Dynamical evidence for intermediate mass black holes in old globular clusters

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**ABSTRACT**

We present an indirect dynamical evidence, based on the measure of the core to half mass radius ratio ($r_c/r_h$), that a significant fraction of globular clusters in our galaxy with an age greater than 10 half-mass relaxation times ($t_{rh}$) host an intermediate mass black hole (IMBH). In fact, after a few $t_{rh}$ much of the memory about the details of initial conditions is erased and $r_c/r_h$ is determined solely by the balance of the stellar encounters energy production in the core with the dissipation due to global expansion of the cluster. Here we compare the observed values of $r_c/r_h$ from a sample of 57 galactic globular cluster, selected to be dynamically old and not strongly influenced by tidal forces, with the theoretical expectation for the $r_c/r_h$ ratio based on analytical models and detailed numerical simulations. The simulations of the evolution of star clusters considered include combinations of single stars, primordial binaries and IMBH. For at least half of the clusters in our sample the observed $r_c/r_h$ ratio appears to be too large to be explained without invoking the presence of an IMBH at the center of the system.

**Key words:** black hole physics — stellar dynamics — globular clusters: general — methods: n-body simulations

1 INTRODUCTION

The existence and detection for both stellar mass black holes as well as supermassive black holes are commonly accepted and recognized in the astronomical community (Ho 1999; Kormendy & Gebhardt 2001). Intermediate mass black holes (IMBH), i.e. black holes with mass in the $10^3 - 10^4 M_\odot$ range, possibly formed by runaway mergers of massive stars (Portegies Zwart et al. 2004) are expected to fill in the mass range from stellar to supermassive BH. Indeed, evidence for the presence of IMBH in star cluster is accumulating, but so far there are only two globular clusters - M15 and G1 - that present observational evidence for the existence of IMBH (Gebhardt et al. 2002, 2005; van der Marel et al. 2002; Gerssen et al. 2003). The cases for the presence of IMBH are based on the challenging determination of the precise kinematics of the systems, that has to be compared with detailed dynamical models in order to infer the presence of the central BH. At the level of velocity dispersion and full line of sight velocity dispersion profiles, the direct dynamical influence of an IMBH is limited within the sphere of influence of the BH, hard to resolve observationally (but see the recent high-resolution HST survey carried out by Noyola & Gebhardt 2006). In addition, the intrinsic three dimensional density (and velocity dispersion) cusp around the central BH is smeared out when the data are projected along the line of sight.

However, even if the direct dynamical signature of a central BH is hard to detect observationally, the presence of an IMBH at the center of a collisional stellar systems has profound consequences on the global dynamics. In particular the visual appearance of a globular clusters hosting an IMBH is that of a system with a sizeable core (Baumgardt et al. 2005; Trenti et al. 2006b) and not that of very concentrated cluster, as could be naively inferred from the existence of the Bahcall-Wolf $1/r^{1.7}$ density cusp (Bahcall & Wolf 1976) within the sphere of influence of the BH. This is because the presence of a central BH and of its compact density cusps generates, through stellar encounters, a sufficient amount of energy to halt the core collapse, if the initial conditions start with a wide core, or to fuel a core expansion if the initial configuration is centrally concentrated. In Trenti et al. (2006b) we have shown that star clusters with a central IMBH starting from a variety of initial stellar density profiles (i.e. King models with concentration parameter $W_0 = 3, 5, 7, 11$ as well as Plummer models) all evolve toward a common value of the core to half mass radius ratio ($r_c/r_h \approx 0.3$) on a timescale of a few initial half-mass relaxation times. This is indeed not surprisingly: a similar behavior is observed when primordial binaries are the responsible driver for energy production in the center of the cluster (Trenti, Heggie & Hut 2006, in agreement with the theoretical expectation formulated by Vesperini & Chernoff 1994). As binaries alone are less effi-
cient at producing energy than an IMBH, in this case we have \( r_c/r_h \lesssim 0.06 \) (where the upper limit is reached in presence of a number density of at least 50% binaries in the core) for a typical globular cluster. Only when no or little binaries are present there can be a deep core collapse, ending when \( r_c/r_h \lesssim 0.02 \) (see, e.g. Heggie, Trenti & Hut 2006).

