TESTING THE KERR BLACK HOLE HYPOTHESIS

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It is thought that the final product of the gravitational collapse is a Kerr black hole and astronomers have discovered several good astrophysical candidates. While there is some indirect evidence suggesting that the latter have an event horizon, and therefore that they are black holes, a proof that the space-time around these objects is described by the Kerr geometry is still lacking. Recently, there has been an increasing interest in the possibility of testing the Kerr black hole hypothesis with present and future experiments. In this paper, I briefly review the state of the art of the field, focussing on some recent results and work in progress.

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1. Introduction

General Relativity (GR) is our current theory of gravity and so far there are no clear observational evidences in disagreement with its predictions. However, the theory has been tested only in certain regimes; in particular, only for weak gravitational fields. Even in the famous observation of the decay of the orbital period of the binary system PSR1913+16, due to the emission of gravitational waves, the gravitational potential is $|\varphi| \sim M/r \sim 10^{-6}$; that is, $|\varphi| \ll 1$.

On the other hand, we know GR breaks down in some extreme situations: the theory allows for the existence of space-time singularities, where predictability is lost, and regions with closed time-like curves, where causality can be violated. GR seems also to be incompatible with quantum mechanics, and probably for this reason we do not have yet a quantum theory of gravity.

One of the most intriguing predictions of GR is that the collapsing matter produces singularities in the space-time, at least in the sense of time-like or null geodesic incompleteness. There is no theorem restricting the nature of these singularities,\textsuperscript{a} that is, it is not known whether they are spacelike, timelike or null.

\textsuperscript{a}Throughout the paper, I use units in which $G_N = c = 1$.

\textsuperscript{b}This conclusion requires the following assumptions: i) the validity of the Einstein’s equations with non-positive cosmological constant, ii) the energy condition $T^\sigma_\sigma \geq 2 T_{\mu\nu} t^\mu t^\nu$ for any vector field $t^\nu$ such that $t^\nu t_\nu = -1$, where $T_{\mu\nu}$ is the matter energy-momentum tensor and the metric

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but there are only two options: the singularities are hidden behind an event horizon, and the final product of the collapse is one or more black holes (BHs), or the singularities are naked. Since space-times with naked singularities have several kinds of pathologies, one assumes the weak cosmic censorship conjecture, according to which all the singularities of gravitational collapse must be hidden within BHs.

Numerical simulations support this conjecture. Surprisingly, it turns out that in 4-dimensional GR there is only one uncharged BH solution, the Kerr metric, which is completely specified by two parameters, the mass, \(M\), and the spin angular momentum, \(J\), of the object. This is the celebrated “no-hair” theorem, although, strictly speaking, a Kerr BH has “two hairs”, \(M\) and \(J\). The condition for the existence of the event horizon is \(|a| \leq 1\), where \(a = J/M^2\) is the (dimensionless) spin parameter. For \(|a| > 1\), the Kerr metric does not describe a BH but a naked singularity, which is forbidden by the weak cosmic censorship conjecture. Astrophysical BHs presumably form from the gravitational collapse of matter (stars or clouds), while the no-hair theorem demands that the space-time is time independent. However, one can see that any deviation from the Kerr solution is quickly radiated through the emission of gravitational waves (but see Ref. [10]).

Numerical simulations show that a BH rapidly goes “bald” even when the initial deviations from the Kerr background are large (see e.g. [11]). Let us also notice that the Kerr metric is not an exact solution of the Einstein’s equations only, but it is common to many other theories of gravity; however, in general, there may not be a uniqueness theorem as in GR.

