Noiseless Quantum Transmission of Information via Aharonov - Bohm Effect

Jian-Zu Zhang

Institute for Theoretical Physics, East China University of Science and Technology, Box 316, Shanghai 200237, P. R. China

Abstract

The possibility of quantum transmission of information via the induced fractional angular momentum by the Aharonov - Bohm vector potential is revealed. Its special advantage is that it is noiseless: Stray magnetic fields of environments influence the energy spectrum of the ion, but cannot contribute the fractional angular momentum to cause noise.

Quantum teleportation [1–3] allows the quantum state of a system to be transported from one location to another, without moving through the intervening space. One of its main problems is how to reduce noise which originates from couplings of the system with uncontrollable environments, though several ways to reduce noise have been designed. In this paper by means of the Aharonov-Bohm (AB) effect [4, 5] we show the possibility of quantum transmission of information via this effect.

The AB effect is purely quantum mechanical phenomenon which has been received much attention for years [6–9]. Experiments [10–13] showed that the interference spectrum of charged particles in a multiply connected region of the space, where the field strength is zero everywhere, suffered a shift according to the quantum phase, i.e. the amount of the loop integral of the magnetic vector potential around an unshrinkable loop. It is noticed that the AB effects is due to the non-trivial topology of a multiply connected region of the space where the magnetic field strength is vanishing [8]. There are lots of works concerning the fractional angular momentum: Investigated from crossed electric and magnetic fields [7,14]; Concerned in AB dynamics and their fractional statistics (see the reviews [15–20] and
references therein); Originated from Spatial noncommutativity [21]; and in connection with the quantum optics [22]. Recently a “spectator” mechanism [23] of an induced fractional angular momentum on ions of the AB vector potential was revealed. The “spectator” mechanism shows that when there is a “spectator” magnetic field the AB vector potential in the well defined limit induces a fractional angular momentum at the full quantum mechanical level. This opens a new way of the quantum transmission of information which is investigated in this paper. The special advantage of this type of the transmission via the AB effect is that it is noiseless. Stray magnetic fields of environments influence the energy spectrum of the ion, but cannot change the fractional angular momentum to cause noise. This effect explores far-reaching consequences of the vector potential in quantum theory: The vector potential itself has physical significant meaning and becomes effectively measurable not only in shifts of interference spectra originated from quantum phases but also in physical observables.

We consider three regions in the ($x_1$, $x_2$) plane. (1) A circle $I$ of radius $a_0$ is centered at the origin 0 of the coordinates. A homogeneous magnetic field $B^{(0)}$ along the $z$-axis is concentrated inside the circle $I$: Inside the circle ($\rho_0 \equiv (x_1^2 + x_2^2)^{1/2} < a_0$) $B^{(0)}_{m,z} = B_0$, ($\rho_0 \geq a_0$) $B^{(0)}_{\text{out}} = 0$. The corresponding vector potential $A^{(0)}$ is (Henceforth the summation convention is used): Inside the circle ($\rho_0 < a_0$) $A^{(0)}_{m,i} = -B_0 \epsilon_{ij} x_j / 2$, $A^{(0)}_{m,z} = 0$; Outside the circle ($\rho_0 \geq a_0$)

$$A^{(0)}_{\text{out},i} = -B_0 a_0^2 \epsilon_{ij} x_j / 2 \rho_0^2 (i, j, k = 1, 2), A^{(0)}_{\text{out},z} = 0. \quad (1)$$

