Another Type of Weakly Closed sets on Semi $\alpha$-Regular

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http://dx.doi.org/10.22147/jusps-A/290903

Acceptance Date 11th August, 2017,   Online Publication Date 2nd September, 2017

Abstract

In this paper we introduce and study the new class of closed sets called semi-$\alpha$-regular weakly closed (briefly $s\alpha rw$-closed) set in the topological spaces. This new class of set lies between the class of $\alpha$-closed sets and the class of generalised semi closed sets. We study the fundamental properties of this class of sets.

Key words: Semi-closed set, $\alpha$-closed sets, $rw$-closed, $arw$-closed set, gs-closed set, $s\alpha rw$-closed set and topological space.

2010 Mathematics Classification: 54A05, 54A10

1. Introduction

In 1970 Levine\textsuperscript{10}, first introduced the concept of generalized closed (briefly g-closed) sets were defined and investigated. Regular open sets and rw-open sets have been introduced and investigated by Stone\textsuperscript{16} and Benchalli\textsuperscript{3} respectively. Levine\textsuperscript{10,11}, Sundaram and Sheik john\textsuperscript{17} and many mathematicians have been, introduced and investigated semi open sets, generalized closed sets, regular semi open sets, $o$-closed sets, semi generalized closed sets, weakly generalized closed sets, strongly generalized closed sets, generalized pre-regular closed sets, regular generalized closed sets, and generalized $\alpha$-generalized closed sets respectively. Maki \textit{et.al.}\textsuperscript{12,13} introduced and studied generalized $\alpha$-closed sets and $\alpha$-generalized closed sets. S.S. Benchalli \textit{et.al.}\textsuperscript{3} studied $o\alpha$-closed sets in topological spaces. Recently, R. S. Wali and Mendalgeri\textsuperscript{20,22} introduced and studied the concepts of $\alpha$-regular w-closed (briefly $arw$-closed) sets in topological spaces. In this paper we define new generalization of closed set called Semi $\alpha$ regular weakly closed (briefly $s\alpha rw$-closed) set which lies between $\alpha$-closed set and gs-closed set. Also we study their fundamental properties.

2. Preliminaries:

\textit{Definition 2.1 :} A subset $A$ of $X$, is called \textbf{semi-open} set\textsuperscript{11} if $A \subseteq \text{cl}(\text{int}(A))$ and \textbf{semi-closed set} if
A subset A of X, is called pre-open set if $A \subseteq \text{int} \,(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.

Definition 2.2: A subset A of X, is called $\alpha$-open set if $A \subseteq \text{int} \,(\text{cl}(A))$ and $\alpha$ -closed set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

Definition 2.4: A subset A of X, is called semi-pre open set if $\beta \text{-open}[1]$ if $A \subseteq \text{cl} \,(\text{int}(\text{cl}(A)))$ and a semi-pre closed set if $\beta \text{-closed})$ if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

Definition 2.5: A subset A of X, is called regular open set if $A = \text{int}(\text{cl}(A))$ and a regular closed set if $A = \text{cl}(\text{int}(A))$.

Definition 2.7: A subset A of X, is called Regular semi open set if there is a regular open set $U$ such that $U \subseteq A \subseteq \text{cl}(U)$.

Definition 2.8: A subset A of a topological space $(X, \tau)$ is called 1. $\alpha$-generalized closed set (briefly $\alpha$-g-closed) if $\alpha \text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in X.
2. Regular generalized closed set (briefly rg-closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in X.
3. Generalized semi-pre closed set (briefly gsp-closed) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in X.
4. $\omega\alpha$- closed set if $\alpha \text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\omega$-open in X.
5. Generalized $\omega\alpha$-closed set (briefly $\alpha$rw-closed) set if $\alpha \text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega\alpha$-open in X.
6. Generalized regular closed (briefly gr-closed) set if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in X.
7. $\alpha$-regular weakly closed (briefly $\alpha$rw-closed )set if $\alpha \text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\alpha$-rw-open in X.

The complement of the above mentioned closed sets are their open sets.

3. weakly-closed sets in terms of Semi $\alpha$-regular :

Definition 3.1: A subset A of X is called a semi $\alpha$- regular weakly closed set (briefly $\text{srw}$-closed set) if $\text{Scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in X. We denote the collection of all $\text{srw}$-closed sets in X by $\text{srw}(X)$.

Theorem 3.2: Every $\alpha$-closed set in X is $\text{srw}$-closed set but converse is not true.

