Classifying and probing flavor transition mechanisms of astrophysical high energy neutrinos

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Abstract. The future neutrino telescopes are expected to identify the flavors of astrophysical neutrinos and therefore determine the flavor ratio. The flavor ratio of astrophysical neutrinos observed at the Earth depends on both the initial flavor ratio at the source and flavor transitions taking place during the propagations of these neutrinos. We propose a model-independent way to parameterize the above flavor transitions, including standard oscillations and beyond. A systematical way is also described to probe those mechanisms taking advantage of \( R \equiv \phi_\mu / (\phi_e + \phi_\tau) \) and \( S \equiv \phi_e / \phi_\tau \), the observables in neutrino telescope measurements.

1. Introduction

The recent development of neutrino telescopes [1, 2, 3, 4, 5] has inspired numerous efforts of studying flavor mixing mechanism of neutrinos during their propagations with astrophysical neutrinos as the beam source (for references, see [6]). Some flavor transition models predict rather different neutrino flavor ratios at the Earth compared to those predicted by the standard neutrino oscillations [7]. In this article, we propose a scheme to parameterize flavor transition mechanisms of astrophysical neutrinos propagating from the source to the Earth. As will be shown later, such a parametrization is very convenient for classifying flavor transition models which can be tested by future neutrino telescopes.

To study flavor transition mechanisms, we describe the neutrino flavor composition at the source by a normalized flavor ratio \( \Phi_0 = (\phi_{0,e}, \phi_{0,\mu}, \phi_{0,\tau})^T \) satisfying the condition

\[
\phi_{0,e} + \phi_{0,\mu} + \phi_{0,\tau} = 1,
\phi_{0,\alpha} \geq 0, \text{ for } \alpha = e, \mu, \tau,
\]

(1)

where neutrinos and antineutrinos are treated identical and summed over. The same convention will be used for the flavor composition at the Earth. The flavor transition occurring between the source and the Earth is represented schematically by the transition matrix \( P_{\alpha\beta} \) such that

\[
\Phi = P \Phi_0,
\]

(2)

where \( \Phi = (\phi_e, \phi_\mu, \phi_\tau)^T \) is the flux of neutrinos reaching the Earth and our convention implies \( P_{\alpha\beta} \equiv P(\nu_\beta \rightarrow \nu_\alpha) \).

¹ It is more convenient to treat \( \Phi_0 \) as a column vector.
For astrophysical neutrinos traversing a vast distance $L$, relative phases between flavors are washed out since $\delta m^2 \gg L/E$. Therefore, the transition matrices are given by

$$P_{\alpha\beta}^{\text{osc}} = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2,$$

(3)

for neutrino oscillation, and

$$P_{\alpha\beta} = \sum_{f, \text{ stable}} (|U_{\beta f}|^2 + \sum_{i, \text{ unstable}} |U_{\beta i}|^2 Br_{i\to f}) |U_{\alpha f}|^2,$$

(4)

for the case of neutrino decay scenarios, with $|U_{\alpha i}|$ the element of the mass-flavor mixing matrix.

### 2. Classifying flavor transition mechanisms

The fact that $P^{\text{osc}}$ is well approximated by the tri-bimaximal matrix leads to a convenient parametrization of $\Phi_0$ by [6]

$$\Phi_0 = \frac{1}{3} V_1 + aV_2 + bV_3,$$

(5)

where $V_1 = (1, 1, 1)^T$, $V_2 = (0, -1, 1)^T$ and $V_3 = (2, -1, -1)^T$. The ranges for $a$ and $b$ are $-1/3 + b \leq a \leq 1/3 - b$ and $-1/6 \leq b \leq 1/3$. The expression in Eq. (5) implicitly assumes that an astrophysical source only produces ordinary neutrinos. The vector $V_1$ gives the normalization of the neutrino flux since the sum of the elements in $V_1/3$ is equal to unity. The vector $bV_3$ determines the electron neutrino fraction $\phi_{0,e}$ or equivalently the sum of muon and tau neutrino fractions $\phi_{0,\mu} + \phi_{0,\tau}$ according to the value of $b$. However this vector preserves the difference $\phi_{0,\mu} - \phi_{0,\tau}$. Finally the vector $aV_2$ determines $\phi_{0,\mu} - \phi_{0,\tau}$ while preserves $\phi_{0,\mu} + \phi_{0,\tau}$ or equivalently $\phi_{0,e}$. The neutrino flavor ratio at the Earth is accordingly written as

$$\Phi = (AQA^{-1}) \Phi_0 = \kappa V_1 + \rho V_2 + \lambda V_3.$$

(6)

