New Supernova Limit on Large Extra Dimensions

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If large extra dimensions exist in nature, supernova (SN) cores will emit large fluxes of Kaluza-Klein gravitons, producing a cosmic background of these particles with energies and masses up to about 100 MeV. Radiative decays then give rise to a diffuse cosmic $\gamma$-ray background with $E_\gamma \lesssim 100$ MeV which is well in excess of the observations if more than 0.5–1% of the SN energy is emitted into the new channel. This argument complements and tightens the well-known cooling limit from the observed duration of the SN 1987A neutrino burst. For two extra dimensions we derive a conservative bound on their radius of $R \lesssim 0.9 \times 10^{-4}$ mm, for three extra dimensions it is $R \lesssim 1.9 \times 10^{-7}$ mm.

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1. Introduction.—The Planck scale of about $10^{19}$ GeV, relevant for gravitation, is very much larger than the electroweak scale of about 1 TeV of the particle-physics standard model. A radical new approach to solving this notorious hierarchy problem holds that there could be large extra dimensions, the main idea being that the standard-model fields are confined to a 3+1 dimensional brane embedded in a higher dimensional bulk where only gravity is allowed to propagate. This concept immediately puts stringent constraints on the size of the extra dimensions because Newton’s law holds at any scale which has thus far been observed, i.e. down to about 1 mm. Extra dimensions can only appear at a smaller scale.

For simplicity we follow the usual practice and assume that the $n$ new dimensions form an $n$-torus of the same radius $R$ in each direction (see, however, Ref. for a more general model). The Planck scale of the full higher dimensional space, $M_{n+4}$, can be related to the normal Planck scale, $M_A = 1.22 \times 10^{19}$ GeV, by Gauss’ law:

$$M_{n+4}^2 = R^n M_A^{n+2}. \quad (1)$$

Therefore, if $R$ is large then $M_{n+4}$ can be much smaller than $M_A$. If this scenario is to solve the hierarchy problem then $M_{n+4}$ must be close to the electroweak scale, i.e. $M_{n+4} \lesssim 10–100$ TeV. This requirement already excludes $n = 1$ because $M_{n+4} \simeq 100$ TeV corresponds to $R \simeq 10^8$ cm. However, $n \geq 2$ remains possible, and particularly for $n = 2$ there is the intriguing perspective that the extra dimensions could be accessible to experiments probing gravity at scales below 1 mm.

Thus far the most restrictive constraint on $M = M_{n+4}$ arises from the observed duration of the supernova (SN) 1987A neutrino burst. If large extra dimensions exist, then the usual 4D graviton is complemented by a tower of Kaluza-Klein (KK) states, corresponding to the new available phase space in the bulk. These KK gravitons would be emitted from the SN core after collapse, compete with neutrino cooling, and shorten the observable signal. This argument has led to the tight bound $R \lesssim 0.66$ $\mu$m ($M \gtrsim 31$ TeV) for $n = 2$ and $R \lesssim 0.8$ mm ($M \gtrsim 2.75$ TeV) for $n = 3$ [10]. Cosmological considerations lead to similar constraints, but the cosmological uncertainties are arguably larger than those connected with the SN bound.

We presently derive a related SN bound which is independent of the low-statistics SN 1987A neutrino signal and which is significantly tighter. The KK gravitons emitted by all core-collapse SNe over the age of the universe produce a cosmological background of these particles. Later they decay into all standard-model particles which are kinematically allowed; for the relatively low-mass modes produced by a SN the only channels are KK $\rightarrow 2\gamma$, $e^+e^-$ and $\nu\bar{\nu}$. The relevant decay rates are $\tau_{2\gamma} = \frac{1}{2} \tau_{e^+e^-} = \tau_{\nu\bar{\nu}} \simeq 6 \times 10^9$ yr ($m/100$ MeV)$^{-3}$ [10]. Therefore, over the age of the universe a significant fraction of the produced KK modes has decayed into photons, contributing to the observed diffuse cosmic $\gamma$-ray background. We calculate the present-day contribution to the MeV $\gamma$-ray background for different numbers and radii of the large extra dimensions. It turns out that the measured cosmic $\gamma$-ray background constrains the KK emission from SNe more tightly than the SN 1987A neutrino signal and thus provides the most restrictive limit on large extra dimensions.

