Dynamic measurement of the lost motion of precision reducers in robots and the determination of optimal measurement speed

Hang XU*, Zhaoyao SHI*, Bo YU* and Hui WANG*
*Beijing Engineering Research Center of Precision Measurement Technology and Instruments, Beijing University of Technology,
Pingleyuan 100, Beijing, China, 100124
E-mail: yubo@bjut.edu.cn

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Abstract
A precision reducer is the core component in an industrial robot and largely determines the performance of a robot. Lost motion is a key index of the performance of precision reducers and its measurement and evaluation are the basis for improving the performance of precision reducers. The dynamic measurement of the lost motion is affected by load torque and speed, but there is no uniform standard or scientific basis for measurement speed, thus leading to deviations in measurement results. In this paper, the principle of the dynamic measurement of the lost motion of precision reducers was firstly introduced and the method of separating the geometric lost motion based on optimal measurement speed was put forward. Then, according to the Stribeck friction effect of precision reducers, the method of determining the optimal measurement speed was proposed and the calculation model of the optimal measurement speed was established. Finally, taking the RV reducer as an example, the measurement experiment was carried out. The friction torque of the RV reducer was the least and the mean value of the measured geometric lost motion was the least under the optimal measurement speed, thus verifying the proposed method.

Keywords: Precision reducer, Lost motion, Dynamic measurement, Optimal measurement speed, Stribeck curve

1. Introduction

A precision reducer is the core component of an industrial robot and directly affects the performance of a robot. RV reducer and harmonic reducer are widely used in robotics field (Shi et al., 2018a). RV reducer has the higher rigidity and is mostly applied in the parts of heavy loads such as base, arm and shoulder, whereas harmonic reducer is usually applied in forearm, wrist and hand (Xu L., 2016; Pham et al., 2018).

Lost motion is a key index of the transmission accuracy of precision reducers. It refers to the lag of output shaft on the rotation angle when the movement direction of precision reducers changes. When the input shaft changes its motion direction, there will be a lag in the angle of the output shaft due to the lost motion. The two shafts will lose contact within a short period, thus resulting in a sudden interruption of the output and the nonlinear motion transfer relationship. However, a precision reducer applied in industrial robots often makes reciprocating motion, which will lead to frequent deviations of the input and output angles of the system. The precision and dynamic performance of the whole robot will be seriously affected. Therefore, it is necessary to further improve the performance of precision reducers. The measurement and evaluation of the lost motion are the basis for improving the performance of precision reducers.

At present, the measurement of the lost motion of precision reducers is mainly based on the hysteresis curve method, a static measurement method. Seyfferth et al. (1995) considered the influences of nonlinear stiffness and friction and established the hysteresis model of harmonic reducer based on odd function polynomial and hyperbolic function. Dhaouadi et al. (2000, 2003) established the hysteresis model of harmonic reducer based on odd function polynomial and coulomb friction model. Tjahjowidodo et al. (2006, 2013) analyzed the nonlinear torsion characteristic...
of harmonic drive based on Maxwell model. Preissner et al. (2012) proposed Preisach model to describe the hysteresis characteristic of harmonic reducer and used MRC model to identify the weight function. Ruderman et al. (2012, 2014) established the hysteresis model of harmonic reducer based on Bouc-Wen-like model and 2SEP friction model. Anh et al. (2017) established the hysteresis model of cycloidal reducer by using the finite element analysis method and obtained the lost motion of single-stage cycloid reducer. Tsai et al. (2017) used LTCA method to analyze the loaded tooth contact performance of a cycloid planetary gear reducer.

The hysteresis curve method is mainly used to measure the lost motion of precision reducers. The lost motion is defined as the difference between the mean value of the forward and reverse torsion angles under the ±3% rated torque on the hysteresis curve, as shown in Fig. 1. Sun et al. (2018) designed a single-stage cycloidal reducer and measured the lost motion of thirty positions by means of locking the input shaft and loading on output shaft. Shi et al. (2018b) developed a RV reducer comprehensive performance tester and measured the lost motion of forty positions of RV reducer.

![Fig. 1 Lost motion.](image)

In general, in the static measurement of lost motion, a single measurement can only get the lost motion at one position. Due to the clearance between transmission elements, the magnitude of lost motion varies with the meshing position of gear pair. The static measurement usually requires multi-point measurement. Therefore, the static measurement has the low efficiency and cannot meet the measurement requirements of industrial applications.