Given this theoretical framework, the observed core to half mass radius ratio appears indeed a powerful indicator of the dynamical configuration of the core of an “old” globular cluster, i.e. a globular cluster that has evolved for about 5 to 10 half-mass relaxation times. \( r_c/r_h \) is in addition a relatively easy photometric measurement, that can be applied even to extra-galactic globular clusters, without worrying about the need of a very accurate measure. In fact, the combination of numerical simulations and theoretical modeling predicts that this ratio increases by about a factor 3 moving from clusters with single stars only to clusters with a sizeable population of binaries; when an IMBH is present \( r_c/r_h \) is 4 to 5 times larger than in the case when only binaries are present and 12 to 15 times larger than when the cluster contains single stars only.

In this letter we present a preliminary application of the use of \( r_c/r_h \) as an indirect indicator for the presence of an IMBH in the cores of 57 galactic globular clusters that satisfy a conservative dynamical age criterion, i.e. their half-mass relaxation time is shorter than \( 10^9 \text{yr} \). In particular, we show that the observed core to half mass radius ratio in more than half of the clusters in the sample is so large that the only general dynamical explanation can be given in terms of the presence of a central BH. The paper is organized as follows: in Sec.2 we summarize the current numerical and theoretical understanding of the evolution of \( r_c/r_h \), in Sec.3 we present the sample of galactic globular clusters that we use in this analysis and we discuss their properties in terms of our theoretical expectations. Finally we conclude in Sec.4.

2 GLOBAL EVOLUTION OF A CLUSTER WITH BINARIES AND IMBH

As discussed in quantitative detail in a series of paper devoted to the dynamical evolution of star clusters with primordial binaries and IMBHs (Heggie, Trenti & Hut 2006; Trenti, Heggie & Hut 2004; Trenti et al. 2006b), the long term evolution of a globular cluster is driven by the heat flow generated in the core of the system toward the halo (see also Vesperini & Chernoff 1994). Three main processes can be identified at the base of this heat flow, that regulates the evolution of \( r_c/r_h \) (shown in Fig.1 for representative simulations of our numerical simulations program). (1) When only single stars are present the core behaves as a self-gravitating systems with negative specific heat (e.g. see Heggie & Hut 2003) and a thermal collapse happens leading to a core contraction. The core contraction proceeds for about 10 – 15 half mass relaxation times for equal mass particles, while it is significantly faster when a mass spectrum is considered (Chernoff & Weinberg 1990). The collapse lasts until the central density is so high that a few binaries are dynamically formed by three body encounters. Given the high central density these binaries interact efficiently with single stars and become progressively tighter and tighter providing enough energy to halt the collapse and create a core bounce. When the number of stars in the system is large enough (\( N \gtrsim 10^5 \)) binary activity may cause a temporary temperature inversion in the core and subsequent core expansion. The process is eventually repeated in a series of “gravothermal oscillations” (Sugimoto & Bettwieser 1983) with the core radius oscillating around a value roughly 2% of the half mass radius. (2) When a sufficient fraction (i.e. greater than a few percent) of stars in the cluster has initially a companion with typical separations below 10AU (that is “hard binaries” that have a binding energy of at least a few times the total energy of the cluster), energy generation due to existing binaries is much more efficient than that due to dynamically formed pairs. Here the equilibrium size of the core is larger than when only single stars are present. The system may even show a core radius expansion if the initial conditions are too concentrated. The precise value of the equilibrium core radius has a moderate dependence on the number of stars in the systems and on the number density of binaries in the core: \( r_c \approx 5\% \, r_h \) for a typical globular cluster (\( \mathcal{N} = 3 \times 10^5 \)) with about 40% of binaries in the core (Heggie, Trenti & Hut 2006; Trenti, Heggie & Hut 2006; Fregeau & Rasio 2006; Vesperini & Chernoff 1994). This quasi-equilibrium evolutionary phase lasts until there is a sufficient number of binaries in the core, that is until the binary fraction in the cluster is reduced at a few percent level. In fact, binaries in the outer part of the cluster tend to sink toward the core for mass segregation, so the core binary fraction is replenished until the total binary fraction is almost exhausted. (3) When an IMBH with mass of the order of 1% of the total mass of the cluster is present, a yet more efficient energy production channel is available in the center of the system via dynamical interactions within the influence sphere of the BH (Trenti et al. 2006b, see also Baumgardt et al. 2004a,b). The equilibrium value of the core radius in this case is significantly larger: \( r_c/r_h \approx 0.3 \) independently from the initial binary fraction. Even in this case there is an expansion of the core radius on a relaxation timescale if the initial conditions are too concentrated.

