Empirical p-n interactions, the synchronized filling of Nilsson orbitals, and emergent collectivity

R B Cakirli
Department of Physics, Istanbul University, Istanbul, Turkey
E-mail: rburcu@istanbul.edu.tr

Abstract. The onset of collectivity and deformation, changes to the single particle energies and magic numbers and so on are strongly influenced by, for example, proton (p) and neutron (n) interactions inside atomic nuclei. Experimentally, using binding energies (or masses), one can extract an average p-n interaction between the last two protons and the last two neutrons, called $\delta V_{pn}$. We have studied $\delta V_{pn}$ values using calculations of spatial overlaps between p and n Nilsson orbitals, considering different deformations, for the $Z=50-82$, $N=82-126$ shells, and comparison of these theoretical results with experimental $\delta V_{pn}$ values. Our results show that enhanced valence p-n interactions are closely correlated with the development of collectivity, shape changes, and the saturation of deformation in nuclei. We note that the difference of the Nilsson quantum numbers of the last filled Nilsson p and n orbitals, has a special relation, $0[110]$, in which they differ by only a single quantum in the z–direction, for those nuclei where $\delta V_{pn}$ is largest for each $Z$ in medium mass and heavy nuclei. The synchronised filling of such orbital pairs correlates with the emergence of collectivity.

1. Introduction and empirical proton-neutron (p-n) interactions

Magic numbers, the onset of deformation, configuration mixing, single particle energies etc can be interpreted in terms of proton(p)-neutron(n) interactions. Heavy nuclei with many possible interactions between nucleons are more complicated than lighter nuclei. Thus, to simplify this, it is often easier to consider only (number of) valence nucleons outside of a closed shell, which is a good approximation to gain an idea about the origins of the structure of nuclei. In these proceedings, empirical p-n interactions for heavy nuclei will mainly be discussed by considering only the number of valence nucleons. Before this, how one can extract empirical p-n interactions using masses (binding energies) will briefly be explained.

Equation (1), a double difference of binding energies, filters most of the interactions between nucleons and gives an average p-n interaction between the last two protons and the last two neutrons, called $\delta V_{pn}$. Its interpretation close to closed shells is obtained by looking at the Shell Model orbits for the unpaired last proton and the last neutron, and the overlap between their wave functions by considering their shell model quantum numbers, nl. For more details see Refs. [1,2].

$$
\delta V_{pn} (Z,N) = 1/4 \left[ (B(Z,N) - B(Z,N-2)) - (B(Z-2,N) - B(Z-2,N-2)) \right]
$$

1 To whom any correspondence should be addressed.
where $B$ is the binding energy. To interpret $\delta V_{pn}$ for deformed nuclei (near mid-shell), one has to take into account the deformed shell model, that is, Nilsson orbitals (see below).

2. The synchronized filling of Nilsson orbitals, and emergent collectivity

Figure 1 shows experimental $\delta V_{pn}$ values against neutron number for light nuclei (left) and heavy nuclei (middle) and a color coded $R_{4/2}$ plot (right) for the $Z=50-82$ and $N=82-126$ shells. It is clearly seen that $\delta V_{pn}$ has spikes at $N=Z$ in the left panel where both proton and neutron orbits fill exactly the same orbits. These spikes with large p-n interactions have been interpreted [3] by Wigner’s SU(4) spin-isospin symmetry [4]. Such a large p-n interaction is not expected in heavy nuclei due to the Coulomb force and spin-orbit force which brings unique parity orbits into each shell. However, although p-n interactions in heavy nuclei are reduced in magnitude, one can still see large p-n interactions at the same (or similar) number of valence protons ($Z_{val}$) and valence neutrons ($N_{val}$). One example is $^{168}\text{Er}$ which has 18 valence protons and 18 valence neutrons and it has a maximum in $\delta V_{pn}$ in the $N=100$ region (see Refs. [5,6] for details and for more examples). Figure 1 (middle) has some sharp zig-zags. These comprise nuclei with both $N_{val}=Z_{val}$ and odd-Z, odd-N values due to the absence of pairing effects on the last nucleons. One example here is $N=97$ ($N_{val}=15$) for $^{65}\text{Tb}$ ($Z_{val}=15$). The average p-n interaction has a peak when $N_{val}=Z_{val}$. Similar examples are also seen for Ho and other nuclei. The right panel of fig. 1 shows the locus of the maximum $\delta V_{pn}$ values shown with white circles on an $Z-N$ contour plot for $R_{4/2}$ which indicates the changes in structure from spherical (blue) through deformed (red). The black line is placed to indicate $N_{val}=Z_{val}$. As seen in this plot, the nuclei for which we have peaks in the experimental $\delta V_{pn}$ values, lie where the saturation of collectivity starts so these maxima in $\delta V_{pn}$ have a clear link to the evolution of collectivity [6].

