DILUTION OF COSMOLOGICAL DENSITIES
BY SAXINO DECAY

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Abstract

Saxino decay can generate significant cosmological entropy, and hence dilute theoretical estimates of the present mass density of a given particle species. The dilution factor depends on the saxino and axion masses, and is constrained by the requirement that saxino decay should not affect nucleosynthesis, as well as by the usual requirement that the axion density be less than the critical density. The latter constraint is evaluated carefully, under both the Harari-Sikivie and Davis proposals about the emission spectrum from axionic strings. Uncertainties are carefully evaluated, points of principle are addressed, and with an eye to future numerical simulation the spacing and typical oscillation wavelength of the strings are represented by parameters varying in the range 1 to 3. Within the constraints, the entropy dilution varies from 1 to $10^{-4}$. Only saxinos originating from thermal equilibrium are considered, so that more dilution might arise from non-thermal saxinos.
1 Introduction

According to current ideas, the present mass density $\Omega$ of the universe is practically equal to 1 (in units of the critical density), baryons contributing $\Omega_B \sim 0.04$ to 0.07, and one or more species of dark matter particle making up the rest. Given a suitable model of the fundamental interactions one can calculate the present density $\Omega_X$ of a given stable particle species. The calculation usually proceeds in two stages \[1\]. First one calculates the particle number density at some initial epoch, defined by its temperature $T_i$, after which the particle number is conserved. Then the number density is evolved forward to the present epoch when the (photon) temperature is $T_0 = 2.74 \text{ K}$.

As long as thermal equilibrium is maintained, the forward evolution can be performed using entropy conservation. Entropy can however be generated by non-equilibrium processes, and this lowers the prediction for $\Omega_X$. A particular example is the decay of a particle species which dominates the energy density of the universe during some era prior to its decay. The Standard Model does not contain such a particle species, but its as yet unknown extension might.

Any acceptable extension must include a natural explanation of CP conservation by the strong interaction. Although it has recently been seen to be rather vulnerable to Planck scale corrections \[3, 4\], the most promising explanation is still a spontaneously broken global symmetry of the type proposed by Peccei and Quinn \[4, 5\]. Along with this symmetry comes a Goldstone boson, the axion \[5, 6\]. The axion is practically stable and has extremely weak interactions, constituting cold dark matter. From accelerator physics and astrophysics its mass has an upper bound $m_a < 10^{-3} \text{ eV}$, and from the cosmological requirement $\Omega_a < 1$ its mass has a lower bound $m_a > m_{\text{min}}$. The lower bound $m_{\text{min}}$ is difficult to calculate, but with no entropy production it is probably not far below $10^{-3} \text{ eV}$. In that case there is only a narrow allowed window for $m_a$, and the axion must make up a significant fraction of the dark matter.

Despite the fact that no supersymmetric particle has yet been observed, it is fair to say that an increasing number of particle theorists feel that an acceptable extension of the Standard Model should also respect (low energy) supersymmetry \[8\]. One reason for this feeling is the successful prediction of $\sin^2 \theta_W$ by supersymmetric grand unified models, and another is the fact that the rival technicolour models are coming under pressure from ever more accurate measurements, notably at CERN. Supersymmetry requires \[9\] that the known particle species have as yet undiscovered superpartners, with masses of order 0.1 to 1 TeV. It also requires that the axion be accompanied by a spin 1/2 partner called the axino, and a spin 0 partner called the saxino \[10, 11\]. The axino might have the typical mass mentioned above, in which case it would decay (perhaps with significant entropy production), or it might be much lighter in which case it could constitute ‘warm’ dark matter. (More complicated possibilities arise if there is a very light gravitino \[13\], which are discounted in the present paper.) The saxino, on the other hand, is definitely expected to have a mass in the usual range, 100 GeV $\lesssim m_{\text{sax}} \lesssim 1 \text{ TeV}$.

Kim \[12\] has pointed out that saxino decay could generate significant entropy, and in the present paper the amount of entropy production is carefully calculated as a function of the axion mass $m_a$, the saxino mass $m_{\text{sax}}$ and the maximum temperature after inflation (‘reheat’ temperature) $T_{\text{reheat}}$. Then, the cosmologically allowed region of this three parameter space is
delineated, by the requirement that saxino decay must not interfere with nucleosynthesis, as well as by the requirement that the axion density satisfies $\Omega_\alpha < 1$. As described in the conclusion, the results have a number of cosmological implications.

Throughout the units are $\bar{\hbar} = c = 1$, $m_{Pl} = 1.2 \times 10^{19}$ GeV is the Planck mass, $a(t)$ is the scale factor of the universe at time $t$, $H = \dot{a}/a$ is the Hubble parameter, and $H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \bar{\hbar}$ is its present value.

\section{Entropy and particle densities}

For reference we recall some basic facts about radiation (relativistic particles) in the early universe \[1\]. According to the Standard Model the radiation is in thermal equilibrium when the temperature $T$ exceeds 1 MeV, with practically zero chemical potential. Accordingly its density is

$$\rho = \frac{\pi^2}{30} g_* T^4$$

where $g_*$ is the effective number of relativistic particle species. Its entropy density $s$ is

$$s = \frac{4}{3} \frac{\rho_r}{T} = \frac{2\pi^2}{45} g_* T^3$$

With three massless neutrino species, $g_* = 10.75$ for $1 \text{ MeV} \lesssim T \lesssim 100 \text{ MeV}$, $g_* = 17.25$ for $100 \text{ MeV} \lesssim T \lesssim \Lambda_{\text{QCD}}$ and $g_* = 61.75$ for $\Lambda_{\text{QCD}} \lesssim T \lesssim 2 \text{ GeV}$ where $\Lambda_{\text{QCD}} \simeq 200 \text{ MeV}$. At higher temperatures the Standard Model predicts a leveling out to $g_* \sim 100$ above $T \sim 100 \text{ GeV}$. However, extensions of the Standard Model have additional particles, leading to a bigger value of $g_*$, and in the minimal supersymmetric extension $g_*$ levels out at $g_* = 229$.

