Chiral symmetry restoration and the linear form of baryonic Regge trajectories

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Abstract

It has recently been suggested that the observed structure of parity doublets, seen in the spectrum of highly excited baryons, may be due to the effective restoration of chiral symmetry for these states. This chiral symmetry restoration high in the spectrum is consistent with the concept of quark-hadron duality. Moreover, if QCD dynamics implies the linear Regge trajectories for highly excited baryons, then the MacDowell symmetry requires the parity doubling to appear, which shows that linear Regge trajectories and chiral symmetry restoration are well compatible. On the contrary, in the low-energy part of baryon spectrum the parity doublets are absent because of spontaneous chiral symmetry breaking. Then the MacDowell symmetry implies that there should be no linear parallel Regge trajectories. Experimental data shows that this is indeed the case.

It is known since 20 years that the low-energy properties of nucleon, such as its mass, are intimately related to the phenomenon of spontaneous breaking of chiral symmetry (SBCS). At the same time the physics of the highly excited baryons (hadrons) is intuitively attributed to the confining property of QCD. The most significant phenomenological support to the latter is represented by the linear-like behaviour of hadron Regge trajectories. While, in the case of mesons, the Regge trajectories are indeed approximately parallel straight lines, this is by far not true for baryons. In Fig. 1 (where we have shown only a part of positive and negative parity Regge trajectories) there are clearly no linear parallel trajectories in the low-energy part of the baryon spectrum (compare trajectories for, e.g. the positive parity $N(939), 1/2^+$, and negative parity $N(1535), 1/2^-$). This fact can naturally be interpreted as that low in the spectrum it is the physics that is not directly related to confining property of QCD is crucially important. Indeed, the whole low-lying spectrum can be reproduced in the context of the chiral constituent quark model, where apart from a phenomenological confining potential the residual interaction between constituent quarks is mediated by chiral (Goldstone boson) fields. Both the constituent quarks and chiral fields can be considered as effective degrees of freedom that are induced by SBCS.
Any constituent quark model in light baryons that rely on a phenomenological confinement potential predicts a lot of states in the region $M \sim 2$ GeV, that are not observed (the so-called missing states). The striking feature of experimental baryon spectrum, however, is the systematical appearance of the parity doublets in this region. The systematical parity doubling cannot be explained in potential constituent quark model and probably indicates the onset of a new physical regime. If this phenomenon is experimentally confirmed, then the most natural explanation would be that a parity doubling should be a manifestation of some fundamental symmetry of the underlying QCD theory. Recently, it has been suggested that the observed parity doublets could be explained as due to the restoration of almost perfect $SU(2)_L \times SU(2)_R$ global chiral symmetry of QCD Lagrangian in the $u, d$ sector.

The chiral symmetry is known to be spontaneously broken by the QCD vacuum, and, as a consequence, there are no parity doublets in the low-energy spectrum. Indeed, in the Nambu-Goldstone mode of chiral symmetry the constituent quarks are adequate degrees of freedom and the quark model works well (see, for example, a recent calculation of covariant formfactors of the nucleon). However, if the effects of SBCS become less important at increasing energy in the baryon spectrum (i.e. chiral symmetry is effectively restored), the constituent quarks become inadequate. In this case the physical states should fall into representations of the chiral group $SU(2)_L \times SU(2)_R$, that have been called chiral-parity multiplets. Depending on the specific representation the multiplets can be either parity doublets in $N$ and $\Delta$ spectra that are not connected to each other, or parity doublets that belong to the same parity-chiral multiplet and hence are degenerate in mass (existing data probably support the latter case).

