An Efficient Evaluation Method for Automobile Shells Design Based on Semi-supervised Machine Learning Strategy

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Abstract. Automobile is one of the important modes of transportation for human travel in today's society. Batch production in various countries in the world has also promoted the transformation of production concepts. At present, the development of the automobile industry is developing towards the trend of intelligence, personalization and sharing. Car appearance in a variety of ways, not every design is reasonable. Therefore, the main purpose of this article is to establish a scientific evaluation standard in order to large-scale test the quality of a variety of car shells design. The scientific nature is mainly reflected in combination the fluid-solid coupling knowledge and machine learning in this article, which can analyze the force of different shells in the flow field, and put out the cloud map information such as the stress, pressure and velocity of the shell. At last, analyze the best test samples and store them in the database, and then using semi-supervised heuristic algorithm to perform the sample training, the ultimate goal is to make the evaluation system more robust. The trained model can correctly evaluate each personalized car shape and give a reasonable score, which is convenient for car manufacturers to make best decision with personalized demand and scientific production.

1. Introduction
It is also a very complex fluid-solid coupling problem for the traditional automobile shell streamline design [1,2], and the most commonly used method to solve the fluid-solid coupling is the finite element method [3]. It covers multidisciplinary knowledge, aerodynamics, structural mechanics, PDE numerical solutions, aesthetics, etc. Objects with streamlined surfaces are designed to reduce air resistance, causing minimal disturbance to the flow of fluid such as water or air. Turbulence caused by air separation can affect the flow of air to the vehicle parts behind the rearview mirror. In addition, more downforce is created when air flows smoothly over the wings. Therefore, the length of the vehicle should be reasonably optimized to reduce turbulence at high speed. Literature [4] proposed a numerical method to optimize automobile windshield. In particular, a simplified elastic model with geometric nonlinearity is considered. The temperature distribution is controlled to match the desired shape and minimize a quadratic functional. Literature [5] proposed a new method for designing free-form surface features of vehicle streamline. Align a streamline network on the base shape by performing stroke constrained mesh parameters, outline streamlines are used as curve brackets to create 3D free streamlines.
In an era where the amount of available data is increasing exponentially, labeling unsupervised data will cost a lot. In fact, in many daily scenarios, labeled data is not easy to obtain. And semi-supervised learning greatly reduces data labeling costs. Machine learning has supervised, unsupervised, and reinforced learning modes, and semi-supervised learning is also a very important type of learning in the field of artificial intelligence. Semi-supervised learning algorithm represents the intermediate transition area between supervised and unsupervised algorithms. It will be used to solve many prediction and classification problems [6,7], the application of semi-supervised machine learning to car shell design is also a new research idea. In this paper, the specific design framework and theoretical basis are given. This study is a typical example of combining scientific methods with practical production value. It is also a new method of combining machine learning with fluid-solid coupling. It has broad prospects for development and is worthy of further research.

2. Semi-supervised machine learning
Semi-supervised GAN (SGAN for short) is a rewrite of the standard generative adversarial network model, the discriminator is trained to predict whether the designed personalized car shell meets the standard, or according to the corresponding evaluation score interval, which category should be divided. Allow it to learn discriminative features from the image, even for unlabeled data is also adaptable. Although most SGAN models can generate images similar to the data set through a well-trained discriminator, the discriminator can still build a classifier on the same data set through transfer learning, allowing supervisory tasks to never benefit from supervised training. Since most of the image features have been learned, the training time and accuracy of classification will be better. Here are two key points: In the unsupervised mode, it is necessary to distinguish between the real image and the generated image. In the supervised mode, an image needs to be classified into several classes, just like in a standard neural network classifier. A well-designed semi-supervised GAN (Generative Adversarial Network) requires only 25 training samples to achieve an accuracy of more than 90% on the MNIST dataset.

