Holographic Ricci dark energy: Current observational constraints, quintom feature, and the reconstruction of scalar-field dark energy

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In this work, we consider the cosmological constraints on the holographic Ricci dark energy proposed by Gao et al. [Phys. Rev. D 79, 043511 (2009)], by using the observational data currently available. The main characteristic of holographic Ricci dark energy is governed by a positive numerical parameter $\alpha$ in the model. When $\alpha < 1/2$, the holographic Ricci dark energy will exhibit a quintomlike behavior; i.e., its equation of state will evolve across the cosmological-constant boundary $w = -1$. The parameter $\alpha$ can be determined only by observations. Thus, in order to characterize the evolving feature of dark energy and to predict the fate of the universe, it is of extraordinary importance to constrain the parameter $\alpha$ by using the observational data. In this paper, we derive constraints on the holographic Ricci dark energy model from the latest observational data including the Union sample of 307 type Ia supernovae, the shift parameter of the cosmic microwave background given by the five-year Wilkinson Microwave Anisotropy Probe observations, and the baryon acoustic oscillation measurement from the Sloan Digital Sky Survey. The joint analysis gives the best-fit results (with 1$\sigma$ uncertainty): $\alpha = 0.359^{+0.024}_{-0.025}$ and $\Omega_{m0} = 0.318^{+0.026}_{-0.024}$. That is to say, according to the observations, the holographic Ricci dark energy takes on the quintom feature. Finally, in light of the results of the cosmological constraints, we discuss the issue of the scalar-field dark energy reconstruction, based on the scenario of the holographic Ricci vacuum energy.

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I. INTRODUCTION

The astronomical observations over the past decade imply that our universe is currently dominated by dark energy that leads to an accelerated expansion of the universe (see, e.g., Refs. [1, 2, 3]). The combined analysis of cosmological observations suggests that the universe is spatially flat and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although we can affirm that the ultimate fate of the universe is determined by the feature of dark energy, the nature of dark energy as well as its cosmological origin remain enigmatic at present (for reviews see, e.g., [4]). However, we still can propose some candidates to interpret or describe the properties of dark energy. The most obvious theoretical candidate of dark energy is the cosmological constant $\lambda$ [5], which always suffers from the “fine-
tuning” and “cosmic coincidence” puzzles. The fine-tuning problem asks why the vacuum energy density today is so small compared to typical particle scales. The vacuum energy density is of order $10^{-47}\text{GeV}^4$, which appears to require the introduction of a new mass scale 14 or so orders of magnitude smaller than the electroweak scale. The second difficulty, the cosmic coincidence problem, says: Since the energy densities of vacuum energy and dark matter scale so differently during the expansion history of the universe, why are they nearly equal today? To get this coincidence, it appears that their ratio must be set to a specific, infinitesimal value in the very early universe. Theorists have made lots of efforts to try to resolve the cosmological-constant problem, but all of these efforts turned out to be unsuccessful.

Numerous other candidates for dark energy have also been proposed in the literature, such as an evolving canonical scalar field [6] usually referred to as quintessence, the phantom energy [7] with an equation of state smaller than $-1$ violating the weak energy condition, the quintom energy [8, 9] with an equation of state evolving across $-1$, and so forth.

Actually, the cosmological-constant (or dark energy) problem is in essence an issue of quantum gravity because the cosmological constant (or the dark energy density) is inevitably related to the vacuum expectation value of some quantum fields within the cosmological context. Therefore, in principle, we cannot entirely understand the nature of dark energy before a complete theory of quantum gravity is established. However, although we are lacking a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. By far, the holographic principle is widely believed as a fundamental principle for the theory of quantum gravity that is being established. Hence, it is believed that the holographic principle may play a significant role in shedding light on the nature of the cosmological constant/dark energy.

Currently, an interesting attempt for probing the nature of dark energy within the framework of quantum gravity is the so-called “holographic dark energy” proposal [10, 11, 12]. It is well known that the holographic principle is an important result of the recent research for exploring the quantum gravity (or string theory) [13]. This principle is enlightened by investigations of the quantum property of black holes. In a quantum gravity system, the conventional local quantum field theory will break down because it contains too many degrees of freedom that will lead to the formation of a black hole breaking the effectiveness of the quantum field theory. To reconcile this breakdown with the success of local quantum field theory in describing observed particle phenomenology, some authors proposed a relationship between the ultraviolet (UV) and the infrared (IR) cutoffs due to the limit set by the formation of a black hole. The UV-IR relation in turn provides an upper bound on the zero-point energy density. In other words, if the quantum zero-point energy density $\rho_{\text{vac}}$ is relevant to an UV cutoff, the total energy of the whole system with size $L$ should not exceed the mass of a black hole of the same size, and thus we have $L^3\rho_{\text{vac}} \leq LM_{\text{Pl}}^2$. This means that
the maximum entropy is of the order of $S^{3/4}_{BH}$. When we take the whole universe into account, the vacuum energy related to this holographic principle \[13\] is viewed as dark energy, usually dubbed holographic dark energy (its density is denoted as $\rho_{de}$ hereafter). The largest IR cutoff $L$ is chosen by saturating the inequality so that we get the holographic dark energy density \[12\]

$$\rho_{de} = 3c^2M^2_{Pl}L^{-2},$$

(1)

where $c$ is a numerical constant, and $M_{Pl} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass. If we take $L$ as the size of the current universe, for instance the Hubble radius $H^{-1}$, then the dark energy density will be close to the observational result.

However, if one takes the Hubble scale as the IR cutoff, the holographic dark energy seems not to be able of leading to an accelerating universe. The possibilities of the particle and the event horizons as the IR cutoff were subsequently discussed by Li \[12\], and it was found that only the event horizon acting as the IR cutoff can give a viable holographic dark energy leading to an accelerating universe. The holographic dark energy model based on the event horizon as the IR cutoff has been widely studied \[14, 15\] and found to be consistent with the observational data \[16, 17\].

