Polarized parton distributions from NLO QCD analysis of world DIS and SIDIS data

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**Abstract**

The combined analysis of polarized DIS and SIDIS data is performed in NLO QCD. The new parametrization on polarized PDFs is constructed. The uncertainties on PDFs and their first moments are estimated applying the modified Hessian method. The especial attention is paid to the impact of novel SIDIS data on the polarized distributions of light sea and strange quarks. In particular, the important question of polarized sea symmetry is studied in comparison with the latest results on this subject.

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Since the observation of the famous spin crisis in 1987 [1] one of the most intriguing and still unsolved problems of the modern high energy physics is the nucleon spin puzzle. The key component of this problem, which attracted the great both theoretical and experimental efforts during many years is the finding of the polarized parton distributions functions (PDFs) in nucleon.

The analysis of data on inclusive polarized DIS enables us to extract such important quantities as the singlet \( \Delta \Sigma(x, Q^2) \) and nonsinglet \( \Delta q_3(x, Q^2) \), \( \Delta q_8(x, Q^2) \) combinations of the polarized PDFs, and, thereby, the sums of valence and sea PDFs \( \Delta q + \Delta \bar{q} \equiv \Delta q_V + 2\Delta \bar{q} \). Besides, dealing with DIS data, the gluon helicity distribution \( \Delta G(x, Q^2) \) is determined due to the evolution in singlet sector and weak dependence on \( \Delta G \) of the polarized structure function \( g_1 \) in NLO QCD. However, even for singlet combinations \( \Delta u + \Delta \bar{u}, \Delta d + \Delta \bar{d}, \Delta s + \Delta \bar{s} \), considered as well-determined within DIS, we still meet the problem: there are only two equations corresponding to inclusive asymmetries \( A_1 \) measured on proton and deuteron targets from which we should determine three unknown combinations \( \Delta q_3(x), \Delta q_8(x) \) and \( \Delta \Sigma(x) \) (or, alternatively, \( \Delta u + \Delta \bar{u}, \Delta d + \Delta \bar{d}, \Delta s + \Delta \bar{s} \)). So, it is unavoidable to involve some additional assumptions performing the fitting procedure for the purely inclusive DIS data. Moreover, DIS data can not help us to solve the important problem of valence and sea PDFs separation.

The basic process which enables us to solve these problems is the process of semi-inclusive DIS (SIDIS). However, until recently the quality of the polarized SIDIS data was rather poor, so that its inclusion in the analysis did not helped us [2] to solve the main task of SIDIS measurements: to extract the polarized sea and valence PDFs of all active flavors. Only in 2004 the first polarized SIDIS data with the identification of produced

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[^4]: As long as no neutrino factory is built and no hyperdense polarized target is created, we cannot study the DIS processes with the neutrino beam, which would enable us to find the polarized valence and sea quark distributions separately.
hadrons (pions and kaons) were published [3]. These data were included in the global QCD analysis in Ref. [1]. Further, COMPASS [5] presented the polarized SIDIS data (without identification of hadrons in the final state), in particular, in the low $x$ region unaccessible for HERMES. This data were included in the latest parametrization of Ref. [3]. Recently, the new data on the SIDIS asymmetries $A_\pi^\pm$, $A_d^K\pm$ were published [7] by the COMPASS collaboration. It is of importance that this data cover the most important and badly investigated low $x$ region. In this paper we include this data in the new global QCD analysis of all existing polarized DIS and SIDIS data. The elaborated parametrization on the polarized PDFs in some essential points differs from the parametrization of Ref. [6] (see below). Nevertheless, the results on PDFs obtained with both parametrizations are compatible within the errors.

Constructing the new parametrization we tried to be as close as possible to our previous NLO QCD analysis of pure inclusive DIS data [8]. Namely, to properly describe the DIS data we, besides $\Delta \Sigma$, $\Delta q_3$, $\Delta q_8$ combinations at the initial scale $Q_0^2 = 1\, \text{GeV}^2$

$$\Delta \Sigma = \eta_{\Delta \Sigma} \int_0^1 x^{2\alpha_{\Delta \Sigma}} (1-x)^{\beta_{\Delta \Sigma}} \, dx$$

$$\Delta q_3 = \eta_{\Delta q_3} \int_0^1 x^{\alpha_{\Delta q_3}} (1-x)^{\beta_{\Delta q_3}} (1+\gamma_{\Delta q_3}) \, dx,$$

$$\Delta q_8 = \eta_{\Delta q_8} \int_0^1 x^{\alpha_{\Delta q_8}} (1-x)^{\beta_{\Delta q_8}} (1+\gamma_{\Delta q_8} x + \delta_{\Delta q_8} \sqrt{x}) \, dx.$$  

