Time-clustering Behaviors of Urban Fires

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Abstract

It has been found that many systems are characterized by scaling behavior. Time-scaling scale-invariant approaches are used to analyze the temporal distribution of urban fire sequences recorded in the city of Wuhan (China) in the present paper. The results of the coefficient of variation show that there is a clustering distribution in the occurrence sequence of urban fire in Wuhan. And with the study of Fano factor and Allan factor for the urban fire sequences in Wuhan, it is found that the urban fire sequences have obviously time-scaling behaviors. These reveal that the point process of urban fires is a fractal process with a high degree of time-clusterization of the events. The results will be helpful to estimate urban fire risk.

1. Introduction

In recent years, many attentions have focused on the dynamic process in natural systems in which the self-similar processes are involved, such as forest fire [1-7], family name distribution [8], earthquake [9-10], climate changes [11-13] and so on. In these systems, the output signals are usually nonstationary and have atypical statistical properties. Results show that the frequency-size distribution of forest fire obeys power-law relation over many magnitudes in different countries [6, 14-15]. The power-law relation indicates that fires with different sizes are generated by a same mechanism.

Researches focused on the time sequence in some systems also show power-law relation. For example, Boffetta et al. found quiescent times between successive bursts of solar flares activity display a power law distribution [16]. Some other systems such as rainfall [17-18], market price [19] and forest fire [3,20] have similar distribution too.

The nonlinear mechanisms, which control the underlying interactions in the fire process, make fire sequences nonstationary. Urban fire system is a complexity system involves natural, social and economic, in which influencing factors are various and complex. So exploring the temporal characteristics of urban fire sequence is challenging in fire safety research. In this paper we apply several time-scaling methods to the urban fire sequence of Wuhan in China to study their long-range correlation.

2. Data and Methods

2.1. Data

To analyze the time-scaling properties of urban fire sequence, the fire data used in this paper include the fire events occurred in Wuhan from 2003 to 2006. Wuhan is located in north latitude 29°58′-31°22′, east longitude 113°41′-115°05′,
and it covers an area of 8494 km². As the largest city of China’s central and western regions, Wuhan is one of China’s economic centers. There are 10.02 million resident populations in 2011.

2.2. Methods

2.2.1. Representations of the city-fire process

An urban fire sequence can be viewed as a realization of a stochastic point process, which is defined by the set of the event times and can describe the events that occur at some random locations or areas in time. Such representation was used in several event systems, such as earthquake [21], solar flares [22] and so on.

Some statistical methods can be used to investigate the statistical behaviors of a temporal point process. When a number of its relevant statistics exhibit scaling, this indicates that there are clusters of points in represented phenomenon over a relatively large set of timescales. The common standard method to investigate the presence of clustering in a time series is the power spectral density, which expresses how the power concentrated at various frequency bands. The power spectral density is calculated by Fourier transform of data. The power spectral density of a process with scaling properties follows \( S(f) \propto f^{-\alpha} \), with the so-called fractal exponent \( \alpha \) measuring the strength of clustering. Some of the statistics show power law in a point process with clustering behavior. Thus power law can be used to describe its properties [23] and estimating the fractal exponent \( \alpha \). Poisson process is a very important and most common distribution of random point process sequence. It describes that the occurrences are independent and do not exist correlation.

Urban fire system is a complexity system with multiple influential factors. A chronological point process which consists of a series of fire incidents is not necessarily a completely random Poisson distribution. It may be a time series that have a certain time relevance and show time-clustering phenomenon. To getting a discrete time point process sequence, pretreatment is needed before studying of city fire point process sequence. Usually there are two equivalent methods: (1) time interval sequence (2) interval counting value sequence. In the first representation, a discrete-time series is formed by the rule \( t_i = I_i - I_{i-1} \), where \( t_i \) indicates the time of event numbered by the index \( i \). In the second representation, the time axis is divided into equally spaced contiguous counting windows of duration \( T \) to produce a sequence of counts \( \{ N_k(T) \} \), where \( N_k(T) \) represents the number of events falling into the \( k \)th window of duration \( T \). This method has been used to study the time-clustering feature of the earthquake.

