Application of sensitivity analysis in building energy simulations: combining first- and second-order elementary effects methods.

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Abstract

Sensitivity analysis plays an important role in the understanding of complex models. It helps to identify the influence of input parameters in relation to the outputs. It can also be a tool to understand the behavior of the model and can then facilitate its development stage. This study aims to analyze and illustrate the potential usefulness of combining first and second-order sensitivity analysis, applied to a building energy model (ESP-r). Through the example of an apartment building, a sensitivity analysis is performed using the method of elementary effects (also known as the Morris method), including an analysis of the interactions between the input parameters (second-order analysis). The usefulness of higher-order analysis is highlighted to support the results of the first-order analysis better. Several aspects are tackled to implement the multi-order sensitivity analysis efficiently: interval size of the variables, the management of non-linearity and the usefulness of various outputs.

\textbf{Keywords:} Energy demand, Buildings, Sensitivity analysis, Morris method, Elementary effects, Building thermal simulation

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1. Introduction

Energy consumption related to the building sector is recognized as a major part of the total energy consumption worldwide (37% of the final energy consumption in the EU in 2004) \[1\] and consequently a significant source of greenhouse gas emissions \[2\]. The growth in population, building services and comfort levels guarantees that this tendency will continue in the forthcoming years. Many tools have been developed to model the energy consumption in buildings (EnergyPlus, TRNSYS, ESP-r), particularly for end uses such as space heating and cooling, ventilation and lighting. In most cases, such models take into account coupling between phenomena (e.g. interactions between occupancy, micro-climate, envelope, and HVAC...) through coupling of different specialized sub-models and by using a large number of diverse input variables.

Sensitivity analysis can help in understanding the relative influence of input parameters on the output \[3\]. In the field of building energy models, combining sensitivity analysis and simulations tools can be useful as it helps to rank the input parameters (or family of parameters) and then to select the most appropriate to be considered, depending on the objective of the modeling. For example, this is particularly interesting when the modeling objective is related to the building design (e.g. sketch stage of the design, modeling retrofitting scenarios according to the only available input data) or when it is to define archetypes. Another application is in the development stage of the tools, and more precisely the definition of possible simplification of models or in the validation of assumptions in the selection of input parameters that must be considered. In these cases, and depending on the objectives of the tool developed, some sub-models and their corresponding input data may become secondary. A solution consists of using a detailed model in the upstream stage, combined with a sensitivity analysis in order to rank the set of parameters and identify the coupling between them. Then, the selection of the most important variable helps to define the structure of the simplified model.

In this study, we propose combining the implementation of ESP-r \[4\] with two
sensitivity analysis techniques: the Morris method and an extension of this methodology for the analysis of interactions between the parameters. In section 2, sensitivity analysis methods are quickly reviewed. Then, the elementary effects method and its second-order variant are described, together with the apartment building test case (section 3). Finally, results are presented and discussed for the two methods used (section 4).

2. Background

2.1. Sensitivity analysis: current approaches

Sensitivity analysis methods have been studied by many authors in the past decades as they have demonstrated their strength in many sectors. Throughout this period new methods and improvements have been developed, offering different solutions depending on the objective. Hamby proposed an inventory of techniques for parameter sensitivity analysis which he divided into three different categories:

- Sensitivity analysis methods assessing the influence of individual parameters. These include Differential Sensitivity Analysis, One-at-a time sensitivity measures, Factorial Design, Sensitivity Index, Importance Factors, and Subjective Sensitivity Analysis.

- Parameter sensitivity analysis utilizing random sampling methods (simple random sampling, Monte Carlo, Latin Hypercube). In this group are listed the methods: Scatter plots, Importance Index, ‘Relative Deviation’, ‘Relative Deviation Ratio’, Pearson’s $r$, Rank Transformation, Spearman’s $\rho$, Partial Correlation Coefficient, Regression, and Standardized Regression techniques.

- Sensitivity tests involving segmented input distributions: the Smirnov test, the Cramer Von-Mises test, the Mann-Whitney test, and the squared-ranked test.

The author then applied these different methods to a case study related to the nuclear industry, in order to compare them in terms of reliability, computational requirements and ease of implementation. The study identified the One-at-a-time method as being the simplest but pointed out that it becomes time-intensive with large numbers of parameters. Saltelli et al. also describe the
different sensitivity analysis techniques. For these authors, techniques can be divided between global and local methods. Local methods are commonly based on the estimation of partial derivatives in order to obtain a qualitative analysis of the importance of each factor on the output response for a limited subset and particular values of the input variables. Global methods vary all the parameters and try to obtain information for a subset of input variables in a wider domain. Global methods can also be divided into quantitative and qualitative techniques. Santner et al. [10] developed the parallel between the physical experiments and the concept of computational experiments as it is understood in this study. In particular, these authors described the added value of sensitivity analysis in such an experiment. This was taken up by Saltelli et al. [9] who presented the One-at-a time sampling for sensitivity analysis for multiple parameters.

