REBUILDING THE CEPHEID DISTANCE SCALE. I. A GLOBAL ANALYSIS OF CEPHEID MEAN MAGNITUDES

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ABSTRACT

We develop a statistical method for using multicolor photometry to determine distances using Cepheid variables, including the effects of temperature, extinction, and metallicity, and apply it to $UBVRIJK$ photometry of 694 Cepheids in 17 galaxies. We derive homogeneous distance, extinction, and uncertainty estimates for four models, starting from the standard extragalactic method and then adding the physical effects of temperature distributions and extinction distributions, requiring positive definite extinctions and metallicity. While we find general agreement with published distances when we make similar systematic assumptions, there is a clear problem in the standard distances because they require Cepheids with negative extinctions, particularly in low-metallicity galaxies, unless the mean LMC extinction exceeds $E(B-V) \geq 0.20$. The problem can be explained by the physically expected metallicity dependence of the Cepheid distance scale, where metal-poor Cepheids are hotter and possibly fainter at $V$ and $I$ than metal-rich Cepheids, or by large systematic errors in Cepheid photometry. For $V$ and $I$ we found that the mean magnitude change is $-0.14 \pm 0.14$ mag dex$^{-1}$ and the mean color change is $0.13 \pm 0.04$ mag dex$^{-1}$, with the change in color dominating the change in distance. The effect on Type Ia supernova estimates of the Hubble constant is dramatic because most were found in the metal-poor galaxies with the bluest Cepheids. The Type Ia multicolor light-curve shape method estimate for formally rises from $69 \pm 8$ to $80 \pm 6$ km s$^{-1}$ Mpc$^{-1}$ with the metallicity correction.

Subject headings: Cepheids — distance scale — galaxies: distances and redshifts

1. INTRODUCTION

Cepheid variables are fundamental to most extragalactic distance estimates, determinations of the Hubble constant, and models for the structure of the Galaxy because the Cepheid period-luminosity (PL) relations are accepted as one of the most accurate primary distance indicators. In the last few years, the number of extragalactic Cepheids has exploded owing to the two large extragalactic surveys using the Hubble Space Telescope (HST) (the Extragalactic Distance Scale Key Project: Freedman et al. 1994; Ferrarese et al. 1996; Graham et al. 1997; Kelson et al. 1996; Silbermann et al. 1997; and the Type Ia Supernova Calibration Project (SNIa): Saha et al. 1994, 1995, 1996a, 1996b, 1997) and the microlensing surveys of the Magellanic Clouds (e.g., Beaulieu et al. 1995; Welch et al. 1997). The goal of the HST projects is to determine the Hubble constant with 10% (0.2 mag) accuracy, which requires uncertainties in the Cepheid distance estimates, including all systematic uncertainties, that are still smaller. In order to reach these goals, the observational projects (see the review by Freedman 1997) have expanded the range of filters used to study Cepheids, particularly into the infrared, introduced systematic corrections for extinction, and tried to find better empirical tests for the effects of metallicity on Cepheid distance estimates.

The physical basis for Cepheids as distance estimators rests on two foundations (see the reviews by Feast & Walker 1987; Madore & Freedman 1991; and Tanvir 1996). The first foundation is stellar structure, which closely correlates the mass, radius, luminosity, temperature, and oscillation period of a star, so that in any localized region of the H-R diagram there is a period-luminosity-color (PLC) relation stating that the luminosity can be determined from the color (temperature) and the period (mass and radius) of oscillation. The second foundation is that the physics of the oscillations limits the Cepheids to a narrow range of luminosity and temperature called the instability strip. As a result, any projection of the three-dimensional PLC space onto a two-dimensional subspace produces a tightly correlated relation. In particular, the period-luminosity (PL) correlations are defined by projecting over the temperature/color distribution. Both the PLC relations and the location of the instability strip are expected to be functions of composition (e.g., Stothers 1988; Stift 1990; Chiosi, Wood, & Capitanio 1993), although there is great debate about the magnitude of the dependence and its measurement (e.g., Caldwell & Coulson 1986, 1987; Freedman & Madore 1990; Gould 1994; Stift 1995; Sasselov et al. 1996). The sense, however, is that metal-rich Cepheids are cooler and possibly brighter in $V$ and $I$ than metal-poor Cepheids at fixed period.

The goal of any analysis of Cepheid data is to determine accurately the distance to the Cepheid and its uncertainty, after compensating for the effects of period, temperature, composition, and extinction. Existing Cepheid analysis methods are divided into extragalactic and Galactic approaches. For extragalactic systems, the analysis must determine the common distance of an ensemble of Cepheids, usually based on two-color, low-accuracy photometry, with poor phase coverage. The standard approach (see Madore & Freedman 1991) uses only the correlation between luminosity and period (the PL relations) to estimate the distance modulus and the mean extinction. Gould (1994) pointed out that the method is statistically inefficient because it ignores the strong correlations in the residuals (i.e., the PLC relations). Galactic analyses (e.g., Caldwell & Coulson 1986, 1987; Caldwell & Laney 1991; Laney & Stobie 1993, 1994) must estimate the distances and extinctions of individual Cepheids using three-color photometry and PLC relations because the Cepheids no longer lie at a
common distance. The standard extragalactic approach explicitly ignores the PLC relations, and the standard Galactic approach (in some senses) ignores the instability strip.

In the following sections we develop a self-consistent physical and mathematical analysis of multicolor Cepheid mean magnitudes as the first in a series papers reanalyzing the Cepheid distance scale. Our approach differs from the standard approaches in three major ways. The first difference is that we analyze all the data simultaneously rather than one galaxy at a time. The use of Cepheids as distance indicators is predicated on the homogeneity of their physical properties, so either all the data can be self-consistently and simultaneously analyzed or we must reject the Cepheids as distance indicators. Moreover, the distance estimates (and other physical parameters) are very highly correlated. Any change in the distance or extinction estimate for one Cepheid galaxy requires a simultaneous change in the values for all Cepheid galaxies. The second difference is that we avoid the false dichotomy between the Galactic and extragalactic analysis procedures and demonstrate how to reconcile the two approaches. The resulting statistical method has greater statistical efficiency than traditional methods; one can model all measured colors simultaneously; and allows for better control, treatment, and understanding of systematic problems such as individual Cepheid extinctions and the effects of metallicity. The resulting scheme, although based on a physical model, closely resembles the empirical treatments of Gould (1994) and Sasselov et al. (1996). The third difference is that we explore the important systematic errors that can affect the Cepheid distance estimates, particularly the physics of extinction, temperature, and metallicity. While treatments of extragalactic Cepheid distances generally recognize the existence of these systematic uncertainties, they are rarely included in the distance estimates or their uncertainties. We develop our analysis method in § 2 and compare it to the existing techniques. In § 3 we examine the Cepheid correlations (PL, PLC relations, etc.) and the resulting distance estimates. We summarize our results, their shortcomings, and possible solutions in § 4.

2. CEPEHID DISTANCE ESTIMATES

We assume that the bolometric luminosity and the bolometric correction are linear functions of the period $P = \log \frac{P}{P_0}$, the effective temperature $T = \log \frac{T_\odot}{T_0}$, and the (logarithmic) metallicity $Z$, so that the (intensity mean) apparent magnitude $V_k$ of a Cepheid in band $k$, at distance modulus $m$, with extinction $E$ is

$$V_k = V_{ok} + a_k p + \beta_k t + \gamma_k Z + \mu_m + E R_k , \quad (1)$$

where $V_{ok}$ is the magnitude zero-point vector, $R_k$ is the reddening vector, $d_k \equiv 1$ is the distance vector, and $P_0 = 1.4$ and $T_\odot$ are reference periods and temperatures. We assume that the slopes $a_k$ and $\beta_k$ are independent of metallicity, although theory predicts a weak dependence (see Stothers 1988; Stift 1990, 1995; Chiosi et al. 1993). If we neglect the dependence of bolometric corrections on surface gravity, $\alpha_k = \alpha_t$. We call equation (1) a period-luminosity-color (PLC) relation, although standard PLC relations (see Feast & Walker 1987) replace the effective temperature by a color. We neglect additional variables such as the Cepheid’s age, the instability strip crossing number, the helium abundance, and nonlinearities in the PLC relation (see Caldwell & Coulson 1986), so we assume that the relations defined by equation (1) have an intrinsic width of $\sigma_{PLC}$.

Formally, if we know the precise values of the vectors $a_k$, $\beta_k$, $\gamma_k$, and $R_k$, the vectors are not degenerate with $d_k$ (distance), and we possess accurate photometry of the Cepheid in a sufficiently large number of bands, then the PLC relations can be used to determine the distance to a particular Cepheid. Unfortunately, since we must determine $\alpha_k$, $\beta_k$, and $\gamma_k$ as we proceed from a small number of colors, it is difficult to use the PLC relations to determine distances without adding additional constraints. The additional constraints used in standard Cepheid analyses (e.g., Madoe & Freedman 1991) correspond to adding Gaussian priors on the distances, temperatures, and extinctions. We now develop a general mathematical description for fitting Cepheid magnitudes that includes the standard methods as subcases or limits of a more general model.

