Entropy Index as a Measure of Heartbeat Irregularity

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Abstract

A method is proposed to analyze the heartbeat waveform that can yield a reliable characterization of the structure after only a few pulses. The measure suggested is entropy index that is related to the one found effective in describing chaotic behaviors in a wide variety of physical systems. When applied to the ECG data that include ventricular fibrillation, the index is shown to change drastically within a few pulses. Wavelet analysis is used to exhibit different scaling behaviors in different phases.

Concepts from the theory of nonlinear dynamics and statistical physics have been applied to the study of nonstationary time series, such as human heartbeats [1, 2, 3, 4] and brain electrical activities[5, 6]. Those analyses generally involve the use of data recorded over a long period of time. In this paper we propose a method of analysis that needs only a short duration of the time series data, as short as, say, ten heartbeats, for example. Conceptually,
the method is interesting because the proposed measure is related to the study of fluctuations in a diverse range of physical problems, e.g., hadron production in high-energy collisions [7, 8], classical chaotic systems [9], and phase transition [10]. The underlying universality in all those applications is rooted in the attempt to quantify the fluctuations of spatial patterns, for which an effective measure found is the entropy index [7, 9]. The application to the ECG time series, as we shall discuss here, provides a diagnostic tool that is both simple and efficient.

A major effort initiated by physicists to analyze the human heartbeat time series is to study the fluctuation of time intervals between the $R$ pulses. The study of such fluctuations is motivated by the possible analogy with critical behaviors in statistical systems, where fluctuations at all length (time) scales occur. While that is certainly an interesting area of investigation, the data required run in excess of $10^4$ heartbeats [3, 4]. There are, however, a great deal of information about the heartbeat time series that is discarded when the focus is only on the interbeat time intervals. As is well known, the structure of the time series between beats changes drastically when a heart goes into fibrillation [11]. The question is how to quantify that structure in an efficient manner so that the numerical value of an appropriate measure can be determined after a few beats. Of course, there is no need for such a measure if one has at hand the data for both before and during fibrillation, anymore than the need for a smoke detector when a house is actually on fire. However, the availability of a numerical measure of the cardiac activity is clearly a useful tool, especially for patients with irregular behaviors of the heart.

The analysis that we propose has its origin in the study of spatial patterns associated with the final state of particles in momentum space detected at the end of each event in high-
energy collisions [7]. For each event the factorial moments are used to describe the pattern; those moments have the virtue of filtering out the statistical fluctuations [12]. The nature of the fluctuations of those moments from event to event is quantified by an index $\mu$, which is larger when the fluctuation is larger. For a heartbeat time series, we partition it into many segments of short duration (e.g., 2 sec), regard each segment as a pattern, and characterize each pattern by studying the fluctuations from bin to bin. Since statistical fluctuation does not have the same meaning in the heartbeat problem as for particle production, we shall not use the factorial moments. In their place we shall employ the wavelet analysis [13, 14, 15], which is natural for a problem that has sharp spikes and low bumps. The corresponding entropy index is then a measure of the fluctuation of the normalized wavelet coefficients at various scales of resolution.

In Fig. 1 we show the digitized electrocardiogram data that we shall analyze. The data were provided by Minh [16], recorded at the Stanford University Medical School, when a patient’s heart went into ventricular fibrillation, followed by a defibrillation process. The three phases (normal, abnormal and recovery) are clearly identifiable visually in Fig. 1. In Fig. 2 are shown in more detail the structures between the pulses in the normal and abnormal phases. Evidently, the minor peaks and dips between the major spikes (called $R$ “waves” [11]) behave very differently in the two phases. To capture and characterize those differences is therefore our task.

In the digitized data of Fig. 1 there are roughly 240 points between two successive $R$ pulses in the normal phase, which spans about 3600 points. There are approximately 2500 points in the abnormal phase. Given the data, we divide the time series into segments of 512 points each, calling each segment $S_n$, with $n = 1, \ldots, 7$ belonging to the normal phase,
\( n = 8, \ldots, 12 \) to the abnormal phase, and the rest \( n = 13, \ldots, 16 \) to the recovery phase. The number of segments in a particular phase is not important, since the fluctuation of the patterns from segment to segment within a phase is not large. Thus for an ordinary time series that does not include a change of phase, one may have only 10 - 20 heartbeats in a diagnostic test. That should be sufficient for the proposed analysis to be performed.

Note that each segment has \( 2^9 \) points. The importance of that number to be an integer power of 2 will become self-evident, as we perform the wavelet analysis whose resolution improves by powers of 2. To have more points in a longer segment will not improve the analysis because the information to be extracted lies with the shape of the waveform between and around the pulses. To have less points would shorten the range of resolutions and inhibit the establishment of a convincing scaling behavior.

