Investigating Data-driven systems as digital twins: Numerical behavior of Ho-Kalman method for order estimation.

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Abstract

System identification has been a major advancement in the evolution of engineering. As it is by default the first step towards a significant set of adaptive control techniques, it is imperative for engineers to apply it in order to practice control. Given that system identification could be useful in creating a digital twin, this work focuses on the initial stage of the procedure by discussing simplistic system order identification. Through specific numerical examples, this study constitutes an investigation on the most “natural” method for estimating the order from responses in a convenient and seamless way in time-domain. The method itself, originally proposed by Ho and Kalman and utilizing linear algebra, is an intuitive tool retrieving information out of the data themselves. Finally, with the help of the limitations of the methods, the potential future outlook is discussed, under the prism of forming a digital twin.

1 Introduction

Adaptive control has been quite popular over the last fifty years [19] [9], with a variety of methodologies available [24]. As a matter of fact, as early as 1955, the adaptive techniques have been reported to be widely utilized in industry and this can be come across in literature [2]. The comparative advantage, being the lack of the model, has helped in creating huge related literature. Even nowadays, with Industry 4.0-like movements across the Globe being the main streams of digitalization trends in industry [23] [16], the cognitive functionalities of automation (exploiting CPS & IoT) have been integrated to a great extent and the use of adaptive control techniques has been spread even more. Also, there have been reported works [3], where the well-established technology of adaptive system identification has been presented as an underlying technology for a digital twin.

Applications of adaptive control can be found literally everywhere. From domestic applications [11], to engineering [17] and manufacturing [5], it is highly evident that adaptive control is very useful. Indicatively, recently, identification techniques had been used to model a system response originating from Partial Differential Equations [18] and attempting to control it in an empirical, yet adaptive way. In the case of a digital twin formation, automatic operation is highly important, so the identification phase is of utmost importance.

A brief, yet full, review on the State-of-The-Art on methods of identification techniques - and more specifically on the issue of choosing the order of the model -
reveals initially the use of empirical methods such as trial and error, estimation utilizing the frequency domain, and MAP method. Works have been done on the choice of the most suitable methodology. Furthermore, the covariances matrix and the residual whiteness are two more methods that are often discussed. Moreover, the set of BIC / AIC / GIC methodologies is another set of methods highly utilized; there has also been a practical comparison between RSS and BIC. It is worth mentioning at this point that the latter method(s) implies the integration of the concept of information.

What seems to be missing, however, is a numerical illustration on the simplest, intuitive way to extract such information (meaning the order of the system) from data (the responses themselves). To this end, this work attempts to investigate numerically a simple method for the estimation of the system order, in time domain, utilizing the linear dependence between the sampled data. The concept of the rank of the matrix is utilized as the tool to perform this, as originally suggested by Ho & Kalman.

2 Method

As briefly aforementioned, the method investigated here is based on the fact that linear systems response values at time \( n \) (in the case of discrete systems) are linear combinations of previous values at time \( n - k \), for some \( n, k \in \mathbb{N} \). Therefore, the concept of linear independence is exploited, through the concept of ranks of matrices. To achieve this a matrix is formed, given a response \( y[n] \), containing translated versions of the response, as shown in Eq. (1):

\[
\tilde{Y}_{N \times N} = \begin{bmatrix}
y[0] & y[1] & \ldots & y[N - 1] \\
y[1] & y[2] & \ldots & y[N] \\
\vdots & \vdots & \ddots & \vdots \\
y[N - 1] & y[N] & \ldots & y[2N - 2]
\end{bmatrix}
\]

The order of the system \( S \) that had \( y[n] \) as a response, is expected to be equal to the rank of this matrix, namely \( \rho_{N \times N} = \rho(\tilde{Y}_{N \times N}) \). Even in the marginal case where the \( N \) is taken to be equal to \( M + 1 \), where \( M \) is the order of the system, it is evident that the rank of the matrix is equal to the order of the system:

\[
\tilde{\Phi}_{N \times N} = \begin{bmatrix}
y[0] & \ldots & y[N - 2] & \sum_{n=0}^{N-2} a_n y[n] \\
y[1] & \ldots & y[N - 1] & \sum_{n=1}^{N-1} a_n y[n] \\
\vdots & \vdots & \ddots & \vdots \\
y[N - 1] & \ldots & y[2N - 3] & \sum_{n=N-1}^{2N-3} a_n y[n]
\end{bmatrix}
\]

\[
\rho(\tilde{\Phi}_{N \times N}) = N - 1 = M
\]

In the next sections, the numerical performance of this algorithm is investigated with respect the complexity of the system; the order itself, the system structure and potential noise interfering.

