Evaluation of dendrite morphology using fractal dimension and dimensionless perimeter in unidirectionally solidified Al-Si Alloys

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Abstract. The dendrite morphology of unidirectionally solidified Al-Si alloys was evaluated by measuring the fractal dimension and dimensionless perimeter of dendrites. In an unidirectional solidification experiment, columnar crystals grew from a bottom chill and columnar to equiaxed transition (CET) occurred at the upper part of an ingot. Then, equiaxed crystals were formed at the top of the ingot. Different dendrite morphology was observed in longitudinal, transverse and oblique sections, however, the fractal dimension or dimensionless perimeter of the dendrites in the sections with same local solidification time showed same values, and continuously decreased with increase in the local solidification time through columnar, CET and equiaxed regions. It can be considered that the fractal dimension and dimensionless perimeter of dendrites are controlled by local solidification time and irrespective of dendrite morphology. This result demonstrated the potential of the fractal dimension and dimensionless perimeter as a parameter for estimating local solidification time of an ingot in which the measurement of SDAS is difficult.

1. Introduction
A dendrite structure is the most frequently observed structure during the solidification of an alloy. Dendrite pattern characterize the micro segregation pattern and mechanical properties of materials. Hence, quantitative evaluation of dendrite morphology is very important for predicting the properties of mushy zone composed of dendrite arrays such as interdendritic fluidity, i.e. the permeability. To evaluate the dendrite morphology, secondary dendrite arm spacing (SDAS) is commonly used for a columnar structure, however, the measurement of SDAS is relatively difficult because a specimen must be cut parallel to the growth direction of columnar dendrites. For equiaxed crystals, the measurement of SDAS is usually difficult and the grain size is often used to evaluate the morphology of the equiaxed structure. Development of a unified method is desirable to evaluate the dendrite morphology of both columnar and equiaxed crystals.

It is known that complex dendrite pattern exhibits self-similar structure, i.e. the fractal structure. Several attempts have been carried out for applying the fractal theory to describe the morphology of materials structure1-4). Yang et al.4) calculated the fractal characteristic of dendrite and cellular structures of a Ni-based superalloy under various cooling conditions. Sanyal et al.5) described the mushy zone of an alloy as a network of continuous fractal structure, and they calculated the permeability of the mushy zone based on the fractal dimensions of the dendrite network. Fractal dimension may be an effective parameter for representing the complexity of dendrite array of an alloy in comparison with the SDAS. The authors also tried to use fractal dimension for evaluating the
morphology of dendrite structure\textsuperscript{(6-8)}. In addition, the authors also tried to use a new parameter “dimensionless perimeter” to evaluate the dendrite morphology and estimated the permeability of the mushy zone based on a phase-field simulation\textsuperscript{(6-8)}.

The first aim of this study is to measure the fractal dimension and dimensionless perimeter of experimentally observed real dendrites and second aim is to examine the characteristics and possibility of the fractal dimension and dimensionless perimeter for evaluating the dendrite morphology of solidified alloys based on a unidirectional solidification experiment.

2. Experiment
Al-3, 5 and 7 mass\%Si alloys were used for the unidirectional solidification experiment. A mold made of an adiabatic material with 30mm in inner diameter and 200mm in height was preheated to 1000K, then the mold was placed on a water-cooled chill box. A molten Al-Si alloy with super heat of 100K was poured into the mold. Temperatures were measured by six thermocouples set at the center of the mold along with the distance from the water-cooled chill surface. Local solidification time at each position were obtained from recorded cooling curves. The local solidification time deceased with the increase in the distance from the chill surface. After the completion of the solidification, an ingot was cut at transverse section and a half of the ingot was etched by an acid reagent and macro grain structure was revealed. Other half ingot was cut into several pieces and polished, etched and dendrite structures at longitudinal, transverse and oblique sections were revealed. Experimental apparatus was shown in figure 1.

![Figure 1. Experimental apparatus and microstructure observation procedure.](image)

The fractal dimensions of observed dendrites at each section were measured by using the box counting method\textsuperscript{9}. Procedure of the box counting method is as follows: an area including dendrites is divided into square boxes with the size of $r$. Then, the number of boxes, $N$, in which the S/L interface is included, is counted. Then, the size of the box, $r$, is changed and the same procedure is repeated. If following relationship holds true between the number of the boxes, $N$, and the size of the box, $r$, the geometry of dendrite shows fractal characteristic.

$$N(r) = r^D$$  \hspace{1cm} (1)

The exponent, $D$, in the equation (1) is called "fractal dimension" and its value can be obtained from the slope of the plot line of $\log r$ vs. $\log N$. In the measurement of the fractal dimension, 8 box sizes were used and total number of boxes was changed from 32 to 42658 during the measurement.