2.2.2. Temporal sequence analysis method

(1) Coefficient of variation \((C_v)\)

Coefficient of variation is a statistic that measures variation degree of the observed values. It’s a commonly used means to evaluate the clustering behavior of a point process. It is defined as follows:

\[
C_v = \frac{\sigma_T}{\langle \tau \rangle}
\]

where \( \langle \tau \rangle \) is the mean of the interevent times, \( \sigma_T \) is its standard deviation. In a Poisson process (completely random), \( C_v = 1 \), while a clustered process is characterized by \( C_v > 1 \). The greater the \( C_v \), the more obviously the clustering feature.

(2) Fano factor \((FF)\)

Variation coefficient method can only test the existence of the sequence clustering phenomenon, but it does not give information about the timescale ranges of the clustering. Fano factor is a parameter to study scaling behavior of event sequence, which is a function of timescale \( T \). The \( FF \) is defined as the variance of the counts for a specified timescale \( T \) divided by the mean number of events in that time-scale. The \( FF \) is written as follows:

\[
FF(T) = \frac{\langle N_k(T)^2 \rangle - \langle N_k(T) \rangle^2}{\langle N_k(T) \rangle}
\]

where \( N_k(T) \) is the number of events falling into the \( k \)th window of duration \( T \), and \( \langle \ldots \rangle \) calculates the average value. Change the value of \( T \), and then we can get a series of corresponding \( FF(T) \). For a random Poisson process, two fires are irrelevant, and for any timescale \( T \) the \( FF(T) \) are approximately equal to 1. If a sequence of points has fractal characteristics, \( FF(T) \) increases as a power-law function of the timescale \( T \), and it satisfies \( FF(T) \propto T^{\alpha} \) where scaling exponent meets \( 0 < \alpha \leq 1 \).
(3) Allan factor (AF)

Though Fano factor can identify whether a sequence has a fractal self similarity, but its fractal index ranges from 0 to 1. To study a sequence whose fractal index greater than 1, the Allan factor (AF) can be used. AF is a powerful means to analyze the sequence of time point process, which is defined as follows:

$$AF(T) = \frac{\left\langle (N_{i+1}(T) - N_i(T))^2 \right\rangle}{\left\langle N_i(T) \right\rangle^2}$$  \hspace{1cm} (3)

For a random Poisson process, Allan factor equals 1 in all time scales. If the sequence of time point process has fractal self similarity characteristics within a certain time scales, AF complies with the following equation:

$$AF(T) \propto T^\beta$$ \hspace{1cm} (4)

where scaling exponent meets $0 < \beta < 3$. Compared with the Fano factor, the difference of successive counts is considered in Allan factor, thereby reducing the effect of possible non-stationary of the point process. Furthermore, AF can be applied to systems in which fractal index is greater than 1. So it is applicable to a wider range.

3. Data processing and results analysis

Firstly we calculate the Coefficient of variation of the urban fires. Taking the data in 2006 for example, we can plot fire interval figure as shown in Fig. 1. With the eq.(1), it is obtained that the interval sequence’s variation coefficient $C_v$ is 2.04 in 2006. Similarly we also get the variation coefficient in 2003, 2004, 2005 which are respectively 1.09, 1.21 and 1.11. If all the fire data during 2003-2006 are taken into consideration, variation coefficient is 1.66. It can be seen that the variation coefficients of fire sequence in Wuhan City during 2003-2006 all meet $C_v > 1$. The coefficient of variation is an important indicator to measure the cluster, thus Wuhan’s City fire time series are not entirely random Poisson process, but have a certain cluster distribution characteristics.

According to the interval division by using interval counting method in certain range of time scales on the fire data, we get the fire data interval counting sequence $\{N_i(T)\}$. Here the time window $T$ ranges from 30 minutes to $P/10$ with an incremental value of 30 minutes each time, where $P$ is the entire length of time sequence. According to this new sequence $\{N_i(T)\}$, we calculated values of $FF(T)$ and $AF(T)$ with different time window $T$.