2.1 Comparison to other methods & Correlation to information

For reasons of completeness, this simple method should be compared against other ones. So, to this end, the following response is utilized. The investigated method gives out explicitly (and correctly) an order of 5, as shown in Fig. 1. However, AIC-based order
estimation gives out 8, while Covariant Matrix Method leads to incocclusive results, as showed in Table I (a potential adoption of order 3 could take place).

\[ y_5[n] = \frac{1}{7} \sum_{k=1}^{7} (-1)^{k+1} \sin \left(\frac{2\pi k}{3}\right)e^{-\frac{k}{10}} \]  

(3)

This small numerical example has pointed out the numerical superiority of this algorithm - in a case where the method is applicable in its current form. Interestingly enough, the whole point of modelling with a differences equation is of course to be able to reproduce a sequence by a finite (smaller) number of numbers. This slightly reminds of Chaitin’s work on linking compression with theory (the concept of statistical inference is also relevant). The only difference herein is that instead of utilizing bits, one tries to compress numbers into numbers, regardless of digits. The rank of the responses matrix (even in its full infinite version) is an index of such a complexity (information). So, transformation metrics related to invertibility can also be used, such as the determinant or the eigenvalues distribution. Alternatively, the Lagrangian (or instead a custom Liapunov function) of the system can be used as a different metric. Such a function is of degree higher than linear, thus there is link to correlation matrix method as well.

3 Numerical Behaviour & Applicability

So, in the context of finding the numerical limitations of this simple method, various systems have been studied in terms of system order identification. In this section particularly, the applicability and the limitations of the method are shown and discussed through specific paradigms.

Figure 1. AIC values as a function of the system order (for system of Eq. 8).

Table 1. Covariance Matrix Results

| Order of System | Determinant of Covariance Matrix |
|-----------------|----------------------------------|
| 2               | 0.00106278                       |
| 3               | 1.96022 × 10^{-12}              |
| 4               | −5.85109 × 10^{-26}            |
| 5               | −4.11625 × 10^{-40}            |
| 6               | 6.85315 × 10^{-55}             |
| 7               | 1.71849 × 10^{-68}             |
| 8               | 1.78185 × 10^{-82}             |
3.1 Simple Numerical Examples

To begin with, a first-order system - that would give a response of the form \( y_1[n] = B e^{-Q n} \) - is utilized. Thus, a responses matrix of dimensions \( 5 \times 5 \) would be given by \( \tilde{Y}_{1}^{5 \times 5} \).

\[
\tilde{Y}_{1}^{5 \times 5} = B \begin{bmatrix}
1 & e^{-Q} & e^{-2Q} & e^{-3Q} & e^{-4Q} \\
 e^{-Q} & e^{-2Q} & e^{-3Q} & e^{-4Q} & e^{-5Q} \\
 e^{-2Q} & e^{-3Q} & e^{-4Q} & e^{-5Q} & e^{-6Q} \\
 e^{-3Q} & e^{-4Q} & e^{-5Q} & e^{-6Q} & e^{-7Q} \\
 e^{-4Q} & e^{-5Q} & e^{-6Q} & e^{-7Q} & e^{-8Q}
\end{bmatrix}
\] (4)

Even using symbolic matrices, without specific values, the rank of the matrix, for various dimensions, is equal to 1, as also computationally shown in Fig. 2, for a specific value of \( Q \). This is easily proved, as each row (or column) is the product of the previous one with \( e^{-Q} \).

