Universal random codes: Capacity regions of the compound quantum multiple-access channel with one classical and one quantum sender

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We consider the compound memoryless quantum multiple-access channel (QMAC) with two sending terminals. In this model, the transmission is governed by the memoryless extensions of a completely positive and trace preserving map which can be any element of a prescribed set of possible maps. We study a communication scenario, where one of the senders shares classical message transmission goals with the receiver while the other sends quantum information. Combining powerful universal random coding results for classical and quantum information transmission over point-to-point channels, we establish universal codes for the mentioned two-sender task. Conversely, we prove that the two-dimensional rate region achievable with these codes is optimal. In consequence, we obtain a multi-letter characterization of the capacity region of each compound QMAC for the present transmission task.

I. INTRODUCTION

Real-world communication situations usually involve more than two communication parties. This rather general statement typically leads to highly nontrivial coding scenarios, when translated to information theory. A very basic situation in this field of topics is, when two or more sending parties are connected to a receiver via a multiple-access channel (MAC). The rate and performance one of the sending parties can achieve is in general strongly connected to the rates of the other parties, and finding code constructions which allow to determine the set of rate tuples which are asymptotically achievable is a task of some piquancy. Early results which determined the average-error capacity region for classical message transmission over a memoryless classical MAC are due to Ahlswede [2] and Liao [23]. Among others, these works stimulated a vital research in classical information theory (see [6], Chapter 14 for an overview). In case of the maximal error, the capacity region is still unknown while Dueck gave an example of a MAC where average and maximal error capacity regions are different [16].

Regarding the setting, were the senders are connected to the receiver by a memoryless quantum multiple-access channel (QMAC), the first notable results include the paper [31]. Therein the region of achievable rate pairs was determined for the case that all senders aim to convey classical messages. For other scenario where some of the senders aim to send quantum information, the achievable rates where characterized in [35], while the quantum capacity region (i.e. the set of rate achievable rate pairs if all senders send quantum information) was derived in [21, 35]. However, all of the mentioned results were proven under the assumption, that the transmission c.p.t.p. map which governs the channel transmission is perfectly known to the senders as well as to the receiver. In this work, we try to impose more realistic assumptions on the channel model. We consider the compound memoryless QMAC, where the communication parties have no precise knowledge of the channel, but instead have only a priori knowledge of a set of channels, in which the generating map is contained. The consequence of their imprecise knowledge is, that they have to use “universal” codes, which perform well on all channels in the set.

The compound channel model which already has been studied in classical Shannon theory since the 1960s. In the domain of quantum Shannon theory, past research activities regarding compound quantum channels where mostly concentrated on point to point quantum channels, leading to determination of the classical capacity [13, 18, 30], and the genuine quantum capacities [5, 9] of a quantum channel. Regarding compound quantum channels with having more than two users, the research was concentrated on private classical message transmission over wiretap channels, i.e. channels having one sender but two receiving parties. Suitable codes for this situation were developed in [11] generalizing techniques introduced for the case of classical compound wiretap channels in [10] to the quantum setting.
The first codes for classical message transmission over compound classical-quantum MACs were provided in [20].

In this article, we aim to extend the scope of multi-user quantum Shannon theory in the direction of models having more than one sender and involving channel uncertainty. We consider a QMAC with two sending parties $A$ and $B$ in the “hybrid” situation, where $A$ pursues the target of transmitting classical messages while sender $B$ aims for entanglement transmission.

A. Outline

In Section III, we provide ourselves with precise definitions regarding the channel model and codes used in this work. Therein, we also state Theorem 6 which is the main result of this work which is a multi-letter characterization of the capacity region of the compound QMAC with a classical and a quantum sender. Section IV is of rather technical nature. We introduce random classical message transmission and entanglement transmission codes which are crucial ingredients for our reasoning. Section V contains the proofs of Theorem 6. In Section VA we construct suitable universal hybrid codes for the QMAC. These are obtained by combining ideas from [35] with the universal random codes from the previous section. By providing the converse part of Theorem 6 in Section V B we complete our proof.

B. Related work

The capacity regions of a perfectly known QMAC with one classical and one quantum sender (and moreover also the genuine quantum capacity regions of that channel model) where determined by Yard et al. in [35]. The strategy used therein to derive codes being sufficient to prove the coding theorem is as follows. By combining known random coding results for classical message transmission from [26], [29], and entanglement transmission [14] for single-user quantum channels in a sophisticated way, the authors constructed random codes for classical and quantum coding over the QMAC. Combining random codes to simultaneously achieve different transmission goals was long standard in classical multi-user Shannon theory, and can, in the quantum case traced back to [32], where the capacity regions of quantum multiple-access channels was determined in case that all senders wish to transmit classical messages. That a strategy in the mentioned manner is successful also in situations where classical and quantum transmission goals are to be accomplished simultaneously was first demonstrated in [14]. Therein, hybrid codes are constructed which allow to transmit classical and quantum information over a memoryless point-to-point quantum channel at the same time. The capacity region for simultaneous transmission of classical and quantum information was also shown to exceed the obvious region which to be achievable by time-sharing strategies for some quantum channels.

In both cases, suitable random codes already exist in the literature without having been exploited simultaneously transmission yet. In case of universal classical message transmission, independent first results can be found in [27], [18], and [13]. In this work, we exploit the recent and very powerful techniques which were added to the aforementioned results in [30]. The random entanglement transmission codes we use in this work where developed in [8], [9]. The reader may note, that the approach pursued in [14] to derive good random quantum codes seems to be not suitable in case of compound quantum channels, as the discussion in Section VII of [11] suggests. The random codes derived in [8] stem from generalization of the codes in [28] which where derived in spirit of the so-called decoupling approach to the quantum capacity (see also [25] for a similar application of the decoupling idea.)

II. NOTATION AND CONVENTIONS

All Hilbert spaces which appear in this work are finite dimensional over the field of complex numbers equipped with the standard euclidean scalar product. For a Hilbert space $\mathcal{H}$, $\mathcal{L}(\mathcal{H})$ denotes the set of linear maps (or matrices), while $\mathcal{S}(\mathcal{H})$ denotes the set of density matrices, and $\mathcal{U}(\mathcal{H})$ the set of unitaries on $\mathcal{H}$. For an alphabet $\mathcal{X}$ (which we always assume to be of finite cardinality), we denote the simplex of probability distributions on $\mathcal{X}$ by $\mathcal{P}(\mathcal{X})$. With a second Hilbert space $\mathcal{K}$, we denote by $\mathcal{C}^{1}(\mathcal{H},\mathcal{K})$ the set of completely positive (c.p.) trace non-increasing maps while $\mathcal{C}(\mathcal{H},\mathcal{K})$ is the notation for completely positive and trace preserving (c.p.t.p.) maps. For positive semi-definite matrices $a,b \in \mathcal{L}(\mathcal{H})$, we use the definition

$$F(a,b) = \|\sqrt{a} \sqrt{b}\|_1$$
for the (quantum) fidelity. For a c.p.t.p. map $\mathcal{N} \in \mathcal{C}(\mathcal{H}_A, \mathcal{H}_B)$ and a density matrix $\rho$, we use the entanglement fidelity defined by

$$F_e(\rho, \mathcal{N}) := \langle \Psi, \text{id}_B \otimes \mathcal{N}(|\Psi\rangle\langle\Psi|) \Psi \rangle,$$

where $\Psi$ is any purification of $\rho$. The von Neumann entropy of a state $\rho$ is defined by $S(\rho) := -\text{tr}(\rho \log \rho)$, and we will use in this work several entropic quantities which derive from it. For a bipartite state $\rho \in \mathcal{S}(\mathcal{H}_A \otimes \mathcal{H}_B)$, \[I_c(A|B,\rho) := S(\rho_B) - S(\rho),\]  
(1) defines the coherent information of $\rho$, while \[I(A;B,\rho) := S(\rho_A) + S(\rho_B) - S(\rho)\]  
(2) is the quantum mutual information. We will employ the usual notation for systems, which have classical and quantum subsystems. E.g.

$$\rho_{XB} := \sum_{x \in \mathcal{X}} p(x) |x\rangle \langle x| \otimes \rho_x$$

represents the preparation of a bipartite system, where one system is classical (with preparation being a probability distribution $p \in \mathcal{P}(\mathcal{X})$) while $\rho_x$ is a density matrix for each outcome $x$ of $\mathcal{X}$ (the random variable with probability distribution $p$). For a set $A \subset \mathbb{R}_+^3 \times \mathbb{R}_+^3$, we denote the closure of $A$ by $\text{cl}A$. Moreover, we define for each $l \in \mathbb{N}$ the set $\frac{1}{l}A$ by

$$\frac{1}{l}A := \{(x, y) : (x, y) \in A\}.$$ 

For each $\delta > 0$ we moreover set

$$A_\delta := \{x \in \mathbb{R}_+^3 \times \mathbb{R}_+^3 : \exists y \in A : |x - y| \leq \delta\}.$$ 

### III. BASIC DEFINITIONS AND MAIN RESULT

In this section we provide precise definitions of codes and capacity regions considered in this work. Let with three Hilbert spaces $\mathcal{H}_A$, $\mathcal{H}_B$, $\mathcal{H}_C$, under control of communication parties labelled by $A$, $B$, and $C$. While $A$, and $B$ act as senders for the channel, $C$ is designated as receiver. Let $\mathcal{M} \subset \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C)$ be a set of c.p.t.p. maps. If not otherwise specified, we do not assume further properties of the set (we especially do not demand $\mathcal{M}$ to be a finite set.)

