Theoretical Approach for the Calculation of the Pressure Drop in a Multibranch Horizontal Well with Variable Mass Transfer

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ABSTRACT: In this study, the pressure drop obtained from physical experiments and theoretical approaches of a single horizontal wellbore is reviewed and a comprehensive wellbore pressure-drop model is derived for a multibranch well. We propose a new coupling model for fluid flow in multibranch wells and reservoirs. Based on this coupling model, we introduce a theoretical approach for the calculation of the pressure drop in a multibranch horizontal well with variable mass transfer. To facilitate the understanding of the physical model, the entire coupling model was divided into three parts: (1) the pressure-drop model of the wellbore, (2) the reservoir inflow model, and (3) the coupling model. By incorporating the acceleration, friction, mixing, and gravity pressure drops, a coupling model with a finite-conductivity multibranch horizontal well was developed. Newton—Raphson iterations and Visual Basic programming were employed to solve the coupling model and to obtain the pressure and inflow rate of the wellbore. The wellbore pressure-drop model was verified by comparing it with different models for the same case study, which has been previously introduced in a different research work. Furthermore, the forecast and sensitivity analysis were conducted, and then the results are discussed. In the proposed new model, several factors are considered, including the wellbore structure, the wellbore completion method, the wellbore, and the fluids and formation properties. The presented approach can be used as a valuable tool to analyze the influence of the pressure drop on the productivity of complex-structured wells and vice versa, and to quantitatively investigate the various pressure drops in wellbores, including the friction, acceleration, mixing, confluence, and gravity pressure losses.

1. INTRODUCTION

Currently, a series of complex production wells are applied to certain challenging reservoirs to maximize oil recovery.1−6 With the development of drilling techniques, multibranch horizontal well technology can improve well productivity by expanding reservoir drainage.7−11 Moreover, the application of this technology can prevent the transportation of large volumes of fluid along a single long horizontal borehole, which would result in pressure losses in the wellbore and would cause a decrease in well productivity.12−16 Therefore, branch wells with complex structures can have a higher production rate with a lower production pressure drop. More specifically, the application of multibranch horizontal wells to bottom water reservoirs appears to be promising in minimizing water cresting or coning.17,22 Depending on whether the radial inflow on the wellbore exists or not, the flow of the wellbore is divided into two types: the variable mass flow and the constant mass flow. The wellbore type of constant mass flow in common pipes was first applied in the field of engineering fluid mechanics, e.g., water head loss caused by frictional resistance in water conveyance pipelines. Since Dikken23 introduced the theory of variable mass flow in the wellbore and reservoir seepage flow to a horizontal well, extensive experimental,24−28 numerical simulation,29,30 and theoretical31−34 studies have been conducted on the flow pressure drop in single horizontal wellbores. Asheim et al.27 investigated the friction correlation for a horizontal wellbore, which included the acceleration pressure drop caused by the continuous fluid radial inflow along the wellbore. Su and Gudmundsson35 conducted research based on experiments that were conducted on a perforated pipe geometrically similar to a wellbore casing (12 SPF, 60 phasing). Their results revealed that under the conditions of the experimental environment, the total pressure drop between the case of single head loss and the case of multiple head losses is not very significant.
drop was induced by the wall friction and the mixing effects, namely, the perforation roughness and the acceleration, by 80, 15, and 5%, respectively. In the model developed by Ouyang,36 the total pressure drop consisted of the pressure losses induced by the friction, acceleration, gravitation, and radial inflow. In this model, each pressure drop was quantitatively analyzed in detail. The empirical formula of the model of Wang26 is not available when the inflow ratio is outside a certain value range. Among the aforementioned models, the models of Ouyang and Wang are commonly utilized as reservoir/wellbore-coupling flow models. However, the aforementioned models are focused on single horizontal wells; hence, wellbore pressure drops for multibranch horizontal wells require further investigation. Moreover, for the pressure drop in a multibranch horizontal wellbore, both the coupling effect of the wellbore flow and the inflow from the reservoir formation and the mutual interference of the branch and the main wellbore should be considered. Therefore, the variable mass flow in a multibranch wellbore is more complex than that in a single horizontal well, as summarized in the following three points.

(1) In physical experiments in pipelines, the radial inflow rate of the perforation on the pipe wall is typically set as a constant. However, in the coupling model, the radial inflow rate from the reservoir will affect the pressure distribution and the pressure drop in the wellbore. Consequently, the pressure distribution in the wellbore will affect the radial inflow rate from the reservoir. In fact, the radial inflow from the reservoir and the pressure drop along the wellbore are coupled with one another; therefore, the types of variable mass flow are significantly different from the experiments on constant radial inflow rates.57

(2) The coupling model of multibranch wells shows whether the main wellbore is produced or not, as well as the different completion methods of the main and the branch wellbores. Therefore, the actual wellbore pressure-drop model may involve the combination of the conventional constant mass flow without radial inflow and the variable mass flow with different completion modes, such as the open hole, perforation, and slot. Thus, the pressure-drop calculation may involve different types of pressure-drop models.

