Robustness and scalability of p-bits implemented with low energy barrier nanomagnets

Justine L. Drobitch and Supriyo Bandyopadhyay
Department of Electrical and Computer Engineering
Virginia Commonwealth University, Richmond, VA 23284, USA

Abstract
Probabilistic (p-) bits implemented with low energy barrier nanomagnets (LBMs) have recently gained attention because they can be leveraged to perform some computational tasks very efficiently. Although more error-resilient than Boolean computing, p-bit based computing employing LBMs is, however, not completely immune to defects and device-to-device variations. In some tasks (e.g. binary stochastic neurons for machine learning and p-bits for population coding), extended defects, such as variation of the LBM thickness over a significant fraction of the surface, can impair functionality. In this paper, we have examined if unavoidable geometric device-to-device variations can have a significant effect on one of the most critical requirements for probabilistic computing, namely the ability to “program” probability with an external agent. We found that the programming ability is fortunately not lost due to reasonable device-to-device variations. This shows that probabilistic computing with LBMs is robust against small geometric variations, and hence will be “scalable”.
I. INTRODUCTION

Probabilistic p-bits are random bits which fluctuate between 0 and 1 [1]. They are neither deterministic bits used in classical Boolean logic, nor qubits which are coherent superposition of 0 and 1. Probabilistic computing with p-bits encoded in the magnetization states of low energy barrier nanomagnets (LBMs) is extremely energy-efficient and far more error-resilient than energy-efficient Boolean computing with nanomagnets, which is normally very error-prone [2]. Computing with p-bits has also been shown to excel in certain tasks such as combinatorial optimization [3], invertible logic [4] and integer factorization [5].

A popular platform for implementing p-bits is a low barrier nanomagnet (LBM) with two degenerate energy minima separated by a low energy barrier on the order of the thermal energy kT [1]. In such a nanomagnet, the magnetization will fluctuate between the two orientations corresponding to the two degenerate energy minima because of thermal fluctuations. If we take a snapshot of the magnetization at any instant of time, it will point in a random direction. However, if its component along one of the two orientations is positive, then we will interpret the magnetization to represent the bit 1, while if it is negative, we will interpret it as bit 0. The bit will thus always fluctuate between 0 and 1 (sometimes 0 and sometimes 1) and act as a p-bit.

If the energy barrier is symmetric between the two degenerate minima, then bits 0 and 1 will be generated with equal probability. However, one can change that by passing a spin polarized current through the nanomagnet with spins polarized along one of the two orientations. This will bias the probability, either toward 0 or toward 1, depending on the current’s magnitude and spin polarization (say, for example, 30% probability of 0 and 70% of 1 for a current of magnitude 1 A with spins polarized in the direction representing bit 1). Such an approach provides a means to “program” the probability, which is the basis of probabilistic computing. It is also the basis of binary stochastic neurons frequently used in stochastic neural networks and machine learning.

The programmability (or “control”) will be lost if the magnitude of the current needed for a particular probability distribution (e.g. 30% for 0 and 70% for 1) varies significantly from one nanomagnet to another because of small variations in the nanomagnet’s lateral dimensions or thickness. This will be debilitating for probabilistic computing and, at best, limit the number of p-bits that can be harnessed to build a “p-circuit”, thereby making p-bits suffer from similar limitations on scalability that afflict qubits. It is this problem that we study. In the past, we have shown that extended defects in an LBM (e.g. thickness variation over a significant fraction of the surface) will radically alter the auto-correlation function of the magnetization fluctuation in time [6] and the fluctuation rate [7], which will, respectively, affect applications in, say, binary stochastic neurons for machine learning [8] and population coding [9]. However, these are less serious than losing control over the probability because the latter is crucial to probabilistic computing. Therefore, it is critical to examine the effect of device-to-device variations caused by fabrication imperfections on the ability to control probability in LBMs.

