Noise-assisted preparation of entangled atoms

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We discuss the generation of entangled states of two two-level atoms inside an optical cavity. The cavity mode is supposed to be coupled to a white noise with adjustable intensity. We describe how the entanglement between the atoms inside the cavity arise in such a situation. The entanglement is maximized for intermediate values of the noise intensity, while it is a monotonic function of the spontaneous rate. This resembles the phenomenon of stochastic resonance and sheds more light on the idea to exploit white noise in quantum information processing.

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Entanglement shared between distant sites is a valuable resource for quantum information processing [1, 2, 3]. Stimulated by this discovery, there are a lot of attention have been focused on how to create, manipulate and exploit entanglement from both sides of experimental and theoretical studies [4]. From a practical point of view, the main task to create and exploit entanglement is how to minimize the impact of noise that is not able to be isolated from our system in any real experimental scenario. This interaction between the system of interest and the noise which usually models surroundings of the system results in a decoherence process. As a consequence the entangled system may end up in a mixed state that would be no longer useful for any quantum information processing. It is therefore important for practical realization of quantum information processing protocols to engineer mechanisms to prevent, minimize, or use the impact of environmental noise.

Numerous proposals have been made for preventing, minimizing or using the environmental noise, for example, loop control strategies, that use an ancillary system coupling to the quantum processor to better the performance of the proposals [5, 6], quantum error correction [7] uses redundant coding to protect quantum states against noisy environments. This procedure is successful as long as the error rate is sufficiently small. It wastes a number of qubits and quantum gates, and then limit its implementation by present available technology. A more economic approach consists of exploiting the existence of so-called decoherence-free subspace that are completely insensitive to specific types of noise [8]. This approach tends to require fewer additional resources, but is only applicable in specific situations. The seminal idea that dissipation can assist the generation of entanglement has been put forward recently [9, 10, 11]. In a system consisting of two distinct leaky optical cavities, it was shown that the entanglement is maximized for intermediate values of the cavity damping rates and the intensity of the white noise, vanishing both for small and for large values of these parameters [11]. In fact, this idea appeared first in Ref. [9] for two atoms inside an optical cavity and it shows that cavity decay can assist the preparation of maximally entangled atoms, without cavity decay, the reduced state of the two-atom system would be in an inseparable mixture at all times, but not in a maximally entangled one.

In the latter case, the photon leakage leads to the undesired parts of the global wave-function to decay and therefore such terms are eliminated for sufficiently large times. This requires that the overlapping between the initial state and the desired maximally entangled state must not be zero, hence one of the two atoms prepared in its excited state initially is necessary. This point makes our proposed scheme here for preparation of entangled two atoms different from the above one.

In this letter, we put the idea in Ref. [11] forward to a two-atom system, we use noise to play a constructive role in quantum information processing. We will concentrate on the problem of creating entangled atoms when only incoherent sources are available and demonstrate that, indeed, controllable entanglement can arise in this situation.

Our system consists of two two-level atoms inside a leaky optical cavity. As depicted in figure 1, we will re-

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FIG. 1: Schematic diagram for the preparation of entangled two atoms. The atom-cavity system is prepared initially in the ground state $|g\rangle_a|g\rangle_b$ with no photon populated in the cavity. The atoms become entangled and the entanglement is maximized for intermediate values of the noise intensity.
fer to atom $a$ and atom $b$ when the context requires us to
differentiate them, but otherwise they are supposed
to be identical. We denote the atomic ground and ex-
cited states by $|g⟩_i$ and $|e⟩_i$, respectively, and call $2Γ$
($Γ_a = Γ_b = Γ$) the spontaneous emission rate from the
upper level. We assume that the distance between the
atoms is much larger than an optical wavelength, there-
fore dipole-dipole interaction can be neglected. The cav-
ity mode is assumed to be resonant with the atomic tran-
sition frequency, and we will denote the cavity decay rate by $2κ$.
For the sake of generality we allow the coupling
between each atom and the cavity mode, $g_i$, to be dif-
ferent. We suppose the cavity is driven by an external
thermal field (white noise) whose intensity will be char-
acterized in terms of an effective photon number $n_T$, but
we do not want to specify the noise. The relaxation of the
atom-cavity system can take place through two channel,
at rate $2κ$ (cavity decay) and $2Γ$ (spontaneous decay).
The master equation governing the time evolution of the
global system is given by (setting $ℏ = 1$)

$$\dot{ρ} = -i[H, ρ] + L(ρ),$$

(1)

where the Hamiltonian $H$ describes the internal energies
of the cavity and the two atoms as well as the atom-cavity
 coupling. The Liouvillean $L(ρ)$ describes the atom
decay and the interaction of the cavity mode with the noise. As
no external coherent driving is present, the Hamiltonian reads

$$H = \frac{ω}{2}σ_+^a + \frac{ω}{2}σ_+^b + ωf a^† a + \sum_{i=a,b} g_i(|g⟩_i⟨e|a^† + h.c.),$$

