A mixed integer programming model for forest harvest scheduling problem

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Abstract. The Forest Harvest Scheduling Problem (FHSP) and forest planning problems in general are combinatorial optimization problems. Briefly, the problem is to develop a forest management plan for a large area of forest satisfying multiple constraints and balancing competing objectives. Traditionally, mathematical programming has been and still is the most widely used technique for solving such problems. For the general forest harvesting problem, there exist a number of LP and MIP models. This paper study a restricted version of the FHSP, called the Clear-cut Scheduling Problem (CCSP) as a test case. We attempt to model and to solve the CCSP using a MIP model. We will not model the problem as a LP since the literature survey shows that LP-based strategies are difficult to interpret and may be impossible to implement, largely due to their inability to express spatial relationships.

1. Introduction

Forest management and planning is complex, involving the application of many scarce and diverse resources to the production and maintenance of multitude of products and services from the forest over a relatively long period of time. The forest manager hopes to produce a balanced mix of products and services, with the mixture depending upon the landowner's objectives. Although many objectives are complementary in nature, others are competitive, with some mutually exclusive. As a result, allocating the resource manager's scarce and diverse resources the alternative and possibly competitive products and services becomes a complex problem.

Timber management planning is an integral part of managing a forest. There are two tasks of timber management planning; establishing harvest schedules (cutting budgets) and developing a regulated forest. The harvest-scheduling problem involves determining what, where, when, and how much to cut in order to ensure a smooth transition from an unregulated to a regulated forest structure, while at the same time meeting short-term requirements, objectives, and constraints. A regulated forest is a forest with age and size classes represented in such a proportion that a stable periodic yield of products and services may be obtained over time [1]. The regulation problem involves selecting and developing a long-term, steady state forest structure, the regulated forest [2].
Governments and private sectors worldwide manage an immense area of forest. The United States Department of Agriculture alone manages a forest area almost twice the size of Germany. Forest harvest planning and scheduling problems are two important things of this forest management. Recently, both governments and the public worldwide are demanding sustainable harvesting practices for these areas, which would take into account not only economics but also the preservation of biodiversity, and the esthetic, and recreational value of the forest. Long term forest harvest scheduling allows communities, governmental, and non-governmental organizations to check if sustainable forestry is actually being practiced. Simplified, the problem is to assign forest treatment types and times to treatment units (often called stands) in a given forest area over a long period of time. This problem called the Forest Harvest Scheduling Problem. An optimization version of the problem involves optimizing objective criteria such as economical, esthetic, and recreational values, which are subject to multiple constraints. Traditionally, mathematical programming models and solution techniques have been extensively used to generate such schedules, and are still the most widely used method (for example, [3], [4], [5]. Artificial Intelligence (AI) techniques such as the rule-based expert systems, and the local search, have also been applied to forest management (for example, [6], [7], [8]. The above techniques have difficulties in modeling a wide variety of spatial, temporal, and visual constraints and/or in performing the (usually conflicting) multi-criteria optimization that is inherent to the problem. In addition, the size of a real life problem is often beyond the practical capacity of these methods.

Misund[9] have suggested the integrated use of Geographic Information Technology (GIT) and certain AI techniques (constraint reasoning and local search) to model and solve such problem. They describe their approach through a simpler version of the problem where only clear-cutting is allowed. The problem is called the Clear-Cut Scheduling Problem (CCSP). CCSP is modeled as a Constraint Satisfaction Problem (CSP) [10] and a solution is iteratively improved using Tabu search [11]. Although the authors give some reasons for their choice of Tabu search as the iterative improvement technique, it is not all that clear whether the traditional method using a mixed integer linear programming (MILP) model can be effective.

2. Methodology

First we define the problem using such as definition through the symbols. Next we will form the model formulation into Mixed Integer Linear Programming Problem and then solve it with LINDO solver.

