Toward discovering low-lying $P$-wave excited $\Sigma_c$ baryon states

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In this study, by combining the equal spacing rule with recent observations of $\Omega_c(X)$ and $\Xi_c(X)$ baryons, we predict the spectrum of the low-lying $\lambda$-mode $1P$-wave excited $\Sigma_c$ states. Furthermore, their strong decay properties are predicted using the chiral quark model and the nature of $\Sigma_c(2800)$ is investigated by analyzing the $\Lambda\pi$ invariant mass spectrum. The $\Sigma_c(2800)$ structure observed in the $\Lambda\pi$ mass spectrum was found to potentially arise from two overlapping $P$-wave $\Sigma_c$ resonances, $\Sigma_c(2813)/2^-$ and $\Sigma_c(2840)/5^-$. These resonances have similar decay widths of $\Gamma \sim 40$ MeV and predominantly decay into the $\Lambda\pi$ channel. The $\Sigma_c(2755)/1^-$ state is likely to be a very narrow state with a width of $\Gamma \sim 15$ MeV, with its decays almost saturated by the $\Lambda\pi$ channel. Additionally, evidence of the $\Sigma_c(2755)/1^-$ resonance as a very narrow peak may be seen in the $\Lambda\pi$ invariant mass spectrum. The other two $P$-wave states, $\Sigma_c(2746)/1^-$ and $\Sigma_c(2796)/1^-$, are relatively narrow states with similar widths of $\Gamma \sim 30$ MeV and predominantly decay into $\Sigma_c\pi$ and $\Sigma_c\pi$, respectively. We hope our study can provide useful references for discovering these low-lying $P$-wave states in forthcoming experiments.

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I. INTRODUCTION

Over the past several years, immense progress toward the observations of singly heavy baryons has been achieved at the LHC. In 2017, five extremely narrow $\Omega_c$ states, $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$, were observed in the $\Xi^0\Lambda$ channel by the LHCb Collaboration [1]. In 2018, the LHCb Collaboration observed a new bottom baryon, $\Xi_b(6227)^-$, in both $\Lambda_b^0K^-$ and $\Xi_b^0\pi^-$ decay modes [2] and two new resonances, $\Sigma_b(6097)^+$, in the $\Lambda_b^0\pi^+$ channels [3]. In 2020, the LHCb Collaboration also observed four new $\Omega_b(X)$ states, $\Omega_b(6316)^+$, $\Omega_b(6340)^-$, $\Omega_b(6340)^-$, and $\Omega_b(6350)^-$, in the $\Xi_b^0K^-$ mass spectrum [4]; three new $\Xi_b(X)$ states, $\Xi_b(2923)^0$, $\Xi_b(2939)^0$, and $\Xi_b(2965)^0$, in the $\Lambda_b^0K^-$ mass spectrum [5]; and a new $\Xi_b(6227)^0$ state in the $\Xi_b^0\pi^+$ channel [6]. Also in 2020, the CMS Collaboration observed a broad enhancement around 6070 MeV in the $\Lambda_b^0\pi^+\pi^-$ invariant mass spectrum [7], which was confirmed by subsequent LHCb experiments with high statistical significance [8]. More recently, the CMS Collaboration observed a new excited beauty strange baryon, $\Xi_b(6100)^-$, decaying to $\Xi_b^0\pi^+\pi^-$ [9].

These newly observed resonances provide opportunities for establishing an abundant singly heavy baryon spectrum. For a singly heavy baryon, there are two kinds of excitations, “$P$-mode” and “$\lambda$-mode”. The $P$-mode excitation appears within the light diquark, while the $\lambda$-mode excitation occurs between the light diquark and the heavy quark. In the heavy quark limit, $m_Q \to \infty$, no mixing occurs between the $\lambda$- and $P$-mode excitations due to a strong suppression of the spin-dependent interactions by the heavy quark mass $m_Q$ [10–12]. The $P$-mode excitation energy should be notably larger than that of the $\lambda$-mode [12–17], which can be understood using the simple harmonic oscillator model, as the frequency $\omega_P$ of the $P$-mode is larger than the frequency $\omega_\lambda$ of the $\lambda$-mode. The lower $\lambda$-mode excitation energy indicates that the $\lambda$-mode excitations should be more easily formed than $P$-mode excitations.

In the literature, the masses for the $1P$-wave $\lambda$-mode $\Omega_c$, $\Omega_b$, $\Xi_b$, $\Xi_b^0$, and $\Sigma_b$ baryon states are predicted to be $\sim 2.95 - 3.10$ GeV [10, 12, 18–32], $\sim 6.30 - 6.38$ GeV [10, 12, 28–37], $\sim 2.85 - 3.03$ GeV [10, 13, 24–31], $\sim 6.15 - 6.25$ GeV [10, 13, 28–31, 34], and $\sim 6.07 - 6.20$ GeV [10–12, 28–31, 34], respectively. The $1P$-wave $\rho$-mode states lie $70 - 150$ MeV above the $\lambda$-mode states according to the quark model predictions in Refs. [11–15]. It should be mentioned that the $\rho$-mode excitation energy calculated within QCD sum rules is slightly lower than that of the $\lambda$-mode in some other cases [25, 34, 38–43]. The newly observed singly heavy baryons, $\Omega_c(X)$, $\Omega_b(X)$, $\Xi_b(X)$, $\Xi_b^0(6227)^0$, and $\Sigma_b(6097)^+$, are just within the predicted mass ranges of the $1P$-wave excitations. Furthermore, prompted by the newly observed singly heavy baryon states and combined with mass spectrum, the strong decay properties have been studied using the QCD sum rules [38–45], $3P_0$ model [13, 19, 46–52], chiral quark model [16, 17, 53–56], and heavy quark effective theory [18, 57].

