Extended MSSM in Supersymmetric SO(10) Grand Unification

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We apply the perturbative grand unification due to renormalization to distinguish TeV-scale relics of supersymmetric SO(10) scenarios. With rational theoretical constraints taken into account, we find that for the breaking pattern of either SU(5) or Pati-Salam only extra matter \[16\] supermultiplet of SO(10) can appear at TeV scale, apart from MSSM spectrum.

I. INTRODUCTION

The discovery of standard model (SM)-like Higgs \[1, 2\] provides a new portal to TeV-scale new physics at the LHC in the forthcoming years. Among other things, such new physics models may reveal the “nature” of SM-like Higgs, and offer a novel mechanism to stabilize divergence involving SM Higgs. For those interesting scenarios in the literature, in this paper we are restricted to the idea of supersymmetry (SUSY). Specifically, we will utilize the grand unification (GUT) \[3\], which is one of the most beautiful features delivered by SUSY, to distinguish TeV-scale relics of SUSY GUT models. For reviews on this subject see e.g. \[4, 5\].

In the viewpoint of unification, the minimal supersymmetric standard model (MSSM) can be embedded into conventional SU(5) \[6–8\], SO(10) \[9, 10\] or other GUT models with gauge groups of higher ranks. In the light of our previous study on SU(5) \[11\], we will continue to explore the TeV-scale relics of SUSY SO(10) unification. Comparing with SU(5), the low-energy effective theories of SO(10) are more complex. The first major reason is that there may be multiple intermediate scales between the weak and GUT scale. The second reason is that since a lot of higher-dimensional representations of SO(10) trivially satisfy gauge anomaly free condition, the constraint imposed by this condition is much weaker in SO(10). Earlier studies on low-energy effective theory which is consistent with perturbative SUSY SO(10) unification are based on specific motivations such as Higgs mass \[12\] and neutrino physics \[13–15\].

Instead of particular phenomenological concerns, we will take a systematic analysis on the low-energy effective theory. In order to simplify the analysis on extra matter beyond the MSSM spectrum, we will explore SO(10) scenarios with the following theoretical features.

- The SO(10) unification is strictly perturbative.
- In the chain of gauge symmetry breaking

\[
SO(10) \xrightarrow{H_1} G_1 \xrightarrow{H_2} G_2 \cdots \xrightarrow{H_n} G_{\text{SM}},
\]

where \(G_{\text{SM}}\) refers to the SM gauge group. When the Higgs component fields responsible for two nearby steps of gauge symmetry breaking can be contained in a single Higgs supermultiplet, these two Higgs supermultiplets will be identified as the same one. Otherwise, they differ from each other \[1\].

- In order to avoid dangerous mixings among Higgs vevs \(\langle H_i \rangle\), all of \(H_i\) are forbidden to directly couple to each other.
- In order to avoid dangerous masses or mixing effects, neither the MSSM fields nor extra matters are allowed to directly couple to any Higgs supermultiplets \(H_i\) in Eq. \(1\). \[2\]

The main reason for the last point is that the vacuum expectations (vevs) of \(\langle H_i \rangle\) would result in large matter masses or large mixing effects if they were directly coupled to either MSSM fields or extra matters, which would lead to them playing no role at weak scale. For example, coupling a \(54\), which can break SU(5) to SM gauge group, to MSSM Higgs \(10_H\) through interaction

\[
54 \times 10_H \times 10_H\]

yields unfavorable Higgsino mass for a large vev of SM singlet in \(54\).

Theoretical constraints above have been partially imposed in the literature to our knowledge. However, they have never been combined together to derive a systematic analysis on the low-energy effective theory. The paper is organized as follows. In Sec.II, we discuss the extra matter supermultiplets which are consistent with our starting points in two well known patterns of gauge symmetry breaking. In Sec.III, we examine the perturbative unification with these representations. Finally, we conclude in Sec.IV.

II. REPRESENTATIONS

According to our starting points, in this section we investigate the representation of extra matter which can
directly couple to SM Higgs $10_H$ in the following two patterns of gauge symmetry breaking,

A : SO(10) \( \rightarrow SU(5) \rightarrow G_{\text{SM}}, \)

B : SO(10) \( \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow G_{\text{SM}}. \)

Pattern A \[\text{16, 10}\] is a two-step breaking with $SU(5)$ subgroup, and pattern B \[\text{21, 22}\] is a three-step breaking referred to Pati-Salam model \[\text{21}\].

