Aircraft flight control using method of robustness aimed at uncertainty

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Abstract—Method of $\mu$ synthesis based on self-contained structured singular value theory provides a resolution for control system with uncertain character. It makes the control system to have enough robustness and optimal control performance. When the aircraft is flying in the dense aerosphere, the autopilot of the aircraft is facing such a control system with a lot of uncertain elements. To obtain the optimal robustness in such case, the method of $\mu$ synthesis is used to design the autopilot in this paper. Also the result of simulation is tested and analysed. And the final analysis indicates the adopted controller has an effective result to the uncertainty.

1. Introduction

It provides satisfactory performance for systems with model errors and uncertain changes, which is the original motivation of the feedback control system. Feedback only needs to be introduced if the uncertainty in the system features makes its performance meet the requirements. If a model with uncertainty description is given, and it is considered that the model appropriately expresses the basic characteristics of the object, the next step in the controller's design is to determine which structure can achieve the desired performance, and the pre-filter input signal (or open-loop control) can change the dynamic response of the model set, but it cannot reduce uncertainty. The qualitative impact, if the uncertainty is too great to meet the expected control performance, the feedback structure is needed. However, feedback structure alone does not guarantee reducing the impact of uncertainty. There are still many obstacles to using feedback to reduce the impact of uncertainty, because for any reasonable set of models that represent the uncertainty of physical systems, it will become larger at a sufficiently high frequency and the phase is completely unknown, so it must be These frequencies minimize closed-loop gain as much as possible to avoid dynamic instability of high-frequency systems\cite{1-3}.

What's worse, the feedback system actually increases the uncertainty and sensitivity of the system in a highly uncertain frequency band. That is to say, due to the limitations that it is difficult to fully describe the establishment of a reasonable set of physical systems models, the closed-loop model set cannot be used to make the closed-loop model set an rational subset of the open-loop model set, but it can be achieved by using feedback to significantly reduce the uncertainty of certain important signals. At the same time, the increase in uncertainty about other signals is very small \cite{4}. Therefore, the core of feedback design is to compromise around how to reduce the comprehensive impact of uncertainty. But this undoubtedly makes the performance of the control system compromised, because usually compromise considerations will lead to the conservativeness of the design, how to target uncertainty,
so that the control system design can resist these uncertainties (i.e. robust) and achieve better control performance, which is counterproductive in this case. The design core of the feed system. Because of the above innate shortcomings, traditional classical control has developed rapidly since the 1990s and applied it to engineering design\[^{5-7}\].

The missile is controlled in a dense atmosphere. Due to the complexity of its control system modeling, its autopilot design has to face a variety of uncertainties, such as inaccurate aerodynamic modeling, aerodynamic uncertainty, wind interference, sensor modeling error, etc. (of which the uncertainty of aerodynamic characteristics is most important) \[^{8}\]. Traditional design methods reduce the impact of uncertainty by improving conservative design with stable margins, but this also reduces the performance of autopilots. Robust control gives a better solution to it \[^{9}\]. Although \(H_\infty\) control is based on the robust design of modern control theory, its inherent conservative design is still robust, and the Method of \(\mu\) synthesis is based on the complete structural singular value theory, which can achieve the most robust design of the control system, which is automatic driving. The goal of the design of the driving instrument is to be achieved. Therefore, this paper adopts the Method of \(\mu\) synthesis to design the missile self-driving instrument. First of all, the control algorithm of the D-K iteration of the Method of \(\mu\) synthesis is introduced, and then the mathematical model of the longitudinal channel driver of the missile is converted and described accordingly, and the traditional small disturbance is linearized. The dynamic model is converted into the form of state space. Through the design of the \(\mu\) comprehensive controller using the robust control toolbox of Matlab, the design controller is simulated and analyzed.

2. \(\mu\) synthesis method
The Method of \(\mu\) synthesis is based on the structural singular value theory. It can not only effectively and conservatively judge the impact of perturbation in the worst case, but also analyze the robust stability and robust performance of the control system when there is structural uncertainty in different expression forms\[^{10}\]. Compared with other robust control methods, it has the following advantages:

- The structured uncertainty can be completely and directly introduced into controller design, without changing it into a class of larger uncertainties, which leads to unnecessary conservatism;
- The \(\mu\) methods can also analyze robust performance and synthesis. It's not like the optimization method that can only analyze and design controllers with robust stability.

