1. Inductive Method and scenarios

To illustrate the procedure of obtaining the scattering matrix of hexagonal waveguide meshes with an arbitrary number of Tunable Basic Units (TBUs), we describe here, for each scenario, the associated interconnection diagram and the resulting matrix after appending to a waveguide mesh composed of n-1 tri-lattices (or n-1 order mesh) an additional tri-lattice element to form an n order mesh.

**Scenario 0**: This is the simplest case and the starting point in the generation of a new mesh design. Here, only one out of the 6 ports of the new tri-lattice (Latt N) is connected to the n-1 order mesh. As shown in Supplementary Figure 1.a the addition of the new tri-lattice increases the number of mesh ports by 4, and correspondingly, the number of rows and columns in the scattering matrix. The interconnection diagram shown in Supplementary Figure 1b illustrates the signal flow possibilities inside the n-1 order mesh and between this mesh and the newly added tri-lattice through the interface node x=P. This interconnection scheme defines a system of equations associated to node x that can be solved, rendering equations (1) that provide the matrix coefficients that characterize the new waveguide mesh ports.

The resulting matrix, shown in Supplementary Figure 1c, can be decomposed in four submatrices: The first submatrix (Submatrix 1) is related to the connections between the n-1 order mesh ports excluding the ones that will be interconnected to the new lattice (in this case port P). The second (Submatrix 2) relates the new inputs in Latt N to the output ports of mesh n-1. The third (Submatrix 3) relates the input ports in mesh n-1 with the new outputs pors in Latt N. Finally, Submatrix 4 describes the connections of the inputs/output ports of Latt N.

Submatrix 1 coefficients: \[ h_{ij} = X = h_{ij}^{n-1}. \]

Submatrix 2 coefficients: \[ h_{ij}^{(P,...,P+4)} = GB'. \] (S1)

Submatrix 3 coefficients: \[ h_{ij}^{(P,...,P+4)} = TS, \]

Submatrix 4 coefficients: \[ h_{ij}^{(P,...,P+4)} = Th_{ij}G + IntCon. \]

where IntCon represents the internal connections given by the scattering matrix of the newly added trilattice Latt n.

**Scenario 1**: Here, the addition of the new tri-lattice (Latt N) increases the number of mesh ports by two but the number of complete hexagonal cells does not increase, as shown in Supplementary Figure 2.a. Supplementary Figures 2.b. and 2.c illustrate the associated interconnection diagram to be solved and the resulting matrix for the n order mesh respectively. In this case, the resulting equations are more complex since two interface nodes \(x=P-1\) and \(y=P\) are required.

Solving the system of equations related to nodes \(x=P-1\) and \(y=P\) renders equations (2) that provide the matrix coefficients that characterize the new waveguide mesh ports and the four submatrices:
**Fig. S1. Scenario 0.** (a) Connection scheme of the additional trilattice with mesh n-1, (b) interconnection graph diagram with the labelled contributions, (c) resulting matrix sections. S0: x = P. Direct contributions inside lattice N ports are not included in the graph.

\[ h_{xy} = x = h_{xy}^{n-1}, \]

\[ h_{x(P-1,...,P-2)} = B'G + DP, \]

\[ h_{y(P-1,...,P-2)} = OE + TS, \]

\[ h_{y(P-1,...,P-2)P-1,...,P-2} = T(h_{x}G + PM) + O(h_{y}P + GN). \]

**Scenario 2:** Here, the addition of the new tri-lattice increases the number of ports by two and the number of complete hexagonal cells by one as shown in Supplementary Figure 3.a. In this case, the signal flow diagram in Supplementary figure 3.b includes the possibility of recirculation between the interfacing nodes x=P-1 and y=P and the newly added trilattice unit (Latt N) as shown by connections V and W. The procedure is similar to the two previous cases by solving the system of equations associated to the nodes y and x.

Solving the system of equations related to nodes x=P-1 and y=P renders equations (3) that provide the matrix coefficients that characterize the new waveguide mesh ports and the four submatrices:

\[ h_{x} = x + \begin{bmatrix} DW & h_{x}^{n-1}VE + (1-VN)S & \vdots \\
B'V & h_{x}^{n-1}WS + (1-MW)E' & \vdots \\
(1-VN)(1-MW) - h_{xy}^{n-1}h_{yx}^{n-1}VW & \vdots \end{bmatrix} \]

\[ h_{y} = y + \begin{bmatrix} O & 0 & \vdots \\
UV & h_{y}^{n-1}P + Wh_{yx}^{n-1}P + (1-MW)N & \vdots \\
(1-VN)(1-MW) - h_{xy}^{n-1}h_{yx}^{n-1}VW & \vdots \end{bmatrix} \]

\[ h_{yP-1,...,P-2} = UG + F'P + \]

\[ + \begin{bmatrix} F'W & \vdots \\
UV & \vdots \\
(1-VN)(1-MW) - h_{xy}^{n-1}h_{yx}^{n-1}VW & \vdots \end{bmatrix} \]

\[ h_{x(P-1,...,P-2)P-1,...,P-2} = T(h_{x}G + PM) + O(h_{y}P + GN). \]

**Fig. S2. Scenario 1.** (a) Connection scheme with mesh n-1, (b) interconnection graph diagram with the labelled contributions, (c) resulting matrix sections. S1: x = P-1, y = P. Direct contribution inside lattice N ports is not included in the graph.
Fig. S3. Scenario 2. (a) Connection scheme with mesh n-1, (b) interconnection graph diagram with the labelled contributions, (c) resulting matrix sections. \( x = P - 1, y = P \). Direct contribution inside lattice N ports is not included in the graph.

