The Baryon Spectrum and Chiral Dynamics

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Abstract

The fine structure of the low energy part of the nucleon and strange hyperon spectra, which are formed of single states without parity doublets, may be understood in terms of an $SU(3)$ flavor-symmetric quark-quark interaction that describes chiral pseudoscalar boson exchange. The model predicts the fine structure splittings within 10-30\% of their empirical values and provides an explanation of the reversed ordering of the lowest positive and negative parity resonances in the nucleon and the $Λ$ hyperon spectra.

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The spectra of the nucleons and the strange hyperons separate into a low energy sector formed of single states without parity partners and a high energy sector formed of near degenerate parity doublets, the latter feature being particularly evident in the spectrum of the Λ hyperon. This is interpreted as a consequence of the (approximate) chiral symmetry of QCD being realized in the hidden (Nambu-Goldstone) mode in the low energy (and low temperature) sector, whereas it is realized in the explicit (Wigner-Weyl) mode in the high energy (and high temperature) sector. Instead of by parity doubling of the spectrum the hidden mode of chiral symmetry is revealed by the existence of the nonet of low mass pseudoscalar (Goldstone) bosons and constituent quarks. This "chiral" pseudoscalar octet (the \( \eta' \) decouples because of the \( U(1) \) anomaly [1]) will mediate interactions between the constituent quarks and thus leads to fine structure in the baryon spectrum, the gross features of which are caused by the confining interaction. The simplest representation of the interaction that is mediated by the chiral octet would be

\[
H_\chi \sim -C_\chi \sum_{i<j} \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \cdot \vec{\sigma}_i \cdot \vec{\sigma}_j.
\]  

(1)

Here the \( \{ \vec{\lambda}_i^F \} \)s are flavor \( SU(3) \) matrices and the \( i, j \) sums run over the constituent quarks. The form of this interaction is an immediate generalization of the spin-spin component of the pseudoscalar (pion) exchange interaction, the tensor component of which in the present context is insignificant. The coefficient \( C_\chi \) represents an averaged radial matrix element. A refined version of the interaction (1) would contain a \( SU(3)_F \) flavor symmetry breaking term to account for the mass splitting within the pseudoscalar octet, that arises from the explicit chiral symmetry breaking in QCD.

The chiral field interaction should be contrasted in form with the color-magnetic interaction [2]

\[
H_c \sim -\alpha_s \sum_{i<j} \vec{\lambda}_i^C \cdot \vec{\lambda}_j^C \cdot \vec{\sigma}_i \cdot \vec{\sigma}_j \delta(\vec{r}_{ij}),
\]  

(2)

where the \( \{ \vec{\lambda}_i^C \} \)s are color \( SU(3) \) matrices, and which should be important in the region of explicit chiral symmetry at short distances and high energy. It is in fact this color-magnetic interaction, which has been used in earlier attempts to describe the baryon spectra with the constituent quark model [3,4]. Although many of the qualitative and some of the quantitative features of the fine structure of the baryon spectra can be described by the interaction (2) a
number of outstanding features have proven hard to explain in this approach. The most obvious one of these is the different ordering of the (confirmed) positive and negative parity resonances in the spectra of the nucleon and the Σ hyperon on the one hand and the Λ hyperon on the other, and in particular the difficulty in describing the low mass of the Λ(1405) resonance. A second such feature is the absence of empirical indications for the spin-orbit interaction that should accompany the color-magnetic interaction (2). We shall show below that the chiral pseudoscalar interaction (1) provides a simpler description of the fine structure of the low energy baryon spectra, that automatically implies the reversal of the ordering of the even and odd parity states between the nucleon and Λ hyperon spectra. Moreover we show that when the effective coupling strength $C_\chi$ in (1) is determined by the the average splitting between baryon states that have the same orbital and spin-flavor symmetry a quite satisfactory description of the fine structure of the whole low lying baryon spectrum is achieved already in lowest order. Finally no spin-orbit force problem appears as pseudoscalar interactions have no spin orbit component. This then suggests that it is the chiral field interaction (1), which plays the dominant role in ordering the baryon spectrum in the region of hidden chiral symmetry.

Consider first the lowest $3 \frac{1}{2}$ baryon states with strangeness 0 and -1, which are listed in Table 1. The symmetry classification of these states is denoted by the Young pattern $[f]$, which is the permutational symmetry with respect to the relevant transformation group. The color singlet structure $[111]_C$, which is common to all states is suppressed in Table 1. The subindex $X$ refers to the spatial, $S$ to the spin, $F$ to the flavor and $FS$ to the intermediate $SU(6)_{FS} \supset SU(3)_F \times SU(2)_S$ symmetry. The latter is required for the unique characterization of the state. The Pauli principle requires antisymmetry with respect to permutation of all quark coordinates - i.e. the an overall permutation symmetry $[111]_{XCSF}$. Note that as the color-($[111]_C$), spin- ($[21]_S$) and color-spin- symmetries ($[21]_{CS}$) of the nucleon and Λ states listed in Table 1 are identical the color-magnetic interaction (2) cannot split them in different directions.

