Masses of 2S single heavy baryons using non-perturbative parameters in HQET

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Abstract

We have employed heavy quark effective theory (HQET) to determine the masses of $n = 2$, S-wave charm and bottom baryons. The HQET parameters $\tilde{\Lambda}$, $\lambda_1$, and $\lambda_2$ are calculated for $n = 1$ using the masses of S-wave baryons. The behavior of ratio of mass terms of $n = 1$ mesons and baryons containing these parameters are also studied by varying the bottom quark mass. The HQET symmetry of $\tilde{\Lambda}$ is used to find the parameters and masses for $n = 2$ S-wave baryons. The variation of mass of 2S baryons with the non-perturbative parameters $\lambda_1$ and $\lambda_2$ is discussed. The Regge trajectories are also plotted in the $(n, M^2)$ plane using masses of $n = 1$ and 2 charm and bottom baryons. The Regge trajectories are parallel and equidistant lines in the $(n, M^2)$ plane.

1 Introduction

Heavy light systems containing a single heavy quark are an active area of research due to continuous experimental observations. The radial excitation of these heavy baryons lies in the same mass regions as many of the recently observed baryons. Despite new observations [1, 2, 3, 4] of baryons, the spectrum for radially excited charm and bottom baryons is not much explored. In PDG [5], we find only few radially excited charm and bottom baryons. The states Λ(3000) and Λ(3055) were identified in decay channel $\Lambda^+_c K^−\pi^+$ [11]. The same state Ξ(2970)$^+$ was identified in decay modes $\Xi_c(2645)\pi$ [12], $\Xi_c^+\pi$ [13] by Belle collaboration and $\Xi_c(2455)K$ by BaBar Collaboration [14]. In 2021, Belle [15] identified the spin parity of the $\Xi_c(2970)$ to be $\frac{3}{2}^+$ using 980 fb$^{-1}$ data sample collected by the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider with the light degrees of freedom $s_1 = 0$. The assignment of $\Xi_c(2970)$ was theoretically studied by [8, 9, 10, 16, 17, 18, 19] and supported by Belle [15] to be $\Xi_c(25)$ . In 2017, LHCb [20] observed five new narrow excited $\Omega_c$ states in the $\Xi_c^+K^−$ mass spectrum with the sample of pp collision data corresponding to an integrated luminosity of 3.3 fb$^{-1}$, collected by the LHCb experiment. The states were $\Omega_c(3000)^0$, $\Omega_c(3055)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$. LHCb Collaboration in 2021 [21], suggested assignments to the four observed resonances $\Omega_c(3000)^0$, $\Omega_c(3055)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$ to be the $\lambda$-mode excitation with $J^P = \frac{1}{2}^−$, $\frac{3}{2}^−$, $\frac{3}{2}^+$ and $\frac{5}{2}^−$ respectively. The absence of $\Omega_c(3119)^0$ indicated that it can be the first radial excitation $\Omega_c(2S)$ with spin $\frac{1}{2}^+$ or $\frac{3}{2}^+$ [10, 22] or the $\rho$-mode excitation of P-wave [9].

