Modelling and Control of Single Arm Exoskeleton for Upper Limb Rehabilitation Application using Hybrid Hierarchical Sliding Mode and PID Controllers

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Abstract. Assistive and rehabilitative robotics technologies have seen increasing adoptions in the industry and medical applications alike. This paper presents the dynamic modelling and control scheme of a single arm robotic exoskeleton that is aimed for rehabilitation and physiotherapy of upper limbs. The dynamic model was developed using Euler-Lagrange formulation and the obtained equations allow close analysis on different sources of dynamic torques, which exhibit highly non-linear behaviour. A hybrid hierarchical control scheme employing sliding mode (SMC) and PID controllers are proposed. Simulation were setup using Simulink to evaluate the ability of the proposed control scheme. The simulation results are presented and they show that the proposed control scheme was able to control and track the desired trajectory for multiple degrees-of-freedom.

1. Introduction
An exoskeleton is an electromagnetic device worn outside human limbs to augment or sense the movement of the limbs of interest. Exoskeletons have been widely used for healthcare, physiotherapy and military purposes, among a few. Based on the works of [1-2], these areas of applications can be classified into: assistive robotics [3], tele-robotic manipulation [4] and rehabilitation robotics [5].

Assistive robotics was the primary area of application explored by researchers in the early implementations of exoskeletons spearheaded by General Electric Company in the 1960s [6]. Exoskeletons assisted users (military or civilians alike) in transporting heavy loads and amplifying muscle power to perform strenuous tasks [7].

Further developments of exoskeleton expanded tele-robotic manipulation and medical applications. In tele-robotics, exoskeleton has been used to conveniently record and observe skilled and complex human movements that can be replicated and reproduced by robot manipulators, thus maximizing the functionality of industrial robots that has to otherwise be programmed through teaching pendants/devices [8].

In medical industry, traumatic circumstances lead to impairment of human motor system. The integration of bilateral movement practices to rehabilitate and repair motor function has been subject of research [9]. There has been a great potential in employing wearable exoskeletons to enable the aforementioned rehabilitative practices. For instance, [10] reported that 1130 persons out of 100000 persons suffered from upper extremity injuries per year in United States from 1 January to 31
December 2009. The same survey also reported significant injuries (dislocations, fracture, strain and sprains) at the shoulder, elbow and wrist joints.

1.1. Related Works on Upper Limb Exoskeleton Actuation Methods
Several studies were dedicated in developing exoskeletons for arm and upper limb rehabilitations, each with different actuation methods targeted at different joints. [11] uses twisted string actuation module integrated with a force sensor that is mounted at the shoulder of the patients or users. One end of twisted string is connected with the human’s arm whereas the other end is attached with a DC motor inside the module. The changes of the length in twisted string are due to the rotation of the DC motor. From there, human arm’s motion is controlled by the changes of length in twisted string depending on the modes of operation. The advantages of this particular actuation are its high precision and sensitivity, although in terms of installation, this actuation requires large amount of space and therefore installed at the back of users rather than attaching them directly to the targeted joints.

An arm exoskeleton based on pneumatic force-feedback system was developed by [12]. The system consists of double acting cylinders, integrated controller and a pair of on-off valves. PWM (Pulse Width Modulation) is used to control the operation of the valves. Thermodynamic and flow equations of the charging and discharging processes were employed to determine the suitable on-off time and duty cycle to obtain the required cylinder force and position. Pneumatic-driven actuators, however, still suffers from relatively low bandwidth compared to electric actuators, especially for exoskeleton applications.

Another upper limb exoskeleton was developed to demonstrate human-robot cooperation control system for augmentation purpose. The inverse dynamics produces singularities which were overcome with damped least square method to minimize output of three-axis force sensor mounted at the handle [13]. Also, [14] proposed an impedance modelling framework to design of active exoskeleton control through formulation of assistance ratio, resistance ratio and human comfort ratio in the frequency domain.

1.2. Prior Works on Control Systems of Exoskeletons
There exist also several approaches in synthesizing control systems of an exoskeleton. In general, exoskeleton control systems can be classified into model-based control systems and hierarchy-based control systems. For model-based control, the model of an exoskeleton was obtained through mathematical modelling, system identification and neural networking [1]. In hierarchy-based control systems, the controllers can be classified into three hierarchies, namely the task controller, the human-exoskeleton interaction (high-level) controller, and the low-level controller. For task controller, the highest level hierarchy, control the overall task to be performed by the exoskeleton. The high-level controller, in the context of exoskeleton control system, are impedance controller or admittance controllers to determine the level of interactions between human and exoskeleton in terms of the forces. Lastly, low-level controller is the closed-loop controller for physical parameter such as position or force of interest [15].

