A theory of interaction semantics

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I dedicate this article to Bernd Finkbeiner

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Abstract

The aim of this article is to delineate a theory of interaction semantics and thereby provide a proper understanding of the "meaning" of the exchanged characters within an interaction.

The idea is to describe the interaction (between discrete systems) by a mechanism that depends on information exchange, that is, on the identical naming of the "exchanged" characters — by a protocol. Complementing a nondeterministic protocol with decisions to a game in its interactive form (GIF) makes it interpretable in the sense of an execution. The consistency of such a protocol depends on the particular choice of its sets of characters. Thus, assigning a protocol its sets of characters makes it consistent or not, creating a fulfillment relation. The interpretation of the characters during GIF execution results in their meaning.

The proposed theory of interaction semantics is consistent with the model of information transport and processing, it has a clear relation to models of formal semantics, it accounts for the fact that the meaning of a character is invariant against renaming and locates the concept of meaning in the technical description of interactions. It defines when two different characters have the same meaning and what an "interpretation" and what an "interpretation context" is as well as under which conditions meaning is compositional.

1 Introduction

In our normal live, the concepts of understanding and meaning have proven to be extremely powerful. Intuitively, we say that the signs exchanged in a conversation have a meaning that is to be understood by the participants. What do we mean by that? What are the benefits to talk this way? This problem, posed this way or another, has puzzled philosophers and scientists since ancient times.

Meaning is also a key concept for our understanding of our natural language. I understand natural language as a facilitation mechanism for inter-subjective or social interactions. In this sense it is a pragmatic solution to the circular
or "chicken-or-the-egg"-problem that on the one hand a purposeful interaction requires mutual understanding and on the other hand establishing a mutual understanding requires purposeful interaction. Another well described and increasingly technically well solved circular problem of this kind is that of "simultaneous localization and mapping (SLAM)" problem (e.g. [TBF05]). To determine my own position in a terrain I need a map and to determine the map I need to know my own position. Following the SLAM acronym, I propose to speak of the "simultaneous interaction and understanding (SIAU)" problem. The obvious solution to such problems is iterative: an internal model is increasingly improved by empirically collecting data. The intelligence of the solution then rests in the way, the relevant aspects of the external world are internally represented and in the update mechanism.

I am not going to propose an iterative algorithm by which we can solve the SIAU problem for technical systems. Nor am I going to dwell on the issue of the possible inner structure of a language as a solution to the problem of limited expressiveness of pure sign-orientation. Instead, the contribution of this article is to make a proposition how to describe interactions, understanding and meaning from a technical perspective so that, perhaps, in a future step, some iterative approach might solve the SIAU problem.

From an engineering perspective we are talking of at least three different languages when dealing with interactions: there are at least two languages we use to talk about the interaction. These are our normal (engineering) language and our formal programming languages. And there is the interaction itself which I also view as a language. Actually, this is our language of interest. To distinguish it from the others, I name it "interaction language".

In my opinion, a theory of interaction semantics or the semantics of the interaction language should fulfill certain requirements. It should at least

1. be consistent with the model of information transport and processing;
2. clarify the relationship to models of formal semantics.
3. account for the fact that the meaning of a character is invariant against renaming
4. locate the concept of meaning in our usual technical description of interactions;
5. define when two different characters have the same meaning;
6. define what is "interpretation" and what is an "interpretation context";
7. explain under which conditions meaning is compositional, and when it is not.

[BDR16] abbreviated it as "I-language". However, This term is already used as "I"- versus "E"-language by Noam Chomsky in an attempt to distinguish between the "I"(intensional)-language, referring to the internal linguistic knowledge, and the "E"(extensional)-language, referring to the observable language people actually produce [Ara17].
These criteria also provide a good base to compare my approach with other approaches to define interaction semantics.

The structure of the article is as following: In section 2 I set the stage by recapitulating the concepts of information and of formal semantics. The key idea then is to proceed in four steps: In section 3 I describe the interaction (between discrete systems) by a mechanism that depends on information exchange, that is, on the identical naming of the "exchanged" characters — a protocol. Next, I look for a decisive property of such a protocol, namely its consistency, that depends on the particular choice of its sets of characters, also named its alphabets. Then, in section 4 I first introduce the decision concept to make the protocol executable in a functional sense, despite its nondeterministic character. Secondly, I define a fulfillment relation where the assignment of the set of alphabets to a protocol makes it consistent or inconsistent. This approach requires the definition of an interpretation function of the protocol and its constituents, namely its characters. This interpretation is the execution of the protocol and the interpretation of the characters during execution results in their meaning. Section 5 provides a brief overview of the relevant work of others. I conclude this article in the final section 6 with a summarizing discussion and some speculations.

2 Information and meaning

The first cornerstone of my theory of interaction semantics is its relation to information theory.

It is the focus on the distinguishability of state values that creates "information" which can be "transported" between systems and processed within systems [Sha48, Sha49]. A character in the sense of information theory is a unique name for a physical state value that can be distinguished from all the other state values, this state can take. We have to provide these names as an additional alphabet of our engineering language to be able to talk about physical state values only in so far as we can distinguish them from other state values — and this kind of information can be transported.

As per theory only information can be transported, it follows immediately that meaning is not transported. In a sense, this makes "meaning" somehow magical. What strange thing can be transmitted while not being transported? From my point of view, magical imagination is essentially an erroneous attribution of meaning: An amulet is attributed an effect that it does not have; spells are recited which are effect free; objects are thought to transfer properties on their owners that they do not have; etc. (see also [Grü10]). In his model of cognitive development of children the developmental psychologist Jean Piaget describes how actually every child in the so-called "pre-operational" phase from 3-7 years thinks magically. Clouds rain because they are sad, the monsters wait in the dark cellar, .... Only in the course of our cognitive development do we humans develop realistic thinking — which allows us to fully understand a local model of attributing meaning by processing and its consequences.
In fact, from the point of view of the presented model of interaction semantics, all models that directly assign meaning to the characters themselves are archaic in a certain sense as they naively ignore the relevance of the local interpretation of the character receiver.

