Simultaneous Analysis of Common and Rare Variants in Complex Traits: Application to SNPs (SCARVA\textsubscript{snp})

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Abstract: Advances in technology and reduced costs are facilitating large-scale sequencing of genes and exomes as well as entire genomes. Recently, we described an approach based on haplotypes called SCARVA\textsuperscript{1} that enables the simultaneous analysis of the association between rare and common variants in disease etiology. Here, we describe an extension of SCARVA that evaluates individual markers instead of haplotypes. This modified method (SCARVA\textsubscript{snp}) is implemented in four stages. First, all common variants in a pre-specified region (eg, gene) are evaluated individually. Second, a union procedure is used to combined all rare variants (RVs) in the index region, and the ratio of the log likelihood with one RV excluded to the log likelihood of a model with all the collapsed RVs is calculated. On the basis of previously-reported simulation studies,\textsuperscript{1} a likelihood ratio \( \geq 1.3 \) is considered statistically significant. Third, the direction of the association of the removed RV is determined by evaluating the change in \( \lambda \) values with the inclusion and exclusion of that RV. Lastly, significant common and rare variants, along with covariates, are included in a final regression model to evaluate the association between the trait and variants in that region. We apply simulated and real data sets to show that the method is simple to use, computationally efficient, and that it can accurately identify both common and rare risk variants. This method overcomes several limitations of existing methods. For example, SCARVA\textsubscript{snp} limits loss of statistical power by not including variants that are not associated with the trait of interest in the final model. Also, SCARVA\textsubscript{snp} takes into consideration the direction of association by effectively modelling positively and negatively associated variants.

Keywords: complex traits, rare and common variants

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Introduction

Biological and empirical evidence suggests that rare variants (RVs) may account for a significant proportion of the genetic component of several disorders including common complex diseases. It is also believed that better insight into the role of RVs will directly inform our understanding of disease pathophysiology, and that if RVs display sufficiently high penetrance RVs may then have predictive value. However, association analyses of RVs present many challenges, including diminished power, potential biases, and the need for a large sample size. Several of the currently available rare variant analysis methods collapse or group RVs. While this approach may help alleviate the problem of small numbers, it may dilute or mask the direction of association by including variants with no effect or that have an effect in a different direction. Recently, Lin and Tang proposed a general framework for detecting disease associations with rare variants in sequencing studies. They employed a score-type statistic (hereafter referred to as the SCORE-TEST). The SCORE-TEST is more powerful and efficient than other methods, but it could not completely resolve the problem of analyzing variants with no effect. Additionally, this method could not simultaneously consider the combined effect of common variants (CVs) and RVs on a trait in a given genomic region. The Sequence Kernel Association Test (SKAT), in contrast, can simultaneously evaluate CVs and RVs. It is a score-based variance-component test that employs a regression method to test for association of variants within a specified region. As with the SCORE-TEST, SKAT could not account for the direction of association of individual variants or exclude variants that are not associated with the trait of interest. As such, implementation of this method produces a single \( P \)-value for all variants in the region, making it uninformative regarding the specific variants or sets of variants within that region are responsible for observed associations. In this article, we present a modified version of the SCARVA technique, which is haplotyped-based, for use with marker-level data in a given region (which could be defined by a gene, target sequence, window size, or pathway involving multiple genes). A variant is considered rare with a minor allele frequency (MAF) < 5% and with an allele count \( \geq 5 \) (RVs with allele counts < 5 are removed from the analysis). Common variants (CVs) are modeled separately to determine the association of each with the phenotype. RVs, however, are combined using a union method, and the combined effect of all RVs are modeled to overcome RV-associated diminished power. After eliminating variants of no effect, a final regression model is constructed with collapsed positively-associated RVs, collapsed negatively-associated RVs, and covariates (including CVs).

Let \( Y = (y_1, \ldots, y_n)' \) be the quantitative trait outcome of \( n \) unrelated individuals, with covariates \( X = (X_1, \ldots, X_n)' \), where each \( X_i \) is a row vector of covariates. \( H = (h_1, \ldots, h_m) \) is the observed genotypes for \( n \) individuals and \( h_j = (h_{j1}, \ldots, h_{jl}) \) is the genotype of the \( j \)-th individual at the \( k \)-th loci. Each variant is assumed to be biallelic. Suppose \( m \) of the \( k \) genotypes \( h_{i1}, \ldots, h_{im} \) are common and \( l \) of them \( h_{i1}, \ldots, h_{il} \) are rare \( (m + l = k) \). Each of the observed \( h_j \) is one of the \((h_{i1}, \ldots, h_{im}) := H\) or one of \((h_{i1}, \ldots, h_{il}) := H'\) \((i = 1, \ldots, n)\).

