MSSM inflation and cosmological attractors

M.N. Dubinin, E.Yu. Petrova, E.O. Pozdeeva, and S.Yu. Vernov
Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University,
Leninskiye Gory 1, 119991, Moscow, Russia

Abstract

Inflationary scenarios motivated by the Minimal Supersymmetric Standard Model (MSSM) where five scalar fields are non-minimally coupled to gravity are considered. The potential of the model and the function of non-minimal coupling are polynomials of two Higgs doublet convolutions. We show that the use of the strong coupling approximation allows to obtain inflationary parameters in the case when a combination of the four scalar fields plays a role of inflaton. Numerical calculations show that the cosmological evolution leads to inflationary scenarios fully compatible with observational data for different values of the MSSM mixing angle β.

Keywords: Inflation; Minimal Supersymmetric Standard Model (MSSM); non-minimal coupling.

1 Introduction

The existence of an extremely short and intense stage of accelerated expansion in the early Universe (inflation) provides a simple explanation of the astrophysical data, including the fact that at very large distances the Universe is approximately isotropic, homogeneous and spatially flat [1, 2, 3, 4]. The inflationary models are the most reasonable models for the evolution of the early Universe, because they generate the primordial density perturbations finally initiating the formation of galaxies and large-scale structure [5, 6, 7, 8].

The high-precision measurements by the Planck space telescope show [9, 10] that the non-Gaussian perturbations are small. Therefore, the single-field inflationary models are realistic. At the same time predictions of simplest inflationary models with minimally coupled scalar field include sufficiently large values of r, the tensor-to-scalar ratio of density perturbations, and are ruled out by the Planck data. Inflationary scenarios with a minimally coupled scalar field can be improved by adding a tiny non-minimal coupling of the inflaton field to gravity [11, 12]. Note that quantum corrections to the action of the scalar field minimally coupled to gravity induce non-minimal coupling term [13, 14], proportional to $R\phi^2$ (where $R$ is the curvature and $\phi$ is the inflaton field). Results of one-loop calculations in a weak gravitational field demonstrate [15, 16, 17] that

---

*E-mail: dubinin@theory.sinp.msu.ru
†E-mail: petrova@theory.sinp.msu.ru
‡E-mail: pozdeeva@www-hep.sinp.msu.ru
§E-mail: svernov@theory.sinp.msu.ru
in order to renormalize the theory of a scalar field in curved space-time it is necessary to introduce an induced gravity term proportional to $R\phi^2$. It follows that one should add non-minimal coupling terms in the inflationary models to take into account quantum properties of the inflaton.

The idea of a supersymmetric (SUSY) inflationary model has been proposed in 1980s \cite{18, 19}. In particular, a number of advantages of simplified SUSY Grand Unified Theories (GUTs) in comparison with non-supersymmetric GUTs such as a naturally longer period of exponential expansion and better stability of the effective Higgs potential with respect to radiative corrections due to a cancelation of loop diagrams has been noted in \cite{18}. The possibility to describe the inflation using particle physics models attracts a lot of attention (for a review see \cite{20}) because it is considered as a fundamental step towards the unification of physics at all energy scales. In numerous models the role of the inflaton has been performed by the Standard Model (SM) Higgs boson \cite{21, 22, 23, 24, 25, 26, 27, 28, 29, 30} or a boson in GUTs \cite{31, 32, 33} or a scalar boson in supersymmetric models \cite{19, 34, 35, 36, 37}, see reviews \cite{38, 39}.

New particles consistent with restrictions on the new physics imposed by the LHC data provide extensive opportunities to improve significantly the Higgs-driven inflationary model. Multifield scenarios under consideration are based on a general observation that redefined fields in the Einstein frame practically coincide with primary fields in the Jordan frame at the low energy scale of the order of superpartners mass scale $M_{\text{SUSY}}$, reproducing the two-Higgs doublet potential, while at the scale higher than the GUT scale, the form of the potential in the redefined fields can be flat to ensure the slow-roll approximation in inflationary scenarios.

In our paper \cite{40}, we have considered the model with two Higgs doublets coupled to gravity non-minimally. Convolutions of the Higgs doublets up to dimension-four in the fields form the well-known MSSM two-Higgs doublet potential which is rewritten in the mass basis of scalar fields \cite{41}. This potential includes three massless Goldstone bosons and five massive Higgs bosons. Working in the physical gauge, when Goldstone bosons are not taken into account, we consider inflationary scenarios that include five Higgs bosons. It was shown \cite{10} that inflationary scenarios with suitable parameters $n_s$ and $r$ are possible at the scale corresponding to the Hubble parameter $H \sim 10^{-5}M_{\text{Pl}}$. In \cite{40} we restrict ourselves to the special case when one of the vacuum expectation values is equal to zero, so, the corresponding mixing angle $\beta = \pi/2$ (the so-called Higgs basis of scalar fields). In the present work, it is shown that inflationary scenarios compatible with observational data are possible for other values of the mixing angle $\beta$. As a rule \cite{42}, the limits on $\tan \beta$ coming from models of low-energy supersymmetry are assumed to be $1 < \tan \beta \leq m_{\text{top}}/m_h \simeq 35$, however a rough lower bound corresponding to a Higgs boson–top quark coupling in the perturbative region can be of about $\tan \beta \geq m_{\text{top}}/600$ GeV \cite{43} and the latest results of ATLAS and CMS collaborations \cite{44} show that regions of large $\tan \beta$ at the 95% confidence level (CL) should be excluded \cite{45}.

