The Weibull functional form for the energetic particle spectrum at interplanetary shock waves

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Abstract. Transient interplanetary shock waves are often associated with high energy particle enhancements, which are called energetic storm particle (ESP) events. Here we present a case study of an ESP event, recorded by the SEPT, LET and HET instruments onboard the STEREO B spacecraft, on 3 October 2011, in a wide energy range from 0.1 MeV to ~ 30 MeV. The obtained particle spectrum is found to be reproduced by a Weibull like shape. Moreover, we show that the Weibull spectrum can be theoretically derived as the asymptotic steady state solution of the diffusion loss equation by assuming anomalous diffusion for particle velocity. The evaluation of Weibull’s parameters obtained from particle observations and the power spectral density of the turbulent fluctuations in the shock region, support this scenario and suggest that stochastic acceleration can contribute significantly to the acceleration of high energetic particles at collisionless shock waves.

1. Introduction
It is generally accepted that collisionless shock waves are strong sources of high energy particles. In the heliosphere the main sources for the acceleration of energetic particles are CME driven shocks, which are thought to accelerate solar energetic particle (SEPs) during gradual SEP events [1], and more generally transient interplanetary shocks which can be associated with enhancements of energetic charged particles [2, 3, 4, 5, 6] called ESP events. They can present a wide variety of different types, ranging from large particle intensity enhancements lasting several hours to small spikes lasting only a few minutes. It is also well established that energetic particles are accelerated in association with corotating interaction regions [7, 8]; they form when a stream of fast solar wind emerging from a coronal hole, overtakes slow solar wind, emitted at an earlier time. As a consequence of solar rotation these different speed plasmas became radially aligned and interact. This interaction creates a compression region, corotating with the Sun, and can strengthen to form a forward shock at the leading edge and a reverse shock at the trailing edge.

The association of energetic charged particles with collisionless shocks suggests an acceleration mechanism that is associated with the shock itself. The mechanism of diffusive shock acceleration (DSA) [9, 10, 11] has received the most attention as it naturally predicts a power law spectrum, where the spectral index ($\gamma$) is related only to the shock compression ratio ($R$), regardless of species, as $\gamma = 1/2[(R + 2)/(R - 1)]$. Nevertheless, it has been shown that the majority of the ESP events, measured in the period 1997 - 2001 at energies $\geq$47 keV, do not follow the prediction [12], although the agreement is better at low energies (47-187 keV) at strong shocks.
(having Mach number $Ma > 3$ and $R > 2.5$ [13]). A variety of effects may truncate the power law behaviour, such as adiabatic deceleration in expanding blast waves, shock lifetime comparable to particle acceleration time, shock size comparable to particle diffusion lengths. Ellison and Ramaty 1985 [14] argued that the power law spectra should roll over at high energies due to increasing diffusion coefficient with energy and assumed an exponential function, modulating the power law. On the other hand, it has been shown [15] that curving proton spectra could be better fitted by the Bessel functions, predicted in the frame of the stochastic acceleration. Nevertheless, both the Ellison-Ramaty and the Bessel functions do not provide a reasonable fit up to the highest observed energies [18]. The so-called double power law shape has been used [16] (following [17] for fitting gamma-ray burst spectra), which is identical to the Ellison-Ramaty form below the a transition energy; then, there is a smooth transition at higher energies to a second power law. It was shown that all of the proton spectra during the 16 Ground Level Enhancements (GLEs) of Solar Cycle 23 exhibit spectral breaks at energies ranging from $\sim 2$ to $\sim 46$ MeV and all are well fitted by a double power-law shape up to relativistic energies. Nevertheless, there have been few attempts to explain why the spectral breaks result in double power-law spectra. Afanasiev et al 2014 [19] studied the effect of an additional process that can modify the power-law spectral form produced by the diffusive shock acceleration: the stochastic re-acceleration of energetic protons by enhanced Alfvénic turbulence in the downstream region of a shock wave. They model stochastic acceleration of protons downstream from the shock by performing Monte Carlo simulations with self-consistently treated wave-particle interactions. The modelled spectra for different values of the intensity of magnetic fluctuations have a broken power law dependence on particle rigidity. The spectra were fitted at high rigidities by power law functions, with power law index values at high rigidities from 9.8 to 4.5, which fully covers the range of values into which the majority of the observed power-law indices for GLEs falls.

