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On global dynamics of COVID-19 by using SQIR type model under non-linear saturated incidence rate

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Received 18 June 2020; revised 25 July 2020; accepted 27 August 2020
Available online 31 August 2020

Abstract This paper investigates a new mathematical SQIR model for COVID-19 by means of four dimensions; susceptible, quarantine, infected and recovered (SQIR) via Non-linear Saturated Incidence Rate. First of all the model is formulated in the form of differential equations. Disease-free, endemic equilibriums and Basic Reproduction Number are found for the said model. Local Stability is analyzed through Jacobean Matrix while Lyapunov Function is constructed for the study of Global Stability of the Model. Using nonstandard finite difference method, numerical results are simulated. By Simulation, we mean how protection, exposure, death and cure rates affect the Susceptible, Quarantined, Infected and recovered population with the passage of time.

1. Introduction

COVID-19 is a new chain of corona group of virus that had not been identified in humans history before December 2019. For the first time COVID-19 was found in Wuhan, China, in December 2019, and has spread to various urban areas in China as well as round about 213 different countries of the world. It has since been declared as an outbreak by World Health Organization (WHO). According to the data reported by “WHO (World Health Organization)”, by March 31, 2020, the reported laboratory confirmed, affected humans reached more than millions including more than 439,500 death cases recorded. Some researchers have also claimed that there are other sources of this corona virus including dogs, pangolin, etc. As per recoded data the death rate is different in different countries. Currently the highest death rate has been observed in Italy, Spain, Iran, UK and USA. The number of confirmed cases growing on a very fast track on daily basis and has been declared a worldwide pandemic disease.

On 31st of December 2019, the WHO reported a novel corona virus (2019-nCoV) in Wuhan City, Hubei Province of China in humans. It was named as severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) by International Committee on Taxonomy of Viruses on 11th of February, 2020. Firstly, this outbreak was identified in Wuhan with most early cases being reported in the city and later spread to the
other countries in an alarming rate and became a lethal disease. There are different schools of thought behind the origin of the COVID-19 - some says that it might be bat origin (see [9]), some says that it might be related to a seafood market exposure (see [10]). If we observe International travel of any form has been a potential reason for the fast spread of the COVID-19 [10,11]. So, immigration has a severe impact on the severity of spreading of the COVID-19.[25–27]. Recently, the whole world has been suffering due to a novel coronavirus pandemic and it was named by "Novel Coronavirus Infectious Disease (NCOVID-19)" which was claimed to outbreak first in Wuhan, central China (see [1]). It has been stated (fact) that the origin of NCOVID-19 is the transmission from animal to human as many infected cases claimed that they had been to a local fish and wild animal market in Wuhan in November (we refer [2–7,28]). Soon, some researchers confirmed that the transmission also happens person to person (see [13–16]). In present situation this pandemic has been produced very harmful effect on the health, economics and social life of the whole globe. In the whole world researchers, policy makers and doctors are struggling how to control this serious pandemic so that the lives of maximum people may be secured [17–22]. They observed this disease from their own point of view. Most people infected with this lethal virus can be identified with fever, respiratory problems, sore throat, cough and pains. To control over this pandemic we feel to study SQIR model [8,12].

2. Model formulation

In this part of our work we present the basic model for NCOVID-19 by dividing the total population into four classes. S(t) susceptible, Q(t) Quarantine, I(t) infected and R (t) represent Recovered class. The dynamic of model governs in the form a differential Eq. (1) given below

\[
\begin{align*}
\frac{dS(t)}{dt} &= \beta S(t)\frac{Q(t)}{1+bQ(t)} - (d_0 + x)S(t) \\
\frac{dQ(t)}{dt} &= \alpha S(t) - (\mu + d_0)Q(t) - a\beta I(t)Q(t) \\
\frac{dI(t)}{dt} &= \frac{\beta S(t)I(t)}{1+bQ(t)} + a\beta I(t)Q(t) - (d_0 + \gamma)I(t) \\
\frac{dR(t)}{dt} &= \gamma I(t) + \mu Q(t) - d_0 R(t).
\end{align*}
\]

(1)