Problem Definition

Sustainable forest management requires the forest harvesting actions scheduled for a long period of time to obey a variety of constraints. For management purposes, a forest landscape is divided into basic treatment units, often equivalent to stands. A stand is a forested area considered homogeneous with respect to a selected set of properties. We will restrict our study to even-aged stands, that is, stands containing trees of the same or almost the same age. We will call the time period of the schedule as scheduling horizon. It is a common practice to divide the scheduling horizon into a number of time periods, each period equivalent to, 5 - 10 years.

Definition 1 (Stand) A stand $S_i$ is an area which is considered homogeneous with respect to certain characteristics. The set of $n$ stands comprises a forest $\mathcal{F}$, such that:

1. $\mathcal{F} = \bigcup_{i=1}^{n} S_i$
2. $S_i \cap S_j = \emptyset$, for all $i \neq j$

For each stand $S_i$,

- $LH(S_i)$: time of most recent harvest
- $EARLY(S_i)$: minimum duration between harvests
- $LATE(S_i)$: maximum duration between harvests
OPT(S_i): optimal time between harvests
MH(S_i, H): time required by tree in the stand to grow
AREA(S_i): area of the stand
GROWTH(S_i, y): volume that may be harvested y years after the last year.

Definition 2 (Scheduling Horizon) a scheduling horizon \( \mathcal{H} \) is an contiguous set of \( m \) periods \( \{P_1, \ldots, P_m\} \), where each \( P_i \), is of a certain length in years.

Definition 3 (Treatment) a treatment unit (stand) is given a number of management options over the scheduling horizon. We will call individual management options a treatment. A set of treatment types \( T = \{T_0, \ldots, T_t\} \) is available for each stand. We let \( T_0 \) to be null treatment or "let grow" where a stand is left without any treatment, and \( T_1 \) to be the clearcut treatment.

Definition 4 (Forest Harvest Scheduling Problem (FHSP)) given an area of forest \( F \) divided into \( n \) stands, \( \{S_1, \ldots, S_n\} \), find a schedule \( S = \{A_1, \ldots, A_n\} \) over scheduling horizon satisfying a set of constraints \( C = \{C_1, \ldots, C_t\} \). Here, \( A_i \) is the set of activities represented by a set of tuples \( \langle P_{ij}, T_{ij} \rangle \) , where each tuple represents treatment period and treatment type for the stand \( S_i \). The constraints are usually adjacency constraint, harvest interval constraint etc.

Definition 5 (Clear-cut Scheduling Problem (CCSP)) the clear-cut Scheduling Problem is a restricted FHSP. Only two possible treatment types are allowed, \( T_0 \) and \( T_1 \) ("let grow" and clear-cut). The clear-cut treatment is scheduled only once for each stand during the entire scheduling horizon.

Figure 1. An example of a forest area divided into several stands

A CCSP can be visualized as assigning values to each of \( h_i \), \( 1 \leq i \leq n \), where \( h_i \) is the scheduled harvesting (clear-cut) period for the stand \( S_i \), rest of the periods are assigned the null treatment. The value of each \( h_i \) is picked from its domain \( D_i = \{1, \ldots, m\} \) where \( m \) is the total number of periods, such that the problem constraints (defined shortly) are satisfied.

**Constraints:** constraints can be generally divided into two categories - hard and soft. The hard constraints are defined as those constraints that must be satisfied whereas the soft constraints can be relaxed to satisfy the hard constraints or to adjust the objective function value.

**Hard Constraint \( C_1 \): Minimum height constraint:** all the neighbors of the stand to be harvested must have an average tree height of at least \( X \) meters. For CCSP,

\[
\forall i \in \{1, \ldots, n\}: \forall j \in I(i): \left( h_i < h_j - MH(S_i, X) \right) \lor \left( h_i \geq h_j + MH(S_j, X) \right) \tag{1.1}
\]

where \( I(i) \) is index set for neighbors of \( S_i \).
Soft Constraint Cz: Harvest Interval: preferred harvest interval based on economical and ecological considerations. A lower and upper threshold is given for each stand, as well as the optimal harvest time relative to the last clear-cut. For CSP,

∀\(i\in\{1, ..., n\}: EALRY(S_i) \leq h_i - LH(S_i) \leq LATE(S_i) \) (1.2)

