Critical exponents in spin glasses: numerics and experiments

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We give an overview of numerical and experimental estimates of critical exponents in Spin Glasses. We find that the evidence for a breakdown of universality of exponents in these systems is very strong.

I. INTRODUCTION

It is well known that at a continuous phase transition, striking critical behaviour is observed. As the transition temperature is approached from above or below, there are power law singularities in a number of physical parameters (the specific heat, the susceptibility,...). On very general grounds it can be shown that various critical exponents which govern the singularities are related to each other through scaling relationships. Even more remarkable is the fact that systems which are very different from each other at the microscopic level can be arranged into universality classes: within a given class all members have strictly identical exponents. Classes are defined by a restricted number of parameters - basically the space dimension and the number of components of the order parameter of each system. For standard second order transitions this behaviour can be fully understood, and the exponents calculated \textit{a priori} thanks to renormalization group theory. This is one of the outstanding achievements of statistical physics (see Fisher [1] for an enlightening historical survey).

At standard ferromagnetic transitions, the exponents follow mean field behaviour down to dimension 4 (the upper critical dimension). At dimensions \( d \) lower than 4, the exponents have been calculated though the \( \epsilon \) expansion, where \( \epsilon = 4 - d \). There are well behaved series in increasing powers of \( \epsilon \) which allow one to give renormalization group estimates of the exponents. The values can be compared with those measured from high precision numerical simulations, and contact can be made with the exact values at dimension 2 (where for the Ising system, the exponents have been known exactly for more than 50 years [2]). Renormalization group values, numerical values, and experimental values are all in excellent agreement with each other. The only exponent whose value is not quite so thoroughly established is the dynamic exponent \( z \); even here disagreements between different estimates are small. Universality is a general rule in systems with standard second order transitions, except for a restricted class of two dimensional systems with conflicting interactions. Here it has been shown analytically as well as numerically that the exponents vary continuously as a function of the ratio of the competing interactions [3,4]. This behaviour can be explained for these particular systems in terms of marginal operators in the renormalization scheme [5].

In the Spin Glass (SG) context, for a long time it was by no means obvious that there were well defined phase transitions at all in real three dimensional materials, or even in 3d model systems. Although the cusp temperature is clearly marked experimentally, the specific heat shows no visible singularity, and the susceptibility does not diverge in the region of the cusp temperature. A very important step forward was the realization in the in the late 1970s that the appropriate parameter to measure is not the standard linear susceptibility but the non-linear susceptibility \( \chi_{nl} \). The Miyako group in Sapporo in pioneering work [6] showed that there is a divergence of \( \chi_{nl} \) at the cusp temperature; this work and that of other groups [7–12] convinced the community that there is indeed a \textit{bona fide} transition in a SG. With the existence of a transition established, estimates were found of the critical exponents. Numerical work soon followed [13–15]. It was important to measure these values in order to compare with the renormalization group approach. Does the combination of frustration and randomness which characterize SGs modify the basic physics of transitions in a fundamental way or not? This has turned out to be a long story, which is still not finished. Unfortunately, as no clear theoretical guidelines appeared, the enthusiasm for the subject dropped and even the empirical ground rules were not fully established. Our aim here is to show that this study, both numerical and experimental, is well worth pursuing.

Before looking in detail at the data, we can first note that the exponents which come out of the numerical or experimental analyses are very different from the clas-
physical ferromagnetic values. For the latter, in dimension 3 (for either Ising, XY, or Heisenberg spins) the exponents $\alpha$ and $\eta$ are numerically each of the exponents can in principle be measured independently through temperature dependences and $\eta$ is far from zero; experimental values range from +0.4 to −0.5 and simulation values are always distinctly negative. $z$ is strong — generally around 6. The major differences compared with the standard second order transition values already indicate that the spin glass transition lies in a quite different category.

In Ising Spin Glasses (ISGs) the upper critical dimension is 6 and one could imagine that from dimension 6 down, a similar renormalization group approach would be valid mutatis mutandis as for the ferromagnets. In fact things are much more complicated. The $\epsilon$ expansion has been calculated to order three [16], but the successive factors in increasing powers of epsilon grow rapidly so it is not at all obvious how the total series will sum. Valiant efforts have been made over many years to set the theory on a firm footing using field theory [17], but so far the only clear result is to confirm that the leading term in $6 - d$ from the $\epsilon$ expansion is correct. After that, numerous badly contolled terms proliferate and theory is of little practical help in predicting exponents even at $d = 5$.