$A_{\text{out}}^{(0)}$ is a vector potential of the AB type. At $\rho_0 = a_0$ the potential $A^{(0)}_{\text{out}}$ passes continuously over into $A^{(0)}_{\text{in}}$. (2) A circle $II$ of a radius $a_c$ is centered at the point $C$ of coordinators $x_C = (x_C, 0, 0)$. Here $x_C$, $a_0$ and $a_c$ satisfy $x_C > a_0 + a_c$. Inside the circle $II$ ($\rho_c \equiv [(x_1 - x_C)^2 + x_2^2]^{1/2} < a_c$) there is a homogeneous magnetic field $B^{(c)}_{m,z} = B_c$ along the $z$-axis, and outside the circle $II$ ($\rho_c \geq a_c$) $B^{(c)}_{\text{out}} = 0$. The corresponding vector potential $A^{(c)}$ is: Inside the circle $II$ ($\rho_c < a_c$) $A^{(c)}_{m,i} = -B_c \epsilon_{ij} (x_j - x_{C,j}) / 2$, $A^{(c)}_{m,z} = 0$; and outside the circle $II$ ($\rho_c \geq a_c$) $A^{(c)}_{\text{out},i} = -B_c a_c^2 \epsilon_{ij} (x_j - x_{C,j}) / 2 \rho_c^2 (i, j, k = 1, 2), A^{(c)}_{\text{out},z} = 0$. At the circle $II$ ($\rho_c = a_c$) the potential $A^{(c)}_{\text{out}}$ passes continuously over into $A^{(c)}_{\text{in}}$. (3) An intervening region $III$ is an area outside the circle $I$ and $II$ where the coordinates ($x_1, x_2$) of a point $P$ satisfy
both conditions \( \rho_0 \geq a_0 \) and \( \rho_c \geq a_c \). In the region III the magnetic fields \( B^{(0)} = B^{(c)} = 0 \), but there are two vector potentials of the AB type: \( A^{(0)}_{\text{out}} \) and \( A^{(c)}_{\text{out}} \).

**Induced Fractional Angular Momentum by the AB Vector Potential** – We consider an ion with mass \( \mu \) and charge \( q (> 0) \) constrained in the circle II where the vector potentials are \( A^{(c)}_{\text{in},i} \) and \( A^{(0)}_{\text{out},i} \). In the following in \( A^{(c)}_{\text{in},i} \) we don’t consider the constant term \( B_c \epsilon_{ij} x_{C,j} / 2 \) which can be gauged away by a gauge transformation \( A^{(c)}_{\text{in},i} \rightarrow A^{(c)}_{\text{in},i} + \partial_i \chi = -B_c \epsilon_{ij} x_i x_{C,j} / 2 \). The Hamiltonian of the charge particle is

\[
H(x_1, x_2) = \frac{1}{2\mu} \left( p_i + \frac{1}{2}\mu \omega_c \epsilon_{ij} x_j + \mu \omega_0 a_0^2 \epsilon_{ij} x_i / 2\rho_0^2 \right)^2, \tag{2}
\]

where \( \omega_c = q B_c / \mu c \) and \( \omega_0 = q B_0 / \mu c \) are the cyclotron frequencies corresponding to, respectively, the magnetic fields \( B^{(c)}_m \) and \( B^{(0)}_m \). This Hamiltonian can be rewritten as

\[
H(x_1, x_2) = (K_1^2 + K_2^2) / 2\mu \]

where

\[
K_i \equiv p_i + \frac{1}{2}\mu \omega_c \epsilon_{ij} x_j + \mu \omega_0 a_0^2 \epsilon_{ij} x_i / 2\rho_0^2, \tag{3}
\]

is the mechanical momenta corresponding to the vector potentials \( A^{(c)}_{\text{in},i} \) and \( A^{(0)}_{\text{out},i} \). The commutation relations between \( K_i \)'s are

\[
[K_i, K_j] = i\hbar \mu \omega_c \epsilon_{ij}. \tag{4}
\]

One point that should be emphasized is that the AB vector potential \( A^{(0)}_{\text{out},i} \) does not contribute to the commutator \([K_i, K_j]\). In the above \( p_i = -i\hbar \partial / \partial x_i \) are the canonical momenta, which satisfy \([p_i, p_j] = 0\). They are different from the mechanical momenta \( K_i \).

