Title: Topological superconductivity in metal/quantum-spin-ice heterostructures

Date: Jun 08, 2017  02:00 PM

URL: http://pirsa.org/17060045

Abstract: Superconductivity research has traditionally been discovery driven. Of course, Tc is a non-universal quantity that cannot be predicted, hence off-limits to theorists. Nevertheless, it must be possible to reach intelligent predictions for superconductors that are interesting for reasons other than high Tc per se. Of particular interest are topological superconductors under pursuit as a platform for quantum computing. Here, I will present the strategy of using the spin-spin correlation of quantum spin ice to achieve topological superconductivity at the interface between metal and quantum spin ice.
“Topological superconductivity in metal/quantum-spin-ice heterostructures”

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Perimeter 6.8.2017
Majorana bound state in odd-parity SC
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Vortices of $p+ip$ SF $\rightarrow$ zero modes at the core
Kopnin and Salomaa PRB (1991)

Zero modes are Majorana
Majorana bound state in odd-parity SC

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- Zero modes are Majorana
  
  - BdG qps $\gamma_i^\dagger = u\psi_i^\dagger + v\psi_i$ $\gamma_i^\dagger(E_n) = \gamma_i(-E_n)$
  
  - zero mode: $\gamma_i^\dagger(0) = \gamma_i(0)$
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  - BdG qp’s \(\gamma_i^\dagger = u\psi_i^\dagger + v\psi_i \quad \gamma_i^\dagger(E_n) = \gamma_i(-E_n)\)
  - zero mode: \(\gamma_i^\dagger(0) = \gamma_i(0)\)
  - non-local q-bit: \(c = \gamma_1 + i\gamma_2\)
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- Majorana + vortex composite $\rightarrow$ non-Abelian statistics

![Diagram of Majorana bound state]
fractionalization and Topo degeneracy

Fractional charge $e^* = e/q$

$N_g = q^g$ e.g., $N_1 = 3$
fractionalization and Topo degeneracy

Fractional charge $e^* = e/q$

$N_g = q^g$ e.g., $N_1 = 3$

2n Non-abelian vortices

$N_{2n} = 2^{n-1}$ for MR state or p+ip SF
Anyons

- Fractional charge $e^*$
- Fractional statistics (spin) $\theta$
- n- **nonabelian** qp state $\Leftrightarrow$ set of Qubits

$$\Psi(x_1, \cdots, x_n) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_{d(n)} \end{pmatrix}$$
Anyons

- Fractional charge $e^*$
- Fractional statistics (spin) $\theta$
- n- nonabelian qp state $\Rightarrow$ set of Qubits

\[ \Psi(x_1, \cdots, x_n) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_{d(n)} \end{pmatrix} \quad \text{exchange of qp's: rotation} \]

in $d(n)$ dim Hilbert space

\[ \Psi(x_1 \leftrightarrow x_3) = M \Psi(x_1, \cdots, x_n) \]
\[ \Psi(x_1 \leftrightarrow x_2) = N \Psi(x_1, \cdots, x_n) \]
Q. Topological Superconductor material?

Review:
Kallin & Berlinsky, Rep. Prog. Phys. (2016),
Alicea, Rep. Prog. Phys (2012)
Q. Topological Superconductor material?

**Bulk**

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Bulk  1D proximity

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Bulk

1D proximity

2D proximity?

Review:
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Design Strategy for intrinsic odd-parity superconductor?

• Manipulate the pairing interaction: target non-phononic mechanism
Wanted: non-phononic mechanism

Dope a Quantum spin liquid

P.W. Anderson

RVB singlet
Wanted: non-phononic mechanism

Dope a Quantum spin liquid

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RVB singlet  Cooper pair singlet
Wanted: non-phononic mechanism

Use Quantum paramagnet
Wanted: non-phononic mechanism

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Wanted: non-phononic mechanism

Use Quantum paramagnet

- Characteristic energy scales:
  \[ E_F, E_{F}, J_{ex}, J_K \]

- Perturbative limit:
  \[ J_K / E_F << 1 \]

- Spin-fermion model
Spin-fermion model for $J_{ex} = 0$

Kondo-Singlet

Doniach (1977)
Spin-fermion model for $J_{ex}=0$

RKKY interaction

Kondo-Singlet

Doniach (1977)
Spin-fermion model for $J_{ex}$ + Frustration

For $J_{RKKY} \sim J_K^2N(0) < J_{ex}$, no AFM

Kondo-Singlet + RVB
singlet + Cooper pair

Coleman & Andrei (1989)
Senthil, Vojta, Sachdev (2003)
How to predictively materialize SC|QPM?
How to predictively materialize SC|QPM?