In 2020, LHCb collaboration [23] observed a new baryon state in the $\Lambda_b^0\pi^+\pi^−$ mass spectrum with mass $m_{\Lambda_b^0}=6072.3±2.9±0.6±0.2$ MeV and natural width $\Gamma = 72±11±2$ MeV. The LHCb suggested that this new state may be assigned as $\Lambda_b^0(2S)$ resonance, the first radial excitation of the $\Lambda_b^0$ baryon. This resonance was assigned as $\Lambda_b(25)$ state in the QCD sum rules [19, 24]. Using $^3P_0$ model [25], authors considered $\Lambda_b(6072)$ as tentative assignments $\Lambda_b(2S)$, $\Sigma_b(1P)$, and $\rho$-mode excitation of $\Lambda_b(1P)$. Considering the decay widths, they assigned the $\Lambda_b(6072)$ as the lowest $\rho$-mode $\Lambda_b(1P)$ resonance. LHCb Collaboration [26] in 2018, observed a new $\Xi_b^-$ resonance with mass, $m_{\Xi_b^-(6227)}=6226.9±2.0±0.3±0.2$ MeV and decay width, $\Gamma_{\Xi_b^-(6227)}=18.1±5.4±1.8$ MeV in both the $\Lambda_b^0K^−$ and $\Xi_b^0\pi^−$ invariant mass spectra. The resonance was compatible with assignments as $\Xi_b(1P)^−$ [27, 28] and $\Xi_b(2S)$ [8].
We are using the heavy quark effective theory (HQET) to study the masses of radially excited charm and bottom baryons. The $n = 1$ S-wave charm and bottom baryons are used as input to compute the masses of $n = 2$ S-wave charm and bottom baryons. The symmetry of HQET parameters is used. The leading order non-perturbative parameters of HQET up to $1/m_Q$ are $\overline{\Lambda}$, $\lambda_1$ and $\lambda_2^Q$. In limit $m_Q \to \infty$, the $1/m_Q$ term in HQET Lagrangian vanishes. At the order of $m_Q$, all hadrons get the contribution to mass from $\overline{\Lambda}$. $\lambda_1$ gives the kinetic energy of the heavy quark and $\lambda_2$ shows the chromomagnetic interaction of the heavy quark. The $\overline{\Lambda}$ parameter comes from the leading term of Lagrangian. We expect it to have a significant contribution to the mass of heavy-light hadrons. The higher order parameters $\lambda_1$ and $\lambda_2$ have a smaller contribution in mass. These parameters have been well studied for charm and bottom mesons. Using the data of inclusive semileptonic decay $B \to Xl\nu_e$ from CLEO [29], the authors in Ref. [30] computed $\overline{\Lambda} = 0.39 \pm 0.11$ GeV and $\lambda_1 = -0.19 \pm 0.10$ GeV$^2$. They also computed the bottom and charm quark masses in $\overline{MS}$ scheme as $\overline{m}_b(m_b) = 4.45$ GeV and $\overline{m}_{c}(m_c) = 1.28$ GeV. In Ref. [31, 32], authors determined $\lambda_1 = -0.27 \pm 0.10 \pm 0.04$ GeV$^2$ for $B$ decays. The lattice QCD [33] was employed to compute the non-perturbative parameters, $\overline{\Lambda} = 0.68^{+0.02}_{-0.12}$ GeV and $\lambda_1 = -(0.45 \pm 0.12)$ GeV$^2$. There are studies related to these parameters for baryons also. Using sum rules within the framework of HQET in Ref. [34], parameters $\overline{\Lambda}$, $\lambda_1 = 0.79 \pm 0.05$ GeV for $\Lambda_Q$ baryons and $\overline{\Lambda}_2 = 0.96 \pm 0.05$ for $\Sigma_Q$ baryons were calculated. Also, they computed heavy quark masses to be $m_c = 1.43 \pm 0.05$ GeV and $m_b = 4.83 \pm 0.07$ GeV. The authors in Ref. [35] computed the parameters $\overline{\Lambda}_A = 0.81$ GeV and $\lambda_{A,1} = -0.26$ GeV$^2$ using the $m_b = 4.71$ [36] GeV. Using sum rules [37], the parameter $\overline{\Lambda} = 0.73 \pm 0.07$ GeV and $\overline{\Lambda}_2 = 0.90 \pm 0.14$ GeV are computed. These non-perturbative parameters can be used to find the masses of excited states.

This paper is organised in the following order: In sec. 2, a brief overview of the theoretical framework of HQET is given. In sec. 3, the parameters of HQET are analyzed, and role of heavy quark mass is shown with respect to mass terms of hadrons. In sec. 4, the masses of $n = 2$ baryons for S-wave are calculated. The conclusions are given in the sec. 5.

## 2 Framework

The hadrons containing a single heavy quark ($Q$) are studied using HQET. The single heavy quark is considered to be much heavier than the light quarks surrounding them. In the limit $m_Q \to \infty$, the spins of light quark get decoupled from heavy quark spin. In mesons ($Q\bar{q}$), the spin of light quark ($s_q$) couples with the orbital angular momentum ($l$) to give a total light spin ($s_l = s_q \pm l$). This total light spin then couples with heavy quark spin ($s_Q$) to give the total spin of mesons ($J = s_Q \pm s_l$). Spins $s_Q$ and $s_l$ are $\frac{1}{2}$ only as quarks are fermions. The total spin $J$ forms a doublet. These doublets are degenerate in the taken limit $m_Q \to \infty$. For baryons($QQq$), the spin of light quarks couples to form a spin ($s_q = 0, 1$). For ground state ($l = 0$), $s_q$ spin couples further with heavy quark spin ($s_Q$) to give the total spin $J = \frac{1}{2}$ for $s_Q = 0$, and $J = \frac{3}{2}$ & $\frac{5}{2}$ for $s_Q = 1$. As in mesons, the baryons with $s_Q = 1$ are degenerate in the limit $m_Q \to \infty$. By taking the effects of heavy quark mass ($m_Q$) to be finite, this degeneracy is broken. The states with $s_Q = 0$ and $J = \frac{1}{2}$ are denoted by $\Lambda$ ($Qud$) and $\Xi$ ($Qus$ and $Qds$). States with $s_Q = 1$ and $J = \frac{3}{2}$ are denoted by $\Sigma$ ($Quu$, $Qud$ and $Qdd$), $\Xi'$ ($Qus$ and $Qds$) and $\Omega$ ($Qss$). And states with $s_Q = 1$ and $J = \frac{5}{2}$ are denoted by $\Sigma^*$ ($Quu$, $Qud$ and $Qdd$), $\Xi^{*}$ ($Qus$ and $Qds$) and $\Omega^*$ ($Qss$). The baryons containing the two light quarks can be represented using SU(3) symmetry by $3 \otimes 3 = \overline{3} \oplus 6$. These multiplets are shown in Fig. 1. The HQET Lagrangian comes from QCD Lagrangian by expanding QCD Lagrangian in terms of $\frac{1}{m_Q}$. The heavy quarks symmetry breaking effects come from the higher terms of HQET Lagrangian which depends on the heavy quark mass ($m_Q$). The HQET Lagrangian is given as

