Spin versus charge noise from Kondo traps

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(Dated: March 31, 2015)

Magnetic and charge noise have common microscopic origin in solid state devices, as described by a universal electron trap model. In spite of this common origin, magnetic (spin) and charge noise spectral densities display remarkably different behaviours when many-particle correlations are taken into account, leading to the emergence of the Kondo effect. Our numerical renormalization group results indicate that while spin noise is a universal function of the Kondo temperature, charge noise remains well described by single-particle theory even when the trap is deep in the Kondo regime. This difference survives even in the presence of disorder, showing that noise can be more manageable in devices that are sensitive to magnetic (rather than charge) fluctuations and that the signature of the Kondo effect can be observed in spin noise spectroscopy experiments.

PACS numbers: 72.70.+m, 75.20.Hr

The tunneling of conduction electrons into local charge traps is a prevalent phenomena in solid state physics. Traps can be realized by artificial structures such as quantum dots [1], or by natural “unwanted” defects such as dangling bonds [2] and bound states in metal/oxide interfaces [3]. It has long been recognized that trap fluctuation causes charge noise in electronic devices, with the signature of individual traps being observed with a fluctuation causes charge noise in electronic devices, with interfaces [3]. It has long been recognized that trap fluctuations as dangling bonds [2] and bound states in metal/oxide interfaces [4, 5] and an ensemble of them causing $1/f$ noise in large structures [6]. Here we address the fundamental question of how the electron spin alters trap noise.

One of the greatest developments of interacting electron physics was the discovery that a local trap interacting with a Fermi sea gives rise to the Kondo effect, the formation of a many-body singlet with conduction electron spins screening out the local trap spin [7]. The signatures of the Kondo effect in transport phenomena are well studied, but key issues related to dynamics have only been addressed recently with the emergence of modern Numerical Renormalization Group (NRG) algorithms [8]. It is particularly interesting to find out whether trap noise will impact devices that are sensitive to magnetic fluctuations as opposed to charge, e.g. spin-based or spintronic devices [9, 10], in the same way that it affects conventional charge-based devices. Recent measurements of intrinsic magnetic flux noise in superconducting quantum interference devices do indeed confirm that trap spin fluctuation is the dominant source of noise [11][12]. Moreover, novel developments in spin noise spectroscopy [13] open several possibilities for the detection of correlated spin fluctuations in quantum dot systems.

Given these interesting prospects, the question that we address here is the qualitative difference between charge and spin noise of a “Kondo trap” interacting with a Fermi sea, which we define as a local charge trap in the Kondo regime. Our starting point is the Anderson model [15] for a trapping-center interacting with a Fermi sea, $H = H_{\text{band}} + H_{\text{imp}} + H_{\text{hyb}}$ with $H_{\text{band}} = \sum_{k,\sigma} c_{k\sigma}^\dagger n_{k\sigma}$, $H_{\text{hyb}} = \sum_{k,\sigma} V_{k} (c_{k\sigma}^\dagger d_{\sigma} + c_{k\sigma} d_{\sigma})$, and $H_{\text{imp}} = \epsilon_{d} (n_{\uparrow} + n_{\downarrow}) + U n_{\uparrow} n_{\downarrow}$. In the above, $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) is a creation (destruction) operator for a conduction electron with wavevector $k$ and spin $\sigma = \uparrow, \downarrow$, $n_{k\sigma} = c_{k\sigma}^\dagger c_{k\sigma}$ counts the number of band electrons in state $k, \sigma$, and energy $\epsilon_{k\sigma}$. Similarly, the operators $d_{\sigma}^\dagger$ and $d_{\sigma}$ create and destroy an impurity electron with spin $\sigma$, respectively, with $n_{\sigma} = d_{\sigma}^\dagger d_{\sigma}$ being the number operator for electrons with spin $\sigma$ occupying the impurity state with energy $\epsilon_{d}$.

Our goal is to calculate the impurity spin ($S_{s}(\omega)$) and charge ($S_{c}(\omega)$) noise spectral densities, defined by:

$$S_{v=s,c}(\omega) = \int_{-\infty}^{\infty} dt \frac{e^{i\omega t}}{2\pi} \langle \left( \hat{O}_{s}(t) - \langle \hat{O}_{s} \rangle \right) \left( \hat{O}_{c}(0) - \langle \hat{O}_{c} \rangle \right) \rangle,$$

where the impurity spin and charge operators are given by $\hat{O}_{s} = S_{z} = (n_{\uparrow} - n_{\downarrow})/2$ and $\hat{O}_{c} = (n_{\uparrow} + n_{\downarrow})$, respectively. It can be readily verified that $S_{s}(\omega)$ and $S_{c}(\omega)$ obey the sum rule: $\int_{-\infty}^{\infty} S_{s}(\omega) d\omega = \langle \hat{O}_{s}^{2} \rangle - \langle \hat{O}_{s} \rangle^{2}$.