In previous works by the authors, a new index "dimensionless perimeter" of dendrites was introduced for evaluating the morphology of dendrites based on a 2D phase-field simulation\textsuperscript{(6-8)}. 
"dimensionless perimeter" was calculated as follows; first, the perimeter of a simulated dendrite \( L_1 \) is measured. Then the perimeter \( L_2 \) of a circle, whose area is same as the simulated dendrite, is measured. The "dimensionless perimeter" of the dendrite was defined as the ratio of \( L_1/L_2 \). The dimensionless perimeter increases when dendrite morphology becomes complex. Therefore, dimensionless perimeter is another index for showing the complexity of dendrite shape. The measurement of the perimeter is easy for a simulated dendrite, however, the measurement of the perimeter is difficult for an experimentally observed dendrite at the cutting surface of a sample.

In this study, we extended the definition of "dimensionless perimeter" to real structures observed in the cutting surface of a sample as shown in figure 2.

\[
L = \frac{L_1}{L_2} \\
S_1: \text{whole area of primary dendrites} \\
L_1: \text{whole perimeters of primary dendrites} \\
L_2: \text{perimeter of the circle having the area } S_1
\]

**Figure 2.** Procedure for measuring dimensionless perimeter of dendrites in an experimentally observed structure.

Whole perimeters of dendrites observed in the section of a sample were measured and summed up to \( L_1 \). Whole areas of primary dendrites were measured and summed up to \( S_1 \). Then the perimeter \( L_2 \) of the circle, whose area is same as \( S_1 \), is measured. The "dimensionless perimeter", \( L \), of dendrites is defined as the ratio of \( L_1/L_2 \).

3. Results and discussion

3.1 Solidified structures

Figure 3 shows solidified structures of a unidirectionally solidified Al-5mass%Si alloy ingot. Grain macrostructure is shown in the left side of the figure 3. Columnar crystals grew from a bottom chill and columnar to equiaxed transition (CET) occurred at the upper part of the ingot. Dendrite structures at longitudinal and transverse sections are shown in the right side of the figure 3. In the longitudinal section of a columnar structure region, columnar dendrites and secondary dendrite arms can be clearly seen. On the other hand, in the transverse section, dendrite morphology seems to be equiaxed structure. In the CET region, both columnar and equiaxed dendrites are observed at the longitudinal section, however, only equiaxed morphology is observed at the transverse section. In the equiaxed region, only equiaxed morphology is observed both at the longitudinal and transverse sections. It is clear that different morphology is observed depending on the observed section in a sample.
3.2 Fractal dimension and dimensionless perimeter of dendrites

Figure 4 shows changes in the fractal dimension (Fig.4 (a)) and dimensionless perimeter (Fig.4 (b)) along with local solidification time (corresponds to the distance from the chill). Dendrite structures at longitudinal, transverse and oblique sections at the local solidification time of about 55s (positions enclosed by a circle in upper figures (a) and (b)) are shown in figure 4(c).

Fractal dimension and dimensionless perimeter continuously decreased with increase in local solidification time through columnar, CET and equiaxed regions. In the figure (c), different dendrite morphology is observed in each section. However, in spite of different morphology, it is clear that the
fractal dimensions or dimensionless perimeters of each section at the same local solidification time show same values. This result demonstrates that local solidification time can be estimated by measuring fractal dimension or dimensionless perimeter in an arbitrary section, even in a case that the measurement of SDAS is difficult.

3.3 Relation between fractal dimension and SDAS
Secondary dendrite arm spacing (SDAS), $\lambda_2$, can be measured only at the longitudinal section in a columnar dendrite region. Figure 5 shows relationship between fractal dimension, $D_f$, and SDAS, $\lambda_2$, measured in the columnar dendrite region in the unidirectionally solidified Al-5mass%Si alloy ingot.

![Figure 5](image)

**Figure 5.** Relationship between fractal dimension, $D_f$, and secondary dendrite arm spacing (SDAS), $\lambda_2$, measured at the columnar region of the longitudinal section in the unidirectionally solidified Al-5mass%Si alloy ingot.

Negative-correlation is shown between the fractal dimension, $D_f$, and secondary dendrite arm spacing (SDAS), $\lambda_2$. It seems that the measurement of the fractal dimension of dendrites is relatively easy in comparison with the measurement of the SDAS. By obtaining a relationship between fractal dimension and SDAS, SDAS may be presumed from the fractal dimension even in a case that the measurement of SDAS is difficult.

