Effects of volume corrections and resonance decays on cumulants of net-charge distributions in a Monte Carlo hadron resonance gas model

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Abstract

The effects of volume corrections and resonance decays (the resulting correlations between positive charges and negative charges) on cumulants of net-proton distributions and net-charge distributions are investigated by using a Monte Carlo hadron resonance gas (MCHRG) model. The required volume distributions are generated by a Monte Carlo Glauber (MCG1b) model. Except the variances of net-charge distributions, the MCHRG model with more realistic simulations of volume corrections, resonance decays and acceptance cuts can reasonably explain the data of cumulants of net-proton distributions and net-charge distributions reported by the STAR collaboration. The MCHRG calculations indicate that both the volume corrections and resonance decays make the cumulant products of net-charge distributions deviate from the Skellam expectations: the deviations of $S\sigma$ and $K\sigma^2$ are dominated by the former effect while the deviations of $\omega$ are dominated by the latter one.

1. Introduction

Recently, tremendous experimental and theoretical efforts have been made to determine the phase diagram of Quantum Chromodynamics (QCD) \[1,2\]. Among various observables, the cumulants of net-charge distributions and net-proton distributions, measured by beam energy scan (BES) program from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) with a wide range of collisional energies from $\sqrt{s_{NN}} = 7.7$ GeV to $\sqrt{s_{NN}} = 200$ GeV \[3,4\], that are expected to provide us some crucial information about the critical end point (CEP) of QCD phase diagram. Besides the fluctuation data and theoretical studies on critical fluctuations \[4,5\], the theoretical baselines from non-critical statistical fluctuation studies are also important \[6,7\].

The hadron resonance gas (HRG) models have been employed to investigate the theoretical baselines of multiplicity distributions in various previous studies \[8,9\]. Especially in Ref. \[10\], Nahrgang and her collaborators split the resonance decay contributions into two parts: an average part and a probabilistic part, but ignore the correlations generated from resonance decays. This method is appropriate for the net-proton distributions discussed in Ref. \[10\], because no correlation between baryons and anti-baryons can be generated from resonance decays \[10\]. However, the correlations between positive charges and negative charges should be taken into account in study of the net-charge fluctuations, because lots of positive-negative charge pairs are generated from resonance decays.

For the data reported by the STAR collaboration \[11\], on the other hand, the volume corrections play significant role on the cumulants (cumulant products) of net-charge distributions \[12,13\]. Therefore, besides the information of chemical potential and temperature investigated in previous studies, the volume information is also important for theorectic baseline studies \[10\]. Meanwhile, with the volume information, one can study the centrality dependence of multiplicity distributions in relativistic heavy ion collisions.

In this work, with the volume information generated by a Monte Carlo Glauber (MCG1b) model, I propose a Monte Carlo hadron resonance gas (MCHRG) model to study the effect of volume corrections and resonance decays on the cumulants of net-proton distributions and net-charge distributions in a transparent way. Instead of primordial particles used in previous studies, the acceptance cuts can be applied to the decay products in the MCHRG simulations. Based on these advantages, the MCHRG model can be used to give more realistic baselines for the cumulants of multiplicity distributions than previous HRG models.

2. Monte Carlo hadron resonance gas model

In the MCHRG model, the multiplicities of different particle species in each event are randomly generated by Poisson distributions, and the Poisson parameters are calculated by \[10\]:

$$\lambda_i = V \frac{T_{ch} g_i m_i^2}{(2\pi)^2} \exp \left( \frac{\mu_i}{T_{ch}} \right) K_0 \left( \frac{m_i}{T_{ch}} \right),$$  \hspace{1cm} (1)$$

where $g_i$ is degeneracy factor, $m_i$ is particle mass, $T_{ch}$ is chemical freeze-out temperature. The chemical potential $\mu_i = B \mu_B + S \mu_S + Q \mu_Q$ with the baryon number $B_i$, strangeness number $S_i$, charge number $Q_i$ and the corresponding chemical potentials $\mu_B$, $\mu_S$, $\mu_Q$. $K$ is modified Bessel function. Note that I have neglected the effects of quantum statistics on multiplicity fluctuations. To study the effect of resonance decays, I use 319 primordial particle species as inputs and 26 stable particle species after performing Monte Carlo resonance decays, as the particle species...
used in [29]. The resonance decay channels are taken from [33] and the contributions from weak decays are not taken into account in the present study.

