Utilization of nonlinear vibrations of soft pipe conveying fluid for driving underwater bio-inspired robot

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Abstract  Creatures with longer bodies in nature like snakes and eels moving in water commonly generate a large swaying of their bodies or tails, with the purpose of producing significant frictions and collisions between body and fluid to provide the power of consecutive forward force. This swaying can be idealized by considering oscillations of a soft beam immersed in water when waves of vibration travel down at a constant speed. The present study employs a kind of large deformations induced by nonlinear vibrations of a soft pipe conveying fluid to design an underwater bio-inspired snake robot that consists of a rigid head and a soft tail. When the head is fixed, experiments show that a second mode vibration of the tail in water occurs as the internal flow velocity is beyond a critical value. Then the corresponding theoretical model based on the absolute nodal coordinate formulation (ANCF) is established to describe nonlinear vibrations of the tail. As the head is free, the theoretical modeling is combined with the computational fluid dynamics (CFD) analysis to construct a fluid-structure interaction (FSI) simulation model. The swimming speed and swaying shape of the snake robot are obtained through the FSI simulation model. They are in good agreement with experimental results. Most importantly, it is demonstrated that the propulsion speed can be improved by 21\% for the robot with vibrations of the tail compared with that without oscillations in the pure jet mode. This research provides a new thought to design driving devices by using nonlinear flow-induced vibrations.

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1 Introduction

Pipes conveying fluid as key components are widely employed in industrial applications, e.g., heat transfer tubes in nuclear industry, liquid-fuel pipes in rocket engine, and production risers in ocean engineering. As flow-induced vibrations of fluid-conveying pipes can cause unexpected deformations and even result in fatigue failure to the whole engineering structures, nonlinear vibrations of pipes conveying fluid have become a hot topic in research fields of fluid-structure interaction (FSI) as well as dynamic systems since the last few decades\cite{1–4}. The literature on this topic is very extensive and still constantly expanding. The rich dynamics of pipes conveying fluid have been discussed in a famous book by Païdoussis\cite{5}. Afterwards, Païdoussis\cite{6} further discussed the radiation of studying pipes conveying fluid problems into other areas of applied mechanics. Recently, significant studies on dynamics of pipes conveying fluid manifested the importance of nonlinear analysis in engineering applications\cite{7–11}.

It is known that, when the flow velocity in supported pipes is successively increased, the system may become unstable, and divergence instability would occur at a critical flow velocity\cite{5}. Unlike the supported pipes conveying fluid which are conservative in the absence of dissipation, however, a cantilevered pipes conveying fluid is a nonconservative system, and for a sufficiently high flow velocity, it would lose stability by flutter\cite{12}. The instability phenomenon was observed in experiments\cite{13}. After the first study of Bourrières\cite{14} on the stability of cantilevered pipes conveying fluid, Benjamin\cite{15–16} examined the dynamics of articulated cantilevers conveying fluid, but with a discussion of the continuous system. Païdoussis\cite{17} and Gregory and Païdoussis\cite{18} extended Benjamin’s work to cases of continuously flexible pipes conveying fluid. They determined the conditions of instability via quasi-analytical and numerical solutions to the partial differential equation. These solutions were also compared with experimental results. Recently, statics and dynamics of slightly curved cantilevered pipes conveying fluid with different initial shapes were explored by Zhou et al.\cite{19} through employing the absolute nodal coordinate formulation (ANCF). In the study of Ye et al.\cite{11}, the vibration characteristics of a slightly curved pipe conveying supercritical fluid were presented. The results showed that the supercritical dynamics of the pipe is sensitive to the initial curvature. Farokhi and Erturk\cite{20} developed a three-dimensional geometrically exact model for cantilevered pipes conveying fluid. The dynamical response of the cantilever was examined in detail in the primary resonance region, highlighting the effect of one-to-one internal resonance between the in-plane and out-of-plane motions.

