Weak Localization Effect in Superconductors by Radiation Damage

Mi-Ae Park
Department of Physics, University of Puerto Rico at Humacao, Humacao, PR 00791

Yong-Jihn Kim
Department of Physics, Bilkent University, 06533 Bilkent, Ankara, Turkey

Abstract

Large reductions of the superconducting transition temperature $T_c$ and the accompanying loss of the thermal electrical resistivity (electron-phonon interaction) due to radiation damage have been observed for several A15 compounds, Chevrel phase and Ternary superconductors, and NbSe$_2$ in the high fluence regime. We examine these behaviors based on the recent theory of weak localization effect in superconductors. We find a good fitting to the experimental data. In particular, weak localization correction to the phonon-mediated interaction is derived from the density correlation function. It is shown that weak localization has a strong influence on both the phonon-mediated interaction and the electron-phonon interaction, which leads to the universal correlation of $T_c$ and resistance ratio.

PACS numbers: 74.20.-z, 74.40.+k, 74.60.Mj, 74.90+n
I. INTRODUCTION

Much attention has been paid to experimental and theoretical investigations of the radiation effects on superconductors. For the practical applications of the superconductors in the magnet coils of a fusion reactor, the radiation response of the materials is important because they are subjected to irradiation. In theoretical side, the disorder effects in superconductors caused by irradiation are interesting. The radiation effects in elemental type II superconductors showed that the superconducting transition temperature, $T_c$, does not change significantly for relatively high-fluence irradiations. The slight reduction was attributed to the reduction of the gap anisotropy. Subsequent annealing leads to the partial recovery of the $T_c$ changes, implying the importance of the microscopic details of the disorder structures.

We note that most elemental type II superconductors are radiation-tolerant. For instance, the He-4 dose which resulted in $\Delta \rho_o \sim 90 \mu \Omega \text{cm}$ in Nb$_3$Ge causes $\Delta \rho_o \sim 2 \mu \Omega \text{cm}$ in Nb. Here $\rho_o$ denotes the residual resistivity. On the other hand, A15 compounds show the universal large reductions of $T_c$ and the critical currents $I_c$ for high-fluence irradiations. The residual resistivity $\rho_o$ also increases over $\sim 100 \mu \Omega \text{cm}$, indicating that A15 compounds are radiation-susceptible. Note that the layered compound NbSe$_2$, Ternary superconductors LuRh$_4$B$_4$ and ErRh$_4$B$_4$, and Chevrel phase superconductors, such as PbMo$_6$S$_8$, PbMo$_6$S$_7$, and SnMo$_5$S$_8$ also show the large $T_c$ reduction in the high fluence regime.

In this paper we explain the microscopic mechanism underlying the universal large reductions of $T_c$ and $I_c$ in A15 compounds and other materials. In particular, we stress the experimental observation of the correlation between the electrical-resistance ratio and $T_c$. Testardi and his coworkers considered Nb-Ge, V$_3$Si, and V$_3$Ge for a variety of samples produced with differing chemical composition, preparation conditions, and with varying amounts of $^4$He-induced defects. They found a close relation between the resistance ratio and $T_c$ for those samples irrespective of the manner how the disorder was achieved. Furthermore, they found that decreasing $T_c$ is accompanied by the decrease of the thermal electrical resistivity (electron-phonon interaction). It was also reported that tunneling experiments in Nb$_3$Ge and Nb-Sn clearly show a decrease of the electron-phonon coupling constant $\lambda$ accompanying the decrease of $T_c$ with disorder.

However, previous theoretical studies focused not on the changes of the electron-phonon interaction but on the smearing of the electronic density of states near the Fermi level $N(E_F)$, the microscopic details of the disorder, the gap anisotropy, and the enhancement of the Coulomb repulsion. It is understandable that a consistent explanation of the existing experimental data was not possible in those prior theories. Recently, Kim and Overhauser pointed out that Anderson’s theorem is valid only to the first power in the impurity concentration and the phonon-mediated interaction decreases exponentially by Anderson localization, in agreement with the above experimental findings. As expected, it was shown that the same weak localization correction terms occur in both the conductivity and the phonon-mediated interaction. Based on the reduced phonon-mediated interaction, we explain the universal reductions of $T_c$ and $I_c$, and the universal correlation of $T_c$ and resistance ratio in Sec. III.

Several comments are in order. (1) It is obvious that both impurity doping and irradiation (or implantation) can be used to study the disorder effects in superconductors and metals. In particular, compensation of the $T_c$ reduction caused by magnetic impurities has
been observed as a consequence of both radiation damage and ordinary impurity doping.\textsuperscript{31–34} This compensation phenomenon has been predicted by Kim and Overhauser.\textsuperscript{35,36} Recently, it has also been observed that impurity doping and/or ion-beam-induced damage in high Tc superconductors cause the metal-insulator transition and thereby suppress $T_c$.\textsuperscript{37–40} These reductions may also be understood by the weak localization effect on superconductors. The only difference is the strong renormalization of the impurity potential due to strong electron-electron interaction in high $T_c$ superconductors. (2) Although Anderson’s theorem suggests no change of the electron-phonon interaction due to disorder how strong it is, there is overwhelming experimental evidence for the decrease of the electron-phonon interaction in the strongly disordered samples and in the high fluence regime. For instance, tunneling,\textsuperscript{16–18} specific heat,\textsuperscript{41} XPS,\textsuperscript{42} loss of the thermal electrical resistivity,\textsuperscript{4} and the correlation of $T_c$ and resistance ratio\textsuperscript{4,8,9,11} manifest the decrease of the electron-phonon interaction when the electrons are weakly localized. Weak localization leads to the decrease of the amplitude of the electron wavefunction. As a result, the phonon-mediated matrix elements are also decreasing.\textsuperscript{28,29} (3) Tunneling data do not show any enhancement of the Coulomb repulsion.\textsuperscript{16–18} In addition, the loss of the thermal electrical resistivity with decreasing $T_c$ and the universal correlation between $T_c$ and resistance ratio can not be explained in terms of the increase of the Coulomb interaction. (4) Irradiation also leads to the strong $T_c$ reductions in Chevrel phase materials, such as PbMo$_6$S$_8$, PbMo$_6$S$_7$, and SnMo$_5$S$_8$,\textsuperscript{3,43,44} and NbSe$_2$,\textsuperscript{43} at fluences above $\sim 10^{18}\text{n/cm}^2$. These materials are more radiation-sensitive than A15 compounds. It is clear that the origin of the strong $T_c$ reduction is not related to the microscopic details of the disorder, but related to the universal nature of the electronic state in the irradiated samples.

