Universal Mass Texture and Quark-Lepton Complementarity

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Abstract. Recent measurements of the neutrino and quark mixing angles satisfy the empirical relations called quark-lepton complementarity. This empirical data suggests the existence of a correlation between the mixing matrices of leptons and quarks. In this work, we examine the possibility that this correlation between the mixing angles of quarks and leptons originates in the similar hierarchy of quarks and charged lepton masses and the seesaw mechanism type I that gives mass to the Majorana neutrinos. We assume that the similar mass hierarchies of charged lepton masses and quark masses allows one to represent all the mass matrices of Dirac fermions in terms of a four zeros Fritzsch texture.

1. Introduction

In the last few years, the neutrino oscillations between different flavour states were measured in a series of experiments with atmospheric neutrinos [1], solar neutrinos [2, 3], neutrinos produced in nuclear reactors [4] and accelerators [5]. As a result of the global combined analysis including all dominant and sub dominant oscillation effects, the difference of the squared neutrino masses and the mixing angles in the lepton mixing matrix, $U_{PMNS}$, were determined:

$$\Delta m^2_{21} = 7.67^{+0.67}_{-0.21} \times 10^{-5} \text{eV}^2, \quad \theta_{12}^l = 34.5^o \pm 1^o$$

$$\Delta m^2_{32} = \begin{cases} -2.37 \pm 0.15 \times 10^{-3} \text{eV}^2, & (m_{\nu_2} > m_{\nu_1} > m_{\nu_3}) \\ +2.46 \pm 0.15 \times 10^{-3} \text{eV}^2, & (m_{\nu_3} > m_{\nu_2} > m_{\nu_1}) \end{cases}$$

$$\theta_{23}^l = \left(42.3^{+5.1}_{-3.3}\right)^o, \quad \theta_{13}^l < 7.9^o$$

at 90% confidence level [6–8]. The CHOOZ experiment [9] determined an upper bound for the $\theta_{13}$ mixing angle. It was soon realized [10] that the solar mixing angle $\theta_{12}^l$ and the Cabibbo angle $\theta_{12}^q$, which is the corresponding angle in the quark sector, satisfy an interesting and intriguing numerical relation,

$$\theta_{12}^l + \theta_{12}^q = 45^0 + 1.5^o \pm 1.45^o,$$

with $\theta_{12}^q = 13^o \pm 0.05^o$. Equation (2) relates the 1-2 mixing angles in the quark and lepton sectors, it is commonly called the quark-lepton complementarity relation (QLC) and, if not accidental, it could imply a quark-lepton symmetry (for a recent review see [11]) or a quark-lepton unification [12–15].
A second QLC relation that relates the atmospheric and 2-3 mixing angles, is also satisfied.

$$\theta_{23}^l + \theta_{23}^q = \left(44.67^{+5.15}_{-3.35}\right)^{\circ}. \quad (3)$$

However, this is not as interesting as (2) because $$\theta_{23}^q$$ is only about two degrees, and the corresponding QLC relation would be satisfied, within the errors, even if the angle $$\theta_{23}^l$$ had been zero, as long as $$\theta_{23}^{MNS}$$ is close to the maximal value $$\pi/4$$.

A third possible QLC relation is not realized at all, or at least not realized in the same way, since is less than ten degrees.

$$\theta_{13}^l + \theta_{13}^q < 8.8^\circ. \quad (4)$$

In this short note we will focus our attention on understanding the nature of the QLC relation.

2. Universal Fritzsch texture of quarks and leptons

The quark and lepton flavour mixing matrices, $$U_{PMNS}$$ and $$V_{CKM}$$, arise from the mismatch between diagonalization of the mass matrices of $$u$$ and $$d$$ type quarks and the mismatch in the diagonalization of the mass matrices of charged leptons and left handed neutrinos,

$$U_{MNS} = U_l U_{\nu}^\dagger, \quad V_{CKM} = U_u U_d^\dagger. \quad (5)$$

Therefore, to get predictions for the flavour mixing angles and CP violating phases, we should specify the mass matrices.

