Investigation of hypersonic conic flows generated by a magnetoplasma light-gas gun equipped with Laval nozzle

P P Khramtsov, V A Vasetskiy, U M Hryshchanka, M V Doroshko, M Yu Chernik, A I Makhnach and I A Shikh
Heat and Mass Transfer Institute of the National Academy of Sciences of Belarus, P Brovka Street 15, Minsk 220072, Belarus
E-mail: emerald@tut.by

Abstract. In this paper, a new method of producing hypersonic flows is proposed and results of an experimental study of hypersonic flow over cones with half-angles $\tau_1 = 3^\circ$ and $\tau_2 = 12^\circ$ are presented. The Mach numbers of studied incident flows were $M_1 = 18$ and $M_2 = 14.4$, respectively. The use of a light-gas gun, where an accelerating channel was replaced by a Laval nozzle, made it possible to obtain a hypersonic gas outflow with optical density sufficiently high for flow visualization and diagnostics by optical methods. The flow structure was visualized by means of schlieren method with the use of a straight Foucault knife. Shadowgraphs were recorded by a high-speed camera with a frame rate of 300,000 fps and an exposure time of 1 $\mu$s. The Mach number for the incident flow was calculated from the shock wave inclination angle on shadowgraphs.

1. Introduction
Study of a high-speed impact, the resistance of materials to high speed solid particles, hypersonic flows over axisymmetric bodies is closely associated with the development of aviation and space technology. Development of light-gas guns as the most effective devices for accelerating projectiles of a relatively large mass up to velocities comparable with the orbital velocity is an extensive area of research. Some types of guns where the acceleration channel is replaced by a nozzle can be effectively used for the study of hypersonic flows over bodies, since the working gas parameters needed for high-speed throwing and hypersonic flow producing are identical. The construction designs of such a high-speed throwing system and operation conditions are described in [1–4]. Since the 40s of the last century, a large number of theoretical, experimental, and computational data have been obtained by researchers all over the world that relate to the field of producing hypersonic flows and studying the characteristic features of flows over various test bodies [5–20]. With the development of computer technology in addition to experimental methods of studying hypersonic flows, numerical simulation methods have come into widespread use [21–23]. It should be noted, however, that experimental studies of hypersonic flows over the models are still preferred. Due to the importance of the problems to be solved flight tests are often used for higher reliability of the experimental data. To determine the Mach number, values of total and static pressure are usually measured with the aid of Pitot-Prandtl probe, but in a large number of research works measurements were not carried out because
of their complexity. Moreover, pressure measurement by means of a Pitot–Prandtl probe and temperature measurement by a thermocouple are invasive and distort the real structure of flow. These measurement techniques complicate the design and assembling of the test objects due to the need to place measuring elements inside and on the surface of studied models, therefore many researchers used the Mach number calculated from geometrical dimensions of the Laval nozzle [5–8, 10, 13, 16].

In hypersonic aerodynamic facilities, preferable methods of investigation are optical methods with high-speed cameras used as recording devices. The optical methods of visualization of supersonic flows include a method of a laser knife, smoke visualization, visualization by means of spark discharge, etc [8, 10, 11, 16]. A large group of contactless optical methods based on the relationship between the optical and thermodynamic properties of the medium (shadow methods, interference methods) makes it possible to carry out measurements in hypersonic flows with high accuracy. For the visualization and measurement of the temperature field on the model surface, temperature-sensitive coating as an alternative to thermocouple measurements is used [14, 15].

The problems of the material resistance to a high-speed impact and the diagnostics of hypersonic flow are often interrelated, therefore the goal of this work was to create a universal compact test facility, equipped with an optical system for the flow structure visualization and flow parameters investigation, which enables us to conduct both ballistic tests and studies of hypersonic flow over objects of any complexity.

