Transporting non-Gaussianity from sub to super-horizon scales

David Mulryne

based on arXiv:1302.4636

(also see earlier work with Daniel Wesley, David Seery, Gemma Anderson, arXiv:1008.3159, arXiv:1203.2635, and forthcoming work with David Seery, Mafalda Dias, Jonny Frazer, Joe Elliston)
Transport introduction

First developed as alternative to $\delta N$ (e.g. Lyth and Rodriguez 2005), based on flow of probability to provide insight to evolution in multi-field models.

Provides set of coupled ODEs for the correlations of inflationary perturbations - easy numerical algorithm in contrast to $\delta N$ or In-In.

Extended to sub-horizon scales and quantum correlation functions in recent paper. Mulryne 2013

Now lots of data (Planck, Ade et al. 2013) - but nearly all (multi-field) models must be tested against it numerically.

Currently we are implementing equations in a code to be publicly released - will calculate full power, bi- (and tri-) spectrum.

This talk will introduce the (sub-horizon) transport equations and some preliminary results.
Transport basics

- We care about the Fourier space correlation functions:

\[
\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 P(k_1) \delta^3(k_1 + k_2)
\]

\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3)
\]

- Where for inflation

\[ P(k) \approx Ak^{-3} \]

- And for vanilla inflation

\[ f_{nl}(k_1, k_2, k_3) \sim \text{slow roll parameters} \]

- Cosmological perturbation theory (e.g. review of Malik and Wands 2008), provides evolution equations for perturbations (curvature/isocurvature or fields).

- Would prefer evolution equations for the correlations of these perturbations.
Transport derivation

Consider M scalar fields with perturbations:

\[ x_{\alpha'} = \{\delta \phi_{\alpha'}, \delta \dot{\phi}_{\beta'}\} \]

Consider their evolution equations in compact notation (after promoting to operators)

\[ \frac{dx_{\alpha'}}{dt} = u_{\alpha' \beta'} x_{\beta'} + \frac{1}{2!} u_{\alpha' \beta' \gamma'} \left( x_{\beta'} x_{\gamma'} - \langle x_{\beta'} x_{\gamma'} \rangle \right) + \ldots \]

Where, e.g.

\[ \delta \varphi_{1'} = \delta \varphi_1(k_1) \]

And

\[ u_{\alpha' \beta'} = (2\pi)^3 u_{\alpha \beta}(k_\alpha) \delta(k_\alpha - k_\beta), \quad u_{\alpha' \beta' \gamma'} = (2\pi)^3 u_{\alpha \beta \gamma}(k_\alpha, k_\beta, k_\gamma) \delta(k_\alpha - k_\beta - k_\gamma) \]
Transport derivation

Now consider correlation functions

\[ \Sigma_{\alpha' \beta'} = \langle x_{\alpha'} x_{\beta'} \rangle, \quad \alpha_{\alpha' \beta' \gamma'} = \langle x_{\alpha'} x_{\beta'} x_{\gamma'} \rangle \]

Where

\[ \Sigma_{\alpha' \beta'} = (2\pi)^3 \delta(k_{\alpha} + k_{\beta}) \Sigma_{\alpha \beta}(k_{\alpha}) \]
\[ \alpha_{\alpha' \beta' \gamma'} = (2\pi)^3 \delta(k_{\alpha} + k_{\beta} + k_{\gamma}) \alpha_{\alpha \beta \gamma}(k_{\alpha}, k_{\beta}, k_{\gamma}) \]

And employ Ehrenfest’s theorem

\[ \frac{d\langle O \rangle}{dt} = \left\langle \frac{dO}{dt} \right\rangle \]

We arrive at

\[ \frac{\Sigma^r_{\alpha \beta}(k_{\alpha})}{dt} = u_{\alpha \gamma}(k_{\alpha}) \Sigma^r_{\gamma \beta}(k_{\alpha}) + u_{\beta \gamma}(k_{\alpha}) \Sigma^r_{\gamma \alpha}(k_{\alpha}) \]
\[ \frac{d\alpha_{\alpha \beta \gamma}(k_{\alpha}, k_{\beta}, k_{\gamma})}{dt} = u_{\alpha \lambda}(k_{\alpha}) \alpha_{\lambda \beta \gamma}(k_{\alpha}, k_{\beta}, k_{\gamma}) + u_{\alpha \lambda \mu}(k_{\alpha}, k_{\beta}, k_{\gamma}) \Sigma^r_{\lambda \beta}(k_{\beta}) \Sigma^r_{\mu \gamma}(k_{\gamma}) \]
\[ -\frac{1}{3} u_{\alpha \lambda \mu}(k_{\alpha}, k_{\beta}, k_{\gamma}) \Sigma^i_{\lambda \beta}(k_{\beta}) \Sigma^i_{\mu \gamma}(k_{\gamma}) + \text{cyclic} \]
Transport, further properties

Many further attractive properties, for example:

Easy to convert to the statistics of $\zeta$. (Used for example in Dias, Frazer and Liddle 2013)

Evolution can be decomposed into equations for ‘shapes’ such as local $f_{\text{NL}}$, $T_{\text{NL}}$ and $g_{\text{NL}}$. Anderson, Mulryne and Seery (2012)

Geometrical decomposition and interpretation. Seery, Mulryne, Frazer and Ribeiro (2012)

Moreover, integral solutions to transport hierarchy in terms of ‘$\Gamma$’ matrices are possible – connect to the integral solutions of the In-In formalism. Seery, Mulryne, Frazer and Ribeiro (2012), Mulryne (2013)

They turn out to be Taylor coefficients of $\delta N$ style expansion, and satisfy ODEs – providing a differential formulation of $\delta N$. (see recent transport papers and earlier work of Yokoyama et al. 2007)
Transport numerical algorithm

- **Step 1.** Derive the $u$ coefficients for the model at hand (multi-field canonical/non-canonical, curved field space etc).
- **Step 2.** Calculate the initial conditions (perhaps Bunch-Davis) – integral solutions can be used to fix these at arbitrary times (at or long before horizon crossing).
- **Step 3.** Solve the ODEs for the correlations of the field perturbations. If want the bi-spectra for example, one evolution for each triangle of $k$ scales.
- **Step 4.** Convert to any other quantity of interest (zeta correlations – power/bi-spectra – fnl.....)
Transport examples (super-horizon)

Consider potential

\[ V = M^4 \left[ \frac{1}{2} m_1^2 \phi_1^2 + \frac{1}{2} g^2 \phi_1^2 \phi_2^2 + \frac{\lambda}{4} (\phi_2^2 - v^2)^2 \right] \]
Transport examples (sub-horizon)

Consider potential

\[ V = \frac{1}{2} m_1 \phi_1^2 + \frac{1}{2} m_2 \phi_2^2 \]

(preliminary results, future publication with Dias, Frazer, Mulryne, Seery)
Transport conclusions

- Transport techniques provide a suite of methods for the calculation of the correlations (power, bi-, trispectrum) of inflationary perturbations.

- In particular evolve correlations from sub- to super-horizon scales in a numerically convenient way.

- For the bi-spectrum one triangle (point on bispectrum), takes a couple of seconds on a laptop for a simple two field model - so can be used to build up a picture of bispectrum in reasonable time scale.

- In due course we will release code, initially for canonical multi-field models up to bispectrum, hopefully followed by non-canonical and other generalizations.

- arXiv:1008.3159, arXiv:1203.2635, arXiv:1302.4636