It has also been suggested in ref. that the concept of quark-hadron duality naturally implies the chiral symmetry restoration. The phenomenon of quark-hadron duality is well established in many processes, e.g. in $e^+e^- \rightarrow$ hadrons, where we have a direct experimental access to creation of the quark-antiquark pair by the electromagnetic current. According to this concept, the spectral density $\rho(s)$ should be dual to the polarization operator calculated at the free quark loop level (up to perturbative corrections) at asymptotically large $s$. For the process $e^+e^- \rightarrow$ hadrons the ”asymptotic regime” sets up at $s \sim 2-3$ GeV$^2$. In the case of baryons, unfortunately, there are no experimentally accessible currents that can create three quarks at some space-time point and connect them to baryons. Nevertheless, one can construct such currents theoretically, and these currents are widely used in QCD sum rules or lattice calculations to extract properties of low-lying baryons directly from QCD. The quark-hadron duality, applied to the present case, would mean that in the asymptotically high part of the baryon spectrum the baryon spectral density should be dual to the one which is calculable in perturbation theory; hence the chiral symmetry should be manifest in the spectral density, because there is no chiral symmetry breaking in perturbation theory.

The pure perturbative contribution is dual to a continuous spectrum (i.e. very dense spectrum of overlapping resonances), so that there should be no isolated resonances. Clearly, as one goes up in the spectrum the nonperturbative contributions to the spectral density should smoothly decrease, approaching eventually regime where a free quark loop is dominant. If
the nonperturbative effects related to SBCS die out earlier than those ones responsible for confinement, then the high-energy part of baryon spectrum should show isolated baryon resonances that do not feel effects of SBCS and consequently fall into parity-chiral multiplets.

So the question arises whether it is possible or not possible to design nonperturbative effects in such a way that the spectral density at large \( s \) is still nontrivial (i.e. it feels some nonperturbative contributions) and, at the same time, becomes chirally symmetric. The answer is probably positive. The aim of this note is to show that QCD nonperturbative dynamics that is responsible for linear Regge trajectories in the high-energy baryon spectrum together with the assumption of analyticity in relativistic quantum theory (that is behind Regge-pole philosophy), is indeed compatible with effective chiral symmetry restoration.

Let us define the current (interpolating field) \( \eta(x) \) that creates three quarks in a color singlet state at the space-time point \( x \). Then, the two-point correlator \( \langle 0 | T(\eta(x), \bar{\eta}(0)) | 0 \rangle \) describes the propagation of three quarks from the point 0, where they are created from the vacuum, to the point \( x \), where they annihilate into the vacuum. On their way from 0 to \( x \), these three quarks interact with each other and with the vacuum fields. This correlation function contains complete information about all baryons that couple to the current \( \eta \).

To be specific, let us consider the example of the Ioffe current \( \eta_N = \epsilon_{abc} (u^a T C \gamma_\mu u^b) \gamma_\mu \gamma_5 d^c \). i.e. one of the currents that couples to isodoublet \( J = 1/2^+ \) and \( J = 1/2^- \) baryons. Here \( a, b, c \) are the colour indices for \( u \) and \( d \) quarks. This current couples to isodoublet \( J = 1/2^+ \) and \( J = 1/2^- \) baryons. With respect to \( SU(2)_L \times SU(2)_R \) transformation properties, it belongs to the \((1/2, 0) \oplus (0, 1/2)\) representation. The correlation function can be rewritten in the closure representation as a sum over all possible baryonic states that can be created by the given current. Hence, in the zero-width approximation the correlator is given by

\[
\Pi(q) = i \int d^4xe^{iqx} \langle 0 | T(\eta(x), \bar{\eta}(0)) | 0 \rangle = \sum_{n_+, n_-} \left[ \lambda^2_{n_+} \frac{\gamma_\mu q^\mu + m_{n_+}}{q^2 - m^2_{n_+} + i \varepsilon} + \lambda^2_{n_-} \frac{\gamma_\mu q^\mu - m_{n_-}}{q^2 - m^2_{n_-} + i \varepsilon} \right],
\]

(1)

where \( m_{n_+} \) and \( m_{n_-} \) are masses of positive and negative parity states and the constants \( \lambda_{n_+} \) and \( \lambda_{n_-} \) parametrize the strengths with which the given baryonic states couple to the current:

\[
< 0 | \eta | n_+(q) >= \lambda_{n_+} u(q),
\]

(2)

\[
< 0 | \eta | n_-(q) >= \lambda_{n_-} \gamma_5 u(q),
\]

(3)

where \( u(q) \) is a generic Dirac spinor for a baryon with momentum \( q \).

