Spin wave excitation patterns generated by spin torque oscillators

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Abstract
Spin torque nano-oscillators (STNO) are nanoscale devices that can convert a direct current into short wavelength spin wave excitations in a ferromagnetic layer. We show that arrays of STNO can be used to create directional spin wave radiation similarly to electromagnetic antennas. Combining STNO excitations with planar spin waves also creates interference patterns. We show that these interference patterns are static and have information on the wavelength and phase of the spin waves emitted from the STNO. We describe a means of actively controlling spin wave radiation patterns with the direct current flowing through STNO, which is useful in on-chip communication and information processing and could be a promising technique for studying short wavelength spin waves in different materials.

Keywords: spin waves, spin torque oscillators, interference patterns

(Some figures may appear in colour only in the online journal)

1. Introduction

Controlling magnetization dynamics in ferromagnetic (FM) thin films is important to a new generation of wave-computing and on-chip communication devices working at high frequency and low power. Spin wave devices may complement digital semiconductor technologies and offer new possibilities for memory capacity and computational performance of particular importance as semiconductor devices miniaturization approaches fundamental limits. The main requirements for wave computation include localized sources and detectors of coherent waves in continuous or patterned propagating media. Nanometer scale electrical contacts to ferromagnetic thin films can carry sufficient current density to generate a high frequency dynamic response of the magnetic moments in FM films [1, 2], resulting in emission of short wavelength spin waves [3].

Emission of spin waves from nanopoint contacts, so called spin torque nano-oscillators (STNO), has been predicted theoretically [3] and recently has also been demonstrated experimentally [4, 5]. Spin wave radiation from a single STNO may not be symmetric and may instead be directional due to spin wave band structures or due to dipolar fields, external fields, anisotropy film fields, or Oersted fields generated by the current in the contact [6]. Magnetic properties of thin films can be tailored to create preferred propagating directions and frequency bandgaps, for example as done in magnonic crystals [7, 8]. Polycrystalline perpendicularly magnetized films may have, however, symmetric radiation patterns. In addition, STNO can be frequency and phase locked to external oscillatory signals or to other STNO frequencies under certain conditions [9–18].

While applications in computation and information storage usually require a unique reference signal (a clock) that times the system and synchronizes it, wave computation offers asynchronous or polysynchronous operation [19–21]. STNO, like other wave sources, can encode information in a carrier signal by modulating combinations of amplitude, frequency, and phase. Nonlinear effects of amplitude and frequency modulation in STNO have been studied both experimentally and theoretically [18, 22–25]. However, little work as been done thus far on phase modulation.

In this paper we investigate spin wave patterns created by STNO and their interactions with background oscillations in
perpendicularly magnetized films. We describe STNO as spin wave antennas, derive expressions for the radiation diagrams, and discuss how to actively control the radiation patterns. We also discuss how to encode information in spin wave radiation.

2. Spin torque nano-oscillators

Spin-polarized currents flowing throughout a magnetic thin film exert a torque on the background magnetization called a spin-transfer torque [1–3, 26, 27]. These polarized electrical currents encounter resistance when crossing a magnetic material that depends on the relative orientation between the current spin polarization and the film magnetization; this is the phenomenon of giant magnetoresistance (GMR) [28, 29]. STNO are nanoscale electrical contacts to a ferromagnetic material that depends on the relative orientation between the current spin polarization direction of the applied current, \( \mathbf{m}_p \). The function \( \beta(\mathbf{r}) \) is a Heaviside function defining the sizes and locations of the point contacts. \( \beta(\mathbf{r}) \) also depends on the current intensity, the layer thickness and the spin polarization [21]. We consider steady state conditions, situations in which the current has been turned on for a certain time. The free magnetic layer where spin dynamics are excited is considered thin compared to the magnetic exchange length, \( \lambda_{ex} \). Therefore, we disregard variations in the magnetic moment across the film thickness (z direction).

We consider a case where the free layer is perpendicularly magnetized, which is the case where a propagating spin wave mode was first described [3]. Since the magnetization vector \( \mathbf{m} = \frac{\mathbf{M}}{\mu} \) lies on the unit sphere, we focus onore the lateral behavior in the free layer. We write the components of \( \mathbf{m} \) as

\[ \mathbf{m} = (m_x, m_z) \quad m = m_x + im_y, \quad m_z = \sqrt{1 - |m|^2}. \]

The out-of-plane component, \( m_z \), or the absolute value of the in-plane magnetization component, \( |m|^2 = 1 - m_z^2 \), describes the amplitude of the excitation and is a non-oscillating quantity. The in-plane components, \( m_x \) and \( m_y \), contain the frequency and phase information of the excitations.

