Parity-time symmetry-enhanced simultaneous magnon and photon blockade in cavity magnonic system

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Abstract

In this work, we consider a parity-time (PT) symmetric cavity magnonic system involving the magnon–photon interaction with small magnon Kerr nonlinearity. Moreover, we investigate the effect of PT-symmetry phase on both the magnon and photon blockade. We show that the PT-symmetry phase, which is achievable by properly selecting the system parameters, can relax the large Kerr nonlinearity requirement for magnon blockade. Consequently, simultaneous perfect magnon and photon blockade can be easily obtained even in the presence of a small value of magnon Kerr nonlinearity. The outstanding feature of the selected scheme is the occurrence of simultaneous perfect magnon and photon blockade with only a small value of magnon Kerr nonlinearity. While photon blockade can be easily distinguished experimentally, the experimental realization of magnon statistics and consequently magnon blockade is still a challenge. The prominent feature of the PT-symmetric cavity magnonic system can relax this challenge by following the magnon blockade criteria via the photon statistics.

Keywords: photon statistics, photon blockade, magnon blockade, parity-time symmetry systems

(Some figures may appear in colour only in the online journal)

1. Introduction

In the last decade, hybrid magnonic systems based on spin wave collective excitations in ferromagnetic materials, like yttrium iron garnet (YIG) with very high spin density and low magnon dissipation rate have attracted a great deal of attention [1–8]. The prominent feature of magnons, the ability of coherent interaction with optical and microwave photons [1–8], phonons [9], and superconducting qubits [10] is providing vast novel platforms for applications in quantum technologies such as quantum information processing [11–16], and quantum sensing [17, 18]. Magnons, the elementary excitations of magnetically ordered systems, that their interaction with microwave photons in the strong and even ultrastrong coupling regimes has been demonstrated [1, 2], can have long lifetimes and long coherence times [19–21], which makes them suitable for different applications in quantum information. In the last decade, different quantum behaviour of magnons such as magnon-magnon or magnon-photon-phonon entanglement [11–16], magnon and phonon squeezing [22], bell state generation [23–25] as well as magnon blockade [26–30], as a pure quantum phenomenon have been theoretically studied. Magnon blockade, which similar to photon and phonon blockade in cavity quantum electrodynamics [31] and cavity optomechanics [32], is essentially originated from the anhamonicity in energy eigenvalues of the system, has been theoretically proposed in various hybrid magnonics systems [26–30]. By increasing the nonlinearity of the magnon mode and subsequently the anhamonicity of the energy level

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spacing, the second magnon excitation is blocked once the first one has been excited. This quantum characteristic can lead to a single-magnon source with nonclassical magnon antibunching statistics in which the excitation of two and multi magnons is suppressed. Evidently, in conventional magnon blockade as the anharmonicity of the eigenernergy spectrum is larger, the magnon blockade is stronger and a perfect magnon blockade requires a large value of magnon nonlinearity, such as magnon Kerr nonlinearity in the system.

On the other hand, recent years have witnessed of striking developing practical techniques from the $\mathcal{PT}$ symmetry structures [33–40]. $\mathcal{PT}$ symmetric systems have non-Hermitian Hamiltonian and their prominent feature is that by properly selecting the system parameters, they can have purely real eigenvalues [41]. Where the system breaks the $\mathcal{PT}$ symmetry, is known as exceptional point (EP) in which the pair of eigenvalues collide. Transition from an unbroken $\mathcal{PT}$-symmetric phase to a spontaneous $\mathcal{PT}$ symmetric broken phase has enabled applications such as low power optical isolation [34, 35], single mode microcavity laser [36, 37], loss-induced or gain induced transparency [38, 39], enhanced photon blockade [42], and etc. In recent years, the influence of $\mathcal{PT}$ symmetry on the quantum properties such as quantum enhanced sensing [17], magnon induced transparency and amplification [43] as well as magnon blockade [28] in hybrid magnonic systems has attracted lots of attention.

In [28], only magnon blockade effect in a $\mathcal{PT}$-symmetric-like-three-mode cavity magnomechanical system involving the magnon-photon and magnon–phonon interaction is studied. It is shown that in the $\mathcal{PT}$ symmetry phase and by properly opting the system parameters a perfect magnon blockade can occur. In the present contribution, we study a $\mathcal{PT}$-symmetric cavity magnonic system involving the magnon–photon interaction. We study the effect of $\mathcal{PT}$ symmetry phase on both the magnon and photon statistics and show that simultaneous magnon and photon blockade can occur. While studying photon statistics and hence photon blockade can be experimentally realized [31], experimentally realization of magnon statistics and subsequently magnon blockade is still challenging. Consequently, in our simple considered scheme with coupled cavity-magnon modes, the outstanding feature of simultaneous magnon and photon blockade paves a way to experimentally distinguish magnon blockade. In fact, photon blockade which can be experimentally realized by studying photon statistics, can be considered as a criteria for magnon blockade. Our results moreover show that magnon–phonon interaction, which is considered in [28], is an unnecessary interaction for the enhanced magnon blockade in the $\mathcal{PT}$ symmetry phase.

