Deformation Models of Concrete Strength Calculation in the Edition of Russian and Foreign Norms

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Abstract. The correlation between the calculation models as amended by Russian and foreign regulatory documents is established by comparing the values of the integral parameters of the diagrams and the ultimate forces obtained by calculating the reinforced concrete element according to the deformation model. As integral parameters of concrete deformation diagrams, it is recommended to use the areas of the areas limited to the branches of diagrams and coefficients of plenitude of diagrams. Mathematical models have been developed that establish a common form for calculating the unambiguous values of deformations at the base points of concrete diagrams, taking into account the accepted functional relationships and the rules for their use according to the tables of normative documents. Analytical modeling of integral parameters allows us to exclude from the calculation of the strength of an element the procedure for numerically summing stresses along elementary strips in a section and solving nonlinear equations by successive approximations.

1. Introduction
The regulatory documents [1,2,3,4] recommend different types of concrete deformation diagrams and analytical dependencies that establish the relationship between relative deformations and stresses 

\[ \varepsilon_b - \sigma_b \]

under single-axle compression and tension. A curvilinear diagram that has an ascending and descending deformation branch fully meets the physical properties of concrete and the experimental test data for standard concrete samples. When you describe curved diagrams of concrete deformation under compression, the editors of domestic and foreign publications [5,6,7,8,9] propose to use the following points: at the top of the diagram on the ascending branch; at the end of the falling branch, in which the deformations reach their maximum values. The analytical dependencies of the diagram descriptions that are different in structure, the difference in the method for determining deformations and the values of the calculated concrete compressive strengths at base points in the edition of regulatory documents, leads to a mutual discrepancy between the values of ultimate forces in the calculations of reinforced concrete elements in strength. There are difficulties in the comparative evaluation of the efficiency of computational models. In calculations by the deformation model, the numerical integration of stresses in the selected elementary strips of concrete over the thickness of the element and the solution of nonlinear equations to satisfy the condition of equilibrium of forces by successive approximation (iterations) method is a laborious procedure in the calculations of complex
2. The purpose and objectives of the research
To develop a mathematical model for calculating deformations at the base points of concrete diagrams, taking into account the accepted functional relationships and the rules for their purpose according to the tables of normative documents. Include the integral parameters of concrete diagrams in the calculation method for the deformation model and establish the relationships between the ultimate forces for the respective classes of concrete in terms of compressive strength. To propose a simplified method for calculating the strength of an element, excluding the procedure for numerical integration of stresses over the thickness and solving nonlinear equations by the iteration method.

3. Research Methodology
In normative documents [2,3], the class of concrete in axial compression strength is indicated by the letter C and numbers, for example, C12 / 15 (in front of the line, the value of regulatory resistance $f_{ck}$ is the compressive strength of cylinders 150 mm in diameter and 300 mm in height, tested in age 28 days; after the line $f_{c,cube}^G$ there is the value of the guaranteed strength of the concrete cube 150×150 mm with an availability of 0.95). Russian standards are based on the strength of the cube, so the principle hereinafter (Table 1) establishes the correspondence between classes C and B (for example, concrete class B15 corresponds to class C12 / 15, etc.).

In the concrete class of compressive strength B and C, the normative concrete axial compression resistance $R_{bn}$ (prismatic strength) and $f_{ck}$ (cylinder strength) are respectively determined from the tables of regulatory documents. The calculated values of concrete resistance $R_b$ and $f_{cd}$ (Table 1) are calculated by dividing the standard concrete resistance to compression, respectively, $R_{bn}$ by the reliability coefficient for concrete under compression $\gamma_b = 1.3$ and $f_{ck}$ the safety coefficient for concrete $\gamma_c = 1.5$. When evaluating the deformation properties of concrete in [2,3], the average values of compressive strength $f_{cm}$ are introduced.

Concrete compression diagrams are constructed in the coordinates "$\varepsilon_b(e_c) - \sigma_b(f_c)$" (in parentheses are denotations of deformations and stresses adopted in [2,3]). In curvilinear diagrams in strength calculations, the base points are $\varepsilon_b(e_{c1})$, $R_b(f_{cd})$ : the top of the diagram on the ascending branch, with coordinates; at the end of the falling branch with the maximum strain and coordinates $\varepsilon_{bu}(e_{cu1})$, $\sigma_{bu}(f_{cu})$.

