Angle-resolved photoelectron energy spectrum from the high-order above-threshold ionization process in IR+XUV two-color laser fields

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Abstract
The angle-resolved photoelectron energy spectrum (PES) of high-order above-threshold ionization (HATI) spectrum in IR+XUV two-color laser fields has been investigated. We found that the quantum feature corresponding to the absorption of the XUV photon is well illustrated by a peculiar dip structure in the second plateau of the HATI spectrum. By the channel analysis, we show that the angle-resolved PES is attributed to the coherent summation over contributions of different channels, and the dip structure in the spectrum is directly related to the absorption of one XUV photon of the ionized electron during the laser-assisted collision (LAC) process with its parent ion in the two-color laser fields. Moreover, by employing the saddle-point approximation, we obtain the classical energy orbit equation, and find that the dip structure comes from the fact that the LAC is limited at a certain direction by the momentum conservation law as the electron absorbs one XUV photon during the collision, where the probability of the HATI gets its minimum value. Finally, we find that the interference pattern in the whole spectrum is attributed to the interference of different orbits at collision moments $t_0$ and $2\pi/\omega - t_0$ in the LAC process.

Keywords: high-order above-threshold ionization, IR+XUV two-color laser fields, angle-resolved photoelectron energy spectrum, the frequency-domain theory

(Some figures may appear in colour only in the online journal)

1. Introduction

The above-threshold ionization (ATI) has attracted much more interest and become one of the most active topics in strong-field atomic and molecular physics [1, 2] since its first observation [3]. In this process, an electron can absorb photons in excess of the minimum photon number necessary to overcome the ionization potential. It was found recently that, the very low energy spectrum of the ionized electron is attributed to the forward scattering of electron to its parent ion, i.e. the effect of the coulomb potential [4–6] during the ionization. In general, the photoelectron spectrum may be
classified by direct ATI part and high-order ATI (HATI) part, where the HATI spectrum is explained by the well-known three-step model [7]: the electron is ionized through tunneling, and then accelerated, and finally driven back to collide with parent ion and scatter elastically off in an intense infrared (IR) laser field, where the electron can obtain more kinetic energy in this process. The HATI process in an IR laser field has been extensively investigated and has been made great achievement. It is well known that plateau structure is a common phenomenon in the HATI process [8–10]. It was found that the position of the resonant structure of rare-gas atoms does not shift with the change of the laser intensity [11–14] and the resonantlike enhancement of molecules is attributed to the channel-closing mechanism in the HATI process [15–17]. The bicircular HATI processes of atom and molecule were discussed in detail [18, 19]. Furthermore, it was proved that the angle-resolved photoelectron energy spectra (PES) of atoms and molecules provide a sensitive perspective to further analyze the HATI process [20–26], especially the laser-driven scattering process of an electron and the image of the molecular geometrical structure.

With the rapid development of laser technology [27, 28], the combined IR and extreme ultraviolet (XUV) laser fields have offered an effective tool to investigate the electronic dynamics of atoms and molecules [28–30]. For example, the control of the electron-correlation for double ionization [31, 32], the molecular dissociative ionization [33, 34] and the real-time observations of valence electron motion [35, 36] were investigated in IR+XUV two-color laser fields. Furthermore, it was found that a sideband structure is presented in the photoelectron spectrum as well as the Auger electron spectrum [37–39], which can be in favor of the investigation of the properties of pulses reversely. Also, these sidebands have been applied in investigating the molecular orbitals of O2, H2O and N2 [40]. Additionally, the ATI spectrum presents a steplike structure when the XUV photon energy is much higher than the atomic ionization potential [41, 42] and the dependence of ATI spectrum on polarization direction of two-color laser fields is identified [43]. Recently, the ATI process of atoms and ions in vortex XUV Bessel beam in the presence of a strong IR laser field was investigated [44]. It was shown that the ATI photoionization spectra present seven different dichroism signals, which may open up avenues for future investigations of the interaction of atom and molecule with vortex laser field. Moreover, the rescattering process of atom in XUV+IR two-color laser fields has been attracted extensive attentions in recent years. It was found the interference structures, resulting from the interference between multiple collisions and directly ionized electrons [45–47]. Also it is demonstrated that when the recolliding electrons of the IR field revisit the parent ion, they can absorb an XUV photon, yielding high-energy electrons [48]. More recently, Peng et al shows that the Coulomb potential may play essential roles in both the electron rescattering and recapturing processes [49].