At the observational level, there are at least two classes of astrophysical BH candidates: stellar-mass objects in X-ray binary systems (mass \(M \approx 5 - 20 M_\odot\)) and super-massive objects in galactic nuclei (\(M \sim 10^5 - 10^9 M_\odot\)). The existence of a third class of objects, intermediate-mass BHs with \(M \sim 10^2 - 10^4 M_\odot\), is still controversial, because their detections are indirect and definitive dynamical measurements of their masses are still lacking. All the BH candidates are supposed to be Kerr BHs because they cannot be explained otherwise without introducing new physics. The stellar-mass objects in X-ray binary systems are too heavy to be neutron or quark stars for any reasonable matter equation of state. At least some of the super-massive objects in galactic nuclei are too massive, compact, and old to be clusters of non-luminous bodies. There is also observational evidence that the surface of the BH candidates does not emit any radiation, even when it is hit by the accreting gas. This fact has been interpreted as an indirect proof of the existence of the Kerr BHs.
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of an event horizon (see however Ref. [19] for different interpretations). The existence of an event horizon is also theoretically required for the stability of any rotating very compact object. However, for systems close to the formation of an event horizon, the instability time scale would be very long and they could still be consistent with observations. Examples of compact objects with a “quasi-event horizon” can be found, for instance, in.

The aim of this paper is to provide a short review on the current attempts to test the Kerr nature of the astrophysical BH candidates, focusing the attention on some selected recent results. The content of the manuscript is as follows. In Sec. 2 I review the basic idea to test the nature of the BH candidates and the theoretical frameworks proposed in the literature. In Sec. 3 I show that non-Kerr BHs may have some fundamental properties that are different from the ones expected for a BH in GR. In Sec. 4 I review the possibilities offered by present and future experiments to test the Kerr black hole hypothesis. The conclusions are reported in Sec. 5.

2. Measuring deviations from the Kerr geometry

A framework within which to test the Kerr BH hypothesis was first put forward by Ryan [24], who considered a general stationary, axisymmetric, asymptotically flat, vacuum space-time. Such a generic space-time can be used to describe the gravitational field around a central object, whatever its nature, and its metric can be expressed in terms of the mass moments $M_\ell$ and current moments $S_\ell$ [25]. Assuming reflection symmetry, the odd $M$-moments and even $S$-moments are identically zero, so that the non-vanishing moments are the mass $M_0 = M$, the mass quadrupole $M_2 = Q$ and the higher-order even terms $M_4, M_6, \ldots$, as well as the angular momentum $S_1 = J$, the current octupole $S_3$ and the higher-order odd terms $S_5, S_7, \ldots$. In the case of a Kerr BH, all the moments $M_\ell$ and $S_\ell$ are locked to the mass and angular momentum by the following relation:

$$M_\ell + iS_\ell = M \left( \frac{J}{M} \right)^\ell. \quad (1)$$

By measuring at least three non-trivial multiple moments of the space-time around an astrophysical BH candidate (e.g. $M$, $J$, and $Q$), one over-constrains the theory and can test the Kerr nature of the object.

A post-Newtonian approach with generic $M$, $J$, and $Q$ can do the job only if we have the possibility of measuring with excellent accuracy the geometry relatively far from the compact object, where the contribution of higher-order moments is negligible. The advantage of a post-Newtonian approach is that we do not need specific assumptions about the space-time close to the BH. The disadvantage is that very accurate measurements are necessary, while they may be out of reach in realistic astrophysical situations.

The differences between a Kerr and a non-Kerr BH are more and more evident if we probe the space-time closer to the object, where gravity is strong. However,
we have to consider a specific metric and take all the special and general relativistic effects into account. In this case, there are basically two approaches:

(1) We assume that the space-time around the compact object can be described by an exact solution of the Einstein’s vacuum equations which, because of the no-hair theorem, cannot be a BH. The idea on the basis of this choice is that this metric can correctly describe the exterior gravitational field around the compact object and holds up to the “surface” of the object. The latter should cover all the pathological features (e.g. space-time singularities and regions with close time-like curves) of the space-time.

(2) We consider a metric with a regular event horizon: this is a true BH but, as follows from the no-hair theorem, it is not a solution of the Einstein’s vacuum equations. The advantage of this approach is that the position of the event horizon is well defined, while in the first approach there is still some arbitrariness to be fixed.

In both cases, we eventually have a metric more general than the Kerr solution and that includes the Kerr space-time as special case. So, we have a compact object characterized by a mass $M$, a spin angular momentum $J$, and one or more “deformation parameters”, say $\delta$ (or $\{\delta_i\}$ with $i = 1, 2, \ldots$), measuring (hopefully generic) deviations from the Kerr geometry. When $\delta = 0$, the object is a Kerr BH. One can then consider a particular astrophysical phenomenon, compare the theoretical predictions in this general space-time with the observational data, and constrain the deformation parameter $\delta$. If the observations demand $\delta = 0$, they confirm the Kerr BH hypothesis.

