Generalized uncertainty principle, quantum gravity and Hořava-Lifshitz gravity

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Abstract

We investigate a close connection between generalized uncertainty principle (GUP) and deformed Hořava-Lifshitz (HL) gravity. The GUP commutation relations correspond to the UV-quantum theory, while the canonical commutation relations represent the IR-quantum theory. Inspired by this UV/IR quantum mechanics, we obtain the GUP-corrected graviton propagator by introducing UV-momentum $p_i = p_0(1 + \beta p_0^2)$ and compare this with tensor propagators in the HL gravity. Two are the same up to $p_0^4$-order.
1 Introduction

Recently Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1], which may be regarded as a UV complete candidate for general relativity. At short distances the theory of $z = 3$ Hořava-Lifshitz (HL) gravity describes interacting nonrelativistic gravitons and is supposed to be power counting renormalizable in (1+3) dimensions. Recently, the HL gravity theory has been intensively investigated in [2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28]. The equations of motion were derived for $z = 3$ HL gravity [29 30], and its black hole solution was first found in asymptotically anti-de Sitter spacetimes [30] and black hole in asymptotically flat spacetimes [31].

It seems that the GUP-corrected Schwarzschild black hole is closely related to black holes in the deformed Hořava-Lifshitz gravity [32 33]. Also, the GUP provides naturally a UV cutoff to the local quantum field theory as quantum gravity effects [34 35].

On the other hand, one of main ingredients for studying quantum gravity is the GUP, which has been argued from various approaches to quantum gravity and black hole physics [36]. Certain effects of quantum gravity are universal and thus, influence almost any system with a well-defined Hamiltonian [37]. The GUP satisfies the modified Heisenberg algebra [38]

$$[x_i, p_j] = i\hbar \left( \delta_{ij} + \beta q^2 \delta_{ij} + 2\beta q_i q_j \right), \quad [x_i, x_j] = [p_i, p_j] = 0$$

where $q_i$ is considered as the momentum at high energies and thus, it can be interpreted to be the UV-commutation relations. Here $q^2 = p_i p_i$. In this case, the minimal length which follows from these relations is given by

$$\delta x_{\min} = \hbar \sqrt{5\beta}. \quad (2)$$

On the other hand, introducing IR-canonical variable $p_{0i}$ with $x_i = x_{0i}$ through the replacement

$$p_i = p_{0i} \left( 1 + \beta q_0^2 \right), \quad (3)$$

these variables satisfy canonical commutation relations

$$[x_{0i}, p_{0j}] = i\hbar \delta_{ij}, \quad [x_{0i}, x_{0j}] = [p_{0i}, p_{0j}] = 0. \quad (4)$$

Here $p_{0i}$ is considered as the momentum at low energies with $p_0^2 = p_{0i} p_{0i}$. It is easy to show that Eq. (1) is satisfied to linear-order $\beta$ when using Eq. (4). Hence, the replacement (3) could be used as an important low-energy window to investigate quantum gravity phenomenology up to linear-order $\beta$. 

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It was known for deformed HL gravity that the UV-propagator for tensor modes $t_{ij}$ take a complicated form Eq. (32), including up to $p^6_0$-term from the Cotton bilinear term $C_{ij}C_{ij}$. We have explored a connection between the GUP commutator and the deformed HL gravity [39]. Explicitly, we have replaced a relativistic cutoff function $K(p_2^2/\Lambda^2)$ by a non-relativistic density function $D_\beta(p^2)$ to derive GUP-corrected graviton propagators. These were compared to (32). It was pointed out that two are qualitatively similar, but the $p^5_0$-term arisen from the crossed term of Cotton and Ricci tensors did not appear in the GUP-corrected propagators. Also, it was unclear why the $D_2$ GUP-corrected tensor propagator (not the $D_3$ GUP-corrected propagator) is similar to the UV-propagator derived from the $z = 3$ HL gravity.

