A Transmuted Survival Model With Application

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Abstract. In this paper, a new survival model is being defined. This model is adaptable for produce a new lifetime distribution for analyzing positive data. We define a new lifetime models called the Transmuted-Survival-Exponential distribution (TSE). Essential functions associated with the proposed distribution are obtained. Some structural properties of the new models are investigated. The new formula may serve as a flexible transmuted to other distributions for modeling survival data arising in several fields of science such as hydrology, biostatistics, metrology and engineering

1. Introduction

Model made of random variability has important and wide domain of the selection of a distribution from a rich library of distributional models. Sometime there is no use of add or multiply components for the random part of a model and get a developed model specially when they are defined in the common way by CDF or PDF. Beside the CDF or PDF leads to mixtures of distributions rather than new distributions. Thus there is no model building kit available for the random component parallel to that for the deterministic component. Models maker are therefore have to use readymade distributional models from the library.

[1] represents his lifetime model depending on survival function, [2] used the relationship between two survival analysis to develop the AFT (accelerated failure time) models such that:

\[ S_1(t) = S_2(c t) \text{ for all } t > 0 \text{, where constant } c > 0 \]

Starting with a survival function \( S(t) \), [3] introduced "general method of adding a parameter into a family of distributions in particular, starting with a survival function \( S(t) \), one-parameter family of survival function with application to the Exponential and Weibull families". [4] gave new approach for building statistical distributions based on transmutation maps that use functional composition of the cumulative distribution function and invers of cumulative distribution function and [5][6][7][8][9] apply the transmutation maps on different types of distribution.

The new distribution (or formula) depends firstly, on survival function \( S(t) \) which is defined by

\[ S(t) = P(t \leq T) \quad , t \geq 0 \]

\[ = 1 - F(T) \]

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Where $T$ is a continues random variable (failure time), the distribution function $F(t)$ describe the probability the time to event ($T$) is smaller or equal compared to a fixed time ($t$) and is given as:

$$F(t) = P(t \leq T)$$

$f(t)$ is the (probability density function) probability that the failure time occurs at exactly time $t$ (out of all possible times) and is given as

$$f(t) = \lim_{\Delta t \to 0} \frac{Prob(t \leq T \leq t + \Delta t)}{\Delta t}$$

Secondly, on computing a new $S(t)$ from the following transmuted formula:

$$S(t) = (1 + \lambda)S_*(t)^2 - \lambda S_*(t) \quad \lambda > 0 \quad (1)$$

Where $S_*(t)$ is the survival function of the baseline distribution and by differentiation law

$$dS(t) = -dF(t) = -f(t)$$

We have

$$-f(t) = -2(1 + \lambda)S_*(t)f_*(t) + \lambda f_*(t)$$

$$f(t) = 2(1 + \lambda)S_*(t)f_*(t) - \lambda f_*(t)$$

$$f(t) = f_*(t)[2(1 + \lambda)S_*(t) - \lambda] \quad (2)$$

According to [10], a function $f(\cdot)$ that is defined as $f: R \to [0, \infty]$ is a probability density function if and only if

1) $f(x) \geq 0 \text{ for all } x > 0$

2) $\int_{-\infty}^{\infty} f(x)dx = 1$

Therefore, the first property is satisfied for all $x > 0$. The second property is shown below:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{0}^{\infty} f(x)dx = 2(1 + \lambda)\int_{0}^{\infty} S_*(x) f_*(x)dx - \lambda \int_{0}^{\infty} f_*(x)dx$$

Now, put $u = F_*(x) \cdot S_*(x) = 1 - u$ and $du = dF_* = f(x)dx$

$$2(1 + \lambda)\int_{0}^{1} S_*(x) f_*(x)dx = 2(1 + \lambda)\int_{0}^{1} (1 - u) \, du$$

$$= 2(1 + \lambda) \left[ 1 - \frac{1}{2} \right]$$

$$= 2 + 2 \lambda - 1 - \lambda$$

So that $\int_{0}^{\infty} f(x)dx = 1$

Therefore equation (2) is a density function.
2. Transmuted-Survival-Exponential distribution (TSE)

The cdf, pdf and survival function of Exponential distribution are respectively

\[ f_e(t) = \gamma e^{-\gamma t} \quad \gamma > 0, t > 0 \]
\[ F_e(t) = 1 - e^{-\gamma t} \]
\[ S_e(t) = e^{-\gamma t} \]