(3) In the coupling model of multibranch horizontal wells, apart from the common pressure drop in the wellbore, the fluid from the branch wellbore will flow into the main wellbore and cause the confluence pressure drop.

The accuracy of the wellbore pressure-drop model for multibranch horizontal wells has a significant impact on well productivity and production performance. However, based on the aforementioned three points, it is still difficult to directly conduct physical experiments on the variable mass flow on the multibranch horizontal wellbore. In this paper, a systematic analysis of the physical experiments and the theoretical calculation methods of the pressure drop in a single horizontal wellbore available in the literature will be presented; moreover, a comprehensive wellbore pressure-drop model suitable for multibranch horizontal wells will be derived. Then, we will propose a new coupling model for fluid flow in multibranch wells and reservoirs. Based on this coupling model, we will introduce the theoretical approach for the calculation of the pressure drop in a multibranch horizontal well with a variable mass transfer.

Moreover, a case study will be presented to illustrate the forecast and the sensitivity analysis, as well as to discuss the results of the proposed coupling model.

2. METHODOLOGY

The present work is organized as follows: (1) description of the coupling model, including the conceptual model, assumptions, definitions, mathematical model, and solutions;18−23 (2) verification of the proposed coupling model by comparing it with different models using the same case as in;36,57 and (3) the study of the impact of the parameters related to pressure losses on the pressure performance of the coupling model.55,33

Figure 1. Schematic diagram of the completion method of the multibranch wellbore.
3. MODEL DEVELOPMENT

3.1. Conceptual Model and Assumptions. Figure 1 shows the completion of a four-branch horizontal well. We assume that the main wellbore of the model is partially perforated and that certain sections are not connected with the reservoir. The branch wells with open-hole or perforation completion are subdivided into full perforations (branch L2) and partial perforations (branch L1). The completion methods of this well vary and can be combined with different existing methods. The microsegment method is commonly used for coupling models, particularly for the case of a variant flow in the wellbore. It can be employed to divide the entire wellbore into N microsegments to calculate the pressure drop and the radial inflow of the wellbore; each segment can be treated as a single horizontal well. The microsegment method includes several factors, including different completion methods, the convergence pressure drop in the branch flow to the main wellbore, the coupling influence of the reservoir, and the wellbore flow. Based on the microsegment method, the pressure drops of microsegments at different wellbore positions can be divided into the following three basic-unit wellbore models.

3.2. Pressure-Drop Model of Wellbore 1. Wellbore 1 is the casing without perforation, as shown in Figure 1. No radial inflow exists in the microwellbore segment; therefore, the pressure drop along this segment can be calculated using the constant mass flow model of a conventional pipe. Only the friction resistance factor is considered for the wellbore pressure drop unless the horizontal pipe is tilted; in this case, the gravity loss should be considered. The key to calculating the friction resistance is the determination of the wall friction coefficient. According to the flow regime of the fluid in the pipe, the formula for the calculation of the friction coefficient is distinguished between that of the laminar flow and that of the turbulent flow. The calculation method for the wall friction coefficient of a laminar flow is shown in eq 1.

\[ f_0 = \frac{16.0}{N_{Re}}, \quad N_{Re} \leq 2000 \]  

The Colebrook-White equation yields the friction coefficient of the turbulent flow in a pipe, as shown in eq 2.

\[ f_0^{-0.5} = \left( -4.0 \log \left( \frac{e}{3.7D} + \frac{1.255}{f_0^{0.5} N_{Re}} \right) \right), \quad N_{Re} > 2000 \]  

Equation 2 is an implicit function of \( f_0 \); an iterative algorithm method, such as the Newton-Raphson, the secant method, or the parabola method, can be utilized to solve this equation. Moreover, various explicit formulas exist that can be used to solve the Colebrook-White equation. Then, the pressure drop of the main wellbore can be obtained using eq 3.

\[ \Delta p_{wall,0,i} = \frac{8f_0}{\pi^2 d_i^4} q_{w,i}^2 \Delta L_i \]  

3.3. Pressure-Drop Model of Wellbore 2. 3.3.1. Composition of Pressure Drop. The microsegment of Wellbore 2 with radial inflow by perforation or open-hole completion is shown in Figure 1. If the wellbore is completed by an open hole, perforation, or slotting, radial inflow will exist in this microsegment. Therefore, the variable mass flow pressure-drop model coupled with the reservoir flow should be adopted to calculate the pressure drop. The detailed derivation of the variable mass flow mode is shown in Appendix 1. It illustrates that the calculation of the frictional pressure drop in the wellbore inflow is obviously different from the case of the constant mass flow, where only the friction pressure loss is considered. It may be observed that the pressure drop in the variable mass flow in the wellbore model is composed of four parts, as shown in eq 4, namely, the frictional pressure drop, the acceleration pressure drop, the mixing pressure drop caused by the radial inflow, and the gravity pressure drop.

\[ \Delta p = \Delta p_{acc} + \Delta p_{wall,0} + \Delta p_{mix} + \Delta p_g \]  

The acceleration pressure drop generated by the radial inflow from the reservoir can be calculated using eq 5.

\[ \Delta p_{acc} = \frac{32\rho q_{w,i}^2}{\pi^3 d_i^4} \Delta L_i \]  

