STACKING SPECTRAS IN PROTOPLANETARY DISKS: DETECTING INTENSITY PROFILES FROM HIDDEN MOLECULAR LINES IN HD 163296

Hsi-Wei Yen1, Patrick M. Koch1, Haoyu Baobab Liu2, Evaria Puspitangingrum3, Naomi Hirano1, Chin-Fei Lee1, and Shigehisa Takakuwa1,4

1 Academia Sinica Institute of Astronomy and Astrophysics, P.O. Box 23-141, Taipei 10617, Taiwan
2 European Southern Observatory (ESO), Karl-Schwarzschild-Str. 2, D-85748 Garching, Germany; hyen@eso.org
3 Department of Astronomy, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jl. Ganesha 10, Bandung 40132 Indonesia
4 Department of Physics and Astronomy, Graduate School of Science and Engineering, Kagoshima University, 1-21-35 Korimoto, Kagoshima, Kagoshima 890-0065, Japan

Received 2015 December 2; revised 2016 May 16; accepted 2016 October 3; published 2016 December 1

ABSTRACT

We introduce a new stacking method in Keplerian disks that (1) enhances signal-to-noise ratios (S/Ns) of detected molecular lines and (2) makes visible otherwise-undetectable weak lines. Our technique takes advantage of the Keplerian rotational velocity pattern. It aligns spectra according to their different centroid velocities at their different positions in a disk and stacks them. After aligning, the signals are accumulated in a narrower velocity range as compared to the original line width without alignment. Moreover, originally correlated noise becomes decorrelated. Stacked and aligned spectra thus have a higher S/N. We apply our method to Atacama Large Millimeter/Submillimeter Array (ALMA) archival data of DCN (3–2), DCO+ (3–2), N2D+ (3–2), and H2CO (3,0,3–2,0), (3,2,2–2,1), and (3,1,2–2,0) in the protoplanetary disk around HD 163296. As a result, (1) the S/Ns of the originally detected DCN (3–2), DCO+ (3–2), H2CO (3,0,3–2,0), and N2D+ (3–2) lines are boosted by a factor of ≥4–5 at their spectral peaks, implying one order of magnitude shorter integration times to reach the original S/N; and (2) the previously undetectable spectra of the H2CO (3,2,2–2,1) and (3,1,2–2,0) lines are materialized at more than 3σ. These dramatically enhanced S/Ns allow us to measure intensity distributions in all lines with high significance. The principle of our method can be applied not only to Keplerian disks but also to any systems with ordered kinematic patterns.

Key words: ISM: individual objects (HD 163296) – line: profiles – protoplanetary disks

1. INTRODUCTION

Studying the physical and chemical structures of protoplanetary disks is essential to understanding the environment of planet formation (e.g., Dutrey et al. 2014). With the Atacama Large Millimeter/Submillimeter Array (ALMA), protoplanetary disks can be observed with resolutions <0″1 (e.g., ALMA Partnership et al. 2015). Although molecular lines can be simultaneously observed with ALMA, their images are typically generated at resolutions of a few times 0″1, lower than continuum images (e.g., ALMA Partnership et al. 2015; Qi et al. 2015), because the sensitivity of line observations is limited by their narrow line widths (on the order of 10 km s−1). Therefore, intensity distributions of molecular lines cannot be directly compared with (sub)millimeter continuum data. Besides, even with ALMA, there are faint molecular lines that remain difficult to detect. In order to advance our understanding of physical and chemical structures in protoplanetary disks, it is thus, important to develop techniques to enhance signal-to-noise ratios (S/Ns) of molecular-line data and reveal their intensity distributions at higher angular resolutions.

An optimal filtering technique is introduced in Dutrey et al. (2007) to enhance the S/N of integrated intensities of molecular-line emission in Keplerian disks. Their technique, however, relies on a best-fit line profile that needs to be derived first from model fitting. This is then adopted to weigh velocity channels for integration. Here, we introduce a new method to enhance the detection of molecular lines while conserving the total integrated flux with minimum assumptions on line profiles. Our method takes advantage of the ordered and symmetric Keplerian rotational velocity pattern. In a Keplerian disk, spectra at different positions have different centroid velocities. Different from directly stacking spectra from different positions in a disk—which is not coherently adding signals—our method first aligns these spectra based on their expected Keplerian velocities and then azimuthally and/or radially stacks them. With this alignment, the total integrated flux of the stacked spectrum remains unchanged but is redistributed into a narrower velocity range, as compared to stacking without prior alignment. As a result, the peak intensity increases, and thus, the S/N is boosted.