Optimization Criteria: a feasible solution is an assignment of problem variables such that all the constraints are satisfied. An optimization version of the CCSP (and the FHSP) arises when various optimization criteria are maximized (or minimized) in the solution. We are interested in solving the optimization version of the problem with the following optimization criteria:

\(O_1\): Optimal Harvest: time the actual time between harvests should be as close to the optimal as possible. For CCSP,

\[
\min \sum_{i=1}^{n} \left( \left| OPT(S_i) - (h_i - LH(S_i)) \right| \right) \] (1.3)

\(O_2\): Even Harvest Volume: the estimated harvested volume \(EHV_p(S)\) for any period \(p\) should be as close to the average harvested volume in a schedule \(S\), \(AHV(S)\). In other words, the variance between the harvest volumes should be minimized.

\[
\min \sum_{p=1}^{m} \left( \left| EHV_p(S) - AHV(S) \right| \right) \] (1.4)

\(O_3\): Old Forest: A specified minimum area of old forest (AOF), that is, the total area of the stands with average age above a certain threshold, should be maintained over the scheduling horizon. Let \(OLD(p, S)\) be the area of old forest in period \(p\).

\[
\min \sum_{y=1}^{m} \max(0, AOF - OLD(p, S)) \] (1.5)

Note that the optimal would be 0 when the area of old forest is greater than or equal to AOF.

\(O_4\): Visual Impact: Minimize the visual damage of clear-cutting relative to a set of viewpoints. Let \(VIS(S_i, p, V)\) denote visual impact from stand \(S_i\) in period \(p\) relative to viewpoint \(V\). Note that the following equation only assumes one viewpoint, whereas in the real case there is usually a set of viewpoints.

\[
\min \min_{p=1}^{m} \left( \sum_{i=1}^{n} VIS(S_i, p, V) \right) \] (1.6)

The minimum height constraint is enforced so that a large area of forest is not clear-cut simultaneously causing a great damage to the regeneration process and the wildlife habitats.

Technique of Optimization

There are two major classes of forest harvest scheduling algorithms. One is based on mathematical programming and the other on heuristic techniques (with or without using simulation in both cases). The former is a global procedure which finds an optimal solution to the forest management model whereas the latter is usually a local procedure which iteratively improves the solution without any guarantee of finding the optimal one. Mathematical programming is the technique that is commonly used in practice and consequently a large percentage of the research activity is devoted to it. There are also some algorithms which do not fall under these two categories, and that we will discuss under the miscellaneous methodology.
Mathematical Proggramming
The general for est planning problem, where harvesting is an important process, has been studied for some time. Early forest management models [12], [13] The LP models are sometimes used together with a growth and treatment simulator. The simulator is used to find all possible treatment schedules for all the stands. In most cases, each stand is treated independently and the necessary information is provided as the simulation proceeds.

Mixed Integer Programming Model
We abandoned the real variables $h_i, \ldots, h_n$, and started with a binary $(0,1)$ variable $x_{ij}$ for each stand $S_i$, where $i = 1, \ldots, n$ and $j = 1, \ldots, m$, where $n$ is the number of stands and $m$ is the number of periods. It may seem counter-intuitive to replace some compact-looking constraints with a large number of constraints with binary variables. But the reason it works well is that the CCSP, in this formulation, has node-packing as a subproblem. Therefore, we can expect to benefit from the clique constraints that are likely to quicken the MIP solution process. Furthermore, this formulation has proven to work well in practice. Our MIP model consists of $n$ stands, $\{S_1, \ldots, S_n\}$. Each harvest period $p$ for a stand $S_i$ is represented by a binary variable $x_{ip}$. We assume that each stand is harvested only once throughout the scheduling horizon. Now, we formulate the constraints and the objective functions for the CCSP as stated in the problem definition.

Constraints: in addition to constraints C1 and C2, we have an additional constraint such that every stand is harvested exactly once during the scheduling horizon.

C1: Minimum height constraint: for all $t$ from $1 \ldots m$,
$$x_{it} + \sum_{t-MHSn\leq t\leq M} x_{jn} \leq 1, \forall \text{ neighbours } (S_i, S_j).$$

C2: Harvest interval constraint: simply do not include the binary variables associated with the undesirable periods for all stands in the model.