Recent observations [20, 21] have shown that the observed spectra at lower energies ($\leq 100$ MeV) in several SEP, ESP and CIR events can be reproduced through the Weibull function [22]. Hence, they argued that the Weibull distribution (WD) can be associated with shock acceleration and explained it in terms of a stochastic multiplicative process. The WD (also known as the two-parameter stretched exponential - see [23]) is a sub-exponential distribution interpolating smoothly between the exponential distribution and power law family.

Here, we analyze the 3 October 2011 ESP event, show that the Weibull function reproduces the observed spectrum with a very good agreement and try to get insight on implications for particle acceleration. We present the ESP event as well as the solar wind conditions in section 2. A theoretical derivation of the WD is presented in section 3, and its consistency with the observed WD parameters and physical conditions of the turbulent medium is discussed in section 4. Conclusions are drawn in section 5.

2. 3 October 2011 ESP event

Transient and corotating shocks are systems where particles are assumed to be locally accelerated by the shock, as they are usually associated with energetic particle enhancements, although these might contain particles trapped around the shock that were accelerated before the shock arrival at the observing point. However, in ESP events the characteristics of both the injected particles and the medium can be directly observed. Hence it is possible in principle to assess what conditions are necessary for shock acceleration to occur and what role irregularities in the ambient solar wind and/or shock surface may play in producing and modifying shock-accelerated particle populations.

We selected a transient interplanetary shock for which particle enhancements were detected up to high energies and occurred on unperturbed background. On 3 October 2011 at 22:23 UT a strong forward shock was observed by the STEREO B spacecraft (located at 1.08 AU, -98.09° 1.08° heliographic longitude and latitude, respectively), with prominent plasma signatures, along
with upstream and downstream waves. Figure 1 illustrates the solar wind parameters, where the shock can be identified from the abrupt changes in their time behavior.

![Figure 1. Time history of solar wind plasma parameters and magnetic field magnitude, as recorded on board the STEREO B spacecraft between 21:00 UT and 24:00 UT on 3 October 2011. From top to bottom, the proton density $n_p$, Temperature $T$, velocity $v$, the magnetic field intensity $|B|$.](image)

In concomitance, a particle enhancement was recorded by the Solar Electron Proton Telescope (SEPT [24]), the Low Energy Telescope (LET [25]) and the High Energy Telescope (HET [26]) in the energy range $0.1 - 30$ MeV. Data used to study this event are 1 minute averaged proton fluxes measured by the three instruments\(^1\) in 39 energy differential channels. Left panel of Figure 2 shows that the event occurs on a quiet background and the intensities start to rise sharply at the shock passage, the proton peak being at 22:23 UT. The energy spectrum was derived for the time interval 22:14 - 22:31 UT around the proton peak and the best fit was performed by using the function $dJ/dE = C \times N(E) \times E^{1/2}$, which takes into into account the conversion from the particle density $N(E)$ to the differential flux, where $N(E)$ is the Weibull distribution:

$$N(E) = E^{\beta - 1} e^{-\left(\frac{E}{E_r}\right)^\beta}$$  \hspace{1cm} (1)

The obtained values for the best-fit parameters are: $C \sim 2 \times 10^5 cm^{-2}s^{-1}sr^{-1}MeV^{-1}$, $\beta = [0.50 \pm 0.07]$ and $E_r = [95 \pm 50]$ keV. Right panel of Figure 3 shows that the Weibull like function well reproduces the observed energy spectra. A very good agreement can be observed

\(^1\) Available at:
http://www.srl.caltech.edu/STEREO/index.html
http://www2.physik.uni-kiel.de/stereo/browseplots/
between the Weibull shape and the experimental data over the wide energy range 0.3 - 30 MeV spanning around two orders of magnitude. Nevertheless, at lower energies the fit departs from the actual spectrum. On the other hand, the best fit performed by using the Ellison-Ramaty form has the best agreement at lower energies ( < 10 MeV).