The resulting patterns created by an STNO (or from arrays of STNO) are mostly controlled by the wave term in the effective field (equation (2)). The equation for the magnetization amplitude \( m \) (from equation (1) and after normalization) is a nonlinear Schrödinger equation [21, 30],

\[ \frac{i}{\hbar} \frac{\partial m}{\partial t} = (1 + i\omega)\nabla^2 m - f(|m|^2)m + ig(|m|^2)m, \quad (3) \]

where \( f \) and \( g \) are nonlinear functions of the amplitude, \( |m|^2 \) (expressions for \( f \) and \( g \) are given in [21]).

A linear approximation of equation (3) has the form

\[ \frac{i}{\hbar} \frac{\partial m}{\partial t} = (1 + i\omega)\nabla^2 m + \omega_k m - i\alpha \omega_k m + i\beta(\mathbf{r})m, \quad (4) \]
where $\omega_k$ is the internal frequency of the oscillator. This equation is a good starting point for studying interference patterns from different point sources. However, phase locking and stabilization of patterns in the presence of noise require analysis of equation (3).

For a single nanocontact we consider a solution of the form $m(\rho, t) = \phi(\rho)e^{i\omega t}$, where $\rho$ is the radial variable,

$$(1 + i\alpha) \left( \partial_{\rho\rho}\phi + \frac{1}{\rho} \partial_{\rho}\phi \right) + (\omega - \omega_k)\phi + i(\beta(\rho) - \alpha \omega)\phi = 0.$$  

(5)

Slonczewski determined a linear solution using Bessel functions and appropriate boundary conditions at the nanocontact boundary [3].

4. Material parameters

We now discuss typical material parameters for STNO. We have performed our calculations with transition metal thin films in mind. We used Permalloy’s most relevant parameters: $M_s \approx 860$ kA m$^{-1}$, $\lambda_{ex} \approx 5.8$ nm, $f \approx 1–30$ GHz for $H \approx 1$ T, and the damping constant, $\alpha \approx 0.01$. Spin waves are expected to propagate tens of microns in such films.

Other candidates for yielding short wavelength spin wave radiation patterns are magnetic semiconductors, where a semiconductor host is magnetically doped by transition metal impurities (e.g., Ga$_{1-x}$Mn$_x$As with $x < 1–10\%$) [31], and Heusler alloys, i.e. ferromagnetic metal alloys based on a Heusler phase (e.g., Cu$_2$MnAl) [32]. Magnetic semiconductors and Heusler alloys have different magnetization saturation values but the resonant frequencies are of the same order (~GHz); damping parameters might vary depending on the compound or the fabrication method (some Heusler alloys have been reported to have damping constants one order of magnitude smaller than those of transition metals).

5. Radiating STNO antennas

The spin waves radiating from a single STNO out of the point contact (in a linear approximation) is given in terms of a Hankel function, which is a linear combination of Bessel functions of the first and second kind:

$$m(\rho, t) = H_0^2(\rho)e^{\omega t}$$  

(6)

where the radial component, $\rho$, has been normalized with the wavelength, $\lambda$, which is defined by the size of the contact, $r$, with $\lambda = 5\pi r$ being the first set of solutions of equation (5) [30]. For large $\rho$ ($\rho \gg 2\pi$), the Hankel function is approximately

$$H_0^2(\rho) \approx \frac{2}{\pi \rho} e^{\rho} e^{-i\rho} = A \frac{e^{-i\rho}}{\rho}.$$  

(7)