The relationship between Fano factor and the time window $T$ is shown in Fig. 2. From this figure, it can be seen that in dual-logarithm coordinates system, when the time window meets $T > 1440$ min, fire sequences of Wuhan have a good linear relationship between the Fano factor and time window $T$, and the slope of fitting line is 0.897 with a correlation coefficient 0.995. It reveals that $FF(T) \sim T$ has the fractal phenomenon. And in the same time $\alpha > 0$ means the correlation exists between the fire events. When time window meets $T < 1440$ min, with the time window $T$ approaches 0, the value of $FF(T)$
approaches 1, and the fractal exponent $\alpha$ tends to 0. When the time scale is smaller, the point process sequence of urban fire can be approximated as a Poisson process.

![Graph showing Fano factor for urban fires in Wuhan from 2003 to 2006](image1)

**Fig. 2.** Fano factor of the time sequence of Wuhan’s city fires from 2003 to 2006

The results of Allan factor is shown in Fig. 3. It can be seen that there are three main characteristic time scales:

1. $T < 240$ min: The value of Allan factor is close to 1. According to the Eq.(4), the slope $\beta$ approaches 0, which means that there is no fractal phenomenon in this range. The urban fire point process sequence can be approximated as a Poisson process.

2. $240$ min < $T < 10080$ min: In this time scale, Allan factor is greater than 1. That means when the time window is greater than 240 min, the fire point process sequence of Wuhan is no longer a completely random Poisson process. There is a certain relevance between fires, revealing the emerging of temporal clustering.

![Graph showing Allan factor for urban fires in Wuhan from 2003 to 2006](image2)

**Fig. 3.** Allan factor of the time sequence of Wuhan’s city fires from 2003 to 2006

3. $T > 10080$ min: In this range of time scales, $AF(T) \sim T$ show the approximate linear relation in dual-logarithm coordinates system. And the slope of the fitting line is 1.560 with the correlation coefficient 0.925. This results shows that there are very obvious temporal clustering phenomenon when the time scale is larger than 10080 min. It can be said that the fire time sequence of Wuhan city has good fractal non-scaling during 2003-2006.

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To investigate the stability of the cluster in the urban fires, we also calculate the Fano factor and Allan factor in different periods, such as in 2003, 2004-2006 and 2005-2006. The results are shown in Tab. 1. From this table it can be seen that there is a good linear relationship between $FF$ and $T$ during $T>1440\text{min}$. When $T$ is larger than $10080\text{min}$, the AF shows a linear increasing with increase of $T$ irrespective of the period of 2003, 2004-2006 or 2005-2006. These results shows the stability of fractal non-scaling in the urban fire system.

Table 1. Slope of fitting line for $AF(T)$ and $FF(T)$ during different periods

| Parameters | Range of time scale | Time periods of the fire sequence |
|------------|---------------------|----------------------------------|
|            | $T<240\text{min}$   | 2003    | 2004-2006 | 2005-2006 |
| Slope of $AF(T)$~$T$ | 240min$<$T$<=$10080min | -       | -         | -         |
|            | $T>10080\text{min}$ | 0.662   | 1.465     | 1.309     |
| Slope of $FF(T)$~$T$ | $T>1440\text{min}$ | 0.516   | 0.993     | 0.892     |

4. Conclusions

A sequence of urban fires can be seen as a stochastic point process, where each event is mainly characterized by occurrence time and size of the loss. The analysis performed in the present study aimed to investigate the time-scaling properties in the urban fire sequence recorded in Wuhan (China). Using different representations of the fire process (interevent times and series of counts) and several statistical measures, time-scaling behavior in the analyzed sequence is detected, and this suggests the presence of time-clusterization of the fire events. The good agreement between results obtained using FF method and AF method suggests the robustness of the time clustering.

Due to the complexity of city fires and cities, it is very difficult to explain exactly the complex dynamics of city fires. Nevertheless, in this paper we presented detailed statistical results of city fires that, we believe, could be helpful to describe and, possibly, understand their dynamics.

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