The PLC relations contain no information about which stars pulse, and the most important prior information is the location and width of the instability strip. The standard extragalactic method uses the instability strip by averaging the distribution over temperature and using only the PL relations to determine distances. We parametrize the instability strip by a period-color (PC) relation defining the temperature distribution for stars at a fixed period using a likelihood function,

$$L(t|p) \propto \exp \left[ -\frac{(t - \delta_Z Z - \delta_p p)^2}{2\sigma_{PC}^2} \right] \quad (2)$$

including a possible shift in the location of the instability strip with metallicity (e.g., Stift 1990; Chiosi et al. 1993). We assume that the width of the instability strip is constant, although the observed narrowing of the strip at short periods (e.g., Fernie 1990) could be modeled by making $\sigma_{PC}$ a function of period (e.g., Gould 1994). We chose to parametrize the instability strip with a PC relation rather than a luminosity-color relation (i.e., the H-R diagram) because it is distance independent and uses the well-defined period as the independent variable instead of the luminosity. From these two assumptions we can derive all the standard relations used to study Cepheids and their correlations. For example, the PL relation in band $V_k$ is

$$\frac{1}{\int L(t|p)dt} \langle V_k \rangle = \langle V_{ok} + a_k p + \gamma_k Z + \mu m + E \rangle R_k , \quad (3)$$

where $a_k = a_t + \beta_t \delta_p$ and $\gamma_k = \gamma_t + \beta_t \delta_Z$ and the dispersion in the relation is $\sigma^2_{PLC} \equiv \beta^2_k \sigma^2_{PC} \approx \beta^2_k \sigma^2_{PC}$. From here on we use the deviation of the temperature from the expected mean, $\delta t = t - \delta_Z Z - \delta_p p$, which also shifts the period and metallicity vectors to the $a_t$ and $\gamma_t$ appearing in the PL relation (3). We cannot independently

\footnote{Gaussian priors are the standard statistical assumption when the true distribution is unknown. The use of Gaussian prior does not force the distribution of the variable to be Gaussian unless there is too little information in the data being fit to make a better estimate. Most standard Cepheid models implicitly make the assumption that the temperature distribution is Gaussian in building their statistical models, but our formalism forces us to explicitly define our priors.}
determine $\alpha, \gamma, \delta_2$, and $\delta_p$ without absolute temperature references. We measure distances, extinctions, and metallicities relative to the LMC values of $\langle \mu \rangle_{\text{LMC}}, \langle E \rangle_{\text{LMC}}$, and $Z_{\text{LMC}} = [\text{O/H}]_{\text{LMC}}$. We use the oxygen abundance for the metallicity variable because it is the only abundance available for most of the extragalactic systems. We can also constrain the models using prior information on the distance modulus, extinction, and metallicity. In external galaxies we can use the constraint that all Cepheids lie at the same distance, but this assumption begins to fail for the LMC and may be a poor assumption for the SMC (e.g., Caldwell & Coulson 1986; Caldwell & Laney 1991). We allowed the Magellanic Clouds to be tilted relative to the line of sight and to have a finite thickness. Distances and extinctions were fit for each Galactic Cepheid. We simultaneously fit a flat rotation curve model to obtain estimates of the solar radius $R_0$ and circular velocity $\Theta_0$. We fit the rotation curve simultaneously with the Cepheid model because it is the only way to correctly include the strong correlations in the Cepheid distance estimates on the rotation curve parameters. The rotation curve model has little effect on the Cepheid model because a radial velocity is a poor distance indicator compared with the calibrations forced by setting the LMC distance. We model the priors using Gaussians with mean values of $\langle \mu \rangle$ for the distance modulus, $\langle E \rangle$ for the extinction, and $\langle Z \rangle$ for the metallicity, with dispersions of $\sigma_\mu, \sigma_E,$ and $\sigma_Z$ for each galaxy or Cepheid. The relative calibration of the standard and the HST $V$ and $I$ magnitudes is uncertain at the level of 0.05 mag (Hughes et al. 1994), so we include two HST calibration variables constrained by a Gaussian prior of width 0.05 mag in the likelihood. The temperature $\sigma_{VPC}$ and PLC $\sigma_{PLC}$ widths were the same for all galaxies.

We can divide our analysis into two parts. First, we estimate the parameters of the individual Cepheids ($\mu, \delta t, E$, and $Z$) given the current parameters of the global model. Second, we optimize the parameters $(V_{0,k}, \alpha_k, \beta_k, \gamma_k, \ldots)$ of the global model. For a particular Cepheid we have mean magnitude measurements $V_{mk}$ with uncertainties $\sigma_{mk}$ in each of $k = 1, \ldots, N$ bands and a known period $P$. If we define $\sigma_k^2 = \sigma_{mk}^2 + \sigma_{PLC}^2$, then the log-likelihood for the model to fit the measured mean magnitudes $V_{mk}$ of a particular Cepheid is

$$-2 \ln L = \sum_{k=1}^{N} \left( \frac{(V_{mk} - V_k)^2}{\sigma_k^2} + \frac{\delta t^2}{\sigma_{VPC}^2} + \frac{(E - \langle E \rangle)^2}{\sigma_E^2} \right) + \frac{(\mu - \langle \mu \rangle)^2}{\sigma_\mu^2} + \frac{(Z - \langle Z \rangle)^2}{\sigma_Z^2} + \ln |S^{-1}|$$

(4)

up to a constant, where the covariance matrix is

$$S_{ij}^{-1} = \sigma_i^2 \delta_{ij} + \sigma_d^2 d_i d_j + \beta_i \beta_j \sigma_{PC}^2 + R_i R_j \sigma_{E}^2 + \gamma_i \gamma_j \sigma_{Z}^2,$$

(5)

and the indices run over the filters included in the calculation. We must use the determinant of the covariance matrix $S$ in the likelihood if we are to determine simultaneously the properties of the individual Cepheids and their global statistical relations.

We can relate our model to standard analyses by breaking the calculation into two sections: first, deriving the deviations in the properties of individual Cepheids from the mean, and second, determining the PLC relation vectors and the mean properties of the Cepheids. If we measure the magnitude residuals $\Delta V_k = V_{mk} - \langle V_k \rangle$ relative to the mean PL relations (eq. [3]) for the parent galaxy, define the deviation of the Cepheid parameters from the mean parameters by $x$ and the vector-weighted residuals by $v$,

$$v = \begin{pmatrix} \delta t \\ E - \langle E \rangle \\ \mu - \langle \mu \rangle \\ Z - \langle Z \rangle \end{pmatrix}$$

(6)

and define the matrix $C$ by

$$C = \begin{pmatrix} \sum \frac{\beta_k^2}{\sigma_k^2} + \frac{1}{\sigma_{PC}^2} & \sum \frac{\beta_k \beta_k'}{\sigma_k \sigma_k'} & \sum \frac{\beta_k d_k}{\sigma_k} & \sum \frac{\beta_k \gamma_k}{\sigma_k} \\ \sum \frac{\beta_k R_k}{\sigma_k} & \sum \frac{R_k^2}{\sigma_k^2} + \frac{1}{\sigma_{E}^2} & \sum \frac{R_k d_k}{\sigma_k} & \sum \frac{R_k \gamma_k}{\sigma_k} \\ \sum \frac{\beta_k d_k}{\sigma_k} & \sum \frac{R_k d_k}{\sigma_k} & \sum \frac{d_k^2}{\sigma_k^2} + \frac{1}{\sigma_{E}^2} & \sum \frac{d_k \gamma_k}{\sigma_k} \\ \sum \frac{\beta_k \gamma_k}{\sigma_k} & \sum \frac{R_k \gamma_k}{\sigma_k} & \sum \frac{d_k \gamma_k}{\sigma_k} & \sum \frac{\gamma_k^2}{\sigma_k^2} + \frac{1}{\sigma_{Z}^2} \end{pmatrix}$$

(7)

then $x = C^{-1}v$ and the covariance matrix of the parameter estimates for a fixed PLC relation is $C^{-1}$. With the optimization of these variables, the contribution of the Cepheid to the likelihood becomes

$$L \propto |S|^{-1/2} \exp \left(-\frac{1}{2} \Delta V^T S \Delta V \right),$$

(8)

where $\Delta V$ is the vector of residuals relative to the mean PL relations and $S$ is the covariance matrix defined in equation (5).

We obtain the standard extragalactic method (see Madore & Freedman 1991) if the four priors have zero width ($\sigma_{PC} = \sigma_\mu = \sigma_E = \sigma_Z = 0$) and there is no metallicity dependence ($\gamma_k = 0$). The covariance matrix $S_{ij} = \delta_{ij} (\sigma_{mi}^2 + \sigma_{PLC}^2)$ is diagonal. Some extragalactic Cepheid distances are derived by using reddening free distances or determining individual extinctions (e.g., Freedman & Madore 1990; Freedman, Wilson, & Madore 1991; Freedman et al. 1992; Tanvir et al. 1995; Saha et al. 1996a, 1996b, 1997), which corresponds to allowing $\sigma_E \rightarrow \infty$. If we optimize the widths of the priors, then the method matches that of Gould (1994) or Sasselov et al. (1996) in using the covariance matrix of the residuals to build a better statistical model of the data. The advantage of our formalism is that it derives from a physical model and we gain new physical insights into the system from the variables making up the covariance matrix. The disadvantage is that if our physical model is incorrect or it is missing important sources of correlated variance in the data, it is not as optimal or correct a statistical approach as using a purely empirical covariance matrix. If we limit the model to two or three filters and use broad priors, we recreate the normal Galactic approach (e.g., Caldwell & Coulson 1986, 1987; Laney & Stobie 1986, 1993, 1994; Fernie 1990; Fernie et al. 1995) with the color dependence of the stan-
standard PLC relations appearing indirectly through the temperature variable.

Three classes of data and parameters enter the problem. The first class consists of the period and the composition. Here we know the dependent variable ($p$ or $Z$) and seek to determine the state vectors ($\alpha'$ and $\gamma'$). Given an adequate range for the dependent variables in an external galaxy, the state vectors are well-determined and nondegenerate. In particular, the standard PL relations (e.g., Madore & Freedman 1991; Laney & Stobie 1994; Tanvir 1996) determine $\alpha'$ based on the Cepheids in the LMC. In the second class, consisting of the extinction and the distance, we know the state vectors ($R_k$ and $d_j$) and would like to determine the dependent variable ($E$ and $\mu$). The distance vector is known exactly ($d_i \equiv 1$), and the extinction vector $R_k$ is known approximately (see §2.2). With accurate measurements in a sufficient number of bands, we can determine the distances and extinctions for individual Cepheids. The uncertainties will depend on how well we can determine the effects of temperature and metallicity, but the formalism will correctly include these uncertainties in the distance and extinction estimates.