Similar attempts have been made to optimize S/Ns of integrated line intensities in debris disks. In Matrà et al. (2015), at a given position, only the velocity channel corresponding to the expected projected Keplerian velocity at that position was added to estimate the upper limit of the CO (3–2) integrated intensity in the Fomalhaut ring. Marino et al. (2016) varied the spatial and velocity ranges for integration, and found that the integrated velocity range optimizing the S/N of the CO (2–1) integrated intensity in the HD 181327 debris ring is consistent with the expected Keplerian velocity. Stacking techniques are also applied in extragalactic studies. In galaxy samples, spectra of individual galaxies can be redshift-aligned and then stacked to enhance a line detection yielding sample-averaged properties (e.g., Delhaize et al. 2013). In a single resolved galaxy, molecular-line spectra from different positions can be aligned using observed mean velocities of H I at the same positions as a proxy. They can then be stacked to measure gas properties at different positions (e.g., Caldú-Primo et al. 2013; Higdon et al. 2015). In this article, we focus on a protoplanetary disk and start with a Keplerian rotational velocity pattern. We demonstrate our method using ALMA archival data of the
protoplanetary disk around HD 163296, and we provide a
detailed analysis of the method with a semi-analytical
comparison and an understanding of how exactly the S/N is
boosted and what improvement can be expected.

2. NEW STACKING METHOD

Our method makes use of the Keplerian rotational velocity
pattern, which is derived with the inclination (i), the position
angle of the disk major axis (PA), and the stellar mass
(Ms). Note that Ms, i, and PA can be measured from
continuum and molecular-line images with sufficient S/N. We
first correct for the plane-of-sky projection and compute the
expected Keplerian velocity at every pixel in the disk.
Assuming that the disk is geometrically thin, the projection
correction is
\[
\begin{align*}
x &= \Delta \cos \sin PA + \Delta \cos \cos PA, \\
y &= \Delta \delta \sin PA - \Delta \cos PA,
\end{align*}
\]  
where x and y are the positional offsets along the major and
minor axes, and \( \Delta \alpha \) and \( \Delta \delta \) are the R.A. and decl.
offsets with respect to the central star. The deprojected radius (\( r \)) in
the disk plane is computed as
\[
r = \sqrt{x^2 + \left(\frac{y}{\cos i}\right)^2},
\]
and the projected Keplerian velocity (\( V_k \)) at the position (\( \Delta \alpha, \Delta \delta \)) as
\[
V_k = \sqrt{\frac{GM_\star}{r}} \cdot \cos \theta \cdot \sin i,
\]
where G is the gravitational constant and \( \cos \theta = x/r \). We first
align spectra (\( \equiv I_\lambda \)) from different positions (\( \Delta \alpha_j, \Delta \delta_j \)) by
shifting them by their \( V_{k,j} (\Delta \alpha_j, \Delta \delta_j) \). We then stack them, that is,
does not hold coherently in velocity,
\[
\sum_{j=1}^{N} I_V (\Delta \alpha_j, \Delta \delta_j, V - V_{k,j}).
\]
where V is the relative velocity with respect to the systemic
velocity of the disk, and N is the number of pixels within the
radial and azimuthal ranges adopted for stacking. Figure 1
illustrates the stacking process. Additionally, since the
Keplerian rotational velocity pattern is mirror-symmetric with
respect to the disk minor axis, spectra from one side of the
minor axis are first reversed with respect to their \( V_k \) before
being stacked with spectra from the other side.\(^5\) Alignment is
not possible for small r when \( V_k \propto r^{-0.5} \) is beyond the velocity
band. Such pixels can still be stacked, but the alignment cannot
be applied.

In our approach, we coherently stack (i.e., add after aligning)
spectra from different positions in a Keplerian disk. In the
following we derive approximate formulae for the achieved S/N
gain in this process. We explain the two effects that make

\(^5\) For blended hyperfine components, our method can still be applied using
their major components to align their overall line profiles. However, the S/N of
minor hyperfine components is not fully enhanced because of their inaccurate
alignment due to slight frequency differences and their line profiles that are not
symmetric with respect to \( V_k \) of their major components. Nevertheless, the
overall S/N can be enhanced. Nonblended components can simply be treated
as individual lines.

this possible, namely (1) narrower line widths and (2) the
decorrelation of initially correlated noise. In order to be
explicit, we illustrate our calculations for azimuthally averaged
spectra.

