Small radii of neutron stars as an indication of novel in-medium effects

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At present, neutron star radii from both observations and model predictions remain very uncertain. Whereas different models can predict a wide range of neutron star radii, it is not possible for most models to predict radii that are smaller than about 10 km, thus if such small radii are established in the future they will be very difficult to reconcile with model estimates. By invoking a new term in the equation of state that enhances the energy density, but leaves the pressure unchanged we simulate the current uncertainty in the neutron star radii. This new term can be possibly due to the exchange of the weakly interacting light U-boson with appropriate in-medium parameters, which does not compromise the success of the conventional nuclear models. The validity of this new scheme will be tested eventually by more precise measurements of neutron star radii.

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Neutron star (NS) is a unique place to test fundamental forces at the extremes of matter density, gravity and magnetic fields. Unfortunately, uncertainties in both the Equation of State (EOS) of super-dense nuclear matter and the strong-field gravity strongly interplay with each other in determining observational properties of neutron stars, for the latest review, see, e.g., [1]. For instance, in a simple version of modified gravity where the non-Newtonian gravity exists, neutron stars could have very different structures compared to predictions using Einstein’s General Relativity (GR) theory of gravity [2–4]. The radius of a neutron star is one of the most important observables sensitive to the underlying nuclear EOS and gravity theories used. Currently, within GR the radius of a canonical NS has been predicted to be roughly from 11 to 15 km [5–9] depending on the EOS used. Provided the third family of compact stars known as strange stars exist, their radii could be as small as 7 or 8 km [10–12], although these models normally predict star masses much smaller than the masses of observed massive neutron stars. Thus, the measurement of NS radii plays a very important role in resolving several issues in fundamental physics. Unfortunately, the extraction of NS radii from observations still suffers from large systematic uncertainties [13] involved in the distance measurements and theoretical analyses of the light spectrum [6, 14–16]. Consequently, a wide range of the radius with the mass around 1.4 $M_\odot$ has been reported [16–24]. In particular, using the thermal spectra from quiescent low-mass X-ray binaries (qLMXBs) Guillot and collaborators extracted NS radii of $R_{NS} = 9.4 \pm 1.2$ km [24]. Another recent comprehensive study of spectroscopic radius measurements suggest that for a 1.5 $M_\odot$ NS the extracted radii are $10.8^{+0.5}_{-0.4}$ km [25]. It is important to note that at the moment no consensus has been reached yet on the extracted NS radii. For instance, Bogdanov found a 3-$\sigma$ lower limit of 11.1 km on the radius of the PSR J0437-4715 [26], and Poutanen et al. got a lower limit of 13 km for 4U 1608-52 [27]. Whether the radii of canonical neutron stars can be as low as 10 km have been an interesting topic of hot debate during the last few years.

Whereas the situation has been significantly improved over the last few years, the systematic errors have been hindering severely the accurate determination of the NS radii from astrophysical observations. However, if the existence of NSs with small radii is firmly established, they would pose a severe challenge to the current models of the nuclear EOSs. While it is not very difficult to satisfy the maximum mass constraint, to our best knowledge, no nuclear models available with best-fit parameters to date can reproduce the small NS radius constraint. There are only a few microscopic models or approaches that—either disregard some of the nuclear physics constraints [5] or adjust the high-density part of the EOS using various polytropes to match the whole density profile obtained from observation [28]—can account for both constraints. From the nuclear physics standpoint, the small NS radius requires certain softness of the EOS of the isospin-asymmetric nuclear matter. As one of the basic blocks of the EOS of asymmetric matter, nuclear symmetry energy around 1-2 times the saturation density of nuclear matter plays a dominating role in determining the radii of neutron stars [6]. A softening of the symmetry energy can lead to an appreciable decrease of the NS radius [7]. However, with the maximum NS mass held approximately at a constant, most non-relativistic and relativistic models that are facilitated with soft symmetry energies can only bring the radii of canonical NSs down to about 12-13 km [8, 9, 17, 21]. Moreover, it becomes very difficult to further reduce the NS radius by further softening the symmetry energy. In particular, one would then encounter the stability problem in the NS matter
EOS when the symmetry energy becomes too soft \cite{3}. To further reduce the NS radius, one could imagine to reduce the pressure of the isospin-symmetric part of the EOS in the intermediate density region. But the space in so doing is actually limited by the saturation properties of nuclear matter and the constraint on the EOS of dense nuclear matter extracted from studying nuclear collective flow in high energy heavy-ion reactions \cite{29}. Furthermore, as we can see from the empirical relation \( R_p^{-1/4}(\rho_B) \approx C(\rho_B) \) between the NS radii \( R_p \) and the pressure \( p(\rho_B) \) with \( C(\rho_B) \) being a constant at a given baryon density \( \rho_B \) \cite{7}, such a reduction is also rather inefficent, since the isospin-symmetric EOS contribution to the total NS matter pressure \( p(\rho_B) \) is relatively small in the relevant density region, where this empirical relation holds. Moreover, even if the significant reduction were allowed for the total pressure, the limited decrease of the NS radius would be at the cost of a large reduction of the NS maximum mass, because the significantly reduced pressure needs the corresponding reduction of the NS mass to balance the gravity therein, and also because the NS mass is the total energy integrated in a nutshell with the reduced radius. A significant reduction in the maximum mass is certainly disfavored by the recently discovered massive neutron stars of \( 2M_\odot \) that require a stiff EOS \cite{30,31}.

