Magnetic Flux Transport in Radiatively Inefficient Accretion Flows and the Pathway toward a Magnetically Arrested Disk

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Received 2022 July 28; revised 2023 January 6; accepted 2023 January 19; published 2023 February 23

Abstract

Large-scale magnetic fields play a vital role in determining the angular momentum transport and generating jets/outflows in accreting systems, yet their origins remain poorly understood. We focus on radiatively inefficient accretion flows (RIAFs) around black holes (BHs), and conduct 3D general-relativistic magnetohydrodynamic simulations using the Athena++ code. We first reconsider that the magnetorotational instability driven dynamo in the RIAF alone does not spontaneously form a magnetically arrested disk (MAD), conducive for strong-jet formation. We next investigate the other possibility, where the large-scale magnetic fields are advected inward from external sources (e.g., the companion star in X-ray binaries and the magnetized ambient medium in active galactic nuclei). Although the actual configurations of the external fields could be complex and uncertain, they are likely to be closed. As a first study, we treat them as closed field loops of different sizes, shapes, and field strengths. Unlike earlier studies of flux transport, where the magnetic flux is injected into the initial laminar flow, we inject the magnetic field loops into the quasi-stationary turbulent RIAF in inflow equilibrium, then follow their evolution. We find that a substantial fraction (\sim 15\%-40\%) of the flux injected at large radii reaches the BH, with a weak dependence on the loop parameters, except when the loops are injected at high latitudes, away from the midplane. The relatively high efficiency of the flux transport observed in our study hints that a MAD might easily be formed relatively close to the BH, provided that a source of the large-scale field exists at larger radii.

\textit{Unified Astronomy Thesaurus} concepts: Accretion (14); Magnetic fields (994); Relativistic fluid dynamics (1389); Relativistic disks (1388); Black holes (162)

1. Introduction

Astrophysical accretion disks influence systems over a wide range of scales, spanning from planet formation to galaxy evolution. They also energize the most powerful sources in the universe. For example, disks orbiting around stellar-mass black holes (BHs) and neutron stars are considered to be among the most luminous X-ray sources in the sky (Remillard & McClintock 2006). Active galactic nuclei (AGNs), powered by the accretion of matter onto supermassive BHs at the centers of galaxies, are not only the most powerful sources, but the energy that is released by an AGN also provides feedback to the entire galaxy and determines its evolution (Silk & Rees 1998; Harrison 2017; Morganti 2017).

Broadly speaking, accretion occurs via three different modes: (i) geometrically thin and optically thick Keplerian disks (standard disks; Novikov & Thorne 1973; Shakura & Sunyaev 1973); (ii) geometrically thick and optically thin radiatively inefficient accretion flows (RIAFs; Chakrabarti 1989; Narayan & Yi 1994; Blandford & Begelman 1999); and (iii) geometrically and optically thick slim disks (Abramowicz et al. 1988). Slim disks accrete matter at a super-Eddington rate, while the mass accretion rates $\dot{m}$ are sub-Eddington for both standard disks ($10^{-2} \lesssim \dot{m} / \dot{m}_{\text{Edd}} \lesssim 1$) and RIAFs ($\dot{m} / \dot{m}_{\text{Edd}} \lesssim 10^{-4}$).

In this paper, we focus on RIAFs, with disks likely spanning most of their time (such as the disks around Sgr A* and the supermassive BH in M87; e.g., Yuan & Narayan 2014) and with dynamics that are relatively simple compared to the other two states.

The structures and evolutions of rotationally supported accretion disks are primarily determined by the process of angular momentum transport. The current consensus is that the magnetorotational instability (MRI; Balbus & Hawley 1991) gives rise to angular momentum transport and vigorous turbulence in a fully ionized accretion flow (e.g., in X-ray binaries, or XRBs, the inner parts of AGN disks, and sufficiently ionized parts of protoplanetary disks). The MRI becomes more efficient in angular momentum transport if the accretion disk is threaded by a net vertical magnetic flux. This has been observed in local shearing-box simulations, where the presence of a net vertical magnetic flux enhances the MRI turbulence and hence the angular momentum transport (Bai & Stone 2013). Additionally, net flux threading the disk helps to launch the winds/outflows (Bai & Stone 2013; Suzuki et al. 2014).

A large-scale magnetic field close to the central accretor (a BH or a neutron star) is a necessary ingredient for jet production in accreting systems (Blandford & Znajek 1977; Blandford & Payne 1982). It has been proposed that a RIAF saturated with strong poloidal magnetic flux close to the BH would provide an ideal condition for jet production (Bisnovatyi-Kogan & Ruzmaikin 1974; Esin et al. 1997; Fender et al. 1999; Narayan et al. 2003; Meier 2005). This idea has been verified in numerical simulations (Igumenshchev et al. 2003; Narayan et al. 2012). The studies
The MAD model predicts a correlation among the mass accretion rate, the magnetic flux threading the BH, and the jet power, which has been found to be in agreement with observations of radio-loud AGNs (Ghisellini et al. 2014; Zamaninasab et al. 2014). Recent polarization studies of M87 at 230 GHz from Event Horizon Telescope observations (Event Horizon Telescope Collaboration et al. 2021a, 2021b; Yuan et al. 2022) have also inferred the presence of a dynamically important near-horizon-organized poloidal magnetic flux that is consistent with general-relativistic magnetohydrodynamic (GRMHD) models of MAD.

What could be the possible source of the magnetic flux close to the BH? Most of the numerical simulations of MAD start with a strong enough large-scale poloidal flux that is eventually brought close to the BH and gets accumulated by flux freezing (Tchekhovskoy et al. 2011; McKinney et al. 2012). However, the source of the large-scale field is not entirely obvious (see also Begelman et al. 2022). It could potentially be generated in the disk itself, by a dynamo action (Bugli et al. 2014; Mattia & Fendt 2020; Vourelis & Fendt 2021; Mattia & Fendt 2022), or it could be advected in from some external sources (Cao 2011; Li & Cao 2021).

The efficiency of the dynamo action in generating the coherent and strong large-scale poloidal field that is required to produce strong jets has been found to be different for different numerical simulations of RIAFs. The \( \alpha \) – effect (responsible for the generation of the poloidal field by a dynamo) has been found to be weak in simulations that start with small poloidal magnetic loops (Hogg & Reynolds 2018a; Dhang & Sharma 2019; Dhang et al. 2020). The quasi-stationary states of these simulations are in a weakly magnetized regime, popularly known as “standard and normal evolution” (SANE; Narayan et al. 2012). However, recent simulations with a very strong (with a gas-to-magnetic pressure ratio \( \beta \approx 5 \)) and coherent initial toroidal field have shown that the production of large-scale poloidal field loops of scale height \( H \times R \) eventually led to a MAD (Liska et al. 2020). Therefore, it is worth noting that simulations need to start with a strong and coherent large-scale field (either poloidal or toroidal) in order to achieve a MAD.

In addition to in situ generations of a magnetic field by a dynamo process, it might be possible that an initially weak field supplied to the disk—from the outer part of the disk or from a companion star, in the case of XRBs, or from the ambient medium, in the case of AGNs—can in principle be amplified by flux freezing. Flux accumulation near the BH depends on the relative efficiency between the inward advection by the accretion flow and the outward diffusion due to turbulent resistivity (Lubow et al. 1994). Additionally, turbulent pumping can also cause the outward transport of the large-scale magnetic field in a dynamo-active accretion flow (Dhang et al. 2020). However, a few studies have proposed that vertical magnetic field accretion can be efficient in the hot tenuous surface layer (the coronal region, where the radial velocity is comparatively higher compared to that in the midplane) in a hot accretion flow (Beckwith et al. 2009). It is also interesting to note that simulations of large-scale accretion flows around the galactic center fed by the magnetized winds of Wolf–Rayet stars also show the efficient inward transport of the magnetic field toward the center (Ressler et al. 2020a, 2020b).

This paper studies the magnetic flux transport in a fully turbulent RIAF, in contrast to previous studies, where the magnetic flux is injected as part of the initial laminar condition. First, therefore, we run a simulation to attain a quasi-stationary RIAF in the SANE regime (weakly magnetized). We then inject the external magnetic flux on top of the existing magnetic field in this turbulent SANE RIAF. In the latter part of the paper, we refer to this as the “Initial RIAF” run. It is customary to use net vertical magnetic flux threading the disk to investigate the flux transport (Beckwith et al. 2009; Zhu & Stone 2018; Mishra et al. 2019). However, we argue that the geometry of the external magnetic field is likely to be closed. In this paper, as a first step, we use magnetic field loops, as the simplest possible forms of the external magnetic field, studying their transport in the turbulent RIAF and the possibility of saturating the BH with magnetic flux toward the MAD regime.

