Non-universal Soft SUSY Breaking and Dark Matter

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A brief review of non-universalities in the soft SUSY breaking sector of supergravity grand unification is given. The effects of these non-universalities on the analysis of event rates in neutralino nucleus scattering are discussed. An analysis is also given of the implications of the simultaneous imposition of proton stability and dark matter constraints. One finds that these constraints put an upper limit on the gluino mass.

1 Introduction

Supergravity unified models depend typically on three arbitrary functions: the gauge kinetic energy function $f_{\alpha\beta}$, the Kahler potential, and the superpotential. The minimal supergravity model is based on a flat Kahler potential, and on a gauge kinetic energy which is independent of the heavy fields. These assumptions lead to universal soft supersymmetry breaking parameters. Under the constraints of radiative breaking of the electro-weak symmetry one may choose these parameters to be the universal scalar mass $m_0$, the universal gaugino mass $m_{1/2}$, the universal trilinear coupling $A_0$, and $\text{tan}\beta = v_2/v_1$, where $v_2$ gives mass to the up quarks, and $v_1$ gives mass to the down quarks. However, in general the Kahler potential need not be flat, and the presence of the non-flat Kahler will lead to non-universalities in the scalar sector. Similarly, the universality of the gaugino masses can be broken by the Planck scale corrections. In the following we will focus on the non-universalities in the scalar sector. Here in general one can expand the Kahler potential in terms of the visible fields so that

$$K = \kappa^2 K_0 + K_0^a Q_a Q^b + (K^{ab} Q_a Q_b + h.c.) + .. \quad (1)$$

where $Q_a$ are the fields of the visible sector, $Q^a = Q_a^\dagger$, and the quantities $K_0, K_0^a, ..$ etc depend on the fields in the hidden sector. As mentioned already in minimal supergravity unification one makes the assumption of a flat
Kahler, i.e., $K^a = K(h, h^\dagger)\delta^a$ (where $h$ are the hidden sector fields) and vanishing non-holomorphic terms beyond the quadratic order. This assumption then leads to universal soft SUSY breaking at the SUSY breaking scale. However, due to dynamics at the Planck scale one will have in general a non-flat Kahler potential which will lead to non-universalities. There are, however, rather stringent constraints on non-universalities from flavor changing neutral currents (FCNC). One sector where the non-universalities are not stringently constrained by FCNC is the Higgs sector. Here one may parametrize the non-universalities at the GUT scale by

$$m^2_{H_1}(M_G) = m^2_0(1 + \delta_1), \quad m^2_{H_2}(M_G) = m^2_0(1 + \delta_2)$$

where a reasonable limit on $\delta_i$ is $|\delta_i| \leq 1$ ($i=1,2$). However, it was pointed out in ref. that the non-universalities in the Higgs sector and in the third generation sector are strongly coupled. Thus one should also include non-universalities in the third generation sector in a manner analogous to the non-universalities in the Higgs sector, i.e., $m^2_{\tilde{Q}_L} = m^2_0(1 + \delta_3)$, $m^2_{\tilde{U}_R} = m^2_0(1 + \delta_4)$, where again a reasonable limit on the $\delta_3, \delta_4$ is $|\delta_i| \leq 1$ ($i=3,4$).

Non-universalities affect low energy physics in important ways. One of the parameters which controls low energy physics is the Higgs mixing parameter $\mu$. One can analytically display the effects of non-universalities on $\mu^2$ for the case of small $\tan\beta$ when the effects of b quark couplings can be ignored. One finds in this case that the non-universality correction to $\mu$ can be written in the form

$$\Delta \mu^2 = m^2_0 \frac{1}{t^2 - 1} (\delta_1 - \delta_2 t^2 - \frac{D_0 - 1}{2} (\delta_2 + \delta_3 + \delta_4) t^2) + \frac{3}{5} t^2 + \frac{1}{1 - 1} S o p$$

Here $D_0 = (1 - \frac{m_f}{m_H})^2$, $m_f \simeq 200 \sin\beta$ GeV, $t \equiv \tan\beta$, $S_0 = Tr(Y m^2)$, and $p=0.0446$. The $Tr(Y m^2)$ term is the anomaly term which vanishes for the universal case since $Tr(Y)=0$, but more generally it makes a contribution although this contribution is relatively small.

