An Novel Method for Object Tracking in Sensor Network Combination with Trilateration and UT Transform

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Abstracts—Positioning and tracking are the key technologies of wireless sensor networks. Trilateration is an important method for the localization of wireless sensor network nodes. This method uses three anchor nodes to determine the location of the target. In this paper, UT transform and trilateration measurement method are combined to obtain the statistics of target position, and then use it as the virtual observation of Kalman filter to realize dynamic target tracking. The simulation results verify the performance of the method proposed in this paper.

1. Introduction
The wireless sensor network is composed of a large number of low-cost sensor nodes with perception, computing and wireless communication capabilities. Its main function is to intelligently perceive the external environment and provide the processed information to users [1-2]. In wireless sensor networks, Target Tracking is one of the most important and classic applications. Whether in the military or civilian fields, the need for target tracking is becoming increasingly strong. The research in this application field appears as a multidisciplinary course, and the aspects involved include signal processing, network architecture, distributed algorithms, and MEMS (Microelectro Mechanical Systems) sensor technology. Due to the easy distribution of sensor networks, sensors can be placed in areas where there is no prior infrastructure preparation for tasks such as environmental monitoring, security monitoring, and battlefield information acquisition. Outdoor target tracking usually uses satellites for surveys, but indoors and other remote areas, satellite signals are weak, and they can not locate and track targets in a timely and effective manner. The sensor has the characteristics of small size, light weight, mobility, easy deployment, and strong real-time performance. It is suitable for applications in military, environmental monitoring, medical and other fields [3].

The current tracking filtering algorithm [4, 5] mainly includes linear filtering and nonlinear filtering. Among the former, the most studied and most commonly used in practice is Kalman filtering. For nonlinear and non-Gaussian models, extended Kalman filter and unscented Kalman filter based on Kalman filter can get good filtering results, and particle filter algorithm provides an effective solution.
Literature [6] proposed a three-dimensional underwater target tracking algorithm, which uses modified extended Kalman gain to track the target. Literature [7-8] applied particle filter and UIF and other methods to realize the target tracking by sensor network. In addition, literature [9] and literature [10] both use particle filtering to select the node with the optimal observation geometric position, and then use the joint distance weight as the information unit to track the target. However, particle filter algorithms also have problems such as particle degradation and sample depletion. Research on filter algorithms with better comprehensive performance is a major challenge to further accelerate the practical process of wireless sensor networks.

2. Trilateration combined with UT Transform

2.1. Three-sided positioning principle
The positioning principle of the trilateral algorithm is: sequentially measure the distance between the mobile node and 3 non-collinear beacon nodes, and make 3 circles with the positions of the 3 beacon nodes as the center and the corresponding distance as the radius. As shown in Figure 1, if there is no error in the ranging, the three circles intersect at one point, which is the position coordinate of the mobile node.

\[ (x-x_a)^2 + (y-y_a)^2 = d_a^2 \]  
\[ (x-x_b)^2 + (y-y_b)^2 = d_b^2 \]  
\[ (x-x_c)^2 + (y-y_c)^2 = d_c^2 \]  

Figure 1 Diagram of Trilateral Measurement Method

The trilateration method [11] is a distance-based positioning algorithm. The algorithm is described as follows: Suppose the coordinates of the target position is \((x, y)\), and the coordinates of the three known points A, B, and C are respectively \((x_a, y_a)\), \((x_b, y_b)\), \((x_c, y_c)\). Their distances to target are \(d_a\), \(d_b\), and \(d_c\) respectively. Then the following equations can be obtained:

\[ (x-x_a)^2 + (y-y_a)^2 = d_a^2 \]  
\[ (x-x_b)^2 + (y-y_b)^2 = d_b^2 \]  
\[ (x-x_c)^2 + (y-y_c)^2 = d_c^2 \]  

Formula (1) minus formula (3), formula (2) minus formula (3) and then simultaneous equations:

\[ 2(x-x_a)x + 2(y-y_a)y = x_a^2 - x^2 + y_a^2 - y^2 + d_c^2 - d_a^2 \]  
\[ 2(x-x_b)x + 2(y-y_b)y = x_b^2 - x^2 + y_b^2 - y^2 + d_c^2 - d_b^2 \]  

The coordinates \((x, y)\) of target can be solved.

2.2. UT transformation
UT transformation is a transformation method to determine sampling points proposed in the unscented Kalman filter technology. After UT transformation is used, the UKF algorithm does not need to solve the Jacobian matrix. The basic idea of UT transformation is to estimate the recursive mean and variance of the state of the system. First, the unscented transformation (UT) is used to determine a set of sample points through the UT method, and then the set of sample points is used to approximate the calculation.
Observe the mean and variance of the posterior probability of the state estimation under the observation state, so as to achieve the purpose of UT transformation [12]. Unscented Transformation (UT) is a method of predicting the mean and variance of a nonlinear system, which allows the Sigma point to be scaled to any size. Choose a set of certain probability distributions that match the sample with the mean and covariance (not necessarily the Lihtian distribution). These samples can be scaled proportionally by adding a scaling factor. This method guarantees the mean and variance of the second-order precision of the mean and covariance, and also provides the same performance. As a filter that can achieve second-order accuracy, it does not need to calculate the Jacobian matrix [13].