Figure 2. Time history of the proton flux for a selected number of energy channels, as recorded on board the STEREO B spacecraft on 3 October 2011. Black vertical lines delimit the time interval around the shock arrival (dashed line), in which the spectrum (shown in right panel) is obtained.

Figure 3. Time averaged spectrum over the time interval 22:14 - 22:31 UT on 3 October 2011. Blue (green) curve is the Weibull (Ellison-Ramaty) function used to fit the spectrum. Data errors are inside the marker size.

3. Derivation of the Weibull distribution

The Weibull distribution can be derived in the framework of stochastic acceleration. As in the majority of astrophysical particle acceleration schemes, we consider a particle population confined in an acceleration region for a characteristic time (leaky box model) and acquire energy through a succession of small increments and can therefore be treated as diffusing and/or being convected through momentum space, along with a spatial diffusion associated with these changes. This can be described through a diffusion loss equation in the presence of energy losses and injection of fresh particles from sources, also including the diffusion of particles in momentum or phase space as described by [27]. Let us assume an isotropic distribution of particles $N(E, \vec{x}, t)$, for which $4\pi^{-1}N(E, \vec{x}, t)\Delta E$ is the number of particles per unit volume and per unit solid angle (at given time $t$ and point $\vec{x}$) with energies in the range $E$ to $E + \Delta E$. The diffusion loss equation at different points in space reads:

$$\frac{\partial N}{\partial t} = D\nabla^2 N + \frac{\partial(b(E)N(E))}{\partial E} + \frac{1}{2} \frac{\partial^2 (d(E)N(E))}{\partial E^2} - \frac{N}{\tau} + Q$$

where the five terms on right-hand side account respectively for: spatial diffusion of the particles ($D$ is the scalar diffusion coefficient), the mean drift of the particles ($b(E) = -\frac{dE}{dt}$ represents the average acceleration rate), and the broadening of the energy distribution due to energy space diffusion ($d(E) = \frac{d((\Delta E)^2)}{dt}$), particles leakage from the acceleration region ($\tau(E)$ is a characteristic escape time) and supply of fresh particles from sources $Q(E, \vec{x}, t)$. We can derive a stationary solution of eq. 2 when the spatial diffusion is not considered (i.e., isotropic particle distribution), no particles sources are present and the escape time $\tau$ is independent from particle energy. Under these conditions, eq. 2 reduces to:

$$\frac{\partial(b(E)N(E))}{\partial E} - \frac{N}{\tau} + \frac{1}{2} \frac{\partial^2 (d(E)N(E))}{\partial E^2} = 0$$

(eq. 3)
This equation describes the original Fermi’s acceleration mechanism (second order) when seeking solutions of power law form [28].

Here we assume that the particles energy increases according to a power law of the time $t'$ as follows:

$$E(t') = E_\tau (\frac{t'}{\tau})^{1/\beta}$$

where the constant $E_\tau = E(\tau)$ represents the energy of a particle after a typical time $\tau$; eq. 4 can be written in terms of the dimensionless quantities $\epsilon = \frac{E}{E_\tau}$ and $t = \frac{t'}{\tau}$ as $\epsilon(t) = t^{1/\beta}$. Such assumption is quite natural within the framework of Fermi’s acceleration mechanism, where the particle velocity undergoes diffusion due to the stochastic nature of the energy gain. For instance, [29] developed a minimal stochastic model for Fermi’s acceleration mechanism and produced an acceleration process characterized by anomalous diffusion in the phase space, taking into account only the general features of the diffusion process in phase space, i.e. that a particle may absorb kinetic energy (accelerate) through collisions against moving scatterers. The relative importance of the broadening of the energy distribution and the mean drift energy gain can be estimated by the ratio $R$ between the first and third term on left-hand side of eq. 3; expressing the time growth of moments as $\langle E^n \rangle \sim t^{q(n)}$, $R$ reads:

$$R = \frac{d(E)}{b(E)} \sim \frac{\langle E^2 \rangle}{\langle E \rangle^2} \sim t^{(\xi-1)/2/\beta} \sim \epsilon^{2(\xi-1)}$$

when $\xi = q(2) - q(1)$. In the hypothesis that $q(n)$ is a convex function ($q(2) < q(1)$, $\xi < 0$), the $R$ time decrease is enough rapid that asymptotically ($E >> E_\tau$) the spreading of the energy distribution due to Fermi’s acceleration can be neglected with respect to the mean drift term, already on timescale of the order of $\tau$. Note that the required time evolution of the distribution moments was found for the tracer particle radial positions in an anomalous (superdiffusive) transport mechanism in a plasma turbulence [30]. Taking into account the above approximation eq. 3 reduces:

$$N = -\frac{1}{\beta} \frac{\partial(\epsilon^{1-\beta}N)}{\partial \epsilon}$$

A straightforward integration yields the solution:

$$N(\epsilon) = A e^{\beta-1} e^{-\epsilon^\beta}$$

where $A$ is an integration constant. Therefore the particles are found to be distributed according to a Weibull statistics. Note that in turbulent plasmas the theoretical escape time from the acceleration region due to the spatial transport is a power law in energy, that is $\tau(E) \sim E^{-\gamma}$ [31]. If we assumed such $\tau(E)$, the resulting (softer) spectrum would differ from that in eq. 7 just for the replacement $(E/E_\tau)^\beta \rightarrow \beta/((\beta + \delta)(E/E_\tau)^{\beta+\delta}$ in the exponential factor. We verified that in the present case study such a correction to Weibull’s spectrum ($\delta = 0$) results to be negligible for energies lower than several tens of MeV. Therefore, in spite of extreme simplicity, a constant escape time proves to be a reasonable approximation.

4. Wavelet analysis of the turbulent fluctuations

Let us now see if this scenario is plausible in case the acceleration region is a collisionless shock wave and if that obtained estimates of the parameters are congruent with the physical picture of the ESP event.

If the acceleration region is a collisionless shock, turbulence can provide efficient particle scattering [32, 27, 33] and it can be responsible for momentum diffusion [31, 32, 29, 34, 35]. Hence, we analyzed the variation of the magnetic fluctuations across the shock, by using the high
time resolution data (0.125 s) of the magnetic field intensity recorded by the MAG experiment on board STEREO B. Top left panel of Figure 4 depicts the time profile of the interplanetary magnetic field (IMF) intensity.

In order to understand how the turbulence spectrum varies with time, we applied the wavelet analysis (WA, [36]) to the IMF intensity, as the WA decomposes a time series into the time-frequency \((t - f)\) space. The middle panel of Figure 4 shows the Wavelet Power Spectrum \(|W(f, t)|^2\) (WPS) of the observed IMF intensity, which indicates that turbulent fluctuations are especially enhanced at the shock front at all frequencies. The variance \(\sigma^2 = \int_{f_1}^{f_2} |W(f, t)|^2\) is represented in the bottom panel of Figure 4, where it can be observed an increase of the magnetic energy at the shock, followed by a second peak still in the region (delimited by the vertical lines) of the particle peak flux. Note that the \(\sigma\) time profile is similar to the one of the particle flux as observed for the high energy channels (see left panel of Figure 2).

If we consider the Weibull parameters obtained through the fit \((\beta=0.5\) and \(E_\tau=0.95\) keV) and assume the scheme presented in section 3 to be valid, we expect from eq. 4 that the greatest observed particle energy of 30 MeV is reached after a time \(T_{\text{high}}=18\tau\). In case of nearly local acceleration, \(T_{\text{high}}\) must be of order of the time width of particle enhancement, that, in present case, is about 20 min, implying an upper limit for \(\tau\sim 1\) min. Hence, we investigate the turbulence spectrum across the shock only at small temporal scales (under \(2\tau = 2\) min), which are smaller than the confinement time and thus are the important ones for particle acceleration. Therefore we detrended the total field intensity by using the Empirical Mode Decomposition (EMD) technique [37], through which a time series \(X(t)\) is decomposed into a finite number \(m\) of oscillating modes, each of them having their own time-scale and represents a zero mean oscillation experiencing amplitude and frequency modulations. The signal can be partially reconstructed in a selected time range (in the present case \(< 2\) min) through partial sums of a proper number of modes. The magnetic field intensity reconstructed at time scales \(< 2\) min is shown in the top right panel of Figure 5.