So, if we want to talk about meaning in accordance with information theory, it has to emerge from processing of the transported information. It also follows that the meaning of the exchanged information of the interaction language must be invariant against renaming them within our engineering language (assuming no naming conflicts).

Another insight is that not everything that is distinguishable - i.e. every piece of information - in an interaction is also of equal relevance or significance. Actually the ancient Greek word "σημαντικός" (semantikos) is usually translated as "significant" or "meaningful". I think that the concept of meaning in its core is about distinguishing the relevant from the irrelevant — which shows the importance, but also the indeterminacy of this concept. Fig. 1 illustrates the relation between physical states, information and attributing meaning.

What determines the significance of an exchanged information? Again, intuitively speaking, the consequences resulting from its processing within a given cut of the world. Again, we arrive at the transformational behavior of the systems and thereby at a local theory of interaction semantics, were the meaning of a character is determined by its local processing, i.e. by its transformation in a given system.

2.1 The concept of meaning in formal languages

To better understand the subsequent model of interaction semantics, I consider it helpful to examine the model of semantics of formal languages, also called "calculi".

Based on Alfred Tarski [Tar35], formal languages are structured according to a certain scheme. First, the syntax is defined, consisting of a set of allowed characters together a set of rules describing which expressions may be formed. Then, in a second step, the semantics is defined by an interpretation function determining the meaning of the allowed expressions by mapping them to entities that are legitimately assumed to exist and can be talked about in normal language.

I illustrate this briefly with the example of the propositional calculus. Under the assumption that we already know what a proposition is and that we can
make elementary statements, the calculus describes how one can obtain further statements from elementary statements by and, or and negation operations. In order to ensure the distinction between the expressions attributed to the calculus and those attributed to our normal language, I put all calculus expressions in quotation marks.

The colloquial expressions that we use to formulate the rules of syntax and semantics deserve special attention. To formulate propositional logic, we have to use so called "propositional forms". Syntactically, propositional forms correspond to the calculus expressions, but they contain special variables as placeholders for calculus expressions. I write down propositional forms like calculus expressions in quotation marks, but symbolize the special variables with a prefixed $ sign, in order to be able to distinguish them reliably from the variables which are part of the calculus.

The allowed characters of the propositional calculus are determined by the alphabet \{"w","f"\}, the set of operator characters \{"\lor","\land","\neg"\}, as well as the set of variables for propositions \(V = \{p,q,\text{etc.}\}\).

The rules for building propositions are:

1. "w" and "f" are propositions;
2. Each variable is a proposition;
3. Are $a$ and $b$ propositions, then $\neg a$, $a \lor b$ and $a \land b$ are also propositions.

The interpretation of a proposition "$a", \mathcal{I}_b("a")", provides its meaning and consists of

1. an assignment of truth values to all variables: \(b : V \to \{\text{true, false}\}\), where true and false are expressions of our colloquial engineering language we hopefully fully comprehend.
2. a recursive rule that determines the meaning of the proposition:
   
   (a) \(\mathcal{I}_b("w") = \text{true}; \mathcal{I}_b("f") = \text{false}\);
   
   (b) \(\mathcal{I}_b("p") = b("p")\);
   
   (c) \(\mathcal{I}_b("\neg a") = \text{false}[\text{true}]\) if \(\mathcal{I}_b(a) = \text{false}[\text{true}]\);
   
   (d) \(\mathcal{I}_b("a \lor b") = \text{true}, \text{if } \mathcal{I}_b(a) = \text{true or } \mathcal{I}_b(b) = \text{true}\);
   
   (e) \(\mathcal{I}_b("a \land b") = \text{true, if } \mathcal{I}_b(a) = \text{true and } \mathcal{I}_b(b) = \text{true}\.\)

Because the interpretation maps our formulas to truth values, we can also define a "fulfillment"-relation |= where an assignment b fulfills a formula "$a", or, symbolically, \((b,"a") \in |=\), or, in the usual infix notation, \(b \models "a"\) iff \(\mathcal{I}_b("a") = \text{true}\). In this case we call the interpretation \(\mathcal{I}_b\) also a "model" of "$a".

One interesting aspect of the semantics of formal calculi is its compositional character. By that I mean that the meaning of composed terms is the result of a function on the meaning of its parts, or formally:
Proposition 1. The meaning $I_b(p)$ of a proposition $p$ is either elementary, that is, given by definition, or composed in the sense that if the proposition is of the form $p = p(p_1, \ldots, p_n)$ then there exists an operator $op$ such that $I_b(p) = op(I_b(p_1), \ldots, I_b(p_n))$. I also say that the meaning attribution of the propositional calculus is "compositional".

This proposition is obviously true by the construction rules of the meaning of composed propositions.

3 The description of systems and their interactions

To describe the interaction of systems, I follow the approach of [Rei20]. A system separates an inner state from the state of the rest of the world, the environment. A state in this sense is a time dependent function, taking a single out of a set of possible values, the alphabet $A$, at a given time [IECfl]. I prefer to speak of "state function" and "state value". The key idea is that these time-varying values are not independent, but some of them are uniquely related by an additional function: the system function. This system function thereby separates the state functions of a system from the state functions of the rest of the world. It also gives the system’s state functions their input-, output-, or inner character. Such a relation logically implies causality and a time scale.

Depending on the class of system function or time, different classes of systems can be identified. However, we will focus on discrete systems.