## 2 The MSSM-inspired Higgs potential

Let us consider the MSSM-inspired cosmological model which is described by the following action

$$S = \int d^4x \sqrt{-g}[f(\Phi_1, \Phi_2)R - \delta^{ab}g^{\mu\nu}\partial_\mu \Phi_a^* \partial_\nu \Phi_b - V(\Phi_1, \Phi_2)], \quad (1)$$

where $\Phi_a$, $a = 1, 2$ are the Higgs doublets and the function $f$ is taken to be

$$f(\Phi_1, \Phi_2) = \frac{M_{\text{Pl}}^2}{2} + \xi_1 \Phi_1^* \Phi_1 + \xi_2 \Phi_2^* \Phi_2. \quad (2)$$
Here $M_{Pl} \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass, the constants $\xi_1$ and $\xi_2$ are positive and dimensionless. Such form of the function $f$ follows from the requirement of renormalizability for quantum field theories in curved space-time \[13,14,15,17\], where non-minimal couplings appear as renormalization counterterms for scalar fields. We assume that vacuum expectation values for scalar fields are negligibly small in comparison with $M_{Pl}$. Note that non-minimal interaction in the form of Eq. (2) has been considered in the framework of the general two-Higgs-doublet model with discrete $Z_2$ symmetry in \[46\].

The most general renormalizable two-doublet effective potential can be written as \[43,47,48\]:

$$V(\Phi_1, \Phi_2) = V_2(\Phi_1, \Phi_2) + V_4(\Phi_1, \Phi_2),$$  

(3)

where

$$V_2 = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - [\mu_{12}(\Phi_1^\dagger \Phi_2) + h.c.],$$  

$$V_4 = \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \lambda_6(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + h.c.\right].$$  

(5)

Two Higgs doublets of the MSSM can be parameterized using the $SU(2)$ states

$$\Phi_1 = \begin{pmatrix} 1 \sqrt{2} & -i \omega_1^+ \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} 1 \sqrt{2} & -i \omega_2^+ \end{pmatrix},$$  

(6)

where $\omega_{1,2}^+$ are complex scalar fields, $\eta_{1,2}$ and $\chi_{1,2}$ are real scalar fields.

The dimensionless factors $\lambda_i (i = 1, ..., 7)$ at the tree level are expressed, using the $SU(2)$ and $U(1)$ gauge couplings $g_2$ and $g_1$, in the form \[49,50\]:

$$\lambda_1^{\text{tree}} = \lambda_2^{\text{tree}} = \frac{g_1^2 + g_2^2}{8}, \quad \lambda_3^{\text{tree}} = \frac{g_2^2 - g_1^2}{4}, \quad \lambda_4^{\text{tree}} = -\frac{g_2^2}{2}, \quad \lambda_5^{\text{tree}} = 0.$$

(7)

At the superpartners mass scale $M_{SUSY}$ renormalization group evolved tree-level quartic couplings $\lambda_i$ can be evaluated using the collider data for $g_1$ and $g_2$ at $m_{top}$ scale. Indeed, the gauge boson masses at tree level $m_Z = v \sqrt{g_1^2 + g_2^2}/2$, $m_W = v g_2/2$ ($v = \sqrt{v_1^2 + v_2^2} = (G_F \sqrt{2})^{-1/2}$, where $G_F$ is the Fermi constant). Substituting pole masses $m_Z = 91.2$ GeV, $m_W = 80.4$ GeV, and $v = 246$ GeV we obtain $g_1 = 0.36$ and $g_2 = 0.65$ at the $m_{top}$ scale.

The mass basis of scalars is constructed in a standard way \[49,50\]. The $SU(2)$ eigenstates $\omega_\alpha^\pm$, $\eta_a$ and $\chi_a (a = 1, 2)$ are expressed through mass eigenstates of the Higgs bosons (two CP-even scalars $h$, $H_0$, pseudoscalar $A$ and two charged bosons $H^\pm$) and the Goldstone bosons $G^0$, $G^\pm$ by means of two orthogonal rotations with angles $\alpha$ and $\beta$. In this paper, we use the unitary gauge $G^0 = G^\pm = 0$, therefore,

$$\eta_1 = \cos(\alpha)H_0 - \sin(\alpha)h, \quad \eta_2 = \sin(\alpha)H_0 + \cos(\alpha)h,$$

$$\chi_1 = -\sin(\beta)A, \quad \chi_2 = \cos(\beta)A, \quad \omega_1^\pm = -\sin(\beta)H^\pm, \quad \omega_2^\pm = \cos(\beta)H^\pm.$$

(9)

In the following the $h$-boson is identified as the observed $125.09 \pm 0.24$ GeV scalar state \[51\], so the 'alignment limit' of the MSSM is used where the $h$-boson couplings to the gauge bosons and fermions are SM-like in agreement with the LHC data. In this limiting case $\beta - \alpha \approx \pi/2$. Masses
of Higgs bosons are negligibly small compared with the Planck mass. At tree level the mass $m_A$ and $t_\beta \equiv \tan \beta = v_2/v_1$ can be chosen as the input parameters which fix the dimension-two parameters $\mu_1^2, \mu_2^2$ and $\mu_2^2$ of the Higgs potential (see [40] for detail). Insofar as during inflation the condition $V_4 \gg V_2$ is respected, it is a good approximation to consider $V_4$ as the potential of the model neglecting the dimension-two convolutions. We use this approximation to get inflationary scenarios.

In the paper [40], we have considered the case of mixing angle $\beta = \pi/2$, then $\tan \beta$ is infinite. In this paper, the case of an arbitrary $\beta$ with a finite value of $t_\beta$ is investigated. Such a situation is typical for various MSSM scenarios which are used to describe the LHC data. If the radiative corrections are included, a number of such effective field theories with the boundary conditions fixed at $m_{\text{top}}$ scale have been considered. A discussion of ‘benchmark scenarios’ which are denominated as $m_{h_{\text{max}}}, m_{h^0_{\text{mod+}}}, m_{h^0_{\text{mod-}}, \text{light stop}, \text{light stau and } \tau \text{-phobic}}$ and restrictions on the MSSM parameter space consistent with the LHC data can be found in [52]. Specific case of parametric scenarios where $\tan \beta \sim 1$ is analysed in [53].