The correlator (1) contains the chiral even and odd terms

\[
\Pi(q) = \Pi^{even}(q^2) q_\mu \gamma^\mu + \Pi^{odd}(q^2),
\]

(4)

3
which behave differently under chiral transformations; while the former conserves chirality, the latter one violates it. Indeed, under a axial rotation exp(\(i\pi/3\gamma_5/2\)) one obtains

\[
\langle 0 | T (\eta(x), \bar{\eta}(0)) | 0 \rangle = -\gamma_5 \langle 0 | T (\eta(x), \bar{\eta}(0)) | 0 \rangle \gamma_5.
\] (5)

This property implies that in the chiral symmetric phase the chiral odd term must vanish and there must be a one-to-one correspondence between the positive and negative parity baryonic states that are degenerate in mass and equally coupled to the current:

\[
m_{n_+} = m_{n_-},
\] (6)

\[
\lambda_{n_+} = \lambda_{n_-}.
\] (7)

Converse is also valid, the conditions \((3)\) and \((6)\) imply that the chiral odd term is zero in the correlation function, which means that the correlation function is chirally invariant. Actually, this is a simple and well known manifestation of the old statement that if the \(SU(2)_L \times SU(2)_R\) symmetry is realized in the Wigner mode, the hadrons should fall into multiplets of the chiral group that contain degenerate states of positive and negative parity \[9\].

We have considered for simplicity the current that belongs to the \((1/2, 0) \oplus (0, 1/2)\) representation. However, one can similarly construct currents that belong to other representations of the chiral group \[10\]. The current that belongs to the \((1/2, 1) \oplus (1, 1/2)\) representation couples at the same time to both positive and negative parity states with the same spin in \(N\) and \(\Delta\) spectra. In the Wigner mode of chiral symmetry, the chiral odd term in the correlator must vanish, which is equivalent to the statement

\[
m_{n_+} = m_{n_-} = m_{\Delta n_+} = m_{\Delta n_-},
\] (8)

\[
\lambda_{n_+} = \lambda_{n_-} = \lambda_{\Delta n_+} = \lambda_{\Delta n_-}.
\] (9)

If the conjecture of Ref. \[3\] is correct, then the highly-lying states in \(N\) and \(\Delta\) spectra should be strongly coupled to the latter current.

In the real world, chiral symmetry is spontaneously broken. As a consequence, the spectrum does not consist of systematical parity doublets. For example, it is the nonvanishing quark condensate in the vacuum state that leads to the nonzero value of the chiral odd term in the correlator and hence is responsible for the splitting between \(N(939), 1/2^+,\) and \(N(1535), 1/2^-,\) states (for a recent study in the context of QCD sum rules see \[6\] and for lattice calculations see \[7\]).

However, even if the chiral symmetry is spontaneously broken in the vacuum ( hence, in the low-lying states), one might expect that it becomes effectively restored at higher energies. Indeed, at large space-like momenta \(q^2 < 0,\) where the operator product expansion (OPE) \[11\] is applicable, the correlator is dominated by free quark loop plus perturbative
corrections. The nonperturbative contributions from the quark condensates, that are the only contributions to the chiral odd term, are suppressed by powers of $q^2$. For instance, using the same Ioffe current like in expressions (1)-(4), the OPE up to dimension dim=6 operators is 

$$
\Pi^{\text{odd}}(q^2) = -\frac{1}{4\pi^2}\langle\bar{q}q\rangle q^2 \ln(-q^2),
$$

(10)

$$
\Pi^{\text{even}}(q^2) = \frac{\ln(-q^2)}{32\pi^2} \left( \frac{q^4}{2\pi^2} + \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a \rangle \right) + \frac{2}{3q^2}\langle\bar{q}q\rangle^2.
$$

(11)

Hence, upon analytical continuation from the deep space-like region (where the language of quarks and gluons is applicable) to the time-like region, $s = q^2 > 0$ (where the current creates baryons), in the high-energy spectral density (that is imaginary part of the correlator in the time-like region) the chiral odd term becomes small with respect to the chiral even one, because it is suppressed by the same powers of $s$ in the time-like region as of $q^2$ in the space-like one. This implies that the splitting between parity partners of the high-energy spectrum will become small with respect to the mass of baryons.

