Phonon thermal Hall effect in strontium titanate

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It has been known for more than a decade that phonons can produce an off-diagonal thermal conductivity in presence of magnetic field. Recent studies of thermal Hall conductivity, $\kappa_{xy}$, in a variety of contexts, however, have assumed a negligibly small phonon contribution. We present a study of $\kappa_{xy}$ in quantum paraelectric SrTiO$_3$, which is a non-magnetic insulator and find that its peak value exceeds what has been reported in any other insulator, including those in which the signal has been qualified as ‘giant’. Remarkably, $\kappa_{xy}(T)$ and $\kappa(T)$ peak at the same temperature and the former decreases faster than the latter at both sides of the peak. Interestingly, in the case of La$_3$CuO$_4$ and $\alpha$-RuCl$_3$, $\kappa_{xy}(T)$ and $\kappa(T)$ peak also at the same temperature. We also studied KTaO$_3$ and found a small signal, indicating that a sizable $\kappa_{xy}(T)$ is not a generic feature of quantum paraelectrics. Combined to other observations, this points to a crucial role played by antiferrodistortive domains in generating $\kappa_{xy}$ of this solid.

In most insulators, thermal conductivity can be understood with reasonable accuracy by picturing phonons as carriers of heat scattered either by other phonons or by defects and boundaries \footnote{[32,33]}. An impressive agreement between experimental data near room temperature and \textit{ab initio} solutions of the Peierls-Boltzmann equation has been achieved in the last few years \footnote{[1,2]}. Since phonons are neutral quasi-particles lacking magnetic moment, one may assume that their path is not affected by a magnetic field and therefore, in contrast to magnons and electrons, they cannot give rise to a transverse response. However, experiments carried out more than a decade ago \footnote{[3]} showed that there is a finite measurable phonon Hall effect. The appearance of a finite transverse thermal gradient upon application of a longitudinal heat current, implied a finite $\kappa_{xy}$ in Tb$_3$Ga$_3$O$_{12}$, a paramagnetic insulator \footnote{[4,5,11]}. The experimental observation motivated numerous theoretical studies \footnote{[5,11] invoking a variety of possible sources of phonon Hall effect including spin-phonon coupling, phonon Berry curvature, skew scattering, or simply ionic bonding \footnote{[11]}} that $\kappa_{xy}$ can vary from one sample to another, the sample dependence is reminiscent of what was reported in LaCuO$_4$ \footnote{[21]}. However, at $T = 15T$, $\kappa_{xy}$ remains 400 times smaller than $\kappa_{xx}$ and calling this a ‘giant’ thermal Hall effect \footnote{[17,21]} does not appear as an illuminating choice.

The study of three different crystals showed that while the peak $\kappa_{xy}$ can vary from one sample to another, the overall temperature dependence remains the same. This sample dependence is reminiscent of what was reported in $\alpha$-RuCl$_3$ \footnote{[18-20]}. Comparing the temperature dependence of $\kappa$ and $\kappa_{xy}$ in SrTiO$_3$, it becomes clear that they both peak at the same temperature, but the decrease in $\kappa_{xy}$ is sharper both below and above the peak temperature. We note that in both $\alpha$-RuCl$_3$ \footnote{[20]} and La$_2$CuO$_4$ \footnote{[21]} $\kappa$ and $\kappa_{xy}$ peak at the same temperature. We also studied KTaO$_3$, another quantum paraelectric with no antiferrodistortive transition and found that its $\kappa_{xy}$ is much smaller. This observation indicates a crucial role played by polar domain walls of SrTiO$_3$ in generating $\kappa_{xy}$. This hypothesis is strengthened by detailed study of how the amplitude of the signal in the same sample is affected by its thermal history after trips across the 105 K structural transition.

A member of the perovskite ABO$_3$ family, SrTiO$_3$ avoids a ferroelectric instability thanks to the quantum fluctuations \footnote{[22]}. Upon the introduction of a tiny amount of mobile electrons, this wide-gap insulator turns to a dilute metal \footnote{[23]} subject to a superconducting instability \footnote{[24,25]} and displaying non-trivial charge transport at room temperature \footnote{[26]}. Its cubic crystal structure at room temperature is lost below 105K \footnote{[28]}. This structural transition is antiferrodistortive (AFD): neighboring TiO$_6$ octahedra tilt clockwise and anti-clockwise. As a

During the past few years, thermal Hall effect was studied in magnetic insulators \footnote{[27-29]} spin-liquid candidates \footnote{[30-31]} and multi-ferroics \footnote{[32]}. These studies of $\kappa_{xy}$ mostly assumed a marginal contribution by phonons and the detected signal was often (but not always \footnote{[16]}) attributed to magnetic excitations. More recently, $\kappa_{xy}$ has been measured in the Kitaev spin-liquid candidate $\alpha$-RuCl$_3$ \footnote{[18-20]} and in cuprates \footnote{[21]}. In both cases, the observed signal was assumed to exceed significantly what could be purely a phononic contribution.

