Detection Prospects of Local Super-Massive Black Holes Based on the Sloan-Digital Sky Survey

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ABSTRACT

We use the Sloan-Digital Sky Survey quasar catalog to statistically infer the local abundance of black holes heavier than $10^8 \, M_{\odot}$, which allows us to estimate the detection prospect of super-massive black holes by future observational campaigns. We find that the upcoming James Webb Space Telescope (JWST) and the Extremely Large Telescope (ELT) should be able to resolve, with integral field spectroscopy techniques, the gravitational influence of $\sim 10^3$ black holes within a sphere of $\sim 50$ Mpc. A Very-Long Baseline (VLB) observatory with one receiver placed in a geostationary orbit, is predicted to capture $\sim 10$ images of the silhouette of a black hole, similar to the image of M87* recently performed by the Event Horizon Telescope.

Keywords: Astrophysical black holes (98), Supermassive black holes (1663), Quasars (1319), Very long baseline interferometers (1768), Space telescopes (1547)

1. INTRODUCTION

Super-massive black holes (BHs) are at the top of a broad BH mass distribution, with BHs mass ranging across roughly ten orders of magnitude. In order to reach such an extreme variety in mass, BHs need to grow from their original seeds, formed at $z \sim 20 - 30$ (Loeb & Furlanetto 2013), via gas accretion and mergers (Pacucci & Loeb 2020).

Super-massive BHs are very rare, those with mass $\sim 10^{10} \, M_{\odot}$ are expected to be $\sim 1000$ times more scarce than those with mass $\sim 10^6 \, M_{\odot}$ (Habouzit et al. 2020). Their abundance also decreases significantly with redshift and supermassive BHs at $z \sim 0$ are roughly a 100 times more prevalent than at $z \sim 4$ (Cao 2010; Ueda et al. 2014; Habouzit et al. 2020).

Currently the most massive BH observed, labeled TON 618, has a mass $\sim 6.6 \times 10^{10} \, M_{\odot}$ estimated from the H$\beta$ emission line (Shemmer et al. 2004). So far, only about 30 BH at the $\sim 10^{10} \, M_{\odot}$ scale were observed. The mass of those BHs is surprisingly close to the theoretical maximal mass predicted for a BH growing by luminous accretion of matter (King 2016; Inayoshi & Haiman 2016). It is important to note that this mass limit, around $5 \times 10^{10} \, M_{\odot}$ with some variations depending on the accretion physics, is valid only for BHs that grow predominantly by active accretion of gas from an accretion disk. There is no theoretical upper bound on the mass of a BH that grows by mergers (e.g., King 2016, see also Pacucci et al. 2017; Woods et al. 2019 for maximum mass considerations in the high-$z$ Universe).

The presence of a BH in a given volume of space can be inferred using several methods. Gas accretion from a stable accretion disk (and corona) emits copious amounts of radiation, which can then be observed in a very broad range of frequencies in the electromagnetic spectrum, typically from radio frequencies all the way to hard X-rays. Local supermassive BHs are, however, challenging to detect via their emitted radiation, as most of them are no longer in an active accretion phase, due to a significant decrease in available cold gas at low redshift (Power et al. 2010). In fact, the X-ray luminosity emitted by ultra-massive BHs was shown to decrease significantly in X-ray surveys at $z < 1$ compared to those at higher redshift (Ueda et al. 2014).

Gravitational waves, with the Laser Interferometer Space Antenna (LISA) to play a fundamental role in the next decade, can also shed light on the presence of BHs. The mergers of BHs can emit a large amount of energy in the form of gravitational waves. Remarkably, heavy super-massive BHs mergers will be challenging to detect even with LISA, due to a decreased sensitivity at masses larger than $\sim 10^9 \, M_{\odot}$ (Amaro-Seoane et al. 2017).
A BH can also have significant gravitational influence on nearby objects. In fact, the effect of the gravitational field of a BH on nearby stars has been spectacularly used to detect and accurately determine the mass of a super-massive BH at the center of our own galaxy (Genzel et al. 2003; Ghez et al. 2008). Furthermore, in a pioneering work, Kormendy & Richstone (1995) applied for the first time dynamical search techniques based on the effects of BH on nearby gas and stars, to central BHs in 7 nearby galaxies, including M31, M32 and M87. Since then, the mass of many super-massive BHs have been measured dynamically using integral-field spectroscopy (IFS) techniques (see, e.g., McConnell et al. 2012 and the more recent Liepold et al. 2020). Additionally, the gravitational field of a BH can produce gravitational lensing on the light of background sources (Paczynski 1996).