Let \( I \) be the indicator function. The saturated model would be...
\[ y_i = \mu + \lambda \sum_{j=1}^{l} I(h_i = h_j') + \sum_{j=1}^{m} \alpha_j I(h_i = h_j') + X_i \beta + \epsilon_i, \quad (i = 1, \ldots, n) \]  

where \( \alpha_j \) is the effect of \( j \)-th associated CV \( h_j' \); \( \lambda \) is the accumulated effects of the associated RVs; \( \beta = (\beta_1, \ldots, \beta_m)' \) is the effects of all the covariates, and the \( \epsilon_i \)'s are i.i.d. with \( E(\epsilon_i) = 0 \) and \( \text{Var}(\epsilon_i) = \sigma^2 \), which is unknown and is estimated. We model the effects of all the rare variants with a single parameter \( \lambda \), instead of modeling each of the individual effects with the \( \lambda_i \)'s as suggested by others.\(^9\) To simplify notations, let \( \alpha = (\alpha_1, \ldots, \alpha_m) \), \( \theta = (\mu, \lambda, \alpha, \beta)' \), \( n_1 = (1, \ldots, 1)' \) of length \( n \), \( 0_n = (0, \ldots, 0)' \) of length \( n \), \( I \) be the identity matrix of dimension \( n \), \( Z = (1_{1n}, U, V, X) \), and \( \epsilon = (\epsilon_1, \ldots, \epsilon_n)' \), where \( U = (u_1, \ldots, u_n)' \), \( V = (v_1, \ldots, v_n)' \), \( X = (X_{1n}' \ldots, X_{ln}') \), with \( u_i = \sum_{j=1}^{l} I(h_i = h_j') \) (\( i = 1, \ldots, n \)) and \( v_i = I(h_i = h_j') \).

Then (1) is re-written as
\[ Y = Z \theta + \epsilon, \quad E(\epsilon) = 0_n, \quad \text{Var}(\epsilon) = \sigma^2 I_n. \]  

The proposed approach for the identification of CVs and RVs that are associated with the trait of interest uses the same basic technique as the SCARVA method.\(^1\) To fit the saturated model (2), the least squares estimate \( \hat{\theta} \) of \( \theta \) under model (2) is
\[ \hat{\theta} = (\hat{\mu}, \hat{\lambda}, \hat{\alpha}', \hat{\beta}') = (Z'Z)^{-1}Z'Y \]
and the estimated variance is
\[ \hat{\sigma}^2 = \frac{1}{n-2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2, \quad \hat{y}_i = z_i \hat{\beta}, \]

where \( z_i = (1, u_i, v_i, X_i') \) is the \( i \)-th row of \( Z \), and \( v_i \) is the \( i \)-th row of \( V \). The algorithm contains the following steps.

**Step 1: Analysis of CVs**

Here we test the significance of the coefficient \( \alpha_j \) (\( j = 1, \ldots, m \)) of each CV separately. Of note, the least squares estimate is equivalent to the maximum likelihood estimate under the normal model. Let \( \phi(\cdot) \) be the density function of the standard normal distribution, and \( \ell(\theta) \) be the log-likelihood of the data under \( \phi(\cdot) \). The hypothesis that the \( j \)-th CV is not associated with the trait is represented by \( H_j: \alpha_j = 0 \). Let \( z_{j-1} = (1, U, V_{j-1}, X_j) \), \( z_j = (z_{j-1}, \ldots, z_{j-n}) \), where \( V \) is \( V \) with the \( j \)-th column removed, and let \( \theta_{j-1} = (\hat{\mu}_{j-1}, \hat{\lambda}_{j-1}, \hat{\alpha}_{j-1}, \hat{\beta}_{j-1}) = (Z_{j-1}Z_{j-1})^{-1}Z_{j-1}Y \) be the least squares estimate of \( \theta_j = (\hat{\mu}, \hat{\lambda}, \hat{\alpha}, \hat{\beta})' \) under \( H_j \), where \( \alpha_{j-1} \) is \( \alpha \) with the \( j \)-th component removed, and the estimation of variance under \( H_j \) is \( \hat{\sigma}_{j-1}^2 = (1/(n-2)) \sum_{i=1}^{n} (y_i - \hat{y}_{j-1})^2 \), \( \hat{y}_{j-1} = z_{j-1} \hat{\beta} \).