When different sources have a common oscillation frequency and very similar wavelengths, one can effectively describe the resulting activity patterns from STNO arrays with

$$m(\rho, \theta, t) \approx D(\theta)H_0^2(\rho) \exp(i\omega t)$$  

(8)

where $D(\theta)$ is the energy distribution of the radiated waves [21, 33]. Although single devices may have a radiation pattern with some asymmetries, the resulting energy distributions from combining multiple sources may still be written as a single-oscillator radiation multiplied by a spatial energy distribution, $D(\theta)$, known as a radiation diagram. Figure 2 shows the contour plots of the radiated spin waves for the quantity $m$ from a single (a), a double (b), and a triple (c) STNO array. Simulation results are shown for a 1 × 1 $\mu$m$^2$ film with point contacts 40 nm in diameter, and separation between contacts of 100 nm ($\approx \lambda = 2\pi/k$). The larger plots (left-hand side) are contour plots of the radiated spin waves corresponding to the amplitude of the excitation, $|m|$, and the smaller 3D graphs (right-hand side) correspond to the amplitude, $|m|$, and one of the oscillating components, $m_\phi$.
the simplest case, two oscillators phase lock and oscillate at
the same frequency, having a relative phase determined by the
difference in internal frequencies [21]. The internal frequency
of an STNO (i.e., $\omega_1$, $\omega_2$, etc; see equation (4)) depends on
the applied current and the local fields at the contact.

STNO can also be patterned with different shapes (ovals,
rectangles, etc). However, simulations of equation (1) showed
that the resulting interference patterns are mostly isotropic. A
critical physical distance between contacts (of the order of the
wavelength) is required to create interference patterns.

In the following subsections we derive expressions for
the radiation diagrams for simple cases of aggregated STNO;
and we calculate the radiation patterns for sets of two and
three STNO and also for groups of STNO that have different
oscillation phases.

5.1. Radiation from two STNO

We consider a simple case for a radiation pattern from two
STNO separated by a distance $2a$ and located as shown in
figure 3. One can write the radial component, $\rho'$, in the
original system of each STNO for any given point $(\rho, \theta)$
assuming that the new reference system is shifted $(a, b)$ from
the original one, where $b$ is an arbitrary shift (this follows from
trigonometric identities):

$$
\rho' (a, b, \theta, \rho) = \rho - \text{sgn}(a) \sqrt{a^2 + b^2} \\
\times \cos (\theta - \arctan(b/a)),
$$

(9)

where we have considered that $\rho \gg \sqrt{a^2 + b^2}$. Following
equation (9) we obtain $\rho' = \rho - a \cos \theta$ for STNO 1 and
$\rho' = \rho + a \cos \theta$ for STNO 2. The solution of equation (4),
$m(\rho, \theta, t)$, for the radiation is a combination of those for
STNO 1 and STNO 2:

$$
m(\rho, \theta, t) = e^{i\omega t} \left( H_{0}^{(2)} (\rho - a \cos \theta) + H_{0}^{(2)} (\rho + a \cos \theta) \right)
\approx e^{i\omega t} \frac{A}{\rho} \left( e^{-i|\rho - a \cos \theta|} + e^{-i|\rho + a \cos \theta|} \right)
\approx 2e^{i\omega t} H_{0}^{(2)} (\rho) \cos (a \cos \theta),
$$

(10)

which is basically the solution for a single oscillator
multiplied by a radiation function,

$$
D(\theta) = 2 \cos (a \cos \theta),
$$

(11)

which depends only on the angle and the distance between the
contacts.

The upper panels in figure 4 show the spatial dependence
of the excitation amplitude, $|m(\rho, \theta, t)|$, from two STNO
positioned as shown in figure 3 (the amplitude $|m|$ is time
independent). The lower panels show the corresponding
radiation diagram, $D(\theta)$, calculated with equation (11). We
have chosen two representative cases: distance, $d = 2a$,
between the STNO being either half of a wavelength or a full
wavelength (0.5$\lambda$, or $\lambda$).

Radiation patterns are determined by spin wave
interference from multiple sources. For two STNO, changing
the phase of one of the sources, $e^{i\omega t} \rightarrow e^{i(\omega t + \psi)}$, changes the
resulting radiation pattern. Radiation patterns for two STNO

with a phase difference $\psi$ can be calculated and the radiation
function becomes

$$
D(\theta) = 2e^{i\psi/2} \cos (a \cos \theta + \psi/2).
$$

(12)

Figure 5 shows the radiation diagrams calculated from
equation (12) for different phase shifts.

