CP violation in $B_d \to \phi K_S$ in SUSY GUT with right-handed neutrinos

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Abstract

CP violation in the $B_d$ system is discussed in the supersymmetric grand unified theory (GUT) with the right-handed neutrinos. Above the GUT scale, the right-handed down-type squarks couple to the right-handed neutrinos. Due to the renormalization group effect, flavor violations in the lepton sector may be transferred to the right-handed down-type squark mass matrix, which affects the CP violation in the $B$ decay. Taking into account this effect, we compare the CP violation in $B_d \to \psi K_S$ and $B_d \to \phi K_S$ processes. We will find that a significant difference is possible between the CP violating phases in two decay processes.
Many efforts have been made to understand the CP violation. In particular, experimentally, B-factories are now being to measure the CP violation in the $B$ system and to obtain insights into the physics behind the CP violation. One of the most important purposes of the $B$-factories is to check the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{\text{CKM}}$ using the $B_d \rightarrow \psi K_S$ decay mode; in the standard model, the phase $\arg(-[V_{\text{CKM}}]_{cd}[V_{\text{CKM}}]_{cb}/[V_{\text{CKM}}]_{td}[V_{\text{CKM}}]_{tb})$ is determined with $B_d \rightarrow \psi K_S$ [1]. Since the phase in the CKM matrix is the only source of the CP violation in the standard model, such a measurement will provide severe constraints on the CP violation in other processes.

There can be, however, extra contribution to the CP violation from a new physics beyond the standard model. Since the standard model suffers from the so-called hierarchy problem, i.e., the stability of the Higgs mass parameter against the radiative corrections, we are forced to introduce a new physics at the electroweak scale. Furthermore, the deficits of the atmospheric and solar neutrino fluxes strongly suggest non-vanishing masses and mixings in the neutrino sector. Since the new physics is, in general, CP violating, the $B$-factories may be able to observe phenomena related to the new CP violating interactions.

In this letter, we consider supersymmetric grand unified theory (GUT) with the right-handed neutrinos as the new physics. In this model, the hierarchy problem is solved by the cancellation of the quadratic divergences between bosonic and fermionic loops due to the supersymmetry (SUSY), while the neutrino masses are generated by the seesaw mechanism [2]. In such a model, the right-handed down-type squarks interact with the right-handed neutrinos and colored Higgs. Of course, such interactions are negligible for the low energy physics at the tree level since the colored Higgs is as heavy as the GUT scale.

However, they become important through the renormalization group (RG) effect [3]. (Similar effects have been studied for the low energy flavor violating processes [4, 5, 6].) In Ref. [5], it was discussed that the right-handed neutrinos affects the structure of the flavor and CP violations in the mass matrix of the right-handed down-type squarks in SUSY GUT. In particular, SUSY contribution to the $\epsilon_K$ parameter can be as large as the currently measured value if we adopt an $O(1)$ phase in the neutrino Yukawa matrix. In this letter, we consider different effects, that is, CP violation in the decay processes of the $B$-mesons. We will see that the SUSY contribution to the decay amplitude of the $B_d \rightarrow \phi K_S$ process can be large, and that the phase in the decay amplitude can be as large as $O(0.1)$ which is much larger than the standard model prediction. We will emphasize that phases in the GUT Lagrangian play a crucial role for this enhancement.

Let us first introduce the model we consider. To make our point clearer, we consider a SUSY $SU(5)$ model with the right-handed neutrinos. The superpotential of the model is

$$W_{\text{GUT}} = \frac{1}{8} \Psi_i [Y_U]_{ij} \Psi_j H + \Psi_i [Y_D]_{ij} \Phi_j \bar{H} + N_i [Y_N]_{ij} \Phi_j H + \frac{1}{2} N_i [M_N]_{ij} N_j, \quad (1)$$

where $\Psi_i$, $\Phi_i$, and $N_i$ are $10$, $\bar{5}$, and singlet matter fields of $SU(5)$ in $i$-th generation, while

#1 We neglect the strong CP problem.

#2 In Ref. [3], similar effect was also discussed. However, in this article, one of the most important effects, i.e., the effect of the phases in the GUT models, was neglected.
$H$ and $H$ are Higgs fields which are in $\mathbf{5}$ and $\mathbf{\overline{5}}$ representations, respectively. Here, $M_N$ is the Majorana mass matrix for the right-handed neutrinos, and for simplicity, we adopt the universal structure:

$$[M_N]_{ij} = M_{\nu_R} \delta_{ij}. \quad (2)$$

The Yukawa matrix $Y_U$ is a complex symmetric matrix while $Y_D$ and $Y_N$ are complex matrix. We take the basis where the Yukawa matrices become

$$Y_U = V_T^Q \hat{\Theta} Q Y_Q, \quad Y_D = \hat{Y}_D, \quad Y_N = \hat{Y}_N V_L \hat{\Theta}_L, \quad (3)$$

where $\hat{\Theta}$'s are real diagonal matrices:

$$\hat{Y}_U = \text{diag}(y_u, y_c, y_t), \quad \hat{Y}_D = \text{diag}(y_d, y_s, y_b), \quad \hat{Y}_N = \text{diag}(y_{\nu_1}, y_{\nu_2}, y_{\nu_3}), \quad (4)$$

while $\hat{\Theta}$'s are diagonal phase matrices:

$$\hat{\Theta}_Q = \text{diag}(e^{i \phi_1(Q)}, e^{i \phi_2(Q)}, e^{i \phi_3(Q)}), \quad \hat{\Theta}_L = \text{diag}(e^{i \phi_1(L)}, e^{i \phi_2(L)}, e^{i \phi_3(L)}), \quad (5)$$

where phases obey the constraints $\phi_1(Q) + \phi_2(Q) + \phi_3(Q) = 0$ and $\phi_1(L) + \phi_2(L) + \phi_3(L) = 0$. Furthermore, $V_Q$ and $V_L$ are unitary mixing matrices parameterized by three mixing angles and one CP violating phase.

The quarks and leptons in the standard model are embedded in the $SU(5)$ multiplets as

$$\Psi_i \simeq \{Q, \ V_Q^T \hat{\Theta}_Q^+ \hat{\Theta}_Q \hat{U}, \ \hat{\Theta}_L \hat{E}\}_i, \quad \Phi_i \simeq \{\hat{D}, \ \hat{\Theta}_L^+ \hat{L}\}_i, \quad (6)$$

where $Q_1(3, 2)_{1/6}$, $U_1(\mathbf{\overline{3}}, 1)_{-2/3}$, $D_1(\mathbf{\overline{3}}, 1)_{1/3}$, $L_1(1, 2)_{1/2}$, and $E_1(1, 1)_1$ are quarks and leptons in $i$-th generation with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge quantum numbers as shown. With the embedding (6), the superpotential for the light particles is

$$W_{\text{SSM}} = Q_i \left[ V_T^Q \hat{Y}_U \right]_{ij} \bar{U}_j H_u + Q_i \left[ \hat{Y}_D \right]_{ij} \bar{D}_j H_d + \tilde{E}_i \left[ \hat{Y}_E \right]_{ij} \bar{L}_j H_d + N_i \left[ \hat{Y}_N V_L \right]_{ij} \bar{L}_j H_u + \frac{1}{2} \mu \nu \bar{N}_i \bar{N}_i, \quad (7)$$

with $H_u$ and $H_d$ being the up- and down-type Higgs fields, respectively. In the $SU(5)$ limit, $\hat{Y}_D = \hat{Y}_D$ although this relation does not hold for the first and second generations. We expect that some mechanism, like higher dimensional operator suppressed by the Planck scale, fixes this problem. We assume such a mechanism does not affect the following analysis. In Eq. (7), the unitary matrix $V_Q$ becomes the CKM matrix: $V_Q \simeq V_{\text{CKM}}$. Furthermore, the neutrino mass matrix after the seesaw mechanism is given by

$$[m_{\nu L}]_{ij} = \frac{(H_u)^2}{M_{\nu_R}} \left[ V_L^T \hat{Y}_N^2 V_L \right]_{ij} = \frac{v^2 \sin^2 \beta}{2M_{\nu_R}} \left[ V_L^T \hat{Y}_N^2 V_L \right]_{ij}, \quad (8)$$

#3 We assume that other chiral multiplets, in particular, ones which are responsible for the $SU(5)$ breaking, do not affect the following argument.
where \( v \simeq 246 \text{ GeV} \), and \( \beta \) parameterizes the relative size of the Higgs vacuum expectation values: \( \tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle \). Therefore, with Eq. (2), \( V_L \) plays the role of the neutrino mixing matrix \( \frac{7}{7} \). In this letter, we mainly consider the neutrino mass matrix suggested by the atmospheric neutrino flux deficit \( \frac{8}{8} \), and also by the large angle MSW solution to the solar neutrino problem \( \frac{9}{9} \):

\[
\begin{align*}
\begin{pmatrix}
0.91 & -0.30 & 0.30 \\
0.42 & 0.64 & -0.64 \\
0 & 0.71 & 0.70
\end{pmatrix}
\end{align*}
\] (9)

Notice that, due to Eq. (8), the neutrino Yukawa couplings \( y_\nu \)’s increase as \( M_{\nu R} \) becomes larger with the fixed light neutrino masses. With a too large \( M_{\nu R} \), the neutrino Yukawa coupling becomes non-perturbative below the Planck scale. With Eq. (9), it happens when \( M_{\nu R} \gtrsim 10^{15} \text{ GeV} \). Therefore, we only consider the cases with \( M_{\nu R} \approx 10^{15} \text{ GeV} \).