In this work, we propose a unified treatment of quarks and leptons. Lepton and quark mass matrices could have the same mass texture from a universal flavour symmetry (exact at a certain energy scale). Imposing a flavour symmetry has been successful in reducing the number of parameters of the Standard Model. In particular, a permutational $$S_3$$ flavour symmetry and its sequential explicit breaking, allows us to represent the mass matrices as a modified Fritzsch texture:

$$M_i^{(F)} = \begin{pmatrix}
0 & A_i & 0 \\
A_i^* & B_i & C_i \\
0 & C_i & D_i
\end{pmatrix} \quad i = u, d, l, \nu. \quad (6)$$

Some reasons to propose the validity of the modified Fritzsch texture as a universal mass texture for all fermions in the theory are the following:

(i) The idea of $$S_3$$ flavour symmetry and its explicit breaking has been realized as a modified Fritzsch texture in the quark sector to interpret the strong mass hierarchy of up and down type quarks [16].

(ii) The quark mixing angles and the CP violating phase appearing in the $$V_{CKM}$$ mixing matrix were computed as explicit, exact functions of the four quark mass ratios $$(m_u/m_t, m_c/m_t, m_d/m_b, m_s/m_b)$$, one symmetry breaking parameter $$Z^{1/2} = \left(\frac{81}{32}\right)^{1/2}$$ and one CP violating phase $$\phi_{u-d} = 90^\circ$$, in very good agreement with experiment [17].

(iii) Since the mass spectrum of the charged leptons exhibits a similar hierarchy to the quark’s one, it would be natural to consider the same $$S_3$$ symmetry and its explicit breaking for the charged lepton mass matrix.

(iv) As for the Dirac neutrinos, we have no direct information about the absolute values or the relative values of the neutrino masses, but the Fritzsch texture can be incorporated in a $$SO(10)$$ neutrino model [18]. Therefore it would be sensible to assume that the Dirac neutrinos have a mass hierarchy similar to that of the u-quarks and it would be natural to take for the Dirac neutrino mass matrix also a modified Fritzsch texture.
3. Invariance of the Fritzsch Texture

The left handed Majorana neutrinos naturally acquire their mass through an effective seesaw mechanism type I of the form

\[ M_{\nu_L} = M_{\nu_L} M_R^{-1} M_{\nu_D}^T, \]  

(7)

where \( M_{\nu_D} \) and \( M_R \) denote the Dirac and right handed Majorana neutrino mass matrices. From our conjecture of a universal \( S_3 \) flavour symmetry it follows that \( M_R \) could have the same texture as that of \( M_{\nu_L} \) and \( M_D \). Then, it is straightforward to show that \( M_{\nu_L} \) has also the same modified Fritzsch texture [19].

The hermitian mass matrix (6) may be written in terms of a real symmetric matrix \( \bar{M}^{(F)} \) and a diagonal matrix of phases \( P \) as follows

\[ M^{(F)} = P^\dagger \bar{M}^{(F)} P. \]  

(8)

Then the seesaw mechanism type I takes the form

\[ M^{(F)}_{\nu_L} = M^{(F)}_{\nu_D}(M^{(F)}_{\nu_R})^{-1}M^{(F)T}_{\nu_D} = P^\dagger \bar{M}^{(F)}_{\nu_D} P_D \left( P^\dagger \bar{M}^{(F)}_{\nu_R} P_R \right)^{-1} \left( P^\dagger \bar{M}^{(F)}_{\nu_D} P_D \right)^T. \]  

(9)

The symmetry of \( M_{\nu_L}^{(F)} \) fixes the right handed Majorana neutrino phases, \( \phi_{\nu_R} = n\pi \). Thus the mass matrix of right handed Majorana neutrino is real and symmetric and has a Fritzsch texture.

