Stability of Plate Girder Web with Longitudinal Stiffeners, According to EN 1993–1–5

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Abstract. In this paper, the method to determine critical plate buckling stresses for slender, longitudinally stiffened plates is presented, according to EN 1993–1–5. The panel consists of a thin plate stiffened with one or two stiffeners is considered. The elastic critical plate buckling stress is computed based on the column buckling stress of a stiffener strut on an elastic foundation. The gross cross-sectional area of this strut is composed of the gross cross-sectional area of the stiffener and the cross-sectional area of adjacent parts of contributive plating. The adjacent parts of the contributive plating are a proportion of the subpanel width when the latter is fully in compression or a proportion of the depth of the compression zone of the plating subpanel when the direct stress in the latter changes from compression to tension. Included numerical example provides procedure for the calculation of adjacent parts of the contributive plating and their critical stresses. The results obtained by using EN 1993–1–5 rules and Finite Element Method (FEM) software are compared.

1. Introduction

Modern plate girders are shaped like light beams with a slender web which can lose plate stability in places of large normal or shear stresses. After losing the local stability of web the compressed elements preserve their ability to transmit the load. This ability is called ‘post - critical capacity’. The figure 1 shows a typical reaction of slender plates that are compressed.

![Figure 1. Post – critical response of slender plates in compression [3]](image-url)
The range of geometrically perfect pre-critical and post-critical capacity are easy to differentiate. However, non-ideal plates can gradually move from one range of capacity to another. In case of plates that are heavily imperfect, the limit between range of the pre-critical and post-critical capacity can be almost invisible.

Usually attaining the critical stress by plate does not use all of its capacity. It can even increase, up to the point of plastic weakness. In the post-critical state redistribution of compressing stress can be seen and possibly result in decreasing of stress in the non-effective zone of the plate section. In the same time however, stress along its stiffened edges can increase figure 2. Reduction of stress is a result of a partial loss of stiffness of the longitudinal plate in the non-effective zone. The growth of the plasticized areas in the edge regions precludes more redistribution of stress: the plate reaches its limit load capacity.

Taking into consideration the non-linear stress distribution can result in too complex and even impractical calculation procedures. Therefore, there are two simplified methods to considering plate buckling proposed in [2]. It is illustrated in figure 2.

The first method determines the resistance of a cross-section by "effective widths" of its various plate elements in compression, where the reduction of stiffness and strength due to local plate buckling is reflected by a reduced section with "holes" (non-effective zone) in the cross-sectional area, which is supposed to be stressed until the flanges reach yielding.

The second method determines the resistance of a cross-section by limiting the stresses in its various plate elements without considering "holes" by using "reduced stress limits" due to local buckling. Values of reduction section and reduction stress are shown below:

\[ F_{ult} = \int_{0}^{b} \sigma_{act} dx = b_{eff} f_{y} = b \sigma_{lim} = \rho b f_{y} \]  \hspace{1cm} (1)

Therefore,

\[ \rho = \frac{b_{eff}}{b} = \frac{\sigma_{lim}}{f_{y}} \]  \hspace{1cm} (2)

The formula above defines a so called plate buckling reduction factor for both methods. The "effective cross-section" method is presented in Chapter 4 of [2]. Chapter 10 of that same norm includes a description of the basics of "reduction stress" method. Therefore, the plate buckling reduction

![Figure 2. Basic ideas of reduced cross section method and reduced stress method [3]](image-url)
factor is crucial in the design standard evaluation of stability plate elements. Its value depends mostly on critical stress in Euler sense.

The theoretical basics of a method to define critical stress for a non-static form of a plate type of slender webs with one or two longitudinal stiffeners in the compressed zone can be found in the Chapter 2 of this paper. The compressed on elastic foundation strut model plays the main role in this case. An example calculation in Chapter 3 illustrates in detail the creation of effective parts of longitudinal stiffeners web, as well as the way of evaluating their stability. The results of Eurocode 3 estimates of the critical stress values have been compared with analogical results delivered by a FEM software.