The Tomimatsu-Sato space-time was the first non-Kerr solution of the Einstein’s vacuum equations discovered\textsuperscript{26,27,28,29}. It is stationary, axisymmetric, and asymptotically flat and it has three free parameters (mass, spin, and deformation parameter). However, the metric is quite complicated, there is no analytic form for any value of the deformation parameter, and deviations from the Kerr geometry are not very generic (for instance, the Kerr solution is recovered even for $|a_*| = 1$, independently of the value of the deformation parameter). A metric more useful to test the Kerr BH hypothesis is the Manko-Novikov solution\textsuperscript{30,31}. It is still a stationary, axisymmetric, and asymptotically flat exact solution of the Einstein’s vacuum equations, but it has an infinite number of free parameters: here the object has a mass $M$, a spin angular momentum $J$, and arbitrary mass multipole moments $M_\ell$ ($\ell = 1, 2, \ldots$), while the current multipole moments $S_\ell$ of order higher than 1 are fixed by the mass multipole moments. Some authors have also proposed approximated solutions of the Einstein’s vacuum equations: these metrics are solutions in GR only up to some order of one or more expansion parameters. This is the case, for instance, of the “quasi-Kerr” metric proposed by Glampedakis and Babak\textsuperscript{32} and of the “bumpy BHs” introduced by the MIT group\textsuperscript{33,34,35}.

In order to be able to probe the geometry of the space-time very close to a BH candidate and study very rapidly-rotating objects (for which possible deviations
from the Kerr metric should produce stronger effects, see next section), it may be more convenient to use a metric describing a BH, without the ambiguity related to the position of the “surface” of the object. These space-time can be seen as BHs in alternative theories of gravity, even if we do not know the gravity theory they would belong to. The first theoretical framework in this direction was proposed in Ref. 36; these objects are called “bumpy BHs in alternative theories of gravity”. Another family of metrics describing non-GR BHs was introduced by Johannsen and Psaltis in Ref. 37.

3. Properties of non-Kerr black holes

If the astrophysical BH candidates are not Kerr BHs, some of their fundamental properties are likely very different from the ones predicted by GR. If we know some generic features of non-Kerr BHs, we can hopefully figure out generic observational signatures to test the nature of the astrophysical candidates.

3.1. Spin parameter

A fundamental limit for a Kerr BH is the bound $|a_\ast| \leq 1$. This is just the condition for the existence of the event horizon. In absence of a horizon there would be a naked singularity, which is forbidden by the weak cosmic censorship conjecture. Even if it is not yet clear if naked singularities can be created in Nature, and therefore if the weak cosmic censorship conjecture can be violated (see e.g. Ref. 38 and references therein), it is apparently impossible to make a star collapse with $|a_\ast| > 1$ or to overspin an already existing BH to $|a_\ast| > 1$. Moreover, even if it were possible to create a Kerr naked singularity, the space-time would be unstable and it should quickly decay. The same would be true if the singularity were replaced by a very compact object arising from new physics, independently of its nature, because of the ergoregion instability.

The story changes if the compact object is not a Kerr BH: in this case, the maximum value of $|a_\ast|$ may be either larger or smaller than 1, depending on the geometry of the space-time around the compact object, on the exact nature of the compact object, and on the gravity theory. Generally speaking, objects with $|a_\ast| > 1$ are quite common in the Universe and even the value of the spin parameter of the Earth is about $10^3$. However, a priori it is not obvious if it is possible to create even a compact object with $|a_\ast| > 1$.

