Flow Characterization in Triply Periodic Minimal Surface (TPMS)-Based Porous Geometries: Part 1—Hydrodynamics

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Received: 26 April 2022 / Accepted: 30 October 2022 / Published online: 22 November 2022
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Abstract
The modeling of flow and heat transfer in porous media systems has always been a challenge, and the extended Darcy transport models are used for macro-level analysis. However, these models are subjected to the limitations depending upon the porous geometry such as pore size, pore type, effective porosity, tortuosity, permeability, and the flow characteristics. The forced convective flow of an incompressible viscous fluid through a channel filled with four different types of porous geometries constructed using the Triply Periodic Minimal Surface (or TPMS) model is presented in this study. Four TPMS lattice shapes, namely Diamond, I-WP, Primitive, and Gyroid, are created with same volume fraction of solid subdomain as 0.68 (or void fraction as 0.32). Using different configurations for the solid subdomain by treating it as (a) solid, (b) fluid, and (c) porous zone, three different classes of porous structures are further generated for each TPMS lattice. The present study is executed with the objective to investigate the effect of shape–morphology, tortuosity, microporosity, and effective porosity on permeability and inertial drag factor. A pore-scale direct numerical simulation approach is performed for the first two types of porous media by solving the Navier–Stokes equations. The specific microporosity is quantitatively induced in the solid subdomain where Darcy–Forchheimer equation is solved, whereas the Navier–Stokes equations are solved for the void subdomain in the third type of porous media. The results reveal that Darcy flow regime exists up to the mean velocity value of \( U < 0.0025 \) m/s (Re < 10) for all the cases discussed here, and it deviates at the higher mean velocity. The conductance to the flow shown by Darcy number has the maximum and minimum values for the Primitive Type 2 and I-WP Type 1 cases. The inertial drag coefficient is minimum in Diamond lattice and maximum in Primitive lattice at lower porosity (0.32), while Primitive lattice has minimum and I-WP lattice has maximum value of inertial drag coefficient for higher porosity (~ 1).

Keywords Darcian and non-Darcian regimes · Pore-scale simulation · TPMS lattice · Ordered porous structures

List of Symbols
\[ a \quad \text{Unit cell size in } x \text{ direction (m)} \]
Specific interfacial area \((m^{-1})\)  
Cross-sectional area \((A = L^2)\) \((m^2)\)  
Unit cell size in \(y\) direction \((m)\)  
Unit cell size in \(z\) direction \((m)\)  
Level-set constant  
Pore or particle size \((m)\)  
Darcy number \(\left( Da = \frac{K}{L^2} \right) \)  
Permeability \((m^2)\)  
Channel width or characteristic length \((m)\)  
Mass flow rate \((kg/s)\)  
Normal distance from the surface \((m)\)  
Pore level pressure \((Pa)\)  
Average pressure \((Pa)\)  
Channel size-based Reynolds number \(\left( \frac{\rho UL}{\mu} \right) \)  
Pore-scale-based Reynolds number \(\left( \frac{\rho UL}{\mu} \right) \)  
Brinkman-scale-based Reynolds number \(\left( \frac{\rho U \sqrt{K}}{\mu} \right) \)  
Pore level velocity in \(x\) direction \((m/s)\)  
Pore level velocity in \(y\) direction \((m/s)\)  
Pore level velocity in \(z\) direction \((m/s)\)  
Mean velocity in \(x\) direction \((m/s)\)  
Mean velocity in \(y\) direction \((m/s)\)  
Volume of void subdomain \((fluid\ zone)\) \((m^3)\)  
Volume of solid subdomain \((m^3)\)  
Mean velocity in \(z\) direction \((m/s)\)  
\(X\)-Direction distance \((m)\)  
\(Y\)-Direction distance \((m)\)  
\(Z\)-Direction distance \((m)\)  
\(x\)  
\(y\)  
\(z\)  
Lattice size in \(x\) direction \((m)\)  
Lattice size in \(y\) direction \((m)\)  
Lattice size in \(z\) direction \((m)\)  
Coefficient of cubic term  
Coefficient of quadratic term  
Porosity  
Porosity of the lattice  
Micro-porosity  
Effective porosity of the lattice  
Dynamic viscosity of the fluid \((kg/m\cdot sec)\)  
Effective viscosity of porous medium \((kg/m\cdot sec)\)  
Density of the fluid \((kg/m^3)\)  
Tortuosity  
Property of solid subdomain in the lattice  
Property of void subdomain in the lattice
1 Introduction

The interconnected network of pores of various shapes and sizes characterizes porous structures in general (Whitaker 1967). When fluid travels through this porous network, it encounters varied levels of stagnation, separation, recirculation, and attachment within the pores, resulting in a superior mixing or dispersion effect in the porous medium (Vafai 1986; Carbonell and Whitaker 1984; Kaviany 1995; Vafai and Amiri 1998). The increased heat and mass transport in porous media is extremely useful in mini-to-microscale channels due to the tortuous nature of the path and a very high surface area-to-volume ratio (Hadim 1994). In these mini-scaled systems, turbulent flow regimes are difficult to achieve, and the flow is largely in the laminar regime; nevertheless, the utilization of porous media in these systems is advantageous (Ward 1964; Hooman and Gorji-Bandpy 2005). These flow systems can be found in a variety of applications, including electronic device heat management, nuclear reactors, solar receivers, and avionics, to mention a few.

One of the popular thermal management devices, loop heat pipe (LHP), as shown in Fig. 1 is composed primarily of three components: an evaporator, a condenser, and an adiabatic section (liquid and vapor lines, where no heat transfer occurs), out of which evaporator is the most crucial component to design, build, and control the operation due to existence of multi-phase flow through porous media (Maydanik 2005; Launay et al. 2007; Ali et al. 2021).

As shown in Fig. 1b, the evaporator of a heat pipe consists of porous structures, which provide the sufficient capillary pressure to drive the flow in the heat pipes and also enhance the heat transfer in the evaporator. Generally, three basic types of porous structures are utilized in heat pipes: grooved, wick, and sintered. The grooves are highly ordered patterns which are inexpensive to manufacture but functionally poor for capillary action and heat transfer. The sintered porous structures are the most expensive to manufacture due to their high stochasticity and, however, have the best heat transfer and capillary action capabilities. Moreover, wick structures are optimum choice for intermediate performance and manufacturing (Deng et al. 2013; Ergun and Orning 1949).