We define canonical variables \( Q = K_1 / \mu \omega_c \) and \( \Pi = K_2 \). They satisfy \([Q, \Pi] = i\hbar \delta_{ij}\). The Hamiltonian \( H(x_1, x_2) \) is rewritten as one of a harmonic oscillator, \( H(x_1, x_2) = H(Q, \Pi) = \Pi^2 / 2\mu + \mu \omega_c^2 Q^2 / 2 \). Its eigenvalues are \( \mathcal{E}_n = \hbar \omega_c(n + 1/2) \). The lowest one is \( \mathcal{E}_0 = \hbar \omega_c / 2 \). From the lowest eigenvalue \( \mathcal{E}_0 \) we estimate that the size \( a_c \) of the circle II should satisfy

\[
a_c \geq (c\hbar / q B_c)^{1/2}. \]

It is worth noting that \( A^{(0)}_{\text{out},i} \) does not contribute to energy spectra.

The limiting case of the Hamiltonian \( H \) in Eq. (2) approaching its lowest eigenvalue is interesting. In this limit the system has non-trivial dynamics \([24, 25]\). The Lagrangian corresponding to \( H \) is

\[
L = \frac{1}{2} \mu \dot{x}_i \dot{x}_i - \frac{1}{2} \mu \omega_c \epsilon_{ij} \dot{x}_i \dot{x}_j - \mu \omega_0 a_0^2 \epsilon_{ij} \dot{x}_i \dot{x}_j / 2\rho_0^2. \tag{5}
\]
In this limit the Hamiltonian $H$ reduces to $H_0 = \hbar \omega_c/2$. The Lagrangian corresponds to $H_0$ is

$$L_0 = -\frac{1}{2} \mu \omega_c \epsilon_{ij} \ddot{x}_i \dot{x}_j - \frac{\mu \omega_0 a_0^2 \epsilon_{ij} \dot{x}_i \dot{x}_j}{2 \rho_0^2} - \frac{1}{2} \hbar \omega_c. \quad (6)$$

**Constraints** – For the reduced system $(H_0, L_0)$ the canonical momenta are

$$p_i = \frac{\partial L_0}{\partial \dot{x}_i} = -\frac{1}{2} \mu \omega_c \epsilon_{ij} x_j + \frac{\mu \omega_0 a_0^2 \epsilon_{ij} \dot{x}_j}{2 \rho_0^2}. \quad (7)$$

Eq. (7) does not determine velocities $\dot{x}_i$ as functions of $p_i$ and $x_j$, but gives relations among $p_i$’s and $x_j$’s. According to Dirac’s formalism of quantizing constrained system, such relations are the primary constraints [25, 26]

$$\varphi_i \equiv p_i + \frac{1}{2} \mu \omega_c \epsilon_{ij} x_j + \frac{\mu \omega_0 a_0^2 \epsilon_{ij} x_j}{2 \rho_0^2} = 0. \quad (8)$$

These constraints should be carefully treated. The subject can be treated simply by the symplectic method in [27, 28]. In this paper we work in the formalism of the Dirac brackets. The Poisson brackets of the constraints (8) are

$$C_{ij} = \{\varphi_i, \varphi_j\} = \mu \omega_c \epsilon_{ij}. \quad (9)$$

From Eq. (9), $\{\varphi_i, \varphi_j\} \neq 0$, it follows that the conditions of the constraints $\varphi_i$ holding at all times do not lead to secondary constraints.

$C_{ij}$ defined in Eq. (9) are elements of the constraint matrix $C$. Elements of its inverse matrix $C^{-1}$ are $(C^{-1})_{ij} = -\epsilon_{ij}/\mu \omega_c$. The corresponding Dirac brackets of $\{\varphi_i, x_j\}_D$, $\{\varphi_i, p_j\}_D$, $\{x_i, x_j\}_D$, $\{p_i, p_j\}_D$ and $\{x_i, p_j\}_D$ can be defined. The Dirac brackets of $\varphi_i$ with any variables $x_i$ and $p_j$ are zero so that the constraints (8) are strong conditions. It can be used to eliminate dependent variables. If we select $x_1$ and $x_2$ as the independent variables, from the constraints (8) the variables $p_1$ and $p_2$ can be represented by, respectively, the independent variables $x_2$ and $x_1$ as

$$p_1 = -\frac{1}{2} \mu \omega_c x_2 - \frac{\mu \omega_0 a_0^2 x_2}{2 \rho_0^2}, \quad p_2 = \frac{1}{2} \mu \omega_c x_1 + \frac{\mu \omega_0 a_0^2 x_1}{2 \rho_0^2}. \quad (10)$$