Based on the mass spectrum and strong decay analyses, the $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, and $\Omega_c(3119)$ structures may be explained with the $1P$-wave $\lambda$-mode $\Omega_c$ states [18–22], although there are different explanations about some of the resonances, such as $\Omega_c(3090)$ and $\Omega_c(3119)$, which may be explained with radially excited $(2S$-wave) $\Omega_c$ states [45, 56–58]. It should be mentioned that the recent LHCb measurements show that the spin assignment of the four observed states $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, and $\Omega_c(3090)$ consistent with $\lambda$-mode excitations with quantum numbers $J = 1/2, 3/2, 3/2$ and $5/2$ [59]. It is interesting to notice that among various models, only the predicted $JP$ quantum numbers in our previous work [56] are
consistent with the above-mentioned scenario as pointed in
the recent review of charmed baryon physics [60]. Similarly,
the new Ω_b(X) states, Ω_b(6316)−, Ω_b(6330)−, Ω_b(6340)−,
and Ω_b(6350)−, can be assigned to the 1P-wave λ-mode
Ω_b states [35–37, 46], although the Ω_b(6316)− may be a p-
mode excitation, as suggested in [43]. The new Ξ_c(X) states,
Ξ_c(2923)0, Ξ_c(2939)0, and Ξ_c(2965)0, are also good can-
didates for the 1P-wave λ-mode Ξ′_c states belonging to 6r, as
suggested in the literature [13, 38, 54], although different
explanations exist for some resonances, such as Ξ_c(2939)0
and Ξ_c(2965)0, which may be candidates of the 1P-wave ρ-
mode excitations [13], and Ξ_c(2965)0, which may be the J^P = 1/2+ Ξ(25) state [52, 61]. Additionally, the Σ_c(6097)+ and
Ξ_c(6227)0 resonances are good candidates for the 1P-wave
λ-mode singly bottom baryons [16, 41–44, 48–51, 53, 62].
Finally, some unconventional interpretations, such as molecular
or pentaquark, were also proposed in the literature for the
newly observed resonances, Ω_c(X) [63–74], Ω_c(X) [75],
Ξ_c(X) [76, 77], and Ξ_c(6227)− [78–81]. As a whole, for a fairly
complete λ-mode P-wave spectrum in the Ω_c, Ξ′_c, Ξ_c, Ω_c,
and Σ_c families may be established with discovery of the se-
ties of heavy baryons at the LHC. Based on our previous
work [16, 17, 53–56], we provide a quark model classification
of these newly observed resonances, summarized in Table I.

LHC experiments have demonstrated the capability for the
discovery of heavy baryons. Therefore, the missing λ-mode
P-wave Σ_c baryon states are likely to be discovered by for-
coming LHC experiments. The Σ_c mass spectrum has been
studied theoretically using various approaches, such as the rel-
ativized quark model [11], relativistic quark model [28, 29,
82], non-relativistic quark model [10–12, 24, 62, 83, 84],
lattice QCD [26, 27], QCD sum rules [25, 32, 85], and more.
Some quark model predictions of the masses for the λ-mode
S-wave and P-wave Σ_c states are collected in Table II [11, 28–30, 83, 84]. Using the heavy-quark-light-
diquark approximation, the masses of the λ-mode P-wave Σ_c
states in the relativistic quark model are predicted to be ap-
proximately 2.71–2.81 GeV [28, 29], which is consistent with
that of the non-relativistic quark model [83]. With the hyper-
central approximation, the λ-mode P-wave Σ_c states in the
non-relativistic quark model are predicted to be approximate-
ly 2.79–2.84 GeV [84]. By strictly solving the three
body problem without the diquark and hypercentral approxi-
mations, the masses of the λ-mode P-wave Σ_c states are pre-
dicted to be approximately 2.76–2.82 GeV and 2.80–2.84
GeV in the relativized quark model [11] and non-relativistic
quark model [12], respectively. The masses for the two ρ-
mode P-wave Σ_c states with J^P = 1/2− and 3/2− are predicted to be ≈ 2.85 – 2.91 GeV [11, 12], which are approximately 70
MeV larger than the highest λ-mode excitation. Considering
the mass, the Σ_c(2800) resonance [86] observed in the Λ_cπ
final states by the Belle and BABAR Collaborations [87, 88]
may be experimental signals of the P-wave Σ_c states. The
case of Σ_c(2800) as the ρ-mode P-wave excitations should be
excluded as the Λ_cπ decay channel is forbidden [89]. There
are some discussions on the nature of Σ_c(2800) in the lit-
erature [17, 25, 28, 31, 40, 60, 89–93]; however, these in-
volve strong model dependencies. For example, the spin-
parity (J^P) numbers were suggested to be J^P = 3/2− within
the heavy hadron chiral perturbation theory approach [60, 91],
J^P = 3/2− or J^P = 5/2− in the 3P0 model [90], and J^P = 1/2−
or 3/2− using the QCD sum rule approach [25, 40]. In our
previous study, it was found that Σ_c(2800) might favor the
J^P = 3/2− state [Σ_c 2P_{3/2}^−] or the J^P = 5/2− state [Σ_c 4P_{5/2}^−]
in the L-S coupling scheme [17].

In this study, we revisit the λ-mode P-wave Σ_c baryon
states. The main aims are as follows: (i) the spectrum
was classified in the L-S coupling scheme in our previous
work [17], where configuration mixing between two different
states with the same J^P numbers, which may be caused by
antisymmetric spin-orbit forces, is not considered. This con-
figuration mixing may notably affect some of our predictions,
thus, we include this effect here by adopting the j·j coupling
scheme. (ii) We hope to provide more reliable predictions for
the λ-mode P-wave Σ_c baryon states by combining the infor-
mation from the most recent observations of the Ξ_c(X) and
Ω_c(X) states.

This paper is organized as follows. In Sec. II, we pro-
vide a quark model classification of the singly heavy baryon
states and the mass analysis of the λ-mode 1P-wave Σ_c states
by incorporating the recent observations of the singly-heavy
baryons. Then, according to our chiral quark model calcula-
tions, their strong decay properties are discussed in Sec. III.
To determine the contributions of the P-wave Σ_c states to
the experimentally observed Σ_c(2800) resonance [86], we fur-
ther analyze the Λ_cπ invariant mass spectrum measured by
BABAR [88] in Sec. IV. Finally, we summarize our results in
Sec. V.

![FIG. 1: (Color online) q_1 q_2 Q_3 system with λ- or ρ-mode excitations.](image-url)

**II. MASS SPECTRUM ANALYSIS**

### A. Quark model classification

For a singly heavy baryon system q_1 q_2 Q_3, shown in Fig.1, it is convenient to introduce two Jacobi coordinates,

\[ \rho = \frac{1}{\sqrt{2}}(r_1 - r_2), \]

\[ \lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3), \]

where r_1 and r_2 are coordinates for the light quarks q_1 and
q_2, respectively, while r_3 is the coordinate for the heavy quark
TABLE I: Quark model classification of the newly observed singly heavy baryon resonances based on our previous work [17, 53–56]. This table is taken from [16].

