String Derived Exophobic $SU(6) \times SU(2)$ GUTs

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Abstract

With the apparent discovery of the Higgs boson, the Standard Model has been confirmed as the theory accounting for all sub-atomic phenomena. This observation lends further credence to the perturbative unification in Grand Unified Theories (GUTs) and string theories. The free fermionic formalism yielded fertile ground for the construction of quasi-realistic heterotic-string models, which correspond to toroidal $Z_2 \times Z_2$ orbifold compactifications. In this paper we study a new class of heterotic-string models in which the GUT group is $SU(6) \times SU(2)$ at the string level. We use our recently developed fishing algorithm to extract an example of a three generation $SU(6) \times SU(2)$ GUT model. We explore the phenomenology of the model and show that it contains the required symmetry breaking Higgs representations. We show that the model admits flat directions that produce a Yukawa coupling for a single family. The novel feature of the $SU(6) \times SU(2)$ string GUT models is that they produce an additional family universal anomaly free $U(1)$ symmetry, and may remain unbroken below the string scale. The massless spectrum of the model is free of exotic states.
1 Introduction

With the imminent discovery of the Higgs boson looming another piece in the particle puzzle falls into place. Confirmation of its scalar identity is another vital goal, which may, however, require a precision machine to study its properties in greater detail. The discovery of additional scalar particles has similarly eluded experiments to date and pushes the scale of supersymmetric particles, possibly beyond the LHC scale. Nevertheless, the discovery of a fundamental Higgs boson lends further credence to the prevailing picture of unifying the observed gauge and matter structure in a Grand Unified Theory, or, ultimately in string theory.

This unification in turn gives rise to new puzzles. Among the most perplexing is that of proton longevity. Indeed, in the Standard Model proton stability is only guaranteed due to the existence of accidental global symmetries at the renormalisable level. Proton instability becomes especially acute in theories of quantum gravity that are not expected to preserve global symmetries. In the context of string constructions it was proposed that the proton stability problem may indicate the existence of an additional $U(1)$ symmetry at a comparatively low scale that serves as a proton lifeguard [1]. The caveat, however, is that other phenomenological requirements constrain possible $U(1)$ symmetries to be broken at the high, or intermediate, scale. Thus, failing to provide adequate suppression of proton decay mediating operators. In general, it is found that keeping an additional $U(1)$ symmetry unbroken in string constructions is highly nontrivial.

To explore such questions in detail one must construct phenomenological string models. The free fermionic formulation [2] of the heterotic string provided a fertile framework to study quasi–realistic string vacua. Three generation models in this construction preserve the $SO(10)$ embedding of the Standard Model states [3, 4, 5, 6, 7]. The existence of models in this class which produce solely the spectrum of the Minimal Supersymmetric Standard Model has been further demonstrated. The free fermionic models correspond to $Z_2 \times Z_2$ orbifold compactifications with discrete Wilson lines [8, 9]. The $SO(10)$ symmetry is broken directly at the string level to one of its subgroups, which included to date: the flipped $SU(5)$ (FSU5) [3]; the standard–like models (SLM) [4]; the Pati–Salam models (PS) [5, 10]; the left–right symmetric models [6]; and the $SU(4) \times SU(2) \times U(1)$ models (SU421) [7]. Of those, it was shown that the SU421 models in this class do not produce realistic spectra [7].

The proposition of a proton lifeguard extra $U(1)$ symmetry in these models was studied in ref. [1]. The $U(1)$ symmetry that can play this role in the free fermionic models is a combination of $U(1)_{B-L}$, which is embedded in $SO(10)$ and a family universal combination of three $U(1)$s in the Cartan subalgebra of the observable $E_8$, and which are external to $SO(10)$. We shall refer to this $U(1)$ combination as $U(1)_\zeta = U(1)_1 + U(1)_2 + U(1)_3$. The caveat, however, is that in the FSU5, SLM and PS free fermionic modes the $U(1)_\zeta$ combination is anomalous and hence must be broken near the string scale. In the LRS string models it is anomaly free, but has
mixed $SU(2)_{L}^{2} \times U(1)_{\zeta}$ anomalies [11]. This necessitates the introduction of additional lepton doublets to render the spectrum anomaly free, which requires also additional colour triplets to facilitate gauge coupling unification [12].

In this paper we study a new class of string models that can produce the desired proton lifeguard $U(1)$ combination. Particularly, in these models $U(1)_{\zeta}$ is anomaly free. The models under consideration possess an $SU(6) \times SU(2)$ unbroken symmetry at the string level and arise from the breaking of $E_{6}$ to this maximal subgroup. Thus, the $U(1)_{\zeta}$ combination is anomaly free by virtue of its $SU(6)$ embedding. It should be noted that the viability of an extra $U(1)$ down to low scales may result in additional phenomenological issues that may need to be addressed. For example, the augmentation of the spectrum by lepton doublets in the model of ref. [11] requires additional colour triplets that may, a priori, lead to proton decay mediating operators.

To construct a full model with a viable $U(1)$ down to the low scale is beyond the scope of this paper. Therefore, the possibility also exist to break the symmetry directly to the Standard Model at the high scale. We present an example of a three generation free fermionic $SU(6) \times SU(2)$ GUT model and explore its phenomenological viability.

Our paper is organised as follows: in section 2 we review the field theory structure of the $SU(6) \times SU(2)$ models. In section 3 we discuss the construction of $SU(6) \times SU(2)$ heterotic–string models and present an exemplary three generation free fermionic model. In section 4 we present the cubic level superpotential of the model and explore its phenomenology. Section 5 concludes the paper.