Proof: Let A be $\alpha$-closed set in X. Let U be any $\alpha$-open set in X s.t $A \subseteq U$. Since A is $\alpha$-closed, we have $\text{Scl}(A) = A \subseteq U$, and $\text{Scl}(A) \subseteq U$. Hence A is $\text{srw}$-closed set in X.

Example 3.3: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then the set $A = \{b, d\}$ is $\text{srw}$-closed set but not $\alpha$-closed in X.

Theorem 3.4: Every $\text{srw}$-closed set is $\text{gs}$-closed set in X but converse is not true.

Proof: Let A be $\text{srw}$ -closed set in X. Let U be any open set in X s.t $A \subseteq U$. Since every open set is $\omega\alpha$-open set and A is $\text{srw}$-closed set, we have $\text{Scl}(A) \subseteq U$, $\text{Scl}(A) \subseteq U$, U is open in X. Hence A is $\text{gs}$-closed set in X.

Example 3.5: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then the set $A = \{b\}$ is $\text{gs}$ closed set but not $\text{srw}$ -closed set in X.

Remark 3.6: The Union of two $\text{srw}$ - closed subsets of X need not be $\text{srw}$ -closed set in X.

Example 3.7: Let $X = \{a, b, c, d\}$ be with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{a\}$ and $B = \{b, c\}$ be two $\text{srw}$ -closed subsets of X. But $A \cup B = \{a, b, c\}$ which is not contained in $\text{srw}$ -closed set in X. Hence union of two $\text{srw}$ -closed sets is not $\text{srw}$ -closed set in X.

Remark 3.8: The intersection of two $\text{srw}$ -closed sets in X is generally not an $\text{srw}$ -closed set in X.
Example 3.9: Let $X = \{a, b, c, d\}$ be with topology $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{b, c\}$ and $B = \{b, d\}$ be two $srw$-closed subsets of $X$. But $A \cap B = \{b\}$ which is not contained in $srw$-closed set in $X$. Hence intersection of two $srw$-closed sets is not $srw$-closed set in $X$.

Theorem 3.10: If a subset $A$ of topological space $X$ is an $srw$-closed set in $X$ then $S\text{Cl}(A) - A$ does not contain any non empty $r_\omega$-closed set in $X$.

Proof: Let $A$ be $srw$-closed set in $X$ and suppose $F$ be a non empty $r_\omega$-closed subset of $S\text{Cl}(A) - A$. $F \subseteq S\text{Cl}(A) - A \Rightarrow F \subseteq S\text{Cl}(A) \cap (X - A) \Rightarrow F \subseteq X - F$ and $X - F$ is $ro$-open set and $A$ is an $srw$-closed set, $S\text{Cl}(A) \subseteq X - F \Rightarrow F \subseteq X - S\text{Cl}(A)$ -- (1) & $F \subseteq X - A \Rightarrow A \subseteq X - F$ and $X - F$ is $ro$-open set in $X$. However the converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ then the set $A = \{b, c\}$ does not contain non empty $r_\omega$-closed set in $X$, but $A$ is not $srw$-closed set in $X$.

4. Characterisation of is $srw$-closed set:

Theorem 4.1: If $A$ is regular open and $agr$-closed then $A$ is $srw$-closed set in $X$.

Proof: Let $A$ be regular open and $agr$-closed in $X$, Let $U$ be any $ro$-open set in $X$ s.t. $A \subseteq U$ Since $A$ is regular open and $agr$-closed in $X$, by definition, $S\text{Cl}(A) \subseteq A$ then $S\text{Cl}(A) \subseteq A \subseteq U$ Hence $A$ is $srw$-closed set in $X$.

Remark 4.2: If $A$ is both regular open and $srw$-closed, then $A$ need not be $agr$-closed in general as seen from the following example.

Example 4.3: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{b, c\}$ is both regular open and $srw$-closed but not $agr$ closed in $X$.

Theorem 4.4: If $A$ is $\omega$-open and $\omega\text{ag}$-closed then $A$ is $srw$-closed set in $X$.

Proof: Let $A$ be $\omega$-open and $\omega\text{ag}$-closed in $X$, Let $U$ be any $ro$-open set in $X$ s.t. $A \subseteq U$. Since $A$ is $\omega$-open and $\omega\text{ag}$-closed in $X$, by definition, $S\text{Cl}(A) \subseteq A$ then $S\text{Cl}(A) \subseteq A \subseteq U$. Hence $A$ is $srw$-closed set in $X$.

Remark 4.5: If $A$ is both $\omega$-open and $srw$-closed, then $A$ need not be $\omega\text{ag}$-closed in general, as seen from the following example.

Example 4.6: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{a\}$ is both $\omega$-open and $srw$-closed but not $\omega\text{ag}$-closed in $X$.

