Comment on “Distinguishing Classical and Quantum Models for the D-Wave Device”

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The SSSV model [1] is a simple classical model that achieves excellent correlation with published experimental data on the D-Wave machine’s behavior on random instances of its native problem [2], thus raising questions about how “quantum” the D-Wave machine is at large scales. In response, a recent preprint by Vinci et al. [3] proposes a particular set of instances on which the D-Wave machine behaves differently from the SSSV model. In this short note, we explain how a simple modeling of systematic errors in the machine allows the SSSV model to reproduce the behavior reported in the experiments of [3].

In the SSSV model [1] for the D-Wave machine, qubits are modeled as classical magnets coupled through nearest-neighbor Coulomb interaction and subject to an external magnetic field. Moreover, the finite temperature of the device is modeled by performing a Metropolis update at each step. The results in [1] showed that the model shows excellent correlation with published data about the input-output behavior of the D-Wave machine on randomly chosen input instances [2]. Nevertheless is it possible that there are other classes of input instances on which the D-Wave machine exhibits “truly quantum” behavior? This is a question of central importance in the evaluation of the D-Wave architecture.

An affirmative answer requires exhibiting a regime in which classical models such as SSSV fail to reproduce the behavior of the D-Wave machine. Of course the SSSV model is extremely rudimentary, and was not meant to be an exact model for the D-Wave machine. For example, it makes no attempt to model details of the D-Wave machine such as errors in control of external fields and interaction strengths. So any such exhibited regime must either be sufficiently robust so that it can be argued that detailed modeling of the machine is unnecessary, or it must differentiate the behavior of D-Wave from reasonable elaborations of the SSSV model. Of course for the regime to be meaningful, there should also be a plausible computational benefit to the phenomenon in question.

A recent preprint by Vinci et al. [3] reports that the behavior of the D-Wave machine and the SSSV model differ on a particular set of instances. Fig. 1 depicts the problem Hamiltonian used in the experiments of [3]. All couplings are ferromagnetic, whereas there is a local z-field applied in the + direction for the four “core” spins, and in the − direction for the four “peripheral” spins. Formally, the Hamiltonian is defined as $H = -\sum_i h_i \sigma_i^z - \sum_{i<j} J_{ij} \sigma_i^z \sigma_j^z$. The local field $h_i$ is set to be 1 if $i$ is a core spin, and −1 otherwise. The coupling strength $J_{ij} = 1$ for every edge $i \sim j$. Figure is from [3].
Hamiltonian,
\[ H_I = \sum_{i} V h_{iz} + \sum_{(i,j) \in E} J_{ij} z_i z_j, \] (2)
and the time-dependent functions \( A(t) \) and \( B(t) \) of \( \alpha \), the magnetic field.

The eigenvalues of all Pauli matrices \( \{V \} \) are \(+1\), which is why there is only a single additional (isolated) ground state where all spins have \(+1\).

One special property of this Hamiltonian is that it has a number of excited states. During this relaxation phase, the arguments leading to these distinct predictions.

Subject to this Hamiltonian, \( n \) is increased. This is a manifestation of the growth, with \( N \), in the number of excited states connected to the isolated ground states, or just the “cluster-states”, or “cluster”.

There is always a finite temperature effect, and hence one should consider both tunneling and thermalization, and the final state reached is stable. Also shown are the attenuated configuration if it lowers the energy and accepting it probabilistically, i.e., \( \arg \min \).

The sixth eigenvalue of \( \{V \} \) is \(+1\) as in Fig. 2. As shown in Fig. 3, they find that the machine and the adiabatic quantum master equation prefer the isolated ground state \( (P_I/P_C > 1) \) when \( \alpha \) is small, whereas the SSSV model always prefers the clustered ground state \( (P_I/P_C < 1) \) at all values of \( \alpha \).

It is illuminating to examine more closely the small \( \alpha \) regime, where D-Wave and SSSV differ. Since in this regime the coupling strength is very small, this may be thought of as the “classical regime” where the machine is expected to be driven mostly by thermal noise rather than quantum effects. Moreover, as can be seen in Fig. 2 when \( \alpha \) is small, it is only after the transverse field \( A(t) \) has almost completely died out that the problem Hamiltonian becomes strong enough to be able to overcome the system temperature, therefore effectively making the annealing schedule trivial.

We also note that when \( \alpha \) is small, the effects of systematic errors in the machine, such as imperfections in the calibration of the annealing schedule, will also become more dominant. Since the SSSV model does not

Figure 3: Experimental and numerical results from [3]. DW2, ME, SA, SD, and SSSV represent D-Wave Two, quantum adiabatic master equation, simulated annealing, Smolin-Smith model [4], and SSSV model respectively. \( P_{GS} \) indicates the probability of finding one of the seventeen ground states.

Figure 2: The solid curves represent the annealing schedule of D-Wave Two. Dotted blue curves represent the effective annealing schedule for cases \( \alpha = 0.2834 \) and \( \alpha = 0.1099 \). The dotted black line represents the system temperature. Figure is from [3].
Figure 4: Simulations results for the modified SSSV model. The model produces a signature similar to that of the D-Wave machine or quantum adiabatic master equation from Fig. 3. The model was simulated for 1,500 steps at the system temperature of $T = 0.22\text{GHz}$. Ten thousand runs were performed for each value of $\alpha$.

attempt to model such systematic error, it is not surprising that it may fail to predict the machine’s behavior in this regime. In fact, we are able to demonstrate that a simple modeling of systematic errors completely alters the SSSV model’s behavior in this regime, so that it then reproduces the qualitative signature of the machine’s behavior shown in [3].

Fig. 4 shows the simulation results of the modified SSSV model in which there is a small independent Gaussian error in the calibration of the local field applied to each spin. To be more precise, the time-dependent Hamiltonian is defined as $H(t) = A(t) \sum_i \sin \theta_i - \sum_i (B(t) \cdot \alpha \cdot h_i + \epsilon_i) \cos \theta_i - \sum_{i<j} B(t) \cdot \alpha \cdot J_{ij} \cos \theta_i \cos \theta_j$ where $\epsilon_i \sim N(0, 0.24)$.

We make no further attempt to improve the quantitative fit of these graphs (since detailed physical modeling of the machine is infeasible at the present time due to the limited access to the machine’s internal mechanism), beyond noting that the set of examples in [3] does not appear to provide a robust regime, in the sense described above, where the results of the D-Wave machine diverge from SSSV.

In a strict sense, establishing that a phenomenon is truly “quantum” at a large scale is extremely challenging, since it involves ruling out all possible classical explanations. While this is not practically feasible, it is difficult to overemphasize the importance of carefully ruling out a range of classical models. Specifically, we hope that this note demonstrates the value of carefully considering elaborations of the rather rudimentary SSSV model while investigating how well it matches the behavior of a complex machine like D-Wave.

Acknowledgments

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We note that introducing similar Gaussian errors on the couplings does not seem to affect the simulation results.
Further simulations of the modified SSSV model reveal that it reproduces various other signatures suggested in [3]. For instance, Fig. 5a exhibits a good qualitative resemblance with the experimental data presented in Fig. 14 of [3]. Figs. 5b, 5c, and 5d show that the behavior demonstrated in Fig. 4 persists as the problem size scales up, which is consistent with the experimental results from Fig. 10 of [3].