In eq 4, \( \Delta p_{wall,0} \) is the frictional pressure drop between the fluid and the wall of the wellbore, ignoring the radial inflow. \( \Delta p_{mix} \) is the mixing pressure drop caused by the intermixing of the inflow fluid in the radial direction with the wellbore flow in the main direction. Based on Appendix 1, \( \Delta p_{wall,0} \) and \( \Delta p_{mix} \) can be calculated together as \( \Delta p_{wall,0} \) by replacing \( f_0 \) by \( f_i \) in eq 3. The apparent friction coefficient with radial inflow, \( f_v \), can be calculated as per refs 36, 45, and 46 for open-hole completion and as per the expressions presented in our previous studies for perforation wellbores, which was offered by Wang’s work as shown in eq 6.

\[ \Delta p_{mix} = \begin{cases} 4.0703 \Delta p_{acc} / R, & 0 < R \leq 2 \\ 6.3371 \Delta p_{acc} / R, & 2 < R \leq 20 \\ 10.7520 \Delta p_{acc} / R, & R = 20 \end{cases} \]  

where \( R \) is the ratio of the main flow rate to the radial inflow rate.

The fourth term on the right-hand side of eq 4 is the gravity pressure drop, which is generated by the inclined and noncurved well trajectory, and can be calculated using eq 7.

\[ \Delta p_g = \rho g \cos \gamma \Delta L_i \]  

Based on the aforementioned four types of pressure drop, the total pressure drop generated by the segment with radial inflow can be calculated using eq 8.

\[ \Delta p_i = p_{w,i} - p_{w,i-1} = \left( \frac{32\rho q_{w,i}^2}{\pi^3 d_i^4} + \frac{8f_i}{\pi^2 d_i^4} q_{w,i}^2 + \rho g \cos \gamma \right) \Delta L_i \]  

The flow rate of the cross-section at different positions of the wellbore can be obtained using eq 9.

\[ q_{w,i} = \sum_{j=1}^{N} q_{w,j} \Delta L_j \]  

3.3.2. Apparent Friction Coefficient. The calculation method of the friction coefficient with radial inflow is different from the laminar flow, with different completion methods.
formula\(^{36}\) to calculate the apparent friction coefficient for a laminar flow regime, as shown in eq 10.

\[
f_f = f_0 \left( 1 + 0.04304N_{Re_w}^{0.6142} \right)
\]  

(10)

For the turbulent flow, based on the experimental tests of Olson and Eckert, the apparent flow friction coefficient for a turbulent flow\(^{36,45,46}\) can be obtained through regression, as shown in eq 11.

\[
f_f = f_0 \left( 1 - 0.0153N_{Re_w}^{0.3978} \right)
\]  

(11)

3.3.2.2. Apparent Friction Coefficient of Perforation Completion. Based on the experimental data of Yuan\(^{37,48}\) and Jiang, we developed a new correlation (eq 12) to predict the apparent friction factor of a horizontal wellbore with different completion shot densities, phasing, main flow Reynolds numbers, and radial inflow rates.\(^{37}\)

\[
f_f(\theta_f, \varphi, d, N_{Re}) = \left[ (0.0035q_0 + 1.866)\varphi + \frac{1300}{q_0} - 20 \right] \frac{2d_{max}}{q} + f(N_{Re})
\]  

(12)

The new regression equation, eq 12, has been applied to analyze the example of Ouyang, which is a real horizontal well in an oil reservoir.\(^{36}\) Figure 2 shows the results of different models for the same single horizontal well. The model presented in this paper was derived from the experimental data presented in ref \(^{47–50}\). The derivation process of this model was different from that of Asheim. However, the results were almost the same when both models were applied to the practical example of Ouyang. The results of the model presented in this paper lie in-between those obtained from other models and presented a good agreement with the measured data, as shown in Figure 2 (Table 1).

![Figure 2. Comparison of the cumulative pressure drop using different models for a single horizontal well.](https://dx.doi.org/10.1021/acsomega.0c03971)

3.4. Pressure-Drop Model of Wellbore 3. Wellbore 3 is the main wellbore segment that has confuence flow from a branch wellbore, as shown in Figure 1. In this model, it is assumed that the fluid flow in the wellbore is one-dimensional and no mass transfer is considered. The branch confuence flow is isothermal and is at a steady state. Based on the study of Xu\(^{30}\) on the mixing loss of perforated borehole inflow, Yang and Schulte\(^{31,52}\) calculated the pressure drop in the confuence that had been caused by the confuence flow from a branch wellbore. This model can be used in the case where the diameter of the branch is not equal to that of the main wellbore. The streamline of model 3 is shown in Figure 1;

| no. | \(\theta_f\) (deg) | \(\varphi\) (1/m) | \(f(N_{Re})\) |
|-----|-----------------|-----------------|--------------|
| 1   | 90              | 16.4            | 0.873N_{Re}^{0.335} |
| 2   | 180             | 16.4            | 0.622N_{Re}^{0.295} |
| 3   | 360             | 16.4            | 0.651N_{Re}^{0.314} |
| 4   | 90              | 32.8            | 0.160N_{Re}^{0.170} |
| 5   | 180             | 32.8            | 0.360N_{Re}^{0.206} |
| 6   | 360             | 32.8            | 0.755N_{Re}^{0.200} |
| 7   | 90              | 65.6            | 1.471N_{Re}^{0.413} |
| 8   | 180             | 65.6            | 0.452N_{Re}^{0.270} |
| 9   | 360             | 65.6            | 1.078N_{Re}^{0.310} |

the momentum equation can be used to obtain the momentum change of control Unit I, eq 13.

\[
F_x = \rho q v_p \cos \theta = \rho q \left( A_3 \right) \cos \theta
\]  

(13)

where control Unit I is the heel point of the branch wellbore.