In this paper, we present a study of thermal Hall effect in SrTiO$_3$ crystals, a quantum paraelectric \footnote{[22]} with a variety of remarkable properties \footnote{[23]}. We found a sizable $\kappa_{xy}$ in this solid. Since phonons are the unique heat carriers in this non-magnetic band insulator, it is hard to see how carriers other than phonons can cause the observed $\kappa_{xy}$. The magnitude of the observed signal is twice larger than what was reported in LaCuO$_4$ \footnote{[21]}. However, at $T = 15T$, $\kappa_{xy}$ remains 400 times smaller than $\kappa_{xx}$ and calling this a ‘giant’ thermal Hall effect \footnote{[17,21]} does not appear as an illuminating choice.

The study of three different crystals showed that while the peak $\kappa_{xy}$ can vary from one sample to another, the overall temperature dependence remains the same. This sample dependence is reminiscent of what was reported in $\alpha$-RuCl$_3$ \footnote{[18-20]}. Comparing the temperature dependence of $\kappa$ and $\kappa_{xy}$ in SrTiO$_3$, it becomes clear that they both peak at the same temperature, but the decrease in $\kappa_{xy}$ is sharper both below and above the peak temperature. We note that in both $\alpha$-RuCl$_3$ \footnote{[20]} and La$_2$CuO$_4$ \footnote{[21]} $\kappa$ and $\kappa_{xy}$ peak at the same temperature. We also studied KTaO$_3$, another quantum paraelectric with no antiferrodistortive transition and found that its $\kappa_{xy}$ is much smaller. This observation indicates a crucial role played by polar domain walls of SrTiO$_3$ in generating $\kappa_{xy}$. This hypothesis is strengthened by detailed study of how the amplitude of the signal in the same sample is affected by its thermal history after trips across the 105 K structural transition.
Quantifying thermal Hall effect in SrTiO$_3$:

(a) Set-up for measuring longitudinal and transverse thermal differences ($\Delta T_x = T_1 - T_2$, $\Delta T_y = T_3 - T_2$) generated by a longitudinal thermal current. (b) A photograph of the sample and the set-up. The heater and the heat sink were connected to two sides of the sample and at the same level. Three thermometers were mounted near the middle of the sample. (c) Field dependence of $\Delta T_y$ and $\Delta T_x$ at $T = 24\,K$ (labeled as sample temperature $T$ below), $\Delta T_y$ has been shifted vertically to cancel an unavoidable misalignment offset. Note that $\Delta T_x$ is even dominant and $\Delta T_y$ is odd dominant in magnetic field. (d) Extracted thermal Hall conductivity $\kappa_{xy}$ and field-induced change in thermal conductivity $\Delta \kappa = \kappa(\mu_0 H) - \kappa(0)$ as function of field. The latter signal is noisier, because the measurement of $\Delta T_x$ has not been done in differential mode.

FIG. 1. 

consequence, the tiny tetragonal distortion generates significant anisotropy in charge transport [29]. In absence of strain, three possible domains can be present [29]. The polar walls between these micrometric domains have been a subject of numerous studies [30–32].

Previous studies of heat transport in this solid [33–35] uncovered two remarkable features. In a pioneer study, Steigmeier [34] determined the temperature dependence of the thermal conductivity (which peaks to 30 W/K-m at $T \approx 20\,K$), and found that below the maximum, it depends on the applied electric field. More recently, Martelli and co-workers [35] found that thermal conductivity decreases faster than $T^3$ below the peak. Such a behavior has only been observed in a handful of solids and attributed to the Poiseuille flow of phonons [36], triggered by abundant normal (i.e. momentum-conserving) collisions among phonons. Soft phonons, either those associated with the antiferrodistortive transition [37,38], or modes corresponding to the aborted ferroelectricity [39], are suspected to drive these unusual features of heat transport [35]. This may also be the case of the observation reported in the present paper.

