A new method of electronic refrigeration based on resonant Fowler-Nordheim emission is proposed and analyzed. In this method, a bulk emitter is covered with a few nm-thick film of a widegap semiconductor, creating an intermediate step between electron energies in the emitter and in vacuum. An external electric field tilts this potential profile, forming a quantum well, and hence 2D electron subbands at the semiconductor-vacuum boundary. Alignment of the lowest subband with the energy levels of the hottest electrons of the emitter (a few $k_B T$ above its Fermi level) leads to a resonant, selective emission of these electrons, providing emitter cooling. Calculations show that cooling power as high as $10^4 \text{ W/cm}^2$ (at 300 K), and temperatures down to 10 K may be achieved using this effect.

The idea of using thermionic transport of electrons over an energy barrier for cooling has been repeatedly discussed in the literature (see, e.g., \cite{1,2}). If the barrier height is a few times the thermal spread $k_B T$, the thermionic current may be quite substantial, with only the hot fraction of electrons being removed from the conductor. Unfortunately, the practical implementation of this idea runs into problems.

A barrier of the necessary height ($\sim 200 \text{ meV}$ for 300 K, and proportionally lower for lower temperatures) may be readily implemented in solid state structures, in particular using composite semiconductors with the necessary conduction band edge offset. However, even if the barriers are relatively thick, the back flow of heat to the cooled conductor is prohibitively high \cite{1,2}. Multilayer structures proposed to overcome this effect \cite{3} seem very complex and promise only a little cooling power. (Only at very low, millikelvin, temperatures where electron-phonon coupling is very weak, has efficient cooling been demonstrated using thermionic transfer over the superconductor energy gap \cite{4,5}.)

Even a very narrow (submicron) vacuum gap can effectively quench the back heat flow, reducing it to radiation-limited levels of the order of 0.1 W/cm$^2$ (at 300 K). Unfortunately, in this case the energy barrier height is determined by the conductor workfunction which is too high ($\gg k_B T$) for most materials. A natural way to enforce electron transfer through a relatively high barrier is to apply a strong electric field ($\sim 10 \text{ MV/cm}$), inducing Fowler-Nordheim tunneling through the barrier. However, in typical situations the tunneling through the initially uniform barrier pulls out electrons within a relatively broad energy range that results in heating rather than in cooling (the “Nottingham effect” \cite{6}).

We propose to limit the energy range of transferred electrons using resonant tunneling in a simple structure (Fig. 1) where the bulk emitter (a metal or a heavily doped semiconductor) is covered with a thin (a few nm) layer of a widegap semiconductor. While at zero voltage the electron potential energy profile of this structure has two steps (Fig. 1(a)), its tilting by the applied electric field creates a triangular-shape potential well at the semiconductor film surface (Fig. 1(b)). Quantization of the energy of electron motion perpendicular to the surface results in discrete levels (subbands for the full energy) localized at the surface. If the electric field aligns the lowest subband with energy levels of the hottest electrons in the emitter (a few $k_B T$ above the Fermi level), resonant tunneling of these electrons to vacuum may lead to very efficient heat removal, and hence to emitter cooling \cite{7}.

Our proposal hinges on several ideas put forward earlier. First of all, numerous experiments indicate that Fowler-Nordheim emission from bulk cathodes is frequently enhanced by resonant tunneling through localized surface states arising from its unintentional contamination (see, e.g., Refs. \cite{8}). Cooling of the nanoclusters using this effect was proposed in Ref. \cite{9}. (Cooling of 2D electron gas based on the resonant tunneling through specially fabricated quantum dots was proposed even earlier \cite{10}). However, to extend cooling to macroscopic objects, a large number of surface nanoparticles should be used in a single device. In this case, unavoidable spread of the size and shape of these particles would result in fluctuations of the resonant level positions, preventing their proper alignment with the Fermi level of the emitter, unless nanoscale fabrication with atomic precision is used - a very distant prospect indeed. In contrast, our suggestion involves only planar structures and does not require nanofabrication.

