A tracking algorithm of hypersonic target based on dynamic model and square-root cubature Kalman filter

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Abstract. Because of the complexities of target motion features, the tracking of hypersonic target in near space is difficult. In this paper, a new adaptive tracking algorithm based on dynamic model and improved square-root cubature Kalman filter (SRCKF) is proposed. According to the target movement characteristics, the improved adaptive piecewise constant Jerk model is proposed based on the dynamic model. The improved square-root cubature Kalman filter which the model mismatch detection function is introduced to reduce the state estimation error. The simulation results indicate that the proposed method which combines the model and the estimation algorithm has higher accuracy and stability on maneuvering target tracking than other existing methods. The research provides a feasible solution to the further improvement of the real time tracking accuracy of near-space hypersonic target.

1. Introduction

As near-space hypersonic target (NSHT) \cite{1-2} with characteristics of expansive flight cross-domain, high velocity and complex aerodynamic parameter variation, it produce serious challenge to air defense and anti-missile systems \cite{3-4}. So, effective tracking models and algorithms for such target according to its characteristics are of great significance. Some feasible suggestions were proposed concerning the tracking sensor and algorithms of the hypersonic target tracking in near space \cite{5}. By considering the skip flying characteristic in cruise segment, Xiao S. et al. \cite{6} presented a modified model for turning maneuvering. However, the paper did not analyze the effectiveness of other maneuvering models. Wu N. et al. used interacting-multiple-model (IMM) method to estimate the motion state along with aerodynamic parameters \cite{7}. However, the model has complex interaction and large amounts of computing. Li H. N. et al. proposed the dynamic hybrid model set for hypersonic target tracking \cite{8}. However, the analytical solution structure is difficult to be implemented for linear filter design.

At present, the dynamic model based on gravity turning frame and aerodynamic pressure is widely used for space target and ballistic target \cite{9}. Therefore, this paper tries to build the dynamic model of NSHT according to its dynamic characteristics at first. To adapt to the nonlinear motion characteristics of NSHT, a piecewise adaptive jerk tracking model of NSHT is built after referring to adaptive algorithm of reentry target presented in \cite{10}. At last, the improved square-root cubature Kalman filter (SRCKF) is used to accomplish the state estimation \cite{11-13}.
2. The dynamic model of NSHT
The model of tracking maneuvering reentry target can be written as [9]:
\[
\begin{align*}
\dot{x} &= -0.5\rho(z)\alpha_d - 0.5\rho(z)\dot{\alpha}_d - 0.5\rho(z)\frac{v_y}{V^2} a_x, \\
\dot{y} &= -0.5\rho(z)\alpha_t + 0.5\rho(z)\dot{\alpha}_t - 0.5\rho(z)\frac{v_y}{V^2} a_x, \\
\dot{z} &= -0.5\rho(z)\alpha_w + 0.5\rho(z)\dot{\alpha}_w - g
\end{align*}
\]
(1)

where, \(\ddot{x}, \ddot{y}, \ddot{z}\) are components of target acceleration along the three axes respectively in East-North-Up (ENU) coordinate system; \(\rho(z)\) is relative atmospheric density function; \(\alpha_d, \alpha_t\) and \(\alpha_w\) are components of aerodynamic parameter vector along the three axes in Velocity Turn Climb (VTC) coordinate system; \(V = \sqrt{x^2 + y^2 + z^2}\) is flight speed of the target, \(V_g = \sqrt{x^2 + y^2}\). The more detailed description of the meaning of the above parameters is reported in literature [9].

Because the glide distance is far and the overload caused by aerodynamic force is relatively small of NSHT, using flat Earth and exponential atmospheric model bring serious problem [7]. So, this paper adopts a model with spherical Earth considering the Coriolis force and fits the atmospheric model to track NSHT. The final aerodynamic model of NSHT is:
\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} = 
\begin{bmatrix}
\frac{1}{2}\rho(z)\dot{\alpha}_d & \frac{v_x}{V^2} & \frac{v_y}{V^2} & \frac{v_z}{V^2} \\
0 & \frac{\mu}{r^3} & 0 & 0 \\
0 & 0 & \frac{\mu}{r^3} & 0
\end{bmatrix}
\begin{bmatrix}
\alpha_d \\
\alpha_t \\
\alpha_w
\end{bmatrix} + 
\begin{bmatrix}
\alpha_{z1} \\
\alpha_{z2} \\
\alpha_{z3}
\end{bmatrix} + 
\begin{bmatrix}
0 \\
-2\omega_x\sin B - 2\omega_z\cos B \\
-2\omega_y\sin B
\end{bmatrix}
\]
(2)