2.2. Principles of the elementary effects method

The Morris method is derived from OAT (One-factor-at-a-time) screening methods to identify the subset of the main important input factors among a large number of $k$ input parameters in a model. This method characterizes the sensitivity of a model with respect to its input variables through the concept of elementary effects, which are approximations of the first order partial derivatives of the model [5]. These elementary effects are estimated at various sampled points, randomly selected on a $p$-values regular grid, defining a relevant design of computational experiments. The average and standard deviations of elementary effects enable negligible and influencing variables to be sorted and linear and non-linear influences to be distinguished. In some respect, this method can be considered intermediate between a local sensitivity analysis and global quantitative methods described above. It is a general approach (model-independent), which achieves a good compromise between accuracy and efficiency. Applications can be found in a number of fields including Environmental Modeling and Agriculture [11], Biophysics [12] and Nuclear Engineering [13]. However, in spite of its advantages, its applications still remain limited. Other methods, such as variance-based sensitivity indices (VBM), have been
Although they generally provide better information to distinguish non-linearities and interactions, the computational cost is much higher: a variance-based analysis for a 12-input parameter model requires at least 14,000 runs of the model, about one hundred times the cost of a first-order Morris analysis (and still ten times more than a second-order Morris analysis).

2.3. Experience with the elementary effects methods for building thermal simulation

Some studies have tested the advantages of the Morris method applied to building energy simulations. Breesch and Janssens implemented it to identify the most important parameters that cause uncertainty in the predicted performances of natural night ventilation. They used a two-zone model in the thermal simulation tool TRNSYS coupled with an infiltration model COMIS. This analysis revealed that the internal heat gains, local outdoor temperature and the diurnal internal convective heat transfer coefficient were the parameters with the greatest impact on thermal comfort. Brohus et al. applied the methodology to reduce a set of 75 parameters used to obtain an accurate output energy consumption distribution. The Morris method was also used as a first indication of correlation or non-linear effects between the parameters. Finally, this method was compared with the Fourier Amplitude Sensitivity Testing method (FAST). The results of both analysis helped in evaluating a safety factor for the annual energy consumption at the design level. De Witt compared the Morris method with the sequential bifurcation technique using a mono-zone office of 81 parameters as a model. Both techniques found the same set of important parameters (12) which explained 94% of the variability of the model output defined as the number of hours of overheating. Corrado and Mechi analyzed the heating and cooling needs of a two-storey single-family house in Turin with the Morris method to calculate the uncertainties in energy rating. The sensitivity analysis showed that only 5 of 129 factors were responsible for most of these uncertainties: the indoor temperature, the air change rate, the number of occupants, the metabolism rate and the equipment heat gains. Heiselberg et al. identified the most important design parameters in relation to a building’s
performance with a focus on the optimization of sustainable buildings. They found that the mechanical ventilation rate in winter and lighting control were the most influential parameters in an office building of 7 floors.

The extension of the Morris method for second- and upper-order analysis has still not been applied in the area of building energy simulations despite its advantages stated in other analysis and its low computational cost compared to more sophisticated techniques like variance-based and FAST methods.

3. Methodology

3.1. The elementary effects method

The building thermal model can be represented by a function \( y(x) \) where \( y \) is the output variable of interest (scalar) and \( x \) is a vector of real input variables with \( k \) coordinates, each input variable being defined within the range of a continuous interval. Input variables are transformed into reduced dimensionless variables in the interval \((0; 1)\) as \( x'_i = \frac{x_i - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \).

\( x_{\text{min}} \) and \( x_{\text{max}} \) are the minimum and maximum of the input variable \( x_i \), respectively. The domain of the vector \( x \) is then a hypercube \( H^k \) with unit length, a subset of \( \mathbb{R}^k \). For each reduced input variable, only discretized values are considered, using a \( p_i \) values regular grid (with step \( \frac{1}{p_i-1} \)): 0; \( \frac{1}{p_i-1} \); \( \frac{2}{p_i-1} \);...; 1.

Although a single grid value is generally used for all the variables, it is possible to use a specific one for each input variable \( x_i \). This enables qualitative input variables with two levels to be incorporated, represented in the Morris design by a 2-value grid, while keeping a more precise grid for continuous input variables.

A simulation trajectory is defined as a sequence of \((k + 1)\) points in this hypercube, with each point differing from the preceding one by only one coordinate. In a trajectory, each input parameter only changes once with pre-defined step \( \triangle_i \). The function \( y \) (i.e. the simulation model) is evaluated for every point in the trajectory. The first point of a trajectory is randomly selected. At each step in a trajectory, the coordinate to be modified is also randomly selected. Thus various trajectories differ by their starting point and by the order of modified coordinates. In order to ensure a strict equal probability for each value, Morris
suggested taking \( p \) even and \( \Delta_i = \frac{p}{2(p-1)} \). So, if the initial value \( x_i \) is lower than 0.5, the final value is \( x_i + \Delta_i \); if it is greater than 0.5, the final value is \( x_i - \Delta_i \). Taking this \( \Delta_i \) value corresponds to a uniform distribution over the discrete grid values in intervals \([0;1]\) for all input variables over the hypercube \( H^k \).