The strain values at the base points $\varepsilon_{c1}$ and $\varepsilon_{cu1}$ in the stress calculations for concrete class for compression C are normalized [3] and are presented in the table (Table 6.1). The dependence stress – strain is constructed through the current values of relative strains $\eta_c = \varepsilon_c / \varepsilon_{c1}$ ($|\varepsilon_c| \leq |\varepsilon_{cu1}|$) and the stress value $f_c$ takes its maximum value $\eta_c = 1$ at the top of the diagram: $f_c = f_{cd}$ - in the calculations for the first limiting state and $f_c = f_{ck}$ - in the calculations for the second limiting state.

In normative documents [1] the magnitude of maximum strains $\varepsilon_{bu}$ is normalized.

Deformations $\varepsilon_{b1}$ at the top of the diagram, in contrast to [2,3], are assigned not according to the tables of norms, but are calculated by the formula, which takes into account the class and type of
concrete. The falling branch of the diagram is limited by the relative stress level \( \eta_b = \eta_{bu} = 0.85 \) and \( \eta_{bu} = 1 \) for high-strength concrete. By transforming the formula for describing the diagram, calculations can be performed both through stress and through deformation.

### Table 1. Calculated parameters of concrete deformation diagrams.

| Class of concrete | C12 | C25 | C35 | C50 | C60 | C70 | C80 | C90 |
|-------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( f_{cl} \), MPa | 8.0 | 16.7 | 23.3 | 33.3 | 39.2 | 42.6 | 47.6 | 50.2 |
| \( \varepsilon_{c1} \) [%] | 1.9 | 2.16 | 2.3 | 2.48 | 2.58 | 2.67 | 2.76 | 2.83 |
| \( \varepsilon_{cu1} \) [%] | 3.5 | 3.5 | 3.47 | 3.35 | 3.24 | 3.11 | 2.98 | 2.83 |
| \( S_{dc} \) | 24.9 | 50.37 | 67.75 | 89.4 | 99.2 | 101.6 | 105.1 | 102.9 |
| \( \omega_{dc} \) | 0.89 | 0.86 | 0.845 | 0.8 | 0.78 | 0.768 | 0.744 | 0.725 |
| \( \varepsilon_{cc} \) [%] | 1.89 | 1.95 | 1.96 | 1.94 | 1.9 | 1.84 | 1.79 | 1.71 |
| \( M_{c,ult} \) | 309 | 630 | 861 | 1191 | 1367 | 1450 | 1560 | 1590 |
| \( kN*m\) | B15 | B30 | B45 | B60 | B75 | B85 | B95 | B105 |
| \( K_b \), MPa | 8.5 | 17.0 | 25.0 | 33.0 | 39.0 | 42.5 | 45.75 | 49.0 |
| \( \varepsilon_b \) [%] | 1.9 | 2.18 | 2.36 | 2.5 | 2.62 | 2.68 | 2.75 | 2.8 |
| \( \varepsilon_{bu} \) [%] | 3.5 | 3.5 | 3.44 | 3.31 | 3.2 | 3.04 | 2.92 | 2.8 |
| \( S_{db} \) | 26.04 | 50.9 | 71.0 | 87.2 | 95.1 | 97.6 | 98.2 | 97.6 |
| \( \omega_{db} \) | 0.875 | 0.855 | 0.826 | 0.8 | 0.762 | 0.755 | 0.735 | 0.711 |
| \( \varepsilon_{bc} \) [%] | 1.88 | 1.948 | 1.95 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| \( M_{b,ult} \) | 321 | 636 | 916 | 1173 | 1341 | 1422 | 1482 | 1528 |
| \( kN*m\) | B15 | B30 | B45 | B60 | B75 | B85 | B95 | B105 |

The parameters of the diagram in the edition of normative documents [2,3] were investigated in experiments with monotonically increasing compression strains, at a speed of \( \varepsilon_c^* \approx 0.015 \% / s \). It is assumed that for a given test mode of concrete samples of prisms and cylinders for compression, the nonlinear properties of concrete, for the corresponding concrete classes B and C, are manifested equally, and deformations at the base points have the same values \( \varepsilon_b = \varepsilon_{c1} ; \varepsilon_{bu} = \varepsilon_{cu1} \).

