Gravitational scalar–tensor theory

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Abstract
We consider a new form of gravity theories in which the action is written in terms of the Ricci scalar and its first and second derivatives. Despite the higher derivative nature of the action, the theory is ghost-free under an appropriate choice of the functional form of the Lagrangian. This model possesses 2 + 2 physical degrees of freedom, namely 2 scalar degrees and 2 tensor degrees. We exhaust all such theories with the Lagrangian of the form $f(R, \nabla R, \Box R)$, where $R$ is the Ricci scalar, and then we show some examples beyond this ansatz. In the course of the analysis, we prove the equivalence between these examples and a subclass of generalized bi-Galileon theories.

Keywords: modified gravity, scalar–tensor theory, dark energy

1. Introduction

Two major mysteries in modern cosmology are inflation and dark energy. Scalar–tensor theory is often invoked to describe those two phenomena (see e.g. for reviews [1, 2]). A conventional definition of scalar–tensor theory will be given as follows: theory of a scalar field (or fields) coupled to gravity.

On the other hand, there is an interesting mathematical tool, namely conformal (Weyl) transformation, which is the transformation of a metric where the metric is scaled by a spacetime-dependent factor, $\Omega(x)$ as $g_{\mu\nu} \rightarrow \Omega^2(x) g_{\mu\nu}$. It is widely known that, under a conformal transformation, a canonical scalar field theory coupled to the Einstein gravity is mapped to a so-called $f(R)$ theory where $R$ represents the Ricci scalar (for a review of $f(R)$...
theory see e.g. [3]). Under this transformation, the degree of freedom of a scalar field in the original theory is replaced by the functional degree of freedom of $f(R)$. This is the reason $f(R)$ theory is also dubbed as a scalar–tensor theory while at first glance, no apparent scalar field exists in the action.

There were significant recent developments in scalar–tensor gravity theories, for example, the rediscovery of Horndeski’s theory [4–6] which is the most general single scalar field theory with second-order differential equations with respect to the metric and the scalar field where only $2 + 1$ degrees of freedom can propagate. This theory is further extended keeping the same number of physical degrees of freedom as standard scalar–tensor theories or the Horndeski’s theory, while apparent higher derivative terms appear in the action in general [7–12] (for further investigations of those theories see also [13–21]). Then, a natural question will arise: ‘Can we reformulate those generalized scalar–tensor theories in terms of the metric and its derivatives only, without using a scalar field in the action?’

To investigate a class of such theories, we shall consider a theory of gravity whose action is composed of the Ricci scalar and its derivatives:

$$S = \frac{1}{\kappa^2} \int \! \! \! \text{d}^4x \sqrt{-g} f(R, (\nabla R)^2, \Box R),$$

(1.1)

where $(\nabla R)^2 = g^{\mu \nu} \nabla_\mu \nabla_\nu R$ and $\kappa^2 = 8\pi G$ represent the gravitational constant, while we suppress it in other equations for brevity, unless otherwise stated. Usually, those higher derivative terms associated with derivatives of the Ricci scalar introduce new and ghostly degrees of freedom through their higher derivative nature, which is commonly called ‘Ostrogradski instability’ [22–24]. However, as we show in this paper, this model does not suffer from such instabilities. Indeed, there exists a theory in which only healthy $2 + 2$ dynamical degrees of freedom can propagate without ghost or Ostrogradski instabilities under a particular and appropriate choice of the functional form of the Lagrangian. We then further extend the theory by including higher derivatives of the Ricci scalar. These theories can be understood as gravitational counterparts (dual) of standard multi-scalar–tensor theories composed of the metric and scalar fields. In fact, this is the reason we dub this theory as ‘Gravitational scalar–tensor theory’ because it is constructed only in terms of gravitational language, namely metric and its derivatives. To the best of our knowledge, to date, these kinds of derivative terms have not been intensively investigated in the literature [25, 26]. And hence this study can potentially open up a new direction in the study of gravity theory.

