The interplay between strong electron correlation and band topology is at the forefront of condensed matter research\textsuperscript{1–3}. As a direct consequence of correlation, magnetism enriches topological phases and also has promising functional applications\textsuperscript{4,5}. However, the influence of topology on magnetism remains unclear, and the main research effort has been limited to ground state magnetic orders. Here we report a novel order above the magnetic transition temperature in magnetic Weyl semimetal (WSM) CeAlGe. Such order shows a number of anomalies in electrical and thermal transport, and neutron scattering measurements. We attribute this order to the coupling of Weyl fermions and magnetic fluctuations originating from a three-dimensional Seiberg-Witten monopole, which qualitatively agrees well with the observations. Our work reveals a prominent role topology may play in tailoring electron correlation beyond ground state ordering, and offers a new avenue to investigate emergent electronic properties in magnetic topological materials.
The effect of strong electron correlation greatly enriches the phase diagram of topological quantum matter\textsuperscript{1–7}. In bulk-gapped phases such as topological insulators (TIs) and topological superconductors (TSCs), interactions lead to the broader symmetry-protected topological (SPT) phases\textsuperscript{8,9}, which preserve symmetry but have distinct topological properties. As a prominent example, in non-interacting 3D TIs, the 2D Dirac surface states can only be gapped by breaking time-reversal symmetry (TRS), yet with the presence of strong interactions, the surface Dirac states can be gapped out while preserving TRS\textsuperscript{10}. In gapless metallic phases, correlation effects may be enhanced\textsuperscript{11}, and can lead to novel phenomena including topological Kondo insulators\textsuperscript{12,13}, magnetic WSMs\textsuperscript{14–17} and Weyl-Kondo phases\textsuperscript{18}. Of particular practical interest are magnetic WSMs, due to their variety of applications in thermoelectric energy harvest\textsuperscript{19,20}, optoelectronics\textsuperscript{21,22}, non-reciprocal optics\textsuperscript{23}, and spintronic devices\textsuperscript{24}.

In this work, we study the interplay between band topology and magnetism in magnetic WSM CeAlGe. CeAlGe is a member of the non-centrosymmetric WSM RAiGe (R = La, Ce, Pr) family\textsuperscript{25–28} which shares the same body-centered tetragonal structure (space group $I4_1md$) with the prototypical type-I WSM TaAs family\textsuperscript{29}. CeAlGe is an ideal platform to study such topology-magnetism interplay due to the dual TRS and inversion symmetry breaking\textsuperscript{26}, singular angular magnetoresistance\textsuperscript{30}, and the presence of an incommensurate topological magnetic meron-antimeron structure\textsuperscript{31}. By performing combined electrical and thermal transport and inelastic neutron scattering measurements above and below the magnetic transition of $T_N = 4.5$K, we find a new order that emerges in a narrow temperature region near $T = 12.8$K. In this region, a number of unusual phenomena emerge, including zero magneto-resistance (MR) up to a precision of 0.2\%, maximally suppressed magneto-thermal conductivity, maximal dilation, and distinct magnetic excitations from inelastic neutron spin-wave measurements. These observations can be explained by the coupling between topological Weyl nodes and helical magnetism, which gives rise to a Seiberg-Witten monopole phase. First, the narrow temperature region originates from the stringent solution space of the Seiberg-Witten monopole. Second, the zero MR is a consequence of the renormalized quasiparticle dispersion. Finally, the calculated spin correlation agrees qualitatively with neutron spin-wave analysis that shows ferromagnetic-like coupling in the magnetic fluctuation phase, causing maximized dilation and strong field-dependence of magneto thermal conductivity. Our findings point towards the emergence of unusual topological phases from the confluence of relativistic fermions and magnetism in correlated electronic systems.

We first establish CeAlGe’s electronic and magnetic structure. CeAlGe becomes ferrimagnetic below $T_N = 4.5$K at which point the two inequivalent Ce sites with different magnitude magnetic moments order oppositely along the $a$-direction (Fig. 1a)\textsuperscript{30}. Density functional theory (DFT) bandstructure
calculations along $\Sigma_0$-N-$\Sigma$, shown in Figs. 1b-1d, demonstrate the modulation of Weyl-fermion band-structures at various magnetic orderings, confirming the coupling of band topology to magnetism (Fig. 1f). The locations of Weyl nodes within the Brillouin zone are displayed in Fig. 1e. Electrical transport confirms a magnetic transition temperature $T_N = 4.5$K (Fig. 1g), consistent with the heat capacity peak positions (Fig. 1h). The magnetic phase transition peak is shifted towards higher temperatures and broadened as the external field increases. To isolate the magnetic contribution of the heat capacity, we subtract the phonon contribution using the non-magnetic LaAlGe as reference (Fig. 1h). The resulting magnetic entropy (Fig. 1h inset) saturates near $0.75k_B \ln 2$ as high as $T = 20$K at $B = 9$T, which originates from the isolated spin doublets in the Ce ions.

Next we turn to the electrical and thermal transport phase diagrams. Figs. 2a-2b show the longitudinal electrical resistivity $\rho_{xx}$ and thermal conductivity $\kappa_{xx}$ phase diagrams, where the phase boundaries (green dotted lines) are obtained by setting $\partial^2 \rho_{xx}/\partial B^2 = 0$ and $\partial^2 \kappa_{xx}/\partial B^2 = 0$, respectively. Identifying inflection points in transport properties is powerful for phase boundary analysis in correlated systems\textsuperscript{32}. Thermal transport is further assigned to five regions (regions I-V in Fig. 2b), as thermal transport can reliably depict the quasiparticle transport even in an electrically insulating phase when compared to electrical transport. At temperatures below $T_N = 4.5$K, by comparing with prior neutron diffraction reports\textsuperscript{30}, the three low-temperature orderings (regions I-III in Fig. 2b) can be assigned as ferrimagnetic (region I), canted magnetic (region II), and fully polarized ordering (region III), summarized in Fig. 2c. The transport data at $T = 1.8$K is further plotted in Fig. 2d. In the ferrimagnetic phase (region I), electrical conductivity increases due to the suppressed electron spin-flip scattering. Near $B = 3.3$T, the Ce sites with smaller magnetic moment become fully aligned, entering into the canted magnetic phase (region II in Fig. 2d). Above $B = 6.2$T, both Ce sites are field-polarized, resulting in the fully-polarized ordering (region III in Fig. 2d). In this region, the electrical conductivity decreases due to enhanced electron cyclotron orbital motion. On the other hand, thermal conductivity increases at regions II and III due to the larger magnon contribution, whereas the drop below $B = 3$T indicates a violation of the Wiedemann-Franz law due to collective behaviors. Above the magnetic transition, there are two regions, IV and V, which can be understood as the field-polarized phase and the spin fluctuation phase, respectively, resulting from the competition of thermal fluctuation and the field alignment of magnetism. The phase dominated by thermal fluctuations is centered around $T = 10 - 15$K at low field, in the same region where we identify a phase boundary in the electrical transport as shown in Fig. 2a.