The dynamic measurement of lost motion refers to the continuous and dynamic measurement under the similar conditions to the running state of a precision reducer. It has the advantages of high measurement efficiency and rich information. Its main realization way is the bidirectional transmission error method. Shi (2008) deeply analyzed the development of the gear single-side meshing measurement technology, pointed out its application in gear clearance measurement, and provided a theoretical basis for the dynamic measurement of the lost motion of precision reducers. Slamani et al. (2013) measured the backlash of ABB IRB 1600 industrial robot by using the bidirectional transmission error method. The dynamic measurement of lost motion is affected by load torque and measurement speed, but there is no uniform standard or scientific basis for determining the measurement speed, thus affecting the accuracy evaluation of precision reducers and the classification of precision grade.

This paper aims at the dynamic measurement of lost motion of precision reducers. Firstly, the principle of the dynamic measurement of lost motion of precision reducers was expounded and the separation method of geometric lost motion under the no-load and optimal measurement speed was put forward. Then, the calculation model of optimal measurement speed was established. Finally, taking the RV reducer as an example, the measurement experiment was carried out. The experimental results verified the proposed method.

2. Dynamic measurement principle and decomposition of lost motion

2.1 Dynamic measurement principle

The dynamic measurement of lost motion of precision reducers is based on the bidirectional transmission error method and the measurement principle is shown in Fig. 2. In this method, the forward transmission error curve of precision reducers is firstly measured under no-load or steady load conditions. Then the input shaft is rotated at a certain angle and then reversed to the original position in order to eliminate the influence of the clearance in the transmission system. The reverse transmission error curve of the reducer is measured in the same way. The difference between the reverse transmission error curve and the forward transmission error curve is called the lost motion curve.
and can be calculated from Eq. (1) (Gear et al. 2006). The lost motion curve obtained by dynamic measurement is shown in Fig. 2(c).

\[ \delta(\theta) = TE_r(\theta) - TE_f(\theta) \]  

(1)

where \( \delta(\theta) \) indicates the lost motion curve; \( TE_r(\theta) \) indicates the forward transmission error curve; \( TE_f(\theta) \) indicates the reverse transmission error curve.

Transmission error refers to the difference between the actual rotation angle of the output shaft of the precision reducer and the theoretical rotation angle and can be calculated from Eq. (2). The angles of input and output shafts are acquired by the data acquisition system according to Fig. 2(a) in real time. The measured forward and reverse transmission error curves are shown in Fig. 2(b).

\[ TE(\theta) = \theta_{\text{out}} - \frac{\theta_{\text{in}}}{R} \]  

(2)

where \( \theta_{\text{out}} \) is the actual angle of the output shaft; \( \theta_{\text{in}} \) is the angle of the input shaft; \( R \) is the speed ratio of the precision reducer.

2.2 Decomposition of lost motion

The lost motion curves of different precision reducers are different. The same precision reducer has different lost
motion curves under different measurement conditions caused by the internal structure of precision reducers, load torque, measurement speed and other factors. Therefore, according to the factors affecting the lost motion of precision reducers, the lost motion is decomposed into the geometric lost motion caused by internal factors and the elastic lost motion caused by external factors in this study. Geometric lost motion is mainly caused by tooth shape, tolerance and backlash. Elastic lost motion is mainly caused by external factors, such as measurement speed, load torque, and temperature. Since the dynamic measurement of lost motion is carried out within a relatively short period of time, the influence of temperature can be approximately ignored. Therefore, the lost motion of precision reducers can be expressed as:

$$\delta = F(\delta_g, \delta_e)$$  \hspace{1cm} (3)

where $\delta_g$ is geometric lost motion; $\delta_e$ is elastic lost motion.

### 2.3 Separation of geometric lost motion

Geometric lost motion can be used to characterize the machining and manufacturing accuracy of precision reducers and is the evaluation basis of transmission accuracy of precision reducers. The key to measure and objectively evaluate geometric lost motion is to reduce the interference of external factors and realize the separation of geometric lost motion. The influence of elastic deformation caused by load torque can be avoided under the no-load condition and the influence of friction torque of precision reducers can be effectively reduced by choosing a reasonable measurement speed. In order to reduce the influence of friction torque of precision reducers, the separation method of geometric lost motion based on optimal measurement speed is proposed in this study.