In the third case, the temperature, we know neither the dependent variable $t$ nor the state vector $\beta_k$. One immediate consequence is a mathematical degeneracy under a rescaling of the temperature by $t \rightarrow \xi t$ and the temperature related variables by $\beta_k \rightarrow \xi^{-1} \beta_k$, $\delta_p \rightarrow \xi \delta_p$, $\delta_Z \rightarrow \xi \delta_Z$, and $\sigma_{PC} \rightarrow \xi \sigma_{PC}$. No observable (i.e., magnitude) depends on the rescaling, and it means that we cannot set an absolute temperature scale. A more fundamental problem is that values found for variables such as the $t$-\(\beta_k\) pair may not have physical meaning assigned to them in the mathematical model.

The minimization procedure will simply use them to absorb as much variance as possible from the residuals, and if $\beta_k$ is unconstrained, it assumes the value of the most important unmodeled principal component of the true covariance matrix. Only if the primary source of the variance is the temperature distribution at fixed period will $t$ represent the physical temperature. In essence, we are defining the covariance matrix in terms of a few principal components defined by the state vectors, and we assert that the $\beta_k$ principal component represents temperature. The degeneracy affects only our interpretation of the variables, and the statistical model will still reproduce the correct observational PLC/PC/PL relations. For practical purposes we solved the degeneracy problem by using a prior for the $\beta_k$ derived from fitting Cepheid light curves, which we discuss in Kochanek (1998). The difficulty in estimating $\beta_k$ (or its equivalent slope in standard PLC relations) and separating temperature from extinction has lead the extragalactic Cepheid community to avoid modeling the temperature distribution (e.g., see the critique of PLC relations in Madore & Freedman 1991).

2.1 Data

We simultaneously analyzed the Cepheids of 17 galaxies (see Table 1), including only Cepheids with periods between 7 and 80 days. We used all available Johnson $UBV$, Kron-Cousins $RI$, and Glass-Carter $JHK$ mean magnitudes for the Cepheids, because the systems with many-color photometry (the Galaxy, LMC, SMC, M31, M33, and NGC 300) offer the best hope of separating the effects of distance, temperature, extinction, and metallicity. With two-color photometry there is no simple means of separating the effects of temperature and extinction, while with eight-color photometry it should be possible. We assigned magnitude uncertainties of 0.05 mag for the Galactic, LMC, and SMC Cepheids, and 0.10 mag for the $HST$ Cepheids and the poorly sampled ground-based data on M31, M33, and NGC 300. Estimates of the distances and extinctions were little affected by changes in the estimated measurement errors. Although we spot-checked any of the inferred magnitudes and periods for the Cepheids, we used the published periods and magnitudes in our analysis.

Galaxy.—We used the Cepheids with radial velocities in Pont, Burki, & Mayor (1994) and periods longer than 7 days. Intensity mean magnitudes were checked from the data compilations of the McMaster Cepheid Photometry and Radial Velocity Data Archive by D. L. Welch and Berdnikov (1987, 1996, private communication) and matched existing $B$ and $V$ tabulations (e.g., Fernie et al. 1995). We dropped the Cepheids AA Ser, XZ Car, and SU Cru from the Pont et al. (1994) sample as outliers.

LMC.—We used the $UBV$ data from J. A. R. Caldwell (1996, private communication, hereafter Caldwell 1996, the $JHK$ data from Laney & Stobie (1986, 1993, 1994), which includes earlier data by Welch et al. (1987), and the $R$ data from Madore (1985). The Caldwell (1996) mean magnitudes include earlier data whose sources are reviewed in Madore (1985). We rejected HV 2301, HV 2378, and HV 2749 from our fits as outliers in the likelihood. Caldwell & Laney (1991) and earlier authors also report that these three Cepheids have abnormal properties.

M33.—We use the $BVR$ data from Freedman et al. (1991) excluding 21979, 23764, and B1. The photometry for V12 in Table 5 of Freedman et al. (1991) is shifted to the left by one column. None of the remaining Cepheids stood out in the likelihood distribution.

SMC.—We used the $BVR$ data from Caldwell (1996) and the $JHK$ data from Laney & Stobie (1993, 1994). We rejected HV 854, HV 1369, HV 1438, HV 1482, HV 1484, and HV 1695 as outliers in the likelihood. Caldwell & Laney (1991) and earlier authors report HV 1369, HV 1484, HV 1636, and HV 1641 as outliers. We find nothing peculiar about HV 1636, and HV 1641 was not included in the sample from Caldwell (1996).

M31.—We use the $BVR$ data from Freedman & Madore (1990; Freedman 1996, private communication). We rejected the 18.5 day period Cepheid in Baade’s field III.

NGC 300.—We use the $BVR$ data from Freedman et al. (1992). We rejected V21 as an outlier. Freedman et al. (1992) comment that the light curve of V21 is not well determined.

M81.—We use the $VI$ data from Freedman et al. (1994). We left the magnitude calibrations unchanged. We rejected C8 as an outlier.

M100.—We use the $VI$ data from Ferrarese et al. (1996). There is a typographical error in the distance modulus given in the abstract of Ferrarese et al. (1996, see erratum), where the correct value is $\mu = 31.04 \pm 0.17$. We added the 0.05 mag “long-exposure correction” to the tabulated Cepheid magnitudes (Hughes et al. 1994). We rejected the two shortest period Cepheids, C68 and C70, as outliers.

M101.—We use the $VI$ data from Kelson et al. (1996) including all the $I$ photometry (the “weak photometry restriction”). The light-curve data tables of Kelson et al. (1996) have the time and magnitude columns in different orders. We rejected C28 as an outlier.
TABLE 1

CEPHEID DATA

| Galaxy      | Group  | $N_i$ | Bands     | $E_V$ | [O/H] |
|-------------|--------|-------|-----------|-------|-------|
| Galaxy      | ...    | 124   | $UBVRiJHK$| ...   | 0.30  |
| LMC         | ...    | 71    | $UBVRiJHK$| 0.063 | 0.00  |
| SMC         | ...    | 78    | $UBVRiJHK$| 0.043 | -0.35 |
| M33         | Local  | 11    | $BVRI$    | 0.045 | 0.58 ± 0.04 |
| M31         | Local  | 30    | $BVRI$    | 0.080 | 0.28 ± 0.27 |
| NGC 300     | Sculptor | 13 | $BVRI$ | 0.025 | 0.05 ± 0.15 |
| M81         | M81 | 31 | $VI$ | 0.038 | 0.32 ± 0.07 |
| M101        | M101 | 29 | $VI$ | -0.008 | 0.02 ± 0.00 |
| IC 4182     | ...    | 19    | $VI$    | -0.015 | -0.35 |
| NGC 5253    | Centaurus | 8 | $VI$ | 0.048 | -0.25 |
| NGC 925     | NGC 1023 | 73 | $VI$ | 0.065 | 0.29 ± 0.02 |
| M96         | Leo I | 7 | $VI$ | 0.015 | 0.69 |
| NGC 3351    | Leo I | 46 | $VI$ | 0.013 | 0.94 ± 0.03 |
| NGC 4536    | Virgo | 32 | $VI$ | -0.005 | 0.00 |
| M100        | Virgo | 51 | $VI$ | 0.010 | 0.84 ± 0.04 |
| NGC 4496A   | Virgo | 56 | $VI$ | 0.003 | 0.00 |
| NGC 4639    | Virgo | 14 | $VI$ | 0.013 | 0.10 |

Note.—The metallicities are from Zaritsky et al. (1994), except for M96 (Oey & Kennicutt 1993) and M31 (Blair et al. 1982). Metallicities for the galaxies not included in Zaritsky et al. (1994) were estimated from the metallicity-type or metallicity-magnitude relations. The uncertainty on the $[O/H]$ value for the cepheids in that galaxy based on the metallicity gradients if known. The foreground extinction estimates $E_V = E(B - V)_f$ are from Burstein & Heiles (1984).

2.2. Extinction

We require an extinction vector $R_V = A_V/E(B - V)$ defined for all eight filters. Our standard vector is $R_{ok} = (5.05, 4.31, 3.30, 2.73, 2.07, 0.95, 0.64, 0.39)$ for the $UBVRiJHK$ filters based on the Cardelli, Clayton, & Mathis (1989) model for the extinction curve. We fixed $R_V = 3.3$ to match the Key Project, but as emphasized by Cardelli et al. (1989), the difference between $R_V = 3.1$ and $R_V = 3.3$ is mainly in the absolute normalization of the $R_V$ vector and the estimated $E(B - V)$. The shape of the extinction vector changes very little. The Type Ia project uses $A_V/A_f = 1.7$ instead of $A_V/A_f = 1.6$. The earlier M31 (Freedman & Madore 1990) and M33 (Freedman et al. 1991) studies used $R_V = 3.1$ combined with the Cardelli et al. (1989) extinction curve, while $R_V = 3.3$ was used for NGC 300 (Freedman et al. 1992). Laney & Stobie (1993) estimated that the JHK values were $(0.82, 0.49, 0.30)$, and the differences compared with Cardelli et al. (1989) model gives some indication of the typical systematic uncertainties in estimating extinction coefficients. We do not include the temperature variations in the extinction coefficient used by many of the Galactic Cepheid models (e.g., Caldwell & Coulson 1986; Laney & Stobie 1986, 1993, 1994; Feast & Walker 1987; Fernie 1990).