| $|n^{((n+1)/2)}_{J^P}|$ | $|J^P, j\rangle$ (nl) | $\Omega_\epsilon$ states | $\Xi_\epsilon$ states | $\Sigma_\epsilon$ states | $\Omega_b$ states | $\Xi_b$ states | $\Sigma_b$ states |
|------------------------|---------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $|1^{-+}1_{J^P}\rangle$ | $|J^P = \frac{1}{2}^+, 1\rangle$ | $\Omega_\epsilon(2695)$ | $\Xi_\epsilon(2578)$ | $\Sigma_\epsilon(2455)$ | $\Omega_b(6046)$ | $\Xi_b(5935)$ | $\Sigma_b(5810)$ |
| $|1^{-+}2_{J^P}\rangle$ | $|J^P = \frac{1}{2}^-, 1\rangle$ | $\Omega_\epsilon(2770)$ | $\Xi_\epsilon(2645)$ | $\Sigma_\epsilon(2520)$ | $\Omega_b(6316)$ | $\Xi_b(5955)$ | $\Sigma_b(5830)$ |
| $|P_{1J}^1\rangle_1$ | $|J^P = \frac{1}{2}^-, 1\rangle$ (1P) | $\Omega_\epsilon(3000)$ | $\Xi_\epsilon(2880)$ | $\Omega_b(6300)$ | $\Xi_b(6227)$ | $\Sigma_b(6097)$ |
| $|P_{1J}^2\rangle_2$ | $|J^P = \frac{1}{2}^-, 2\rangle$ (1P) | $\Omega_\epsilon(3065)$ | $\Xi_\epsilon(2939)$ | $\Omega_b(6310)$ | $\Xi_b(6277)$ | $\Sigma_b(6067)$ |
| $|P_{1J}^3\rangle_3$ | $|J^P = \frac{1}{2}^-, 3\rangle$ (1P) | $\Omega_\epsilon(3065)$ | $\Xi_\epsilon(2939)$ | $\Omega_b(6310)$ | $\Xi_b(6277)$ | $\Sigma_b(6067)$ |

TABLE II: Predicted mass spectrum of 1S-wave and λ-mode 1P-wave $\Sigma_\epsilon$ states belonging to the $6_F$ multiplet in various quark models. The $\Sigma_\epsilon$ states are denoted by $|J^P, j\rangle$ in the $j$-$j$ coupling scheme, where $j$ stands for the total angular momentum quantum number of the two light quarks. The unit of mass is MeV.

| $|J^P, j\rangle$ | Ref. [28] | Ref. [29] | Ref. [12] | Ref. [11] | Ref. [83] | Ref. [84] | Observed state |
|----------------|----------|----------|----------|----------|----------|----------|----------------|
| $|J^P = \frac{1}{2}^-, 1\rangle$ | 2439 | 2443 | 2460 | 2440 | 2456 | 2452 | $\Sigma(2455)$ |
| $|J^P = \frac{1}{2}^-, 1\rangle$ | 2518 | 2519 | 2523 | 2495 | 2515 | 2501 | $\Sigma(2520)$ |
| $|J^P = \frac{1}{2}^-, 0\rangle$ | 2795 | 2713 | 2802 | 2765 | 2702 | 2832 | $\Sigma(2755)$ |
| $|J^P = \frac{1}{2}^-, 1\rangle$ | 2805 | 2799 | 2826 | 2770 | 2765 | 2841 | $\Sigma(2746)$ |
| $|J^P = \frac{1}{2}^-, 1\rangle$ | 2761 | 2773 | 2807 | 2770 | 2785 | 2812 | $\Sigma(2796)$ |
| $|J^P = \frac{1}{2}^- 2\rangle$ | 2799 | 2798 | 2837 | 2805 | 2798 | 2822 | $\Sigma(2813)$ |
| $|J^P = \frac{1}{2}^- 2\rangle$ | 2790 | 2789 | 2839 | 2815 | 2790 | 2790 | $\Sigma(2840)$ |

TABLE III: Classification of the λ-mode 1P-wave singly heavy baryon states belonging to $6_F$ in the $j$-$j$ coupling scheme. The states in the $j$-$j$ coupling scheme are denoted by $|J^P, j\rangle$.

| $|J^P, j\rangle$ | $J^P$ | $j$ | $\ell_\rho$ | $\ell_\rho$ | $L$ | $s_\rho$ | $s_\rho$ | $S$ |
|----------------|-------|-----|---------|---------|-----|-------|-------|-----|
| $|J^P = \frac{1}{2}^-, 0\rangle$ | $\frac{1}{2}$ | 0 | 1 | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $|J^P = \frac{1}{2}^-, 1\rangle$ | $\frac{1}{2}$ | 1 | 0 | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $|J^P = \frac{1}{2}^- 1\rangle$ | $\frac{1}{2}$ | 1 | 0 | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $|J^P = \frac{1}{2}^- 2\rangle$ | $\frac{1}{2}$ | 2 | 0 | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |

$Q_3$. The orbital/radial excitation appearing between the light quarks $q_1$ and $q_2$ with a Jacobi coordinate $\rho$ is denoted by "$p$-mode", while the excitation appearing between the light diquark $q_1q_2$ and heavy quark $Q_3$ with a Jacobi coordinate $\lambda$ is denoted by "$\lambda$-mode".

The heavy baryon containing a heavy quark violates the SU(4) symmetry. However, the SU(3) symmetry between the other two light quarks ($u$, $d$, or $s$) is approximately kept. According to this symmetry, heavy baryons containing a single heavy quark belong to two different SU(3) flavor representations: the symmetric sextet $6_F$ and antisymmetric antitriplet $3_F$ [17]. For charmed baryons $\Lambda_c$ and $\Xi_c$ belonging to $3_F$, the

FIG. 2: Mass spectra of the 1S and 1P-wave $\Omega_\epsilon$, $\Xi_\epsilon$, and $\Sigma_\epsilon$ baryon states belonging to $6_F$, predicted by combining the equal spacing rule with the recent observations of the $\Omega_\epsilon(X)$ and $\Xi_\epsilon(X)$ baryons.
antisymmetric flavor wave functions can be written as

\[
\phi_3^c = \begin{cases} 
\frac{1}{\sqrt{2}} (ud - du)c & \text{for } \Lambda_c^+, \\
\frac{1}{\sqrt{2}} (us - su)c & \text{for } \Xi_c^+, \\
\frac{1}{\sqrt{2}} (ds - sd)c & \text{for } \Omega_c^0.
\end{cases}
\]  

(3)

For the charmed baryons belonging to \(6_F\), the symmetric flavor wave functions can be written as

\[
\phi_6^c = \begin{cases} 
uc & \text{for } \Sigma^+_c, \\
\frac{1}{\sqrt{2}} (ud + du)c & \text{for } \Sigma^+_c, \\
ddc & \text{for } \Sigma^0_c, \\
\frac{1}{\sqrt{2}} (us + su)c & \text{for } \Xi^+_c, \\
\frac{1}{\sqrt{2}} (ds + sd)c & \text{for } \Xi^0_c, \\
ssc & \text{for } \Omega^0_c.
\end{cases}
\]  

(4)