Assuming the alignment limit with $\beta = \pi/2 + \alpha$, when the branching ratios of the observed resonance with the mass at 125 GeV are SM-like, and using Eqs. (8) and (9), we obtain

$$ (\Phi_1^\dagger \Phi_1) = \frac{1}{2(t_\beta^2 + 1)} \left[ (A^2 + H_0^2 + 2H^- H^+) t_\beta^2 + 2H_0 h_v t_\beta + h_v^2 \right], $$

$$ (\Phi_2^\dagger \Phi_2) = \frac{1}{2(t_\beta^2 + 1)} \left[ h_v^2 t_\beta^2 - 2H_0 h_v t_\beta + A^2 + H_0^2 + 2H^- H^+ \right], $$

$$ (\Phi_1^\dagger \Phi_2) = \frac{1}{2(t_\beta^2 + 1)} \left[ h_v (H_0 + iA) t_\beta^2 + (h_v^2 - A^2 - H_0^2 - 2H^- H^+) t_\beta + (iA - H_0) h_v \right], $$

where $h_v = h + v$.

Using Eqs. (10)–(12), the potential $V_4$ can be written in terms of the scalar fields:

$$ V_4 = \frac{1}{32(t_\beta^2 + 1)^2} \left\{ (g_1^2 + g_2^2) (t_\beta^2 - 1)^2 h_v^4 - 8 (g_1^2 + g_2^2) t_\beta (t_\beta^2 - 1) H_0 h_v^2 + [(2(2H^- H^+ - A^2 - H_0^2)g_2^2 - 2(A^2 + H_0^2 + 2H^- H^+)g_1^2) t_\beta^4 + ((4A^2 + 20H_0^2 + 8H^- H^+)g_1^2 + 4(A^2 + 5H_0^2 + 6H^- H^+)g_2^2) t_\beta^2 + 2(2H^- H^+ - A^2 - H_0^2)g_2^2 - 2(A^2 + H_0^2 + 2H^- H^+)g_1^2) h_v^2 + 8(g_1^2 + g_2^2) t_\beta (t_\beta^2 - 1) (A^2 + H_0^2 + 2H^- H^+) H_0 h_v + (g_1^2 + g_2^2) (t_\beta^2 - 1)^2 (A^2 + H_0^2 + 2H^- H^+)^2 \right\}. $$

The function $f$ in terms of the scalar fields can be written as

$$ f = \frac{M_{\text{Pl}}^2}{2} + \frac{\xi_1}{2(t_\beta^2 + 1)} \left[ (A^2 + H_0^2 + 2H^- H^+) t_\beta^2 + 2H_0 h_v t_\beta + h_v^2 \right] + \frac{\xi_2}{2(t_\beta^2 + 1)} \left[ h_v^2 t_\beta^2 - 2H_0 h_v t_\beta + A^2 + H_0^2 + 2H^- H^+ \right]. $$

(13)
3 Cosmological attractors

In this section, we show how the idea of cosmological attractors \[54, 55, 56, 57, 58, 59, 60, 61, 62\] which is based on an observation that the kinetic term in the Jordan frame practically does not affect the slow-roll parameters (so-called 'strong coupling regime') allows to get approximate values of the inflationary parameters \(n_s\) and \(r\) in the analytic form.

The potential \(V\) depends on five real scalar fields
\[
\phi^1 = \frac{H^+ + H^-}{\sqrt{2}}, \quad \phi^2 = \frac{H^+ - H^-}{\sqrt{2i}}, \quad \phi^3 = A, \quad \phi^4 = H_0, \quad \phi^5 = h_v.
\]
(14)

The action defined by Eq. (1) with \(f\) and \(V\) written in terms of the scalar fields \(\phi^I\) can be transformed to the following action in the Einstein frame [63] (see also [30, 64]):
\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{2} g_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - W \right],
\]
(15)

where
\[
G_{IJ} = \frac{M_{Pl}^2}{2 f(\phi^K)} \left[ \delta_{IJ} + \frac{3 f_{IJ}}{f(\phi^K)} \right], \quad W = \frac{M_{Pl}^4}{4 f^2},
\]
\(f_{IJ} = \partial f/\partial \phi^I\). Metric tensors in the Jordan and in the Einstein frames are related by the equation
\[
g_{\mu\nu} = \frac{2 f(\phi^I)}{M_{Pl}^2} \tilde{g}_{\mu\nu}.
\]

By definition the strong coupling regime of the field system takes place if the following inequality is respected
\[
\delta_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J \ll \frac{3}{f(\phi^K)} f_{IJ} f_{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J.
\]
(16)

Using Eq. (15) in this approximation, we get
\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{g^{\mu\nu}}{2} \partial_\mu \Theta \partial_\nu \Theta - \frac{M_{Pl}^4 V_4}{4 f^2} \right],
\]
(17)

where
\[
\Theta = \sqrt{\frac{3}{2} M_{Pl} \ln \left( \frac{f}{f_0} \right)}
\]
(18)

and \(f_0\) is a positive constant with the same dimension as \(f\). If we choose \(f_0 = M_{Pl}^2/2\), then \(\Theta = 0\) corresponds to zero values of all scalar fields. We assume that \(h_v\) is marginal and consider the inflationary scenario in the case \(\phi^5 = h_v = 0\) during inflation. Note that in [40] this scenario has been considered at \(\beta = \pi/2\) (denominated in [40] as the 'case A') which gives suitable inflationary parameters. One can observe that in the strong coupling regime the system can be described by the reduced action with only one scalar field \(\Theta\). Indeed, from (13) we get
\[
f = \frac{M_{Pl}^2}{2} + \frac{(\xi_1 t_0^2 + \xi_2)(A^2 + H_0^2 + 2H^+H^-)}{2 (t_0^2 + 1)}.
\]
(19)
It follows that the potential $V_4$ can be expressed as a function of $f$:

$$V_4 = \frac{1}{32}(g_1^2 + g_2^2) (t_H^2 - 1)^2 \left( \frac{2f - M_{Pl}^2}{\xi t_H^2 + \xi} \right)^2. \tag{20}$$

In the Einstein frame the potential is

$$W = \frac{M_{Pl}^4 V_4}{4f^2} = \frac{(g_1^2 + g_2^2) M_{Pl}^4}{32 \left( \xi t_H^2 + \xi \right)^2} (t_H^2 - 1)^2 \left( 1 - e^{-\sqrt{\frac{3}{2} M_{Pl}}} \right)^2. \tag{21}$$