2 The supersymmetric $SU(6) \times SU(2)$ Models

In this section we briefly summarise the field theory content of the $SU(6) \times SU(2)$ models. This gauge group is a maximal subgroup of $E_{6}$. The matter states of the model are obtained therefore from the 27 representation of $E_{6}$ decomposed under the $SU(6) \times SU(2)$ subgroup. We note that we have two options of embedding the Standard Model into this subgroup. Namely, we can choose the electroweak $SU(2)_{L}$ to reside inside $SU(6)$ or, alternatively, we can identify it with the $SU(2)$ which is orthogonal to $SU(6)$. The field theory model building and symmetry breaking patterns of the $SU(6) \times SU(2)$ GUT models were discussed in ref. [13, 14], as well as the possibility of preserving an unbroken $U(1)$ symmetry to low scales [15]. The advantage of having an $SU(6)$ GUT in respect to proton lifetime [16], as well as the flexibility it affords in regard to gauge coupling unification [14], has also been examined. The novelty in our paper is the derivation of a string model that can realise the $SU(6) \times SU(2)$ GUT model. Further details of the field theory construction can be found in [14, 15], and will be further explored in a future publication. The non–trivial aspect From the point of view of the string model building is that $U(1)_{\zeta}$ is anomaly free. Turning to the field theory content, the 27 representation of $E_{6}$ decomposes as

$$27 = (15, 1) + (\bar{6}, 2) \quad (2.1)$$
Our string models will be constructed in the following as an enhancement of the Pati–Salam heterotic string models. It is instructive to note the decomposition in the Pati–Salam and Standard–Model subgroups. In this regard the 27 representation of $E_6$ decomposes under $SO(10) \times U(1)$ as

$$27 = (16, -\frac{1}{2}) + (10, 1) + (1, -2),$$

(2.2)

where the 16 and 10 are the spinorial and vectorial representations of $SO(10)$, respectively. Under the Pati–Salam subgroup these decompose into representations of $SU(4) \times SU(2)_R \times SU(2)_L \times U(1)_{\zeta}$ as

$$(16, -\frac{1}{2}) = F_L + F_R = (4, 2, 1, -\frac{1}{2}) + (\bar{4}, 1, 2, -\frac{1}{2})$$

$$(10, 1) = D + h = (6, 1, 1, 1) + (1, 2, 2, 1)$$

$$(1, -2) = S = (1, 1, 1, -2).$$

The decomposition of the 27 under $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L \times U(1)_{\zeta}$ is shown in table 1.

| Field | $SU(3)_C \times SU(2)_L$ | $U(1)_C$ | $U(1)_L$ | $U(1)_{\zeta}$ |
|-------|--------------------------|---------|---------|----------------|
| $Q$   | 3                        | 2       | $+\frac{1}{2}$ | 0               | $-\frac{1}{2}$ |
| $L$   | 1                        | 2       | $-\frac{3}{2}$ | 0               | $-\frac{1}{2}$ |
| $u$   | 3                        | 1       | $-\frac{1}{2}$ | $-1$ | $-\frac{1}{2}$ |
| $d$   | 3                        | 1       | $-\frac{1}{2}$ | $+1$ | $-\frac{1}{2}$ |
| $e$   | 1                        | 1       | $+\frac{3}{2}$ | $+1$ | $-\frac{1}{2}$ |
| $N$   | 1                        | 1       | $+\frac{3}{2}$ | $-1$ | $-\frac{1}{2}$ |
| $h^u$ | 1                        | 2       | 0         | $+1$ | +1 |
| $h^d$ | 1                        | 2       | 0         | $-1$ | +1 |
| $D$   | 3                        | 1       | $-1$      | 0               | +1 |
| $\bar{D}$ | 3                   | 1       | $+1$      | 0               | +1 |
| $S$   | 1                        | 1       | 0         | 0               | $-2$ |

Table 1: Decomposition of the 27 representation of $E_6$ under $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L \times U(1)_{\zeta}$, where $U(1)_C = \frac{3}{2}U(1)_{B-L}$ is the Baryon minus Lepton number and $U(1)_L = 2U(1)_{3R}$ is the diagonal generator of $SU(2)_R$.

If we choose the electroweak $SU(2)_L$ gauge group to be the one external to $SU(6)$, then the Standard Model matter and Higgs representations are embedded in

$$\mathcal{F}^i_L = (6, 2)^i = F^i_L + h^i = (Q + L + h^u + h^d)^i$$

(2.3)

$$\mathcal{F}^i_R = (15, 1)^i = F^i_R + D^i + S^i = (u + d + e + N + D + \bar{D} + S)^i$$

(2.4)
where the index \( i = 1, 2, 3 \) is a generation index. The weak hypercharge combination is given by

\[
U(1)_Y = \frac{1}{3} U(1)_C + \frac{1}{2} U(1)_L
\]  

(2.5)

In this case the heavy Higgs states in the model are in the \((15, 1)\) and \((\overline{15}, 1)\) representations of \(SU(6) \times SU(2)\). We need two pairs, \(H + \overline{H}\) and \(\mathcal{H} + \overline{\mathcal{H}}\) to generate the symmetry breaking down to the Standard Model gauge group \(SU(3)_C \times SU(2)_L \times U(1)_Y\). The \(H + \overline{H}\) fields obtain VEVs in the direction of the \(N\) and \(\overline{N}\) components. These VEVs breaks the gauge symmetry to \(SU(3) \times SU(2) \times U(1)_Y \times U(1)_Z\). The \(\mathcal{H} + \overline{\mathcal{H}}\) fields get VEVs in the direction of the \(S\) and \(\overline{S}\) components, which break the gauge symmetry down to the Standard Model. The first set of VEVs has to be sufficiently large to generate a seesaw mechanism and light neutrino masses, whereas the second set of VEVs can be at a lower scale. The fields in the \((6, 2)\) representation of \(SU(6) \times SU(2)\) contain the electroweak Higgs bi–doublets that are required to break the \(SU(2)_L \times U(1)_Y\) down to \(U(1)_{\text{e.m.}}\).