Although the holographic model based on the event horizon is successful in fitting the current observational data, the model is suffering from some serious conceptual problems. As discussed in Ref. \[18\], the event horizon may lead to an obvious drawback concerning the causality. Since the event horizon is a global concept of space-time, and the density of dark energy is, however, a local quantity, a question naturally arises: Why should a local quantity be determined by a global one? In addition, the event horizon is determined by the future evolution of the universe, leading to a puzzle of why the current density of dark energy is determined by the future evolution of the universe rather than the past of the universe. Moreover, the future event horizon can exist only under the condition that the future evolution of the universe is always in an acceleration phase, and thus it appears that a causality problem is encountered, posting a challenge to the model.

To avoid the causality problem, it was proposed in Ref. \[18\] that the age of the universe can be chosen as the length measure, instead of the horizon distance of the universe. In this case, by combining the general relativity and the uncertainty relation in quantum mechanics, the energy density of quantum fluctuations of space-time can be viewed as dark energy, and this model is consistent with the observational data provided that the unique parameter is taken to be a number of order unity. A new version of this model replacing the age of the universe by the conformal time of the universe was also discussed in Ref. \[19\], in order to avoid some internal inconsistencies in the original model. For further studies on this model, see, e.g., \[20\].

Furthermore, inspired by the above ideas on the holographic dark energy, Gao et al. \[21\] proposed to
consider another interesting possibility: The length scale, namely, the IR cutoff, in the holographic model may be given by the average radius of the Ricci scalar curvature |R|\(^{-1/2}\), so in this case the density of the holographic dark energy is \(\rho_{de} \propto R\). This is the so-called “holographic Ricci dark energy” model. See also, e.g., [22], for extensive studies. The studies on its phenomenological properties show that this model works fairly well in explaining the observations such as the cosmic acceleration, and it could also help to understand the cosmic coincidence problem. The model is free of the causality problem and the age problem that plague the holographic model based on the future event horizon. However, it should be pointed out that the physical motivation for the Ricci model is still obscure in Ref. [21].

Recently, however, Cai, Hu, and Zhang [23] investigated the causal entropy bound in the holographic framework, providing us with an appropriate physical motivation for the holographic Ricci dark energy. The causal entropy bound for a spatial region in a cosmological setting is given by assuming that the maximal black hole in the universe is formed by gravitational collapse with the “Jeans” scale of perturbations, beyond which the black hole cannot form very likely. Therefore, the Jeans scale of perturbations in the universe gives a causal connection scale \(R_{CC}\) that is naturally to be chosen as an IR cutoff in the holographic setup. For gravitational perturbations, \(R_{CC}^{-2} = \text{Max}(H + 2H^2, -H)\) for a flat universe. It turns out that only the case with the choice \(R_{CC}^{-2} = H + 2H^2\) (proportional to the Ricci scalar \(R\) of the Friedmann-Robertson-Walker space-time in this case), could be consistent with the current cosmological observations.

Like the Li model of the holographic dark energy, the main characteristic of the holographic Ricci dark energy is also governed by the numerical parameter \(c\) in the model. In particular, when \(c^2 < 1/2\), the holographic Ricci dark energy will exhibit a quintomlike behavior; i.e., its equation of state will evolve across the cosmological-constant boundary \(w = -1\). The parameter \(c\) can be determined only by observations. Thus, in order to characterize the evolving feature of dark energy and to predict the fate of the universe, it is of extraordinary importance to constrain the parameter \(c\) by using the observational data. Note that hereafter we will use the redefined parameter \(\alpha\) with \(\alpha = c^2\) as in Ref. [21]. In this paper, we will use the observational data currently available to constrain the parameters in the model of holographic Ricci dark energy.

On the other hand, the scalar-field dark energy models are often viewed as an effective description of the underlying theory of dark energy. However, the underlying theory of dark energy cannot be achieved before a complete theory of quantum gravity is established. We can, nevertheless, speculate on the underlying theory of dark energy by taking some principles of quantum gravity into account. The holographic models of dark energy are no doubt tentative in this way. We are now interested in, if we assume the holographic Ricci vacuum energy scenario as the underlying theory of dark energy, how the scalar-field model can be used to effectively describe it. We will also address this issue in light of the fitting results to the observational
This paper is organized as follows: In Sec. II, we review the model of holographic Ricci dark energy and discuss the basic characteristics of the model. In Sec. III, we perform constraints on the holographic Ricci dark energy model by using the up-to-date observational data sets. In Sec. IV, we discuss the issue of the reconstruction of the scalar-field dark energy model from the observations, according to the scenario of the holographic Ricci vacuum energy. Finally, we give the concluding remarks in Sec. V.

II. THE MODEL OF HOLOGRAPHIC RICCI DARK ENERGY

In this section, we briefly review the model of the holographic Ricci dark energy. We first consider the general case with an arbitrary spatial geometry in the Friedmann-Robertson-Walker (FRW) universe, and then in practice we focus only on the spatially flat case as motivated by the inflation.

