An Efficient Approach to Achieve Compositionality using Optimized Multi-Version Object Based Transactional Systems

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Abstract

In the modern era of multi-core systems, the main aim is to utilize the cores properly. This utilization can be done by concurrent programming. But developing a flawless and well-organized concurrent program is difficult. Software Transactional Memory Systems (STMs) are a convenient programming interface which assist the programmer to access the shared memory concurrently without worrying about consistency issues such as priority-inversion, deadlock, livelock, etc. Another important feature that STMs facilitate is compositionality of concurrent programs with great ease. It composes different concurrent operations in a single atomic unit by encapsulating them in a transaction.

Many STMs available in the literature execute read/write primitive operations on memory buffers. We represent them as Read-Write STMs or RWSTMs. Whereas, there exist some STMs (transactional boosting and its variants) which work on higher level operations such as insert, delete, lookup, etc. on a hash-table. We refer these STMs as Object Based STMs or OSTMs.

The literature of databases and RWSTMs say that maintaining multiple versions

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ensures greater concurrency. This motivates us to maintain multiple version at higher level with object semantics and achieves greater concurrency. So, this paper proposes the notion of Optimized Multi-version Object Based STMs or OPT-MVOSTMs which encapsulates the idea of multiple versions in OSTMs to harness the greater concurrency efficiently. For efficient memory utilization, we develop two variations of OPT-MVOSTMs. First, OPT-MVOSTM with garbage collection (or OPT-MVOSTM-GC) which uses unbounded versions but performs garbage collection scheme to delete the unwanted versions. Second, finite version OPT-MVOSTM (or OPT-KOSTM) which maintains at most $K$ versions by replacing the oldest version when $(K + 1)^{th}$ version is created by the current transaction.

We propose the OPT-MVOSTMs for hash-table and list objects as OPT-HT-MVOSTM and OPT-list-MVOSTM respectively. For memory utilization, we propose two variants of both the algorithms as OPT-HT-MVOSTM-GC, OPT-HT-KOSTM and OPT-list-MVOSTM-GC, OPT-list-KOSTM respectively. OPT-HT-KOSTM performs best among its variants and outperforms state-of-the-art hash-table based STMs (HT-OSTM, ESTM, RWSTM, HT-MVTO, HT-KSTM) by a factor of 3.62, 3.95, 3.44, 2.75, 1.85 for workload W1 (90% lookup, 8% insert and 2% delete), 1.44, 2.36, 4.45, 9.84, 7.42 for workload W2 (50% lookup, 25% insert and 25% delete), and 2.11, 4.05, 7.84, 12.94, 10.70 for workload W3 (10% lookup, 45% insert and 45% delete) respectively. Similarly, OPT-list-KOSTM performs best among its variants and outperforms state-of-the-art list based STMs (list-OSTM, Trans-list, Boosting-list, NOrec-list, list-MVTO, list-KSTM) by a factor of 2.56, 25.38, 23.57, 27.44, 13.34, 5.99 for W1, 1.51, 20.54, 24.27, 29.45, 24.89, 19.78 for W2, and 2.91, 32.88, 28.45, 40.89, 173.92, 124.89 for W3 respectively. OPT-MVOSTMs are generic for other data structures as well. We rigorously proved that OPT-MVOSTMs satisfy opacity and ensure that transaction with lookup only methods will never return abort while maintaining unbounded versions.

**Keywords:** Software Transactional Memory Systems, Optimized, Lazyrb-list, Hash-Table, List, Object, Multi-version, Compositionality, Opacity, Keys
1. Introduction

Nowadays, multi-core systems are in trend which necessitated the need for concurrent programming to exploit the cores appropriately. Howbeit, developing the correct and efficient concurrent programs is difficult. Software Transactional Memory Systems (STMs) are a convenient programming interface which assist the programmer to access the shared memory concurrently using multiple threads without worrying about consistency issues such as deadlock, livelock, priority-inversion, etc. STMs facilitate one more feature compositionality of concurrent programs with great ease which makes it more approachable. Different concurrent operations that need to be composed to form a single atomic unit is achieved by encapsulating them in a transaction. In this paper, we discuss various STMs such as read-write STMs (or RWSTMs), object based STMs (or OSTMs) available in the literature along with the benefits of OSTMs over RWSTMs. After that, we motivated from multi-version RWSTMs and propose multi-version object based STMs (or MVOSTMs) [1] which maintain multiple versions and improves the concurrency further. Later, we made a couple of modifications (discussed in Section 4, Section 5 and Section 7) to optimize the MVOSTMs and propose optimized MVOSTMs (or OPT-MVOSTMs).

Read-Write STMs: There exists a lot of popular STMs in the literature such as ESTM [2], NOrec [3] which executes read/write operations on transaction objects or t-objects. We represent these STMs as Read-Write STMs or RWSTMs. RWSTMs typically export the following methods: (1) \texttt{t.begin}: which begins a transaction with a unique identity, (2) \texttt{t.read} (or \texttt{r}): which reads the value of t-object from shared memory, (3) \texttt{t.write} (or \texttt{w}): which writes the new value to t-object in its local memory, (4) tryC: which validates the values written to t-objects by the transaction and tries to commit. If all the updates made by the transaction is consistent then updates reflect to the shared memory and transaction returns commit, and (5) tryA: which returns abort on any inconsistency.

Object based STMs: There are few STMs available in the literature which executes higher level operations such as insert, delete, lookup on hash-table. We represent these STMs as Object based STMs or OSTMs. The concept of Boosting by Herlihy et al. [4], the optimistic variant by Hassan et al. [5] and recently HT-OSTM system by Peri et al.
are some examples that demonstrate the performance benefits achieved by *OSTMs*. Peri et al. [6] showed that *OSTMs* provide greater concurrency than RWSTMs while reducing the number of aborts.

**Benefits of **OSTMs** over RWSTMs**: To show the benefits of OSTMs, we consider a hash-table based STM system which invokes insert (or *ins*), lookup (or *lu*) and delete (or *del*) method. Each hash-table consists of B buckets with the elements in each bucket arranged in the form of a linked-list. Figure 1 (a) represents a hash-table with the first bucket containing keys \(\langle k_3, k_6, k_8 \rangle\). Figure 1 (b) shows the execution by two transactions \(T_1\) and \(T_2\) represented in the form of a tree. \(T_1\) performs lookup operations on keys \(k_3\) and \(k_8\) while \(T_2\) performs a delete on \(k_6\). The delete on key \(k_6\) generates read on the keys \(k_3, k_6\) and writes the keys \(k_6, k_3\) assuming that delete is performed similar to delete operation in lazy-list [7]. The lookup on \(k_3\) generates read on \(k_3\) while the lookup on \(k_8\) generates read on \(k_3, k_8\). Note that in this execution \(k_6\) has already been deleted by the time lookup on \(k_8\) is performed.

In this execution, we denote the read-write operations (leaves) as layer-0 and \(lu, \text{del}\) methods as layer-1. Consider the history (execution) at layer-0 (while ignoring higher-level operations), denoted as \(H_0\). It can be verified this history is not opaque [8]. This is because, between the two reads of \(k_3\) by \(T_1, T_2\) writes to \(k_3\). It can be seen that if history \(H_0\) is input to an RWSTMs one of the transactions between \(T_1\) or \(T_2\) would be aborted to ensure opacity [8]. Figure 1 (c) shows the presence of a cycle in the conflict graph of \(H_0\).

Now, consider the history \(H_1\) at layer-1 consists of \(lu, \text{del}\) methods, while
ignoring the read/write operations since they do not overlap (referred to as pruning in [9, Chap 6]). These methods work on distinct keys \( (k_3, k_6, \text{and } k_8) \). They do not overlap and are not conflicting. So, they can be re-ordered in either way. Thus, \( H1 \) is opaque with equivalent serial history \( T_1T_2 \) (or \( T_2T_1 \)) and the corresponding conflict graph shown in Figure 1 (d). Hence, a hash-table based \( OSTM \) system does not abort any of \( T_1 \) or \( T_2 \). This shows that \( OSTM \)s can reduce the number of aborts and provide greater concurrency.

**Multi-Version Object Based STMs:** Some of the \( OSTM \)s such as [4], [5], [6] exploits the advantages of it. In this paper, we propose and analyze \textit{Optimized Multi-version Object Based STMs} or \textit{OPT-MVOSTMs} along with the rigorous correctness proof. This work is motivated by the observation that databases and \textit{RWSTM}s achieves greater concurrency by storing multiple versions corresponding to each t-object [10]. Specifically, maintaining multiple versions can ensure that more read operations succeed because the reading operation will obtain an appropriate version to read. Our goal is to analyze the benefit of \textit{OPT-MVOSTMs} over both single version \textit{OSTM}s and multi-version \textit{RWSTM}s.

**The potential benefit of OPT-MVOSTMs over OSTM and multi-version RWSTM:**

We now illustrate the advantage of \textit{OPT-MVOSTMs} as compared to single-version \textit{OSTM}s (\textit{SV-OSTMs}) using the hash-table object with \( B \) buckets having the same operations as discussed above: \textit{ins}, \textit{lu}, \textit{del}. Figure 2 (a) represents a history \( H \) with two concurrent transactions \( T_1 \) and \( T_2 \) operating on a hash-table \( ht \). \( T_1 \) first tries to perform a \textit{lu} on key \( k_3 \). But due to the absence of key \( k_3 \) in \( ht \), it obtains a value of \textit{null}. Then \( T_2 \) invokes \textit{ins} method on the same key \( k_3 \) and inserts the value \( v_3 \) in \( ht \). Then \( T_2 \) deletes the key \( k_2 \) from \( ht \) and returns \( v_0 \) implying that some other transaction had previously inserted \( v_0 \) into \( k_2 \). The second method of \( T_1 \) is \textit{lu} on the key \( k_2 \). With this execution, any \textit{SV-OSTM} system has to return abort for \( T_1 \)’s \textit{lu} operation to ensure
correctness, i.e., opacity. Otherwise, if $T_1$ would have obtained a return value $v_0$ for $k_2$, then the history would not be opaque anymore. This is reflected by a cycle in the corresponding conflict graph between $T_1$ and $T_2$, as shown in Figure 2(c). Thus to ensure opacity, $SV-OSTM$ system has to return abort for $T_1$'s lookup on $k_2$.

In an $OPT-MVOSTMs$ based on hash-table, denoted as $OPT-HT-MVOSTM$, whenever a transaction inserts or deletes a key $k$, a new version is created. Consider the above example with an $OPT-HT-MVOSTM$, as shown in Figure 2(b). Even after $T_2$ deletes $k_2$, the previous value of $v_0$ is still retained. Thus, when $T_1$ invokes $lu$ on $k_2$ after the delete on $k_2$ by $T_2$, $OPT-HT-MVOSTM$ return $v_0$ (as previous value). With this, the resulting history is opaque with equivalent serial history being $T_1T_2$. The corresponding conflict graph is shown in Figure 2(d) does not have a cycle.

Thus, $OPT-MVOSTM$ reduces the number of aborts and achieve greater concurrency than $SV-OSTMs$ while ensuring the compositionality. We believe that the benefit of $OPT-MVOSTM$ over multi-version $RWSTM$ is similar to $SV-OSTM$ over single-version $RWSTM$ as explained above. $OPT-MVOSTM$ is a generic concept which can be applied to any data structure. In this paper, we have considered the hash-table and list based $OPT-MVOSTMs$ as $OPT-HT-MVOSTM$ and $OPT-list-MVOSTM$ respectively. If the bucket size $B$ of hash-table becomes 1 then hash-table based $OPT-MVOSTMs$ boils down to the list based $OPT-MVOSTMs$.