In Sec. II, we briefly review the experimental results of the radiation damage effects on A15 compounds. The universal large reduction of $T_c$, the accompanying decrease of thermal electrical resistivity, and the correlation of $T_c$ and resistance ratio will be emphasized. In Sec. III, weak localization correction on the phonon-mediated interaction is derived. The resulting $T_c$ decrease will be compared with experiments in Sec. IV.

II. RADIATION EFFECTS IN A15 COMPOUND SUPERCONDUCTORS: UNIVERSAL $T_C$ REDUCTION AND RESISTANCE RATIO

Several A15 compounds have been investigated, including Nb$_3$Sn\textsuperscript{5–7,11–13,20}, Nb$_3$Al\textsuperscript{6,7,10}, Nb$_3$Ge\textsuperscript{4,6–8,11,13}, Nb$_3$Ga\textsuperscript{6,7}, Nb$_3$Pt\textsuperscript{24}, V$_3$Si\textsuperscript{4,7,11}, and V$_3$Ge\textsuperscript{4,11}. Both high-energy neutron\textsuperscript{5,6,10} and other energetic charged particles, such as protons,\textsuperscript{14} $\alpha$ particles,\textsuperscript{4,8,11,13} oxygens,\textsuperscript{12} and electrons\textsuperscript{20} were used to irradiate a variety of A15 compounds. Table I summarizes the experimental results of the irradiation effects on A15 compound superconductors, Chevrel phases, and NbSe$_2$. Note that the large $T_c$ reductions are found in not only A15 compound superconductors but also Chevrel phase superconductors, Ternary superconductors, and NbSe$_2$, implying the universality of the phenomenon.

The response of the superconducting properties of A15 compounds to irradiation can be classified into the behavior at low fluences and at higher fluences. In the low-fluence regime, little or no change in $T_c$ occurs, while universal large reductions of $T_c$ are observed for higher fluences. We focus on the universal $T_c$ reduction in this paper. The boundary between the two regimes depends on the irradiating particles, since the heavy ions give
rise to more severe radiation damage. For instance, the low-fluence regime corresponds to neutron fluence \(< \sim 10^{18} \text{n/cm}^2\) and \(^4\text{He}\) fluence \(< \sim 10^{15} \text{\(^4\text{He}/\text{cm}^2\)}\). For much more higher fluences the saturation of \(T_c\) is often found. It is noteworthy that the saturated \(T_c\) state is accompanied by a saturated value of the residual resistivity \(\rho_o\). Accordingly, the classification based on the residual resistivity (not the fluence) may be more appropriate.

In terms of the residual resistivity, the low-fluence regime corresponds to \(\rho < \sim 10^{\mu \Omega \text{cm}}\) irrespective of the irradiating particles. The saturations of \(T_c\) and the residual resistivity are easily understood in this classification scheme.

From Table I, it is clear that the universal \(T_c\) reduction is not crucially dependent on any specific irradiating particle, any specific material and any specific defect. Many people noticed that the universal \(T_c\) reduction is governed by the total residual resistivity \(\rho_o\) due to the radiation damage and the inherent damage present in the sample. Furthermore, the close relation between the \(T_c\) decrease and resistance ratio was established in Nb – Ge, V\(_3\)Si, and V\(_3\)Ge. The relation was also noticed in Nb – O solid solutions. Testardi and his coworkers reported that the correlation of \(T_c\) and resistance ratio is independent of all sputtering conditions, film thickness, composition, and radiation damage. This result implies that the defects produced during the irradiation are similar in their effect on \(T_c\) to those produced during the film growth process. Consequently, the correlation of \(T_c\) and resistance ratio is also universal. Until the resistance ratio is about 5, \(T_c\) does not change much. When it is smaller than 2, \(T_c\) drops quickly. Finally superconductivity disappears if resistance ratio is around 1. Testardi et al. also found that decreasing \(T_c\) is accompanied by the loss of the thermal electrical resistivity (electron-phonon interaction), which indicates the significant role of defects in both the superconducting and normal-state behavior. This finding, consistent with the correlation of \(T_c\) and resistance ratio, predicts the complete destruction of superconductivity for resistance ratio less than 1 because of the complete loss of the electron-phonon interaction.

Other evidence for the decrease of the electron-phonon interaction in the high fluence regime is the following: (1) By channeling measurements in single-crystal V\(_3\)Si, Testardi et al. also found that the radiation damage leads to the large increase of resistivity and the reduction of the electron-phonon interaction. (2) Viswanathan and Caton reported the correlation of \(T_c\) and residual resistivity in neutron-irradiated V\(_3\)Si. (3) Tsuei, Molnar, and Coey did a comparative study on the superconducting and the normal-state properties of the amorphous and the crystalline phases of Nb\(_3\)Ge. They found that the drastic reduction of \(T_c\) is due to the changes in the strength of electron-phonon interaction. Pollak, Tsuei, and Johnson did an XPS study of the crystalline and amorphous phases of Nb\(_3\)Ge and found that the crystalline phase has a higher \(T_c\) because of the enhancement of the electron-phonon coupling.