The compound memoryless quantum multiple access channel (QMAC) generated by $\mathcal{M}$ (the compound QMAC $\mathcal{M}$ for short) is given by the family $\{M^{\otimes n} : M \in \mathcal{M}\}_{n=1}^{\infty}$ of transmission maps. The above definition is interpreted as a channel model, where the transmission statistics for $n$ uses of the system is governed by $M^{\otimes n}$, where $M$ can be any member of $\mathcal{M}$. We designate $A$ as sender transmitting classical messages. In this work, we consider two different coding scenarios which differ in the quantum transmission task $B$ performs. Adopting the terminology from [53] consider Scenario I (B aims to perform entanglement generation), Scenario II (B aims to perform entanglement transmission). For the rest of the section we fix a set $\mathcal{M} = \{M^{\otimes n}\}_{n=1}^{\infty}$.

**Definition 1** (Scenario-I code). An $(n,M_1,M_2)$-Scenario-I code for the compound QMAC $\mathcal{M}$ is a family $\mathcal{C} = (V(m), \Psi, D_m)_{m=1}^{M_1}$, where with an additional Hilbert spaces $\mathcal{F}_A \cong \mathcal{F}_C \cong \mathcal{C}^{M_2}$

- $V : [M_1] \to \mathcal{S}(\mathcal{H}_A^{\otimes n})$ is a classical-quantum channel, i.e. $V(m) \in \mathcal{S}(\mathcal{H}_A^{\otimes n})$ for each $m \in [M_1]$.
- $\Psi \in \mathcal{F}_A \otimes \mathcal{H}_A^{\otimes n}$ is a pure state.
- $D_m \in \mathcal{C}^1(\mathcal{H}_C^{\otimes n} \otimes \mathcal{C}^{M_1} \otimes \mathcal{F})$ for each $m \in [M_1]$ such that $\sum_{m=1}^{M_1} D_m$ is a quantum channel.

In the situation, where $B$ and the receiver perform entanglement transmission over the QMAC, we define
The following theorem is the main result this work.

We next define the performance functions of the codes introduced above

\[ P^I(C, M^{\text{SN}}, m) := F \left( (m | m) \otimes \Phi, \id_F \otimes D \circ M^{\text{SN}}(V(m) \otimes \Phi) \right), \]

and set for \( X \in \{I, II\} \)

\[ P^I(C, M) := \frac{1}{M_1} \sum_{m=1}^{M_1} P^I(C, M, m). \]

**Definition 2** (Scenario-II code). An \((n, M_1, M_2)\)-Scenario-II code for the compound QMAC \( \mathcal{M} \) is a family \( \mathcal{C} = (V(m), E, D_m)_{m=1}^{M_1} \), where with an additional Hilbert spaces \( \mathcal{F}_A = \mathcal{F}_C = \mathcal{C}^{M_2} \)

- \( V : [M_1] \rightarrow \mathcal{S}(\mathcal{H}_A^{\text{SN}}) \) is a classical-quantum channel.
- \( E \in \mathcal{C}(\mathcal{F}_B, \mathcal{H}_B^{\text{SN}}) \)
- \( D_m \in \mathcal{C}(\mathcal{H}_C^{\text{SN}}, \mathcal{C}^{M_1} \otimes \mathcal{F}) \) for each \( m \in [M_1] \), such that \( \sum_{m=1}^{M_1} D_m \) is trace preserving.

We define the performance functions of the codes introduced above

\[ P^I(C, M^{\text{SN}}, m) := F \left( (m | m) \otimes \Phi, \id_F \otimes D \circ M^{\text{SN}}(V(m) \otimes \Phi) \right), \]

and set for \( X \in \{I, II\} \)

\[ P^I(C, M) := \frac{1}{M_1} \sum_{m=1}^{M_1} P^I(C, M, m). \]

**Definition 3** (Achievable rates). Let \( X \in \{I, II\} \). A pair \((R_1, R_2)\) of non-negative numbers is called an achievable Scenario-X rate for the compound QMAC \( \mathcal{M} \), if for each \( \epsilon, \delta > 0 \) exists a number \( n_0 = n_0(\epsilon, \delta) \), such that for each \( n > n_0 \) there is an \((n, M_1, M_2)\)-Scenario-X code \( \mathcal{C} \) for \( \mathcal{M} \) such that the conditions

1. \( \frac{1}{n} \log M_i \geq R_i - \delta \) for \( i \in \{1, 2\} \), and
2. \( \inf_{m \in S} P^X(C, M^{\text{SN}}) \geq 1 - \epsilon \)

are simultaneously fulfilled. We define the Scenario-X capacity region of the compound QMAC \( \mathcal{M} \) by

\[ \mathcal{C}Q^X(\mathcal{M}) := \{(R_1, R_2) \in \mathbb{R}_+^2 : (R_1, R_2) \text{ achievable Scenario-X rate for } \mathcal{M}\}. \]

As an operational fact, following directly from the above definitions, we have the following.

**Fact 4.** For each \( \mathcal{M} \subset \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C), \mathcal{C}Q^I(\mathcal{M}) \) and \( \mathcal{C}Q^I(\mathcal{M}) \) are compact convex subsets of \( \mathbb{R}^2 \).

**Fact 5.** Let \( \mathcal{M} \subset \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C) \). It holds \( \mathcal{C}Q^I(\mathcal{M}) \subset \mathcal{C}Q^I(\mathcal{M}) \).

**Proof.** Let for an arbitrary but fixed blocklength \( n \in \mathbb{N} \) \( C := (V(m), E, D_m)_{m=1}^{M_1} \) be an \((n, M_1, M_2)\) Scenario-II code. Let for fixed \( \mathcal{M} \in \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C), m \in [M_1] \), a spectral decomposition of \( \id_F \otimes E(\Phi) \) given by

\[ \id_F \otimes E(\Phi) = \sum_{i=1}^{N} \lambda_i \Psi_i. \]

It holds for each \( m \in [M_1] \)

\[ F(|m \rangle \langle m| \otimes \Phi, \id_F \otimes D \circ M^{\text{SN}} \circ \id_H^{\text{SN}} \otimes E(V(m) \otimes \Phi)) = \sum_{i=1}^{N} \lambda_i F(|m \rangle \langle m| \otimes \Phi, \id_F \otimes D_m \circ M^{\text{SN}}(V(m) \otimes \Psi_i)). \]

If now \( j \) is any index such that \( F(|m \rangle \langle m| \otimes \Phi, \id_F \otimes D_m \circ M^{\text{SN}}(V(m) \otimes \Psi_j)) \) is maximal, the \((n, M_1, M_2)\) Scenario I code \( \tilde{C} := (V(m), \Psi_j, D_m)_{m=1}^{M_1} \) suffices \( P^I(\tilde{C}, M^{\text{SN}}) \geq P^I(C, M^{\text{SN}}) \) by the inequality in [3].

To enable us for concise statement of the main result, we introduce some more notation. Fix Hilbert spaces \( \mathcal{K}_A, \mathcal{K}_B, \mathcal{K}_C \), and an alphabet \( \mathcal{X} \). For given probability distribution \( p \), c.p.t.p. map \( T \in \mathcal{C}(\mathcal{K}_A \otimes \mathcal{K}_B, \mathcal{K}_C) \), cq channel \( V : \mathcal{X} \rightarrow \mathcal{S}(\mathcal{K}_A) \), pure state \( \Psi \in \mathcal{S}(\mathcal{K}_B^{\text{SN}}) \), we define an effective cq state

\[ \omega_{XBC} := \omega(T, p, V, \Psi) := \sum_{x \in \mathcal{X}} p(x) |x \rangle \langle x| \otimes \id_{K_B} \otimes T(\mathcal{V}(x) \otimes \Psi), \]

and a region

\[ C^{(1)}(T, p, V, \Psi) := \{(R_1, R_2) \in \mathbb{R}_+^2 : R_1 \leq I(X; C, \omega) \land R_2 \leq I(B\mid CX, \omega)\}. \]

The following theorem is the main result this work.
Theorem 6. Let $\mathcal{M} \subset \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C)$. It holds

$$CQ^I(\mathcal{M}) = CQ^H(\mathcal{M}) = \inf_{I=1}^\infty \bigcup \bigcup \bigcup 1 \in \mathcal{C}(\mathcal{M}^{\otimes n}, p, \psi, V, \psi)$$

(7)

Remark 7. The terms on the rightmost sides of the inclusion chains in the above theorem do not need convexification. The corresponding fact for the capacity region of the perfectly known QMAC was already proven in \cite{55}. For the reader’s convenience, we give an argument for the present case in Appendix B.

The proof of the equalities in Eq. (7) is split in several parts. Note, that the inequality $CQ^H(\mathcal{M}) \subset CQ^I(\mathcal{M})$ is Fact~\cite{55}. That the rightmost term in (7) is a subset of $C^I$ is the statement of Proposition\cite{12}. To complete the proof, we show, that $C^I$ is smaller than the rightmost term in Proposition\cite{17}.