2. New gravitational scalar–tensor theory

Let us investigate the nature of a gravity theory described by the action of the form (1.1). In the case of the standard $f(R)$ gravity, we invoke an equivalent Lagrangian linear in $R$ instead of a non-linear function of $R$ by introducing a Lagrange multiplier and the associated auxiliary field:

$$\int \! \! \! \text{d}^4x \sqrt{-g} f(R) = \int \! \! \! \text{d}^4x \sqrt{-g} [f(\phi) - \lambda(\phi - R)].$$

(2.1)

In the presence of derivatives of $R$, it is not trivial whether we can simultaneously replace derivatives of $R$ with the corresponding derivatives of $\phi$. Interestingly, we can do so as it is verified here. In the case of $f(R, (\nabla R)^2, \Box R)$, first let us introduce a set of Lagrange multipliers $(\lambda, \lambda_1, \lambda_2)$ and the associated auxiliary fields $(\phi, X, B)$ to reduce the order of derivatives:
\[
S = \int d^4x \, \sqrt{-g} \left[ f(\phi, X, B) - \tilde{\lambda}(\phi - R) - \tilde{\Lambda}_1 (X - (\nabla R)^2) - \tilde{\Lambda}_2 (B - \Box R) \right]. \tag{2.2}
\]

Equivalently, one can rewrite this action as
\[
S = \int d^4x \, \sqrt{-g} \left[ f(\phi, X, B) - \lambda(\phi - R) - \Lambda_1 (X - (\nabla \phi)^2) - \Lambda_2 (B - \Box \phi) \right], \tag{2.3}
\]
where \( \lambda = \tilde{\lambda} - \nabla^\mu [\tilde{\Lambda}_1 \nabla_\mu (\phi + R)] + \Box \tilde{\Lambda}_2, \) \( \Lambda_1 = \tilde{\Lambda}_1, \) \( \Lambda_2 = \tilde{\Lambda}_2. \) This replacement is justified since the transformation from \((\tilde{\lambda}, \tilde{\Lambda}_1, \tilde{\Lambda}_2)\) to \((\lambda, \Lambda_1, \Lambda_2)\) is regular and invertible. Since the action does not include any derivative terms of \( L_1 \) and \( L_2 \), variation of the action with respect to those variables yields constraint equations rather than dynamical equations of motion, which can be plugged back into the action without changing the nature of the theory:
\[
S = \int d^4x \, \sqrt{-g} \left[ f(\phi, (\nabla \phi)^2, \Box \phi) - \lambda(\phi - R) \right]. \tag{2.4}
\]

This verifies that we can indeed replace all the derivatives of \( R \) with those of \( \phi \) under the replacement of \( R \) by \( \phi \) and the introduction of the Lagrange multiplier. It should be noted that due to the presence of derivative terms of \( \phi \), the extremization of the action with respect to \( \phi \) gives a dynamical equation of \( \phi \) rather than a constraint equation, which cannot be plugged back into the action. And hence now \( \lambda \) should be treated as one of dynamical fields. This is in sharp contrast to the case of \( f(R) \) where \( \lambda \) can be determined by \( f \), that is \( \lambda = df(\phi)/d\phi \).

Now, higher derivative terms except for \( \lambda R \) only come from \( \Box \phi \). In order to reduce the order of derivatives, it is convenient to (re)introduce a Lagrange multiplier and the associated auxiliary field as
\[
S = \int d^4x \, \sqrt{-g} \left[ f(\phi, (\nabla \phi)^2, B) - \lambda(\phi - R) - \Lambda (B - \Box \phi) \right]. \tag{2.5}
\]
Given the fact that the action does not depend on the derivative of \( B \), variation of the action with respect to \( B \) gives a constraint equation,
\[
\Lambda = f_B. \tag{2.6}
\]
Hereafter, subscript \( B \) to \( f \) denotes a partial derivative with respect to it. The constraint \( (2.6) \) enables us to eliminate \( \Lambda \) in the action by plugging it back into the action, provided that
\[
f_{BB} = 0. \tag{2.7}
\]
Under (and only under) this condition, substituting \( (2.6) \) to the action and then varying the resulting action with respect to \( B \) leads to the original relation \( B = \Box \phi \). We thus first consider the case with \( (2.7) \).

The action except for the \( \lambda R \) term reduces to a first-order form after integration by parts,
\[
S = \int d^4x \, \sqrt{-g} \left[ (\lambda R - g^{\mu\nu} \partial_\mu f_B \partial_\nu \phi + f - f_B B - \lambda \phi) \right]. \tag{2.8}
\]
If \( f_{BB} = 0 \) then we can define a new variable \( \varphi \) as
\[
\varphi \equiv f_B, \tag{2.9}
\]
and consider \((g_{\mu\nu}, \lambda, \phi, \varphi)\) as basic variables. This is justified since the transformation from \((g_{\mu\nu}, \lambda, \phi, B)\) to \((g_{\mu\nu}, \lambda, \phi, \varphi)\) is locally invertible because of non-vanishing \( f_{BB} \). Moreover, it is convenient to move to the Einstein frame via a conformal transformation. Under the assumption that tensor modes have positive kinetic terms, i.e.
\[
\lambda > 0, \tag{2.10}
\]
after a conformal transformation such that \( g_{\mu\nu} = \Omega^2 \delta_{\mu\nu} \) with \( \Omega^{-2} = 2\lambda \) the action can be cast into the form of the action of multi-scalar fields coupled to the Einstein gravity:
Here, we have introduced a new variable $\chi$ via

