To Enjoy the Morning Flower in the Evening——Where is the Subtlety of Quantum Mechanics? *

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Abstract

Why does the \( i = \sqrt{-1} \) appear essentially in the quantum mechanics? Why are there operators and noncommutativity (the uncertainty relation) in the quantum mechanics? Why are these two aspects closely related and indivisible? In probing these problems, a new point of view is proposed tentatively.

Dirac, who had made prominent contribution to the establishment and development of Quantum Mechanics (QM), said in 1970 that in the past he deemed the “noncommutativity” characterized by the Planck constant \( \hbar \) being the main feature of QM, but after 1970 he tended to consider the existence of probability amplitude, i.e., the wave function as the most important one. He further pointed out that where the appearance of phase in wave function looks the most subtle.

In C.N. Yang’s thinking, both two above features are very important and form the revolutionary progress with far-reaching meaning in the description of nature. Moreover, he pointed out that these two aspects are related intimately and there is an imaginary unit \( i = \sqrt{-1} \) appearing in them [1].

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1 Why $i = \sqrt{-1}$ appears in QM?

Being one of the founders of QM, Schrödinger proposed his famous equation by writing six papers successively. Among them all the weal and woe are closely around an important problem whether it is necessary to introduce the imaginary unit $i = \sqrt{-1}$?[1] Eventually, when he made up his mind to write it in, he brought an epoch-making reform into physics.

It is well known that in classical physics or electric engineering the use of $i$ is rather convenient. If the alternative current is denoted by $I(t) = I_0 e^{i\omega t}$, it is quite easy to do calculation in calculus. Finally one has to pick the real part of result corresponding to the measured quantity.

However, the introduction of $i$ in QM is by no means a matter of convenience but a need in essence. If discarding $i$, the Schrödinger equation would reduce into a classical equation similar to that describing the heat conduction or particle diffusion, it would be totally impossible to describe the so-called “wave particle dualism” of microscopic particles.

Today we already understand that the most important or subtle aspect in QM is the phase of wave function, which in turn must be expressed by $i$. Though the Born statistical interpretation was in conformity with the experiments, the phase was erased and the $i$ disappeared too. Despite the brilliant success of QM, its fundamental interpretation remains still in a rather obscure status. This is a seldom phenomenon in the history of natural science. How long time need we further to wait for? Let me try to propose some personal understanding for the reference of our readers.

2 The phase transformation of wave function

One used to discuss the phase transformation of wave function $\psi$ in QM as follows:

$$\psi \rightarrow \psi' = e^{i\theta} \psi$$

$$\text{Re}\psi \rightarrow \text{Re}\psi' = \cos \theta \text{Re}\psi - \sin \theta \text{Im}\psi$$

$$\text{Im}\psi \rightarrow \text{Im}\psi' = \sin \theta \text{Re}\psi + \cos \theta \text{Im}\psi$$
There are three interesting properties in the wave function $\psi$:

(a) The real part $\text{Re}\psi$ and imaginary part $\text{Im}\psi$ are all real numbers, but both of them are nonobservable.

(b) The distinction between $\text{Re}\psi$ and $\text{Im}\psi$ is relative but necessary. One side regards the existence of other side as the premise of the existence of itself. In other words, they are indivisible, any one side can not exist alone. A young man Jia Baoyu (in the classic Chinese novel “Dream of the red mansion” by Cao Xueqin and Gao e around 1760) caught an idea one day. He wrote as follows:

You won’t be you if I wasn’t born.
One can’t understand her from her alone.

(c) $\text{Re}\psi$ and $\text{Im}\psi$ can transform into each other as shown in Eq.(2) while keeping $|\psi|^2 \longrightarrow |\psi'|^2 = |\psi|^2$ invariant.