$$\mathcal{L} = \overline{Q}e (iv.D)Q - \overline{Q}e \frac{D^2}{2m_Q}Q - a(\mu)g\overline{Q}e \sigma_{\mu\nu}G^{\mu\nu}Q$$

where, $D^2 = D^\mu - D.\partial^\mu$ and $D^\mu = \partial^\mu - igA^\mu$ is the covariant derivative. $v$ is the heavy quark velocity. In the limit $m_Q \to \infty$, the heavy quark velocity is the velocity of hadron. $Q_v$ is effective heavy field. $G^{\mu\nu}$ is the gluon field strength tensor. Only the first term survives in the limit $m_Q \to \infty$. Higher terms contain $\frac{1}{m_Q}$ factor and thus breaks the heavy quark symmetry.

The masses of the hadrons can be obtained by using heavy quark symmetry. All hadrons containing a single
(a) Baryons with $s_q = 0$ and $J^P = \frac{1}{2}^+$.

The flavor $\bar{3}$ representation of SU(3).

(b) Baryons with $s_q = 1$ and $J^P = \frac{1}{2}^+$ and $3^2\frac{1}{2}^+$. The baryons with $J^P = \frac{3}{2}^+$ are denoted with * in superscript. The flavor $6$ representation of SU(3).

Figure 1: The different multiplets of baryons with a heavy charm ($c$) quark. Similar multiplets are for baryons with heavy bottom ($b$) quark.

Heavy heavy quark (Q) are degenerate at order $m_Q$, and have same mass $m_Q$. At order $m_Q^0$, the hadron masses gets a contribution from the first term of the Lagrangian as shown in Eq. (1).

\[
\Lambda \equiv \frac{1}{2} \left\langle H^{(Q)} \right| H_0 \left| H^{(Q)} \right\rangle \quad (2)
\]

\[
2\lambda_1 = - \left\langle H^{(Q)} \right| \overline{Q}_v D^2_v Q_v \left| H^{(Q)} \right\rangle \quad (3)
\]

\[
16(S_Q s_l) \lambda_2^Q = a(\mu) \left\langle H^{(Q)} \right| \overline{Q}_v g\sigma_{\alpha\beta} G^{\alpha\beta} Q_v \left| H^{(Q)} \right\rangle \quad (4)
\]

where in Eq. (2), $H_0$ is the $\frac{1}{m_Q}$ order term of the Hamiltonian of HQET obtained from the first term of Lagrangian. $\Lambda$ is a HQET parameter. It has the same value for all particles in a spin-flavor multiplet. We denote $\bar{\Lambda}_H$ for $D(D^*)$ and $B(B^*)$ mesons, $\bar{\Lambda}_A$ for $\Lambda_{c(b)}$ baryons, $\bar{\Lambda}_E$ for $\Xi_{c(b)}$ baryons, $\bar{\Lambda}_I$ for $\Sigma_{c(b)}^{(*)}$ baryons and, $\bar{\Lambda}_\Omega$ for $\Omega_{c(b)}^{(*)}$ baryons. In Eq. (3), $\lambda_1$ is HQET parameter independent of $m_Q$ but is different for different multiplets of baryons. In Eq. (4), $\lambda_2^Q$ is HQET parameter depends on $m_Q$ through the dependence of $a(\mu)$ on $m_Q$. In the leading logarithmic approximation

\[
a(\mu) = \left[ \frac{\alpha_s(m(Q))}{\alpha_s(\mu)} \right]^{9/(33-2N_q)} \quad (5)
\]

where, $N_q$ is the number of light quark flavors. The $\lambda_2$ matrix element transform like $s_Q s_l$ under the spin symmetry, as $\overline{Q}_v g\sigma_{\alpha\beta} G^{\alpha\beta} Q_v$ has the same transformation property. The operator $\overline{Q}_v \sigma Q_v$ is the heavy quark spin [38]. Using $s_Q s_l = (J^2 - s_Q^2 - s_l^2)/2$, the mass equations for hadrons can be written. The masses of charm and bottom mesons in terms of HQET parameters are given in Eq. (6).