2.1. Narrower Line Widths

Without any alignment, the line width of an azimuthally
averaged spectrum (\( \Delta V_{\text{int}} \)) at a radius \( r \) in a Keplerian disk is
approximately
\[
\Delta V_{\text{int}} (r) \sim 2 \times V_k (r, \theta) \sim 2 \sqrt{\frac{GM_\star}{r}} \sin i,
\]
where \( \theta = 0 \) is adopted because \( V_k \) reaches its maximum along
the major axis. The factor of two results from \( V_k \) having
opposite signs (blueshifted and redshifted) with respect to the
systemic velocity on the two sides of the disk. The line width of
an azimuthally averaged spectrum after alignment in velocity
(\( \Delta V_k \)) is related to its intrinsic line width (\( \Delta V_{\text{int}} \)) and line
broadening due to beam convolution (\( \Delta V_{\text{conv}} \)) over positions
with different \( V_k \). Here, \( \Delta V_{\text{int}} \) is a combination of thermal and
turbulent line widths together with the integration along the line
of the sight passing through different scale heights, where the
rotational velocities are different. Observations of a sample of
protoplanetary disks show that the typical 1\( \sigma \) line width of
\( \Delta V_{\text{int}} \) is
\[
\Delta V_{\text{int}} \sim (0.1 - 0.3) \times \left(\frac{r}{100 \text{ au}}\right)^q \text{ km s}^{-1},
\]
where \( q \) ranges from \(-0.3\) to \(-0.1\) (e.g., Piétu et al. 2007).
Here, \( \Delta V_{\text{conv}} \) can be described as
\[
\Delta V_{\text{conv}} \sim A_b \times \frac{\partial^2}{\partial r \partial \theta} V_k \sim \frac{A_b \sqrt{GM_\star}}{2r^{2.5}} \sin \theta \sin i,
\]
where \( A_b \) is the area of the synthesized beam. Then \( \Delta V_k \)
is estimated as
\[
\Delta V_k \sim 4 \Delta V_{\text{int}} + \Delta V_{\text{conv}},
\]
where the factor of four results from considering the line width
covering most of the total integrated flux, that is, the full-
width zero-intensity (FWZI) line width, which is approximately
\( 4 \Delta V_{\text{int}} \).

2.2. Decorrelation of Correlated Noise in Interferometric
Images

Interferometric images are generated by Fourier-transforming
visibilities. Thus, in an interferometric image, pixels within
a synthesized beam area are not independent but are correlated,
while pixels in different velocity channels are independent
from each other. Therefore, when averaging over an area (\( A_{\text{ave}} \))
without alignment in velocity, the number of independent
pixels is approximately \( A_{\text{ave}} / A_b \). On the contrary, when
computing an average spectrum after velocity alignment,
averaging is performed over pixels that originally were in
different velocity channels. As a result, the number of
independent pixels increases. Generally, the number of
independent pixels increases when the difference in \( V_k \) between
nearby pixels is larger than the velocity channel width \( \Delta v \). The
spatial scale \( A_{\text{dep}} \) where the difference in \( V_k \) becomes larger
than $dv$ can be derived as

$$A_{\text{dep}} = \frac{rdv}{\partial \theta \partial \theta} V_k \sim \frac{2r^{2.5}dv}{\sqrt{GM_b \sin \theta \sin \imath}},$$

and only pixels within $A_{\text{dep}}$ are correlated after alignment. Hence, the number of independent pixels when averaging over $A_{\text{ave}}$ after alignment becomes $A_{\text{ave}}/A_{\text{dep}}$. We note that $A_{\text{dep}}$ has a limited range,

$$A_{\text{pix}} \leq A_{\text{dep}} \leq A_b,$$

where $A_{\text{pix}}$ is the area of one pixel. This is because the minimum scale in an image is one pixel, and in interferometric images pixels separated by more than the synthesized beam size are inherently not correlated.

