Spatiotemporal Stabilization of Locally PT-symmetric Semiconductor Lasers

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**Abstract:** We provide a feasible and compact scheme to control and stabilize the spatiotemporal dynamics of BAS lasers. The proposal is based on the ability of non-Hermitian potentials with given local symmetries to manage the flow of light. A local PT-symmetric configuration allows to control, enhance and localize the generated light. We impose a pump modulation, with a central symmetry axis which induces in-phase gain and refractive index modulations due to the Henry factor. Both modulations are, in turn, spatially dephased by an appropriate index profile to yield to a local PT-symmetry within the modified BAS laser. Such local PT-symmetry potential induces an inward mode coupling, accumulating the light generated from the entire active layer at the central symmetry axis, which ensures spatial regularization and temporal stability. By an exhaustive exploration of the modulation parameters, we show a significant improvement of the intensity concentration, stability and brightness of the emitted beam. This approach produces a two-fold benefit: light localization into a narrow beam emission and the control over the spatiotemporal dynamics, improving the laser performance.

Broad Area Semiconductor (BAS) amplifiers and lasers are prevalent and trustworthy light sources, used for many applications ranging from biomedicine [1] to telecommunications [2]. Their main advantage is the compactness and the high conversion efficiency, while their major drawback is the relatively low spatial and temporal quality of the emitted beam [3,4]. The fundamental phenomenon that induces spatiotemporal instabilities is Modulation Instability (MI) [5]. This intrinsic instability and the nonlinear modal interaction lead to complex spatiotemporal dynamics and filamentation, a disruption of the field into multiple filaments, limiting possible applications [6,7]. The stabilization of these devices is achieved by considering different schemes, some of which propose the introduction of spatial [8,9] or spatiotemporal modulations [10]. Besides, another study introduces a spatial non-Hermitian potential in BAS amplifiers, with a double spatial modulation of refractive index and pump, leading to a substantial improvement of the spatial quality of the beam [11]. Also, vertical-external-cavity surface-emitting lasers with external flat mirrors can be stabilized by applying a periodic spatiotemporal modulation of the pump current [12]. More recently, attention was paid on a specific kind of non-Hermitian systems, holding PT-symmetry [13]. In such materials, the complex refractive index fulfills: \( n(x) = n^*(-x) \) i.e., the real part representing the refractive index is symmetric while the imaginary part representing gain-loss is antisymmetric in space. In periodic PT-symmetric media index and gain-loss modulations are dephased by a quarter of the wavenumber of the modulation. Such systems hold maximum asymmetric mode coupling at the PT-symmetry breaking point, or exceptional point, when the gain-loss and refractive index modulation amplitudes are balanced [14,15]. In optics, global or local PT-symmetry lead to unconventional beam dynamics arising precisely from the asymmetric mode coupling, such as the unidirectional light transport following arbitrary vector fields [16,17]. Therefore, we here propose that local PT-symmetry could also be applied to regularize the spatiotemporal dynamics of BAS lasers, while improving stability through unidirectional mode coupling.
In this letter, we apply a local PT-symmetric potential to control the spatiotemporal dynamics of BAS lasers by localizing the light generated in the entire active layer into a narrow central beam. The typical complex emission from BAS sources is unstable both in space and in time domains, as shown on Figs. 1.a. The proposal is to regularize and control the emission in modulated BAS, as shown in Fig. 1.b, leading to a narrow beam emission which is expected to be useful for a large variety of practical applications.

Fig. 1. BAS emission, schematic illustration: (a) complex irregular spatial pattern emitted from a conventional BAS source, (b) bright and narrow beam emission from a modified BAS with local PT-symmetry with pump and effective refractive index modulations as shown in (c). (c) Scheme of the transverse spatial distribution of the local PT-symmetric potential, with balanced gain (in brown) and effective refractive index (in blue) modulations, for different configurations. A possible implementation could consist on a simple patterning of the electrodes [19] for the pump modulation and using well-known techniques as metallo-organic molecular beam epitaxy [20] to structure the substrate.