We should not overlook these three properties, which go beyond the “atomism” familiar to western science community. This is why many physicists feel QM being too abstract. Actually, this kind of concept already existed in Chinese philosophy. If we set $\text{Re}\psi$ and $\text{Im}\psi$ in correspondence with “yin” (feminine or negative) and “yang” (masculine or positive), then the complex expression of wave function, its phase transformation and Eq.(2) are nothing but the mathematical description of the theory of “Yuan-qi” (the primary gas, see Ref[2]). They also correspond precisely to the principles of “contradiction theory” in philosophy. The opposition of “yin” and “yang” comprises the “repulsiveness” of contradiction while their mutual dependence and transformation property comprise the “identity” of contradiction.

3 Subtlety in the use of $i = \sqrt{-1}$

The introduction of $i = \sqrt{-1}$ with the property $i^2 = -1$ reflects exactly the law of opposition and mutual transformation of two sides in contradiction. The use of mathematical language in physics is the most precise one, it is nearly no substitute. Suppose there is no $i$ in QM, the situation would become very serious. People would regard the wave function as some quantity describing the direct observable. That would be totally wrong. In fact, the development of physics to the stage of QM has been penetrating into the essence of matter. The essence is nothing but contradictions, which are invisible and untouchable. The introduction of $i$ in QM just prevents people
from creating the extravagant hopes which regard the wave function as direct observable. On the other hand, it undertakes the mission to search for the essence of matter successfully. Hence, the quantum theory is actually a theory of contradiction. Then the question arises: "What contradiction does the wave function in QM represent?"

Notice that the Schrödinger equation is always a linear and homogeneous equation for $\psi(x, t)$ in various external field $V(x)$ (in one dimensional space). This comprises an acute contrast with the classical mechanics.

Assume that $V(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$. In classical mechanics, $x$ denotes the position coordinate of particle. Then the Newtonian equation $F = ma$ with the external force $F = -\frac{dV}{dx}$ and acceleration $a = \frac{d^2x}{dt^2}$ leads to equation

$$m\frac{d^2x}{dt^2} + a_1 + 2a_2x + 3a_3x^2 + \ldots = 0 \quad (3)$$

If $a_1 \neq 0, a_2 = a_3 = \ldots = 0$, Eq.(3) is linear but inhomogeneous. If $a_1 = 0, a_2 \neq 0, a_3 = a_4 = \ldots = 0$, Eq.(3) is linear and homogeneous. If in general case, $a_3 \neq 0, \ldots$, Eq.(3) is nonlinear.

The reason of appearance of nonlinear equation in classical mechanics lies in the fact that being the coordinate of particle, $x$ forms directly the passive (dependent) variable of an ordinary differential equation with one active (independent) variable $t$ (time) only. Things are quite different in QM, where the $x$ in wave function $\psi(x, t)$ is not the coordinate of particle, but a flowing coordinate describing the position in space and forms the active variable together with $t$. The Schrödinger equation with $\psi$ as its passive variable is always a linear and homogeneous partial differential equation.

The above fact enlightens us to interpret the wave function $\psi(x, t)$ in QM as the abstract representation of contradiction of the particle with its environment and $V(x)$ as the potential energy describing the strength of interaction between them[3]. Then the linear and homogeneous property of equation can be explained as the feature of contradiction interaction. The interaction between particle and its environment is not realized via the external force pushing forward but an interaction of percolation type—the external cause works only via the internal cause.
4 How large is an electron?

Some reader might think that “What a contradiction? It is invisible and so can be talked about at one’s disposal. I don’t believe in it.” Good, the problem is just in nowhere but the extravagant hope to see the essence directly whereas the latter, i.e., the contradiction is just invisible before its excitation and transmutation. What we see are phenomena, which can not be identified as the essence. The latter is always hiding inside and don’t exhibit outside. It only displays itself clearer and clearer with the increase of excitation energy. Nonetheless, theoretical analysis is needed for handling the essence.

