Subwavelength gratings for creation and focusing of cylindrical vector beams

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Abstract. In this paper we derive the equations for the subwavelength grating vector of polarization element that creates the arbitrary order radially polarized beam. We show that the element can be implemented with different shape for different orientations of the input and output polarization. There are under consideration different essential cases of the first and the second order polarization. The second-order element can be sun- or mira- shape and axicon shape. Intermediate shapes are also possible.

1. Introduction
Cylindrical vector beams of different orders are of wide interest in various areas of optics, such as multiplexed optical data transmission [1], amplitude-polarization modulation of focal distributions [2] for micromanipulation, microscopy, laser ablation, imaging of exoplanets [3], inverse energy flux generation [4].

At the moment there are several methods for the formation of cylindrical vector beams. The main approaches are polarization transformations of the initial beam with liquid-crystalline polarization modulators [5], subwavelength gratings [6, 7], using superposition of vector beams [8]. However, for CO2 laser (wavelength 10.6 μm) subwavelength gratings are more appropriate because they are transmissive for this range.

It is shown in [9, 10] that it is possible to form vector beams using thin lamellar structures, which essentially are subwavelength gratings. In [9], authors propose to form radially and azimuthally polarized beams of the first order using subwavelength gratings of a curved profile. It is shown numerically in [10, 11] that polarization-phase modulation of an initial beam can be implemented with the aid of subwavelength gratings. In particular, in [10, 11] the authors obtain focused radially and azimuthally polarized vortical beams using sectorial elements. The phase jump by π is provided by orienting subwavelength gratings grooves at an angle of 90° in neighboring Fresnel zones. In the article [7] sectorial subwavelength gratings are proposed for the formation of vector beams with radial and azimuthal polarization of arbitrary orders.

2. Subwavelength grating vector for cylindrical vector beam creation
The subwavelength grating works as a uniaxial crystal. Fast axis of the crystal is perpendicular to grating’s grooves and slow axis is oriented along the grating grooves. Thus, half-wave and quarter-wave plate can be manufactured as subwavelength gratings with a certain height.
Jones matrix of the half-wave plate in its own coordinate system (x-axis is oriented as fast axis) has the following form:

\[
M_{z/2}^{\text{eigen}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]  
(1)

If the coordinate system is rotated by the angle \( \varphi \) relative to the polarizing plate, than Jones matrix of the half-wave plate takes the form:

\[
M_{z/2} = M_{\text{rot}}^{-1} M_{z/2}^{\text{eigen}} M_{\text{rot}} = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{pmatrix}.
\]  
(2)

Let us consider electric vector \( \mathbf{E} \):

\[
\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}.
\]  
(3)

If the electric vector is oriented along the x-axis (\( E_y = 0 \)), the Jones matrix works as a rotation matrix that rotates the electric vector by an angle of \( 2\varphi \). Using that fact we can design a subwavelength grating with curved grooves that creates a cylindrical vector beam with an arbitrary order.

The electric vector of the \( m \)-order radially polarized vector beam in cylindrical coordinate system has the following view (Fig. 1):

\[
\mathbf{E}_{\text{out}} = \mathbf{E}_{m}^{\text{rad}}(r, \varphi) = A(r) \begin{pmatrix} \cos(m-1)\varphi \\ \sin(m-1)\varphi \end{pmatrix}.
\]  
(4)

where \( (r, \varphi) \) are cylindrical coordinates and \( A(r) \) is an amplitude.

Fig. 1. Mutual arrangement of coordinate axis, electric vectors, and subwavelength grating vector.
Rotating coordinate system by the angle \( \phi_0 \) we obtain new coordinate system \((r, \phi')\) where \( \phi' = \phi - \phi_0 \). In new coordinate system the electric vector takes the following form:

\[
E_{out} = E_{in}^{rad}(r, \phi') = A(r) \begin{pmatrix} \cos(m-1)(\phi' + \phi_0) \\ \sin(m-1)(\phi' + \phi_0) \end{pmatrix}.
\]

Let us notice that the m-order azimuthally polarized beam has an expression:

\[
E_{out} = E_{m}^{Az}(r, \phi) = A(r) \begin{pmatrix} \sin(m-1)\varphi \\ -\cos(m-1)\varphi \end{pmatrix}.
\]

Thus, rotating the radially polarized beam by the angle \( \frac{\pi}{2(m-1)} \) we can obtain the azimuthal polarization. The interesting case is the first order of polarization. When \( m = 1 \) we can not obtain the radial polarization from azimuthal and vice versa.

If the angle between the \( x' \)-axis and initial electric-field vector \( E_{in} \) is \( \varphi_{in} \) (as in Fig. 1) than the subwavelength grating vector \( K \) in the cylindrical coordinate system is

\[
K(r, \phi') = \begin{pmatrix} \cos \left( \frac{(m-2)\varphi' + (m-1)\varphi_0 + \varphi_{in}}{2} \right) \\ \sin \left( \frac{(m-2)\varphi' + (m-1)\varphi_0 + \varphi_{in}}{2} \right) \end{pmatrix}.
\]

And in Cartesian coordinate system is

\[
K(x', y') = \begin{pmatrix} \cos \left( \frac{m\varphi' + (m-1)\varphi_0 + \varphi_{in}}{2} \right) \\ \sin \left( \frac{m\varphi' + (m-1)\varphi_0 + \varphi_{in}}{2} \right) \end{pmatrix}.
\]

From the equation (8) we can derive that changing the meaning of \( \frac{(m-1)\varphi_0 + \varphi_{in}}{2} \) we just rotate the element. The case of \( m = 2 \) is essential because for different term \( \frac{\varphi_0 + \varphi_{in}}{2} \) the element has its own shape and can not be obtained from another with rotation.

3. Conclusions

In this work, we derive the formula for the subwavelength grating vector for polarizing element that can be used for creation of cylindrical vector beams with arbitrary order. We show that the element can be implemented with different shape for different orientations of the input and output polarization.

Acknowledgements

This work was supported by the Russian Science Foundation (Grant No. 17-12-01258).

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