BAS semiconductor sources, amplifiers and lasers, are generally described by stationary models including field and carriers [21] or, alternatively, by mean field models including temporal evolution [22]. A different approximation for BAS amplifiers is given by the optical field propagation along the semiconductor, while carrier and field dynamics is neglected. Previous studies on local PT-symmetry were similarly performed considering strongly simplified stationary models and only considering forward field propagation in a paraxial approximation [17].

These simplifications become inadequate for the spatial regularization and temporal stabilization of BAS lasers that requires a spatiotemporal calculation of field and carriers. Thus, we develop a new integration scheme taking advantage of the different time scales of the cavity round trip time and carrier’s relaxation time. Every integration step combines the field propagation in one cavity round trip assuming constant carriers and the temporal integration of carriers, considering an already stabilized field. The following BAS source representation obtained from well-known models [21, 22], allows us to integrate along the cavity the field amplitude envelope, $A$, —composed by the forward, $A^+$, and backward, $A^-$, fields—, and to integrate carriers, $N$, in time:
\[
\pm \frac{\partial A^\pm}{\partial z} = \frac{i}{2k_0n} \frac{\partial^2 A^\pm}{\partial \xi^2} + s[(1-i\hbar)N-(1+\alpha)]A^\pm + i\Delta n(x,z)k_0A^\pm
\]
\[
\frac{\partial N}{\partial t} = \gamma(-N-(N-1)|A|^2 + p_0 + \Delta p(x,z) + D\nabla^2 N)
\]
where \(|A|^2 = |A^+|^2 + |A^-|^2\)

and where \(k_0\) is the wavevector, \(n\) the refractive index, \(s\) a parameter inversely proportional to the light matter interaction length, \(h\) the Henry factor or linewidth enhancement factor of the semiconductor, \(\alpha\) corresponds to losses, \(\gamma\) is the carriers relaxation rate, \(p_0\) is the pump, and \(D\) is the carrier diffusion. The polarization of the semiconductor is adiabatically neglected (as typically considered for class B lasers). Finally, \(\Delta p(x,z)\) and \(\Delta n(x,z)\) represent the pump and index modulations, respectively. We assume a symmetric harmonic transverse modulation of the pump in the form: \(\Delta p(x,z) = m_1 \sin(q_x|x|+\Phi)\) where \(q_x = 2\pi/d_x\), being \(d_x\) the transverse period, and \(m_1\) the amplitude of the pump modulation \((z\) is the propagation direction). Such pump modulation, with symmetry axis at \(x=0\), induces in turn an in-phase refractive index modulation trough the Henry factor, \(h\). Therefore, the introduced harmonic refractive index \(\Delta n(x,z) = m_2 \cos(q_x|x|+\Phi) + m_3 \sin(q_x|x|+\Phi)\) is written in two terms, where the second one is intended to compensate the induced index modulation and the first to render the refractive index in quadrature with the gain modulation. The symmetry axis at \(x = 0\) divides the system in two half-spaces, both holding a global PT-symmetry. Such potential is expected to couple the transverse modes asymmetrically, promoting the inward coupling, enhancing and localizing light at axis. Hence, the modulated BAS system depends on four parameters, namely the three modulations amplitudes \((m_1, m_2, m_3)\) and the phase, \(\Phi\) that controls the character of the center — in Fig. 1c the modulation profiles correspond to: \(\Phi = 0\) (left), \(\Phi = \pi/4\) (center) and \(\Phi = \pi/2\) (right) —.

We preliminarily analyze a BAS amplifier with real parameters as a proof of concept of the regularization scheme based on local PT-symmetry, before considering the management of radiation from laser sources.