The Dirac brackets of $x_1$ and $x_2$ is

$$\{x_1, x_2\}_D = \frac{1}{\mu \omega_c}. \quad (11)$$
We introduce new canonical variables \( q = x_1 \) and \( p = \mu \omega_c x_2 \). Their Dirac bracket is \( \{q, p\}_D = 1 \). According to Dirac’s formalism of quantizing a system which is associated with a number of primary constraints, the corresponding quantum commutation relation is \([q, p] = i\hbar\).

Angular Momentum of the Reduced System – The Hamiltonian \( H \) in Eq. (2) possess a rotational symmetry in \((x_1, x_2)\) plane. The \(z\)-component of the orbital angular momentum \( J_z = \epsilon_{ij} x_i p_j \) commutes with \( H \). They have common eigenstates. Now we consider the quantum behavior of the angular momentum in the reduced system \((H_0, L_0)\). Using Eq. (10) to replace \( p_1 \) and \( p_2 \) by, respectively, the independent variables \( x_2 \) and \( x_1 \), then using new canonical variables \( p \) and \( q \) to replace \( x_2 \) and \( x_1 \), the orbital angular momentum \( J_z \) is rewritten as

\[
J_z = \frac{q}{2\pi c} \Phi_0 + \frac{1}{\omega_c} \left( \frac{1}{2} \mu p^2 + \frac{1}{2} \mu \omega_c^2 q^2 \right).
\] (12)

Here \( \Phi_0 = \pi a_0^2 B_0 \) is the flux of the magnetic field \( \mathbf{B}_{in}^{(0)} \) inside the circle \( \mathbf{I} \) which comes from the second term of \( p_1 \) and \( p_2 \) in Eq. (10). That is, \( \Phi_0 \) is only contributed by the AB vector potential \( \mathbf{A}_{in}^{(0)} \) in Eq. (1). We introduce an annihilation operator \( A = \sqrt{\mu \omega_c / 2\hbar} q + i\sqrt{1/2\hbar \mu \omega_c} p \) and its conjugate one \( A^\dagger \). The operators \( A \) and \( A^\dagger \) satisfies \([A, A^\dagger] = 1\). The eigenvalues of the number operator \( N = A^\dagger A \) is \( n = 0, 1, 2, \cdots \). Using \( A \) and \( A^\dagger \) to rewrite \( J_z \), we obtain \( J_z = q\Phi_0 / 2\pi c + \hbar (A^\dagger A + 1/2) \). The zero-point angular momentum of \( J_z \) is \( J_0 = \hbar / 2 + q\Phi_0 / 2\pi c \). In the above the term

\[
J_{AB} = \frac{q}{2\pi c} \Phi_0
\] (13)

is the fractional zero-point angular momentum of the ion \([29]\) induced by the AB vector potential \( \mathbf{A}_{out}^{(0)} \). The magnetic flux \( \Phi_0 \) can be continuously changed. This leads to \( J_{AB} \) taking fractional values and the ground state of the angular momentum being infinitely degenerate.

We notice that two vector potentials \( \mathbf{A}_{in}^{(c)} \) and \( \mathbf{A}_{out}^{(0)} \) play different roles: \( \mathbf{A}_{in}^{(c)} \) contributes to energy spectra, but does not contribute to the fractional angular momentum \( J_{AB} \); On the other hand, \( \mathbf{A}_{out}^{(0)} \) does not contribute to energy spectra, but contributes to \( J_{AB} \).

One point that should be emphasized is that \( J_{AB} \) is only contributed by the the AB vector potential \( \mathbf{A}_{out}^{(0)} \) in Eq. (1). The structure of \( \mathbf{A}_{out}^{(0)} \) is special. Any other types of vector
potentials cannot contribute to $J_{AB}$.