Consider the FRW universe with the metric
\[ ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \]
where \( k = 1, 0, -1 \) for closed, flat, and open geometries, respectively, and \( a(t) \) is the scale factor of the universe with the convention \( a(t_0) = 1 \). The Friedmann equation is
\[ H^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{k}{a^2}, \]
where \( H = \dot{a}/a \) is the Hubble parameter, the dot denotes the derivative with respect to the cosmic time \( t \), and the summation runs over various cosmic components. If we focus only on the late-time evolution of the universe, the radiation component \( \rho_{\text{rad}} \) is negligible, and then the cosmic contents include the matter component \( \rho_m \) and the dark energy component \( \rho_{\text{de}} \). The Ricci scalar
\[ R = -6 \left( H + 2H^2 + \frac{k}{a^2} \right), \]
determines, as suggested by Gao et al. [21], the density of dark energy:
\[ \rho_{\text{de}} = \frac{3\alpha}{8\pi G} \left( H + 2H^2 + \frac{k}{a^2} \right) = -\frac{\alpha}{16\pi G} R, \]
where \( \alpha \) is a positive numerical constant to be determined by observations. Comparing to Eq. (1), we see that if we identify \( L^{-2} \) with \( -R/6 \), we have \( \alpha = c^2 \). This is the so-called holographic Ricci dark energy model. This model was originally viewed as lacking physical reasoning [21]. Thanks to the work of Cai et al. [23], the Ricci model gets an appropriate physical mechanism or reasoning for which such a dark energy could be motivated. Assuming the maximal black hole in the universe is formed through gravitational collapse data.
of perturbations in the universe, then the Jeans scale of the perturbations gives a causal connection scale \( R_{CC} \) that is naturally to be chosen as an IR cutoff in the holographic setup. For gravitational perturbations, 
\[ R_{CC}^{-2} = \text{Max}(\dot{H} + 2H^2, -H) \]
for a flat universe. It turns out that only the case with the choice 
\[ R_{CC}^{-2} = \dot{H} + 2H^2 \]
(proportional to the Ricci scalar \( \mathcal{R} \) of the FRW space-time in this case) could be consistent with the current cosmological observations. So, the Ricci dark energy can be viewed as originating from taking the causal connection scale as the IR cutoff in the holographic setting.

Now the Friedmann equation can be rewritten as
\[
H^2 = \frac{8\pi G}{3} \rho_{m0} e^{-3x} + (\alpha - 1)ke^{-2x} + \alpha \left( \frac{1}{2} \frac{dH^2}{dx} + 2H^2 \right),
\]
where \( x = \ln a \) and the subscript “0” denotes the present values of various variables, hereafter. This equation can be further rewritten in the following form:
\[
E^2 = \Omega_{m0} e^{-3x} + (1 - \alpha)\Omega_{k0} e^{-2x} + \alpha \left( \frac{1}{2} \frac{dE^2}{dx} + 2E^2 \right),
\]
where \( E \equiv H/H_0 \), \( \Omega_{m0} = 8\pi G \rho_{m0}/(3H^2) \) and \( \Omega_{k0} = -k/H_0^2 \). Solving this equation, one obtains
\[
E(a)^2 = \Omega_{m0} a^{-3} + \Omega_{k0} a^{-2} + \frac{\alpha}{2 - \alpha} \Omega_{m0} a^{-3} + f_0 a^{-(4\alpha - 2)},
\]
where \( f_0 \) is an integration constant. Using the initial condition \( E_0 = E(t_0) = 1 \), the integration constant \( f_0 \) is determined as
\[
f_0 = 1 - \Omega_{k0} - \frac{2}{2 - \alpha} \Omega_{m0}.
\]

In Eq. (8), it is easy to identify the contribution of dark energy (the last two terms on the right hand side); consequently, we can define
\[
\bar{\Omega}_{de} \equiv \frac{\rho_{de}}{\rho_{c0}} = \frac{\alpha}{2 - \alpha} \Omega_{m0} a^{-3} + f_0 a^{-(4\alpha - 2)},
\]
where \( \rho_{c0} = 3H_0^2/(8\pi G) \) is the present critical density of the universe. From this expression, one can see that the parameter \( \alpha \) plays a significant role for the evolution of the Ricci dark energy. When \( 1/2 \leq \alpha \leq 1 \), the equation of state of dark energy will evolve in the region of \(-1 \leq w \leq -1/3\). In particular, if \( \alpha = 1/2 \) is chosen, the behavior of the holographic Ricci dark energy will be more and more like a cosmological constant with the expansion of the universe, such that ultimately the universe will enter the de Sitter phase in the far future. When \( \alpha < 1/2 \), the holographic Ricci dark energy will exhibit a quintomlike evolution behavior (for “quintom” dark energy, see, e.g., [8] and references therein), i.e., the equation of state of holographic Ricci dark energy will evolve across the cosmological-constant boundary \( w = -1 \) (actually, it evolves from the region with \( w > -1 \) to that with \( w < -1 \)). That is to say, the choice of \( \alpha < 1/2 \) makes
the Ricci dark energy today behave as a phantom energy that leads to a cosmic doomsday ("big rip") in the future. Thus, as discussed above, the value of $\alpha$ determines the destiny of the universe in the holographic Ricci dark energy model. On the other hand, from Eq. (10), one can easily infer that the Ricci dark energy could track the evolution of the nonrelativistic matter in the early times, which can help to alleviate the cosmic coincidence problem.

![Graph](image)

FIG. 1: The evolution of the equation of state parameter for the holographic Ricci dark energy. Here we take $\Omega_{m0} = 0.3$ and show the cases for $\alpha = 0.3, 0.4, 0.5, \text{ and } 0.6$. Clearly, the cases with $\alpha \geq 1/2$ behave like a quintessence, and the cases with $\alpha < 1/2$ behave like a quintom.

Of course, one can also derive the usual fractional density of dark energy,

$$\Omega_{de} \equiv \frac{\rho_{de}}{\rho_c} = \frac{\mathcal{Q}_{de}}{E^2},$$

where $\rho_c = 3H^2/(8\pi G)$ is the critical density of the universe. Furthermore, from the energy conservation equation $\dot{\rho}_{de} + 3H(1 + w)\rho_{de} = 0$, one can obtain the equation of state for Ricci dark energy

$$w(z) = -1 + \frac{(1 + z)}{3} \frac{d\mathcal{Q}_{de}}{dz},$$

where $z = (1/a) - 1$ is the redshift.

Since the current observations strongly favor a spatially flat universe that is also supported by the inflation theory, hereafter the discussions will be restricted to the case of $\Omega_{k0} = 0$ (or $k = 0$).

