Nucleosynthesis of elements in gamma ray bursts engines

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ABSTRACT

Aims. We consider the gamma ray bursts central engine powered by the collapse of a massive rotating star or compact binary merger. The engine is a hot and dense accretion disk, composed of free nucleons, electron-positron pairs, and Helium, and cooled by neutrino emission. Significant number density of neutrons in the inner disk body will provide conditions for neutron rich plasma in the GRB outflows or jets. The extreme energetics of the observed bursts, detectable from cosmological distances, implies that they must be connected with gravitational potential energy released by accretion onto a compact star. The typical mass of the accreting, newly born black hole is therefore on the order of a few Solar masses, while the accretion rates might exceed 1 M⊙s⁻¹.

Methods. The GRB central engine is modelled hydrodynamically in the frame of a dense and hot disk which accretes with high rate (up to 1 Solar mass per second) onto a maximally spinning, stellar mass black hole. The synthesis of heavy nuclei up to Germanium and Gallium is then followed by the nuclear reaction network.

Results. The accretion at high rate onto a Kerr black hole feeds the engine activity and establishes conditions for efficient synthesis of heavy nuclei in the disk. These processes may have important observational implications for the jet deceleration process and heavy elements observed in the spectra of GRB afterglows.

Key words. accretion, accretion disks; black hole physics; gamma ray burst; nuclear reactions, nucleosynthesis, abundances

1. Introduction

Gamma ray bursts are transient sources of extreme brightness observed on the sky with isotropic distribution. The extreme energetics of the observed bursts detectable from cosmological distances implies that they must be connected with gravitational potential energy released by accretion onto a compact star. The short bursts (T90 < 2 s) are likely related with mergers of binary compact star systems, such as binary neutron stars (NS-NS) or black hole-neutron star (BH-NS) binaries, while the long duration events are believed to originate from collapsing massive stars. The typical mass of the accreting, newly born black hole is therefore on the order of a few Solar masses, while the accretion rates might exceed 1 M⊙s⁻¹.

In the collapsar model, the black hole surrounded by the part of the fallback stellar envelope helps launching relativistic jets (Woosley 1993, MacFadyen & Woosley 1999). These polar jets give rise to the gamma rays, produced far away from the ‘engine’ in the circumstellar region (see e.g., the reviews by Zhang & Mészáros 2004, Piran 2004).

The process of nucleosynthesis of heavy elements in the central engines of Gamma Ray Bursts has recently been studied in a number of works. The evolution of abundances calculated by Banerjee & Mukhopadhyay (2013) shown that synthesis of rare elements, such as 31P, 39K, 43Sc and 35Cl and other uncommon isotopes. These elements, produced in the simulations at outer parts of low M accretion disks (i.e. 0.001-0.01 M⊙s⁻¹), have been discovered in the emission lines of some long GRB X-ray afterglows, however are yet to be confirmed by future observations.

In this article, we consider the nucleosynthesis of elements in the accretion disk itself for higher initial accretion rates, as appropriate for type I collapsars or neutron star mergers and short GRBs. We account for the neutrino opacities and nuclear equation of state in dense matter. For the accretion disk model, we use the equation of state introduced in Janiuk et al. (2007) and subsequently adopted in Janiuk & Yuan (2010) to describe the disk around a Kerr black hole, with arbitrary spin a. Contrary to the previous work, which analytically accounted for the total pressure that consists of gas, radiation and completely degenerate electrons (Popham et al. 1999), with subsequent addition of the neutrino pressure (Di Matteo et al. 2002), we compute the EOS of matter numerically, from the nuclear reaction balance. Our approach allows for a partial degeneracy of all the species, as well as partial trapping of neutrinos, as well as Kerr black hole solutions.

We calculate the profiles of density and temperature, as well as electron fraction in the converged steady state model of an accreting torus in the GRB engine. We then follow the nucleosynthesis process and we determine the abundances of the heavy elements isotopes.

