A Realistic Solvable Model for the Coulomb Dissociation of Neutron Halo Nuclei

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Received: March 30, 2022/ Revised version: March 30, 2022

Abstract. As a model of a neutron halo nucleus we consider a neutron bound to an inert core by a zero range force. We study the breakup of this simple nucleus in the Coulomb field of a target nucleus. In the post-form DWBA (or, in our simple model CWBA (“Coulomb Wave Born Approximation”)) an analytic solution for the $T$-matrix is known. We study limiting cases of this $T$-matrix. As it should be, we recover the Born approximation for weak Coulomb fields (i.e., for the relevant Coulomb parameters much smaller than 1). For strong Coulomb fields, high beam energies, and scattering to the forward region we find a result which is very similar to the Born result. It is only modified by a relative phase (close to 0) between the two terms and a prefactor (close to 1). A similar situation exists for bremsstrahlung emission. This formula can be related to the first order semiclassical treatment of the electromagnetic dissociation. Since our CWBA model contains the electromagnetic interaction between the core and the target nucleus to all orders, this means that higher order effects (including postacceleration effects) are small in the case of high beam energies and forward scattering. Our model also predicts a scaling behavior of the differential cross section, that is, different systems (with different binding energies, beam energies and scattering angles) show the same dependence on two variables $x$ and $y$.

PACS. 25.70.De Coulomb excitation – 25.60.-t Reactions induced by unstable nuclei – 25.60.Gc Breakup and momentum distributions

1 Introduction

Breakup processes in nucleus-nucleus collisions are complicated, in whatever way they are treated. They constitute at least a three-body problem, which is further complicated due to the long range Coulomb force. Exact treatments (like the Faddeev-approach) are therefore prohibitively cumbersome. On the other hand, many approximate schemes have been developed in the field of direct nuclear reactions, and these approaches have been used with considerable success \cite{1}. In this context we wish to investigate a realistic model for the Coulomb breakup of a neutron halo nucleus. With the operation of exotic beam facilities all over the world, these reactions (previously restricted essentially to deuteron induced reactions) have come into focus again. The Coulomb breakup of these nuclei is of interest also for nuclear astrophysics, since the breakup cross section can be related to the photodissociation cross section and to radiative capture reactions relevant for nuclear astrophysics \cite{2}.

An important benefit of the present model is that it can be solved analytically in the DWBA (CWBA) approximation. Thus it constitutes an ideal “theoretical laboratory” to investigate the physics of breakup reactions, certain limiting cases and its relation to other models like the semiclassical approximation, which is mainly used in the interpretation of experiments. Especially the effect of postacceleration (to be explained in more detail below) can be studied in a unique way in this approach.

2 Description of Theoretical Model

We consider the breakup of a particle $a = (c + n)$ (deuteron, neutron-halo nucleus) consisting of a loosely bound neutral particle $n$ and the core $c$ (with charge $Z_c$) in the Coulomb field of a target nucleus with charge $Z$.

$$a + Z \rightarrow c + n + Z.$$  \hspace{1cm} (1)

As a further simplification the $a = (c + n)$ system is assumed to be bound by a zero range force. The bound-state wave function of the system is given by

$$\phi_0 = \frac{\sqrt{\kappa} \exp(-\kappa r)}{2\pi r},$$  \hspace{1cm} (2)

where the quantity $\kappa$ is related to the binding energy $E_{\text{bind}}$ of the system by

$$E_{\text{bind}} = \frac{\hbar^2 \kappa^2}{2\mu}, \quad \mu = \frac{m_c m_n}{m_n + m_c}.$$  \hspace{1cm} (3)
In the post-form CWBA the T-matrix for the reaction Eq. (3) can be written as

$$T = \langle \chi_{q_a}^{(-)} \psi_{q_a} | V_{nc} | \chi_{q_c}^{(+)} \phi_0 \rangle$$

$$= D_0 \int d^3 R \chi_{q_a}^{(-)}(R)e^{-i\mathbf{q}_a \cdot \mathbf{R}} \chi_{q_c}^{(+)}(R),$$

with the "zero range constant" $D_0$ given by $D_0 = \frac{e^2}{2\hbar^2} \sqrt{8\pi v_a}$. The initial state is given by the incoming Coulomb wave function $\chi_{q_a}^{(+)}$ with momentum $q_a$ and the halo wave function $\phi_0$. The final state is given by the independent motion of the core described by the outgoing Coulomb wave function $\chi_{q_c}^{(-)}$ in the Coulomb field of the target nucleus $Z$ with asymptotic momentum $q_c$ and the free neutron with momentum $q_n$, described by a plane wave. In these wave functions the Coulomb interaction is taken into account correctly to all orders. In our model there is no resonance structure in the $c + n$ continuum. This is clearly observed in low energy deuteron breakup, in the corresponding experiments, see Fig. 2 and also, e.g., [7,8].