In Refs. 43, 44, I considered compact objects characterized by three free parameters (mass $M$, spin parameter $a_\ast$, and quadrupole moment $Q$) and I studied the evolution of $a_\ast$ as a consequence of the accretion process from a thin disk. It is indeed well known that accretion from a thin disk is a quite efficient mechanism to spin-up a compact body. In the standard theory of thin disks (Novikov-Thorne model), the disk is on the equatorial plane, the gas particles move on nearly geodesic circular orbits, and the inner radius of the disk is at the innermost stable circular orbit (ISCO). When the gas reaches the ISCO, it quickly plunges into the
BH and crosses the event horizon, with negligible emission of additional radiation. So, the BH changes its mass by $\delta M = E_{\text{ISCO}} \delta m$ and its spin by $\delta J = L_{\text{ISCO}} \delta m$, where $E_{\text{ISCO}}$ and $L_{\text{ISCO}}$ are respectively the specific energy and the $z$-component of the specific angular momentum of a particle at the ISCO radius, while $\delta m$ is the gas rest-mass. When the Novikov-Thorne model can be applied, the evolution of the spin parameter is thus governed by the following equation:

$$\frac{da_*}{d\ln M} = \frac{1}{M} \frac{L_{\text{ISCO}}}{E_{\text{ISCO}}} - 2a_*.$$ \hfill (2)

Recent numerical simulations support this model for thin disks around Kerr BHs [47,48] (see however Ref. [49]). The equilibrium spin parameter is reached when the right hand side of Eq. (2) is zero. For a Kerr BH, this occurs when $a_{eq}^* = 1$. Including the small effect of the radiation emitted by the disk and captured by the BH, one finds the famous Thorne’s limit $a_{eq}^* = 0.99850$. On the other hand, if the compact object is more oblate than a Kerr BH, the equilibrium spin parameter is larger than 1, and its value increases as the object becomes more and more oblate. The situation is more complicated for objects more prolate than a Kerr BH, and the value of the equilibrium spin parameter can be either larger or smaller than 1. The result of Refs. [43,44,45] is thus that compact objects with $|a_*| > 1$ can be created, at least in principle. Depending on the exact nature of the compact object and on the gravity theory, there is still the possibility that the equilibrium spin parameter predicted by Eq. (2) can never be reached, because the compact object becomes unstable at a lower value of $|a_*|$. For instance, a similar situation occurs for neutron stars: the accretion process can spin them up to a frequency $\sim 1$ kHz, but the existence of unstable modes prevents these objects from rotating at higher frequencies [51].

3.2. Event horizon

The event horizon of a BH is defined as the boundary of the causal past of future null infinity. The spatial topology of the event horizon at a given time is the intersection of the Cauchy hypersurface at that time with the event horizon. In the Kerr space-time, the spatial topology of the event horizon is a 2-sphere. However, the topology does not change even if the space-time is not exactly described by the Kerr geometry, e.g. like in the case of a BH surrounded by a disk of accretion. The Hawking’s theorem indeed ensures that, in 4-dimensional GR, the spatial topology of the event horizon must be always a 2-sphere in the stationary case, under the main assumptions of asymptotically flat space-time and validity of the dominant energy condition [52]. In the non-stationary case, BHs with a toroidal spatial topology can form, but the hole must quickly close up, before a light ray can pass through [53]. Numerical simulations confirm the theoretical results [54,55].

Non-Kerr BHs may instead have topologically non-trivial event horizons, as they are not solutions of the Einstein’s equations. In Ref. [56], I have even argued that
topologically non-trivial event horizons may be a quite common feature for rapidly-rotating non-Kerr BHs, providing two explicit examples in which the horizon of the BH changes topology above a critical value of the spin parameter. The basic mechanism is the following. A Kerr BH has an outer horizon of radius $r_+$ and an inner horizon of radius $r_-$. As $|a_+|$ increases, $r_+$ decreases, while $r_-$ increases. For $|a_+| = 1$, there is only one horizon ($r_+ = r_-$) and, for $|a_+| > 1$, there is no horizon. In general, however, the outer and the inner horizons may not have the same shape. If this is the case, when $|a_+|$ increases, the two horizons approach each other, but eventually they merge together forming a single horizon with non-trivial topology. Interestingly, such rapidly-rotating BHs can be easily created. Indeed, the topology transition can be potentially induced by the accretion of material from a thin disk, which should be a quite common event in the Universe.