2. Neutronisation in the hyperdense matter

The central engines of GRBs are dense and hot enough to allow for the nuclear processes leading to neutron excess (i.e., neutron to proton number density ratio above 1). The reactions of electron and positron capture on nucleons and neutron decay must...
establish nuclear equilibrium. These reactions are:

\[ p + e^- \rightarrow n + \nu_e \]
\[ p + \bar{\nu}_e \rightarrow n + e^+ \]
\[ p + e^- + \bar{\nu}_e \rightarrow n \]
\[ n + e^+ + \nu_e \rightarrow p + \nu_e \]
\[ n + \nu_e \rightarrow p + e^- \]

(1)

The ratio of protons to nucleons must satisfy the balance between their number densities and reaction rates:

\[ n_p (\Gamma_{p+e^-\rightarrow n+\nu_e} + \Gamma_{p+\nu_e\rightarrow n+e^-} + \Gamma_{p+e^-+\bar{\nu}_e\rightarrow n}) = n_0 (\Gamma_{n+e^-\rightarrow p+\nu_e} + \Gamma_{n+p+e^-+\nu_e}\rightarrow p+e^- + \bar{\nu}_e) \]

(2)

The reaction rates are the sum of forward and backward rates and are given by the following formulae (Reddy et al. 1998; Kohri et al. 2005):

\[ \Gamma_{p+e^-\rightarrow n+\nu_e} = \frac{1}{2\pi^2} |M|^2 \int_q^\infty dE_eE_e^2p_e(Q - E_e)^2f_e(1 - b_sf_{\nu_e}), \]
\[ \Gamma_{p+\nu_e\rightarrow n+e^-} = \frac{1}{2\pi^2} |M|^2 \int_q^\infty dE_eE_e^2p_e(Q - E_e)^2f_e(1 - b_sf_{\nu_e}), \]
\[ \Gamma_{n+e^-\rightarrow p+\nu_e} = \frac{1}{2\pi^2} |M|^2 \int_q^\infty dE_eE_e^2p_e(Q - E_e)^2f_e(1 - b_sf_{\nu_e}), \]
\[ \Gamma_{n+p+e^-+\nu_e}\rightarrow p+e^- + \bar{\nu}_e = \frac{1}{2\pi^2} |M|^2 \int_q^\infty dE_eE_e^2p_e(Q - E_e)^2f_e(1 - b_sf_{\nu_e}). \]

Here \( Q = (m_p - m_p)c^2 \) is the (positive) difference between neutron and proton masses and \( |M|^2 \) is the averaged transition rate which depends on the initial and final states of all participating particles. For nonrelativistic and noninteracting nucleons, \( |M|^2 = G_F^2 \cos^2 \theta_C (1 + 3g_A^2) \), where \( G_F \approx 1.136 \times 10^{-50} \text{ erg cm}^3 \) is the Fermi weak interaction constant, \( \theta_C (\sin \theta_C = 0.231) \) is the Cabibbo angle, and \( g_A = 1.26 \) is the axial-vector coupling constant. \( f_e \) and \( f_{\nu_e} \) are the distribution functions for electrons and neutrinos, respectively. The chemical potential of neutrinos is zero for the neutrino transparent matter. When neutrinos are trapped, the factor \( b_s \) reflects the percentage of the partially trapped neutrinos.

In addition, two other conditions that need to be satisfied are the conservation of the baryon number, \( n_n + n_p = n_b \times X_{nuc} \) and charge neutrality (Yuan 2005):

\[ n_e = n_e - n_{e^+} = n_p + n_{e^-}. \]

(3)

The above condition includes formation of Helium nuclei. Therefore, the net number of electrons is equal to the number of free protons plus the number of protons in Helium, given by:

\[ n_e^0 = 2n_{He} = (1 - X_{nuc}) \frac{n_b}{2}. \]

(4)

2.1. Chemical potential of neutrinos

In the neutrino transparent regime, the neutrinos are not thermalized and the chemical potential of neutrinos is negligible (see e.g. Beloborodov 2003). In the neutrino opaque regime, when neutrinos are trapped, the chemical equilibrium condition yields:

\[ \mu_e + \mu_{\nu_e} = \mu_p + \mu_{\bar{\nu}_e}. \]

The chemical potential of neutrinos depends on how much neutrinos and anti-neutrinos are trapped, and assuming that the number densities of the trapped neutrinos and anti-neutrinos are the same, \( \mu_{\nu_e} = 0 \).

