Vibration of composite thin-walled beams with variable open cross section by a high order implicit algorithm

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Abstract:
In this work, we study the forced nonlinear vibrations with large amplitude and large torsion of composite thin-walled beams with open variable cross sections under external dynamic loads using a high order implicit algorithm. The used algorithm is based on the temporal and spatial discretizations, the homotopy transformation, Taylor series expansion and the continuation technique. A 3D beam element with two nodes and seven degrees of freedom per node is adopted. The obtained results are compared with those computed by the industriell Abaqus code.

Keywords: Composite thin-walled beam, open variable cross section, large displacement, large torsion, forced nonlinear vibrations, Newmark implicit scheme, Taylor series, homotopy transformation.

1 Introduction

For about a century, many industrial sectors have sought to strengthen and at the same time lighten the structural solutions to optimize the efficiency and cost of structures. Among them are the civil, mechanical, naval and aerospace industries. This has led to an increasing use of thin-walled open-section structures such as cold-formed steel girders, steel and concrete girders, ship hulls, trapezoidal steel sheets and other structures in which a dimension is small compared to other dimensions. The use of rigid beams that have a slight weight encourages industrials to use composite beams with open and variable sections.

To ensure the use of thin-walled composite beams, it is important to study their behavior when it is subjected to external dynamic loading. Several efforts have been devoted to this research domain. Many finite element models have been proposed to study the linear and nonlinear dynamic responses of these thin-walled structures with constant and variable open sections under these external loadings.

This work deals with the study of forced nonlinear vibrations with large amplitude and large torsion of composite thin-walled beam with open variable cross sections under external dynamic loads. The free vibrations of composite beams have been studied in [1] using the thin tube theory for analyzing thin-walled composite beams. This study is focused on an analysis of free vibrations of a composite thin-walled beam with open and variable cross section. We propose in this work to study the nonlinear forced vibration of this structure type. The influence of the section and the material effect on the vibrational behavior of thin-walled beam is also studied. The nonlinear dynamic behavior of this structure type is investigated by a high order implicit algorithm [2]. This algorithm combines the discretization procedures in space and time respectively, homotopy technique, Taylor series representation and a continuation procedure.

2 Governing dynamic equations and resolution strategy

Consider a composite 3D thin-walled beam of length \( L \) and open variable cross section of area \( A(x) \). The used rectangular cartesian co-ordinates system is \( Gxyz \) with origin at \( G \). \( G_y \) and \( G_z \) are the principal bending axes and \( G_x \) is the axial axis (see figure 1). The co-ordinates of shear point \( C \) located in \( Gyz \) plane are \( y_c, z_c \). An arbitrary point \( M \) of a section is defined by its coordinates \((y, z, \omega)\); with \( \omega \) is the sectoriel coordinate. The displacement components \( u_M, v_M \) and \( w_M \) at point \( M \) are expressed by the following relations [3]:

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3.1 Linear case

In this section, we study the linear free vibration of composite thin-walled beam with variable open cross section. Indeed we consider a clamped-free bi-symmetrical composite beam of length \( L = 40 \text{cm} \), of an I-section varying in web height and flange width linearly, while all other dimensions remain constant (see figure 2-(a)). All numerical analyses performed in this section are the same data used in [1]. The geometric characteristics of the extreme end sections of the beam are shown in the figure 2-(b). The beam is composed of two material types: flange and web. The mechanical characteristics of the web are \( E_w = 7.010^3 \text{KN/cm}^2 , G_w = 2.610^3 \text{KN/cm}^2 \) and \( \rho_w = 2.85t/m^3 \) and the mechanical characteristics of the flange are: \( E_f = 2.110^4 \text{KN/cm}^2 , G_f = 8.0510^3 \text{KN/cm}^2 \) and \( \rho_f = 7.85t/m^3 \). The considered beam is meshed in 20 3D beam elements. In Table (1), the ten first eigenfrequencies of the free vibrating beam are presented and compared with those obtained by the industrial Abaqus code and those given in [1].

![Geometrical characteristics of a I section composite thin-walled beam with variable open cross section and temporal evolution of external dynamic load](image)

| \( \omega_i \) | present study | Abaqus |
|---|---|---|
| 1 | 282.5493 | 284.1088 | 285.3457 |
| 2 | 974.6288 | 979.8491 | 981.7391 |
| 3 | 2244.756 | 2254.054 | 2255.276 |
| 4 | 4127.814 | 4132.001 | 4132.765 |
| 5 | 6633.401 | 6640.321 | 6641.566 |
| 6 | 9762.938 | 9765.432 | 9766.941 |
| 7 | 13516.330 | 13519.899 | 13520.189 |
| 8 | 17892.780 | 17895.911 | 17896.744 |
| 9 | 22891.272 | 22896.100 | 22897.243 |
| 10 | 28510.183 | 28512.002 | 28512.581 |

Table 1: Comparison of eigenfrequencies \( \omega_i \) of the composite beam

According to the table (1), we notice that the results are in good agreement with those calculated by the industrial Abaqus code and those given in [1].

3.2 Nonlinear case

In order to study the forced nonlinear vibrations of composite thin-walled beam with variable open cross sec-
tion, we consider the same beam studied in (section 3.1). This beam is subjected to two temporal eccentric loadings \( F_y(t) \) and \( F_y(t) \) applied at point \( O \); with \( F_y(t) = 600K N x(t), F_y(t) = 100K N x(t) \) \( \lambda(t) \) is that plotted in figure 2-(c), eccentricities of these loadings are: \( e_y = -0.5cm \) and \( e_z = 4.5cm \). Firstly, we study the influence of the variation of the section on the forced nonlinear vibrations behavior. To do this, we take three values \( \alpha_1 = 0^\circ, \alpha_2 = 32.0054^\circ \) and \( \alpha_3 = 48.3665^\circ \) with the corresponding heights \( h_{A1} = 9cm, h_{A2} = 14cm, h_{A3} = 18cm \) and \( h_B = 9cm \). The nonlinear dynamic analysis is made in the time range \([0, 1s]\) with a time step \( \Delta t = 10^{-3}s \). The problem have been solved by the high order implicit algorithm with a truncation order \( p = 12 \) and a tolerance parameter \( \varepsilon = 10^{-6} \). The obtained results are compared with those obtained by the industrial Abaqus code. Indeed this comparison shows that when the ratio \( r_i \) increases the effect becomes important and the amplitude decreases. In addition, we notice that if the material used in the flange is more rigid than the other used in the web the amplitude decreases and the beam becomes more rigid and in contrary if the material of the web is more rigid. The influence of composite sections is remarkable, thus, the amplitude of the forced vibrations decreases in a considerable manner.

4 Conclusion

This work is dedied to investigate numerically the behavior dynamic of thin-walled composite beams with open variable cross section subjected to external arbitrary loadings by the high order implicit algorithm. The obtained results are compared with those calculated by the industrial Abaqus code. Indeed this comparison shows that these results are consistent and illustrate the performance of this algorithm.

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