2. Experimental equipment and measurement techniques

The majority of experimental schemes that realize hypersonic outflow of gas at high Mach numbers are built on the basis of the shock tube. At the end wall of the shock tube a block of diaphragms and a Laval nozzle are installed [24]. Heated gas behind the reflected shock wave flows out of the nozzle at hypersonic speed. Because of the high degree of gas expansion in the nozzle, designed for a high Mach number, the density of the net flow turns out to be rather low, which extremely complicates visualization of the flow by optical methods. When light passes through optical inhomogeneity, it is possible to estimate the angles of deflection $\varepsilon_x$ and $\varepsilon_y$ of a probing beam (in the case of small deflection angles) by the expressions [25, 26]

$$
\varepsilon_x = \frac{1}{n_0} \int \frac{\partial n(x,y,z)}{\partial x} \, dz, \quad \varepsilon_y = \frac{1}{n_0} \int \frac{\partial n(x,y,z)}{\partial y} \, dz.
$$

Here the gradients of the gas refraction index $\partial n(x,y,z)/\partial y$ and $\partial n(x,y,z)/\partial x$ turn out to be very small in the case of light gas flows out into vacuum. Thus, for example, when the light gas is helium, the residual pressure in the vacuum chamber is 1 Torr and the characteristic scale of a streamlined body is $\sim 10^{-2}$ m, the refraction index gradients are not greater than $5 \times 10^{-6}$ m$^{-1}$. This complicates the use of shadow and interference visualization methods, and in such cases one has to use methods of multiple-pass interferometry, multi-beam interferometry, interferometry in a polarized light, holographic methods in order to increase the sensitivity of measurements [27].

On the other hand, the use of a light-gas launcher where accelerating channel is replaced by a Laval nozzle allows one to obtain a hypersonic flow with a high optical density of the outflowing gas and to apply a schlieren method for its visualization. The hypersonic flow facility studied in the present work is a modified light-gas launcher used by us for ballistic tests is described in detail in [28]. As a light-gas gun, this facility was capable of accelerating a bearing-steel projectile of diameter 2.5 mm to speeds 2.5–4 km/s.

Experiments on hypersonic blowing of cones were carried out in the vacuum chamber with a residual gas pressure of 1 Torr. The scheme of the experimental facility is presented in figure 1. In the experiments, the light-gas section was filled with helium up to a pressure of 40 bar. The piston for adiabatic compression of the light gas was set in motion as a result of erosion.
magnetoplasma accelerator discharge. A capacitor bank of $1.2 \times 10^3 \, \mu\text{F}$ was charged to a voltage of 4.5 kV. Directly in front of the confuser part of the Laval nozzle, five brass diaphragms with a thickness of 100 $\mu\text{m}$ each were placed. The rupture of the diaphragms occurred when the pressure in the light-gas section reached $\sim 1.6 \text{kbar}$. The compression rate and the temperature of the working gas can be estimated approximately from the Poisson equation:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \left(\frac{P_2}{P_1}\right)^{\gamma - 1/\gamma},$$

where $T$ is the temperature, $V$ is the volume, $P$ is the pressure, and $\gamma = 5/3$ is the specific heat ratio for a monatomic gas. Subscript 1 refers to the initial state of gas and subscript 2, to the state of the gas on diaphragm rupture.

In this experiment, at the moment of diaphragm rupture, the compression rate of the working gas is about 10 and the temperature is about 1500 K. After the rupture of the diaphragms, further compression of the gas continues with simultaneous gas outflow through the critical cross section of Laval nozzle. The maximum compression rate that can be calculated as the ratio of light-gas section volume to the nozzle confuser volume is about 50 for the present experimental facility.

The geometrical parameters of the Laval nozzle for the Mach number 18 were calculated by the formula [29, 30]

$$\frac{A}{A^*} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \frac{1}{M} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma + 1}{2(\gamma - 1)}},$$

where $A$ is the area of nozzle exit and $A^*$ is the area of the nozzle throat.

