Optimal Private Payoff Manipulation against Commitment in Extensive-form Games

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Abstract

To take advantage of strategy commitment, a useful tactic of playing games, a leader must learn enough information about the follower’s payoff function. However, this leaves the follower a chance to provide fake information and influence the final game outcome. Through a carefully contrived payoff function misreported to the learning leader, the follower may induce an outcome that benefits him more, compared to the ones when he truthfully behaves.

We study the follower’s optimal manipulation via such strategic behaviors in extensive-form games. Followers’ different attitudes are taken into account. An optimistic follower maximizes his true utility among all game outcomes that can be induced by some payoff function. A pessimistic follower only considers misreporting payoff functions that induce a unique game outcome. For all the settings considered in this paper, we characterize all the possible game outcomes that can be induced successfully. We show that it is polynomial-time tractable for the follower to find the optimal way of misreporting his private payoff information. Our work completely resolves this follower’s optimal manipulation problem on an extensive-form game tree.
1 Introduction

In extensive-form games, game trees capture the sequential interaction among players. While the interaction structure of a game is predetermined, a player may still be able to commit to a strategy first at every turn of her move rather than choose strategies simultaneously. This makes her the “conceptual leader” and potentially gain more utility. Such an idea is originally captured in von Stackelberg (1934). With the prediction that the follower will best-respond to the commitment, the leader can commit to a strategy that optimizes her own utility. The pair of a leader’s optimal commitment and a follower’s corresponding best response is called strong Stackelberg equilibrium (SSE), with the assumption that the follower breaks ties in favor of the leader to assure its existence (Letchford and Conitzer, 2010).

Generally, the leader can gain potentially more in SSE than in Nash Equilibrium (NE), since she can always choose her NE strategy for the follower to best-respond to, so as to reach NE as a feasible Stackelberg solution. Due to its modelling of conceptually asymmetric roles, SSE in extensive-form games has inspired a wide range of applications, such as modelling sequential defenses and attacks in security games (Nguyen et al., 2019b), addressing execution uncertainties in airport securities (Fave et al., 2014), and so on (Sinha et al., 2018; Cooper et al., 2019; Durkota et al., 2019).

However, to achieve this “first-mover” advantage, the leader needs to have enough knowledge about the follower’s payoff function. Various learning methods have been proposed to elicit the follower’s payoff information through interaction (Letchford et al., 2009; Blum et al., 2014; Haghtalab et al., 2016; Roth et al., 2016; Peng et al., 2019). A practical example is big data discriminatory pricing, as a result of learning application to the economics. According to different prices, consumers will best-respond by buying or not buying. Then platforms can construct user profiles via data collection, and set specialized prices for the same products to different customers (Mahdawi, 2018; Chang et al., 2021).

Such learning process then leaves the follower a chance to misreport his private payoff information. He can strategically act as possessing an alternative payoff function and induce a new game. This may improve his real utility on the SSE of the newly induced game. An example is shown in Figure 1 where the leader commits to a pure strategy. When the follower reports his real payoff function, he gets a value of 3 on the SSE of the game, as shown in Figure 1a. When he reports a payoff function shown in Figure 1b, his real utility rises to 4 on the SSE of the induced game.

Back to the above practical example, there are evidences that customers are aware of the price discrimination and beginning their manipulation, like changing their credit cards (Mahdawi, 2018). We note that even though the leader is fully rational, she might temporarily fail to realize such strategic behaviors. Such cases may happen when the leader is playing games with huge amounts of followers simultaneously, e.g., games between e-commerce platforms and consumers. And a rational follower would definitely seize and benefit from this chance. Actually, as will be further illustrated in the related work, the learning and manipulation of private information has shown increasing interests in both learning and game theory communities.

In this paper, we study how a follower optimally misleads the leader through strategic reporting. Specifically, we model the whole process into two phases: in the first phase, the follower reports a payoff function to the leader, which is a simplification of leader’s learning follower’s payoff information through interaction; in the second phase, the leader computes an SSE in an extensive-form game with perfect information, according to a predetermined game tree, the leader’s publicly-known payoff function and the follower’s reported payoff function.

Two settings of commitment are studied: we first consider the setting where the leader only commits to a pure strategy. We then proceed to the more general and complicated setting where the leader commits to a behavioral strategy. We will regard a probability distribution over leaf nodes of the game tree as the game outcome that the follower aims to finally induce through strategic reporting, because of that 1) different strategy profiles may lead to the same distribution; and that 2) it is the distribution that plays a decisive role in calculating the players’ expected utilities.
When the follower acts according to his true function $U_F$.

(b) When the follower acts according to $\tilde{U}_F$.

Figure 1: An example how the follower can gain more utility by strategically reporting an alternative payoff function, when the leader only commits to pure strategies. The follower acts at the root node, while the leader acts at other internal nodes. The first row below the game tree shows the leader’s payoff at each leaf node, while the second row shows the follower’s. Blue arrows denote the SSEs in the games. Fig. 1a shows results when the follower reports his true payoff function $U_F$, while Fig. 1b shows that when he reports a payoff function $\tilde{U}_F$. He gains 1 more utility by strategic reporting.

For both settings, we develop four key results as follows:

1. We characterize all the probability distributions that can be led to by one SSE of an induced game, made up of the leader’s true payoff function and a follower’s proposed payoff function, as called inducibility (Theorem 3.5, Theorem 4.3).

2. We propose polynomial-time algorithms for the follower to find an optimal distribution that yields his highest utility among all the inducible ones, and construct a follower’s payoff function that induces this distribution (Theorem 3.7, Theorem 4.9).

3. Ultimately, the bar on the follower’s strategic reporting is raised: we characterize all the probability distributions that there is a way of reporting to make it the unique outcome, led to by all induced SSEs (Theorem 5.2, Theorem 5.3), as called strong inducibility. We give polynomial-time algorithms to find an optimal distribution for the follower or a near-optimal distribution when the optimal one does not exist (Theorem 5.4, Theorem 5.5).

4. We compare the optimal utilities in one game that a follower can get among different settings, especially those under inducible and strongly inducible distributions, respectively: we fully characterize the games where these two values are equal (Theorem 5.7). We say such games satisfy the property Utility Supremum Equivalence.

Figure 2 summarizes our main results by their relationships with different settings. In more detail, under the inducibility and pure commitment case, we show that a distribution can be successfully induced so long as the leader’s utility is no less than her max-min value. Beyond this setting, max-min value is not enough for such characterization, due to the best responses’ subgame perfect properties on extensive-form games. While conditions are becoming stricter and more complicated, we show that the follower can gain no less utility under behavioral commitment case than under pure commitment case. Algorithmically, for the inducibility and pure commitment case, the time complexities of the above algorithms are optimal.

**Technique Overview**

Our technique exploits the sequential structure of extensive-form games, modeled by game trees.

First, we abstract from strategy profiles and treat probability distributions on leaf nodes of game trees as the game outcomes the follower aims to induce via strategic reporting: While SSE are defined
by strategy profiles, multiple strategy profiles can lead to the same distribution over leaf nodes. It is the latter that really influences players’ expected utilities, and thus is what players really care. To realize a desired distribution, the easiest-to-analyze way for the leader is to commit to minimax strategies on the subgames she does not want the follower to choose. However, there can be other commitments that is hard to describe but also lead to the same outcome. These nonessentials, as we will show, can be ignored by focusing on distributions instead of strategy profiles. In particular, this greatly simplifies our algorithms for finding optimal (strongly-)inducible distributions.

Second, we use recursive conditions on game trees to avoid non-convexity issues: when we analyze the leader’s optimal commitment (when payoff functions are known) and the follower’s optimal strategic reporting, consider the sets of all possible distributions that can be achieved by commitments and induced by strategic reporting, respectively. Since the follower always best-responds with pure strategies, these sets are not convex. Thus, traditional convex programming approaches, e.g., linear programming, are not applicable in our problem. Instead, based on the sequential structure and the subgame perfect property of best responses, specially owned by extensive-form games, we are able to characterize the aforementioned sets and design algorithms with recursion.

Third, we successfully identify a subclass of “succinct” distributions on leaf nodes, whose support size is at most 2: when we want to design algorithms for optimal deception or to find a sufficient and necessary condition whether a distribution can be uniquely induced, this subclass of “succinct” distributions are easier to analyze, and maintain the same supremum properties with the universal set. That is, for each distribution that can be (uniquely) induced, one can always find a member of this subclass, that is also (uniquely) induced and yields no less utilities for both players. With this discovery, we are able to reduce an optimization problem among arbitrary distributions to that among a clearer-to-analyze subclass of distributions, design algorithms for behavioral commitment settings and characterize games that achieve “Utility Supremum Equivalence” property under strong inducibility.

Other Related Work

There is an increasing interest in the learning and manipulation of private information. For example, to commit to an optimal auction that maximizes revenue, the seller needs to know the buyers’ value distributions and thus how buyers will best-respond to the mechanism (Myerson, 1981). Since the distributions are actually the buyers’ private information, they may benefit from pretending a different one by manipulating the bid data (Tang and Zeng, 2018; Deng et al., 2020; Chen et al., 2022). In this paper, the private information is the follower’s payoff function. Our work also relates to the adversarial learning in the machine learning community (Lowd and Meek, 2005; Barreno et al., 2010), which considers the problem that the training data may be manipulated maliciously, leading to inaccurate prediction. Thorough studies on extensive-form games with perfect information would help design more algorithms robust to such data manipulation behavior.

Previous work on the follower’s strategic reporting on SSE mainly focuses on normal-form games
and security games. As for the normal-form games, Gan et al. (2019b) studied the case where the follower chooses from a finite set of payoff functions to disguise. Birmpas et al. (2021) considered the case where the follower can pretend any payoff function. The characterization condition in their work is also maximin value, which coincides with our results on pure commitment and inducibility setting. However, maximin value is not enough in other settings considered in our paper, which attributes to the sequential structures of extensive-form games.

Although any extensive-form game can be transferred into a normal-form game, results in Birmpas et al. (2021) cannot directly be applied to ours. First, such transformation has an exponential blow-up in size. Second, compared to normal-form games, the sets of all game outcomes that can be induced are not even convex, thus linear programming used in Birmpas et al. (2021) fails in our case, and new techniques are needed.

Security games are a special case of Stackelberg games with compact representation, where the defender (the leader) allocates security resources to several targets and the attacker (the follower) chooses one to attack. Gan et al. (2019a) considered the case where the attacker is able to imitate a different type, while Nguyen and Xu (2019) studied the case where the attacker can imitate any payoff function. Their results on ways of imitation show similarities of ours, while totally different techniques are used due to different game structures. Nguyen et al. (2019a) considered the case in which the attacker with dynamic payoffs can manipulate attacks in repeated security games. Nguyen et al. (2020) took the different bounded rationality level of the attacker into consideration. Other studies on the asymmetric information in security games considered how the defenders manipulate when they possess more information (Yin et al., 2013; Xu et al., 2015; Rabinovich et al., 2015; Guo et al., 2017; Schlenker et al., 2018).

We note that the Stackelberg equilibria on normal-form games and security games can be remodeled as the subgame perfect equilibria (SPE) of a two-layer extensive-form games with perfect information. In this game, the leader acts at root node and her action space there is all her mixed strategies, which is infinite. Our results do not necessarily imply theirs, as we focus on games where the players’ action space at each node is finite.

Paper Organization

Section 2 introduces the preliminaries needed throughout this paper. Section 3 presents the results of the case when the leader commits to pure strategies. Section 4 proceeds to the case when the leader commits to behavioral strategies. Section 5 provides the results on studying outcomes that can be uniquely induced. Section 6 summarizes. All omitted proofs are in the appendix.

2 Preliminaries

We consider two-player extensive-form games with perfect information on the second phase:

**Definition 2.1 (Extensive-form Game).** A two-player extensive-form game with perfect information is a tuple \((T_{root}, N, P, \{U_i\}_{i \in N})\), where

- \(T_{root} = (H \cup Z; \text{Child})\) is a rooted game tree, in which
  - \(\text{root}\) is the root node;
  - \(H\) is the set of decision nodes (internal nodes);
  - \(Z\) is the set of terminal nodes (leaf nodes);
  - \(\text{Child} : H \rightarrow 2^{H \cup Z}\) assigns each internal node \(v \in H\) the set of children nodes of \(v\), i.e., the set of available actions at \(v\);
\( N = \{L, F\} \) is the set of players. \( L \) (\( F \)) denotes the Conceptual Leader (Follower, respectively).

\( \mathcal{P} : H \rightarrow N \) assigns each internal node \( v \in H \) the unique player to make decision at \( v \).

\( U_i : Z \rightarrow \mathbb{R} \) is the payoff function of player \( i \in N \).

Since the players \( N \) and assignment \( \mathcal{P} \) keeps the same throughout our analysis, we simply denote an extensive-form game by \((T_{\text{root}}, U_L, U_F)\). Let \( \mathcal{P}^{-1}(i) = \{v \in H : \mathcal{P}(v) = i\} \) be the set of decision nodes designated to player \( i \in \{L, F\} \). A pure strategy \( \pi_i \in \Pi_i := \prod_{v \in \mathcal{P}^{-1}(i)} \text{Child}(v) \) chooses exactly one action at each node for player \( i \) to act. A behavioral strategy \( \delta_i \in \Delta_i := \prod_{v \in \mathcal{P}^{-1}(i)} \Delta(\text{Child}(v)) \) chooses a probability distribution over the action set \( \text{Child}(v) \) at each node \( v \in \mathcal{P}^{-1}(i) \). Note that a pure strategy is also a behavioral strategy.

Define player \( i \)'s utility under strategy profile \((\delta_L, \delta_F)\) to be
\[
U_i(\delta_L, \delta_F) := \sum_{z \in Z} U_i(z) p(\delta_L, \delta_F)(z)
\]
where \( p(\delta_L, \delta_F)(z) \) denotes the probability that the game ends in \( z \) under \((\delta_L, \delta_F)\). Then we can define the best response set of the follower to be\(^1\)
\[
\text{BR}(\delta_L) = \arg\max_{\pi_F \in \Pi_F} U_F(\delta_L, \pi_F).
\]

The key solution concept of our paper is the strong Stackelberg equilibrium (SSE). We consider two settings in extensive-form games. In one setting, the leader is only allowed to commit to pure strategies. Then we proceed to the general setting when the leader commits to behavioral strategies.

**Definition 2.2 (Strong Stackelberg Equilibrium).** Given a game \((T_{\text{root}}, U_L, U_F)\),

A strong Stackelberg equilibrium with pure commitment is defined as
\[
(\pi_L, \pi_F) \in \arg\max_{\pi'_L \in \Pi_L, \pi'_F \in \text{BR}(\pi'_L)} U_L(\pi'_L, \pi'_F);
\]

A strong Stackelberg equilibrium with behavioral commitment is defined as
\[
(\delta_L, \pi_F) \in \arg\max_{\delta'_L \in \Delta_L, \pi'_F \in \text{BR}(\delta'_L)} U_L(\delta'_L, \pi'_F).
\]

We note that the definitions of SSEs imply that the follower breaks ties in favor of the leader, and then their existence in these two settings is guaranteed (Letchford and Conitzer, 2010). An example in Figure 3 shows that, in contrast to SPEs in games with perfect information, the leader can gain more by committing to behavioral strategies in SSEs: when the leader only commits to pure strategies, the best utility she can get is 2, while she can get expected utility of 2.8 by behavioral commitment.

**Equilibrium manipulation via strategic reporting** While any strategy profile \((\delta_L, \pi_F)\) realizes a unique distribution over leaf nodes \( p(\delta_L, \pi_F) \in \Delta(Z) \), a probability distribution may correspond to zero or multiple strategy profiles, as the follower always best-responds with pure strategies. We call a distribution realizable if it corresponds to at least one strategy profile and give it a characterization in Section 4.2. We will simply call \((\delta_L, \pi_F)\) a strategy profile of distribution \( p \in \Delta(Z) \) if \( p(\delta_L, \pi_F) = p \).

Since players’ expected utilities are actually determined by these distributions, we will study all distributions that the follower can report a payoff function and induce an SSE leading to it. We use “inducibility”, originally proposed in Birmpas et al. (2021), to describe such distributions.

\(^1\)Since there must be a pure strategy best response for any leader’s (pure or behavioral) commitment, we assume the follower always chooses a pure strategy best response (Letchford and Conitzer, 2010).
We characterize all the inducible leaf nodes with pure commitment. To put it simply, a leaf node is inducible if and only if the leader’s utility at the node is at least as much as the maximin value of the node. Specifically, if \( \delta_L(\pi) \) is an SSE with pure commitment, then \( \pi \) is inducible, if it satisfies

\[
\forall z \in Z, \quad U(z, \pi)|_{Z \setminus \{z\}} \geq U(z, \pi)\bigg|_{\{z\}}
\]

We call such a \( \pi \) a function defined on a subset \( S \subseteq (H \cup Z)_v \) for node \( v \in H \). For \( v \in \text{Child}(v) \), define its restriction on subtree \( T_w \) of \( T_v \), \( f|_{T_w} \), to be:

\[
\text{Dom } f|_{T_w} = S \cap (H \cup Z)_w, \\
f|_{T_w}(x) = f|_v(x), \quad \forall x \in S \cap (H \cup Z)_w
\]

When \( f = \text{BR} \), to simplify notations, noticing that any strategy profile defined on a subtree can be regarded as the restriction of one defined on the whole game tree, we will only use \( \pi_F \in \text{BR}(\delta_L)|_v \) to denote that \( \pi_F|_v \) is a best response of \( \delta_L|_v \) at subtree \( T_v \). We call such \( (\delta_L, \pi_F)|_v \) feasible in game \((T_v, U_L|_v, U_F|_v)\).

We will omit the \( U_i \)’s and simply say a distribution (strategy profile) is inducible (feasible, respectively) at \( T_v \) when there is no confusion.

### 3 Optimally Reporting under Pure Commitment

This section presents results of the follower’s strategic reporting when the leader only commits to pure strategies. When \( \delta_L \) is a pure strategy \( \pi_L, \delta_L, \pi_F \) leads to a distribution that put all probability on one leaf node and is equivalent to the one \( z \in Z \) that \( p(\pi_L, \pi_F)(z) = 1 \). Thus we will directly talk about the inducibility of leaf nodes. We first give some important properties for pure commitment with respect to the maximin values in Section 3.1. We characterize all the inducible leaf nodes with pure commitment in Section 3.2. To put it simply, a leaf node is inducible if and only if the leader’s utility is no less than her maximin value. We then give a linear-time algorithm to find an optimal inducible leaf node and construct a corresponding follower’s payoff function in Section 3.3.
3.1 Relating the Maximin Value with Pure Commitments

We first define the maximin value of an extensive-form game as follows.