Cepheid distances are sensitive to the assumed structure of the extinction vector $R_V$, but most existing treatments of Cepheid distances treat the extinction as a known, understood property of the interstellar material even while different analyses select different models. Exceptions are Laney & Stobie (1993, 1994) and Sasselov et al. (1996), who systematically varied the $R_V$ or tried to determine them from the data, and Freedman & Madore (1990) and Freedman et al. (1991), who examined the effects of using standard $R_V = 3.1$ or $3.3$ or extremal models on the distance estimates. We will use a fixed extinction model for all the Cepheids, but we allow the coefficients to adjust themselves to best fit the data, and the uncertainties in the extinction vector will be included in the distance uncertainties. We constrain the extinction vector $R_V$ to fit the Cardelli et al. (1989) extinction vector $R_{ok}$ by adding a Gaussian prior to the likelihood $\propto \exp \left(-\frac{(R_{ok} - R_V)^2}{2\sigma_R^2}\right)$ with $\sigma_R = 0.1$ and $R_{ov} \equiv R_V$. The uncertainty roughly matches the uncertainties in the Cardelli et al. (1989) model and the differences between various extinction models used for Cepheid distances. The extinction is not a free variable because it must be positive definite and larger than the estimated foreground extinction (taken from Burstein & Heiles 1984, see Table 1).
Freedman & Madore (1990) and Freedman et al. (1992) were the first to face this problem when they found a negative estimated extinction for the Cepheids in M31 (Baade IV field) and NGC 300, although Sasselov et al. (1996) were the first to use the positivity of the extinction as a constraint when estimating the metallicity dependence of the Cepheid distances from the EROS sample (Beaulieu et al. 1995) of LMC and SMC Cepheids. Most treatments have either when estimating the metallicity dependence of the Cepheid period origin to log We discuss only the results for M31. Both of these potential problems could lead to an underestimate of the strength of any metallicity effect.

2.3. Metallicity

There is no debate about the qualitative effects of metallicity on the Cepheids: high-metallicity Cepheids are cooler and possibly brighter in $V$ and $I$ than low-metallicity Cepheids at fixed period. The debate centers only on the magnitude of the effect on distance estimates. The flux is reduced in the blue due to line blanketing and increased in the red and infrared by backwarming. Theoretical estimates suggest the effect is moderate, about $(\gamma_p + \gamma_I)/2 \approx 0.14$ mag dex$^{-1}$ and $(\gamma_B + \gamma_I)/2 = 0.16$ mag dex$^{-1}$ for the mean $B$- and $V$-band luminosity and color (Stothers 1988). Chiosi et al. (1993) find a weaker effect of about $(\gamma_p + \gamma_I)/2 = -0.05$ to 0.13 mag dex$^{-1}$ in the mean magnitude, and $(\gamma_V - \gamma_I) = 0.05$ to 0.09 mag dex$^{-1}$ in the $V-I$ color. An obvious signature of a metallicity dependence in the observational data should be a correlation of Cepheid extinction estimates with the metallicity of the parent galaxy.

Recent attempts to measure the metallicity effects either examined M31 or compared the LMC and SMC. Freedman & Madore (1990) examined the Cepheids in M31 where the work of Blair, Kirshner, & Chevalier (1982) implied a steep metallicity gradient. They found a distance change with metallicity of $\delta u \approx (0.32 \pm 0.21)$ mag dex$^{-1}$, where the numerical estimate of their gradient is due to Gould (1994). The weak significance of the Freedman & Madore (1990) estimate is used by the Key Project as the basis for neglecting the effects of metallicity in distance estimates pending an analysis of metal rich and metal poor Cepheids in M101. Gould (1994) also reanalyzed the M31 data including the empirical covariance matrix of the residuals to find a larger correction of $(0.88 \pm 0.16)$ mag dex$^{-1}$, although the numerical value depended on the colors used. The low-metallicity Baade IV field required a large negative extinction, suggesting that the metallicity requires a color term, as emphasized by Stift (1995) and Sasselov et al. (1996), unless the mean LMC extinction is significantly underestimated (Freedman et al. 1992; Böhm-Vitense 1997). Sasselov et al. (1996) analyzed the EROS sample of fundamental and overtone Cepheids in the LMC and SMC (Beaulieu et al. 1995) using a method based on Gould (1994) but including both distance and color terms for the metallicity. Unlike the M31 studies, where the estimate depends on Cepheids of differing metallicity at a common distance, the Sasselov et al. (1996) value relies on positivity of extinction and estimates of the foreground and internal extinctions of the LMC and SMC to estimate the effect. They find a correction of $0.4_{-0.2}^{+0.1}$ mag dex$^{-1}$, in the mean magnitude and $(0.20 \pm 0.02)$ mag dex$^{-1}$ in the $V-I$ color. The color dependence is similar to the Caldwell & Coulson (1986, 1987) and Gieren et al. (1993) estimates for the Galactic Cepheids, of $(B-V) \propto (0.29 \pm 0.05)$ mag dex$^{-1}$ and $(V-I) \propto (0.20 \pm 0.05)$ mag dex$^{-1}$.

We use the logarithmic abundance of oxygen relative to the LMC (see Table 1, $\Delta Z = [\text{O}]/[\text{H}]$) as our metallicity variable because it is the only measured abundance for most of the Cepheid galaxies. Where possible we include the spatial gradients of the metallicity from Zaritsky, Kennicutt, & Huchra (1994) and assign metallicities to the Cepheids based on their positions in the galaxy (M33, M31, NGC 300, M81, M101, NGC 925, NGC 3351, M100). Where no gradient was available, we used a mean metallicity for the whole system (LMC, SMC, IC 4182, NGC 5253, M96, NGC 4536, NGC 4496A, NGC 4536, NGC 4496A, and NGC 4695). We assigned the Galactic Cepheids a metallicity 0.3 higher than that of the LMC, and we decided not to force a radial metallicity gradient in our current model. For consistency with Freedman & Madore (1990) and Gould (1994), we used the Blair et al. (1982) gradient for M31 rather than the Zaritsky et al. (1994) gradient. Zaritsky et al. (1994) averaged the Blair et al. (1982) and Dennefeld & Knuth (1981) data even though Blair et al. (1982) strongly disagreed with Dennefeld & Knuth’s (1981) results. We also find an anomalously high metallicity estimate for the M33 Cepheids compared with M31. Both of these potential problems could lead to an underestimate of the strength of any metallicity effect.

3. STATISTICAL MODELS OF CEPHEID MAGNITUDES AND DISTANCES

In this section we build the full model for the Cepheid mean magnitudes, starting from the standard extragalactic method in model 0. Since the residuals are roughly parallel to the extinction vector, model 1 allows all Cepheids to have individually estimated extinctions. In model 2 we allow the Cepheids to have a distribution in temperature at fixed period and enforce the positivity of the extinction on the models. Finally, we estimate the effects of metallicity in model 3. We used a flat rotation curve model for the Galaxy to obtain estimates for the solar radius $R_0$ and circular velocity $\Theta_0$. The LMC and SMC are tilted relative to the line of sight, have zero width, and their relative distance is determined between the (bar) centers. The tilt is optimized as part of the solution and agrees well with Caldwell & Laney (1991). To facilitate comparisons with the standard PL relations of Madore & Freedman (1991), we assume an LMC distance modulus of 18.5 mag and a mean LMC extinction of $E(B-V) = 0.10$, but we have shifted the period origin to log $P_0 = 1.4$. We discuss only the results for the Johnson $UBV$, Kron-Cousins $RI$, and Glass-Carter $JHK$ bands.
3.1. Model 0: The Standard Method

We start our analysis by considering the Cepheid correlation functions in the absence of metallicity \((\chi_k = 0)\), scatter in the temperature \((\beta_k = 0)\), and allow only the Galactic Cepheids to have individually determined extinctions. Model 0 treats all the extragalactic Cepheids using the standard extragalactic method. The parameters for the PLC relations and their uncertainties are summarized in Table 2, and the derived distances and mean extinctions for the extragalactic samples are summarized in Table 3. Our zero points and period slopes are in general agreement with Freedman & Madore (1990). The zero points are approximately 0.04 mag and 0.02 mag fainter in the critical \(V\) and \(I\) bands, but the differences lie well within the standard uncertainties. The PLC relation slopes \(k_e\) are uniformly shallower by 0.05–0.10, although the agreement is again consistent with the uncertainties in the individual estimates. The zero points and slopes agree with recent studies by Laney & Stobie (1993) and Tanvir (1996). The best-fit extinction vector differs from the nominal Cardelli et al. (1989) vector by \(0.15 \pm 0.08, -0.01 \pm 0.06, -0.07 \pm 0.05, -0.05 \pm 0.05, +0.10 \pm 0.06, +0.02 \pm 0.06, \) and \(0.08 \pm 0.06\) for \(U, B, R, I, J, H,\) and \(K\), respectively. The changes in the coefficients are not a simple change in the \(R_V\) value of the Cardelli et al. (1989) model, and the final uncertainties in the \(R_V\) are less than the uncertainties in the prior. The ratio \(A_V/A_I = 1.63 \pm 0.05\) lies between the Key Project \((A_V/A_I = 1.6)\) and SNIa \((A_V/A_I = 1.7)\) values. The shifts in the Glass-Carter \(JHK\) coefficients were expected, because we lacked the true mean effective wavelengths for these filters and had simply set their values to those for the CTIO \(JHK\) filters. However, the sign of the shift is opposite to that found by Laney & Stobie (1993), who found smaller extinction coefficients using a different analysis technique.

Our distance estimates generally agree with the published values (see Tables 3 and 4, and Figs. 1 and 2a) with a few exceptions. We find that M31, M33, and NGC 300 are closer than the published values by \(-0.11, -0.12,\) and \(-0.05\) mag, respectively. In M33 the reason is the significantly higher extinction estimate of \(E(B-V) = 0.16\) instead of 0.10. Both earlier Cepheid distance estimates for M33 (Freedman 1985; Madore et al. 1985) and other models tested in Freedman et al. (1991) match our higher extinction estimates. Our agreement with the Key Project estimates is excellent, with a mean shift of \(-0.05\) mag, similar to that found by Tanvir (1996) due to zero-point recalibration. Our error and extinction estimates are also in good agreement, except for M81, where we find a significantly higher mean extinction of 0.075. For M96, we agree with the values for the distance and mean extinction found by Tanvir et al. (1995), but we derive much larger uncertainties of 0.28 (vs. 0.16) mag for the distance and 0.12 (vs. 0.03) for the extinction. We found significant disagreements with the Type 1a project in the distance estimates, extinction estimates and uncertainties. On average our distances were \(-0.09\) mag smaller, the extinctions were 0.04 larger, and our distance uncertainties were generally twice as large. The results significantly change the zero point of the multicolor light-curve shape (MLCS) SNIa distance scale (Riess, Press, & Kirshner 1996), since the distances to the calibrators 1972E in NGC 5253, 1981B in NGC 4536, and 1990N in NGC 4639 are revised from \(28.08 \pm 0.10\) to \(27.70 \pm 0.32\) mag, \(31.10 \pm 0.13\) to \(30.97 \pm 0.22\) mag, and \(32.03 \pm 0.22\) to \(32.11 \pm 0.32\) mag, respectively. The revised MLCS estimate of the Hubble constant is \(69 \pm 8\) km s\(^{-1}\) Mpc\(^{-1}\), an 8\% increase from the original estimate of \(64 \pm 6\) km s\(^{-1}\) Mpc\(^{-1}\).