A trajectory enables a coefficient of variation for each input variable \( i \), called an elementary effect (EE) to be evaluated. It is computed between the two points of the trajectory where this input variable \( i \) is modified (equation 1)

\[
EE_i = \frac{y(x + \epsilon_i \Delta_i) - y(x)}{\Delta_i}
\]  

(1)

\( \epsilon_i \) is a vector of zeros but with its \( i \)-th component equal to \( \pm 1 \). Each trajectory, with its \((k+1)\) simulations, provides an estimate of the \( k \) elementary effects. A set of \( r \) different random trajectories (with index \( t \)) is defined in the hypercube of input variables; it provides \( r \) estimates \( EE_{it} \) of elementary effects related to each input variable \( i \), at the cost of \( r \times (k+1) \) simulations. The average and the standard deviation of the elementary effects are computed for each input variable \( i \) (equations 2 and 3):

\[
\mu_i = \frac{1}{r} \sum_{t=1}^{r} EE_{it}
\]

(2)

\[
\sigma_i = \sqrt{\frac{1}{(r-1)} \sum_{t=1}^{r} (EE_{it} - \mu_i)^2}
\]

(3)

Campolongo et al. [21] recommend using the average of absolute elementary effects (equation 4) rather than the usual average, since some elementary effects can eliminate each other in non-monotonic models.

\[
\mu_i^* = \frac{1}{r} \sum_{t=1}^{r} |EE_{it}|
\]

(4)

The criterion \( \mu_i^* \) is a good indicator to classify input variables by order of importance, despite the fact that information about the sign of the elementary effects is lost. Moreover, the standard deviation of the elementary effects is a relevant
indicator of non-linearity in input parameters of the model or interactions with other parameters involved in the model. By plotting both statistical indicators, the Morris method identifies the inputs that can be considered to have an effect:

1. Negligible (low average, low standard deviation)
2. Linear and additive (high average, low standard deviation)
3. Non-linear or involved in interactions with other input parameters (high standard deviation).

3.2. The second-order interactions sensitivity analysis

If parameters show a marked non-linear behavior and a significant influence in the model, a subsequent experiment with only these inputs is recommended. Campolongo and Braddock [6, 22] proposed an extension of the Morris method which enables the calculation of at least second-order effects with a reasonable computational cost. This method is based on the calculation of the equivalent of a second-order derivative of the model and optimizes the computational experiment by using the solution of the “handcuffed prisoner problem” [23]. Thus, the number of evaluations of the model required to obtain one second-order elementary effect is optimized (about $k^2r$). Campolongo and Braddock [6] defined a second-order elementary effect (equation 5):

$$ EE_{ij} = \left| \frac{SEE_{ij}}{\Delta_j} - \frac{EE_i}{\Delta_i} \right| $$

(5)

where $SEE_{ij}$ characterizes the influence due to the change of both factors $i$ and $j$ at the same trajectory (equation 6):

$$ SEE_{ij} = \frac{y(x + e_i\Delta_i + e_j\Delta_j) - y(x)}{\Delta_i\Delta_j} $$

(6)

A further extension to the analysis of third-order (or higher) interactions was proposed by Campolongo and Braddock [6]. Unfortunately the computational cost of this analysis can be prohibitive if the model is too complex. One practical solution is to perform a second-order experiment first and then an analysis of
third-order effects only in the group with the highest values of standard deviation in the second-order analysis.

Statistics can be applied to second-order elementary effects so that the results of the mean, absolute mean and standard deviation of the effects can be interpreted in the same way as in the first-order Morris method \[6\]. In addition, as the standard-deviation of the first-order provides information about second- and higher-order interactions between the parameters, the second-order extension can give valuable data about the interactions of third- and upper-order degrees. The combination of both methods at the same intervals enables the importance of a parameter in the first, second, and higher orders to be classified. This combination gives a better interpretation of complex models and can be applied to the construction of reduced models based only on the most important parameters.

3.3. Test case: an apartment building

In order to explore the potential of the elementary effects method, a rather complex multi-zone building was chosen as a test case. It is a seven-storey residential building with 32 dwellings and an estimated population of between 70 and 80 inhabitants (Figure 1). It has an approximate floor area of 3500 \(m^2\), and the average yearly energy needs for heating are 67.59 \(kWh/m^2\). The characteristics of this building are given in Table 1. It is connected to a heating network and no cooling equipment is available. The glazing ratio is 30%, composed mainly of double glazed windows. All exterior walls are insulated with 10 cm of insulating material. The building is divided into 24 thermal zones (two heated and non-heated zones per floor).

The building was modeled using the energy calculation software ESP-r \[4\]. The choice of ESP-r was driven by its detailed sub-models as well as the numerous input variables considered in this tool, allowing many possible situations of influential factors to be investigated. The \(p\)-dimensional grid was defined as \(p=10\) for all experiments, so each reduced input variable could take the discretized values 0, 0.111, 0.222, ..., 0.889, 1 in the reduced interval \([0; 1]\) and simulations
were made with regularly spaced values between a minimum and a maximum. The numerical experiments were classified into two different sets (A, B) based on the number of parameters involved in the analysis (see Table 2 for the definition of all input variables and their range).

In the set of experiments (A), a first-order effects sensitivity analysis was performed with 24 parameters, representing different choices in building design. These design parameters were the corrections for building height (input 1), width (2) and length (3) as well as the building rotation (24), insulation thickness (23) and glazing ratio (12-15). Corrections of the weather parameters used in the analysis were added in a first attempt to quantify the importance of the environment of the building in the different outputs studied (19-22).

For some parameters, common values for all heated zones except one were taken: heat gains due to occupants (8), the set point temperature (4), the ventilation rate (10) and the difference in the set point temperature between day and night (6). The objective was to see the influence of these common parameters compared to the climatic and design factors that also affect the entire building.

In addition, to determine the relative influence of these parameters in a single thermal zone, apartment E42 (which is located on the fourth floor) was chosen arbitrarily to change these parameters only in this specific zone. However, the specific results from this apartment are not presented here, as they do not affect the results for the building as a whole (parameters 5-7-9-11).

