The Dirac cone underlies many unique electronic properties of graphene and topological insulators, and its band structure—two conical bands touching at a single point—has also been realized for photons in waveguide arrays, atoms in optical lattices, and through accidental degeneracy. Deformation of the Dirac cone often reveals intriguing properties; an example is the quantum Hall effect, where a constant magnetic field breaks the Dirac cone into isolated Landau levels. A seemingly unrelated phenomenon is the exceptional point, also known as the parity–time Dirac cone into isolated Landau levels. A seemingly unrelated phenomenon such as loss-induced transparency, unidirectional transmission or reflection, and lasers with reversed pump dependence, or single-mode operation, provide a complete picture of this system, ranging from an analytic model and numerical simulations to experimental observations; taken together, these results illustrate the role of radiation-induced non-Hermiticity that bridges the study of EPs and the study of Dirac cones.

We start by showing that non-Hermiticity from radiation can deform an accidental Dirac point into a ring of EPs. First, consider a two-dimensional photonic crystal (Fig. 1a inset), where a square lattice (periodicity $a$) of circular air holes (radius $r$) is introduced in a dielectric material. This is a Hermitian system, as there is no material gain or loss and no open boundary for radiation. By tuning a system parameter (for example, $r$), one can achieve accidental degeneracy between a quadrupole mode and two degenerate dipole modes at the $\Gamma$ point (centre of the Brillouin zone), leading to a linear Dirac dispersion due to the anti-crossing between two bands with the same symmetry.

The accidental Dirac dispersion from the effective Hamiltonian model (see equation (1) below with $\gamma_d = 0$) is shown as solid lines in Fig. 1a, agreeing with numerical simulation results (symbols). In the effective Hamiltonian we do not consider the dispersionless third band (grey line) owing to symmetry arguments (Supplementary Information Section 1), although this third band cannot be neglected in certain calculations, including the Berry phase and effective medium properties.

Next, we consider a similar, but open, system: a photonic crystal slab (Fig. 1b inset) with finite thickness $h$. With the open boundary, modes within the radiation continuum become resonances because they radiate by coupling to extended plane waves in the surrounding medium. Non-Hermitian perturbations need to be included in the Hamiltonian to account for the radiation loss. To the leading order, radiation of the dipole mode can be described by adding an imaginary part $-i\gamma_d$ to the Hamiltonian, while the quadrupole mode does not radiate owing to its symmetry mismatch with the plane waves. Specifically, at the $\Gamma$ point the system has $C_2$ rotational symmetry (invariant under $180^\circ$ rotation around the $z$ axis), and the quadrupole mode does not couple to the radiating plane wave because the former has a field profile $E(r)$ that is even under $C_2$ rotation, $E(r) = \tilde{O}_{C_2}E(r)$, whereas the latter is odd, $E(r) = -\tilde{O}_{C_2}E(r)$. The effective Hamiltonian is

$$H_{\text{eff}} = \begin{pmatrix} \omega_0 & v_g |k| \\ v_g |k| & \omega_0 - i\gamma_d/2 \end{pmatrix}$$

