Event horizons and holography

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Abstract

We consider the microcanonical ensemble of black holes in gravitational theories in asymptotically anti-de Sitter spacetime with a conformal field theory dual. We argue that typical black hole states show no violations of general covariance on the horizon.

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I. INTRODUCTION

For black holes in asymptotically anti-de Sitter spacetime there are two natural choices of vacua compatible with the symmetries. One such vacuum is the analog of the Boulware vacuum [1]: positive frequency field modes far from the black hole, defined with respect to the timelike Killing vector, annihilate the vacuum state. Another natural choice is the analog of the Hartle–Hawking vacuum [2], where positive frequency is defined with respect to time translations of smooth global slices. This choice of vacuum gives rise to entanglement between the left and right asymptotic regions of the maximally extended AdS–Schwarzschild Penrose diagram.

Such geometric states can be viewed either in an entangled tensor-product of two CFTs associated with the two asymptotic regions, or they may be viewed as a density matrix in a single CFT. In the following, we will restrict our attention to the second option.

One immediately runs into an issue at the semiclassical level: a normalizable scalar field mode in Schwarzschild anti-de Sitter contains both an ingoing and an outgoing flux at the horizon. The outgoing flux produces a divergent energy density as seen by an infalling observer crossing the future horizon. At first sight, this seems to imply that the typical finite energy states will be singular geometries, and moreover the horizon will appear singular as it is approached from the outside.

Analyzing this problem from the point of view of the dual quantum field theory in the microcanonical ensemble, Marolf–Polchinski (MP) [3] studied the number operator for Kruskal-like modes, those natural from the viewpoint of a freely falling observer. They argued in any eigenstate of such normalizable modes, this number operator would always be of order 1. This then implies that typical black holes always have violations of general covariance near the horizon.

II. SEMI-CLASSICAL APPROACH

Let us re-examine this argument in more detail. The $b$-modes are eigenstates of the timelike Killing vector at infinity, which is closely associated with the CFT Hamiltonian. They argued that these modes provide a complete basis for the energy eigenstates, along with a set of possible other labels/modes. This is certainly correct in the semiclassical limit,
where one quantizes the modes around the black hole geometry, ignoring issues of back-reaction. The Penrose diagram for the anti-de Sitter Schwarzschild black hole is shown in figure 1.

A bulk scalar field in the right patch (R) may be decomposed as

$$\phi = \sum_k b_{R,k} \phi_{R,k} + b_{R,k}^\dagger \phi_{R,k}^*$$

where $k$ schematically represents the set of labels. We refer to these modes as the $b$-modes, and they annihilate the Schwarzschild vacuum state $b_{R,k}|0\rangle_S = 0$.

If the CFT description is to include a description of the black hole interior, one must also consider a set of operators representing the fields in the left patch (L), which propagate into the upper patch in figure 1. Both sets of modes are needed to provide a complete description of the field in the upper patch. The full decomposition, valid in all coordinate patches is then

$$\phi = \sum_k b_{L,k} \phi_{L,k} + b_{L,k}^\dagger \phi_{L,k}^* + b_{R,k} \phi_{R,k} + b_{R,k}^\dagger \phi_{R,k}^*$$

and the Schwarzschild vacuum state is also annihilated by the left $b$-modes $b_{L,k}|0\rangle_S = 0$. Note that we define the mode functions, $\phi_{L/R}$, in the above such that they only have support in the appropriate (left/right) region.
One may also choose to decompose the field with respect to Kruskal modes, which are analytic across the horizon
\[
\phi = \sum_k a_k \phi_{K,k} + a_k^\dagger \phi_{K,k}^* .
\]
We refer to these modes as the \( a \)-modes. These modes annihilate the Kruskal vacuum \( a_k |0\rangle_K = 0 \). These modes look rather complicated when decomposed into frequencies with respect to the timelike Killing vector at infinity. However as shown in [4], these may be rewritten in terms of another set of operators \( d_{L,k} \) and \( d_{R,k} \) that annihilate \( |0\rangle_K \) but are simply related to the \( b \)-modes
\[
\phi = \sum_k (2 \sinh (\beta \omega_k / 2))^{-1/2} \left( d_{R,k} \left( e^{\beta \omega_k / 2} \phi_{R,k} + e^{-\beta \omega_k / 2} \phi_{L,-k}^* \right) + d_{L,k} \left( e^{-\beta \omega_k / 2} \phi_{R,-k}^* + e^{\beta \omega_k / 2} \phi_{L,k} \right) \right) + \text{h.c.}
\]
where \( \beta \) is the inverse Hawking temperature of the black hole and \( \omega_k \) is the positive frequency associated with the mode labeled by \( k \). These operators are related to the \( b \)-mode operators by a Bogoliubov transformation
\[
b_{L,k} = (2 \sinh (\beta \omega_k / 2))^{-1/2} \left( e^{\beta \omega_k / 2} d_{L,k} + e^{-\beta \omega_k / 2} d_{L,-k}^\dagger \right)
b_{R,k} = (2 \sinh (\beta \omega_k / 2))^{-1/2} \left( e^{\beta \omega_k / 2} d_{R,k} + e^{-\beta \omega_k / 2} d_{R,-k}^\dagger \right)
\]
which allows the different vacua to be related via
\[
|0\rangle_K = \prod_k \exp \left( e^{-\beta \omega_k / 2} b_{L,k}^\dagger b_{R,k}^\dagger \right) |0\rangle_S = \prod_k \sum_{n_k=0}^{\infty} e^{-\beta \omega_k n_k / 2} |N_{b,L,k} = n_k \rangle \times |N_{b,R,k} = n_k \rangle
\]
where \( N_{b,L,k} \) and \( N_{b,R,k} \) are the number operators for the \( b \)-modes. It is important to keep in mind the energy in terms of the \( b \)-mode eigenstates takes the form
\[
E = N_{b,L,k} \omega_k - N_{b,R,k} \omega_k
\]
so semiclassically any energy level has infinite degeneracy and the entropy in the microcanonical ensemble diverges. Obviously this computation ignores back-reaction, and should not be trusted. Likewise the finite Bekenstein–Hawking entropy of the black hole cannot be obtained in this kind of semiclassical quantum field theory in curved spacetime approximation.