$OPT-HT-MVOSTM$ and $OPT-list-MVOSTM$ use an unbounded number of versions for each key. To address this issue, we develop two variants for both hash-table and list data structures (or DS): (1) A garbage collection method in $OPT-MVOSTMs$ to delete the unwanted versions of a key, denoted as $OPT-MVOSTM-GC$. Garbage collection gave an average performance gain of 16% over $OPT-MVOSTM$ without garbage collection in the best case. Thus, the overhead of garbage collection scheme is less than the performance improvement due to improved memory usage. (2) Placing a limit of $K$ on the number versions in $OPT-MVOSTM$, resulting in $OPT-KOSTM$. This gave an average performance gain of 24% over $OPT-MVOSTM$ without garbage collection in the best case.

Experimental results show that $OPT-HT-KOSTM$ performs best among its variants and outperforms state-of-the-art hash-table based STMs (HT-OSTM, ESTM, RWSTM,
HT-MVTO, HT-KSTM) by a factor of 3.62, 3.95, 3.44, 2.75, 1.85 for workload W1 (90% lookup, 8% insert and 2% delete), 1.44, 2.36, 4.45, 9.84, 7.42 for workload W2 (50% lookup, 25% insert and 25% delete), and 2.11, 4.05, 7.84, 12.94, 10.70 for workload W3 (10% lookup, 45% insert and 45% delete) respectively. Similarly, \textit{OPT-list-KOSTM} performs best among its variants and outperforms state-of-the-art list based STMs (list-OSTM, Trans-list, Boosting-list, NOrec-list, list-MVTO, list-KSTM) by a factor of 2.56, 25.38, 23.57, 27.44, 13.34, 5.99 for W1, 1.51, 20.54, 24.27, 29.45, 24.89, 19.78 for W2, and 2.91, 32.88, 28.45, 40.89, 173.92, 124.89 for W3 respectively. To the best of our knowledge, this is the first work to explore the idea of using multiple versions in \textit{OSTMs} to achieve greater concurrency.

\textbf{Contributions of the paper:}

- We propose a new notion of optimized multi-version objects based STM system as \textit{OPT-MVOSTM} in Section 4. In this paper, we develop it for list and hash-table objects as \textit{OPT-list-MVOSTM} and \textit{OPT-HT-MVOSTM} respectively. \textit{OPT-MVOSTM} is generic for other data structures as well.

- For efficient space utilization in \textit{OPT-MVOSTMs} with unbounded versions, we develop \textit{Garbage Collection} for \textit{OPT-MVOSTM} (i.e. \textit{OPT-MVOSTM-GC}) and bounded version \textit{OPT-MVOSTM} (i.e. \textit{OPT-KOSTM}).

- Section 6 shows that \textit{OPT-list-MVOSTM} and \textit{OPT-HT-MVOSTM} satisfy standard correctness-criterion of STMs, \textit{opacity} [8].

- Experimental analysis of both \textit{OPT-list-MVOSTM} and \textit{OPT-HT-MVOSTM} with state-of-the-art STMs are present in Section 7. Proposed \textit{OPT-list-MVOSTM} and \textit{OPT-HT-MVOSTM} provide greater concurrency and reduces the number of aborts as compared to \textit{MVOSTMs}, \textit{SV-OSTMs}, single-version \textit{RWSTMs} and, multi-version \textit{RWSTMs} while maintaining multiple versions corresponding to each key.

\textit{Roadmap:} The paper is organized as follows. We describe our building system model in Section 2. In Section 3 we formally define the graph characterization of opacity. Section 4 represents the \textit{OPT-MVOSTMs} design and data structure. Section 5 shows the
working of \textit{OPT-HT-MVOSTMs} and its algorithms. We formally prove the correctness of \textit{OPT-MVOSTMs} in Section 6. In Section 7 we show the experimental evaluation of \textit{OPT-MVOSTMs} with state-of-art-STMs. Finally, we conclude in Section 8.

2. Building System Model

Our assumption follows [11, 6] in which the system consists of a finite set of \( p \) processes, \( p_1, \ldots, p_n \), accessed by a finite number of \( n \) threads in a completely asynchronous fashion and communicates each other using shared keys (or objects). The threads invoke higher level methods on the shared objects and get corresponding responses. Consequently, we make no assumption about the relative speeds of the threads. We also assume that none of these processors and threads fail or crash abruptly.

**Events and Methods:** We assume that the threads execute atomic events and the events by different threads are (1) read/write on shared/local memory objects, (2) method invocations (or \texttt{inv}) event and responses (or \texttt{rsp}) event on higher level shared memory objects.

Within a transaction, a process can invoke layer-1 methods (or operations) on a hash-table \( t\text{-object} \). A hash-table\( (ht) \) consists of multiple key-value pairs of the form \( \langle k, v \rangle \). The keys and values are respectively from sets \( \mathcal{K} \) and \( \mathcal{V} \). The methods that a thread can invoke are: (1) \texttt{t.begin}(i): begins a transaction and returns a unique id to the invoking thread. (2) \texttt{t.insert}(ht, k, v): transaction \( T_i \) inserts a value \( v \) onto key \( k \) in \( ht \). (3) \texttt{t.delete}(ht, k, v): transaction \( T_i \) deletes the key \( k \) from the hash-table \( ht \) and returns the current value \( v \) for \( T_i \). If key \( k \) does not exist, it returns \texttt{null}. (4) \texttt{t.lookup}(ht, k, v): returns the current value \( v \) for key \( k \) in \( ht \) for \( T_i \). Similar to \texttt{t.delete}, if the key \( k \) does not exist then \texttt{t.lookup} returns \texttt{null}. (5) \texttt{tryC}(i): which tries to commit all the operations of \( T_i \) and (6) \texttt{tryA}(i): aborts \( T_i \). We assume that each method consists of an \texttt{inv} and \texttt{rsp} event.

We denote \texttt{t.insert} and \texttt{t.delete} as update methods (or \texttt{upd}\_method or \texttt{up}) since both of these change the underlying data structure. We denote \texttt{t.delete} and \texttt{t.lookup} as return-value methods (or \texttt{rv}\_method or \texttt{rv}) as these operations return values from \( ht \). A method may return \texttt{ok} if successful or \texttt{A}(abort) if it sees an inconsistent state of \( ht \).
Formally, we denote a method $m$ by the tuple $\langle evts(m), <_m \rangle$. Here, $evts(m)$ are all the events invoked by $m$ and the $<_m$ a total order among these events.

**Transactions:** Following the notations used in database multi-level transactions\[9\], we model a transaction as a two-level tree. The layer-0 consist of read/write events and layer-1 of the tree consists of methods invoked by a transaction.

Having informally explained a transaction, we formally define a transaction $T$ as $\langle evts(T), <_T \rangle$. Here $evts(T)$ are all the read/write events at layer-0 of the transaction. $<_T$ is a total order among all the events of the transaction.

We denote the first and last events of a transaction $T_i$ as $T_i.firstEvt$ and $T_i.lastEvt$. Given any other read/write event $rw$ in $T_i$, we assume that $T_i.firstEvt <_T rw <_T T_i.lastEvt$. All the methods of $T_i$ are denoted as $methods(T_i)$. We assume that for any method $m$ in $methods(T_i)$, $evts(m)$ is a subset of $evts(T_i)$ and $<_m$ is a subset of $<_T$. We assume that if a transaction has invoked a method, then it does not invoke a new method until it gets the response of the previous one. Thus all the methods of a transaction can be ordered by $<_T$. Formally, $(\forall m_p, m_q \in methods(T_i) : (m_p <_T m_q) \vee (m_q <_T m_p))$, here $m_p$ and $m_q$ are $p_{th}$ and $q_{th}$ methods of $T_i$ respectively.

**Histories:** A history is a sequence of events belonging to different transactions. The collection of events is denoted as $evts(H)$. Similar to a transaction, we denote a history $H$ as tuple $\langle evts(H), <_H \rangle$ where all the events are totally ordered by $<_H$. The set of methods that are in $H$ is denoted by $methods(H)$. A method $m$ is incomplete if $inv(m)$ is in $evts(H)$ but not its corresponding response event. Otherwise, $m$ is complete in $H$.

Coming to transactions in $H$, the set of transactions in $H$ are denoted as $txns(H)$. The set of committed (resp., aborted) transactions in $H$ is denoted by $committed(H)$ (resp., $aborted(H)$). The set of live transactions in $H$ are those which are neither committed nor aborted and denoted as $live(H) = txns(H) - committed(H) - aborted(H)$. On the other hand, the set of terminated transactions are those which have either committed or aborted and is denoted by $term(H) = committed(H) \cup aborted(H)$.

The relation between the events of transactions & histories is analogous to the relation between methods & transactions. We assume that for any transaction $T$ in...
txns(H), evts(T) is a subset of evts(H) and <_T is a subset of <_H. Formally, (∀T ∈ txns(H) : (evts(T) ⊆ evts(H)) ∧ (<_T⊆<_H)).

We denote two histories H_1, H_2 as equivalent if their events are the same, i.e., evts(H_1) = evts(H_2). A history H is qualified to be well-formed if: (1) all the methods of a transaction T_i in H are totally ordered, i.e. a transaction invokes a method only after it receives a response of the previous method invoked by it (2) T_i does not invoke any other method after it received an affine response or after tryC(ok) method. We only consider well-formed histories for OPT-MVOSTM.

A method m_{ij} (j^{th} method of a transaction T_i) in a history H is said to be isolated or atomic if for any other event e_{pq}r (r^{th} event of method m_{pq}) belonging to some other method m_{pq} of transaction T_p, either e_{pq}r occurs before inv(m_{ij}) or after rsp(m_{ij}).

Sequential Histories: A history H is said to be sequential (term used in [12, 13]) if all the methods in it are complete and isolated. From now onwards, most of our discussion would relate to sequential histories.

Since in sequential histories all the methods are isolated, we treat each method as a whole without referring to its inv and rsp events. For a sequential history H, we construct the completion of H, denoted T_H, by inserting tryA_k(aff) immediately after the last method of every transaction T_k ∈ live(H). Since all the methods in a sequential history are complete, this definition only has to take care of completed transactions.

Consider a sequential history H. Let m_{ij}(ht, k, v/nil) be the first method of T_i in H operating on the key k as H.firstKeyMth((ht, k), T_i), where m_{ij} stands for j^{th} method of i^{th} transaction. For a method m_{ix}(ht, k, v) which is not the first method on k of T_i in H, we denote its previous method on k of T_i as m_{ij}(ht, k, v) = H.prevKeyMth(m_{ix}, T_i).

