Design of Model-Based Gain Scheduling Controllers for Nonlinear Systems

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Abstract. The design of controllers for nonlinear systems has always been a challenge due to their complex dynamic structures that have ill-defined models with inherent uncertainties. This article investigates the implementation of PID controllers, using a gain scheduling technique that allows dynamic tuning of their gains. The methodology introduced in this paper is model-based, where sub-models for the system to be controlled are created, based on the output, as a scheduling parameter. Dominant dynamics of the nonlinear models are encapsulated into a linearized version that has a dynamic nature to effectively cover the whole range of operation. A pole-placement technique is used to design the PID dynamic gain. The proposed technique is shown to be effectively applied to robotic manipulators, process industry, and automotives.

1. Introduction
Designing controllers for nonlinear systems has always been a challenging task, especially for applications having uncertain parameters or working in changing environments. Many control strategies were reported in the literature; among them adaptive, robust, optimal, and intelligent systems. Gain scheduling is one of the most important techniques that are capable of embedding the nonlinearities into some linear time-varying parameters that depend on the states of the systems, its inputs and outputs, and/or the surrounding environment [1]. The two most famous techniques in applying gain scheduling are the linear parameter varying (LPV) and the Takagi-Sugeno (TS) paradigms [2]. Initially, LTI controllers were designed to correspond to several operating points that cover the whole range of the system’s working conditions [3], for which the gains were scheduled to achieve certain performance criteria. Lately, LPV gain scheduling systems were introduced and proved to be more successful for many applications, especially for designing PID controllers [4].

In [4], a Lyapunov-based technique was used to design a controller with guaranteed stability for the complete range of its parameters. The controller was tested on three numerical examples; one of them is a realistic model for a magnetic levitation system. The results showed that the performance of the designed controller can be better than conventional robust controllers. Nonlinear model predictive control (MPC) was used to design self-tuning PID controllers for nonlinear systems that optimize a certain performance index in [5]. Integrated quadratic forms of the tracking error and manipulated variables for the PID gains represented the performance index. The controller was tested on a first order system, incorporated with PI controller, and the results showed that the proposed method outperforms the conventional fixed gains PI controller. Another gain scheduling PID controller, with back calculation integrator anti-windup, was designed in [6] to improve the performance of a first order tank system, while preventing instability. The controller parameters were determined by trial and error at dif-
ferent operating points. The results showed that the performance of the designed controller is better than the classical PID controller. Yet, another gain scheduling controller, with piecewise affine model (PWA), was designed in [7] that relies on the identification of the working points. The PWA model is made to reduce the computational complexity of the corresponding gain scheduling matrix inequalities. The scheduling parameter was designed to be the same as the measured output. The controller was tested on a nonlinear drum boiler benchmark model. The results are considered to be a major contribution to this field, with possible extensions for further research.

Gain scheduling was applied to a wide variety of applications, as an alternative to classical PID controllers. It was used to replace traditional tuning methods for the gains of the PID controllers. In [8], it was applied to a continuous stirred tank reactor (CSTR) model, using a nonlinear PID controller that is based on local linear models and nonlinear model predictive controller. In [9], it was successfully applied to second and third order processes on a real-time digital servo system, using an improved auto-tuning scheme for Ziegler-Nichols tuned PID controllers. In [10], it was applied to a two-axis pneumatic artificial muscle robot arm model, using online tuning gain scheduling MIMO neural dynamic DNN-PID control. In [11], it was applied to a double star induction machine, using a fuzzy logic PI controller. In [12], it was applied to an active suspension system of a vehicle model, using state derivative feedback. It also incorporated soft computing via designing a neural-based PI controller for a CSTR in [13]. In this paper, gain scheduling is explored to design model-based controllers for two case studies that utilize two different forms of linearization. The first case study deals with explicit linearization, where local approximation is used to replace the nonlinear terms with their equivalent linear ones. The design is based on applying pole placement to force the closed-loop system to follow a prescribed model that has a dominant critically damped performance. A single degree of freedom (DoF) robotic manipulator is used to analyze this technique. The operating point is used to design the gain scheduler. In the second case study, an implicit linearization is made to convert a very complex CSTR model into a much simpler LTI first order model. Adaptive estimation is employed to find the parameters of the linearized model, which are indirectly used to design a PID-like controller. Stability and performance for both case studies are investigated for different operating conditions.