In this paper, we investigate the HATI process of an atom in IR+XUV two-color laser fields by employing the frequency-domain theory, where the energy of the XUV photon is much larger than the atomic ionization potential. In this theory, the ionization process can be treated as quantum transitions between two states of the entire system consisting of the atom and the laser field, where the laser field, as a part of the whole system, is treated as a quantized-field. In this work, the laser field at initial state is described as a Fock state for simplicity. In order to establish the relationship between the frequency-domain theory and the time-domain description, there are many works to examine the correspondence between the quantum-mechanical and semiclassical description of a system in which an atom is exposed to an intense field [50–53]. These results indicate that these two methods can obtain the same results for the dynamic processes of an atom in laser fields, including high-order harmonic generation (HHG), HATI and nonsequential double ionization (NSDI) etc. Moreover, compared with time-domain theory [15–19, 54], this theory may have three advantages: (i) the key characteristic of this approach is time-independent, hence it can save a lot of computational time; (ii) this approach provides a different window to understand the physical processes of strong-field, especially for recollision process, such as HATI [55, 56], HHG [50, 57] and NSDI [58–61], which can be decoupled into a direct ATI followed by a laser-assisted collision (LAC) or laser-assisted recombination so as to investigate the role of these subprocesses separately; (iii) all of the recollision processes can be investigated under a unified theoretical frame, which is facilitate to analyze the relationship among all these recollision processes. Here we present the angle-resolved PES of the HATI process with multiplateau structure, where there exists a dip structure in the second plateau. With the help of the channel analysis, one can see that the interference pattern comes from the contributions of different channels. Furthermore, the HATI can be treated as a two-step process: an ATI followed by a LAC process [26, 55]. It is found that the dip structure is attributed to the fact that one XUV photon is absorbed by the atom in the LAC process. Also, by using the saddle-point approximation, we obtain the classical energy orbit equation, which illustrates the formation of interference pattern. Atomic units are used throughout unless otherwise stated.

2. Theoretical method

The Hamiltonian of an atom in two-color linearly polarized laser fields is [26, 55]

\[
H = H_0 + U + V, 
\]

where \( H_0 = \frac{-i\nabla^2}{2} + \omega_1 N_1 + \omega_2 N_2 \) is the Hamiltonian of a free electron-laser system with the photon number operator \( N_i \) for the laser field of frequency \( \omega_i (i = 1, 2) \), \( U \) is the interaction potential between an electron and the nucleus. Finally, \( V \) is the electron–photon interaction potential, which can be expressed by \( V = -[( -i \nabla) \cdot \mathbf{A}_1 (-\mathbf{k}_1 \cdot \mathbf{r}) + (-i \nabla) \cdot \mathbf{A}_2 (-\mathbf{k}_2 \cdot \mathbf{r}) + \mathbf{A}_1 (-\mathbf{k}_1 \cdot \mathbf{r}) \cdot (-i \nabla) + \mathbf{A}_2 (-\mathbf{k}_2 \cdot \mathbf{r}) \cdot (-i \nabla)]/2 + [\mathbf{A}_1 (-\mathbf{k}_1 \cdot \mathbf{r}) + \mathbf{A}_2 (-\mathbf{k}_2 \cdot \mathbf{r})]^2/2 \), where the vector potential is \( \mathbf{A}_i (-\mathbf{k}_i \cdot \mathbf{r}) = g_i [\mathbf{\tilde{e}}_i \exp (i \mathbf{k}_i \cdot \mathbf{r}) a_i + c.c.] \) with \( g_i = (2\omega_i V_i)^{-1/2} \), \( \mathbf{k}_i \) is the wave vector, \( \mathbf{\tilde{e}}_i \) is the polarization, and \( a_i (a_i^\dagger) \)
is the annihilation (creation) operator, and \( V_s \) is the normalization volume of the laser field with the frequency \( \omega_s \) for \( s = 1, 2 \).

