Atom transistor from the point of view of nonequilibrium dynamics

Z Zhang¹, V Dunjko² and M Olshanii²
¹ Department of Physics and Astronomy, State University of New York at Stony Brook, Stony Brook, NY 11794, USA
² Department of Physics, University of Massachusetts Boston, Boston, MA 02125, USA
E-mail: Maxim.Olchanyi@umb.edu

Keywords: atom transistor, thermalization, ergodicity, mean field, semiclassical methods, onset of chaos

Abstract
We analyze the atom field-effect transistor scheme (Stickney et al 2007 Phys. Rev. A 75 013608) using the standard tools of quantum and classical nonequilibrium dynamics. We first study the correspondence between the quantum and the mean-field descriptions of this system by computing, both \textit{ab initio} and by using their mean-field analogs, the deviations from the Eigenstate Thermalization Hypothesis, quantum fluctuations, and the density of states. We find that, as far as the quantities that interest us, the mean-field model can serve as a semi-classical emulator of the quantum system. Then, using the mean-field model, we interpret the point of maximal output signal in our transistor as the onset of ergodicity—the point where the system becomes, in principle, able to attain the thermal values of the former integrals of motion, albeit not being fully thermalized yet.

1. Introduction
In atomtronics, in an analogy to the conventional electronics, a transistor is a nonlinear device where a small atom current or atom number controls a large current. The existing proposals for an atom transistor can be divided into open and closed architectures. The former [1–3] is conceptually close to the conventional electronic devices (the work [2] also contains a design for a diode). The latter, in turn, can be separated into the schemes based on an adiabatic population transfer protocol [4–6] (along with the similar atom diode proposals [7, 8]), and the schemes where the (large) base current is induced by the difference in chemical potentials between the ‘electrodes’ (this is similar to bosonic Josephson currents; see [9] and the references therein). The base current can be controlled by either the internal state of a localized impurity [10–12] or by the number of (strongly interacting) atoms in the middle site [13, 14]. This last scheme is nothing else but an atomic version of the field-effect transistor (FET), and is the subject of the present study. In this scheme, the large atomic current from the ‘source’ to the ‘drain’ is controlled by a small number of atoms in the ‘gate’. The interatomic repulsion in the gate is so strong that the chemical potential is large even when the number of atoms in the gate is small.

Our principal result is that the point of maximal output signal in the atomic FET corresponds to the onset of ergodicity—the point where the system becomes, in principle, able to attain the thermal values of the former integrals of motion, albeit not being fully thermalized yet. We show this in two steps: first, we establish that a mean-field model can serve as a semi-classical emulator for the nonequilibrium quantum dynamics of the system. We do this by comparing the quantum \textit{ab initio} results—for the deviations from the Eigenstate Thermalization Hypothesis, quantum fluctuations, and the density of states—to their mean-field analogs. Then we use the mean-field model to study the transistor output signal threshold. We begin by introducing the quantum system and its mean-field model.

2. The quantum system of interest and its mean-field model
The paradigmatic atom transistor is realized in a tight-binding model for three coupled bosonic wells on a line (the ‘source’, the ‘gate’, and the ‘drain’). For a derivation, we refer the reader to [13] (see also [15]). The Hamiltonian of the quantum system is
where the indices $\eta = 's', 'g' \text{ and } 'd'$ stand for the source, the gate, and the drain, respectively; $\varepsilon_{\eta}$ is the one-body energy at site $\eta$, while $U_{\eta}$ is the strength of the on-site two-body interactions for the site $\eta$; finally, $J_{\eta\eta'}$ is the hopping constant between sites $\eta$ and $\eta'$. The operators $\hat{b}_{\eta}^{\dagger}$ and $\hat{b}_{\eta}$ are the usual bosonic creation and annihilation operators for site $\eta$, satisfying the commutation relations $[\hat{b}_{\eta}^{\dagger}, \hat{b}_{\eta'}^{\dagger}] = \delta_{\eta\eta'}$ and $[\hat{b}_{\eta}, \hat{b}_{\eta'}] = \left[\hat{b}_{\eta}^{\dagger}, \hat{b}_{\eta'}^{\dagger}\right] = 0$. For $N$ particles, the dimension of the Hilbert space is $\binom{N + 2}{2}$. The main observables of interest are the relative occupations of the source, gate, and drain: $\hat{n}_{\eta} \equiv \hat{N}_{\eta}/N$, where $\hat{N}_{\eta} = \hat{b}_{\eta}^{\dagger}\hat{b}_{\eta}$; absent the hoppings, they are conserved quantities.