with complex eigenvalues

$$\omega_{\pm} = \omega_0 - i\gamma_d/2 \pm v_g \sqrt{|k|^2 - k_z^2}$$

where $\omega_0$ is the frequency at accidental degeneracy, $v_g$ is the group velocity of the linear Dirac dispersion in the absence of radiation, $|k|$ is the magnitude of the in-plane wavevector $(k_x, k_y)$, and $k_z = \gamma_d/2v_g$. Here, one of the three bands is decoupled from the other two and is not included in equation (1) (see Supplementary Information Section 2). In equation (2), a ring defined by $|k| = k_z$ separates the space
Figure 1 | Accidental degeneracy in Hermitian and non-Hermitian photonic crystals. a, Band structure of a two-dimensional photonic crystal consisting of a square lattice of circular air holes. Tuning the radius $r$ leads to accidental degeneracy between a quadrupole band and two doubly degenerate dipole bands, resulting in two bands with linear Dirac dispersion (red and blue) and a flat band (grey). b, c, The real (b) and imaginary (c) parts of the eigenvalues of an open, and therefore non-Hermitian, system: a photonic crystal slab with finite thickness, $h$. By tuning the radius, accidental degeneracy in the real part can be achieved, but the Dirac dispersion is deformed owing to the non-Hermiticity. The analytic model predicts the real (imaginary) part of the eigenvalue stays as a constant inside (outside) a ring in the wavevector space, indicating two flat bands in dispersion, with a ring of exceptional points (EPs) where both the real and the imaginary parts are degenerate. The orange shaded regions correspond to the inside of the ring. In the upper panels of a–c, solid lines are predictions from the analytic model and symbols are from numerical simulations: red squares represent the band connecting to the quadrupole mode at the centre; blue circles represent the band connecting to the dipole mode at the centre; and grey crosses represent the third band that is decoupled from the previous two due to symmetry. The three-dimensional plots in the lower panels are from simulations.

into two regions: inside the ring ($|k| < k_\text{c}$), Re($\omega_\text{c}$) are dispersionless and degenerate; outside the ring ($|k| > k_\text{c}$), Im($\omega_\text{c}$) are dispersionless and degenerate. In the vicinity of $k_\text{c}$, Im($\omega_\text{c}$) and Re($\omega_\text{c}$) exhibit square-root dispersion (also known as branching behaviour*) inside and outside the ring, respectively. Exactly on the ring ($|k| = k_\text{c}$), the two eigenvalues $\omega_\text{c}$ are degenerate in both real and imaginary parts; meanwhile, the matrix $H_{\text{eff}}$ becomes defective with an incomplete eigenspace spanned by only one eigenvector $(1, -i)^T$ that is orthogonal to itself under the unconjugated 'inner product', given by $a^T b$ for vectors $a$ and $b$. This self-orthogonality is the definition of EPs; hence, here we have not just one EP, but a continuous ring of EPs. We call it an exceptional ring.

Figure 1b, c shows the complex eigenvalues of the photonic crystal slab structure calculated numerically (symbols), which closely follow the analytic model of equation (2) shown as solid lines in the figure. In Supplementary Fig. 1, we show that the two eigenvectors indeed coalesce into one at the EP, which is impossible in Hermitian systems (also see Supplementary Information section III). When the radius $r$ of the holes is tuned away from accidental degeneracy, the exceptional ring and the associated branching behaviour disappear, as shown in Supplementary Fig. 2. Several properties of the photonic crystal slab contribute to the existence of this exceptional ring. Owing to periodicity, one can probe the dispersion from two degrees of freedom, $k_x$ and $k_y$, in just one structure. The open boundary provides radiation loss, and the $C_2$ rotational symmetry differentiates the radiation loss of the dipole mode and of the quadrupole mode.

We can rigorously show that the exceptional ring exists in realistic photonic crystal slabs, not just in the effective Hamiltonian model. Our proof is based on the unique topological property of EPs: when the system parameters evolve adiabatically along a loop encircling an EP, the two eigenvalues switch their positions when the system returns to its initial parameters$^{7,21,25}$, in contrast to the typical case where the two eigenvalues return to themselves. Using this property, we numerically show, in Supplementary Fig. 3 and Supplementary Information section IV, that the complex eigenvalues always switch their positions along every direction in the $k$ space, and therefore prove the existence of this exceptional ring. As opposed to the simplified effective Hamiltonian model, in a real photonic crystal slab, the EP may exist at a slightly different magnitude of $k$ and for a slightly different hole radius $r$ along different directions in the $k$ space, but this variation is small and negligible in practice (Supplementary Information section V).