The frequency-domain theory based on the Hamiltonian equation (1) enables us to treat an atom-laser interaction process as a genuine scattering process in the isolated system that consists of an atom and photons. Since the total energy of the system is conserved throughout the interaction process, the formal scattering theory [62] can be applied. The S-matrix element between the states \( \psi_i \) and \( \psi_f \) is [62]

\[
S_{if} = \langle \psi_f | \psi_i \rangle,
\]

where

\[
\psi_i^+ = \psi_i + \frac{1}{E_i - H + i\epsilon} V \psi_i,
\]

and

\[
\psi_f^+ = \psi_f + \frac{1}{E_f - H - i\epsilon} U \psi_f.
\]

Physically, \( \psi_i^+ \) is the state at \( t = 0 \) which develops from a precollision state \( \psi_i \) in the remote past, while \( \psi_f^+ \) is the state at \( t = 0 \) which evolves to a postcollision state \( \psi_f \) in the remote future. The S-matrix element can be expressed as

\[
S_{if} = \delta_{if} - 2\pi i \delta(E_f - E_i) T_{if}.
\]

Here

\[
T_{if} = \langle \psi_f | V | \psi_i \rangle + \langle \psi_f | U \frac{1}{E_i - H + i\epsilon} V | \psi_i \rangle
\]

is the transition matrix element. In above equation, the first and second terms represent the direct ATI and HATI processes, respectively. It is noticed that we neglected the contributions of multiple rescattering processes since their contributions are very little. Therefore, the transition matrix can be simply expressed as \( T_{if} \). The direct ATI process in IR+XUV two-color laser fields has been investigated [42, 43]. In this paper, we will mainly focus on the HATI process. Here, \( \psi_i = \{| \phi_i \rangle \otimes | \phi_l \rangle \otimes | \phi_0 \rangle \} \) with the initial energy \( E_i = -I_0 + (l_1 + 1/2)\omega_1 + (l_2 + 1/2)\omega_2 \), where \( | \phi_0 \rangle \) is the ground state of an atom and \( | \phi_l \rangle \) is the Fock state of the laser field with photon number \( l_j \) for \( j = 1, 2 \). The final state \( | \psi_f \rangle = | \psi_{p,m,n} \rangle \) is the quantized-field Volkov state in the two-color laser field [61, 63]

\[
| \psi_{p,m,n} \rangle = V_{l_1}^{-1/2} \exp \left( i \left( (\mathbf{p}_f \cdot \mathbf{r}_f + \mathbf{p}_i \cdot \mathbf{r}_i) + m \right) \right)
\]

\[
\times \sum_{j_1 = -m_2 - m_1}^{\infty} \sum_{j_2 = -m_1}^{\infty} \tilde{R}_{j_1,j_2}(\gamma_i)^{j_1} \exp \left( -i \left( j_2 \mathbf{k}_2 \cdot \mathbf{r} + \phi_2 \right) \right)
\]

\[
\times \left( j_1 \mathbf{k}_1 \cdot \mathbf{r} + \phi_1 \right)
\]

with the energy of the final state \( E_f = \mathbf{p}_f^2/2 + (m_1 + 1/2)\omega_1 + (m_2 + 1/2)\omega_2 \). In the above, \( V_{l_1} \) is the normalization volume, \( \mathbf{p}_f \) is the momentum of the ionized electron. For intense laser fields, the photon number is very large and hence the laser field should be described in terms of a classical vector potential. Under the large photon number limit (i.e. \( m_1 \to \infty, \mathbf{p}_f \to 0, \mathbf{p}_i \to \mathbf{p}_0 \to \Lambda_0 \) [64], the ponderomotive potential divided by the photon energy reduces to \( U_{\text{p}} = \frac{e^2 m_0}{\omega^2} \to \frac{E_0}{\omega^2} \) [65], and the intensity of laser field \( I_0 = \frac{m_0 \omega}{c} \to \frac{2m_0 \omega^3}{c^2} \) [51], where \( \Lambda_0 \) is the half amplitude of the corresponding classical field for \( j = 1, 2 \). Further, the ponderomotive potential can be written as \( U_{\text{p}} = \omega_0 u_{\text{p}} \). Also, it is generally known that a Fock state is related to its photon number while the phase is entirely undetermined. Meanwhile, the time-dependent phase of the classical field is very important in dynamic of interaction with laser and atom [7, 66]. In our theory, the phase is retained in the quantized-field Volkov state. In equation (7), \( \phi_i \) is the initial phase of the two laser fields with \( j = 1, 2 \). The correspondence of the quantized-field and classical field had been discussed in papers [51, 64, 65, 67, 68]. The quantized-field Volkov state may describe the entangled between the photonic Fock states and the description of the electron motion being approximated by the plane wave. By employing the Fock state, we may directly derive the transition amplitude by using the energy conservation law for the atom-laser system. The term \( \tilde{R}_{j_1,j_2}(\gamma_i) \) in equation (7) is the generalized Bessel function, which can be written as [63]

