Measurement of the CKM Angle $\gamma$

M. Whitehead$^1$ on behalf of the LHCb collaboration

$^1$ CERN, CH-1211 Geneva 23, Switzerland
E-mail: mwhitehe@cern.ch

Abstract. A summary of constraints on the CKM angle $\gamma$ from recent LHCb analyses are discussed. The results of a combination of several LHCb measurements of $\gamma$ are presented, giving $\gamma = (70.9^{+7.1}_{-8.5})^\circ$, where the single uncertainty is a combination of statistical and systematic sources. This represents the most precise measurement of the angle $\gamma$.

1. Introduction

Measurements of $CP$ violation are important to understand the matter-antimatter asymmetry of the universe. In the quark sector, $CP$ violation is included in the Standard Model (SM) of particle physics through a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2]. The parameter $\gamma$ is the least precisely measured angle of the CKM unitarity triangle, where the area of the triangle is proportional to the amount of $CP$ violation in the SM [3]. Theoretically $\gamma$ is very clean ($\Delta \gamma / \gamma \approx 10^{-7}$), because it can be measured through tree-level processes only [4]. Discrepancies between the value of $\gamma$ from loop dominated processes and a precise direct measurement would infer the presence of New Physics effects.

Several methods are available to measure $\gamma$ using decays such as $B^\pm \rightarrow D K^\pm$, where $D$ is either a $D^0$ or $\bar{D}^0$ meson. The GLW method [5, 6] considers decays of the $D$ meson to $CP$ eigenstates, such as the $CP$ even final states $K^+K^-$ and $\pi^+\pi^-$. The ADS approach [7, 8] requires favoured and doubly Cabibbo suppressed (DCS) decays, for example $D \rightarrow K^\pm \pi^\mp$. A third method, known as GGSZ [9], uses $D$ decays to self conjugate final states such as $K^0_S\pi^+\pi^-.$

Three recent LHCb analyses are discussed; determining $\gamma$ with $B^\pm \rightarrow D K^\pm$ with $D \rightarrow K^\pm \pi^\mp$, $K^+K^-$, $\pi^+\pi^-$, $\pi^+\pi^-\pi^+\pi^-$ and $K^+\pi^+\pi^-\pi^-$ decays [10] (Sec. 2), measuring $CP$ violation using $B^0 \rightarrow D K^* (892)^0$ decays where $D \rightarrow K^0_S\pi^+\pi^-$ [11] (Sec. 3) and constraining $\gamma$ using $B^0 \rightarrow D K \pi$ decays with $D \rightarrow K^+K^-$ and $D \rightarrow \pi^+\pi^-$ [12] (Sec. 4). The results of a combination of LHCb $\gamma$ measurements is discussed in Sec. 5 [13].

2. Analysis of $B^\pm \rightarrow D K^\pm$ Decays

The GLW/ADS analysis of the decay $B^\pm \rightarrow D K^\pm$, using 2- and 4-body $D$ decays is documented in Ref. [10]. Candidate $B^\pm \rightarrow D \pi^\pm$ decays are used as a high statistic control channel, although results are also given for these channels. Data samples are selected using multivariate analyses to remove background candidates.

Figure 1 shows the $B$ candidate invariant mass distribution for the 2-body DCS $D$ meson decays, where $CP$ violation is clearly visible by eye in the $B^\pm \rightarrow D K^\pm$ channel, and has a
Figure 1. Invariant mass distribution of $B^\pm \to D K^\pm$ decays with the DCS decay $D \to \pi^\pm K^\mp$. Figure taken from Ref. [10], where the fit model is described.