Facing currently with this severe theoretical issue, we explore in this work a new possibility to soften the EOS: adding a new term in the EOS that enhances the energy density while keeping the pressure unchanged. In so doing, the enhancement of the energy density requires some shrinkage in NS radius without reducing significantly the NS maximum mass. Since the pressure is \( p = \rho_B^2 \partial(\epsilon/\rho_B)/\partial \rho_B \), to keep the pressure unchanged we modify the energy density \( \epsilon \) by

\[
\epsilon = \epsilon_0 + C_L \rho_B, \tag{1}
\]

where \( \epsilon_0 \) is the base energy density given by any model. The second term \textit{linear in density} is an addition to the base EOS with \( C_L \) being the amending coefficient. As we shall explain, this modification to the EOS of isospin-asymmetric nuclear matter can be realized by considering the interaction added by a vector boson with appropriate in-medium parameters.

In principle, the modification given in Eq. (1) could be added to any nuclear model. In this work, we just demonstrate the effects using several typical relativistic mean-field (RMF) models. The RMF models under consideration include the SLC, SLCd \cite{32}, SL3 \cite{9} and NL3040 \cite{33}. The SL3 and the NL3040 have similarly stiff EOSs at high density and both give large NS maximum masses of more than \( 2.6M_\odot \), while the SLC and the SLCd feature the same EOS of symmetric matter within the constraints obtained from analyzing the collective flow in relativistic heavy-ion collisions \cite{29}. The only difference between the SLC and the SLCd is that the latter has a softer symmetry energy. In a similar fashion, the NL3040 was also built from the original NL3 to feature a softer symmetry energy. The stiffness of the symmetry energy is normally measured by its density slope at the saturation density of nuclear matter \( \rho_0 \), \( L = 3\rho_0 \left( \partial E_{\text{sym}}(\rho_B)/\partial \rho_B \right) \rho_0 \). The value of \( L \) for the SLC and SL3 is 92.3 and 97.1 MeV, while it is 61.5 and 45.0 MeV for the SLCd and NL3040, respectively. For a comparison, it is interesting to note that currently the most probable value of \( L \) is in the range of \( 40 \lesssim L \lesssim 70 \) MeV according to recent analyses of various terrestrial experiments and astrophysical observations, see, e.g., Refs. \cite{34–40} and Ref. \cite{41} for a comprehensive review. Thus, the SLC and SL3 are obviously too stiff while the SLCd and NL3040 are consistent with the existing constraints in terms of their \( L \) values. Nevertheless, they are all appropriate for the purposes of this study.

Shown in Fig. 1 are two examples of the EOS, i.e., pressure versus energy density, with the SLCd and SL3 parameter sets. It is seen that at a constant pressure, the amending term can soften the EOS considerably, i.e., reducing the slope of the pressure with respect to the energy density, especially with the SLCd. However, the relative effect of the amending term goes down with
the increasing density because it is just linear in density while the EOS of usual nuclear models evolves generally with the density squared. We emphasize that the EOS softening scheme considered here is quite different from the usual mechanisms mentioned in the introduction. In particular, typical phase transitions to matter with new degrees of freedom normally reduce the maximum mass dramatically, but often keep the NS radius more or less the same because the phase transitions usually occur in the small inner core of NSs. Of course, exceptions may exist when the new degrees of freedom, such as the $\Delta$ resonances, can emerge at a relatively low density [42]. It is interesting to note that by using Lindblom’s inversion algorithm [43] Chen and Pickarewicz were recently able to obtain a softened EOS from the given small NS radii [44].