So far, everything seems to work well. However, in reality, the sampled values of the responses contain noise, either from measurement, or from sampling itself. Therefore, in this section, noise is going to be regarded, as this is the case in all measured responses. To simulate this, a uniform random number is added after sampling the response, which is regarded in continuous time. Supposing that the continuous-time response is a simplified version of the above one, sampling is applied. In the case where the amplitude of the noise is relatively small, then, as shown in Fig. 3, the convergence is rather rapid. However, if the noise amplitude is increased by one order of magnitude (same Fig.), then the convergence becomes much slower. Oddly enough, the unitary signal \( \hat{f}(t) = 1 \) has been added to the response on purpose. It has been observed that if the mean value of the response is increased by an offset, then the method converges much faster. Also, the adoption of a row-echelon form of the responses matrix also seems to accelerate the convergence of the method. Furthermore, to see the effect of the poles proximity, since a second order system is also concerned, the following response is regarded, given that \( \delta p = 2^{-q} \).

\[
y_2[n] = 0.5 e^{-n/p} + (0.5 + \delta p)e^{-n/(p+\delta p)}
\] (5)

The elaboration of such a system has as a goal to study the numerical limitations of the method, as the system tends to be a double-pole system in the limit of \( q \) approaching infinity. Simultaneously, the effect of the dominance of one pole is studied. The results are shown in Fig. 3. Evidently, the rank remains equal to 2, for values of \( q < q_0 \) and \( N \in \{2, \ldots, 8\} \), depending also on SNR value.
3.2 Performance on Systems of higher order

To move on to higher order systems, the (arbitrarily chosen) following response consisting of \( N_0 \) terms is considered:

\[
y_N[n] = \frac{1}{N_0} \sum_{k=1}^{MN_0} f_0(-n/s_k)
\]  

(6)

This, has a result, the diagram of \( \rho_{N \times N} \) as a function of \( N \) is given in Fig. 4 and 5, for \( f_0(t) = \sin t \) and \( f_0(t) = e^{-t} \), respectively. It is quite interesting that in the case of sinusoidal functions, some sort of numerical effect takes place, driving the rank estimation evolution (as a function of responses matrix size) to converge to the value of \( N \) at a “faster” rate. This should be investigated to a further extent, through the consideration of a case of a system of even larger order. This is not the case when dealing with exponential functions, probably due to bad condition number of the responses matrix. Also, since often there can be responses of high order [21], i.e. close to 100 [20], similar case have also been included here.

3.3 Non-homogeneous Systems

Moving on to a different kind of complexity, one can form a matrix for a solution for a non-homogeneous system, such as in the case of \( y'(t) + 0.9y(t) = e^{-t/8} \). The rank of a 10 × 10 responses matrix would be equal to \( P + Z = 2 \), where the \( Z \) is the number of
Figure 5. The rank of the responses matrices as a function of the responses matrix dimensions, for the case of $f_0[n] = e^{-n}$.

Zeros and $P$ the number of poles (1 and 1 respectively). If one augments this matrix to be $11 \times 10$ or $10 \times 11$, padding with (translated) excitation function values to the bottom or to the right, as shown in the equation below, then the rank remains equal to 2. This indicates that the order of the differential equation is equal to $P$ (equal to 1 in this case), as the input and the output have terms linearly dependent.

$$
\hat{Y}_{\text{avg}}^{N \times N} = \begin{bmatrix}
y[0] & y[1] & \ldots & y[N] \\
\ldots & \ldots & \ldots & \ldots \\
y[N-1] & y[N] & \ldots & y[2N-1] \\
u[0] & u[1] & \ldots & u[N]
\end{bmatrix}
$$

(7)

4 Sum-up & Future Outlook

The simplest and most intuitive method for estimating the order of the model by the responses themselves in time-domain, utilizing the rank of the responses matrix, has been reviewed as a candidate method for forming automatically a digital twin. It has been proved to be quite successful in many cases, with the efficiency of the algorithm depending highly on the order of the problem as well as the available response dataset size. However, its extension towards practical rank estimation algorithms or iterative decomposition of responses is required so that it is able to handle highly noisy data. This iterative method, may be loosely correlated to the Gram-Schmidt procedure, however, one must have in mind the issue of orthogonality of signals. Furthermore, the extensibility of the Ho-Kalman method in non-linear systems should be further investigated.

It seems that as far as the formation of a digital twin concerned, the use of such a method is promising, however, in presence of noise, the method has to be combined with the use of another methodology.

References

1. Matlab model structure. [https://www.mathworks.com/help/ident/ug/model-structure-selection-determining-model-order-and-input-delay.html](https://www.mathworks.com/help/ident/ug/model-structure-selection-determining-model-order-and-input-delay.html).