To further investigate the region of the thermal transport phase diagram where thermal fluctuations...
dominate, we focus on the transport near \( T = 12.8 \text{K} \). In this temperature region, we observe several anomalous features in the electronic and thermal transport. At 12.8K, the magnetoresistance switches from being positive to negative for all measured values of applied magnetic field (Figs. 3a, d). For temperatures in the vicinity of \( T = 12.8 \text{K} \), as displayed in the inset of Fig. 3a, the magnetoresistance is almost exactly zero up to \( B = 9 \text{T} \) with a precision lower than 0.2%. Remarkably, at the same temperature that the magnetoresistance is zero, the thermal conductivity is most suppressed by external magnetic field (Fig. 3b). Such disparity between the electrical and thermal conductivities suggests that magnetic excitations are the dominant contribution to the thermal conductivity. Applying a magnetic field opens a gap in the low energy magnon branch, decreasing the magnetic contribution to the thermal conductivity\(^{33}\). Furthermore, we observe that the low temperature dilation of CeAlGe reaches a local maximum at \( T = 12.8 \text{K} \) (Fig. 3c). As the dilation is related to magnetic order-lattice coupling, the distinct dilation at \( T = 12.8 \text{K} \) further suggests the enhanced role of magnetism. The combined behavior of thermal transport, electrical transport and dilation near \( T = 12.8 \text{K} \) point to the emergence of a novel order in the proximity of this temperature region.

The novel order in this temperature region is further corroborated through spin excitations revealed with inelastic neutron scattering. Our results indicate that ferromagnetic-like correlations persist above the magnetic transition temperature. As shown in Fig. 4, we identify dispersive spin wave features at temperatures above and below the magnetic transition. To simulate the spin waves for a given temperature and magnetic field, we input the magnetic unit cell (antiferromagnetic or ferromagnetic) and define parameters \( J_1 \) and \( J_2 \) as the interlayer and the intralayer magnetic exchange interactions, respectively. We extract these parameters by fitting the simulated spin-wave dispersions to the data and minimizing the squared error. For the regions with the anticipated magnetic structure (ferrimagnetic at 1.8K and 0T; field-aligned at 1.8K, 10K and 8T), we find \( J_1 = (-0.27 \pm 0.08)\text{meV} \) and \( J_2 = (0.60 \pm 0.05)\text{meV} \). Unexpectedly, at 10K and 0T the spin-wave dispersion suggests greater spin alignment with \( J_1 = (-0.25 \pm 0.05)\text{meV} \) and \( J_2 = (0.80 \pm 0.05)\text{meV} \). We note that the intralayer ferromagnetic exchange \( J_2 \) is stiffer at this point than in other regions of the phase diagram. Although the net magnetization at this temperature is zero, the ferromagnetic-like order is likely due to local domains of coherent fluctuating moments, or paramagnons. The ferromagnetic-like order is further consistent with the thermal conductivity data in Fig. 3b, since greater spin alignment generally leads to a larger magneto-thermal conductivity difference. The distinct magnetism in this region is also consistent with the dilatometry (Fig. 3c), which reaches a local maximum because of the emergent magnetic order and enhanced magnetism-lattice coupling. The presence and stiffening of spin waves detected with inelastic neutron scattering further characterize this region of phase space with anomalous transport.
To support the transport and neutron scattering, we develop an effective field theory for the coupling between the magnetization $M$ and Weyl-fermion field $\psi$. The effective Hamiltonian $H_{\text{eff}}$ can be written as (further details in Supplementary Information I),

$$H_{\text{eff}} = \int d^3r \left( -i \left( \psi^\dagger \sigma \cdot \nabla \psi + \frac{1}{2} K \psi^\dagger M \cdot \sigma \psi \right) + \frac{D}{4} M \cdot \nabla \times M + \frac{J}{2} (\nabla M)^2 \right),$$  

(1)

where $K$, $D$ and $J$ denote the Kondo coupling, Dzyaloshinskii-Moriya interaction, and exchange coupling constants, respectively\textsuperscript{34,35}. When coupled to helical magnetism, $M = \frac{Q}{K} (\sin(Qz), \cos(Qz), 0)$, a prototypical magnetization mimicking the real incommensurate magnetic ordering in CeAlGe\textsuperscript{30,31}, the ground state solution contains zero energy modes that are dual to the electrical charge in the $SU(2)$ supersymmetric Yang-Mills theory and have a non-trivial Seiberg-Witten topological invariant\textsuperscript{36}. In particular, such a Seiberg-Witten monopole phase only exists under the constraint $Q = K = \frac{1}{2J} (1 + D)$.

When the fluctuations above the transition temperature become strong enough with sufficiently large $Q$, the constraint can be met and the helical magnetization manifests itself within locally correlated magnetic domains. This explains why the observed anomalous behavior at $T \sim 12.8K$ does not appear within the vicinity of the transition temperature $T_N$ but only appears in a narrow, elevated temperature range. The renormalized quasiparticle dispersion of the low-energy bands is shown to have a saddle shape, leading to a momentum-dependent transverse effective mass that gives rise to zero MR. However, energy can still be transported through the collective magnetic degrees of freedom, consistent with the thermal conductivity’s strong response in this phase. In addition, the corresponding spin-spin correlation function can be written as $\langle s_x(z)s_x(z') \rangle \propto \sin(Qz)\sin(Qz')$, which is weaker than a long-range ferromagnet ordering but stronger than a paramagnet. This is consistent with the neutron scattering fitting where $J_2$ is enhanced from 0.6meV to 0.8meV when entering into the possible Seiberg-Witten monopole fluctuation regime. This model exposes the responses of local moments and topological Weyl-fermions when they are free to interact with each other.

Our observations in thermal and electrical transport measurements, and inelastic neutron scattering, supported with quantum field-theoretical calculations, provide strong evidence for coupling between Weyl fermions and fluctuating magnetic moments in CeAlGe. These findings mark a step towards understanding how electron topology impacts microscopic interactions such as magnetic exchange, and conversely, how magnetic interactions affect topological states. Given the crucial role that magnetic fluctuations play in correlated quantum materials, such as unconventional superconductors, we anticipate broader novel responses to arise from the interactions between magnetic fluctuations and electronic band topology. In particular, our work sheds light on the pathway of generating high-
temperature topological states above the magnetic transition temperature.