The most general cubic level superpotential involving the \(SU(6) \times SU(2)\) charged fields is given by

\[
W = \lambda_{ijk} F^i_L F^j_R F^k_L + \lambda_{ij} F^i_R F^j_R H + \lambda_{ij} F^i_R F^j_R \mathcal{H} + \lambda_{ijk} F^i_L F^j_L F^k_L + \lambda_{ij} F^i_L F^j_L H + \lambda_{ijk} F^i_L F^j_L \mathcal{H} + \lambda^6 HH\mathcal{H} + \lambda^7 \overline{HH}\overline{HH}
\]  

(2.6)

We note that some of these couplings may need to vanish for viable phenomenological realisation. The first term generates the fermion Yukawa couplings to the electroweak Higgs fields. The second and third terms may generate mass terms for some of the colour triplet fields. The sixth and seventh term generate the heavy Higgs superpotential. The fourth term couples the VEV of \(S\) to the electroweak bi–doublets. Hence, if this VEV is at the low scale, it provides an explanation for the suppression of the supersymmetric \(\mu\)–term. The scale of the \(\mu\)–term is associated then with the scale of \(U(1)_Z\) breaking. The neutrino seesaw mechanism may be generated by coupling the right–handed neutrino components to light \(SU(6) \times SU(2)\) singlet fields, i.e.

\[
\lambda_{ij} F^i_R \overline{H} \phi^j + \lambda_{ijk} \phi^i \phi^j \phi^k
\]  

(2.7)

This scenario therefore requires the introduction of three additional \(SU(6) \times SU(2)\) singlets. Alternatively, the seesaw mass terms may be obtained from dimension five terms

\[
\lambda_{ij} F^i_R F^j_R \overline{HH}
\]  

(2.8)

Turning to the proton decay mediating operators, we note that the terms arising from the quartic \(16^4\) as well as those arise from dimension four operators in the MSSM are forbidden in this model by \(U(1)_\xi\). This is because we identified the Standard

\[\text{footnote} U(1)_c = 3/2 U(1)_{B-L} ; U(1)_L = U(1)_{\tau_R}.\]
Model states as arising solely from the 16 representations of $SO(10)$ and all these states carry the $U(1)_{\zeta}$ charge, as dictated by the $E_8$ charge assignment. Therefore, both the dimension four and five operators are forbidden. Preservation of an unbroken $U(1)_{Z'}$, which is a combination of $U(1)_{B-L}$ and $U(1)_{\zeta}$, then guarantees that the proton decay mediating operators, as well as the Higgs $\mu$-term are adequately suppressed in this model. A more elaborate field theory analysis of the model will be presented elsewhere.

The alternative to embedding the $SU(2)_R$ in $SU(6)$, as outlined above, is to embed $SU(2)_L$ in $SU(6)$ and $SU(2)_R$ external to it. In this case the Standard Model matter and Higgs states are embedded in

\[ F^i_R = (\bar{6}, 2)^i = F^i_R + h^i = (u + d + e + N + h^u + h^d)^i \]  
\[ F^i_L = (15, 1)^i = F^i_L + D + S^i = (Q + L + D + \bar{D} + S)^i \]

where the index $i = 1, 2, 3$ is a generation index. It is seen that the effect is to flip the $F^i_L$ and $F^i_R$ representations between the two representations in (2.3) and (2.4). The weak hypercharge combination is still given by eq. (2.5) in terms of the generators of the Cartan Subalgebra of the observable $E_8$. The consequence is that with the assignment in (2.9) and (2.10) two states of the heavy Higgs representations are in the $(\bar{6}, 2)$ and $(6, 2)$, denoted by $H$ and $\bar{H}$, which contain the VEVs in the direction of the $N$ and $\bar{N}$ components. The two other heavy Higgs states are, as before, in the $(15, 1)$ and $(\bar{15}, 1)$, denoted by $H + \bar{H}$, contain the VEVs in the direction of the $S$ and $\bar{S}$ components. Hence, in this case the general cubic level superpotential involving the $SU(6) \times SU(2)_R$ charged fields is given by

\[ W = \lambda_{ijk}^1 F^i_R F^j_L F^k_R + \lambda_{ijk}^2 F^i_R F^j_R H + \lambda_{ij}^3 F^i_L F^j_L H + \lambda^4 H H H + \lambda^5 H H H \]  

The terms in (2.11) are given in terms of the assignment in (2.9) and (2.10). The seesaw term can be taken from (2.8), but with the modified field assignment in (2.9) and (2.10).

We note that in the $SU(6) \times SU(2)_L$ string models the heavy Higgs states may arise solely from the untwisted sector, whereas the $SU(6) \times SU(2)_R$ string models also require heavy string states that arise from twisted sectors.

3. $SU(6) \times SU(2)$ Heterotic–String GUT Models

In this section we present a three generation $SU(6) \times SU(2)$ string GUT model. The model is constructed in the free fermionic formulation by using the classification method developed in refs. [17, 18, 19, 10]. The set of basis vectors is identical to the one used in the classification of the Pati–Salam heterotic–string models [10]. The difference between the two cases is that in [10] the vector bosons that enhance the observable $SO(16)$ gauge symmetry to $E_8$ are projected out, whereas here they are retained. Therefore, the $SU(6) \times SU(2)$ gauge symmetry is obtained here as
enhancement of the $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_1 \times U(1)_2 \times U(1)_3$ observable gauge symmetry arising from the untwisted sector, to $SU(6) \times SU(2)_L \times U(1)'_1 \times U(1)'_2$. We then follow a fishing algorithm in which we generate random choices of GSO projection coefficients. We fish out the choices that satisfy the enhancement criterion and additional phenomenological criteria, like the existence of three generations and absence of exotics.