IV. UNIVERSAL RANDOM CODES FOR MESSAGE AND ENTANGLEMENT TRANSMISSION

In this section, we state and discuss some universal random coding results for entanglement transmission and classical message transmission over single-sender channels. Most of the statements below, are already implicitly contained in the literature. However, the random nature of the codes where not explicitly stressed. Some additional properties which are connected to these random codes are revealed below, and may be useful in their own right.

A. Classical message transmission

For the reader’s convenience, we first introduce some terminology. A map $V : \mathcal{X} \rightarrow \mathcal{S}(\mathcal{H})$ with a (finite) alphabet $\mathcal{X}$ and a Hilbert space $\mathcal{H}$ is called a classical-quantum (cq) channel $\mathcal{A}$.

An $(n,M)$ classical message transmission code for $\mathcal{U}$ is a family $\mathcal{C} := \{u_m, D_m\}_{m=1}^M$, where $u_m \in \mathcal{X}^n$, and $D_m \in \mathcal{L}(\mathcal{K})$ for each $m \in [M]$, with the additional property, that $\sum_{m=1}^M D_m = \mathbb{1}_{\mathcal{K}}$, if $n$ instances of the cq channel $\mathcal{W}$ and the code $\mathcal{C}$ are used for classical message transmission, we use the average transmission error

$$\overline{\epsilon}(\mathcal{C}, \mathcal{W}^{\otimes n}) := \frac{1}{M} \sum_{m=1}^M \text{tr}(\mathbb{1}_{\mathcal{K}} - D_m \mathcal{W}^{\otimes n}(u_m))$$

as error criterion. The following proposition states existence of universal random message transmission codes for each given set of classical-quantum channels. Its proof can be extracted from \cite{30}, where it was proven using the properties of quantum versions of the Renyi entropies together in combination with the Hayashi-Nagaoka random coding lemma \cite{19}.

Proposition 8 (Universal random cq codes without state knowledge \cite{30}, Theorem 4.18). Let $\mathbb{I} := \{\mathcal{W}_t : \mathcal{X} \rightarrow \mathcal{S}(\mathcal{K}_C) : t \in T\}$ be a set of classical-quantum channels, and $q \in \mathcal{P}(\mathcal{X})$. For each $\delta > 0$ and large enough $n \in \mathbb{N}$ there exists an $(n,M)$-random message transmission code $\mathcal{C}(U) = \{(U_{m}, D_{m}(U))\}_{m=1}^M$, which fulfills the following conditions.

1. $U = (U_1, \ldots, U_M)$ is an independent family of random variables, each with distribution $p^{\otimes n}$,
2. $\frac{1}{2} \log M \geq \inf_{t \in T} I(X; C, \tau_t) - \delta$, where $\tau_t := \sum_{x \in \mathcal{X}} p(x) |x^{X}\rangle \langle x^{X}| \otimes \mathcal{W}_t(x)$, and
3. $\mathbb{E} \sup_{t \in T} \overline{\epsilon}(\mathcal{C}, \mathcal{W}_t) \geq 2^{-nc}$,

where $c > 0$ is a constant dependent on $\delta$.

B. Entanglement transmission

In this paragraph we introduce universal coding results for the task of entanglement transmission, which were implicitly proven already in \cite{8,9}. For a given quantum channel $\mathcal{N} \in \mathcal{C}(\mathcal{K}_A, \mathcal{K}_B)$, an $(n,M)$ entanglement transmission code is a pair $\mathcal{C} = (\mathcal{E}, \mathcal{D})$, where with a Hilbert space $\mathcal{F}$ of dimension $M$, $\mathcal{E} \in \mathcal{C}(\mathcal{F}, \mathcal{H}_A^{\otimes n})$, and $D \in \mathcal{C}(\mathcal{H}_B^{\otimes n}, \mathcal{F})$ are c.p.t.p. maps. The performance of the code $\mathcal{C}$ is then measured by the entanglement
fidelity $F_e(\pi, N)$, where $\pi$ is the maximally mixed state on $\mathcal{F}$. Their strategy to derive universal entanglement transmission codes for compound quantum channels was to generalize the decoupling lemma from \cite{28} to achieve a one-shot bound for the performance in case of a finite set of channels for a fixed code subspace. A subsequent randomization over unitary transformations of that encoding led to random codes with achieving arbitrarily close to coherent information minimized over all possible channel states, given maximally mixed state on the input space. Further approximation using the so-called BSST lemma \cite{5} approximating asymptotically each state by a sequence of maximally mixed state and a net approximation on the set of channels allowed to input mixed state to the channel after encoding. It is a tensor product of the maximally mixed state appearing in the coheren information terms lower-bounding the rate.

**Proposition 9** (cf. \cite{8}, Lemma 9, \cite{12}). Let $\bar{1} := \{N_i\}_{i \in T} \subset \mathcal{C}(\mathcal{K}_A, \mathcal{K}_B)$ be a set of c.p.t.p. maps, $G \subset \mathcal{K}_A$ a subspace of $\mathcal{H}$, $\delta > 0$. For each large enough $n$ exists an $(n, M)$ random entanglement transmission code $\mathcal{C}_n := (\mathcal{E}_u, \mathcal{D}_u)$, $u \in A \subset \mathcal{U}(\mathcal{G}^{\otimes n})$, $|A| < \infty$ such that

1. $\frac{1}{n} \log M \geq \inf_{i \in T} I_\epsilon(A; B, \sigma_i) - \delta$, where $\sigma_i := \text{id}_{\mathcal{H}_A} \otimes N_i(\langle \psi | \langle \psi \rangle)$ with $\psi$ being a purification of $\rho$,

2. $\frac{1}{|A|} \sum_{u \in A} \mathcal{E}_u(\pi, \sigma) \mathcal{D}_u = \pi^{\otimes n}$, and

3. $\frac{1}{|A|} \sum_{u \in A} \inf_{i \in T} F_e(\pi; \mathcal{D}_u \circ N^{\otimes n} \circ \mathcal{C}_u) \mathcal{D}_u \geq 1 - 2^{-nc}$

with a constant $c > 0$.

**Remark 10.** In \cite{7}, actually a continuous random code distributed according to the Haar measure on the unitary group on the encoding subspace was constructed. To obtain the above finite version the Haar measure is replaced by a finitely supported measure which forms a so-called unitary design \cite{17}. This replacement is detailed in \cite{12}.

We notice, that in earlier work on perfectly known quantum channels (see e.g. \cite{14}, \cite{13}, \cite{22}) usually a different type of random code was used. Instead of employing the random entanglement transmission codes from \cite{28} or \cite{25} based on the decoupling approach, the entanglement generation codes of \cite{14} were used. These arise from a clever reformulation of private classical codes for a classical-quantum wiretap channel. In Appendix D in \cite{14}, these codes where moreover further developed to approximately reproduce $\rho^{\otimes n}$ for a given $\rho$ by the random encoding.

However, we remark here, that establishing the results on this paper by generalizing the random codes in \cite{14} is not very auspicious. As it was already noted in \cite{11}, that the method of generating entanglement generation codes from private classical codes employed in \cite{14} seems not to carry over to the case of channel uncertainty.

**Remark 11.** By applying the Proposition 9 to the special case $|\bar{1}| = 1$, we obtain an alternative to the random codes from \cite{14} in case of a perfectly known quantum channel. Alternatively one could also take the direct route to prove such a result and derive such codes directly from the original works \cite{28}, \cite{25} on the perfectly known channel.)

V. PROOFS

**A. Inner bounds to the capacity regions**

In this paragraph, we prove the achievability part of Theorem 6 i.e. the following statement.

**Proposition 12** (Inner bound for the capacity region for uninformed users). Let $\mathcal{M} \subset \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C)$. It holds

$$
CQ^{\mathcal{M}}(\mathcal{M}) \supset \text{cl} \left( \bigcup_{l=1}^{\infty} \bigcup_{p, V, \Psi} \bigcap_{M \in \mathcal{M}} \frac{1}{l} \mathcal{C}^{(1)}(M^{\otimes l}, p, V, \Psi) \right)
$$

(8)
The main technical steps for proving the above assertion is done in the proof of the following proposition.

**Proposition 13.** Let \( I := \{T_s\}_{s \in S} \subset C(K_A \otimes K_B, K_C) \), \( \Phi \in S(K_B \otimes K_B) \) a pure maximally entangled state, \( p \in \mathcal{P}(\mathcal{X}) \), \( V : \mathcal{X} \rightarrow \mathcal{S}(K_A) \) a channel having only pure outputs. For each \( \delta > 0 \) exists a number \( n_0 \) such that for each \( n > n_0 \) we find an \((n, M_1, M_2)\) Scenario-II code \( C \) with

1. \( \frac{1}{n} \log M_1 \geq \inf_{s \in S} J(X; C, \omega_s) - \delta \),
2. \( \frac{1}{n} \log M_2 \geq \inf_{s \in S} J(B|CX, \omega_s) - \delta \), and
3. \( \inf_{s \in S} p^{I1}(C, T_s) \geq 1 - 2^{-nc} \)

where \( c \) is a strictly positive constant. The entropic quantities on the right hand sides of the first two inequalities are evaluated on the states

\[ \omega_s := \omega(T_s, p, V, \Psi) = \sum_{x \in \mathcal{X}} p(x) \langle x | x \rangle \otimes \text{id}_{K_B} \otimes T_s(V(x) \otimes \Phi) \quad (s \in S) \]

Before we give a proof of the above proposition, we state a net approximation result which is used therein. We use the diamond norm \( \| \cdot \|_\diamond \) defined on the set of maps from \( \mathcal{L}(\mathcal{H}) \) to \( \mathcal{L}(\mathcal{K}) \) by

\[ \|N\|_\diamond := \max_{a \in \mathcal{L}(\mathcal{H} \otimes \mathcal{H})} \|\text{id}_\mathcal{H} \otimes N(a)\| \quad (N \in \mathcal{L}(\mathcal{H})). \]

We will use

**Lemma 14 (8, Lemma 5.2).** Let \( I \subset C(\mathcal{H}, \mathcal{K}) \). For each \( \theta > 0 \) there is a set \( I_\theta \subset I \) such that the following conditions are fulfilled

1. \( |I_\theta| \leq (6/\theta)^{2(\dim K \cdot \dim \mathcal{H})^2} \), and
2. for each \( N \in I \) exists \( N' \in I_\theta \) such that \( \|N - N'\|_\diamond \leq \theta \).