Then, the quantities $\Delta u + \Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta s = \Delta \bar{s}$ are determined as:

$$\Delta u + \Delta \bar{u} = \frac{1}{6} (3\Delta q_3 + \Delta q_8 + 2\Delta \Sigma), \quad \Delta d + \Delta \bar{d} = \frac{1}{6} (3\Delta q_3 - \Delta q_8 + 2\Delta \Sigma),$$

$$\Delta s = \Delta \bar{s} = \frac{1}{6} (\Delta \Sigma - \Delta q_8).$$

The gluon PDF is parametrized as

$$\Delta G = \eta_{\Delta G} \int_0^1 x^{\alpha_{\Delta G}} (1-x)^{\beta_{\Delta G}} (1+\gamma_{\Delta G} x) \, dx.$$  

It is easy to see that in our parametrization the coefficients $\eta$ are just the first moments of the respective local quantities. In particular, the advantage of $\Delta q_3$ and $\Delta q_8$ parametrization in the form (2), (3) is that with the such choice it is very convenient to apply and control the $SU_f(2)$ and $SU_f(3)$ sum rules

$$\eta_{\Delta q_3} \equiv \Delta_3 q_3 \equiv \int_0^1 dx \Delta q_3(x) = F + D = 1.269 \pm 0.003,$$

$$\eta_{\Delta q_8} \equiv \Delta_8 q_8 \equiv \int_0^1 dx \Delta q_8(x) = 3F - D = 0.586 \pm 0.031.$$  

Besides, the such inheritance with Ref. [8] allow us to clearly see the impact of SIDIS data on the results of pure inclusive DIS data analysis.

Further, to properly describe the SIDIS data we, besides $\Delta \Sigma$, $\Delta q_3$ and $\Delta q_8$, parametrize the sea PDFs of $u$ and $d$ flavors:

$$\Delta \bar{u} = \eta_{\Delta \bar{u}} \int_0^1 x^{2\alpha_{\Delta \bar{u}}} (1-x)^{\beta_{\Delta \bar{u}}} \, dx, \quad \Delta \bar{d} = \eta_{\Delta \bar{d}} \int_0^1 x^{\alpha_{\Delta \bar{d}}} (1-x)^{\beta_{\Delta \bar{d}}} \, dx.$$  


Then, $\Delta u$ and $\Delta d$ are determined from Eqs. (11) and (9), while the valence PDFs are determined by $\Delta u_{\nu} = \Delta u - \Delta \bar{u}$ and $\Delta d_{\nu} = \Delta d - \Delta \bar{d}$. Thus, all polarized PDFs are completely determined within the parametrization.

Comparing DIS sector, Eqs. (11)-(3), (6) with the respective parametrizations from Ref. [8], one can see some distinctions. These are additional factors $\gamma_{\Delta q_3} x$, $\gamma_{\Delta q_8} x$ in Eqs. (2), (3) which are introduced to provide the better flexibility of the parametrizations on the respective quantities, required by the inclusion of SIDIS data. Besides, we introduce the additional factors $\delta_{\Delta q_8} \sqrt{x}$ in Eq. (3) and $\gamma_{\Delta G} x$ in Eq. (6) to provide the possibility of sign-changing scenarios for $\Delta s$ and $\Delta G$, respectively (see below). We consider two options for handling of $SU_f(2)$ and $SU_f(3)$ sum rules. In the first case we apply the constraints (7) and (8) putting $\eta_{\Delta q_3}$ and $\eta_{\Delta q_8}$ equal to the central values of the respective constants. In the second case we allow $\eta_{\Delta q_3}$ and $\eta_{\Delta q_8}$ to vary within the uncertainties for $F + D$ and $3F - D$. It is of importance that, as we will see below, both options produce almost the same results.

We analyze the inclusive $A_1$ and semi-inclusive $A_1^h$ asymmetries. The inclusive asymmetry reads

$$A_{1p} = \frac{g_{1p}(x, Q^2)}{F_{2p}(x, Q^2)/\{2x(1 + R(x, Q^2))\}},$$

(10)

where the polarized structure function $g_{1p}$ in NLO QCD looks as

$$2g_{1p} = \sum_{q, q'} \left\{ \Delta q + \frac{\alpha_s(Q^2)}{2\pi} \left[ \Delta C_q \otimes \Delta q + \frac{1}{f} \Delta C_g \otimes \Delta G \right] \right\}.$$  

(11)

Throughout this paper we use the $\overline{MS}$ factorization scheme. The respective coefficient functions $\Delta C_{q,g}$ can be found in Ref. [9]. The semi-inclusive asymmetries, besides $x$ and $Q^2$, depend also on hadronic variable $z$. As usual, we apply the semi-inclusive asymmetries

$$A_{1p}^h(x, Q^2) = \frac{\int_{0.2}^{1} dz g_{1p}^h(x, Q^2, z)}{\int_{0.2}^{1} dz F_{2p}^h(x, Q^2, z)/\{2x(1 + R(x, Q^2))\}}$$