where, \(r, v, w\) are Earth’s core vector path, speed vector, and rotational angular speed of Earth; \(h\) is target altitude in spherical model; \(\mu\) is the earth’s gravitational coefficient; \(\rho\) is atmosphere density at target altitude; \(r_t\) is radius of the Earth; \((B,L,H)\) are latitude, longitude, and altitude of ground-based observation radar respectively; \(x, y, z\) are target coordinates in ENU coordinate system.

3. Piecewise-Constant Jerk model
3.1. The Piecewise-Constant acceleration model
According to the idea of tracking reentry target in literature [9] , the position, the velocity, and the aerodynamics parameter can be obtained by the Piecewise-Constant Acceleration (PCA) model when the observation intervals is small. Under ENU coordinate system, the PCA model discretization can be stated as:
\[
X_{k+1} = F_{CV} X_k + G_A d(X_k, p_k) + W^A_k
\]
(3)

where \(X_k = (x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k)^T\) is state vector of target under ENU; \(p_k\) is aerodynamics parameter vector quantity; \(F_{CV}\) is uniform model state transition matrix; \(G_A\) is input acceleration matrix; \(W^A_k\) is Gaussian noise with covariance \(Q^A\); \(d(X_k, p_k) = [\dot{x}_k, \dot{y}_k, \dot{z}_k]^T\) is acceleration vector of target.

According to equation (3), when the errors of aerodynamics parameters, target position and velocity increases, the error of acceleration will increase. Position and velocity estimation errors will increase with the increasing acceleration error. If the status model contains acceleration recursion model, the filter could amend the acceleration estimation with innovation directly, thus improving the robustness of dynamic model and weakening the dependence on position and velocity of the dynamic model.
3.2. The Piecewise-Constant Jerk model

Suppose that the jerk value of NSHT is uniform in the sampling interval, according to random model approximate thought [14], taking random error into acceleration recursion equation, the acceleration recursion equation is defined by:

\[
\begin{bmatrix}
\dot{x}_{k+1} \\
\dot{y}_{k+1} \\
\dot{z}_{k+1}
\end{bmatrix} =
\begin{bmatrix}
\dot{x}_k \\
\dot{y}_k \\
\dot{z}_k
\end{bmatrix} +
\begin{bmatrix}
\ddot{x}_k \\
\ddot{y}_k \\
\ddot{z}_k
\end{bmatrix} +\mathbf{w}_k
\quad (4)
\]

where \( \mathbf{w}_k \) is defined as the acceleration vector quantity at time \( k \) and the error which is generated by the random error of jerk acceleration at time \( k+1 \), \( \mathbf{w}_k = \left[ \ddot{x}_k + \dddot{x}_k T \dddot{y}_k + \dddot{y}_k T \dddot{z}_k + \dddot{z}_k T \right]^T \).

From equation (4), the subsection uniform Jerk model status equation under ENU coordinate system is generated by

\[
X_{k+1} = \mathbf{F}_{\text{ca}} X_k + \mathbf{G}_J(X_k, p_k) + \mathbf{W}_k^{\text{ca}}
\quad (5)
\]