Furthermore, the heavy-quark symmetry as an approximation is commonly adopted for the study of the singly heavy baryons. In the heavy-quark symmetry limit, the quark model states may favor the \(j-j\) coupling scheme [10]:

\[
| J^P, j \rangle = \left| \left( \ell_P \ell_3 \right)_L, s_P, s_Q \right\rangle_{J^P},
\]

(5)

where \(\ell_P\) and \(\ell_3\) correspond to the quantum numbers of the orbital angular momentum \(\ell_P\) within the light diquark and the orbital angular momentum \(\ell_3\) between the light diquark and the heavy quark, respectively; \(s_P\) and \(s_Q\) correspond to quantum numbers of the spins \(s_P\) and \(s_Q\) of the light diquark and heavy quark, respectively; \(L\) stands for the quantum number of the total orbital angular momentum \(L = \ell_P + \ell_3\); \(j\) is the quantum number of the total angular momentum \(j = \ell_P + \ell_3 + s_P\) of the light diquark, which is conserved in the heavy quark symmetry limit; and \(J\) is the quantum number of the total angular momentum \(J = j + s_Q\) of the heavy baryon system. The parity of the state is determined by \(P = (-1)^{\ell_P \ell_3 j}\). In the \(j-j\) coupling scheme, there are five \(\lambda\)-mode \(P\)-wave states belonging to \(6_F\): \(| J^P = \frac{1}{2}^-, 0 \rangle, | J^P = \frac{3}{2}^-, 1 \rangle, | J^P = \frac{1}{2}^+, 1 \rangle, | J^P = \frac{3}{2}^+, 2 \rangle\), and \(| J^P = \frac{1}{2}^+, 2 \rangle\). Their corresponding quantum numbers are displayed in Table III.

The states within the \(j-j\) coupling scheme are linear combinations of the configurations within the \(L-S\) coupling scheme, in which the quark model configurations are constructed by

\[
| ^{2S+1}L_J \rangle = \left| \left( \ell_P \ell_3 \right)_L, s_P, s_Q \right\rangle_{J^P},
\]

(6)

where \(S\) stands for the quantum number of the total spin angular momentum \(S = s_P + s_Q\). In the \(L-S\) coupling scheme, there are also five \(\lambda\)-mode \(P\)-wave states: \(| 1^2 P_{\frac{1}{2}}^\pm \rangle, | 1^2 P_{\frac{3}{2}}^\pm \rangle, | 1^4 P_{1}\rangle, | 1^4 P_{2}^\pm \rangle, \), and \(| 1^4 P_{2}^\pm \rangle\). The relationship between the \(j-j\) and \(L-S\) coupling schemes is given by [10]

\[
\left| \left( \ell_P \ell_3 \right)_L, s_P, s_Q \right\rangle_{J^P} = (-1)^{L+J+\frac{1}{2}} \sqrt{2J + 1} \sum_S \sqrt{2S + 1} \left\{ \begin{array}{ccc} L & s_P & j \\ S & J & S \end{array} \right\} \left| \left( \ell_P \ell_3 \right)_L, s_P, s_Q \right\rangle_{J^P}. \]

(7)

The heavy quark symmetry may suggest that there are configuration mixing between singly heavy baryons states with the same \(J^P\) numbers in the \(L-S\) coupling scheme. In the heavy quark limit, the mixing angles are determined by Eq. (7). The two \(J^P = 1/2^+\) states, \(| J^P = \frac{1}{2}^-, 0 \rangle\) and \(| J^P = \frac{1}{2}^-, 1 \rangle\), in the \(j-j\) scheme, are mixed states between \(| 1^2 P_{\frac{1}{2}}^- \rangle\) and \(| 1^4 P_{\frac{1}{2}}^\pm \rangle\) of the \(L-S\) coupling scheme with a mixing angle of \(\phi \approx 35^\circ\). The two \(J^P = 3/2^-\) states, \(| J^P = \frac{3}{2}^-, 1 \rangle\) and \(| J^P = \frac{3}{2}^-, 2 \rangle\), are mixed states via \(| 1^2 P_{\frac{3}{2}}^- \rangle\) and \(| 1^4 P_{\frac{3}{2}}^\pm \rangle\) mixing with a relatively small angle of \(\phi \approx 24^\circ\).

B. Mass analysis

For \(\Lambda_Q, \Sigma_Q\), and \(\Omega_Q(Q = c/b)\) systems containing two light quarks with an equal mass \(m_q\) and one heavy quark with a mass \(m_Q\), considering the simplified case of the harmonic oscillator potentials, the oscillator frequencies \(\omega_\lambda\) and \(\omega_\rho\) for the \(\lambda\)- and \(\rho\)-mode excitations satisfy the relation [12, 16, 17, 89]

\[
\frac{\omega_\lambda}{\omega_\rho} = \sqrt{\frac{1}{3} + \frac{2m_q}{3m_Q}}.
\]

(8)

This relation approximately holds for \(\Xi_Q\) \((Q = b, c)\) baryons as well, as the masses of \(m_{q, d}\) and \(m_c\) may be considered to be approximately equal in the SU(3) limit. From Eq. (8), when \(m_Q \gg m_q\), the \(\lambda\)-mode excited energy is smaller than that of the \(\rho\) mode, \(\omega_\lambda < \omega_\rho\). In the heavy quark limit, \(m_Q \to \infty\), there is no mixing between the \(\lambda\)- and \(\rho\)-mode excitations [10–12]. This is due to the spin-dependent interaction, which causes the mixing, being suppressed by a factor of \(1/m_Q\). For the singly charm and bottom baryons, the \(\lambda\) and \(\rho\)-modes are well separated [10, 12]. For example, Yoshida et al. showed the probability of the \(\lambda\)- and \(\rho\)- modes of \(J^P = 1/2^-\) for the lowest \(\Sigma_Q\) and \(\Lambda_Q\) as a function of the heavy quark mass \(m_Q\) in Fig. 10 of their article [12]. One sees that the lowest state is almost purely in the \(\lambda\) mode at \(m_Q \geq 1.5\) GeV. It should be mentioned that in the SU(3) limit, i.e., \(m_Q = m_q\), the excited energies of the \(\lambda\) and \(\rho\) modes degenerate, \(\omega_\lambda = \omega_\rho\), while the \(\lambda\) and \(\rho\) modes in the light baryon systems are largely mixed. The relatively small excitation energy of the \(\lambda\)-mode indicates that a \(\lambda\)-mode excitation should be more easily formed than a \(\rho\)-mode excitation. This may explain why most of the newly observed singly heavy baryons, \(\Omega_c(X), \Omega_b(X), \Xi_c(X), \Sigma_c(6097)\), and \(\Xi_b(6227)\), may favor the \(\lambda\)-mode excitations, as predicted in the literature [16, 17, 19, 38–40, 42–46, 48–56].