\[
m_D = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2}{2m_c} \quad (6a)
\]

\[
m_{D^*} = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_c} + \frac{\lambda_2}{2m_c} \quad (6b)
\]

\[
m_B = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} - \frac{3\lambda_2}{2m_b} \quad (6c)
\]

\[
m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} + \frac{\lambda_2}{2m_b} \quad (6d)
\]
The masses of S-wave baryons containing single charm and bottom quarks are given in Eq. (7) and (8).

\begin{align}
    m_{Λ_1} &= m_c + \bar{Λ}_1 - \frac{λ_{Λ,1}}{2m_c} \\
    m_{Σ_1} &= m_c + \bar{Σ}_1 - \frac{λ_{Σ,1}}{2m_c} - \frac{2λ_{Σ,2}}{m_c} \\
    m_{Σ_2'} &= m_c + \bar{Σ}_2' - \frac{λ_{Σ,1}}{2m_c} + \frac{λ_{Σ,2}}{m_c} \\
    m_{Λ_2} &= m_b + \bar{Λ}_2 - \frac{λ_{Λ,2}}{2m_b} \\
    m_{Σ_2} &= m_c + \bar{Σ}_2 - \frac{λ_{Σ,1}}{2m_b} - \frac{2λ_{Σ,2}}{m_b} \\
    m_{Σ_2'} &= m_b + \bar{Σ}_2' - \frac{λ_{Σ,1}}{2m_b} + \frac{λ_{Σ,2}}{m_b}
\end{align}

We have used Λ(Σ), Ξ(Ξ'), and Ω in subscript to denote for presence of zero, one, and two strange(s) quark in hadrons respectively. The SU(3) symmetry is broken for u, d, s quarks as, s quark is much heavy than u and d quarks. So, the parameters taken are also different for baryons containing different number of strange quarks.

### 3 Analysis of non-perturbative parameters

The non-perturbative parameters (\(\bar{Λ}, λ_1, λ_2\)) of HQET are useful to find masses, form factors, decay width, etc [38]. These parameters are shown in the above mass equations Eqs. (6), (7) and (8). The masses of heavy-light hadrons are very much dependent on the nature of these non-perturbative parameters. These parameters are well studied for heavy-light mesons as discussed in Sec 1. Here, we have also computed the values of all \(\bar{Λ}, λ_1\) and \(λ_2(Q)\) parameters using masses of charm and bottom mesons. The masses of charm and bottom mesons are taken from PDG [5] and some theoretical papers as given in Table 1. We have computed the HQET parameters with \(m_c = 1270\) MeV and \(m_b = 4180\) MeV as shown in Table 2 and with \(m_c = 1290\) MeV and \(m_b = 4670\) MeV as shown in Table 3. We find a drastic difference between the values of HQET parameters up to \(\frac{1}{m_Q}\) corrections by changing the heavy quark masses \((m_Q)\) shown in Tables 2 and 3. This demands a further investigation for the nature of these parameters and their contribution to masses. The values of parameters for mesons estimated by previous studies are reproduced by taking the heavy quark masses \(m_c = 1290\) MeV and \(m_b = 4670\) MeV [40].

We can have a look at the contributions of the parameters in mass by changing the mass of charm and bottom quarks. For \(m_c = 1270\) MeV and \(m_b = 4180\) MeV, term containing \(λ_1\) in \(D\) and \(D^*\) mesons, has a contribution of 615.35 MeV, and term containing \(λ_2^5\) in \(D\) meson has a contribution of -105.46 MeV and in \(D^*\) meson has 35.153 MeV. For term containing \(λ_1\) in \(B\) and \(B^*\) mesons, their contribution is about 186.96 MeV and term containing \(λ_2\) in \(B\), contribution is 34.00 MeV and in \(B^*\) meson, contribution is only 11.33
Λ parameter changes from 1320 MeV quark same in all hadrons. Thus, the Fig. 2 confirms this assumption.

