Analytical modelling and power density optimisation of a single phase dual active bridge for aircraft application

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Abstract: A design procedure for the dual active bridge (DAB) converter is presented, which aims to optimised power density and computational effort. When designing a DAB, the selection of circuit design parameters such as switching frequency, leakage inductance and semiconductor technologies is a complex question when targeting losses and weight minimisation of the final design. In this study, analytical models of the operating waveforms, the losses and the weight of all DAB components are developed. The proposed design algorithm is used for designing a 3 kW high-frequency DAB for an aircraft DC power system.

1 Introduction

The concept of the more electric aircraft promotes the use of electrical instead of traditional hydraulic, pneumatic or mechanical systems [1, 2]. The advantages are reduced cost, reduced fuel consumption, lower weight and less environmental impact. This results in an increase in the power rating of the aircraft power system and imposes the use of high-voltage DC buses. Hence, DC/DC converters will play an important role in the management of electrical power in future aircraft. The ASPIRE project, part of the Clean Sky 2 Joint Undertaking, aims to design an electrical power system for next-generation aircraft. One goal of the ASPIRE project is to design an ultra-light and efficient isolated bidirectional DC/DC converter, interfacing the 28 V and 270 V DC buses at 3 kW rated power. The dual active bridge (DAB) is a suitable topology [3] for such application.

The design of the DAB implies the choice of many converter parameters such as switching frequency, leakage and magnetising inductances, semiconductor devices and magnetic materials in order to minimise the total losses and weight of the converter. Such large numbers of design parameters make the design of the DAB a tedious and complex process.

Existing analytical modelling-based design approaches rarely consider modelling of power losses or weight of converter components [4–8]. Moreover, many DAB designs are very specific to the target application [9–11]. In this paper, an efficient design procedure based on analytical modelling of the DAB is proposed and applied to optimise the converter for the highest power density with minimal computational effort. The proposed optimisation technique, visualised in Fig. 1, consists of three steps. It starts with the calculation of the current and voltage waveforms and other electrical variables of the DAB, e.g. the RMS current. In the second step, these variables are used to assess converter losses. The third layer of the optimisation technique is an analytical model of the component weights used to assess the power density.

The paper is organised in six sections. Section 2 gives an overview of the DAB. Section 3 introduces the analytical model of the DAB operating waveforms. In Sections 4 and 5, loss and weight analytical models are given. Finally, Section 6 gives an example design of the DAB for the ASPIRE project.

2 Single phase dual active bridge

The DAB was first proposed in [3] and its generic single phase variant is depicted in Fig. 2. The single phase DAB consists of two active H-bridges and a high frequency transformer. \( V_P \) and \( V_S \) denote the primary and secondary side DC voltages, respectively. The original modulation strategy is called single phase shift (SPS) modulation, which is depicted in Fig. 3 [3]. Both H-bridges operate at 50% duty cycle, but phase shifted in time by the angle \( \phi \). The output voltages of the...
primary and secondary side H-Bridge are denoted $v_P(t)$ and $v_S(t)$, respectively. Using the transformer turns ratio $n$, all secondary side quantities are referred to the primary side and indicated by a dash. The waveforms $v_P(t)$ and $v_S(t)$ are shown in Fig. 3a. The voltage difference $v_P(t) - v_S(t)$ drops across the leakage inductance $L$ of the transformer and results in a quasi-square wave AC current in the transformer, $i_L(t)$ (see Fig. 3c). As shown in Fig. 2, the input currents of the primary and secondary side H-Bridge are denoted by $i_{L,P}(t)$ and $i_{L,S}(t)$, respectively. They are plotted in Figs. 3d and e, respectively. The current ripples are filtered by the DC-link capacitors $C_P$ and $C_S$. The average currents provided by the primary and secondary side DC buses are denoted by $I_{L,P}$ and $I_{L,S}$, respectively. The transferred power $P$ is given in (1), where $\phi$ denotes the phase shift angle between $v_P(t)$ and $v_S(t)$ and $f_{sw}$ denotes the switching frequency [3]

$$P = \frac{V_P V_S}{2\pi f_{sw} L} \cdot \phi(\pi - |\phi|)$$

Maximum power is transferred for a phase shift angle of $\pi/2$. The turn-off transition of the devices in both H-Bridges is generally hard-switched, whereas for the turn-on transition, zero voltage switching (ZVS) is achieved for most operating points.