If the occupations of all sites are large, one expects the system to be describable by a classical mean-field model [13]. The heuristic recipe for constructing such a model is well known: take the right-hand side of (1), and replace each operator $\hat{b}_{\eta}$ by the classical variable $Q_{\eta}$, and each $\hat{b}_{\eta}^{\dagger}$ by $-i\Pi_{\eta}$ (recall that in a classical field theory with complex-valued fields, the canonical momentum conjugate to the field $\psi$ is $i\psi^{*}$); call the result the classical Hamiltonian $H(Q_{s}, Q_{g}, Q_{d}, \Pi_{s}, \Pi_{g}, \Pi_{d})$. The equations of motion for the variables $Q_{\eta}$ are given by $Q_{\eta} = \partial H/\partial \Pi_{\eta}$, where, once the partial derivatives are taken, we replace each $\Pi_{\eta}$ by $iQ_{\eta}$. There are also the equations of motion for the variables $\Pi_{\eta}$, $\Pi_{\eta} = -\partial H/\partial Q_{\eta}$. However, after the replacement $\Pi_{\eta} \rightarrow iQ_{\eta}$, the equations of motion for the $\Pi$'s become simply the complex conjugates of the equations for the $Q$'s, so that in the end, there are just three independent equations. Since the mean-field theory should not depend on $N$ (formally it is an $N \rightarrow \infty$ limit), one should in principle divide $H$ by $N$ and make the replacements $Q_{\eta} \rightarrow \sqrt{N} b_{\eta}, \Pi_{\eta} \rightarrow \sqrt{N} \eta_{\eta}/N$ and $U_{\eta} \rightarrow Z_{\eta}/N$. To obtain a good mean-field limit as one increases the system size, the products $U_{\eta}N$ should be held fixed. Then the resulting Hamiltonian, a function of the $b_{\eta}$ and $\pi_{\eta}$, does not depend on $N$. In these variables, the three equations of motion are

$$\dot{b}_{\eta} = \varepsilon_{\eta} b_{\eta} + U_{\eta}N |b_{\eta}|^{2} b_{\eta} + J_{g} (\delta_{\eta g} b_{g} + \delta_{\eta b} b_{b}) + J_{gd} (\delta_{\eta g b_{d}} + \delta_{\eta d} b_{b}),$$

where $\eta = 's', 'g', 'd'$, such that it should be kept in mind that the quantum counterpart of $b_{\eta}$ is $\hat{b}_{\eta}/\sqrt{N}$. Stickney et al [13] have verified that the mean-field results for time evolution agree, in the appropriate parameter regimes, with the quantum ones (though in their model, the source is treated as always highly populated). The following publications also studied atomic transport in three-well configurations: [16–19].