**Definition 3.1 (Mi).** Let \( v \in H \cup Z \). We define \( M_i(v) \) for \( i \in N \) recursively as follows:

\[
M_i(v) = \begin{cases} 
U_i(v), & v \in Z; \\
\max_{w \in \text{Child}(v)} M_i(w), & v \in P^{-1}(i); \\
\min_{w \in \text{Child}(v)} M_i(w), & v \in P^{-1}(-i).
\end{cases}
\]

We relate \( M_i(v) \) to values players can get by leader’s optimal commitment. Specifically, Proposition 3.2 shows that \( M_i(\text{root}) \) is the least value players can guarantee themselves on SSEs.

**Proposition 3.2.** Given a leaf node \( z \in Z \), if there exists an SSE with pure commitment of game \((T_{\text{root}}, U_L, U_F)\) leading to \( z \), then \( U_L(z) \geq M_L(\text{root}) \) and \( U_F(z) \geq M_F(\text{root}) \).

The following Proposition 3.3 shows that the value of an SSE in a zero-sum extensive-form game, i.e., the optimal value the leader can get by committing, is equal to \( M_L(\text{root}) \).

**Proposition 3.3.** Given a zero-sum extensive-form game \((T_{\text{root}}, U_L, U_F = -U_L)\), we have

\[
\max_{\pi_L \in \Pi_L, \pi_F \in \text{BR}(\pi_L)} U_L(\pi_L, \pi_F) = M_L(\text{root}).
\]

**Proposition 3.4** characterizes leaf nodes that the leader can lead the game to via pure commitment.

**Proposition 3.4.** For leaf node \( z \in Z \) in game \((T_{\text{root}}, U_L, U_F)\), there exists a feasible strategy profile \((\pi_L, \pi_F \in \text{BR}(\pi_L))\) leading to \( z \) if and only if \( U_F(z) \geq M_F(v) \) for every \( v \in P^{-1}(F) \) that is on the path from \( \text{root} \) to \( z \).

3.2 Leader’s Maximin Value Yields all Inducible Leaf Nodes

In this section, we first characterize all inducible leaf nodes with pure commitment.

**Theorem 3.5.** Leaf node \( z \in Z \) is inducible with pure commitment if and only if \( U_L(z) \geq M_L(\text{root}) \).

Comparing our results with the characterization of all feasible pure strategy profiles in Proposition 3.4, one can find out that with the ability of exploiting his private utility information, the follower has the actual advantage when the leader only commits to pure strategies. Moreover, he gains more power through strategically reporting than committing: To induce a leaf node, the follower only needs to check the leader’s utility at \( \text{root} \), while to check if it can be achieved by commitment, the leader has to consider the follower’s maximin values in every subgame with positive probabilities to be played. While this comparison of power is not that obvious in the behavioral commitment case, as we will show in Section 4.6, the follower can generally gain more under behavioral commitment case than under pure case, via misreporting optimally.

**Proof of Theorem 3.5.** The necessity is immediately from Proposition 3.2. A leaf node \( z \) is inducible means that for some payoff function, \( \hat{U}_F \), an SSE with pure commitment of game \((T_{\text{root}}, U_L, \hat{U}_F)\) leads to \( z \). Thus \( U_L(z) \geq M_L(\text{root}) \). We prove the sufficiency by structural induction over the game tree.

**Inductive Base:** When \( \text{root} \in Z \), the only leaf node in \( Z \) is \( \text{root} \), then \( U_L(\text{root}) = M_L(\text{root}) \).

Since the strategy set of \( T_{\text{root}} \) is empty, \( \text{root} \) is inducible with pure commitment at \( T_{\text{root}} \).

**Inductive Step:** When \( \text{root} \in H \), suppose that leaf node \( z \in Z \) satisfies \( U_L(z) \geq M_L(\text{root}) \).

When \( P(\text{root}) = L \), \( U_L(z) \geq M_L(\text{root}) = \max_{v \in \text{Child}(\text{root})} M_L(v) \). Let \( v \in \text{Child}(\text{root}) \) be \( z \)'s ancestor, then \( U_L(z) \geq M_L(v) \). So \( z \) is inducible at \( T_v \) by the inductive hypothesis. Let \( \hat{U}_F|_v \) be
a payoff function defined on $T_v$ that an SSE of subgame $(T_v, U_L|_{v}, \tilde{U}_F|_{v})$ leads to $z$, and extend it to $T_{\text{root}}$ as follows:

$$\tilde{U}_F(z') = \begin{cases} 
\tilde{U}_F|_{v}(z') & z' \in Z_v \\
-U_L(z') & z' \in Z \setminus Z_v.
\end{cases}$$

We prove that there is an SSE with pure commitment of game $(T_{\text{root}}, U_L, \tilde{U}_F)$ leading to $z$. Consider any pure commitment $\pi_L \in \Pi_L$ of the leader. If $\pi_L(\text{root}) = v$, since an SSE of $(T_v, U_L|_{v}, \tilde{U}_F|_{v})$ leads to $z$, the leader’s utility by commitment is upper bounded by $U_L(z)$. The leader can simply choose the same commitment in the SSE of $(T_v, U_L|_{v}, \tilde{U}_F|_{v})$ that leads to $z$ and get $U_L(z)$. If $\pi_L(\text{root}) = v' \neq v$, then by Proposition 3.3, the leader’s utility is upper bounded by

$$\max_{\pi_L \in \Pi_L, \pi_F \in \text{BR}(\pi_L)|_{v'}} U_L(\pi_L, \pi_F)|_{v'} = M_L(v') \leq U_L(z).$$

So there is an SSE with pure commitment in $(T_{\text{root}}, U_L, \tilde{U}_F)$ leading to $z$.

When $P(\text{root}) = F$, let $v_1 \in \text{Child}(\text{root})$ be the ancestor of $z$ and $v_2 \in \arg\min_{v' \in \text{Child}(\text{root})} M_L(v')$. Note that $v_1$ may equal $v_2$ and we first consider the case $v_1 \neq v_2$. Construct $\tilde{U}_F$ as follows:

$$\tilde{U}_F(z') = \begin{cases} 
-M_L(v_2) + 1 & z' = z; \\
-M_L(v_2) - 1 & z' \notin Z_{v_2} \land z' \neq z; \\
-U_L(z') & z' \in Z_{v_2}.
\end{cases}$$

(2)

For any pure commitment $\pi_L \in \Pi_L$ of the leader, if the follower cannot achieve $z$ under $\pi_L$, he will choose $v_2$ at root as his best response, obtaining a utility of at least $-M_L(v_2)$. Then the leader’s utility is upper bounded by $M_L(v_2) \leq U_L(z)$ by Proposition 3.3. If the follower can achieve $z$ under $\pi_L$, his best response is to achieve $z$ and get $-M_L(v_2) + 1$. The leader then gets $U_L(z)$, which is the best she can get. Thus there is an SSE with pure commitment of $(T_{\text{root}}, U_L, \tilde{U}_F)$ leading to $z$.

If $v_1 = v_2$, $U_L(z) \geq M_L(v_1)$ holds, then $z$ is inducible at $T_{v_1}$ by the inductive hypothesis. Let $\tilde{U}_F|_{v_1}$ be a payoff function that induces it at $T_{v_1}$.

Define $\tilde{m}_F(v_1) = \min_{z' \in Z_{v_1}} \tilde{U}_F|_{v_1}(z')$, and extend $\tilde{U}_F|_{v_1}$ to $T_{\text{root}}$ as follows:

$$\tilde{U}_F(z') = \begin{cases} 
\tilde{U}_F|_{v_1}(z') & z' \in Z_{v_1}; \\
\tilde{m}_F(v_1) - 1 & z' \in Z \setminus Z_{v_1}.
\end{cases}$$

(3)

For whatever strategy the leader commits to, the follower will always choose $v_1$ at root, and get at least $\tilde{m}_F(v_1) > \tilde{m}_F(v_1) - 1$. Then what matters is the strategies’ restrictions on $T_{v_1}$. Since an SSE in subgame $(T_{v_1}, U_L|_{v_1}, \tilde{U}_F|_{v_1})$ leads to $z$, there is also an SSE in game $(T_{\text{root}}, U_L, \tilde{U}_F)$ leading to $z$.

This finishes the proof. \hfill \Box

We note that our proof of Theorem 3.5 is constructive. With a slight modification, we can actually construct a payoff function in time $O(|H| + |Z|)$ for any inducible leaf node.

**Corollary 3.6.** For any inducible leaf node $z \in Z$, we can construct a payoff function $\tilde{U}_F$ that induces $z$ in time $O(|H| + |Z|)$.

### 3.3 Algorithm for Optimally Reporting

With the characterization of all inducible leaf nodes (Theorem 3.5), we can compute an optimal inducible leaf node and its follower’s payoff function in linear time$^2$: compute $M_L(\text{root})$ and traverse all leaf nodes that satisfy conditions in Theorem 3.5.

$^2$In this paper, we will use random access machine model to determine the algorithms’ time complexities.
Theorem 3.7. Computing an inducible leaf node $z^*$ and a follower’s payoff function $\tilde{U}_F^*$ such that

- a strategy profile $(\pi_L, \pi_F)$ of $z^*$ is an SSE with pure commitment of game $(T_{\text{root}}, U_L, \tilde{U}_F^*)$;
- $U_F(z^*) \geq U_F(z)$ for all inducible leaf node $z$;

can be solved in time $\Theta(|H| + |Z|)$.

4 Optimally Reporting under Behavioral Commitment

This section presents results of the follower’s strategic reporting when the leader commits to behavioral strategies. Due to the game tree structure of extensive-form games, the choice of strategies at a node is closely related to players’ utilities in the subgames. Thus maximin value on root is not enough for characterizing all inducible distributions. We show the characterization in Theorem 4.3. Its constructive proof provides an efficient way to find a follower’s payoff function for an inducible distribution.

We note that the proof of Theorem 3.5 cannot be directly applied to the behavioral case. When the leader acts at the root node, different from pure commitment case, $U_L(\delta_L, \pi_F)$ is a convex combination of $U_L(\delta_L, \pi_F)|_v$ for $v \in \text{Child}(\text{root})$. There is no reason for the leader to put positive probabilities on those nodes that lead to less utilities. Thus the conditions no longer hold for the case $P(\text{root}) = L$, and neither does the inductive analysis when $P(\text{root}) = F$.

We use two examples in Figure 4 to further illustrate the necessities of additional conditions in Theorem 4.3. In each example, a strategy profile of a distribution is presented with red arrows. Both distributions satisfy conditions in Theorem 3.5 but are not inducible in the behavioral case.

![Figure 4](image-url)

(a) Example 1 for the necessities of Condition (1)(a). (b) Example 2 for the necessities of condition (2)(b).

Figure 4: Two examples illustrating the necessities of additional conditions in Theorem 4.3. Red arrows in each subfigure represent a strategy profile of a distribution $p$ that satisfies the conditions in Theorem 3.5 but is not inducible with behavioral commitment. That is, $U_L(p) \geq M_L(\text{root})$. As for the distribution and the game shown in Fig. 4a, $p(z_1) = p(z_3) = \frac{1}{2}$. When the leader acts at root, going left with probability 1 is always a dominant strategy. There is no need for her to commit to strategies which are mixed at root, although she can get utilities greater than $M_L(\text{root}) = 4$. In Fig. 4b, $p(z_3) = p(z_5) = \frac{1}{2}$. For any payoff function $\tilde{U}_F$ that makes the red strategy profile feasible, by Lemma 4.4, going to $z_3$ at $F_2$ is the follower’s best response at subgame $T_{F_2}$. Then by going left at $L_1$ and $L_2$, the leader can get $8 > 5$, and the red strategy profile will never be an SSE no matter the follower’s reported payoff function.

We provide a polynomial-time algorithm to find an optimal inducible distribution with behavioral commitment in Section 4.5. Actually, while the characterization conditions are complicated, Theorem 4.3 helps us find optimal inducible distributions with succinct structure, which we call “Y-shape” as defined in Definition 4.6. Furthermore, we show that the follower can gain no less utility by strategic reporting with behavioral commitment than with pure commitment in Section 4.6.
4.1 Additional Notations

Given distribution \( p \in \Delta(Z) \), define \( \text{Supp}(p) := \{ z \in Z : p(z) > 0 \} \). With a slight abuse of notation, we recursively extend it onto tree \( T_{\text{root}} \) as follows,

\[
p(v) = \begin{cases} 
p(v), & v \in Z; \\
\sum_{w \in \text{Child}(v)} p(w), & v \in H.
\end{cases}
\]  

(4)

For each \( v \in H \cup Z \), \( p(v) \) denotes the probability of reaching \( v \) from \( \text{root} \) to realize \( p \) on \( Z \).

For the incoming inductive proofs, we define the restriction of \( p \) on subtree \( T_v \), for \( v \) with \( p(v) > 0 \), to be \( p|_v \in \Delta(Z_v) \) and \( p|_v(z) := p(z)/p(v), \forall z \in Z_v \). When \( v = \text{root} \), we simply use \( p \) instead of \( p|_\text{root} \). The notions of inducibility and utility on subgame \( (T_v, U_{|L_v}, U_{|F_v}) \) are defined similarly.

For a distribution \( p|_v \in \Delta(Z_v) \) and \( w \in H_v \), we use

\[
\text{Supp}(p|_v, w) := \{ x \in \text{Child}(w) : p|_v(x) > 0 \}
\]

to denote the set of \( w \)'s children that are reached with positive probabilities from \( v \) to realize \( p|_v \).

For a strategy profile \( (\delta_L, \pi_F)|_v \) on subtree \( T_v \), we use \( p(\delta_L, \pi_F)|_v \) to denote the its distribution on \( Z_v \). Similarly, for \( w \in H_v \), we use

\[
\text{Supp}(\delta_L, \pi_F)|_v, w) := \{ x \in \text{Child}(w) : p(\delta_L, \pi_F)|_v(x) > 0 \}
\]

to denote the set of \( w \)'s children that are reached with positive probabilities from \( v \) under \( (\delta_L, \pi_F)|_v \).

4.2 Relating the Maximin Value with Behavioral Commitments

We extend results on properties for pure commitments in Section 3.1 to behavioral commitments.

**Proposition 4.1.** If strategy profile \( (\delta_L, \pi_F) \) is an SSE with behavioral commitment of game \( (T_{\text{root}}, U_{|L}, U_{|F}) \), then \( U_{|L}(\delta_L, \pi_F) \geq M_L(\text{root}) \) and \( U_{|F}(\delta_L, \pi_F) \geq M_F(\text{root}) \).

The proof of Proposition 4.1 is similar to that of Proposition 3.2. Just notice that the forms of the follower’s strategies minimizing the leader’s utility or maximizing his own utility, given the leader’s strategy, do not depend on whether the leader’s strategy is pure or behavioral: given the utilities players can get in the subgames, choosing the node with the leader’s lowest utility (the follower’s highest utility, respectively) is always the correct action.

Using similar proof to the one of Proposition 3.3, we have the following Proposition 4.2, which shows that the leader gains no advantage in committing to behavioral strategies in zero-sum games.

**Proposition 4.2.** Given a zero-sum extensive-form game \( (T_{\text{root}}, U_{|L}, U_{|F} = -U_{|L}) \), we have

\[
\max_{\delta_L \in \Delta_L, \pi_F \in \text{BR}(\delta_L)} U_{|L}(\delta_L, \pi_F) = M_L(\text{root}).
\]

We also note the following simple fact:

**Fact 1.** \( p \in \Delta(Z) \) is realizable if and only if \( |\text{Supp}(p, v)| = 1 \), for every \( v \in \mathcal{P}^{-1}(F) \) with \( p(v) > 0 \).

4.3 Characterizing all Inducible Distributions by Subtree Recursion

The characterization of all inducible distributions under behavioral commitment is more complicated than that under pure commitment. For \( \text{root} \in Z \), the game tree only has one terminal node and the only distribution \( p \) that \( p(\text{root}) = 1 \) is inducible. For general extensive-form games, we have the following theorem.
Theorem 4.3. A realizable distribution on leaf nodes \( p \in \Delta(Z) \) is inducible if and only if

1. if \( \mathcal{P}(\text{root}) = L \), all of the following conditions are met:
   (a) \( U_L(p)|_{v,S} \) are the same at \( T_v \) for all \( v \in \text{Supp}(p, \text{root}) \);
   (b) \( U_L(p) \geq M_L(\text{root}) \);
   (c) \( p|_v \) is inducible at \( T_v \), for all \( v \in \text{Supp}(p, \text{root}) \).

2. if \( \mathcal{P}(\text{root}) = F \), let \( \text{Supp}(p, \text{root}) = \{v\} \), then at least one of following two conditions are met:
   (a) \( U_L(p) \geq \min_{v' \in \text{Child(\text{root})), v' \neq M_L(v')} \);  
   (b) \( p|_v \) is inducible at \( T_v \).

We note that condition (1)(a) in Theorem 4.3 deals with the case that the leader may mix at \( \text{root} \) among several children nodes that yield same utilities for her. Recall that a strategy profile \( (\delta_L, \pi_F)|_v \) is feasible with respect to \( (T_v, U_F|_v) \) if \( \pi_F \in \text{BR}(\delta_L)|_v \) and we simply say it is feasible at \( T_v \) when there is no confusion. Before proving Theorem 4.3, we present a necessary lemma on the properties of feasible strategy profiles.

Lemma 4.4. Given strategy profile \( (\delta_L, \pi_F)|_v \), defined on subgame \( (T_v, U_L|_v, U_F|_v) \) for \( v \in H \):

1. If \( (\delta_L, \pi_F)|_w \) is feasible at \( T_w \), \( (\delta_L, \pi_F)|_w \) is also feasible at \( T_w \), for any \( w \in \text{Supp}(\delta_L, \pi_F)|_v \).
2. If \( (\delta_L, \pi_F)|_w \) is feasible at \( T_w \), for any \( w \in \text{Supp}(\delta_L, \pi_F)|_v \), then \( (\delta_L, \pi_F)|_v \) is feasible at \( T_v \).
3. If \( (\delta_L, \pi_F)|_v \) is feasible and \( U_L(\delta_L, \pi_F)|_v = \max_{\pi_F' \in \text{BR}(\delta_L)} U_L(\delta_L, \pi'_F)|_v \), then \( U_L(\delta_L, \pi_F)|_w = \max_{\pi_F' \in \text{BR}(\delta_L)} U_L(\delta_L, \pi'_F)|_w \), for any \( w \in \text{Supp}(\delta_L, \pi_F)|_v \).