C3: At least one harvest per plan
$$\sum_m x_{im} = 1, \forall i$$

Objective Criteria: All except the objective criterion 0 is forni~late~ads followis.

$O_1$: Optimal Harvesting Time: the actual time between harvests should be as close to the optimal as possible. For all $i$ in $1 \ldots n$.
$$\sum_j x_{ij} * (|j - OPT(i)|) + \alpha_i \geq 0$$

Where $j$ goes from $1, \ldots, m$. Here $\alpha_i$ is penalty for not harvesting stand $i$ in its optimal harvest time. Let $P_{opt} = \sum_i \alpha_i$. Then, the optimal harvesting time objective is achieved by minimizing $P_{opt}$ in the objective function.

$O_2$: Even Harvest Volume: the estimated harvested volume $EHV_p(S)$ for any period $y$ should be as close to average harvested volume in a schedule $S$, $AHV(S)$. In other words, the variance between the harvest volun~es should be minimized. Let $v_{ij}$ be estimated volume of forest obtained by cutting stand $S_i$ in period $j$. Then, $V_j = \sum_{i=1}^n x_{ij} v_{ij}, \forall j$ is total volume harvested in period $j$. Set constrains $V_{\min} \leq V_j$ and $V_{\max} \geq V_j, \forall j = 1, \ldots, n$
Then objective is to minimize $V_{\text{max}} - V_{\text{min}}$ which effectively minimizes the variance between volumes harvested, making an overall schedule with even consumption throughout the planning horizon.

$O_3$: Old Forest: a specified minimum area of old forest (AOF), that is, stands with average age above a certain threshold, should be maintained throughout the scheduling horizon. Let $OLD(p, S)$ be the area of old forest in period $p$. Let $Olt = 1$ if stand $i$ is not cut before it becomes old in period $t$, otherwise $Olt = 0$. That is,

$$\sum_{i \leq j \leq t - 1} x_{ij} + O_{it} = 1, \quad \forall i$$

Let $\theta_t$ be the area of old forest at period $t$, which is $\sum_i O_{it} * v_{it}, \forall i$. We want to keep $\theta_t$ as close to AOF as possible. That is, for all $t$ in $1, ..., m$

$$\theta_t - AOF + \gamma_t \geq 0, \quad \gamma_t \geq 0$$

Then, the objective function will minimize $\sum_t \gamma_t$

$O_4$: Visual Impact: this is not put into the MIP model.

This type of nociel can be extended to handle alternative treatments. For an additional $t$ treatments, $t(nm) - nm$ variables will be added to the model. The same order of constrains will also be added, of type $x_{ij} + \sum_{k=2}^{nm} x_{ijk} \leq 1$. which means that one treatment type inhibits other treatments for a certain number of periods. The assumption that only one treatments is allowed per scheduling horizon can also be dropped.

3. Result and Discussion

First we describe empirical results for random data, with only the adjacency and harvest interval constraints ($C_1$ and $C_2$), using a public domain MIP solver LINDO. Next, we present the results obtained from solving instances based on the ECOPLAN prototype data [9], using a commercial MIP solver cplex version 4.0.4 (CPLEX n.d.) with all the constraints and the objective criteria as described in our MIP.

Initial Experiment using LINDO

We generated problem instances for a variable number of stands, but the planning horizon was left constant at 100 periods with each period equivalent to 1 year. The number of stands were varied from 5 to 100. The results using LINDO workstation is as shown in Table 1. The solution time varied from 5 minutes to 2 hours.

| N  | Object Value | remark   |
|----|--------------|----------|
| 5  | 5            | Optimal  |
| 10 | 10           | Optimal  |
| 20 | 20           | Optimal  |
| 30 | 30           | Optimal  |
| 40 | 40           | Optimal  |
| 50 | 50           | Optimal  |
| 60-95 | N/A    | Memory fault |
| 100| 100          | Optimal  |
The results show that for almost half the instances lp-solve exits abnormally with a memory fault. The rest are solved to optimality. For our randomly generated data, the neighboring stands for any given stand were low resulting in a loose adjacency constraint structure. This may explain why instances with 100 stands were solved to optimality. Another reason is the domain size of 100 periods. Although some of the periods were disallowed by the harvest interval constraint (Cz), every stand has still between 75 to 100 domain values to pick from. In fact, when the number of periods were reduced to 20, the solver was not very successful. Furthermore, memory faults caused by more than half the instances lead us to choose a more powerful MIP solver cplex.