Let \( \chi_i^2 \) be the centered chi-squared distribution with 1 degree of freedom. If \( H_j \) is true, then, approximately,
\[ 2(\ell(\hat{\theta}, \hat{\sigma}^2) - \ell(\hat{\theta}_j, \hat{\sigma}_{j-1}^2)) \sim \chi_i^2. \]

Given a significance level of \( \delta \), if
\[ \Lambda_j := 2(\ell(\hat{\theta}, \hat{\sigma}^2) - \ell(\hat{\theta}_j, \hat{\sigma}_{j-1}^2)) > \chi_i^2(1-\delta), \]
we reject \( H_j \). When \( \chi_i^2(1-\delta) \) is the \( (1-\delta) \)-th upper quantile of \( \chi_i^2 \), we accept \( H_j \). After testing all the \( \alpha_j \)'s (\( j = 1, \ldots, m \)), remove all the non-significant components of \( \alpha \) (the reduced term will still be denoted by \( \alpha \)). Let \( H^\prime \) be the collection of all the significantly associated CVs, and let \( V \) and \( Z \) denote their counterparts with the corresponding columns removed. Re-fit the model in equation (2) with the current \( Z \) to get a new estimate of \( \theta \) \( \hat{\theta} = (Z'Z)^{-1}Z'Y \). Linear regression models are used to evaluate CVs (MAF \( \geq 0.05 \)) within a given region. Those CVs with \( P < 0.05 \) after Bonferroni correction are added to the covariate matrix.

**Step 2: Genetic coding of RVs**

RVs (those with MAF \( < 0.05 \) and observed minor allele counts \( \geq 5 \)) are coded as dominant and recessive variables. For each coding, collapsed RV parameters are made from the union of all RVs (ie, all recessively-coded RVs are collapsed, all dominantly-coded RVs are collapsed, and the two codings are summed to create a set of additively-coded RVs, then collapsed).
Step 3: Analysis of RVs
Associated RVs can either be positively or negatively associated with the trait; let \( R^+ \) and \( R^- \) denote collections of these two types of RVs. The association of the positively- and negatively-associated RVs are modeled using different coefficients. First, we test the statistical significance of each RV \( h_j^* \) and its effect based on the \( Z \) estimated in Step 2. Let \( \hat{H}_j^* \) be the hypothesis that \( h_j^* \) is not associated with the outcome. Similarly, let \( \hat{Z}_j^* = (1, U_{-j}, V, X) \), where \( U_j = (u_{j,1}, \ldots, u_{j,n})' \), \( u_{j,i} = \sum_{k=1,k\neq j}^n I(h_k = h_k^*) \) \( (i = 1, \ldots, n) \).

Let \( \hat{\theta}_j = (\hat{\mu}_{-j}, \hat{\sigma}_{-j}, \hat{\beta}_j) = (Z_j', Z_{-j})'Z_j'Y \) be the least squares estimate of \( \theta \) under \( H_j^* \). The variance under \( H_j^* \) is estimated as \( \hat{\sigma}_{-j}^2 = (1/(n-2)) \sum_{j=1}^n (y_j - \hat{y}_{-j})^2 \), where \( \hat{y}_{-j} = z_{-j}\hat{\theta}_{-j} \) (the same notation was used in Step 2). The hypothesis \( H_j^* \) is not nested within the full model, hence we cannot use the chi-square test (as in Step 2). Instead we version of the Bayesian information criterion (BIC)\(^\text{10}\) is used. Let \( m_j \) be the number of associated parameters under \( H_j^* \), using this criterion. The model under \( H_j^* \) is preferred if \( l(\hat{\theta}_{-j}, \hat{\sigma}_{-j}^2) - (m_j/2) \log(n) \) is the largest among all \( j = 1, \ldots, l \). Here \( m_j \) is the same for all values of \( j \), thus, we pick the RVs \( h_j^* \)'s as associated for those \( j \)'s where \( l(\hat{\theta}_{-j}, \hat{\sigma}_{-j}^2) \) is larger than the others. Let \( \delta_j = l(\hat{\theta}_j, \hat{\sigma}_j^2) - l(\hat{\theta}_{-j}, \hat{\sigma}_{-j}^2) \) \((j = 1, \ldots, m)\) and \( \hat{\delta} = m^{-1} \sum_{j=1}^m \delta_j \). We reject \( H_j^* \) if there is a big relative increase in \( \delta_j \), ie, if
\[
\frac{\delta_j}{\hat{\delta}} > \gamma.
\]

Based on Yuan et al.,\(^\text{1}\) the following values for \( \gamma \) are preferred:
\( \gamma = 1.3 \) and 1.5 to represent significant and very significant, respectively.