In what follows, it will be convenient to rewrite the mean-field model in terms of the variables

$$\vec{I} = (I_s, I_g, I_d)$$

and

$$\vec{\phi} = (\phi_s, \phi_g, \phi_d),$$

defined through $Q_{\eta} = \sqrt{I_{\eta}} e^{i\phi_{\eta}}$ (correspondingly we have $b_{\eta} = \sqrt{\eta_{\eta}} e^{i\phi_{\eta}}$, where $n_{\eta} = I_{\eta}/N$). It is perhaps in these variables that the connection to the quantum system is most easily understood, because it is given through the quantum expectation values: $I_{\eta} = \langle \hat{b}_{\eta}^{\dagger} \hat{b}_{\eta} \rangle$ and $\phi_{\eta} = \phi_{\eta} = \arg(\langle \hat{b}_{\eta}^{\dagger} \hat{b}_{\eta'} \rangle)$. The mean-field Hamiltonian becomes

$$H(\vec{I}, \vec{\phi}) = \sum_{\eta = 's', 'g', 'd'} \left( \varepsilon_{\eta} I_{\eta} + \frac{U_{\eta} I_{\eta}^{2}}{2} \right) + 2J_{g} \sqrt{I_{g} I_{d}} \cos(\phi_{g} - \phi_{b}) + 2J_{gd} \sqrt{I_{g} I_{d}} \cos(\phi_{g} - \phi_{d});$$

note that when the hopping terms are zero, $\vec{I}$ and $\vec{\phi}$ are the action-angle variables for this Hamiltonian.

In general, the purpose of the atom FET transistor is to control the current of atoms from the source to the drain using small variations in the chemical potential of the gate, $\mu_{g} = \frac{\partial}{\partial N_{g}} H_{g}(N_{g}) = \varepsilon_{g} + U_{g} N_{g}^{2}$ (recall that the number of atoms in the gate is $N_{g} = I_{g} = \langle \hat{b}_{g}^{\dagger} \hat{b}_{g} \rangle$). The chemical potential, in turn, is controlled by the relative population $n_{g} = N_{g}/N$. According to the original proposal [13], the transition of atoms between the source and the gate plays the role of a ‘bottleneck,’ activated only when the source and the gate chemical potentials become close in value. In contrast, the gate–drain link is made insensitive to the gate–drain chemical potential difference, transmitting every atom that happens to appear at the gate. This can be achieved by providing a comparatively large on-site interaction strength for the gate, subsequently detuned from the energy of the source. One then chooses the hopping constants and the on-site energies in such a way that the source–gate transition exhibits a narrow resonance, and the gate–drain transition a broad one. In particular, to obtain the narrow resonance, the parameters must be set such that the magnitude of the source–gate hopping constant is much less than the magnitude of the source–gate detuning corresponding to an empty gate (see also figure 4). We will be using the following representative set of parameters:
Experimentally, the on-site two-body interactions can be separately tuned by separately changing the strength of confinement in each well: the two-body interaction scales as $\omega$, where $\omega$ is the frequency of the confinement [15]. To get a ratio of 100 in interaction strengths, one would need a trapping frequency ratio of $10^4$, which is achievable in 1D experiments (a longitudinal confinement of as little as 1 Hz, and a transverse confinement of as much as 10 kHz).

$$\varepsilon_s = 0; \quad \varepsilon_g = -1.3; \quad \varepsilon_d = 0.5;$$
$$U_s N = 0; \quad U_g N = 100; \quad U_d N = 0;$$
$$J_{sg} = -0.1; \quad J_{gd} = -1.$$ (4)

3. The mean-field model as a semi-classical emulator for quantum nonequilibrium dynamics

The central objects in quantum nonequilibrium dynamics of isolated quantum systems are the deviations from the eigenstate thermalization hypothesis, quantum fluctuations, and the density of states [20–24]. We now establish that they can be emulated by their classical counterparts from the mean-field model, by comparing to the exact quantum ab initio results. Figure 1(a) shows the energy spectrum $E_\alpha$ of the transistor for the set of parameters in (4). Here and below, $\alpha$ is the index of the eigenstate $|\alpha\rangle$ of the Hamiltonian in (1): $H |\alpha\rangle = E_\alpha |\alpha\rangle$. The spectrum, bounded from both the below and the above, contains 496 eigenstates. Note that the density of states decreases with the energy. On the upper end of the spectrum, the dominant contribution to the energy is provided by the interactions between the atoms in the gate; this energy increases quadratically with $n_g$, leading to the decreasing (with energy) energy spacings.
3.1. Density of states
Most of the spectrum turns out to be within the limits of applicability of the semiclassical approximation. To verify this assertion, we check that the Weyl’s law holds in our system. According to the Weyl’s law, if one computes how the number of quantum states below an energy $E$ depends on the energy $E$, and then takes the smooth envelope of this dependence, the result is proportional to the classical phase space volume occupied by these states:

$$\left( \# \text{ of states with } E_n \leq E \right) \approx \frac{1}{(2\pi)^m \hbar^m} \int d^mI \, d^m\phi \Theta\left[ E - H\left( I, \phi \right) \right] \delta\left( I_{\text{tot}} - N \right),$$

(5)

where $I$, $\phi$, and $H(I, \phi)$ are as in (3), $I_{\text{tot}} = I_1 + I_g + I_d$ is the norm, $\Theta(x)$ is the Heaviside step-function, the delta-function constrains the number of particles to be $N$, and $\int d^mI \, d^m\phi$ is the integral over the whole phase space defined by the variables $I$ and $\phi$, i.e.,

$$\int_{0}^{2\pi} d\phi_g \int_{0}^{\infty} dL_g \int_{0}^{2\pi} d\phi_s \int_{0}^{\infty} dL_s \int_{0}^{2\pi} d\phi_d \int_{0}^{\infty} dL_d \delta(L_{\text{tot}} - N).$$

Figure 1(b) shows the inverse of the dependence in (5), computed using a Monte-Carlo method: 496 points of the phase space defined by the variables $I$ and $\phi$ were sampled uniformly from within the narrow shell (of width $w = 0.005N$) corresponding to a window of the values of the norm, centered around $N_{\text{norm}} = N$. That is: the variables $I_1$, $I_g$, and $I_d$ were each drawn from the uniform distribution on $\left[0, N + w/2\right]$ (one can show that the statistics are independent on the upper limit of this interval, provided it is $N + w/2$ or more); if the sum $I_1 + I_g + I_d$ (which is just the norm) turned out to lie in the window $\left[N - w/2, N + w/2\right]$, then the angles $\phi_s$, $\phi_g$, and $\phi_d$ were each drawn from the uniform distribution on $\left[0, 2\pi\right]$. These values for the variables $I$ and $\phi$ were then plugged into the Hamiltonian in (3), and the result divided by $N$. Once 496 such numbers were produced, they were sorted in increasing order; the result is plotted in figure 1(b).

Notice that for this calculation, we have chosen as many Monte-Carlo realizations as there are quantum eigenstates of the system. A priori, one number is not related to the other, and more realizations would produce a smoother semi-classical curve. However, it can be argued that the spectrum in figure 1(b) is a semi-classical emulation of the quantum spectrum in figure 1(a), in the sense that the fluctuations of the eigenenergies around a smooth envelope have at least the same order of magnitude as their semiclassical counterparts. This should be so provided the quantum system is not too far from being integrable, as can be seen by the following argument: on the one hand, the Monte-Carlo method used to generate the classical spectrum produces energy values through a Poisson process; on the other hand, it is known that in quantum integrable systems, eigenenergies are distributed as if they were the outcome of a Poisson process [25, 26]. In contrast, quantum-ergodic systems with time-reversal invariance exhibit the type-1 Wigner–Dyson statistics [25, 26]. In that case, the standard deviation for the spacing between two neighboring levels is $1/\sqrt{(4/\pi)} - 1 \approx 2$ times lower than in the case of Poisson statistics.