To investigate this issue, we have carried out stochastic Landau-Lifshitz-Gilbert simulations to study magnetization fluctuations in an LBM (with in-plane magnetic anisotropy) at room temperature in the presence of a spin polarized current injected perpendicular to the plane of the LBM. These simulations allow us to generate the probability of bit 1 (encoded in the magnetization state of the LBM) as a function

Corresponding author: Supriyo Bandyopadhyay (sbandy@vcu.edu).
of the spin polarized current magnitude and polarization, and examine how this probability function varies with small variations in the nanomagnet’s lateral dimensions and thickness. Our results show that the probability function is insensitive to reasonable variations. This is reassuring since it establishes that probabilistic computing with p-bits is not impaired by reasonable device-to-device variation and hence a large number of p-bits can be harnessed for p-circuits, meaning that p-bits are largely scalable.

II. SIMULATIONS

We consider an elliptical cobalt nanomagnet of nominal thickness 6 nm, major axis 100 nm and minor axis 99.7 nm (Fig. 1). This nanomagnet has in-plane magnetic anisotropy and because it has very small eccentricity (nearly circular), the shape anisotropy energy barrier separating the two stable orientations along the major axis (easy axis) is only 1.3 kT at room temperature. We follow the time evolution of the magnetization in this nanomagnet in the presence of thermal noise and a spin-polarized current injected perpendicular to plane with spin polarization along the major axis by solving the stochastic Landau-Lifshitz-Gilbert equation:

\[
\frac{d\vec{m}(t)}{dt} = -\gamma \vec{m}(t) \times \vec{H}_\text{total}(t) + \alpha \left( \vec{m}(t) \times \frac{d\vec{m}(t)}{dt} \right) \\
+ a\vec{m}(t) \times \left( \eta \vec{I}_s(t) \mu_\parallel \times \vec{m}(t) \right) + b \frac{\eta \vec{I}_s(t) \mu_\parallel}{qM_s\Omega} \times \vec{m}(t) 
\]

where

\[
\vec{m}(t) = m_x(t) \hat{x} + m_y(t) \hat{y} + m_z(t) \hat{z} \\
\vec{H}_\text{total} = \vec{H}_\text{demag} + \vec{H}_\text{thermal} \\
\vec{H}_\text{demag} = -M_s N_{d-xx} m_x(t) \hat{x} - M_s N_{d-yy} m_y(t) \hat{y} - M_s N_{d-zz} m_z(t) \hat{z} \\
\vec{H}_\text{thermal} = \frac{2\alpha kT}{\gamma (1 + \alpha^2) \mu_0 M_s \Omega (\Delta t)} \left[ G_{(0,2)}^x(t) \hat{x} + G_{(0,2)}^y(t) \hat{y} + G_{(0,2)}^z(t) \hat{z} \right]
\]

Fig. 1: (a) A slightly elliptical nanomagnet with in-plane magnetic anisotropy into which a spin-polarized current is injected perpendicular-to-plane. The spin polarization is along the major axis. The nanomagnet’s dimensions are: major axis = 100 nm, minor axis = 99.7 nm and thickness = 6 nm. (b) The potential energy profile as a function of the in-plane magnetization orientation.
The last term in the right hand side of Equation (1) is the field-like spin transfer torque and the second to last term is the Slonczewski torque. The coefficients $a$ and $b$ depend on device configurations and following [10], we will use the values $a = 1$, $b = 0.3$. Here $\dot{m}(t)$ is the time-varying magnetization vector in the nanomagnet normalized to unity, $m_x(t)$, $m_y(t)$ and $m_z(t)$ are its time-varying components along the x-, y- and z-axis, $\hat{H}_{demag}$ is the demagnetizing field in the soft layer due to shape anisotropy and $\hat{H}_{rand}$ is the random magnetic field due to thermal noise. The different parameters in Equation (1) are: $\gamma = 2\mu_0\mu_0/h$ (gyromagnetic ratio), $\alpha$ is the Gilbert damping constant, $\mu_0$ is the magnetic permeability of free space, $M_s$ is the saturation magnetization of the magnetostrictive soft layer, $kT$ is the thermal energy, $\Omega$ is the volume of the nanomagnet given by $\Omega = (\pi/4)a_1a_2a_3$, $a_1 =$ major axis, $a_2 =$ minor axis, and $a_3 =$ thickness, $\Delta t$ is the time step used in the simulation (0.1 ps), and $G^{x}_{(a)}(t)$, $G^{y}_{(a)}(t)$ and $G^{z}_{(a)}(t)$ are three uncorrelated Gaussians with zero mean and unit standard deviation [11]. The quantities $N_{d_{xx}}, N_{d_{yy}}, N_{d_{zz}} \left[N_{d_{xx}} + N_{d_{yy}} + N_{d_{zz}} = 1\right]$ are calculated from the dimensions of the nanomagnet following the prescription of ref. [12]. We assume that the charge current injected into the nanomagnet is $\dot{I}_{z}(t)$ and that the spin polarization in the current is $\eta$. The spin current is given by $\eta\dot{I}_{z}(t) = \eta\dot{I}_{x}(t)\hat{z}$ where $\hat{z}$ is the unit vector along the major axis as shown in Fig. 1. The various parameters for the simulation are given in Table I.