Figure 2. Invariant mass distribution of $B^\pm \to D K^\pm$ decays with the DCS decay $D \to \pi^\pm K^\mp \pi^\pm \pi^-$. Figure taken from Ref. [10], where the fit model is described.

significance of about $8\sigma$. Similarly, Fig. 2 shows similar distributions for the 4-body DCS decays of the $D$ meson. In this case the $CP$ violation effect is visible, but not significant, with the current statistics. In addition, $CP$ violation is observed at the $5\sigma$ level using the $D \to K^+ K^-$ and $D \to \pi^+ \pi^-$ modes combined. The 4-body decay $\pi^+ \pi^- \pi^+ \pi^-$ is also studied for the first time. There are 22 observables that are determined from the yields of the $B^+$ and $B^-$ decays for each $D$ decay final state, which are used as an input to the $\gamma$ combination described in Sec. 5.
A model dependent GGSZ analysis of 

is not discussed in the following. Candidates are selected using a boosted decision tree to
detailed in Ref. [11]. Note that a model independent analysis has also been performed [14],

Figure taken from Ref. [11], where the fit model is described. The vertical lines show the signal mass window and components are as described in the legend. Figure 3.

Invariant mass distribution of

Figure 3. Invariant mass distribution of \( B^0 \to DK^*(892)^0 \) decays where \( D \to K_S^0\pi^+\pi^- \), the vertical lines show the signal mass window and components are as described in the legend. Figure taken from Ref. [11], where the fit model is described.

3. Study of \( B^0 \to DK^*(892)^0 \) Decays

A model dependent GGSZ analysis of \( B^0 \to DK^* \) decays with \( D \to K_S^0\pi^+\pi^- \) by LHCb is detailed in Ref. [11]. Note that a model independent analysis has also been performed [14], but is not discussed in the following. Candidates are selected using a boosted decision tree to reject background candidates. A fit is performed to the \( B \) candidate invariant mass, as shown in Fig. 3, to obtain the signal and background yields in the mass window \( \pm 25 \text{MeV}/c^2 \) around the \( B^0 \) mass.

For candidates in the signal mass window, an amplitude fit is performed to the \( D \to K_S^0\pi^+\pi^- \)
Dalitz plot distribution for \( B^0 \) and \( \bar{B}^0 \) candidates. The amplitude model is taken from a BaBar analysis, described in Ref. [15]. The observables \( x_\pm \equiv r_B \cos(\delta_B \pm \gamma) \) and \( y_\pm \equiv r_B \sin(\delta_B \pm \gamma) \) are determined to be

\[
\begin{align*}
x_- &= -0.15 \pm 0.14 \pm 0.03 \pm 0.01, \\
y_- &= 0.25 \pm 0.15 \pm 0.06 \pm 0.01, \\
x_+ &= 0.05 \pm 0.24 \pm 0.04 \pm 0.01, \\
y_+ &= -0.65^{+0.24}_{-0.23} \pm 0.08 \pm 0.01.
\end{align*}
\]

Here the first uncertainty is statistical, the second experimental systematic and the third model dependent systematic. The hadronic parameters \( r_B \) and \( \delta_B \) are the ratio and strong phase difference between the favoured and suppressed \( B \) decay amplitudes. All observables are consistent with zero at roughly 2\( \sigma \), and are therefore consistent with \( CP \) conservation.

4. Dalitz Plot Analysis of \( B^0 \to DK^+\pi^- \) Decays

Reference [12] describes the GLW-Dalitz analysis of the \( B^0 \to DK^+\pi^- \) channel where \( D \to K^+K^- \) and \( D \to \pi^+\pi^- \). In addition, the favoured \( D \to K\pi \) decay is used as a high statistics control channel. Signal candidates are separated from backgrounds using an artificial neural network, and the data are binned in the output variable of the network. This approach preserves the maximum signal yield without sacrificing purity in each bin. Projections of the fits to the \( B \) candidate invariant mass distributions are shown in Fig. 5 for the three \( D \) decay modes.