In semi-supervised, the discriminator model is updated to predict \( k + 1 \) classes, where \( k \) is the number of classes in the prediction problem, and an additional class label is added for a new "fake" class. It involves simultaneous training discriminator models for unsupervised classification tasks and supervised classification tasks. When a training sample has a label, the weight of the discriminator will be adjusted, otherwise, the classification task will be ignored, and the discriminator will adjust the weight to better distinguish the real image from the generated image. Although SGAN is allowed to be unsupervised training allows the model to learn very useful feature extraction from a very large unlabeled data set. Supervised learning allows the model to use the extracted features and use them for classification tasks. The result is a classifier that can be used in standards like MNIST achieve excellent results on the problem, even if it is trained on very few labeled samples (tens to hundreds). SGAN cleverly combines the aspects of unsupervised and supervised learning.

![Diagram of semi-supervised learning and training process](image)

**Fig. 1** Description diagram of the semi-supervised learning and training process
The ability of semi-supervised learning to combine the overfitting and "unfitting" of supervised learning and unsupervised learning, respectively, creates a new model that can perform classification tasks excellently. In addition to classification tasks, semi-supervised algorithms have many other uses, such as enhanced clustering and anomaly detection. You only need to mark a small part of the data, about hundreds or so, you can get a good prediction effect, and the effect of SOTA can be achieved in standard tasks. Semi-supervised learning algorithm training data Some are labeled, but most are unlabeled. "Unsupervised data" can enhance model generalization and prediction performance.

2.1. Generation the semi-supervised training mode

As for the supervised generative model, first needs to estimate the conditional probability density function of each class, and then to obtain the probability of each class according to the Bayesian formula. The semi-supervised learning generative model is a generalization of this method, namely, it is assumed that labeled samples and unlabeled samples have the same or similar probability distribution, that is, clustering hypothesis. In this way, unlabeled probability samples can be associated. Unlabeled data are missing variables and can be solved using EM algorithm. Assuming that each class of samples obeys probability distribution \( p(x | y; \theta) \), where \( \theta \) is the parameter of the probability density function. If the unlabeled samples and the labeled samples come from the same probability distribution, adding these unlabeled samples as training samples can essentially enhance the accuracy of the model. The probability distribution of unlabeled samples is normally a Gaussian mixture model, assuming that each type obeys a normal distribution. The joint density function of the sample labels can be represented by the conditional probability density function of the class:

\[
p(x, y | \theta) = p(y | \theta) p(x | y, \theta)
\]

(1)

Here \( \theta \) is the parameter to be determined, obtained by training with labeled and labeled samples, i. e., the following optimization problem can be written as formular (2):

\[
\max_{\theta} \ln(p(x_i, y_i | \theta)) + \lambda \ln p(x_i^{labeled} | \theta)
\]

(2)

Among them, \( \lambda \) is the manually set parameter and \( x_1, x_2, \ldots, x_j \) is the sample tag, \( x_1^{labeled}, x_2^{labeled}, \ldots, x_j^{labeled} \) is an unlabeled sample, it corresponding log-likelihood function is:

\[
\ln p(x_i, y_i | \theta) = \sum_{j=m}^{l} \ln \sum_{y_j} p(x_i | y_j, \theta)
\]

(3)

For the pressure and stress cloud map generated by the car shell in the fluid-solid coupling field are the key analysis objects, the graphs are used to represent the labeled and unlabeled algorithms, so that each graph in the sample belongs to an undirected graph with weights. The vertex of the graph is there label information, and the weight of the edge is the similarity between samples. The purpose of the classification function is to accurately evaluate the scores of different car shells. The prediction results also need to meet the continuity assumption, which can be achieved by introducing periodic terms. The core idea of the algorithm is that if two samples are similar in manifold, their labels should also be similar. Manifolds regularization makes the predictive values of \( x_i \) and \( x_j \) of \( f(x_i) \) and \( f(x_j) \) similar for the two samples, the training target can be expressed as:

\[
\min_f \frac{1}{l} \sum_{i=1}^{l} L(f(x_i), y_i) + \lambda_x \| f \|^2_U + \lambda_y \sum_{j=M}^{l} \| \nabla f(x) \|^2 dp(x)
\]

(4)