In the free fermionic formulation the 4–dimensional heterotic–string, in the light-cone gauge, is described by 20 left–moving and 44 right–moving real fermions. The models are constructed by choosing different phases picked up by fermions $(f_A, A = 1, \ldots, 44)$ when transported around non–contractible loops of the world–sheet torus. Each model corresponds to a choice of fermion phases consistent with modular invariance that can be generated by a set of basis vectors $v_i, i = 1, \ldots, n$

$$v_i = \{\alpha_i(f_1), \alpha_i(f_2), \alpha_i(f_3), \ldots\}$$

describing the transformation properties of each fermion

$$f_A \rightarrow -e^{i\pi \alpha_i(f_A)} f_A, \ A = 1, \ldots, 44$$

(3.1)

The basis vectors span a space $\Xi$ which consists of $2^N$ sectors that give rise to the string spectrum. Each sector is given by

$$\xi = \sum N_i v_i, \ N_i = 0, 1$$

(3.2)

The spectrum is truncated by a generalised GSO projection whose action on a string state $|S>$ is

$$e^{i\pi v_i \cdot F_S} |S> = \delta_S c_{v_i} |S>,$$

(3.3)

where $F_S$ is the fermion number operator and $\delta_S = \pm 1$ is the space–time spin statistics index. Different sets of projection coefficients $c_{v_i} = \pm 1$ consistent with modular invariance give rise to different models. Summarising: a model can be defined uniquely by a set of basis vectors $v_i, i = 1, \ldots, n$ and a set of $2^{N(N-1)/2}$ independent projections coefficients $c_{v_i}, i > j$.

The free fermions in the light-cone gauge in the usual notation are: $\psi^\mu, \chi^i, y^i, \omega^i, i = 1, \ldots, 6$ (left–movers) and $\bar{y}^i, \bar{\omega}^i, i = 1, \ldots, 6, \psi^A, A = 1, \ldots, 5$, $\bar{\eta}^B, B = 1, 2, 3, \bar{\phi}^\alpha, \alpha = 1, \ldots, 8$ (right–movers). The class of models we investigate,
is generated by a set of thirteen basis vectors $B = \{v_1, v_2, \ldots, v_{13}\}$, where

$$v_1 = 1 = \{\psi^\mu, \chi^{1,\ldots,6}, y^{1,\ldots,6}, \omega^{1,\ldots,6}\},$$

$$v_2 = S = \{\psi^\mu, \chi^{1,\ldots,6}\},$$

$$v_{2+i} = e_i = \{y^i, \omega^i, \bar{y}^i, \bar{\omega}^i\}, \; i = 1, \ldots, 6,$$

$$v_9 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56}, \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^{1}, \bar{\psi}^{1,\ldots,5}\},$$

$$v_{10} = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56}, \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^{2}, \bar{\psi}^{1,\ldots,5}\},$$

$$v_{11} = z_1 = \{\bar{\phi}^{1,\ldots,4}\},$$

$$v_{12} = z_2 = \{\bar{\phi}^{5,\ldots,8}\},$$

$$v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}.$$

The first twelve vectors in this set are identical to those used in [18, 19] for the classification of $Z_2 \times Z_2$ heterotic–string models with an $SO(10)$ GUT group. The basis vector $v_{13}$ is the vector that breaks the $SO(10)$ GUT symmetry to $SO(6) \times SO(4)$.

The second ingredient that is needed to define the string vacuum are the Generalised GSO (GGSO) projection coefficients that appear in the one–loop partition function, $c_{ij}$, spanning a $13 \times 13$ matrix. Only the elements with $i > j$ are independent, and the others are fixed by modular invariance. A priori there are therefore 78 independent coefficients corresponding to $2^{78}$ distinct string vacua. Eleven coefficients are fixed by requiring that the models possess $N = 1$ supersymmetry. An explicit choice of GGSO projection coefficients that produces a model with $SU(6) \times SU(2)$ GUT group is given by the following GGSO coefficients matrix :

$$\begin{pmatrix}
1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
\end{pmatrix}$$

where we introduced the notation $c_{ij} = e^{i\pi(v_i|v_j)}$. 

8
The gauge group of the model is obtained from the untwisted sector and the $x$–sector [20], which arises from the vector combination

$$x = 1 + S + \sum_{i=1}^{6} e_i + z_1 + z_2 = \{\psi_1^{1,\ldots,5}, \bar{\eta}^{1,2,3}\}. \quad (3.6)$$

The untwisted sector gives rise to space–time vector bosons transforming under the group symmetry:

- **observable**: $SO(6) \times SU(2) \times SU(2) \times U(1)^3$
- **hidden**: $SO(4)^2 \times SO(8)$

where the three $U(1)$ factors, $U(1)_{1,2,3}$, in the observable gauge group are generated by the complex world–sheet fermions $\bar{\eta}^{1,2,3}$. The $x$–sector produces space–time vector bosons transforming under the untwisted group symmetry as:

$$\left(4, 2, 1, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right) + \left(\bar{4}, 2, 1, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right). \quad (3.7)$$

Hence, the $SO(6) \times SU(2) \times U(1)$ group symmetry is enhanced to $SU(6)$. The $U(1)$ combination which is embedded in $SU(6)$ is given by

$$U(1)_6 = U(1)_1 + U(1)_2 - U(1)_3 \quad (3.8)$$

and the two orthogonal combinations are given by

$$
\begin{align*}
U(1)'_1 & = 2U_1 - U_2 + U_3 \quad (3.9) \\
U(1)'_2 & = U_2 + U_3. \quad (3.10)
\end{align*}
$$

An important feature of this model, as compared to models in which the observable gauge symmetry is not enhanced, is that the $U(1)_6$ combination is automatically anomaly free. On the other hand, of the two orthogonal combinations, one combination may, in general, be anomalous. Additional space–times vector bosons may arise from the sectors [18, 19, 10]

$$G = \left\{ \begin{array}{cccc}
z_1, & \bar{z}_2, & z_1 + z_2, & \alpha, \\
\alpha + z_2, & \alpha + z_1 + z_2, & \alpha + x, & \alpha + x + z_1 \end{array} \right\} \quad (3.11)$$

and may further enhance the four dimensional gauge group. Here we impose the condition that the additional space–time vector bosons arising from the sectors in (3.11) are projected out.

