Resonances $\phi(1020)$ and $\phi(1680)$ contributions for the three-body decays $B^+ \to D_s^+ K \bar{K}$

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We study the resonances $\phi(1020)$ and $\phi(1680)$ contributions for the three-body decays $B^+ \to D_s^+ K \bar{K}$ in the perturbative QCD approach. The branching ratios for $B^+ \to D_s^+ \phi(1020) \to D_s^+ K^+ K^-$ and $B^+ \to D_s^+ \phi(1020) \to D_s^+ K^0 \bar{K}^0$ are predicted to be $(1.53 \pm 0.17 \pm 0.12 \pm 0.10) \times 10^{-7}$ and $(1.02 \pm 0.09 \pm 0.08 \pm 0.05) \times 10^{-7}$, respectively. The decay $B^+ \to D_s^+ \phi(1680)$ with $\phi(1680)$ decays into $K^+ K^-$ or $K^0 \bar{K}^0$, has the branching fraction $(6.94^{+0.90+1.29+0.35}_{-0.81-1.62-0.21} \pm 0.86) \times 10^{-9}$, which is about 5% of the result for $B^+ \to D_s^+ \phi(1020) \to D_s^+ K^0 \bar{K}^0$.

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The rare decay $B^+ \to D_s^+ \phi(1020)$ presents a very clean channel for us to test the annihilation contribution in the Standard Model (SM). This decay process has been extensively studied on theoretical and experimental sides during the past decades, with the predictions for its branching fraction in the range of $1.30 \times 10^{-7}-1.88 \times 10^{-6}$ in the SM [1–5]. In addition, the small branching ratio makes this process probably sensitive to the parameters of the physics beyond SM and its direct CP violation which is expected to be zero in SM could also be produced in the new physics models [2, 3]. The search for $B^+ \to D_s^+ \phi(1020)$ was performed by CLEO [6] and BABAR Collaborations [7] years ago, but no significant signal has been observed. The first evidence for this decay was found with greater than $3\sigma$ significance by LHCb Collaboration with the measured branching fraction $(1.87^{+1.25}_{-0.73} \pm 0.19 \pm 0.32) \times 10^{-6}$ [8]. Recently, in their work [9], LHCb set a limit as $B(B^+ \to D_s^+ \phi(1020)) < 4.9 (4.2) \times 10^{-7}$ at 95\% (90\%) confidence level in the analysis of the three-body decay $B^+ \to D_s^+ K^+ K^-$ for this two-body subprocess, which is roughly one order smaller than the previous result in [8]. One should note that the $\phi(1020)$ meson is usually reconstructed within $K \bar{K}$ final states in the experimental analysis [8–13], but treated as a stable particle in the aforementioned theoretical studies.

Three-body hadronic $B$ meson decays are much more complicated than the two-body cases partly because of the three-body effects and rescattering effects [14–16] and also because of entangled resonant and nonresonant contributions. The resonant contributions in the three-body decays are related to the low energy scalar, vector and tensor resonant states, and could be isolated from the total decay amplitudes and studied in the quasi-two-body framework [17–19]. At the edge of the Dalitz plot [20], the three final state particles are quasi-aligned in the rest frame of the $B$ meson, while two of them move collinearly and recoil against the third meson. The factorization for the two-body decays is still valid for this part of the phase space. Then the relevant decay processes can be represented as $B \to R h_3 \to h_1 h_2 h_3$ where $h_3$ represents the bachelor particle moves in the opposite direction and the $h_1 h_2$ pair proceeds by the intermediate resonance $R$. The studies on a series of charmless three-body hadronic $B$ meson decays have been accomplished based on the QCD factorization (QCDF) [21–33] and the perturbative QCD (PQCD) approach [17, 34–45].