### 2.3. Approximate Formulae for S/N Enhancement after Stacking with Alignment

Since the total integrated intensity over all velocity channels remains unchanged after alignment, the peak intensity of azimuthally averaged spectra after alignment in velocity increases approximately as $\Delta V_{\text{na}}/\Delta V_a$. Noise, on the contrary, decreases as the square root of the number of independent measurements after averaging, that is, by $\sqrt{A_b/A_{\text{dep}}}$. As a result, the S/N of azimuthally averaged spectra at the peak after alignment is boosted by

$$\frac{\Delta V_{\text{na}}}{\Delta V_a} \times \frac{\sqrt{A_b}}{A_{\text{dep}}},$$

Figure 1. Schematic illustration of stacking three spectra having different centroid velocities (color curves) at different positions (color dots) in a Keplerian disk (orange oval). When aligning the spectra, the signals are shifted to the same velocity, and the line width becomes narrower. As a result, the peak intensity, and thus the S/N, increases after stacking. Without alignment, the signals remain at different velocities and do not coherently add after stacking, and the S/N is not enhanced.
while the S/N of the total integrated intensity or the mean intensity per channel of azimuthally averaged spectra is enhanced after alignment by
\[
\frac{\Delta V_{na}}{\Delta V_a} \times \frac{A_b}{A_{dep}},
\]
where \(\Delta V_{na}\) and \(\Delta V_a\) are explicitly derived in Section 2.1. The S/N enhancement in azimuthally averaged spectra after alignment \(R_{na}\) can then be estimated as
\[
R_{na} \sim \frac{\Delta V_{na}}{4\Delta V_{int} + \Delta V_{conv}} \times \frac{A_b}{A_{dep}}.
\]
At outer radii, the velocity gradient \((\partial, V_k)\) is small, \(4\Delta V_{int} \gg \Delta V_{conv}\), and thus
\[
R_{na} \sim \frac{\Delta V_{na}}{4\Delta V_{int}} \times \frac{A_b}{A_{dep}} \propto r^{-3/4}.
\]
At inner radii, where \(\Delta V_{conv} \gg 4\Delta V_{int}\),
\[
R_{na} \sim \frac{\Delta V_{na}}{\sqrt{\Delta V_{conv}}} \times \frac{A_b}{A_{dep}} \propto r^{-0.25}.
\]
Therefore, the S/N enhancement increases with smaller radii and flattens at larger radii. The improvement is small when \(V_k\) is beyond the velocity band and when \(r\) is small compared to the beam size.

The S/N enhancement further depends on the accuracy of \(V_k\) at individual positions. Therefore, the (unknown) scale heights of molecular distributions in a disk, uncertainties in stellar mass and disk orientation, and coarse velocity resolutions lead to more inaccurately aligned spectra, leaving the signal spread over a wider velocity range even after alignment. Hence, these effects reduce the S/N of stacked spectra after alignment, and a maximum S/N is achieved when correct stellar mass and disk orientation are adopted. Consequently, this method can also be used to estimate proper coordinates and disk orientation.

Since the S/N enhancement is related to the ratio of line widths of spectra with and without alignment, this method is more effective for disks originally having wider line widths, that is, disks with larger stellar masses and closer to edge-on. Stacking with alignment can be performed over any meaningful combination of radial and azimuthal ranges as long as the chosen area leads to the desired S/N. Hence, this method can also be used to extract intensity profiles (when stacked azimuthally in radial bins) or azimuthally asymmetric distributions (when stacking azimuthally distinct sectors).

3. DEMONSTRATION CASE: HD 163296

HD 163296 is a Herbig Ae star at a distance of 122 pc (van den Ancker et al. 1998). It is surrounded by a disk of several hundred astronomical units showing a clear Keplerian rotation with a central stellar mass of \(\sim 2.5 M_{\odot}\) (from CO, HCO\(^+\), and their isotopes; Mannings & Sargent 1997; Isella et al. 2007; Hughes et al. 2008; de Gregorio-Monsalvo et al. 2013; Mathews et al. 2013; Rosenfeld et al. 2013). Inclination and position angle are measured to be \(\sim 45^\circ\) and \(\sim 130^\circ\), respectively (Isella et al. 2007). Its three-dimensional structure is imaged in CO with ALMA (de Gregorio-Monsalvo et al. 2013; Rosenfeld et al. 2013). Observations in \(^{13}\)CO, DCO\(^+\), and N\(_2\)H\(^+\) have revealed the location of the CO snow line at \(r \sim 90\) au (Qi et al. 2011, 2015; Mathews et al. 2013). The ALMA DCO\(^+\) and N\(_2\)H\(^+\) images clearly show ring-like structures (Mathews et al. 2013; Qi et al. 2015). Several additional molecular lines toward HD 163296 were selected by ALMA but remain marginally or not at all detected. This disk is, thus, an excellent proof-of-concept target.

