Phenomenology of SIDIS unpolarized cross sections and azimuthal asymmetries

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Abstract

I review the phenomenology of unpolarized cross sections and azimuthal asymmetries in semi-inclusive deeply inelastic scattering (SIDIS). The general theoretical framework is presented and the validity of the Gaussian model is discussed. A brief account of the existing analyses is provided.

1 Introduction

It has been known since the early years of quantum chromodynamics that azimuthal asymmetries in unpolarized processes are generated by gluon radiation and splitting. The observation of these asymmetries was in fact proposed as a test of perturbative QCD [1]. The first experimental results date back to the Eighties and showed that unpolarized semi-inclusive deeply inelastic scattering (SIDIS) indeed displays non vanishing azimuthal modulations of the type $\cos \phi_h$ and $\cos 2\phi_h$ [2, 3].

1Invited review talk at “Transversity 2011”, Veli Lošinj, Croatia.
While the pQCD processes account for these asymmetries at large transverse momenta, in the low- \( P_{h\perp} \) region it is the intrinsic transverse motion of quarks that plays a key rôle (for a recent review see [4]). There are two non-perturbative sources of azimuthal asymmetries in unpolarized processes: 1) the Cahn effect [5, 6], a purely kinematic correction due to the quark transverse momentum; 2) the Boer-Mulders effect [7], arising from the correlation between the transverse momentum and the transverse spin of quarks inside an unpolarized nucleon.

## 2 Theoretical framework

We will consider unpolarized SIDIS, \( l(\ell) + N(P) \to l(\ell') + h(P_h) + X(P_X) \), in the current fragmentation region. At low \( P_{h\perp} \) the cross section of this process factorizes into a transverse-momentum dependent quark distribution (TMD) and a fragmentation function [8].

Let us consider the transverse kinematics in a reference frame where the virtual photon and the nucleon are collinear (we call it \( \gamma^* N \) collinear frame, see fig. 1). The transverse momenta are denoted as follows: \( k_{\perp} \), transverse momentum of the initial quark; \( p_{\perp} \), transverse momentum of the hadron w.r.t. to the fragmenting quark; \( P_{h\perp} \), transverse momentum of the hadron w.r.t. to the \( \gamma^* N \) axis. Neglecting \( 1/Q^2 \) corrections, these momenta are related by

\[
p_{\perp} \simeq P_{h\perp} - z_h k_{\perp}.
\]  

The SIDIS structure functions are actually expressed in terms of the quark transverse momentum \( k_T \) defined in the frame where the produced hadron \( h \) and the nucleon are collinear (\( hN \) collinear frame). However, if one neglects \( 1/Q^2 \) corrections, the transverse momenta in the \( \gamma^* N \) and \( hN \) collinear frame
coincide: \( k_\perp \simeq k_T \).

The general expression of the cross section for unpolarized SIDIS is \[9, 10\]

\[
\frac{d\sigma}{dxdydz_h d\phi_h dP_{h\perp}^2} = \frac{2\pi \alpha_{em}^2}{x_B y Q^2} \left\{ (1 - y + \frac{1}{2} y^2) F_{UU,T} + (1 - y) F_{UU,L} + (2 - y) \sqrt{1 - y} \cos \phi_h F_{UU}^{c\cos \phi_h} + (1 - y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right\}.
\]

The azimuthal asymmetries are defined as:

\[
A^{\cos \phi_h} = 2 \langle \cos \phi_h \rangle = 2 \frac{\int d\phi_h \cos \phi_h d\sigma}{\int d\phi_h d\sigma},
\]

and similarly for \( A^{\cos 2\phi_h} \).