The matter spectrum in the model arises from untwisted and twisted sectors. As the basis vectors, eq. (3.4), that generate the model are identical to those that generate the Pati–Salam models of [10] the sectors of the models are identical. The difference is that in the $SU(6) \times SU(2)$ models the states from the different sectors
coalesce into representations of the enhanced group. Since the sector producing the enhanced symmetry is the $x$-sector, it entails that for every sector $\alpha$, which produces massless states that transform under the untwisted group symmetry, there is a corresponding sector $\alpha + x$, which completes the states to representations of the enhanced $SU(6) \times SU(2)$ group. The enumeration of all of the sectors that produce massless states in these models, and the type of states that they can a priori give rise to, was presented in ref. [10]. The full massless spectrum of the string model generated by the basis vectors and GGSO projection phases in eq. (3.5) is shown in tables 2, 3 and 4, where we define the vector combination $b_3 \equiv b_1 + b_2 + x$. In table 2 we list the untwisted gauge and and matter multiplets. The untwisted matter multiplets consist of the three pairs of $15 + \overline{15}$. Therefore, there is no net chirality arising from the untwisted sector, as expected due to the $Z_2 \times Z_2$ orbifold structure. Due to the symmetric boundary conditions assigned to the internal world–sheet fermions that correspond to the compactified six dimensional torus, the untwisted $(\overline{6}, 2) \oplus (6, 2)$ are projected out by the GGSO projections.

The twisted matter states generated in the string vacuum of eq. (3.5) produce the needed spectrum for viable phenomenology. It contains three chiral generation plus an additional generation and anti–generation that can get a heavy mass along a flat direction. These pair of additional states is an artifact of the fishing algorithm that was used to fish the particular model presented here. However, models that do not contain the additional states are phenomenologically viable provided that only states in the $(15, 1)$ and $(\overline{15}, 1)$ are used as heavy Higgs representations. In that case the untwisted matter states give rise to the heavy Higgs multiplets. Electroweak bi–doublets are obtained in the model from the twisted sectors. The model additionally contains $SU(6) \times SU(2)$ singlet states, some of which transform in non–trivial representations of the hidden sector gauge group.

Exotic representations may arise in the string models in the $(6, 1)$, $(\overline{6}, 1)$ and $(1, 2)$ representations of $SU(6) \times SU(2)$. Such exotic states carry fractional electric charge and are severely constrained by experimental observations. They are generic in heterotic–string models that preserve the canonical GUT embedding of the weak hypercharge [21, 22]. All the exotic states are projected in our model by the choice of GGSO projection coefficients, eq. (3.5) that define the model, rather than by giving them mass along flat direction in the effective low energy field theory [23].

4 The superpotential

Using the methodology of ref. [24] for the calculation of renormalisable and nonrenormalisable terms, we calculate the cubic level superpotential of our $SU(6) \times SU(2)$ string model. The trilevel superpotential is given by
The string vacuum contains one anomalous $U(1)$. In our model the anomalous 

$$W_{SM} = \frac{F_1 F_2 F_4 + f_1 f_2 F_4 + f_3 f_4 F_4 + f_1 f_2 f_3 F_3 + F_1 F_5 + F_2 F_6}{g \sqrt{2}}$$

$$+ \frac{F_4 F_4 F_7 + \bar{f}_1 \bar{f}_1 \bar{F}_5 + \bar{f}_2 \bar{f}_2 \bar{F}_6 + \bar{f}_4 \bar{f}_4 \bar{F}_7 + F_3 F_5 + \bar{f}_3 \bar{f}_3 \bar{F}_5}{g \sqrt{2}}$$

$$+ \frac{f_1 f_1 \bar{F}_7 + \bar{F}_4 \bar{F}_4 \bar{F}_7 + \bar{F}_5 \bar{F}_5 \chi_1 + \bar{F}_6 \bar{F}_2 \chi_2 + \bar{F}_6 \bar{F}_3 \chi_3 + 2 \bar{F}_7 \bar{F}_4 \chi_4}{g \sqrt{2}}$$

$$+ \frac{F_7 \bar{F}_4 \chi_5 + \chi_1 \chi_4 + F_5 \bar{F}_6 \bar{F}_7 + \bar{F}_5 \bar{F}_6 \bar{F}_7 + \Phi_{12} \bar{\Phi}_{34} \Phi_{56} + \bar{\Phi}_{12} \Phi_{34} \bar{\Phi}_{56}}{g \sqrt{2}}$$

$$+ \frac{\bar{F}_5 \bar{F}_6 \bar{\Phi}_{56} + \bar{F}_5 \bar{F}_7 \bar{\Phi}_{34} + \bar{F}_5 \bar{F}_6 \Phi_{56} + \Phi_{12} \Phi_{34} \Phi_{12} + \bar{\Phi}_{12} \Phi_{56} + \Phi_{12} \Phi_{56} + \Phi_{12} \Phi_{12}}{g \sqrt{2}}$$