(12)

integrated over the cut $z > 0.2$, which corresponds to the current fragmentation region. The semi-inclusive structure functions $g_{1p}^h$ in NLO QCD are given by

$$2g_{1p}^h = \sum_{q, q'} e_q^2 \Delta q \{ 1 + \frac{\alpha_s(Q^2)}{2\pi} \Delta C_{qq} \otimes D^h_q + \{ \sum_{q, q'} e_q^2 \Delta q \} \otimes \frac{\alpha_s}{2\pi} \Delta C_{gg} \otimes D^h_g \} + \Delta G \otimes \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \{ \sum_{q, q'} e_q^2 D^h_q \}.$$  

(13)

The respective Wilson coefficients $\Delta C_{qq,gg,gg}$ can be found in [10]. Of great importance for the SIDIS data analysis is the choice of parametrization on the fragmentation functions. We use here the latest NLO parametrization from Ref. [11]. Calculating $F_2$ in Eq. (11) and $F_2^h$ in Eq. (12) we use parametrization for $R$ from [12] and the recent NLO parametrization on unpolarized PDFs from Ref. [13]. For the $\alpha_s(Q^2)$ calculation we apply the same procedure as in Ref. [13] (i.e. $\alpha_s(Q^2)$ in $\overline{MS}$ scheme is calculated just as in [13] with $\alpha_s(M_Z^2) = 0.1145$). The deuteron structure functions $g_{1d}$ and $g_{1d}^h$ are calculated applying $g_{1d}^h = (g_{1p}^h + g_{1n}^h)(1 - \frac{3}{2} \omega_d)/2$ with $\omega_d = 0.058$ (see, for instance [8]).
In our analysis the positivity constraint $|\Delta q| < q$ and $|\Delta G| < G$ holds with the precision 0.001.

Of importance is the correct application of the factor $1 + \gamma^2 = 1 + 4M^2 x^2 / Q^2$ in the analysis – see the discussion on this question in Ref. [14]. When one rewrites the asymmetry $A_1$ in terms of $F_2$ instead of $F_1$ then this factor precisely cancel out in the ratio of $g_1(1 + \gamma^2)$ and $F_1 = F_2(1 + \gamma^2)/2x(1 + R)$, so that we just arrive at right-hand side of Eq. (10). The same cancellation holds in SIDIS case, so that one arrives at Eq. (12). At the same time, sometimes the inclusive data is tabulated in the form $A_1 = (1 + \gamma^2)g_1/F_1$. In this case we fit the experimental values $g_1/F_1$ by the $g_1/[(1 + \gamma^2)F_2/2x(1 + R)]$.

One of the main conditions of the successful global QCD analysis is the robust program for the DGLAP solution. The respective program should be fast, well tested and should provide a good precision of DGLAP solution. The elaborated in Ref. [8] program for the polarized DIS data analysis, based on inverse Mellin transformation method, satisfies to all these requirements. So, we again use here this program package, properly modifying it in accordance with the peculiarities of SIDIS data (calculation of SIDIS structure functions and asymmetries in the space of Mellin moments are included).

All procedures of the global QCD analysis are based on the construction of the effective $\chi^2$ function that describes the quality of the fit to data for a given set of varying parameters $\{a_i\}$. In the case of polarized DIS and SIDIS analysis one usually uses the $\chi^2$ function in the form (see, for instance, [6] for detail)

$$\chi^2 = \sum_i \left( \frac{A_{\text{exp}} - A_{\text{theor}}(\{a_i\})}{\delta(A_{\text{exp}})} \right)^2,$$

where $A_{\text{exp}}$ is the measured value of the asymmetry, $\delta(A_{\text{exp}})$ is its uncertainty, $A_{\text{theor}}$ is its theoretical estimation. For the minimization of $\chi^2$ function we use the MINUIT package [16].

For our analysis we collected all accessible in literature polarized DIS and SIDIS data. We include the inclusive proton data from Refs [17, 18, 19, 20, 15], inclusive deuteron data from Refs [21, 17, 18, 22, 20, 15] and inclusive neutron data from Refs [23, 24, 25, 26]. The semi-inclusive data (asymmetries $A_{p,d,\pi}^{K^+,K^-}$, $A_{p,d}^{\pi^+,\pi^-}$, $A_d^{K^+,K^-}$) are taken from Refs [3, 27, 5] and, besides, we include the latest COMPASS data from Ref [7]. In total we have 232 points for the inclusive polarized DIS and 202 points for semi-inclusive polarized DIS. If we fix $\eta_{\Delta q_3}$ and $\eta_{\Delta q_8}$ by the center values of $F + D$ and $3F - D$ in sum rules (7), (8), then for 16 fit parameters $\chi_0^2|\text{inclusive} = 188.4$ and $\chi_0^2|\text{semi-inclusive} = 194.8$ for DIS and SIDIS data, while $\chi_0^2|\text{total} = 383.9$ for the full set of data (434 points). On the other hand, if $\eta_{\Delta q_3}$ and $\eta_{\Delta q_8}$ are varied within the errors on $F + D$ and $3F - D$, then for 18 fit parameters the resulting $\chi^2$ values are: $\chi_0^2|\text{inclusive} = 188.2$, $\chi_0^2|\text{semi-inclusive} = 194.8$ with $\chi_0^2|\text{total} = 383.7$. Thus, one can conclude that the fit quality is quite good: $\chi_0^2/D.O.F. \simeq 0.84$.