The mass formulae in Eq. (6), (7) and (8), assumes the nature of heavy below for hadrons. Thus the mass of bottom quark
\[ m_\lambda \] the mass of the heavy quark. Most of the theoretical predictions and their averages give
c to be
\[ m_\lambda \] charm quark to be
\[ m_\lambda \] MeV, the ratio
\[ \frac{\lambda_1}{m_\lambda} \] provide us the behaviour of terms containing these parameters. The
\[ m_\lambda \] and heavy quark masses (\( m_c = 1270 \) MeV and \( m_b = 4180 \) MeV) given in PDG [5].

| \( \Lambda_H \) | \( \lambda_1 \) | \( \lambda_2^5 \) | \( \lambda_2^6 \) |
|---|---|---|---|
| 1320 | \( 1.56 \times 10^6 \) | \( 8.93 \times 10^4 \) | \( 9.48 \times 10^4 \) |
| \( \Lambda_{H,s} \) | \( \lambda_{1,s} \) | \( \lambda_{2,s}^5 \) | \( \lambda_{2,s}^6 \) |
| 1405 | \( 1.52 \times 10^6 \) | \( 9.13 \times 10^4 \) | \( 10.22 \times 10^4 \) |

| \( \Lambda_H \) | \( \lambda_1 \) | \( \lambda_2^5 \) | \( \lambda_2^6 \) |
|---|---|---|---|
| 1959 | \( 1.58 \times 10^6 \) | \( 4.95 \times 10^4 \) | \( 5.22 \times 10^4 \) |
| \( \Lambda_{H,s} \) | \( \lambda_{1,s} \) | \( \lambda_{2,s}^5 \) | \( \lambda_{2,s}^6 \) |
| 2068 | \( 1.66 \times 10^6 \) | \( 7.81 \times 10^4 \) | \( 5.64 \times 10^4 \) |

Table 2: Computed values of non-perturbative parameters for mesons in HQET using meson masses given in Table 1 and heavy quark masses (\( m_c = 1270 \) MeV and \( m_b = 4180 \) MeV) given in PDG [5].

| \( \Lambda_H \) | \( \lambda_1 \) | \( \lambda_2^5 \) | \( \lambda_2^6 \) |
|---|---|---|---|
| 627 | \( -0.15 \times 10^6 \) | \( 9.07 \times 10^4 \) | \( 10.59 \times 10^4 \) |
| \( \Lambda_{H,s} \) | \( \lambda_{1,s} \) | \( \lambda_{2,s}^5 \) | \( \lambda_{2,s}^6 \) |
| 713 | \( -0.19 \times 10^6 \) | \( 9.28 \times 10^4 \) | \( 11.41 \times 10^4 \) |

| \( \Lambda_H \) | \( \lambda_1 \) | \( \lambda_2^5 \) | \( \lambda_2^6 \) |
|---|---|---|---|
| 1266 | \( -0.13 \times 10^6 \) | \( 5.03 \times 10^4 \) | \( 5.84 \times 10^4 \) |
| \( \Lambda_{H,s} \) | \( \lambda_{1,s} \) | \( \lambda_{2,s}^5 \) | \( \lambda_{2,s}^6 \) |
| 1373 | \( -0.05 \times 10^6 \) | \( 7.93 \times 10^4 \) | \( 6.30 \times 10^4 \) |

Table 3: Computed values of non-perturbative parameters for mesons in HQET using masses given in Table 1 and heavy quark masses (\( m_c = 1290 \) MeV and \( m_b = 4670 \) MeV) from Ref. [40].

MeV. We find a significant decrease in contributions from terms containing \( m_b \) than from terms containing \( m_c \). This is easily justified by the mass difference of \( c \) and \( b \) quarks.

The contributions to masses by taking the quark masses \( m_c = 1290 \) MeV and \( m_b = 4670 \) MeV will provide us the behaviour of terms containing these parameters. The \( \Lambda \) parameter changes from 1320 MeV to 627 MeV. The contribution of terms containing \( \lambda_1 \) in \( D \) and \( D^* \) is 57.36 MeV and, in \( B \) and \( B^* \) is 15.84 MeV. Now things get interesting as the contributions from terms having \( \lambda_2 \) remains same as in the previous case. This can be explained as \( \lambda_2 \) given rise to the hyperfine splitting of hadrons. So, \( \lambda_2 \) contributes in the mass difference of doublets i.e., between \( J^P = 0^- \) and \( J^P = 1^- \) states of mesons. So, This difference remains same irrespective of the quark masses. Although, value of \( \lambda_2 \) changes by changing the masses of \( c \) and \( b \) quarks, the contribution of the terms containing \( \lambda_2 \) in mass formulae remains same.