To demonstrate the existence of the exceptional ring in such a system, we fabricate large-area periodic patterns in a Si$_3$N$_4$ slab ($n = 2.02$ in the visible spectrum, thickness $180 \text{ nm}$) on top of $6 \mu\text{m}$ of silica ($n = 1.46$) using interference photolithography$^{24}$. Scanning electron microscope (SEM) images of the sample are shown in Fig. 2a, featuring a square lattice (periodicity $a = 336 \text{ nm}$) of cylindrical air holes with radius $109 \text{ nm}$. We immerse the structure into an optical liquid with a specified refractive index that can be tuned; accidental degeneracy in the Hermitian part is achieved when the liquid index is selected to be $n = 1.48$. We perform angle-resolved reflectivity measurements (set-up shown in Fig. 2b) between $0^\circ$ and $2^\circ$ along the $\Gamma \rightarrow X$ direction and the $\Gamma \rightarrow M$ direction, for both s and p polarizations. Details of the sample fabrication and the experimental setup can be found in Supplementary Information section VI. The measured reflectivity for the relevant polarization is plotted in the upper panel of Fig. 2c, showing good agreement with numerical simulation results (lower panel), with differences coming from scattering due to surface roughness, inhomogeneous broadening, and the uncertainty in the measurements of system parameters. The complete experimental result for both polarizations is shown in Supplementary Fig. 4; the third and dispersionless band shows up in the other polarization, decoupled from the two bands of interest.

The peaks of reflectivity (dark red colour in Fig. 2c) follow the linear Dirac dispersion; this feature disappears for structures with different radii that do not reach accidental degeneracy (experimental results in...
Supplementary Fig. 8b, we plot the values of part, consistent with the simplified model in equation (1). In Supplementary Information section VII and Supplementary Fig. 6 (TCMT), we show that when matrix\( A \) and matrix \( B \) are both Hermitian

\[
H = \begin{pmatrix} \omega_1 & K \\ K & \omega_2 \end{pmatrix} - i \begin{pmatrix} \gamma_1 & \gamma_1 \\ \gamma_2 & \gamma_2 \end{pmatrix}
\]

and diagonalizing

\[
\begin{pmatrix} \omega_+ & 0 \\ 0 & \omega_- \end{pmatrix}
\]

As before, we use \( \omega_\pm \) to denote the complex eigenvalues of the Hamiltonian \( A - iB \). Physically, matrix \( A \) describes a lossless system, and matrix \( -iB \) adds the effects of loss. In \( B \), the diagonal elements are loss rates (in our system, they come primarily from radiation), and the off-diagonal elements arise from overlap of the two radiation patterns, also known as external coupling of resonances via the continuum. Modelling the reflectivity using temporal coupled-mode theory (TCMT), we show that when matrix \( B \) is dominated by radiation, the reflection peaks occur near the eigenvalues \( \Omega_{1,2} \) of the Hamiltonian part \( A \) and are independent of the anti-Hermitian part \(-iB\) (see Supplementary Information section VII and Supplementary Fig. 6 for details). Therefore, the linear Dirac dispersion observed in the measured data of Fig. 2c (dark red) indicates that we have successfully achieved accidental degeneracy in the eigenvalues of the Hamiltonian part, consistent with the simplified model in equation (1). In Supplementary Fig. 8b, we plot the values of \( \Omega_{1,2} \) extracted from the reflectivity data through a more rigorous data analysis using TCMT (described below); the linear dispersion is indeed observed. We note that when there is substantial non-radiative loss or material gain in the system, the reflection peaks no longer follow the eigenvalues of the Hamiltonian part (see Supplementary Information section VIII and Supplementary Fig. 7).