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\(^2\) Laney & Stobie (1994) found zero points of \(V_\odot = 13.28 \pm 0.09, 11.90 \pm 0.06, 11.47 \pm 0.06, 11.38 \pm 0.06,\) and mag and PL slopes of \(x_V = -2.87 \pm 0.07, -3.31 \pm 0.05, -3.42 \pm 0.05,\) and \(-3.44 \pm 0.05\) in the \(V, J, H,\) and \(K\) bands for a mean LMC modulus of 18.5 mag.

\(^3\) Tanvir (1996) found zero points of \(V_\odot = 13.24 \pm 0.04\) and \(12.45 \pm 0.03\) and PL slopes of \(x_V = -2.774 \pm 0.083\) and \(-3.039 \pm 0.059\) for \(V\) and \(I\).
| Model  | $-2\ln L$ | Vector | $U$       | $B$      | $V$      | $R$      | $I$      | $J$      | $H$      | $K$      |
|--------|-----------|--------|-----------|----------|----------|----------|----------|----------|----------|----------|
| MF1    | 4277      | $V'$   | $14.03 \pm 0.08$ | $13.24 \pm 0.07$ | $12.80 \pm 0.05$ | $12.41 \pm 0.04$ | ...      | ...      | ...      | ...      |
|        |           | $\alpha$ | $-2.43 \pm 0.14$ | $-2.76 \pm 0.11$ | $-2.94 \pm 0.09$ | $-3.06 \pm 0.07$ | ...      | ...      | ...      | ...      |
|        |           | $\sigma_{PLC}$ | 0.36       | 0.27     | 0.22     | 0.18     | ...      | ...      | ...      | ...      |
| MF2    | 7418      | $V'$   | $14.03 \pm 0.16$ | $13.23 \pm 0.12$ | $12.80 \pm 0.11$ | $12.40 \pm 0.09$ | $11.89 \pm 0.07$ | $11.50 \pm 0.06$ | $11.43 \pm 0.05$ | ...
|        |           | $\alpha$ | $-2.53 \pm 0.28$ | $-2.88 \pm 0.20$ | $-3.04 \pm 0.17$ | $-3.14 \pm 0.17$ | $-3.31 \pm 0.11$ | $-3.37 \pm 0.10$ | $-3.42 \pm 0.09$ | ...
|        |           | $\sigma_{PLC}$ | 0.40       | 0.29     | 0.25     | 0.21     | 0.16     | 0.14     | 0.13     | ...
| 0      | 4277      | $V'$   | $14.54 \pm 0.05$ | $14.05 \pm 0.03$ | $13.28 \pm 0.03$ | $12.83 \pm 0.03$ | $12.45 \pm 0.03$ | $11.89 \pm 0.02$ | $11.53 \pm 0.02$ | $11.45 \pm 0.02$ |
|        |           | $\alpha$ | $-2.06 \pm 0.14$ | $-2.30 \pm 0.08$ | $-2.62 \pm 0.05$ | $-2.81 \pm 0.06$ | $-2.95 \pm 0.05$ | $-3.22 \pm 0.05$ | $-3.31 \pm 0.05$ | $-3.35 \pm 0.04$ |
|        |           | $\sigma_{PLC}$ | 0.27       | 0.29     | 0.27     | 0.15     | 0.21     | 0.13     | 0.11     | 0.10     | 0.01     |
| 1      | 7418      | $V'$   | $14.60 \pm 0.02$ | $14.06 \pm 0.01$ | $13.29 \pm 0.01$ | $12.86 \pm 0.01$ | $12.44 \pm 0.01$ | $11.86 \pm 0.01$ | $11.49 \pm 0.01$ | $11.41 \pm 0.01$ |
|        |           | $\alpha$ | $-1.83 \pm 0.12$ | $-2.25 \pm 0.05$ | $-2.63 \pm 0.07$ | $-2.79 \pm 0.03$ | $-2.95 \pm 0.04$ | $-3.16 \pm 0.03$ | $-3.26 \pm 0.03$ | $-3.30 \pm 0.03$ |
|        |           | $\sigma_{PLC}$ | 0.13       | 0.04     | 0.27     | 0.15     | 0.21     | 0.13     | 0.11     | 0.10     | 0.01     |
| 2-15   | 7340      | $V'$   | $14.34 \pm 0.03$ | $13.82 \pm 0.02$ | $13.13 \pm 0.02$ | $12.75 \pm 0.02$ | $12.38 \pm 0.02$ | $11.87 \pm 0.02$ | $11.53 \pm 0.02$ | $11.46 \pm 0.02$ |
|        |           | $\alpha$ | $-1.95 \pm 0.12$ | $-2.31 \pm 0.09$ | $-2.67 \pm 0.07$ | $-2.80 \pm 0.06$ | $-2.96 \pm 0.05$ | $-3.15 \pm 0.04$ | $-3.25 \pm 0.03$ | $-3.29 \pm 0.03$ |
|        |           | $\sigma_{PLC}$ | 0.03       | 0.01     | 0.00     | 0.01     | 0.01     | 0.01     | 0.06     | 0.07     | 0.01     |
| 3-15   | 7270      | $V'$   | $14.36 \pm 0.03$ | $13.84 \pm 0.02$ | $13.02 \pm 0.02$ | $12.74 \pm 0.02$ | $12.36 \pm 0.02$ | $11.88 \pm 0.02$ | $11.53 \pm 0.02$ | $11.46 \pm 0.02$ |
|        |           | $\alpha$ | $-1.86 \pm 0.12$ | $-2.28 \pm 0.09$ | $-2.66 \pm 0.07$ | $-2.81 \pm 0.06$ | $-2.97 \pm 0.05$ | $-3.15 \pm 0.04$ | $-3.26 \pm 0.04$ | $-3.30 \pm 0.04$ |
|        |           | $\sigma_{PLC}$ | 0.23       | 0.02     | 0.26     | 0.19     | 0.21     | 0.14     | 0.14     | 0.03     | ...
| Priors | 2-15      | $R'$   | $5.05 \pm 0.10$ | $4.31 \pm 0.10$ | $4.10 \pm 0.07$ | $3.63 \pm 0.07$ | $3.15 \pm 0.07$ | $2.67 \pm 0.07$ | $2.19 \pm 0.07$ | $1.71 \pm 0.07$ |
|        |           | $\beta'$ | $-1.86 \pm 0.05$ | $-1.40 \pm 0.05$ | $-1.27 \pm 0.05$ | $-1.00 \pm 0.05$ | $-0.64 \pm 0.05$ | $-0.30 \pm 0.05$ | $-0.19 \pm 0.05$ | $-0.07 \pm 0.05$ |

**Note:** All correlations assume an LMC distance modulus of 18.5 mag. The mean LMC extinction is $E(B-V) = 0.10$ for MF1, MF2, model 0, and model 1, and it is $E(B-V) = 0.15$ for models 2-15 and 3-15. MF1 and MF2 are the Madore & Freedman (1991) PLC relations for the LMC after shifting the period origin to log $P_0 = 1.4$ from 1.0. The uncertainties in MF1 and MF2 the zero points may be exaggerated by the shift in the period origin because we did not possess the correlations in the $V_0 - \alpha_0$ errors.
consistent treatments of the covariance between distance uncertainties in the distance. compares two absolute

| Table 3: Cepheid Distances and Extinctions |
|-------------------------------------------|
| Galaxy          | Variable | Published | Model 0 | Model 1 | Model 2-15 | Model 3-15 |
|------------------|----------|-----------|---------|---------|------------|------------|
| Galaxy          | $R_0$    | $\Theta_0$ (km s$^{-1}$) |         |         |            |            |
| LMC             | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 0.08    | $\pm 0.0$ | $\pm 0.10$ | $\pm 0.15$ |
| SMC             | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 0.04    | $\pm 0.01$ | $\pm 0.03$ | $\pm 0.09$ |
| M33             | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 0.19    | $\pm 0.03$ | $\pm 0.06$ | $\pm 0.10$ |
| NGC 300         | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 0.93    | $\pm 0.12$ | $\pm 0.15$ | $\pm 0.17$ |
| M81             | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.01    | $\pm 0.12$ | $\pm 0.15$ | $\pm 0.20$ |
| M101            | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.04    | $\pm 0.13$ | $\pm 0.15$ | $\pm 0.21$ |
| IC 4182         | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.08    | $\pm 0.15$ | $\pm 0.18$ | $\pm 0.22$ |
| NGC 5253        | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.11    | $\pm 0.16$ | $\pm 0.20$ | $\pm 0.24$ |
| NGC 925         | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.13    | $\pm 0.17$ | $\pm 0.21$ | $\pm 0.25$ |
| M96             | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.14    | $\pm 0.18$ | $\pm 0.22$ | $\pm 0.26$ |
| NGC 3351        | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.22    | $\pm 0.20$ | $\pm 0.24$ | $\pm 0.29$ |
| NGC 4536        | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.22    | $\pm 0.22$ | $\pm 0.26$ | $\pm 0.30$ |
| M100            | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.30    | $\pm 0.24$ | $\pm 0.28$ | $\pm 0.32$ |
| NGC 4496A       | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.30    | $\pm 0.26$ | $\pm 0.30$ | $\pm 0.34$ |
| NGC 4639        | $\mu - \mu_{MC}$ | $\langle E(B-V) \rangle$ | 1.30    | $\pm 0.28$ | $\pm 0.32$ | $\pm 0.36$ |

Note.—The published extinction estimates for M33, M31, IC 4182, NGC 5253, NGC 4536, NGC 4496A, and NGC 4639 are not directly comparable to our estimates because the authors used different extinction models. $?^*$ indicates that no estimate was included in the published results. The value for the solar radius $R_0$ is the consensus value from Reid (1993), and the value for the circular velocity of the Sun, $\Theta_0$, combines the $R_0$ estimate with the proper motion of Sgr A* (Backer 1996). The relative distance of the LMC and the SMC and the extinction values are from the review by Westerlund (1990), although Jacoby, Walker, & Ciardullo (1990) find higher values of 0.13 for the LMC and 0.06 for the SMC based on Balmer decrements in planetary nebulae. Our distances and the Westerlund (1990) value for the SMC are larger than the Caldwell & Laney (1991) values because of differences in defining the Cloud centers. The distance errors are very strongly correlated and cannot be treated as independent, random uncertainties.

