On $\mathcal{N} = 1$ AdS$_3$ solutions of Type IIB

Achilleas Passias$^1$ and Daniël Prins$^{2,3}$

$^1$ Département de Physique, École Normale Supérieure, Université PSL, CNRS, 24 Rue Lhomond, 75005 Paris, France

$^2$Institut de Physique Théorique, Université Paris Saclay, CNRS, CEA, F-91191 Gif-sur-Yvette, France

$^3$Dipartimento di Fisica, Università di Milano–Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy
and
INFN, sezione di Milano–Bicocca

Abstract

We study $\mathcal{N} = 1$ supersymmetric AdS$_3 \times M_7$ backgrounds of Type IIB supergravity, with non-vanishing axio-dilaton, three-form and five-form fluxes, and a “strict” $SU(3)$-structure on $M_7$. We derive the necessary and sufficient conditions for supersymmetry as a set of constraints on the torsion classes of the $SU(3)$-structure. Given an Ansatz for the three-form fluxes, the problem of also solving the equations of motion involves a “master equation”, which generalizes ones that have previously appeared in the literature.
1 Introduction

Recently, there has been renewed interest in supersymmetric \( \text{AdS}_3 \times M_7 \) backgrounds of Type IIB supergravity dual to \((0,2)\) superconformal field theories (SCFTs) in two dimensions \([1,2]\). In particular, the authors of \([1,2]\) studied such backgrounds with only five-form flux \([3]\), and showed the existence of the geometric dual of \(c\)-extremization in two-dimensional \((0,2)\) SCFTs \([4,5]\).

Motivated by the expectation that a geometric dual of \(c\)-extremization should exist for more general backgrounds than the ones considered in \([1,2]\), we aim to provide, as a first step, a systematic classification of supersymmetric \( \text{AdS}_3 \times M_7 \) backgrounds of Type IIB supergravity dual to two-dimensional \((0,2)\) SCFTs.\(^1\) Such a classification was initiated by the author of \([3]\), with the backgrounds mentioned in the previous paragraph (see also \([8]\)). In \([9]\), this class was extended to also admit a three-form flux satisfying certain conditions, whereas in \([10]\) instead a varying axio-dilaton was included. In this note we extend this classification program further, by allowing for both varying axio-dilaton and magnetic three-form fluxes. We restrict to the case that \(M_7\) is equipped with a “strict” \(SU(3)\)-structure, which is equivalent to requiring that the two Majorana supersymmetry parameters on \(M_7\) are orthogonal. Our classification includes as special cases the ones by \([1,3]\). The necessary and sufficient conditions for supersymmetry are phrased as restrictions on the torsion classes of the \(SU(3)\)-structure, which in seven dimensions is determined by a real one-form \(v\), a real two-form \(J\), and a complex decomposable three-form \(\Omega\). The vector dual to \(v\) foliates \(M_7\), and we find that the transverse six-dimensional space \(M_6\) is conformally symplectic.

\(^1\)A similar expectation for the geometric dual of \(a\)-maximization in four dimensions \([6]\) was explored in \([7]\) using generalized geometry.
On $\text{AdS}_3 \times M_7$, a solution to the supersymmetry equations also solves the equations of motion if and only if the Bianchi identities are satisfied by the fluxes (see for example [11]). By making an Ansatz for the three-form fluxes in our solution to the supersymmetry equations, we reduce the problem of finding a solution to the Bianchi identities, and hence the equations of motion, to two conditions: a “master equation” (5.11), which is a partial differential equation for the conformally Kähler metric on $M_6$, and existence of a primitive $(1,2)$-form satisfying (5.7). Similar master equations (and solutions thereof) associated with Bianchi identities appeared in [3,9,10], and the one presented here reduces to the ones of [3,9,10] in the appropriate limits.\footnote{See [12,13] for more solutions dual to two-dimensional $(0, 2)$ SCFTs.} The relation of these classes of solutions, and the corresponding master equations is depicted in Figure 1. Solutions to the aforementioned conditions, as well as more general Ansätze will be reported in future work.

The rest of this note is organized as follows. In section 2, we present the supersymmetry equations as a set of equations involving a pair of polyforms on $M_7$. In section 3, we introduce an $SU(3)$-structure in seven dimensions, and parameterize the polyforms in terms of it. In section 4, we derive a set of necessary and sufficient conditions for supersymmetry as restrictions on the torsion classes of the $SU(3)$-structure, and also give expressions for the fluxes in terms of the latter. A summary at the end of this section is included. Section 5 presents a class of solutions to the equations of motions following an Ansatz, as described earlier. Our conventions and certain technical details are included in the appendix.