Among them, \( l \) is a labeled training sample and \( M \) is the prevalence where the sample predicted values lie. The first term is the labeled classification loss, and the second term is the prediction function regularization term, which used to control the complexity of the prediction function. The third term is the manifolds regularization term, implementing the manifold assumption that the predicted values for labeled and unlabeled samples are on the same prevalence. Where the \( \lambda_x, \lambda_y \) is the regularization term coefficient. The manifolds regularization term is then represented using the Laplacian quadratic to form the Eq.(5).
\[ f^T L f = \sum_{i=1}^{l+u} \sum_{j=1}^{l+u} w_{ij} (f(x_i) - f(x_j))^2 \approx \int_M \| \nabla_M f(x) \|^2 \, dp(x) \]  

In Eq.(5), \( u \) is the number of training samples and \( w_{ij} \) is the similarity between \( x_i \) and \( x_j \), a method also called Laplace mapping, indicating that when the similarity of two samples in the original space is \( w_{ij} \), the corresponding loss function value after the mapping is \( \int_M \| \nabla_M f(x) \|^2 \, dp(x) \).

2.2. Network building
Let \( n \) label samples \( (x^k, y^k) \), \( k = 1,2,\ldots,n \) were assumed for each min-batch, \( m \) samples \( x^{k'} \) \( k' = 1,2,\ldots,m \) are no label, the grade category \( C_i = 5 \) of the car shape design is recorded as follows, among them, \( C_i = 1 \) indicates the score range on \([0,20]\), \( C_i = 2 \) indicates the \([20,40]\) points, \( C_i = 3 \) indicates the \([40,60]\) points, in the same way, \( C_i = 5 \) indicates the score range on \([80,100]\). \( x^k \) is the output value predicted by the network to labeled samples, \( y^k \) is the predicted output value of the network to no label samples. The loss function is the sum of losses with labeled and unlabeled samples:

\[ L = \frac{1}{n} \sum_{k=1}^{n} \sum_{i=1}^{C} L(y^k_i, f_i^k) + \alpha(t) \frac{1}{m} \sum_{k'=1}^{m} \sum_{i'=1}^{C} L(y^{k'}_{i'}, f_i^{k'}) \]  

(6)

Where \( y^{k'} \) is pseudo-label, the previous trained neural network predictions on label-free samples.

\[ y^{k'} = \begin{cases} 
1 & \text{i = arg max}_i f_i(X) \\
0 & \text{other} 
\end{cases} \]  

(7)

The parameter \( \alpha(t) \) is dynamically adjusted in the iterative process. When the labeled sample is not accurately predicted, the weight value will decrease. With the improvement of accuracy, the value of parameter \( \alpha(t) \) will gradually increase to a stable value. For semi-supervised learning, building a network can build a ladderNet-type network for training. The network has a cross-layer connected decoder network, and at the same time, random noise and a cost function \( C_i \) that minimizes the reconstruction error are added to each layer of the encoder network. These white noises obey the normal distribution \( \mathcal{N}(0,\sigma^2) \). Then, the supervised loss function and the unsupervised loss function are added to the highest layer of the encoder, and finally the network construction task is realized through multiple training and parameter tuning.

3. Fluid-Solid coupling model

3.1. Fluid mechanics
To study fluids at the micro level, Hamilton equations or molecular dynamics methods can be used to describe the motion of microscopic molecules, but this method requires a huge amount of calculation. Boltzmann equations can also be used to describe the motion of microscopic particles so that the virtual microscopic particles are orthogonal. The mesh line motion of the car uses the collision and migration of particles to simulate the motion of the fluid, but it not only satisfies the conservation of mass, momentum, and energy. In literature[8,9], the fluid motion of the car shell belongs to the macroscopic level. Physical quantities such as speed, pressure, etc. are basic variables. According to Navier-Stokes equations, the law of fluid motion is studied.Navier-Stokes equations are nonlinear partial differential equations, which are difficult to find analytical solutions. As for finite elements are
often used. The equal discretization method obtains the algebraic equation system, and the numerical solution is obtained through the iterative method.