With the embedding \( \frac{10}{10} \), all the GUT scale phases drop off from the Yukawa interactions among the light fields. However, the phases \( \phi(L) \) and \( \phi(Q) \) remain in the colored Higgs vertices. Above the GUT scale \( M_{\text{GUT}} \), interaction among \( N, \tilde{D}, \) and \( H_C \) is effective in the following form

\[
W_{\text{GUT}} = N_i \left[ \hat{Y}_N V_L \hat{\Theta}_L \right]_{ij} \tilde{D}_j H_C + \cdots = \sum_{i,j} y_n \left[ V_L \right]_{ij} e^{i\phi(L)} N_i \tilde{D}_j H_C + \cdots .
\] (10)

Although the effect of the colored Higgs is negligible to the \( B \) physics at the tree level, the interaction (10) is potentially important since it affects the soft SUSY breaking mass parameters through the RG effect. The simplest way to estimate the effect is to assume the universality of the scalar mass at the reduced Planck scale \( M_* \approx 2.4 \times 10^{18} \text{ GeV} \). This corresponds to adopting the model so-called mSUGRA. With this framework, we will estimate the possible size of the SUSY contribution to the CP violation in the \( B \) system.

In this approach, the soft SUSY breaking parameters are parameterized by four free parameters: the universal scalar mass \( m_0 \), the universal \( A \)-parameter \( a_0 \), the gaugino mass \( m_{\tilde{g}_5} \), and the \( B \)-parameter which is determined by the condition for the proper electroweak symmetry breaking. With the universal scalar mass at the reduced Planck scale, we run all the parameters down to the electroweak scale and evaluate physical quantities using the parameters at the electroweak-scale. Of course, the scalar mass parameters may be non-universal at the reduced Planck scale, and if so our results can be modified. Since the off-diagonal elements of the scalar mass matrix is the most important for our discussion, however, we expect that the mSUGRA approach gives us a reasonable and conservative estimation in most of the cases. The CP violation in the \( B \) system we will discuss will be more enhanced if the non-universal contribution is larger than the running effect. If two contributions are comparable, the signal can be smaller due to an accidental cancellation. However, such a cancellation requires a tuning of the parameters, and we neglect this possibility.

Next, we estimate the size of the flavor violating off-diagonal elements in the down-type squark mass matrix. Relevant part of the soft SUSY breaking terms is given by

\[
\mathcal{L}_{\text{soft}} = -\left[ m_0^2 \right]_{ij} \tilde{Q}_i \tilde{Q}_j^* - \left[ m_{\tilde{D}}^2 \right]_{ij} \tilde{D}_i \tilde{D}_j^* - \left( [A_D]_{ij} \tilde{Q}_i \tilde{D}_j H_d + \text{h.c.} \right)
\]
\[-\frac{1}{2} (m_{G3} \tilde G \tilde G + \text{h.c.}) , \tag{11}\]

where \( \tilde Q_i \) and \( \tilde D_i \) are scalar components of the corresponding chiral superfields while \( \tilde G \) is the gluino field. The gluino mass is given by \( m_{\tilde G} = |m_{G3}| \). With the mSUGRA boundary condition, the off-diagonal elements in the mass matrix of \( \tilde D \) are approximately given by

\[
[m^2_{\tilde D}]_{ij} \approx -\frac{1}{8\pi^2} \left[ Y^\dagger_N Y_N \right]_{ij} (3m_0^2 + a_0^2) \log \frac{M_*}{M_{\text{GUT}}} \approx -\frac{1}{8\pi^2} e^{-i(\phi_1^{(L)} - \phi_2^{(L)})} \sum_k y_{\nu k}^2 \left[ V^*_L \right]_{ki} \left[ V_L \right]_{kj} (3m_0^2 + a_0^2) \log \frac{M_*}{M_{\text{GUT}}} , \tag{12}\]

and hence non-vanishing off-diagonal elements are generated due to the neutrino Yukawa matrix. Furthermore, in general, these off diagonal elements have unknown \( O(1) \) phases. For a more precise calculation, we solve the RG equation numerically. With \( m_0 \gg m_{G3} \) and \( a_0 \), the ratios of the off-diagonal to the diagonal elements are approximately given by

\[
\left| \frac{[m^2_{\tilde D}]_{13}}{[m^2_{\tilde D}]_{11}} \right| \approx 6 \times 10^{-4} \times \left( \frac{M_{\nu R}}{10^{14} \text{ GeV}} \right) , \quad \left| \frac{[m^2_{\tilde D}]_{23}}{[m^2_{\tilde D}]_{11}} \right| \approx 2 \times 10^{-2} \times \left( \frac{M_{\nu R}}{10^{14} \text{ GeV}} \right) , \tag{13}\]

where we used the neutrino masses and mixings given in Eq. (9). The neutrino Yukawa couplings are proportional to \( M^{1/2}_\nu \), and hence the RG induced off-diagonal elements are approximately proportional to \( M^{1/2}_\nu \). Similarly, the top Yukawa coupling generates those of the left-handed down-type squarks:

\[
[m^2_{\tilde Q}]_{ij} \approx -\frac{1}{8\pi^2} y_t^2 \left[ V_{\text{CKM}} \right]_{ti} \left[ V^*_{\text{CKM}} \right]_{tj} (3m_0^2 + a_0^2) \left( 3 \log \frac{M_*}{M_{\text{GUT}}} + \log \frac{M_{\text{GUT}}}{M_{\text{weak}}} \right) , \tag{14}\]

where \( M_{\text{weak}} \) is the electroweak scale. The phases in \( [m^2_{\tilde Q}]_{ij} \) is governed by that in the CKM matrix in the mSUGRA case. Numerically, we obtain

\[
\left| \frac{[m^2_{\tilde Q}]_{13}}{[m^2_{\tilde Q}]_{11}} \right| \approx 2 \times 10^{-3} , \quad \left| \frac{[m^2_{\tilde Q}]_{23}}{[m^2_{\tilde Q}]_{11}} \right| \approx 9 \times 10^{-3} . \tag{15}\]

Since the off-diagonal elements \( [m^2_{\tilde Q}]_{13} \) and \( [m^2_{\tilde Q}]_{13} \) are coefficients of \( \Delta B \neq 0 \) operators, they change the standard model predictions to the mixing and decay of the \( B \)-mesons. In Fig. 4 we show the Feynman diagrams contributing to \( \Delta B = 2 \) and \( \Delta B = 1 \) processes.

Before discussing the \( B_d \to \phi K_S \) process, let us consider \( B_d \to \psi K_S \) which is the primary target of the \( B \)-factories. In the standard model, CP violation in this process is from the \( B_d \bar B_d \) mixing (with the standard parametrization of the CKM matrix). Once the CP asymmetry in \( B_d \to \psi K_S \) is measured, the phase \( \arg(-[V_{\text{CKM}}]_{cd}[V^*_{\text{CKM}}]_{cb}/[V_{\text{CKM}}]_{td}[V^*_{\text{CKM}}]_{tb}) \) is determined.

If there is a new source of the flavor and CP violations, however, this prediction is changed. In the SUSY GUT with the right-handed neutrinos, the sd-down sbottom mixing
Figure 1: Feynman diagrams contributing to the $\Delta B = 2$ and $\Delta B = 1$ processes. The solid lines are quarks with $d_i = d$ and $s$, the dashed ones squarks, the wiggled ones with straight lines gluinos, and the wiggled line is the gluon. The “dot” $\bullet$ on the squark lines represents the mass insertion. For the box diagrams, diagrams with crossing gluino lines also exist.

through $[m^2_{\tilde{D}}]_{13}$ and $[m^2_{\tilde{Q}}]_{13}$ becomes an origin of the $B_d \bar{B}_d$ mixing. The SUSY contribution to the $B_d \bar{B}_d$ mixing is discussed in Ref. [10] where it was pointed out that $[m^2_{\tilde{Q},\tilde{D}}]_{13}/[m^2_{\tilde{Q},\tilde{D}}]_{11}$ has to be larger than $O(10^{-2})$ to change the standard model prediction significantly. In the present case, these ratios are at most $O(10^{-3})$ with the largest possible value of the right-handed neutrino mass, $M_{\nu_R} \sim 10^{15}$ GeV. Therefore, we expect that the SUSY contribution to the $B_d \bar{B}_d$ mixing is smaller than the future experimental sensitivity. Indeed, we calculated $\Delta \phi_{\text{mix}}^{B_d \rightarrow \bar{B}_d}$, the SUSY contribution to the phase in the $B_d \bar{B}_d$ mixing amplitude. We found that $\Delta \phi_{\text{mix}}^{B_d \rightarrow \bar{B}_d}$ is typically $O(0.1 \%)$ of the standard model contribution, which is too small to be seen in the experiments.

The CP violation in the process $B_d \rightarrow \psi K_S$ is also affected if the phase in the decay amplitude is changed. For the process $B_d \rightarrow \psi K_S$, however, the standard model contribution to the decay amplitude is at the tree level while the SUSY ones are at the one-loop level. As a result, the SUSY contribution to the decay amplitude is negligible. In this scenario, the CP violation in the decay $B_d \rightarrow \psi K_S$ is likely to be consistent with the standard model prediction. For $B_d \rightarrow \psi K_S$, the SUSY contribution is relatively insignificant because this process has a tree level decay amplitude in the standard model.

For processes without tree level decay amplitude, the SUSY contribution may be more significant. The process $B_d \rightarrow \phi K_S$ is such a process [11, 12]. At the quark level, $\Delta B = 1$ operators contributing to $B_d \rightarrow \phi K_S$ have a structure like $(\bar{s}b)(\bar{s}s)$. Such operators are induced only at the one-loop level in the standard model.

