Collapse of a Bose gas: Kinetic approach

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Abstract. We have analytically explored the temperature dependence of critical number of particles for the collapse of a harmonically trapped attractively interacting Bose gas below the condensation point by introducing a kinetic approach within the Hartree–Fock approximation. The temperature dependence obtained by this easy approach is consistent with that obtained from the scaling theory.

Keywords. Thermodynamical, statistical and static properties of condensates; Ultracold and trapped gases; matter waves.

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1. Introduction

For an ultracold Bose gas, interparticle interaction is characterized by s-wave scattering length ($a_s$) which can be tuned arbitrarily by the Feshbach resonance method [1]. For attractive ($a_s < 0$) interaction, a harmonically trapped Bose gas tends to increase its density in the central region of a trap. This tendency, of course, is opposed by the quantum and thermal fluctuations. If the number of atoms is greater than the critical number ($N_c$), then the central density increases strongly, and the zero-point and thermal fluctuations are no longer able to avoid the collapse of the gas. Consequently, the gas becomes unstable even for two-body interaction.

The stability and collapse of the Bose–Einstein condensates with negative scattering lengths have already been observed in the clouds of ultracold $^7$Li [2] and $^{85}$Rb [3] for temperatures ($T$) close to zero or well below the condensation point ($T_c$). Soon after the observation, many theories for the collapse have been proposed for $T \rightarrow 0$ [4–10], as well as for $T > 0$ [11–14]. The remarkable one among these (theories) was given by Baym and Pethick [4]. They proposed a scaling theory for $T = 0$, and we generalized their theory for $0 \leq T \leq T_c$ within the Hartree–Fock (H–F) approximation [14]. In the generalized theory, different parts of the free energy (grand potential) of our system were scaled by a parameter which reduces the length scale of the system as a result of attractive...
interaction; and a critical number for the collapse was eventually calculated from a critical condition of existence of a metastable minimum of the grand potential [14]. In this brief report we shall calculate the same, but in a kinetic approach which is supposed to be the easiest way.

This time, for calculating the critical number, we shall not start from the free energy, but shall adopt a mere kinetic theory like approach based on the energy and pressure of the system. We shall start from the H–F energy of the system, and shall pick up the kinetic energy and interaction energy parts of the H–F energy. While the kinetic energy of the particles causes an outward pressure, the attractive interaction causes an inward pressure. For critical number of particles, magnitudes of the two pressures would be the same. Beyond the critical number of particles, the inward pressure would be larger than the outward one, and as a consequence, the whole system would collapse. Thus, we shall calculate the critical number, and shall show its temperature dependence. Our present technique is easier than that of the already existing theory [4,14] because outward and inward pressures appear as the first-order derivative of the two parts of energy with respect to the effective volume of the system, and the already existing technique involves a second-order derivative of the grand potential with respect to the scaling parameter for obtaining the critical condition of existence of its metastable minimum.

2. Qualitative result

Before going into the details of the kinetic approach, let us estimate the critical number by qualitative manner. Our system consists of a large number \( N \) of Bose particles, each of which is a three-dimensional isotropic harmonic oscillator with angular frequency \( \omega \) and mass \( m \). The system, of course, is in thermodynamic equilibrium with its surroundings at temperature \( T \). For \( T \to 0 \), all the particles occupy the ground state, and the system can be well described by the ground state wave function \( \Psi_0(\mathbf{r}) = \sqrt{N/ (l^3 \pi^{3/2})} e^{-r^2/2l^2} \) in the position \((\mathbf{r})\) space, where \( l = \sqrt{\hbar/m\omega} \) is the confining length scale of the oscillators [15]. Thus, for \( T = 0 \), the density of the condensed particles is given by [15]

\[
 n_0(\mathbf{r}) = |\Psi_0(\mathbf{r})|^2 = \frac{N}{l^3 \pi^{3/2}} e^{-r^2/2l^2}. \tag{1}
\]

On the other hand, the number density of the excited particles (in the absence of interaction) is given by [14,16]

\[
 n_T(\mathbf{r}) = \frac{1}{\lambda^3_T} g_{3/2}(e^{-m\omega^2 r^2/2k_B T}), \tag{2}
\]

where \( \lambda_T = \sqrt{2\pi \hbar^2/mk_B T} \) is the thermal de Broglie wavelength and \( g_{3/2}(x) = x + x^2/2^{3/2} + x^3/3^{3/2} + \cdots \) is a Bose–Einstein function of the real variable \( x \).

Let us consider the attractive interaction potential as \( V_{\text{int}}(\mathbf{r}) = g \delta^3(\mathbf{r}) \), where \( g = -4\pi \hbar^2 a/m \) is the coupling constant and \( a = -a_s \) is the absolute value of the s-wave scattering length [16–18]. Typical two-body interaction energy for \( N \) number of particles is \( \sim N^2 g/2l^3 \). Due to this, the gas tends to increase the density at the central region of the trap. Well below \( T_c \) (i.e. for \( T \to 0 \)), this tendency is resisted by the zero-point motion of the atoms. At the critical situation, the energy \((3N\hbar\omega/2)\) for the zero-point