, transitions between scalar and axial-vector diquarks are possible: charged currents – \{dd\} ↔ \{ud\}, \{ud\} ↔ \{uu\}; and neutral currents – \{dd\} ↔ \{dd\}, \{uu\} ↔ \{uu\}. Once again, in the isospin symmetry limit, the coupling strengths are identical in magnitude. Using Eqs. (6), (7) in Fig. 3, one obtains:

\[ \Gamma^{aa}_{5\mu,\rho\rho}(p_d, k_d) = \left[ k_{aa}^{uu} \epsilon^{uu}_{\mu\rho\rho}(p_d + k_d) \right] \]

where \( \epsilon^{uu} \) is in (9) and (10) is the computed \( Q^2 = 0 \) values of the couplings are

\[ \kappa_{ax}^{uu} = 0.75, \kappa_{ps}^{uu} = \kappa_{ax}^{uu} m_N/|2M_E| = 1.34, \]

where the second equation expresses a Ward-Green-Takahashi identity, which we verified numerically; and, emulating the electromagnetic current construction, \( d(x) = 1/(1 + x) \) is introduced to express diquark compositeness via form factor suppression on \( Q^2 > 0 \). Notably, using dipole suppression instead, i.e. \( d(x)^2 \), no prediction in any image drawn herein changes by more than the line width because, in all cases, diagram (1) in Fig. 2 is both dominant and hard, whereas all weak-boson–diquark interactions are soft and subdominant. (This is discussed further below.) Note, too: \( \Gamma_{ax}^{uu} = -\Gamma_{ay}^{uu} \).

There are also transitions between axial-vector diquarks: charged currents – \{dd\} ↔ \{ud\}, \{ud\} ↔ \{uu\}; and neutral currents – \{dd\} ↔ \{dd\}, \{uu\} ↔ \{uu\}. In the isospin symmetry limit, the coupling strengths are identical in magnitude. Using Eqs. (6), (7) in Fig. 3, one obtains:

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3. Axial Form Factor

Using the elements described in Sec. 2, one can evaluate the sum of diagrams in Fig. 2 and, employing an appropriate
Table 1: Selected $Q^2 = 0$ properties of the nucleon’s axial-vector form factor, $G_A$, compared with RL-truncation three-body Faddeev equation results [31], empirical determinations [4, 13, 14, 17], and lattice-QCD (lQCD) computations [57–59]. Ref. [59] reports results from two different parametrisations of their simulation outputs. "—" in any position indicates no information available in the cited source for the associated quantity. The listed uncertainty in our predictions reflects the impact of ±5% variations in the diquark masses in Eq. (5). For comparison, the proton’s measured electromagnetic radius yields $m_N/r^2_m = 4.00$ [4].

|            | $g_A$ | $m_N(r^2_A)^{1/2}$ | $m_A/m_N$ |
|------------|-------|--------------------|------------|
| Herein     | 1.25(03) | 3.25(04)           | 1.23(03)   |
| Faddeev3 [31] | 0.99(02) | 2.63(06)           | 1.32(03)   |
| Exp [4]    | 1.2756(13) | —                  | —          |
| Exp [13]   | —       | 3.02(11)           | 1.15(04)   |
| Exp [14]   | —       | 3.23(72)           | 1.15(08)   |
| Exp [17]   | —       | 2.41(31)           | 1.44(18)   |
| lQCD [57]  | 1.21(3)(2) | 2.45(08)(03)     | 1.41(04)(02) |
| lQCD [58]  | 1.30(6)   | 3.57(30)           | 0.97(16)   |
| lQCD_d [59]| 1.23(3)   | 2.48(15)           | 1.39(09)   |
| lQCD_s [59]| 1.30(9)   | 3.19(30)           | 1.09(11)   |

spinor projection matrix, obtain the nucleon’s axial form factor, $G_A(Q^2)$. In Table 1, we list predictions for selected $G_A(Q^2)$ characteristics in comparison with other determinations, with

$$
(r^2_A) = -\frac{6}{G_A(0)} \frac{d}{dQ^2} G_A(Q^2) \bigg|_{Q^2=0} ^{Q^2=\infty}.
$$

