Self-consistent approach to deformation of intruder states in neutron-deficient Pb and Po

N. A. Smirnova1,2, P.-H. Heenen3, G. Neyens3
1 Instituut voor Kern- en Stralingsfysica, University of Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium
2 Vakgroep Subatomaire en Stralingsfysica, Universiteit Gent, Proeftuinstraat 86, B-9000 Gent, Belgium
3 Service de Physique Nucléaire Théorique, Université Libre de Bruxelles, C.P. 229, B-1050 Bruxelles, Belgium

(Dated: November 3, 2021)

We present systematic calculations of the properties of 11− isomers in neutron-deficient 184−198Pb and 188−200Po. These states are based on the π(h9/213/2)K∗=11− configuration. They are calculated in the framework of the Hartree-Fock-Bogoliubov method with a Skyrme interaction and density-dependent pairing force. The energies and deformations of the 11− states are compared to those of the intruder 0+ states in both Pb and Po isotopes. In the most neutron-deficient Po isotopes, the calculations predict, below a weakly oblate 11− state, another oblate 11− state which is even more deformed than the intruder oblate 11− state in their Pb isotones. The energies and quadrupole moments of the 11− isomers, corresponding to a weakly oblate nuclear shape, are in fair agreement with the available experimental data.

PACS numbers: 21.60.Jz,21.10.Dr,21.10.Re,21.10.Ky,27.80.+w

The neutron-deficient nuclei around the Z = 82 spherical shell gap. In a shell model picture [9, 10], the energy of these multi-particle-multi-hole (mp-mh) configurations is lowered by the quadrupole-quadrupole and pairing interactions. In mean-field models [11, 12], these states are of particular interest since the measurement of their static electromagnetic moments provides direct information on both their single-particle structure and their deformation. Several isomeric states are known in Pb isotopes. In particular, 11− levels have been observed from 188Pb to 199Pb (see, e.g. Ref. [13]). They are not known in heavier Pb, most probably because they are not yrast. These states are interpreted as a two-quasiparticle (2qp) excitation based on the π(h9/213/2)K∗=11− configuration as confirmed in 196Pb by the measurement of its g-factor [14]. Experimental information on deformation is extracted from the collective bands based on the excited 0+ states and 11− states [16, 17]. Recent data on static quadrupole moments for 11− states in 194,196Pb [18,19] support the mean-field interpretation of these states as weakly deformed oblate configurations.

The experimental excitation energies of the 11− states and of the 0+ states in Pb isotopes are shown in the left part of Fig. 1. They are known in heavier Pb, most probably because they are not yrast. These states are interpreted as a two-quasiparticle (2qp) excitation based on the π(h9/213/2)K∗=11− configuration as confirmed in 196Pb by the measurement of its g-factor [14]. Experimental information on deformation is extracted from the collective bands based on the excited 0+ states and 11− states [16, 17]. Recent data on static quadrupole moments for 11− states in 194,196Pb [18,19] support the mean-field interpretation of these states as weakly deformed oblate configurations.

The neutron-deficient nuclei around the Z = 82 spherical shell gap. In a shell model picture [9, 10], the energy of these multi-particle-multi-hole (mp-mh) configurations is lowered by the quadrupole-quadrupole and pairing interactions. In mean-field models [11, 12], these states are of particular interest since the measurement of their static electromagnetic moments provides direct information on both their single-particle structure and their deformation. Several isomeric states are known in Pb isotopes. In particular, 11− levels have been observed from 188Pb to 199Pb (see, e.g. Ref. [13]). They are not known in heavier Pb, most probably because they are not yrast. These states are interpreted as a two-quasiparticle (2qp) excitation based on the π(h9/213/2)K∗=11− configuration as confirmed in 196Pb by the measurement of its g-factor [14]. Experimental information on deformation is extracted from the collective bands based on the excited 0+ states and 11− states [16, 17]. Recent data on static quadrupole moments for 11− states in 194,196Pb [18,19] support the mean-field interpretation of these states as weakly deformed oblate configurations.

The experimental excitation energies of the 11− states and of the 0+ states in Pb isotopes are shown in the left part of Fig. 1. Both exhibit a similar behavior. The steady decrease of energy with decreasing neutron number indicates that, in a shell model picture, the residual quadrupole and pairing interactions, or, in mean-field models, the deformation effects, are maximal at the middle of the neutron sub-shell.