The number operator relevant for an infalling observer can be defined as the number operator built from the Kruskal mode number operators
\[
N_{d,k} = d_{L,k}^\dagger d_{L,k} + d_{R,k}^\dagger d_{R,k}
\]
and this annihilates \(|0\rangle_K\). In fact, everything we say applies to each term in the above separately. We will utilize this expression momentarily.

### III. EUCLIDEAN QUANTUM GRAVITY APPROACH

This framework imposes periodicity in imaginary time to formulate the canonical ensemble. The microcanonical ensemble is then defined via a Laplace transform of the canonical ensemble. In the gravitational sector, the correct Bekenstein–Hawking black hole entropy is obtained, along with a contribution due to a thermal bulk field modes. This contribution can be viewed as computing the entropy of the reduced density matrix obtained by starting with the pure state \((1)\) and tracing over the left-modes. For sufficiently large total energies, the microcanonical ensemble is dominated by purely the black hole entropy contribution, with a negligible term coming from the bulk modes.

### IV. ADS/CFT APPROACH

There is strong evidence the CFT is able to correctly reproduce the Bekenstein–Hawking contribution to the entropy in the large mass limit. The entropy is reproduced up to an overall constant that is difficult to determine precisely, because the CFT is strongly coupled in the limit that it is dual to a gravitational phase.

This approach must also yield significant corrections to the approach of section II. Let us focus on the case of the four-dimensional bulk spacetime theory for the sake of definiteness. The boundary of the theory is \(S^2 \times \mathbb{R}\), with the \(\mathbb{R}\) factor corresponding to the time coordinate. Because the spatial sections are compact spheres, the energy spectrum of the conformal field theory becomes discrete. This induces a particular cutoff on the spectrum of the bulk theory.

The argument of MP proceeds by assuming that the \(b_R\)-mode number eigenstates provide approximate energy eigenstates in the exact theory dual to the CFT. Presumably they are also assuming the states also carry \(b_L\)-mode quantum numbers, as well as other possible labels. The exact bulk spectrum should be discretized in some way, to match that of the CFT. Within any such a number eigenstate, the expectation value of \((2)\) is greater than or equal to 1. This leads to a bound on the average of \(N_d\) in the microcanonical ensemble that is also greater than or equal to 1. One then concludes that general covariance
is violated. Moreover, each $b$-mode number eigenstate leads to a divergent stress energy tensor outside the horizon, as described in [7].

However the discussion of section II shows there is a counting problem, once one tries to diagonalize the $N_d$ operator using $b_R$-mode eigenstates. There are far more semiclassical modes than there are exact quantum states. It is therefore wishful thinking that the analog of the $N_d$ operator in the exact theory can be approximately diagonalized in terms of such eigenstates. The typicality argument of MP for firewall states therefore breaks down.

Therefore if one wants to take seriously the matching between the finite entropy of the microcanonical ensemble $S = S(M)$ and the finite entropy of the CFT, then the Hilbert space of the exact quantum description must be much smaller than the infinite dimensional Hilbert subspace one has in the semiclassical description, at fixed total energy.

Since this is ultimately determined by the properties of a strongly coupled conformal field theory and the bulk-boundary dictionary, for the moment we are free to postulate what properties the exact modes should have. In particular, we expect if a black hole is formed by sending matter in from the boundary of empty anti-de Sitter spacetime, that the bulk field states should be determined by following unitary evolution from the vacuum up into the black hole region. This implies a correlation between particular $b_R$-modes and $b_L$-modes. We further postulate that a good approximation to the energy eigenstates are actually the $a$-modes, equipped with a cutoff. In the presence of this cutoff, one cannot simply rediagonalize and use the $b$-mode number eigenstates. Expressed in terms of the $b$-modes, only very particular complex superpositions correspond to exterior bulk states in the exact theory, such as the vacuum state $|1\rangle$, and states obtained by acting with the operators $d_{L}^\dagger$ and $d_{R}^\dagger$.

It should be noted, that while the $d_{R}^\dagger$ operators contain a component inside the horizon of the black hole, this contribution is suppressed by a factor $e^{-\beta \omega_k}$ relative to the component outside. This is sufficient for these modes to be used to define a local effective field theory outside the horizon, with some proper distance cutoff scale in the bulk close to the Planck length.

Another argument in favor of using the $d_{L}^\dagger$ and $d_{R}^\dagger$ modes comes from the lattice black hole analysis of [8]. There it was found that a short distance regulator appropriate for a freely falling frame, does not commute with the Hamiltonian associated with the timelike Killing vector. Thus $b$-mode eigenstates are not preserved under time evolution. The $d$-modes avoid
this issue, by including entanglement between the interior and exterior. Tracing over the interior component then leads to an approximately thermal density matrix for the $b$-modes, which is indeed preserved under time evolution.

V. CONCLUSIONS

We have argued that the result of MP [3], that a typical black hole in a theory with a dual gauge theory sees a violation of general covariance at the horizon, relies on expanding the Hilbert space well-beyond that of the exact theory. Because the exact states will only be close to very special states in this expanded Hilbert space, the typicality statement fails. We have given examples of sequences of states in the microcanonical ensemble where general covariance is expected to hold near the horizon, and conjecture that such states provide a good approximation to the exact states of the quantum theory.

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