After an electron is captured by a proton, a Hydrogen atom is formed. An amount of energy 13.6eV, called as binding energy, is delivered. On the contrary, if an amount of energy larger than B is injected into the Hydrogen atom, it will be ionized. According to the theory of Special Relativity (SR), a free electron has a “rest energy” $E_0 = m_e c^2 = 0.511 MeV$.

Attention must be paid to the difference between the binding state of an electron in a Hydrogen atom and its free state. However, because $B/E_0$ is far less than 1, people often neglect the difference between these two kinds of electron. An electron in free space is described by plane wave function whereas its wave function of 1s state binding in Hydrogen atom reads $\psi(r) = e^{-r/a}/(\pi a^3)^{1/2}$ with Bohr radius $a = 0.0529nm$ and $r$ being the distance measured from the nucleus (proton).

People often deemed the electron as a point-like particle. If with this view, it would be very difficult to get rid of the concept that the electron is in some orbital motion inside the Hydrogen atom, let alone to explain why it will not emit the electromagnetic wave. After the establishment of QM, people cognized that the stationary state as shown by the 1s wave function corresponds to some “de Broglie stationary wave”, which does not emit the electromagnetic wave and will never fall into the nucleus.

According to the uncertainty relation in QM, the smaller the size of electron wave packet is compressed, the higher its kinetic energy will be, i.e., the stronger its tendency against the compression will be. If the charge number of nucleus increases from 1 to $Z$, then $a \rightarrow a/Z$ in the 1s wave function of this Hydrogen-like atom implies the spreading radius of electron shrinking to a factor of $Z$, while the binding energy enhances to a factor of $Z^2$.

I wish to raise a bold conjecture that an electron is something arbitrary in
size. When it is bound in 1s state of Hydrogen-like atom, an electron shows a spherical distribution with radius $a/Z$. When it is under the collision of other high energy particle, an electron behaves itself like a point particle with radius less than, say, $10^{-16} \text{cm}$.

5 Two basic points of view in epistemology

The above claim is based on the following two points of view.

The existence state of an object is depending on its environment. Certain (many body) environment determines its property, e.g., its mass, stability (life), etc. The prominent merit of physics in 20th century lies in the fact that it enables us begin to analyse such kind of problem by means of the theory like QM.

When we wish to cognize the state of an object, we have to resort to some measurement. At that time the state of object changes abruptly. Hence the state of object exhibiting in the measurement is strongly depending on the style of measurement, i.e., on the device of changing method and its strength, etc. Though the study of measurement theory in physics is far from enough, but one thing is beyond any doubt that there is intervention of apparatus and subject into the measurement. No cognition can be talked about without subject. An information is created by subject and object in common. No any data exists before the change in the state of an object occurs in the measurement.

Therefore, in our point of view, it is groundless to identify the tiny radius of electron shown in the high energy collision as its size when it is in low energy state. On the other hand, we should not look at the $x$ or $p$ in the wave function (e.g., the plane wave function or that in 1s state) as the position or momentum of electron before the measurement.

In nonrelativistic QM, a wave packet of freely moving particle will spread unceasingly, a phenomenon difficult to understand even in the Born statistical interpretation. In our point of view, the wave function $\psi(x, t)$ is not the probability amplitude in distribution of position $x$ of particle but the distribution of “contradiction field” under the interaction between the particle and its environment. One supposes that at $t = 0$ the wave function $\psi(x, 0)$ has a distribution as a wave packet, and assumes again that $V(x) = 0$ i.e., there is no interaction between particle and environment. Then being the “con-
tradiction field”, the wave packet has no way but spreads and approaches to zero. It is totally reasonable, no any difficulty exists in understanding.

6 What we learn from the EPR paradox?

Einstein had made prominent contribution to the establishment of quantum theory, but he had doubts about QM especially the uncertainty relation. He doubted first of the consistency of QM, then of its completeness. In 1935, Einstein, Podolsky and Rosen wrote a paper to raise the question whether the description of QM on the ”physical reality” is complete. They tried to explore the possible incompleteness of QM via ideal experiments. (see e.g., Ref [4]).

