Time lags of flaring AGN

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Abstract. In order to distinguish the leptonic from the hadronic origin of the non-thermal emission of a flaring AGN we present a semi-analytical model that describes the temporal development of the emergent particles (i.e. photons and neutrinos) based on their leptonic and hadronic origin respectively. The approach starts with the transport equation of the injected relativistic particles and takes spatial diffusion and continuous energy losses into account. On the one hand, a relativistic electron pick-up is considered, that leads to synchrotron, as well as external Compton emission and on the other hand, a relativistic proton pick-up, which results in high energy photons and neutrinos by inelastic proton-proton collisions. The paper ends up in the temporal development of the emergent photon and neutrino intensities of AGN flares in hadronic and leptonic interaction scenarios and gives useful predictions of flare durations and time lags between photons of different wavelength and high energy neutrinos.

1. Introduction

The primary hadronic or leptonic nature of the non-thermal radiation production in AGN is still an unsolved problem in high energy astroparticle physics and the detection of extragalactic high energy neutrinos could become the smoking gun evidence. Currently, several multifrequency campaigns on flaring blazars have shown a strong correlation between the optical, the X-ray and the TeV emission ([1], [2], [3]), that indicates the same origin of this non-thermal radiation. Consequently, there are four effects determining the temporal behaviour of the emission volume: (1) The injection condition of relativistic particles, its subsequent (2) spatial diffusion and (3) continuous cooling processes, as well as (4) propagation effects on the generated particles (i.e. photons and neutrinos respectively) in the emission knot.

Taking these four processes into account, we calculate the synchrotron and external Compton (EC) radiation by relativistic electrons, as well as the emission of high energy photons and neutrinos by inelastic p-p interactions. Hence, the calculations offers a systematic study of the temporal behaviour of a flaring AGN based on the leptonic respectively hadronic nature of the primary particles. Due to the mathematical complexity, this first approach neglects the radiation processes by synchrotron self-Compton (SSC) and photohadronic interactions. However, we show that these effects are also negligible at a certain range of parameters.

We consider a spherical emission knot of radius $R = 10^{15} R_{15}$ cm that is Doppler boosted towards the observer and consists of a non-relativistic plasma of electrons and protons of density $N_{b} = 10^{10} N_{10} \text{cm}^{-3}$. Using such a high particle density in the emission volume, we obtain significant pion production by p-p interactions ([4]) and a diffusive buildup of synchrotron photons.
in the interior, as the optical depth $\tau = \sigma_T N_b R > 1$.

2. Kinetic equation of charged particles

All physical quantities are calculated in a coordinate system comoving with the radiation source.

2.1. Relativistic electrons and protons

The differential number density of relativistic electrons $n_e(r, \gamma, t)$ and protons $n_p(r, \gamma, t)$ respectively obeys the transport equation of [5] and takes spatial diffusion, continuous energy losses and a separable injection term $q_1(\gamma, t)q_2(r)$ into account:

$$\frac{\partial n_{e,p}(r, \gamma, t)}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 D_1(\gamma) \frac{\partial n_{e,p}(r, \gamma, t)}{\partial r} \right) + \frac{\partial}{\partial \gamma} [\gamma |n_{e,p}(r, \gamma, t)|] + q_1(\gamma, t)q_2(r).$$

(1)

Here we idealized that the spatial diffusion is independent of its solid angle, as the influence of anisotropy decreases with increasing time and hence decreasing Lorentz factor. The ultrarelativistic electrons and protons respectively are instantaneously injected at the time $t = t_0$ into the emission knot with a particle density $q_0 = 10^{-4}$ $q_{-4}$ cm$^{-3}$ and an initial Lorentz factor $\gamma_0 = 10^6 \gamma_6$. This particle beam excites electrostatic turbulences, whereby the particles’ energy is quickly changed until a plateau distribution in $P_{||}$ is established ([6]), so that the injected particles can be considered as plateau distributed and we obtain

$$q_1(\gamma, t) = \frac{q_0}{\gamma_0 - 1} H[\gamma_0 - \gamma] \delta(t - t_0).$$

