Flight control of tethered kites and winch control for autonomous airborne wind energy generation in pumping cycles

Michael Erhard* and Hans Strauch

SkySails GmbH, Luisenweg 40, D-20537 Hamburg, Germany

(10 September 2014)

Energy harvesting based on tethered kites makes use of the advantage, that these airborne wind energy systems are able to exploit higher wind speeds at higher altitudes. The setup, considered in this paper, is based on the pumping cycle, which generates energy by winching out at high tether forces, driving an electrical generator while flying crosswind and winching in at a stationary neutral position, thus leaving a net amount of generated energy. The economic operation of such airborne wind energy plants demands for a reliable control system allowing for a complete autonomous operation of cycles. This task involves the flight control of the kite as well as the operation of a winch for the tether.

The focus of this paper is put on the flight control, which implements an accurate direction control towards target points allowing for eight-down pattern flights. In addition, efficient winch control strategies are provided. The paper summarises a simple comprehensible model with equations of motion in order to motivate the approach of the control system design. After an extended overview on the control system, the flight controller parts are discussed in detail. Subsequently, the winch strategies based on an optimisation scheme are presented. In order to demonstrate the real world functionality of the presented algorithms, flight data from a fully automated pumping-cycle operation of a small-scale prototype setup based on a 30 m² kite and a 50 kW electrical motor/generator is given.

Keywords: Airborne wind energy; Flight control; Kite Power; Tethered kites

*Corresponding author. Email: michael.erhard@skysails.de
1. Introduction

Since the first proposal of energy generation using tethered wings more than thirty years ago (Loyd, 1980), a great interest in this kind of renewable energy source has emerged, especially during the last decade. The application of tethered wings or kites appear very attractive, as they combine high achievable forces in crosswind flight together with the possibility of easily venturing into higher flight altitudes in order to increase the energy harvesting result due to the higher wind speeds at these higher altitudes. The different concepts can be grouped together by using the term ‘airborne wind energy’, for an overview see e.g. (Fagiano & Milanese, 2012). An extended summary on geometries, theory oriented research activities, realised prototype system and planned setups can be found in the recent textbook on airborne wind energy (Ahrens, Diehl, & Schmehl, 2013).

The economic operation of airborne wind energy plants demands for reliable and fully automatic operation of the power generation process. Thus, numerous theoretical control proposals (Baayen & Ockels, 2012; De Lellis, Saraiva, & Trotino, 2013; Diehl, 2007) as well as experimental implementations have been published (Erhard & Strauch, 2013a; Fagiano, Milanese, & Piga, 2011; Ilzhöfer, Houska, & Diehl, 2001) as well as experimental implementations.

The paper is organised as follows: starting with a brief background information can be found in (Fritz, 2013). An extended description of involved components and prerequisites are motivated in Sect. 3. After summarising the equations of motion in Sect. 4, an overview on the complete control setup is presented in Sect. 5. Sections 6–10 present details of the single controller parts and illustrate their principle of operation by discussing experimental flight data results.

In this manuscript, the complete control design shall be presented, based on the equations of motion of a simple model (Erhard & Strauch, 2013a), which describes the steering behaviour of the kite as well as the kinematics, and has been extended for changing tether lengths in (Erhard & Strauch, 2013c). The single design steps towards a robust pattern eight-down flight are discussed in detail and the applicability is illustrated by the discussion of real flight data results.

In this paper, we will report on the control system for complete autonomous power cycles with a small-scale 50 kW prototype system using our 30 m² kite. The focus is put on flight control of efficient dynamical pattern-eight trajectories, which are crucial in order to obtain an optimal power generation output. A distinguishing feature of the pattern-eight flight trajectories is the option of flying them in two ways. From the performance point of view, one would prefer the so called eight-down trajectories as those significantly increase the power output. However, the performance advantage comes along with drawbacks of temporarily flying ahead the surface and the need for proper curve flights, which pose special requirements to the flight control system. As a consequence, we extended our previously published control system (Erhard & Strauch, 2013a) in order to combine it with target point concepts similar to (Erhard et al., 2013) and (van der Vlugt, Peschel, & Schmehl, 2013). For the overall power generation control, a compact description with three states and simple winch control strategies for the different phases have been added, which already yield remarkable results.

In this section, a general overview on the setup and the operation principle for power generation will be given. An extended description of involved components and background information can be found in (Fritz, 2013).

2. Implemented prototype and power generation

2.1 Setup

A picture of our small-scale prototype is shown in Fig. 1. The ram-air kite of 30 m² is steered by a control pod directly located under the kite, pulling steering lines by the steering actuator located in the control pod. This geometry allows for a single main towing line, consisting of 6 mm diameter high-performance Dyneema® rope, which connects the flying system to the ground station and is used to transfer the aerodynamic forces. The prototype features 300 m installed line of tether length on the main winch attached to a 50 kW electrical motor/generator-combination.
In order to support research and development projects, the prototype is equipped with several sensors. Although the specific choice of sensors and signal preprocessing is crucial for the whole control design, its discussion would go far beyond the scope of this paper. However, in order to allow for a proper understanding of the subsequent sections, a short summary on the most important sensors is given in Table 1. In the following, measured sensor quantities are indicated by the subscript ‘$m$’. For a detailed overview on the sensors for flight control of tethered kites, the interested reader is referred to (Erhard & Strauch, 2013b) and to (Fagiano, Huynh, Bamieh, & Khammash, 2014), (Ranneberg, 2013) for application examples of fusion algorithms.

### 2.2 Power generation cycle

This subsection focuses on the applied power generation principle. A typical flight trajectory during operation is sketched in Fig. 2. The power generation is done in cycles, which consist of the following three phases:

1. **Power Phase**: In the power generation phase, the kite is flown dynamically in pattern-eight configuration, which induces high line forces. Meanwhile the line is winched out, driving an electrical generator producing energy.
2. **Transfer Phase**: When a certain line length is reached, the transfer phase brings the kite to a neutral position.
3. **Return Phase**: During the return phase, the line is winched in, operating the generator as motor while the kite is kept at a neutral wind window position. This phase consumes a certain amount of the energy produced in phase 1. As the tether force at neutral position is much lower than during dynamic flight, only a minor fraction of the generated energy of phase 1 is needed leaving a considerable net amount of generated energy. When the lower line length threshold is reached, the whole cycle repeats starting at (1).

This periodic winching process is also called pumping cycle or yo-yo operation configuration.

![Figure 1. Small scale prototype system for kites of sizes ranging from 20–40 m$^2$ (30 m$^2$ shown here). The main winch with motor/generator is located in the ground station. A tether line of length typically in the range 150–300 m transfers the forces from the flying system. A distinguishing feature of the latter is the control pod located under the kite, which allows for a single towing rope. The actuator in the control pod pulls certain lines in order to steer the kite.](image)

![Figure 2. Flight trajectory for the power generation cycle. 3D view of experimental flight data.](image)
Finally, we would like to remark, that our current kites are flown with constant angle of attack during all phases and there is no de-powering feature for the return phase implemented as e.g. in (van der Vlugt et al., 2013). Therefore, the return phase is accomplished by winching the kite directly against the wind. At first sight, this strategy seems to be inefficient as it suggests slow winching speeds in order to keep the tether forces and thus the needed energy low. However, the opposite is the case, as by winching in, the air flow at the kite and subsequently the tether forces are even reduced by increasing the winch speed as shown in Sect. 4.5 making this power generation scheme competitive.