The parameters involved in the system (1) are described as in Table 1. Flow chart for the model (1) is

| Table 1 Description of parameters of the model. |
|-----------------------------------------------|
| Parameters                  | The physical interpretation                      |
| S(t)                        | Susceptible compartment                          |
| Q(t)                        | Quarantine compartment                            |
| I(t)                        | Infected compartment                              |
| R(t)                        | Recovered compartment                             |
| a                           | Contact rate                                      |
| \(\mu\)                     | Recovery Rate from quarantine                     |
| \(d_0\)                     | Death due to corona                               |
| b                           | The saturation constant                           |
| \(\beta\)                   | Recruitment rate                                  |
| \(\beta_1\)                 | Disease contact rate                               |
| \(\gamma\)                  | Recovery rate from infected compartment           |
| \(\beta_2\)                 | Interaction of infected and quarantine            |
| \(x\)                       | Susceptible goes to quarantine                    |

The addition of all equations of system (1)

\[
\frac{dN(t)}{dt} = \beta - d_0 N(t).
\]

(2)

Where \(N(t) = S(t) + Q(t) + I(t) + R(t)\). From (2), we get

\[
\lim_{t \to \infty} \sup N(t) \leq N_0.
\]

With \(\lim_{t \to \infty} \sup N(t) = N_0\) if and only if \(\lim_{t \to \infty} \sup I(t) = 0\). From the 1st equation of the model (1), it show

\[
0 \leq \lim_{t \to \infty} \sup S(t) \leq S_0.
\]

And second equation follow

\[
0 \leq \lim_{t \to \infty} \sup Q(t) \leq Q_0.
\]

So from the above, we can see that if \(N > N_0\), then \(\frac{dN(t)}{dt} < 0\). We can write

\[
\Phi = (S(t), Q(t), I(t), R(t)) \in \mathbb{R}_+^4 : S(t) + Q(t) + I(t) + R(t) \leq N_0; S \leq S_0, Q \leq Q_0
\]

3. Equilibria

For the equilibrium of the model (1), we consider it’s existence. Corresponding to some values of parameters there exists a disease-free equilibrium for system (1) denoted by \(E_0 = (S^0, Q^0, 0, 0)\).

\[
E_0 = (S^0, Q^0, 0, 0) = \left( \frac{\beta}{\mu + d_0}, \frac{\alpha \beta}{(\mu + d_0)(x + d_0)}, 0, 0 \right)
\]

Endemic equilibria

\[
\begin{align*}
S' &= \frac{\beta S(I(t))}{1+bQ(t)} - (d_0 + x)S(t) \\
Q' &= \alpha S(t) - (\mu + d_0)Q(t) - a\beta I(t)Q(t) \\
I' &= \frac{\beta S(t)I(t)}{1+bQ(t)} + a\beta I(t)Q(t) - (d_0 + \gamma)I(t) \\
R' &= \gamma I(t) + \mu Q(t) - d_0 R(t)
\end{align*}
\]

4. Derivation of the basic reproductive number \(R_0\)

As \(R_0\) is one of the important factor, which shows us how the disease is spread and control in a population. In order to find the expression for model (1), let \(\chi = (I(t), Q(t))\), then it follow from model (1)

\[
\frac{d\chi}{dt} = \mathcal{F} - \gamma',
\]

where

\[
\mathcal{F} = \left( \frac{\beta S(I(t))}{1+bQ(t)} + a\beta I(t)Q(t) \right) 0
\]

and

\[
\gamma' = \left( \frac{(\gamma + d_0)I(t)}{(\mu + d_0)Q(t)} \right)
\]

for the disease-free equilibrium Jacobian of \(\mathcal{F}\) is
and Jacobi of $\mathcal{V}$ to deduce the disease-free equilibrium is given by

$$V = \begin{pmatrix} \gamma + d_0 & 0 \\ 0 & \mu + d_0 \end{pmatrix}.$$ 

Hence

$$V^{-1} = \frac{1}{(\gamma + d_0)(\mu + d_0)} \begin{pmatrix} \mu + d_0 & 0 \\ 0 & \gamma + d_0 \end{pmatrix}.$$ 

We have

$$FV^{-1} = \begin{pmatrix} \beta_1 S^0 + a \beta_2 Q^0 & 0 \\ 0 & \gamma \end{pmatrix}.$$ 

Hence the required $R_0$ is given by

$$R_0 = \frac{\beta_1 (\mu + d_0 + a \beta_2)}{\mu + d_0}.$$ 

On the basis of (3), we have the following result

**Theorem 4.1.** (i) There is no positive equilibrium of system if $R_0 \leq 1$. (ii) There is a unique positive equilibrium $E^* = (S^*(t), Q^*(t), R^*(t))$ of the model (1), called the endemic equilibrium, if $R_0 > 1$.