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Methods

**Synthesis of CeAlGe single crystals and powder.** We successfully synthesized high-quality single crystals of CeAlGe through the Al self-flux method. A mixture of Ce powder (Strem Chemicals, 99.9%), Al beads (Sigma-Aldrich, 99.9%) and Ge powder (Beantown Chemical, 99.999%) were weighed in a molar ratio of 1:10:1 in a glovebox and placed into a crucible. The mixture-filled crucible was flame-sealed in an evacuated quartz tube and was subsequently heated up to 1100°C from room temperature at a rate of 80°C/h. Afterwards, the mixture dwelled for 20 hours and subsequently cooled to 700°C at a rate of 3°C/h. This was followed by several days of annealing at this temperature after which centrifugation was performed to remove the excess flux. The resulting products of CeAlGe single crystals approximately half-centimeter large and have a metallic luster with lattice constants $a = 4.29\,\text{Å}$ and $c = 14.74\,\text{Å}$ as measured with powder X-ray diffraction. 10g of CeAlGe in powder form were also prepared via a solid-state reaction for the time-of-flight neutron scattering experiments. The Ce, Al and Ge powders were weighed in a 1:1:1 molar ratio and placed in a crucible which was flame-sealed in an evacuated quartz tube. The materials were calcined at 700°C. The resulting products were ground and flame-sealed in quartz tube to be annealed at 700°C for several days.

**Electrical and thermal transport.** To perform electrical and thermal transport experiments, the single crystals of RAlGe ($R = \text{La, Ce}$) were thinned along the $c$-axis to a thickness of 0.265mm and 0.370mm, respectively. The magneto-transport measurements (longitudinal resistivity and thermal conductivity) were acquired using a Physical Property Measurement System (PPMS) Dynacool (Quantum Design) using a diagonal offset probe geometry. A more detailed description of these measurements, along with the data processing, can be found in the Supplementary Information.

**Dilatometry.** Sample dilation was measured with an ultrasensitive differential capacitive dilatometer produced by Quantum Design\textsuperscript{37} inserted into PPMS Dynacool. At each temperature, data was taken at both positive and negative magnetic field and the resulting dilatometry was symmetrized.

**Inelastic neutron scattering (INS).** Inelastic time-of-flight neutron scattering experiments were carried out on the fine-resolution Fermi chopper spectrometer SEQUOIA\textsuperscript{38,39} at the Spallation Neutron Source (SNS) in Oak Ridge National Laboratory (ORNL). We utilized co-aligned single crystals of CeAlGe on an aluminum plate using X-ray diffraction with a total mass of 2.8g. Data were collected with different incident energies $E_i = 4, 12, 30, 60, 240, 1000\,\text{meV}$ at base temperature (1.5K) and at 10K which is above $T_N$. The runs at $E_i = 4$ and 12meV were performed on the high flux setting of the instruments whereas the other incident energies were carried out on the high resolution setting enabling measurements downwards to 2% energy resolution. An external magnetic field was applied.
along the c-axis of the CeAlGe single crystals with measurements performed at 0, 1, 3, and 8T.

Inelastic triple-axis neutron scattering experiments were carried out on the polarized triple-axis spectrometer PTAX (HB-1) and on the triple-axis spectrometer TAX (HB-3) at the High Flux Isotope Reactor in Oak Ridge National Laboratory (ORNL). In these triple-axis experiments, we also used co-aligned CeAlGe single crystals with a total mass of 4g. We used a fixed $E_f = 14.7\text{meV}$ with $48^\circ$-$40^\circ$-$40^\circ$-$120^\circ$ collimation and Pyrolytic Graphite filters to eliminate higher-harmonic neutrons. Measurements were performed using closed-cycle refrigerators at base temperature (1.5K) and at a temperature of 10K in tandem with applied magnetic fields along the a- and c-axis of the single crystals of 0, 1, 3, 5, and 8T. Measurements of the spin waves described in the main text were performed along (h10), (h02), and (hh0). The bulk of the measurements concentrated on the optical phonon dispersion of CeAlGe along (h00) at an energy range between 10meV and 25meV.

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Author contributions

ML conceived and supervised the project. NCD, TN, and FH performed the transport measurements with the support from ML. FH, NCD and TN synthesized the materials with support from QTN. TN, NCD, FH, NA, ML performed neutron scattering measurements with help from TJW, MBS, AIK, SC, JFB. NCD, TN, ZC and LKN performed magnetometry. TN, FH, NA, ML performed X-ray scattering measurements with help from AA. TN, NA, QTN, AAP performed Raman measurements with support from DBG and SH. XL, YY, and GB developed theory. ZZ performed ab initio calculations. NCD, TN, and ML wrote the paper with input from all authors.

Competing financial interests

The authors declare no competing financial interests.

Data availability

The data that support the plots within this paper and other findings of this study are available from the corresponding authors upon reasonable request.
Figure 1 | Crystal structure and characterization of CeAlGe. a, CeAlGe crystallizes in space group $I4_1md$ (190) which has broken inversion symmetry. Below $T_N = 4.5K$, CeAlGe becomes ferrimagnetic with the Ce moments on each sublattice pointing in opposite directions along the $a$ direction. b-d, Evolution of bandstructures from the (b) paramagnetic, (c) ferrimagnetic and (d) ferromagnetic phases. e, The location of Weyl points within the Brillouin zone. W1, W2, and W3 nodes are indicated by blue, yellow, and orange, respectively. f, Schematic of the interaction between local magnetic moments and linearly-dispersive Weyl fermions. g, Longitudinal resistivity $\rho_{xx}$ from 2K to 300K. h, Low temperature heat capacity of CeAlGe and LaAlGe (blue dashed line) with extracted magnetic entropy contribution (inset).
Figure 2 | Transport phase diagrams and low temperature magnetic orderings. 

a, Estimation of the electrical resistivity from the second derivative of $\rho_{xx}$ with respect to $B$. Between 10-15K at low field, the longitudinal resistivity barely changes.

b, Estimation of thermal transport phase diagram from second derivatives of $\kappa_{xx}$ with respect to $B$. Dotted lines mark transitions between distinct phases. Centered around 10-15K, a distinct regime arises in the thermal transport at the same point in phase space where we observe anomalous electronic transport.

c, Magnetic phases can be identified from the phase diagrams as regions where thermal fluctuations compete with intrinsic magnetic order and external field alignment.

d, Electronic and thermal conductivity field-dependent behavior at 1.8K follows the anticipated trend from magnetic structure configurations changing with external field.
Figure 3 | Anomalous transport at 12.8K.  

a, Near 12.8K, the magnetoresistance is almost exactly zero for all measured values of magnetic field (vicinity of 12.8K shown in inset).  

b, At this temperature, the thermal conductivity is greatly suppressed with increasing magnetic field.  

c, Low temperature dilation shows a local maximum at 12.8K.  

d, Magnetoresistance exhibits a crossover with magnetic field from positive to negative values around 12.8K.
Figure 4 | Low energy magnetic excitations from inelastic neutron scattering. a-b, Magnon dispersion measured along \([h00]\) at a temperature of 10K at magnetic fields of 0T and 8T, respectively. c-d, Similar to a-b, but below the magnetic transition temperature at 1.8K. Simulated spin wave dispersions (white) with ferromagnetic intralayer and antiferromagnetic interlayer couplings for the given magnetic structure are overlaid. The presence of magnons at 10K indicates the role of short range magnetic correlations even above the magnetic transition temperature.
Fluctuation-driven, topology-stabilized order in a correlated nodal semimetal: Supplemental Information

Contents

A Electronic transport 1
B Thermal transport 4
C Electrical and thermal phase diagrams 6
D Magnetization 6
E Spin wave dispersion from inelastic neutron scattering 7
F Crystal field excitations 8
G Phonon dispersion from inelastic X-ray scattering 10
H Raman spectroscopy 11
I Effective field theory on Weyl fermion-magnetism coupling 12

A Electronic transport

We perform a thinning-down process of the RAlGe (R = La, Ce) single crystals along the $c$-axis to a thin slab down to thicknesses of 0.265mm and 0.370mm, respectively, as it is difficult to perform electric and thermal transport measurements on the as-grown single crystals. The contacts of the electric and thermal transport probes were made using silver epoxy H20E.