Very recently, Païdoussis et al.\cite{21} considered a hanging cantilevered pipe discharging fluid with the external fluid flowing upwards in an annulus covering the upper part of the pipe. The developed theoretical model achieves good agreement with experimental observations. Therefore, dynamical behaviors of pipes, with supported ends, clamped-free ends, or with unusual boundary conditions; articulated rigid pipes or continuously flexible pipes; pipes conveying incompressible or compressible fluid, with steady flow velocity; linear, nonlinear, and chaotic dynamics, are the main research goals and going on all the time\cite{22–24}.

Nevertheless, understanding the dynamical mechanism for pipes conveying fluid is not an end. Importantly, effective controlling methods need to be proposed based on the mechanism to meet requirements in engineering applications. In this way, passive and active strategies were put forward to suppress flow-induced vibrations\cite{25–29}. For example, Tani and Sudani\cite{25}
applied a sub-optimal control law for suppressing the vibrations of pipes conveying fluid using motor-controlled tendons. Yau et al.\cite{26} used piezoelectric actuators and employed the quantitative feedback theory to actively control the nonlinear vibrations of a constrained pipe conveying fluid. Sugiyama et al.\cite{27} utilized an electronic valve to control the stability of pipes conveying fluid. The valve was used to adjust the flow velocity through a feedback on-off control.

To suppress the flutter instability of cantilevered pipes conveying fluid, many scholars\cite{28–31} have done a series of significant works. For example, an adaptive approach was used to control the flutter instability of cantilevered pipes conveying fluid by Tsai and Lin\cite{29}. Phononic crystals were employed to suppress vibrations of pipes conveying fluid by Yu et al.\cite{30–31}, who found that gaps with Bragg scattering and the locally resonant mechanism might exist in a fluid-conveying pipe system. Dai et al.\cite{32} analyzed a cantilevered pipe composed of two different materials and found that the stability of the system might be controlled by changing the length ratio between the two pipe segments. Based on the work of Rinaldi and Paidoussis\cite{33}, Wang and Dai\cite{34} further considered the dynamics of fluid-conveying pipes fitted with an additional Y-shaped end-piece consisting of two symmetric elbows, which can be used to control the stability and vibration properties of pipes conveying fluid, for both supported and clamped-free boundary conditions. Yang et al.\cite{35} initiated a numerical examination of the nonlinear responses of pinned-pinned pipes with an attached nonlinear energy sink (NES). It was shown that the NES can robustly dissipate a major portion of the vibrational energy of the pipe. In addition, Zhou et al.\cite{36} proposed to design suitable passive control strategies to control nonlinear vibrations of a cantilevered pipe conveying fluid.

Nonetheless, it is noted that vibration responses can also be efficiently harnessed as well as controlled in engineering applications\cite{37}. In particular, significant studies have been done to utilize flow-induced vibrations for energy harvesting in recent years\cite{38}. The harvested electrical energy can be potential for powering micro electromechanical system (MEMS) or wireless sensors. As to flow-induced vibrations of pipe conveying fluid, a few studies considered to employ nonlinear oscillations as a kind of driving force\cite{39–41}. Paidoussis\cite{39} was the first to propose and test an underwater propulsor based on flutter of cantilevered fluid-conveying plate. It was found that the propulsion by plate undulation generated through internal flow was a feasible proposition, and it could be more efficient than propulsion by fluid jet alone. Afterwards, Hellum et al.\cite{40} reported a fluid-conveying and fluid-immersed pipe model to propose a submersible, which uses a combination of jet action and flutter instability induced tail motion to produce thrust. The equations of motion for an immersed fluid-conveying pipe affixed to a rigid body were derived, and the stability and thrust generated by the fluttering tail were investigated. The results showed that a fluttering flexible tail was capable of generating higher propelled speed than a dimensionally identical rigid tail, which is consistent with those in numerical studies on the propulsion of flapping flexible swimmers in fluid\cite{42–45}.