3.2. Deviations from the eigenstate thermalization hypothesis
Figure 2(a) explores the central object in the eigenstate thermalization theory [20–23], namely, the expectation values of the relevant observables. In our case, the relevant observable is the occupation of the drain. We refer to this expectation value, when plotted as a function of $t$, as the relative occupation of the drain:

$$\langle n_d(t)\rangle_l \equiv \lim_{t_{\text{max}} \to \infty} \frac{1}{t_{\text{max}}} \int_0^{t_{\text{max}}} dt \; n_d(t).$$

(6)

They are shown in figure 2(b); each point was generated using a point in figure 1(b) as the initial condition. That is: for each point in figure 1(b), we took the values of $I$ and $\phi$ that were used to produce it (via the Hamiltonian in (3)); these values were used as the initial conditions $I(t = 0)$ and $\phi(t = 0)$ to compute the classical trajectory $I(t)$ and $\phi(t)$, via the equations of motion generated by the Hamiltonian in (3); finally, using (6), this trajectory was used to compute a point in figure 2(b). The overall behaviors of the quantum and classical expectation values stand in good correspondence.

3.3. Quantum fluctuations
The quantum fluctuations of $n_d$ in the eigenstates $|\alpha\rangle$:

$$\sigma\left[ \hat{n}_d \right]_{\alpha} \equiv \sqrt{\langle |\alpha\rangle \left( \hat{n}_d \right)^2 |\alpha\rangle - \left( \langle |\alpha\rangle \hat{n}_d |\alpha\rangle \right)^2},$$

(7)

are shown in figure 3(a). In accordance with the conjecture expressed in [28], their proper classical analogue is the classical temporal variance.
shown in figure 3(b). Again, there is substantial agreement between the quantum and classical plots.

4. Atom transistor from the point of view of quantum nonequilibrium dynamics: the point of maximal output signal as the point at which the system becomes able to reach thermal equilibrium values

Having verified that the mean-field theory can emulate the important quantities of quantum nonequilibrium dynamics (and keeping in mind that the time evolution of the site occupations agrees well with quantum mean-field and quantum dynamics [13]), we now examine the properties of the output signal of the transistor.

Recall that the source–gate resonance is set to be narrow, so that the source and the gate will exchange atoms only when their chemical potentials are close in value. In our case 

\[ \mu_s = \varepsilon_s \quad \text{and} \quad \mu_g = \varepsilon_g + U_g N n_g, \]

and with parameters as in (4), the resonance occurs at 

\[ n_g = 0.013. \]

Figure 4 shows the detuning of the source–gate transition as a function of the relative gate occupation.

In all numerical experiments below, the initial value of the drain occupation \( n_d \) is zero or very close to zero.

In studies of transistor behavior, the conventional figure of merit is the current to the drain, characterized by the initial slope in the dependence of \( n_g \) versus time (in our case, the slope of the red line between \( t = 0 \) and \( t \approx 20 \), in figure 5(b)). For example, figure 6 of [13] in effect shows the current in the drain: because their initial population of the drain is 0, it follows that to obtain the current, one just needs to divide the results they plot by the time interval \( \tau = 20 \).

Below, we will use an integral figure of merit—the infinite time average of the drain occupation, \( \langle n_d \rangle_t \)—to facilitate the analysis of the system from the point of view of ergodicity or its absence. Our figure of merit is thus the net ‘charge’ transferred (i.e. the proportion of atoms transferred), rather than the current.
At zero \( n_g(t = 0) \), only a minimal conductance is expected, and this is what we observe (figure 5(a)). To the contrary, at the resonant point, \( n_g(t = 0) = 0.013 \), the transistor is supposed to provide the maximal source–drain conductance, and this is indeed what we find (figure 5(b)).

