Dual Simplex Method Based solution for a Fuzzy Transportation Problem

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Abstract This article focuses on solving the fuzzy transportation problem using the dual approach of the simplex. This method uses a strong ranking technique for the iterative values of the trapezoidal fuzzy numbers within the fuzzy transportation problem. Because of approach, the fuzzy transportation problem (in which cost, demand, and supply appear as trapezoidal fuzzy numbers) is transformed into a crisp valued transportation problem. LPP (Linear Programming Problem) and employed dual simplex procedure for solving. The earlier task of obtaining the optimal solution through the IBFS technique and therefore the fuzzy MODI method is reduced here and the optimal solution is obtained directly by the dual simplex procedure.

Keywords: Fuzzy transportation problem (FTP), Trapezoidal fuzzy numbers (TFN), Robust ranking techniques, Fuzzy version of MODI method (FMODI), Dual simplex method.

1. INTRODUCTION

H. Arsham and A. B. Kahn[1] present a Simplex type Algorithm for common transportation problems that acts as an alternative procedure for existing problems. SAUL I. Gass[2] solve the transportation problem. Nagoor Gani and K Abdul Razak[3] present a two-stage cost-minimizing fuzzy transportation problem where the supply and demand are given as trapezoidal fuzzy numbers and a fuzzy solution is obtained using a parametric approach. Nagoor Gani, A, Edward Samuel, D. Anuradha[4] present Arsham - Khan’s simplex algorithm applied for solving fuzzy transportation problems with the indefinite output and required constraints. This algorithm is shown and described briefly with numerical examples to prove its efficiency. R. Parvathi and C. Malathi[5] solve the given Intuitionistic Fuzzy Linear Programming Problems (IFLPPs) using the intuitionistic fuzzy simplex method and do not transform them into crisp linear programming problems. Amarpreet Kaur and Amit Kumar[6] propose a novel algorithm to solve a particular fuzzy transportation problem where the transportation costs are denoted as common trapezoidal fuzzy numbers. Nagoor Gani, S. Abbas[7] proposes a method to find an optimal solution for an intuitionistic fuzzy transportation problem and the key characteristics is it needs only simple arithmetical calculations rather than the huge number of iterations.

2. DEFINITIONS AND NOTATIONS

2.1. Fuzzy sets and membership functions: Let it be assumed that X is a collection of objects represented in general by x, and F the fuzzy set in X is said to be a set of ordered pair F and fuzzy set of F in X which is a set of real numbers a set of ordered pairs $F = \{(x, \mu_F(x)) \mid x \in X\}$, where $\mu_F(x)$ is said to be the fuzzy set membership function and it maps every element of X between the values 0 to 1.

2.1.1. $\alpha$–cut and strong $\alpha$–cut: Fuzzy set having $\alpha$–cut or $\alpha$–level set is said to be crisp set and is denoted by $F_{\alpha} = \{x \in X/\mu_F(x)\geq\alpha\}$. $F_{\alpha} = \{x \in X/\mu_F(x)>\alpha\}$ defines the strong $\alpha$–cut.

2.1.2. Fuzzy number: F is said to be a fuzzy set that is defined on R, which is a set of real numbers R, and is said to be a number of its membership function $\mu_F: R \rightarrow [0, 1]$ with the below mentioned properties:
a) \( x \in \mathbb{R} \) exists such that \( \mu_F(x) = 1 \) and hence \( F \) is normal.

b) For every \( x_1, x_2 \in \mathbb{R} \), \( \mu_F[\lambda x_1 + (1-\lambda)x_2] \geq \min \{\mu_F(x_1), \mu_F(x_2)\} \). \( \lambda \in [0, 1] \), so \( F \) is convex.

c) \( F \) meant to be upper semi-continuous.