A note of caution is due here because our arguments rely on the assumption that KK modes can only decay into standard model particles on one brane. If there are other branes present the KK modes can also decay into standard model particles on these, and the limit is weakened accordingly. The argument also relies on the fact that KK modes cannot decay into other KK modes with smaller bulk momenta. In models where the extra dimen-
sions are toroidally compactified this assumption holds because requiring both energy and momentum conservation leads to zero decay probability. In other spaces which are not translational invariant this argument will in general not be valid.

2. Supernova Rate.—The first ingredient of our calculation is the SN rate in the universe which is usually expressed in terms of the “SN unit” or SNU, corresponding to 1 SN per 100 years per $10^{10} L_{B^0}$ with $L_{B^0}$ the solar luminosity in the blue spectral band. Moreover, the average cosmic luminosity density in the blue is $h 1.93^{+0.8}_{-0.6} \times 10^6 L_{B^0} \text{ Mpc}^{-3}$ where as usual $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$. Therefore, in good approximation 1 SNU corresponds to a spatial density of $h 2 \times 10^{-5} \text{ yr}^{-1} \text{ Mpc}^{-3}$.

The observed present-day SN rate depends on the morphological type of the host galaxy. Elliptical and S0 galaxies host hardly any core-collapse SNe while in spiral galaxies of type Sbc–Sd the rate is $h^2 (1.8 \pm 0.6) \text{ SNU}$ according to [1] and $h^2 3.9 \text{ SNU}$ according to [3]. The rate is similar in Sm, Irr and Pec galaxies, but only about half as large in types S0a–S0b. While early-type galaxies (E and S0) are less frequent than spirals, they are brighter in the blue. Assuming pulsars contribute at least half the blue luminosity density and in view of the quoted SN rates we adopt a nominal density of

$$R_{SN} = h^3 2 \times 10^{-4} \text{ yr}^{-1} \text{ Mpc}^{-3}$$

for the present-day rate of core-collapse SNe. This value is almost identical to the one adopted in [14] who apparently used $h = 0.5$.

In the past, especially during the first 1 Gyr after galaxy formation, the SN rate must have been much larger due to the much larger rate of early star formation. This issue has been broadly discussed in calculations of the cosmic $\bar{\nu}_e$ relic flux from SNe that might be detectable in neutrino observatories such as Super-Kamiokande. Equation (3) corresponds to an integrated $\bar{\nu}_e$ flux at Earth of about $2 \text{ cm}^{-2} \text{ s}^{-1}$, assuming $h = 0.75$, a cosmic age of 12 Gyr, and the emission of $2 \times 10^{57} \bar{\nu}_e$ per SN. The larger rates of star formation in the past increase this flux prediction by factors of up to 100. The cosmological “bench-mark case” of Ref. [15] predicts 44 $\text{ cm}^{-2} \text{ s}^{-1}$ for the integrated $\bar{\nu}_e$ flux at Earth while [16] find an upper limit of $54 \text{ cm}^{-2} \text{ s}^{-1}$, but think that a realistic value is perhaps a factor of ten smaller. This lower estimate agrees closely with [17] who find an integrated flux of about $10 \text{ cm}^{-2} \text{ s}^{-1}$ per neutrino flavor.

In our case of slowly decaying particles, the early SN rate is even more important because the KK gravitons from early SNe have the longest time to decay and thus contribute more to the cosmic $\gamma$-ray background. Therefore, with the constant rate Eq. (3) we likely underestimate the contribution to the cosmological $\gamma$-ray background by at least a factor of 10, and perhaps by as much as a factor of 100.

3. Emission from SN Core.—The next ingredient of our calculation is the emission of KK gravitons from a hot and dense SN core after collapse. The main process is nucleonic bremsstrahlung $N + N \to N + N + \text{KK}$ which has been discussed in detail in Ref. [1]. Besides the density, temperature and chemical composition of the nuclear medium, the energy-loss rate $Q$ into KK states depends on the mass $m$ of these particles. The mass of the KK gravitons appears as a consequence of their momentum in the compactified extra dimensions and as such is a discrete parameter, but the mode spacings are so small that we may treat $m$ as a continuous parameter characterizing the tower of KK states that can be produced by the medium.