\[
\tilde{R}_{j_1,j_2}(\gamma_i) = \sum_{\gamma_{1,2}} J_{n_1+n_2} \left( \gamma_{1,2} \right) J_{n_1} \left( \gamma_{1,2} \right) J_{n_2} \left( \gamma_{1,2} \right) \times J_{n_1+n_2} \left( \gamma_{1,2} \right) J_{n_1} \left( \gamma_{1,2} \right) J_{n_2} \left( \gamma_{1,2} \right),
\]

where

\[
\gamma_1 = 2 \frac{\mu_{\text{p}}}{\omega_1} \mathbf{p} \cdot \hat{\mathbf{e}}_1,
\]

\[
\gamma_2 = 2 \frac{\mu_{\text{p}}}{\omega_2} \mathbf{p} \cdot \hat{\mathbf{e}}_2,
\]

\[
\gamma_3 = \frac{1}{2} u_{\text{p}},
\]

\[
\gamma_4 = \frac{1}{2} u_{\text{p}},
\]

\[
\gamma_5 = 2 \frac{\mu_{\text{p}} \omega_2 u_{\text{p}}}{\omega_1 + \omega_2},
\]

\[
\gamma_6 = 2 \frac{\mu_{\text{p}} \omega_1 u_{\text{p}}}{\omega_1 + \omega_2},
\]

and \( J_m(t) \) is the Bessel function of order \( m \). Also, \( \mathbf{p} \) is the momentum of the ionized electron, \( \hat{\mathbf{e}}_j \) is the unit vector representing the polarization direction of laser field with \( j = 1, 2 \).

In the HATI process, an intermediate state can be expressed as \( | \psi_{p,m,n} \rangle \) with the energy \( E_0 = \mathbf{p}_f^2/2 + (m_1 + 1/2)\omega_1 + (m_2 + 1/2)\omega_2 \). Here by applying the strong-field approximation in the HATI process, and the completeness relation of the intermediate states together with the initial and final states, the second term of equation (6) can be expressed as [55]

\[
T_{\text{HATI}} = -i \pi \sum_{p,m,n} \langle \psi_{p,m,n} | V | \psi_{p,m,n} \rangle
\]

\[
\times \langle \psi_{p,m,n} | \psi_{p,m,n} \rangle \delta(E_f - E_i).
\]

Here, we have noted that the term \( \langle \psi_{p,m,n} | V | \psi_{p,m,n} \rangle \) stands for the ATI process, where an electron is ionized directly by two-color laser fields, and the term \( \langle \psi_{p,m,n} | V | \psi_{p,m,n} \rangle \) stands for the LAC process, where the ionized electron collides with the parent ion and scatters off by a LAC. According to equation (10), the HATI can be described as a direct ATI followed by an LAC.
process with all HATI channels summed up coherently. Therefore, the HATI transition indicates that ATI transition provides a weighting amplitude for LAC transition in each HATI channel. The ATI and LAC are physical processes. It is worth noting that \( \mathbf{p}_e \) is momentum of electron ionized in the ATI process. By using equation (7), the transition matrix for the HATI can be expressed as

\[
T_{\text{HATI}} = -\frac{i}{8\pi^4\alpha^2} \sum_{s_1, q_1} \sum_{s_2, q_2} |p|_r \times [(u_{p_1 - s_1})\omega_1 + (u_{p_2 - s_2})\omega_2] \\
\times \exp[(i\omega_1 + q_1\phi_1)(i\omega_2 + q_2\phi_2)] \\
\times \int \sin\theta_i d\theta_i d\phi_i r_{s_1 j} q_{n i} (\zeta_{\theta_i})^n (\zeta_{\phi_i})^n \sum_{q_1, q_2} (\zeta_f - \zeta_e) \phi_f(\mathbf{p}_e) \int d\mathbf{r} e^{-i\mathbf{p}_r \cdot \mathbf{U}(\mathbf{r})} e^{i\mathbf{p}_e \cdot \mathbf{r}},
\]