As a predictive theory is lacking, we are forced to turn to numerical and experimental methods so as to establish the empirical values of the exponents. The empirical results show clear violation of the Universality rules.

II. NUMERICAL RESULTS

In the following discussion we will concentrate mainly on the systems which have been studied the most fully - ISGs on (hyper)cubic lattices with random unbiased interactions between nearest neighbours.

The definitions of the critical exponents are familiar, with appropriate modifications for spin glasses to take into account the fact that that the order parameter is the Edwards Anderson parameter. The specific heat exponent is $\alpha$, the order parameter exponent below the ordering temperature is $\beta$, the spin glass susceptibility exponent is $\gamma$. The non-linear magnetization at the ordering temperature exponent is $\delta$, with

$$M_{nl} \sim H^{2/\delta}$$  \hspace{1cm} (1)

The correlation length exponent is $\nu$, and the exponent for the form of the correlation function at the ordering temperature is $\eta$. The relaxation time dynamic exponent is $z$. The scaling relationships between these exponents are $\alpha + 2\beta + \gamma = 2$, $d\nu = 2 - \alpha$, $\gamma = (2 - \eta)\nu$, $\nu(d - 2 + \eta) = 2\beta$ and $\delta = (d + 2 - \eta)/(d - 2 + \eta)$. Numerically each of the exponents can in principle be measured independently through temperature dependences on large samples, though frequently they are measured through using finite size scaling relationships. Experimentally, $\gamma, \alpha, \delta$ and the combination $\nu \eta$ can be measured directly while $\beta, \nu$ and $\eta$ can only be obtained through scaling. There are many other useful relationships; for instance the relaxation of the autocorrelation function at $T_g$ has an exponent

$$x = (d-2+\eta)/2z$$  \hspace{1cm} (2)

In addition to the standard critical exponents other exponents can be defined, in particular the stiffness exponent $\theta$. This is not a critical exponent but is defined by the size dependence of changes in energy with boundary conditions [18]. For a spin glass energy measurements can be made with periodic and anti-periodic boundary conditions. The variation of the sample to sample fluctuations of the energy differences scale as $L^g$. If the zero temperature $\theta$ is positive, then the ordering temperature is greater than zero.

A first type of quasi-numerical approach is furnished by high temperature expansions [13,19,20]. The method consists of an extrapolation from a finite number of exact terms in the high temperature series expansion of some thermodynamic function to its asymptotic coefficients. The asymptotic form of the series contains the information on the singularities of the function. The extrapolation is not exact, but excellent results have been obtained in regular systems. The situation is less favourable in disordered systems. Careful analysis of the spin glass susceptibility from a series with a large number of terms (up to 20 in dimension 3) provides a set of estimates for the values of $T_g$ and $\gamma$ obtained from different approximant functions. If the series expansion was infinite, the method would become exact but in practice the limited length of the series means that the estimates are not perfect. The exponent results become less accurate as one gets further from the upper critical dimension. For dimensions 5 and 4 the longest series give high quality estimates which can be used as independent yardsticks to compare with the Monte Carlo data which we will discuss below. The method does not have the same problems (such as thermalization) which are encountered in Monte Carlo simulations, but considerable know-how is needed to calculate long series and to extract reliable exponent estimates from the raw series results. Up to now the only specific ISG series expansion results published are for binomial ($\pm J$) interactions.

For dimension 5 a reliable and accurate ordering temperature and set of exponents is given by [21]. For dimensions 4 and 3 the results quoted in [19] are more transparent; it is clear that the approximant data points cluster satisfactorily with a strong correlation between estimates for $T_g$ and those for $\gamma$.

The most widely used technique for determining critical exponents numerically has been that of Monte Carlo
Many efforts have been made to measure ISG critical exponents accurately, despite considerable technical difficulties. For measurements exploiting the finite size scaling method [14], each sample is first annealed numerically until the spin system can be judged to be in thermal equilibrium; then the fluctuations in the autocorrelation function \( q(t) = \langle S_i(0)S_i(t) \rangle \) are measured. The precautions necessary are described in [14]. Long enough times must be used for each part of the procedure, and the time scale defining "long enough" depends on the size \( L \) and the temperature \( T \). The larger the sample and the lower the temperature the longer the time scales, so it becomes very difficult to obtain significant data on large samples at low temperatures. Sophisticated update methods have been developed which alleviate this problem to some extent [21]. One must be sure that measurements have been done over a sufficient number of independent samples. Even if numerically high quality data has been obtained, there may be intrinsic corrections to finite size scaling which mean that the scaling rules (which are valid asymptotically for large sizes) may not yet hold exactly for the range of sizes studied.