Very recently, the observational techniques at our disposal to study BHs have been extended by the Event Horizon Telescope (EHT) collaboration, which used Very-Long Baseline (VLB) interferometry to study the shadow of a BH on scales comparable to its Schwarzschild radius (Event Horizon Telescope Collaboration et al. 2019, hereafter EHT19).

In this study we focus on super-massive BHs heavier than $10^8 \, M_\odot$. We infer their local abundance from the statistics of quasars derived from the SDSS DR7 (Schneider et al. 2010; Shen et al. 2011), together with the work Shankar et al. (2010), which calculated the fraction of black holes that shine as optically luminous quasars at a given time, denoted by $f_{\text{opt}}$. This allows us to investigate the local abundance of super-massive BHs as a function of the angular size of their shadows (Sygge 1966; Luminet 1979), or of their gravitational influence radii (Peebles 1972). This information enables us to predict how likely will it be for current and future VLB observatories to resolve a BH shadow image similar to that of M87+ (EHT19). It also allows us to predict the number of BHs that could indirectly be observed via their gravitational effect on nearby stars and gas, via IFS with the James Webb Space Telescope (JWST) and, in the more distant future, with the Extremely Large Telescope (ELT).

2. DATA

We restrict our attention to SDSS quasars with virial mass $M_\star$ estimated as $M_\star > 10^8 \, M_\odot$, following Shen et al. (2011). Note that virial masses in this database may be over-estimated by a factor $\sim 3$ towards the high-end of the mass distribution (Shen et al. 2008, see also Appendix A for the effect on our results). We will make use of Shankar et al.’s (2010) measurement of $f_{\text{opt}}$, which is a redshift dependent quantity, for consistency with their analysis we restrict our analysis to the same redshift interval as they did, $0.4 \leq z \leq 2.5$. To simplify our analysis (see Sec. 3 for further details) we seek the largest (co-moving) sphere that can reside entirely inside this redshift interval, within which the number density SDSS quasars is distributed uniformly. We find this sphere to have a co-moving radius of 2300 Mpc and to be centred at $z = 1.3$, RA = 180 degrees and Dec = 32.4 degrees. These cuts on the data leaves us with a total of 50478 quasars and an observed co-moving volume of $\sim 5.1 \times 10^{40}$ Mpc$^3$, resulting in a quasar-quasar co-moving mean separation of $\sim 62$ Mpc. This mean co-moving distance between quasars is consistent with results from previous high-$z$ quasar surveys (see, e.g., Fan et al. 2001 and Pacucci & Loeb 2019 about how this distance could be over-estimated).

3. METHOD

We assume the location of Earth in the universe is arbitrary. As such, the distribution of BHs seen by an observer located on Earth should be equivalent to that seen from any random point within the SDSS observed volume. Our methodology therefore proceeds as follows: We calculate the quasar mass function (QMF) seen from a random point in the SDSS observed volume and interpret it as an estimate of the QMF at $z \sim 1.4$. Then, correcting for the fraction of halos that host luminous quasars, $f_{\text{opt}}$, we obtain an estimate for the BH mass function (BHMF) at the same redshift. Bearing in mind that the activity of AGNs drops rapidly after $z \lesssim 1$ (Ueda et al. 2014), we treat the this BHMF a faithful estimate of the local ($z \sim 0$) BHMF. Integrating the resulting BHMF over a certain range of BH mass and/or distance, we then predict the total number of super-massive BHs with an angular size larger then a certain value. Using those results we predict the number of BHs a current or future observational campaign may find, per mass interval, distance interval and as a function of resolution. We note here that the contribution of BHs with a mass lower than roughly $10^8 \, M_\odot$ to our analysis is under-estimated. This is due to an incomplete selection of BHs resulting from flux limitations (see. Richards et al. 2006; Vestergaard & Osmer 2009; Kelly et al. 2010.) Those BHS however do not drastically effect our analysis since their angular size is typically too small to be observed in the techniques we discuss.