From (9), it also follows that the Fritzsch mass texture is invariant under the operation of see-saw mechanism type I.

\[ M_{\nu_L} = \begin{pmatrix} 0 & a_{\nu_L} e^{in\pi} & 0 \\ a_{\nu_L} e^{in\pi} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix}, \]  

(10)

\[ a_{\nu_L} = \left| \frac{a_{\nu_R}}{c_{\nu_R}} \right|^2, \]

\[ b_{\nu_L} = \frac{c^2_{\nu_L} + c^{-2}_{\nu_L}}{d_{\nu_L}} + \frac{c^2_{\nu_L} - c^{-2}_{\nu_L}}{d_{\nu_L}} \left| \frac{a_{\nu_R}}{c_{\nu_R}} \right|^2 e^{i2\phi_{\nu_D}} + 2 \left( b_{\nu_D} - \frac{c_{\nu_D}}{d_{\nu_D}} \right) \frac{a_{\nu_R}}{c_{\nu_R}} \cos(\phi_{\nu_D} + n\pi), \]

\[ c_{\nu_L} = \frac{c_{\nu_D}}{d_{\nu_D}} e^{i2\phi_{\nu_D}} + \frac{a_{\nu_R}}{c_{\nu_R}} \left( \frac{c_{\nu_D}}{d_{\nu_D}} - \frac{c_{\nu_D}}{d_{\nu_D}} \right) e^{i(\phi_{\nu_D} + n\pi)}, \]

\[ d_{\nu_L} = \left| \frac{c_{\nu_R}}{d_{\nu_R}} \right|^2. \]

Since \( b_{\nu_L} \) and \( c_{\nu_L} \) are complex, in general, the left handed Majorana neutrino mass matrix, \( M_{\nu_L} \), is complex symmetric but not hermitian.

4. Mixing Matrices as Functions of the Fermion Masses

When the unitary matrices that diagonalize the mass matrices \( M_{i}^{(F)} \) \( (i = u, d, e) \) are written in polar form, \( U_i = O_i^T P_i \) and \( M_{i}^{(F)} = P_i^T \bar{M} I P_i \), the expressions (5) for the mixing matrices take the form

\[ U_{PMNS} = O_u^T P^{(l-u)} O_e K, \quad V_{CKM} = O_d^T P^{(u-d)} O_d, \]  

(11)

where \( O_i \) are the orthogonal matrices that diagonalize the real symmetric mass matrices \( \bar{M}_i^{(F)} \) and \( P^{(u-d)} = \text{diag} \left[ 1, e^{i\phi}, e^{i\Phi} \right], P^{(l-u)} = \text{diag} \left[ 1, e^{i\Phi}, e^{i\phi} \right] \), where \( \phi = \phi_u - \phi_d \), and \( \Phi = \Phi_t - \Phi_{\nu} \), are the matrices of the Dirac phases and \( K \) is the diagonal matrix of the Majorana phases.
We reparametrized the matrices $\tilde{M}_i$ in terms of their eigenvalues. The orthogonal matrices are then expressed in terms of the mass eigenvalues of $M_i$:

$$O_1 = \begin{pmatrix}
\tilde{m}_2 f_{11}^{\frac{1}{3}} & \tilde{m}_1 f_{12}^{\frac{1}{3}} & \tilde{m}_1 \tilde{m}_2 f_{13}^{\frac{1}{3}} \\
\tilde{m}_1 (1-\delta_i) f_{11}^{\frac{1}{3}} & \tilde{m}_1 (1-\delta_i) f_{12}^{\frac{1}{3}} & (1-\delta_i) f_{13}^{\frac{1}{3}} \\
-\tilde{m}_1 f_{21}^{\frac{1}{3}} & -\tilde{m}_1 f_{22}^{\frac{1}{3}} & f_{23}^{\frac{1}{3}}
\end{pmatrix},$$

(12)

$$f_{i1} = 1 - \tilde{m}_{i1} - \delta_i, \quad f_{i2} = 1 + \tilde{m}_{i2} - \delta_i, \quad f_{i3} = \delta_i,$$

(13)

$$D_{1i} = (1 - \delta_i)(\tilde{m}_{i1} + \tilde{m}_{i2})(1 - \tilde{m}_{i1}),$$

(14)

$$D_{2i} = (1 - \delta_i)(\tilde{m}_{i1} + \tilde{m}_{i2})(1 + \tilde{m}_{i2}),$$

(15)

$$D_{3i} = (1 - \delta_i)(1 - \tilde{m}_{i1})(1 + \tilde{m}_{i2}),$$

(16)

$$\tilde{m}_{i1,2} = \frac{m_{i1,2}}{m_{i3}},$$

(17)

the small parameters $\delta_i$ are also functions of the mass ratios and the symmetry breaking parameter $Z^{1/2}$.