$$\lambda \equiv e^{\sqrt{g} \chi},$$

which has a canonically normalized kinetic term. It is easy to see that the kinetic matrix has one negative eigenvalue, meaning the existence of a ghost. This is a consequence of the cross term $\nabla^\mu \phi \nabla_\mu $ in the absence of $\nabla_\mu \phi \nabla^\mu \phi$. On the other hand, $\chi$ has the standard kinetic term $\nabla^\mu \phi \nabla_\mu $ and no cross term. Because of this structure, the $3 \times 3$ kinetic matrix has two positive eigenvalues and one negative eigenvalue. Therefore, irrespective of the functional form of $f$, there is one ghostly mode owing to this structure of the Lagrangian.

Here, comments are in order. Since now $\lambda$ and correspondingly $\chi$ are dynamical fields, strictly speaking, it is not a priori guaranteed that (2.10) is satisfied all the time through the evolution of the system. However, in fact, if the condition is initially satisfied, it always holds at least in the Einstein frame. Since now $\lambda$ is one of dynamical fields, one can always choose initial conditions such that this condition is satisfied. Then, as is clear from (2.12), $\lambda = 0$ corresponds to infinity in the moduli space where $\chi$ approaches negative infinity and hence the condition (2.10) will never be violated at least with the finite lapse of time (in the Einstein frame).

Next, let us investigate an interesting case with $f_{BB} = 0$. In this case, since $B$ or correspondingly $\Box R$ linearly enters in the Lagrangian, we can rewrite $f$ in a simpler manner as

$$f(R, (\nabla R)^2, \Box R) = \mathcal{K}(R, (\nabla R)^2) + \mathcal{G}(R, (\nabla R)^2) \Box R. \quad (2.13)$$

Here, it should be noted that if $\mathcal{G}$ is a function of $R$ solely, the second term can be absorbed into the first one after the integration by parts

$$\mathcal{K}(R, (\nabla R)^2) + \mathcal{G}(R) \Box R = \mathcal{K}(R, (\nabla R)^2) + (\text{tot. der.}), \quad \mathcal{K}(R, (\nabla R)^2) \equiv \mathcal{K}(R, (\nabla R)^2) - \mathcal{G}'(R)(\nabla R)^2. \quad (2.14)$$

Hence, this case is equivalent to the case with $\mathcal{G} = 0$. In any case, irrespective of whether $\mathcal{G}$ depends on $(\nabla R)^2$ or not, after following the same steps as before, one obtains the following action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} \tilde{g}^{\mu \nu} \left( \partial_\mu \chi \partial_\nu \chi + \frac{2}{3} e^{2\sqrt{\tilde{g}} \chi} G \partial_\mu \chi \partial_\nu \phi \right) \right. $$

$$+ \left. \frac{1}{4} e^{-2\sqrt{\tilde{g}} \chi} G \Box \phi + \frac{1}{4} e^{-2\sqrt{\tilde{g}} \chi} K - \frac{1}{4} e^{-2\sqrt{\tilde{g}} \chi} \phi \right], \quad (2.15)$$

where it is understood that

$$K = K(\phi, 2 e^{\sqrt{\tilde{g}} \chi} \tilde{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi), \quad G = G(\phi, 2 e^{\sqrt{\tilde{g}} \chi} \tilde{g}^{\mu \nu} \partial_\mu \phi \partial_\nu \phi). \quad (2.16)$$

One of the most remarkable differences from the previous case is that now both $\phi$ and $\chi$ fields can have healthy kinetic terms at least by properly choosing the functional form of $K$ and $G$. Although there is a cross term, $\nabla_\mu \chi \nabla^\mu \phi$, in the presence of non-vanishing $G$, this term does not necessarily cause a problem since both $K$ and $G$ depend on $\nabla_\mu \phi \nabla^\mu \phi$. Due to this crucial difference in the structure of the Lagrangian, this case will end up with a healthy theory.
without Ostrogradski or ghost instabilities, which is actually confirmed by perturbative analysis around some background e.g. a cosmological one. In the absence of $\mathcal{G}$ as well as in the case of (2.14), the cross term, $\nabla_{\mu} \nabla_{\nu} \phi$ disappears and hence this case will also be healthy since both $\phi$ and $\chi$ fields can have healthy kinetic terms under an appropriate choice of the functional form of $\mathcal{K}$.