In this work, we investigate a close connection between GUP and deformed HL gravity. At high energies, we assume that the UV-propagator takes the conventional form $G_{UV}(\varpi, p^2)$ in Eq. (44), whereas at low energies, the IR-propagator takes the conventional form $G_{IR}(\varpi, p^2_0)$ in Eq. (35). It is very important to understand how the UV-propagator is related to the IR-propagator in the non-relativistic gravity theory. We find a GUP-corrected graviton propagator by applying (3) to $G_{UV}(\varpi, p^2)$ and compare it with the UV-tensor propagator (32) in the HL gravity. Two are the same up to $p^4_0$-order, although the $p^5_0$-term arisen from a crossed term of Cotton tensor and Ricci tensor is still missed in the GUP-corrected graviton propagator. This indicates that a power-counting renormalizable theory of the HL gravity is closely related to the GUP.

## 2 $z = 3$ HL gravity

Introducing the ADM formalism where the metric is parameterized

$$ds^2_{ADM} = -N^2dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt),$$

the Einstein-Hilbert action can be expressed as

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{g}N \left[ K_{ij}K^{ij} - K^2 + R - 2\Lambda \right],$$

where $G$ is Newton’s constant and extrinsic curvature $K_{ij}$ takes the form

$$K_{ij} = \frac{1}{2N} \left( \dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right).$$

Here, a dot denotes a derivative with respect to $t$. An action of the non-relativistic renormalizable gravitational theory is given by [1]

$$S_{HL} = \int dt d^3x \left[ \mathcal{L}_K + \mathcal{L}_V \right],$$

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where the kinetic terms are given by
\[
\mathcal{L}_K = \frac{2}{\kappa^2} \sqrt{g} N K_{ij} G^{ijkl} K_{kl} = \frac{2}{\kappa^2} \sqrt{g} N \left( K_{ij} K^{ij} - \lambda K^2 \right),
\]
with the DeWitt metric
\[
G^{ijkl} = \frac{1}{2} \left( g^{ik} g^{jl} - g^{il} g^{jk} \right) - \lambda g^{ij} g^{kl}
\]
and its inverse metric
\[
G_{ijkl} = \frac{1}{2} \left( g^{ik} g^{jl} - g^{il} g^{jk} \right) - \frac{\lambda}{3\lambda - 1} g^{ij} g^{kl}.
\]

The potential terms is determined by the detailed balance condition as
\[
\mathcal{L}_V = -\frac{\kappa^2}{2} \sqrt{g} N E^{ij} g_{ijkl} E^{kl} = \sqrt{g} N \left\{ \frac{\kappa^2 \mu^2}{8(1 - 3\lambda)} \left( \frac{1}{4} R^2 + \Lambda_W R - 3 \Lambda_W^2 \right) \right. \\
- \frac{\kappa^2}{2\eta^4} \left( C_{ij} - \frac{\mu \eta^2}{2} R_{ij} \right) \left( C_{ij} - \frac{\mu \eta^2}{2} R_{ij} \right) \right\}.
\]

Here the \( E \) tensor is defined by
\[
E^{ij} = \frac{1}{\eta^2} C^{ij} - \frac{\mu}{2} \left( R^{ij} - \frac{R}{2} g^{ij} + \Lambda_W g^{ij} \right)
\]
with the Cotton tensor \( C_{ij} \)
\[
C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left( R^{ij} - \frac{1}{4} R \delta^{ij} \right).
\]

Explicitly, \( E_{ij} \) could be derived from the Euclidean topologically massive gravity
\[
E^{ij} = \frac{1}{\sqrt{g}} \frac{\delta W_{TMG}}{\delta g_{ij}}
\]
with
\[
W_{TMG} = \frac{1}{\eta^2} \int d^3x \epsilon^{ikl} \left( \Gamma^m_{il} \partial_j \Gamma^l_{km} + \frac{2}{3} \Gamma^m_{il} \Gamma^l_{jm} \Gamma^m_{kn} \right) - \mu \int d^3x \sqrt{g} (R - 2 \Lambda_W),
\]
where \( \epsilon^{123} = 1 \).