Based on a force which acts on an object is opposed by an equal and opposite reaction, Unit II on the main wellbore are reacted the force, \(F_x = -F_x\). Moreover, control Unit II is coated by \(p_1, p_2, p_3,\) and \(F_x\). Based on the momentum conservation of control Unit II, eq 14 can be obtained.

\[
p_1A - p_2A - F_x = \rho q_4 v_4 - \rho q_1 v_1
\]  

(14)

where control Unit II is the confuence point of the main wellbore.

According to the continuity equation, the relationship between \(v_1\) and \(v_3\) can be expressed using eq 15.

\[
Av_1 + q = Av_3
\]  

(15)

According to the energy conservation principle, the relationship between the upstream pressure and the downstream pressure can be expressed as eq 16.

\[
\frac{P_1 - P_4}{\rho g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_{14}
\]  

(16)

Combining eqs 13 with 16 and ignoring the height difference in the horizontal wellbore, the pressure drop can be obtained using eq 17.

\[
p_1 - P_4 = \frac{\rho}{A} (Q_4 v_4 - Q_1 v_1 - q v_p \cos \theta)
\]  

(17)

According to the wellbore pressure-drop model, the pressure-drop vector for each microsegment of the multibranch horizontal well can be expressed as eq 18.

\[
\Delta P = [\Delta P_{11}, \Delta P_{12}, ..., \Delta P_{31}, ..., \Delta P_{i1}, ..., \Delta P_{mn}]^T
\]  

(18)

Based on the principle of continuous pressure, the pressure vector for each microsegment of the borehole can be expressed as eq 19.

\[
P = [p_{11}, p_{12}, ..., p_{13}, ..., p_{mn}]^T
\]  

(19)

3.5. Reservoir Inflow Model. Considering a bottom water reservoir as a study case, Figure 1 shows the schematic diagram of a complex branch horizontal well. The branches of the horizontal well are randomly distributed in three-dimensional space. The complex structure with any number of branches, arbitrary configurations, and orientations in three-dimensional...
3.6 Model Coupling and Solution. By substituting the pressure equation, eq 19, into the reservoir inflow equation, eq 25, the coupled nonlinear equations can be obtained. Regarding the aforementioned coupling model of nonlinear equations, the number of equations that compose the coupled equations is equal to the number of microsegments. The unknowns are the flow rates and the pressures of each microsegment. The Gaussian iteration can be employed to solve this problem by means of computer programming using a Visual Basic code. The flow vector, $Q$, and the pressure vector, $P$, of the microsegment are solved to obtain the inflow and the corresponding wellbore pressure of the microsection of the horizontal well. Therefore, the flow profile, the pressure-drop profile, the cumulative flow, and the cumulative pressure-drop
profile of the horizontal wellbore can be obtained. Figure 3 illustrates the workflow and the method to obtain a solution from the coupling model.

3.7. Forecast and Sensitivity Analysis. A case study will be presented to illustrate the forecast and the sensitivity analysis, as well as to discuss the result of the proposed coupling model. A schematic of the subdivision of the branch well and of the numbering of the microsegments is shown in Figure 4. The reservoir parameters, the fluid properties, and the wellbore parameters are listed in Table 2.

According to the previously described coupling model, by setting a pressure difference of 1 MPa between the bottom constant-pressure boundary and the heel point of the main wellbore, the pressure drop and the radial inflow rate of each microsegment of the wellbore can be obtained via the proposed coupling model, as shown in Figures 5 and 6, respectively. As illustrated, at the confluence of the four main wellbore segments (14, 28, 42, and 58), the inflow rates of these microsegments decrease owing to the convergence pressure drop, which was caused by the branch inflow and could not be ignored. Figure 7 shows the Reynolds number and the cumulative pressure drop; as may be observed, from the toe to the heel of the main wellbore, the cumulative flow rate increase results in a quicker cumulative pressure drop, where the final cumulative pressure drop in the main horizontal wellbore is 160 kPa.

If the total length of the wellbore is known, the variables that may affect the pressure drop in the multibranch wellbore...
include the wellbore structure, the completion methods and parameters, and the properties of the rock and the fluids. To identify the degree to which each factor affects the prediction result and the applicable scope of the model, a sensitivity analysis was conducted for certain factors. The influence of the wellbore pressure drop in a branch well with a complex structure is shown in Figures 8–11.