where \( s_1 = l_1 - n_1 \) and \( s_2 = l_2 - n_2 \) are the number of photons absorbed from the first and second laser fields in the ATI process, \( q_1 = m_1 - n_1 \) and \( q_2 = m_2 - n_2 \) are the number of photons absorbed from the first and second laser fields in the LAC process. In above expression, \( |p|_r = \sqrt{2[s_1\omega_1 + s_2\omega_2 - u_{p_1}\omega_1 - u_{p_2}\omega_2 - I_1]} \) is the magnitude of momentum for electron ionized from the nucleus in the ATI, \( \theta_i \) is the angle between the emission direction of the electron ionized from the atom and the laser polarization, \( \phi_i \) is the azimuth angle. In equation (11), the integral \( A(p) = \int d\mathbf{r} e^{-i\mathbf{p}_r \cdot \mathbf{U}(\mathbf{r})} e^{i\mathbf{p}_e \cdot \mathbf{r}} \) can be rewritten as \( A(p) = \frac{\alpha^2}{\alpha^2 + (\mathbf{p}_e - \mathbf{p}_r)^2} \) by using the short-range atomic potential \( U(\mathbf{r}) = \exp(-\alpha r)/r \). With the help of the direct or rectangular integration method, we can obtain the convergent result in the case of \( \alpha = 1.0 \).

3. Numerical results

In this section we will consider the HATI process of an atom in an IR+XUV two-color laser field. The atomic ionization threshold is \( I_p = 12.1 \) eV. The frequencies of the IR and XUV laser fields are \( \omega_1 = 1.165 \) eV and \( \omega_2 = 50\omega_1 \), and their intensities are \( I_1 = I_2 = 1.0 \times 10^{13} \) W cm\(^{-2} \). The polarization directions of the two-color laser fields are the same and their initial phases are set to zero for simplify.

3.1. The angle-resolved photoelectron energy spectrum of the HATI process

Figure 1(a) presents the HATI spectra for different \( \theta_i \) and (b) the angle-resolved PES of HATI probability of an electron in the HATI process, where \( \theta_i \) is the angle between the laser polarization direction and the emission direction of the final ionized electron after collision with the parent ion. One can see that the HATI spectrum, as well as, the angle-resolved PES shows a multiplateau structure, where the probability of the first plateau is much higher than that of the second plateau by about six orders of magnitude. This result leads to that the interference between the two plateaus is not obvious. Furthermore, it is found that the probability distribution shown in figure 1(b) is symmetry about the line defined by \( \theta_i = 90^\circ \), where the width of the spectrum decreases (increases) with the value of \( \theta_i \) as \( \theta_i \leq 90^\circ \) (\( \theta_i > 90^\circ \)). Especially, the width of the spectrum shows narrowest around \( \theta_i = 90^\circ \). Also, we have noted that there is a narrow dip structure in the second plateau of the angular distribution. In what follows, we will explain the formation of the interference patterns and the emergence of the dip structure on the spectrum.

According to the previous investigations [32, 42, 58, 59], we know that the XUV laser field plays a crucial role in the ionization process. Figure 2 shows the angle-resolved PESs of the atom absorbing one (figure 2(a)) and two (figure 2(b)) XUV photons in the HATI process. Comparing figure 1 with figures 2(a) and (b), one can find that the first and second plateaus in figure 1 are separately attributed to the processes of atom absorbing one and two XUV photons. Therefore, the probability of the first plateau is much larger than that of the second plateau.

Based on the frequency-domain theory, the HATI can be decoupled into a two-step process, i.e. an ATI followed by an LAC. The atom may absorb XUV photons in both ATI and LAC processes. Hence we now define the channel as \((s_2, q_2)\), where \( s_2 \) and \( q_2 \) are the number of the XUV photons absorbed by an atom in the ATI and LAC processes, respectively. Firstly, we consider the first plateau of the spectrum where the electron absorbs one XUV photon in the HATI process. Figure 3 shows the angle-resolved PES of the HATI process for channel \((1, 0)\). The channel \((0, 1)\) is not shown because of its ignorable contributions to the HATI process. One can see that the channel \((1, 0)\) dominates almost all contributions to the first plateau. This can be understood as follows: since one XUV photon energy (i.e. 58.25 eV) is much larger than the atomic ionization threshold (i.e. 12.1 eV) in our calculation, the electron may have enough energy to be ionized by absorbing one XUV photon in the ATI process, as a result, the probability of the first step in HATI increases dramatically comparing with that of the monochromatic IR laser case; then the ionized electron may absorb more IR photons in LAC process and form the first plateau in the HATI spectrum.

Next, we consider the second plateau shown in figure 2(b). We find that there are two channels \((1, 1)\) and \((2, 0)\) to make dominate contributions to the second plateau, as shown in figure 4, where these two channels provide comparable contributions to the plateau. It is mentioned that the channel \((0, 2)\) is not shown because of its ignorable contributions to the HATI process. Furthermore, it is found that the dip structure only comes from the contribution of channel \((1, 1)\), while the angular distribution of channel \((2, 0)\) is similar to that of channel \((1, 0)\). For the channel \((1, 1)\), the electron absorbs another XUV photon in the LAC process; but for channel \((2, 0)\), the electron does not absorb any XUV photon in the LAC process. These results indicate that absorbing XUV photon in the LAC process can change the angular distribution of the HATI process, especially can form the dip structure.