The SUSY contribution to the decay amplitude for $B_d \rightarrow \phi K_S$ is from the penguin and box diagrams. (See Fig. 1.) The $\Delta B = 1$ effective Lagrangian has the following form:

$$L_{\text{eff}} = C_{RR}(\bar{s}^a \gamma^\mu P_R b^a)(\bar{s}^b \gamma^\mu P_R b^b)$$
\[ +C_{RL}^V(s^a\gamma^\mu P_Rb^a)(s^b\gamma^\mu P_Ls^b) + C_{RL}^S(s^a P_Rb^a)(s^b P_Ls^b) \]
\[ +m_b C_{RM}^{(DM)} T_{ab} s^a [\gamma^\mu, \gamma^\nu] P_L b^b G_{\mu\nu}^A + (L \leftrightarrow R) + \text{h.c.}, \]  
(16)

where \( m_b \) is the bottom quark mass, \( G_{\mu\nu}^A \) is the gluon field strength, \( T_{ab}^A \) is the \( SU(3)_C \) generator and the indices \( a \) and \( b \) are the color indices. We calculate the SUSY and the standard model contributions to the coefficients. In our analysis, we only consider the dominant contribution from the squark-gluino loops, and the expressions for the SUSY contribution are given in the Appendix. Then, we estimate the decay amplitude:

\[ \mathcal{M}_{B_d \rightarrow \phi K^0} = \langle \phi K^0 | \mathcal{L}_{\text{eff}} | B_d \rangle. \]  
(17)

In our calculation, no QCD corrections below the electroweak scale are included. Then, we adopt the factorization approximation to obtain

\[ \frac{\mathcal{M}_{B_d \rightarrow \phi K^0}}{2(p_B \epsilon_\phi) m_0^2 f_\phi F_{+}^{BK}} = \frac{1}{4} \left[ \left( 1 + \frac{1}{N_C} \right) C_{RR} + C_{RL}^V - \frac{1}{2 N_C} C_{RL}^S \right] + \frac{1}{2} \left( 1 - \frac{1}{N_C} \right) g_3 \kappa_{DM} C_{RM}^{(DM)} + (L \leftrightarrow R), \]  
(18)

where \( g_3 \) is the \( SU(3)_C \) gauge coupling constant, \( N_C = 3 \), and the following relations are used

\[ \langle \phi(p_\phi, \epsilon_\phi) | s^a \gamma^\mu s^a | 0 \rangle = m_0 f_\phi \epsilon_\phi^\mu, \]  
(19)
\[ \langle K^0(p_K) | s^a \gamma^\mu b^a | B_d(p_B) \rangle = F_+^{BK}(p_B + p_K)^\mu + F_-^{BK}(p_B - p_K)^\mu. \]  
(20)

Furthermore, \( \kappa_{DM} \) is an \( O(1) \) coefficient from the hadronization of the chromo-dipole moment operator. We estimate \( \kappa_{DM} \) using relations derived from the quark model [13] and the heavy quark effective theory [14]:

\[ \kappa_{DM} \simeq \frac{m_b^2}{2 q^2} \left[ \frac{9}{8} + O(m_\phi/m_b^2) \right], \]  
(21)

where \( q \) is the momentum transfer in the gluon line. Typically, \( q^2 = \frac{1}{2} (m_B^2 - \frac{1}{2} m_\phi^2 + m_K^2) \) [13], which gives \( \kappa_{DM} \simeq 1.2 \) [12]. In our following discussion, we will present results with several values of \( \kappa_{DM} \) to show the uncertainty related to \( \kappa_{DM} \).

Now, we discuss the SUSY contribution to the decay amplitude \( \mathcal{M}_{B_d \rightarrow \phi K^0}^{(SUSY)} \). There are two types of contributions, i.e., one proportional to \( m_D^2 \) [23] and the other proportional to \( m_Q^2 \) [32]. In the mSUGRA approach, \( m_D^2 \) is generated by the neutrino Yukawa matrix and is proportional to \( e^{i(\phi_1^{(L)} - \phi_2^{(L)})} \). The phase in the CKM matrix is independent of \( \phi^{(L)} \)'s in the SUSY \( SU(5) \) model. Since the standard model contribution to the decay amplitude \( \mathcal{M}_{B_d \rightarrow \phi K^0}^{(SM)} \) is proportional to \( [V_{CKM}]_{ts}[V_{CKM}]_{tb} \), the right-handed down-type squark contribution may have, in general, an arbitrary phase relative to the standard model one. On the contrary, in mSUGRA, \( m_Q^2 \) is approximately proportional to \( y_t^2 [V_{CKM}]_{ts}[V_{CKM}]_{tb} \). Furthermore, with
the model parameters we will use below, \([|m_Q^2|]_{32}\) becomes much smaller than \([|m_D^2|]_{23}\) since
\(|V_{\text{CKM}}^a|_{32} \ll |V_{\text{L}}|_{32}\). Therefore, when \(y_{3\alpha} \approx y_\ell\), the SUSY contribution is dominated by the
one proportional to \([m_D^2]_{23}\) and \(M^{(\text{SUSY})}_{B_d \to \phi K^0}\) is approximately proportional to \(e^{i(\phi_3^{(L)} - \phi_2^{(L)})}\). Of
course, if we consider different model, \([m_Q^2]_{32}\) may also contribute.