An important parameter which can be deduced from the equilibrium fluctuations is the spin glass susceptibility \( \chi_{SG} = \langle q^2 \rangle \), directly related to the non-linear susceptibility. The finite size scaling form is

\[
\chi_{SG} = L^{2-\eta} f(L^{1/\nu}(T - T_g))
\]

Precisely at \( T_g \), as a function of size

\[
\chi_{SG}(L) \sim L^{2-\eta}
\]

\( T_g \) can be determined quite accurately as the highest temperature at which \( \log(\chi_{SG}/L^2) \) varies linearly with \( \log(L) \) up to large \( L \).

These expressions ignore corrections to finite size scaling. There should be a further factor so that for instance the spin glass susceptibility is multiplied by \( [1 - L^{-\omega} f_L(L^{1/\nu}(T - T_g)) + \ldots] \). The correction to scaling exponent \( \omega \) has been estimated at around 2.8 in the binomial ISG in dimension 3 [22], as against 0.9 for the 3d Ising ferromagnet and 1.6 for 3d site percolation [28]. It turns out that the strength of the correction to scaling can change dramatically from one system to another.

Many authors estimate the critical temperature \( T_g \) through the Binder cumulant method [14]. The cumulant for a given \( T \) and \( L \) is defined by a dimensionless combination of moments of the autocorrelation fluctuations averaged over a large number of samples:

\[
g_L = 1/2(3 - < q^4 > / < q^2 >^2)
\]

for Ising spins. This cumulant is defined so as to go from zero for a high temperature Gaussian distribution of \( q(t) \) values, to 1 for a unique low temperature state. (Other related cumulants can be defined.) For a continuous transition with a critical temperature \( T_g \), \( g_L(T_g) \) should be independent of size \( L \), with values fanning out as a function of \( L \) above and below \( T_g \). Once \( T_g \) has been established accurately, the exponents can be estimated by plotting the whole set of data for \( g_L(T) \) and for \( \chi_{SG}(L,T) \) in an appropriate scaling form.

In practice, sample to sample fluctuations in \( < q^4 > \) are strong so very large numbers of samples must be measured (the lack of self averaging is very much worse for \( < q^4 > \) than for \( < q^2 > \)). The crossing point can be ill determined because the \( g_L(T) \) curves do not fan out appreciably at temperatures lower than the crossing point. When this is the case, relatively minor corrections to finite size scaling can modify the apparent crossing temperature. (Correction factors of the form given above should apply to both \( < q^4 > \) and \( < q^2 > \).) Finally, the values of the exponents deduced from the scaling plots, strongly correlated with the \( T_g \) estimate, frequently vary very steeply with the apparent value of \( T_g \). In conclusion, the exponent estimates obtained in this traditional way must be treated with considerable caution, even when large scale numerical efforts have been made. Accurate and reliable results can be obtained only in favourable circumstances.

When calculations have been made to large sizes, it is possible to use the scaling rules for the spin glass susceptibility and for the spin correlation length to estimate the critical temperature and the exponents by extrapolations to infinite size [24]. This method should be reliable, as corrections to finite size scaling are kept well under control.

Alternative techniques which have been less widely used rely (at least partly) on dynamic measurements. In massive simulations on large samples (64³ spins) Ogielski [15] studied the relaxation of \( q(t) \) as a function of temperature. An advantage of this method is that a strict thermal equilibrium state is not necessary; as long as the anneal has been made over a time \( \tau_a \) much longer than the subsequent measuring time over which \( q(t) \) is studied, the measured \( q(t) \) curve will be the true thermal equilibrium form (see for instance [25]). Ogielski assumed the standard critical behaviour for \( q(t) \), which is

\[
q(t) = t^{-\frac{x}{\tau(T)}}
\]

He estimated \( T_g \) from the divergence of the relaxation time \( \tau(T) \). With \( T_g \) in hand he estimated the critical exponents from a combination of dynamic and equilibrium measurements.