Real-time Order and Serial Histories: Given a history H, <_H orders all the events in H. For two complete methods m_{ij}, m_{pq} in methods(H), we denote m_{ij} <^M_H m_{pq} if rsp(m_{ij}) <_H inv(m_{pq}). Here MR stands for method real-time order. It must be noted that all the methods of the same transaction are ordered. Similarly, for two transactions T_i, T_p in term(H), we denote (T_i <^T_H T_p) if (T_i,lastEvt <_H T_p.firstEvt). Here TR stands for transactional real-time order.
We define a history \( H \) as **serial** \(^{14}\) or **t-sequential** \(^{13}\) if all the transactions in \( H \) have terminated and can be totally ordered w.r.t \( \prec_{TR} \), i.e. all the transactions execute one after the other without any interleaving. Intuitively, a history \( H \) is serial if all its transactions can be isolated. Formally, \( \langle (H \text{ is serial}) \implies (\forall T_i \in \text{txns}(H) : (T_i \in \text{term}(H)) \wedge (\forall T_i, T_p \in \text{txns}(H) : (T_i \prec_{H} T_p) \lor (T_p \prec_{H} T_i))) \rangle \). Since all the methods within a transaction are ordered, a serial history is also sequential.

**Valid Histories:** A \( rv \), method (\( t\_\text{delete} \) and \( t\_\text{lookup} \)) \( rv_{ij} \) on key \( k \) is valid if it returns the value updated by any of the previously committed transaction that updated key \( k \). A history \( H \) is said to be valid if all the \( rv \), methods of \( H \) are valid.

**Legal Histories:** We define the **legality** of \( rv \), methods on sequential histories which we use to define correctness criterion as opacity \(^{8}\). Consider a sequential history \( H \) having a \( rv \), method \( rv_{ij}(ht,k,v) \) (with \( v \neq \text{null} \)) as \( j^{th} \) method belonging to transaction \( T_i \). We define this \( rv \), method to be legal if:

**Rule 1** If the \( rv_{ij} \) is not the first method of \( T_i \) to operate on \( \langle ht, k \rangle \) and \( m_{ix} \) is the previous method of \( T_i \) on \( \langle ht, k \rangle \). Formally, \( rv_{ij} \neq H.\text{firstKeyMth}(\langle ht, k \rangle, T_i) \wedge (m_{ix}(ht,k,v') = H.\text{prevKeyMth}(\langle ht, k \rangle, T_i)) \) (where \( v' \) could be null).

Then,

- If \( m_{ix}(ht,k,v') \) is a \( t\_\text{insert} \) method then \( v = v' \).
- If \( m_{ix}(ht,k,v') \) is a \( t\_\text{lookup} \) method then \( v = v' \).
- If \( m_{ix}(ht,k,v') \) is a \( t\_\text{delete} \) method then \( v = \text{null} \).

In this case, we denote \( m_{ix} \) as the last update method of \( rv_{ij} \), i.e.,
\[
m_{ix}(ht, k, v') = H.\text{lastUpdt}(rv_{ij}(ht, k, v)).
\]

**Rule 2** If \( rv_{ij} \) is the first method of \( T_i \) to operate on \( \langle ht, k \rangle \) and \( v \) is not null. Formally, \( rv_{ij}(ht,k,v) = H.\text{firstKeyMth}(\langle ht, k \rangle, T_i) \wedge (v \neq \text{null}) \). Then,

- There is a \( t\_\text{insert} \) method \( t\_\text{insert}_{pq}(ht,k,v) \) in \( \text{methods}(H) \) such that \( T_p \) committed before \( rv_{ij} \). Formally, \( \exists t\_\text{insert}_{pq}(ht,k,v) \in \text{methods}(H) : \text{tryC}_p \prec_{MR} H \text{rv}_{ij} \).

- There is no other update method \( up_{xy} \) of a transaction \( T_x \) operating on \( \langle ht, k \rangle \) in \( \text{methods}(H) \) such that \( T_x \) committed after \( T_p \) but before \( rv_{ij} \). Formally, \( \exists \text{up}_{xy}(ht,k,v'') \in \text{methods}(H) : \text{tryC}_p \prec_{MR} H \text{rv}_{ij} \).
In this case, we denote \( \text{tryC}_p \) as the last update method of \( rvm_{ij} \), i.e., \( \text{tryC}_p(ht,k,v) = H.\text{lastUpdt}(rvm_{ij}(ht,k,v)) \).

**Rule 3** If \( rvm_{ij} \) is the first method of \( T_i \) to operate on \( \langle ht,k \rangle \) and \( v \) is null. Formally, 
\[ rvm_{ij}(ht,k,v) = H.\text{firstKeyMth}(\langle ht,k \rangle, T_i) \land (v = \text{null}). \]
Then,

(a) There is \( t\text{_delete} \) method \( t\text{_delete}_{pq}(ht,k,v') \) in \( \text{methods}(H) \) such that \( T_p \) committed before \( rvm_{ij} \). Formally, \( \exists t\text{_delete}_{pq}(ht,k,v') \in \text{methods}(H) : \text{tryC}_p \prec MR_H rvm_{ij} \). Here \( v' \) could be null.

(b) There is no other update method \( up_{xy} \) of a transaction \( T_x \) operating on \( \langle ht,k \rangle \) in \( \text{methods}(H) \) such that \( T_x \) committed after \( T_p \) but before \( rvm_{ij} \). Formally, \( \neg \exists up_{xy}(ht,k,v'') \in \text{methods}(H) : \text{tryC}_p \prec MR_H \text{tryC}_x \prec MR_H rvm_{ij} \).

In this case, we denote \( \text{tryC}_p \) as the last update method of \( rvm_{ij} \), i.e., 
\[ \text{tryC}_p(ht,k,v) = H.\text{lastUpdt}(rvm_{ij}(ht,k,v)) \.]
3. Graph Characterization ofOpacity

To prove that an STM system satisfies opacity, it is useful to consider graph characterization of histories. In this section, we describe the graph characterization of Guerraoui and Kapalka [11] modified for sequential histories.

Consider a history $H$ which consists of multiple version for each t-object. The graph characterization uses the notion of version order. Given $H$ and a t-object $k$, we define a version order for $k$ as any (non-reflexive) total order on all the versions of $k$ ever created by committed transactions in $H$. It must be noted that the version order may or may not be the same as the actual order in which the versions of $k$ are generated in $H$. A version order of $H$, denoted as $\ll_H$ is the union of the version orders of all the t-objects in $H$.

Consider the history $H3$ as shown in Figure 3: $lu_1(k_{x,0}, null)$, $lu_2(k_{x,0}, null)$, $lu_1(k_{y,0}, null)$, $lu_3(k_{z,0}, null)$, $ins_1(k_{x,1}, v_{11})$, $ins_3(k_{y,2}, v_{21})$, $ins_4(k_{y,3}, v_{31})$, $ins_1(k_{z,1}, v_{12})$, $c_1$, $c_2$, $lu_4(k_{x,1}, v_{11})$, $lu_4(k_{y,2}, v_{21})$, $ins_3(k_{z,3}, v_{32})$, $c_3$, $lu_4(k_{z,1}, v_{12})$, $lu_5(k_{x,1}, v_{11})$, $lu_6(k_{y,2}, v_{21})$. Using the notation that a committed transaction $T_i$ writing to $k_x$ creates a version $k_{x,i}$, a possible version order for $H3 \ll_H$ is: $\langle k_{x,0} \ll k_{x,1} \rangle, \langle k_{y,0} \ll k_{y,2} \ll k_{y,3} \rangle, \langle k_{z,0} \ll k_{z,1} \ll k_{z,3} \rangle$.

We define the graph characterization based on a given version order. Consider a history $H$ and a version order $\ll$. We then define a graph (called opacity graph) on $H$ using $\ll$, denoted as $OPG(H, \ll) = (V, E)$. The vertex set $V$ consists of a vertex for
each transaction $T_i$ in $H$. The edges of the graph are of three kinds and are defined as follows:

1. *rt*(real-time) edges: If the commit of $T_i$ happens before beginning of $T_j$ in $H$, then there exist a real-time edge from $v_i$ to $v_j$. We denote set of such edges as $rt(H)$.

2. *rvf*(return value-from) edges: If $T_j$ invokes $rv$ method on key $k_1$ from $T_i$ which has already been committed in $H$, then there exists a return value-from edge from $v_i$ to $v_j$. If $T_i$ is having $upd$ method as insert on the same key $k_1$, then $ins_i(k_{1,i}, v_{i1}) < H c_i < H rvm_j(k_{1,i}, v_{i1})$. If $T_i$ is having $upd$ method as delete on the same key $k_1$, then $del_i(k_{1,i}, null) < H c_i < H rvm_j(k_{1,i}, null)$. We denote set of such edges as $rvf(H)$.

3. *mv*(multi-version) edges: This is based on version order. Consider a triplet with successful methods as $up_i(k_{1,i}, u), rvm_j(k_{1,i}, u), up_k(k_{1,i}, v)$, where $u \neq v$. As we can observe it from $rvm_j(k_{1,i}, u), c_i < H rvm_j(k_{1,i}, u)$, if $k_{1,i} \ll k_{1,k}$ then there exist a multi-version edge from $v_j$ to $v_k$. Otherwise ($k_{1,k} \ll k_{1,i}$), there exist a multi-version edge from $v_k$ to $v_i$. We denote set of such edges as $mv(H, \ll)$.

We now show that if a version order $\ll$ exists for a history $H$ such that it is acyclic, then $H$ is opaque.

![Figure 4: OPG($H_3, \ll H_3$)](image)
Using this construction, the $OPG(H3, \ll_{H3})$ for history $H3$ and $\ll_{H3}$ is given above is shown in Figure 4. The edges are annotated. The only mv edge from $T_4$ to $T_3$ is because of $t$-objects $k_y, k_z$. $T_4$ lookups value $v_{12}$ for $k_z$ from $T_1$ whereas $T_3$ also inserts $v_{32}$ to $k_z$ and commits before $lu_4(k_{z,1}, v_{12})$.

Given a history $H$ and a version order $\ll$, consider the graph $OPG(H, \ll)$. While considering the $rt$ edges in this graph, we only consider the real-time relation of $H$ and not $H$. It can be seen that $\ll_{RT_H} \subseteq \ll_{RT_H}$ but with this assumption, $rt(H) = rt(H)$. Hence, we get the following property.

**Property 1.** The graphs $OPG(H, \ll)$ and $OPG(H, \ll)$ are the same for any history $H$ and $\ll$.

**Definition 1.** For a t-sequential history $S$, we define a version order $\ll_S$ as follows: For two version $k_{x,i}, k_{x,j}$ created by committed transactions $T_i, T_j$ in $S$, $(k_{x,i} \ll_S k_{x,j} \Leftrightarrow T_i <_S T_j)$.

Now we show the correctness of our graph characterization using the following lemmas and theorem.

**Lemma 2.** Consider a legal t-sequential history $S$. Then the graph $OPG(S, \ll_S)$ is acyclic.

**Proof:** We numerically order all the transactions in $S$ by their real-time order by using a function $ord$. For two transactions $T_i, T_j$, we define $ord(T_i) < ord(T_j) \Leftrightarrow T_i <_S T_j$. Let us analyze the edges of $OPG(S, \ll_S)$ one by one:

- **rt edges:** It can be seen that all the rt edges go from a lower ord transaction to a higher ord transaction.

- **rvf edges:** If $T_j$ lookups $k_z$ from $T_i$ in $S$ then $T_i$ is a committed transaction with $ord(T_i) < ord(T_j)$. Thus, all the rvf edges from a lower ord transaction to a higher ord transaction.