Similar coexisting structures based on intruder configurations are known experimentally above and below the Z = 82 shell closure, in the neutron-deficient Hg, Pt and Po isotopes [2, 13]. The energy systematics of the low-energy excited 0+ states in neutron-deficient 200−196Po [13, 20] strongly resembles that in Pb. The observation of bands based on these states seems to indicate a strong mixing between different shapes. The 11− isomers have been identified from the semi-magic 210Po down to 194Pb [18]. In contrast to the Pb case, their excitation energy is nearly constant as a function of the neutron number.

Previous theoretical studies of these isotopes have been mainly devoted to the description of the 0+ intruder states, in particular in Pb. A large variety of models has been used: a schematic spherical shell model [5, 7, 14, 15] and its symmetry-based boson truncations [22], the Strutinsky approach with a Woods-Saxon potential [11, 14, 23], and several variants of self-consistent mean-field methods [24, 25, 26, 27]. A comprehensive study of the low-energy spectroscopy and ground-state properties of light Po isotopes has been presented within several schematic approaches in Ref. [28] (see review [13] for more references). On the other hand, the only systematic study of isomers in these neutron-deficient nuclei is based on cranked Nilsson-Strutinsky type calculations with a schematic Woods-Saxon potential and is devoted to 186−200Pb [11, 16]. The quadrupole moments of 11− intruder states in Pb and Po down to 194Pb and 198Po have also been analyzed recently within the particle-core coupling model [18]. The deformation properties of the 11− state in 196Pb have been discussed in the framework of the tilted-axis cranking model [18]. The
magnitude of the $11^-$ nuclear deformation is indeed a basic ingredient in this model for the description of magnetic rotational bands in both even-$A$ and odd-$A$ neutron-deficient Pb isotopes. However, up to now, no fully microscopic investigation of these isomeric configurations has been performed.

In this letter, we present the results of the first fully self-consistent description of the properties of $11^-$ intruder isomers in $^{184-198}$Pb and in $^{188-200}$Po based on the Hartree-Fock-Bogoliubov (HFB) method. Our aim is to analyze the nature of these states and to study the evolution of their deformation as a function of neutron number. The simultaneous consideration of Pb and Po isotopes will allow also to determine the effect of the $Z = 82$ spherical shell closure on the deformation trends. Since several Po isotopes have extremely shallow energy surfaces as a function of the axial quadrupole moment, we have also performed selected configuration-mixing calculations to validate the construction of isomers on an oblate configuration.

All our calculations have been performed with the same set of nucleon-nucleon effective interactions. In the mean-field channel, we use the Skyrme parameterization Sly4 and in the pairing channel, a density-dependent zero-range force mainly active on the nuclear surface as introduced in Ref. 31. The HFB equations have been solved on a three-dimensional cubic mesh for reflection symmetric nuclear shapes with the method presented in Refs. 32, 33 and include an approximate particle-number projection on the correct proton and neutron numbers. The isomeric states are constructed fully self-consistently as $2\nu$ excitations as in Ref. 37, selecting the quasiparticles whose dominant components in the HF basis are the $9/2^-$ and $13/2^+$ particle states.

In Pb isotopes, we have taken for the energy and the deformation of the first excited $0^+$ states those of the oblate minimum of the deformation energy curves. Configuration-mixing calculations have shown that this approximation is well justified, even when the potential well of the oblate minimum is very shallow. The topology of the energy curves for Po isotopes is more complex, with several shallow minima on both prolate and oblate sides. The properties of the first excited $0^+$ state have therefore been determined by performing configuration-mixing calculations with respect to the axial quadrupole collective coordinate. To avoid the ambiguity resulting from the fact that the states obtained from the configuration-mixing calculations do not have the same mean-particle number, we have included in the configuration mixing a projection on the correct proton and neutron numbers.

This set of methods has already been applied with success to several other problems, such as the description of neutron-rich nuclei, of superdeformed rotational bands and of heavy nuclei.

