Electromagnetic Trihybrid Ellis Nanofluid Flow Influenced with a Magnetic Dipole and Chemical Reaction Across a Vertical Surface

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ABSTRACT: The purpose of this study is to evaluate the augmentation of thermal energy transfer in trihybrid Ellis nanofluid flow in the occurrence of magnetic dipole passes over a vertical surface. The ternary hybrid nanofluid is prepared by the dispersion of ternary nanoparticles (Al₂O₃, SiO₂, and TiO₂) in the Carreau Yasuda fluid. The velocity and heat transportation has been examined in the existence of the Darcy Forchheimer influence and heat source/sink. The phenomena of fluid flow have been mathematically designed for energy and fluid velocity in the form of a nonlinear partial differential equation (PDE)-based system. The system of PDEs is further refined to the set of ordinary differential equations via suitable similarity substitutions. The acquired dimensionless equations are numerically solved with the help of the HAM. It has been noticed that the energy contour is enhanced versus the variation of viscous dissipation and heat generation. A significant contribution of a magnetic dipole is observed to elevate the production of the thermal energy field, and an opposite trend is noticed versus the flow profile. The accumulation of Al₂O₃, SiO₂, and TiO₂ nanomaterials in the base fluid “engine oil” improves the velocity and energy profiles.

1. INTRODUCTION

The analysis of nanofluid flow over a vertical surface with heat transaction properties has received great interest due to its major engagement in industrial requirements and recent developments. Hosseinzadeh et al. scrutinized the micropolar MHD (magneto-hydrodynamics) fluid flow moving across an upright plate with three distinct base fluids: ethylene glycol, ethylene glycol/water, and water. The thermal gradient of lamina-shaped nanoparticles (nps) in water-based fluids is 38.09 percent advanced tendency than common nps, according to the findings. Reddy et al. designated an arithmetical analysis of an entropy generation on unsteady ferrofluid flow and mixed convection over an infinite vertical surface. Fluid flow across a vertical surface has a variety of uses in geothermal technology, petroleum extraction, oil drilling, and barrier properties, according to the study. Haq et al. explored the energy conduction of an unsteady nanoliquid flow with the MHD effect across an indefinite vertical sheet. Kumar et al. evaluated the effects of Dufour and thermal radiation on an MHD boundary layer flow across a permeable media. Algehyne et al. reported the bioconvection generated by the MHD flow of a hybrid ferrofluid consisting of nanomaterials through a vertical substrate. The influence of the porosity component and buoyancy ratio on the velocity trajectory has been noticed. The energy transformation characteristic improves dramatically as Eckert number and thermal absorption/generation levels rise. Dadheech et al. demonstrated the development of entropy in nanofluid flow over permeable and vertical plates. Kodi et al. evaluated the nano fluid flow with energy transport properties, influenced by chemical reactivity. Shah et al. evaluated the thermophoresis effect, concentration diffusivity, and variable viscosity over a vertical plate. Shah et al. examined the effect of the AB noninteger derivative with trihybrid nano particulates suspension across a vertical surface. Hussain et al. numerically reported the effectiveness of thermal features over a nonuniform vertical surface with heat generation through Carreau fluid. Some related recent studies have been reported.

Fluid flow can be classified as Newtonian or non-Newtonian. Various natural fluids, such as air and water, have been considered to be Newtonian fluids for research purposes. When fluid flow defied Newton’s model, there was a strong demand for a new concept. Numerous theories were put to the

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test in order to completely categorize the nature of these non-Newtonian viscoelastic substances, but the issue remained. The important feature of Ellis fluid is that it can indicate viscous fluid behavior at small shear stresses and Ostwald de Waele fluid behavior at elevated shear stresses. The Ellis fluid displays shear thinning and thickening characteristics at low, moderate, and high stress rates. Kheyfets and Kieweg demonstrated three-dimensional simulations of the constant-volume, gravity-driven unrestricted surface flow of an Ellis fluid by using thin film lubrication estimation. Javed et al. examined the calendaring of the Ellis fluid using the lubrication approximation theory. Celli et al. evaluated the onset of convective instability within a horizontal porous layer coated with a non-Newtonian fluid. They assumed fluid is shear—thinning, and the Ellis model defines its apparent viscosity. Sajid et al. investigated the flow in the renal tubule using Ellis fluid model. Rooman et al. investigated the entropy formation and its influences on Ellis fluid flow. They considered a porous horizontal stretching cylinder with an MHD effect. Shah et al. investigated, at a low Reynolds number, the mathematical model associated with bacterial gliding mechanism using the Ellis fluid model.

