ANN Based Design Parameter Estimation for Structural Systems

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Abstract. Estimation of the probability of failure of multi-dimensional structural systems is expensive from the computation perspective. To decrease the burden of computation, one can use simple approximation methods like Surrogate models, Kriging model, Support vector machine, Artificial neural network, and more based on the suitability for the problems. In Surrogate or Response surface modeling, the limit state function of any system is suitably approximated by making use of known mathematical models like polynomials, exponentials, etc. During the construction of surrogates, variables in the model should be well known prior to the approximation. In practical consideration, the design parameters are the unknowns that need to be evaluated before reliability-based design. Inverse Response surface procedure is proposed in the paper to address the above-mentioned issue. The procedure developed is the combination of adaptive Response surface method with appropriate experimental design i.e. Halton low discrepancy sequence sampling technique for evaluating the probability of failure or reliability index and an Artificial neural network is utilised as an inverse reliability procedure for design optimisation. The method gives an accurate result and the efficiency is increased for the same number of iterations in comparison to the work of David Lehky and Martina Somodikova [1] with Latin hypercube sampling as experimental design.

1. Introduction
To reach the expected level of probability of failure during any design is a challenging task. Uncertainties arise at every stage of the process in the design stages. For example live load, dead load, material properties, and many more in case of the structural design. Consideration of these uncertainties during reliability evaluation or engineering design is very essential. Reliability evaluation needs the help of forwarding approaches for evaluating the probability of failure or reliability index. In contrast, during an engineering design inverse approach is required for estimating the design variables or parameters to arrive at target reliability levels.

For any structure, the probability of failure ($P_f$) or reliability index ($\beta$) is estimated at a particular limit state. In complex systems, the evaluation of $P_f$ or $\beta$ becomes an expensive task given computation. To reduce the burden due to computation approximate metamodels are utilised without accuracy compromise. These approximate methods use a simpler function in place of Limit State Function (LSF) which consumes less time for computation. Estimation of $P_f$ or $\beta$ is carried out with the help of classical simulation methods like the Monte-Carlo technique. Some of the popular metamodeling techniques are the Response surface method (RSM) [1], Support vector machine [2], Kriging metamodel [3], and Artificial neural network (ANN) [4]. The inverse reliability approach for design parameter estimation has been carried out by many researchers by different techniques like Newton-Raphson iterative
algorithm [5][6], decomposition technique [7], ANN [8][9][10]. To increase the accuracy for estimating the design parameter for required $P_f$ or $\beta$ with reducing the computation time, the inverse metamodeling approach is proposed by using RSM with a polynomial model for a forward approach and ANN for inverse approach. Sampling procedure based on Halton Low discrepancy sequence (LDS) is utilised here due to its less discrepancy between samples generated for any parameter.

2. Response surface approximation

Let $\mathbf{Y}$ denote the vector of input variables to represent the limit state function in equation (1).

$$ h(\mathbf{Y}) \leq 0 $$

The LSF in equation (1) is replaced with equation (2) a polynomial function of second-order as a surrogate model

$$ \tilde{h}(\mathbf{Y}) = a + \sum_{i=1}^{n} b_i Y_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} Y_i Y_j $$

where $a, b, c_{ij}$ are the unknown coefficients with $i, j = 1, 2, \ldots, n$. Number of numerical experiments are performed by utilising appropriate experimental design for the input variables to evaluate the regression coefficients. The total number of experiments essential for evaluation of coefficients in equation (2) is $1 + n + n(n + 1) / 2$. The process can be further simplified by utilising polynomial function without considering mixed terms $Y_i Y_j$ as stated in equation (3) shown by Bucher and Bourgund [11] and also shown that accuracy is not much affected by neglecting the mixed terms.

$$ \tilde{h}(\mathbf{Y}) = a + \sum_{i=1}^{n} b_i Y_i + \sum_{i=1}^{n} c_i Y_i^2 $$

By neglecting mixed terms the number of evaluations of experiments decreases to $2n + 1$. Assuming uncorrelated normal variables $\tilde{h}(\mathbf{Y})$ is used to get $Y_D$ and the middle point $Y_M$ is obtained as follows.

$$ Y_M = \bar{Y} + (Y_D - \bar{Y}) \frac{\tilde{h}(\bar{Y})}{\tilde{h}(\bar{Y}) - \tilde{h}(Y_D)} $$

where $\bar{Y}$ denotes the mean point and $Y_D$ is the design point evaluated for the surface $\tilde{h}(\mathbf{Y}) = 0$. The new center point $Y_M$ is utilised to update the model in equation (3) and $4n + 3$ the number of evaluations has to be carried out in each iteration. After the number of iterations, an accurate response surface will be obtained which can be replaced with LSF of the given problem.

