Baryogenesis through Collapsing String Loops in Gauged Baryon and Lepton Models

Henry Lew\textsuperscript{(a)} and Antonio Riotto\textsuperscript{(b)(c)}

\textsuperscript{(a)} Physics Department, Purdue University, West Lafayette, IN 47907-1396, U.S.A.
\textsuperscript{(b)} International School for Advanced Studies, SISSA, via Beirut 2-4, I-34014 Trieste, Italy.
\textsuperscript{(c)} Istituto Nazionale di Fisica Nucleare, Sezione di Padova, I-35100 Padua, Italy.

Abstract

A scenario for the generation of the baryon asymmetry in the early Universe is proposed in which cosmic string loops, predicted by theories where the baryon and/or lepton numbers are gauged symmetries, collapse during the friction dominated period of string evolution. This provides a mechanism for the departure from thermal equilibrium necessary to have a non-vanishing baryon asymmetry. Examples of models are given where this idea can be implemented. In particular, the model with the gauge symmetry $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$ has the interesting feature where sphaleron processes do not violate the baryon and lepton numbers so that no wash out of any initial baryon asymmetry occurs at the electroweak scale.

\textsuperscript{1}email: lew@purdd.hepnet@LBL.Gov
\textsuperscript{2}email: riotto@tsmi19.sissa.it
1. Introduction

The basic requirements for the generation of the baryon asymmetry of the Universe (BAU) \[1\] are: (1) baryon number violation, (2) \(C\) and \(CP\) violation and (3) a departure from thermal equilibrium. In the past, models of Grand Unified Theories (GUTs) in conjunction with the standard hot big model of cosmology were found to have the above necessary ingredients to explain the observed matter-antimatter asymmetry of the Universe \[2\]. However typical GUTs require relatively high symmetry breaking scales (usually not too far away from the Planck scale) making direct testing of such theories inaccessible to currently known experimental techniques. It would be interesting to find particle physics models which can realise the baryogenesis scenario and can at the same time be tested experimentally in the near future. Recent attempts to do this include baryogenesis at the electroweak scale using just the standard model (SM) of particle physics \[3\]. In this paper we will explore an alternative mechanism for generating the BAU and give examples of particle physics models which can implement such a scenario.

This mechanism of generating the BAU is based on the collapse of cosmic string loops. A variant of this mechanism has been studied in Ref. \[4\]. This will be described in section 2. Then in the following sections we will give examples of particle physics models which can realise this mechanism. The first example will be a model with gauged baryon and lepton numbers. The crucial aspect of this model is that since the baryon and lepton symmetries are anomaly free then the sphaleron processes will not violate these symmetries and hence any initial asymmetry produced will not be washed out. This is in contrast with many of the GUT scenarios for the generation of the BAU where the wash out of the initial asymmetry is a common problem. The
second example we will consider is a model where the \((B - L)\) symmetry is gauged. The right-handed neutrinos in this model are responsible for establishing an initial lepton number asymmetry. The lepton number asymmetry is then converted to a baryon number \((B)\) asymmetry via the sphaleron. In both of these models the ultimate source of the asymmetry comes from particle production as a result of the collapse of cosmic string loops.

We conclude this section by commenting on the motivation for the particular particle physics models we have chosen to discuss. We feel that these models are interesting simply from the particle physics point of view, let alone their implications for cosmology. Firstly, these models deal with physics beyond the SM at scales well below the GUT scale. They are simple in the sense that they rely only on the concepts central to the theoretical framework of the SM. They are (i) gauge symmetry and (ii) spontaneous symmetry breaking. The success enjoyed by the SM in describing nature up to energies hitherto explored by experiment has promoted these two concepts to a status of fundamental importance. So much so, that in the case of symmetries, any symmetry which is not gauged is viewed with suspicion by some. For example, even though global symmetries (continuous or discrete) appear in many particle physics models, including the SM, they are considered to be less than fundamental or at best “accidental”. Since the origin of symmetries is still an open question, such a theoretical prejudice might prove to be wrong in the future. Nevertheless, in this paper we will pursue this theoretical prejudice, perhaps to an extreme, by gauging all or some combination of the known classical global symmetries of the SM.
2. Collapsing string loops mechanism

In this section we will describe a mechanism by which the collapse of cosmic string loops can provide a source of baryon or lepton number. The analysis will have some similarities to that given in Ref. [4] but because of the different scales involved there will be some important differences.