According to Ref.\cite{46}, interestingly, the biological snake moving in water tends to ignore the fin form but follows the form of sinusoidal meandering swing. The dynamic force for continuous forward motion of the snake in fluid mainly results from collision and friction between the snake body and fluid\cite{47}. This movement mode is similar to high-order mode vibrations of cantilevered flexible beams. As a result, the above few studies inspire the present work, which focuses on designing an underwater bionic snake robot driven by high-order nonlinear vibrations of a soft pipe conveying fluid. The objective of this research is to develop an effective FSI model by combining theoretical modeling with computational fluid dynamic (CFD) analysis to simulate and characterize the forward movement of bionic snake robot, which is compared with experimental measurements. To this end, the concept of designing a bio-inspired snake robot is proposed, and its propulsion mechanism is elaborated in Section 2. In Section 3, the head of robot is fixed, and then the nonlinear vibrations of the soft tail are measured by experiments
and compared with theoretical results to verify the theoretical modeling based on the ANCF. In the following, as the head is free, the moving forward speed of the robot is measured and compared with that predicted by the FSI model. Finally, some important conclusions are drawn in Section 4.

2 Concept design and propulsion mechanism

The concept schematic of underwater bio-inspired snake robot is designed as shown in Fig. 1(a). It simply consists of a rigid body as the head and a soft pipe conveying fluid as the tail. The soft tail is clamped to the head. \( OXY \) denotes the inertial reference frame, and \( Oxy \) is a body-fixed reference frame. It is assumed that the robot is moving in the \( XY \)-plane. The head is symmetric with respect to the \( xz \)-plane and is uniform along its length. A corollary to this assumption is that both the head and tail are symmetric with respect to the \( xz \)-plane. The soft tail sways like the Euler-Bernoulli beam due to the fact that the aspect ratio of the pipe is large enough and assumed to be in-extensible during oscillations. In addition, the internal flow in the tail is steady and has a uniform velocity profile. When the internal flow velocity \( u \) is beyond a critical value \( u_{cr} \), a flutter of the soft tail occurs, and displays periodic high-order mode vibration behaviors. In this way, the designed robot can be propelled forward by virtue of nonlinear vibrations of the tail in water.

The propulsion mechanism of the robot can be described in Fig. 1(b). As the tail starts to vibrate in the fluid field, it generates friction and collision between the tail and the fluid gradually forming shedding vortex in the wake of the tail. The higher the vibration amplitude

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Fig. 1 (a) Concept design of bio-inspired snake robot; (b) moving processes of designed snake robot in water; and (c) snapshot of biological snake swimming in water\(^{[48]}\) (color online)
is, the larger the vortex shedding is. Then, the tail sways to the other side and forms a new shedding vortex with an opposite direction. These alternative and periodic shedding vortexes produce lift and drag forces on the robot. As a result, it yields fluid forces in the $x$- and $y$-directions, which can be used to illustrate the propulsion of a designed robot. In fact, the tail of the robot swaying like a snake generates thrust through periodic swing pectorals. Figure 1(c) shows the snapshot of a real snake swimming in water\cite{38}. It is observed that during swimming, swaying of the snake's tail is quite similar to high-order mode vibrations of the designed robot.

3 Results and discussion

In this section, experimental measurements, corresponding theoretical modeling, and CFD simulations are conducted to realize and explore the vibration characteristics and propulsion performance of the designed bio-inspired robot. Firstly, in order to clearly and accurately extract measured data from experiments, the head of the robot is fixed. In this case, vibration characteristics of the tail are compared between experimental and theoretical results. In the next, the head is free, and hence the robot can be well propelled and move forward in water at a swimming speed. Subsequently, an FSI model is developed to simulate the moving process of the robot, which offers a further discussion on the propulsion mechanism.