**Table I: Parameters used in the simulations**

| Parameters                | Values          |
|---------------------------|-----------------|
| Saturation magnetization ($M_s$) | $1.1 \times 10^6$ A/m |
| Gilbert damping ($\alpha$)       | 0.01           |
| Temperature (T)            | 300 K          |
| Spin polarization ($\eta$)  | 0.3, 0.7       |
| Major axis ($a_1$)         | 100 nm         |

We start the simulation for any given magnitude and polarization of the spin polarized current with the initial value $m_x(0) = m_y(0) = 0$; $m_z(0) = 1$, i.e. the magnetization is initially pointing in one direction along the minor axis. We run the simulation for 10 ps and note the final value of $m_z$. If it is positive, then we interpret the magnetization state to represent the bit 1, while if it is negative, we interpret it as bit 0. One would measure the $m_z$ component with a magneto-tunneling junction (MTJ) whose hard layer is magnetized
in one direction along the z-axis, and hence the resistance of the MTJ will be a measure the $m_z$ component. The resistance, of course, will not be binary and vary continuously between the high and low values since $m_z$ component will vary continuously between -1 and +1. Hence, a threshold function is used in probabilistic computing to interpret all positive $m_z$ component as bit 1 and all negative component as bit 0.

Fig. 2: The probability of bit 1 as a function of spin polarized current for three different nanomagnet thicknesses of 5, 6 and 7 nm. The major axis is 100 nm and the minor axis is 99.7 nm. The results are plotted for two different degrees of spin polarization in the current: 30% and 70%. The variation in the probability at any given current is reduced at higher spin polarization. Positive value of the current corresponds to spin polarization in the +z-direction and negative values correspond to spin polarization in the –z-direction.
III. RESULTS

We run 10,000 simulations of the magnetization dynamics for each value of spin polarized current (in steps of 0.1 mA) and calculate the fraction of simulations where the final state after 10 ps represents the bit 1. That fraction is the probability of bit 1 or \( P(1) \). If we had monitored the bit as a function of time, this would have been the probability of observing the bit as 1, based on ergodicity. Obviously, \( P(0) \) is always 1 – \( P(1) \). In Fig. 2, we show \( P(1) \) as a function of the magnitude and spin polarization of the spin polarized current for three different nanomagnet thicknesses of 5 nm, 6 nm and 7 nm. These nanomagnets vary in thickness by 1 nm which is a reasonable variation since they are usually fabricated on substrates with surface roughness of 0.3 nm. Positive current corresponds to spin polarization along the +z-axis and negative current corresponds to polarization along the –z-axis. We plot the results for two different degrees of spin polarization \( \eta \) in the current: 30% and 70%.

![Fig. 3: The probability of bit 1 as a function of spin polarized current for three different nanomagnet minor axis dimensions of 98, 99 and 99.7 nm. The major axis dimension is fixed at 100 nm and the thickness is 6 nm. The results are plotted for two different degrees of spin polarization in the current: 30% and 70%. As in Fig. 2, the variation is reduced at higher spin polarization.](image-url)
In Fig. 3 we show $P(l)$ as a function of the magnitude and spin polarization of the spin polarized current for three different minor axis dimensions of 99.7, 99 and 98 nm (the major axis is fixed at 100 nm and the thickness is fixed at 6 nm). Again, we show the plots for two different degrees of spin polarization $\eta$ in the current: 30% and 70%.