In the previous works [46–52], the $S$- and $P$-wave $\pi\pi$, $KK$ and $D\pi$ resonance contributions to several three-body $B \to Dh_1 h_2$ decays have been studied within the PQCD approach, and most of the theoretical predictions are in good agreement with the available experimental results. In this work, we shall study the contributions from the subprocesses $\phi(1020, 1680) \to K^+ K^-$ and $\phi(1020, 1680) \to K^0 \bar{K}^0$ to the three-body decay $B^+ \to D_s^+ K^+ K^-$ within PQCD approach. In our framework for the quasi-two-body decays, the two-meson distribution amplitudes are introduced to describe the interactions between the meson pair associated with the resonance, the relevant decay amplitude $A$ for the quasi-two-body decays $B \to DR \to Dh_1 h_2$ concerned in this work can be expressed as [34, 35]

$$A = \Phi_B \otimes H \otimes \Phi_D \otimes \Phi_{h_1 h_2},$$

where $H$ is the hard kernel and $\Phi_B (\Phi_D, \Phi_{h_1 h_2})$ represents the $B$ meson ($D$ meson, $h_1 h_2$ pair) distribution amplitude.

In the rest frame of $B$ meson, we could define the momenta of the $B$ meson, the kaon pair which generated from the intermediate states $\phi(1020, 1680)$, and the $D$ meson in the light-cone coordinates as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, \theta_T), \quad p = \frac{m_B}{\sqrt{2}}(1-r^2, \eta, \theta_T), \quad p_3 = \frac{m_B}{\sqrt{2}}(r^2, 1-\eta, \theta_T),$$

where the mass ratio $r = m_D/m_B$ and $m_{B(D)}$ is the mass for $B(D)$ meson. The variable $\eta$ is defined as $\eta = s/[(1-r^2)m_B^2]$ with the invariant mass square $s = p^2 = m_{KK}^2$ for the kaon pair. The momenta of the light quarks in the corresponding states

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where the momentum fractions can be written as 

\[
F_{\phi} = \frac{1}{2N_c} \left[ \sqrt{s} f_L \phi^0(z, s) + f_L \phi^1(z, s) + \sqrt{s} \phi^s(z, s) \right],
\]

with the distribution amplitudes

\[
\phi^0(z, s) = \frac{3F_K(s)}{\sqrt{2N_c}} z(1 - z) \left[ 1 + a_2^2 \frac{C_2^{3/2}}{2N_c} (1 - 2z) \right],
\]

\[
\phi^s(z, s) = \frac{3F_K(s)}{2\sqrt{2N_c}} (1 - 2z),
\]

\[
\phi^f(z, s) = \frac{3F_K(s)}{2\sqrt{2N_c}} (1 - 2z)^2.
\]

The Gegenbauer polynomial \(C_2^{3/2}(t) = 3(5t^2 - 1)/2\) and the Gegenbauer moment \(a_2^2 = 0.18 \pm 0.08\) as the same employed in Ref. [54] for the two-body \(B\) decays. When concern only the resonance contributions, the relation between the kaon time-like form factors \(F_s^{K^+K^-}\), \(F_s^{K^0\bar{K}^0}\) and the kaon electromagnetic form factors \(F_{\phi}^{K\bar{K}}\) can be written as [27, 45]

\[
F_K(s) = F_s^{K^+K^-}(s) = F_s^{K^0\bar{K}^0}(s) = -3F_{\phi}^{K\bar{K}}(s) = - \sum_\phi c_{\phi}^{K}\text{BW}_{\phi}(s).
\]

For the \(F_{\phi}^{K\bar{K}}(s)\) in the distribution amplitudes, we adopt the relation \(F_{\phi}^{K\bar{K}}(s) \approx (f_{\phi}^{K\bar{K}}/f_{\phi}) F_K(s)\) [45] with the ratio \(f_{\phi}^{K\bar{K}}/f_{\phi} = 0.75\) [55]. The parameters \(c_{\phi}^{K}\) have been fitted to the data in Refs. [56–58], we adopt the values \(c_{\phi}^{K}(1020) = 1.038\) and \(c_{\phi}^{K}(1680) = -0.150 \pm 0.009\) [58] as those are discussed and chosen in Ref. [45].

