Effect of solid body aspect ratio on natural convection of nanofluid in a square cavity

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Abstract. The goal of this numerical study is to understand the free convection flow and transfer of heat in a differentially heated cavity having solid block in the centre. The coupled equations of momentum, mass and energy are governed the mathematical model. The governing equations are solved via utilizing the finite difference technique. The outcomes are performed for various combinations of aspect ratios of the solid block, nanoparticles volume fraction and Rayleigh numbers. The outcomes are shown in the form of isotherms, streamlines, and Nusselt numbers. It is concluded that the increment of average Nusselt number is detected when the nanoparticles volume fraction and Rayleigh number increase.

1. Introduction

There are several studies devoted to consider the issues of the free convection in cavities containing geometries like square, rhombic and rectangles in several fields. On the other hands, some numerical and experimental techniques were searched of enhancing heat transition in cavities [1-5]. De et al. [6] considered the free convection heat transition of a heated tilted square cylinder in a cavity. The problem of free convection in a square cavity with solid body placed at the corner position was presented by Costa et al [7]. The nature convection in a differentially heated square enclosure with a polygon was analyzed by Saleh et al. [8]. They got that the rate of heat transition rises with the increment of size of the solid polygon, until it arrives its maximum value. Sivasankaran et al. [9] employed the finite volume technique to explore the convective flow and heat transition of nanofluids with various nanoparticles in a square cavity. The study got that the average Nusselt number highly depends upon the nanoparticles types. In addition, Sivasankaran et al. [10] investigated numerically the impact of sinusoidal heating on double-diffusive mixed convective flow in a lid-driven square enclosure. The finite-volume technique used to find the solution of the unsteady governing equations. Sivasankaran et al. [11] investigated the slip effect and Newtonian heating on mixed convective flow in a porous medium. The study found that the temperature and velocity got increment on an increment in the Biot number.

The effects of Dufour and Soret on the double-diffusive convective boundary layer flow of nanofluid were investigated by Ruhaila et al. [12]. The study observed that the rate of the heat transition increments when the parameter of Soret increases and it decreases when the parameter of Dufour increments. The impacts of the Soret and Dufour on MHD mixed convection flow on a vertical plate inserted in a porous medium were studied by Niranjan et al. [13]. Mahmoodi et al [14] utilized the finite volume technique to study the subject of free convection flow of liquid and transition of heat in Cu-water nanofluid with square bodies at middle. The study reported that the average Nusselt number increases when the nanoparticles volume fraction increases, for each Rayleigh numbers, excluding for Ra=10⁴. Bouchta et al. [15] examined numerically the natural convection flow and heat transition of...
(Cu, Al₂O₃ and TiO₂)-water nanofluids in the annuli of two partially-heated square enclosure. The study reported that an increment of the average Nusselt number is witnessed by the increase of the volume fraction of the nanoparticles and the Rayleigh number. Further, efforts are still underway, in particular at experimental and numerical level to include the overall marketing advantages of nanofluids. They can intensify their readiness for the market via providing several researches in various configurations and several nanoparticles types as possible [16-20].

The goal of this work is to consider the issue of natural convection flow and heat transfer in a square cavity which having Cu-water nanoliquid and containing a solid object placed at the centre of the cavity. The present work is performed by changing the dimension of inner body between 0.2 to 0.6 and different volume part of the nanoparticles for various Rayleigh number ($10^3 \leq Ra \leq 10^6$).

2. Mathematical formulation

A schematic perspective of two-dimensional square enclosure with width, W and height, H ($H = W$) having an adiabatic object of height, $l_y$ and width, $l_x$ is placed at its center as shown in figure 1, which presents the physical configuration of current problem. The length or height of inner body varies from 0.2 to 0.6. The left side wall is kept at a high temperature $T_h$, whilst, the right side wall is assumed cooled $T_c$. The bottom and top walls of the cavity is considered as perfectly insulated. The enclosure is loaded with nanoliquid of Cu-water. The gravity acts in the downward trend. The velocities, $u$ and $v$ are defined in $x$ and $y$ directions, respectively.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  \hspace{1cm} (1)

\[
\mu_n\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = \left(\frac{\rho_0 \beta_n}{\rho_n}\right) g(T - T_c) + \frac{\rho_0 \beta_n}{\rho_n} g(T - T_c)
\]  \hspace{1cm} (2)

\[
\frac{\partial v}{\partial x} + \frac{\partial P}{\partial y} = \frac{1}{\rho_n} \left[ \frac{\mu_n}{\rho_n} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\rho_0 \beta_n}{\rho_n} g(T - T_c) \right]
\]  \hspace{1cm} (3)

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_n \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  \hspace{1cm} (4)

\[
-\rho_0 \beta_n \frac{\partial T}{\partial y} = \frac{\partial T}{\partial y} - \frac{\partial T}{\partial y}
\]  \hspace{1cm} (5)

\[
\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T}{\partial y^2}
\]  \hspace{1cm} (6)

Figure 1. A schematic diagram of the cavity.