## 2 Supersymmetry equations

We start with a general bosonic background of Type IIB supergravity invariant under $SO(2,2)$. The ten-dimensional metric is a warped product of a metric on AdS$_3$ and a metric on a seven-dimensional Riemannian manifold $M_7$:

\[ g_{10} = e^{2A} g_{\text{AdS}_3} + g_{M_7}, \]  

\( \text{Sec. 5} \)

\( \phi = 0 \)

\( H = 0 \)

\( [9] \)

\( [10] \)

\( H = 0 \)

\( \phi = 0 \)

\( [3] \)

\( \phi = 0 \)

\( H = 0 \)

Figure 1: Depiction of the relation between classes of solutions. $\phi$ is the dilaton, and $H$ the NSNS flux.
where $A$ is a function on $M_7$. Conforming to the $SO(2, 2)$ symmetry, the NSNS field-strength $H_{10d}$ and the RR field-strengths $F_{10d}$, with $F_{10d}$ denoting their sum in the democratic formulation, are decomposed as

$$H_{10d} = \kappa e^{3A} \text{vol}_{\text{AdS}3} + H, \quad F_{10d} = e^{3A} \text{vol}_{\text{AdS}3} \wedge \star_7 \lambda(F) + F. \quad (2.2)$$

The magnetic fluxes $H$ and $F = \sum_{p=1,3,5,7} F_p$, are forms on $M_7$. The operator $\lambda$ acts on a $p$-form $F_p$ as $\lambda(F_p) = (-1)^{[p/2]} F_p$. The RR field-strengths are subject to $dH_{10d}F_{10d} = 0$, which decomposes as

$$dH(e^{3A} \star_7 \lambda(F)) + \kappa F = 0, \quad dH F = 0, \quad (2.3)$$

where $d_H \equiv d - H \wedge$. We will refer to the first set of equations as equations of motion for $F$, and to the second one as the Bianchi identities.

In order to study the restrictions imposed by supersymmetry on the above bosonic background, we decompose the supersymmetry parameters of type IIB supergravity, $\epsilon_1$ and $\epsilon_2$ under $\text{Spin}(1, 2) \times \text{Spin}(7) \subset \text{Spin}(1, 9)$:

$$\epsilon_1 = \zeta \otimes \chi_1 \otimes \left( \frac{1}{1 - i} \right), \quad \epsilon_2 = \zeta \otimes \chi_2 \otimes \left( \frac{1}{1 - i} \right). \quad (2.4)$$

Here, $\chi_1$ and $\chi_2$ are Majorana Spin(7) spinors; $\zeta$ is a Majorana Spin(1, 2) spinor satisfying the Killing equation:

$$\nabla_\mu \zeta = \frac{1}{2} m \gamma_\mu \zeta, \quad (2.5)$$

where the real constant parameter $m$ is related to the AdS$_3$ radius $L_{\text{AdS}3}$ as $L_{\text{AdS}3}^2 = 1/m^2$. The above decomposition follows the requirement for $\mathcal{N} = 1$ supersymmetry.

The necessary and sufficient conditions for preserving $\mathcal{N} = 1$ supersymmetry can be derived following the derivation for Type IIA supergravity in the appendix of [14], with straightforward modifications. They are expressed in terms of bispinors $\psi_\pm$ defined by

$$\chi_1 \otimes \chi_2^\dagger \equiv \psi_+ + i \psi_. \quad (2.6)$$

Following the Fierz expansion of $\chi_1 \otimes \chi_2^\dagger$, and application of the Clifford map which maps anti-symmetric products of gamma matrices to forms, $\psi_+/\psi_-$ become polyforms on $M_7$, of even/odd degree.