In the modern era, hybrid nanofluid has been heavily used to boost heat conductivity. Renewable energy, microelectronics, emollients, automotive industry, electronic cooling, generators, nuclear coolant system, ships, and biomedicine development are all the usages of hybrid nanofluids. investigated, at a low Reynolds number, the mathematical model associated with bacterial gliding mechanism using the Ellis fluid model.

Ahmadian et al. observed the Maxwell hybrid nanoliquid flow moving across two horizontal spinning discs. According to the findings, the disc-expanding action opposes the flow behavior. Zhang et al. addressed the 3D computational formulation of an unstable Ag–MgO/water hybrid peristaltic transport with energy and momentum transfer induced by a moving wavy revolving disc. The topology of a whirling disc is hypothesized to affect velocities and thermal heat transfer in a beneficial way. Sepehrnia et al. addressed the rheologic conduct and active transport through engine oil. Bilal et al. reported the joint upshot of magnetism and electrohydrodynamics on hybrid nanofluids flow. The electric component enhances the velocity boundary layer while decreasing the temperature contour. Pattanaik et al. analyzed the flow characteristics of a 2D conducting hybrid nanofluid passing via an exponentially extending porous substrate. The flow outlines rise as the thermal buoyancy variable increases. The influence of Hall current, magnetic flux, and heat radiation on hybrid nanofluid flow across the top of a rotating disc was investigated by Lv et al. Their goal was to improve the energy propagation rate for mechanical manufacturing applications. Nourinia et al. reviewed the ZnO-CP/MMT hybrid nanoliquid to improve the performance of base fluid for commercial applications by incorporating nanomaterials. Gal et al. performed a numerical analysis of the CNTs and water-based nanoliquid in a 3D cavity with mixed convection. By raising the obstacle opening angle and the CNT volume fraction, an improvement in energy transfer was seen. Recently many researchers have made a significant contribution to the field of hybrid nanofluids.

A magnetic dipole is a measurement of a magnet’s magnetic intensity and orientation, as well as any other item that generates a magnetic force. Gowda et al. experimented with the magnetic dipole influence on 2D ferromagnetic fluid flow across a flat flexible surface. Kumar et al. addressed the energy allocation through hybrid nanoliquid flow across an extending sheet while accounting for magnetic dipoles. The intensification in the magnetic integrand decreases the velocity
of fluids, whereas the thermal properties of the liquids show the opposite tendency. The efficiency of heat and mass transport slows as the ferromagnetic parameter rises. Gul et al. investigated the hybrid nanocomposites flow moving over an extending sheet, as well as the function of the magnetic dipole on nanofluid flow. Shuaib et al. depicted the nanoliquid flow across two orthogonal whirling discs, with the simultaneous impact of an electromagnetic force on the nanoliquid flow, as well as energy and mass transport properties. Copper oxide nanoparticles have been found to have a good influence on molecular transmitting power and can be employed for refrigeration. Shoaib et al. have devised numerical computing and have assessed a novel hydrodynamic fluid flow model in the context of a magnetic dipole. Isa et al. examined the K$_2$CO$_3$—Glycine nanofluid for the dynamical analysis of environmental cleanliness from carbon dioxide. Some recent literature related to fluid flow under the upshot of magnetic dipole has been documented.

Nanotechnology has been the subject of research due to its wide range of potential applications in engineering and biomedical fields. The insertion of a trihybrid nanomaterials mixture (Al$_2$O$_3$, SiO$_2$, and TiO$_2$) in Ellis’s liquid is studied in the present analysis within the context of magnetic dipole, heat source, and Darcy Forchheimer medium. The phenomena of fluid flow have been mathematically designed for energy and fluid velocity in the form of a nonlinear system of partial differential equations. The system of Partial differential equations (PDEs) is further simplified to the set of ordinary differential equations (ODEs) through suitable resemblance substitutions. The obtained dimensional equations are mathematically resolved with the help of the semi-analytical technique HAM. In the next section, the flow scenario has been formulated, unraveled, and discoursed.

2. MATHEMATICAL FORMULATION

The heat transition is studied through two-dimensional Ellis’s fluid over an extending sheet. The Ellis fluid is immersed along with the engine oil (base fluid). Three different sorts of nano particulates (SiO$_2$, TiO$_2$, and Al$_2$O$_3$) are dispersed in the engine oil. The wall surface is assumed to be stretchable to generate motion in fluidic molecules. The motion in fluidic atoms is generated due to the fluctuation of the wall. The magnetic dipole is supposed in the center and placed horizontally. The flow dynamics are elaborated in Figure 1.