As a first approximation we calculate the noise spectral densities using Hartree-Fock (HF) decomposition based on writing expectation values into products of spectral functions [15]. The advantage of HF is that it becomes exact in the $U = 0$ non-interacting limit [16]. The result for charge noise is

$$S_{c}(\omega) = \sum_{\sigma=\uparrow,\downarrow} \int d\epsilon A_{\sigma \sigma}(\epsilon) A_{\sigma \sigma}(\epsilon - \omega)[1 - f(\epsilon)] f(\epsilon - \omega),$$

and for the spin noise we get simply $S_{s}(\omega) = \frac{1}{2} S_{c}(\omega)$, i.e., in the HF approximation magnetic noise is simply $\frac{1}{2}$ times the charge noise. In Eq. (2) we set $\hbar = k_{B} = 1$, with
S(ω) = 1/[exp((ε − ε_F)/T) + 1] denoting Fermi functions, and
\[ A_{↑↑}(ε) = \frac{\Gamma/\pi}{(ε - \epsilon_d)^2 + \Gamma^2}, \]
\[ A_{↓↓}(ε) = \frac{\Gamma/\pi}{(ε - \epsilon_d - U)^2 + \Gamma^2}, \]
representing spectral functions for the trap with spin ↑ and ↓, respectively. The energy scale \( \Gamma \equiv \pi \rho V_d^2 \) models the rate for escape of a trap electron into the Fermi sea, with \( \rho \) the energy density at the Fermi level, and \( V_{dk} \equiv V_d \) a \( k \)-independent coupling between trap and Fermi sea. Note that Eqs. (3a) and (3b) break the local spin symmetry by assuming the energy for the ↑ and ↓ trap states are \( \epsilon_d \) and \( \epsilon_d + U \), respectively. This result is well known to be incorrect, in that it misses Kondo physics, i.e. the screening of impurity spin by the electron gas spins.

We shall compare this approach to non-perturbative NRG calculations of the noise spectra, that take into account local spin symmetry and the formation of the Kondo singlet. The NRG algorithm calculates, within some well-controlled approximations [8], the many-body spectrum for the Anderson model [8,16]. Conduction electrons are assumed to have a continuum spectrum, forming a metallic band with a half-bandwidth \( D \). Using the Lehmann representation, we write an exact expression for the spin and charge noise:
\[ S_i(ω) = \sum_{m,n} \frac{e^{-\beta E_m}}{Z} \left| \langle n|\hat{O}_i|m\rangle \right|^2 \delta(ω-E_{nm}) - (\langle \hat{O}_i \rangle)^2 \delta(ω), \]
where \( Z \) is the partition function, \( |m\rangle \) are (many-body) eigenstates of the Hamiltonian with energy \( E_m \) (\( E_{nm} \equiv E_n - E_m \)) and \( \langle n|\hat{O}_i|m\rangle \) are the many-body matrix elements of the local operator \( \hat{O}_i \). Note that Eq. (4) implies \( S_i(ω) = e^{-ω/T} S_i(ω) \) as required by our assumption of thermal equilibrium.

At zero temperature, the first term in Eq. (4) can be computed from the NRG spectral data [8,17,18] down to arbitrarily small non-zero frequencies \( |ω| > 0 \). The spectral weight at \( ω = 0 \) and the fulfillment of the sum-rules can be obtained by calculating the expectation values \( \langle \hat{O}_i \rangle \) and \( \langle \hat{O}_i^2 \rangle \) with NRG. Since we will be interested in the large frequency regime and our spectral functions obey well-defined sum rules, we have chosen to use the “Complete Fock Space” (CFS) approach [19] to calculate \( S_i(ω > 0) \) at zero temperature. This choice has two important features: (i) the \( T = 0 \) spectral functions are sum-rule-conserving by construction and (ii) broadening artifacts in the high frequency regime, which can mask the correct power-law behavior, are minimized.

Figure 1 shows the calculated charge noise in the case \( \epsilon_d = -U/2 \) for several different \( U \). Remarkably, HF remains a good approximation to charge noise even at large \( U \). We interpret this result to be evidence that charge noise is dominated by single particle processes even when the trap is deep in the Kondo regime \( U \gg \Gamma \) for \( T = 0 \).