where \( X_k = \left[ x_k, \dot{x}_k, \ddot{x}_k, y_k, \dot{y}_k, \ddot{y}_k, z_k, \dot{z}_k, \ddot{z}_k \right]^T \) is state vector; \( \mathbf{F}_{\text{ca}} = \text{blkdiag}(F, F, F) \) is state transition matrix; \( \mathbf{G}_J = \text{blkdiag}(\mathbf{G}_J, \mathbf{G}_J, \mathbf{G}_J) \) is jerk input matrix, and \( \mathbf{G}_J = \mathbf{G}_J = \left[ T^3/6, T^2/2, T \right]^T \); \( p_k \) is aerodynamics parameter vector and it has the same meaning as in equation (4); \( J(X_k, p_k) = \left[ x_k, y_k, z_k \right]^T \) is jerk of the target; \( \mathbf{W}_k^{\text{ca}} \) is Gaussian noise of covariance matrix \( \mathbf{Q}_k^{\text{ca}} \); \( \mathbf{Q}_k^{\text{ca}} = \text{blkdiag}(\sigma_{\text{el}}^2, \sigma_{\text{el}}^2, \sigma_{\text{el}}^2 \mathbf{q}) \), \( \sigma_{\text{el}}^2 \), \( \sigma_{\text{el}}^2 \), and \( \sigma_{\text{el}}^2 \) are instantaneous variance of the acceleration in \( x \)-, \( y \)- and \( z \)- direction, respectively. \( F \) and \( q_{\text{ca}} \) are defined as: \( F = [1 \ T \ T^2 0 1 0 0 0 1] \), \( q_{\text{ca}} = [0 0 0 0 0 q_{\text{ca}} T] \) where \( q_{\text{ca}} \) is steady state precision adjust factor; according to equation (2), if \( \bar{y} = \frac{y}{V} \), \( \bar{z} = \frac{z}{V} \), and \( J(X_k, p_k) \) is computed by

\[
\begin{bmatrix}
\dot{X} \\
\dot{Y} \\
\dot{Z}
\end{bmatrix} = \frac{1}{2} \rho (b) V^2 \sin \beta \frac{\dot{V}}{V} \left[ \dddot{X} \dddot{Y} \dddot{Z} \right] + \frac{3 \mu_x}{r^3} \frac{\dot{V}}{V} \left[ \dddot{X} \dddot{Y} \dddot{Z} \right] + \frac{3 \mu_y}{r^3} \frac{\dot{V}}{V} \left[ \dddot{X} \dddot{Y} \dddot{Z} \right] + \frac{3 \mu_z}{r^3} \frac{\dot{V}}{V} \left[ \dddot{X} \dddot{Y} \dddot{Z} \right] + \begin{bmatrix}
\dddot{X} \\
\dddot{Y} \\
\dddot{Z}
\end{bmatrix}
\quad (6)
\]

Where \( \dddot{X} = \frac{\dot{V}}{V} + \frac{\dot{V}}{V} \), \( \dddot{Y} = \frac{\dot{V}}{V} + \frac{\dot{V}}{V} \), \( \dddot{Z} = \frac{\dot{V}}{V} + \frac{\dot{V}}{V} \).

Suppose \( \sigma_{\text{el}}^2 = [\sigma_{\text{el}}^2 \sigma_{\text{el}}^2 \sigma_{\text{el}}^2] \), the instantaneous variance of acceleration is generated by

\[
\sigma_{\text{el}}^2 = \text{diag} \left( \dddot{X} \dddot{Y} \dddot{Z} + 2T^2 \dddot{X} \dddot{Y} \dddot{Z} + 2T \dddot{X} \dddot{Y} \dddot{Z} \right)
\quad (7)
\]

Where \( \dddot{X} = \left[ \dddot{X} \dddot{Y} \dddot{Z} \right] \) is acceleration estimation vector quantity, \( \dddot{X} = \left[ \dddot{X} \dddot{Y} \dddot{Z} \right] \) is jerk estimation error vector quantity. When ignoring the product of jerk and acceleration error, the variance expectation of acceleration and jerk are similar to the instantaneous variance, so equation (7) can be defined by

\[
\sigma_{\text{el}}^2 = \text{diag} \left( C_a E[\dddot{X} \dddot{Y} \dddot{Z}] + T^2 C_J E[\dddot{X} \dddot{Y} \dddot{Z}] \right)
\quad (8)
\]

Where \( C_a \) and \( C_J \) are respectively the acceleration and jerk variance conversion coefficient. The recursion of aerodynamic parameters in the segmented uniform jerk model can be achieved by calculating \( E[\dddot{X} \dddot{Y} \dddot{Z}] \) and \( E[\dddot{X} \dddot{Y} \dddot{Z}] \) separately using different filtering algorithms.
4. Improved square-root cubature Kalman filter

4.1. Square root cubature Kalman filter

The SRCKF has small computational load and is more suitable for real-time calculation [11-13]. The SRCKF directly calculates the square root of the covariance matrix, it solves the issue of easy divergence of CKF algorithm and improves the accuracy and stability of filtering.