The paper is organized as follows. In section 2, the system and Hamiltonian are described. Magnon and photon blockade in the $\mathcal{PT}$-symmetric cavity magnonic system are investigated in section 3. Then, in section 4 we represent the physical description of magnon and photon blockade in the $\mathcal{PT}$-symmetric phase. The experimental realization of magnon and photon blockade in the considered scheme are discussed in section 5. Finally, the summary, conclusion, and outlooks are mentioned in section 6.

2. The system and Hamiltonian

Figure 1 shows a schematic of a cavity magnonic system, in which a single YIG sphere is coupled to a microwave cavity. The Hamiltonian of such a system is given by

$$\hat{H} = \hat{H}_a + \hat{H}_m + \hat{H}_{\text{int}},$$

where $\hat{H}_a$ is free energy of the cavity mode. Assuming that the cavity is optimized to support a single mode, the cavity mode is described by $\hat{H}_a = \hbar \omega_a \hat{a}^\dagger \hat{a}$. $\omega_a$ denotes the cavity mode frequency and $\hat{a} (\hat{a}^\dagger)$ represents the photon annihilation (creation) operator. In order to determine the energy of the YIG sphere, the Zeeman energy, demagnetization energy, as well as magnetocrystallian anisotropy energy have to be included. Consequently, the Hamiltonian of the YIG sphere is described by [44]

$$\hat{H}_m = -\int_{V_m} \mathbf{M} \cdot \mathbf{B}_0 d\mathbf{V} - \frac{\mu_0}{2} \int_{V_m} \mathbf{M} \cdot (\mathbf{H}_{\text{de}} + \mathbf{H}_{\text{an}}) d\mathbf{V},$$

where $V_m$ is the volume of the YIG sphere, $\mathbf{M}$ is the sample magnetization, $\mathbf{B}_0$ is the static magnetic field applied in the $z$ direction, and $\mu_0$ is the magnetic permeability of free space. The first term elucidates Zeeman energy, while the second part represents demagnetization and magnetocrystallian anisotropy energies with $\mathbf{H}_{\text{de}}$ and $\mathbf{H}_{\text{an}}$ being the demagnetization field induced by the static magnetic field and the anisotropic field caused by the magnetocrystallian anisotropy in YIG. In the proposed scheme, the YIG sphere is placed inside a microwave cavity with uniform external bias magnetic field, which is applied to the YIG sphere to produce a homogeneous magnonic mode, the so-called Kittel mode. For such a homogeneous magnonic mode, by using the Holstein–Primakoff representation the Hamiltonian can be written in terms of the magnon annihilation and creation operators $\hat{m}$ and $\hat{m}^\dagger$ with frequency $\omega_m$ as [44]

$$\hat{H}_m = \hbar \omega_m \hat{m}^\dagger \hat{m} + \hbar \chi (\hat{m}^\dagger \hat{m})^2.$$  

The second term describes the magnon Kerr effect originating from the magnetocrystallian anisotropy in the YIG sphere. The coefficient $\chi$ characterizing the strength of the nonlinear magnon effect is given by $\chi = \frac{\mu_0 N \gamma \chi_{\text{an}} \chi_{\text{ac}}^2}{\mu_0 N \gamma_{\text{ac}}^2}$, where $\chi_{\text{an}}$ is the first order anisotropy constant of YIG, $\gamma = g_s \mu_B$ is the gyromagnetic ratio with $g_s$ being the Lande factor and $\mu_B$ denoting the Bohr magneton. Finally, the magnon-cavity mode interaction Hamiltonian in the second quantization can be written as $\hat{H}_{\text{int}} = \hbar g (\hat{a} \hat{m}^\dagger + \hat{a}^\dagger \hat{m})$, in which by assuming $g \ll \omega_a, \omega_m$, is simplified under the rotating wave approximation [2]. The strength of magnon-photon coupling is given by $g = \frac{2g_s}{\sqrt{2N}} \sqrt{2N}$ where $s$ and $N$ respectively stand for the spin angular momentum on each unit cell of the magnetic material and the number of unit cells with spin $s = 5/2$ in the YIG sphere [10]. It worth emphasizing that in contrast to the Kerr nonlinear coefficient, which is inversely proportional to the YIG volume ($\chi \propto \frac{1}{V_m}$), the coupling strength is proportional to the square root of the YIG volume ($g \propto \sqrt{N} \propto \sqrt{V_m}$). Consequently, reducing the size of the YIG sphere leads to the Kerr nonlinear coefficient.
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cavity mode and there is gain instead (figure
such an interaction in which the coupling strength of the
with an external magnetic field
where it has been supposed that the magnonic mode is driven
including the dissipation processes, the system can be effect-
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posed that the two magnonic and photonic modes in the system
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are dissipated with
increment and coupling strength decrement. Finally, it is sup-
posed that the two magnonic and photonic modes in the system
are dissipated with
κ
and
κ
m
(κ
a
, κ
m
 > 0), respectively. By
including the dissipation processes, the system can be effect-
vely described by the following non-Hermitian Hamiltonian