In the \(P\)-wave \(\Sigma_c\) states, there are two \(\rho\)-mode excitations: \(J^P = 1/2^-\) and \(J^P = 3/2^-\), and five \(\lambda\)-mode excitations: \( \Sigma_c| J^P = \frac{1}{2}^-, 0 \rangle, \Sigma_c| J^P = \frac{1}{2}^-, 1 \rangle, \Sigma_c| J^P = \frac{3}{2}^-, 1 \rangle, \Sigma_c| J^P = \frac{3}{2}^-, 2 \rangle, \) and \(\Sigma_c| J^P = \frac{3}{2}^-, 3 \rangle\). In this study, we focus on the \(\lambda\)-mode \(P\)-wave \(\Sigma_c\) baryon states. There is a high likelihood that they will be discovered in future experiments, as many \(\lambda\)-mode-like states have been observed in the \(\Omega_c, \Xi_c, \Xi'_c, \Omega_b, \) and \(\Sigma_c\) families in recent years. Many theoretical approaches, such as the relativized quark model [11], relativistic quark model [28, 29, 82], non-relativistic quark
model [10, 12, 24, 62, 83, 84], lattice QCD [26, 27], QCD sum rules [25, 32, 85], and more, have been adopted in the literature to calculate the mass spectrum. In Table II, we display some masses for the $\lambda$-mode $P$-wave $\Sigma$ states predicted within the heavy-quark-light-diquark approximation in both the relativistic quark model [28, 29] and non-relativistic quark model [83], using the hypercentral approximation in the non-relativistic quark model [84] and by strictly solving the three body problem in relativized quark model [11] and non-relativistic quark model [12]. The masses predicted using various approaches using different approximations are comparable to each other at $\sim 2.71 - 2.84$ GeV. There is an obvious gap between the $\rho$- and $\lambda$-mode $P$-wave $\Sigma$ states. The masses for the two $\rho$-mode $P$-wave $\Sigma$ states with $J^P = 1/2^-$ and $3/2^-$ are predicted to be $\sim 2.85 - 2.91$ GeV [11, 12], which are $\sim 70$ MeV larger than the highest $\lambda$-mode excitation.

The $\Sigma_c(2800)$ structure observed in experiments [87, 88] is just within the predicted mass range of $\lambda$-mode $1P$-wave excitations. However, the predicted mass order and mass splitting between the $P$-wave spin multiplets is different for various models. Fortunately, one can determine the mass order and mass splitting for the $1P$-wave $\Sigma$ states from the newly observed $\Xi_c(X)$ and $\Omega_c(X)$ states. In our previous work [56], the $\Omega_c(3000), \Omega_c(3050), \Omega_c(3065)$, and $\Omega_c(3090)$ resonances were predicted to be the $\lambda$-mode $1P$-wave states with $J^P = 1/2^+, J^P = 3/2^+, J^P = 3/2^-$, and $J^P = 5/2^-$, respectively. The newly observed states, $\Xi_c(2923), \Xi_c(2939)$, and $\Xi_c(2965)$, could be the flavor partners of $\Omega_c(3050), \Omega_c(3065)$, and $\Omega_c(3090)$, respectively [16, 54, 56].

The equal spacing rule [94, 95] perfectly holds for the newly observed $\Xi_c(X)$ and $\Omega_c(X)$ states, i.e.,

$$m[\Omega_c(3050)] - m[\Xi_c(2923)] \approx m[\Omega_c(3066)] - m[\Xi_c(2939)] \approx m[\Omega_c(3090)] - m[\Xi_c(2965)] \approx 125$ MeV.$$

The equal spacing rule also perfectly holds for the $J^P = 3/2^+$ charmed ground states:

$$m[\Omega_c(2770)] - m[\Xi_c(2645)] \approx m[\Xi_c(2645)] - m[\Sigma_c(2520)] \approx 125$ MeV.$$

For the $J^P = 1/2^-$ charmed ground states, the equal spacing rule also holds: $m[\Omega_c] - m[\Xi_c] \approx m[\Xi_c] - m[\Sigma_c] \approx 120$ MeV. Thus, the equal spacing rule is potentially universal for the charmed baryon states. Based on this, we predict that for the charmed baryon sector, the flavor partners of the four $\Omega_c(X)$ states, $\Omega_c(3000), \Omega_c(3050), \Omega_c(3065)$, and $\Omega_c(3090)$, are likely to be $\Omega_c(3000)$: $\Xi_c(2873), \Sigma_c(2746)$, $\Omega_c(3050)$: $\Xi_c(2923), \Sigma_c(2796)$, $\Omega_c(3065)$: $\Xi_c(2939), \Sigma_c(2813)$, $\Omega_c(3090)$: $\Xi_c(2965), \Sigma_c(2840)$, respectively. The states labeled with “?” are yet to be discovered by current experiments. Finally, the equal spacing rule can be further confirmed using other experimentally observed baryon and meson states. For example, the equal spacing rule holds:

(i) for the $J^P = 3/2^+$ light ground baryon states,

$$m[\Omega(1672)] - m[\Xi(1530)] \approx m[\Xi(1530)] - m[\Sigma(1385)] \approx m[\Sigma(1385)] - m[\Delta(1232)] \approx 145$ MeV;

(ii) for the well-established light unflavored $m\bar{m}$ ($n = u, d$) and $s\bar{s}$ meson states,

$$m[\phi(1020)] - m[\omega(782)] \approx m[h_1(1380)] - m[h_1(1170)] \approx m[f_2(1270)] - m[f_2(1520)] \approx 240$ MeV;

(iii) and for the $D$ and $D_s$ meson states,

$$m[D_s(1680)] - m[D(1865)] \approx m[D(2112)] - m[D^*(2070)] \approx m[D_s(2536)] - m[D_s(2420)] \approx m[D_s(2573)] - m[D_s(2460)] \approx 105$ MeV.$$

Less is known about the $P$-wave state $|J^P = 1/2^-, 0\rangle$ with the only hint coming from the recent LHCb experiments with a broad structure $\Xi_c(2880)$ found in the $\Lambda_c^+ K^-$ mass spectrum with a small significance [5]. This $\Xi_c(2880)$ may be a candidate for $|J^P = 1/2^-, 0\rangle$ in the $\Xi'_c$ family. Thus, according to the equal spacing rule [94, 95], the masses of the $\Sigma_c J^P = 1/2^-, 0\rangle$ and $\Omega_c J^P = 1/2^-, 0\rangle$ state are predicted to be 2755 and 3005 MeV, respectively. The two $P$-wave $J^P = 1/2^-$ states, $|J^P = 1/2^-, 0\rangle$ and $|J^P = 1/2^-, 1\rangle$, may be largely overlapping. It is worth mentioning that for the $\lambda$-mode $P$-wave states there is no place for $\Omega_c(3119)$, which was observed by LHCb [1]. The $\Omega_c(3119)$ may be a candidate for the $\lambda$-mode $2S$ states with $J^P = 1/2^+$ or $J^P = 3/2^-$ [56]. Combining this assumption with the equal spacing rule, the masses of the $\lambda$-mode $2S$ state of $\Sigma_c$ and $\Xi_c$ are predicted to be approximately 2869 and 2994 MeV, respectively.