3.1 Robot with fixed head

At first, a series of experiments are conducted to evaluate the oscillation trait of the robot’s tail for different internal flow velocities. The head of the robot is fixed, and a high-speed camera is used to shot the tail’s oscillations. It is noted that the tail is fabricated by a soft pipe with polyethylene materials. The tail is initially straight and flexible with a length of 40 cm. The inner and outer diameters are 6 mm and 8 mm, respectively. The experimental setup includes a small water pump, a single tail, and an adaptor that connects the tail with head. The role of the water pump is to pump water from the surrounding environment to the fixed end of the tail, and then the water is ejected from the tail’s free end. Clearly, the tail suffers from flutter as the internal flow is beyond a critical value. In this experiment, it is observed that when the water pump is powered on, the soft tail undergoes periodic vibrations due to internal fluid flows, which can be seen in Fig.2. Accordingly, the live snapshots are recorded in the act of tail’s vibrating in one cycle under an internal flow velocity of $3.04 \text{ m} \cdot \text{s}^{-1}$, as depicted in Fig. 2. Moreover, by inspecting this figure, it can be found that as time goes on, the tail is alternatively vibrating from one to the other side and displays a second mode oscillation, that is, swaying. Indeed, this swaying motion resembles that of a snake’s tail swimming in water. During a full vibration cycle, the tail displays a wave propagation behavior, which is different from the classical second mode of a cantilevered beam.

It is noted that the thrust of the robot depends on the vibration amplitude and frequency of the tail, which is determined by the internal flow velocity. This is because the flow velocity has a big impact on vibration characteristics of the tail. While the internal flow velocity can be controlled by adjusting the outlet velocity of the water pump. In this way, by regulating outlet velocity, three different flow velocities are recorded, namely, $3.04 \text{ m} \cdot \text{s}^{-1}$, $3.18 \text{ m} \cdot \text{s}^{-1}$, and $3.25 \text{ m} \cdot \text{s}^{-1}$. Table 1 gives the tip-end vibration amplitude and frequency of the tail for these three different values of flow velocity. It is seen that when the flow velocity is increased from $3.04 \text{ m} \cdot \text{s}^{-1}$ to $3.25 \text{ m} \cdot \text{s}^{-1}$, the vibration amplitude and frequency are both increased. Especially for the vibration amplitude, it is increased from 19.17 cm to 29.78 cm. This indicates that a small increment of flow velocity can result in a large enhancement of vibration amplitude. This trend is consistent with those predicted by previous studies\cite{5}.

Subsequently, we propose to build a theoretical model for better understanding of the tail’s nonlinear vibration behaviors. The ANCF is employed to establish a nonlinear dynamic theo-
Fig. 2  Live snapshots of tail vibrating in one full cycle when internal flow velocity is 3.04 m·s\(^{-1}\) (color online)

Table 1  Tip-end vibration amplitude and frequency of tail for different values of internal flow velocity

| Flow velocity/(m·s\(^{-1}\)) | Tip-end vibration amplitude/cm | Vibration frequency/Hz |
|-----------------------------|-------------------------------|------------------------|
| 3.04                        | 19.17                         | 0.50                   |
| 3.18                        | 28.33                         | 0.70                   |
| 3.25                        | 29.78                         | 0.80                   |

Theoretical model for the designed tail, which is considered to be slender and only vibrates in the XY-plane. It should be mentioned that the present theoretical study qualitatively explains the vibration behaviors of the tail of robot compared with experimental data. Therefore, the damping from the surrounding fluid domain is not considered here. In this way, the 2-node planar ANCF elements are chosen to discretize the tail. Accordingly, the extended Lagrange equation introduced by Irschik and Holl\(^{[49]}\) is used to derive the nonlinear governing equations of this system. The equation can be written as follows:

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \int_{S} \mathbf{d}a \cdot (v_F - v_P) \frac{\partial T'}{\partial \dot{q}} - \int_{S} \mathbf{d}a \cdot \left( \frac{\partial v_F}{\partial q} - \frac{\partial v_P}{\partial q} \right) T' = \mathbf{Q},
\]

where \(T\) denotes the total kinetic energy of the system, \(q\) and \(\dot{q}\) are, respectively, the generalized coordinate vector and the velocity vector; \(\mathbf{Q}\) represents the vector of generalized forces including the generalized elastic force and the non-conservative force. The two surface integrals shown in Eq. (1) should be only evaluated at the tail element boundaries \(S\). In addition, \(v_F\) and \(v_P\), respectively, denote the absolute velocities of the internal fluid and pipe. \(T'\) represents the kinetic energy per unit volume of the fluid; \(\mathbf{d}a\) is an oriented surface element on \(S\).