The distance from the target nucleus to the breakup point is denoted by $r$.

![Fig. 1. A schematic view of Coulomb-breakup, as adapted from [4].](Image)

![Fig. 2. Comparison of calculations and measurement for the deuteron breakup coincidence cross section on $^{197}$Au at $E_d = 12$ MeV (Fig. 4 of [8]).](Image)

On the other hand the 1st order semiclassical Coulomb excitation theory was widely applied in the past years to the Coulomb dissociation of high energy neutron halo nuclei, see, e.g., [13]. The theory corresponds to the "prior form", mentioned above. The question of higher order electromagnetic effects was studied recently in [12] within this framework. These effects were found to be small, for zero range as well as finite range wave functions of the $a = (c+n)$-system. It seems interesting to note that postacceleration effects arises through higher order electromagnetic effects in straight line semiclassical theories, see [13]. Through the interference of 1st and 2nd order amplitudes even a "post-deceleration" can arise, as was seen in that paper.

In this work we want to establish the relation between the apparently very different post-form CWBA and semiclassical theory. It was recently noticed [13] that in the limit of Coulomb parameters $\eta_a = ZcZe^2/\hbar v_a \ll 1$ (i.e. in the Born approximation), where $v_a$ denotes the velocity of particle $a$ ($v_a = \hbar k_a/m_a$), both theories give the same result. Expanding the Coulomb wave functions up to first order in the Coulomb fields one finds
We observe that (for $\eta = 0$) this parameter $\zeta(0)$ is found to be negative and $-\zeta(0) \gg 1$ for beam energy large compared to the binding energy and for perpendicular momentum transfers $q_\perp \gg 2\eta q_\parallel$ (nonadiabatic case), where $q_\perp = \omega/v$ with $\hbar \omega = E_{\text{bind}} + E_{\text{rel}}$ and where the relative energy between $c$ and $n$ is $E_{\text{rel}} = \frac{k^2 p^2}{2m}$ with the relative momentum given by $q = \frac{m_c q_n - m_n q_c}{m_c + m_n}$. It was already noticed in the numerical evaluation of the process that, due to $-\zeta(0) \gg 1$ that the hypergeometric series does not converge and an analytic continuation had to be used. Here we use this fact to our advantage and make a linear transformation to get the argument of the hypergeometric function close to 0. The transformation we are using leads to the argument of the hypergeometric function $z = \frac{1}{1 - \zeta(0)}$ (Eq. 15.3.7 of [18]). In this respect our approach differs from the one used in the bremsstrahlung case, where a transformation giving an argument close to one is used. Using only the lowest order term in the hypergeometric series one obtains after some algebra (up to an overall phase)

$$T \approx 4\pi D_0 f_{\text{coul}} e^{-\frac{2}{\zeta} \xi} \left[ e^{-i\phi} \frac{1}{q_n^2 - (q_a + q_c)^2} + e^{+i\phi} \frac{1}{m_a q_n^2 - (q_a - q_c)^2} \right]$$

Hereby, the relative phase is $\phi = \sigma_0(\eta_c) - \sigma_0(\eta_a) - \sigma_0(\xi) - \xi/2 \log |\zeta(0)|$. The $\sigma_0(\eta) = \arg \Gamma(1+i\eta)$ is the usual Coulomb phase shifts, and $\xi = \eta_c - \eta_a$. The correspondence to the Born result is clearly seen. One only has a finite prefactor $e^{-\frac{2}{\zeta} \xi}$ and a relative phase $e^{\pm i\phi}$ between the two terms. The phase $\phi$ obviously is $O(\xi)$. Since $\xi \sim v_c \eta_a$ the quantity $\xi$ is usually very small and so is $\phi$ for the cases of [10]. The prefactor is also well known in the semiclassical theory, where it accounts for the replacement of the “Coulomb bended” trajectories with the straight line trajectories. Both corrections vanish in the limit $\xi \to 0$ and the result coincides with the usual Born approximation (even if $\eta_a$ and $\eta_c$ are not small).