Following [Rei20], we describe the behavior of a (possible projection of a) discrete system by input/output-transition systems (IOTs) of the following form\footnote{Another name in the literature is "transducer" [Sak09], because this machine translates a stream of incoming characters into a stream of outgoing characters.}:

**Definition 1.** An input/output transition system (IOT) $A$ is given by the tuple $A = (I, O, Q, (q_0, o_0), \Delta)$ with $I$ and $O$ are the possibly empty input and output alphabets and $Q$ is the non empty set of internal state values, $(q_0, o_0)$ are the initial values of the internal state and output and $\Delta_A \subseteq I^* \times O^* \times Q \times Q$ is the transition relation describing the behavior of a discrete system.

A general execution fragment of an IOA is a sequence of 3-tuples, listing the values that the input, output and state functions of the corresponding system have at the considered times: $(i_0, o_0, p_0), (i_1, o_1, p_1), \ldots, (i_n, o_n, p_n)$. In the arrow notation, a single 3-tuple is written as $i^o \rightarrow p^i / o^i \rightarrow p^i$. Thus, in the arrow notation an execution fragment is written as $i_0^o / o_0^i \rightarrow p_0^i i_1^o / o_1^i \rightarrow p_1^i \cdots i_n^o / o_n^i \rightarrow p_n^i$.

3.1 System interactions

In our model, interaction simply means that information is transmitted. Accordingly, the description of interaction is based on the use of equal characters in the
sending and receiving systems such that the state values of an output component of a transition of a "sender" system are reproduced in the input component of the "receiver" system and serve there as input of a further transition (see Fig. 2).

Figure 2: Interaction between two systems in which the output character of a "sender" system is used as the input character of a "receiver" system. Interaction therefore means the coupling of the two transition systems of sender and receiver based on the "exchanged" character.

We call such a state function that serves as output as well as input of two systems a "Shannon state function". It is an idealized Shannon channel as it has no noise and no delays.

3.2 Protocols

In the following we focus on systems which interact with multiple other systems in a stateful and nondeterministic way. In the literature there have been many names coined for these kind of systems, some examples are "processes" (e.g. [MPW92]), "reactive systems" (e.g. [HP85]), "agents" (e.g. [Pos07]) or "interactive systems" (e.g. [Rei20]). Their interactions are described by protocols [Hol91].

While in deterministic interactions the purpose of composition is simply the construction of super-systems, in nondeterministic interactions, things are different. We need an additional criterion for success, the so-called acceptance component \(\text{Acc}\). We thus get from IOTs to I/O automata (IOA) by adding an additional acceptance component to our transition system structure. This acceptance component depends on the success model. For finite calculations with a desired end, \(\text{Acc}_{\text{finite}}\) consists of the set of final state values. For infinite calculations of a finite automaton there are differently structured success models. One of them is the so-called Muller acceptance, where the acceptance component is a set of subsets of the state value set \(Q\), i.e. \(\text{Acc}_{\text{Muller}} \subseteq \mathcal{P}(Q)\). An execution (see below) is considered to be successful whose finite set of infinitely often
traversed state values is an element of the acceptance component (e.g. [Far01]).

Given the IOAs of all interacting systems of interest, such that all their input and output state functions represent Shannon states, their product IOA is again an IOA and together with the set of Shannon states represents a protocol. It is self-contained or closed in the sense, that it has neither any external inputs nor outputs any character.

**Definition 2.** A protocol is a pair \((A, C)\) of a set of IOAs, also called "roles" \(A = A_1 \ldots A_n\) that represent the behavior of \(n\) discrete systems and a set of coupling Shannon signals \(C\) that connect the output components with the input components, such that all inputs are provided by the output of one of the roles and no output goes somewhere else ("closure"-property).

To simplify our further considerations, I assume all characters to have at most one component unequal the empty character \(\epsilon\). I define the execution of a protocol recursively as follows:

**Definition 3.** Let \(\mathcal{P}\) be a protocol with the roles \(A_1 \ldots A_n\) that represent the behavior of \(n\) discrete systems and \(C\) be a set of coupling signals that connect the output components with the input components of \(\mathcal{P}\). There are no extra external input characters.

The current values of \(i, o\) and \(q\) are indicated by a \(*\), the values calculated in the current step by a \(+\).

1. **Initialization (time \(j = 0\)):** \((q^*, o^*) = (q_0, o_0)_{\mathcal{P}}.\)

2. **Loop:** Determine for the current state \(q^*\) the set of all possible transitions. If this set is empty, end the calculation.

3. **Determine input character \(i^*\):** Proceed in the following sequence:
   
   (a) If the current output character \(o^* \in O_{\mathcal{P}}\) has the value \(v \neq \epsilon\) in its \(k\)-th component, i.e. \(o^* = \epsilon[v,k]\), and \(o^*\) is part of a feedback signal \(c = (k,l)\) to the input component \(0 \leq l \leq n\), then set \(i^* = \epsilon[v,l]\). If otherwise \(o^*\) is not part of a feedback signal, terminate the calculation with an error.
   
   (b) Otherwise, if there are spontaneous transitions for \(q^*\), select \(i^* = \epsilon\) as the current input character.

   (c) otherwise finish the calculation.

4. **Transition:** With \(q^*\) as current state value and \(i^*\) as current input character select a transition \(t = (i^*, o^*, q^*, q^+) \in \Delta_{\mathcal{P}}\) and so determine \(o^+\) and \(q^+\). If there is no possible transition at this point, terminate the calculation with an error.

5. **Repetition:** Set \(q^* = q^+\) and \(o^* = o^+\) and jump back to 2.
In the special case of a protocol and the Shannon state \((k,l)\), the \(k\)-th component of the output character of the current step is always the \(l\)-th component of the input character of the next step, which translates to \((i_t)_l = (o_t)_k\) for every point in time \(t\). From a formal point of view, this is also true for the empty character \(\epsilon\). Thus, we can write an execution fragment of a protocol
\[
/o_0 \rightarrow p_0 \rightarrow o_1 \rightarrow p_1 \rightarrow \cdots \rightarrow o_n \rightarrow p_n
\]
also as a sequence of pairs of state values and characters:
\[
(c_0, p_0) \rightarrow (c_1, p_1) \rightarrow \cdots \rightarrow (c_n, p_n)
\]
where \(c_t = (i_t)_l = (o_t)_k\).