We take the tail (survival) distribution of exponential distribution and by using transmutation (1) we get the Transmuted-Survival-Exponential distribution:

\[ S_{TSE}(t) = (1 + \lambda)e^{-2\gamma t} - \lambda e^{-\gamma t} \quad (3) \]

The CDF of this distribution is:

\[ F_{TSE}(t) = 1 - S_{TSE}(t) \]
\[ F_{TSE}(t) = 1 - (1 + \lambda)e^{-2\gamma t} + \lambda e^{-\gamma t} \quad (4) \]

Then the pdf of the new distribution from the derivative of the CDF distribution where \( \gamma > 0 \), is the scale parameter

\[ f_{TSE}(t) = 2\gamma(1 + \lambda)e^{-2\gamma t} - \lambda\gamma e^{-\gamma t} \quad (5) \]

Or we can write (5) in the following:

\[ f_{TSE} = \gamma e^{-2\gamma t}(2 + \lambda(2 - e^{-\gamma t})) \quad (6) \]

It is easy to show that the new distribution satisfy the following properties

\[ f_{TSE}(t) \geq 0 \quad \text{and} \quad \int_{0}^{\infty} f_{TSE}(t) \, dt = 1 \]

The following graph figure (1.a & 1.b) show some shapes of the pdf of the TSE distribution for selected values of the parameters \( \lambda, \gamma \).
Figure 1.a: The pdf TSE distribution with different values of $\gamma$ and fixed $\lambda$.

Figure 1.b: The probability density function of TSE distribution with different values of $\lambda$ and fixed $\gamma$.

3. The Shapes of New distribution

We discuss the shapes of the CDF, pdf function of the new (TSE) distribution, as follows:

$$\lim_{t\to0} f_{TSE}(t, \gamma, \lambda) = \lim_{t\to0} 2\gamma(1 + \lambda)e^{-2\gamma t} - \lambda\gamma e^{-\gamma t}$$
$$= \lim_{t\to0} 2\gamma(1 + \lambda)\lim_{t\to0} e^{-2\gamma t} - \lambda\gamma\lim_{t\to0} e^{-\gamma t}$$
$$= 2\gamma(1 + \lambda) - \lambda\gamma$$
\[ = \gamma(2 + \lambda) \]

While

\[ \lim_{t \to \infty} f_{TSE}(t, \gamma, \lambda) = 0 \]

The limit of cdf of TSE:

\[ \lim_{t \to 0} F_{TSE}(t, \gamma, \lambda) = \lim_{t \to 0} 1 - (1 + \lambda) e^{-2\gamma t} + \lambda e^{-\gamma t} \]

\[ = 1 - 1 - \lambda + \lambda \]

\[ = 0 \]

And it is clear that \( \lim_{t \to \infty} F_{TSE}(t, \gamma, \lambda) = 1 \)

4. Statistical Properties

This section presents formulas for the reliability, hazard, moments, moment generating functions and quantiles functions of the distribution.

4.1 Reliability function

The reliability function of TSE distribution \( R(t) \) is given by

\[ R_{TSE}(t) = 1 - F_{TSE}(t) \]

\[ R_{TSE}(t) = 1 - (1 - (1 + \lambda) e^{-2\gamma t} + \lambda e^{-\gamma t}) \]

\[ R_{TSE}(t) = (1 + \lambda) e^{-2\gamma t} - \lambda e^{-\gamma t} \]

Note that \( R_{TSE}(t) \) is our transmuted formula (1).
Figure 2: The reliability function of TSE distribution with different values of $\gamma$ and fixed $\lambda$.

4.2 Hazard function

Hazard function, which is sometime called “force of mortality” is defined by

$$h(t) = P(T < t + dt | T \geq t)$$

$$= -\frac{S'(t)}{S(t)}$$

$$= -\frac{d \ln(S(t))}{dt}$$

$$= \frac{2\gamma(1+\lambda)e^{-2\gamma t} - \gamma\lambda e^{-\gamma t}}{(1+\lambda)e^{-2\gamma t} - \lambda e^{-\gamma t}}$$

Hence the hazard transmuted-survival-exponential distribution is:

$$h_{TSE}(t) = \frac{2\gamma(1+\lambda) - \gamma\lambda e^{\gamma t}}{(1+\lambda) - \lambda e^{\gamma t}}$$
4.3 Quantile function

