Examining Two-dimensional Luminosity–Time Correlations for Gamma-Ray Burst Radio Afterglows with VLA and ALMA

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Abstract

Gamma-ray burst (GRB) afterglow emission can be observed from sub-TeV to radio wavelengths, though only 6.6% of observed GRBs present radio afterglows. We examine GRB radio light curves (LCs) to look for the presence of radio plateaus resembling the plateaus observed at X-ray and optical wavelengths. We analyze 404 GRBs from the literature with observed radio afterglow and fit 82 GRBs with at least five data points with a broken power-law model, requiring four parameters. From these, we find 18 GRBs that present a break feature resembling a plateau. We conduct the first multiwavelength study of the Dainotti correlation between the luminosity $L_\gamma$ and the rest-frame time of break $T_\nu^\ast$ for those 18 GRBs, concluding that the correlation exists and resembles the corresponding correlation at X-ray and optical wavelengths after correction for evolutionary effects. We compare $T_\nu^\ast$ for the radio sample with $T_\nu^\ast$ values in X-ray and optical data, finding significantly later break times in the radio. We propose that this late break time and the compatibility in slope suggest either a long-lasting plateau or the passage of a spectral break in the radio band. We also correct the distribution of the isotropic energy $E_{iso}$ versus the rest-frame burst duration $T_{90}$ for evolutionary effects and conclude that there is no significant difference between the $T_{90}$ distributions for the radio LCs with a break and for those without.

Supporting material: machine-readable table

1. Introduction

Gamma-ray bursts (GRBs) are observed at all wavelengths, from high-energy gamma rays to radio. GRBs are characterized by an energetic “prompt emission” of $\gamma$-rays followed by a much longer period of lower-energy emission called the “afterglow,” lasting from hundreds of seconds to years. X-ray, optical, and radio afterglows are not observed equally: X-ray afterglows have been detected in $\sim$66% of observed GRBs when we consider the full sample of detected GRBs including those that were not observed by the Neil Gehrels Swift Observatory (Swift), optical afterglows in 38%, and radio afterglows in only 6.6% of all known GRBs (Greiner 2021).12 Indeed, some GRBs are too faint to be detected in the radio (Chandra & Frail 2012). With the new Square Kilometer Array (SKA) facilities (Bij de Vaate et al. 2021) and SKA pathfinder (Johnston et al. 2007; Schinckel et al. 2011) we will be able to observe the radio afterglows of more GRBs. Out of the total number of GRBs observed in the radio, we count that the majority (152) are observed by the Very Large Array (VLA).

Swift light curves (LCs) resulting from GRB afterglows have highlighted complicated features inconsistent with a simple power-law decay (Sakamoto et al. 2007; Zhang et al. 2009). Analysis of X-ray LCs has shown the existence of plateaus, or a flattening in the afterglow emission between the prompt emission and the subsequent afterglow decay (Sakamoto et al. 2007; Dainotti et al. 2013b; Fraija et al. 2020, 2021b). These plateaus have also been confirmed in optical LCs (Dainotti et al. 2020b).

An interesting two-dimensional correlation between the luminosity, $L_\gamma$, and the rest-frame end time of the plateau, $T_\nu^\ast$, known as the Dainotti correlation, was discovered more than a decade ago (Dainotti et al. 2008, 2011a, 2013b, 2015a, 2016, 2017b, 2017b, 2020a, 2020b; Srinivasaragavan et al. 2020) and has been proposed as a tool to standardize the plateau sample of GRBs. These plateaus are thought to be produced by continuous energy injection. One proposed explanation is accretion falling back onto a black hole, where energy is released into the external shock, interacts with the surrounding medium, and is injected into the observed afterglow (Liang et al. 2007; Oates et al. 2012). Another interpretation involves the spin-down luminosity from a newborn magnetar providing the continuous energy injection (Duncan & Thompson 1992; Usov 1992; Thompson & Duncan 1993;...
Zhang & Mészáros 2001; Metzger et al. 2011; Rowlinson et al. 2014; Rea et al. 2015; Stratta et al. 2018; Fraija et al. 2021a.

Using the magnetar scenario, Dainotti et al. (2013b, 2020b) showed that this relation can be seen in both X-ray and optical wavelengths with a slope of \( \approx -1 \). This relation in X-rays has been used to build a GRB Hubble diagram with redshift values up to \( z > 8 \) (Cardone et al. 2009, 2010; Dainotti et al. 2013b; Postnikov et al. 2014). If this correlation also exists in the radio, it could reveal information about the underlying GRB emission mechanisms and prove a step toward the standardization of the varied GRB population.