We have seen that the $T$-matrix in the case of large Coulomb parameters $\eta_a$ and $\eta_c$ corresponds to the Born result (small Coulomb parameter) in the sudden (or nonadiabatic) case $q_\perp \gg 2\eta q_\parallel$. We note that the derivation of Eq. [10] only depends on the condition $-\zeta(0) \gg 1$ (and not on the values of the $\eta$’s). For $\eta_a, \eta_c \gg 1$ one can define a classical path for both $a$ and $c$ in the initial and final state and Eq. [10] can be related to the semiclassical approach (see the discussion below following Eq. [10]). We expect to find a connection between the semiclassical theory and the adiabatic case ($q_\perp < 2\eta q_\parallel$) and the fully quantal expression for the $T$-matrix. For the adiabatic regime the well known exponential decrease with the adiabacity parameter is observed in the numerical calculations. In this case, the inequality $-\zeta(0) \gg 1$ is not generally satisfied. We are presently working to see how the semiclassical limit can be obtained with analytical methods in this case also. Such a method would also be valid in both the adiabatic and nonadiabatic case as long as $\eta_a, \eta_c \gg 1$.

A similar situation is encountered in the theory of bremsstrahlung and Coulomb excitation, see Section II E of [13]. There a fully quantal expression for the differential cross-section for dipole Coulomb excitation is given in II E.62. It looks similar to the corresponding expression...
for Coulomb breakup (see [3]). The semiclassical "variant" of this formula is found in II E.57. It is noted there that it can be obtained from the quantal expression by letting at the same time $\eta_\parallel$ and $\eta_\perp$ go to infinity and perform a confluence in the hypergeometric functions.

3 Scaling Properties

In many experimental situations the Coulomb push $q_{\text{coul}}$ is small. Having found that the full CWBA results agrees with the Born result for small scattering angles, we can expand Eq. (6) or Eq. (8) with $\phi = \xi = 0$ for small values of $q_{\text{coul}}$. We obtain

$$T = f_{\text{coul}} \frac{2D_0}{\pi^2} \frac{m_n^2 m_e^2}{m_0^3} \frac{2q \cdot q_{\text{coul}}}{(\kappa^2 + q^2)^2}. \tag{9}$$

This result is in remarkable agreement with the usual 1st order treatment of electromagnetic excitation in the semiclassical approximation.

In the semiclassical approach the scattering amplitude is given by the elastic scattering (Rutherford) amplitude times an excitation amplitude $a(b)$, where the impact parameter is related to the $q_\perp$ and $\eta$, see above. The absolute square of $a(b)$ gives the breakup probability $P(b)$, in lowest order (LO). It is given by

$$\frac{dP_{LO}}{dq} = \frac{16\eta^2}{3\pi \kappa} \frac{x^4}{(1 + x^2)^4}. \tag{10}$$

where the variable $x$ is related to the relative momentum between $n$ and $c$ by $x = \frac{2}{\kappa}$ and $y$ is a strength parameter given by

$$y = \frac{2Z_n e^2}{\hbar v_n a_{bc}}. \tag{11}$$

This formula shows very interesting scaling properties: Very many experiments, for neutron halo nuclei with different binding energy, beam energy, scattering angles (or $q_\parallel$ and $q_\perp$) all lie on the same universal curve! (Corrections for finite values of $\xi_{\text{eff}} = \omega b/v = \xi(\theta) = 2\eta_\parallel q_\parallel/q_\perp$ should also be applied, according to [14].) It will be interesting to see in future calculations under what conditions (beam energy, ...) one finds deviations from this simple scaling behavior. E.g., postacceleration effects will lead to such scaling violations.

4 Conclusion and Outlook

The present model can be seen as a "theoretical laboratory", which allows to study analytically, as well as, numerically the relation between quantal and semiclassical theories, and the importance of postacceleration effects. We mention that from an experimental point of view, the postacceleration effects are not fully clarified, see, e.g., [12, 20] ("postacceleration") and on the other hand [21] ("no postacceleration"). Finally, let us mention recent work on the electromagnetic dissociation of unstable neutron-rich oxygen isotopes [22, 23]. These authors deduce photoneutron cross-sections from their dissociation measurements. If the neutrons are emitted in a slow evaporation process in a later stage of the reaction, the question of postacceleration is not there. On the other hand, for the light nuclei there is some direct neutron emission component and the present kind of theoretical analysis further proves the validity of the semiclassical approach used in [22].

Postacceleration effects are also of importance for the use of Coulomb dissociation for the study of radiative capture reactions of astrophysical interest. We expect that our present investigations will shed light on questions of postacceleration and higher order effects in these cases also.

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