3 Analytical model of SPS modulation

Before analysing the losses and the weight of the DAB, the analytical model of SPS modulation is derived from two current waveforms, namely the current $i_L(\omega_m t)$ in the AC link and the magnetising current $i_M(\omega_m t)$. Due to waveform symmetries (see Fig. 3), it is sufficient to regard half a switching period. The following conventions are made, which are visualised in Fig. 4. For positive phase shifts $\phi \geq 0$, the rising edge of the primary voltage $v_P(t)$ is located at $\omega_m t = 0$ and the rising edge of the secondary voltage $v_S(t)$ is delayed to $\omega_m t = \phi$. For negative phase shifts $\phi < 0$, the rising edge of the secondary voltage is located at $\omega_m t = 0$ and the rising edge of the primary voltage is delayed to $\omega_m t = |\phi|$.

3.1 Line current and derived quantities

The current waveform $i_L(\omega_m t)$ is obtained by integrating the output voltage difference of the H-bridges, $v_P(t) - v_S(t)$. For keeping the model simple, ohmic resistances and the magnetising inductance $L_m$ are neglected. The resulting current waveform as well as its initial values at $\omega_m t = 0$ and $\omega_m t = |\phi|$ are given as follows:

$$i_L(\omega_m t) = \begin{cases} i_L(0) + \text{sgn}(\phi) \frac{V_P + V_S}{\omega_m L} \omega_m t & \text{for } 0 \leq \omega_m t < |\phi| \\ i_L(|\phi|) + \frac{V_P - V_S}{\omega_m L} \omega_m t - |\phi| & \text{for } |\phi| \leq \omega_m t < \pi \end{cases}$$

$$i_L(|\phi|) = -\frac{1}{2} \left( \frac{V_P + V_S}{\omega_m L} \phi + \frac{V_P - V_S}{\omega_m L} (\pi - |\phi|) \right)$$

$$i_L(|\phi|) = -\frac{1}{2} \left( \frac{V_P + V_S}{\omega_m L} \phi - \frac{V_P - V_S}{\omega_m L} (\pi - |\phi|) \right)$$

Depending on the sign of the phase shift angle $\phi$, the two H-bridges either switch at the time instant $\omega_m t = 0$ or $\omega_m t = |\phi|$. For example, for $\phi \geq 0$, the primary H-bridge switches at $\omega_m t = 0$ and for $\phi < 0$, it switches at $\omega_m t = |\phi|$. Therefore, it is important to calculate the current which has to be switched off by the H-bridges, regardless of the respective time instant. For the primary and the secondary H-bridges, the turn-off currents are given in (5) and (6), respectively:

$$I_{sw,P} = -\frac{1}{2} \left( \frac{V_P + V_S}{\omega_m L} |\phi| + \frac{V_P - V_S}{\omega_m L} (\pi - |\phi|) \right)$$

$$I_{sw,S} = \frac{1}{2} \left( \frac{V_P + V_S}{\omega_m L} |\phi| - \frac{V_P - V_S}{\omega_m L} (\pi - |\phi|) \right)$$

From (5) and (6), the conditions for ZVS of the primary and secondary side H-Bridge, respectively, are derived. At the
switching instant, the current has to flow in the antiparallel diodes of the switches to achieve ZVS. This is formulated as follows:

\[
I_{sw,p} < 0 \Rightarrow V_p > V_S \left(1 - \frac{2|\phi|}{\pi}\right) \quad (7)
\]

\[
I_{sw,s} > 0 \Rightarrow V_s > V_p \left(1 - \frac{2|\phi|}{\pi}\right) \quad (8)
\]

As \(|\phi| \leq \pi/2\), the second term in (7) and (8) is always positive. It is clear that if the ratio of the DC voltages does not match the transformer turns ratio \(n\), i.e. \(V_p \neq V_s\), the ZVS conditions will be violated at low load [12].

