Application of fractal theory in diagnostics of composite materials

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Abstract. The article presents the results of studies to identify the features of diagnostic acoustic signals when defects are detected in composite materials using the methods of the theory of fractals. The use of the analysis of the spectral power density (PSD) and wavelet spectra of these signals did not lead to the detection of significant quantitative differences in the acoustic signals during the process of sample diagnostics (panel 4 mm thick). Acoustic signals from the standpoint of analyzing the characteristics of multifractal spectra were analyzed. It is shown that the defect state of the sample corresponds to the monofractal characteristic of the multifractal spectrum.

1. Introduction
The safety of technical systems depends on many factors. One of them is technical diagnostics. To ensure the effectiveness of diagnostics, it is necessary to develop computer methods for studying the obtained diagnostic data. The widespread use of composite materials that have their own distinctive mechanical and structural characteristics from metals and alloys requires the development of non-destructive testing methods that take into account these features. Therefore, the task was set to study the possibility of using the methods of the theory of fractals for analyzing diagnostic data and obtaining criteria for distinguishing acoustic diagnostic signals with similar characteristics of the signal power distribution density (PSD), which is often required when determining defects in a composite material during diagnostics by acoustic methods.

2. Methods
The purpose of this work is to study the features of acoustic signals in the diagnosis and detection of defects in composite materials, to determine the most sensitive numerical parameters that depend on the presence of defects.

In experiments on diagnostics of damage in a sample made of composite material (CM), a low-frequency acoustic AD-701 flaw detector was used, designed for non-destructive testing of multilayer structures and products made of laminated plastics, both in production and operation conditions. Acoustic control was carried out at 5 points of the sample - one control point (1) and 4 control points, where the amplitude values of the diagnostic signal were higher than the control value. Sample - panel 4 mm thick, 600 mm wide, 800 mm high.

The impedance method that was used makes it possible to detect defects such as delamination and non-gluing in non-metallic coatings and products made of laminated plastics, lying at a depth of 15-20...
mm. This method is used to control joints that have several layers: to detect discontinuities in adhesive and soldered joints, to control the quality of the fit of studs, pins, axles and other parts installed with an interference fit. This control does not require bilateral access to the product and the application of a couplant on its surface [1,2].

3. Results
In figure 1 shows the wavelet spectra $C(f,n)$ of acoustic signals measured at control points 1, 2 and 4 [3]. It is difficult to evaluate them and find differences in their wavelet spectra. You can give the qualitative characteristics of the frequency spectrum, their variability. But nothing more.

![Wavelet spectra of acoustic diagnostic signals at control points: 1 (a), 2 (b) and 4 (c).](image)

The clustering of wavelet spectra $C(f,n)$ of acoustic signals by the k-means method at all diagnosed points did not show any difference in the number of clusters. The number of clusters in all signals was 10. A change in the number of clusters would indicate the presence of differences in the structure of the signal.

Also, for the wavelet spectra of acoustic signals, the Minkowski coupling function $\chi$ was calculated, which quantitatively estimates the coupling of the wavelet spectrum image $C(f,n)$ by the values of the energy $E$ of the frequency components in the signal

$$\chi = \frac{E_W - E_B}{n},$$

where $E_W$, $E_B$ - symbols of the energy values of the frequency components in relation to a certain energy level $E$, which are determined from the following condition: $E_W < E$, $E_B > E$; $n$ is the number of pixels in the wavelet spectrum image.

Figure 2 shows the Minkowski connectivity functions for points 1 and 4. The difference between these functions is that for point 1 there is more significant connectivity (+, -) at the level of values close to $10 \cdot 10^{-3}$, while at point 4 it is less - $7 \cdot 10^{-3}$.

It is known that during the transition of systems from one state to another, i.e. as the bifurcation point is approached, the susceptibility to noise increases in the system, with soft modes making the main contribution. The faint noise inherent in any real system is amplified. Small low-frequency perturbations become fundamental modes (order parameters), and high-frequency ones become subordinate to them. The growth of soft modes near the bifurcation of the stationary regime is universal. Thus, one can judge about the approach of a bifurcation (catastrophe) in advance by the growth of soft modes in the spectrum of its noise [4].
The resulting frequency response near the bifurcation can be described in the form of the dependence \( S(f) \approx f^{-\beta} \), which describes power-law self-similar laws. The power spectra (PSD) (squares of amplitudes of the Fourier transform), often called noise, are especially adherent to simple homogeneous power laws of the \( f^{-\beta} \) type. Among the noise, white noise with a spectral index \( (\beta = 0) \) is widely known. Depending on the value of the index \( \beta \), noise is distinguished: white, pink \( (f^{-1}) \), brown \( (f^{-2}) \) and black \( (f^{-3}) \) \[5\]. This type of power-law dependence of the spectral characteristic is monofractal - a term used in the theory of fractals.