3.3. Accretion process

The geometry of the space-time around astrophysical BH candidates can be probed by studying the properties of the electromagnetic radiation emitted by the gas of accretion. It is thus important to figure out clearly the accretion process itself and have under control all the astrophysical processes.

If the gas around the compact object has negligible angular momentum, a Bondi-like accretion can correctly describe the evolution of the system. For example, this may be the case of a BH accreting from the interstellar medium or of one belonging to a binary system in which the companion is massive and has a strong stellar wind. Here, the accretion process is basically determined by the balance between the gravitational force, which attracts the gas towards the central body, and the gas pressure, which increases as the gas becomes more and more compressed and hampers the accretion process. If the gravitational force is stronger, the compact object can swallow a larger amount of matter without problems. If the gravitational force is weaker, the accretion process is more difficult and the production of outflows is favored. Around naked singularities, the gravitational force is typically much weaker (indeed, unlike a BH, a naked singularity cannot trap light rays) and can be even repulsive. For more details, see Refs. 57, 58, 59, 60, 61, 62.

The formation of an accretion disk around the compact object is possible when the gas around the body has significant angular momentum. The disk is geometrically thin and optically thick when the gravitational energy of the falling gas can be efficiently radiated away. Thin disks are described by the Novikov-Thorne model. One of the key ingredients is the existence of an ISCO: circular orbits inside the ISCO are radially unstable and therefore, as the gas reaches the ISCO, it quickly plunges onto the compact object and crosses the event horizon, without significant emission of additional radiation. The evolution of the spin parameter of the BH is given by Eq. (2) and the radiative efficiency $\eta$, defined by $L_{\text{acc}} = \eta \dot{M}$, where $L_{\text{acc}}$ is the luminosity due to the accretion process and $\dot{M}$ is the mass accretion rate, is simply $\eta = 1 - E_{\text{ISCO}}$. 
If the geometry around the compact object is not described by the Kerr metric, one can find even other scenarios. In particular, if the object is more prolate than a Kerr BH, circular orbits on the equatorial plane may be even vertically unstable (in the Kerr background, all the circular orbits on the equatorial plane are vertically stable) and, in addition to an outer region with stable circular orbits delimited by the ISCO, one may find regions with stable circular orbits even closer to the compact object. At least for the subclass of Manko-Novikov space-times studied in [63], there are four qualitatively different final stages of accretion:

1. The ISCO is radially unstable, and the gas plunges into the compact object remaining roughly on the equatorial plane and without emitting significant radiation. This is the same scenario as in the Kerr case.
2. The ISCO is radially unstable and the gas plunges, but does not reach the compact object. Instead, it gets trapped between the object and the ISCO, forming a thick disk.
3. The ISCO is vertically unstable, and the gas plunges into the compact object outside the equatorial plane and without emitting significant radiation.
4. The ISCO is vertically unstable and the gas plunges, but does not reach the compact object. Instead, it gets trapped between the object and the ISCO and forms two thick disks, above and below the equatorial plane.

The scenarios (2) and (4) occur only in a limited range of the parameters of these space-times. Nevertheless, they have quite peculiar features and may be likely tested with future observations. Because of the presence of a thick disk inside the ISCO, the evolution of $a_*$ is given by Eq. (2) with $E_{\text{inner}}$ and $L_{\text{inner}}$ replacing respectively $E_{\text{ISCO}}$ and $L_{\text{ISCO}}$, where $E_{\text{inner}}$ and $L_{\text{inner}}$ are the specific energy and the $z$-component of the specific angular momentum at the inner edge of the thick disk, and the radiative efficiency becomes $\eta = 1 - E_{\text{inner}}$.

4. Observational tests

Generally speaking, the Kerr BH hypothesis can be tested by using the same techniques through which astronomers can measure the spin parameter of a BH, assuming the geometry of the space-time is described by the Kerr metric. In what follows, I discuss in some details the three approaches of which I have some experience, while other possibilities are just mentioned in the last subsection.