Many existing exoskeletons do not rely on using a single hierarchy of controller. An integrated cooperation between different hierarchies of controllers to perform better dynamic performance are used instead. In the development of exoskeleton system, most of the low-level controllers employed are PD controller, sliding mode controller [16] and H-∞ controller [17].

Sliding mode control (SMC) is a robust control system that forces a system trajectory into desirable phase trajectory and maintain system stability in the error state space. There are two transition stages of phase trajectory which are reaching phase and sliding phase. The controller consists of two control actions: the corrective controller and the equivalent controller [18]. Different reaching laws determine the chattering vulnerability of controlled system and different sliding surface dominates the stability of the system.

Recent work by [19] incorporated adaptive neural network controller and SMC to perform trajectory tracking for a 2-DOF upper limb exoskeleton. This work successfully demonstrated the
robustness of SMC for trajectory tracking while using neural network controller to perform real-time estimations of unknown system parameters. A blend of PD, fuzzy logic controller and SMC was recently devised by [20] for control of wheelchair-bound 2-DOF upper limb exoskeleton. The technique presented by the authors was able to produce smooth trajectory tracking as control input chattering was successfully reduced by the fuzzy logic controller.

This work presents the dynamic modelling and control system design of a planar 3-DOF (degree-of-freedom) single arm exoskeleton using Hybrid Hierarchical PID and Sliding Mode Controllers. Section 2 discusses the dynamic modelling of the exoskeleton. Section 3 presents the proposed hierarchy-based control system design based on SMC and PID control schemes. Section 4 presents the simulation results and discussions.

2. Mathematical Modelling of the Single Arm Exoskeleton Mechanical System

Considering that common upper limb injuries happen in shoulder, elbow and wrist joints, as discussed in the previous Section, this work chose the mechanical configuration of a planar 3-DOF single arm exoskeleton with shoulder, elbow and wrist joints. Figure 1 shows the free-body diagram of the proposed mechanical configuration.

![Figure 1. Free-body diagram of the 3-DOF planar single-arm exoskeleton](image)

Point O refers to the shoulder joint, which is also the origin, with +x-axis pointing downward and +y-axis pointing horizontally to the right. The potential energy at point O is taken as zero. The elbow and wrist joints are denoted as Points B and D.

Link 1, rotating about Point O with a joint angular displacement $\theta_1$, is modelled as a rigid body with a mass $m_1$, consisting of the mass of the upper arm and the mass of the actuator for the elbow joint and its related mechanism. The center-of-mass (COM) of Link 1 is denoted as Point A, with the moment of inertia $J_1$ taken about the link’s COM.

Link 2, rotating about Point B with an angular displacement $\theta_2$, has a mass of $m_2$, consists of the mass of the forearm and the mass of the actuator for the wrist joint and its related mechanism. Link 2 has a moment of inertia $J_2$ taken about its COM at Point C. Link 3, rotating about Point D with an angular displacement $\theta_3$, has a mass of $m_3$ consisting of the mass of the hand. Link 3 has a moment of inertia $J_3$ taken about its COM at Point E. The dimensions associated with this model are described in figure 1.

To model the dynamic behaviour of the 3-DOF planar exoskeleton, Euler-Lagrange dynamic formulation is used. Here, the system Lagrangian $L$, is given by [21] as the difference of the total kinetic energy $K$ and total potential energy $P$ of the system:

$$ L = K - P $$
The kinetic energy of each Link \( i \) is the sum of its rotational kinetic energy and translational kinetic energy:

\[
K_i = \frac{1}{2} J_i \dot{\theta}_i^2 + \frac{1}{2} m_i V_{i,COM}^2 , \text{ for } i = 1, 2, 3 \tag{2}
\]

The square of each translational velocity is given by:

\[
V_{1,COM}^2 = \dot{x}_A^2 + \dot{y}_A^2 , \quad V_{2,COM}^2 = \dot{x}_C^2 + \dot{y}_C^2 , \quad V_{3,COM}^2 = \dot{x}_E^2 + \dot{y}_E^2 \tag{3}
\]

The total potential energy from all the three links is given as below with trigonometric functions written in abbreviated forms, i.e. \( S_{ji} = \sin(\theta_j + \theta_k + \theta_l) \) and \( C_{ji} = \cos(\theta_j + \theta_k + \theta_l) \):

\[
P = -m_1 g l_{1c} C_1 - m_2 g l_{2c} C_{12} - m_3 g l_{3c} C_{123} \tag{4}
\]

The three joints are revolute joints and thus they are actuated by the net external torques of each joint, \( T_i \). In Euler-Lagrange mechanics, the equations of motion for all three joints can then be written as:

\[
T = \begin{bmatrix} T_1 & T_2 & T_3 \end{bmatrix}^T = \begin{bmatrix} \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} & \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} & \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_3} - \frac{\partial L}{\partial q_3} \end{bmatrix}^T
\tag{5}
\]