In the IR limit, comparing \( \mathcal{L}_0 \) with Eq.(6) of general relativity, the speed of light, Newton’s constant and the cosmological constant are given by
\[
c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda_{cc} = \frac{3}{2} \Lambda_W.
\]

The equations of motion were derived in [29] and [30]. We would like to mention that the IR vacuum of this theory is anti-de Sitter (AdS\( _4 \)) spacetimes. Hence, it is interesting to
take a limit of the theory, which may lead to a Minkowski vacuum in the IR sector. To this end, one may deform the theory by introducing \( \mu^4 R \) (\( \mathcal{L}_V = \mathcal{L}_V + \sqrt{g}N \mu^4 R \)) and then, take the \( \Lambda_W \to 0 \) limit \([31]\). We call this the deformed HL gravity without detailed balance condition. This does not alter the UV properties of the theory, while it changes the IR properties. That is, there exists a Minkowski vacuum, instead of an AdS vacuum. In the IR limit, the speed of light and Newton’s constant are given by

\[
e^2 = \frac{\kappa^2 \mu^4}{2}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \lambda = 1.
\]  

(18)

The deformed HL gravity has an important parameter \([31]\)

\[
\omega = \frac{8\mu^2(3\lambda - 1)}{\kappa^2},
\]

(19)

which takes the form for \( \lambda = 1 \)

\[
\omega = \frac{16\mu^2}{\kappa^2}.
\]

(20)

Actually, \( \frac{1}{2\omega} \) plays the role of a charge in the Kehagias-Sfetsos (KS) black hole with \( \lambda = 1 \) and \( K_{ij} = C_{ij} = 0 \) \([32]\) derived from the Lagrangian

\[
\mathcal{L}_V^{\lambda=1} = \sqrt{g}N \mu^4 \left( R + \frac{3}{4\omega} R^2 - \frac{2}{\omega} R_{ij}R_{ij} \right).
\]

(21)

and a spherically symmetric metric ansatz. Furthermore, it was shown that the entropy of KS black hole could be explained from the entropy of GUP-corrected Schwarzschild black hole when making a connection of \( \beta \to \frac{1}{\omega} \) \([33]\).

### 3 GUP-quantum mechanics

A meaningful prediction of various theories of quantum gravity (string theory) and black holes is the presence of a minimum measurable length or a maximum observable momentum. This has provided the generalized uncertainty principle which modifies commutation relations shown by Eq. (1). A universal quantum gravity correction to the Hamiltonian is given by

\[
\mathcal{H}_{UV} = \frac{p_i^2}{2m} + V(x_i) = \frac{p_0^2}{2m} + V(x_0) + \frac{\beta}{m} p_0^4 + \frac{\beta^2}{2m} p_0^6 + \mathcal{H}_{IR} + \mathcal{H}_1
\]

(22)

\[
\equiv \mathcal{H}_{IR} + \mathcal{H}_1
\]

(23)

with

\[
\mathcal{H}_{IR} = \frac{p_0^2}{2m} + V(x_0), \quad \mathcal{H}_1 = \frac{\beta}{m} p_0^4 + \frac{\beta^2}{2m} p_0^6.
\]

(24)
We note that Eq. (23) may be used for a perturbation study with \( p_0 = -i\hbar d/dx_0 \). We see that any system with a well-defined quantum (or even classical) Hamiltonian \( \mathcal{H}_{IR} \), is perturbed by \( \mathcal{H}_1 \) near the Planck scale. In this sense, the quantum gravity effects are in some sense universal. Some examples were performed in \([37, 40, 41, 42]\). It turned out that the corrections could be interpreted in two ways when considering linear-order perturbation \( \mathcal{H}_1 = \beta m p_4^0 \): either that for \( \beta = \beta_0 l_{1p}^2/2\hbar^2 \) with \( \beta_0 \sim 1 \), they are exceedingly small, beyond the reach of current experiments or that they predict upper bounds on the quantum gravity parameter \( \beta_0 \leq 10^{34} \) for the Lamb shift.