The conclusion can be found in Chapter 4.

2. Model with one or two longitudinal stiffeners in the compression zone

The elastic critical plate buckling stress is computed based on the column buckling stress of a stiffener strut on an elastic foundation. The gross cross-sectional area of this strut is composed of the gross cross-sectional area of the stiffener and the cross-sectional area of adjacent parts of contributive plating, according to the rules given in [2]. When the latter is fully in compression the adjacent parts of contributive plating are a proportion of the width $b_1$ respectively $2/(5 - \psi)$ at the latter with extreme stress and $(3 - \psi) (5 - \psi)$ at the latter of the second subpanel. When the latter is fully in compression the adjacent parts of contributive plating are a proportion $0.4$ of the depth $b_c$ of the compression zone of the plating subpanel when the direct stress in the latter changes from compression to tension, figure 3.

![Figure 3. Model when a single longitudinal stiffener in the compression zone [2]](image)

The differential stability equation of a simple supported beam at the ends under the compressive force $N$ on an elastic foundation $k$ is:

$$ EJ_{st,1} \frac{d^4w}{dx^4} + N \frac{d^2w}{dx^2} + kw = 0 $$

(3)

Where: $J_{st,1}$ is second moment of area of the gross cross-section about the relevant axis of bending.

The expected solution of equation (3) is presented below:

$$ w = w_0 \sin \frac{m \pi x}{a} $$

(4)

Where $m$ is half sine waves of length $a/m$. The critical load is formulated by substituting (4) into the (3):

$$ N_{cr, st} = \frac{m^2 \pi^2 E J_{st,1}}{a^2} + \frac{ka^2}{m^2 \pi^2} $$

(5)
This equation gives the critical load as a function of \( m \), which depends on the properties of both the stiffener and the elastic foundation. By gradually increasing \( k \), one reaches a condition where \( N \) becomes smaller for \( m = 2 \) than for \( m = 1 \); then the buckled stiffener has an inflection point in the middle. Increasing \( k \) furthermore leads to more half waves (\( m = 3, 4, ... \)) and \((m - 1)\) intermediate inflexion points. Then, \( k \) being given, there is a length at which, for each value of the integer larger than 1, minimises the value of \( N \); it is drawn from the condition \( \partial N_{cr, st}/ \partial m = 0 \). The latter gives:

\[
\bar{a} = m \pi \frac{\sqrt{EJ_{st,1}}}{k}
\]

The minimum value of critical load when \( m > 1 \), i.e when \( a > \bar{a} \), is:

\[
N_{cr, st, min} = 2\sqrt{kEJ_{st,1}}
\]

The rigidity of the elastic foundation is developed by equilibrium of the pin-ended plate strip of thickness \( t \), unit width and span \((b_1 + b_2)\), figure 4.

![Figure 4. Line load plate strip](image)

The reaction of the stiffener on the plating included as a unit transverse line load plate strip. The gross load are resulted from a different location of the stiffener with the latter of the subpanel \((b_1 \text{ and } b_2)\). The ratio of the rigidity factor of the foundation is equal to the maximum deflection of the plate strip.

\[
1 = \frac{b_1^2 b_2^2}{3 bD}
\]

where

\[
D = \frac{E t^3}{12(1 - \nu^2)}
\]

is the flexural rigidity of the plate. Substituting (8) with (9) into the (5) for \( m = 1 \) leads to:

\[
N_{cr, st} = \frac{\pi^2 EJ_{st,1}}{a^2} + \frac{E t^3 b a^2}{4\pi^2(1 - \nu^2)b_1^2 b_2^2}
\]
In [2] minimum value of the critical load defined for $m = 1$:

$$a_c = \bar{a}(m = 1) = 4,33 \sqrt[4]{\frac{J_{st,1}b_1^2b_2^2}{bt^3}}$$

(11)

