Mathematical Modeling of Creep Induced by Machining Residual Stresses

Timothy W. Spence\textsuperscript{a} and Makhlouf M. Makhlouf\textsuperscript{b,\*}

\textsuperscript{a}BAE Systems Electronics and Integrated Solutions, P.O.Box 868, NHQ3-2145, Nashua, NH 03061, USA
\textsuperscript{b}Department of Mechanical Engineering, Worcester Polytechnic Institute, Worcester, MA 01609, USA

Abstract

Dimensional stability of materials is of critical importance in fabricating precision components such as those used in optical systems. An important source of dimensional instability is residual stresses introduced into the surface of parts during machining. In this paper, a creep model is developed and used to describe how these surface stresses affect the overall geometry of a component as it creeps over time and temperature. The model can be used for two purposes: (1) to predict long term storage effects on part geometry for purposes of assessing part reliability, and (2) to design short term, moderate temperature stress relief treatments for components. The model is verified by applying it to components manufactured from aluminum alloy 6061-T6.

Keywords: Dimensional stability; creep; mathematical modelling; machining-induced residual stress; aluminum alloy

1. Background

An important cause of dimensional instability in metallic components is residual stress induced by machining [1-5]; and although stress relief procedures have been developed for many alloys [6], dimensional instability caused by surface stresses due to machining operations has not been thoroughly characterized. Therefore, there is a need for a means that allows predicting the magnitude of the dimensional changes that occur in metallic components over time and temperature ranges. In addition,
there is a need for a comprehensive understanding of the kinetics of the strains that develop in the material as a function of machining-induced residual stresses. A model that describes how surface stresses affect the shape of a part over time and temperature is an extremely useful tool in predicting whether a precision part will maintain its critical dimensions after exposure to temperature or not. The model can also guide development of stress relief procedures that provide the required reduction in stress without subjecting the material to extended temperature soaks that may reduce its strength. Hence, the main objective of this work is to develop a mathematical model that allows predicting the creep behaviour of metallic components due to machining residual stresses, specifically high precision components made with aluminium alloy 6061-T6.

2. Development of the mathematical model

Machining operations produce surface significant stresses in machined components. Material that is in a state of sustained stress will be subject to creep. There are two unique characteristics to this situation: (1) Creep models have a stress term that is raised to an exponent that is larger than 1 for metals. Consequently, if a metallic specimen has a non-uniform stress, the more stressed regions will creep at exponentially higher rates than the less stressed regions; and (2) As the material strains, the stress level becomes proportionally lower. These two characteristics serve as the basis for the proposed creep model. The temperature range of interest is -55°C to +125°C. This range is selected because it is typical for a wide variety of parts that operate over what is generally considered the full military temperature range. The typical magnitude of residual stress caused by machining operations is 100-200MN/m² [2, 6]. Considering aluminium alloys, this temperature and stress ranges put the creep mode in the range of both the Dislocation Glide and the Dislocation Creep mechanisms [7]. The simplest creep model that is applicable for Dislocation Glide and Dislocation Creep, and also accounts for both stress and temperature is the Power Law [8]. However, the Power Law Creep model best represents Secondary Creep. In the case described herein, the constantly changing stress magnitude (due to creep strain) will keep the creep mode in the Primary Creep regime. Nevertheless, the strains are expected to be small enough that a straight line approximation over the range of interest should introduce only a small error.

The Power Law Creep equation for crystalline materials [8] is given by Eq. (1)

\[ \dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{\sigma^m}{k} \frac{q}{d^q} e^{\frac{Q}{RT}} \]  

In Eq. (1), the variables that affect the strain rate \( \dot{\varepsilon} \) are stress \( \sigma \), average grain diameter \( d \), and absolute temperature \( T \). The coefficients \( k \), the exponents \( m \) and \( q \), and the activation energy \( Q \) have values that
depend on the material and the creep mechanism. Grain boundaries are not a major factor in Power Law Creep for crystalline materials and so the strain rate is not significantly affected by grain size and we assume \( q = 0 \) [8], so that Eq. (1) becomes

\[
\dot{\epsilon} = \frac{k}{T} \sigma^m e^{\frac{Q}{RT}}
\]  

(2)

A unique condition of creep strain due to residual (not applied) stress in an un-restrained material is that stress is not constant but is a function of strain. Given Hooke’s Law: \( \epsilon_i = \frac{1}{E} \left[ \sigma_i - v (\sigma_2 + \sigma_3) \right] \), and if \( \sigma_2 = \sigma_3 = 0 \), then \( \sigma_1 = \sigma = E \epsilon \). A stress function is then defined with a linear relationship to strain and where the value of stress reduces to zero as strain reaches its final magnitude \( (\epsilon_f) \) as shown in Eq. (3).

\[
\sigma = \sigma_{in} \left(1 - \frac{\epsilon}{\epsilon_f}\right)
\]

(3)

Substituting Eq. (3) into Eq. (2), and integrating from an initial strain of zero at an initial time of zero, yields an expression that defines strain as a function of initial stress \( (\sigma_{in}) \), final strain \( (\epsilon_f) \), stress exponent \( (m) \), activation energy \( (Q) \), a constant \( (k) \), and time \( (t) \) as shown in Eq. (4)

\[
\frac{\epsilon - \epsilon_f}{\epsilon_f} = \int_{t=0}^{t} k e^{\frac{Q}{RT}} dt = \int_{t=0}^{t} \frac{\sigma_{in} - \epsilon_{in} \epsilon}{\epsilon_f} \cdot \frac{Q}{RT} \cdot \frac{1}{t} \cdot \left[ \left( \frac{m-1}{m} \right) \frac{\sigma_{in}}{\epsilon_f} + \left( \frac{\sigma_{in}}{\epsilon_f} \right)^{m-1} \right]^{\frac{1}{1-m}}
\]