We use GRMHD simulations to study the magnetic flux transport. The usage of the general-relativistic approach is crucial to our work. Many of the important diagnostics that we use in our work involve computing fluxes (the mass, angular momentum, and magnetic flux) at the event horizon of the BH. However, Newtonian MHD simulations suffer from the effects of inner boundary conditions, which artificially affect the evolution of the flow and the magnetic flux close to the BH. This is avoided in GRMHD, by placing the inner boundary within the event horizon, so that the computation domain is causally disconnected from the inner boundary. We also neglect radiation physics in our GRMHD simulations, as radiation is supposed to play an insignificant role in determining the dynamics of RIAFs with low accretion rates (see also Dexter et al. 2021).

The paper is organized as follows. In Section 2, we discuss the solution method and the physical setup of the RIAF simulations. In Section 3, we discuss the evolution of the flow, the convergence, and the magnetic state of the Initial RIAF run. We describe the method of flux injection and its results in Section 4. Finally, the key points from the results are discussed and summarized in Sections 5 and 6.

2. Method

We performed two sets of simulations. In the first set, we performed a simulation to achieve a fully turbulent quasi-stationary RIAF that forgets the initial conditions. This is labeled as the “Initial RIAF” run. In the second set of simulations, we restart the Initial RIAF run at late times, inject external magnetic flux, and study its evolution in terms of the different parameters that are associated with the injected magnetic field loops. In this section, we describe the formulations of the simulations and their setups. We note that the initial condition of the Initial RIAF run is specified in this section. The parameters related to the restart setup will be described in Section 4.

2.1. Equations Solved

We solve the ideal GRMHD equations,

\[
\partial_t (\sqrt{-g} \rho u^j) + \partial_a (\sqrt{-g} j^{aj}) = 0, \tag{1}
\]

where \( j^{aj} \) is the magnetic flux density tensor, \( u^j \) is the fluid velocity, \( \rho \) is the mass density, and \( g \) is the determinant of the metric tensor.
in spherical-like Kerr–Schild coordinates \((t, r, \theta, \phi)\), with \(G = c = M\) = 1. All the length scales and timescales in this work are expressed in units of the gravitational radius \(r_g = GM/c^2\) and \(t_g = r_g/c\), respectively, unless stated otherwise. Here, \(g_{\mu\nu}\) and \(g\) are metric coefficients and metric determinant, respectively. Following convention, the Greek indices run through \([0,1,2,3]\), while \(i\) denotes a spatial index. Equations (1), (2), (3), and (4) describe the conservation of the particle number, the conservation of the energy momentum, source-free Maxwell equations, and the no magnetic monopole constraint, respectively. Here,

\[
T^\mu\nu = (\rho + b^2)u^\mu u^\nu + \left(\rho_{\text{gas}} + \frac{b^2}{2}\right)g^{\mu\nu} - b^\mu b^\nu
\]

is the stress–energy tensor and

\[
F^{\mu\nu} = b^\mu u^\nu - b^\nu u^\mu
\]

is the dual of the electromagnetic field tensor, given that \(\rho\) is the comoving rest-mass density, \(\rho_{\text{gas}}\) is the comoving gas pressure, \(u^\mu\) is the coordinate frame four-velocity, \(\Gamma = 5/3\) is the adiabatic index of the gas, \(h = 1 + \Gamma/(\Gamma - 1)\rho_{\text{gas}}/\rho\) is the comoving enthalpy per unit mass, and \(B^\mu = \rho_{\text{gas}}^i b^i\) is the magnetic field in the coordinate frame. The four-magnetic fields \(b^\mu\) are related to the three-magnetic fields \(B^i\) as

\[
b^i = g_{\mu i} B^\mu \quad (i = 0, r, \theta, \phi)\]

For diagnostics, we also use the magnetic field components \((B_r, B_\theta, B_\phi)\), defined in a spherical polar-like quasi-orthonormal frame as

\[
B_r = B^1, \quad B_\theta = r B^2, \quad B_\phi = r \sin \theta B^3.
\]

The GRMHD code Athena++ (White et al. 2016) to perform the simulations. We employ the HLLE solver (Einfeldt 1988), with a third-order piecewise parabolic method (Colella \\& Woodward 1984) for spatial reconstruction. For time integration, a second-order-accurate van Leer integrator is used with the Courant–Friedricks–Lewy (CFL) number 0.3. We use a constrained transport (Gardiner \\& Stone 2005; White et al. 2016) update of the face-centered magnetic fields to maintain the no magnetic monopole condition.

2.2. Initial Condition

We initialize a geometrically semithick disk of aspect ratio \(\epsilon_{\text{in}} = H_G/R = 0.23\) embedded in a hot corona. Here, \(H_G\) is the Gaussian scale height. The rest-mass density distribution of the initial disk is given by

\[
\rho_d(r, \theta) = e^{-\gamma/2H_G^2} \left(\frac{r_m}{R}\right)^{\epsilon_d} \delta(r_m);
\]

and the gas pressure is given by

\[
P_{\text{gas, d}} = \rho_d c^2_{\text{s,d}} = \rho_d \cdot \epsilon_d^2 \left(\frac{M}{R}\right)
\]

Here, \(z = r \cos \theta, R = r \sin \theta\), and \(\delta(r_m) = 1/[1 + e^{-r_m/R}]\) is a tapping function with an inner disk radius \(r_m = 15\). We consider \(q_d = -1.5\) and the mass of the BH to be \(M\) = 1. It is to be noted that the Gaussian scale height \(H_G\) is related to the density-weighted scale height

\[
H = \int \sqrt{-g} \frac{\rho_{\text{gas}}}{\rho} d\theta d\phi
\]

as \(H = \sqrt{2/\pi} H_G\), and hence the disk aspect ratio \(\epsilon = \sqrt{2/\pi} \epsilon_{\text{in}}\).

The disk is surrounded by an atmosphere defined by

\[
\rho_c = \rho_{\text{in}} \left(\frac{r_m}{r}\right)^{q_c} \quad \rho_{\text{gas,c}} = \rho_c \left(\frac{M}{R}\right)
\]

with \(q_c = -1.5\) and \(\rho_{\text{in}} = 10^{-5}\). The tenuous atmosphere is static, while the gas within the disk \((\rho \geq \rho_d)\) is rotating with a Keplerian speed, given by

\[
\eta = \frac{r}{r - 2 R^{3/2}}
\]

in the Boyer–Lindquist coordinates. Also note that although two regions are set up separately and are only in approximate equilibrium, as the system evolves and becomes MRI-active, the dynamics of the atmosphere become completely overwhelmed by the internal dynamics within the RIAF, and are insensitive to the initial prescriptions in the atmosphere.

In order to attain a quasi-stationary weakly magnetized RIAF (SANE; Narayan et al. 2012), we initialize the multiple magnetic field loops (see Figure 1) using the vector potential (Penna et al. 2013)

\[
A_\phi = \begin{cases} Q \sin [f(r) - f(r_m)], & Q > 0 \\ 0, & \text{otherwise.} \end{cases}
\]

Here,

\[
Q = C_B \sin^3 \theta \left(\frac{p_1}{p_2} - p_{\text{cut}}\right)
\]

\[
f(r) = \left(r^{2/3} + \frac{15}{8 \lambda_B^2} \right) \frac{1}{\lambda_B};
\]

with \(p_1(r, \theta) = p_{\text{gas}}(r, \theta) - p_{\text{gas}}(r_0, \pi/2)\) and \(p_2(r) = p_{\text{gas}}(r, \pi/2) - p_{\text{gas}}(r_0, \pi/2)\). The vector potential \(A_\phi\) vanishes for \(r > r_0 = 200\). We choose \(C_B = 0.5, p_{\text{cut}} = 0.4,\) and \(\lambda_B = 0.75\), giving rise to an average plasma beta \(\beta = (p_{\text{gas}})/(p_{\text{mag}}) = 800\) for the initial disk (the averaging is done over the region within one scale height of the disk), with \(p_{\text{mag}}\) being the magnetic pressure.

2.3. Numerical Setup

We perform all the simulations of RIAFs around a nonspinning BH (spin parameter \(a = 0\)). The computational domain spans over \(r \in [1.94, 300]\), \(\theta \in [0, \pi]\), \(\phi \in [0, 2\pi/3]\). It is to be noted that one grid point is inside the event horizon, in the radial direction, at the root level. This allows a causally
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Figure 1. Pictorial description of the grid and the initial hydrostatic and magnetic conditions of the simulation. Each block represents a meshblock of size 16 × 14 in the poloidal plane, in the r, \( \theta \) directions, respectively. Each meshblock has 16 grid points in the azimuthal direction. The disk aspect ratio is 1/2.3, which is shown by the white dashed lines. The streamlines describe the initial poloidal magnetic field lines, with average plasma \( \beta \approx 800 \).

disconnected inner boundary. Radial grids are spaced logarithmically, while meridional grids are compressed toward the midplane using

\[
\theta = \theta_u + \frac{1 - s}{2} \sin 2\theta_u,
\]

with \( s = 0.49 \), which gives rise to \( \Delta \theta_{\text{pole}}/\Delta \theta_{\text{eq}} \approx 3.0 \). Uniform grids are employed in the azimuthal direction. To improve the effective resolution, we use two levels of static refinements with a root grid resolution of 160 × 56 × 32, giving rise to \( r : r \Delta \theta : r \Delta \phi = 1 : 2 : 3 \) at the equator in the Newtonian limit. Hence, the number of grid points increases by a factor of 2 in each direction for each level of refinement. While the first level of refinement covers \( r_{L1} \in [1.94, 180] \), \( \theta_{L1} \in [\pi/3, 2\pi/3] \), the second level of refinement is applied to the region \( 8 < r < 140, 4\pi/9 < \theta < 5\pi/9 \), such that the number of \( \theta \) cells per scale height is \( H/r \Delta \theta \approx 40 \) in the quasi-stationary state. Hence, the effective resolution at the finest level of refinement will be \( 640 \times 224 \times 128 \).