Now we give the detailed proof of Theorem 4.3. Though the conditions in Theorem 4.3 are more complicated, the constructions of payoff functions yield similar intuitive idea. That is,

- show stronger conflicts of interests where the leader can gain more utilities;
- use constant-sum subgames and games with constant worst utilities to restrict the feasible strategy profiles for the leader to consider to commit to;

Proof of Theorem 4.3. We first prove the case \( \mathcal{P}(\text{root}) = L \).

The necessity: Let \( p \) be inducible and \( (\delta_L, \pi_F) \) the strategy profile of \( p \) that is an SSE of game \((T_{root}, U_L, \hat{U}_F)\) for some payoff function \( \hat{U}_F \), then condition (b) holds by Proposition 4.1.

For condition (a): If \( U_L(\delta_L, \pi_F)|_{v_1} > U_L(\delta_L, \pi_F)|_{v_2} \) for some \( v_1, v_2 \in \text{Supp}(\delta_L, \pi_F, \text{root}) \), then for any follower’s payoff function \( \hat{U}_F' \) that makes \( (\delta_L, \pi_F)|_{v_i} \) feasible at \( T_v \), \( \forall v \in \text{Supp}(\delta_L, \pi_F, \text{root}) \), the leader can always increase the probability of choosing \( v_1 \) and decrease the probability of choosing \( v_2 \) at \( root \) to gain higher utility than \( U_L(\delta_L, \pi_F) \), which means \( (\delta_L, \pi_F) \) is not an SSE.

For condition (c): If \( p|_{v_0} \) is not inducible at \( T_{v_0} \), for some \( v_0 \in \text{Supp}(\delta_L, \pi_F, \text{root}) \), then for any \( \hat{U}_F' \) that makes \( (\delta_L, \pi_F)|_{v_0} \) feasible at \( T_{v_0} \), there always exists a feasible strategy profile \( (\delta'_L, \pi'_F)|_{v_0} \) such that \( U_L(\delta'_L, \pi'_F)|_{v_0} > U_L(\delta_L, \pi_F)|_{v_0} = U_L(\delta_L, \pi_F) \), where the second equality holds by condition (a). Extend \( (\delta'_L, \pi'_F)|_{v_0} \) to \( T_{root} \) by letting \( \delta'_L|_{root} = v_0 \). \( (\delta'_L, \pi'_F)|_{v_0} \) is feasible at \( T_{root} \) by Lemma 4.4, and,

\[
U_L(\delta'_L, \pi_F) = U_L(\delta'_L, \pi'_F)|_{v_0} > U_L(\delta_L, \pi_F)|_{v_0} = U_L(\delta_L, \pi_F).
\]

Thus \( (\delta_L, \pi_F) \) is not SSE of game \((T_{root}, U_L, \hat{U}_F)\) for any \( \hat{U}_F \), leading to a contradiction.

The sufficiency: Suppose \( p \) satisfies the three conditions. Then \( p|_v \) is inducible at \( T_v \) for all \( v \in \text{Supp}(p, \text{root}) \); there exists a strategy profile \( (\delta_L, \pi_F)|_v \) of \( p|_v \) and a payoff function \( \hat{U}_F|_v \) at \( T_v \), such that \( (\delta_L, \pi_F)|_v \) is an SSE of subgame \((T_v, U_L|_v, \hat{U}_F|_v)\).
Extend $\tilde{U}_F|_v s (v \in \text{Supp}(p, root))$ to $T_{root}$ as follows:

$$\tilde{U}_F(z) = \begin{cases} 
\tilde{U}_F|_v(z), & z \in Z_v \wedge v \in \text{Supp}(p, root); \\
-U_L(z), & z \in Z_v \wedge (v \in \text{Child}(root) \setminus \text{Supp}(p, root)). 
\end{cases}$$  \hspace{1cm} (5)

Extend $(\delta_L, \pi_F)|_v s$ to $T_{root}$ by letting $(\delta_L, \pi_F)(root) = \sum_{v \in \text{Child}(root)} p(v)v$, and $(\delta_L, \pi_F)|_v$ be any feasible strategy profile in $(T_v, U_L|_v, \tilde{U}_F|_v)$ for $v \in \text{Child}(root) \setminus \text{Supp}(p, root)$. Then $(\delta_L, \pi_F)$ corresponds to $p$ and is feasible in game $(T_{root}, U_L, \tilde{U}_F)$ by Lemma 4.4. At $T_v s$ for $v \in \text{Child}(root) \setminus \text{Supp}(p, root)$, by Proposition 4.2, the leader can get at most $M_L(v) \leq M_L(root) \leq U_L(p) = U_L(\delta_L, \pi_F)$ via commitment, which is also the best she can get at $T_v s$ where $v \in \text{Supp}(p, root)$. Thus the leader’s optimal commitment is to put all non-zero probabilities only on nodes $v \in \text{Supp}(p, root)$ and that $M_L(v) = U_L(\delta_L, \pi_F)$, and get $U_L(\delta_L, \pi_F)$. Since $(\delta_L, \pi_F)$ is feasible at $T_{root}$, $(\delta_L, \pi_F)$ is an SSE with behavioral commitment of game $(T_{root}, U_L, \tilde{U}_F)$, and thus $p$ is inducible.

When $P(root) = F$, first we prove the necessity part.

**The necessity:** Suppose $U_L(p) < \min_{v' \in \text{Child}(root), v' \neq v} M_L(v')$. If $p|_v$ is not inducible at $T_v$, then for any $p$’s strategy profile $(\delta_L, \pi_F)$ and function $\tilde{U}_F|_v$, that makes $(\delta_L, \pi_F)|_v$ feasible at $T_v$, there exists a feasible $(\delta'_L, \pi'_F)|_v$ that $U_L(\delta'_L, \pi'_F)|_v > U_L(\delta_L, \pi_F)|_v$.

Then for any $\tilde{U}_F$ that makes $(\delta_L, \pi_F)$ feasible at $T_{root}$, let $\delta'_L|_v$ be the leader’s strategy in the aforementioned strategy profile with respect to $\tilde{U}_F|_v$, and extend it to $\delta'_L$ at $T_{root}$ by letting it equal the leader’s strategy in an SSE of subgame $(T_v, U_L|_v, \tilde{U}_F|_v)$ at $T_v$, for $v' \in \text{Child}(root)$, $v' \neq v$; and equal $\delta'_L|_v$ at $T_v$. Then for $\pi'_F \in \text{BR}(\delta'_L)$ that maximizes $U_L(\delta'_L, \pi'_F)$ among the follower’s strategies in $\text{BR}(\delta'_F)$, if $\pi'_F(root) = v$, $U_L(\delta', \pi'_F) \geq U_L(\delta'_F, \pi'_F)|_v > U_L(\delta_L, \pi_F)|_v = U_L(\delta_L, \pi_F)$. If $\pi'_F(root) = v' \neq v$, the leader can get at least $M_L(v') > U_L(\delta_L, \pi_F)$, contradicting to the assumption that $p$ is inducible.

**The sufficiency:** Case 1: $U_L(p) \geq \min_{v' \in \text{Child}(root), v' \neq v} M_L(v')$. Consider a strategy profile $(\delta_L, \pi_F)$ of $p$, which equals the leader’s minimax strategy and the follower’s maximin strategy on nodes $h \in H$ that $p(h) = 0$. Let

$$v_0 \in \arg \min_{v' \in \text{Child}(root), v' \neq v} M_L(v'),$$

$$m_0 = \min_{v' \in \text{Child}(root), v' \neq v} M_L(v'),$$

$$M = \max \{\max_{z \in Z} U_L(z) + 1, 1\}$$

$$A(\delta_L, \pi_F) := \{z \in Z : p(\delta_L, \pi_F)(z) > 0\} = \text{Supp}(p).$$

We define payoff function, $\tilde{U}_F$, as follows:

$$\tilde{U}_F(z) = \begin{cases} 
-U_L(z), & z \in A(\delta_L, \pi_F); \\
-U_L(z) + m_0 - U_L(\delta_L, \pi_F), & z \in Z_{v_0}; \\
-2M, & z \notin Z_{v_0} \wedge z \notin A(\delta_L, \pi_F). 
\end{cases}$$  \hspace{1cm} (6)

Then $(\delta_L, \pi_F)|_{v_0}$ is an SSE of subgame $(T_{v_0}, U_L|_{v_0}, \tilde{U}_F|_{v_0})$, and the leader’s (follower’s) utility is $m_0 (-U_L(\delta_L, \pi_F)$, respectively).

First, $\pi_F \in \text{BR}(\delta_L)$ in game $(T_{root}, U_L, \tilde{U}_F)$: At any node $h \in P^{-1}(F)$, if the follower chooses another child $v'$ of $h$ instead of $(\delta_L, \pi_F)(h)$, since all the leaf nodes in $T_{v'}$ does not belong to $A(\delta_L, \pi_F)$, the follower will get at most $\max \{-U_L(\delta_L, \pi_F), -2M\} = -U_L(\delta_L, \pi_F)$.

Now we prove $U_L(\delta_L, \pi_F)$ is the best the leader can get via commitment, and thus $(\delta_L, \pi_F)$ is an SSE of the game $(T_{root}, U_L, \tilde{U}_F)$. $p$ is inducible.

First for all feasible $(\delta'_L, \pi'_F \in \text{BR}(\delta'_L))$, we have

$$\tilde{U}_F(\delta'_L, \pi'_F) \geq \tilde{U}_F(\delta_L, \pi_F) = -U_L(\delta_L, \pi_F),$$  \hspace{1cm} (7)
since otherwise the follower can always get higher utility by choosing \(v_0\) at root, yielding a leader’s utility of \(m_0 \leq U_L(\delta_L, \pi_F)\).

If \(A(\delta_L', \pi_F') \subseteq A(\delta_L, \pi_F)\), then the leader’s utility is the negation of the follower’s utility, and thus is at most \(U_L(\delta_L, \pi_F)\).

If \(A(\delta_L', \pi_F') \neq A(\delta_L, \pi_F)\), suppose with probability \(\alpha \in (0,1)\), the game ends in \(Z \setminus A(\delta_L, \pi_F)\), then
\[
\hat{U}_F(\delta_L', \pi_F') = -2\alpha M - (1 - \alpha)U \geq \hat{U}_F(\delta_L, \pi_F),
\]
for some \(U \in \mathbb{R}\) representing the leader’s expected utility if the game ends in \(A(\delta_L, \pi_F)\), then
\[
U_L(\delta_L', \pi_F') < \alpha M + (1 - \alpha)U \leq 2\alpha M + (1 - \alpha)U \leq -\hat{U}_F(\delta_L, \pi_F) = U_L(\delta_L, \pi_F).
\]

**Case 2:** \(p|_v\) is inducible at \(T_v\). Then there exists a strategy profile \((\delta_L, \pi_F)|_v\) of \(p|_v\) and an \(\hat{U}_F|_v\) at \(T_v\), such that \((\delta_L, \pi_F)|_v\) is an SSE of subgame \((T_v, U_L|_v, \hat{U}_F|_v)\). Define \(\hat{m}_F = \min_{z \in Z_v} \hat{U}_F(z)\). Construct payoff function, \(\hat{U}_F\), as follows:
\[
\hat{U}_F(z) = \begin{cases} 
\hat{U}_F|_v(z) & z \in Z_v; \\
\hat{m}_F - 1 & z \notin Z_v.
\end{cases}
\]
Then whatever strategy the leader commits to, the follower will always choose \(v\) at root. Extend \((\delta_L, \pi_F)|_v\) to \(T_{root}\) by letting \((\delta_L, \pi_F)(\text{root}) = v\). Since \(U_L(\delta_L, \pi_F)\) is the highest value the leader can get by commitment, \((\delta_L, \pi_F)\) is an SSE of game \((T_{root}, U_L, \hat{U}_F)\).

The proof of Theorem 4.3 actually gives a polynomial-time algorithm to construct a payoff function for each inducible distribution, as shown in Corollary 4.5.

**Corollary 4.5.** For any inducible distribution over leaf nodes \(p \in \Delta(Z)\), we can construct a payoff function \(\hat{U}_F\) that induces \(p\) in \(O(|H| \cdot |Z|)\) time.

### 4.4 Optimal Inducible Distributions with Succinct Structures

Although inducible distributions are much more complicated, in this section, we show that an optimal inducible distribution can have succinct structure, as we call “Y-shape”.

**Definition 4.6 (Y-shape).** A distribution \(p \in \Delta(Z)\) is called “Y-shape” if \(|\text{Supp}(p)| \leq 2\).

Intuitively, a Y-shape realizable distribution corresponds to strategy profiles, under which the nodes with positive probabilities of being reached from root node are in a “Y” shape. This means that the leader will mix two actions at at most one decision node, so the players’ utilities is a mixture of utilities at at most two leaf nodes. A demonstration of Y-shape distributions is in Figure 5.

![Figure 5: Examples to illustrate Y-shape distributions.](image-url)
Now we utilize Theorem 4.3 to show that for any inducible distribution \( p \), there is always a dominant “Y-shape” inducible distribution, in the sense that the follower can always get no less utility on it. Thus there exists an optimal “Y-shape” inducible distribution. Corollary 4.7 helps us give a concise algorithm for optimally reporting.

**Corollary 4.7.** For any extensive-form game \((T_{root}, U_L, U_F)\), there exists a “Y-shape” inducible distribution \( p^* \in \Delta(Z) \), such that \( U_F(p^*) \geq U_F(p) \) for any inducible distribution \( p \).

To simplify the following proofs, we present a non-recursive characterization of Y-shape inducible distributions here.

**Corollary 4.8.** A Y-shape realizable distribution \( p \) is inducible if and only if

1. if \( |\text{Supp}(p)| = 2 \), and \( U_L(z_1) \neq U_L(z_2) \), for \( z_1, z_2 \in \text{Supp}(p) \), then:
   
   (a) \( z_1 \) and \( z_2 \) have common ancestors \( v \in \mathcal{P}^{-1}(F) \) and \( w \in \text{Child}(v) \) such that
   
   \[ U_L(p) \geq \min_{w' \in \text{Child}(v), w' \neq w} M_L(w'); \]

   (b) \( U_L(p) \geq M_L(u), \forall u \in \mathcal{P}^{-1}(L) \) on the path from root to \( v \);

2. if \( |\text{Supp}(p)| = 1 \) or \( U_L(z_1) = U_L(z_2) \) for \( z_1, z_2 \in \text{Supp}(p) \), let \( v \) be the least common ancestor of \( z_1 \) and \( z_2 \), then

   \[ U_L(p) \geq M_L(u), \forall u \in \mathcal{P}^{-1}(L) \] on the path from root to \( v \);

4.5 **Algorithm for Optimally Reporting**

Now we give the algorithm for optimally reporting under behavioral commitment in Algorithm 1.

**Theorem 4.9.** Computing an inducible distribution \( p^* \in \Delta(Z) \) and a payoff function \( \tilde{U}_F^* \) such that

- a strategy profile \((\delta_L, \pi_F)\) of \( p^* \) is an SSE with behavioral commitment of game \((T_{root}, U_L, \tilde{U}_F^*)\);

- \( U_F(p^*) \geq U_F(p) \) for all inducible distribution \( p \);

can be solved in polynomial time.

**Proof of Theorem 4.9.** We prove that Algorithm 1 can find the optimal inducible distribution \( p^* \) and the follower’s payoff function \( \tilde{U}_F^* \) in Theorem 4.9 in \( O(|H| \cdot |Z|^2) \) time.

**Correctness.** The key observation of Corollary 4.7 shows that it suffices for us to find an optimal Y-shape inducible distribution \( p^* \), which dramatically simplifies our algorithm design.

Algorithm 1 enumerates all subtrees \( T_v \) where \( v \in H \cup Z \) and find all the inducible distributions that may be optimal, for all pairs \((z_1, z_2) \in E(v)\). The correctness then follows from Corollary 4.8.

**Complexity.** Firstly, for each \( v \in H \cup Z \), we have \(|E(v)| \leq |Z|^2\). So the complexity of line 2 would be \( O(|H| \cdot |Z|^2) \). We then analyze the complexity of the procedure OptimalReport. For each \( v \in H \cup Z \), the loop in line 16 will run at most \( O(|Z|^2) \) times, so the total complexity of the procedure OptimalReport would be \( O(|H| \cdot |Z|^3) \). Finally, after finding the optimal distribution \( p^* \), by Corollary 4.5, we can construct a payoff function that induces \( p^*, \tilde{U}_F^* \), in \( O(|H| \cdot |Z|) \) time.

This completes the proof. \( \square \)
Algorithm 1: Optimally Reporting with Behavioral Commitment

**Input:** An extensive-form game \((T_{\text{root}}, N, \mathcal{P}, \{U_i\}_{i \in N})\).

**Output:** A Y-shape inducible distribution \(p^*\) and a payoff function \(U_F^*\) such that (1) \(U_F(p^*) \geq U_F(p)\) for all inducible distributions \(p\); (2) an SSE with behavioral commitment of game \((T_{\text{root}}, U_L, U_F^*)\) correspond to \(p^*\).

1. **Procedure** \(\text{Main}()\)
   2. For each \(v \in H \cup Z\), let \(E(v)\) be the set of all pairs of leaf nodes \((z_1, z_2)\) with \(z_1 \neq z_2\) in subtree \(T_v\), such that the least common ancestor of \(z_1\) and \(z_2\) belongs to \(\mathcal{P}^{-1}(L)\).
   // This step will run in \(O(|H| \cdot |Z|^2)\) time.
3. Initialize \(p^*\) to be a distribution such that it corresponds to an SSE on game \((T_{\text{root}}, U_L, U_F)\).
4. Run \(\text{OptimalReport}(root, -\infty)\).
   // By \(-\infty\) we mean that we initiate the second parameter to be sufficiently small.
5. Construct a follower’s payoff function \(U_F^*\) by Corollary 4.5.
6. **return** \(U_F^*\) and \(p^*\).

7. **Procedure** \(\text{OptimalReport}(v, L_{\text{minval}})\)
   8. if \(v\) is a leaf node then
      9. if \(U_L(v) \geq L_{\text{minval}}\), let \(p(v) = 1\).
       10. if \(U_L(p) \geq U_L(p^*)\), then let \(p^* = p\).
       **return**
   11. for \(w \in \text{Child}(v)\) do
       12. \(\text{OptimalReport}(w, \max(L_{\text{minval}}, M_L(v)))\).
       13. if \(\mathcal{P}(v) = F\) then
           14. Let \(\text{Threshold} \leftarrow \max(\min_{w' \in \text{Child}(v), w' \neq w} M_L(w'), L_{\text{minval}})\).
           for \((z_1, z_2) \in E(w)\) do
               17. if \(U_L(z_1) < \text{Threshold}\) and \(U_L(z_2) < \text{Threshold}\) continue.
               18. Find \([a, b] \subseteq [0, 1]\) s.t. \(\forall \alpha \in [a, b], \alpha U_L(z_1) + (1 - \alpha) U_L(z_2) \geq \text{Threshold}\).
               19. Find \(\alpha \in \arg\max_{a' \in [a, b]} (a' U_L(z_1), U_L(z_2)) + (1 - a')(U_F(z_1), U_L(z_2))\).
                  // \((a_1, b_1) < (a_2, b_2)\) if and only if either \(a_1 < a_2\) or \(a_1 = a_2\) and \(b_1 < b_2\).
               20. Let \(p(z_1) = \alpha\) and \(p(z_2) = 1 - \alpha\).
               21. if \(U_L(p) \geq U_L(p^*)\), then let \(p^* = p\).