Extending the Standard Model does not usually have any other effect on the formulas that we are reviewing in this section, except for the possibility of entropy production that is our central concern.

In thermal equilibrium the entropy $S \propto a^3 s$ in a comoving volume is constant. This implies that

$$a^3 g_* (T) T^3 = \text{constant}$$

After the neutrinos decouple at $T \sim 1 \text{ MeV}$ they are not in thermal equilibrium but their entropy continues to be conserved, and the same is true of the photons when they too decouple at a much later epoch. Thus, according to the Standard Model the total entropy in a comoving volume is conserved, and a standard calculation \[1\] gives the present entropy density $s_0 = 2938 \text{ cm}^{-3}$. This allows one to calculate the present mass density of any particle species $X$, whose number density is conserved after some epoch $T_i > 1 \text{ MeV}$, given the number density at that epoch. Indeed, in units of the present critical density $3H_0^2 m_{Pl}^2 / 8\pi$ it is obviously given by

$$\Omega_X = \frac{8\pi}{3} \frac{m s_0}{m_{Pl}^2 H_0^2} \frac{n_i}{s_i}$$

This relation has been used to calculate the baryon density $\Omega_B$ and the density $\Omega_X$ of various dark matter candidates, in terms of theoretically calculated initial number densities.
The above analysis may fail if there is a non-relativistic particle species with a lifetime in the range \( t_i < \tau < t_0 \). However, the required modification is very simple provided that the decay products thermalise quickly on the Hubble timescale. Then the radiation density and entropy density are still given by Eqs. (1) and (2), and the only effect of the decay is to reduce \( \Omega_X \) by a factor \( \Delta = S_0/S_i \), the increase in the entropy of a comoving volume

\[
\Omega_X = 2.44h^{-2} \frac{m}{10\text{ eV}} \frac{n_i}{s_i} \Delta^{-1}
\]  

(6)

Neither the Standard Model or its minimal supersymmetric extension contains such a particle species, but for extensions which include Peccei-Quinn symmetry the saxino can be such a species, as we now discuss.

3 Entropy production from saxino decay

The supersymmetric generalisation of the usual axion-gluon interaction is

\[
\mathcal{L} = \frac{\alpha_c}{16\pi f_a} \Phi W_i W^i
\]  

(7)

where the superfields are \( \Phi \) corresponding to the axion and saxino (spin 0) and the axino (spin 1/2), and \( W_i \) corresponding to the gluon (spin 1) and the gluino (spin 1/2). This reproduces the usual axion-gluon interaction, and also gives a saxino-gluon interaction and other interactions,

\[
\mathcal{L} = \frac{\alpha_c}{8\pi f_a} (\bar{\psi}_a G_{\mu\nu} G^{\mu\nu} + \bar{\psi}_{\text{sax}} G_{\mu\nu} G^{\mu\nu} + \ldots)
\]  

(8)

The saxino is kept in thermal equilibrium by reactions like \( q \bar{q} \leftrightarrow sg \) and \( gg \leftrightarrow sg \) above a temperature \( T_{\text{decoup}} \),

\[
T_{\text{decoup}} = 10^{11} \text{ GeV} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^2 \left( \frac{\alpha_c}{1} \right)^{-3}
\]  

(9)

At an energy scale of order \( 10^{11} \text{ GeV} \), \( \alpha_c \simeq 1/20 \) in the minimal supersymmetric standard model \( 14 \), so that

\[
T_{\text{decoup}} \simeq 8 \times 10^{11} \text{ GeV} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^2
\]  

(10)

After decoupling the saxino number density \( n_{\text{sax}} \) is given as a fraction of the entropy density by

\[
r \equiv \frac{n_{\text{sax}}}{s} = \frac{1}{3.6g_s} = 1.2 \times 10^{-3}
\]  

(11)

(setting \( g_s = 229 \) corresponding to the minimal supersymmetric standard model).

In order to achieve thermal equilibrium the reheat temperature must satisfy \( T_{\text{reh}} > T_{\text{decoup}} \). Unfortunately, even the order of magnitude of \( T_{\text{reh}} \) is unknown. Bounds on the cmb anisotropy, as well as the COBE detection, imply that it is less than \( 10^{10} \text{ GeV} \) \( 15 \). If there is a gravitino with mass \( m_g < 1 \text{ TeV} \) nucleosynthesis implies that it is less than

\footnote{Since Peccei-Quinn symmetry is assumed the extension cannot in fact be minimal, but this estimate of \( \alpha_c \) and the estimates of \( g_s \) used later will hopefully still be adequate.}
$10^{13}$ GeV, but that bound rapidly goes away as $m_g$ is increased and provides no additional constraint if $m_g \sim 10$ TeV [17].

In what follows we leave $T_{\text{reh}}$ as a free parameter, and assume that if $T_{\text{reh}} < T_{\text{decoup}}$ the saxino density is negligible, considering only saxinos of thermal origin. A significant non-thermal saxino density might originate as an inflationary fluctuation or be radiated from axionic strings, possibilities which should certainly be investigated.

Continuing the story of the thermal saxinos, they become non-relativistic at a temperature $T \sim m_{\text{sax}}$, and if they live long enough they dominate the energy density of the universe below the temperature

$$T_{\text{saxeq}} = \frac{4}{3} r m_{\text{sax}} = 1.6 \text{ GeV} \left( \frac{m_{\text{sax}}}{1 \text{ TeV}} \right)$$  \hspace{1cm} (12)$$

(The subscript ‘saxeq’ indicates that the saxino density is equal to that of the radiation at this temperature.) The dominant decay mode of the saxino is into two gluons, with lifetime $\Gamma^{-1}$ given by [12]

$$\Gamma = \frac{\alpha_s^2 m_{\text{sax}}^3}{128 \pi^3 f_a^2}$$  \hspace{1cm} (13)$$

and saxino domination will occur for a significant period if $t_{\text{saxeq}} \ll \Gamma^{-1}$.