Concerning planar structures, Fowler-Nordheim tunneling via the resonant subbands was predicted long ago \cite{11} and then observed in several systems \cite{12,13}. The emission via resonant subbands at the outer surface of a widegap semiconductor in a strong electric field was predicted in Ref. \cite{14}. The Fowler-Nordheim emission coupled to the electron resonance in the vacuum gap was considered in Ref. \cite{15} (the implementation of this effect would, however, require an impractically fine uniformity of the gap). However, the possibility of heat removal was not mentioned in any of these publications.

The objective of this work was a quantitative analy-
sis of the cooling effect in the system shown in Fig. 1, using a simple but natural model. First of all, we assume the interfaces to be perfectly plane. In this case the electron motion in the direction of tunneling (x-axis) and in the perpendicular direction (along the interfaces) are separated. Neglecting band bending due to quantum well charging (i.e. assuming its shape to be triangular) we have the following well known result for the electron eigenenergies (see, e.g., [21]):

\[ E = E_x + E_\perp, \quad E_x = U - eEd + E_n, \]
\[ E_n = (-a_n) (e^2E^2h^2/2m)^{1/3} \]

(1) (2)

where all energies are relative to the emitter Fermi level, \( U \) is the initial energy step (Fig. 1), \( E \) is the electric field in the film, \( d \) is the film thickness, \( E_\perp = \hbar^2k_{\perp}^2/2m \), \( m \) is the electron effective mass in the conduction band of the film, and \( a_n \) is the sequence of Airy function zeros: \( a_0 = -2.34, a_1 = -4.09, \ldots, a_n \to -[3\pi(n + 3/4)]^{2/3} \).

In absence of energy relaxation, the level filling probability \( p = p_n(E_\perp) \) may be found from the stationary solution of the usual master equation, giving \( p = f\gamma_L/(\gamma_L + \gamma_R) \), where \( f = f(E) \) is the Fermi distribution of the emitter electrons, and \( \gamma_L \) and \( \gamma_R \) are the rates of electron escape from the quantum well into conductor and into vacuum, respectively. These rates may be calculated as \( \gamma_L,R = \nu D_{L,R} \), where \( \nu \) is the “attempt frequency”, \( \nu = [2\int dz/e(x)]^{-1} = E_n/2\hbar|a_0|^{3/2} \), and \( D_{L,R} \) are transparencies of the left and right triangular barriers [Fig. 3(b)]. Within the WKB approximation (neglecting the image charge effects),

\[ \ln D_L = -\frac{4\sqrt{2m}}{3\epsilon_0Eh} (eEd - E_n)^{3/2}, \]
\[ \ln D_R = -\frac{4\sqrt{2m_0}}{3\epsilon_0Eh} (\Phi - U - E_n - \Delta E)^{3/2}. \]

(3) (4)

Here the shift \( \Delta E = (\hbar^2k_{\perp}^2/2)(m^{-1} - m_0^{-1}) \) is due to the possible difference between \( m \) and the electron mass \( m_0 \) in vacuum, \( \Phi \) is the work function of the bulk emitter, and \( E_0 \) is the electric field in vacuum. The relation between this field and \( E \) includes the 2D charge density \( \sigma \) of the electrons accumulated in the well:

\[ \epsilon_0E_0 = \epsilon_0E + \sigma, \]

(5)

(\( \epsilon \) is the dielectric constant of the semiconductor film). The charge density \( \sigma \), as well as the resonant current density \( j \) and thermal flow \( q \), may be calculated as

\[ \sigma = \sum_{n,k} e_p, \quad j = \sum_{n,k} e \gamma_R p, \quad q = \sum_{n,k} E \gamma_R p. \]

(6)

When the quantized level is above the emitter Fermi level, the typical spread of \( E_\perp \) for the electrons in the subband is of the order of temperature \( T \) (here and below measured in energy units). Hence, assuming that the barriers are much higher than \( T \), we can neglect the term \( \Delta E \) in Eq. (4). Then integrating over \( E_\perp \), we get:

\[ j = e\rho \sum_n T \ln(1 + e^{-E_n/T}) \frac{\gamma_L \gamma_R}{(\gamma_L + \gamma_R)}, \]
\[ q = \rho \sum_n \left[ E \frac{\gamma_L \gamma_R}{(\gamma_L + \gamma_R)} \right]^2 + T^2 \ln(1 + e^{-E_n/T})^2 \]

(7) (8)

where \( \text{Li}_2(z) = \sum_{n=1}^\infty z^n/n^2 \) is the dilogarithm function and \( \rho = m/\pi h^2 \) is the 2D density of states per unit area.