I call an execution fragment which starts with an initial state a "run".

And I call an execution fragment of a protocol that is started by a character resulting from a spontaneous transition and goes on until the output becomes empty an "interaction chain". Thus, as \(c_0 = c_n = \epsilon\), an interaction chain is characterized by
\[
(c_0, p_0) \rightarrow (c_1, p_1) \rightarrow \cdots \rightarrow (c_{n-1}, p_{n-1}) \rightarrow p_n
\]
As can be seen from the error conditions in the execution rule, a protocol must fulfill certain consistency conditions to make sense. It has to be "well-formed" in the sense that for each transition with a sent character \(o\) unequal to \(\epsilon\) in at least one component, a corresponding receiving transition must exist. Is must not contain infinite chains of interaction, i.e. it must be "interruptible". And for each run, the acceptance condition is fulfilled. So, I define:

**Definition 4.** A protocol is called...

1. "well formed" if each input character determined in step 2 can be processed in step 3.

2. "interruptible" if each interaction chain remains finite.

3. "accepting" if for each run the acceptance condition is fulfilled.

A protocol that is well formed, interruptible, and accepting is called consistent.

### 3.2.1 Example: The single-track railway bridge

To illustrate the protocol concept we give the simple example of a single-track railway bridge drawn from [Alu15]. As is shown in Fig. 3, two trains, \(Z_1\) and \(Z_2\), must share the common resource of a single-track railway bridge. For this purpose, both trains interact with a common controller \(C\), which must ensure that there is no more than one train on the bridge at any one time.

The interaction between each train and the controller is described by a protocol. For this we need to describe both the train and the controller in terms of the role they play in the interaction. For both the train and the controller we choose a model of 3 state values, which we call \(Q_{Z_1,2/C} = \{\text{away, wait, bridge}\}\) for each train as well as for the controller. The input alphabet of the trains \(I_{Z_1,2} = \{\text{go}\}\) is the output alphabet of the controller \(O_C\) and the output alphabet of the trains \(O_{Z_1,2} = \{\text{arrived, left}\}\) is the input alphabet of the controller \(I_C\).

In Fig. 4 the protocol is shown. It can be seen that the interaction by a Shannon state restricts the transition relation. The protocol between train and controller is complete as no further external characters occur. It is well formed
Figure 3: A single-track railway bridge crossed by two trains. To avoid a collision on the bridge, both trains interact with a central controller.

Figure 4: Presentation of the protocol between train and controller for the problem of the single-track railway bridge. Initially, both controller and train are in the away state. When a train arrives, it signals arrived to the controller. This sign must now be processed by the controller, the controller in turn changes to its wait state. The controller releases the track with go and the train signals the controller with away that it has left the bridge again. The interaction is successful when both the train and the controller go through their three states infinitely often.

as for each sent character there is a processing transition at the right time. And finally, it is consistent as it has only finite interaction chains and it fulfills its acceptance condition.

Please note that the correctness, we could also say the truth, of the representation of the state of the train in the controller depends on the correctness of the protocol.
4 Interaction semantics

As said before, the choice of alphabets is prima facie arbitrary. However the mechanism of information transport requires a naming convention across the interacting systems. The consistency of a protocol therefore depends on the choice of the alphabets in the descriptions of the interacting systems.

To investigate the meaning of the characters further, I introduce a fulfillment relation for protocols which relates the choice of the alphabets to the consistency of the protocols. So, we are looking for an interpretation of a protocol under the assignment of a set of alphabets for all its roles. This interpretation is the protocol’s execution, i.e. the transition steps under the rules given by the protocol, where the new state and output value are calculated from the old state and input value.

However, in general, a protocol has a nondeterministic transition relation and therefore no interpretation function can be given mapping state values plus input characters onto state values and output characters. To complete a nondeterministic transition relation I therefore introduce the concept of decisions: decisions determine the behavior, i.e. the transitions which would otherwise be indeterminate (see [Rei20]).

4.1 Decisions

According to the two mechanisms that give rise to nondeterminism of transitions, we can distinguish two classes of decisions: spontaneous decisions that determine the spontaneous transitions without input character and selection decisions that determine a selection if for a given input character and state value several transition could be selected.

Decisions in this sense are very similar to information and can be seen as a further, ”inner” input alphabet $D$, which complements the input alphabet $I$ of an IOT $\mathcal{A}$ according to Def. 1 to $I' = I \times D$ such that a complemented transition relation $\Delta'$ becomes deterministic. They are enumerated by an alphabet and their names are relevant only for their distinction. In contrast to ordinary input characters, whose main characteristic is to appear in other output alphabets and that are allowed to appear in different transitions, we name all decisions of a corresponding transition system differently and different from all input and output characters and internal state values, so that we can be sure that they really do determine all transitions.