Results using CPLEX solver

The Norwegian ECOPLAN project's prototype data was used to test our MIP model using cplex MIP solver. This data set is different than the one used with LINDO in that the former has realistic adjacency relations and the number of periods for each stand. The area of each stand was generated randomly to occur between the largest and smallest area in the actual data set since we were not able to get the data set with the real area information. All the instances were run on LINDO.

The ECOPLAN prototype data for 494 stands was divided into smaller instances of n stands where n = {25, 50, 100, 200, 300, 400, 494}. Each of these instances was tested with 5, 10, 15 and 20 periods (m) as the scheduling horizon. Furthermore, each of these instances was used to generate 5 different sets with different objective criteria, S1, ..., S5, as input for the cplex solver. Among these sets, S1 contains data for constraint satisfaction only. That is, only the adjacency constraint (C1), the harvest interval constraint (C2), and one harvest per scheduling horizon (C3) were present (besides the integrality constraints) without any of the objective criteria. The objective value was set to be n for a data set with n stands. All other sets also contain these constraints but the objective criteria differ in each of the sets. S2 contains objective O1 (optimal harvest time), S3 contains objective O2 (even harvest volume), S4 contains objective O3 (old forest), and S5 contains all three objective criteria (O1, O2, and O3).

Table 2 shows the cplex output for the ECOPLAN prototype data when the green-up age for adjacency constraint was set to 3 periods, with each period being equal to 5 years. The table shows that it was difficult to satisfy most of the instances. There were no integer feasible solution for all of the data sets with 5 periods. Note that if an integer feasible solution does not exist for the set S1, then no integer solution exists for any other sets. Furthermore, if no integer feasible solution exists for a data set with n stands and m periods, no integer feasible solution exists for any of the other data sets with m periods and stands greater than n. Only the data sets with pairs (25, 10), (25, 20), and (50, 10) were solved to optimality, where each pair represents (n, m).

The entries in the table marked with a "***" may seem awkward, for example, why is there no integer solution for the instance with 25 stands and 15 periods when there exists an optimal solution for the instance with 25 stands and 10 periods? The reason is that during the data generation, if the allowable harvest interval for a stand goes beyond the scheduling horizon, it is relaxed so that the stand may be allowed to be harvested during the last 5 periods. This was done so that every stand is harvested once. Thus, in the case of the instance with 10 periods, some stands got a larger harvest interval than in the case with 15 periods.

| Set | n   | m   | read | C   | V   | opt | Int.Sol | time | iter | BB nodes | remark      |
|-----|-----|-----|------|-----|-----|-----|---------|------|------|----------|-------------|
| S1  | 25  | 5   | .11  | 190 | 125 | N   | N/A    | .08  | 0    | 0        | no int sol  |
| S2  | 25  | 10  | .20  | 296 | 250 | Y   | 25      | .18  | 42   | 5        |             |
| S3  | 25  | 10  | .21  | 537 | 501 | Y   | 103     | .48  | 227  | 8        |             |
| S4  | 25  | 10  | .12  | 326 | 262 | Y   | 70035   | 18.27| 6941 | 105      |             |
| S5  | 25  | 10  | .20  | 573 | 532 | Y   | 202177  | 2.69 | 1248 | 75       |             |
Careful analyses of the solver output for the unsatisfiable instances revealed that the harvest interval constraint for some stands was too tight, that is, the stands could be harvested in only 1 or 2 periods. This caused the solver to conclude integer infeasibility relatively quickly. We were interested in testing the MIP solver for larger instances with a number of objective criteria. Therefore, we relaxed the harvest interval constraint so that a stand can be harvested in any period during the scheduling horizon. This is not an unrealistic relaxation since the minimization of the objective $O_1$ essentially has the equivalent effect on the solution. Also, to increase the number of periods, we only generated data with 20 periods.