If \( h_j^* \) is significant by the above method, and \( \hat{\lambda}_{-j} < \hat{\lambda} \), then removing \( h_j^* \) resulted in underestimate of the total effect; thus we can deduce \( h_j^* \in R^- \).

Thus, we can identify all the positively and negatively associated rare variants. Now let \( U = (U^+, U^-)' \), with \( U^+ = (u^+_1, \ldots, u^+_n) \) as \( u^+_i = \sum_{h_i \in R^+} I(h_i = h_i^*) \), \( U^- = (u^-_1, \ldots, u^-_n) \) as \( u^-_i = \sum_{h_i \in R^-} I(h_i = h_i^*) \), and \( V \) as after Step 2, \( Z = (1, U, V, X)' \), \( \hat{\lambda} = (\hat{\lambda}^+, \hat{\lambda}^-) \) and \( \theta \) be the corresponding components for \( Z \).

Briefly, first, run regression models of the combined RVs. Second, a reduced model of the RV is implemented by removing one RV at a time and noting the \( \lambda \) and log likelihood values for the reduced models. Then, calculate the \( \lambda \) difference and ratio of the likelihood of full model to that of the reduced model with one of the RV removed at a time. If the ratio of the log likelihood of the full to the reduced models is \( \geq 1.3 \), the RV is considered to be associated with the trait. If the \( \lambda \) difference between the full and the reduced model is positive, the excluded RV is negatively-associated; otherwise the RV is positively-associated.

Step 4: Combining associated RVs
Combine all positively-associated RVs into one group, with coefficient \( \hat{\lambda}^+ \), and \( H^+ \) and \( \hat{\lambda}^- \) and \( H^- \) for all the negatively-associated RVs. \( H^+ \) is the set of all significantly-associated CVs. The final model is
\[
y_i = \mu + \hat{\lambda}^+ \sum_{h_i \in H^+} I(h_i = h_i^*) + \hat{\lambda}^- \sum_{h_i \in H^-} I(h_i = h_i^*) + \sum_{h_i \in H^+} \alpha_i I(h_i = h_i^*) + X_i \beta + \epsilon_i, \quad (i = 1, \ldots, n)
\]

Step 5: Final model
Fit the final linear regression model (3), and evaluate the positively- and negatively-associated RVs, along with the significantly-associated CVs. Let \( \mathbb{H}^+ : \hat{\lambda}^+ = 0 \); \( \mathbb{H}^- : \hat{\lambda}^- = 0 \), and \( \mathbb{H}^- : \alpha_i = 0 \) for \( h_i^* \in H^- \). Chi-squared test statistics can be used to find and report the \( P \)-values of the null hypotheses.

Type I Error and Power
Let \( H_0 : \lambda = \alpha = 0, (j = 1, \ldots, m) \) be the null hypothesis that there is no association of the variants. Let \( \hat{\theta} \) be the MLE of \( \theta \) under the full model, \( \hat{\sigma}^2 \) be the corresponding variance estimator. \( \hat{\theta}_0 \) be the MLE of \( \theta \) under \( H_0 \), and \( \hat{\sigma}_0^2 \) be the corresponding variance estimator. When \( H_0 \) is true, we have, asymptotically,
\[
\Lambda_0 = 2(l(\hat{\theta}, \hat{\sigma}^2) - l(\hat{\theta}_0, \hat{\sigma}_0^2)) - \chi_{m+1}^2,
\]
where \( \chi_{m+1}^2 \) is the chi-squared distribution with \( (m+1) \) degrees of freedom. Thus, assuming the data is generated under \( H_0 \), for a given significance level \( \delta \), the type I error is approximated by...
\[ Y = y(\delta) = P_{H_0}(\Lambda_0 \geq \chi^2_{m+1}(1 - \delta)) = \delta. \]

Thus, the test is asymptotically unbiased.