![Figure 3. Principle of pattern-eight flight. Note that the eight can be flown in two ways indicated by the triangular shaped arrows on the eight trajectories. The eight-up pattern (a) suffers from low aerodynamic forces $F_a$ in the outer regions further reduced by gravity $F_g$, leading to low total forces $F_t$. In the center region, the higher aerodynamic forces are even increased by gravity leading to huge variations of the force with respect to time as shown in (b). For the eight-down variant (c), the variations of gravity and aerodynamic forces compensate partly, leading to a more regular total force, compare (d).](image-url)

### 3. Effective power pattern

Optimisation of power output means basically maximisation of traction forces, which are generated by dynamical pattern-eight flight. An important distinguishing feature is, that the pattern-eight can be flown in two ways as illustrated in Fig. 3. Note the triangles drawn on the trajectories in the figures indicating the flight directions. Comparing the eight-up and eight-down configurations with respect to maximum energy generation, the eight-down variant is clearly favourable due to the better compensation of gravity by aerodynamic forces as shall be explained in the following. The highest aerodynamic forces occur in the center of the wind window. The eight-up pattern, shown on the left hand side, significantly suffers from flying up against gravity in the outer regions with low aerodynamic forces. In order to improve performance, one can take advantage of gravity in the outer regions with lower aerodynamic forces by flying down and using the high-force region in the center to fly up against gravity. This is achieved by the eight-down pattern shown on the right hand side. A further advantage of these eights-down is the significantly reduced variation in tether force as well as a reduction of the required minimum wind speed for stable operation. Both benefits make the pattern-eight-down concept very attractive for power generation, especially for pumping cycle concepts, based on power generation by winching, which leads to a significant reduction of the apparent wind speed.

However, this very effective concept of pattern-eight-down comes along with certain drawbacks, which should not be concealed. First, during every curve flight, the kite runs through an extended phase while heading directly onto the surface or into the power zone. Hence, a short failure or even a moderate perturbation of the control system will inevitably end up in a crash onto the surface or may lead to a destructive overload in the power zone. Second, in contrast to the eight-up variant, a step-wise approach to the operational high force pattern, which starts with small eights at low forces and gradually increases the size based on small direction changes and thus sensing the environmental conditions and system parameters, is hardly possible. The reason is that the low force eight-down variant would require very well controlled, narrow curves.

As a consequence, there are certain demands on the control system in addition to general reliability and availability specifications. A simple approach to pattern control (Erhard & Strauch, 2013a, 2013c), which produces the pattern implicitly by evolution, rather than by guiding of a pre-defined trajectory, turned out to be quite robust in a broad range of sometimes unknown environmental conditions. Such a strategy determines the set of parameters by starting at save, low-force operating points and adapting these parameters during operation gradually. Unfortunately, this simple strategy, although successfully employed in kite towing operation, is hardly suitable for pattern-eight-down generation. The given challenge of automated pattern-eight-down flight demands that a certain level of trajectory guidance must be achieved.

We consider the following two properties as essential. First, the flight towards a target point (Fagiano et al., 2013) must be controlled quite accurately in order...
Figure 4. Coordinate system. The distance between origin and kite is given by the tether length $l$.

to always keep the kite in the desired region of the wind window and especially in order to compensate for perturbations due to side gusts. Second, the curve flight, which is basically steered by a trapezoidal signal on the actuator, must be well engineered because a major part of the steering speed of the control actuator has to be used explicitly to allow for reasonable small curve radii. In addition, the final target direction of a curve flight should be reached quite accurately with absence of significant overshoots.

Although gravity effects are important for the specific choice of the flight pattern for performance reasons, they are neglected in the following as they could be regarded as uncertainties in the dynamics, which are compensated by the control feedbacks. A thorough modelling in order to cover low wind performance is subject to current research activities and will be published elsewhere.

4. Plant description

In this section, the dynamic model of the system shall be presented. The extended derivation of the model as well as system identification and comparing the model to experimental data is done in (Erhard & Strauch, 2013c). The model is based on four state variables $x = [\varphi, \theta, \psi, l]$ as shown in Fig. 4. The position of the kite $r$ is given in polar coordinates by the angles $\varphi, \theta$ in combination with the tether length $l$ and could be written as

$$ r = l \begin{pmatrix} \cos \theta \\ -\sin \varphi \sin \theta \\ \sin \theta \sin \varphi \\
\end{pmatrix} \quad (1) $$

The orientation of the kite is parameterised by the angle $\psi$, which could be defined as angle to the wind, i.e. $\psi = 0$ corresponds to directly heady against the wind. The tether of our model could be regarded as rigid rod, hence the complete orientation of the kite is determined by the angles $\varphi, \theta, \psi$. An ambient constant and homogeneous wind field with wind speed $v_w$ along the $x$-direction is assumed.

4.1 Model assumptions

In order to derive the equations of motions, some assumptions are made, which shall be summarised and justified in the following:

- The aerodynamic forces are assumed to be large compared to system masses. This allows for some simplifications. First, the rope can be implemented as simple tether (mass-less and infinitely thin rod). Second, all acceleration effects can be neglected assuming the system is always in an equilibrium of forces. Especially the latter assumption significantly reduces the equations of motion from second to a first order system.
- The aerodynamics of the kite is reduced to the glide ratio number $E$, which describes the ratio of the air flow between roll- and yaw axis. Further, it is assumed, that there is no air flow component in pitch-direction (no side-slip).
- The steering behaviour of the kite is described by a simple turn-rate law, which has been introduced empirically and shown experimentally (Erhard & Strauch, 2013a), (Jehle & Schmehl, 2014), but also can be nicely derived from first principles (Fagiano et al., 2013).

4.2 Equations of motion

The kite system input is described by the following control input vector $u = [\delta, v_{\text{winch}}]$. The deflection $\delta$ determines the steering input applied by the actuator of the control pod to the kite. As winch dynamic is not subject to this paper, it is assumed, that $v_{\text{winch}}$ directly determines the change of tether length $l$.

Taking into account the assumptions of the previous subsection, the following equations of motion can be derived:

$$ \dot{\psi} = g_k v_a \delta + \dot{\varphi} \cos \theta \quad (2) $$
$$ \dot{\theta} = \frac{v_a}{l} \left( \cos \psi - \tan \theta \frac{\tan \varphi}{E} \right) - \frac{l}{l} \tan \varphi \quad (3) $$
$$ \dot{\varphi} = -\frac{v_a}{l} \sin \varphi \sin \psi \quad (4) $$
$$ \dot{l} = -v_{\text{winch}} \quad (5) $$

The system parameter $g_k$ quantifies the response of the kite due to a steering deflection. The value $v_a$ represents
In order to resolve this issue, (6) is inserted into (3)–(4). This is also done explicitly or implicitly when using these equations for numerical simulations. However, there is a major reason, why this is done only partially here. As we aim at designing a control algorithm for a real system, the capability of reliably measuring certain quantities becomes important, as opposed to a pure simulation. Measuring \( v_a \) turns out to be comparably easy by an on-board anemometer in the control pod, while the determination of \( v_w \) is quite involved. The straight-forward approach of using an anemometer next to the ground station would require, that the wind field is constant and homogeneous. But this is definitely not the case for airborne wind energy devices making use of higher wind speeds at higher altitudes. Thus, knowing \( v_a \) by measurement instead of using (6) could be regarded as kind of generalisation of the equations of motion by taking into account local (measured) effects. Hence, using \( v_a \) instead of \( v_w \) whenever possible is preferred in order to enhance the robustness of the control system.