This general picture has been obtained with somewhat idealized N-body simulations that employed single particles only and did not take into account stellar evolution. Therefore it is important to assess if the results that we obtain can be biased by our assumptions. Modest deviations from our numerical expectations will not impair the usefulness of \( r_c/r_h \) as diagnostic tool to infer the presence of IMBH, as \( r_c/r_h \) is expected in this case to be larger by a factor 4 to 5 with respect to the case where only binaries are present.

To ensure that no major bias is present we examine critically the idealizations in the numerical runs:

(i) Mass spectrum. The choice to use equal mass stars is probably the most significant simplification introduced in our simulations. When a realistic mass spectrum is present, massive stars segregate on a relaxation timescale toward the core of the system and speed up the core collapse by about one order of magnitude, i.e. the collapse takes about \( 1 - 2 \, t_h \) (Chernoff & Weinberg 1990). In principle, the equilibrium value of the core radius could be changed significantly. However simulations of the dynamics of star clusters with primordial binaries that include a realistic mass spectrum have been recently performed with a Monte Carlo code by Fregeau & Rasio (2006) and their results on \( r_c/r_h \) are fully consistent with our more idealized runs. Similarly, realistic simulations of globular clusters with an IMBH that include a mass spectrum and stellar evolution (but without binaries) have been carried out by Baumgardt et al. (2004a) and again the presence of a large core radius is confirmed. The analysis of the final configuration at \( t = 12 \text{Gyr} \) of two largest runs by Baumgardt et al. (2004b) - with \( N = 131072 \) - leads to \( r_c/r_h \approx 0.27 \) for the run with a Kroupa (2001) IMF truncated at 100 \( M_\odot \) and \( r_c/r_h \approx 0.35 \) when the IMF is truncated at 30 \( M_\odot \).

(ii) Stellar evolution. Our runs do not take into account the ef-
fects of stellar evolution, that limits the lifetime of massive stars and may lead to the coalescence of tight binaries. These effects may possibly lead to a more rapid depletion of the binary population (see Ivanova et al. 2005), so that a star cluster would burn out its binary reservoir earlier than expected from our simulations. If this is the case, then the core undergoes a deep core contraction, like in the case where only single stars are present. The direction of the evolution is fortunately in the right direction to avoid an observational bias when we are interested, as in this paper, to focus on large values of the core to half mass radius. For simulations with an IMBH, the study of Baumgardt et al. (2004b) (see point above), that included stellar evolution, guarantees instead that the bias due to stellar evolution is not important (see also Fig.1 in Baumgardt et al. 2004b), where the Lagrangian radii, that is the radii enclosing a fixed fraction of the total mass of the system, expand steadily starting from a $W_0 = 7$ model).