As signalled above, the listed uncertainties in our values express the impact of varying the diquark masses in Eq. (5). The results obtained from the independent variations are combined with weight determined by the relative strength of scalar and axial-vector diquark contributions to $g_A = G_A(0)$, i.e. approximately 4:1. Notably, scalar and axial-vector diquark variations interfere destructively, e.g. reducing $m_{ud}$ increases $g_A$, whereas $g_A$ decreases with the same change in the axial-vector mass.

Our prediction for the momentum dependence of $G_A$ is drawn in Fig. 4. On the domain depicted, an accurate interpolation of the central result is provided by

$$
G_A(x) = \frac{1.25 - 0.22x}{1 + 1.24x + 0.0052x^2}
$$

or, almost equally well, by a dipole characterised by the mass $M_A = 1.23 \times m_N$. Computed as explained above, the lighter blue band surrounding our result expresses the impact of ±5% variations in the diquark masses that define the Faddeev kernel.

In Table 2, referring to Fig. 2, we list the relative strengths of each diagram’s contribution to the nucleon’s axial charge. Diagram (1), with the weak-boson striking the dressed-quark in association with a spectator scalar diquark, is overwhelmingly dominant. On the other hand, as anticipated following Eqs. (7), diagrams (5) and (6) are identically zero in this case.

**Flavour Separation of $g_A$** — With some reflection, it becomes apparent that the fractions in the top row of Table 2 can readily be translated into a flavour decomposition of the nucleon’s axial charge; hence, the strength of $u$ and $d$ quark contributions to the nucleon light-front helicity. For $u$-quarks and $d$-quarks, we find that the helicity parallel-antiparallel differences are $g^u_d = 0.86 g_A$ and $g^d_d = -0.14 g_A$. Hence, the nucleon’s light-front helicity is overwhelmingly invested in the $u$-quark: $g^u_d/g^d_d = -0.16(2)$. This result is consistent with the estimates made in Refs. [60] using other means of analysing the nucleon’s Faddeev amplitude. It is notable that $g^u_d/g^d_d = -0.25$ in nonrelativistic quark models with correlated wave functions [61].

The difference between that value and our prediction highlights the impact of strong diquark correlations in the nucleon wave function: with high probability, the nucleon’s $d$ quark is sequestered within a $[ud]$ diquark, which does not participate in weak interactions.

In contrast, contemporary lQCD analyses report values for this ratio that are larger in magnitude than the quark model result, locating a greater portion of the nucleon’s axial charge with the $d$-quark: $g^u_d/g^d_d = -0.40(2)$ [62]; $g^d_d/g^u_d = -0.58(3)$ [63]. Notably, whilst $g_A$ is a conserved charge, invariant under QCD evolution [64–67], the separation into component contri-
Table 2: Referring to Fig. 2, separation of $G_A(0)$ and $G_P(0)$ into contributions from various diagrams, listed as a fraction of the total $Q^2 = 0$ value. Diagram (1): $(J)_g^A$ – weak boson strikes dressed-quark with scalar diquark spectator; and $(J)_g^A$ – weak boson strikes dressed-quark with axial-vector diquark spectator. Diagram (2): $(J)_g^{AA}$ – weak boson interacts with axial-vector diquark with dressed-quark spectator. Diagram (3): $(J)_g^{AA+AS}$ – weak boson mediates transition between scalar and axial-vector diquarks, with dressed-quark spectator. Diagram (4): $(J)_g^A$ – weak boson strikes dressed-quark “in-flight” between one diquark correlation and another. Diagrams (5) and (6): $(J)_g^A$ – weak boson couples inside the diquark correlation amplitude. The listed uncertainty in these results reflects the impact of ±5% variations in the diquark masses in Eq. (5), e.g. 0.71 $\pm$ 0.01.