The friction torque of precision reducers decreases with the increase in the speed when the speed is low. When the speed is greater than a certain value, the friction torque increases with the increase in the speed, showing a typical Stribeck effect, as shown in Fig. 3 (Stribeck 1902; Hei 2015). It can be seen that the forward and reverse Stribeck curves of precision reducers have a wave trough point $A^+$ and a wave peak point $A^-$, respectively. Taking the forward curve as an example, when the speed $\omega \leq \omega^+_f$, the friction torque of precision reducers decreases with the increase in the speed. When the speed $\omega \geq \omega^+_f$, the friction torque of precision reducers increases with the increase in the speed. When the speed $\omega = \omega^+_f$, the precision reducer is subjected to the smallest friction torque. Therefore, the friction torque is the smallest when the speed $\omega = \omega^+_f$, and this speed is the optimal measurement speed for the forward transmission error measurement. Similarly, $\omega = \omega^-_f$ is the optimal measurement speed for the reverse transmission error measurement.

When measuring the lost motion of precision reducers based on the bidirectional transmission error method, it is necessary to set the forward and reverse speeds of the driving motor in advance. However, the Stribeck curve measurement is influenced by the measurement accuracy and the data fitting algorithm and there is a certain deviation between $\omega^+_f$ and $\omega^-_f$. Under the premise of ensuring measurement accuracy, in order to control the motor conveniently, the average value of $\omega^+_f$ and $\omega^-_f$ is taken as the optimal measurement speed of geometric lost motion in this study, shown in Eq. (4).
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\[ \omega_\alpha = \frac{1}{2} \left( |\omega_1^\alpha| + |\omega_2^\alpha| \right) \]  \hfill (4)

Then geometric lost motion can be expressed as:

\[ \delta_{\text{g}}(\theta) = G(T_0, \omega_\alpha) \]  \hfill (5)

where \( G(T_0, \omega_\alpha) \) is the dynamic lost motion under the no-load condition \( T_0 \) and the speed \( \omega_\alpha \). \( T_0 \) indicates that the load on the output side is 0Nm.

3. Calculation model of optimal measurement speed

According to Strubeck friction theory, the friction characteristics of precision reducers can be divided into the following four stages: (I) static friction stage, (II) boundary lubrication stage, (III) partial fluid lubrication stage, and (IV) full fluid lubrication stage, as shown in Fig. 4. When the speed of the precision reducer is zero, the precision reducer is ready to start and it is in the static friction stage. The friction is static friction independent of the speed and mainly caused by the elastic deformation of the precision reducer. With the increase in external force, there is a low relative velocity between the contact surfaces and the lubricating oil film cannot be established between the surfaces. The friction is mainly determined by the impurity characteristics of the boundary layer and caused by the shearing force between solids. At this time, the precision reducer is in the boundary lubrication stage. With the further increase in the speed, the liquid film formed between the contact surfaces is thicker and thicker, but the contact surface is not completely separated by the lubricant and there is still a region of solid contact, indicating that it has entered the stage of partial fluid lubrication. When the speed continues to increase, the liquid film between the objects is formed completely, and there is no region of solid contact. However, with the increase in the relative speed, the viscous friction, which is proportional to the velocity, is the dominant factor. At this time, the friction mainly depends on the velocity and the viscosity coefficient of the lubricant and it enters the stage of full fluid lubrication (Marton et al. 2006).

\[ T_f^+ (\omega) = T_c^+ + (T_c^+-T_c^-) \cdot e^{-(\omega_0 \cdot \omega)} + B^+ \cdot \omega \quad \omega > 0 \]

\[ T_f^- (\omega) = -T_c^- - (T_c^+ - T_c^-) \cdot e^{-(\omega_0 \cdot \omega)} + B^- \cdot \omega \quad \omega < 0 \]  \hfill (6)

where \( T_f^+ (\omega) \) and \( T_f^- (\omega) \) respectively indicate the forward and reverse friction torques; \( T_c^+ \) and \( T_c^- \) respectively denote forward and reverse Coulomb friction torques; \( T_c^+ \) and \( T_c^- \) respectively denote forward and reverse static friction torques; \( \omega_0^+ \) and \( \omega_0^- \) respectively denotes forward and reverse Strubeck speeds; \( B^+ \) and \( B^- \) respectively denote forward and reverse viscous friction coefficients.