Consider the Abelian Higgs model (which is the simplest example of a model in which cosmic strings can form as a result spontaneous symmetry breaking [5]) where a complex Higgs field develops a vacuum expectation value (VEV) \( \langle H \rangle = \sigma e^{i\theta} \). The arbitrary phase, \( \theta = \theta(x) \), can vary in different regions of space. For \( H \) to be single valued \( \theta \) must change by an integer multiple of \( 2\pi \) around a closed loop. When the loop is shrunk to a point \( \theta \) becomes undefined so that there exists a point where \( \langle H \rangle = 0 \), i.e., thin tubes of false vacuum get trapped somewhere inside the loop. Such tubes of false vacuum or “strings” must either be closed or infinite in length so that the closed loop cannot be contracted to a point without encountering the tubes of false vacuum. Hence these strings are topologically stable.

Once the cosmic strings have been produced their evolution in time is demarcated by two periods. Initially the strings will be moving in a dense medium so that their motion is strongly damped and this is usually referred to as the friction dominated period. After this time the friction becomes negligible and the strings can move at relativistic speeds. The evolution of strings is then mainly determined by the expansion of the Universe, gravitational radiation of string loops and the intercommuting of intersecting strings. The friction dominated period lasts from the time of the phase transition, \( t_c \), until the time \( t_* \approx (G\mu)^{-1}t_c \), where \( G \) is Newton’s constant and \( \mu \approx \sigma^2 \) is the mass per unit length of the string. In terms of temperatures,
\[ T_s \simeq \frac{\sigma}{M_p} T_c, \] where \( M_p \) is the Planck mass and \( T_c \sim \sigma \) is the critical temperature of string formation. For instance, if the phase transition takes place at scales less than \( \sigma \sim 10^{10} \) GeV then the friction dominated period will not end before the temperature drops below about 10 GeV. Since in this paper we are interested in relatively low symmetry breaking scales, only the friction dominated period of string evolution will be relevant. The situation depicted here is therefore different from that described in Ref. [4] where the breaking scale is very close to the GUT scale and the friction dominated period lasts for a much shorter time. In such a case the typical scale is the coherence length for a long string network and the fact that its expansion is catching up with the Hubble radius must be used.

Also note that since the gravitational effects of cosmic strings depend on the dimensionless quantity \( G\mu \), and \( \sigma \ll M_p \), then we expect that the usual cosmological effects associated with strings (e.g. seeding for structure formation) will be negligible.

The basic idea for a nonzero \( B \) and/or \( L \) produced from strings is that when strings collapse to a size comparable to its width the microphysical forces of the string become important. As a result the energy of the string will be released in the form of particle production. We will assume for simplicity that all the energy released from the collapsed string goes into the quanta of the massive Higgs field. The energy of \( N_Q \) quanta of mass \( M_h \simeq \lambda^{\frac{1}{2}}\sigma \) produced from the collapse of a single string loop is \( N_Q\lambda^{\frac{1}{2}}\sigma \), where \( \lambda \) is the quartic self-coupling of the Higgs field. The energy of the string loop is given by \( \mu \beta R \) where \( \beta \sim 2\pi \) and \( R \) is the loop radius. Equating the two expressions for the energy for \( R \sim w \) results in \( N_Q \simeq \beta \lambda^{-1} \), where the thickness of the string \( w \simeq \lambda^{-\frac{1}{2}}\sigma^{-1} \) was used.

We will now proceed to estimate the number of loops contributing to
particle production. The size distribution of loops, for loops of size $l$, is

$$dn \sim \xi_c^{-4+\alpha} l^{-\alpha} dl,$$

(1)

where $\xi_c \simeq (\lambda \sigma)^{-1}$ is the correlation length of the Higgs field and $\alpha$ was found from Monte Carlo simulations of string systems to take on values from 2 to 2.5 [3]. Integrating Eq. (1) gives

$$n(t_w) \simeq \frac{\xi_c^{-4+\alpha}}{1-\alpha} \left\{ l(t_c)^{(1-\alpha)} - l(t_w)^{(1-\alpha)} \right\},$$

(2)

where $t_w$ is the time when $l(t_w) = w$ and $l(t_c)$ is expected to be the largest sized loop that can collapse to $l(t_w)$ in time.