The flow parameters (pressure, velocity, etc.) of the fluid particle at any point in the moving fluid change with time is called unsteady flow. The Navier-Stokes equation with time term is given below:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla P + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}$$

(8)

It can also be expressed as follows:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \frac{\partial \mathbf{u}}{\partial x_j}\right) = -\frac{\partial p}{\partial x_j} + \mu \nabla^2 u_j + \rho g_j$$

(9)

The gravity effects can be incorporated throughout the basin when the fluid density can be incorporated into the pressure term

$$\bar{\rho} = \rho - \rho g \cdot \mathbf{x}$$

(10)

In two-dimensional plane fluids, the transient Navier-Stokes equation is formed as follows:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \frac{\partial \mathbf{u}}{\partial x} + \frac{\partial \mathbf{u}}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial^2 \mathbf{u}}{\partial y^2}\right)$$

(11)

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{u} \cdot \frac{\partial \mathbf{v}}{\partial x} + \frac{\partial \mathbf{v}}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 \mathbf{v}}{\partial x^2} + \frac{\partial^2 \mathbf{v}}{\partial y^2}\right)$$

(12)

Based on the assumption of continuum, the basic equations of fluid motion are established, which has a wide range of adaptability. Strictly speaking, this system of equations is usually not closed. That is, there are more unknowns in the equations than the number of equations. In order to find a theoretical solution, some assumptions that are in line with or close to reality must be put forward according to the situation. If the number of equations and unknowns are equal, the problem is solvable. So the continuous equations are added:

$$\frac{\partial \rho}{\partial t} + \nabla \rho \cdot \mathbf{u} = 0$$

(13)

When dealing with practical problems, the density approximation of the fluid is sometimes regarded as invariant, that is $\frac{d\rho}{dt} = 0$, it was called incompressible fluid. The so-called constant density means that with the change of pressure and temperature, but the density changes only slightly. In most cases, the influence of compressibility of the liquid can be ignored, and then we can obtain:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(14)

As for the time term of the Navier-stokes equation, the velocity of the spatial term adopts the quadratic Lagrange basis function, and the pressure term can be approximated using the linear element basis function.

$$\rho\left(\frac{\mathbf{u}^n - \mathbf{u}^n_t}{\tau} + \mathbf{u}^n \cdot \nabla \mathbf{u}^{n+1}\right) = -\nabla P^{n+1} + \mu \nabla^2 \mathbf{u}^{n+1} + \mathbf{F}^{n+1}$$

(15)

The finite element method is then discrete, first using the finite element basis function linearized approximation for the velocity and pressure in the Navier-stokes equation:

$$\mathbf{u} = \sum_{i=1}^{N_t} u_i(t) \phi_i^u, \quad p = \sum_{i=1}^{N_t} p_i(t) \phi_i^p$$

(16)

Among these, $u_i(t), v_i(t), \quad j = 1,2,\ldots, N_1$ represents the velocity vector values at the corresponding each node, $p_i(t)$ is the value of the pressure vector at the corresponding node.
Considering the fluid on the two-dimensional region $\Omega$, the boundaries are represented by $\Gamma$, with the Navier-stokes equation corresponding to the weak form of:
\[
\iint_\Omega \varphi^n \rho \frac{\partial u}{\partial t} + u \cdot \nabla u \, dx \, dy = \iint_\Omega \varphi^n (-\nabla P + \mu \nabla^2 u) \, dx \, dy
\]  
(17)

The discrete form for the continuity equation is as follows Eq.(18).
\[
\iint_\Omega \varphi^n (\nabla \cdot u) \, dx \, dy = 0
\]  
(18)

As for the left of the equation (16), combined with the Gauss divergence theorem, the following form can be obtained:
\[
\iint_\Omega \varphi^n \rho \frac{\partial u}{\partial t} + u \cdot \nabla u \, dx \, dy = \oint_\Gamma \varphi^n P \, ds - \mu \oint_\Gamma \varphi^n n \cdot \nabla u \, ds +
\iint_\Omega \nabla \varphi^n P \, dx \, dy - \mu \iint_\Omega \nabla \varphi^n \cdot \nabla u \, dx \, dy
\]  
(19)