**Scenario 3**: In this case as shown in Supplementary Figure 4.a, the addition the new tri-lattice does not increase the number of ports, since it connects three ports to the previous mesh and the number of complete cells is increased by one. Here, the interconnection diagram involves three interfacing nodes \( x, y, z \), depicted in Supplementary Figure 4.b. The procedure to obtain the coefficients of the different submatrices is similar to the three previous cases. Final expressions are included in (4).

\[
\begin{align*}
\xi_z &= C(MK + H_y E) + L(NE + H_y K), \\
\xi_1 &= 1 - FE - IK - H_z z_2, \\
\xi_i &= \left( C(1 - BC - JL) + \right) \\
&\quad \left( SC + E L \right) + z_1 z_2, \\
\xi_3 &= \left( 1 - BC - JL \right) \\
&\quad \left( SC + E L \right) Hz z_1 \\
\xi_4 &= \left( 1 - BC - JL \right) \\
&\quad \left( BC + J L \right) Hz z_2,
\end{align*}
\]

**SM1**

\[
\begin{align*}
z_i &= \frac{C(1 - BC - JL) + (SC + E L) Hz z_1}{(1 - BC - JL) + (BC + J L) Hz z_2}, \\
z_4 &= \frac{(SC + LE + z_2 z_1)}{(1 - CB - LJ)}.
\end{align*}
\]

**SM2**

\[
\begin{align*}
h_{ij} &= X + (DK + B'E) z_i + R z_i. \\
\xi_z &= C(MK + H_y E) + L(NE + H_y K), \\
\xi_1 &= 1 - FE - IK - H_z z_2, \\
\xi_i &= \left( 1 - BC - JL \right) \\
&\quad \left( IP + FG \right), \\
\xi_3 &= \left( 1 - BC - JL \right) \\
&\quad \left( H + Ch x G + CMP \right), \\
\xi_4 &= \left( 1 - BC - JL \right) \\
&\quad \left( BC + J L \right) Hz z_2,
\end{align*}
\]

**SM3**

\[
\begin{align*}
z_i &= \frac{C(1 - BC - JL) + (SC + E L) Hz z_1}{(1 - BC - JL) + (BC + J L) Hz z_2}, \\
z_4 &= \frac{\beta_1 + \alpha_1 z_2}{(1 - BC - JL)}, \\
z_3 &= \frac{(1 - BC - J L) \beta_2 - H_z z_1 \beta_1}{(H_z z_1 \alpha_1 - (1 - BC - JL) \alpha_2)}, \\
z_2 &= \frac{(1 - BC - J L) \beta_1 - H_z z_1 \beta_1}{(H_z z_1 \alpha_1 - (1 - BC - JL) \alpha_2)}, \\
z_1 &= \frac{(1 - BC - J L) \beta_2 - H_z z_1 \beta_1}{(H_z z_1 \alpha_1 - (1 - BC - JL) \alpha_2)}, \\
z_0 &= \frac{(1 - BC - J L) \beta_1 - H_z z_1 \beta_1}{(H_z z_1 \alpha_1 - (1 - BC - JL) \alpha_2)},
\end{align*}
\]

\[
\begin{align*}
y_3 &= (S + B L y_3 + \chi_2 z_3) / (1 - BC), \\
x_3 &= (S + B L y_3 + \chi_2 z_3) / (1 - BC), \\
h_{ij} &= O y_3 + A z_3 + T x_3.
\end{align*}
\]
\[
\chi_1 = NE + H_y K_y, \\
\chi_2 = MK + H_x K_x, \\
\chi_3 = 1 - IK - FE,
\]
\[
\alpha_1 = - \chi_1 (H_y P + JH + NG) + \ldots, \\
\chi_1 (H_x G + BH + MP),
\]
\[
\alpha_2 = (1 - BC) \chi_1 + JC \chi_2, \\
\alpha_3 = - \chi_1 (H_x G + BH + MP) - \ldots, \\
\chi_1 (H_x H + IP + FG),
\]
\[
\alpha_4 = \chi_1 CH_x \chi_1 (1 - BC), \\
\beta_1 = (1 - JL) \chi_x + BL \chi_x, \\
\beta_2 = \chi x L + \chi x H_x L,
\]
\[
y_3 = (\alpha_3 \alpha_2 - \alpha_1 \alpha_4) / (\alpha_4 \beta_1 + \alpha_1 \beta_2),
\]
\[
x_3 = (-y_3 * \beta_2 + \alpha_3) / \alpha_3,
\]
\[
z_3 = \left( y_3 (1 - JL) \right) / \chi_1,
\]
\[
or
\]
\[
z_3 = \left( H_x H + IP + FG + \right) \chi_3 / \chi_3
\]
\[
H_{[\ldots,2]}(P_{[\ldots,2]} \ldots) = \\
Oy_3 + Az_3 + T x_3 + IntCont.
\]

After the definition of each scenario and the presentation of the equations that must be used for each case, we propose the following procedure to progress sequentially. First, one must know the waveguide mesh shape or distribution of the TBUs. Then, a procedure involving the segmentation of the arrangement into tri-lattices is performed. If the targeted arrangement cannot be done, it is possible to define and place dummy TBUs in the external perimeter to provide the discretization of the mesh into an integer number of tri-lattices. These will be transparent and no-loss auxiliary components. Then, the practitioner defines/load the TBU settings to be considered in the simulation. These can involve arbitrary parameters like losses, internal coupling factors, phase configurations, and any arbitrary TBU architecture and configuration in general. With these data the computation of the scattering matrix of each tri-lattice is straightforward. Next, the inductive procedure is performed, where the next waveguide mesh scattering matrix is computed as a function of the scattering matrix of the previous step and the one of the new adjacent tri-lattice. An identification of the next scenario is mandatory to choose between Eqs (S1-S4), depending on the number of ports that are interconnected. For each scenario the ports to be connected must be identified in both the previous mesh and the new tri-lattice.

H = \text{procedure} \text{getSMatrixMesh}

Require: Mesh definition
Require: Mesh segmentation into tri-lattices (1-N)
Require: getSMatrix of TBUs (1-3N)
Require: getSMatrix of Tri-lattices (1-N)
\]

\[
H(1) = H^H(1);
\]

for n from 2 to N do
\]

s = defineScenario \( (H^H(n)) \);
\]

case s equal to 0
\]

\[
H(n) = f(H(n-1), H^H(n)) \% \text{Eqs (S1)}
\]

case s equal to 1
\]

\[
H(n) = f(H(n-1), H^H(n)) \% \text{Eqs (S2)}
\]

case s equal to 2
\]

\[
H(n) = f(H(n-1), H^H(n)) \% \text{Eqs (S3)}
\]

case s equal to 3
\]

\[
H(n) = f(H(n-1), H^H(n)) \% \text{Eqs (S4)}
\]

end case
\]

end for
\]

H = H(N);

2. Procedure to develop the model for different waveguide mesh topologies.

Waveguide meshes can be implemented using different unit cell geometries [1],[2] (square, hexagonal, triangular, etc.). These have been benchmarked against a series for photonic figures of merit and the hexagonal topology features the best performance in terms of most of them [1]. Nevertheless, it might be advisable for other reasons to implement the mesh following alternative unit cell geometries. The inductive method described here can also be applied in those cases to obtain the overall waveguide mesh scattering matrix provided that a suitable building block is identified. Supplementary Figure 5 highlights different building block alternatives for each of the main mesh geometries. Depending on the waveguide mesh topology and the chosen TBU, we can straightforwardly identify the different connection scenarios and proceed to solve their associated system of equations in the same way as is done in this paper for the tri-lattice and the hexagonal topology.