The slow-roll approximation from a simple single-field model can be used to get inflationary parameters. In the spatially flat Friedman–Lemaitre–Robertson–Walker (FLRW) universe with the interval

$$ds^2 = -dt^2 + a^2(t) \left( dx_1^2 + dx_2^2 + dx_3^2 \right),$$

where $a(t)$ is the scale factor, the slow-roll parameters are

$$\epsilon = \frac{M_{Pl}^2}{2} \left( \frac{W_\Theta}{W} \right)^2 = \frac{4 e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \left( 2e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} - 1 \right)}{3 \left( 1 - e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)^2},$$

$$\eta = M_{Pl}^2 W_\Theta W = \frac{4e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \left( 2e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} - 1 \right)}{3 \left( 1 - e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)^2} = \frac{4 \left( 2 - e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)}{3 \left( 1 - e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)^2},$$

where primes denote derivatives with respect to $\Theta$.

It is convenient to express the inflationary parameters

$$n_s = 1 - 6\epsilon + 2\eta = 1 - \frac{8 e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \left( 1 + e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)}{3 \left( 1 - e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)^2}, \tag{22}$$

$$r = 16\epsilon = \frac{64 e^{-\frac{2\sqrt{3} \Theta_{Pl}}{M_{Pl}}}}{3 \left( 1 - e^{-\frac{\sqrt{3} \Theta_{Pl}}{M_{Pl}}} \right)^2}. \tag{23}$$

Using Eq. (13), we get

$$n_s = 1 - \frac{8 M_{Pl}^2 (M_{Pl}^2 + 2f)}{3 (M_{Pl}^2 - 2f)^2}, \quad r = \frac{64 M_{Pl}^4}{3 (M_{Pl}^2 - 2f)^2}. \tag{24}$$

Note that these expressions for $n_s(f)$ and $r(f)$ do not depend on $t_H$ and coincide with the corresponding formulae for $\beta = \pi/2$ (when $t_H = \infty$) which has been obtained in [40]. At the same time it is not correct to say that the corresponding inflationary scenarios do not depend on $\beta$, the
direction to potential minima in the \((v_1, v_2)\)-plane, because the Hubble parameter is expressed through \(f\) as follows:
\[
H_j^2 \approx \frac{M_{\text{Pl}}^2 (g_1^2 + g_2^2) \left( t_\beta^2 - 1 \right)^2 (M_{\text{Pl}}^2 - 2f)^2}{384 \left( \xi_1 t_\beta^2 + \xi_2 \right)^2 f^2}.
\]
(25)

One can see that inflationary scenarios are excluded at \(\beta = \pi/4\).

The temperature data of the Planck full mission and first release of the polarization data at large angular scales \([10]\) constrain the spectral index of curvature perturbations to
\[
n_s = 0.968 \pm 0.006 \ (68\% \text{ CL}),
\]
and restrict the tensor-to-scalar ratio from above
\[
r < 0.11 \ (95\% \text{ CL}).
\]
(27)

Using the observable value of \(n_s = 0.968\), we obtain from Eq. (21) the corresponding value of \(f = 43.14M_{\text{Pl}}^2\), so the Hubble parameter is expressed in a compact form
\[
H_j^2 \approx 0.01 \frac{\left( t_\beta^2 - 1 \right)^2 M_{\text{Pl}}^2 (g_1^2 + g_2^2)}{\left( \xi_1 t_\beta^2 + \xi_2 \right)^2 f^2}.
\]
(28)

As mentioned in the Introduction, there is an upper bound on the Hubble parameter during inflation \([10]\): \(H < 3.6 \times 10^{-5}M_{\text{Pl}}\). Using Eq. (28) with given values of the model parameters, one can easily check whether this upper bound is respected. More precise numerical calculations are essentially simplified when suitable initial values of \(f\) can be easily found in the strong coupling approximation.

4 Numerical calculations in the Einstein frame formulation

Varying the Einstein frame action \([15]\) with respect to \(g_{\mu\nu}\) and fields, one can get the following equations for the spatially flat FLRW metric \([64]\):
\[
H^2 = \frac{1}{3M_{\text{Pl}}^2} \left( \frac{\dot{\sigma}^2}{2} + W \right), \quad \dot{H} = -\frac{1}{2M_{\text{Pl}}^2} \dot{\sigma}^2,
\]
(29)
\[
\ddot{\phi}_I + 3H \dot{\phi}_I + \Gamma^I_{JK} \dot{\phi}_J \dot{\phi}_K + \mathcal{G}^{IJK} W_{,K} = 0,
\]
(30)
where \(\Gamma^I_{JK}\) is the Christoffel symbol for the field-space manifold with the metric \(\mathcal{G}_{IJ}, W_{,K} = \partial W/\partial \phi^K\) and \(\dot{\sigma}^2 = \mathcal{G}_{IJ} \dot{\phi}_I \dot{\phi}_J\).

During inflation the scalar factor \(a\) is a monotonically increasing function. So, we can use the number of e-foldings \(N_e = \ln(a/a_e)\), where \(a_e\) is the value of the scalar factor at the end of inflation, as a new measure of time. Due to the relation \(d/dt = H d/dN_e\), Eqs. (29) and (30) can be written in the following form \([40]\):
\[
H^2 = \frac{2W}{6M_{\text{Pl}}^2 - (\sigma')^2}, \quad \frac{d \ln H}{dN_e} = -\frac{1}{2M_{\text{Pl}}^2} (\sigma')^2,
\]
(31)
Figure 1: Evolution of the fields and the Hubble parameter as functions of the number of e-foldings $N^*_e = -N_e$ during inflation in scenarios A1 (left) and C1 (right), see Table 1. The vertical line corresponds to $N^*_e = 65$.