We measured the thermal Hall effect by using a one-heater-three-thermometers method as shown in Fig. 1a,b (See the Supplemental Material [39], for more details). Fig. 1c-d shows the data at 24 K. As seen in Fig. 1c, $\Delta T_x$ is an even and $\Delta T_y$ is an odd function of magnetic field. $\Delta T_y$, shifted vertically to zero by a tiny quantity due to unavoidable misalignment between $T_2$ and $T_3$, has opposite signs for positive and negative magnetic fields, which implies $\kappa_{xy}(\mu_0 H) = -\kappa_{xy}(-\mu_0 H)$, as one would expect for the off-diagonal component of the conductivity tensor. On the other hand, $\Delta T_x$ is finite at zero field and increases symmetrically with applied magnetic field implying $\kappa(\mu_0 H) = \kappa(-\mu_0 H)$.

The field dependence of thermal Hall conductivity $\kappa_{xy}$ and the field-induced change in thermal conductivity $\Delta \kappa = \kappa(\mu_0 H) - \kappa(0)$ at $T = 24\,K$ are plotted in Fig. 1. The magnitude of $\kappa_{xy}$ attains -80 mW/K-m, twice larger than the maximum observed in cuprates [21]. As seen in the figure, however, this is four times smaller than the field-induced change in longitudinal thermal conductivity, $\Delta \kappa$, itself about one percent of total signal. We note that Jin et al. [40] have recently reported on a similar field-induced decrease in the lattice thermal conductivity of another band insulator.

The temperature dependence of $\kappa_{xy}$ in SrTiO$_3$ is shown in Fig. 2a. As seen in the figure, in a magnetic field of 12 T, it peaks to -0.08 W/K-m at $\approx 20\,K$ and falls rapidly at both sides of this peak. Panel b presents a comparison of the temperature dependence of $\kappa_{xy}$ and $\kappa$, with the former multiplied by a factor $\alpha = -450$. Both peak at...
the same temperature, but the decrease in $\kappa_{xy}$ is faster on either sides of the maximum. As found previously [35], $\kappa$ in SrTiO$_3$ follows $T^3$ (with $\beta$ slightly larger than 3) below the peak temperature. $\kappa_{xy}$ decreases even more sharply in this regime, and it almost follows a $T^4$ temperature dependence. With warming, the drop in the transverse signal is slightly sharper than the drop in the longitudinal one.

A phenomenological picture of $\kappa$ equates it with a product of specific heat, $C$, velocity, $v$, and mean-free-path, $\ell$. This should be summed over different phonon modes, indexed $\lambda$:

$$\kappa = \frac{1}{\nu} \sum_{\lambda} C^\lambda v^\lambda \ell^\lambda$$

(1)

Here, $\nu$ is a dimension-dependent normalisation factor. Usually, the variation of sound velocity with temperature is negligible. Indeed, the experimentally measured elastic moduli of SrTiO$_3$ [41] (and therefore its sound velocity) changes by less than a few percent in our temperature range of interest. The thermal evolution of the mean-free-path and the specific heat, on the other hand, is strong and opposite to each other (see Fig. 2). Therefore, in an insulator $\kappa$ peaks at a temperature where the global phonon trajectory (i.e. phonon population times phonon mean-free-path) is maximal. This temperature has a physical significance. Thermal conductivity is most vulnerable to the introduction of point defects near this peak temperature [41]. Our observation that $\kappa_{xy}$ peaks at this very temperature is a source of information on what causes the transverse signal. Phenomenologically, a finite $\kappa_{xy}$ implies either an off-diagonal (temperature-independent) velocity or an off-diagonal (temperature-dependent) mean-free-path. Therefore:

$$\kappa_{xy} = \frac{1}{\nu} \sum_{\lambda} C^\lambda (v^\lambda \ell_{xy}^\lambda + v^\lambda \ell^\lambda_{xy})$$

(2)

Presumably, $\ell_{xy}$ and $v_{xy}$ are both much smaller than their longitudinal counterparts as sketched in Fig. 2 and Fig. 3. Therefore, the fact that $\kappa_{xy}$ peaks at the same temperature, but the decrease in $\kappa_{xy}$ is faster on either sides of the maximum. As found previously [35], $\kappa$ in SrTiO$_3$ follows $T^3$ (with $\beta$ slightly larger than 3).
temperature but decreases faster may be ascribed to one of the right-hand terms of equation 2 or their combination.