We use a pure inflow boundary condition (\( u^i \leq 0 \)) at the radial inner boundary, while at the radial outer boundary, primitive variables are set according to their initial radial gradients. The magnetic fields in the inner ghost zones are copied from the nearest computation zone. On the other hand, the magnetic fields at the outer ghost zones are set according to \( B_r, B_{\phi} \propto r^{-2} \), while keeping \( B_\theta \) unchanged from the last computation zone. Polar and periodic boundary conditions are used at the meridional and azimuthal boundaries, respectively.

We would also like to mention that floor values are used on different variables for numerical stability. Pressure and density are maintained throughout the simulation, following

\[
\begin{align*}
 p_{\text{gas, floor}} &= \max(10^{-4} r^{-5/2}, 10^{-10}), \\
 \rho_{\text{floor}} &= \max(2 \times 10^{-4} r^{-3/2}, 10^{-7}).
\end{align*}
\]

Additionally, we also constrain the following variables: \( \beta > 0.001 \); magnetization \( \sigma = 2p_{\text{mag}}/\rho < 100 \); and the Lorentz factor \( \gamma < 50 \). It is to be noted that in the saturated state, with magnetic pressure support, the floor is mainly applied in funnels (regions close to both poles) close to the BH. This is the case for all GRMHD simulations. However, it is to be noted that the floor values do not affect the results of our RIAF simulations, which are weakly magnetized.

We will introduce various diagnostics as we discuss the simulation results, with the data being averaged in different ways. Here, for future reference, we mention that the symbol “\( \cdot \)” is reserved for the azimuthally averaged mean quantities, while any additional averaging (e.g., vertical or time averaging) of the quantities will be indicated by “\( \langle \cdot \rangle \)” in this paper.

3. Evolution of the Initial RIAF Run

To begin with, we would like to investigate the plausibility of the conversion of a weakly magnetized RIAF (SANE) into a highly magnetized one (MAD), due to a dynamo action. Therefore, we perform an Initial RIAF simulation, as previously mentioned. We run the simulation for the time \( t = 1.2 \times 10^5 \), to probe whether a SANE to MAD conversion occurs. In this section, we describe the evolution of the Initial RIAF toward stationarity, its convergence, and its magnetic state.

3.1. Flow Evolution of the Initial RIAF

The top and bottom panels of Figure 2 show the time evolution of the flow for the Initial RIAF run. We particularly focus on the initial stage of evolution of the RIAF in the top panel of Figure 2, where we show how the rest-mass density (\( \rho \)) and the poloidal magnetic field \( B_r = B_{\phi} + B_\theta \) vary in time. In contrast, the bottom panel of Figure 2 focuses on the evolution of the mean (\( \phi \)-averaged; for a definition, see Equation (27)) magnetic fields at late stages.

The first panels in the top and bottom rows of Figure 2 show the initial magnetic condition—poloidal field loops of alternate signs with average \( \beta_{\text{av}} = \langle p_{\text{gas}} \rangle / \langle p_{\text{mag}} \rangle = 800 \), aiming to achieve a weakly magnetized RIAF (SANE; Narayan et al. 2012) in the quasi-stationary phase. The shear in the accretion flow converts the poloidal field into the toroidal field, while MRI amplifies the poloidal field. Therefore, both the poloidal and toroidal fields grow exponentially in a dynamical time \( t_{\text{dyn}} \approx 1/2 \times R^2/\Omega \). As a result, MRI grows faster in the disk close to the BH. Hence, the disk close to the BH breaks up earlier, compared to that further away. After few dynamical times, the system likely enters the nonlinear regime, under the influence of parasitic instabilities (Goodman & Xu 1994) or due to different super-Alfvénic rotational instabilities (Goedbloed & Koppens 2002), and finally, MHD turbulence is fully developed throughout the disk.

The second panel in the bottom row of Figure 2 shows magnetic structure at the time \( t = 10,050 \), when the MHD turbulence is fully developed throughout the region of interest \( (r < 120) \). However, it is worth noting that the system still remembers the initial field geometry, as indicated by the
alternate signs of the mean toroidal fields at different radii. As time evolves, alternate-polarity fields reconnect, and the accretion flow gradually removes the signature of the initial field geometry, as can be seen in the third panel of bottom row of Figure 2. Around the time $t = 40,200$ (the fourth panel in the bottom row of Figure 2), the accretion flow largely forgets its magnetic initial condition, and the magnetic fields generated due to an in situ dynamo start to dominate. Finally, the subsequent disk evolution is self-regulated, with a combination of MRI turbulence, dynamo action, and angular momentum transport. One point to be noted is that in the quasi-stationary phase, the toroidal magnetic field is always the dominant component, comprising almost 85% of the total magnetic field energy. We have also found (but not shown in figures) that this ratio of the toroidal to poloidal magnetic field energy modestly decreases toward the surface and within the ISCO.

Figure 2. Time evolution of the Initial RIAF run. The colors and streamlines in the top panels show the evolutions of the rest-mass density ($\rho$) and the poloidal magnetic field $B_p = B_r + B_\theta$, respectively, at an azimuthal angle $\phi = 0$. The MRI grows over a few dynamical timescales and saturates into turbulence afterward. The bottom panels show the evolution of the mean ($\phi$-averaged) poloidal ($\bar{B}_p = \bar{B}_r + \bar{B}_\theta$) and toroidal ($\bar{B}_t$) magnetic fields with time in the Initial RIAF run. The colors show the mean toroidal field, while the streamlines describe the mean poloidal field lines. Poloidal field loops of alternate polarity thread the initial disk. Shortly afterward, the toroidal magnetic field becomes the dominant component, due to the background shear. The accretion flow largely forgets the initial field configuration around $t = 40,200$, entering a quasi-stationary phase, with an MRI-driven dynamo in action.

3.2. Convergence

Before analyzing the simulation results, we first verify that our simulations have achieved proper numerical convergence.
Numerical convergence implies that physically important observables (e.g., the mass accretion rate) should not change significantly with the change in numerical resolution. Ideally, we are supposed to run simulations with different resolutions and compare the results to find the minimum grid resolution that is required to achieve convergence. However, the GRMHD simulations that we performed were computationally quite expensive. Therefore, to test the convergence, we calculated different numerical metrics that had been found to be useful in defining the convergence of the MRI turbulence in earlier studies (Sorathia et al. 2012; Hawley et al. 2013). In this work, we focus on the quality factors

\[
Q_\theta = \frac{2\pi}{\Omega} \frac{|\vec{b}_\theta|}{\sqrt{\overline{\omega}_{\text{tot}}}} \frac{1}{dx^\theta},
\]

\[
Q_\phi = \frac{2\pi}{\Omega} \frac{|\vec{b}_\phi|}{\sqrt{\overline{\omega}_{\text{tot}}}} \frac{1}{dx^\phi},
\]

and the magnetic tilt angle

\[
\theta_B = -\frac{\vec{b}^\beta B^\phi}{P_{\text{mag}}},
\]

measured in an orthonormal fluid frame (White et al. 2019). Here, the angular velocity is defined as \(\Omega(r, \theta) = \Omega^i \hat{e}^i / \rho^i\) and total entropy is given by \(s_{\text{tot}}(r, \theta) = (\rho + 1)H^i (1 - P_{\text{gas}} + P_{\text{mag}}) / \rho^i\). The line elements are given by \(dx^\beta = g^\beta_\mu e^\mu_\beta dx^\mu_{BL}\), \(dx^\phi = g^\phi_\mu e^\mu_\phi dx^\mu_{BL}\), where \(dx^\mu_{BL} = [0, \Delta r, \Delta \theta, \Delta \phi - ar/(r^2 - 2Mr + a^2)\Delta \theta]\). The quality factors \(Q_\theta\) and \(Q_\phi\) provide information about the number of cells across the fastest-growth mode in the \(\theta\) and \(\phi\) directions, respectively, while \(\theta_B\) measures the magnetic field anisotropy, a key factor behind the angular momentum transport. A magnetic tilt angle above a critical value confirms the transition from the linear growth of MRI to saturated turbulence (Pessah 2010). Earlier studies have suggested that the toroidal and poloidal resolutions are coupled and that the product of the quality factors \(Q_\theta Q_\phi \gtrsim 200–250\) is a good indicator of convergence in the MRI simulations (Narayan et al. 2012; Sorathia et al. 2012; Dhang & Sharma 2019; Porth et al. 2019). In the meantime, we note that a unique feature of the magnetic tilt angle is that it does not change with an increasing resolution for converged simulations; \(\theta_B\) shows a narrow range of values, \(10^2–14^\circ\), for the converged runs (e.g. Sorathia et al. 2012; Hogg & Reynolds 2018b; Dhang & Sharma 2019) and turns out to be a better indicator of convergence.