(2)

Additionally, electromagnetic turbulences (i.e. Alfvén waves) are excited, which result in pitch angle scattering of the injected charged particles with an energy dependent diffusion coefficient $D_1(\gamma)$. We assume an isospectral Kolmogorov turbulence with a spectral index $q = \{1, \ldots, 3\}$, so that

$$D_1(\gamma) = \frac{c e_p^q \gamma^\beta}{3},$$

(3)

where $\beta = 2 - q$ and in the full diffusion limit the mean free path $l_{e,p} \gamma_0^\beta \ll R$. In addition the relativistic electrons cool by synchrotron and EC energy losses with the loss rate

$$|\gamma| = D_s \gamma^2, \quad D_s = \frac{4}{3} \frac{\sigma_T}{m_e c^2} \left( U_B + \frac{4}{3} \delta^2 U_{ph} \right) = 1.3 \cdot 10^{-9} (b^2 + l_{EC}) \text{ s}^{-1}.$$  

(4)

Here we assume a magnetic field of constant strength $B = 1$ bGauss, that is randomly distributed on scales larger than the Larmor radii of the injected electrons. Furthermore, $l_{EC} = 0.093 L_{G6} \tau_{-2} \zeta_{pc}^2$ refers to the EC losses due to an isotropic external photon field ([7]) of energy density $U_{ph}$, which depends on the luminosity $L_{ad} = 10^{46} L_{G6}$ erg s$^{-1}$ of the accretion disk, the scattering optical depth $\tau_{sc} = 10^{-2} \tau_{-2}$ and the extension $z_{sc} = 1 z_{pc} \text{ pc}$ of the scattering gas, as well as the Doppler factor $\delta = 10 \delta_1$ of the emission volume. Initial SSC energy losses are negligible compared to the synchrotron energy losses if the injection parameter is less than unity ([8]), i.e. $q_0 < 10^9 R_{15}^{-1} \gamma_0^{-2}$. The cross-section of electromagnetic interactions of protons is much smaller than that of electrons with the same Lorentz factor, as the Thomson cross-section for a proton $\sigma_{T,p} = 3 \times 10^{-7} \sigma_{T,e}$, where $\sigma_{T,p}$ is the Thomson cross-section for an electron. Thus, the relevant interaction processes are on the one hand interactions with cosmic photons and on the other hand interactions with cosmic matter fields. We focus on the latter and especially on the energy loss by pion production:

$$|\gamma|_{pp}^{\gamma p} \approx D_c H[\gamma - 1.30] \gamma, \quad D_c = 7 \times 10^{-16} (N_b/1 \text{ cm}^{-3}) \text{ s}^{-1}.$$  

(5)
Due to the thermal radiation field (of the accretion disk or the Broad Line Region) with a photon energy $\theta = 10 \theta_1$ eV the production of $N$ pions by photohadronic interactions occur only for ultrarelativistic protons with a minimum Lorentz factor of ([9])

$$\gamma_{\text{min}} \approx \frac{N m_{\pi} c^2}{2 \theta} \left( 1 + \frac{N m_{\pi}}{2m_p} \right) \approx 7.3 \cdot 10^6 N \theta_1^{-1}. \tag{6}$$

Therefore, the photohadronic energy losses are negligible for protons with a Lorentz factor $\gamma_6 < 7.3 N \theta_1^{-1}$. Using the spatial boundary conditions for the particle propagation in the spherical emission volume ([5]):