As illustrative examples, we plot in Figs. 1 and 2 the selected evolutions of the holographic Ricci dark energy. Figure 1 shows the evolution of the equation of state $w(z)$, and Fig. 2 shows the evolution of the
FIG. 2: The evolution of the fractional densities $\Omega_{de}(z)$ and $\Omega_{m}(z)$. Also, we plot the cases for $\Omega_{m0} = 0.3$ and $\alpha = 0.3$, 0.4, 0.5, and 0.6. At first sight, one finds that at early times of roughly $z > 2$ the density contribution of dark energy can occupy roughly 20% – 30%. However, it should be pointed out that in this epoch the dark energy behaves like dust matter, so, effectively speaking, the matter density contribution is still $\sim 100\%$, namely, $\Omega_{m}^{(\text{eff})} \sim 1$.

fractional densities $\Omega_{de}(z)$ and $\Omega_{m}(z)$. In both figures, we plot the cases for $\Omega_{m0} = 0.3$ and $\alpha = 0.3$, 0.4, 0.5 and 0.6. From Fig. [1] it is clear that the cases with $\alpha \geq 1/2$ always evolve in the region of $w \geq -1$, whereas the cases with $\alpha < 1/2$ behave as a quintom whose equation of state $w$ crosses the cosmological-constant boundary $-1$ during the evolution. Also, at early times, roughly $z > 2$, the equation of state of the Ricci dark energy approaches 0; i.e., in this model the dark energy behaves like dust matter during most of the epoch of matter domination. This tracking behavior can help to alleviate the cosmic coincidence problem of dark energy. In Fig. [2] one finds that at early times of roughly $z > 2$ the density contribution of dark energy can occupy roughly 20% – 30%. However, it should be pointed out that in this epoch the dark energy behaves like dust matter, so, effectively speaking, the matter density contribution is still $\sim 100\%$, namely, $\Omega_{m}^{(\text{eff})} \sim 1$, almost the same as the $\Lambda$CDM model. It has been shown in Ref. [21] that the structure formation in this model is very similar to that in the $\Lambda$CDM model.

III. CURRENT OBSERVATIONAL CONSTRAINTS

In this section, we constrain the parameters in the holographic Ricci dark energy model and analyze the evolutionary behavior of this dark energy by using the latest observational data of type Ia supernova (SNIa)
combined with the information from cosmic microwave background (CMB) and large scale structure (LSS) observations.

A. Cosmological constraints from SNIa

First, we consider the latest 307 Union SNIa data, the distance modulus \( \mu_{\text{obs}}(z_i) \), compiled in [24]. The SCP (Supernova Cosmology Project) “Union” SNIa compilation brings together data from 414 SNe drawn from 13 independent data sets, of which 307 SNe pass usability cuts. All SNe were fit using a single lightcurve fitter and uniformly analyzed. All analyses and cuts were developed in a blind manner, i.e. with the cosmology hidden. We shall analyze the holographic Ricci dark energy model in light of the Union sample of SNIa in this subsection.

The theoretical distance modulus is defined as

\[
\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0, \tag{13}
\]

where \( \mu_0 \equiv 42.38 - 5 \log_{10} h \), \( h \) is the Hubble constant \( H_0 \) in units of \( 100 \text{ km/s/Mpc} \), and

\[
D_L(z) = (1 + z) \int_0^z \frac{dz'}{E(z'; \theta)} \tag{14}
\]

is the Hubble-free luminosity distance \( H_0 d_L \) (here \( d_L \) is the physical luminosity distance) in a spatially flat FRW universe, where \( \theta \) denotes the model parameters.

The \( \chi^2 \) for the SNIa data is

\[
\chi^2_{\text{SN}}(\theta) = \sum_{i=1}^{307} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \theta)]^2}{\sigma_i^2}, \tag{15}
\]

where \( \mu_{\text{obs}}(z_i) \) and \( \sigma_i \) are the observed value and the corresponding 1\( \sigma \) error for each supernova, respectively. The parameter \( \mu_0 \) is a nuisance parameter, but it is independent of the data points and the data set. Following [25], the minimization with respect to \( \mu_0 \) can be made trivially by expanding the \( \chi^2 \) of Eq. (15) with respect to \( \mu_0 \) as

\[
\chi^2_{\text{SN}}(\theta) = A(\theta) - 2 \mu_0 B(\theta) + \mu_0^2 C, \tag{16}
\]

where

\[
A(\theta) = \sum_{i=1}^{307} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \theta)]^2}{\sigma_i^2}, \tag{17}
\]

\[
B(\theta) = \sum_{i=1}^{307} \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, \theta)}{\sigma_i^2}, \tag{18}
\]
\[ C = \sum_{i=1}^{307} \frac{1}{\sigma_i^2}. \]  

(19)

Evidently, Eq. (15) has a minimum for \( \mu_0 = B/C \) at

\[ \chi^2_{SN}(\theta) = A(\theta) - \frac{B(\theta)^2}{C}. \]  

(20)

Since \( \chi^2_{SN,\text{min}} = \chi^2_{SN,\text{min}} \), instead minimizing \( \chi^2_{SN} \) one can minimize \( \tilde{\chi}^2_{SN} \) which is independent of the nuisance parameter \( \mu_0 \). Obviously, the best-fit value of \( h \) can be given by the corresponding \( \mu_0 = B/C \) at the best fit.

FIG. 3: SCP Union sample of 307 SNIa residual Hubble diagram comparing to the holographic Ricci dark energy model with best-fit values for parameters. The dark-yellow solid line represents the best fit for SNIa alone analysis with \((\alpha, \Omega_{m0}) = (0.394, 0.304)\); the green dashed line represents the best-fit for SNIa+CMB+BAO joint analysis with \((\alpha, \Omega_{m0}) = (0.359, 0.318)\). The data and model are shown relative to the case of \((\alpha, \Omega_{m0}) = (0.394, 0.304)\).