The supersymmetry restrictions take the form of the following system of equations:

$$2mc_\pm = -c_\pm \kappa, \quad (2.7a)$$

$$d_H(e^{A-\phi} \psi_+) = \frac{1}{16} c_- F, \quad (2.7b)$$

$$d_H(e^{2A-\phi} \psi_-) + 2mc^A \psi_+ = \frac{1}{16} c_+ e^{3A} \star_7 \lambda(F), \quad (2.7c)$$

$$(\psi_+, F)_7 = \frac{m}{2} e^{-\phi} \text{vol}_7. \quad (2.7d)$$

---

3We work in the string frame.
Here $c_{\pm}$ are constants defined by the norms of $\chi_1$ and $\chi_2$:

$$c_{\pm} \equiv e^{\mp A}(||\chi_1||^2 \pm ||\chi_2||^2).$$  \hspace{1cm} (2.8)

Furthermore, $(\psi_+, F)_7 \equiv (\psi_+ \wedge \lambda(F))_7$, with $(\cdot)_7$ denoting the restriction to the seven-form component.

In this work we will consider backgrounds with zero electric component for $H_{10d}$ i.e. $\kappa = 0$. Supersymmetry then dictates $c_- = 0$, or equivalently $||\chi_1||^2 = ||\chi_2||^2$. The system of supersymmetry equations thus becomes:

$$d_H(e^{A-\phi} \psi_+) = 0, \hspace{1cm} (2.9a)$$
$$d_H(e^{2A-2\phi} \psi_-) + 2me^{A-\phi} \psi_+ = \frac{1}{8} e^{3A} \ast_7 \lambda(F), \hspace{1cm} (2.9b)$$
$$(\psi_+, F)_7 = \frac{m}{2} e^{-\phi} \text{vol}_7. \hspace{1cm} (2.9c)$$

Without loss of generality we have set $c_+ = 2$ i.e. $||\chi_1||^2 = ||\chi_2||^2 = e^A$.

## 3 Supersymmetry and $G$-structures

A nowhere-vanishing Majorana spinor $\chi$ on $M_7$ defines a $G_2$-structure for $TM_7$. A pair of nowhere-vanishing Majorana spinors $\chi_1, \chi_2$ define a $G_2 \times G_2$-structure on the generalized tangent bundle $TM_7 \oplus T^*M_7$. If $\chi_1, \chi_2$ are parallel, the $G_2 \times G_2$-structure reduces to a $G_2$-structure, whereas if $\chi_1, \chi_2$ are orthogonal it reduces to a “strict” $SU(3)$-structure. This can be illustrated by the decomposition of $\chi_2$ in terms of $\chi_1$ (taking $\chi_1, \chi_2$ to be of equal norm):

$$\chi_2 = \sin \theta \chi_1 - i \cos \theta v m \gamma^m \chi_1, \hspace{1cm} (3.1)$$

where $v$ is a real one-form with $||v|| = 1$, and $\theta \in [0, \pi/2]$. As $\theta$ varies from 0 to $\pi/2$, the $G_2 \times G_2$-structure varies from a “strict” $SU(3)$-structure, to an “intermediate” $SU(3)$-structure, to a $G_2$-structure. In this work we will consider the first case, i.e. $\theta = 0$.

An $SU(3)$-structure on $M_7$ is defined by a real one-form $v$, a real two-form $J$, and a complex decomposable three-form $\Omega$, all nowhere-vanishing, satisfying

$$v_m J = v_m \Omega = 0, \hspace{0.5cm} \Omega \wedge J = 0, \hspace{0.5cm} \frac{i}{8} \Omega \wedge \Omega = \frac{1}{3!} J \wedge J \wedge J. \hspace{1cm} (3.2)$$

These forms can be expressed as bilinears in terms of the spinors $(\chi_1, \chi_2)$; see appendix A for our conventions. The one-form $v$ gives a foliation of $M_7$ with leaves $M_6$; accordingly, we define the volume form as

$$\text{vol}_7 \equiv \frac{1}{3!} v \wedge J \wedge J \wedge J$$

and locally decompose the metric on $M_7$ as

$$g_{M_7} = v \otimes v + g_{M_6}. \hspace{1cm} (3.3)$$

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\textsuperscript{4} \hspace{0.5cm} X \omega_k \equiv \frac{1}{k+1!} X^m \omega_{m_1...m_{k-1}} dx^{m_1} \wedge ... \wedge dx^{m_{k-1}}.$
Existence of an $SU(3)$-structure ensures that all forms on $M_7$ decompose into irreducible representations of $SU(3)$. In particular, the local $k$-forms with no component along $v$ can be decomposed into primitive $(p,q)$-forms.\footnote{A primitive $k$-form $\omega^{(k)}$ satisfies $J_\pm \omega^{(k)} = 0$ for $k = 2,3$, whereas $k$-forms with $k = 0,1$ are primitive by definition. The $(p,q)$ decomposition of $k$-form $\omega$ is defined by}