To realize the recursive estimation of the hypersonic target aerodynamic parameter $p$ in the adjacent space, the segment jerk model is extended as a state estimation parameter, and the extended jerk motion model is shown in equation (9):

$$
\begin{bmatrix}
X_{k+1} \\
p_{k+1}
\end{bmatrix} =
\begin{bmatrix}
F_j & 0 \\
0 & I
\end{bmatrix}
\begin{bmatrix}
X_k \\
p_k
\end{bmatrix} +
\begin{bmatrix}
G_i \\
0
\end{bmatrix}
J_k +
\begin{bmatrix}
W_{k}^j \\
W_{pk}
\end{bmatrix}
$$

(9)

where $W_{pk}$ is parameter of Wiener model process noise, $Q_{pk}$ is covariance matrix. Combine equation (9) with measure equation, and the result is as follows,

$$
\begin{bmatrix}
X_k^a \\
Z_k
\end{bmatrix} = f (X_{k-1}^a) + W_{k}^a
$$

(10)

where $X_{k-1}^a = (X_{k-1}^T, p_{k-1}^T)^T$, $W_{k}^a = [W_{k-1}^T, W_{p,k-1}^T, h(X_{k}^a)]$ is a non-linear function between the measured value and the state value, and the measurement noise $V_k$ is zero-mean Gaussian white noise vector with variance $R_k$.

Suppose the posterior probability distribution of the known state estimation $p(x_{k,i} | z_{k,i-1})~ N(x_{k,i}^a, \hat{x}_{k,i}^a, P_{k,i})$, the corresponding covariance is $P_{k,i}^a$ and the expanded model progress noise covariance matrix is $Q_{k,i}^a = \text{blkdiag}(Q_{k,i}^a, Q_{a_k})$, then $S_{k,i}^a = \text{chol}(P_{k,i}^a)$ is a Cholesky decomposition. The SRCKF algorithm based on expanded status model is defined as follows:

4.1.1. Calculate the basic cubature points and the corresponding weights. The basic cubature point and the corresponding weight is:

$$
\xi_i = \sqrt{\frac{m}{2}} [1], \quad w_i = \frac{1}{m}, \quad i=1,2,\ldots,m
$$

(11)

where cubature point number is $m$, and let $m=2n_s$ according to the third-degree cubature principle; $n_s$ is dimension of state; $[1]_m$ represents the i-th line of the complete symmetric point set $[1]$.

4.1.2. Time updates. Calculate the m cubature points of the current state $(i=1,2,\ldots,m)$, $m=2n$

$$
X_{k,i-jk-1}^a = S_{k,i-jk-1}^a \xi_i + \hat{x}_{k,i-jk-1}^a
$$

(12)

Calculate the predicted value of the volume point through the nonlinear state transfer function

$$
X_{k,i}^{a(r)} = f (X_{k-1,i}^a)
$$

(13)

Estimate the predicted state (SRCKF uses equal weights) in conjunction with the weights and volume point predictions.

$$
\hat{x}_{k,i}^a = \frac{1}{m} \sum_{i=1}^{m} X_{k,i}^{a(r)}
$$

(14)

The square root of the estimated covariance matrix

$$
S_{k,i}^a = \text{Tri}(\{Z_{k,i,j}^a\})
$$

(15)

where $Q_{k,i}^a = S_{k,i}^a S_{k,i}^{T}$, and $Z_{k,i,j}^a = \frac{1}{\sqrt{m}} [X_{k,i}^{a(r)} - \hat{x}_{k,i}^a, X_{k,i}^{a(r)} - \hat{x}_{k,i}^a, \ldots, X_{k,i}^{a(r)} - \hat{x}_{k,i}^a]$. 

The algorithm $S = \text{Tria}(A)$ means that the matrix $A$ is first QR-decomposed, and a normal orthogonal matrix $B$ and an upper triangular matrix $C$ are obtained. Let $S=C^T$, and the resulting $S$ is an upper triangular matrix.