\[
\hat{H} = \hbar (\omega_a - i\kappa_a) \hat{a} \hat{a}^\dagger + \hbar (\omega_m - i\kappa_m) \hat{m} \hat{m}^\dagger + \hbar \chi (\hat{m}^\dagger \hat{m})^2 
+ \h g (\hat{a} \hat{m}^\dagger + \hat{a}^\dagger \hat{m}) + \h \Omega \left( \hat{m}^\dagger e^{-i\omega_0 t} + \hat{m} e^{i\omega_0 t} \right),
\]

(4)

where it has been supposed that the magnonic mode is driven
with an external magnetic field [44]. The last term describes
such an interaction in which the coupling strength of the
magnon driving field (whose frequency and magnitude are
respectively given by
\omega
a
and
\delta
a
) with the magnonic mode of
the YIG sphere is
Ω = \frac{g}{2} B, where \eta = \frac{g}{2 \sqrt{3} N}.

In the following, inspired by several studies based on
\mathcal{PT}-symmetric devices [33–39, 42, 43], we suppose that the
photonic mode is amplified such that there is no loss for the
cavity mode and there is gain instead (figure 1). In this case, the
photonic mode dissipation rate is negative (κ
a
 < 0), while in
order to provide the \mathcal{PT}-symmetric conditions in the system,
the magnonic mode dissipation rate is still positive (κ
m
 > 0).

By the substitution of κ
a
 = −κ
m
 in equation (4), it can be found
that this Hamiltonian under the condition of a very
weak nonlinearity, \chi \ll 1, and switching off the driving field,
Ω = 0, satisfies the \mathcal{PT}-symmetric. The Hamiltonian with
positive dissipation rates (κ
a
, κ
m
 > 0) under the above men-
tioned conditions can be written as

\[
\hat{H} = \left( \hat{a}^\dagger \hat{m}^\dagger \right) \left( \begin{array}{c}
\omega_a - i\kappa_a \\
g \\
\omega_m - i\kappa_m \end{array} \right) \left( \begin{array}{c}
\hat{a} \\
\hat{m} \end{array} \right),
\]

(5)

which its eigenvalues are

\[
\omega_{1,2} = \frac{1}{2} \left( \omega_a + \omega_m - i (\kappa_a + \kappa_m) \right) 
\pm \frac{1}{2} \sqrt{4g^2 - [i (\omega_a - \omega_m) + (\kappa_a - \kappa_m)]^2}.
\]

(6)

In equation (6), the two eigenfrequencies, \omega_{1,2}, depend
on \omega_a, \omega_m, κ_a, κ_m, and g. This equation evidently shows
that to obtain purely real eigenvalue spectra, it is required
that ω_a = ω_m \equiv ω, κ_a = −κ_m and g > κ_m which can be well
established in the system [45]. Achieving ω_a = ω_m is experi-
mentally possible and in such a case that the magnon fre-
nency is close to the photon frequency, the strong interaction
between magnon and cavity photon mixes each degree of free-
dom and creates the hybridized states of the cavity magnon-
polariton [45]. The second condition, κ_a = −κ_m, characterizes
the balanced gain and loss for the two coupled modes, cavity
photon and magnon. In this case, the eigenfrequencies reduces
to \omega_{1,2} = \omega_a \pm \sqrt{g^2 - \kappa_m^2}. Consequently, three different scen-
arios can be studied: (i) if \kappa_m \equiv \kappa_m, the two frequencies col-
ides to the central frequency \omega_a, (ii) if \kappa_m > \kappa_m, the system is in
the unbroken \mathcal{PT}-symmetric phase, and finally (iii) if \kappa_m < \kappa_m the
eigenfrequencies become complex and the \mathcal{PT}-symmetric
is spontaneously broken. Consequently, g = \kappa_m often refer-
ing to the spontaneous \mathcal{PT} symmetric point, is called EP. In
order to obtain more insight, the real and imaginary part of
\omega_{1,2} = (\omega_{1,2} - \omega)/\kappa_m versus the normalized detuning, Δ/κ_m,
normalized cavity-magnon coupling strength, g/κ_m, and
the normalized cavity mode dissipation rate, κ_a/κ_m, are respect-
ively represented in figures 2–4, which all show that under
the above mentioned conditions the real and imaginary parts
of the eigenvalues coincide. In more details, the blue-dotted,
cyan solid line, red solid line, and black dotted in figures 2–4
respectively represent Re(\hat{\omega}_1), Im(\hat{\omega}_1), Re(\hat{\omega}_2), and Im(\hat{\omega}_2).
Figure 2 shows the behaviour of these parameters versus the
normalized detuning δ/κ_m, with δ \equiv \omega_a - \omega_m, and under
the selection of κ_a/κ_m = −1, g/κ_m = 1. As is clearly seen, by
selecting δ = 0, or in other words, opting the same frequencies
for the cavity and magnon modes (as one of the main
requirements of the \mathcal{PT}-symmetric cavity magnonic system)
the real and imaginary parts of the eigenfrequencies coincide.
To see the effect of the coupling strength, g, on the beha-
uour of the eigenvalues, the real and imaginary parts of \hat{\omega}_{1,2}
versus the normalized coupling strength g/κ_m are represented
in figure 3. The choice of a \mathcal{PT}-symmetric cavity magnonic
system, represented by selecting parameters δ \equiv \omega_a - \omega_m = 0,
and κ_a = −κ_m, can lead to certain results. One of these results
is the achievement of eigenfrequencies coincidence, where
the eigenfrequencies of the system align or coincide with
each other. Finally, figure 4, which indicates the behaviour of
the real and imaginary parts of \hat{\omega}_{1,2} versus the normalized cav-
ity dissipation rate κ_a/κ_m, clearly elucidate that in a gain-loss