The properties of fluid are constant except the density variation in term of buoyancy by Boussinesq approximation. Moreover, the present study supposes that the dissipation of viscous is negligible. By the conservation law for mass, energy, and momentum, the employed equations are given as below;
The conditions of boundary for equations as below:

\begin{align*}
\text{on each solid boundaries} & : u = v = 0, \\
\text{on} \ x = 0, \ 0 \leq y \leq H & : T = T_h, \\
\text{on} \ x = W, \ 0 \leq y \leq H & : T = T_c, \\
\text{on} \ Y = 0, \ Y = H, \ 0 \leq x \leq W & : \frac{\partial T}{\partial Y} = 0.
\end{align*}

The representing condition can be changed over to the dimensionless frame by utilizing the following parameters of dimensionless:

\begin{align}
X &= \frac{x}{W}, \quad Y = \frac{y}{H}, \quad V = \frac{vH}{\alpha_f}, \quad U = \frac{uW}{\alpha_f}, \quad P = \frac{pWH}{\rho_\text{nf}\alpha_f^2}, \quad \Theta = \frac{T - T_c}{T_h - T_c}, \\
\Psi &= \frac{\psi P_r}{v}, \quad \Omega = \frac{\omega WHP_r}{v}, \quad \Theta = \frac{T - T_c}{T_h - T_c}.
\end{align}

The governing equations in dimensionless forms are:

\begin{align}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0, \\
U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial X} + \frac{\mu_\text{nf}}{\rho_\text{nf}\alpha_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right), \\
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial X} + \frac{\mu_\text{nf}}{\rho_\text{nf}\alpha_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \left( \frac{\rho_\text{nf}}{\rho_\text{nf} \beta_f} \right) \text{Ra Pr} \Theta, \\
U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} &= \frac{\alpha_\text{nf}}{\alpha_f} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right).
\end{align}

The conditions of boundary for equations as below:

\begin{align*}
\text{on the left} & : U = V = 0, \ \Theta = 1, \\
\text{on the right wall} & : U = V = 0, \ \Theta = 0, \\
\text{on the adiabatic body} & : U = V = 0, \ \frac{\partial \Theta}{\partial n} = 0.
\end{align*}

The vorticity and stream functions are taken as:

\begin{align}
U &= \frac{\partial \Psi}{\partial Y}, \quad V = -\frac{\partial \Psi}{\partial X}, \quad \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega.
\end{align}

Employing equations (11) and (5), the governing equations and the boundary conditions (10) with regard to the dimensionless variables are;
has been taken for the calculations. A converged solution is obtained.

\[ \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} = -\Omega, \]  

(12)

\[ \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} = \frac{\rho_{nf} \alpha_{nf}}{\mu_{nf}} \left[ \frac{\partial^2 \Psi}{\partial Y \partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} \right] - \frac{(\rho \beta)_{nf}}{\rho_{nf}} \left[ R_d Pr \frac{\partial \Theta}{\partial X} \right], \]  

(13)

\[ \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} = \frac{\alpha_f}{\alpha_{nf}} \left( \frac{\partial \Psi}{\partial Y} \frac{\partial \Theta}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \Theta}{\partial Y} \right), \]  

(14)

under the constraint conditions:

on the solid walls : \( \Psi = 0, \)

on \( x = 0, \) \( 0 \leq y \leq 1 \) : \( \Theta = 1, \) \( \Omega = -\frac{\partial^2 \Psi}{\partial X^2}, \)

on \( x = 1, \) \( 0 \leq y \leq 1 \) : \( \Theta = 0, \) \( \Omega = -\frac{\partial^2 \Psi}{\partial X^2}, \)

on \( Y = 0, \) \( Y = 1, \) \( 0 \leq X \leq 1 \) : \( \frac{\partial \Theta}{\partial Y} = 0, \) \( \Omega = -\frac{\partial^2 \Psi}{\partial X^2}. \)

The general transfer of heat over the cavity is a significant parameter in the enforcements of engineering. The local and averaged Nusselt number are calculated on the side walls via utilizing the following equations:

\[ Nu = \left( \frac{k_{nf}}{k_f} \right) \frac{\partial \Theta}{\partial X} \bigg|_{X=0} \]  

(16)

\[ Nu_{avg} = \int_0^1 NudY \bigg|_{X=0} \]  

(17)