The right end of Equation (16) contains the first term of the pressure
\[
\iint_\Omega \varphi^n \nabla P \, dx \, dy = \iint_\Omega \nabla (\varphi^n P) \, dx \, dy - \iint_\Omega \nabla \varphi^n P \, dx \, dy = -\oint_\Gamma \varphi^n P \, ds - \iint_\Omega \nabla \varphi^n P \, dx \, dy
\]  
(20)

The right end of the equation (16) contains the second term of the velocity $u$
\[
\iint_\Omega \varphi^n (\nabla \cdot u) \, dx \, dy = \oint_\Gamma \varphi^n n \cdot \nabla u \, ds - \iint_\Omega \nabla \varphi^n \cdot \nabla u \, dx \, dy
\]  
(21)

The integration of the continuity equation over each triangular element can be written as:
\[
A_iu^e + A_jv^e = 0
\]  
(22)

\[
A_1 = \iint_{\Gamma^1} \varphi^n \left( \frac{\partial \varphi^n}{\partial x} \right) dx \, dy = \int_{-1}^1 \int_{-1}^1 [\varphi^n \left( \frac{\partial \varphi^n}{\partial x} \right)] \, J | \, d\xi \, d\eta
\]  
(23)

The integration of the equation of motion over the triangular element is:
\[
\iint_{\Gamma^1} \varphi^n \rho \frac{\partial u}{\partial t} dx \, dy = \rho \left( \iint_{\Gamma^1} \varphi^n \varphi^n dx \, dy \right) u' = Mu'
\]  
(24)

\[
\iint_{\Gamma^1} \varphi^n \rho \frac{\partial v}{\partial t} dx \, dy = \rho \left( \iint_{\Gamma^1} \varphi^n \varphi^n dx \, dy \right) v' = Mv'
\]  
(25)

Meanwhile, $M = \iint_{\Gamma^1} \varphi^n \varphi^n dx \, dy$. Similarly, dealing with computational expressions with pressure can be obtained as follows:
\[
Mu' + B_{11}u^e + B_{12}v^e + C_1P^e = -F_1^e
\]  
(26)

\[
Mv' + B_{21}u^e + B_{22}v^e + C_2P^e = -F_2^e
\]  
(27)

Then, assembling each element matrix yields the total system of discrete algebraic equations as follows:
\[
Mu' + B_{11}u + B_{12}v + C_1P = -F_1
\]  
(28)

\[
Mv' + B_{21}u + B_{22}v + C_2P = -F_2
\]  
(29)

\[
A_iu^e + A_jv^e = 0
\]  
(30)

Among these, $u', v'$ represents the first derivative of the velocity about time along the $x$ direction and the $y$ direction, respectively, which can be written as using the forward difference approximation:
\[
u' = \frac{v^{n+1} - v^n}{\Delta t}
\]  
(31)
Both the velocity component \( u, v \), as well as the pressure term \( P \) are unknown. To improve the numerical approximation, we introduce the factor \( \theta \) which can reduce the numerical iteration error, the weight factor \( \theta \) combination the previous moment \( n + 1 \) and the latter \( n \):

\[
\begin{align*}
  u &= \theta u_{n+1} + (1 - \theta) u_n \\
  v &= \theta v_{n+1} + (1 - \theta) v_n \\
  P &= \theta P_{n+1} + (1 - \theta) P_n
\end{align*}
\]

(32) (33) (34)

Now, bring \( u, v, P \) into the equation (28)–(30) to get an algebraic equation with a time-term iteration \( Kx = b \), where the vector \( x = (u_{n+1}, v_{n+1}, p_{n+1})^T \) is to be solved.

\[
K = \begin{pmatrix}
  M + \Delta t B_{11} & \Delta t B_{12} \theta & -\Delta t C_1 \theta \\
  \Delta t B_{21} \theta & M + \Delta t B_{22} & -\Delta t C_2 \theta \\
  A_1 & A_2 & 0
\end{pmatrix}
\]

(35)