4.1.3. Measurement update ($k=1, 2, \ldots$). Calculate the update state cubature points ($i=1,2,\cdots,m$)

$$X_{i,k|k-1}^u = S_{i,k-1}^{u} + \tilde{X}_{i,k|k-1}^u$$

Calculate the predicted measurement cubature points

$$Z_{i,k|k-1} = h(X_{i,k|k-1}^u)$$

Estimate predictive measurements

$$\tilde{Z}_{i,k|k-1} = \frac{1}{m} \sum_{i=1}^{m} Z_{i,k|k-1}$$

Estimate of innovation covariance matrix is

$$S_{zz,k|k-1} = \text{Tria}([Z_{i,k|k-1} S_{R,k}])$$

where $R_k = S_{k,k}^T S_{k,k}$, $Z_{i,k|k-1} = \frac{1}{\sqrt{m}} [Z_{1,k|k-1} - \tilde{Z}_{k|k-1}, Z_{2,k|k-1} - \tilde{Z}_{k|k-1}, \ldots, Z_{m,k|k-1} - \tilde{Z}_{k|k-1}]$

Estimate the cross-covariance matrix

$$P_{zz,k|k-1} = X_{k|k-1}^* Z_{k|k-1}^T$$

where $X_{k|k-1} = \frac{1}{\sqrt{m}} [X_{1,k|k-1} - \tilde{x}_{k|k-1}, X_{2,k|k-1} - \tilde{x}_{k|k-1}, \ldots, X_{m,k|k-1} - \tilde{x}_{k|k-1}]$

Estimate the SRCKF filter gain

$$W_k = (P_{zz,k|k-1} / S_{zz,k|k-1}) / S_{zz,k|k-1}$$

Based on the new measurement $z_k$ at time $k$, the system state is updated

$$\tilde{x}_{k|k} = \tilde{x}_{k|k-1} + W_k (z_k - \tilde{z}_{k|k-1})$$

The square root factor of the error covariance matrix is updated

$$S_{k|k} = \text{Tria}([X_{k|k-1} W_{k} Z_{k|k-1} W_{k} S_{R,k}])$$

4.1.4. Calculation of instantaneous variance of acceleration. To achieve the recursion of the aerodynamic parameters in the homogeneous jerk model, the instantaneous variance of acceleration needs to be calculated. $\text{diag}(E[\tilde{x}_i \tilde{x}_i^T])$ in equation (8) can be obtained directly from the state covariance matrices associated with the output of the k-time filter, that is $\text{diag}(E[\tilde{x}_i \tilde{x}_i^T]) = [P_i(x, \dot{x}, \ddot{x}), P_i(y, \dot{y}, \ddot{y}), P_i(z, \dot{z}, \ddot{z})]^T$.

$$\vec{x} = J(x, \dot{x}, y, \dot{y}, z, \dot{z}, p)$$

where $\vec{x} = [x \ y \ z]^T$ is jerk vector. Construct the variable $x_n = [x, \dot{x}, y, \dot{y}, z, \dot{z}, p]^T$ and the state covariance matrix $P_n$, then $\vec{x} = J(x_n)$.

The state estimation value $\tilde{x}_n$ and covariance $P_n$ are known at time $k$, and the $E[\tilde{x}_i \tilde{x}_i^T]$ calculation method based on the SRCKF is given by:

$$E[\tilde{x}_i \tilde{x}_i^T] = E(J(\tilde{x}_n + S_{i,n} \xi_i) - J(\tilde{x}_n)) (J(\tilde{x}_n + S_{i,n} \xi_i) - J(\tilde{x}_n))^T$$

where $S_n = \text{chol}(P_n)$ is Cholesky decomposition $P_n$; $S_{i,n}$ is $i$-th line of $S_n$; $n_i$ is dimension number of $x_n$, $\xi_i$ is number $i$ basic cubature point. The covariance of jerk estimation error vector at time $k$ can be calculated based on equation (11), (14), (16), (19), and (20).
4.2. Model state error adaptive estimation
When the model is mismatched, the state error of the target tracking will worsen, or even diverge. Therefore, the state error coefficient of the target tracking is estimated by using the model mismatch detection function $D_k$ in real time, and the state error coefficient is transformed into the variance transformation coefficients $C_s$ and $C_t$ to drive the change of state covariance.