4.6 Follower Can Get No Less Actual Utility under Behavioral Commitment

Though the characterization conditions of inducible distributions become much more complicated, however, as we show in Proposition 4.10, via optimally inducing an inducible distribution, the follower can get no less actual utility under behavioral commitment than that under pure commitment.

**Proposition 4.10.** Let \(P(\Pi_L \times \Pi_F)\) \((P(\Delta_L \times \Pi_F))\) be the set of all inducible leaf nodes (inducible distributions, respectively). Then

\[
\max_{p \in P(\Delta_L \times \Pi_F)} U_F(p) \geq \max_{z \in P(\Pi_L \times \Pi_F)} U_F(z).
\]
5 Raise the Bar: Induce the Unique Outcome

So far, we have characterized all the inducible distributions on leaf nodes, i.e., the possible outcomes that can be led to by an induced SSE. However, while all SSEs yield the same utilities for the leader, there is still the equilibrium selection problem for the follower. An example is shown in Figure 6a that different SSEs yield dramatically different utilities for the follower.

Thus in this section, we proceed to a more restricted condition for strategic reporting: When can a distribution be the only outcome that all SSEs of an induced game lead to?

Formally, we define strong inducibility, which is originally proposed in Birmpas et al. (2021).

Definition 5.1 (Strong Inducibility). A distribution \( p \in \Delta(Z) \) is strongly inducible with pure (behavioral) commitment in game \((T_{\text{root}}, U_L, U_F)\) if there exists a payoff function \( \tilde{U}_F \), such that all SSEs with pure (behavioral, respectively) commitments of game \((T_{\text{root}}, U_L, \tilde{U}_F)\) correspond to \( p \).

We characterize all the strongly inducible outcomes and give related algorithmic results with different commitments in Section 5.1. We then compare the optimal utilities the follower can get in one game under inducible and strongly inducible distributions, respectively. We give a characterization of games where these two values are equal in Section 5.2.

5.1 Characterization and Near Optimality

We first give the characterization of strong inducibility with pure commitment.

Theorem 5.2. Given leaf node \( z \in Z \), let \( v \in \text{Child}(\text{root}) \) be the ancestor of \( z \), then \( z \) is strongly inducible with pure commitment if and only if

1. if \( P(\text{root}) = L \), then all of the following conditions are met:
   
   (a) \( U_L(z) > \max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \);
   
   (b) \( z \) is strongly inducible at \( T_v \).

2. if \( P(\text{root}) = F \), then at least one of following two conditions are met:
   
   (a) \( U_L(z) > \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \);
   
   (b) \( z \) is strongly inducible at \( T_v \).

As for the behavioral commitment case, we here give a sufficient and necessary condition for all Y-shape strongly inducible distributions. Actually, this suffices for us to give algorithms and find the characterization in the next section.

Theorem 5.3. A “Y-shape” realizable distribution on leaf nodes \( p \in \Delta(Z) \) is strongly inducible if and only if

1. if \( P(\text{root}) = L \), then \( |\text{Supp}(p, \text{root})| = 1 \). Let \( \text{Supp}(p, \text{root}) = \{v\} \), then all of the following conditions are met:
   
   (a) \( U_L(p) > \max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \);
   
   (b) \( p|_v \) is strongly inducible at \( T_v \).

2. if \( P(\text{root}) = F \), let \( \text{Supp}(p, \text{root}) = \{v\} \), then \( U_L(z_1) \neq U_L(z_2) \) if \( \text{Supp}(p) = \{z_1, z_2\} \); and at least one of following two conditions are met:
   
   (a) \( U_L(p) > \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \);
(b) \( p|_{T_v} \) is strongly inducible at \( T_v \).

The strict inequalities in Theorem 5.3 show that different from the case of inducibility, strongly inducible distributions may not even exist: e.g., games that the leader has a constant payoff function. Furthermore, if strongly inducible distributions do exist, there may not be an optimal one (see an example in Figure 6a); even if an optimal strongly inducible distribution exists, the follower may get far less utility than he does under optimal inducible distributions (see an example in Figure 6b).

We show it is polynomial-time tractable to decide under which aforementioned case a game is, and find an (near-)optimal solution if it exists.

**Theorem 5.4.** It is polynomial-time tractable to

1. decide if a strongly inducible leaf node exists;
2. if so, find an optimal strongly inducible leaf node and construct a payoff function that induces it.

Denote the set of all strongly inducible distributions as \( SP(\Delta_L \times \Pi_F) \), we have,

**Theorem 5.5.** It is polynomial-time tractable to

1. decide if a strongly inducible distribution exists;
2. if so, decide if an optimal strongly inducible distribution exists;
3. if so, find an optimal strongly inducible distribution; if not, for any \( \epsilon > 0 \), find an \( \epsilon \)-optimal strongly inducible distribution \( p^* \) such that
   \[
   U_F(p^*) \geq \sup_{p \in SP(\Delta_L \times \Pi_F)} U_F(p) - \epsilon.
   \]

*In both cases, construct a payoff function that induces the distribution in polynomial time.*

Similarly, we can show that the follower can get no less utility by optimally strategically reporting under behavioral commitment case, than under pure commitment case. Denote the set of all strongly inducible leaf nodes as \( SP(\Pi_L \times \Pi_F) \), and we have,

**Proposition 5.6.** \( \sup_{p \in SP(\Delta_L \times \Pi_F)} U_F(p) \geq \sup_{z \in SP(\Pi_L \times \Pi_F)} U_F(z) \).
5.2 Characterization of When Follower Gains Arbitrarily The Same Optimally in the Two Senses of Inducibility

Note that the follower’s optimal utility through strategic reporting can be dramatically different between the case of inducibility and of strong inducibility. We are interested in what kind of games can the follower’s optimal values in these two cases are arbitrarily close. That is, denote an optimal inducible distribution as \( p^* \in \arg \max_{p \in P(\Delta_L \times \Pi_F)} U_F(p) \). We wish to find a sufficient and necessary condition for a game to satisfy the following “Utility Supremum Equivalence” (USE) property \((P)\) with behavioral commitment:

\[
\sup_{p \in SP(\Delta_L \times \Pi_F)} \frac{U_F(p)}{U_F(p^*)} = 1 \tag{P}
\]

We note that, due to the finiteness of all (strongly) inducible leaf nodes with pure commitment, a game satisfies USE with pure commitment if and only if one of the optimal inducible leaf nodes is also strongly inducible.

As for the behavioral commitment case, we first provide two conditions. Theorem 5.7 shows that these two conditions fully characterize property \(P\).

**Condition 1.** There exists an optimal “Y-shape” inducible distribution \( p^* \), such that, \(|\text{Supp}(p^*)| = 2 \) and \( U_L(z_1) \neq U_L(z_2) \) where \( \text{Supp}(p^*) = \{z_1, z_2\} \).

**Condition 2.** One of the following conditions are met for some optimal “Y-shape” inducible distribution \( p^* \):

1. \( p^* \) is strongly inducible;
2. For some \( z^* \in Z \) be the leaf node that \( p^*(z^*) > 0 \), there exists a node \( v \) on the path from root to \( z^* \) such that
   (a) \( P(v) = F \), and \( U_L(z^*) \geq \min_{w \in \text{Child}(v), w \neq w(v,z^*)} M_L(w) \), where \( w(v,z^*) \in \text{Child}(v) \) is \( z^* \)’s ancestor;
   (b) \( U_L(z_1) > U_L(z^*) \) for some \( (z_1, z^*) \in E(v) \), i.e., \( z_1 \in Z_v \) is a leaf node that its least common ancestor with \( z^* \) belongs to \( P^{-1}(L) \).

**Theorem 5.7.** Game \( (T_{\text{root}}, U_L, U_F) \) satisfies property \(P\) if and only if it satisfies Condition 1 or Condition 2.

One important observation for proving Theorem 5.7 is that: recalling that we can find an optimal inducible distribution that is Y-shape in polynomial time, and characterize all the Y-shape strongly inducible distributions, denote the set of all Y-shape strongly inducible distributions as \( SYP(\Delta_L \times \Pi_F) \), and we consider another property \((P')\):

\[
\sup_{p \in SYP(\Delta_L \times \Pi_F)} \frac{U_F(p)}{U_F(p^*)} = 1 \tag{P'}
\]

Lemma 5.8 shows that these two properties are equivalent.

**Lemma 5.8.** Game \( (T_{\text{root}}, U_L, U_F) \) satisfies property \(P\) if and only if it satisfies property \(P'\).
6 Conclusion and Future Works

We study how the follower strategically reports his payoff function to gain optimal utility, when facing a learning leader who constantly interact with the follower and collect his utility data. We characterize all the game outcomes that can be successfully induced under different settings, and give efficient algorithms to find the optimal way of strategically reporting. We completely resolve this problem in extensive-form games with perfect information. One of the future work is to study such follower’s strategic behavior in the computation of other solution concepts or in other settings. Though in some game settings, e.g., the computation of Stackelberg equilibria in extensive-form games with imperfect information is NP-hard (Letchford and Conitzer, 2010), and the approximation of Nash equilibria in normal-form games is PPAD-complete (Daskalakis et al., 2009; Chen et al., 2009), it is still interesting to study such follower’s strategic behavior in such settings.

Another future work would be to consider how to counteract such manipulation. Actually, when the follower may misreport to exercise his right of privacy, we assume he is in a higher cognitive hierarchy than the leader. Since the follower only needs to best-respond to the leader’s commitment, it is not necessary for the leader to hide her own private payoff information, if he is not aware of the follower’s strategic behavior. How about these two players being in the same hierarchy, or the leader’s hierarchy is higher than the follower’s? It is worth mentioning that if we also allow the leader to report a fake payoff function, i.e., think about a new game built from the original Stackelberg game where both players’ strategies are which payoff functions to announce. Then it is an NE that the leader reports her true function and the follower optimally strategically reports according to our results. A complete equilibrium analysis might help us find a better way to come up with a countermeasure.
A Omitted Proofs from Section 3.1

A.1 Proof of Proposition 3.2

Proposition 3.2. Given a leaf node \( z \in Z \), if there exists an SSE with pure commitment of game \( (T_{\text{root}}, U_L, U_F) \) leading to \( z \), then \( U_L(z) \geq M_L(\text{root}) \) and \( U_F(z) \geq M_F(\text{root}) \).

Let \( U_L(\pi_L, \pi_F)|_z = U_L(z) \) for \( z \in Z \) to simplify the notation. To make the proof clear, we first prove two lemmas.

Lemma A.1. 1. (a) Given the leader’s pure strategy \( \pi_L \), the following follower’s pure strategies minimize the leader’s utility,

\[
\pi'_F(v) \in \arg\min_{w \in \text{Child}(v)} U_L(\pi_L, \pi'_F)|_w, \quad \forall v \in \mathcal{P}^{-1}(F); \tag{11}
\]

(b) Assuming that the follower always minimizes the leader’s utility, using strategies defined by Equation (11), the following leader’s pure strategies maximize her own utility,

\[
\pi_L(v) \in \arg\max_{w \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_w, \quad \forall v \in \mathcal{P}^{-1}(L). \tag{12}
\]

2. (a) Given the leader’s pure strategy \( \pi_L \), the following follower’s pure strategies maximize his own utility,

\[
\pi'_F(v) \in \arg\max_{w \in \text{Child}(v)} U_F(\pi_L, \pi'_F)|_w, \quad \forall v \in \mathcal{P}^{-1}(F); \tag{13}
\]

(b) Assuming that the follower always maximizes his own utility, using strategies defined by Equation (13), the following leader’s pure strategies minimize the follower’s utility,

\[
\pi'_L(v) \in \arg\min_{w \in \text{Child}(v)} U_F(\pi'_L, \pi_F)|_w, \quad \forall v \in \mathcal{P}^{-1}(L). \tag{14}
\]

Proof. We only prove (1) here, the proof of (2) is analogous.

First we prove (1)(a). Intuitively, starting at any strategy \( \pi'_F \) of the follower, changing any action at any node \( v \in \mathcal{P}^{-1}(F) \), to the one leading to the least value of the leader in the subgame \( T_v \) always decreases the leader’s utility.

Given the leader’s strategy \( \pi_L \in \Pi_L \), we will prove that \( U_L(\pi_L, \pi'_F)|_v \geq U_L(\pi_L, \pi_F)|_v \) for any \( \pi'_F \in \Pi_F \) and any \( v \in H \cup Z \) by induction.

Inductive Base: When \( v \in Z \), \( U_L(\pi_L, \pi'_F)|_v = U_L(v) = U_L(\pi_L, \pi_F)|_v \), thus \( U_L(\pi_L, \pi'_F)|_v \geq U_L(\pi_L, \pi_F)|_v \) holds.

Inductive Step: When \( v \in H \), assume the inductive hypothesis holds, i.e., \( U_L(\pi_L, \pi'_F)|_{w'} \geq U_L(\pi_L, \pi_F)|_{w'} \) for any \( w' \in \text{Child}(v) \).

If \( \mathcal{P}(v) = L \), suppose \( \pi_L(v) = w \), then \( U_L(\pi_L, \pi'_F)|_{w'} = U_L(\pi_L, \pi_F)|_{w'} \geq U_L(\pi_L, \pi_F)|_{w} = U_L(\pi_L, \pi'_F)|_{w}. \)

If \( \mathcal{P}(v) = F \), suppose \( \pi'_F(v) = w \), then \( U_L(\pi_L, \pi'_F)|_{w} = U_L(\pi_L, \pi_F)|_{w} \geq U_L(\pi_L, \pi_F)|_{w} \geq \min_{w' \in \text{Child}(v)} U_L(\pi_L, \pi'_F)|_{w'} = U_L(\pi_L, \pi_F)|_{w}. \)

Then we prove (1)(b). Let \( \pi_L \in \Pi_L \) be the leader’s strategy defined by Equation (12), and \( \pi_F \in \Pi_F \) the corresponding follower’s strategy defined by Equation (11), we prove for any leader’s strategy \( \pi'_L \in \Pi_L \) and the corresponding follower’s strategy \( \pi'_F \in \Pi_F \) defined by Equation (11), \( U_L(\pi'_L, \pi_F)|_v \geq U_L(\pi_L, \pi_F)|_v \) for any \( v \in H \cup Z \).

Inductive Base: When \( v \in Z \), \( U_L(\pi'_L, \pi_F)|_v = U_L(v) = U_L(\pi'_L, \pi'_F)|_v \).

Inductive Step: When \( v \in H \), assume the inductive hypothesis holds, i.e., \( U_L(\pi'_L, \pi_F)|_{w'} \geq U_L(\pi'_L, \pi_F)|_{w} \) for any \( w' \in \text{Child}(v) \).

If \( \mathcal{P}(v) = L \), suppose \( \pi'_L(v) = w \), then

\[
U_L(\pi_L, \pi_F)|_v = \max_{w' \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_{w'} \geq U_L(\pi'_L, \pi_F)|_{w} \geq U_L(\pi'_L, \pi_F)|_{w} = U_L(\pi'_L, \pi'_F)|_v.
\]
If \( P(v) = F \), since \( U_L(\pi_L, \pi_F)|_{w'} \geq U_L(\pi_L', \pi_F')|_{w'} \) for any \( w' \in \text{Child}(v) \), we have
\[
U_L(\pi_L, \pi_F)|_v = \min_{w' \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_{w'} \geq \min_{w' \in \text{Child}(v)} U_L(\pi_L', \pi_F')|_{w'} = U_L(\pi_L', \pi_F')|_v.
\]

\[\square\]

**Lemma A.2.** 1. Strategy profile \((\pi_L, \pi_F)\) defined by Equation (11) and (12) has the following form:
\[
\pi_L(v) \in \arg\max_{w \in \text{Child}(v)} M_L(w)
\]
(15)
\[
\pi_F(v) \in \arg\min_{w \in \text{Child}(v)} M_L(w)
\]
(16)
and \( U_L(\pi_L, \pi_F) = M_L(\text{root}) \).

2. Strategy profile \((\pi_L, \pi_F)\) defined by Equation (13) and (14) has the following form:
\[
\pi_L(v) \in \arg\min_{w \in \text{Child}(v)} M_F(w)
\]
(17)
\[
\pi_F(v) \in \arg\max_{w \in \text{Child}(v)} M_F(w)
\]
(18)
and \( U_F(\pi_L, \pi_F) = M_F(\text{root}) \).

**Proof.** We only prove the first statement here, the proof of the second statement is similar. Besides the form of the strategy profiles, we will also prove that under that strategy profile \((\pi_L, \pi_F)\) defined by Equation (11) and (12), \( U_L(\pi_L, \pi_F)|_v = M_L(v) \) for all \( v \in H \cup Z \). Thus \( U_L(\pi_L, \pi_F) = M_L(\text{root}) \).

**Inductive Base:** When \( v \in Z \), \( U_L(\pi_L, \pi_F)|_v = U_L(v) = M_L(v) \). The action space is empty here, the statements hold.

**Inductive Step:** When \( v \in H \), assume the inductive hypothesis holds and \((\pi_L, \pi_F)\) in Equation (11) and (12) have been defined in its descendant nodes.

If \( P(v) = L \), \( \pi_L(v) \in \arg\max_{w \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_w = \arg\max_{w \in \text{Child}(v)} M_L(w) \) by the inductive hypothesis, and \( U_L(\pi_L, \pi_F)|_v = \max_{w \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_w = \max_{w \in \text{Child}(v)} M_L(w) = M_L(v) \).