There are only tree operators contribute to the decay amplitude of the decays \(B^+ \rightarrow D_s^+ \phi(1020, 1680) \rightarrow D_s^+ K\bar{K}\), which can be written as

\[
A(B^+ \rightarrow D_s^+ \phi \rightarrow D_s^+ K\bar{K}) = \frac{G_F}{\sqrt{2}} V_{ub}V_{cs} \left[ \frac{C_1}{3} + C_2 \right] F_{aD}^{LL} + C_1 M_{aD}^{LL},
\]

where \(G_F\) represents the Fermi coupling constant, \(V_{ij}\) are the CKM matrix elements and \(C_{1,2}\) mean the Wilson coefficients. The symbols \(F_{aD}^{LL}\) and \(M_{aD}^{LL}\) are the amplitudes from the factorizable and nonfactorizable annihilation diagrams shown in Fig. 1, respectively, with the specific expressions given by

\[
F_{aD}^{LL} = 8\pi C_F m_B f_B f_D x_3 z_3 b_d b_d b_d \phi_D \times \left\{ \left[ (r^2 - 1) [\eta(\eta + r^2 - 1) - (1 - \eta)^2 x_3] \phi_0 + 2r \sqrt{\eta} (1 - r^2) [1 + \eta + (1 - \eta) x_3 - r^2] \phi_s \right] \times E_a(t_a) h_a(z, x_3, b_3, b_3, b_3, b_3) S_l(x_3) - \left[ (1 - \eta) (r^4 - z + 1) + r^2 (1 - \eta - 2z) + z - 2rr_c + 2r^3r_c \phi_0 \right] + \sqrt{\eta} (1 - r^2) [r(2z + 2r^2 (1 - z) - rr_c) (\phi_s + \phi_l) + (1 - \eta) (2r - r_c)(\phi_s - \phi_l)] \right\} \times E_a(t_b) h_b(z, x_3, b_3, b_3, b_3) S_l(z),
\]
\[
M_{1D}^{4L} = 32\pi C_F m_B^4 / \sqrt{6} d x_B d z d x_d d b_d b_d d b_d d b_d \phi_d \\
\times \{ [(\eta + r^2 - 1)(1 - \eta)(r^2(z - x_3) - x_B - z) + r^2 - \eta] \phi_0 + r \sqrt{\eta(1 - r^2)}[(z(1 - r^2) + x_B) \\
\times (\phi_s + \phi_t) + (1 - \eta)x_3(\phi_s - \phi_t) + 2\phi_s] E_n(t_c) h_c(x_B, z, x_3, b, b_B) \\
- [(1-\eta + r^2)[(1-\eta)x_3 - n z] + x_B n \phi_0 + r \sqrt{\eta(1-r^2)}[(1-\eta)x_3(\phi_s + \phi_t) \\
+ ((1 - r^2)z - x_B)(\phi_s - \phi_t)] E_n(t_d) h_d(x_B, z, x_3, b, b_B)},
\]
(11)

with the color factor \(C_F = \frac{4}{3}\). The explicit expressions of the hard functions \(h_i\), the evolution factors \(E(t_i)\) and the threshold resummation factor \(S_t\) can be found in Ref. [46].

In the numerical calculation, we adopt the following input parameters [59], with the QCD scale, masses, decay constants and full widths in units of GeV,

\[
\Lambda_{f=4}^{(f=4)} = 0.25, \quad m_B = 5.279, \quad m_{D_s} = 1.968, \quad m_{K^{\pm}} = 0.494, \quad m_{K^0} = 0.498, \\
m_{s} = 0.048, \quad m_c = 1.275, \quad f_B = 0.189, \quad f_{D_s} = 0.249, \quad \tau_B = 1.638 \text{ ps}, \\
m_{\phi(1020)} = 1.019, \quad \Gamma_{\phi(1020)} = 0.00425, \quad m_{\phi(1680)} = 1.680, \quad \Gamma_{\phi(1680)} = 0.150.
\]
(12)

For the Wolfenstein parameters \((A, \lambda, \bar{\rho}, \bar{\eta})\) of the CKM mixing matrix, we use the values \(A = 0.836 \pm 0.015\), \(\lambda = 0.22453 \pm 0.00044\), \(\bar{\rho} = 0.122^{+0.018}_{-0.017}\), \(\bar{\eta} = 0.355^{+0.012}_{-0.011}\) [59].

