Pinning down the mass of Kepler-10c: the importance of sampling and model comparison

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ABSTRACT
Initial radial velocity (RV) characterization of the enigmatic planet Kepler-10c suggested a mass of \( \sim 17 \, M_\oplus \), which was remarkably high for a planet with radius \( 2.32 \, R_\oplus \); further observations and subsequent analysis hinted at a (possibly much) lower mass, but masses derived using RVs from two different spectrographs (HARPS-N and HIRES) were incompatible at a 3\( \sigma \) level. We demonstrate here how such mass discrepancies may readily arise from suboptimal sampling and/or neglecting to model even a single coherent signal (stellar, planetary or otherwise) that may be present in RVs. We then present a plausible resolution of the mass discrepancy, and ultimately characterize Kepler-10c as having mass \( 7.37^{+1.32}_{-1.19} \, M_\oplus \), and mean density \( 3.14^{+0.63}_{-0.55} \, g \, cm^{-3} \).

Key words: methods: data analysis – techniques: radial velocities – stars: activity – stars: individual: Kepler-10 – planetary systems.

1 INTRODUCTION
Kepler-10 (KOI-72; hereafter K-10 for short) is a slowly rotating, Sun-like star that exhibits little stellar activity (Dumusque et al. 2014, hereafter D14). It is known to host at least two planets, viz. Kepler-10b and Kepler-10c.

Stony-iron world Kepler-10b (hereafter K-10b) – with orbital period 0.84 d, radius \( 1.48 \, R_\oplus \) and mass \( \sim 4 \, M_\oplus \) – was the first unambiguously rocky exoplanet to be discovered, and also the first super-Earth discovered around a Sun-like star that exhibits little stellar activity (Dumusque et al. 2014, hereafter D14). It is known to host at least two planets, viz. Kepler-10b and Kepler-10c.

Kepler-10c (hereafter K-10c) – with orbital period 45.29 d and radius \( 2.32 \, R_\oplus \) – has proven more enigmatic. Following its discovery and statistical validation as a planet (Batalha et al. 2011), D14 used 148 HARPS-N radial velocities (RVs) spanning two observing seasons to infer a mass of \( 17.2 \pm 1.9 \, M_\oplus \). Given K-10c’s radius, this was a striking result. Most planets with radii \( 2.0–2.5 \, R_\oplus \) have masses significantly lower than \( 17 \, M_\oplus \), with a weighted mean mass of \( 5.4 \, M_\oplus \) (Weiss & Marcy 2014); Weiss & Marcy’s empirical mass-radius relation for planets between 1.5 and \( 4 \, R_\oplus \), viz. \( M_p/M_{\oplus} = 2.69 (R_p/R_{\oplus})^{0.93} \), predicts a mass of \( 5.8 \, M_\oplus \) for K-10c. D14 interpreted the composition of K-10c as being mostly rock by mass, and regarded the planet as being the first evidence of a class of more massive solid planets with longer orbital periods.

Weiss et al. (2016, hereafter W16) built on the work of D14, adhering closely to the techniques employed by the latter authors, but adding 72 RVs from Keck-HIRES to the analysis, resulting in a combined RV baseline of 6 yr. Since it has been well established that both the HIRES and HARPS-N spectrometers are independently capable of accurate and precise measurement of low-amplitude planetary signals, it was a great surprise when W16 inferred a mass for K-10c of \( 5.69^{+3.10}_{-2.90} \, M_\oplus \) (fitted RV semi-amplitude \( K_c = 1.09 \pm 0.58 \, m \, s^{-1} \)) using the HIRES RVs alone, which was incompatible with D14’s estimate of \( 17.2 \pm 1.9 \, M_\oplus \) (\( K_c = 3.26 \pm 0.36 \, m \, s^{-1} \)) using the HARPS-N RVs alone.

W16 concluded that some additional, time-correlated signal (possibly from stellar activity or additional planets) was present and led to the discrepant mass estimates for K-10c. This claim was supported by (i) the fact that masses inferred using RVs from either instrument were found to be time dependent, and (ii) >5\( \sigma \) evidence for transit timing variations (TTVs) of K-10c (Kipping et al. 2015). W16 found that dynamical solutions including a third planet candidate, KOI-72.X, were very strongly favoured over a two-planet solution (based on Bayesian information criterion differentials); the TTVs and RVs were consistent with KOI-72.X having an orbital period of 24, 71 or 101 d, with 101 d being strongly favoured over the other periods. W16 inferred a likely mass of \( \lesssim 7 \, M_\oplus \) for KOI-72.X, based on the best solutions from a partial exploration of the dynamical parameter space, with the parameters of K-10b fixed.