One can also exploit the critical behaviour of strictly non-equilibrium dynamics. Suppose a spin glass sample is initially at infinite temperature (so the spins have random orientations); it is then annealed from this configuration to the critical temperature \( T_g \). The non-equilibrium spin glass susceptibility at a time \( t \) after the start of the anneal will increase as

\[
h = (2 - \eta)/z
\]

where
Analagous non-equilibrium dynamic parameters have been studied extensively in a large number of regular systems [27]. This non-equilibrium scaling behaviour has been established on a very firm theoretical basis [28]. An obvious practical advantage for numerical work is that no preliminary anneal is required.

Now it is clearly possible to combine the measurements of the dynamic relaxation exponent \(x(T)\) and \(h(T)\) at a series of test temperatures \(T_i\) to obtain a sequence of apparent exponents \(\eta(T_i)\) and \(z(T_i)\):

\[
\eta = \frac{(4x + (2 - d)h)}{(2x + h)} \quad (8)
\]

\[
z = \frac{d}{(2x + h)} \quad (9)
\]

The \(\eta(T_i)\) can be compared with independent \(\eta(T_i)\) estimates from the equilibrium spin glass susceptibility (equation (9)). Consistency dictates that at the true \(T_g\) the two estimates must coincide. This leads to estimates of \(T_g\), \(\eta\) and \(z\) which turn out in practice to be precise (the crossing is clean) and virtually free from pollution by corrections to finite size scaling [29]. Once these parameters are well established, the other exponents such as \(\nu\) can be determined from conventional scaling plots with only one unknown parameter. We will refer to this method as the "three scaling rule" technique.

The low dimensions present special cases. First, in dimension 1 where critical temperatures are certainly always zero for short range interactions, the stiffness exponent \(\theta\) is exactly \(-1\) for continuous distributions of interactions and 0 for \(\pm J\) interactions [18]. In dimension 2 it is also well established that the ordering temperature \(T_g = 0\) [14] (except possibly when the interactions are \(\pm J\) [24]). From the definition of \(\eta\), when \(T_g\) is zero and for a unique ground state (corresponding to any continuous distribution of interactions such as the Gaussian distribution) there is an additional scaling rule [14]

\[
\eta = 2 - d \quad (10)
\]

As fewer spins are involved for a given size \(L\) than at high dimension, it is easier to cover a wide range of \(L\) for finite size scaling in Monte Carlo simulations. Even better, sophisticated numerical techniques exist to find exact ground states for systems up to large sizes [33]. (Recently exact ground states in dimension 3 have also been obtained to quite large \(L\) [32]).

### III. EXPONENT VALUES

We will concentrate on the values of \(\eta\), and start from the low dimensions.

For the 2d binomial ISG, \(\eta\) has been estimated to be \(0.20 \pm 0.02\) [14]. Curiously, even if the \(T_g\) is small but non-zero, the estimated value of \(\eta\) is very similar [28].

For the Gaussian distribution, from the values of the stiffness exponent \(\theta(d)\) as a function of dimension [18, 31, 33], we can estimate the lower critical dimension \(d_l\), the dimension at which \(\theta = 0\), figure 1. Because the Gaussian distribution is continuous, as \(d_l\) is close to 2.50, \(\eta(d_l)\) must be close to \(-0.50\), from equation (10).

![Figure 1](image1.png)

**FIG. 1.** Stiffness exponent \(\theta\) as function of dimension for ISGs with Gaussian interactions. Dimension 1 [18], dimension 2 [31], dimension 3 [33].

Numerous estimates have been given of the exponents for ISGs in dimension 3 with binomial or Gaussian interaction distributions. We have summarized the situation in [22], where we find that there are strong deviations from finite size scaling for the binomial case and where we obtain reliable exponent estimates principally from the three scaling rule method. Recent data on the binomial case established by extrapolation to infinite size [24] are consistent with the values of [22]. Other estimates relying on the Binder cumulant method are probably affected by corrections to finite size scaling and so are less reliable.

We will use the results obtained in dimension 4 to demonstrate the coherence of the different methods when these are used carefully, and the inescapable conclusion that the values of the exponents change with the form of the interaction distribution.

![Figure 2](image2.png)
FIG. 2. Binder cumulant data for the binomial \((\pm J)\) interaction ISG in dimension 4, (A.P. Young, unpublished). Sizes \(L = 4, 6, 8, 12\). The curves should intersect at the critical temperature.

Figure 2 shows high precision data for the Binder cumulant in the binomial case \([34]\). There is a clean intersection of the curves at \(T = 2.00 \pm 0.01\).