The differential branching fractions \(B\) for the quasi-two-body decays \(B \to D\phi(1020, 1680) \to DKK\) can be written as [23, 44, 45]
\[
\frac{dB}{d\eta} = \frac{\tau_B}{12\pi^3 m_B^4} |A|^2.
\]
(13)

The magnitudes of the momenta for \(K\) and \(D\) in the center-of-mass frame of the kaon pair are written as
\[
q = \frac{1}{2} \sqrt{s - 4m_K^2},
\]
\[
q_D = \frac{1}{2} \sqrt{s} \sqrt{(m_B^2 - m_D^2)^2 - 2(m_B^2 + m_D^2)s + s^2}.
\]
(14)
(15)

By employing the decay amplitudes as given in Eq. (10)-(11) and the differential branching fractions in Eq. (13), integrating over the full \(KK\) invariant mass region \(2m_K \leq \sqrt{s} \leq (m_{B_D} + m_{D_s})\) for the resonant components, we obtain the branching ratios

\[
B(B^{+} \to D_{s}^{+} \phi(1020) \to D_{s}^{+} K^{+} K^{-}) = (1.53 \pm 0.17(\omega_B)^{+0.12}_{-0.12}(a_{2d})^{+0.07}_{-0.07}(C_{D_s})) \times 10^{-7},
\]
\[
B(B^{+} \to D_{s}^{+} \phi(1680) \to D_{s}^{+} K^{0} K^{0}) = (1.02^{+0.13}_{-0.09}(\omega_B)^{+0.12}_{-0.08}(a_{2d})^{+0.06}_{-0.06}(C_{D_s})) \times 10^{-7},
\]
(16)

where the first error is come from the uncertainty of the \(B\) meson shape parameter \(\omega_B = 0.40 \pm 0.04\) GeV, the second error comes from the Gegenbauer coefficient \(a^{0}_{2d} = 0.18 \pm 0.08\) in the kaon-kaon distribution amplitudes and the last one is induced by \(C_{D_s} = 0.4 \pm 0.1\) for \(D_s\) meson wave function. The errors come from the uncertainties of other parameters are small and have been neglected. Under the narrow-width approximation, the two-body branching fraction for \(B \to D\phi(1020)\) can be extracted from the quasi-two-body prediction with the relation

\[
B(B \to D\phi(1020) \to DKK) \approx B(B \to D\phi(1020)) \cdot B(\phi(1020) \to K\bar{K}),
\]
(17)

Utilizing the decay rate \(B(\phi(1020) \to K^{+} K^{-}) = 0.492\) [59], we have the two-body branching fractions \(B(B^{+} \to D_{s}^{+} \phi(1020)) = (3.11 \pm 0.47) \times 10^{-7}\). The corresponding experimental results are given as

\[
B(B^{+} \to D_{s}^{+} \phi(1020)) \begin{cases} < 3 \times 10^{-4} & \text{CLEO [6],} \\ < 1.9 \times 10^{-6} & \text{BABAR [7],} \\ (1.87^{+1.25}_{-0.73} \pm 0.19 \pm 0.32) \times 10^{-6} & \text{LHCb [8],} \\ (1.2^{+1.6}_{-1.4} \pm 0.8 \pm 0.1) \times 10^{-7} & \text{LHCb [9].} \end{cases}
\]
(18)

The branching fraction predicted in this work is consistent with the experiment data and limits. The branching fraction for the two-body decay \(B^{+} \to D_{s}^{+} \phi(1020)\) has been calculated in [1, 4] within PQCD approach, with the results consistent with our prediction within errors. Comparing with the relatively large branching ratios predicted by other works [2, 3, 5], the measured result by LHCb in [9] is more closer to the PQCD prediction in this work.
The ratio between the branching fractions of the decays $B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^0 \bar{K}^0$ and $B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^+ K^-$ is defined as

$$ R_1 = \frac{B(B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^0 \bar{K}^0)}{B(B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^+ K^-)} \approx 0.67. \quad (19) $$

Based on the Eq. (17), we have

$$ R_1 \approx \frac{B(\phi(1020) \rightarrow K^0 \bar{K}^0)}{B(\phi(1020) \rightarrow K^+ K^-)}. \quad (20) $$

Then we estimate $B(\phi(1020) \rightarrow K^0 \bar{K}^0) = 0.33$ with $B(\phi(1020) \rightarrow K^+ K^-) = 0.492$ [59], which is agree with $B(\phi(1020) \rightarrow K^0 \bar{K}^0) = 0.340 \pm 0.004$ in the Review of Particle Physics [59].