3. Further Examples on Single Wavelength Analysis

One of the main benefits of the method proposed resides in its powerful and flexible behavior. To further illustrate its versatility, here we include more examples where multiple 2x2 and 4x4 linear transformations are implemented by more than one circuit programmed over the same waveguide mesh in the case of single-wavelength operation. Moreover, we test the flexibility of the method to characterize the performance of waveguide meshes under realistic conditions by showing how it can be employed to evaluate the impact of having non-ideal components.
Fig. S4. Scenario 3. (a) Connection scheme with mesh $n-1$, (b) interconnection graph diagram with the labelled contributions, (c) resulting matrix sections. $x = P-2$, $y = P-1$, $z = P$. Direct contribution inside lattice $N$ ports is not included in the graph.

Fig. S5. Possible building blocks for the implementation of the inductive method described in the paper in different waveguide mesh cell topologies. The highlighted structures identify unitary elements upon which the given mesh topology can be constructed by suitable addition of them. (a) Square waveguide mesh, (b) Hexagonal waveguide mesh, (c) Triangular waveguide mesh.

Linear transformation examples
A wide variety of signal processing operations involve mode transformations, which can be described in terms of multiple input/multiple output linear optics transformations given by an $N \times N$ unitary matrix $U$, [3,4]. These include, among others, switching and broadcasting, mode combiners and splitters, and quantum logic gates. We programmed the waveguide mesh to demonstrate several $2 \times 2$, $3 \times 3$ and $4 \times 4$ linear transformations. These are relevant examples of signal processing tasks that are needed in different applications and the results are shown in Supplementary Figs. 6-8.

$2 \times 2$ Transformations (Hadamard, Pauli-y, z): Supplementary Figure 6a shows an example of three simultaneously programmed $2 \times 2$ transformations. These matrices, specified in (4), are the Y-, Z- Pauli gates and the Hadamard matrix. Supplementary Fig. 6a illustrates the configuration of the considered waveguide mesh. First, the circuit location is chosen for the three transformations (coloured background). Then, the access to the structures is provided by properly configuring the TBUs. The configuration of the phase and coupling factor of each TBU concerning the interferometric part is provided by the adaptation to the hexagonal waveguide mesh [5] of the original rectangular interferometer transformation, [4]. The resulting configuration coefficients are included in Supplementary Table 6.

$$
H_{p,y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad H_{had} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad H_{p,z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
$$

Supplementary Figure 6b illustrates the targeted circuit layouts and the access ports in the waveguide mesh (red ink). Supplementary Figure 6c illustrates the computed amplitude and phase response of the overall mesh at the centre wavelength. A light-dashed line surrounds the targeted ports, resulting in a perfect implementation of the desired transformations and the additional undesired input/output port relationships that are enabled by the waveguide mesh programming.
Fig. S6 Scalable Analysis Method application to single wavelength operation of waveguide mesh configuration for universal linear interferometers. (a) Mesh architecture and configuration for simultaneously implementing three 2x2 linear transformations. (b) Equivalent circuit layouts with indication of the input and output ports in red ink. (c) Moduli and phases of all the 40x40 matrix coefficients when the 2x2 transformations are programmed to implement a Pauli-Y, Hadamard and Pauli-Z, respectively.

4x4 Transformations DFT & CNOT: Here, we increase the complexity of the circuits by programming a larger size transformation. Supplementary Fig. 7a illustrates the implementation example of a linear transformation of a Controlled-NOT (C-NOT) gate and simultaneously, a Discrete Fourier Transform (DFT) operator, both described by unitary 4x4 matrices, as specified in (5). The CNOT gate is a 2 bit gate where one bit acts as a control bit while the other is the information bit. If the control bit is “0” then the logical value of the information bit at the input port is unchanged at the output port. If the control bit is “1”, the logical value of the information bit at the input port is swapped (from 0 to 1 or from 1 to 0) at the output port. The CNOT gate is the fundamental building block of quantum information circuits. In photonics the CNOT gate is implemented using a dual rail approach. This means that the logical bit is implemented using 2 waveguides and the information bit is implemented using another 2 waveguides. The presence of light in one waveguide of the pair indicates a logical zero, while its presence in the other indicates a logical 1. In this way a photonic implementation of the CNOT gate requires 4 input and 4 output ports and its operation a 4x4 unitary matrix.

The coupling and phase coefficients of each TBU are computed and specified in Supplementary Table 7. We can program the mesh to implement these gates in a very compact layout. Again, the results illustrated in Supplementary Fig. 7c match with the targeted values of amplitude and phase and again, the 40x40 waveguide mesh scattering matrix provides information related to the additional undesired input/output port relationships that are enabled by the waveguide mesh programming.

Non-unitary Linear transformations: The Single Value Decomposition (SVD) can be used to decompose a non-unitary linear operator into two unitary operators and an intermediate diagonal matrix, [6]. For example, a linear transformation characterizes the operation of an optical hybrid used in coherent detection. This functionality can be described by a two by two operator described in (6). Then, the application of SVD results in three- matrices U, S, V’ in (7), that satisfy $H_{Hybrid} = USV’$.

$$H_{DFT} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -i & -i \\ 1 & -1 & -1 \\ 1 & i & -i \end{pmatrix}, \quad H_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (S5)$$

$$H_{Hybrid} = \begin{pmatrix} 1 & 1 \\ 1 & -j \end{pmatrix}. \quad (S6)$$

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-0.875\pi} & e^{-0.375\pi} \\ e^{0.875\pi} & e^{0.375\pi} \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad V' = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}. \quad (S7)$$
For its implementation using the hexagonal waveguide mesh, we program the three matrices in a cascaded configuration V’-S-U as illustrated in the Supplementary Figure 8. The values of each TBU are provided by Supplementary Table 8. The SVD decomposition applied to the 2x4 Hybrid transformation in the main document is specified in (8), together with the corresponding values in Supplementary Table 1.

![Diagram](image)

**Fig. S7 Scalable Analysis Method application to single wavelength operation of waveguide mesh configuration for universal linear interferometers.** (a) Mesh architecture and configuration for simultaneously implementing two 4x4 linear transformations. (b) Equivalent circuit layouts with indication of the input and output ports in red ink. (c) Moduli and phases of all the 40x40 matrix coefficients when the 2x2 transformations are programmed to implement a CNOT (Blue background) and Discrete Fourier Transform operator (Ochre background).