After the terms shown in Eq. (1) are determined, a series of operations are performed. Then, we can obtain the nonlinear dynamic equation of the tail element, which can be found in the following form:

\[
M_e^* \ddot{q}^* + C_e^* \dot{q}^* + K_e^* q^* + N_e^*(q^*) = 0.
\]
respectively; and $N_e^*(q^*)$ represents the vector of nonlinear terms. These matrices and the vector of nonlinear terms can be given as follows:

$$
\begin{align*}
M_e^* &= \int_0^l S^T S \, dx, \quad C_e^* = u\sqrt{\beta} \int_0^l (S^T S' - S'^T S) \, dx + u\sqrt{\beta}L(S^T S|_{x=l} - S^T S|_{x=0}), \\
K_e^* &= -\left(u^2 + \frac{1}{2}\Pi_0\right) \int_0^l S'^T S' \, dx + u^2L(S^T S'|_{x=l} - S^T S'|_{x=0}), \\
N_e^*(e) &= \frac{1}{2}\Pi_0 \int_0^l S'^T S' q^* q^* S'^T S' q^* \, dx \\
&\quad + \int_0^l \left(\frac{((\tilde{I}S')^T S'' + S''^T (\tilde{I}S')^T S'')q^* q^* (\tilde{I}S')^T S' q^*}{(q^* S'^T S' q^*)^3} + \frac{3S'^T S' q^* q^* (\tilde{I}S')^T S'' q^*}{(q^* S'^T S' q^*)^4}\right) \, dx.
\end{align*}
$$

(3)

In addition, in order to obtain the above non-dimensional expressions, we introduce the following quantities:

$$
\tau = \left(\frac{EI}{M + m}\right)^{\frac{1}{2}} \frac{t}{L^2}, \quad q^* = \frac{q}{L}, \quad u = \left(\frac{M}{EI}\right)^{\frac{1}{2}}UL, \quad \beta = \frac{M}{M + m}, \quad \Pi_0 = \frac{A_P L^2}{I},
$$

(4)

where $EI$ is the bending stiffness of the tail, $M$ and $m$ are mass per unit for fluid and tail, respectively, $L$ denotes the length of tail, $U$ is the internal flow velocity, and $A_P$ is the cross-section area of the tail. According to the concept of the traditional finite element method, these matrices and vector for tail elements can be assembled into the corresponding global matrices and vector, and then we can have the nonlinear governing equation of the whole tail system,

$$
M\ddot{e} + C\dot{e} + Ke + Ne(e) = 0,
$$

(5)

where $M$, $C$, and $K$ are the mass, damping, and stiffness matrices for the whole tail structure, respectively; and $N(e)$ represents the assembled vector of nonlinearities. Then, based on this equation, the nonlinear vibration behaviors of the tail can be determined with the aid of fourth-order Runge-Kutta integration algorithm.

The corresponding experimental results for vibration responses of the tail are obtained through graphic processing, as depicted in Fig. 3(a). $x$-axis and $y$-axis denote the position along the tail length and vibration amplitude, respectively. They both are the ratio of the length to the tail. It is noted that the tail displays a second mode vibration, and the peak vibration amplitude for the free end of the tail is near 0.5L. Based on theoretical modeling, the oscillation shape of the tail is acquired, as shown in Fig. 3(b). Inspecting Fig. 3, it is noted that although the vibration mode of the pipe predicted by the theoretical model can be qualitatively compared with experimental measurements, there is a clear difference between experimental and theoretical results, especially for the vibration amplitude at the free end of the pipe. The reason for this difference is that the theoretical modeling does not consider the effect of external fluid surroundings at the free end.