If \( P(v) = F \), \( \pi_F(v) \in \arg\min_{w \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_w = \arg\min_{w \in \text{Child}(v)} M_L(w) \) by the inductive hypothesis, and \( U_L(\pi_L, \pi_F)|_v = \min_{w \in \text{Child}(v)} U_L(\pi_L, \pi_F)|_w = \min_{w \in \text{Child}(v)} M_L(w) = M_L(v) \). \[\square\]

**Proof of Proposition 3.2.** Since there is an SSE leading to \( z \), we have
\[
U_L(z) = \max_{\pi_L' \in \Pi_L} U_L(\pi_L', \pi_F) \geq \max_{\pi_L' \in \Pi_L} \min_{\pi_F' \in \Pi_F} U_L(\pi_L', \pi_F').
\]
(19)
By Lemma A.1 and Lemma A.2, the right hand side of Equation (19) equals \( M_L(\text{root}) \).
\[
U_F(z) = \max_{\pi_F' \in \Pi_F} U_F(\pi_L, \pi_F') \geq \min_{\pi_L' \in \Pi_L} \max_{\pi_F' \in \Pi_F} U_F(\pi_L', \pi_F').
\]
(20)
By Lemma A.1 and Lemma A.2, the right hand side of Equation (20) equals \( M_F(\text{root}) \). \[\square\]

**A.2 Proof of Proposition 3.3**

**Proposition 3.3.** Given a zero-sum extensive-form game \((T_{\text{root}}, U_L, U_F = -U_L)\), we have
\[
\max_{\pi_L \in \Pi_L, \pi_F \in \text{BR}(\pi_L)} U_L(\pi_L, \pi_F) = M_L(\text{root}).
\]
Proof. Since $U_F = -U_L$, we have $M_F(root) = -M_L(root)$. Let $(\pi^*_L, \pi^*_F)$ be an SSE with pure commitment of $(T_{root}, U_L, -U_L)$, we have
\[
U_L(\pi^*_L, \pi^*_F) = \max_{\pi_L \in P_L, \pi_F \in BR(\pi_L)} U_L(\pi_L, \pi_F) \geq M_L(root),
\]
\[
U_F(\pi^*_L, \pi^*_F) = -U_L(\pi^*_L, \pi^*_F) \geq M_F(root) = -M_L(root),
\]
which means
\[
M_L(root) \leq \max_{\pi_L \in P_L, \pi_F \in BR(\pi_L)} U_L(\pi_L, \pi_F) \leq M_L(root)
\]
\[
\Rightarrow \max_{\pi_L \in P_L, \pi_F \in BR(\pi_L)} U_L(\pi_L, \pi_F) = M_L(root).
\]

A.3 Proof of Proposition 3.4

Proposition 3.4. For leaf node $z \in Z$ in game $(T_{root}, U_L, U_F)$, there exists a feasible strategy profile $(\pi_L, \pi_F \in BR(\pi_L))$ leading to $z$ if and only if $U_F(z) \geq M_F(v)$ for every $v \in \mathcal{P}^{-1}(F)$ that is on the path from root to $z$.

Proof. For the sufficiency, suppose that $z \in Z$ satisfies $U_F(z) \geq M_F(v)$ for all nodes $v$ on the path from root to $z$. For node $v \in H$ that is on the path from root to $z$, we use $w(v, z) \in Child(v)$ to denote the child of $v$ that is on the path from root to $z$. Consider the following commitment of the leader
\[
\pi_L(v) = \begin{cases} w(v, z), & v \in \mathcal{P}^{-1}(L) \text{ is on the path from root to } z; \\ \arg \min_{w \in Child(v)} M_F(w), & \text{otherwise}, \end{cases}
\]
and the following strategy of the follower
\[
\pi_F(v) = \begin{cases} w(v, z), & v \in \mathcal{P}^{-1}(F) \text{ is on the path from root to } z; \\ \arg \max_{w \in Child(v)} M_F(w), & \text{otherwise}. \end{cases}
\]

Notice that $(\pi_L, \pi_F)$ will lead to the leaf node $z$, so we only need to prove $\pi_F \in BR(\pi_L)$. This is mainly based on the following observation: for each $v \in \mathcal{P}^{-1}(F)$ that is not on the path from root to $z$, we have
\[
\max_{\pi'_F \in P_F} U_F(\pi_L, \pi'_F)|_v = M_F(v), \tag{21}
\]
and for each $v \in \mathcal{P}^{-1}(F)$ that is on the path from root to $z$, we have
\[
\max_{\pi'_F \in P_F} U_F(\pi_L, \pi'_F)|_v = U_F(z). \tag{22}
\]

The proof of Equation 21 can be directly derived by Lemma A.2. We then prove Equation 22 for every $v \in \mathcal{P}^{-1}(F)$ that is on the path from root to $z$. Intuitively, we can observe the Equation 22 by a simple induction over the tree structure. If $\pi'_F(v) = w(v, z)$, then $U_F(\pi_L, \pi'_F)|_v \leq U_F(z)$ by the inductive hypothesis. If $\pi'_F(v) = w' \neq w(v, z)$, then $U_F(\pi_L, \pi'_F)|_v = M_F(w')$. By the condition $U_F(z) \geq M_F(v) = \max_{w' \in Child(v)} M_F(w')$ and thus Equation 22 holds on $v$. Thus
\[
\max_{\pi'_F \in P_F} U_F(\pi_L, \pi'_F) = U_F(\pi_L, \pi_F).
\]

For the necessity part, suppose that $(\pi_L, \pi_F \in BR(\pi_L))$ leads to $z \in Z$. Suppose, for the sake of contradiction, that there is a node $v \in \mathcal{P}^{-1}(F)$ on the path from root to $z$ such that $U_F(z) < M_F(v)$. Then the follower can construct a new strategy $\pi'_F$, where $\pi'_F(v) = w \in \arg \max_{w' \in Child(v)} M_F(w')$, $\pi'_F \in BR(\pi_L)|_w$ and $\pi'_F(v') = \pi_F(v')$ for other undefined nodes $v' \in \mathcal{P}^{-1}(F)$. Then by Equation (20), $U_F(\pi_L, \pi'_F)|_v = U_F(\pi_L, \pi_F)|_v \geq M_F(w) = M_F(v)$, which means $U_F(\pi_L, \pi'_F)|_v \geq M_F(v) > U_F(\pi_L, \pi_F)|_v$, contradicting to our assumption. \qed
B Omitted Proofs from Section 3.3

B.1 Proof of Theorem 3.7

Theorem 3.7. Computing an inducible leaf node $z^*$ and a follower’s payoff function $\hat{U}^*_F$ such that

- a strategy profile $(\pi_L, \pi_F)$ of $z^*$ is an SSE with pure commitment of game $(T_{root}, U_L, \hat{U}^*_F)$;
- $U_F(z^*) \geq U_F(z)$ for all inducible leaf node $z$;

can be solved in time $\Theta(|H| + |Z|)$.

Proof. We prove that Algorithm 2 is a linear-time algorithm to find the $z^*$ and $\hat{U}^*_F$ in Theorem 3.7.

Correctness. By Theorem 3.5, $SF$ is the set of all inducible leaf nodes. So $z^*$ satisfies $U_F(z^*) \geq U_F(z)$ for all inducible leaf nodes $z$. By the proof of Theorem 3.5, we know that there is an SSE with pure commitment of game $(T_{root}, U_L, \hat{U}^*_F)$ leading to $z^*$. So the correctness follows.

Complexity. We note that $M_L(root)$ can be computed in $O(|H| + |Z|)$ time. Calculating the set $SF$ and picking the leaf node $z^*$ would run in $O(|Z|)$ time. By Corollary 3.6, we can construct the follower’s payoff function in $O(|H| + |Z|)$ time. So the complexity of Algorithm 2 is $\Theta(|H| + |Z|)$. □

---

Algorithm 2: Optimally Reporting with Pure Commitment

Input: An extensive-form game $(T_{root}, N = \{L, F\}, P, \{U_L, U_F\})$.

Output: An inducible leaf node $z^*$ and a payoff function $\hat{U}^*_F$ satisfying (1) a strategy profile of $z^*$ is an SSE with pure commitment of game $(T_{root}, U_L, \hat{U}^*_F)$, (2) $U_F(z^*) \geq U_F(z)$ for all inducible leaf nodes $z$.

1. Calculate a set of leaf nodes $SF = \{z \in Z | U_L(z) \geq M_L(root)\}$).
2. Pick a leaf node $z^* \in SF$ such that $z^* \in \arg \max_{z \in SF} U_F(z)$.
3. Construct a follower’s payoff function $\hat{U}^*_F$ via Corollary 3.6.
4. return $\hat{U}^*_F$ and $z^*$.

C Omitted Proofs from Section 4.2

C.1 Proof of Lemma 4.4

Lemma 4.4. Given strategy profile $(\delta_L, \pi_F)|_v$ defined on subgame $(T_v, U_L|_v, U_F|_v)$ for $v \in H$:

1. If $(\delta_L, \pi_F)|_w$ is feasible at $T_w$, $(\delta_L, \pi_F)|_w$ is also feasible at $T_v$, for any $w \in \text{Supp}((\delta_L, \pi_F)|_v, v)$.
2. If $(\delta_L, \pi_F)|_w$ is feasible at $T_w$, for any $w \in \text{Supp}((\delta_L, \pi_F)|_v, v)$, then $(\delta_L, \pi_F)|_v$ is feasible at $T_v$.
3. If $(\delta_L, \pi_F)|_w$ is feasible and $U_L(\delta_L, \pi_F)|_v = \max_{\pi_F'[w] \in \text{BR}(\delta_L)|_w} U_L(\delta_L, \pi_F')|_v$, then $U_L(\delta_L, \pi_F)|_w = \max_{\pi_F'[w] \in \text{BR}(\delta_L)|_w} U_L(\delta_L, \pi_F')|_w$, for any $w \in \text{Supp}((\delta_L, \pi_F)|_v, v)$.

Proof. 1. Since if not, suppose there exists $\pi_F'|_w$ such that $U_F(\delta_L, \pi_F')|_w > U_F(\delta_L, \pi_F)|_w$ for some $w \in \text{Supp}((\delta_L, \pi_F)|_v, v)$, then define the follower’s strategy $\pi_F'|_v$ at $T_v$ to be equal to $\pi_F'|_w$ at $T_w$, and $\pi_F'|_v$ on other undefined nodes. Then $U_F(\delta_L, \pi_F')|_v > U_F(\delta_L, \pi_F)|_v$, contradicting to that $\pi_F \in \text{BR}(\delta_L)|_w$.

2. Since for any follower’s strategy $\pi_F'|_v$ at $T_v$,

$$U_F(\delta_L, \pi_F')|_w \leq U_F(\delta_L, \pi_F)|_w, \quad \forall w \in \text{Supp}((\delta_L, \pi_F)|_v, v),$$

the convex combination of $\{U_F(\delta_L, \pi_F')|_w \}_{w \in \text{Supp}((\delta_L, \pi_F)|_v, v)}$ is at most the same combination of $\{U_F(\delta_L, \pi_F)|_w \}_{w \in \text{Supp}((\delta_L, \pi_F)|_v, v)}$.
3. Suppose \( (\delta_L, \pi_F)|_v(v) = \sum_{w' \in \text{Child}(v)} \alpha_{w'} w' \) and, for the sake of contradiction, for some \( w \in \text{Supp}(\delta_L, \pi_F)|_v(v) \), \( \pi_F' \in \arg \max_{\pi_F' \in \text{BR}(\delta_L)} U_L(\delta_L, \pi_F')|_w \) satisfies \( U_L(\delta_L, \pi_F')|_w > U_L(\delta_L, \pi_F)|_w \). Extend \( \pi_F'|_w \) to \( T_v \) by letting it equal \( \pi_F'|_v \) on other undefined nodes. By 1 and 2, \( (\delta_L, \pi_F)|_v(v) \) is feasible and \( U_L(\delta_L, \pi_F)|_v(v) = \sum_{w' \in \text{Child}(v)} \alpha_{w'} U_L(\delta_L, \pi_F')|_{w'} > U_L(\delta_L, \pi_F)|_v(v) \), leading to a contradiction. □

C.2 Proof of Corollary 4.5

**Corollary 4.5.** For any inducible distribution over leaf nodes \( p \in \Delta(Z) \), we can construct a payoff function \( \tilde{U}_F \) that induces \( p \) in \( O(|H| \cdot |Z|) \) time.

**Proof.** Use backward induction to calculate \( \tilde{U}_F \) by Equation 5, Equation 6 and Equation 10. Note that for each \( h \in H \), the Equation 5 and Equation 6 will use at most \( |Z| \) steps to update \( \tilde{U}_F \), so the total complexity is \( O(|H| \cdot |Z|) \). □

D. Omitted Proofs from Section 4.4

D.1 Proof of Corollary 4.7

**Corollary 4.7.** For any extensive-form game \( (T_{\text{root}}, U_L, U_F) \), there exists a "Y-shape" inducible distribution \( p^* \in \Delta(Z) \), such that \( U_F(p^*) \geq U_F(p) \) for any inducible distribution \( p \).

**Proof.** We prove by structural induction over the game tree that for any inducible distribution \( p \in \Delta(Z) \) on game \( (T_{\text{root}}, U_L, U_F) \), there exists a Y-shape inducible distribution \( p^* \in \Delta(Z) \) such that \( U_F(p^*) \geq U_F(p) \) and \( U_L(p^*) \geq U_L(p) \).

**Inductive Base:** When root \( \in Z \), the only distribution \( p \in \Delta(Z) \) satisfies \( |\text{Supp}(p)| = 1 < 2 \), and is inducible.

**Inductive Step:** Suppose that root \( \in H \) and \( p \in \Delta(Z) \) is inducible.

- **When \( P(\text{root}) = F \):** Since \( p \) is realizable, let \( v \in \text{Child}(\text{root}) \) be the only child of \( \text{root} \) that \( p(v) > 0 \). If \( p|_v \) is inducible at \( T_v \), then by the inductive hypothesis, we know that there exists a Y-shape inducible distribution \( p^* \in \Delta(Z_v) \) such that \( U_F(p^*)|_v \geq U_F(p)|_v \) and \( U_L(p^*)|_v \geq U_L(p)|_v \). Extend \( p^*|_v \) to \( T_{\text{root}} \) by letting \( p^*(z) = 0 \) for each \( z \in Z \setminus Z_v \), then \( p^* \) is Y-shape and inducible at \( T_{\text{root}} \).

- **When \( P(\text{root}) = L \):** Pick node \( v \) such that

\[
\begin{align*}
\arg\max_{v' \in \text{Supp}(p, \text{root})} U_F(p)|_{v'}. \\
\end{align*}
\]

By **Theorem 4.3** (2)(a), \( p^* \) is inducible, and satisfies \( U_F(p^*) \geq U_F(p) \).

This completes the proof.
D.2 Proof of Corollary 4.8

In the following proofs, given \( z_1, z_2 \in Z \) and \( \alpha \in [0, 1] \), we use \( p_{z_1, z_2} \) to denote the distribution \( p \in \Delta(Z) \) that \( p(z_1) = \alpha \) and \( p(z_2) = 1 - \alpha \).

**Corollary 4.8.** A Y-shape realizable distribution \( p \) is inducible if and only if

1. if \( |\text{Supp}(p)| = 2 \), and \( U_L(z_1) \neq U_L(z_2) \), for \( z_1, z_2 \in \text{Supp}(p) \), then:
   
   \( a \) \( z_1 \) and \( z_2 \) have common ancestors \( v \in \mathcal{P}^{-1}(F) \) and \( w \in \text{Child}(v) \) such that
   
   \[ U_L(p) \geq \min_{w' \in \text{Child}(v), w' \neq w} M_L(w'); \]

   \( b \) \( U_L(p) \geq M_L(u), \forall u \in \mathcal{P}^{-1}(L) \) on the path from root to \( v \);

2. if \( |\text{Supp}(p)| = 1 \) or \( U_L(z_1) = U_L(z_2) \), let \( v \) be the least common ancestor of
   
   \( z_1 \) and \( z_2 \), then
   
   \[ U_L(p) \geq M_L(u), \forall u \in \mathcal{P}^{-1}(L) \) on the path from root to \( v \);

**Proof.** First notice that for a Y-shape distribution \( p \) that \( |\text{Supp}(p)| = 2 \) to be realizable, let \( \text{Supp}(p) = \{ z_1, z_2 \} \), then the least common ancestor of \( z_1 \) and \( z_2 \) must belong to \( \mathcal{P}^{-1}(L) \), which we denote as LCA. Now we first consider Case (1):

**The necessity:** Suppose for the sake of contradiction, condition (a) is not satisfied:

If \( z_1 \) and \( z_2 \) do not have common ancestors belonging to \( \mathcal{P}^{-1}(F) \), then for any \( \alpha \in [0, 1] \), any strategy profile of \( p_{z_1, z_2} \) is feasible for the leader to choose, whatever the follower’s payoff function is. W.L.O.G., let \( U_L(z_1) < U_L(z_2) \), then the pure strategy profile leading to \( z_2 \) always gains the leader higher utility. Thus \( p \) will never correspond to an SSE under any follower’s payoff function.

Now suppose for any common ancestors \( v \in \mathcal{P}^{-1}(F) \) and \( w \in \text{Child}(v) \) of \( z_1 \) and \( z_2 \),

\[ U_L(p) = U_L(p)|_v < \min_{w' \in \text{Child}(v), w' \neq w} M_L(w'), \]

then by Theorem 4.3, \( p|_u \) has to be inducible at \( T_u \) for any node \( u \) on the path from root to LCA. When \( u = LCA \in \mathcal{P}^{-1}(L) \), that \( U_L(z_1) \neq U_L(z_2) \) contradicts to condition (1)(a) of Theorem 4.3.

Condition (b) follows from condition (1)(b) of Theorem 4.3.

**The sufficiency:** By condition (2)(a) of Theorem 4.3, \( p|_u \) is inducible at \( T_u \). Suppose \( p|_u \) is inducible at \( T_u \) for ancestor \( u \) of \( v \), we prove that for \( v \)’s ancestor \( t \) that \( u \in \text{Child}(t) \), \( p|_t \) is inducible at \( T_t \): the case when \( t \in \mathcal{P}^{-1}(F) \) follows from condition (2)(b) of Theorem 4.3; when \( \mathcal{P}^{-1}(L) \), conditions (1)(a) are satisfied and (1)(b)(c) of Theorem 4.3 follows from condition (b).

