Analysis on the two-dimensional effect of long-rod penetration

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Abstract. The two-dimensional (2D) effect is observed in experiments and simulations of long-rod penetration. Different nose shapes may occur in the penetration of long rods made by different materials, which will have an effect on the penetration performances. Based on the 2D model established in authors’ previous work, combining with the experimental and simulation results, the 2D effect of long-rod penetration is analyzed in present paper. According to the analysis, the basic assumption of the 2D model, the nose shape of long rods keeps invariant during the primary phase, is verified. In addition, the 2D model can briefly reflect the 2D effect shown in experiments and simulations.

1. Introduction

In long-rod penetration, the rod may deform into different nose shapes during the penetration process and thus the penetration performance will be affected. In Magness and Farrand’s experiment [1], the penetration performance of depleted uranium (DU) rods is 10% higher than that of tungsten heavy alloy (WHA) (its density and strength are very close to DU) rods against armor steel targets at velocities around 1.2-1.9 km/s. They suggested that the difference in their penetration performances are caused by the different nose shapes during the penetration process because of their different material properties. The adiabatic shearing property of DU makes the nose of DU rods shaper than that of WHA rods. Rosenberg and Dekel [2] found in their experiments that the titanium alloy (Ti6Al4V) has the same adiabatic shearing property as DU, thus titanium alloy rods also keep sharp nose during the penetration process. In addition, Rong et al. [3] and Chen et al. [4] found tungsten fiber/metal glass matrix (WF/MG) composite has the similar “self-sharpening” behavior. Based on simulation and theoretical analysis, Li et al. [5] analyzed the “self-sharpening” effect and suggested its mechanism. To theoretically analyze the 2D effect of long-rod penetration, a 2D model is established by Jiao and Chen [6] by constructing the nose-shape function and determining the resistances of rods with different nose shapes during the hypervelocity penetration process.

In this paper, results of ballistic experiments conducted by Rong et al. [3] and Chen et al. [4] and corresponding simulation conducted by Li et al. [5] are analyzed using the 2D model. In addition, the simulation of WHA rods penetrating 4340 steel and SiC ceramic is conducted to analyze the influence of target material on the 2D effect of long-rod penetration.
2. Geometrical models and constitutive models

2.1. Geometrical models
The geometrical sizes of rods and targets in the simulations are same as those in corresponding to ballistic experiments conducted by Rong et al. [3] and Chen et al. [4], respectively. The 30CrMnMo steel target is 50 mm in thickness, and the corresponding long rod is 88 mm in length and 8 mm in diameter, denoted as “small rod”, whose initial geometry of the rod is shown in figure 1(a). The Q235 steel target is 250 mm in thickness, and the corresponding long rod is 206 mm in length and 19 mm in diameter, denoted as “large rod”, whose initial geometry of the rod is shown in figure 1(b) and its initial nose shape is semi-spherical.

![Figure 1](image1.png)

(a) Small rod [3]  (b) Large rod [4]

Figure 1. The initial geometry of WF/MG composite long rods.

In this paper, results of ballistic experiments conducted by Rong et al. [3] and Chen et al. [4] and corresponding simulation conducted by Li et al. [5] are analyzed using the 2D model. In addition, the simulation of WHA rods penetrating 4340 steel and SiC ceramic is conducted to analyze the influence of target material on the 2D effect of long-rod penetration. The FEM simulations are implemented in the LS-DYNA commercial software [5]. Due to the axis-symmetry property of the penetrating condition, the FEM simulation is set as the axis-symmetrical model. The tungsten fibers (WFs) in WF/MG composite rod are about 300 μm in diameter and around 80% in volume fraction. Thus, the WF/MG composite rod has the similar density and strength as the WHA rod. As can be seen from the figure 2, the WFs are distributed uniformly in the MG matrix. For comparative analysis, the corresponding geometrical models of WHA rods are the same as that of the composite ones, while the materials in both the fiber and the matrix are defined as tungsten instead.

![Figure 2](image2.png)

(a) Small rod  (b) Large rod

Figure 2. Geometrical model of the WF/MG composite rods [5].

2.2. Constitutive models of materials
The materials used in the analysis in present paper are tungsten, steel, and metal glass. Constitutive models and parameters of these materials are briefly introduced.