Note, in the MSSM the SM fermion matters are described by $\text{16}$ of SO(10) with index $i = 1 - 3$, and the SM Higgs is contained in the $\text{10}_H$ of SO(10). In particular, $\text{16}_i$ contain three-generation right-hand neutrinos, whereas $\text{10}_H$ is composed of $\text{5}_H$ and $\overline{\text{5}}_H$ of $SU(5)$ which contain the two Higgs doublets of MSSM and two color triplets.

A. $SU(5)$

In this pattern of symmetry breaking $H_1$ should contain an $SU(5)$ singlet, there are two candidates $H_1 = \{\text{16}, \text{126}\}$. The second Higgs $H_2$ should contain a 24 of $SU(5)$, which corresponds to three potential choices $H_2 = \{\text{45, 54, 210}\}$. Since $H_1 \neq H_2$, we take the rational that the splitting between these two broken scales is large.

With potential assignments on $H_1$ and $H_2$ above, there are six sets of combinations. In each case, may exist four types of dangerous gauge-invariant superpotentials which violate the last two starting points in the Sec.I:

\[
\begin{align*}
H_1 &\times N \times N; \\
H_2 &\times N \times N; \\
H_1 \times H_2 &\times N; \\
H_1 \times H_1 &\times H_2, \quad H_2 \times H_2 \times H_1; \\
M &\text{ should contain a singlet of } SU(5), \quad \text{and } \text{SU}(3) \times U(1)_{B-L} \rightarrow U(1)_Y.
\end{align*}
\]

where MSSM matter field $N = \{\text{16}, \text{10}_H\}$. In Eq.2, the first three types of superpotentials tend to seed Dirac or Majorana masses to MSSM matters or MSSM Higgs doublets; and the last type of gauge invariant superpotentials yields dangerous mixings between vevs of $H_1$ and $H_2$.

Firstly, one finds a dangerous operator $16 \times 16 \times 10_H$ of the first type in Eq.2 which disfavors the choice $H_1 = \text{16}$. Secondly, due to dangerous operator $126 \times 210 \times 10_H$ the set of $H_1 = \text{126}$ and $H_2 = \text{210}$ is also disfavored. Therefore, we are left with two combinations $H_1 = \text{126}$ and $H_2 = \{\text{45, 54}\}$. In the case $(H_1, H_2) = (\text{126}, \text{54})$, there exists a dangerous operator $126 \times 126 \times 54$. In the last case $(H_1, H_2) = (\text{126}, \text{45})$, denote the new matter supermultiplets with $\mathcal{M}$, we find that unsafe superpotentials exclude $\mathcal{M} = \{\text{10}, \text{16}, \text{120}, \text{144}, \text{144}, \text{210}\}$, leaving us only two possibilities,

\[
\begin{align*}
16_M &\times 16_M \times 10_H, \quad 16_M \times 16_i \times 10_H.
\end{align*}
\]

In compared with breaking pattern A, there is another pattern of two-step breaking \(^3\)

\[
SO(10) \rightarrow G_{\text{SM}} \times U(1) \rightarrow G_{\text{SM}}.
\]

In this case, the potential choices are $H_1 = \{\text{45, 210}\}$ and $H_2 = \{\text{16, 126}\}$. According to Eq.2, dangerous operator $16 \times 16_i \times 10_H$ excludes the case $H_2 = \text{16}$. Moreover, a dangerous operator $210 \times 126 \times 10_H$ excludes $H_1 = \text{210}$. Therefore, there is only a viable combination $(H_1, H_2) = (\text{45, 126})$, in which case the extra matter $\mathcal{M}$ is similar to those of $SU(5)$ subgroup.

B. Pati-Salam

In this pattern of symmetry breaking, $H_1$ should contain a singlet of $SU(4)_c \times SU(2)_L \times SU(2)_R$, which has two choices $H_1 = \{\text{54, 210}\}$. $H_2$ should contain a singlet of $SU(3)_c$ and $U(1)_{B-L}$, which is a 15 of $SU(4)_c$. There are two representations $H_2 = \{\text{45, 210}\}$ of SO(10) which include such a 15. Finally, $H_3 = \text{16}$ offers the breaking of $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$.