The response of the critical current \(I_c\) also depends on fluence. For low-fluence regime, \(I_c\) decreases first with fluence, and then increases with increasing fluence, implying the importance of the flux-pinning mechanism. For higher fluences, the universal reduction of \(T_c\) drives down \(I_c\). The \(I_c\) drop is field independent.
III. WEAK LOCALIZATION CORRECTION TO PHONON-MEDIATED INTERACTION

In the presence of an impurity potential $U_o$, the Hamiltonian is given by

$$H = \int dr \sum_{\alpha} \Psi_{\alpha}^\dagger \left[ \frac{p^2}{2m} + U_o(r) \right] \Psi_{\alpha}(r) - V \int \Psi_{\uparrow}^\dagger(r) \Psi_{\downarrow}^\dagger(r) \Psi_{\downarrow}(r) \Psi_{\uparrow}(r) dr,$$

where $\Psi_{\alpha}^\dagger(r)$ and $\Psi(r)$ are creation and annihilation operators for electrons. In terms of the exact scattered states $\psi_n(r)$, we expand the field operator $\Psi_{\alpha}(r)$ as

$$\Psi_{\alpha}(r) = \sum_n \psi_n(r) c_{n\alpha},$$

where $c_{n\alpha}$ is a destruction operator of the electron. Upon substituting Eq. (2) into Eq. (1), we find

$$H = \sum_n \epsilon_n c_{n\alpha}^\dagger c_{n\alpha} - \sum_{nn', n \neq n'} V_{nn'} c_{n\uparrow}^\dagger c_{n'\downarrow}^\dagger c_{n\downarrow} c_{n'\uparrow},$$

where

$$V_{nn'} = V \int \psi_{n'}^\ast(r) \psi_n^\ast(r) \psi_n(r) \psi_{n'}(r) dr.$$

Here $\epsilon_n$ is the normal state eigenenergy and $\bar{n}$ denotes the time reversed partner of the scattered state $n$. Eq. (4) was first obtained by Ma and Lee.\textsuperscript{54}

A. Anderson’s Theorem

By a unitary transformation between the scattered states and the plane wave states, $\psi_{n\alpha} = \sum_{\vec{k}} \phi_{\vec{k}\alpha} \langle \vec{k} | n \rangle$, Eq. (4) can be rewritten as,\textsuperscript{26}

$$V_{nn'} = V \sum_{\vec{k}, \vec{k}', \vec{q}} < -\vec{k}' | n > < \vec{k} | n > < \vec{k} - \vec{q} | n' > - < -\vec{k}' - \vec{q} | n' >^\ast.$$

Anderson\textsuperscript{27} assumed that the transformed BCS part in Eq. (6) plays a much more important role and each individual matrix element of the remaining interaction is so small as to be safely disregarded. Then the normalization condition of the scattered states leads to

$$V_{nn'} \cong V_{nn'}^{BCS} = V \sum_{\vec{k}, \vec{k}'} | < \vec{k} | n > |^2 | < n' | \vec{k}' > |^2$$

which is the essence of Anderson’s theorem.
However, the remaining term,
\[
V_{nn'}^{\text{non-BCS}} = V \sum_{\vec{k} \neq \vec{k}', \vec{q}} < -\vec{k} | n > < \vec{k} | n > ^* < \vec{k} - \vec{q} | n' > < -\vec{k}' - \vec{q} | n' > ^*,
\] (8)
can not always be ignored. As Anderson suggested, the above term is indeed negligible in low-fluence regime, where residual resistivity is smaller than 10\( \mu \Omega \) cm. In fact, low-fluence regime corresponds to the dirty limit where \( 1/k_F \ell < 0.1 \). \( k_F \) and \( \ell \) denotes Fermi wave vector and mean free path, respectively. Whereas for higher fluences, the remaining term contribute significantly. In this regime, electron wave functions are weakly localized. Note that weak localization yields the well-known weak localization correction to the conductivity.

Now we calculate \( V_{nn'} \) including both the BCS and non-BCS terms. To do that, we use Eq. (4) (not Eq. (6)) which is more transparent physically. In the dirty limit, the exact eigenstates \( \psi_n(\vec{r}) \) can be approximated by the incoherent superpositions of plane-wave states suggested by Thouless,\(^{55}\) which leads to the Boltzmann conductivity. The wavefunction, with energy \( \hbar^2 k_n^2/2m \), is written as
\[
\psi_n(\vec{r}) = \sum_{\vec{k}} a_n^\vec{k} e^{i\vec{k} \cdot \vec{r}}.
\] (9)
The amplitudes \( a_n^\vec{k} \) are assumed to be independent normally-distributed random variables with the variance,
\[
\overline{a_n^\vec{k} a_n^\vec{k}'} \approx \delta_{nn'} \delta_{\vec{k}\vec{k}'} \frac{\pi}{k_n^2 \ell} \left( \frac{1}{k_n^2 \ell^2} + \frac{1}{4 \ell^2} \right),
\] (10)
for large \( k_n \ell \).