The quantile function or inverse cumulative distribution function returns the value $t$ such that

$$t = Q(p) = F^{-1}(p) \quad \text{where} \quad 0 < p < 1$$

Using equation (4) the quantile function of TSE distribution may be expressed within implicit form as

$$p = 1 - (1 + \lambda)e^{-2\gamma t} + \lambda e^{-t}$$

$$1 - p = (1 + \lambda)e^{-2\gamma t} + \lambda e^{-t}$$

$$\ln(1 - p) = -\gamma t + \ln [(1 + \lambda)e^{-t} + \lambda]$$

Which is nonlinear equation can be solved for $t$ numerically.

Figure 3: The hazard function of TSE distribution with different values of $\lambda$ and fixed $\gamma$.
4.4 Moments

Theorem 1: The rth moment \( M'_r \) of TSE distribution about the origin is

\[
M'_r = \frac{\Gamma(r+1)}{(2\gamma)^{r-1}}
\]

(7)

Therefore the rth moment \( M'_r \) of TSE distribution about the means

\[
M_r = \sum_{i=0}^{r} (-1)^i \binom{r}{i} M'_{r-i}(M'_1)^i
\]

(8)

Proof:

\[
E(T^r) = \int_0^\infty f_{TSE}(x) \, dx = \int_0^\infty t^r [2(\lambda + 1) \gamma e^{-2\gamma t} - \lambda \gamma e^{-\gamma t}] \, dt
\]

\[
= \int_0^\infty t^r 2(\lambda + 1) \gamma e^{-2\gamma t} \, dt - \int_0^\infty t^r \lambda \gamma e^{-\gamma t} \, dt
\]

\[
= \frac{\lambda + 1}{(2\gamma)^{r-1}} \int_0^\infty (2\gamma t)^{(r+1)-1} e^{-2\gamma t} \, dt - \frac{\lambda}{(\gamma)^{r-1}} \int_0^\infty (t)^{(r+1)-1} e^{-\gamma t} \, dt
\]

\[
= \frac{\lambda + 1}{(2\gamma)^{r-1}} \Gamma(r+1) - \frac{\lambda}{(\gamma)^{r-1}} \Gamma(r+1)
\]

\[
= \frac{\Gamma(r+1)}{(2\gamma)^{r-1}}
\]

So that \( E(t - \mu)^r = \) Therefore the rth moment \( M'_r \) of TSE distribution about the mean is

\[
M_r = \sum_{i=0}^{r} (-1)^i \binom{r}{i} M'_{r-i}(M'_1)^i
\]

Since

\[
E(T - \mu)^r = \sum_{i=0}^{r} (-1)^i \binom{r}{i} (T)^{r-i}(\mu)^i . \, \mu = E(T)
\]

The mean of the new distribution is (when \( r = 1 \)):

\[
E(T) = \Gamma(2)
\]

And, the second moment of the TSE distribution is (when \( r = 1 \)):

\[
E(T) = \frac{\Gamma(3)}{(2\gamma)^2}
\]

Then, the variance of the TSE distribution is:
\[ \text{var}(T) = E(T^2) - (E(T))^2 \]

\[ \text{var}(T) = \frac{\Gamma(3)}{(2\gamma)^2} - \Gamma(2)^2 \]

The Coefficient of Variation CV is given by:

\[ CV = \frac{\sqrt{\text{var}(T)}}{E(T)} = \frac{\Gamma(3)}{(2\gamma)^2} - \Gamma(2)^2 \]

\[ CV = \frac{\Gamma(2)}{\Gamma(2)^2 - \Gamma(2)^2} \]

The Coefficient of skewness CS is given by:

\[ CS = \frac{E(t - \mu)^3}{\sigma^3} = \frac{\sum_{i=0}^{3}(-1)^i\binom{3}{i}M_{3-i}(M_i)}{\sigma^3} \]

Or \[ CS = \frac{E(t^3) - 3E(t)\sigma^2 - \mu^3}{\sigma^3} \]

Where \( \sigma \) is the squared root of the variance and \( \mu \) is the mean \((E(T))\). So by using theorem 1:

\[ CS = \frac{\Gamma(4)}{(2\gamma)^3} - 3\Gamma(2) \left( \frac{\Gamma(3)}{(2\gamma)^2} - \Gamma(2)^2 \right) - \Gamma(2)^3 \]

\[ \frac{(\Gamma(3))}{(2\gamma)^2} - \Gamma(2)^2^{3/2} \]