Theorem 4.7: If $A$ is open and $\alpha\text{g}$-closed then $A$ is $srw$-closed set in $X$.

Proof: Let $A$ be open and $\alpha\text{g}$-closed in $X$, Let $U$ be any $ro$-open set in $X$ s.t. $A \subseteq U$. Since $A$ is open and $\alpha\text{g}$-closed in $X$, by definition, $\text{Cl}(\text{int}(A)) \subseteq A$ then $\text{Cl}(\text{int}(\text{Cl}(A))) \subseteq A \subseteq U$. Hence $A$ is $srw$-closed set in $X$.

Remark 4.8: If $A$ is both open and $srw$-closed, then $A$ need not be $\alpha\text{g}$-closed in general, as seen from the following example.

Example 4.9: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{b, c\}$ is both open and $srw$-closed but not $\alpha\text{g}$-closed in $X$.

Theorem 4.10: If $A$ is regular open and $\text{rgw}$-closed then $A$ is $srw$-closed set in $X$.

Proof: Let $A$ be regular open and $\text{rgw}$-closed in $X$, Let $U$ be any $ro$-open set in $X$ s.t. $A \subseteq U$. Since $A$ is regular open and $\text{rgw}$-closed in $X$, by definition, $\text{Cl}(\text{int}(A)) \subseteq A$ then $\text{Cl}(\text{int}(\text{Cl}(A))) \subseteq A \subset U$. Hence $A$ is $srw$-closed set in $X$.

Remark 4.11: If $A$ is both regular open and $srw$-closed, then $A$ need not be $\text{rgw}$-closed in general, as seen from the following example.

Example 4.12: Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let $A = \{a\}$ is both regular open and $srw$-closed but not $\text{rgw}$-closed in $X$. 
Remark 4.13: If A is both open and s\alpha rw -closed, then A need not be wg - closed in general, as seen from the following example.

Example 4.14: Let X={a, b, c, d} with the topology \( \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\} \). Let A={b,c} is both open and s\alpha rw-closed but not wg-closed in X.

Remark 4.15: If A is both open and s\alpha rw -closed, then A need not be gp - closed in general, as seen from the following example.

Example 4.16: Let X={a, b, c, d} with the topology \( \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\} \). Let A={b, c} is both open and s\alpha rw-closed but not gp-closed in X.

Theorem 4.17: If a subset A of a topological space X in both regular open and s\alpha rw -closed then it is \alpha-closed.

Proof: Suppose a subset A of a topological space X is regular open and s\alpha rw-closed, As every regular open is ro-open. Now A \subseteq A then definition of s\alpha rw-closed, SCl(A) \subseteq A and also A \subseteq SCl(A) then SCl(A)=A. Hence A is \alpha-closed.

Theorem 4.18: If a subset A of a topological space X is both regular semi open and gprw-closed then it is s\alpha rw-closed.

Proof: Let A be an regular semi open and gprw-closed set in X. Let A \subseteq U and U be ro-open in X. Now A \subseteq A by hypothesis pcl(A) \subseteq A then we know that pcl(A) \subseteq Scl(A) \subseteq A, hence Scl(A) \subseteq U therefore A is s\alpha rw-closed set in X.

Remark 4.19: If A is both regular semi open and s\alpha rw -closed, then A need not be gprw - closed in general, as seen from the following example.

Example 4.20: Let X={a, b, c, d} with the topology \( \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\} \). Let A={b,c} is both regular semi open and s\alpha rw-closed but not gprw-closed in X.

Theorem 4.21: If a subset A of a topological space X is both regular semi open and rgw-closed then it is s\alpha rw-closed.

Proof: Let A be an regular semi open and rgw-closed set in X. Let A \subseteq U and U be ro-open in X. Now A \subseteq A by hypothesis cl(int(A)) \subseteq A then we know that cl(int(A)) \subseteq cl(int(cl(A))) \subseteq A, hence Scl(A) \subseteq U therefore A is s\alpha rw-closed set in X.

Remark 4.22: If A is both regular semi open and s\alpha rw-closed, then A need not be rgw - closed in general, as seen from the following example.

Example 4.23: Let X={a, b, c, d} with the topology \( \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\} \). Let A={b,c} is both regular semi open and s\alpha rw-closed but not rgw-closed in X.

Conclusion

In the present work, a new class of sets called s\alpha rw -Closed sets in Topological spaces is introduced and some of their properties are studied. This new class of sets widens the scope to do further research in the areas like Bitopological Spaces, Soft topological spaces and Fuzzy Topological Spaces.

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