![Diagram](image)

**Fig. S8 Scalable Analysis Method application to single wavelength operation of waveguide mesh configuration for universal linear interferometers.** (a) Mesh architecture and configuration for a 2x2 Hybrid operator described by cascading two 2x2 linear transformations and a phase and amplitude tapping array. (b) Equivalent circuit layout with indication of the input and output ports in red ink. (c) Moduli and phases of all the 40x40 matrix coefficients when the 2x2 transformation is programmed to implement an optical hybrid operator.
Analysis for non-ideal components: The non-ideal behaviour of the TBUs leads to additional losses, scattering, and errors when setting the values for the phases and coupling factors. In the particular case of the implementation and analysis of linear transformations, we can use a simple model that assumes equal insertion loss for every TBU and normal coupling factor and phase drift. In the particular case of the example illustrated in Supplementary Figures 7, here, we illustrate the test for transformations (CNOT and DFT4) displayed in the example of Supplementary Figure 7. Here, we illustrate the test for two different $\sigma_K$ equal to 0.5% and 1%, whereas $\sigma_\phi$ is varied between 0 and 0.5%.

The medians of the Fidelity distributions are:

| $\sigma_K$ | Fidelity of CNOT | Fidelity of DFT4 |
|------------|------------------|------------------|
| 0.5%, 0%   | 98.00 %          | 99.45 %          |
| 0.5%, 0.5% | 97.13 %          | 98.14 %          |
| 1.0%, 0%   | 96.11 %          | 98.86 %          |
| 1.0%, 0.5% | 95.34 %          | 98.44 %          |

Note that these results tend to confirm the findings reported by Burgwal and co-workers [7] in the sense that given a linear transformer topology, the fidelity is dependent on the application. Note as well that our method of analysis can include simultaneously the effect of errors in both the TBU power splitting ratio and phase shift and the TBU model could be substituted by any desired alternative.

Synthesis and analysis of photonic integrated circuits: As mentioned in the main document, the more powerful and versatile characteristics of the analytic method are unleashed when using it for spectral characterization. Here, the wavelength (or frequency) dimension is added and truly spectral transfer functions are immediately provided in a few seconds. Moreover, the method can be readily applied in the analysis of complex setups resulting either from the simultaneous programming of multiple independent circuits over the mesh or from involved multistage resonant filters. We provide here two application examples to each case illustrated in Supplementary Figures 10 and 11 respectively. The case illustrated in Supplementary Figure 10 correspond to the simultaneous programming of three different ring cavities (of different cavity lengths) in the same waveguide mesh. The method generates a 40x40 matrix of transfer functions, each one spanning 1000 wavelengths. However, for practical purposes the only interesting transfer functions are $h_{22,20}$, $h_{7,8}$ and $h_{3,1}$, which can be recovered form the
main system matrix by using an input vector given by \( I = (i_1, i_2, \ldots, i_{40}) \) where \( i_k = 0 \) unless \( k = 3, 8, 20 \) and \( i_3 = i_8 = i_{20} = 1 \). Note that the relevant transfer functions are exactly recovered and moreover, the remaining undesired contribution can also be retrieved (although they are not shown in the figure). In addition, the method allows to investigate the effect of changing individual phase shifters in the ring cavities.

Supplementary Fig. 11 illustrates the model application to a complex 2D resonant structure (in this case a three stage SCISSOR each one composed of a two cavity CROW. The schematic is depicted in Supplementary Figure 11b. Once configured by employing the coefficients in Supplementary Table 10, we obtained the transmission and reflection responses shown in Supplementary Fig. 11 c1. We can see the effect of switching off columns of CROWs by modifying the corresponding coupling coefficients to \( K = 0 \). The different traces for switching on 1, 2 and three second-order CROWs are illustrated in Supplementary Figure 11 c2. The method generates again the 40x40 matrix of transfer functions, each one spanning 1000 wavelengths. The interesting transfer functions are \( h_{33,13} \) and \( h_{32,13} \), which characterize the transmitted and reflected signals respectively and can be recovered form the main system matrix by using an input vector given by \( I = (i_1, i_2, \ldots, i_{40}) \) where \( i_k = 0 \) unless \( k = 13 \) and \( i_{13} = 1 \). Again, the scalable method provides a fast and exact determination of the transfer functions even for this particularly involved structure where both longitudinal and lateral coupling and recirculations are allowed.

5. Relation between Overall TBU Coupling/Phase variation and internal Couplers variation in the balanced Mach-Zehnder configuration.

The Monte-Carlo test performed in this work consider the variation of the whole coupling and phase response of a generic TBU, rather than the variation in the inner couplers and phase-modulators in a balanced Mach-Zehnder configuration. This decision is due to the fact that the TBU architecture is not constrained to the MZI configuration and several alternatives can be employed. To compare the variability in both approaches, we have performed series of 1000-instances Monte-Carlo Test of the MZI-based TBU architecture considering different standard deviation of the inner couplers. These are defined by two independent normal distributions with mean 0.5 (blue-trace). For comparison, we also consider the effect of having the same coefficient in the input and output coupler (red-trace). We show that the numbers employed during the paper of \( \sigma_K \) equal to 0.5% and 1% are in good agreement to the ones achievable with the state-of-the-art fabrication and design techniques.

6. Tables with the coupling and phase configuration for each programmed PIC in the document

Tables with coupling and phase values. For the implementation of linear transformations, we have employed the adaptation of the rectangular interferometer approach [4], to be programmed over a hexagonal waveguide mesh performed in [5].
Fig. S10 Scalable Analysis Method application to full spectral analysis of a waveguide mesh implementing three Optical Ring Resonator filters composed of cavity lengths equal to 6, 10, 12 TBUs. (a) Mesh architecture and configuration for simultaneously implementing the three filters. (b) Equivalent circuit layouts with indication of the input and output ports in red ink (upper). (c1) Transmission response of the 6-BUL ORR, (c2) Transmission Response of the 10-BUL ORR and (c4) the 12-BUL ORR. (c3) Tunability response of 6-BUL ORR for $\phi_c=0$, 0.5, 1 and 1.5, respectively. The coupling factors are included in Supplementary Table 9.