Figure 4 shows radial profiles of the average (averaged over \(\phi, \theta, \text{ and time}\) quality factors \(\langle Q_\theta \rangle, \langle Q_\phi \rangle\)) and the magnetic tilt angle \(\langle \theta_B \rangle\) close to the midplane of the disk for the Initial RIAF run. The meridional average is done over \(t = (5–10) \times 10^4\). The simulation is well resolved for \(r < 120\) and the resolvability starts declining afterward.

Figure 3. Azimuthal structures of the rest-mass density and toroidal magnetic field in the disk midplane for the Initial RIAF run. Elongated structures in the \(\phi\) direction are observed, which are expected for a shear-dominated accretion flow.
where the area element is given by
\[ dS_r = \sqrt{-g} \, d\theta \, d\phi \]
and the integration is performed over all \( \theta \) and \( \phi \).

The MAD parameter \( \tilde{\phi}_{BH} \) is a dimensionless number that is found to be useful in characterizing the magnetic states of the simulations. Earlier studies have suggested that an accretion flow attains a MAD state once \( \tilde{\phi}_{BH} \) reaches a critical value \( \tilde{\phi}_{BH,c} \approx 40 \) at the event horizon (Tchekhovskoy et al. 2011). Additionally, \( j_{\text{net}} \) also displays a highly sub-Keplerian nature at the event horizon for MAD simulations, where the angular momentum transport is highly efficient due to the large-scale Maxwell stress. On the contrary, \( j_{\text{net}} \) maintains a slightly sub-Keplerian value at the event horizon in the SANE simulations (Narayan et al. 2012). Moreover, \( j_{\text{net}} \) has been shown to be a good indicator of convergence in the MRI-active turbulent accretion flow. For a converged simulation, \( j_{\text{net}} \) maintains a sub-Keplerian value inside the ISCO, with a nondecreasing trend in time throughout the simulation (Hawley et al. 2013; Dhang & Sharma 2019).

The top and bottom panels of Figure 5 shows the time evolution of \( j_{\text{net}} \) and \( \tilde{\phi}_{BH} \) at two different radii—at the ISCO and at the event horizon. We also plot the time variation of the signed flux threading the event horizon \( (r_H = 2) \) of the BH in the northern hemisphere. \( \Phi_{\text{NH}}(t_H) \) (Equation (29)), for future reference in Section 4. The value of \( \tilde{\phi}_{BH} \) always remains around 1, which is well below the value \( (>40) \) required for the MAD state. Such a low value of \( \tilde{\phi}_{BH} \) implies that the magnetic state of the Initial RIAF run is in the SANE regime. A slightly sub-Keplerian value of the specific angular momentum \( j_{\text{net}} \) is also an indicator of the SANE magnetic state of our Initial RIAF simulation.

### 3.4. Inflow Equilibrium

We will inject external magnetic flux into the quasi-stationary turbulent RIAF to study the magnetic flux transport (for fuller details, see Section 4). Therefore, it is important to find out the inflow equilibrium radius—the radius within which the flow attains a quasi-stationary state—for the Initial RIAF run. Following Narayan et al. (2012), we investigate the variation of the average mass accretion rate \( \langle \dot{m}(r) \rangle \) with time, to find out the inflow equilibrium radius. Spatial averages are done over all \( \theta \) and \( \phi \). We use five different intervals—\( \Delta t_0 = (3.75-7.5) \times 10^3; \Delta t_1 = (7.5-15) \times 10^3; \Delta t_2 = (1.5-3) \times 10^3; \Delta t_3 = (3-6) \times 10^3; \) and \( \Delta t_4 = (6-12) \times 10^3 \)—to perform the time average. The left panel of Figure 6 shows \( \langle \dot{\phi}_{\text{BH}} \rangle \) at different time intervals for the Initial RIAF run. It can be inferred from the radial profiles of \( \dot{\phi}_{\text{BH}} \) that the inflow equilibrium radius for the Initial RIAF run reaches \( r_{\text{eq}} \approx 60 \) at late times. This is the radius that guides us in determining the injection radius for the external magnetic field loops.

The right panel of Figure 6 shows the radial variation of the disk aspect ratio \( \epsilon = H/r \) in the quasi-stationary state. Although, overall, the disk aspect ratio \( \epsilon \) fluctuates over time at the \( \lesssim 20\% \) level, when averaged over \( \Delta t_4 \), the disk aspect ratio \( \epsilon \) slowly increases with the increasing radius, until the inflow equilibrium radius and its value lie around \( \epsilon = 0.25 \) for \( r > 20 \), where general-relativistic effects are negligible. Such a variation of the scale height in our simulation is also in agreement with that observed in previous GRMHD simulations of the SANE RIAF (e.g., Narayan et al. 2012).

### 3.5. Large-scale Magnetic Field and Dynamo

We find that our Initial RIAF simulation is in the SANE state and that an MRI dynamo generates the large-scale magnetic fields and governs the magnetic field evolution at late times, as discussed in Section 3.1. To characterize the dynamo action, it is customary to visualize the spatiotemporal variation of the mean magnetic field. We define the mean magnetic field as the azimuthally averaged field

\[ \bar{B}_i(r, \phi) = \frac{1}{\phi_{\text{ext}}} \int_0^{\phi_{\text{ext}}} B_i(r, \phi) \, d\phi, \]

where \( \phi_{\text{ext}} \) is the extension in the \( \phi \) direction and \( i \in (r, \theta, \phi) \).

Figure 7 shows the variation of the mean radial \( B_r(R_0, \theta, t) \) (top
expressed in units of local orbit at time $t_0 = 60$ for the Initial RIAF run. Right: disk aspect ratios $\epsilon = H/r$ for the Initial RIAF run in the quasi-stationary state. Simultaneously, we also plot its initial radial profile, to study the change over time. The time averages are performed over $\Delta t_4 = (6-12) \times 10^4$. 

Figure 7. Butterfly diagrams: spacetime plots of the mean radial $B_r(r = 60, \theta, t)$ (top) and toroidal $B_\phi(r = 60, \theta, t)$ (bottom) for the Initial RIAF run. Time is expressed in units of local orbit at $t = 60$. Both the radial and toroidal fields show irregularities, which are typical features of a dynamo in a geometrically thick RIAF. 

Panel) and the mean toroidal field $\bar{B}_\phi(R_0, \theta, t)$ (bottom panel) with latitude $(90^\circ - \theta)$ and time at the radius $R_0 = 60$. This is also known as the butterfly diagram. Both the radial and toroidal fields show irregular behaviors in their butterfly diagrams. Additionally, the radial field is less coherent compared to the toroidal field, as observed in earlier studies of the dynamo in the SANE RIAF (Hogg & Reynolds 2018b; Dhang et al. 2020). This intermittent dynamo cycle in the RIAF is in contrast to the very regular dynamo cycles that have been observed in a thin Keplerian disk (e.g., see Flock et al. 2012). The irregularity in the dynamo cycle arises because of the slightly sub-Keplerian angular velocity of the geometrically thick RIAF (Dhang & Sharma 2019).

Earlier studies have found that while a large-scale dynamo generates large-scale magnetic fields at high latitudes, a fluctuation dynamo dominates close to the disk midplane, suppressing the production of the large-scale magnetic field there (Dhang & Sharma 2019). This can be qualitatively understood by looking at the large and coherent magnetic structures (especially for the toroidal fields) at high latitudes, while noticing the patchier distributions near the disk midplane $(90^\circ - \theta = 0^\circ)$ in the butterfly diagrams in Figure 7 and also in the last two panels of Figure 2. However, it should be emphasized that although the MRI dynamo does produce a large-scale magnetic field, it is not strong enough to create a MAD, which is conducive for strong jets (Section 3.3). This inefficiency is likely to be due to the weak $\alpha$-effect (Dhang et al. 2020), which is responsible for poloidal field generation. Additionally, the strong turbulent pumping that is present in an MRI-active RIAF tends to prevent the accumulation of a large-scale magnetic field near the BH, as suggested in Dhang et al. (2020).