The solution of equation (4) takes the form of:

\[
T = H(q)\dot{q} + C(q, \dot{q})\dot{q} + G(q) \tag{6}
\]

where \( q = [\theta_1 \ \theta_2 \ \theta_3]^T \) are the joint variables and \( H(q) \) and \( C(q, \dot{q}) \) are in this case 3x3 matrices. Equation (6) shows the different forms of torques in the system. The first term in the RHS of equation (6), \( H(q)\dot{q} \), signifies inertial torques, second term \( C(q, \dot{q})\dot{q} \) signifies torques emerging from the Coriolis and centrifugal accelerations, and third term \( G(q) \) signifies the torques due by gravitational forces. The elements of \( H(q) \), \( C(q, \dot{q}) \) and \( G(q) \) are derived as shown in equations (7) and (8):

\[
H(q) = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} , \quad C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} , \quad G(q) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \tag{7}
\]
3. Hybrid Hierarchical PID and SMC Controllers

3.1. Proposed Control System

The proposed control system employs hierarchical control system to optimize controlling multiple links simultaneously to provide smooth motions according to the intention of the user (wearer) of the exoskeleton. Figure 2 illustrates the overall strategy of the control system.

In this strategy, the high-level controller that consists of trajectory generator and sliding mode controller (SMC) provide the desired torque to be produced by each link actuator on the joint. Each link actuator is then PID controlled to provide closed-loop control to track the desired torque generated by the SMC.

3.2. High-level Controller: Trajectory Generation

The trajectory generator aims to generate optimum motion trajectory for wearer that uses the proposed exoskeleton for rehabilitation. Here, the optimization in this context covers the reduction of jerk (time derivative of acceleration) and generation of trajectory that complies with wearer’s intention.

To achieve the objective above, a saturation function was applied to the domain of jerk-time function. To do so, first, the derivative of angular acceleration was evaluated to obtain the jerk of the system. Secondly, the saturation function defined was applied to the angular jerk motion. Finally, the saturated jerk was integrated accordingly to give desired angular acceleration, angular velocity and angular position. This method also has the advantage of filtering unwanted noise signal from the system and reconstructing a controlled jerk motion.

\[
H_{11} = J_1 + m_1 l_{1c}^2 + m_2 \left(l_2^2 + l_2 c^2 + 2l_1 l_2 C_2 \right) + m_3 \left(l_1^2 + l_2^2 + l_3 c^2 + 2l_1 l_2 C_2 + 2l_1 l_3 C_3 + 2l_2 l_3 C_{23} \right)
\]

\[
H_{12} = H_{21} = m_2 \left(l_2^2 + l_2 c^2 + l_1 l_2 C_2 \right) + m_3 \left(l_1^2 + l_2^2 + l_3 c^2 + 2l_1 l_3 C_3 + l_2 l_3 C_{23} \right)
\]

\[
H_{13} = H_{31} = m_3 \left(l_1^2 + l_2^2 + l_3 c^2 \right), \quad H_{22} = J_2 + m_2 l_{2c}^2 + m_3 \left(l_1^2 + l_3 c^2 + 2l_2 l_3 C_3 \right)
\]

\[
H_{23} = H_{32} = m_3 \left(l_1^2 + l_2 l_3 C_3 \right), \quad H_{33} = J_3 + m_3 l_{3c}^2
\]

\[
C_{11} = -2\dot{\theta}_1 \left(m_2 l_{1c} S_2 + m_3 l_{1c} S_{23} \right) - 2\dot{\theta}_3 \left(m_2 l_{3c} S_3 + m_3 l_{3c} S_{23} \right)
\]

\[
C_{12} = -\dot{\theta}_3 \left(m_3 l_{2c} S_2 + m_3 l_{3c} S_{23} \right) - 2\dot{\theta}_1 \left(m_2 l_{3c} S_3 + m_3 l_{3c} S_{23} \right)
\]

\[
C_{13} = -\dot{\theta}_1 \left(m_3 l_{3c} S_3 \right), \quad C_{21} = 2\dot{\theta}_3 \left(m_2 l_{3c} S_3 \right)
\]

\[
C_{22} = \dot{\theta}_3 \left(m_3 l_{2c} S_2 + m_3 l_{3c} S_{23} \right) + 2\dot{\theta}_1 \left(m_2 l_{3c} S_3 \right), \quad C_{23} = \dot{\theta}_1 \left(m_3 l_{3c} S_3 \right), \quad C_{33} = 0
\]

\[
G_1 = m_1 g l_{1c} S_1 + m_2 g \left(l_1 S_1 + l_2 S_{12} \right) + m_3 g \left(l_1 S_1 + l_2 S_{12} + l_3 S_{123} \right)
\]

\[
G_2 = m_2 g l_{2c} S_{12} + m_3 g \left(l_2 S_{12} + l_3 S_{123} \right), \quad G_3 = m_3 g l_{3c} S_{123}
\]

