Convolutional Geometric Matrix Completion

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Abstract
Geometric matrix completion (GMC) has been proposed for recommendation by integrating the relationship (link) graphs among users/items into matrix completion (MC). Traditional GMC methods typically adopt graph regularization to impose smoothness priors for MC. Recently, geometric deep learning on graphs (GDLG) is proposed to solve the GMC problem, showing better performance than existing GMC methods including traditional graph regularization based methods. To the best of our knowledge, there exists only one GDLG method for GMC, which is called RMGCNN. RMGCNN combines graph convolutional network (GCN) and recurrent neural network (RNN) together for GMC. In the original work of RMGCNN, RMGCNN demonstrates better performance than pure GCN-based method. In this paper, we propose a new GMC method, called convolutional geometric matrix completion (CGMC), for recommendation with graphs among users/items. CGMC is a pure GCN-based method with a newly designed graph convolutional network. Experimental results on real datasets show that CGMC can outperform other state-of-the-art methods including RMGCNN.

1. Introduction
Recommender systems (Koren et al., 2009) have been widely deployed in lots of applications, such as item recommendation on shopping web site, friend recommendation on social web site and so on. There are two main kinds of methods for recommender systems, content-based filtering (Pazzani & Billsus, 2007) and collaborative filtering (Breese et al., 1998). Content-based filtering methods recommend new items that are most similar to users' historical favorite items. Collaborative filtering methods use collective ratings to make new recommendations by similar rating patterns between users or items.

By regarding rows as users, columns as items and entries as ratings on items by users, the task of recommender systems can be formulated as a matrix completion (MC) problem (Candès & Recht, 2009; 2012). MC has attracted lots of attention in recent years. MC models aim to predict the missing entries in a matrix given a small subset of observed entries. Under the low-rank setting, (Candès & Recht, 2009; 2012) have proved that matrix can be exactly recovered given sufficiently large number of observed entries, although it is a NP-hard problem. One efficient solution for MC problem is to adopt matrix factorization (MF) techniques (Monti et al., 2017).

In many real applications, besides the rating matrix which contains the ratings on items by users, other side information is also available. Typical side information includes attributes of users/items and the relationship (link) graphs between users/items. Therefore, there have appeared a few works to incorporate the attributes of users/items to boost the performance of matrix completion models (Jain & Dhillon, 2013; Xu et al., 2013). Furthermore, geometric matrix completion (GMC) models (Li & Yeung, 2009; Kalofolias et al., 2014; Rao et al., 2015; Monti et al., 2017) have also been proposed for recommendation by integrating the relationship (link) graphs among users/items into matrix completion. For example, the methods in (Li & Yeung, 2009; Agarwal & Chen, 2009; Adams et al., 2010; Porteous et al., 2010; Ma et al., 2011; Cai et al., 2011; Menon et al., 2011; Kalofolias et al., 2014; Rao et al., 2015) propose to encode the structural (geometric) information of graphs via graph Laplacian regularization (Belkin & Niyogi, 2001; 2003) which tries to impose smoothness priors on latent factors (embeddings) of users/items. These graph regularization based methods have shown promising performance in real applications.

Recently, geometric deep learning techniques (Bruna et al., 2014; Gori et al., 2005; Li et al., 2016; Henaff et al., 2015; Sukhbaatar et al., 2016; Defferrard et al., 2016; Kipf & Welling, 2017) are proposed to learn meaningful representations for geometric structure data, such as graphs and manifolds. In particular, geometric deep learning on graphs (GDLG) (Defferrard et al., 2016; Monti et al., 2017) has been proposed to solve the GMC problem, showing
better performance than existing GMC methods including graph regularization based methods. To the best of our knowledge, there exists only one GDLG method for GMC, which is called recurrent multi-graph convolutional neural network (RMGCNN) (Monti et al., 2017). Based on spectral graph convolution framework (Defferrard et al., 2016), RMGCNN defines two-dimensional graph convolutional filters to process multi-graphs. The graph embeddings extracted by the two-dimensional graph convolutional filters are fed into a Long Short-Term Memory (LSTM) recurrent neural network (RNN) (Hochreiter & Schmidhuber, 1997) to perform diffusion process, which is actually feature transformation. After that, the final embeddings are used to do matrix completion task. A factorized (matrix factorization) version, called separable RMGCNN (sRMGCNN), is also proposed in (Monti et al., 2017) for efficiency improvement. RMGCNN combines graph convolutional network (GCN) and recurrent neural network (RNN) together for GMC. Experimental results in (Monti et al., 2017) show that the GCN part and RNN part can improve the performance of matrix completion simultaneously. However, matrix completion with pure GCN, named MGCNN in (Monti et al., 2017), is shown to be worse than RMGCNN in experiments.