3. Artificial neural network-based inverse reliability method

Inverse reliability analysis deals with the evaluation of design parameters for required reliability. A reliability-based design optimisation problem can be generally stated as in equation (5).

$$ \text{Given: } P_{f,j} \text{ or } \beta_j \quad \text{Find: } \mathbf{d} \text{ and/or } \mathbf{r} \quad \text{Subject to: } Z_j = h(\mathbf{Y}, \mathbf{d}, \mathbf{r}) = 0 \text{ for } j = 1, 2, \ldots, m $$

$Z_j$ indicates safety margins, $P_{f,j}$ represents target failure probabilities or $\beta_j$ depicts reliability indices, where $j = 1, 2, \ldots, m$ is the number of LSFs. $\mathbf{d} \& \mathbf{r}$ represents deterministic and random variables parameter vector respectively. Also $\mathbf{Y} = \{Y_1, Y_2, \ldots, Y_n\}$ represents the vector of basic random variables. ANN is utilised in the present work as a surrogate model for the unknown inverse function. ANN builds relation between reliability indicators $\mathbf{I} = \mathbf{B}$ (or $\mathbf{I} = \mathbf{P}_I$) and the design parameters $\mathbf{P} = (\mathbf{d} \cup \mathbf{r})$ as in equation (6) based on preliminary data generated from suitable experimental design [12].
In the present work, the Halton LDS sampling technique has been considered for the preparation of the training set. LDS technique is chosen here since it generates values in the required region with a low discrepancy between the nearest values in the domain in comparison with other experimental designs [13]. The values generated from sampling are used for ANN training, testing, and validation. ANN architecture as shown in figure 1 consists of non-linear hidden layers and linear output layers. The design parameters corresponding to the required reliability level can be evaluated using a trained ANN model.

\[ P = f_{ANN}^{-1}(I) \]  

(6)

Figure 1. Artificial neural network architecture.

Figure 2. Flowchart representing the iterative procedure of the methodology.

4. Inverse response surface method

The response surface approach is used to replace the original LSF by a much simpler mathematical model as detailed in Section 2. Construction of the Response surface requires to know the design variables prior to the process. To get these values, the inverse approach i.e. ANN is utilised to generate an unknown surrogate model. The proposed method uses ANN with the Halton LDS sampling technique for better efficiency. Figure 2 represents the iterative scheme followed in the present work for upgrading the Response surface along with the inverse reliability analysis [6, 14].

a) The approximation by polynomial function is constructed in the direct procedure using the initial design parameter values.
b) By making use of the Response surface constructed in (a) the reliability indicators for a set of design parameters (formed from Halton LDS) are generated.

c) ANN-based inverse reliability analysis is carried out. The set of design parameters and reliability indicators are used for training ANN and an ANN surrogate model is obtained.

d) A new estimate of the design point is obtained from the ANN model in the inverse approach.

e) A new point is estimated from the design point by utilising equation (4). The new point is used to update the Response surface.

f) The process from (a) to (e) is repeated until the desired or acceptable level of design parameter is obtained.

5. **Numerical example**

Let the explicit nonlinear function [15] in equation (7) represent an LSF:

\[
h(Y) = \exp[0.4(Y_i + 2) + 6.2] - \exp(200 - Y)\]

\[
= \frac{\exp(0.4Y_i + 6.2)}{\exp(200)}
\]

where - \(Y_i\) is a random variable and \(d\) is a certain unknown number.

Numerical simulation is performed on equation (7) for a target value of \(d = 5.163\) 10 million samples. Reliability Index (\(\beta\)) is defined as the shortest distance of the origin to the failure region and can be obtained as the ratio of mean to standard deviation. The reliability index value obtained is \(\beta = 2.688\). Table 1 shows the parameter randomisation of \(d\). The values of mean, standard deviation, and boundary values are used in the RSM construction and to conduct the inverse approach.

| Variable | \(\mu\) | \(\sigma\) | Range |
|----------|---------|-----------|-------|
| \(d\)    | 6.0     | 1.155     | 4.0-8.0 |

The design parameter \(d\) in equation (7) is estimated by the proposed methodology. The ANN architecture in this case consists of two non-linear neurons in the hidden layer and a linear output layer associated with \(d\). The values of \(\beta\) and \(d\) obtained in the iterative procedure are tabulated in table 2. The methodology proposed utilising the Halton LDS method provides efficient results nearer to the target values of \(d\) in comparison to the use of the LHS method by Lehky and Martina Somodikova [1] as depicted in figure 3.

| Parameter | Identification | Target value |
|-----------|---------------|--------------|
| \(d\)     | Trial 1       | Trial 2      | Trial 3      | Target value |
|           | 5.471         | 5.199        | 5.160        | 5.163        |
| \(\beta\) | 2.660         | 2.674        | 2.687        | 2.688        |

![Figure 3. Results showing comparison between LHS and LDS.](image)
6. Conclusion

The iterative procedure adopted in the work along with the Halton LDS sampling technique as an experimental design increase the quality of response surface in few iterations, resulting in improving the accuracy of estimation of design parameter in comparison with LHS sampling technique. A direct approach coupled with inverse analysis cannot yield good results without the iterative procedure. Original LSF and initial values of the design parameter range selected govern the efficiency of the results from the procedure. For non-linear and more complex problems the second-order polynomial may not be sufficient, higher-order polynomials can be utilised in such cases. The response surface approach is one of the suitable ways in comparison to other methods in terms of computation effort acceptability. For accuracy assurance, one should make use of the iterative procedure while utilising inverse response surface approaches like ANN.

7. References

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