Conductance of an Artificial Atom in Strong Magnetic Fields

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Abstract

The conductance resulting from resonant tunneling through a droplet of $N \sim 30$ electrons is used to measure its chemical potential $\mu_N$. Abrupt shifts of $\mu_N$ occur at sharply defined values of the magnetic field, at which the state of the droplet changes. These are used to study part of the phase-diagram of the droplet in strong magnetic fields; we find evidence for a new phase in the spin polarized regime. We make a detailed comparison between theory and experiment: Hartree-Fock provides a quantitative description of the measurements when both spin-split states of the lowest orbital Landau level are occupied and a qualitative one in the spin polarized regime.

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Recent experiments [1,2] have demonstrated the possibility of measuring the chemical potential \( \mu_N \) of a droplet of \( N \) electrons confined by an external potential, an artificial atom. Abrupt shifts of \( \mu_N \) occur at values of the magnetic field \( B \) at which the ground state (GS) of the droplet changes. These results have stimulated calculations of the \( B-N \) phase diagram, in which each phase is designated by the quantum numbers of the GS; the changes in \( \mu_N(B) \) happen at the phase boundaries. Because exact numerical calculations are possible only for \( N \leq 6 \) [3], approximate methods [5–7] have been used for larger \( N \) to account for electron-electron interactions. The strong magnetic field regime is the appealing place to test these approximations, because the most intriguing aspects of the phase-diagram occur at these fields. In particular, MacDonald et al. [6], and Chamon et al. [7] have independently predicted the existence of new phases of a spin polarized droplet in a parabolic potential. These phases are especially interesting because any transition in the spin polarized regime is the consequence of many-body phenomena that cannot be explained by a single-electron picture.

In this Letter, we present detailed measurements of a portion of the phase diagram in strong magnetic field. We propose a new systematic approach for comparing the experimental results with the models. Using this approach, we find that Hartree-Fock (HF) [6,7] provides a quantitative description when both spin states of the lowest orbital Landau level (LL) are occupied, whereas a semi-classical model (SC) [5] does not, indicating that exchange plays an important role. We also find evidence for a new phase in the spin polarized regime, which is described qualitatively by HF.

The device that we study is of the type described by Meirav et al. [8]. It consists of a two-dimensional electron gas (2DEG) in an inverted GaAs/Al\(_x\)Ga\(_{1-x}\)As heterostructure with electrostatic gates above and below it. The bottom gate is a highly conducting substrate of \( n^+ \) doped GaAs. A positive bias, \( V_g \), applied to the bottom gate varies the density of the 2DEG. On the top surface of undoped GaAs, two metallic (TiAu) gates are lithographically patterned with a double constriction. Applying a negative bias to these top gates depletes the 2DEG 100nm underneath them, confining the electrons to an island be-
tween the constrictions. Current flows through the resulting electron droplet via the tunnel barriers caused by the constrictions. The top gate geometry of the device under investigation has been examined with an Atomic Force Microscope. The constrictions are poorly defined, but we estimate that the region between them is roughly $500 \times 500\text{nm}^2$. According to the simulation of the device by Kumar et al. [9], the external confinement potential of the droplet is approximately parabolic. Although all results presented here are for this one structure, we have observed similar features in samples of different geometries.

The negative bias on the top gate is maintained constant during the experiment and the bottom gate voltage is varied in a narrow range near $V_g = 160 \pm 1\text{mV}$, for which the electron density of the 2DEG regions outside the constriction is almost constant at $1.3 \pm 0.01 \times 10^{11}\text{cm}^{-2}$. The conductance $G$ of the device as a function of $V_g$ at $B = 0\text{T}$ is shown in the lower inset of Fig.1. It consists of quasi-periodic sharp peaks ($\Delta V_g = 1.2\text{mV}$), crudely described by the coulomb blockade mechanism [10]. In this model, when the bottom gate voltage is set between peaks, transport is suppressed by the charging energy $U \sim 0.66\text{meV}$ necessary to add an electron to the droplet. Each period thus corresponds to the addition of one electron to the droplet. At resonance, the electrochemical potential of the droplet, $\mu_N - e\alpha V_g$, is aligned with the Fermi energy of the leads and current flows; current requires a fluctuation of the charge on the droplet. Thus, the value of $V_g$ at which the peak occurs provides a measure of $\mu_N$. At $T= 0$, $\mu_N = E_N - E_{N-1}$, where $E_N$ is the energy of the $N$-electron GS.