In this paper, we propose a new GMC method, called convolutional geometric matrix completion (CGMC), for recommendation with graphs among users/items. CGMC is a pure GCN-based method. The contributions of CGMC are listed as follows:

- **In CGMC, a new graph convolutional network is designed, by taking only the first two terms of Chebyshev polynomials in spectral graph convolution (Defferrard et al., 2016) and adopting weighted policy to control the contribution between self-connections and neighbors for graph embeddings.**

- **Because the roles of users in the rating matrix and user graph are different, the latent factors (embeddings) to represent users for rating matrix and those for user graph should also have some difference, although the users are the same. Hence, in CGMC, a fully connected layer is added to the output of GCN to project the user graph embeddings to a compatible space for rating matrix. Similar operations are also performed for items.**

- **CGMC integrates GCN and MC into a unified deep learning framework, in which the two components (GCN and MC) can give feedback to each other.**

- **Experimental results on real datasets show that CGMC can outperform other state-of-the-art methods including RMGCNN. Hence, with properly designed network architecture for graph convolution, our work shows that pure GCN-based method can also achieve the best performance.**

The following content is organized as follows. Section 2 briefly discusses some related work. Section 3 presents the details of CGMC. Section 4 shows experimental results. Section 5 concludes the paper.

### 2. Related Work

In this section, we introduce the related work of CGMC, including matrix completion (MC), geometric matrix completion (GMC), geometric deep learning on graphs (GDLG), and GDLG based GMC.

#### 2.1. Matrix Completion

Suppose $M \in \mathbb{R}^{m \times n}$ is a rating matrix, with $m$ being the number of users and $n$ being the number of items. Given a subset of the entries $M_{ij}, (i, j) \in \Omega, |\Omega| \ll mn$. Matrix completion problem aims to estimate $M_{ij}, \forall(i, j) \notin \Omega$. It is formulated as follows (Candès & Recht, 2009; 2012; Cai et al., 2010):

$$\min_{Z} \|P_{\Omega}(M - Z)\|_{F}^{2} + \lambda_{Z} \|Z\|_{*},$$

where $\|Z\|_{*}$ is the nuclear norm of the matrix $Z$, $P_{\Omega}$ is the projection operator, where $[P_{\Omega}(M)]_{ij} = M_{ij}$ if $(i, j) \in \Omega$, else $[P_{\Omega}(M)]_{ij} = 0$.

One solution to solve the MC problem is to reformulate it as the following matrix factorization (MF) problem:

$$\min_{W, H} \|P_{\Omega}(M - WH^{T})\|_{F}^{2} + \lambda_{w} \|W\|_{F}^{2} + \lambda_{h} \|H\|_{F}^{2},$$

where $W$ and $H$ are latent factor representation for users and items, respectively.

#### 2.2. Geometric Matrix Completion

Geometric matrix completion (GMC) (Li & Yeung, 2009; Agarwal & Chen, 2009; Adams et al., 2010; Porteous et al., 2010; Ma et al., 2011; Cai et al., 2011; Menon et al., 2011; Kalofolias et al., 2014; Rao et al., 2015) has been developed to exploit the relationship (link) graph among users/items to assist the matrix completion process. One kind of GMC methods is to adopt graph Laplacian for regularization. GRALS (Rao et al., 2015) is one representative of this kind, which is formulated as follows:

$$\min_{W, H} \|P_{\Omega}(M - WH^{T})\|_{F}^{2} + \lambda_{w} \|W\|_{F}^{2} + \lambda_{h} \|H\|_{F}^{2}$$