The rest of this paper is organized as follows. Section 2 addresses the need for applying gain scheduling to control nonlinear systems. The first case study is covered in Section 2.1, while the second case study is covered in Section 2.2. Section 3 summarizes the results obtained in this paper and provides a comprehensive conclusion regarding the findings of the current study, in addition to recommendations for best practices in the future.

2. Applying gain scheduling to nonlinear systems
Gain scheduling was first employed in military applications to design autopilot systems for advanced jet aircrafts. It successfully adapted to fast changes in altitude and speed, while reducing the mathematical complexity in dealing with sophisticated nonlinear dynamic models. Ever since, it played an important role in controlling robotics, industrial processes, and automotive applications [1-13].

Two case studies are now introduced; the first one addresses a planar one DoF robotic manipulator. This system has a simple nonlinearity that could be locally linearized at the mid-point of the operating range, resulting in an approximated LTI second order Dynamics, for which an analytical PID controller is designed, using a pole-placement technique. The second case study corresponds to a complex industrial CSTR that has a nonlinear fourth order model. Linearization is done via capturing the dominant dynamics of the system, using adaptive estimation for which the stability of the closed loop system is established, using a Lyapunov-based technique. Similarities and differences between the two presented case studies are investigated in Section 3.

2.1. The one DoF robotic manipulator
Figure 1-a illustrates the layout of the system to be considered. Without loss of generality, this system can be extended to describe multi-link planar robots, using either the Lagrange-Euler formulation or the Recursive Newton-Euler methods [14]. The proposed controller is shown in figure 1-b.
Assuming the mass to be uniformly distributed along the robot arm, the following second order non-linear model is obtained:

\[
\frac{1}{3} m L^2 \ddot{\theta} + d \dot{\theta} + \frac{1}{2} mg L \sin(\theta) = u
\]  

(1)

where \( m \) and \( L \), are the mass and length of the robot arm, respectively. In addition, \( d \) and \( g \) represent the coefficient of viscous friction at the joint and the gravitational acceleration, respectively. The control signal and the rotational angle are represented by \( u \) and \( \theta \), respectively. For all the simulations in this paper, the following default values, for the system parameters, were assumed: \( m = 1 \) kg, \( L = 1 \) m, \( d = 1 \) Nm.s, and \( g = 10 \) m/s². The initial values for both the angular velocity and acceleration were assumed zero. Moreover, the initial value for \( u \) was set, according to \( \theta_i \), which is the initial value of \( \theta \).

Using linearization for the range \( \theta_i \leq \theta \leq \theta_f \) results in:

\[
\sin(\theta) \equiv (\sin(\theta_m - \theta_m \cos(\theta_m)) + \theta_m \cos(\theta_m) = a + b \theta, \quad \theta_m = \frac{\theta_f + \theta_i}{2}
\]  

(2)

where \( \theta_f \) is the final value of \( \theta \), which should correspond to the desired value \( \theta_d \) and \( \theta_m \) is the midpoint of the linearized range. Both \( a \) and \( b \) are constants. Equation (1) is now approximated as:

\[
\frac{1}{3} m L^2 \ddot{\theta} + d \dot{\theta} + \frac{1}{2} mg L \theta = \frac{1}{2} mg L a
\]  

(3)

Let the control signal be a combination of feedforward/I-PD control, such that:

\[
u = \frac{1}{2} mg L a + \frac{K_f}{3} \left( \theta_d - \theta \right) dt - K_p \theta - K_i \dot{\theta}
\]  

(4)

resulting in the following closed-loop linearized ODE:

\[
\frac{1}{3} m L^2 \ddot{\theta} + d \dot{\theta} + \frac{1}{2} mg L \theta = \frac{K_f}{3} \left( \theta_d - \theta \right) dt - K_p \theta - K_i \dot{\theta}
\]  

(5)

which leads to the following closed-loop third order LTI system:

\[
\dot{\theta} + \frac{3}{m L^2} (d + K_p) \dot{\theta} + \frac{3}{m L^2} \left( \frac{1}{2} mg L + K_f \right) \theta + \frac{3}{m L^2} K_i \theta \equiv \frac{3}{m L^2} K_i \theta_d
\]  

(6)
Applying pole-placement methods, the suggested controller can force the close-loop system to have a dominant second order dynamics, with a critically damped response, if the characteristic equation of the system satisfies the condition of equation (7):

$$\ddot{\theta} + \frac{3}{mL^2}(d + K_D)\dot{\theta} + \frac{3}{mL^2}\left(\frac{1}{2}mgLb + K_p\right)\theta + \frac{3}{mL^2}K_i\theta \approx (s + p)(s + q)^2$$