Mpc$^{-1}$. Some of the differences may be due to our consistent treatment model (the Type Ia projects uses the Madore & Freedman (1991) PL relation based on $R_V/R_F = 1.6$ but fit the data using $R_V/R_F = 1.7$), and our use of statistically consistent treatments of the covariance between distance and extinction. For example, in NGC 5253 Saha et al. (1995) give large uncertainties in the extinction (see Table 3) that are statistically inconsistent with the small uncertainties in the distance. Figure 2 compares two absolute distance estimates, the expanding photosphere method for Type II supernovae (Eastman, Schmidt, & Kirshner 1996) and physical models of Type Ia supernovae (Höflich & Khokhlov 1996), and two relative distance estimates, surface brightness fluctuations (Tonry et al. 1996) and the MLCS/SNIa (Riess et al. 1996) method, to the Cepheid distances (see Table 4).

Magnitude selection biases may be important in model 0 because of the large scatter in the PL relations. Biases have been extensively discussed in the literature, most recently by Tanvir (1996), along with debates on how to optimally
perform model fits (e.g., fitting inverse PL relations as in Kelson et al. 1996). We can avoid the biases almost completely by using PLC relations to model the correlations in the residuals and to reduce the intrinsic scatter. Since these approaches are also physically more useful, we will not discuss selection biases further.

3.2. Model 1: Scatter in the Extinction

The magnitude residuals of all the extragalactic Cepheids are highly correlated and lie along the extinction direction (see Fig. 3). The importance of allowing individual extinctions is glaringly obvious if we compare the residuals for the Galactic and extragalactic samples—for the Galactic Cepheids there are 624 measured mean magnitudes for 121 Cepheids with an rms residual of 0.06 mag, while for the extragalactic Cepheids there are 1617 magnitudes for 545 Cepheids with an rms residual of 0.29 mag. The independent distance and extinction estimates for the Galactic Cepheids dramatically reduce their residuals compared to the rest of the sample even though a large subset of the Galactic Cepheids have five- to eight-color photometry while most of the extragalactic Cepheids have only two-color photometry.

In model 1 we allow all Cepheids an independent extinction variable, as we have already used for the Galactic Cepheids in model 0. Each Cepheid $i$ in galaxy $k$ was assigned extinction $E_{\mu} + \Delta E_{\mu}$, where $E_{\mu}$ is the mean extinction and the correction $\Delta E_{\mu}$ is forced to have zero mean for each galaxy and is limited by a Gaussian prior whose width $\sigma_{E\mu}$ is simultaneously optimized. We need the prior because most of the extragalactic Cepheids have only two filters, and as we start assigning each Cepheid individual extinctions, temperatures and metallicities we will overfit the data if we do not include the covariance matrix (eqs. 4 and 5) and optimize the priors. Some extragalactic distance estimates have used individual extinction estimates (e.g., Tanvir et al. 1995) or reddening-free magnitude estimates (e.g., Freedman & Madore 1990; Freedman et al. 1991, 1992; Saha et al. 1996a, 1996b, 1997), although the final distance estimates are usually based on the standard PL relations. These individual extinction estimates correspond to taking the limit $\sigma_{E\mu} \rightarrow \infty$ in equations (4) and (5), and they will overestimate the width of the extinction distribution in noisy data.

The changes in the Cepheid relations and distances are summarized in Tables 2 and 3 and Figure 1. In model 1 the rms residuals for the extragalactic Cepheids drop to 0.09 mag from 0.29 mag in model 0. The rms residuals for the LMC and SMC (0.06 and 0.08 mag) are smaller than the other extragalactic systems (0.10 mag) even though the average Cloud Cepheid was measured in five filters. The removal of the extinction-correlated residuals leads to large reductions in the PLC relation widths (which represent uncorrelated errors), but the residuals are still correlated (particularly $UJHK$) and the PLC widths are still broader than expected given the estimated measurement errors. The zero points and slopes show little change from model 0 and have reduced uncertainties, but the estimated extinction vector has changed considerably in the infrared, suggesting that it has absorbed some of the variance due to temperature as well as extinction. In particular, the distance of the Sun from the Galactic center ($R_{\odot}$) is strongly affected by the change in the IR extinction coefficients (from 7.5 to 6.7 kpc).

As Figure 3 illustrates, the extinction and temperature are very nearly parallel for $V$ and $I$, so it is very difficult to separate cleanly the two physical terms in noisy two-color data. Madore & Freedman (1991) question whether the two variables can be accurately separated even with more accurate three-color data used for Galactic Cepheid extinction estimates, although their position is strongly rejected by Laney & Stobie (1993, 1994). Moreover, the $HST$ Cepheid

| Galaxy    | EPM $\mu - 18.5$ | SNLa $\mu - 18.5$ | SBF $\mu - \mu_{M31}$ | SNLa/MLCS $\mu - \mu_{SN1981B}$ |
|-----------|-----------------|-------------------|------------------------|-------------------------------|
| M31       | ...             | ...               | $\pm 0.06$             | ...                           |
| M81       | ...             | ...               | $3.26 \pm 0.08$        | ...                           |
| M101      | $10.85 \pm 0.28$| ...               | ...                    | ...                           |
| IC 4182   | ...             | $9.77 \pm 0.44$   | ...                    | ...                           |
| NGC 5253  | $(9.56 \pm 0.2)$| $9.51 \pm 0.35$   | $3.65 \pm 0.10$        | $-3.17 \pm 0.09$             |
| NGC 925   | $(11.63 \pm 0.25)$| ...               | $5.53 \pm 0.09$        | ...                           |
| NGC 397   | $(12.38 \pm 0.33)$| ...               | $(5.76 \pm 0.06)$      | ...                           |
| NGC 4536  | $(12.38 \pm 0.33)$| ...               | $(5.76 \pm 0.06)$      | ...                           |
| NGC 4566  | $12.89 \pm 0.41$| ...               | $\pm 0.07$             | ...                           |
| M100      | $12.37 \pm 0.51$| ...               | ...                    | ...                           |
| NGC 4639  | $13.00 \pm 0.62$| ...               | $0.66 \pm 0.14$        | ...                           |

Note.—Absolute distances for the Cepheid galaxies based on the expanding photosphere method (EPM: Eastman et al. 1996) or physical models of Type Ia supernovae (SNLa: H"oftl"ich & Khokhlov 1996), and relative distances based on the surface brightness fluctuation method (SBF: Tonry et al. 1996) and the MLCS method for Type Ia supernovae (SNLa/MLCS: Riess et al. 1996). The SBF distances are relative to M31, and the MLCS distances are relative to SN 1981B in NGC 4536. Values without parentheses are distances to the Cepheid galaxy, while values in parentheses are for galaxies in the same group.
data may have correlated systematic errors due to crowding, differences in analysis methods, and the construction of the $I$ mean magnitude using the $V$ light curves for interpolation that model 1 interprets as extinction variations. Such systematic errors have no effect on the distance estimates, since correlated residuals must be modeled for a correct statistical treatment, but they may strongly affect our interpretation of the scatter in terms of extinction. Note, however, that the spread in color in the extragalactic systems is quite comparable to the spread in the Magellanic Clouds (Fig. 3), and it would be truly astonishing if the extinction from a large sample of objects randomly chosen from spiral galaxies failed to show significant scatter.

Other than the value of $R_0$, the distances are little changed from model 0 (see Fig. 1), with a mean shift of less than 0.01 mag. The typical distance and mean extinction uncertainties, however, are roughly half those of model 0, although the reduction is most dramatic for galaxies with small numbers of Cepheids (e.g., M96). The change is largely due to the reduced rms magnitude residual in estimating the intrinsic Cepheid luminosities. In model 0 the statistical uncertainty is $\sim 0.29/N^{1/2}$ mag where the numerical coefficient is the combination of the measurement errors and the PLC relation width, while in model 1 the typical uncertainty is $\sim 0.10/N^{1/2}$ mag. The total uncertainties are larger because of calibration uncertainties in the PLC relations and the $HST$ magnitudes that are unaffected by statistical averaging. Fitting the correlated residuals also makes our model significantly less sensitive to magnitude selection biases than model 0, although the general agreement of the results suggest they were not of great importance. The MLCS estimate of the Hubble constant becomes $72 \pm 6$ km s$^{-1}$ Mpc$^{-1}$, slightly higher than in model 0. Figure 2 compares the Cepheid distances with the distances in Table 4.

