Meson-meson scattering in the massive Schwinger model — a status report*

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We discuss the possibility of extracting phase shifts from finite volume energies for meson-meson scattering, where the mesons are fermion-antifermion bound states of the massive Schwinger model with SU(2) flavour symmetry. The existence of analytical strong coupling predictions for the mass spectrum and for the scattering phases makes it possible to test the reliability of numerical results.

1. INTRODUCTION

After the successful applications of Lüscher’s method\textsuperscript{1} to the elastic scattering of elementary scalar and fermionic particles (see, e.g.,\textsuperscript{2}), we have chosen the massive Schwinger model (QED\textsubscript{2}) with 2 flavours to determine scattering phases in meson-meson scattering. Similar to four-dimensional QCD, mesons occur because of confinement. This two-dimensional model also exhibits charge screening and it possesses a non-trivial vacuum structure. Moreover, there exist analytical predictions which enable us to test the numerical results.

2. LÜSCHER’S RELATION IN d=2

Lüscher’s relation connects elastic scattering phases in infinite volume and two-particle energies in finite volumes. In 2 dimensions it has the simple form

\[ 2\delta(k) = -kL \pmod{2\pi}, \] (1)

Using the energy-momentum relation, \( k \) can be calculated from the mass and the two-particle energies, which are accessible to Monte Carlo simulations.

3. THE SCHWINGER MODEL

The massive Schwinger model with 2 flavours has the following continuum Euclidean action:

\[ S_{\text{cont}} = \int d^2x \left\{ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{f=1}^{2} \bar{\psi}^f(x) \left( \partial^2 + m + ieA(x) \right) \psi^f(x) \right\}. \] (2)

Bosonization of the model leads in the strong coupling limit to the Sine-Gordon model, plus corrections\textsuperscript{4}. Since the particle spectrum of the Sine-Gordon model is known analytically, one can derive that the particle spectrum of the massive two-flavour Schwinger model consists of a pseudoscalar isotriplet (“pion”) with G-parity G=+1 and a mass \( m_\pi \) according to

\[ \frac{m_\pi}{e} = 2.066 \left( \frac{m}{e} \right)^{2/3} \] (3)

and a scalar isosinglet with G=+1 and mass \( m_S = \sqrt{3}m_\pi \)\textsuperscript{4}. In the Sine-Gordon model also the elastic scattering phases have been calculated\textsuperscript{4} and will form the basis for our numerical tests.

From the corrections to the Sine-Gordon model only the “\( \eta \)”-particle (a pseudoscalar isosinglet, G=−1) with mass \( m_\eta \approx \sqrt{2}e/\sqrt{\pi} \) is known\textsuperscript{4}.

The lattice formulation of the Schwinger model with staggered fermions and compact Wilson ac-
Energy eigenstates are classified according to irreducible representations of the group of symmetry transformations leaving the time slices fixed. The representation \( \tilde{\Delta}^{\sigma_1 \sigma_1 \sigma_1} \) of the lattice time slice group (LTS) is characterized by the quantum numbers \( \sigma_1 \), \( \sigma_I \) and \( \sigma_C \), which correspond to shift in space, inversion and charge conjugation on the lattice. Restriction of an irreducible CTS representation to the subgroup LTS will in general lead to reducible representations. For the one-(two-)particle operators, which transform according to the representations \( \tilde{D} \) of rank 2 (4), every lattice symmetry sector couples to two (four) continuum sectors, as shown in Table 1.

| LTS     | CTS     |
|---------|---------|
| \( \Delta^{\sigma_1 \sigma_1 \sigma_1} \) | \( \tilde{\Delta}^{\sigma_1 \sigma_1 \sigma_1} \) (rank 2) | \( \tilde{\Delta}^{\sigma_1 \sigma_1 \sigma_1} \) (rank 4) |
| \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) |
| \( \sigma_1 \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \) | \( \sigma_P \) | \( \sigma_G \) |
| \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) | \( \bar{\Delta} \) |
| \( \sigma_1 \sigma_I \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) | \( \sigma_1 \sigma_C \) | \( \sigma_P \) | \( \sigma_G \) |

\[ S_{\text{lat}} = \beta \sum_{\mu} (1 - \text{Re}(U_{\mu})) + \sum_{x, \mu} \frac{1}{2} \eta_\mu(x) \left\{ \bar{\chi}(x) U_{\mu}(x) \chi(x + e_\mu) - \bar{\chi}(x + e_\mu) U_{\mu}^*(x) \chi(x) \right\} + m \sum_x \bar{\chi}(x) \chi(x). \]

4. **FIRST NUMERICAL RESULTS**

We first verified the analytical prediction for the relation of the pion mass to the bare mass of the fermions (see Fig. 1). We have good agreement between the analytical formula and our simulations: for \( \beta = 4 \) we extract an exponent of 0.689(10) as compared to 2/3 of eq. (4). The deviation between the measured points and the analytical curve decreases with increasing \( \beta \), which can be explained by the fact that the continuum limit corresponds to \( \beta \to \infty \). This is supported by additional measurements for higher values of...
Figure 2. The particle spectrum of the Schwinger model for $\beta = 7$ ($24 \times 32$). The lowest state corresponds to the “pion”.

$\beta$, up to $\beta = 10$, for which the points tend towards the analytically predicted curve.

In Fig. 2 we show the particle content of the model found by us after the investigation of almost all lattice symmetry sectors. Unfortunately we are not yet able to identify numerically the analytically predicted singlet states mentioned above.

With increasing $\beta$, a problem related to the topological properties of the model arises: the tunnelling rate between sectors of different topological charges decreases strongly, so that we are facing an ergodicity problem. To estimate its impact we analyzed the dependence of the pion mass on the topological charge

$$Q = \frac{1}{2\pi} \sum_P \Theta_P$$

of the configurations by comparing measurements on configurations associated with different topological charges (see Fig. 3). Within the error bars, no significant deviation is visible.

Because of the unexpected complexity of the one-particle spectrum for the massive Schwinger model we have not yet been able to determine scattering phases satisfactorily.

Figure 3. Dependence of $m_\pi$ on the topological charge $Q$ at $\beta = 1$ and $m = 0.1$ ($16 \times 32$). Measurements on configurations with definite topological charge are compared to the measurement on all configurations (indicated by the lines).

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