It can be seen from Eq. (6) that the forward and reverse friction functions are continuous and can be respectively derived, so the coordinates of the points \( A^+ \) and \( A^- \) can be solved by the first order differential equation of Eq. (6). The first order differential equation of Eq. (6) is shown in Eq. (7). If Eq. (7) is equal to zero, the optimal measurement forward and reverse speed can be obtained by Eq. (8). The optimal measurement speed of geometric lost motion of precision reducers can be obtained by substituting Eq. (8) into Eq. (4), as shown in Eq. (9).

\[ T_f^+ (\omega) = T_c^+ + (T_c^+-T_c^-) \cdot e^{-(\omega_0 \cdot \omega)} + B^+ \cdot \omega \quad \omega > 0 \]

\[ T_f^- (\omega) = -T_c^- - (T_c^+ - T_c^-) \cdot e^{-(\omega_0 \cdot \omega)} + B^- \cdot \omega \quad \omega < 0 \]  \hfill (7)

\[ \frac{d\omega}{dt} = T_f^+ (\omega) / J - T_f^- (\omega) / J \]

\[ \omega_{\text{opt}}^+ = \text{max} \left( \frac{d\omega}{dt} \right) \]  \hfill (8)

\[ \omega_{\text{opt}}^- = \text{min} \left( \frac{d\omega}{dt} \right) \]

\[ \delta_{\text{g}}(\theta) = G(T_0, \omega_{\text{opt}}) = G(T_0, \omega_{\text{opt}}^+ \text{ or } \omega_{\text{opt}}^-) \]  \hfill (9)
4. Experimental study

Taking RV reducer as an example, based on the RV reducer comprehensive performance tester developed by our research group, the actual dynamic measurement experiment of the lost motion was carried out. The rated torque of the measured RV reducer is 784 Nm and the speed ratio is 121. The number of the cycloidal gear teeth is 39 and the number of the pin gears is 40.

Our research group developed a RV reducer comprehensive performance tester (Fig. 5). In the dynamic measurement of lost motion, the RV reducer was installed on the precision support and the input shaft was connected with input components (including motor, torque sensor, etc.) via precision coupling and driven by a servo motor. The measurement speed was controlled by the measuring and controlling software in the speed mode to ensure the speed stability in the measurement. The rated speed of the motor was 3000 r/min. The output flange shaft was connected with output components (including torque sensor, magnetic powder brake, etc.) via precision coupling. The load was applied on the output shaft by a magnetic powder brake and the load torque was adjusted by a programmable power supply. The rated torque of the magnetic powder brake was 2000 Nm. The working states of servo motor and magnetic powder brake were controlled by an industrial control computer. The angle measurements of input and output shafts were achieved with the high precision circular grating and the resolutions reached 2.06' and 1.13'. The dual-channel torque sensor with the precision of 0.1%FS was adopted in torque measurements. Based on the zero position signal of the circular grating, the transmission error at the same position can be obtained in the forward and reverse directions and the dynamic measurement of lost motion can be realized through the analysis and processing in measurement software.

(a) Structure diagram.
4.1 Striebeck curve measurement and parameter identification

In order to accurately obtain the Striebeck curve of the precision reducer, the friction torque measurement experiments from low speed to high speed were carried out. The driving motor works in the speed mode and the fluctuation of speed is small and can be ignored. Therefore, the friction torque of the precision reducer is equal to the driving torque of the motor, which is acquired with the torque sensor at the input shaft in real time, and the mean value filtering method is used to process the obtained torque signal.

The forward and reverse friction torques of the RV reducer at different speeds are respectively shown in Table 1 and Table 2. The Striebeck curve of RV reducer is fitted by least square method. The fitted Striebeck curve is shown in Fig. 6.