Inserting Eq. (9) into Eq. (4) we obtain
\[
V_{nn'} = V \int |\psi_{n'}(\vec{r})|^2 |\psi_n(\vec{r})|^2 d\vec{r}
\approx V \int | \sum_{\vec{k}} a_n^\vec{k} e^{i\vec{k} \cdot \vec{r}} |^2 | \sum_{\vec{k}'} a_n'^{\vec{k}'} e^{i\vec{k}' \cdot \vec{r}} |^2 d\vec{r}
= V \int \sum_{\vec{k}} |a_n^\vec{k}|^2 \sum_{\vec{k}'} |a_n'^{\vec{k}'}|^2 d\vec{r}
= V.
\] (11)
We have made use of Eq. (10) which eliminated the non-BCS term. As a result, Anderson’s theorem is proven for large \( k_n \ell \) under the assumption.

B. Weak Localization Correction

For the high fluence regime, we may use the weakly localized wavefunctions suggested by Kaveh and Mott.\(^{56,57}\) For three dimensions, the weakly localized wavefunctions consist of power-law and extended wavefunctions,
\[
\psi_{\vec{k}}(\vec{r}) = A e^{i\vec{k} \cdot \vec{r}} + B \frac{e^{i\vec{k} \cdot \vec{r}}}{r^2},
\] (12)
where

\[ A^2 = 1 - 4\pi B^2 \left( \frac{1}{\ell} - \frac{1}{L} \right), \quad B^2 = \frac{3}{8\pi k_F^2} \left( \frac{1}{k_F \ell} \right). \]  \hspace{1cm} (13)

\( L \) denotes inelastic diffusion length. We then write an eigenstate \( \psi_n \) as

\[ \psi_n(r) = \sum_k a_k^n \psi_k(r) \]

\[ = \sum_k a_k^n (A e^{ikr} + B e^{ikr} r^2). \]  \hspace{1cm} (14)

Comparing to the Thouless’ wavefunction Eq. (9), Kaveh and Mott’s wavefunction includes the power-law component which originates from the diffusive motion of the electrons. Since the power-law wavefunction \( 1/r^2 \) does not contribute to the current, the conductivity is reduced as

\[ \sigma^{3d} = \sigma_B A^4 \]

\[ = \sigma_B \left[ 1 - \frac{3}{(k_F \ell)^2} \left( 1 - \frac{\ell}{L} \right) \right]. \]  \hspace{1cm} (15)

Similar situation occurs in the phonon-mediated interaction. The power-law component does not contribute to the phonon-mediated matrix element either. The reason is the following: since the power-law component peaks at some point, its contribution to the bound state of Cooper pairs far from the point is almost negligible. This is analogous to the insensitivity of the localized (bound) state with the change of the boundary conditions.\(^{58}\) Accordingly, substitution of Eq. (14) into Eq. (4) leads to the weak localization correction to the phonon-mediated interaction,

\[ V_{nn'} = V \int \sum_k |a_k^n|^2 \sum_{k'} |a_{k'}^{n'}|^2 |\psi_k(r)|^2 |\psi_{k'}(r)|^2 dr \]

\[ = V \int |\psi_k(r)|^2 |\psi_{k'}(r)|^2 dr \]  \hspace{1cm} (16)

\[ \equiv VA^4 \]

\[ = V \left[ 1 - \frac{3}{(k_F \ell)^2} \left( 1 - \frac{\ell}{L} \right) \right]. \]  \hspace{1cm} (17)

We have made use of the fact that Eq. (16) does not depend on \( \vec{k} \) or \( \vec{k}' \).

We can also derive the weak localization correction term based on the diffusive density correlation for the eigenstates. In the presence of impurities, the correlation function has a free-particle form for \( t < \tau \) (scattering time) and a diffusive form for \( t > \tau. \)\(^{25}\) As a result, for \( t > \tau \) (or \( r > \ell \)), one finds\(^{59–62}\)

\[ R = \int_{t > \tau} |\psi_n(r)|^2 |\psi_m(r)|^2 dr \]

\[ = \sum_{\vec{q}} |<\psi_n| e^{i\vec{q} \cdot \vec{r}} |\psi_m>|^2 \]

\[ = \sum_{1/L < q < 1/\ell} \frac{1}{\pi \hbar N_o(E_F) Dq^2} \]

\[ = \frac{3}{2(k_F \ell)^2} \left( 1 - \frac{\ell}{L} \right). \]  \hspace{1cm} (18)

(19)
where $N_o(E_F)$ and $D$ are the density of states and the diffusion constant, respectively. Then, the contribution from the free-particle-like density correlation is

$$V_{nm} = \int_{t<\tau} |\psi_n(r)|^2|\psi_m(r)|^2 dr$$

$$\cong VA^4$$

$$= V[1 - \frac{3}{(k_F\ell)^2}(1 - \frac{\ell}{L})].$$  \hspace{1cm} (20)

with $A^2 = 1 - R$.\textsuperscript{59} Since, only the free-particle-like density correlation contributes to the phonon-mediated interaction as explained above, we find the same weak localization correction to both the conductivity and the phonon-mediated matrix element.

The BCS $T_c$ equation is now,

$$T_c = 1.13\omega D e^{-1/\lambda_{eff}},$$  \hspace{1cm} (21)

where

$$\lambda_{eff} = N_oV[1 - \frac{3}{(k_F\ell)^2}(1 - \frac{\ell}{L})].$$  \hspace{1cm} (22)

The initial slope of $\Delta T_c$ is

$$\Delta T_c \cong \frac{1}{\lambda} \frac{3}{(k_F\ell)^2}(1 - \frac{\ell}{L})$$

$$\propto \rho_o^2.$$  \hspace{1cm} (23)

which is in good agreement with experiments.\textsuperscript{13,63–65}

\section*{C. Strong-coupling Theory}

In the strong-coupling theory,\textsuperscript{66,67} the electron-phonon coupling constant is defined by

$$\lambda = 2 \int \frac{\alpha^2(\omega)F(\omega)}{\omega} d\omega$$

$=$ \hspace{1cm} (24)

$$N_o \frac{<I^2>}{M <\omega^2>}.$$  \hspace{1cm} (25)