Metal glass (MG) has high shear sensitivity, i.e., below the glass transition temperature and under a high stress, highly localized shear bands are much easily to occur in MG [7]. Because of this property, MG rods can keep sharp during the penetration process, i.e., the “self-sharpening” behavior similar to DU. Furthermore, due to the high density and good plasticity of tungsten (W), low density MG (about 6 g/cm³) is usually reinforced by WFs when used as the kinetic energy penetrator material. In the
current study, reinforced WFs are added to Zr-based MG matrix, and the density and strength of WF/MG composite is close to that of WHA because the volume fraction of WFs is around 80%. Based on the free volume model [8, 9] and the coupled thermo-mechanical model [10, 11], Li et al. [12, 13] further developed a modified couple thermo-mechanical model and a failure criterion of critical free volume concentration, which is used in simulation conducted by Li et al. [5]. In their FEM calculation process, the “element killing” algorithm is invoked to delete and remove the failed element from the geometrical model when the failure criterion is met. The free volume concentration is considered in the failure criterion, i.e., the material is failed when the critical free volume concentration is met.

The tungsten and steel analyzed in present paper all belong to the traditional crystalline alloys, and their mechanical behaviors are described by the Johnson-Cook constitutive model [14] combined with the accumulative damage failure criterion [15]. Moreover, the Gruneisen equation of state is employed to calculate pressure in the penetration process. Reference to tests of corresponding materials, parameters of corresponding equations are listed in table 1. Similarly, when the failure criterion is met, the “element killing” algorithm is invoked to delete and remove the failed element from the geometrical model.

Table 1. Parameters of Johnson-Cook model for different materials.

| Materials | \( \rho \) (g/cm\(^3\)) | \( E \) (GPa) | \( \nu \) (-) | \( C_J \) (kg•K) | \( T_f \) (K) | \( T_m \) (K) | \( \dot{E} \) (s\(^{-1}\)) |
|-----------|----------------|--------------|-------------|----------------|-------------|-------------|----------------|
| 95W       | 17.450         | 410         | 0.28       | 134            | 300         | 1752        | 1             |
| 30CrMnMo  | 7.850          | 200          | 0.29       | 477            | 300         | 1793        | 1             |
| Q235      | 7.800          | 200          | 0.29       | 477            | 300         | 1793        | 1             |
| 4340      | 7.830          | 200          | 0.29       | 477            | 300         | 1793        | 1             |

| Materials | \( A \) (GPa) | \( B \) (GPa) | \( n \) (-) | \( C \) (-) | \( m \) (-) | \( C_0 \) (m/s) | \( S \) (-) |
|-----------|---------------|---------------|-------------|-------------|-------------|----------------|-------------|
| 95W       | 1.650         | 0.450         | 0.12        | 0.016       | 1.00        | 3850           | 1.44        |
| 30CrMnMo  | 1.200         | 0.310         | 0.26        | 0.014       | 1.03        | 4578           | 1.38        |
| Q235      | 0.235         | 1.050         | 0.25        | 0.015       | 1.03        | 4578           | 1.36        |
| 4340      | 0.792         | 0.510         | 0.26        | 0.014       | 1.03        | 4578           | 1.38        |

| Materials | \( \gamma \) (-) | \( \alpha \) (-) | \( D_1 \) (-) | \( D_2 \) (-) | \( D_3 \) (-) | \( D_4 \) (-) |
|-----------|-----------------|-----------------|--------------|--------------|--------------|--------------|
| 95W       | 1.58            | 0               | 3.0          | 0            | 0            | 0            |
| 30CrMnMo  | 1.67            | 0.47            | 3.2          | 0            | 0            | 0            |
| Q235      | 1.65            | 0.45            | 3.2          | 0            | 0            | 0            |
| 4340      | 1.67            | 0.47            | 3.2          | 0            | 0            | 0            |

3. Analysis on the 2D effect of long rod penetration
Because of the “self-sharpening” property of WF/MG composite, the rod can keep sharp nose during the penetration process, different from the “mushroom” nose for rods made by other materials such as WHA. Different target resistance is caused by the difference in nose shapes, and finally affect the penetration performance. Experimental studies are conducted by Rong et al. [3] and Chen et al. [4], and corresponding simulation analysis is made by Li et al. [5]. In this section, results of the above experiments and simulations will be theoretically analyzed using the 2D model of long-rod penetration. Corresponding parameters and result data are listed in table 2.

3.1. Small rod penetrating 30CrMnMo steel targets
The results of experiment of small rod penetrating 30CrMnMo steel targets [3] and corresponding simulation [5] is firstly analyzed, in which the variation of nose shapes during the penetration process is emphatically discussed.