With this assumption, the evolution of the matter and radiation densities can easily be worked out. Approximate analytic results are available on the assumption that $g_*$ is constant \[13, 14, 20, 21, 22\], which are now briefly recalled. The temperature corresponding to $t_{\text{decay}}$ is

$$T_{\text{decay}} = 0.55 g_*^{-1/4} \sqrt{\Gamma m_P}$$  \hspace{1cm} (14)$$

and the saxino dominates the energy density until this epoch. The density of the ‘new’ radiation from saxino decay becomes equal to that of the ‘old’ radiation at a temperature $T_=$, given by

$$T_5 = T_{\text{decay}}^4 T_{\text{saxeq}}$$  \hspace{1cm} (15)$$

Before this epoch there is no significant entropy generation, but after it the entropy in a comoving volume is proportional to $T^{-5}$. This continues until the epoch $T_{\text{decay}}$, after which the saxino density becomes negligible and entropy production stops. Thus, if $T_i < T_=$ the entropy generation factor in Eq. (6) is

$$\Delta = \left( \frac{T_i}{T_{\text{decay}}} \right)^5 \quad (T_i < T_=)$$  \hspace{1cm} (16)$$

It depends strongly on $T_i$ because only part of the entropy is generated after $T_i$. On the other hand, if $T_i > T_=$, all of the entropy is generated after $T_i$, and $\Delta$ is given by

$$\Delta_{\text{total}} = \left( \frac{T_=}{T_{\text{decay}}} \right)^5 = \frac{T_{\text{saxeq}}}{T_{\text{decay}}} \quad (T_i > T_=)$$  \hspace{1cm} (17)$$

The last equality follows from Eq. (15). It implies incidentally that the new radiation has caused the ratio $\rho_{\text{sax}}/\rho_r$ to fall to a value $\simeq 1$, just before it falls off exponentially at the epoch $T_{\text{decay}}$. 

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These results are valid if $g_*$ is constant during the interval $T_{\text{decay}} < T < T_{\text{saxeq}}$. Setting $g_* = 10.75$, appropriate to the range $1 \text{ MeV} < T < 100 \text{ MeV}$, gives

$$T_{\text{decay}} = 52 \text{ MeV} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{-1} \left( \frac{m_{\text{sax}}}{1 \text{ TeV}} \right)^{3/2}$$

(18)

The three temperatures $T_{\text{decay}} < T = T_{\text{saxeq}}$ are plotted in Figure 1. If the saxino is heavy, $T_{\text{decay}}$ can exceed the temperature $T = \Lambda_{\text{QCD}}$ at which $g_*$ rises sharply to 62, but using this value would not change the results much. We have not investigated the intermediate case $T_{\text{decay}} < \Lambda_{\text{QCD}} < T_{\text{saxeq}}$, but assume that the results above are still valid to sufficient accuracy.

Substituting Eq. (18) into Eq. (17) gives the maximum entropy generation factor which can be supplied by saxino decay,

$$\Delta_{\text{total}} = 31 \left( \frac{f_a}{10^{12} \text{ GeV}} \right) \left( \frac{m_{\text{sax}}}{1 \text{ TeV}} \right)^{-1/2} \left( \frac{m_{\text{sax}}}{1 \text{ TeV}} \right)^{-1/2}$$

(19)

The saxino must decay before it has a significant effect on nucleosynthesis. According to Scherrer and Turner [21] a safe bound is $t < 10 \text{ s}$, which is equivalent to $T_{\text{decay}} > 3 \text{ MeV}$, or

$$\left( \frac{f_a}{10^{12} \text{ GeV}} \right) \simeq 170 \left( \frac{m_{\text{sax}}}{1 \text{ TeV}} \right)^{3/2}$$

(20)

The region forbidden by this constraint is indicated in Figure 1. The corresponding constraint on the entropy generation factor is

$$\Delta_{\text{total}} < 5.2 \times 10^3 \left( \frac{m_{\text{sax}}}{1 \text{ TeV}} \right)$$

(21)

A further constraint can arise from the requirement $\Omega_a < 1$, which is our main concern for the rest of the paper, as is now investigated.

4 The axion density

The axion density has been the subject of numerous investigations. The situation as it was understood in 1989 is described in [1, 7], and further work has been done since then [22, 23, 24, 25, 26, 27, 3]. There are essentially two possibilities regarding axion cosmology. One is that the Peccei-Quinn field has been in its vacuum (ie., that PQ symmetry has been spontaneously broken) ever since the observable universe left the horizon during inflation. In that case axion cosmology is quite complicated as will be briefly recalled in Section 4.3.

More likely is the opposite case, where Peccei-Quinn symmetry is restored in the early universe, through either finite temperature effects (after inflation) or the quantum fluctuation (during inflation). In that case strings form at the epoch when the symmetry is spontaneously broken, and the strings oscillate until domain walls form when the axion mass switches on at $T \simeq 1 \text{ GeV}$. If the string network ‘scales’ as one expects, $\Omega_a$ can in principle be calculated uniquely in terms of $f_a$ (up to the entropy generation factor $\Delta^{-1}$), at least if the strings radiate nothing but axions. To do a proper calculation requires numerical simulations which are perhaps tractable while the strings are oscillating but which would be
very difficult after domain walls form. In the absence of such simulations, $\Omega_a$ has been estimated in the literature on the basis of simplifying assumptions. The first of these, used by essentially all investigators, is that axion number is conserved after the domain walls form, so that Eq. (3) gives $\Omega_a$ in terms of the axion number density just before wall formation. The second assumption is a statement about the shape of the spectrum of axions emitted by the strings. Two alternative proposals exist, one by Davis [29] and one by Harari and Sikivie [30]. In what follows these two proposals are critically assessed, and in each case $\Omega_a$ is carefully estimated. The treatment represents an advance on previous ones (except for [26] which is discussed below) in two respects. First, the spectrum of all axions present at a given epoch is calculated (not just that of those being emitted), which enables important points of principle to be addressed. Second, the result is given as a function of the string spacing, as well as (for Davis’ proposal) the string oscillation frequency. Correspondingly there appear in the result two parameters $\gamma$ and $\beta$, which are expected to lie in the range 1 to a few, and which should be calculable in the foreseeable future from numerical simulations. Pending such a calculation, we estimate $\Omega_a$ by allowing them to vary between 1 and 3.