Figure 3. \(\eta(T)\) data for the binomial interaction ISG in dimension 4. \(\eta_1\) (squares) is from the spin glass susceptibility (A.P. Young, unpublished) and \(\eta_2\) (circles) is from the \(h\) and \(x\) effective exponent, see text. The intersection should correspond to the critical temperature \(T_g\) and \(\eta(T_g)\).

Figure 3 shows the intersection of the two curves for \(\eta(T_i)\) used in the three scaling method described above \([33]\). The intersection is at precisely the same temperature, validating this technique. From the intersection point we can deduce \(T_g\) and \(\eta = 0.30 \pm 0.02\). As the slope of the curve for \(\eta(T)\) for the estimate from the dynamic exponents \(x\) and \(h\) is much weaker than from that deduced from the equilibrium finite size scaling spin glass susceptibility, the precision on the value of \(\eta\) is much higher using this method rather than working only with the pure equilibrium susceptibility and Binder cumulant data. The series expansion data \([19]\) is in quite independent agreement with these Monte Carlo results. From the results plotted in Figure 3 of \([33]\), for this value of \(T_g\) one would expect \(\gamma = 2.1 \pm 0.1\), or \(\nu = 0.92 \pm 0.05\) from the scaling relation. This value of \(\nu\) is perfectly consistent with direct scaling of the Monte Carlo spin glass susceptibility data.

FIG. 4. Binder cumulant data for the Gaussian interaction ISG in dimension 4. Sizes \(L = 4, 6, 10\).

Figure 4 shows Binder cumulant data for the 4d Gaussian case; it can be seen that the intersection point lies at \(T = 1.785 \pm 0.01\). Figure 5 shows the three scaling rule intersection. Again the agreement is excellent, but the value of \(\eta\) at the intersection point \(\eta = -0.44 \pm 0.02\), is considerably higher than for the binomial case.

FIG. 5. \(\eta(T)\) data for the Gaussian interaction ISG in dimension 4. Squares correspond to the SG susceptibility, circles to the \((h,x)\) method. The \(\eta\) value at the intersection is clearly different from that of the binomial interaction distribution.

Figure 6 shows a direct plot of \(\log(\chi_{SG}/L^2)\) against \(\log(L)\). This has to be straight at \(T_g\). It can be seen that \(T_g\) must lie just below \(T = 1.79\), and \(\eta\) can be estimated from the corresponding slope. We can note that there is excellent agreement point by point between the data of \([30]\) and the present results wherever comparisons can be made. Using a scaling plot we find \(\nu = 1.08 \pm 0.10\).
The upper straight line represents the leading term in the $\epsilon$ expansion: $\eta(d) = -(6 - d)/3 + ...$. It is clear that the data demonstrate that the exponents for the two systems, binomial and Gaussian, follow regular curves as a function of dimension. Independent results are consistent with each other. However the values at each dimension are different for the two sets of interactions so universality is clearly violated. Results for other sets of interactions confirm this variation.

Critical exponents have also been measured in the Ising phase diagram as a function of the ratio $p$ of the number of ferromagnetic to antiferromagnetic bonds. In the ferromagnetic region to the right of the Nishimori line, the static exponents do not appear to vary with $p$, but the dynamic exponent $z$ does change continuously and very significantly.

We can note that the Migdal-Kadanoff renormalization approach (which should be exact for a hierarchical lattice) has been used to measure effective ordering temperatures and exponents for four different ISG interaction distributions. For dimension 3, a diamond hierarchical lattice, and a renormalization factor $b = 2$, the ordering temperatures are in excellent agreement with Monte Carlo estimates on cubic lattices. However the Migdal-Kadanoff calculations lead to a universal saddle point critical temperature and exponent values. This result seems to be closely related to the hypotheses intrinsic to the Migdal Kadanoff method.

### IV. SPIN GLASS EXPERIMENTS

Estimates of exponents have been made by many experimental groups, in general using slightly different protocols and on different materials. The non-linear susceptibility is defined as

$$\chi_{nl} = \chi_0 - M/H$$

where $\chi_0$ is the zero field magnetic susceptibility. At $T_g$ $\chi_{nl}$ should behave as $H^{2/3}$ and above $T_g$

$$M = \chi_0 H + \chi_2 H^3 + \chi_4 H^5 + ...$$

with $\chi_2$ diverging as $t^{-\gamma}$, where $t$ is the reduced temperature. The Suzuki equation of state is