To find the explicit time dependence of the loop size, consider the motion of the string in the friction dominated epoch. The typical damping time on the motion of the string is

$$t_d \sim \frac{\Delta P/l}{F/l} \simeq \frac{\mu}{n} \simeq \frac{\sigma^2}{k_n T^3},$$

(3)

where the change in momentum per unit length of the string is $\Delta P/l \sim \mu v$ and the force per unit length on the string is $F/l \sim n v$. $v$ is the speed of the string relative to the surrounding medium and $n = k_n T^3$ is the number density with

$$k_n = \frac{\zeta(3)}{\pi^2} \sum_{i=bosons} g_i \left( \frac{T_i}{T} \right)^3 + \frac{3}{4} \sum_{i=fermions} g_i \left( \frac{T_i}{T} \right)^3,$$

(4)

where $\zeta(x)$ is the Riemann zeta function and $g_i$ is the number of degrees of freedom of the particles in the dense medium. Therefore the characteristic loop size at a time $t$ is

$$l(t) \sim t_d \simeq \frac{\sigma^2}{k_n} \left( \frac{t}{\kappa M_p} \right)^{\frac{3}{2}},$$

(5)
where the time-temperature relation of \( t = \kappa M_p T^{-2} \) for an expanding Universe in the radiation dominated epoch was used. The rate at which the loops collapse can be estimated by assuming

\[
\frac{dl}{dt} \sim -\frac{dt_d}{dt}.
\]

This then gives

\[
l(t_c) \simeq w + \frac{\sigma^2}{k_n} \left\{ \left( \frac{t_w}{\kappa M_p} \right)^{\frac{2}{3}} - \left( \frac{t_c}{\kappa M_p} \right)^{\frac{2}{3}} \right\},
\]

with \( l(t_w) = w \). Substituting this into Eq. (4) results in

\[
n(t_w) \simeq \frac{\xi e^{-4+\alpha}}{(1-\alpha)} \left\{ \left[ w + \frac{\sigma^2}{k_n} \left\{ \left( \frac{t_w}{\kappa M_p} \right)^{\frac{2}{3}} - \left( \frac{t_c}{\kappa M_p} \right)^{\frac{2}{3}} \right\} \right]^{(1-\alpha)} - w^{(1-\alpha)} \right\},
\]

This is the number of loops per unit volume that will collapse to size \( w \).

However, not all of these loops will contribute to generating a nonzero \( B \) or \( L \) since the decays of the Higgs particles will be compensated by their inverse decays at temperatures above the freeze-out temperature, \( T_F \), (corresponding to a time, \( t_F \)) which is of the order of the Higgs boson mass. The relevant number density of loops is then given by

\[
n = \int_{t_F}^{t_{\text{min}}} f(t_w) \left[ \frac{a(t_w)}{a(t_F)} \right]^3 dt_w,
\]

where

\[
f(t_w) \equiv \frac{dn(t_w)}{dt_w},
\]

and \( a(t) \propto t^{\frac{1}{2}} \propto T^{-1} \) is the usual cosmic scale factor in the radiation dominated epoch of the Universe. The limits of integration correspond to their respective temperatures, i.e.,

\[
t_{\text{min}} \longleftrightarrow T_{\text{min}} \simeq k_n^{-\frac{1}{2}} \lambda^\frac{3}{2} \sigma,
\]

\[
t_F \longleftrightarrow T_F \simeq \lambda^\frac{1}{2} \sigma,
\]

(11)
$T_{\text{min}}$ is obtained by requiring $l(t) \geq w$ (see Eq. (4)). To simplify the evaluation of $n$, notice that the ratio

$$\left( \frac{a(t_{\text{min}})}{a(t_F)} \right)^3 = \left( \frac{T_F}{T_{\text{min}}} \right)^3 \simeq k_n \lambda. \quad (12)$$

This is expected to be of order one for “typical” values of the parameters (e.g. $k_n \sim 10$ and $\lambda \sim 0.1$). Therefore the scale factor ratio shouldn’t vary too rapidly for $t_F \leq t \leq t_{\text{min}}$ and hence will be neglected. Then Eq. (4) becomes

$$n \simeq \frac{\lambda^{\frac{4}{3}(1-\alpha)}}{(1 - \alpha)} \left\{ 2^{(1-\alpha)} - \left(1 + \frac{1}{k_n \lambda}\right)^{(1-\alpha)} \right\} \sigma^3. \quad (13)$$

So the number density of heavy particles produced after $t_F$ is given by $N_{Qn}$ and this leads to a baryon number (or lepton number depending on the model under consideration) given by

$$B \ (\text{or} \ L) \simeq \frac{N_{Qn}}{s} \epsilon \simeq \frac{45 \beta}{2\pi^2 g_* (1 - \alpha)} \left\{ 2^{(1-\alpha)} - \left(1 + \frac{1}{k_n \lambda}\right)^{(1-\alpha)} \right\} \epsilon, \quad (14)$$

where $\epsilon$ is the particle physics model dependent CP-violation factor and $s \simeq \frac{2}{45\pi^2 g_* \lambda^{\frac{4}{3}} \sigma^3}$ is the entropy density evaluated at $T_F$.