For the first numerical experiment (A), rather broad intervals were chosen for all input variables, representing the diversity of the characteristics of apartment buildings in an urban context on a large scale (e.g. at national level), regarding size, insulation, glazing ratio, and ventilation. In a similar way, broad ranges were taken for variables concerning the building environment (particularly the climate) and occupancy. Nevertheless, the values adopted remain representative of the variations or uncertainties usually considered. It was a deliberate choice to start with such a “maximum-variability” scenario in order to be representative of apartment buildings at a national level. One possible application could be to identify the data to be collected with high priority for a good description of the
building stock in a region or a country.

In the second-order Morris extension experiments (set B), only 12 of 24 parameters from the previous experiment were selected. This choice was made due to the high computational cost of the analysis.

4. Results and discussion

4.1. General remarks on the presentation of results

The results of the elementary effects analysis of building thermal simulations are presented as scatter plots with a point for each input variable $i$: the x-axis represents the absolute average ($\mu_i*$) and the y-axis represents the standard-deviation of the elementary effects ($\sigma_i$) (see, for example, Figures 3 to 6).

The absolute average ($\mu_i^*$) was introduced above as a measure of importance for the input factor $i$. This information can be complemented by the ratio ($\sigma_i/\mu_i^*$) as an indicator of linearity (or non-linearity), as justified below.

First, looking at one input variable $i$: if all estimates of elementary effects $EE_{it}$ have the same sign, we can say that this input factor $i$ has a monotonic effect on the response $y_i$ increasing or decreasing, depending on the sign of the elementary effects. In this case, $\mu_i^*$ is equal to the absolute value of $\mu_i$. The reverse is also true: if $\mu_i^* = \text{abs}(\mu_i)$, the effects of input variable $i$ are monotonic.

Using well-known statistical properties, if elementary effects are assumed to be normally distributed, 95% of $EE$-estimates are within the range $(\mu_i \pm 1.96 \sigma_i)$. As a consequence, if $\sigma_i/\mu_i$ is smaller than 0.10, most elementary effects (95%) are in a range $\pm 20\%$ around $\mu_i$: the elementary effects are almost constant and the input variable $i$ has an almost linear effect on the model. A true linear response correspond to $\sigma_i/\mu_i = 0$.

If the ratio $\sigma_i/\mu_i$ is smaller than 0.5, most elementary effects (95% with the normal assumption) have the same sign and the model response can be considered as monotonic with respect to the input factor $i$. Thus, in this case, it can be considered that $\mu_i \approx \text{abs}(\mu_i)$ and $\sigma_i/\mu_i^* \approx \sigma_i/\text{abs}(\mu_i)$. This fact justifies using the ratio $\sigma_i/\mu_i^*$ as an indicator for almost linear (if $< 0.1$) or monotonic influences (if $< 0.5$).
As the distribution of elementary effects may be far from normal, another justification is presented in Figure 2 with a scatter plot of \( \sigma_i/\mu^*_i \) vs. \( \sigma_i/abs(\mu_i) \) for all first-order analyzes performed within the present work (sets A and B1, B2).

This diagram shows that for \( \sigma_i/abs(\mu_i) \) smaller than one, most of the points are on the bisector \( \sigma_i/\mu^*_i = \sigma_i/abs(\mu_i) \), indicating a monotonic (or almost monotonic) behavior in a wider interval than expected (not only < 0.5). For highly scattered elementary effects (\( \sigma_i/abs(\mu_i) > 1 \)), a non-monotonic behavior is clearly established and, in this case, \( \sigma_i/\mu^*_i \) stays in the interval between 1 and 2. The absolute average \( \mu^*_i \) is very different from the average \( abs(\mu_i) \) and is more influenced by the standard deviation \( \sigma_i \).

So, by plotting three straight lines of slopes \( \sigma/\mu^* = 0.1, 0.5 \) and 1, respectively, we can graphically identify in the elementary effects scatter plot, those factors which are almost linear (below the line \( \sigma/\mu^* = 0.1 \)), monotonic (\( 0.5 > \sigma/\mu^* > 0.1 \)) or almost monotonic (\( 1 > \sigma/\mu^* > 0.5 \)), and those factors with marked non-monotonic non-linearities or interactions with other factors (\( \sigma/\mu^* > 1 \)).

Defining these four zones also provides a means of checking the results of the sensitivity analysis if the results contradict what is understood from the physical point of view.

For some of the figures (for example Figs. 5c, 5d, 6c and 6d), a slightly different presentation of data has been used with the ratio \( \sigma_i/\mu^*_i \) on the vertical axis (the different domains defined above as linear, monotonic and highly non-linear being distinguished by horizontal lines). This presentation provides exactly the same information but is sometimes easier to read as the different points are better separated.

### 4.2. Computational experiment A: first-order analysis results

For the first set of numerical experiments (set A), several characteristics for output results were examined using the first-order elementary effects analysis: yearly heating loads (Fig. 3a), outputs derived from the latter (heating load per m\(^3\); logarithmic transformation) (Fig. 3b - 3d), heating power (with the example of the power exceeded during 1000 hours/year) (Fig. 4b) and the
summer comfort factor, using the average internal temperature in July-August (Fig. 4a).