In order to explain the interference patterns shown in figure 4, we can consider the HATI as such a process: the ionized electron with certain momentum \( \mathbf{p}_e \) in the ATI
process collides with its parent ion, and then gets its final Volkov state in the IR+XUV two-color laser fields. According to our theory, the interference pattern may be attributed to the coherent summation of the contributions from different subchannels. Hence we further define the subchannel as (\(s_1|s_2\), \(q_1\)) within channel (\(s_2, q_2\)), where \(s_1\) is the number of IR photons absorbed (\(s_1 > 0\)) or emitted (\(s_1 < 0\)) in the ATI process.

We now consider the interference pattern of channel (1, 1) shown in figure 4(a). Figures 5(a)–(d) present the channel contributions for channel (1, 1) with certain emission angles of \(p_n\) at \(\theta_n = 20^\circ, \theta_n = 40^\circ, \theta_n = 65^\circ, \) and \(\theta_n = 160^\circ\), respectively. \(\theta_n\) is the angle between the electron momentum \(p_n\) and the polarization direction of the laser fields. From figures 5(a)–(c), one can find that there exists a narrow dip structure in each graph, and the dip structure moves toward the line defined by \(\theta_f = 90^\circ\) with the increase of \(\theta_n\). Furthermore, comparing figures 5(a) and (d), one can see that the two patterns are symmetrical, which leads to the formation of symmetrical structure shown in figure 4(a). Here the results of the electron

**Figure 1.** (a) The HATI spectra for different \(\theta_f\) and (b) the angle-resolved PES of an electron in the HATI process, where \(\theta_f\) is the angle between the laser polarization and the emission direction of the ionized electron (on logarithmic scale).

**Figure 2.** The angle-resolved PES of the atom absorbing (a) one and (b) two XUV photons in the HATI process (on logarithmic scale).

**Figure 3.** Channel contribution of the angle-resolved PES for channel (1, 0) (on logarithmic scale).
emitted along the laser polarization direction in ATI process are not shown because their contributions are small comparing with other results. This phenomenon can be easily understood as follows: since the energy of one XUV photon is very large, the ionized electron can obtain larger energy in ATI process by absorbing XUV photons. If the electron is emitted along the laser polarization direction, the probability is very small to collide with parent ion in the laser field, which leads to the lower ionization probability for HARTI process. In order to further explain interference pattern in more detail, we present the sub-channel contributions for \((1, 1)\) (figures 5(a1)–(d1)), \((2, 0)\) (figures 5(a2)–(d2)) and \((12, 1)\) (figures 5(a3)–(d3)) with certain emission angles of \(\theta_n = 20^\circ\) (figures 5(a1)–(d1)), \(\theta_n = 40^\circ\) (figures 5(a2)–(d2)) and \(\theta_n = 65^\circ\) (figures 5(a3)–(d3)) and \(\theta_n = 160^\circ\) (figures 5(a4)–(d4)).
\( \theta_n = 160^\circ \) (figures 5(a1)–(d3)). It is shown that there is a waist in each of the interference patterns for every subchannel. From figures 5(a1) to (a3), one can see that the waist in each graph gradually moves toward the line defined by \( \theta_l = 0^\circ \) with the increase of the number of the IR photons \( s_1 \) absorbing by the electron in the ATI process. Comparing figure 5(a) with figures 5(a1)–(a3), it is found that the interference pattern shown in figure 5(a) comes from the summations of the contributions from different subchannels. Moreover, one can find that the minimum value appears at the waist for subchannel \( \{ l, n, 1 \} \). Hence these minimum values of the subchannels lead to the formation of the dip structure, rather than the narrowest distribution. It is demonstrated in the above analysis, we have calculated the angle-resolved PESs of different partial subchannels, and find that only the coherent summation of contributions of all subchannels can lead to the narrowest distribution around \( \theta_l = 90^\circ \) presented in figure 5(a). To further demonstrate the above analysis, we have calculated the angle-resolved PESs of different partial subchannels; and find that only the coherent summation of contributions of all subchannels lead to the waist for subchannel \( \{ l, n, 1 \} \). These results are obviously different from the interference of the contributions of all subchannels, while the dip structure is from the direct summation over the minimum value of the waists for different subchannels.