For the numerical analysis of transport phenomena in the porous media, there are two approaches which are popularly followed: direct and indirect modeling. In the
indirect modeling, actual porous geometry is not constructed; instead, the fluid domain is virtually assigned as the porous zone using associated viscous and inertial resistances to it, and then, the porous media flow simulations (popularly known as Darcy advection models (Kaviany 1995)) are performed (Nield and Bejan 2013; Givler and Altabelli 1994; Liu et al. 1994). This approach is sufficiently effective in hydrodynamic studies of single-phase flow though porous media, and based on the flow rate regimes or porosity values, different models are used (refer to Eq. 14, 15, 16 in Appendix) (Breugem 2007; Hommel et al. 2018; Whitaker 1986). However, in case of multiphase flow in porous media, this approach may not be as much effective to handle the complicated relationship of relative permeability and capillary pressure with the saturation (Valvatne et al. 2005; Wang 1997; Blunt et al. 2013). On the other hand, in the direct modeling, the porous structure (or solid matrix) is digitally constructed as a CAD model and the pore-scale simulation is performed over this geometry (Wang 1997; Dawe et al. 1987; Cromwell et al. 1984). Both single-phase flow and multiphase flow can be effectively and accurately handled due to lesser involvement of models. Furthermore, on considering the direct modeling approach, two kinds of solid matrix ‘ordered and random’ are generally used by the researchers. Geometrical reconstruction of random porous structures is a challenging task in and of itself, and it also necessitates very expensive equipment such as X-ray CT scanners and MRI machines, making the entire process uneconomical (Wang et al. 1984; Dong and Blunt 2009; Elkins and Alley 2007; Gao et al. Oct. 2018; Hill et al. 2001). Processing scanned tomographic images into a 3D digital model is similarly time-consuming and error-prone, resulting in a geometric representation that differs from the original porous structures. On the other hand, with the advent of 3D printing ordered solid matrix is preferred due to controlled performance of the system and repeatability; thus, many researchers have attempted to create simplified ordered structures (both two and three dimensional) to resemble with the porous media. In some cases, mostly circular- or square-shaped obstructions were used in both patterned and random distributions (Chen et al. 2020; Ranjan et al. 2012), whereas for other cases, spherical-shaped particles were used in simple cubic (SC), face-centered cubic (FCC), body-centered cubic (BCC), and hexagonal closed packing (HCP) arrangements (Yang et al. 2013; Yang et al. 2016; Bodla et al. 2010). The errors associated with the geometry will therefore hamper the system’s performance and may not provide the repeatability. As a result, when compared to random structures, using ordered porous structures for analysis and application is both cost-effective, accurate, and performance consistent (Krishnan et al. 2005; Al-Ketan and Abu Al-Rub 2019).

The pore-scale numerical simulations in the ordered porous structures have been performed for the flow in order to extract the parameters for the macro-scale modeling and also to determine the limit for the applicability of the Darcy law. TPMS-based ordered porous geometries are used, and numerical analyses are performed in the present work to characterize the quantities such as permeability, inertial drag factor, and also the flow range up to which the Darcian regime prevails. The manuscript is organized as follows: Sect. 2 defines the problem statement, followed by Sect. 3 which contains the governing equations and boundary conditions for pore-scale simulations. The solution methodology is described in Sect. 4, the results and discussion are presented in Sect. 5, and lastly, the present work’s conclusions are presented in Sect. 6. The appendix describes the equations, some relationships, and parameters for the macro-scale modeling of the flow in porous medium.
2 Problem Statement

| TPMS lattice name | Geometry of TPMS lattice | Lattice structure of a unit cell | Interior surface of a unit cell |
|-------------------|--------------------------|---------------------------------|--------------------------------|
| Diamond           | ![Diamond Geometry]      | ![Diamond Lattice]              | ![Diamond Interior]           |
| I-WP              | ![I-WP Geometry]        | ![I-WP Lattice]                 | ![I-WP Interior]             |
| Primitive         | ![Primitive Geometry]   | ![Primitive Lattice]            | ![Primitive Interior]         |
| Gyroid            | ![Gyroid Geometry]      | ![Gyroid Lattice]               | ![Gyroid Interior]           |

Fig. 2 Solid subdomain and interior surface of four porous structures used in the study based on TPMS lattices

In this study, the hydrodynamics of the flow within the channel of square-cross section filled with porous medium has been characterized by using the pore-scale simulations. The ordered porous structures are generated using Triply Periodic Minimal Surfaces (TPMS) method, and four different TPMS lattices, namely Diamond, I-WP, Primitive, and Gyroid, are developed as shown in Fig. 2. Furthermore, the entire domain is divided into two
subdomains, one of which, ‘void subdomain,’ is always treated as a fluid zone, while the other, ‘solid subdomain,’ is treated as (a) a solid zone, (b) a fluid zone separated from the ‘void subdomain’ by an interface wall, and (c) a porous zone with specified microporosity. The lattices created are described below:

### 2.1 TPMS Lattice and their Level-Set Equations

A minimal surface is a surface with a mean curvature of zero, is known as a TPMS, and is infinite and repetitive in three dimensions. To develop a minimal surface, different mathematical approaches were proposed, such as nodal surface approximation, and phase field approach. The level-set approximation strategy, on the other hand, is the simplest and most widely used method and has been used in the present work (Al-Ketan and Abu Al-Rub 2021; Jung et al. 2007).

Level-set equations are a group of trigonometric functions in three dimensions that satisfy the equality condition \( f(X, Y, Z) = C \). Four sets of equations are given below and can produce four different lattices (Al-Ketan et al. 2018; Hetsroni et al. 2006).

1. **Schwarz-Diamond (Diamond)**
   \[
   f(X, Y, Z) = \cos X \cos Y \cos Z - \sin X \sin Y \sin Z
   \]

2. **Schoen-IWP (I-WP)**
   \[
   f(X, Y, Z) = 2(\cos X \cos Y + \cos Y \cos Z + \cos Z \cos X) - (\cos 2X + \cos 2Y + \cos 2Z)
   \]

3. **Schwarz-Primitive (Primitive)**
   \[
   f(X, Y, Z) = \cos X + \cos Y + \cos Z
   \]

4. **Schoen-Gyroid (Gyroid)**
   \[
   f(X, Y, Z) = \sin X \cos Y + \sin Y \cos Z + \sin Z \cos X
   \]

where \( X = 2a\pi x, Y = 2b\pi y, Z = 2c\pi z \) and \( a, b, c \) are constants related to the unit cell size in corresponding \( x, y, z \) directions. Iso-surfaces divide space into two subvolumes when the level-set equation is assessed at \( C = 0 \). By changing the value of \( C \), we can generate the lattice of desired porosity. The domain is divided in two sub-volumes as solid subdomain and void subdomain which are separated by the minimal surface. Using above equations, an STL file containing information of surfaces is generated by open-source software MS-Lattice (Al-Ketan and Abu Al-Rub 2021) which uses the surface minimization algorithm in MATLAB environment. The lattices which are used in the present study are shown in Fig. 2. The details of different treatments of each lattice are shown in Fig. 3.

