The anisotropy effect on jet using AdS/CFT Correspondence

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Abstract.
We study the collective flow effects on the jet of light quark in a strongly coupled plasma. We consider the finite temperature static strongly coupled anisotropic plasma which has been generated by a space dependent axion term. By computing the falling string solution in this background, we find that the jet quenching increases in both longitudinal and transverse direction in comparison with the static isotropic medium. Although, the enhancement of quenching is larger in the beam direction.

1. Introduction
The experiments at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) have produced a strongly-coupled matter which is a deconfined state of QCD, Quark-Gluon Plasma (QGP) [1, 2, 3, 4]. While lattice QCD is the proper tool for understanding the static equilibrium thermodynamics of such strongly coupled plasma, it does not allow us to calculate its dynamical evolution in heavy-ion collision. Recently, a novel tool called "the AdS/CFT correspondence" [5, 6, 7, 8, 9] provides valuable insight into the strongly coupled plasma.

Although ideal hydrodynamics successfully describes many properties of QGP, further studies propose that the medium must be anisotropic in the early times during the formation of QGP [10, 11, 12, 13, 14, 15]. These models proposed that within this time period, the pressure of the medium along the transverse direction may surpass the pressure in the longitudinal direction. The anisotropic geometries have been studied using the gauge/gravity duality [16, 17]. In [16], the dual theory is an anisotropic $\mathcal{N} = 4$ SYM plasma with rotational symmetry in the $x-y$ directions, while the $z$ direction corresponds to the beam direction. The drag force and jet quenching parameter in this anisotropic medium were calculated [18, 19, 20]. The results show that the energy loss of partons depends on the relative orientation between the anisotropic direction and the velocity of the quark. Also, pQCD calculations on jet energy loss show that the resulting medium-induced gluon radiation does not depend solely on the energy density of the medium, but also on the collective flow. Their results show that flow effects lead to a characteristic breaking of the rotational symmetry of the average jet energy [21, 22]. In this paper, we studied the effect of anisotropy on light quark using the AdS/CFT correspondence. We found that the maximum stopping distance is very sensitive to the anisotropy parameter and is smaller in the direction of beam.
2. Anisotropic background

In this section we review the anisotropic background introduced in [16] which is the gravity dual to a deformation of $N = 4$ SYM by adding a $\theta$ term to the action that depends linearly on one of the three spatial coordinates $z$ as $\theta = 2\pi n_{D7} z$, where $n_{D7}$ can be thought as the density of D7-branes homogeneously distributed along the anisotropic direction. The type IIB supergravity solution presented in this paper is static and anisotropic with finite temperature. Moreover it is regular on and outside the horizon and asymptotically approaches AdS$_5$.

The supergravity solution in the string frame is given by [16]

$$ds^2 = \frac{L^2}{u^2} \left( -\mathcal{F} dt^2 + dx^2 + dy^2 + \mathcal{H} dz^2 + \frac{du^2}{\mathcal{F}} \right) + L^2 e^{\phi/2} ds^2_{S^5}$$

$$\phi = \phi(u), \quad \chi = a z,$$

where $\phi$ and $\chi$ are the dilaton and axion fields respectively, and the anisotropy parameter is $a = \frac{g_s M n_{D7}}{4\pi}$. The metric functions are $\mathcal{F}, \mathcal{B}$ and $\mathcal{H}$ which depend on the holographic radial coordinate, $u$ the radius of horizon, $u_H$ and the anisotropy parameter, $a$. The boundary of space is at $u = 0$ and the horizon located at $u = u_H$ where the blackening function is zero, $\mathcal{F}(u_H) = 0$.