5.2. Radiation from three STNO

The radiation function for an array of three STNO arranged as
shown in the inset of figure 6 can be written as

$$
D(\theta) = e^{i[2\omega/\sqrt{3} \cos (\theta - \pi/6)]} + e^{-i[2\omega / \sqrt{3} \cos (\theta + \pi/6)]}
+ e^{-i[2\omega / \sqrt{3} \sin \theta]}
\approx 2 \cos \left( \frac{2a}{\sqrt{3}} \cos \theta \right) e^{-i\pi/6} + e^{-i[\frac{2\omega}{\sqrt{3}} \sin \theta]},
$$

(13)

which results in a pattern that radiates in six directions (see
figure 6). Combinations with different phases allow rotation
of the pattern and other complicated spin wave excitation
patterns.

5.3. Radiation from n STNO

For a general case with $n$ oscillators located at arbitrary
positions one can find the expression for the radiation
functions by adding the functions describing the individual
radiation patterns for each of the oscillators.

6. The effect of Oersted fields

The presence of Oersted fields caused by the current flow
through a point contact can drastically influence the magnetic
response in STNO systems [34, 35]. Calculations have shown
that under certain conditions a collimated spin wave beam
is observed [35]. In the case where the magnetized film has
a component in the film plane (i.e., the applied field is not
perpendicular to the film plane) the local applied fields on
opposite sides of the nanocontact (say above and below) will
differ in magnitude. The Oersted fields will add, in one case,
and subtract, in the other, to the applied field. This can modify
the dispersion relations and result, in some cases, in corral
propagating waves or localized bullets [35].
Figure 4. Upper panels show a plot $|r(\rho, \theta, t)|$ for two STNO as in figure 1; on the left-hand side, the STNO are separated by one wavelength, $\lambda$, while on the right, the STNO are separated by half of a wavelength, $\lambda/2$. Plots correspond to a $1 \mu m \times 1 \mu m$ area with point contacts 40 nm in diameter. Lower panels show the corresponding radiation diagram, $D(\theta)$, calculated with equation (11). The radiation diagrams are plotted in a two-dimensional plot where $x = D(\theta) \cos(\theta)$ and $y = D(\theta) \sin(\theta)$.

Figure 5. Radiation diagram, $D(\theta)$, calculated with equation (12) for $\psi = 0, \pm \pi/2$ and $\pi$. The STNO are separated by half of a wavelength, $\lambda/2$. The radiation diagrams are plotted in a two-dimensional plot where $x = |D(\theta)| \cos(\theta)$ and $y = |D(\theta)| \sin(\theta)$.

However, here we are considering a perpendicular magnetized film and we aim at showing that the radiation interference from arrays of STNO is not significantly modified by the Oersted fields. When the film magnetization is fully perpendicular to the film plane (having a large applied field in the perpendicular direction) the effect of the Oersted
Figure 6. The left-hand-side panel shows a plot of $|m(\rho, \theta, t)|$ for three STNO as in figure 4; the plots correspond to a $1 \, \mu m \times 1 \, \mu m$ area with point contacts of 40 nm in diameter. The right-hand-side panel shows the corresponding radiation diagram, $D(\theta)$, calculated with equation (13). The separation between contacts is equal to the wavelength, $\lambda$.

Figure 7. Radiation patterns from a single STNO with (right-hand-side panels) and without (left-hand-side panels) Oersted fields. An Oersted field of a maximum value of 10 mT is considered with an applied field of 1.1$M_s$. The Oersted field distribution is calculated from the approximation of an infinitely long wire with the radius of the nanocontact. The plots correspond to a $600 \times 600 \, nm^2$ area with point contacts of 50 nm in diameter.

fields does not change the oscillation frequency of magnetic moments located at a given distance from the nanocontact; the radial symmetry is apparently conserved, although we might encounter variations in the oscillation phases.

We consider Oersted fields created by the point contact as if they were infinite wires, having a maximum of the order of 10 mT, which is a reasonable estimation for a thin ($\sim 2 \, nm$) layer that requires small currents ($\sim 5 \, mA$). We assume that the fields decay as $1/r$ outside the nanocontact—and increase as $r$ inside. We thus neglect corrections due to the finite size of the electrical contacts and correction due to the field caused by the electrical current in the electrode plates above and below. We consider equation (3) with the perturbation of the Oersted fields in the effective field, $H_{\text{eff}}$ in equation (2), in arrays of STNO. The value for the applied magnetic field is 1.1$M_s$.