The local PT-symmetric BAS amplifier is modelled by the system of equations (1), only assuming a forward propagating field, i.e. \(A^- = 0\). The spatiotemporal integration of the unmodulated BAS exhibits both inhomogeneous spatial and unstable temporal behaviors (Figs 2.a and 2.b). Introducing the above-mentioned pump and refractive index periodic modulations with PT-symmetric profiles, the BAS radiation emission may be regularized to enhance and spatially localize light at the symmetry center, \(x = 0\), resulting into a bright and narrow BAS amplifier emission (Fig. 2.c). Moreover, in addition to the spatial regularization, the field becomes temporally stable, as shown in Fig. 2.d. Therefore, local PT-symmetry has a direct impact on both spatial and temporal stability.

To assess the performance of the proposed modulated BAS amplifier, we evaluate the axial concentration factor, as a figure of merit of light localization, defined as the central intensity, \(I_0=I(x=0)\), over the averaged intensity: \(c = I_0/\langle I(x) \rangle\). We explore the parameter space of the index modulations amplitudes \(m_2\) and \(m_3\), for a fixed value of the pump modulation amplitude, \(m_1\), and a fixed phase, \(\Phi = 0\). We expect that for particular parameters, the inward coupling localize light around the symmetry axis, while the central index maximum contributes to induce a guiding effect. The results are summarized on Fig. 2.e. In this scheme, depending on the sign of the product \(m_1m_2\), we may expect regions of inward or outward mode coupling. The inward coupling \((m_1m_2 < 0)\) leads to an accumulation of the field around the center, which is maximized for a particular value of the counterbalancing amplitude; \(m_3\) (being also \(m_1m_3 < 0\)). Note that the maximum energy localization is expected for balanced gain and refraction index modulations in a local PT-symmetric potential [17]. Indeed, for \(m_1 > 0\), we observe the maximum confinement for positive \(m_2\) and negative \(m_3\) values, see Fig. 2.e. For \(m_2 < 0\) the concentration factor is below 1 indicating the field at the center is lower than the average,
consistent with the outward coupling. Finally, Fig. 2.f depicts the transverse beam intensity profile for three particular situations, corresponding to the same values for \( m_1 \) and \( m_3 \) but for positive, zero and negative values of \( m_2 \). We may observe that when the modulations induce an inward coupling light is localized and strongly enhanced at the center. On the contrary, when index and gain modulations are in phase (\( m_2 = 0 \)) the coupling is symmetric, and the field is homogeneously distributed, and when coupling is outwards (\( m_2 < 0 \)) intensity at the center is reduced.

The BAS laser is also described by the system of equations (1) where the field components, \( A^+ \) and \( A^- \), are related by the corresponding boundary conditions \( A^+(x, z=L, t) = r_L A^-(x, z=L, t) \) and \( A^-(x, z=0, t) = r_{0A} A^+(x, z=0, t) \). Where \( L \) is the semiconductor length and \( r_{0A} \) are the corresponding reflection coefficients of the cavity mirrors at \( z = 0/L \), respectively, see Fig. 3.a.
We start by assessing the laser emission in the simplest homogeneous and stationary state for the unmodulated case. The solution only depends on mirror reflectivities, homogeneous pump, losses and s parameter. Assuming: \( \nabla^2 N = 0; \frac{\partial I}{\partial x} = 0, \Delta n = 0 \) and \( \Delta p = 0 \), the adiabatic state of carriers becomes: \( N = N_0 + \frac{1}{1 + |d|^2} \), a function of the field intensity \( I^\pm \equiv |A^\pm|^2 = A^\pm A^{\pm*} \). Therefore, forward and backward field intensities \( I^\pm(z) \), the total intensity \( I = I^+ + I^- \) and the output intensity \( I_{\text{out}} \), are evaluated by the propagation equation: 

\[
\pm \frac{\partial I^\pm}{\partial z} = \frac{1}{2s(0 - 1 - s^2 + \alpha + 1)} \ln(I^\pm(z) - p_0 + \alpha + 1) - \alpha \ln(I^\pm(z)) \]

with analytic solution: 

\[
g[I^\pm(z)] = g(I^\pm(0)) \pm \beta(z), \quad \text{where } g[I^\pm(z)] = (p_0 - 1) \ln(I^\pm(z) - p_0 + \alpha + 1) - \alpha \ln(I^\pm(z)) \quad \text{and } \beta(z) = 2s\alpha(1 + \alpha - p_0)z.
\]

The resulting pump threshold, total intensity inside the cavity (Fig.3a,b) output intensity (Fig.3c) as well as carrier distributions determines the working regime of the unmodulated BAS laser, which will be considered as initial conditions in the spatiotemporal integration of the PT-symmetry BAS laser and further used for comparison.