Fig. S11. (a) Waveguide mesh configuration for the implementation of a three stage SCISSOR where each stage is composed of a second order CROW. Note that both lateral and longitudinal recirculations are allowed in this structure. (b) Circuit layout, (c1) Spectral Response (moduli) for equal value of the resonator coupling constants $K=0.07$, (c2) Spectral Response (moduli) when switching off the third and both third and second CROW units.
**Supplementary Table 1:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 3 in the main document. DFT and a 2x4 Optical Hybrid respectively.

| TriLatt | KA | KB | KC | TA/\pi | TB/\pi | TC/\pi |
|---------|----|----|----|--------|--------|--------|
| 1       | 1  | 0  | 1  | 0      | 0      | 0      |
| 2       | 1  | 0  | 1  | 0      | 0      | 1      |
| 3       | 1  | 0  | 1  | 0      | 0      | 0      |
| 4       | 1  | 0  | 1  | 0      | 0      | 1      |
| 5       | 1  | 0  | 1  | 0      | 1      | 0      |
| 6       | 1  | 0.5| 1  | 0      | 0      | -1     |
| 7       | 1  | 0  | 1  | 0      | 1      | 0      |
| 8       | 1  | 0.5| 1  | 0      | 0      | 1.5    |
| 9       | 1  | 0.5| 1  | 0      | 0      | -0.4097|
| 10      | 1  | 0  | 1  | 0      | 0      | 0      |
| 11      | 1  | 0.6667| 1  | 0      | 0      | 1      |
| 12      | 1  | 0  | 1  | 0      | 0      | 0      |
| 13      | 1  | 0.6667| 1  | 0      | 0      | -0.4548|
| 14      | 1  | 0  | 1  | 0      | 1      | 0      |
| 15      | 1  | 0.5| 1  | 0      | 0      | -0.5   |
| 16      | 1  | 0  | 1  | 0      | 1      | 0      |
| 17      | 1  | 0.75| 1  | 0      | 0      | 0.0452 |
| 18      | 1  | 0.75| 1  | 0      | 0      | 0      |
| 19      | 1  | 0  | 1  | -0.5   | 0      | 0      |
| 20      | 1  | 0  | 1  | -0.5   | 0      | 0.5    |
| 21      | 1  | 0  | 1  | 0      | 0      | 0      |
| 22      | 1  | 0.6667| 1  | 0      | 0      | -1     |
| 23      | 1  | 0  | 1  | 0      | 1      | 0      |
| 24      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 25      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 26      | 1  | 0  | 1  | -1.2952| 0      | -0.7952|
| 27      | 1  | 0  | 1  | -2.0452| 0      | -0.5452|

**Supplementary Table 2:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 3 in the main document. Three-way beamsplitter and a 4x4 Hadamard matrix respectively.

| TriLatt | KA | KB | KC | TA/\pi | TB/\pi | TC/\pi |
|---------|----|----|----|--------|--------|--------|
| 1       | 1  | 0  | 1  | 0      | 0      | 0      |
| 2       | 1  | 0  | 1  | 0      | 0      | 1      |
| 3       | 1  | 0  | 1  | 0      | 0      | 0      |
| 4       | 1  | 0  | 1  | 0      | 0      | 1      |
| 5       | 1  | 0  | 1  | 0      | 1      | 0      |
| 6       | 1  | 0.5| 1  | 0      | 0      | -1.33334|
| 7       | 1  | 0  | 1  | 0      | 1      | 0      |
| 8       | 1  | 0.5| 1  | 0      | 0      | 0      |
| 9       | 1  | 0.5| 1  | 0      | 0      | 0      |
| 10      | 1  | 0  | 1  | 0      | 0      | 0      |
| 11      | 1  | 0.6667| 1  | 0      | 0      | 1      |
| 12      | 1  | 0  | 1  | 0      | 0      | 0      |
| 13      | 1  | 0.6667| 1  | 0      | 0      | 0.4548 |
| 14      | 1  | 0  | 1  | 0      | 1      | 0      |
| 15      | 1  | 0.5| 1  | 0      | 0      | -0.5   |
| 16      | 1  | 0  | 1  | 0      | 1      | 0      |
| 17      | 1  | 0.75| 1  | 0      | 0      | 0.0452 |
| 18      | 1  | 0.75| 1  | 0      | 0      | 0      |
| 19      | 1  | 0  | 1  | -0.5   | 0      | 0      |
| 20      | 1  | 0  | 1  | -0.5   | 0      | 0.5    |
| 21      | 1  | 0  | 1  | 0      | 0      | 0      |
| 22      | 1  | 0.6667| 1  | 0      | 0      | -1     |
| 23      | 1  | 0  | 1  | 0      | 1      | 0      |
| 24      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 25      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 26      | 1  | 0  | 1  | -1.2952| 0      | -0.7952|
| 27      | 1  | 0  | 1  | -2.0452| 0      | -0.5452|

| TriLatt | KA | KB | KC | TA/\pi | TB/\pi | TC/\pi |
|---------|----|----|----|--------|--------|--------|
| 1       | 1  | 0  | 1  | 0      | 0      | 0      |
| 2       | 1  | 0  | 1  | 0      | 0      | 1      |
| 3       | 1  | 0  | 1  | 0      | 0      | 0      |
| 4       | 1  | 0  | 1  | 0      | 0      | 1      |
| 5       | 1  | 0  | 1  | 0      | 1      | 0      |
| 6       | 1  | 0.5| 1  | 0      | 0      | -1.33334|
| 7       | 1  | 0  | 1  | 0      | 1      | 0      |
| 8       | 1  | 0.5| 1  | 0      | 0      | 0      |
| 9       | 1  | 0.5| 1  | 0      | 0      | 0      |
| 10      | 1  | 0  | 1  | 0      | 0      | 0      |
| 11      | 1  | 0.6667| 1  | 0      | 0      | 1      |
| 12      | 1  | 0  | 1  | 0      | 0      | 0      |
| 13      | 1  | 0.6667| 1  | 0      | 0      | 0.4548 |
| 14      | 1  | 0  | 1  | 0      | 1      | 0      |
| 15      | 1  | 0.5| 1  | 0      | 0      | -0.5   |
| 16      | 1  | 0  | 1  | 0      | 1      | 0      |
| 17      | 1  | 0.75| 1  | 0      | 0      | 0.0452 |
| 18      | 1  | 0.75| 1  | 0      | 0      | 0      |
| 19      | 1  | 0  | 1  | -0.5   | 0      | 0      |
| 20      | 1  | 0  | 1  | -0.5   | 0      | 0.5    |
| 21      | 1  | 0  | 1  | 0      | 0      | 0      |
| 22      | 1  | 0.6667| 1  | 0      | 0      | -1     |
| 23      | 1  | 0  | 1  | 0      | 1      | 0      |
| 24      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 25      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 26      | 1  | 0  | 1  | -1.2952| 0      | -0.7952|
| 27      | 1  | 0  | 1  | -2.0452| 0      | -0.5452|
**Supplementary Table 3:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 4 in the main document. SCISSOR filter composed of 5 cascaded rings. Case 1: $K=0.2$ in all the rings and ring resonances are slightly detuned, $\phi_1=-0.12$, $\phi_2=-0.06$, $\phi_3=0$, $\phi_4=0.06$, $\phi_5=-0.12$, to reduce the filter bandpass and main to secondary sidelobes. Second case coupling constants are apodized $K_1=0.39$, $K_2=0.47$, $K_3=0.55$, $K_4=0.63$, $K_5=0.71$, and the ring resonances strongly detuned $\phi_1=-0.4$, $\phi_2=-0.2$, $\phi_3=0$, $\phi_4=0.2$, $\phi_5=0.4$.