The effective porosity in Type 1 is same as original, which is equal to the void fraction of 0.32 of the lattice. In case of Type 2, effective porosity is approximately equal to 1 as two subdomains are only separated by zero thickness wall. However, they are different from a normal channel flow (without porous inserts), as this zero-thickness wall will allow the growth of boundary layer over it; therefore, it offers the resistance to flow. By the induction of microporosity \( (\varepsilon^* = 0.32) \) in Type 3 lattice, the effective porosity increases to 0.54. The pore-scale simulation has been performed on the pores of the lattice for first two lattices, and porous media modeling (Darcy–Forchheimer–Brinkman...
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| Description                        | Type 1                                      | Type 2                                      | Type 3                                      |
|------------------------------------|---------------------------------------------|---------------------------------------------|---------------------------------------------|
| Treatment of solid subdomain in   | Solid zone (No equation)                    | Fluid zone (Navier-Stokes equation)         | Porous zone (Darcy-Forchheimer equation)    |
| the solver                         |                                             |                                             |                                             |
| Physical representation of the    | ![Solid zone](image)                        | ![Fluid zone](image)                       | ![Micro porous zone](image)                 |
| porous medium created              |                                             |                                             |                                             |
| Treatment of the interstitial      | Impermeable wall (Fluid from void subdomain| Impermeable wall (Fluid between two         | Permeable wall (Fluid from void subdomain   |
| surface between two subdomains     | can not penetrate into solid zone)          | subdomains can not penetrate through        | can penetrate into porous zone)             |
|                                   |                                             | interface or separating wall)               |                                             |
| Effective porosity calculation     | $\varepsilon = \frac{V_f}{V_f + V_s}$      | $\varepsilon = \frac{1}{1 + \left(\frac{V_s}{V_f}\right)}$ | $\varepsilon = \frac{V_f + \varepsilon^* V_s}{V_f + V_s}$ |
| ($\varepsilon_0$ is the original  | $\varepsilon = \frac{1}{1 + \left(\frac{V_s}{V_f}\right)}$ | $\varepsilon \to 1$                         | $\varepsilon = \frac{V_f + \varepsilon^* \left(V_f / \varepsilon_0 - V_f\right)}{V_f + V_s}$ |
| porosity or void fraction)         | $\varepsilon = \varepsilon_0$              | $\varepsilon \to 1$                         | $\varepsilon = \varepsilon_0 + \varepsilon^*(1 - \varepsilon_0)$ |
|                                   | $\varepsilon_{eff} \approx 0.32$           | $\varepsilon_{eff} \to 1$                  | $\varepsilon_{eff} = 0.54$                  |

Fig. 3 Description and effective porosity calculation in three different types of treatments used for the solid subdomain

Extended Darcy model with nonlinear inertial terms is used as the reference model to calculate the flow properties of porous media, such as

\[ \dot{m} = \mu LRe \]

model) has been used in the microporous region, and flow parameters in the microporous zone have been obtained from experimental results of Hetsroni (2006) and are shown in Table A3 (Hetsroni et al. 2006). Therefore, by different treatments of solid zone in the TPMS lattice, three different levels of porosity are obtained as low porosity to super-porosity and a total of 12 scenarios of flow in porous media have been considered in the present work.

Usually, three different kinds of Reynolds number definitions have been in use in the literature, as (a) based on Brinkman screening length ($Re_K = \rho U K^2 / \mu$), (b) based on pore size ($Re_p = \rho U L / \mu$), and (c) based on channel size ($Re = \rho U L / \mu$), and keeping the flow scenario in the loop heat pipe, the present work is performed for different values of mass flow rate (which is related as $\dot{m} = \mu LRe$) and for three sets of porosity values $\varepsilon_{eff} \approx 0.32, 0.54, \text{ and } \sim 1$. Extended Darcy model with nonlinear inertial terms is used as the reference model to calculate the flow properties of porous media, such
as permeability (or Darcy number) and inertial drag coefficients through estimation of pressure developed across the lattice.

3 Governing Equations

3.1 Governing Equations for Flow at Pore Scale

At pore scale, the 3D Navier–Stokes equations are solved for above lattices with the assumptions of steady, incompressible, and laminar flow. The fluid is Newtonian with constant thermo-physical properties ($\rho = 998.2 \text{ kg m}^{-3}$ and $\mu = 0.001003 \text{ kg m}^{-1} \text{sec}^{-1}$). Due to small size of the pores through which the fluid is being transported, the gravitational force is neglected. The equations which are solved for the fluid zones are written as follows:

$$\nabla \cdot \vec{v} = 0 \quad (5)$$

$$\rho (\vec{v} \cdot \nabla) \vec{v} = -\nabla p + \nabla \cdot \left[ \mu \left( \nabla \vec{v} + (\nabla \vec{v})^T \right) \right]; \quad \text{for } \vec{v} = \vec{u} + \vec{v} + \vec{w}$$

where the $\vec{v}$ and $p$ are the pore-scale velocity and pressure, respectively. $\rho$ and $\mu$ are the fluid density and dynamic viscosity, respectively.

3.2 Governing Equations for Flow in Microporosity (Only in Case of Type 3)

The flow in microporous region (solid subdomain in Type 3) is modeled by using extended Darcy equation, as given below:

$$\nabla \cdot \vec{V} = 0 \quad (7)$$

$$\nabla P = -\frac{\mu}{K} \vec{V} - \frac{\rho \beta}{\sqrt{K}} \vec{V} |\vec{V}|; \quad \text{for } \vec{V} = \vec{U} + \vec{V} + \vec{W}$$

where $\rho$ and $\vec{V}$ are Eularian extrinsic phase averaged pressure and velocity. The linear term is dominant in the low flow rates, called Darcian regime, whereas the nonlinear terms are dominant in the relatively higher flow rates, called as non-Darcian regime.

3.3 Boundary Conditions

Figure 4 shows the representation of TPMS lattice with a translational periodic boundary condition between inlet and outlet to resemble with a channel of infinite length or hydrodynamically developed flow condition (Patankar et al. 1977). The other boundaries, i.e., top, bottom, front, and back, are impermeable wall of the channel. The level-set wall which separates two subdomains (solid & void) is internal walls, and no-slip boundary conditions are used on them. These walls are treated as permeable walls (or interior) for microporosity case (Type 3). The mathematical representation of boundary conditions is summarized in Table 1. The inlet–outlet pair is assigned with translational
periodic boundary conditions, which allows the pressure gradient to remain constant along the periodic length while keeping the velocity profile identical in the streamwise direction. All the impermeable walls are assigned with the no-slip boundary condition.