Let the Haar wavelet \( \psi^H_{jk}(t) \) be defined by

\[
\psi^H_{jk}(t) = \psi^H(2^jt - k) ,
\]

where \( \psi^H(t) = 1 \) for \( 0 \leq t < 1/2, = -1 \) for \( 1/2 \leq t < 1 \), and = 0 otherwise. For any scalar function \( f(t) \) defined in \( 0 \leq t \leq 1 \), the wavelet coefficient after a discrete wavelet transform is

\[
w_{jk} = (\psi^H_{jk}, f) = \int dt \psi^H_{jk}(t)f(t) .
\]

By virtue of the properties of \( \psi^H_{jk}(t) \), which is zero for \( t \) outside the interval \( [k2^{-j}, (k + 1)2^{-j}] \), \( w_{jk} \) selects a narrow sector of \( f(t) \) that depends on the scale factor \( j \) and shift variable \( k \). Thus with appropriate values of \( j \) and \( k \), \( w_{jk} \) can identify spikes in \( f(t) \).

We use (2) to analyze the waveforms of various segments \( S_n \) of our ECG time series separately. Since each segment has 512 points, we consider the range of \( j \) values from 0 to
8, so that at the highest resolution two neighboring points are resolved by the transform. The shift $k$ can vary from 0 to $2^j - 1$. For the purpose of our use of the wavelet coefficients below, we want to avoid negative values by taking the absolute value of the transform, i.e., for the $n$th segment,

$$w_{jk}^{(n)} = \left| \left\langle \psi_{jk}^H, S_n \right\rangle \right|,$$

(3)

where we have mapped the 512 points on the time axis to the interval $0 \leq t \leq 1$. With the definition in (3), the average (over all $k$ at fixed $j$)

$$\left\langle w_{jk}^{(n)} \right\rangle = 2^{-j} \sum_{k=0}^{2^j-1} w_{jk}^{(n)}$$

(4)

is always positive definite. We now can define a normalized wavelet coefficient

$$z_{jk}^{(n)} = \frac{w_{jk}^{(n)}}{\left\langle w_{jk}^{(n)} \right\rangle},$$

(5)

which measures the fluctuation of $w_{jk}^{(n)}$ from the average. This is an important step that combines both the local and global properties of the waveform in a segment, since $z_{jk}^{(n)}$ is sensitive to the values of $w_{jk}^{(n)}$ in all bins. Moreover, note that in the ratio (3) the normalization of the Haar wavelet $\psi_{jk}^H(t)$ defined in (1) is unimportant.

In order to quantify the fluctuations of $z_{jk}^{(n)}$ from bin to bin, we now define

$$K_j^{(n)} = \left\langle z_{jk}^{(n)} \ln z_{jk}^{(n)} \right\rangle,$$

(6)

where the angular brackets denote an average over $k$ as defined in (4). $K_j^{(n)}$ is not far from being the entropy. If we define $p_{jk}^{(n)} = 2^{-j} z_{jk}^{(n)}$ with $\sum_k p_{jk}^{(n)} = 1$, we can define the entropy as

$$S_j^{(n)} = -\sum_k p_{jk}^{(n)} \ln p_{jk}^{(n)}.$$

(7)
It then follows that

\[ S_j^{(n)} = j \ln 2 - K_j^{(n)} \]  \hspace{1cm} (8)

In the study of problems of this type that have fluctuations at all scales, we look for scaling behavior as an organizing feature. The quantities that are expected to possess scaling behaviors are the moments

\[ C_p^{(n)}(M) = \langle (z_{jk}^{(n)})^p \rangle , \]  \hspace{1cm} (9)

where the dependence on \( j \) may be expressed in terms of the number of bins, \( M \), via \( M = 2^j \). Clearly, we have from (3)

\[ K_j^{(n)} = \frac{d}{dp} C_p^{(n)} \Big|_{p=1} . \]  \hspace{1cm} (10)

Thus, if \( C_p^{(n)} \) has the scaling behavior

\[ C_p^{(n)}(M) \propto M^{\psi_p^{(n)}}, \]  \hspace{1cm} (11)

as the resolution is increased (i.e., higher \( M \)), then it follows from (10) and (11) that

\[ K_j^{(n)} \propto \mu_j^{(n)} \ln M = \mu_j^{(n)} j \ln 2 , \]  \hspace{1cm} (12)

where \( \mu_j^{(n)} = \frac{d}{dp} \psi_p^{(n)} \Big|_{p=1} \). Our entropy index is defined by

\[ \sigma^{(n)} = 1 - \mu^{(n)} , \]  \hspace{1cm} (13)

which follows naturally from (8) and (12).