When the branch angle changed from 15 to 90°, the accumulated pressure-drop profiles and the productivity of the aforementioned branch horizontal wells were obtained and are shown in Figures 8 and 9, respectively. Figure 9 shows that when the branch angle is 15°, the radial inflow of each segment is the lowest owing to the fact that the pay zone controlled by the well is the smallest and that the mutual interference for each wellbore segment is the highest; therefore, the value of the cumulative pressure drop in the main wellbore is the lowest. When the branch angle is 90 and 45°, the total inflow (productivity) does not significantly differ between the two cases; therefore, no obvious difference exists in the cumulative pressure drop between these two cases, as shown in Figure 8.

The wellbore pressure drop is mainly affected by the flow regime in the wellbore. The pressure drop in the turbulent regime is obviously higher than that in the laminar regime. A different mobility ratio and a different borehole diameter were selected for the analysis of the effect of the wellbore pressure drop and productivity. As shown in Figure 10, the lower the mobility ratio is, the higher the inflow rate from the reservoir, the faster the flow velocity in the wellbore, and the greater the accumulated pressure drop. Correspondingly, the difference in the productivity caused by the wellbore pressure drop becomes greater.

The cumulative pressure drop and the productivity of the multibranch horizontal well are shown in Figure 11; depending on the well diameter, the pressure drop may either be considered or not considered. The smaller the wellbore diameter, the greater the fluid resistance in the wellbore and the greater the cumulative pressure drop, consequently the greater the decrease in the productivity caused by the wellbore pressure drop. For the case where the inner diameter of the wellbore is 0.05 m, the productivity for which the pressure drop is considered is approximately 85% of the productivity in the case where the pressure drop is not considered.

Figures 10 and 11 show the differences between the analytical model (based on the potential and the ignored pressure drop) and the coupling model (based on the discrete flow of the microsegment). It may be concluded that the pressure drop has a significant influence on the well
productivity when the well has a small diameter and the reservoir has a high mobility ratio.

4. RESULTS AND DISCUSSION

We set the fluid viscosity at 15 mPa·s (mobility ratio of 10) and the inner diameter of the wellbore at 0.05 m to discuss the composition of the pressure drop in wellbores. As shown in Figure 12, the Reynolds number increases from 0 at the toe to 5500 at the heel of the main wellbore. At each branch confluence point, the Reynolds number jumps to the next level because of the inflow from the previous branch wellbore; the friction pressure drop, which is related to the Reynolds number, increases as well. Regarding the pressure-drop composition in the microsegment, which is not at the confluence, the friction pressure drop accounts for most of the total pressure drop; it is followed by the acceleration pressure drop and the mixing pressure drop. The frictional, acceleration, mixing, and confluence pressure drop in the main wellbore is 93.3, 4.9, 1.2, and 0.6% of the total cumulative pressure drop, respectively.

Figure 12. Composition of the pressure drop and the Reynolds number in the main wellbore microsegment.

Figure 13 demonstrates that the pressure drop in each branch is almost the same and that the friction pressure drop is the main pressure loss. However, the proportion of the friction pressure in the total friction pressure is obviously smaller than that in the total friction pressure of the main wellbore. The Reynolds number gradually increases from the toe to the heel of each branch wellbore, and all types of pressure drops gradually increase. However, the interference of the main wellbore causes a decrease in the radial inflow rate; hence, all types of pressure drops in the first microsegment of each branch decrease.

The microsegment number influences the result. We investigated the influence of the microsegment number (length of microsegment) and compared it with the reservoir numerical simulation results. We used the ECLIPSE 100 to simulate the cases of microsegments of 15, 75, and 150. Taking the 75 microsegment numerical model as an example, the reservoir model is shown in Figure 14. The grid system is 35 × 15 × 33. The block size, DX = 10 m, DY = 10 m, the DZ in the oil zone (z < 33) is 1 m, in the water zone (z = 33) is 10 m. Therefore, the reservoir model size is similar to the data given in Table 2. We also attached a big enough Fetkovich Aquifer (volume is 10^{12} m^{3}) under the water cell to simulate the constant pressure of bottom water. The four-branch horizontal well in the grid model is shown in Figure 15.

The simulation results and the theoretical results are shown in Figure 16. The numerical simulation of ECLIPSE 100 only considers the frictional pressure loss and uses the Fanning friction factor to calculate the frictional pressure. Based on the Reynolds number Re, in the "uncertain region" (2000 < Re < 4000), E100 uses a linear interpolation between the values at Re = 2000 and 4000.53 Our model not only considers the frictional pressure loss but also takes other kinds of pressure losses into account. Thus, our results have bigger pressure losses than the simulation results and the productivity is smaller than that of simulation results. Compared with the reservoir numerical simulation, the coupling model proposed in the present work can calculate the pressure and flow profile of the wellbore efficiently (Figure 16) and also show that the productivity and the pressure drop affected by the microsegment number can be ignored when the segment number is above 70 for the case presented in this paper. The case study shows that the accuracy of the calculation results is reasonable and reliable. The theoretical method can conveniently and rapidly evaluate the parameters that influence the multibranch horizontal wells (main and branch wellbore structure, tubing size, and perforation parameters), as well as the influence of the reservoir parameters on the pressure drop and the productivity. It can serve as a preliminary guide to optimize the design of multibranch well structures and of tubing parameters in actual applications.