Then we consider Case (2):

**The necessity** follows from condition (1)(b) of Theorem 4.3.

**The sufficiency:** since \( p \) is realizable, \( v \) either belongs to \( \mathcal{P}^{-1}(L) \) (when \( z_1 \neq z_2 \)) or is \( z_1 \) (when \( z_1 = z_2 \), \( |\text{Supp}(p)| = 1 \)). By checking conditions of Theorem 4.3, \( p|_v \) is inducible at \( v \). Suppose \( p|_u \) is inducible at \( T_u \) for ancestor \( u \) of \( v \), for \( v \)’s ancestor \( t \), that \( u \in \text{Child}(t) \), \( p|_t \) is inducible at \( T_t \) by Theorem 4.3. \( \square \)

E Omitted Proofs from Section 4.6

E.1 Proof of Proposition 4.10

**Proposition 4.10.** Let \( P(\Pi_L \times \Pi_F) \) \( (P(\Delta_L \times \Pi_F)) \) be the set of all inducible leaf nodes (inducible distributions, respectively). Then

\[ \max_{p \in P(\Delta_L \times \Pi_F)} U_F(p) \geq \max_{z \in P(\Pi_L \times \Pi_F)} U_F(z). \]
Proof. Recalling that any leaf node \( z \) is equivalent to a distribution \( p \) on leaf nodes that puts all probability on that leaf node, it suffices to show that \( P(\Pi_L \times \Pi_F) \subseteq P(\Delta_L \times \Pi_F) \). For \( p \in P(\Pi_L \times \Pi_F) \), we show by induction on the game tree that \( p \) also satisfies conditions in Theorem 4.3.

**Inductive Base:** When \( \text{root} \in Z \), the only realizable distribution \( p \) where \( p(\text{root}) = 1 \) is both inducible with pure and behavioral commitment.

**Inductive Step:** When \( \text{root} \in H \), let \( \text{Supp}(p, \text{root}) = \{ v \} \). We assume the inductive hypothesis holds in subtrees and consider any \( p \in P(\Pi_L \times \Pi_F) \).

When \( P(\text{root}) = L \), by the definition of \( P(\Pi_L \times \Pi_F) \), condition (1)(a) and (b) of Theorem 4.3 are satisfied. Then \( U_L(p)_v = U_L(p) \geq M_L(\text{root}) = \max_{v' \in \text{Child}(\text{root})} M_L(v') \geq M_L(v) \). Thus \( p|_v \) is inducible at \( T_v \) with behavioral commitment by the inductive hypothesis, which means condition (1)(c) is satisfied. So \( p \) is inducible with behavioral commitment in game \( (T_{\text{root}}, U_L, U_F) \).

When \( P(\text{root}) = F \), \( U_L(p) \geq M_L(\text{root}) = \min_{v' \in \text{Child}(\text{root})} M_L(v') \).

If \( v \in \arg \min_{v' \in \text{Child}(\text{root})} M_L(v') \), then \( U_L(p)_v = U_L(p) \geq M_L(v) \). \( p|_v \) is inducible with both pure and behavioral commitment by the inductive hypothesis in subgame \( (T_v, U_L|_v, U_F|_v) \), satisfying condition (2)(b) of Theorem 4.3. Otherwise, \( U_L(p)_v \geq \min_{v' \in \text{Child}(\text{root})} M_L(v') \), satisfying condition 1. Thus \( p \) is inducible with behavioral commitment in game \( (T_{\text{root}}, U_L, U_F) \).

\( \square \)

**F  Omitted Proofs from Section 5**

**F.1  Proof of Theorem 5.2**

**Theorem 5.2.** Given leaf node \( z \in Z \), let \( v \in \text{Child}(\text{root}) \) be the ancestor of \( z \), then \( z \) is strongly inducible with pure commitment if and only if

1. if \( P(\text{root}) = L \), then all of the following conditions are met:
   
   (a) \( U_L(z) > \max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \);

   (b) \( z \) is strongly inducible at \( T_v \).

2. if \( P(\text{root}) = F \), then at least one of following two conditions are met:

   (a) \( U_L(z) > \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \);

   (b) \( z \) is strongly inducible at \( T_v \).

**Proof.** The proof is based on structural induction.

**Inductive Base:** When \( \text{root} \in Z \), the strategy set is empty and \( \text{root} \) is strongly inducible.

**Inductive Step:** When \( \text{root} \in H \), we first prove the necessities.

When \( P(\text{root}) = L \): assume that there exists \( v' \in \text{Child}(\text{root}) \) that \( v' \neq v \) and \( U_L(z) \leq M_L(v') \) for the sake of contradiction. Then the leader can always get at least \( M_L(v') \) by committing to a pure strategy \( \pi_L \) such that \( \pi_L(\text{root}) = v' \). So \( z \) cannot be the unique outcome of all SSEs of game \( (T_{\text{root}}, U_L, U_F) \) for any \( U_F \).

Now assume that \( z \) is not strongly inducible at \( T_v \). Then for any \( U_F \), if the leader optimally commits to some pure strategy \( \pi_L \) that \( \pi_L(\text{root}) = v \), then \( z \) cannot be the unique outcome of all induced SSEs. On the other hand, if the leader optimally commits to some pure strategy \( \pi_L \) that \( \pi_L(\text{root}) \neq v \), then \( z \) cannot be committed to by any induced SSE, leading to a contradiction.

When \( P(\text{root}) = F \): We assume \( U_L(z) \leq \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \) and prove that \( z \) is strongly inducible at \( T_v \).

We claim that to make \( z \) the unique outcome of SSEs, \( U_F \) must satisfy that \( \forall (\pi_L, \pi_F \in \text{BR}(\pi_L)) \in (\Pi_L \times \Pi_F), \pi_F(\text{root}) = v \). Otherwise, the leader must have a commitment \( \pi_L' \) and a corresponding
follower’s best response \( \pi_F' \in \text{BR}(\pi_L') \) such that \( \pi_F'(\text{root}) = v' \neq v \). Then the leader can get at least \( M_L(v') \geq U_L(z) \) by committing to \( \pi_L' \), contradicting to that \( z \) is the unique outcome.

Now we suppose \( \tilde{U}_F \) satisfies that for any \( (\pi_L, \pi_F) \in (P_L \times P_F) \) if \( \pi_F \in \text{BR}(\pi_L) \), then \( \pi_F(\text{root}) = v \). This reduces the problem to the subtree \( T_v \). So if \( z \) is strongly inducible on \( T_{\text{root}} \), it must be strongly inducible on \( T_v \).

Now we prove the **sufficiencies**. When \( P(\text{root}) = L \): Let \( \tilde{U}_F|_v \) be a payoff function such that all SSEs of subgame \( (T_v, U_L|_v, \tilde{U}_F|_v) \) lead to \( z \). We extend it to \( T_{\text{root}} \) by setting \( \tilde{U}_F(z) = -U_L(z) \) for all \( z \in Z \setminus Z_v \). Then if the leader commits to a strategy \( \pi_L' \) that \( \pi_L'(\text{root}) = v' \neq v \), the best he get is \( M_L(v') < U_F(z) \), thus his optimal commitment is to choose \( v \) at \( \text{root} \). Thus all SSEs of game \( (T_{\text{root}}, U_L, \tilde{U}_F) \) correspond to \( z \).

When \( P(\text{root}) = F \): If \( (2)(a) \) holds, suppose that \( v' \in \text{Child}(\text{root}), v' \neq v \) satisfies \( U_L(z) > M_L(v') \). Construct the follower’s payoff function, \( \tilde{U}_F \), as follows:

\[
\tilde{U}_F(z') = \begin{cases} 
- M_L(v') + 1 & z' = z; \\
- M_L(v') - 1 & z' \not\in Z_{\text{root}} \land z' \neq z; \\
- U_L(z') & z' \in Z_{\text{root}}. 
\end{cases}
\]

Since \( U_L(z) > M_L(w) \), the best choice for the leader is to commit to a pure strategy that can achieve \( z \). Thus all SSEs lead to \( z \).

If \( (2)(b) \) holds, let \( \tilde{U}_F|_v \) be a payoff function such that all SSEs of subgame \( (T_v, U_L|_v, \tilde{U}_F|_v) \) lead to \( z \). We simply extend it by setting \( \tilde{U}_F(z) = \min_{z' \in Z} -\tilde{U}_F(z) \) on \( Z \setminus Z_v \). Then the follower will always best-respond to choose \( v \) at root, and thus all SSEs of game \( (T_{\text{root}}, U_L, \tilde{U}_F) \) lead to \( z \).

This finishes the proof. \( \square \)

**F.2 Proof of Theorem 5.3**

**Theorem 5.3.** A “Y-shape” realizable distribution on leaf nodes \( p \in \Delta(Z) \) is strongly inducible if and only if

1. if \( P(\text{root}) = L \), then \( |\text{Supp}(p, \text{root})| = 1 \). Let \( \text{Supp}(p, \text{root}) = \{v\} \), then all of the following conditions are met:

   (a) \( U_L(p) > \max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \); 

   (b) \( p|_v \) is strongly inducible at \( T_v \).

2. if \( P(\text{root}) = F \), let \( \text{Supp}(p, \text{root}) = \{v\} \), then \( U_L(z_1) \neq U_L(z_2) \) if \( \text{Supp}(p) = \{z_1, z_2\} \); and at least one of following two conditions are met:

   (a) \( U_L(p) > \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \); 

   (b) \( p|_v \) is strongly inducible at \( T_v \).

**Lemma F.1.** Given game \( (T_{\text{root}}, U_L, U_F) \) and realizable distribution \( p \in \Delta(Z) \), there exists a feasible strategy profile \( (\delta_L, \pi_F) \in \text{BR}(\delta_L) \) of \( p \) if and only if \( U_L(p)v \geq M_F(v) \) for any \( v \in \{v' \in P^{-1}(F) : p(v') > 0\} \).

**Proof.** The sufficiency: Consider the following commitment of the leader

\[
\delta_L(v) = \begin{cases} 
\sum_{w \in \text{Child}(v)} \frac{p(w)}{p(v)} w, & p(v) > 0; \\
\arg \min_{w \in \text{Child}(v)} M_F(w), & \text{otherwise}; 
\end{cases}
\]
and the following strategy of the follower

\[
\pi_F(v) = \begin{cases} 
\sum_{w \in \text{Child}(v)} \frac{p(w)}{p(v)}w, & p(v) > 0; \\
\arg\max_{w \in \text{Child}(v)} M_F(w), & \text{otherwise.}
\end{cases}
\]

Notice that \((\delta_L, \pi_F)\) correspond to \(p\), and since \(p\) is realizable, \(\pi_F\) is pure. We prove that for any \(v \in \mathcal{P}^{-1}(F)\) that \(p(v) > 0\), \(\max_{\pi_F \in \Pi_F} U_F(\delta_L, \pi_F)|_v = U_F(\delta_L, \pi_F)|_v = U_F(p)|_v\). For \(w \in \text{Child}(v)\) that \(p(w) = 0\), by Lemma A.2, we have \(\pi_F \in \text{BR}(\delta_L)|_w\), and the follower get at most \(M_F(w)\) at \(T_w\). Since \(U_F(p)|_v \geq M_F(v) = \max_{v' \in \text{Child}(v)} M_F(v')\), the follower’s choosing \(w\) that \(p(w) > 0\) will lead to the best utility for the follower at \(v\).

**The necessity:** Suppose, for the sake of contradiction, that there is a \((\delta_L, \pi_F \in \text{BR}(\delta_L))\) of \(p\) and a node \(v \in \mathcal{P}^{-1}(F)\) that \(p(v) > 0\) satisfying \(U_F(p)|_v < M_F(v)\). Then the follower can construct a new strategy \(\pi_F^*\), where \(\pi_F^*(v) = w \in \arg\max_{v' \in \text{Child}(v)} M_F(w')\), \(\pi_F^* \in \text{BR}(\delta_L)|_w\), and \(\pi_F^*(v') = \pi_F(v')\) for other nodes \(v' \neq v\). By Equation (19), \(U_F(\delta_L, \pi_F^*)|_v \geq M_F(w) = M_F(v) > U_F(\delta_L, \pi_F)|_v\), contradicting to that \((\delta_L, \pi_F \in \text{BR}(\delta_L))\) by Lemma 4.4.

**Proof of Theorem 5.3.** We prove by structural induction over the game tree.

**Inductive Base:** When root \(\in Z\), the only realizable distribution \(p\) where \(p(\text{root}) = 1\) is strongly inducible, and all the conditions are satisfied.

**Inductive Step:** When \(\text{root} \in H\), we first prove the necessities.

When \(\mathcal{P}(\text{root}) = L\): consider a strongly inducible distribution \(p\), then \(p\) is also inducible. Suppose \(\text{Supp}(p, \text{root}) = \{v_1, v_2\} \subseteq \text{Child}(\text{root})\). By the conditions of inducibility \(U_L(p)|_{v_i} = U_L(p)\) for \(i \in [2]\). Extend \(p|_{v_i}\) to \(p_t\) on \(T_{\text{root}}\) by \(p_t(z) = 0\) for \(z \in Z \setminus Z_{v_i}\), then \(U_L(p_t) = U_L(p)|_{v_i} = U_L(p)\).

For any \(U_F\) that makes a \((\delta_f, \pi_F)\) of \(p\) SSE and \(i \in [2]\), consider an extension of \((\delta_f, \pi_F)|_{v_i}\) to \(T_{\text{root}}\), \((\delta_f', \pi_F')\), that \(\delta_f' = \delta_f\) and \(\pi_F' = \pi_F\) on \(T_{\text{root}}\), \((\delta_f', \pi_F')\) is also feasible by Lemma 4.4 and \(U_L(\delta_f', \pi_F') = U_L(\delta_f, \pi_F)\).

Thus \((\delta_f', \pi_F')\) is an SSE of game \((T_{\text{root}}, U_L, U_F)\), leading to a contradiction.

Now let \(\text{Supp}(p, \text{root}) = \{v\}\) and suppose \(U_L(p) = \max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v')\), then for any payoff function that makes a strategy profile of \(p\) SSE, by Proposition 4.1, the leader can always gain at least \(\max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v')\) by choosing at root from \(\arg\max_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v')\) and thus not all SSEs correspond to \(p\).

If \(p|_{v_i}\) is not strongly inducible at \(T_{v_i}\), then for any \(U_F|_{v_i}\) on \(T_{v_i}\) that makes a strategy profile \((\delta_f, \pi_F)|_{v_i}\) of \(p|_{v_i}\) SSE, there is always another \(p'|_{v_i}\), such that \(p'|_{v_i} \neq p|_{v_i}\), and one of its strategy profile \((\delta_f', \pi_F')|_{v_i}\) is also an SSE. Extend \((\delta_f', \pi_F')|_{v_i}\) to \((\delta_f', \pi_F')\) on \(T_{\text{root}}\) by letting \((\delta_f', \pi_F')|_{v_i} = v\), then \((\delta_f', \pi_F')\) corresponds to \(p'|_{v_i}\) on \(T_{v_i}\) and \(p|_{v_i} = 0\) otherwise. Then \((\delta_f', \pi_F')\) is also an SSE, leading to a contradiction.

When \(\mathcal{P}(\text{root}) = F\): first suppose for a Y-shape strongly inducible distribution \(p\), \(\text{Supp}(p) = \{z_1, z_2\}\), and \(U_L(z_1) = U_L(z_2)\). Consider any \(U_F\) that makes a strategy profile \((\delta_f, \pi_F)\) of \(p\) an SSE, and W.L.O.G., let \(U_F(z_1) \leq U_F(z_2)\). Then \(U_F(p)|_v \leq U_F(z_2)\) for any \(v \in H\) on the path from root to \(z_2\) and \(U_L(p) = U_L(z_2)\). Since \((\delta_f, \pi_F)\) is an SSE and so is feasible, \(z_2\) satisfies conditions in Lemma 1.1. There exists a strategy profile \((\delta_f', \pi_F' \in \text{BR}(\delta_f'))\) yielding \(z_2\). Thus \((\delta_f', \pi_F')\) is also an SSE, leading to a contradiction.

For a strongly inducible distribution \(p\) that \(U_L(p) \leq \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v')\), we show that \(p|_{v_i}\) is strongly inducible at \(T_{v_i}\). For any \(U_F\) that makes a strategy profile \((\delta_f, \pi_F)\) of \(p\) SSE in game \((T_{\text{root}}, U_L, U_F)\) and any feasible strategy profile \((\delta_f', \pi_F' \in \text{BR}(\delta_f'))\) in game \((T_{\text{root}}, U_L, U_F)\), it must be that \(\pi_F'(\text{root}) = v\), since otherwise the leader can always gain at least \(\min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \geq U_L(\delta_f, \pi_F)\) by committing to \(\delta_f\) and lead to a different distribution. Then the problem reduces to the strong inducibility on subgame \(T_{v_i}\), thus \(p|_{v_i}\) is strongly inducible at \(T_{v_i}\).
Now we prove the sufficiency. Specifically, we construct the follower’s payoff function as in the proof of Theorem 4.3, the only difference is that when \( p|_v \) is strongly inducible at subgame \( T_v \), we utilize the corresponding \( \tilde{U}_F|_v \) at \( T_v \) that all SSEs of subgame \( (T_v, U_L|_v, \tilde{U}_F|_v) \) correspond to \( p|_v \) at \( T_v \).

We show that all SSEs of game \((T_{root}, U_L, \tilde{U}_F)\) correspond to \( p \).

When \( P(root) = L \): First, by Theorem 4.3, there is an SSE \((\delta_L, \pi_F)\) of game \((T_{root}, U_L, \tilde{U}_F)\) that leads to \( p \). Consider any SSE \((\delta'_L, \pi'_F)\) of game \((T_{root}, U_L, \tilde{U}_F)\), then \((\delta'_L, \pi'_F)(root) = v \), since choosing any other child node makes the leader get at most \( \max_{v' \in \text{Child}(root), v' \neq v} M_L(root) < U_L(p) = U_L(\delta_L, \pi_F) \). Then since all SSEs of \((T_v, U_L|_v, \tilde{U}_F|_v)\) correspond to \( p|_v \), all SSEs of game \((T_{root}, U_L, \tilde{U}_F)\) correspond to \( p \).

When \( P(root) = F \): Case 1: When \( U_L(p) > \min_{v' \in \text{Child}(root), v' \neq v} M_L(v') \). Consider any other SSE \((\delta'_L, \pi'_F) \in BR(\delta'_L)) \). First if \( \pi'_F(root) = v_0 \), then the most the leader can get is \( m_0 < U_L(\delta_L, \pi_F) \). When \( \pi'_F(root) \neq v_0 \), by the proof of Theorem 4.3, to make \( U_L(\delta'_L, \pi'_F) = U_L(\delta_L, \pi_F) \), we have \( A(\delta'_L, \pi'_F) \subseteq A(\delta_L, \pi_F) \). Since the only distribution that has \( A(\delta_L, \pi_F) \) as its support and make the leader’s gain \( U_L(\delta_L, \pi_F) \) is \( p \), thus \((\delta'_L, \pi'_F)\) correspond to \( p \).

Case 2: When \( p|_v \) is strongly inducible at \( T_v \). Since the follower will always choose \( v \) at \( root \), and all SSEs of \((T_v, U_L|_v, \tilde{U}_F|_v)\) correspond to \( p|_v \), all SSEs of game \((T_{root}, U_L, \tilde{U}_F)\) correspond to \( p \).