| TriLatt | KA | KB | KC | TA/π | TB/π | TC/π |
|---------|----|----|----|------|------|------|
| 1       | 1  | 1  | 1  | 0    | 0    | 0    |
| 2       | 1  | 1  | 1  | 0    | 0    | 0    |
| 3       | 0  | K1 | 1  | 0    | 0    | 0    |
| 4       | 1  | 0  | 0  | 0    | 0    | $\phi_1$ |
| 5       | 1  | 1  | 1  | 0    | 0    | 0    |
| 6       | 0  | 0  | 1  | 0    | $\phi_2$ | 0    |
| 7       | 1  | K2 | 0  | 0    | 0    | 0    |
| 8       | 0  | 1  | 0  | 0    | 0    | 0    |
| 9       | 1  | 1  | 1  | 0    | 0    | 0    |
| 10      | 1  | 1  | 1  | 0    | 0    | 0    |
| 11      | 0  | 1  | 0  | 0    | 0    | 0    |
| 12      | 0  | K3 | 1  | 0    | 0    | 0    |
| 13      | 1  | 0  | 0  | 0    | 0    | 0    |
| 14      | 1  | 1  | 1  | 0    | 0    | 0    |
| 15      | 0  | 0  | 1  | 0    | $\phi_4$ | 0    |
| 16      | 1  | K4 | 0  | 0    | 0    | 0    |
| 17      | 0  | 1  | 0  | 0    | 0    | 0    |
| 18      | 1  | 1  | 1  | 0    | 0    | 0    |
| 19      | 1  | 1  | 1  | 0    | 0    | 0    |
| 20      | 0  | 1  | 0  | 0    | 0    | 0    |
| 21      | 0  | K5 | 1  | 0    | 0    | 0    |
| 22      | 1  | 0  | 0  | 0    | $\phi_5$ | 0    |
| 23      | 1  | 1  | 1  | 0    | 0    | 0    |
| 24      | 1  | 1  | 1  | 0    | 0    | 0    |
| 25      | 1  | 1  | 1  | 0    | $-0.5$ | 0    |
| 26      | 0  | 1  | 0  | 0    | 0.5  | 0    |
| 27      | 1  | 1  | 1  | 0    | 0    | 0    |

**Supplementary Table 4:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 5 in the main document. MZI loaded with four ORRs. The values in the table are considered as the average values for the normal distributions in the Monte-Carlo configurations.

| TriLatt | KA | KB | KC | TA/π | TB/π | TC/π |
|---------|----|----|----|------|------|------|
| 1       | 1  | 1  | 1  | 0    | 0    | 0    |
| 2       | 1  | 1  | 1  | 0    | 0    | 0    |
| 3       | 1  | 0.5| 1  | 0    | 0    | 0    |
| 4       | 1  | 1  | 1  | 0    | 0    | 0    |
| 5       | 1  | 1  | 1  | 0    | 0    | 0    |
| 6       | 0  | 0  | 1  | 0    | 0.3819 | 0    |
| 7       | 1  | 0.59| 1  | 0    | 0    | 0    |
| 8       | 0  | 0.59| 1  | 0    | 0    | 0    |
| 9       | 1  | 0  | 0    | 0    | -0.382 | 0    |
| 10      | 1  | 1  | 1  | 0    | 0    | 0    |
| 11      | 0  | 1  | 0    | 0    | 0    | 0    |
| 12      | 1  | 0  | 0    | 0    | 0    | 0    |
| 13      | 0  | 1  | 0    | 0    | 0    | 0    |
| 14      | 1  | 1  | 1  | 0    | 0    | 0    |
| 15      | 0  | 0  | 1    | 0    | 0.581 | 0    |
| 16      | 1  | 0.999| 0  | 0.112| 0    | 0    |
| 17      | 0  | 0.999| 1  | 0    | -0.112 | 0    |
| 18      | 1  | 0  | 0    | 0    | -0.581 | 0    |
| 19      | 1  | 1  | 1  | 0    | 0    | 0    |
| 20      | 0  | 1  | 0    | 0    | 0    | 0    |
| 21      | 1  | 0.5| 1  | 0    | 0    | 0    |
| 22      | 0  | 1  | 0    | 0    | 0    | 0    |
| 23      | 1  | 1  | 1  | 0    | 0    | 0    |
| 24      | 1  | 1  | 1  | 0    | 0    | 0    |
| 25      | 1  | 0  | 1    | 0    | 0    | 0    |
| 26      | 0  | 1  | 0    | 0    | 0    | 0    |
| 27      | 1  | 1  | 1  | 0    | 0    | 0    |
**Supplementary Table 5:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 6 in the main document. Waveguide mesh implementing simultaneously a 3 stage CROW with cavity lengths (6 BULs) and a MZI. The values in the table are considered as the average values for the normal distributions in the Monte-Carlo configurations. The values in red are associated to the optimization implementation.