$$+ \gamma_{w} \text{tr}(W^{T}L_{W}W) + \gamma_{h} \text{tr}(H^{T}L_{H}H),$$

where $L_{W}$ and $L_{H}$ are the normalized graph Laplacian of user graph $A$ and item graph $B$, respectively. $L_{W} = I -$
A variant of spectral graph convolutional network (GCN) is proposed in (Kipf & Welling, 2017). We call it GCN-kw in this paper. GCN-kw is a simplified version of the GCN in (Defferrard et al., 2016) by assuming $K = 1$ and $\lambda_{\text{max}} \approx 2$:

$$g_\theta \ast x = \theta_0 x + \theta_1 (L - I) x = \theta_0 x - \theta_1 D^{-\frac{1}{2}} GD^{-\frac{1}{2}} x,$$

where $D$ denotes the diagonal degree matrix of $G$. Then, by constraining $\theta = \theta_0 = -\theta_1$, we have:

$$g_\theta \ast x = \theta (I + D^{-\frac{1}{2}} GD^{-\frac{1}{2}}) x.$$

2.4. GDLG based GMC

To the best of our knowledge, RMGCNN (Monti et al., 2017) is the only work which has applied geometric deep learning on graphs (GDLG) for GMC. RMGCNN adopts GCN (Defferrard et al., 2016) to extract graph embeddings for users and items, and then combines with recurrent neural network (RNN) to perform diffusion process. The factorized version of RMGCNN (Monti et al., 2017) is shown as follows:

$$\min_{\theta, \sigma} ||P_\Omega (M - W_{\theta, \sigma}(H_{\theta, \sigma}))^2 ||_F$$

$$+ \frac{\mu}{2} (||W_{\theta, \sigma}||_{\text{F}}^2 + ||H_{\theta, \sigma}||_{\text{F}}^2),$$

where $W_{\theta, \sigma}$ and $H_{\theta, \sigma}$ are the graph embeddings extracted by GCN and RNN for users and items respectively, $G_u$ and $G_c$ are graphs on users and items respectively, $T$ denotes the graph embedding matrices for $\lambda_{\text{max}}$, and $\| \cdot \|_{\text{F}}$ represents the Frobenius norm.

3. Convolutional GMC

In this section, we present the details of our new GDLG-based GMC method, called convolutional geometric matrix completion (CGMC). CGMC is a pure GCN-based method. CGMC shows that GMC with only GCN can outperform the GCN+RNN method RMGCNN to achieve the state-of-the-art performance.

CGMC is formulated as follows. Firstly, a new GCN is proposed to extract graph embedding, which is called convolutional graph embedding (CGE) in this paper, for user/item
representation. Then, a fully connected layer is added to the output of GCN to project user/item graph embeddings to a compatible space for rating matrix. After that, GCN and MC are integrated into a unified deep learning framework to get CGMC.

3.1. Convolutional Graph Embedding (CGE)

Here, we propose a new GCN to get the convolutional graph embedding (CGE) for graph node representation.

By taking $K = 1$ in the spectral graph convolution of (4), we have:

$$g_0 \ast x = \theta_0 x + \theta_1 \tilde{L}x$$
$$= \theta_0 x + \theta_1 \left(\frac{2}{\lambda_{\max}} LH\right)x$$
$$= \theta_0 x + \theta_1 \left(\frac{2}{\lambda_{\max}}(I - S) - I\right)x$$
$$= \left(\theta_0 + \theta_1 \left(\frac{2}{\lambda_{\max}} - 1\right)I\right)x$$
$$= \left(\theta_0 + \theta_1 \left(\frac{2}{\lambda_{\max}} - 1\right)\right)x$$

where we let $S = D^{-\frac{1}{2}}GD^{-\frac{1}{2}}$ and $G$ is the link matrix of the graph with $N$ nodes. Since $\theta_0$, $\theta_1$ are free parameters, and there is no constraints between the coefficients of $I$ and $S$, we can let $\alpha_0 = \theta_0 + \theta_1 \left(\frac{2}{\lambda_{\max}} - 1\right)$, $\alpha_1 = -\theta_1 \frac{2}{\lambda_{\max}}$. Then, we have

$$g_0 \ast x = (\alpha_0 I + \alpha_1 S)x.$$  \hspace{1cm} (9)

Furthermore, $\alpha_0$, $\alpha_1$ are still free parameters. We let $\alpha_0 = \theta \sigma_0$, $\alpha_1 = \theta \sigma_1$, and get

$$g_0 \ast x = \theta (\sigma_0 I + \sigma_1 S)x.$$  \hspace{1cm} (10)

Here, $\sigma_0$ and $\sigma_1$ can be explained as a weight controlling the contribution between self-connections and neighbors.