$$+ \frac{\bar{\Phi}_{12}}{g \sqrt{2}} \chi_4$$

Table 2: Untwisted matter spectrum.
| Sector          | Field | $SU(6) \times SU(2)_R$ | $U(1)_Y$ | $U(1)_A$ | $SO(4)_1 \times SO(4)_2 \times SO(8)$ |
|-----------------|-------|------------------------|----------|----------|-------------------------------------|
| $S + b_1 + e_4$ | $F_1$ | (15,1)                 | 1        | 0        | (1,1,1)                             |
| $\oplus$        | $\chi_1$ | (1,1)                 | -3       | 0        | (1,1,1)                             |
| $S + b_1 + e_4 + x$ | $\zeta_a, a = 1,2$ | (1,1)       | 0        | 1        | (1,1,1)                             |
| $\bar{\zeta}_a, a = 1,2$ | (1,1) | 0        | -1       | (1,1,1) |
| $S + b_1 + e_4 + e_6$ | $\bar{f}_1$ | (17,2)                | 1        | 0        | (1,1,1)                             |
| $S + b_1 + e_4 + e_6 + x$ | $F_2$ | (15,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\oplus$        | $\chi_2$ | (1,1)                 | -\frac{1}{2} | -\frac{1}{2} | (1,1,1)                             |
| $S + b_2 + e_2$ | $\zeta_a, a = 3,4$ | (1,1)       | -\frac{1}{2} | -\frac{1}{2} | (1,1,1)                             |
| $\bar{\zeta}_a, a = 3,4$ | (1,1) | -\frac{1}{2} | -\frac{1}{2} | (1,1,1) |
| $S + b_2 + e_2 + e_6$ | $\bar{f}_2$ | (17,2)                | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_2 + e_2 + e_6 + x$ | $F_3$ | (15,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\oplus$        | $\chi_3$ | (1,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_2 + e_1 + e_6$ | $\zeta_a, a = 5,6$ | (1,1)       | -\frac{1}{2} | -\frac{1}{2} | (1,1,1)                             |
| $\bar{\zeta}_a, a = 5,6$ | (1,1) | -\frac{1}{2} | -\frac{1}{2} | (1,1,1) |
| $S + b_2 + e_1$ | $\bar{f}_3$ | (17,2)                | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_2 + e_1 + x$ | $F_4$ | (15,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\oplus$        | $\chi_4$ | (1,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_3 + e_1 + e_2$ | $\zeta_a, a = 7,8$ | (1,1)       | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\bar{\zeta}_a, a = 7,8$ | (1,1) | -\frac{1}{2} | \frac{1}{2} | (1,1,1) |
| $S + b_3 + e_1 + e_2 + e_6$ | $F_5$ | (15,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\oplus$        | $\chi_5$ | (1,1)                 | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_3 + e_1 + e_2 + e_1$ | $\zeta_a, a = 9,10$ | (1,1)       | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\bar{\zeta}_a, a = 9,10$ | (1,1) | -\frac{1}{2} | \frac{1}{2} | (1,1,1) |
| $S + b_3 + e_4$ | $f_1$ | (6,2)                  | \frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_3 + e_4 + x$ | $\bar{f}_4$ | (17,2)                | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_3 + e_4 + e_1$ | $\tilde{\zeta}_{11}$ | (1,1)       | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\bar{\zeta}_{11}$ | (1,1) | -\frac{1}{2} | \frac{1}{2} | (1,1,1) |
| $S + b_3 + e_4 + e_1 + x$ | $\tilde{\zeta}_{12}$ | (1,1)       | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $S + b_3 + e_4 + e_6$ | $\zeta_{11}$ | (1,1)       | -\frac{1}{2} | \frac{1}{2} | (1,1,1)                             |
| $\bar{\zeta}_{12}$ | (1,1) | -\frac{1}{2} | \frac{1}{2} | (1,1,1) |

Table 3: Observable twisted matter spectrum.
| Sector | Field | $SU(6) \times SU(2)_R$ | $U(1)_4^\prime$ | $U(1)_A$ | $SO(4)_1 \times SO(4)_2 \times SO(8)$ |
|--------|-------|-----------------|----------------|----------|-----------------|
| $S + b_1 + x + e_4 + e_5$ | $H_{11}$ | (1, 1) | 0 | 1 | (4,1,1) |
| $S + b_1 + x + e_4 + e_5 + e_6$ | $H_{12}$ | (1, 1) | 0 | 1 | (1,4,1) |
| $S + b_1 + x + e_3 + e_4 + e_6$ | $Z_1$ | (1, 1) | 0 | 1 | (1,1,8v) |
| $S + b_2 + x + e_1 + e_2 + e_5$ | $H_{12}$ | (1, 1) | $\frac{3}{2}$ | $\frac{1}{2}$ | (4,1,1) |
| $S + b_2 + x + e_1 + e_2 + e_5 + e_6$ | $H_{22}$ | (1, 1) | $\frac{3}{2}$ | $\frac{1}{2}$ | (1,4,1) |
| $S + b_2 + x + e_5 + e_6$ | $H_{23}$ | (1, 1) | $\frac{3}{2}$ | $\frac{1}{2}$ | (4,1,1) |
| $S + b_2 + x + e_5$ | $H_{24}$ | (1, 1) | $\frac{3}{2}$ | $\frac{1}{2}$ | (1,4,1) |
| $S + b_3 + x + e_1 + e_3 + e_4$ | $Z_2$ | (1, 1) | $-\frac{3}{2}$ | $\frac{1}{2}$ | (1,1,8v) |
| $S + b_3 + x + e_3 + e_4$ | $Z_3$ | (1, 1) | $\frac{3}{2}$ | $-\frac{1}{2}$ | (1,1,8v) |
| $S + b_1 + x + z_2 + e_3 + e_4 + e_5 + e_6$ | $Z_4$ | (1, 1) | 0 | 1 | (1,1,8v) |
| $S + b_1 + x + z_2 + e_3 + e_4 + e_5$ | $Z_5$ | (1, 1) | 0 | 1 | (1,1,8v) |
| $S + b_1 + x + z_1 + e_3 + e_4 + e_5 + e_6$ | $H_{125}$ | (1, 1) | 0 | 1 | (2,2,1) |
| $S + b_1 + x + z_1 + e_3 + e_4 + e_5$ | $H_{126}$ | (1, 1) | 0 | 1 | (2,2,1) |
| $S + b_1 + x + z_1 + e_3 + e_4 + e_6$ | $H_{127}$ | (1, 1) | 0 | 1 | (2,2,1) |
| $S + b_1 + x + z_1 + e_3 + e_4$ | $H_{128}$ | (1, 1) | 0 | 1 | (2,2,1) |

Table 4: Twisted hidden matter spectrum.
The anomalous \( U(1) \) is a combination of the \( U(1)_{1,2,3} \). It is orthogonal to the combination of \( U(1)_{1,2,3} \) that is embedded in \( SU(6) \). Consistently, the family universal \( U(1) \) combination, which is embedded in \( SU(6) \), is anomaly free. This is an important distinction in comparison to the FSU5, SLM and PS string models of refs. [3], [4] and [5], respectively, in which the family universal combination of \( U(1)_{1,2,3} \) is anomalous. Therefore, in these models the family universal combination must be broken at the string scale and cannot remain unbroken down to low scales. By contrast, the LRS models [6] produce models in which all three \( U(1) \)s are anomaly free, whereas in the \( SU(6) \times SU(2) \) models the family universal combination is anomaly free by virtue of its embedding in \( SU(6) \). Of the two orthogonal combinations, given by eqs. (3.9) and (3.10), the first is anomaly free, whereas the second is anomalous.