One of the so-called EPR experiments was performed by C.S.Wu et al in 1975. A positronium, i.e., the binding state of $e^+$ and $e^-$ will annihilate into two photons (see Ref [5]):

$$e^+ + e^- \rightarrow \gamma_1 + \gamma_2$$

(4)

The positronium begins at a rest state with spin zero. According to the laws of conservation of momentum and angular momentum, two photons emitted along two opposite directions must be either both in right polarization state $| R_1 R_2 \rangle$ or both in left polarization state $| L_1 L_2 \rangle$. The QM reveals that the final state after decay is a linear superposition of the above two states:

$$| F \rangle = | R_1 R_2 \rangle - | L_1 L_2 \rangle$$

(5)

There are two observers, say, Bob and Alice. Bob (Alice) performs the measurement at the west (east) side. Both of them use the linear polarizer to detect the light. A photon with circular polarization, either left or right, will have 50% of probability in either $x$ polarization or $y$ polarization.. According to the calculation of QM, the probability amplitude that Bob has measured the $\gamma_1$ in $x$ polarization state $| x_1 \rangle$ while Alice measured the $\gamma_2$ in $y$ polarization state $| y_2 \rangle$ reads

$$\langle x_1 y_2 | F \rangle = i$$

(6)
which is understandable. However, the QM predicts that the probability amplitude for both measurements being in \( x \) (or \( y \)) polarization equals zero:

\[
\langle x_1 x_2 \mid F \rangle = \langle y_1 y_2 \mid F \rangle = 0 \tag{7}
\]

From EPR’s point of view, the above result seems a violation in the belief of “physical reality” and so is difficult to be accepted. Consider the original appointment was made to measure the process (6). After \( \gamma_1 \) and \( \gamma_2 \) are separated to a long distance, Bob suddenly changes his mind and measures the \( y \) polarization of \( \gamma_1, | y_1 \rangle \). It seems that Bob’s change should not influence the outcome of Alice’s measurement, i.e., the zero amplitude shown in Eq.(7) can hardly be understood. However, the prediction of QM does show that in this case Alice can not find the \( \gamma_2 \) in \( y \) polarization state. Bob’s act does influence Alice’s result far away. In the past 60 years, especially after the study of Bell’s Inequality, the accuracy of EPR experiments has been improved year after year with the distance between Bob and Alice increase even to 10 Kilometers [6]. The experiments have been verifying the correctness of prediction of QM and exclude the existence of so-called “local hidden variable”.

In our point of view, the study on EPR paradox in 60 years has been telling us one important thing that the quantities appear in the calculation of QM like the position, the momentum, the angular momentum or spin orientation, etc., are all not the direct observables. To be precise, they do not really exist before the measurement. For example, in the entangled two-photon state, Eq.(5), before the quantum coherence is destroyed by the measurement, no one of these two photon is in left polarization or in right polarization. Similarly, to talk about \( x \) polarization or \( y \) polarization is meaningless before the measurement. The reason why we felt strange about EPR paradox is nothing but that we might look at the ”description” in QM too seriously.

7 The quantum state, wave function, operator and the uncertainty relation

According to the rigorous notation of Dirac, a state of microscopic particle is denoted by an abstract state vector \( | \Psi \rangle \), which even does not contain the
time \( t \) in the Heisenberg picture, let alone \( x \) or \( p \). Then the problem of the choice of representation arises. One may choose the basic vector of position \( |x, t\rangle \) to get the wave function in configuration space, i.e., in \( x \) representation as

\[
\Psi(x, t) = \langle x, t | \Psi \rangle \tag{8}
\]

Alternatively, one may choose the basic vector of momentum \( |p, t\rangle \) to get the wave function in momentum space, i.e., in \( p \) representation as

\[
\Phi(p, t) = \langle p, t | \Psi \rangle \tag{9}
\]

Neither \( x \) nor \( p \) is more fundamental than the other one. Actually, they are not really existing in the original state \( |\Psi\rangle \) as the position or momentum of particle. At most, we could say that \( x \) or \( p \) would be the position or momentum of the particle found by us if the measurement on the state \( |\Psi\rangle \) were really made.