It can be proved that the fractional zero-point angular momentum induced by the AB vector potential cannot be gauged away by a gauge transformation [23]. It is a real physical observable.

**Dynamics in the Intervening Region** – In this region the magnetic fields $B_{\text{out}}^{(0)} = B_{\text{out}}^{(c)} = 0$, but there are two vector potentials of the AB type $A_{\text{out},i}^{(0)}$ and $A_{\text{out},i}^{(c)}$. The Hamiltonian of an ion in this region is $\tilde{H}(x_1, x_2) = \left(\tilde{K}_1^2 + \tilde{K}_2^2\right)/2\mu$ where $\tilde{K}_i$ is the mechanical momenta

$$\tilde{K}_i \equiv p_i + \mu \omega_0 a_0^2 \frac{\epsilon_{ij} x_j}{2\rho_0^2} + \mu \omega_c a_c^2 \frac{\epsilon_{ij} (x_j - x_{C,j})}{2\rho_c^2}. \quad (14)$$

The Lagrangian corresponding to $\tilde{H}$ is

$$\tilde{L} = \frac{1}{2} \mu \dot{x}_i \dot{x}_i - \mu \omega_0 a_0^2 \frac{\epsilon_{ij} \dot{x}_i x_j}{2\rho_0^2} - \mu \omega_c a_c^2 \frac{\epsilon_{ij} \dot{x}_i (x_j - x_{C,j})}{2\rho_c^2} \quad (15)$$

The $\tilde{K}_i$’s commute each other

$$[\tilde{K}_i, \tilde{K}_j] = 0. \quad (16)$$

Behavior of $\tilde{H}$ is similar to a Hamiltonian of a free particle. Its spectrum is a continuous one.

We emphasize again that vector potentials $A_{\text{out}}^{(0)}$ and $A_{\text{out}}^{(c)}$ of the AB type do not contribute to the commutators between $\tilde{K}_i$’s. This does not lead to them contributing to energy spectra of charged particles either.

We consider the limiting case of $\tilde{H}$ approaching to some constant energy $\tilde{E}_k$: $\tilde{H} \to \tilde{H}_0 = \tilde{E}_k$. The Lagrangian corresponding to $\tilde{H}_0$ is

$$\tilde{L}_0 = -\mu \omega_0 a_0^2 \frac{\epsilon_{ij} \dot{x}_i x_j}{2\rho_0^2} - \mu \omega_c a_c^2 \frac{\epsilon_{ij} \dot{x}_i (x_j - x_{C,j})}{2\rho_c^2} - \tilde{E}_k. \quad (17)$$

From $\tilde{L}_0$ it follows that the canonical momenta is

$$\tilde{p}_i = \frac{\partial \tilde{L}_0}{\partial \dot{x}_i} = -\mu \omega_0 a_0^2 \frac{\epsilon_{ij} \dot{x}_i x_j}{2\rho_0^2} - \mu \omega_c a_c^2 \frac{\epsilon_{ij} \dot{x}_i (x_j - x_{C,j})}{2\rho_c^2}. \quad (18)$$

Eq. (18) does not determine velocities $\dot{x}_i$ as functions of $\tilde{p}_i$ and $x_j$, but gives the following primary constraints

$$\tilde{\varphi}_i \equiv \tilde{p}_i + \mu \omega_0 a_0^2 \frac{\epsilon_{ij} x_j}{2\rho_0^2} + \mu \omega_c a_c^2 \frac{\epsilon_{ij} (x_j - x_{C,j})}{2\rho_c^2} = 0. \quad (19)$$
Here the special feature is that the corresponding Poisson brackets between $\tilde{\phi}_i$'s are identically zero,

$$\tilde{C}_{ij} = \{\tilde{\phi}_i, \tilde{\phi}_j\} \equiv 0.$$  \hspace{1cm} (20)

Because of $\{\hat{\phi}_i, \hat{\phi}_i\}$ identically vanishing, it follows that the conditions of the constraints $\tilde{\phi}_i$ holding at all times lead to secondary constraints $\tilde{\phi}_i^{(2)} = -\mu \omega^2 p x_i$. The Poisson brackets $\{\tilde{\phi}_i^{(2)}, \tilde{\phi}_j\} = 0$, $\{\tilde{\phi}_i^{(2)}, \tilde{\phi}_j^{(2)}\} = 0$, and $\{\tilde{\phi}_i^{(2)}, \tilde{H}_0\} = 0$, so that persistence of the secondary constraints $\tilde{\phi}_i^{(2)}$ in course of time does not lead to further secondary constraints $\tilde{\phi}_i^{(3)}$.