The peak value of the AC link current \(i_L\) is denoted by \(I_L\),

\[
i_L = \max \{i_L(t)\} = \frac{V_p - V_S}{2\omega_{ac}L} + \frac{(V_p + V_S) - |V_p - V_S|}{2\omega_{ac}L} (9)
\]

By squaring (2) and integrating, the RMS value of the AC link current, \(I_L\), is also derived

\[
I_L = \frac{1}{\omega_{ac}L} \left[\frac{\pi^2}{12}(V_p - V_S)^2 + V_p V_S \left(\frac{\phi}{2} - \frac{8}{12\pi} |\phi|\right)\right] \quad (10)
\]

The input currents of the two H-bridges, \(i_{L,P}(t)\) and \(i_{L,S}(t)\), as shown in Figs. 3d and e, are obtained by changing the sign of \(i_L\) in a way such that it accounts for the switching state of the respective H-bridges. From the waveforms, the average DC currents \(I_{L,P}\) and \(I_{L,S}\) are obtained by averaging

\[
I_{L,P} = \frac{V_S}{\omega_{ac}L} \phi (\bar{x} - |\phi|) \quad (11)
\]

\[
I_{L,S} = \frac{V_p}{\omega_{ac}L} \phi (\bar{x} - |\phi|) \quad (12)
\]

Multiplying the currents (11) and (12) with the respective DC voltage yields the power transfer equation (1). If pure DC currents are assumed at the two ports of the DAB, the current ripple of \(i_{L,P}(t)\) and \(i_{L,S}(t)\) are assumed to flow into the DC-link capacitors \(C_P\) and \(C_S\). The ripple charges \(\Delta Q_P\) and \(\Delta Q_S\) are derived from the areas enclosed by the waveforms \((i_{L,P}(\omega_{ac}t) - I_{L,P})\) and \((i_{L,S}(\omega_{ac}t) - I_{L,S})\). The results are shown in (see (13)) (see (14))

\[
E_{CP} = \frac{1}{2} CP (V_p + \Delta V_p)^2 = \frac{V_p^2 \Delta Q_P}{2 \Delta V_p} + V_p \Delta Q_p + \frac{\Delta V_p \Delta Q_P}{2} \quad (15)
\]

\[
E_{CS} = \frac{1}{2} CS (V_s + \Delta V_s)^2 = \frac{V_s^2 \Delta Q_S}{2 \Delta V_s} + V_s \Delta Q_S + \frac{\Delta V_s \Delta Q_S}{2} \quad (16)
\]

3.2 Magnetising current and derived quantities

The transformer is modelled by its T-equivalent circuit, which is shown in Fig. 5. The inductances \(L_{ps}\) and \(L_{ds}\) describe the leakage inductances of the transformer on both the primary and secondary windings, so that \(L = L_{ps} + L_{ds}\). The ratio of leakage inductances \(r\), the ratio of voltages \(d\) and the transformer utilisation factor \(\lambda\) are defined as follows

\[
r = \frac{L_{ps}}{L_{ds}} \quad (17)
\]

\[
d = \frac{V_p}{V_s} \quad (18)
\]

\[
\lambda = 1 - \frac{|V_p - r V_S|}{V_p + r V_S} = 1 - \frac{|d - r|}{d + r} \quad (19)
\]

Using definitions (17)–(19) and the assumption that the leakage inductances are much smaller than \(L_{m}\), the virtual voltage \(v_m\) on the
magnetising inductance can be calculated from the circuit in Fig. 5 as in (20)

\[ v_{\text{pad}}(t) = \frac{\nu_p(t) + \nu_m(t)}{1 + r} = \begin{cases} \frac{\nu_p(t)}{2} & \text{for } r = 0 \\ \frac{\nu_p(t) + \nu_m(t)}{2} & \text{for } r = 1 \\ \nu_m(t) & \text{for } r \to \infty \end{cases} \]