On the basis of Ref. [1] we have calculated the differential emissivity $dQ/dE$ and $dQ/dm$ assuming that the nucleons are non-relativistic and non-degenerate. This approximation is only marginally valid in a hot SN core, but it has no serious bearing on our results. A more complete treatment of the nuclear medium is important for the absolute emission rate, but for the moment we are only interested in the spectrum which will not significantly depend on the degree of nucleon degeneracy. In Fig. 1 we show $dQ/dE$ and $dQ/dm$ for $n = 2$ and 3. Naturally, most KK states are produced near their kinematic limit; their average speed is about 0.7, i.e. typically they are only mildly relativistic. After integrating over all masses, the average energy of the emitted particles is $\langle E \rangle = 4.25 T$ and $5.42 T$ for $n = 2$ and 3, respectively, while conversely, after integrating over all energies, the average mass is $\langle m \rangle = 2.72 T$ and $3.89 T$ for $n = 2$ and 3.

The total number of KK gravitons produced in a SN collapse is $N_{KK} = f_{KK} E_{tot} / \langle E \rangle$, where $E_{tot}$ is the total SN energy, taken to be $3 \times 10^{53}$ erg, and $\langle E \rangle$ is the average energy of the emitted KK modes. The fraction $f_{KK}$ of the total energy emitted in KK modes is the main parameter to be constrained by our new argument.

4. Cosmic Background of Decay Photons.—Finally we turn to calculating the present-day photon number density $n_\gamma$ produced by the decay of KK gravitons from all past SNe. Formally, this density can be written as

$$\frac{dn_\gamma}{dt} = \int_0^t dt' \frac{d^2 n_\gamma}{dt'dt},$$

where $\epsilon$ is the photon energy and $t_0$ the age of the universe. In a standard cosmological model this is

$$\frac{dn_\gamma}{dt} = \frac{2}{3} H^{-1}_0 \int_0^\infty dz F(z, \Omega_M, \Omega_{\Lambda}) R_{SN} N_{KK} \times \int_2^{z_\gamma} dm \frac{dN_\gamma}{dm} \frac{dN_{KK}}{dm} [1 - e^{-t_{2z}/\tau}],$$

where $\epsilon' = (1 + z) \epsilon$ with $z$ the cosmic redshift, $t_2$ is the cosmic time corresponding to redshift $z$, and $\tau$ is the total KK lifetime which depends on the KK mass $m$. Notice that for $\tau \lesssim t_0$ this lifetime disappears from this equation.
because essentially all KK modes will have decayed at the present epoch. The factor $2/3$ arises because the radiative decay rate is $1/3$ of the total, and because 2 photons are produced per radiative decay.

Further, $dN_\gamma/d\epsilon$ is the normalized photon energy distribution from the decay of KK modes, $N_{KK}$ the total number of KK modes emitted in a given SN, and $dN_{KK}/dm$ their normalized mass spectrum. We will always use the limit of non-relativistic KK particles where $dN_\gamma/d\epsilon = \delta(\epsilon - m/2)$. This simplification is conservative because we calculate $N_{KK}$ on the basis of the average emitted energy $\langle E \rangle$ while the decay photons take each only half of the mass. On the other hand, we ignore the Lorentz factor for the decay rate. These two simplifications introduce small errors which go in opposite directions.

Finally, $F(z, \Omega_M, \Omega_\Lambda) = [\Omega_M(1+z)^5 + \Omega_\Lambda(1+z)^2]^{-1/2}$ with $\Omega_M$ and $\Omega_\Lambda$ being the relative contributions of matter and a cosmological term to the cosmic mass-energy density. We will use the standard values $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 75$ km s$^{-1}$ Mpc$^{-1}$, but our results do not depend much on the choice of background cosmology.

The present-day diffuse flux from the decaying KK modes obtained in this way is shown in Fig. 2 for $n = 2$ and 3, assuming a SN core temperature of 30 MeV and $f_{KK} = 1$, i.e. that all of the energy is emitted as KK gravitons. For comparison we show a fit to the diffuse $\gamma$-flux observed by the EGRET instrument, $j(\epsilon) = 2.26 \times 10^{-3}$ cm$^{-2}$ s$^{-1}$ ster$^{-1}$ MeV$^{-1}(\epsilon/$MeV$)^{-2.07}$ [18].