Electronic resistance $R$ is measured for both LaAlGe and CeAlGe as a function of magnetic field using the electric transport option (ETO) of a Physical Property Measurement System (PPMS, Quantum Design) within an applied magnetic field interval of 0T to 9T and a temperature interval of $\sim$1.8K to 300K. These electronic resistivity measurements are performed using a standard six-probe configuration with the longitudinal and transverse resistivity probes connected to independent measurements channels. As the contacts are manually fabricated with silver epoxy, the measured data may exhibit effects of asymmetry with magnetic field due to slight misalignments of the silver contacts. Accordingly, we mitigate this effect of possible contact misalignment by averaging the longitudinal $\rho_{xx}$ and
transverse $\rho_{xy}$ resistivities using the following equations

$$\rho_{xx}(B) = \frac{\rho(+B) + \rho(-B)}{2}$$

$$\rho_{xy}(B) = \frac{\rho(+B) - \rho(-B)}{2} \left( \frac{L}{W} \right),$$

where $\rho(+B)$, $\rho(-B)$ indicate the measured resistivity at positive and negative values of the magnetic field, respectively, while $L$ and $W$ designates the length and the width of the sample. These values of $\rho_{xx}$ are used in the second-order derivative plots of Figure 2a. The plot of $\rho_{xx}(B)$ at different temperatures for CeAlGe is shown in Figures 1g (and in the vicinity of 12.8K in Figure 3d) whereas that for LaAlGe can be observed in Figure S1. The measured longitudinal resistivity $\rho_{xx}$ is symmetric with respect to external magnetic field and does not show any signs of Shubnikov-de Haas (SdH) oscillations up to 9T at the lowest measured temperature of $\sim$1.8K. From our data, the longitudinal resistivity $\rho_{xx}$ of LaAlGe demonstrates normal metallic behavior which is in sharp juxtaposition to the behavior seen in CeAlGe, which is described in the main text.

![Figure S1 | Longitudinal resistivity of LaAlGe.](image)

Figure S1 | Longitudinal resistivity of LaAlGe. (left) Longitudinal resistivity $\rho_{xx}$ with respect to temperature up to 300K from 0T to 9T. (right) Magnetoresistance (MR) plotted with respect to $\mu_0H/\rho_0$ at temperatures ranging from 2K to 300K.

We quantify the magnetoresistance (MR) of the samples using the following definition

$$MR = \frac{\rho(B) - \rho(B = 0)}{\rho(B = 0)} \times 100\%.$$
The temperature dependence and the applied magnetic field dependence of the MR for CeAlGe is plotted in the main text in Figure 3a and in Figure 3d, respectively. The field-dependence of the MR for LaAlGe is plotted in Figure S1 as a MR versus $\mu_0 H/\rho(B = 0)$ at different temperatures. The MR of LaAlGe increases quadratically with $\mu_0 H/\rho(B = 0)$ which is again consistent with normal metallic behavior. LaAlGe appears to exhibits single-band behavior at low temperatures as evidenced by the universal scaling of MR at these temperatures.

Furthermore, we show data of the measured Hall (transverse) resistivity $\rho_{xy}$, which is antisymmetric with external magnetic field, in Figure S2 for LaAlGe and CeAlGe. The magnitudes of the transverse resistivities $\rho_{xy}$ of both compounds are quantitatively comparable and exhibit a linear increase in magnitude with magnetic field. For CeAlGe, there is a slight rise in the magnitude of $\rho_{xy}$ at low temperatures with decrease in temperature.

![Figure S2 | Transverse resistivity of RAlGe (R = La, Ce). Transverse resistivity $\rho_{xy}$ with respect to applied magnetic field along the c-direction of LaAlGe (left) and of CeAlGe (right) samples up to 9T from 2K to 300K.](image)

The carrier concentrations and mobilities of LaAlGe and CeAlGe were extracted using a one-band model as Fig. S1 suggests single-band behavior for LaAlGe. The Hall coefficient for both materials is positive, thereby indicating that the samples are hole-dominated as seen in Figure S3. The carrier concentrations of both samples are comparable, especially at low temperatures, with LaAlGe having an overall larger carrier concentration. CeAlGe has a larger hole mobility at lower temperatures, but decreases below that of LaAlGe at larger temperatures.
B Thermal transport

The thermal conductivity of the CeAlGe sample is measured with the thermal transport option (TTO) of the PPMS (Quantum Design). Similar to the electric transport measurements of Section A, we perform a thinning-down process of the sample to a thickness of 0.260mm for the purpose of facilitating the measurement. The contacts are fabricated using silver epoxy H20E.

With the device geometry shown in Figure S4a, the ends of the thinned crystal serve as the heat source and the heat sink, with thermometers positioned along the transverse direction to record the temperature difference. The magnetic field is applied perpendicular to the device, along the c-axis of the crystal. The resulting total thermal conductivity is slightly asymmetric with respect to positive and negative fields due to the effect of possible contact misalignment. To correct for this effect, the longitudinal and transverse components of the thermal resistivities are averaged over as follows

$$\rho_{th,xx}(B) = \frac{\kappa(+B) + \kappa(-B)}{2\kappa(+B)\kappa(-B)}$$

$$\rho_{th,xy}(B) = \frac{\kappa(+B) - \kappa(-B)}{2\kappa(+B)\kappa(-B)} \left( \frac{L}{W} \right)$$,

where $\kappa$ is the measured total thermal conductivity and $L$, $W$ represent the length- and width-wise distance, respectively, between the two thermometers measuring the hot and cold ends of the sample.
The longitudinal ($\kappa_{xx}$) and transverse ($\kappa_{xy}$) thermal conductivity are subsequently calculated using

$$\kappa_{xx} = \frac{\rho_{th,xx}^2}{\rho_{th,xx}^2 + \rho_{th,xy}^2}$$

$$\kappa_{xy} = \frac{\rho_{th,xy}^2}{\rho_{th,xx}^2 + \rho_{th,xy}^2}.$$  

**Figure S4 | Thermal transport of CeAlGe.**  

**a,** Device geometry for the thermal transport option (TTO) of the PPMS. Heat is injected along the longitudinal direction with thermometers placed along the transverse direction. The magnetic field is applied perpendicular to the sample, in the $c$-direction. **b,** Transverse thermal conductivity $\kappa_{xy}$ versus magnetic field at different temperatures. **c,** Longitudinal thermal conductivity $\kappa_{xx}$ versus temperature at 0T and 9T. The temperature derivative of $\kappa_{xx}$ at low temperatures is plotted as an inset. **d,** Magneto-thermal conductivity with respect to applied magnetic field at different temperatures.