While figure 5(b) may be reminiscent of oscillations in a bosonic Josephson junction, there are some key differences. First, in Josephson oscillations, as the population gets transferred, the on-site energies get shifted. In our case, in contrast, the gate occupation remains almost the same throughout. Second, if we were seeing Josephson oscillations, then once the population of the drain reached 0.5, it would continue to increase all the way to 1, instead of turning around and decreasing back to zero (as it does in our case). On the other hand, when...
The figure shows the temporal variation of $n_t$ and $n_\text{g}$ versus $n_\text{d}$. The slope of this dependence, $16$, is identified as gain. There are two ways to see that this really is gain rather than, say, drain leakage. First, if one takes the average of the cluster of points corresponding to the lower values of gate population (say, $n_\text{g}$ between 0.001 and 0.005) and the average corresponding to the higher values (say, $n_\text{g}$ between 0.005 and 0.012), these two points will be, to a good approximation, colinear with the origin. Second, recall that what we are studying is the ability of the values of interest to assume thermal values (represented in figure 6 by the solid horizontal line at the vertical position just below 0.3). Thermal values constitute a fundamental plateau, which cannot be overcome. Now note that even for the cluster of points corresponding to the lower values of gate population ($n_\text{g}$ between 0.001 and 0.005), the observable of interest reaches 30% of that plateau, which again shows that this is not mere leakage.

In connection with comparisons to the thermal values, it should be realized that such comparisons are meaningful even when one is not interested in the thermodynamic limit (and we are not; see below). Namely, in the context of studying the onset of equilibration and thermalization, there are two major reasons why the dynamics of a system might not drive it (all the way) to the thermal equilibrium: one is a lack of ergodicity, and the other is insufficient size (‘mesoscopicity’). In a system of our size, the two are not really distinguishable.

The quantum system we are trying to emulate has 200 particles, resulting in a 20 301-dimensional Hilbert space. As mentioned in the caption of figure 2(b), however, only about 1/20th of the whole available phase space—corresponding to the lowest 1/20th portion of the possible energy values—are relevant for the transistor operation. This places an upper limit of 0.05 on the sampled values of $n_\text{g}$ (recall that the interactions in the gate are really strong). Beyond that, the energy is completely dominated by the energy of the gate. The dynamics becomes simply that of one very strong nonlinear oscillator—the gate—and the rest is completely dominated by it. Moreover, even though we sampled $n_\text{g}$ up to the value of 0.05, in figure 6 we only plot values up to 0.014:

![Figure 5. Dynamics of the source (solid, blue, topmost), gate (solid, green, bottommost) and drain (dashed, red) occupations for low and high output signals. (a) Low output signal (low values of $n_\text{g} = N_\text{g}/N$ in figures 4 and 6). (b) High output signal (value $n_\text{g} = N_\text{g}/N$ close to the resonant value of 0.013, according to figures 4 and 6).](image-url)
because the source–gate resonance is a narrow one, the gain simply dies out for larger values of \( n_g \). Yet another reason for concentrating only on the lower values of \( n_g \) is that we wanted to see how one can control gain with small populations of the gate, which, after all, is the purpose of the transistor.

The reason we limit our number of particles to 200 (and thus, the number of different realizations to 1015) is that this way, the statistical noise present in figure 6, being an emulation of quantum noise, is an indication of what would be seen in an actual experiment.

We emphasize that increasing the number of atoms to values much larger than 200 would not reduce the scatter seen in the plot. It would merely increase the number of points in the plot, where all the points already present would stay. And since these already-present points were randomly selected, their scatter is a good indication of the scatter that would be seen if more points were added. We realize that there is a strong intuition that increasing the number of atoms should reduce the scatter, so let us explain in more detail why it would not.

The main point to realize is that in our case, the \( N \rightarrow \infty \) limit is the mean-field limit, which is not the same thing as the thermodynamic limit. The latter is, in fact, not even applicable to our system, because it would require the size of the system (the number of lattice sites \( M \)) to increase to infinity as well, while keeping constant the density \( N/M \). For us, however, the number of lattice sites is fixed to \( M = 3 \). So, as far as the underlying quantum system, the effect of increasing the number of particles is to make it behave more in accordance with the mean-field model; but the mean-field model is precisely what we used to produce the results in figure 6. (In this connection, recall that the purpose of section 3 was to verify that, for our system, the mean-field model can emulate the quantum properties relevant for nonequilibrium dynamics.) Thus, for the purposes of figure 6, ‘increasing the number of particles’ corresponds merely to increasing the number of points (i.e. realizations) we are sampling; each individual point is already computed in the \( N \rightarrow \infty \) limit, the mean-field limit. Incidentally, increasing the number of points would also make it possible to have multiple points appearing at one and the same value of \( n_g \).