The HD 163296 data analyzed here are retrieved from the ALMA archive (project code: 2013.1.01268.S). Observations were done with 31 to 33 antennas during the cycle 2 observing period on 2014 July 27–29. HD 163296 was observed for 3.6 hours in total. The pointing center was \(\alpha\) (J2000) = 17h56m21s28, \(\delta\) (J2000) = –21°57′22′′4. The correlator was configured in the Frequency Division Mode. DCO\(^+\) (3–2; 216.113 GHz), DCN (3–2; 217.239 GHz), N\(_2\)D\(^+\) (3–2; 231.322 GHz), and H\(_2\)CO (3\(_0\),3–2\(_0\),3; 218.222 GHz), (3\(_2\),2–2\(_1\),2; 218.476 GHz), and (3\(_1\),1–2\(_2\),0; 218.76 GHz) were observed simultaneously with a spectral resolution of 61 kHz. Each spectral window had a bandwidth of 58.6 MHz. Calibration of the raw visibility data was performed with the standard reduction script for cycle 2 data using tasks in Common Astronomy Software Applications (CASA) and without self-calibration. Calibrated visibilities were Fourier-transformed with natural weighting and CLEANed with the CASA task “clean” at a velocity resolution of 0.1 km s\(^{-1}\). The angular resolutions of the images are \(\sim 0.5\) × \(0.4\). The noise levels are 2.4 mJy Beam\(^{-1}\) in the DCO\(^+\) and DCN, 2 mJy Beam\(^{-1}\) in the H\(_2\)CO, and 2.9 mJy Beam\(^{-1}\) in the N\(_2\)D\(^+\) image.

4. RESULTS AND DISCUSSION

Figure 2 presents the total integrated intensity (moment 0) maps of the six molecular lines from the archival data. DCO\(^+\) (3–2) and H\(_2\)CO (3\(_0\),3–2\(_0\),3) are clearly detected at S/N \(\geq 10\), showing ring-like structures. Such structures are also seen in ALMA DCO\(^+\) (4–3) and N\(_2\)H\(^+\) (3–2) observations of HD 163296 (Mathews et al. 2013; Qi et al. 2015), and H\(_2\)CO (3\(_1\),2–2\(_1\),1) and (4\(_1\),4–3\(_1\),3) are suggested to be showing ring-like structures from observations with the Submillimeter Array (Qi et al. 2013). DCN (3–2) and N\(_2\)D\(^+\) (3–2) are also detected in our moment 0 maps at S/N \(\sim 5\). DCN (3–2) is offset from the center and is more compact than DCO\(^+\) (3–2) and H\(_2\)CO (3\(_0\),3–2\(_0\),3). N\(_2\)D\(^+\) (3–2) displays several clumpy components, hinting at a ring-like structure. The H\(_2\)CO (3\(_2\),2–2\(_1\),2) and (3\(_2\),1–2\(_2\),0) maps show clear detections. Figure 3 exhibits azimuthally averaged spectra without alignment in velocity (black histograms). The radial ranges for averaging are adopted to be \(R < 300\) au \((\sim 27''5)\) for DCO\(^+\) (3–2) and H\(_2\)CO (3\(_0\),3–2\(_0\),3), (3\(_2\),2–2\(_1\),1), (3\(_2\),1–2\(_2\),0), \(R < 200\) au \((\sim 16''6)\) for DCN (3–2), and \(100 < R < 300\) au \((\sim 0''8–2''5)\) for N\(_2\)D\(^+\) (3–2). As we show below, these radial ranges are the regions where the emission lines primarily originate from. The spectra of DCO\(^+\) (3–2) and H\(_2\)CO (3\(_0\),3–2\(_0\),3) are clearly detected at S/N \(\geq 10\). They show double peaks, the characteristic line profile of Keplerian rotation. The spectra of DCN (3–2) and N\(_2\)D\(^+\) (3–2) are seen at \(<4\sigma\). H\(_2\)CO (3\(_2\),2–2\(_1\),1) and (3\(_2\),1–2\(_2\),0) are detected in neither their moment 0 maps, nor their spectra, nor their image cubes.