According to experimental and theoretical results, the tail of the designed robot could indeed exhibit swaying behaviors in water. However, whether this kind of swaying can truly result in propulsion force in the $x$- and $y$-directions, it requires a further clarification. To this end, a CFD simulation based on the ANSYS workbench (Fluent) is performed to determine the propulsion force. In this way, a simulation model with a 160 cm $\times$ 100 cm 2D rectangle fluid domain is constructed. In the simulation, we use the $k$-epsilon model, and the time step is set
to be 0.01 s. The dynamic mesh technology is applied. It should be stated that the tail of the robot is assumed to be a cantilevered beam model, and the vibration responses are controlled by inputting the experimental data shown in Fig. 3(a).

The vorticity diagram for vortex shedding process of the tail vibrating in water at different moments are plotted in Fig. 4. The black line indicates the tail structure, and the white background represents the surrounding fluid domain. $T_0$ means the time for one period of vibration. $x$-axis and $y$-axis denote the coordinate of tail along its length and the corresponding vibration amplitude, respectively. By inspecting Fig. 4, during one full cycle (from $t = 0$ to $t = T_0$), it is observed that when the tail sways to positive $y$-direction (e.g., $t = 0$), vortex shedding produces a positive vorticity (red color). While the negative vorticity (blue color) is generated as the tail is swaying to negative $y$-direction (e.g., $t = \frac{1}{2}T_0$). In addition, the size of the shedding vortex is becoming larger when the tail is swaying from one side to the other. For example, it is varying from $t = \frac{1}{4}T_0$ to $t = \frac{1}{2}T_0$ and from $t = \frac{3}{4}T_0$ to $t = \frac{1}{2}T_0$, as seen in Fig. 4. In this way, the positive and negative vortexes are alternately shedding in the wake of the tail, resulting in periodic lift and drag forces. Indeed, such fluid forces are detected through CFD simulation in the $x$- and $y$-directions, which are plotted in Fig. 5. It is noted that the $x$-axis means the time in ten cycles, and the $y$-axis denotes the fluid force coefficients $C_x$ and $C_y$. The coefficients $C_x$ and $C_y$ are obtained by $F_x = \frac{1}{2} \rho v^2 L C_x$ and $F_y = \frac{1}{2} \rho v^2 L C_y$. The fluid forces $F_x$ and $F_y$ can be calculated by integrating alone the length of the tail. $\rho$ is the density of water, $v$ is the oscillation velocity of the free end of the tail, and $L$ is the length of the tail. Clearly, the fluid forces acting on the tail change periodically. The black dot dashed lines in Fig. 5 indicate the mean value varying with time going on. However, at the beginning of tail’s swaying, there is a fluctuation of the fluid force. After two cycles, the fluid force tends to be stable. Inspecting Fig. 5(a), the mean value of $C_x$ is above zero in stable state. This indicates that the swaying of tail behavior produces a thrust force in positive $x$-direction when the head of the robot is fixed. While the mean value of $C_y$ is nearly zero, as seen in Fig. 5(b), this means that the robot will not move forward in the $y$-direction. However, the head of the robot may turn left and right during moving forward if the head is free. Consequently, it is believed that such a swaying of tail is capable of generating a stable thrust in the $x$-direction for the designed robot.

3.2 Robot with free head

We next address a major challenge for bionic snake robot, that is, achieving a freely moving forward. Except for the soft tail, a head made of plastic material is designed. It should be mentioned that it is difficult to fabricate and get balance if the outline shape of head is designed to be elliptic shown in Fig. 1(a). To overcome this, the head is fabricated to be T-shape so that
it keeps a good balance in water. In this way, the head can always keep neutrally buoyant during moving forward. The length of the head is 18 cm. The overall weight is 230 g. The top and front views for the head are shown in Fig. 6(a). Finally, by controlling the internal flow velocity, the tail can display periodic vibrations and deformations, so as to promote the robot to propel freely. Furthermore, since the internal fluid will discharge from the free end of the tail, this jet action may also provide a synchronous propulsive force. The live snapshots for robot moving forward in water are illustrated in Fig. 6(b). The head is in white color, and the tail is in yellow color. Experiments show that the robot can freely advance in a straight line. It should be mentioned that although the robot is in free boundary condition, its tail is fixed to the head. Thus, the tail can be deemed to be in fixed-free boundary. This can be verified by observing