### 4.1 Axion cosmology

First we need some basic properties of the axion [3, 4]. It is a practically stable particle, which is massless for $T \gg \Lambda_{\text{QCD}} \simeq 200 \text{ MeV}$. At lower temperatures its interaction with the gluon leads to an effective potential

$$V = (79 \text{ MeV})^4 (1 - \cos \theta)$$

(22)

where $\theta = \psi_a/f_a$ and $\psi_a$ is the canonically normalised axion field. (We ignore a possible generalisation of the above equations, $\theta \to N\theta$ and $f_a \to f_a/N$ with $N$ an integer, because it probably leads to cosmologically unacceptable domain walls.) Small oscillations around the minimum of this potential correspond to free axions with mass $m_a$ given by $f_a m_a = 79 \text{ MeV}$, or

$$\frac{m_a}{10^{-6} \text{ eV}} = 6.2 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{-1}$$

(23)

Accelerator physics and astrophysics provide a lower limit $m_a \lesssim 10^{-3} \text{ eV}$, corresponding to $f_a \gtrsim 10^{10} \text{ GeV}$ [11].

**The case of a homogeneous axion field**

As already mentioned, axion cosmology depends crucially on whether or not there are axionic strings. Before turning to the former case which is our main concern, we focus on the simplest possible example of the latter case. Namely, we assume that the axion field is spatial homogeneous. In this case an accurate estimate of $\Omega_a$ is provided by the calculation of Turner [32], which we briefly recall. It will form the basis of our later estimates for the more complicated string case.

The axion mass switches on gradually as the epoch $T = \Lambda_{\text{QCD}}$, becoming significant at the epoch $\tilde{t}$ defined by

$$m_a(\tilde{t}) = 3H(\tilde{t})$$

(24)
Before this epoch, the homogeneous axion field is time independent, and afterwards it oscillates around $\theta = 0$. If $|\tilde{\theta}| \ll \pi$ (a tilde on any quantity will always denote its value at the epoch $\tilde{t}$), the oscillations are harmonic with angular frequency $\tilde{m}$ and amplitude proportional to $a^{-3/2}m_a^{-1/2}$, corresponding to the presence of zero momentum non-interacting axions with number density

$$\tilde{n} = \frac{1}{2} \tilde{m} f_a^2 \tilde{\theta}^2$$  \hspace{1cm} (25)

(Note that $n \propto a^{-3}$ corresponding to conserved axion number [33].) The present axion density is therefore given by Eq. (33) once $\tilde{T}$ is known. While the axion mass is switching on it is given by

$$m_a(T)/m_a = .077(\Lambda_{\text{QCD}}/T)^{3.7}$$  \hspace{1cm} (26)

Taking $h = .5 \pm .1$ and $\Lambda_{\text{QCD}} = (200 \pm 50)$ MeV this leads to

$$\tilde{T} = .87 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{-1.18} \text{ GeV}$$  \hspace{1cm} (27)

and

$$\Omega_a = 0.9 \times 10^{+5} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.18} \Delta^{-1} \tilde{\theta}^2$$  \hspace{1cm} (28)

The quoted uncertainty is essentially the one evaluate by Turner as arising from the uncertainties in $g_*$ and $m_a(T)$, slightly increased to allow for additional uncertainty arising from the values of $h$ and $\Lambda_{\text{QCD}}$. (Bearing in mind the age of the universe we here discount the possibility $h \simeq 1$, which according to Eq. (33) would multiply $\Omega_a$ by a factor .25.) To this level of accuracy, the expression is valid for $|\tilde{\theta}| \ll .9\pi$, only the case $|\tilde{\theta}| \simeq \pi$ requiring special treatment [32, 27].

The entropy production factor $\Delta$ is equal to 1 if $T_{\text{reh}} < T_{\text{decoup}}$ or if $T_{\text{saxeq}} < T_{\text{decay}}$ where the four temperatures are defined in the last section. Otherwise it is given by Eqs. (16) and (17), with $T_i = \tilde{T}$. From the left hand column of Figure 1, one sees that unless the saxino is rather heavy $\tilde{T}$ is bigger than $T_\ast$, so that the total entropy generation factor is experienced by the axions, given by Eq. (19).

Eqs. (27) and (28) are derived under the standard assumption that $\tilde{T}$ occurs during radiation domination. From Figure 1 one sees that if the saxino is rather heavy $\tilde{T}$ can occur during (saxino) matter domination, in which case there are well defined correction factors [19]. In the present context these are not very significant, and will be ignored.

The expression for $\Omega_a$ given by Eqs. (16), (17) and (28) does not agree with that of Kim [12]. The reason is that he used formulae developed by Lazarides et al [19] in a different context, where the decaying object is not a saxino and where $\tilde{T}$ falls in the interval $T_{\text{decay}} < T < T_\ast$ throughout the relevant portion of parameter space. Figure 1 illustrates that this is far from true in the present case.