The effect of the chiral field interaction (1) on the $J = \frac{1}{2}$ in Table 1 can be calculated by algebraic methods if the difference in the radial structure of the $[3]_X$ and $[21]_X$ baryon states is neglected. That would in fact be an exact result for the totally symmetric spatial state, for which the spatial and flavor-spin matrix elements factorize completely, and should be a sufficiently adequate approximation for the present purposes. As the spin-flavor part of (1) is an
invariant of the $SU(6)$ group in the reduction $SU(6) \supset SU(3) \times SU(2)$ we have, using a result originally developed for the color-magnetic interaction [5],

$$< [f]^{SU(6)}|[f]^{SU(3)}|[f]^{SU(2)}| \sum_{i<j} \tilde{X}_i \cdot \tilde{X}_j \tilde{\sigma}_i \cdot \tilde{\sigma}_j|[f]^{SU(6)}|[f]^{SU(3)}|[f]^{SU(2)}>$$

$$= 4C_2^{(6)} - 2C_2^{(3)} - \frac{4}{3}C_2^{(2)} - 8N,$$

(3)

where $N$ is the number of quarks and

$$C_2^{(n)} = < [f]^{SU(n)}|C_2^{(n)}|[f]^{SU(n)}> =$$

$$\frac{1}{2}[f'_1(f'_1 + n - 1) + f'_2(f'_2 + n - 3) + ...$$

$$+ f'_{n-1}(f'_{n-1} - n + 3)] - \frac{1}{2n}(\sum_{i=1}^{n-1} f'_i)^2.$$  

(4)

Here $n$ is rank of the corresponding group, $f'_i \equiv f_i - f_n$ and $f_i$ denotes the length of the $i-th$ row of the corresponding Young pattern. With this result we find that the matrix elements (3) for the states $|21\rangle_{SF}|21\rangle_F|21\rangle_S>$, $|21\rangle_{SF}|111\rangle_F|21\rangle_S>$, $|3\rangle_{SF}|21\rangle_F|21\rangle_S>$ and $|21\rangle_{SF}|21\rangle_F|3\rangle_S>$ are 2, 8, 14 and $-2$ respectively. These are the only ones required for the baryon states in Table 1.

The effective coupling constant $C_\chi$ in the chiral interaction (1), which represents an averaged radial matrix element, may be determined by the mass difference of any two baryon states that have the same radial structure in the $SU(3)$ harmonic oscillator model (see below), the same spin- flavor symmetry but different spin- or (and) flavor symmetries. Such are eg. the nucleon and the $\Delta_{33}$, the $\Lambda(1405)$ and $\Sigma(1750)$ resonances, the $\Sigma$ and the $\Sigma(1385)$ and the $\Xi$ and the $\Xi(1530)$. The values for $C_\chi$ obtained by these mass differences are 29.3, 34.5, 19.0 and 21.2 MeV respectively. The spread in these values is a reflection of the neglect of the orbital structure of the states and the explicit flavor dependence of averaged matrix element $C_\chi$ in (1). We shall choose for $C_\chi$ the average of these values:

$$C_\chi \sim 26 \text{ MeV.}$$

(5)

In order to estimate the mass differences between the baryon states with different orbital excitation we adopt the $SU(3)$ oscillator version of the constituent quark model, in which the confining interaction is harmonic. In this
model the oscillator parameter $\hbar \omega$ may be extracted from the mass differences between the first excited $\frac{1}{2}^+$ states and the ground states of the baryons as long as the radial structure of the interaction (1) is neglected. This yields $\hbar \omega \sim 250$ MeV. In the $N$ and $\Sigma$ sectors the mass difference between the excited $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states in Table 1 will then be

\begin{align*}
N : \quad m(\frac{1}{2}^+) - m(\frac{1}{2}^-) &= 250 \text{ MeV} - C_\chi(14 - 2) \\
&= -62 \text{ MeV}, \quad (6a) \\
\Sigma : \quad m(\frac{1}{2}^+) - m(\frac{1}{2}^-) &= 250 \text{ MeV} - C_\chi(14 + 2) \\
&= -166 \text{ MeV}, \quad (6b)
\end{align*}

whereas it for the $\Lambda$ system should be

\begin{equation}
\Lambda : \quad m(\frac{1}{2}^+) - m(\frac{1}{2}^-) = 250 \text{ MeV} - C_\chi(14 - 8) = 94 \text{ MeV}. \tag{7}
\end{equation}