Our goal is to determine the existence of radio plateaus and examine the two-dimensional Dainotti correlation in the radio. To our knowledge, this is the first such analysis with radio data, and thus the most complete multiwavelength study of this relation. Our paper is organized as follows. In Section 2, we describe our data sample. In Section 3, we discuss the multiwavelength Dainotti correlation, the distribution of the isotropic energy in the prompt emission, \( E_{\text{iso}} \), the rest-frame time duration of the prompt emission, \( T_{90} \), the correlation between \( E_{\text{iso}} \) and the rest-frame end time of the plateau emission, \( T_{0}^{*} \), and its corresponding luminosity, \( L_{0} \), as well as a comparison of \( T_{0}^{*} \) in the X-ray, optical, and radio. In Section 4, we discuss the implications of the results achieved and present our conclusions.

2. Data Selection

We take our sample from all published radio afterglows in the literature, mainly observed by the VLA. The largest portion of our data comes from Chandra & Frail (2012), consisting of 304 radio afterglows observed from 1997 to 2011. We extend our search to 2020, gathering an additional 100 GRBs from the literature for a total sample of 404 GRBs. We also note that of our search to 2020, gathering an additional 100 GRBs from the literature for a total sample of 404 GRBs. We also note that of our data comes from Chandra & Frail 2012.

We then put our sample through a filtering process to obtain LCs useful for our analysis. We first reject all radio observations that report only upper limits, bringing our sample to 211 GRBs. To attempt a fit to the radio LCs, we require at least five observations at the same frequency, discarding 127 GRBs that do not fit that criterion. We further discard two GRBs without known redshift, leaving us with a final sample of 82 GRBs.

Where the data are available, we consider multiple LCs within different frequency bands for one GRB. We therefore attempt to fit 202 LCs in total from our sample of 82 GRBs—on average, we fit LCs at 2–3 frequencies for each GRB, though some have more LCs (e.g., GRB 030329 has nine). We perform the fit with a broken power-law (BPL) model, using the equation

\[
F(t) = \begin{cases} 
F_a \left( \frac{t}{T_a} \right)^{-\alpha_1} & t < T_a \\
F_a \left( \frac{t}{T_a} \right)^{-\alpha_2} & t \geq T_a,
\end{cases}
\]

where \( F_a \) is the flux at the end of the plateau emission in erg cm\(^{-2}\) s\(^{-1}\), \( T_a \) is the observed frame time in seconds at the end of the plateau emission, and \( \alpha_1 \) and \( \alpha_2 \) refer to the temporal power-law decay indices before and after the break, respectively. The \( \alpha_1 \) index is important because it determines the flatness of the plateau. We compute the flux in these units for ease of comparison to X-ray and optical data.

To compile our final sample, we first discard 75 LCs because their data are too scattered to be fitted with a BPL. Then, we discard 18 LCs because they do not support the shape of the BPL with a plateau (i.e., their \( |\alpha_1| > 0.5 \), or they can be fitted with a simple power law, etc.). In addition, 70 LCs do not fulfill the Avni (1978) prescription regarding the \( \Delta \chi^2 \) analysis, namely, we varied \( \Delta \chi^2 \) so that the 1\( \sigma \) bounds could be determined assuming that the shape of \( \Delta \chi^2 \) is a parabola for each of the parameters involved in the fitting; see Avni (1978) for additional details.

With this selection we are left with 39 LCs that display a clear break—however, we further reject nine that are too steep to present a plateau, with \( |\alpha_1| > 0.5 \), and 12 from repeated GRBs. Thus, we find 18 LCs that resemble a plateau with \( 0 < |\alpha_1| < 0.5 \). We report the best-fit parameters for the sample of 18 plateau GRBs in Table 1. All the errors quoted in this paper and gathered in Table 1 are calculated to 1\( \sigma \). We show the LCs of the plateau sample in Figure 1.

All of these 18 GRBs are classified as long GRBs, with one (GRB 020903) classified as an X-ray flash, one (GRB 141121A) considered X-ray-rich (XRR), and two with supernova associations (Dainotti et al. 2017b). We classify GRBs with supernova (SN) associations according to the convention presented in Hjorth & Bloom (2012) in relation to the GRB–SN e connection, with GRB 030329 and GRB 980425 classified as “type-A” or SN-A, indicating strong spectroscopic evidence for the association.