Even when including a third planet in their models, however, W16 were not able to reconcile the HIRES and HARPS-N masses for K-10c, so settled on a ‘compromise’ mass for K-10c of \( 13.98 \pm 1.79 \, M_\oplus \). We suggest the observed 3\( \sigma \) incompatibility between the HARPS-N and HIRES estimates for K-10c’s mass points to an inadequate model under which at least one (if not both) of the inferred masses is incorrect, and that the true mass need not lie in the middle of the two incompatible mass posteriors.

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2 DOUBLE TROUBLE: IMPERFECT MODEL MEETS INADEQUATELY SAMPLED SIGNAL

To shed light directly on the effects of (i) suboptimal sampling and (ii) inference based on an imperfect physical model, consider synthetic RV data sets \( \{ t_i, y_i \}_{i=1,2,\ldots,N} \) generated as follows:

\[
y_i = \sum_{j=1}^{M} K_j \sin \left( \frac{2\pi t_i}{P_j} + \phi_j \right);
\]

\( y_i \) may be interpreted as the combined RV signal at time \( t_i \) due to \( M \) planets on zero-eccentricity orbits around a star. For planet \( j \), the associated RV amplitude \( K_j \) would be determined by the planet’s mass and inclination (assuming known stellar mass); \( P_j \) would correspond to the planet’s orbital period; and \( \phi_j \) would be determined by the planet’s orbital phase in some coordinate system.

Suppose we set \( M = 3 \), and let the periods of the three mock planets be \( P_1 = 0.84 \text{ d}, P_2 = 45.29 \text{ d}, P_3 = 101.36 \text{ d} \). \( P_1 \) and \( P_3 \) correspond to the known orbital periods of K-10b and K-10c, respectively, while \( P_2 \) correspond to the known orbital periods of K-10b and K-10c, respectively, while \( P_2 \) would be determined by the planet’s orbital period and \( \phi_2 \) would be determined by the planet’s orbital phase in some coordinate system.

In Fig. 1 we present the results of this fitting exercise, showing ML estimates for \( K_2 \), based on synthetic data comprising three sine waves, with four different sampling patterns [left to right, corresponding to sampling patterns (i)–(iv) listed in Section 2] and fitted with a two-sine model (upper panels) and a three-sine model (lower panels). \( \phi_3 \) axis compressed to save space.

\[ \text{Colour scale: } K_2, \text{ inferred RV semi-amplitude of 'Kepler-10c' [m s}^{-1}]. \]

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When combining the HIRES and HARPS-N observations, \( K_2 \) interpolates the values predicted by the separate data sets, yet may still differ from \( K_2 = 2.5 \, \text{m s}^{-1} \) by up to \( \pm 0.65 \, \text{cm s}^{-1} \).

(ii) Notably, even when using the inadequate \( M = 2 \) model but the uniform observing cadence, \( |K_2 - K_2| < 5 \, \text{cm s}^{-1} \) for \( \forall \phi_j \).

(iii) Equally notably, when using the correct \( M = 3 \) model, the inferred mass for the second planet is relatively insensitive in the estimated for the HARPS-N data set (\( \sigma \)).

(iv) Notably, even when using the inadequate \( M = 2 \) model but the uniform observing cadence, \( |K_2 - K_2| < 5 \, \text{cm s}^{-1} \), yet even larger differences will result when using the uniform observing cadence, the resultant uniform phase coverage allows remarkably robust inference about \( K_2 \) to be made: e.g. \( |K_2 - K_2| < 7 \, \text{cm s}^{-1} \) \( \forall \phi_j \), even with the \( M = 2 \) model.

Thus far we have established the prevalence of sizeable differences in \( K_2 \) when fitting simplistic synthetic signals with HARPS-N versus HIRES sampling, yet even larger differences will result when including in our synthetic data such details as photon noise, quasi-periodic stellar activity signals, instrumental noise, multiple undetected planets, planets with non-circular orbits, possible dynamical interactions between planets and more. For example, adding to our synthetic data white Gaussian noise at a level consistent with that estimated for the HARPS-N data set (\( \sigma \sim 2 \, \text{m s}^{-1} \)), then repeating the previous test, results in HARPS-N/HIRES discrepancies for \( K_2 \), of up to \( 3 \, \text{m s}^{-1} \) under the \( M = 2 \) model, and up to \( 2.3 \, \text{m s}^{-1} \) under the \( M = 3 \) model; see Fig. 2, available online.