Equations (7), (8) do not include the components of current \( j' \) and heat flow \( q' \) which are due to nonresonant, direct tunneling through the complete energy barrier. For this process, the barrier transparency may be calculated as \( D = D_L D_R \). A standard WKB calculation yields:

\[ j' = \frac{e\rho}{2\pi^2 h^2} \frac{D_L^2 D_R^2}{|\sin t|} T \ln(1 + e^{-E_n/T})^2 \]
\[ q' = -\frac{\rho}{2\pi^2 h^2} \frac{D_L^2 D_R^2}{|\sin t|^2} T \ln(1 + e^{-E_n/T})^2 \]

(9) (10)

where \( 1/E_0 = d(-\ln D)/dE = 1/E_0 + 1/eE_0 \),

\[ E_0^L = e\hbar E/2(2m)^{1/2}[U^{1/2} - \max(0, U - eEd)^{1/2}], \]
\[ E_0^R = e\hbar E/2(2m_0)^{1/2}[\Phi - eE_0]^{1/2}, \]

\[ \ln D_0^L = -\frac{4(2m)^{1/2}}{3eEh} [(U - E_x)^{3/2} - \max(0, U - E_x - eEd)^{3/2}], \]
\[ \ln D_0^R = -\frac{4(2m_0)^{1/2}}{3eE_0h} \max(0, \Phi - eE_0 - E_x)^{3/2}. \]

(Notice that these formulas may be used even if \( E_x > U - eEd \). The exact calculation in this case would give an extra factor on the order of unity, however, it can be neglected within the accuracy of WKB approximation.)

The well-known factor \( 2\pi t/\sin t \) shows that our approximation, based on the linear expansion of \( \ln D \) near the Fermi surface, can be used only at \( T < E_0 \). (At higher temperatures the transport at large energies prevails.) At “low” temperatures we are discussing, the nonresonant tunneling always provides heating of the emitter, although it changes to cooling at the inversion temperature \( T_{inv} = E_0/2 \).

Figure 2 shows one of the results of our calculations using Eqs. (7)-(10). The cooling power first increases exponentially with the field, because the lowest subband is aligned with more and more populated hot electron levels, and then drops sharply as soon as the subband approaches the Fermi level (at larger fields \( q \) becomes negative). Just before this drop, the cooling power reaches a maximum, in this case, as high as \( 3 \times 10^3 \) W/cm² at \( T = 300 \) K.

The maximum values of \( q \), as well as the corresponding values of \( j' \) and \( q' \), for several other parameter sets
are listed in Table I. From Fig. 2, one can see that the suitable range for the electric field \( E \) shrinks rapidly as emitter temperature goes down. Nevertheless, our model indicates that \( q + q' \) may be positive (i.e., cooling is still possible) for temperatures as low as 10 K.

Let us discuss how realistic our model is. First of all, Eqs. (1) are valid only if the energy relaxation in the well is much slower than \( \gamma_L, \gamma_R \). We have also neglected the resonant subband broadening due to tunneling, but it was monitored through our calculations, so that the results are presented for only such parameters that the broadening is negligible in comparison with the important energy scale, \( T \). One more possible source of deviations from our model is electron scattering in the well and during tunneling. However, these processes cannot lower the lowest subband deeper into the well, and so can hardly affect the process of hot electron extraction. Next, we have implicitly assumed that the Fermi energy of the bulk emitter is much larger than all the considered energies. If this condition is not satisfied, the results would change, but not significantly.

Despite the used assumptions, we expect that for smooth films the overall accuracy of our results for \( j, q, j', q' \) for a given applied field is limited mainly by that of the WKB approximation \( 20 \% \) and can be characterized by a numerical factor of the order of two or three. On the logarithmic scale at which we are working (Fig. 2) this is good accuracy indeed. Since the results show that the resonant emission cooling at temperatures above ~100 K may prevail over the nonresonant heating in a relatively broad range of electric field, and their ratio may be very high, we are confident that the net cooling of the emitter may be achieved. However, the estimate of the lowest achievable temperature (10 K) may be more vulnerable.

