Bottom Baryon Decays in Non-relativistic Quark Model

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The reactions \( \Sigma_b^0 \to \Lambda_b \pi, \Sigma_b \to \Lambda_b \pi, \) and \( \Xi_b^0 \to \Xi_b \pi, \) are studied in the \(^3P_0\) non-relativistic quark model with all the model parameters fixed in the sector of light quarks. The theoretical predictions for the decay widths \( \Gamma_{\Sigma_b^0 \to \Lambda_b \pi} \) and \( \Gamma_{\Sigma_b \to \Lambda_b \pi} \) are consistent with the experimental data of the CDF Collaboration. Using as an input the recent mass of \( \Xi_b \) and the theoretical predictions mass of \( \Xi_b^* \), a narrow decay width about 1 MeV is predicted for the bottom baryon \( \Xi_b^* \). The work suggests that the \(^3P_0\) quark dynamics is of independence of environments where heavy quarks may or may not be a component of baryons.

I. INTRODUCTION

The first bottom baryon \( \Lambda_b \), with the \( udb \) configuration and a mass around 5640 MeV, was reported by UA1 Collaboration at CERN in late 1990s \(^1\). Later the \( \Lambda_b \) was confirmed by other experiments such as DELPHI Collaboration \(^2\), ALEPH Collaboration \(^3\), and CDF Collaboration \(^4\) with neutral charge and mass between 5614 to 5668 MeV. In 2005, the mass of \( \Lambda_b \) was further measured to be 5619.7 MeV by the CDF Collaboration at Fermilab \(^5\). Very recently five new bottom baryons, \( \Sigma_b^*(s) \) and \( \Xi_b^*(s) \) were reported by the CDF Collaboration at Fermilab \(^6,7\) in proton-antiproton collisions at \( \sqrt{s}=1.96 \) TeV.

The decay processes \( \Sigma_b^0 \to \Lambda_b \pi \) and \( \Sigma_b \to \Lambda_b \pi \) have been studied by combining the chiral dynamics and the MIT bag model \(^8\), and the theoretical results for the decay widths of the reactions are consistent with the experimental data. More recently, the strong decays of \( \Sigma_b^{(*)} \) and \( \Xi_b^{(*)} \) are studied in the \(^3P_0\) quark model, as a byproduct of the work \(^9\) which concentrates on the strong decays of charmed baryons. However, the limited consistency of the theoretical results with the experimental data make it rather difficult to conclude whether the \(^3P_0\) quark dynamics, with all the model parameters fixed in the light quark sector, is applicable to the sector of bottom baryons.

II. \( \Sigma_b^{(*)} \) AND \( \Xi_b^{(*)} \) DECAY IN THE \(^3P_0\) QUARK DYNAMICS

We study here the decay processes \( \Sigma_b^{(*)} \to \Lambda_b \pi \) and \( \Xi_b^{(*)} \to \Xi_b \pi \) in the \(^3P_0\) quark model. There is no experimental data for the masses of \( \Xi_b \), but one may make a reasonable estimation by averaging the predictions of recent theoretical works \(^10,11,12,13,14,15\). The theoretical predictions are indeed very close each other, and the averaged value for the \( \Xi_b \) mass is 5967 MeV.