The upshot is that using an inadequate physical model, and/or suboptimal sampling, can lead to incorrect conclusions about the masses of planets whose other properties are well constrained – and even when we have hundreds of RVs at our disposal. Moreover, through our choice of real sampling patterns, and realistic values for \( K_1 \) and \( P_j \), we have provided a plausible explanation for why \( \text{W16} \) obtained discrepant masses for K-10c using HIRES versus HARPS-N RVs. Specifically, the real K-10 RVs likely contained not only K-10b and K-10c’s signals, but one or more other coherent signals (KOI-72.X, a stellar signal, etc., as indeed adduced by \( \text{W16} \)) that interfered constructively or destructively with the signals of the known planets. The suboptimality of the phase coverage is easily checked by phasing the HIRES or HARPS-N observation times to the orbital period of K-10c; see Fig. 3. In principle, accounting for the other signals jointly with those of the known planets (i.e. using a more appropriate physical model), and/or obtaining more observations to provide more complete phase coverage of K-10c’s signal, could have mitigated the discrepancy.

3 RECONCILING THE MASS ESTIMATES

We noted \( >3 \sigma \) evidence for linear correlations (\( \rho \sim 30 \) per cent) between the published HARPS-N RVs and (i) \( \log R'_{\text{HK}} \) index and (ii) bisector inverse slope (BIS) measurements; we did not find any similarly significant correlations in the HIRES RVs.

Whereas the models of \( \text{D14} \) and \( \text{W16} \) did not accommodate possible stellar activity signals in the RVs, we used the GP framework of \( \text{Rajpaul et al. (2015, hereafter R15)} \) to model jointly all available RV, \( \log R'_{\text{HK}} \) (in the case of HARPS-N) or \( \delta_{\text{HIRES}} \) (in the case of HIRES), and BIS time series, for a total of 660 data points. As in \( \text{R15} \), we adopted a quasi-periodic covariance kernel, and non-informative priors were placed on all GP hyperparameters. GP amplitude parameters were also constrained to be smaller than the total variation seen in a given time series, and of the overall GP period we required \( P > 20 \, \text{d} \) (based on \( \text{D14} \)’s lower limits on K-10’s stellar rotation period). We additionally allowed at least two but up to five possible planetary signals in the RVs, modelled with Keplerian functions. We constrained the periods and periapsis passage times of two of the Keplerians to be consistent with the most precise values inferred from K-10b and K-10c’s transits (Holczer et al. 2016; Morton et al. 2016), but left the other parameters free, with priors identical to those in \( \text{W16} \)’s eccentric two-planet model. We adopted analogous uninformative priors for all parameters of the additional possible planets, ensuring only that planet periods did not overlap. Finally, we used the \text{MULTINEST} nested-sampling algorithm (Feroz & Hobson 2008; Feroz, Hobson & Bridges 2009; Feroz et al. 2013) to obtain a full joint posterior distribution for each model’s parameters (and marginal posteriors for parameters of interest), and to compute a Bayesian evidence (\( \mathcal{Z} \)) for each model.

Of the numerous models we considered, we found only one in which estimates for all planet parameters were consistent within \( 1 \sigma \) between the HARPS-N, HIRES and merged data sets: viz. a model including three planets, all with orbits consistent with circular, plus correlated noise. Significantly, this model was also favoured over others by Bayesian model comparison tests, and the period for the third planet in our model was \( 102 \pm 1 \, \text{d} \) in accord with the \( \text{W16} \)’s favoured period for KOI-72.X (based on both analytical considerations and dynamical modelling), despite us not including this as prior information in our model. We summarize the marginal posteriors for this favoured model’s planet parameters in Table 1; masses (for all three planets) and mean densities (for the transiting planets) were derived using the same stellar mass and planet radii as in \( \text{W16} \).
Additionally, we note the following. First, for the HIRES, HARPS-N and merged data sets, three-planet models were strongly favoured over two-planet models ($\Delta \ln Z > 10$), which were in turn favoured over four- and five-planet models. Secondly, we obtained consistent parameters for all planets when splitting either the HIRES or HARPS data sets in two; presumably W16 found discrepant results because neither a third planet nor a nuisance signal model was included when performing the same test. Thirdly, a zero-amplitude GP component was favoured for the HIRES RVs; the latter two cases suggested a GP period of $P = 55 \pm 1 \text{ d}$. We interpreted the third finding as evidence of the HARPS-N RVs and merged data sets, three-planet models were strongly consistent with those of D14 and W16. Our mass estimate for K-10c ($3.14^{+0.63}_{-0.56} \text{ M}_\oplus$), however, is significantly lower than those from D14 and W16 ($17.2 \pm 1.9$ and $13.98 \pm 1.79 \text{ M}_\oplus$, respectively); accordingly, we also infer a significantly lower mean density of $\rho_c = 3.14^{+0.63}_{-0.56} \text{ g cm}^{-3}$. This implies a composition that is either consistent with a low-density, solid planet with a significant fraction of volatiles in the form of e.g. water or methane, or a planet with a dense core and an extended gaseous envelope. K-10c would interfere strongly over time-scales of several months (an envelope with period 248 d would be predicted if the nuisance signal were sinusoidal).  

