Resistivity effects in surface superconductivity of thin films in strong magnetic fields

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Phase slips creation in the thin film in perpendicular magnetic filed with edge superconductivity is studied. These centers are due to thermal activation of the order parameter below superconducting temperature transition leading to the suppression of the superconductivity. The corresponding resistance is calculated. The Aslamazov- Larkin correction to the conductivity above the critical magnetic field destroying the surface superconductivity is studied. Such structures could be applied as a new system for the study of the phase slip phenomenon in one- dimensional superconducting wires.

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As it was first shown by Saint- James and de Gennes [1], superconductivity can nucleate in the thin surface superconducting sheath to magnetic fields higher than the bulk critical field $H_{c2} < H < H_{c3} \approx 1.69H_{c2}$.

Thin films reveal the most simple picture of surface superconductivity. In particular, this case was studied experimentally in papers [2, 3], where the temperature dependencies of resistivity of thin Nb films were measured. Effects of the surface inhomogeneities and the sample shape, properties of mesoscopic size superconductors in surface superconductivity regime were investigated both theoretically and experimentally in papers [4, 5].

However, it is of definite interest to consider the limit of the extremely thin superconducting film, where superconductivity persists in the quasi- one- dimensional edge layer.

It is well known that the fluctuations of the order parameter play an important role in the physics of low-dimensional superconductors (thin films, wires). At temperatures above the critical temperature of the superconducting transition $T_c$, fluctuations lead to the enhancement of the conductivity [6], while below $T_c$, they destroy the long- range order and lead to the finite resistance of the system.

In the vicinity of $T_c$ for example in thin superconducting wires with the diameter smaller than the coherence length thermal activation of the phase slips centers locally destroys the superconductivity [7, 8, 9, 10]. Phase slip event is of the order of the coherence length $\xi(T)$ at which the amplitude of the order parameter vanishes at one point while the phase difference between the opposite sites of this point is $\pi$.

The observed resistance of the extremely thin superconducting wires at temperatures $T \ll T_c$ [11, 12] is argued to be caused by the phase slip events due to quantum tunneling of the order parameter [13].

In the present paper we will show that similar fluctuation effects appear in the edge superconductivity of thin superconducting film in the magnetic field perpendicular to the plane of the film. To the best of our knowledge the question of the fluctuations of the order parameter in the vicinity of the phase transition in thin films with edge superconductivity still remains open.

We will give the detailed analysis of the sample resistivity dependencies on temperature and magnetic field. The equation for Aslamazov- Larkin correction to the conductivity at fields higher than $H_{c3}(T)$ will be also obtained.

Let us consider a thin superconducting film with the magnetic field applied perpendicular to the surface of the film (see Fig.1). We will study the case of the 2-type superconductor under the surface superconductivity condition, when the Ginzburg- Landau parameter $\kappa \gg 1$. Thus, we will not take into account the magnetic field modulations due to supercurrents. The size of the film is such that $d \ll \xi(T) \ll L$, where $d$ and $L$ is the width and length of the film. The Ginzburg- Landau equation in dimensionless variables could be written in the form

$$(i \nabla + A)^2 \Psi = \Psi(1 - |\Psi|^2) \quad (1)$$

where $\Psi$ is the complex order parameter, length is measured in units of the coherence length $\xi(T) = (\hbar \pi D / 8 (T_c - H))^{1/2}$, $A$- vector potential measured in units $\frac{e}{2\mu_0 D}$, $D$- diffusion coefficient.

Since the superconductivity in this regime exists only in the thin edge layer of the film we can treat each edge independently. Then let $y$ axis to be applied along the corresponding edge, $x$ axis directed to the bulk of the film. Magnetic field is applied along the $z$-axis. Vector potential is chosen in the Landau gauge, $A = (0, H x, 0)$. The boundary condition for the order parameter at the $(0, y)$ edge of the film is given as

$$\frac{d\Psi}{dx}|_{x=0} = 0 \quad (2)$$

at the same time, the order parameter vanishes at the bulk of the film. We will search for the solution of the nonlinear Ginzburg- Landau equation (1) in the form

$$\Psi_0(x, y) = \gamma g(x) e^{iky} \quad (3)$$

where $\gamma$ is some constant, $g(x)$ is a function subject to the condition $\int_0^\infty g^2(x) dx = 1$. Then the equation (1) can be reformulated as

$$V(x) = |\gamma g_0(x)|^2, E_0 = 0 \quad (4)$$
where $g_0(x)$ is the eigenfunction corresponding to the lowest eigenvalue $E_0$ of the equation

$$-rac{d^2}{dx^2} + (Hx - k)^2 - 1 + V(x) \ g(x) = Eg(x) \quad (5)$$

In order to solve equation (5) we can use the perturbation theory for $\gamma \ll 1$. The eigenfunctions are then expressed through the parabolic cylinder function $F_{n}^\kappa \bigl( \sqrt{2H} - k \sqrt{\frac{2}{H}} \bigr)$, where $a = \frac{E_n - H + 1}{2H}$.