The optimal values of our fit parameters are presented in Table 1 for both options, with fixed and varied $\eta_{\Delta q_3}$ and $\eta_{\Delta q_8}$. As it is seen from Table 1 the results are almost the same in both cases: the differences in parameters are less than 1%.

Our calculations show that the fit quality does not decrease if we cancel the extra parameters setting $\beta_{\Delta q_8}$ equal to $\beta_{\Delta q_3}$, since their values occur very close to each other.

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5We treat $(\delta(A_{\text{exp}}))^2$ in Eq. (14) as the quadratic sum of the statistical and systematical errors (see discussion on this question in Ref. [6]).
Table 1: Optimal values of the global fit parameters at the initial scale $Q_0^2 = 1 \text{GeV}^2$.

| Parameter | $\Delta \Sigma$ | $\Delta q_3$ | $\Delta q_8$ |
|-----------|----------------|--------------|--------------|
| $\alpha$  | 1.0227 (1.0216) | -0.6342 (-0.6380) | -0.7916 (-0.7827) |
| $\beta$   | 3.3891 (3.3873) | 3.1418 (3.1398) | $= \beta_{\Delta q_3}$ |
| $\gamma$  | 0.0 (fixed) | 23.9180 (24.2240) | 36.8400 (37.1990) |
| $\delta$  | 0.0 (fixed) | 0.0 (fixed) | -13.7480 (-13.8390) |
| $\eta$    | 0.3846 (0.3850) | 1.2660 (1.2690) | 0.6170 (0.5860) |

| Parameter | $\Delta G$ | $\Delta \bar{u}$ | $\Delta d$ |
|-----------|------------|----------------|------------|
| $\alpha$  | 0.9040 (0.9154) | -0.3506 (-0.3412) | 0.2802 (0.2852) |
| $\beta$   | $= \beta_{\Delta \bar{u}}$ | 15.0 (fixed) | $= \beta_{\Delta \bar{d}}$ |
| $\gamma$  | -5.6703 (-5.6317) | 0.0 (fixed) | 0.0 (fixed) |
| $\delta$  | 0.0000 (fixed) | 0.0 (fixed) | 0.0 (fixed) |
| $\eta$    | -0.1828 (-0.1813) | 0.0672 (0.0670) | -0.0792 (-0.0794) |

when we try to find the best fit. Besides, we use the equality $\beta_{\Delta \bar{u}} = \beta_{\Delta \bar{d}} = \beta_{\Delta G}$ (just as in Ref. [6]) since the polarized data at $x > 0.6 \div 0.7$ is just absent, while in the region $0.4 \div 0.7$ the statistical errors are too large to feel the difference in values of these parameters.

Certainly, the construction of the best fit should be accompanied by the reliable method of uncertainties estimation. We choose the modified Hessian method [28], [29] which well works (as well as the Lagrange multipliers method – see [6] and references therein) even in the case of deviation of $\chi^2$ profile from the quadratic parabola, and was successfully applied in a lot of physical tasks.

Let us recall that the standard Hessian method is based on the assumption that the global $\chi^2$ is quadratic near the minimum $\chi^2_0$:

$$\chi^2 = \chi^2_0 + \sum_{i,j=1}^M H_{ij}(a_i - a_i^0)(a_j - a_j^0).$$

(15)

Here $a_i^0$ are the fit parameters values at the global minimum and $H_{ij}$ are the elements of the Hessian matrix – the matrix of second derivatives of $\chi^2$ in the minimum. Then, the uncertainty on any physical quantity $F$ which depends on $M$ fit parameters $\{a_i\}$, can be estimated applying

$$(\delta F)^2 = \Delta \chi^2 \sum_{i,j=1}^M \frac{\partial F}{\partial a_i} H_{ij}^{-1} \frac{\partial F}{\partial a_j}.$$

(16)

The Hessian method in the simple form (16) is implemented in the MINUIT program [16] ("Hesse procedure") and it perfectly works if $\chi^2$ has the parabolic profile near the minimum. However, in practice we often meet the problems which decrease the reliability of this simple version of Hessian method, and main of them is the deviation of $\chi^2$ from parabolic form – see [28] for detail. To deal with asymmetric $\chi^2$ profiles we apply the modification [28], [29] of the standard Hessian method, which is proved to be well working even in the strongly asymmetric cases. Shortly, the essence of procedure is following. First, one starts with still symmetric errors $\pm \delta F$ on the physical quantity $F$, which one finds via
Figure 1: The scheme illustrating the search of asymmetric errors within the modified Hessian method.