In the extended parton model, the structure functions appearing in eq. (2) are given by convolutions of transverse momentum dependent quark distributions and fragmentation functions. Up to order \( 1/Q \) one has

\[
F_{UU,T} = C \left[ f_1 D_1 \right],
\]

\[
F_{UU,\text{Cahn}}^{\cos \phi_h} = \frac{2M}{Q} C \left[ \frac{\hat{h} \cdot k_T}{M} f_1 D_1 \right],
\]

\[
F_{UU,\text{BM}}^{\cos \phi_h} = \frac{2M}{Q} C \left[ -\frac{(\hat{h} \cdot k'_T) k_T^2}{M h_1^+ H_1^+} \right],
\]

\[
F_{UU,\text{BM}}^{\cos 2\phi_h} = C \left[ -\frac{2(\hat{h} \cdot k_T)(\hat{h} \cdot k'_T) - k_T \cdot k'_T}{MM_h} h_1^+ H_1^+ \right],
\]

where \( C \left[ w f D \right] \) is a transverse momentum convolution.

Equation (5) represents the Cahn contribution to the \( \cos \phi_h \) structure function. It originates from the elementary lepton-quark cross section in presence of transverse momenta, which reads

\[
d\hat{\sigma} \sim \frac{1}{y^2} \left[ \left( 1 - 4 \frac{k_\perp}{Q} \sqrt{1 - y} \cos \varphi \right) + \left( 1 - 4 \frac{k_\perp}{Q} \frac{\cos \varphi}{\sqrt{1 - y}} \right) \right] + \mathcal{O} \left( \frac{k_\perp^2}{Q^2} \right).
\]

Equations (6) and (7) are the Boer-Mulders (BM) contributions to the azimuthal modulations of unpolarized SIDIS. They couple the Boer-Mulders function \( h_1^+ \) to the Collins function \( H_1^+ \) \[11\], which describes the fragmentation of transversely polarized quarks into an unpolarized hadron. Notice
that the BM effect contributes to $\cos \phi_h$ at order $1/Q$ (i.e., at twist three) and to $\cos 2\phi_h$ at leading twist.

At twist three one should also take into account quark-gluon interactions, which give rise to the so-called “tilde” distributions. Thus $F_{UU}^{\cos \phi_h}$ gets the additional term [10]

$$F_{UU}^{\cos \phi_h} = \frac{2M}{Q} C \left[ -\frac{\hat{h} \cdot k_T}{M_h} \left( x\hat{h}H_1^h + \frac{M_h}{M} f_1 \frac{\tilde{D}_1^\perp}{z} \right) \right.$$

$$\left. -\frac{\hat{h} \cdot k_T}{M} \left( x\hat{f}_1^\perp D_1 + \frac{M_h}{M} h_1^h \frac{\tilde{H}}{z} \right) \right]. \quad (9)$$

We recall however that TMD factorization is not proven beyond leading twist, so eq. (9) must be taken with a grain of salt.

The Cahn effect produces at order $1/Q^2$ (twist four) a further contribution to the $\cos 2\phi_h$ structure function,

$$F_{UU,Cahn}^{\cos 2\phi_h} = \frac{M^2}{Q^2} C \left[ \frac{2(\hat{h} \cdot k_T)^2 - k_T^2}{M^2} f_1 D_1 \right]. \quad (10)$$

This expression incorporates only part of the $1/Q^2$ kinematic corrections. Besides them, there are also dynamical $1/Q^2$ corrections arising from twist-four TMD’s and fragmentation functions. Therefore, $F_{UU,Cahn}^{\cos 2\phi_h}$ is just an approximate estimate of the full $O(1/Q^2)$ structure function.