The red histograms in Figure 3 display azimuthally averaged spectra with alignment, following our method, where averaging is over the same radial and azimuthal ranges as above. The black histograms show the original spectra with a line width of \(\sim 6\) km s\(^{-1}\) for DCO\(^+\) (3–2) and H\(_2\)CO (3\(_0\),3–2\(_0\),3). After applying our method, the signals are accumulated in a much narrower velocity range of \(\sim 1\) km s\(^{-1}\). As a result, the peak
intensities of the spectra increase by a factor of ~5, thus boosting their S/N. Fluxes integrated over the full velocity range of the new spectra of DCO$^+$ (3–2) and H$_2$CO (3$_{0,3}$–2$_{0,2}$) are measured to be $12.9 \pm 0.1$ and $4.9 \pm 0.1$ mJy beam$^{-1}$ km s$^{-1}$, respectively, and are consistent with those from the original spectra ($11.4 \pm 0.2$ and $4.5 \pm 0.3$ mJy beam$^{-1}$ km s$^{-1}$). DCN (3–2) and N$_2$D$^+$ (3–2) are seen more clearly at $>10\sigma$ in the averaged spectra with alignment. Strikingly, H$_2$CO (3$_{2,2}$–2$_{2,1}$) and (3$_{2,1}$–2$_{2,0}$)—which are not detected at all in the moment 0 maps nor in the spectra in the original data—are now clearly visible.
In Figure 4(a), we compare radial intensity profiles derived with three different methods: (1) from the original moment 0 maps, (2) from azimuthally averaged spectra without alignment, and (3) from azimuthally averaged spectra with alignment. The profiles from all three methods have the same radial bins and are averaged over the full azimuthal range of $2\pi$. Each radial bin is half of the synthesized beam size. The intensity profiles extracted from the moment 0 maps are derived by computing mean values of pixels in each radial bin, with their uncertainties estimated as

$$\frac{\sqrt{N_0} \cdot \sigma \cdot dv}{\sqrt{A_{\text{ave}}/A_b}},$$

where $N_0$ is the number of velocity channels included to generate the moment 0 maps, $\sigma$ is the noise per velocity.
Figure 5. Reconstructed moment 0 maps of DCO$^+$ (3–2), DCN (3–2), N$_2$D$^+$ (3–2), and H$_2$CO (3$_{0,3}$–2$_{0,2}$), (3$_{2,1}$–2$_{2,0}$), and (3$_{2,1}$–2$_{2,0}$) in HD 163296 from intensity profiles from our method of azimuthally averaging (a) over 2π and (b) over azimuthally distinct sectors with alignment.
channel in the original image cubes, and $A_{\text{ave}}$ corresponds to the area of each radial bin. For the intensity profiles derived from azimuthally averaged spectra with and without alignment, we first stack spectra only from pixels in the same radial bin with and without alignment, respectively. Then we compute the integrated intensity and divide it by the number of the pixels in the radial bin to derive a mean integrated intensity at each radius. Their uncertainties are estimated as

$$\sigma_i \cdot \Delta v \cdot \sqrt{N}, \quad (17)$$

where $\sigma_i$ is the noise per channel in the averaged spectra, and $N$ is the number of integrated velocity channels. As explained in Section 2, $\sigma_i$ and $N$ of the averaged spectra with alignment are smaller than those without alignment because of noise decorrelation and narrower line widths.

Since the total integrated flux is conserved in all three methods, intensity profiles necessarily need to be consistent within uncertainties. This is, indeed, observed for DCO$^+$ (3–2) and DCN (3–2), which are detected at high significance in the original data. In contrast, the two weakest lines, H$_2$CO (3$_{2,2}$$-$2$_{2,1}$) and (3$_{2,1}$$-$2$_{2,0}$), seem to show differences. This is because their intensities are comparable to their uncertainties when directly working with their moment 0 maps or their averaged spectra without alignment. These intensity profiles are, thus, highly uncertain. Here, method (3) is clearly superior, greatly reducing uncertainties and being the only one of the three methods that is able to measure radial features.