The prediction for the branching ratio involves $\phi(1680)$ is

$$ B(B^+ \rightarrow D_s^+ \phi(1680) \rightarrow D_s^+ K^+ K^-) = (6.94^{+0.90}_{-0.81}(\omega_B)+1.29(a_{2\phi})+0.35(C_{D_s}) \pm 0.86(c_K)) \times 10^{-6}, \quad (21) $$

with the last error comes from the coefficient $c_K(\phi(1680)) = -0.150 \pm 0.009$ in the form factor $F_K$. Different from the decay modes with the subprocesses $\phi(1020) \rightarrow K^0 \bar{K}^0$ and $\phi(1020) \rightarrow K^+ K^-$, the decay $B^+ \rightarrow D_s^+ \phi(1680) \rightarrow D_s^+ K^0 \bar{K}^0$ almost has the same branching fraction as the decay $B^+ \rightarrow D_s^+ \phi(1680) \rightarrow D_s^+ K^0 \bar{K}^0$ because of the ratio $B(\phi(1680) \rightarrow K^0 \bar{K}^0) \approx 1$ [45]. From another perspective, the main portion of the related branching ratios come from the region around the pole mass of the resonant states, the lower limit of integration $2m_K$ is close to the pole mass of $\phi(1020)$ but relatively far away from the one of $\phi(1680)$ which makes the branching ratios of the decay $B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K K$ more sensitive to the mass of kaon. We can define the ratio $R_2$ between the branching fractions for $\phi(1680) \rightarrow K^+ K^-$ and $\phi(1020) \rightarrow K^+ K^-$ as

$$ R_2 \approx \frac{B(\phi(1680) \rightarrow K^+ K^-)}{B(\phi(1020) \rightarrow K^+ K^-)} \approx \frac{B(B^+ \rightarrow D_s^+ \phi(1680) \rightarrow D_s^+ K^+ K^-)}{B(B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^+ K^-)} \approx 0.05, \quad (22) $$

which is consistent with the result 0.06 obtained from the fit fractions $(70.5 \pm 0.6 \pm 1.2)\%$ and $(4.0 \pm 0.3 \pm 0.3)\%$ for the contributions of $\phi(1020)$ and $\phi(1680)$ in $B^0 \rightarrow J/\psi K^+ K^- \bar{K}^0$ decay [12]. With the branching ratio $B(B^+ \rightarrow D_s^+ K^+ K^-) = (7.1 \pm 0.5 \pm 0.6 \pm 0.7) \times 10^{-6}$ presented by LHCb [9], one has the percent at about 2.2% of the total branching fraction for the quasi-two-body decay $B^+ \rightarrow D_s^+ \phi(1680) \rightarrow D_s^+ K^+ K^-$, which is expected to be tested in the future experiments.

To sum up, we studied the contributions for the $K^+ K^-$ and $K^0 \bar{K}^0$ originated from the intermediate states $\phi(1020)$ and $\phi(1680)$ in the three-body decays $B^+ \rightarrow D_s^+ K \bar{K}$. The branching ratios for $B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^+ K^- B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^0 \bar{K}^0$ are predicted to be $(1.53 \pm 0.10 \pm 0.12 \pm 0.07) \times 10^{-7}$ and $(1.02 \pm 0.13 \pm 0.12 \pm 0.06 \pm 0.05) \times 10^{-7}$, respectively, in this work. The branching ratio extracted from the quasi-two-body result for the two-body decay $B^+ \rightarrow D_s^+ \phi(1020)$ agrees with the existing experiment data within errors. The decay $B^+ \rightarrow D_s^+ \phi(1680)$ with $\phi(1680)$ decays into $K^+ K^-$ or $K^0 \bar{K}^0$, has the branching fraction $(6.94^{+0.90}_{-0.81}+1.29+0.35 \pm 0.86) \times 10^{-6}$, which is about 5% of the result for $B^+ \rightarrow D_s^+ \phi(1020) \rightarrow D_s^+ K^+ K^-$. 

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