### 4. The calculated dependences of the calculation of deformations at base points

When implementing calculations in software systems, it is more convenient to use analytical dependencies in which the functional relationship is preserved when assigning normalized parameters to the tables. Deformations \( \varepsilon_{c1} \) at maximum compression stress increase with increasing concrete strength and Meyer (1998) proposed a mathematical model for their calculation:

\[
\varepsilon_{c1} = 1.6(f_{cm}/10MPa)^{0.25}/1000, \quad (1)
\]

where \( f_{cm} = f_{ck} + \Delta f \) (\( \Delta f = 8 \text{MPa} \)).
The ultimate compressive strain of concrete $\varepsilon_{cul1}$, normalized in tabular form [2,3], is proposed to calculate by the formula:

$$
\varepsilon_{cul1} = \varepsilon_{cl1} \left(1 - \frac{f_{cm} - f_{cm}^*}{81\text{MMPa}} \left(\frac{10\text{MMPa}}{f_{cm}}\right)^{0.2}\right),
$$

(2)

where $f_{cm}^*$ is the fixed value of the average concrete strength for the concrete class, in which the descending branch is excluded from the calculation and the equalities are satisfied: $|\varepsilon_{cl1}| = |\varepsilon_{cul1}|$ and $f_{cd} = f_{cu}$ (assumed $= 98$ MPa).

For heavy concrete, analytical dependencies of determining deformations at base points $\hat{\varepsilon}_b$ and $\varepsilon_{bu}$ (Tab.1), Uniform in structure with (1) and (2), are introduced:

$$
\hat{\varepsilon}_b = 1,75 \left(\frac{B}{10\text{MMPa}}\right)^{0.2}/1000; \quad \varepsilon_{bu} = \hat{\varepsilon}_b \left(1 - \frac{B - B^*}{98\text{MMPa}} \left(\frac{10\text{MMPa}}{B}\right)^{0.2}\right),
$$

(3)

where $B^*$ is a fixed class of concrete, in which the descending branch is excluded from the calculation and the equalities are satisfied: $|\hat{\varepsilon}_b| = |\varepsilon_{bu}|$ and $\sigma_{bu} = R_b$ (assumed $B^* = 105$ MPa).

In working with charts, a general rule follows. If during the construction of diagrams, deformations are assigned and stresses are calculated, then the maximum values of deformations are limited by values $\varepsilon_{cul1}$ (2) and $\varepsilon_{bu}$ (3). If stresses are assigned and deformations are calculated [1,4], then the minimum stress values on the falling branch are limited by the relative stress value $\eta_{bu}$ calculated by the formula:

$$
\eta_{bu} = 1 + \beta_b \frac{B - B^*}{B + B^*},
$$

(4)

Where $\eta_{bu} = \sigma_{bu} / R_b$, here $B^*$ is a fixed class of concrete, in which the falling branch of concrete is excluded from the calculation ($B^* = 105$ MPa is assumed).

The values of deformations at the base points are determined according to the rules of norms depending on the average stresses $f_{cm}$ in the formula (1, 2) and concrete class B in the formula (3). This means that the strain values at the base points can be used in the calculations for the limiting states of both the first and second groups. According to the analytical dependencies presented in the normative documents [1,2,3,4], concrete diagrams are constructed "$\varepsilon_b(\varepsilon_c) - \sigma_b(f_c)$". The branches of these diagrams pass through the base points, the values of which are calculated from the expressions (1), (2), (3) and (4). The outlines of the concrete diagrams correspond to the outlines of the stress diagrams in the compressed zone of the element.

An energy model is used to calculate the strength of a reinforced concrete element using material deformation diagrams [7]. In the strength calculations, the complete concrete diagram is taken (Fig. 2a) and the area of the area bounded by the branches of the diagram $S_{db}$ ($S_{dc}$) remains constant. An integral characteristic of a concrete diagram is the coefficient of completeness of the diagram $\omega_{db}$ ($\omega_{dc}$), which characterizes the deviation of the actual area of the curved diagram $S_{db}$ ($S_{dc}$) from the area of the rectangle $S_{db}^*$ ($S_{dc}^*$) that describes the diagram by base points. The area of the complete
diagram $S_{db} (S_{dc})$ for each class of concrete is calculated by numerical methods or using graphical computer programs (Table 1). The normalized values of ultimate strains $\varepsilon_{bu} (\varepsilon_{cu1})$ are calculated by formulas (2) and (3). Coefficients of completeness of the plot $\omega_{db} = S_{db}/S^*_db$ and $\omega_{dc} = S_{dc}/S^*_dc$ are calculated by the formulas

$$\omega_{db} = 0.71 - 0.2 \frac{B - B^*}{B^*}; \quad \omega_{dc} = 0.724 - 0.2 \frac{f_{cm} - f_{cm}^*}{f_{cm}^*},$$

(5)

where $B$ is a fixed class of concrete, in which the descending branch is excluded from the calculation and the equals are satisfied: $|\varepsilon_b| = |\varepsilon_{bu}|$ and $\sigma_{bu} = R_b$ (assumed = 105 MPa); $-f_{cm}^*$ is a fixed value of the average concrete strength for the concrete class, in which the descending branch is excluded from the calculation and the equals are satisfied: $|\varepsilon_{c1}| = |\varepsilon_{cu1}|$ and $f_{cd} = f_{cu}$ (assumed = 98 MPa).