2. J. Aseltine, A. Mancini, and C. Sarture. A survey of adaptive control systems. *IRE Transactions on Automatic Control*, 6(1):102–108, 1958.
3. H. Brandtstädter, C. Ludwig, L. Hübner, E. Tsouchnika, A. Jungiewicz, and U. Wever. Digital twins for large electric drive trains. In *2018 Petroleum and Chemical Industry Conference Europe (PCIC Europe)*, pages 1–5. IEEE, 2018.

4. R. Brincker, L. Zhang, and P. Andersen. Modal identification from ambient responses using frequency domain decomposition. In *Proc. Proc. of the 18*International Modal Analysis Conference (IMAC), San Antonio, Texas, 2000.

5. K. P. Burnham and D. R. Anderson. Multimodel inference: understanding aic and bic in model selection. *Sociological methods & research*, 33(2):261–304, 2004.

6. G. J. Chaitin. Meta math! the quest for omega. *arXiv preprint math/0404335*, 2004.

7. B. Choi. *ARMA model identification*. Springer Science & Business Media, 2012.

8. G. Chryssolouris. *Manufacturing systems: theory and practice*. Springer Science & Business Media, 2013.

9. G. A. Dumont and M. Huzmezan. Concepts, methods and techniques in adaptive control. In *American Control Conference, 2002. Proceedings of the 2002*, volume 2, pages 1137–1150. IEEE, 2002.

10. F. J. Fabozzi, S. M. Focardi, S. T. Rachev, and B. G. Arshanapalli. *The basics of financial econometrics: Tools, concepts, and asset management applications*. John Wiley & Sons, 2014.

11. D.-M. Han and J.-H. Lim. Design and implementation of smart home energy management systems based on zigbee. *IEEE Transactions on Consumer Electronics*, 56(3), 2010.

12. B. Ho and R. E. Kalman. Effective construction of linear state-variable models from input/output functions. *at-Automatisierungstechnik*, 14(1-12):545–548, 1966.

13. C. Kapsalas, J. Sakellariou, P. Koustoumpardis, and N. Aspragathos. An arx-based method for the vibration control of flexible beams manipulated by industrial robots. *Robotics and Computer-Integrated Manufacturing*, 52:76–91, 2018.

14. S. Karimian-Azari, J. R. Jensen, and M. G. Christensen. Fundamental frequency and model order estimation using spatial filtering. In *Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on*, pages 5964–5968. IEEE, 2014.

15. A. D. McQuarrie and C.-L. Tsai. *Regression and time series model selection*. World Scientific, 1998.

16. D. Mourtzis, E. Vlachou, and N. Milas. Industrial big data as a result of iot adoption in manufacturing. *Procedia CIRP*, 55:290–295, 2016.

17. S. A. Nivison and P. Khargonekar. A sparse neural network approach to model reference adaptive control with hypersonic flight applications. In *2018 AIAA Guidance, Navigation, and Control Conference*, page 0842, 2018.

18. A. Papacharalampopoulos, J. Stavridis, P. Stavropoulos, and G. Chryssolouris. Cloud-based control of thermal based manufacturing processes. *Procedia CIRP*, 55:254–259, 2016.
19. D. Seborg, T. Edgar, and S. Shah. Adaptive control strategies for process control: a survey. *AIChE Journal*, 32(6):881–913, 1986.

20. N. Spanos, J. Sakellariou, and S. Fassois. Vibration–response–only statistical time series shm methods: a critical assessment via a lab–scale wind turbine jacket foundation structure and two sensor types. In *Proceedings of ISMA 2016–international conference on noise and vibration engineering*, pages 4081–4095, 2016.

21. P. Stavropoulos, A. Papacharalampopoulos, E. Vasiliadis, and G. Chryssolouris. Tool wear predictability estimation in milling based on multi-sensorial data. *The International Journal of Advanced Manufacturing Technology*, 82(1-4):509–521, 2016.

22. P. Stoica and Y. Selen. Model-order selection: a review of information criterion rules. *IEEE Signal Processing Magazine*, 21(4):36–47, 2004.

23. D. Trotta and P. Garengo. Industry 4.0 key research topics: A bibliometric review. In *Industrial Technology and Management (ICITM), 2018 7th International Conference on*, pages 113–117. IEEE, 2018.

24. O. Yechiel and H. Guterman. A survey of adaptive control. *Int Rob Auto J*, 3(2):00053, 2017.