We note that two GRBs, GRB 020903 and GRB 120326A, have very large error bars in the \( \alpha_2 \) parameter—thus, we do not consider them in the subsequent analysis of the Dainotti correlation. However, we still include them in the plateau sample, because the sample size is small for the computation of the Efron & Petrosian method. In addition, those GRBs meet all other criteria for acceptance of the fit. A plot of \( \alpha_2 \) versus \( \alpha_1 \) for the sample of 16 GRBs with a plateau in the correlation analysis is shown in the left panel of Figure 2. The same relation for the plateau sample plus the nine additional GRBs with \( |\alpha_1| > 0.5 \), hereafter referred to as the “break” sample (because they can be reliably be fitted with a BPL model, but the slope is too steep to be considered a plateau), is shown in the right panel of Figure 2.

From the observed radio flux, we then compute the luminosity \( L_a \) at the time of break, \( T_a \), using the equation

\[
L_a = 4\pi D_L^2(z) F_a(T_a) K,
\]

where \( F_a \) is the observed flux at \( T_a \), \( D_L(z) \) is the luminosity distance assuming a flat \( \Lambda \)CDM model with \( \Omega_M = 0.3 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), and \( K \) is the \( k \)-correction:

\[
K = \frac{1}{(1 + z)^{\beta + 1}},
\]

with \( \beta \) as the radio spectral index of the GRB (Chandra & Frail 2012). The \( \beta \) values were gathered from the literature; where no value existed, the average of the existing values, \( \beta = 0.902 \), was assigned, with the average of the known uncertainties \( \sigma_{\beta} = 0.17 \).
3. Results

3.1. Luminosity–Time Correlation

For the LCs resembling a plateau, we examine the Dainotti correlation between $L_{\alpha}$ and the rest-frame time of break $T_{a} = T_{a}/(1+z)$ (the star denotes the rest frame), similar to the work in Dainotti et al. (2017a, 2020b) (Figure 3). For a review of the afterglow correlations see Dainotti & Del Vecchio (2017), Dainotti & Amati (2018), and Dainotti et al. (2018). We use the Bayesian D’Agostini method with the cobaya Python package to obtain our fitting parameters. Uncertainties are given to $1\sigma$. The luminosity–time (Dainotti) correlation in the radio is defined as

$$\log L_{\alpha, \text{radio}} = C_{\alpha} + a_{\text{rad}} \times \log T_{\alpha}^*,$$

where $C_{\alpha}$ is the normalization constant and $a_{\text{rad}}$ is the slope determined by the linear fit. Using the subsample of 16 GRBs from the full plateau sample, we find best-fit parameters $C_{\alpha} = 55.42 \pm 3.91$ and $a_{\text{rad}} = -2.34 \pm 0.66$. An ANOVA test gives a $p$-value that this correlation is drawn by chance of $p = 0.005$, and the Spearman $\rho$ coefficient for this correlation is $\rho = -0.6$, indicating that the correlation is significant.

We compare the results of the $L_{\alpha} - T_{a}$ correlation in the radio to the corresponding correlations in the X-ray and optical (Dainotti et al. 2013b, 2020b). We take our sample of 222 X-ray LCs from Dainotti et al. (2013b, 2021c, in preparation) and our sample of 131 optical LCs from Dainotti et al. (2020b, 2021d, in preparation). To our knowledge, this is the first time such a comparison has been considered. We find a slope of $a_{\alpha} = -1.25 \pm 0.07$ in the X-ray and $a_{\text{opt}} = -0.97 \pm 0.07$ in the optical. The radio slope agrees with these values within $2.1\sigma$.

However, for a comparison that accounts for biases and redshift evolution, we apply the Efron–Petrosian (EP) method (Efron & Petrosian 1992) to recover the intrinsic slopes of the full sample of 18 plateau GRBs due to the paucity of the data (Dainotti et al. 2013b, 2015a, 2015b). To this end, we need to mimic the evolution of the variables with redshift with a simple function of redshift, $f(z) = (1+z)^{\delta}$, where $\delta$ is the slope of the evolutionary function determined by the EP method through the computation of a modified version of the Kendall $\tau$ statistics. The slope $\delta$ is found when $\tau = 0$, corresponding to the removal of the evolution. We update the analysis of the evolution in the X-ray sample (M. G. Dainotti et al. 2021c, in preparation) and we find $T_{a} = T_{a}^*/(1+z)^{\delta_{T_{a}}}$ where $\delta_{T_{a}} = -1.2 \pm 0.28$ and $L_{\alpha} = L_{\alpha}/(1+z)^{\delta_{L_{\alpha}}}$ where $\delta_{L_{\alpha}} = 2.4 \pm 0.65$. For an updated optical sample (M. G. Dainotti et al. 2021d, in preparation), we find $\delta_{T_{a}} = -2.1 \pm 0.60$, which agrees with the X-ray value within $1.5\sigma$, and $\delta_{L_{\alpha}} = 3.97 \pm 0.45$, which agrees within $2.4\sigma$. This is more likely due to a difference in sample size than a result of an underlying physical process.