### 3.4 Tortuosity

Tortuosity is a geometrical feature of porous media that quantify the straightness of the fluid channel formed by interconnecting pores. It is a dimensionless ratio of the length of the actual path to the length of the minimum path between two reference points in the flow direction (refer Eq. 17 in appendix). In the absence of any obstacle, its value for a channel must be unity; however, in case of a porous medium, depending upon the

![Fig. 4 Representation of TPMS lattice with the translational periodic condition in one direction](image-url)
morphology of the solid matrix, the value could be significantly larger than unity. In the present study, the tortuosity of the lattices is calculated by using the lengths of fluid trajectories in the interconnected voids and are mentioned in following table. Table 2 shows the tortuosity values of four lattices for void subdomain. Moreover, in Type 2 case, the solid subdomain is treated as a fluid zone, and therefore, corresponding values of tortuosity are also calculated and shown in the same table.

4 Solution Methodology

4.1 Mesh Generation Process

In the MS-Lattice software, four lattices with void subdomain’s volume fraction as 32% are generated and geometry information is stored in STL format. The lattice is generated for unit cell of size 1 mm and repeated till 4 mm length in three directions and stored in the STL file, and this STL file is then imported into the ANSYS® ICEM-CFD software to create the mesh of void and solid subdomains inside the bounding surfaces of lattice. The topology is properly checked at each step to ensure the connectivity of different surfaces and to avoid the negative volume during mesh generation. Unstructured tetrahedral mesh is employed for grid generation to deal with the complexity of the geometry as shown in the figure below.

4.2 Setting of Simulation Model

The 3-D steady-state and incompressible governing equations are solved by using ANSYS® FLUENT with pressure-based solution approach. SIMPLE algorithm is used for pressure–velocity coupling, with standard scheme for discretization of the pressure and second-order upwind scheme for the momentum. The iterations are performed up to the solution and reached a convergence level of $10^{-6}$ for continuity and $10^{-8}$ for the $x$, $y$, and $z$-momentum equations, respectively (Fluent Tutorial Guide 2018).

4.3 Grid Sensitivity Tests

The grid sensitivity test is performed in empty channel of square cross-section by taking four different meshes, coarser (Mesh 1) to finer (Mesh 4), to verify the result is grid independent or not. Non-uniform finer grids are used near the wall compared to the core to resolve the gradients with reasonable accuracy. Minimum orthogonality and maximum aspect ratio are two parameters which provides the idea of the quality of mesh generated. Fig. 5 shows the mesh generated for one of the cases in the different lattices. The aspect ratio measures the ratio of longest edge to the shortest edge in a tetrahedral cell, whereas orthogonality is the measure of cosine of angle between the line joining cell centers and the normal of shared face. In ideal situation, a good-quality mesh has value of orthogonality as well as aspect ratio to be equal to unity. However, the acceptable values of minimum orthogonality and maximum aspect ratio are 0.1 and 35, respectively (Fluent Tutorial Guide 2018). The details of mesh used in this study are shown in Table 3. It is also showing the results of the grid sensitivity tests performed on these grids using the pressure gradient as the measuring parameter. Last column in this table is showing the consecutive change in the pressure gradient from its previous value.
Table 3  Details of grids and results of grid dependency study

| S. no. | No. of nodes | Minimum orthogonality | Maximum aspect ratio | Pressure gradient (Pa/m) | Difference between consecutive cases (%) |
|--------|--------------|------------------------|----------------------|--------------------------|------------------------------------------|
| Mesh 1 | 681          | 0.4                    | 8.2                  | 559.5                    | -                                        |
| Mesh 2 | 3301         | 0.4                    | 8.8                  | 534.1                    | 4.47                                     |
| Mesh 3 | 47,540       | 0.4                    | 8.9                  | 520.6                    | 2.62                                     |
| Mesh 4 | 63,200       | 0.4                    | 9.7                  | 521.7                    | 0.19                                     |

Fig. 5  Display of mesh in the void and solid subdomains for the four lattices, namely: a Diamond b I-WP c Primitive, and d Gyroid
The pressure gradients are compared for the mass flow rate at $4 \times 10^{-4}$ kg. In addition to that, velocity and axial pressure profiles are also compared as shown in Fig. 6. The difference in the value of pressure gradient between Mesh 1 and Mesh 2 is 4.47%, while between Mesh 2 and Mesh 3 it is 2.62%, and between Mesh 3 and Mesh 4 it is 0.19%. The relative difference in the profiles of velocity and pressure variation is also negligibly small between the ‘Mesh 3’ grid and the ‘Mesh 4.’ Therefore, based on this test, Mesh 3 is chosen for further simulations in the present study.

4.4 Validation of CFD Model

The above governing equation, boundary condition, and solution algorithm are validated for the flow in a mini square cross-sectional channel with periodic boundary condition between the inlet and outlet pair. The results of the numerical calculations are compared to empirical correlations given by Bahrami (Bahrami et al. 2005) and Shah (Shah 1978) as shown in Eq. (9) and (10), respectively. For this pressure drop per unit length is plotted and compared for different Reynolds number, ranging from $Re = 0.01$ to 100 (corresponding to $\dot{m} = 4 \times 10^{-8}$ to $4 \times 10^{-4}$ kg). The present results as shown in Fig. 7 match perfectly and therefore indicate the accuracy of the present numerical model.

\[
\frac{\Delta p}{L} = 4\left(\frac{1}{3} - \frac{64}{\pi^5} \tanh \left(\frac{\pi}{2}\right)\right)^{-1} \frac{\mu}{\rho A^2 \dot{m}}
\]  

(9)

\[
\frac{\Delta p}{L} = \left(\frac{56.92}{2}\right) \frac{\mu}{\rho A^2 \dot{m}}
\]  

(10)

Fig. 6  a Velocity profiles, and b Periodic pressure drops for four different grids used in the grid sensitivity test
5 Results and Discussions

The pore-scale simulations have been performed for the flow in the TPMS lattice of twelve cases as described in Sect. 2 for mass flow rate ranging from $4 \times 10^{-8}$ to $4 \times 10^{-4}$ kg/s. The direct numerical simulations have been used for the first two types of treatments, while both direct numerical modeling for pore scale and macroscale modeling for the microporosity have been used for Type 3 treatment. The results shown in this section correspond to the macroscopic pressure drop for different values of overall mass flow rate imposed via Reynolds number, as described in Sect. 2.1. Essential flow properties of a porous media, permeability, and inertial drag factors are calculated and compared.

It has been observed that the relation between the pressure drop and the average velocity is linear for a very small flow rate ($Re_p \ll 1$) which is known as the Darcy law, and it is nonlinear for a relatively high value of the flow rate ($Re_p > 1$) (Forchheimer 1901; Chen et al. 1987). Both quadratic and cubic behaviors have been reported in different zones of $Re_K$ and concluded that the inertial term is initially cubic and takes the quadratic for with the increase in $Re_K$ (Rojas and Koplik 1998). However, it has been reported by many researchers using the homogenization theory (Skjetne and Auriault 1999) that the order of first non-linear inertia term is cubic (Skjetne and Auriault 1999; Sanchez-Palencia 1980; Adler et al. 2012, 2013; Mei and Auriault 1991). Therefore, in the present work, it has been attempted to verify the validity of cubic relation between the pressure gradient and mean velocity data by fitting a third-order polynomial, as shown in the following equation.

$$\frac{\Delta P}{L} = AU + BU^2 + CU^3$$

(11)

In this third-order polynomial when compared with the extended-Darcy model as reported in Rojas and Koplik (1998) (Eq. 12), the values of three parameters such as permeability ($K$) and the nonlinear drag coefficients ($\beta$ and $\alpha$) are calculated as shown in Eq. 13.
The present numerical simulation data are fitted with a cubic polynomial to obtain the flow parameters by comparing the coefficients.