\[
\frac{d\phi^I}{dN_e} = \psi^I, \quad \frac{d\psi^I}{dN_e} = -\left(3 + \frac{d\ln H}{dN_e}\right) \psi^I - \Gamma^I_{JK} \psi^J \psi^K - \frac{1}{H^2} g^{IK} W'_K, \quad (32)
\]

where $(\sigma')^2 = H^2 (\dot{\sigma})^2$. So, we get the following system of ten first order equations

\[
\begin{align*}
\frac{d\phi^I}{dN_e} &= \psi^I, \\
\frac{d\psi^I}{dN_e} &= -\left(3 - \frac{(\sigma')^2}{2M^2_{Pl}}\right) \psi^I - \Gamma^I_{JK} \psi^J \psi^K - \frac{6M^2_{Pl} - (\sigma')^2}{2W} g^{IK} W'_K. \quad (33)
\end{align*}
\]

In order to calculate the spectral index $n_s$ and the tensor-to-scalar ratio $r$, the slow-roll parameters are introduced analogously to the single-field inflation

\[
\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta_{\sigma\sigma} = M^2_{Pl} \frac{M_{\sigma\sigma}}{W}, \quad M_{\sigma\sigma} \equiv \dot{\sigma}^K \dot{\sigma}^J (D^K D_J W), \quad (34)
\]

where $\sigma^I = \dot{\phi}^I / \dot{\sigma}$ is the unit vector in the field space and $D$ denotes a covariant derivative with respect to the field-space metric: $D_I \phi^J = \partial_I \phi^J + \Gamma^J_{IK} \phi^K$.

In the case of mixing angle $\beta = \pi/2$ suitable inflationary scenarios have been found in [40], where various combinations of scalar fields $\phi^1, ..., \phi^5$ play the role of inflaton. The goal of this paper is to show that inflationary scenarios for finite values of $t_\beta$ are also possible and to find the corresponding values of inflationary parameters.

As mentioned above, the approximation $V \simeq V_4$ is used at $h_v \approx 0$. In this case, the formulae obtained in the strong coupling approximation are validated. Numerical calculations show (Fig. 1) that all scalar fields monotonically decrease before and during inflation, so the function $f$ is also a monotonically decreasing function. For this reason, such initial values of scalar fields are chosen that the corresponding values of $f$ are greater than $43.14 M^2_{Pl}$. Thus, the strong coupling approximation simplifies the choice of initial conditions for numerical calculations. The list of initial conditions giving acceptable inflationary scenarios is presented in Table 1.

The spectral index $n_s$ and tensor-to-scalar ratio $r$ at the time when a characteristic scale is of the order of the Hubble radius (50–65 e-foldings before the end of inflation) can be expressed via
A

B

C

D

Table 1: The parameters of the model and the initial field values for numerical calculations.

Table 2: The values of function $f$ and the Hubble parameter $H$ at $N_e = -65$, together with the tensor-to-scalar ratio $r$ and spectral index $n_s$, obtained by Eq. (35), for successful inflationary scenarios.

5 Summary

In this paper, a MSSM-inspired extension of the original Higgs-driven inflation [22, 23, 24, 25, 26, 27, 28] is constructed using the two-Higgs doublet potential, given by Eq. (3). During inflation the quadratic part of this potential is negligibly small, so the potential can be approximated by its fourth order part $V_4$. Assuming that the field $h_v$ is negligibly small during inflation, we simplify the potential in a way suitable for calculation of transparent symbolic and numerical results for the main observables: the spectral index $n_s$ and the tensor-to-scalar ratio $r$. The inflationary scenarios under consideration incorporate four non-minimally coupled scalar fields. It is shown that the considered model can be mapped to the single-field model with the effective inflaton field defined by Eq. (18) using the strong coupling approximation. In this approximation the
inflationary parameters do not depend on the mixing angle $\beta$. On the other hand, numerical calculations in the strong coupling regime are realized in an inflationary scenarios with different values of model parameters and initial conditions. Such models share very close results for the spectral index and the tensor-to-scalar ratio in combination with negligible non-Gaussianity, which are in good agreement with the latest experimental data. A generalization of the MSSM-inspired model analysed in [40] for the case of finite values of $\tan \beta$ is found.

In conclusion, we should note that an important point beyond our analysis is the stability of results with respect to radiative corrections and the renormalization group evolution of $\xi_i$. The general case of renormalization group improved (RG-improved) effective potential for gauge theories in curved spacetime when a generalization of Coleman–Weinberg resummation for the flat space is introduced can be found in [66], where several applications to explicit field theory models have been presented. A number of simple supersymmetric models with running coupling of scalar field to gravity have been analysed. The simplest Wess–Zumino model effective potential along a flat direction in de Sitter space [67] demonstrates the power-like running of $\xi$ typical for $\lambda \phi^4$ field theory [68]. The RG-improved effective action in curved space coupled with the classical gravity includes the term $\xi(\tilde{t}) R \phi^2$ which is analogous to the representation of Eq. (2), $\tilde{t} \equiv \log(\phi^2/\mu^2)$, where $\mu$ is the renormalization scale. Inflation with different evolution of $\xi$ defined by exponent $\exp(c g^2 \tilde{t})$ has been observed in supersymmetric finite GUTs [69], where $g$ is the gauge coupling in the matter sector and the numerical constant $c$ defined by the symmetry group of the grand unification theory can be negative, positive or zero. The initial conditions for the inflaton field which are discussed in [67, 69] respect definite restrictions (for example, $|c| g^2 \sim 10^{-3}$ in the $SU(2)$ gauge theory with $SU(N)$ global invariance considered in [69]) in order to have successful inflationary scenarios. So, supersymmetric finite GUTs in curved spacetime are recognized as interesting candidates in connection with the inflationary universe scenarios and possible applications to the cosmological constant problem. The renormalization group corrections have been analyzed in different single-field inflationary scenarios motivated by different nonsupersymmetric models [26, 27, 28, 70, 71, 72]. For a slow-roll it is essential to have nearly flat effective potential in the region of the field amplitudes of the order of $M_{Pl}$. While the quantum gravity corrections are expected to be not important, the corrections induced by the SM fields and the superpartner fields (see, for example, [73] and references therein) require careful analysis which is dependent on the MSSM parametric scenario under consideration.