Since 210 contains both a singlet of $SU(4)_c \times SU(2)_L \times SU(2)_R$ and a 15 of $SU(4)_c$, according to the second starting point both $H_1$ and $H_2$ are identified as 210. In this case, the three-step breaking is approximately two-step. With $H_1 = H_2 = 210$ we are left with a single choice $(H_1, H_2, H_3) = (210, 210, 16)$, where similar to previous discussions about $SU(5)$ there are four viable choices for extra matters,

\[
\begin{align*}
16_M \times 16_M \times 10_H, \quad 10_M \times 54_M \times 10_H, \\
16_M \times 144_M \times 10_H, \quad 144_M \times 144_M \times 10_H.
\end{align*}
\]

Note, unlike in pattern A, extra matter supermultiplets in Eq.[4] are allowed to directly couple to $H_3$. Because the broken scale of $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ can be close to TeV scale (see, e.g. [25, 30]).

III. PERTURBATIVE UNIFICATION

With the theoretical constraints in the Introduction, we have clarified that a single or two 16 supermultiplets are allowed in pattern A, whereas two 16s, a 10 with 54, a 16 with 144 or a pair of vector-like 144 may appear in the pattern B. Now, we examine whether any of them are consistent with the first constraint - perturbative unification.

We start with the one-loop renormalization group equations (RGEs) for SM gauge coupling constant,

\[
\frac{d}{dt}g_i^{-1} = -\frac{b_i}{2\pi},
\]

\(^3\) The author thanks the referee for reminding us this case.
where \( \tau \equiv \ln \mu \) and coefficients \( b_i = (b_{U(1)_Y}, b_{SU(2)_L}, b_{SU(3)_c}) \) are determined by \([31, 32]\),

\[
   b_i = -\left\{ \frac{11}{3} C_i^2(G) - \frac{4}{3} \kappa \cdot T(r_f_i) - \frac{1}{6} T(r_s_i) \right\}. \quad (6)
\]

Here, \( C_i^2(G) \) is the quadratic Casimir invariant, and \( T(r) \) refers to Dynkin index that depends on details of the representation \([5]\).

**A. SU(5)**

In the case of SU(5) subgroup there are two intermediate scales \( \Lambda_{\text{SUSY}} \) and \( \Lambda_5 \) between \( M_Z \) and \( \Lambda_{10} \), corresponding to SUSY and SU(5) breaking, respectively. The \( b_i \) coefficients are given by

\[
   b_i = \begin{cases} 
   \left( \frac{41}{10}, -\frac{19}{6}, -7 \right), & \mu \in [M_Z, \Lambda_{\text{SUSY}}] \\
   \left( \frac{33}{5} + \delta b_1(M), 1 + \delta b_2(M), -3 + \delta b_3(M) \right), & \mu \in [\Lambda_{\text{SUSY}}, \Lambda_5] \\
   \delta b_5(M), & \mu \in [\Lambda_5, \Lambda_{10}] 
   \end{cases} \quad (7)
\]

and \( \delta b_i(M) \) refer to contributions to \( b \) coefficients due to extra matter. In particular, \( \delta b_i(16_M) = (2, 2, 2) \) and \( \delta b_i(16_M + \bar{16}_M) = (4, 4, 4) \), respectively in Eq. (7). Fig. 1 shows the plots of RG running of SM gauge coupling constants according to Eq. (7). It reveals that for \( \Lambda_{\text{SUSY}} = 1 \) TeV the SU(5) unification occurs at \( \Lambda_5 \simeq 10^{15.3} \) GeV. Moreover, the SO(10) unification in both cases can occur at \( \Lambda_{10} \) large than \( 10^{18} \) GeV. Comparing \( \Lambda_5 \) with \( \Lambda_{10} \), one finds that there is indeed sufficient splitting between them, which verifies previous arguments.