Here $F(\omega)$ is the phonon density of states, and $<I^2>$ and $<\omega^2>$ are the average over the Fermi surface of the square of the electronic matrix element and the phonon frequency, respectively.\textsuperscript{67} For a homogeneous system with the Einstein model, it is written as

$$\lambda_o = \frac{N_o \frac{I^2}{M\omega_D^2}}{BCS \text{ theory}}$$

In the presence of impurities, weak localization leads to a correction to $\alpha^2$ or $<I^2>$, (disregarding the changes of $F(\omega)$ and $N_o$). From Eq. (25), one finds\textsuperscript{68}
\[ \lambda = N_o \frac{I^2}{M \omega_D^2} < \int |\psi_n(r)|^2 |\psi_m(r)|^2 dr >, \]  

which agrees with the BCS theory,

\[ \lambda_{eff} = N_o V < \int |\psi_n(r)|^2 |\psi_m(r)|^2 dr >. \]  

D. Resistance Ratio

According to Matthiessen’s rule, resistivity \( \rho(T) \) caused by static and thermal disorder is additive, i.e.,

\[ \rho(T) = \rho_o + \rho_{ph}(T), \]  

where \( \rho_{ph} \) is mostly due to electron-phonon scattering. At high temperatures, the phonon limited electrical resistivity is given by\(^{69}\)

\[ \rho_{ph}(T) = \frac{4\pi mk_B T}{ne^2 \hbar} \int \frac{\alpha_{tr}^2 F(\omega)}{\omega} d\omega, \]  

where \( \alpha_{tr} \) includes an average of a geometrical factor \( 1 - \cos \theta_\ell \). Assuming \( \alpha_{tr}^2 \approx \alpha^2 \), we obtain

\[ \rho_{ph}(T) \approx \frac{2\pi mk_B T}{ne^2 \hbar} \lambda_{eff} \]

\[ \approx \frac{2\pi mk_B T}{ne^2 \hbar} N_o \frac{I^2}{M \omega_D^2} [1 - \frac{3}{(k_F \ell)^2}], \]  

Note that decreasing \( T_c \) is accompanied by the loss of the thermal resistivity \( \rho_{ph}(T) \), in good agreement with experiment.\(^4\) The ternary superconductor LuRh\(_4\)B\(_4\),\(^{15}\) also shows the same behavior. The room temperature resistance ratio is then written as

\[ \frac{\rho(300K)}{\rho_o} = \frac{\rho_o + \rho_{ph}(300K)}{\rho_o} \approx 1 + \frac{2\pi \tau \times 300K}{\hbar} \lambda_{eff}. \]  

When \( \lambda_{eff} \) goes to zero, the system is not superconducting and resistance ratio becomes 1, which is in good agreement with experiments.\(^{4,8,9,11,46}\) More details will be published elsewhere.

IV. COMPARISON WITH EXPERIMENTS

Wiesmann et al.\(^{13}\) examined the \( T_c \) change of vapor-deposited Nb\(_3\)Ge and Nb\(_3\)Sn as a function of \( \alpha \) particle fluence. The 2.5-MeV \( \alpha \) particles irradiated the samples, which were held at 30K. The samples were then cooled and both \( T_c \) and the residual resistivity \( \rho_o \)
were measured. Figure 1 shows the dependence of \( T_c \) on \( \rho_o \) in Nb$_3$Ge and Nb$_3$Sn. Thin lines are our theoretical results obtained from Eqs. (21) and (22). We find good agreement between theory and experiment. The Debye temperature and \( T_{co} \) (for the pure sample) are \( \omega_D = 302K \), \( T_{co} = 23K \) and \( \omega_D = 290K \), \( T_{co} = 18K \) for Nb$_3$Ge and Nb$_3$Sn, respectively. In the absence of experimental data for the inelastic diffusion length, we used the same value of \( L = \sqrt{D\tau_i} = \sqrt{\ell} \times 387\AA/T \) for both materials.\(^70\) Here \( \tau_i \) means the inelastic scattering time. Since it is very difficult to evaluate \( k_F\ell \) accurately,\(^71\) we assumed that \( \rho = 100\mu\Omega cm \) corresponds to \( k_F\ell = 3.65 \) and 3.60 for Nb$_3$Ge and Nb$_3$Sn with the same value of \( k_F = 0.3\AA^{-1} \). These values also give a good fitting to the dependence of \( T_c \) on the residual resistivity in impurity-doped samples.\(^72\) This is persuasive evidence that the \( T_c \) behavior is not crucially dependent on any specific defect, rather its behavior is governed by the residual resistivity.

Testardi and his coworkers\(^9\) prepared about 130 Nb-Ge films and examined the dependence of \( T_c \) on resistivity, resistance ratio, chemical composition, and sputtering conditions. The Nb/Ge ratios were in the range of \( \sim 2.4 - 5.5 \) and film thickness were about 2,000\( \AA \sim 3,500\AA \). Only films which show the width of the superconducting transition less than \( \sim 2 - 3K \) were chosen to insure the macroscopic homogeniety of the samples. They found a universal correlation of \( T_c \) and resistance ratio irrespective of all sputtering conditions, composition, and specific nature of the disorder.\(^4,8,9,11,73\) Figure 2 (a) presents a sampling of \( T_c \)-vs-resistance-ratio data for 130 Nb-Ge films by them. The correlation between \( T_c \) and the resistance ratio is obvious. Resistance ratios less than 1 were generally found in films which are not superconducting, which agrees with theoretical expression, Eq. (33). Our theoretical curve, which was obtained from Eqs. (21), (22) and (33), is also shown in the same figure. We again find good agreement between the theoretical curve and experiment. Since A15 compounds show deviations from Matthiessen’s rule possibly due to saturation,\(^4,74\) we adjusted \( \rho_{ph}(T) \) to fit experimental values\(^4\) at 300\( K \) in the following manner: \( \rho_{ph}(300K) \cong 100\mu\Omega cm \times \lambda_{eff}(1 - 2.4/k_F\ell) \). We used the same values for \( k_F, L, \) and \( \omega_D \) as in Fig. 1. But we found that \( T_{co} = 24K \) for the pure Nb$_3$Ge gives better fitting, which supports that sputtered films may have not yet achieved the highest possible \( T_c \)’s.\(^9\)