The largest problem we see with the experimental implementation of resonant emission cooling is the necessary film uniformity. In fact, Table I shows that at 300 K the effect is stable with respect to substantial (~20\%) variations of \( d \). However, to achieve cooling to 100 K, film thickness variations should not exceed ~4\%, i.e., about one monolayer. (Thickness fluctuations require the electric field to be decreased below the optimal value in order to be sure that we have not stepped into the heating region on any considerable fraction of the emitter area.)

While the proposed device potentially offers very large cooling power, its efficiency (more exactly, the “coefficient of performance”, COP) may be relatively low. For our system, COP may be presented as \( (q + q')/V(j + j') \), where \( V \) is the applied voltage. Even if the vacuum gap \( d_0 \) is as small as 10 nm, the necessary voltage \( V = Ed + Ed_0d_0 \) exceeds 10 V, giving COP about \( 10^{-3} \) at \( T = 300 \) K. To increase the COP, the electric field may be provided by a micromachined “grid” electrode very close to the surface, followed by another, much more distant grid at approximately the same voltage, and a collector at lower voltage, so that electrons are decelerated before the landing (see, e.g., \( 20 \)).

To summarize, we have proposed a new method of electronic refrigeration using the resonant Fowler-Nordheim tunneling in a fairly simple structure. If the experiment confirms our theory, this device may be invaluable for heat removal from electronic chips, as well as for the integration of advanced low-temperature devices with room-temperature circuits.

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FIG. 1. The energy diagram of the proposed device: (a) in absence of bias and (b) at finite electric field. Resonant tunneling via quantized levels above the Fermi energy removes the hot fraction of electrons, thus cooling the emitter.

FIG. 2. Solid lines: the resonant current density $j$, the corresponding cooling power density $q$, nonresonant current $j'$, and the corresponding heating power $-q'$ as functions of the applied electric field $E$ for $\Phi = 4$ eV, $U = 1$ eV, $m = 0.5m_0$, $m_c = m_0$, $\epsilon = 5$, and $d = 2.5$ nm, at $T = 300$ K. The dashed lines show the cooling power $q$ at $T = 100$ K and $T = 30$ K (the curve for $-q'$ does not change significantly when the temperature is lowered).

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| $\Phi$ (eV) | $U$ (eV) | $m$ ($m_0$) | $d$ (nm) | $T$ (K) | $E$ (MV/cm) | $j$ (kA/cm$^2$) | $q$ (W/cm$^2$) | $-q'$ (W/cm$^2$) |
|------------|-----|-----------|---------|-------|----------|--------|--------|--------|
| 4          | 1   | 0.5       | 2.5     | 300   | 7.2      | 90     | 3000   | 8      |
|            |     |           |         |       |          |        |        |        |
|            | 100 | 7.2       | 30      | 300   | 8        |
|            | 30  | 7.2       | 7       | 20    | 7        |
| 4          | 1   | 0.5       | 2.7     | 300   | 6.4      | 30     | 1000   | 0.8    |
|            |     |           |         |       |          |        |        |        |
|            | 100 | 6.4       | 10      | 100   | 1.0      |
|            | 30  | 6.4       | 3       | 9     | 0.9      |
|            | 10  | 6.4       | 1       | 1     | 0.9      |
| 4          | 1   | 0.2       | 7       | 300   | 6.8      | 400    | 10000  | 900    |
| 5          | 1.5 | 0.2       | 7       | 300   | 7.4      | 20     | 900    | 10     |
|            | 100 | 7.4       | 8       | 90    | 15       |
Fig. 1

(a) Schematic diagram of a field emitter. The emitter is surrounded by a widegap semiconductor and vacuum. The work function $\Phi$ and the field $E_x \sim k_B T$ are shown.

(b) A close-up view of the emitter showing the vacuum and the vacuum region.

Fig. 2

Graph showing the temperature dependence of the emission currents $q$, $q'$, and the current density $j$, $j'$ as a function of the electric field $E$. The parameters $d = 2.5 \text{ nm}$, $\Phi = 4 \text{ eV}$, $m = 0.5 m_0$, $U = 1 \text{ eV}$, $m_e = m_0$, $\epsilon = 5$, and the temperatures 300 K, 100 K, and 30 K are indicated.