The best fit for the analysis of the SCP Union sample of 307 SNIa happens at \( \alpha = 0.394, \Omega_{m0} = 0.304, \) and \( h = 0.704 \), with \( \chi^2_{\text{min}} = 310.682 \). The Union sample is illustrated on a residual Hubble diagram with respect to our best-fit universe in Fig. 3. Next, we show the probability contours at 68.3% and 95.4% confidence levels for \( \alpha \) versus \( \Omega_{m0} \) in Fig. 4 from the constraints of the SNIa data. The 1\( \sigma \) and 2\( \sigma \) fit values for the model parameters are \( \alpha = 0.394^{+0.152}_{-0.106} \) (1\( \sigma \)) and \( \Omega_{m0} = 0.304^{+0.091}_{-0.131} \) (2\( \sigma \)).

We see that the best-fit value for parameter \( \alpha \) is 0.394, smaller than 0.5, leading the holographic Ricci dark energy to behave as a quintom with equation of state evolving across \( w = -1 \). Moreover, the parameter \( \alpha \) in 1\( \sigma \) range, \( 0.288 < \alpha < 0.546 \), is also basically smaller than 0.5, albeit the 1\( \sigma \) upper bound slightly larger.
FIG. 4: Probability contours at 68.3% and 95.4% confidence levels in the $(\Omega_m^0, \alpha)$ plane, for the holographic Ricci dark energy model, from the constraints of the SCP Union SNIa data. The fit values for model parameters with one-sigma errors are $\alpha = 0.394^{+0.152}_{-0.106}$ and $\Omega_m^0 = 0.304^{+0.091}_{-0.131}$. A point denotes the best fit; at the best fit, we have $\chi^2_{\text{min}} = 310.682$ and $h = 0.704$.

than 0.5, indicating the quintom nature for the holographic Ricci dark energy. To see the constraints on the evolution of the equation of state from the SNIa data, we show in Fig. 5 the corresponding $w(z)$ with 1σ uncertainty. The present value of the equation of state $w_0$, with 1σ error, is $w_0 = -1.215 \pm 0.308$.

From Figs. 4 and 5, we see that the SNIa data alone do not seem to be sufficient to constrain the holographic Ricci dark energy model strictly. The confidence region of the $\Omega_m^0 - \alpha$ plane is rather large; say, the 2σ ranges for the parameters are $\alpha \in (0.235, 0.684)$ and $\Omega_m^0 \in (0.056, 0.441)$. To break the degeneracy of the parameters, we seek to find other observations as complements to the SNIa data. So, in the next subsection, we shall make a combined analysis of SNIa, CMB, and LSS for the model of holographic Ricci dark energy.

B. Cosmological constraints from SNIa, CMB, and BAO

In this subsection, we further perform constraints on the model of holographic Ricci dark energy by combining the observations from SNIa, CMB and LSS. For the CMB data, we use the CMB shift parameter $R$; for the LSS data, we use the parameter $A$ of the baryon acoustic oscillation (BAO) measurement. In fact,
FIG. 5: Constraints on the evolution of the equation of state $w(z)$ from the SNIa data. The central thick solid line represents the best fit, and the light gray contour represents the 1σ confidence level around the best fit. The present value of the equation of state $w_0$, with 1σ error, is $w_0 = -1.215 \pm 0.308$. Errors are calculated by the Fisher matrix approach.

It is commonly believed that both $R$ and $A$ are nearly model-independent and contain essential information of the full CMB and LSS BAO data (however, see also, e.g., [26, 27, 28]).

The shift parameter $R$ is given by [29, 30]

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_{\text{CMB}}} \frac{dz'}{E(z')}$$

(21)

where the redshift of recombination $z_{\text{CMB}} = 1090$ has been updated in the Wilkinson Microwave Anisotropy Probe (WMAP) five-year data [31]. The shift parameter $R$ relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at $z_{\text{CMB}}$, and the angular scale of the first acoustic peak in the CMB power spectrum of temperature fluctuations [29, 30]. The value of the shift parameter $R$ has been updated by WMAP5 [31] to be $1.710 \pm 0.019$ independent of dark energy model. The parameter $A$ of the measurement of the BAO peak in the distribution of Sloan Digital Sky Survey (SDSS) luminous red galaxies is defined as

$$A \equiv \Omega_{m0}^{1/2} E(z_{\text{BAO}})^{-1/3} \left[ \frac{1}{z_{\text{BAO}}} \int_0^{z_{\text{BAO}}} \frac{dz'}{E(z')} \right]^{2/3},$$

(22)

where $z_{\text{BAO}} = 0.35$. The SDSS BAO measurement [32] gives $A = 0.469(n_s/0.98)^{-0.35} \pm 0.017$ (independent of a dark energy model), where the scalar spectral index is taken to be $n_s = 0.960$ as measured by WMAP5.
[31]. We notice that both $R$ and $A$ are independent of $H_0$; thus these quantities can provide a robust constraint as a complement to SNIa data on the holographic Ricci dark energy model.

![Probability contours at 68.3% and 95.4% confidence levels in the ($\Omega_{m0}, \alpha$) plane, for the holographic Ricci dark energy model, from the joint analysis of the SNIa, CMB, and BAO observations. The fit values for model parameters with one-sigma errors are $\alpha = 0.359^{+0.024}_{-0.025}$ and $\Omega_{m0} = 0.318^{+0.026}_{-0.024}$. A point denotes the best fit; at the best fit, we have $\chi^2_{\text{min}} = 324.317$ and $h = 0.711$.]

Here, we pause for a while to make some additional comments on the utilization of the SDSS baryon acoustic peak. Actually, about whether or not the baryon acoustic peak should be used to constrain models of dark energy that behave differently to a cosmological constant, there is still some debate [27, 28]. The reason comes from the assumption of a constant equation of state made in the reconstruction from redshift space to comoving space required to accurately identify the position of the acoustic peak [32]. For an alternative dark energy model where the equation of state is a function of redshift, actually, it would be expected that the change in the position of the acoustic peak is small [28]. Although so, it is indeed difficult to quantify the correction without detailed study for each model in question. However, it should also be pointed out that the SDSS baryon acoustic peak has been adopted by the majority of the cosmology community in placing constraint on dark energy models. Therefore, in this paper, we do use the parameter $A$ of the BAO to constrain the parameter space of the holographic Ricci dark energy model, believing that it is indeed nearly model-independent.