4. Transport of External Magnetic Field Loops

In this section, we study the accretion of the external magnetic flux injected on top of the fully turbulent SANE state that was obtained in Section 3. Our aim is to investigate whether or not the system can bring the external magnetic flux that is available at the outer radii all the way to the central BH, which may eventually lead to a MAD state. While the actual configuration of the external field is unknown and could be complex, we anticipate that it is likely to be closed. Therefore, instead of the commonly used net vertical field, we inject poloidal magnetic field loops of different strengths and radial and vertical sizes, as shown in Figure 8, and study their transport. As controlled experiments, these field loops confined between the radii $r_l$ and $r_u$ are prescribed by

$$A_{\phi,t} = \sqrt{\frac{2 p_{\text{gas}}(r, \pi/2)}{C_l} \left[ \frac{\rho(r, \theta')}{\rho(r, \pi/2)} - \delta l \right] \sin[\kappa(R - r_l)]},$$

where $p_{\text{gas}}$ and $\rho$ are the initial pressure and density profiles, respectively. $\theta' = \theta + \theta_{\text{shift}}$, $\kappa = \pi/(r_{l} + r_{l})$, and $r_{l} = (r_{l} + r_{l})/2$. A vanishing $\theta_{\text{shift}}$ implies that the loop
Note. $\theta_{\text{shift}}$ and $\Phi_{l,\text{max}}$ are the tilt of the loop, with respect to the midplane, and the total magnetic flux in the injected loop, in code units, respectively. The magnetic field loops are injected at $t = 8.06 \times 10^4$ and run until $t = t_{\text{end}}$.

center is at the midplane, while a positive value of $\theta_{\text{shift}}$ indicates that the loop is off-center. The vertical size of the loop is set by $\beta_l$. The magnetization of the loop is controlled by the parameter $C_l$ and characterized by $\beta_l = \langle p_{\text{gas}} \rangle / \langle p_{\text{mag},l} \rangle$, where $\langle p_{\text{gas}} \rangle$ and $\langle p_{\text{mag},l} \rangle$ are the gas pressure of the Initial RIAF at the time of the loop injection and the magnetic pressure of the injected loop, respectively. Additionally, note that the average is performed within the loop. We choose $r_{l,1}$ to be the inflow equilibrium radius $r_{\text{eq}} = 60$, with different values of $r_{l,2}$, as tabulated in Table 1.

We restart the Initial RIAF run at $t = 8.06 \times 10^4$, inject the external field loops (Equation (28)), then run until $t = t_{\text{end}}$, as tabulated in Table 1. It is to be noted that we also run the Initial RIAF simulation longer, to compare it with the simulations with injected magnetic field loops. We inject loops of different strengths, with a wide range of plasma $\beta$, ranging from $\beta_l = 7000$ (weak, but stronger than the pre-existing mean fields that are produced by the MRI dynamo) to $\beta_l = 70$ (a very strong field that is typically used in MAD simulations, but of a much larger size than those used in our simulations). We also explore the effects of other parameters, such as the radial and vertical sizes and the injection latitudes of the loops, on the flux transport process, while considering loops with fiducial plasma $\beta$ values of $\beta_l = 3500$ and $\beta_l = 1500$, respectively. Additionally, we study the transport of large magnetic loops with strengths (of $\beta_l = 12,200$) similar to those of the mean fields that are produced by the MRI dynamo in the quasi-stationary phase of the Initial RIAF run. The configurations of the injected field loops from all these restarts are illustrated in Figure 8.

### 4.1. Diagnostics

Before discussing the results in detail, we define the following quantities that are used to discuss the transport of the external magnetic flux.

#### Table 1

| Name            | $C_l$   | $\delta_l$ | $r_{l,1}$ | $r_{l,2}$ | $z_l$ | $\beta_l$ | $\theta_{\text{shift}}$ | $\Phi_{l,\text{max}}$ | $t_{\text{end}}/10^5$ |
|-----------------|---------|------------|-----------|-----------|-------|-----------|------------------------|---------------------|-------------------|
| Initial RIAF    | ...     | ...        | ...       | ...       | ...   | ...       | ...                    | ...                 | ...               |
| $\beta_{7000}$  | $10^{-1}$| 0.2        | 60        | 90        | 1.5 H | 7000      | 0°                     | 1.48                | 1.2               |
| $\beta_{3500}$  | $5 \times 10^{-4}$ | 0.2 | 60 | 90 | 1.5 H | 3500 | 0° | 2.01 | 1.2 |
| $\beta_{1500}$  | $2.21 \times 10^{-4}$ | 0.2 | 60 | 90 | 1.5 H | 1500 | 0° | 2.93 | 1.2 |
| $\beta_{700}$   | $10^{-3}$ | 0.2 | 60 | 90 | 1.5 H | 700 | 0° | 4.28 | 1.2 |
| $\beta_{3500}$  | $1.168 \times 10^{-3}$ | 0.0016 | 60 | 90 | 2.5 H | 3500 | 0° | 7.95 | 1 |
| $\beta_{1500}$  | $5 \times 10^{-4}$ | 0.0016 | 60 | 90 | 2.5 H | 1500 | 0° | 13.14 | 1.2 |
| $\beta_{3500,\text{tall}}$ | $1.429 \times 10^{-5}$ | 0.2 | 60 | 120 | 1.5 H | 3500 | 0° | 2.04 | 1 |
| $\beta_{1500,\text{tall}}$ | $6.12 \times 10^{-5}$ | 0.2 | 60 | 120 | 1.5 H | 1500 | 0° | 2.97 | 1 |
| $\beta_{3500,\text{big}}$ | $5 \times 10^{-4}$ | 0.2 | 60 | 120 | 1.5 H | 12,200 | 0° | 4.42 | 1 |
| $\beta_{1500,\text{big}}$ | $5 \times 10^{-4}$ | 0.2 | 60 | 120 | 1.5 H | 12,200 | 0° | 1.70 | 1.1 |
| $\beta_{3500,\text{offc}}$ | $5.88 \times 10^{-4}$ | 0.2 | 60 | 90 | 1.5 H | 3500 | 15° | 1.56 | 1 |
| $\beta_{1500,\text{offc}}$ | $2.55 \times 10^{-4}$ | 0.2 | 60 | 90 | 1.5 H | 1500 | 15° | 2.32 | 1 |

Figure 8. The different types of external field loops injected at $t = 8.06 \times 10^4$. The white dashed lines mark $z = 1.5H$ above and below the midplane. For details, see Table 1.
The radial magnetic flux threading the \( r = \text{constant} \) surface in the northern hemisphere:

\[
\Phi_{\text{NH}}(r) = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{\phi_{\text{tot}}} 4\pi B_r(r, \theta, \phi) \sqrt{-g} \, dr \, d\phi; \quad (29)
\]

the vertical flux threading the midplane region:

\[
\Phi_{\text{mid}}(r) = \int_{r=r_H}^{r} \int_{\phi=0}^{\phi_{\text{tot}}} \frac{4\pi}{r} B_\theta(r, \theta) \sqrt{-g} \, dr \, d\phi;
\]

and the total flux available for accretion at different \( r \) in the northern hemisphere:

\[
\Phi_{\text{tot}}(r) = \Phi_{\text{NH}}(r_H) + \Phi_{\text{mid}}(r), \quad (31)
\]

where \( r_H = 2r_g \) is the event horizon radius of the BH. We also define a normalized flux, representing the efficiency of the flux transport, which is defined by

\[
f_B = \frac{\Phi_{\text{NH}}(r_H)}{\Phi_{l,\text{max}}}. \quad (32)
\]

Here, \( \Phi_{l,\text{max}} \) is the total flux at the loop center at the time of injection or at the beginning of the simulation (only for the Initial RIAF run; see also Table 1).

### 4.2. Results for the Fiducial Parameter—Plasma \( \beta \)

First, we discuss the dependence of the flux transport and the emergent accretion properties on the strength of the loops, defined by the plasma \( \beta \) (see also Table 1 and Figure 8). We start by discussing the qualitative picture of the evolution of the magnetic flux injected between the radii \( r = 60 \) and \( r = 90 \) on top of the existing magnetic field in the quasi-stationary Initial RIAF. It is worth noting that the plasma \( \beta \) of the total (mean + fluctuation) magnetic field is \( \beta_{\text{tot}} = 70 \), while that of the mean field alone is \( \beta_{\text{mean}} = 12,250 \), for the Initial RIAF run. Figure 9 shows the evolution of the external magnetic field loops of three different \( \beta \)s. The colors show the intensity of the mean radial field \( B_r \), while the streamlines describe the mean poloidal fields \( \vec{B}_\phi = \vec{B}_r + \vec{B}_\theta \).

The top panels show the time evolution of the weakly magnetized loop with strength \( \beta_l = 7000 \). The injection of the weak external magnetic field loops re-excites the MRI in the accretion flow and enhances the accretion stresses (see Figure 14), leading to higher mass accretion rates (see Figure 13). The poloidal flux slowly drifts toward the BH, and a fraction of the injected flux accumulates near the BH (which is quantitatively shown in Figure 11). We see an increase in the radial magnetic flux threading the BH when compared to that of the Initial RIAF run. This can be comprehended by comparing the snapshots at \( t = 80,601 \) (the flux level close to the BH does not change significantly in the quasi-steady state of the Initial RIAF, as shown in Figure 2) and, in the last panel, at \( t = 10^5 \).