The magnitude of \( \kappa_{xy} \) in strontium titanate is two orders of magnitude larger than what was reported for TbsGaO\(_3\)\(_{12}\) \(3\). This raises a natural question: can proximity to a ferroelectric (FE) quantum criticality \(42\) play a role in generating a large phonon thermal Hall effect? In order to answer this question, we investigated heat transport in KTaO\(_3\). This insulator, like SrTiO\(_3\), is close to a FE transition, but its low-temperature electric permittivity is five times smaller \(43\).

In agreement with what was reported before for KTaO\(_3\) \(44\) and SrTiO\(_3\) \(45\) \(46\), we found that the amplitude of the peak in longitudinal thermal conductivity is comparable (30-35 W/K-m) in the two perovskites (see Fig. 3). On the other hand, the amplitude of the thermal Hall conductivity is very different. In KTaO\(_3\), \( \kappa_{xy} \) is more than one order of magnitude smaller than in SrTiO\(_3\) (Fig. 3). Let us note that even in the case of longitudinal thermal conductivity, there are remarkable differences between these two solids. Around T\(\approx\)5K, thermal conductivity is sharply decreasing (displaying a faster than cubic temperature dependence) in SrTiO\(_3\) but is increasing (presenting an additional bump, as seen in the inset of Fig. 3) in KTaO\(_3\). In other words, the consequences of anharmonicity for longitudinal heat transport is qualitatively different in these two apparently similar solids. Structurally, the most notable difference is the absence of the AFD transition in cubic KTaO\(_3\) \(45\), in contrast to its presence in SrTiO\(_3\). This is our first evidence that this peculiar structural transition plays a role in setting the amplitude of \( \kappa_{xy} \).

The correlation between the position of peaks in longitudinal and transverse response in SrTiO\(_3\) and KTaO\(_3\) led us to put under scrutiny the published data in two other insulators. As seen in Fig. 3, according to the available data, there is a similar correlation between \( \kappa_{xy}(T) \) and \( \kappa(T) \) in both La\(_2\)CuO\(_4\) \(21\) \(46\) and in \( \alpha\)-RuCl\(_3\) \(20\). In all cases, \( \kappa_{xy} \) and \( \kappa \) peak at almost the same temperature and the decrease in \( \kappa_{xy} \) is sharper (or in one case almost equal) to the decrease in \( \kappa \). We notice that this correlation, which was not reported before, indicates a major role played by the principal heat carriers in setting the transverse response.

Our additional measurements build up the case for a prominent role played by AFD domains. The results are shown in Fig. 4. First of all, we studied three different SrTiO\(_3\) samples, provided by two different companies. As shown in Fig. 4-a-b, all three samples show a sizable \( \kappa_{xy} \), but different amplitudes. Two samples in which the magnitude of \( \kappa \) is almost the same (panel a), display...
a threefold difference in their peak of $\kappa_{xy}$ (panel b, d). As seen in panel (c), the field-induced decrease in $\kappa$ is roughly the same in the two samples.

In a second set of measurements, we repeated our measurements of $\kappa_{xy}$ on the same sample after warming it above $T_{AFD} = 105$ K and cooling it back again. As seen in Fig. 4e-f (for more details, see the Supplemental Materials [33]), warming above the AFD transition temperature can change the magnitude of $\kappa_{xy}$ ($T=24$ K) in the same sample.

Buckley et al. [30] observed needle-like structural domains below 105 K in SrTiO$_3$ and “found almost no memory of the domain patterns under repeated heating and cooling through the transition point” [30]. The typical size of the observed domains was a micron, comparable to the apparent phonon mean-free-path extracted from longitudinal thermal conductivity and specific heat [35]. An intimate link between domain configuration and the amplitude of $\kappa_{xy}$ would explain why the amplitude of $\kappa_{xy}$ can be different after thermal cycling above $T_{AFD}$, wiping out the previous configuration of domains. Obviously, the sample dependence of the signal and its virtual absence in KTaO$_3$ also find natural explanations.