2 Analysis of Event Rates in Dark Matter Detectors

One of the interesting features of supersymmetric models is that with R parity invariance they produce a lowest supersymmetric particle which is absolutely stable and hence a candidate for cold dark matter (CDM). Further in supergravity unification one finds that over most of the parameter space of the model
the lightest neutralino is also the LSP. The neutralino is in general a combination of two neutral gaugino states and two neutral Higgsino states, i.e., one has

$$\chi^0 = n_{11} \tilde{W}_3 + n_{12} \tilde{B} + n_{13} \tilde{H}_1 + n_{14} \tilde{H}_2$$  \hspace{1cm} (4)

where $\tilde{W}_3$ is the neutral component of the SU(2) gauginos, $\tilde{B}$ is the U(1) gaugino, and $\tilde{H}_i$ (i=1,2) are the neutral Higgsinos. The analysis of relic density has been discussed extensively in the literature. For some recent references see \textsuperscript{11}, \textsuperscript{12}, \textsuperscript{13}.

One can put reasonably stringent bounds on the relic density. Using the current astro-physical data one finds that the relic density is constrained by the following limits

$$0.1 < \Omega h^2 < 0.4$$  \hspace{1cm} (5)

There are a variety of techniques discussed in the literature for the detection of supersymmetric dark matter. We shall focus here on the direct method which utilizes the scattering of CDM from target nuclei\textsuperscript{14}, \textsuperscript{15}, \textsuperscript{16}, \textsuperscript{17}, \textsuperscript{18}, \textsuperscript{19}. Event rate analyses for this scattering have been discussed within MSSM and also for the standard supergravity unified models with four soft SUSY breaking parameters and including the $b \to s + \gamma$ experimental constraint\textsuperscript{20} which has a significant effect on dark matter analyses. More recently the effect of non-universalities on the event rates has also been investigated. In Fig.1 we exhibit the result of the analysis for event rates using a xenon target. One finds that in the region of the neutralino mass $< 65$ GeV the maximum and the minimum of event rates can vary by a factor of 10 due to the effect of non-universalities. However, in
the region of the neutralino mass \( > 65 \text{ GeV} \) the effect of the non-universalities on the maximum and the minimum event rate curves is relatively smaller. This phenomenon occurs in part due to the relatively large contribution that the Landau pole arising from the top Yukawa coupling makes in this region. The analysis of Fig.1 shows that the event rates can vary over a wide range from few events/kg.d to \( 10^{-5} \text{ events/kg.d} \). The current sensitivity of the detectors is \( \mathcal{O}(1) \text{ event/Kg.d} \). Thus one needs detectors more sensitive by a factor of \( 10^{3} \) or more to sample a majority of the parameter space of the supergravity models.

3 Effects of Proton Stability and Dark Matter Constraints

We discuss now the implications of imposing simultaneously the constraints of proton stability and relic density. As is well known in SUSY unified theories the dominant proton decay proceeds via dimension five operators generated via the exchange of color triplet Higgsinos. The dominant decay mode here is the mode \( p \rightarrow \bar{\nu}K \) and theoretically one finds that

\[
\Gamma(p \rightarrow \bar{\nu}K^+) = \sum_{i=e,\mu,\tau} \Gamma(p \rightarrow \bar{\nu}_iK^+) = \left( \frac{\beta_p}{M_{H_3}} \right)^2 F \tag{6}
\]

Here \( M_{H_3} \) is the Higgs triplet mass and \( \beta_p \) is the matrix element of the three quark operator between the vacuum and the proton state. Currently the best evaluation of \( \beta_p \) comes from lattice gauge calculations which gives \( \beta_p = 5.6 \times 10^{-3} \text{ GeV}^3 \). The term \( F \) in Eq.(6) depends on a variety of factors. These include quark masses and CKM factors, SUSY spectrum which enters in dressing loop diagrams needed to convert dimension five operators into dimension six four fermion operators, and chiral Lagrangian factors needed to convert the four fermion operators into an effective Lagrangian involving mesons and baryons.

In Fig.2 we give an analysis of the maximum lifetime for the decay mode \( p \rightarrow \bar{\nu}K \) for the minimal supergravity model as a function of the gluino mass for different choices of naturalness for the parameter \( m_0 \). The analysis is done by integrating over the remaining parameters of the model, i.e., \( A_0 \) and \( \tan \beta \). The results of the analysis can be compared to the current experimental lower limit of

\[
\tau(p \rightarrow \bar{\nu}K) > 1 \times 10^{32} \text{ yr} \tag{7}
\]

It is expected that Super Kamionkande will reach a sensitivity of

\[
\tau(p \rightarrow \bar{\nu}K) > 2 \times 10^{33} \text{ yr} \tag{8}
\]
Figure 2: Analysis of the maximum lifetime for the mode $p \rightarrow \bar{\nu}K$ for various choices of the naturalness assumption on $m_0$, i.e., 1 TeV (solid), 1.5 TeV (dashed-dot), 2.0 TeV (dashed) for the minimal SU(5) model. The lower horizontal line is the current experimental lower limit and the upper horizontal line is the lower limit expected from Super K. (Taken from ref.[29]).