The effect of the phase, \( \Phi \), is more critical for the case of the local PT-symmetric BAS laser, due to the feedback introduced by the cavity. In this case, \( \Phi = \pi/4 \) turns out to be the optimum situation for field concentration at the center because gain-maxima are located closer to the center while an index relative maximum is still present at \( x = 0 \) preserving an index guiding effect. While for \( \Phi = 0 \) and \( \Phi = \pi/2 \), larger values of \( m_2 \) and \( m_3 \) are needed to obtain an effective inward coupling, for \( \Phi = \pi/4 \) the effect is maximized for lower modulation amplitudes. The values of the rest of parameters (\( \alpha, h, s, q_t, k_0, D \)) are the previously considered for the BAS amplifier. We introduce the same pump and refractive index modulations of the amplifier case to induce a PT-symmetric potential, fixing the amplitude of the pump harmonic modulation to \( m_1 = 0.5, m_2 \) and \( m_3 \) are adjusted to simultaneously compensate the (in phase) refractive index modulation induced by the Henry factor and to introduce the shifted refractive index modulation. Indeed, we observe that the BAS laser radiation can be regularized, and the field concentrated within a spatially stable central narrow beam as shown in Fig. 4.a. Moreover, the laser emission may also be stabilized in time, as shown on Fig. 4.b. The intricate spatial patterns and unstable temporal behavior of the complex dynamics of the same unmodulated BAS laser, provided in the inset in Figs. 4.a and 4.b, are now regularized.

Next, we perform a comprehensive exploration of the axial concentration, the results are mapped in Fig. 4.c. We observe regions of maximal localization for \( m_2 > 0 \) and \( m_3 < 0 \), when the coupling is inwards, analogous to the case of the amplifier, achieving a significant
concentration. We note that the required $m_2$ and $m_3$ amplitudes are smaller than for the amplifier. This is attributed to the long effective cavity length $L/(1-r_0)(1-r_L)$ with a value about $10^2 L$ for the considered reflectivities, see Fig. 4d. Finally, it is worth to remind that since we here assume $\Phi = \pi/4$ the area of maximal axial concentration, mapped on Fig. 4c, is slightly shifted as compared to Fig. 2.e.

Fig. 4. (a) Localized spatial intensity distribution (in W/cm$^2$) within the modulated BAS laser and (b) temporal evolution of the narrow beam emission from the modified BAS with $m_1 = 0.5$, $m_2 = 0.0048$ and $m_3 = -0.0143$ with $p_0 = 1.23$, $\Phi = \pi/4$, $r_0 = 0.99$ and $r_L = 0.9$. The insets depict the inhomogeneous laser spatial intensity pattern and unstable laser emission of the unmodulated case, respectively. (c) Axial concentration factor map for $m_1 = 0.5$, in $(m_2, m_3)$ parameter space. (d) Intensity distribution transverse cuts, at $z = L$, normalized to the mean intensity, for three representative points corresponding to: (I) $m_2 = 0.0143$, $m_3 = -0.0143$, (II) $m_2 = 0.0048$, $m_3 = -0.0143$, (III) $m_2 = 0.0048$, $m_3 = -0.0072$. (e) Temporal stability map of the PT-symmetric BAS laser: amplitude of the temporal intensity oscillations. (f) Temporal evolution of the emission for the previous three representative points, (I) and (III) achieve a temporally stable regime while (II) corresponding to a pulsed regime. All other integration parameters are the same as in Fig. 2.