**Definition 5.** Be $\mathcal{A}$ an IOT and $D$ an alphabet. The transition system $\mathcal{A}'$ is called a ”decision system” to $\mathcal{A}$ and the elements of $D$ ”decisions”, if $I \cap D = \emptyset$, $O \cap D = \emptyset$, $Q \cap D = \emptyset$, and $\Delta' \subseteq (I' \times D) \times O' \times Q \times Q$ with $(i, o, p, q) \in \Delta'$ if $(i, o, p, q) \in \Delta$ and for $d$ applies:
\[ d = \begin{cases} 
\epsilon, & \text{if there's no further transition } (i^*, o^*, p^*, q^*) \in \Delta \\
\text{so selected that } \Delta' \text{ is deterministic, i.e. } \Delta' \text{ determines the function } \\
f': I^* \times D \times Q \to O^* \times Q \text{ with } (o, q) = f'(i, d, p). \\
\text{For two transitions } t'_1, t'_2 \in \Delta' \text{ it holds } t'_1 \neq t'_2 \Rightarrow d_1 \neq d_2. \\
\text{Additionally, } \Delta' \text{ is the smallest possible set.} 
\end{cases} \]

Obviously, the set of decisions for an already deterministic IOT is empty. In Fig. [5] I illustrate the decision notion with the train-controller protocol. To determine the actions of train and controller three decisions are necessary. The train has to decide when to arrive and when to leave ("IArrive" and "ILeave") and the controller has to decide when it let the train go ("ILetYouGo").

I call the decision automaton to a consistent protocol also a "game in interactive form (GIF)". For a GIF we can modify the protocol execution rule such that the selection choice becomes determined by some possible decision.

An execution fragment of a GIF is like the execution fragment of the protocol, but extended by the additional decisions: \((c_0, d_0, p_0) \rightarrow (c_1, d_1, p_1) \rightarrow \cdots \rightarrow (c_n, d_n, p_n). As an interaction chain is started by an empty character and ends by an empty output, we now can say that an interaction chain is always triggered by a spontaneous decision.

A run \(r\) is then the result of an interpretation of the GIF \(A\) under the assignment of the alphabets and some input sequence of decisions \(seq\): \(r = interp_b(A, seq). To simplify notation, I further on drop the subscript \(b\) of the assignment.

### 4.2 The meaning of an exchanged character

The consistency requirements for a GIF are the same as for its corresponding protocol. But now, as explained in the last section, we have an interpretation function which is defined by the extended transition relation of the corresponding protocol and which operates on the incoming characters.

With these considerations a transition of the GIF defines the meaning of the input character \(i\) with respect to some start value \(p\) and possibly some decision \(d\) — which is the new state value \(q\) with the possibly generated output character \(o\):

\[ (o, q) = f((i, d), p) =: interp_{d, p}(i) \tag{1} \]

With this definition, we also stick to our initial requirement for a good theory of interaction semantics, to locate the concept of meaning in our usual technical description of interactions. But we can take this though even further, as we

\[ \text{Actually this "game" still lacks the utility function, a game in the traditional game theoretic sense has.} \]
know that any outgoing character only enforces another transition, leading to an extended interpretation of the same character. Marking the number of steps an interpretations represent by a superscript index, I can write:

\[ \text{interp}^{(2)}(d_1, d_2, p_2) = \text{interp}^{(1)}(\text{interp}^{(1)}(d_1, p_1)(i_1)) \]

This deliberation leads us to define the meaning of every character in an interaction chain as the state value of the endpoint of this interaction chain. As a result, I drop the output character from the definition of meaning. It’s sole function is to trigger the next transition, until the final transition of the interaction chain is reached without any further output character.

As the choice of the current transition for a given input character is determined by decisions, we can say that the meaning of a character exchanged within an interaction chain is determined by the decisions taken along this interaction sequence. Thus, I define:

**Definition 6.** Let \( \mathcal{P} \) be a consistent protocol and \( \hat{\mathcal{P}} \) a corresponding GIF. Let us also consider a finite interaction chain \((c_1, d_1, q_1), \ldots, (c_{m-1}, d_{m-1}, q_{m-1}), q_m\) of \( \hat{\mathcal{P}} \) where \( c_1 \) resulted from a spontaneous transition and only the last transition of the chain is without output. Then the meaning of \( c_j \) for \( 1 \leq j < m \) in the state \( q_j \) under the decisions \( d_j, \ldots, d_{m-1} \) is the state \( q_m \).

That is, we define the function

\[ \text{interp}(d_j, \ldots, d_{m-1}, q_j)(i_j) := \text{interp}^{(m-j)}(d_j, \ldots, d_{m-1}, q_j)(i_j) = q_m \]

With this definition, two characters \( c \neq \epsilon \) and \( c' \neq \epsilon \) have the same meaning with respect to the states \( p \) and \( p' \) and two decision sequences \( d \) and \( d' \), if \( \text{interp}_d(c) = \text{interp}_{d'}(c') \), that is, if the final states of the two interaction chains, where the characters together with the state values are part of, are the same. The state values \( p \) and \( p' \) do not have to be the same. I also write \( (c, p) \sim_{\hat{\mathcal{P}}} (c', p') \).

### 4.3 The meaning of a decision

We had introduced decisions as an "inner" input alphabet. So it is obvious to be interested in the question to what extent the introduced meaning of input characters can be transferred to the decision concept.

There is a notable differences between decisions and characters. The same character can occur in different transitions and therefore can have different meanings, depending on the transition it is attached to. In contrast, decisions are unique. Hence, a meaning definition will partition all decisions into equivalence classes of equal meaning.

The idea is to look at the state that can be reached by a single decision. This leads immediately to the notion of \( \epsilon \)-closure in the decision space and to construct a somehow "reduced" decision automaton in a procedure similar to \( \epsilon \)-elimination for determining a deterministic from a nondeterministic finite automaton.