2.1.3. Trapezoidal fuzzy numbers: A fuzzy number \( \tilde{F} = (L, a, b, U) \) is said to be a trapezoidal fuzzy number if its membership function is given by the following expression.

\[
\mu_{\tilde{F}}(x) = \begin{cases} 
\frac{x-L}{a-L}, & L \leq x \leq a \\
1, & a \leq x \leq b \\
\frac{U-x}{U-b}, & b \leq x \leq U \\
0, & \text{otherwise}
\end{cases}
\]

![Figure 1](image)

2.1.4. Robust’s Ranking Technique: Robust’s ranking technique satisfies compensation, linearity, and additive properties. The results obtained through this technique are proved to be consistent with human perception. If \( \tilde{F} \) is a fuzzy number then the ranking is defined by:

\[
\text{R}(\tilde{F}) = \int_0^1 (0.5) (a_x^L, a_x^U) d\alpha, \quad (a_x^L, a_x^U) = \{(a-L) \alpha + L, \ U-(U-b)\alpha\}, \ \alpha \in [0, 1].
\]

Where \( (a_x^L, a_x^U) \) is the \( \alpha \)-cut of the trapezoidal fuzzy number \( \tilde{F} \). Here, index \( \text{R}(\tilde{F}) \) is termed as the representative value of the fuzzy number \( \tilde{F} \) and index \( \text{R}(h_{ij}) \) is given.

2.1.5. Mathematical formulation of fuzzy transportation problem: Let the fuzzy transportation be with \( r \) sources and \( s \) destinations with trapezoidal fuzzy numbers, where \( g_i, (g_i \geq 0) \) denote the source availability at \( i \) and \( f_j, (f_j \geq 0) \) denote the destination requirement at \( j \). Let \( h_{ij} \) represent each unit fuzzy transportation cost calculated from source \( i \) to destination \( j \) and \( x_{ij} \) be the total number of fuzzy units that are proposed to be transported from source \( i \) to destination \( j \). Feasible path and optimal cost transportation is the required solution for this problem.

The mathematical model of a balanced fuzzy transportation problem is as follows.

Minimize \( z = \sum_{i=1}^{r} \sum_{j=1}^{s} h_{ij} x_{ij} \) \hspace{1cm} (1)

Subject to \( \sum_{j=1}^{s} x_{ij} = g_i ; i = 1, 2, \ldots, r \) \hspace{1cm} (2)

\( \sum_{i=1}^{r} x_{ij} = f_j ; j = 1, 2, \ldots, s \) \hspace{1cm} (3)
\[ x_{ij} \geq 0 \text{ for } i = 1, 2, \ldots, r; \quad j = 1, 2, \ldots, s. \]  

(4)

\[ \sum_{i=1}^{r} g_{i} = \sum_{j=1}^{s} f_{j} \text{ for } i = 1, 2, \ldots, r; \quad j = 1, 2, \ldots, s \]

A solution is said to be fuzzy feasible if it has the set of positive allocations \( x_{ij} \) that is equivalent to the row and the column restrictions. A solution is known to be a fuzzy degenerate feasible solution if the number of positive allocations in a fuzzy solution is less than \( r+s-1 \). Minimization of total fuzzy transportation cost makes the fuzzy feasible solution as a fuzzy optimal solution.

3. METHODOLOGY

3.1. Fuzzy version of MODI method (FMODI)

Step 1: A fuzzy transportation table (FTT) is made with the formulated fuzzy transportation problem. Next, the fuzzy cost of transportation is given within the table allocation. Later the approximate fuzzy cost of the transportation problem is applied by Robust’s ranking method. The balance status of the FTP is tested and if found unbalanced it’s balanced.

Step 2: Using the IBFS technique (VAM), an initial basic feasible solution (IBFS) is found.

Step 3: An IBFS is given for a fuzzy transportation problem. The answer is present as non-allocated and allocated cells of FTT. Assign the variables \( e_{i}, i=1, 2, 3, 4, \ldots, p \) & \( f_{j}, j=1, 2, 3, \ldots, q \) and columns correspondingly. For all occupied cells, find the values of \( e_{i} \) and \( f_{j} \) using the connection \( h_{ij} = e_{i} + f_{j} \).

Select either \( e_{i}, \)’s or \( f_{j} \)’s to 0 that’s associated with the row or column of the transportation table which has the very best number of allocated cells. Opportunity cost \( \delta_{ij} \) is calculated for all unoccupied cells using the formulae: \( \mu_{ij} = h_{ij} - (e_{i} + f_{j}) \) for all \( i \) and \( j \).

Step 4: Examine the sign of every \( \mu_{ij} \)

If \( \mu_{ij} > 0 \), then current basic feasible solution under the test is perfect.

If \( \mu_{ij} = 0 \), then there’s also an alternate optimal solution to the transportation problem.