In our GCN, we constrain $\sigma_0 + \sigma_1 = 1$ and $\sigma_0$, $\sigma_1 \in [0, 1]$. For convenience, we denote $\sigma = \sigma_0$, and get

$$g_0 \ast x = \theta (\sigma I + (1 - \sigma)S)x.$$  \hspace{1cm} (11)

The eigenvalues of $\sigma I + (1 - \sigma)S$ are in $[-1, 1]$, which can be easily verified according to Lemma 1.7 in (Chung, 1997). Hence, repeated application of the above filter won’t result in numerical instability. Due to the flexibility of $\sigma$, we treat it as a hyper-parameter and tune it based on a validation set.

When the input signal is multi-dimensional, denoted by $V \in \mathbb{R}^{N \times r}$ with $N$ being the number of nodes and $r$ being the dimensionality, we can get the formulation of multi-dimensional graph convolution as follows. We use $v_j$ to denote the $j$-th column of $V$, which is the $j$-th input signal.

$$\hat{v}_i = \sum_{j=0}^{r-1} \Theta_{ji} (\sigma + (1 - \sigma)S)v_j \quad i = 0, 1, \ldots, q - 1$$

where $q$ is the dimensionality of the output signal, $\Theta_{ji}$ is the filter parameter of the $i$-th output signal defined on the $j$-th input signal, $\Theta \in \mathbb{R}^{r \times q}$. Then we can get,

$$\hat{V} = (\sigma I + (1 - \sigma)S)V\Theta,$$  \hspace{1cm} (12)

which transforms the node representation from $V \in \mathbb{R}^{N \times r}$ to $\hat{V} \in \mathbb{R}^{N \times q}$ though one-layer graph convolution with the convolution parameter $\Theta$.

By stacking the above formulation to multiple layers, we can get a deep model for CGE. This is formulated as follows:

$$V^{(\ell+1)} = f((\sigma I + (1 - \sigma)S)V^{(\ell)}\Theta^{(\ell)})$$  \hspace{1cm} (13)

where $V^{(\ell)}$ is the output signal of the $\ell$-th layer, $\Theta^{(\ell)}$ is the convolution parameter of the $\ell$-th layer, and $f(\cdot)$ is an activation function.

3.2. Model of CGMC

Our CGMC can also be used for the nuclear norm regularization formulation in (1), by adopting similar techniques in RMGCNN (Monti et al., 2017). However, as pointed out by (Monti et al., 2017), the nuclear norm regularization formulation is time-consuming. Hence, in this paper, we adopt the MF formulation in (2) for our CGMC.

Suppose $X \in \mathbb{R}^{m \times r_1}$ denotes the input user features, $Y \in \mathbb{R}^{n \times r_2}$ denotes the input item features, with $m$ and $n$ being the number of users and items respectively, $r_1$ and $r_2$ being the feature dimensionality for users and items respectively. If $X$ or $Y$ is not available, we set $X = I$ or $Y = I$. $A$ and $B$ are user graph and item graph. Then the CGE for users and items can be generated by applying (13) to graph $A$ and graph $B$:

$$\hat{L}_W = \sigma I + (1 - \sigma)D_{W}^{-\frac{1}{2}}AD_{W}^{-\frac{1}{2}}$$
$$\hat{L}_H = \sigma I + (1 - \sigma)D_{H}^{-\frac{1}{2}}BD_{H}^{-\frac{1}{2}}$$

$$X^{(\ell+1)} = f(\hat{L}_W X^{(\ell)} W_\ell)$$
$$Y^{(\ell+1)} = f(\hat{L}_H Y^{(\ell)} H_\ell)$$  \hspace{1cm} (14)

where $D_{W}$ and $D_{H}$ are diagonal degree matrices of $A$ and $B$ respectively, $X^{(\ell)}$ and $Y^{(\ell)}$ are the output feature representation of the $\ell$-th layer, $X^{(0)} = X$ and $Y^{(0)} = Y$, $f(\cdot)$ is an activation function, here we take $f(\cdot) = \tanh(\cdot)$, $W_\ell$ and $H_\ell$ are convolution parameters which play the same role as $\Theta^{(\ell)}$ in (13).