The $x$-axis and $y$-axis are taken along the horizontal direction and vertical direction. Thermal energy transfer is considered as absorption and generation into the trihybrid nanofluid. The basic modeled phenomena are articulated as:

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0
\]

\[
\rho T \Phi \left( \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} \right)
= - \frac{\partial p}{\partial x} + \frac{1}{1 + \frac{\mu}{\nu \phi}} \left( \frac{\partial \mu}{\partial x} \right) + \mu_0 M \frac{\partial H}{\partial x}
+ \sigma_\Phi (E_0 B_0 - B_0^2 u)
\]

\[
(\rho C_p) T \Phi \left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right) + \left( \frac{\partial H}{\partial x} + \nu \frac{\partial H}{\partial y} \right) \frac{\partial M}{\partial T}
= k T \Phi \frac{\partial^2 T}{\partial y^2}
\]

\[- \lambda_x \left( \frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} \right) + \tau_x \left( \frac{D_B}{T} \frac{\partial C}{\partial y} + \frac{D_T}{T_0} \left( \frac{\partial T}{\partial y} \right)^2 \right)
\]

\[
u \frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_0} \frac{\partial^2 T}{\partial y^2} - \lambda_{x}(C - C_w)
\]

\[
\phi = \theta (\eta) \approx \phi (\eta), \eta = \sqrt{\frac{x}{y}}
\]

\[
\left(1 + (2 - \alpha) (\beta f')^{y-1} \right) f'' + \lambda_{Th} f' - f'' = 0
\]

\[
\left( \frac{\sigma_{Th} T H_a}{\sigma_\Phi} \right) E''_1 - \frac{2 \phi f'}{(\eta + y)}^{1/2}
+ \frac{1}{\epsilon} \left( \frac{\sigma_{Th} T H_a}{\sigma_\Phi} \right) E''_1 - \frac{2 \phi f'}{(\eta + y)}^{1/2}
+ \frac{1}{\epsilon} \left( \frac{\sigma_{Th} T H_a}{\sigma_\Phi} \right) E''_1 - \frac{2 \phi f'}{(\eta + y)}^{1/2}
\]

\[
\phi'' + \left( \frac{\rho C_p}{k T} \right) \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
+ \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
+ \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
\]

\[
\phi'' + \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
+ \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
\]

\[
\phi'' + \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
+ \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
\]

\[
\phi'' + \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
+ \frac{4 f}{(\eta + y)^3} \left[ \frac{k_f}{k_{Th}} \right] \frac{k_f}{k_{Th}} \frac{k_f}{k_{Th}} \left[ E''_1 \phi' - f'' \phi'' \right]
\]
The reduced boundary conditions are
\[ f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1 \]
\[ f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \]  
(10)
where, \( \epsilon = (1 - \varphi)\gamma_{1} + (1 - \varphi)\gamma_{2} + (1 - \varphi)^{2.5} \).

The non-dimensional parameters appearing in this study are defined as
\[ \beta_{1} = \frac{1}{\tau_{0}} \sqrt{\frac{\mu u_{w}}{\rho_{T}}}, \quad Ha^{2} = \frac{B_{0}^{2} \sigma u_{w}}{\mu_{T}}, \quad \beta = \frac{\gamma^{*} M_{p} \mu_{0}}{2 \mu_{T}}, \quad E_{1} = \frac{E_{0}}{B_{0} u_{w}}, \quad Pr = \frac{\delta_{k}(\rho C_{p})}{k_{f}}, \quad Ht = \frac{Q_{0}}{s(\rho C_{p})}, \]
\[ \delta_{k} - \frac{\lambda_{2,5}}{(\rho C_{p})}, \quad Ec = \frac{C_{p}(T_{w} - T_{w})}{C_{p}(T_{w} - T_{w})}, \quad Nb = \frac{\tau D_{p}(C_{w} - C_{w})}{\theta_{0}(\rho C_{p})}, \quad Sc = \frac{\delta_{k}}{D_{p}}, \quad R_{c} = \frac{k_{1}}{s} \]  
(11)

3. PHYSICAL QUANTITIES OF INTEREST

The physical quantities of engineering interest are as follows
\[ c_{l} = \frac{\tau_{w}}{\rho_{T} u_{w}^{2}}, \quad N_{u} = \frac{x q_{w}}{k_{f}(T_{w} - T_{w})} \]  
(12)
where \( \tau_{w} \) and \( q_{w} \) are the surface shear stress and heat flux, defined as
\[ \tau_{w} = \left[ \frac{1}{1 + \left( \frac{c_{l}}{\varphi} \right)^{a-1}} \right] \frac{\partial \varphi}{\partial y} \quad \text{and} \quad q_{w} = -k_{Thnf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]  
(13)
The dimensionless form of eq 11 is
\[ Re_{x}^{1/2} C_{l} = \frac{f^{*}(0)}{e + (\beta f^{*})^{a-1}}, \quad Re_{x}^{1/2} N_{u} = \frac{k_{Thnf} \phi(0)}{k_{f}} \]  
(14)

4. SOLUTION METHOD

We should use Homotopy Analysis Method (HAM) with the following processes to rectify eqs 7–9 underneath the constraints of eqs 10. The solutions with the auxiliary factors \( \lambda \) modify and regulate the solution convergence.