In Figure 4(b) we compare gains in S/N for the intensity profiles from the three different methods. Only data points at radial bins where all three methods show a detection at more than 3$\sigma$ are plotted. DCO$^+$ (3–2) and DCN (3–2) show ratios in the S/N of intensity profiles from averaged spectra with alignment that are ~4–6 times higher than those from moment 0 maps, and they are ~2–4 times higher than those from averaged spectra without alignment. Our results further show that only with the boost in S/N provided by our method of alignment can the intensity profiles of the weaker H$_2$CO (3$_{2,2}$$-$2$_{2,1}$) and (3$_{2,1}$$-$2$_{2,0}$) lines be measured at high significance. Additionally, the integrated intensities computed from averaged spectra with alignment tend to be higher than those from method (1) and (2). This is because our method with alignment can detect fainter emission, originally embedded in the noise, due to the effect of decorrelation. Furthermore, our method gives a better constraint on the inner and outer radii of the N$_2$D$^+$ (3–2) ring, where emission is only marginally detected with method (1) and (2). In conclusion, our method of alignment can measure intensity profiles of weak molecular lines at high significance, providing constraints on size, width, and depth of cavities, gaps, and rings (if present) in protoplanetary disks where molecular-line intensity is low.

We further compute the expected S/N enhancement (Equation 13) to compare with our observational results (Figure 4b). We adopt $\Delta V_{\text{int}} \sim 0.3 \times (r/100 \text{ au})^{-0.3}$. For the S/N enhancement compared to method (1), we substitute $\Delta V_{\text{int}}$ (which is a function of radius) with a constant velocity width of 9.3 km s$^{-1}$, which is the velocity width we integrated to generate the moment 0 maps. The S/N enhancements in DCO$^+$ (3–2), DCN (3–2), N$_2$D$^+$ (3–2), and H$_2$CO (3$_{0,3}$$-$2$_{0,2}$) are consistent with expectations, except at radii smaller than the synthesized beam size where the observed enhancement is lower, and Equation (13) is not valid any longer.

For a visual impression, we reconstruct moment 0 maps from the radial profiles (Figure 5a), noting that the actual emission distributions appear not to be fully axisymmetric (Figure 2). The intensity distributions of H$_2$CO (3$_{2,2}$$-$2$_{2,1}$) and (3$_{2,1}$$-$2$_{2,0}$) are similar to those of DCO$^+$ (3–2) and H$_2$CO (3$_{0,3}$$-$2$_{0,2}$), and the intensities peak at radii around ~100 AU. In addition, all three H$_2$CO transitions show flat distributions at radii beyond $\gtrsim$200 au. This is different from DCO$^+$ (3–2) and DCN (3–2), which show more steeply declining profiles. The ring-like structure in DCN (3–2) is possibly located closer (at a radius of ~50 AU) to the center than those in DCO$^+$ and H$_2$CO, while the ring-like structure of N$_2$D$^+$ is further outside than all the other lines at around 200 au. This trend is consistent with the observational results in DCO$^+$ (4–3) and N$_2$H$^+$ (3–2) in HD 163296 (Mathews et al. 2013; Qi et al. 2015), which show N$_2$H$^+$ (3–2) having a larger inner cutoff radius than DCO$^+$ (4–3). Similarly, we can also measure mean integrated intensities of azimuthally distinct sectors to investigate asymmetric intensity distributions. We divide the outer disk at $r \gtrsim 0''45$ (the beam size) into azimuthally and radially distinct sectors, each sector spanning 1.5 times the beam size along the radial axis and 45$^\circ$ in azimuth. The inner disk inside $r < 0''45$ is still azimuthally averaged over $2\pi$ because position angles of individual pixels cannot be computed accurately for such small radii with this resolution. For each sector, we align and stack spectra from every pixel within its radial and azimuthal range, divide the stacked spectra by the number of pixels included, and measure the total integrated intensity. The reconstructed moment 0 maps are presented in Figure 5b. In the original moment 0 map (Figure 2), DCO$^+$ (3–2) is brighter toward the northwest and the southeast, and N$_2$D$^+$ (3–2) shows brighter clumps toward north and south. These features are also identified in the reconstructed maps (Figure 5b). The original bright DCN (3–2) emission toward the east is mostly in the innermost bin that is now averaged over $2\pi$. Hence, this feature is not clear in the reconstructed map (Figure 5b). A fainter emission toward the southwest is now revealed in H$_2$CO (3$_{0,3}$$-$2$_{0,2}$), and the previous nondetections now show patchy detections in H$_2$CO (3$_{2,2}$$-$2$_{2,1}$) and (3$_{2,1}$$-$2$_{2,0}$). The detailed interpretation of the different distributions of these molecular lines is beyond the scope of the present article and is not discussed here.