Then we repeated the WA on the signal of magnetic field intensity reconstructed at time scales \(< 2\) min, to determine the properties of the turbulence, such as its WPS (middle right panel of Figure 5) and its variance (bottom right panel of Figure 5), in the vicinity of the shock. An enhanced turbulent energy is observed in the downstream region. Moreover, a relationship between the variation of the energetic particle intensity (see left panel of Figure 2) and the variation of the magnetic fluctuations across the shock, is apparent at \(< 2\) min scales.

5. Conclusions
The energy spectrum obtained for the 3 October 2011 ESP event was found to be reproduced by the Weibull functional form, as previously found [20, 21] in case of several SEP events, as well as in their ESP phase, at the Earth’s shock passage, and at forward shocks of two corotating interaction regions. This result confirms that the Weibull distribution can be associated with shock acceleration. We propose a theoretical derivation of the Weibull spectrum, involving the effect of stochastic acceleration at shock waves, in case of momentum anomalous diffusion. This scenario seems to be also consistent with the observation of enhanced magnetic fluctuations (as expected [38, 39, 40, 41]) around the shock front, which is assumed to be the particle acceleration region. First we note that magnetic irregularities (hydromagnetic waves, turbulence) in a moving fluid background, can produce isotropy of the particle distribution through the particle-wave interaction as well as momentum superdiffusion, so that the energy of the turbulent field in the downstream and/or the upstream regions could be efficiently transferred into particle energy through the second order Fermi acceleration. In addition, a second order acceleration process acting in the vicinity of the shock wave has been shown to be efficient [42] both in terms of energy budget and acceleration time scale, due to the presence of high amplitude MHD turbulence. Finally, [35] proposed that due to efficient momentum diffusion of particles in the
downstream region of the shock, the acceleration is dominated by the second order acceleration mechanism and the particle spectra become flatter than in the original treatment of diffusive shock acceleration. Accordingly, the derived accelerated particle distribution can be expressed as a Weibull function with $\beta < 1$, i.e. having a slower decay with respect to the power law modulated by an exponential function, proposed by Ellison and Ramaty to reproduce the roll over at high energies in the frame of the diffusive shock acceleration. In particular, the considered ESP spectrum is better reproduced by the Weibull function at high energies (greater than $E_\tau$), and by the Ellison-Ramaty one at lower energies.

Both the spectrum and the flux time profile of the considered ESP event suggest that diffusive shock acceleration and stochastic acceleration should be taken into account simultaneously. First, we note that the acceleration time scale (< 2 min) derived from our interpretation of the Weibull parameters is comparable with that expected from DSA. Moreover, the energetic particle flux profiles indicate that they are diffusing across the shock. Therefore, it is natural to assume they experience energy gain by interaction with the shock as well. In particular, the particle profiles at low energy are consistent with those expected from DSA, while at high energies seem to be related to the variations of the magnetic energy, represented by the variance of the power spectrum of the magnetic field intensity. Hence, stochastic acceleration seems to give a significant contribution at high energies.

We conclude that the stochastic acceleration of protons in the downstream region of a near-perpendicular shock can be an additional mechanism for producing the observed particle energy spectra associated with interplanetary shocks. We suggest that stochastic acceleration may act as a re-acceleration of energetic protons by enhanced turbulence in the downstream region of the shock, as proposed by [19] for relativistic energies in case of GLE events. Further studies about the microphysical processes in the shock region are needed, e.g., the turbulence level, the nature and geometry of small-scale dissipative structures of turbulence, which can be complicated and possibly fractal, and how they define the particle diffusion regimes [44].
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