Generation of ocean internal wave with vertical-slice hydrodynamic simulation

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Abstract. The instability of the density of seawater causes the formation of internal waves in the sea. This phenomenon is difficult to observe visually but can be studied with a hydrodynamic model approach. This study investigates the internal waves caused by underwater obstructions (seamounts) and the stratified density with a two-dimensional marine hydrodynamic model. Four scenarios are simulated in studying internal waves. Two scenarios used one barrier with uniform density and stratified density (based on the Brunt Vaisala stability frequency $N^2 = 5 \times 10^{-4} \text{s}^{-2}$). Meanwhile, the other two scenarios use two barriers with uniform density and stratified density (based on the Brunt Vaisala stability frequency $N^2 = 5 \times 10^{-4} \text{s}^{-2}$). Based on the simulation results, it is known that the density conditions have a significant effect on the dynamics of ocean currents. In hydrodynamic simulations of one and two barriers with varying or stratified densities, current resonances and density resonances are formed underwater topography. Meanwhile, in uniform density, the currents formed the rotor, cavity, turbulence, and resonance with the underwater topography. Thus the current dynamics are stronger in the case of uniform density than in the stratified density. It has implications for differences in the mixing of suspended matter in the sea. So this study can be useful in the study of sediment transport, upwelling, the thermocline layer, energy from internal waves, and the distribution of plankton or fish larvae.

1. Introduction
Internal waves are the phenomenon of pycnocline turbulent gravitational waves along the oceans which appear under the sea, making it difficult to see [1-4]. These internal waves' existence is caused by the density of seawater, which is not constant [5, 6]. Internal waves can also be induced by wind, tides, or even ship movement and have significant implications for physical and biological processes [7-9], such as suspended sediment, upwelling, thermocline layers, energy from internal waves, and distribution of plankton or fish larvae [10, 11]. Internal waves produce tidal waves and geostrophic flows [12, 13]. In this research, internal wave dynamics are studied using numerical models. This model can simulate breaking waves at the sea interface, so it is suitable for use.

Internal wave research has been conducted by Matthews et al. [14] to investigate the behavior of internal waves hidden in the ocean's deep layers. Rizal et al. [3] examine the internal wave response in detail in a finite channel. Internal wave measurements can be predicted by direct observation or measurement on the pycnocline or thermocline. Pycnocline is a layer where the density of seawater changes rapidly with thickness, while the thermocline is a layer of seawater that has a temperature that changes rapidly with depth [15]. The use of sensors to measure internal waves results in varying temperatures, seawater salinity, and current speeds.
Internal waves can be described using an experiment in the laboratory or observing them from the air or space using remote sensing technology [7]. Internal waves affect the dynamics of upwelling, downwelling, and the thermocline layer [15, 16]. This research aims to simulate simple internal waves by considering the level of seawater stratification and underwater topography.

2. Materials and Methods

2.1. Basics equation

This research was conducted by simulating a two-dimensional non-hydrostatic numerical model. This model was developed by Kämpf [17] from the Navier-Stokes equation with equations of advection and pressure gradient due to seawater density. The basic equation used in the model is as follows [18]:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho_o} \frac{\partial (p+q)}{\partial x} \tag{1}
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho_o} \frac{\partial q}{\partial z} \tag{2}
\]

\[
\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + w \frac{\partial p}{\partial z} = \frac{\partial}{\partial x} \left( K_o \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial p}{\partial z} \right) \tag{3}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{4}
\]

\[
\frac{\partial q}{\partial x} = - \rho_o g \frac{\partial (h<\omega>)}{\partial x} \tag{5}
\]

\[
\frac{\partial p}{\partial x} = - \rho_o \frac{\partial \eta_o}{\partial x} \tag{6}
\]

The stability equation for the Brunt-Väisälä frequency stratification [17] is as follows:

\[
N^2_{BV} = - \frac{g}{\rho_o} \frac{\partial \rho}{\partial z} \tag{7}
\]

where \( u \) and \( w \) are the horizontal and vertical current velocities (m/s), \( \rho_o \) is the reference density (kg/m³), \( p \) is the dynamic pressure (kg/m²), \( q(x, z, t) \) is the non-hydrostatic pressure (kg/m²), \( h(x, t) \) is the depth of fluid (m), \( N^2_{BV} \) is the Brunt-Väisälä frequency, \( \Delta t \) is the change in time (s), \( \Delta x \) and \( \Delta z \) are the horizontal and vertical distances, in units (m).

2.2. Model Setup

In studying internal waves, four numerical simulations are applied:

Stratified density (density at the surface is 1021 kg/m³ while the bottom layer is calculated based on the stability of the Brunt Vaisala Frequency with \( N^2 = 5 \times 10^{-4} \) s⁻² and one underwater barrier (65 m high and 100 m wide).

Uniform density 1021 kg/m³ with one underwater barrier. The height and width of the barrier are 55 m and 100 m.