### 5.1 Type 1: Solid Subdomain is Treated as Solid Zone

In this case, the solid subdomain of lattice is treated as solid zone and the fluid flow is taking place only through the void subdomain which is 32% (porosity $\varepsilon = 0.32$) of the total cubical volume. Figure 8 shows the fluid path in the four lattices using flow trajectories at mass flow rate $4 \times 10^{-4}$ kg/sec. The lattices with high tortuosity, i.e., Diamond and Gyroid, have relatively low value of pore velocity 0.270 and 0.254 m/s, respectively, whereas the lattices with low tortuosity, i.e., I-WP and Primitive, have relatively high pore velocity, i.e., 0.455 and 0.548 m/s, respectively, as shown in Fig. 8. Table 4 shows the maximum velocity (within the lattice) and pressure gradient for different mean velocity through the channel. It can be seen that the linear relationship between the velocity and pressure gradient is maintained up to lower flow rate ($U < 0.0025$ m/s or corresponding $Re < 10$), and it deviates at higher flow rate.

The drop in pressure with different mean velocity is shown in Fig. 9. In these plots, the Darcy regime is identified by the constant value of slope, observed in the low velocity range. Maximum value of slope is observed in I-WP lattice, followed by Diamond and Primitive lattices, and has minimum slope for the Gyroid lattice. The coefficient of quadratic drag term is significantly smaller and, therefore, negligible than the cubic drag term. The coefficient of cubic term has higher values for Primitive and Gyroid lattices, also visible by the curvatures of their corresponding fitted curves. Moreover, the Diamond and I-WP lattices have one-order lower values of the cubic coefficients.

Table 5 depicts the permeability and the coefficients of non-Darcy terms ($\alpha$ and $\beta$) for all lattices of Type 1 treatment for the solid subdomain. The I-WP lattice shows the minimum Darcy number as $Da = 1.77 \times 10^{-5}$, followed by Diamond and Primitive lattices as $2.48 \times 10^{-5}$ and $2.74 \times 10^{-5}$, respectively, and maximum for the Gyroid lattice as $Da = 3.91 \times 10^{-5}$. Coefficient of quadratic drag ($\beta$) is many orders smaller than the coefficient of cubic drag ($\alpha$) and therefore can be safely neglected from the fitted equation. Moreover, the cubic drag coefficient $\alpha$ has maximum and minimum values in case of Gyroid and Diamond lattices, respectively.

The pressure drop monotonically increases with the mass flow rate for all the lattices as shown in Fig. 10. It is observed that the I-WP and Gyroid lattice have, respectively, highest and lowest pressure drop for all mass flow rate studied in the present work. This indicates the I-WP lattice offers maximum resistance to flow followed by Diamond and Primitive lattices and minimum for the Gyroid lattice.

\[
\frac{\Delta P}{L} = \frac{\mu}{K} U + \frac{\beta \rho}{\sqrt{K}} U^2 + \frac{\alpha \rho^2}{\mu} U^3
\]  

(12)

\[
K = \frac{\mu}{A}; \quad \beta = \frac{B \sqrt{K}}{\rho}; \quad \alpha = \frac{C \mu}{\rho^2}
\]

(13)
5.2 Type 2: Solid Subdomain is Treated as Fluid Zone

For this type, solid subdomain of the lattice is treated as the fluid zone; thus, the two flow regions are separated by a zero-thickness wall which may break the boundary layer growth of the top, bottom, front, and back walls which encloses the channel, while
Table 4  Maximum velocity and pressure gradient at different mass flow rates for Type 1 treatment of solid subdomain

| Mean velocity × 10⁻⁵ (m/s) | Diamond | I-WP | Primitive | Gyroid |
|---------------------------|---------|------|-----------|--------|
|                           | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) |
| 0.25                      | 2.8     | 0.0061 | 5.3       | 0.0081 | 6.7     | 0.0042 |
| 2.5           | 28      | 0.061  | 53        | 0.081  | 67      | 0.042  |
| 25            | 280     | 0.61   | 530       | 0.81   | 670     | 0.42   |
| 250           | 2800    | 6.1    | 5200      | 8.2    | 6600    | 4.4    |
| 2500          | 27,000  | 74.2   | 45,000    | 100.3  | 54,000  | 76     |
|               |         |        |           |        | 25,000  | 58.3   |
allowing the boundary layer growth on the interface walls which separates the two subdomains. Therefore, almost the entire volume is effectively open for the flow, and this type of super-porosity is common in foams, particularly metal foams. Figure 11 depicts the fluid trajectories for the lattices with Type 2 treatment of the solid subdomain. Since the solid and void subdomains are completely separated by the internal walls, the tortuosity value of void subdomain will not change and remain same as Type 1. In case of Diamond (Fig. 11a), due to the distorted shape of the interstitial wall which separates the two flow regions, the overall flow path is also tortuous in shape. In I-WP lattice (Fig. 11b), the flow path is observed to be following a combination of straight and helical path for two subdomains. In Primitive lattice (Fig. 11c), the flow path is found to be relatively straighter in both subdomains. Lastly in Gyroid lattice (Fig. 11d), for both subdomains, solid and void, the fluid paths are found to be following helical trajectories. The straightness of fluid trajectory also indicates the lower reduction in the maximum value of pore velocity; therefore, it is observed in Fig. 11 and also in Table 7 that the lattices with overall high tortuosity, i.e., Diamond, I-WP, and Gyroid, have relatively low value of max. pore velocity (0.075, 0.078, and 0.090 m/s, respectively), whereas the Primitive lattice has relatively high max. pore velocity as 0.137 m/s.

The pressure gradient variation with mean velocity is shown in Fig. 12 for four lattices with Type 2 treatment. It is observed that Diamond lattice has the maximum value of slope, followed by I-WP and Gyroid lattices, and Primitive lattice has the minimum.