![Figure 1](image-url)
the overall shapes or patterns of the Nilsson diagrams are so similar this sequential filling of highly overlapping pairs of orbitals satisfying $0[110]$ gives large $\delta V_{pn}$ values (see fig. 1 (middle)) along the line corresponding to saturation in $R_{d2}$ (right) and following $N_{val} \sim Z_{val}$ line, even as the deformation increases. As a result, this synchronized filling of $0[110]$ relation is directly related to onset of deformation.

![Figure 2](image-url)

**Figure 2.** Nilsson diagrams for the proton $Z= 50-82$ and neutron $N= 82-126$ shells. Based on Refs. [6,7].

In order to understand these interactions in a simple way, we calculated [6] spatial overlaps of proton and neutron Nilsson wave functions with three deformations, namely $\varepsilon = 0.05, 0.22, 0.3$ [6,7]. The calculated overlaps are compared with the experimental $\delta V_{pn}$ results in fig. 3. The left panel is experimental $\delta V_{pn}$ values, theoretical spatial overlaps for the nuclei where the data exist are shown in the middle panel. All the calculated nuclei are illustrated in the right panel of fig. 3.

Since the proton shell has 32 nucleons and the neutron shell 44 nucleons, the figures are designed as square. Thus, along the diagonal line in fig. 3, nuclei will fill similar orbitals so large overlaps (large $\delta V_{pn}$) are expected along this line. The upper black line corresponds to $N_{val}= Z_{val}$ nuclei.

Overall agreement between the data and theory is very good. There are some disagreements such as in the upper right in fig. 3. However, this is not surprising because this disagreement may happen due to a $\gamma$-soft structure that our calculations do not take into account [7]. In addition, we obtain large $\delta V_{pn}$ values with a $0[101]$ relation (not $0[110]$) around $Z= 52-64$ with $N= 92-108$ ($5/2[413]$ $p$ - $5/2[512]$ $n$ and $1/2[420]$ $p$ - $1/2[521]$ $n$ orbitals). Mass measurements in this region will help us to understand the $p$-$n$ interactions more, in particular in the future at facilities such as FAIR or FRIB.
3. Conclusion

Double differences of binding energies, $\delta V_{pn}$, extracted from high-precision mass measurements, are discussed from both experimental and theoretical perspectives by focusing on the $Z=50-82$, $N=82-126$ shells. Reflecting the clear spikes in $\delta V_{pn}$ for light nuclei, studies of $\delta V_{pn}$ for heavy nuclei are discussed in terms of the number of valence nucleons. We obtained a systematic behavior (large $\delta V_{pn}$ values) when a nucleus has equal (or almost equal) numbers of valence protons and neutrons and noticed a consistent difference between proton and neutron Nilsson orbital quantum numbers, namely $0[110]$. Calculated spatial overlaps between proton and neutron Nilsson orbitals are presented and compared with the experimental results. The sequential filling of orbitals with the $0[110]$ relation for equal numbers of valence protons and neutrons leads to a maximizing in the growth of collectivity along the $N_{val} = Z_{val}$ line.

4. References

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