Acknowledgments

The authors are thankful to M.V. Sumin and I.P. Volobuev for useful discussions. This work was partially supported by Grant No. NSh-7989.2016.2 of the President of the Russian Federation. The research of E.O.P. and E.Yu.P. was supported in part by Grant No. MK-7835.2016.2 of the President of the Russian Federation.

References

[1] A. D. Linde, Particle Physics and Inflationary Cosmology, Harwood, Switzerland (1990).

[2] D. H. Lyth and A. Riotto, Particle Physics Models of Inflation and the Cosmological Density Perturbation, Phys. Rep. 314 (1999) 1.
[3] D. S. Gorbunov and V. A. Rubakov, *Introduction to the Theory of Early Universe*, Krasand, Moscow (2010) [in Russian];
D. S. Gorbunov and V. A. Rubakov, *Introduction to the theory of the early universe: Cosmological perturbations and inflationary theory*, World Scientific (2011) 489 pp.;
D. S. Gorbunov and V. A. Rubakov, *Introduction to the Theory of the Early Universe: Hot Big Bang Theory* (2nd Edition) World Scientific (2017) 596 pp.

[4] M. Yu. Khlopov, S. G. Rubin, *Cosmological Pattern of Microphysics in the Inflationary Universe*, Kluwer Academic Publishers, Dordrecht (2004)

[5] A.D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Phys. Lett. B 108 (1982) 389.

[6] V. F. Mukhanov and G. V. Chibisov, *Vacuum energy and large-scale structure of the Universe*, Sov. Phys. JETP 56 (1982) 258.

[7] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, Phys. Lett. B 91 (1980) 99;
A. A. Starobinsky, *Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations*, Phys. Lett. B 117 (1982) 175.

[8] J. M. Bardeen, P. J. Steinhardt and M. S. Turner, *Spontaneous creation of almost scale-free density perturbations in an inflationary universe*, Phys. Rev. D 28 (1983) 679.

[9] P. A. R. Ade et al., Planck Collaboration, *Planck 2013 results. XXII. Constraints on inflation*, Astron. Astrophys. 571 (2014) A22 [arXiv:1303.5082 [astro-ph.CO]].

[10] P. A. R. Ade et al., Planck Collaboration, *Planck 2015 results. XX. Constraints on inflation*, Astron. Astrophys. 594 (2016) A20 [arXiv:1502.02114 [astro-ph.CO]].

[11] F. Bezrukov, D. Gorbunov, *Light inflaton after LHC8 and WMAP9 results*, J. High Energy Phys. 1307 (2013) 140 [arXiv:1303.4395]

[12] R. Kallosh, A. Linde, *Superconformal generalization of the chaotic inflation model $\frac{1}{4} \phi^4 - \frac{1}{2} \phi^2 R$, J. Cosmol. Astropart. Phys. 1306 (2013) 027 [arXiv:1306.3211]

[13] N. A. Chernikov and E. A. Tagirov, *Quantum theory of scalar fields in de Sitter spacetime*, Ann. Inst. Henri Poincare A9 (1968) 109.

[14] E. A. Tagirov, *Consequences of field quantization in de Sitter type cosmological models*, Annals Phys. 76 (1973) 561.

[15] C.G. Callan, S.R. Coleman, R. Jackiw, *A New improved energy — momentum tensor Annals Phys. 59* (1970) 42

[16] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, England, 1982

[17] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, *Effective Action in Quantum Gravity*, IOP Publishing, Bristol (1992).
[18] J. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis, Primordial supersymmetric inflation, *Nucl. Phys.* **B221** (1983) 524.

[19] B. A. Ovrut and P. J. Steinhardt, Supersymmetric inflation, baryon asymmetry and the gravitino problem, *Phys. Lett.* **B147** (1984) 263.

[20] D. H. Lyth and A. Riotto, Particle physics models of inflation and the cosmological density perturbation, *Phys. Rept.* **314** (1999) 1 [arXiv:hep-ph/9807278].

[21] J. L. Cervantes-Cota and H. Dehnen, Induced gravity inflation in the standard model of particle physics, *Nucl. Phys.* **B442** (1995) 391 [arXiv:astro-ph/9505069].

[22] F. L. Bezrukov and M. Shaposhnikov, The Standard Model Higgs boson as the inflaton, *Phys. Lett.* **B659** (2008) 703 [arXiv:0710.3755 [hep-th]].

[23] A. O. Barvinsky, A. Y. Kamenshchik and A. A. Starobinsky, Inflation scenario via the Standard Model Higgs boson and LHC, *J. Cosmol. Astropart. Phys.* **0811** (2008) 021 [arXiv:0809.2104 [hep-ph]].

[24] F. L. Bezrukov, A. Magnin, M. Shaposhnikov and S. Sibiryakov, Higgs inflation: consistency and generalisations, *J. High Energy Phys.* **1101** (2011) 016 [arXiv:1008.5157 [hep-ph]].

[25] F. L. Bezrukov, The Higgs field as an inflaton, *Class. Quant. Grav.* **30** (2013) 214001 [arXiv:1307.0708 [hep-ph]].

[26] A. De Simone, M. P. Hertzberg and F. Wilczek, Running Inflation in the Standard Model, *Phys. Lett.* **B678** (2009) [arXiv:0812.4946 [hep-ph]].

[27] F. L. Bezrukov, A. Magnin and M. Shaposhnikov, Standard Model Higgs boson mass from inflation, *Phys. Lett.* **B675** (2009) 88 [arXiv:0812.4950 [hep-ph]].

[28] A. O. Barvinsky, A. Yu. Kamenshchik, C. Kiefer, A. A. Starobinsky and C. F. Steinwachs, Higgs boson, renormalization group, and cosmology, *Eur. Phys. J. C**712** (2012) 2219 [arXiv:0910.1041 [hep-ph]].

[29] K. Allison, Higgs $\xi$-inflation for the 125-126 GeV Higgs: a two-loop analysis, *J. High Energy Phys.* **1402** (2014) 040 [arXiv:1306.6931 [hep-ph]].