The observation of the four bottom baryons \( \Sigma_b^{(*)\pm} \) in the \( \Lambda_b \pi \) invariant mass spectrum make it possible to explore whether the \(^3P_0\) non-relativistic quark dynamics is independent of environments which may or may not have heavy quarks involved. In this work we study the decay processes \( \Sigma_b^0 \to \Lambda_b \pi, \Sigma_b \to \Lambda_b \pi, \) and \( \Xi_b^* \to \Xi_b \pi \) in the \(^3P_0\) quark dynamics with all the model parameters fixed by reactions in the light quark sector. The paper is arranged to calculate the widths of the \( \Sigma_b^{(*)} \) and \( \Xi_b^{(*)} \) strong decays in Section II and to give our discussion and conclusions in Section III.
defined as
\[ T = \langle \Psi_f | V_{68} | \Psi_i \rangle \]  
(1)
where \( \Psi_f \) and \( \Psi_i \) are respectively the final and initial states of the reactions. \( V_{68} \) is the quark-antiquark \( ^3P_0 \) vertex, taking the form
\[ V_{ij} = \lambda \hat{\sigma}_{ij} \cdot (\vec{p}_i - \vec{p}_j) \hat{C}_{ij} \hat{F}_{ij} \delta(\vec{p}_i + \vec{p}_j) \]
\[ = \lambda \sum_{\mu} \sqrt{\frac{4\pi}{3}} (-1)^{\mu} \sigma_\mu y_{1\mu}(\vec{p}_i - \vec{p}_j) \hat{C}_{ij} \hat{F}_{ij} \delta(\vec{p}_i + \vec{p}_j) \]  
(2)
with
\[ \hat{\sigma}_{ij} = \frac{\hat{\sigma}_i + \hat{\sigma}_j}{2} \]
\[ y_{1\mu}(\vec{p}) \equiv |\vec{p}| Y_{1\mu}(\vec{p}) \]  
(3)
where \( \vec{p} \equiv |\vec{p}| \), \( \sigma_i \) are Pauli matrices and \( Y_{1\mu}(\vec{p}) \) are the spherical harmonics. \( \vec{p}_i \) and \( \vec{p}_j \) are the momenta of quark and antiquark which pumped out from vacuum, and \( \hat{C}_{ij} \) and \( \hat{F}_{ij} \) are respectively the color and flavor operators projecting a quark-antiquark pair to vacuum in the color and flavor spaces. The derivation and interpretation of the quark-antiquark \( ^3P_0 \) dynamics may be found in literatures [16, 17].

\[ \Sigma^{(*)} \rightarrow \Lambda_b \pi \text{ and } \Xi^{*} \rightarrow \Xi_b \pi \text{ in the } ^3P_0 \text{ quark model.} \]

The spin-flavor wave functions of the baryons involved may be constructed in the framework of the flavor SU(4) and spin SU(2) symmetries. The explicit forms of the spin-flavor functions are given in Appendix A. As for the spatial wave functions which depend on the strong interaction, we just adopt the conventional Gaussian form which result from the spherical harmonics oscillator interaction. We have, for example, for \( \pi \) meson
\[ \psi(\pi) = N_m \exp \left[ -\frac{b^2}{8} \left( \vec{p}_1 - \vec{p}_2 \right)^2 \right] \]  
(5)
and for \( \Sigma_b \)
\[ \psi(\Sigma_b) = N_B \exp \left[ -\frac{a^2}{2} \left( \frac{\vec{p}_2 - \vec{p}_3}{\sqrt{2}} \right)^2 \right] \cdot \exp \left[ -\frac{a^2}{2} d^2 \left( \frac{\vec{p}_2 + \vec{p}_3 - 2 m_\pi \vec{p}_1}{\sqrt{6}} \right)^2 \right] \]  
(6)
with \( N_m = b^{3/2} / \pi^{3/4} \), \( N_B = 3^{3/4} a^{3/2} / \pi^{3/2} \), \( m_\pi = m_q / m_b \) and \( d = 3(1 + 2m_\pi) \) where \( m_\pi \) and \( m_b \) are respectively the masses of the constituent \( u(d) \) and \( b \) quarks. The parameters \( b \) and \( a \) in Eqs. (5) and (6) are linked to the sizes of meson and baryon, respectively.