The real part of the complex eigenvalues of the Hamiltonian, \( \text{Re}(\omega_\pm) \), behave very differently from the reflectivity peaks. Simulation results (solid white lines in the lower panel of Fig. 2c) show \( \text{Re}(\omega_-) \) is dispersionless at small angles with a branch-point singularity around \( 0.31^\circ \)—consistent with the feature predicted by the simplified Hamiltonian in equation (2). In Fig. 2d, we compare the reflectivity spectra from simulations (with peaks indicated by red arrows) with the corresponding complex eigenvalues at three representative angles (0.8° in blue, 0.31° in green and 0.1° in magenta). At 0.31°, the two complex eigenvalues are degenerate, indicating an EP; however, the two reflection peaks do not coincide since they represent the eigenvalues of only the Hamiltonian part of the Hamiltonian, which does not have degeneracy here. The dip in reflectivity between the two peaks (marked as black arrows in Figs 2 and 3) is the coupled-resonator-induced transparency (CRIT) that arises from the interference between radiation of the two resonances\(^{26}\), similar to electromagnetically induced transparency. Qualitatively, the peak locations of the measured reflectivity spectrum reveal the eigenvalues of the Hamiltonian part, \( A \), and the linewidths of the peaks reveal the anti-Hermitian part, \(-iB\); diagonalizing \( A - iB \) yields the eigenvalues \( \Omega_{1,2} \), as illustrated in equation (3). To be more quantitative, we use TCMT and account for both the direct

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**Figure 2** Experimental reflectivity spectrum and accidental Dirac dispersion. a, SEM images of the photonic crystal samples: side view (upper panel) and top view (lower panel). b, Schematic drawing of the measurement set-up. Linearly polarized light from a super-continuum source is reflected off the photonic crystal slab (‘sample’) immersed in an optical liquid, and collected by a spectrometer (SP). The incident angle \( \theta \) is controlled using a precision rotating stage. BS, beam splitter. c, Reflectivity spectrum of the sample measured experimentally (upper panel) and calculated numerically (lower panel) along the \( \Gamma \to X \) and the \( \Gamma \to M \) directions. The peak location of reflectivity reveals the Hermitian part of the system, which forms Dirac dispersion due to accidental degeneracy. In the lower panel, white solid lines indicate the real part of the eigenvalues; spectra and eigenvalues at three representative angles (marked by dashed lines and circles) are shown in d. d, Three line cuts of reflectivity \( R \) from simulation results. Also shown are the complex eigenvalues (open circles) calculated numerically. At large angles (0.8°), the two resonances are far apart, so the reflectivity peaks (red arrows) are close to the actual positions of the complex eigenvalues. However, at small angles (0.3°, 0.1°), the coupling between resonances cause the resonance peaks (red arrows) to have much greater separations in frequency compared to the complex eigenvalues. The black arrows mark the dips in reflectivity that correspond to the coupled-resonator induced transparency (CRIT, see text for details).
band structures and the density of states; this effect becomes most
prominent near EPs. The photonic crystal slab described here provides a simple-to-realize platform for studying the influence of EPs on light–matter interaction, such as for single particle detection and modulation of quantum noise. The two-dimensional flat band can also provide a high density of states and therefore high Purcell factors. The strong dispersion of loss in the vicinity of the Γ point can improve the performance of large-area single-mode photonic crystal lasers.

Repeating the fitting procedure for the reflectivity spectrum measured at different angles, we obtain the dispersion curves for all complex eigenvalues, which are plotted in Fig. 3b. Along both directions in k space (Γ → X and Γ → M), the two bands of interest (shown in blue and red) exhibit the EP behaviour predicted in equation (2): for \( |k| < k_c \) the real parts are degenerate and dispersionless; for \( |k| > k_c \) the imaginary parts are degenerate and dispersionless; for \( |k| \) in the vicinity of \( k_c \) branching features are observed in the real or imaginary part. In Fig. 3c, we plot the eigenvalues on the complex plane for both the Γ → X and Γ → M directions. We can see that in both directions, the two eigenvalues approach each other and become very close at a certain \( k \) point, which is a clear signature of the system being very close to an EP.

We have shown that non-Hermiticity arising from radiation can significantly alter fundamental properties of the system, including the band structures and the density of states; this effect becomes most
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