The anomalous \( U(1)_A \) is broken by the Green–Schwarz–Dine–Seiberg–Witten mechanism [25] in which a potentially large Fayet–Iliopoulos \( D \)-term \( \xi \) is generated by the VEV of the dilaton field. Such a \( D \)-term would, in general, break supersymmetry, unless there is a direction \( \hat{\phi} = \sum \alpha_i \phi_i \) in the scalar potential for which \( \sum Q^i_\alpha |\alpha_i|^2 < 0 \) and that is \( D \)-flat with respect to all the non–anomalous gauge symmetries along with \( F \)-flat. If such a direction exists, it will acquire a VEV, cancelling the Fayet–Iliopoulos \( \xi \)-term, restoring supersymmetry and stabilising the vacuum. The \( D \)-term flatness constraints in our model, assuming zero VEVs for the hidden sector fields, are given by:

\[
U(1)_A': D_1 = -6|\chi_1|^2 + 3|\chi_2|^2 + 3|\chi_3|^2 + 3|\chi_4|^2 - 3|\chi_5|^2 + 3(|\zeta_1|^2 - |\tilde{\zeta}_1|^2) + 3(|\zeta_2|^2 - |\tilde{\zeta}_2|^2) + 3(|\zeta_3|^2 - |\tilde{\zeta}_3|^2) + 3(|\zeta_4|^2 - |\tilde{\zeta}_4|^2) + 3(|\zeta_5|^2 - |\tilde{\zeta}_5|^2) + 3(|\zeta_6|^2 - |\tilde{\zeta}_6|^2)
\]

\[
+ 3(|\zeta_7|^2 - |\tilde{\zeta}_7|^2) + 3(|\zeta_8|^2 - |\tilde{\zeta}_8|^2) + 3(|\zeta_9|^2 - |\tilde{\zeta}_9|^2) + 3(|\zeta_{10}|^2 - |\tilde{\zeta}_{10}|^2) + 3(|\zeta_{11}|^2 - |\tilde{\zeta}_{11}|^2) + 3(|\zeta_{12}|^2 - |\tilde{\zeta}_{12}|^2)
\]

(4.2)

\[
+ 6(|\Phi_{34}|^2 - |\tilde{\Phi}_{34}|^2) + 6(|\Phi_{56}|^2 - |\tilde{\Phi}_{56}|^2)
\]

\[
+ 2|\bar{f}_1|^2 - |\bar{f}_2|^2 - |\bar{f}_3|^2 + |f_1|^2 - |f_2|^2
\]

\[
+ 2|F_1|^2 - |F_2|^2 - |F_3|^2 - |F_4|^2 - |\bar{F}_4|^2 = 0,
\]

\[
U(1)_A: D_A = -3|\chi_2|^2 - 3|\chi_3|^2 - 3|\chi_4|^2 - 3|\chi_5|^2 + 2(|\zeta_1|^2 + 2|\zeta_2|^2 - 2|\tilde{\zeta}_1|^2 - 2|\tilde{\zeta}_2|^2)
\]

\[
+ (|\zeta_3|^2 - |\tilde{\zeta}_3|^2 - |\tilde{\zeta}_4|^2) + (|\zeta_5|^2 - |\tilde{\zeta}_5|^2 - |\tilde{\zeta}_6|^2 - |\tilde{\zeta}_7|^2)
\]

\[
+ (|\zeta_8|^2 - |\tilde{\zeta}_8|^2 - |\tilde{\zeta}_9|^2) + (|\zeta_{10}|^2 - |\tilde{\zeta}_{10}|^2 - |\tilde{\zeta}_{11}|^2 - |\tilde{\zeta}_{12}|^2)
\]

(4.3)

\[
+ 2(|\tilde{\Phi}_{34}|^2 - |\Phi_{34}|^2) + |\bar{f}_2|^2 + |\bar{f}_3|^2 + |f_1|^2 - |f_2|^2
\]

\[
+ |F_2|^2 + |F_3|^2 - |F_4|^2 - |\bar{F}_4|^2 + \frac{3g_{\text{string}}^2 M^2}{8\pi^2} = 0.
\]
In eq. (4.3) $g$ is the gauge coupling in the effective field theory. The set of $F$–flatness constraints is obtained by requiring

$$\left\langle \frac{\partial W}{\partial \eta_i} \right\rangle = 0, \quad \forall \eta_i.$$ (4.4)

Let us briefly discuss the phenomenology of our model. The model’s spectrum consists of $7 \times (15,1) + 4 \times (\overline{15},1)$, namely $F_1, F_2, F_3, F_4, F_5, F_6, F_7, \tilde{F}_4, \tilde{F}_5, \tilde{F}_6, \tilde{F}_7$, and $4 \times (6,2) + (6,2)$, namely $\tilde{f}_1, \tilde{f}_2, \tilde{f}_3, \tilde{f}_4, f_1$ and a number of $SU(6) \times SU(2)$ singlets.