**Fully-Connected Layer after CGE** For a specific user, he/she plays a role in the user graph, and he/she also plays another role in the rating matrix. These two roles are different. Intuitively, the latent factors (embeddings) to represent these two different roles of this user should also have some difference. Items also have similar property.
To capture the difference between these two roles, a fully connected layer is added to the output of GCN to project the CGE to a compatible space for rating matrix. The formulation is as follows:

\[
\hat{W} = f(LW + 1b_W), \\
\hat{H} = f(YH + 1b_H)
\]  

(15)

where \(X^{(L)}\) and \(Y^{(L)}\) are the output user features and item features of the CGE with \(L\) layers, \([W_L, b_W]\) and \([H_L, b_H]\) are parameters of the fully connected layer for user CGE and item CGE, \(f(\cdot) = \tanh(\cdot)\).

This is one key difference between our method and other methods like RMGCNN. In our experiments, we will verify that this fully connected layer will improve the performance of CGE.

**Objective Function** With the projection by the fully connected layer, CGMC is formulated as follows:

\[
\min_{\mathcal{W}, \mathcal{H}} \|P_\Omega(M - \hat{W}\hat{H}^\top)\|_F^2 + \frac{\lambda}{2} \mathcal{L}_{\text{reg}}
\]  

(16)

where \(\mathcal{W}\) denotes \([W_0, W_1, \cdots, W_L, b_W]\) and \(\mathcal{H}\) denotes \([H_0, H_1, \cdots, H_L, b_H]\), \(\mathcal{L}_{\text{reg}}\) is \(\ell_2\)-norm regularization on parameters in CGMC:

\[
\mathcal{L}_{\text{reg}} = \sum_{l=0}^{L} \|W_l\|_F^2 + \|H_l\|_F^2.
\]  

(17)

From (16), it is easy to find that CGMC seamlessly integrates GCN and MC into a unified deep learning framework, in which GCN and MC can give feedback to each other for performance improvement.

### 3.3. Learning

We adopt alternating minimization scheme to alternately optimize the parameters \(\mathcal{W}\) and \(\mathcal{H}\). Adadelta (Zeiler, 2012) is adopted as our optimization algorithm.

In our training process, we have tried two different kinds of mini-batch sampling policies, user/item sampling and rating sampling. User/item sampling means that one user/item is randomly sampled with probability \(p\), and all ratings of the sampled user/item are kept for training. Rating sampling means that a rating is randomly sampled for training with probability \(p\). In our experiments, these two kinds of policies behave similarly. We adopt user/item sampling policy in our following experiments.

For each training iteration, we first perform user sampling policy to get a mask \(\tilde{\Omega} \in \mathbb{R}^{m \times m}\), \(\tilde{\Omega}\) is a diagonal matrix, with \(\tilde{\Omega}_{ii}\) being 1 with probability \(p\), being 0 with probability \(1 - p\). Then the gradients of \(\mathcal{W}\) can be computed as follows:

\[
\nabla \mathcal{W} = 2\tilde{\Omega}(P_\Omega(M - \hat{W}\hat{H}^\top))\hat{H}
\]

\[
\nabla \mathcal{W}_L = (X^{(L)})^\top(\nabla \mathcal{W} \odot (1 - \hat{W}^2)) + \lambda W_L
\]

\[
\nabla \mathcal{W}_{b_W} = (\nabla \mathcal{W} \odot (1 - \hat{W}^2))\mathcal{I}
\]

\[
\nabla \mathcal{W}_L = \tilde{L}_W^\top\left(\nabla X^{(\ell+1)} \odot (1 - (X^{(\ell+1)})^2)\right) \mathcal{I}
\]

\[
\nabla \mathcal{W}_{b_W} = \tilde{L}_W^\top X^{(\ell)} \left(\nabla X^{(\ell+1)} \odot (1 - (X^{(\ell+1)})^2)\right) \mathcal{I}
\]

\[
\lambda W_L
\]

\[
\lambda \mathcal{W}_L
\]

where \(\hat{W}^2\) denotes element-wise square of matrix \(\hat{W}\). Gradients of \(\mathcal{H}\) can also be derived similar to \(\mathcal{W}\), which are omitted here.