Eq. (16) rewritten [28] in terms of eigenvectors and eigenvalues of the diagonalized Hessian matrix. Simultaneously, one calculates the values of fit parameters \{a_i\} corresponding to \(F + \delta(F)\) and \(F - \delta(F)\) and, thereby, the respective \(\chi^2\) values. To obtain asymmetric errors [29] corresponding to the real \(\chi^2\) profile one varies \(\Delta \chi^2\) in Eq. (16) (rewritten in terms of eigenvectors and eigenvalues) and calculates the respective values of the parameters, finding the intersections of \(\chi^2(F)\) curve with the straight line \(\chi^2 = \chi^2_0 + \Delta \chi^2\), where \(\Delta \chi^2\) is already fixed quantity (\(\Delta \chi^2 = 1\) or \(\Delta \chi^2 = 18.065\) here – see below). Then the differences of \(F\) values in these intersections with \(F = F_0\) in the global minimum just determine \(F + \delta^{(+)}(F)\) and \(F - \delta^{(-)}(F)\), where \(\delta^{(\pm)}(F)\) are in general asymmetric uncertainties on the physical quantity \(F\) – see Fig. 1.

Besides, very important question arises about choice of \(\Delta \chi^2\) determining the uncertainty size. The standard choice is \(\Delta \chi^2 = 1\), just as we did before in Ref. [8] (see also [6] and references therein). However, the such choice of \(\Delta \chi^2\) can lead to underestimation of uncertainties, as it was argued in Ref. [31]. The alternative choice of \(\Delta \chi^2\) (see, for example, Refs. [31], [32] and references therein) is based on the equation

\[
P = 0.68 = \int_0^{\Delta \chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{x}{2}\right)^{(M/2)-1} \exp \left(-\frac{x}{2}\right) dx,
\]

where P = 0.68 (1σ deviation) is the probability to find the values of all \(M\) fitting parameters inside the hypervolume determined by the condition \(\chi^2 \leq \chi^2_0 + \Delta \chi^2\). In our case (17 parameters) the \(\Delta \chi^2\) value calculated from Eq. (17) is equal to 18.065. We calculate the uncertainties for both \(\Delta \chi^2 = 1\) and \(\Delta \chi^2 = 18.065\) options.

The local distributions together with their uncertainties are presented in Fig. 2. The first moments of PDFs together with their uncertainties are presented in Table 2. As it was mentioned above, the results for both scenarios with fixed and varied \(\eta_{\Delta q_3}\) and \(\eta_{\Delta q_8}\) are almost the same (the differences in fitting parameters are less than 1%). That is why we present all figures and Table 2 only for the option with the varied \(\eta_{\Delta q_3}\) and \(\eta_{\Delta q_8}\).

Let us now discuss the obtained parametrization. First, one can see that the results on the first moments \(\Delta_1 \Sigma \equiv \eta_{\Delta \Sigma}\) and \(\Delta_1 G \equiv \eta_{\Delta G}\) are very close to the respective results

6For the respective calculations we apply the program ITERATE by J. Pumplin [30].
Figure 2: Best fit PDFs (solid lines) together with their uncertainties for $\Delta \chi^2 = 1$ (inner bands) and $\Delta \chi^2 = 18.065$ (outer bands).

(scenario with $\Delta G < 0$) obtained in Ref. [8] in the case of pure inclusive DIS. Indeed, even with the more smaller errors, corresponding to the option $\Delta \chi^2 = 1$, at $Q^2 = 3 \text{ GeV}^2$ we have $\Delta_1 \Sigma = 0.372^{+0.008}_{-0.014}$ and $\Delta_1 G = -0.161^{+0.096}_{-0.146}$ in this paper and $\Delta_1 \Sigma = 0.329^{+0.004}_{-0.012}$ and $\Delta_1 G = -0.181^{+0.042}_{-0.031}$ in Ref. [8], respectively. Notice that the obtained value $\Delta_1 \Sigma = 0.372^{+0.008}_{-0.014}$ is even more close to the respective NLO value $a_0(Q^2 = 3 \text{ GeV}^2) = 0.35 \pm 0.03$ obtained by COMPASS [8] directly (without fitting procedure), from the first moment of the measured structure function $g_1d$. From Table 2 one can see also that in the more pessimistic case $\Delta \chi^2 = 18.065$, $\Delta_1 G$ value is just zero within the errors – we will return to this question below. The impact of SIDIS data on the $\Delta G(x)$ shape will be also discussed some later. What concerns nonsinglet combinations $\Delta q_3(x)$ and $\Delta q_8(x)$, we see (compare Table 4 with Table 3 from Ref. [8]) that, even despite the small difference in initial scales ($Q_0^2 = 1 \text{ GeV}^2$ here and $Q_0^2 = 3 \text{ GeV}^2$ in Ref. [8]), the values of $\Delta q_3(x)$ are very
Table 2: Estimations of the uncertainties on the first moments of polarized PDFs for two options of $\Delta \chi^2$ choice.