The following are the initial suppositions
\[ f_{0}(\eta) = (1 - e^{-a}), \quad \theta_{0}(\eta) = e^{-\eta}, \quad \phi_{0}(\eta) = e^{-\eta} \]  
(15)
The linear operators are assumed to be \( \sigma_{\theta} \sigma_{\phi} \theta_{\phi} \)
\[ \sigma(f) = f^{*} - f', \quad \sigma(\theta) = \theta^{*} - \theta, \quad \sigma(\phi) = \phi^{*} - \phi, \]
which have the following characteristics
\[ \sigma_{c}(c_{i} + c_{j} e^{-\eta} + c_{j} e^{-\eta}) = 0, \quad \sigma_{\phi}(c_{i} \phi^{*} + c_{j} e^{-\eta}) = 0, \quad \sigma_{\phi}(c_{j} e^{\eta} + c_{j} e^{-\eta}) = 0 \]  
(16)
where in general solution \( c_{i}(i = 1–7) \) are constants.

The resultant nonlinear operatives \( N_{o} N_{\theta} N_{\phi} \) are specified as
\[ N_{o}[f(\eta; p), \theta(\eta; p)] = \left( 1 + \frac{2 - \alpha}{(\beta f^{*}(\eta; p))^{a-1}} \right) \left( 1 + \frac{2 - \alpha}{(\beta f^{*}(\eta; p))^{a-1}} \right)^{2} \]  
\[ \frac{\partial^{3} f(\eta; p)}{\partial \eta^{3}} + \frac{1}{\sigma_{\theta} \theta_{\phi} \theta_{\phi}} \left( f(\eta; p) \frac{\partial f(\eta; p)}{\partial \eta} - \left( \frac{\partial f(\eta; p)}{\partial \eta} \right)^{2} \right) \]  
(18)
\[ N_{o}[f(\eta; p), \theta(\eta; p), \phi(\eta; p)] = \frac{2 \beta f(\eta; p)(\theta(\eta; p) - \epsilon_{i})}{k_{Thnf} (\eta + \gamma)^{2}} \]  
(19)
\[ N_{o}[f(\eta; p), \theta(\eta; p), \phi(\eta; p)] = \frac{2 \beta f(\eta; p)}{(\eta + \gamma)^{2}} \]  
(20)

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The literature describes the fundamental concept of HAM, and the zero-order problems from eqs 7–9 are

\((1 - p)\sigma_{0}[f(\eta; p) - f_{0}^{\prime}\eta] = p\chi_{0}N_{0}[f(\eta; p), \theta(\eta; p)]\)

\((21)\)

\((1 - p)\sigma_{0}\theta(\eta; p) = \theta_{0}(\eta)\)

\((22)\)

\((1 - p)\sigma_{0}\phi(\eta; p) - \phi_{0}(\eta)\)

\((23)\)

The equivalent boundary conditions are

\[f(\eta; p)]_{\eta=0} = 0, \quad \frac{\partial f(\eta; p)}{\partial \eta}]_{\eta=0} = 1, \quad \frac{\partial \theta(\eta; p)}{\partial \eta}]_{\eta=0} = 0, \quad \frac{\partial \phi(\eta; p)}{\partial \eta}]_{\eta=0} = 0,\]

\[(24)\]

where \(p \in [0, 1]\) denotes the embedding parameter, \(\chi_{0}, \chi_{0}, \chi_{0}\), are utilized to control the solution’s convergence. At \(p = 0\) and \(p = 1\) we get:

\[f(\eta; 1) = f(\eta), \quad \theta(\eta; 1) = \theta(\eta), \quad \phi(\eta; 1) = \phi(\eta),\]

\[(25)\]

Inflating \(f(\eta; p), \theta(\eta; p), \phi(\eta; p), \chi(\eta; p)\) in Taylor’s series about \(p = 0\)

\[f(\eta; p) = f_{0}(\eta) + \sum_{m=1}^{\infty} f_{m}(\eta)p^{m},\]

\[\theta(\eta; p) = \theta_{0}(\eta) + \sum_{m=1}^{\infty} \theta_{m}(\eta)p^{m},\]

\[\phi(\eta; p) = \phi_{0}(\eta) + \sum_{m=1}^{\infty} \phi_{m}(\eta)p^{m}.\]

\[(26)\]

where

\[f_{m}(\eta) = \frac{1}{m!}\frac{\partial f(\eta; p)}{\partial \eta} \bigg|_{p=0}, \quad \theta_{m}(\eta) = \frac{1}{m!}\frac{\partial \theta(\eta; p)}{\partial \eta} \bigg|_{p=0},\]

\[\phi_{m}(\eta) = \frac{1}{m!}\frac{\partial \phi(\eta; p)}{\partial \eta} \bigg|_{p=0}.\]

\[(27)\]