1. a finite density at the centre of the knot; and
2. an exclusively outward directed particle flux at the boundary surface of the knot;

we finally obtain the differential number density of relativistic electrons

$$n_e(\gamma, t, r) = \frac{q_0}{\gamma_0 - 1} H \left[ \frac{\gamma}{1 - D_\gamma(t - t_0)} \right] H \left[ \frac{\gamma}{1 - D_\gamma(t - t_0)} - 1 \right]$$

$$\times \frac{H[t - t_0]}{(1 - D_\gamma(t - t_0))^2} \sum_{k=1}^{\infty} b_k \frac{\sin(\lambda_k r) E_{e,k,\beta}(\gamma, t)}{r}, \tag{7}$$

with

$$E_{e,k,\beta}(\gamma, t) = \begin{cases} \exp \left( \frac{\lambda_k^2 c t_\gamma \gamma^{\beta - 1}}{3(\beta - 1) D_e} \left( 1 - \frac{1}{(1 - D_\gamma(t - t_0))^2} \right) \right); & \text{for } \beta \neq 1, \\ (1 - D_\gamma(t - t_0)) \frac{c t_\gamma \lambda_k^2}{D_e}; & \text{for } \beta = 1, \end{cases} \tag{8}$$

as well as the differential number density of relativistic protons

$$n_p(\gamma, t, r) = \frac{q_0}{\gamma_0 - 1} H[t - t_0] H[\gamma_0 - \gamma \exp(D_c(t - t_0))]$$

$$\times \exp(D_c(t - t_0)) \sum_{k=1}^{\infty} b_k \frac{\sin(\lambda_k r) E_{p,k,\beta}(\gamma, t)}{r}, \tag{9}$$

where

$$E_{p,k,\beta}(\gamma, t) = \begin{cases} \exp \left( -\frac{\lambda_k^2 c t_p \gamma^\beta}{3(\beta - 1) D_e} (\exp(\beta D_e(t - t_0)) - 1) \right); & \text{for } \beta \neq 0, \\ \exp \left( -\lambda_k^2 c t_p(t - t_0)/3 \right); & \text{for } \beta = 0. \end{cases} \tag{10}$$

Here the eigenvalues are

$$\lambda_k \simeq \frac{k \pi}{R}, \quad k = 1, 2, 3, ..., k_{\text{max}}, \tag{11}$$

and the expansion coefficient is calculated by

$$b_k \simeq \frac{2}{R} \int_0^R dr q_2(r)r \sin(\lambda_k r). \tag{12}$$

Based on mathematical convenience, we consider a homogeneous particle injection into the whole emission volume ($q_2(r) = 1$), since we have no a priori knowledge about the true location of the relativistic particle source. However, the solution for more complex spatial injection assumptions are simply obtained by changing the expansion coefficient (12).
3. Leptonic emission
In this section we consider the radiation resulting from the relativistic electrons of Eq. (7) and neglect any further relativistic electrons in the emission region.

3.1. Synchrotron radiation
First, we calculate the synchrotron photon emission generated by the relativistic electron density distribution (7). The broad continuum of the synchrotron power, that is described by an integral over the modified Bessel function of the second kind $K_{5/3}(x)$ can be approximated in vacuum ([10], [11]) by

$$P(\nu, \gamma) \simeq P_0 \left(\frac{\nu}{\nu_s \gamma^2}\right)^\frac{1}{3} \exp\left(-\frac{\nu}{\nu_s \gamma^2}\right),$$

with $P_0 = 2.647 \times 10^{-10} \text{ eV s}^{-1} \text{ Hz}^{-1}$ and the characteristic frequency $\nu_s \gamma^2 = 3eB\gamma^2/(4\pi m_e c)$. Hence, the spontaneous synchrotron emission coefficient $j_s(r, \nu, t)$ that results from an isotropic relativistic electron distribution $n_e(r, \gamma, t)$ yields

$$j_s(r, \nu, t) = \frac{P_0}{4\pi} \left(\frac{\nu}{\nu_s}\right)^\frac{1}{3} \int_1^\infty d\gamma \gamma^{-\frac{2}{3}} \exp\left(-\frac{\nu}{\nu_s \gamma^2}\right) n_e(r, \gamma, t).$$