As can be observed from the figure of residual rod (figure 3), the nose of WHA rod is upset and the “mushroom” shape is obviously presented; the nose of WF/MG composite rod presents ogive shape after shear deformation and failure, and the maximum radius of the nose \( R \) is close to the rod (tail) radius \( r \). A preliminary conclusion can be drawn that the nose of WF/MG composite rod is sharper than that of WHA rod during the penetration process because of the higher shear sensitivity of
WF/MG composite. WF/MG composite rod penetrates deeper than WHA rod at same impact velocity, according to Table 2.

Table 2. Corresponding parameters and results in experiments and simulations.

| Target thickness (mm) | Target material | \(\sigma_t\) (GPa) | \(\rho_r\) (g/cm\(^3\)) | Rod geometry \(L*D\) (mm*mm) | Rod material | \(\rho_p\) (g/cm\(^3\)) | \(V_o\) (km/s) | Penetration depth (mm) | Residual rod length (mm) |
|----------------------|-----------------|--------------------|--------------------------|-----------------------------|-------------|--------------------------|----------------|-----------------------|-------------------------|
| 50 (Small) | 30CrMnMo | 1.200 | 7.85 | 88*9 (Small) | 95W | 1.650 | 17.45 | 0.853 | 23.4 | 22.2 | 14.1 | 13.6 |
| 250 (Large) | Q235 | 0.235 | 7.80 | 206*19 (Large) | 95W | 1.650 | 17.45 | 1.300 | 215 | 207.1 | 25 | 25.3 |

The simulation results of final penetration depth and residual rod length show excellent agreement with experimental data. Moreover, geometry of residual rods in simulation is also close to figure 3. Therefore, simulation results especially detailed information such as nose shapes, pressures and velocities can be used to analyze the variation of nose shapes during the penetration.

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![Figure 3](image)

(a) WHA rod, \(V_o = 0.853\) km/s
(b) WF/MG composite rod, \(V_o = 1.066\) km/s

Figure 3. Residual rods after penetrating 30CrMnMo steel target [3].

The variations of nose shapes of rods made by different materials when impact on 30CrMnMo steel targets at two velocities are shown in figure 4 and figure 5. According to figure 4, the nose of WHA rod upsets at the initial stage of penetration process; subsequently, the nose remains semi-spherical shape while debris is peeling off from the edge of nose. As is observed from figure 5, the nose of WF/MG composite rod also upsets at the initial stage, however, the upsetting degree is lower than that of WHA rod. Meanwhile, a small amount of material fails due to backward bending and shear. During the following penetration process, bending tungsten fibers are squeezed laterally and sheared by the target and the material continuously peels off from the edge of the rod nose. The nose continuously sharpens and finally remains the ogive shape. In addition, it can be clearly seen from the figure that the length of the rod is significantly shortened due to the continuous peeling (erosion) of the material of the rod, which is the typical feature of long-rod penetration.

The shape of crater can also reflect the change of nose shape during penetration process. Comparing figure 4 with figure 5, it is observed that, at the initial stage of penetration process, rods made by two kinds of materials are upset to form a wide crater, and the crater diameter of composite projectile is relatively smaller. In the subsequent penetration process, the crater is gradually narrowed. The diameter of WHA rod (2r) is obviously smaller than that of its crater, while the diameter of WHA rod is close to that of its crater. In addition, comparing the crater morphology at \(t=120.6\) ms and \(t=200\)
μs in the two figures, it can be found that there is still has some kinetic energy remaining in target after loaded by the rod nose. The crater diameter further increases as the body continues to move forward.

Curve fitting is applied to rod noses in figure 4 and figure 5, thus dimensionless parameters of nose shapes at three moments of the penetration process are obtained, as shown in table 4. As can be seen from the table, the diameter ratio of the rod $\eta$ is relatively larger at the initial stage of penetration, and gradually decreases and eventually stabilizes during the subsequent penetration. Meanwhile, the nose shape factor $N^*$, another dimensionless parameter reflecting the 2D effect of long-rod penetration exhibits a similar variation law as $\eta$ during the penetration process, i.e., $N^*$ is relatively larger at the initial stage of penetration and gradually decreases and eventually stabilizes during the subsequent penetration. Corresponding to Table 3, the variation degree of $\eta$ and $N^*$ from $t = 120.6$ μs to $t = 200$ μs (the primary phase of long-rod penetration) is much smaller than that from $t = 14$ μs to $t = 120.6$ μs.