**Proof.** Fix \( \delta > 0 \), and set

\[ R_1 := \inf_{s \in S} J(X; C, \omega_s) \quad \text{and} \quad R_2 := \inf_{s \in S} J(B|XC, \omega_s). \]

We assume that \( R_1 - \delta \) and \( R_2 - \delta \) are both non-negative, otherwise the results follow either by trivial coding or reduction to a case of channel coding with a single sender. We define the state \( \pi_B := \text{tr}_{K_B} \Phi \), and a classical-quantum channel \( T_{n,s} : \mathcal{X} \rightarrow \mathcal{S}(\mathcal{H}_C) \) with outputs

\[ T_{n,s}(x) := T_s(V(x) \otimes \pi_B) \quad (x \in \mathcal{X}) \]

for each \( s \in S \). If we fix the blocklength \( n \) to be sufficiently large, we find by virtue of Proposition 8 a random \((n, M_1)\) message transmission code \( C_A(U) := (U_m, D_m(U))_{m=1}^{M_1} \) for the compound classical-quantum channel \( \{T_{n,s}\}_{s \in S} \), where \( U = (U_1, \ldots, U_{M_1}) \) is an i.i.d. random family with generic distribution \( p^\otimes n \), rate \( \frac{1}{n} \log M_1 \geq J(X; C, \omega_s) - \delta = R_1 - \delta \), and expected average message transmission error

\[ \mathbb{E} \parallel C_A(U), T_{n,s} \parallel_{\diamond} \leq 2^{-nc_1} \]

where \( c_1 \) is a strictly positive constant. Moreover we define for each \( s \) a c.p.t.p. map \( T_{B,s} \in C(K_B, K_B \otimes \mathcal{C}^{\mathcal{X}}) \) by

\[ T_{B,s}(\tau) := \sum_{x \in \mathcal{X}} p(x) T_s(V(x) \otimes \tau) \otimes \langle x | x \rangle \quad (\tau \in \mathcal{L}(K_B)). \]

Under the assumption of large enough blocklength \( n \), Proposition 9 assures us, that there exists a random \((n, M_2)\) entanglement transmission code \( C_B(A) := (E_A, D_A) \) for the compound quantum channel \( \{T_{B,s}\}_{s \in S} \) where \( A \) is supported on a finite set \( A \), and which has rate

\[ \frac{1}{n} \log M_2 \geq \inf_{s \in S} J(B|CX, \omega_s) - \delta = R_2 - \delta, \]
such, that with a positive constant $c_2 > 0$ the expected entanglement fidelity can be bounded as

$$
\mathbb{E}_A \left[ \inf_{x \in \mathcal{X}} \mathcal{F}_e(\pi_{xf}, \overline{D}_A \circ T_{B,s}^{\otimes m} \circ \mathcal{E}_A) \right] \geq 1 - 2^{-nc_2}.
$$

(12)

In addition, the expected density matrix resulting from the random encoding procedure is maximally mixed, i.e. $\mathbb{E}_A \mathcal{E}_A(\pi_{xf}) = \mathbb{1}_{B}.\,$ For each pair $(u, \alpha) \in \mathcal{X}^{\Lambda_{M_1}} \times A$ of realizations of $(U, A)$, we define an $(n, M_1, M_2)$ Scenario-II code $C(u, \alpha) := (V(u_m), \mathcal{E}_A, D_{m,au})_{m=1}^{M_1}$ using the decoding operations

$$
D_{m,au}(x) := \overline{D}_a \circ \check{D}_m(u)(x) \quad (x \in \mathcal{L}(K^{\otimes n})).
$$

with $\check{D}_m(u) := \frac{1}{\sqrt{B}} D_m(\cdot) \frac{1}{\sqrt{B}}$. Each of the codes defined above already is a Scenario-II code of suitable rates for classical message and entanglement transmission. To complete the proof of the proposition, we will lower-bound the the expected Scenario-II fidelity of the random code $C(U, A)$. Fix, for the moment the channel state $s$. Let $T_{s,sa}^\prime$ be the cq channel defined by the states

$$
T_{s,sa}^\prime(x^u) := T_{s}^{\otimes n}(V^{\otimes n}(x^u) \otimes \mathcal{E}_a(\pi_{xf})) \quad (a \in A, x^u \in \mathcal{X}^n).
$$

(13)

Averaging over the random choice of $x$ the transmission statistics of $T_{s,sa}^\prime$ is reproduced. Indeed, for each $x^u \in \mathcal{X}^n$

$$
\mathbb{E}_A T_{s,sa}^\prime(x^u) = T_{s}^{\otimes n}(V^{\otimes n}(x^u) \otimes \mathbb{E}_A(\pi_{xf})) = T_{s,sa}^\prime(x^u).
$$

(14)

Since the average transmission error is an affine function of the cq channel, we have for each $u \in \mathcal{X}^{\Lambda_{M_1}}$

$$
\mathbb{E}_{U} \mathbb{E}_A \mathfrak{C}(u) T_{s,sa}^\prime(u) = \mathbb{E}_{U} \mathfrak{C}(u) T_{s,sa}^\prime(u) \leq 2^{-nc_1}.
$$

(15)

Define the cq channel $T_{s,sa}^\prime$ defined by

$$
T_{s,sa}^\prime(x^u) := \text{id}_{K_B}^{\otimes n} \otimes T_{s}^{\otimes n}(V^{\otimes n}(x^u) \otimes (\text{id}_{K_B}^{\otimes n} \otimes \mathcal{E}_a(\Phi))) \quad (x^u \in \mathcal{X}^n, a \in A).
$$

(16)

(note that the reduction of $T_{s,sa}^\prime(x^u)$ is, in fact, $T_{s,sa}^\prime(x^u)$.) For each classical message $m \in [M_1]$, it holds

$$
\mathbb{E}_A \mathbb{E}_{U} \text{tr} \left( (\text{id}_{K_B}^{\otimes n} \otimes \check{D}(U)) T_{s,sa}^\prime(U_m) \right) = \mathbb{E}_A \mathbb{E}_{U} \text{tr} \check{D}(U) T_{s,sa}^\prime(U_m) \geq 1 - 2^{-nc_1}.
$$

(17)

The above inequality stems from the bound in (15) together with the observation, that by symmetry of the random selection procedure for the codewords, the expectation of the one-word message transmission error does not depend on the individual message $m$. If we define

$$
\gamma_s(u, \alpha) := 1 - \text{tr}(\text{id}_{K_B}^{\otimes n} \otimes \check{D}(u)) T_{s,sa}^\prime(u_m)
$$

we have by (18)

$$
\mathbb{E}_A \mathbb{E}_{U} \gamma_s \leq 2^{-nc_1}.
$$

(19)

Define for each realization $(u, \alpha)$ of $(U, A)$

$$
\Gamma_{mm'}^{(s)}(u, \alpha) := \text{id}_{K_B}^{\otimes n} \otimes D_m(U) T_{s,sa}^\prime(u_m) \quad (m, m' \in [M_1]).
$$

(20)

$$
\Gamma_m^{(s)}(u, \alpha) := \sum_{m'=1}^{M_1} \Gamma_{mm'}^{(s)}(u, \alpha) \otimes |u_{m'}\rangle \langle u_{m'}|, \quad \text{and}
$$

$$
\Gamma^{(s)}(x^u, \alpha) := \text{id}_{K_B}^{\otimes n} \otimes T_{s}^{\otimes n}(V^{\otimes n}(x^u) \otimes \mathcal{E}_a(\Phi)) \otimes |x^u\rangle \langle x^u| \quad (x^u \in \mathcal{X}^n).
$$

(22)

Note, that if $\check{U}$ is an $\mathcal{X}^n$-valued random variable with distribution $p^{\otimes n}$, that

$$
\mathbb{E}_{\check{U}} \Gamma^{(s)}(\check{U}, \alpha) = (\text{id}_{K_B}^{\otimes n} \otimes T_{B,s}^{\otimes n} \circ \mathcal{E}_a(\Phi))
$$

(23)

holds by definition of $T_{B,s}^\prime$. By the gentle measurement lemma (see Lemma [15] in Appendix [3]), we have

$$
||\text{id}_{K_B}^{\otimes n} \otimes (\text{id}_{K_C}^{\otimes n} - \check{D}(u)) T_{s,sa}^\prime(u_m)||_1 \leq 3 \sqrt{\gamma_s(u, \alpha)}.
$$