The secondary restrictions \(\lambda_{0}, \lambda_{0}, \lambda_{0}\) are selected in such a tactic that the series (26) converges at \(p = 1\); exchanging \(p = 1\) in eq 26, we acquire

\[f(\eta) = f_{0}(\eta) + \sum_{m=1}^{\infty} f_{m}(\eta),\]

\[\theta(\eta) = \theta_{0}(\eta) + \sum_{m=1}^{\infty} \theta_{m}(\eta),\]

\[\phi(\eta) = \phi_{0}(\eta) + \sum_{m=1}^{\infty} \phi_{m}(\eta).\]

\[(28)\]

The \(m\)th-order problem fulfills the following:

\[\sigma_{m}[f_{m}(\eta) - \chi_{m}f_{m-1}(\eta)] = \chi_{m}R_{m}(\eta),\]

\[\sigma_{m}[\theta_{m}(\eta) - \chi_{m}\theta_{m-1}(\eta)] = \chi_{m}R_{m}(\eta),\]

\[\sigma_{m}[\phi_{m}(\eta) - \chi_{m}\phi_{m-1}(\eta)] = \chi_{m}R_{m}(\eta).\]

\[(29)\]

The corresponding boundary conditions are

\[f_{m}(0) = f_{m}^{\prime}(0) = \theta_{m}(0) = \phi_{m}(0) = 0\]

\[f_{m}(\infty) = \theta_{m}(\infty) = \phi_{m}(\infty) = 0\]

\[(30)\]

Here

\[R_{m}^{\alpha}(\eta) = \left(\frac{1 + (2 - \alpha)(\beta f_{m-1}^{\prime}(\eta))^{\alpha-1}}{1 + (\beta f_{m-1}^{\prime}(\eta))^{\alpha-1}}\right) f_{m-1}^{\prime} + \frac{k_{f}}{k_{Th} \sigma_{f}} \sum_{k=0}^{m-1} f_{m-1-k} \sum_{i=0}^{k} \left(\sum_{j=0}^{k} \frac{j!}{\eta^{j}} \frac{f_{m-1-k}^{j} \theta_{m-1-k}^{j}}{(k-j)!} \right)\]

\[= \frac{1}{\sigma_{f}^{2}} \left(\frac{\sigma_{Th} f_{m-1}^{\prime} + \sigma_{f} R_{Th} f_{m-1}^{\prime} - E_{1}}{(\eta + \gamma)^{2}}\right)\]

\[= \frac{1}{\sigma_{f}^{2}} \left(\frac{\sigma_{f}^{2} f_{m-1}^{\prime} + \sigma_{Th} f_{m-1}^{\prime} - E_{1}}{(\eta + \gamma)^{2}}\right)\]

\[(31)\]

\[R_{m}^{\beta}(\eta) = \theta_{m-1}^{\beta} + \left(\frac{\sigma_{Th} f_{m-1}^{\prime} + \sigma_{f} R_{Th} f_{m-1}^{\prime} - E_{1}}{(\eta + \gamma)^{2}}\right)\]

\[= \frac{1}{\sigma_{f}^{2}} \left(\frac{\sigma_{f}^{2} f_{m-1}^{\prime} + \sigma_{Th} f_{m-1}^{\prime} - E_{1}}{(\eta + \gamma)^{2}}\right)\]

\[(32)\]

\[R_{m}^{\phi}(\eta) = \phi_{m-1}^{\phi} + \frac{N_{f}}{N_{b}} \left(\sigma_{Th} f_{m-1}^{\phi} - R_{Th} \phi_{m-1}^{\phi}\right)\]

\[(33)\]
5. RESULTS AND DISCUSSION

This section expresses the physical mechanism and trend behind each Figure and Table. Figure 1a,b illustrates the flow configuration under the influence of magnetic dipole and synthesis of ternary nanofluid with the dispersion of $\text{Al}_2\text{O}_3$, $\text{SiO}_2$, and $\text{TiO}_2$ nanoparticles in engine oil.