As an example, let us consider the simple case with $\mathcal{K} = -(\nabla R)^2/2$ and $\mathcal{G} = 0$. In this case, the equivalent action (2.15) is reduced to

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \kappa^2 R - \frac{1}{2} g^\mu_\nu \partial_\mu \chi \partial_\nu \chi 
- \frac{1}{4} e^{-\sqrt{\frac{G}{k^2}}} g^{\mu_\nu} \partial_\mu \phi \partial_\nu \phi 
- \frac{1}{4} \kappa e^{-\sqrt{\frac{G}{k^2}}} \phi \right], \quad (2.17)$$

where we have recovered the gravitational constant for clarity and $\tilde{\chi}$ and $\tilde{\phi}$ are introduced by

$$\chi \equiv \kappa \tilde{\chi}, \quad \phi \equiv \kappa \tilde{\phi}. \quad (2.18)$$

This system is simply the Einstein gravity with the two minimally-coupled scalar fields $\chi$ and $\phi$, whose $2 \times 2$ kinetic matrix is manifestly positive definite. Therefore, it is obvious that there is no ghost in the system, at least at the time and length scales sufficiently shorter than the curvature radius [27].

3. Further extensions

In the previous section, we have investigated theories whose action is of the form (1.1) and showed that the theory can be free from ghost only if $f$ is of the form (2.13). This exhausts all possible theories with $2 + 2$ physical degrees of freedom without ghost if we start with the ansatz (1.1) for the action. In this section, we shall further extend the theory so as to include more higher derivative terms. We shall not provide a systematic search. Instead, we shall show some explicit examples of such theories.

In order to keep the same number of degrees of freedom as in the previous section, we implement the structure of the generalized Galileon theory, or Horndeski’s theory. This suggests that the $\Box R^2$ term be accompanied by the term $(\nabla_R, \nabla_R)^2 = (\nabla_\mu, \nabla_\mu)(\nabla^\mu, \nabla^\mu)$. Then, in terms of three arbitrary functions of $R$ and $(\nabla_R)^2$, $(\mathcal{K}, \mathcal{G}, \mathcal{Q})$, let us consider the action of the form $S = \int d^4x \sqrt{-g} f$, where

$$f = \mathcal{K}(R, (\nabla_R)^2) + \mathcal{G}(R, (\nabla_R)^2) \Box R + \mathcal{Q}(R, (\nabla_R)^2) R 
- 2 \frac{\partial \mathcal{Q}}{\partial (\nabla_R)^2} (R, (\nabla_R)^2) \Box (\nabla_R)^2 - (\nabla_\mu, \nabla_\mu)^2. \quad (3.1)$$

By introducing a Lagrange multiplier and an auxiliary field to partly replace $R$, the action is rewritten as

$$S = \int d^4x \sqrt{-g} \{ \mathcal{K}(\phi, X) + \mathcal{G}(\phi, X) \Box \phi + \mathcal{Q}(\phi, X) R 
- 2 \frac{\partial \mathcal{Q}}{\partial X} (\phi, X) (\Box \phi)^2 - (\nabla \nabla \phi)^2 \} - \lambda (\phi - R), \quad (3.2)$$

where $X = (\nabla \phi)^2$. This is a special case of the generalized bi-Galileons [28–30]. Therefore, the number of physical degrees of freedom in this theory is $2 + 2$.

Next, let us add a term corresponding to the so-called quintic Horndeski term. For this purpose, it is convenient to rewrite $G_{\mu\nu} \nabla^\mu \nabla^\nu R$ in terms of $R$ and its derivatives, where $G_{\mu\nu}$ is
the Einstein tensor. First, by the definition of the Riemann tensor we have

$$(\nabla_\alpha \nabla_\gamma - \nabla_\gamma \nabla_\alpha) \nabla_\alpha \psi = R^\delta_{\alpha \beta \gamma} \nabla_\delta \psi,$$

(3.3)

for an arbitrary scalar $\psi$. By contracting this with inverse metric $g^{\alpha \gamma}$ and taking its divergence, one obtains

$$\nabla^\alpha (\nabla_\alpha \Box - \Box \nabla_\alpha) \psi = R_{\alpha \beta} \nabla^\alpha \nabla^\beta \psi + (\Box R)_{\alpha \beta} \nabla^\alpha \nabla^\beta \psi = R_{\alpha \beta} \nabla^\alpha \nabla^\beta \psi + \frac{1}{2} \nabla_\alpha R \nabla^\alpha \psi,$$