Next we consider the interference pattern of channel \( (2, 0) \) shown in figure 4(b). Similarly, figures 6(a)–(d) present the channel contributions for channel \( (2, 0) \) with \( \theta_n = 20^\circ \), \( \theta_n = 40^\circ \), \( \theta_n = 60^\circ \) and \( \theta_n = 160^\circ \), respectively. One can see that there is no dip structure whatever direction that the ionized electron emits along with in the ATI process. Similarly, here the results for \( \theta_n = 0^\circ \) and \( \theta_n = 180^\circ \) are not shown because their contributions are very small. In order to understand more deeply the interference patterns of the subchannel PESs shown in figures 5 and 6, we now focus on the analysis of the LAC process by the saddle-point approximation. Under our present calculation condition, the Bessel function in equation (11) can be reduced as

\[
\hat{n}_q (\zeta_f - \zeta_n) = J_q (\zeta_{n1} - \zeta_{f1}) J_q (\zeta_{n2} - \zeta_{f2}).
\]  

(12)

For absorbing a certain number of XUV photons \( q_2 \), the Bessel function \( J_q (\zeta_{n1} - \zeta_{f1}) \) can be expressed in an integral form

\[
J_q (\zeta_{n1} - \zeta_{f1}) = \frac{\omega_1}{2\pi} \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} dt \exp \{ i(\zeta_{n1} - \zeta_{f1}) \sin(\omega_1 t)
- q_1 \omega_1 t \},
\]  

(13)

where \( T_1 = 2\pi/\omega_1 \). On the other hand, the IR laser field can be treated as a classical field, \( A_{cl} (t) = \tilde{E}_l / \omega_1 \cos(\omega_1 t) \), where \( \tilde{E}_l \) is the amplitude of the laser’s electric field and \( \tilde{E}_l \) is the polarization direction. Therefore, the classical action of an electron is

\[
S_{cl} (t, p) = \frac{1}{2} \int_0^t \left( p^2 + U_{cl} (t) \right) dt
= \frac{1}{2} \left( p^2 + U_{cl} (t) \right) t + \frac{H_{cl} \sin(\omega_1 t)}{\omega_1} \cdot \tilde{E}_1
+ \frac{1}{2} u_{cl} \sin(2\omega_1 t),
\]  

(14)

where \( U_{cl} = E_{cl}^2 / (4\omega_1^2) \) is the ponderomotive energy in the IR laser field. By using equation (14) and the energy conservation \( E_i = E_{n1} \) in the ATI process, equation (13) can be cast into

\[
J_q (\zeta_{n1} - \zeta_{f1}) = \frac{\omega_1}{2\pi} \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} dt \exp \{ i(\zeta_{n1} - \zeta_{f1}) \sin(\omega_1 t)
+ \frac{1}{2} u_{cl} \sin(2\omega_1 t),
\]  

(15)

where \( f (t) = S_{cl} (t, p_f) - S_{cl} (t, p_f) + q_2 \omega_1 t \). By using the saddle-point approximation, equation (15) can be rewritten as

\[
J_q (\zeta_{n1} - \zeta_{f1}) = \frac{2\omega_1}{\sqrt{\pi f (t_0)}} \cos [ F (t_0) - \pi/4],
\]  

(16)

where \( F (t_0) = \left( \zeta_{n1} - \zeta_{f1} \right) \sin(\omega_1 t_0) - q_1 \omega_1 t_0 \), and the saddle-point \( t_0 \) satisfies \( \cos \omega_1 t_0 = \zeta_{n1} - \zeta_{f1} = \frac{n_1}{n_2} = \frac{\zeta_{f1} - \zeta_{f2}}{\zeta_{f1} - \zeta_{f2}} \). The final energy of ionized electron \( E_f \) can be written as

\[
E_f = \sqrt{2U_{p1} \cos^2(\omega_1 t_0) \cos^2 \theta_f + E_{n1} + 2\sqrt{2E_{p1} U_{p1}} \cos(\omega_1 t_0) \cos \theta_n + q_2 \omega_2 - \sqrt{2U_{p1} \cos(\omega_1 t_0) \cos \theta_f}^2}.
\]  

(18)
$E_n$ is the energy of the ionized electron in the ATI process. Therefore, the final energy $E_f$ of ionized electron is determined by the above energy orbital equation (18) in the LAC process.