### Table 1 Measurement results of forward friction torque of RV reducer

| Speed (°/s) | Friction torque (Nm) | Speed (°/s) | Friction torque (Nm) | Speed (°/s) | Friction torque (Nm) |
|-------------|----------------------|-------------|----------------------|-------------|----------------------|
| 24          | 0.3474               | 180         | 0.2021               | 600         | 0.2286               |
| 36          | 0.2781               | 240         | 0.2008               | 720         | 0.2408               |
| 48          | 0.2473               | 300         | 0.2057               | 900         | 0.2580               |
| 60          | 0.2350               | 360         | 0.2084               | 1000        | 0.2686               |
| 90          | 0.2084               | 420         | 0.2137               | 1200        | 0.2889               |
| 120         | 0.2088               | 500         | 0.2199               | 1500        | 0.3187               |

### Table 2 Measurement results of reverse friction torque of RV reducer

| Speed (°/s) | Friction torque (Nm) | Speed (°/s) | Friction torque (Nm) | Speed (°/s) | Friction torque (Nm) |
|-------------|----------------------|-------------|----------------------|-------------|----------------------|
| -24         | -0.1626              | -180        | -0.0637              | -600        | -0.0902              |
| -36         | -0.1211              | -240        | -0.0622              | -720        | -0.1055              |
| -48         | -0.1002              | -300        | -0.0688              | -900        | -0.1224              |
| -60         | -0.0840              | -360        | -0.0747              | -1000       | -0.1235              |
| -90         | -0.0665              | -420        | -0.0774              | -1200       | -0.1474              |
| -120        | -0.0637              | -500        | -0.0935              | -1500       | -0.1884              |
Fig. 6 Fitted Stribeck curve of RV reducer.

The parameters of the forward and reverse Stribeck curve obtained by the least square fitting are shown in Table 3 and Table 4. The RSS (residual sum of squares) of forward fitting is 3.64609E-5 and the Adj. R-Square (adjusted coefficient of determination) is 0.97998. The RSS (residual sum of squares) of reverse fitting is 2.60655E-5 and the Adj. R-Square (adjusted coefficient of determination) is 0.98091. The two parameters indicate that both the forward and reverse fitting can achieve the ideal fitting effect.

Table 3 Stribeck curve forward fitting parameters of RV reducer.

| Parameters | $T^+$ | $T^-$ | $\omega^+_f$ | $B^+$ | RSS          | Adj. R-Square |
|------------|--------|--------|--------------|-------|--------------|---------------|
| Values     | 0.18081| 0.55604| 0.48697      | 0.00505| 3.64609E-5  | 0.97998       |

Table 4 Stribeck curve reverse fitting parameters of RV reducer.

| Parameters | $T^-$ | $T^+$ | $\omega^-_f$ | $B^-$ | RSS          | Adj. R-Square |
|------------|-------|--------|--------------|-------|--------------|---------------|
| Values     | 0.04203| 0.28943| -0.55287     | 0.00519| 2.60655E-5  | 0.98091       |

By substituting the parameters of the fitted Stribeck curve into Eqs. (8) and (9), the optimal measurement speed can be calculated as

$$\begin{align*}
\omega_f^+ &= 2.4483 \text{rad} / \text{s} \\
\omega_f^- &= -2.4641 \text{rad} / \text{s} \\
\omega_a &= 2.4562 \text{rad} / \text{s} = 141^\circ / \text{s}
\end{align*}$$

(10)

4.2 Measurement of geometric lost motion

At first, the bidirectional transmission error curve of RV reducer is obtained under the no-load condition and the optimal measurement speed, $\omega_a = 141^\circ / \text{s}$, as shown in Fig. 7. The geometric lost motion curve is shows in Fig. 8. The mean value of geometric lost motion can be calculated as:

$$\overline{\delta_g} = MEAN(\delta_g(\theta)) = 0.5087 \text{ arcmin}$$

(11)
Then the measurement speed is changed while other conditions remain unchanged. The lost motion of the RV reducer is measured from low speed to high speed. The mean value of lost motion at different speeds is shown in Table 5 and the mean value of the lost motion is shown in Fig. 9. When the speed is less than the optimal measurement speed, the mean value of lost motion decreases with the increase in the speed. When the speed is larger than the optimal measurement speed, the mean value of the lost motion increases with the increase in the speed. At the optimal measurement speed, the mean value of geometric lost motion is the smallest because the friction torque of the precision reducer is the smallest at the optimal measurement speed. Therefore, the influences of external factors can be minimized under the no-load condition and optimal measurement speed and the geometric lost motion can be separated.
4.3 Loading experiment

At the optimal measurement speed, the load torque of the output was adjusted by adjusting the current of the programmable power supply. The current was set to 0.5 A, 1 A and 1.5 A and the corresponding load torque was 73 Nm, 249 Nm and 482 Nm.