By taking the ratio of masses from term containing \( \lambda_1 \) and \( \Lambda \) can be seen to estimate the contributions of different mass terms. When the heavy quark masses are taken as \( m_c = 1270 \) MeV and \( m_b = 4180 \) MeV, the ratio \( \frac{2m_c}{\Lambda} = 46.6\% \) and \( \frac{2m_b}{\Lambda} = 14.2\% \). For heavy quark masses \( m_c = 1290 \) MeV and \( m_b = 4670 \) MeV, the ratio \( \frac{2m_c}{\Lambda} = 9.1\% \) and \( \frac{2m_b}{\Lambda} = 2.52\% \). The values of ratio given above are absolute values. We have shown in Fig. 2, the above ratios with the change of mass of bottom quark and keeping the mass of charm quark to be \( m_c = 1290 \) MeV. This simply shows that the parameter \( \lambda_1 \) changes sign while changing the mass of the heavy quark. Most of the theoretical predictions and their averages give \( \lambda_1 \) to be negative for hadrons. Thus the mass of bottom quark \( m_b \) entering the hadrons is greater than 4620 MeV. In Fig. 2, the mass term ratio changes rapidly for \( \Lambda_Q \) baryons in lower mass regions of \( m_b \). The point when ratio goes below \( m_b \) axis is same in both figures of Fig. 2. This may indicate that the heavy quark behaves almost same in both mesons and baryons. The mass formulae in Eq. (6), (7) and (8), assumes the nature of heavy quark same in all hadrons. Thus, the Fig. 2 confirms this assumption.
The heavy quark masses used are $m_c$ and $m_b$ taken from Table 5. The non-perturbative parameters are calculated from the mass formulae given in Eqn. (7) and (8). The masses of 1S heavy baryons shown in Table 4 are taken from PDG [5]. The mass of $\Omega_b$ is taken from Ref. [41] and all other masses are taken from PDG.

### 4 Masses of S-wave $n = 2$ baryons

The masses of 1S heavy baryons shown in Table 4 are taken from PDG [5]. The $\Omega_b^*$ mass has been taken from the Ref. [41]. The masses are used to find HQET non-perturbative parameters for $n = 1$, as given in Table 5. The non-perturbative parameters are calculated from the mass formulae given in Eqn. (7) and (8).

The heavy quark masses used are $m_c = 1290$ MeV and $m_b = 4670$ MeV. The same heavy quark masses are taken as for mesons. To calculate masses for $n = 2$ S-wave charm and bottom baryons the following HQET symmetry is used:

$$\bar{\Lambda}_{\Xi(\xi')} - \bar{\Lambda}_{\Lambda(\Sigma)} = \bar{\Lambda}_{\Xi(\xi')} - \Lambda_{\Lambda(\Sigma)}$$

The Eq. (9) gives the difference of parameters for strange and non-strange hadrons is similar for higher radially excited states. As parameter $\bar{\lambda}_{\Lambda,1}$ and $\bar{\lambda}_{\Lambda,2}$ comes from higher order correction term $\left(1/m_Q\right)$. To estimate the nature of these parameters for higher excited states, the same case in mesons can be used. As mesons are well studied sector under the same framework, we can get a better understanding of these non-perturbative parameters for $n = 2$ case. Table 2 suggests that on going from $n = 1$ to 2 the parameter $\bar{\Lambda}$ is increased as expected, whereas the parameters $\lambda_1$ and $\lambda_2$ are decreased. Extending this behaviour to baryons using the Eq. (10), which is the difference of mass of $\Lambda_c$ (given by Eq. (7a)) for $n = 2$ and 1.

$$m_{\Lambda_c(2S)} - m_{\Lambda_c} = \bar{\Lambda}_A - \bar{\Lambda}_A - \frac{\bar{\lambda}_{\Lambda,1}}{2m_c}$$

where, $m_{\Lambda_c(2S)} = 2766.6 \pm 2.4$ is the mass of radially excited $\Lambda_c$ with $n = 2$ taken from PDG [5]. The parameter $\lambda_1$ in case of mesons, does not change significantly from $n = 1$ and 2. We take $\bar{\lambda}_{\Lambda,1} = -0.19 \times 10^6$
Table 5: Computed parameters for $n = 1$ S-wave baryons. The parameters $\lambda$ are in units MeV. $\lambda_1$ and $\lambda_2$ are in MeV$^2$ units.