For an optically thin source, the synchrotron intensity on the surface ($r = R$) at a time $t$ and an energy $\epsilon = h\nu$ is given by ([12])

$$I_{Sy}(R, \epsilon, t) = \frac{1}{2R} \int_0^R dr' \int_{R-r'}^{R+r'} ds \rho_{Sy}(r', \epsilon, t - \frac{s}{c}) \frac{j_s(r', \epsilon, t - \frac{s}{c})}{s},$$

with the omnidirectional photon production rate

$$\rho_{Sy}(r', \epsilon, t - \frac{s}{c}) = 4\pi \exp\left(-\frac{g(x)s}{R}\right) j_s(r', \epsilon, t - \frac{s}{c}),$$

where the exponential term expresses the survival probability of traveling the distance $s = \sqrt{R^2 + r'^2 - 2Rr'\mu'}$ from the spherical coordinates $r'$, $\mu' = \cos \phi'$ to the boundary surface of the knot ([5]). Due to the synchrotron cooling of the relativistic electrons the amplitude of the emergent intensity, as well as the total flare duration and the time of maximal synchrotron flare emission increase with decreasing photon energy $\epsilon$ (see Fig 1). In the full diffusion limit ($R \gg \gamma_0 l_e$) the spatial diffusion of the relativistic electrons has no influence on the temporal behaviour of the emergent synchrotron intensity.

3.2. External Compton radiation
Secondly, we consider the inverse Compton scattering of low energy photons by the relativistic electron density distribution (7). Since we assume a low number density of injected particles [$q_0 < 10^9 R_{15}^{-1} \gamma_0^{-2}$] to keep the SSC losses negligible, the seed photons for the EC scattering result from an external photon field like the accretion disk of the AGN. Thus we adopt an isotropic and temporal constant number density of thermal photons

$$n_{Ph}(\epsilon) = n_0 \left(\exp\left(\frac{\epsilon}{\theta}\right) - 1\right)^{-1}$$

with a disk temperature $\theta = 10\theta_1 \text{ eV}$ and $n_0 = 34 L_{46} \tau_{-2} \gamma_0^{-2} \text{ cm}^{-3}$. Using the $\delta$-function approximation for inverse Compton scattering ([13]) in the limit $\gamma \gg 1 \gg \epsilon/(m_e c^2)$ the

$$\rho_{Sy}(r', \epsilon, t - \frac{s}{c}) = 4\pi \exp\left(-\frac{g(x)s}{R}\right) j_s(r', \epsilon, t - \frac{s}{c}),$$

where the exponential term expresses the survival probability of traveling the distance $s = \sqrt{R^2 + r'^2 - 2Rr'\mu'}$ from the spherical coordinates $r'$, $\mu' = \cos \phi'$ to the boundary surface of the knot ([5]). Due to the synchrotron cooling of the relativistic electrons the amplitude of the emergent intensity, as well as the total flare duration and the time of maximal synchrotron flare emission increase with decreasing photon energy $\epsilon$ (see Fig 1). In the full diffusion limit ($R \gg \gamma_0 l_e$) the spatial diffusion of the relativistic electrons has no influence on the temporal behaviour of the emergent synchrotron intensity.
Figure 1. Temporal development of the emergent synchrotron intensity at an energy $\epsilon = 1 \text{ eV}$ (solid line), $\epsilon = 100 \text{ eV}$ (dashed line) and $\epsilon = 1 \text{ keV}$ (dotted line). Furthermore, we used $\beta = 0$, $l_e = 10^{10} \text{ cm}$, $R_{15} = 1$, $\gamma_6 = 1$, $q_{-4} = 1$ and $b^2 + l_{EC} = 1$.