The stratified density (the density at the surface is 1021 kg/m³ while the lower layer is calculated based on the stability of the Brunt Vaisala Frequency with \( N^2 = 5 \times 10^{-4} \) s⁻²) and two submarine barriers (height and width of the first barrier 55 m and 100 m, while the height and width of the second barrier are 65 m and 100 m.

Uniform density of 1021 kg/m³ with two barriers. The height and width of the first barrier are 55 m and 100 m. The height and width of the second barrier are 65 m and 100 m.

In all scenarios, the canal length and depth are 500 m and 100 m, respectively, where \( \Delta x = \Delta y = 5 \) m. In all scenarios, a sea-level gradient of 0.5 m is given as open boundary values. The value at the lateral boundary is cyclic with the equation as follows:

\[
\text{Forcing term} = - g \frac{\partial \eta_o}{\partial x} \tag{8}
\]

where \( \partial \eta_o / \partial x \) a is the defined ambient sea level gradient, the reference density \( (\rho_o) \) used is 1028 kg/m³. Numerical simulation is run for 5 hours with time step \( \Delta t = 30 \) s. The simulation configuration can be seen in Figure 1 as follows:
3. Results and Discussions

3.1. One barrier numerical simulation with stratified density

The numerical simulation results of the internal waveform of a barrier with stratified density are shown in Figure 2 (a - e). Figure 2 (a) when $t = 1$ h at a depth of 20 - 60 m at position $x = 150 - 250$ m, the current and density resonate with the topography [19] with a current velocity of 0.2 m/s. Because the current hits the barrier at a depth of 60 - 100 m, the current is pushed upwards towards the top of the barrier and causes the current above the barrier to be disturbed. Meanwhile, Figure 2 (b) when $t = 2$ hours shows that the current speed increases over the barrier by 0.5 - 0.7 m/s. Current and density resonate along the channel. Then in Figure 2 (c - e), when $t = 3$ hours to 5 hours, there is significant turbulence in currents and densities along the channel. It is due to the increasingly high current speed. The current speed increases when $t = 3$ hours until $t = 5$ hours of simulation from 0.5 - 1.2 m/s to 2 - 2.3 m/s. It is due to the change in density, which is not constant [5]. The density undergoes mixing above and in front of the barrier.

Figure 2. (a) Current and stratified density conditions with one seabed barrier, when the simulation time $t = 1$ hour, (b) $t = 2$ hours, (c) $t = 3$ hours, (d) $t = 4$ hours and (e) $t = 5$ hours
3.2. One barrier numerical simulation with uniform density
The numerical simulation of a barrier with uniform density is shown in Figure 3 (a - e). In Figure 3 (a-c) when \( t = 1 \) h to \( t = 3 \) h, the current forms a rotor [19] in front of the barrier at a depth of 30-90 m. When \( t = 1 \) hour, the current velocity is 0.3 m/s. As time increases, the current moves faster over the barrier. When \( t = 2 \) hours at position \( x = 200 - 450 \) m a cavity is formed [19] in front of the barrier. When \( t = 3 \) hours at position \( x = 350 - 450 \) m, the current forms the rotor, and the current velocity over the barrier can reach 0.7 m/s. Then in Figure 3 (d), when \( t = 4 \) hours, the current forms a rotor in front of the barrier at position \( x = 270 - 450 \) m at a depth of 40 - 90 m. The current velocity over the barrier reaches 1.4 m/s. Furthermore, in Figure 3 (e), when \( t = 5 \) hours, speed, current forms the rotor in front of the barrier at position \( x = 200 - 450 \) m, and the current speed gets stronger to 2.2 m/s.

3.3. Two barriers numerical simulation with stratified density
The numerical simulation results of the internal waveform of two barriers with stratified density are shown in Figure 4 (a - e). In Figure 4 (a - b), when \( t = 1 \) hour to 2 hours, the current and density resonate with the topography along the channel. The current velocity increases from 0.2 m/s to 0.5 m/s as time increases over the barrier. Density when \( t = 3 \) hours to \( t = 5 \) hours, turbulence occurs, as shown in Figure 4 (c - e). The currents at each layer of depth along the channel resonate behind the barrier, above the barrier, and in front of the barrier. The current velocity in front of the barrier reaches 1.5 m/s.

3.4. Two barriers numerical simulation with uniform density
The numerical simulation of two uniform density barriers is different from the scenario of two barriers of stratified density. This numerical simulation does not show any change in stratified density. The results of the observations are shown in Figure 5 (a - e). In Figure 5 (a), when \( t = 1 \) hour, the current has formed a rotor in front of the barrier at position \( x = 200 \) m - 350 m. Whereas in Figure 5 (b), when \( t = 2 \) hours, the current resonates behind the barrier at a depth of 20 - 90 m and resonates in front of the barrier at 30 - 70 m. Furthermore, in Figure 5 (c), when \( t = 3 \) hours of simulation, the current forms a rotor in front of the barrier with a current speed of 0.3 - 1.5 m/s at position \( x = 350 - 450 \) m. Then in Figure 5 (d...
- c), when $t = 4$ hours to 5 hours, the current velocity increases to 3.2 m/s, the current turbulences behind and in front of the barrier.