The metric functions are parameterized by two parameters, the dilaton field value at the horizon $\phi_h$ and $u_H$ or equivalently by $T$ and $a/T$. At small $a/T$, the entropy density scales as in the isotropic case $S_{iso} = \frac{c_{ent}}{T^3} N_c^2 T^3$, while at large $a/T$ it scales as $S_{aniso} = c_{ent} N_c^2 a^{1/3} T^{8/3}$, where $c_{ent}$ is a constant. For small $a/T$ the metric functions and the radius of the horizon are known as some expansions around the black D3-brane solution [16],

$$\mathcal{F} = 1 - \frac{u^4}{u_H^4} + a^2 \mathcal{F}_2 + \mathcal{O}(a^4),$$

$$\mathcal{B} = 1 + a^2 \mathcal{B}_2 + \mathcal{O}(a^4),$$

$$\log \mathcal{H} = \frac{a^2 u_H^2}{4} \log \left( 1 + \frac{u^2}{u_H^2} \right) + \mathcal{O}(a^4),$$

$$u_H = \frac{1}{\pi T} + \frac{5 \log 2 - 2}{48 \pi^3 T^3} a^2 + \mathcal{O}(a^4),$$

in which

$$\mathcal{F}_2 = \frac{1}{24 u_H^2} \left[ 8 u_H^2 (u_H^2 - u^2) - 10 u^4 \log 2 + 3 u_H^4 + 7 \log \left( 1 + \frac{u^2}{u_H^2} \right) \right],$$

$$\mathcal{B}_2 = -\frac{u_H^2}{24} \left[ \frac{10 u^2}{u_H^2 + u^2} + \log \left( 1 + \frac{u^2}{u_H^2} \right) \right].$$

The pressures and energies which are useful on making a naive connection with the weak coupling results can be found from the expectation values of the stress energy tensor and read

$$E = 3e_1 + a^2 e_2 + \mathcal{O}(a^4), \quad P_\perp = e_1 + a^2 e_2 + \mathcal{O}(a^4), \quad P_\parallel = e_1 - a^2 e_2 + \mathcal{O}(a^4),$$

where

$$e_1 = \frac{N_c^2 T^2}{32} + \mathcal{O}(a^4), \quad e_2 = \frac{N_c^2 T^2}{32}.$$

where $N_c$ is the number of colors. Notice that

$$P_\parallel < P_\perp.$$
3. Holographic jets in anisotropic background

The goal of this section is to study the effect of anisotropy of the medium on a light quark. The physical setup of interest is one of a back-to-back jet pair created in a quark-gluon plasma. We therefore consider configurations for which the string is created at a point and expands in space-time such that the two endpoints of the string move away from each other; the total spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the rest frame of the plasma (equivalent to the rest frame for the whole string) one half of the spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the space-time such that the two endpoints of the string move away from each other; the total spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the rest frame of the plasma (equivalent to the rest frame for the whole string) one half of the spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the space-time such that the two endpoints of the string move away from each other; the total spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the rest frame of the plasma (equivalent to the rest frame for the whole string) one half of the spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the space-time such that the two endpoints of the string move away from each other; the total spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the rest frame of the plasma (equivalent to the rest frame for the whole string) one half of the spatial momentum of the string vanishes. With an appropriate choice of coordinates, in the space-time such that the two endpoints of the string move away from each other; the total spatial momentum of the string vanishes.

The physical setup of interest is one of a back-to-back jet pair created in a quark-gluon plasma. The goal of this section is to study the effect of anisotropy of the medium on a light quark. We seek IC such that the string is long-lived, has most of its energy and momentum concentrated of motion everywhere on the worldsheet [23, 24, 25]. The Polyakov action for the string has the form

\[ S_P = -\frac{T_0}{2} \int d^2\sigma \sqrt{-\eta} \eta^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}. \]  