Before presenting the results, it is important to recall the definition of elementary effects: one value of the elementary effect for the input variable \( i \) corresponds to the output variation when the input \( i \) moves from the minimum (0) to the maximum (1). For example, in Figure 3a, the average elementary effect for the building height (input 1) has a value of 470 000 kWh/year. This is the average influence of a modification of building height from 9 m to 27 m (nominal height is 18 m, corrected by \( \pm 50\% \)). For the building considered and the wide intervals taken for input parameters, the yearly energy needs are within the range of 57 587 kWh/year to 788 894 kWh/year.

The first analysis based on the yearly heating load (Fig. 3a) highlights the dimensional parameters (numbered 1, 2 and 3), indicating a typical size effect: building size is the major influence on heating demand. Next, a second group of input parameters appears with a significant influence: set point temperature (4), ventilation rate (10) and insulation thickness (23). The remaining parameters can be classified as less important, although not negligible for a number of them (6, 8, 12, 13, 19, 20, 21, 24) which will be discussed below in this section. Only one parameter has an almost linear effect (occupant free heat gains, 8) with \( \sigma/\mu \) close to 0.1. All other parameters show a non-linear influence and/or interactions with other parameters (\( \sigma/\mu > 0.5 \) for many of them). Nevertheless, they generally stay within the monotonic zone (\( \sigma/\mu < 1 \)) with the notable exception of insulation thickness (23).

An attempt to eliminate the size effect is made through the analysis of annual heating load per cubic meter, i.e. load divided by the three-dimensional parameters (Fig. 3c). The second group of parameters identified in the former case (4, 10 and 23) now becomes predominant, with a behavior closer to linear for the set point temperature (4) and ventilation rate (10) (\( \sigma/\mu \) equals 0.3 and 0.2, respectively).

A third group of parameters is emphasized by this presentation, including factors depending on occupants (free heat gains, 8, and heating set point night reduc-
tion, 6), solar gains (diffuse, 19, and direct, 21, radiation potential; building rotation, 24; glazing ratios on main facades, 12 and 13) and climatic sensitivity related to the building environment (external temperature, 20). Compared with Fig 3a all these parameters coincide with those already identified as secondary but non-negligible.

The importance of dimensional parameters (1, 2 and 3) is clearly reduced, but they remain among the non-negligible variables and contribute to the high variability of the model ($\sigma/\mu$ close to 1).

A logarithmic transformation is applied to the output to identify interactions caused by possible multiplicative effects (Figure 3b for annual heating load and 3d for heating load per cubic meter). Logarithmic transformation is often used for the statistical analysis of positive outputs with variations of several orders of magnitude or when multiplicative phenomena take place. If the response function can be expressed as a multiplicative function of various inputs $x_1, x_2, x_3...$ as $y(x) = f_1(x_1) \times f_2(x_2) \times f_3(x_3)...$, the elementary effects of $y$ vs. input variables $x_1, x_2, x_3...$ are highly influenced by interactions between $x_1, x_2, x_3...$. The standard deviations of their respective elementary effects can be expected to have high values. Taking the logarithm of $y(x)$ separates the terms $ln(y(x)) = ln(f_1) + ln(f_2) + ln(f_3)...$ and eliminates interactions: for the logarithm output, the standard deviation of EE depends only on the curvature of function $ln(f_1); ln(f_2); ln(f_3)...$ Another advantage of the natural logarithmic transformation is the interpretation of the elementary effects as rates of change of the output when the value remains low (typically <0.5). For instance, regarding the input variable 'night and day difference temperature' (6), the average elementary effect (expressed in logarithm) is 0.35 (here a decrease, i.e. -0.35, Figure 3d). This means that a full-scale change in this input (from 0 K to 8 K) leads to a 35% variation in space heating, on average.

The logarithmic transformation provides consistent results with the former ones: the same input parameters are identified as important and non-important with about the same ranking and similar clusters. The use of a logarithmic transformation emphasizes some factors with partial multiplicative effects, mainly
temperature-related parameters: set point (4), night reduction (6) and external temperature (20). Their $\sigma/\mu$ ratios are both reduced significantly when compared with Fig. 3a and 3b and are close to 0.1.

Other parameters remain non-linear with the logarithmic transformation, such as parameters related to the glazing ratio (12, 13) and rotation (24) or insulation thickness (23). It can be noted that the two presentations (Fig. 3b and 3d) in the logarithmic transformation lead to exactly the same coordinates for all input parameters, except the dimensional ones (1, 2 and 3).

As a partial conclusion, all the presentations show consistent results in terms of predominant parameters. The high variability of almost all the parameters encourages the use of a methodology to analyze the effects of interactions and non-linearities. It is notable that the most important parameters in the analysis (ventilation and set point temperature) are in accordance with a similar analysis carried out by Brohus et al. [17] for a single-family detached house model with 71 input parameters. The glazing ratio appears to be less important for the model output of energy demand for heating. Similar conclusions were also reported by Gasparella et al. [24] who did an analysis and modeling of various types of glazing in a high insulated building with a high percentage of double glazing.