Figure 2. Temporal development of the emergent EC intensity at an energy $E_{\gamma} = 1 \text{ GeV}$ (solid line), $E_{\gamma} = 10 \text{ GeV}$ (dashed line) and $E_{\gamma} = 100 \text{ GeV}$ (dotted line). Furthermore, we used an accretion disk temperature of $\theta_1 = 1$ (left panel) and $\theta_1 = 10$ respectively, as well as $n_0 = 1 \text{ cm}^{-3}$, $\beta = 0$, $l_e = 10^{10} \text{ cm}$, $R_{15} = 1$, $\gamma_6 = 1$, $q_{-4} = 1$ and $b^2 + l_{EC} = 1$.

spontaneous EC emission coefficient yields

$$j_c(r, E_{\gamma}, t) = \frac{E_{\gamma} c \sigma T}{12 \pi^2 m_e c^2} \int_0^\infty d\epsilon n_{ph}(\epsilon) \int_0^{m_e c^2/\epsilon} d\gamma n_\gamma(r, \gamma, t)^{2} \delta \left(E_{\gamma} - \gamma^2 \epsilon\right). \quad (18)$$

Analogous to the previous section the emergent intensity of EC photons at a time $t$ and an energy $E_{\gamma}$ is calculated by

$$I_{EC}(R, E_{\gamma}, t) = \frac{1}{2} \frac{R}{R_{15}} \int_0^{R_{15} + R} dr' r' \int_{R - r'}^{R + r'} ds \frac{\rho_{EC}(r', E_{\gamma}, t - \frac{s}{c})}{s}, \quad (19)$$

with the omnidirectional photon production rate

$$\rho_{EC}(r', E_{\gamma}, t - \frac{s}{c}) = 4\pi \exp \left(-s/R \right) j_c \left(r', E_{\gamma}, t - \frac{s}{c}\right), \quad (20)$$

where the exponential term expresses the probability of escaping the emission volume ([5]). Fig. 2 shows that the temporal development of the emergent EC intensity is similar to the case
Figure 3. Temporal development of the emergent intensity of gammas (red), muon-neutrinos (magenta) and electron-neutrinos (orange) by inelastic p-p interactions with a background particle density of $N_{10} = 10$ (left panel) and $N_{10} = 0.1$ (right panel) respectively. We considered the emission at an energy $E_l = 0.1$ TeV (solid line), $E_l = 1$ TeV (dashed line) and $E_l = 100$ TeV (dotted line) and used $\beta = 0$, $l_p = 10^{10}$ cm, $R_{15} = 1$, $\gamma_0 = 1$ and $q_{-4} = 1$.

of synchrotron radiation and the time of maximal flare emission, as well as the flare duration increase with decreasing gamma energy $E_\gamma$. The spatial diffusion has no influence on the light-curves in the full diffusion limit. With increasing disk temperature the amplitude of the light curves increase and the time of maximal flare emission shifts to later times (especially at low gamma energies $E_\gamma$).

4. Hadronic emission

In this section we neglect the electrons, as well as the photon field and consider only the relativistic protons of Eq. (9) and their interactions with the non-relativistic protons of the background plasma. According to Eq. (6) the pion production by photohadronic interactions is negligible. However, using the minimal Lorentz factor $\gamma_{\text{min}}^{e^+e^-} = 5.1 \cdot 10^4 \theta_{\text{I}}^{-1}$ for photo-pair production ([9]), the emission knot has to be considered at a distance

$$z_{pc} > 1.6 \cdot 10^{-4} \gamma_{\text{I}} \Delta_{46}^{1/2} \sigma_{\text{e}^+e^-}^{\text{pp}}(E_p) n_{\gamma}(r, E_p, t),$$

in order to neglect (in comparison to the inelastic proton-proton losses) the photo-pair production by the scattered radiation field $n_{ph}$ of the accretion disk.