**Deformation energy curves**

The deformation energy curves obtained for $^{194-198}$Pb and $^{198-198}$Po isotopes are shown in Figs. 2 and 3, respectively. These curves have been obtained by HF+BCS calculations with a constraint on the axial mass quadrupole moment $q \equiv q_{20} = (2z^2 - x^2 - y^2)$. Our formalism for configuration mixing calculations is indeed up to now limited to the mixing of HF+BCS wave functions and does not permit to use HFB ones. For this reason, we shall base our discussion of the deformation properties on the HF+BCS energy curves. We have checked that very similar curves are obtained within the constrained HFB method and that the trend in the $0^+_2$ excitation energy as a function of the neutron number is the same. In order to make a connection with simpler models, we have labelled orbitals by approximate Nilsson indices.

For all Pb isotopes (Fig. 2), the ground state corresponds to the spherical minimum. With decreasing neutron number, two structures appear, first as shoulders, then as secondary minima on the oblate (starting from $A = 192$) and prolate sides (starting from $A = 188$). The oblate minimum occurs around $q = -10$ b which is the deformation corresponding to the crossing between the down-sloping $9/2[514]$ mean-field orbital and the up-going orbital $1/2[440]$. On the prolate side, the minimum at $q \sim 20$ b corresponds to the deformed shell gap due to the $1/2[530]$, $1/2[541]$ and $3/2[532]$ intruder orbitals and the rising of the $1/2[400]$, $3/2[402]$ and $11/2[505]$ orbitals. For $^{186}$Pb, the excitation energy of the prolate minimum significantly overestimates the experimental data. This discrepancy is irrelevant for the description of the isomeric states which are constructed on the oblate minimum. The energy of this prolate state is very sensitive to the details of the mean-field method. Several other mean-field calculations, based either on the HF+BCS method and other Skyrme interactions, or on the HFB method with the Gogny force, predict a proton minimum lower in energy than the oblate one.

As we have explained in the introduction, for the Pb isotopes, we have approximated the energy of the oblate $0^+$ state by the energy of the oblate minima. These energies are compared with the experimental data in the left part of Fig. 1. The decrease of the HF+BCS energies with decreasing neutron number is in good agreement with the data, although the experimental energies are overestimated. This overestimation is even larger by approximately 300 keV in HFB calculations.

The deformation energy curves for Po isotopes (Fig. 3) exhibit a complicated coexistence of several shallow minima. To check whether these minima are stable against triaxial deformations, we have performed selected calculations including constraints on the triaxial quadrupole moment. These calculations show that the symmetric minima corresponding to $q \sim \pm 8$ b ($\beta \sim \pm 0.1$) obtained for all Po isotopes are separated by a very low barrier of about 100 keV in the $\gamma$ plane. They are therefore probably unstable against $\gamma$ vibrations. The shoulder that appears in $^{198}$Po...
for a deformation around $-15 \text{ b}$ becomes a deep well for lighter isotopes which is the ground state for $^{192,190}\text{Po}$. In $^{188}\text{Po}$, the oblate minimum is nearly degenerate in energy with a prolate minimum at $q \sim 19 \text{ b}$. The Strutinsky calculations of Refs. [11, 23, 28] agree qualitatively with our results. It is clear from this discussion that a study of the low lying levels of Po isotopes must take into account the vibrational motion with respect to the axial quadrupole mode.

Properties of low-lying oblate $0^+$ states: configuration-mixing calculations for Po isotopes

We have performed configuration-mixing calculations with respect to the axial quadrupole moment for the Po isotopes within the generator coordinate method (GCM), as was presented in Refs. [32, 33]. The basis for the calculations is a set of wave functions $\Phi(q)$ obtained by HF+BCS calculations with a constraint on the axial quadrupole moment. These wave functions $\Phi(q)$ are projected on the correct number of protons and neutrons. Collective wave functions are then constructed as a linear superposition of the projected non-orthogonal basis functions. The weights corresponding to the different quadrupole moments are determined by diagonalizing a discretized version of the Hill and Wheeler equation [31, 32]. This procedure provides as many eigenstates as discretization points of the quadrupole moment. The lowest eigenvalue corresponds to the ground state with a wave function spread over the axial quadrupole moment. As we shall see, depending on the structure of this wave function, it represents either a vibration within a well-defined well or a mixing of mean-field states with different shapes. In the same way, the other eigenvalues of the Hill and Wheeler equation represent excited states. Depending upon their shape, they can be classified into three main categories. Some states are mainly located in a well that is clearly separated from the ground state (as it is the case for a super-deformed well). Other states can be $\beta$ vibrations with respect to a lower state in the same potential well. Finally, some states can represent a mixing of states corresponding to different shapes. We have not included triaxial deformations in these configuration-mixing calculations. Some Po isotopes being soft against the $\gamma$ degree of freedom, these deformations may affect some of our results. However, their inclusion would considerably increase the computation time, and since our main purpose is to calculate the $11^-$ isomers, we have limited the configuration mixing to axial shapes.