(iii) Number of particles. Our direct simulations have been limited by CPU speed to employ only up to $\approx 20000$ particles. The typical number of stars in globular clusters is larger by at least one order of magnitude. Given our choice to use equal mass particles and no stellar evolution, our results are scale-free and can be adapted to any physical value for the mass and radius scales. From theoretical considerations by Vesperini & Chernoff (1994) when primordial binaries are present $r_c/r_h$ is expected to have a modest dependence on the number of particles $N$, that is $r_c/r_h \propto 1/\log(0.11N)$. Simulations with $N$ from 512 to 16384 indeed verify that the scaling is approximately consistent with the Vesperini & Chernoff (1994) model (Heggie, Trenti & Hut 2006). If anything $r_c/r_h(N)$ seems to decrease slightly faster than expected. In addition the simulations by Fregeau & Rasio (2006) employ a realistic number of particles and their core radii are consistent with our results extrapolated using the Vesperini & Chernoff (1994) model. When a IMBH is present we do not expect a significant scaling of $r_c/r_h$ with $N$ as confirmed by our analysis of the two $N = 131072$ runs of Baumgardt et al. (2004b). Even if a log $N$ scaling were to be assumed despite the indication from the simulations by Baumgardt et al. (2004b), the core radius value extrapolated from our runs with a IMBH would still be such that $r_c/r_h \gtrsim 0.15$ for a realistic number of particles.

(iv) Tidal field. The runs with an IMBH in Trenti et al. (2006b) do not take into account the presence of a tidal field. This is however not expected to introduce a significant bias: $r_c/r_h$ in runs with primordial binaries varies at the 10% level with or without inclusion of the galactic tidal field (Trenti, Heggie & Hut 2006).

(v) Spherical symmetry. Our initial conditions are all spherically symmetric. We do not expect this to be a problem as globular clusters are typically very close to spherical symmetry. In addition, the precise details of the starting configuration are not important on a collisional timescale. To be sure, in the analysis of the next section we analyze a subsample of the clusters selected by excluding objects that departs from spherical symmetry, obtaining the same $r_c/r_h$ distribution as in the parent sample.

(vi) Rotation. The simulations that we consider have zero net angular momentum, i.e. no rotating model is studied. Observational evidence of rotation in globulars is weak and the velocity dispersion tensor displays a mild anisotropy at most (Meylan & Heggie 1997). Therefore it would be very surprising if the comparison with observations turns out to be strongly biased by not including rotation in simulations.

2.1 Core radius definition

Finally, before moving on to compare our theoretical expectations for $r_c/r_h$ with the data, we have to consider a yet different source of uncertainty. Numerical simulations give us mass-defined core and half-mass radii, while luminosity-based quantities are derived from observations. A discrepancy can therefore arise (1) if the luminosity profile does not track accurately the mass profile and/or (2) if the core radius is differently defined in the two cases.

For the first issue we have to consider that the luminosity of a globular cluster is dominated by stars in the giant branch, that are more massive than the average star of an old globular cluster. The most luminous stars are more centrally concentrated due to mass segregation than the average, so if a bias is introduced, it goes in the direction of reducing the observed core radius with respect to a mass weighted definition. However Chernoff & Weinberg (1990) derive only modest color gradients due to this effect. In any case it does not affect the interpretation of large observed cores.

The second issue, that is a difference between the observational and theoretical core radius definition is also not expected to be a major problem. In our simulations we adopted the density weighted definition of the core radius from Casertano & Hut (1985), Eq. IV.3:

$$r_c \equiv \frac{\langle |\vec{z}|\rho \rangle_M}{\langle \rho \rangle_M} = \frac{\sum_i r_i \rho_i m_i}{\sum_i \rho_i m_i}$$

where the sum is carried over all the stars in the simulation, $r_i$ is the distance of the i-th star from the center of the system, $m_i$ its mass and $\rho_i$ the stellar density computed at the star position. This definition for $r_c$ is closely aligned to the standard observational practice of defining the core radius as the radius $r_\nu$ where the luminosity surface density has dropped to half its central value.

3 DATA SAMPLE

As a preliminary application of the $r_c/r_h$ diagnostic to infer the presence of an IMBH, we consider the compilation of galactic globular clusters properties by Harris (1996), revised in February 2003. For each globular cluster in the tables of Harris (1996) we extract the following quantities: (1) core radius ($r_c$); (2) half-mass radius ($r_h$); (3) tidal radius ($r_t$); (4) half-mass relaxation time ($t_r$); (5) ellipticity ($e$); (6) cluster luminosity - i.e. the absolute visual magnitude - ($M_V$).