The evaluation of the transition amplitudes is straightforward for all the decay processes, and it is found that only the \( l = 1 \) partial wave gives contributions. The partial wave transition amplitudes take the general form
\[ T_{1M} = \lambda \cdot f_1 \cdot f_2 \cdot f_3 \]  
(7)
with \( f_1, f_2 \) and \( f_3 \) resulting respectively from the spin, spatial and color-flavor sectors. Detailed calculations lead to
\[ f_1 = C(S_i M_i; 1M; S_f M_f) \left[ \begin{array}{ccc} 1 & 1/2 & S_i \\ 1/2 & 1/2 & 1 \\ S_f & 0 & S_f \end{array} \right] \]  
(8)
\[ f_2 = \frac{16 \pi^5 \left( (9m_\pi + 3)a^2 + b^2(2m_\pi + 1) \right)}{9a^3 (3a^2 + b^2)^{5/2} (2m_\pi + 1)} \]  
(9)
where \( S_i \) and \( S_f \) are respectively the spins of the initial and final baryons, being \( \frac{3}{2} \) for \( \Sigma_b^{(*)} \) and \( \Xi^{*} \), and \( \frac{1}{2} \) for \( \Lambda_b, \Xi_b \) and \( \Sigma_b \). \( M_i \) and \( M_f \) are the corresponding spin magnetic moments. The first and second factors in Eq. (8) are respectively the C-G coefficient and square \( \theta_j \) symbol. The factor \( f_3 \) in Eq. (7) takes the values as
\[ f_3 = \begin{cases} \frac{1}{\sqrt{2}}, & \Sigma_b^{(*)} \rightarrow \Lambda_b \pi^\pm \\ \frac{1}{\sqrt{2}}, & \Sigma_b^{(*)} \rightarrow \Lambda_b \pi^\pm \\ \frac{1}{\sqrt{2}}, & \Xi_b^- \rightarrow \Xi_b^0 \end{cases} \]  
(10)
The decay width of the processes \( \Sigma_b^{(*)} \rightarrow \Lambda_b \pi \) and \( \Xi_b \rightarrow \Xi_b \pi \) takes the form in terms of the partial wave transition.
TABLE I: Summary of input parameters which are fixed by other processes

|       |       |
|-------|-------|
| $\lambda$ | 3.1 |
| $a$ | 3.1 GeV$^{-1}$ |
| $b$ | 3.85 GeV$^{-1}$ |
| $m_{u(d)}$ | 330 MeV |
| $m_s$ | 550 MeV |
| $m_\Lambda$ | 4200 MeV |
| $M_{\Lambda_b}$ | 5619 MeV |
| $M_{\Sigma_b^-}$ | 5816 MeV |
| $M_{\Sigma_b^0}$ | 5808 MeV |
| $M_{\Sigma_b^+}$ | 5837 MeV |
| $M_{\Xi_b^-}$ | 5829 MeV |
| $M_{\Xi_b^0}$ | 5793 MeV |
| $M_{\Xi_b^+}$ | 5967 MeV |

where $k$ is the final momentum at the rest frame of the initial particle, $M_B$ the mass of the initial baryon, and $E_1$ and $E_2$ are the energies of the two final particles.

In addition to the quark masses, one also needs to determine, prior to our evaluation of the decay widths of $\Sigma(+)\Sigma$ and $\Xi^*$, the effective strength parameter $\lambda$ of the $3P_0$ quark vertex and the baryon and meson size parameters $a$ and $b$. We take for the $u$ and $d$ quarks the widely used constituent quark mass $m_u = m_d = 330$ MeV, and for the $s$ quark $m_s = 550$ MeV. For the $b$ quark we use the $\overline{MS}$ mass $m_b = 4.2$ GeV evaluated by the Particle Data Group [19]. The meson size parameter $b$ in the work is determined to be 3.85 GeV$^{-1}$ by the reaction $\rho \to e^+e^-$ as in the work [20] while the value of the baryon size parameter $a$ is taken to be 3.1 GeV$^{-1}$ which corresponds to a 0.6 fm quark core of ground state baryons [17,21].