We may also apply this decomposition to the exterior derivatives of the $SU(3)$-structure $\{v, J, \Omega\}$ itself. Doing so, we find a parameterization in terms of torsion classes. These constitute the components of the intrinsic torsion of the $SU(3)$-structure expressed in irreducible representations of $SU(3)$. Specifically, we have (see for example \cite{[15]})

\begin{align}
\rho &= RJ + T_1 + \text{Re}(\overline{\upsilon} \cdot \Omega) + v \wedge W_0 , \\
\rho = & 3 \text{ Im}(\upsilon \wedge \Omega) + W_3 + W_4 \wedge J + v \wedge \left( \frac{2}{3} \text{ Re} EJ + T_2 + \text{Re}(\overline{\upsilon} \cdot \Omega) \right) , \\
\rho = & W_1 J \wedge J + W_2 \wedge J + \upsilon \wedge \Omega + v \wedge (E \Omega - 2V_2 \wedge J + S) .
\end{align}

The real scalar $R$ and the complex scalars $E$ and $W_3$ transform in the $\mathbf{1}$ representation of $SU(3)$. The complex $(1,0)$-forms $V_1$, $V_2$ and $W_5$ transform in the $\mathbf{3}$, and the real one-forms $W_0$ and $W_4$ in the $\mathbf{3} \oplus \overline{\mathbf{3}}$. The real primitive $(1,1)$-forms $T_1$ and $T_2$, and the complex primitive $(1,1)$-form $W_3$ transform in the $\mathbf{8}$. Finally, the real primitive $(2,1) + (1,2)$-form $W_3$ transforms in the $\mathbf{6} + \overline{\mathbf{6}}$, and the complex primitive $(2,1)$-form $S$ in the $\mathbf{8}$.

In order to solve the supersymmetry equations, we parameterize the polyforms $\psi_{\pm}$ as defined in (2.6) in terms of the $SU(3)$-structure data. Making use of (3.1), (A.2), (A.3) we find that in the general case,

\begin{align}
\psi^G_{\pm} &= \frac{1}{8} e^A \left[ \text{Im}(e^{i\theta J} + v \wedge \text{Re}(e^{i\theta \Omega})) \right] , \\
\psi^{-G}_{\pm} &= \frac{1}{8} e^A \left[ v \wedge \text{Re}(e^{i\theta J} + \text{Im}(e^{i\theta \Omega})) \right] ,
\end{align}

for $||\chi_1||^2 = ||\chi_2||^2 = e^A$. As stated earlier, we will study the case of a strict $SU(3)$-structure for which $\theta = 0$ and hence

\begin{align}
\psi_{+} &= \frac{1}{8} e^A \left[ \text{Im}(e^{i\theta J} + v \wedge \text{Re}(\Omega)) \right] , \\
\psi_{-} &= \frac{1}{8} e^A \left[ v \wedge \text{Re}(e^{i\theta J} + \text{Im}(\Omega)) \right] .
\end{align}

Substituting the above expressions in the supersymmetry equations (2.9), we will derive the restrictions on the intrinsic torsion of the $SU(3)$-structure imposed by supersymmetry.
4 A class of solutions to the supersymmetry equations

In this section, we derive a class of solutions to the supersymmetry equations (2.9) by inserting the strict SU(3)-structure polyforms (3.7).

The first constraint (2.9a) yields

\[ d\left( e^{2A-\phi}J \right) = 0 , \]  
\[ d\left( e^{2A-\phi}v \wedge \text{Re}\Omega \right) - e^{2A-\phi}H \wedge J = 0 , \]  
\[ d\left( e^{2A-\phi}J \wedge J \wedge J \right) + 3! e^{2A-\phi}H \wedge v \wedge \text{Re}\Omega = 0 . \]

These in turn determine

\[ 0 = W_1 = W_3 = V_2 = T_2 , \]
\[ 2dA - d\phi = -W_4 - 2 \frac{3}{2} \text{Re}Ev . \]