![Graphs showing pressure gradient variation with mean velocity for different lattices.](image)
**Table 5** Porous media characteristics for Type 1 treatment of solid subdomain

| Lattice name | Porosity, $\varepsilon$ | Tortuosity, $\tau$ | Permeability, K (m$^2$) | Darcy number, Da | Quadratic coefficient, $\beta$ | Cubic coefficient, $\alpha$ |
|--------------|--------------------------|---------------------|--------------------------|------------------|-------------------------------|-------------------------------|
| Diamond      | 0.32                     | 1.325               | $3.98 \times 10^{-10}$   | $2.48 \times 10^{-5}$ | $3.83 \times 10^{-8}$           | 0.72                          |
| I-WP         | 0.32                     | 1.051               | $2.83 \times 10^{-10}$   | $1.77 \times 10^{-5}$ | $3.70 \times 10^{-8}$           | 0.78                          |
| Primitive    | 0.32                     | 1.033               | $4.39 \times 10^{-10}$   | $2.74 \times 10^{-5}$ | $8.05 \times 10^{-8}$           | 1.29                          |
| Gyroid       | 0.32                     | 1.256               | $6.24 \times 10^{-10}$   | $3.91 \times 10^{-5}$ | $3.19 \times 10^{-8}$           | 1.18                          |
slope. Moreover, the coefficient of cubic term has largest value for I-WP lattice, then followed by Gyroid and Diamond lattices, and has the smallest value for Primitive lattice.

Table 6 shows Darcy number and coefficients of non-Darcy terms, $\alpha$ and $\beta$ for Type 2 treatment of lattices. The mean tortuosity is volume-weighted average of individual tortuosity of void and solid subdomains. Darcy number has minimum value ($2.29 \times 10^{-4}$) for Diamond lattice, followed by I-WP and Gyroid ($3.20 \times 10^{-4}$ and $3.26 \times 10^{-4}$, respectively), and has the maximum value ($4.48 \times 10^{-4}$) for Primitive lattice. The cubic drag coefficient, $\alpha$, in Type 2 treatment is in general one order smaller than the Type 1 case, as observed in previous section. It has largest and smallest values in I-WP and Primitive lattices, respectively. In comparison with the Type 1 lattice, here in Type 2 lattice, pressure drops are significantly reduced, and the Darcy number is increased by one order (ref. Tables 5 and 6).

Table 7 presents the maximum velocity and pressure gradient for Type 2 lattices. On comparing the pore velocity for mean velocity of 0.025 m/s, it is observed that that the largest value of velocity is obtained for Primitive lattice, which may be due to its comparatively smaller tortuosity by which it offers lower resistance (since it also has maximum permeability and minimum drag factor) to the flow. Furthermore, Diamond and I-WP lattices are showing nearly similar value of max. velocity to be 0.075 and 0.078 m/s, respectively. Lastly, the Gyroid lattice is showing smallest value of pore velocity (0.09 m/s) at the same mean velocity, i.e., 0.025 m/s.

The variation in pressure drop with mass flow rate and channel size-based Reynolds number for all the lattices with Type 2 treatment of solid subdomain is depicted in Fig. 13. The Diamond and Primitive lattices show the highest and the lowest pressure drops, respectively, for the entire range of the Reynolds number, which means that their resistance to pass the fluid through the tortuous path provided by the complicated thin-walled interior is, respectively, maximum and minimum. This is also clearly visible in their permeability values. For the remaining two lattices, I-WP and Gyroid, the drop in pressure or flow resistance is almost same for the entire range of flow rates.
| Lattice name | Porosity, $\varepsilon$ | Mean tortuosity, $\tau$ | Permeability, $K$ (m$^2$) | Darcy number, Da | Quadratic term coefficient, $\beta$ | Cubic term coefficient, $\alpha$ |
|--------------|--------------------------|--------------------------|--------------------------|-----------------|-------------------------------|-------------------------------|
| Diamond      | ~1                       | 1.223                    | $3.67 \times 10^{-9}$    | $2.29 \times 10^{-4}$ | $1.15 \times 10^{-7}$ | 0.053                         |
| I-WP         | ~1                       | 1.099                    | $5.11 \times 10^{-9}$    | $3.20 \times 10^{-4}$ | $1.51 \times 10^{-7}$ | 0.075                         |
| Primitive    | ~1                       | 1.036                    | $7.17 \times 10^{-9}$    | $4.48 \times 10^{-4}$ | $3.06 \times 10^{-7}$ | 0.021                         |
| Gyroid       | ~1                       | 1.268                    | $5.22 \times 10^{-9}$    | $3.26 \times 10^{-4}$ | $1.35 \times 10^{-7}$ | 0.055                         |
5.3 Type 3: Solid Subdomain is Treated as Porous Zone

In this case, the solid subdomain of the lattices is considered as the porous zone with
Table 7 Maximum velocity and pressure gradient at different mass flow rates for Type 2 treatment of solid subdomain

| Mean velocity × 10⁻⁵ (m/s) | Diamond          | I-WP            | Primitive        | Gyroid           |
|---------------------------|------------------|-----------------|------------------|------------------|
|                           | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity × 10⁻⁵ (m/s) | Pressure gradient (kPa/m) |
| 0.25                      | 0.71             | 0.00067         | 0.71             | 0.00046          | 1.5             | 0.00033          | 0.87             | 0.00044          |
| 2.5                       | 7.1              | 0.0067          | 7.1              | 0.0046           | 15              | 0.0033           | 8.7              | 0.0044           |
| 25                        | 71               | 0.067           | 71               | 0.046            | 150             | 0.033            | 87               | 0.044            |
| 250                       | 710              | 0.67            | 710              | 0.46             | 1500            | 0.33             | 870              | 0.45             |
| 2500                      | 7500             | 7.7             | 7800             | 6.1              | 14,000          | 3.8              | 9000             | 5.6              |
microporosity value of 0.32, with the consideration that the pore size in the solid subdomain is very small (microporosity) compared to the pore scale of the lattice. The pore-scale numerical simulation (Navier–Stokes equations) is performed for the void subdomain of the lattice, while porous media modeling (Darcy-Forchheimer equations) is considered for

![Figure 12](image1.png) Variation in dimensional pressure gradient with mean velocity for a Diamond, b I-WP, c Primitive, and d Gyroid lattices with Type 2 treatment for the solid subdomain

![Figure 13](image2.png) Variation in pressure drop with mass flow rate and channel size-based Reynolds number for lattices with Type 2 treatment of solid subdomain

![Graph](image3.png)
| Name of the lattice | Flow in the lattice | Enlarge view of flow in a pore |
|---------------------|---------------------|-----------------------------|
| **(a) Diamond** \( (\tau_{\text{void}} = 1.325) \) | ![Flow in Diamond lattice](image) | ![Enlarge view of flow in a pore](image) |
| **(b) I-WP** \( (\tau_{\text{void}} = 1.051) \) | ![Flow in I-WP lattice](image) | ![Enlarge view of flow in a pore](image) |
| **(c) Primitive** \( (\tau_{\text{void}} = 1.033) \) | ![Flow in Primitive lattice](image) | ![Enlarge view of flow in a pore](image) |
| **(d) Gyroid** \( (\tau_{\text{void}} = 1.256) \) | ![Flow in Gyroid lattice](image) | ![Enlarge view of flow in a pore](image) |

**Fig. 14** Flow trajectories in four lattices and their unit cell or pore for Type 3 treatment for mass flow rate \( 4 \times 10^{-4} \text{ kg/s} \)
the solid subdomain assigned with the microporosity. It must be noted that the internal surface between void subdomain and solid subdomain is made permeable in the present case by treating it as interior and allowing the fluid to pass through it. The simulation results for this case are presented below.