[30] R. N. Greenwood, D. I. Kaiser and E. I. Sfakianakis, Multifield Dynamics of Higgs Inflation, *Phys. Rev. D**87** (2013) 064021 [arXiv:1210.8190 [hep-ph]].

[31] A. O. Barvinsky and A. Yu. Kamenshchik, Quantum scale of inflation and particle physics of the early universe, *Phys. Lett. B**332** (1994) 270 [arXiv:gr-qc/9404062].

[32] J. L. Cervantes-Cota and H. Dehnen, Induced gravity inflation in the SU(5) GUT, *Phys. Rev. D**51** (1995) 395 [arXiv:astro-ph/9412032].

[33] M. B. Einhorn and D. R. T. Jones, GUT Scalar Potentials for Higgs Inflation, *J. Cosmol. Astropart. Phys.* **1211** (2012) 049 [arXiv:1207.1710 [hep-ph]].

[34] G. R. Dvali, Natural inflation in SUSY and gauge mediated curvature of the flat directions, *Phys. Lett.* **B387** (1996) 471 [arXiv:hep-ph/9605445].
[35] L. Alvarez-Gaume, C. Gomez and R. Jimenez, Minimal inflation scenario, \textit{J. Cosmol. Astropart. Phys.} \textbf{1103} (2011) 027 [arXiv:1101.4948 [hep-th]].

[36] M. Einhorn, D.R.T. Jones, Inflation with non-minimal gravitational couplings in supergravity, \textit{J. High Energy Phys.} \textbf{03} (2010) 026 [arXiv:0912.2718 [hep-ph]].

[37] S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Jordan Frame Supergravity and Inflation in NMSSM, \textit{Phys. Rev. D} \textbf{82} (2010) 045003 [arXiv:1004.0712 [hep-th]].

[38] K. Enqvist and A. Mazumdar, Cosmological consequences of MSSM flat directions, \textit{Phys. Rept.} \textbf{380} (2003) 99 [arXiv:hep-ph/0209244].

[39] A. Sagnotti and S. Ferrara, Supersymmetry and Inflation, \textit{PoS PLANCK} \textbf{2015} (2015) 113 [arXiv:1509.01500 [hep-th]].

[40] M. N. Dubinin, E. Yu. Petrova, E. O. Pozdeeva, M. V. Sumin and S. Yu. Vernov, MSSM-inspired multifield inflation, \textit{J. High Energy Phys.} \textbf{1712} (2017) 036 [arXiv:1705.09624 [hep-ph]].

[41] P. Fayet, Supergauge invariant extension of the Higgs mechanism and a model for the electron and its neutrino, \textit{Nucl. Phys.} \textbf{B90} (1975) 104.

[42] G. F. Giudice and G. Ridolfi, Constraints on the Minimal $N=1$ Supergravity Theory From Electroweak Symmetry Breaking, \textit{Z. Phys. C} \textbf{41} (1988) 447;
M. Olechowski and S. Pokorski, Hierarchy of Quark Masses in the Isotopic Doublets in N=1 Supergravity Models, \textit{Phys. Lett. B} \textbf{214} (1988) 393.

[43] H. E. Haber and R. Hempfling, The renormalization-group improved Higgs sector of the Minimal Supersymmetric Model, \textit{Phys. Rev. D} \textbf{48} (1993) 4280 [arXiv:hep-ph/9307201].

[44] CMS Collaboration, Search for a neutral MSSM Higgs boson decaying into $\tau\tau$ with 12.9 fb$^{-1}$ of data at $\sqrt{s}=13$ TeV, CMS-PAS-HIG-16-037 (2016);
ATLAS Collaboration, Search for Minimal Supersymmetric Standard Model Higgs Bosons H/A in the $\tau\tau$ final state in up to 13.3 fb$^{-1}$ of $pp$ collisions at $\sqrt{s}=13$ TeV with the ATLAS Detector, ATLAS-CONF-2016-085 (2016).

[45] A. D. Medina and M. A. Schmidt, Enlarging Regions of the MSSM Parameter Space for Large $\tan\beta$ via SUSY Decays of the Heavy Higgs Bosons, \textit{J. High Energy Phys.} \textbf{1708} (2017) 095 [arXiv:1706.04994 [hep-ph]];
A. Djouadi, L. Maiani, A. Polosa, J. Quevillon and V. Riquer, Fully covering the MSSM Higgs sector at the LHC, \textit{J. High Energy Phys.} \textbf{1506} (2015) 168 [arXiv:1502.05653 [hep-ph]].

[46] J. Gong, H.M. Lee and S.K. Kang, Inflation and dark matter in two-Higgs-doublet models, \textit{J. High Energy Phys.} \textbf{1204} (2012) 128 [arXiv:1202.0288 [hep-ph]].

[47] E. Akhmetzyanova, M. Dolgopolov and M. Dubinin, Higgs bosons in the two-doublet model with CP violation, \textit{Phys. Rev. D} \textbf{71} (2005) 075008 [arXiv:hep-ph/0405264].

[48] E. Akhmetzyanova, M. Dolgopolov and M. Dubinin, Violation of CP invariance in the two-doublet Higgs sector of the MSSM, \textit{Phys. Part. Nucl.} \textbf{37} (5) (2006) 677.
[49] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Low energy parameters and particle masses in a supersymmetric grand unified model, *Prog. Theor. Phys.* **68** (1982) 927.

[50] R. A. Flores and M. Sher, Higgs masses in the Standard, multi-Higgs and supersymmetric models, *Ann. Phys. (N.Y.)* **148** (1983) 95.

[51] Aad G. et al. (ATLAS Collaboration) Measurement of the Higgs boson mass from the $H \to \gamma\gamma$ and $H \to ZZ^\ast \to 4l$ channels in pp collisions at center-of-mass energies of 7 and 8 TeV with the ATLAS detector, *Phys. Lett. B* **716** (2012) 1–29 [arXiv:1207.7214 [hep-ex]]; Chatrchyan S. et al. (CMS Collaboration), Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC, *Phys. Lett. B* **716** (2012) 30–61, [arXiv:1207.7235 [hep-ex]].