(7)

where (assuming $p = 10$ and $q = 1$):

$$(s + p)(s + q)^2 \approx (s + q)^2, p \gg q$$

(8)

Matching the coefficients of both sides of equation (7), results in the following control gains:

$$K_p = \frac{ml^2}{3}q(2p + q) - \frac{mgL}{2}b = 7 - 5b$$

$$K_i = \frac{ml^2}{3}pq^2 = 10/3$$

$$K_D = \frac{ml^2}{3}(p + 2q) - d = 3$$

(9)

and the following state space model for the closed-loop system:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \theta_d - \theta \end{bmatrix} \\
\dot{x}_1 = \frac{3}{mL^2}x_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} = \left[ \frac{3}{mL^2}(u - \frac{1}{2}mgL\sin(x_1) - dx_2) \right]$$

(10)

where:

$$u = 5a + K_x x_1 - K_p x_1 - K_D x_2$$

(11)

For initialization of equations (10) and (11), $x_1(0) = \theta$, $x_2(0) = 0$, and $x_3(0) = [u(0) - 5a + K_P \theta]/K_i$, and $u(0) = 5\sin(\theta)$. Running the simulation, for the system in figure 1-b, results in the response shown in figure 2. The output is illustrated in figure 2-a, where it is shown that the actual response is almost identical to that of the reference model, with the dynamics of equation (8). No bias was obtained, and the linearization process was very effective, despite the wide difference between the initial and final values of the output. The relatively slow performance of the robotic manipulator is illustrated in figure 2-b, showing the small range of the velocity. Based on equation (4), the control signal has four different components, as illustrated in figure 4-c. The control signals, corresponding to the integral, proportional, and the derivative actions are illustrated by $u_i$, $u_p$, and $u_d$, respectively. The compensating control signal, $u_c$, corresponds to the offset caused by the linearization action.

Figure 2. Response of the closed-loop system using linearized gain scheduling (pole-placement).
2.2. A continuous stirred tank reactor (CSTR) with complex dynamics:

In this section, we extend the previous analysis to a heavily nonlinear model that describes the behavior of a CSTR. Figure 3-a illustrates a typical layout for the CSTR, for which the mathematical model is given in equations (12) and (13), assuming the output to be the reactor temperature. Complete definitions for the variables and the parameters of the CSTR could be found in [15]. Figure 3-b illustrates the layout of the linearized model and its proposed controller.

![Layout of the CSTR system and its control block diagram.](image)

The nonlinear states of the CSTR and their derivatives are given by:

\[
\dot{X} = \begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} = \begin{bmatrix}
\frac{F}{V} (C_A - x_1) - \frac{e}{\rho C_P} \left( \frac{x_2}{V} \right) x_1^2 \\
\frac{F}{V} (T_i - x_2) - \frac{\Delta H_2}{\rho C_p} \left( \frac{x_1}{V} \right) x_1^2 \\
\frac{UA}{V C_P} (x_2 - x_3) - \frac{F_{\text{cool}}}{V C} x_3 \\
\frac{1}{\tau_T} \left( x_4 - T_i + \frac{T_C}{\Delta T_T} x_4 \right)
\end{bmatrix}
\]  

(12)

where:

\[
X = [x_1 \ x_2 \ x_3 \ x_4]^T = [C_A \ T \ T_C \ b]^T \quad \text{and} \quad y = [0 \ 1 \ 0 \ 0]^T X
\]  

(13)

Traditionally, most industrial processes use normalized variable; consequently, the measured signal, \(x_4\), and the control signal, \(u\), are calculated in a percentage format such that 0% corresponds to a fully open valve position and a temperature of 80 °C and 100% corresponds to a fully closed valve position and a temperature of 100 °C. The nominal dynamics of the slow temperature process of the CSTR could be encapsulated in a simple first order linear model that has a single parameter, \(a\). This is depicted in equation (14):

\[
\dot{y} + ay = u \quad \text{and} \quad u = \dot{a}y + K_P (y_d - y)
\]  

(14)

where \(a\) is assumed constant within the dynamic range of the desired output and \(K_P\) is a control gain. Linearization techniques that was carried out for the previous model of the one DoF robotic manipulator is now replaced with an adaptive technique that requires estimating the value of \(a\), as shown in the
equation of the control signal, $u$. Adopting the control strategy, outlined in [15], the following simple parameter update law for $a$ could be used:

$$\dot{\hat{a}} = -K_a(y - y_d)$$

(15)

where $K$ is another control gain that is used to adjust the convergence rate of the estimation algorithm. The complete dynamics of the linearized closed-loop system and its asymptotic performance is given in equation (16), where $y(\infty)$ represents the final steady state value of $y$.