Figure 1. (a) A schematic of a \mathcal{PT}-symmetric cavity magnonic system, in which by changing the position of the YIG sphere the
magnon-photon coupling strength can be controlled. (b) A Diagram of coupled cavity-magnon mode, in which the cavity mode is amplified
with κ_a while the magnon mode is dissipated with κ_m.
coupled cavity magnonic system the real and imaginary parts of the eigenfrequencies collide.

It should be mentioned that $g = \kappa_m$ is not the only EP in such a system. Another selection as $g = \frac{1}{2} |\kappa_\delta - \kappa_m|$ leads to the eigenfrequencies coincidence can also be known as the EP, when $\omega_a = \omega_m$, and $\kappa_a \neq \kappa_m$. Evidently, the threshold of the symmetry breaking strongly depends on the cavity and magnon mode dissipation rates. Several studies such as $PT$-symmetric enhanced sensing [17], magnon induced transparency and amplification [43] as well as magnon blockade [28] in hybrid magnonic systems and etc in the last decade, have motivated us to investigate the behaviour of the magnon and photon statistics and, hence, magnon and photon blockade in the EP of a $PT$-symmetric cavity magnonic system.

3. Magnon and photon blockade in $PT$-symmetric cavity magnonic system

By rotating the Hamiltonian, equation (4), in the frame with the frequency of the magnonic mode driving field, $\omega_d$, we obtain

$$H_{rot} = h \left( \Delta_a - i \kappa_a \right) \hat{a}^\dagger \hat{a} + h \left( \Delta_m - i \kappa_m \right) \hat{m}^\dagger \hat{m} + h \chi \left( \hat{m}^\dagger \hat{m} \right)^2$$

$$+ h g \left( \hat{a}^\dagger \hat{m} + \hat{a} \hat{m}^\dagger \right) + h \kappa_d \left( \hat{m}^\dagger + \hat{m} \right),$$  

(7)

in which $\Delta_a \equiv \omega_a - \omega_d$. Evidently, in the cavity magnon polariton with $\omega_a = \omega_m$ we obtain the same frequency detunings $\Delta_a = \Delta_m \equiv \Delta$.

The magnon (photon) statistics and consequently magnon (photon) blockade effect is characterized by the equal-time second order correlation function

$$g_m^{(2)}(0) = \frac{\langle \hat{m}^\dagger \hat{m} \hat{m} \hat{m}^\dagger \rangle}{\langle \hat{m}^\dagger \hat{m} \rangle^2} = \frac{\sum_n \sum_{m=1}^{\infty} P_m n(n-1) P_n}{\left( \sum_n n P_n \right)^2},$$  

(8)

$$g_a^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2} = \frac{\sum_n \sum_{m=1}^{\infty} P_m n(n-1) P_n}{\left( \sum_n n P_n \right)^2}. $$  

(9)

The equal time second order correlation functions, $g_m^{(2)}(0)$, represents the quantum statistics of magnon (photon) excitations such that $g_m^{(2)}(0) > 1$ and $g_m^{(2)}(0) = 1$ respectively correspond to the super-Poissonian and Poissonian statistics. While for a sub-Poissonian statistics as a non-classical feature $g_m^{(2)}(0) < 1$ should be provided. $g_m^{(2)}(0) < 1$ expresses the unwilling of simultaneous magnon (photon) excitation and hence $g_m^{(2)}(0) \rightarrow 0$ is referred to a perfect magnon (photon) blockade.