Next, we discuss the transport of the moderately strong magnetic field loops with \( \beta_l = 1500 \), as shown in the middle panels of Figure 9. The magnetic flux reaches the BH in a shorter time compared to that of the weak-field case, with \( \beta_l = 7000 \). This is due to the stronger accretion stresses produced (see Figure 14) in the accretion flow, due to the injection of stronger magnetic field loops. The radial magnetic field strength in the polar region is found to be stronger than the weak-field case at late times. This is due to the larger amount of flux that is associated with the loop with \( \beta_l = 1500 \) than with the loop with \( \beta_l = 7000 \).

Finally, we examine the transport of the strong-field loops, with \( \beta_l = 70 \), similar to the strength of the total (mean + fluctuation) magnetic field in the quasi-stationary phase of the Initial RIAF. Unlike the previous two weak-field cases, the injected loops here are strong, with the most unstable wavelengths comparable to the disk’s scale height, and this drives strong channel flows over the entire vertical extent of the disk. It further generates a spike in the large-scale Maxwell stress (see Figure 14), producing a strong inflow of mass and magnetic flux. The magnetic flux reaches the BH very quickly and fills the polar region. The system remains in a strongly turbulent state until the end of the simulation.

The qualitative pictures discussed above are representative of all our simulations. Close-in views of the flows and the magnetic field structures at late times for these three representative runs, along with the Initial RIAF run, are shown in Figure 10. The snapshots show that most of the advected flux is concentrated in the low-density laminar funnel region (the polar region), with a different sign across the midplane. At the same time, the turbulent disk midplane has small patches of magnetic field of both polarities. The animations of the other runs with external magnetic field loops, along with the Initial RIAF run, can be viewed in a YouTube playlist. In the upcoming subsections, we quantify the different metrics of magnetic flux transport in greater detail, for all the runs that we performed as listed in Table 1.

#### 4.2.1. Evolution of Magnetic Flux

The left panel of Figure 11 shows the time evolution of the magnetic flux through the event horizon in the northern hemisphere \( \Phi_{\text{NH}}(r_H) \), for runs with different strengths of injected magnetic field loops, which are also compared with the Initial RIAF run. It is clear to see that the injection of an external loop enhances the amount of flux at the event horizon. In the runs with low \( \beta_l \leq 1000 \), there is a transient rise of \( \Phi_{\text{NH}} \), due to the fast transport from the strong MRI channel flow. In the more extreme case of \( \beta_l = 70 \), the transient phase is so extreme that it leads to a strong initial spike in \( \Phi_{\text{NH}} \), followed by a gradual decline toward a more steady flux level. For other runs with \( \beta_l \geq 1000 \), the buildup of the magnetic flux in the BH horizon is more gradual, and the buildup is slower for runs with higher \( \beta_l \). It is worth noting that none of our simulations with injected loops reach the MAD state, with the MAD parameter ranging from \( \phi_{\text{BH}} = 2 \) to \( \phi_{\text{BH}} = 10 \).

We show the spatiotemporal variation of the total flux \( \Phi_{\text{tot}}(r, t) \) available for accretion in the northern hemisphere in Figure 12, to obtain a more complete picture of the flux transport at different radii. Each panel of Figure 12 describes the evolution of the radial profile of \( \Phi_{\text{tot}} \) over time for runs with different \( \beta_l \). The first panel in the top row corresponds to the Initial RIAF run. It again demonstrates that the system forgets its initial magnetic field configuration after the time \( 3 \times 10^4 \)–\( 4 \times 10^4 \). The rest of the panels show the spatiotemporal evolution of \( \Phi_{\text{tot}}(r, t) \) for the other runs, after we inject external magnetic field loops. In accordance with Figure 11, we see that there are two regimes of flux transport, depending on the strength of the injected loop. With very strong external flux (\( \beta_l = 70, 200 \)), the external flux is quickly transported both inward and outward, which is characteristic of channel flows with flow directions alternating over height, as seen in the last two panels in the
Figure 9. Evolution of the external magnetic field loops injected into a turbulent quasi-stationary RIAF. The top, middle, and bottom panels show the evolutions of the magnetic field loops of plasma $\beta = 7000$ (weak field strength), $\beta = 1500$ (moderate field strength), and $\beta = 70$ (strong field strength), respectively. The colors represent the mean radial field $\bar{B}_r$, while the poloidal field distribution $\bar{B}_p = \bar{B}_r + \bar{B}_\theta$ is described by the streamlines. Note that a fraction of the injected magnetic flux reaches the BH.
bottom row of Figure 12. The channel flows lead to the initial transient transport of a large fraction of the initial flux into the BH, followed by a subsequent relaxation and diffusion toward a more steady flux level.

With weak external flux ($\beta \lesssim 3500$), the initial external magnetic flux gradually diffuses, while being advected inward. In the end, a fraction of the flux overcomes the diffusion to reach the BH, which we discuss more quantitatively in the next subsection. In between these two regimes, there lies the case of a moderately strong external flux ($\beta \approx 700, 1500$), for which the flux transport by the channel flows diffuses before reaching the BH, and subsequent transport is likely mediated by a combination of advection and diffusion.

4.2.2. Efficiency of Transport

Up to now, we have considered the total flux as the primary diagnostic, irrespective of the amount of flux that is associated
with the injected loops. However, it is worth noting that different loops have different amounts of flux. Therefore, a normalized flux $f_{fb}$ as defined in Equation (32), would be a better indicator of the efficiency of the flux transport.

The right panel of Figure 11 shows the time variation of the fraction $f_{fb}$ for different runs. In the regime of the very strong injected flux ($\beta = 70, 200$), the efficiency is quite high (up to ~50%) during the initial phase, when channel flows dominate. Later, the efficiency goes down to around 15%–20%. In the weak-field regime ($\beta \geq 3500$), despite the flux at the BH being accumulated gradually, the efficiency of the flux transport is more or less similar, at around 15%–20%. For comparison, we also show the result for the Initial RIAF run, where we define $\Phi_{t,\text{max}}$ by calculating the flux at a radius $r = 75$ at $t = 0$. Finally, it is interesting to note that the flux transport appears to be more efficient, reaching about 20%–40%, when the field strength lies between the two regimes, i.e., for $\beta_f = 700$ and $\beta_f = 1500$.

4.2.3. Effects on Mass Accretion Rate and Accretion Stresses

In this subsection, we study how the injection of the external magnetic flux influences accretion properties, such as the mass accretion rate and accretion stresses. Figure 13 shows the time history of the mass accretion rate at the event horizon for runs with a range of $\beta_f$. We further show in Figure 14 the spacetime plot of the azimuthally and vertically (over one scale height) averaged total accretion stress $\langle W_{\text{tot}} \rangle$, which is a combination of the Maxwell and Reynolds stresses defined in the orthonormal fluid frame (see Section 3.2), as

$$W_{\text{Max}} = 2 p_{\text{max}} \dot{r} \dot{\phi} - b_r b_{\phi},$$

$$W_{\text{Rey}} = \left( \rho + \frac{\gamma}{\gamma - 1} p_{\text{gas}} \right) \dot{r} \dot{\phi} + \rho \sqrt{-g} d\theta d\phi,$$

$$\langle W_{\text{tot}} \rangle = \langle W_{\text{Max}} \rangle + \langle W_{\text{Rey}} \rangle,$$

where the Maxwell stress is the dominant component.

The fresh injection of external field loops reignites the linear MRI, leading to higher accretion stresses and hence an increase in the mass accretion rate. We find that with a high field strength in the loop (i.e. $\beta_f \lesssim 200$), there is substantially enhanced accretion stress, leading to a rapid, strong, and transient increase in the accretion rate. The stresses are reduced after the transient phase, but are still much stronger than those in the Initial RIAF run within the simulation time, reflecting the prolonged influence of the initial flux loop. Simulations with $\beta_f \gtrsim 3500$ show only a modest increase in accretion stress and accretion rates compared to the Initial RIAF run, indicating that the injection of the external flux has only a minor impact on the disk turbulence. For simulations with intermediate $\beta_f$, there is a modest enhancement of the accretion stress, resulting in a modest enhancement of the accretion rate. We also note that after $t = 1.1 \times 10^9$, despite having a higher level of magnetic flux (compared to the Initial RIAF run; see Figure 12), the mass accretion rates in the runs with injected loops are very similar to that of the Initial RIAF run. This occurs due to the quick depletion of mass supply in the disk at earlier times, due to the enhanced stresses in the runs with the injected loops.