Based on the derived gradients, we adopt back propagation (BP) to learn the parameters of CGMC. The learning process is summarized in Algorithm 1, where \(\eta\) is the learning rate.

### 3.4. Comparison to Related Work

The most related work to our CGMC is RMGCNN (sRMGCNN) (Monti et al., 2017) and GCN-kw in (Kipf & Welling, 2017). Here we discuss the difference between them and our CGMC.

As mentioned above, sRMGCNN is a factorized (MF) version of RMGCNN. Because we only focus on the factorized version in this paper due to its efficiency, RMGCNN in this paper refers to sRMGCNN unless otherwise stated. CGMC is different from RMGCNN in the following aspects. Firstly, CGMC adopts a different GCN to extract graph embeddings, and the newly designed GCN in CGMC is better than that in RMGCNN which will be verified in experiments. Secondly, RMGCNN adopts both GCN and RNN for GCM, while our CGMC adopts only GCN without RNN. Thirdly,
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A fully connected layer is introduced in our CGMC for space compatibility.

CGMC is different from GCN-kw in the following aspects. Firstly, GCN-kw is proposed for semi-supervised learning, and it has not been used for MC. Secondly, the GCN in CGMC adopts weighted policy to control the contribution between self-connections and neighbors for graph embedding, while the self-connections and neighbors in GCN-kw contribute equally for graph embedding. Hence, the GCN in CGMC is more flexible than GCN-kw. Thirdly, the GCN in CGMC will not get into numerical instability, while GCN-kw (Kipf & Welling, 2017) has numerical instability problem if no further operation is performed. Although GCN-kw is not proposed for GCM, we adapt it for GCM in this paper and find that our CGMC achieves better performance than GCN-kw based method in our experiment.

4. Experiment

We evaluate the proposed model CGMC and other baselines on collaborative filtering datasets. Our implementation is based on PyTorch with a NVIDIA TitanXP GPU server. PyTorch is only used to call GPU interfaces. The gradient computation and BP learning procedure are implemented by ourselves rather than calling the auto-gradient interface in PyTorch.

4.1. Datasets

As in RMGCNN (Monti et al., 2017), we evaluate CGMC and other baselines on four real datasets: Movielens-100K1 (ML-100K), Douban, Flixster, YahooMusic. For fair comparison, the dataset size and training/test data partition are exactly the same as those in RMGCNN (Monti et al., 2017). In particular, the latter three datasets are 3000 × 3000 subsets of Douban, Flixter, YahooMusic that are preprocessed and provided by (Monti et al., 2017)2. Statistics of datasets are presented in Table 1.

4.2. Settings and Baselines

Settings As in RMGCNN (Monti et al., 2017), the graph information is constructed from user/item features. Therefore, we implement featureless version of CGMC in our experiments, where we set \( X, Y = I \). For each dataset, we randomly sample instances from training set as validation set that has the same number as test set. We repeat the experiments 5 times and report the mean of results. On all the datasets, we adopt a version of single graph convolution layer for CGMC to compare with baselines. We learn CGMC according to Algorithm 1. The optimization algorithm we use is Adadelta (Zeiler, 2012) and the maximum number of iterations is set to 5,000.

The regularization parameter \( \lambda \) is selected from \([10^{-4}, 10^{6}]\), \( \sigma \) is selected from \([0, 0.2, 0.4, 0.6, 0.8, 1.0]\). We use the validation set to tune these two hyper-parameters. For all datasets, we set \( r_1 = r_2 = 20, p = 0.5, \eta = 0.5 \). For ML-100K, Douban, and Flixster, the output dimensionality of the fully connected layer \( d = 32 \). \( d = 128 \) for YahooMusic because its rating level is relatively large. We do not tune these hyper-parameters, although fine-tuning with validation set might further improve the performance of CGMC. For baselines, we adopt the hyper-parameters that achieve the best results. As in RMGCNN (Monti et al., 2017), root mean square error (RMSE) is adopted as metric for evaluation. The smaller the RMSE is, the better the performance will be.