The eigenvalue of the linear equation (5) corresponding to the ground state of the system is a function of $k$ and has a minimum where

$$E_0 = |k - k_0(H)|^2 - \epsilon + \gamma^2 \int_0^\infty g_0^n(x)dx \quad (6)$$

where $\epsilon = 1 - H/H_c3$ and $k_0(H) \sim \sqrt{H}$. First two terms in the equation (6) can be obtained from the boundary condition $\frac{d}{dx} D_0 \bigl( \sqrt{2H} - k \sqrt{\frac{2}{H}} \bigr) |_{x=0}$.

The ground state eigenvalue condition $E_0 = 0$ describes the superconducting transition point. The choice of $k = k_0(H)$ corresponds to the zero current case, further we will treat only this case. Finally, we obtain

$$\gamma^2 = \epsilon \left( \int_0^\infty dxg_0^n(x) \right)^{-1} \quad (7)$$

We also provide the numerical solution for the nonlinear Ginzburg-Landau equation. Fig. 2 shows the zero current case $k = k_0(H)$ solution for different values of the magnetic field. Indeed, the order parameter is localized in the vicinity of the edge of the film at distance of the order of the magnetic length. The amplitude of the order parameter vanishes with increasing the magnetic field. The deviations from the solution obtained by the perturbation theory are small up to $H \approx H_{c2}$.

For practical purpose it is convenient to approximate the solution for the $g(x)$ by the function

$$\tilde{g}(x) = \left( \frac{4bH}{\pi} \right)^{1/4} e^{-\frac{bHx^2}{2}} \quad (8)$$

where $b = \sqrt{1 - 2/\pi}$ and $H_{c3} = 1/b \simeq 1.66$, that differs from the exact one only by the $2\%$, while $k_0(H) = \sqrt{H/b\pi}$. Using equations (3) and (8) we find the approximate solution to the nonlinear Ginzburg-Landau equation

$$\tilde{\Psi}_0(x, y) = (\epsilon \sqrt{2})^{1/2} e^{-H_{c3}x^2} e^{ik_0y} \quad (9)$$

It is well known that phase slip events in 1D superconducting wires as well as vortices creation in superfluid liquid at $T \leq T_c$ are due to thermal activation of the order parameter. The order parameter switches between the metastable states of the superconductor changing phase by $2\pi$. The probability of such process is governed by the Arrhenius law $\propto \exp(-\Delta F/T)$, where $\Delta F$ is the energy barrier separating these states. Solution for the phase slip event corresponds to the saddle point of the barrier.

We have found the numerical solution for the phase slip center in the edge layer of the thin film by analyzing the time-dependent Ginzburg-Landau equation (for the review see [19]) with the periodic boundary conditions on the order parameter. The solution for the amplitude of the order parameter is shown in Fig. 3 for different values of $x$ at the magnetic field $H = 1.6, k = 0.93$, which corresponds to $\epsilon = 0.1$. In order to derive the approximate analytical solution for the phase slip center we will search for the order parameter in the form

$$\Psi_1(x, y) = \gamma e^{ik_0y} \sum_{n \geq 0} C_n(y) g_n(x), \quad \text{where the summation is over the set of eigenfunctions of the equation (6)}$$

In the zero-mode regime taking into account only $n = 0$, ****
we find
\[ \frac{d^2 C}{dy^2} + \epsilon C(1 - C^2) = 0 \]  
(10)

This equation is similar to the Ginzburg- Landau equation for the one- dimensional wire at zero applied current. Solving equation (10) we obtain the expression for the order parameter corresponding to phase slip event in surface superconductivity
\[ \Psi_1(x, y) = \Psi_0(x, y) \tanh \left( y \sqrt{\epsilon/2} \right) \]  
(11)

Taking into account solution (11) we conclude according to [7] with the expression for the resistivity of the thin film superconducting edge layer.
\[ R = \frac{\pi \hbar^2 \Omega}{2e^2 T} \exp(-\Delta F/T) \]  
(12)

where
\[ \Delta F = \frac{b \sqrt{2H_c^2(T)}}{16\pi \kappa^2} [\ell_{H,3}^d] \int dx dy \left( |\Psi_0|^4 - |\Psi_1|^4 \right) \]  
(13)

is the saddle-point free energy barrier increment, \( \Omega = (L/\xi(T))(e^{3/2}/\tau_{GL})(\Delta F/T)^{1/2} \) is the attempt frequency, \( L \) is the length of the edge superconducting layer, \( \tau_{GL} = [\pi \hbar/8(T_{c,3}(H) - T)] \) is the relaxation time and \( \kappa \) is the GL parameter.

Using equations (9) and (11) at \( H < H_{c,3}(T) \) we find
\[ \Delta F = \frac{b \hbar^2 H_c^2(T)}{12\sqrt{\pi \kappa^2} [\ell_{H,3}]^d} \epsilon^{3/2} \]  
(14)

where \( d \) is the film thickness, \( \ell_{H,3} = \sqrt{\hbar c/eH_{c,3}(T)} \) is the magnetic length.