The energies of the first few states obtained by the diagonalization of the Hill and Wheeler equation are shown for selected Po isotopes in the upper part of Fig. 4, together with the deformation energy curves. The triangles representing the energy of the configuration-mixed states are positioned at their average quadrupole moment. The amplitudes of the collective eigenfunctions (which are related to the mixing coefficients by an integral transformation [43]) are plotted in the lower part of the figure for the three lowest states obtained for each isotope. The wave functions of these states are spread over a large range of quadrupole moments. In $^{200,198,196}\text{Po}$, the ground state and the first excited state are a mixing of an oblate and a prolate configuration, resulting in an average small oblate deformation. This mean quadrupole moment is close to the one of the first shallow oblate minimum of the deformation energy curve. In the three isotopes, the second excited $0^+$ state has an average quadrupole moment $q \sim -10 \text{ b}$, corresponding to a weakly oblate shape. While it has only a slightly dominant weight in the ground-state wave function of $^{200,198,196}\text{Po}$, this oblate configuration becomes the ground state in $^{194,192,190}\text{Po}$, with still a spreading over a large range of quadrupole moments. In $^{190}\text{Po}$, it is nearly degenerate with a predominantly prolate configuration. These results agree well with the interpretation of the experimental data that the ground states of $^{200–196}\text{Po}$ isotopes are mixed but nearly spherical, the ground state of $^{196}\text{Po}$ is mixed, with a large oblate component and that of $^{192}\text{Po}$ is predominantly oblate [13, 14]. Experiments also suggest a strong mixing between oblate and prolate shapes in the $^{196}\text{Po}$ ground state (13, 14).

In the right part of Fig. 1, we compare the three first eigenstates of the configuration-mixing calculation with the available experimental data. The states that have the largest oblate mean deformation are indicated by filled triangles. The experimental excited $0^+$ states are known in $^{200,198,196}\text{Po}$ [20] and are interpreted as $4p-2h$ excitations in the spherical shell model (see Refs. [13, 20, 14] and references therein) or as states with an oblate deformation in mean-field models. The experimental trend is well reproduced by the most oblate states, although it is not the first excited state. Note however that the experimental energies are intermediate between those of the two first excited states. This may indicate that the interaction that we use leads to a too large mixing between the deformed configurations. Such a result could be modified by the inclusion of triaxial deformations, but in a way which can hardly be anticipated.

From the deformation energy curves for the Po isotopes, the $11^-$ states can be constructed on either of the two oblate minima. The configuration-mixing analysis clearly favors the slightly oblate minimum for the heavier isotopes. For the lighter isotopes ($^{194–188}\text{Po}$), the mean deformation of the configuration-mixed ground state is intermediate between the two. For these nuclei, we shall therefore explore whether an $11^-$ isomer can be constructed for both deformation regions.

Properties of the $11^-$ isomers

The $11^-$ states in the Pb and Po isotopes have been constructed as a $2qp$ excitation on an HFB vacuum. As in Ref. [35], the $2qp$ state is calculated fully self-consistently, which means that effects such as core polarisation due to the time-odd terms of the interaction and the conservation of the number of particles or changes of deformation are taken into
account. Since the single-particle basis, which is formed by the eigenstates of the Hartree-Fock Hamiltonian, does not diagonalize the \(qp\) basis, the \(qp\) excitations cannot be defined exactly by single-particle states. We have always chosen the \(2qp\) wave functions which have the largest overlaps with the lowest \(9/2^-\) and \(13/2^+\) single-particle states. In practice, these overlaps are always larger than 0.95. As in previous calculations of \(1qp\) and \(2qp\) excited states [27, 35], the isomeric state has been obtained axial in all cases, although triaxial deformations are allowed by the symmetries of our computer code.