| TriLatt | KA  | KB  | KC  | TA/$\pi$ | TB/$\pi$ | TC/$\pi$ |
|---------|-----|-----|-----|----------|----------|----------|
| 1       | 1   | 1   | 1   | 0        | 0        | 0        |
| 2       | 1   | 1   | 1   | 0        | 0        | 0        |
| 3       | 1   | 1   | 1   | 0        | 0        | 0        |
| 4       | 1   | 1   | 1   | 0        | 0        | 0        |
| 5       | 1   | 1   | 1   | 0        | 0        | 0        |
| 6       | 0.15| 0   | 1   | 0        | 0        | 0        |
| 7       | 0   | 0   | 0   | 0        | 0        | 0        |
| 8       | 1   | 0   | 1   | 0        | 0        | 0        |
| 9       | 1   | 1   | 1   | 0        | 0        | 0        |
| 10      | 0   | 0   | 1   | 0        | 0        | 0        |
| 11      | 0   | 0   | 0.2 | 0        | 0        | 0        |
| 12      | 1   | 1   | 1   | 0        | 0        | 0        |
| 13      | 0   | 1   | 1   | 0        | 0        | 0        |
| 14      | 1   | 1   | 1   | 0        | 0        | 0        |
| 15      | 0.2 | 0.15| 0   | 0        | 0        | 0        |
| 16      | 0   | 0   | 0   | 0        | 0        | 0        |
| 17      | 1   | 0   | 1   | 0        | 0        | 0        |
| 18      | 1   | 0   | 1   | 0        | 0        | 0        |
| 19      | 1   | 1   | 1   | 0        | 0        | 0        |
| 20      | 0   | 1   | 0   | 0        | 0        | 0        |
| 21      | 1   | 1   | 0   | 0        | 0        | 0        |
| 22      | 0   | 0   | 0   | 0        | 0        | 0        |
| 23      | 0.5 | 1   | 1   | 0        | 0        | 0        |
| 24      | 1   | 0   | 1   | 0        | 0        | 0        |
| 25      | 0   | 0   | 1   | 0        | 0        | 0        |
| 26      | 1   | 0   | 1   | 0        | 0        | 0        |
| 27      | 1   | 0.5| 1   | 0        | 0        | 0        |

**Supplementary Table 6:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 6 in the supplementary document. 2x2 Pauli-y, Pauli-z, and Hadamard transformations.

| TriLatt | KA  | KB  | KC  | TA/$\pi$ | TB/$\pi$ | TC/$\pi$ |
|---------|-----|-----|-----|----------|----------|----------|
| 1       | 1   | 1   | 1   | 0        | 0        | 0        |
| 2       | 1   | 1   | 1   | 0        | 0        | -0.5     |
| 3       | 1   | 0   | 1   | 0        | 0        | 0        |
| 4       | 1   | 0   | 1   | 0        | 0        | 0        |
| 5       | 1   | 0   | 1   | 0        | 0        | 0        |
| 6       | 1   | 0   | 1   | 0        | 0        | 0        |
| 7       | 1   | 0   | 1   | 0        | 0        | -0.5     |
| 8       | 1   | 0   | 1   | 0.5     | -1       | 0        |
| 9       | 1   | 0   | 1   | 0        | 0        | 0.5      |
| 10      | 1   | 1   | 0   | 0        | 0        | 0        |
| 11      | 1   | 0   | 1   | 1        | 0        | -1.5     |
| 12      | 1   | 0.5 | 1   | 0        | 0        | 0        |
| 13      | 1   | 0   | 1   | 1.5     | 0        | 1        |
| 14      | 1   | 0   | 1   | 0        | 0        | 0        |
| 15      | 1   | 0   | 1   | 0        | 0        | 0        |
| 16      | 1   | 0   | 1   | 0.5     | 0        | 0        |
| 17      | 1   | 0   | 1   | 0        | 0        | 0.5      |
| 18      | 1   | 0   | 1   | 0        | 0        | 0        |
| 19      | 1   | 0   | 1   | 0        | 0        | 0        |
| 20      | 1   | 0   | 1   | 0        | 0        | 0        |
| 21      | 1   | 0   | 1   | 0        | 0        | 0        |
| 22      | 1   | 0   | 1   | 0        | 0        | 0        |
| 23      | 1   | 0   | 1   | 0        | 0        | 0        |
| 24      | 1   | 0   | 1   | 0        | 0        | 0        |
| 25      | 1   | 0   | 1   | 0        | -0.5     | 0        |
| 26      | 1   | 0   | 1   | 0        | 0.5      | 0        |
| 27      | 1   | 0   | 1   | 0        | 0        | 0        |
**Supplementary Table 7:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 7 in the supplementary document. DFT4 and CNOT transformations.

| TriLatt | KA | KB | KC | TA/\pi | TB/\pi | TC/\pi |
|---------|----|----|----|--------|--------|--------|
| 1       | 1  | 0  | 1  | 0      | 0      | 0      |
| 2       | 1  | 0  | 1  | 0      | 0      | 1      |
| 3       | 1  | 0  | 1  | 0      | 0      | 1      |
| 4       | 1  | 0  | 1  | 0      | 0      | 1      |
| 5       | 1  | 0  | 1  | 0      | 0      | 1      |
| 6       | 1  | 1  | 1  | 0      | 0      | -1     |
| 7       | 1  | 0.5| 1  | 0      | 0      | -1.5   |
| 8       | 1  | 0.5| 1  | 0      | 0      | 0      |
| 9       | 1  | 0  | 1  | 0      | 0      | 0      |
| 10      | 1  | 0  | 1  | 0      | 0      | 0      |
| 11      | 1  | 0  | 1  | 0      | 0      | -1     |
| 12      | 1  | 0  | 1  | 0      | 0      | 1      |
| 13      | 1  | 0.6667| 1 | 0   | 0     | -0.75  |
| 14      | 1  | 0  | 1  | 0      | 0      | 1      |
| 15      | 1  | 0  | 1  | 0      | 0      | 0      |
| 16      | 1  | 0  | 1  | 0      | 0      | 0      |
| 17      | 1  | 0.75| 1  | 0      | 0      | -1.25  |
| 18      | 1  | 0.75| 1  | 0      | 0      | -0.5   |
| 19      | 1  | 0  | 1  | 0      | 0      | 0      |
| 20      | 1  | 0  | 1  | 0      | 0      | 1      |
| 21      | 1  | 0  | 1  | 0      | 0      | 1      |
| 22      | 1  | 0.3334| 1 | 0   | 0     | -0.5   |
| 23      | 1  | 0  | 1  | 0      | 0      | 1      |
| 24      | 1  | 0  | 1  | -2     | 0      | 0      |
| 25      | 1  | 0  | 1  | -2     | 0      | -1     |
| 26      | 1  | 0  | 1  | -1.75  | 0      | -2     |
| 27      | 1  | 0  | 1  | -2.25  | 0      | -1     |