Figure 14. Reservoir model and the Grid system.
5. CONCLUSIONS

(1) The radial inflow from the reservoir and the pressure drop along the wellbore was coupled with one another. In this work, we systematically analyzed physical experiments on the pressure drop, as well as the theoretical calculation methods for a single horizontal wellbore. Moreover, we derived a comprehensive wellbore pressure-drop model, which was suitable for multibranch horizontal wells of complex wellbore structures and of different completion types.

(2) In the proposed model, several factors may be considered, including the wellbore structure, the wellbore completion method, and the properties of fluids and formations, which significantly affected the wellbore pressure drop. In addition, this method can be utilized for the case in which the complex structure well has multibranch and complex structural branches in irregular distributions with different completion methods.

(3) The theoretical method can be used conveniently and rapidly to evaluate the influence of the parameters of the multibranch horizontal wells and the reservoir on the pressure drop and productivity. It can be used to analyze the influence of the pressure drop on the productivity of the complex-structured wells and vice versa, and to quantitatively investigate the composition of various wellbore pressure drops, namely, the friction, acceleration, mixing, confluence, and gravity pressure losses. It may serve as a preliminary guide for the optimization of the design of multibranch well structures and of tubing parameters.

■ PRESSURE-DROP MODEL OF VARIABLE MASS FLOW IN MICROSEGMENT

17 shows the flow element control unit for a horizontal wellbore at a certain inclination angle. Assume that the fluid in the control unit is an incompressible fluid, the one-dimensional single-phase flow, constant temperature of the system and there is no energy exchange between the fluid and the pipe wall.

Mass Conservation Equation

The microsegment is selected in 17 as the research object. According to the law of mass conservation, the increase of the mass in the body is equal to the difference between the mass of inflow and outflow, as is shown in eq AA1.

\[
\int_{CS} \rho(u_n) \, dA + \frac{\partial}{\partial t} \int_{CV} \rho \, dV = 0
\]  

(A1)

where \( V \) is the volume of the microsegment, \( m^3 \), and \( S \) is the circumference of the inner cross-section of the wellbore, \( m \).

According to 17, eq A1 can be rebuilt as eq A2.

\[
\rho_2 A_2 U_2 - \rho_1 A_1 U_1 - \Delta x \rho q + \frac{\partial}{\partial t} (\rho \bar{A} \cdot \Delta x) = 0
\]

(A2)

where \( \bar{A} \) is the average cross-section area of the microsegment.
Let $\Delta x \to 0$, then, $\bar{A} \to A$, and eq AA2 can be written in the differential form as eq AA3.

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho A U) = \rho q_i$$  \hspace{1cm} \text{(A3)}$$

**Momentum Conservation Equation**

According to the law of momentum conservation, the resultant force on the control unit is equal to the difference momentum between the inflow and outflow of the control unit. It can be expressed as eq AA4.

$$\int_{CS} \rho(u \cdot n) \, dA + \frac{\partial}{\partial t} \int_{CV} \rho \, dV = \sum F$$  \hspace{1cm} \text{(A4)}$$

The momentum correction factor $\beta$ is used to correct the uneven flow in the wellbore section. For turbulent flow, $\beta$ can take a value of 1.0, then the first term on the left of eq AA4 can be written as eq AA5.

$$\int_{CS} \rho(u \cdot n) \, dA = \frac{1}{\beta} \rho A U_2^2 - \frac{1}{\beta} \rho A U_1^2 \approx \rho A U_2^2 - \rho A U_1^2$$  \hspace{1cm} \text{(A5)}$$

The momentum change in the controlled body is shown in eq AA6.

$$\frac{\partial}{\partial t} \int_{CV} \rho \, dV = \frac{\partial}{\partial t} (\rho A U \Delta x)$$  \hspace{1cm} \text{(A6)}$$

The force received by the control unit is shown in eq AA7.

$$\sum F = p A_1 - p A_2 - \rho g A \Delta x \sin \theta_g - \tau_w S \Delta x$$  \hspace{1cm} \text{(A7)}$$

where $\tau_w = \frac{6u^2}{2}$ is the shear stress on the wall, N/m².

**Pressure-Drop Model of Variable Mass Flow in the Wellbore**

Substituting eqs AA5, AA6, and AA7 into eq AA4, eq AA8 can be obtained.

$$p A_1 - p A_2 - \rho g A \Delta x \sin \theta_g - \tau_w S \Delta x$$

$$= \rho A U_2^2 - \rho A U_1^2 + \frac{\partial}{\partial t} (\rho A U \Delta x)$$  \hspace{1cm} \text{(A8)}$$

Let $\Delta x \to 0$, $\bar{A} \to A$, then eq AA8 can be written as eq AA9.

$$- \frac{\partial (\rho A)}{\partial x} - \rho g A \sin \theta_g - \tau_w S = \frac{\partial (\rho A U)}{\partial t} + \frac{\partial (\rho A U^2)}{\partial x}$$  \hspace{1cm} \text{(A9)}$$