For the radio data, the luminosity limit has been determined using a method described in Dainotti et al. (2021b), in which a complete “parent” sample of GRBs with known peak flux are compared to a subsample of GRBs with known peak flux and known redshift. Here, the peak radio flux is defined as the highest flux observed in the LC, which coincides with $T_{a}$ for the majority of the “break” sample. We therefore take the same convention for the parent sample.
Figure 1. LCs accepted into the plateau sample from BPL fitting.
The two-sample Kolmogorov–Smirnov (KS) test is then used to quantify the probability, as a function of flux limit, that the subsample is pulled from the parent sample. We find >90% probability for all flux limits, with an observed increase in \( p \)-value beginning at \( f_{\text{lim}} = -17.8 \), in units of erg cm\(^{-2}\) s\(^{-1}\), and reaching a plateau at 100% probability at \( f_{\text{lim}} = -17.2 \). Therefore, we choose \(-17.2\) as the radio luminosity limit and find \( \delta \alpha = -1.94 \pm 0.86 \), which agrees with the value in the X-ray and optical within \( 1\sigma \), and \( \delta \ell_u = 3.15 \pm 1.65 \), which also agrees within \( 1\sigma \).

After applying the EP method using this prescription for limiting luminosity, we find the slope for the Dainotti correlation in the radio as \( a_{\text{rad}} = -0.26 \pm 0.71 \). This is compatible with the corresponding correlation in the X-ray, with a corrected slope of \( a_X = -1.02 \pm 0.07 \), within \( 1.07\sigma \), and the corrected slope in the optical, \( a_{\text{opt}} = -0.79 \pm 0.06 \), within \( 0.75\sigma \). However, we note that two GRBs, GRB 980425 and GRB 111005A, appear to be outliers, with lower radio luminosity and redshift than the rest of the sample. If those GRBs are removed from the radio correlation, the slope of the corrected correlation becomes \( a_{\text{rad}} = -0.45 \pm 0.47 \). This value of the correlation, which we consider as the intrinsic value, has been corrected for the effects of selection bias and redshift evolution and does not consider systematically different, low-luminosity GRBs. This slope agrees with the slope of the corrected correlation in the X-ray within \( 1.26\sigma \) and the corrected optical correlation within \( 0.72\sigma \). We stress here that these two outliers are the closest GRBs at the smallest redshift—an order of magnitude less than the redshift of GRB 030329.

It is highly likely that these two GRBs are off-axis GRBs and this is the reason why their luminosities are substantially lower than those expected at their specified redshift; for additional details see Ryan et al. (2015). The radio Dainotti correlation for the plateau sample is shown without the corrections from the EP method in the upper left panel of Figure 3, and with the correction (and removal of outliers) in the upper right panel. The multiwavelength correlations in the X-rays, optical, and radio, without corrections (left) and with corrections and removal of outliers (right), are shown in the lower panels. The X-ray, optical, and radio data are shown in red, blue, and purple, respectively.

Looking at the distribution of radio data within the luminosity–time correlation by class (Figure 3), we observe no particular clustering of any type of GRBs.

3.2. \( E_{\text{iso}} – T^*_9 \) Distribution and the Presence of LC Breaks

To further investigate the behavior of the GRBs that present a break, we examine whether there is a relation between the existence of the break and the energy and duration of the prompt emission properties of a GRB. Thus, we investigate the distribution of the isotropic energy, \( E_{\text{iso}} \), versus the rest-frame burst duration, \( T^*_9 = T_{90} / (1 + z) \), for a subsample of 80 of the 82 GRBs considered for fitting, for which we could either find a value of \( E_{\text{iso}} \) in the literature or compute the value from the literature; where no value could be found (GRBs 050509C and 170105A), we compute \( E_{\text{iso}} \) using the following equation:

\[ E_{\text{iso}} = 4\pi D_L^2(z)SK, \]