Figures 2–6 display the velocity outlines $f'(\eta)$ versus the influence of fluid parameter $\beta_1$, ferrohydrodynamic interaction term $\beta$, Hartman number $Ha_2$, magnetic dipole $\gamma$, and local electric parameter $E_1$, respectively. Figures 2 and 3 determine that the velocity contour improves with the influence of fluid parameter $\beta_1$, while diminishes by the action of ferrohydrodynamic interaction term $\beta$. Physically, the stretching velocity of the plate intensifies with the impact of fluid constraint, which reassures the fluid particles to stream fast, as a result, the velocity field $f'(\eta)$ improves as presented in Figure 2. The magnetic effect and density of fluid enhance the variation of the ferrohydrodynamic interaction factor. Actually, the magnetic effect (due to Lorentz force) and density of the fluid, both stipulate hurdles to the flow field, which bases the decline of momentum boundary layer $f'(\eta)$.

Figures 4 and 5 govern that the velocity curve $f'(\eta)$ augments with the outcome of Hartman number $Ha_2$, while degenerating with the consequence of the magnetic dipole parameter. Physically, the rising influence of the Hartmann number boosts the stretching velocity of the vertical plate as well as lessens the density of base fluid, which reasons for such a state, as observed in Figure 4. The magnetic dipole generates resistive effects, which contest the flow stream and drops its velocity $f'(\eta)$, as publicized in Figure 5. Figure 6 shows that the velocity profile magnifies with the growing upshot of local electric constraint $E_1$. The magnetic effect and stretching velocity of the vertical surface both exposed an inverse trend against the electric force, as a product the velocity field $f'(\eta)$ boosts.

Figures 7–13 highlighted the behavior of energy contour$\theta$-$(\eta)$ versus the upshot of ferrohydrodynamic interaction number $\beta$, Hartman number $Ha_2$, thermal relaxation parameter $\nu$, and Lorentz number $Le$, respectively. In general, the variation of energy increases with the ferrohydrodynamic interaction term $\beta$, which conforms to the increment of velocity field $f'(\eta)$ and thermal profile $\theta(\eta)$, as presented in Figure 7. The magnetic effect helps in increasing the fluid temperature, which leads to an increment in energy, as observed in Figure 8. Additionally, with the growing of magnetic dipole $\gamma$, the fluid temperature increases, which corroborates the augmentation in the velocity field, as illustrated in Figure 9. The Lorentz number, on the other hand, shows that the velocity increases with the Lorentz number, while the energy contour diminishes, as depicted in Figure 10. The thermal relaxation parameter $\nu$ and Lorentz number $Le$ exert a significant influence on the energy profile $\theta(\eta)$, as illustrated in Figure 11. The velocity field $f'(\eta)$ improves with the thermal relaxation parameter $\nu$, whereas the energy contour $\theta(\eta)$ reduces, as observed in Figure 12. Finally, the interaction of ferrohydrodynamic and thermal relaxation parameters is described in Figure 13, which shows that the velocity field $f'(\eta)$ improves with the thermal relaxation parameter $\nu$, while the energy contour $\theta(\eta)$ decreases, as illustrated in Figure 13.
\[\delta, \text{ Eckert number } Ec, \text{ viscous dissipation term } \lambda_1, \text{ heat source number } Ht, \text{ and local electric parameter } E_1, \text{ respectively.}\]

Figures 7 and 8 exposed that the energy field diminutions with the influence of ferrohydrodynamic interaction factor augments with the difference of Hartman number \( Ha^2 \). As discussed earlier, the magnetic effect and density of fluid improve versus the variation of the ferrohydrodynamic interaction factor, which resists the flow field and generates friction force; as a result, the fluid energy field \( \theta(\eta) \) declines. On the other hand, the rising frequency of the Hartmann number expands the energy profile \( \theta(\eta) \) as exposed in Figure 8. Figures 9 and 10 uncovered that the energy field contracts with the impact of thermal relaxation parameter \( \delta_e \) and enriches with the variation of Eckert number Ec. Additional heat is produced due to the upshot of the Eckert number because the specific heat capacity of fluid declines with the influence of the Eckert number, which causes an elevation in the energy profile \( \theta(\eta) \).

Figures 11 and 12 exposed that the energy profile enriches with the influence of thermal viscous dissipation term \( \lambda_1 \) and heat source number Ht. The heat source parameter works as a
heat generating agent in the fluid, so the rising impact of the heat source term boosts the energy distribution as shown in Figure 12. Figure 13 shows that the action of the local electric parameter also magnifies the energy profile because the magnetic effect and stretching velocity of the vertical surface both exposed an inverse relation to the electric force, and as a consequence, the energy field $\theta(\eta)$ boosts.