The Morris method analysis was applied to the average internal temperature in July and August (Figure 4a) and to the power in the monotonic curve at hour 1000 (Figure 4b). The objective was to show the applicability of this method to different kinds of output. By comparing Figures 3b and 4b, we can conclude that there is a strong relationship between the output of heating needs in ln(kWh/year) and the monotonic curve of power. Only the set point temperature difference between night and day (6) becomes less important in terms of average elementary effects but with a higher standard deviation. On the contrary, the comfort criterion (Figure 4a) shows a very different behavior. The most important parameter is related to the external temperature with a highly linear effect. The next important parameters (with a high $\mu_i\sigma$) are related to the environment or the geometry of the building. According to Figure 4a, the glazing ratio of side B (parameter 13) seems to have more importance in
the variability of the average temperature than the other sides (parameters 12, 14 and 15). This implies that parameter 13 could have a major impact on the comfort level of the building. The same conclusions can be made by analyzing the geometrical parameters (1, 2 and 3). Thus, the Morris method could also serve as a diagnostic tool to help designers to review the impact of the building on comfort levels and to analyze in more depth the sensitivity of the building in terms of solar gains.

4.3. Computational experiment B: first- and second-order results

4.3.1. Data used in experiment B

In order to make the analysis done with the first-order approach in experiment A more precise, experiments B1 to B4 were performed, focusing on the study of the second-order interactions. Due to the high computational cost of second order analysis (5760 runs with $r = 10$), a subset of only 12 from 24 initial input parameters was selected (see Table 2). Parameters with negligible influence were omitted and important parameters (from experiment A) were kept with exceptions for some parameters playing similar roles: among size-related parameters (1, 2, 3), the height was omitted and among the glazing ratio of the facades (11, 12, 13, 14), only the one for facade A (11) was kept. Furthermore, the analysis was done for two interval sizes for the parameters: a large one (the same as the one used previously for the first-order experiment) and a small one (Table 2). These small intervals may be considered representative of the level of knowledge or uncertainties of the main parameters in the situation of a detailed energy audit (with measurements being made inside the building). Small intervals can be considered for parameters known approximately while large intervals can be used for totally unknown parameters.

Two different outputs are presented: the annual heating needs and the natural logarithm of the annual heating needs per cubic meter. Before running the second-order experiment, a first-order analysis is made, limited to the 12 input variables selected. So experiment B actually incorporates 4 numerical experiments: B1 (1st order, large interval, same interval as experiment A), B2 (2nd
order, large interval), B3 (1st order, small interval) and B4 (2nd order, small interval).

4.3.2. First order - large interval (B1)
A comparison between the two first-order experiments B1 (12 parameters: Fig. 5a and 5b) and A (24 parameters: Fig. 3a and 3d) demonstrates that the decrease or increase in the number of parameters involved in the sensitivity analysis does not significantly affect the ranking between the parameters. When comparing the two sets of results, one should keep in mind that differences may come from two sources:

- the Morris Method is basically a Monte Carlo approach with random generation, so changes may come from sampling differences,

- reducing the number of variables (with unchanged intervals) results in fewer interactions between input variables, so a lower standard deviation for elementary effects can be expected. This trend is actually observed for most of the variables for the two output responses considered, but with a limited decrease in the standard deviation.

4.3.3. Second order (B2)
For the second-order analysis (experiment B2, Fig. 5c and 5d), it can be seen by comparing the x-axis of Figures 5a and 5c that the second-order influences are not negligible. Figure 5c (with logarithmic response) shows that the average of the second-order elementary effect of the highest influence interaction (3;8) is almost 0.2, exceeding some significant first-order influences of parameters (e.g. 2 and 24) (Figure 5a). This influence becomes even more evident in the analysis of annual heating needs in kWh (Figures 5b and 5d). The average second-order elementary effect of the highest influence (3,2) exceeds first-order averages of elementary effects in all parameters with the exception of parameters 2, 3, and 4.

Size parameters must be emphasized as being the main source of interactions; width (2) and length (3) are included in the five highest second-order elementary effects in the two presentations. For the annual heating load (Fig. 5d), their
own interaction (2-3) is the main one followed by their interaction with the set point temperature (4) and the ventilation rate (10). These results are fully consistent with the first-order analysis (Fig. 5b) which pinpoints parameters 2, 3, 4, 10 and 23 (insulation) as the major influences and highly scattered elementary effects. In the logarithmic transformation, insulation thickness is clearly the factor with the highest sigma values (highly scattered elementary effects) so it is not surprising to see interactions with size parameters among the main second-order interactions. High interactions with occupant heat gain (8) in effects (3; 8) and (2; 8) are more surprising. A possible interpretation could be that free heat gains have an almost additive influence on the space heating load as indicated by the fact that parameter 8 is the only one in the linear zone in the first-order analysis (Figs. 3a and 5b). Dividing by the building volume introduces high interactions with size parameters while the logarithmic transformation introduces non-linearity for such additive phenomena: with the different transformations (Fig. 3b, 3c and 3d) input parameter 8 is no longer in the linear zone. No other factors are involved in the significant second-order interactions with occupants. Taking into account the size effect of occupants would suggest using occupants/m$^3$ (or occupants/m$^2$of – floor – area) as an input parameter instead of just occupants in further work.