![Figure 6](image)

**Figure 6.** Relationship between cube root of local solidification time, $t^{1/3}$ and fractal dimension, $D_f$ in the Al-5mass%Si alloy ingot.

Figure 6 shows the relationship between the cube root of local solidification time, $t^{1/3}$, and fractal dimension, $D_f$, in the unidirectionally solidified Al-5mass%Si alloy ingot. Negative-correlation is realized between cube root of local solidification time, $t^{1/3}$, and fractal dimension, $D_f$. It is well known
that the secondary dendrite arm spacing SDAS, \( \lambda_2 \), is proportional to the cube root of local solidification time \( t^{1/3} \). Negative-correlation is shown between fractal dimension, \( D_f \), and SDAS, \( \lambda_2 \), as shown in the figure.5. Hence, the relationship between the cube root of local solidification time, \( t^{1/3} \), and fractal dimension, \( D_f \), shown in the figure 6 can be understood. The results shown in the figures 5 and 6 suggest that there a close relationship between fractal dimension, \( D_f \), and SDAS, \( \lambda_2 \). Further study will be desired.

3.4 Effect of initial solute content

Figure 7 shows changes in the fractal dimension (Fig.7 (a)) and dimensionless perimeter (Fig.7 (b)) along with local solidification time in unidirectionally solidified Al-3, 5 and 7mass\%Si alloys. Dendrite structures in the longitudinal section at the solidification time of about 100s (positions enclosed by an ellipse in upper figures (a) and (b)) are shown in figure 7 (c).

Fractal dimension and dimensionless perimeter basically increased with increase in initial solute content. This result means that the dendrite morphology of an alloy with higher solute content is more complex than that of an alloy with lower solute content in the case if solidification time is same. This result can be explained based on the “constitutional undercooling”. The degree of the “constitutional undercooling” of an alloy with higher solute content is larger than that of an alloy with lower solute content. High degree of the “constitutional undercooling” makes \( S/L \) interface unstable and produces more complex dendrite morphology. However, the fractal dimension of Al-7mass\%Si alloy decreased quickly along with increase in local solidification time and shows lowest value when local solidification time becomes longer. This result may be attributed to following two reasons. First reason is that the range of fractal dimension in two dimensional is relatively narrow from 1 to 2. Hence, the effect of error arising from the measurement of fractal dimension is relatively high in comparison with that of the dimensionless perimeter. The range of the dimensionless perimeter is relatively wide from 5 to 30 as shown in the figure 7. Therefore, it is expected that the effect of the error arising from the
measuring process of the dimensionless perimeter is relatively small. Second reason may be considered as follows. The dendrite morphology of Al-7mass%Si alloy shown in the figure 7 (c) is relatively simple. The amount of eutectic of Al-7mass%Si alloy is largest of about 49% in the three alloys. The volume of liquid surrounding primary crystals may affect primary dendrite morphology. Further study about this will be desired.

It should be noted that the permeability of solidifying alloys was estimated by regarding the dimensionless perimeter as the tortuosity factor of interdendritic liquid channels. Relatively high value of the dimensionless perimeter made such calculations possible. From the point of view for engineering applications, it seems that dimensionless perimeter has more advantage than fractal dimension due to its relatively higher value in comparison with that of fractal dimension.

4. Conclusions

Unidirectional solidification experiments were carried out for Al-3, 5, and 7mass%Si alloys. Fractal dimension and dimensionless perimeter of dendrites observed at longitudinal, transverse and oblique sections in the solidified ingot were measured along with the distance from the bottom chill. Obtained results are summarized as follows;

1) Different dendrite morphology was observed at longitudinal, transverse and oblique sections, however, the fractal dimension or dimensionless perimiter of the dendrites in each section with same local solidification time showed same values

2) The fractal dimension and dimensionless perimeter of the dendrites in each section continuously decreased with increase in local solidification time through columnar, CET and equiaxed regions.

3) Negative-correlation is shown between the fractal dimension, \(D_f\), and secondary dendrite arm spacing (SDAS), \(\lambda_s\).

4) Negative-correlation is realized between cube root of local solidification time, \(t^{1/3}\), and fractal dimension, \(D_f\).

5) If local solidification time is same, the dendrite morphology of an alloy with higher solute content is more complex than that of an alloy with lower solute content. However, the fractal dimension of Al-7mass%Si alloy decreased quickly along with increase in local solidification time and showed lowest value when local solidification time becomes longer.

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