The model mismatch detection function is:

$$D_k = v_k S_{ss}^{-1} v_k$$  \hspace{1cm} (26)

where $v_k$ is innovation, $S_{ss}$ is innovation covariance of the filter output. $C_s$ and $C_t$ are defined by,

$$C_s = \begin{cases} 
q_k D_k > 3 \\
q_k D_k \leq 3
\end{cases}, \quad C_t = \begin{cases} 
q_k D_k > 3 \\
q_k D_k \leq 3
\end{cases}$$  \hspace{1cm} (27)

where $q_s$ and $q_t$ are designed parameter, which can be obtained by simulation. Therefore, considering the model mismatch caused by target maneuver, the acceleration variance calculation method of equation (8) is modified as:

$$\sigma_{ak}^2 = \begin{cases} 
\text{diag} \left( q_k D_k E[\ddot{x}_k \ddot{x}_k^T] + T^2 q_t D_k E[\ddot{x}_k \ddot{x}_k^T] \right) & D_k > 3 \\
\text{diag} \left( q_k E[\ddot{x}_k \ddot{x}_k^T] + T^2 q_t E[\ddot{x}_k \ddot{x}_k^T] \right) & D_k \leq 3
\end{cases}$$  \hspace{1cm} (28)

After the above derivation, the state covariance, the covariance of the process noise and the model mismatch detection function are correlated. When the target aerodynamic parameter changes cause the model mismatch, and the increase of covariance of the process noise will increase the state covariance. Mutual stimulation between state covariance and the covariance of the process noise will increase the gain of algorithm filter greatly which reduces the state estimation error.

5. Simulation results and analysis

5.1. The simulation scene
Finally, to validate the validity of proposed algorithm, the boost-glide p hypersonic vehicle is simulated [3-4]. Suppose the observing location of the radar and the moving path of the target are located to the horizon, the horizon is perpendicular to X-Z plane. The maneuvering only performs vertical climbing and diving. Resetting the target initial state as: $x_0 = 432$ km, $z_0 = 88$ km, $V_0 = 2290$ m/s, the intersection angle of initial velocity direction and forward direction of X is $\eta_0 = 190^\circ$. Figure 1 show the reentry path of maneuvering, the target velocity, and the accelerated velocity; Figure 2(a) (b) show the change laws of target aerodynamic parameter resistance and the lift force ($\beta_s = 1/\alpha_s, \beta_l = 1/\alpha_l$).

![Figure 1. The state change of target.](image-url)
5.2. The simulation results and analysis

Setting the radar standard deviation of range and angle measuring noise respectively to 50 m and 0.01°, and sample interval is T=0.1 s. Tracking algorithm process noise variance is set to $\sigma_{a,k}^2 = \sigma_{\delta,k}^2 = 5$; the algorithm’s initial resistance force parameter and variance are $\beta_{\mu_0} = 70000 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$ and $\sigma_{\mu_0}^2 = 6.5 \times 10^4 \text{ kg}^2 \cdot \text{m}^2 \cdot \text{s}^{-4}$; The initial lift parameter and variance are $\beta_{\nu_0} = 73000 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$ and $\sigma_{\nu_0}^2 = 5 \times 10^4 \text{ kg}^2 \cdot \text{m}^2 \cdot \text{s}^{-4}$.

To verify the efficiency of the proposed algorithm, the tracking effect of NSHT adopt three algorithms are compared. (1) The proposed algorithm, the piecewise constant jerk adaptive model based on the dynamic model and the SRCKF; (2) APCA, based on the segmented uniform jerk adaptive model and the SRCKF in [10]; (3) AUKF, the dynamic model and the adaptive tracking algorithm based on UKF used in [7].

According to the above assumptions, the three algorithms were 100 Monte Carlo simulations. The position and velocity root mean square (RMS) errors of the three algorithms are shown in figure 3 and figure 4.
Figure 3 shows the RMS error of the position and velocity estimation of the hypersonic target. The aerodynamic parameters of the target are unchanged at 0-90 s and 136-225 s, and the performance of the two algorithms is basically the same. In the 91-135 s interval, the target reentry process occurs maneuver. The state estimation error of the two algorithms shows that the tracking performance of the proposed algorithm is optimal; the state estimation error of the proposed algorithm is increased in tracking more maneuverable target. But the sensitivity of the proposed algorithm to the target maneuver ability change is lower than that of the other algorithms.