The transverse thermal conductivity $\kappa_{xy}$ of CeAlGe is shown in Figure S4b which demonstrates a linear increase in magnitude with applied magnetic field. Figure S4c shows the longitudinal thermal conductivity $\kappa_{xx}$ as function of temperature and its first derivative with respect to temperature (inset) at zero field and at 9T, demonstrating the magnetic transition near 5K.

Furthermore, in analogy to the (electronic) magnetoresistance (MR) defined for electric transport, we define the magneto-thermal conductivity $MC_{th}$ by the following equation

$$MC_{th} = \frac{\Delta \kappa_{xx}(B)}{\kappa_{xx}(B = 0)} = \frac{\kappa_{xx}(B) - \kappa_{xx}(B = 0)}{\kappa_{xx}(B = 0)} \times 100\%.$$
As shown in Figure S4d, the magneto-thermal conductivity for CeAlGe is negative above the magnetic transition, but exhibits two kinks at temperatures below the onset of ferrimagnetic order as a result of a spin alignment with the applied magnetic field. This effect is discussed further in main text and described in Figure 2.

C Electrical and thermal phase diagrams

Transport phase diagrams are computed from data taken at different temperatures and magnetic fields that have been interpolated with a triangular mesh. For each transport quantity, numerical first and second derivatives are taken with respect to temperature or magnetic field. Figure S5 shows the resulting phase diagrams for the derivatives with respect to field of the electronic resistivity and thermal conductivity, respectively. The construction of the phase diagrams in Figure 2 of the main text focuses on the second derivative in which the phase boundaries correspond to the extrema in the first-derivative phase diagrams.

![Figure S5 | Transport phase diagrams. a, Temperature and field dependence of the first derivative of longitudinal electrical resistivity $\rho_{xx}$ with respect to field. b, Temperature and field dependence of the first derivative of longitudinal thermal conductivity $\kappa_{xx}$ with respect to field. Red indicates positive derivative while blue indicates negative derivative.](image)

D Magnetization

We measure the DC magnetization of a sample with mass 7 mg in a Magnetic Property Measurement System (MPMS3, Quantum Design). DC magnetization is measured from $-7$T to $7$T at temperature intervals of 0.5K from 2K to 15K. The phase diagrams shown in Figure S6 are computed with a triangular interpolation scheme based on the data.
E  Spin wave dispersion from inelastic neutron scattering

For inelastic neutron scattering (INS) data at different temperatures and magnetic fields, spin wave dispersions are fit to peak locations. Starting with the raw data shown in Figure S7a, the peak locations are found by fitting Gaussian peaks to line-cuts of the INS spectrum taken at different energies, with the standard deviation of the fitted Gaussian peak taken to be the error. This process is demonstrated in Figure S7d-S7g. The resulting peaks are overlaid on top of the raw data as shown in S7b.

Once the peaks are identified, we use the MATLAB library SPINW\textsuperscript{2} to simulate the spin wave dispersions while extracting interlayer and intralayer coupling parameters. SPINW uses linear spin wave theory to numerically simulate spin excitations for the general Hamiltonian

\[
H = \sum_{i,j} S_i J_{ij} S_j + \sum_i S_i A_i S_i + B \sum_i g_i S_i,
\]

where exchange coefficients $J_{ij}$ couple spins $S_i$ and $S_j$, $A_i$ is the anisotropy, $B$ is the magnetic field, and $g_i$ is the g-factor tensor. Given an input magnetic structure and corresponding crystallographic space group, along with the coefficients of the Hamiltonian, SPINW calculates the spin dispersion along any direction in the Brillouin zone.

To model the spin waves in CeAlGe for comparison to INS data, we use the space group of CeAlGe, $I\overline{4}1md$, and define $\mathbf{S}$ to have magnitude of 1/2 at each Ce site. The magnetic ordering is defined to be either ferromagnetic (FM), with all spins pointing along the $c$-axis, or antiferromagnetic (AFM), with spins pointing along the $a$ direction, ordered oppositely along the $c$-axis. The interlayer exchange $J_1$ is defined to be negative tending towards an AFM ground state, and intralayer exchange $J_2$ is defined...
to be positive tending towards spin alignment within each $ab$ plane. The magnetic field is defined to point along the $c$-direction and has a magnitude of 0 for the 0T data but a magnitude of 5T and 6T for the 10K and 1.8K, respectively, at 8T data. This difference between experimental and simulated magnetic field originates from the variation of the total magnetic moment with temperature and field.

For each INS spectrum we input the magnetic structure as either FM or AFM, and calculate the spin wave dispersions for $J_1 \in [-0.20, -0.25, -0.30, -0.35, -0.40]$ and $J_2 \in [0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80]$ (all values in meV). Under each set of conditions, the resulting error between the spin wave calculation and the experimentally-identified points in the INS spectrum is computed. By minimizing the error, we find that $(J_1, J_2) = (-0.25, 0.80), (-0.35, 0.55), (-0.25, 0.60), (-0.20, 0.65)$ for $(T, B) = (10K, 0T), (1.8K, 0T), (10K, 8T), (1.8K, 8T)$ respectively. To find the values of $J_1, J_2$ in the normal regions of the phase diagram, we average them to find $J_1 = -0.27 \pm 0.08$ meV and $J_2 = 0.60 \pm 0.05$ meV.

F Crystal field excitations

We perform time-of-flight INS measurements on single crystals of CeAlGe at different combinations of temperature (1K, 10K) and external magnetic field (0T, 1T, 8T) using an incident neutron energy
of $E_i = 240\,\text{meV}$ as shown in Fig. S8a to obtain direct information on the crystal electric field (CEF) excitations within this compound. These crystal fields excitations are also observable in an INS measurement performed using $E_i = 1000\,\text{meV}$ incident neutrons, seen in Fig. S8b. The presence of CEF effects has been previously studied in other cerium intermetallics\textsuperscript{3–5} as originating from a splitting of the 6-fold $J = 5/2$ Ce\textsuperscript{3+} multiplet into three doublets. Indeed, six prominent CEF excitation lines can be observed in the wavevector-energy intensity plots that do not disperse with magnitude of the wavevector $|Q|$ and are not drastically affected by temperature nor external magnetic field. In Fig. S8c, we show a crude fit of the neutron scattering intensity (integrated over $|Q|$) using Gaussian peaks and a uniform background to extract the energies of these CEF excitations. The values of the crystal field excitations are as follows: 102.6(3)\,\text{meV}, 116.7(2)\,\text{meV}, 138.2(3)\,\text{meV}, 161.1(5)\,\text{meV}, 171.9(2)\,\text{meV}, and 181.1(1)\,\text{meV}.