|                  | $\Delta \chi^2 = 1$            | $\Delta \chi^2 = 18.065$ |
|------------------|--------------------------------|---------------------------|
| $\Delta \Sigma$  | $0.3846^{+0.0059}_{-0.0122}$  | $0.3846^{+0.0342}_{-0.0389}$ |
| $\Delta u + \Delta \bar{u}$ | $0.8640^{+0.0028}_{-0.0049}$  | $0.8640^{+0.0114}_{-0.0084}$  |
| $\Delta d + \Delta \bar{d}$ | $-0.4020^{+0.0048}_{-0.0048}$  | $-0.4020^{+0.0115}_{-0.0120}$  |
| $\Delta s = \Delta \bar{s}$ | $-0.0387^{+0.0014}_{-0.0024}$  | $-0.0387 +0.0061_{-0.0065}$  |
| $\Delta G$       | $-0.1828^{+0.0720}_{-0.1090}$  | $-0.1828^{+0.1063}_{-0.2831}$  |
| $\Delta \bar{u}$ | $0.0672^{+0.0263}_{-0.0270}$  | $0.0672^{+0.0643}_{-0.0737}$  |
| $\Delta \bar{d}$ | $-0.0792^{+0.0191}_{-0.0238}$  | $-0.0792^{+0.0795}_{-0.0830}$  |

similar for both parametrizations. At the same time, the values of $\Delta q_s(x)$ differ much more essentially, which occurs due to the impact (first of all on $\Delta s$ – see below) of the additional SIDIS data.

Let us now compare our results on $\Delta G$ with the respective results from Refs. [6] and [31]. The such comparison seems to be reasonable because among the number of all parametrizations applying the DIS data, the SIDIS data is included only in the sequel of papers Ref. [2], [4], [6]. At the same time, though SIDIS data is not included in the analysis of Ref. [31], the RHIC $\pi^0$ production data is added there (just as in Ref. [6]), which should provide significant impact on $\Delta G$ in the RHIC $x$ region [0.05, 0.2].

![Figure 3](image_url)

Figure 3: Obtained parametrization on $\Delta G$ (solid line) in comparison with the respective parametrizations from Ref. [6] (dashed line) and Ref. [31] (dot-dashed line) at $Q^2_0 = 1 \text{ GeV}^2$.

The results of comparison are presented in Fig. 3. From this figure one can see that the best fit results on $\Delta G$ presented in this paper and in Ref. [6] are very close to each other while they are significantly differ (in both $\Delta G < 0$ and $\Delta G > 0$ regions) from the respective result of Ref. [31], even for the same sign-changing scenario for $\Delta G$. On the other hand, comparing the results on the first moment $\Delta_1 G \equiv \eta_G$, one can see that the central values of this quantity are almost the same for all three parametrizations: these are $-0.183$, $-0.118$ and $-0.120$ in this paper, Refs. [6] and [31], respectively. Moreover, including the uncertainties in comparison (see Table 2 and Table IV in Ref. [6]), one can see that the central values of this quantity are almost the same for all three parametrizations: these are $-0.183$, $-0.118$ and $-0.120$ in this paper, Refs. [6] and [31], respectively.

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7In in this paper we are mainly interested in the study of the SIDIS data impact on the polarized PDFs (especially on the still poorly known sea quark PDFs). The influence of RHIC $\pi^0$ production data on our parametrization will be considered some later.
Figure 4: Obtained parametrization on $\Delta s$ (solid line) in comparison with the respective parametrizations from Ref. [6] (dashed line) and [8] (dot-dashed line), $Q^2 = 1\, GeV^2$.

Ref. [31]), we see that the first moment $\Delta_1 G$ is just zero within the errors for all three parametrizations. Thus, we can conclude that with the present quality of the data it is hardly possible to realize the first moment $\Delta_1 G$ zero or not. To answer this question and also to distinguish between the different shapes of $\Delta G(x)$, still more precise data is necessary.