The test cones were rigidly fixed coaxially with the nozzle at a distance of 20 mm from the nozzle exit section. For visualization of the patterns of hypersonic flow over a test cone the schlieren method was used. The width of the slit was 0.16 mm. A Foucault knife was placed in the focal plane of the receiving part of the shadow device so as to shade a half-image of the slit. A 150-W halogen lamp was used as a light source. The focal length of the shadow device was 1000 mm. Registration of the shadow patterns was carried out using a high-speed Photron Fastcam camera (300 000 fps) with an exposure time of 1 $\mu\text{s}$ (figure 2).
3. Experimental results and discussion
An analysis of the sequence of shadow patterns shows that at the initial period of time the flow is unstable and contains some particles formed as a result of diaphragm destruction, with its own conical shock around each particle (see figure 2). This period lasts for not more than 300 µs. During the next 1.3 ms the flow velocity increases, and the nozzle passes gradually into a stationary operating regime. At a maximum flow velocity, the shadow patterns show the sharpest angle of shock wave inclination. Figure 3 presents shadow patterns of hypersonic flow over cones with half-angles $\tau_1 = 3^\circ$ (a) and $\tau_2 = 12^\circ$ (b).

Figure 2. Shadowgraph images of hypersonic flow over a cone with half-angle $\tau_2 = 12^\circ$.

Figure 3. Shadowgraph images of hypersonic flows over cones with half-angle $\tau_1 = 3^\circ$ (a) and $\tau_2 = 12^\circ$ (b).
The relationship between the Mach number and the shock wave inclination angle in supersonic flow over a cone is described in detail in [18,19]. Calculation of the Mach number was based on the measurement of shock angle observed in shadow patterns (see figure 3) using a formula [20], which is valid for $M \gg 1$:

$$M = \csc \sigma \sqrt{\frac{2(B + C - \sec \sigma)}{(\gamma + 1)\left(B + C + \frac{\cos \sigma}{\sin^2 \sigma}\right) - (\gamma - 1)(B + C - \sec \sigma)},}$$

(4)

where

$$B = -\ln\left(\frac{\sin \tau}{1 - \cos \tau}\right) - \frac{\cos \tau}{\sin^2 \tau}, \quad C = \ln\left(\frac{\sin \sigma}{1 - \cos \sigma}\right),$$

where $\tau$ is the half-angle of the cone, $\sigma$ is the shock wave inclination angle, and $M$ is the free-stream Mach number.

The calculated values of the Mach number were equal to $M_1 = 18$ for hypersonic flow over the cone with half-angle $\tau_1 = 3^\circ$ and $M_2 = 14.4$ for the cone with $\tau_2 = 12^\circ$.

4. Analysis of shadowgraphs and calculation of helium density field for hypersonic flow over a cone

As is known from the theory of schlieren technique [25], a relative change in the light intensity $\Delta I/I_0$ in the geometrical optics approximation is directly proportional to the magnitude of the deflection angle of light rays $\varepsilon$ in the optical inhomogeneity and is defined by the ratio

$$\frac{\Delta I}{I_0} = \frac{\varepsilon f}{d},$$

(5)

where $\Delta I = I - I_0$, $d$ is the slit width, $f$ is the focal length of the optical system, $I_0$ is the light intensity in the absence of optical inhomogeneity, and $I$ is the light intensity in the presence of optical inhomogeneity.

Since the streamlined body has an axial symmetry, the assumption of axisymmetric structure of flow was used in processing the shadow patterns. We introduce the coordinate system so that the $x$ axis be normal to the edge of the knife, the $z$ axis be directed along the probe light beam, and the $y$ axis be directed along the axis of symmetry, with the origin coinciding with the cone tip. In this case, the angles of light deflection are associated with the distribution of refraction index $\varepsilon(x)$ inside the inhomogeneity ($R$ is the inhomogeneity radius) by the Abel equation [25,26]:

$$\varepsilon(x) = 2 \int_x^R \frac{d \ln n(r)}{dr} \frac{xdr}{\sqrt{r^2 - x^2}}.$$

(6)

For further calculations we used the inversion of the Abel equation, where the quantity $d \ln n/dr$ is expressed explicitly as

$$\frac{d \ln n(r)}{dr} = \frac{1}{\pi} \frac{d}{dr} \int_r^R \frac{\varepsilon(x)dx}{\sqrt{x^2 - r^2}}.$$

(7)

where $r$ is the current radius.