By integrating the voltage of (20), the magnetising current waveform \( i_p(\omega_{\text{pad}}t) \) is obtained

\[ i_p(\omega_{\text{pad}}t) = \begin{cases} i_p(0) + \text{sgn}(\nu_p(t)) \frac{\nu_p(t) + \nu_m(t)}{1 + r} \omega_{\text{pad}} t & \text{for } 0 \leq \omega_{\text{pad}} t < |\nu| \\ i_p(|\nu|) + \text{sgn}(\nu_m(t)) \frac{\nu_p(t) - |\nu|}{1 + r} \omega_{\text{pad}} t & \text{for } |\nu| \leq \omega_{\text{pad}} t < \pi \end{cases} \]

The flux density in the transformer core is given by \( B = L_{\text{pad}} \psi \) (see Fig. 3b). In analogy to (9), the peak magnetising current \( I_p \) and the peak flux linkage \( \psi \) are calculated as

\[ i_p(0) = -\frac{1}{2} \left( \frac{V_p + V_S}{1 + r} \omega_{\text{pad}} \right) \nu_p(t) + \frac{V_p + V_S}{1 + r} \omega_{\text{pad}} \nu_m(t) \]

\[ i_p(|\nu|) = \frac{1}{2} \left( \frac{V_p - V_S}{1 + r} \omega_{\text{pad}} \right) \nu_p(t) - \frac{V_p + V_S}{1 + r} \omega_{\text{pad}} \nu_m(t) \]

From (25), it becomes clear that the maximum peak flux linkage in the transformer core, \( \psi_{\text{max}} \), is given as follows:

\[ \psi_{\text{max}} = \frac{V_p + V_S}{2(1 + r) \omega_{\text{pad}}} \pi \]

The flux density \( B \) cannot be calculated as the number of turns and the core area are still unknown. However, if a reasonable value is assumed for the maximum flux density \( B_{\text{max}} \), the per-unit utilisation of the transformer core can be derived as given by (27), based on the utilisation factor \( \lambda \) defined in (19).

\[ \frac{\dot{B}}{B_{\text{max}}} = \frac{\psi}{\psi_{\text{max}}} = 1 - \frac{|\nu|}{\pi} \]

### 4 Analytical loss model

A loss model is required to evaluate efficiency and the cooling effort. For the DAB under design, MOSFETs are used. The conduction losses are calculated using the on-state resistances \( R_{\text{DS,}P} \) and \( R_{\text{DS,S}} \) of the MOSFETs in the primary and secondary side H-bridges, respectively. The AC link current always flows through two switches of each H-bridge simultaneously. Also the transformer contributes to conduction losses with its total equivalent copper resistance \( R_{\text{Cu}} \). High-frequency loss components due to skin and proximity effects are neglected for simplicity. Therefore, the total conduction losses of the converter, \( P_{\text{cond,tot}} \), are expressed as a function of a total equivalent resistance \( R_{\text{tot}} \)

\[ R_{\text{tot}} = 2R_{\text{DS,}P} + 2n^2R_{\text{DS,S}} + R_{\text{Cu}} \]

\[ P_{\text{cond,tot}} = R_{\text{tot}} \cdot I_i^2 \]
\[ P_{\text{FOM}} = 2^{\alpha + \beta} \left( -\frac{\Delta}{\mu_1} B_{\text{max}} \right) \left( 1 - \lambda \frac{\Delta}{d + r} \right) \]

where \( \lambda = 1 - \left( \frac{\Delta}{d + r} \right)^{\alpha} \).

The transformer core volume can be calculated using the stored energy \( E_{\text{mc}} \). It is expressed by the peak flux linkage and the magnetising current on the one hand and by the peak flux density on the other hand.

\[ E_{\text{mc}} = \frac{1}{2} I_{\text{rms}}^2 \mu_1 B_{\text{max}} = \frac{1}{2} I_{\text{max}} \mu_1 B_{\text{max}} \]

\[ \Rightarrow V_{\text{FOM}} = \frac{1}{2} I_{\text{rms}}^2 \mu_1 B_{\text{max}} \]

### 5 Analytical weight model

In this section, all components of the DAB as well as the losses are mapped to weights. Losses affect the weight in terms of the size of the heat sink. Its thermal resistance is denoted \( R_{\text{hlHS}} \). Research [14] proposes a figure of merit, which is a material constant, to estimate the weight of a heat sink \( m_{\text{HS}} \)

\[ FOM_{\text{HS}} = \frac{1}{m_{\text{HS}} \cdot R_{\text{hlHS}}} \]

Knowing the losses from the model in Section 4 and the maximum permissible temperatures, (36) can be used to translate the required thermal resistance of the cooling system into its weight.