We can, therefore, define a new representation of the flavor transition matrix $Q = A^{-1}PA$ with

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix},$$

(7)

the transformation matrix. In this representation, it is easy to show that

$$\kappa = \frac{1}{3} Q_{11} + aQ_{12} + bQ_{13},$$

$$\rho = \frac{1}{3} Q_{21} + aQ_{22} + bQ_{23},$$

$$\lambda = \frac{1}{3} Q_{31} + aQ_{32} + bQ_{33},$$

(8)

Compared to $P$, the matrix $Q$ is very convenient for classifying flavor transition models. First of all, flux-conserving models must give $\kappa = 1$, irrespective of the initial flavor composition. This implies $(Q_{11}, Q_{12}, Q_{13}) = (1, 0, 0)$. Second, for those models which do not seriously break the $\nu_{\mu} - \nu_{\tau}$ symmetry, one can show that $(Q_{21}, Q_{22}, Q_{23}) \simeq (0, 0, 0)$ and $(Q_{12}, Q_{22}, Q_{32}) \simeq (0, 0, 0)$. Hence, under these two assumptions, one can simply use the values for $Q_{31}$ and $Q_{33}$ to classify flavor transition models.
Considering the standard three-flavor neutrino oscillations, we obtain, in the limit $\sin^2 \theta_{13} = 0$ and $\sin^2 \theta_{23} = 0.5$, $Q^{\text{osc}}_0 = A^{-1} P_0^{\text{osc}} A$ with $[6]$

\[
Q^{\text{osc}}_0 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & (4 - 3\omega)/4
\end{pmatrix}, \tag{9}
\]

in the limit $\sin^2 \theta_{13} = 0$ and $\sin^2 \theta_{23} = 0.5$, with $\omega = \sin^2 2\theta_{12}$. For a general flavor transition matrix $P$, it remains useful to write $Q = A^{-1} P A$ with the same matrix $A$, despite $Q$ is no longer diagonal. In this case $P^{\text{osc}} = P_0^{\text{osc}} + P_1^{\text{osc}} + \cdots$ where $P_1^{\text{osc}}$ is the leading order correction in powers of $\cos 2\theta_{23}$ and $\sin \theta_{13}$. We have

\[
P_1^{\text{osc}} = \begin{pmatrix}
0 & \epsilon & -\epsilon \\
\epsilon & -\epsilon & 0 \\
-\epsilon & 0 & \epsilon
\end{pmatrix}, \tag{10}
\]

where $\epsilon = \omega \cos 2\theta_{23}/4 + \sqrt{\omega(1 - \omega)} \sin \theta_{13} \cos \delta/2$ with $\delta$ the $CP$ phase. Taking into account the correction term $P_1^{\text{osc}}$, we obtain $Q^{\text{osc}} = Q_0^{\text{osc}} + Q_1^{\text{osc}}$ where

\[
Q_1^{\text{osc}} = A^{-1} P_1^{\text{osc}} A = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & -3\epsilon \\
0 & -\epsilon & 0
\end{pmatrix}. \tag{11}
\]

We next consider the neutrino decay models in which only the heaviest neutrino decays. To simplify our discussions, we set $\sin^2 \theta_{12} = 1/3$ (i.e., $\omega = \sin^2 2\theta_{12} = 8/9$) while allowing $\theta_{23}$ and $\theta_{13}$ to deviate from $\pi/4$ and 0 respectively. For the normal hierarchy, we have $Q^{\text{dec}} = Q^{\text{dec}}_0 + Q^{\text{dec}}_1$ where $Q^{\text{dec}}_0$ is the leading term obtained in the limit $\sin^2 \theta_{23} = 0.5$ and $\sin \theta_{13} = 0$ while $Q^{\text{dec}}_1$ is the first order correction to the above limit. We find

\[
Q^{\text{dec}}_0 = \frac{1}{6} \begin{pmatrix} 4 + 2(r + s) & 0 & 2 - 2(r + s) \\ 0 & 0 & 0 \\ 1 + r & 0 & 1 - r \end{pmatrix}, \tag{12}
\]

and

\[
Q^{\text{dec}}_1 = \begin{pmatrix}
\frac{1}{12} (Q^{\text{dec}}_1)_{12} & 0 \\
0 & (Q^{\text{dec}}_1)_{23} \\
\frac{1}{32} (Q^{\text{dec}}_1)_{32} & 0
\end{pmatrix}, \tag{13}
\]

where $r$ and $s$ are the branching ratios for decay modes $\nu_3 \to \nu_2$ and $\nu_3 \to \nu_1$, respectively.