Upon decomposing the NSNS field-strength \( H \) with respect to the SU(3)-structure as

\[ H = H^R \text{Re}\Omega + H^I \text{Im}\Omega + \left( H^{(1,0)} + H^{(0,1)} \right) \wedge J + H^{(2,1)} + H^{(1,2)} \]
\[ + v \wedge \left( H_v^{(1,1)} + H_v^{(0,1)} j \Omega + H_v^{(1,0)} \Omega \right) , \]

where \( H^{(2,1)} \) and \( H_v^{(1,1)} \) are primitive, we also find expressions for several of the components in terms of torsion classes from (4.1). Using (A.7), we find:

\[ H^I = -\frac{1}{3} \text{Re}E , \quad H^{(1,0)} = V_1 , \]
\[ H_v^{(1,1)} = -\text{Re}W_2 , \quad H_v^{(0,1)} = 0 , \quad H_v^{(1,0)} = \frac{1}{2j} \left( W_v^{(1,0)} + W_0^{(1,0)} - W_5 \right) . \]

The exterior derivatives of the the SU(3)-structure tensors now read

\[ dv = RJ + \text{Re}(\overline{V_1} j\Omega) + T_1 + v \wedge W_0 , \]
\[ dJ = d(2A - \phi) \wedge J , \]
\[ d\Omega = W_2 \wedge J + (\overline{W_5} + E v) \wedge \Omega + v \wedge S . \]

We define a rescaled metric \( g_{M^7} = e^{-2A+\phi}g_{M^7} \) and rescale the SU(3)-structure tensors accordingly as \( \{ v, J, \Omega \} = \{ e^{-A+\phi/2}v, e^{-2A+\phi}J, e^{-3A+3\phi/2} \} \) to obtain

\[ d\tilde{v} = R\tilde{J} + \text{Re}(\overline{V_1} \tilde{\Omega}) + \tilde{T}_1 + \tilde{v} \wedge \tilde{W}_0 , \]
\[ d\tilde{J} = 0 , \]
\[ d\tilde{\Omega} = \tilde{W}_2 \wedge \tilde{J} + (\overline{\tilde{W}_5} + i \text{Im} \tilde{E} \tilde{v}) \wedge \tilde{\Omega} + \tilde{v} \wedge \tilde{S} , \]

where

\[ \tilde{W}_0 = W_0 + \frac{1}{2} W_4 , \quad \tilde{W}_5 = W_5 - \frac{3}{2} W_4 . \]
and
\begin{align}
R &= e^{A - \frac{\phi}{2}} \tilde{R}, \quad \text{Im} E = e^{A - \frac{\phi}{2}} \text{Im} \hat{E}, \quad V_1 = \tilde{V}_1, \\
W_2 &= e^{-A + \frac{\phi}{2}} \tilde{W}_2, \quad T_1 = e^{-A + \frac{\phi}{2}} \tilde{T}_1, \quad S = e^{-2A + \phi} \tilde{S}.
\end{align}
(4.8)

We note that the condition \( d \tilde{J} = 0 \) means that the six-dimensional leaves \( M_6 \) transverse to \( \tilde{v} \) admit a symplectic structure.

Turning to the second constraint (2.9b) we obtain:
\begin{align}
e^{3A} \star_7 F_7 &= 0, \quad (4.9a) \\
e^{3A} \star_7 F_5 &= d\left(e^{3A-\phi} v\right) + 2me^{2A-\phi} J, \quad (4.9b) \\
-e^{3A} \star_7 F_3 &= d\left(e^{3A-\phi} \text{Im}\Omega\right) - e^{3A-\phi} H \wedge v + 2me^{2A-\phi} v \wedge \text{Re}\Omega, \quad (4.9c) \\
e^{3A} \star_7 F_1 &= -\frac{1}{2} d\left(e^{3A-\phi} v \wedge J \wedge J\right) - e^{3A-\phi} H \wedge \text{Im}\Omega - \frac{1}{3} me^{2A-\phi} J \wedge J \wedge J. \quad (4.9d)
\end{align}

From these equations, employing (4.4) and (4.5) and the set of identities (A.5), (A.6), we can obtain expressions for the magnetic RR fluxes \( F_p, p = 1, 3, 5, 7 \). We give these in (4.14) in the summary below.