**B. Pati-Salam**

In the case of Pati-Salam model there are two intermediate scales \( \Lambda_{\text{SUSY}} \) and \( \Lambda_R \) between \( M_Z \) and \( \Lambda_{10} \), which denotes SUSY and SU(2)_R × U(1)_B−L breaking scale, respectively. In this case the coefficients \( b_i \) are given by,

\[
   b_i = \begin{cases} 
   \left( \frac{41}{10}, -\frac{19}{6}, -7 \right), & \mu \in [M_Z, \Lambda_{\text{SUSY}}] \\
   \left( \frac{33}{5} + \delta b_1(M), 1 + \delta b_2(M), -3 + \delta b_3(M) \right), & \mu \in [\Lambda_{\text{SUSY}}, \Lambda_R] \\
   (6 + 2 + \delta b_{B−L}(M), 0 + 2 + \delta b_{2L}(M), 0 + 2 + \delta b_{2R}(M), -3 + 2 + \delta b_3(M)), & \mu \in [\Lambda_R, \Lambda_{10}] 
   \end{cases} \quad (8)
\]

for the RG scale between \( M_Z \) and \( \Lambda_R \), and

\[
   b_i = (6 + 2 + \delta b_{B−L}(M), 0 + 2 + \delta b_{2L}(M), 0 + 2 + \delta b_{2R}(M), -3 + 2 + \delta b_3(M)), \quad \mu \in [\Lambda_R, \Lambda_{10}] \quad (9)
\]

for the RG scale between \( \Lambda_R \) and \( \Lambda_{10} \). Above the RG scale \( \Lambda_R \), MSSM matters and Higgs field \( H_3 = 16 \) con-
FIG. 2: One-loop RGEs for SM gauge coupling $\alpha_i^{-1}$ (green), $\alpha_{2L}^{-1}$ (blue), $\alpha_3^{-1}$ (red) and $\alpha_{B-L}^{-1}$ (black) for extra matter $16_i$. For $A_{SU(2)_R}$ one obtains $A_R \simeq 10^{15.5}$ GeV and $A_{10} \simeq 10^{16.7}$ GeV. The RG running of $SU(2)_R$ gauge coupling constant between $A_R$ and $A_{10}$ coincides with that of $SU(2)_L$.

 contributes to $\delta b_1 = (6, 0, 0, -3)$ and $\delta b_2 = (2, 2, 2, 2)$ in Eq. (9), respectively. Regardless of what extra matter appears above $A_{SU(2)}$ and what kind of Higgs $H_{1,2,3}$ above $A_R$, $SO(10)$ unification at the one-loop level yields $\ln(A_{10}/A_R) \simeq 2.53 \pm 0.08$, which uniquely determines $A_R$ once the content of extra matters is identified.

Take a pair of $16_i$ for example, they contribute to $\delta b_1(M) = (4, 4, 4)$ and $\delta b_2(M) = (4, 4, 4)$ in Eq. (8) and Eq. (9), respectively, which gives rise to $A_{SU(2)} \simeq 10^{15.5}$ GeV, unified gauge coupling $\alpha^{-1} \simeq 5.45$, and $A_{10} \simeq 10^{16.7}$ GeV. Fig 2 shows the RG running of SM gauge coupling constant, which offers us perturbative $SO(10)$ unification. Note, the RG running of $SU(2)_R$ gauge coupling constant between $A_R$ and $A_{10}$ coincides with that of $SU(2)_L$, and as required $\alpha^{-1}_Y$ is equal to $\frac{3}{5}\alpha^{-1}_{2R} + \frac{2}{5}\alpha^{-1}_{B-L}$ at RG scale $A_R$.

Repeat the analysis for other choices on extra matters in Eq. (11). We find that in these cases $b$ coefficients such as $\delta b_1(10_M + 54_M) = (11.5, 11.5, 11.5, 11.5)$ are always too large to support the idea of perturbative unification.

IV. CONCLUSION

In the forthcoming years we will enter into a new era of precise Higgs physics, which means that studying new physics through the Higgs portal will become very interesting. In this paper, we have utilized perturbative unification due to renormalization to explore the low energy effective theory of SUSY $SO(10)$ scenarios. With the rational theoretical constraints taken into account, we find that for the breaking pattern of either SU(5) or Pati-Salam only $16$ supermultiplet can appear at TeV scale apart from the MSSM spectrum.

The quarks or leptons in the $16$ supermultiplet(s) can be either chiral or vector-like. Note, vector-like fermion mass requires addition of SM singlet (with vev of order TeV) which does not affect our discussions. While the chiral case has been excluded, the vector-like quarks or leptons are smoking guns in these SUSY $SO(10)$ scenarios. Moreover, the neutral fermions of singlet or doublets of the $16$ supermultiplet can serve as dark matter totally, or partially with the neutralinos of the MSSM.

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