At present, the effective approximation method of \(\mu\) synthesis is "D-K iteration". Assuming that \(K\) is a state or output feedback matrix, \(M\) is a generalized nominal object matrix, \(\Delta\)is a structural uncertainty matrix, and \(\mathcal{S}(\Delta(j\omega))<1, \forall \omega \in R\) is satisfied. According to the \(\mu\) theorem, it can be seen that the sufficient and necessary conditions for robust stability of the system:

The feedback controller \(K\) can stabilize the closed loop of the object and constitute a stable nominal system, which is the \(K\) can calm the \(M\);

\[\mu_\gamma(M(K)) < 1, \forall \omega \in R\]

Where the \(M(K)\) can be expressed \(F_r(M, K)\) as the lower linear fractional transformation form of \(M\) and \(K\), and according to the fundamental property theorem of \(\mu\), the above sufficient and necessary conditions can be transformed into:

\[F_r(M, K) \in RH_{-\infty}, \inf \mathcal{S}(D^{-1} F_r(M, K)D^{-1}) < 1\]

The purpose of Method of \(\mu\) synthesis design is to find the regular controller \(K\) so that the above conditions can be established, where the scale matrix \(D\) can be selected as a stable minimum phase matrix and

\[D(s)\Delta(s) = \Delta(s)D(s)\]

Can be satisfied.

According to the conversion condition (2), by repeatedly solving \(K\) and \(D\), a satisfactory \(\mu\) value is finally obtained. Considering the optimal performance problem, the \(\mu\) synthesis problem is further transformed from condition (2) to
If $D$ is fixed, then

$$\min_k \| D F_i (M, K) D^{-1} \|_{\infty}$$

This is a standard control problem; if $K$ is fixed, it is

$$\inf (, ) \inf \| D F_i (M, K) D^{-1} \|_{\infty}$$

This is a convex optimization problem about $D$.

It can be seen that the basic idea of the D-K iteration of the optimization $\mu$ synthesis problem is to fix $D$ first and obtain the minimized $K$; then fix $K$ to obtain the minimized $D$; then fix $D$ to minimize $K$, and so on, finally obtain the optimal $D$ and $K$. The description of the D-K iterative algorithm is as follows:

1. select the initial scale matrix $D (S)$;
2. fix $D (S)$, solve formula (3) for control problem, get the controller $K(S)$;
3. fix $K(S)$, solve the convex optimization problem of the solution (4), and get the scale matrix $\bar{D}(s)$;
4. Compare $D(S)$ and $\bar{D}(s)$. If the difference between the two meets the set value requirements, the iteration ends, and the resulting controller $K(S)$ is the optimal controller; otherwise, let $D(s) = \bar{D}(s)$, return step (2), continue a new round of iteration, as shown in Figure 1.

![Fig.1 D-K iteration algorithm program](image)

Although the D-K iteration does not guarantee global optimal solution, the controller obtained by iteration is usually very effective, which is also what needs to be improved by D-K iteration.

3. Integrated Control Design of Aircraft and Simulation

Longitudinal autopilot design of an aircraft (i.e. missile) usually adopts the method of small disturbance linearization, so as to obtain the transfer function of each dynamic variable, and then design the transfer function through parameter design. Because this paper adopts a robust optimal control method, the traditional dynamic model of small disturbance linearization is converted into the form of state space, so as to facilitate the design and analysis of $\mu$ synthesis. The longitudinal motion of the missile
can be approximated by a small perturbation equation. We describe the transfer function of the longitudinal motion of the missile in the form of an equation of state as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

\[\text{\textbf{(5)}}\]

\[
x = \begin{bmatrix}
\alpha \\
\phi
\end{bmatrix}, u = \delta_p, y = \begin{bmatrix}
A_{\alpha m} \\
q_m
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
\frac{1}{V_{s0}} \left[ \frac{\tilde{\Omega}SC_{\alpha u0}}{\frac{mV_{s0}}{\overline{x}}} - A_{X0} \right] & 1 \\
0 & \frac{\tilde{\Omega}SC_{\alpha u0}}{I_{YY}}
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
\frac{\tilde{\Omega}SC_{\alpha u0}}{mgI_{YY}} - \frac{\tilde{\Omega}SdC_{\alpha u0}\overline{x}}{I_{YY}} & 0 \\
0 & 1
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
\frac{\tilde{\Omega}SC_{\alpha u0}}{mg} - \frac{\tilde{\Omega}SdC_{\alpha u0}\overline{x}}{I_{YY}} & 0 \\
0 & 1
\end{bmatrix},
\]

\[
D = \begin{bmatrix}
\frac{\tilde{\Omega}SC_{\alpha u0}}{mg} - \frac{\tilde{\Omega}SdC_{\alpha u0}\overline{x}}{I_{YY}} & 0 \\
0 & 1
\end{bmatrix}
\]