Finally, the third constraint (2.9c) reads
\[ F_5 \wedge J - F_3 \wedge v \wedge \text{Re}\Omega - \frac{1}{3!} F_1 \wedge J \wedge J \wedge J = 4me^{-A-\phi} \text{vol}_7, \quad (4.10) \]
and plugging in the expressions for the RR fields we conclude that
\[ 3R + 6me^{-A} + 4H^R + 2\text{Im}E = 0. \quad (4.11) \]

4.1 Summary

Let us summarize our results. The differential constraints imposed on the \( SU(3) \)-structure by supersymmetry are:
\begin{align}
dv &= R J + \text{Re}(\overline{V}_1) \tilde{\Omega} + T_1 + v \wedge W_0, \quad (4.12a) \\
dJ &= d(2A - \phi) \wedge J, \quad (4.12b) \\
d\Omega &= W_2 \wedge J + (\overline{W}_5 + E v) \wedge \Omega + v \wedge S. \quad (4.12c)
\end{align}

The expression for the NSNS field is:
\begin{align}
H &= -\frac{1}{4} \left( 3R + 6me^{-A} + 2\text{Im}E \right) \text{Re}\Omega - \frac{1}{3} \text{Re} E \text{ Im}\Omega + 2\text{Re} V_1 \wedge J + 2\text{Re}(H^{(2,1)}) \\
&\quad + v \wedge \left( -\text{Re} W_2 + \text{Im} \left( (W_4^{(1,0)} + W_0^{(1,0)} - W_5)^{\overline{\Omega}} \right) \right). \quad (4.13)
\end{align}
The expressions for the RR fields are:

\[
e^{\phi}F_1 = (2\text{Im}E + 4me^{-A})v + 2\text{Im}(X_1^{(1,0)}) ,
\]
\[
e^{\phi}F_3 = \frac{1}{4}(-2me^{-A} + 3R - 2\text{Im}E)\text{Im}\Omega - 2\text{Im}V_1 \wedge J + v \wedge \text{Im}W_2
- 2\text{Im}(H^{(2,1)}) - \text{Re}S + X_3 \wedge (v \wedge \text{Re}\Omega) ,
\]
\[
e^{\phi}F_5 = \frac{1}{2}(R + 2me^{-A})v \wedge J \wedge J - \text{Im}(X_5^{(1,0)}) \wedge J \wedge J
- v \wedge J \wedge T_1 + 2v \wedge \text{Re}V_1 \wedge \text{Im}\Omega ,
\]
\[
e^{\phi}F_7 = 0 ,
\]

with

\[
X_1 \equiv dA + W_0 + 3W_4 - 2(W_5 + \overline{W}_5) ,
\]
\[
X_3 \equiv dA - W_4 + W_5 + \overline{W}_5 ,
\]
\[
X_5 \equiv dA - W_0 - W_4 .
\]

5 A new class of solutions

We make the following Ansatz:

\[
H + ie^{\phi}F_3 = 2H^{(1,2)} ,
\]
and recall that $H^{(1,2)}$ is primitive. This leads to \(v \wedge dA = 0 = v \wedge d\phi\) and the following restrictions on the torsion classes:

\[
0 = \text{Re}E = V_1 = W_2 = S = W_5 - W_0^{(1,0)} - W_4^{(1,0)} ,
\]
\[
\text{Im}E = -2me^{-A} , \quad R = -\frac{2}{3}me^{-A} , \quad W_0 = -dA , \quad W_4 = -2dA + d\phi .
\]

We thus have

\[
dv = -\frac{2}{3}me^{-A}J + T_1 - v \wedge dA ,
\]
\[
dJ = (2dA - d\phi) \wedge J ,
\]
\[
d\Omega = (-3dA + d\phi - 2ime^{-A}v) \wedge \Omega ,
\]
or in terms of the rescaled $SU(3)$-structure

\[
d\tilde{v} = -\frac{2}{3}me^{-2A+\phi/2}\tilde{J} + \tilde{T}_1 - \tilde{v} \wedge \left(2dA - \frac{1}{2}d\phi\right) ,
\]
\[
d\tilde{J} = 0 ,
\]
\[
d\tilde{\Omega} = \left(-\frac{1}{2}d\phi - 2ime^{-2A+\phi/2}\tilde{v}\right) \wedge \tilde{\Omega} .
\]
From the differential equations for \( \{ \tilde{J}, \tilde{\Omega} \} \) we conclude that \( \tilde{M}_6 \) is Kähler. In what follows we will introduce the exterior derivative \( d_6 \) on \( \tilde{M}_6 \), Dolbeault operators \( \partial, \bar{\partial} \) so that \( d_6 = \partial + \bar{\partial} \), and \( d_6^c = i(\bar{\partial} - \partial) \). The remaining RR fields read

\[
F_1 = -d_6^c e^{-\phi} ,
F_5 = \frac{2}{3} n e^{6A+3\phi/2} \tilde{\nabla}_6 \vdot J^2 + \frac{1}{2} d_6^c (e^{-4A+\phi}) \wedge \tilde{J} \wedge \tilde{J} - e^{-4A+\phi} \vdot \tilde{J} \wedge \tilde{T}_1 .
\]  