(24)
Taking expectations on both sides of the inequality in (24), we arrive at
\[ E_{\Lambda} E_{U} |\text{id}_{K_B}^{\otimes n} \otimes (\text{id}_{C_\Lambda}^{\otimes n} - \hat{D}_m(U)) T_{A,m}^{\otimes n} (U_m)\|_1 \leq 3 E_{\Lambda} E_{U} \sqrt{\gamma_s(u,\alpha)} \leq 3 \sqrt{E_{\Lambda} E_{U} \gamma_s(u,\alpha)} \leq 3 \sqrt{2^{-nc}}, \]
where the second inequality above is by Jensen’s inequality together with concavity of the square-root function. The last inequality is by the estimate in (19). As a consequence of these bounds, we have
\[ E_{U} E_{\Lambda} \|\tilde{I}^{(s)}_m (U,\Lambda) - \Gamma^{(s)}(U_m,\Lambda)\|_1 \leq 3 \cdot \sqrt{2^{-nc} + 2^{-nc}} \leq 4 \cdot \sqrt{2^{-nc}}, \]
for each message \( m \in [M_1] \). Moreover, we can bound
\[ E_{\Lambda} E_{U} F(\Phi, \text{id}_{K_B}^{\otimes n} \otimes \tilde{D}_A (\hat{I}^{(s)})) \geq E_{\Lambda} E_{U} \left| F(\Phi, \text{id}_{K_B}^{\otimes n} \otimes \tilde{D}_A (\hat{I}^{(s)})) - \|\text{id}_{K_B}^{\otimes n} \otimes \tilde{D}(\Gamma^{(s)}(U,\Lambda) - \Gamma^{(s)}(U_m,\Lambda))\|_1 \right| \]
\[ \geq E_{\Lambda} E_{U} \left| F_{\epsilon}(\Phi, \tilde{D}_A \circ T_{B,s}^{\otimes n} \circ \mathcal{E}_A) - \|\Gamma^{(s)}(U,\Lambda) - \Gamma^{(s)}(U_m,\Lambda)\|_1 \right| \]
\[ \geq E_{\Lambda} F_{\epsilon}(\Phi, \tilde{D}_A \circ T_{B,s}^{\otimes n} \circ \mathcal{E}_A) - 4 \cdot \sqrt{2^{-nc}}, \]
\[ \geq 1 - 2^{-nc} - 4 \cdot \sqrt{2^{-nc}}. \] (26)

The first line above is by application of Lemma 13 which can be found in Appendix A. The second is by using the equality in (23) together with monotonicity of the trace distance under taking partial traces. The third is by (25). The last estimate comes from (12). Putting all the estimates together, we can bound the expected Scenario-II fidelity. We have for each \( m \in [M_1] \)
\[ E_{U} E_{\Lambda} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}, m) = F(\{n\} \otimes \Phi, \text{id}_{K_B}^{\otimes n} \otimes D_{U,A} \circ T_{s}^{\otimes n} \circ (\text{id}_{K_B}^{\otimes n} \otimes \mathcal{E}_A)(V^{\otimes n}(U_m) \otimes \Phi)) \]
\[ \geq 1 - \|\{n\} \otimes \Phi - \text{id}_{\Sigma_{\otimes n}} \circ T_{s}^{\otimes n} (V^{\otimes n}(U_m) \otimes \mathcal{E}_A(n))\|_1 \]
\[ \geq 1 - 2 E(C, T_{A,\Lambda}^{'}, 3) - 3 (1 - F(\Phi, \text{id}_{K_B}^{\otimes n} \otimes \text{tr}_{\mathcal{M}_1} \circ D_{U,A} \circ T_{s}^{\otimes n} \circ (\text{id}_{K_B}^{\otimes n} \otimes \mathcal{E}_A)(V^{\otimes n}(U_m) \otimes \Phi)) \]
\[ \geq 1 - 2 E(C, T_{A,\Lambda}^{'}, 3) - 3 (1 - F(\Phi, E_{\Lambda} \text{id}_{K_B}^{\otimes n} \otimes \tilde{D}_A (\mathcal{E}_A(\hat{I}^{(s)}(U_m)))))) \]
\[ \geq 1 - 2^{-nc} + 2 \cdot 2^{-nc} + 4 \sqrt{2^{-nc} + 2^{-nc}}. \]
(27)

The first inequality is by Lemma 20 to be found in Appendix A. The second inequality is by ... . The third inequality is by inserting the bounds from (15) and (26). Consequently, we have for each \( s \in S \)
\[ E_{U} E_{\Lambda} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}) \geq 1 - 2^{-n^c \tilde{c}} \] (28)
with \( \tilde{c} := \min\{c_1, c_2\} \) provided, that \( n \) is large enough. The inequality in (28) in fact provides an individual lower bound on the expected Scenario-II fidelity for each channel state \( s \). Since we aim for a lower bound on the expected worst-case fidelity over the set \( S \), we include another step of approximation. We derive from the individual bounds on the expected Scenario-II fidelity of the random code for each \( s \) a universal bound, i.e. a bound on the expected worst-case Scenario-II fidelity of the code. We assume, for each \( n \in \mathbb{N}, S_n \) to be a subset of \( S \), such that \( |T_s|_{s \in S_n} \) is a \( n \)-net for the original set of channels generating the transmission with \( \delta_n := 2^{-n^c} \), i.e. to each \( s \in S \) exist an \( s \in S_n \), such that \( |T_s - T_s|_{s \in S_n} \). We assume moreover, that the cardinality of each of these sets is bounded by \( |S_n| \leq 2^{n^c \tilde{c}} \). Note, that such sets indeed exist by Lemma 14. We have
\[ E_{U} E_{\Lambda} \left( \frac{1}{|S|} \sum_{s \in S} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}) \right) = \frac{1}{|S|} \sum_{s \in S} E_{U} E_{\Lambda} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}) \geq 1 - 2^{-n^c \tilde{c}} \]
(29)
by (28), which implies,
\[ E_{U} E_{\Lambda} \min_{s \in S_n} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}) \geq 1 - |S_n| \cdot 2^{-n^c \tilde{c}} \geq 1 - 2^{-n^c \tilde{c}}. \]
(30)
for each sufficiently large \( n \). The rightmost inequality above follows from the cardinality bound in (28). By continuity of \( P^{\otimes n} \), we have
\[ E_{U} E_{\Lambda} \inf_{s \in S} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}) \geq E_{U} E_{\Lambda} \inf_{s \in S} P^{\otimes n}(C(U,\Lambda), T_{s}^{\otimes n}) - n \delta_n \geq 1 - 2^{-nc} \]
with a strictly positive constant \( c \). We are done.
Next, we prove a generalization of Proposition 13, i.e., we drop the condition, of $\Phi$ being a maximally entangled state.

**Proposition 15.** Let $T := \{T_{ij} \in S \mid C \in (K_A \otimes K_B \otimes K_C) \}$, $\Psi \in S(K_B \otimes K_B)$, a pure state, $p \in P(\mathcal{X})$, $V : \mathcal{X} \to S(K_A \otimes K_B)$ a channel with pure outputs. For each $\delta > 0$ there exists a constant $n_0$ such that for each $n > n_0$ we find an $(n, M_1, M_2)$ Scenario-II code $C$ with

1. $\frac{1}{n} \log M_1 \geq \inf_{s \in S} I(X; C, \omega_s) - \delta$,
2. $\frac{1}{n} \log M_1 \geq \inf_{s \in S} I(BCX, \omega_s) - \delta$, and
3. $\inf_{s \in S} \mathbb{P}_1(C, \pi) \geq 1 - 2^{-nc}$

where $c$ is a strictly positive constant.

To prove the above statement, we will invoke Proposition 13 together with an elementary approximation argument. Note, that for each $\rho \in S(\mathcal{H})$, $l \in \mathbb{N}$, $\rho^{\otimes l}$, can be written in the form

$$\rho^{\otimes l} = \sum_{i=1}^{D} q(i) \pi_i,$$

with maximally mixed states $\pi_1, \ldots, \pi_D$ supported on pairwise mutually orthogonal subspaces, $q \in P([D])$, and $D \leq \frac{l(l+1) \dim \mathcal{H}}{t}$. This can be seen by using the spectral decomposition together with the fact, that the spectrum of $a^{\otimes l}$ has its cardinality upper-bounded by $(l+1)^{\dim \mathcal{H}}$ for each $a \in L(\mathcal{H})$. The following lemma formalizes a very basic fact about approximation by empirical distributions.

**Lemma 16.** Let $q \in P(\mathcal{X})$ a probability distribution. There exists a constant, for each $t \in \mathbb{N}$, $t > 2/\min_{x:p(x)>0} p(x)$ exist integers $N_x \in \mathbb{N}$, $x \in \mathcal{X}$, such that $N_x$ is zero if $p(x)$ vanishes, and

- $\forall x \in \mathcal{X} : |p(x) - N_x/t| < |\text{supp}(p)|/t$, and
- $\forall x \in \text{supp}(p) : N_x \geq C_p \cdot t$

with a constant $C_p > 0$.