| Parameter | Units | HIRES Median | ±σ | HARPS-N Median | ±σ | Merged Median | ±σ |
|-----------|-------|--------------|----|----------------|----|--------------|----|
| $K_b$ | m s$^{-1}$ | 2.39 | 0.30 | 2.33 | ±0.16 | 2.32 | ±0.21 |
| $P_b$ | d | 0.83748 | ±0.00003 | 0.837501 | ±0.000005 | 0.837501 | ±0.0000004 |
| $\sqrt{\sigma_b} \cos \omega_b$ | – | 0.000 | ±0.003 | 0.000 | ±0.003 | 0.000 | ±0.004 |
| $\sqrt{\sigma_b} \sin \omega_b$ | – | 0.000 | ±0.003 | 0.000 | ±0.003 | 0.000 | ±0.004 |
| $m_b$ | M$_\oplus$ | 3.33 | ±0.40 | 3.25 | ±0.22 | 3.24 | ±0.28 |
| $\rho_b$ | g cm$^{-3}$ | 5.65 | ±0.94 | 5.51 | ±0.73 | 5.48 | ±0.78 |
| $K_c$ | m s$^{-1}$ | 1.27 | ±0.42 | 1.64 | ±0.42 | 1.41 | ±0.25 |
| $P_c$ | d | 45.2948 | ±0.0008 | 45.2940 | ±0.0008 | 45.2946 | ±0.0008 |
| $\sqrt{\sigma_c} \cos \omega_c$ | – | 0.1 | ±0.2 | 0.0 | ±0.1 | 0.0 | ±0.1 |
| $\sqrt{\sigma_c} \sin \omega_c$ | – | 0.0 | ±0.2 | 0.1 | ±0.1 | 0.0 | ±0.1 |
| $m_c$ | M$_\oplus$ | 5.87 | ±2.20 | 8.59 | ±2.19 | 7.37 | ±1.32 |
| $\rho_c$ | g cm$^{-3}$ | 2.50 | ±0.98 | 3.66 | ±0.80 | 3.14 | ±0.55 |
| $K_X$ | m s$^{-1}$ | 1.30 | ±0.51 | 0.84 | ±0.16 | 0.85 | ±0.24 |
| $P_X$ | d | 102 | ±1.79 | 101 | ±1.19 | 102 | ±1.14 |
| $\sqrt{\sigma_X} \cos \omega_X$ | – | 0.1 | ±0.2 | −0.1 | ±0.1 | −0.1 | ±0.1 |
| $\sqrt{\sigma_X} \sin \omega_X$ | – | −0.1 | ±0.2 | 0.0 | ±0.1 | 0.0 | ±0.1 |
| $m_X$ | M$_\oplus$ | 8.93 | ±3.50 | 5.80 | ±1.20 | 5.90 | ±1.70 |
| $K_{GP}$ | m s$^{-1}$ | 0.09 | ±0.22 | 1.46 | ±0.17 | 1.68 | ±0.25 |
| $P$ | d | 63 | ±10 | 55 | ±1 | 55.5 | ±0.8 |
| $\lambda_d$ | – | 1.3 | ±0.6 | 0.33 | ±0.04 | 0.32 | ±0.02 |
| $\lambda_e$ | d | 330 | ±100 | 86 | ±4 | 90 | ±6 |

3 The small posterior uncertainty of ±1 d may simply indicate that 55 d is the only GP period that does a reasonable job of modelling some (possibly complex) combination of nuisance signals. Regardless, given the similarity of the 55 d period to K-10c’s orbital period, the nuisance and planet signals interfere strongly over time-scales of several months (an envelope with period 248 d would be predicted if the nuisance signal were sinusoidal).
and from our Keplerian solution was of the order \(1 \text{ cm s}^{-1}\) over 101 d. As this is two orders of magnitude below the RV noise floor, we concluded that full dynamical modelling would have yielded no constraints beyond those already derived by W16.