We now perform a joint analysis of SNIa, CMB, and BAO on the constraints of the holographic Ricci
FIG. 7: Constraints on the evolution of the equation of state $w(z)$, from the joint analysis of the SNIa, CMB, and BAO observations. The central thick solid line represents the best fit, and the light gray contour represents the 1σ confidence level around the best fit. The present value of the equation of state $w_0$, with 1σ error, is $w_0 = -1.370 \pm 0.081$. Errors are calculated by the Fisher matrix approach. The quintom feature with $w = -1$ crossing characteristic for the holographic Ricci dark energy can be explicitly seen in this plot.

dark energy model. The total $\chi^2$ is given by

$$\chi^2 = \chi^2_{\text{SN}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}},$$

where $\chi^2_{\text{SN}}$ is given by Eq. (20) for SNIa statistics and $\chi^2_{\text{CMB}} = [(R - R_{\text{obs}})/\sigma_R]^2$ and $\chi^2_{\text{BAO}} = [(A - A_{\text{obs}})/\sigma_A]^2$ are contributions from CMB and BAO, respectively.

The main fitting result is shown in Fig. 6 In this figure, we show the contours of 68.3% and 95.4% confidence levels in the $\Omega_{m0} - \alpha$ plane. It is clear to see that the combined analysis of SNIa, CMB, and BAO data provides a fairly tight constraint on the holographic Ricci dark energy model, compared to the constraint from the SNIa data alone. The fit values for the model parameters with 1- and 2-σ errors are $\alpha = 0.359^{+0.024}_{-0.025}$ (1σ) $^{+0.040}_{-0.040}$ (2σ) and $\Omega_{m0} = 0.318^{+0.026}_{-0.024}$ $^{+0.043}_{-0.038}$ (2σ) with $\chi^2_{\text{min}} = 324.317$. At the best fit, we have $h = 0.711$. We also show the best-fit case of SNIa+CMB+BAO analysis on the residual Hubble diagram with respect to the best-fit case of SNIa alone analysis in Fig. 5 As a comparison, we also fit the spatially flat $\Lambda$CDM model to the same observational data. It is found that, for the $\Lambda$CDM model, we have $\chi^2_{\text{min}} = 313.742$ for the best-fit parameter $\Omega_{m0} = 0.270$.

From Fig. 6 we see that, according to the joint analysis of the observational data, the holographic Ricci...
dark energy takes on the nature of a quintom, since the parameter $\alpha$ is less than 0.5, say, in the $2\sigma$ range, $\alpha \in (0.319, 0.399)$. This result completely rules out the probability of $\alpha > 0.5$ and clarifies the ambiguity in the analysis of SNIa alone. So, the joint analysis definitely concludes that the holographic dark energy behaves as a quintom. The resulting $w(z)$ with $1\sigma$ error is shown in Fig. [7]. In this figure, the quintom feature with the $w = -1$ crossing characteristic for the holographic Ricci dark energy can be explicitly seen. The present value of $w_0$, with a $1\sigma$ error, is $w_0 = -1.370 \pm 0.081$.

IV. THE RECONSTRUCTION OF SCALAR-FIELD DARK ENERGY

As explained by Cai et al. [23], the Ricci dark energy takes the causal connection scale in the universe as the IR cutoff in the holographic setting. When taking the holographic principle into account, the vacuum energy will acquire a dynamical property that its equation of state is evolving, as shown in the previous sections. The current available observational data imply that the holographic Ricci vacuum energy behaves as quintom-type dark energy. Presently, we adopt the viewpoint that the scalar-field models of dark energy are effective theories of an underlying theory of dark energy. If we regard the scalar-field model as an effective description of such a holographic vacuum theory, we should be capable of using the scalar-field model to mimic the evolving behavior of the dynamical vacuum energy and reconstructing this scalar-field model according to the fits of the observational data sets. In this section, we shall discuss this issue.

A. Motivation for reconstruction

It is well known that the cosmological-constant/dark energy problem is an UV problem. However, when considering the holographic property of gravity, the UV regime is related to the IR regime. Thanks to the UV-IR relation, the dark energy problem can be converted to an IR problem. This is the key point of the holographic dark energy proposal. In this view, the UV-IR relation provides an upper bound on the zero-point energy (vacuum energy) density, and consequently the vacuum energy becomes a dynamical dark energy.

Actually, the dynamical dark energy scenario is an alternative proposal to the cosmological-constant scenario. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential. Actually, this mechanism is enlightened to a great extent by the inflationary cosmology. As we have known, the occurrence of the current accelerating expansion of the universe is not the first time for the expansion history of the universe. There is significant observational evidence strongly supporting that
the universe underwent an early inflationary epoch, over sufficiently small time scales, during which its expansion rapidly accelerated under the drive of an “inflaton” field which had properties similar to those of a cosmological constant. The inflaton field, to some extent, can be viewed as a kind of dynamically evolving dark energy. Hence, the scalar-field models involving a minimally coupled scalar field are proposed, inspired by inflationary cosmology, to construct dynamically evolving models of dark energy. The only difference between the dynamical scalar-field dark energy and the inflaton is the energy scale that they possess. Famous examples of scalar-field dark energy models include quintessence \[6\], \(K\)-essence \[33\], tachyon \[34\], phantom \[7\], ghost condensate \[35\] and two-field quintom \[8\], and so forth.

Generically, there are two points of view on the scalar-field models of dynamical dark energy. One viewpoint regards the scalar field as a fundamental field of the nature. The nature of dark energy is, according to this viewpoint, completely attributed to some fundamental scalar field which is omnipresent in supersymmetric field theories and in string/M theory. The other viewpoint supports that the scalar-field model is an effective description of an underlying theory of dark energy. On the whole, it seems that the latter is the mainstream point of view. Since we regard the scalar field model as an effective description of an underlying theory of dark energy, a question arises: What is the underlying theory of the dark energy? Of course, hitherto, this question is far beyond our present knowledge, because we cannot entirely understand the nature of dark energy before a complete theory of quantum gravity is established.