4.1. Radiative efficiency

The radiative efficiency $\eta$ is defined by the relation $L_{\text{acc}} = \eta M$, where $L_{\text{acc}}$ is the accretion luminosity and $\dot{M}$ is the mass accretion rate. In the case of a BH, $\eta$ may be even extremely small, because the gas can cross the event horizon before it can radiate away the energy of the gravitational potential. For example, in a spherically symmetric and adiabatic accretion onto a Schwarzschild BH (Bondi accretion), $\eta \sim$
High values of the radiative efficiency can be easily obtained in presence of a thin accretion disk, where \( \eta = 1 - E_{\text{ISCO}} \) for a Schwarzschild BH \((a_s = 0)\), we find \( \eta = 0.057 \), while for a maximally-rotating Kerr BH and a corotating disk \((a_s = 1)\), \( \eta = 0.42 \). When the compact object has a solid surface, the picture is instead much more complicated, as the gas may radiate additional energy when it hits the surface of the body; anyway, this does not seem the case for the BH candidates\(^{17,18}\).

In general, it is difficult to estimate the radiative efficiency of a BH candidate, because it is not possible to measure the mass accretion rate. However, one can estimate the mean radiative efficiency of active galactic nuclei (AGN) by using the Soltan’s argument\(^{64}\), which relates the mean energy density in the contemporary Universe radiated by the super-massive BHs with the today mean mass density of these objects. In the final result, there are definitely several sources of uncertainty. However, a reliable lower bound is thought to be \( \eta > 0.15 \)\(^{65,66}\), especially if one restrict the attention to the most massive objects.

For a Kerr BH, since the radiative efficiency is \( \eta = 1 - E_{\text{ISCO}} \) and increases as \( a_s \) increases, \( \eta > 0.15 \) corresponds to \( a_s > 0.89 \). If we do not assume that the super-massive objects in galactic nuclei are Kerr BHs, it is possible to get a constraint on the mean deformation parameter of AGN. In Ref.\(^{67}\), I considered a subclass of the Manko-Novikov space-times, in which the deformation parameter was the quadrupole moment \( Q \) of the object. Defining the dimensionless parameter \( q \) as \( Q = -(1 + q)a_s^2M^3 \) (for \( q = 0 \), we recover exactly the Kerr solution), observations require:

\[
-2.01 < q < 0.14 ,
\]

Let us notice that, in the case of a self-gravitating fluid like a neutron star, one would expect \( q > 1 \). In other words, BH candidates are much stiffer than ordinary matter.

Since the most efficient way to spin a compact object up is through the accretion process from a thin disk, the maximum possible value of the spin parameter of a super-massive object at the center of a galaxy is given by the equilibrium spin parameter \( a_{\text{eq}}^* \) of Eq. (2). The exact value depends on the deformation parameter. If we require that a BH candidate must be able to have \( \eta > 0.15 \) with \( a_s \leq a_{\text{eq}}^* \), we find the maximum value of the spin parameter for the super-massive objects in galactic nuclei\(^{68,69}\):

\[
|a_s| \lesssim 1.2 ,
\]

which is basically independent of the choice of the theoretical framework. This argument cannot be applied to the stellar-mass BH candidates because the value of the spin parameter of the latter reflects the one at the time of their formation; that is, it is determined by more complex physics involving the gravitational collapse.

\(^{d}\)As discussed in Subsec. \(^{68}\), \( \eta = 1 - E_{\text{inner}} \) in those special cases in which the gas forms a thick disk between the ISCO and the BH.
As shown in this subsection, the estimate of the radiative efficiency can already be used to provide interesting constraints on the nature of the BH candidates. However, the constraints are weak and presumably they cannot be significantly improved in the near future, as there are several sources of uncertainty. On the other hand, the approaches discussed in the next subsections are more complicated, but much more promising to get stronger and more robust bounds.

4.2. Continuum fitting method

The X-ray spectrum of stellar-mass BH candidates has often a soft component (< 10 keV), which is thought to be the thermal spectrum of a geometrically thin and optically thick disk of accretion. In the Novikov-Thorne model, the observed spectrum of a thin disk around a BH depends only on the background metric, the mass accretion rate, the distance of the observer, and the viewing angle. Assuming the Kerr background, one can measure the spin parameter from the observational data. This technique is called continuum-fitting method and at present it has been used to estimate the spin of a few stellar-mass BH candidates \(^70\)\(^71\)\(^e\). Basically, one has to get independent measurements of the mass of the object, its distance from us, and the inclination angle of the disk, and then it is possible to fit the soft X-ray component of the source and deduce \(a^*\) and \(\dot{M}\). The key-point is that there is a one to one correspondence between the value of \(a^*\) and the one of the radiative efficiency \(\eta\).