We then proceed to build a uniform and homogeneous data set that includes only old globular clusters to ensure that these systems are well relaxed by two body encounters. As a first step we therefore exclude from the analysis:

(i) All the global clusters for which one of these values is not quoted (except for $e$);

(ii) All the global clusters with $t_r > 10^9$ yr. Assuming a typical globular cluster age of $10^{10}$ yr, this ensures that all the objects in the sample are at least 10 half mass relaxation times old. Most of the clusters excluded from the sample fail this test;

(iii) All the cluster that are under the influence of a strong tidal field effects, i.e. those for which $r_t/r_h < 4$. This last selection criterion is mainly a precaution to exclude systems where $r_c/r_h$ may start fluctuating as $r_t$ approaches $r_h$ (see Trenti, Heggie & Hut 2006). Only four clusters that pass the previous selections are excluded due to a small tidal radius.

After applying our selection criteria we are left with a sample
of globular clusters. The results shown in Fig. 2 do not change significantly when this correction is applied: the average value is \( \langle \tilde{r}_c/\tilde{r}_h \rangle = 0.27 \) and the median value of the distribution is 0.22, only marginally smaller than without the correction. The presence of an IMBH is therefore still needed to explain the large value of the core radius for at least half the objects in the sample.

Could this conclusion be biased by the fact that the observed globular clusters are for some (improbable) reasons not yet well relaxed, so that the equilibrium value for \( r_c/r_h \) has not yet been reached after \( \approx 10 \, t_{wh} \)? To address this point we enhance our selection algorithm to select only objects that are at least \( 20 \, t_{wh} \) old, i.e. with \( t_{wh} < 5 \cdot 10^8 \, yr \). This reduces our sample to 25 clusters whose average core radius is still \( \langle r_c/r_h \rangle = 0.36 \) (marginally larger than the whole sample value), while the median is at \( r_c/r_h = 0.21 \) (marginally smaller than the whole sample analysis). If the \( \text{Vesperini & Chernoff} \, (1994) \log (0.11 N) \) correction is applied we have \( \langle r_c/r_h \rangle = 0.28 \) and median value of \( r_c/r_h = 0.17 \). The net effect of the correction is to reduce the mean and the median values, as by requiring \( t_{wh} \leq 5 \cdot 10^8 \, yr \) we are biased toward selecting smaller clusters. Even for this reduced sample more than half of the objects have \( r_c/r_h > 0.15 \), so the need for a central IMBH remains strong.

As an additional test to ensure absence of observational biases, we have further refined the reduced sample with \( t_{wh} < 5 \cdot 10^8 \, yr \) to exclude globular clusters that depart from spherical symmetry, i.e. those clusters with a quoted \( e > 0.1 \), where the ellipticity ratio \( e \) is defined in terms of the axis ratio \((e = 1 - a/b)\). This leaves 20 objects in the sample and does not change the results: \( \langle r_c/r_h \rangle = 0.38 \) (median 0.23), while \( \langle r_c/r_h \rangle = 0.29 \) (median 0.20).

All the progressively selective cuts that we applied to the galactic globular cluster system provide a consistent picture where about half of the objects have a significantly large core radius, i.e. \( r_c/r_h \geq 0.2 \). If an IMBH turns out to be absent in these systems, then a different dynamical explanation for their large core radii must be found. If this is indeed the case for the whole sample then a major rethinking of our knowledge of the dynamics of globular clusters is needed. Nevertheless, a few individual objects in the sample may turn out to have a large core radius for peculiar reasons, especially if they have been strongly perturbed and are therefore not as dynamically old as their relaxation time would imply. One example of a peculiar object that passes all our selection criteria is the cluster with the largest \( r_c/r_h \) ratio, Pal 13, which is considered a very unusual object, either due a strong tidal heating during the last perigalacticon passage or due to the presence of a sizeable dark matter halo \( \text{Côté et al.} \, (2002) \).