For more realistic simulations of acceptance cuts, the transverse momentum ($p_T$) spectra of primordial particles are simulated by the blast-wave model [34] of reference multiplicity if the data becomes available [20]. Since I do not focus on the details of strangeness fluctuations, the chemical freeze-out parameter $m_S$ is just obtained from $m_S = d_S/(1 + e_S \sqrt{SN})$ with $d_S = 0.214$ and $e_S = 0.161$ [33]. The chemical freeze-out parameter $m_Q$ is determined by the multiplicity ratio of positive charges and negative charges. The mean multiplicity of net-charges is the remainder of two large values, i.e. mean multiplicity of positive and negative charges, which are two orders of magnitude larger than the remainder. To give a good description of centrality dependence of mean multiplicity of net-charges, therefore, a centrality-dependent $\mu_Q$ is parameterized as

$$\mu_Q = \mu_{Q0} + a_Q \tanh \left[ 0.4(\sqrt{p_T} - 8) \right],$$

with $\mu_{Q0} = -4.7$ MeV and $a_Q = 1.5$ MeV. Besides $\mu_Q$, in general, all the chemical and kinetic freeze-out parameters are centrality-dependent for more precise constraint. For simplicity in this work, all of them except $\mu_Q$ are set to constants to roughly reproduce the mean multiplicity of positive charges, negative charges, net-charges [3], protons, anti-protons and net-protons [10] with some specific acceptance cuts. Note that the main conclusions obtained in the present study are almost independent of the selection of collision energy and model parameters, and the impact of Glauber parameters on cumulant calculations has been investigated in my previous study [20].

I apply the same acceptance cuts as used in experiment [3, 9]. More specifically, $|y| < 0.5, 0.2 < p_T < 2.0$ GeV for fluctuation measures ($\pi^\pm$, $K^\mp$ and $p/\pi$ after removing protons and anti-protons with $p_T < 0.4$ GeV) and $1.0 < |y| < 0.5, 0.2 < p_T < 2.0$ GeV for reference particles (total charged hadrons) in the net-charge case; $|y| < 0.5, 0.4 < p_T < 0.8$ GeV for fluctuation measures ($p/\pi$) and $|y| < 1.0, 0.2 < p_T < 2.0$ GeV for reference particles ($\pi^\pm$ and $K^\mp$) in the net-proton case. Here $\eta$ is pseudo-rapidity.

3. Results and Discussions

Figure 1 shows the centrality-dependent cumulants of net-proton distributions and net-charge distributions for Au+Au collisions at $\sqrt{s_{NN}} = 39$ GeV. To study the effect of volume corrections and resonance decays on cumulants of multiplicity distributions, some other baselines are also shown in the figures: one is the Skellam baseline, which is obtained by

$$c_1^\pm = c_3^\pm = c_4^\pm = c_5^\pm,$$

the other one is the independent product (IP) baseline, which is obtained by

$$c_n^{IP} = c_n^+ + (-1)^n c_n^-.$$

Here $c_n^+$ and $c_n^-$ are the cumulants of proton (positive charge) distributions and anti-proton (negative charge) distributions in the net-proton (net-charge) case calculated by the MCChRG model. According to the Central Limit Theorem (CLT), the cumulants