3.1. Estimating the Local Black Hole Mass Function

The QMF, $\Phi_Q$, estimated directly from the SDSS data, is defined as the number of quasars (active and luminous galactic nuclei) per co-moving volume per log BH mass interval $^1$

\[
\Phi_Q(M_\star, z) = \frac{d^2 N_Q}{dV_C(z) \, d \log M_\star},
\]

where $V_C(z)$ is the co-moving volume of a sphere of co-moving radius $D_C(z)$. As mentioned in Sec. 2, we are dealing with a spherical region within which we find a constant

$^1$ Throughout this Letter we use log base 10.
co-moving quasars number density. Estimating the QMF is therefore achieved by calculating the mean number of quasars seen by a random observer in a given mass interval and dividing it by the observed volume. Our selection of random observers covers a spherical region of radius 1000 Mpc, each observes a maximal distance of 1300 Mpc (corresponding to a redshift $z \approx 0.32$). See the Appendix for additional details on the method used for this calculation.

The local BHMF, $\Phi$, is defined as the number of BHs, active or dormant, per co-moving volume per log BH mass interval. The BHMF is related to the QMF via

$$\Phi_Q = f_{\text{opt}} \Phi,$$

where $f_{\text{opt}}$ is the fraction of black holes that shine as optically luminous quasars at a given time. We adopt $f_{\text{opt}}$ as calculated in Shankar et al. (2010). There, based on Shen et al.’s (2009) measurements of the luminosity-dependent clustering of quasars in SDSS DR5 (York et al. 2000; Adelman-Mccarthy et al. 2007; Schneider et al. 2007), $f_{\text{opt}}$ is derived by assuming a mean relation between quasar luminosity and its host halo mass, and matching the co-moving number density of quasars to that of halos, corrected for the duty cycle. For our interest we shall use a fiducial, somewhat conservative, value of $f_{\text{opt}} \approx 2 \times 10^{-3}$, which Shankar et al. (2010) find to be consistent with both their analysis and continuity equation modelling of the BH population (Cavaliere et al. 1971; Small & Blandford 1992; Shankar et al. 2009). The resulting local BHMF is shown in Fig. 1. The drop of the mass function at masses above $M_* \sim 10^9 M_\odot$ is physical (Kelly & Merloni 2012), the suppression below $M_* \sim 10^7 M_\odot$ is attributed to the flux limitation selection explained earlier in this section.

3.2. The Distribution of Black Holes of a Given Angular Size

We define the angular size of a BH shadow, $\theta_{\text{shad}}$ (Johannsen & Psaltis 2010; Psaltis 2019), and gravitational influence angular size, $\theta_{\text{grav}}$ (Peebles 1972) as

$$\theta_{\text{shad}} = \frac{5G M_*}{c^2 D_A(z)} \quad \text{and} \quad \theta_{\text{grav}} = \frac{G M_*}{\sigma^2 D_A(z)},$$

respectively, where $D_A(z)$ is the angular diameter distance to a BH at redshift $z$ and $\sigma$ is the velocity dispersion around the BH, for which we phenomenologically assume the $M_* - \sigma$ relation from Kormendy & Ho (2013). Whenever possible we will address the two angles collectively as $\theta_*$. Starting from a BHMF, the differential number of BH seen with angular size $\theta_* > \theta$ is given by

$$dN_* (\theta) = \Theta (\theta_* - \theta) \Phi (M_*) dV_C(z) d \log M_*,$$

where $\Theta$ is the Heaviside function. Since each random observer in our analysis observes a small redshift interval, we have explicitly dropped the BHMF redshift dependence in the equation above. Integrating this differential over $V_C$, $M_*$ or both, results in the number of BHs with angular size larger than $\theta$, per log mass, per co-moving volume, or in total, respectively. Since the luminosity of distant objects drops as $(1 + z)^{-4}$, we will integrate the mass function only up to $z_{\text{max}}$ so that we do not count sources that are too faint. As mentioned earlier, in our analysis $z_{\text{max}} \approx 0.32$, within this redshift range $D_A(z)$ is a monotonically increasing function of $z$, therefore $\theta$ is determined uniquely for a given $D_A$. Under these restrictions we find

$$\frac{dN_* (\theta_* > \theta)}{d \log M_*} = V_C (\bar{z}) \Phi (M_*)$$

$$\frac{dN_* (\theta_* > \theta)}{dV_C} = \int_{M_{\text{min}}}^{\infty} \Phi (M_*) d \log M_*$$

where $\bar{z} = \min [z_{\text{max}}, z_{\text{th}}]$ such that $D_A(z, \theta)$ satisfies $\theta_* = \theta$ for a fixed $M_*$, while $M_{\text{min}}$ satisfies $\theta_* = \theta$ for a fixed $D_A$ (see. Eq. 3 and bear in mind that $\sigma^2$ is a function of the BH mass.) The total number of BHs can be obtained by integrating Eq. (5) over $M_*$ or Eq. (6) over $V_C$.