3. Solution procedure

The equations (12), (13), (14) subjected to the conditions of boundary (15) are solved by employing the finite difference technique. The function of stream (12) is solved by utilizing Successive Over Relaxation (SOR) technique whilst the equation of vorticity (13) and equation of energy (14) are solved by employing Successive Under Relaxation (SUR) technique. As stated by Oztop et al. [20] (SUR) is utilized because of the nature of non-linear for the equations especially for the equation of vorticity at high \( Ra. \) Uniform grids in \( X - \) and \( Y - \) directions are employed in all computational procedures of this work. The grid size \( 101 \times 101 \) has been taken for the calculations. A converged solution is obtained by utilizing the relation as follows:

\[ \Xi = \sum_{i,j} \left( \psi_{i+1,j}^{n+1} - \psi_{i,j}^{n} \right) < 10^{-6}, \]

where \( \psi \) is either \( \Psi, \Omega, \) or \( \Theta. \)

The Nusselt number along the side-wall has computed by using the forward difference approximation while the integration of numerical (i.e. trapezoidal rule) is employed to compute the averaged Nusselt number at the sidewalls. Based on the preceding literature by using heated square enclosure with a constant temperature at the left wall the code of the implemented computerized is validated.
4. Results and discussion

Figure 2. Streamlines (left) and isotherms (right) for pure fluid $\varphi = 0\%$ and nanofluid $\varphi = 4\%$ with $Ra = 10^6$ at (a) $lx = ly = 0.2$, (b) $lx = ly = 0.4$, (c) $lx = 0.2$, $ly = 0.4$, (d) $lx = 0.4$, $ly = 0.2$, (e) $lx = 0.6$, $ly = 0.2$, (f) $lx = 0.2$, $ly = 0.6$. 
In this present work the outcomes for the isotherms, streamlines, (local and average) Nusselt number have been presented. The outcomes are offered graphically to display the conduct of free convection steady of Cu–water nanofluid. This work focuses on the impacts of the volume part of the nanoparticles which are ranging from 0 to 0.04, the varies of the Rayleigh number in the range \((10^3 \leq Ra \leq 10^6)\) and the dimension of inner body is, changed between 0.2 to 0.6 on the fluid flow and transition of heat in the cavity.

Fig. 2 shows the streamlines and the isotherms for the six cases of changing the dimension of the solid object for the enclosure having pure water \((\varphi = 0.0)\) and Cu–water nanofluid \((\varphi = 0.04)\) for \(Ra = 10^6\). We found that nearly the same behaviour for (water) pure fluid and nanofluid Cu–water. The heated fluid escalates along the left wall, after that drives out horizontally, is cooled and inclines at the nearby areas of the right wall, subsequently, clockwise whirlpool is improved in the enclosure for all cases. The whirlpools of the secondary clockwise are improved in the right and left solid object side by transfer of the primary whirlpool and solid body existence. Moreover, the isotherms in Fig. 2 present that the adiabatic body size does not impact the temperature distribution significantly.

Variation of the local Nusselt number with the Rayleigh numbers at various volume part of the nanoparticles and different dimension of inner body, are displayed in Fig. 3 and Fig. 4. We can conclude that with an increment the nanoparticles volume fraction the transition of heat is increased. Also, similarity with respect to the volume part of the nanoparticles, the average Nusselt number of the heat source variations at the various Rayleigh numbers, are shown up in Fig. 5. The Rayleigh numbers range is considered.

![Fig. 3](image-url)

**Figure 3.** Local Nusselt number with various dimension of inner object with \(\varphi=0\%\) at various Rayleigh numbers for (a) \(Ra = 10^3\) (b) \(Ra = 10^4\) (c) \(Ra = 10^5\) (d) \(Ra = 10^6\).
Figure 4. Local Nusselt number with various dimension of inner object with $\varphi=4\%$ at various Rayleigh numbers for (a) $Ra = 10^3$ (b) $Ra = 10^4$ (c) $Ra = 10^5$ (d) $Ra = 10^6$.

5. Conclusion

The present work investigates the transfer of heat of nanofluid Cu–water in a differentially heated square enclosure with a solid object located in its centre by employing the finite different method. A parametric study was done and impacts of the Cu nanoparticles volume part, the Rayleigh number, and the different length or height of inner body on the fluid flow are studied and the outcomes were had as following.

In all examined cases, the rate of the heat transition increases by increment of the Rayleigh number, when the volume part of the nanoparticles is kept constant. The average Nusselt number increases by increment in the nanoparticles volume fraction, when the number of Rayleigh is kept constant. Change the dimension of inner object does not impact heat transition rate when the number of Rayleigh and nanoparticles volume fraction are constant.
Figure 5. Averaged Nusselt number with different dimension of inner object for various Rayleigh number (a) $Ra=10^3$, (b) $Ra=10^4$, (c) $Ra=10^5$, (d) $Ra=10^6$.

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