The procedure for analyzing the data should now be clear and straightforward. For each segment \( S_n \), use (3) - (8) to determine \( K_j^{(n)} \) for \( j = 0, \ldots, 8 \). As an illustration of the result,
we show in Fig. 3 $K_8^{(n)}$ vs $n$ for $j = 8$. Evidently, $K_8^{(n)}$ is quite stationary at around 2.1 for $n = 1, \cdots, 7$, which are the segments in the normal phase. Then at $n = 8$, $K_8^{(n)}$ drops down to below 1 and stays below for the remaining segments of the abnormal phase, $n = 8, \cdots, 12$. The fluctuations from segment to segment are not significant within one or the other of the two phases. Since $j = 8$ is the highest resolution that the data allow, it provides the most dramatic changes of $K_j^{(n)}$ in the transitions between phases. At lower $j$ the spikes in the waveform are smeared by the wavelet transform, with the consequence that the changes in $K_j^{(n)}$ between phases become less pronounced. In that sense $K_8^{(n)}$ itself can serve as a measure of cardiac regularity.

To capture the information contained in $K_j^{(n)}$ at lower $j$, we investigate the scaling behavior (11), which implies a linear dependence of $K_j^{(n)}$ on $j$, as given in (12). To have an average measure over all segments in a particular phase, we define the following averages

$$K_j^N = \frac{1}{7} \sum_{n=1}^{7} K_j^{(n)}, \quad K_j^A = \frac{1}{5} \sum_{n=8}^{12} K_j^{(n)}$$

(14)

for the normal and abnormal phases, respectively. Beginning with $n = 13$, the recovery phase commences. Since $K_j^{(n)}$ in the recovery phase is time dependent, a similar average is less meaningful, although it can analogously be defined. In Fig. 4 we show $K_j^N$ and $K_j^A$ vs $j$, which clearly exhibit linear behavior. The straightline fits give

$$\mu_{N,A} = \frac{1}{\ln 2} \frac{\partial}{\partial j} K_j^{N,A}.$$  

(15)

Their numerical values obtained are $\mu_N = 0.54$ and $\mu_A = 0.12$. The corresponding values of the entropy indices are then

$$\sigma_N = 0.46, \quad \sigma_A = 0.88.$$  

(16)
If the same procedure is followed for the recovery phase, the corresponding entropy index is \( \sigma_R = 0.58 \), which is a coarse summary of the transitory change from \( \sigma_A \) back to \( \sigma_N \). Eq. (16) exhibits the numerical result of this work. Whereas \( \sigma_A \) may vary, depending on the nature of the cardiac abnormality, \( \sigma_N = 0.46 \) can be regarded as the standard number for a normal heartbeat. To register the state of cardiac health in terms of \( \sigma \) is clearly useful.

The increase of \( \sigma \) in the transition from the normal to abnormal phase signifies the increase of disorder in the waveform. That is a feature that is visually obvious from Figs. 1 and 2. We now have a quantitative measure of that disorder. In the normal phase the fluctuation from bin to bin is relatively small despite the large, but regular, spikes, whereas the irregularity in the abnormal phase generates large fluctuations.

It is pertinent to remark that from the fluctuations of beat-to-beat intervals studied over very long periods it has been inferred that the normal heartbeat is chaotic [17]. Since stochastic disorder is not the same as chaotic behavior, there is no obvious conflict between that conclusion and ours. Nevertheless, it would be useful to point out here the possible source of the difference in interpretations. Because the emphasis in this paper is on the characterization of ECG waveforms in short periods, we have considered only a few segments with detailed analysis of the bin-to-bin fluctuations. To study chaotic behavior, we would have to consider segment-to-segment fluctuations over a long period. That happens to be the type of analysis done earlier with the \( \mu \) index (for event-to-event fluctuations) for both classical-chaotic time series [9] and quantum systems involving particle production [7]. In fact, it was found that \( \mu \) can play the role of the Lyapunov exponent. Applying similar method to data collected over very long periods, it should be possible to make more elaborate analysis, not just on the beat-to-beat intervals, but on the fluctuation of the interpulse
structure.

To illuminate the dual properties of stochasticity and chaoticity would be highly interesting. Here we present only the method of analysis that can quantify the disorder aspect of the ECG waveform structure. Multichannel analysis and finding predictive signatures in correlated data are examples of other problems well worth further investigation.

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**Figure Captions**

**Fig. 1** Time series of human heartbeats that includes a period of ventricular fibrillation.

Each unit on the horizontal scale is 1/256 s; the vertical scale has arbitrary unit.

**Fig. 2** Details of Fig. 1 in (a) the normal phase, and (b) the abnormal phase.

**Fig. 3** $K_j^{(n)}$ at $j = 8$ for various segments $S_n$.

**Fig. 4** Scaling behaviors of $K_j$ for the normal and abnormal phases.