New class of self-similar solutions for plasma expansion admitting monoenergetic ion spectra

Naveen Kumar* and Alexander Pukhov
Institut für Theoretische Physik I, Heinrich-Heine-Universität Düsseldorf, D-40225, Germany

We report a new class of self-similar solutions for plasma expanding into vacuum that allows for quasi-monoenergetic ion spectra. A simple analytical model takes into account externally controlled time-dependent temperature of the hot electrons. When the laser temporal profile is tailored properly, the quasi-neutral self-similar expansion of the plasma results in ion concentration in the phase-space at a particular velocity thus producing a quasi-monoenergetic spectrum. We prove this analytical prediction using a 1D particle-in-cell (PIC) simulation where the time-dependent plasma temperature is controlled by two laser pulses shot at a foil with a suitable time delay between them.

PACS numbers: 52.38.Kd, 41.75.Jv, 52.65.-y

Generation of highly energetic ions and protons beams from the laser interaction with thin foil targets in relativistic regime \((I \lambda^2 > 10^{18} \text{ W cm}^{-2} \mu\text{m}^2, \text{where} \ I \text{ is} \ \text{the} \ \text{intensity} \ \text{of} \ \text{the} \ \text{laser} \ \text{and} \ \lambda \ \text{its} \ \text{wavelength})\) is one of the fast developing research fields \([1, 2, 3, 4, 5, 6]\). Particularly interesting is the generation of quasi-monoenergetic ion beams, because these have a number of important potential applications in medical physics, inertial confinement fusion, compact ion accelerators \([7, 8, 9, 10]\).

When the laser pulse strikes a thin solid density foil, it heats electrons at the front surface of the target. These hot electrons traverse the target and leave it at the rear side. A space charge is build up that leads to a huge electrostatic potential. This, in turn, accelerates the background ions to high energies. This mechanism is known as target normal sheath acceleration (TNSA) \([3, 4, 5, 6, 7, 8, 9, 10]\). From theoretical point of view the generation of energetic ions from thin foil targets has been studied well in past based on the approach of self-similar expansion of plasma \([11, 12, 13]\). In this model, the plasma fluid is quasi-neutral, ions and electrons both have similar flow on the ion sound time scale. In the works \([14, 15]\) also kinetic effects at the front of the expanding plasma were included in this self-similar model for different geometries.

All the self-similar models up to now have predicted broad band ion spectra, decaying exponentially at high energies. Yet, most of practical applications require monoenergetic ion beams. A straightforward way to overcome this difficulty is to select a group of ions with the same energy, e.g. using ions with the same initial conditions. Indeed, when a monolayer of protons, or a proton-rich dot is placed on the back surface of a higher-Z target, then all the light protons are accelerated nearly to the same energy \([3, 8]\), because all the protons in the monolayer feel same electric field. Still, the number of accelerated protons out of the monolayer and thus the efficiency of the process are very limited. Recently, it was also predicted that ion acceleration from multi-species targets may result in peaked spectra \([9, 10]\).

In this Letter, we report a new type of self-similar solution for the quasi-neutral vacuum-plasma expansion that does allow for monoenergetic spikes in the ion spectrum. As we will see, this new self-similar solution requires a suitable time-dependent temperature of laser-generated hot electrons. Experimentally, this corresponds to a tailored temporal profile of the driving laser pulse, because the temperature of the hot electrons is a definite function of the laser intensity \([14, 15]\). The rising electron temperature with time is accompanied by a rising electrostatic field inside the expanding plasma. Consequently, this results in a higher acceleration rate of ions at later times. These rear ions catch up with the leading ions, which began to accelerate earlier. When this catching up occurs, a bunch of ions concentrated in phase space is formed. If driven too harsh, this catching up may result in “wave breaking” \([18]\). However, when the temperature is changed gently, one may stay “just at the verge” of wavebreaking. This is exactly the solution we present here.

Under the quasi-neutral assumption \(i.e \ n_e \approx Z n_i\) where \(n_e\) and \(n_i\) are the electron and ion densities repectively, and \(Z\) is the ionization state, the dynamics of the plasma expansion is governed by the continuity equation

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial r}(n_i v_i) = 0,
\]

and the Euler equation for the ion fluid

\[
\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial r} + \frac{Z}{n_i m_i} \frac{\partial}{\partial r} (n_i T_e(t)) = 0.
\]

Here \(v_i\) is the ion fluid velocity, \(m_i\) is the ion mass, \(T_e(t) = T_0(t) (n_e(0,t)/n_e(0,0))^{-\gamma-1}\) is the local electron temperature and \(\gamma\) is the expansion polytropic index for electrons. We also assume that the plasma electrons obey the Boltzmann statistics.