Although we are lacking a quantum gravity theory today, we can, nevertheless, speculate on the underlying theory of dark energy by taking some principles of quantum gravity into account. Needless to say, the holographic models of dark energy are an interesting tentative in this way. Since the holographic principle is taken into account, the holographic models possess some significant features of an underlying theory of dark energy. Now, we are interested in, if we assume the holographic Ricci dark energy as the underlying theory of dark energy, how the scalar-field model can be used to describe it. In Sec. \(III\) we have constrained the holographic Ricci dark energy model using the latest observational data. Hence, in turn, if there is a low-energy effective scalar-field describing the Ricci dark energy, the scalar-field model can be reconstructed in light of the constraint results from the observations. For the works in this way, see, e.g., \[36, 37\].

**B. Reconstructing a single-scalar-field quintom model from the observations**

The nomenclature quintom is suggested in the sense that its behavior resembles the combined behavior of quintessence and phantom. Thus, a simple realization of a quintom scenario is a model with the double fields of quintessence and phantom \[8\]. The cosmological evolution of such a model has been investigated
in detail. It should be noted that such a quintom model would typically encounter the problem of quantum instability inherited from the phantom component.

For the single real scalar-field models, the transition of crossing $-1$ for $w$ can occur for the Lagrangian density $p(\phi, X)$, where $X$ is a kinematic term of a scalar-field $\phi$, in which $\partial p/\partial X$ changes sign from positive to negative, and thus we require nonlinear terms in $X$ to realize the $w = -1$ crossing [38]. When adding a high derivative term to the kinetic term $X$ in the single-scalar-field model, the energy-momentum tensor is proven to be equivalent to that of a two-field quintom model [39].

In addition, it is remarkable that the generalized ghost condensate model of a single real scalar field is a successful realization of the quintomlike dark energy [37, 40]. In Ref. [41], a dark energy model with a ghost scalar field has been explored in the context of the runaway dilaton scenario in low-energy effective string theory. The authors addressed for the dilatonic ghost condensate model the problem of vacuum stability by implementing higher-order derivative terms and showed that a cosmological model of quintomlike dark energy can be constructed without violating the stability of quantum fluctuations. Furthermore, a generalized ghost condensate model was investigated in Refs. [37, 40] by means of the cosmological reconstruction program. In what follows we will focus on the generalized ghost condensate model. We shall use this scalar field model to effectively describe the holographic Ricci dark energy, and perform the reconstruction for such a scalar model. For the reconstruction of dark energy models, see, e.g., [36, 37, 40, 42, 43, 44, 45, 46, 47, 48].

First, let us consider the Lagrangian density of a general scalar field $p(\phi, X)$, where $X = -\partial_{\mu} \phi \partial_{\nu} \phi / 2$ is the kinetic energy term. Note that $p(\phi, X)$ is a general function of $\phi$ and $X$, and we have used a sign notation $(-, +, +, +)$. Identifying the energy-momentum tensor of the scalar field with that of a perfect fluid, we can easily derive the energy density $\rho_{\text{de}} = 2X p_X - p$, where $p_X = \partial p / \partial X$. Thus, in a spatially flat FRW universe, the dynamic equations for the scalar field are

$$3H^2 = \rho_m + 2X p_X - p, \quad (24)$$

$$2\dot{H} = -\rho_m - 2X p_X, \quad (25)$$

where $X = \dot{\phi}^2 / 2$ in the cosmological context. Here we have used the unit $M_{\text{Pl}} = 1$ for convenience. Also, for convenience, we introduce the quantity $r = E^2 = H^2 / H_0^2$. We find from Eqs. (24) and (25) that

$$p = [(1 + z) r' - 3r] H_0^2, \quad (26)$$

$$\dot{\phi}^2 p_X = \frac{r' - 3\Omega_m (1 + z)^2}{r(1 + z)}. \quad (27)$$
where the prime denotes a derivative with respect to $z$. The equation of state for dark energy is given by

$$w = \frac{p}{\phi^2 p_X - p} = \frac{(1 + z)r' - 3r}{3r - 3\Omega_{m0}(1 + z)^3}. \quad (28)$$

Next, if we establish a correspondence between the holographic Ricci vacuum energy and the scalar field dark energy, we should choose a scalar-field model in which crossing the cosmological-constant boundary is possible. So, let us consider the generalized ghost condensate model proposed in Ref. [40], with the Lagrangian density

$$p = -X + h(\phi)X^2, \quad (29)$$

where $h(\phi)$ is a function in terms of $\phi$. The dilatonic ghost condensate model [41] corresponds to a choice $h(\phi) = c e^{\lambda \phi}$. From Eqs. (26) and (27) we obtain

$$\phi'^2 = \frac{12r - 3(1 + z)r' - 3\Omega_{m0}(1 + z)^3}{r(1 + z)^2}, \quad (30)$$

$$h(\phi) = \frac{6(2(1 + z)r' - 6r + r(1 + z)^2\phi'^2)}{r^2(1 + z)^4\phi^4} \rho_{c0}^{-1}, \quad (31)$$

where $\rho_{c0} = 3H_0^2$ represents the present critical density of the universe. The generalized ghost condensate describes the holographic Ricci vacuum energy, provided that

$$r(z) = \frac{2}{2 - \alpha} \Omega_{m0}(1 + z)^3 + f_0(1 + z)^{4 - \frac{3}{2}}, \quad (32)$$

where $f_0 = 1 - 2\Omega_{m0}/(2 - \alpha)$.