### 4 CONCLUSIONS

In this paper we propose the use of the observed ratio of the core to half mass radius as a powerful indirect dynamical indicator for the presence of an IMBH at the center of old stellar cluster. A number of theoretical and numerical investigations combined together \( \text{Vesperini & Chernoff} \, (1994); \text{Baumgardt et al.} \, (2004b); \text{Fregeau & Rasio} \, (2004); \text{Heggie, Trenti & Hut} \, (2006); \text{Trenti, Heggie & Hut} \, (2006); \text{Trenti et al.} \, (2006b) \) strongly support the idea that after about 5 to 10 half-mass relaxation times the value of \( r_c/r_h \) in a globular cluster assumes significantly different values depending only on whether the core contains single stars \( r_c/r_h \approx 0.02 \), a large fraction of hard binaries \( r_c/r_h \approx 0.05 \) or an IMBH \( r_c/r_h \approx 0.3 \). The details of the initial conditions, such as the initial density profile, do not affect this quantity. By an-

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**Figure 1.** Evolution of \( r_c/r_h \) in a series of runs with equal mass stars from [Heggie, Trenti & Hut] \, (2006b); Trenti, Heggie & Hut \, (2006); Trenti et al. \, (2006b) starting from a variety of initial conditions that include single stars only (red line: Plummer model, \( N=8192 \)), primordial binaries with galactic tidal field (green line: King model \( W_0 = 7 \% \) & 10\% binaries, \( W_0 = 11 \% \) & 20\% binaries, \( N = 16384 \)) and with IMBH (black line: \( m_{mbh} = 1.4\% M_{tot} \) & 10\% binaries, Plummer model, \( N=8192 \); \( m_{mbh} = 3\% M_{tot} \) no binaries, King model \( W_0 = 11 \); \( N=4096 \)). When a realistic mass spectrum is considered for single stars runs the core collapse is much faster (\( \approx 2 \, t_{wh} \), see [Chernoff & Weinberg] \, (1998)).
Figure 2. Main panel: distribution of the $r_c/r_h$ ratio in the main sample of 57 old galactic globular clusters (selected to have $t_{rh} < 10^6\text{yr}$). Small panel: $\xi \cdot r_c/r_h$ (corrected to rescale it to a reference number of particles $N_{ref} = 3 \cdot 10^5$) for a sub-sample (20 objects) of the main sample, selected by imposing $t_{rh} < 5 \cdot 10^6\text{yr}$ and $e < 0.1$. Both panels show a significant number of clusters with $r_c/r_h \gtrsim 0.2$.

analyzing the distribution of $r_c/r_h$ in a sample of 57 galactic globular clusters, selected to ensure that they are well collisionally relaxed, we conclude that there is a strong indirect evidence for the presence of an IMBH in at least half of the objects in the sample.

A globular cluster that hosts an IMBH represents a laboratory where dynamical interactions between hierarchical systems take place frequently, leading in particular to the formation of triples (and even quadruples) systems and to the ejections of a number of high velocity stars, with some able to reach ejection velocities up to several hundreds km/s (see Trenti et al. 2006b).

Intriguingly, some objects in the sample have a core radius that could possibly be too large ($r_c/r_h \gtrsim 0.5$) to be explained by the presence of a single IMBH. It could therefore be possible that some of these globular clusters host a binary IMBH. In this case, however, the heating may be so efficient (e.g. see Yu & Tremaine 2003 for an estimate of the interaction rate of a binary BH with single stars) that it is not clear whether the system is able to survive for even a few relaxation times. Therefore an accurate numerical modeling is critically required before any conclusion about this speculation can be drawn.

Our conclusions on the presence of singles IMBHs appear instead to be robust. We have discussed a number of possible biases in the comparison of numerical simulations with the observations, but while some of the idealizations introduced in the modeling may induce limited changes in $r_c/r_h$, we could not identify a possible major bias. An error of the order at least 300% would be required to be able to explain the observed distribution of $r_c/r_h$ without invoking the systematic presence of an IMBH in half of the objects in the sample.

Clearly more detailed numerical simulations will be required to evaluate and confirm the presence of IMBH in specific clusters of the sample. In particular it would be extremely interesting the conversion of N-body snapshots into synthetic observations that could therefore be directly analyzed like images acquired by a telescope, avoiding an indirect comparison between theoretical and observed quantities.

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