Because of $\tilde{C}_{ij} \equiv 0$, the inverse matrix $\tilde{C}^{-1}$ does not exist. The Dirac brackets $\{\tilde{\phi}_i, x_j\}_D$, $\{\tilde{\phi}_i, p_j\}_D$, $\{\tilde{\phi}_i^{(2)}, x_j\}_D$, $\{\tilde{\phi}_i^{(2)}, p_j\}_D$, $\{x_i, x_j\}_D$, $\{p_i, p_j\}_D$, and $\{x_i, p_j\}_D$ cannot be defined.

According to Dirac’s formalism of quantizing a system with constraints, there is no way to establish dynamics at the quantum mechanical level.

Properties of the AB vector potentials in the intervening region is summarized as follows. The intervening region is multiply connected. As is well known, due to the non-trivial topology in this region, the interference spectrum of charged particles suffered a shift according to the quantum phase, i.e. the amount of the loop integral of the AB vector potential around an unshrinkable loop. But unlike in the region II with a “spectator” magnetic field, in the intervening region the two AB vector potentials do not contribute to physical observables:

(i) The AB vector potentials $A^{(0)}_{\text{out},i}$ and $A^{(c)}_{\text{out},i}$ appear in the mechanical momenta $\tilde{K}_i$ of Eq. (14) and the corresponding Hamiltonian $\tilde{H}$, but do not contribute to the commutators between $\tilde{K}_i$'s. Therefore, they do not contribute to the energy spectrum. The spectrum of the Hamiltonian $\tilde{H}$ is a continuous one of a free particle.

(ii) In the limit of the Hamiltonian $\tilde{H}$ approaching some constant the system has not a non-trivial dynamics survived at the full quantum mechanical level. Therefore, the two AB vector potentials $A^{(0)}_{\text{out}}$ and $A^{(c)}_{\text{out}}$ cannot contribute to the fractional angular momentum $J_{AB}$ by means of the constraints $\tilde{\phi}_i$'s of Eq. (19).

**Quantum Transmission of Information via Aharonov - Bohm effect** – Now we elucidate the application of the above results in the quantum transmission of information.

The situation in the circle II is different from the intervening region because of the “spectator” magnetic field $B^{(c)}_{\text{in}}$. The vector potentials $A^{(c)}_{\text{in}}$ guarantees that in Eq. (9) the
Poisson brackets of the constraints $\varphi_i$ are well defined, which lead to a non-trivial dynamics surviving at the full quantum mechanical level in the limit of the Hamiltonian approaching its one of eigenvalues. The vector potential $\mathbf{A}^{(0)}_{\text{out}}$ does not contribute to energy spectra, but in this case it contributes to the fractional angular momentum $J_{AB}$ by means of the constraints $\varphi_i$'s. It is clear that though the vector potential $\mathbf{A}^{(c)}_{\text{in}}$, like a “spectator”, does not contribute to $J_{AB}$, it plays essential role in guaranteeing non-trivial dynamics at the quantum mechanical level.

If at a moment $t$ we adjust the magnetic field $\mathbf{B}^{(0)}_{\text{in}}(t)$ in the circle I, though the magnetic field $\mathbf{B}^{(0)}_{\text{out}}$ is zero outside the circle I everywhere. According to the continuous condition of vector potentials, $\mathbf{A}^{(0)}_{\text{in}}$ passes continuously over into $\mathbf{A}^{(0)}_{\text{out}}$ on the boundary between the circle I and the outer region. At some later time $t + T$ a fractional angular momentum $J_{AB}(t + T)$ induced by the AB vector potential $\mathbf{A}^{(0)}_{\text{out}}(t + T)$ in the circle II will be changed correspondingly. Information encoded in variations of $\mathbf{B}^{(0)}_{\text{in}}$ in the circle I is transmitted, moving through the intervening region III, to $J_{AB}$ in the circle II.