Tab.1 The meaning of mainly variables

| variable | Meaning description | variable | Meaning description |
|----------|---------------------|----------|---------------------|
| \(V_{s0}\) | Missile speed | \(C_{\alpha u0}\) | Derivation of angle of attack with pitch coefficient |
| \(m\) | Missile quality | \(C_{\alpha u0}\) | Derivation of angle of attack with pitch torque coefficient |
| \(I_{YY}\) | Turn inertia of pitch channel | \(C_{\alpha f,\alpha}\) | Derivation of rudder deviation by pitch force coefficient |
| \(\overline{x}\) | Distance from center of mass to inertial conduction | \(C_{\alpha f,\alpha}\) | Derivation of rudder deviation with pitch torque coefficient |
| \(A_{X0}\) | Axial acceleration | \(\bar{\Omega}\) | Dynamic pressure |
| \(S\) | Reference area | \(d\) | Reference length |

The block diagram of the missile control system based on \(\mu\) synthesis is shown in Figure 2. The purpose is to design a \(\mu\) integrated controller under the influence of various noise \(W\) and uncertainties so that the actual overload of the missile can correctly track overload instructions.

Using D-K iteration to get the controller of the autopilot, which is also an equation of state. After 3 iterations, the controller is of order 28.
| Iteration | Controller J | Maximum value of controller command $\mu$ |
|-----------|--------------|------------------------------------------|
| 1         | 14           | 1.203                                    |
| 2         | 18           | 1.044                                    |
| 3         | 28           | 0.807                                    |

In Figure 3(a), the transient response of the closed-loop system with the step signal of the designed $\mu$ controller is shown, and the amplitude of the step signal corresponds to the normal acceleration change of $\pm$ 15g. The overshoot is less than 1%, and the establishment time is less than 1 s.

For the same reference signal, the transient response of pitch rate is shown in Figure 3(b).

In Figure 4, we show the disturbance transient response of the closed-loop system for a step reference signal with an amplitude of $\pm$ 15g, considering various perturbation factors. As can be seen, in different perturbation conditions, the autopilot is stable.
4. Conclusion
This paper adopts the establishment of a longitudinal missile body model based on state space, and adopts the $\mu$ synthesis method to design the autopilot. Compared with the traditional classical control method, the structural singular value method solves the robustness problem unconservatively for controlled objects with various bounded and non-parameter uncertainties. And the $\mu$ analysis method is more common than the gain margin and phase margin method in classical control theory. The $\mu$ comprehensive method can not only solve the parameter change problem in the dynamic controller, but also deal with the uncertainty in the controlled object model and the parameter perturbation at any position in the loop. In any case that makes the closed-loop system unstable, the stability and robustness of the structure singular value can be quantitatively and unconservatively given through the analysis of its singular value. It can be seen from the simulation results that the controller designed by $\mu$ synthesis can not only ensure the robust stability of the closed-loop system, but also ensure the robust performance of the system under perturbation. So the $\mu$ synthesis designed controller is beyond the traditional control methods, that is the advantages of $\mu$ synthesis.

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References
[1] Jing Zhou, Lantao Xing, Changyun Wen. Adaptive Control of Dynamic Systems with Uncertainty and Quantization[M]. CRC Press:2021-09-23.
[2] Yanzhou Li, Yongkang Lu, Yuanqing Wu, Shenghuang He. Robust cooperative control for micro/nano scale systems subject to time-varying delay and structured uncertainties. The International Journal of Advanced Manufacturing Technology, 2019,105(4):4863-4873.
[3] A S Faskhodi, A Fakharian. Output Feedback Robust Siding Mode Controller Design for Wind Turbine. Journal of Electrical Engineering & Technology, 2019,14(6):2477-2485.
[4] R T Reichert, Robust Autopilot Design Using u-Synthesis, Proceedings of the American Control Conference, San Diego, 1990, CA:2368-2373.
[5] Wang Na, Dang Miao. Robust Tracking Control of Multijoint Robots Based on Adaptive Fuzzy Observer. Journal of Engineering Science & Technology Review. 2020, 13(4):154-161.
[6] H F Ghavidel, A A Kalat. Robust control for MIMO hybrid dynamical system of underwater vehicles by composite adaptive fuzzy estimation of uncertainties[J]. Nonlinear Dynamics, 2017,89(4):2347-2365.
[7] V H Nguyen, Tran, T A Thanh. Novel Hybrid Robust Control Design Method for F-16 Aircraft Longitudinal Dynamics. Mathematical Problems in Engineering. 2020, 22(9):1-10.
[8] Doyle J C, Analysis of Feedback Systems with Structured Uncertainties, IEEE Proceedings. Part D.1982, 129(6):242-250.
[9] Li Chenfu. Adaptive Robust bank to turn missile autopilot design using neural networks [J]. Journal of guidance control and dynamics, 1997, 20(2).
[10] Sadr. N. E, Momeni. H. R. Fuzzy sliding mode control for missile autopilot design. Proceeding of the IEEE Grid and Cooperative Computing Conference and Exhibition. 2011: 453-456
[11] Ma Yueyue. Research on high maneuver flight control method of Agile Missile at high angle of attack [D]. Beijing University of technology, 2016