For constant tether lengths \( l = 0 \), it can be shown that \( \dot{\vartheta} \leq \arctan E \) and thus (2)–(4) could be used with \( v_a \) as measurement input. However, as soon as winching in is allowed, \( \dot{\vartheta} = \pi/2 \) is in the usual operation condition and (3) is no longer defined due to the singularity in \( \tan \dot{\vartheta} \). In order to resolve this issue, (6) is inserted into (3)–(4). The resulting complete set of equations of motions reads then:

\[
\begin{align*}
\dot{\psi} &= g_k \, v_a \, \delta + \phi \cos \vartheta \\
\dot{\vartheta} &= \frac{v_w}{l} \left( E \cos \vartheta \cos \varphi - \sin \vartheta \right) - \frac{l}{E} \cos \varphi \\
\dot{\phi} &= -\frac{v_w}{l} \cos \vartheta \left( E \sin \vartheta \right) \sin \varphi. \\
\dot{l} &= -v_{\text{winch}}
\end{align*}
\]

\[\text{(10)}\]

4.3 Motion on a sphere and crossterm

Due to the motion on a sphere, covering significant angular ranges in short time, the curvature of the state space has certain effects on the kinematics. Regarding the inertial turn-rate sensor aligned along the yaw axes, one would obtain a turn rate of \( \psi_m = 0 \) for the tethered motion in absence of other external forces. For a steering deflection, the turn rate is described by the turn-rate law as \( \psi_m = g_k \, v_a \, \delta \). Comparing this expression to (2)

\[
\psi = g_k \, v_a \, \delta + \phi \cos \vartheta
\]

one recognises the term \( \phi \cos \vartheta \), which implements a cross-coupling between \( \psi \) and \( \varphi, \dot{\vartheta} \). This term is defined as crossterm

\[
\psi_{\text{ct}} = \dot{\phi} \cos \vartheta
\]

Equation (11) then reads

\[
\psi = \psi_m + \psi_{\text{ct}}
\]

In other words, the crossterm \( \psi_{\text{ct}} \) represents the difference between the turn rate w.r.t. the inertial system \( \psi_m \) and the time derivative of \( \psi = d/dt \varphi \). Note, that the prime is always used to indicate turn rates w.r.t. an inertial system.

In a cascaded controller topology one would like to setup the chain as \( \varphi \rightarrow \psi \rightarrow \delta \) and implement both controllers as two cascaded SISO blocks for simplicity and robustness reasons. This could be done by assuming \( \varphi \approx \psi_m \), which is an appropriate approximation for a range of other applications. However, for highly dynamical pattern flight, consideration of the crossterm is desired by using (11) explicitly for the controller design. For theoretical simulations \( \psi_{\text{ct}} = \phi \cos \vartheta \) simply could be used to accomplish the task. Unfortunately, measuring \( \phi \) accurately and reliably turns out to be cumbersome, therefore a quantity based on controller states and sensor values rather than derivatives of sensor values is preferred. Using the model relation (10) yields:

\[
\psi_{\text{ct}} = \phi \cos \vartheta = -\frac{v_a}{l \tan \vartheta} \sin \varphi.
\]

Details on controller implementations and performance discussions will be presented in Sect. 7.

4.4 Flight direction

In order to navigate on the sphere, the flight direction \( \gamma \) is defined as follows:

\[
\gamma \equiv \arctan(-\phi \sin \vartheta, \dot{\vartheta}).
\]

As depicted in Fig. 5, \( \gamma \) denotes the angle between the 'latitude' line with \( \varphi = \text{const} \) through the current position \( r \) and the direction of the kinematic motion \( \dot{r} \). Direct
measurement of the flight direction is subject to high noise caused by the time derivatives of $\varphi_m$, $\dot{\varphi}_m$ and sensitive to slack line effects in the towing rope. Therefore directly processing $\gamma$ in a control loop, where $\gamma$ is computed from sensor values using (15), should be avoided. An alternative is to control the flight direction indirectly by $\psi$.

In order to obtain the corresponding relation, (8) and (9) are inserted into (15)

$$
\gamma = \arctan \left( \sin \psi, \cos \psi - \frac{1}{E} \frac{v_w \sin \dot{\varphi}}{v_w \cos \dot{\varphi} - \dot{l}} \right). \quad (16)
$$

The inversion of this relation is done by using the relation

$$
\psi = \arctan(r \sin \gamma, c_1 + r \cos \gamma) \quad (17)
$$

were the quantities $r$ and $c_1$ can be determined as:

$$
c_1 = \frac{1}{E} \frac{v_w \sin \dot{\varphi}}{v_w \cos \dot{\varphi} - \dot{l}} \quad (18)
$$

$$
r = \sqrt{1 - c_1^2 \sin^2 \gamma - c_1 \cos \gamma} \quad (19)
$$

The difference between $\gamma$ and $\psi$, which is referenced to the air flow $\dot{\mathbf{r}} - \dot{\mathbf{e}}_{\text{yaw}} - v_w \mathbf{e}_x$, is thus determined by the background wind vector. In crosswind flight with $|\dot{\mathbf{r}}| \gg v_w$, the difference is small compared to the range of directions in the pattern-eight. For an accurate direction control, this difference should be considered, however.

### 4.5 Winching

In order to examine the effect of winching, the steady state for constant winching speed $\dot{l}$ is calculated by setting $\dot{\varphi} = 0$ in (8). Some trigonometric manipulations yield for the equilibrium wind window angle

$$
\vartheta_0^{\text{winch}} = \vartheta_0 - \arcsin \left[ \frac{\cos \psi}{\sqrt{\cos^2 \psi + (1/E^2 \dot{l})^2 v_w}} \right] \quad (20)
$$

with the zenith position for zero winch speed of $\vartheta_0 = \arctan(E \cos \psi)$. The corresponding curve is shown in Fig. 6. Note, that reeling in at windward position in order to decrease the tether force is explicitly exploited in the cycle scheme presented here.

In order to further illustrate this effect of winching, the side view of a cycle trajectory is plotted in Fig. 7. Finally, we would like to comment on the winching-in phase. In order to keep the kite in a stable flight configuration, a certain minimum air path speed $v_{a,\text{min}}$ is required. Assuming $v_a > v_{a,\text{min}}$ and resolving (6) w.r.t. $\dot{l}$ yields:

$$
\dot{l} < -\frac{v_{a,\text{min}}}{E} + v_w \cos \dot{\varphi} \quad (21)
$$

It can be seen, that for angles $\varphi > \pi/2$, a certain winch speed $\dot{l} < 0$ is needed in order to keep the stable flight configuration. Before stopping the winch, the angle $\varphi$
Figure 7. Side view of trajectory for one power cycle. Note, that during the return phase, the kite is flown in a windward position in order to further reduce the tether force, compare Fig. 6.

has to be reduced by e.g. flying into the wind window as is done by starting the power phase.

5. Control design overview

In this section, a brief overview on the complete control system is given. Details of the controller parts and presentation of experimental results will be given in the subsequent sections.

5.1 Design philosophy

Before diving into the details of the control system, we would like to summarise the design ideas and principles in order to provide a kind of justification for the described setup in the rest of the paper. Examining the equations of motion (2)–(5), a possible control approach could be based on the concept of dynamic inversion or model predictive control (Lucia & Engell, 2014). In the field of tethered kite control, such approaches have been applied to much more complex theoretical models than ours (Williams, Lansdorp, & Ockels, 2008), (Gros & Diehl, 2013) and the principal functionality successfully demonstrated in simulations. However, successful applications of these involved algorithms on real prototypes has not been reported yet.