In summary, ignoring unknown higher-twist terms, the two azimuthal asymmetries of unpolarized SIDIS are symbolically given by:

$$\langle \cos \phi_h \rangle = \frac{1}{Q} \text{Cahn} + \frac{1}{Q} \text{BM}, \quad \langle \cos 2\phi_h \rangle = \text{BM} + \frac{1}{Q^2} \text{Cahn}. \quad (11)$$

### 3 The Gaussian approach

In most phenomenological analyses, the TMD’s and fragmentation functions are parametrized by Gaussians:

$$f_1(x, k_1^2) = \mathcal{A} f_1(x) e^{-\frac{k_1^2}{k_1^2}}, \quad D_1(z, p_1^2) = \mathcal{B} D_1(z) e^{-\frac{p_1^2}{p_1^2}}. \quad (12)$$

Here $x$ is the fraction of the nucleon light-cone momentum carried by the quark, and $z$ is the fraction of the light-cone momentum of the struck quark.
carried by the final hadron. These two variables coincide with $x_B$ and $z_h$, respectively, modulo $1/Q^2$ corrections. We stress that there is a strong assumption – with no fundamental basis – behind the Gaussian model, namely that distributions factorize in $x$ and $k_\perp$.

In principle, one has to distinguish the Gaussian widths $k_\perp^2$ and $p_\perp^2$ from the average squared momenta, defined as

$$\langle k_\perp^2 \rangle \equiv \int d^2 k_\perp k_\perp^2 f_1(x, k_\perp^2), \quad \langle p_\perp^2 \rangle \equiv \int d^2 p_\perp p_\perp^2 D_1(z, p_\perp^2).$$

(13)

Average values coincide with the Gaussian widths,

$$\langle k_\perp^2 \rangle = k_\perp^2, \quad \langle p_\perp^2 \rangle = p_\perp^2,$$

only if we integrate over $k_\perp$ and $p_\perp$ between 0 and $\infty$.

Bounds on $x \equiv k^-/P^-$ and $k_\perp$ can be obtained in the quark parton model by inserting a set of physical intermediate states in the quark-nucleon amplitudes [12]. Since the momentum of these states is

$$P_{\mu} = \left( xM^2 - \frac{k_\perp^2}{2xP^-}, (1 - x)P^-, -k_\perp \right),$$

(14)

the condition $M_n^2 \geq 0$ yields

$$k_\perp^2 \leq x(1 - x)M^2.$$  

(15)

This is the bound on the truly intrinsic quark transverse momentum [13] (for recent discussions see [14, 15]). Numerically the upper limit (15) is $k_\perp^2 < 0.25$ GeV$^2$. Clearly, the average squared momentum $\langle k_\perp^2 \rangle$ must be smaller than this value. On the other hand, various phenomenological analyses give much larger figures, $\langle k_\perp^2 \rangle \sim 0.25 - 0.40$ GeV$^2$.

Is there a contradiction between these results? The answer is that the bound (15) actually refers to a “static” nucleon ($Q^2 = 0$), whereas the value of $\langle k_\perp^2 \rangle$ extracted from experiments effectively accounts for the “non-intrinsic” transverse momentum, radiatively generated at a given $Q^2$. As a consequence, $\langle k_\perp^2 \rangle$ is not a fixed, universal, quantity.

Consider now the unpolarized and angle-independent structure function:

$$F_{UU} = \sum_a e_a^2 \int d^2 k_\perp \int d^2 p_\perp \delta^2(p_\perp + z_hk_\perp - P_{h\perp}) f_a^u(x_B, k_\perp^2) D_a^u(z_h, p_\perp^2) \quad \quad (16)$$

Using Gaussian parametrizations and integrating between 0 and $\infty$, one gets

$$F_{UU} = \sum_a e_a^2 f_a^u(x_B) D_a^u(z_h) e^{-P_{h\perp}^2/P_{h\perp}^2} \pi P_{h\perp}^2 \quad \quad (17)$$
Figure 2: The ratio $R(P_{h\perp}) = d\sigma_{UU}(P_{h\perp})/d\sigma_{UU}(0)$: Gaussian model vs. CLAS data (from [16]).

with

$$P_{h\perp}^2 = p_{h\perp}^2 + z_h^2 k_{h\perp}^2$$  \hspace{1cm} (18)

This is a popular relation appearing in many analyses (where it is often quoted as a relation between average momenta). It must used with some caution. In fact: i) it is invalidated by a possible cutoff on $k_{h\perp}^2$; ii) it has (complicated) $1/Q^2$ corrections. iii) due to experimental cuts, $P_{h\perp}^2$ differs from the measured $\langle P_{h\perp}^2 \rangle$.