**Figure 4.** (a) Current conditions and stratified density with two seabed barriers, when the simulation time $t = 1$ hour, (b) $t = 2$ hours, (c) $t = 3$ hours, (d) $t = 4$ hours and (e) $t = 5$ hours

**Figure 5.** (a) Current conditions and uniform density with two seabed barriers, when the simulation time $t = 1$ hour, (b) $t = 2$ hours, (c) $t = 3$ hours, (d) $t = 4$ hours and (e) $t = 5$ hours
Based on the results of the four simulations, the average current velocity data is obtained, which is presented in Table 1 below.

### Table 1. The average current velocity in four simulation cases

| Current velocity and stratified density conditions with one seabed barrier | t = 1 hour | t = 2 hour | t = 3 hour | t = 4 hour | t = 5 hour |
|---|---|---|---|---|---|
| Current velocity and uniform density conditions with two seabed barrier | 0.2 m/s | 0.5 m/s | 1 m/s | 1.7 m/s | 2 m/s |
| Current velocity and stratified density conditions with one seabed barrier | 0.2 m/s | 0.4 m/s | 0.8 m/s | 1 m/s | 1.3 m/s |
| Current velocity and uniform density conditions with two seabed barrier | 0.5 m/s | 0.8 m/s | 1.4 m/s | 2.5 m/s | 3 m/s |

Table 1 above shows that the simulation with uniform density conditions has a faster current speed than the simulation with stratified density conditions. This result is significant to know, namely, to make it easier to map areas with such density properties. The dynamics of upwelling, downwelling, and thermocline layers can be known [15, 16].

### 4. Conclusion

Numerical simulations with one and two barriers with stratified densities show that current turbulence and density form at the seawater interface (internal waves). Current turbulence and density are seen over the barrier at t = 3 h to t = 5 h. Meanwhile, in the numerical simulation, one and two barriers with uniform density currents form the rotor, cavity, turbulence, and the underwater topography. It is due to the influence of current velocity. The speed of a weak current is 0.2 m/s, and the speed of a strong current is 3.2 m/s. Simulations with uniform density conditions have a faster current speed than simulations with stratified density conditions. These results will make it easier to map areas with such density properties so that the dynamics of upwelling, downwelling, and the thermocline layer can be known.

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### References

[1] Scott R B, Goff J A, Garabato A C N and Nurser A J 2011 *J. Geophys. Res.* **116** C09029.
[2] Preusse M, Peeters F, and Lorke A 2010 *Limnol. Oceanogr.* **55** 2353.
[3] Rizal S, Iskandar T, Muhammad, Haditiar Y, Ilhamsyah Y, Setiawan I and Sofyan H 2019 *J. Eng. Sci. Technol.* **14** 2836.
[4] Craig W, Guyenne P and Sulem C 2012 *Journal of Fluid Mechanics* **710** 277
[5] Chen Z W, Xie J, Wang D, Zhan J M, Xu J and Cai S 2014 *Journal of Geophysical Research. Ocean* **119** 7029.
[6] Palmer M R, Stephenson G R, Inall M E, Balfour C, Dusterhus A and Green J A M 2015 *Journal of Marine System* **144** 57.
[7] Hermansyah H and Jaharuddin 2011 *Jurnal Matematika dan Aplikasinya* **10** 1.
[8] Shanmugam G 2013 AAPG Bulletin 97 799.
[9] Walter R K, Stastna M, Woodson C B and Monismith S G 2016 Continental Shelf Research 117 100.
[10] Marshall J and Speer K 2012 Nature Geoscience 5 171.
[11] Walter R K, Squib M E, Woodson C B, Koseff J R and Monismith S G 2014 Journal of the Geophysical Research Ocean 119 8709.
[12] Waterman S, Garabato A C N and Polzin K L 2013 Journal of Physical Oceanography 43 259
[13] Garabato A C N, Nurser A J G, Scott R B and Goff J A 2013 Journal of Physical Oceanography 43 647.
[14] Matthews J P, Aiki H, Masuda S, Awaji T and Ishikawa Y 2011 J. Geophys. Res. 116 C05007.
[15] Irmasyithah N, Haditiar Y, Ikhwan M, Wafdan R, Setiawan I and Rizal S 2019 Thermocline studies using CMEMS data in the Andaman Sea during October 2017 IOP Conf. Ser.: Earth Environ. Sci. 348 1 012064.
[16] Ardila D, Haditiar Y, Ikhwan M, Wafdan R, Sugianto S and Rizal S 2019 Ocean eddy detection in the Andaman Sea during April 2008 IOP Conf. Ser.: Earth Environ. Sci. 348 1 012063.
[17] Kämpf, Jochen. 2010. Advanced Ocean Modelling, Using open-source software. Berlin, Germany. Springer Science and Business Media.
[18] Rizal S, Wafdan R, Haditiar Y, Ramli M and Halfiani V 2020 J. Eng. Sci. Technol. 15 1056
[19] Stull R B 2011 Meteorology for scientists and engineers (United States of America: Univ. of British Columbia).