I first define the \( \epsilon \)-decision closure of a state \( q \):
Definition 7. Let $\mathcal{P}$ be a consistent protocol and $\hat{\mathcal{P}}$ a corresponding GIF. The $\epsilon$-decision closure of a state $q$ is the set of all states that are accessible from $q$ without further decisions under the I/O-coupling mechanism of the protocol, including $q$ itself. That is $h_\epsilon(q) = \{p \in Q | p = q \text{ or: if there is a } p' \in h_\epsilon(q) \text{ and there exists } i \in I \text{ and } o \in O \text{ s.t. } (i, \epsilon, o, p', p) \in \Delta_{\hat{\mathcal{P}}}\}$

Please remember that if the decision is $\epsilon$, such an input character always exists. With these sets of state values we can find the reduced decision automaton as following:

Definition 8. Let $\mathcal{P}$ be a consistent protocol and $\hat{\mathcal{P}}$ a corresponding GIF. Then I call the automaton $\mathcal{B}$, constructed by the following rules, the "reduced decision automaton" or "game in its decision form (GDF)" of $\mathcal{P}$:

- $Q_\mathcal{B}$ is the set of all closures of $\epsilon$-decisions of $\hat{\mathcal{P}}$.
- The input alphabet is the set of all decisions $I_\mathcal{B} = D_{\hat{\mathcal{P}}}$.
- The transition relation $\Delta_\mathcal{B} \subseteq I_\mathcal{B} \times Q_\mathcal{B} \times Q_\mathcal{B}$ is defined by: $(d, p, q) \in \Delta_\mathcal{B}$ if and only if there exists a reachable $p' \in p$ and some $i \in I_\mathcal{P}$, $o \in O_\mathcal{P}$, and $q' \in q$ such that $(i, d, o, p', q') \in \Delta_{\hat{\mathcal{P}}}$.
- The acceptance component $Acc_\mathcal{B}$ is defined by: $p \in Q_\mathcal{B}$ is an element of $Acc_\mathcal{B}$ if and only if at least one $p' \in p$ is an element of $Acc_{\hat{\mathcal{P}}}$.
- The initial state $q_0\mathcal{B}$ is defined as $q_0\mathcal{B} = h_\epsilon(q_0\hat{\mathcal{P}})$.

Figure 5: The transitions of the reduced decision automaton or GDF of the train-controller protocol are shown in blue. The train decides that it arrives and leaves ("IArrive" and "ILeave") and the controller decides when it let the train go ("ILetYouGo")
The resulting reduced decision automaton or GDF is deterministic with $D$ as its input alphabet. Please note, that it relates to the product state space of the protocol. With a GDF, the definition of the meaning of a decision is fairly straightforward:

**Definition 9.** Let $\mathcal{P}$ be a consistent protocol and $\mathcal{B}$ a corresponding GDF. With $p \xrightarrow{d} q$, the meaning of the decision $d$ is given by $\text{interp}_p(d) := q$.

The resulting equivalence classes of equivalent decisions correspond, in my opinion, to our intuitive understanding of decisions. Intuitively, we judge decisions by their consequences rather independently of the initial situation. Whether a child hears a violin and decides "to play the violin", or whether it sees a picture and — we are already talking like this — also decides "to play the violin". Now, we can now formulate this more precisely that both decisions are different but have the same meaning and are therefore in this sense equivalent.

It’s interesting that the input- and output characters do no longer appear in the GDF. They only contribute implicitly as being part of the coupling mechanism between the interacting systems.

I illustrate the transformation of a consistent protocol to a GDF in Fig. 5 for the train protocol.

### 4.4 Composition of meaning in the interaction language

The question is, whether the meaning of characters and decisions as we have defined it in the previous sections is compositional in the same sense as it was the semantics of formal calculi.

**Definition 10.** The meaning of two consecutive characters $c_1, c_2$ is compositional if an operator $\text{op}$ exists such that for the interpretation function for two consecutive characters holds $\text{interp}(d_1, d_2) = \text{op}(\text{interp}(d_1), \text{interp}(d_2))$.

In the case of characters we focus on the operation of concatenation and want to know whether the interaction semantics of consecutive occurring characters (or strings) can be deduced solely from the semantics of the characters itself.

The first thing we have to clarify is what we mean be "consecutively occurring". From the perspective of a single system, two characters occur consecutively if one succeeds the other as input of the system. However, from an interaction perspective, two characters occur consecutively, if they are consecutively exchanged between two systems.

According to our definition, two characters that are consecutively exchanged between two systems within a single chain of interaction have the same meaning.

For two characters that are consecutively received by a system, their composite meaning is only defined, if the meaning of the first character is the state value that the meaning of the second character relates to.

The meaning of two successive decisions is defined as expected as the target state of the second transition of the reduced decision automaton.
We see that the compositionality of the meaning of characters and decisions depends on the well-definedness of the transition relation of the different automata as its defining context. In settings where the transition relation, i.e. the interaction context itself, becomes the object of consideration, compositionality of meaning is destroyed.

5 Other work

As I have already indicated in the beginning, this article touches scientific, engineering as well as philosophical aspects. Gerard Holzmann [Hol91] already noted the similarities between protocols and natural language. Closest to the ideas elaborated in this article seems to me the work of Carolyn Talcott [Tal97] as well as the article of Tizian Schröder and Christian Diedrich [SD20].

Carolyn Talcott [Tal97] uses the term "interaction semantic" of a component to denote the set of sequences of interactions in the sense of input or output messages or silent steps in which it might participate. She composes her components of multisets of so called actors with unique addresses where the actor semantics could be either internal transitions as a combination of execution and message delivery steps or interaction steps with an exchange of messages. In her formalism she takes into account that the interaction semantics must be invariant against renaming of addresses, state and message values but she neither addresses any semantic fulfillment relation nor the concept of the "meaning" of a single exchanged character. In summary, her approach is very similar to the π-calculus [MPW92] but her addresses refer to actors with state and not to stateless channels and the interactions are asynchronous.

Tizian Schröder and Christian Diedrich [SD20] published an approach that is very similar to mine in several but not all respects. They view the semantics of the exchanged characters within an interaction as being provided by its processing. Like [Rei10] they use a discrete system with a system function \( f \), mapping the two sets of input and internal state values onto a set of output and internal state values as processing model. Instead of using transition systems or automata to describe the behavior of these systems, they use the functional representation for both the system under consideration as well as for the environment. Both, the system as well as the environment receive additional (not considered or "rest") inputs resulting in nondeterministic, stateful and asynchronous behavior towards each other (although it remains unclear where this additional input is supposed to come from, possibly from some "unconsidered environment"). They define as the semantics \( \text{Sem} \) of a considered input character \( u_{\text{cons}} \) the set of all possible pairs of output characters and new internal state values provided by the system function, operating on \( u_{\text{cons}} \), the current internal state value and any possible \( u_{\text{rest}} \).