1The mean multiplicity of positive charges ($M_+$) and negative charges ($M_-$) are extracted from the data of mean multiplicity of net-charges $M_{Q0}$ and the Skellam baselines of $S_\sigma$ reported by the STAR collaboration [11], by using the relations $M_Q = M_+ - M_-$. and $(S_\sigma)_{Q0} = (M_+ - M_-)/(M_+ + M_-)$. For more precise constraint, the parameters $x$ and $h$ can be determined by the distribution of reference multiplicity if the data becomes available [34, 33].
The reasons are twofold: First, in the absence of critical fluctuations, the volume corrections on cumulants of net-proton distributions at $\sqrt{s_{NN}} = 39\text{GeV}$ can be neglected. Second, the resonance decay processes make no contributions to the correlation between protons and anti-protons [25]. Therefore, for the study of net-proton distributions, the MCHRG model and the HRG model established in Ref. [26] are almost the same. Note that the MCHRG model give more realistic simulations of acceptance cuts for the fluctuation measures, but it is more computational expansive due to the high statistics in Monte Carlo simulations. In general the MCHRG model can reasonably reproduce the data of net-proton distributions, but, for more precise predictions, more non-critical effects need to be investigated.

The situation is very different for the net-charge fluctuations shown in Fig. 1(e-h). The volume corrections play a significant role for the cumulants of net-charge distributions reported by the STAR collaboration. As I have explained in Ref. [18,20], such difference comes from the different magnitude of reduced cumulants $c_n \propto \langle n_{\text{part}} \rangle$, $\sigma \propto \sqrt{\langle n_{\text{part}} \rangle}$, $S \propto 1/\sqrt{\langle n_{\text{part}} \rangle}$ and $\kappa \propto 1/\langle n_{\text{part}} \rangle$. The CLT baselines are shown in Fig. 1 with red-solid curves, which follow the general trend of the MCHRG baselines and experimental data.

The MCHRG baselines, Skellam baselines and IP baselines are almost the same for the first four cumulants of net-proton distributions shown in Fig. 1(a-d). The reasons are twofold: First, in the absence of critical fluctuations, the volume corrections on cumulants of net-proton distributions at $\sqrt{s_{NN}} = 39\text{GeV}$ can be neglected. Second, the resonance decay processes make no contributions to the correlation between protons and anti-protons [25]. Therefore, for the study of net-proton distributions, the MCHRG model and the HRG model established in Ref. [26] are almost the same. Note that the MCHRG model give more realistic simulations of acceptance cuts for the fluctuation measures, but it is more computational expansive due to the high statistics in Monte Carlo simulations. In general the MCHRG model can reasonably reproduce the data of net-proton distributions, but, for more precise predictions, more non-critical effects need to be investigated.

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corrections, resonance decays, and more realistic simulations of acceptance cuts, the MCHRG model can reasonably reproduce the skewness and kurtosis of net-charge distributions reported by the STAR collaboration [3], but show obvious deviations for the variances. The results indicate that the correlations between positive and negative charges from other sources are required to quantitatively reproduce the data of variances of net-charge distributions, which are beyond the scope of the MCHRG model proposed in this work. The skewness and kurtosis of net-charge distributions seem insensitive to these correlations and its deviations from Skellam baselines are dominated by the effect of volume corrections.

To explore in depth the effects of volume corrections and resonance decays on the cumulants of net-charge distributions individually, I further calculate the cumulant products

\[ \omega = c_2/c_1, \quad S = c_3/c_2, \quad \kappa = c_4/c_2 \]  

in three different cases: (1) MCHRG simulations with MC-Glb volume distributions discussed before. (2) Similar to case (1) but for the primordial particles before resonance decays, (3) MCHRG simulations in a fixed volume with volume \( V = 1540 \text{fm}^3 \) and \( \mu_Q = -4.7 \text{MeV} \). Obviously, case (2) only include the effect of volume corrections, while case (3) only include the effect of resonance decays and the latter cumulants are independent of centrality.