Taking subchannels (1|−9, 1) and (2|−9, 0) with $\theta_n = 20^\circ$ as examples, figure 7 presents the subchannel contributions and corresponding energy orbits for (1|−9, 1) (figure 7(a)) and (2|−9, 0) (figure 7(b)). The insets in figure 7 show the details of the waists. One can find that the energy orbits in figure 7 agree well with the numerical results, which indicates that the patterns come from the interference between these different orbits. We have noted that the energy of the electron with minimum value at $\theta_f = 0^\circ$ increases with $\theta_f$, while the energy with maximum value at $\theta_f = 0^\circ$ decreases with the angle. Therefore all of these energy orbits intersect at the waist, and the positions of these orbits exchange as $\theta_f$ increases through the waist from 0° to 180°. It tells us that the energy that the ionized electron obtains from the IR laser field...
in the LAC process varies with the emission angle $\theta_e$. Especially, the ionized electron obtains the maximum energy when it is emitted along the opposition direction of laser polarization. These results indicate that the IR laser field plays an important role in the LAC process. In other words, the IR laser field can accelerate or decelerate the electron in the LAC process, which leads to the formation of plateau in HATI process. Furthermore, one can see that the range of energy distribution becomes wider as the number of the IR photons that electron absorbs or emits in the LAC process increases. However, it is noteworthy that there exists corresponding relationship between $q_1 = 0$ and $P_{\parallel} = P_{\parallel, \perp}$ at the waist, which indicates that, the momentum of the electron parallel to the laser polarization keeps constant before and after the collision, and also the vector potential of IR laser field is equal to zero in the LAC process. For the waist of the subchannel $(1|-9, 1)$, the electron absorbs one XUV photon and zero IR photon in the LAC process, but the momenta parallel to the laser polarization before and after collision still remain constant, hence we have $\frac{1}{2}(p_{\parallel, \perp})^2 = \frac{1}{2}(p_{\parallel, \perp})^2 + q_{2} \omega_{2}$, where $p_{\parallel, \perp}$ is the momentum of the electron perpendicular to the laser polarization direction after (before) the collision. As we know that the absorption of the XUV photon is a multiphoton absorption process, hence the electron’s momentum may change in all directions by absorbing one XUV photon during the collision. However in the above case, the electron’s momentum only changes along the direction perpendicular to the laser polarization, resulting in that the probability of the absorption process during the collision gets a minimum value. Hence the probability at the waist becomes minimum value on the spectrum for subchannel $(1|-9, 1)$, as shown in the inset in figure 7(a), leading to the dip structure for channel $(1, 1)$. On the contrary, for the case of subchannel $(2|-9, 0)$, since electron does not absorb any IR or XUV photon at the waist in the LAC process, its momentum keeps constant before and after collision, i.e. $p_r = p_r$. Hence the probability at the waist keeps a certain value for subchannel $(2|-9, 0)$, as shown in the inset in figure 7(b), leading to the fact that there is no dip structure for channel $(2, 0)$.

Furthermore, by using channel analysis, one can find that the interference pattern shown in figure 1 is from the coherent summation of many subchannels. By employing the saddle-point approximation, we may find that the interference pattern is attributed to the term $\cos[F(t_0) - \pi/4]$ in equation (16) for $p_r$, along $\theta_e = 20^\circ$. The inset presents the details at waist (on logarithmic scale).

4. Conclusions

Based on the frequency-domain theory, we have investigated the HATI process of an atom in IR-XUV two-color laser fields. It has been shown that the angle-resolved PES presents a multiplateau structure, where the probability of the first plateau is much larger than that of the second plateau and there is a dip structure in the second plateau. In this theory, the HATI can be treated as a two-step process: an ATI followed by an LAC. With the help of channel analysis, it has been found that the angle-resolved PES is attributed to the coherent summation over contributions of different channels, and the dip structure is attributed to the XUV photon absorption by the ionized electron in the LAC process. Furthermore, it has been demonstrated that the formation of the narrowest distribution results from the quantum interference of the contributions of all subchannels, while the dip structure is caused by the direct summation over the minimum values at the waists of all subchannels. Moreover, by using the saddle-point approximation, we have obtained an energy equation of classical orbits and have shown that the predictions of the destructive interference positions by the classical
energy orbits agrees well with the quantum interference pattern. Additionally, it has been demonstrated that the interference pattern of HATI process comes from the interference of different orbits at collision moments $t_0$ and $2\pi/\omega_1 - t_0$ in the LAC process.

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