To select a unique result out of this set, they define the set of "decisions" \( \text{Dec} \) of some \( u_{\text{rest}} \) as the set of all possible pairs of output characters and new internal state values provided by the system function, operating on \( u_{\text{rest}} \), the current internal state value and any possible \( u_{\text{cons}} \). They view the internal
state value $x$ as the context of this decision. Now, they claim that the $(x, y)$ realized by the system in its internal state $x$ in response to the input $u_{cons}$ is the intersection $Sem_x(u_{cons}) \cap Dec_x(u_{rest})$. But this claim seems to depend on whether the system function is a bijection or not, as the simple example in Tab. 1 shows:

| $u_{cons}$ | 0 0 0 0 1 1 1 1 |
| $u_{rest}$ | 0 0 1 1 0 0 1 1 |
| $x$ | 0 1 0 1 0 1 0 1 |
| $x'$ | 1 * * * 1 * 1 * |
| $y'$ | 0 * * * 1 * 0 * |

Table 1: An example system function for a system as described in [SD20]. The values of the mapping which are irrelevant to the example have been marked with a '*'.

Just consider $u_{cons} = 1$ and $x = 0$ where we have $Sem_0(1) = \{(1,1),(1,0)\}$ and for $u_{rest} = 0$ and the same $x = 0$, we have $Dec_0(0) = \{(1,1),(1,0)\}$, resulting in $Sem_0(1) \cap Dec_0(0) = \{(1,1),(1,0)\}$ which has more than one element. So, in summary, their key proposal to use the system function to define the semantics of the exchanged character is very much aligned with the ideas of this article. However, according to my understanding, their decision concept is not consistent.

Then there exists extensive research where the iterative character of acquiring knowledge about interaction semantics is already investigated. This could be on an evolutionary timescale (e.g. [BEJvR11] for a brief overview) or on an online-timescale. An example for the latter is Sida I. Wang, Percy Liang and Christopher D. Manning [WLM16] who explore the idea of language games in a learning setting, which they call interactive learning through language game (ILLG). A human wishes to accomplish a certain configuration of blocks, but can only communicate with a computer, who performs the actual actions. The computer initially knows nothing about language and therefore must learn it from scratch through interaction, while the human adapts to the computer’s capabilities. The objective is to transform a start state into a goal state, but the only action the human can take is entering an utterance.

Researchers have also addressed the relation between meaning, knowledge, and logic in the context of interactions in the sense of games or dialogues under the notions of “dialogical logic” [LL78] or “game-theoretical semantics” [HS97]. The former focuses more on real human discourse while the latter focus is more on model-oriented analysis of the logical meaning of linguistic sentences and its relation to certain rule-governed human activities. The basic idea of Hintikka’s evaluation game is that as a proof, a Verifier tries to find a winning strategy in a two person game against a Falsifier such that a given first order formula $\phi$ is true in a given Model $\mathcal{M}$ under some assignment of the variables. Negation, conjunction and disjunction are translated into role switches and choice attributions.
Language philosophy has a long tradition to reflect on the concept of meaning. In modern times, it was the late Ludwig Wittgenstein [Wit53] who stressed the function of language as a tool for interaction with his famous remark "For a large class of cases of the employment of the word 'meaning' — though not for all — this word can be explained in this way: the meaning of a word is its use in the language" (paragraph 43).

Based on his impression of the interactive nature of language, David Lewis [Lew69] was the first to introduced game theory to analyze social conventions and in particular to analyze the conventional use of language expressions. He viewed mutual understanding in an exchange of characters as a coordination problem and introduced signaling games as an analytical instrument. In a signaling game, a sender sends some message as a function of its state such that the receiver gains knowledge about this state and becomes capable to choose an action that is beneficial for both. Then, a convention is a solution of such a coordination problem which contains at least two proper coordination equilibria. Karl Warneryd [Wä93] showed that such conventions arises naturally in evolutionary settings.

Herbert P. Grice [Gri89] emphasized the interactive character of meaning by noting that to understand an utterance is to understand what the utterer intended to convey - and that what has been traditionally understood as its "meaning" is only loosely related to that. Quite recently, K.M. Jaszczolt proposed that to understand the concept of meaning one has to investigate "not the language system and its relation to context but principally the context of interaction itself, with all its means of conveying information" ([Jas16] pp.12-13).

Also related is the field of computational semantics as it is concerned with computing approximations of the meanings of linguistic objects such as sentences, text fragments, and dialogue contributions (e.g., [BM99, Bol20]).

6 Discussion

The aim of this article was to delineate a theory of interaction semantics and en passant provide a concrete understanding of the meaning of characters within an interaction. The key idea was to define a fulfillment relation for the assignment of the alphabets of a protocol, whose interaction mechanism depends on the identical naming of the "exchanged" characters and use the emerging interpretation function to define the "meaning" sought of these characters.

Just assume for a moment, that this approach is complete nonsense — Would it be irrelevant? For sure, we can assume that reading it would then leave the capacity of any esteemed reader to say something meaningful invariant. So despite of being nonsense, it would enfold a certain significance by serving as a good example for the delightful fact, that we do not have to comprehend the meaning of meaning correctly to say something meaningful — as we (fortunately) do not have to comprehend the mathematical concept of induction to be able to count.