The dynamics of the system can be determined by numerically solving the master equation $\frac{d}{dt} \rho = -i[H_{rot}, \rho] + L\rho$, in which $\rho$ is the system density matrix, and $H_{rot}$ is given by

$$H_{rot} = h \Delta_a \hat{a}^\dagger \hat{a} + h \Delta_m \hat{m}^\dagger \hat{m} + h \chi \left( \hat{m}^\dagger \hat{m} \right)^2 + h g \left( \hat{a}^\dagger \hat{m} + \hat{a} \hat{m}^\dagger \right) + h \kappa_d \left( \hat{m}^\dagger + \hat{m} \right).$$  

(10)

In this approach, the dissipation rates are not included in the Hamiltonian but are described by the Liouvillian operators as $L\rho = \kappa_a (2\hat{a}^\dagger \hat{a} \hat{m}^\dagger \hat{m} - \hat{a} \hat{a}^\dagger \hat{m} - \hat{a}^\dagger \hat{a} \hat{m}^\dagger \hat{m}) + \kappa_m (2\hat{m}^\dagger \hat{m} - \hat{m}^2 - \hat{m} \hat{m}^\dagger - \hat{m}^\dagger \hat{m})$, in which as is considered through this study, the thermal photon and magnon excitation are zero, $n_{th}^{(m)} = 0.$
Another approach is analytically solving the Schrödinger equation \( \hat{H}_m \langle \psi \rangle = \hat{H}_m |\psi \rangle \). To analytically determine the magnon (photon) statistics as the main criteria of magnon (photon) blockades, we consider the weak driving limit \( \Omega_d \ll \chi, g, \Delta \), in which the state of the system can be truncated in a few excitation subspace as

\[
|\psi \rangle = C_{00} |00 \rangle + C_{10} |10 \rangle + C_{01} |01 \rangle + C_{11} |11 \rangle + C_{20} |20 \rangle + C_{02} |02 \rangle + C_{12} |12 \rangle + C_{21} |21 \rangle + C_{22} |22 \rangle ,
\]

with \( C_{mn} \) representing the probability amplitude of \( m \) magnon and \( n \) photon number excitations in the system. The probability amplitudes \( C_{mn} \) in the state \( |\psi \rangle \) can be obtained by solving the Schrödinger equation \( i \hbar \frac{\partial}{\partial t} |\psi \rangle = \hat{H}_m |\psi \rangle \). To obtain the probability amplitudes in the steady state, we substitute the truncated state, equation (11), into the Schrödinger equation. Consequently, we obtain

\[
\begin{align*}
\dot{C}_{10} &= C_{10} (\Delta_m - i\kappa_m) + \chi C_{10} + g C_{01} + \Omega \sqrt{2} C_{20} + \Omega, \\
\dot{C}_{01} &= C_{01} (\Delta_m - i\kappa_a) + g C_{10} + \Omega C_{11}, \\
\dot{C}_{11} &= C_{11} (\Delta_m - i\kappa_a) + C_{11} (\Delta_m - i\kappa_m) + \chi C_{11} + \sqrt{2} g (C_{02} + C_{20}) + \Omega \sqrt{2} C_{10} + \Omega C_{01}, \\
\dot{C}_{20} &= 2 C_{20} (\Delta_m - i\kappa_m) + 4 g C_{20} + \sqrt{2} g C_{11} + \sqrt{2} \Omega C_{10}, \\
\dot{C}_{02} &= 2 C_{02} (\Delta_m - i\kappa_a) + g \sqrt{2} C_{11} + \Omega C_{12}, \\
\dot{C}_{12} &= C_{12} (\Delta_m - i\kappa_a) + 2 C_{12} (\Delta_m - i\kappa_m) + \chi C_{12} + 2 g C_{11} + \Omega C_{22} + \Omega C_{02}, \\
\dot{C}_{21} &= 2 C_{21} (\Delta_m - i\kappa_m) + C_{21} (\Delta_m - i\kappa_a) + 4 \chi C_{21} + 2 g C_{12} + \sqrt{2} \Omega C_{11}, \\
\dot{C}_{22} &= 2 C_{22} (\Delta_m - i\kappa_a) + 2 C_{22} (\Delta_m - i\kappa_m) + 4 \chi C_{22} + \Omega \sqrt{2} C_{12},
\end{align*}
\]

which are equal to zero in the steady state.