The experimental and theoretical excitation energies of the \(11^-\) states are compared in Fig. 1 for the Pb and Po isotopes. In the Pb isotopes, the energy of the \(11^-\) states decreases with decreasing neutron number, both in experiment and theory. The experimental trends are quite well reproduced by our calculation, although the excitation energy is overestimated by 600 to 800 keV. This overestimation is most probably related to a better description in the HFB formalism of the vacuum (corresponding to the ground state) than of \(qp\) excitations. Our calculation shows that the excitation energy of the \(11^-\) state is the lowest in \(^{190,188}\)Pb and increases in \(^{186,184}\)Pb for which there are no experimental data yet.

For the Po isotopes, the \(11^-\) states are known down to \(^{194}\)Po. The isomer has been identified in \(^{192}\)Po [44], but its energy could not be obtained from the data. In \(^{200-196}\)Po, a minimum corresponding to an \(11^-\) state could be constructed only for small oblate deformations. The theoretical excitation energies are in good agreement with the data. In the case of \(^{194-188}\)Po, \(2qp\) states have been obtained for both oblate deformations. The isomers corresponding to smaller (larger) absolute values of the quadrupole moment are shown by filled (empty) diamonds in Fig. 1 (right part). The excitation energies of both solutions are close to each other. These states have to be further mixed by a configuration-mixing calculation. One should not expect a significant change in the excitation energy of the lowest mixed eigenstate, the ground state being lowered in a similar way by configuration mixing.

The energies of the \(11^-\) states are nearly constant as a function of the neutron number and the isomers occur at a lower energy than in Pb. The \(11^-\) energies do not follow the deep decreasing trend as the intruder \(0^+\) energies. This suggests that the structure of the \(11^-\) isomers is rather similar for all \(Z = 84\) isotopes, at least down to \(^{196}\)Po. In lighter Po isotopes, the \(11^-\) states built on the most oblate minimum become lower in energy than the weakly oblate ones, and configuration-mixing calculations would be necessary to get more accurate predictions on the \(11^-\) energy and deformation.

In the terminology of phenomenological models, the constant energy of the \(11^-\) states in heavier Po isotopes, indicates that the self-consistency effects due to the \(2qp\) excitation do not affect strongly the core. This is not the case for the Pb isotopes, in which the \(Z = 82\) magic number is broken by the \(2qp\) excitation, inducing an additional core deformation, as also demonstrated by the trend in the intrinsic quadrupole moments.

The calculated intrinsic charge quadrupole moments of the \(11^-\) states in Pb and Po are plotted in Fig. 5. For comparison, there are also given two available data points for \(^{194,196}\)Pb extracted from the measured spectroscopic quadrupole moments in the assumption of the axial symmetry and \(K = 11\) [18]. The theoretical results are in good agreement with the experimental values. The deformation parameters \((\beta_2, \beta_4)\) defined as in Ref. [46] vary for Pb isotopes from \((-0.15, -0.014)\) for \(^{198}\)Pb to \((-0.19, -0.018)\) for \(^{184}\)Pb. The quadrupole moments of the intruder isomers in the Pb-isotopes are significantly larger than those of the \(11^-\) states corresponding to the weakly oblate shape in the heaviest Po isotopes. This reflects the influence of core polarisation due to the breaking of the \(Z = 82\) core in the Pb isotopes.

In lighter Po isotopes, two candidates for an \(11^-\) isomer have to be mixed by a configuration-mixing calculation. It is difficult to predict the quadrupole moment of the mixed state which could have an intermediate value between those of the two mean-field states. From these results, one can infer a smooth change from a slightly deformed oblate shape of \(11^-\) states in the heaviest Po to a more deformed oblate shape in the lightest. Experimental observation of the rotational bands above these isomers or a direct measurement of the spectroscopic quadrupole moments would be helpful to clarify the situation. For the unambiguous \(11^-\) states in the heavier Po isotopes, the deformation parameters \((\beta_2, \beta_4)\) vary from \((-0.114, -0.007)\) for \(^{200}\)Po to \((-0.117, -0.007)\) for \(^{196}\)Po.