For a lowest order estimate these numbers agree well enough with the empirical values -95 MeV, -90 MeV and +195 MeV respectively. The agreement with the last value improves if it is replaced by the difference of 137 MeV between the $\Lambda(1600)$ and the average mass of the $\Lambda(1520)$ and $\Lambda(1405)$ as would be proper because of the neglect of the spin-orbit interactions between the quarks. That the $SU(2)_I \times SU(2)_S$ version of the chiral field interaction (1) may be of importance for the explanation of the splitting between the positive and negative parity nucleon resonances has in fact been noted earlier [6]. In fact this represents an explanation of the whole low-lying part of the baryon spectrum where all resonance masses are within 10% of their empirical values where known. It is readily seen that if the parameter $C_\chi$ in (1) is allowed to be different in the nucleon and strange hyperon sectors, as one would expect it to be by the considerable empirical mass splitting within the octet of Goldstone bosons, the fit to the empirical spectra can be much improved above the present qualitative level. The present description of the baryon spectrum should of course also be refined by a proper treatment of the anharmonicity of the confining potential, which is linear in $r$ rather than quadratic, as well as by consideration of the spatial behavior of the chiral field interaction and the mass difference between the constituent $u,d$ and $s$ quarks [3,4].
A short digression on the sign of the chiral interaction (1) is in order. This corresponds to that of the usual pion exchange potential at short distances, where the interaction is attractive in completely symmetric spin-isospin states and repulsive in antisymmetric states (the relevant term is either represented as a $\delta$-function or as a regularizing term, which dominates at short ranges $\leq 1$ fm). This should be dominant for the baryon states, which are confined within ranges of $\sim 0.8$ fm. The argument generalizes directly to $SU(3)_F$. In that case one has the matrix elements

$$< [f_{ij}]_F \times [f_{ij}]_S | \vec{\lambda}^F_i \cdot \vec{\sigma}_i \cdot \vec{\sigma}_j | [f_{ij}]_F \times [f_{ij}]_S > = \begin{cases} \frac{4}{3} [2]_F, [2]_S : [2]_{FS} \\ \frac{8}{3} [11]_F, [11]_S : [2]_{FS} \\ -4 [2]_F, [11]_S : [11]_{FS} \\ -\frac{8}{3} [11]_F, [2]_S : [11]_{FS} \end{cases}$$

(8)

Symmetrical $FS$ pair states thus experience an attractive interaction at short range, whereas antisymmetrical ones experience repulsion. This explains why the $[3]_{SF}$ state in the $N(1440)$ and $\Sigma(1660)$ positive parity resonances experiences a much larger attractive interaction than the mixed symmetry state $[21]_{SF}$ in the $N(1535)$ and $\Sigma(1750)$ resonances. Consequently the masses of the $J^P = \frac{1}{2}^+$ states $N(1440)$ and $\Sigma(1660)$ are shifted down relative to the two other ones, which explains the reversal of the otherwise expected "normal ordering".

The situation is different in the case of the $\Lambda(1405)$ and $\Lambda(1600)$, as the flavor state of the $\Lambda(1405)$ is totally antisymmetric. Because of this, even with the mixed $[21]_{SF}$ state, the $\Lambda(1405)$ experiences a gain in attractive energy, which is comparable to that of the $\Lambda(1600)$ and thus the ordering suggested by the confining oscillator interaction is not reversed.

The chiral interaction (1) also, of course, predicts the ordering of the of the strangeness -2 and -3 hyperon states. If the mass of the light and strange constituent quarks were equal the ordering of the $\Sigma$ and $\Xi$ hyperons should coincide with that in the combined nucleon and $\Delta$ spectrum. As a consequence the $\Xi(1690)$ would be a $\frac{1}{2}^+$ state. The ordering of the spectrum of the $\Omega^-$ should be the same as that of the $\Delta$ and hence the $\Omega^-(2250)$ should be a $\frac{3}{2}^+$ state.

These results suggest that the baryon spectrum can be understood in the following way. At low energies the chiral symmetry of the underlying QCD is realized in the hidden mode, and the spectrum consists of states without nearby
parity partners. In the nucleon case this part of the spectrum is formed by the nucleon and the \( \Delta(1232) \) and in the \( S = -1, I = 0 \) sector it is formed by the \( \Lambda \), and the negative parity states \( \Lambda(1405) \) and \( \Lambda(1520) \). In this low energy region the gross structure of the spectrum is caused by the confining interaction, and (most of) the fine structure by the interaction (1) that is mediated by the octet of pseudoscalar Goldstone bosons, which are associated with the hidden mode of chiral symmetry. The high energy part of the baryon spectrum on the other hand, which is formed of near degenerate parity doublets (or more generally multiplets), reveals the explicit Wigner-Weyl mode of chiral symmetry, which is due to the indistinguishability between left- and right-handed massless quarks in QCD. The remaining small splitting of the degeneracy between the parity partners is then due to the small mass of the current quarks and the gradually vanishing hidden mode of chiral symmetry. This is so as the dynamical masses of the constituent quarks arise as a consequence of the spontaneous breaking of chiral symmetry [7,8]. Similarly the shell gap \( \sqrt{h\omega} \) that arises from the confining force in the oscillator model becomes obscured by the chiral interaction (1), which mixes the states in adjacent oscillator shells.