3.1 Tensor modes for deformed \( z = 3 \) HL gravity

The field equation for tensor modes propagating on the Minkowski spacetimes is given by \([24]\)

\[
\ddot{t}_{ij} - \frac{\mu^4 \kappa^2}{2} \Delta t_{ij} + \frac{\mu^2 \kappa^4}{16} \Delta^2 t_{ij} - \frac{\mu \kappa^4}{4\eta^2} \epsilon_{ilm} \partial^l \Delta t^j m - \frac{\kappa^4}{4\eta^4} \Delta^3 t_{ij} = T_{ij}
\]

with external source \( T_{ij} \) and the Laplacian \( \Delta = \partial_i^2 \rightarrow -p_0^2 \). We could not obtain the covariant propagator because of the presence of \( \epsilon \)-term. Assuming a massless graviton propagation along the \( x^3 \)-direction with \( p_0 = (0, 0, p_3) \), then the \( t_{ij} \) can be expressed in terms of polarization components as \([25]\)

\[
t_{ij} = \begin{pmatrix} t_+ & t_x & 0 \\ t_x & -t_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

Using this parametrization, we find two coupled equations for different polarizations

\[
\ddot{t}_+ - \frac{\mu^4 \kappa^2}{2} \Delta t_+ + \frac{\kappa^4 \mu^2}{16} \Delta^2 t_+ + \frac{\kappa^4 \mu}{4\eta^2} \partial_3 \Delta^2 t_x - \frac{\kappa^4}{4\eta^4} \Delta^3 t_+ = T_+,
\]

\[
\ddot{t}_x - \frac{\mu^4 \kappa^2}{2} \Delta t_x + \frac{\kappa^4 \mu^2}{16} \Delta^2 t_x - \frac{\kappa^4 \mu}{4\eta^2} \partial_3 \Delta^2 t_+ - \frac{\kappa^4}{4\eta^4} \Delta^3 t_x = T_x.
\]

In order to find two independent components, we introduce the left-right base defined by

\[
t_{L/R} = \frac{1}{\sqrt{2}} \left( t_+ \pm it_x \right)
\]

where \( t_L(t_R) \) represent the left (right)-handed modes. After Fourier-transformation, we find two decoupled equations

\[
-\omega^2 t_L + c^2 p_0^2 t_L + \frac{\kappa^4 \mu^2}{16} (p_0^2)^2 t_L - \frac{\kappa^4 \mu}{4\eta^2} p_3 (p_0^2)^2 t_L + \frac{\kappa^4}{4\eta^4} (p_0^2)^3 t_L = T_L,
\]

\[
-\omega^2 t_R + c^2 p_0^2 t_R + \frac{\kappa^4 \mu^2}{16} (p_0^2)^2 t_R + \frac{\kappa^4 \mu}{4\eta^2} p_3 (p_0^2)^2 t_R + \frac{\kappa^4}{4\eta^4} (p_0^2)^3 t_R = T_R.
\]
We have UV-tensor propagators with $\omega = 16\mu^2/\kappa^2$

$$t_{L/R} = -\frac{T_{L/R}}{\omega^2 - c^2\left(p_0^2 + \frac{2}{n} p_0^4 + \frac{8}{\eta^4}\mu_\omega p_0^6 + \frac{128}{\eta^4\kappa^2\omega^2} p_0^6\right)},$$  \hspace{1cm} (32)

We note that the left-handed mode is not allowed because it may give rise to ghost ($-\frac{8c^2}{\eta^4\mu_\omega} p_0^4$), while the right-handed mode is allowed because there is no ghost ($\frac{8c^2}{\eta^4\mu_\omega} p_0^4$). At this stage, we mention that $p_0(=\sqrt{p_0^4})$ is a magnitude of momentum $p_0$ but not a time component $\omega$.

Finally, we find UV-propagators in the four dimensional frame with $p^\mu = (\omega, 0, 0, p_3)$ as

$$t_{L/R} = -\frac{T_{L/R}}{\omega^2 - c^2\left(p_3^2 + \frac{2}{n} p_3^4 + \frac{8}{\eta^4}\mu_\omega p_3^6 + \frac{128}{\eta^4\kappa^2\omega^2} p_3^6\right)}, \hspace{1cm} (33)$$

\section{GUP-corrected propagator}

It is known for deformed HL gravity that the UV-propagator for tensor modes $t_{ij}$ take a complicated form shown in Eq. (32), including up to $p_0^6$-term from the Cotton bilinear term $C_{ij}C_{ij}$.