The power will depend on the magnitude of the effect size. Let \( \eta = (\lambda^2 + \alpha^2_1 + \cdots + \alpha^2_m)/\bar{\delta} \), and \( \Lambda_0 \) as given above. Assuming the data are generated not from \( H_0 \), but from \( H_1 \), then asymptotically,

\[ \Lambda_0 \sim \chi^2_{m+1, \eta}, \]

where \( \chi^2_{m+1, \eta} \) is the chi-squared distribution with \((m + 1)\) degrees of freedom (where \( m \) is number of associated CVs) and non-centrality parameter \( \eta \). The power for a given \( \delta \) at \( \eta \) is approximated by

\[ \rho = \rho(\delta, \eta) = P_{H_1}(\Lambda_0 \geq \chi^2_{m+1}(1 - \delta)) \approx P(\chi^2_{m+1, \eta} \geq \chi^2_{m+1}(1 - \delta)). \]

For a given \( \delta \) and \( \eta \), \( \rho(\delta, \eta) \) can be determined using a table of the non-central chi-squared distribution. Figure 1 gives the \( \rho(\delta, \eta) \) values when \( \eta \) goes from 1 to 20, \( \delta = 0.05, 0.025 \) and 0.01, and \( m = 2, 4, 6, 8, \) and 10.

**Simulation Study**

We conducted a range of simulations based on a set of 4,000 observed quantitative traits, covariates, and corresponding alleles within a given region; for brevity, we present the results from one of these simulation exercises. Ten CVs and 10 RVs were simulated, with frequencies of \((p; q) = (p_1, \ldots, p_{10}; q_1, \ldots, q_{10}) = (0.075, 0.115, 0.130, 0.060, 0.220, 0.085, 0.105, 0.050, 0.015, 0.095; 0.008, 0.007, 0.006, 0.005, 0.005, 0.008, 0.009, 0.007, 0.008, 0.009)\), where \( p_1, \ldots, p_{10} \) denote the frequencies of CVs 1 to 10 and \( q_1, \ldots, q_{10} \) denote the frequencies of RVs 1 to 10. For the RVs, let \( H^c = R^c \cup R^c \), with \( R^c = \{h^c_i, h^c_j, h^c_k\} \) (representing all positively-associated RVs) and effect sizes \((\lambda^c, \lambda^c, \lambda^c) = (0.42, 0.52, 0.62)\), and with \( R = \{h_i, h_j\} \) (representing all negatively-associated RVs) with effect sizes \((\lambda_i, \lambda_j) = (-0.53, -0.49)\). For the CVs, we define the collection of associated variants as \( H^c = \{h^c_i, h^c_j\} \), with effects \( \alpha_3 = 0.45 \), and \( \alpha_5 = 0.50 \), thus \( \alpha_j = 0 \) \((j \neq 3, 7)\). Covariates are contained in \( X = (x_1, x_2, x_3) = (gender, age, Body Mass Index (BMI)) \), where gender has the value of 0 or 1 with 0.5 probability of each, age (years) is uniformly distributed [10,70], and BMI values are uniformly distributed [12,42]. The effect sizes of covariates are \( \beta = (\beta_1, \beta_2, \beta_3) = (0.0167, 0.008, 0.120) \). Given the genotypes and covariates, the quantitative trait follows the normal \( N(1.5, 2) \) distribution.

Using SCARVAsnp, the joint analysis detected significant associations for two CVs \((v_3 \) and \( v_5\), \( P < 0.0001 \) for both), positively-associated RVs \((u_2, u_3, u_{10}) \), \( P < 0.0001 \) and negatively-associated RVs \((u_5, u_3, P < 0.0001) \) as displayed in Table 1.

**Data Analysis for DHS Application**

We used the SCARVAsnp method to analyze the association of genotypes to plasma triglyceride (TG) in the Dallas Heart Study (DHS).\(^\text{11} \) In the DHS, angiotopoietin-like (ANGPTL) genes 3, 4, and 5 were sequenced in 3551 participants. Multiple rare non-synonymous (NS) sequence variants in these genes have been reported to be associated with lower plasma TG levels\(^\text{11} \) based on a Wilcoxon rank-sum test (with adjustments for age and gender). Using SCARVAsnp, the joint analysis detected significant associations were displayed in Table 2. For comparison, we analyzed the

![Figure 1](image-url). Power (Y-axis) and non-centrality parameters (X-axis) for different values of \( \delta \) (from left to right, panels represent \( \delta = 0.05, 0.025, \) and 0.01). **Note:** The degrees of freedom (df) are indicated by the color of the lines.
DHS data set using the recently published SCORE-TEST method\(^2\) as well as SKAT.\(^8\) We implemented the \(T_5\), \(F_p\), \(V_T\), and \(T_{\text{max}}\) variations of the SCORE-TEST for the analysis of the simulated and real datasets, but for brevity, we present only the results of the \(T_5\). The results of these analyses are displayed in Figure 2 (SCORE-TEST) and Table 3.