The CP violation in the decay process \(B_d \to \phi K_S\) is determined by the sum of the mixing
and decay phases:

\[
\phi_{B_d \to \phi K_S}^{\text{total}} = \phi_{B_d \to \phi K_S}^{\text{mix}} + \phi_{B_d \to \phi K_S}^{\text{decay}}
= \phi_{B_d \to \phi K_S}^{\text{mix}} + \arg \left[M^{(\text{SM})}_{B_d \to \phi K^0} + M^{(\text{SUSY})}_{B_d \to \phi K^0}\right].
\]

(22)

The phase in the mixing is universal for the two processes \(B_d \to \psi K_S\) and \(B_d \to \phi K_S\). In
addition, the standard model predicts very small decay phases for these decay modes. As
a result, in the standard model, \(\phi_{B_d \to \phi K_S}^{\text{total}}\) should be almost the same as the CP violating
phase measured in \(B_d \to \psi K_S\). However, in the model we consider, the decay phase can be
different in two cases. In order to estimate the SUSY contribution to the decay phase, we
calculate the quantity

\[
\Delta \phi_{B_d \to \phi K^0}^{\text{decay}} = \tan^{-1} \left(\frac{|M^{(\text{SUSY})}_{B_d \to \phi K^0}|}{|M^{(\text{SM})}_{B_d \to \phi K^0}|}\right).
\]

(23)

Notice that \(|M^{(\text{SUSY})}_{B_d \to \phi K^0}|\) is almost independent of the phases \(\phi^{(L)}\)'s, and that \(\Delta \phi_{B_d \to \phi K^0}^{\text{decay}}\) is the
maximal possible correction to the decay phase for a given set of model parameters (except
the GUT phases). Such a maximal value is obtained when the phases are chosen such that
\(\phi_3^{(L)} - \phi_2^{(L)} \approx \arg[M^{(\text{SM})}_{B_d \to \phi K^0}] + \pi/2\).

In Fig. 2, we plot \(\tan[\Delta \phi_{B_d \to \phi K^0}^{\text{decay}}]\) as a function of the lightest down-type squark mass
\(m_{\tilde{d}_1}\). We plot the results with \(m_{\chi} = 500\) GeV, \(a_0 = 0\) and \(\kappa_{\text{DM}} = 0, 0.5, 1, 1.5, \) and
2. Let us discuss the behavior of the SUSY contribution. As mentioned in the Appendix,
there are three classes of contributions: box, color-charge form factor, and chromo-dipole
contributions. We found that, within our approximation, box and color-charge form factor
contributions have almost the same size but the opposite sign in most of the parameter space
we studied. As a result, there is a significant cancellation between these two contributions.
In particular, with the parameter we used for Fig. 2, an exact cancellation occurs when
\(m_{\tilde{d}_1} \approx 600\) GeV. The chromo-dipole contribution is comparable to the others when \(\kappa_{\text{DM}} \sim 1\).
However, this contribution is very sensitive to \(\kappa_{\text{DM}}\), and hence the final result strongly
depends on the value of \(\kappa_{\text{DM}}\). From Fig. 4 we see that \(\Delta \phi_{B_d \to \phi K^0}^{\text{decay}}\) can be as large as \(O(0.1)\)
with the reasonable value of \(\kappa_{\text{DM}} \sim 1\), although more precise calculation of \(\Delta \phi_{B_d \to \phi K^0}^{\text{decay}}\) requires better understanding of the hadronic matrix elements.

Here, we should comment on the process \(\mu \to e\gamma\). With the parameters we used, \(\text{Br}(\mu \to e\gamma)\)
becomes larger than the current upper bound \(1.2 \times 10^{-11}\) \([3]\) when \(m_{\tilde{d}_1} \lesssim 900\) GeV.
However this is rather a model-dependent statement since all the mixings in \(V_L\) affect \(\text{Br}(\mu \to e\gamma)\). If the 3-1 element of \(V_L\) is \(\sim 0.1\), or if we adopt a non-universal right-handed neutrino
mass matrix, $\text{Br}(\mu \to e\gamma)$ may become smaller. In addition, with the neutrino mass matrix suggested from the large angle MSW solution with small mass splitting (i.e., so-called “LOW” solution) or with those suggested from the small angle MSW or vacuum oscillation solutions to the solar neutrino problem (although they are now statistically disfavored [9]), $\text{Br}(\mu \to e\gamma)$ is more suppressed. On the contrary, $\Delta\phi^{\text{decay}}_{B_d \to \phi K^0}$ is sensitive only to the 2-3 mixing in the neutrino sector. Therefore, we do not exclude the possibility of $m_{\tilde{d}_1} \gtrsim 900$ GeV in Fig. 2. Of course, even with $m_{\tilde{d}_1} \gtrsim 900$ GeV, $\Delta\phi^{\text{decay}}_{B_d \to \phi K^0}$ can be $O(0.1)$.