3. Gauged baryon and lepton numbers

From an experimental point of view, baryon and lepton number appear to be good symmetries, at least at the classical level. An interesting possibility for physics beyond the SM is to gauge the baryon and/or lepton numbers. However, no long distance effects associated with baryon or lepton number have been observed. So if they are gauge symmetries, then they must be spontaneously broken.
To construct a model where $B$ and/or $L$ are gauged, the fermion sector of the SM needs to be extended so that these symmetries are anomaly-free. We know of two ways to do this. One way is to add a mirror set of fermions which have the same quantum numbers as the SM fermions but with opposite chiralities. In this case the anomalies cancel trivially. The other way is to add “exotic” generations to the SM fermions. The multiplet structure of an exotic generation can be defined as:

\[
\begin{align*}
\text{leptons:} & \quad N_R(Y) \oplus (N - 1)_L(Y - 1) \oplus (N - 1)'_L(Y + 1) \oplus (N - 2)_R(Y); \\
\text{quarks:} & \quad N_R \left( \frac{1}{3}Y \right) \oplus (N - 1)_L \left( -\frac{1}{3}Y - 1 \right) \\
& \quad \oplus (N - 1)'_L \left( -\frac{1}{3}Y + 1 \right) \oplus (N - 2)_R \left( -\frac{1}{3}Y \right),
\end{align*}
\]

where $N$ is the $N$-dimensional representation of $SU(2)_L$ and $Y$ is the hypercharge. One of the simplest anomaly-free structures of the gauge group

\[
G_{BL} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L
\]

corresponds to four ordinary doublet-type and two exotic triplet-type generations. Note that it is also possible to use the same methodology to construct models where either $B$ or $L$ are anomaly-free only. For further details see Ref. [8].

Having noted that it is possible to gauge $B$ and/or $L$, we now consider a model with the gauge group given by Eq. (16) where we will apply the results of section 2 to generate the BAU. The fermion sector consists of the SM fermions together with a set of either mirror fermions or exotic generations to cancel the potential $B$ and $L$ anomalies. The usual electroweak symmetry breaking and fermion mass generation are facilitated by the SM Higgs doublet. In addition, two complex SM Higgs singlets, $H_1$ and $H_2$, are introduced.
to spontaneously break $U(1)_B$ and $U(1)_L$ when they acquire nonzero VEVs, $\langle \sigma e^{i\theta} \rangle$ and $\langle \eta e^{i\phi} \rangle$, respectively. The Higgs fields transform under the gauge group $G_{BL}$ as

$$H_1 \sim (1,1,0)(2,0) \quad \text{and} \quad H_2 \sim (1,1,0)(0,2),$$

(17)

where the first set of parentheses give the SM quantum numbers and the second set gives the $B$ and $L$ numbers. From phenomenological considerations, it is expected that the scales of $B$ and $L$ breaking lie above the electroweak scale. Since $B$ and $L$ are spontaneously broken the formation of cosmic strings will result and particle production from the collapse of these string loops will follow the scenario described in section 2. In this section we will only be interested in what happens in the quark sector for the generation of the BAU and thus we will not consider the lepton sector any further. So far a nonzero VEV for $H_1$ only breaks the gauged $B$ symmetry, leaving behind a residual global $B$ symmetry. To also break the global symmetry so that there is baryon number violation in the quark sector, the following vectorlike pair of fermions are introduced. Their transformation properties under $G_{BL}$ are defined by their Yukawa interactions with $H_1$:

$$\mathcal{L}_{Yuk}^Q = \lambda_L H_1 \bar{Q}_L Q_L^c + \lambda_R H_1 \bar{Q}_R Q_R^c + 2D \bar{Q}_L Q_R + \text{H.c.},$$

(18)

so that

$$Q_L \sim (8,1,0)(1,0) \quad \text{and} \quad Q_R \sim (8,1,0)(1,0).$$

(19)

Finally, we need to introduce one more scalar field into the model. This is a charged colored scalar field, $\Delta$, which does not get a nonzero VEV. Its purpose is to transfer the $B$ asymmetry produced in the $Q_{L,R}$ sector to the ordinary quark sector. The transformation properties of $\Delta$ are determined
by the interaction term,

\[ \mathcal{L}_{\Delta \gamma_k}^\Delta = \lambda \bar{Q}_L \Delta d_R + \text{H.c.,} \]

so that \( \Delta \sim (3, 1, \frac{2}{3})(\frac{2}{3}, 0) \), where \( d_R \) is the usual right-handed \( d \)-type quark field.