The weight of the input and output capacitors is proportional to the area product \( A_p \) and winding window area \( A_w \), by a factor \( K_W \)

\[ m_{\text{T}} = K_W \cdot A_p^{1/4} \]

Describing the window area in terms of the copper fill factor \( \kappa_{\text{Cu}} \), the number of turns \( N \), the peak current density \( J_{\text{rms}} \) and the maximum AC link RMS current \( A_{\text{max}} \), and describing the core area in terms of maximum flux linkage \( \psi_{\text{max}} \) and maximum flux density \( B_{\text{max}} \), the area product is found

\[ A_p = A_{\text{C}} \cdot A_{\text{w}} = \frac{\psi_{\text{max}}}{N f_{\text{sw}} \cdot k_{\text{Cu}} B_{\text{max}} J_{\text{rms}}} = \frac{2P_{\text{max}}}{4 f_{\text{sw}} k_{\text{Cu}} B_{\text{max}} J_{\text{rms}}} \]

An alternative to this approach is to research existing transformers and find empirical relationships of weight and power.

### 6 Optimisation example

Combining the equations from the previous sections connects basic design parameters to the weights of the DAB components and enables optimisation of power density. As an example, a preliminary design of the DAB of the ASPIRE project is presented in this section. Table 1 lists the converter parameters. The analysis is performed for a wide range of switching frequencies and different MOSFETs, including modern SiC and GaN devices. For minimisation of the conduction losses, the maximum phase shift is reduced below the theoretical limit of \( \pi/2 \). This decides the leakage inductance according to (1).

For estimating the switching losses, the equations from [13] are used. Moreover, the thermal figure of merit for an aluminum cooling system including fan is needed. For the results of [16], a value of \( FOM_{\text{HS}} = 15 \text{ W/}(\text{kgK}) \) is a reasonable choice. Additionally, for the filter capacitors \( C_p \) and \( C_s \), a research of ceramic capacitors with suitable voltage ratings gave power densities of 41 J/kg and 19 J/kg, respectively. For the transformer, an efficiency of 99% is assumed, yielding a copper resistance of \( R_{\text{Cu}} = 0.2 \Omega \). A ratio of \( r = 1 \) for the leakage inductances is assumed. Planar transformers with similar power ratings from different companies have been researched and the following empirical weight model has been fitted to the collected data:

\[ m_{\text{T}} \approx 1.59 \frac{P}{V_{sw}^{1/2}} \frac{\kappa_{\text{Cu}}}{f_{\text{sw}}} \]

The researched transformers weights as well as the empirical model from (39) are shown in Fig. 7. The magnetic material parameters \( k, a \) and \( \beta \) are obtained from the loss density graphs from the datasheet. Moreover, a net weight of 140 g for the printed circuit board (PCB), the hardware, the semiconductors and additional circuitry as gate drivers and sensors is assumed.

In Fig. 8, the theoretical power density for this example design is shown. The proposed target design is indicated by the dot. It reaches a power density of 6 kW/kg at \( f_{\text{sw}} = 250 \text{ kHz} \). Finally, Fig. 9 shows the weight and the loss breakdown of the design which is indicated by the dot in Fig. 8. Generating these results in MATLAB requires few seconds of computation time.

### 7 Conclusion

In this paper, analytical models of the operating waveforms, the losses and the component weights of a single phase DAB are developed and used in a design optimisation algorithm. The design approach is easy to implement and is computationally efficient. The developed design optimisation algorithm is a useful tool for the designer to obtain initial and fast designs of DAB and to identify the impact of specific design decisions on the power density of the DAB.
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