Substitution of the expressions (12) and (13)-(16) in (11) allows us to express the mixing matrices $U_{PMNS}$ and $V_{CKM}$ as explicit functions of the masses of quark and lepton masses.

5. Quark-Lepton Complementarity

The resulting theoretical expression for the quark mixing angles written to first order in $m_u/m_c$ and $m_d/m_s$, is

$$\sin^2 \theta_{12}^q \approx \frac{\tilde{m}_d}{m_s} + \frac{\tilde{m}_u}{m_c} - 2 \frac{\tilde{m}_d \tilde{m}_u}{m_c m_s} \cos \phi,$$

(18)

$$\tan \theta_{23}^q \approx \left( \sqrt{\delta_d} - \sqrt{\delta_u} \right),$$

(19)

$$\tan \theta_{13}^q \approx \sqrt{\frac{\tilde{m}_u}{m_c} \left( \sqrt{\delta_d} - \sqrt{\delta_u} \right)}.$$

(20)

Taking for the quark masses the values $m_u = 2.75\text{MeV}$, $m_c = 1310\text{MeV}$, $m_d = 6.0\text{MeV}$, $m_s = 120\text{MeV}$ [20] and maximal CP violation, $\phi = 90^o$ [17], we reproduce the numerical value of the quark mixing angles

$$\theta_{12}^q = 12.8^o, \quad \theta_{23}^q = 1.4^o, \quad \theta_{13}^q = 0.07^o,$$

(21)

in very good agreement with the latest analysis of the experimental data [21].

The theoretical expression for the lepton mixing angles are derived in a similar way. We obtain

$$\tan^2 \theta_{12}^\ell = \frac{\tilde{m}_1}{m_2} + \frac{\tilde{m}_e}{m_\mu} - 2 \frac{\tilde{m}_1 \tilde{m}_e}{m_\mu m_\mu} \cos \Phi,$$

(22)

$$\tan \theta_{23}^\ell \approx \left( \sqrt{\delta_e} - \sqrt{\delta_\mu} \right),$$

(23)

$$\tan \theta_{13}^\ell \approx \sqrt{\frac{m_\mu}{m_e} \left( \sqrt{\delta_\mu} - \sqrt{\delta_e} \right)}.$$

(24)
In the absence of experimental information, we assumed that CP violation is also maximal in the lepton sector \( i.e. \Phi = 90^\circ \). Taking for the masses of the left handed Majorana neutrinos a normal hierarchy with the numerical values \( m_{\nu_1} = 4.4 \times 10^{-3} \text{ eV} \) and \( m_{\nu_2} = 9 \times 10^{-3} \text{ eV} \), \( m_{\nu_3} = 5 \times 10^{-2} \text{ eV} \) and for the charged lepton masses the values \( m_e = 0.5109 \text{ MeV} \), \( m_\mu = 105.685 \text{ MeV} \) and \( m_\tau = 1776.99 \text{ GeV} \) \([20]\), we obtain the following numerical values

\[ \theta_{12}^l \approx 33.9^\circ, \quad \theta_{23}^l \approx 41.5^\circ, \quad \theta_{12}^l \approx 3.58^\circ. \]  

(25)

We may now address the question of the meaning of the quark-lepton complementarity relations, as expressed in eq(2). The previous theoretical analysis allows us to calculate,

\[
\tan \left( \theta_{12}^l + \theta_{12}^q \right) = 1 + \Delta_{12}^{\text{th}},
\]

(26)

\[
\Delta_{12}^{\text{th}} = \left( \frac{\tilde{m}_s + \tilde{m}_c}{m_s} \right)^{1/2} \left( \frac{\tilde{m}_d + \tilde{m}_u}{m_d} \right)^{1/2} \left[ 1 + \left( \frac{m_s}{m_d} \right)^{1/2} \left( \frac{m_c}{m_u} \right)^{1/2} \right] + \left[ 1 + \left( \frac{m_s}{m_u} \right)^{1/2} \left( \frac{m_c}{m_d} \right)^{1/2} \right] - \left( 1 + \frac{\tilde{m}_s}{\tilde{m}_d} \right)^{1/2} \left( 1 + \frac{\tilde{m}_c}{\tilde{m}_u} \right)^{1/2}.
\]