Relaxing the Kerr BH hypothesis, one can probe the geometry around the stellar-mass BH candidates \(^72\) (see also Refs. \(^73\)\(^74\)\(^75\)\(^76\)\(^77\)\(^78\)\(^79\) for tests of more specific models). Generally speaking, current X-ray data are not so good to break the degeneracy between the spin parameter and the deformation parameter, but it is only possible to constrain a combination of the two. However, the thermal spectrum of a thin disk around a rapidly-rotating Kerr BH can be hardly mimicked by a compact object very different from a Kerr BH. So, if we observe a spectrum that seems to be generated around a very rapidly-rotating Kerr BH, we can constrain the deformation parameter, independently of the value of its spin. In principle, we could also discover deviations from the Kerr geometry if we find that the thermal spectrum of the disk is too hard even for a Kerr BH with \(a^* = 1\). It may be interesting to test this possibility with the spectrum of the high-spin BH candidate GRS1915+105 \(^80\), when future more accurate measurements of the distance to this object will be available. However, significant work has still to be done, especially in the case of fast-rotating objects, before using the continuum fitting method to probe the geometry around the BH candidates \(^72\).

\(^e\)For super-massive BHs, the disk temperature is lower (the effective temperature scales like \(M^{-0.25}\)) and this approach cannot be applied.
4.3. Direct imaging

The capability of very long baseline interferometry (VLBI) has improved significantly at short wavelength and it is now widely believed that within 5-10 years it will be possible to observe the direct image of the accretion flow around nearby supermassive BH candidates with a resolution comparable to their event horizon. If the disk is optically thin (which is always the case for sufficiently high frequencies) and geometrically thick, it will be possible to observe the BH “shadow”, i.e. a dark area over a bright background. While the intensity map of the image depends on the details of the accretion process, the contour of the shadow is determined exclusively by the geometry of the space-time around the compact object. The observation of the shadow can thus be used to test the Kerr BH hypothesis, as first suggested in and further explored in.

The contour of the shadow is the photon capture surface as seen by a distant observer. As light rays are bent by the gravitational field of a massive object, the size of the shadow is always larger than the one of the photon capture surface. In the case of rotating objects, in general the shadow is not symmetric with respect to the axis of the spin of the BH, because the capture radius for corotating photons is smaller than the one for counterrotating ones; the effect is maximum for an observer on the equatorial plane and goes to zero for one along the z-axis. For generic space-times, the contour of the shadow is computed by considering the photons crossing perpendicularly the image plane of the distant observer and integrating numerically backward in time the geodesic equations. All the points on the image plane of the observer whose trajectories cross the BH horizon make the shadow.

As discussed in Subsec. 3.2, rapidly-rotating non-GR BHs may have a topologically non-trivial event horizon. One can see that in these space-times the central singularity is naked. In other words, in these scenarios it is easy to overspin a BH and violate the weak cosmic censorship conjecture. If such rapidly-rotating BHs exist, a distant observer may see also the central singularity or, more likely, the quantum gravity region replacing the central singularity.