and ICARUS may reach a sensitivity of $3 \times 10^{33}$ yr. From Fig.2 we find that for the case when the naturalness on $m_0$ is chosen to be 1 TeV the parameter space of the minimal model for gluino masses $\geq 300$ GeV will be exhausted if Super K can reach their expected sensitivity given by Eq.29. For the choice of naturalness of $m_0 = 1.5$ TeV, one finds that the parameter space of the minimal model for $m_{\tilde{g}} \geq 750$ GeV will be exhausted. For the case of the naturalness of $m_0 = 2$ TeV one finds that the region of the gluino masses up to 1 TeV is still allowed. For the case of ICARUS the constraints will be even more stringent.

Next we discuss the effect of imposing the relic density constraint. The analysis is given in fig.3. In this case one finds that the gluino mass range consistent with proton stability and relic density constraint is severely limited. For the case when the naturalness on $m_0$ is chosen to be 1 TeV, the Kamiokande limit on the gluino mass consistent with p stability and relic density constraint falls significantly below 500 GeV. For the case when the naturalness on $m_0$ is increased to 5 TeV, one still finds that the gluino mass lies below 500 GeV. The reason for this stringent constraint is not difficult to understand. It arises due to the fact that for gluino masses greater than 500 GeV, one is in the region beyond where the annihilation of neutralinos via the Z boson and the Higgs boson exchange take place. In this region the relic density is governed by the
Figure 3: The maximum life of the $p \to \bar{p}K$ mode as a function of the gluino mass with the relic density constraint $0.1 < \Omega h^2 < 0.4$ for the naturalness constraint on $m_0$ of 1 TeV (solid) and 5 TeV (dashed). The horizontal lines are as in Fig. 2. (Taken from ref. [29]).

Sfermion exchange diagrams and an efficient annihilation requires a small value of $m_0$, with typical values of $m_0 \leq 100$ GeV. However, a small value of $m_0$ tends to destabilize the proton since proton decay has a dependence of the type $m_{\tilde{g}} m_0$. Thus the combined p stability and relic density constraints appear to put a stringent limit on the gluino mass. In this sense the minimal model in the region of gluino masses $m_{\tilde{g}} > 500$ GeV with the imposition of the relic density constraints is similar to the so-called "unflipped no-scale" models which also have problems with p stability.

The analysis has been given here for the minimal SU(5) supergravity model. However, one expects a similar analysis to hold for a class of non-minimal models as well.