$$\dot{y} + (a - \hat{a} + K_p)y = K_p y = K_p y_d \Rightarrow y(\infty) = y_d$$

(16)

To analyse the stability and the performance of the closed-loop system, near the equilibrium point, the following state space structure is used, where $x_1$ and $x_2$ correspond to $y$ and $\hat{a}$, respectively.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -(a + K_p)x_1 + x_2 + K_p y_d \\ K_p (y_d - x_1) \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -K_p & y_d \\ -K_p y_d & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(17)

where the linearized version was evaluated at the equilibrium points; $x_1 = y_d$ and $x_2 = a$. The test for stability of the linearized version of equation (17) yields:

$$\begin{bmatrix} \lambda + K_p & -y_d \\ K_p y_d & \lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 + K_p \lambda + K_p y_d^2 = 0$$

(18)

where $\lambda$ is the Eigen value for the characteristic equation of the linearized second order system. The closed-loop system is shown to be stable for any positive values for $K_p$, $K$, and $y_d$. Moreover, the performance of the closed-loop system can be approximated by a standard second order dynamics with the following parameters:

$$\omega_n = y_d \sqrt{K_p} \quad \text{and} \quad \zeta = \frac{K_p}{2 y_d \sqrt{K_p}}$$

(19)

where $\omega_n$ and $\zeta$ stand for the natural damping frequency and the damping ratio, respectively. Equation (19) can be used as a guide for tuning the controller parameters, $K_p$ and $K$.

It is clear that the system has an explicit integral action that guarantees arriving at the desired output, $y_d$, provided that the estimation of $a$ is unbiased. The augmented dynamics of both the output and the parameter estimator result in second order dynamics, which is typical when applying a PI controller to a first order process. Moreover, the gain $K$ can be thought of as a derivative controller as it can manipulate the damping effect of the closed-loop system. Thus, the proposed controller can be thought of an adaptive PID controller, for which the gains are scheduled, according to the dynamics and the operating range of the output. Figure 4 illustrates the response of the adaptive gain scheduling system, assuming $y(0) = 88 \, ^\circ C$, $y_d = 90 \, ^\circ C$, $K_p = 2$, $K = 0.01$, and $\hat{a}(0) = 0.5$.

![Figure 4](image_url)

Figure 4. Response of the closed-loop system using linearized gain scheduling (adaptive estimation).
The output, in figure 4-a is shown to be satisfactory, with minimum overshoot and a settling time that is comparable with the 1000 s dominant time constant of the temperature process [15]. The estimation of $a$, illustrated in figure 4-b, is shown to settle down to a final value of approximately 0.6344, in a reasonable time that could be made shorter via manipulating the gain $K$. Finally, the control signal is shown to be feasible, with no saturation, as illustrated in figure 4-c.

3. Discussion and conclusion

The first case study had a simple nonlinear structure, for which explicit linearization was possible. The linearization affected only the gravitational torque of the robotic manipulator, which should result in an offset between the actual and desired values for the output. However, careful implementation and initialization of the PID controller resolved this issue and resulted in excellent tracking of the reference model. Although no adaptive tuning was done, using the error between the desired model and the actual model, the linearized system was able to result in a stable and satisfactory performance. For wider ranges of the operating conditions, deterioration in the performance will be observed. For such case, the operating range can be divided into smaller sub-ranges to improve the linearization accuracy. Only the proportional gain needed scheduling for this case study; however, for different performance criteria, the remaining gains for the integral and the derivative actions could be scheduled as well. The proposed technique could be extended to multi-link robots, after employing decoupling techniques, but at the expense of greater mathematical effort [16].

The second case study had a more complex structure that is of higher order and greater nonlinearity. Linearization-based gain scheduling was changed to be adaptive, for which the time-varying parameter of the linearized system is directly estimated. The PID-like structure of the controller needed only two gains to be tuned, which is superior, compared to traditional PID controller [17-19].

Two different methods for applying gain scheduling to nonlinear systems were introduced. Both designs exhibited a PID-like behaviour, with much easier tuning effort, compared to classical PID controllers. Based on the system to be controlled, either explicit or implicit linearization could be adopted to capture the dominant behavior of the system, while using an ODE of the same order, or much less. Only the output was used as a scheduling parameter in this paper; however, the proposed two techniques could be easily extended to include other variables as well, whether corresponding to the system itself or the working environment.

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