4.4. Other tests

In addition to the continuum fitting method, another famous technique among astronomers to measure the spin parameter of BH candidates is the approach of the relativistic iron line. Basically, one sees a broad spectral line which is interpreted as fluorescent iron Kα emission from cool gas in the accretion disk. The rest energy of the iron line is 6.4 keV, while the observed line is broad because it is affected by Doppler boosting, frame dragging, and gravitational redshift. The advantage with the continuum fitting method is that this technique can be applied either to stellar-mass and super-massive BH candidates. However, the physics is more complicated, we have no way to predict a priori the intrinsic surface brightness profile of the Kα line, and even the basic model is subject to critiques. A preliminary study to use the Kα line to test the Kerr BH hypothesis is reported in Ref.
The geometry of the space-time around BH candidates can also be mapped by studying the orbital motion of individual stars. The latter are massive objects, so they move along the geodesics of the space-time and, unlike the gas particles, their motion is not affected by the presence of electromagnetic fields. However, they are usually relatively far from the BH candidate, where gravity is weak, and therefore very accurate measurements are necessary. In the literature, there are two proposals in this direction. The study of the motion of a radio pulsar around a stellar-mass BH companion\textsuperscript{94} for the time being, however, no BH-pulsar binary system is known. Astrometric monitoring of stars orbiting at mpc distances from SgrA$^*$\textsuperscript{95,96}, in principle, it would indeed be possible to measure the spin $J$ and the mass-quadrupole moment $Q$ of the super-massive BH candidate at the center of the Galaxy. While future infrared experiments could be able to observe such short period stars, it is not clear if the idea can work, because the presence of unknown and unseen objects closer to the BH candidate would also affect the motion of these stars and spoil the measurement.

Another interesting proposal was put forward in Ref.\textsuperscript{97} and uses the available optical data of the BL Lacertae object OJ287. The system is modeled as a spinning primary BH with an accretion disk and a non-spinning less massive secondary BH. If we assume that the observed outbursts from 1913 to 2007 arise when the secondary BH crosses the accretion disk and we fit the data, we can constrain the spin and the quadrupole moment of the primary BH.

Future gravitational wave astronomy may open new possibilities and test the nature of the BH candidates with excellent accuracy. That should be possible in at least two ways:

**Observations of extreme-mass ratio inspirals (EMRIs).** These are systems consisting of a stellar-mass compact object orbiting a super-massive BH candidate. Since future space-based gravitational waves antennas like LISA (or a similar ESA-led mission) will be able to follow the stellar-mass object for millions of orbits around the super-massive BH candidate, the space-time around the latter can be mapped with very high accuracy. Any deviation from the Kerr geometry will lead to a phase difference in the gravitational waveforms that grows with the number of observed cycles. The technique is very promising, it is not very sensitive to the exact field equations of the gravity theory, and it has been studied in details by many authors\textsuperscript{32,98,99,100,101,102,103}.

**Observations of quasi-normal modes (QNMs).** Ground-based detectors like the Einstein Telescope will be able to observe the QNMs of stellar-mass objects, while the ones of super-massive BH candidates will require space-based missions like LISA. For a Kerr BH, the frequencies of these modes depend only on its mass $M$ and spin $J$. The identification of at least three modes can thus be used to test the nature of a BH candidate. As the QNMs are more sensitive to the specific field equations of the gravity theory, the interpretation of the data is more complicated with respect to the first approach of the EMRIs. For more details, see Refs.\textsuperscript{104,105,106,107}.
5. Conclusions

Up to now, GR has successfully passed all experimental tests, but the theory has never been checked under extreme conditions, where new physics can more likely appear. In particular, we currently believe that the final product of the gravitational collapse is a Kerr BH, but we have no observational confirmations. Astronomers have already discovered several good astrophysical candidates and recently there has been an increasing interest in the possibility of testing the Kerr nature of these objects with present and future experiments. If the BH candidates are not the BH predicted by GR, they likely have different fundamental properties; for instance, they may violate the bound $|a^*| \leq 1$, they may have a topologically non-trivial event horizon, and even the accretion process may present peculiar features. For the time being, one can probe the geometry around a BH candidate by using the techniques developed to estimate the spin parameter of these objects under the assumption they are Kerr BHs; the continuum fitting method and the study of relativistic lines are the two most popular approaches. With the already available X-ray data, we can test the Kerr BH hypothesis, at least when all the astrophysical processes are well understood. In the future, we can hope to get more reliable and precise measurements with the advent of new observational facilities: VLBI arrays will be able to observe the shadow of nearby super-massive BHs, thus detecting the apparent photon capture surface of the space-time, ground-based gravitational wave detectors like the Einstein Telescope will be able to test the Kerr BH hypothesis by detecting the QNMs of stellar-mass BHs, and space-based gravitational wave antennas like LISA or a similar ESA-led mission will be able to observe the gravitational waves emitted by EMRIs and test the nature of super-massive BHs.

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