The trajectories of fluid in the lattices at mass flow rate of $4 \times 10^{-4}$ kg/s (mean velocity 0.025 m/s corresponding to $Re = 100$) are illustrated in Fig. 14. Two subdomains are present in the lattice: first is void subdomain, in which the fluid flows without additional resistance (infinite permeability), whereas the second subdomain is assigned as the porous zone with induced viscous and inertial resistances caused due to micro-porosity; therefore, it has some finite permeability. The trajectory of the fluid path in the void subdomain is helical in Diamond lattice. However, in the microporous region, the magnitude of the flow velocity is less as compared to its void subdomain. Further, the shape of flow path in the void subdomain is approximately straight, while the flow path is zig-zag in the microporous zone of I-WP lattice. In Primitive lattice, the shape of flow path is similar to the I-WP lattice, in which the flow paths are almost straight and zig-zag, in the void and solid (microporous zone) subdomain, respectively, making it to have less tortuous. Lastly, the flow path in the void subdomain takes a helical shape in Gyroid lattice, and porous zone shows smaller magnitude of the velocity similar to the Diamond lattice, and therefore, overall flow path is more tortuous. The tortuosity in case of Type 3 lattice is similar to the Type 1 for the void subdomain. Since, for the solid subdomain (or microporous region), porous media flow modeling is used and pore-scale simulation

Fig. 15 Variation in dimensional pressure gradient with mean velocity for a Diamond, b I-WP, c Primitive, and d Gyroid lattices with Type 3 treatment for the solid subdomain
is not performed, actual path of the fluid and tortuosity are not possible to determine in the microporous region of the present case, and thus, only the tortuosity of void subdomain is shown. Similar to the Type 1, again the lattices with high tortuosity, i.e., Diamond and Gyroid, has relatively low value of pore velocity 0.226 m/s, whereas the lattices with low tortuosity, i.e., I-WP and Primitive, have relatively high pore velocity i.e., 0.383 m/s and 0.464 m/s, respectively, as shown in Fig. 14.

The variation in pressure gradient with mean velocity for Type 3 treatment is presented in Fig. 15. The slope (coefficient of linear term) in this treatment is one order higher than the previous case (Type 2) and is has similar order of pressure drop as observed in Type 1 case. Maximum value of slope is observed in I-WP lattice, followed by Diamond and Primitive lattices, and the Gyroid lattice has the minimum value. Primitive and Gyroid lattices have higher, whereas Diamond and I-WP lattices have lower values of cubic term coefficient which is also visible qualitatively by the curvature of their fit curves.

Porous media properties for all lattices are presented in Table 8 in Type 3 treatment of solid subdomain. The Darcy number is minimum for the I-WP lattice ($Da = 2.44 \times 10^{-5}$), followed by Diamond and Primitive lattices ($3.31 \times 10^{-5}$ and $3.69 \times 10^{-5}$, respectively), and has the maximum value $Da = 4.88 \times 10^{-5}$ for the Gyroid lattice. Cubic drag coefficient, $\alpha$, has lowest value as 0.48 for Diamond lattice and highest value as 0.92 for Primitive lattice. It is similar in magnitude of orders as the Type 1 case.

The maximum velocity and pressure gradient for different values of mean velocity are shown in Table 9. It is worth to note that the magnitude of maximum velocity is the highest in Type 1 and the lowest in Type 2 while, for Type 3, the magnitude of the maximum velocity lies in between.

The pressure drop with different orders of mass flow rate for Type 3 treatment of solid subdomain is shown in Fig. 16. Pressure drop is observed to be maximum and minimum for the I-WP and Gyroid lattices, respectively, for the entire range of mass flow rate. Pressure drop in case of Primitive lattice is same as the Gyroid lattice for the low flow rate, $\dot{m} < 4 \times 10^{-5}$ kg/s (or Re $< 10$), but sharply increases to match with the I-WP lattice when the flow rate reaches to the value of $\dot{m} \sim 4 \times 10^{-4}$ kg/s (Re $\sim 100$).

### 5.4 Comparison Between Three Treatments of Solid Subdomain

The I-WP Type 1 lattice has the highest pressure drop and thus the lowest flow conductance of all 12 cases as seen by its lowest permeability value (refer to Fig. 17a, b). Primitive Type 2 has the lowest pressure drop and the highest flow conductance. Due to the increased fraction of volume available for the flow, Type 2 lattices have a 1-order

| Lattice name | Porosity, $\epsilon$ | Tortuosity, $\tau$ | Permeability, $K$ | Darcy number, $Da$ | Quadratic coefficient, $\beta$ | Cubic coefficient, $\alpha$ |
|--------------|---------------------|-------------------|------------------|------------------|-----------------|------------------|
| Diamond      | 0.54                | 1.325             | $5.29 \times 10^{-10}$ m$^2$ | $3.31 \times 10^{-5}$ | $4.19 \times 10^{-8}$ | 0.48             |
| I-WP         | 0.54                | 1.051             | $3.91 \times 10^{-10}$ m$^2$ | $2.44 \times 10^{-5}$ | $6.35 \times 10^{-8}$ | 0.56             |
| Primitive    | 0.54                | 1.033             | $5.91 \times 10^{-10}$ m$^2$ | $3.69 \times 10^{-5}$ | $5.29 \times 10^{-8}$ | 0.92             |
| Gyroid       | 0.54                | 1.256             | $7.82 \times 10^{-10}$ m$^2$ | $4.88 \times 10^{-5}$ | $6.54 \times 10^{-8}$ | 0.87             |

Springer
| Mean velocity $\times 10^{-5}$ (m/s) | Diamond | I-WP | Primitive | Gyroid |
|-----------------------------------|---------|------|-----------|--------|
|                                  | Max. pore velocity $\times 10^{-5}$ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity $\times 10^{-5}$ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity $\times 10^{-5}$ (m/s) | Pressure gradient (kPa/m) | Max. pore velocity $\times 10^{-5}$ (m/s) | Pressure gradient (kPa/m) |
| 0.25                              | 2.3     | 0.0046 | 4.3       | 0.0059 | 5.8       | 0.0033 | 2.3       | 0.0029 |
| 2.5                               | 23      | 0.046  | 43        | 0.059  | 58        | 0.033  | 23        | 0.029  |
| 25                                | 230     | 0.46   | 430       | 0.58   | 580       | 0.33   | 230       | 0.29   |
| 250                               | 2300    | 4.6    | 4300      | 6.0    | 5600      | 3.4    | 2300      | 3.1    |
| 2500                              | 22,000  | 54.9   | 38,000    | 72.7   | 46,000    | 56.2   | 23,000    | 45.4   |
lower pressure drop and a 1-order higher permeability. Furthermore, due to the increased cross-sectional area available for the flow, the maximum velocity for Type 2 lattice is reduced by one order compared to Type 1 and Type 3 lattices (refer Tables 4, 7, and 9). When the coefficient of cubic drag ($\alpha$) for all scenarios is evaluated, the highest value of 1.28 is obtained for Primitive Type 1, whereas the lowest value is observed in Primitive Type 2 as 0.02.

