Logarithmic Companding Technique to Improve Performance of SC-FDMA Systems

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Abstract

Objectives: This paper proposes logarithmic companding technique to improve PAPR and BER performance of conventional Single Carrier Frequency Division Multiple Access (SC-FDMA) system. Methods/Analysis: The lower signal envelope fluctuations in SC-FDMA signal results in low peak to average power ratio, in contrast to that of OFDMA. But still, there is a need to minimize PAPR in SC-FDMA systems. Although there are many techniques to minimize PAPR in SC-FDMA systems they are highly complex or need side information to be transmitted. This paper proposes a companding technique based on a logarithmic function which provides better PAPR reduction when compared to $\mu$ law companding. Findings: Simulation results demonstrate that proposed method provides better PAPR and BER performance when compared to conventional SC-FDMA system and $\mu$ law companding.

Keywords: Companding, LTE, PAPR Reduction, SC-FDMA

1. Introduction

3rd Generation Partnership Project (3GPP) implemented Single Carrier Frequency Division Multiple Access as an alternative to Orthogonal Frequency Division Multiple Access (OFDMA) in Long Term Evaluation (LTE) for uplink communications. The lower signal envelope fluctuations in SC-FDMA signal results in low PAPR in contrast to that of OFDMA. But still, there is a need to minimize PAPR in SC-FDMA systems. When PAPR is high, power amplifier should be operated in linear region which degrades the efficiency of power amplifier.

In literature, several methods have been studied to minimize PAPR in SC-FDMA systems. Some of them include pulse shaping techniques, selected mapping technique (SLM), Partial Transmit Sequence (PTS) and pre-coding techniques. But these techniques are highly complex, or they need excess bandwidth for transmission of side information. For simplicity non-linear companding techniques has been used in OFDMA to reduce PAPR and provide better BER performance. They also have low implementation complexity, does not require any bandwidth expansion. Several companding methods have been proposed to minimize PAPR in OFDMA systems. These methods can also be implemented in SC-FDMA systems for PAPR reduction.

When $\mu$ law companding is used to minimize PAPR in SC-FDMA systems, the companding coefficient affects the performance of PAPR and BER. As companding coefficient increases, PAPR performance is improved but BER performance is degraded. The author selected optimum value for companding coefficient as $\mu = 4$. When $\mu = 4$, PAPR is improved by about 3.25 dB at CCDF= $10^{-4}$ for Localized SC-FDMA. The power function companding technique has better PAPR reduction performance when compared to raised-cosine like companding technique. We can see that there is not much improvement in PAPR by using these methods. The improvement is less than 3.5 dB. These are the only companding methods that have been considered in the literature to minimize PAPR in single carrier FDMA systems.

This paper proposes a new companding technique based on logarithmic function to minimize PAPR in single carrier FDMA systems. Unlike other companding...
methods, proposed method also improves the BER performance of conventional SC-FDMA system. Also, this method is very simple and do not need any side information to be transmitted.

Section II introduces SC-FDMA system model. Proposed system model with logarithmic companding is described in Section III. PAPR and BER results of proposed system along with conventional SC-FDMA system are discussed in Section IV. Few conclusions are presented in Section V.

2. SC-FDMA System Model

Conventional SC-FDMA system model is shown in Figure 1. Firstly the data is encoded and modulated by using QPSK or QAM techniques. Then the modulated data is grouped into blocks of $N$ symbols, $X_n$ and passed through $N$-point DFT. The resulting signal $X_k$ is given by

$$X_k = \sum_{n=0}^{N-1} X_n e^{-j \frac{2 \pi}{N} nk}, k = 0, 1, ..., N - 1$$  \hspace{1cm} (1)

These $N$ symbols are then mapped into $M$ subcarriers ($N< M$, $M=QN$). For subcarrier mapping in SC-FDMA system, we have two techniques namely, Localized FDMA (LFDMA) and Distributed FDMA (DFDMA). Interleaved FDMA (IFDMA) is one realization of DFDMA. In LFDMA, each terminal transmits its symbols over adjacent subcarriers whereas in IFDMA the entire signal band is used to spread the subcarriers equidistantly. If localized subcarrier mapping is used then $\bar{X}_l$ is given by

$$\bar{X}_l = \begin{cases} X_k(l/Q), & (0 \leq l \leq N - 1) \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (2)

where $0 \leq l \leq M - 1$. If interleaved subcarrier mapping is used then $\bar{X}_l$ is given by

$$\bar{X}_l = \begin{cases} X_k(l), & (0 \leq l \leq N - 1) \\ 0, & (N \leq l \leq M - 1) \end{cases}$$  \hspace{1cm} (3)

After subcarrier mapping, $\bar{X}_l$ is passed through $M$-point IDFT and the resulting time domain complex signal $\bar{x}_m$ is given by

$$\bar{x}_m = \frac{1}{M} \sum_{l=0}^{M-1} \bar{X}_l e^{j \frac{2 \pi ml}{M}}, m = 0, ..., M - 1$$  \hspace{1cm} (4)

Then cyclic prefix is added to it and transmitted through the channel. The corresponding inverse operations will be carried at the receiver side to extract the data.