\( \square \)

F.3 Corollaries of Theorem 5.5 Necessary for the Following Proofs

Actually, one can find out that the proofs of necessities in Theorem 5.3 almost apply to arbitrary strongly inducible distributions, except one condition.

Corollary F.2. If a realizable distribution \( p \in \Delta(Z) \) is strongly inducible, then

1. if \( P(root) = L \), then \( |\text{Supp}(p, root)| = 1 \). Let \( \text{Supp}(p, root) = \{v\} \), then all of the following conditions are met:

   (a) \( U_L(p) > \max_{v' \in \text{Child}(root), v' \neq v} M_L(v') \);

   (b) \( p|_v \) is strongly inducible at \( T_v \).

2. if \( P(root) = F \), let \( \text{Supp}(p, root) = \{v\} \), then at least one of following two conditions are met:

   (a) \( U_L(p) > \min_{v' \in \text{Child}(root), v' \neq v} M_L(v') \);

   (b) \( p|_v \) is strongly inducible at \( T_v \).

Proof. When \( P(root) = L \): Consider a strongly inducible distribution \( p \), then \( p \) is also inducible. Suppose \( \text{Supp}(p, root) = \{v_1, \ldots, v_k\} \subseteq \text{Child}(root) \) and \( k \geq 2 \). By the conditions of inducibility \( U_L(p)|_{v_i} = U_L(p) \) for \( i \in [k] \). Extend \( p|_{v_i} \) to \( p_i \) on \( T_{root} \) by letting \( p_i(z) = p|_{v_i}(z) \) for \( z \in Z_{v_i} \) and \( p_i(z) = 0 \) otherwise, then \( U_L(p_i) = U_L(p)|_{v_i} = U_L(p) \).

For any \( \tilde{U}_F \) that makes a strategy profile \((\delta_L, \pi_F)\) of \( p \) SSE and \( i \in [k] \), consider an extension of \((\delta_L, \pi_F)|_{v_i} \) to \( T_{root}, (\delta'_L, \pi'_F) \), that \((\delta'_L, \pi'_F)(root) = v_i, (\delta'_L, \pi'_F)|_{v_i} = (\delta_L, \pi_F)|_{v_i} \) and is defined arbitrarily on other nodes. Then \((\delta'_L, \pi'_F)\) corresponds to \( p_i \), is also feasible by Lemma 4.4 and \( U_L(\delta'_L, \pi'_F) = U_L(\delta_L, \pi_F) \). Thus \((\delta'_L, \pi'_F)\) is an SSE of game \((T_{root}, U_L, \tilde{U}_F)\), leading to a contradiction.

Now let \( \text{Supp}(p, root) = \{v\} \) and suppose \( U_L(p) = \max_{v' \in \text{Child}(root), v' \neq v} M_L(v') \), then for any payoff function that makes a strategy profile of \( p \) SSE, by Proposition 4.1, the leader can always gain at least \( \max_{v' \in \text{Child}(root), v' \neq v} M_L(v') \) by choosing at \( root \) from \( \text{arg max}_{v' \in \text{Child}(root), v' \neq v} M_L(v') \) and thus all SSEs correspond to \( p \).

If \( p|_v \) is not strongly inducible at \( T_v \), then for any \( \tilde{U}_F|_v \) on \( T_v \) that makes a strategy profile \((\delta_L, \pi_F)|_v \) of \( p|_v \) SSE, there is always another \( p'|_v \), such that \( p'|_v \neq p|_v \), and one of its strategy profile \((\delta'_L, \pi'_F)|_v \) is also an SSE. Extend \((\delta'_L, \pi'_F)|_v \) to \((\delta'_L, \pi'_F)\) on \( T_{root} \) by letting \((\delta'_L, \pi'_F)(root) = v \), then \((\delta'_L, \pi'_F)\)
corresponds to \( p' \) that \( p'|_v \) on \( T_v \) and \( p'(z) = 0 \) otherwise. Then for any \( \tilde{U}_F \) that makes \( (\delta_L, \pi_F) \) an SSE, \( (\delta'_L, \pi'_F) \) is also an SSE, leading to a contradiction.

**When \( P(\text{root}) = F \):** First suppose for a Y-shape strongly inducible distribution \( p \), \( \text{Supp}(p) = \{z_1, z_2\} \), and \( U_L(z_1) = U_L(z_2) \). Consider any \( \tilde{U}_F \) that makes a strategy profile \( (\delta_L, \pi_F) \) of \( p \) an SSE, and W.L.O.G., let \( \tilde{U}_F(z_1) \leq \tilde{U}_F(z_2) \). Then \( \tilde{U}_F(p)|_v \leq \tilde{U}_F(z_2) \) for any \( v \in H \) on the path from root to \( z_2 \) and \( U_L(p) = U_L(z_2) \). Since \( (\delta_L, \pi_F) \) is an SSE and so is feasible, \( z_2 \) satisfies conditions in **Lemma F.1**, and there exists a \( (\delta'_L, \pi'_F) \in \text{BR}(\delta_L) \) leading to \( z_2 \). Thus \( (\delta'_L, \pi'_F) \) is also an SSE, leading to a contradiction.

For a strongly inducible distribution \( p \) that \( U_L(p) \leq \min_{v' \in \text{Child}(\text{root}), v' \neq v} M_L(v') \), we show that \( p|_v \) is strongly inducible at \( T_v \). For any \( \tilde{U}_F \) that makes a strategy profile \( (\delta_L, \pi_F) \) of \( p \) SSE in game \((T_{\text{root}}, U_L, \tilde{U}_F)\) and any feasible strategy profile \( (\delta'_L, \pi'_F) \in \text{BR}(\delta'_L) \) in game \((T_{\text{root}}, U_L, \tilde{U}_F)\), it must be that \( \pi'_F(\text{root}) = v \), since otherwise the leader can always gain at least \( \min_{w' \in \text{Child}(\text{root}), w' \neq v} M_L(v') \) \( \geq U_L(\delta_L, \pi_F) \) by committing to \( \delta'_L \) and result in a different distribution. Then the problem reduces to the strong inducibility on subgame \( T_v \), thus \( p|_v \) is strongly inducible at \( T_v \).

And to facilitate the following proofs, we state the non-recursive characterization for Y-shape strongly inducible distributions.

**Corollary F.3.** A “Y-shape” realizable distribution \( p \) is strongly inducible if and only if

1. if \( |\text{Supp}(p)| = 2 \), let \( \text{Supp}(p) = \{z_1, z_2\} \), then
   (a) \( U_L(z_1) \neq U_L(z_2) \);
   (b) \( z_1 \) and \( z_2 \) has common ancestors \( v \in \mathcal{P}^{-1}(F) \) and \( w \in \text{Child}(v) \) such that
   \[
   U_L(p) > \min_{w' \in \text{Child}(v), w' \neq w} M_L(w');
   \]
   (c) \( U_L(p) > M_L(u) \) for \( u \in \mathcal{P}^{-1}(L) \) on the path from root to \( v \);

2. if \( |\text{Supp}(p)| = 1 \), let \( \text{Supp}(p) = \{z\} \), then
   \[
   U_L(p) > \max_{v \in \text{Child}(u), v \neq v(u, z)} M_L(v);
   \]
   for \( u \in \mathcal{P}^{-1}(L) \) and \( v(u, z) \in \text{Child}(u) \) on the path from root to \( z \).

**Proof.** First notice that for a Y-shape distribution \( p \) to be realizable and \( |\text{Supp}(p)| = 2 \), let \( \text{Supp}(p) = \{z_1, z_2\} \), then the least common ancestor of \( z_1 \) and \( z_2 \) must belong to \( \mathcal{P}^{-1}(L) \), which we denote as \( \text{LCA} \). Now we first consider **Case (1)**:

**The necessity:** Condition (a) follows from **Theorem 5.3**. Suppose for the sake of contradiction, condition (b) is not satisfied: by the proof of **Corollary 4.8**, \( z_1 \) and \( z_2 \) must have at least one common ancestor in \( \mathcal{P}^{-1}(F) \). Now suppose for any common ancestors \( v \in \mathcal{P}^{-1}(F) \) and \( w \in \text{Child}(v) \) of \( z_1 \) and \( z_2 \),
\[
U_L(p) \leq \min_{w' \in \text{Child}(v), w' \neq w} M_L(w'),
\]
then by the conditions in **Theorem 5.3**, \( p|_u \) has to be strongly inducible at \( T_u \) for any \( u \) on the path from root to \( \text{LCA} \). When \( u = \text{LCA} \in \mathcal{P}^{-1}(L) \), that \( |\text{Supp}(p|_{\text{LCA}})| = 2 \) leads to a contradiction.

Condition (c) follows from condition (1)(b) of **Theorem 5.3**.

**The sufficiency:** By condition (2)(a) of **Theorem 5.3**, \( p|_v \) is strongly inducible at \( T_v \). Suppose \( p|_u \) is strongly inducible at \( T_u \) for ancestor \( u \) of \( v \), we prove that for \( v \)’s ancestor \( t \), that \( u \in \text{Child}(t), p|_t \)
is strongly inducible at $T_i$: the case when $t \in \mathcal{P}^{-1}(F)$ follows from condition (2)(b) of Theorem 5.3; when $t \in \mathcal{P}^{-1}(L)$, conditions (1)(b) are satisfied and (1)(a) of Theorem 5.3 follows from condition (c).

Then we consider Case (2): The necessity follows from condition (1)(b) of Theorem 5.3.

**The sufficiency:** By checking conditions of Theorem 5.3, $p|_u$ is strongly inducible at $T_u$. Suppose $p|_u$ is strongly inducible at $T_u$ for ancestor $u$ of $v$, for $v$’s ancestor $t$, that $u \in \text{Child}(t)$, $p|_u$ is strongly inducible at $T_t$ by Theorem 5.3.

Finally, we show that for any strongly inducible distribution $p \in \Delta(Z)$, there always exists a dominant $Y$-shape strongly inducible distribution $p^*$.

**Corollary F.4.** For any extensive-form game $(T_{\text{root}}, U_L, U_F)$ and any strongly inducible distribution $p$, there exists a $Y$-shape strongly inducible distribution $p^* \in \Delta(Z)$, such that $U_F(p^*) \geq U_F(p)$ and $U_L(p^*) \geq U_L(p)$.

**Proof.** We show by structural induction over the game tree.

**Inductive Base:** When $\text{root} \in Z$, the only distribution $p$ that $p(\text{root}) = 1$ is strongly inducible and is $Y$-shape.

**Inductive Step:** When $\text{root} \in H$, suppose the inductive hypothesis holds.

When $\mathcal{P}(\text{root}) = L$: Since $p$ satisfies the conditions in Corollary F.2, let $\text{Supp}(p, \text{root}) = \{v\}$, then $p|_v$ is strongly inducible at $T_v$. By the inductive hypothesis, there is a $Y$-shape strongly inducible distribution $p^*_v$, on $T_v$ satisfies $U_L(p^*_v) \geq U_L(p)|_v$, and $U_F(p^*_v)|_v \geq U_F(p)|_v$. Let $p^*$ be its extension to $T_{\text{root}}$ that $p^*(z) = 0$ for $z \in Z \setminus Z_v$, then

$$U_L(p^*) \geq U_L(p) \geq \max_{v' \in \text{Child}(\text{root}), v' \neq \text{root}} M_L(v').$$

Thus $p^*$ is strongly inducible and $U_F(p^*) \geq U_F(p)$, $U_L(p^*) \geq U_L(p)$.

When $\mathcal{P}(\text{root}) = F$: Case 1: When $U_L(p) > \min_{v' \in \text{Child}(\text{root}), v' \neq \text{root}} M_L(v')$. If $p$ is not $Y$-shape, then $|\text{Supp}(p)| > 2$, and $U_F(p)$ is a mixture of utilities of more than two leaf nodes. There exists a mixture of two leaf nodes $z_1$ and $z_2$ in the set $\{z | z \in Z, p(z) > 0\}$, say $p^*$, satisfying $U_L(p^*) \geq U_L(p) > \min_{v' \in \text{Child}(\text{root}), v' \neq \text{root}} M_L(v')$ and $U_F(p^*) \geq U_F(p)$.

If $U_L(z_1) \neq U_L(z_2)$, then $p^*$ is strongly inducible. Else if $U_L(z_1) = U_L(z_2) = U_L(p_1)$, suppose $U_F(z_1) \leq U_F(z_2)$, then distribution $p^{**}$ that puts all probability on $z_2$ satisfies $U_F(p^{**}) = U_F(z^*) \geq U_F(p)$. $p^{**}$ is $Y$-shape, satisfies all conditions in Theorem 5.3 and thus is strongly inducible.

Case 2: When $p|_{\text{root}}$ is strongly inducible at $T_{\text{root}}$. By the inductive hypothesis, there exists a $Y$-shape strongly inducible distribution $p^*_v \in \Delta(Z_v)$ satisfying $U_L(p^*_v)|_v \geq U_L(p)|_v$ and $U_F(p^*_v)|_v \geq U_F(p)|_v$. Let $p^*$ be its extension to $T_{\text{root}}$ that $p^*(z) = 0$ for $z \in Z \setminus Z_v$, then $p^*$ is $Y$-shape and strongly inducible.

**F.4 Proof of Theorem 5.4**

**Theorem 5.4.** It is polynomial-time tractable to

1. decide if a strongly inducible leaf node exists;
2. if so, find an optimal strongly inducible leaf node and construct a payoff function that induces it.

**Proof.** Note that we can enumerate each leaf node to check if it satisfies the conditions in Theorem 5.2. If there is no leaf node satisfying such conditions, then we conclude that there does not exist a strongly inducible leaf node. Otherwise, pick the strongly inducible leaf node with maximal follower’s utility and construct a follower’s payoff function to induce it by Theorem 5.2. This process can be solved in $O((|H| + |Z|)^2)$ time.
F.5 Proof of Theorem 5.5

Theorem 5.5. It is polynomial-time tractable to
(1) decide if a strongly inducible distribution exists;
(2) if so, decide if an optimal strongly inducible distribution exists;
(3) if so, find an optimal strongly inducible distribution; if not, for any \( \epsilon > 0 \), find an \( \epsilon \)-optimal strongly inducible distribution \( p^* \) such that
\[
U_F(p^*) \geq \sup_{p \in SP(\Delta_L \times \Pi_F)} U_F(p) - \epsilon.
\]
In both cases, construct a payoff function that induces the distribution in polynomial time.

Proof. We first use a lemma to prove that if \( SP(\Delta_L \times \Pi_F) \neq \emptyset \), either there exists an optimal strongly inducible distribution, or for any \( \epsilon \), an \( \epsilon \)-optimal one exists

Lemma F.5. For game \( (T_{root}, U_L, U_F) \), if \( SP(\Delta_L \times \Pi_F) \neq \emptyset \), then either

1. there exists a distribution \( p^* \in SP(\Delta_L \times \Pi_F) \), such that
\[
U_F(p^*) = \sup_{p \in SP(\Delta_L \times \Pi_F)} U_F(p)
\]
or

2. for any \( \epsilon > 0 \), there exists \( p(\epsilon) \in SP(\Delta_L \times \Pi_F) \), such that
\[
U_F(p(\epsilon)) \geq \sup_{p \in SP(\Delta_L \times \Pi_F)} U_F(p) - \epsilon
\]

Proof of Lemma F.5. By Corollary F.4, it suffices to consider all the Y-shape strongly inducible distributions, which we denote as \( SYP(\Delta_L \times \Pi_F) \).

Note that, to prove the lemma, it suffices to prove that there exists a finite number of open intervals \( \{(a_i, b_i)\}_{i \in [k]} \) and a finite number of points \( \{r_i\}_{i \in [s]} \), such that \( U_F(SYP(\Delta_L \times \Pi_F)) = (\bigcup_{i \in [k]}(a_i, b_i)) \cup \bigcup_{i \in [s]}\{r_i\} \). For each \( z \in Z \), define
\[
M(z) := \max_{u \in P^{-1}(L) \text{ and } v(u, z) \in \text{Child}(u)} \max_{v \in \text{Child}(u), v \neq v(u, z)} M_L(v)
\]
on the path from \( \text{root} \) to \( z \)

then by Corollary F.3, \( SYP_1 := \{z \in Z : U_L(z) > M(z)\} \) consists of all strongly inducible distributions that have support size of 1.

Consider \( (z_1, z_2) \in E(\text{root}) \), that is, the least common ancestor of \( z_1, z_2 \in Z \) belongs to \( P^{-1}(L) \). And we consider those that \( U_L(z_1) \neq U_L(z_2) \). For any of their common ancestors \( v \in P^{-1}(F) \) and \( w \in \text{Child}(v) \), define
\[
M(v, (z_1, z_2)) := \max u \in P^{-1}(L) \min_{w' \neq w \in \text{Child}(v)} M_L(u),
\]
on the path from \( \text{root} \) to \( v \)

\[
M(z_1, z_2) := \min_{v \in CAF(z_1, z_2)} M(v, (z_1, z_2)),
\]

given \( z_1, z_2 \in Z \) and \( \alpha \in [0, 1] \), recall that \( p_{\alpha, z_1, z_2} \) denotes the distribution \( p \in \Delta(Z) \) that \( p(z_1) = \alpha \) and \( p(z_2) = 1 - \alpha \). Then \( SYP_{(z_1, z_2)} := \{p_{\alpha, z_1, z_2} : \alpha \in (0, 1), U_L(p_{\alpha, z_1, z_2}) > M(z_1, z_2)\} \) denotes all the strongly inducible distributions that has support \( \{z_1, z_2\} \). \( U_F(SYP_{(z_1, z_2)}) \) is an open interval.
Since $E(root)$ and $Z$ are finite sets, and

$$U_F(SYP(\Delta_L \times \Pi_F)) = \left( \bigcup_{z \in SYP_1} U_F(z) \right) \cup \left( \bigcup_{(z_1, z_2) \in E(root)} U_F(SYP_{(z_1, z_2)}) \right),$$

$U_F(SYP(\Delta_L \times \Pi_F))$ is a union of a finite number of of open intervals and a finite set. This finishes the proof.

Now we finish the proof of Theorem 5.5.

The algorithm first calculates the sets $SYP_1$ and $SYP_{(z_1, z_2)}$ for each $(z_1, z_2) \in E(root)$ as in the proof of Lemma 5.5.

Let $C := \sup \left( \bigcup_{z \in SYP_1} U_F(z) \right) \cup \left( \bigcup_{(z_1, z_2) \in E(root)} U_F(SYP_{(z_1, z_2)}) \right)$.