By considering the truncated state, equation (11), under the assumption of weak driving limit, the equal-time second order correlation functions, \( g_m^{(2)}(0) \) and \( g_a^{(2)}(0) \) can be reduced as follows

\[
\begin{align*}
g_m^{(2)}(0) &= \sum m (m-1) P_n \left( \sum m P_m \right)^2 \\
&\approx \frac{2(C_{20} + C_{21} + C_{22})^2}{(|C_{10} + C_{11}|^2 + 2|C_{20} + C_{21} + C_{22}|^2)}, \\
g_a^{(2)}(0) &= \sum n (n-1) P_n \left( \sum n P_m \right)^2 \\
&\approx \frac{2|C_{02} + C_{12} + C_{22}|^2}{(|C_{01} + C_{11}|^2 + 2|C_{02} + C_{12} + C_{22}|^2)}.
\end{align*}
\]

The results of the analytical approach as well as the ones obtained by numerically solving the master equation using QuTiP [46] are represented in figures 5–7. In order to obtain more insights about the behaviour of the second order magnon and photon correlation functions, \( g_m^{(2)}(0) \) and \( g_a^{(2)}(0) \), in the proposed \( PT \)-symmetric scheme and also see the effect of the magnon detuning, \( \Delta \), the second order correlation functions of magnons and photons \( g_m^{(2)}(0) \) and \( g_a^{(2)}(0) \) versus the normalized detuning, \( \Delta/\kappa_m \), are illustrated in figures 5(a) and (b). In these figures, figures 5(a) and (b), two different configurations are considered: (i) a coupled of dissipated cavity-magnon mode, i.e. \( \kappa_a = \kappa_m (g = \kappa_m) \) and (ii) a coupled dissipative magnon mode and amplified cavity mode, i.e. \( \kappa_a = - \kappa_m (g = \kappa_m) \). As is evident from both the analytical approximated results as well as the exact numerical ones, by selecting coupled dissipative modes \( (\kappa_a = \kappa_m) \), the minimum value of \( g_m^{(2)}(0) \) and \( g_a^{(2)}(0) \) is about 0.9 which can be achieved by tuning \( \Delta/\kappa_m \approx 1.5 \). For the \( PT \)-symmetric coupled modes, i.e. \( \kappa_a = - \kappa_m, g = \kappa_m \), a strong magnon and photon antibunching effect can be obtained, which are considered as perfect magnon and photon blockade because the two magnon as well as photon excitations are strongly suppressed in comparison with the single excitations when the driving field is on resonance with magnon mode, i.e. \( \Delta/\kappa_m = 0 \).

In order to clarify the importance of the EP and its effect on the quantum magnon and photon statistics, the second order correlation functions \( g_m^{(2)}(0) \) and \( g_a^{(2)}(0) \), versus the normalized coupling strength \( g/\kappa_m \), are respectively represented in figures 6(a) and (b). As is mentioned above, \( g/\kappa_m = 1 \) with \( \kappa_a = - \kappa_m \) is not the only EP of the system, \( g = \kappa_m - \kappa_m \) with \( \kappa_a \neq \kappa_m \) can also be considered as an EP, that its influences are represented in figure 6. In this figure, three different cases have been considered: (i) a coupled of decayed cavity and magnon mode with the same dissipation rates \( \kappa_a = \kappa_m \), (ii) a coupled dissipative magnon and amplified cavity modes with the same value of dissipation rates \( \kappa_a = - \kappa_m \), and finally (iii) a coupled of dissipative magnon and amplified cavity modes with different values of dissipation rates \( \kappa_a = - 3 \kappa_m \). To obtain these results, we have selected the resonance detuning, \( \Delta/\kappa_m = 0 \). As is evident, for \( \kappa_a = \kappa_m, g_m^{(2)}(0) \rightarrow 1 \) and \( g_a^{(2)}(0) \rightarrow 1 \) except that for small value of the coupling strength \( 0 < g/\kappa_m < 1 \), which leads to a small antibunching. The behaviour of the two gain-loss coupled cavity magnon-modes is completely different. By choosing \( g/\kappa_m = 1 \) and \( \kappa_a = - \kappa_m \) or \( g/\kappa_m \approx 2 \) and \( \kappa_a = - 3 \kappa_m \) a strong magnon and photon antibunching and subsequently magnon and photon blockade can be achieved. As will be clarified in the next section, the strong antibunching in the \( PT \)-symmetric phase and EP is arising from the huge nonlinearity induced by the EP.