$$\chi_{nl} = t^\beta g(H^2/t^{\beta+\gamma})$$

Both d.c. and a.c. magnetization techniques have been used. For instance Monod and Bouchiat and Bouchiat used d.c. entirely, taking care to stay in a field range where the non-linear magnetization remained less than 10% of the linear magnetization so as not to corrupt the data. Svedlindh et al first analysed low field and low frequency a.c. measurements to fit to $\gamma$ and $T_g$. (The...
ordering temperature was in agreement with the low d.c. field cusp temperature). They then used the equation of state with d.c. measurements up to moderate fields with $\beta$ as the only fit parameter. In a second set of experiments they measured the field and frequency dependence of the a.c. susceptibility and obtained an estimate of the dynamics exponent $z\nu$. In a sophisticated experiment, Lévy measured the Fourier transform spectrum of the magnetization response to a 0.1 Hz field, picking up a series of non-linear susceptibilities from the different harmonics. He could deduce accurate values of static and dynamic exponents.

The experiments have to be performed with care, particular attention being paid to the proper identification of the critical temperature and to the necessity to remain in a suitably low field range throughout. Recent measurements in which the exponent $\delta$ was measured in a range of different materials using one single protocol gave excellent confirmation of earlier data by other teams (except for the case of AuFe where an early experiment had given values of exponents out of line with all other results). These results validate the earlier measurements and show that the considerable spread of values of exponents reported for different materials, is not due to artefacts in the different measuring procedures. As in the numerical data it is evident that the expected universality of exponents breaks down.

The finite temperature $T_c$ values for real material spin glasses have been somewhat of a mystery for some years. These systems are Heisenberg, and reliable numerical work has demonstrated that Heisenberg spin glasses in dimension 3 should have zero temperature ordering. Kawamura has made the interesting suggestion that the ordering process in real life Heisenberg materials is basically a chiral spin glass ordering. This ordering would not be visible directly to magnetization experiments if there were no anisotropy. However in all real systems random anisotropy (of the DM type) is always present, and by coupling the chiral degrees of order with the magnetism an anisotropy, however weak, reveals the chiral order.

The critical exponents for pure Heisenberg chiral ordering in dimension 3 have been estimated numerically. The best values are around $\nu = 1.25$ and $\eta = +0.7$. It can be noted that $\nu$ is similar to the Monte Carlo Ising values which we have presented, while $\eta$ is strongly positive rather than negative as seen in the numerical Ising work. A plausible hypothesis is one in which the exponents change progressively from chiral-like for weak anisotropy to Ising-like for strong anisotropy. On this scenario, the value of $\nu$ should remain relatively stable for all the materials, while the value of $\eta$ should vary progressively, becoming gradually more negative as anisotropy increases.

So far the qualitative trend for systems where both anisotropy and exponents have been measured is in excellent agreement with this picture. All the experimental exponent values fall within the chiral limit at one end and the Ising limit at the other end. For the three alloy systems AgMn, CuMn and AuFe the anisotropies are weak, moderate and strong respectively. The $\nu$ values are similar, near 1.3, while the $\eta$ values are about 0.4, 0.1 and −0.1. The trend of exponent values is clearly in the sense predicted by the scenario.

V. CONCLUSION

The main lesson which can be drawn from this overview of numerical and experimental exponent data in spin glasses is that transitions in these glassy systems are quite different from those in regular systems with standard second order transitions. The values of the exponents are far from those in regular systems and the breakdown of Universality is manifest in carefully analysed data.

The statistical physics community has been very loth to accept evidence against universality because the renormalization scenario appears to give such an appealingly general picture of behaviour at transitions. However the fact that it has proved extremely difficult on the field theory level to produce predictions for spin glasses is a strong indication that unexpected behaviour cannot be excluded a priori.

What possible mechanism could lead to this breakdown? It has been found analytically that Ising spin systems with ferromagnetic interactions on hierarchical lattices show no universality. For the spherical model on graphs of non-integer dimension, the exponents vary continuously with the spectral dimension of the graph. We can speculate that in spin glasses the effective dimension of the system at criticality could depend on the form of the interaction distribution.

It would be unfortunate if this phenomenon was left unexplored because of preconceptions as to the physical laws which should hold for complex systems. If it could be accepted that critical behaviour is much richer in glassy systems than at conventional second order transitions without frustration, an important new field of investigation should open out. What control parameters affect the exponents and why? What are the implications for the physics of the glass transition in the most general sense?

[1] M.E. Fisher, Rev. Mod. Phys. 70 653 (1998)
[2] L. Onsager, Phys. Rev. 62 117 (1944)