At high energies, we assume that the UV-propagator takes the conventional form

$$G_{UV}(\omega, p^2) = \frac{1}{\omega^2 - c^2 p^2}, \hspace{1cm} (34)$$

whereas at low energies, the IR-propagator takes the conventional form

$$G_{IR}(\omega, p_0^2) = \frac{1}{\omega^2 - c^2 p_0^2}. \hspace{1cm} (35)$$

Considering (33), the UV-propagator (34) takes the form

$$G_{UV}(\omega, p_0^2) = \frac{1}{\omega^2 - c^2\left(p_0^2 + 2\beta p_0^4 + \beta^2 p_0^6\right)}. \hspace{1cm} (36)$$

The GUP-corrected tensor propagator is determined by

$$t_{ij}^{GUP} = -G_{UV}(\omega, p_0^2)T_{ij} = -\frac{T_{ij}}{\omega^2 - c^2\left(p_0^2 + 2\beta p_0^4 + \beta^2 p_0^6\right)}, \hspace{1cm} (37)$$

where scaling dimensions are given by $[\beta] = -2$, $[\omega] = 3$, and $[c] = 2$ for the $z = 3$ HL gravity. \textit{This is exactly the same form as the UV-tensor propagator (32) up to $p_0^4$} when using the replacement of $\beta \rightarrow 1/\omega$ which was derived for entropy of the Kehagias-Sfetsos black hole without the Cotton tensor $(C_{ij} = 0)$ [33]. However, considering terms beyond $p_0^4$ ($p_0^6$ and $p_0^8$), we could not make a definite connection between two propagators even though highest space derivative of sixth order are found in both propagators. Explicitly, the $p_0^6$-term is absent for the GUP-corrected propagator and coefficients in the front of $p_0^6$ are different. Two coefficients are the same for $\eta^4 = 128/\kappa^2$.  

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4 Discussions

We have explored a close connection between generalized uncertainty principle (GUP) and deformed Hořava-Lifshitz (HL) gravity. It was proposed that the GUP commutation relations describe the UV-quantum theory, while the canonical commutation relations represent the IR-quantum theory. Inspired by this UV/IR quantum mechanics, we obtain the GUP-corrected graviton propagator by introducing UV-momentum of \( p_i = p_0i(1 + \beta p_0^2) \) with \( p_0 \) the IR momentum. We compare this with tensor propagators in the HL gravity. Two are the same up to \( p_0^4 \)-order, but the \( p_0^5 \)-term arisen from the crossed term of Cotton and Ricci tensors did not appear in the GUP-corrected propagators.

Importantly, we confirm that the deformed HL gravity with \( \omega \) parameter contains effects of quantum gravity implied by the GUP with the linear-order of \( \beta \) when using a relation of \( \beta = 1/\omega \). This means that the deformed \( z = 2 \) HL gravity without Cotton tensor could be well described by the GUP [2]. This Lagrangian is given by

\[
\tilde{L}_{z=2} = \sqrt{g}N \left[ \frac{2}{\kappa^2} \left( K_{ij}K_{ij} - \lambda K^2 \right) + \mu^4 \left( R + \frac{1}{2\omega} \frac{4\lambda - 1}{3\lambda - 1} R^2 - \frac{2}{\omega} R_{ij}R_{ij} \right) \right]. \tag{38}
\]

The tensor propagator is derived from the above Lagrangian on the Minkowski background where Ricci-square term \( R^2 \) does not contribute to the bilinear term of \( t_{ij}t_{ij} \). Hence, it is easily shown that \( \frac{2}{\omega} p_0^4 \)-term in the tensor propagator comes from \( R_{ij}R_{ij} \)-term. On the other hand, the modified Heisenberg commutation relation (1) is satisfied to linear-order \( \beta \) when calculating the GUP-corrected propagator (37). Therefore, it is valid that the deformed \( z = 2 \) HL gravity without Cotton tensor is well explained by the GUP.

However, it needs a further study in order to make a clear connection between \( z = 3 \) HL gravity and the GUP with second-order of \( \beta (\beta^2) \) because the former contains the Cotton tensor \( C_{ij} \) and the replacement is obscure.

Acknowledgement

This work was supported by Basic Science Research Program through the National Science Foundation (KRF) of Korea funded by the Ministry of Education, Science and Technology (2009-0086861).

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