Now notice that regardless of how many realizations we take, it remains the case that we are sampling the entire microcanonical energy window (between \( E_{\text{min}}/N \approx 0.08 \) and \( E_{\text{max}}/N \approx 0.13 \), out of the full available energy range of \( E_{\text{min,full}}/N = E_{\text{min}}/N \approx 0.08 \) and \( E_{\text{max,full}}/N = 47 \)). In the language of \([13]\), we are sampling all the various phase differences between the gate and the drain. Thus, the scatter in our figure 6 mostly corresponds to the difference between the solid and the dashed lines in figure 6 in \([13]\), and this difference has nothing whatsoever to do with the number of realizations. Our procedure additionally introduces some noise in the initial drain occupation, which, we would argue, makes our procedure better able (compared to \([13]\)) to describe the initial non-coherent states of the transistor.

With these clarifications out of the way, let us present our main result. Unexpectedly, we find that even at the resonant point, the average drain occupation does not reach its ensemble (i.e. thermal) average, which is represented by the short horizontal line in figure 2. Instead, as the system approaches the resonant point

---

**Figure 6.** Time average of the drain occupation of the atom transistor (dashed line), along with its temporal standard deviation (error bars), in the mean-field approximation, as a function of the initial occupation of the gate. 1015 initial conditions were uniformly distributed inside a phase space volume corresponding to the lowest (in energy) 1/20 of the total available phase space volume. The corresponding window of energies is between \( E_{\text{min}}/N \approx 0.08 \) and \( E_{\text{max}}/N \approx 0.13 \), out of the full available energy range of \( E_{\text{min,full}}/N = E_{\text{min}}/N \approx 0.08 \) and \( E_{\text{max,full}}/N = 47 \). Subsequently, only the initial conditions corresponding to low initial conditions \( \langle N_g(t = 0) \rangle < 0.05N \) of drain occupation were selected. A linear fit (dashed line) gives an estimate of the gain, \( \beta \equiv \langle N_d(t) \rangle / N_d(t = 0) \), as \( \beta \approx 16 \). Solid horizontal line reflects the average over all the realizations in the window, with no selection of the initial values of the drain occupation \( N_d \). Note that only when the initial gate occupation approaches the resonant value of \( n_g = .013 \) (see figure 4), the mobility in the phase space becomes sufficient for the drain occupation to reach its thermal value (solid line), at least at some instances of time. The system parameters are the same as in figure 2(b).
\( n_d(t = 0) = 0.013 \), the ensemble average starts falling within the range of the temporal fluctuations; not enough to be predominantly within the range, but enough for this effect to be detectable.

5. Conclusion

In this work, we studied the atom FET transistor scheme suggested in [13, 14] using the tools of quantum non-equilibrium dynamics [27]. We first justified the applicability of the semiclassical approximation applied to the standard measures of the non-equilibrium. We then focused, using a semiclassical model (equivalent to a mean-field one), on the initial conditions with zero drain occupation, and used the gate occupation as a knob that controls further dynamics. Instead of the traditional source-to-drain current, as the transistor output we chose the potential levels with that of the source, the drain becomes populated. One may expect that at this point, the occupation, no atoms are transmitted to the source. To the contrary, at a point where the gate occupation is still far below its thermal value. Instead, we found that the resonant regime is characterized by the temporal fluctuations of the drain occupation become capable of ‘touching’ the thermal values for some periods of time, without spending much time there. The overall lesson is that ergodicity is not a necessary condition for the operation of an atom transistor (while the conventional semiconductor devices do operate close to thermal equilibrium): this conclusion may allow one to broaden the search for an optimal configuration of an atom transistor.