Fig. 4 Vortex shedding process of tail vibrating in water during one full cycle (color online)
Fig. 5  Fluid force coefficient in (a) x- and (b) y-directions varying with time in ten cycles (color online)

Fig. 6  Live snapshots of robot move forward in water when internal flow velocity is $3.25 \text{ m} \cdot \text{s}^{-1}$ (color online)
the snapshots in Fig. 6(b) that the tail displays a clear second mode vibration behavior during swimming, which is similar to that observed in Fig. 2. For example, the swaying shapes of the tail, between $t = 1$ s in Fig. 6(b) and $t = 1.63$ s in Fig. 2, between $t = 1.6$ s in Fig. 6(b) and $t = 0.45$ s in Fig. 2, between $t = 4$ s in Fig. 6(b) and $t = 1.45$ s in Fig. 2, and between $t = 4.5$ s in Fig. 6(b) and $t = 2$ s in Fig. 2, have a qualitative comparison. This is because the second mode oscillation of pipe produces fluid forces acting on the robot to propel it forward.

To further illustrate the propulsion mechanism of robot freely moving forward in water, an FSI model is developed to simulate the vortex shedding process which is addressed in Fig. 7. In the simulation, the head is rigid, and has no deformations. The data for tail’s oscillation shape are offered according to the theoretical model based on the ANCF. This model is validated in Subsection 3.1. In this way, we combine the theoretical analysis with CFD simulation to establish the FSI model. Inspecting Fig. 7, the large and significant vortex is alternately shedding...
in the wake of the tail. Similarly, the negative and positive vorticities are formed periodically in blue and red colors. In addition, the vortex is also becoming larger with the increase in the vibration amplitude. For example, the robot is moving ahead from $t = \frac{1}{4}T_0$ to $t = \frac{3}{4}T_0$ and from $t = \frac{1}{2}T_0$ to $t = \frac{5}{2}T_0$. Interestingly, it is noted that there are also negative and positive vortexes shedding from the front of the head, and they are asymmetric. This states that the head turns left and right periodically during swimming forward. Indeed, this left and right rotation phenomenon is observed in experiments shown in Fig. 6(b) but not apparent.

Different from simulations in the case of fixed head, there is a clear trailing vortex in the fluid field, as depicted in Fig. 8(a). Then, the fluid forces $F_x$ and $F_y$ in the $x$- and $y$-directions varying with time are obtained by integration along the surface of the robot. Likewise, the coefficients of $C_x$ and $C_y$ are calculated through $F_x$ and $F_y$. The results are plotted in Figs. 8(b) and 8(c). The black dot dashed line represents the mean value. Inspecting Fig. 8(b), at the beginning of swimming, the mean fluid force in the $x$-direction is negative. This states that the fluid force is large enough to propel robot forward in negative $x$-direction and the robot is swimming in acceleration. After about four cycles, the mean fluid force is close to zero, this demonstrates that the mean swimming speed tends to be stable. It is clearly embodied in Fig. 8(d) that the mean swimming speed is increased at the beginning, and then gradually arrives at a stable value. As to the fluid force in the $y$-direction shown in Fig. 8(c), the value becomes zero very quickly. It signifies the fluid force acting on the robot in the $y$-direction reaches a balance soon, so that the robot will not move in the $y$-direction. Therefore, the robot is swimming in a straight line in

![Figure 8](image-url)