We now examine effects of the amending term on the radii of neutron stars. As in Ref. [20], here we consider the simplest model of neutron stars consisting of just neutrons, protons and electrons. Shown in Fig. 2 are the mass-radius (MR) trajectories of neutron stars with the amending term within various RMF models. The amending coefficient is exemplified as $C_L = 100, 200$ and $300$ MeV, and the results with original models are displayed with $C_L = 0$. Comparing results of the original models, we see that the softening of the symmetry energy may reduce the NS radius by as large as 1.5 km for a canonical NS when $L$ is reduced from 97.1 to 45 MeV. The space for further reducing the slope parameter $L$ is small, and in fact, the further reduction in $L$ has a very limited effect in decreasing the NS radius. Moreover, it is seen that even with the significant softening of the symmetry energy within the original models the NS radius is still far above $9.4 \pm 1.2$ km extracted by Guillot et al. [24]. If such small radii are established, it is then interesting to see that the amending term can indeed further reduce the NS radius. Obviously, the role of the amending term is similar in all models: the larger the amending coefficient is, the more is the reduction of the NS radius. With the same amending coefficient in different models, the shifted magnitude is also similar. Typically, the amending coefficient, varying up to 300 MeV, can cause a reduction of about 3 km in the NS radius. With the amending coefficient of $C_L = 300$ MeV, we see that the MR trajectories with the SLC and SLCd fall into the regime extracted by Guillot et al. [20]. We see from Fig. 2 that the NS maximum mass is still not reached in the SLC and SLCd models before reaching the causality boundary, because the allowed nucleon density has a maximum in the construction of such models to meet the chiral limit [9]. The removal of such a limiting density may bring the NS maximum mass closer to the causality limit. With the larger amending coefficient, the MR trajectories for the SL3 and NL3040 can be very close to the upper margin of the extracted regime. Notably, we see that the MR trajectories are not clearly away from the causality constraint, though the amending coefficient causes the decrease of the NS maximum mass. Here, the moderate reduction of

![FIG. 2: (Color online) The mass-radius trajectories of neutron stars with RMF models: SLC, SLCd, SL3 and NL3040. In each panel, the amending coefficient $C_L$ is taken to be the values 0, 100, 200, 300 MeV, respectively. The slope parameter of the symmetry energy is also labeled in each panel.](image)

In the RMF framework, the amending term in Eq. (1) can be understood as a specific in-medium effect. Similar to the analysis in Refs. [45–47], the amending term, incorporated into the vector potential, leads to the density-dependent coupling constant of the vector ($\omega$) meson

$$g_\omega^2(\rho_B) = (g_\omega \omega + 2C_L)m_\omega^2/\rho_B.$$  \hspace{1cm} (2)

This relation indicates that the larger $C_L$ is responsible for the stronger density dependence of the coupling constant, and with the increase of density, the in-medium effect decreases with the growing $\omega$. Fig. 3 shows the density-dependent coupling constant for the models considered. For a comparison, the $g_\omega(\rho_B)$ obtained from the Dirac-Brueckner (DB) potential of Bonn A is also shown in the figure. It is seen that the density dependence, similar to the one from the DB potential, is needed to produce a significant reduction of the NS radius. We can infer, indeed, that the density dependence here is stronger than that from the DB potential, because the latter owns a partial cancelation of the density dependencies between the scalar and vector potentials [45–47]. In addition, we see that the in-medium effect of all cases tend to vanish at high densities. This means that the decrease of the NS radius is dominated by the modification of the amending term to the low-density EOS. For the dropping of the NS
maximum mass, it can then hopefully be cured by modifying the high-density component of the EOS of nuclear matter.

\[
\epsilon_{UB} = \frac{1}{2} \left( \frac{g_u}{m_u} \right)^2 \rho_B^2, \tag{3}
\]

with \( g_u / m_u \) being the ratio of the U-boson coupling constant and its mass.