Signal mass windows are defined to be 5246.6–5309.9\( \text{MeV}/c^2 \), 5246.9–5310.5\( \text{MeV}/c^2 \) and 5243.1–5312.3\( \text{MeV}/c^2 \) for \( D \to K^+\pi^- \), \( K^+K^- \) and \( \pi^+\pi^- \) samples, respectively. For candidates in the signal windows, a simultaneous Dalitz plot fit is performed to the three \( D \) decay samples, following the method described in Refs. [16, 17]. Terms for \( CP \) violation are included in the
\textbf{Figure 4.} Dalitz plot (a) and projections of the amplitude fit on to the invariant mass squares (b, c and d) for $D \to K^0_S \pi^+ \pi^-$ decays from the $B^0 \to DK^*(892)^0$ sample. The labels $m_+$, $m_-$ and $m_0$ represent $m(K^0_S \pi^+)$, $m(K^0_S \pi^-)$ and $m(\pi^+ \pi^-)$ respectively. Figure taken from Ref. [11], where the amplitude model is described.

$K^*(892)^0$ amplitudes for the $CP$ even $D \to K^+ K^-$ and $D \to \pi^+ \pi^-$ modes. The observables determined from the amplitude fit, as defined above, are found to be

\begin{align*}
x_- &= -0.02 \pm 0.13 \pm 0.14, \\
y_- &= -0.35 \pm 0.26 \pm 0.41, \\
x_+ &= \quad 0.04 \pm 0.16 \pm 0.11, \\
y_+ &= -0.47 \pm 0.28 \pm 0.22,
\end{align*}

where the first uncertainty is statistical and the second systematic. These results are consistent with no $CP$ violation.

\section{5. Combination of LHCb $\gamma$ Measurements}

The combination of several LHCb measurements using a frequentist approach is described in Ref. [13], which is an update of the previous combination [18], including the new results described in Secs. 2–4. The combination takes input from $B \to DK$-like decay modes only, using a total of 71 observables from 10 LHCb analyses. The result of the one dimensional PLUGIN scan [19] for $\gamma$ is shown in Fig. 7 (left), the (right) plot shows the breakdown of the result in terms of the different species of $B$ mesons. It is clear that the $B^{\pm}$ decay modes currently dominate the sensitivity, though the other contributions are non-negligible. The confidence intervals $\gamma \in [62.4, 78.0]^\circ$ at 68\% CL and $\gamma \in [62.4, 78.0]^\circ$ at 95\% CL are set and $\gamma$ is measured to be

$$\gamma = (70.9^{+7.1}_{-8.5})^\circ,$$

where the uncertainty includes both statistical and systematic effects. This is currently the most precise measurement of $\gamma$, improving on the previous LHCb combination by approximately two degrees.
Figure 5. Invariant mass distribution of $B^0 \rightarrow DK^+\pi^-$ decays with (a) $D \rightarrow K^+\pi^-$, (b) $D \rightarrow K^+K^-$ and (c) $D \rightarrow \pi^+\pi^-$. The fit components are as described in the legend and the figure is from Ref. [12].

Figure 6. Projections of the Dalitz plot fit on $m(K\pi)$ for (a) $\bar{B}^0$ and (b) $B^0$ candidates. The fit components are as described in the legend. Figure taken from Ref. [12].
Figure 7. (Left) 1-CL curve for the angle $\gamma$ obtained using the PLUGIN method, with the central value (solid vertical line), 1$\sigma$ uncertainties (dashed vertical lines), and 68.3% and 95.5% CLs (horizontal dotted lines) shown. (Right) a breakdown to show the contributions of the different $B$ meson species: $B^+$ (blue), $B^0$ (yellow), $B_s^0$ (orange) and the full combination (green). Figure taken from Ref. [13].

6. Summary
The latest measurements of the CKM angle $\gamma$ from LHCb are presented. The combination of results is consistent with the expected LHC Run 1 sensitivity. To improve the precision of the measurement, new decay modes can be added and updates can be included to existing measurements using Run 1 data. For example, information from $B \rightarrow D\pi$-like decays can be included. In the longer term the full Run 2 data sample will provide more than double the current available statistics, with an aim to reduce the uncertainties to around 4\degree.

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