Theoretical scenarios for phonon thermal Hall effect [5][11] either invoke skew scattering of heat carriers or let the magnetic field generate a transverse velocity. Let us have a look to our results in either of these schemes. One may be tempted to attribute the observed $\kappa_{xy}$ to skew scattering of phonons by the AFD domain walls, which according to a number of experiments [31][32] are polar. However, the skew-scattering picture would have a hard time to explain the disconnection between the field-induced decrease in $\kappa$ and the finite $\kappa_{xy}$ (Fig. 4, d). Alternatively, one may point to the fact that the slight tetragonal distortion leads to quasi-degenerate acoustic phonon modes and it has been suggested [47] that the acoustic phonons hybridize with the transverse optical phonons. Thanks to these features, the magnetic field may become able to couple to titanium-oxygen ionic bonds [11] and generate a transverse velocity. Presumably, this should crucially depend on the relative orientation of the magnetic field and each of the three tetragonal domains; hence a dependence on precise domain configuration. Note that Ab initio theoretical calculations find imaginary frequencies [45] for strontium titanate. Two recent theoretical studies succeeded in finding real phonon frequencies [49][50]. However, the focus of both was the cubic state and the phonon spectrum below the AFD transition remains unknown. Future theoretical studies may fill this void. Future experiments may use strain to control the configuration of domains.

In summary, phonons in SrTiO$_3$ can generate a $\kappa_{xy}$ larger than what was reported in any other insulator. This is not generic to all quantum paraelectric solids and appear to be intimately linked to the occurrence of antiferrodistortive (AFD) transition in SrTiO$_3$. We find that not only in SrTiO$_3$, but also in other insulators $\kappa_{xy}$ and $\kappa$ peak at the same temperature. The observation appears as a clue to identify carriers and collisions which generate the transverse signal. In the case of SrTiO$_3$, two experimental observations point to the role of tetragonal domains in generating the signal.

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Supplemental Material for “Phonon thermal Hall effect in strontium titanate”

S1. SAMPLES AND MEASUREMENT TECHNIQUE

Single crystals of SrTiO\textsubscript{3} and KTaO\textsubscript{3} were obtained commercially. The dimensions of the SrTiO\textsubscript{3} samples (#1 and #2 from SurfaceNet, #3 from CrystTech) were 5mm × 5mm × 0.5mm and their orientation along (100). The KTaO\textsubscript{3} sample had a dimension of 10mm × 6.5mm × 0.5mm. Three thermometers (Cernox-1030) were used to simultaneously measure the longitudinal and transverse thermal gradient. A one-heater-three-thermometers technique was employed. The heat flow was applied along the x-axis and the magnetic field oriented along the z-axis. Then the longitudinal (∆T\textsubscript{x} = T\textsubscript{1}-T\textsubscript{2}) and the transverse (∆T\textsubscript{y} = T\textsubscript{3}-T\textsubscript{2}) temperature difference were detected simultaneously using three resistive chips (Cernox-1030), measuring T\textsubscript{1}, T\textsubscript{2} and T\textsubscript{3}. The longitudinal (κ) and the transverse (κ\textsubscript{xy}) thermal conductivity were obtained using:

\[
\kappa = \frac{Q}{(\Delta T_x/l)(w \cdot t)}, \quad (S1)
\]

\[
\kappa_{xy} = \kappa \cdot \frac{\Delta T_y/w}{\Delta T_x/l}. \quad (S2)
\]

Here, Q, l, w, t are the heat power, length between T\textsubscript{1} and T\textsubscript{2}, the sample width and thickness respectively. All the measurements were performed in a PPMS (Physical Property Measurement System) within a stable high-vacuum sample chamber. The thermal gradient in the sample was produced through a 1kΩ chip resistor aligned by a DC current source (Keithley 6220). The DC voltage on the heater was measured through a Keithley 2000. Three lock-in amplifiers (SR830) were used to simultaneously measure the longitudinal and transverse thermal gradient. A one-heater-three-thermometers technique was employed. The heat flow was applied along the x-axis and the magnetic field oriented along the z-axis. Then the longitudinal (∆T\textsubscript{x} = T\textsubscript{1}-T\textsubscript{2}) and the transverse (∆T\textsubscript{y} = T\textsubscript{3}-T\textsubscript{2}) temperature difference were detected simultaneously using three resistive chips (Cernox-1030), measuring T\textsubscript{1}, T\textsubscript{2} and T\textsubscript{3}. The longitudinal (κ) and the transverse (κ\textsubscript{xy}) thermal conductivity were obtained using:

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S2. SUPPRESSION OF THE BACKGROUND SIGNAL

As seen in Fig. [S1] there is a small change in the transverse thermal gradient (∆T\textsubscript{y}) induced by the magnetic field even in absence of an applied heat. This background signal has two origins. First of all, the resistive chips have a finite magnetoresistance. This can be reduced by choosing two very similar thermometers. The other source of this background is the misalignment between transverse electrodes. Both effects (magneto-resistance and misalignment) are expected to be symmetric with the magnetic field, in contrast to the intrinsic thermal Hall effect signal. As illustrated in Fig. [S1] (black circles), the measured signal is dominantly odd. To extract the purely asymmetric component of the raw data, we anti-symmetrized it: (∆T\textsubscript{y} - ∆T\textsubscript{y} (red star) and without (black circle) the heat power at 24K. (b) ∆T\textsubscript{y} (left axis) and κ\textsubscript{xy} (right axis) signal after the asymmetrization.