3.3. Model 2: Temperature and the Positivity of the Extinction

Figure 4 shows the mean extinctions relative to the foreground extinction and the width of the extinction distribution as a function of the metallicity of the host galaxy. The model 1 solutions are unphysical because many of the galaxies have either negative mean internal extinctions (SMC, NGC 300, IC 4182, and NGC 4639) or extinction distributions extending to negative internal extinctions. Even for the LMC and the SMC, where we have many filters, good accuracy, and no systematic problems such as the $HST$
calibration terms, there are Cepheids with extinctions less than the expected foreground extinction. Some correlation
between metallicity and extinction should exist because the
metals are required to form the dust, but this cannot explain
the robustly negative internal extinctions of the low-
metallicity galaxies. The simplest solution to the problem
(Freedman et al. 1992; Böhm-Vitense 1997) is simply to
raise the mean LMC extinction from the standard
metallicity galaxies. The simplest solution than the expected foreground extinction. Some correlation
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distribution increases the required LMC extinction to
the interpretation of the scatter in color as an extinction
derive mean LMC extinctions of only 0.07 (e.g., &Caldwell
combination of the Cepheid colors and other estimates of
polarization, interstellar absorption lines, and color
for its period, constrained by a Gaussian prior of width
each Cepheid a temperature deviating by
perature distribution in biasing the extinctions by giving
assigned reduced extinctions.
distinguishable and hotter-than-average Cepheids are
temperature distribution at Ðxed period contributes to the
solution. As et al. also noted, the tem-Freedman (1992)
found that we could not stably estimate without includ-
determine uniquely from the mean magnitudes. Indeed, we
Freedman the temperature-state vector is hard to1991),
tions, and as is frequently noted in the extragalactic
in arguing for higher than generally accepted acceptance
extinctions for the LMC and SMC, advocates an increase to
only <E> ~ 0.13 based on H i column densities, optical
polarization, interstellar absorption lines, and color
excesses. Thus the trivial solution to the negative extinction
problem of simply raising <E>LMC can only be a partial
solution. As Freedman et al. (1992) also noted, the tem-
perature distribution at fixed period contributes to the
negative extinction problem if the two variables are not
distinguishable and hotter-than-average Cepheids are
assigned reduced extinctions.
In model 2 we try to minimize the effects of the tem-
perature distribution in biasing the extinctions by giving
each Cepheid a temperature deviating by δt from the mean
for its period, constrained by a Gaussian prior of width σpc
corresponding to the width of the instability strip (see eq.
[2]). We also force the extinctions to be positive by adding
the extra terms to the likelihood function discussed in § 2.2.
Since there is some uncertainty in the extinction normal-
ization, we explore three models with <E>LMC = 0.10, 0.15,
and 0.20 labeled by model 2-10, 2-15, and 2-20, that corre-
spond to low, slightly high, and very high estimates for the
mean extinction. Equivalently, the three models can be
regarded as weakening the positive extinction constraint by
shifting the threshold for the negative extinction terms to
E(B − V) = 0.00, −0.05, and −0.10 at a fixed LMC extinct-
ion of 0.10. We adopt model 2-15 as our standard, and the
resulting PLC vectors, distances, and extinctions are pre-
sented in Tables 2 and 3. In model 2 we must determine the
temperature vector βk as well as the temperature correc-
tions, δt, and as is frequently noted in the extragalactic
critiques of the Galactic Cepheid methods (e.g., Madore &
Freedman 1991), the temperature-state vector βk is hard to
determine uniquely from the mean magnitudes. Indeed, we
found that we could not stably estimate βk without includ-
ing a prior based on the variation of color with phase.
Based on an analysis of Cepheid light curves (Kochanek
1998), we estimated a model β0k (see Table 2), assumed that
the uncertainty in the coefficients was 0.05, and added a
Gaussion prior for the deviation of βk from β0k to the likeli-
hood. We should note, however, that many of the Galactic
analyses use a very similar approach to calibrating the PLC
relations (e.g., Laney & Stobie 1986, 1993, 1994). The need
to make such a strong assumption about βk combined with
(or due to) the limited two-color photometry on most of the
extragalactic systems is a primary limitation for the remain-
der of our analysis, but the simultaneous optimization of the
prior widths keeps us from overfitting the data.
We find a width for the instability strip of σpc ≥ 0.18 ±
0.01 in all three models, which corresponds to a FWHM
in the V − I (B − I) colors at fixed period of 0.08 (0.14) mag.
Unlike the previous two models, model 2 gives nonzero
values for HST calibration variables (V, ΔI) of
(0.05 ± 0.04, −0.04 ± 0.04), (0.03 ± 0.04, −0.03 ± 0.04),
(0.02 ± 0.04, −0.03 ± 0.04) for models 2-10, 2-15, and 2-20,
respectively. In all three cases the calibration uncertainties
provide a significant part of the solution to the negative
extinction problem. Note that the PLC zero points for
model 2-15 are not directly comparable to the previous
models because of the change in the extinction. The infrared
extinction coefficients are now closer to the priors than in
model 1 and agree with the Laney & Stobie (1993) esti-
mates, and the estimated value of R0 = 7.7 ± 0.3 kpc is
again consistent with other estimates (Reid 1993).
Standard Cepheid distances are biased distance estimators
because they treat positive and negative extinctions equally.
The sense of the bias is always to overestimate the
distances, because lower extinctions correspond to higher
distances. Random photometry errors can and will produce
negative extinctions for objects with positive extinctions,
but such problems affect only the mathematics of imple-
menting the positive extinction condition, not the need for
it. Any procedure to impose the physical condition that the
extension is positive must reduce the likelihood for large
distances and thereby drive the best-fit distance estimate
downward. It is also a global bias, because the Cepheid
distance and extinction estimates are tightly correlated by
the PLC relations. When we force a galaxy with strongly
negative extinction estimates like NGC 300 to have a higher
extension and a lower distance, the correlations caused by
all the Cepheids sharing the same estimate of their intrinsic
colors will also force galaxies like M33 with strictly positive
extensions to higher extinctions and lower distances as
well. Adjusting the extinction of one galaxy without adjusting
the extinction of all galaxies is physically and statistically
incorrect, unless the reason for the adjustment is a systematic
error in the data for that particular galaxy. The negative
extinction problems should not be solved “locally.”
As Figure 5 shows, model 2 simply drives the galaxies to
lower distances and higher extensions, unless it is reasonably
unlikely that any Cepheids have negative extinctions. The
mean distances shift by −0.20, −0.14, and −0.05 mag for
models 2-10, 2-15, and 2-20, respectively, with the low LMC
extinction model 2-10 requiring the largest shift. The effect
is strongest on the low-metallicity galaxies, which means the
changes in the MLCS calibrations are considerable. The
MLCS Hubble constant estimates are 80 ± 6, 78 ± 6, and
72 ± 6 km s−1 Mpc−1 for the three models compared with 72 ± 6 km s−1 Mpc−1 for model 1. Because the effects of the
bias are global, even a relatively high extinction galaxy
like M100 is driven to lower distances. Scaling from the
Mould et al. (1995) estimate of 80 km s−1 Mpc−1, which
becomes 82 km s−1 Mpc−1 in model 1, the Hubble constant
shifts to H0 = 90, 88, and 84 km s−1 Mpc−1 in models 2-10,
2-15, and 2-20, respectively.5 Despite the large shifts, the
agreement with other distance indicators (see Fig. 2) is no
worse than in model 1, if only because the absolute distance

5 The uncertainties in the H0 estimate from M100 are dominated by the
systematic uncertainties in the model for Virgo (Mould et al. 1995).
indicators have low accuracy while the relative distances are
almost unchanged. The goodnesses of fit for model 2 are
considerably worse than model 1, even though we have
allowed separate temperature estimates for the Cepheids.
The likelihood function has increased by
0.03, 0.04, 0.15 mag dex$^{-1}$ for models 3-10, 3-15, and 3-20,
respectively, and the mean changes in color, $y_V - y_I$, are
0.15 ± 0.03, 0.13 ± 0.04, and 0.15 ± 0.07, respectively.
The less strongly we restricted the permitted range for the
extinctions, the more uncertain the metallicity dependence.
As pointed out by Sasselov et al. (1996), the positivity of the
extinction is an important component in the quantitative
determination of composition effects. The HST calibration
uncertainties still represent a major problem, and they con-
tribute much of the solution to maintaining positive extinc-
tions. The calibration variables ($\Delta V, \Delta I$) are (0.09 ± 0.04,
-0.07 ± 0.04), (0.05 ± 0.04, -0.05 ± 0.04), and (0.03 ± 0.04,
-0.03 ± 0.04) for the three solutions. The overall structure of the metallicity vector $\gamma_I$ matches theo-
retical expectations. Metal-rich Cepheids show a decreased
flux in $U$ and $B$, and then a gradually increasing flux in the
not justified by the quality of the data. If low-metallicity
Cepheids are blue ($y_V - y_I > 0$), then adding the metallicity
term can solve the extinction problem without driving the
distances of all galaxies downward as in model 2. The
metallicity terms will not, however, change the embar-
rassingly low distances to the low-metallicity galaxies in
model 2, because their distances are still reduced by
$(R_V + R_I)/2 \sim 2.7$ times the change in the extinction.

Our solutions generically made the metal-rich Cepheids
gerder than the metal-poor Cepheids, as shown in Figure 6.
The best-fit solution changes little with the assumptions
about the LMC extinction, and the absence of a metallicity
dependence is ruled out at greater than 95% confidence.
The mean luminosity changes in the $V$ and $I$ bands,
$(y_V + y_I)/2$, are $-0.13 \pm 0.09$, $-0.15 \pm 0.14$, and
$-0.17 \pm 0.15$ mag dex$^{-1}$ for models 3-10, 3-15, and 3-20,
respectively, and the mean changes in color, $y_V - y_I$, are
0.15 ± 0.03, 0.13 ± 0.04, and 0.15 ± 0.07, respectively.

3.4. Model 3: Metallicity

The model 2 solutions are not very attractive because of
the large downward shifts in the distances, and the associa-
tion of unphysical extinctions with low metallicity in
Figure 4 strongly suggests the need for a metallicity depen-
dence in the Cepheid distance. In model 3 we add a metal-
licity dependence to the Cepheid zero-point vector $\gamma_I$, as in
the model of Sasselov et al. (1996). The formal trend of
extinction with metallicity (neglecting NGC 300 and NGC 4639)
corresponds to a color dependence of $V-I \propto
(0.09 \pm 0.01)\Delta[O/H]$, which is comparable to the color
variations predicted in theoretical models (e.g., Stothers
1988; Stift 1990; Chiosi et al. 1993) and previous experimental
estimates (e.g., Gieren et al. 1993; Sasselov et al. 1996).
There are, however, two discrepant points in Figure 4, NGC
300 and NGC 4639, both of which are significantly bluer
than the trend. For NGC 4639 the large uncertainties in the
extinction can probably explain the discrepancies. NGC
300 is peculiar, and we drop it from our subsequent
analysis—in doing so, we are assuming that the blue color of
the Cepheids is either an artifact of the Freedman et al.
(1992) data or that the H II region abundance estimates are
incorrect.