Three CVs and 15 RVs in \(\text{ANGPTL}3\) were analyzed (Table 2). CV ANG3-008357 (MAF = 0.40) was associated with TG levels (\(P = 2.17 \times 10^{-7}\) in the joint analysis). There were 3 RVs with ratio values \(\geq 1.3\): ANG3-005308 and ANG3-005424 were negatively associated with TG (\(P = 2.86 \times 10^{-7}\) and ANG3-004520 was positively associated with TG levels (\(P = 1.99 \times 10^{-7}\)). RVs ANG3-005308, ANG3-005424, and ANG3-004520 also had the top scores (4.61, 2.73 and 2.10, respectively) using the SCORE-TEST method, and variants in this gene were associated with TG using the SKAT method as well (\(P = 3.28 \times 10^{-7}\)).

Four CVs and 28 RVs were included in the analysis of \(\text{ANGPTL}4\) (Table 2). Two CVs (ANG4-010707 and ANG4-009155) were associated with TG levels (\(P = 2.75 \times 10^{-5}\) and \(1.54 \times 10^{-2}\), respectively). Five RVs (ANG4-006052, ANG4-009191, ANG4-001175, ANG4-010620, and ANG4-006175) had high ratio values (consistent with the top scores from SCORE-TEST, Figure 2). All 5 RVs were negatively associated with TG levels (\(P\)-value = \(1.00 \times 10^{-20}\), Table 2). An association with this gene was also identified using the SKAT method (\(P = 3.78 \times 10^{-23}\)).

Four CVs and 30 RVs were included in the analysis of \(\text{ANGPTL}5\). No CVs were associated with TG levels. Four RVs (ANG5-014661, ANG5-011617, ANG5-022751, and ANG5-012530) were negatively

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**Table 1. Association analysis of the simulated sequence data set using SCARVAsnp.**

| Type of variant | Sig. CVs/RVs | Single | Joint analysis | \(P\)-value |
|----------------|-------------|--------|----------------|------------|
| \(\nu^*(n = 10)\) | \(\nu_2\) | \(<0.0001\) | 0.46 (0.02) | \(<0.0001\) |
| \(\nu_7\) | \(<0.0001\) | 0.47 (0.02) | \(<0.0001\) |
| \(u^{**}(n = 10)\) | Rare (+) | \(u_2\) | 1.38 | \(<0.0001\) |
| | \(u_3\) | 1.30 | \(<0.0001\) |
| | \(u_{10}\) | 2.36 | 0.61 (0.04) | \(<0.0001\) |
| Rare (−) | \(u_5\) | 1.92 | \(<0.0001\) |
| | \(u_8\) | 1.69 | −0.57 (0.05) | \(<0.0001\) |

**Notes:** Sig. CVs/RVs—statistically significant common and rare variants. *Total number of CVs analyzed; **total number of RVs analyzed. Rare (+): Positively-associated RVs. Rare (−): Negatively-associated RVs. \(\beta\): regression coefficients for CVs. \(\lambda\): regression coefficients for collapsed RV terms.

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**Figure 2.** The distribution of the ratio of the log likelihood values from SCARVAsnp (blue line), and the distribution of scores from SCORE-TEST (red line) for rare variants in \(\text{ANGPT4}\) in the Dallas Heart Study (DHS).
Table 2. Results of the DHS sequence data for 3 lipid genes using SCARVAsnp.