A prototype setup and the task of developing an operationally robust control system, imposes a different emphasis and leads to a different controller structure. The plant is subject to huge perturbations due to wind gusts and with uncertainties about wind conditions over the range of flight altitudes for one cycle. In order to successfully tackle these real world and industrial challenges, one prefers certain control topologies and design principles, which will be summarised in the following:

- An important step is to split up the system into separate parts which can be described, to a large degree, by analytic equations, related to intuitive physical models. For each subsystem, it is much easier to design robust, but still simple and linear controllers, augmented by nonlinear elements like limiters, which can be easily inserted and tuned. This approach naturally leads to a cascaded controller topology.
- Implement a feedforward/feedback structure in order to achieve the bandwidth needed for tailored curve flight and to capture the major nonlinearities in the feedforward path. The feedforward paths also allow for a proper shaping of signals according to system constraints and can be easily added to a cascaded design.
- Force the system only when necessary. As a consequence, we do not try to follow an exact predefined trajectory at all cost. The achievable control bandwidth of the inner loop would not allow for very high accuracy, when taking robustness as major design consideration into account. Instead we implemented the simple target point tracking scheme proposed in (Fagiano et al., 2013), which supports a kind of natural evolution of the eight-pattern, but keeps the pattern reliably in the desired region of the wind window on the other hand side.

The implemented overall control system structure is drawn in Fig. 8. The control strategy is based on flight control towards target points (TPs) while switching to a subsequent target point, before the current target point is reached (Fagiano et al., 2013). This algorithm, which also controls the winch, is called winch and cycle control. Compared to (Fagiano et al., 2013), where a simple heading control has been implemented, we present a cascaded flight control setup based on earlier work (Erhard & Strauch, 2013a) providing accurate heading control as well as shaped curve flight, which are both prerequisites for robust eight-down flight.
5.2 Winch and cycle control

This part is responsible for the overall control of the power generation cycle and the computation of the winch speed set value $v_{\text{winch}}$.

1. The cycle control in its simplest, albeit not fully optimised, version can be based on three target points only. The geometry of these target points is indicated in Fig. 2. Switching between target points is triggered by geometric conditions, i.e. by approaching condition to the active target point as well as based on the line length $l$ in order to obtain the periodic repetitions of cycles consisting of power and return phases. Details will be given in Sect. 9.

2. In order to implement an efficient winch control algorithm, the overall optimisation problem involving complete cycles has to be tackled as suggested in (Horn, Gros, & Diehl, 2013). However, as wind conditions in different flight altitudes are not exactly known and subject to gusts and significant variations, simpler approaches are desirable at least for first proof-of-concept flight tests. Performing a rudimentary numerical optimisation with our model (2–6), we identified a simple relation, which allows for the computation of the winch speed based on the geometric condition of wind window position and wind speed only. Although this approach is one of the most simplest, it is performing surprisingly well as it is capable of operating all phases and the energy production seems not to be much away from the achievable optimum. Details will be given in Sect. 10.

5.3 Flight control

The flight control design is composed of three cascaded controllers as shown in Fig. 5. In the following, these blocks are described from right to left.

1. Inner loop ($\dot{\psi}$-controller): Yaw axis stabilisation with set point turn rate $\psi'_s$. The plant behaviour $\delta \rightarrow \dot{\psi}$ is based on the turn rate law

\[
\dot{\psi}' = g_k v_a \delta
\]  

(22)

Note, that the inertial turn-rate sensor (yaw axis) measures the turn rate $\dot{\psi}_m'$ which corresponds to $\dot{\psi}'$ and not $\dot{\psi}$.

2. Outer loop ($\psi$-controller): $\psi_s$ angle control (related to flight direction). The plant behaviour

\[
\psi'_s \rightarrow \psi \text{ given by:}
\]

\[
\psi = \psi'_s + \psi_{ct} = \psi'_s + \phi \cos \vartheta
\]  

(23)

It should be remarked, that the controller deals with multiple input values $[\psi_s, \psi_{ct}] \rightarrow \psi'_s$ where $\psi_{ct}$ involves further quantities given by (14). However, it is appropriate to assume $\psi \approx \dot{\psi}'$ for the initial design and add the crossterm as compensating correction.

3. Guidance: Control flight towards target point. The flight direction is based on wind window position, hence the guidance computes

\[
[\varphi_m, \vartheta_m, \varphi_{TP}, \vartheta_{TP}] \rightarrow \psi_s
\]  

(24)

The guidance is done by computing the flight direction $\gamma$ towards a target point and subsequently $\psi_s$ by inverting (16). It should be finally noted, that due to the switching of target points, discontinuities are imposed on $\psi_s$. As these steps $\psi_s$ should result in well defined and controlled curve flights, a proper shaping has to be performed. In our design this is done in the $\psi$-controller as described in Sect. 7 in detail.

6. Controller for yaw rate $\dot{\psi}'$

The complete setup of the $\dot{\psi}'$-controller is shown in Fig. 9. The controller is based on the feedforward and feedback parts marked by the dashed grey boxes. The plant behaviour is based on the turn-rate law, compare (22)

\[
\dot{\psi}' = g_k v_a \delta
\]  

(25)

In order to obtain a linear plant behaviour with stationary parameters, the dependence on $v_a$ is eliminated by the $1/K_{\psi}$ block in the feedback controller with $K_{\psi} = g_k v_a$. This creation of a meta-actuator is based on the assumption, that $K_{\psi}$ changes slowly compared to the $\dot{\psi}'$ dynamics. Due to the proportional nature of the linearised plant ($\dot{\psi}' = K_{\psi} \delta$), a PI-controller is used for the control task in addition with a low-pass to suppress unwanted frequency components. Further, a limiter is applied to the error signal $\varphi_e$ for safety reasons. It should be mentioned here, that the source of the turn rate measurement $\psi_m'$ is a single inertial turn-rate sensor aligned in yaw axis of the flying system.

The feedforward command is computed based on $\delta_{ff} = \psi_s$ and shaped by a limiter and rate limiter, in order to
Figure 9. Setup of the ψ-controller based on a feedforward and feedback structure. The limiter and rate limiter in the feedforward shape the signal according to limited steering range and speed of the actuator in the control pod. Note the $1/K_\psi$-block in order to obtain a linear plant behaviour.

take into account the limited deflection range and steering speed of the control pod, respectively. It should be mentioned, that gravity leads to an additional term in the turn-rate law, which reads (Erhard & Strauch, 2013a, 2013c)

$$\dot{\psi}' = K_\psi \delta + M \frac{\cos \theta_s \sin \psi_s}{v_a}$$

with a system weight-dependent parameter $M$. Note, that the angles $\theta_s$ and $\psi_s$ are defined with respect to a different coordinate system. For details on definition and origin, the reader is kindly referred to (Erhard & Strauch, 2013a, 2013c). In order to compensate for the gravity term, the quantity

$$T_1 = \frac{M \cos \theta_s \sin \psi_s}{K_\psi v_a}$$

can be fed into the feedforward block. It should be noted, that the $T_1$ input feature is given for sake of completeness only. For usual operating conditions, the gravity compensation could be switched off ($T_1 = 0$) and the neglected effect in the feedforward path is easily dealt with by the feedback path. In addition, as $M$ depends on the weight of the flying system, a dependence on $l$ should be taken into account. The accurate compensation of this term, taking into account varying line lengths, is subject of current research and will be published elsewhere.

7. Controller for yaw angle $\psi$

The complete setup of the $\psi$-controller is given in Fig. 10. Like the inner loop controller described in the previous section, it features a feedforward/feedback structure. As the plant dynamics, apart from the crossterm correction, is of integrator type, the feedback is implemented as a proportional controller with preceding low pass filter and limiter for safety reasons.