*Electronic mail: kumar@tp1.uni-duesseldorf.de
\begin{align}
n_e(r, t) & = n_e(t) \exp(e \phi / T_e), \quad (3) \\
\end{align}

where \( n_e(t) \) is the time-dependent electron density, \(-e\) is the electron charge, and \( \phi(r, t) \) is the electrostatic potential. The ions are assumed to be cold initially.

It is well known that the above system of equations permits the following self-similar solution \( [13] \)

\[ v_i(\xi) = \dot{R} \xi, \quad \xi = x/R, \]

\[ n_e \approx Z n_i = n_{00}(R_0/R) N_i(\xi), \quad N_i(0) = 1, \quad (4) \]

where \( R(t) \) is the time dependent characteristic size of the plasma, \( \xi \) is the similarity coordinate \( (x \) is the position coordinate), and \( N_i \) is a positive unknown function. The subscripts \( i \) and \( e \) refer to the ions and electrons respectively. The rationale behind this self-similar solution is that a linear velocity-radius relation is the correct limit for the asymptotic stage of the expansion, whenever the \( R(t) \) greatly exceeds the initial value \( R_0 = R(0) \) \( [13] \). We may see that this solution rules out the possibility of generation of quasi-monoenergetic ion beams because the ion velocity scales linearly with the similarity coordinate \( \xi \).

In this Letter, we introduce a more general self-similarity ansatz and assume that the plasma expansion scales as

\[ v_i(\xi) = \dot{R} f(\xi), \quad \xi = x/R, \]

\[ n_e \approx Z n_i = n_{00}(R_0/R) N_i(\xi), \quad N_i(0) = 1, \quad (5) \]

where \( f(\xi) \) is a function which is yet to be determined. Because the new solution must satisfy the equation of continuity, we have from Eq. \( [11] \)

\[ f(\xi) = \xi - C / N_i(\xi). \quad (6) \]

Here \( C \) is a constant of integration. Physically, the fluid velocity \( v_i \) must be monotonic. Consequently, the solution \( [13] \) has a meaning only in the range \( \xi < \xi_{\text{max}} \) where \( \xi_{\text{max}} \) marks the front end of the plasma expansion in vacuum. We require that \( C < \xi_{\text{max}} N_i(\xi_{\text{max}}) \). This restriction means that \( C \) is small, because \( N_i \) decays exponentially with \( \xi \). Thus, the new solution is hardly distinguishable from the classic one at small \( \xi \). Only approaching \( \xi_{\text{max}} \) the solution is modified significantly. For the sake of accuracy, we mention that our solution leads to a non-zero fluid velocity at the origin, \( v_i(0) = -C \dot{R} \). This velocity, however, is exponentially small (just as \( C \)) and has no great physical meaning. It is well known, that self-similar solutions are not well defined at the origin and must be considered at some distance from the origin only \( [11] \).

One easily retrieves the old similarity ansatz \( [13] \) by setting \( C = 0 \) in \( [13] \). Thus, all the previous self-similar solutions for plasma expansion into vacuum were missing the second term in \( [6] \). Yet, this term is very important. It is the second term in \( [6] \) with the non-zero constant \( C \) that allows for a new class of self-similar solutions and gives the option for quasi-monoenergetic ion spectra. The condition to have a monoenergetic spike in the ion spectrum is the presence of a stationary point in the expansion velocity, \( d v_i(\xi_m)/d \xi = 0 \), that gives

\[ C \frac{d N_i(\xi_m)}{d \xi} = -N_i^2(\xi_m). \quad (7) \]

This ion beam produces a spike in the energy spectrum around \( \xi_m = m_i \nu_m^2 / 2 \).