Figure 4 shows the RMS of drag parameters and lift parameters. In the 0-90 s interval, the accuracy of the proposed algorithm for aerodynamic parameter estimation is higher than that of APCA algorithm; in 91-135 s interval, during the transition phase of aerodynamic parameters, the APCA is sensitive to the change of the target aerodynamic parameters, which makes the state estimation error increased. The AUKF is also sensitive to the parameter variation, but with a relatively small fluctuation. The error is relatively stable; the proposed algorithm is not sensitive to the rapid changes in parameters, state estimation mainly depends on the correction of filtering algorithm, thus the state estimation error is relatively small. Table 1 shows the tracking effect of the three algorithms. The AUKF and the proposed algorithm can estimate the state with higher accuracy and are more efficient in tracking than APCA algorithm. Among them, the state estimation error and tracking validity of the proposed algorithm are the highest, but the APCA is the lowest, and AUKF falls in between.

Table 1. Comparison of algorithm performance.

| Types of algorithm | State Estimation Mean Error ( $\sigma_r = 50m$ $\sigma_\theta = 0.01^\circ$) | tracking validity | calculating time[s] |
|--------------------|------------------------------------------|-------------------|----------------------|
|                    | Position/ [m] | Velocity /[m·s$^{-1}$] |                      |                      |
| APCA               | 48.0488       | 17.8909             | 0.8504               | 0.3719               |
| AUKF               | 37.3966       | 15.2694             | 0.9174               | 0.5288               |
| proposed           | 34.3372       | 15.0608             | 0.9777               | 0.5407               |

To analyze the effect of measurement noise increase on the performance of the algorithm, the standard deviation of distance and angle measurement noise is increased to 4 times, that is, the standard deviation of the range noise is increased to 200 m and the angle noise standard deviation is increased to 0.04°.

Figure 5 shows the RMS of each algorithm after increasing the measurement noise. Compared with figure 3, with the increase of measurement error, the tracking performance of APCA algorithm seriously deteriorates, while the effect on the tracking performance of AUKF and proposed algorithm is relatively limited. This is due to the particularity of the hypersonic target motion, which makes the
model of APCA use the flat earth and exponential atmospheric model without considering the dynamic model of the earth rotation, which leads to a significant increase of tracking error. Although both the AUKF and the proposed algorithm adopted an improved dynamic model, the improved adaptive SRCKF is used to improve the filtering accuracy and tracking stability in high-noise environment.

Figure 5. The target state estimation RMS error ($\sigma_r = 200m$, $\sigma_\theta = 0.04^\circ$).

Figure 6 shows the RMS error of the estimated aerodynamic parameters of each algorithm after the increase of measurement noise. Compared to figure 4, the increase of measurement noise greatly affects the estimation of the drag parameter, while the influence on lift parameter estimation is relatively small. This is because the drag parameter varies in a small range, and the increase of measurement noise leads to the rapid decrease of the drag parameter filtering correction ability. The range of lift parameter is larger, the measurement noise increase does not affect the lift parameter filtering correction ability greatly.

Table 2 shows the average error of the state estimation and the tracking validity of the three algorithms. Compared with figure 5 and table 1, the tracking validity of AUKF and proposed algorithm significantly improves with the increase of measurement noise, and the improvement of tracking efficiency of APCA algorithm is not obvious. The tracking validity of AUKF and proposed algorithm promotes fastest and the state estimation error of proposed algorithm is the slowest.
Based on the above analysis, the proposed algorithm has significant advantages over APCA and AUKF when tracking hypersonic targets. Due to the adoption of the dynamic model, which is more suitable to the hypersonic target, and the adaptation of the process noise, the parameter estimation performance is improved while improving the state estimation performance, the steady-state tracking accuracy of the proposed algorithm is improved while greatly improving the maneuvering target tracking capability.

### 6. Conclusions

In this paper, the dynamic model of NSHT is modeled based on the analysis on aerodynamic characteristics of NSHT for the movement features of NSHT. Thus, the piecewise adaptive jerk tracking model is established to accomplish the track of NSHT with the improved SRCKF. The simulation results show that the proposed method is more precise in hypersonic tracking than the segmental uniform acceleration model. Moreover, the proposed method increases the convergence performance and stability for the state tracking, especially in case of target mutation, and this algorithm also has strong robustness for the model parameter selection.

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