If one or more \( \mu_{ij} < 0 \), then the present solution under the test isn’t optimal. during this case, we processed to subsequent step.

Step 5: A graph is drawn plotting the unoccupied cells that have the highest negative cost. Commence this closed graph with the chosen unoccupied cell and provides (+α) sign to the present cell, mark a route alongside the columns or rows to an occupied cell, and there give (-α) sign. These proceeds to all or any the occupied cells of rows and columns. Later the corners are marked with a plus (+α) sign and minus (-α) sign alternatively. The path is closed back to the chosen unoccupied cell.

Step 6: Within the closed path, the corner cell with “-α” and therefore the smallest value is chosen which smallest value is allocated to a specific unoccupied cell. That value is added to another occupied cell that has the “+α” sign and the same is subtracted from the “-α” signed occupied cell. A completely unique enhanced solution is obtained through the allocation of units to the unoccupied cells supported step 5. The overall transportation cost is calculated. The obtained solution is tested for further optimality and also the process continues till \( \mu_{ij} \geq 0 \) for all unoccupied cells.

3.2. Dual simplex method (DSM)

Step 1: Fuzzy Transportation Table (FTT) is made with the formulated FTP. Next, the fuzzy cost (supply and demand) of transportation is given within the table allocation. Later the approximate fuzzy cost of
(supply and demand) the transportation problem is applied by Robust’s ranking method. Next, the Balance status of the FTP is tested and if found unbalanced it’s balanced.

Step 2: Balanced fuzzy transportation table is converted into the LPP problem and using Balanced FTP’s both supply and demand are written here and their mathematical model of dual simplex method balanced fuzzy transportations equations are:

\[
\text{Minimize } z = \sum_{i=1}^{r} \sum_{j=1}^{s} h_{ij} x_{ij}
\]

Subject to \( \sum_{j=1}^{s} x_{ij} \geq g_{i} ; i = 1,2,\ldots,r \) (supply constrains) \( \tag{5} \)

\( \sum_{i=1}^{r} x_{ij} \geq f_{j} ; j = 1,2,\ldots,s \) (demand constrains) \( \tag{6} \)

\( x_{ij} \geq 0 \) for \( i = 1,2,\ldots,r ; j=1,2,\ldots,s \).

Step 3: The objective function of the given linear programming problem is usually considered as maximum. within the case of the objective function being minimal, it's converted to maximum by changing the objective function sign.

Step 4: Transform the inequality constraints of the kind \( \geq \) into inequality constraints of the kind \( \leq \) by changing the sign in the whole function. Next, Introduce the slack variables for converting all the inequality constraints into equality constraints and obtain the initial basic solution.

Step 5: An initial simplex table is understood as a “dual simplex table” is ready. For each column \( Zj - Hj \) is calculated.

a). An answer obtained is alleged to be an optimal basic feasible solution if all \( Zj - Hj \geq 0 \) and every one \( bi \) are non-negative. b). Step 5 is repeated if all \( Zj - Hj \geq 0 \) and a minimum of one among the \( bi \) is negative. c). This method is alleged to be failed if all \( Zj - Hj \leq 0 \).

Step 6: A row is claimed to be a key or pivotal row if it contains the most negative ‘\( bi \)’ and the respective basic variable is off from the present solution. The key or pivotal row elements are viewed. a). The matter doesn't seem to possess a feasible solution if all elements are non-negative. b). Find at least one negative element and its ratio to the corresponding elements of the \( Hj - Zj \) row to that element. Next, the ratio associated with the positive or zero elements of the given pivotal row is ignored and therefore the smallest ratio is chosen. The respective column is that the pivotal column and therefore the related variable is that the entering variable. This pivotal element is created unity within the successive dual simplex table. the same as the Simplex method row operation is administered and every one other element present within the pivot column are made zero and this step is repeated until an optimal feasible solution is reached.