| $\Lambda$ | $\lambda_{\Lambda,1}$ | $\Lambda$ | $\lambda_{\Xi,1}$ | $\Xi$ | $\lambda_{\Xi,1}$ |
|----------|-----------------|----------|-----------------|--------|-----------------|
| 931      | $-0.17 \times 10^6$ | 1103     | $-0.19 \times 10^6$ |       |                 |
| 1136     | $-0.18 \times 10^6$ | $\lambda_{\Xi,2}$ | $2.78 \times 10^4$ | $\lambda_{\Xi,2}$ | $3.02 \times 10^4$ |
| 1256     | $-0.20 \times 10^6$ | $\lambda_{\Xi',2}$ | $2.89 \times 10^4$ | $\lambda_{\Xi',2}$ | $2.93 \times 10^4$ |
| 1380     | $-0.19 \times 10^6$ | $\lambda_{\Omega,1}$ | $3.04 \times 10^4$ | $\lambda_{\Omega,2}$ | $5.59 \times 10^4$ |

(a) Mass of $\Lambda_c(2S)$ with difference of $\lambda_1$ for $n = 2$ and 1

(b) Mass of $\Lambda_c(2S)$ with difference of $\lambda_1$ for $n = 2$ and 1

Figure 3: The variation of masses of $\Lambda_Q(2S)$ with $\lambda_{\Lambda,1,ud}$

which is smaller than $\lambda_{\Lambda,1}$ from Table 5. The parameter $\lambda_{\Lambda}$ can be calculated for $n = 2$ from Eq. (10) as shown in Table 6. Now using these parameters in the Eq. (11), we get mass for $m_{\Lambda_c(2S)}$, shown in Table 7. In Fig. 3, the mass of $\Lambda_Q(2S)$ is varying with the difference of $\lambda_{\Lambda,1} - \lambda_{\Lambda,1}$ parameters. The variation of difference of parameters shows the dependence and effect of these parameters on the masses of radially excited baryons. The mass of $\Lambda_Q$ baryon is affected slightly by the variation of parameters from $n = 1$ to 2.

The above behaviour of excited baryon masses is expected in heavy quark symmetry. The same behaviour is also expected for the parameter $\lambda_2$ as shown in the Fig. 4 for the $\Sigma_Q$ baryon. The $\Sigma_Q$ has $J^P = \frac{3}{2}^+$, this make a positive contribution of $\lambda_2$ parameter as given in Eq. (7) and (8), thus a positive slope in Figs. 4b and 4d. The Figs. 4a and 4c are similar to Figs. 3a and 3b as the masses of baryons are changing with approximate same values. The masses of other baryons $\Xi_Q$ and $\Omega_Q$ also show similar results. The dependence on parameters of baryon masses justify the heavy quark symmetry. The values of parameters $\lambda_1$ and $\lambda_2$ are taken by keeping these considerations.

$$m_{\Lambda_c(2S)} - m_{\Lambda_b} = \lambda_{\Lambda} - \lambda_{\Lambda_{1,ud}} \frac{\lambda_{\Lambda,1} - \lambda_{\Lambda,1}}{2m_b}$$  \hspace{1cm} (11)

For $\Xi_c$ and $\Xi_b$, we invoke the symmetry given by Eq. (9) to get $\lambda_{\Xi} = 1574$ as shown in Table 6. Also, we take $\lambda_{\Xi,1} = -0.21 \times 10^6$, which preserves the difference of $\lambda_{\Lambda,1}$ and $\lambda_{\Lambda,1}$. Using Eqs. (13) equations similar to Eqs. (10) and (11), we can now get masses for $\Xi(2S)_c$ and $\Xi(2S)_b$ for $J^P = \frac{1}{2}^+$ as shown in Table 7.

$$m_{\Xi_Q(2S)} - m_{\Xi_Q} = \lambda_{\Xi} - \lambda_{\Xi_{1,ud}} \frac{\lambda_{\Xi,1} - \lambda_{\Xi,1}}{2m_Q}$$  \hspace{1cm} (12)

$$m_{\Xi_Q(2S)} - m_{\Xi_Q} = \lambda_{\Xi} - \lambda_{\Xi_{1,ud}} \frac{\lambda_{\Xi,1} - \lambda_{\Xi,1}}{2m_Q}$$  \hspace{1cm} (13)
For the 6 multiplet of SU(3) group we take mass \( m_{\Sigma(2S)} = 2901 \text{ MeV} \) from Ref. [17]. We also take \( \lambda_{\Sigma, 1} = -0.20 \times 10^6 \text{ MeV}^2 \), \( \lambda_{\Sigma, 2} = 2.0 \times 10^4 \text{ MeV}^2 \) and \( \lambda_{\Sigma, 2} = 2.2 \times 10^4 \text{ MeV}^2 \). The values are close to their \( n = 1 \) counterparts from Table 5. Then we can use the Eq. (14) to find \( \lambda_{\Sigma, ud} \), shown in Table 6.