5. Local stability

To derive the results which reveal “disease free and endemic equilibrium” for our developed model, we take the given reduced form of our model (1) as

$$\frac{dS(t)}{dt} = \beta - \frac{\beta S(t) Q(t)}{(1 + b F(t))^2} - (d_0 + \gamma) S(t)$$

$$\frac{dQ(t)}{dt} = \gamma S(t) - (d_0 + \gamma) Q(t) - a \beta_2 Q(t) I(t)$$

$$\frac{dI(t)}{dt} = a \beta_2 Q(t) I(t) - (d_0 + \gamma) I(t)$$

Subject to initial condition $S(0) = S_0 \geq 0, Q(0) = Q_0 \geq 0, I(0) = I_0 \geq 0$.

Here we established the coming result.

**Theorem 5.1.** At $E^0$, the disease free equilibrium of the model (4) is locally asymptotically stable under the condition $R_0 < 1$.

**Proof.** The jacobian matrix at $E^0$ is given by

$$M^0 = \begin{pmatrix} -(d_0 + \mu) & 0 & \beta_1 S^0 \\ \gamma & -(d_0 + \mu) & a \beta_2 Q^0 \\ 0 & 0 & R_0 - 1 \end{pmatrix}.$$ 

The auxiliary equation of $M^0$ is given by

$$\lambda^3 + \lambda^2 c_1 + \lambda c_2 + c_3 = 0,$$

where

$$c_1 = (d_0 + \mu)(\gamma + d_0)(1 - R_0) > 0$$

$$c_2 = (d_0 + \mu)(\gamma + d_0)(1 + (\gamma + d_0)(1 - R - 0) > 0$$

$$c_3 = (d_0 + \mu)(\gamma + d_0)(1 - R_0) > 0.$$ 

We have

$$c_1 c_2 - c_3 = (d_0 + \mu)(\gamma + d_0)((\gamma + d_0)^2 + (d_0 + \mu)(\gamma + d_0))$$

$$\times [(\gamma + d_0) + 1](1 - R_0) > 0.$$ 

From (5) the Routh-Hurtwitz criteria is satisfied as $c_1 > 0, c_2 > 0, c_3 > 0$ and $c_1 c_2 - c_3 > 0$ if $R_0 < 1$. Hence the negativity of real parts of the concerned eigen values are ensured for (4). Hence the locally asymptotically stablling of the corresponding equilibrium point for (4) at $E^0$ has reached.

Further, locally asymptotically stability corresponding to $R^0 > 1$ about $E^*$ the model (4) is derived in next result.

**Theorem 5.2.** Under the condition $R^0 > 1$, at $E^*$ our model (4) is locally asymptotically stable, otherwise unstable.

**Proof.** At the endemic equilibrium $E^*$ the jacobian matrix of the system (4) is

$$M_1 = \begin{pmatrix} -\gamma - d_0 & -\frac{\beta S^0(t)}{1 + b F^0(t)} & 0 \\ \alpha & -\frac{\beta S^0(t)}{1 + b F^0(t)} & -\frac{\beta S^0(t)}{1 + b F^0(t)} \\ -\frac{\beta S^0(t)}{1 + b F^0(t)} & a \beta_2 \Gamma(t) - \frac{\beta S^0(t)}{1 + b F^0(t)} & a \beta_2 Q^0(t) \end{pmatrix}.$$ 

The given matrix $M_1$ is received after performing some operations

$$M_1 = \begin{pmatrix} -\gamma - d_0 & \beta \Gamma(t) & a \beta_2 Q^0(t) - (\gamma + d_0) \\ \alpha & -\beta \Gamma(t) & a \beta_2 Q^0(t) \\ -\beta \Gamma(t) & a \beta_2 \Gamma(t) - \beta \Gamma(t) & a \beta_2 Q^0(t) \end{pmatrix}.$$ 

We calculate trace and determinant of $M_1$

$$\text{tra} M_1 = -(\gamma + d_0) - \frac{a \beta_2 \Gamma(t)(1 + b F^0(t)) + S^0(t)}{1 + b F^0(t)^2} - a \beta_2 Q^0(t)(1 + \Gamma(t)) < 0$$

and

$$\text{det} M_1 = (d_0(\mu + d_0)(\gamma + d_0) + a d_0 \beta_2 \Gamma(t)(\gamma + d_0)b_2 \Gamma(t)$$

$$+ (ad_0 \beta_2 Q^0(t) + d_0(\gamma + d_0)(d_0 \Gamma(t)(a \beta_2 - \beta_1)))) > 0.$$ 

the determinant of $M_1 > 0$ if $a \beta_2 - \beta_1 > 0$. Therefore negative real part of the eigen values at $E^*$ may be obtained. Hence, the endemic equilibrium $E^*$ of the model (4) is locally asymptotically stable under the condition $R_0 > 1$.