4 INSTRUMENTAL CONSIDERATIONS

W16 detrended the HIRES RVs by removing correlations between RVs and instrumental parameters, RV uncertainties and spectrum signal-to-noise ratio. The RVs published in W16 are these detrended RVs; the published RV uncertainties also already have jitter applied. We obtained both the pre-detrending RVs and the uncertainties without jitter from Weiss (personal communication), and re-ran the analyses described in Section 3. As before, we ended up favouring a three-planet plus correlated noise model strongly over all competing models, and the posterior distributions for the parameters of all three Keplersian were consistent (\(< 1 \sigma\)) with those obtained when using the detrended HIRES RVs.

To explore the possibility of the HARPS-N data reduction pipeline contributing to the discrepancy, we applied a novel, template- and mask-free approach we are developing (paper in preparation) for extracting RVs from observed spectra. We model each observed spectrum non-parametrically, with shifts between all possible pairs of spectra included as parameters in the modelling (in addition to possible telluric, stellar activity and instrumental effects). Interestingly, we found that when modelling HARPS-N RVs extracted with our own pipeline versus the HARPS-N pipeline, our inferred RV semi-amplitude for K-10b was unchanged, but we reliably inferred \(K_1 < 2 \text{ m s}^{-1}\) even without a correlated noise (GP) component in our model. This suggests the possibility that at least part of the signal confounding K-10c’s signal might be instrumental rather than stellar (and would explain why the same nuisance signal is absent from the HIRES RVs); given the preliminary nature of our pipeline, however, further investigation is required.

5 DISCUSSION AND CONCLUSIONS

Previous studies (e.g. Dawson & Fabrycky 2010) have explored the impact of irregular time sampling on planet period estimation; here we have demonstrated that a failure to account for one or more coherent signals (whether of stellar, planetary or instrumental origin) in RV data, and/or uneven phase coverage, can confound attempts to infer the masses of planets with known periods. We used synthetic data with sampling based on real observations to demonstrate how such difficulties could arise when characterizing planets in a system analogous to K-10; tests such as the ones we presented may readily be applied to other systems, to test the sensitivity of planet characterization to sampling and model selection.

By accounting for a time-correlated (stellar or instrumental) signal present in the HARPS-N K-10 RVs, as well as a likely third planet in the system, we were able to achieve full consistency between the Keplerian solutions for the HIRES, HARPS-N and combined RVs. The third planet included in our model has properties consistent with K-10c’s TTVs; and although our model is more complex than the one used by W16 to model the RVs, it was nevertheless favoured over simpler models in Bayesian model comparison testing. While our proposed resolution of the K-10c mass discrepancy is a plausible one, it appears that (many) more RVs will be required for a definitive characterization of the K-10 system.

Whereas W16 suggested a strategy of employing a long observing baseline compared to time-correlated noise influences, we suggest it’s also important to focus on obtaining more complete phase coverage of the relevant signals. As we demonstrated in Section 2, good phase coverage can permit robust inference about known planets, even when using a demonstrably inadequate physical model. While uniform cadence might not be feasible or desirable, e.g. to avoid aliasing, a long observing baseline and approximately uniform cadence would lead to good phase coverage even of planets with unknown orbital periods (see Appendix A, online, for more details). And while W16 suggested that a long baseline would help to average out spurious signals that may arise from stellar activity, we suggest it is strongly preferable to model these nuisance signals, as it is difficult to know a priori how these nuisance signals might interfere with signals of interest. Baselines and cadence aside, it seems all but essential to implement a variety of physical models (to account for varying numbers of possible planets, nuisance signals, etc.), and to compare systematically the evidence for the competing models.

Our findings may also have relevance to archival RV data sets, and indeed, this is not the first example of a system where inference has turned out to be extremely sensitive to both sampling and model choice, despite the availability of a large number of RVs (Rajpaul, Aigrain & Roberts 2016). Then again, K-10 might have been a relatively pathological case; as W16 noted, there were various hints (TTVs, K-10c mass discrepancy, etc.) that existing characterizations of the system were inadequate. Looking to the future, with a new generation of RV spectrographs with expected precisions of \(10 \text{ cm s}^{-1}\) soon to come online, optimized sampling strategies and careful model selection will clearly both be essential if these spectrographs are to be used for accurate characterization of small planets, especially those in potentially multiplanet systems. Moreover, it would be prudent to coordinate observations made by different teams with different telescopes, to minimize ‘redundant’ observations that do not contribute to improved coverage of a given planet’s orbital phases.

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Figure 2. As for Fig. 1, but now with noisy synthetic data comprising four rather than three sine waves.

Figure 4. Mass–radius relation for planets smaller than 3.2 $R_{\oplus}$, and mass determinations better than 20 per cent precision.

Appendix A. On uniform cadence and phase coverage.

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