- **mv edges:** Consider a successful rv_method $rvm_j(k_x, u)$ and a committed transaction $T_k$ writing $v$ to $k_x$ where $u \neq v$. Let $c_i$ be $rvm_j(k_x, u)$’s lastWrite. Thus,
Thus, we have that $\text{ord}(T_i) < \text{ord}(T_j)$. Now there are two cases w.r.t $T_i$: (1) Suppose $\text{ord}(T_k) < \text{ord}(T_i)$. We now have that $T_k \ll T_i$. In this case, the mv edge is from $T_k$ to $T_i$. (2) Suppose $\text{ord}(T_i) < \text{ord}(T_k)$ which implies that $T_i \ll T_k$. Since $S$ is legal, we get that $\text{ord}(T_j) < \text{ord}(T_k)$. This case also implies that there is an edge from $\text{ord}(T_j)$ to $\text{ord}(T_k)$. Hence, in this case as well the mv edges go from a transaction with lower ord to a transaction with higher ord.

Thus, in all the three cases the edges go from a lower ord transaction to higher ord transaction. This implies that the graph is acyclic.

**Lemma 3.** Consider two histories $H, H'$ that are equivalent to each other. Consider a version order $\ll_H$ on the t-objects created by $H$. The mv edges $\text{mv}(H, \ll_H)$ induced by $\ll_H$ are the same in $H$ and $H'$.

**Proof:** Since the histories are equivalent to each other, the version order $\ll_H$ is applicable to both of them. It can be seen that the mv edges depend only on events of the history and version order $\ll$. It does not depend on the ordering of the events in $H$. Hence, the mv edges of $H$ and $H'$ are equivalent to each other.

Using these lemmas, we prove the following theorem.

**Theorem 4.** A valid history $H$ is opaque iff there exists a version order $\ll_H$ such that $\text{OPG}(H, \ll_H)$ is acyclic.

**Proof:** (if part): Here we have a version order $\ll_H$ such that $G_H = \text{OPG}(H, \ll)$ is acyclic. Now we have to show that $H$ is opaque. Since the $G_H$ is acyclic, a topological sort can be obtained on all the vertices of $G_H$. Using the topological sort, we can generate a t-sequential history $S$. It can be seen that $S$ is equivalent to $\overline{H}$. Since $S$ is obtained by a topological sort on $G_H$ which maintains the real-time edges of $H$, it can be seen that $S$ respects the rt order of $H$, i.e $\preceq_H^{RT} \subseteq \prec_S^{RT}$.

Similarly, since $G_H$ maintains return value-from (rvf) order of $H$, it can be seen that if $T_j$ lookups $k_x$ from $T_i$ in $H$ then $T_i$ terminates before $lu_j(k_x)$ and $T_j$ in $S$. Thus, $S$ is valid. Now it remains to be shown that $S$ is legal. We prove this using
contradiction. Assume that $S$ is not legal. Thus, there is a successful $rv$-method $rvm_j(k_x, u)$ such that its lastWrite in $S$ is $c_k$ and $T_k$ updates value $v(\neq u)$ to $k_x$, i.e $up_k(k_x, v) \in evts(T_k)$. Further, we also have that there is a transaction $T_i$ that inserts $u$ to $k_x$, i.e $up_i(k_x, u) \in evts(T_i)$. Since $S$ is valid, as shown above, we have that $T_i \prec^S T_k \sim^S T_j$.

Now in $\ll_H$, if $k_{x,i} \ll_H k_{x,k}$, then there is an edge from $T_k$ to $T_i$ in $G_H$. Otherwise ($k_{x,i} \ll_H k_{x,k}$), there is an edge from $T_j$ to $T_k$. Thus, in either case, $T_k$ can not be in between $T_i$ and $T_j$ in $S$ contradicting our assumption. This shows that $S$ is legal.

(Only if part): Here we are given that $H$ is opaque and we have to show that there exists a version order $\ll$ such that $G_H = OPG(H, \ll)(= OPG(\overline{H}, \ll)$, Property[1] is acyclic. Since $H$ is opaque there exists a legal t-sequential history $S$ equivalent to $\overline{H}$ such that it respects real-time order of $H$. Now, we define a version order for $S$, $\ll_S$ as in Definition[1]. Since the $S$ is equivalent to $\overline{H}$, $\ll_S$ is applicable to $\overline{H}$ as well. From Lemma[2] we get that $G_S = OPG(S, \ll_S)$ is acyclic. Now consider $G_H = OPG(\overline{H}, \ll_S)$. The vertices of $G_H$ are the same as $G_S$. Coming to the edges,

- rt edges: We have that $S$ respects real-time order of $H$, i.e $\prec^H \subseteq \prec^S$. Hence, all the rt edges of $H$ are a subset of $S$.

- rvf edges: Since $\overline{H}$ and $S$ are equivalent, the return value-from relation of $\overline{H}$ and $S$ are the same. Hence, the rvf edges are the same in $G_H$ and $G_S$.

- mv edges: Since the version-order and the operations of the $H$ and $S$ are the same, from Lemma[3] it can be seen that $\overline{H}$ and $S$ have the same mv edges as well.

Thus, the graph $G_H$ is a subgraph of $G_S$. Since we already know that $G_S$ is acyclic from Lemma[2] we get that $G_H$ is also acyclic.

4. OPT-MVOSTMs Design and Data Structure

This section describes the design and data structure of optimized MVOSTMs (or OPT-MVOSTMs). Here, we propose hash-table and list based OPT-MVOSTMs as OPT-HT-MVOSTM and OPT-list-MVOSTM respectively. OPT-MVOSTMs are generic for
other data structure as well. OPT-HT-MVOSTM is a hash-table based OPT-MVOSTM that explores the idea of multiple versions in OSTM for hash-table object to achieve greater concurrency. The design of OPT-HT-MVOSTM is similar to HT-MVOSTM consisting of $B$ buckets. All the keys of the hash-table in the range $\mathcal{K}$ are statically allocated to one of these buckets.

Each bucket consists of linked-list of nodes along with two sentinel nodes head and tail with values $-\infty$ and $+\infty$ respectively. The structure of each node is as $\langle$ key, lock, marked, vl, nnext $\rangle$. The key is a unique value from the set of all keys $\mathcal{K}$. All the nodes are stored in increasing order in each bucket as shown in Figure 5(a), similar to any linked-list based concurrent set implementation [7, 15]. In the rest of the document, we use the terms key and node interchangeably. To perform any operation on a key, the corresponding lock is acquired. marked is a boolean field which represents whether the key is deleted or not. The deletion is performed in a lazy manner similar to the concurrent linked-lists structure [7]. If the marked field is true then key corresponding to the node has been logically deleted; otherwise, it is present. The vl field of the node points to the version list (shown in Figure 5(b)) which stores multiple versions corresponding to the key. The last field of the node is nnext which stores the address of the next node. It can be seen that the list of keys in a bucket is as an extension of lazy-list [7]. Given a node $n$ in the linked-list of bucket $B$ with key $k$, we denote its fields as $n.key$ (or $k.key$), $n.lock$ (or $k.lock$), $n.marked$ (or $k.marked$), $n.vl$ (or $k.vl$), $n.nnext$ (or $k.nnext$).

The structure of each version in the vl of a key $k$ is $\langle$ts, val, rvl, max$r_{vl}$, vnext $\rangle$ as shown in Figure 5(b). The field ts denotes the unique timestamp of the version. In
our algorithm, every transaction is assigned a unique timestamp when it begins which is also its id. Thus ts of this version is the timestamp of the transaction that created it. All the versions in the vl of k are sorted by ts. Since the timestamps are unique, we denote a version, ver of a node n with key k having ts j as n.nl[j].ver or k.nl[j].ver. The corresponding fields in the version as k.nl[j].ts, k.nl[j].val, k.nl[j].rvl, k.nl[j].max_rvl, k.nl[j].vnext.

The field val contains the value updated by an update transaction. If this version is created by an insert method t_insert(ht, k, v) by transaction Ti, then val will be v. On the other hand, if the method is t_delete(ht, k, v) then val will be null. In this case, as per the algorithm, the node of key k will also be marked. OPT-HT-MVOSTM algorithm does not immediately physically remove deleted keys from the hash-table. The need for this is explained below. Thus an rv_method (t_delete or t_lookup) on key k can return null when it does not find the key or encounters a null value for k.

The field rvl stands for return value list which is a list of all the transactions that executed rv_method on this version, i.e., those transactions which returned val. The first optimization in OPT-HT-MVOSTM to reduce the traversal time of rvl, we have used max_rvl which contains the maximum ts of the transaction that executed rv_method on this version. The field vnext points to the next available version of that key.

In order to increase the efficiency and utilize the memory properly, We propose two variants of OPT-HT-MVOSTM as follows: First, we apply garbage collection (or GC) on the versions and propose OPT-HT-MVOSTM-GC. It maintains unbounded versions in vl (the length of the list) while deleting the unwanted versions using garbage collection scheme. Second, we propose OPT-HT-KOSTM which maintains the bounded number of versions such as K and improves the efficiency further. Whenever a new version ver is created and is about to be added to vl, the length of vl is checked. If the length becomes greater than K, the version with lowest ts (i.e., the oldest) is replaced with the new version ver and thus maintaining the length back to K.

We propose OPT-list-MVOSTMs while considering the bucket size as 1 in OPT-HT-MVOSTM. Along with this, we propose two variants of OPT-list-MVOSTM as OPT-list-MVOSTM-GC and OPT-list-KOSTM which applies the garbage collection scheme in unbounded versions and bounded K versions for list based object respectively similar
Marked Version Nodes: \textit{OPT-HT-MVOSTM} stores keys even after they have been deleted (the version of the nodes which have \textit{marked} field as true). This is because some other concurrent transactions could read from a different version of this key and not the \textit{null} value inserted by the deleting transaction. Consider for instance the transaction \(T_1\) performing \(lu_1(ht, k_2, v_0)\) as shown in Figure 2(b). Due to the presence of previous version \(v_0\), \textit{OPT-HT-MVOSTM} returns this earlier version \(v_0\) for \(lu_1(ht, k_2, v_0)\) method. Whereas, it is not possible for \textit{HT-OSTM} to return the version \(v_0\) because \(k_1\) has been removed from the system by delete method of higher timestamp transaction \(T_2\) than \(T_1\). In that case, \(T_1\) would have to be aborted. Thus as explained in Section 1, storing multiple versions increases the concurrency.

To store deleted keys along with the live keys (or unmarked node) in a lazy-list will increase the traversal time to access unmarked nodes. Consider Figure 6, in which there are four keys \(\langle k_2, k_4, k_8, k_{11}\rangle\) present in the list. Here \(\langle k_2, k_4, k_8\rangle\) are marked (or deleted) nodes while \(k_{11}\) is unmarked. Now, consider accessing the key \(k_{11}\) by \textit{OPT-HT-MVOSTM} as a part of one of its methods. Then \textit{OPT-HT-MVOSTM} would have to unnecessarily traverse the marked nodes to reach key \(k_{11}\).

This motivated us to modify the lazy-list structure of nodes in each bucket to form a skip list based on red and blue links. We denote it as \textit{red-blue lazy-list} or \textit{lazyrb-list}. This idea was earlier explored by Peri et al. in developing \textit{OSTMs} \[6\], \textit{lazyrb-list} consists of nodes with two links, red link (or RL) and blue link (or BL). The node which is not marked (or not deleted) are accessible from the head via BL. While all the nodes including the marked ones can be accessed from the head via RL. With this modification, let us consider the above example of accessing unmarked key \(k_{11}\). It can be seen that \(k_{11}\) can be accessed much more quickly through BL as shown in Figure 7. Using the idea of \textit{lazyrb-list}, we have modified the structure of each node as \(\langle\text{key}, \text{lock}, \text{marked},\)
Further, for a bucket $B$, we denote its linked-list as $B.lazyrb$-list.