The baryon spectrum suggests that the phase transition between the Nambu-Goldstone and Wigner-Weyl mode of chiral symmetry is gradual, as the mass difference between the nearest neighbours with opposite parity falls to zero only gradually with increasing resonance energy. The clearest signal for this is that while the splitting within the \( \Lambda(1600) - \Lambda(1670) \) parity doublet is still 70 MeV, the splittings within the \( J^P = \frac{1}{2}^\pm \) and \( J^P = \frac{3}{2}^\pm \) \( \Lambda \)-resonance parity doublets around 1800 MeV are only 10 MeV. This is an indication of the amorphic (disordered) structure of the QCD vacuum and its quark condensate. The implication would then be that there is a gradual chiral restoration phase transition. The gradual nature of the phase transition also appears in the instanton [9] liquid model of the QCD vacuum [8,10]. If the onset of the parity doubling in the resonance spectrum is taken to be at about 500 MeV above the ground state, as suggested by the mass difference between the \( \Lambda \) and the lowest parity doublet formed by the \( \Lambda(1600) \) and the \( \Lambda(1670) \), the approximate transition energy would then by of the order 500 MeV. The absence of structure in the baryon spectrum above 2 GeV excitation energy shows that both the Nambu-Goldstone mode and confinement have totally disappeared in that energy range.

The present results indicate that the role of the color-magnetic interaction (2), which is arises from gluon exchange and which should be important in the Wigner-Weyl mode, for the ordering of the baryon spectrum is small. If it is
included in the model as a phenomenological term the value of the effective coupling strength $\alpha_S$ should be much smaller than the values $\sim 1$ that have been typically employed [3,4]. The present suggestion for the decoding of the baryon spectrum is of course very different from that in refined versions of the chiral bag model, that attempt to explain the fine structure in terms of the color-magnetic model, and which can predict the low mass of the $\Lambda(1405)$ only with the extreme conclusion that it has almost no flavor singlet component [11]. Closer in spirit, are the extended Nambu-Jona-Lasinio [12] and chiral soliton models [13,14] in which the low mass of the $\Lambda(1405)$ resonance also emerges naturally. In these the Pauli principle at the quark level is however missing and the interaction between the constituents is mediated by (implicit) heavy meson exchanges rather than by pseudoscalar bosons.

It should finally be pointed out that the chiral field interaction (1) is not expected to be important in the heavy flavor charm and bottom sectors, because of the large breaking of chiral symmetry caused by the large masses of the charm and bottom quarks. Further application of the the pseudoscalar octet mediated interaction should take into account the flavor symmetry breaking caused by the mass splitting within the octet as well as the spatial dependence of the interaction, which - at least in the case of two-baryon systems - should be important [15,16].

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Table 1

The structure of the lowest confirmed $J = \frac{1}{2}$ baryon states

| $N$ | $N(1440)$ | $N(1535)$ |
|-----|------------|------------|
| $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{1}{2}^-$ |
| $[3]_X$ | $[3]_X$ | $[21]_X$ |
| $[3]_{SF}$ | $[3]_{SF}$ | $[21]_{SF}$ |
| $[21]_F$ | $[21]_F$ | $[21]_F$ |
| $[21]_S$ | $[21]_S$ | $[21]_S$ |

| $\Lambda$ | $\Lambda(1405)$ | $\Lambda(1600)$ |
|-----------|------------------|------------------|
| $\frac{1}{2}^+$ | $\frac{1}{2}^+$ | $\frac{3}{2}^+$ |
| $[3]_X$ | $[21]_X$ | $[3]_X$ |
| $[3]_{SF}$ | $[21]_{SF}$ | $[3]_{SF}$ |
| $[21]_F$ | $[111]_F$ | $[21]_F$ |
| $[21]_S$ | $[21]_S$ | $[21]_S$ |

| $\Sigma$ | $\Sigma(1660)$ | $\Sigma(1750)$ |
|-----------|------------------|------------------|
| $\frac{1}{2}^+$ | $\frac{3}{2}^+$ | $\frac{1}{2}^-$ |
| $[3]_X$ | $[3]_X$ | $[21]_X$ |
| $[3]_{SF}$ | $[3]_{SF}$ | $[21]_{SF}$ |
| $[21]_F$ | $[21]_F$ | $[21]_F$ |
| $[21]_S$ | $[21]_S$ | $[3]_S$ |