4 Transverse-momentum dependence of cross sections

The Gaussian Ansatz for the unpolarized SIDIS cross section,

$$\frac{d\sigma_{UU}(P_{h\perp})}{dz\, dP_{h\perp}^2} = \frac{d\sigma_{UU}(0)}{dz\, dP_{h\perp}^2} e^{-P_{h\perp}^2/P_{h\perp}^2},$$  \hspace{1cm} (19)

has been tested by Schweitzer, Teckentrup and Metz [16] in the CLAS kinematics ($x_B = 0.24$, $z_h = 0.34$, $Q^2 = 2.4$ GeV$^2$) [17]. As shown in fig. 2, the description of data is very good, with a Gaussian width $P_{h\perp}^2 = 0.17$ GeV$^2$.

The results of a Gaussian analysis of the $P_{h\perp}^2$ distributions measured by COMPASS are displayed in fig. 3 (left) [18]. The fitted $P_{h\perp}^2$ is found to have a mild dependence on $x_B$ and to increase with $Q^2$. A check of the relation (18) has been also performed, with the result shown in fig. 3 (right). One
Figure 3: Left: $P_{h\perp}^2$ distributions fitted by Gaussians (COMPASS data) [18]. Right: Fitted $P_{h\perp}^2$ (called $\langle p_T^2 \rangle$ in the figure) vs. $z_h^2$ (COMPASS data) [18].

Figure 4: Left: Dependence of $\langle P_{h\perp}^2 \rangle$ on the lepton-nucleon c.m. energy squared of various experiments (from [16]). Right: Dependence of $\langle P_{h\perp}^2 \rangle$ on the photon-nucleon c.m. energy squared $W^2$ (COMPASS data, from [18]).

sees a clear departure from a linear dependence of $P_{h\perp}^2$ on $z_h^2$. The dashed and dot-dashed curves in the figure are fits of the type

$$ P_{h\perp}^2 = z_h^{0.5} (1 - z_h) p_T^2 + z_h^2 k_{\perp}^2 $$

Finally, it has been noticed in [16] that the average transverse momentum of hadrons extracted in different experiments (JLab, HERMES, COMPASS) has an approximately linear rise in $s$ (fig. 4 left). This energy dependence is confirmed by a COMPASS analysis [18], which shows that $\langle P_{h\perp}^2 \rangle$ increases with $W^2$ (fig. 4 right), which may be an indication of a possible broadening of $\langle k_{\perp}^2 \rangle$ and $\langle p_{\perp}^2 \rangle$. 

7
5 Azimuthal asymmetries

The first analysis of the $\cos \phi_h$ asymmetry in SIDIS in terms of the Cahn effect was presented in 2005 by Anselmino et al. [19]. They fitted the EMC data [2] with a Gaussian model, obtaining the values $k_{\perp}^2 = 0.25 \text{ GeV}^2$, $p_{\perp}^2 = 0.20 \text{ GeV}^2$, but did not consider the Boer-Mulders term, which is of the same order as the Cahn contribution.

The $\cos 2\phi_h$ asymmetry was extensively studied in [20, 21]. Both the leading-twist Boer-Mulders contribution and the twist-4 Cahn term were included in the analysis. Signs and magnitudes of the Boer-Mulders function, extracted from a fit to HERMES [22] and COMPASS [23] preliminary data, were found to be in agreement with the theoretical expectations (based on the impact-parameter approach, lattice studies and large $N_c$ arguments):

$$h_{\perp}^u \sim Z f_{\perp}^u T, \quad h_{\perp}^d \sim -Z f_{\perp}^d T.$$  

In particular, it was pointed out that, since the Cahn effect is the same for $\pi^+$ and $\pi^-$, and the favorite and unfavorable Collins functions are related by $H_{\perp}^{\text{fav}} \approx -H_{\perp}^{\text{unf}}$, as shown by the fits to the Collins effect in SIDIS, a signature of the Boer-Mulders effect is $\langle \cos 2\phi_h \rangle_{\pi^-} > \langle \cos 2\phi_h \rangle_{\pi^+}$. Another interesting output of the analysis in [20, 21] is that the Cahn contribution turns out to be relatively large in spite of being $O(1/Q^2)$. An example of the fits presented in [21] is shown in fig 5.