The Coefficient of kurtosis CK is given by

\[ CK = \frac{E(T - \mu)^4}{\sigma^4} \]

Where

\[ E(T - \mu)^4 = E(T^4) - 4E(T)E(T^3) + 6E(T^2)E(T)^2 - 3E(T)^4 \]

And \( \sigma^4 = \text{var}(T)^2 \). Hence

\[ CK = \frac{\Gamma(5)}{(2\gamma)^4} - 4\Gamma(2)^3 \left( \frac{\Gamma(4)}{(2\gamma)^3} - 3\Gamma(2)^2 \right) + \frac{\Gamma(6)}{2\gamma} \Gamma(2)^3 \]

\[ \left( \frac{\Gamma(5)}{(2\gamma)^4} - 3\Gamma(2)^4 \right) \]

**4.5 Moment Generating Function of TSE**

Now, we drive the moment generating function of TSE.
Theorem 2  The (mgf) of $z$, $M(z) = E(e^{zt})$, it is given by:

$$
M(z) = \frac{\lambda + 1}{z - 2\gamma} - \frac{\lambda}{z - \gamma}
$$

proof

$$
M(z) = E(e^{zt}) = \int_0^\infty e^{zt} f_{TSE}(t) dt
$$

$$
= \int_0^\infty e^{zt} \left[ (\lambda + 1) e^{-2\gamma t} - \lambda e^{-\gamma t} \right] dt
$$

$$
= \int_0^\infty \left[ (\lambda + 1) e^{(z-2\gamma)t} - \lambda e^{(z-\gamma)t} \right] dt
$$

$$
= (\lambda + 1) \int_0^\infty e^{(z-2\gamma)t} dt - \lambda \int_0^\infty e^{(z-\gamma)t} dt
$$

$$
= \left. \frac{\lambda + 1}{z - 2\gamma} e^{(z-2\gamma)t} \right|_0^\infty - \left. \frac{\lambda}{z - \gamma} e^{(z-\gamma)t} \right|_0^\infty
$$

Hence

$$
M(z) = \frac{\lambda + 1}{z - 2\gamma} - \frac{\lambda}{z - \gamma}
$$

4.6 Order Statistics

In statistical theory and application, order statistics make their importance in many areas like statistical inference estimates of some unknown parameters, which are the best in some sense (efficient, robust) or satisfies useful properties (sufficient, simple and convenient for applications), have the form of order statistics or can be expressed as functions of order statistics.

Let $T_1, T_2, \cdots, T_n$ be a random sample of size $n$ from TSE distribution with a cdf $F(t, \Xi)$ and pdf $f(t, \Xi)$ given (4, 5) respectively, where $\Xi = (\lambda, \gamma)$. Let $T_{1:n} \leq T_{2:n} \leq \cdots \leq T_{k:n}$ represent the order statistics taken from the sample. The pdf of $T_{k:n}$ is given by [11]:

$$
f_{k:n}(t; \Xi) = \frac{1}{B(k, n-k+1)} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} f(t, \omega) (F(t, \omega))^{k+i-1}
$$

$$
f_{k:n}(t; \Xi) = \frac{1}{B(k, n-k+1)} \sum_{i=0}^{n-k} (-1)^i 2\gamma(1 + \lambda)e^{-2\gamma t}
$$

$$
- \lambda ye^{-\gamma t} \left( 1 - (1 + \lambda)e^{-2\gamma t} + \lambda e^{-\gamma t} \right)^{k+i-1}
$$
If \( k = 1 \) then the pdf of nth order statistics = \( \min \{ T_1, T_2, \cdots, T_n \} \)

is defined as

\[
f_{1,n}(t; \Xi) = \frac{1}{B(1,n)} \sum_{i=0}^{n-1} (-1)^i 2\gamma(1 + \lambda)e^{-2\gamma t} - \lambda\gamma e^{-\gamma t} (1 - (1 + \lambda)e^{-2\gamma t} + \lambda e^{-\gamma t})^{k+1-1}
\]