Have I met all the requirements any "good" theory of interaction semantics should fulfill that I listed in my introduction?
First, it is not only consistent with the model of information transport and processing, but essentially depends on it. The idea to name the value of physical states by names whose only characteristic is to make them distinguishable is one of the key elements of my construct. It in fact paved the way to use the approach of formal semantics to define a fulfillment relation to define the semantics of interaction. Here two additional ideas unfold their effect, namely to view the interpretation of a protocol as its execution, and to identify the consistency of a protocol as its essential property for the concretization of what I meant with "invariance against renaming of the characters". The latter actually fits nicely to the "consistency management" Johann van Bentham refers to [vB+08]. Thereby the concept of meaning of an exchanged character could be quite naturally identified in our technical description of interactions and also the relation of equal meaning of two such characters. The transition relation of the GIF became the "interpretation context", and, very importantly, suggested a special role of our decision making capabilities for our interpretation within our interaction, namely that the receiver has the opportunity to decide in a defined frame about the meaning of a received character.

What could be possible consequences of my theory of interaction semantics? First, game theory becomes the theory of the meaning of decisions. However, the notion of decision is a complex one, as I introduced it only as a fiction to fill up the void left by the nondeterministic interaction. As this void could also be filled by other interactions, a subject has to coordinate, decisions in this sense can be viewed as a concept to enable the isolated consideration of consistent interactions of a subject that is in fact only partially separable. Only looking at a subject as a whole leverages the full potential of this notion, as only this holistic view leads to the important question, what might be a genuine "free" or internally determined decision in contrast to an effectively externally determined one. Thus, the focus of game theory could shift from exploring strategies in individual interactions by optimizing ad hocly assumed utilities more to the problem of coordination of multiple interactions, for example how to preserve the freedom of decision while coordinating multiple interactions.

In the area of computer science, the enormous growth of the internet in the past was mainly due to semantically agnostic transport protocols for documents like HTTP, FTP, SMTP, etc., leaving the essential problems of semantic interoperability within the sphere of the human mind. Nowadays, however, technical information-processing systems are more and more integrated into interactions on a content-level with a certain degree of autonomy, greatly increasing the interest in clear and systematic concepts of semantic interoperability. Based on the ideas of this theory of interaction semantic, Tizian Schröder and myself [RS20] proposed a simple classification of interactions according to their information transport and processing characteristics, which allows for a sound layering of computer applications. In contrast to other interface theories as proposed for example by Luca de Alfaro and Thomas A. Henzinger, who wrote in [DAH01] that "Nondeterminism in interfaces, however, seems unnecessary and is expensive ..., the resulting interface notion emphasizes the importance of non-deterministic, game-like interfaces in the form of protocols to achieve
semantic interoperability in non-hierarchical interaction networks.

However, I think that the power of the meaning concept in computer science will unfold its full potential if we start to tackle the "simultaneous interaction and understanding (SIAU)" problem with iterative algorithms, mentioned in the introduction. In analogy to the solution of the SLAM problem, this requires us to explore how to represent easy context identification and switching capabilities and language-expressible vague knowledge properly structured to improve it iteratively, based on the speaker’s experience. And not in the sense of a "speech-collage" were a system learns how to formulate sentences in a way that it becomes difficult to distinguish them from those generated by a knowledgeable system by some less knowledgeable system — although this mechanism also seems to be not uncommon even among humans.

Within linguistics, there is a long tradition to distinguish between semantics and pragmatics. Semantics is viewed as the study of the relation between syntactical and real world entities in a sense of the literal meaning of language expressions, following by and large Gottlob Frege’s principle of compositionality. While pragmatics is viewed as the theory of language use, dealing with the way context can influence our understanding of language expressions (e.g. Sza09). I think, that my explanations argues against such a distinction, but rather for a model that first, emphasizes necessary local interpretation contexts in a given interaction, which might be hierarchically structured and which might be changed on the fly, and that secondly emphasizes the role of internal states, both for representing and for acting.

From a philosophical point of view, the theory of interaction semantics implies that: without interpretation, the world is meaningless. Actually, this statement has (at least) 2 connotations. Concretely it says (or means) that meaning is attributed by interpretation and without such a mapping which we declare as interpretation, there is no meaning. More abstractly it says that the notion of meaning depends necessarily on the notion of interpretation as defined by the theory.

Based on the presented concept of meaning, one could speculate that the "flow of thought" we introspectively experience when we think abstractly is based on "anticipated interactions" which would bind our capability to think abstractly reciprocally and thereby tightly to our ability to express ourselves language-wise, just as our ability to imagine playing an instrument like a violin depends on our years of practice of this instrument.

One could further speculate that sense and sensibility are inseparable if we understand our sensibility as a mode of understanding. If we are calm or angry, if we hate or love, we essentially interpret our world differently. Our emotions modulate our intuition about what is relevant or not (see e.g. SL18 for an overview, how emotion and cognition interact).

Actually, this theory of meaning relates state values to state values. There are other theories doing so, like physics. So, I think, the most important consequence of this theory of meaning is to show that talking about meaning is not something special, almost magical, or only philosophical, but it is just another way to talk (and think) about some phenomena, in this case our interactions.
This implies some potential, namely to derive powerful concepts, but it also implies some limitations. We can talk about the physics of an asteroid impact on Earth or we can talk about the meaning of such an event for the existence of humanity. In the latter case it is us who interprets, that is, makes some distinctions about the relevance of a "physical" phenomenon by choosing a certain context. Do bacteria attribute meaning if they follow a gradient of some soluble indicator substance? Yes and no. No in the sense that they do not have a theory of meaning and can articulate what they are doing, but yes in the sense that we can describe what they do in the framework of our theory of meaning relating states to states, separating the relevant from the irrelevant in a chosen context.

A unifying understanding of interaction semantics and meaning could therefore provide a common conceptual framework such that scientists of natural sciences and humanities as well as engineers could understand each other more easily, especially with the advent of the cyber-physical systems that are just on the doorstep.

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