With an increase in the class of concrete, the curvature of the diagram decreases, approaching the elastic (Table 1), however $\omega_{db} > 0.5$. The value of the ultimate bending moment perceived by the section of the element $M_{ult}$ is determined relative to a fixed zero line:

$$M_{ult} = \frac{S_{db}}{\chi} b z_b + R_s A_{st} z_s + \sigma^l s A^l s z^l s.$$  

(6)

The force distances in the reinforcement $N^l s$, $N_s$ and concrete $N_b$ to the neutral axis, respectively, are:

$$z^l s = \frac{\varepsilon^{(k)} b - \alpha^l \chi^{(k)}}{\chi^{(k)}}; \quad z_s = \frac{\chi^{(k)} h_0 - \varepsilon^{(k)} b}{\chi^{(k)}}; \quad z_b = \frac{W_{db}}{\chi^{(k)} S_{db}} = \frac{\varepsilon_{bc}}{\chi^{(k)}},$$

(7)

where $W_{db} = \sum_{i=1}^n A_{b,j} \varepsilon_{b,j} = \sum_{i=1}^n \sigma_{b,j} \Delta \varepsilon_{b,j} \varepsilon_{b,i}$ is the moment numerically equal to the sum of the product of the areas of the elementary areas in the concrete diagrams at the distances of the center of gravity to the stress axis $\sigma_b$; $\varepsilon_{bc} = W_{db} / S_{db}$ is deformation at the level of the center of gravity of the diagram $O_3$; $\chi^{(k)}$ is the curvature of the element after fulfilling the equilibrium condition at the k-th iteration.

Studies indicate that the relative values of strain at the center of gravity of the diagrams to strain at the top of the diagrams $\eta_{bc} = \varepsilon_{bc} / \varepsilon_b$ ($\eta_{cc} = \varepsilon_{cc} / \varepsilon_{c1}$) with increasing concrete class B and average concrete strength $f_{cm}$, there are monotonically decreasing functions of the form:

$$\eta_{bc} = \left( \frac{0.75 M_P a}{B} \right)^{0.1} - 0.29 \frac{B - B^*}{B^*}; \quad \eta_{cc} = \left( \frac{0.65 M_P a}{B} \right)^{0.1} - 0.35 \frac{f_{cm} - f_{cm}^*}{f_{cm}^*},$$

(8)

where the parameters $B^*$ and $f_{cm}^*$ are taken from (2,3).
Modeling the parameters of the diagrams by analytical dependencies allows us to exclude from the calculations of the strength of elements the procedure for the numerical integration of the areas of elementary areas and the solution of nonlinear equations by the iteration method. The calculation of the strength of the element, taking into account the proposed dependencies, is performed in the following sequence:

- assigned a class of concrete, section, reinforcement $A_s, A_s'$;
- for a given class of concrete, the completeness coefficient of the concrete diagram $\psi_{db}$ is calculated by formula (5), for the area of a rectangular diagram $S_{db}^*$, the area of the region $S_{db}$ bounded by the branches of the diagram is calculated;
- the equation of equilibrium of forces in the section is converted into a quadratic equation relative to the actual height of the compressed zone $x$:
  \[ \chi^2 \frac{S_{db}b}{\varepsilon_{bn}} + \chi (\varepsilon_{bn} A_s' - R_s A_s) - a' \varepsilon_{bn} E_s A_s' = 0; \] (9)
- according to formula (6), taking into account (7), the moment value in the limiting state is calculated, where the force distances to the neutral axis are determined not with the parameter $\chi^{(k)}$ obtained by the sequential approximation procedure, but by solving the quadratic equation for the height of the compressed zone (9) and calculating the element curvature $\chi$.

5. Conclusion
Given the reduction to the same strain values at the base points of the curved diagrams of concrete deformation, the effectiveness of the calculation of ultimate forces in the Russian and foreign standards is determined by the normalized values of the calculated concrete resistances for classes B and C. Analytical modeling of the integral parameters of the concrete compression deformation diagrams allows excluding the calculation of strength procedures for numerically summing stresses over elementary strips in a section and solving nonlinear equations.

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