Let us now discuss the impact of SIDIS data on the polarized strangeness in nucleon. It is of importance because, after the appearance of the first results on $\Delta s$ extraction from SIDIS data performed by HERMES [3], we met the puzzle with the positive $\Delta s$ in the middle $x$ HERMES region $0.023 < x < 0.6$, while the total moment $\Delta_1 s$ definitely should be negative [33] in accordance with the sum rule [8]. Thus, to satisfy this requirement, $\Delta s(x)$ should possess the compensating negative behavior in the unaccessible for HERMES low $x$ region $0 < x < 0.023$, i.e. the sign-changing scenario for $\Delta s$ should be realized. Looking at Fig. 2 we see that this is indeed the case and we produce the best fit namely for the sign-changing $\Delta s$ scenario, as well as in Ref. [6]. However, we see also the distinction in the $\Delta s$ shape, in comparison with Ref. [6] – see Fig. 4. Namely, while within parametrization [6] $\Delta s$ changes the sign one time, within our parametrization $\Delta s$ changes the sign twice. It seems that this distinction occurs due to the inclusion of the latest COMPASS semi-inclusive data [7], which allow to better fix $\Delta s$ shape. This is illustrative to compare the NLO results on $\Delta s$ obtained here with the respective results of direct $\Delta s$ extraction in LO by COMPASS – see Fig. 5. We see very similar $\Delta s$ behavior in both cases. Certainly, one should be careful comparing LO and NLO results. However, the LO and NLO results do not differ too drastically, so that the such comparison is very useful and allow us to make at least qualitative conclusion about the PDFs behavior (shape of distributions). The new COMPASS data on the kaon asymmetries in the wide Bjorken $x$ region $[0.003, 0.7]$ will be available in the nearest future (the paper in

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8Notice that within this paper we apply the sign-changing scenario for $\Delta G(x)$ and do not consider scenario $\Delta G(x) > 0$ which was considered in Refs. [8] and [31]. For this scenario the first moment of $\Delta G$ is of opposite sign and larger in absolute value, about $+0.3 \div +0.4$ instead of $-0.1 \div -0.2$. Nevertheless, the such scenario also produce quite acceptable $\chi^2$ value. The such arbitrariness even in the sign of $\Delta_1 G$ once again tell us that we still need more data to properly fix $\Delta G$ and that even inclusion of $\pi^0$ production data (Refs. [6], [31]) still poorly enables us to solve the problem.

9Notice that after inclusion of the latest COMPASS data to our fit $\chi^2/D.O.F.$ value becomes small only if we allow $\Delta s$ to change the sign twice (due to the additional parameter $\delta_{\Delta q s} \sqrt{x}$).
Figure 5: Obtained NLO parametrization on $\Delta s$ (solid line) in comparison with the COMPASS results [7] obtained in LO QCD (points with error bars), $Q^2 = 3\, GeV^2$.

preparation). Thus, one can hope to eventually fix the strangeness in nucleon.

As it was discussed above, the exclusive task for SIDIS is the finding of sea $\Delta \bar{q}$ (and, consequently, valence $\Delta q_V = \Delta q - \Delta \bar{q}$) PDFs in nucleon. We again compare our results on $\Delta \bar{u}$ and $\Delta \bar{d}$ with the respective results from Ref. [6]. Comparing the central values (best fit values) of $\Delta \bar{u}$ and $\Delta \bar{d}$ one can see that they are quite similar – see Fig. [6]. At the same time, there are also some distinctions, and main of them are connected with the sum $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ and its first moment $\Delta \bar{u} + \Delta \bar{d}$. The point is that recently, analyzing

Figure 6: Obtained parametrization on $\Delta \bar{u}$ and $\Delta \bar{d}$ (solid lines) in comparison with the respective parametrization of Ref. [6] (dashed lines), $Q^2 = 1\, GeV^2$.

the SIDIS data on $h^\pm$ production COMPASS obtained rather surprising result [5] that the sum $[\Delta_1 \bar{u} + \Delta_1 \bar{d}](Q^2 = 10\, GeV^2)$ is just zero within the errors (see Table 2 in Ref. [5])

$$[\Delta_1 \bar{u} + \Delta_1 \bar{d}]_{COMPASS} = 0.0 \pm 0.04 \pm 0.03.$$  \ (18)