| $(J)_g^A$ | $(J)_g^{AA}$ | $(J)_g^{AA+AS}$ | $(J)_g$ |
|-----------|--------------|-----------------|---------|
| $G_A(0)$  | 0.714, 0.0042, 0.025, 0.13 0, 0.072 0, 0 | 0 0 0 0 0 0 |
| $G_P(0)$  | 0.74 0, 0.076 0, 0.025 0, 0.13 0, 0.2 0, 0.19 0 |

Figure 5: Upper panel – A. Comparison of our prediction for $g_A^*$ (blue asterisk) with an empirical value [74] (purple circle) and a collection of IQCD results: red triangle [57], green diamond [58], and cyan crosses [59]. An uncertainty-weighted average of the theory results is depicted by the grey line and associated band: $g_A^*$ = 8.49(14). Lower panel – B. Analogous comparison for the ratio $g_A^*/g_A$, including the value computed from results in Ref. [31] (black star). Again, the grey line and associated band depicts an uncertainty-weighted average of the theory results: $g_A^*/g_A = 6.77(10)$. Using the empirical result for $g_A = 1.2756(13)$, this average value corresponds to $g_A^* = 8.63(13)$, which is drawn as the gold line and associated band in the upper panel.

4. Induced Pseudoscalar Form Factor

$G_P(Q^2)$ in Eq. (1) can be obtained following the procedure described in Sec. 3 simply by changing the spinor projection matrix. The value at $Q^2 = 0.88 m_N^2$, where $m_N$ is the mass of the $\mu$-lepton, defines the nucleon’s induced pseudoscalar charge:

$$g_P^* = \frac{m_\mu}{2m_N} G_P(Q^2 = 0.88 m_N^2),$$

(14)

which can be measured in $\mu p$ capture reactions [75], e.g. $\mu + p \rightarrow n + n$. We obtain $g_P^* = 8.80(23)$, a value that is consistent with both measurements and computations using other methods [59, Fig. 16]: $8 \lesssim g_P^* \lesssim 9$. This is illustrated in Fig. 5A, which depicts our prediction along with the value reported from a precision $\mu$-capture experiment [74] and a collection of IQCD results [57–59].

It is worth remarking here that after rescaling to correct for the $g_A$ underestimate – see Table 1, the Ref. [31] RL-truncation three-body Faddeev equation analysis yields $g_A^* = 8.59(13)$. The agreement between this value and the uncertainty-weighted average in Fig. 5A suggests that some systematic theory uncertainties cancel in the ratio $g_A^*/g_A$. So, in Fig. 5B, we depict our prediction for $g_A^*/g_A = 7.04(22)$ along with that calculated using inputs from Ref. [31], viz. $g_A^*/g_A = 6.77(15)$, the value computed from a $\mu$-capture experiment [74] and the empirical value of $g_A$, plus the ratios obtained from IQCD results [57–59]. Evidently, working with this ratio, there is some improvement in agreement between theory predictions.

Our prediction for the momentum dependence of $G_P$ is drawn in Fig. 6. On the domain depicted, an accurate interpolation of the central result is provided by

$$G_P(x) = \frac{40.1 - 2.11 x - 1.84 x^2}{0.127 + 7.75 x + 11.9 x^2},$$

(15)
The model-uncertainty band surrounding this curve is practically invisible on the ordinate scale necessary to draw the figure.