The feedforward part is more involved, since it has to meet the following requirements. As already introduced, the switching of target points imposes steps on the set value $\psi_s$. These step discontinuities are related to commanded changes of flight direction and thus have to be implemented as properly curved flights, which should take into account limits on steering deflection $\delta_i$ and steering speed $\dot{\psi}_i$. The resulting flown curve radii must be small enough for efficiency reasons and for a safe fitting into a limited space of the wind window. In addition, the crossterm correction $\dot{\psi}_{ct}$ has to be considered according to (23). These shaping demands for $\psi_s$ are implemented by an internal loop, which will be discussed in the following.

The control pod is modeled by a limiter and a rate limiter for steering deflection $\delta_i$ and steering speed $\dot{\psi}_i$, respectively. The integrator implements $\psi_c = \int dt (\dot{\psi}_f + \dot{\psi}_{ct})$, which reflects the plant dynamics, compare to (23). The scaling from rates to steering and back is done by the $1/K_\psi$ and $K_\psi$ blocks, respectively. The feedback scaling function $f(x)$ implements the inverse of the modeled plant in order to achieve time optimal following of $\psi_c = \psi_s$. In order to determine $f(x)$, the process starting with an initial deflection $\delta_i > 0$ and steering with speed $-\dot{\psi}_i$ to a target deflection $\delta_t$ is considered. At target deflection, attainment of set point is assumed, i.e. $\psi_c = \psi_s$ and $\dot{\psi}_c = \dot{\psi}_s$. The latter implies

$$K_\psi \delta_i + \dot{\psi}_{ct} = \psi_s$$

Inserting the steering $\delta_i = \delta_{i0} t$ and resolving w.r.t. $t$ yields:

$$t = \frac{\delta_t - (\psi_s - \dot{\psi}_{ct})/K_\psi}{\delta_p}$$
The corresponding $\Delta \psi$ of this process can be computed as

$$\Delta \psi = \int_0^t dt' K_\psi (\delta_i - \delta p)$$  \hspace{1cm} (30)$$

and inserting (29) yields

$$\Delta \psi = \frac{(\delta_i + \psi_{ct}/K_\psi)^2}{2 \delta_p}$$  \hspace{1cm} (31)$$

This $\Delta \psi$ is interpreted as error, resolving this equation w.r.t. $\delta_i$ and reads

$$\delta_i = \sqrt{\frac{2 \delta_p \Delta \psi}{K_\psi} - \frac{(\psi_{ct} - \psi_s)}{K_\psi}}$$  \hspace{1cm} (32)$$

As $\delta_i$ is deflection related to this error, it is an appropriate feedback value. Comparing (32) to the diagram and generalising by consideration of $\delta_i < 0$ results in the following

$$f(x) = \text{sign}(x) \sqrt{2 \delta_p |x|}$$  \hspace{1cm} (33)$$

For constant of step-wise $\psi_s$ input signals, one would chose

$$\psi_{ct}^* = \psi_{ct}$$  \hspace{1cm} (34)$$

For $\psi_s$ inputs based on target points, $\psi_{ct}^* = 0$ is the appropriate choice as can be reasoned as follows: when heading to a target point and the final course is reached, the motion could be regarded approximately as free 'inertial' motion, which implies, compare (12) and text below, $\psi_s \approx \psi_{ct}$. Inserting into (34) implies $\psi_{ct}^* = 0$.

8. Flight direction control

As already introduced, pattern generation is accomplished by navigating towards target points. The basic principle is sketched in Fig. 11. Based on the great-circle navigation, which determines the shortest connection between two given points on the sphere, the direction from the current position $\phi_m, \theta_m$ to the target point $\phi_{TP}, \theta_{TP}$ can be computed by

$$\gamma_s = \arctan \left( \frac{\sin(\phi_m - \phi_{TP})}{\cos \theta_m \cos(\phi_{TP} - \phi_m) - \cot \theta_{TP} \sin \theta_m} \right)$$  \hspace{1cm} (35)$$

Having determined the flight direction $\gamma_s$, the set value for $\psi_s$ is computed by inversion of (17). For sake of stability, the distance from current position to the target should not fall below a certain minimum value. This is achieved by switching to another target point or moving away the current target point, respectively, as will be explained in Sect. 9.

In order to perform a curve flight, the target point is switched, as from TP1 to TP2 in this example. This switching leads to a step in $\psi_s$. However, due to the shaping in the $\psi$-feedforward as explained in Sect. 7, a smooth and well-controlled curve will be commanded.
The nominal steering deflection value $\delta_s$ determines the radius of the curve, which can be estimated by comparing the approximation for the tangential speed $\psi_m' \approx \nu_a / r_{\text{curve}}$ with (25)

$$r_{\text{curve}} = \frac{1}{g_k \delta_s}$$

Finally, the issue of unwrapping the course angles shall be explained. Using (36) one would obtain for the directions in Fig. 11 e.g. $\gamma_1 = 1.0 \text{ rad}$ and $\gamma_2 = -1.0 \text{ rad}$, respectively. However, the curve $\gamma = 1.0 \rightarrow (-1.0)$ would be clockwise and not counter-clockwise as needed for the drawn figure-eight. Hence $\gamma_2 = (2\pi - 1.0)$ has to be chosen. Alternatively, the same figure could be parameterised by $\gamma_1 = (-2\pi + 1.0)$ and $\gamma_2 = -1.0$. The modulo-$2\pi$ offset could be freely chosen as initial condition, but has to be kept constant during pattern operation.

9. Cycle control

In this section, the flight control generating the pattern-eight as well as steering the kite during the return phase is presented. In order to illustrate the target point method, projections of the flight trajectory on the unit sphere including target points are shown in Fig. 12.

The general principle of the target points is basically to control the flight direction heading towards an active target point. The dynamic pattern is generated by switching to another target point before the currently active target point is reached. The eight-down for the power phase is guided by the two target points TP1 and TP2, compare Fig. 12 with the coordinates chosen symmetrically with respect to the vertical axis as follows: $\varphi_{\text{TP2}} = -\varphi_{\text{TP1}}$ and $\vartheta_{\text{TP2}} = \vartheta_{\text{TP1}}$. The trigger condition for switching to the subsequent target point is defined with respect to the 'angular' distance on the unit sphere as follows:

$$\left(\varphi_m - \varphi_{\text{TP_i}}\right)^2 \sin^2 \vartheta_{\text{TP_i}} + \left(\vartheta_m - \vartheta_{\text{TP_i}}\right)^2 \leq \sigma^2$$

This condition is graphically illustrated in Fig. 13. The value $\sigma$ has been chosen empirically and kept constant for the results presented here. However, in order to optimise the pattern, a dependence $\sigma = \sigma(l)$ could be introduced.

For the transfer and return phases, the kite is flown towards target point TP3, compare Fig. 12. In contrast to the power phase, no switching is performed during these phases. In order to get a reasonable feedback, the target point should not be chosen too far away. In addition, it has to be made sure, that the target point is never reached. Hence its elevation value is chosen dynamically dependent on the current position

$$\vartheta_{\text{TP3}} = \max(\pi / 2, \vartheta_m + \Delta \vartheta)$$

with a typical value $\Delta \vartheta = 0.3 \text{ rad}$. The azimuth coordinate has to be chosen as a compromise. For $\varphi_{\text{TP3}} = 0$, no influence of gravity on the steering behaviour would be present, compare (26), but the tether tension would be maximally reduced by gravity. For values $\varphi_{\text{TP3}} > 0$, the influence of gravity on tether force is reduced allowing for lower operational wind speeds, but the effect of gravity on steering increases and sufficient space above the surface has to be left for the manoeuvres. As a consequence, the chosen value is typically $\varphi_{\text{TP3}} = 0.4 \text{ rad}$.