**Supplementary Table 8:** Coupling coefficients and phase coefficients normalized to $\pi$ for the results showed in Fig. 8 in the supplementary document. Hybrid 2x2 trasformation

| TriLatt | KA | KB | KC | TA/\pi | TB/\pi | TC/\pi |
|---------|----|----|----|--------|--------|--------|
| 1       | 0  | 1  | 0  | 0      | 0      | 0      |
| 2       | 1  | 1  | 1  | 0      | 0      | 0      |
| 3       | 1  | 0.5| 1  | 0      | 0      | -1.75  |
| 4       | 1  | 1  | 1  | 0      | 0      | 0      |
| 5       | 1  | 1  | 1  | 0      | 0      | 0      |
| 6       | 1  | 1  | 1  | 0      | 0      | 0      |
| 7       | 1  | 0  | 1  | -0.75  | -1.5   | 0      |
| 8       | 1  | 0  | 0.1716 | 0   | -0.5   | 0.25   |
| 9       | 1  | 1  | 1  | 0      | 0      | 0      |
| 10      | 1  | 1  | 1  | 0      | 0      | 0      |
| 11      | 1  | 1  | 1  | 0      | 0      | 0      |
| 12      | 1  | 0.5| 1  | 0      | 0      | -0.5   |
| 13      | 1  | 1  | 1  | 0      | 0      | 0      |
| 14      | 1  | 1  | 1  | 0      | 0      | 0      |
| 15      | 1  | 1  | 1  | 0      | 0      | 0      |
| 16      | 1  | 0  | 1  | 0.125  | 0      | 0      |
| 17      | 1  | 0  | 1  | 0      | 0      | -0.125 |
| 18      | 1  | 1  | 1  | 0      | 0      | 0      |
| 19      | 1  | 1  | 1  | 0      | 0      | 0      |
| 20      | 1  | 1  | 1  | 0      | 0      | 0      |
| 21      | 1  | 0  | 1  | 0      | 0      | 0      |
| 22      | 1  | 1  | 1  | 0      | 0      | 0      |
| 23      | 1  | 1  | 1  | 0      | 0      | 0      |
| 24      | 1  | 1  | 1  | 0      | 0      | 0      |
| 25      | 1  | 0  | 1  | 0      | -0.5   | 0      |
| 26      | 1  | 0  | 1  | 0      | 0.5    | 0      |
| 27      | 1  | 1  | 1  | 0      | 0      | 0      |
Supplementary Table 9: Coupling coefficients and phase coefficients normalized to π for the results showed in Fig. 10 in the supplementary document. Multiple Rings with different cavity lengths.

| TriLatt | KA  | KB  | KC  | TA/π | TB/π | TC/π |
|---------|-----|-----|-----|------|------|------|
| 1       | 0   | 0.35| 1   | 0    | 0    | 0    |
| 2       | 1   | 1   | 1   | 0    | 0    | 0    |
| 3       | 1   | 0   | 0   | 0    | 0    | 0    |
| 4       | 1   | 1   | 1   | 0    | 0    | 0    |
| 5       | 1   | 1   | 1   | 0    | 0    | 0    |
| 6       | 1   | 1   | 0   | 0    | 0    | 0    |
| 7       | 0   | 1   | 1   | 0    | 0    | 0    |
| 8       | 0   | 0   | 1   | 0    | 1.5  | 0    |
| 9       | 0   | 0.23| 0   | 0    | 0    | 0    |
| 10      | 1   | 1   | 1   | 0    | 0    | 0    |
| 11      | 1   | 1   | 1   | 0    | 0    | 0    |
| 12      | 1   | 1   | 1   | 0    | 0    | 0    |
| 13      | 0   | 1   | 0   | 0    | 0    | 0    |
| 14      | 0   | 1   | 0   | 0    | 0    | 0    |
| 15      | 1   | 1   | 1   | 0    | 0    | 0    |
| 16      | 1   | 1   | 1   | 0    | 0    | 0    |
| 17      | 0   | 1   | 1   | 0    | 0    | 0    |
| 18      | 1   | 1   | 0   | 0    | 0    | 0    |
| 19      | 1   | 1   | 1   | 0    | 0    | 0    |
| 20      | 1   | 1   | 1   | 0    | 0    | 0    |
| 21      | 1   | 0   | 1   | 0    | 1    | 0    |
| 22      | 1   | 1   | 1   | 0    | 0    | 0    |
| 23      | 1   | 1   | 0.41| 1    | 0    | 0    |
| 24      | 1   | 1   | 1   | 0    | 0    | 0    |
| 25      | 1   | 1   | 1   | 0    | 0    | 0    |
| 26      | 1   | 1   | 0   | 0    | 0    | 0    |
| 27      | 0   | 1   | 1   | 0    | 0    | 0    |

Supplementary Table 10: Coupling coefficients and phase coefficients normalized to π for the results showed in Fig. 11 in the supplementary document. Three-stage SCISSOR of two-CROWs.

| TriLatt | KA  | KB  | KC  | TA/π | TB/π | TC/π |
|---------|-----|-----|-----|------|------|------|
| 1       | 0   | 0   | 0   | 0    | 0    | 0    |
| 2       | 0   | 0.07| 1   | 0    | 0    | 0    |
| 3       | 0   | 0.07| 0   | 0    | 0    | 0    |
| 4       | 1   | 0.07| 0   | 0    | 0    | 0    |
| 5       | 0   | 0   | 0   | 0    | 0    | 0    |
| 6       | 1   | 0   | 0   | 0    | 0    | 0    |
| 7       | 0   | 0   | 0   | 0    | 0    | 0    |
| 8       | 0   | 0   | 0   | 0    | 0    | 0    |
| 9       | 0   | 0   | 1   | 0    | 0    | 0    |
| 10      | 0   | 0   | 0   | 0    | 0    | 0    |
| 11      | 0   | 0.07| 1   | 0    | 0    | 0    |
| 12      | 0   | 0.07| 0   | 0    | 0    | 0    |
| 13      | 1   | 0.07| 0   | 0    | 0    | 0    |
| 14      | 0   | 0   | 0   | 0    | 0    | 0    |
| 15      | 1   | 0   | 0   | 0    | 0    | 0    |
| 16      | 0   | 0   | 0   | 0    | 0    | 0    |
| 17      | 0   | 0   | 0   | 0    | 0    | 0    |
| 18      | 0   | 0   | 1   | 0    | 0    | 0    |
| 19      | 0   | 0   | 0   | 0    | 0    | 0    |
| 20      | 0   | 0.07| 1   | 0    | 0    | 0    |
| 21      | 0   | 0.07| 0   | 0    | 0    | 0    |
| 22      | 1   | 0.07| 0   | 0    | 0    | 0    |
| 23      | 0   | 0   | 0   | 0    | 0    | 0    |
| 24      | 1   | 0   | 0   | 0    | 0    | 0    |
| 25      | 0   | 0   | 0   | 0    | 0    | 0    |
| 26      | 0   | 0   | 0   | 0    | 0    | 0    |
| 27      | 0   | 0   | 1   | 0    | 0    | 0    |

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