4.2.4. Disk and Flow Structures

In this subsection, we further examine how the external magnetic flux changes the disk structure and flow properties. We start by considering the radial profiles of the surface density, defined as

$$\Sigma(r, \phi) = \frac{1}{\rho_{\text{ext}}(\phi)} \int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} \rho \sqrt{-g} d\theta d\phi,$$

and the radial velocity $\langle u_r(r) \rangle$, averaged within one scale height about the midplane. The results for the Initial RIAF run and the runs with external field loops, where the time average is done over $t = 9 \times 10^7$–$10^8$, are shown in Figure 15.

In the Initial RIAF run, we see that the accretion velocity approaches the freefall velocity ($v_{ff} = \sqrt{2/r}$) within the ISCO, while the accretion velocity ranges between 0.01 and 0.5 of the Keplerian velocity farther out, until the radius of inflow equilibrium. Upon imposing an external field, the higher accretion stresses lead to higher accretion velocities. The enhancement can be by up to a factor of ~10 in the strong-field case, with $\beta_f = 70$ at the representative radii of $r \sim 30–60r_s$, while for the weak-field runs (e.g., $\beta_f \gtrsim 3500$), the accretion velocity is only enhanced by a modest factor of ~2. We also

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6 We note that the standard definition (36) asymptotes to $\rho v^2 d\Omega$ at large radii, with an extra $r$ factor compared to the Newtonian definition (assuming that the surface density is defined by integrating along spherical shells).
after injections of magnetic ISCO. The vertical black dashed line in the top left panel note that the pro

Figure 13. Time evolution of the mass accretion rate $m_{in}$ at the event horizon, after injections of magnetic field loops of different strengths, defined by the plasma $\beta$.

note that the profile of $\langle u^i(r) \rangle$ evolves over time, accompanying the magnetic flux transport, but qualitatively, the profiles shown in Figure 15 are representative over the duration of our simulations.

The altered accretion velocity profile $\langle u^i(r) \rangle$ further modifies the surface density profile. Generally, after imposing an external field loop, the surface density becomes steeper compared to the surface density profile in the Initial RIAF run, though the deviation is only modest. The surface density profile also evolves over time. We note that earlier RIAF simulations of the SANE state have already indicated that there might not be a universal power law for the surface density profile and other flow properties (White et al. 2020). When supplied with external magnetic flux in the outer disk, our results suggest additional surface density variations during the process of magnetic flux transport. In other words, the dynamics of RIAFs are dependent on the magnetized mass reservoir at larger radii.

4.3. Results for Other Parameters

In this section, we assess the robustness of our fiducial simulation results by considering different geometries for the injected field loops. In particular, we change the loop sizes (both vertical and radial) and the injection latitudes. We focus on loops of fiducial strength $\beta_l = 3500$ and $\beta_l = 1500$, respectively. Additionally, we study the transport of a large loop of similar strength ($\beta_l = 12,200$) to mean poloidal fields that are produced by the MRI dynamo in the Initial RIAF run in the quasi-stationary phase. Figure 16 shows the time evolution of the radial magnetic flux threading the event horizon in the northern hemisphere $\Phi_{NH}(r_H)$ (top) and the flux transport efficiency (bottom) for these additional simulations.

4.3.1. Vertical and Radial Sizes

The left panels of Figure 16 compare the flux transport for taller loops of vertical size $zl = 2.5\Delta H$ with loops of similar strength, but fiducial size $zl = 1.5\Delta H$. We find that the vertical size of the loops does not affect the amount of flux reaching the BH, while the efficiency of the flux transport remains almost unaltered following the change of the vertical size of the loops.

Similarly, the middle panels of Figure 16 compare the flux transport between loops of different radial sizes, where we consider larger loops of radial size $\Delta r_l = 60$, as opposed to the fiducial radial size of $\Delta r_l = 30$. We observe that a larger amount of flux reaches the BH for the larger loops, which is reasonable, because more magnetic flux is available in these loops compared to their smaller counterparts. However, the fraction of the flux reaching the BH remains similar for both the smaller and larger loop cases with the same plasma $\beta_l$. This result also holds for our additional run with $\beta_l = 12,200$. This indicates that the efficiency of the flux transport remains unaffected by the radial extent of the injected loops.

4.3.2. Injection Latitude

In addition to studying the effects of the strength and size of the loops on the transport process, we also consider injecting off-centered loops with plasma $\beta_l = 3500$ and $\beta_l = 1500$, to examine whether loop injection away from the midplane facilitates flux transport or not. The comparison with our fiducial injection prescription is shown in the right panels of Figure 16. Surprisingly, the injection of off-centered loops leads to a distinctly lower flux level at the event horizon. While it is not entirely clear why this is the case, we speculate that it is related to stronger magnetic reconnection occurring in the off-centered case, which leads to more considerable destruction of
the magnetic field during the interplay between the injected field and the dynamo-generated background field.

The stark difference between the magnetic field evolution in the off-centered and the fiducial cases can be also seen by comparing the movies describing the magnetic field loop evolutions for the runs \( \beta_{1500} \) and \( \beta_{1500, 0.2} \), respectively.

5. Discussion

5.1. Inefficiency of Dynamo in SANE/RIAF

We threaded the initial geometrically semithick disk \( (H/R \approx 0.2) \) with small magnetic field loops of alternating polarity and attained a quasi-stationary weakly magnetized RIAF (SANE; see Figure 5), which does not remember the initial field geometry (see Sections 3.1 and 3.3). An MRI dynamo is responsible for generating and sustaining magnetic fields (both small-scale and large-scale) in the quasi-stationary RIAF (Hawley et al. 2013; Hogg & Reynolds 2018a). A large-scale dynamo does operate (Dhang & Sharma 2019) and generate large-scale magnetic fields in the weakly magnetized RIAF (see the final two panels of Figure 2), but this is not efficient enough to produce strong magnetic fields that can convert a SANE to a MAD. This result aligns with earlier works, which found that the dynamo action in a SANE RIAF does not lead to jet formation (Beckwith et al. 2008; Narayan et al. 2012). Earlier works (Hogg & Reynolds 2018a; Dhang et al. 2020) with different numerical setups investigating MRI dynamos arrived at the conclusion that the dynamo action is weak in a geometrically thick RIAF. Hence, the inefficiency of the jet formation is likely to be attributed to insufficient poloidal field generation (a weak \( \alpha \)-effect) and strong turbulent pumping that transports a large-scale magnetic field radially outward in a RIAF (Dhang et al. 2020).

Recently, Liska et al. (2020) reported that if the simulation is started with an unusually strong \( (\beta \approx 5) \) and coherent toroidal magnetic field, the MAD state can be achieved at late times. They argued that an MRI dynamo could produce strong poloidal field loops of size \( H \approx R \) from the very strong and coherent initial toroidal field. The farther away the creation location is, the larger the loops are. Most of the loops move outward, while a few “lucky” loops that are created at large radii are somehow arrested and stretched inward, leading to the MAD state. However, how the accretion disk could in the first place possess such a coherent initial toroidal field of the same polarity spanning several decades in radii remains questionable.

Overall, we reaffirm that the MRI dynamo in the standard SANE state does not spontaneously generate a strong coherent large-scale poloidal field to turn the disk into the MAD state. In the absence of an initial poloidal field, achieving the MAD state may require an unusually strong and coherent toroidal field, which may be unpractical in reality.

5.2. Plausible Sources of External Magnetic Fields

In this work, we have considered the possibility that the disk acquires an external poloidal field in the form of field loops of different sizes and shapes. What could be the source of such external field loops, though? While definitive evidence is lacking, we speculate that accreting such external field loops could be plausible in a variety of systems.

In the hard state of XRBs, a RIAF close to the BH has been proposed as being connected to an outer thin disk (Esin et al. 1997; Done et al. 2007), which can supply large-scale magnetic flux to the inner RIAF. The outer thin disk can in principle harbor a large-scale magnetic field, due to an efficient dynamo action (Flock et al. 2012; Gressel & Pessah 2015) or due to the coronal accretion of magnetic flux (Guilet & Ogilvie 2012) from the companion/donor star, or a combination of both. The donor stars in low-mass XRBs are likely to be either K- or M-type dwarf stars (Fragos & McClintock 2015) or evolved stars (e.g., as in GRS 1915+1105). The donor stars in the XRBs are supposed to be tidally locked to the rotation periods of the binaries, with orbital periods of hours to days (Coriat et al. 2012). These fast-rotating dwarf stars show vigorous magnetism, with surface magnetic fields of strengths \( (\sim 10^3 \text{G}) \) that are similar to sunspots (West et al. 2008; Davenport 2016). Additionally, in the active region, the magnetic field is one order of magnitude stronger than the average stellar magnetic field. Magnetized matter from the donor star passes through the first Lagrange point \( (L_1) \) and enters the Roche lobe of the primary (accretor) almost ballistically, circularizing at the circularization radius (Frank et al. 2002). We speculate that the
mass loss from the L1 nozzle may proceed through a chain of mass blobs that are encircled by field loops (e.g., as also considered in Ju et al. 2017), which may be amplified and become quasi-axisymmetric during the circularization process. Thus, if this external flux can be brought in through the outer thin Keplerian disk, then it may further feed the inner RIAF, where flux transport is efficient, and saturate the BH.