Table 3. Results obtained for Set 2 and 4 using cplex MIP solver

| n  | C   | V   | $O_1$ | time | CC  | nodes | C   | V   | $O_3$ | time | CC  | nodes |
|----|-----|-----|-------|------|-----|-------|-----|-----|-------|------|-----|-------|
| 25 | 610 | 500 | *60   | 2    | 13  | 2     | 115 | 1041| *180  | 46   | 36  | 87    |
| 50 | 1740| 1000| *45   | 17   | 64  | 54    | 2781| 2041|       | 350  | 47  | 102   |
| 75 | 1926| 1500| *83   | 37   | 67  | 57    | 4251| 5041|       | 380  | 116 | 112   |
| 100| 3640| 2000| *115  | 53   | 87  | 37    | 5681| 4041|       | 202  | 22  | 120   |
| 150| 5540| 3000| *203  | 130  | 123 | 91    | 8581| 6041|       | NA   | 7200| 115   |
| 200| 11482| 8001| *307  | 2538 | 245 | 3776  | 1161| 8041|       | NA   | 7200| 101   |
| 150| 14781| 10001| 367 | 7200 | 303 | 9090  | 15021| 10041|       | 2016 | 297 | 189   |
| 300| 18264| 12001| 468 | 7200 | 291 | 11488 | 18541| 12041|       | NA   | 7200| 123   |
| 350| 220041| 14001| 527  | 7200 | 426 | 5033  | 22361| 14041|       | NA   | 7200| 382   |
| 400| 25081| 16001| 601  | 7200 | 420 | 5014  | 25441| 16041|       | NA   | 7200| 179   |
| 450| 28264| 18001| 669  | 7200 | INT |       | 28741| 18041|       | NA   | 7200| 56    |

N: number of stands
M: number of periods
Read: problem read time
C: number of constraints
V: number of variables
Opt: is solution optimal?,
int sol: integer solution,
time: cpu time (in secs),
iter: number iterations,
BB nodes: number of BB nodes tried
Table 4. Results obtained for Set 3 and 5 using cplex MIP solver

| n      | Set 3 |          |          |          | Set 5 |          |          |          |
|--------|-------|----------|----------|----------|-------|----------|----------|----------|
|        | C     | V        | O₃       | CC nodes |       | C        | V        | O₁       | O₂       | O₃       | CC nodes |
| 25     | 670   | 522      | 114.8    | 219      | 36213 | 1687     | 1564     | 125      | 73.8     | 180      | 144      | 9765     |
| 50     | 1800  | 1022     | 361.4    | 541      | 6387   | 3795     | 3064     | 221      | 323      | *0       | 272      | 4907     |
| 75     | 2770  | 1522     | 552.2    | 480      | 1553   | 5746     | 4564     | 209      | 504      | *0       | 280      | 1326     |
| 100    | 3700  | 2022     | 317.6    | 610      | 394    | 7655     | 6064     | 355      | 452      | *0       | 284      | 492      |
| 150    | 5600  | 3022     | 366      | 658      | 933    | 11516    | 9064     | 511      | 556      | *0       | 377      | 525      |
| 200    | 7700  | 4022     | 455.2    | 382      | 506    | 15583    | 12064    | no int   | NA       | NA       | 165      | 190      |
| 150    | 10040 | 5022     | 202.6    | 502      | 514    | 19882    | 15064    | no int   | NA       | NA       | 215      | 150      |
| 300    | 12560 | 6022     | no int   | 269      | 93     | 24365    | 18064    | no int   | NA       | NA       | 287      | 236      |
| 350    | 15380 | 7022     | no int   | 332      | 30     | 29142    | 21064    | no int   | NA       | NA       | 359      | 197      |
| 400    | 17460 | 8022     | no int   | 228      | 61     | 24064    | 24064    | no int   | NA       | NA       | 389      | 4        |
| 450    | 19680 | 9022     | INT      |          |        |          |          |          |          |          |          |          |