Variation of the Polyakov action with respect to the embedding functions \( X^\mu \) leads to the equation of motion

\[ \partial_a [\sqrt{-\eta} \eta^{ab} G_{\mu\nu} \partial_b X^\nu] = \frac{1}{2} \sqrt{-\eta} \eta^{ab} \frac{\partial G_{\mu\nu}}{\partial X^b} \partial_a X^\nu \partial_b X^\rho \]

\[ \iff \quad \nabla_a \Pi^a_{\mu} = -\frac{T_0}{2} \eta^{ab} \frac{\partial G_{\mu\nu}}{\partial X^b} \partial_a X^\nu \partial_b X^\rho, \]

where \( \Pi^a_{\mu} \) are the canonical momentum densities associated with the string that are obtained from varying the action with respect to the derivatives of the embedding functions. In order to optimize the performance of the numerical integrator, we choose a worldsheet metric of the form [23, 24, 25]

\[ \| \eta_{ab} \| = \begin{pmatrix} -\Sigma(x, u) & 0 \\ 0 & \Sigma(x, u)^{-1} \end{pmatrix}, \]

where \( \Sigma \) is called a stretching function, which can be a function of \( x(\tau, \sigma) \) and \( u(\tau, \sigma) \). In fact, the choice of worldsheet metric is a choice of gauge. A common choice is conformal gauge with \( \Sigma = 1 \). We choose \( \Sigma \) such that the singularities in the equations of motion are cancelled.

In order to solve the equations of motion we need a self-consistent initial conditions (IC) for the string profile which obey the constraint equation,

\[ -\mathcal{F}(u)^2 \mathcal{B} i^2 + \mathcal{F}(u) \dot{i}^2 + \mathcal{H} \mathcal{F} \dot{z}^2 + \dot{u}^2 = 0. \]

The \( \sigma \) derivatives of \( X^\mu \) are initially zero for the string with point-like IC. So, in order to satisfy (11) we just need to choose IC that satisfy \( \dot{X}^2 = 0 \) and obey the boundary condition for open string. We seek IC such that the string is long-lived, has most of its energy and momentum concentrated.
near its endpoints, and produces stable numerical solutions. We choose the following IC for the string in this background

\[
\begin{align*}
\dot{x}(0, \sigma) &= A u_c \cos(\sigma) \sin(\phi), \\
\dot{z}(0, \sigma) &= A u_c \cos(\sigma) \cos(\phi), \\
\dot{u}(0, \sigma) &= u_c \sqrt{f(u_c) (1 - \cos 2\sigma)}, \\
\dot{t}(0, \sigma) &= u_c \sqrt{\frac{A^2 \cos^2(\sigma) (H(u_c) \cos^2(\phi) + \sin^2(\phi)) + (1 - \cos(2\sigma))^2}{B(u_c)}},
\end{align*}
\]

where \(u_c\) and \(A\) are free parameters that can be related to the energy and momentum of the dual quark in the field theory (see below). These IC yield a string profile that is symmetric about \(r = 0\) at all times, because \(\dot{x}(0, \sigma)\) and \(\dot{z}(0, \sigma)\) are antisymmetric about \(\sigma = \pi/2\) while \(\dot{u}(0, \sigma)\) is symmetric.

We realize that the equations of motion remain well behaved everywhere on the worldsheet by choosing any stretching function of the following form,

\[
\Sigma(x, u) = \left( \frac{1 - u/uh}{1 - u_c/uh} \right)^a \left( \frac{u_c}{u} \right)^b
\]

Now we can solve the equation of motion Eq. (9) numerically with Mathematica’s NDSolve to obtain the embedding functions \(X^\mu\) as a function of \((\tau, \sigma)\). We choose the values of \(a\) and \(b\) case by case; \(a\) and \(b\) are in the range of \(1\) to \(3\). The shape of a representative string solution for the \(0\) component of the spacetime momentum of the string along the \((x, z)\) direction and falls toward the horizon. Numerical results show that the string in the anisotropic direction is suppressed with respect to the isotropic direction even at small anisotropy parameter.