The right-end term \( b \) vector can be recorded as:

\[
b = \begin{pmatrix}
  Mu_{n+1} - \Delta t B_{11}(1 - \theta) u_n - \Delta t B_{12}(1 - \theta) v_n + \Delta t C_1(1 - \theta) p_n - \Delta t F_1 \\
  Mu_{n+1} - \Delta t B_{21}(1 - \theta) u_n - \Delta t B_{22}(1 - \theta) v_n + \Delta t C_2(1 - \theta) p_n - \Delta t F_2 \\
  0
\end{pmatrix}
\]

(36)

Then, iterative calculations can be performed only given the initial moment speed \( u_0, v_0 \) and pressure \( p_0 \), the final numerical results are obtained \( x = (u_{n+1}, v_{n+1}, p_{n+1})^T \). The flow in which the flow parameters (pressure and velocity, etc.) of the fluid particles at any point in the moving fluid do not change with time is called steady flow. The recursive formula for the transient solution is obtained, and the study of solving the steady flow is much simpler.

3.2. Solid mechanics

Investigation a particle in an undeformed and deformed elastomer with the satisfying relationship between the component of the displacement \( u = (u_x, u_y) \), strain tensor and the displacement component:

\[
\varepsilon_{ki} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k} \right) \quad k, l = 1, 2
\]

(37)

If the deformation material is a linear elastic medium, the stress and deformation relation is:

\[
\sigma_{ij} = \alpha \lambda \delta_{ij} + 2 \mu \varepsilon_{ij}
\]

(38)

Among these, \( \lambda \) and \( \mu \) are the Lame constant and \( \delta_{ij} \) is the kronecker function

\[
\delta_{ij} = \begin{cases} 
  0 & \text{if } i \neq j \\
  1 & \text{if } i = j
\end{cases}
\]

(39)

Satisfaction the relation \( f = n \cdot \sigma \) between Cauchy tensor and surface force density, \( n = (n_x, n_y) \) is the normal vector of the microelement face, the Hamilton operator is \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \), and \( \nabla \cdot \sigma \) is the divergence of the stress tensor, the physical density \( \rho \), \( b = (b_x, b_y) \) acting on an object is defined as the volume force divided by the volume. Based on the equilibrium conditions, separate equilibrium equations along the x,y-axis direction can be derived.

Considering the two dimensional linear elastic equation:
\[
\begin{cases}
- \nabla \cdot \sigma(u) = b & \text{in } \Omega \\
u = g & \text{on } \partial \Omega
\end{cases}
\]

(40)

For the numerical solution of the elasticity of the plane problem, the finite element method is still selected [10]. First, the displacement field variables are linearized and approximated. Using the quadratic Lagrange basis function, it can be obtained \( u_i = (u_i, v_i)^T \). The \( N \) in the formula represents the number of nodes in the entire discrete region.

\[
u(x, y) = \sum_{i=1}^{N} u_i \phi_i(x, y) \quad i = 1, 2, \ldots, N
\]

(41)

Using the Galerkin principle to discrete the equation (40), and the weak form are given:

\[
\iint_{\Omega} \phi_i(x, y) (\nabla \cdot \sigma + b) dxdy = 0
\]

(42)

Using the Gauss's divergence theorem again, it can be obtained

\[
\iint_{\Omega} \phi \nabla \cdot \sigma dxdy = \iint_{\Omega} \nabla \cdot (\phi \sigma) dxdy - \iint_{\Omega} \nabla \phi \cdot \sigma dxdy
\]

(43)

It simply divides the area and transforms it into a line integration

\[
\iint_{\Omega} \phi \nabla \cdot \sigma dxdy = \int_{\Gamma} \phi n \cdot \sigma dl - \iint_{\Omega} \nabla \phi \cdot \sigma dxdy
\]

(44)

For the upper equation, if the boundary surface force \( f = n \cdot \sigma \), \( n \) is a unit normal vector perpendicular to the boundary inward, and the \( i \) is the arc length corresponding to the boundary \( \Gamma \), then the upper equation can be further converted to:

\[
\iint_{\Omega} \nabla \phi_i \cdot \sigma dxdy = \int_{\Gamma} \phi_i f dl + \iint_{\Omega} \phi_i b dxdy
\]

(45)