For the intermediate regime of partially trapped neutrinos, the Boltzmann equation should be solved. To simplify this problem, we use here the "gray body" model with a blocking factor \( b = \sum_{\nu_{\alpha}^\mu, \bar{\nu}_{\alpha}^\mu} b_{\nu_{\alpha}^\mu}. \) (Sawyer 2003; Janiuk et al. 2007). The neutrino distribution function is then:

\[ f_{\nu_e}(p) = \frac{b_1}{\exp(p/c\sqrt{T}) + 1} = b_1f_{\nu_e}, \quad (0 \leq b_1 \leq 1). \]

(5)

When neutrinos are completely trapped, the blocking factor is \( b_1 = 1 \).

2.2. Electron fraction and nucleosynthesis

In Figure[1] we show the contours of the constant electron fraction on the temperature-density plane. This value, calculated as

\[ Y_e = (n_e - n_{e^+})/n_b, \]

(6)

is defined by the net number of electrons, equal to the number density difference between electrons and positrons. From the charge neutrality condition, it is equal to the number of free protons plus the number of electrons in Helium nuclei, if they are formed. Therefore, the electron fraction is modified here by the Helium synthesis process.

The contour of \( Y_e \) = 0.5 corresponds to the helium number density of \( n_{He} = 1/4(n_b - 2n_p) \). If the neutrons do not dominate over protons, the Helium density would vanish here. The top-right region of this figure represents the contours with \( n_{He} < 1/4(n_b - 2n_p) \), where helium abundance is smaller and the electron-positron pairs are produced.

The above defined electron fraction is different from the equilibrium value defined as \( Y_e = 1/(1 + n_{e^-}/n_p) \), given by the ratio of number densities of protons to nucleons. The reason is due to the presence of electron-positron pairs and Helium nuclei in the plasma. This ‘proton fraction’ is plotted in figure[2]
The established chemical balance between the neutronisation reactions leads to the neutron excess, therefore in the temperature and density range shown in the Figure the free proton number density is mostly \( n_p < 1/2(n_p + n_n) \).

### 3. Accretion hydrodynamics

We consider the vertically integrated accretion disk around a black hole of mass \( M \) and dimensionless spin parameter \( a \) \cite{Janiuk&Yuan2010}. The surface density of the disk is \( \Sigma = H\rho \), where \( \rho \) is the baryon density and height is given by \( H = c_\nu/\Omega_K \). Here the sound speed is defined by \( c_\nu = \sqrt{P/\rho} \), \( \Omega_K = c^3/(GM(a + r/r_\text{s})^{3/2}) \) is the Keplerian angular velocity, \( r_\text{s} = GM/c^2 \) is the gravitational radius, and \( P \) is the total pressure. We note that, at very high accretion rates, the disk becomes geometrically 'slim' \((H \sim -0.3 - 0.5r)\) in regions where neutrino cooling becomes inefficient and advection dominates.

For the disc viscous stress we use the standard \( \alpha \) viscosity prescription of \cite{Shakura&Sunyaev1973}, where the stress tensor is proportional to the pressure:

\[
\tau_{\nu} = -\alpha P \tag{8}
\]

and we take a fiducial value of \( \alpha = 0.1 \).