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1. A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49, 970 (1982).
2. For reviews see P.Nath, R. Arnowitt and A.H.Chamseddine, “Applied N = 1 Supergravity” (World Scientific, Singapore, 1984); H.P. Nilles, Phys. Rep. 110, 1 (1984); R. Arnowitt and P. Nath, Proc. of VII J.A. Swieca Summer School ed. E. Eboli (World Scientific, Singapore,(1994).
3. S. K. Soni and H. A. Weldon, Phys. Lett. B126, 215(1983); V. S. Kaplunovsky and J. Louis, Phys. Lett. B306, 268(1993).
4. R. Arnowitt and P. Nath, Phys. Rev. D56, 2833(1997).
5. C.T. Hill, Phys. Lett. B135, 47(1984); Q. Shafi, C. Wetterich, Phys. Rev. Lett. 52, 875(1984); L. Hall and U. Sarid, Phys. Rev. Lett. 70, 2673(1993); P. Langacker and N. Polonsky, Phys. Rev. D47, 4028(1993).
6. T. Dasgupta, P. Mimasas, and P. Nath, Phys. Rev. D52, 5366(1995); D. Ring, S. Urano and R. Arnowitt, Phys. Rev. D52, 6623(1995); S. Urano, D. Ring and R. Arnowitt, Phys.Rev. Lett. 76, 3663(996); P. Nath, Phys. Rev. Lett. 76, 2218(1996).
7. D. Matalliotakis and H.P. Nilles, Nucl.Phys.B435, 115(1995); M. Olechowski and S. Pokorski, Phys.Lett. B344, 201(1995); N. Polonski and A. Pomarol,Phys.Rev.D51,6532(1995).
8. V. Berezinsky, A. Bottino, J. Ellis, N. Forrengo, G. Mignola, and S. Scopel, Astropart. Phys.5, 1(1996);ibid, 5, 333 (1996).
9. P. Nath and R. Arnowitt, Phys. Rev. D56, 2820(1997).
10. For a review see G. Jungman, M. Kamionkowski and K. Greist, Phys. Rep. 267,195(1995); E.W. Kolb and M.S. Turner, “The Early Universe” (Addison-Wesley, Redwood City, 1989); P. Nath and R. Arnowitt, Proc. of the Workshop on Aspects of Dark Matter in Astrophysics and Particle Physics, Heidelberg, Germany 16-20 September, 1996.
11. J.L. Lopez, D.V. Nanopoulos, and K. Yuan, Phys. Lett. B267,219(1991); J. Ellis and L. Roskowski, Phys. Lett. B283, 252(1992); M. Kawasaki and S. Mizuta, Phys. Rev. D46, 1634(1992); M. Drees and M.M. Nojiri, Phys. Rev. D47, 376(1993); G.L. Kane, C. Kolda, L. Roskowski, and J.D. Wells, Phys. Rev.D49, 6173(1993).
12. K. Greist and D. Seckel, Phys. Rev. D43, 3191 (1991); P. Gondolo and G. Gelmini, Nucl. Phys. B360, 145 (1991).
13. R. Arnowitt and P. Nath, Phys. Lett. B299, 103(1993); Phys. Rev. Lett. 70, 3696(1993); Phys. Rev. D54, 2374(1996); M. Drees and A. Yamada, Phys. Rev. D53, 1586(1996); H. Baer and M. Brhlick, Phys. Rev. D53, 597(1996); V. Barger and C. Kao, hep-ph/9704403.
14. M.W. Goodman and E. Witten, Phys. Rev. D31, 3059(1983); K. Greist, Phys. Rev. D38, (1988)2357; D39,3802(1989)(E); J. Ellis and R. Flores, Phys. Lett. B300,175(1993); R. Barbieri, M. Frigeni and G.F. Giudice, Nucl. Phys. B313,725(1989); M. Srednicki and R.Watkins,
Phys. Lett. B225, 140 (1989); R. Flores, K. Olive and M. Srednicki, Phys. Lett. B237, 72 (1990).
15. A. Bottino et al., Astropart. Phys. 1, 61 (1992); 2, 77 (1994).
16. V.A. Bednyakov, H. V. Klapdor-Kleingrothaus and S.G. Kovalenko, Phys. Rev. D50, 7128 (1994).
17. R. Arnowitt and P. Nath, Mod. Phys. Lett. A10, 1257 (1995).
18. P. Nath and R. Arnowitt, Phys. Rev. Lett. 74, 4592 (1995); R. Arnowitt and P. Nath, Phys. Rev. D54, 2374 (1996).
19. L. Bergstrom and P. Gondolo, Astropart. Phys. 5:263-278 (1996); J.D. Vergados, J. Phys. G22, 253 (1996).
20. M.S. Alam et al. (CLEO Collaboration), Phys. Rev. Lett. 74, 2885 (1995).
21. R. Arnowitt and P. Nath, Phys. Lett. B336, 395 (1994); F. Borzumati, M. Drees and M.M. Nojiri, Phys. Rev. D51, 341 (1995).
22. P. Nath, J. Wu and R. Arnowitt, Phys. Rev. D52, 4169 (1995).
23. R. Bernabei et al., Phys. Lett. B389, 757 (1996).
24. D. Cline, Nucl. Phys. B (Proc. Suppl.) 51B, 304 (1996); P. Benetti et al., Nucl. Inst. and Method for Particle Physics Research, A307, 203 (1993).
25. S. Weinberg, Phys. Rev. D26 (1982) 287; N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533; S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. 112B (1982) 133; J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B202 (1982) 43; B.A. Campbell, J. Ellis and D.V. Nanopoulos, Phys. Lett. 141B (1984) 299; S. Chadha, G.D. Coughlan, M. Daniel and G.G. Ross, Phys. Lett. 149B (1984) 47.
26. R. Arnowitt, A.H. Chamseddine and P. Nath, Phys. Lett. 156B (1985) 215; P. Nath, R. Arnowitt and A.H. Chamseddine, Phys. Rev. 32D (1985) 2348; J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B402 (1993) 46.
27. R. Arnowitt and P. Nath, Phys. Rev. D49 (1994) 1479.
28. M.B. Gavela et al., Nucl. Phys. B312, 269 (1989).
29. R. Arnowitt and P. Nath, hep-ph/9801246.
30. Particle Data Group, Phys. Rev. D50, 1173 (1994).
31. Y. Totsuka, Proc. XXIV Conf. on High Energy Physics, Munich, 1988, Eds. R. Kotthaus and J.H. Kuhn (Springer Verlag, Berlin, Heidelberg, 1989).
32. ICARUS Detector Group, Int. Symposium on Neutrino Astrophysics, Takayama, 1992.
33. P. Nath and R. Arnowitt, Phys. Lett. B289 (1992) 308.