Regarding the possible linearity of second order effects, no pair of parameters in Figures 5c and 5d are in the linear zone ($\sigma/\mu < 0.1$), which means that all pairs of parameters have non-linear effects or are combined with other parameters in the higher order. This can be compared to the first-order analysis for which only parameter 4 (set point temperature) falls in the almost linear zones (Fig. 5a). Continuing the analysis of parameter 4, Figure 5c emphasizes the strong interactions of this parameter with the size of the building (parameters 2 and 3). These interactions remain important even if the size effect is reduced by considering the heating needs per cubic meter as the output. Parameter 23 (insulation thickness of external walls) appears in all experiments (Figure 5) with high variability in the first order as well as in the second order. The non-linear influence of the insulation thickness can explain this high variability.
In contrast, the first-order elementary effect of set point temperature (4) (non-monotonic for the annual heating needs) moves to the linear zone after logarithmic transformation per cubic meter, with the highest influence. The standard deviation remains non-negligible if compared with other factors and a second-order interaction with size parameters appears even with the logarithm, but other interactions are limited.

4.3.4. First order - small interval (B3)

The same kind of analysis in Figure 5 can be found in Figure 6 where a small interval is considered. The main consequence of reducing the interval size of parameters (by a factor 10 for most of input parameters) is that almost all parameters are now in the linear zone for the first and second order. The set point temperature is again the most important parameter in this experiment with the highest average and standard deviation of the first-order elementary effects (all outputs in Fig. 6). The importance of set point temperature is highlighted by the fact that its variation interval is kept rather wide in the experiment (reduction by a factor 4 only if compared to the large interval). In the reverse, insulation thickness has now almost no influence on the output, due to a choice of rather well insulated wall (between 9 and 10 cm) for which the precise thickness is of little importance.

4.3.5. Second order - small interval (B4)

As first-order analysis results show almost a linear behavior for most of input parameters, one could consider that second-order analysis could be of little interest. Actually, results can be considered as “second-order” magnitude: Second-order average elementary effects are smaller than 0.3% in the logarithmic output, e.g.. Moreover, they are close to the standard deviation of the first order analysis in the two presentations -kWh or Ln(kWh/m³)-, showing that scattering of first order EE are mainly caused by interactions between input parameters and not by quadratic or other non-linear influence. For the yearly energy output (kWh; Fig. 6d), the four factors dominating the first order analysis (2, 3, 4 and 10) are encountered again: the six main second-order EEs correspond to their six
pairs of double interactions, with interactions between size parameters (2, 3) and set-point (4) being the largest values. In the logarithmic output (Fig. 6c), the interactions between size parameters (2, 3) and occupant heat gains (8) analyzed in section 4.3.3 for the large interval are encountered again. Size parameters are not involved in any significant interactions. But other interactions involving the set point temperature (4), the most influential parameter, are also identified with night-reduction set point (6) and occupant heat gains (8). For the first time in our analysis, input parameters related to a specific thermal zone of the building are shown as non negligible, with interaction between set point temperature in the whole building (4) and in this specific zone (5). Significant heat transfer between zones could explain this phenomenon which could become significant in situations where building characteristics are precisely known and important uncertainties could be related to diverse inhabitant behaviors in the various zones of the building. Standard deviations of second order elementary effects (in the logarithmic output) are always smaller than 0.02%, so third- or upper-order analysis will bring no useful information.

5. Conclusions

In this work, first- and second-order sensitivity analyzes have been combined. The Morris method and its extension have been applied to a building thermal simulation case study, using ESP-r to calculate the output. Among the results of this approach, the first order-analysis has demonstrated that, even if a precise ranking between the input parameters is not relevant, they can be split into different families in order to discuss their importance. This first order analysis helps to identify possible non-linearity or interactions of higher orders. The usefulness of considering various forms for the output function (kWh/year, kWh/year.m³, ln(kWh/year.m³)) has been explored, in particular to reduce the number of variables affected by these non-linearities or interactions. For instance, specific values per m³ (or per m²) reduce the correlation to the size-related parameters. Moreover, considering the logarithm of the output helps to identify the origin of some of the non-linearity. It is worth noting that investigat-
ing various model outputs does not require additional simulation runs: the same set of simulation trajectories is used and only complementary post-processing is needed. This remark can be extended to any transformed output calculated from model output and input variables. For those parameters still remaining in the area of high standard deviation, it has been demonstrated how the implementation of the second-order sensitivity analysis can be helpful to sort variables and to specify their interaction in pairs. It has also been shown how the usefulness of the upper-order analysis is amplified when combined with the different forms for the output. A new way of presenting results from the Morris method has been proposed to classify parameters (or couples of parameters in the case of second-order analysis) according to their sensitivity: linear-, monotonic-, almost monotonic or highly non-linear/interaction-of-higher-order). The value of sensitivity analysis using the elementary effects method has been clearly established. In any case, for a given building being simulated with a specific modeling tool, no general sensitivity can be derived: the results depend on the input parameters, with fixed values or varying values (and for the latter, their variation range). Thus, sensitivity analysis must be performed for each particular situation, in relation to modeling goals, with a careful choice of variation intervals. Its conclusions are valid only for this particular situation.

The choice of variation interval for each input parameter must be related to either a constrained range for decision variables or an uncertainty domain for exogenous variables.

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Fig. 1. The apartment building case study.

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Fig. 3. Estimated absolute average \( \mu^* \) and standard deviation \( \sigma \) of the first-order elementary effects for different sets of outputs related to energy consumption (see Table 2 experiment A1). The numbers on the plot are the parameter indices and the lines represent the slope \( \sigma/\mu^* \) at the values 0.1, 0.5 and 1.

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Fig. 6. Estimated first- and second-order elementary effects for two outputs related to annual heating needs in kWh and in ln(kWh/year/m\(^3\)) with a small interval between the parameters and \( r = 10 \) (Table 2). The lines represent the slope \( \sigma/\mu \) at the values 0.1, 0.5 and 1.