(27)

\[
\Delta_{23}^{\text{th}} = 1 - \frac{(\sqrt{\delta_d} - \sqrt{\delta_u}) [(\sqrt{\delta_d} - \sqrt{\delta_u}) + 1] - (\sqrt{\delta_d} - \sqrt{\delta_u})}{(\sqrt{\delta_d} - \sqrt{\delta_u}) (\sqrt{\delta_d} - \sqrt{\delta_u})}.
\]

(29)

\[
\tan \left( \theta_{13}^l + \theta_{13}^q \right) = \frac{\sqrt{\tilde{m}_s} (\sqrt{\delta_c} - \sqrt{\delta_u}) \sqrt{\tilde{m}_d} (\sqrt{\delta_u} - \sqrt{\delta_c})}{1 - \sqrt{\tilde{m}_s} \tilde{m}_d (\sqrt{\delta_c} - \sqrt{\delta_u}) (\sqrt{\delta_u} - \sqrt{\delta_c})}.
\]

(30)

After substitution of the numerical value of the mass ratios of quarks and leptons in (27), we obtain,

\[
\theta_{12}^q + \theta_{12}^l = 45^\circ + 1.7^\circ.
\]

(31)

\[
\theta_{23}^q + \theta_{23}^l = 45^\circ - 0.8^\circ.
\]

(32)

\[
\theta_{13}^q + \theta_{13}^l \approx 4^\circ.
\]

(33)

in very good agreement with the experimental values.

According with the previous analysis, quark-lepton complementarity arises from the combined effect of two factors:

(i) The strong mass hierarchy of the Dirac fermions produces in small and very small mass ratios of \( u \) and \( d \)-type quarks and charged leptons. The quark mass hierarchy is reflected in a similar hierarchy of small and very small quark mixing angles.

(ii) The normal seesaw mechanism gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio \( m_{\nu_1}/m_{\nu_2} \) and allows for large \( \theta_{12}^l \) and \( \theta_{23}^l \) mixing angles.
6. The effective Majorana mass

One of the most fundamental problems of the physics of neutrinos is the question of the nature of massive neutrinos. A direct way to reveal the nature of massive neutrinos is to investigate processes in which the total lepton number is not conserved [22]. The matrix element of this processes are proportional to the effective Majorana neutrino masses, which are defined as

$$\langle m_{ll} \rangle \equiv \sum_{j=1}^{3} m_{\nu_j} U_{ij}^2, \quad l = e, \mu, \tau,$$

where $m_{\nu_j}$ is the neutrino Majorana mass and $U_{ij}$ are the elements of lepton mixing matrix $U_{PMNS}$.

The magnitudes square of the effective Majorana neutrino masses ec(34) are

$$|\langle m_{ll} \rangle|^2 = \sum_{j=1}^{3} m_{\nu_j}^2 |U_{lj}|^4 + 2 \sum_{j<k}^{3} m_{\nu_j} m_{\nu_k} |U_{lj}|^2 |U_{lk}|^2 \cos 2(w_{lj} - w_{lk}),$$

where

$$w_{lj} = \arg \{U_{lj}\}.$$  

The theoretical expression for the squared magnitudes of effective Majorana neutrino masses of electron and muon neutrinos are:

$$|\langle m_{ee} \rangle|^2 \approx \frac{2m_{e1}^2 + m_{\nu_1} m_{\nu_2} \left( \frac{m_{\nu_1}}{m_{\nu_2}} - 2 \frac{m_{\nu_2}}{m_{\nu_1}} \left(1 - \frac{m_{\nu_1}}{m_{\nu_2}} \right) \cos \Phi \right) \cos 2(w_{e1} - w_{e2})}{(1 + \frac{m_{\nu_1}}{m_{\nu_2}})^2 (1 + \frac{m_{\nu_2}}{m_{\nu_1}})^2},$$