It is important to make a minor comment on the energy and momentum of the string. Since \(G^{\mu\nu}\) depends only on \(u\), for \(\mu\) corresponding to \((t, \vec{x})\) we have

\[
\nabla_\mu \Pi^\mu_{\mu} = 0. \tag{14}\n\]

Hence the corresponding momentum densities \(\Pi^\mu_{\mu}\) are conserved Noether currents on the worldsheet associated with the invariance of the action under spacetime translations. The \(\Pi^a_{\mu}\) describe the flow of the \(\mu\) component of the spacetime momentum of the string along the \(a\) direction on the worldsheet [26]. So the total energy of the string at any time is constant and equal to the initial energy of the string

\[
E_{\text{string}} = -\lambda \int_0^{\pi} d\sigma \sqrt{-\eta} \Pi^\tau_{\tau}(0, \sigma) = \frac{\lambda}{2\pi} \int_0^{\pi} d\sigma \frac{F(u_c)}{u_c \Sigma(u_c)} \sqrt{\frac{A^2 \cos^2(\sigma) (H(u_c) \cos^2(\phi) + \sin^2(\phi)) + (1 - \cos(2\sigma))^2}{B(u_c)}}. \tag{15}\n\]

Also, the total momentum in \(x\) and \(z\) direction is conserved and can be calculated by the string IC as follows

\[
P_{\tau x} = \frac{\sqrt{\lambda} A \sin(\phi)}{2\pi u_c \Sigma(u_c)}, \quad P_{\tau z} = \frac{\sqrt{\lambda} A H(u_c) \cos(\phi)}{2\pi u_c \Sigma(u_c)}. \tag{16}\n\]

In the above integral, we used the fact that the total momentum of the string in \(x - z\) direction is zero and it is completely symmetric about the point \(\sigma = \pi/2\). So, the momentum of the quark is equal to the momentum of the anti-quark in the opposite direction. We can see that the two parameters \(A\) and \(u_c\) in the IC of the string determine the energy and momentum of the string.
Figure 1. A typical falling string profile moving in the $x - z$ plane obtained numerically. The string is created at a point at $u_c = 0.1 u_h$ and evolves to an extended object. The endpoints of the string move away from each other and fall toward the horizon.

Figure 2. The ratio of the stopping distances in the anisotropic geometry to the $AdS - Sch$ metric for different values of $a/T$. Red dots show the ratio for $x_{Aniso} = x_{transverse}$, while the blue dots stand for $x_{Aniso} = x_{longitudinal}$. Here we fix the temperature of both backgrounds and the string has the same energy and virtuality in both backgrounds.
completely. It is useful to note that the virtuality of the corresponding jet in the field theory is defined as $Q^2 \equiv E_q^2 - P_q^2$, where $P_q$ is the total spatial momentum of the quark. In order to have a better understanding of the jet quenching of a light probe in the anisotropic plasma we compare the stopping distance of a light quark moving in the anisotropic metric with the same string moving in the AdS – $Sch$ metric. To compare the anisotropic effect, we fix the temperature of the both backgrounds and the string moving in both background has the same energy as well. We find that even at the small anisotropy, $a/T < 1$ the stopping distance is smaller in both the transverse and the longitudinal direction which indicates stronger jet quenching of light probes traveling through the anisotropic medium, see figure 2.

4. Discussion

In this paper, we have studied the effect of anisotropy on an energetic jet traveling in the QGP. We have considered a point-like initial condition string which moves through the anisotropic background with high temperature or small anisotropy. The equation of motion for the string has been solved numerically and is plotted in figure 1 for a typical string. We have assumed that the string moves in the $x-z$ plane with an angle $\phi = \pi/3$ with respect to the direction of beam, $z$. Our results show that the string is highly suppressed in the longitudinal direction. In order to compare the effect of anisotropy, we have calculated the maximum stopping distance of an energetic jet traveling in a strongly coupled anisotropic plasma with and without anisotropy, figure 2. We have seen that even at small anisotropic metric, the stopping distance decreasing for both longitudinal (blue dots) and transverse direction (red dots) with respect to the static isotropic medium. Although, the quenching is stronger along the longitudinal direction.

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