Simple isotropic metal

1. \( <S> = 0 \)
2. Dynamic spin fluctuation \( <S_i S_j> \)
3. Well understood
Emergent Vector Field in Spin Ice
Quantum fluctuations in spin-ice-like Pr$_2$Zr$_2$O$_7$

K. Kimura$^1$, S. Nakatsuji$^{1,2}$, J.-J. Wen$^3$, C. Broholm$^{3,4,5}$, M.B. Stone$^5$, E. Nishibori$^6$ & H. Sawa$^6$

- Elastic neutron: pinch points (spin-ice like)
- Inelastic neutron: over 90% weight
Effective Continuum Theory

\[ H_c = \sum_{k\alpha} \left( \frac{\hbar^2 k^2}{2m} - E_F \right) \psi_\alpha^\dagger(k) \psi_\alpha(k) \]

\[ H_K(t) = J_K v_{\text{cell}} \sum_{a\alpha\beta} \int d^2r \psi_\alpha^\dagger(r) \sigma^a_{\alpha\beta} \psi_\beta(r) S_a(r_\perp = r, z = 0, t) \]
Effective Continuum Theory

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- Integrate out spins >> Effective e-e interaction
Effective Continuum Theory

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- Integrate out spins >> Effective e-e interaction

\[ H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int \text{d}^2 r \text{d}^2 r' S_{a}(r, t) (S_{a}(r, 0, t) S_{b}(r', 0, t')) s_{b}(r', t') \]
Effective Continuum Theory

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- Integrate out spins >> Effective e-e interaction

\[ H_{\text{int}}(t) = -\frac{(J_K^2 v_{\text{cell}}^2)}{2\hbar} \sum_{ab} \int dt' \int d^2r d^2r' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t') \]

\[ s_a(\mathbf{r}, t) = \sum_{\alpha\beta} \psi^\dagger_\alpha(\mathbf{r}, t) \sigma_{\alpha\beta}^a \psi_\beta(\mathbf{r}, t) \]
Hydrodynamic model for dynamic susceptibility

\[
\chi_{ab}(\mathbf{q}, \omega) = \frac{\chi^T}{1 - i\omega/\Omega} \left( \delta_{ab} - \frac{q_a q_b}{q^2} \right) + \frac{\chi^L}{1 - i\omega/\Omega + q^2\xi^2} \frac{q_a q_b}{q^2}
\]
Unusual Gauge-Matter Coupling

- Minimal Coupling
  \[
  \vec{j}(q) \cdot \vec{A}(q) = e \sum_k \vec{A}(q) \cdot \frac{k}{m} \psi_{k+\frac{3}{2},\alpha}^\dagger \psi_{k-\frac{3}{2},\alpha} 
  \]

- Repulsion against Cooper pairing
Unusual Gauge-Matter Coupling

• Minimal Coupling

\[ j(q) \cdot A(q) = e \sum_{k} A(q) \cdot \frac{k}{m} \psi_{k+\frac{3}{2}, \alpha} \psi_{k-\frac{3}{2}, \alpha} \]

• Repulsion against Cooper pairing

\[ - \sum_{p_1, p_2, q, \alpha} D(q) \frac{(p_1 \times q) \cdot (p_2 \times q)}{m^2} \psi_{p_1+q, \alpha} \psi_{p_1, \alpha} \psi_{p_2-q, \beta} \psi_{p_2, \beta} \]
Unusual Gauge-Matter Coupling

- Minimal Coupling

\[ j(q) \cdot A(q) = \sum_{k, \alpha} A(q) \cdot \frac{\psi_{k+\frac{3}{2}, \alpha} \psi_{k+\frac{3}{2}, \alpha}^\dagger}{m} \]

- Repulsion against Cooper pairing

\[ - \sum_{p_1, p_2, q} D(q) \frac{(p_1 \times \hat{q}) \cdot (p_2 \times \hat{q})}{m^2} \]