The total pressure is contributed by free nuclei, electron-positron pairs, helium, radiation and the trapped neutrinos. The fraction of each species is determined by self-consistently solving the balance of the nuclear reaction rates.

\[
P = P_{\text{nuc}} + P_{\text{He}} + P_{\text{rad}} + P_{\nu} . \tag{9}
\]

where

\[
P_{\text{nuc}} = P_{e^-} + P_{e^+} + P_{\nu} + P_{\bar{\nu}} \tag{10}
\]

with

\[
P_{k} = 2 \sqrt{2} \left( m_i c^2 \right)^4 \beta_i^2 \left[ F_{3/2}(\eta_i, \beta_i) + \frac{1}{2} \beta_i F_{5/2}(\eta_i, \beta_i) \right] . \tag{11}
\]

Here \( F_k \) are the Fermi-Dirac integrals of the order \( k \), and \( \eta_i \), \( \eta_i \), and \( \eta_i \) are the reduced chemical potentials of electrons, protons and neutrons in units of \( kT \), respectively, \( \eta_i = \mu_i/kT \). The reduced chemical potential (or degeneracy parameter) of positrons is \( \eta_{e^+} = -\eta_e - 2/\beta_e \). The relativity parameters of the species \( i \) are defined as \( \beta_i = kT/m_i c^2 \).

The pressure of non-relativistic and non-degenerate Helium is given by:

\[
P_{\text{He}} = n_{\text{He}} kT, \tag{12}
\]

where the density is

\[
n_{\text{He}} = \frac{1}{2} n_0 (1 - X_{\text{nuc}}) , \tag{13}
\]

and the fraction of free nucleons scales with density and temperature as

\[
X_{\text{nuc}} = 295.5 \nu_{10}^{-3/4} \nu_{9/8}^{9/8} \exp(-0.8209/T_{11}^3). \tag{14}
\]

Here \( T_{11} \) the temperature in the units of \( 10^{11} \) K and \( \nu_{10} \) is density in the units of \( 10^{10} \) g cm\(^{-3} \) \cite{Qian&Woosley1996, Popham&etal1999, Janiuk&etal2007}.

The radiation pressure is negligible in comparison with other terms in the GRB central engines. However, when neutrinos are trapped in the disc, the neutrino pressure is non-zero. Following the treatment of photon transport under the two-stream approximation \cite{Popham&Narayan1995, DiMatteo&etal2002}, we have

\[
P_{\nu} = \frac{7 \pi^2 (kT)^4}{8 15 3(hc)} \sum \frac{\beta_i \tau_{\alpha,\beta}^\nu}{\tau_{\nu,\alpha}^\nu} + \frac{1}{\sqrt{3}} \left( \frac{\tau_{\alpha,\beta}^\nu}{\tau_{\nu,\alpha}^\nu} \right) \tag{15}
\]

where \( \tau_{\nu} \) is the scattering optical depth due to the neutrino scattering on free neutrons and protons and \( \tau_{\alpha,\beta}^\nu \) and \( \tau_{\nu,\alpha}^\nu \) are the absorptive optical depths for electron and muon neutrinos. The absorption of electron neutrinos is determined by the reactions inverse to all their production processes: electron-positron capture on nucleons, electron-positron pair annihilation, nucleon bremsstrahlung and plasmon decay. The absorption of muon neutrinos is governed by the rates of pair annihilation and bremsstrahlung reactions, and the contribution from tau neutrinos is the same as that from muon neutrinos. The scattering optical depth is the Rosseland mean opacity, derived for all the neutrinos from their cross section of scattering on nucleons \cite{DiMatteo&etal2002}.

Throughout the calculations we adopt fiducial values of parameters: black hole mass of \( M = 3M_\odot \), dimensionless spin \( a = 0.9 \), viscosity parameter \( \alpha = 0.1 \).

We solve the hydrodynamical balance to establish the structure of the accretion disk, adopting the standard mass, energy and momentum conservation equations. The viscous heating, \( Q^\nu = 3/2 \alpha \Omega H \), is balanced by the advective cooling, \( Q_{\text{adv}} = \Sigma v T dS/dt \), photodissociation of alpha particles (if they form) and the radiative and neutrino cooling, \( Q_c \). Therefore, we calculate the stationary disk configuration from:

\[
F_{\text{tot}} = Q_{\text{nuc}}^\nu + Q_{\text{adv}} + Q_{\text{rad}} + Q_{\nu} + Q_{\text{photo}} . \tag{16}
\]