| Genes | SNPs          | MAF | Sig. CVs/RVs | $P$ ratio | Joint model |
|-------|---------------|-----|--------------|-----------|-------------|
| ANGPTL3 | ANG3-008357 | 0.40 | Common       | $5.44 \times 10^{-10}$ | $2.17 \times 10^{-7}$ |
|        | (v* = 3, u** = 15) |     |              |           |             |
|        | ANG3-005424 | 0.01 | Rare (−)     | 2.56      |             |
|        | ANG3-005308 | 0.02 | Rare (−)     | 5.21      | $2.86 \times 10^{-7}$ |
|        | ANG3-004520 | 0.01 | Rare (+)     | 1.86      | $1.99 \times 10^{-2}$ |
| ANGPTL4 | ANG4-010707 | 0.06 | Common       | $1.50 \times 10^{-7}$ | $2.75 \times 10^{-5}$ |
|        | (v = 4, u = 28) |     |              |           |             |
|        | ANG4-009155 | 0.28 | Common       | $5.09 \times 10^{-3}$ | $1.54 \times 10^{-2}$ |
|        | ANG4-006052 | 0.03 | Rare (−)     | 3.88      |             |
|        | ANG4-009191 | 0.03 | Rare (−)     | 4.44      |             |
|        | ANG4-001175 | 0.04 | Rare (−)     | 4.08      |             |
|        | ANG4-010620 | 0.04 | Rare (−)     | 3.98      |             |
|        | ANG4-006175 | 0.04 | Rare (−)     | 3.61      | $1.00 \times 10^{-20}$ |
| ANGPTL5 | ANG5-014661 | 0.01 | Rare (−)     | 1.69      |             |
|        | (v = 4, u = 30) |     |              |           |             |
|        | ANG5-011617 | 0.02 | Rare (−)     | 2.67      |             |
|        | ANG5-022751 | 0.02 | Rare (−)     | 1.58      |             |
|        | ANG5-012530 | 0.04 | Rare (−)     | 1.32      | $2.20 \times 10^{-4}$ |
|        | ANG5-026244 | 0.01 | Rare (+)     | 3.22      |             |
|        | ANG5-012581 | 0.01 | Rare (+)     | 2.59      |             |
|        | ANG5-017106 | 0.03 | Rare (+)     | 5.58      | $6.66 \times 10^{-6}$ |

Notes: *Total number of CVs analyzed; **total number of RVs analyzed. Rare (+): Positively-associated RVs. Rare (−): Negatively-associated RVs. Abbreviations: DHS, Dallas Heart Study; Sig. CVs/RVs, statistically significant common and rare variants; $P$ ratio, $P$ values for CVs or ratio values for RVs.

associated ($P = 2.24 \times 10^{-4}$), and 3 RVs (ANG5-026244, ANG5-012581, and ANG5-017106) were positively associated with TG levels ($P = 6.66 \times 10^{-6}$; Table 2). The 7 RVs also received the top scores using the SCORE-TEST method, and this gene was also associated with TG when the SKAT method was used ($P = 2.01 \times 10^{-7}$).

While all of these methods were identified an association between each of these genes and TG, there are notable differences in the inferences possible given the output given from each method. SCARVAsnp provides a separate $P$-value for the union of all positively-associated RVs, the union of all negatively-associated RVs, and for each CV (Table 3). In contrast, the SCORE-TEST provides one $P$-value for all RVs evaluated in the region, and SKAT produces a single $P$-value all CVs and RVs together. Thus, SCARVAsnp supports conclusions

Table 3. Results of simulated and DHS data comparing SCARVAsnp, SCORE-TEST, and SKAT.

| Data sets | Total # of SNPs | SCARVAsnp | SCORE-test ($T_5^{***}$) | SKAT**** |
|-----------|-----------------|-----------|--------------------------|----------|
| Simulated | $v^* = 10$ | $v_3 < 0.0001$ | $v_3 < 0.0001$ | Rare (+) $< 0.0001$ | Rare (−) $< 0.0001$ | 2.02 $\times 10^{-66}$ |
|           | $u^{**} = 10$ |            |                         |          | 0.000044 |          |
| ANGPTL3   | $v = 3$       | ANG3-008357 = $2.17 \times 10^{-7}$ | $1.99 \times 10^{-2}$ | $2.86 \times 10^{-7}$ | ANG4-010707 = $2.75 \times 10^{-5}$ | ANG4-009155 = $1.54 \times 10^{-2}$ | 3.28 $\times 10^{-7}$ |
|           | $u = 15$     |            |                         |          | 0.008470 |          |
| ANGPTL4   | $v = 4$       | ANG4-009175 = $2.75 \times 10^{-5}$ | 3.78 $\times 10^{-23}$ |          | ANG5-009175 = $1.54 \times 10^{-2}$ | 0.000001 |
|           | $u = 28$     |            |                         |          |          |          |
| ANGPTL5   | $v = 4$       | $P$-value $> 0.05$ | Rare (+) $= 6.66 \times 10^{-6}$ | 2.01 $\times 10^{-7}$ | Rare (−) $= 2.20 \times 10^{-4}$ | 0.015066 |
|           | $u = 30$     |            |                         |          |          |          |