Finally, to see how the second order correlation functions, \( g_m^{(2)}(0) \) and \( g_a^{(2)}(0) \), explicitly depend on the magnon Kerr effect, which leads to a nonlinearity in the system as the main requirement for blockade, \( g_m^{(2)}(0) \) as well as \( g_a^{(2)}(0) \) versus the normalized nonlinearity \( \chi/\kappa_m \) are represented in figures 7(a) and (b). For \( \chi = 0, g_m^{(2)}(0) = g_a^{(2)}(0) = 1 \), which shows the Poissonian statistics and means that there is no magnon and photon blockade in the system. This result affirms that the nonlinear interaction is a necessary condition for the blockade effect. The results, which are obtained by selecting \( g/\kappa_m \approx 2 \) in two different configurations: (i) a coupled dissipative cavity-magnon modes with different dissipation rates
$\kappa_a = 3 \kappa_m$ and (ii) a coupled of dissipative magnon and amplified cavity mode with different dissipation rates $\kappa_a = -3 \kappa_m$, show that an EP can relax the requirement of strong Kerr nonlinearity such that even by a small magnon Kerr nonlinearity, strong magnon and photon excitation suppression is achieved. It is worth emphasizing that in figures 5–7, the numerical and analytical results have a high agreement which shows the validity of the utilized approximation in equation (11). If we truncate the quantum state $|\psi\rangle$ over larger quantum number excitations, we can obtain more exact results which have more consistency with the numerical ones.
Moreover, we consider a series of super mode operators. First, we define a photon bunching suppression and, hence, magnon and photon occurrence can increase. Consequently, the probability of magnon (photon) blockade is blockade once the first one has been excited (figure 8). The anharmonicity of energy level spacing increases by the Kerr nonlinearity increment.

4. Physical description of magnon and photon blockade in the \(P\bar{T}\)-symmetric phase of cavity magnonic system

Before clarifying the effect of \(P\bar{T}\)-symmetric phase of a cavity magnonic system on the magnon and photon blockade, we physically describe how an induced anharmonicity in the energy levels by the Kerr nonlinearity can lead to magnon (photon) blockade. To this end, we consider an arbitrary bosonic mode, which can be a single magnon or cavity mode. As is schematically shown in figure 8, in the absence of Kerr nonlinearity the energy levels are equally spaced. In this case, as the same energies are required to excite the first and second magnon (photon) to the system, after exciting one magnon (photon) to the system, the second one can also be easily excited. Consequently, once there is no nonlinearity in the system, one magnon (photon) can be excited after another and subsequently magnon (photon) blockade is not probable. By increasing the anharmonicity of the energy level spacing, which can be induced by adding the Kerr nonlinearity to the system [47], the second magnon (photon) excitation is blockade once the first one has been excited (figure 8). Consequently, the probability of magnon (photon) blockade occurrence can increase.

In order to obtain a physical intuition about the magnon and photon bunching suppression and, hence, magnon and photon blockade, we rewrite the Hamiltonian, equation (5), in terms of super mode operators. First, we define

\[
P \equiv \begin{pmatrix}
\omega_a - i\kappa_a & g \\
g & \omega_m - i\kappa_m
\end{pmatrix},
\]

which its eigenvalues are represented in equation (6). Moreover, we consider a \(2 \times 2\) transformation matrix as

\[
Q \equiv \begin{pmatrix}
Q_{11} & Q_{12} \\
Q_{21} & Q_{22}
\end{pmatrix} = (|q_1\rangle \langle q_1|)
\]

such that \(PQ = QD\) with

\[
D = \begin{pmatrix}
\omega_1 & 0 \\
0 & \omega_2
\end{pmatrix}.
\]

Consequently, the Hamiltonian [5] can be rewritten as follows

\[
\hat{H} = (\hat{a}^\dagger \hat{m}^\dagger) QQ^{-1} \hat{P} Q Q^{-1} (\hat{a} \hat{m})
= (\hat{A}^\dagger \hat{M}^\dagger) \begin{pmatrix}
\omega_1 & 0 \\
0 & \omega_2
\end{pmatrix} (\hat{A} \hat{M}),
\]

with the definition \(\begin{pmatrix}
\hat{A}^\dagger \hat{M}^\dagger
\end{pmatrix} = Q^{-1} (\hat{a} \hat{m}) \). Now, we consider the Kerr nonlinear term \(\hat{H}_d = h\chi (\hat{m}^\dagger \hat{m})^2\), in equation (7) and rewrite that in terms of the modified super mode operators \(A\) and \(M\). In more details, we substitute \(\hat{m} = Q_{21} \hat{A} + Q_{22} \hat{M}\) in the Kerr nonlinear term and approximately obtain:

\[
\hat{H}_d \approx h\chi A^\dagger AA^\dagger A + h\chi M^\dagger MM^\dagger M + h\chi A^\dagger A \hat{A} \hbar M^\dagger M
\]

in which the nonlinear coefficients are given by

\[
\chi_1 = \chi |Q_{21}|^4,
\]
\[
\chi_2 = \chi |Q_{22}|^4,
\]
\[
\chi_3 = 4\chi |Q_{21}|^2 |Q_{22}|^2.
\]

The coefficients \(Q_{21}\) and \(Q_{22}\), which are generally complex, can be straightforwardly obtained by considering the condition \(PQ = QD\).