**Proof.** Excluding a trivial case, we assume $p > 1$. Let $x_0$ be any member of suppp. Fix $m \in \mathbb{N}$ large enough, define $N_x := [m \cdot p(x)]$ for each $x \neq x_0$, and $N_{x_0} := m - \sum_{x \neq x_0} N_x$. That the first claim of the lemma holds follows from these definitions. For the second claim, notice, that for all $x \in \text{supp}(p)$, it holds $N_x \geq m \cdot p(x) - 1 \geq m \cdot C_p$, where $C_p := \min_{x:p(x)>0} p(x)/2$.

**Proof of Proposition 15.** Fix $l \in \mathbb{N}$ such that $\dim \mathcal{H} \cdot \log (l+1) \leq \frac{\delta}{t}$, and let $\sum_{i=1}^{D} q(i) \pi_i$ be a decomposition of $\rho^{\otimes l}$ as in Eq. (32). Fix for each $i$ a maximally entangled state $\Phi_i$ which purifies $\pi_i$. and define a state $\omega_{s,i} := \omega_s(T^{\otimes l}_i, \rho^l, V^{\otimes l}, \Phi_i)$

for each $s, i$. Note, that $\omega_i = \sum_{i=1}^{D} q(i) \omega_{s,i}$ holds. Using an approximation of $q$ by numbers $N_1, \ldots, N_D$ according to Lemma 16 we conclude, that if $t$ is any appropriately large number

$$I_c(BCX, \omega_s) = \frac{1}{l} I_c(BCX, \omega_s^{\otimes l})$$

$$\leq \sum_{i=1}^{D} q(i) I(B^l|X^l, \omega_{s,i}) + \frac{\text{dim} \log (l+1)}{t}$$

$$\leq \frac{1}{l} \sum_{i=1}^{D} N_i I(B^l|X^l, \omega_{s,i}) + \frac{\delta}{2}$$

does hold. The first of the above inequalities is by almost-convexity of the von Neumann entropy. Let for each $i \in [D]$, $C_i := (V_i(m), E_i, D^{(i)}_m)_{m=1}^{M_i}$ be an $(N_i, M_{i,1}, M_{i,2})$-Scenario-II code for $T$, such that

$$\frac{1}{N_i} \log M_{i,1} \geq \inf_{s \in S} I(X^l; C^l, \omega_{s,i}) - \frac{\delta}{4} := R_{i,j},$$

(37)
\[
\frac{1}{N_i} \log M_{i,2} \geq \inf_{s \in S} I_s(B^i) C^i(X^i, \omega_{s,i}) - \frac{\delta}{4l} := R_{2,i}
\]  
(38)

and

\[
\inf_{s \in S} P^I(C_s, T_s^{lN}) \geq 1 - 2^{-N c_i} \geq 1 - 2^{-lN c_i}
\]
(39)

with \(c_i > 0, c := \min\{c_1, \ldots, c_D\}\). Note, that such codes exist, if we choose \(l\) large enough, since the second claim of Lemma 16 guarantees long enough blocklengths \(N_i\). By concatenation, we build the \((l \cdot t, M_1, M_2)\) Scenario II code \(C := (V(m), E, D_m)_{m=1}^M\) for \(T\) with

\[
M_1 = \prod_{i=1}^D M_{i,1} \quad \text{and} \quad M_2 = \prod_{i=1}^D M_{2,i},
\]
(40)

where we defined, with any bijection \(i : [M_1] \to \prod_{i=1}^D M_{i,1},\)

\[
V(m) := \bigotimes_{i=1}^{M_1} V_i(t_i(m)) \quad \text{and} \quad D_m := \bigotimes_{i=1}^{M_2} D^{(i)}_m(m).
\]
(41)

For the rates of this code, it holds

\[
\frac{1}{l} \log M_1 = \frac{1}{l} \sum_{i=1}^D \frac{1}{t} \log M_{i,1}
\]
(42)

\[
= \frac{1}{l} \sum_{i=1}^D \frac{N_i}{t} \frac{1}{N_i} \log M_{i,1}
\]
(43)

\[
\geq \sum_{i=1}^D \frac{N_i}{t} \frac{1}{t} I(X^i; C^i, \omega_{s,i}) - \frac{\delta}{4l}
\]
(44)

\[
\geq \sum_{i=1}^D q(i) \frac{1}{t} I(X^i; C^i, \omega_{s,i}) - \frac{\delta}{4l} - \frac{\delta}{2}
\]
(45)

\[
\geq I(X; C, \omega) - \frac{\delta}{4l} - \frac{\delta}{2}.
\]
(46)

The first inequality above is by (37), the second by choice of \(N_1, \ldots, N_D\). The last line is by convexity of the quantum mutual information. Moreover, we have

\[
\frac{1}{lm} \log M_1 = \frac{1}{l} \sum_{i=1}^D \frac{1}{m} \log M_{i,1}
\]
(47)

\[
= \frac{1}{l} \sum_{i=1}^D \frac{N_i}{m} \frac{1}{N_i} \log M_{i,1}
\]
(48)

\[
\geq \sum_{i=1}^D \frac{N_i}{m} \frac{1}{m} I(B^i X^i; C^i, \omega_{s,i}) - \frac{\delta}{4l}
\]
(49)

\[
\geq I(B; X, C, \omega) - \frac{\delta}{4l} - \frac{\delta}{2},
\]
(50)

where the last inequality is by (36). To evaluate the Scenario-II fidelity of \(C\) regarding \(T\), we have for each \(s \in S\)

\[
P^I(C, T_s^{lN}) = \frac{1}{M_1} \sum_{m=1}^{M_1} F(|m\rangle \langle m| \otimes \Phi_{m} \otimes D_m \otimes T^{(l)}_s(\Phi_{m}))
\]
(51)

\[
= \prod_{i=1}^D \left( \frac{1}{M_{1,i}} \sum_{t_i(m)=1}^{M_{1,i}} F(|t_i(m)\rangle \langle t_i(m)| \otimes \Phi_{t_i,m} \otimes D^{(i)}_{t_i(m)} \otimes T^{(l)}_s(\Phi_{t_i,m})) \right)
\]
(52)

\[
\geq (1 - 2^{lN c_i})^D
\]
(53)

\[
\geq 1 - D 2^{-lN c_i}.
\]
(54)
The last inequality is Bernoulli’s. We are done.

Finally can prove the coding theorem[12]

Proof of Proposition[12] Since $CQ^H(M)$ is closed by definition (see Fact[1]), it suffices to show the inclusion

$$CQ^H(M) \supset \left\{ \frac{1}{l} \sum_{i=1}^{\infty} \sum_{p,V,\Psi,M \in M} \frac{1}{l} \mathcal{C}^{(1)}(M^{a,}\ p,\ V,\ \Psi) \right\}. \quad (55)$$

We fix $l \in \mathbb{N}$, a finite alphabet $\mathcal{A}$, a cq channel $W \in \mathcal{CQ}(\mathcal{A})$ with pure-state outputs, a pure state $\Psi \in \mathcal{S}(\mathcal{A}^{\otimes l} \otimes K^{\otimes l})$, and a probability distribution $p(x)$ on $\mathcal{A}$. We show

$$\mathcal{C} = \left\{ \sum_{s,l} \frac{1}{l} \mathcal{C}^{(1)}(M^{a,}\ p,\ V,\ \Psi) \right\}, \quad (56)$$

which, with subsequent maximization proves the inclusion in (55). Fix $\delta > 0$, and let $n > l$ be a blocklength written as $n = a \cdot l + b$ with $a, b \in \mathbb{N}$, $0 \leq b < k$. Let $n$ (and consequently $a$) be large enough, to find, using Proposition[12] with $T := M^{a,}$, an $(a, M_1, M_2)$ Scenario-II code

$$\tilde{C} = (\tilde{W}(m), \tilde{E}, \tilde{D}_{m \in M}) \quad (57)$$

with

$$\frac{1}{a} \log M_1 \geq \inf_{s \in S} I(X; C^1, \omega_{s,l}) - \frac{\delta}{2}, \quad \text{and} \quad \frac{1}{a} \log M_2 \geq \inf_{s \in S} I(B^{l} \mid X C^1, \omega_{s,l}) - \frac{\delta}{2}. \quad (58)$$

The informational quantities on the right hand sides of the above inequalities are evaluated on the states

$$\omega_{s,l} := \sum_{x \in \mathcal{X}} p(x)\mathcal{D}_{x} \otimes M_{s,l}(W(x) \otimes \Psi), \quad (s \in S)$$

and moreover,

$$\inf_{s \in S} P^H(\tilde{C}, M^{\omega_{s,l}}) \geq 1 - 2^{-l \omega - \delta}. \quad (59)$$

holds with a constant $\omega > 0$. We define another $(n, M_1, M_2)$ Scenario-I code $C := (W(m), C, D_{m \in M})$ with

$$W(m) := \tilde{W}(m) \otimes \pi_{K^{a}}^{\otimes l}$$

$$C(x) := \tilde{E}(x) \otimes \pi_{K^{a}}^{\otimes l} \quad (x \in \mathcal{C}(F)), \quad \text{and}$$

$$D_{m}(y) := \tilde{D}_{m} \otimes \pi_{K^{a}}^{\otimes l}(y) \quad (y \in \mathcal{C}(H_{C}^{\otimes l})).$$