- Spin-ice/electron

\[ J_K \sum_{r, \alpha, \beta} \psi_{r, \alpha}^\dagger \sigma_{\alpha, \beta} \psi_{r, \beta} \cdot \left[ \vec{\nabla} \times \vec{A}(r) \right] \]
Unusual Gauge-Matter Coupling

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\[ J_K \sum_{r, \alpha, \beta} \psi_{r, \alpha} \bar{\sigma}_{\alpha \beta} \psi_{r, \beta} \cdot \left[ \nabla \times A(r) \right] \]

Electrons are not magnetic monopoles

- Attractive equal-spin interaction!

\[ -J_K^2 \frac{D(q) \cdot (\bar{\sigma}_{\alpha \beta} \times \hat{q}) \cdot (\bar{\sigma}_{\alpha' \beta'} \times \hat{q})}{m^2} \]
Selection Rule Dictated Odd-Parity

- Pair binding problem with dipole-dipole interaction

\[ V_{dd} = \frac{1}{r^3} [\vec{S}_1 \cdot \vec{S}_2 - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})] \propto R^{(2)}(r_1, r_2) \cdot S^{(2)}(s_1, s_2) \]
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- Wigner-Eckart thm: \[ \langle l'|T^{(r)}|l \rangle = 0 \text{ unless } |r - l| \leq l' \leq (r + l) \]
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- Wigner-Eckart thm: \( \langle l' | \mathcal{T}^{(r)} | l \rangle = 0 \) unless \( |r - l| \leq l' \leq (r + l) \)

C: Cooper pair |\uparrow\downarrow> \rightarrow |\uparrow\downarrow> = |S=0>

D: Cooper pair |\uparrow\downarrow> = |S=1>

quadrupolar mode (s=2)

\[ |2-1| \leq 1 \leq 2+1 \]
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Dealing with **interacting** electrons?

\[
H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2h) \sum_{ab} \int dt' \int d^2r d^2r' \langle S_a(r, 0, t) S_b(r', 0, t') \rangle s_b(r', t')
\]

- Separation of scale: \( \omega_s / E_F << 1 \)
  
  ➞ “Migdal theorem”

- Dimensionless ratio: \( \lambda \sim N(0)V \sim J_K^2 N(0) / J_{\text{ex}} < 1 \)

- Full problem \( \approx \)
  
  solving the **BCS mean-field theory**
Dealing with interacting electrons?

\[ H_{\text{int}}(t) = -\left( J_K^2 v_{\text{cell}}^2 / 2\hbar \right) \sum_{ab} \int dt' \int d^2r d^2r' \langle s_a(r, t) | s_b(r', 0, t') \rangle s_b(r', t') \]

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- Full problem \( \approx \) solving the BCS mean-field theory

  \[ T_C \sim \omega_s e^{-1/\lambda} \]
Leading channels

\[ \begin{align*}
J_z = 0 & : (k_x + i k_y) |\downarrow\downarrow \rangle + (k_x - i k_y) |\uparrow\uparrow \rangle \\
J_z = 1 & : \frac{(k_x \pm i k_y) |\uparrow\downarrow \rangle + |\downarrow\uparrow \rangle}{\sqrt{2}}
\end{align*} \]
Criteria for Metal
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• Structural
  ‣ Lattice match
    ➞ $A_2B_2O_7$
  ‣ No orphan bonds
Criteria for Metal

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  ‣ Lattice match
     ➔ $A_2B_2O_7$
  ‣ No orphan bonds
Wave function penetration

![Wave function penetration diagram](image-url)
Band structure for the Proposal

$\text{Pr}_2\text{Zr}_2\text{O}_7/\text{Y}_2\text{Sn}_{2-x}\text{Sb}_x\text{O}_7 \ (111)$

$x=0.2$
Band structure for the Proposal

$Pr_2Zr_2O_7/Y_2Sn_{2-x}Sb_xO_7$ (111)

$x=0.2$
TSC in Metal/Quantum-Spin-Ice Heterostructures

- Topological superconductor riding on QSL
TSC in Metal/Quantum-Spin-Ice Heterostructures

- Topological superconductor riding on QSL
- Selection Rule Dictated Intrinsic Topo SC.
Acknowledgements

Jian-huang She  Choonghyun Kim  Criag Fennie  Michael Lawler