Baselines For ML-100K, we compare CGMC with baselines that utilize information of user/item features. User/item features are constructed in the same way as (Rao et al., 2015) and the user/item graphs are constructed via \( k \)-nearest neighbors measured by Euclidean distance of features. On this dataset, we compare CGMC with MC (Candes & Recht, 2012), IMC (Jain & Dhillon, 2013; Xu et al., 2013), GMC (Kalofolias et al., 2014), GRALS (Rao et al., 2015), RMGCNN (Monti et al., 2017). MC learns the full matrix with a nuclear norm regularization. IMC utilizes the features of users and items to formulate an inductive matrix model for approximating the target. GMC learns a full matrix that approximates the observed rating matrix and constrains the full matrix by applying graph Laplacian regularization on it. GRALS learns the factorized matrices of the target by applying graph Laplacian regularization on the factorized matrices.

For Douban, Flixster, YahooMusic, we compare CGMC with MC, GRALS, and RMGCNN. For MC, GRALS and RMGCNN, we present results of min-max normalized version and non-normalized version. Min-max normalized version means re-scaling the predictions to the range of the rating level before each training iteration, just as that in RMGCNN (Monti et al., 2017). We implement both min-max normalized version and non-normalized version of MC by ourselves. Because the results of non-normalized version of GRALS have been reported in (Monti et al., 2017), we only implement the min-max normalized version of GRALS3 by ourselves. For RMGCNN, the code of min-max normalized version is directly from (Monti et al., 2017), and we use the code provided by (Monti et al., 2017) to implement the non-normalized version.

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1https://grouplens.org/datasets/movielens/
2https://github.com/fmonti/mgcnn
3We adopt Adadelta to optimize the min-max normalized version of GRALS.
4.3. Result

The results on ML-100K are reported in Table 2, where the results of baselines are directly copied from (Monti et al., 2017). Because the training/test data partition of this paper is exactly the same as that in (Monti et al., 2017), the comparison is fair. From Table 2, we can find that our CGMC outperforms all the other baselines, including graph regularization methods and GDLG-based methods, to achieve the best performance.

The results on Douban, Flixster and YahooMusic are reported in Table 3. Once again, we can find that CGMC outperforms other state-of-the-art baselines to achieve the best performance. Moreover, we can conclude from Table 3 that the min-max normalization before each training iteration boosts the performance of baselines. However, min-max normalization will hurt the training speed and cannot scale well. Our CGMC achieves the best results under both settings.

4.4. Effect of Fully-Connected Layer

To demonstrate the effectiveness of the fully-connected layer in GCN proposed by us, we remove the fully-connected layer after GCE. The CGMC variant without fully connected layer is denoted as CGMC-0, and CGMC is with fully-connected layer. We compare CGMC-0 to CGMC under the conditions where GCN grows from 1 layer to 4 layers. The results are shown in Table 4.

From Table 4, we can observe that with different number of layers for GCN, the improvements of CGMC over CGMC-0 are significant. These results verify the effectiveness of the fully-connected layer in CGMC.

4.5. Effect of Weighted Policy in GCN

To demonstrate the effectiveness of the weighted policy proposed in our newly designed GCN, we replace our GCN in CGMC by the GCN-kw (Kipf & Welling, 2017). The resulting model is denoted as GMC-GCN-kw. We design two variants of GMC-GCN-kw. GMC-GCN-kw denotes the variant of our CGMC by only replacing our GCN by GCN-kw, with all other parts fixed. It means that GMC-GCN-kw also includes a fully connected layer which is proposed by us. GMC-GCN-kw-0 denotes a variant of GMC-GCN-kw without the fully connected layer.

The results are reported in Table 5. We can observe that CGMC performs better than GMC-GCN-kw on all datasets. The results show the effectiveness of adopting weighted policy to control the contribution between self-connections and neighbors for graph embeddings. Compared with GMC-GCN-kw, the performance improvement of GMC-GCN-kw once again verifies the effectiveness of the fully connected layer proposed by us.

4.6. Sensitivity to Hyper-parameters

In CGMC, \(\lambda\) and \(\sigma\) are two important hyper-parameters. Here, we study the sensitivity of these two hyper-parameters on Flixster and YahooMusic.

The results are presented in Figure 1. With respect to \(\lambda\), we can see that CGMC behaves well in a wide range of \(\lambda\). As for \(\sigma\), we observe that CGMC is a little sensitive to \(\sigma\), which depends on the quality of graphs. By using the validation techniques, we can always find a suitable hyper-parameter for CGMC in our experiments.