3. Proposed System Model

SC-FDMA system model with companding technique is shown in Figure 2. Here the data $\bar{x}_m$ is passed through compander $h(x)$ before adding CP to it. The companding scheme is described by

$$\tilde{x}_m = h(\bar{x}_m)$$  \hspace{1cm} (5)

where $\bar{x}_m$ is the signal before companding process, $\tilde{x}_m$ is the companded signal. At the receiver side, the inverse operations will be carried out and the data $\tilde{r}_m$ is passed through decompander $h^{-1}(x)$ before applying $M$-point DFT.

$$\tilde{r}_m = h^{-1}(\tilde{x}_m)$$  \hspace{1cm} (6)
where $\tilde{m}_r$ is the received signal after removing CP, $\tilde{m}_d$ is the decompanded signal. Companding and Decompanding are the additional blocks that have been added to the conventional system.

### 3.1 Logarithmic Companding

Logarithmic companding function is given by

$$h(x) = \log \left( k \left| x \right| + 1 \right) \sgn(x)$$ \hspace{1cm} (7)

where $\sgn$ is signum function and 'k' is positive number controlling the amount of companding. The logarithmic decompanding function at the receiver side is given by

$$h^{-1}(x) = \left| \exp \left( \frac{1}{k} \right) - 1 \right| \sgn(x)$$ \hspace{1cm} (8)

### 4. Results and Discussion

Here, the performance of proposed single carrier FDMA system with logarithmic companding technique is compared with conventional single carrier FDMA system. Table 1 lists the simulation parameters.

| Parameter                  | Description                        |
|----------------------------|------------------------------------|
| Simulation method          | Monte Carlo                        |
| Subcarriers spacing        | 9.765625 kHz                       |
| System bandwidth           | 5 MHz                              |
| Channel coding             | 1/2 rate Convolutional code        |
| Modulation type            | QPSK                               |
| N                          | 128                                |
| M                          | 512                                |
| Subcarrier mapping         | Localized and Interleaved          |
| CP length                  | 20 samples                         |
| Channel model              | Vehicular-A outdoor                |
| Channel estimation         | Perfect                            |
| Equalization               | MMSE                               |

Figure 3 shows complementary cumulative distribution function curves (CCDF) of proposed system with logarithmic companding for different values of k. Here we used raised cosine (RC) pulse shaping technique with a roll-off factor, $\alpha = 0.22$. We can observe that PAPR value is minimum at $k=100$ and PAPR value increases as 'k' decreases for both IFDMA and LFDMA subcarrier mapping techniques.

![Figure 3. PAPR performance of proposed system with logarithmic companding for different values of k.](image)

Bit Error Rate (BER) curves for proposed system with logarithmic companding for different values of 'k' are shown in Figure 4. We can observe BER performance is better when $k=100$. So we can choose $k=100$ as the opti-
The CCDF curves for proposed system with companding technique and conventional system with pulse shaping have been presented in Figure 5. Clearly, we can observe that PAPR has been improved for both LFDMA and IFDMA due to logarithmic companding technique. At CCDF=10^{-4} there is an improvement of about 7.71 dB in LFDMA and about 5.86 dB in IFDMA due to logarithmic companding. Clearly, logarithmic companding technique provides better PAPR reduction when compared to \( \mu \) law companding.

### Table 2. PAPR at CCDF=10^{-4}, with RC Pulse Shaping

| System | Original PAPR (dB) | \( \mu \) law PAPR (dB) | Logarithmic PAPR (dB) |
|--------|--------------------|--------------------------|-----------------------|
| LFDMA  | 8.24               | 5.04                     | 0.53                  |
| IFDMA  | 6.29               | 3.77                     | 0.43                  |

The BER curves for proposed system with companding technique and conventional system have been presented in Figure 6. We can observe that BER performance has been improved for both LFDMA and IFDMA systems due to logarithmic companding technique.

### Figure 6. BER performance of the proposed system with logarithmic companding and SC-FDMA system.

#### 5. Conclusions

In this paper a companding technique based on logarithmic function is proposed. The results show that proposed system with logarithmic companding technique provides better PAPR reduction when compared to other companding methods and conventional SC-FDMA system. At CCDF=10^{-4} there is an improvement of about 7.71 dB in LFDMA and about 5.86 dB in IFDMA due to logarithmic companding. Also, the proposed companding technique provides better BER performance than conventional SC-FDMA system. The proposed system is not complex and do not require any additional side information to be transmitted like PTS, SLM and pulse shaping methods. Hence the proposed logarithmic companding technique can be used to minimize PAPR of conventional SC-FDMA system and further enhance its BER performance.
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