We begin by considering the effect of magnetic field on a single conductance peak. That is, we measure $\mu_N(B)$ at constant $N$. The value of the gate voltage at which the $N$th conductance peak occurs is plotted as a function of magnetic field between 1 and $5\text{T}$ in Fig.1. McEuen et al. recognized that the change in behavior near $1.6\text{T}$ results from the depopulation of all but the lowest orbital LL [5]. The step-like behavior of the peak position above $1.6\text{T}$ can be thought of as resulting from the transfer of electrons between the two spin-split states of the lowest orbital LL [5]. Each step corresponds to a change in the quantum numbers of the GS, for example, the total spin of the droplet. The number of steps above
1.6T is proportional to \( N \), the number of electrons in the droplet, but the proportionality constant depends on the shape of the charge distribution.

A novel way to characterize the data in Fig.1 is to examine the separation in \( B \) of the upward steps. The peak conductance as a function of \( B \) has a sharp minimum at each of these steps \([4]\). The \( n \)th minimum precisely determines \( B_n \), the field for the \( n \)th step. (The \( B_n \) are indicated by arrows in Fig.1.) We plot in Fig.2a the quantity \((B_n - B_{n-1})^{-1}\) as a function of \( B_n \). Because each step corresponds to the flip of a spin, one may think of \((B_n - B_{n-1})^{-1}\) as being roughly proportional to the spin susceptibility. A fit to the form 

\[
y(B) = y_0 \left( \frac{B - B'}{B'} \right)^\epsilon
\]

gives \( B' = 1.7 \pm 0.02T \) and \( \epsilon = -0.41 \pm 0.06 \) for our data; the solid curve in Fig.2a shows the fit. The same functional form also fits the experimental data for two other devices with different geometries \((500 \times 700 \text{nm}^2 \text{ and } 450 \times 900 \text{nm}^2)\) and larger numbers of steps (\( \sim 25 \) and 35); we find \( \epsilon = -0.37 \pm 0.1 \) for all three devices.

Plotted in Fig.2c is the result obtained when \((B_n - B_{n-1})^{-1}\) is determined using \( \mu_N \) of the SC model of McEuen \textit{et al.} \([3]\). For a parabolic potential with cylindrical symmetry, 

\[
V(r) = m^* \omega_0^2 r^2/2,
\]

the SC spatial density of electrons \( \rho(r) \) is approximately that of classical electrons in zero magnetic field 

\[
\rho(0) \sqrt{1 - (r/R)^2},
\]

except near \( r \) corresponding to integer filling factors \( \nu = 2\pi \ell^2 \rho(r) \) where the electrons form an incompressible liquid. \( m^* \) is the effective mass of electrons in GaAs, \( \omega_0 \) is the oscillator frequency, \( R \) is the radius of the droplet, and \( \ell \) is the magnetic length. In this picture, the change in behavior at \( B = 1.6T \) corresponds to a filling factor \( \nu = 2 \) at \( r = 0 \). At fixed \( B \), \( \rho(r) \) is uniquely determined by \( N \) and \( \hbar \omega_0 \). We adjust \( N \) so that the calculated \( \mu_N(B) \) has the same number of steps as observed experimentally. With \( N \) fixed, we adjust \( \hbar \omega_0 \) to match the value of \( B' = 1.7T \), at which the transfer of electrons between the two spin states of the lowest LL begins in our experiment. Using this procedure, we find \( N = 42 \) and \( \hbar \omega_0 = 1.8 \text{meV} \). As seen in Fig.2c the SC model predicts values of \((B_n - B_{n-1})^{-1}\) which are roughly the same size as the measured ones. However, it does not predict the upward curvature of \((B_n - B_{n-1})^{-1}\) near \( B' \). In the SC calculation, the last spin flip occurs at 

\[
B_I = 3.23T = 1.9B'.
\]