The complete state diagram for the overall cycle control is shown in Fig. 14. Note, that during the power phase, switching of target points is triggered.
by geometrical conditions while begin and end of the transfer/return-phases are determined by line length limits, which are in our case \( l_{\text{start}} = 130 \text{ m} \) and \( l_{\text{transfer}} = 270 \text{ m} \). Finally, it should be remarked that the transition from states 1 to 3 is due to the assumption \( \varphi_{\text{TP3}} > 0 \). The according diagram for negative values \( \varphi_{\text{TP3}} < 0 \) follows straight forward from symmetry considerations.

10. Winch control and power generation

First it should be remarked, that the setup of an electrical motor/generator attached to a frequency converter involves internal control loops, which are not subject to this paper as they have been tuned according to the respective user manuals. However, one feature is worth mentioning. The current control loop of the frequency converter is used to limit the maximal tether load and avoid overload of the kite by setting the maximal current accordingly. This is a very effective mechanism, as the current control loop is very fast and therefore, apart from inertia effects of the moving elements of winch and motor/generator, depowering is done as fast as possible.

In the following, we focus on computing the set value for the winch speed \( v_{\text{winch}} \) in order to operate efficient power cycles.

It should first be noted, that a computation of the winch speed for optimal power output calls for solving an optimisation problem considering complete power cycles, which is quite involved and subject to current research activities (Costello, Francois, & Bonvin, 2013). (Horn et al., 2013). However, these extended models are far beyond the scope of getting a small-scale prototype setup operational in order to prove, that fully automatic power generation is feasible. Hence, simpler approaches are needed, which compute the winch speed based on the system state as e.g. given in (Fechner & Schmehl, 2014), which proposes a feedforward implementation for constant force.

The main idea is to separate flight and winch control on the kinematic level. This is accomplished by flying the pattern geometrically, e.g. guided by target points defined on the unit sphere as primary control and commanding the winch speed dependent on the current pattern. This is also the control strategy chosen when operating the prototype by two human operators. The pilot flies the pattern-eight and static positions, respectively, while the winch operator commands the winch speed accordingly. This task comprises basically winching out during the pattern and winching in during the static flight position, respectively. Hence, the goal is to find simple controllers for the winch speed for the different phases, which will be be considered separately in the following.

It has to be remarked, that for sake of clarity, only the basic functionalities are given, which were used for the presented experimental results. They may lack robustness w.r.t. untypical wind situations or temporary free (quasi untethered, i.e. a non stretched line) flight. Operational extensions are subject to current development and would go far beyond the scope of this paper.

10.1 Power phase winch control

For the power phase, the thumb-rule using 1/3 of the projected wind speed could be used (Fagiano & Milanese, 2012; Loyd, 1980). Further optimisations have been proposed (Luchsinger, 2013), which suggest a different factor \( (1/a) < (1/3) \) by taking into account the return phase. Thus the set point for the winch speed is given by

\[
\dot{l}_s = \frac{1}{a} v'_w \cos \vartheta
\]

(39)

The \( v'_w \) value denotes the wind speed at flight position. Note, that \( v'_w \) is hardly measurable directly. However making use of (34), which could be regarded as kind of plant \( l \rightarrow v_a \) with

\[
v_a = v'_w E \cos \vartheta - \dot{l}_s E.
\]

(40)

and the simple proportional feedback

\[
\dot{l}_s = \frac{v_a}{(a - 1)E}
\]

(41)

it can be shown, that this simple loop fulfills the requirement (39) as the stationary value reads \( l \rightarrow v'_w (\cos \vartheta) / a \).
The setup is drawn schematically in Fig. [15] In order to allow for a realistic simulation, the main practical constraints of limited speed \( l_{\text{max}} \) and acceleration \( l_{\text{max}} \) of the winch as well as delay \( \tau \) and low pass behaviour of the system have been added to the figure, but will not be further discussed here. Note, that due to the loop setup, gusts on \( v_w' \) are anticipated by \( l \) leading to an efficient and consistent behaviour of the winch during the power phase.

### 10.2 Transfer and return phase winch control

In view of overall efficiency, winching in as fast as possible at tether forces as low as possible would be desirable. In order to achieve this, tuning the glide ratio has been proposed and performance modeling for such systems has been presented (Luchsinger, 2013). (Fechner & Schmehl, 2013). In contrast, our system is operated at constant glide ratio, but also allows for low-force return phases as discussed in Fig. [5]. In order to understand how to choose the winch speed for efficient transfer and return phases, an optimisation problem based on our model has been solved.

The simulation, which is sketched in Appendix A, suggests a remarkable simple law for choosing the winch speed \( l_s \) as function of the windwindow angle \( \vartheta \). The simulated results are plotted in Fig. [16]. Comparing winch speed and wind window angle, a linear dependence could be suspected which would suggest the following ansatz for the winch controller as given in Fig. [17].

\[
f(\vartheta) = \begin{cases} 
  a_{\text{lower}}(\vartheta - \vartheta_0) & \text{for } \vartheta < \vartheta_0 \\
  a_{\text{upper}}(\vartheta - \vartheta_0) & \text{for } \vartheta \geq \vartheta_0 
\end{cases}
\]

The constant parameters are typically chosen as follows: \( \vartheta_0 = 1.05 \text{ rad} \), \( a_{\text{lower}} = -0.55 \) and \( a_{\text{upper}} = -0.65 \). While the \( a \) values directly follow from the simulation, the \( \vartheta \) value can be modified slightly for practical operation in order to take into account line slack during the transfer phase. The winch speed limits are chosen as \( a_{\text{limit,in}} = -0.5 \) in order to limit the wind window angle to approx. \( \vartheta < 1.9 \text{ rad} (\approx 110 \text{ deg}) \) during the return phase, compare (20). The limit \( a_{\text{limit,out}} = 0.3 \) is motivated by the rule of thumb given above. For \( v_w \), either the anemometer at the ground station, or a wind estimation algorithm output is used. In summary, the winch controller consists of a simple function mapping the wind window angle \( \vartheta_m \) to \( v_{\text{winch}} \), scaled by the wind speed \( v_w \).

### 10.3 Pattern restart winch control

A special situation is given while flying from the static windward position heading to target point TP3 back into the dynamical pattern. As our optimisation is based on circular orbits, which are quite different to the curve-down manoeuvre from TP3 to TP1, the simulation results are not suitable for deriving an appropriate control law. First experimental tests have shown, that the transfer phase controller of the previous section can be applied for the restart, albeit the significant force peaks for lower \( \vartheta \) values should be avoided. The testing of improved controllers is subject to current research activi-
ties, a possible restart control phase is drawn in Fig. 16 as dash-dotted line.

11. Experimental flight results

In this section, experimental flight data of our small prototype, using a 30 m² kite at $v_w \approx 8$ m/s wind speed, will be discussed.

(1) Inner loop ($\psi'$-controller): Performance results for the inner loop $\psi'$-controller are given in Fig. 18. In order to review the controller performance, the set ($\psi'_s$) and reference ($\psi'_c$) values for the turn rate are compared to the measured turn rates $\psi'_m$. The excellent agreement of the measured turn rate shows, that system constraints are met. Note, that the major part of the steering originates from the feedforward part, while the feedback part contributes only slight corrections to the steering. This further validates the turn-rate law (25) and (26) on which the feedforward path is based.

(2) Outer loop ($\psi$-controller): The respective controller signals are shown in Fig. 19. Note, that although switching of target points introduces steps to the commanded $\psi_s$, the output quantity $\psi_c$ is properly shaped subject to limited steering deflection $\delta$, and speed $\delta_p$. An accurate analogy of the estimated orientation $\psi_m$ to the reference value $\psi_c$ can be observed, resulting in the fact, that dynamics is mainly controlled by the feedforward part. Hence, a proper design of the inner loop as well as validity of the dynamics given in Sect. 4.3 can be stated.