4.2 The string scenario

The above calculation provides a basis for calculating $\Omega_a$ on the assumption that there are axionic strings, if axion number is conserved after domain walls form (the validity of this assumption is briefly addressed at the end of the present section). By looking at the typical
spatial and temporal variation of $\theta$ it was argued in [26] that the walls form at about the temperature $\approx \tilde{T}$, and this estimate will be used in what follows. (More precisely it was argued that the formation temperature is lower by an insignificant factor $\gamma^{-1/6}$ where $\gamma$ is the string spacing introduced below.) Substituting Eq. (25) into Eq. (28) gives an expression for $\Omega_a$ in terms of the number density $\tilde{n}$ just before wall formation.

$$\Omega_a = 0.9 \times 10^{\pm 5} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.18} \left( 2\tilde{m}_a^{-2}f_a^{-2} \right) \Delta^{-1} \tag{29}$$

To estimate $\tilde{n}$, the simplest assumption is that $\theta$ is as homogeneous as it can be, subject to the requirement that it changes by $2\pi$ as one goes around a string. This implies that the typical value of $\theta$ is of order 1 radian, and to a rough approximation it also implies that the space and time dependence of $\theta$ can be ignored in the equation of motion. After wall formation there is an a roughly homogeneous axion field oscillating with initial amplitude $\tilde{\theta} \sim 1$, giving $\tilde{n} \approx \frac{\sqrt{2}\tilde{m}_a^2 f_a^2}{\Delta}$ and

$$\Omega_a \sim 1 \times \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.18} \Delta^{-1} \tag{30}$$

As we now discuss, this simplest estimate was queried by Davis [29], who argued that axion radiation from strings plays a crucial role.

**Energy loss from the string network**

Assume that there is a network of axionic strings, with at least one piece of string passing through a typical Hubble volume. On scales bigger than the horizon the string network expands with the universe, and on smaller scales string annihilation occurs. We assume that the string network has the ‘scaling’ property, whereby it looks the same at each epoch when viewed on the Hubble scale. To be precise, we assume that the energy density of the string network can be written

$$\rho_{st} = H^2 \mu \gamma^2 \tag{31}$$

where $\mu$ is the string energy per unit length and $\gamma$ is time independent. Roughly speaking, $\gamma$ is the string spacing in units of the Hubble distance $H^{-1}$.

The energy per unit length $\mu$ is dominated by the axion field around the string, rather than by the string core. The latter has thickness $f_a^{-1}$ and energy per unit length $f_a^2$, but a static axion field out to a distance $R$ gives energy per unit length $2\pi f_a^2 \ln(R f_a)$. As we shall discuss in a moment the axion field far away from the string is oscillating, but as a rough estimate we can set $R$ equal to the string spacing $(H \gamma)^{-1}$. Then $\mu$ has only logarithmic time dependence, and at the relevant epoch $T \sim 1 \text{ GeV}$ it is given by $\mu = 2\pi f_a^2 \eta$ where $\eta = \ln(f_a/\gamma H)$. With $f_a \sim 10^{12} \text{ GeV}$ and $\gamma \sim 1$ this gives the estimate $\eta \approx 70$, which we use in what follows.

From Eq. (31) one can calculate the rate at which the string network loses energy [29]. As in the homogeneous case we assume radiation domination throughout the string oscillation era. One then has $H = 1/t$ and Eq. (31) becomes

$$\rho_{st} = \frac{\pi}{2} \gamma^2 f_a^2 \eta t^{-2} \tag{32}$$
The condition that comoving strings lose no energy is $\rho_{\text{st}} \propto a^{-2}$. To maintain the scaling solution, energy conservation therefore requires that a comoving volume $a^3$ of the string network emits energy at a rate $a^3 R$ where $R$ is given by

$$ R = -a^{-2} \frac{d(a^2 \rho_{\text{st}})}{dt} = \frac{\pi}{2} \gamma^2 f_a \eta t^{-3} \quad (33) $$

All of this energy is assumed to be gained by the axion field. In the absence of the string network this field would correspond to a collection of massless non-interacting axions with energy density $\rho_a \propto a^{-4}$, so energy conservation requires

$$ a^{-4} \frac{d(a^4 \rho_a)}{dt} = R = \frac{\pi}{2} \gamma^2 f_a \eta t^{-3} \quad (34) $$

### Estimating the axion number density

So far everything is rigorous, given the scaling assumption. To go further one has to introduce the concept of an ‘axion’ with well defined momentum and energy, as opposed to just an ‘axion field’. Such an axion corresponds to field configuration which is a plane wave, and this clearly requires that the wavelength is much less than the string spacing, or equivalently that the wavenumber $\omega = k/a$ is much bigger than

$$ \omega_{\text{min}} = \frac{k_{\text{min}}}{a} = 2\pi \gamma H \quad (35) $$

For a plane wave with amplitude $\theta$, the energy density $\rho_a$ is related to the number density $n$ by $\rho_a = \omega n_a$ where $\omega = k/a$ is the angular frequency. Let us define the spectrum $P_a$ of the axion energy density as the contribution to it from unit interval of $\ln k$,

$$ \rho_a = \int_{k_{\text{min}}}^{\infty} P_a(t, k) \frac{dk}{k} \quad (36) $$

then

$$ n = \int_{k_{\text{min}}}^{\infty} P_a(t, k) \left( \frac{a}{k} \right) \frac{dk}{k} \quad (37) $$

One can also define the spectrum of the axions which are being emitted during some small time interval $dt$, which we write as $dt P_{\text{emis}}$. It is related by energy-momentum conservation to the axion spectrum (cf. Eq. (34)),

$$ P_{\text{emis}}(t, k) = a^{-4} \frac{\partial}{\partial t} \left( a^4 P_a(t, k) \right) \quad (38) $$

Integrating over $k$ gives the energy conservation constraint Eq. (34),

$$ \int_{k_{\text{min}}}^{\infty} P_{\text{emis}}(t, k) \frac{dk}{k} = \frac{\pi}{2} \gamma^2 f_a^2 \eta t^{-3} \quad (39) $$

Given a hypothesis about its shape, this equation determines $P_{\text{emis}}$, and integrating Eq. (38) then gives $P_a$ which gives the number density through Eq. (37).
Two proposals have been made about the shape of the spectrum. According to Davis [29, 34], axion emission is caused by smooth oscillations of the string, with wavelength roughly $\beta^{-1}$ times the string spacing, where $\beta$ is between 1 and a few. This corresponds to

$$P_{\text{emis}}(t,k) = f(t)\delta(k - k_*(t))$$

(40)

where

$$\omega_* \equiv \frac{k_*}{a} = \beta \omega_{\text{min}} = \frac{\pi \gamma \beta}{t}$$