In fact, the SC value of the ratio \( B_I/B' \) is almost independent of \( N \) and \( \hbar \omega_0 \). Although we find a step at \( 2B' \) in our
measurement, there is also an additional step at $B_c = 3.75T = 2.2B'$ (filled circle in Fig.1) not predicted by the SC model. For all devices studied we find a step at $2B'$, marking the complete depopulation of the higher energy spin state, and a step at larger field, in the spin polarized regime [11]. For one device, we have explored the phase diagram beyond $2.7B'$ and have found evidence for other steps [12].

The step at $B_c$ behaves in a way that is very different from those between $B'$ and $2B'$. By examining successive peaks in $G$ vs. $V_g$ (lower inset of Fig.1), i.e. probing the droplet at successive $N$, we find that each step in $\mu_N$ (Fig.1) shifts to higher $B$ when another electron is added to the droplet. We have averaged the shift over four consecutive conductance peaks and have plotted its inverse $[B_n(N) - B_n(N - 1)]^{-1}$ in Fig.3a for each of the steps in Fig.1. This quantity measures the slope of the phase boundary ($\partial N/\partial B$). It is clear from Fig.3a, that the step at $B_c$ has a larger slope than those at lower $B$.

The temperature dependence of the step at $B_c$ is also peculiar. Figs.4a and b show that the features between 1.7 and 3.4T disappear by 500mK as $T$ is increased. This behavior is now well understood [11]. In clear contrast, the height of the step at $B_c = 3.75T$ does not change with temperature up to 800mK, our measurement limit.

To compare our results with a more sophisticated theory, we have performed a HF calculation of $\mu_N(B)$, choosing the states of the symmetric gauge as the complete basis set, with the Hilbert space truncated to the two spin states of the lowest LL. Because of exchange, the HF $\rho(r)$ is more compact than the SC $\rho(r)$ with larger incompressible regions, smaller compressible regions and a more rapid decrease with $r$ near the edge of the droplet [7].

In Fig.2b, we plot $(B_n - B_{n-1})^{-1}$ as a function $B_n$ extracted from $\mu_N(B)$ for the HF model with $N = 27$ electrons and $\hbar\omega_0 = 2.1$meV. As with the SC model, these parameters are chosen to match the number of steps in the peak position and the experimental value of $B'$, respectively. Like the value of $\hbar\omega_0$ used for the SC model (1.8meV), the $\hbar\omega_0$ that fits the HF calculation is in agreement with the value $2 \pm 1$meV calculated by Kumar et al. from the sample geometry [11]. The HF value of $N = 27$, however, is different from that ($N = 42$)
which fits the SC model. This discrepancy is a result of the difference in shape of the charge distribution. In the HF model, the number of steps in $\mu_N(B)$ is equal to $N/2$ because the two spin states are equally occupied at $B'$, and half the electrons flip their spin as the field is increased between $B'$ and $B_I$.

It is obvious from Fig.2b that the HF calculation is in excellent quantitative agreement with the experiment (Fig.2a). This is particularly impressive since there are no other fitting parameters once $N$ and $\hbar \omega_0$ are fixed. In particular, the HF model predicts correctly the apparent divergence of $(B_n - B_{n-1})^{-1}$ near $B'$, in clear contrast with the SC model. A fit to the HF results with $y(B)$ gives $\epsilon = -0.43 \pm 0.03$, which is the same as the experimental value within the errors. The apparent divergence of $(B_n - B_{n-1})^{-1}$ in Fig.2a suggests that because of exchange the two spin states of the lowest LL are equally occupied at $B'$ in our droplet. This is consistent with another experimental observation: a new step in $\mu_N(B)$ is added between $B'$ and $2B'$ for every two electrons added, implying that the two spin states of the lowest LL of our droplet are equally populated with increasing $N$.