(3) Guidance: In order to evaluate the flight direction control, the direction to the active target point is compared to the measured flight direction as plotted in Fig. 20. The measured directions are com-

![Figure 18](image1.png)

**Figure 18.** Principle of operation of the $\psi'$ controller illustrated by signals of experimental flight data. Upper plot: comparison of set value $\psi'_s$ (solid), reference value $\psi'_c$ (dashed) and measured value $\psi'_m$ (dotted). Lower plot: feedforward (solid) and feedback (dashed) outputs of the $\psi'$ controller.

![Figure 19](image2.png)

**Figure 19.** Principle of operation of the $\psi$ controller illustrated by signals of experimental flight data. The commanded $\psi_c$ features steps, which are shaped by the feedforward block to get $\psi_c$. The measured value $\psi_m$ is shown for comparison.

![Figure 20](image3.png)

**Figure 20.** Measured flight directions compared to target point (TP) directions. The noise figure of $\gamma_m$ originates from the difference equation (13). Note, that $\gamma_m$ is plotted unfiltered for comparison only and not used for the control. During the return phase, the direction becomes moreless undefined due to the static flight.
computed by differences $\Delta \varphi, \Delta \vartheta$ between consequent samples using (15), i.e.

$$
\gamma_m = \arctan(-\Delta \varphi \cos \vartheta, \Delta \vartheta)
$$

For the return phase with insignificant kinematic motion, $\gamma_m$ becomes moreless undefined and unsuitable as direct control input. In order to handle this issue, a regularisation, as proposed in Zgraggen, Fagiano, & Morari (2013), could be used. In contrast, we decided to base the direction control on $\psi$ as introduced in Sect. 4.4. The excellent agreement of directions during the dynamic flight apart from the continuous $\gamma_m$ following the step in $\gamma$ during curves, demonstrates, that the flight direction control works satisfactorily and relation flights characterised by a step in $\gamma_m$ and relation (16) between $\gamma$ and $\psi$ holds.

Finally, we would like to close this section with a discussion of experimental power generation data as shown in Fig. 21. As reference value for the generated power, the Loyd’s limit is used, which corresponds to a continuous crosswind flight and optimal winch speed for that situation. This limit is calculated assuming, that the airpath speed in crosswind condition is determined by the back wind reduced by winching and the glide ratio as $v_a = E(v_w - l)$. Using (A3) and determining the maximum by varying $l$, one finds the above mentioned thumb rule $l = v_w / 3$ and for the power:

$$
P_{Loyd} = \frac{\rho C_R A 4E^2}{27} v_w^3
$$

For our flight test conditions $A = 0.7 \cdot 30 \text{m}^2$, $\rho = 1.2 \text{kg/m}^3$, $C_R = 1.0$, $E = 5.0$ and $v_w = 7.5 \text{m/s}$ (estimated at mean flight altitude), we obtain $P_{Loyd} \approx 19.7 \text{kW}$. Evaluating a typical cycle, as shown in Fig. 21, yields an average power of $\bar{P} = 3.5 \text{kW}$, hence $\bar{P} = 0.18P_{Loyd}$, which is about 60% of the expected value due to simulation results given in $\Box$. Taking into account, that the simulation does not consider the significant losses due to curve flights and real conditions coming along with gusts, the theoretical simulation based on the simple model and the experimental finding can be regarded as consistent. Hence, although neither precision measurements nor an extended optimisation of the power output were in the scope of the presented prototype setup, the presented model as well as the controller setup could be regarded as solid bases for further development steps. Especially the trajectory of recycling back into the power phase, but also the whole power phase are candidates for significant improvements in the near future.

12. Summary and future work

We have presented a complete control setup for autonomous power generation flying tethered kites at constant glide ratio in the pumping cycle scheme. The control structure design is based on and motivated by a simple model for the system dynamics. The discussed flight data with a small-scale demonstrator illustrates, that the implemented target point control allows to fly reliably the pattern eight-down, which is an ambitious scheme, but allows for significantly higher power generation efficiency.

The winch control has been implemented as simple state-feedback and tailored for an efficient transfer phase. Although it is quite rudimentary, it can be utilised for all phases with not fully optimised albeit remarkable energy generation results.

With respect to future work, there are two major fields for further developments. First, a long-term autonomous operation requires a high level of robustness of the control system under extreme environmental conditions, which basically involve extreme gusts and temporary untethered states. Especially, the development of estimation and filtering algorithms of sensor values is a research field on its own, which has been kept out of
this paper in order to focus on the general scheme and control. Extended results will be published elsewhere. Second, it should be noted, that the whole field of optimising the power output has only been scratched by the surface in this paper. An extended understanding of optimisation criteria, which also take into account different wind conditions at different altitudes, has to be established. A further major challenge will arise from the implementation of robust operational algorithms smartly adapting to varying environmental conditions. Currently, only few theoretical proposals (Diwale, Lymeropoulos, & Jones, 2014) and experimental results on that issue have been reported, see e.g. the maximum power point searching stepping algorithm (Zgraggen, Fagiano, & Morari, 2014), which adapts to varying wind directions.

References

Ahrens, U., Diehl, M., & Schmehl, R. (Eds.). (2013). Airborne wind energy. Springer Berlin Heidelberg. doi: 10.1007/978-3-642-39965-7

Baayen, J. H., & Ockels, W. J. (2012). Tracking control with adaption of kites. IET Control Theory and Applications, 6(2), 182-191.

Costello, S., Francois, G., & Bonvin, D. (2013). Real-Time Optimization for Kites. In Proceedings of the 5th IFAC-PSYCO Workshop (pp. 64–69).

De Lellis, M., Saraiva, R., & Trofino, A. (2013, Dec). Turning angle control of power kites for wind energy. In Decision and control (CDC), 2013 IEEE 52nd annual conference on (p. 3493-3498). doi: 10.1109/CDC.2013.6760419

Diehl, M. (2001). Real-time optimization for large scale nonlinear processes (PhD thesis). University of Heidelberg, Germany.

Diwale, S. S., Lymeropoulos, I., & Jones, C. (2014). Optimization of an airborne wind energy system using constrained gaussian processes. In IEEE multi-conference on systems and control.

Erhard, M., & Strauch, H. (2013a, Sept). Control of towing kites for seagoing vessels. Control Systems Technology, IEEE Transactions on, 21(5), 1629-1640. doi: 10.1109/TCST.2012.2221093

Erhard, M., & Strauch, H. (2013b, July). Sensors and navigation algorithms for flight control of tethered kites. In Control conference (ECC), 2013 European (p. 998-1003).

Erhard, M., & Strauch, H. (2013c). Theory and experimental validation of a simple comprehensible model of tethered kite dynamics used for controller design. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 141-165). Springer Berlin Heidelberg. doi: 10.1007/978-3-642-39965-7_8

Fagiano, L., Huynh, K., Bamieh, B., & Khammash, M. (2014, May). On sensor fusion for airborne wind energy systems. Control Systems Technology, IEEE Transactions on, 22(3), 930-943. doi: 10.1109/TCST.2013.2269865

Fagiano, L., & Milanese, M. (2012, June). Airborne wind energy: An overview. In American control conference (ACC), 2012 (p. 3132-3143).