On the left hand side, we have here the vertically integrated viscous heating rate per unit area, over a half thickness \( H \), is given by the global parameters of the model, i.e. black hole mass and accretion rate:

\[
F_{\text{tot}} = \frac{3GM M}{8 \pi r^2} f(r) \tag{17}
\]

where \( f(r) \) denotes the boundary condition around the spinning Kerr black hole \cite{Bardeen&etal1972, Ruffert&Herold1995}. 
In the advective cooling term, the entropy gradient is assumed constant with radius and on the order of unity. The entropy is the sum of four components, corresponding to the pressure terms:

$$S = S_{\text{nucl}} + S_{\text{He}} + S_{\text{rad}} + S_{\nu}. \quad (18)$$

Here

$$S_{\text{nucl}} = S_{e^-} + S_{e^+} + S_{p} + S_{n} \quad (19)$$

and the contributions from electrons, positrons, protons and neutrons are

$$S_{i} = \frac{1}{k} (\epsilon_i + P_i) - n_i \eta_i \quad (20)$$

with

$$\epsilon_i = \frac{2 \sqrt{2} (m_{\text{He}} c^2)^4}{3 \pi^2} \beta_i^{5/2} [F_{5/2}^\text{He}(\eta_i, \beta_i) + \beta_i F_{3/2}^\text{He}(\eta_i, \beta_i)] \quad (21)$$

with $P_i$ being the pressure components of the species, $n_i$ the number densities and $\eta_i$ the reduced chemical potentials.

If the Helium nuclei form, their entropy is given by:

$$S_{\text{He}} = n_{\text{He}} \left[ 5 \frac{3}{2} \log (m_{\text{He}} kT/(hc)^2) - 1 - \log n_{\text{He}} \right]. \quad (22)$$

for $n_{\text{He}} > 0$.

The cooling term due to photodisintegration of $\alpha$ particles has a rate:

$$Q_{\text{photo}} = 4 \pi a \rho v_{1}\frac{dX_{\text{nucl}}}{dr} \quad (23)$$

where

$$q_{\text{photo}} = 6.28 \times 10^{28} \rho_{10} v_{1} \frac{dX_{\text{nucl}}}{dr} \quad (24)$$

and $X_{\text{nucl}}$ is the mass fraction of free nucleons.

The radiative cooling $Q_{\text{rad}} = (3P_{\text{rad}}c)/(4\pi) = (11\tau T^4)/(4\pi \Sigma)$, is in practice negligible term in comparison with other terms, for our assumed global flow parameters. The neutrino cooling rate is high, if only the neutrinos are not completely trapped. In the neutrino optically thick disk, their cooling rate is given by

$$Q_{\nu} = \frac{7}{3} \tau T^4 \sum_{i=e,\mu,\tau} v_{i} \frac{1}{\sqrt{2}}  \frac{1}{1 + \frac{1}{\sqrt{2} + \frac{1}{\tau_{\text{e}}} + \frac{1}{\tau_{\mu}}} + \frac{1}{\tau_{\tau}}} \quad (25)$$

where we include the absorption and scattering optical depths for all three neutrino flavours, following their emission processes as in Jianiku et al. (2007).

4. Results

4.1. Mass fraction of free nucleons in the inner disk

In Figure 3 we plot the mass fraction of free nucleons in the accretion disk powering the central engine of gamma ray burst. The Figure shows two values of accretion rate, $M = 0.1 M_{\odot}/s$ and $M = 1.0 M_{\odot}/s$. We adopt the black hole mass of 3 $M_{\odot}$.

For a given accretion rate, converged solution for $X_{\text{nucl}}$ is not sensitive to the black hole spin value and its only effect is on the location of marginally stable orbit. The disk is located closer to the black hole gravitational radius, $r_g = GM/c^2$, for large spin value.

Depending on the accretion rate, we find the regions of Helium formation in the disk. For a given black hole spin, $a = 0.9$, the smaller $M = 0.1 M_{\odot}/s$ solution results in a constant maximum value of $X_{\text{He}} = 1.0$ below the radius of about 100 $r_g$. For our assumed mass of the black hole it is about 450 km. Outside this radius, the Helium nuclei start forming and above ~ 300 $r_g$ the mass fraction of free nucleons drops to zero and the accreting flow is dominated by Helium. For larger value of accretion rate, $M = 1.0 M_{\odot}/s$, this outer zone dominated by Helium nuclei is shifted outwards, to above ~ 630 $r_g$, while below ~ 250 $r_g$, there is no Helium nuclei.