FIG. S1. (a) ∆T\textsubscript{y} as a function of the magnetic field with (red star) and without (black circle) the heat power at 24K. (b) ∆T\textsubscript{y} (left axis) and κ\textsubscript{xy} (right axis) signal after the asymmetrization.

S3. DEPENDENCE ON HEAT POWER

Combining equations S1 and S2, we get:

\[
\kappa_{xy} = \kappa^2 \cdot \frac{\Delta T_y \cdot t}{Q}. \quad (S3)
\]

\[
Q = \kappa^2 \cdot \frac{\Delta T_y \cdot t}{\kappa_{xy}}. \quad (S4)
\]

According to the formula S4, the transverse temperature difference ∆T\textsubscript{y} is expected to be linear with the heat power. Fig. [S2] presents data obtained with three different heat powers (Q=20.8 mW, 33.3 mW and 46.0 mW) applied to the same sample kept at the temperature of 24K. As expected, κ\textsubscript{xy} is the same and ∆T\textsubscript{y} is a linear function of the heat power (Fig. [S2]).
As discussed in the main text, we observed a very small \( \kappa_{xy} \) signal in KTaO\(_3\), about 30 times smaller than in SrTiO\(_3\). \( \kappa_{xy} \) of KTaO\(_3\) has a peak about 3 mW/K-m, which is 1/12000 of \( \kappa \). We swept the field from +12T to -12T at different temperatures. In Fig. S3a-d, four typical curves are shown. After extracting the \( \kappa_{xy} \) value at 12T, we determined the temperature dependence of \( \kappa_{xy} \), shown in Fig. S3e. We note that \( \kappa_{xy} \) of KTaO\(_3\) is positive at its peak value, in contrast to SrTiO\(_3\) and La\(_2\)CuO\(_4\). Error bars in Fig. S3e are large. This is because, we needed to apply a quite large heat power (50-80mW above 20K) to detect a \( \kappa_{xy} \) of such a small magnitude in KTaO\(_3\). This large heat power will generate a large thermal gradient (1K/mm) and difference (3K be-
FIG. S5. The field dependence of the thermal conductivity in two SrTiO$_3$ samples, $\Delta\kappa = \kappa(\mu_0 H) - \kappa(0)$. 

S5. THERMAL HISTORY MEASUREMENTS

We reported four sequences of thermal measurement in the main manuscript (Fig. 4f). Each sequence of measurement was separated from the previous one by a warming process to room temperature and staying in air for several days. What we report here is a second set of thermal history measurements, in which each thermal cycle has two different warming temperature points, 300K and 90K, above and below the AFD transition temperature (105K) respectively. After an initial measurement of $\kappa_{xy}$ by sweeping field from +12T to -12T at 24K (labelled as thermal circling sequence 0), two other field sweeps and measurements were performed at 24K. In each case the sample was warmed to 300K and to 90K and was kept in high-vacuum for hours (see Fig. S4a). Such thermal circles were repeated four times and the results are shown in Fig. S4b-c. It’s obvious that warming to 300K can significantly change the $\kappa_{xy}(T = 24K)$, but not warming to 90K. This observation points out to the role of AFD domains in setting the magnitude of $\kappa_{xy}$. Interestingly, after three cycles, $\kappa_{xy}$ is gradually stabilized and no more affected by the thermal cycling.

S6. FIELD DEPENDENT THERMAL CONDUCTIVITY IN STO

We also studied the field dependence of the thermal conductivity in two SrTiO$_3$ samples. The results, shown in Fig. S5, are roughly similar in the two samples. In the $20K < T < 30K$ temperature range, a magnetic field of 12 T induces a change ($\Delta\kappa$), which is about one percent of total $\kappa$. The magnitude of $\Delta\kappa$ is larger than the off-diagonal response.