For the inverted hierarchy, we denote $r$ and $s$ as branching ratios for the decay modes $\nu_2 \to \nu_1$ and $\nu_2 \to \nu_3$, respectively. We obtain $Q^{\text{dec}} = Q^{\text{dec}}_0 + Q^{\text{dec}}_1$ with

\[
Q^{\text{dec}}_0 = \frac{1}{6} \begin{pmatrix} 4 + 2(r + s) & 0 & 0 \\ 0 & 0 & 0 \\ r - s & 0 & 2 \end{pmatrix}, \tag{14}
\]

and

\[
Q^{\text{dec}}_1 = \begin{pmatrix}
\frac{1}{12} (Q^{\text{dec}}_1)_{12} & 0 \\
0 & (Q^{\text{dec}}_1)_{23} \\
\frac{1}{32} (Q^{\text{dec}}_1)_{32} & 0
\end{pmatrix}. \tag{15}
\]

The nonzero elements of $Q^{\text{dec}}_1$ and $Q^{\text{dec}}_1$ are given in Table I in [8].
3. Probing the elements of $Q$ by measuring flavor ratios of astrophysical neutrinos

We have shown that the flavor transitions of astrophysical neutrinos can be parameterized by the matrix $Q$ in a model independent way. As we have argued earlier, the $Q$ matrix is very convenient for classifying flavor transition models. It is clear that the knowledge of the neutrino flavor ratio at the source is crucial for probing the matrix $Q$.

Experimentally, the measurement of muon track to shower ratio [9] in a neutrino telescope such as IceCube can be used to extract the flux ratio $R \equiv \phi_{\mu}/(\phi_{e} + \phi_{\tau})$. If shower signals are investigated further in detail, $S \equiv \phi_{e}/\phi_{\tau}$ can also be extracted. As in [8], one can in principle disentangle $Q_{31}$ and $Q_{33}$ by measuring $R$ from two different sources. If both $R$ and $S$ can be measured, the structure of the $Q$ matrix can be explored in more detail and the flavor transition models are constrained. Assume the flux is conserved for neutrinos coming from pion and damped muon sources, we find

$$q_{12} = (1 - 2R_{\pi})(1 + R_{\pi})^{-1},$$
$$q_{23} = (R_{\mu} - R_{\pi})(1 + R_{\pi})^{-1}(1 + R_{\mu})^{-1},$$
$$l_{12} = -1 + 3[(1 + R_{\pi})(1 + S_{\pi})]^{-1},$$
$$l_{23} = 6\left( [(1 + R_{\pi})(1 + S_{\pi})]^{-1} - [(1 + R_{\mu})(1 + S_{\mu})]^{-1} \right),$$

(16)

where $q$’s and $l$’s are linear combinations of the elements of the $Q$ matrix

$$q_{12} = (Q_{21} - Q_{22}) + (Q_{31} - Q_{32}), \quad l_{12} = (Q_{21} - Q_{22}) - (Q_{31} - Q_{32}),$$
$$q_{23} = (Q_{22} + Q_{23}) + (Q_{32} + Q_{33}), \quad l_{23} = (Q_{22} + Q_{23}) - (Q_{32} + Q_{33}).$$

(17)

Solving Eq. (16) with $R$ and $S$ measurements can reduce the number of unknown elements in the $Q$ matrix from six to two and hence greatly constrains the transition models.

4. Conclusion

We have described a parametrization of the flavor transitions of propagating astrophysical neutrinos by the matrix $Q$, which is related to the usual flavor transition probability matrix $P$ by $Q = A^{-1}PA$ where $A$ is given by Eq. (7). We have argued that it is much easier to classify flavor transition models by the $Q$ matrix parametrization, where the relevant matrix elements are $Q_{31}$ and $Q_{33}$ in the exact $\nu_{\mu} - \nu_{\tau}$ symmetry limit. We have also described that, with the future measurements of $R$ and $S$, one can constrain the flavor transition using the $Q$ matrix.

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