5. Working of OPT-HT-MVOSTM

OPT-HT-MVOSTM exports $t._begin$, $t._insert$, $t._delete$, $t._lookup$, and $tryC$ methods as explained in Section 2. Among them $t._delete$, $t._lookup$ are return-value methods (or rv.methods) while $t._insert$, $t._delete$ are update methods (or upd.methods). We treat $t._delete$ as both rv.method as well as upd.method. The rv.methods return the current value of the key. The upd.methods, update to the keys are first noted down in the local log, $txLog$. Then in the $tryC$ method after successful validations of these updates are transferred to the shared memory. We now explain the working of each method as follows:

$t._begin()$: A thread invokes a new transaction $T_i$ using this method. The transaction $T_i$ local log $txLog_i$ is initialized at Line 2. This method returns a unique id to the invoking thread by incrementing an atomic counter at Line 3. This unique id is also the timestamp of the transaction $T_i$. For convenience, we use the notation that $i$ is the timestamp (or id) of the transaction $T_i$.

Algorithm 1 $t._begin()$: It provides the local log and unique id to each transaction.

| Procedure $t._begin()$ |
|-------------------------|
| 1. procedure $t._begin()$ |
| 2. $txLog_i$ ← new $txLog()$. |
| 3. $t._id$ ← get&inc($counter$). |
| 4. return $t._id$. |
| 5. end procedure |

rv.methods: It can be either $t._delete(ht, k, v)$ or $t._lookup(ht, k, v)$. Both these methods return the current value of key $k$. Algorithm 2 gives the high level overview of these methods. First, the algorithm checks to see if the given key is already in the local log, $txLog_i$ of $T_i$ (Line 7). If the key is already there then the current rv.method is not the first method on $k$ and is a subsequent method of $T_i$ on $k$. So, we can return the value of $k$ from the $txLog_i$.

If the key is not present in the $txLog_i$, then OPT-HT-MVOSTM searches into shared memory. Specifically, it searches the bucket to which $k$ belongs to. Every key in the range $\mathcal{K}$ is statically allocated to one of the $B$ buckets. So the algorithms search
for $k$ in the corresponding bucket, say $B_k$ to identify the appropriate location, i.e.,
identify the correct predecessor or pred and current or curr keys in the lazyrb-list of $B_k$ without acquiring any locks similar to the search in lazy-list [7]. Since each key has two links, RL and BL, the algorithm identifies four node references: two pred and two curr according to red and blue links. They are stored in the form of an array with preds[0] and curr[1] corresponding to blue links; preds[1] and curr[0] corresponding to red links. If both preds[1] and curr[0] nodes are unmarked then the pred, curr nodes of both red and blue links will be the same, i.e., preds[0] = preds[1] and curr[0] = curr[1]. Thus depending on the marking of pred, curr nodes, a total of two, three or four different nodes will be identified. Here, the search ensures that 

$$\text{preds[0].key} \leq \text{preds[1].key} < k \leq \text{curr[0].key} \leq \text{curr[1].key}.$$

Next, the re-entrant locks on all the pred, curr keys are acquired in increasing order to avoid the deadlock. Then all the pred and curr keys are validated by rv.-Validation() in Line 12 as follows: (1) If pred and curr nodes of blue links are not marked, i.e, (¬preds[0].marked) && (¬curr[1].marked). (2) If the next links of both blue and red pred nodes point to the correct curr nodes: (preds[0].BL = curr[1]) && (preds[1].RL = curr[0]) at line 74.

If any of these checks fail, then the algorithm retries to find the correct pred and curr keys. It can be seen that the validation check is similar to the validation in concurrent lazy-list [7].

Next, we check if $k$ is in $B_k$.lazyrb-list. If $k$ is not in $B_k$, then we create a new node $n$ for $k$ as: ⟨key = $k$, lock = false, marked = true, vl = ver, nnext = φ⟩ and insert it into $B_k$.lazyrb-list such that it is accessible only via RL. This node will have a single version ver as ⟨ts = 0, val = null, rvl = i, max_rvl = i, vnext = φ⟩. Here invoking transaction $T_i$ is creating a version with timestamp 0 to ensure that rv.methods of other transactions will never abort. As we have explained in Figure 2(b) of Section 4 even after $T_2$ deletes $k_2$, the previous value of $v_0$ is still retained. Thus, when $T_1$ invokes $lu$ on $k_2$ after the delete on $k_2$ by $T_2$, OPT-HT-MVOSTM will return $v_0$ (as previous value). Hence, each rv.method will find a version to read while maintaining the infinite version corresponding to each key $k$. marked field sets to true because it access by RL only. In rvl and max_rvl, $T_i$ adds the timestamp as $i$ in it and vnext is initialized to
empty value. Since val is null and the n, this version and the node are not technically inserted into Bk.lazyrb-list.

If k is in Bk.lazyrb-list then, k is the same as currs[0] or currs[1] or both. Let n be the node of k in Bk.lazyrb-list. We then find the version of n, verj which has the timestamp j such that j has the largest timestamp smaller than i (timestamp of Ti). Add i to verj’s rvl (Line 24). maxrvl maintains the maximum timestamp among all rv_method read from this version at Line 26. Then release the locks, update the local log txLogi in Line 29 and return the value stored in verj.val in Line 31.

Algorithm 2 rv_method: It can be either t_delete(ht, k, v) or t_lookup(ht, k, v) on key k that maps to bucket Bk of hash-table ht.

6: procedure rv_method(ht, k, v)
7: if (k ∈ txLogi) then
8: Update the local log and return val.
9: else
10: Search in lazyrb-list to identify the preds[] and currs[] for k using BL and RL in bucket Bk.
11:Acquire the locks on preds[] and currs[] in increasing order.
12:if (!rv_validated) then
13:Release the locks and goto Line 10.
14:end if
15:if (k /∈ Bk.lazyrb-list) then
16:Create a new node n with key k as: ⟨key = k, lock = false, marked = true, vl = ver, nnext = φ⟩.
17:Create the version ver as: ⟨ts = 0, val = null, rvl = i, maxrvl = i, vnext = φ⟩.
18:Insert n into Bk.lazyrb-list such that it is accessible only via RLs. ▷ n is marked
19:Release the locks; update the txLogi with k.
20:return null.
21:end if
22:Identify the version verj with ts = j such that j is the largest timestamp smaller than i.
23:Add i into the rvl of verj.
24:if (verj, maxrvl < i) then
25:Set verj, maxrvl to i.
26:end if
27:retVal = verj.val.
28:Release the locks; update the txLogi with k and retVal.
29:return retVal.
30:end procedure

\texttt{t_insert():} This is another optimization done in OPT-HT-MVOSTMs to identify the early abort which prevents the work done by aborted transactions and saves time. The actual effect of the \texttt{t_insert()} comes after the successful \texttt{tryC} method. First, \texttt{t_insert()} searches the key k in the local log, txLogi of Ti at Line 34. If k does not exist in the txLogi, then it identifies the appropriate location (pred and curr) of key k using BL and RL (Line 35) in the lazyrb-list of Bk without acquiring any locks similar to rv_method explained above.
Next, it acquires the re-entrant locks on all the \textit{pred} and \textit{curr} keys in increasing order. After that, all the \textit{pred} and \textit{curr} keys are validated by \texttt{tryC\_Validation} in Line 37 as follows: (1) It does the \texttt{rv\_Validation()} as explained above in the \texttt{rv\_method}. (2) If key \textit{k} exists in the $B_k.lazyrb$-list and let \textit{n} as a node of \textit{k}. Then algorithm identifies the version of \textit{n}, \textit{ver} with the timestamp \textit{j} such that \textit{j} has the largest timestamp smaller than \textit{i} (timestamp of $T_i$) at Line 85. If $max_{rel}$ of \textit{ver} is greater than timestamp \textit{i} at Line 86 then it returns \texttt{Abort} in Line 38.

\texttt{tryC\_Validation()} in \texttt{t\_insert()} identifies the early abort of invalid transaction. The advantage of doing the early validation to save the significant computation of long running transaction which will abort in the future. Consider Figure 8 where two transaction $T_1$ and $T_2$ working on key $k_5$. In Figure 8(a), $T_1$ aborts in \texttt{tryC} (delayed validation) because higher timestamp $T_2$ committed. But in Figure 8(b), $T_1$ validates the \texttt{t\_insert()} instantly by looking into the $max_{rel}$ of $k_5$ as shown in Figure 8(c) and save its computation and returns abort.

\textbf{Algorithm 3} \texttt{t\_insert()}: Actual insertion happens in the \texttt{tryC}.

\begin{verbatim}
33: procedure \texttt{t\_insert()}
34: if (k \notin txLog) then
35: Search in lazyrb-list to identify the preds[] and currss[] for k using BL and RL in bucket $B_k$.
36: Acquire the locks on preds[] and currss[] in increasing order.
37: if (!\texttt{tryC\_Validation()}()) then
38: return \texttt{Abort}. \Comment{Release the locks}
39: end if
40: Release the locks.
41: else
42: Update the local log.
43: end if
44: end procedure
\end{verbatim}

\textbf{upd\_methods}: It can be either \texttt{t\_insert}(ht, k, v) or \texttt{t\_delete}(ht, k, v). Both the methods create a version corresponding to the key \textit{k}. The actual effect of \texttt{t\_insert} and \texttt{t\_delete} in shared memory will take place in \texttt{tryC}. Algorithm 4 represents the high level overview
of \textit{tryC}.

Initially, to avoid deadlocks, the algorithm sorts all the \texttt{keys} in increasing order which are present in the local log, \texttt{txLog}. In \textit{tryC}, \texttt{txLog} consists of \texttt{upd\_methods} (\texttt{t\_insert} or \texttt{t\_delete}) only. For all the \texttt{upd\_methods} (\texttt{opn}_i) it searches the key \(k\) in the shared memory corresponding to the bucket \(B_k\). It identifies the appropriate location \((\texttt{pred} \text{ and } \texttt{curr})\) of key \(k\) using BL and RL. (Line 50) in the lazyrb-list of \(B_k\) without acquiring any locks similar to \texttt{rv\_method} explained above.

Next, it acquires the re-entrant locks on all the \texttt{pred} and \texttt{curr} keys in increasing order. After that, all the \texttt{pred} and \texttt{curr} keys are validated by \textit{tryC\_Validation} in Line 52 as explained in \texttt{t\_insert}().

\begin{algorithm}
\caption{\textit{tryC}(\(T_i\)): Validate the \texttt{upd\_methods} of the transaction and then commit.}
\begin{algorithmic}
\Procedure{tryC}(\(T_i\))
\State /*Operation name (\texttt{opn}) which could be either \texttt{t\_insert} or \texttt{t\_delete} */
\State /*Sort the keys of \texttt{txLog}, in increasing order */
\ForAll{\((\texttt{opn}_i \in \texttt{txLog}_i)\)}
\If{\((\texttt{opn}_i = \texttt{t\_insert}) \text{ or } (\texttt{opn}_i = \texttt{t\_delete})\)}
\State Search in lazyrb-list to identify the \texttt{preds[]} and \texttt{currs[]} for \(k\) using BL and RL in bucket \(B_k\).
\State Acquire the locks on \texttt{preds[]} and \texttt{currs[]} in increasing order.
\If{\((\textit{tryC\_Validation()}\)\)
\State return \texttt{Abort}.
\EndIf
\EndIf
\EndFor
\EndProcedure
\end{algorithmic}

If \textit{tryC\_Validation} is successful then each \texttt{upd\_methods} exist in \texttt{txLog}_i will take the effect in the shared memory after doing the \texttt{intraTransValidation()} in Line 58. If two \texttt{upd\_methods} of the same transaction have at least one common shared node among its recorded \texttt{pred} and \texttt{curr} keys, then the previous \texttt{upd\_method} effect may
overwrite if the current `upd.method` of `pred` and `curr` keys are not updated according to the updates are done by the previous `upd.method`. Thus to solve this we have `intraTransValidation()` that modifies the `pred` and `curr` keys of current operation based on the previous operation in Line 58.