5. Conclusion

In this paper, we propose a novel geometric matrix completion method, called convolutional geometric matrix completion (CGMC), for recommender systems with relationship (link) graphs among users/items. To the best of our knowledge, CGMC is the first work to show that pure
Table 3. Performance (RMSE) on Douban, Flixster and YahooMusic. Douban and YahooMusic only use single graph. Flixster uses both user graph and item graph. Flixster-U only uses user graph. (·) denotes that the result is implemented by ourselves, and other results are directly copied from (Monti et al., 2017). Left/right denotes the results of non-normalized/min-max normalized versions of the model.

| METHOD             | DOUBAN     | FLIXSTER   | FLIXSTER-U | YAHOO MUSIC |
|--------------------|------------|------------|------------|-------------|
| MC                 | (0.8476)/ (0.7544) | (1.5338)/ (0.9709) | (1.5338)/ (0.9709) | (52.0102)/ (24.1944) |
| GRALS (RAO ET AL., 2015) | 0.8326/(0.7537) | 1.3126/(0.9722) | 1.2447/(0.9751) | 38.0423/(24.0744) |
| RMGCNN (MONTI ET AL., 2017) | (1.1541)/0.8012 | (0.9700)/1.1788 | (2.8095)/0.9258 | (45.6049)/22.4149 |
| CGMC               | 0.7298/0.7308 | 0.8822/0.8853 | 0.9006/0.8876 | 19.3751/20.0032 |

Table 4. The influence of the fully-connected layer. CGMC-0 denotes CGMC without fully connected layer, and CGMC denotes CGMC with fully connected layer.

| METHOD             | ML-100K | DOUBAN | FLIXSTER | FLIXSTER-U | YAHOO MUSIC |
|--------------------|---------|--------|----------|------------|-------------|
| CGMC-0/CGMC (1 LAYER) | 0.997/0.894 | 0.9580/0.7298 | 1.0868/0.8822 | 37.0064/19.3751 |
| CGMC-0/CGMC (2 LAYERS) | 0.905/0.897 | 0.7421/0.7384 | 0.9024/0.8961 | 36.7636/19.5726 |
| CGMC-0/CGMC (3 LAYERS) | 0.913/0.904 | 0.7570/0.7368 | 0.9099/0.8976 | 36.7769/19.7722 |
| CGMC-0/CGMC (4 LAYERS) | 0.917/0.911 | 0.7653/0.7447 | 0.9211/0.9113 | 36.7769/19.7722 |

Table 5. The influence of our weighted policy in the GCN of CGMC.

| METHOD             | ML-100K | DOUBAN | FLIXSTER | FLIXSTER-U | YAHOO MUSIC |
|--------------------|---------|--------|----------|------------|-------------|
| GMC-GCN-KW-0 (1 LAYER) | 1.088 | 1.7088 | 1.5279 | 1.5805 | 34.6415 |
| GMC-GCN-KW-0 (2 LAYERS) | 1.049 | 0.7548 | 0.9254 | 1.1116 | 34.4439 |
| GMC-GCN-KW-0 (3 LAYERS) | 1.076 | 0.7728 | 0.9721 | 1.1581 | 34.4976 |
| GMC-GCN-KW-0 (4 LAYERS) | 1.082 | 0.7785 | 1.0049 | 1.1568 | 34.4765 |
| GMC-GCN-KW | 1.010 | 0.7372 | 0.8886 | 0.9368 | 19.8627 |
| CGMC               | 0.894 | 0.7298 | 0.8822 | 0.9006 | 19.3751 |

Figure 1. Sensitivity to hyper-parameters. The first row presents the effect of $\lambda$, and the second row presents the effect of $\sigma$.

Graph convolutional network (GCN) based methods can achieve the state-of-the-art performance for GMC, as long as a proper GCN is designed and a fully connected layer is adopted for space compatibility. Experimental results on four real datasets show that CGMC can outperform other state-of-the-art baselines, including the RMGCNN (Monti et al., 2017) which is a combination of GCN and RNN.

We believe that other techniques developed in graph signal processing can also be applied in matrix completion with graph information and it is left for future work.

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