Acknowledgments

We are grateful to Dana Anderson and Alex Zozulya for numerous in-depth discussions on the subject. This work was supported by grants from National Science Foundation (PHY-1402249) and the Office of Naval Research (N00014-12-1-0400).

References

[1] Seaman B T, Kramer M, Anderson D Z and Holland M J 2007 Phys. Rev. A 75 023615
[2] Pepino R A, Cooper J, Anderson D Z and Holland M J 2009 Phys. Rev. Lett. 103 140405
[3] Pepino R A, Cooper J, Meier D, Anderson D Z and Holland M J 2010 Phys. Rev. A 82 013640
[4] Vaishnav J Y, Ruseckas J, Clark C W and Juzeliūnas G 2008 Phys. Rev. Lett. 101 265302
[5] Bensenyi A, Fernández-Vidal S, Bagadá J, Corbalán R, Picón A, Roso L, Birkl G and Mompert J 2010 Phys. Rev. A 82 013604
[6] Schlagheck P, Malet F, Cremon J C and Reimann S M 2010 New J. Phys. 12 063020
[7] Ruschhaupt A and Muga J G 2004 Phys. Rev. A 70 061604
[8] Ruschhaupt A, Muga J G and Raizen M G 2006 J. Phys. B: At. Mol. Phys. 39 3833
[9] Sakmann K, Streltsov A I, Alon O E and Cederbaum L S 2009 Phys. Rev. Lett. 103 220601
[10] Micheli A, Daley A J, Jaksh D and Zoller P 2004 Phys. Rev. Lett. 93 140408
[11] Daley A J, Clark S R, Jaksh D and Zoller P 2005 Phys. Rev. A 72 043618
[12] Micheli A and Zoller P 2005 A single atom mirror for 1D atomic lattice gases 2006 Phys. Rev. A 73 043613
[13] Stickney J A, Anderson D Z and Zozulya A A 2007 Phys. Rev. A 75 013606
[14] Caliga S C, Straatsma G E, Zozulya A A and Anderson D Z 2013 A matterwave transistor oscillator arXiv:1208.3109
[15] Jaksh D, Bruder C, Cirac J I, Gardiner C W and Zoller P 1998 Phys. Rev. Lett. 81 3108–11
[16] Rab M, Cole J H, Parker N G, Greentree A D, Hollenberg L C M and Martin A M 2008 Phys. Rev. A 77 061602
[17] Graefe E M, Korsch H J and Witthaut D 2006 Phys. Rev. A 73 013617
[18] Nesterenkov V, Novikov A, de Souza Cruz F and Lapolli E 2009 Laser Phys. 19 616–24
[19] Schlagheck P, Malet F, Cremon J C and Reimann S M 2010 New J. Phys. 12 065020
[20] Deutsch M 1991 Phys. Rev. A 43 2046–9
[21] Srednicki M 1994 Phys. Rev. E 50 888–901
[22] Rigol M, Dunjko V and Olshanii M 2008 Nature 452 854–8
[23] Dunjko V and Olshanii M 2013 Thermalization from the perspective of eigenstate thermalization hypothesis Annual Review on Cold Atoms ed K W Madison et al (Singapore: World Scientific)
[24] Polkovnikov A, Sengupta K, Silva A and Vengalattore M 2011 Rev. Mod. Phys. 83 863–83
[25] Bohigas O 1991 Random matrix theories and chaotic dynamics Chaos and Quantum Physics ed M-J Gianoni et al (Amsterdam: North-Holland)
[26] Guhr T, Müller–Groeling A and Weidenmüller H A 1998 Phys. Rep. 299 189–425
[27] Olshanii M 2015 Phys. Rev. Lett. 114 060401
[28] Srednicki M 1996 J. Phys. A: Math. Gen. 29 L75