N: number of stands  
C: number of constraints  
O₁: optimal harvest time value  
O₃: old forest objective value (≡ opt)  
O₂: even consumption obj. Value  
time: CPU running time (sec)  
CC: number of clique cuts applied  
Nodes: number of BB node tried  
INT: interrupted

Data instances with \( n = \{75, 100, 150, 200, 250, 300, 350, 400, 250, 494\} \) was generated for all 5 sets \( S₁ \) to \( S₅ \). While solving some instances it was observed that the solver was taking more than 2 hours just to find an initial solution. These instances were rerun with the solver’s rounding heuristic turned on. The solver used a sophisticated rounding heuristic to find the first integer solution. In addition, the solver had also clique and cover cuts options to improve performance of the branch and bound phase.

In Table 3 smaller instances have been solved to optimality. The objective value for \( O₁ \) denotes the total number of periods off the optimal harvesting period for all the stands. For example, if two stands were harvested 5 periods either before or after the optimal, and the rest were scheduled on the optimal harvesting time, then the value of \( O₁ \) is 10. The value of the objective \( O₃ \) (in thousands of square kilometers) is the total area of forest not above the old forest threshold for all the periods. The old forest threshold was set to 10% of the total area for all the periods in our experiments, which comes to 1440 square kilometers (in thousands). The solver could not find optimal solutions for the set \( S₂ \) starting from \( n = 250 \). For the set \( S₄ \), no integer solution was found within the time limit of 2 hours for \( n \) starting from 150.

Table 4 shows results for the sets \( S₃ \) and \( S₅ \). The value of the objective \( O₂ \) (in thousands of square kilometers) is the difference between the biggest and the smallest harvest volume during the scheduling horizon. All the instances were stopped after 2 hours of execution, that is, no optimal solution was found. No integer solution for the set \( S₃ \) was found for values of \( n \) starting from 300. For the set \( S₅ \), this value was 200.

In all the instances, the number of clique cuts generated was high. This means that turning this option on during solving was probably a good idea since these cuts give tighter bounds and help branch and bound algorithm to find an integer optimal solution faster. After the elimination of the harvest interval constraint, the solver was able to find the optimal solution for all of the instances in \( S₁ \), that is, the adjacency constraint was satisfied. When the objective criteria were added, the solver required more time to find the optimal or an integer solution. From the results, it seems that the
addition of the even harvest volume objective makes the NIP much harder since both the sets \( S_3 \) and \( S_5 \) that contain the objective were not solved to optimality even for smaller instances. It remains to be seen how good these solutions are since currently we have no means of comparing them. One measure could be the optimal solution of the full problem with \( n = 454 \). However, running the problem for 33 hours did not even produce an integer feasible solution.

These results show that the solver is capable of finding integer solutions to the ECOPLAN prototype data when the harvest interval constraint is relaxed and no objective criteria is in the model. Also, an optimal solution was found for smaller instances (usually 25 and 50) even after the addition of all of the objective criteria.

4. Conclusion

From our experimental results we can conclude that our MIP model can be used for the CCSP and that small instances are solved to optimality if only some objective criteria are used. For larger instances the harvest interval constraint was too tight and had thus to be relaxed. Then, satisfying the adjacency constraint was easy. It was when we started introducing some additional objectives, that the solving started to become time consuming. In particular, the even flow objective seemed to be hard to optimize. The clique and cover cuts option in the solver was turned on while solving these instances. A high number of clique cuts was generated for all the instances suggesting tighter bounds for the branch and bound phase. However, the largest instance with all the objective criteria was not even solved for an integer feasible solution after 33 hours of execution. Furthermore, CPLEX is considered to be the state-of-the-art among the current MIP solvers. The MIP can be made more efficient if the problem decomposition technique called column generation can be utilized. However, modelling the constraints and the objectives to use this technique is not a trivial task. Thus, our MIP model alone is not realistic for solving large practical FHSP.

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