(4)

Within the temperature ranges considered in this work, increasing temperature caused an increase in \( \epsilon_f \). The theoretical maximum creep strain that would be recovered during isothermal temperature exposures would be in accordance with Hooke’s Law (if the initial stress is completely converted to strain); but in reality, the theoretical maximum strain is not reached, so an expression is developed to predict \( \epsilon_f \) as shown in Eq. (5)

\[
\epsilon_f = B \frac{\sigma_{in} T^n}{E}
\]

(5)

Substituting Eq. (5) into Eq. (4) yields the Creep Model Equation

\[
\epsilon = B \frac{\sigma_{in} T^n}{E} - BT^n \left[ \frac{(m-1)E}{B} \frac{k}{T^{m+1}} e^{\frac{-Q}{RT}} \right]^{\frac{1}{1-m}}
\]

(6)
3. Measurements

Coupons were machined on one side to cause them to curve when subjected to heat. The magnitude of curvature of these specimens was monitored over an extended period of time at different temperatures. The bimetallic strip model [9] was used to convert these measured curvatures to strains in the machined layer. The creep parameters $m$, $Q_c$, and $k$ in Eq. (6) were obtained by fitting the strain vs. time and temperature data to curves by means of curve fitting software; the final strain, $\varepsilon_f$, was determined experimentally. Several 2.5 × 15.25 × 0.15 cm coupons were machined from 6061-T6 aluminum alloy. Since the machining operations introduce a stressed layer on all the surfaces of these coupons, the surface layer of the coupons which was between 0.0127 to 0.0152 cm deep (based on [5]) was removed by etching for 10 minutes with a 20% sodium hydroxide solution at 40°C. Procedures for machining the coupons in order to introduce residual stresses as well as procedures for measuring the resulting residual stresses and the temperature-dependant modulus of elasticity of the material are described in detail in reference [10]. Table 1 shows the measured average stresses and Table 2 shows the elastic modulus of 6061-T6 alloy at various temperatures. In Table 1, the stress which is found just below the surface of the coupon is highlighted.

Table 1. Measured stress distribution in 6061-T6 coupons that have been milled.

| Depth (cm) | Measured stress (MN/m²) | Gradient (MN/m²) | Relaxation (MN/m²) |
|------------|-------------------------|-----------------|-------------------|
| 0.0000     | -92.86 ± 2.72           | -16.67          | -14.63            |
| 0.0005     | -143.54 ± 2.72          | -146.94         | -141.50           |
| 0.0025     | -89.80 ± 2.72           | -165.99         | -152.72           |

Table 2. Elastic modulus of 6061-T6 alloy at various temperatures.

| Temperature (°C) | Modulus of Elasticity (kN/m²) |
|------------------|-------------------------------|
| 25               | 68,815                        |
| 60               | 67,597                        |
| 85               | 66,577                        |
| 125              | 64,684                        |
4. Results and analysis

As mentioned earlier, the creep parameters $m$, $Q_c$, and $k$ in Eq. (6) were obtained by fitting the strain vs. time data to curves by means of curve fitting software. Determining these parameters required the raw data (radius of curvature vs. time) to be first converted into strain vs. time data. This was accomplished by means of the simplified version of the bimetallic strip equation given in Eq. (7) [10] where $t_A$ and $t_B$ are thickness of stressed and un-stressed regions respectively, and $r_1$ and $r_2$ are radii of curvature. Eq. (7) is valid when the modulus of elasticity of both layers is equal and $t_A \ll t_B$.

$$
E_{t_1 \rightarrow t_2} = \frac{t_B}{6t_A} \left( 3t_A + t_B \right) \left( \frac{1}{r_2} - \frac{1}{r_1} \right)
$$

Table 3 lists the calculated creep parameters and Fig. 1 shows a comparison between the measured strain and the strain calculated by Eq. (6) with the creep parameters shown in Table 3. Fig. 2 is a plot of the final strain ($\epsilon_f$) obtained by Eq. (5) with $B = 2.81 \times 10^{-13}$ and $n = 4.4$.

Table 3. Creep parameters for milled 6061-T6 aluminium alloy.

| $Q_c$ (cal/mol) | $m$     | $k$       |
|----------------|---------|-----------|
| 6,907          | 3.583   | 2.717×10^{-21} |

**Fig. 1.** Strain vs. time at different temperatures for 6061-T6 aluminium alloy. Points are the measured values and solid lines are model predictions. △ is at 125°C, □ is at 85°C, and ○ is at 60°C.

**Fig. 2.** Plot used to calculate the values of the constants $B$ and $n$ in Eq. (5).
5. Conclusions

The creep model presented herein can accurately predict creep induced by milling 6061-T6 aluminium alloy and therefore it can be used for forecasting the effects of long term storage, as well as for designing high temperature, short term stress relief treatments for parts made of this alloy. A detailed procedure for designing an effective stress relief regimen is described in detail in reference [10].

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