Figures 14–17 reported the compartment of concentration contour $\phi(\eta)$ versus the upshot of Schmidt number $Sc$, chemical reaction parameter $Rc$, thermophoresis effect $Nt$, and Brownian motion $Nb$, respectively. Figures 14 and 15 show that the mass transfer rate reduces with the effect of the Schmidt number and chemical reaction factor. Physically, the kinetic viscosity of ternary nanofluid enhances, while the molecular diffusion declines with the action of the Schmidt number, which results in the retardation of mass transition rate. Figures 16 and 17 revealed that the concentration outlines dimming with the upshot of the thermophoresis effect and augments with the action of the Brownian motion $Nb$. The Brownian motion of fluid particles inside the fluid generates a retarding effect, which opposes the moving fluid; eventually, the mass transmission rate $\phi(\eta)$ reduces.

Tables 1 and 2 expressed the experimental values and basic thermophysical characteristics of ternary nanoparticles and engine oil, respectively. Tables 3 and 4 revealed the statistical results of skin friction $Re_x^{1/2}C_f$ and Nusselt number $Nu$.

Table 1. Investigational Values of $Al_2O_3$, $SiO_2$, $TiO_2$, and Engine Oil

|          | $K$     | $\sigma$     | $\rho$  |
|----------|---------|---------------|---------|
| Engine oil | 0.144   | $0.125 \times 10^{-11}$ | 884     |
| $Al_2O_3$ | 32.9    | $5.96 \times 10^{7}$   | 6.310   |
| $TiO_2$   | 8.953   | $2.4 \times 10^{8}$    | 4.250   |
| $SiO_2$   | 1.4013  | $3.5 \times 10^{8}$    | 2.270   |
Table 2. Thermochemical Properties of Ternary Hybrid Nanofluids$^{54}$

viscosity

\[
\frac{\rho_{\text{Thnf}}}{\rho} = (1 - \phi_{\text{Al}_2\text{O}_3})^{\frac{1}{2}}(1 - \phi_{\text{TiO}_2})^{\frac{1}{2}}(1 - \phi_{\text{SiO}_2})^{\frac{1}{2}}.
\]

density

\[
\frac{\rho_{\text{Thnf}}}{\rho} = (1 - \phi_{\text{TiO}_2})^{\frac{1}{2}} \left( (1 - \phi_{\text{SiO}_2}) + \phi_{\text{SiO}_2} \frac{\rho_{\text{Al}_2\text{O}_3}}{\rho} + \phi_{\text{TiO}_2} \frac{\rho_{\text{TiO}_2}}{\rho} \right)
\]

\[
(\rho_{\text{cp}})_{\text{Thnf}} = \phi_{\text{Al}_2\text{O}_3}(\rho_{\text{cp}})_{\text{Al}_2\text{O}_3} + (1 - \phi_{\text{Al}_2\text{O}_3})\left(1 - \phi_{\text{TiO}_2}\right)\left(1 - \phi_{\text{SiO}_2}\right)
\]

specific heat

\[
k_{\text{Thnf}} = \frac{k_{\text{Al}_2\text{O}_3} + 2k_{\text{Al}_2\text{O}_3} - 2\phi_{\text{Al}_2\text{O}_3}(k_{\text{Al}_2\text{O}_3} - k_{\text{SiO}_2})}{k_{\text{Al}_2\text{O}_3} + 2k_{\text{Al}_2\text{O}_3} + \phi_{\text{Al}_2\text{O}_3}(k_{\text{Al}_2\text{O}_3} - k_{\text{SiO}_2})} k_{\text{Al}_2\text{O}_3}
\]

thermal conduction

\[
k_{\text{f}} = \frac{k_{\text{Al}_2\text{O}_3} + 2k_{\text{Al}_2\text{O}_3} + \phi_{\text{Al}_2\text{O}_3}(k_{\text{Al}_2\text{O}_3} - k_{\text{SiO}_2})}{k_{\text{Al}_2\text{O}_3} + 2k_{\text{Al}_2\text{O}_3} + \phi_{\text{Al}_2\text{O}_3}(k_{\text{Al}_2\text{O}_3} - k_{\text{SiO}_2})}
\]

electrical conductivity

\[
\frac{\sigma_{\text{Thnf}}}{\sigma} = \frac{1 + \frac{3\left(\frac{\sigma_{\text{Al}_2\text{O}_3}}{\sigma} - 1\right)\phi_{\text{Al}_2\text{O}_3}}{\left(\frac{\sigma_{\text{Al}_2\text{O}_3}}{\sigma} + 2\right) - \left(\frac{\sigma_{\text{Al}_2\text{O}_3}}{\sigma} - 1\right)\phi_{\text{Al}_2\text{O}_3}}}{\sigma_{\text{Al}_2\text{O}_3}}
\]

Table 3. The Quantitive Outputs of Skin Friction $Re_x^{1/2}C_f$ Versus Different Physical Constraints

| $\beta_i$ | $\beta$ | $Ha^2$ | $E_i$ | $Re_x^{1/2}C_f$ |
|----------|---------|--------|------|----------------|
| 0.1      | 0.3     | 0.5    | 0.5  | 0.1            |
| 0.3      | 0.6     | 0.5    | 0.5  | 0.1            |
| 0.5      | 0.4     | 0.5    | 0.5  | 0.1            |

$Re_x^{-1/2}N_u$ versus physical interest entities, respectively. It can be perceived that the rising values of the Hartmann number and electric parameter decline the skin friction, while the influence of the ferrohydrodynamic interaction factor boosts the tendency of skin friction. From Table 4, it can be observed that the upshot of the Eckert and Hartmann number reduces the Nusselt number.