(41)

Inserting this expression into Eq. (38) gives

$$P_a(t,k) = \pi \eta \gamma^2 f_a^2 t^{-2} \theta(k - k_*(t))$$

(42)

and then Eq. (37) gives

$$n = \pi \eta \gamma^2 f_a^2 t^{-2} \int_{k_{\text{min}}}^{\infty} \frac{d\omega}{\omega^2} \ln \left( \frac{\omega}{\omega_{\text{min}}} \right)$$

(43)

and

$$= \frac{\gamma \eta}{\beta} f_a^2 t^{-1}$$

(44)

leading to

$$\Omega_a = 1.2 \times 10^{\pm 5} \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{1.18} \gamma \eta \beta^{-1} \Delta^{-1}$$

(45)

This is only a factor or order $\gamma$ times the simple estimate. Taking $1 < \gamma < 3$ times the simple estimate, it corresponds to

$$\Omega_a = (0.4 \text{ to } 11) \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{1.18} \Delta^{-1}$$

(51)

In other words, taking all of the uncertainties into account Davis’ proposal increases the simple estimate by a factor of order 10 to 1000.

The proposal of Harari and Sikivie [30, 35] is that the emission spectrum is flat,

$$P_{\text{emis}}(t,k) = f(t) \quad \left( \frac{k_{\text{min}}}{a} < \frac{k}{a} < f_a \right)$$

(47)

This gives

$$P_a(t,k) = \pi \gamma^2 f_a^2 t^{-2} \ln \left( \frac{k}{k_{\text{min}}(t)} \right)$$

(48)

$$n = \pi \gamma^2 f_a^2 t^{-2} \int_{k_{\text{min}}}^{\infty} \ln \left( \frac{\omega}{\omega_{\text{min}}} \right) \frac{d\omega}{\omega^2}$$

(49)

$$= f_a^2 \gamma t^{-1}$$

(50)

$$\Omega_a = 1.2 \times 10^{\pm 5} \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{1.18} \Delta^{-1}$$

(51)

This is only a factor or order $\gamma$ times the simple estimate. Taking $1 < \gamma < 3$ it corresponds to

$$\Omega_a = (0.4 \text{ to } 11) \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{1.18} \Delta^{-1}$$

(52)
In the two right hand columns of Figure 1 are shown the bands which correspond to these estimates. The full lines correspond to $T_{\text{reh}} = 10^{16}$ GeV, the biggest possible value. The dotted lines (and their straight continuation) correspond to the absence of supersymmetry, or equivalently to $T_{\text{reh}} < 10^9$ GeV. The allowed window for $m_a$ is shown in the two left hand columns of Figure 2 as a function of $T_{\text{reh}}$.

**Critical assessment of the proposals**

According to both estimates, most of the axions have angular frequency not far above $\omega_{\min}$ (Eqs. (43) and (49)). In other words the condition that their wavelength is much less than the string spacing, necessary for the axion concept to make sense, is not terribly well satisfied. Indeed, if $\gamma = 1$ and (in the Davis case) $\beta = 1$, it is at best marginally satisfied; most axions then have wavelength of order the string spacing. In that case, one might ask why the rough estimate Eq. (30) is not reproduced, which after all started with precisely the assumption that the wavelength was of order the string spacing. A clue to the answer appears when one calculates the mean square axion field [26],

$$f_a^2 \bar{\theta}^2 = \int_{k_{\min}}^{\infty} P_a(t,k) \left(\frac{a}{k}\right)^2 \frac{dk}{k}$$

(53)

(which follows from the fact that a relativistic plane wave with amplitude $\theta$ and frequency $\omega$ has energy density $f_a^2 \theta^2 \omega^2$). The corresponding mean square amplitude of the oscillation in $\theta$ is

$$\bar{\theta}^2 = \frac{\eta}{2\pi \beta^2} \sim \left(\frac{3.3}{\beta}\right)^2 \quad \text{(Davis)}$$

$$\bar{\theta}^2 = \frac{1}{2\pi} \sim (0.4)^2 \quad \text{(Harari-Sikivie)}$$

(54)

(55)

If we set $\bar{\theta}^2 = \bar{\theta}^2$ in Eq. (28) then Eqs. (13) and (31) are roughly reproduced for the the case $\beta = \gamma = 1$ (to be precise the result is a factor $3/8\pi$ smaller). In other words, the essential reason why the Harari-Sikivie proposal reproduces the simple estimate is that it leads to an amplitude $\theta \sim 1$, and the essential reason why the Davis proposal is bigger by a factor of order $\eta$ is that it leads to an amplitude $\theta \sim \eta^{1/2}$.

This last fact is rather worrying. If $\eta$ had been a few orders of magnitude bigger than 70 the amplitude would have been much bigger than $2\pi$. Such an oscillation amplitude would not in itself be unphysical, but the problem would be that it cannot be generated by the smooth, only marginally relativistic string oscillations envisaged in Davis’ proposal; rather, such oscillations generate a typical amplitude $\theta \sim 1$ [34]. Thus Davis’ proposal would be inconsistent with the scaling assumption if $\eta$ were much bigger than its actual value. One wonders what mechanism resolves the conflict when it arises, and whether this mechanism is already beginning to operate in the marginal case that we are dealing with.

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2The above viewpoint is somewhat different from the one taken by the present author in [26]. There, a less careful calculation gave a somewhat larger amplitude. More importantly, the fact that Davis’ string oscillations cannot generate an amplitude $\theta \gg 2\pi$ was not appreciated, and an estimate was made of the axion density resulting from such an amplitude. While this estimate would indeed be valid if such an amplitude were somehow generated, it does not apply to the the case at hand. I am indebted to Rick Davis for a clarifying discussion on this question.
Nevertheless, the statement that the oscillating strings emit radiation with a wavelength of order their spacing seems rather natural and to this extent the proposal of Davis is perhaps a reasonable working hypothesis. By contrast the proposal of Harari and Sikivie is extremely radical, in that it postulates the existence of a mechanism whereby the strings emit radiation impartially with all wavelengths, right down to the string thickness $f_a^{-1} \lesssim 10^{-10} \text{GeV}^{-1}$, no matter how late the epoch. As the string thickness is only $10^{-28}$ of the string spacing by the time radiation finishes, one would like to know something about the mechanism before taking the proposal too seriously.