![FIG. 1. The differential emissivity of KK modes from a non-degenerate, non-relativistic nuclear medium with a temperature of 30 MeV. The dashed lines are $dQ/dm$ and the full lines are $dQ/dE$.](image1)

![FIG. 2. The present day photon flux due to KK mode decay. The dotted line is for $n = 2$ and the dashed for $n = 3$. The solid line shows the best fit to the flux observed by EGRET.](image2)

The predicted flux exceeds the observations by about a factor of 100–200 both for $n = 2$ and 3. Therefore, to avoid a conflict with the data we need to require

$$f_{KK} \lesssim 0.005 - 0.01. \tag{5}$$

This is our main result. The standard SN 1987A cooling limit amounts to the requirement $f_{KK} \lesssim 0.5$ [19] so that our new limit on the energy-loss rate is about two orders of magnitude more restrictive.

This bound is rather insensitive to the assumed temperature of the emitting medium. In Fig. 3 we show the maximum allowed value $f_{\text{max}}$ for $f_{KK}$ as a function of $T$. In the entire plausible range $T \gtrsim 15$ MeV the limit hardly depends on $T$. The reason is that for a larger temperature the average energy of the emitted KK states increases, leading to a decrease of their total number. In addition, the energy of the decay photons is distributed over a broader range of energies, further decreasing the differential flux. Therefore, the predicted photon flux is lower, but reaches to larger energies. On the other hand, the measured $\gamma$-ray flux falls approximately as $\epsilon^{-2}$, canceling the previous effects. When $T$ and thus the typical KK masses become too small, the KK lifetime begins to exceed the age of the universe, decreasing the predicted flux of decay photons.
that the cosmic SN rate and would allow us to improve our limit accordingly.

Finally, we stress that the limit we have derived (as well as all other astrophysical and cosmological limits) can be avoided in some models where compactification of the extra dimensions is non-toroidal [6], or for example in the model of Dvali et al. [20], where a Ricci term on the brane suppresses the graviton emission rates.

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5. Summary.—We have shown that the decay photons from KK gravitons emitted by all past SNe exceeds the measured diffuse $\gamma$-flux unless the fraction of the SN energy released in KK modes is less than about 0.5–1% of the total. This limit is very conservative because we have used the present-day SN rate as representative for the entire cosmic evolution. A more realistic assumption could lead to a photon flux of up to a factor of 10–100 larger than our estimate. Our limit on the fractional SN energy loss is at least two orders of magnitude more restrictive than the usual SN 1987A cooling limit. Of course, this latter argument applies to any invisible energy-loss channel while our new result depends on the relatively fast radiative decay of the KK modes.

Our new argument implies that if the number of extra dimensions $n$ is 2 or 3, their radius $R$ must be about a factor of 10 smaller than implied by the SN 1987A cooling limit. For $n = 2$ the SN 1987A limit is $R \leq 0.71 \times 10^{-3}$ mm [1], so that our limit becomes roughly $R \lesssim 0.9 \times 10^{-4}$ mm. Our limit translates into a lower bound on the energy scale of $M \geq 84$ TeV for $n = 2$. For $n = 3$ the cooling limit is $R \leq 0.85 \times 10^{-6}$ mm [2] while our new limit is $R \lesssim 0.19 \times 10^{-6}$ mm. The energy scale is then bounded by $M \geq 7$ TeV.

Our new bounds appear to be the most restrictive limits on large extra dimensions, except for limits which invoke potentially more uncertain early-universe arguments. Our argument does not depend on the low counting statistics of the SN 1987A neutrino signal, but shares all uncertainties related to calculating the KK graviton emission from a dense nuclear medium. If the relic neutrinos from all past SNe were to be observed in a neutrino observatory, such a measurement would pin down the cosmic SN rate and would allow us to improve our limit accordingly.

FIG. 3. Maximum allowed value for $f_{KK}$ as a function of the SN core temperature. The dotted line is for $n = 2$ and the dashed is for $n = 3$. [1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998) [hep-ph/9803317].
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