$$|\langle m_{\mu\mu} \rangle|^2 \approx \frac{4m_{e1}^2 \left( \cos^2 \Phi + \frac{m_{\nu_1}}{m_{\nu_2}} \cos \Phi \right) + m_{e2}^2 \left(1 + 4 \frac{m_{\nu_1}}{m_{\nu_2}} \frac{m_{\nu_2}}{m_{\nu_1}} \cos \Phi \right)}{(1 + \frac{m_{\nu_1}}{m_{\nu_2}})^2 (1 + \frac{m_{\nu_2}}{m_{\nu_1}})^2},$$

with

$$w_{e1} = \arctan \left\{ \frac{\sqrt{\frac{m_{\nu_1}}{m_{\nu_2}}} \sin \Phi}{1 + \sqrt{\frac{m_{\nu_1}}{m_{\nu_2}}} \frac{m_{\nu_2}}{m_{\nu_1}} \cos \Phi} \right\}, \quad w_{e2} = \arctan \left\{ -\sqrt{\frac{m_{\nu_1}}{m_{\nu_2}}} \sin \beta_1 + \frac{m_{\nu_1}}{m_{\nu_2}} \sin \Phi \right\}.$$
with \( w_{\mu 3} = \Phi + \beta_2 \),

\[
\begin{align*}
    w_{\mu 1} &= \arctan \left\{ \frac{m_{\mu}}{m_{\mu}} \sin \Phi - \frac{m_{\mu}}{m_{\nu 2}} \cos \Phi \right\}, \\
    w_{\mu 2} &= \arctan \left\{ \frac{m_{\mu}}{m_{\mu}} \cdot \frac{m_{\mu}}{m_{\nu 2}} \sin \beta_1 + \sin(\Phi + \beta_1) \right\} \\
    \end{align*}
\]

(40)

Since we have no information about Majorana phases \( \beta_1 \) and \( \beta_2 \), in first approximation we consider this phases equal to zero. Also, we made the assumption of maximal CP violation in the leptonic sector \( (\Phi = 90^\circ) \). Then, we obtain the following numerical value of the effective Majorana neutrino masses

\[
|\langle m_{ee} \rangle| \approx 4.8 \times 10^{-3} \text{eV}, \quad |\langle m_{\mu\mu} \rangle| \approx 4.6 \times 10^{-3} \text{eV}.
\]

(41)

These numerical values are consistent with the very small experimentally determined upper bounds for the reactor neutrino mixing angle \( \theta_{13}^{\nu} \) [23]-[24].

7. Conclusions

In this short communication, we outlined a unified treatment of masses and mixing of quarks and leptons in which the left handed Majorana neutrinos acquire their masses via the seesaw mechanism type I, and the mass matrices of all Dirac fermions have a similar Fritzsch texture and a normal hierarchy. Then the mass matrix of the left handed Majorana neutrino has also a Fritzsch texture of four zeros. In this scheme, we derived exact, explicit expressions for the Cabibbo \( (\theta_{12}^q) \) and solar \( (\theta_{13}^l) \) mixing angles as functions of the quark and lepton masses. The quark-lepton complementarity relation takes the form,

\[
\theta_{12}^q + \theta_{12}^l = 45^\circ + \delta_{12}.
\]

(42)

The correction term, \( \delta_{12} \), is an explicit function of the ratios of quark and lepton masses, given in eq.(27), which reproduces the experimentally determined value, \( \delta_{12} \approx 1.7^\circ \), when the numerical values of the quark and lepton masses are substituted in (27) and maximal violation of CP in the lepton sector is assumed.

Three essential ingredients are needed to explain the correlations implicit in the small numerical value of \( \delta_{12} \):

(i) The strong hierarchy in the mass spectra of the quarks and charged leptons, realized in our scheme through the explicit breaking of the \( S_3 \) flavour symmetry in the Fritzsch mass texture, explains the resulting small or very small quark mixing angles, the very small charged lepton mass ratios explain the very small \( \theta_{13}^{\text{MNS}} \) which, in our scheme, is independent of the neutrino masses.

(ii) The normal seesaw mechanism that gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio \( m_{\nu 1}/m_{\nu 2} \) and allows for large \( \theta_{12}^{\text{MNS}} \) and \( \theta_{23}^{\text{MNS}} \) mixing angles.

(iii) The assumption of maximal CP violation in the lepton sector.

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