In Figure 4 we show the number densities of free particles in the disks with two accretion rate values. Free neutrons are dominant species in a large part of the disk, up to 40 $r_g$ for $M = 0.1 M_{\odot}/s$ and up to 250 $r_g$ for $M = 1.0 M_{\odot}/s$. Outwards of this region, neutrons and protons are synthesizing the Helium nuclei, and the electron-positron pairs are still abundant, keeping the charge neutrality.

For $M = 1.0 M_{\odot}/s$, in the inner 10 $r_g$ the number density of free neutrons is reduced, with a flattened radial profile, and the density of protons is smaller. The reason is that at these density and temperature regime, protons are more degenerate than neutrons.

In Figure 5 we show the electron fraction $Y_e$, calculated as the net number of electrons per baryon (eq. 7). The dashed line in this figure shows for comparison the 'proton fraction', derived from the ratio of free neutrons to protons. If the latter is smaller than 0.5, the free neutrons dominate over protons, which is the case in the innermost 50 – 500 gravitational radii of the disk, depending on its accretion rate. The electron fraction, plotted with the solid line, is changed due to the formation of Helium nuclei. In the outer regions of the disk, the value of electron fraction is saturated at 0.5, while the number densities of neutrons are first slightly less than protons, but increase with radius. The efficient neutronisation eventually allows for the maximum Helium abundance at the outskirts of the disk.
most temperatures therefore may reach 5 or 10 MeV, while the regions, where temperature decreases below 1 MeV, are located in the outer disk parts, above 50 or 100 gravitational radii.

To compute the abundances of heavy elements, we used the thermonuclear reaction network code (http://webnucleo.org). The computational methods are described in detail in Seitenzahl et al. (2008) (see also Meyer 1994, Wallerstein et al. 1997, and Hix & Meyer 2006). The code is using the nuceq library to compute the nuclear statistical equilibria established for the thermonuclear fusion reactions. The abundances are calculated under the constraint of nucleon number conservation and charge neutrality. We used here the correction function to account for degeneracy of relativistic species. We used the data downloaded from JINA website (http://www.jinaweb.org), prepared to study the nuclear masses and nuclear partition functions and to compute nuclear statistical equilibria. The reaction data available on JINA reaclib online database provide the currently best determined reaction rates. The nuclide data are merged with reaction data into a network data file, prepared for astrophysical applications. The network is working well for the temperature ranges about or below 1 MeV and our method is appropriate for the outer parts of accretion disks in GRBs. For hotter astrophysical plasmas, more advanced computations could be appropriate (Kafexhiu et al. 2012).

Once we have computed the accretion disk model for the GRB central engine, the mass fraction of heavy nuclei was then computed at every radius of the disk, given the profiles of density and temperature and electron fraction. In Figures 3 and 7 we show the resulting distributions of the most abundant isotopes of heavy elements synthesized in the accretion disk.

We checked first for these isotopes, whose mass fraction is greater than $10^{-4}$. For the accretion rate of 0.1 $M_{\odot}$s$^{-1}$, the abundance of $^4$He is large up to 260 $r_g$ and then decreases throughout the disk, and there is also some fraction of $^3$He as well as Deuterium and Tritium. The next abundant isotopes are $^{28}$Si - $^{30}$Si, $^{31}$P, $^{32}$S - $^{34}$S, then $^{35}$Cl and $^{36}$Ar - $^{38}$Ar. Further, synthesized are $^{39}$K, $^{40}$Ca - $^{42}$Ca, and $^{44}$Ti - $^{50}$Ti, $^{47}$V - $^{52}$V, $^{48}$Cr - $^{54}$Cr, and $^{51}$Mn - $^{56}$Mn. The most abundant Iron isotopes formed in the disk are $^{52}$Fe through $^{58}$Fe. Cobalt is formed with isotopes $^{54}$Co through $^{60}$Co, and Nickel isotopes are $^{56}$Ni through $^{62}$Ni. The heaviest most abundant isotopes in our disk are $^{59}$Cu through $^{63}$Cu. There is further a smaller fraction of Zinc, $^{60}$Zn - $^{64}$Zn, with mass fraction above $10^{-3}$. These heavy elements are generally produced outside 300-400 $r_g$. Inside this radius, the disk consists of mainly free neutrons and protons, with some fraction of Helium. The mass fraction of free neutrons is smaller than that of protons, and free neutrons disappear above $\sim 300r_g$.