After repeated integration [25,26,31] we obtain the radial distribution of the refraction index:

$$\ln \frac{n(r)}{n_0} \approx \frac{n(r) - n_0}{n_0} = \frac{\Delta n}{n_0} = \frac{1}{\pi} \int_r^R \frac{\varepsilon(x)dx}{\sqrt{x^2 - r^2}},$$

(8)

where $n_0$ is the $a priori$ known value of the refraction index on the line of integration, for example, in the undisturbed flow regions $n_0 = n(R) \cong 1$.

Next, the investigated inhomogeneity was divided into annular zones, the number of which was equal to the number of pixels in the direction of the $x$ axis. The magnitude of the deflection
Figure 4. Calculated density field for hypersonic helium flow over a cone with $\tau_1 = 3^\circ$: (a) the density as a function of a radius $r$ and a coordinate $l$ along the axis of symmetry with the origin coinciding with the cone tip; (b) selected profiles of the density along $r$ at different $l$.

angle $\varepsilon$ within the annular zone $[r_i, r_{i+1}]$ was taken constant. Then expression (8) can be represented as a sum of elementary integrals:

$$\frac{\Delta n(r_j)}{n_0} = \frac{1}{\pi} \sum_{i=j}^{N-1} \varepsilon(r_i) \int_{r_i}^{r_{i+1}} \frac{dr}{\sqrt{r^2 - r_j^2}}.$$

(9)

The refraction-index distributions over the flow field are obtained as a result of independent integration in each cross section perpendicular to the cone symmetry axis. With such processing of shadow patterns, the values of the refraction index in the adjacent cross sections turns out to be unlinked, which leads to the appearance of typical distortions in the distribution of the refraction index. These distortions are due to the high noise component of the experimental light intensity function $I$ along the $y$ axis. To eliminate the random noise, preliminary smoothing of the intensity function in the $y$ direction was performed by the least squares method [32–34]:

$$\int_{y_{\min}}^{y_{\max}} (I_{yy}''(y))^2 dy \rightarrow \min.$$

In this case, the values of the approximating function should not differ from the initial experimental function by more than the value equal to the standard deviation of the probe light intensity $I_0$ in the region of unperturbed flow.

The helium density $\rho$ was calculated with the use of the Gladstone–Dale relation [26,31]

$$n - 1 = K_{He} \rho,$$

(10)

where $K_{He} = 0.19607 \text{ cm/g}^3$ is the Gladstone–Dale constant for helium. The value of $K_{He}$ for the given gas and for the given wavelength is considered to be constant in a wide pressure range [31]. Its value was calculated for the gas under normal conditions from the tabulated values of density and refraction index [26]. Since the choice of the initial values $n_0$ and $\rho_0$ was arbitrary, we can calculate $\Delta n$ from the following relation:

$$\Delta n = K_{He} \Delta \rho.$$

(11)

The calculated results of helium density fields for hypersonic flow over the cones with the half-angles $\tau_1 = 3^\circ$ and $\tau_2 = 12^\circ$ are given in figures 4 and 5. The results of calculation show a rapid change in the helium density in the collision zone near the cone surface. Further, downstream,
Figure 5. Calculated density field for the helium flow over a cone with $\tau_1 = 12^\circ$: (a) the density as a function of $r$ and $l$; (b) selected profiles of the density along $r$ at different $l$.

the density value decreases smoothly according to a law similar to the exponential one. The figures also show the presence of small fluctuations in the helium density at a certain distance from the cone tip, seemingly connected with the loss of stability in the boundary layer. The obtained data are in good qualitative agreement with the results of calculations [18, 19].

5. Conclusion
The use of light-gas ballistic systems, where acceleration channel is replaced by a Laval nozzle, make it possible to realize hypersonic flow with a high Mach number and the gas density sufficient for photometric optical measurements by shadow or interference methods. This facility enables one to study the hypersonic flow over models of any complexity.

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