Figure 7 shows both the envelope, $|m|$, and one of the in-plane components, $m_x$, of the film magnetization in a single STNO; we compare the effect of Oersted fields with the same
Figure 8. Radiation patterns from a double STNO with (right-hand-side panels) and without (left-hand-side panels) Oersted fields. Simulations are done with parameters similar to those for figure 7. We obtain essentially similar interference patterns in the two cases, with a strong enhancement in between the two nanocontacts and preferred radiation directions. The separation between the nanocontacts is chosen at 125 nm (around the wavelength, $\lambda$).

7. Interaction with background oscillations

Spin waves from STNO may also interfere with background spin wave oscillations (e.g., with incoming planar waves). Electromagnetic radiation from electrical antennas can create planar spin wave oscillations in FM thin films [36–38].

Here we consider the interference between a single source (STNO) and a planar wave excitation, sharing the same oscillating frequency. Again we take the asymptotic expression for an STNO radiation (from equation (7)), which is the same as that for a circular wave with wavelength $\lambda_c$:

$$m_c(\lambda_c, \rho, t) = e^{i\omega t}H_0^{(2)}(\rho/\lambda_c) \approx e^{i\omega t}A e^{-i\rho/\lambda_c}$$

and a simple planar wave in the $x$ direction with wavelength $\lambda_p$:

$$m_p(\lambda_p, x, t) = e^{i\omega t}B e^{-ix/\lambda_p}.$$  

Let us assume that there is no damping, so both planar and circular excitations have constant envelopes (both in time and in space). Adding the two signals we obtain

$$m = m_c + m_p$$

$$= e^{i\omega t}(A e^{-i\rho/\lambda_c} + B e^{-ix/\lambda_p})$$

$$= Ae^{i\omega t} e^{-i\rho/\lambda_c} \left(1 + \frac{B}{A} e^{-i(x/\lambda_p - \rho/\lambda_c)}\right).$$  

(16)
Figure 9. Three plots of patterns of spin wave interference between a single STNO and a planar spin wave; \( \lambda_c = \lambda_p \) in the left-hand-side panel, 5\( \lambda_c = \lambda_p \) in the center panel and \( \lambda_c = 5\lambda_p \) in the right-hand-side panel. The upper panels show the patterns of the individual excitations (time-dependent patterns); the amplitude is constant for each individual excitation. The lower panels show the amplitudes of the resulting interference patterns.

Adding the two waves produces a new excitation that varies in space while still being constant in time (notice that the individual excitations had constant envelopes both in time and in space). Three different representative cases are drawn using three different wavelengths. The three cases are plotted in figure 9: \( \lambda_c = \lambda_p \), 5\( \lambda_c = \lambda_p \) and \( \lambda_c = 5\lambda_p \). We first plotted the real parts for both waves, which correspond to the oscillatory component \( m_x \), where we identify the wavelengths, \( \lambda_c \) and \( \lambda_p \) respectively. The resulting pattern of interference between the planar and the STNO spin wave excitations induces a pattern in the amplitude, which is a time-independent pattern that captures the structure of the spin waves from both the point contact and the planar source. The dynamic response of the magnetic moment in the FM thin film is modulated by the wavelengths of the two excitations—a modulation of the energy.

A closer look at equation (16) shows that the interference of a planar and a circular wave creates a new wave excitation in the energy—or the envelope—that has a wavelength that depends on the direction (taking as the origin the point source). Writing the planar excitation in the polar coordinates \( x = \rho \cos(\theta) \) centered at the point source gives

\[
m = m_c + m_p = e^{i\omega t}e^{-i\frac{\phi}{\pi}}A\left(1 + \frac{B}{A}e^{-i\theta\left(\frac{\text{tan}^{-1}\frac{\rho}{\lambda_p}}{\lambda_c} - \frac{\pi}{2}\right)}\right).
\]

The wavelength, \( \lambda_{\text{env}} \), of the modulation envelope depends on the direction, \( \theta \), and is set by

\[
\lambda_{\text{env}}(\theta) = \frac{\lambda_p \lambda_c}{\lambda_c \cos \theta - \lambda_p},
\]

and the strength of the modulation depends on the relative amplitudes, \( A \) and \( B \), of the two waves. The smaller wavelength determines the modulation.