IV. CONCLUSION

Clearly, the plots show that the probability curves are not affected much by variations in either thickness or lateral dimensions. Increasing degree of spin polarization in the current decreases the variation, along with (expectedly) the magnitude of the current needed to pin the bit to either 0 or 1. This is reassuring since it implies that the “control” over p-bits exercised with spin polarized current is not impaired by reasonable device-to-device variations and therefore a fairly large number of p-bits can be harnessed for “p-circuits” in many applications, i.e. p-bits are generally “scalable”. This is in sharp contrast to qubits where only a small number can be entangled for quantum operations (the largest number entangled so far appears to be 53 [13]) because of decoherence. Classical p-bits do not suffer from decoherence and their scalability does not appear to be severely limited by device-to-device variations either. Some specific applications may still be vulnerable to defects [5, 6], but the practicality of implementing p-bits with LBMs is unassailable.

ACKNOWLEDGMENT

This work was supported in part by the U.S. National Science Foundation under grants ECCS-1609303 and CCF-1815033.

REFERENCES

1) K. Y. Camsari, B. M. Sutton and S. Datta, “p-bits for probabilistic spin logic”, Appl. Phys. Rev., 6, 011305, 2019.
2) D. Winters, M. A. Abeed, S. Sahoo, A. Barman and S. Bandyopadhyay, “Reliability of magnetoelastic switching of nonideal nanomagnets with defects: A case study for the viability of straintronic logic and memory”, Phys. Rev. Appl., 11, 034010, 2019.
3) B. Sutton, K. Y. Camsari, B. Behin-Aein, and S. Datta, “Intrinsic optimization using stochastic nanomagnets,” Sci. Rep., 7, 44370, 2017.
4) K. Y. Camsari, R. Faria, B. M. Sutton, and S. Datta, “Stochastic p-bits for invertible logic,” Phys. Rev. X, 7, 031014, 2017.
5) W. A. Borders, A. Z. Pervaiz, S. Fukami, K. Y. Camsari, H. Ohno and S. Datta, “Integer factorization using stochastic magnetic tunnel junctions”, Nature, 573, 390, 2019.
6) M. A. Abeed and S. Bandyopadhyay. “Low energy barrier nanomagnet design for binary stochastic neurons: Design challenges for real nanomagnets with fabrication defects”, IEEE Magnetics Letters, 10, 4504405, 2019.
7) M. A. Abeed and S. Bandyopadhyay, “Sensitivity of the power spectra of thermal magnetization fluctuations in low barrier nanomagnets proposed for stochastic computing to in-plane barrier height variations and structural defects”, SPIN (in press).
8) O. Hassan, R. Faria, K. Y. Camsari, J. Z. Sun, and S. Datta, “Low barrier magnet design for efficient hardware binary stochastic neurons,” *IEEE Magnetics Lett.* **10**, 4502805, 2019.

9) A. Mizrahi, T. Hirtzlin, A. Fukushima, H. Kubota, S. Yuasa and J. Grollier, “Neural-like computing with populations of super-paramagnetic basis functions,” *Nature Commun.*, **9**, 1533, 2018.

10) K. Roy, S. Bandyopadhyay and J. Atulasimha, “Metastable state in a shape anisotropic single-domain nanomagnet subjected to spin-transfer-torque”, *Appl. Phys. Lett.*, **101**, 162405, 2012.

11) K. Roy, S. Bandyopadhyay and J. Atulasimha, “Energy dissipation and switching delay in stress-induced switching of multiferroic nanomagnets in the presence of thermal fluctuations”, *J. Appl. Phys.*, **112**, 023914, 2012.

12) S. Chikazumi, *Physics of Magnetism* (Wiley, New York, 1964).

13) A. Cho, “Google claims quantum computing milestone”, *Science*, **365**, 1364, 2019.