Notes: *Total number of CVs analyzed; **total number of RVs analyzed; ***$P$-value for the set of RVs with MAF < 5%; ****$P$-value for the set of CVs and RVs. Abbreviation: DHS, Dallas Heart Study.
about associations at a finer level of detail than existing methods, and avoids the potential pitfalls of collapsing variants that differ in their level of association (i.e., including variants of no effect) or in the direction of that association.

**Discussion**

We have extended the haplotype-based simultaneous common and rare variant analysis (SCARVA) method to be used with marker-level data (SCARVAsnp). The method is easy to use and could be computationally efficient, as illustrated in the analysis of simulated and real datasets.

In general, RV analysis is challenged by a low number of observations with the relevant genotype, and the resulting decreased power to detect associations. To solve this problem, grouping strategies have been used, which increase the number of observations with relevant genotypes and reduce the number of tests needed by evaluating collapsed markers simultaneously. It is known that not all RVs within a given region have an effect, grouping all RVs regardless of effect may produce an unsatisfactory signal-to-noise ratio. Incorporating information from prior studies or background population variation to grouping strategies has been suggested. In practice, it is unlikely to find this information for a particular disease in a specific population, of significance given that rare variation is less shared across populations than is common variation. The present method is implemented in three stages to deal with issues above. First, RVs are collapsed and the log-likelihood of a model with this collapsed term is compared to the log likelihood of a model with a collapsed term that excludes one of the RVs. After each of the RVs has been evaluated in this manner, the RVs for which the reduced model does not significantly differ from the full model are excluded from the analysis as having no effect. Secondly, RVs are grouped according to the direction of association, with separate groups for positively- and negatively-associated RVs. Thirdly, the group of positively-associated RVs, the group of negatively-associated RVs, associated CVs, and covariates are analyzed together in a joint analysis. This strategy significantly reduces the number of tests, resulting in a large number of degrees of freedom. Our method identified the same RVs as were identified using the SCORE-TEST method for a simulated and a real data set, indicating that the method is as accurate as the SCORE-TEST method for RVs.

In our simulation, CVs ($v_3$ and $v_7$) explained 24% of the total variance, while positive and negative RVs explained only 6% of the total variance alone (combined, CVs and RVs explained 29% of total variance). These findings suggest that the combined analysis of RVs and CVs may be important in explaining the so-called missing heritability from GWAS analyses. A benefit of SCARVAsnp for RV identification is that it treats CVs as covariates, acknowledging the potential contribution of both types of variation in the outcome. Future work will focus on the interaction between CVs and RVs at different levels of linkage disequilibrium.

**Conclusions**

The modified SCARVA method (SCARVAsnp) combines rare variants (RVs) for association analysis to avoid the problem of low number of observations with relevant genotypes. It eliminates variants with no effect within a given region, separately analyzes positively- and negatively-associated RVs, and allows the adjustment for common variants and covariates. SCARVAsnp was used to analyze simulated and well known data sets. In all cases, it identified the RVs that produced the highest scores in the SCORE-TEST, while eliminating the costly effect of analyzing markers with no clear association with the trait of interest. Notably, and in contrast to the SCORE-TEST and SKAT, SCARVAsnp provides $P$-values for the union of all positively- and all negatively-associated RVs separately.

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**Author Contributions**

Conceived and designed the experiments: GC, AY, AA, CR. Analysed the data: JZ, GC, AY. Wrote the first draft of the manuscript: GC, AY, WP, YZ. Contributed to the writing of the manuscript: AB, DS, AA, RC. Agree with manuscript results and conclusions: GC, AY, YZ, AB, JZ, WC, DS, AA, CR. Jointly developed the structure and arguments for the paper: GC, AY, AB, AA, DS, CR. Made critical
revisions and approved final version: GC, AY, AB, CR. All authors reviewed and approved of the final manuscript.

Competing Interests
Author(s) disclose no potential conflicts of interest.

Disclosures and Ethics
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