Because similar feature is observed in simulation results at higher impact velocities (1.076 km/s and 1.066 km/s), it can be believed that the nose shape of the rod remains unchanged during the primary phase (the quasi-steady phase) of long-rod penetration. Therefore, the core assumption of the 2D model established in author’s previous work is proved to be reasonable.

It can also be seen from Table 3 that compared with WHA rod, the nose shape of WF/MG composite rod changes more drastically, especially at the initial stage represented by $t = 14$ μs. The diameter ratio of the rod $\eta$ and the nose shape factor $N^*$ of WF/MG composite rod are both smaller than those of WHA rod during the whole penetration process. The above phenomenon shows that the WF/MG composite has a sharper nose shape during penetration process at the same impact velocity, due to its higher shear sensitivity.
Figure 5. The variation of the nose shape of WF/MG composite rod penetrating at $V_0 = 0.858$ km/s.

Table 3. Parameters of nose shapes of small rods during penetration process.

| Rod material | t=14 μs | t=120.6 μs | t=200 μs |
|--------------|---------|-------------|----------|
|              | η       | N*          | η        | N*       | η        | N*       |
| 95W          | 1.613   | 0.722       | 1.565    | 0.5      | 1.565    | 0.5      |
| WF/MG        | 1.457   | 0.389       | 1.261    | 0.189    | 1.078    | 0.200    |

3.2. Large rods penetrating Q235 steel targets

Base on the results of ballistic test of small rod and the corresponding simulation, the variation of the nose shape during the penetration process is analyzed. In particular, the validity of the assumption of the 2D model is confirmed. However, because only the macroscopic results are given from the ballistic test, the 2D effect of long-rod penetration can only be preliminarily analyzed by numerical simulation results. In experiments of large rods penetrating Q235 steel targets conducted by Chen et al. [4], metallographic analysis is used and the meso-geometric structure of the rod is obtained. The 2D effect can be further analyzed on this base, combining with the 2D model of long-rod penetration.

Two experimental results and one simulation results of the nose shapes of residual rods are shown in figure 6. Curve fitting is applied to these rod noses, and corresponding geometry parameters are listed in table 4. It should be noted that nose shapes during the penetration process are assumed to be similar to those of residual rods. Therefore, nose shapes during the penetration process are approximately presented by nose shapes of residual rods (a) and (b) from experiments at 1.3 km/s. However, the nose shape of residual rod (c) from simulation at 1.766 km/s is used, because the WF/MG composite rod penetrates through the target at this speed and the residual rod has not been measured.

As can be observed from figure 6, there is a spherical nose for WHA rod (a), and ogive noses for WF/MG composite rods (b)(c). At impact velocity of 1.3 km/s, the difference of nose shapes between
(a) and (b) is mainly caused by rod materials. Comparing nose shape (b) with (c), it is found that the diameter ratio $\eta$ of WF/MG composite rods is larger for higher impact velocity.

![Image](a) 95W, 1.3 km/s, Exp  (b) WF/MG, 1.313 km/s, Exp  (c) WF/MG, 1.766 km/s, Sim

**Figure 6.** Curve fitting of noses of residual rods after penetrating Q235 steel targets.

| Num | $r$ (mm) | $R$ (mm) | $S$ (mm) | $h$ (mm) | $\eta$ | $\psi$ | $N^*$ |
|-----|---------|---------|---------|---------|------|------|-----|
| (a) | 9.5     | 13.091  | 13.091  | 13.091  | 1.378| 0.5  | 0.5  |
| (b) | 9.5     | 10.706  | 14.177  | 13.754  | 1.127| 0.673| 0.492|
| (c) | 9.5     | 11.628  | 15.770  | 15.256  | 1.224| 0.678| 0.488|

Combined geometrical parameters of nose shapes with material parameters of rods and targets ($A=6.851$, $B=0.5$), the 2D model can be applied for the theoretical analysis. Figures 8-10 show the penetration performances of rods with different (fitting) nose shapes from experiments and simulations, calculated by present 2D model. It should be noted that the 2D model is applicable for semi-infinite targets, thus the effect of moderately thick targets such as the bulge on the back of targets is not considered. However, penetration depths approach or exceed the thickness of target in experiments and simulations. Particularly, the bulge on the back of targets and penetration through targets are observed in experiments and simulations of WF/MG composite rods penetrate Q235 steel targets. Therefore, the calculation stops when the penetration depth reaches 250 mm for WF/MG composite rods with nose shapes (b) and (c).