We will now proceed to determine the BAU from the collapse of string loops by using the results of section 2. First we will outline the scenario of section 2 as applied to this model. When the Universe cools down to a temperature, \( T_c \sim \sigma \), the \( U(1)_B \) symmetry is spontaneously broken and the formation of cosmic strings is initiated. The string loops collapse rapidly in the friction dominated epoch of the evolution of the string network. The production of Higgs scalars, \( h \) (\( h \) is a real scalar field left over from the Higgs mechanism with its mass given by \( M_h \sim \lambda \frac{1}{2} \sigma \)), is assumed to result from this collapse. This provides a source of these Higgs scalars which are at this point overabundant and hence out of equilibrium. The scalar, \( h \), then decays rapidly into the \( Q \)-type particles since its decay width, of the order of \( M_h \), is much larger than the Universe expansion rate, \( H \sim T^2/M_p \), at the temperatures under consideration and thereby an excess of \( B \) can be generated. The \( Q \)'s in turn get converted to \( d \)-type quarks via their interactions with the \( \Delta \) scalars. \( B \) can be calculated from Eq. (14) once \( \epsilon \) is determined. Hence the remainder of this section is used to show how \( \epsilon \) can be calculated.

To determine \( \epsilon \) we need to write down the interactions of \( h \) with the \( Q \)'s. First write \( Q_{L,R} \) in terms of their mass eigenstates. When \( H_1 \) develops a nonzero VEV the interaction terms of Eq. (18) generates the following mass terms:

\[ \mathcal{L}_{\text{mass}} = \bar{\Psi}_R \mathcal{M} \Psi_L + \text{H.c.,} \]

10
where
\[
\Psi_L = \begin{pmatrix} Q_L \\ Q_R \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} Q_R^c \\ Q_L \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} \lambda_L \sigma & D \\ D & \lambda_R \sigma \end{pmatrix}.
\]

(22)

Note that \(\lambda_{L,R}\) can be made real by a phase redefinition of the fields. The mass matrix, \(\mathcal{M}\), can be diagonalized so that
\[
\mathcal{L}_{\text{mass}} = \bar{\Psi}_R^c \mathcal{D} \Psi'_L + \text{H.c.},
\]

(23)

where \(\mathcal{D} = \text{Diag}(M_1, M_2)\) and the mass eigenstates are given by
\[
\Psi'_L = \begin{pmatrix} \zeta_{1L}^c \\ \zeta_{2R} \end{pmatrix} = U^\dagger \Psi_L, \quad \Psi'_R = \begin{pmatrix} \zeta_{1L} \\ \zeta_{2R}^c \end{pmatrix} = V^\dagger \Psi_R
\]

(24)

with
\[
U = \begin{pmatrix} \cos \theta & \sin \theta \\ -e^{i\delta} \sin \theta & e^{i\delta} \cos \theta \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} \cos \theta & \sin \theta \\ -e^{-i\delta} \sin \theta & e^{-i\delta} \cos \theta \end{pmatrix}.
\]

(25)

The interactions of \(h\) from Eq. (18) can now be written as
\[
\mathcal{L}_{\text{int}} = \lambda_1 h \zeta_{1L}^c \zeta_{1L} + \lambda_2 h \zeta_{2R}^c \zeta_{2R} + \lambda_{12} h \zeta_{2R} \zeta_{1L} + \text{H.c.},
\]

(26)

where
\[
\lambda_1 = \lambda_L \cos^2 \theta + \lambda_R e^{i2\delta} \sin^2 \theta, \\
\lambda_2 = \lambda_L \sin^2 \theta + \lambda_R e^{-i2\delta} \cos^2 \theta, \\
\lambda_{12} = \sin 2\theta \left( \lambda_L - \lambda_R e^{i2\delta} \right).
\]

(27)