(2)

where $a$ represent the cavity mode with frequency $ω_f$.
The Liouvillean is given by

$$L(ρ) = -κ(n_T + 1)(a^† aρ + ρ a^† a - 2aρ a^†)
- κnt(aa^† ρ + ρa a^† - 2a a^† ρ)
- Γ \sum_{i=a,b} (|e⟩_i⟨e|ρ + ρ|e⟩_i⟨e| - 2|g⟩_i⟨e|ρ|e⟩_i⟨g|).$$

(3)

Here $Γ$ describes the atom decay rate and $κ$ stands for the
cavity leakage rate. We do not explicitly specify the white
noise, but its intensity $n_T$ refers to its effective particle
number. To simplify the representation, now we turn to
an interaction picture with respect to $H_0 = \sum_{i=a,b} g_i^2 Σ_i +
ωa a^† a$. After this transformation, the Liouvillean part
remains unchanged, while the Hamiltonian part is now
given by

$$H_I = \sum_{i=a,b} g_i(|g⟩_i⟨e|a^† + h.c.),$$

(4)

where the atom-cavity coupling is on resonance was
assumed. The analytical solution to the equation (1) is
extremely tedious. To make the physical interpretation
clear, we introduce two new effective atomic modes one
of which will be decoupled from the cavity mode. The
two collective atomic modes is given by the following definition

$$σ_+^A = \frac{g_aσ_+^a + g_bσ_+^b}{\sqrt{g_a^2 + g_b^2}}, σ_+^B = \frac{g_bσ_+^a - g_aσ_+^b}{\sqrt{g_a^2 + g_b^2}},$$

(5)

with $σ_i^+ = |e⟩_i⟨g|$, together with $σ_i^- = (σ_i^+)^†$, and $σ_i^z$
represent the pauli operators for the atom $i$. In terms of
these new operators, the Hamiltonian and Liouvillean
part of the master equation are given by

$$H_I = g(|g⟩_A⟨e|a^† + h.c.),$$

(6)

where $g = \sqrt{g_a^2 + g_b^2}$, and

$$L(ρ) = -κ(n_T + 1)(a^† aρ + ρ a^† a - 2a a^† ρ)
- κnt(aa^† ρ + ρa a^† - 2a a^† ρ)
- Γ \sum_{i=A,B} (|e⟩_i⟨e|ρ + ρ|e⟩_i⟨e| - 2|g⟩_i⟨e|ρ|e⟩_i⟨g|).$$

(7)

Note that the sum in the last line of Eq.(7) is taken over the
two NEW modes. The transformation between the
resulting atom, $a$, $b$ and the collective modes $A$, $B$ is clear.
For example, both the resulting atoms $a$ and $b$ in its
ground state $|g⟩_a|g⟩_b$ can be equivalently expressed in terms of
$|g⟩_A|g⟩_B$ and $|e⟩_a|e⟩_b$ likewise. The new master
equation Eq.(7) shows us that we have one mode (mode
$B$) which is completely decoupled from the Hamiltonian
dynamics and is purely damped under the Liouvillean
dynamics, this is a consequence of the transformation from
the resulting atoms to the collective modes. The mode
$B$ will then not be populated in steady state irrespective
of its initial states. In other words, if the mode $B$ is in
its ground state initially, it will remain on that forever.
Therefore, we begin our investigations with both collective
modes $A$ and $B$ in the ground state $|g⟩_A|g⟩_B$. As
the mode $B$ will then never be populated, we disregard
that mode in the following discussions. Apart from the
above assumption, we discuss the entanglement generation
here only for the case of no photon in the cavity
initially, this is relevant to the topics under our consid-
eration, i.e., study the role of the white noise in the en-
tanglement generation. To understand the origin of
the generation of entanglement from white noise, let us begin
by considering a special case of perfect cavity, i.e., $κ = 0$.
For the general form of the reduced density matrix of the
atom system

$$ρ_{atom} = ρ_{ee}|e⟩⟨e| + ρ_{eg}|e⟩⟨g| + ρ_{ge}|g⟩⟨e| + ρ_{gg}|g⟩⟨g|,$$

it is easy to check that $ρ_{eg} = ρ_{ge} = 0$ in this situation,
where $ρ_{xy}(x, y = g, e) = Tr_c(|g⟩⟨g|)|x⟩⟨ρ|$ and $Tr_c$
denotes trace over the cavity mode. Noticing $|e⟩_A⟨e| =
\frac{g_e^2|e⟩_a⟨e| + g_g^2|e⟩_b⟨e|}{g_a^2 + g_b^2}$, and $|g⟩_A⟨g|$ has a similar form, we con-
clude that for perfect cavity with no initial photon inside
the cavity, the atoms remain separable at all the
times. Moreover this conclusion holds even for $\kappa \neq 0$ with $n_T = 0$. In this case, the cavity mode will never be populated if there is no photon in the cavity initially. As the cavity mode act as a data bus, no photon exist inside the cavity means the data bus is idle. This conclusion will be changed if we increase the value of the thermal noise intensity $n_T$. For a limiting case of $g = 0$ and $\Gamma = 0$, the stationary state of the cavity field is no longer the vacuum, instead, it takes \[ \rho_c = \sum_n |n\rangle\langle n| \left( \frac{nT}{1 + nT} \right)^n \frac{1}{1 + nT}. \]

The active data bus indicate that the two atoms inside the cavity will be entangled together.