\[
m_{\Sigma(2S)} - m_{\Sigma} = \lambda_{\Sigma} - \frac{\lambda_{\Sigma, 1} - \lambda_{\Sigma, 1}}{2m_c} - 2\frac{\lambda_{\Sigma, 2} - \lambda_{\Sigma, 2}}{m_c}
\]  

(14)

Now, we are well equipped to find other parameters and masses of the \( n = 2 \) S-wave baryons. The masses of other \( \Sigma \) baryons are calculated by using the above computed value of \( \bar{\Lambda}_{\Sigma} \) and taken values of \( \lambda_{\Sigma, 1}, \lambda_{\Sigma, 2} \) and \( \lambda_{\Sigma, 2} \) in equations similar to Eq. (14). The masses are shown in the Table 7. The parameter \( \bar{\Lambda}_{\Xi} \) can be estimated by using Eq. (9) and the computed parameter \( \bar{\Lambda}_{\Sigma} \). The masses of \( \Xi_Q^{(2S)} \) are then calculated by taking the values of \( \lambda_{\Xi, 1} \) and \( \lambda_{\Xi, 2} \) as shown in Table 6. These parameters are again fixed by the pattern of their corresponding parameters in Table 5. The masses are given by equations similar to Eq. (14). These equations can be formed by taking differences of mass equations of \( \Xi \) given by Eq. (9) for \( n = 2 \) and 1. Similar procedure can be employed to find the parameter \( \bar{\Lambda}_{\Xi} \) and masses of \( \Omega_Q^{(2S)} \). The required parameters \( \lambda_{\Omega, 1} \) and \( \lambda_{\Omega, 2} \) and shown in Table 6, are taken in similar fashion of their analogous parameters in Table 5. The masses of all \( \Omega(2S) \) baryons are shown in Table 7. Using the above masses of \( n = 2 \) baryons, plots can be made similar to Fig. 2. This makes the analysis interesting as the plots look similar in both cases of \( n = 2 \) and \( n = 1 \). This indicates that the role of heavy quark remains same for the radially excited states. This feature of heavy quark mass inside hadron highlights the effectiveness of heavy quark symmetry. Some of the excited states are experimentally observed candidates which are shown in Table 7. The baryons with \( n = 1 \) and \( n = 2 \) can be used to make Regge trajectories. Regge trajectory is a very powerful method.
to analyse the masses of hadrons. The Regge trajectories are defined by the following set of equations

\[ J = \alpha M^2 + \alpha_0 \]

\[ n_r = \beta M^2 + \beta_0 \]

where, \( \alpha_0 \) and \( \beta_0 \) are intercepts and \( \alpha \) and \( \beta \) are slopes of the trajectories. \( n_r \) is related to radial quantum number by \( n_r = n - 1 \). \( M \) and \( J \) are the mass and total angular momentum of the hadrons. The masses of hadrons form a straight line in \((J,M^2)\) and \((n_r,M^2)\) planes. We have plotted the Regge trajectories in the \((n,M^2)\) plane for bottom baryons we have used and calculated in the present work. The trajectories shown in Figs. 5 are nearly parallel for different baryons with same angular quantum numbers. Also, the different trajectories within same plot are equally spaced. These properties of Regge trajectories signifies a good set of masses of hadrons. The non-perturbative parameters of HQET are a very useful method to identify the masses of excited states. The confirmation of these radially excited baryons will help in further fixing the parameters given in Table 6.

## 5 Conclusion

The radially excited heavy baryons are analyzed in the framework of HQET. The non-perturbative parameters of HQET are calculated for \( n = 1 \) heavy baryons. The bottom quark mass in the charm and bottom mesons and baryons is varied to analyze the behaviour of HQET parameters \( \Lambda, \lambda_1 \) in the mass terms of HQET mass formulae. The non-perturbative parameter \( \Lambda \) are calculated for \( n = 2 \) baryons, using the HQET symmetry of the parameters. The masses of \( n = 2 \) heavy baryons are estimated by the behaviour of these non-perturbative parameters. The variations of masses of baryons on parameters \( \lambda_1 \) and \( \lambda_2 \) are also shown. The calculated masses are compared with other theoretical and experimental results. The Regge trajectories...
of $n = 2$ S-wave baryons are also shown for the masses obtained. The Regge trajectories are parallel and equidistant for the same quantum numbers ($J^P$).

6 Acknowledgment

The authors gratefully acknowledge the financial support by the Department of Science and Technology (SERB/F/9119/2020), New Delhi and for Junior Research Fellowship (09/0677(11306)/2021-EMR-I) by Council of Scientific and Industrial Research, New Delhi.

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