For Type 1 porous media, the porosity is 0.32 and permeability is ranging from $2.82 \times 10^{-10}$ to $6.23 \times 10^{-10} m^2$, for which corresponding Darcy number is in the range of $1.77 \times 10^{-5}$ to $3.91 \times 10^{-5}$. Similarly, for Type 2 porous media in which the porosity is almost 1, the permeability is ranging from $3.67 \times 10^{-9}$ to $7.17 \times 10^{-9} m^2$ and corresponding Darcy number value is ranging from $2.29 \times 10^{-4}$ to $4.48 \times 10^{-4}$. Lastly, porosity is 0.54 for Type 3 porous media in which the permeability is ranging between $3.91 \times 10^{-10}$ and $7.82 \times 10^{-10} m^2$, for which corresponding Darcy number is in the range $2.44 \times 10^{-5}$ to $4.88 \times 10^{-5}$. It is observed that the Darcy number (or dimensionless permeability) increases with the porosity, whereas the cubic drag coefficient ($\alpha$) follows the opposite trend and it decreases with the increase in porosity.
6 Conclusions

The flow within porous media and associated properties including permeability, inertial drag factor, and pressure gradient are presented by pore-scale numerical simulation for the TPMS lattices as the porous structures. For this work, four different types of TPMS lattices: Diamond, I-WP, Primitive, and Gyroid, and three different treatments of the solid subdomain of the lattice structure are considered. The solid subdomain of the lattice is computationally treated as solid zone (Type 1), fluid zone (Type 2), and porous zone (Type 3), yielding a total of 12 different types of porous media, each of which is studied for hydrodynamic behavior for range of mass flow rates.

The key findings and conclusions from the present study are:

Fig. 17  a Variation in pressure drop with mass flow rate and channel size-based Reynolds number and b Permeability and coefficients of non-linear terms, for all cases
1. The present simulation results of pressure gradient variation with the mean velocity reinforce the existence of a cubic nonlinear regime after the Darcy regimes ($\dot{m} < 4 \times 10^{-5} \frac{kg}{s}$ or $Re < 10$). The present results show that the cubic nonlinear regime extends for higher pore-scale Reynolds number ($Re_p > 1$ or $Re > 10$) also.

2. The maximum and minimum value of permeability is observed in Primitive Type 2 and I-WP Type 1 lattice, respectively. In general, Type 2 lattices shows highest value of permeability and lowest value of inertial drag coefficient leading to lowest drop in pressure.

3. The Diamond, Primitive, and Gyroid lattices have significant tortuosity in the flow channel for all three types of treatments for their solid zone, whereas the I-WP lattice is comparatively less tortuous.

4. The maximum pore velocity at mass flow rate $\dot{m} = 4 \times 10^{-4} \frac{kg}{s}$ is observed in the Primitive Type 1 lattice as $0.54 m/s$, whereas for the same flow rate, the minimum pore velocity is observed in Diamond Type 2 lattice as $0.075 m/s$.

5. The inertial drag factor or the coefficient of cubic term ($\alpha$) is minimum for Primitive Type 2 lattice and maximum for Primitive Type 1, respectively.

**Appendix**

See Appendix Tables 10 and 11.

**Equation for Momentum Conservation in porous media**

a. Darcy model (for low flow rates, $Re_p \ll 1$)

$$\nabla p = -\frac{\mu}{K} \vec{V}$$  \hspace{1cm} (14)

b. Extended Darcy model (for intermediate to high flow rates)

**Table 10** Different correlations for effective viscosity (Almalki and Hamdan 2016)

| Sr. no. | Name of the correlation               | Effective viscosity                               |
|---------|---------------------------------------|--------------------------------------------------|
| 1       | Einstein’s relation                   | $\mu^* = \mu \left( 1 + \frac{5}{2} \left( 1 - \varepsilon \right) \right) \frac{m^2}{s}$ |
| 2       | Brinkman’s relation                   | $\mu^* = \mu \left( \frac{1}{\varepsilon} \right)$                                        |
| 3       | Breugem’s relation                    | $\mu^* = \mu \left( \frac{1}{2} \left( 1 - \frac{3}{7} \varepsilon \right) \right)$       |
| 4       | Common assumption in high porosity media | $\mu^* = \mu$                                      |

**Table 11** Properties of porous medium used in Type 3 treatment (Hetsroni et al. 2006)

| Property                  | Value                |
|---------------------------|----------------------|
| Porosity, $\varepsilon$   | 0.32                 |
| Permeability, $K$         | $0.215 \times 10^{-10} m^2$ |
| Quadratic drag factor, $\beta$ | 1.23             |
\[ \nabla p = -\frac{\mu}{K} \vec{V} - \frac{\rho \beta}{\sqrt{K}} |\vec{V}| - \frac{\alpha \rho^2}{\mu} \frac{1}{|\vec{V}|^2} \quad (\text{Re}_p > 1) \]  
\hspace{1cm} (15)

c. Brinkman extended Darcy model (for higher flow rates and high porosity, \( \varepsilon \to 1 \))

\[ \nabla p = -\frac{\mu}{K} \vec{V} - \frac{\rho \beta}{\sqrt{K}} |\vec{V}| - \frac{\alpha \rho^2}{\mu} \frac{1}{|\vec{V}|^2} + \mu^* \nabla^2 \vec{V} \]  
\hspace{1cm} (16)

In Eq. (16), \( \mu^* \) is the effective viscosity of the porous medium and some important relations between the porosity and effective viscosity are mentioned in Table 10.

**Calculation of Tortuosity**

Tortuosity is calculated by taking two points in the direction of the flow and measuring actual length of flow path (length of fluid trajectory) and minimum length (length of straight line) between these two points. It is then defined as below:

\[ \tau = \frac{\text{length of actual path}}{\text{length of minimum path}} \]  
\hspace{1cm} (17)

**Acknowledgements** The first two authors Surendra Singh Rathore and Balkrishna Mehta would like to acknowledge the Indian Institute of Technology Bhilai for providing the computational and other peripheral resources through the institute research initiation grant.

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