Now we substitute the formula \( [11] \) in Eq. \( [2] \) and obtain

\[ \frac{d N_i}{d \xi} = \frac{N_i^2[N_i \xi - C(1 + a)]}{[a C^2 - b N_i^2 + 1]}, \quad (9) \]

where we have used the following relations

\[ \dot{R} = \frac{R_0}{a R} = \frac{Z \gamma}{b m_i R_0} \left( \frac{R_0}{R} \right)^\gamma T_0(t), \quad (10) \]

\( a \) and \( b \) are constants. It follows from Eqs. \( (7) \) and \( (9) \) that the density of the monoenergetic ion beam is (in the isothermal case, \( \gamma = 1 \))

\[ N_i(\xi_m) = (C/2b) \left[ \xi_m + (\xi_m^2 - 4b)^{1/2} \right], \quad (11) \]

and its velocity

\[ \nu_m = \dot{R} \left( \xi_m - 2b \left( \xi_m + (\xi_m^2 - 4b)^{-1/2} \right) \right). \quad (12) \]

It is clear that the density and energy of the monoenergetic bunch do not depend on the constant \( C \). Hence, in principle this constant can be chosen freely. The constant \( b \) can be expressed in terms of the constant \( C \) and density of the monoenergetic bunch, which is sought experimentally, as

\[ b = C \left( N_i(\xi_m)(\xi_m - C) / N_i^2(\xi_m) \right). \quad (13) \]
One can draw further conclusions regarding ion acceleration from the self-similar solution. We see from the Eq. [10] that the ion acceleration is inversely proportional to \( R \), which is an increasing function of time. We obtain the expression for \( R \) on integrating the first relation in Eq. [10]

\[
\frac{R}{R_0} = \left(1 + \frac{(a - 1) \nu_f t}{\nu_f R_0} \right)^{\frac{1}{a-1}},
\]

where \( \nu_f = \nu_f(R_0/R_f)^{1/a} \). Here \( \nu_f \) and \( R_f \) are the velocity and size of the expanding plasma, in the case of a time independent electron temperature, respectively. These can be determined from the results of Ref. [13]. Once the expression for \( R \) is obtained, we can determine the required electron temperature profile from the second part of the Eq. [10]:

\[
T_0(t) = \frac{b T_f}{Z \gamma a} \left(1 + \frac{(a - 1) \nu_f t}{\nu_f R_0} \right)^{\frac{2 + a(\gamma - 1)}{a-1}},
\]

where \( T_f = m_i \nu_f^2 \) is the time independent electron temperature. This relation describes the electron temperature variation in terms of the constants \( a \) and \( b \). These constants define how time dependent electron temperature profile must be tailored to produce the monoenergetic ion beam. The idea of controlling the plasma electron temperature has been recently studied by means of Vlasov and PIC simulations [19]. It is found that a suitable temperature variation of the plasma electrons indeed can produce the mono-energetic ions.

An analytical solution of Eq. [9] is hardly possible and we integrate it numerically. Fig. 1 shows \( N_i \) and \( \nu_i \) as a function of \( \xi \) for the parameter set \( a = 5, b = 0.19, C = 3.3636 \times 10^{-6} \), and \( \gamma = 1 \). Fig. 2 describes the self-similar expansion of the quasi-neutral plasma. The most important result is the behaviour of \( \nu_i \) showing flattening with \( \xi \) near \( \xi = 2 \). This flattening manifests the production of monoenergetic ions. The velocity of expansion reaches its maximum at the flattening point. It corresponds also to the position of the ion front, where the self-similar solution has to be terminated. The energy spectrum of ions produced by this self-similar solution is given in Fig. 2. We observe the quasi-monoenergetic bunch formation from the figure.

We have performed a 1D particle-in-cell (PIC) simulation of the quasi-monoenergetic protons generation from a thin foil target with the help of virtual laser plasma laboratory (VLPL) code [20]. In the simulation, we take a thin foil of thickness 1.5A with an initial density ramp and solid density 30n_e, where \( n_e \) is the critical density and \( \lambda \) is the wavelength of the laser pulse.

To tailor the time dependent temperature of the hot electrons, we use two driving laser pulses shot at the target with a time delay between them. The simulation parameters for the laser pulses are: \( a_1 = eA_1/mc^2 = 1, c\tau_1 = 5\lambda, a_2 = eA_2/mc^2 = 3, c\tau_2 = 15\lambda \) and \( \lambda = 0.8 \mu m \), where \( \tau_1 \) and \( \tau_2 \) are the pulse durations, and \( a_{1,2} \) are the normalized vector potentials of the two laser pulses. Here subscripts 1 and 2 represent the first and second laser pulse respectively. The time delay between the two pulses is 60 laser periods. Both laser are \( \gamma-\)
polarized and move in $\hat{x}$ direction. The resolution in $x$-direction is $0.01\lambda$ while 200 numerical particles per cells are chosen to run the simulation.