4. RESULTS

In Fig. 2 we show the main result of this study: the expected number of BHs that can be resolved by a telescope with a given angular resolution. In both panels we demonstrate the dependence of our result on the maximal accessible distance, which is limited by the telescope sensitivity. For

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Footnote:

3 To be precise, the angular size of the BH shadow is given by $AG M_*/c^2 D_A$, where $A$ is a BH spin dependent number, consistent with 5 to within 4% (Johannsen & Psaltis 2010; Psaltis 2019).
The cumulative number of BHs a telescope with angular resolution $\theta_{\text{min}}$ is predicted to to resolve, for different maximal accessible distance ranging in $10 - 1000$ Mpc. The shaded blue/purple bands represent the $1\sigma$ confidence interval. Left: Prediction for BH shadow searches. The green vertical bands indicate, from right to left, the angular resolution of a hypothetical future VLB observation (at frequency of 200 GHz) with one Earth-based receiver and the other one in geostationary orbit, on the Moon and at the Sun-Earth Lagrange point L2, respectively. Right: Prediction for gravitational influence searches. Here the green vertical bands indicate, from right to left, the angular resolution of the JWST and the future ELT (Gardner et al. 2020; Weinberg et al. 2005). On both panels grey bands shows for reference the angular size of M87*, using its shadow size as reported in EHT19 and the velocity dispersion of M87 as reported in Murphy et al. (2014).

To appreciate the number oh BHs next-generation missions could observe, we show on the left panel of Fig. 2 the angular resolution of a VLB observatory at frequency of 200 GHz, with one receiver located on earth and the other in a geostationary orbit, on the Moon or in the Sun-Earth Lagrange point L2. Such VLB observatories are further motivated as they offer novel and exciting tests of general relativity (Johnson et al. 2020; Gralla et al. 2020). On the right panel we show the resolution of the upcoming JWST and the ELT (Gardner et al. 2006; Weinberg et al. 2005).

To gain some qualitative intuition we analyze here the small and large $\theta_{\text{min}}$ behaviour of Fig. 2. The plateau at small resolution is the result of the finite number of BHs contained within a sphere of radius $D_{C,\text{max}}$, we find it to be

$$N_\bullet(\theta_\bullet > 0) \approx 10^3 \left( \frac{2 \times 10^{-3}}{f_{\text{opt}}} \right) \left( \frac{D_{C}}{50 \text{ Mpc}} \right)^3,$$

where we have re-introduced the dependence on $f_{\text{opt}}$ for completeness. This number would in practice be larger when the selection of BHs lighter than $10^9 M_\odot$ is corrected for.

The tail at large resolution on the other hand demonstrates the scaling $N_\bullet \sim \theta_{\text{min}}^{-3}$. There, as a result of the fact that at large resolution we observe only nearby BHs, $D_C \approx D_A$ and one has

$$N_\bullet(\theta_\bullet > \theta_{\text{min}}) \approx \frac{4 \pi G^3}{3 \theta_{\text{min}}^3} \int_{M_{\text{min}}}^{\infty} S^3 (M_\bullet) M_\bullet^2 \Phi(M_\bullet) d \log M_\bullet \approx \left( \frac{27 \mu\text{arcsec}}{\theta_{\text{min}}} \right)^3 \frac{\text{shad}}{\text{grav}},$$

where $S = 5c^{-2}$ for the BH shadow and $S = \sigma v^{-2}$ for the gravitational influence.

In Fig. 3 we show the total number of BH expected to be seen with a telescope of given resolution, per unit log mass and as a function of the BH mass. To demonstrate the effect of the telescope sensitivity we show (in dotted and dashed lines) how the curve changes when the maximal accessible distance $D_{C,\text{max}}$ is changed. At the high-mass-end of the curve the dependence is simply $N_\bullet \sim D_{C,\text{max}}^3$. The sharp features in the curves are a result of the definition of $\xi = \min [\xi_{\text{max}}, \xi_\circ]$ (see Eq. 5); physically, to the left of the kink the relevant volume is limited by resolution $\theta_{\text{min}}$, and to the right by the maximal distance $D_{C,\text{max}}$. We stress again that for $M_\bullet \lesssim 10^9 M_\odot$ the reported results underestimate the actual density of BHs due to the SDSS flux limitation.