The HF calculation predicts that the last spin flip occurs at $B_I = 3.15T = 1.85B'$. Like the SC model, the HF ratio $B_I/B'$ is nearly independent of $N$ or $\hbar \omega_0$. Thus, above $B_I$ the droplet is spin polarized. MacDonald et al. [6], and Chamon et al. [7] showed that there exists a region in the $B$-$N$ phase diagram (sketched in the upper inset of Fig.1) in which, for $N < N_c \sim 100$, the GS of the spin polarized droplet is the maximum density droplet (MDD). In the MDD state, all the single-particle eigenstates of angular momentum index $m = 0, 1, \ldots, N - 1$ are occupied, leading to an approximately constant $\rho(r)$ in the droplet. The MDD is of course the GS of non-interacting electrons at high $B$, but surprisingly it is also the GS in a region of $B$-$N$ even in the presence of repulsive interactions [6,7].

With increasing magnetic field the radius of the MDD decreases, the electrons get closer together, and the interaction energy eventually favors a larger area droplet. HF [7] predicts that, at $B_c$, the edge undergoes a reconstruction and electrons form an annulus at a distance $\sim 2\ell$ away from the central droplet, causing an abrupt upward shift of $\mu_N$ at $B_c$ of roughly the same height as the step at $B_I$ [7]. In the HF calculation, $B_c/B_I$ is almost independent
of $\hbar \omega_0$, but it decreases with increasing $N \ll N_c$ for $N < N_c$.

The excellent quantitative agreement between HF and the experiment for $B \leq 2B'$, strongly suggests that the MDD is formed in our experiment above $2B'$. The HF calculation predicts that the transition to the reconstructed droplet occurs at $B_c = 4.21T$ for our droplet, a value larger than the one observed experimentally. In this regard, it is important to bear in mind that although the HF energy of the MDD is exact because the MDD is an exact eigenstate of the many-body Hamiltonian, the HF energy of the reconstructed droplet is only variational. Therefore, the calculated value of $B_c$ is an upper bound on the true transition field. Indeed, an exact calculation for small $N$ shows that the HF model overestimates $B_c$.

Turning to the slopes of the phase boundaries, one sees in Fig.3 that $[B_n(N) - B_n(N - 1)]^{-1}$ from HF ($\sim 2.2 \times 10^{-3}G^{-1}$) agrees fairly well with experiment ($3 \pm 1 \times 10^{-3}G^{-1}$) between $B'$ and $2B'$. However, at $B_c$, the HF value $3.2 \times 10^{-3}G^{-1}$, is smaller than the experimental value, $8 \pm 1.5 \times 10^{-3}G^{-1}$. The quantities $[B_n(N) - B_n(N - 1)]^{-1}$ at $B_I$ and $B_c$ are the slopes of the phase boundaries in the $B$-$N$ phase diagram between which the MDD is the GS. The fact that $[B_n(N) - B_n(N - 1)]^{-1}$ is larger at $B_c$ than at $B_I$ suggests that the MDD does not exist above some $N_c$. The experimental observation of both a larger value of $[B_n(N) - B_n(N - 1)]^{-1}$ at $B_c$ and a smaller value of $B_c$ than the ones predicted by HF suggests that $N_c$ is smaller than predicted by HF.