4.1. Gamma and neutrino emission by inelastic p-p interactions

The relativistic protons with an energy $E_p = \gamma m_p c^2$ collide inelastically with the non-relativistic protons of the background plasma of constant density $N_b$ and generate neutral and charged ultrarelativistic $\pi$-mesons which decay quasi instantaneously into gamma-rays ($l = \gamma$) and neutrinos ($l = \{\nu_e, \nu_\mu\}$) respectively with an energy $E_l$. Using the energy spectra $F(E_l/E_p, E_p)$ of secondary particles ([14]) its omnidirectional production rate is calculated by

$$\rho_{\gamma}^{pp}(r, E_l, t) = 4\pi \exp(-s/R) c N_b \int_{E_l}^{\infty} \frac{dE_p}{E_p} \sigma_{\pi}^{pp}(E_p) n(r, E_p, t) F(E_l/E_p, E_p), \quad (21)$$

where high energy photons ($x \geq 1$) and neutrinos have the same escape probability $\exp(-s/R)$. 


Consequently, the emergent intensity of gamma-rays and neutrinos by inelastic p-p collisions yields

\[ I_{pp}(R, E_l, t) = \frac{1}{2R} \int_0^R dr' r' \int_{R-r'}^{R+r'} ds \rho_{pp}^\gamma(r', E_l, t - \frac{s}{c}). \]  

(22)

Due to the slower cooling mechanism (5) of the relativistic protons, the time of maximal emission of gammas and neutrinos hardly increases with decreasing energy \( E_l \) (see Fig. 3). However, the flare duration considerably increases with decreasing \( E_l \), as well as with a decreasing number density \( N_b \) of background protons. Except for the amplitudes of the emergent intensities there is only a marginal difference in the emission of gammas and neutrinos. Like in the case of leptonic radiation processes, the spatial diffusion of the relativistic protons has in the full diffusion limit \( (R \gg \gamma_0 l_p) \) no influence on the temporal flare development.

5. Time lags and conclusions

The amplitude of the emergent intensity by the leptonic respectively hadronic scenarios strongly depends on several unknown parameters, like the spatial distribution of seed electrons and protons respectively, as well as the properties of the emission volume \((R, N_b, B)\) or the external photon field \((n_0, \theta)\). However, the temporal development of the emergent intensity is mainly defined by the parameters \( B, \theta \) and \( N_b \) of the particular cooling and emission process. The temporal flare behaviour is not significantly changed by the injection assumption, but a finite injection duration \( \tau_{inj} \) enlarges the flare duration by the time \( \tau_{inj} \). Based on the leptonic respectively hadronic scenario Fig. 4 shows the half-life of the emergent intensity and the time of maximal flare emission at an energy range from 1 eV up to 100 TeV. The flare duration, as well as the time of maximal emission for each radiation scenario increases with decreasing photon energy and/or decreasing cooling parameters \( B \) and \( N_b \) respectively. The EC emission gets increasingly delayed (especially at low \( E_\gamma \)) with increasing disk temperature \( \theta \), however at the smallest \( E_\gamma \) the time of maximal flare emission stays at the beginning of the flare. Furthermore,
there is a plain difference between the hadronic and the leptonic scenario, since the time of maximal emission of gammas and neutrinos by pion production hardly differs by changing the cooling parameter $N_b$ or the energy $E_l$. The minimal total flare duration is limited by the radius of the emission knot due to the light crossing time $R/c$. Another feature of Fig 4 is the small time lag between the neutrino and the gamma emission by p-p interactions. The time of maximal flare emission, as well as the half-life of the flare decrease with increasing energy $E_l$, however, the decay is steeper in case of neutrinos, which results from the bigger difference in the maximal value of its energy spectra $F_l$ at different proton energies (see Fig. 7-9 in [14]). Consequently, the model yields significant time lags between the time of leptonic and hadronic emission maxima, as well as different flare durations of gammas and neutrinos resulting from a hadronic emission scenario.

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