The lost motion curves under different load torques are shown in Fig. 10. The lost motion increases with the increase in load torque due to the large torsional deformation caused by the transmission element under large load torque.
4.4 Spectrum analysis

Based on the FFT transformation, the spectrum of the dynamic lost motion curve is analyzed. The spectrum of the geometric lost motion is shown in Fig. 11, where $f$ and $f_0$ respectively represent the frequency of geometric lost motion and the frequency of output shaft; $f/f_0$ represents the number of variation times of the geometric lost motion within one rotation of the output shaft. The harmonic component is shown in Table 6. The spectral component contains constant term. The constant term is mainly caused by the inherent clearance of the RV reducer and has the greatest influence on geometric lost motion. In the dynamic measurement of lost motion of RV reducer, the pin gear shell is fixed and the planetary frame is the output end. Therefore, in the range of one rotation of the output shaft, the planetary frame rotates in a circle, and the harmonic component 1 is mainly caused by the machining error of planetary frame. The number of cycloidal gear teeth of the measured RV reducer is 39, and the harmonic component 39 is mainly caused by the machining error of cycloidal gear. The number of pin gear of the measured RV reducer is 40, and the harmonic components 40, 80, 120, 160 and 200 are mainly caused by the machining error of pin gear. In the range of one rotation of the output shaft, the input shaft rotates 121 cycles, and the harmonic component 121 is mainly caused by the machining error of input shaft. The harmonic components indicate that the machining and installation errors of planetary frame, cycloid gear, pin gear and input shaft of the RV reducer have great influences on geometric lost motion, and should be considered in the design and manufacturing process.

The spectrum of lost motion at different speeds is shown in Fig. 12 and the main harmonic components are shown in Table 6. The spectrum of lost motion under different load torques is shown in Fig. 13 and the main harmonic components are shown in Table 7. From Table 6 and 7, it can be seen that the speed and load torque only affect the amplitude of the harmonic other than the harmonic component. The harmonic component of the precision reducer is mainly determined by the structure of the precision reducer. In actual measurements, the effect of load torque on lost motion is greater than that of speed.
Fig. 12 Spectrum of lost motion at different speeds.

Fig. 13 Spectrum of lost motion under different loads.
4.5 Discussion

The friction torque of precision reducers decreases with the increase in the speed when the speed is low. The friction torque increases with the increase in the speed when the speed is greater than a certain value, displaying the typical Stribeck friction effect.

The dynamic lost motion curve of the precision reducer increases with the increase in load torque. The dynamic lost motion curve contains the stiffness information of precision reducers. Therefore, the stiffness calculation model of the precision reducer constructed based on the dynamic lost motion curve is important for the extension of the application scope of bidirectional transmission error method.

In the dynamic measurement of lost motion, the speed and load torque only affect the amplitude of the harmonic component of lost motion other than the harmonic component. The speed and load torque are the external factors that affect the lost motion. The harmonic component of lost motion is mainly determined by the internal structure of the precision reducer.

5. Conclusions

In this study, the dynamic measurement principle of the lost motion of precision reducers was introduced and the separating method of geometric lost motion based on optimal measurement speed was put forward. Then the calculation model of optimal measurement speed was established. Through measurement experiments, the following conclusions can be obtained.

The bidirectional transmission error method is a continuous and dynamic measurement method. It has the higher measurement efficiency and is suitable for industrial field applications. It can be used conveniently to analyze the spectrum of the lost motion and realize the error traceability.

The influence of friction is inevitable in the dynamic measurement of lost motion, but the influence can be minimized by choosing a reasonable measurement speed. Under the no-load condition and optimal measurement speed, the measurement and objective evaluation of the lost motion of precision reducers can be realized.

The method proposed in this paper provides a scientific basis for the speed determination in dynamic measurement of the lost motion. The methods proposed in this study for the dynamic measurement of lost motion and the
determination of optimal measurement speed are suitable for the measurement of lost motion of various precision reducers.

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