It is important to note that \(\lambda_{1,2} \) and \(\lambda_{12} \) cannot be made real because the phases of the fields are now fixed in the mass eigenstate basis. These complex couplings are necessary for the nonzero CP-violation parameter, \(\epsilon\), to be generated. From Eq. (26), it can be seen that there are three processes
which contribute to \( \epsilon \). They are (1) \( h \rightarrow \zeta_{1L}\bar{\zeta}_{2R} \), (2) \( h \rightarrow \zeta_{1L}\zeta_{1L} \) and (3) \( h \rightarrow \zeta_{2R}\zeta_{2R} \). The Feynman diagrams for process (1) are shown in Figs. (1) and (2). The corresponding diagrams for processes (2) and (3) are very similar. For example, for process (1), only the diagram in Fig. 2(b) has a complex product of couplings for the interference term between the tree and one-loop amplitudes and hence can contribute to \( \epsilon \). The relevant amplitudes can be written as follows:

\[
\mathcal{A}(h \rightarrow \zeta_{1L}\bar{\zeta}_{2R}) = \lambda_{12}\hat{a}_0 + \lambda_{12}^*\lambda_2\hat{a}_1
\]

\[
\bar{\mathcal{A}}(h \rightarrow \bar{\zeta}_{1L}\zeta_{2R}) = \lambda_{12}^*\hat{a}_0 + \lambda_{12}\lambda_2\hat{a}_1,
\]

(28)

where \( \hat{a}_0 \) and \( \hat{a}_1 \) denote the tree and one-loop contributions respectively. Then

\[
|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2 = -4 \text{Im} (\lambda_1\lambda_2^*\lambda_{12}^*) \text{Im} (\hat{a}_0\hat{a}_1).
\]

(29)

For convenience, define

\[
\Delta(i \rightarrow f) \equiv \Gamma(i \rightarrow f) - \Gamma(\bar{i} \rightarrow \bar{f}),
\]

(30)

where \( i \) and \( f \) represent the initial and final states of the decay process. Then

\[
\frac{\Delta(h \rightarrow \bar{\zeta}_{1L}\zeta_{2R})}{\Gamma(h \rightarrow \text{all})} \approx \frac{4 \text{Im} (\lambda_1\lambda_2^*\lambda_{12}^*) \text{Im} (a_0^*a_1)}{(|\lambda_1|^2 + |\lambda_2|^2 + 2|\lambda_{12}|^2)|a_0|^2}\beta_{12}^3
\]

(31)

Similarly for processes (2) and (3)

\[
\frac{\Delta(h \rightarrow \zeta_{1L}\zeta_{1L})}{\Gamma(h \rightarrow \text{all})} \approx \frac{2 \text{Im} (\lambda_1\lambda_2^*\lambda_{12}^*) \text{Im} (a_0^*a_1)}{(|\lambda_1|^2 + |\lambda_2|^2 + 2|\lambda_{12}|^2)|a_0|^2}\beta_{11}^3
\]

(32)

and

\[
\frac{\Delta(h \rightarrow \zeta_{2R}\zeta_{2R})}{\Gamma(h \rightarrow \text{all})} \approx \frac{-2 \text{Im} (\lambda_1\lambda_2^*\lambda_{12}^*) \text{Im} (a_0^*a_1)}{(|\lambda_1|^2 + |\lambda_2|^2 + 2|\lambda_{12}|^2)|a_0|^2}\beta_{22}^3
\]

(33)

respectively, where \( a_0 \) and \( a_1 \) are the amplitudes \( \hat{a}_0 \) and \( \hat{a}_1 \) with the phase space factor \( \beta_{ij} = \sqrt{1 - (M_i + M_j)^2/M_h^2} \) factored out. Note that \( a_0 \) and \( a_1 \)
are not the same for each of the above processes since $M_1 \neq M_2$ in general. However in the following, to simplify the calculations, we will assume $M_1 \sim M_2$ and much smaller than $M_h$. There is also a set of diagrams where the internal Higgs line is replaced by the massive gauge boson line but these diagrams do not contribute to $\epsilon$ since the product of couplings of the interference terms are real due to the real gauge couplings. So far, all of the $B$-violation is carried by $\zeta_{1,2}$ in the exotic sector but this will be transferred to the ordinary sector via the interactions of Eq. (20), i.e.,

$$L_{\Delta Yuk}^\Delta = \lambda_\Delta \left( \cos \theta \bar{\zeta}_{1L} \Delta d_R + \sin \theta \bar{\zeta}_{2R} \Delta d_R \right) + H.c., \quad (34)$$