Conduct partial derivatives of the second item on the right of eq AA9.

$$- \frac{\partial (\rho A)}{\partial x} - \rho g A \sin \theta_g - \tau_w S$$

$$= \frac{\partial (\rho A U)}{\partial t} + U \frac{\partial (\rho A U)}{\partial x} + \rho A \frac{\partial U}{\partial x}$$  \hspace{1cm} \text{(A10)}$$

Substituting eq AA3 into eq AA10, eq AA11 can be obtained.

$$- \frac{\partial (\rho A)}{\partial x} - \rho g A \sin \theta - \tau_w S = \rho q_i U + \rho A \left( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right)$$  \hspace{1cm} \text{(A11)}$$

For steady flow in the microsegment, $\frac{\partial}{\partial x} \to 0$, eq AA3 can be simplified to eq AA12.

$$\rho A \frac{\partial U}{\partial x} = q_i$$  \hspace{1cm} \text{(A12)}$$

Substituting eq AA12 into eq AA11, eq AA13 can be obtained.

$$\frac{dp}{dx} = -2 \rho q_i \frac{U}{A} - \tau_w \frac{S}{A} - \rho g \sin \theta$$  \hspace{1cm} \text{(A13)}$$

From eq AA13, the first item on the right side is the acceleration pressure drop; the second item is the friction pressure drop, and the third item is the gravity pressure drop. Therefore, the pressure-drop equation of the microsegment can be written as eq AA14.

$$\Delta p = 2 \rho q_i \frac{U}{A} \Delta x + \tau_w \frac{S}{A} \Delta x + \rho g \sin \theta \Delta x$$

$$= \Delta p_{\text{acc}} + \Delta p_{\text{wall}} + \Delta p_{\text{mix}} + \Delta p_{g}$$  \hspace{1cm} \text{(A14)}$$

The radial inflow of variable mass flow, first, changes the friction coefficient of the pipe wall and second, generates acceleration pressure drop by radial inflow. At present, there are two main methods to deal with the friction pressure drop of variable mass flow. One is to determine the size of the total friction coefficient of the wall through the experiment and then use eq AA14 to calculate the pressure drop of the variable mass pipe flow. The second method is to add a mixing pressure drop on eq AA14 and use the ordinary friction coefficient without radial inflow to calculate the friction pressure drop directly. Then, the mixing pressure drop is determined by the experiment, and the total pressure drop can be written as eq AA15.

$$\Delta p = \Delta p_{\text{acc}} + \Delta p_{\text{wall}} + \Delta p_{\text{mix}} + \Delta p_{g}$$  \hspace{1cm} \text{(A15)}$$

### POTENTIAL EQUATION FOR INFINITE FORMATION OF A SINGLE HORIZONTAL WELL

For the infinite, homogeneous, and isotropic formation, the potential function of a single horizontal well under steady and uniform inflow conditions satisfies the Laplace Equation, as shown in eq BB1.

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0, \Phi = \frac{K}{\mu_o} p$$  \hspace{1cm} \text{(B1)}$$

where $\Phi$ and $p$ is the potential and pressure of formation, respectively, MPa, $K$ is the permeability of formation, mD, and $\mu_o$ is the viscosity of the oil, mPa⋅s.

A microsegment $dx_0$ which is taken from the line sink ($x_1, y_0, z_0$) to ($x_2, y_0, z_0$) as shown in Figure 18, can be equivalent to a point source $x_0$ in infinite space. The velocity potential of any point $(x, y, z)$ in the reservoir caused by the $x_0$ point can be written as eq BB2.

![Figure 18. Sketch of a single horizontal well in the infinite formation.](https://dx.doi.org/10.1021/acsomega.0c03971)
\[
\text{d} \Phi = -\frac{q}{4\pi L} \frac{dx_0}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}
\]

where \( L \) is the length of horizontal wellbore, m, and \( q \) is the production rate of the horizontal well, m\(^3\)/d.

According to the theory of Potential Superimposition, the potential generated by the whole horizontal well can be written as eq 

\[
\Phi = \int_{L} d\Phi = -\frac{q}{4\pi L} \int_{x_1}^{x_L} \frac{dx_0}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}} + C
\]

(B3)

The potential equation for the infinite formation of a single horizontal well can be obtained by Integral of eq B3 as shown in eq B4.

\[
\Phi = -\frac{q}{4\pi L} \ln \left( \frac{r_1 + r_2 + L}{r_1 + r_2 - L} \right) + C
\]

(B4)

where \( r_j = \sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2} \), \( j = 1, 2 \).