Poate et al.\(^8\) irradiated superconducting Nb-Ge films by 2-MeV \( \alpha \) particles and found a \( T_c \)-resistance correlation similar to that as-grown films. Figure 2 (b) shows the correlations both for 130 as-grown films\(^9\) and for \( \alpha \) particle irradiated films.\(^8\) They lie nicely within the correlation band. It indicates that the correlation of \( T_c \) of resistance ratio is universal irrespective of how disorder is caused, e.g., by irradiation or substitutional alloying. The correlation was also reported in V$_3$Si and V$_3$Ge.\(^4\)

\[ \text{V. DISCUSSION} \]

It is clear that weak localization effect in superconductors caused by impurity doping or radiation damage should be subjected to further experimental study. In particular, since the same weak localization correction term occurs both in the conductivity and the phonon-mediated interaction, comparative study of the normal and superconducting properties of the samples will be beneficial. It is noteworthy that Fiory and Hebard\(^65\) found that both the conductivity and the transition temperature vary as \( (k_F\ell)^{-2} \) for bulk amorphous InO$_x$.

The anti-localization effect of spin-orbit interaction will provide more insights on weak
localization effect in superconductors. In fact, Miller et al.\textsuperscript{75} found the compensation of $T_c$ decrease in highly disordered superconductors by adding impurities with large spin-orbit scattering.

The loss of the thermal electrical resistivity $\rho_{ph}(T)$ (electron-phonon interaction) with decreasing $T_c$ needs more experimental study. In particular, we may consider samples satisfying the Matthiessen’s rule, where the correlation of $T_c$ and resistance ratio is more physically transparent. We propose to investigate the usual low $T_c$ superconductors near the superconductor-insulator transition.\textsuperscript{76} We expect to find the loss of the thermal electrical resistivity as approaching the insulating regime. Unfortunately, no systematic study is available yet. Note that this behavior may provide a means of probing the phonon-mechanism in exotic superconductors, such as, heavy fermion superconductors, organic superconductors, and high $T_c$ cuprates.

\section*{VI. CONCLUSION}

We have considered irradiation effects on A15 superconductors. The universal large reductions of $T_c$ and $I_c$ due to radiation damage has been explained by the weak localization of electrons. Using weak localization correction to the phonon-mediated interaction derived from the density correlation function, we calculated $T_c$ values which are in good agreement with experimental data. It is shown that weak localization decreases significantly both the electron-phonon interaction and the phonon-mediated interaction, and thereby gives rise to the universal correlation of $T_c$ and resistance ratio.

\section*{ACKNOWLEDGMENTS}

YJK is grateful to Profs. Yun Kyu Bang and Bilal Tanatar for discussions and encouragement. M. Park thanks the FOPI at the University of Puerto Rico-Humacao for release time.
REFERENCES

1. S. V. Vonsovsky, Y. A. Izyumov, and E. Z. Kurmaev, *Superconductivity of Transition Metals*, Springer-Verlag, Berlin, (1982), ch. 9.

2. C. L. Snead, Jr and T. Luhman, *Physics of Radiation Effects in Crystals*, eds. R. A. Johnson and A. N. Orlov, (Elsevier, 1986) ch. 6.

3. A. R. Sweedler, C. L. Snead Jr, and D. E. Cox, in *Treatise on Materials Science and Technology*, Vol. 14, eds Th. Luhman and D. Dew-Hughes (Academic Press, New York, 1979) pp. 349-426.

4. L. R. Testardi, J. M. Poate, and H. J. Levinstein, Phys. Rev. B. **15**, 2570 (1977).

5. R. Bett, Cryogenics, **14**, 361 (1974).

6. A. R. Sweedler, D. S. Schweitzer, and G. W. Webb, Phys. Rev. Lett. **33**, 168 (1974)

7. A. R. Sweedler, D. E. Cox, and L. Newkirk, J. Electron. Mater. **4**, 883 (1975).

8. J. M. Poate, L. R. Testardi, A. R. Storm, and W. M. Augustyniak, Phys. Rev. Lett. **35**, 1291 (1975).

9. L. R. Testardi, R. L. Meek, J. M. Poate, W. A. Royer, A. R. Storm, and J. H. Wernick, Phys. Rev. B **11**, 4304 (1975).

10. A. R. Sweedler and D. E. Cox, Phys. Rev. B **12**, 147 (1975).

11. J. M. Poate, R. C. Dynes, L. R. Testardi, and R. H. Hammond, Phys. Rev. Lett. **37**, 1308 (1976).

12. G. Ischenko, H. Adrian, S. Klaümmunzer, M. Lehmann, P. Müller, H. Neumüller, and W. Szymczak, Phys. Rev. Lett. **39**, 43 (1977).

13. H. Wiesmann, M. Gurvitch, A. K. Ghosh, H. Lutz, K. W. Jones, A. N. Goland, and M. Strongin, J. Low. Temp. Phys. **30**, 513 (1978).