(5.5)

Let us now examine the Bianchi identities. The first Bianchi identity, \( dF_1 = 0 \), enforces

\[
\partial \bar{\partial} e^\phi = 0 ,
\]

(5.6)

which is solved by setting \( \phi = \log(\varphi + \varphi^\dagger) \), with \( \varphi \) holomorphic. Next, the three-form Bianchi identities \( dH = 0 \) and \( dF_3 - H \wedge F_1 = 0 \) yield the constraints

\[
\partial H^{(1,2)} = \partial H^{(1,2)} = \partial \phi \wedge H^{(1,2)} = 0 .
\]

(5.7)

Locally, a particular solution to these constraints, for non-constant dilaton, is given by

\[
H = dB , \quad d(e^\phi B) = d(\star_6 B) = 0 ,
\]

(5.8)

which reduces the problem of finding solutions to existence of a co-closed and conformally closed (1,1)-form \( B \).

In analyzing the Bianchi identity for \( F_5 \), we will invoke the results of [10]. The authors of [10] study supersymmetric solutions which descend from the solutions analyzed here, upon setting \( H^{(1,2)} = 0 \). However, even when \( H^{(1,2)} \neq 0 \), the \( SU(3) \)-structure of [10] and the expressions for \( F_1 \) and \( F_5 \) can be identified with the ones presented in this section. The map identifying the tensors there (left-hand side), with the tensors here (right-hand side) is

\[
P = \partial \phi , \quad Q = -\frac{1}{2} d^c \phi , \quad F^{(2)} = -e^{3A} \star_7 F_5 ,
\]

\[
\Delta = A - \frac{1}{4} \phi , \quad e^{2\Delta} K = \tilde{v} , \quad \bar{g}_6 = m^2 \tilde{g}_6 ,
\]

(5.9)

and in particular, \((4.9b)\) is identified with \((2.58)\) of [10].\(^6\) The authors of [10] showed that the Bianchi identity for \( F_5 \), \( dF_5 = 0 \), amounts to

\[
\nabla^2 (R - 2|\partial \phi|^2) - \frac{1}{2} R^2 + R_{ij} R^{ij} + 2|\partial \phi|^2 R - 4 R_{ij} \partial^i \phi \partial^j \phi + \frac{8}{3} e^{-\phi} H^{(2,1)}_{ij}(H^{(1,2)})^{ij} = 0 ,
\]

(5.10)

which they refer to as the “master equation”. In the above, \( R \) and \( R_{ij} \) respectively the Ricci scalar and the Ricci tensor of \( \tilde{g}_6 \), and contractions are also made using \( \tilde{g}_6 \). This master equation generalizes the one derived in [3] by including a varying axio-dilaton. For the case at hand the Bianchi identity of \( F_5 \) is \( dF_5 = H \wedge F_3 \), and the term on the right-hand side (a “transgression” term) modifies the master equation, which now becomes:

\[
\nabla^2 (R - 2|\partial \phi|^2) - \frac{1}{2} R^2 + R_{ij} R^{ij} + 2|\partial \phi|^2 R - 4 R_{ij} \partial^i \phi \partial^j \phi + \frac{8}{3} e^{-\phi} H^{(2,1)}_{ij}(H^{(1,2)})^{ij} = 0 .
\]

(5.11)

\(^6\)One has to take into account that we work in the string frame whereas the Einstein frame is used in [10]. In addition, we use a different orientation on \( \text{AdS}_3 \).
As noted above, in the limit $H^{(1,2)} = 0$ the present class of solutions and master equation (5.11) reduce to the ones of [10]. Further setting the axio-dilaton to zero, they reduce to the ones studied in [3]. Starting with the latter, the authors of [9] “turned on” a three-form flux $G = -H - iF_3 = -2H^{(1,2)}$, and taking the limit of vanishing axio-dilaton we recover their results. See Figure 1.