**Figure S8 | Crystal field excitations in CeAlGe.** a, Powder averaged neutron intensity plots measured with an incident energy of $E_i = 240\,\text{meV}$ at various combinations of temperature and applied magnetic field. The horizontal and vertical axes denotes momentum $|Q|$ and energy transfers $E$, respectively. The crystal field excitations are observed as horizontal lines (constant energy) with respect to $Q$ between 80\,\text{meV} to 170\,\text{meV}, which do not change significantly with temperature or magnetic field. b, Similar to a, with an incident energy of $E_i = 1000\,\text{meV}$. c, The measured neutron scattering intensity integrated over $|Q|$ obtained from $E_i = 240\,\text{meV}$ data sets for CeAlGe samples at 10K and 0T. The experimental data (black line) is fitted (blue line) with a sum of Gaussian peaks with a uniform background to extract the energies of the crystal field excitations. The individual Gaussian peak contributions are plotted as dashed lines and the value of the peak center is shown in the legend.
G  Phonon dispersion from inelastic X-ray scattering

Inelastic X-ray scattering measurements are performed on the high-energy resolution inelastic x-ray (HERIX) instrument at Section 3-ID at the Advanced Photon Source (APS), Argonne National Laboratory (ANL)\textsuperscript{6–8}. The incident beam energy is 21.657 keV (corresponding to $\lambda = 0.5725\text{Å}$) with an overall energy resolution of 2.1meV.

We measure the phonon modes of a single-crystal sample of LaAlGe that was thinned down to a thickness of $<50\mu m$ using inelastic x-ray scattering. These are measured along high-symmetry directions in the Brillouin zone along $\Gamma$-$\Sigma$-$\Sigma_1$-$Z$-$\Gamma$ and can be seen in as an normalized intensity map in Figure S9. The measured phonon modes, ranging from 0meV to 22.5meV, agree well with \textit{ab initio} calculations shown as grey lines. These measurements serve to identify the energy range of the phonon modes in the INS measurements of CeAlGe, described in Sections F and G, and to distinguish them among other excitations such as magnons and crystal field excitations. As LaAlGe only differs to CeAlGe by the rare earth atom, it is expected that their vibrational modes closely resemble each other and thus, these IXS measurements on LaAlGe serve as a good reference for the INS measurements of CeAlGe.

![Figure S9 | Phonon dispersion of LaAlGe](image)

\textbf{Figure S9 | Phonon dispersion of LaAlGe.} Phonon modes of LaAlGe measured along high-symmetry line $\Gamma$-$\Sigma$-$\Sigma_1$-$Z$-$\Gamma$ in the Brillouin zone plotted as a normalized intensity map. The experimentally-measured phonon modes agrees reasonably well with the \textit{ab initio} calculations shown as grey lines.

The \textit{ab initio} calculations were computed using VASP\textsuperscript{9–11} with projector-augmented-wave (PAW) pseudopotentials and Perdew-Burke-Ernzerhof (PBE) for the exchange-correlation energy functional\textsuperscript{12}. This was optimized with a 6x6x2 Monkhorst-Pack grid of $k$-point samplings. Second- and third-order force constants were calculated using a real space supercell approach with a 3x3x1 supercell and the
PHONOPY package\textsuperscript{13} was using to obtain the second-order force constants.

H Raman spectroscopy

Polarization angle-resolved Raman scattering measurements are performed at the Center for Nanophase Materials Sciences (CNMS) at Oak Ridge National Laboratory (ORNL) on single crystal samples of CeAlGe and LaAlGe. These measurements are performed at $T = 300$K. From the symmetry of the polarized Raman dependence, different phonon modes are identified in CeAlGe and LaAlGe, as plotted in Figs. S10-S11, respectively, as waterfall and polar plots. Notably, several phonon modes exhibits a fourfold rotation symmetry as a function of the polarization angle.

Figure S10 | Polarized Raman spectroscopy of CeAlGe. **a.** Waterfall plot of polarized Raman spectra measured using a 532-nm laser excitation over a range of polarization angles. The spectra have been offset vertically for clarity. Two identified angle-dependent $B_1$ modes at 200 cm$^{-1}$ and 352 cm$^{-1}$, and one angle-independent $A_1$ mode at 367 cm$^{-1}$, are labeled. **b.** Polar plot of three selected modes whose intensity exhibits fourfold rotational symmetry as a function of polarization angle. Two are identified as $B_1$ modes, while the third mode with lower intensity is observed at 240 cm$^{-1}$. **c.** Polar plot of two selected modes with conserved intensity as a function polarization angle. One is identified as an $A_1$ mode, while the other mode with lower intensity is observed at 154 cm$^{-1}$.
Figure S11 | Polarized Raman spectroscopy of LaAlGe. a. Waterfall plot of polarized Raman spectra measured using a 532-nm laser excitation over a range of polarization angles. The spectra have been offset vertically for clarity. Two identified angle-dependent $B_1$ modes at 190 cm$^{-1}$ and 337 cm$^{-1}$, and one angle-independent $A_1$ mode at 353 cm$^{-1}$, are labeled. b. Polar plot of three selected modes whose intensity exhibits fourfold rotational symmetry as a function of polarization angle. Two are identified as $B_1$ modes, while the third mode with lower intensity is observed at 230 cm$^{-1}$. c. Polar plot of two selected modes with conserved intensity as a function polarization angle. One is identified as an $A_1$ mode, while the other mode with lower intensity is observed at 152 cm$^{-1}$.

I Effective field theory on Weyl fermion-magnetism coupling

To understand the unusual transport properties in CeAlGe, we develop an effective field theory according to the noncentrosymmetry of the crystal in addition to the coupling between the fluctuating magnetization and the spin of the Weyl fermion. We find that if the coupling between the magnetization fluctuation and the Weyl fermion is fixed by the Dzyaloshinskii-Moriya and the ferromagnetic interaction strength, an emergent helimagnetic order appears. Furthermore, if the helimagnet is pinned, the fluctuation around this induced order reduces to a Chern-Simons fluctuation.

Model setup and effective Hamiltonian. For simplicity, we consider the model Hamiltonian of a pair of
Weyl points with opposite chiralities that couples to local magnets, i.e.,

\[ H = \int d^3r \sum_{\nu = \pm} \left(-iv_\nu \psi_\nu^\dagger \sigma \cdot \nabla \psi_\nu + K_\nu M \cdot s_\nu \right) + \cdots, \]

where the \( \nu \) subscript labels the chirality of the Weyl point, \( v_\nu \) is the Fermi velocity of the Weyl point, \( M \) is the magnetization fluctuation, \( s_\nu = \frac{1}{2} \psi_\nu^\dagger \sigma \psi_\nu \) is the spin, and \( K_\nu \) is the Kondo coupling between the magnetic field and the spin near the Weyl point. The \( \cdots \) terms include the higher order derivatives and the exchange terms between the local magnetizations. For a noncentrosymmetric system, we integrate out one Weyl fermion that is below the chemical potential, for example, \( \psi_- \). The effective Hamiltonian is given by

\[ H_{\text{eff}} = \int d^3r \left(-i \left( \psi_+^\dagger \sigma \cdot \nabla \psi_+ + K_+ M \cdot s_+ \right) + \frac{D}{4} M \cdot \nabla \times M + \frac{J}{2} (\nabla M)^2 \right), \quad (S1) \]

with \( J \propto K_-^2 \) as the exchange coupling strength and \( D \propto v_- K_-^2 \) being the Dzyaloshinskii-Moriya interaction (DMI) constant\(^{14,15} \). We have set \( \hbar = v_+ = 1 \) and neglected the higher order terms. To simplify the notation, we omit the "+" index in the following derivation. This is a Hamiltonian with a Weyl fermion coupling to the magnetization that includes the DMI and ferromagnetic interactions. Taking the extreme for \( M \) and \( \psi \), we have

\[ -i \sigma \cdot \left( \nabla + i \frac{K}{2} M \right) \psi = 0 \]

\[ s + \frac{D}{2K} \nabla \times M - \frac{J}{K} \nabla^2 M = 0. \quad (S2) \]