We have extended the HF calculation to obtain excited states and thus study the temperature dependence of $\mu_N$. We find that the HF excitation spectrum (proportional to the height of the $\mu_N$ steps) has an energy scale 4 times larger than the experimental one over the entire magnetic field range. Nonetheless, the steps in the region between $B'$ and $B_I$ are predicted to wash out more rapidly with increasing $T$ than the one at $B_c$ in agreement with observation (Fig.4).

Finally, we note that HF also describes the $B$ dependence of the conductance peak height. HF predicts the experimentally observed decrease in peak height just below $B_c$ followed by an increase for $B$ above $B_c$. The increase above $B_c$ is ascribed to the reduced
separation between the edge of the droplet and the leads when the annulus is formed.

The failure of HF to predict the size of the magnetic field window in which the MDD is the GS (Fig.2b) and the dependence of $B_c$ on $N$ (Fig.3b) may indicate that correlations are playing an important role in this transition. The downward step at about 3.5T (Fig.1) is also reminiscent of features predicted to result from correlations [3].

In conclusion, we have made a detailed study of the conductance peak positions in strong magnetic fields. We have focused on that part of the phase diagram in which only the lowest orbital LL with its two spin-split states are occupied. By looking at the increase in magnetic field required to flip each successive spin, we are able to make a quantitative comparison between experiment and theory. We find that HF is in excellent quantitative agreement with experiment at low field. However, when the droplet is spin polarized a new transition occurs which is only qualitatively described by HF.

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[11] We point out that while the results reported here are completely reversible and continuous, we have, from time to time, observed hysteresis and discontinuous behavior in strong magnetic fields.

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FIGURES

FIG. 1. Upper Inset: $B$-$N$ phase diagram of the droplet. The boundaries corresponding to a change of the total spin of the droplet in the $2 > \nu > 1$ regime are omitted; the MDD domain of stability is limited on one side by $B_I(N)$, the boundary of the spin polarized phase, and on the other side by $B_c(N)$, where there is a reconstruction of the charge density. Above $N_c$ the MDD phase is terminated. Lower Inset: Conductance through the island as a function of the bottom gate voltage at $B = 0T$. Main: Position of the $N$th conductance peak as a function of $B$ at $T = 100mK$. We have used a constant factor $\alpha = 0.55$ to convert the bottom gate voltage scale to energy [1,5]. The arrows indicate the minima of the conductance peak height. $B_n$ is the field for the $n$th minimum above 1.6T, with $n = \{1, ..., 14\}$.

FIG. 2. (a) $(B_n - B_{n-1})^{-1}$ vs. the $B_n$ obtained from Fig.1. The error bars represent the spread of the data when the analysis is repeated for other conductance peaks on the same device. (b) and (c) results obtained with the HF ($N = 27$ and $h\omega_0 = 2.1$meV) and SC ($N = 42$ and $h\omega_0 = 1.8$meV) calculations. The solid line is a fit with $y(B) = y_0[(B - B')/B']^\epsilon$ where $B' = 1.7 \pm 0.02T$ and $\epsilon = -0.41 \pm 0.06$ for the experiment and $\epsilon = -0.43 \pm 0.03$ for the HF. $B_I$ indicates the field onset of the spin polarized regime in both models. The solid circle indicates $B_c$. The dashed line in (c) is the constant interaction model [5] (the scale is expanded by a factor 10).

FIG. 3. (a) $[B_n(N) - B_n(N - 1)]^{-1}$ is the slope of the phase boundary; it is measured by looking at the shift in $B$-field of the same $B_n$ between two adjacent conductance peaks (or equivalently when an electron is added to the droplet). The plotted value is the average over 4 consecutive peaks and the error bars are the standard deviations (b) HF and SC values of $[B_n(N) - B_n(N - 1)]^{-1}$ measured from the simulated $\mu_N(B)$.

FIG. 4. Magnetic field dependence of the peak position at 100 and 500mK. (a) and (b) show the behavior below $2B'$, while (c) and (d) show it above $2B'$. In each figure, the peak position is offset for clarity.