The results are shown in Fig. 3. The volume corrections on \( \omega \) of net-charge distributions can be neglected [14], though the volume corrections on \( \omega \) of positive charges and negative charges are important [15, 20]. This is because the mean multiplicity of net-charge distribution is much smaller than the reference multiplicity, though the magnitudes of mean multiplicity of positive and negative charges are the same order of reference multiplicity. The volume corrections play significant role for \( S \) and \( \kappa \), which make their values deviate far away from the Skellam predictions. The resonance decays make \( \omega \) of net-charge distributions smaller than the Skellam predictions, but it make \( S \) and \( \kappa \) larger than the Skellam predictions. From the MCHRG calculations, I find that the deviations of \( \omega \) from Skellam distributions are mainly due to the effect of resonance decays, while the deviations of \( S \) and \( \kappa \) are mainly due to effect of volume corrections.

For the effect of resonance decays, the multi-charged hadrons play special role in study of net-charge distribution [21]. To identify the effect of multi-charged hadrons on net-charge distributions through the resonance decay process, I then calculate the cumulant products of net-charge distributions in case (3) without the decay channels of multi-charged hadrons. The results are shown in red-dotted curves of Fig. 3. The resonance decays of multi-charged hadron enhance the fluctuations of net-charges, as it have been investigated in Ref. [16]. Comparison the results with (black-dashed lines) and without (red-dotted lines) multi-charged hadrons in Fig. 3. I find that the effect of multi-charged hadrons can be neglected for \( \omega \) and \( \kappa \) of net-charge distributions, while they make substantial contributions to \( S \) of net-charge distributions.

The deviations of \( \omega, S \) and \( \kappa \) from data are mainly due to the fact that the MCHRG model fail to quantitatively reproduce the variances of net-charge distributions (see Fig. 3(c)). The results imply that, for more realistic baselines predictions of the first four cumulants of net-charge distributions, some other effects are expected to make relevant contributions to the variances, without significant affecting its skewness and kurtosis.

4. Conclusions

I investigated the cumulants of net charge distributions and net-proton distributions within a Monte Carlo hadron resonance

![Figure 2](Color online) Centrality-dependence of cumulant products \( \omega, S \) and \( \kappa \) of net-charge distributions from the full MCHRG+MC-Glb simulations (blue solid squares) MCHRG+MC-Glb simulations without resonance decays (blue open squares) and MCHRG simulations in a fixed volume with (black dashed lines) and without (red-dotted lines) multi-charged hadrons. The data (red open stars) are taken from [3].
gas (MCHRG) model. To study the centrality dependence of multiplicity distributions, as well as the effect of volume corrections on its cumulants, the volume distributions are generated by a Monte Carlo Glauber (MC-G1b) model. For the net-proton distributions, even with more realistic simulations of acceptance cuts and volume corrections, the MCHRG calculations are consistent with the semi-analytical calculations given in Ref. [12]. However, both the effect of volume corrections, resonance decays, as well as the resulting correlations between positive charges and negative charges, that are important for the cumulants of net-charge distributions. With these effects, the MCHRG calculations provide more realistic baseline predictions for the cumulants of net-charge distributions than previous HRG studies.

Except the variances of net-charge distributions, the MCHRG model can reasonably explain the cumulants of net-proton distributions and net-charge distributions reported by the STAR collaboration. To explore in depth the effect of volume corrections and resonance decays on the cumulants of net-charge distributions individually, I also calculated the cumulant products of net-charge distributions with primordial particles, as well as the cumulant products of net-charge distributions in a fixed volume with and without multi-charged hadrons. The deviations of $\omega$ of net-charge distributions from Skellam expectations are mainly due to the effect of resonance decays, while the deviations of $S\sigma$ and $k\sigma^2$ are mainly due to the effect of volume corrections.

Note that, for more realistic baseline predictions in the future, more additional effects beyond the non-interacting MCHRG model used in this work are in order, e.g., the correlations between positive charges (protons) and negative charges (anti-protons) from the charge (baryon) conservation laws [13], the dynamic evolution in hadronic phase [14], etc. The results in the present study imply that these effects are expected to make substantial contributions to the variances of net-charge distributions, without significant affecting its skewness and kurtosis.

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