We will consider a solution in which the magnitude \( |M| \) is normalized to unity. In the limit where \( J \to 0 \), Eqs. (S2) are known as the Seiberg-Witten equations in three dimensions\(^{16,17} \) and the zero energy solutions of \((\psi, M)\) are called Seiberg-Witten monopoles. Here, we generalize the Seiberg-Witten equations by taking \( J \neq 0 \).

**Zero modes and spectrum in a condensed matter system.** We consider the solutions of Eqs. (S2) that satisfy periodic boundary conditions. The spectrum \( E \) of the fermions is determined by solving the equation of motion under the effective Hamiltonian (S1)

\[ \left(-i \sigma \cdot \nabla + \sigma \cdot \frac{K}{2} M \right) \psi = E \psi. \]

In order to find the zero energy solution, we first focus along one direction, say, the \( z \)-axis, and take
the following ansatz,

\[
\psi_0(r) = \psi_n(z) \propto e^{i(k_z + nQ)z} \begin{pmatrix} 1 \\ -i \frac{2}{Q}(k_z + nQ - E)e^{-iQz} \end{pmatrix},
\]

and \( M = \frac{Q}{K}(\sin(Qz), \cos(Qz), 0) \), where \( k_z \) is the wavevector along the \( z \)-axis, \( Q \) is the magnetic wavevector of the magnetization, and \( n \) labels the \( n \)th magnetic Brillouin zone determined by \( Q \).

Then, through a direct calculation, the spectrum becomes

\[
E_{\pm} = \frac{Q}{2} \pm \sqrt{\left(k_z + \left(n + \frac{1}{2}\right)Q\right)^2 + \frac{K^2}{4}}.
\]

The zero energy occurs when \(|Q| = |K|\), which is consistent with the normalization \(|M| = 1\), and when \( k_z = -(n + \frac{1}{2})Q \). The spin of these zero modes are the same, i.e., \( s_n = s = \frac{1}{2}Q/\kappa M \). The second equation of Eq. (S2) determines \( Q \) as

\[
\frac{Q}{2K} + \frac{DQ^2}{2K} - \frac{JQ^2}{K} = 0 \Rightarrow Q = \frac{1}{2J}(1 + D),
\]

which also gives a constraint condition between \( K \) and \( J/D \). This is the reasoning behind the very narrow temperature region for the phase near 12.8K.

For a pinned helimagnet with \( Q > 0 \) in, say, the \( z \) direction, the dispersion near \( k_z = -(n + \frac{1}{2})Q \) for a given zero mode gives

\[
E_-(k_z) \approx -\frac{1}{|K|} \left(k_z + \left(n + \frac{1}{2}\right)Q\right)^2 = -\frac{1}{|K|}q_z^2.
\]

We now perform the perturbation by taking \( H_1 = k_x \sigma^x + k_y \sigma^y \) as the Hamiltonian for the pinned helimagnet in Eq. (S1) is translational invariant in the \( xy \)-plane. The first order perturbation is zero.

The matrix elements between \( \psi_{n-}(z) \) and \( \psi_{n'-}(z) \), where

\[
\psi_{n-}(z) \propto e^{i(k_z + nQ)z} \begin{pmatrix} 1 \\ -i \frac{2}{Q}(k_z + nQ - E_{n-})e^{-iQz} \end{pmatrix},
\]

are

\[
\langle \psi_{n'-}(z)|H_1|\psi_{n-}(z)\rangle = \delta_{n,n'+1}A_n(k_x - ik_y) + \delta_{n,n'-1}A_{n'}(k_x + ik_y).
\]
The energy differences at \( k_z = -\left(n + \frac{1}{2}\right) Q \) are

\[
E_-(n, k_z) - E_-(n \pm 1, k_z) = \frac{\sqrt{5} - 1}{2} K.
\]

The second order perturbation is written as

\[
E^{(2)}_-(n) = \sum_{n' \neq n} \frac{\langle \psi_{n'}(z) | H_1 | \psi_n(z) \rangle \langle \psi_{n'}(z) | H_1 | \psi_{n'}(z) \rangle}{E_n - E_n'} = \frac{2}{\sqrt{5} - 1} \frac{k_x^2 + k_y^2}{K}.
\]

Thus, the dispersions near \( k_z = -\left(n + \frac{1}{2}\right) Q \) are

\[
E_-(k) = -\frac{1}{K} q_z^2 + \frac{2}{\sqrt{5} - 1} \frac{k_i^2}{K} = -\frac{q_z^2}{2m_z} + \frac{k_i^2}{2m_t} = -\varepsilon_z + \varepsilon_t,
\]

with \( k_i = (k_x, k_y) \). The dispersion is of a saddle shape – namely, in the \( z \)-direction, the dispersion is of hole-type while in the \( xy \)-plane is of particle-type. This signifies that the dynamics in parallel and perpendicular directions to the helimagnet are separated.

**Seiberg-Witten invariant.** The helimagnet direction can be arbitrary: the general helimagnet with a wave vector \( Q \) can be described as

\[
M = \frac{Q}{K} \left( e_1 \sin (Q \cdot r + \varphi_0) \pm e_2 \cos (Q \cdot r + \varphi_0) \right),
\]

where \( e_1 \times e_2 = Q/|Q| \) and \( |Q| = |K| \) for \( Q = Q \cdot r/|r| \).

All possible wave vectors \( Q \) with \( |Q| = |K| \) form a moduli space of the zero modes of the Weyl fermions. The Seiberg-Witten invariant, which is equal to the Euler number of the moduli space, is 2.

**Chern-Simons-type magnetization fluctuations and mass renormalization.** Around the helimagnet, there may be a fluctuation \( a = \frac{K}{2} (\delta M_x, \delta M_y, \delta M_z) \). If the fluctuation is spatially slow-varying, similar to the minimal coupling \( -i \nabla \rightarrow -i \nabla + a \), the first order perturbation of Eq. (S2) results in \( a_z \sigma^z \) and the second order perturbation becomes proportional to \( \propto (-i \partial_x + a_x)^2 + (-i \partial_y + a_y)^2 + a_z^2 \). For the \( a_z \) term, we will only keep the first order in the following.