Therefore $\epsilon$ can be written as

$$\epsilon = \sum_f B_{f'} \frac{\Delta(h \rightarrow f) \Gamma(f \rightarrow f')}{\Gamma(h \rightarrow all) \Gamma(f \rightarrow all)}$$

$$= B_d \left\{ \frac{\Delta(h \rightarrow \zeta_{1L} \zeta_{1L}) \Gamma(\zeta_{1L} \rightarrow \Delta^1 d_R)}{\Gamma(h \rightarrow all) \Gamma(\zeta_{1L} \rightarrow all)} 
+ \frac{\Delta(h \rightarrow \bar{\zeta}_{2R} \bar{\zeta}_{2R}) \Gamma(\bar{\zeta}_{2R} \rightarrow \Delta^1 d_R)}{\Gamma(h \rightarrow all) \Gamma(\bar{\zeta}_{2R} \rightarrow all)} 
+ \frac{1}{2} \frac{\Delta(h \rightarrow \zeta_{1L} \zeta_{1L}) \Gamma(\zeta_{1L} \rightarrow \Delta^1 d_R)}{\Gamma(h \rightarrow all) \Gamma(\zeta_{1L} \rightarrow all) + \Gamma(\bar{\zeta}_{2R} \rightarrow \Delta^1 d_R)} \right\} \quad (35)$$

which simplifies to

$$\epsilon \sim \frac{1}{4\pi} \left( \frac{M_1 - M_2}{M_h^2} \right)^2 \frac{\text{Im} (\lambda_1 \lambda_2^* \lambda_{12}^2)}{|\lambda_1|^2 + |\lambda_2|^2 + 2|\lambda_{12}|^2}, \quad (36)$$

where the branching ratios of the processes, $\zeta_{1L}, \bar{\zeta}_{2R} \rightarrow \Delta^1 d_R$ are unity, the factor $16\pi \text{Im} (a_0^* a_1) / |a_0|^2$ is of order one [3] and $B_d = -\frac{2}{3}$ for the final state baryon number since each process results in the decay product $\Delta^1 d_R$. To get a rough estimate of the size of the couplings required for the observed value of the BAU, let $\lambda_\zeta \equiv \lambda_1 \sim \lambda_2 \sim \lambda_{12}$ and $M_{1,2} \sim O(\lambda_\zeta \sigma)$, so that
\( \epsilon \sim \lambda^4/(16\pi \lambda) \). Substituting this in Eq. (14) gives

\[
B \sim 10^{-3} \frac{\lambda^4}{(1-\alpha)} \left\{ 2^{(1-\alpha)} - \left( 1 + \frac{1}{k_n \lambda} \right)^{(1-\alpha)} \right\} \lambda^4. \tag{37}
\]

For example, by taking \( \lambda \sim 0.1, \alpha = 2.5, \) with \( k_n \sim 10 \) (see Eq. (34)), then \( \lambda_\zeta \sim 10^{-2} \) for the present baryon asymmetry which lies in the range \((4 - 5.7) \times 10^{-11}\) [10]. Therefore if \( M_{1,2} \lesssim 10^3 \text{GeV} \), then the breaking scale for \( B \) is expected to be less than about \( 10^5 \text{GeV} \). Note that no particular limit must be set on the \( L \) breaking scale since now sphalerons violate neither \( B \) nor \( L \) and therefore the interplay between the sphalerons and \( L \)-violating processes is completely harmless for the generated baryon asymmetry. Furthermore, the dangerous \( B \)-violating processes like \( \Delta + \Delta \rightarrow d_R + d_R \) mediated by the \( \zeta \) particles, whose rate is \( \sim \lambda_\Delta^4 T^3 / m_\zeta^2 \), are out of equilibrium for small enough values of \( \lambda_\Delta \). As a result they also do not affect the \( B \) produced from the rapid decays of \( \zeta \rightarrow \Delta + d_R \).

4. The gauged \((B - L)\) model

It is known that the SM with right-handed neutrinos has an anomaly-free \((B - L)\) symmetry. Therefore \((B - L)\) can be gauged. In this section we will consider such a model for generating the BAU using the collapse of string loops mechanism of section 2. The gauge group is

\[
SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}. \tag{38}
\]

To break the \((B - L)\) symmetry and to give the right-handed neutrinos a Majorana mass, we introduce the scalar field, \( \chi \), which is a complex SM Higgs singlet with two units of \((B - L)\). As the temperature in the early
Universe drops to about $T \sim \langle \chi \rangle$ the $(B - L)$ symmetry is spontaneously broken and the formation of cosmic strings begins. These strings will evolve in the friction dominated period where string loops tend to collapse rapidly. The collapse of string loops will result in the production of Higgs particles associated with $\chi$. These Higgs particles will then in turn decay into right-handed neutrinos providing a source of lepton number. By using the results of section 2, $L$ will be given by Eq. (14). The $\epsilon$ parameter can be calculated from the decays of the right-handed neutrino, i.e.