Next, we check if `upd.method` is `t_insert` and `k` is in $B_k.lazyrb-list$. If `k` is not in $B_k$, then create a new node $n$ for `k` as $\{key = k, lock = false, marked = false, vl = ver, vnext = \phi\}$. This node will have two versions `ver` as $\{ts = 0, val = null, vrl = \phi, max_{rel} = \phi, vnext = i\}$ for $T_0$ and $\{ts = i, val = v, vrl = \phi, max_{rel} = \phi, vnext = \phi\}$ for $T_i$. $T_i$ is creating a version with timestamp 0 to ensure that `rv` methods of other transactions will never abort. For second version, $i$ is the timestamp of the transaction $T_i$ invoking this method; `marked` field sets to false because the node is inserted in the `BL`. `vrl`, `max_{rel}`, and `vnext` are initialized to empty values. We set the `val` as $v$ and insert $n$ into $B_k.lazyrb-list$ such that it is accessible via `RL` as well as `BL` and set the lock field to be `true` (Line 52). If `k` is in $B_k.lazyrb-list$ then, `k` is the same as `currs[0]` or `currs[1]` or both. Let $n$ be the node of `k` in $B_k.lazyrb-list$. Then, we create the version `ver` as $\{ts = i, val = v, vrl = \phi, max_{rel} = \phi, vnext = \phi\}$ and insert the version into $B_k.lazyrb-list$ such that it is accessible via `RL` as well as `BL` (Line 64).

Subsequently, we check if `upd.method` is `t_delete` and `k` is in $B_k.lazyrb-list$. Let $n$ be the node of `k` in $B_k.lazyrb-list$. Then create the version `ver` as $\{ts = i, val = null, vrl = \phi, max_{rel} = \phi, vnext = \phi\}$ and insert the version into $B_k.lazyrb-list$ such that it is accessible only via `RL` (Line 67).

Finally, at Line 69 it updates the `pred` and `curr` of `opn_i` in local log, `txLog_i`. At Line 71 releases the locks on all the `pred` and `curr` in increasing order of keys to avoid deadlocks and return `Commit`.

We illustrate the helping methods of `rv.method`, `t_insert()`, and `upd.method` in detail as follows:

**rv.Validation()**: It is called by the `rv.method`, `t_insert()`, and `upd.method`. It identifies the conflicts among the concurrent methods of different transactions. Consider an example shown in Figure 9, where two concurrent conflicting methods of different transactions are working on the same key $k_4$. Initially, at stage $s_1$ in Figure 9(c) both
the conflicting method optimisticly (without acquiring locks) identify the same \textit{pred} and \textit{curr} keys for key \( k_4 \) from \( B_k.lazyrb-list \) in Figure 9(a). At stage \( s_2 \) in Figure 9(c), method \( \text{ins}_1(ht, k_4, v_1) \) of transaction \( T_1 \) acquired the lock on \textit{pred} and \textit{curr} keys and inserted the node into \( B_k.lazyrb-list \) as shown in Figure 9(b). After successful insertion by \( T_1 \), \textit{pred} and \textit{curr} have been changed for \( lu_2(ht, k_4) \) at stage \( s_3 \) in Figure 9(c). So, the above modified information is delivered by \( \text{rv}_\text{Validation} \) method at Line 74 when \((\text{preds}[0].BL \neq \text{currs}[1])\) for \( lu_2(ht, k_4) \). After that again it will find the new \textit{pred} and \textit{curr} for \( lu_2(ht, k_4, v_1) \) and eventually it will commit.

\begin{algorithm}
\begin{algorithmic}
    \Procedure{rv\_Validation}{\label{rv\_Validation}}
    \If{((\text{preds}[0].marked)||(\text{currs}[1].marked)||(\text{preds}[0].BL) \neq \text{currs}[1])||(\text{preds}[1].BL) \neq \text{currs}[0])}
        \Return{false.}
    \Else
        \Return{true.}
    \EndIf
    \EndProcedure
\end{algorithmic}
\end{algorithm}

\textbf{tryC\_Validation():} It is called by \( t\_\text{insert}(), \) and \( \text{upd\_method} \) in \( \text{tryC} \). First, it does the \( \text{rv\_Validation}() \) in Line 81 If its successful and key \( k \) exists in the \( B_k.lazyrb-list \) and let \( n \) as a node of \( k \). Then algorithm identifies the version of \( n \), \textit{ver}\_\textit{j} which has the
timestamp $j$ such that $j$ has the largest timestamp smaller than $i$ (timestamp of $T_i$) at Line[85] If $max_{rvl}$ of $ver_j$ is greater than the timestamp of $i$ then the algorithm returns false (in Line[87]) and eventually, return $Abort$ in Line[38] or Line[53]. Consider an example as shown in Figure[10](a), where second method $ins_1(ht, k_5)$ of transaction $T_1$ returns $Abort$ because higher timestamp of transaction $T_2$ is already present in the $max_{rvl}$ of version $T_0$ identified by $T_1$ in Figure[10](b).

Algorithm 6 tryC Validation(): It maintains the order among the transactions.

```plaintext
 procedure tryC_validation()
   if (!rvValidation()) then
     Release the locks and retry.
   end if
   if ($k \in B_k.lazyrb-list$) then
     Identify the version $ver_j$ with $ts = j$ such that $j$ is the largest timestamp smaller than $i$.
     if ($ver_j.max_{rvl} > i$) then
       return false.
     end if
   end if
   return true.
 end procedure
```

Algorithm 7 intraTransValidation(): It is called by upd_method in tryC. If two upd_methods of the same transaction have at least one common shared node among its recorded pred and curr keys, then the previous upd_method effect may overwrite if the current upd_method of pred and curr keys are not updated according to the updates done by the previous upd_method. Thus to solve this we have intraTransValidation() that modifies the pred and curr keys of current operation based on the previous operation from Line[93] to Line[103]. Consider an example as shown in Figure[11] where two upd_methods of transaction $T_1$ are $ins_{11}(ht, k_4, v_1)$ and $ins_{12}(ht, k_6, v_2)$ in Figure[11](c). At stage $s_1$ in Figure[11](c) both the upd_methods identify the same pred and curr.
from underlying DS as $B_k.lazyrb-list$ shown in Figure 11 (a). After the successful insertion done by first upd_method at stage $s_2$ in Figure 11 (c), key $k_1$ is part of $B_k.lazyrb-list$ (Figure 11(b)). At stage $s_3$ in Figure 11 (c), $ins_{12}(ht, k_6, v_2)$ identified ($preds[0].BL \neq currs[1]$) in intraTransValidation() at Line 93. So it updates the $preds[0]$ in Line 96 for correct updation in $B_k.lazyrb-list$.

![Diagram](image)

(a) Underlying list at stage $s_1$  
(b) Successful insertion of $k_4$ at stage $s_2$  
(c) Two update methods of $T_1$

Figure 11: Illustration of intraTransValidation()

6. Correctness of OPT-MVOSTM

In this section, we will prove that our implementation satisfies opacity. Consider the history $H$ generated by OPT-MVOSTM algorithm. Recall that only the $t.begin$, $rv.method$, $t.insert()$, $upd.method$ (or $tryC$) access shared memory.

Note that $H$ is not necessarily sequential: the transactional methods can execute in an overlapping manner. To reason about correctness, we have to prove $H$ is opaque. Since we defined opacity for histories which are sequential, we order all the overlapping methods in $H$ to get an equivalent sequential history. We then show that this resulting sequential history satisfies method.

We order overlapping methods of $H$ as follows: (1) two overlapping $t.begin$ methods based on the order in which they obtain lock over the $counter$; (2) two $rv.method$ accessing the same key $k$ by their order of unlocking over $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of $k$; (3) an $rv.method$ $rvm_i(k)$ and a $t.insert_j()$, of a transaction $T_j$ accessing the same key $k$, are ordered by their order of unlocking over $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of $k$; (4) an $rv.method$ $rvm_i(k)$ and a $tryC_j$, of a transaction $T_j$ which has written to $k$, are similarly ordered by their order of unlocking over $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of $k$; (5) two $t.insert()$ methods accessing the same key $k$ by their order of unlocking over $\langle preds[0], preds[1], currs[0], currs[1] \rangle$.
of $k$; (6) a $t\_insert_i()$ and a $tryC_j$, of a transaction $T_j$ which has written to $k$, are similarly ordered by their order of unlocking over $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of $k$; (7) similarly, two $tryC$ methods based on the order in which they unlock over $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of same key $k$.

Combining the real-time order of events with above-mentioned order, we obtain a partial order which we denote as $lockOrder_H$. (It is a partial order since it does not order overlapping $rv$ methods on different keys or an overlapping $rv$ method and a $tryC$ which do not access any common key).

In order for $H$ to be sequential, all its methods must be ordered. Let $\alpha$ be a total order or linearization of methods of $H$ such that when this order is applied to $H$, it is sequential. We denote the resulting history as $H^\alpha = \text{linearize}(H, \alpha)$. We now argue about the validity of histories generated by the algorithm.

**Lemma 5.** Consider a history $H$ generated by the OPT-MVOSTM algorithm. Let $\alpha$ be a linearization of $H$ which respects $lockOrder_H$, i.e. $lockOrder_H \subseteq \alpha$. Then $H^\alpha = \text{linearize}(H, \alpha)$ is valid.

**Proof:** Consider a successful $rv$ method $rvm_i(k)$ that returns value $v$. The $rv$ method first obtains the lock on $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of key $k$. Thus the value $v$ returned by the $rv$ method must have already been stored in $k$’s version list by a transaction, say $T_j$ when it successfully returned OK from its $tryC$ method. For this to have occurred, $T_j$ must have successfully locked and released $\langle preds[0], preds[1], currs[0], currs[1] \rangle$ of $k$ prior to $T_i$’s locking method. Thus from the definition of $lockOrder_H$, we get that $tryC_j(ok)$ occurs before $rvm_i(k, v)$ which also holds in $\alpha$.

It can be seen that for proving correctness, any linearization of a history $H$ is sufficient as long as the linearization respects $lockOrder_H$. The following lemma formalizes this intuition.

**Lemma 6.** Consider a history $H$. Let $\alpha$ and $\beta$ be two linearizations of $H$ such that both of them respect $lockOrder_H$, i.e. $lockOrder_H \subseteq \alpha$ and $lockOrder_H \subseteq \beta$. Then, $H^\alpha = \text{linearize}(H, \alpha)$ is opaque if $H^\beta = \text{linearize}(H, \beta)$ is opaque.