As the main purpose of the work is to figure out whether the $3P_0$ quark dynamics is consistently applicable to both the light and heavy quark sectors, we would determine the effective coupling constant $\lambda$ via the process $\Sigma(1385) \to \Lambda(1116)\pi$. Using as an input $b = 3.85$ GeV$^{-1}$, $a = 3.1$ GeV$^{-1}$, $M_{\Sigma^0} = 1383$ MeV, $M_\Lambda = 1116$ MeV, $m_u = 330$ MeV, $m_s = 550$ MeV, and the experimental value $\Gamma_{\Sigma^0 \to \Lambda\pi^+} = 32.0$ MeV, we get the effective coupling constant $\lambda = 3.1$.

Summarized in Table I are all the input parameters for the evaluation of the decay widths of the processes $\Sigma_b^{(*)} \to \Lambda_b\pi$ and $\Xi_b^* \to \Xi_b\pi$. Note that all the parameters are taken from other works. Using as an input the parameters listed in Table I, the decay widths for the reactions $\Sigma_b^{(*)} \to \Lambda_b\pi$ and $\Xi_b^* \to \Xi_b\pi$ are worked out as shown in Table II.

| Reactions       | $3P_0$ results | Data |
|-----------------|-----------------|------|
| $\Sigma_b^+ \to \Lambda_b\pi^-$ | 14.6 | $\sim 15$ |
| $\Sigma_b^0 \to \Lambda_b\pi^+$ | 12.4 | $\sim 15$ |
| $\Sigma_b^- \to \Lambda_b\pi^-$ | 9.0 | $\sim 8$ |
| $\Sigma_b^0 \to \Lambda_b\pi^+$ | 7.1 | $\sim 8$ |
| $\Xi_b^+ \to \Xi_b\pi$ | 1.3 | $-$ |

III. DISCUSSION AND CONCLUSIONS

The reactions $\Sigma_b^* \to \Lambda_b\pi$, $\Sigma_b \to \Lambda_b\pi$, and $\Xi_b^* \to \Xi_b\pi$ are investigated in the $3P_0$ non-relativistic quark model with all the model parameters taken from other sources. The meson size parameter $b$ is fixed by the reaction $\rho(770) \to e^+e^-$ while the baryon size parameter $a$ is taken to give a 0.6 fm quark core of ground state baryons. With $b = 3.85$ GeV$^{-1}$ and $a = 3.1$ GeV$^{-1}$, the effective strength parameter $\lambda$ of the $3P_0$ quark vertex is fixed by the reaction $\Sigma(1385) \to \Lambda(1116)\pi$. The theoretical prediction for the decay width of the process $\Sigma(1385) \to \Lambda(1116)\pi$ depends strongly on the size parameters $a$ and $b$, hence with different values of $a$ and $b$ we surely needs different $\lambda$ to fit the experimental decay width. However, the predictions for the decay widths
of the decay processes $\Sigma_0^* \rightarrow \Lambda_0 \pi$, $\Sigma_0 \rightarrow \Lambda_0 \pi$, and $\Xi_0^* \rightarrow \Xi_0 \pi$ are rather insensitive to the combined parameter set $\{a, b, \lambda\}$. For instance, a 20% variation of $a$ and $b$ results in less than 5% change over the decay widths.

The theoretical predictions for the decay widths of the decay processes $\Sigma_0^* \rightarrow \Lambda_0 \pi$, $\Sigma_0 \rightarrow \Lambda_0 \pi$, and $\Xi_0^* \rightarrow \Xi_0 \pi$ are also insensitive to the quark masses. With different masses of $u(d)$ and $s$ quarks one gets different coupling constants $\lambda$ from fitting to the experimental decay width of the process $\Sigma(1385) \rightarrow \Lambda(1116) \pi$, but the combined effect is very trivial on the theoretical predictions for the decay widths of the reactions $\Sigma_0^* \rightarrow \Lambda_0 \pi$, $\Sigma_0 \rightarrow \Lambda_0 \pi$, and $\Xi_0^* \rightarrow \Xi_0 \pi$. Since the mass of the bottom quark is much larger than the light ones, a 10% variation of the mass $m_b$ about 4.2 GeV gives no observable effect on the theoretical predictions.