In summary, our results demonstrate that, by aligning spectra with different centroid velocities from different positions in a disk and stacking them, we are able to enhance the S/N of molecular-line data and materialize spectra of molecular lines that are originally undetectable. Our method can significantly lower the detection limit of molecular lines and, thus, can be applied to search for faint molecular lines in protoplanetary disks. Furthermore, because of the S/N enhancement, intensity profiles of molecular lines can be measured more accurately and in smaller bins, which is equivalent to achieving higher spatial resolutions. Molecular-line images can also be generated at higher angular resolutions because more weighting can be given to longer baselines, which typically are noisier, but their S/N can be enhanced with this method. Finally, our method can be applied not only to molecular-line data of protoplanetary disks but to any systems having ordered kinematic patterns.

This paper makes use of the following ALMA data: ADS/JAO.ALMA#2013.1.01268.S. ALMA is a partnership of ESO
(representing its member states), NSF (USA), and NINS (Japan), together with NRC (Canada) and NSC and ASIAA (Taiwan), in cooperation with the Republic of Chile. The Joint ALMA Observatory is operated by ESO, AUI/NRAO, and NAOJ. We thank all the ALMA staff supporting this work. PMK acknowledges support from an Academia Sinica Career Development Award. PMK, NH, C-FL, and ST acknowledge support from the Ministry of Science and Technology (MOST) of Taiwan through grants MOST 104-2119-M-001-019-MY3, MOST 105-2112-M-001-026, MOST 104-2119-M-001-015-MY3, and MOST 102-2119-M-001-012-MY3.

REFERENCES

ALMA Partnership, Brogan, C. L., Pérez, L. M., et al. 2015, ApJL, 808, L3
Caldú-Primo, A., Schruba, A., Walter, F., et al. 2013, AJ, 146, 150
de Gregorio-Monsalvo, I., Ménard, F., Dent, W., et al. 2013, A&A, 557, A133
Delhaize, J., Meyer, M. J., Staveley-Smith, L., & Boyle, B. J. 2013, MNRAS, 433, 1398

Dutrey, A., Henning, T., Guilloteau, S., et al. 2007, A&A, 464, 615
Dutrey, A., Semenov, D., Chapillon, E., et al. 2014, in Protostars and Planets VI, ed. H. Beuther et al. (Tucson, AZ: Univ. Arizona Press), 317
Higdon, J. L., Higdon, S. J. U., Martin Ruiz, S., & Rand, R. J. 2015, ApJL, 814, L1
Hughes, A. M., Wilner, D. J., Qi, C., & Hogerheijde, M. R. 2008, ApJ, 678, 1119
Isella, A., Testi, L., Natta, A., et al. 2007, A&A, 469, 213
Mannings, V., & Sargent, A. I. 1997, ApJ, 490, 792
Marino, S., Matrà, L., Stark, C., et al. 2016, MNRAS, 460, 2933
Mathews, G. S., Klaassen, P. D., Juhász, A., et al. 2013, A&A, 557, A132
Matrà, L., Panić, O., Wyatt, M. C., & Dent, W. R. F. 2015, MNRAS, 447, 3936
Piétu, V., Dutrey, A., & Guilloteau, S. 2007, A&A, 467, 163
Qi, C., D’Alessio, P., Oberg, K. I., et al. 2011, ApJ, 740, 84
Qi, C., Oberg, K. I., Andrews, S. M., et al. 2015, ApJ, 813, 128
Qi, C., Oberg, K. I., & Wilner, D. J. 2013, ApJ, 765, 34
Rosenfeld, K. A., Andrews, S. M., Hughes, A. M., Wilner, D. J., & Qi, C. 2013, ApJ, 774, 16
van den Ancker, M. E., de Winter, D., & Tjin A Djie, H. R. E. 1998, A&A, 330, 145