By studying the \(P\bar{T}\)-symmetric conditions in the cavity magnonics system, i.e. \(\omega_a = \omega_m\) and \(\kappa_a = -\kappa_m\), after performing the cumbersome algebraic calculations, one can obtain the nonlinear coefficient as \(\chi_j \propto \chi g^4/(\kappa_m^2 - \kappa_m^2 + \epsilon)^2\) with \(j = 1, 2, 3, \epsilon\). The small quantity originating from the Kerr nonlinearity. The calculation of obtaining the effective Kerr nonlinear coefficients is straightforward but is cumbersome and too lengthy to be represented here.

As is evident from the expression of effective Kerr nonlinearities, in the vicinity of the EP, \(g \approx \kappa_m\), which also is known as the \(P\bar{T}\)-symmetric phase, the effective Kerr nonlinearity of both the magnonic and photonic modes is strikingly enhanced. As is explained above, the generated strong effective Kerr nonlinearity leads to a strong anharmonicity in the energy levels of the system. This interpretation denotes that even by selecting a small Kerr nonlinearity (\(\chi \ll 1\)), which cannot essentially change the energy levels of the system, by choosing the \(P\bar{T}\)-symmetric phase, an effective magnon Kerr nonlinearity can be efficiently enhanced and a strong effective photonic Kerr nonlinearity can be generated. Subsequently, the probability of magnon and photon blockade, which are originated from the induced anharmonicity of the energy level spacing can strongly increase and a perfect magnon and photon blockade are probable.

5. Experimental realization of \(P\bar{T}\)-symmetric cavity magnonic system

The main characteristic of the considered scheme is the \(P\bar{T}\)-symmetry in which by properly opting the system parameters a perfect magnon and photon blockade can be achieved. Experimentally controlling a cavity magnonic system with
$PT$-symmetric properties and the observation of the EP has been reported in [45]. One of the requirements to achieve a $PT$-symmetric system is $\omega_p = \omega_m$, in which the magnon mode frequency is determined by $\omega_m = \gamma_0 + \omega_{m,\text{int}}$, with $\gamma_0$ being the electron gyromagnetic ratio, $B_0$ representing the static magnetic field and $\omega_{m,\text{int}}$ denoting the anisotropic field. Evidently, this requirement can be easily realized by tuning $\omega_{m,\text{int}}$ via the static magnetic field which is utilized to excite the Kittel mode [45]. Another requirement is to satisfy $\kappa_a = -\kappa_m$, in which the cavity mode dissipation rate includes the extrinsic and intrinsic decay rates of the cavity, i.e., $\kappa_a = \kappa_{a,\text{int}} + \kappa_{a,\text{ext}}$ while the magnon mode dissipation rate $\kappa_m$ comes from the surface roughness as well as the impurities and defects in the YIG sphere [45]. In the reported experimental scheme [45] the extrinsic cavity mode dissipation rate is tailored to meet the mentioned requirement. Moreover, to observe the exceptional point, where $\gamma = (\kappa_a - \kappa_m)$ the cavity-magnon coupling strength can be tuned by adjusting the displacement of the YIG sphere through which the amplitude of the magnetic field would be changed. Finally, as the value of the Kerr coefficient strongly depends on the angle between the external magnetic field and the crystalline axes, the small required Kerr nonlinearity can be achieved by choosing a suitable value of this angle [48].

While there is no experimental techniques to directly distinguish the second order correlating function of magnons, $g^{(2)}_m(0)$, as the quantum excitations of magnetically ordered systems in the solid YIG sphere, the detection of emitted photons from the cavity and subsequently the measurement of the photon second order correlation function, $g^{(2)}_p(0)$, would be performed by the Hanbury–Brown–Twiss interferometer [31]. On the other hand, paying attention to the simultaneous behaviour of the second order magnon and photon correlation functions in the selected scheme, the measurement of the photonic second order correlation function, $g^{(2)}_p(0)$, can be considered as a criteria not only for photon but also for magnon blockade.

6. Summary, conclusion and outlooks

In the present contribution, inspired by the effect of the $PT$-symmetry phase in the photonic devices, we proposed to study the magnon and photon blockade in the $PT$-symmetric phase of the cavity magnonic system. To this end, we considered cavity-magnonic system in which by properly controlling the cavity and magnon mode dissipation rates as well as the cavity-magnon coupling strength the $PT$-symmetric phase can be provided. We show that the $PT$-symmetry phase and subsequently the EP can strongly suppress the magnon and photon bunching and consequently leads to perfect magnon and photon blockade even in the presence of very weak magnon Kerr nonlinearity in the system. In fact, the $PT$-symmetry phase can relax the large nonlinearity requirement which is considered as a necessary condition in conventional blockade scenarios. Although there is no photonic nonlinearity in the considered scheme, a perfect photon blockade can occur simultaneously with magnon blockade. Thanks to this simultaneous behaviour of magnon and photon statistics, photon statistics can be experimentally considered as a criteria of the magnon blockade.

It worth emphasizing that although in this work we have considered magnon mode driving filed, the same results can be obtained by driving the cavity mode.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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