In addition, we assess the temporal stability of the emitted narrow beam by mapping the temporal oscillations of the peak intensity. Inspecting Fig. 4e, we observe that not all the spatial concentration region on Fig. 4c is temporally stable, yet there is a wide range of parameters for which we simultaneously achieve full regularization of the emission, spatial field localization and temporal stability. Temporal instabilities arising from pulsed regimes appear for a restricted island of parameters, see Fig. 4f. Interestingly, the laser is temporally stable at the maximum concentration area for the considered pump value, above threshold. Simulations also show that the temporal stability of the localized state is mainly controlled by the balance
between pump and refractive index modulations, namely the value of $m_2$, while the spatial shift between potentials, $m_3$, determines the spatial instability.

We finally analyze the BAS laser general performance in different working conditions - as a function of the input homogeneous pump, $p_0$, and pump modulation amplitude, $m_1$, keeping the index modulation amplitudes, $m_2$ and $m_3$, optimized and proportional to $m_1$. As might be expected, increasing $m_1$, the lasing threshold of the central area decreases leading to two clear different lasing regimes, see Fig. 5a. For small pump powers, lasing is mainly restricted to the central area, leading to a significantly high intensity concentration factor, see Fig. 5b. Interestingly, for sufficiently high modulations of the BAS laser a bright and narrow beam is generated even below the homogenous laser threshold ($p_0 = 1.2$). Above threshold, amplification occurs in all the active material, and localization persists. The mean generated intensity increases with the pump, almost independent of the potential, either for the unmodulated or modulated BAS lasers, see the dotted curves in Fig. 5c. We note that for small modulation amplitudes of the local PT-symmetric potentials the peak intensity $I_0$ grows faster, increasing pump while having less concentration. This is attributed to the existence of several transverse modes for the peak profile, see Fig. 5d. Moreover, the lasing threshold line in Fig. 5a shows clear different slopes in the $p_0$-$m_1$ plane for both peak transverse profiles. The transition between these profiles for a given value of the modulation amplitude, does neither show hysteresis nor bistability, Fig. 5d. These calculations intended to regularize the BAS laser radiation, while performed for a set of parameters, are restricted to realistic values, including actual values of the cavity reflectivity (which however could be generalized using Fig. 3).
To conclude, we propose to use local PT-symmetric potentials to tailor and control the complex spatiotemporal dynamics of BAS amplifiers and lasers. We propose to introduce modulations in the pump and refractive index with a central symmetry axis (at $x = 0$) to induce local PT-symmetry. While the pump modulation is a symmetric harmonic modulation the required double harmonic index modulation is intended to generate a symmetric and local PT-symmetry by introducing a dephased index modulation while compensating the index modulation induced by the pump modulation through the Henry factor (which is in phase with the pump). The field regularization mechanism relies on the inward coupling of the light generated in the active layer, which directs and concentrates light from each of the two half-spaces ($x < 0$ and $x > 0$) towards the center. The maximum localization regimes are first analyzed for a BAS amplifier. Exploring the relative amplitudes of the modulation of the pump and the index, we observe a significant intensity field enhancement and concentration of the emitted beam as compared to the unmodulated amplifier, coinciding with inward PT-symmetric coupling. The results are further numerically confirmed in the more engaged case of the BAS laser, where both forward and backward field components are considered in the model. In this case, the character of the symmetry center (controlled by a general phase, $\Phi$) plays a more important role. When the center holds a high effective refractive index, with two side gain-maxima ($\Phi = \pi/4$), the local PT-symmetry potential induces simultaneously stable and regularized emission for a wide range of modulation amplitudes. It is worth to mention that this area of temporal stability partially coincides with the maximum concentration area in a twofold benefit. The system is studied under different working conditions of input power and mirror reflectivities, observing a substantial filed regularization, especially for pumps below and close to the unmodulated laser threshold. The proposed scheme renders BAS lasers into spatially and temporally stable laser sources emitting a narrow and enhanced beam. Moreover, the proposed scheme is general and can be applicable to other BAS lasers to improve their performance.

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