One point that should be emphasized is that this type of quantum transmission has to satisfy the following condition: In the limit of the Hamiltonian $H$ approaching one of its eigenvalues there is non-trivial dynamics survived at the full quantum mechanical level. Here “the full quantum mechanical level” means that in the defined limit the reduced system can be quantized according to Dirac’s formalism of quantizing a system with constraints. Information encoded in $\mathbf{A}^{(0)}_{\text{out}}$ moves through the intervening region III, but cannot be received by an ion in this region. The reason is: because in the region III in the defined limit the reduced system cannot be quantized according to Dirac’s formalism of quantizing a system with constraints. Thus there is no way to establish dynamics at the full quantum mechanical level. Therefore, the vector potential $\mathbf{A}^{(0)}_{\text{out}}$ cannot contribute to the fractional angular momentum $J_{AB}$ by means of the constraints $\tilde{\varphi}_i$'s of Eq. (19). Such an intervening region may be called the blind area.

A special advantage of this type of transmission via the AB vector potentials is that it is noiseless. It is true that no quantum systems really isolated, and the coupling to the uncontrollable environments produces noise. Here the point is that the fractional angular momentum $J_{AB}$ is contributed only by the second term of Eq. (10), which is related to the
AB vector potential $A_{\text{out},i}^{(0)}$ of Eq. (1). Generally, stray magnetic fields of environments very both in space and time. Their vector potentials are not the AB type of Eq. (1). Therefore, they cannot influence the $J_{AB}$. Specially, the stray magnetic fields $B_{s}^{(0)}$ in the circle I and $B_{s}^{(c)}$ in the II of environments add, respectively, extra terms in Eq. (10) which are related to the vector potentials $A_{s,i}^{(0)}$ and $A_{s,i}^{(c)}$. These terms contribute to the energy spectrum of the reduced system in the required limit, but they do not influence the $J_{AB}$. Another point that should be clarified is that there is no energy transmission via the AB vector potential from the circle I to the circle II. Alternating electromagnetic fields induced by variations of $B_{\text{in}}^{(0)}$ in circle I transmit energy and interact with environments. However, vector potentials of alternating electromagnetic fields are not the AB type. They cannot influence $J_{AB}$ either. The quantum transmission via vector potentials of the AB type is not influenced by stray magnetic fields of the uncontrollable environments and so on. It is noiseless.

The above investigation opens a way of quantum transmission of information via the AB effect, leads to new experimental studies and technological applications. We expect results obtained in this paper to be of importance for the current efforts in the field of quantum communication.

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Ref. [14] investigated the angular momentum $\mathbf{J}$ originated from the Poynting vector produced by crossing the Coulomb field $\mathbf{E}$ of a charged particle with an external magnetic field $\mathbf{B}$,

$$\mathbf{J} = \frac{1}{4\pi c} \int \mathbf{r} \times [\mathbf{E} \times \mathbf{B}(\mathbf{r})] d^3r.$$ 

In cases where the magnetic field is only in the $z$-direction, this angular momentum reduces to

$$J_z = -\frac{q\phi}{2\pi c},$$

where $\phi = \int \int B_z(x_1, x_2) dx_1 dx_2$ is the total magnetic flux. $J_z$ is the angular momentum of the electromagnetic fields. In cases where the magnetic field $B_z$ is produced by an infinitely long solenoid, this angular momentum exists only inside the solenoid. $J_z$ should be distinguished from $J_{\mathbf{AB}}$ of Eq. (13). $J_{\mathbf{AB}}$ is the angular momentum of the ion which is induced by the AB vector potential outside the solenoid. $J_{\mathbf{AB}}$ does not exist inside the solenoid.