4. NUMERICAL EXAMPLE

Example 1. Consider the following fuzzy transportation problem.

| Sources/Destinations | Demand D_1 | Demand D_2 | Demand D_3 | Supply       |
|----------------------|------------|------------|------------|--------------|
| S_1                  | (7,9,11,13)| (9,11,13,15)| (10,12,14,16)| (62,64,66,68),|
| S_2                  | (6,8,10,12)| (6,8,10,12)| (8,10,12,14)| (22,24,26,28),|
| S_3                  | (8,10,12,14)| (9,11,13,15)| (9,11,13,15)| (42,44,46,48),|
**Solution:** The fuzzy transportation problem is given in Table 1.

$$R(7, 9, 11, 13) = \int_0^1 (0.5)(2\alpha + 7 + 13 - 2\alpha)d\alpha = 10 \text{ (by Robust’s ranking technique)}$$

Given fuzzy problem is converted into the crisp value problem using Robust’s ranking technique.

| Sources/ Destinations | D1 | D2 | D3 | Supply |
|-----------------------|----|----|----|--------|
| S1                    | 10 | 12 | 13 | 65     |
| S2                    | 9  | 9  | 11 | 25     |
| S3                    | 11 | 12 | 12 | 45     |
| Demand                | 60 | 37 | 38 |        |

**Table 2. After applying Robust’s ranking**

| Sources/ Destinations | D1 | D2 | D3 | Supply |
|-----------------------|----|----|----|--------|
| S1                    | 10 | 60 | 12 | 5      |
| S2                    | 9  | 9  | 25 | 11     |
| S3                    | 11 | 12 | 7  | 12     |
| Demand                | 60 | 37 | 38 |        |

**Table 3. After applying FMODI**

| Sources/ Destinations | D1 | D2 | D3 | Supply |
|-----------------------|----|----|----|--------|
| S1                    | 10 | 60 | 12 | 5      |
| S2                    | 9  | 9  | 25 | 11     |
| S3                    | 11 | 12 | 7  | 12     |
| Demand                | 60 | 37 | 38 |        |

**Dual simplex method:** LPP is generated from Table 2 that contains the converted FTP.

**Table 4. Simplex Representation of the FTP**

$$\text{Min } z = 10x_{11} + 12x_{12} + 13x_{13} + 9x_{21} + 9x_{22} + 11x_{23} + 11x_{31} + 12x_{32} + 12x_{33}$$

$$x_{i1} + x_{i2} + x_{i3} \geq 65 \quad \text{for } i = 1, 2, 3; j=1,2,3.$$ 

**Computer Output:** Tora Computer Software Package is used for solving the FTP problem. The Dual simplex method is applied for obtaining the solution and the computer results are as follows: Linear programming output summary: Dual simplex Solution, Objective Value = 1425

| variables | x_{11} | x_{12} | x_{13} | x_{21} | x_{22} | x_{23} | x_{31} | x_{32} | x_{33} | SX_{12} |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| values    | 60     | 5      | 0      | 25     | 0      | 0      | 7      | 38     | 0      |        |

Now, dual simplex solution put on Transportation Table Model

| Sources/ Destinations | D1 | D2 | D3 | Supply |
|-----------------------|----|----|----|--------|
| S1                    | 10 | 60 | 12 | 13 65  |
| S2                    | 9  | 9  | 25 | 11 25  |
| S3                    | 11 | 12 | 7  | 12 38  |
| Demand                | 60 | 37 | 38 |        |
Dual simplex method procedure is employed and, the obtained solution is compared with the existing solution as presented in the table.

5. RESULTS AND DISCUSSION

| Method                  | Transportation cost |
|-------------------------|---------------------|
| FMODI                   | 1425                |
| Dual simplex method     | 1425                |
| Optimal solution        | 1425                |

In the above result, we observed that the dual simplex method gives the lowest feasible solution which is equal to the MODI method. It is equal to optimal solution 1425.

| Source/Destination | Demand | S1 | S2 | S3 | D1 | D2 | D3 | Supply |
|--------------------|--------|----|----|----|----|----|----|--------|
| D1                 | 60     |    | 5  |    | 0  |    |    | 65     |
| D2                 | 0      |    | 25 |    | 0  |    |    | 25     |
| D3                 | 0      |    |    | 7  |    | 38 |    | 45     |

The results are obvious that the optimal solution obtained from the FMODI method and therefore the Dual simplex methods are equal. Further significance of the connection is expressed through the comparison of the obtained solutions and found that they’re similar.

6. CONCLUSION

Dual simplex method provides a far better feasible solution than other methods. Application of this dual simplex method proved that it directly yields the optimal solution obtained with the IBFS technique and therefore the fuzzy MODI.

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