By combining the precise experimental data of $\Omega_c(X)$ and $\Xi_c(X)$ with the equal spacing rule [94, 95], we can determine the masses for the $\Sigma_c(X)$ states model-independently as corresponding flavor partners of $\Omega_c(X)$ and $\Xi_c(X)$. Then, according to our previous analysis of the strong decays and masses, the newly observed resonances $\Omega_c(3000), \Omega_c(3050), \Omega_c(3065), \Omega_c(3090)$ can be naturally explained as the $\lambda$-mode $1P$-wave states with $J^P = 1/2^-, J^P = 3/2^-, J^P = 3/2^-$, and $J^P = 5/2^-$, respectively [16, 17, 54, 56]. Finally, combining the masses determined from the equal spacing rule with possible configuration arrangements based on our previous analysis, we predict a mass spectrum for the $\lambda$-mode $P$-wave $\Sigma_c$ states, which are summarized in Table II. For clarity and comparison, we also plot the mass spectra of the $\lambda$-mode $P$-wave $\Omega_c$, $\Xi_c$, and $\Sigma_c$ states in Fig. 2. Finally, it should be emphasized that the equal spacing rule perfectly holds for the $1S$-wave ground charmed baryon states $\Sigma_c^{(*)}, \Xi_c^{(*)}$, and $\Omega_c^{(*)}$, which is illustrated in Fig. 2. Considering the newly observed $\Omega_c(X)$ and $\Xi_c(X)$ resonances as the
$\lambda$-mode $1P$-wave states, from Fig. 2 we observe that the equal spacing rule also holds for these higher excitations. This indicates that the equal spacing rule should hold for all $1P$-wave charmed states. Thus, combining the equal spacing rule with the observations, a fairly reliable and precise prediction of the $P$-wave $\Sigma_c$ states in a certain mass region can be obtained, as displayed in Fig. 2. The masses obtained with the equal spacing rule are comparable with the quark model predictions [11, 12, 28, 29, 83], although a precise prediction cannot be obtained by any quark models due to their typical uncertainties of approximately 10s to 100 MeV.

TABLE IV: Strong decay properties of the $\lambda$-mode $P$-wave $\Sigma_c$ states. The $\Sigma_c$ states are denoted by $[J^P, j]$ in the $j$-$j$ coupling scheme. The units of the partial widths $\Gamma_i$ and masses of the resonances are both MeV.

| State          | $[J^P, j]$ | Channel | $\Gamma_i$ | $B_i$ |
|----------------|------------|---------|------------|-------|
| $\Sigma_c(2746)$ | $[1/2, 1]$ | $\Lambda_c \pi$ | 30.59 | 75.42% |
| $\Sigma_c(2755)$ | $[1/2, 0]$ | $\Sigma_c \pi$ | 7.53 | 18.57% |
| $\Sigma_c(2796)$ | $[3/2, 1]$ | $\Lambda_c \pi$ | 37.63 | 73.96% |

III. STRONG DECAY ANALYSIS

A. The model

Combining with the masses of the $P$-wave $\Sigma_c$ states, the decay properties can provide crucial references in searching for them in future experiments. In this study, we reinvestigate the strong decay properties of $1P$-wave $\Sigma_c$ baryons with the chiral quark model [96], which has been successfully applied to the strong decays of heavy-light mesons, and charmed and strange baryons [14, 16, 17, 53–56, 89, 97–106]. In this model, the nonrelativistic transition operator for a strong decay process by emitting a pseudoscalar meson is adopted as [107–109]

$$H_m^{\mu\nu} = \sum_j \frac{\omega_m}{E_j + M_f} \omega_j \cdot P_f + \frac{\omega_m}{E_i + M_i} \omega_j \cdot P_i$$

$$-\sigma_j \cdot q + \frac{\omega_m}{2\mu_q} \sigma_j \cdot p_j' \Gamma e^{-i q \cdot r},$$

where $(E_i, M_i, P_i)$ and $(E_f, M_f, P_f)$ denote the energy, mass, and three-vector momentum of the initial and final baryons, respectively; $\omega_m$ and $q$ represent the energy and three-vector momentum of the final light pseudoscalar meson; $\sigma_j$ is the Pauli spin vector on the $j$th quark; $p_j'$ is the momentum of the final light pseudoscalar meson; $\mu_q$ is a reduced mass expressed as $1/\mu_q = 1/m_j + 1/m'_j$. The isospin operator $I$ associated with $\pi$ mesons has been defined in Refs. [107–109]. Using the wave functions for the initial and final baryon states, the transition amplitude $M$ of a decay process can be determined. For simplicity, harmonic oscillator wave functions are adopted for the initial and final baryon states in our calculations.

The partial decay widths for a decay process can be calculated using [89, 98]

$$\Gamma = \frac{\delta}{2 f_m} \frac{(E_f + M_f) |q|}{4 \pi M_i} \frac{1}{2J_i + 1} \sum_{J_0, J_f} |M_{J_0, J_f}|^2,$$

where $J_{0, f}$ are the third components of the total angular momenta of the initial and final baryons, respectively, and $\delta$ is a global parameter accounting for the strength of the quark-meson couplings. Here, we fix its value to the same as that in Refs. [89, 98], i.e., $\delta = 0.557$.

For the calculations, the masses of well-established hadrons were taken from the Particle Data Group [86] and the standard quark model parameters have been determined previously [17]. For the harmonic oscillator space-wave functions, $\Psi_m = R_{m_\alpha} Y_{l m_\alpha}$, the harmonic oscillator parameter $\alpha_p$ in the wave functions for a $uu/ud/dd$ system is taken as $\alpha_p = 400$ MeV. Another harmonic oscillator parameter $\alpha_c$ is related to $\alpha_p$ by $\alpha_c = [3m_c/(2m_u + m_c)]1/4\alpha_p$, where $m_q$ and $m_c$ denote the light $u/d$ quark mass and heavy charmed $c$ quark mass, respectively. Here, we take $m_q = 330$ MeV and $m_c = 1480$ MeV.

B. Results and discussions

Using the predicted masses of the $\lambda$-mode $P$-wave $\Sigma_c$ states from the mass spectrum analysis in the above section, their strong decay properties are calculated using the chiral quark model. The predicted results are display in Table IV.