The accretion flow in a low-luminosity AGN is also thought to be a RIAF. In this case, the gas supplied by the ambient medium to the accretion flow is magnetized. It can harbor a large-scale magnetic field, as inferred from the observation of the large-scale poloidal flux in the Galactic center (Nishiyama et al. 2010). Recent numerical simulations by Ressler et al. (2020a, 2020b) have found that the large-scale accretion flow around the galactic center, fed by the winds of Wolf–Rayet stars, can achieve the MAD state, with efficient inward transport of the magnetic field embedded in the accreting material. Their injected magnetic fields have a pure toroidal component with random orientation, thus we may consider that such fields effectively enter the accretion disk in the form of closed field loops from random directions. Our results are in line with their findings, while our controlled experiments provide a further physical basis for better understanding the efficient flux transport around supermassive BHs.

5.3. Transport Efficiency in SANE and the Possibility of Transformation to a MAD

In Section 4, we show the results from the effects of loop injection into a turbulent quasi-stationary SANE RIAF. We observe the simultaneous transport of the magnetic flux inward and outward, due to channel flows (this is more evident in the strong-field case, in Figures 8 and 10). For the strong-field case, the channels quickly transport the field inward, giving very high efficiency, while for weak-field cases, a fraction of the flux reaches the BH, slowly overcoming diffusion. We find that except for off-centered loops, the transport of externally injected magnetic flux loops is relatively efficient, with typically ∼20% of the available flux ending up being accreted to the central BH, regardless of the initial field strength and size.

Note that we find that all of the simulations with injected loops have the MAD parameter φBH ≲ 10. This implies that none of our simulations reach the MAD state, but if this relatively high efficiency of flux transport obtained from our controlled experiments is universal, we can estimate the requirement on the external flux to potentially transform a SANE disk into a MAD.

The MAD parameter φBH (the normalized unsigned flux threading the BH) is related to the magnetic flux (signed) ΦNH threading the northern hemisphere of the BH as follows:

$$\phi_{BH} \approx \frac{\Phi_{NH}(r_H)}{m \, r_f \, c^{1/2}}.$$ (37)

We have found that a certain fraction $f_B = \Phi_{NH}(r_H)/\Phi_{in}$ (Equation (32)) of the injected flux $\Phi_{in}$ reaches the BH. Earlier numerical experiments suggest that a MAD could be achieved if the MAD parameter exceeds a critical value $\phi_{BH,c}$ at the event horizon (e.g., see Tchekhovskoy et al. 2011). This indicates a critical value of the injected flux $\Phi_{in,c}$, which is the plausible minimum flux required for the MAD state, and is given by

$$\Phi_{in,c} = \left(\frac{r_e}{f_B} \right)^{1/2} \phi_{BH,c} \sqrt{m}.$$ (38)

Therefore, the minimum poloidal magnetic field required at the injection location is given by

$$B_{in,c} = \left(\frac{r_e \, c^{1/2}}{2 \pi f_B} \right) \left(\frac{r}{\Delta r} \right) \left(\frac{\phi_{BH,c}}{r^2} \right) \sqrt{m},$$ (39)

where we have estimated, for a loop centered on radius $r$ with a half-width $\Delta r$, that $\Phi_{in,c} \approx 2 \pi B_{in,c} \Delta r$. If we take $\Delta r/r = 0.2$, then the value of $B_{in,c}$, in terms of the Eddington accretion rate $M_{Edd} = 1.5 \times 10^{19} M_{10} \text{ gm s}^{-1}$, is given by

$$B_{in,c} \approx \frac{10^4}{f_B} \phi_{40} \, r_1^{-2} \, m_{-4}^{-1/2} \, M_{10}^{-1/2} \, G,$$ (40)

where $\phi_{40} = \phi_{BH,c}/40$, $r_1 = r/(100 r_g)$, $m_{-4} = m/10^{-4} M_{Edd}$, and $M_{10} = M_{BH}/10 M_\odot$. 


Would this amount of magnetic field be available for accretion at the outer radii of the RIAF? We will estimate the poloidal field strength that is available for accretion in the case of an XRB. In the low hard state, the RIAF close to the BH is proposed to be connected to an outer thin disk. The plausible source of the large-scale magnetic field in the thin disk could be the dynamo action. Another scenario would be the advection of large-scale field loops from the companion star, as discussed in Section 5.2. Independent of the mechanism, we can estimate the characteristic poloidal field strength in the thin accretion disk, given that the accretion is driven by the radial transport of the angular momentum in the disk, as (e.g., Bai & Goodman 2009)

$$-B_r B_{\phi} \approx \frac{m \Omega}{h_0}.$$  \hspace{1cm} (41)

Here, $h_o = \xi H_{thin}$ is the thickness of the disk over which the accretion proceeds. Further, if we assume that $|B_r| \approx 1/5|B_\phi|$ (which has been found to be consistent in the MRI simulations of accretion disks) and $\xi = 6$, then the total radial magnetic field in the thin disk of aspect ratio $\epsilon_{thin} = H_{thin}/R$ is given by

$$B_{r,d} \approx 10^4 \epsilon_{0.05}^{-1/2} M_{100}^{-1/4} M_{1/2} G.$$  \hspace{1cm} (42)

where $\epsilon_{0.05} = \epsilon_{thin}/0.05$. It is to be noted that in the strongly magnetized coronal region of the thin disk, a large share of this estimated total field $B_{r,d}$ will likely be in the mean coherent part of the magnetic field. In reality, the mass accretion rate in the outer thin disk is expected to be higher compared to that in a RIAF (Yuan & Narayan 2014), and hence the total poloidal field will also be higher. Therefore, a comparison of $B_{inc}$ and $B_{r,d}$ leads to the inference that it is quite possible that in an XRB, the outer thin disk reservoir could potentially supply an adequate amount of magnetic flux to the inner RIAF, which may eventually form a MAD close to the BH.

6. Summary

In this paper, we have studied magnetic field generation and transport in a geometrically thick RIAF. We initialize the disk with magnetic field loops of alternate polarity, so that the quasi-stationary RIAF is weakly magnetized, i.e in the SANE regime. In this quasi-stationary turbulent SANE RIAF, we study the transport of the external magnetic flux in the form of loops of different strengths, sizes, and shapes. Here we outline the key findings of our work.

1. We reconfirm that the MRI dynamo in a standard SANE RIAF does not generate a strong coherent large-scale poloidal field to turn the SANE state into the MAD state.
2. The magnetic flux transport is relatively efficient in the SANE RIAF: 15% to 40% of the external magnetic flux that is injected at the outer radii is able to reach the BH.
3. The flux transport efficiency is independent of loop parameters such as strength and size. However, if the loops are injected at high latitudes, rather than at the midplane, the efficiency becomes poor.

We also find that the accretion flow profiles (e.g., the surface density and accretion velocity) are altered as the external magnetic flux is injected into the disk. We propose that the dynamics of the RIAF depend on the magnetized mass reservoir at the outer radii.

Based on our results, we argue that it might be easier to transform a SANE disk to a MAD by supplying external poloidal field loops at the outer disk, provided that the

Figure 16. The time evolution of the amount of radial magnetic flux threading the event horizon in the northern hemisphere $\Phi_{rad}(r_H)$ (top) and its fraction over the injected magnetic flux (bottom) for runs with injected loops of larger vertical size (left), larger radial size (middle), and off-centered injection latitudes (right). The results are compared with runs with fiducial loop geometries and the initial RIAF run with the same level of magnetization ($\beta = 1500$ and 3500).
relatively high efficiency of the flux transport obtained from our controlled experiments is universal. It is to be noted that, as a first study, it is not yet clear which factors determine the magnetic flux transport efficiency of \(~15\%–40\%\) in our work. Additionally, we must mention that we have studied the transport of the external magnetic flux in the quasi-stationary turbulent RIAF in a limited parameter space. For example, we have considered only one injection location, with the inner edge of the loop being at \(r = 60\), whereas, in reality, the loops are supposed to be available for accretion as far as in the disk truncation region in XRBs, or at even larger radii in low-luminosity AGNs. We plan to explore the magnetic flux transport with different configurations and larger dynamical ranges. Furthermore, future work should extend this study to the thin-disk regime, which is applicable to regions beyond the truncation radius in the low/hard states of XRBs, as well as in luminous AGNs.

We thank Ramesh Narayan for his initial input into this project. We also thank Kandaswamy Subramanian and the anonymous referee for constructive suggestions. This research was supported by NSFC grant No. 11873033. The numerical simulations were conducted on TianHe-1 (A) at the National Supercomputer Center in Tianjin, China, and on the Orion cluster at the Department of Astronomy, Tsinghua University.

All the movies of the simulations mentioned in Table 1 are available that this YouTube link.

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