3. High-level controller: Generate the desired trajectory of the links

4. Low-level controller: PID controller: Provide closed-loop control for each link to track the reference torque.
3.3. High-level Controller: Sliding Mode Controller

The SMC takes into account the desired angular positions \( q_d \) of the links from the generated trajectory and calculate the reference/desired torques for each actuator. Consider the following:

\[
s = \dot{e} + \Lambda e \quad (9)
\]

where \( s = [s_1, s_2, s_3]^T \), the position error \( e = [e_1, e_2, e_3]^T = q_d - q \) and a diagonal matrix \( \Lambda = \text{diag}(\lambda_1, \lambda_2, \lambda_3) \). Taking the first time derivative of equation (9) gives:

\[
\dot{s} = \ddot{e} + \Lambda \ddot{e} = \ddot{q}_d - \ddot{q} + \Lambda (\ddot{q}_d - \ddot{q}) \quad (10)
\]

Here, we employ exponential reaching law to allow the sliding surface to reach the sliding mode function exponentially:

\[
\dot{s} = -k_s - \eta \text{sgn}(s) \quad (11)
\]

with diagonal matrices \( k = \text{diag}(k_1, k_2, k_3) \) and \( \eta = \text{diag}(\eta_1, \eta_2, \eta_3) \).

Using equations (6), (10) and (11), the reference torque for each joint can be calculated:

\[
T_e = H(q)(\ddot{q}_d + \Lambda (\ddot{q}_d - \ddot{q}) + k_s + \eta \text{sgn}(s)) + C(q, \dot{q})\dot{q} + G(q) \quad (12)
\]

3.4. Low-level Controller: PID Controller

The use of high-level SMC equation (12) allows for the generation of joint reference torques from the 3-DOF planar exoskeleton dynamics that is mainly non-linear. The link actuators are, however, considered as exhibiting linear behaviours and can be controlled by lower level closed-loop controllers like PID controllers.

Each proposed actuator is a DC motor that is physically actuated by varying the DC Voltage input. Here, each link \( i \) is controlled an incremental algorithm PID controller [22] in discrete mode. The Controller Output (CO) is given as follows:

\[
CO_i[n] = CO_i[n-1] + K_{I,i} T_s e_{T,i}[n] + K_{P,i} (e_{T,i}[n] - e_{T,i}[n-1]) + K_{D,i} \frac{e_{T,i}[n] - 2e_{T,i}[n-1] + e_{T,i}[n-2]}{T_s} \quad (13)
\]

where \( e_{T,i} = T_{d,i} - T_i \) is the link’s torque deviation from its reference/desired value, \( T_i \) is the system sampling time, \( n \) is the index of the latest sample.

4. Results and Discussions

To evaluate its effectiveness, the proposed control scheme was implemented in MATLAB Simulink environment. Here, the simulations were performed in MATLAB Simulink and simulation parameters used are based on typical weights and dimensions of adult arms. In this simulation, trajectory tracking for a step-like position were performed.
Figure 3. Simulated joint positions and velocities of step-like position tracking with the current control scheme

Figure 3 shows the simulation results for step-like position tracking for each joint. For each joint, a displacement of 90° (or π/2 radians) were applied. Due to the jerk reduction of the trajectory generation and sliding mode parameters, abrupt changes of desired angles and velocities are dampened. The position tracking of the three joints exhibit predominantly first order responses, with 10%-to-90% rise times observed to be about 2.2 seconds. It can be seen that while the position tracking performed very well, the velocity tracking took some time to track the desired velocity and exhibited small sustained oscillations even when the positions had already settled to the final values.

5. Conclusion and Future Works
In this paper, the dynamic modelling and control of 3-DOF planar upper limb exoskeleton has been discussed. The derived dynamic equations from Euler-Lagrange formulation serves as the basis to analyse the behaviour of the proposed mechanical configuration. In particular, the combined non-linear effects of inertia, Coriolis, centrifugal forces and gravity to the overall joint torques allow for optimum control strategy to designed.

The proposed control scheme combined the use of smooth trajectory generation, sliding mode and PID controllers that took into consideration of human users. Simulation results show that the proposed control scheme is able to track the desired trajectories for each joint.

Future experiments to evaluate the feasibility of the proposed control scheme can be performed by developing a prototype with human users and to compare its performance with the simulated results. Based on the proposed control scheme, future prototype shall be equipped with sensory systems to
measure joint kinematic states (angular positions, velocities and accelerations) as well as measurements of the joint torques.

On a practical point of view, possible challenge that may arise from developed prototype is the effects of dynamic model inaccuracies on the performance and stability of the sliding mode controllers, which is subject to further investigations.

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