The predictions for the decay widths of the reactions $\Sigma_0^* \rightarrow \Lambda_0 \pi$ and $\Sigma_0 \rightarrow \Lambda_0 \pi$ are in line with the CDF experimental data [6]. One may conclude that the $^3P_0$ quark dynamics is of independence of environments where heavy quarks may or may not be a component of baryons.

Using as an input $M_{\Xi_0} = 5793$ MeV from the experimental data and $M_{\Xi_0^*} = 5967$ MeV derived by averaging the recent theoretical predictions, the work predicts a narrow $\Xi_0^*$ width $\Gamma \approx 1$ MeV.

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**APPENDIX A: SPIN-FLAVOR WAVE FUNCTIONS**

The spin-flavor wave functions of baryons made of $u$, $d$, $s$ and $b$ quarks may be constructed in the framework of the flavor SU(4) and spin SU(2) symmetries. The spin-flavor wave functions $\Psi_{SF}$ for the baryons $\Sigma_0^*(uub)$, $\Sigma_0^*(uub)$, $\Xi_0^*(usb)$, $\Xi_0^*(usb)$, and $\Lambda_0^*(usb)$ are respectively

\[
\Psi_{SF}(\Sigma_0^*) = \phi^S(\Sigma_0^*) \chi^S \\
\Psi_{SF}(\Xi_0^*) = \phi^S(\Xi_0^*) \chi^S \\
\Psi_{SF}(\Sigma_0^+) = \frac{1}{\sqrt{2}} \left[ \phi^\lambda(\Sigma_0^+) \chi^\lambda + \phi^\rho(\Sigma_0^+) \chi^\rho \right] \\
\Psi_{SF}(\Sigma_0^0) = \frac{1}{\sqrt{2}} \left[ \phi^\lambda(\Sigma_0^0) \chi^\lambda + \phi^\rho(\Sigma_0^0) \chi^\rho \right] \\
\Psi_{SF}(\Lambda_0^0) = \frac{1}{\sqrt{2}} \left[ \phi^\lambda(\Lambda_0^0) \chi^\lambda + \phi^\rho(\Lambda_0^0) \chi^\rho \right]
\]

where $\chi^S(\phi^S)$, $\chi^\lambda(\phi^\lambda)$ and $\chi^\rho(\phi^\rho)$ are the symmetric, $\lambda$ type and $\rho$ type spin (flavor) wave functions, respectively.

It is convenient to construct wave functions of baryons or other multi-quark particles in the framework of the Yamanouchi basis and the corresponding projection operators of permutation group. For more details, one may refer to group theory books like [22, 23]. The various flavor wave functions are

\[
\phi^S(\Sigma_0^+) = \frac{1}{\sqrt{3}}(uub + buu + bu) \\
\phi^S(\Xi_0^0) = \frac{1}{\sqrt{6}}(usb + bsu + sbu + sub + usb) \\
\phi^\lambda(\Sigma_0^+) = \frac{1}{\sqrt{6}}(2uub - ubu - buu) \\
\phi^\rho(\Sigma_0^+)= \frac{1}{\sqrt{2}}(buu - ubu) \\
\phi^\lambda(\Xi_0^0) = \frac{1}{2}(ubs + bus - bsu - sbu) \\
\phi^\rho(\Xi_0^0) = \frac{1}{\sqrt{12}}(2sub - 2usb + bus + sbu - usb - bsu) \\
\phi^\lambda(\Lambda_0^0) = \frac{1}{2}(ubd + bud - bdu - dbu) \\
\phi^\rho(\Lambda_0^0) = \frac{1}{\sqrt{12}}(2dub - 2udb + bud + dbu - ubd - dbu)
\]

where $u$, $d$, $s$ and $b$ stand for the flavor wave functions of the corresponding quarks, respectively.

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