Fig. 3 depicts the phase space evolution of protons at the various stages of plasma expansion. The upper panel depicts the stage of the thin foil expansion under the influence of the first laser pulse. It is clear that the single laser pulse leads to the standard self-similar plasma expansion given by Eq. (1) and thus clearly rules out the possibility of concentration of protons in the phase-space. The proton velocity scales linearly with the distance from the target. Later (at $t = 80 \cdot 2\pi/\omega$ where $\omega$ is the carrier frequency of the laser pulse), when the second pulse hits the target (c.f. middle panel of the Fig.), the dynamics of the expanding plasma changes and one can see momenta flattening in the phase space. The momenta flattening forms the monoenergetic protons bunch. Physically, the second laser pulse increases the hot electron temperature that leads to a rising electrostatic field and a larger acceleration of ions at later times. These ions, which form a second group, catch up with the first group of ions, which began to accelerate earlier. This process flattens the velocity profile and forms a bunch of quasi-monoenergetic ions. As the time passes, this bunch of ions accelerates and acquires substantial energy as shown in the lower panel of the Fig. Fig. 4 shows the ion energy spectrum obtained in the PIC simulation. It can be seen that there is a significant number of ions with the peak energy around 4 MeV and the energy spread is a few percent. The number density of quasi-monoenergetic ions can be further enhanced by using higher density targets and longer laser pulses. Using the two-pulse technique - or, more general, a tailored time-dependent laser intensity - it will be possible to produce light as well as heavy ions ($Z > 1$) with monoenergetic spectra. However, the heavy ion case may demand higher laser intensities.

In summary, we have found a new class of self-similar solutions for the quasi-neutral vacuum-plasma expansion that allows for quasi-monoenergetic ion spectra. The quasi-monoenergetic spectrum corresponds to the flattening of the ion momentum dependence on the expansion coordinate. Thus, ions pile up at a particular velocity in the phase space and form a quasi-monoenergetic bunch. This new scheme demands for laser pulses with a tailored temporal profile. To prove realizability of the new scheme, we have performed a 1D PIC simulation using the code VLPL. In the PIC simulation, we use two laser pulses hitting the foil target with a suitable time delay between them. Although the developed model is simple, it catches the essence of the physical process quite well. The proposed scheme works already for moderate laser powers which should make the laser-plasma generation of monoenergetic protons (ions) a stable and well
controlled process.

This work was supported by the DFG through project TR-18.

[1] S. P. Hatchett et al., Phys. Plasmas 7, 2076 (2000).
[2] R. A. Snavely et al., Phys. Rev. Lett. 85, 2945 (2000).
[3] S. C. Wilks et al., Phys. Plasmas 8, 542 (2001).
[4] A. Pukhov, Phys. Rev. Lett. 86, 3562 (2001).
[5] B. M. Hegelich et al., Phys. Rev. Lett. 89, 085002 (2002).
[6] D. Hahs, G. Pretzler, A. Pukhov and J. Mayer-ter-Vehn, Prog. Part. Nucl. Phys. 46, 375 (2001).
[7] B. M. Hegelich et al., Nature 439, 441 (2006).
[8] H. Schwoerer et al., Nature 439, 445 (2006).
[9] S. Ter-Avetisyan et al., Phys. Rev. Lett. 96, 145006 (2006).
[10] A. V. Brantov et al., Phys. Plasmas 13, 122705 (2006).
[11] A. V. Gurevich, L. V. Pariiskaya, and L. P. Pitaevskii, Sov. Phys. JETP 22, 449 (1966).
[12] J. E. Allen and J. G. Andrews, J. Plasma Phys. 4, 187 (1970).
[13] M. Murakami, Y.-G. Kang, K. Nishihara, S. Fujioka and H. Nishimura, Phys. Plasmas 12, 062706 (2005).
[14] P. Mora, Phys. Rev. Lett. 90, 185002 (2003).
[15] M. Murakami and M. M. Basko, Phys. Plasmas 13, 012105 (2006).
[16] S. C. Wilks, W. L. Kruer, M. Tabak, and A. B. Langdon, Phys. Rev. Lett. 69, 1383 (1992).
[17] A. Pukhov, Z.-M. Sheng, and J. Meyer-ter-Vehn, Phys. Plasmas 6, 2847 (1999).
[18] T. Grismayer and P. Mora, Phys. Plasmas 13, 032103 (2006).
[19] A. P. L. Robinson, D. Neely, P. McKenna and R. G. Evans, Plasma Phys. Control. Fusion, 49, 373 (2007).
[20] A. Pukhov, J. Plasma Phys. 61, 425 (1999).