14. C. L. Snead, Jr., J. Nucl. Mater. **72**, 192 (1978).

15. R. C. Dynes, J. M. Rowell, and P. H. Schmidt, in *Ternary Superconductors*, ed. G. K. Shenoy, B. D. Dunlap, and F. Y. Fradin (North-Holland, Amsterdam, 1981), p. 169.

16. K. E. Kihlstrom, D. Mael, and T. H. Geballe, Phys. Rev. B **29**, 150 (1984).

17. D. A. Rudman and M. R. Beasley, Phys. Rev. B **30**, 2590 (1984).

18. J. Geerk, H. Rietschel, and U. Schneider, Phys. Rev. B **30**, 459 (1984).

19. R. C. Dynes and C. M. Varma, J. Phys. F **6**, L215 (1976).

20. M. Gurvitch, A. K. Ghosh, C. L. Snead, Jr., and M. Strongin, Phys. Rev. Lett. **39**, 1102 (1977).

21. H. Wiesmann, M. Gurvitch, A. K. Ghosh, H. Lutz, O. F. Kammerer, and M. Strongin, Phys. Rev. B **17**, 122 (1978).

22. A. K. Ghosh, M. Gurvitch, H. Wiesmann, and M. Strongin, Phys. Rev. B **18**, 6116 (1978).

23. D. F. Farrell and B. S. Chandrasekar, Phys. Rev. Lett. **38**, 788 (1977).

24. A. R. Sweedler, D. E. Cox, and S. Moehelecke, J. Nucl. Mater. **72**, 50 (1978).

25. P. W. Anderson, K. A. Muttalib, and T. V. Ramakrishnan, Phys. Rev. B **28**, 117 (1983).

26. Yong-Jihn Kim and A. W. Overhauser, Phys. Rev. B **47**, 8025 (1993).

27. P. W. Anderson, J. Phys. Chem. Solids **11**, 26 (1959).

28. Yong-Jihn Kim, Mod. Phys. Lett. B, **10**, 555 (1996).

29. Yong-Jihn Kim, Int. J. Mod. Phys. B **11**, 1731 (1997).

30. Mi-Ae Park and Yong-Jihn Kim, Bull. Am. Phys. Soc. **44**, 1589 (1999).

31. M. F. Merriam, S. H. Liu, and D. P. Seraphim, Phys. Rev. **136**, A17 (1964).

32. G. Boato, G. Gallinaro, and C. Rizzuto, Phys. Rev. **148**, 353 (1966).
33 A. Hofmann, W. Bauriedl, and P. Ziemann, Z. Phys. B 46, 117 (1982).
34 M. Hitzfeld and G. Heim, Sol. Sta. Comm. 29, 93 (1979).
35 Yong-Jihn Kim and A. W. Overhauser, Phys. Rev. B 49, 15779 (1994).
36 Mi-Ae Park, M. H. Lee and Yong-Jihn Kim, Physica C 306, 96 (1998).
37 J. M. Valles, Jr., A. E. White, K. T. Short, R. C. Dynes, J. P. Garno, A. F. Levi, M. Anzlowar, and K. Baldwin, Phys. Rev. B 39, 11599 (1989).
38 Y. Li, G. Xiong, and Z. Gan, Physics C 199, 269 (1992).
39 Y. Dalichaouch, M. S. Torikachvili, E. A. Early, B. W. Lee, C. L. Seaman, K. N. Yang, H. Zhou, and M. B. Maple, Solid State Commun. 65, 1001 (1988).
40 J. Fink, N. Nücker, H. Romberg, M. Alexander, M. B. Maple, J. J. Neumeier, and J. W. Allen, Phys. Rev. B 42, 4823 (1990).
41 C. C. Tsuei, S. von. Molnar, and J. M. Coey, Phys. Rev. Lett. 41, 664 (1978).
42 R. A. Pollak, C. C. Tsuei, and R. W. Johnson, Sol. Sta. Com. 23, 879 (1977).
43 A. R. Sweedler et al., in Int. Conf. Radiat. Effects Tritium Technol. Fusion Reactors, Gatlinberg, Tennessee CONF-750989, Vol II. p. 422 (1976).
44 B. S. Brown, J. W. Hafstrom, and T. E. Klippert, J. Appl. Phys. 48, 1759 (1977).
45 A. K. Ghosh, H. Weismann, M. Gurvitch, H. Lutz, O. F. Kammerer, C. L. Snead, A. Goland, and M. Strongin, J. Nucl. Mater. 72, 70 (1978).
46 C. C. Koch, J. O. Scarbrough, and D. M. Kroeger, Phys. Rev. B 9, 888 (1974).
47 L. R. Testardi, J. M. Poate, W. Weber, W. M. Augustyniak, and J. H. Barret, Phys. Rev. Lett. 39, 716 (1977).
48 R. Viswanathan and R. Caton, Phys. Rev. B 18, 15 (1978).
49 B. S. Brown, T. H. Blewitt, D. G. Wozniak, and M. Suenaga, J. Appl. Phys. 46, 5163 (1975).
50 B. S. Brown, T. H. Blewitt, T. L. Scott, and D. G. Wozniak, J. Appl. Phys. 49, 4144 (1978).
51 D. G. Schweitzer and D. M. Parkin, Appl. Phys. Lett. 24, 333 (1974).
52 D. M. Parkin, and D. G. Schweitzer, Nucl. Technol. 22, 108 (1974).
53 C. L. Snead Jr, and D. M. Parkin, Nucl. Technol. 29, 264 (1976).
54 M. Ma and P. A. Lee, Phys. Rev. B 32, 5658 (1985).
55 D. J. Thouless, Phil. Mag. 32, 877 (1975).
56 M. Kaveh and N. F. Mott, J. Phys. C 14, L177 (1981).
57 N. F. Mott and M. Kaveh, Adv. Phys. 34, 329 (1985).
58 D. J. Thouless, Phys. Rep. 13 C, 93 (1974).
59 M. Kaveh, Phil. Mag. 51, 453 (1985).
60 W. L. McMillan, Phys. Rev. B 24, 2739 (1981).
61 B. L. Altshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. 77, 2028 (1979) [Sov. Phys. JETP 50, 968 (1979)]
62 E. Abrahams, P. W. Anderson, P. A. Lee, and T. V. Ramakrishnan, Phys. Rev. B 24, 6783 (1981).
63 J. P. Orlando, E. J. McNiff, Jr., S. Foner, and M. R. Beasley, Phys. Rev. B 19, 4545 (1979).
64 S. J. Bending, M. R. Beasley, and C. C. Tsuei, Phys. Rev. B 30, 6342 (1984).
65 A. T. Fiory and A. F. Hebard, Phys. Rev. Lett. 52, 2057 (1984).
66 G. M. Eliashberg, Zh. Eksp. Teor. Fiz. 38, 966 (1960) [Sov. Phys. JETP 11, 696 (1960)].
67 W. L. Mc Millan, Phys. Rev. B 167, 331 (1968).
68 Yong-Jihn Kim, unpublished.
69 G. Grimvall, *The Electron-Phonon Interaction in Metals*, (North-Holland, Amsterdam, 1981) p.4.
70 P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. 57, 287 (1985).
71 H. Gutfreund, M. Weger, and O. Entin-Wohlman, Phys. Rev. B 31, 606 (1985).
72 Yong-Jihn Kim and K. J. Chang, Mod. Phys. Lett. B12, 763 (1998).
73 H. Lutz, H. Weismann, O. F. Kammerer, and M. Strongin, Phys. Rev. Lett. 36, 1576 (1976).
74 H. Wiesmann, M. Gurvitch, H. Lutz, A. Ghosh, B. Schwarz, M. Strongin, P. B. Allen, and J. W. Halley, Phys. Rev. Lett. 38, 782 (1977).
75 T. A. Miller, M. Kunchur, Y. Z. Zhang, P. Lindenfeld, and W. L. McLean, Phys. Rev. Lett. 61, 2717 (1988).
76 A. M. Goldman and Y. Liu, Physica D 83, 613 (1995).
Table I. Irradiation effects on A15 Compounds, Chevrel phases, and NbSe$_2$