This result was confirmed in the subsequent COMPASS paper [7], where sum $\Delta \bar{u}(x, Q^2 = 3\, GeV^2) + \Delta \bar{d}(x, Q^2 = 3\, GeV^2)$ of the local PDFs was extracted from the measured asymmetries $A_{1d}, A_{1d}^{\pi^\pm}, A_{1d}^{K^\pm}$ in the region $0.004 < x < 0.3$ (see Fig. 4 in Ref. [7]) and occurs to be about zero in the whole this region (the central values occur in both positive and negative vicinities of zero). Thus, at least in the leading order (COMPASS analysis) the sum $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ is about zero in the region $3\, GeV^2 < Q^2 < 10\, GeV^2$, which sheds
new light on our understanding of polarized light quark sea. Namely, the sea is extremely asymmetric ($\Delta \bar{u} \simeq -\Delta \bar{d}$), on the contrary to the assumption of symmetric sea scenario $\Delta \bar{u}(x, Q_0^2) = \Delta \bar{d}(x, Q_0^2)$, applied in the practically all existing parametrizations based on the pure inclusive DIS data analysis. Our analysis shows that the sum $\Delta \bar{u} + \Delta \bar{d}$ is very small quantity in NLO QCD too. It is very illustrative for qualitative comparison to put our best NLO QCD fit for $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ (evolved to the COMPASS $Q^2 = 3 \text{GeV}^2$) on the figure (Fig. 4 in Ref. [7]) showing the COMPASS LO results on this quantity – see Fig. 7. From Fig. 7 it is clearly seen that $\Delta \bar{u}(x) + \Delta \bar{d}(x)$ is very small quantity in both LO and NLO QCD orders. In turn, the first moment $\Delta_1 \bar{u} + \Delta_1 \bar{d}$ for the proposed parametrization is also just zero within the errors

$$[\Delta_1 \bar{u} + \Delta_1 \bar{d}](Q^2 = 1 \text{GeV}^2) = -0.01^{+0.01}_{-0.02},$$

(19)
even in the case $\Delta \chi^2 = 1$. Notice that QCD evolution weakly influences this result. Even at extremely large $Q^2$ value 100 GeV$^2$ (for instance, for COMPASS upper bound on accessible $Q^2$ is about 60 GeV$^2$) the quantity $\Delta_1 \bar{u} + \Delta_1 \bar{d}$ is still very close to zero within the errors, even in the case $\Delta \chi^2 = 1$:

$$[\Delta_1 \bar{u} + \Delta_1 \bar{d}](Q^2 = 100 \text{GeV}^2) = -0.02^{+0.01}_{-0.03}.$$  

(20)

Notice that looking on parametrization of Ref. [6] (see Fig. 3 there) we see that the distributions $\Delta \bar{u}(x)$ and $\Delta \bar{d}(x)$ are also of opposite sign. However, they are significantly differ in their absolute values (just as in the DIS parametrization GRSV2000 [34] – see footnote[10]). As a result, the central value of $[\Delta_1 \bar{u} + \Delta_1 \bar{d}](Q_0^2 = 1 \text{GeV}^2)$ in Ref. [6] (see Table IV there) is $-0.08$ (i.e. almost the same as $\Delta_1 \bar{d}$ central value $-0.11$) instead of $-0.01$ in Eq. (19).

The important remark should be made here. At present, the SIDIS data is of such quality that all above conclusions about sea PDFs should be considered as preliminary. Indeed, looking at Fig. 2 here and Fig. 3 in Ref. [6] we see that the uncertainties on $\Delta \bar{u}$ and $\Delta \bar{d}$ are rather large even for the option $\Delta \chi^2 = 1$, while in the case of $\Delta \chi^2$ determined by Eq. (14) ($\Delta \chi^2 = 18.065$ here) $\Delta \bar{u}$ and $\Delta \bar{d}$ are still the zeros within the errors, and

\footnote{The only exception is asymmetric GRSV2000 parametrization [34]. However, though in this parametrization $\Delta \bar{u}$ and $\Delta \bar{d}$ are also of opposite sign, they are strongly differ in absolute value.}
even just to see them within this option for $\Delta \chi^2$ we need more SIDIS data (first of all the expected COMPASS data).

In conclusion, the new combined analysis of polarized DIS and SIDIS data in NLO QCD is presented. The impact of modern SIDIS data on polarized PDFs is studied, which is of especial importance for the light sea quark PDFs and strangeness in nucleon. The obtained results are in agreement with the latest direct leading order COMPASS analysis of SIDIS asymmetries as well as with the recent global fit analysis in NLO QCD of Ref. 6, where the SIDIS data were also applied. Nevertheless, there also some distinctions concerning, first of all, the polarized quark sea peculiarities. At the same time, the quality of SIDIS data is still not sufficient to make the eventual conclusions about the quantities influenced mainly by SIDIS. In this situation of especial importance becomes the direct $\Delta q$ extraction in NLO QCD, where (just as in LO QCD) the central values of asymmetries and their uncertainties directly propagate to the extracted $\Delta q$ values and their errors. The such NLO QCD method, free of any fitting procedure with a lot of parameters, was elaborated in the sequel of papers 35 (see review 36 for details). At present the respective analysis of the whole existing polarized DIS and SIDIS data is in preparation. In any case, the new COMPASS semi-inclusive data should be available in the nearest future. In particular, we expect $\pi^\pm$, $K^\pm$ data on the proton target in the wide $x$ region (today the only available such data is the HERMES data in the narrow $x$ region and only for $\pi^\pm$ production), which should essentially increase the precision of $\Delta \bar{u}$, $\Delta \bar{d}$ and $\Delta s$ extraction.

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