High in the spectrum the chiral even spectral density is driven by perturbative contributions as well as by contributions from gluonic condensate (the last term in (11) does not contribute to the spectral density). The gluonic condensate parametrizes soft nonperturbative gluonic effects in QCD and does not break chiral symmetry. If these nonperturbative contributions imply the linear Regge trajectories in the high-energy spectrum, then they are indeed compatible with the chiral symmetry restoration, as it will be shown in the following.

The position $\alpha(t)$ of the corresponding Regge pole in the complex angular momentum plane is identified with the physical state once the trajectory $\alpha(t)$ crosses the real values $J = 1/2, 3/2, \ldots$ and, at this point, $t$ is identified with the mass of baryon (for a review of the Regge theory see [13]). A very general requirement of analyticity implies that fermionic (but not bosonic!) Regge trajectories of positive and negative parity must satisfy the MacDowell symmetry [14],

$$
\alpha^+(\sqrt{t}) = \alpha^-(\sqrt{-t}), \; t > 0.
$$

(12)

Hence, at positive $t$ this equation implies that the most general functional form for positive- and negative-parity baryonic Regge trajectories is given by

$$
\alpha^+(t) = \alpha_0 + \alpha_1\sqrt{t} + \alpha_2 t + \ldots,
$$

(13)

$$
\alpha^-(t) = \alpha_0 - \alpha_1\sqrt{t} + \alpha_2 t - \ldots
$$

(14)

Specific QCD dynamics, that is responsible for the given functional form of Regge trajectories, is hidden in the coefficients $\alpha_0, \alpha_1, \ldots$, that in principle should be calculable in terms of QCD degrees of freedom. If QCD nonperturbative dynamics leads to the linear form of
baryonic Regge trajectories in the high-energy part of baryon spectrum, \( i.e. \alpha(t) = \alpha_0 + \alpha't \), then at large positive \( t \), \( \alpha_1 = \alpha_3 = \ldots = 0 \) in (13) and (14), \( i.e. \) the MacDowell symmetry requires that here the positive and negative parity Regge trajectories must coincide,
\[
\alpha^+(t) = \alpha^-(t). 
\] (15)

Hence, it follows that baryons will occur in degenerate doublets of opposite parity. Just the same is implied by the effective chiral symmetry restoration. One then concludes that chiral symmetry restoration is consistent with that QCD nonperturbative dynamics that is responsible for the linear behaviour of Regge trajectories.

On the contrary, in the low-energy spectrum SBCS tells that there must be no degenerate parity doublets, which is consistent with data. This together with the MacDowell symmetry, requires that at small positive \( t \) at least one of the coefficients \( \alpha_1, \alpha_3, \ldots \) in (13) and (14) is not vanishing, which makes Regge trajectories curved. Hence the chiral symmetry breaking effects of QCD together with requirements of analyticity imply that there must be no linear parallel Regge trajectories low in the baryon spectrum. That this indeed the case is well seen from Fig. 1.

In summary, we have shown that the proposed effective restoration of chiral symmetry in the upper part of the baryon spectrum, that explains the systematic appearance of parity doublets, is consistent with the linear form for baryonic Regge trajectories. On the other hand, the absence of parity doublets in the low-lying part of the spectrum, that is attributed to spontaneous breaking of chiral symmetry, implies that there should be no linear parallel Regge trajectories. The experimental data support it.
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Figure captions

Fig.1

Some of the Regge trajectories in the nucleon spectrum. The solid lines represent the positive parity Regge trajectories, while the dashed ones - the negative parity trajectories.