In our point of view, the choice of basic vector \( |x, t\rangle \) or \( |p, t\rangle \) for representation corresponds to the choice of apparatus for \( x \) or \( p \) measurement we are going to use. However, before the measurement is really done, it is just a fictitious one. Hence the corresponding wave function, i.e., the probability amplitude contains the \( i = \sqrt{-1} \), showing its unobservableness. This is why we explain the wave function as a “contradiction field” to stress its unobservableness before the transmutation of contradictions.

Once the measurement is made, it is a changing (operating) process. The transmutation of contradiction occurs immediately. For instance, the variable \( p \) in the plane wave function for free electron \( \Psi \sim \exp\{\frac{i}{\hbar}(p \cdot x - E \cdot t)\} \) is not a directly observable momentum of electron, but showing a potential possibility that under an operation of infinitesimal “space translation” (i.e., under an infinitesimal change in \( x \)), an observable \( p \) would emerge:

\[
-i\hbar \lim_{\Delta x \to 0} \frac{\Psi(x + \Delta x, t) - \Psi(x, t)}{\Delta x} = -i\hbar \frac{\partial}{\partial x} \Psi = p\Psi(x, t) \tag{10}
\]

Now we understand why an observable (like \( p \)) in classical physics evolves into an operator in QM (like \( \hat{p} = -i\hbar \frac{\partial}{\partial x} \)). In our point of view, it is just a reflection of principle in epistemology that “the measurement is nothing but an operation (changing) on the object”.

9
this principle is by no means beginning from QM. We already had it in classical physics. But people often overlooked it. We had discussed as an example the measurement of specific heat under constant volume \( c_v = \frac{\partial U}{\partial T} \bigg|_v \) or that under constant pressure \( c_p = T \frac{\partial S}{\partial T} \bigg|_p \), see Ref [6].

Notice that, however, the operation of space translation during the process for measuring the momentum \( p \) is bound to disturb the measurement of position \( x \). The nonconformity between two kinds of operation leads to the noncommutativity between these two operators and also to the uncertainty relation. Even in classical physics, we would meet the similar situation when measuring the \( c_v \) and \( c_p \) at the same time.

In summary, we have tried to answer the question raised at the beginning of this article: why the two aspects of subtlety—the noncommutativity and the existence of probability amplitude—are correlated intimately. The reason lies in the fact that any observable is nothing but a consequence of transmutation in contradiction induced in a changing process. The ’contradiction” is characterized by \( i = \sqrt{-1} \) and its transmutation characterized by the Planck constant \( \hbar \).

8 A particle is an excitation state of fundamental contradiction in nature

At the level of QM, the contradiction is relatively weak and the change of particle is small. For instance, one often ignores the distinction between the free electron and the electron inside a Hydrogen atom. Once when the energy exceeds \( E_0 \) and so the electron itself could be created or annihilated, i.e., at the level of quantum field theory, we are facing directly the excitation and transmutation of fundamental contradictions in nature.

As an example, the collision between electron \((e^-)\) and positron \((e^+)\) would create a \( J/\psi \), a meson. The rest mass of \( J/\psi \) (3.097GeV/c^2) is 6000 times as much as that of an electron (0.511MeV/c^2). How can one say that \( J/\psi \) is hiding originally inside the \( e^+ \) or \( e^- \)? We prefer to say that \( J/\psi \) is excited from the vacuum. Usually, it is considered as a binding state of a pair of quark-antiquark, \( J/\psi = (c - \bar{c}) \). However, \( c \) or \( \bar{c} \) has never been detected in free state. If the energy is raised into \( 2 \times 1.868GeV \), the ”bond” between \( c \) and \( \bar{c} \) is broken while a new quark pair, \( d \) and \( \bar{d} \), is created from the vacuum.
to form two separated $D$ mesons, $D^+ = (c - \bar{d})$, $D^- = (\bar{c} - d)$. The isolated quark has never been seen and is called as the confinement of quark.