For the model with accretion rate of 1.0 $M_{\odot}$s$^{-1}$, the abundance of $^3$He is large up to about 600 $r_g$ and then decreases. There is also some Deuterium and Tritium, while the next abundant isotopes are $^{28}$Si - $^{30}$Si, $^{31}$P, $^{32}$S - $^{34}$S, $^{35}$Cl, and $^{36}$Ar - $^{38}$Ar. Next, there is $^{39}$K, $^{40}$Ca - $^{42}$Ca, $^{43}$Sc - $^{47}$Sc, $^{44}$Ti - $^{50}$Ti, $^{47}$V - $^{52}$V. Synthesized are also $^{40}$Cr - $^{55}$Cr, $^{50}$Mn - $^{57}$Mn and $^{52}$Fe - $^{59}$Fe. Further heavy elements are $^{54}$Co - $^{61}$Co, and $^{56}$Ni - $^{63}$Ni, and $^{58}$Cu - $^{63}$Cu. The last abundant heavy isotope is $^{62}$Zn, while the abundances of $^{65}$Ga and $^{67}$Ga are about $10^{-6}$. In comparison with the model with small accretion rate presented above, the conditions in the larger accretion rate disk are such that the mass fraction of free neutrons is larger than that of free protons inside $\sim 200r_g$, and comparable to proton mass fraction up to $\sim 500r_g$. In both models, the heavy elements dominate above $\sim 550r_g$. 

Fig. 4. Number density of free particles, as a function of distance in the accreting disk, plotted for neutrons (solid line), protons (dotted line), electrons (short dashed line) and positrons (long dashed line). The steady state models were calculated for $M = 1.0 M_{\odot}$/s (top panel) and $M = 0.1 M_{\odot}$/s (bottom panel). The assumed black hole spin parameter is $\alpha = 0.9$.

Fig. 5. Electron fraction, as a function of distance in the accreting disk. The steady state models were calculated for $M = 1.0 M_{\odot}$/s (top panel) and $M = 0.1 M_{\odot}$/s (bottom panel). The assumed black hole spin parameter is $\alpha = 0.9$. Dashed lines show the corresponding proton fraction.

4.2. Formation of heavy elements in the outer disk

The steady state models were computed for accretion rates of $M = 1.0 M_{\odot}$s$^{-1}$ and $M = 0.1 M_{\odot}$s$^{-1}$, with the black hole dimensionless spin equal to $\alpha = 0.9$. In these conditions, the density range in the disk, up to 1000 gravitational radii, is $3 \times 10^{10}$ - $10^6$ g cm$^{-3}$ or $5 \times 10^{11}$ - $10^7$ g cm$^{-3}$, for accretion rate of 0.1 and 1.0 $M_{\odot}$s$^{-1}$, respectively. The corresponding temperature range is $5 \times 10^{10}$ - $1.5 \times 10^9$ K and $1.2 \times 10^{11}$ - $2 \times 10^9$ K.
5. Outflows

The outflow of gas from the accretion disk surface may be driven by centrifugal force or magnetic field (McKinney 2006; Janiuk et al. 2013). In either case, the flow can be described by a spherical geometry and velocity depending on distance allows to compute trajectories of particles (Surman & McLaughlin 2004). The slowly accelerated outflows will allow for production of heavier elements via the triple-alpha reactions up to Nickel 56, or, if the entropy per baryon is quite low, up to Iron peak nuclei (Woosley 1973). Also, other heavy nuclei such as Sc, Ti, Zn, and Mo, may be produced in the outflows with a moderate abundance (Surman et al. 2006).