In summary, we have, for the first time, applied the HFB method with a Skyrme interaction and a realistic pairing force to the study of the \(11^-\) isomers in a series of neutron-deficient Pb and Po nuclei. The isomers have been constructed self-consistently as \(2qp\) excitations \(\pi(h_9/2^-, i_{13/2}^+)_{11}^-\) - . Their main properties are in good agreement with the available experimental data. We obtain that the energy of \(11^-\) states in Pb isotopes reaches minimum around \(N = 106 - 108\) and increases in lighter isotopes for which there are no data yet. For Po isotopes, the \(11^-\) energy is predicted to remain roughly constant as a function of neutron number. The calculations support the assumption of oblate deformation of these nuclei in their \(11^-\) states, with a steady increase towards neutron-deficient isotopes. The deformation is more pronounced in Pb than in Po, but in lighter Po isotopes the deformation of the \(11^-\) states might become of the same order or even larger than that in the Pb isotopes. We have also studied the properties of the low-lying \(0^+\) states in Po isotopes within the configuration-mixing approach on the basis of HF+BCS wave functions. These calculations show that, in all cases, the lowest \(0^+\) states do not have a well defined deformation, and oblate, spherical and prolate shapes are strongly mixed, both in the ground and the excited \(0^+\) states.
Acknowledgments

We acknowledge discussions with A. Andreyev, D. Balabanski, M. Huyse, K. Van de Vel and P. Van Duppen on experimental data and thank M. Bender for his help in the GCM calculations. N.A.S. thanks Kris Heyde for a careful reading of the manuscript and valuable comments. This work is supported by the DWTC grant IUAP #P5/07.