**Fig. 8** (a) Overall view of vortex shedding for robot moving ahead; fluid force coefficient in (b) $x$- and (c) $y$-directions; and (d) swimming speed of robot varying with time in ten cycles (color online)
the $x$-direction. As for the swimming speed, it also changes with time periodically. This is due to periodical variations of fluid forces. When the internal flow velocity is $3.18 \text{ m} \cdot \text{s}^{-1}$, the stable mean swimming speed is $10.92 \text{ cm} \cdot \text{s}^{-1}$ based on the FSI model. However, the measured moving speed in experiments is $10.79 \text{ m} \cdot \text{s}^{-1}$, as shown in Table 2. The error is $1.2\%$. This suggests that the present FSI model is effective in simulating the swimming behavior of designed robot from both qualitative and quantitative aspects.

| Mode          | Flow velocity/(m $\cdot$ s$^{-1}$) | Swimming speed/(cm $\cdot$ s$^{-1}$) |
|---------------|------------------------------------|--------------------------------------|
| Vibration mode| 3.04                               | 8.39                                 |
|               | 3.18                               | 10.79                                |
| Jet mode      | 3.25                               | 11.15                                |

According to Table 1, it is noted that the internal flow velocity has a great effect on vibration amplitude of tail, and hence the swimming speed of the robot can be affected. Therefore, the mean swimming speed of robot at different internal flow velocities are measured in experiments. The corresponding experimental data are summarized in Table 2. It can be found that when the flow velocity is increased from $3.04 \text{ m} \cdot \text{s}^{-1}$ to $3.25 \text{ m} \cdot \text{s}^{-1}$, the swimming speed is increased from $8.39 \text{ cm} \cdot \text{s}^{-1}$ to $11.15 \text{ cm} \cdot \text{s}^{-1}$. This means that higher vibration amplitude results in a larger moving speed of the robot. It should be mentioned that the designed robot in pure jet mode can also be propelled forward. Subsequently, a further experiment is done to compare the swimming speed of the robot in vibration mode with that in jet mode. In the pure jet mode experiments, the soft tail is not removed, but is hardened to ensure that the deformations will not occur in the tail. When the internal flow velocity is $3.25 \text{ m} \cdot \text{s}^{-1}$, the swimming speed in the jet mode is measured to be $9.18 \text{ cm} \cdot \text{s}^{-1}$, which is lower than that of $11.15 \text{ cm} \cdot \text{s}^{-1}$ in the vibration mode. The propelled speed is enhanced by 21% through designing vibrations of the tail. That is to say, the vibration mode is more efficient than the jet mode. For one thing, utilization of nonlinear vibrations based on the soft pipe conveying fluid is available for improving the propulsion performance. For another one, the present study provides a new idea for designing driving devices.

4 Conclusions

The present research is devoted to proposing a new underwater bio-inspired robot by utilizing the nonlinear vibrations of a soft pipe conveying fluid. Experiments indicate that the designed robot moves ahead by virtue of a second mode vibration of the tail, which is similar to that of a snake swimming forward in a swaying mode. An FSI simulation model based on combining the theoretical modeling with the CFD analysis is constructed to investigate the swimming speed, fluid forces and swaying shape of the robot. The results obtained by the FSI model agrees well with those by experiments. Some important conclusions are drawn as follows.

(i) Based on the ANCF, a theoretical model is established to explore vibration behaviors of the tail. It is shown that the tail displays a second mode oscillation, and the peak amplitude arrives as high as 0.5 times length of the tail, which can be validated by experimental measurements.

(ii) An effective FSI model is developed to simulate the designed robot swimming in water. It is indicated that the swimming speed is gradually increased to $11 \text{ cm} \cdot \text{s}^{-1}$ when the fluid forces are in stable state, which agrees well with experiments as well as the swaying shape.

(iii) The experimental measurement demonstrates that the propulsion speed can be enhanced
by 21% for the robot driven by the second mode vibrations of tail compared with that in the pure jet mode, which suggests that using nonlinear vibrations is more efficient than the pure jet.

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