It holds for all $s, m \in [M_1]$

$$F \left\{ |m| \otimes \Phi, \text{id}_{K^{a}}^{\otimes l} \otimes D \circ M_{s,l} \circ (\text{id}_{K^{a}}^{\otimes l} \otimes \mathcal{C}) (W(m) \otimes \Phi) \right\} \quad (60)$$

$$\geq F \left\{ |m| \otimes \Phi, \text{id}_{K^{a}}^{\otimes l} \otimes D \circ M_{s,l} \circ (\text{id}_{K^{a}}^{\otimes l} \otimes \mathcal{C}) (W(m) \otimes \Phi) \right\}, \quad (61)$$

i.e.

$$P^H(\tilde{C}, M^{\omega_{s,l}}, m) \geq P^H(C, M^{\omega_{s,l}}, m) \geq 1 - 2^{-l \omega - \delta} = 1 - 2^{-l \omega} \quad (62)$$

with $\omega := \omega / (l + 1)$. Moreover, if $n$ is large enough, we have

$$\frac{1}{n} \log M_1 \geq \frac{1}{l} \log M_1 \geq \frac{1}{l} \left( \frac{1}{a} \log M_1 - \frac{\delta}{2} \right) \geq \frac{1}{l} \inf_{s \in S} I(X; C^1, \omega_{s,l}) - \delta. \quad (63)$$

In the same manner, we can also show

$$\frac{1}{n} \log M_2 \geq \frac{1}{l} \inf_{s \in S} I(B^{l} \mid X C^1, \omega_{s,l}) - \delta. \quad (64)$$

With the inequalities in (63), and (64) we have shown

$$\mathcal{C}^{(1)}(M^{a,}\ p,\ V,\ \Psi) \subset CQ^H(M), \quad (65)$$

Since $\delta > 0$ was arbitrary, (55) follows. □
B. Outer bounds to the capacity regions

In this section, we prove the outer bounds to the capacity regions as stated in Theorem\ref{theorem:outer_bound}.  

Proposition 17 (Outer bound to the informed receiver capacity region). Let $\mathcal{M} \subset C(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C)$. It holds

$$CQ^I(\mathcal{M}) \subset \mathcal{C} \left( \bigcup_{l=1}^{\infty} \bigcup_{p, V, \Psi, M \in \mathcal{M}} \frac{1}{l} \mathcal{C}^{(1)}(M^{\otimes l}, p, V, \Psi) \right)$$

(66)

Proof. The proof of the converse is fairly standard, and we give the argument for the reader’s convenience. We will show that the proposed outer bound remains valid even in case, that the receiver is allowed to choose the decoding channel dependent on the channel parameter, i.e. user $C$ has channel state information (CSI). Fix an arbitrary $\delta > 0$. Let $\{C_{n,s}\}_{s \in S}$ with $C_{n,s} := (V(m), \Psi, D_{m,s})_{m=1}^{M_{1,n}}$ be an $(n, M_{1,n}, M_{2,n})$ scenario-II code with C-CSI. We set

$$\inf_{s \in S} P^I(C_{n,s}, M^{\otimes n}_{s}) = 1 - \epsilon_n$$

and $R_{n,i} := \frac{1}{n} \log M_{i,n}$. For $i \in \{1, 2\}$, Define moreover the states

$$\omega_{M_{1,n}}=\omega(M_{1,n}^{\otimes n}, V, \Psi) = \sum_{m=1}^{M_{1,n}} P_M(m|m)\langle m|\otimes (id_{K_B} \otimes \mathcal{M}^{\otimes n})(\phi(m) \otimes \Psi),$$

and $\omega_{M_{1,n}} := (id_{C^{\otimes n,1}} \otimes id_{K_B} \otimes D_s) (\omega_{M_{1,n}}^{\otimes n})$ for each $s \in S$, where $P_M$ is the equidistribution on the message set $[M_{1,n}]$, $\phi(m)$ is a purification of $V(m)$, and $D_s(x) := \sum_{m=1}^{M_{1,n}} \langle m| \otimes D_{m,s}(x)$ for each $s \in S$. We define a pair $(M, \hat{M}_s)$ of classical random variables having joint distribution

$$P_{M\hat{M}_s}(m, m') := \langle m \otimes m', \omega_{M\hat{M}_s} m \otimes m' \rangle.$$

It holds

$$\Pr\left(M \neq \hat{M}_s \right) \leq 2 \sqrt{\epsilon_n} := \hat{\epsilon}_n$$

by (67). We have for each $s \in S$

$$\log M_{1,n} = H(M)$$

$$= I(M; \hat{M}_s) + H(M|\hat{M}_s)$$

$$\leq I(M; \hat{M}_s) + \hat{\epsilon}_n \log M_{1,n} + 1$$

$$\leq I(M; C^{\otimes n}, \omega_{M_{1,n}}^{\otimes n}) + \hat{\epsilon}_n \log M_{1,n} + 1$$

(69)

where first of the above inequalities is by Fano’s lemma, while the second one is Holevo’s bound. With $\Phi$ being the target maximally entangled state of the quantum part of the code, we have, by monotonicity of the fidelity under taking partial traces

$$p^{11}(C_{n,s}, M^{\otimes n}) = \frac{1}{M_{1,n}} \sum_{n=1}^{M_{1,n}} F(|m\rangle \otimes \Phi, \omega_{M\hat{M}_s}^{\otimes n}) \leq F(\Phi, \omega_{M\hat{M}_s}^{\otimes n}),$$

which, combined with (67) allows to bound $\|\Phi - \omega_{M\hat{M}_s}^{\otimes n}\| \leq \hat{\epsilon}_n$. Using the Lemma \ref{lemma:partial_trace} in Appendix A we have

$$|I(B)\hat{B}_s, \Phi) - I(B)\hat{B}_s, \omega_{M\hat{M}_s}^{\otimes n})| \leq 2H(\epsilon_n) + 4\epsilon_n \cdot \log M_{2,n}.$$

(70)

Consequently

$$\log M_{2,n} = I(B)\hat{B}_s, \Phi)$$

$$\leq I(B)\hat{B}_s, \omega_{M\hat{M}_s}^{\otimes n}) + 2H(\epsilon_n) + 4\epsilon_n \cdot \log M_{2,n}$$

$$\leq I(B)\hat{B}_s, \omega_{M\hat{M}_s}^{\otimes n}) + 2H(\epsilon_n) + 4\epsilon_n \cdot \log M_{2,n}$$

$$\leq I(B)\hat{B}_s, \omega_{M\hat{M}_s}^{\otimes n}) + 2H(\epsilon_n) + 4\epsilon_n \cdot \log M_{2,n}$$

(71)
If we now demand $\epsilon_n \to 0$, Eqns. (69) and (71) show, that if $n$ is large enough, the inequalities

$$\frac{1}{n} \log M_{n,1} \leq \inf_{s \in S} \frac{1}{n} I(M; C^n, M_{\text{BC}^n, s}) + \delta,$$

$$\frac{1}{n} \log M_{n,2} \leq \inf_{s \in S} \frac{1}{n} I(B; \hat{M}, M_{\text{BC}^n, s}) + \delta$$

holds, which implies the chain of inclusion relations

$$(R_{1,n}, R_{2,n}) \in \bigcap_{s \in S} \frac{1}{n} C^{(1)}(M_{\text{BC}^n, p, V, \Psi})_\delta$$

$$\subset \bigcup_{p, V, \Psi \in S} \frac{1}{n} C^{(1)}(M_{\text{BC}^n, p, V, \Psi})_\delta$$

$$\subset \bigcup_{l=1}^\infty \bigcup_{p, V, \Psi \in S} \frac{1}{n} C^{(1)}(M_{\text{BC}^n, p, V, \Psi})_\delta$$

$$\subset \text{cl} \left( \bigcup_{l=1}^\infty \bigcup_{p, V, \Psi \in S} \frac{1}{n} C^{(1)}(M_{\text{BC}^n, p, V, \Psi}) \right)_\delta.$$  

Since $\delta > 0$ was arbitrary, we are done.  

\[\square\]

VI. CONCLUSION AND DISCUSSION

In this work, we derived a multi-letter description of the capacity region of the compound QMAC where one sender sends classical messages while the other aims to transmit quantum information. To our knowledge the characterization in Theorem 6 is the first result for coding of the QMAC where channel uncertainty is present and genuine quantum transmission tasks such as entanglement transmission and generation are performed. (The earlier work [20] considered the case where all senders send classical messages.) Further research in this direction hopefully will bring new insights for coding of the compound QMAC also in situations where more senders are present and full-quantum coding is performed.

The argument employed to prove the coding theorem follows a strategy which seems rather common for perfectly known classical as well as quantum multiple-access channels. A clever combination of single-terminal random codes results in a random code for the QMAC which has sufficient performance on the average. This technique was in the quantum settings first used for simultaneous classical and quantum coding over a single sender quantum channel and can be generalized to channels with uncertainty also in this case [27]. By extracting powerful universal random coding results for classical message and entanglement transmission from the literature ([7], [30]), we were able to make successful use of this strategy also in case of channel uncertainty. The mentioned universal random coding results may be applied in further research deriving codes in multi-user quantum information theory. Beside consideration of more general multi-user and coding scenario also an extension of the results to other channel models may be a direction of further research. One possible variation of the compound MAC channel model is when one or more of the users are provided with channel state information (CSI). Such additional knowledge of the system is already known as very relevant from the practical point of view when regarding classical channels. Especially, as CSI might lead substantially larger capacity regions. On the other hand, our results possibly provide the basis for tackling coding for the far more demanding arbitrarily varying QMAC (AVQMAC).