The HERMES and COMPASS Collaborations have recently presented new preliminary data on azimuthal asymmetries in unpolarized SIDIS [24].
While the two measurements are in fairly good agreement for $\langle \cos \phi_h \rangle$, the COMPASS result for the $\cos 2\phi_h$ asymmetry is systematically larger than the corresponding HERMES result.

The CLAS Collaboration at JLab has released final results on the azimuthal modulations [17]. A large discrepancy is found between the data on $\langle \cos \phi_h \rangle$ and the Cahn effect predictions (extrapolated from other analyses). The $\langle \cos 2\phi_h \rangle$ signal is non-zero only in the small-$z_h$ region, where target fragmentation effects are important and may affect the interpretation of data.

From the theoretical viewpoint, the situation is quite unclear, as only partial analyses are presently available. If we simply extrapolate them, we find that the Cahn contribution to $\langle \cos \phi_h \rangle$ is huge and largely overshoots the data, and that the Boer-Mulders contribution to $\langle \cos 2\phi_h \rangle$ is small and apparently goes in the wrong direction, in the sense that it has an opposite sign in $\langle \cos \phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$, and therefore should produce a $\langle \cos \phi_h \rangle\pi_−$ smaller than $\langle \cos \phi_h \rangle\pi_+$, whereas the data show a similar $\pi^+\pi^−$ pattern for both asymmetries.

We have already said that the intrinsic transverse momentum of quarks is kinematically bounded. We expect that the radiatively generated transverse momentum should be limited as well. The effect of an upper bound on $k_\perp$ has been recently explored by Boglione, Melis and Prokudin [26]. They derived a cutoff of the form $k_\perp^2 \leq \eta(x_B)Q^2$ in the $\gamma^*N$ center-of-mass frame. The consequence of setting this bound is that the Cahn contribution gets largely suppressed, while the BM contribution remains almost unaffected. This correction goes in the right direction, but further work is needed to establish a frame-independent condition on $k_\perp$.

### 6 Conclusions

There is nothing magic about the Gaussian approach. It is just a parametrization, which happens to work rather well for cross sections at low $P_{h\perp}$. Since this model lacks a solid basis, one should not be surprised if some simple regularities based on it turn out to fail.

The parameters $\overline{k_\perp^2}$ and $\overline{p_\perp^2}$ in the Gaussian distributions are likely to be $Q^2$ (and $W^2$) dependent. This means that they cannot be fixed once forever and used anywhere. One should do as for normal PDF’s: take a TMD at small $Q^2$ from some model, evolve it and fit the data.
Although we have clear signals of relatively large azimuthal asymmetries, the situation is still unsettled. A combined state-of-the-art fit of $\langle \cos \phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$ would be highly desirable in order to understand the origin of these observables. We should be prepared to the possibility that the scheme (11) might not work. We recall that the corrections to this scheme are of three types: 1) “genuine” (i.e. “tilde”) twist-3 contributions (originating from quark-gluon correlations); 2) further $1/Q^2$ kinematic terms; 3) dynamical twist-4 contributions.

A lesson we have learned is that higher twists are relevant in the presently explored region of SIDIS. Clearly, it would be important to disentangle them from leading twist contributions. In order to do so, a wider $Q^2$ range is needed – an important task for future experiments. In the meanwhile, the $1/Q^2$ contributions should be fully worked out, computing both the kinematic corrections and the target mass effects.

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