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A Conventions & identities

Spin(7)

We work with the Majorana representation of Cliff(7), for which the gamma matrices are imaginary and antisymmetric. The charge-conjugate of a Spin(7) spinor is the complex conjugate, and a Majorana spinor is real. The basis elements of Cliff(7) are related via the identity

$$\gamma_{m_1...m_k} = \frac{i}{(7-k)!} (-1)^{k(k-1)/2} \epsilon_{m_1...m_k m_{k+1}...m_7} \gamma^{m_{k+1}...m_7}.$$  

(A.1)

As discussed in section 3, a pair of nowhere-vanishing Spin(7) Majorana spinors $\chi_1$ and $\chi_2$ define an $SU(3)$-structure $\{v, J, \Omega\}$ in seven dimensions. For the strict $SU(3)$-structure ($\theta = 0$, or equivalently, $\chi_1^T \chi_2 = 0$), we introduce a Dirac spinor $\eta$ as

$$\chi_1 = \frac{1}{\sqrt{2}} (\eta + \eta^*) , \quad \chi_2 = \frac{i}{\sqrt{2}} (\eta^* - \eta) .$$

(A.2)

The bilinears that can be constructed using $\eta$ are $\eta^\dagger \gamma_{m_1...m_k} \eta$ and $\eta^T \chi_1 \chi_2 = 0$. In terms of the $SU(3)$-structure, $\eta$ satisfies

$$\eta^\dagger \eta = e^A , \quad \eta^T \eta = 0 ,$$

$$\eta^\dagger \gamma_m \eta = e^A v_m , \quad \eta^T \gamma_m \eta = 0 ,$$

$$\eta^\dagger \gamma_{mn} \eta = ie^A J_{mn} , \quad \eta^T \gamma_{mn} \eta = 0 ,$$

$$\eta^\dagger \gamma_{mnp} \eta = 3ie^A v_{[m} J_{np]} , \quad \eta^T \gamma_{mnp} \eta = - i e^A \Omega_{mnp} .$$

(A.3)

These can then be used to deduce the expressions (3.7) for the polyforms. In the case of the $G_2$-structure ($\theta = \pi/2$, or $\chi_1 = \chi_2$), we may instead consider $\chi_1 = \chi_2 = \frac{1}{\sqrt{2}} (\eta + \eta^*)$, where $\eta$ satisfies the above equations.
Identities

The SU(3)-structure is normalized as follows:

\[ \Omega_{mpq} \Omega^{npq} = 2^3 (\delta_m^n - iJ_m^n - v_m v^n), \quad \Omega_{mnp} \Omega^{mnp} = 3!2^3, \]

\[ \epsilon_{m_1...m_7} = \frac{7!}{3!2^3} v_{[m_1} J_{m_2 m_3} J_{m_4 m_5} J_{m_6 m_7]} . \]  

(A.4)

Given the above normalization, we derive a number of identities necessary to obtain the RR fields \( F_{1,3,5} \) from their Hodge duals \( \star_7 F_{1,3,5} \). Duals of the SU(3)-structure are given by

\[ \star_7 J = \frac{1}{2} v \wedge J \wedge J , \quad \star_7 (v \wedge J) = \frac{1}{2} J \wedge J , \quad \star_7 \Omega = iv \wedge \Omega . \]  

(A.5)

Duals for arbitrary primitive \((p,q)\)-forms \( \omega^{(p,q)} \) are given by

\[ \star_7 (\omega^{(1,0)} \wedge J) = iv \wedge \omega^{(1,0)} \wedge J , \quad \star_7 (\omega^{(1,0)} \wedge J \wedge J) = 2iv \wedge \omega^{(1,0)} , \]

\[ \star_7 (v \wedge (\omega^{(0,1)} \wedge \Omega)) = -i \omega^{(0,1)} \wedge \Omega , \quad \star_7 (\omega^{(0,1)} \wedge \Omega) = -iv \wedge \omega^{(0,1)} \wedge \Omega , \]

\[ \star_7 \omega^{(1,1)} = -v \wedge J \wedge \omega^{(1,1)} , \quad \star_7 (\omega^{(1,1)} \wedge J) = -v \wedge \omega^{(1,1)} , \]

\[ \star_7 \omega^{(2,1)} = -iv \wedge \omega^{(2,1)} . \]  

(A.6)

We also make use of the identities

\[ (\omega^{(0,1)} \wedge \Omega) \wedge J = -i \omega^{(0,1)} \wedge \Omega , \quad (\omega^{(0,1)} \wedge \Omega) \wedge \Omega = 4 \omega^{(0,1)} \wedge J \wedge J , \]  

(A.7)

in order to obtain the components of \( H \).

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