### Tables

| Year built (Base case) | 1990 |
|------------------------|------|
| Floor area (m\(^2\))   | 3500 |
| Width (m)              | 14.00|
| Length (m)             | 33.40|
| Height (m)             | 18.00|
| U-value external walls (W/m\(^2\)K) | 0.472 |
| U-value internal walls (W/m\(^2\)K)  | 4.400 |
| U-value double-glazing (W/m\(^2\)K)  | 2.811 |
| U-value basement floor (W/m\(^2\)K)  | 0.887 |
| Glazing ratio (%)      | 30   |
| Number of occupants    | 70–80|

Table 1: Main characteristics of the apartment building test.
| N  | Parameter                                                                 | Intervals                  |
|----|---------------------------------------------------------------------------|----------------------------|
| 1  | Building size: correction for height [%]                                  | 50-150                     |
| 2  | Building size: correction for width [%]                                   | 50-150 A,B1,B2 90-100 B3,B4|
| 3  | Building size: correction for length [%]                                  | 50-150 A,B1,B2 90-100 B3,B4|
| 4  | Set point temperature of all apartments except Apt. E42 [°C]              | 17-24 A,B1,B2 20-22 B3,B4  |
| 5  | Set point temperature of Apt. E42 [°C]                                    | 17-24 A,B1,B2 20-22 B3,B4  |
| 6  | Night-day set point temp. diff. affecting all apartments except Apt. E42 [°C] | 0-8 A,B1,B2 0-1 B3,B4     |
| 7  | Night-day set point temperature of Apt. E42 [°C]                          | 0-8 A                      |
| 8  | Occupants affecting all apartments except Apt. E42 [occ./apt.]            | 1-8 A,B1,B2 3-4 B3,B4     |
| 9  | Occupants of Apartment. E42 [occ./apt.]                                   | 1-8 A                      |
| 10 | Ventilation rate affecting all apartments except Apt. E42 [%]              | 40-100 A,B1,B2 80-90 B3,B4|
| 11 | Ventilation rate of Apartment E42 [%]                                     | 40-100 A                   |
| 12 | Glazing ratio A [%]                                                       | 5-50 A,B1,B2 45-50 B3,B4  |
| 13 | Glazing ratio B [%]                                                       | 5-50 A                     |
| 14 | Glazing ratio C [%]                                                       | 5-50 A                     |
| 15 | Glazing ratio D [%]                                                       | 5-50 A                     |
| 16 | Ground reflectivity [%]                                                   | 20-30 A                    |
| 17 | Ground reflectivity in presence of snow (January-December) [%]            | 30-50 A                    |
| 18 | View factor of ground [%]                                                 | 30-40 A                    |
| 19 | Climatic sensitivity:correction for horizontal diffuse solar rad. [-]     | 0.2-1 A,B1,B2 0.9-1 B3,B4 |
| 20 | Climatic sensitivity:correction for external dry bulb temp. [-]           | 0.2-1 A                    |
| 21 | Climatic sensitivity:correction for direct normal solar intensity [-]     | 0.2-1 A,B1,B2 0.2-1 B3,B4 |
| 22 | Climatic sensitivity:correction for wind speed [-]                        | 0.5-1 A                    |
| 23 | Insulation thickness of external walls [mm]                               | 5-100 A,B1,B2 90-100 B3,B4|
| 24 | Building rotation [degrees]                                               | 0-180 A,B1,B2 0-10 B3,B4  |

Table 2: List of parameters used in the Morris method first- and second-order experiments and the different intervals used in the analysis.
Figures

Figure 1

Figure 2
(a) Analysis of the elementary effects related to annual heating needs with a large interval in each parameter and $r = 10$.

(b) Analysis of the elementary effects related to the logarithm of annual heating needs with a large interval in each parameter and $r = 10$.

(c) Analysis of the elementary effects related to annual heating needs per cubic meter with a large interval in each parameter and $r = 10$.

(d) Analysis of the elementary effects related to the natural logarithm of annual heating needs per cubic meter with a large interval in each parameter and $r = 10$.

Figure 3
(a) Analysis of the elementary effects related to a comfort factor defined as the mean temperature in summer (July and August) with a large interval in each parameter and $r = 10$

(b) Analysis of the elementary effects related to power exceeded during 1000 hours/year. Each parameter has a large interval and the analysis has ten elementary effects per parameter ($r = 10$).

Figure 4:
(a) Analysis of the first-order elementary effects related to annual heating needs with a large interval in each parameter and \( r = 10 \).

(b) Analysis of the first-order elementary effects related to annual heating needs with a large interval in each parameter and \( r = 10 \).

(c) Analysis of the second-order elementary effects related to annual heating needs with a large interval in each parameter and \( r = 10 \).

(d) Analysis of the second-order elementary effects related to annual heating needs with a large interval in each parameter and \( r = 10 \).

Figure 5
(a) Analysis of the first-order elementary effects related to annual heating needs with a small interval in each parameter and $r = 10$.

(b) Analysis of the first-order elementary effects related to annual heating needs with a small interval in each parameter and $r = 10$.

(c) Analysis of the second-order elementary effects related to annual heating needs with a small interval in each parameter and $r = 10$.

(d) Analysis of the second-order elementary effects related to annual heating needs with a small interval in each parameter and $r = 10$.

Figure 6