**Case 1:** If $C \in \left( \bigcup_{z \in SYP_1} U_F(z) \right) \cup \left( \bigcup_{(z_1, z_2) \in E(root)} U_F(SYP_{(z_1, z_2)}) \right)$, then we enumerate $z \in SYP_1$ to check if $U_F(z) = C$ and enumerate $(z_1, z_2) \in E(root)$ to check if $C \in U_F(SYP_{(z_1, z_2)})$. Since $C \in \left( \bigcup_{z \in SYP_1} U_F(z) \right) \cup \left( \bigcup_{(z_1, z_2) \in E(root)} U_F(SYP_{(z_1, z_2)}) \right)$, we must find a Y-shape distribution $p$ such that $U_F(p) = C$ and $p$ is strongly inducible.

**Case 2:** Otherwise, suppose that we are given an $\epsilon > 0$. Then we enumerate $z \in SYP_1$ to check if $C - \epsilon \leq U_F(z)$ and enumerate $(z_1, z_2) \in E(root)$ to check if $C - \epsilon \leq \sup(U_F(SYP_{(z_1, z_2)}))$. Since $\left( \bigcup_{z \in SYP_1} U_F(z) \right) \cup \left( \bigcup_{(z_1, z_2) \in E(root)} U_F(SYP_{(z_1, z_2)}) \right)$ is a union of a finite number of open intervals and a finite set, we must find a Y-shape distribution $p$ such that $U_F(p) \geq C - \epsilon$ and $p$ is strongly inducible.

Note that once we get a “Y-shape” distribution, we can construct a corresponding payoff function to induce it according to the proof of Theorem 4.3 by Theorem 5.3.

This finishes the proof.

F.6 Proof of Proposition 5.6

**Proposition 5.6.** $\sup_{p \in SP(\Delta_L \times \Pi_F)} U_F(p) \geq \sup_{z \in SP(\Pi_L \times \Pi_F)} U_F(z)$.

**Proof.** Same as in the proof of proposition 4.10, it suffices to show that $SP(\Pi_L \times \Pi_F) \subseteq SP(\Delta_L \times \Pi_F)$.

Denote the set of all Y-shape strongly inducible distributions as $SY P(\Delta_L \times \Pi_F)$, since $p \in SP(\Pi_L \times \Pi_F)$ is Y-shape, we show by induction that $p$ satisfies conditions in Theorem 5.3.

**Inductive Base:** When $root \in Z$, the only realizable distribution $p$ where $p(root) = 1$ is both strongly inducible with pure and behavioral commitment.

**Inductive Step:** When $root \in H$, we assume the inductive hypothesis holds in subtrees and consider any $p \in SP(\Pi_L \times \Pi_F)$.

**When $P(root) = L$,** by the definition of $SP(\Pi_L \times \Pi_F)$, $|\text{Supp}(p, root)| = 1$, and condition (1)(a) of Theorem 5.3 is satisfied. Let $\text{Supp}(p, root) \setminus \{v\}$, since $p_v$ is strongly inducible with pure commitment at subgame $T_v$, by the inductive hypothesis, it is also strongly inducible with behavioral commitment. Thus condition (1)(b) is satisfied. $p$ is strongly inducible with behavioral commitment at $T_{root}$.

**When $P(root) = F$,** let $\text{Supp}(p, root) = \{v\}$.

If $UL(p) > \min_{v' \in \text{Child}(root), v' \neq v} ML(v')$, then $p$ satisfies condition (2)(a) of Theorem 5.3 and is strongly inducible with behavioral commitment.

If $p|_v$ is strongly inducible with pure commitment at $T_v$, then by the inductive hypothesis, it is also strongly inducible with behavioral commitment. Thus condition (2)(b) is satisfied. $p$ is strongly inducible with behavioral commitment at $T_{root}$. 

□
F.7 Proof of Lemma 5.8

Lemma 5.8. Game $(T_{root}, U_L, U_F)$ satisfies property $P$ if and only if it satisfies property $P'$.

Proof. The necessity follows from that if $P$ then $U_F(p_1) \geq U_F(p^*) - \epsilon$. By Corollary F.4, there exists a Y-shape strongly inducible distribution $p_1$ such that $U_F(p_1) \geq U_F(p^*) - \epsilon$. By Corollary F.3, we argue that with small enough $\epsilon > 2U$, there exists a Y-shape strongly inducible distribution $p_2$ such that $U_L(p_2) \geq U_L(p_1)$, and $U_F(p_2) \geq U_F(p_1)$, which yields $U_F(p_2) \geq U_F(p^*) - \epsilon$.

F.8 Proof of Theorem 5.7

Theorem 5.7. Game $(T_{root}, U_L, U_F)$ satisfies property $P$ if and only if it satisfies Condition 1 or Condition 2.

By analyzing the possible optimal Y-shape inducible distributions a game may possess, we first prove two important lemmas.

Lemma F.6. If game $(T_{root}, U_L, U_F)$ satisfies Condition 1, then it satisfies property $P$.

Proof. W.L.O.G., suppose $p(z_1) = \alpha > 0$ and $U_F(z_1) \leq U_F(z_2)$, thus $U_F(p) = \alpha U_F(z_1) + (1 - \alpha)U_F(z_2)$ and $U_L(p) = \alpha U_F(z_1) + (1 - \alpha)U_F(z_2)$. Since $U_L(z_1) \neq U_L(z_2)$, for any small enough $\epsilon > 0$, there exists $\alpha' > 0$, and distribution $p'$ such that $p(z_1) = \alpha'$ and $p(z_2) = 1 - \alpha'$, satisfying $U_F(p') > U_F(p) - \epsilon$, and $U_L(p') > U_L(p)$.

Furthermore, suppose $v \in P^{-1}(F)$ and $w \in Child(v)$ are common ancestors of $z_1$ and $z_2$ that $U_L(p) \geq \min_{w \in Child(v), w' \neq w} M_L(w')$, the existence of which is assured by Corollary F.3, then

$$U_L(p') > U_L(p) \geq \min_{w \in Child(v), w' \neq w} M_L(w');$$

$$U_L(p') > U_L(p) \geq M_L(u) \text{ for any } u \text{ on the path from root to } v.$$

Thus $p'$ is strongly inducible and game $(T_{root}, U_L, U_F)$ satisfies Property $P$.

Lemma F.7. If game $(T_{root}, U_L, U_F)$ does not satisfy Condition 1, then it satisfies property $P'$ if and only if it satisfies Condition 2.

Given $z_1, z_2 \in Z$ and $\alpha \in [0, 1]$, recall that $p_{\alpha, z_1, z_2}$ denotes the distribution $p \in \Delta(Z)$ that $p(z_1) = \alpha$ and $p(z_2) = 1 - \alpha$.

Proof. The necessity: Now consider the case for all optimal Y-shape inducible distributions $p^*$, $p^*$ is not strongly inducible, and either $|supp(p^*)| = 1$ or $U_L(z_1) = U_L(z_2)$ (which also means $U_F(z_1) = U_F(z_2)$) when $|supp(p^*)| = 2$ and $supp(p^*) = \{z_1, z_2\}$. First noticing that for property $P'$ to be satisfied, for small enough $\epsilon > 0$, there must exist a Y-shape strongly inducible distribution $p$ with $|supp(p)| = 2$ such that $U_F(p) \geq U_F(p^*) - \epsilon$. Let $\epsilon_0 = \min_{z \in Z, U_F(z) \neq U_F(p^*), |U_F(p^*) - U_F(z)|}$. Since leaf nodes with utility $U_F(p^*)$ is not strongly inducible, for $\epsilon < \epsilon_0$, there does not exist a strongly inducible leaf node with utility at least $U_F(p^*) - \epsilon$.

For any $\epsilon < \epsilon_0$, consider an aforementioned Y-shape strongly inducible distribution $p$ with $|supp(p)| = 2$, and $supp(p) = \{z_1, z_2\}$, then $U_L(z_1) \neq U_L(z_2)$, and $U_F(z_1) \neq U_F(z_2)$. W.L.O.G., assume $U_L(z_1) > U_L(z_2)$ and $U_F(z_1) < U_F(z_2)$, then $U_L(p') < U_L(z_1)$ and $U_F(p') \leq U_F(z_2)$. Again we argue that with small enough $\epsilon > 0$, $z_2$ must be optimally inducible: otherwise denote the set of all pairs of leaf nodes $(z'_1, z'_2)$ as $I$, which satisfy that: neither node is optimally inducible, and we define $\epsilon_1 = U_F(p^*) - \max_{(z'_1, z'_2) \in I} \max_{\alpha \in [0, 1]} U_F(p_{\alpha, z'_1, z'_2})$, $p_{\alpha, z'_1, z'_2}$ is inducible. |
then $\epsilon_1 > 0$ and for $\epsilon < \epsilon_1$, there does not exist $(z'_1, z'_2) \in I$ and an inducible distribution $p_{\alpha, z'_1, z'_2}$ with utility at least $U_F(p^*) - \epsilon$. Neither does a strongly inducible one.

Now for any node $z_1$ that its least common ancestor with $z^*$, denoted as $LCA$, belongs to $P^{-1}(L)$ and $U_L(z_1) > U_L(z^*)$, noticing that for distribution $p_{\alpha, z_1, z^*}$, where $\alpha \in (0, 1)$ to be inducible, there must exist a node belonging to $P^{-1}(F)$. If $U_L(z^*) < \min_{w \in \text{Child}(v), w \neq w(v, z^*)} M_L(w)$ for all $v \in P^{-1}(F)$ on the path from root to $z^*$, then for small enough $\alpha > 0, U_L(p_{\alpha, z_1, z^*}) < \min_{w \in \text{Child}(v), w \neq w(v, z^*)} M_L(w)$ and thus is not inducible, let alone strongly inducible. Thus there must exists a node $v$ on the path from root to LCA, such that $U_L(z^*) \geq \min_{w \in \text{Child}(v), w \neq w(v, z^*)} M_L(w)$. This finishes the proof of the necessities.

The sufficiency: that $p^*$ is strongly inducible makes the game satisfies property $P$ and thus $P'$. Now suppose game $(T_{\text{root}}, U_L, U_F)$ satisfies condition (2), then for any $\epsilon > 0$, there exists $\alpha > 0$, such that $U_F(p_{\alpha, z_1, z^*}) > U_F(z^*) - \epsilon$, and $U_L(p_{\alpha, z_1, z^*}) > U_L(z^*)$. Since $z^*$ is inducible, and $U_L(z^*) \geq \min_{w \in \text{Child}(v), w \neq w(v, z^*)} M_L(w)$, thus $p_{\alpha, z_1, z^*}$ is strongly inducible.

Proof of Theorem 5.7. Since the optimal Y-shape inducible distributions either satisfies conditions in Lemma F.7 or in Lemma F.6, and by Lemma 5.8, the theorem is proved.
References

Marco Barreno, Blaine Nelson, Anthony D Joseph, and J Doug Tygar. 2010. The security of machine learning. *Machine Learning* 81, 2 (2010), 121–148.

Georgios Birmpas, Jiarui Gan, Alexandros Hollender, Francisco J Marmolejo-Cossio, Ninad Rajgopal, and Alexandros A Voudouris. 2021. Optimally Deceiving a Learning Leader in Stackelberg Games. *Journal of Artificial Intelligence Research* 72 (2021), 507–531.

Avrim Blum, Nika Haghtalab, and Ariel D Procaccia. 2014. Learning Optimal Commitment to Overcome Insecurity. In *Advances in Neural Information Processing Systems*, Z. Ghahramani, M. Welling, C. Cortes, N. Lawrence, and K. Q. Weinberger (Eds.), Vol. 27. Curran Associates, Inc. https://proceedings.neurips.cc/paper/2014/file/cc1aa436277138f61cda703991069eaf-Paper.pdf

Yenjae Chang, Clifford Winston, and Jia Yan. 2021. Does Uber Benefit Travelers by Price Discrimination? (2021).

Xi Chen, Xiaotie Deng, and Shang-Hua Teng. 2009. Settling the complexity of computing two-player Nash equilibria. *J. ACM* 56, 3 (2009), Art. 14, 57. https://doi.org/10.1145/1516512.1516516

Zhaohua Chen, Xiaotie Deng, Jicheng Li, Chang Wang, and Mingwei Yang. 2022. Budget-Constrained Auctions with Unassured Priors. arXiv preprint arXiv:2203.16816 (2022).

Matt Cooper, Jun Ki Lee, Jacob Beck, Joshua D Fishman, Michael Gillett, Zoë Papakipos, Aaron Zhang, Jerome Ramos, Aansh Shah, and Michael L Littman. 2019. Stackelberg Punishment and Bully-Proofing Autonomous Vehicles. In *International Conference on Social Robotics*. Springer, 368–377.

Constantinos Daskalakis, Paul W. Goldberg, and Christos H. Papadimitriou. 2009. The complexity of computing a Nash equilibrium. *SIAM J. Comput.* 39, 1 (2009), 195–259. https://doi.org/10.1137/070699652

Xiaotie Deng, Tao Lin, and Tao Xiao. 2020. *Private Data Manipulation in Optimal Sponsored Search Auction*. Association for Computing Machinery, New York, NY, USA, 2676–2682. https://doi.org/10.1145/3366423.3380023

Karel Durkota, Viliam Lisý, Branislav Bošanský, Christopher Kiekintveld, and Michal Pechouček. 2019. Hardening networks against strategic attackers using attack graph games. *Computers & Security* 87 (2019), 101578.

F. M. Delle Fave, A.X. Jiang, Z. Yin, C. Zhang, M. Tambe, S. Kraus, and J.P. Sullivan. 2014. Game-theoretic Security Patrolling with Dynamic Execution Uncertainty and a Case Study on a Real Transit System. *Journal of Artificial Intelligence Research* 50 (2014), 321–367.

Jiarui Gan, Qingyu Guo, Long Tran-Thanh, Bo An, and Michael Wooldridge. 2019a. Manipulating a Learning Defender and Ways to Counteract. In *Advances in Neural Information Processing Systems*, H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. Fox, and R. Garnett (Eds.), Vol. 32. Curran Associates, Inc. https://proceedings.neurips.cc/paper/2019/file/c4819d06b0ca810d38506453fcaae9d8-Paper.pdf

Jiarui Gan, Haifeng Xu, Qingyu Guo, Long Tran-Thanh, Zinovi Rabinovich, and Michael Wooldridge. 2019b. Imitative follower deception in stackelberg games. In *Proceedings of the 2019 ACM Conference on Economics and Computation*. 639–657.
Joshua Letchford and Vincent Conitzer. 2010. Computing optimal strategies to commit to in extensive-form games. In Proceedings of the 11th ACM conference on Electronic commerce. 83–92. 1, 2, 1, 6

Joshua Letchford, Vincent Conitzer, and Kamesh Munagala. 2009. Learning and Approximating the Optimal Strategy to Commit To. In Algorithmic Game Theory, Marios Mavronicolas and Vicky G. Papadopoulou (Eds.). Springer Berlin Heidelberg, Berlin, Heidelberg, 250–262. 1

Daniel Lowd and Christopher Meek. 2005. Adversarial learning. In Proceedings of the eleventh ACM SIGKDD international conference on Knowledge discovery in data mining. 641–647. 1

Arwa Mahdawi. 2018. Is your friend getting a cheaper Uber fare than you are? The Guardian (2018).

Roger B. Myerson. 1981. Optimal Auction Design. Math. Oper. Res. 6, 1 (1981), 58–73. https://doi.org/10.1287/moor.6.1.58 1

Thanh H. Nguyen and Haifeng Xu. 2019. Imitative Attacker Deception in Stackelberg Security Games. In Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19. International Joint Conferences on Artificial Intelligence Organization, 528–534. https://doi.org/10.24963/ijcai.2019/75 1

Thanh H. Nguyen, Arunesh Sinha, and He He. 2020. Partial Adversarial Behavior Deception in Security Games. In Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence, IJCAI-20, Christian Bessiere (Ed.). International Joint Conferences on Artificial Intelligence Organization, 283–289. https://doi.org/10.24963/ijcai.2020/40 Main track. 1

Thanh H. Nguyen, Yongzhao Wang, Arunesh Sinha, and Michael P. Wellman. 2019a. Deception in Finitely Repeated Security Games. Proceedings of the AAAI Conference on Artificial Intelligence 33, 01 (Jul. 2019), 2133–2140. https://doi.org/10.1609/aaai.v33i01.33012133 1

Thanh H Nguyen, Amulya Yadav, Branislav Bosansky, and Yu Liang. 2019b. Tackling sequential attacks in security games. In International Conference on Decision and Game Theory for Security. Springer, 331–351. 1

Binghui Peng, Weiran Shen, Pingzhong Tang, and Song Zuo. 2019. Learning optimal strategies to commit to. In Proceedings of the AAAI Conference on Artificial Intelligence, Vol. 33. 2149–2156. 1

Zinovi Rabinovich, Albert Xin Jiang, Manish Jain, and Haifeng Xu. 2015. Information Disclosure as a Means to Security. In Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems (Istanbul, Turkey) (AAMAS ’15). International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 645–653. 1
Aaron Roth, Jonathan Ullman, and Zhiwei Steven Wu. 2016. Watch and Learn: Optimizing from Revealed Preferences Feedback. In Proceedings of the Forty-Eighth Annual ACM Symposium on Theory of Computing (Cambridge, MA, USA) (STOC ’16). Association for Computing Machinery, New York, NY, USA, 949–962. https://doi.org/10.1145/2897518.2897579

Aaron Schlenker, Omkar Thakoor, Hai teng Xu, Fei Fang, Milind Tambe, Long Tran-Thanh, Phebe Vayanos, and Yevgeniy Vorobeychik. 2018. Deceiving Cyber Adversaries: A Game Theoretic Approach. In Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems (Stockholm, Sweden) (AAMAS ’18). International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 892–900.

Arunesh Sinha, Fei Fang, Bo An, Christopher Kiekintveld, and Milind Tambe. 2018. Stackelberg security games: Looking beyond a decade of success. IJCAI.

Pingzhong Tang and Yulong Zeng. 2018. The Price of Prior Dependence in Auctions. In Proceedings of the 2018 ACM Conference on Economics and Computation (Ithaca, NY, USA) (EC ’18). Association for Computing Machinery, New York, NY, USA, 485–502. https://doi.org/10.1145/3219166.3219183

Heinrich von Stackelberg. 1934. Marktform und gleichgewicht. J. Springer.

Haifeng Xu, Zinovi Rabinovich, Shaddin Dughmi, and Milind Tambe. 2015. Exploring Information Asymmetry in Two-Stage Security Games. Proceedings of the AAAI Conference on Artificial Intelligence 29, 1 (Feb. 2015). https://ojs.aaai.org/index.php/AAAI/article/view/9290

Yue Yin, Bo An, Yevgeniy Vorobeychik, and Jun Zhuang. 2013. Optimal deceptive strategies in security games: A preliminary study. In Proc. of AAAI.