This expression is simply the condensation energy of the superconductivity in volume \( \ell_{H,3} \ell_H d \) of thin film superconducting edge layer.

Notice that the width of the edge superconducting layer \( \ell_H \) at \( T \to T_c(H) \) is much smaller than the length of the normal part of the layer \( \ell_{H,3}/\sqrt{\epsilon} \) caused by the thermal activation of the phase slip event, pointing the applicability of the thin wire approximation to the surface superconductivity of thin film.

Phase slip events in 1D superconducting wires at low temperatures are argued to be due to quantum tunneling [11, 12, 13]. The resistivity of the wire is then \( R \propto \exp(-2S) \), where the exponent of the tunneling amplitude is \( S = A \hbar \xi G_c \) and \( A \) is a constant of the order of unity, \( G_c \) is the conductance of the wire of length \( \xi(T) \) [13]. In our case of the edge superconducting layer it is the magnetic length \( \ell_{H,3} \) that governs the tunneling amplitude. The tunneling process is accompanied by the creation of the acoustic plasmons [17] which are responsible for the interaction between the phase slip centers. As a result, this interaction suppresses the tunneling probability and this effect is stronger as smaller the plasmons velocity, i.e. as stronger the Coulomb screening. The dissipation effects are the second factor that leads to decreasing of the probability of the phase slip centers due to quantum tunneling [15].

In contrast to the case of isolated superconducting wire, the Coulomb interactions in the regime of edge superconductivity is screened by the charge of the normal part of the film. This immediately leads to decreasing of the acoustic plasmons velocity. The interaction with the normal part of the film results in the effective relaxation of the order parameter phase fluctuations. Consequently, the resistivity of the surface superconducting layer of the thin film should be lower than the resistivity of the wire, taken under the same conditions.

Let us consider the fluctuations of the order parameter in the case of surface superconductivity at magnetic fields \( H > H_{c,3}(T) \). The corresponding Aslamazov- Larkin correction to the conductivity at magnetic fields higher than the superconducting transition field \( H_{c,3} \) was studied in the paper [18]. It was shown that for the case of two- dimensional surface superconducting layer this correction has the same temperature dependence as for the thin film at \( T > T_c \).

However, in present work we focus on the extreme case of surface superconductivity, when the order parameter is concentrated in the quasi-one- dimensional layer of the thin film.

Fluctuations of the order parameter at magnetic fields \( H > H_{c,3}(T) \) result in additional correction to the conductance which according to [19] is
\[ G = \frac{(2e)^2}{2m} \sum_{\nu} \langle |\phi_{\nu}|^2 \rangle \frac{T_{nu}}{2} \]  
(15)

where summation goes over the set \( \nu = n, k \). The value of fluctuation of the order parameter \( \Psi(x, y) =
while $\tau_{\nu} = \tau_{GL}/E_n(k)$ is the characteristic decay time of the fluctuation. The lowest eigenvalue with $n = 0$ gives the main contribution to the sum and coming from summation to the integration over $k$ we obtain the correction to the conductance of unit length of the layer

$$G \simeq 0.28 \frac{e^2 H_{c3}(0)}{h} \frac{\ell_{H_{c3}}}{|e|^{3/2}}$$  \hspace{1cm} (17)$$

Notice that the functional dependence of the correction $G \propto |e|^{-3/2}$ is similar to the case of one-dimensional superconducting wire.

It is seen also, with decreasing the temperature $H_{c3}(T)$ increases and the value of the Aslamazov-Larkin correction decreases. However, at the same time equation (17) still valid in the interval of the magnetic fields $H < H_{c3}(T)$ that increases with decreasing the temperature.

For the numerical estimations we take typical values $H_{c3} \sim 1$T then $\ell_{H_{c3}} \sim 25$nm. For ultrathin film $d \sim 10$nm, $\kappa = 10$, we estimate at $T_{c3} \sim 1$K for the value $\Delta F /T \sim 10^3|e|^{3/2}$. The probability of the phase slip event becomes negligibly small unless the parameter $\epsilon = 1 - H/H_{c3}$ is of the order of $10^{-2}$.

According [16] the critical current destroying the surface superconductivity could be written as $J_c \sim j_c \ell_{H_{c3}}d$, where $j_c = \frac{1}{2\pi \kappa \epsilon} \frac{\ell_{H_{c3}}(T)}{e}$ is the critical current density of thin wire, we estimate $J_c \sim 20 \mu$A.

To summarize, we have shown that the phase slip phenomenon reveals in edge superconducting layer of thin film in perpendicular magnetic field at $H < H_{c3}(T)$. The corresponding resistance was calculated. The Aslamazov-Larkin correction to the edge superconductivity of thin film at $H > H_{c3}(T)$ have also been obtained. We conclude that such structures could be applied as a new system for the study of the phase slip phenomenon in one-dimensional superconducting wires.

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