Minimum value for $m > 1$, i.e when $a > a_c$ have the form:

$$N_{cr,stm} = 2\sqrt{kEJ_{st,1}} = \frac{1,05E}{b_1^2b_2^2} \sqrt[4]{J_{st,1}t^3b}$$

(12)

If $a > a_c$ then buckling considered strut might be dominated by several half sine waves. Value of critical stress should be determined by the equation:

$$\sigma_{cr,sl} = N_{cr,stm}/A_{st,1} = \frac{1,05E}{A_{st,1} b_1^2 b_2^2} \sqrt[4]{J_{st,1}t^3b}$$

(13)

If $a < a_c$ then the buckling length is forced to be equal to the length. Value of critical stress should be determined by the equation:

$$\sigma_{cr,sl} = \frac{N_{cr,stm}}{A_{st,1}} = \frac{\pi^2EJ_{st,1}}{A_{st,1} a^2} + \frac{Et^3b a^2}{4\pi^2(1-\nu^2)A_{st,1} b_1^2 b_2^2}$$

(14)

The above procedure, fully described for a single stiffener, can be extended to the case of two longitudinal stiffeners in the compression zone as follows [2]. Each of these two stiffeners, considered separately, is supposed to buckle while the other one is assumed to be a rigid support; the procedure for one stiffener in the compression zone is thus applied twice with appropriate values of section properties and distances. Then, as a conservative approach, a fictitious lumped stiffener is substituted for the two individual stiffeners. It is such that:

- Its section properties (cross – sectional area and second moment of area) are the sum of the properties of the individual stiffeners,
- Its location is the point of application of the stress resultant of the respective forces in the individual stiffeners.

After the minimum value of critical load plate girder web has been determined using one of the two methods, it can be used for further calculations. The scope of the next chapter includes an example of using the above procedure to determine the critical load plate girder web with longitudinal stiffeners.

3. Numerical example

The value of critical load plate girder web with longitudinal stiffeners in the elastic strength range should be determined [4].

3.1. Data

Geometrical parameters for the cross – section are listed in table 1.

| $b_{f1}$ | $t_{f1}$ | $b_{f2}$ | $t_{f2}$ | $h_w$ | $t_w$ | $b_{s1}$ | $t_{s1}$ | $h_{w1}$ | $h_{w2}$ | $f_y$ |
|----------|----------|----------|----------|-------|-------|----------|----------|----------|----------|-------|
| mm       |          |          |          |       |       |          |          |          |          | MPa   |
| 400      | 20       | 600      | 40       | 2000  | 8     | 130      | 10       | 500      | 1000     | 235   |

Neutral axis: $h_s = 134.2\ c$
3.2. Normal stresses
Geometric characteristics are calculated ignoring the contribution of longitudinal stiffeners, which are not continuous, table 2.

Table 2. Normal stresses

|      | $\sigma_1$ | $\sigma_{st1}$ | $\sigma_{st2}$ | $\sigma_2$ |
|------|------------|----------------|----------------|------------|
| MPa  | 211        | 132            | 53,8           | −104       |

![Figure 5. Geometrical parameters for web with longitudinal stiffeners](image)

![a) Geometric characteristic stiffener with the adjacent parts of contributive plating](image)

![b) Gross cross section](image)

![Figure 6. Gross cross section](image)
Table 3. Geometrical parameters for the web subpanels

| Web subpanel 1 | Web subpanel 2 | Web subpanel 3 |
|----------------|----------------|----------------|
| \( b_{1,\text{edge}} = 22.6 \, \text{cm} \) | \( b_{2,\text{sub}} = 21.3 \, \text{cm} \) | \( b_{3,\text{sub}} = 13.5 \, \text{cm} \) |
| \( b_{1,\text{inf}} = 26.9 \, \text{cm} \) | \( b_{2,\text{inf}} = 27.7 \, \text{cm} \) | \( b_{3,\text{edge}} = 20.2 \, \text{cm} \) |

Lumped stiffener: \( h_{w,x} = 63.4 \, \text{cm} \)