Table 4. The Statistical Outputs of Nusselt Number $Re_x^{-1/2}N_u$ Versus Different Physical Constraints

| $\delta$ | $Ec$ | $Ht$ | $E_i$ | $Re_x^{-1/2}N_u$ |
|----------|------|------|------|----------------|
| 0.1      | 0.3  | 0.5  | 0.5  | 0.1            |
| 0.3      | 0.3  | 0.5  | 0.5  | 0.1            |
| 0.5      | 0.4  | 0.5  | 0.5  | 0.1            |

6. CONCLUSIONS

We have studied the augmentation of energy transfer in trihybrid Ellis nanofluid flow when a magnetic dipole passes over a vertical surface. The velocity and heat transportation has been examined in the presence of the electromagnetic effect and heat source/sink. The phenomena of fluid flow have been
mathematically designed for energy and fluid velocity in the form of a nonlinear system of PDEs. The system of PDEs is further refined to the set of ODEs through suitable resemblance substitutions. The obtained dimensional equations are numerically answered with the help of HAM. The key findings are:

1. The velocity outline enhances with the effect of fluid parameter $\beta$, while diminish by the action of the ferrohydrodynamic interaction term $\beta$.
2. The accumulation of $Al_2O_3$, $SiO_2$, and $TiO_2$ nanomaterials to the base fluid “engine oil” advances its momentum and energy profiles.
3. The velocity curve $f(\eta)$ augments with the outcome of electric constraint $E_1$ and Hartman number $Ha^2$ while degenerates as a consequence of the magnetic dipole parameter.
4. Energy field diminutions with the influence of the ferrohydrodynamic interaction factor but augments with the difference of the Hartman number $Ha^2$.
5. The energy field contracts with the impact of the thermal relaxation parameter $\delta_t$ and boosts with the variation of the Eckert number $Ec$.
6. The energy profile augments with the influence of the electric parameter, thermal viscous dissipation term $\lambda_t$, and heat source number $Ht$.
7. The mass transfer rate reduces with the effect of the Schmidt number and chemical reaction factor.
8. The concentration outlines dimming with the upshot of the thermophoresis effect and augments with the action of Brownian motion $Nb$.

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**NOMENCLATURE**

- $B_0$ magnetic field intensity ($A m^{-1}$)
- $C$ concentration
- $D_f$ thermophoretic diffusion coefficient
- $E_1$ local electric parameter
- $k_1$ chemical reaction rate
- $H$ Magnetic field
- $M$ magnetization
- $Nt$ thermodiffusion parameter
- $Q_0$ heat source number
- $Re$ local Reynolds number
- $T$ temperature of fluid
- $x,y$ coordinates axis ($m$)

**GREEK LETTERS**

- $\alpha$ material parameter
- $\beta$ ferrohydrodynamic interaction number
- $\gamma$ strength of the magnetic dipole
- $\epsilon$ ratio parameter
- $\theta$ dimensionless temperature
- $\lambda$ thermal relaxation time
- $\dot{\theta}$ kinematic viscosity
- $\rho_C$ heat capacity
- $\phi_1, \phi_2, \phi_3$ volume fraction

**SUBSCRIPTS**

- $f$ fluid
- $hnf$ hybrid nanofluid
- $C_p$ specific heat transfer ($J kg^{-1} K^{-1}$)
- $D_h$ Brownian diffusion coefficient
- $E_0$ electric field intensity
- $Ec$ Eckert number
- $k$ thermal conductivity ($W m^{-1} K^{-1}$)
- $Ha^2$ Hartman number
- $Nb$ Brownian motion parameter
- $Pr$ Prandtl number
- $R_c$ chemical reaction parameter
- $Sc$ Schmidt number
- $u,v$ velocity components ($m s^{-1}$)
- $\alpha$ temperature ratio
- $\beta$ fluid parameter
- $\delta_t$ thermal relaxation parameter
- $\eta$ independent coordinate
- $\lambda_t$ viscous dissipation number
- $\mu_0$ magnetic permeability
- $\rho$ density ($kg m^{-3}$)
- $\sigma$ electrical conductivity
- $bf$ base fluid
- $Thnf$ ternary hybrid nanofluid
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