By assuming a density-dependent U-boson mass of \( m_u^2 = g_u^2 \rho_B / 2C_L \), we realize the linear density dependence of the amending energy density. If the U-boson has a very light bare mass of \( m_{u0} \) in free space, its in-medium mass can be given as

\[
m_u^2 = g_u^2 \rho_B / 2C_L + m_{u0}^2. \tag{4}
\]

In general, the in-medium effect of the boson parameters is associated with the contribution of the intermediate states. In the RMF theory, the in-medium effect is usually attributed to the nonlinearity of the meson self-interactions \([59]\). The in-medium effect for the boson may also be realized by carefully choosing the nonlinear self-interacting terms. We leave this problem for a future study.

Regarding the possible violation of the low-energy nuclear constraints for finite nuclei, one can avoid it by limiting the weak interaction strength of the U-boson. For instance, if \( g_u \) is 0.01, the interaction strength is about two orders of magnitude weaker than the electromagnetic interaction, and such weak interaction is not able to affect properties of finite nuclei. In this sense, the present scheme to invoke the U-boson does not compromise the success of optional nuclear models. The interesting question is what behavior of the U-boson mass will allow such a weak interaction strength. Shown in Fig. 4 is the density-dependent mass of the U-boson as a function of density, for \( g_u = 0.01 \). Here, as an example, the small bare boson mass is taken to be as 0.1 MeV, which is just a free parameter. We see that for the given coefficients \( C_L \), the in-medium mass is just within a few MeV. With the increase of the \( C_L \), the in-medium mass of the U-boson decreases. For \( C_L = 300 \) MeV, the U-boson mass is close to 1 MeV, which is consistent with that considered in Ref. \([4]\).

We should say that the role of the vector boson in softening the EOS eventually is rather pragmatic, albeit putatively accounting for small NS radii. Usually, the vector boson is a source of the pressure, while here it gives zero contribution. From the point of view of nuclear many-body approaches, the repulsion of the vector meson can be softened in the intermediate density region due to the correlation effect from the intermediate-state contributions, and such a softening can be simulated in the RMF theory by invoking the nonlinear self-interacting terms.
At high densities where the intermediate-state contribution is small due to the Pauli blocking the reduction of the in-medium effect of the vector boson will then recover the repulsion which may enable a stiffer EOS at high densities. In this sense, if we use a more complicated density-dependent term in Eq. (1) in the high density region, the corresponding U-boson can then stiffen the high-density EOS and the U-boson mass would be also much closer to a constant at high densities. While we use a simple amending term in Eq. (1) to demonstrate the reduction of the NS radius, it should not represent that the corresponding vector U-boson would simply soften the EOS.

The NS radius can attain larger reduction by further increasing the amending coefficient. In our analysis here we have invoked the special amending term that does not affect the pressure. Generally, other forms could also be optional, as long as they result in the reduction of the NS radius. A favorable form should reduce the low-density pressure and satisfy all constraints on the EOS of nuclear matter at saturation density. To meet these demands, one can consider the light scalar boson. Note that the scalar boson, like the scalar meson, can also modify the mass of baryons. We have checked that the inclusion of such a light scalar boson can again reduce the NS radius by 0.5-1.5 km, depending on the model used. To keep the pressure positive definite, the coupling constant of the scalar boson should be much weaker (e.g., \( g_s = 0.005 \)) than that of the vector boson, and the NS maximum mass is little changed by the scalar boson.

In summary, we have demonstrated that the large uncertainty of the NS radius can be simulated by introducing the amending term that enhances the energy density, but leaves the pressure unchanged. The distinct effect of the amending term on the reduction of the NS radius is rather universal to any nuclear EOS. The new term is linear in density, and the incorporation into the relativistic mean-field potential produces the strong in-medium effect on the vector coupling constant at low densities. The strong in-medium effect can be responsible for the significant reduction of neutron star radii, and may provide an explanation for the small NS radius if it is firmly established. We interpret this novel in-medium effect by the possible exchange of a putative weakly interacting U-boson. By requiring that the low-energy constraints in finite nuclei must not be violated, we find interestingly that the in-medium U-boson mass is just below a few MeV with the interaction strength being two orders of magnitude weaker than the electromagnetic interaction. Finally, the validity of the scheme presented here will be tested eventually by the more precise measurements of the NS radii.

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