Upon comparison with Eq. (S3), the first-quantization effective Hamiltonian for the lower band is given by

\[
H_h = -\frac{1}{2m_z} \left(-i \partial_z - \frac{K}{2}\right)^2 + \frac{1}{2m_t} (-i \nabla_t + a_t)^2 - a_z,
\]

where \( \nabla_t = (\partial_x, \partial_y) \) and \( a_t = (a_x, a_y) \). For the magnetization fluctuation, we neglect the Maxwell-like
term and keep the Chern-Simons term,

\[ H_a = \frac{D}{K^2} a \cdot \nabla \times a. \]

For the Chern-Simons theory, we consider a bare potential caused by impurity \( v(r) = e^2/(\varepsilon \sqrt{r^2 + d_s^2})^{18} \).

Since the Chern-Simons fluctuation is in the \( xy \)-plane, reducing to this plane results in

\[ v(k) = \frac{2\pi e^2}{\varepsilon k_t} e^{-k_i d_s}. \quad \text{(S4)} \]

Now, let us write down the effective field theory for the band including the zero mode

\[
\hat{H} = \int d^3r \chi^\dagger(r) \left(-\frac{1}{2m_z} (-i \partial_z - \frac{K}{2})^2 + \frac{1}{2m_t} (-i \nabla_t + a_t)^2 - a_z + \mu_h \right) \chi(r) \\
+ \int d^3r \frac{D}{K^2} \epsilon_{abc} a_a \partial_b a_c + \int d^3r d^3r' : \chi^\dagger(r) \chi(r') v(|r - r'|) \chi^\dagger(r') \chi(r') :.
\]

Notice that \( a_z \) is not a dynamic field and the variation of \( a_z \) gives a constraint between the density fluctuation and the magnetization fluctuation as

\[ \rho = \chi^\dagger \chi = -\frac{D}{K^2} \nabla_t \cdot a_t \cdot \hat{z}. \]

Integrating over the Lagrange multiplier \( a_z \), we have

\[ L = \int d^3r \left( \chi^\dagger \partial_t \chi + a_1 \partial_t a_2 - a_2 \partial_t a_1 \right) - H, \]

with the Hamiltonian

\[
\hat{H} = \int d^3r \chi^\dagger \left(\frac{1}{2m_z} (-i \partial_z - \frac{K}{2})^2 + \frac{1}{2m_t} (-i \nabla_t + a_t)^2 \right) \chi \\
+ \int d^3r \frac{D}{K^2} (a_x \partial_y a_y - a_y \partial_x a_x) + \frac{D^2}{K^4} \int d^3r d^3r' |\nabla_t \times a_t(r')| v(|r - r'|) |\nabla_t \times a_t(r)|.
\]

Thus, the corresponding actions are

\[
S_{f0} \propto \int d\omega \, dq_z \, dk_x \, dk_y \, \chi^\dagger(k) \left(\omega + \frac{1}{2m_z} q_z^2 - \frac{1}{2m_t} k_t^2 \right) \chi(k) \\
S_{a0} \propto \int d\omega \, dq_z \, dk_x \, dk_y \left( \epsilon_{ij} a_{ij}(\omega, k) \left( i \omega - \frac{D}{K^2} q_z \right) a_{ij}(-\omega, -k) + \frac{\hat{n}}{2m_s} a^2_t + \frac{D^2}{K^2} k_t^2 v(|k|) a_{\perp}(\omega, k) a_{\perp}(-\omega, -k) \right)
\]

16
\[ S_i = - \int d\omega \, H_i = - \int d\omega \, d^3r \, a_t \cdot \frac{i}{2m_t} (\chi^\dagger \nabla_t \chi - \nabla_t \chi^\dagger \chi) \]
\[ S_{ia} = - \int d\omega \, H_{ia} = \frac{D}{K^2} \int d\omega \, d^3r \, (\nabla_t \times a_t) \cdot \hat{z} \]

where \( i, j = \parallel, \perp \) in the second line designates a direction that is parallel to or perpendicular to \( k_t \), respectively. We see that in \( S_{f0} \) and \( S_{a0} \), \( q_z \) can be absorbed by redefining \( \omega' = \omega + \frac{1}{2m_z} q_z^2 \) and \( \omega'' = \omega - \frac{D}{K^2} q_z \). \( q_z \) only appears in the interacting potential \( v(|k|) \). If we use Eq. (S4) to approximate the interaction potential, the action is then reduced to a two-dimensional one. Formally, this is the same as the effective Chern-Simons theory description for the \( \nu = 1/2 \) composite fermion in temporal gauge.\(^\text{19,20} \) According to Halperin et al.\(^\text{18} \), under an RPA calculation, the mass \( m_t \) in the transverse direction is renormalized to
\[ m^* \propto |k_t|^{-1/2}. \quad (S5) \]

Behavior of MR near 12.8K. As shown in the experimental data (Fig. 3a), at 12.8K, the MR is almost exactly zero even in the existence of a strong magnetic field. The behavior of MR near 12.8K indicates the phase transition from Weyl semimetal phase into the Seiberg-Witten monopole phase. Because in the Seiberg-Witten monopole phase, the effective mass \( m^* \propto k_t^{-1/2} \) as shown in Eq. (S5), the cyclotron frequency goes as \( \omega_c^* = eB/m^* \to 0 \) when \( k_t \to 0 \) if an external magnetic field is present, i.e., the Landau level gap \( \hbar \omega_c^* \) is very narrow in long wavelength limit. According to the quadratic dispersion of the fermions in the Seiberg-Witten phase, the Drude formula can be applied. The magnetoconductance
\[ \sigma(B) = \frac{\sigma_0}{1 + \omega_c^* \tau} \approx \sigma_0 \quad (S6) \]
becomes nearly independent of the external magnetic field when \( \omega_c^* \tau \ll 1 \) which ultimately explains the near-zero MR in Fig. 3a at the temperature of 12.8K.

Spin correlation functions. For the zero modes, the magnetization \( \mathbf{M} \) is helical: \( \mathbf{M} = \frac{Q}{K} (\sin(Qz), \cos(Qz), 0) \) with \( |Q| = |K| \). The spin of the zero modes are determined by Eqs. (S2), i.e., \( \mathbf{s} = \frac{1}{2} \frac{Q}{K} \mathbf{M} \). Therefore, if we neglect the fluctuations around the zero modes, the spin correlation functions are
\[ \langle s_x(z)s_x(z') \rangle \propto \sin(Qz) \sin(Qz') \]
\[ \langle s_x(z)s_y(z') \rangle \propto \sin(Qz) \cos(Qz') \]
\[ \langle s_y(z)s_y(z') \rangle \propto \cos(Qz) \cos(Qz') \]
while the others are zero. These spin correlation functions show a short-range ferromagnetic behavior and no constant long-range order which is consistent with the experimental data shown in Fig. 4.
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