We will now show in a numerical way that the envisioned idea we have described above can be realized in the atom-cavity system by properly chosen parameters. We will choose the Wootters concurrence as the entanglement measure \[ c(\rho) = \max \left\{ 0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \right\}, \]

where the $\lambda_i$ are the square roots of the eigenvalues of the non-Hermitian matrix $\hat{\rho}\hat{\rho}^*$ with $\hat{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$ in decreasing order. The Wootters concurrence gives an explicit expression for the entanglement of formation, which quantifies the resources needed to create a given entangled state. The typical behavior of the entanglement in the system is illustrated in figure 2. There we have plotted the amount of entanglement of the joint state of the two atoms as a two-variable function of the intensity of the noise $n_T$ and time $t$. The chosen parameters are $\omega = \omega_f$, $g_a = g_b = 1$, $\kappa = 2$ and $\Gamma = 0.2$. We want to stress that our simulation is presented for Eq.(1), i.e., the original master equation for the atom-cavity system, and as we mentioned above the initial state of the global system is $|g\rangle_a |g\rangle_b |0\rangle_c$ in our simulation, and we cut off the intra-cavity photon number at a value of 5. Note that for any value of $t$ in the region of entanglement $\neq 0$, the behavior of the amount of entanglement between the two atoms is non-monotonic, it increase to a maximum value for an optimal intensity of the noise and then decrease towards zero for a sufficiently large intensity. Physically, to get non-zero amount of entanglement, the cavity mode must be populated( or be excited ) at any value of time $t$. For the two limiting case of either $n_T = 0$ or $n_T \rightarrow \infty$, however, the data bus remains idle for all the times. Thus the amount of entanglement equals zero. It is also worthwhile to study the dependence of entanglement on the intensity of the noise and the cavity decay rates. In figure 3, we present those for $\omega = \omega_f$, $g_a = g_b = 1$, and $\Gamma = 0.2$. It seems that the amount of entanglement is fixed for a specific value of $n_T \kappa$. It is interesting to note that the amount of entanglement behave as a monotonic function of the atom decay rate $\Gamma$. As figure 4 shows, this is quite different from the case presented in Ref. [11], where the cavity decay can assist themselves to entangle together, and the entanglement is generated among themselves then.

The noise-assisted entanglement preparation is somehow reminiscent of the well known phenomenon of stochastic resonance \[ 12,13,14 \], where the response of a system to a periodic force can be enhanced in the presence of an intermediate amount of noise. A related effect that cavity decay can assist the generation of squeezing has been found recently \[ 15 \], there they show that the squeezing effect is enhanced as the damping rate of the cavity is increased to some extent. However, the pumping field amplitude is required to be inversely proportional to the damping rate for the optimal squeezing. This is similar to our results shown in figure 3.

To sum up, we have described an experimental situation where entanglement between two atomic
FIG. 4: Amount of entanglement versus the noise intensity $n_T$ and the atomic spontaneous emission rate $\Gamma$ for a value of time $t = 1/(2g)$. We choose the parameters $g_a = g_b = 1, \omega_f = \omega$ and $\kappa = 2$ for this plot. The entanglement of the two atoms is a monotonic function of $\Gamma$ for any value of $n_T$, this is quite different from the case presented in Ref. [11].

systems can be prepared with assistance of the white noise. The entanglement measured by the Wootters concurrence is maximized for intermediate values of the cavity decay rate and the intensity of the white noise, while it is a monotonic function of the atomic spontaneous emission rate. Recall that the atomic decay itself cannot induce entanglement among the atoms, even if at finite temperatures, we conclude that the coupling between the data bus and the white noise is the origin of the generation of the entanglement. The phenomenon of white noise-assisted entanglement generation is not a rare phenomenon, it resembles the phenomenon of stochastic resonance. However, this discovery [11] is really valuable because it sheds new light on the constructive role that noise may play in quantum information processing. In contrast with the results in Ref. [11], the proposal presented here is for the entanglement generation between two two-level atoms. For such a two-qubit system, any amount of entanglement, even if very small, is distillable [18], and therefore the entangled atoms are useful for quantum information processing. The entanglement among many atoms also can be created by the same manner, the results will be presented elsewhere.

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