In the beginning of 20th century, people were bothered by the puzzle "why an electron will never fall into the nucleus?" Now people feel strange again by the puzzle "why the quark can not escape from the hadron?" It seems that a radical change in the concept of matter structure is inevitable. We can not prevent from querying that as people had underestimated the repulsiveness of contradiction again and again, if we may still underestimate the identity of it today?

In comparing with the discussion on the real and imaginary parts of wave function, we guess that the quark may be some representation of fundamental contradiction. While quark $c$ ($\bar{c}$) has one character, $d$ ($\bar{d}$) has another one, both of them are different from that of $e^-(e^+)$. According to our present cognition, the difference between the wave functions of $e^-$ and $e^+$ is only ascribed to the opposite sign in its phase.

\[
\Psi_{e^-} \sim \exp\left\{\frac{i}{\hbar} (\vec{p} \cdot \vec{x} - E \cdot t)\right\}
\]
\[
\Psi_{e^+} \sim \exp\left\{-\frac{i}{\hbar} (\vec{p} \cdot \vec{x} - E \cdot t)\right\}
\]  

($E > 0$ in both case). Starting from this point of view, we are able to derive the main results of SR. Also we may understand why the wave packet of an electron in Dirac theory will not spread when $v \to c$. Rather, it undergoes a boosting effect accompanying the Lorentz contraction. This is due to the entanglement of two kinds of contradiction field, that of particle and of antiparticle, inside an electron.

9 Will the "Ether" theory revive?

The development of QM enables us try to answer the question raised by Einstein :"what is the physical reality?" It seems to us that the matter needs to be defined at two levels. When an object is independent of the consciousness of mankind and before the measurement is made, it could be called as "thing in itself". In QM it is denoted by a quantum state $|\Psi\rangle$ separated approximately from its environment and contains no information.
Then when it appears as various phenomena, reflecting a series of experimental data in our measurement, it is turned into "thing for us". The wave functions in QM just connects the quantum state to phenomena via fictitious measurements. As Wheeler said: "No phenomenon is phenomenon until it is an observed phenomenon."

In Fig 1 (see the original paper in *Kexue* (Science) Vol. 50 No. 2, 38, 1998 or the similar one in Internet hep-th/9508069), a particle is shown as certain excitation of the vacuum, which may also be called as "Yuan-qi" (the primary gas, [2] ) or "Ether". The lower part of Fig 1 is not directly observable. But after the excitation of particle with momentum ($\vec{p}$) and energy ($E$) as its existence form, we may also say that the space ($\vec{x}$) and time ($t$) are the existence form of the vacuum. An universal constant, the speed of light $c$, exhibits itself as the horizontal link. On the other hand, the vertical excitation is displayed by the quantum operator relationship:

\[
\begin{align*}
\vec{p} &= -i\hbar \nabla, \\
\vec{p}_c &= i\hbar \nabla \\
E &= i\hbar \frac{\partial}{\partial t}, \\
E_c &= -i\hbar \frac{\partial}{\partial t}
\end{align*}
\]

where the Planck constant $\hbar$ exhibits as the vertical link. The left two relations in Eq.(12) are well known in QM whereas the right two with subscript $c$, which belong to the antiparticle (see Eq.(11)), are provided by SR in our point of view. Eq.(12) as a whole are just the DNA in quantum field theory and particle physics inherited from QM and SR each with 50% [8].

It seems to us that the revival of a new "Ether" theory based on "Yuan-qi" is inevitable. It is the time to fuse two kinds of wisdom of both eastern and western. Sooner or later, we will be convinced by the saying in Chinese philosophy: "Oneness of heaven and man".

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