As only strings as opposed to domain walls are involved it would seem feasible to perform numerical simulations. They would hopefully confirm the scaling behaviour, and determine the string spacing $\gamma$ defined by Eq. (31) and emission spectrum, thus allowing one to calculate $\Omega_a$ on the assumption that axion number is conserved after wall formation. We end this section by briefly asking about the validity of that assumption.

After domain wall formation

The string-wall network must annihilate well before the present to avoid cosmological disaster, and in order to make this possible we have assumed that the vacua on the two sides of a wall are identical (Eq. (22)). How does annihilation proceed? Based on (30), the standard assumption in the literature seems to be that gravitational radiation is the only significant process. The underlying idea, inspired by what happens for a loop of gauge string, is that the network has significant structure only on macroscopic scales, of order the Hubble distance and bigger. However, the walls are made out of the axion field and have a thickness of order $1/m_a$, and it therefore seems clear that collisions between pieces of wall will result in the emission of marginally relativistic axions at some level. Since gravity is so weak, this process is presumably the dominant one. As discussed in (24) wall annihilation gives a contribution

$$\Omega_a \sim \left( \frac{t_{\text{ann}}}{t} \right)^{3/2} \left( \frac{f_a}{10^{12} \text{GeV}} \right)^{1.18} \gamma$$

where $t_{\text{ann}}$ is the wall annihilation time. Thus, axions emitted by the walls are probably significant if the Harari-Sikivie hypothesis is correct, but may not be if the Davis hypothesis is correct.

It seems difficult to go beyond these rather crude considerations except through numerical simulations, which after wall formation would become extremely difficult.

4.3 The no-string scenario

So far we have assumed that axionic strings are generated in the early universe, either after inflation by the Kibble mechanism or during inflation by the quantum fluctuation. The criterion for the former possibility is $T_{\text{reh}} \gtrsim f_a$, and from Figure 3 one sees that this condition is rather similar to the condition $T_{\text{reh}} \gtrsim T_{\text{decoup}}$ which corresponds to the presence of thermal saxinos in the universe. The latter condition can hold without the former only if $T_{\text{reh}}$ is rather low, which means that the Kibble mechanism will certainly generate strings if saxinos generate a significant amount of entropy.

The criterion for the generation of strings by the quantum fluctuation is less certain. Considering only the usual non-supersymmetric Peccei-Quinn field and ignoring any inter-
action with the inflaton field or the spacetime curvature, the criterion was estimated in \[28\] to be \(H_1 \gtrsim 1 \times f_a\), where \(H_1\) is the Hubble parameter when the observable universe leaves the horizon. This estimate should be fairly robust as a rough criterion, and may perhaps be thought of as a quasi-thermal effect, due to the Hawking temperature \(H_1/2\pi\). It implies that string production does not occur if \(H_1 < 10^{10} \text{GeV}\) (because \(f_a > 10^{10} \text{GeV}\)). While such a low value of \(H_1\) is possible \[25, 16\], most models of inflation require \(H_1 \sim 10^{13}\) to \(10^{14} \text{GeV}\), in which case the no-string scenario requires \(f_a \sim > 10^{13} \text{GeV}\) (Figure 3).

A rather complete treatment of the no-string scenario has been given in \[27\], and we recall briefly the main results, again ignoring any coupling of the Peccei-Quinn field to other fields. The axion field is homogeneous before the epoch \(\tilde{T}\), except for an inhomogeneity which originates as a vacuum fluctuation during inflation. Writing \(\tilde{\theta} = \bar{\theta} + \delta \tilde{\theta}\), where \(\bar{\theta}\) is the average of \(\tilde{\theta}\) in the observable universe, Eq. (28) for \(\Omega_a\) becomes

\[\Omega_a = 0.9 \times 10^{\pm.5} \left(\frac{f_a}{10^{12} \text{GeV}}\right)^{1.18} \Delta^{-1}(\bar{\theta}^2 + \sigma_{\theta}^2)\]  

(57)

where \(\sigma_{\theta}^2\), the average of \((\delta \tilde{\theta})^2\), is given by

\[\sigma_{\theta} \approx \frac{4}{\pi} \frac{H_1}{f_a}\]  

(58)

The inhomogeneity of the axion field causes a primeval isocurvature density perturbation, with a flat spectrum given by \[27\]

\[P_{\text{iso}} \approx \frac{\Omega_a^2}{16} \frac{4\bar{\theta}^2 \sigma_{\theta}^2 + \sigma_{\theta}^4}{(\bar{\theta}^2 + \sigma_{\theta}^2)^2}\]  

(59)

(It might cause bags of domain wall as well \[23\], but this possibility has yet to be fully explored.) If present, a primeval isocurvature density perturbation contributes to the large scale microwave background anisotropy. Its effect on the rms anisotropy is the same as that of an adiabatic density perturbation whose spectrum \(\delta_H^2\) at horizon entry is given by

\[\delta_H = 6 \times (2/15) P_{\text{iso}}^{1/2}\]  

(60)

The COBE measurement implies \[16\] that \(\delta_H = (1.7 \pm 0.3) \times 10^{-5}\) which corresponds to\[^4\]

\[P_{\text{iso}}^{1/2} < 1.7 \times 10^{-5}\]  

(61)

At fixed \(\bar{\theta}\), Eqs. (27), (54) and (61) forbid a substantial region of the \(m_a-H_1\) plane \[28\]. The forbidden region is minimised for \(\bar{\theta} = 0\), but there is no reason to expect \(\bar{\theta}\) to be very small, unless a very small value is demanded by the requirement \(\Omega_a < 1\). Suppose that one sets \(\bar{\theta} = \min(1\pi, \bar{\theta}_{\Omega})\) where \(\bar{\theta}_{\Omega}\) is the value of \(\bar{\theta}\) which makes \(\Omega_a = 1\), and fixes \(H_1\). Then an interval of \(m_a\) is forbidden, which covers the entire regime \(H_1 < f_a < m_{Pl}\) if \(H_1 > 10^{12} \text{GeV}\) but which shrinks to nothing for \(H_1 < 10^{10} \text{GeV}\).