The two $j = 1$ states, $\Sigma_c(2746)_{[1/2, 1]}$ and $\Sigma_c(2796)_{[3/2, 1]}$, should have very small decay rates into the $\Lambda_c\pi$ final state, as this decay mode is forbidden in the heavy quark symmetry limit. The $\Sigma_c(2746)_{[3/2, 1]}$ and $\Sigma_c(2796)_{[3/2, 1]}$ states have a comparable width of $\Gamma \approx 30$ MeV, and predominantly decay into $\Sigma_c\pi$ and $\Sigma_c\pi$, respectively. To establish these two states, the $\Lambda_c\pi\pi$
\begin{equation}
(\Sigma^p \pi \rightarrow \Lambda_c \pi \pi) \text{ invariant mass spectrum should be observed in future experiments.}
\end{equation}

The two $j = 2$ states, $\Sigma_c(2813)|J^P = \frac{3}{2}^-, 2)$ and $\Sigma_c(2840)|J^P = \frac{5}{2}^-, 2)$, predominantly decay into the $\Lambda_c \pi$ channel with a comparable branching fraction of $\sim 75\%$. Additionally, they have relatively large decay rates into $\Sigma_c \pi$ and $\Sigma_c^* \pi$ final states, with branching fractions of $\sim 10\%$. The decay widths for $\Sigma_c(2813)|J^P = \frac{3}{2}^-, 2)$ and $\Sigma_c(2840)|J^P = \frac{5}{2}^-, 2)$ states are predicted to be $\Gamma \approx 41$ and $51$ MeV, respectively. These two states might contribute to the $\Sigma_c \pi$ invariant mass spectrum. It may be difficult to distinguish the $\Sigma_c(2813)|J^P = \frac{3}{2}^-, 2)$ and $\Sigma_c(2840)|J^P = \frac{5}{2}^-, 2)$ states from the $\Lambda_c \pi$ invariant mass spectrum due to their highly overlapping masses, which will be discussed later. Fortunately, as these two highly overlapping states have different $J^P$ numbers, they may be separated by measuring the angular distributions.

The $j = 0$ state, $\Sigma_c(2755)|J^P = \frac{1}{2}^-, 0)$, may be very narrow state with a width of $\Gamma \approx 15$ MeV. Its decays are likely saturated by the $\Lambda_c \pi$ channel, and the $\Sigma_c \pi$ and $\Sigma_c^* \pi$ decay modes are forbidden in the heavy quark symmetry limit. It might be interesting to search for this narrow state in the $\Lambda_c \pi$ channel.

Considering the case that the masses of the $\lambda$-mode $P$-wave $\Sigma_c$ states may be out of our predictions, in Fig. 3 we plot the strong decay properties as functions of the mass within a possible region. The sensitivities of the decay properties to the mass can be clearly seen from the figure. The uncertainties of the mass for the $\lambda$-mode $P$-wave $\Sigma_c$ states cannot affect our main conclusion.

Finally, the strong decay properties of the two $\rho$-mode $1P$-wave $\Sigma_c$ states with $J^P = \frac{1}{2}^+$ and $J^P = \frac{5}{2}^-$ are determined in this study. The $\rho$-mode $1P$-wave states do not overlap with the $\lambda$-mode $1P$-wave states according to the quark model predictions [11, 12]. Their masses were predicted to be $M = 2909$ and $2910$ MeV in a recent study [12], which is approximately $70$ MeV larger than that of the highest $\lambda$-mode $P$-wave state, $\Sigma_c(2840)|J^P = \frac{5}{2}^-, 2)$. Additionally, the $\Lambda_c \pi$ decay mode is forbidden for the two $\rho$-mode $1P$-wave $\Sigma_c$ states. Taking the mass predicted in Ref. [12], we find the $J^P = \frac{1}{2}^-$ $\rho$-mode state has a width of $\Gamma \approx 78$ MeV, and predominantly decays into $\Sigma_c \pi$ and $\Sigma_c^* \pi$ with branching fractions of $\sim 65\%$ and $\sim 35\%$, respectively, while the $J^P = \frac{5}{2}^-$ state is relatively broad, with a width of $\Gamma \approx 90$ MeV, and predominantly decays into $\Sigma_c \pi$ and $\Sigma_c^* \pi$ with branching fractions of $\sim 25\%$ and $\sim 75\%$, respectively.

IV. INVARIANT MASS SPECTRUM ANALYSIS

To determine the contributions of the $P$-wave $\Sigma_c$ states to the experimentally observed $\Sigma_c(2800)$ structure, we further analyze the $\Lambda_c \pi$ invariant mass spectrum measured by BABAR [88]. In our analysis, we adopt a relativistic Breit-Wigner function to describe the event distribution:

\begin{equation}
\frac{dN}{dm} \propto f + \sum_R \frac{C_R \Gamma_R(m) \sqrt{\Phi_R(m)}}{m^2 - m_R^2 + i m_R \Gamma_R(m)}^2,
\end{equation}

where $m$ and $m_R$ are the invariant mass and the resonance mass, respectively, $\Gamma_R(m)$ is the resonance strong decay amplitude for the $\Lambda_c \pi$ channel, and $\Gamma_R(m)$ denotes the energy-dependent total decay width of a resonance. The decay width of resonance is considered to be saturated by the two-body OZI-allowed strong decay modes. Thus, as an approximation, $\Gamma_R(m)$ is a sum of the energy-dependent partial widths of all two-body OZI-allowed strong decay modes, calculated using the strong decay amplitudes extracted from our chiral quark model. Additionally, $\Phi_R(m)$ is strong decay phase space and $f$ represents the background contributions. In this study, a linear background, $f = \sqrt{a + bm}$, is adopted, where $a \approx 248.0$ MeV and $b \approx -67.0$, which were determined by fitting the background taken in Ref. [88]. Finally, $|C_R|$ is a free parameter set related to the resonance production rates.