In this article, it was checked how the acoustic signal changes in the defect zone according to the type of the multifractal spectrum. Monofractal and multifractal dynamic processes differ in the nature of the power spectral density distributions. If in a monofractal process the distribution of the spectral power density can be described by a function that depends on the frequency \( f \) in the form - \( S(f) \approx f^{-\beta} \), where \( \beta \) is a constant, then in the case of a multifractal process, the PSDs are described by a more complex dependence.

As noted above, a composite specimen was used in defect detection studies. According to the results of the inspection of this panel, 4 points were found (points 2-5, see the table), where the impedance amplitude was higher than the set threshold for point 1.

The multifractality of the process is usually represented by a multifractal spectrum (singularity spectrum) \( f(\alpha) \). Multifractal spectra are characterized by spectral width, asymmetry, curvature \[6\].

The multifractal spectrum of (singularities) \( f(\alpha) \) characterizes the dependence of the number of elements of the covering \( N_\varepsilon \) with different scales \( \varepsilon \) corresponding to points with the singularity exponent equal to some value

\[
N_\alpha(\varepsilon) \sim \varepsilon^{-f(\alpha)}
\]

In terms of meaning, the value \( f(\alpha) \) under the condition \( \alpha = \text{const} \) corresponds to the Hausdorff dimension. In this case of uniform distribution of the measure on the set, the spectrum of singularities is a single point on the plane \((\alpha, f)\), which corresponds to a monofractal process. With a nonuniform distribution of the measure, the function \( f(\alpha) \) has a more complex (bell-shaped) form. The function \( f(\alpha) \) is called the multifractal spectrum, characterized by the values \( D_i \) \((i = -\infty, \ldots, 2, 1, 0, 1, 2, \ldots, \infty)\), the asymmetry and spectrum width \( S \). A decrease in the spectrum width \( S \) leads to the loss of multifractality. When the number of significant fractal dimensions \( D_i \) in the spectrum becomes minimal, it becomes monofractal.

Since the PSD of acoustic signals were visually similar, their multifractal spectra are similar, with the difference that for the defect-free zone, the fractal dimension is determined by the value \( D_0 = 1.6090 \) (the maximum point on the multifractal spectrum, figure 3), and for the defect-free zone

![Figure 2. Minkowski connectivity \( \chi \) for control points: (a) - 1; (b) – 4.](image)
value $D_0 = 1.6356$ (see table 1). These values are close and cannot be an objective criterion for detecting a defect in a research object.

**Table 1.** Data of acoustic signals and their multi-fractal spectra.

| Number of point | Maximum signal amplitude $A_m$ | Fractal dimension $D_0$ | Inhomogeneity index spectrum $\Delta D$ | Width of the multi-fractal spectrum $S$ |
|-----------------|-------------------------------|------------------------|------------------------------------------|----------------------------------------|
| 1               | 0.037                         | 1.6090                 | 0.609                                    | 0.975                                  |
| 2               | 0.038                         | 1.4732                 | 0.423                                    | 1.065                                  |
| 3               | 0.148                         | 1.4957                 | 0.745                                    | 0.950                                  |
| 4               | 0.295                         | 1.6356                 | 0.273                                    | 0.715                                  |
| 5               | 0.140                         | 1.6232                 | 0.550                                    | 1.435                                  |

**Figure 3.** PSD of the acoustic signal: at point 1 (a) and its multifractal spectrum (b).

It turned out that if we compare two points (1 and 4, table 1), they similar in shape of the PSD and to the histograms of the distribution of acoustic signal amplitudes, but they differ significantly in the multifractal spectrum. For point 4 the multifractal spectrum degenerates into a monofractal.

**4. Conclusion**

Thus, the presented results show that the type of multifractal spectrum is related to the presence of a defect or significant inhomogeneity of the material. Defective structures correspond to signals with a monofractal characteristic of the multifractal spectrum, which corresponds to the predominance of conventionally low-frequency components in the signal in relation to high-frequency components in a certain frequency range.

**References**

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