The situation is drastically different for magnetic noise as shown in Fig. 2. While NRG and HF agree with each other in the \( U = 0 \) limit (when HF is exact), as soon as \( U \) becomes non-zero the two methods show opposite results. As \( U \) increases, the single particle noise (HF)
The magnetic noise spectral densities all collapse in the same universal curve and scale as an anomalous power law \( \omega > U \). For \( \omega \ll T_K \), the magnetic noise scales linearly with \( \omega / T_K \), where \( T_K \) is the Kondo temperature. In the case of an ensemble of \( N \) Kondo traps, the noise is expected to be affected by disorder. The usual model for trap disorder [6] is to assume trap tunneling rate \( \Gamma = \Gamma e^{-\lambda} \), with \( \lambda \) the tunneling distance between trap and the Fermi gas uniformly distributed in the interval \([0, \lambda_{\text{max}}]\). This gives rise to the broad distribution of Kondo temperatures shown in Eq. (7).

It is important to note that this high temperature expression also shows universal scaling as a function of \( \omega / T_K \) and \( T / T_K \). Notably, \( \Gamma_s \) increases linearly with \( T \), evidencing a much stronger temperature dependence when compared to the charge noise.

In the case of an ensemble of \( N \) Kondo traps, the noise is expected to be affected by disorder. The usual model for trap disorder [6] is to assume trap tunneling rate \( \Gamma = \Gamma e^{-\lambda} \), with \( \lambda \) the tunneling distance between trap and Fermi sea. We take the limit of high disorder assuming \( \lambda \) to be uniformly distributed in the interval \([0, \lambda_{\text{max}}]\), giving rise to the following distribution of Kondo temperatures,

\[
P(T_K) = \frac{N}{\lambda_{\text{max}} T_K} \left[ \kappa - \log \left( \frac{T_K}{T_K^{\text{min}}} \right) \right],
\]

for \( T_K \in [T_K^{\text{min}}, T_K^{\text{max}}] \), and \( P(T_K) = 0 \) for \( T_K \) outside this interval. Here \( T_K^{\text{max}} \) and \( T_K^{\text{min}} \) are given by the Hal-
dane expression [23] with \( \lambda = 0 \) and \( \lambda = \lambda_{\text{max}} \), respectively. The parameter \( \kappa = -\sqrt{3} \epsilon_d (\epsilon_d + U)/(U T_0) \) characterizes the type of trap (in deriving Eq. (7) we assumed \( \epsilon_d \) and \( U \) are fixed with \( \kappa \ll 1 \), as these parameters are less sensitive to disorder).

Applying this averaging prescription to our spin noise expressions Eqs. (4)–(6b) leads to the behavior shown in Fig. [1]. We find that disordered spin noise displays \( 1/f \) behavior over a temperature range that depends strongly on temperature, in marked contrast to the usually temperature-independent charge \( 1/f \) noise. The additional temperature dependence implies that temperature actually competes against disorder, converting the \( 1/f \) spin noise into a Lorentzian.

In conclusion, we presented a theory of charge and spin noise of a Kondo trap interacting with a Fermi sea. We showed that trap spin noise is qualitatively different from charge noise, in that the former occurs due to many-body processes, while the latter is mainly dominated by single particle tunneling. This difference implies that spin noise has a stronger temperature dependence than charge noise, and that it is controllable by tuning Kondo temperature \( T_K \) rather than trap tunneling rate \( \Gamma \). This difference survives even in the presence of disorder, and shows that magnetic noise may have a narrower \( 1/f \) frequency range than charge noise. Given that \( 1/f \) noise is notoriously difficult to control [23], we reach the conclusion that ubiquitous trap noise can be more manageable in spin or flux-based devices that are sensitive to magnetic fluctuations rather than charge.

These results show that Kondo trap dynamics displays two quite distinct behaviors depending on which property is probed. The experimental methods of charge [11] and spin [14] noise spectroscopy use optical absorption to detect noise via the fluctuation-dissipation theorem (energy absorbed is proportional to frequency times noise). Our results elucidate how Kondo correlations can be observed with these methods. Pure charge absorption does not enable the detection of the Kondo effect; in [11] the formation of the exciton state mixes charge and spin fluctuation, and this feature was critical in enabling their observation of the Kondo effect. For spin noise spectroscopy, universal scaling with Ohmic behavior at \( \omega \ll T_K \) coupled with a \( 1/\omega \log^2 (\omega/T_K) \) tail for \( T_K \ll \omega \ll U \) can be taken as the signature of the Kondo effect, allowing the extension of this technique to probe Kondo correlations.

Acknowledgements. LGDS acknowledges support from Brazilian agencies FAPESP (2013/50220-7), CNPq (307107/2013-2) and PRP-USP NAP-QNano. RdS acknowledges useful discussions with M. Le Dall, H. E. Türeci and I. Žutić, and financial support from the Canadian program NSERC-Discovery and a generous FAPESP-UVic exchange award.

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