Then, bringing the stress tensor into the equation (46) yields two component equations

\[
\iint_{\Omega} \frac{\partial \phi_i}{\partial x} \sigma_{xx} + \frac{\partial \phi_i}{\partial y} \sigma_{xy} dxdy = \int_{\Gamma} \phi_i f_d dl + \iint_{\Omega} \phi_i b_x dxdy
\]

(46)

\[
\iint_{\Omega} \frac{\partial \phi_i}{\partial x} \sigma_{yx} + \frac{\partial \phi_i}{\partial y} \sigma_{yy} dxdy = \int_{\Gamma} \phi_i f_y dl + \iint_{\Omega} \phi_i b_y dxdy
\]

(47)

According to the physical equation, the stress can be expressed by displacement, so the stress can also be linearized by the finite element base, successively obtaining three equations.

\[
\sigma_{xx} = \frac{E}{1 - \nu^2} \sum_{i=1}^{N} \left( u \frac{\partial \phi_i}{\partial x} + v \frac{\partial \phi_i}{\partial y} \right)
\]

(48)

\[
\sigma_{yy} = \frac{E}{1 - \nu^2} \sum_{i=1}^{N} \left( v \frac{\partial \phi_i}{\partial x} + \frac{\partial \phi_i}{\partial y} \right)
\]

(49)

\[
\sigma_{xy} = \frac{E}{2(1 + \nu)} \sum_{i=1}^{N} \left( u \frac{\partial \phi_i}{\partial y} + v \frac{\partial \phi_i}{\partial x} \right)
\]

(50)

After finishing, one can form a system of linear equations \( Au = F + Mb \), and \( A \) is the total stiffness matrix with sparsity and symmetry, \( u \) is the numerical solution of the displacement field component, \( F \) corresponds to the node surface force component, and \( M \) is the overall mass matrix.

4. Numerical example

This numerical example studies the stress, pressure and velocity field under the high velocity flow field. In addition, a standard automobile shell evaluation system can be established, what’s more, we combined flow-solid coupling and machine learning knowledge to facilitate the assessment and rapid screening of design work.
Model parameter: The car length is $L = 3.9m$, height $H = 1.4m$, tire radius $R = 0.3m$, and width $D = 1.5m$ as designed in this numerical example. The car shell is steel products, with its elastic modulus $E = 206Gpa$, Poisson ratio $\nu = 0.3$, density $\rho_s = 7.85 \times 10^3 kg / m^3$. The density of air is $\rho_a = 1.29 \times 10^3 kg / m^3$, the air power viscosity is $\nu_a = 1.79 \times 10^{-5} Pa \cdot s$. We limit the fluid region to a rectangular region of length $L_{air} = 6m$, and height $H_{air} = 9m$.

As for two-dimensional elastic mechanics problems, including displacement and boundary information, it can be abbreviated as:

$$
\begin{align*}
\nabla \cdot \sigma + b &= 0 \quad (x, y) \in \Omega \\
u(x, y) |_\Gamma &= g_1(x, y) \quad (x, y) \in \Gamma \\
v(x, y) |_\Gamma &= g_2(x, y) \quad (x, y) \in \Gamma \\
n_x \sigma_{xx} + n_y \sigma_{xy} &= f_x \quad (x, y) \in \Gamma \\
n_x \sigma_{xy} + n_y \sigma_{yy} &= f_y \quad (x, y) \in \Gamma \\
\end{align*}
$$

(51)

The Navier-stokes equation in 2-dimensional transients, the specific form is as follows:

$$
\begin{align*}
\frac{\partial u}{\partial t} + u \cdot \nabla u = f - \frac{1}{\rho} \nabla P + \nu \nabla^2 u \\
\nabla \cdot u &= 0 \quad (x, y) \in \Omega \\
u(x, y, t_0) &= u_0 \quad (x, y) \in \Gamma \\
v(x, y, t_0) &= v_0 \quad (x, y) \in \Gamma \\
P(x, y, t_0) &= P_0 \quad (x, y) \in \Gamma \\
\end{align*}
$$

(52)