**Proof:** From Lemma 5, we get that both $H^\alpha$ and $H^\beta$ are valid histories. Now let us consider each case
If: Assume that $H^\alpha$ is opaque. Then, we get that there exists a legal t-sequential history $S$ that is equivalent to $H^\alpha$. From the definition of $H^\beta$, we get that $H^\alpha$ is equivalent to $H^\beta$. Hence, $S$ is equivalent to $H^\beta$ as well. We also have that, $\prec^\text{RT}_{H^\alpha} \subseteq \prec^\text{RT}_{S}$. From the definition of $\text{lockOrder}_H$, we get that $\prec^\text{RT}_{H^\alpha} = \prec^\text{RT}_{\text{lockOrder}_H} = \prec^\text{RT}_{H^\beta}$. This automatically implies that $\prec^\text{RT}_{H^\beta} \subseteq \prec^\text{RT}_{S}$. Thus $H^\beta$ is opaque as well.

Only if: This proof comes from symmetry since $H^\alpha$ and $H^\beta$ are not distinguishable.

This lemma shows that, given a history $H$, it is enough to consider one sequential history $H^\alpha$ that respects $\text{lockOrder}_H$ for proving correctness. If this history is opaque, then any other sequential history that respects $\text{lockOrder}_H$ is also opaque.

Consider a history $H$ generated by $\text{OPT-MVOSTM}$ algorithm. We then generate a sequential history that respects $\text{lockOrder}_H$. For simplicity, we denote the resulting sequential history of $\text{OPT-MVOSTM}$ as $H_{to}$. Let $T_i$ be a committed transaction in $H_{to}$ that writes to $k$ (i.e. it creates a new version of $k$).

To prove the correctness, we now introduce some more notations. We define $H_{to}.\text{stl}(T_i, k)$ as a committed transaction $T_j$ such that $T_j$ has the smallest timestamp larger (or stl) than $T_i$ in $H_{to}$ that writes to $k$ in $H_{to}$. Similarly, we define $H_{to}.\text{lts}(T_i, k)$ as a committed transaction $T_k$ such that $T_k$ has the largest timestamp smaller (or lts) than $T_i$ that writes to $k$ in $H_{to}$. Using these notations, we describe the following properties and lemmas on $H_{to}$.

**Property 7.** Every transaction $T_i$ is assigned a unique numeric timestamp $i$.

**Property 8.** If a transaction $T_i$ begins after another transaction $T_j$ then $j < i$.

**Lemma 9.** If a transaction $T_k$ looks up key $k_x$ from (a committed transaction) $T_j$ then $T_j$ is a committed transaction updating to $k_x$ with $j$ being the largest timestamp smaller than $k$. Formally, $T_j = H_{to}.\text{lts}(T_k, k_x)$.

**Proof:** We prove it by contradiction. So, assume that transaction $T_k$ looks up key $k_x$ from $T_i$ that has committed before $T_j$ so, from Property 8, $i < k$ and $k < j$ i.e. $i$ is not largest timestamp smaller than $k$. But given statement in this lemma is $i < j < k$ which
contradicts our assumption. Hence, $T_k$ looks up key $k_x$ from $T_j$ which is the largest timestamp smaller than $k$.

**Lemma 10.** Suppose a transaction $T_k$ looks up $k_x$ from (a committed transaction) $T_j$ in $H_{to}$, i.e. $\{up_j(k_{x,j}, v), rvm_k(k_{x,i}, v)\} \in evts(H_{to})$. Let $T_i$ be a committed transaction that updates to $k_x$, i.e. $up_i(k_{x,i}, u) \in evts(T_i)$. Then, the timestamp of $T_i$ is either less than $T_j$’s timestamp or greater than $T_k$’s timestamp, i.e. $i < j \oplus k < i$ (where $\oplus$ is XOR operator).

**Proof:** We will prove this by contradiction. Assume that $i < j \oplus k < i$ is not true. This implies that, $j < i < k$. But from the implementation of $rv_{method}$ and tryC methods, we get that either transaction $T_i$ is aborted or $T_k$ looks up $k$ from $T_i$ in $H$. Since neither of them are true, we get that $j < i < k$ is not possible. Hence, $i < j \oplus k < i$.

To show that $H_{to}$ satisfies opacity, we use the graph characterization developed above in Section 3. For the graph characterization, we use the version order defined using timestamps. Consider two committed transactions $T_i, T_j$ such that $i < j$. Suppose both the transactions write to key $k$. Then the versions created are ordered as $k_i \ll k_j$. We denote this version order on all the keys created as $\ll_{to}$. Now consider the opacity graph of $H_{to}$ with version order as defined by $\ll_{to}$, $G_{to} = OPG(H_{to}, \ll_{to})$. In the following lemmas, we will prove that $G_{to}$ is acyclic.

**Lemma 11.** All the edges in $G_{to} = OPG(H_{to}, \ll_{to})$ are in timestamp order, i.e. if there is an edge from $T_j$ to $T_i$ then the $j < i$.

**Proof:** To prove this, let us analyze the edges one by one,

- rt edges: If there is an rt edge from $T_j$ to $T_i$, then $T_j$ terminated before $T_i$ started. Hence, from Property 8 we get that $j < i$.

- rvf edges: This follows directly from Lemma 9.

- mv edges: The mv edges relate a committed transaction $T_k$ updates to a key $k$, $up_k(k, v)$; a successful $rv_{method}$ $rvm_j(k, u)$ belonging to a transaction $T_j$ looks up $k$ updated by a committed transaction $T_i$, $up_i(k, u)$. Transactions $T_i, T_k$
create new versions \(k_i, k_k\) respectively. According to \(\preceq_{to}\), if \(k_k \preceq_{to} k_i\), then there is an edge from \(T_k\) to \(T_i\). From the definition of \(\preceq_{to}\) this automatically implies that \(k < i\).

On the other hand, if \(k_i \preceq_{to} k_k\) then there is an edge from \(T_j\) to \(T_k\). Thus, in this case, we get that \(i < k\). Combining this with Lemma \ref{lemma10} we get that \(j < k\).

Thus in all the cases, we have shown that if there is an edge from \(T_j\) to \(T_i\) then the \(j < i\).

**Theorem 12.** Any history \(H_{to}\) generated by OPT-MVOSTM is opaque.

**Proof:** From the definition of \(H_{to}\) and Lemma \ref{lemma5} we get that \(H_{to}\) is valid. We show that \(G_{to} = OPG(H_{to}, \preceq_{to})\) is acyclic. We prove this by contradiction. Assume that \(G_{to}\) contains a cycle of the form, \(T_{c1} \rightarrow T_{c2} \rightarrow \ldots T_{cm} \rightarrow T_{c1}\). From Lemma \ref{lemma11} we get that, \(c1 < c2 < \ldots < cm < c1\) which implies that \(c1 < c1\). Hence, a contradiction. This implies that \(G_{to}\) is acyclic. Thus from Theorem \ref{theorem4} we get that \(H_{to}\) is opaque.

Now, it is left to show that our algorithm is live, i.e., under certain conditions, every operation eventually completes. We have to show that the transactions do not deadlock. This is because all the transactions lock all \(\langle \text{preds}[0], \text{preds}[1], \text{currs}[0], \text{currs}[1] \rangle\) of keys in a predefined order. As discussed earlier, the STM system orders all \(\langle \text{preds}[0], \text{preds}[1], \text{currs}[0], \text{currs}[1] \rangle\) of keys. We denote this order as accessOrder and denote it as \(\prec_{ao}\). Thus \(k_1 \prec_{ao} k_2 \prec_{ao} \ldots \prec_{ao} k_n\).

From accessOrder, we get the following property

**Property 13.** Suppose transaction \(T_i\) accesses shared objects \(p\) and \(q\) in \(H\). If \(p\) is ordered before \(q\) in accessOrder, then lock\((p)\) by transaction \(T_i\) occurs before lock\((q)\). Formally, \((p \prec_{ao} q) \iff (\text{lock}(p) \lt_{H} \text{lock}(q))\).

**Theorem 14.** OPT-MVOSTM with unbounded versions ensures that rv\_methods do not abort.

**Proof:** This is self-explanatory with the help of OPT-MVOSTM algorithm because each key is maintaining multiple versions in the case of unbounded versions. So rv\_method always finds a correct version to read it from. Thus, rv\_methods do not abort.
7. Experimental Evaluation

This section describes the experimental analysis of proposed OPT-MVOSTMs with state-of-the-art STMs. We have three main goals in this section: (1) Analyze the performance benefits of the optimized multi-version object based STMs (or OPT-MVOSTMs) over multi-version object based STMs (or MVOSTMs). (2) Evaluate the benefit of OPT-MVOSTMs over the single-version object based STMs (or OSTMs), and (3) Analyze the benefit of OPT-MVOSTMs over multi-version read-write STMs.

We implement hash-table object and list object as OPT-HT-MVOSTM and OPT-list-MVOSTM described in Section 5. We also consider the extension of this optimized multi-version object STMs to reduce memory usage. Specifically, we consider a variant that implements garbage collection with unbounded versions and another variant where the number of versions never exceeds a given threshold $K$ for both OPT-HT-MVOSTMs and OPT-list-MVOSTMs.

Experimental system: The Experimental system is a large-scale 2-socket Intel(R) Xeon(R) CPU E5-2690 v4 @ 2.60GHz with 14 cores per socket and two hyper-threads (HTs) per core, for a total of 56 threads. Each core has a private 32KB L1 cache and 256 KB L2 cache (which is shared among HTs on that core). All cores on a socket share a 35MB L3 cache. The machine has 32GB of RAM and runs Ubuntu 16.04.2 LTS.

All code was compiled with the GNU C++ compiler (G++) 5.4.0 with the build target x86_64-Linux-gnu and compilation option -std=c++1x -O3.

STM implementations: We have taken the implementation of NOrec-list [3], Boosting-list [4], Trans-list [16], ESTM [2], and RWSTM directly from the TLDS framework\(^3\). And the implementation of MVOSTM [1], OSTM [6] and MVTO [10] from our PDCRL library\(^4\). We implemented our algorithms in C++. Each STM algorithm first creates N-threads, each thread, in turn, spawns a transaction. Each transaction exports $t_{\text{begin}}$, $t_{\text{insert}}$, $t_{\text{lookup}}$, $t_{\text{delete}}$ and $\text{tryC}$ methods as described in Section 2.

Methodology: We have considered three types of workloads: (W1) Li - Lookup

\(^3\)https://ucf-cs.github.io/tlds/
\(^4\)https://github.com/PDCRL/
\(^5\)Code is available here: https://github.com/PDCRL/MVOSTM/OPT-MVOSTM
intensive (90% lookup, 8% insert, and 2% delete), (W2) Mi - Mid intensive (50% lookup, 25% insert, and 25% delete), and (W3) Ui - Update intensive (10% lookup, 45% insert, and 45% delete). The experiments are conducted by varying number of threads from 2 to 64 in power of 2, with 1000 keys randomly chosen. We assume that the hash-table of OPT-HT-MVOSTM has five buckets and each of the bucket (or list in case of OPT-list-MVOSTM) can have a maximum size of 1000 keys. Each transaction, in turn, executes 10 operations which include $t_{\text{lookup}}$, $t_{\text{delete}}$, and $t_{\text{insert}}$ operations. We take an average over 10 results as the final result for each experiment.