The specific implementation steps of UT transformation are as follows:

Suppose the nonlinear function $\alpha = f(\beta)$, the input variable $\beta$ is an n-dimensional random state vector, and it is known that the mean is $\hat{\beta}$ and the variance is $P_\beta$. The first and second moments of the function $\alpha$ can be calculated through the UT transformation technique. The main process is as follows [14].

1. The random state vector is n-dimensional, so calculate $2n+1$ sample points $s_i$ and the corresponding weight $w_i$.

\[
\begin{align*}
    s_0 &= \hat{\beta}, \\
    w_0 &= \lambda / (n + \lambda) \\
    s_i &= \hat{\beta} + \sqrt{(n + \lambda) P_\beta}, w_i = 1 / [(2(n + \lambda)] (i = 1, 2, ..., n) \\
    s_{n+i} &= \hat{\beta} - \sqrt{(n + \lambda) P_\beta}, w_i = 1 / [2(n + \lambda)] (i = n+1, n+2, ..., 2n)
\end{align*}
\]

(3)

In the above formula, $\lambda$ is a fine-tuning parameter. By adjusting the size of this parameter, the sample points can be closer to the true value of the state distribution. This $\lambda$ can not only adjust the size of high-order sample moments, but also adjust the distance from the sample point to the mean.

Represents the square root matrix of the i-th column. And the weight $w_i$, which meets the normalization requirements.

2. Bring each sampling point into the nonlinear equation to obtain the transformed point set.

\[\alpha_i = f(s_i) \quad (i = 1, 2, ..., 2n)\]

(4)

3. Use the new sample points with weights to estimate the mean and variance of the transformed nonlinear function. As shown in the following formula:

\[
\hat{\alpha} = \sum_{i=0}^{2n} w_i \alpha_i
\]

(5)

\[
P_\alpha = \sum_{i=0}^{2n} w_i (\alpha_i - \hat{\alpha}) (\alpha_i - \hat{\alpha})
\]

(6)

It can be seen from the above that after the UT transformation, the mean and variance of the nonlinear system can be estimated. And through the UT transformation technology, the accuracy of the state estimation of the nonlinear system can reach the second order or even the second order. The accuracy of the variance estimation of the system state can reach a third-order quantity, and when the nonlinear system is transformed into a linear system by the Taylor series expansion method used in the extended Kalman filter algorithm, the estimation accuracy of the mean and variance of the system state is equal to the first order.
2.3. Improving trilateral positioning method with UT transform

When the filtering method is used for wireless sensor network tracking and positioning, nonlinear transformation is required. UT transformation is a new method to deal with nonlinear transformation problems. The combination of trilateral measurement and UT transform transforms the tracking from nonlinear filtering to linear filtering.

Based on the previous three-side positioning method model, Formula (2) can be rewritten as following:

\[
\begin{bmatrix}
2x_a - 2x_c, \ 2y_a - 2y_c \\
2x_b - 2x_c, \ 2y_b - 2y_c
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
\frac{d^2 - d_a^2 + x_a^2 + y_a^2 - y_c^2}{d_a^2} \\
\frac{d^2 - d_c^2 + x_c^2 + y_c^2 - y_b^2}{d_c^2}
\end{bmatrix}
\]

(7)

It is assumed that the distance observation \(d_a^*, d_b^*, d_c^*\) is the superposition of real distance \(d_a, d_b, d_c\) and observation noise \(\epsilon_a, \epsilon_b, \epsilon_c\) as follows:

\[
\begin{align*}
d_a^* &= d_a + \epsilon_a \\
d_b^* &= d_b + \epsilon_b \\
d_c^* &= d_c + \epsilon_c
\end{align*}
\]

(8)

If the observation noise \(\epsilon_a, \epsilon_b, \epsilon_c\) is an independent 0-means Gaussian distribution and the variances are \(\sigma_a^2, \sigma_b^2, \sigma_c^2\), then the three real distances \(d_a, d_b, d_c\) become a Gaussian random vector. That is:

\[
f(d_a, d_b, d_c) = 
\frac{1}{(2\pi)^{\frac{3}{2}}|\Sigma_d|^\frac{1}{2}} \exp \left[-\frac{1}{2}(d - m_d)^T \Sigma_d^{-1} (d - m_d) \right]
\]

(9)

Where, \(d = [d_a, d_b, d_c]\), \(m_d = [d_a^*, d_b^*, d_c^*]\),

\[
\Sigma_d = \begin{bmatrix}
\sigma_a^2 & 0 & 0 \\
0 & \sigma_b^2 & 0 \\
0 & 0 & \sigma_c^2
\end{bmatrix}
\]

(10)

Knowing that A, B, and C are not collinear, it can be obtained from formula (7):

\[
M = [d_a, d_b, d_c], \begin{bmatrix}
\sigma_a & 0 & 0 \\
0 & \sigma_b & 0 \\
0 & 0 & \sigma_c
\end{bmatrix}
\]