As can be seen from figure 7, the calculation results obtained by substituting the fitted nose shape into the 2D model can basically reflect the variation trend of tail velocities in the numerical simulation, which illustrates that the above method is applicable for analyzing the 2D effect of long-rod penetration. It is easy to observed that the difference between the model calculation results and the simulation results is mainly reflected at the rear of penetration process and thus the 2D model applicable for semi-infinite targets fails to account for typical effects of medium-thick targets.
Comparing the results of WF/MG composite rods at different initial impact velocities, it can be concluded that the above differences decrease with the increase of the initial impact velocity.

In figure 7, the initial penetration velocity of WF/MG composite rod (0.823 km/s) is evidently higher than that of WHA rod (0.695 km/s) at the initial impact velocity of 1.3 km/s, corresponding to the larger diameter ratio $\eta$ of nose shape (b) than (a). As (b) is sharper than (a), the former rod decelerates slower than the later, and the deceleration degree quantified by the deceleration index $\alpha$ [16] is also smaller. Meanwhile, the duration time of WF/MG composite rod penetration is shorter than that of WHA rod because of the higher penetration velocity.

![Figure 7. Velocity-time curves for penetration of rods with different shapes.](image)

As can be observed from figure 8, penetration performances of WHA rod and WF/MG composite rod are significantly different at the same impact velocity. The penetration performances WF/MG composite rod is higher than that of WHA rod, which is represented by greater slope of the curve of the former rod than the later one. Because densities and strengths of these two materials are very close, the difference in penetration performances is mainly caused by their different nose shapes during the penetration process. Besides, the penetration depth of WHA rod calculated from the 2D model is about 211 mm, slightly lower than the experiment result (215 mm). This difference illustrates the diameter ratio $\eta$ of real nose shape during the penetration process is smaller than that of the assumed nose shape (the nose shape of residual rod in experiment, shown as figure 6(a)). Actually, it is found in experiment that the rod upsets at the rear of penetration process.

![Figure 8. Penetration depth-time curves for penetration of rods with different shapes.](image)
As for the variation of residual rod length shown in Fig. 10, the WHA rod erodes slower than WF/MG composite rod at the same impact velocity. Likewise, it is caused by the smaller diameter radio $\eta$ of later rod than that of the former one. The final residual length of WF/MG composite rod calculated from the 2D model is about 2 mm, approaching to fully eroded, which is seriously different from the experimental result (25 mm). Because present 2D model is applicable for semi-infinite targets, the rod is assumed to penetrate until it is nearly fully eroded. However, the upsetting residual rod is found in experiment, which is a proof of the upset rod at the rear of penetration process.

In figure 9, the calculated residual length of WF/MG composite rod (at 1.313 km/s) from the 2D model and the corresponding experimental result are 54 mm and 40 mm, respectively. When the impact velocity is 1.766 km/s, the calculated result and the simulation result are 43.6 mm and 39.9 mm, respectively. The calculated results are obviously greater than the experimental result and simulation result, because the 2D model applicable for semi-infinite targets does not consider effects of moderately thick targets such as the bulge on the back of targets and penetration through targets which are observed in experiments.

As can be observed from from figure 9, the erosion rate of WF/MG composite rod is greater at higher impact velocity. In fact, the loss of kinetic energy consists of the erosion and the deceleration, and the erosion rate reflects the velocity of the loss of kinetic energy caused by erosion. The diameter radio of nose shape (c) is bigger than that of nose shape (b), which illustrates that the radial flowing of rod material increases and the the loss of kinetic energy accelerates with the increasing of impact velocity.

![Figure 9. Residual rod length-time curves for penetration of rods with different shapes.](image)

**Figure 9.** Residual rod length-time curves for penetration of rods with different shapes.

4. **Conclusion**

In present paper, combining with experimental and simulation results, the 2D effect of long-rod penetration is analyzed based on the 2D model established in authors’ previous work. Through the analysis of the results of ballistic tests of long rods and corresponding simulations, it is found that the nose shape of the long rod remains unchanged during the primary phase of penetration process, thus the validity of the assumption of the 2D model is confirmed. In addition, the 2D effect reflected in experimental and simulation results can be preliminarily explained by the 2D model. The deviation of model calculated results from experimental and simulation results are mainly caused by the difference between the ideal semi-infinite targets and the moderately thick targets. Therefore, long-rod penetration experiments and simulations of more rod-target combinations should be conducted to further analyze phenomenon and mechanism of the 2D effect and establish the 2D theoretical model considering the variation of the nose shape.
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