In model 3 we added the metallicity dependent zero-point
correction $\gamma_I$ to model 2 and repeated the calculation for
the three different mean LMC extinctions labeled by model
3-10, 3-15, and 3-20. As in model 2, we present the full
results only for model 3-15 in Tables 2 and 3. We fit the data
leaving the input metallicities fixed ($\sigma_Z \to 0$), because
attempting to determine the metallicities from the data was

![Figure 5](image_url)

**Figure 5.** Distance comparisons between model 2 and model 1 distances for the low (top), middle (center), and high (bottom) LMC extinction estimates. The horizontal error bar is the error in model 1, while the vertical error bar is the error in model 2.

![Figure 6](image_url)

**Figure 6.** Metallicity corrections to the zero point. Likelihood contours for the change in mean magnitude $(y_V + y_I)/2$ and $V-I$ color $(y_V - y_I)$ compared with other observational and theoretical determinations. The contours are the $1-\sigma (\Delta x^2 = 2.30)$ and $2-\sigma (\Delta x^2 = 6.17)$ confidence intervals for two parameters. The G94 (Gould 1994) and MF90 (Madore & Freedman 1991, as estimated by Gould 1994) estimates included no color variation. CWC93 marks the theoretical estimate for $V-I$ from Chiosi et al. (1993). S88 and St95 mark the theoretical estimates for $B-V$ from Stothers (1988) and Stift (1990, 1995). CC87 marks the semiempirical model for $B-V$ from Caldwell & Coulson (1987). S96 marks the Sasselov et al. (1996) $V$ and $I$ determination from the EROS Cepheids.
redder bands, reaching a plateau in the infrared. The uncertainties in the $\gamma_k$ in Table 2 are dominated by the uncertainties of the mean luminosity change and the uncertainties in the color changes are considerably smaller (see Fig. 6).

Most of the debate about composition effects on extragalactic Cepheid distances has focused on the M31 Cepheids studied by Madore & Freedman (1991) and Gould (1994). These models allowed no color changes due to composition ($\gamma_k = \gamma_j$), while it is clear from theoretical models, previous experimental estimates, and our estimates that the changes in color are the dominant source of changes in distance estimates. Our estimates of the effect are slightly higher than the theoretical estimates of Chiosi et al. (1993). The $B-V$ color change of 0.28 mag dex$^{-1}$ is not as well constrained because most of the galaxies lack $B$ photometry, but it is larger than the theoretical estimates of Stothers (1988) and Stift (1990, 1995). The values match the experimental determination by Sasselov et al. (1996) using the EROS sample of Cepheids in the LMC and SMC and the typical Galactic Cepheid metallicity correction models (e.g., Caldwell & Coulson 1986, 1987; Gieren et al. 1993).

Figure 7 shows the changes in the distances relative to model 1 for the three different assumptions about the LMC extinction. As expected, the metal-poor galaxies (e.g., NGC 5253, IC 4182, NGC 4536) are shifted to lower distances relative to the metal-rich galaxies (e.g., NGC 3351, M33, M100). The mean change in distance depends on the mean extinction of the LMC, with model 3-10 still requiring a significant reduction in the mean distances. The MLCS estimates of the Hubble constant become $85 \pm 6, 80 \pm 6$, and $79 \pm 6$ km s$^{-1}$ Mpc$^{-1}$ for models 3-10, 3-15, and 3-20, respectively, while the estimates based on M100 (Mould et al. 1995) are 83, 78, and 74 km s$^{-1}$ Mpc$^{-1}$, respectively. The metal-poor calibration (MLCS) moves to lower distances and higher Hubble constants, while the metal-rich calibration (M100) moves to higher distances and lower Hubble constants. The comparison with other distance indicators (Fig. 2) is no worse than the other cases, with the exception of NGC 5253.

4. CONCLUSIONS

We have systematically explored the problem of extragalactic Cepheid distances and their primary systematic errors. While the details of some of the model implementations are certainly open to criticism, we have tried to include or illustrate all the principal uncertainties. When we implement the standard extragalactic analysis method (Madore & Freedman 1991) in model 0, we find general agreement with existing distances, with significant corrections only for the Type Ia supernova calibration galaxies. Our revised distances to the SNIa MLCS (Riess et al. 1996) calibrating galaxies produce a revised Hubble constant estimate of $69 \pm 8$ km s$^{-1}$ Mpc$^{-1}$. Our new distance estimates and uncertainties have the advantages of a homogeneous statistical treatment, a standard extinction model, and the inclusion of the full uncertainties in the extinction and Cepheid model on the distances. When we allow each Cepheid an individual extinction, the typical magnitude residual drops from 0.29 to 0.09 mag. As the sadly neglected work of Gould (1994) emphasized, a correct statistical model must include the effects of these correlations on the uncertainties in the distance independent of the physical interpretation for their origin. The danger of confusing extinction, temperature, and correlated systematic errors affects only the interpretation of the model, not the need to account for correlated residuals. Including the correlations does not change the values for the distances and mean extinctions, but it does significantly reduce their uncertainties (see Fig. 1).

In both of these models, many Cepheids require negative intrinsic extinguions for the host galaxies, which is clearly unphysical. The simplest solution to the problem is to raise the mean LMC extinction from the standard value of $<E^*_{V-LMC}> = 0.1$ to $\geq 0.2$ (e.g., Freedman et al. 1992; Böhm-Vitense 1997). However, even the high estimates of the mean extinction in the LMC by Bessell (1991) give only $<E^*_{V-LMC}> \approx 0.13$, and 90% of the LMC supergiants studied by Grieve & Madore (1986) had extinctions less than 0.18, and simply raising the mean LMC extinction is an implausible solution to the problem of negative extinctions. If we reject raising the LMC extinction, then the need for negative extinctions is conclusive evidence for additional Cepheid physics, serious correlated, systematic errors in the Cepheid data, or in the Burstein & Heiles (1984) foreground extinction estimates.

As a general mathematical point, the positivity of extinction means that the standard Cepheid distance estimates are systematically biased by their equal treatment of positive and negative extinctions. The covariance of distance and extinction mean that low-extinction estimates are associated with higher distances, so any requirement for positivity in the extinction will drive the distances to all Cepheid galaxies downward. The bias extends equally to galaxies with and without a negative extinction problem because the extinction estimates and distances are also correlated between galaxies—if you force a reduction in the distance to one galaxy, the correlations lead to a reduction in the distances to all galaxies.

Introducing a finite temperature distribution at fixed period so that blue Cepheids are interpreted as being hotter rather than having lower extinctions (see the discussion in Freedman et al. 1992) cannot solve the problem because the instability strip is narrow and because it cannot change the
mean extinction. When we allow the Cepheids a temperature distribution and force the positivity of the extinction, we find mean decreases in the distances of \(-0.20, -0.14, \) and \(-0.05\) mag for LMC mean extinctions of \(E_{\text{VLMC}} = 0.10, 0.15,\) and 0.20, respectively, as expected from the bias in the standard distance estimates. All Hubble constant estimates systematically rise, although the effect is more dramatic for the low-metallicity Type Ia Project galaxies because of their bluer Cepheids. In particular, the MLCs Hubble constant estimates become 80 \pm 6, 78 \pm 6, and 72 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}, and the M100 (Mould et al. 1995) estimates become 90, 88, and 84 \text{ km s}^{-1} \text{ Mpc}^{-1} in order of increasing LMC extinction.

The extinctions show a rough correlation with the metallicity of the host galaxy, as would be expected from theoretical models predicting that metal-poor Cepheids should be bluer than metal-rich Cepheids (e.g., Stothers 1988; Stift 1990, 1995; Chiosi et al. 1993). When we add a metallicity correction to the Cepheid zero point, similar to the model of Sasselov et al. (1996), we find a change in the mean \(V\) and \(I\) magnitudes of \(-0.14 \pm 0.14\) mag dex\(^{-1}\) and a change in the \(V - I\) color of 0.13 \pm 0.04 mag dex\(^{-1}\). The metallicity terms change little with the assumptions about the LMC extinction. Metal-rich Cepheids become fainter in \(U\) and \(B\), start getting brighter at \(V\), and show the largest increases in the infrared, as expected from line-blanking increasing the opacity toward the blue and back-warming increasing the emission toward the red. The quantitative estimates agree with both the theoretical expectations and other experimental or semiempirical determinations (e.g., Caldwell & Coulson 1986, 1987; Gieren et al. 1993; Stift 1995; Sasselov et al. 1996). The change in color with metallicity is more important than the change in luminosity, so studies focusing only on the change in luminosity (Madore & Freedman 1991; Gould 1994) miss the dominant effect. The metallicity correction solves the negative extinction problems and explains many of the discrepancies between low (Type Ia supernovae in low-metallicity galaxies) and high (other distance indicators in high-metallicity galaxies) estimates of the \(H_0\) (Gould 1994; Sasselov et al. 1996). The MLCs Hubble constant estimates from the low-metallicity galaxies are 85 \pm 6, 80 \pm 6, and 76 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}, while the high-metallicity M100 estimates are 83, 78, and 74 \text{ km s}^{-1} \text{ Mpc}^{-1} as we increase the mean LMC extinction.

There is no question that the models including temperature and metallicity are beginning to push the limits of the validity and accuracy of our current conclusions. I would like to thank many people for material and intellectual help, particularly J. A. R. Caldwell, N. R. Evans, W. L. Freedman, J. Huchra, M. Krockenberger, and D. Sasselov. R. Kirshner, R. Kurucz, A. Saha, and D. Welch offered helpful comments. J. A. R. Caldwell provided his latest compilation of phase-averaged data on the LMC and SMC Cepheids, and W. L. Freedman and A. Gould helped reconstruct the unpublished M31 data.

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