(11)

\[
[x, y] = \begin{bmatrix}
2x_a - 2x_c, \ 2y_a - 2y_c \\
2x_b - 2x_c, \ 2y_b - 2y_c
\end{bmatrix} \begin{bmatrix}
\frac{d^2 - d_a^2 + x_a^2 + y_a^2 - y_c^2}{d_a^2} \\
\frac{d^2 - d_c^2 + x_c^2 + y_c^2 - y_b^2}{d_c^2}
\end{bmatrix}
\]

(12)

Obviously, the second term on the right of the above equation is a two-dimensional random variable and a function of the random vector \(\bar{d}\). Therefore, from formula (3), (4), (5), (6), it can be known after UT transformation. Next, by linear transformation with the first term on the right side of the above formula, the mean and variance of the target position \((x, y)\) can be obtained.

Finally, the mean value and covariance of \([x, y]\) can be obtained as the virtual observation of the trilateral positioning of the sensor network combined with Kalman filtering to achieve positioning and
tracking. For time k, the mean value of [x, y] is expressed as \( Z_k \), covariance of [x, y] is expressed as \( Q_k \).

### 2.4. Kalman filtering

The motion state of the target is three-dimensional, each representing the abscissa, ordinate, and orientation angle relative to a fixed coordinate system, which is recorded as a circle model in this article. The motion model is as follows:

\[
e_{k+1} = F_k x_k + G_k w_k
\]

\( w_k \) is a process measurement noise sequence with a mean value of 0 and a covariance of \( Q_w \).

The distance measurement and the target position are originally non-linear mapping, but the mean and variance of the virtual observation of the target position [x, y] are given above, so Kalman filter can be used. It is generally divided into two parts: time update and observation update. Specific steps are as follows:

- **Time update:**
  - **2.4.1. Pre-estimated state variables**
    \[
    \hat{x}_k = F \hat{x}_{k-1}
    \]
  - **2.4.2. Pre-estimated state variance**
    \[
    P_k^- = F P_{k-1} F^T + Q
    \]

- **Observation update**
  - **2.4.3. Calculate Kalman gain**
    \[
    K = P_k^- H^T (H P_k^- H^T + R)^{-1}
    \]
  - **2.4.4. Update state variables**
    \[
    \hat{x}_k = \hat{x}_k^- + K(z_k - H \hat{x}_k^-)
    \]
  - **2.4.5. Update state variance**
    \[
    P_k = (I - KH) P_k^-
    \]

### 3. Simulation analysis

The simulation platform used in this article is MATLAB R2016a to simulate and verify the improved algorithm. The instantaneous velocity of the vehicle is fixed at 1m/s, and then the yaw rate is fixed at 5deg/s. At each time step, Gaussian white noise is added to the state. The noise is three-dimensional, obeys the mean value of zero, and the covariance is \( \text{diag}([0.01, 0.01, 1\text{deg}^2]) \). When the observation noise is introduced, the Gaussian white noise with a mean value of 0 and a variance of 0.01 is introduced. The positions of the four nodes are (10,0); (10,10); (0,15); (-5,20) respectively, and the maximum distance detection threshold is 20. The three nodes closest to the target are selected each time to track the positioning. The number of particles selected respectively is 100, 500, 900, and the simulation is performed respectively. The number of Monte Carlo simulations per time is 40 times. The results obtained are shown in the following chart:
Table 1  Average running time of two algorithms

| Algorithm | Our method | 100PF   | 500PF   | 900PF   |
|-----------|------------|---------|---------|---------|
| Time      | 7.043147   | 17.261485 | 68.499729 | 125.184469 |

Figure 2  Trajectory diagram

Figure 3  Measurement error comparison chart

Figure 4  Comparison of lateral speed performance estimates
Figure 5 Longitudinal speed performance estimation comparison

1) In order to verify the state estimation effect of several particle filter algorithms on nonlinear problems, select different particle numbers N for simulation. Here N is taken as 100, 500, 900, and the simulation results are shown in the chart:

2) Through observation, it can be found that with the increase of the number of particles, the approximation degree of the state estimation and the actual value of various tracking algorithms is getting higher and higher. As the number of particles increases, the state estimate of the algorithm will be closer to the actual state value. When there are more particles, its overall performance will indeed be better than when there are fewer particles.

3) As the number of particles increases, the running time of the algorithm has been increasing, which shows that blindly increasing the number of particles cannot improve the overall performance of the algorithm. Based on the above chart, it can be found that regardless of the number of particles, the measurement error of the UT-based three-pass measurement algorithm is less than PF, indicating that the improved trilateral measurement algorithm has better estimation accuracy than PF.

4. Conclusion
In this paper, UT transform is applied to wireless sensor network positioning and tracking, combined with the trilateral measurement method, to obtain the location information of the target location. The simulation results show that the method in this paper has lower measurement error and time complexity, and better tracking effect. The conclusions obtained in the article can provide references for the practical application of wireless sensor network positioning and tracking technology.

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