Last, in Fig. 4 we show the total number of BH expected to be detectable by a telescope of a given resolution, per unit log distance and as a function of co-moving distance. The small distance behaviour of those plots is simply given by the
In addition, IFS observations with the upcoming JWST and ELT will be able to dynamically measure the mass locked into BHs within many local galactic nuclei (Gardner et al. 2006).

We conclude by stressing some of the underlying assumptions and caveats of our analysis. First, a word of caution regarding BH mass estimates is warranted. BH mass estimates in the SDSS DR7 are performed via virial mass estimators, based on the emission lines Hβ, MgII, and CIV. These mass estimates are affected by uncertainties, which are possibly large, especially at the high-luminosity end. As shown by Shen et al. (2008), the observed virial BH mass distribution for the SDSS sample used in this study is biased by a factor of ~3, towards the high end of the BH mass distribution.

5. DISCUSSION AND CONCLUSION

Using a statistical approach based on data from the SDSS, we have inferred the local abundance of BHs heavier than $10^9 \, M_\odot$. We have shown that future VLB interferometry observations with longer arm lengths will be able to image scales comparable to the event horizon of many more BHs. In addition, IFS observations with the upcoming JWST and

**Figure 3.** Predicted number of BHs of angular dimension larger than a given $\theta_{\text{min}}$, per log mass interval. **Solid lines** shows the result assuming a maximal accessible co-moving distance of 700 Mpc. The **shaded blue/purple bands** represent the 1σ confidence interval around the solid lines. **Dashed and Dotted lines** demonstrate the sensitivity of our result to the maximal distance that a given telescope can observe. We omit the confidence interval from the dashed and dotted lines for clarity. The sharp feature appears because of two different causes that limit the amount of observables BHs; To the left of the sharp feature the number of BHs is limited by resolution, while to the right it is limited by the maximal observable distance. See also main text.

**Figure 4.** Predicted number of BHs of angular dimension larger than a given $\theta_{\text{min}}$, per log comoving distance interval. The **shaded blue/purple bands** represent the 1σ confidence interval around the solid lines.

dervative of Eq. (7) with respect to $D_C$. The drop at large distances is due to the fact that the further away you observe the more massive the BH needs be in order to be observable for a given resolution, together with the drop of the BHMF at mass heavier than $10^9 \, M_\odot$. 

...
A factor $\sim 2 - 4$ uncertainty is typical for mass estimates of high-$z$ quasars (see, e.g., Unal et al. 2020). While this bias does not significantly affect the results in the present study, the reader should be cautioned that the very high-mass BHs reported in this statistical study could possibly be spurious. For completeness, in Appendix A we study more quantitatively the effect of this bias on the robustness of our results. Fig. 5 in that Appendix shows a “worst casescenario” version of Fig. 2, where all BH mass in the data used are scaled-down by a factor of 3.

Our analysis is based on the duty cycle as estimated in Shankar et al. (2010). A more dedicated study of the duty cycle and its dependence on the BH mass (see, e.g., DeGraf & Sijacki 2017) can improve the accuracy of our analysis. As we adopted a single value for $f_{\text{opt}}$, a strong mass dependence could significantly affect our results.

Another assumption we have taken was that the BHMF calculated at $z \sim 1.4$ is a good estimate of the BHMF at $z \sim 0$. Accounting for the evolution of the BHMF from $z = 1.4$ to $z = 0$ should only increase the number of observed BHs, since the BHs could only grow (in mass and therefore in size) during this time. However, due to the activity drop at $z \lesssim 1$ we expect this effect to be rather small.

As a final note, we wish to emphasize again that due to SDSS flux limitation, our analysis under-predicts the number of BHs lighter than $\sim 10^9 \, M_\odot$. By means of complementing data from the SDSS with other surveys, the resulting number of BHs that are expect to be observed in the local Universe could be higher.

In spite of the fact that a large fraction of central BHs in the local Universe may be dormant and dark, the future is bright. In this study we showed that forthcoming observations with VLB techniques, as well as IFS analysis with the JWST and the ELT will likely greatly expand our census of the local population of super-massive BHs.

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The number of BHs a telescope with angular resolution $\theta_{\text{min}}$ is predicted to to resolve, for different maximal accessible distance ranging in 10 - 1000 Mpc. Solid lines and 1σ bands were calculated assuming a constant scaling by a factor 3 of all BH mass used in the analysis. Dashed lines shows the unscaled results. See the caption of Fig. 2 for further details.