This paper studied a particular moment, therefore, the 2D Navier-stokes equation belongs to the steady state, namely the time term is fixed or known. Then using the finite element method to solve the above two systems of equations, the flow field is coupled to each other, and the horizontal $x$ direction of the current flow field has the initial velocity $u_0 = 20m / s$, the initial velocity in the direction of the vertical component $y$ is $v_0 = 0m / s$. When brought into the fluid equation is solved, the pressure $P$, pressure acts on the surface of the car shell, equivalent to add a external force load $f$ into the elastic equation (51), and then the corresponding displacement, stress, strain and other information can be solved. For these two physical fields using a separate solver, combined with the iterative method, it finally realizes the output of the coupling equation values. The mesh of this example adopts the adaptive mesh, that is, in the change of irregular positions, such as the tire, the car shell has an arc of the transition position, they are the key research areas. The fluid region has $E_f = 3685$ element, vertices nodes are $N_f = 1961$, and the mean quality of mesh is $\eta_1 = 0.897$. The entire small car has elements $E_{car} = 889$, vertices nodes are $N_{car} = 487$, and the mean quality of mesh is $\eta_2 = 0.832$. Below is the corresponding numerical solution cloud diagram.

Fig2. Numerical solution of fluid-solid coupling and stress evaluation effect picture of the vehicle shell based on the establishment of the model in this paper, the numerical solution cloud diagram of the fluid-solid coupling of the car shell is obtained. Fig2.(a) is the velocity distribution of the 2D flow field. We can observe that the wind speed above the front hood and the tires of the car increases sharply. The location where the ground touches the ground is also relatively high, and the wind speed behind the vehicle is very small. The simulation results are consistent with the actual test. Fig2.(b) is the pressure distribution cloud map of the two-dimensional. The location where the pressure is the
most concentrated is on the front hood. Fig2. (c) is the stress generated by the flow field acting on the car shell. The stress is also concentrated near the front tires and hood. The improvement of the geometric appearance of these positions has the most obvious impact on the force of the car.

Fig. 2 Fluid-solid coupling and stress evaluation effect picture of the vehicle shell
Fig2.(d) is the velocity distribution cloud diagram of the 3D flow field. The three-dimensional fluid-solid coupling solution method is similar to that of the two-dimensional, except that a component is added to the dimension, but the calculation time is much longer, and the mesh fineness is also very demanding. The figure is a cloud map of wind speed \( v_y = 40 \text{ m/s} \). Fig2.(e) is the convergence diagram of the error under the 2D fluid-solid coupling separation solver. A total of 40 iterations have been carried out. The convergence speed of velocity and pressure is slightly lower than the convergence speed of displacement. Fig2.(f) is evaluation criteria about 2D fluid-solid coupling model, at the same speed, 5 kinds of car appearances are designed, and the mechanics of each appearance is solved, and then divided into 5 reference levels, which is convenient for further use of semi-supervised machine learning methods for training and the test is also equivalent to the evaluation classification criteria given in this article. The test samples are classified and screened according to these data standards, and then different evaluation levels are obtained for reference.

5. Conclusion
This paper studies the coupled assessment of the flow field and mechanical mechanics of two-dimensional and three-dimensional automobile shells. The Navier-stokes equations and elastic equations are discretized using finite elements to obtain the corresponding variational forms. Finally, the algebraic equations are obtained by assembling. The Numerical results show that the fluid-solid coupling model used in this paper to solve the flow field distribution and stress of the car shell is efficient. The common feature of the numerical results is that the stress and pressure are relatively large on the hood and front tires of the car, and the flow field will increase sharply above the hood. In addition, three-dimensional is more intuitive than two-dimensional, and the amount of flow calculation is much larger than that of two-dimensional. In addition, when machine learning is mainly used for matrix calculation, three-dimensional needs to capture the interest region when comparing the stress’s changement. It is relatively difficult to normalize the size and lock the local of call shell. Another contribution of this paper is to give a scheme for evaluating the force of the car shell in the flow field, and introduce the idea of stress reference standard classification, which provides a clear view for semi-supervised learning optimization target and classification criteria. Our follow-up work will focus on how to enhance the training speed and evaluation accuracy.

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