6. Global stability

In this part of our work, we used Lyapunov function for the study global stability endemic and disease free equilibrium. The stability of disease free is present in the form of following theorem.

**Theorem 6.1.** The disease free equilibrium of the system (4) is globally asymptotically stable if $R_0 < 1$. 
Proof. We construct a Lyapunov function to prove this result.

\[ L = a_1(S(t) - S_0) + a_2(Q(t) - Q_0) + a_3R(t). \]  
\hspace{1cm} (6)

such that \( a_1, a_2, a_3 > 0 \) are constant may be computed later. Taking derivative of (6) with respect to time, one has

\[
\frac{dL}{dt} = a_1 \left( \beta - \frac{\beta_1 S(t) I(t)}{1+b I(t)} - (d_0 + z)S(t) \right) + a_2 \left( zS(t) - (d_0 + \gamma)I(t) \right)
\]  
\[+ a_3 \left( \frac{\beta_1 S(t) I(t)}{1+b I(t)} + a \beta Q(t) I(t) - (d_0 - \gamma)I(t) \right). \]

We get

\[
\frac{dL}{dt} = \frac{\beta_1 S(t) I(t)}{1+b I(t)} (a_3 - a_1) + a \beta Q(t) I(t) (a_3 - a_2)
\]  
\[+ a_1 (z + d_0) S(t) + a_2 z S(t) - a_3 (d_0 + d_0) Q(t). \]

Let assume \( a_1 = a_2 = a_3 = 1 \), we get finally

\[
\frac{dL}{dt} = -d_0 S(t) - d_0 I(t) - \mu Q(t) - d_0 Q(t) - \gamma I(t).
\]

Which implies

\[
\frac{dL}{dt} = -d_0 N(t) - \mu Q(t) - \gamma I(t) < 0.
\]

Hence “globally asymptotically stable” for the considered model with \( R_0 < 1 \) has reached. \( \square \)

Furthermore, We present here a new result for the global stability of the “endemic equilibrium of system” (4) at \( E^* \).

**Theorem 6.2.** The endemic equilibrium \( E^* \) of model (4) is stable globally asymptotically if \( R_0 > 1 \) and \( \mu > d_0 \).

**Proof.** We construct a Lyapunov function to prove this is following:

\[
L_2 = \left( \mu + d_0 \right) (S(t) - S^*) + \frac{(\mu + d_0)(d_0 + z)}{z} Q(t) + \left( \mu + d_0 \right) I(t).
\]  
\hspace{1cm} (7)

Differentiate w.r.t time (7) along with model (4), we get

\[
\frac{dL_2}{dt} = \left( \mu + d_0 \right) S(t) + \frac{(\mu + d_0)(d_0 + z)}{z} Q(t) + \left( \mu + d_0 \right) I(t).
\]

After some arrangement we get

\[
\frac{dL_2}{dt} = \beta (\mu + d_0) - \frac{(\mu + d_0)^2 (d_0 + z)}{z} \frac{Q(t)}{1 + b I(t)} - a \beta Q(t) I(t).
\]

Thus \( \frac{dL_2}{dt} < 0 \), the “endemic equilibrium” \( E^* \) of the system (4) is “stable globally asymptotically”, prove that \( R_0 > 1 \). \( \square \)

7. Numerical results and discussion

We present numerical simulation for system (1) with the used values. We take different initial values corresponding to different classes involved in the model (1) for the month of March in four different localities in Pakistan.