According to our strong decay analysis, three $P$-wave $\Sigma_c$ states, $\Sigma_c(2755)|J^P = \frac{1}{2}^-, 0)$, $\Sigma_c(2813)|J^P = \frac{3}{2}^-, 2)$ and $\Sigma_c(2840)|J^P = \frac{5}{2}^-, 2)$, predominantly decay into the $\Lambda_c \pi$ channel. Due to the unknown production rates of these resonances, seven cases with different relative ratios, $C_{\Sigma_c(2755)}$ : $C_{\Sigma_c(2813)}$ : $C_{\Sigma_c(2840)}$ ratio = 1 : 1 : 1 : 0 : 0 : 0 : 1, are considered to reproduce the $\Lambda_c \pi$ invariant mass spectrum measured by BABAR with our predicted strong decay properties.
for the three resonances [88]. Our results are displayed in Figs. 4 (a)-(g), respectively. Assuming the three resonances with $J^P = 1/2^-, 3/2^-, 5/2^-$ have comparable production rates as displayed in Figs. 4 (a), (d), (e), and (f), the measured $\Lambda_c \pi$ invariant mass spectrum can be described. In these cases, $\Sigma_c(2755)/J^P = 1/2^+$, with $0$ value, would contribute a very narrow peak to the invariant mass spectrum, while the interferences between $\Sigma_c(2813)/J^P = 3/2^-$, $\Sigma_c(2840)/J^P = 5/2^-$ contribute to the main broad peak at approximately $2.8$ GeV. The invariant mass spectrum may also be explained with the two state interferences between $\Sigma_c(2813)/J^P = 3/2^-$, $2$ and $\Sigma_c(2840)/J^P = 5/2^-$, $2$ [see Fig. 4 (g)]. However, due to the large uncertainties, the present data cannot exclude the possibility that the $\Sigma_c(2800)$ structure is caused by a single resonance $\Sigma_c(2813)/J^P = 3/2^-$, $2$ or $\Sigma_c(2840)/J^P = 5/2^-$, $2$ [see Figs. 4 (b) and (c)]. The measurements of the angular distributions are required to separate these two overlapping states.

According to our analysis of the $\Lambda_c \pi$ invariant mass spectrum, the $\Sigma_c(2800)$ structure may be caused by two largely overlapping resonances, $\Sigma_c(2813)/J^P = 3/2^-$, $2$ and $\Sigma_c(2840)/J^P = 5/2^-$, $2$, although the explanation with only one single resonance cannot be excluded. Evidence of the $\Sigma_c(2755)/J^P = 1/2^+$, $0$ resonance, as a very narrow peak, may be seen in the $\Lambda_c \pi$ invariant mass spectrum. More accurate measurements of the $\Lambda_c \pi$ invariant mass spectrum along with the partial wave analysis of the measured angular distributions are crucial for establishing $\Sigma_c(2755)/J^P = 1/2^+$, $0$, $\Sigma_c(2813)/J^P = 3/2^-$, $2$, and $\Sigma_c(2840)/J^P = 5/2^-$, $2$.

V. SUMMARY

In this study, by employing the equal spacing rule, the newly observed $\Xi_c(2923)^0$, $\Xi_c(2939)^+$, and $\Xi_c(2965)^+$ states appear to be flavor partners of $\Omega_c(3050)$, $\Omega_c(3066)$, and $\Omega_c(3090)$, respectively. As the flavor partners of four $P$-wave candidates, $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, and $\Omega_c(3090)$, as suggested in the literature, four $P$-wave $\Sigma_c$ baryon states, $\Sigma_c(2746)$, $\Sigma_c(2796)$, $\Sigma_c(2813)$, and $\Sigma_c(2840)$, are predicted using the equal spacing rule. According to our assignments for the $\Omega_c(X)$ states, $\Sigma_c(2746)$, $\Sigma_c(2796)$, $\Sigma_c(2813)$, and $\Sigma_c(2840)$, may correspond to the $\lambda$-mode $P$-wave states $\Sigma_c/J^P = 1/2^+$, $1$, $\Sigma_c/J^P = 3/2^-$, $1$, $\Sigma_c/J^P = 3/2^-$, $2$, and $\Sigma_c/J^P = 5/2^-$, $2$, respectively, in the heavy quark symmetry limit.

Furthermore, their strong decay properties are predicted using the chiral quark model. It is found that these $1P$-wave $\Sigma_c$ states have relatively narrow widths within the range of $\sim 15$–$50$ MeV. The $\Sigma_c(2813)/J^P = 3/2^-$, $2$ and $\Sigma_c(2840)/J^P = 5/2^-$, $2$ states have comparable decay widths of $\Gamma \sim 40$ MeV and predominantly decay into the $\Lambda_c \pi$ channel. The $\Sigma_c/J^P = 1/2^+$, $0$ state may be a very narrow state with a width of $\Gamma \sim 15$ MeV, and its decays are nearly saturated by the $\Lambda_c \pi$ channel. The $\Sigma_c(2755)/J^P = 1/2^+$, $0$, $\Sigma_c(2813)/J^P = 3/2^-$, $2$, and $\Sigma_c(2840)/J^P = 5/2^-$, $2$ states may be established in the $\Lambda_c \pi$ invariant mass spectrum with more accurate measurements and angular distribution analysis in future experiments. The other two $P$-wave states, $\Sigma_c(2746)/J^P = 1/2^+$, $1$ and $\Sigma_c(2796)/J^P = 3/2^-$, $1$, are relatively narrow states with comparable widths of $\Gamma \sim 30$ MeV, and they mainly decay into $\Sigma_c$ and $\Sigma_c\pi$, respectively. To establish the existence of these two states, the $\Lambda_c \pi$ ($\Sigma_c^{(8)} \pi \to \Lambda_c \pi\pi$) invariant mass spectrum is worth observing in future experiments.

Then, using our predicted decay amplitudes, the $\Lambda_c \pi$ invariant mass spectrum measured by $BABAR$ is further analyzed, which improves our understanding of the nature of $\Sigma_c(2800)$.
It is found that the $\Sigma_{c}(2800)$ structure can be explained with two largely overlapping resonances, $\Sigma_{c}(2813)|J^{P} = \frac{3}{2}^{-}, 2\rangle$ and $\Sigma_{c}(2840)|J^{P} = \frac{3}{2}^{-}, 2\rangle$, although the explanation with only one single resonance cannot be excluded. If the production rate of $\Sigma_{c}(2813)|J^{P} = 1/2^{-}, 0\rangle$ is comparable with that of $\Sigma_{c}(2813)|J^{P} = 3/2^{-}, 2\rangle$ and $\Sigma_{c}(2840)|J^{P} = 5/2^{-}, 2\rangle$, a narrow peak will be observed in the $\Lambda_{c}\pi$ mass spectrum at approximately 2.8 GeV.

Finally, the equal spacing rule appears to perfectly hold for the charmed baryon states. The mass spectrum of the four $P$-wave states, $\Sigma_{c}(2746)$, $\Sigma_{c}(2796)$, $\Sigma_{c}(2813)$, and $\Sigma_{c}(2840)$, is approximately extracted model-independently by combining the recent observations of the charmed baryon, $\Omega_{c}(X)$ and $\Xi_{c}(X)$, with the equal spacing rule. The extracted masses are in the range of the quark model predictions, and should be more precise than the quark model predictions due to the highly precise measurements from the LHCb Collaboration. A reliable determination of the mass spectrum alongside our detailed analysis of the decay properties for these $P$-wave $\Sigma_{c}$ states should provide useful references for observations in future experiments.

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