Since the standard model predicts almost the same mixing and decay phases in the $B_d \to \psi K_S$ and $B_d \to \phi K_S$ processes, $\Delta\phi^{\text{decay}}_{B_d \to \phi K^0} \sim O(0.1)$ should be an interesting signal. Since the uncertainties are expected to be $O(10^{-2})$ in the standard model calculation [10], $\Delta\phi^{\text{decay}}_{B_d \to \phi K^0} \sim O(0.1)$ will be regarded as a sign of a new physics beyond the standard model if observed.

So far, we have discussed the decay of the $B_d$-meson. If the $B_s$-meson is available, the similar analysis is possible. The discussion is almost parallel to the case of $B_d$-meson. For the process $b \to c\bar{c}s$ (resulting in, for e.g., $B_s \to \psi\eta, D_s\bar{D}_s$), a tree amplitude exists and the SUSY contribution to the decay phase is negligible. On the contrary, $b \to s\bar{s}s$ (which induces $B_s \to \phi\eta'$) occurs at the one-loop level and the SUSY contribution can be significant. In the standard model, very small CP asymmetries are expected in these processes. Once the SUSY contribution is taken into account, however, the decay phase of $O(0.1)$ may be induced for $B_s \to \phi\eta'$ since the standard model contribution is one-loop suppressed.

In summary, the SUSY contribution may change the phase in $B_d \to \phi K_S$. In the future,
comparing this process with $B_d \to \psi K_S$, we may see a difference in the decay phases in these two processes, which cannot be explained by the standard model. This suggests the importance to study various decay modes of the $B$-mesons. Since the branching ratio of the process $B_d \to \phi K_S$ is expected to be $O(10^{-5})$, such a study should be challenging at the first stage of the present asymmetric $B$ factories. However, at the second stage, or at the hadron colliders, more $B$ samples are expected. Then, the study of the decay modes with smaller branching ratios is an interesting possibility to look for a signal from the new physics beyond the standard model. Therefore, it is desirable to collect a large number of the $B$-mesons and to study the CP violation in the various decay modes.

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Appendix: Coefficients

In this appendix, we present the expressions for the SUSY contribution to the coefficients of the $\Delta B = 1$ operators:

$$L_{\text{eff}} = C_{RR}(\bar{s}^a \gamma^\mu P_R b^a)(\bar{s}^b \gamma^\mu P_R s^b)$$
$$+ C_{RL}^V (\bar{s}^a \gamma^\mu P_R b^a)(\bar{s}^b \gamma^\mu P_L s^b) + C_{RL}^S (\bar{s}^a P_R b^a)(\bar{s}^b P_L s^b)$$
$$+ m_6 C_{R}^{DM} T_{ab} A^s (\gamma^\mu, \gamma^\nu) P_L b^b G_{\mu\nu}^A + (L \leftrightarrow R) + \text{h.c.} \quad (A.1)$$

We only consider the dominant contribution from the squark-gluino loops, and we use the mass-eigenstate basis.

With the soft SUSY breaking terms given in Eq. (11), the mass matrix of the down-type squarks is given by

$$M_d^2 = \begin{pmatrix}
  m_Q^2 & A_d^T \langle H_d \rangle + \mu Y^T_d \langle H_u \rangle \\
  A_d^T \langle H_d \rangle + \mu Y^T_d \langle H_u \rangle & m_D^2
\end{pmatrix}, \quad (A.2)$$

where $\mu$ is the SUSY invariant Higgs mass (i.e., so-called the $\mu$-parameter). The above mass matrix is diagonalized by the unitary matrix $U_d$

$$[U_d^\dagger M_d^2 U_d]_{AB} = m_{d_A}^2 \delta_{AB}. \quad (A.3)$$

Then, the coupling constant for the $d_i - \tilde{d}_A$-gluino vertex is given by

$$X_{iA}^L = -\sqrt{2} g_3 [U_{d_i A}]^* e^{-i\phi_G}, \quad X_{iA}^R = -\sqrt{2} g_3 [U_{d_i A + 3, A}]^* e^{i\phi_G}, \quad (A.4)$$

#In Eq. (A.2) expression, we omit the Yukawa and $D$-term contributions to the diagonal terms, which are included in our numerical calculation.
where $\phi_G$ is the phase in the gluino mass

$$m_{G3} = |m_{G3}| e^{-2i\phi_G} \equiv m_G e^{-2i\phi_G}. \quad (A.5)$$

In our calculation, we take $\phi_G = 0$ to evade the constraint from the electric dipole moments.