An important issue is that of the generation mass hierarchy. In $SU(6) \times SU(2)$ models all fermions in a generation receive masses from a single invariant in the superpotential. Fermion mass hierarchy requires that only the heavy generation obtains mass term at leading order, whereas the mass terms of the lighter generations are generated at higher orders. As mentioned earlier there are two possible embeddings the electroweak $SU(2)_L$ gauge symmetry in $SU(6) \times SU(2)$. Both embeddings can be realised in our string model. Considering the $SU(6) \times SU(2)_R$ embedding of (2.10)-(2.9) the breaking to the Standard Model gauge group can be realised with a set of $(15,1) + (\overline{15},1)$ and $(6,2) + (\overline{6},2)$ Higgs multiplets. The remaining three $(15,1) + (\overline{15},1)$ multiplets may become massive from cubic mass terms in the superpotential (4.1). There are three candidate fermion generation mass couplings in the superpotential $\tilde{f}_1 \tilde{f}_2 F_4 + f_2 \tilde{f}_4 F_1 + \tilde{f}_4 f_1 F_3$. Assuming $\tilde{f}_1, f_1$ to play the role of one of the heavy Higgs sets, this requires $\zeta_{12} = 0$. We are left with a single fermion generation mass term at tri-level $f_2 \tilde{f}_4 F_1$. This is compatible with the requirement that only one generation becomes massive at tri-level. Masses for the additional $d$–quark like triplets, accommodated in $(15,1)$ and $(\overline{15},1)$ may arise from couplings of the form $F_H^L F_H^L H$. In the case of $SU(6) \times SU(2)_L$ embedding of (2.4)-(2.3), the breaking to the Standard Model gauge group is realized by two sets of $(15,1) + (\overline{15},1)$ Higgs multiplets. The additional two sets of $(15,1) + (\overline{15},1)$ may receive masses from tri-level superpotential terms while the extra $(6,2) + (\overline{6},2)$ set can be removed by assigning a VEV to $\zeta_{12}$ which provides mass to $\tilde{f}_1, f_1$. A single fermion family mass coupling is obtained also in this scenario namely $f_2 \tilde{f}_4 F_1$, consistent with the above mentioned phenomenological requirements. Additional triplets may also acquire heavy masses through superpotential couplings of the type $F_H^L F_H^L H$.

As the string model contains several pairs of electroweak Higgs doublets, we need to ensure that at least one light pair survives in order to provide masses to the chiral fermions. The electroweak Higgs doublets are accommodated in the $(6,2) + (\overline{6},2)$ representations of $SU(6) \times SU(2)$, and correspond to the fields $\tilde{f}_1, \tilde{f}_2, \tilde{f}_3, \tilde{f}_4$ and $f_1$. 15
The Higgs doublets mass matrix is given by:

\[
M_f = \begin{pmatrix}
    F_5 & F_4 & 0 & \tilde{\zeta}_{12} & F_3 \\
    F_4 & F_6 & 0 & 0 & 0 \\
    0 & 0 & F_6 & 0 & F_1 \\
    \tilde{\zeta}_{12} & 0 & 0 & F_7 & 0 \\
    F_3 & 0 & F_1 & 0 & F_7
\end{pmatrix}.
\] (4.5)

The requirement of existence of light Higgs doublets dictates the vanishing of the determinant of the Higgs mass matrix, i.e.

\[
\det(M_f) = [(F_5 F_6 - F_4 F_3^2) F_7 - \tilde{\zeta}_{12}^2 F_6](F_6 F_7 - F_1^2) - F_3 F_6^2 F_7 = 0.
\] (4.6)

We now discuss an example for the flatness conditions, taking the field assignment in eqs. (2.9) and (2.10), for $SU(6) \times SU(2)_R$ scenario. In the solution that we discuss here we will assume that the $SU(6) \times SU(2)$ symmetry is broken to the Standard Model at high scale. The scenario with a light extra $U(1)$ requires first a more detailed effective field theory analysis of the $SU(6) \times SU(2)$ GUT model.

The breaking of $SU(6) \times SU(2)$ is obtained by choosing the pair of $(15, 1) + (\overline{15}, 1)$, for example, $F_3$ and $\bar{F}_4$. The condition that $F_6 \bar{F}_7 = 0$ ensures that these fields are massless. To break the Pati-Salam group, we choose a pair of $(6, 2) + (\overline{6}, 2)$, for example $\bar{f}_2$ and $f_1$. The minimal choice

\[
F_3 = \bar{F}_1 \\
\bar{f}_2 = f_1
\] (4.7)

\[
|\Phi_{12}|^2 = +\frac{1}{2}|\bar{f}_2|^2 + \frac{1}{2}|F_3|^2 + \frac{3 g_{\text{string}}^2 M_P^2}{32\pi^2}
\] (4.9)

with all the other field VEVs vanishing, satisfies all F and D-flatness conditions.

5 Conclusions

In this paper we studied the construction of heterotic–string models with $SU(6) \times SU(2)$ GUT group. As a GUT model this case as been scarcely studied in the literature. As a string model it possesses some interesting properties. In particular it gives rise to an additional anomaly free flavour universal $U(1)$ symmetry. While string models generically produce additional $U(1)$ symmetries, it is often found that the flavour universal combinations, beyond the $SU(10)$ group that gives rise to the Standard Model subgroup, are anomalous. Such anomalous $U(1)$ symmetries must be broken at the string scale, and cannot remain unbroken down to lower scales. It is therefore of further interest to explore in detail the phenomenology of the $SU(6) \times$
SU(2) GUT models, and to study whether the additional $U(1)$ can indeed remain unbroken down to low scales while satisfying all other phenomenological constraints. The motivation to keeping the $Z'$ unbroken down to low scales stems from his possible role in suppressing proton decay mediating operators, as well as that of the $\mu$-term. In this regard, we remark that while substantial amount of phenomenological studies in the literature are devoted to string inspired $Z'$ models, the viability of a low scale $Z'$ in a string derived model is yet to be demonstrated.

In this paper we constructed a three generation heterotic–string with $SU(6) \times SU(2)$ GUT group. We used our recently developed fishing algorithm to obtain a model with pre–conditioned phenomenological properties. By using the free fermionic formalism for the construction of string compactifications, we generate a large space of vacua and use statistical sampling to extract a model with the desired characteristics\footnote{We note that analysis of large sets of string vacua has also been carried out by other groups [26].}. The model contains the heavy and light Higgs representations needed to break the gauge symmetry down to the Standard Model and to generate fermion masses. We extracted the cubic level superpotential, which is compatible with all the symmetries and the string selection rules. By analysing the superpotential we showed that there exist supersymmetric $F$– and $D$–flat directions that produce a fermion mass term at leading order for a single family. The mass terms of the lighter families, which must then arise from higher order terms in the superpotential, are naturally suppressed relative to the heavy family mass term. Additionally, due to pre–imposed constraints in our statistical sampling procedure, our model is free of all massless exotic representations that carry fractional electric charge. The work reported in this paper therefore enlarges the space of phenomenologically viable string vacua to a new class of models.

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