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**Notes**

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### NOMENCLATURE

- \( A \) cross-sectional area of the main wellbore, m\(^2\)
- \( A_3 \) cross-sectional area of the branch wellbore, m\(^2\)
- \( B_0 \) volume factor of crude oil, dimensionless
- \( a_j \) angle between the \( x \)-axis and the direction of the \( j \)-th segment on the \( i \)-th branch, deg
- \( d \) diameter of pipe or wellbore, m
- \( d_{w,ij} \) diameter of the \( i \)-th microsegment of the wellbore, m
- \( f_0 \) distance between each branch, m
- \( f_i \) friction coefficient of the constant mass flow in the pipe, dimensionless
- \( f_i \) apparent friction coefficient of the pipe with radial inflow, dimensionless
- \( F_i \) force which acts on control Unit II, N
- \( F_i' \) force which acts on control Unit I, N
- \( h \) thickness of pay zone, m
- \( h_{t,ij} \) height difference between the entrance and the exit of control Unit II, m
- \( L_0 \) Length of main wellbore, m
- \( L_i \) Length of the \( i \)-th branch wellbore \((i > 0)\), m
- \( L_{ij} \) Length of the \( j \)-th segment on the \( i \)-th branch wellbore, m
- \( M \) any point of the reservoir, dimensionless
- \( \rho_{w,i} \) Reynolds number of the wellbore radial inflow, dimensionless
- \( N_{Re,w} \) \( 10^6 \rho_{w} \), Reynolds number, dimensionless
- \( p_i \) pressure at the constant boundary, Pa
- \( p_{wi} \) pressure of the \( j \)-th segment on the \( i \)-th branch of the multibranch horizontal well, Pa
- \( p_{w,ii} \) bottom hole pressure (BHP) in the \( i \)-th micro-wellbore segment, Pa
- \( p_{w,ii} \) upstream pressure of control Unit II, Pa
- \( p_{w,ii} \) downstream pressure of control Unit I, Pa
- \( p_{w,ii} \) downstream pressure of control Unit II, Pa
- \( p_{w,ii} \) pressure vector of each microsegment of the wellbore, Pa
- \( q_i \) cross-section flow rate of the wellbore, m\(^3\)/s
- \( q_{w,ij} \) cross-section flow rate in the \( i \)-th microsegment, m\(^3\)/s
- \( q_{w,ii} \) radial inflow rate of the wellbore, m\(^3\)/s
- \( q_{w,ii} \) radial inflow rate at the unit length of the \( i \)-th microsegment, m\(^3\)/(s-m)
- \( q_i \) upstream flow rate of control Unit II, m\(^3\)/s
- \( q_{s} \) downstream flow rate of control Unit II, m\(^3\)/s
- \( Q_i \) flow vector of each microsegment of the wellbore, m\(^3\)/s
- \( r_{w,ij} \) wellbore radius of the \( j \)-th segment on the \( i \)-th branch,
- \( v \) average velocity of the pipe cross-section, m/s
- \( v_{i} \) mainstream velocity on the upstream of control Unit II, m/s
- \( v_{i} \) mainstream velocity on the downstream of control Unit II, m/s
- \( v_{i} \) inflow velocity from the branch, m/s
- \( x_{ij} \) heel coordinate of the \( j \)-th segment on the \( i \)-th branch of the wellbore, dimensionless
- \( y_{ij} \) toe coordinate of the \( j \)-th segment on the \( i \)-th branch of the wellbore, dimensionless
- \( z_{ij} \) average vertical coordinate of the \( j \)-th segment on the \( i \)-th branch wellbore, dimensionless
- \( z_{w} \) distance between the wellbore and bottom boundary, m
- \( \rho \) fluid density, kg/m\(^3\)
- \( \mu \) fluid viscosity, mPa·s
- \( \varepsilon \) inner wall roughness of pipe, m
- \( \varphi \) shot density, 1/m,
- \( \gamma \) wellbore inclination angle of the \( i \)-th micro-wellbore segment, deg
\( \theta \) angle between the mainstream and the inflow direction, deg
\( \theta_i \) angle between the main and branch wellbore, deg
\( \Phi_e \) potential of the bottom constant-pressure boundary, Pa
\( \xi_i(M) \) \( \Phi_e(x_i, y_i, \alpha_i, M)/q_{iw} \) potential at point M generated by the production in the unit inflow of the \( j \)th segment on the \( i \)th branch, Pa
\( \xi_i(m, n) \) potential at the \( n \)th segment on the \( m \)th branch generated by the production in the unit inflow of the \( j \)th segment on the \( i \)th branch, Pa
\( \xi_{ij} \) value of \( \xi_i(M) \) when \( M \) is at the bottom constant-pressure boundary, Pa
\( \Delta L_i \) length of the \( i \)th microsegment, m
\( \Delta p \) total pressure drop in each segment of the wellbore, Pa
\( \Delta p_{\text{wall}} \) friction pressure drop, Pa
\( \Delta p_{\text{acc}} \) acceleration pressure drop, Pa
\( \Delta p_{\text{mix}} \) mixing pressure drop, Pa
\( \Delta p_g \) gravity pressure drop, Pa
\( \Delta p_{ij} \) pressure drop in the \( j \)th segment on the \( i \)th branch of the multibranch horizontal well, Pa
\( \Delta P \) pressure-drop vector of each microsegment of the wellbore, Pa
SPF Shot per feet, 1/ft

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\( ij \) the \( j \)th segment on the \( i \)th branch wellbore, dimensionless
\( i \) represents the main wellbore, \( I > 0 \) represents the \( i \)th branch wellbore.

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