Fagiano, L., Milanese, M., & Piga, D. (2011). Optimization of airborne wind energy generators. Int. J. Robust Nonlinear Control. Retrieved from http://dx.doi.org/10.1002/rnc.1808 doi: 10.1002/rnc.1808

Fagiano, L., Zgraggen, A., Morari, M., & Khammash, M. (2013). Automatic crosswind flight of tethered wings for airborne wind energy: Modeling, control design, and experimental results. Control Systems Technology, IEEE Transactions on, PP(99), 1-1. doi: 10.1109/TCST.2013.2279592

Fechner, U., & Schmehl, R. (2013). Model-based efficiency analysis of wind power conversion by a pumping kite power system. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 249-269). Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-39965-7_14 doi: 10.1007/978-3-642-39965-7_14

Fechner, U., & Schmehl, R. (2014). Feed-forward control of kite power systems. Journal of Physics: Conference Series, 524(1), 012081.

Fritz, F. (2013). Application of an automated kite system for ship propulsion and power generation. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 359-372). Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-39965-7_20 doi: 10.1007/978-3-642-39965-7_20

Gros, S., & Diehl, M. (2013). Modeling of airborne wind energy systems in natural coordinates. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 181-203). Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-39965-7_10 doi: 10.1007/978-3-642-39965-7_10

Horn, G., Gros, S., & Diehl, M. (2013). Numerical trajectory optimization for airborne wind energy systems described by high fidelity aircraft models. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 205-218). Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-39965-7_11 doi: 10.1007/978-3-642-39965-7_11

Ilzhöfer, A., Houska, B., & Diehl, M. (2007, November). Nonlinear MPC of kites under varying wind conditions for a new class of large scale wind power generators. Int. J. Robust Nonlinear Control, 17(17), 1590-1599.

Jehle, C. (2012). Automatic flight control of tethered kites for power generation (Master thesis). Technical University
of Munich, Germany.

Jehle, C., & Schmehl, R. (2014). Applied tracking control for kite power systems. AIAA Journal of Guidance, Control and Dynamics. doi: 10.2514/1.62380

Loyd, M. L. (1980). Crosswind kite power. Journal of energy, 4(3), 106-111.

Luchsinger, R. (2013). Pumping cycle kite power. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 47-64). Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-39965-7_3 doi: 10.1007/978-3-642-39965-7_3

Lucia, S., & Engell, S. (2014, June). Control of towing kites under uncertainty using robust economic non-linear model predictive control. In Control conference (ECC), 2014 European (p. 1158-1163). doi: 10.1109/ECC.2014.6862335

Ranneberg, M. (2013, Sep). Sensor setups for state and wind estimation for airborne wind energy converters. Retrieved from arXiv:1309.1029

van der Vlugt, R., Peschel, J., & Schmehl, R. (2013). Design and experimental characterization of a pumping kite power system. In U. Ahrens, M. Diehl, & R. Schmehl (Eds.), Airborne wind energy (p. 403-425). Springer Berlin Heidelberg. Retrieved from http://dx.doi.org/10.1007/978-3-642-39965-7_23 doi: 10.1007/978-3-642-39965-7_23

Williams, P., Lansdorp, B., & Ockels, W. (2008, January). Optimal crosswind towing and power generation with tethered kites. AIAA Journal of Guidance, Control and Dynamics, 31(1), 81-93.

Zgraggen, A., Fagiano, L., & Morari, M. (2013, Oct). Retraction phase control using regularized velocity angle. ETH Zurich, Tech. Rep.,(TR_ZFM_15103). Retrieved from http://control.ee.ethz.ch/~{}aldoz/docs/techRep/ZFM_TR_15103.pdf

Zgraggen, A., Fagiano, L., & Morari, M. (2014). Real-time optimization and adaptation of the crosswind flight of tethered wings for airborne wind energy. Control Systems Technology, IEEE Transactions on, PP(99), 1-1. doi: 10.1109/TCST.2014.2332537

Appendix A. Optimisation of power generation cycles

In order to get a principal idea of how to perform transfer and return phases w.r.t. winch speed efficiently with our setup of constant glide ratio, we have applied a numerical optimisation to our simple model, which will be briefly presented in the following.

The quantity to optimise is the average power of a complete cycle of duration $T$

$$\bar{P} = \frac{1}{T} \int_{0}^{T} dt \dot{l}(t) F(t) \quad (A1)$$

using the expression for the tether force

$$F(t) = \frac{\rho}{2} C_R A_v^2 (t) \quad (A2)$$

with $\rho$ the air density, $A$ the projected kite area and $C_R$ the force coefficient, one obtains for the average power

$$\bar{P} = \frac{\rho C_R A}{2T} \int_{0}^{T} dt \dot{l}(t) v_w^2 (t) \quad (A3)$$

As it is reasonable to vary $\psi(t)$ and $\dot{l}(t)$, these are considered as input functions. As consequence, a functional $\bar{P} : \{\psi(t), \dot{l}(t), T\} \mapsto \bar{P}[\psi(t), \dot{l}(t), T]$ can be defined by solving the equations of motion $(3), (4), (6)$ and using $(A3)$. The optimisation problem can now be stated:

$$\text{maximise } \bar{P}[\psi(t), \dot{l}(t), T] \quad (A4)$$

by variation of $\{\psi(t), \dot{l}(t), T\}$ subject to the following constraints:

- Periodic boundaries: $l(T) = l(0)$ and $\psi(T) = \psi(0)$. Further, the initial condition for $\dot{\varphi}(0)$ must also be periodic $\varphi(0) = \varphi(T)$.
- There is only one reel-in and one reel-out phase, i.e. $\dot{l}(t)$ has only two roots for $0 < t < T$. In addition, the operational range is given, $l_{\min} \leq l \leq l_{\max}$ and these range limits must be reached.
- $a_{\text{limit.in}} \leq \dot{l}(t) \leq a_{\text{limit.out}}$
- $0 \leq \psi(t) \leq \psi_{\max}$. The $\varphi$ motion is free and not subject to periodic boundary condition through cycles. These condition lead to circular orbits (also below the surface) instead of figure-eights during the power phase. The $\psi_{\max}$ value can directly by regarded as force control as it determines the wind window position as discussed in detail in (Erhard & Strauch, 2013a). This rough approximation of figure eights significantly simplifies the optimisation algorithm and is adequate for a first approach. Future work should include proper figure eights and consider losses due to curve flights (Costello et al., 2013).

It should be stated, that the simulation has been performed for $v_w = 10$ m/s. Winch speeds for other wind speeds can be easily obtained by scaling the results with...
Figure A1. Optimisation results for $v_w = 10 \text{ m/s}$. The plots comprise one cycle and single phases are separated by vertical dashed lines. The power (lower subplot) is normalised to the Loyd’s limit $P_{\text{Loyd}}$, compare (43), and the average value is given by $\bar{P} = 0.28P_{\text{Loyd}}$.

$v_w$ accordingly. Simulation curves are shown in Fig. A1. As a result for the transfer and return phase, a similarity of winch speed $v_{\text{winch}}$ and angle $\vartheta$ can be recognised, which motivates the implementation of a winch controller as linear functions $v_{\text{winch}} = v_{\text{winch}}(\vartheta)$, compare Fig. 16. Note, that in the first part of the transfer phase the optimisation yields $v_{\text{winch}} > 0$ and thus suggests utilisation of the high tether forces for energy production. Winching in starts for elevation angles higher than $\vartheta_0 \approx 1.05 \text{ rad}$. Finally, we would like to remark, that the results in the appendix have to be considered as a first step in tackling the cycle optimisation issue. They should be regarded as illustration of the basic idea rather than a complete treatment of the problem. Hence, further extended optimisation results may lead to major modifications of the winch control strategy presented in this paper.