[52] M. Carena, S. Heinemeyer, O. Stal, C.E.M. Wagner, and G. Weiglein, MSSM Higgs Boson Searches at the LHC: Benchmark Scenarios after the Discovery of a Higgs-like Particle, *Eur. Phys. J. C* **73** 2552 (2013) [arXiv:1302.7033 [hep-ph]]

[53] E. Bagnashi, F. Frensch, S. Heinemeyer, G. Lee, S. Liebler, M. Muhlleitner, A. McCarn, J. Quevillon, N. Rompotis, P. Slavich, M. Spira, C. E. M. Wagner, R. Wolf, Benchmarks scenarios for low $\tan\beta$ in the MSSM, CERN Report No. LHCHXSWG-INT-2015-004, 2015 [http://cds.cern.ch/record/2004747/files/LHCHXSWG-INT-2015-004.pdf]

[54] D. Roest, Universality classes of inflation, *J. Cosmol. Astropart. Phys.* **1401** (2014) 007 [arXiv:1309.1285 [hep-th]].

[55] M. Galante, R. Kallosh, A. Linde and D. Roest, A universal attractor for inflation at strong coupling, *Phys. Rev. Lett.* **112** (2014) 011303 [arXiv:1310.3950 [hep-th]].

[56] D.I. Kaiser and E.I. Sfakianakis, Multifield Inflation after Planck: The Case for non-minimal Couplings, *Phys. Rev. Lett.* **112** (2014) 011302 [arXiv:1304.0363 [astro-ph.CO]].

[57] R. Kallosh and A. Linde, Multi-field Conformal Cosmological Attractors, *J. Cosmol. Astropart. Phys.* **1312** (2013) 006 [arXiv:1309.2015 [hep-th]].

[58] P. Binetruy, E. Kiritsis, J. Mabillard, M. Pieroni and C. Rosset, Universality classes for models of inflation, *J. Cosmol. Astropart. Phys.* **1504** (2015) 033 [arXiv:1407.0820 [astro-ph.CO]].

[59] M. Rinaldi, L. Vanzo, S. Zerbini and G. Venturi, Inflationary quasi-scale invariant attractors, *Phys. Rev. D* **93** (2016) 024040 [arXiv:1505.03386 [hep-th]].

[60] E. Elizalde, S. D. Odintsov, E. O. Pozdeeva and S. Yu. Vernov, Cosmological attractor inflation from the RG-improved Higgs sector of finite gauge theory, *J. Cosmol. Astropart. Phys.* **1602** (2016) 025 [arXiv:1509.08817 [gr-qc]]

[61] S. D. Odintsov and V. K. Oikonomou, Inflationary $\alpha$-attractors from $F(R)$ gravity, *Phys. Rev. D* **94** (2016) 124026 [arXiv:1612.01126 [gr-qc]].

[62] T. Kobayashi, O. Seto and T.H. Tatsuishi, Toward pole inflation and attractors in supergravity: Chiral matter field inflation, *PTEP* **2017** (2017) no. 12, 123B04 [arXiv:1703.09960 [hep-th]].
[63] D.I. Kaiser, Conformal Transformations with Multiple Scalar Fields, *Phys. Rev. D* **81** (2010) 084044 [arXiv:1003.1159 [gr-qc]].

[64] D.I. Kaiser, E.A. Mazenc and E.I. Sfakianakis, Primordial Bispectrum from Multifield Inflation with non-minimal Couplings, *Phys. Rev. D* **87** (2013) 064004 [arXiv:1210.7487 [astro-ph.CO]].

[65] K.A. Malik and D. Wands, Cosmological perturbations, *Phys. Rep.* **475** (2009) 1 [arXiv:0809.4944 [astro-ph]].

[66] E. Elizalde, S.D. Odintsov, Renormalization group improved effective potential for gauge theories in curved space-time, *Phys. Lett. B* **303** (1993) 240 [hep-th/9302074]

[67] T. Futamase, M. Tanaka, Chaotic inflation with running nonminimal coupling, *Phys. Rev. D* **60** (1999) 063511 [hep-ph/9704303]

[68] I.L. Buchbinder, S.D. Odintsov, *Lett. Nuovo Cim.* **42** (1985) 379

[69] S. Mukaigawa, T. Muta, S.D. Odintsov, Finite GUTs and inflation, *Int. J. Mod. Phys. A* **13** (1998) 2739 [hep-ph/9709299]

[70] G. Barenboim, E.J. Chun and H.M. Lee, Coleman–Weinberg Inflation in light of Planck, *Phys. Lett. B* **730** (2014) 81 [arXiv:1309.1605 [hep-ph]].

[71] E. Elizalde, S.D. Odintsov, E.O. Pozdeeva, and S.Yu. Vernov, Renormalization-group inflationary scalar electrodynamics and SU(5) scenarios confronted with Planck2013 and BICEP2 results, *Phys. Rev. D* **90** (2014) 084001 [arXiv:1408.1285 [hep-th]]; E.O. Pozdeeva and S.Yu. Vernov, Renormalization-group improved inflationary scenarios, Phys. Part. Nucl. Lett. **14** (2017) 386 [arXiv:1604.02272 [gr-qc]].

[72] T. Inagaki, S.D. Odintsov and H. Sakamoto, Gauged Nambu-Jona-Lasinio inflation, *Astrophys. Space Sci.* **360** (2015) 67 [arXiv:1509.03738 [hep-th]]; T. Inagaki, S.D. Odintsov and H. Sakamoto, Inflation from the finite scale gauged Nambu-Jona-Lasinio model, *Nucl. Phys. B* **919** (2017) 297 [arXiv:1611.00210 [hep-ph]].

[73] M.N. Dubinin and E.Yu. Petrova, Radiative corrections to Higgs boson masses for the MSSM Higgs potential with dimension-six operators, *Phys. Rev. D* **95** (2017) 055021 [arXiv:1612.03655 [hep-ph]].