For the accretion rate of 0.1\(M_{\odot}/s\), our calculations show the significant proton excess in the disk, above \(\sim 250r_g\). The wind ejected at this region may therefore provide a substantial abundance of light elements, Li, Be and B. The high accretion rate disk, on the other hand, will rather produce neutron rich outflows and formation of heavy nuclei via the r-process. Also, as we show here, the outflows ejected from the innermost 100\(r_g\) in the high accretion rate disks will be significantly neutron rich. Therefore these neutron loaded ejecta, accelerated via the black hole rotation, will feed the collimated jets at large distance from the central engine. This will have important implications for the observed GRB afterglows, induced by the radiation drag (Metzger et al. 2008) and collisions between the proton rich and neutron rich shells within the GRB fireball (Beloborodov 2003).
6. Conclusions

We considered here the central engine model for gamma ray bursts, as resulting from the massive rotating star collapse or a compact object merger. The two accretion rates invoked as bursts, as resulting from the massive rotating star collapse or a compact object merger. The two accretion rates are degenerate. The outflows from this region should be neutron rich. The neutrons therefore should be present at large distances within the expanding fireball (i.e. $10^{17}$ cm). Their large kinetic energy will affect the dynamics of the expanding fireball and lead to interaction with the circumburst medium.

Our results are in line with those obtained by other groups working on nucleosynthesis models of GRB engines. In particular, Fujimoto et al. (2004) also compute the nuclear synthesis of elements in the accretion disk up to 1000 gravitational radii, and they find further layers of dominant Oxygen, Silicon and Calcium, with their abundances enhanced by the $\alpha$-capture process. With mass fraction about $\sim 1$, these elements may be present in large collapsar disks. However, the disk is probably not larger in most progenitors. Nevertheless, we obtain similar results for the isotopes synthesized above $\sim 100r_g$ (note that we define $r_g = GM/c^2$), and we also find a trend of shifting the layers outwards with increasing accretion rate. We estimate the transition radius for the subsurface layers to be $r_m \sim M^{-0.25}$. At the outer parts of the disk, Fujimoto et al. (2004) studied the synthesis of light elements such as $^{16}$O, $^{28}$Si, $^{40}$Ca, which mass fractions above 0.0001. We find the mass fraction of $^{16}$O to be less than $10^{-4}$ inside 1000 gravitational radii, while for the other two elements they are about 0.01 at their peak, around few hundred $r_g$. For the heavier elements above the Iron peak, their yields were computed by Surman et al. (2006). Our reaction network results give the isotopes of $^{25}$Mg, $^{26}$Mg, $^{27}$Al, $^{32}$S, $^{34}$S, $^{36}$Ar, $^{38}$Ar, with mass fractions of $10^{-7}$, depending on accretion rate, from the inner to the outer disk region. Fujimoto et al. (2004) and also Surman & McLoughlin (2006) found non-negligible abundances of elements above Nickel, such as isotopes of Copper, Zinc, as well as Galium and Germanium, that could be found in the afterglows of short GRBs. The different properties of central engines in the two classes of bursts that determine the nucleosynthesis, should therefore be also accounted for in the statistical studies of the observed phenomena

The radioactive decay of certain isotopes should be detectable via the emission lines observed by X-ray satellites. Such lines, e.g. the decay of $^{44}$Ti to $^{40}$Ca with emission of hard X-ray photons at 68 and 78 keV have been detected by NuSTAR in case of supernova remnants. The energy band of this instrument (3-80 keV) should allow in principle for finding the X-ray signatures of other elements synthesized in the accretion disks in GRB central engines, like the radioactive isotopes of Cuprum, Zinc, Gallium, Cromium and Cobalt. The heavy elements, once ejected with a wind outflow from the engine, might also give their imprints in the absorption lines of the GRB optical spectra.

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