8. Detecting spin wave activity

Point contacts may also be used as detectors [21] of spin wave activity in a thin film; the respective alignments of the fixed and free layers determine the resistance of the STNO [28, 29]. Thus, low current densities—that do not excite magnetization dynamics—through the point contacts will serve to read the state of the free layer. The magnetization of the two layers can be set in different geometries for different purposes: a geometry where the resistance of the contact is the same for all points of the oscillation trajectory (see figure 10, left-hand-side panel) may serve to sense the energy or the amplitude of the spin wave excitation, \( |m| \) (e.g., detecting amplitude modulation); a geometry where the resistance of the junction varies along the oscillation trajectory (see figure 10(b)) may serve to sense the frequency of the spin wave excitation (e.g., detecting frequency or phase modulation).

The giant magnetoresistance effect is weak when the orientation of one layer tilts only a few degrees, particularly when the layer magnetizations are initially collinear. Adding a magnetic tunnel junction either between the free and polarizing layers or the free layer and a separate magnetic electrode would enhance the magnetoresistance effect [21, 39].
9. Encoding information in the modulation of STNO spin waves

STNO are spin wave sources and detectors; their radiating oscillatory signals may be modulated. Frequency modulation usually offers an ideal method for reducing noise in communications. However, the frequency and the amplitude of the oscillatory signals in STNO depend on the driving current; amplitude and frequency are nonlinearly connected. Experiments have shown the effect of nonlinear frequency modulation in single [22, 25] and double STNO [24] and the effect has been described theoretically in [18, 23].

Pulse amplitude modulation provides a case where the frequency and amplitude are uncoupled; turning the spin wave sources on and off causes a spin wavefront to propagate away from the contacts every time the STNO are pulsed. In [21] it was shown that pulse modulation of an STNO source has a dissipative nature because of the spin wave diffusion (see equation (1)). However, information can still be processed and transmitted using a simple amplitude modulation system [21].

Phase modulation may also be possible in STNO. Whenever a STNO phase locks to another STNO or to an external oscillatory signal, there is an interval of nominal frequencies where the STNO still synchronizes to the reference signal [9–12]. Within this frequency interval oscillators remain synchronized but their phase relative to that of the reference signal changes. In most of the spectroscopic techniques one measures the phase difference between an external signal—used to excite an intrinsic resonance—and the intrinsic resonance (e.g., ferromagnetic resonance). One may combine arrays of synchronized STNO and independently tune their relative phases with respect to that of an external signal. Phase modulation may provide an interesting option for encoding information as its detection is usually simple with the help of a reference signal.

10. Discussion and conclusion

We have studied spin wave interference patterns and directional spin wave radiation in ferromagnetic thin films with STNO having a perpendicularly magnetized free layer. The spatial dependence of the excitation energy in a single STNO might not be symmetric but lacks information about the wavelength of the spin wave excitation. The spatial energy of a planar wave created by an rf antenna also lacks information about its wavelength. The creation of interference patterns captures the information of wavelength from single sources. Arrays of STNO and STNO in combination with planar waves allow energy directionality and control of the spatial energy distribution. Different relative distances and positions of the STNO with respect to their wavelength lead to different radiation diagrams. Patterns can also be controlled by setting the relative phase between the STNO.

Communications and computation are open fields that would welcome using gigahertz excitations in solid state materials. However, a more precise control of these excitations would be needed to achieve the most basic operations. Magnonic crystals use control of the propagating media; here we explored control of radiating spin waves from STNO through interference patterns. Additionally, the cases studied, of arrays of STNO and STNO with planar waves, could serve to block certain wavelengths or to enhance others in different locations (i.e., to create bandgaps).

Another interesting outcome of this study on static interference patterns and radiation diagrams is the potential use in sensing short wavelengths of high frequency spin wave excitations. STNO in a ferromagnet such as Permalloy have excitation frequencies of tens of gigahertz and wavelengths of hundreds of nanometers; the temporal and spatial resolution are challenging and involve synchronization between the measuring equipment and the internal frequency of the STNO at the nanosecond scale and space resolution at the nanometer scale. Spin wave interference patterns from STNO arrays (or from STNO combined with planar waves) create steady patterns of the excitation amplitude (energy) that depend on the wavelength of the single sources [21, 33] and the relative phase; in such a case, a static detecting method could probe the wavelength of the excitations.

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