The right hand column of Figure 2 illustrates the case \(H_1 < 10^{10} \text{GeV}\); to the left of the diagonal line there is no constraint on \(m_a\) except the astrophysical one. As \(H_1\) is increased from \(10^{10} \text{GeV}\) to \(10^{12} \text{GeV}\) a forbidden region rapidly develops and the situation becomes practically the same as for the string scenario.

\[^4\]A weaker constraint \(P_{\text{iso}}^{1/2} < 10^{-4}\) was used, corresponding to the use of pre-COBE quadrupole data, but the difference is not very significant for the present purpose.
5 Conclusions

The calculation presented here is valid under the assumption that the correct extension of the Standard Model respects Peccei-Quinn symmetry and supersymmetry, the latter being implemented without introducing dramatic new features such as a very light gravitino [13]. Subject to this requirement, the calculation shows that saxinos of thermal origin can dilute estimates of cosmological mass densities by a factor of up to $10^4$, and non-thermal saxinos might cause even more dilution. Let us close by briefly considering the implication of a dilution factor for some commonly discussed cases [1], leaving aside the axion which has already been treated.

The baryon density is usually considered these days to be generated at the electroweak transition, and at least the more conservative proposals do not generate significantly more than the observed density (eg. [37]). Accordingly these models could not tolerate a large dilution factor, but as the theoretical situation is still very fluid it is probably too early to say anything very definite.

Apart from the axion a favourite dark matter candidate is a massive neutrino species. Species with mass $m_\nu \lesssim 10$ MeV decouple while still relativistic, and without any dilution factor $\Omega < 1$ requires $m_\nu < 100$ eV. Decoupling occurs just before nucleosynthesis, so this bound is not affected by entropy production (which must finish by that epoch). Species with mass $m_\nu \gtrsim 10$ MeV on the other hand decouple while non-relativistic, and without dilution this opens up an additional allowed interval $m_\nu \gtrsim 1$ GeV. In that case decoupling occurs at $70$ MeV($m_\nu/1$ GeV) [1] so dilution can occur and it does the allowed interval extends to lower masses. From Fig. 5.2 of [1] one learns that with a dilution factor $\Delta \sim 10^4$ the interval extends right down to $m_\nu \sim 10$ MeV.

Finally, consider a particle species with much weaker interactions than the neutrino, so that decoupling occurs at $T \gg 1$ GeV when $g_* \sim 100$. Without entropy dilution the mass needed to make $\Omega = 1$ is $m \sim 1$ keV, but with dilution the mass is increased by a factor $\Delta$ and so could be of order 10 MeV. Such a particle species would constitute cold dark matter, instead of ‘warm’ dark matter as in the usual case.

From these examples one sees that a significant entropy dilution factor could have profound implications. Whether or not such a factor is generated by thermal saxinos depends mainly on whether the reheat temperature is high or low, and we are reminded that an understanding of this quantity is one of the outstanding problems of theoretical cosmology.

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Figure Captions

FIG. 1.—Plots against axion mass. All scales are logarithms to base 10, and in each plot the horizontal axis is \( m_a/1\text{eV} \). The rows correspond to different saxino masses as indicated. The left hand column shows various temperatures in units of 1 MeV, the full line indicating the temperature \( \tilde{T} \) after which axion number is conserved, and the dotted lines indicating the three temperatures \( T_{\text{decay}} < T < T_{\text{axioneq}} \) defined in the text (entropy generation occurs only in the narrow interval \( T_{\text{decay}} < T < T_{\text{axioneq}} \)). The second column gives the entropy production factor \( \Delta \), given by Eq. (17) or (dotted line) Eq. (17). The two right hand columns give the axion density \( \Omega_a \), for \( T_{\text{reh}} = 10^{16}\text{GeV} \) (full lines) and \( T_{\text{reh}} < 10^8\text{GeV} \) (dotted lines and their continuation). In each case a band of values is shown to take into account the uncertainties discussed in the text, and the second case is identical with the no-supersymmetry result. The hatched regions correspond to the nucleosynthesis constraint and (for the two right hand columns) the constraint \( \Omega_a < 1 \).
FIG. 2.—The axion window. In each plot the vertical axis is $\log_{10}(m_a/1\text{eV})$ and the horizontal axis is $\log_{10}(T_{\text{reh}}/1\text{GeV})$. The first row corresponds to the non-existence of the saxino and the others to different saxino masses. The three columns correspond to different possibilities described in the text. The upper cross hatched region is forbidden by accelerator physics and astrophysics. The lower cross hatched region is forbidden either by $\Omega_a < 1$ or by the requirement that saxino decay must not interfere with nucleosynthesis. The latter requirement is relevant only for the second and third rows, giving the right hand horizontal part of the boundary in each plot. In each plot (except on the first row) the diagonal full line and its straight continuation is the line below which thermal saxinos are absent, so that there is no dilution factor (non-thermal saxinos are not considered in the present paper). The diagonal dotted line in the right hand column is the line below which the Kibble mechanism does not produce axionic strings.

FIG. 3.—Various regimes of the $m_a-T_{\text{reh}}$ plane, as discussed in the text. The horizontal line marks the regime $H_1 > f_a$ for the case $H_1 = 10^{13}\text{GeV}$, and it moves up one decade for every decade that $H_1$ moves down.