(3.4)

where in the last equality we have utilized the Bianchi identity. Finally, by replacing $\psi$ with $R$ in the above equation, one arrives at the identity of the form

$$G_{\mu \nu} \nabla^\mu \nabla^\nu R = \nabla^\alpha (\nabla_\alpha \Box - \Box \nabla_\alpha) R - \frac{1}{2} (\nabla R)^2 - \frac{1}{2} R \Box R.$$  

(3.5)

Hence, one can add the following term to (3.1).

$$\mathcal{C}(R, (\nabla R)^2) \left[ \nabla^\alpha (\nabla_\alpha \Box - \Box \nabla_\alpha) R - \frac{1}{2} (\nabla R)^2 - \frac{1}{2} R \Box R \right] + \frac{1}{3} \frac{\partial \mathcal{C}}{\partial (\nabla R)^2} (R, (\nabla R)^2) \left[ (\Box R)^3 - 3 (\Box R) (\nabla_\mu \nabla_\nu R)^2 + 2 (\nabla_\mu \nabla_\nu R)^3 \right],$$

(3.6)

where $(\nabla_\alpha \nabla_\beta R) = (\nabla_\alpha \nabla_\beta) (\nabla^\mu \nabla^\nu R)(\nabla_\mu \nabla_\nu R)$. By introducing the auxiliary field $\phi$ and the Lagrange multiplier $\lambda$, the action is rewritten as

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{K}(\phi, X) + \mathcal{G}(\phi, X) \Box \phi + \mathcal{Q}(\phi, X) R 
- 2 \frac{\partial \mathcal{Q}}{\partial X} (\phi, X) \left[ (\Box \phi)^2 - (\nabla \nabla \phi)^2 \right] - \lambda (\phi - R) 
+ \mathcal{C}(\phi, X) \left[ \nabla^\alpha (\nabla_\alpha \Box - \Box \nabla_\alpha) \phi - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} \phi \Box \phi \right] 
+ \frac{1}{3} \frac{\partial \mathcal{C}}{\partial X} (\phi, X) \left[ (\Box \phi)^3 - 3 (\Box \phi) (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3 \right] \right\},$$  

(3.7)

where $(\nabla_\alpha \nabla_\beta \phi)^3 = (\nabla_\alpha \nabla_\beta \phi)^3 (\nabla^\mu \nabla^\nu \phi)(\nabla_\mu \nabla_\nu \phi)$. Again, this is a special case of the generalized bi-Galileons [28–30] and thus the number of physical degrees of freedom in this theory is $2 + 2$.

A multi-field extension of yet another generalization performed in [7, 8] can also be implemented in a similar way.

4. Summary and discussion

We have considered a new form of scalar–tensor theory where the action is composed of Ricci scalar and its first and second derivatives, which is dubbed as ‘Gravitational scalar–tensor theory’ because it can be written only in terms of gravitational language, that is metric and its derivatives. Surprisingly, despite the higher derivative nature of the Lagrangian, the theory is healthy in the sense that there is no ghost or Ostrogradski instabilities under an appropriate choice of the functional form of the Lagrangian. This model only possesses $2 + 2$ degrees of freedom, namely 2 for scalar degrees of freedom and 2 for tensors. We have also extended the theory so as to include more higher derivative terms, while keeping the same number of degrees of freedom as before and discussed the relation to the multi-field extension of Horndeski’s theory.
In the present paper, we have focused on the inclusion of the Ricci scalar and its derivatives only. It will also be interesting to include derivatives of Riemann tensor to extend the so-called $f$ (Riemann) theory [31].

Finally, we discuss matter, particularly its coupling to gravity. While we have only focused on the gravitational sector, the matter sector must be introduced in reality. In principle, matter field can couple to either the original metric or the conformally related metrics. However, since the action of the gravitational sector of the theory is written in terms of the curvature and its derivatives, we consider it natural to couple matter field to the original metric $g_{\mu\nu}$, i.e. the Jordan frame metric. In this case, the conformal transformation to the Einstein frame, if it exists, affects the matter action. In particular, the gravitational force is mediated by not only the Einstein frame metric but the two scalar fields in general. Also, the behavior of the two scalar fields in the gravity sector may depend on the environment provided by the matter fields. The phenomenology of this modified gravity theory is thus worthwhile investigating in more detail.

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