APPENDIX

A. THE EFFECT OF THE SDSS HIGH LUMINOSITY BIAS

In this short appendix we wish the estimate the sensitivity of our results to the BH mass bias as studied in Shen et al. (2008), where a maximal bias of 3 reported. This bias is not easily corrected for, however, to estimate the sensitivity of our results to that bias, here we shall treat it as universal bias and scale-down all the BH masses used in our analysis by a factor of 3. The effect on the reported BHMF is a trivial shift of the BHMF by a factor of 3 to the left (see Fig. 1). The effect on the total number of BH seen with angular size larger than $\theta_{\text{min}}$ is slightly more involved, the results are shown in Fig. 5, which should be compared with the un-scaled version shown in Fig. 2. The dashed lines in fig. 5 shows for clarity the un-scaled results. Qualitatively, the mass scaling results in a scaling of the $\theta_{\text{min}}$ axis by a factor of 3 in the BH shadow calculation, and by a factor of $\sim 1.82$ in the gravitational influence calculation. The different scaling is a result of the additional BH mass dependence arising from the $M_* - \sigma$ relation in the gravitational influence calculation (Shankar et al. 2019). We therefore conclude that at a fix $\theta_{\text{min}}$ the total number of BHs that can be resolved is diminished at most by a factor of 29 for BH shadow searches, and at most by a factor of $\sim 6$ for the gravitational influence searches.

B. GEOMETRICAL DETERMINATION OF THE OBSERVED NUMBER OF QUASARS

Usually a Monte Carlo (MC) process is sufficient to estimate the distribution of objects in 3D space. One chooses a large number of randomly positioned observers and calculate the distribution of those objects as seen by each observer. The "distribution of distributions" is therefore sampled by the randomly located observers. Here we aim to study the distribution of quasars (and infer that of BHs) with a sufficiently large angular size. To achieve this, we would have to sample our observed volume to a very fine resolution, making the MC process very computationally expensive. To exemplify, in order to probe quasars with of mass $\sim 10^9 M_\odot$ and shadow with angular size $\sim 5 \mu\text{arcsec}$, we would have to use more than $10^7$ observers.

To bypass this difficulty we devise a geometric method for calculating the mean and standard deviation of the number of quasars seen by a random observer with an angular size larger than $\theta$. Each quasar in the SDSS catalogue is assigned with a co-moving position vector $\mathbf{r}_i$ and a mass $M_i$. The number of quasars seen by an observer located at $\mathbf{r}$, with distance smaller than $r_{\text{max}}$ is given by

$$N(\mathbf{r}) = \sum_i \int_{r' < D_{C,\text{max}}} d^3 r' (r', \mathbf{r} - \mathbf{r}_i).$$  

(B1)

The mean number of quasars is then obtained by averaging the above expression over the observed volume $V_O$

$$\bar{N} = \frac{1}{V_O} \int_{V_O} d^3 r N(\mathbf{r}).$$  

(B2)
Since all points closer than $D_{C,max}$ to $r_i$ form a ball of radius $D_{C,max}$ centred at $r_i$, which we denote $B_i$, the above integral is simply given by

$$\bar{N} = \sum_i \frac{V_{O \cap B_i}}{V_O},$$

i.e., the sum of the volume of the intersection of a ball $B_i$ and the entire observed volume. With similar steps, we find that the variance is given by

$$\text{Var}[N] = 2 \sum_{j>i} \frac{V_{O \cap B_i \cap B_j}}{V_O} + \bar{N}_\theta - \bar{N}_\theta^2,$$

where now the intersection is of two balls and the observed region. Higher cumulants can be calculated analogously and require the calculation of volume of intersection of a larger number of balls. We have checked this calculation against MC when the MC converges and the two methods agree.

As described in Sec. 2, we restrict our analysis to spherical observed volumes. One reason for this choice is that the volume of intersection of two and three spheres has been studied in the literature and is given in closed form, allowing for an efficient calculation (Gibson & Scheraga 1987; Chkhartishvili 2001; Petitjean 2013). To avoid biases at the edges of our observed volume, we shall always use observed sphere of radius $R_O$ satisfying $R_O + r_{\text{max}} \leq 2300$ Mpc, such that all quasars influencing our observed region is within our data set.