| Sample       | Irradiating particle | $T_{co}$  | $\Delta T_c$ | Maximum fluence | Reference |
|--------------|----------------------|-----------|--------------|-----------------|-----------|
| Nb$_3$Ge     | $\alpha$ particle    | $\sim 20 K$ | $\sim 8 K$  | $10^{17}\alpha/cm^2$ | 8         |
|              | neutron              | $\sim 20 K$ | $\sim 16 K$ | $5 \times 10^{19}n/cm^2$ | 3         |
| Nb$_3$Sn     | $\alpha$ particle    | $\sim 18 K$ | $\sim 15 K$ | $7 \times 10^{17}\alpha/cm^2$ | 11        |
|              | neutron              | $\sim 18 K$ | $\sim 7 K$  | $2 \times 10^{19}n/cm^2$ | 5         |
|              | electron             | $\sim 17.8 K$ | $\sim 3.8 K$ | $4 \times 10^{20}el./cm^2$ | 20        |
| Nb$_3$Al     | neutron              | $\sim 18 K$ | $\sim 14 K$ | $5 \times 10^{19}n/cm^2$ | 6         |
| Nb$_3$Pt     | neutron              | $\sim 10.6 K$ | $\sim 8.4 K$ | $3 \times 10^{19}n/cm^2$ | 24        |
| V$_3$Si      | $\alpha$ particle    | $16.8 K$   | $\sim 14.5 K$ | $7 \times 10^{17}\alpha/cm^2$ | 11        |
|              | neutron              | $\sim 16.5 K$ | $\sim 13.5 K$ | $2.5 \times 10^{19}n/cm^2$ | 3         |
| V$_3$Ge      | $\alpha$ particle    | $6.5 K$    | $\sim 5.5 K$ | $5 \times 10^{17}\alpha/cm^2$ | 11        |
| PbMo$_6$S$_8$| neutron              | $12.8 K$   | $\sim 8.6 K$ | $1 \times 10^{19}n/cm^2$ | 3         |
| PbMo$_6$S$_7$| neutron              |            | 61%          | $1.5 \times 10^{19}n/cm^2$ | 3         |
| SnMo$_5$S$_8$| neutron              |            | 51%          | $1.5 \times 10^{19}n/cm^2$ | 3         |
| 2H – NbSe$_2$| neutron              |            | $\sim 50\%$ | $3 \times 10^{18}n/cm^2$ | 3         |
FIGURES

FIG. 1. Calculated $T_c$’s vs. residual resistivity $\rho_o$ for Nb$_3$Ge and Nb$_3$Sn. Experimental data are due to Wiesmann et al., Ref. 13.

FIG. 2. (a) Calculated $T_c$’s vs. resistance ratio for Nb-Ge. The data points relate to about 130 films made with various sputtering voltage, deposition conditions, film thickness, crystal structure, and chemical composition. Data are from Testardi et al., Ref. 9.
(b) $T_c$-resistance-ratio correlation band for 130 as-grown films with the values for damaged films superimposed. Data are from Poate et al., Ref. 8.
RESISTANCE RATIO \[ \rho(300K)/\rho(25K) \]
This figure "fig_2b.gif" is available in "gif" format from:

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