Returning to Eq. (4), it is evident that in the neighbourhood of the chiral limit, the right-hand-side is $O(m_N^2)$ and can be neglected; hence, it is a good approximation to write

$$G_P(x) \approx \frac{4}{x + m_N^2} G_A(x).$$

Treating this as an identity, one has the so-called pion pole dominance (PPD) prediction for $G_P$. Looking at Fig. 6, it is evidently a good approximation, as observed empirically in Ref. [76]. One can also illustrate the accuracy of PPD by seeking a least-squares fit to $G_P(x)$ using Eq. (16) and a dipole Ansatz for $G_A(x)$: on $2m_N^2 / m_N^2 < x < 1.8$, which excises a small neighbourhood of the true pion pole, one finds $G_A(x) = 1.25/(1 + x/1.24)^2$, i.e. one effectively reproduces Eq. (13).

In Table 2, referring to Fig. 2, we also list the relative strengths of each diagram’s contribution to the nucleon’s induced pseudoscalar charge. Once again, diagram (1), with the weak-boson striking the dressed-quark in association with a spectator scalar diquark, is overwhelmingly dominant. In this case, however, there is an active cancellation between the contributions from diagrams (4), (5) and (6).

5. Summary and Perspective

Using a Poincaré-covariant quark+diqaurk Faddeev equation treatment of the nucleon and a weak interaction current that ensures consistency with relevant Ward-Green-Takahashi identities, we delivered parameter-free predictions for the nucleon’s axial and induced pseudoscalar form factors, $G_A$ and $G_P$, respectively. In doing so, we unified these features of the nucleon with an array of other properties of baryons and their excitations, e.g. spectra and structural properties, as exposed in elastic and transition form factors.

Relating to the milieu created by modern neutrino experiments, our results may be summarised succinctly as follows. $G_A$ can reliably be represented by a dipole form factor, normalised by the axial charge $g_A = 1.25(3)$ and characterised by a mass-scale $M_A = 1.23(3) m_N$, where $m_N$ is the nucleon mass; and on their common domain, the associated pointwise behaviour matches well with that determined in a recent lattice-QCD (lQCD) study [58]. Based on the analysis in Ref. [7], one may reasonably expect that our prediction for $G_A$ can provide a sound foundation for the calculation of the neutrino-nucleus and antineutrino-nucleus cross-sections that are relevant, e.g. to modern accelerator neutrino experiments.

Regarding $G_P$, our predictions for the induced pseudoscalar charge, $g_P$, and the ratio $g_P/g_A$ are consistent with the values determined from the most recent $\mu$-capture experiment [74]; and its momentum-dependence agrees with data obtained from low-energy pion electroproduction [76] and also the lQCD result in Ref. [58]. Furthermore, we find that the pion pole dominance Ansatz provides a sound estimate of the directly computed result, viz. using a $\Delta L$ measure, the mean relative difference between the curves is 2.7% on $0 \leq Q^2 \leq 1.6m_N^2$.

It is worth highlighting that we find $g_A^\mu / g_A^\pi = -0.16(2)$ at the hadronic scale. This is a significant suppression of the magnitude of the $d$-quark component relative to that found in non-relativistic quark models. The size reduction owes to the presence of strong diquark correlations in our nucleon wave function, with the calculated value reflecting the relative strength of scalar and axial-vector diquarks: the isoscalar–scalar correlations are dominant, but the isovector–axial-vector diquarks have a measurable influence.

Herein, we canvassed physics impacts of our treatment of the nucleon axial current. Unification of this analysis with that of the nucleon pseudoscalar current, Eq. (2); a detailed discussion of the partial conservation of the nucleon axial current and associated Goldberger-Treiman relations; and all technical details relating to current constructions in our quark+diquark approach, including the seagull terms, will be presented elsewhere [56].

A natural next step is to go beyond the quark+diquark approach and use the more fundamental three-quark Faddeev equation treatment of the baryon problem, revisiting the analysis in Ref. [31]. In seeking and following a path toward improving the expressions of emergent hadronic mass in both the Faddeev kernel and interaction current, one could therewith provide continuum predictions for the nucleon’s axial current form factors that posses a more rigorous connection to QCD’s Schwinger functions.

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