### 3.3. Critical stresses

Buckling of the lower stiffener: \( \sigma_{\text{cr},p,1} = 586 \, \text{MPa} \)
Buckling of the upper stiffener: \( \sigma_{\text{cr},p,2} = 1358 \, \text{MPa} \)
Buckling of the lumped stiffener: \( \sigma_{\text{cr},p,x} = 578 \, \text{MPa} \)
Critical plate buckling stresses: \( \sigma_{\text{cr},p} = \min(\sigma_{\text{cr},p,1}, \sigma_{\text{cr},p,2}, \sigma_{\text{cr},p,x}) = 578 \, \text{MPa} \)

### 4. FEM results

The final result will be compared to the numerical solution, which was determined by the finite element method. The static plate scheme simply supported with an edge load was used for the 3.0 m long web section between the transverse stiffeners, figure 8.

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Figure 7. Stiffener 1 and 2 and lumped stiffener

Figure 8. Static plate scheme subjected to pure bending
In the example the Autodesk Robot Structural Analysis Professional 2015 package was used. The area of the plate was discretized with shell elements. Thus, the analysis of spacial instability forms was possible. The calculation model includes 2400 shell elements connected in 3721 junctions. There are 14724 degrees of freedom in this task, figure 9.

![Figure 9. Plate without longitudinal stiffeners](image)

In the numerical analysis the linear buckling option was used. The figure includes an illustration of the first mode of buckling, as well as the value of the critical load. The plate without longitudinal stiffeners buckles with the edge load equal to $39.6 \, MPa$, figure 9. The critical load of this plate obtained from an analytical solution is:

$$
\sigma_{cr,p} = \frac{k \sigma E}{12 \lambda_w^2 (1 - \nu^2)} = \frac{13.24 \cdot \pi^2 \cdot 21000}{12 \cdot 250^2 \cdot (1 - 0.3^2)} = 4.02 \frac{kN}{cm^2} = 40.2 \, MPa \quad (15)
$$

The result of this calculation indicates the good convergence of the numerical method used.

![Figure 10. Plate with one longitudinal stiffener](image)
After placing the stiffener in the compression zone, the plate resistance to buckling increases significantly. The position of the stiffener is also important. We can see that the placing of stiffener in the web area should be conducted with special care. In the case of two stiffeners, figure 11, a significant reduction of the instability zone is noticed, the critical load value increases. However, the determined accordingly to the Eurocode EC3 critical load value $\sigma_{cr,p} = 578 \, \text{MPa}$ significantly exceeds the one determined numerically, figure 11.

This result raises concerns because it will lead to lower values of slenderness parameter, which could later result in a smaller reduction of the gross cross – section to the effective cross- section.

5. Final conclusions
Contemporary codes of design steel structures comprise set of general and bridge rules, but in some cases these rules are alternative. It is therefore difficult to find complete procedures and algorithms to
calculate structural elements. Effective use of Eurocode requires the designer to be well – prepared and having the basic knowledge of the matter.

The theoretical basics of a method to define critical stress for a non-static form of a plate type of slender webs with one or two longitudinal stiffeners in the compressed sphere [2] are presented. The compressed on elastic foundation strut model plays the main role in this case. An example calculation illustrates in detail the creation of effective parts of longitudinal stiffeners web, as well as the way of evaluating their stability. The results of design standards (EC3) estimates of the critical stress values have been compared with analogical results delivered by a FEM software.

The results obtained by using EN 1993–1–5 rules give incorrect estimated values of relevant relative slender, which could later result in a smaller reduction of the gross cross-section to effective cross-section.

References

[1] EN 1993-1-1: 2005+AC: 2006, Eurocode 3: Design of Steel Structures. Part 1.1 General rules and rules for buildings.
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[3] D. Beg, U. Kuhlmann, L. Davaine, B. Braun, “Design of plated structures Eurocode 3: Design of steel structures, Part 1-5 Design of plated structures”. ECCS 2010.
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