[1] K. Heyde et al, Phys. Rep. 102 (1983) 291.
[2] For a review see J.L. Wood et al, Phys. Rep. 215 (1992) 101.
[3] S. Åberg, H. Flocard, W. Nazarewicz, Annu. Rev. Nucl. Part. Sci. 40 (1990) 439.
[4] R.V.F. Janssens and T.L. Khoo, Annu. rev. Nucl. Part. Sci. 41 (1991) 301.
[5] P. Van Duppen et al, Phys. Rev. Lett. 52 (1984) 1974; P. Van Duppen et al, Phys. Lett. B 154 (1985) 354.
[6] N. Bijnen et al, Z. Phys. A 356 (1996) 3; A. N. Andreyev et al, J. Phys. G 25 (1999) 835.
[7] A. N. Andreyev et al, Nature (London) 405 (2000) 430.
[8] A. N. Andreyev et al, Eur. Phys. J. A 6 (1999) 381.
[9] K. Heyde et al, Nucl. Phys. A 466 (1987) 189.
[10] K. Heyde, F. Van Isacker, J.L. Wood, Phys. Rev. C 49 (1994) 559.
[11] R. Bengtsson, W. Nazarewicz, Z. Phys. A 334 (1989) 269.
[12] W. Nazarewicz, Phys. Lett. B 305 (1993) 195.
[13] R. Julian, K. Helariutta, M. Muikku, J. Phys. G 27 (2001) R109.
[14] J. Penninga et al, Nucl. Phys. A 471 (1987) 535.
[15] G.D. Dracoulis et al, Proc. Int. Conf. “Frontiers of Nuclear Structure”, Berkeley, 2002, edit. P.Fallon.
[16] R.M. Clark et al, Nucl. Phys. A 562 (1993) 121.
[17] G.D. Dracoulis et al, Phys. Lett. B 432 (1998) 37.
[18] K. Vyvey et al., Phys. Rev. Lett. 88 (2002) 102502.
[19] K. Vyvey et al., Phys. Lett. B 538 (2002) 33.
[20] N. Bijnen et al, Phys. Rev. Lett. 75 (1995) 4571.
[21] C. De Coster, B. Decroix, K. Heyde, Phys. Rev. C 61 (2000) 067306.
[22] R. Fossier, K. Heyde, G. Thiamova, P. Van Isacker, Phys. Rev. C 67 (2003) 024306.
[23] F. R. May, V. V. Pashekevich, S. Frauenendorf, Phys. Lett. B 68 (1977) 113.
[24] N. Tajima, H. Flocard, P. Bonche, J. Dobaczewski, P.-H. Heenen, Nucl. Phys. A 551 (1993) 409.
[25] R.R. Chasman, J.L. Egido, L.M. Robledo, Phys. Lett. B 513 (2001) 325.
[26] P.-H. Heenen, A. Valor, M. Bender, P. Bonche, H. Flocard, Eur. Phys. J. A 11 (2001) 393.
[27] T. Duguet, M. Bender, P. Bonche, P.-H. Heenen, Phys. Lett B 559 (2003) 201.
[28] A.M. Oros et al., Nucl. Phys. A 645 (1999) 107.
[29] R.M. Clark, A.O. Macchiaveli, Annu. Rev. Nucl. Part. Sci. 50 (2000) 1.
[30] E. Chabanat, P. Bonche, P. Haensel, R. Schaeffer, Phys. Scripta T 56 (1995) 231.
[31] C. Rigollet, P. Bonche, H. Flocard, P.-H. Heenen, Phys. Rev. C 59 (1999) 3120.
[32] J. Terasaki, P.-H. Heenen, H. Flocard, P. Bonche, Nucl. Phys. A 600 (1996) 371.
[33] B. Gall, P. Bonche, J. Dobaczewski, H. Flocard, P.-H. Heenen, Z. Phys. A 348 (1994) 183.
[34] H.J. Lipkin, Ann. Phys. (N.Y.) 9 (1960) 272; Y. Nogami, Phys. Rev. B 134 (1962) 313; H.C. Pradhan, Y. Nogami, J. Law, Nucl. Phys. A 201 (1972) 357.
[35] P.-H. Heenen, R.V.F. Janssens, Phys. Rev. C 57 (1999) 159.
[36] J. Terasaki, H. Flocard, P.-H. Heenen, P. Bonche, Nucl. Phys. A 621 (1996) 706.
[37] J. Terasaki, H. Flocard, P.-H. Heenen, P. Bonche, Phys. Rev. C 55 (1997) 1231.
[38] T. Duguet, P. Bonche, P.-H. Heenen, Nucl. Phys. A 679 (2001) 427.
[39] P. Bonche et al, Nucl. Phys. A 510 (1985) 466.
[40] P.-H. Heenen, P. Bonche, J. Dobaczewski, H. Flocard, Nucl. Phys. A 561 (1993) 367.
[41] D. L. Hill, J. A. Wheeler, Phys. Rev. 89 (1953) 1102.
[42] M. Bender, P.-H. Heenen, and P.-G. Reinhard, Rev. Mod. Phys 75 (2003) 121.
[43] P. Ring, P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980).
[44] K. Helariutta et al, Eur. Phys. J. A 6 (1999) 289.
[45] K. Van de Vel et al, Eur. Phys. J. A 17 (2003) 167.
[46] S. Cwiok, J. Dobaczewski, P.-H. Heenen, P. Magierski, W. Nazarewicz, Nucl. Phys. A 611 (1996) 211.
FIG. 1: Theoretical and experimental energies of the $11^-$ and $0^+_2$ states in $^{184-198}$Pb (left) and in $^{188-200}$Po (right) (see Ref. [13] for the references). The experimental $0^+_2$ in $^{184}$Pb is not indicated, since its nature is not well established [8]. For $^{184}$Pb, the oblate $0^+_2$ state from configuration-mixing calculations is shown for comparison. For the theoretical $0^+_2$ states in Po isotopes, the first three eigenvalues obtained in the configuration-mixing calculations are represented by triangles. The filled triangles correspond to the states with the largest oblate mean deformation. The empty diamonds denote the $11^-$ states based on the second oblate minima for the cases when convergence was achieved.

FIG. 2: Deformation energy curves for $^{194-184}$Pb obtained from the constrained HF+BCS calculations.
FIG. 3: Deformation energy curves for $^{198-188}$Po obtained from the constrained HF+BCS calculations.

FIG. 4: Energies of the GCM eigenstates, together with the HF+BCS deformation energy curves for $^{198-190}$Po (upper part). The amplitude of the corresponding collective GCM eigenfunctions for three lowest states enumerated by $k$ are shown in the lower part of the figure.
FIG. 5: Comparison between the charge quadrupole moment of the $11^-$ states in Pb (triangles) and Po (diamonds). The empty diamonds correspond to the $11^-$ states constructed on the second oblate minima in light Po isotopes. The intrinsic quadrupole moment as extracted from the experiment [13, 14] for $^{196, 198}$Pb are shown by circles with error bars.