**Results:** Figure [12] represents the performance benefit of all the variants of proposed optimized MVOSTM with all variants of MVOSTM for hash-table objects. It shows OPT-HT-KOSTM performs best among all the algorithms (OPT-HT-MVOSTM-GC, OPT-HT-MVOSTM, HT-KOSTM, HT-MVOSTM-GC, HT-MVOSTM) by a factor of 1.02, 1.11, 1.05, 1.07, 1.22 for workload W1, 1.06, 1.09, 1.07, 1.08, 1.15 for workload W2, and 1.01, 1.03, 1.02, 1.03, 1.08 for workload W3 respectively. Along with this, Figure [13] shows the abort count respective algorithms on workload W1, W2, and W3. This represents for less number of threads, the number of aborts are almost same for all the algorithms. But while increasing the number of threads, the number of aborts are least in OPT-HT-KOSTM as compare to others. So, we compare the performance of OPT-HT-KOSTM with the state-of-the-art STMs as shown in Figure [14]. OPT-HT-KOSTM outperforms all the algorithms (HT-OSTM, ESTM, RWSTM, HT-MVTO, HT-KSTM) by a factor of 3.62, 3.95, 3.44, 2.75, 1.85 for W1, 1.44, 2.36, 4.45, 9.84, 7.42 for W2, and 2.11, 4.05, 7.84, 12.94, 10.70 for W3 respectively. The corresponding number of aborts are represented in Figure [15]. Number of aborts are minimum for OPT-HT-KOSTM as compare to other state-of-the-art STMs. Especially, the number of aborts for OPT-HT-KOSTM is almost negligible as compared to HT-OSTM on lookup-intensive workload (W1) because OPT-HT-KOSTM finds a correct version to looks up as shown in Figure [15](a).

The observation of optimized list based MVOSTM is similar as optimized hash-table based MVOSTM. Figure [16] represents the performance benefit of all the variants of proposed optimized MVOSTM with all variants of MVOSTM for list objects. It shows
OPT-list-KOSTM performs best among all the algorithms (OPT-list-MVOSTM-GC, OPT-list-MVOSTM, list-KOSTM, list-MVOSTM-GC, list-MVOSTM) by a factor of 1.14, 1.24, 1.21, 1.20, 1.35 for W1, 1.06, 1.07, 1.12, 1.13, 1.20 for W2, and 1.09, 1.19, 1.11, 1.17, 1.31 for W3 respectively. Along with this, Figure 17 shows the minimum abort count by OPT-list-KOSTM as compare to other algorithms on workload W1, W2, and W3. Hence, we choose the best-proposed algorithm OPT-list-KOSTM and compare with the state-of-the-art list based STMs.

Figure 12: Time comparison among variants of OPT-HT-MVOSTMs and HT-MVOSTMs on hash-table

Figure 13: Abort count among variants of OPT-HT-MVOSTMs and HT-MVOSTMs on hash-table

Figure 18 represents OPT-list-KOSTM outperforms all the algorithms (list-OSTM, Trans-list, Boosting-list, NOrec-list, list-MVTO, list-KSTM) by a factor of 2.56, 25.38, 23.57, 27.44, 13.34, 5.99 for W1, 1.51, 20.54, 24.27, 29.45, 24.89, 19.78 for W2, and 2.91, 32.88, 28.45, 40.89, 173.92, 124.89 for W3 respectively. Similarly, Figure 19 depicts that OPT-list-KOSTM obtained the least number of aborts as compare to others
on the respective workloads.

![Graph](image1.png)  
**Figure 14:** Time comparison of OPT-HT-KOSTM and State-of-the-art hash-table based STMs

![Graph](image2.png)  
**Figure 15:** Abort count of OPT-HT-KOSTM and State-of-the-art hash-table based STMs

![Graph](image3.png)  
**Figure 16:** Time comparison among variants of OPT-list-MVOSTMs and list-MVOSTMs on list
Figure 17: Abort count among variants of OPT-list-MVOSTMs and list-MVOSTMs on list

Figure 18: Time comparison of OPT-list-KOSTM and State-of-the-art list based STMs

Figure 19: Abort count of OPT-list-KOSTM and State-of-the-art list based STMs
Figure 20: Memory consumption among variants of OPT-HT-MVOSTMs and HT-MVOSTMs on hash-table

Figure 21: Memory consumption among variants of OPT-list-MVOSTMs and list-MVOSTMs on list

Figure 22: Optimal Value of K for OPT-HT-KOSTM and OPT-list-KOSTM
As explained in Section 5 for efficient memory utilization, we develop two variations of OPT-MVOSTM. The first, OPT-MVOSTM-GC, uses unbounded versions but performs garbage collection. This is achieved by deleting non-latest versions whose timestamp is less than the timestamp of the least live transaction. OPT-MVOSTM-GC gave a performance gain of 16% over OPT-MVOSTM without garbage collection in the best case which is on workload W1 with 64 number of threads. We did one more optimization in OPT-MVOSTM-GC on the marked node exist in the RL to make it search efficiently. This is achieved by deleting a marked node from RL whose max_rvl of the last version is less than the timestamp of the least live transaction. The second, OPT-KOSTM, keeps at most $K$ versions by replacing the oldest version when ($K + 1)^{th}$ version is created by a current transaction as explained in Section 5. OPT-KOSTM shows a performance gain of 24% over OPT-MVOSTM without garbage collection in the best case which is on workload W1 with 64 number of threads. As OPT-KOSTM has a limited number of versions while OPT-MVOSTM-GC can have infinite versions, the memory consumed by OPT-KOSTM is also less than OPT-MVOSTM-GC. We have integrated these variations in both hash-table based (OPT-HT-MVOSTM-GC and OPT-HT-KOSTM) and linked-list based MVOSTMs (OPT-list-MVOSTM-GC and OPT-list-KOSTM), we observed that these two variations increase the performance, concurrency and reduce the number of aborts as compared to OPT-MVOSTM which does not perform garbage collection.

Memory Consumption by OPT-MVOSTM-GC and OPT-KOSTM: As depicted above OPT-KOSTM performs better than OPT-MVOSTM-GC. Continuing the comparison between the two variations of OPT-MVOSTM we chose another parameter as memory consumption. Here we test for the memory consumed by each variation algorithms in creating a version of a key. We count the total versions created, where creating a version increases the counter value by 1 and deleting a version decreases the counter value by 1. Figure 20 depicts the comparison of memory consumption by all the variants of proposed optimized MVOSTM with all variants of MVOSTM for hash-table objects. OPT-HT-KOSTM consumes minimum memory among all the algorithms (OPT-HT-MVOSTM-GC, OPT-HT-MVOSTM, HT-KOSTM, HT-MVOSTM-GC, HT-MVOSTM) by a factor of 1.07, 1.16, 1.15, 1.15, 1.21 for W1, 1.01, 1.08, 1.06,
1.07, 1.19 for W2, and 1.01, 1.03, 1.02, 1.03, 1.08 for W3 respectively. Similarly, Figure 21 depicts the comparison of memory consumption by all the variants of proposed optimized MVOSTM with all variants of MVOSTM for list objects. OPT-list-KOSTM consumes minimum memory among all the algorithms (OPT-list-MVOSTM-GC, OPT-list-MVOSTM, list-KOSTM, list-MVOSTM-GC, list-MVOSTM) by a factor of 1.01, 1.05, 1.05, 1.04, 1.11 for W1, 1.02, 1.1, 1.1, 1.11, 1.19 for W2, and 1.01, 1.03, 1.05, 1.08, 1.13 for W3 respectively.

Finite version OPT-MVOSTM (OPT-KOSTM): To find the ideal value of $K$ such that performance as compared to OPT-MVOSTM-GC does not degrade or can be increased, we perform experiments on all the workloads (W1, W2, and W3) for both (OPT-HT-KOSTM and OPT-list-KOSTM). Figure 22 (a) and (b) shows the best value of $K$ as 5 for OPT-HT-KOSTM and OPT-list-KOSTM on all the workloads for both hash-table and list objects.

8. Conclusion

With the rise of multi-core systems, concurrent programming becomes popular. Concurrent programming using multiple threads has become necessary to utilize all the cores present in the system effectively. But concurrent programming is usually challenging due to synchronization issues between the threads.

In the past few years, several STMs have been proposed which address these synchronization issues and provide greater concurrency. STMs hide the synchronization and communication difficulties among the multiple threads from the programmer while ensuring correctness and hence making programming easy. Another advantage of STMs is that they facilitate compositionality of concurrent programs with great ease. Different concurrent operations that need to be composed to form a single atomic unit is achieved by encapsulating them in a single transaction.

In literature, most of the STMs are RWSTMs which export read and write operations. To improve the performance, a few researchers have proposed OSTMs [4, 5, 6] which export higher level objects operation such as hash-table insert, delete, and lookup etc. By leveraging the semantics of these higher level operations, these STMs provide greater
concurrency. On the other hand, it has been observed in STMs and databases that by storing multiple versions for each t-object in case of RWSTMs provides greater concurrency \cite{17,10}.

This paper proposed the notion of the optimized multi-version object based STMs (OPT-MVOSTMs) and compares their effectiveness with multi-version object based STMs (MVOSTMs), single-version object based STMs and multi-version read-write STMs. We find that OPT-MVOSTM provides a significant benefit over above-mentioned state-of-the-art STMs for different types of workloads. Specifically, we have evaluated the effectiveness of OPT-MVOSTM for the hash-table and list data structure as OPT-HT-MVOSTM and OPT-list-MVOSTM respectively.

OPT-HT-MVOSTM and OPT-list-MVOSTM use the unbounded number of versions for each key. To utilize the memory efficiently, we limit the number of versions and develop two variants for both hash-table and list data structures: (1) A garbage collection method in OPT-MVOSTM to delete the unwanted versions of a key, denoted as OPT-MVOSTM-GC. (2) Placing a limit of $K$ on the number of versions in OPT-MVOSTM, resulting in OPT-KOSTM. Both these variants (OPT-MVOSTM-GC and OPT-KOSTM) gave a performance gain of over 16% and 24% over OPT-MVOSTM in the best case. OPT-KOSTM consumes minimum memory among all the variants of it. We represent OPT-MVOSTM-GC in hash-table and list as OPT-HT-MVOSTM-GC and OPT-list-MVOSTM-GC respectively. Similarly, We represent OPT-KOSTM in hash-table and list as OPT-HT-KOSTM and OPT-list-KOSTM respectively.

OPT-HT-KOSTM performs best among its variants and outperforms state-of-the-art hash-table based STMs (HT-OSTM, ESTM, RWSTM, HT-MVTO, HT-KSTM) by a factor of 3.62, 3.95, 3.44, 2.75, 1.85 for workload W1, 1.44, 2.36, 4.45, 9.84, 7.42 for workload W2, and 2.11, 4.05, 7.84, 12.94, 10.70 for workload W3 respectively. Similarly, OPT-list-KOSTM performs best among its variants and outperforms state-of-the-art list based STMs (list-OSTM, Trans-list, Boosting-list, NOrec-list, list-MVTO, list-KSTM) by a factor of 2.56, 25.38, 23.57, 27.44, 13.34, 5.99 for W1, 1.51, 20.54, 24.27, 29.45, 24.89, 19.78 for W2, and 2.91, 32.88, 28.45, 40.89, 173.92, 124.89 for W3 respectively. We rigorously proved that OPT-MVOSTMs satisfy the correctness criteria as opacity.
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