$$
\nu_{R_i} \rightarrow F_{L_j} + \Phi \\
\nu_{R_i} \rightarrow \bar{F}_{L_j} + \Phi,
$$

such that

$$
\epsilon = \sum_i \epsilon_i
$$

where $\epsilon_i$ is the difference between particle-antiparticle branching ratios given by

$$
\epsilon_i = \frac{1}{2\pi (hh^\dagger)_{ii}} \sum_j \left( \text{Im} \left[ (hh^\dagger)_{ij} \right]^2 \right) f \left( \frac{m_{\nu_{R_j}}^2}{m_{\nu_{R_i}}^2} \right),
$$

with

$$
f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right].
$$

The $h$'s denote the Yukawa couplings between right-handed and left-handed neutrinos through the SM Higgs doublet $\Phi$ and are assumed to be complex to give a source of CP violation.

The lepton number produced by right-handed neutrino decays is converted to a nonzero baryon number via the $(B - L)$ conserving sphaleron processes. Therefore the BAU is given by

$$
B = \kappa L
$$
\[ \sim \kappa \frac{45/\beta}{2\pi^2 g_*} \frac{\lambda^{\frac{2(2-\alpha)}{2}}}{(1-\alpha)} \left\{ 2^{(1-\alpha)} - \left( 1 + \frac{1}{k_n\lambda} \right)^{(1-\alpha)} \right\} \epsilon, \]  

where \( \kappa \) is a numerical factor of \( O(1) \) and can be easily calculated from Ref. \[12\]. For \( \lambda \sim 10^{-2} \) and \( \alpha = 2.5 \) gives a baryon asymmetry of about \( B \sim 10^{-1} \epsilon \). For the present baryon asymmetry to be of the order of \( 5 \times 10^{-11} \), \( \epsilon \) has to be as large as \( 10^{-10} \).

We also need to check that the initial baryon asymmetry generated by the collapsing string loops and the subsequent decays of right-handed neutrinos is not erased by a combination of other lepton violating interactions and sphaleron processes \[13\]. The lowest dimension \( L \)-violating operator in the effective low energy theory is given by

\[ L_{\Delta L=2} = \frac{m_{\nu_L}}{v^2} F_L \Phi + H.c., \]  

where \( m_{\nu_L} \) is the mass of the left-handed neutrino and \( v \) is the electroweak breaking scale. The interaction rate of these \( \Delta L = 2 \) processes is \( \Gamma_{\Delta L=2} \sim m_{\nu_L}^2 T^3 / (\pi^3 v^4) \). For the survival of the pre-existing asymmetry we require this interaction rate to be less than the expansion rate of the Universe, \( i.e., H \sim T^2 / M_p \), where \( T \) is given by \( T_{\text{min}} \) in Eq. (11). This results in a bound on the \( (B - L) \) breaking scale,

\[ \langle \chi \rangle \gtrsim \frac{\gamma^4}{\pi^3 h^2} k_{n}^{-1/3} \lambda^{1/2} M_p, \]  

where \( h \) is the Yukawa coupling for the Dirac mass term and \( \gamma \) is the one for the Majorana mass. So for natural values of the couplings involved, the bound on \( \langle \chi \rangle \) can easily be made as low as \( 10^{9} \) GeV.

5. Conclusions
In the present paper we have shown that models, in which $B$ and $L$ are gauged symmetries, are interesting not only from the particle physics perspective but also for cosmology – the generation of the baryon asymmetry in the early Universe. Indeed, such models provide a natural implementation of the out-of-equilibrium condition through the collapse of string loops formed during the phase transition at intermediate scales where $B$ and/or $L$ are spontaneously broken. Also, differently from what happens in ref. [4], these loops decay during the friction dominated epoch of string evolution. We would also like to stress that, in the case in which both $B$ and $L$ are gauged, the sphaleron processes do not violate $B + L$ so that there is no wash out of any pre-existing asymmetry and no particular limit must be imposed on the scale of $L$ breaking (other than that from experiment). Finally, in our scenario, the electroweak phase transition need not be first order as required in most non-GUT scenarios for the generation of the BAU [3].

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Figure 1: Tree level diagram for the decay $h \to \zeta_{1L}\bar{\zeta}_{2R}$. 
Figure 2: One-loop diagrams contributing to CP violation in the $h \to \zeta_{1L} \bar{\zeta}_{2R}$ decay; only diagram (b) gives a nonvanishing contribution.