In this model, the channel statistics can be given by an arbitrary element chosen from a prescribed set for each channel use. An interpretation of this model is that the channel map is chosen by a malicious “jamming” party. The capacity region of the corresponding classical channel model was determined in [27], [3]. The code construction established to show achievability in this work allows to achieve each point in the capacity regions with codes with fidelity approaching one exponentially fast. This may allow to use Ahlswede’s robustification and elimination techniques [1] to find good codes in case of the much more AVQMAC.

We remark here, that albeit the characterization of the capacity regions $C^I$ and $C^{II}$ given in Theorem 6 are correct, the description may be improved regarding computational aspects. While our capacity formula is a union over intersections of one-shot regions which are of rectangles of points fulfilling

$$R_1 \leq I(X; C^I, \omega) \quad \text{and} \quad R_2 \leq I_c(B^I) C^I X, \omega)$$

(72)
where \( \omega_i \) may be the state as defined in \( \hat{\mathcal{C}}(\mathcal{M}) \) for each channel state \( s \in S \). The authors of this paper feel, that a description replacing the rectangular one-shot regions with the above given boundaries by pentagons with boundaries

\[
R_1 \leq I(X;C^l,B^l_i,\omega_s), \quad R_1 \leq I(B^l_i;C^l_iX,\omega_s) \quad \text{and} \quad R_1 + R_2 \leq I(X;C^l_iX,\omega_s) + I_e(B^l_i;C^l_iX,\omega_s)
\] (73)
instead. Such a characterization was indicated to be possible in case of the perfectly known QMAC in [35], Chapter VII. Whether or not such a description is indeed possible for each example of a compound QMAC and to what extend it would improve the capacity formula given here is a future research topic. In case of a perfectly known QMAC it was discussed in [34] that standard examples having single-letter capacity regions in the latter characterization also single-letterize in the first characterization.

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Appendix A: Auxiliary results

For the convenience of the reader, some auxiliary standard results used in the text are collected.

Lemma 18 (Gentle measurement lemma [31]). Let \( \rho \in \mathcal{S}(\mathcal{K}), E \in \mathcal{L}(\mathcal{H}), 0 \leq E \leq \mathbb{1}_H \). It holds

\[
\|\sqrt{E}\rho\sqrt{E} - \rho\|_1 \leq 3 \cdot \sqrt{1 - \text{tr}E}\rho.
\]

Lemma 19 ([35]). Let \( \Psi, \rho, \sigma \in \mathcal{S}(\mathcal{K}) \) be states, where \( \Psi \) is pure. Then

\[
F(\Psi, \rho) \geq F(\Psi, \sigma) - \frac{1}{3}\|\rho - \sigma\|_1
\]

Lemma 20 ([35]). Let \( \Psi \in \mathcal{S}(\mathcal{K}_A), \rho \in \mathcal{S}(\mathcal{K}_B), \sigma \in \mathcal{S}(\mathcal{K}_A \otimes \mathcal{K}_B) \). Then

\[
F(\Psi \otimes \rho, \sigma) \geq 1 - \|\rho - \sigma\|_1 - 3(1 - F(\Psi, \sigma_A)).
\]

Lemma 21 ([4],[33]). Let \( \rho, \sigma \in \mathcal{S}(\mathcal{K}_A \otimes \mathcal{K}_B) \). Then

\[
\|\rho - \sigma\|_1 \leq \frac{1}{\epsilon} \left( |I(A)B, \rho) - I(A)B, \sigma)| \leq 6 \epsilon \log \dim K_A + (2 + 4\epsilon)h(2\epsilon/1 + 2\epsilon),
\]

where \( h(x) = -x \log x - (1 - x) \log(1 - x) \), \( s \in (0,1) \) is the binary Shannon entropy.

Appendix B: Convexity of the capacity region formula

In this appendix, we show, that the functional expressions in Theorem 6 do not need further convexification. Following the arguments given in [35] for the case of a perfectly known QMAC, we show, that for given \( \mathcal{M} \subset \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C) \), the set

\[
\hat{\mathcal{C}}_1(\mathcal{M}) := \bigcup_{l=1}^{\infty} \bigcup_{p, V, \Psi} \bigcap_{\mathcal{M} \ni \mathcal{M}} \hat{\mathcal{C}}_1(\mathcal{M}^{(l)}, p, V, \Psi), \quad (B1)
\]
is convex, i.e. we show

**Lemma 22.** Let \( \mathcal{M} \in \mathcal{C}(\mathcal{H}_A \otimes \mathcal{H}_B, \mathcal{H}_C) \). It holds

\[
\text{conv}(\hat{\mathcal{C}}_1(\mathcal{M})) = \hat{\mathcal{C}}_1(\mathcal{M}). \quad (B2)
\]

**Proof.** We have to show, that for each \( \lambda \in (0,1) \), and any two rate pairs \( (R_1^{(i)}, R_2^{(i)}) \in \hat{\mathcal{C}}_1(\mathcal{M}), i = 1, 2 \), the their convex combination \( (\overline{R}_1, \overline{R}_2) \) with \( \overline{R}_1^{(i)} = \lambda R_1^{(i)} + (1 - \lambda)R_2^{(i)} \) for \( i = 1, 2 \) is also a member of \( \hat{\mathcal{C}}_1(\mathcal{M}) \). Assume

\[
(R_1^{(i)}, R_2^{(i)}) \in \bigcap_{\mathcal{M} \ni \mathcal{M}} \hat{\mathcal{C}}_1(\mathcal{M}^{(l)}, p, V, \Psi)
\]

(\ref{B3})
for some $l_i, p_i, V_i, \Psi_i$, i.e. with effective states
\[
\omega_i(M) := \omega(M^\otimes l_i, p_i, V_i, \Psi_i) \quad (i \in 1, 2, M \in \mathbb{N})
\]
according to [5], the equations
\[
R_1 - \epsilon \geq \inf_{M \in \mathbb{N}} I(X_i; C_i, \omega_i), \quad \text{and}
\]
\[
R_2 - \epsilon \geq \inf_{M \in \mathbb{N}} I(B_1 \otimes X_i, \omega_i)
\]
are fulfilled. Fix $\delta > 0$, and let $k, n \in \mathbb{N}$, $0 < k < n$ such that
\[
|\lambda - \frac{k}{n}| < \frac{\delta}{R_1^{(1)} + R_2^{(1)} + R_1^{(2)} + R_2^{(2)}}
\]
Set $t_1 := kl_2$, $t_2 := (n-k)l_1$. With
\[
\alpha(M) := \omega_1(M)^{\otimes k} \otimes \omega_2(M)^{\otimes n-k},
\]
which is unitarily equivalent to
\[
\omega(M^\otimes l_1, l_2, p_1^l \otimes p_2^l, V_1^\otimes l_1 \otimes V_2^\otimes l_2, \Psi_1^\otimes l_1 \otimes \Psi_2^\otimes l_2)
\]
we have
\[
I(X_1^i X_2^j; C_i^{l_i n}, \alpha(M)) = I(X_1^i X_2^j; C_i^{l_i n}, \omega_1(M)^{\otimes k} \otimes \omega_2(M)^{\otimes n-k})
\]
\[
= I(X_1^i X_2^j; C_i^{l_i n}, \omega_1(M)^{\otimes k}) + I(X_2^j; C_i^{l_i n}, \omega_2(M)^{\otimes n-k})
\]
\[
\geq t_1 l_1 R_1^{(1)} + t_2 l_2 R_2^{(2)}
\]
\[
\geq k l_1 l_2 R_1^{(1)} + (n-k) l_1 l_2 R_2^{(2)}
\]
where the second and third equality above are by additivity of the quantum mutual information evaluated on product states, and the inequality is by [5]. Consequently, we have
\[
\frac{1}{l_1 l_2 n} \inf_{M \in \mathbb{N}} I(X_1 X_2; C_i^{l_i n}, \alpha(M)) \geq \frac{k}{n} R_1^{(1)} + (1 - \frac{k}{n}) R_2^{(2)}
\]
\[
\geq R_1^{(1)} + (1 - \lambda) R_2^{(2)} - \delta.
\]
In a similar manner, also the inequality
\[
\frac{1}{l_1 l_2 n} \inf_{M \in \mathbb{N}} I(B_1 X_2; C_i^{l_i n}, X_1^i X_2^j, \alpha(M)) \geq \lambda R_1^{(1)} + (1 - \lambda) R_2^{(2)} - \delta
\]
is verified. By [B15], and [B16],
\[
(\overline{R}_1, \overline{R}_2) \subset \left( \bigcup_{M \in \mathbb{N}} \frac{1}{l_1 l_2} C_i^{(1)}(M^\otimes l_1, l_2, p_1^l \otimes p_2^l, V_1^\otimes l_1 \otimes V_2^\otimes l_2, \Psi_1^\otimes l_1 \otimes \Psi_2^\otimes l_2) \right) \subset \hat{C}_i(\mathbb{M})_{\delta}.
\]
Since $\delta > 0$ can be chosen arbitrarily, the claim of the lemma follows. \[\square\]

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