In Sec. III we have derived cosmological constraints on the holographic Ricci dark energy model from the joint analysis of SNIa, CMB, and BAO observations. Now, one can reconstruct the function $h(\phi)$ for the generalized ghost condensate model in light of the holographic Ricci dark energy and the corresponding fit results of the observational constraints. The reconstruction for $h(\phi)$ is plotted in Fig. 8 using the $1\sigma$ fit results from the joint analysis of SNIa, CMB, and BAO observations. In this figure, the central black solid line represents the best fit, and the red dotted area around the best fit covers the range of $1\sigma$ errors. The errors quoted in Fig. 8 are calculated using a Monte Carlo method where random points are chosen in the $1\sigma$ region of the parameter space shown in Fig. 6. The evolution of the scalar field $\phi(z)$ is also determined by the reconstruction program (see Fig. 9) in which we have fixed the field amplitude at the present epoch ($z = 0$) to be zero: $\phi(0) = 0$. In addition, the reconstructed evolution of $h(z)$ is also shown in Fig. 10. Note that the errors quoted in Figs. 9 and 10 are calculated using the Fisher matrix approach.

The crossing of the cosmological-constant boundary corresponds to $hX = 1/2$. The system can enter the phantom region ($hX < 1/2$) without discontinuous behavior of $h$ and $X$. In addition, as has been
FIG. 8: Reconstruction of the generalized ghost condensate model according to the holographic Ricci dark energy scenario. In this plot, we show the reconstructed function $h(\phi)$, in units of $\rho^{-1}$, corresponding to the joint analysis results of SNIa, CMB, and BAO observations. The central black solid line represents the best fit, and the red dotted area covers the range of 68% errors. The errors are calculated using a Monte Carlo method.

pointed out by Tsujikawa [40], it should be cautioned that the perturbation of the field $\phi$ is plagued by a quantum instability whenever it behaves as a phantom [41]. Even at the classical level, the perturbation becomes unstable for $1/6 < hX < 1/2$, because the speed of sound $c_s^2 = p_X/(p_X + 2X p_{XX})$ will become negative. This instability may be avoided if the phantom behavior is just transient. In fact the dilatonic ghost condensate model can realize a transient phantom behavior (see, e.g., Fig. 4 in Ref. [41]). In this case the cosmological-constant boundary crossing occurs again in the future, after which the perturbation will become stable. Nevertheless, one may argue that the field can be regarded as an effective one so as to evade problems such as stability. In particular, the present focus is how to establish a dynamical scalar-field model on a phenomenological level to describe the possible underlying theory of dark energy, disregarding whether the field is fundamental or not.

V. CONCLUDING REMARKS

The cosmic acceleration observed by distance-redshift relation measurement of SNIa strongly supports the existence of dark energy. The fantastic physical property of dark energy not only drives the current
FIG. 9: Reconstruction of the generalized ghost condensate model according to the holographic Ricci dark energy scenario. In this plot, we show the evolution of the scalar field $\phi(z)$, in units of the Planck mass $M_{\text{Pl}}$ (note that here the Planck normalization $M_{\text{Pl}} = 1$ has been used), corresponding to the joint analysis results of SNIa, CMB, and BAO observations.

FIG. 10: Reconstruction of the generalized ghost condensate model according to the holographic Ricci dark energy scenario. In this plot, we show the evolution of the function $h(z)$, in units of $\rho_{c0}^{-1}$, corresponding to the joint analysis results of SNIa, CMB, and BAO observations.
cosmic acceleration, but also determines the ultimate fate of the universe. However, hitherto, the nature of
dark energy as well as its cosmological origin still remain enigmatic for us. Though the underlying theory
of dark energy is still far beyond our knowledge, it is guessed that the quantum gravity theory shall play a
significant role in resolving the dark energy enigma.

Therefore, one can try to probe the nature of dark energy according to some principles of quantum
gravity. By far, the holographic principle is widely believed as a fundamental principle for the theory of
quantum gravity. So, the holographic models of dark energy become an important attempt for exploring
dark energy within the framework of quantum gravity. It is believed that the holographic models possess
some significant features of an underlying theory of dark energy.

In this paper, we consider the model of holographic Ricci dark energy that can be viewed as originating
from taking the causal connection scale as the IR cutoff in the holographic setting. The main characteristic of
holographic Ricci dark energy is governed by a positive numerical parameter $\alpha$ in the model. In particular,
when $\alpha < 1/2$, the holographic Ricci dark energy will exhibit a quintomlike behavior; i.e., its equation of
state will evolve across the cosmological-constant boundary $w = -1$. The parameter $\alpha$ can be determined
only by observations. Thus, in order to characterize the evolving feature of dark energy and to predict the
fate of the universe, it is of extraordinary importance to constrain the parameter $\alpha$ by using the observational
data.

We have derived, in this paper, the constraints on the holographic Ricci dark energy model from the latest
observational data including the 307 Union sample of SNIa, the CMB shift parameter given by WMAP5,
and the BAO measurement from SDSS. The joint analysis gives the best-fit results (with 1$\sigma$ confidence
level): $\alpha = 0.359^{+0.024}_{-0.025}$ and $\Omega_{m0} = 0.318^{+0.026}_{-0.024}$. That is to say, according to the observations, the holographic
Ricci dark energy takes on a quintom feature.

If we regard the scalar-field model as an effective description of such a theory (holographic Ricci vac-
uum energy), we should be capable of using the scalar-field model to mimic the evolving behavior of the
dynamical vacuum energy and reconstructing this scalar-field model according to the evolutionary behavior
of holographic Ricci dark energy and the fits to the observational data sets. We find the generalized ghost
condensate model is a good choice for depicting the holographic Ricci vacuum energy, since it can easily
realize the quintom behavior. We thus reconstructed the function $h(\phi)$ of the generalized ghost condensate
model using the fit results of the observational data (SNIa + CMB + BAO). We hope that the future high
precision observations (e.g., the SuperNova Acceleration Probe) may be capable of determining the fine
property of the dark energy and consequently reveal some significant features of the underlying theory of
dark energy.
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