We use nonstandard finite difference (NSFD) scheme \([15,24]\) to write the model in difference form as: consider first equation of model (1)

\[
\frac{dS(t)}{dt} = \beta - \frac{\beta_1 S(t) I(t)}{1+b I(t)} - (d_0 + z)S(t).
\]  
\hspace{1cm} (8)

Which is decomposed in NSFD scheme as

\[
S_{j+1} - S_{j} = \beta - \frac{\beta_1 S_{j} I_{j}}{1+b I_{j}} - (d_0 + z)S_{j}.
\]  
\hspace{1cm} (9)

Like (9), we can decomposed the model (1) in NSFD scheme and write the whole system as

\[
S_{j+1} = S_{j} + \beta \frac{S_{j} I_{j}}{1+b I_{j}} - (d_0 + z)S_{j} I_{j}
\]  
\hspace{1cm} (10)

\[
Q_{j+1} = Q_{j} + h \left( \mu + d_0 \right) Q_{j} - a \beta Q_{j} I_{j}.
\]  
\hspace{1cm} (11)

Using the scheme developed in (10) and we plot the model corresponding to the given values in Table 2 as

We have testified our model by using the real data of four provinces of Pakistan \([23]\) including Punjab, Sindh, KPK, Baluchistan and taking the values of parameter of Table 2. We see that as the susceptibility decreasing with high rate in Punjab due to large population as a results quarantined and infected classes are growing as a results recovery rate also raising. The said dynamics has been presented in Figs. 1–4 respectively. Since huge population is also big cause of spreading of disease as there the people are living and working in crowded form in which keeping social distance is rarely considered by the public. In such situation in Pakistan in Punjab and in Sindh the infection is growing with exponential rate which is big threat for the Pakistan for coming few months. At this rate in coming month the infections may raise to infect millions of people in Pakistan and also there is great chance of more fatality cases. Also nearly 64% of the population of Pakistan is consisted on young people. So the recovery rate here recorded up to date is 90%.

8. Conclusion

The behavior of the results got for the projected model shows the pandemic of the lethal NCOVID-19 with convex incidence rate. The said research paper proclaims the fast spreading ratio of NCOVID-19 of the infected people who migrate to healthy area or the healthy people are allowed to enter the infected area. Minimizing the ratio of immigrations of infected people can greatly increase the healthy population and a great number of people can be saved from infection. On the other hand immigration of healthy population towards the infected area can also cause increase in infections. The speedy increasing ratio can be controlled effectively to ensure quarantining those people who are infected from this disease.

In order to monitor its spread, it is important to analyze its outbreak and behavior with different variables. The sole remedy of this disease is to quarantine the infected people. During their stay at quarantine, those who have strong immunities have great chances of being recovered and those who have weak immunities can be shifted for further treatment. In this
If we control the current outbreak in the upcoming few months. Recent studies have shown that the spread of NCOVID-19 is very speedy. It has similarities in comparison to Influenza than SARS. This comparison is evident from the 1918 influenza which had an infection-fatality-rate of 2 percent, which is equalling the level of the case-fatality-rate of NCOVID-19 in Wuhan, China. Though there were no considerable precautionary ideas at the initial stages of the outbreak of NCOVID-19, but later Quarantining the infected people was considered as an effective way to stop the spread. New Zealand effectively succeeded to control the pandemic. Therefore, it is concluded, that Quarantining

| Parameters | Physical description | Numerical value |
|------------|----------------------|-----------------|
| $S(t)$     | Susceptible compartment | 110,012, 47.886051, 35.53, 12.34 in millions |
| $Q(t)$     | Quarantine compartment | 0.050087, 0.049256, 0.016415, 0.007866 in million |
| $I(t)$     | Infected compartment   | 0.050087, 0.049256, 0.016415, 0.007866 in million |
| $R(t)$     | Recovered compartment  | 0.0017560, 0.0024387, 0.009150, 0.002021 in million |
| $a$        | Contact rate          | 0.0034          |
| $\mu$      | Recovery Rate from quarantine | 0.062 |
| $d_0$      | Death due to corona   | 0.019           |
| $b$        | The saturation constant | 0.000761 |
| $\beta$    | Recruitment rate      | 10.7            |
| $\beta_1$  | Disease contact rate  | 0.05            |
| $\gamma$   | Recovery rate from infected compartment | 0.0003 |
| $\beta_2$  | Interaction of infected and quarantine | 0.04 |
| $\alpha$   | Susceptible goes to quarantine | 0.0009 |

Fig. 1 Dynamical behavior in four different places of susceptible class of the considered model.

Fig. 2 Dynamical behavior in four different places of quarantined class of the considered model.
the infected people is the most effective way to controlled the current outbreak.

Declaration of Competing Interest

There exist no competing interest.

Acknowledgments

The second gratefully acknowledge the support of the University of Tabuk, Ministry of Education in Saudi Arabia.

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