Neglecting the left-right mixing in the down-type squark mass matrix, the box contribution is

$$\tilde{C}_{RR}\big|_{\text{Box}} = \frac{1}{16\pi^2 m_G^2} \sum_{AB} X_{sA}^{R*} X_{sA}^{R} X_{sB}^{R*} X_{sB}^{R} \times \left[ \frac{N_C^3 - 2N_C + 1}{4N_C^2} B_1(x_A, x_B) + \frac{N_C^3 - 2N_C + 1}{8N_C^2} B_2(x_A, x_B) \right], \quad (A.6)$$

$$\tilde{C}_{RL}\big|_{\text{Box}} = \frac{1}{16\pi^2 m_G^2} \sum_{AB} X_{sA}^{R*} X_{sA}^{R} X_{sB}^{L*} X_{sB}^{L} \times \left[ -\frac{N_C^2 + 1}{4N_C^2} B_1(x_A, x_B) - \frac{1}{8N_C^2} B_2(x_A, x_B) \right], \quad (A.7)$$

$$\tilde{C}_{LS}\big|_{\text{Box}} = \frac{1}{16\pi^2 m_G^2} \sum_{AB} X_{sA}^{L*} X_{sA}^{L} X_{sB}^{R*} X_{sB}^{R} \times \left[ -\frac{1}{4N_C} B_1(x_A, x_B) + \frac{N_C^2 - 2}{4N_C} B_2(x_A, x_B) \right], \quad (A.8)$$

where the “tilde” denote the SUSY contribution to the coefficients. Here, the functions $B_1$ and $B_2$ are given by

$$B_1(x_A, x_B) = -\frac{x_A^2 \log x_A}{4(x_A - x_B)(x_A - 1)^2} - \frac{x_B^2 \log x_B}{4(x_B - x_A)(x_B - 1)^2} - \frac{1}{4(x_A - 1)(x_B - 1)}, \quad (A.9)$$

$$B_2(x_A, x_B) = -\frac{x_A \log x_A}{(x_A - x_B)(x_A - 1)^2} - \frac{x_B \log x_B}{(x_B - x_A)(x_B - 1)^2} - \frac{1}{(x_A - 1)(x_B - 1)}, \quad (A.10)$$

with $x_A = m_{dA}^2/m_G^2$. Notice that, approximately, the following relation holds in our case:

$$\sum_B X_{sB}^{L*} X_{sB}^{L} B_1(x_A, x_B) \simeq \sum_B X_{sB}^{R*} X_{sB}^{R} B_1(x_A, x_B) \simeq 2g_s^2 B_i(x_A, x_{\tilde{i}}), \quad (A.11)$$

where $B_i = B_1, B_2$.

The contribution from the color-charge form factor are

$$\tilde{C}_{RR}\big|_{\text{CF}} = \frac{N_C - 1}{2N_C} \tilde{C}^{\text{CF}}_R, \quad \tilde{C}_{RL}\big|_{\text{CF}} = -\frac{1}{2N_C} \tilde{C}^{\text{CF}}_R, \quad \tilde{C}_{RL}\big|_{\text{CF}} = -\tilde{C}^{\text{CF}}_R, \quad (A.12)$$
where

$$\tilde{C}^{CF}_R = \frac{g_3^2}{16\pi^2 m_G^2} \sum_A X_{sA}^{R*} X_{bA}^R \left[ -\frac{1}{2N_C} C_1(x_A) + \frac{1}{2} N_C C_2(x_A) \right], \quad (A.13)$$

with

$$C_1(x) = \frac{2x^3 - 9x^2 + 18x - 11 - 6 \log x}{36(1-x)^4}, \quad (A.14)$$

$$C_2(x) = \frac{-16x^3 + 45x^2 - 36x + 7 + 6x^2(2x - 3) \log x}{36(1-x)^4}. \quad (A.15)$$

For the chromo-dipole operator,

$$\tilde{C}^{DM}_R = \frac{g_3}{64\pi^2 m_G^2 m_b} \sum_A \left[ -\frac{1}{2N_C} \left\{ m_b X_{sA}^{R*} X_{bA}^R D_1(x_A) + m_G X_{sA}^{R*} X_{bA}^L D_2(x_A) \right\} + \frac{1}{2} N_C \left\{ m_b X_{sA}^{R*} X_{bA}^R D_3(x_A) + m_G X_{sA}^{R*} X_{bA}^L D_4(x_A) \right\} \right] \quad (A.16)$$

where

$$D_1(x) = \frac{-x^3 + 6x^2 - 3x - 2 + 6x \log x}{6(1-x)^4}, \quad (A.17)$$

$$D_2(x) = \frac{-x^2 + 1 + 2x \log x}{(x-1)^3}, \quad (A.18)$$

$$D_3(x) = \frac{2x^3 + 3x^2 - 6x + 1 - 6x^2 \log x}{6(1-x)^4}, \quad (A.19)$$

$$D_4(x) = \frac{-3x^2 + 4x - 1 + 2x^2 \log x}{(x-1)^3}. \quad (A.20)$$

Other coefficients, $\tilde{C}_{LL}$, $\tilde{C}_{LR}^{V}$, $\tilde{C}_{LR}^{S}$, and $\tilde{C}_{LM}^{DM}$, are obtained by interchanging $L \leftrightarrow R$.

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