If \( k = n \) then the pdf of nth order statistics = \( \max \{ T_1, T_2, \cdots, T_n \} \)

is defined as

\[
f_{n,n}(t; \Xi) = \frac{2\gamma(1 + \lambda)e^{-2\gamma t} - \lambda\gamma e^{-\gamma t} [1 - (1 + \lambda)e^{-2\gamma t} + \lambda e^{-\gamma t}]^{n-1}}{B(n,1)}
\]

### 4.7 Maximum Likelihood Estimators

Let \( \Omega = (\lambda, \gamma)^T \) be the parameter vector of that we want to estimate and \( t_1, t_2, \cdots, t_n \) be random variables of size \( n \) with a pdf given by (5). In this subsection we estimate the parameters \( \lambda, \gamma \) using maximum likelihood estimators method. The likelihood function defined as the common joint probability density function distribution of the data and can be written as follows:

\[
l(t; \lambda, \gamma) = \prod_{i=1}^{n} f_{TSE}(t_i; \lambda, \gamma)
\]

The log-likelihood function for the vector of parameters can be written as,

\[
L = \ln l(t; \lambda, \gamma) = \ln \prod_{i=1}^{n} (2\gamma(1 + \lambda)e^{-2\gamma t_i} - \lambda\gamma e^{-\gamma t_i})
\]

Then, by take the partial derivatives of \( L \) with respect to unknown parameters \( \lambda, \gamma \) as follows:

\[
\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \frac{2\gamma e^{-2\gamma t_i} - \gamma e^{-\gamma t_i}}{2\gamma(1 + \lambda)e^{-2\gamma t_i} - \lambda\gamma e^{-\gamma t_i}}
\]

\[
\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} \frac{2\gamma e^{-2\gamma t_i} - \gamma e^{-\gamma t_i}}{2\gamma(1 + \lambda)e^{-2\gamma t_i} - \lambda\gamma e^{-\gamma t_i}}
\]
\[
\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} \frac{2(1 + \lambda) e^{-2\gamma t_i} [1 - 2t_i]}{2\gamma (1 + \lambda) e^{-2\gamma t_i} - \lambda e^{-\gamma t_i}}
\]

Put \( \frac{\partial L}{\partial \lambda} = 0 \) and \( \frac{\partial L}{\partial \gamma} = 0 \) which is nonlinear equations. The maximum likelihood estimate of \( \lambda \) and \( \gamma \) can be obtained by numerical methods.

5. Applications: breast Cancer Data

Here, we test the following data

\[0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0, 47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0, 154.0.\]

This real data set represent the survival times of 121 cases with breast cancer taken from a large hospital censored from 1929 to 1938. This data set has recently been studied by [11].

We illustrate the proposed model using the above data by comparing TSE distribution with lognormal distribution (LND), log-logistic distribution (LLD) and Exponential distribution (ED). Table 1 indicates that the MLEs of the models parameters. The MLE problem solved by applying Newton-Raphson algorithm within Matlab (2015) program.

Information criteria illustrate how the model is good at explaining the relationship between variables and when we want to know about how good the model, so Table 2 shows that the TSE model has better fit to the set data by using the statistics test: AIC, the BIC and the CAIC. These statistics give lowest values comparing to other fitted models in the table 2.

Table 1: MLEs for the cancer data (standard errors in parentheses)
Table 2. The AIC, CAIC, BIC values for the Cancer data

| Distribution | Estimates |  |  |  |  |
|--------------|-----------|--------|------|--------|------|
|              | $\alpha$  | $\beta$ | $\lambda$ | $\varphi$ | $\mu$ | $\sigma$ |
| $LN(\mu, \sigma)$ | --- | --- | --- | 3.46 | 1.033 |
|                |          |       |       | (0.094) | (0.066) |
| $LL(\alpha, \beta)$ | 1.495 | 30.984 | --- | --- | --- |
|                |        | (0.175) | (4.290) |       |       |
| $E(\lambda)$   | --- | --- | 0.0216 | --- | --- |
| $TSE(\lambda, \gamma)$ | --- | --- | 0.2235 | 0.0092 | --- |
|                |          |        | (0.0135) | (0.0015) |       |

6. Conclusion
In this work we introduced a new lifetime model that can be used to produce statistical distributions work with survival data. Statistical properties of TSE has been given. The maximum likelihood procedure yields nonlinear equation solved numerically. The statistics AIC, BIC and CAIC among different fitted models: ETED, LND, LLD and ED explain that the proposed model has better fit to tested set data.

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