where \( S \) is the fluence, \( D_L^2(z) \) is defined as in Equation (2), and \( K \) is the correction

\[ K = \frac{1}{(1 + z)^{3 - \beta}}, \]

with \( \beta \) as the spectral index of the GRB. The values of \( S \) and \( \beta \) are taken from the Swift/BAT GRB catalog.\(^\text{13} \) We plot the distribution of \( E_{\text{iso}} \) versus \( T^*_9 \), shown in Figure 4. In the upper panels, we color-code the sample according to whether it presents a break—blue refers to the 18 GRBs that present a break resembling a plateau, with \( 0 < |\alpha_1| < 0.5 \); red refers to the nine GRBs that present a break, but with steeper \( |\alpha_1| > 0.5 \), and gray refers to 53 GRBs that do not present a break. In the lower panels, we note from which satellite and instruments the GRBs have been observed: the Fermi Large Area Telescope, the Fermi Gamma Ray Burst Monitor, and Swift/BAT are shown in purple, blue, and cyan, respectively. The left panels show the GRB variables without the correction from the EP method; the right panels show the variables after correction for selection bias and redshift evolution.

\(^\text{13} \) https://swift.gsfc.nasa.gov/results/batgrbcat/index_tables.html
Similar to the analysis performed for the Dainotti relation in the radio shown in the previous section, we correct the $E_{\text{iso}}$ distribution for selection biases and redshift evolution using the EP method, which gives $E_{\text{iso}}' = E_{\text{iso}}/(1 + z)^{\delta_{E_{\text{iso}}}}$ and $T_{90}^* = T_{90}/(1 + z)^{\delta_{T_{90}}}$. The $\delta_{E_{\text{iso}}}$ value agrees with values previously reported in Lloyd-Ronning et al. (2019, 2020) within 1$\sigma$, while the $\delta_{T_{90}}$ agrees within 2.17$\sigma$. We show the corrected distributions in the lower two panels of Figure 4, and find no particular trend or clustering after correction. An examination of the corrected $T_{90}$ distribution alone (Figure 6) shows overlap between the GRBs with a break and those without. A KS test between the two samples yields KS = 0.18 with $p = 0.56$, suggesting that they are drawn from the same parent sample.

### 3.3. Correlation of $E_{\text{iso}}$ with $L_{\alpha}$ and $T_{\alpha}^*$

To better understand the relation of the prompt emission to the radio afterglow, we further analyze the correlation of $E_{\text{iso}}$ with $L_{\alpha}$ and $E_{\text{iso}}$ with $T_{\alpha}^*$, similar to the work done in Dainotti et al. (2011b) in X-rays. For the sample of 16 GRBs with a plateau considered for the $L_{\alpha}$-$T_{\alpha}^*$ correlation without correction, we find that the slope of the correlation between $E_{\text{iso}}$ and $T_{\alpha}^*$ is $-2.14 \pm 0.96$, while the slope of the correlation between $E_{\text{iso}}$ and $L_{\alpha}$ is $0.97 \pm 0.17$. After correction for evolutionary effects and removal of the two low-luminosity GRBs, we find the slope after correction for $E_{\text{iso}}$ versus $T_{\alpha}^*$ is $0.23 \pm 0.7$, while the slope for the corrected correlation of $E_{\text{iso}}$ with $L_{\alpha}$ is $0.61 \pm 0.42$. This indicates that both correlations are susceptible to evolutionary effects—indeed, the correlation between $E_{\text{iso}}$ and $T_{\alpha}^*$ nearly vanishes after correction, indicating that the original result is very likely a result of selection bias and redshift evolution. These results are shown in Figure 5, with $E_{\text{iso}}$ versus $T_{\alpha}^*$ in the top panels and $E_{\text{iso}}$ versus $L_{\alpha}$ in the bottom panels. The left panels show the uncorrected correlation, while the corrected correlation is shown in the right panels.

### 3.4. Comparison of $T_{\alpha}^*$ in the X-Ray, Optical, and Radio

We compare the $T_{\alpha}^*$ distribution between the uncorrected and the corrected X-ray, optical, and radio samples to determine
whether the distributions are drawn from the same parent population (Figure 6), similar to the work presented in Dainotti et al. (2020b). The X-ray sample of 222 GRBs presented in Dainotti et al. (2020a, 2021a) and Srinivasaragavan et al. (2020), and the extended optical sample (131 GRBs versus 102 GRBs presented in Dainotti et al. 2020b) show significant overlap, and a KS test between the two samples without correction gives a value of 0.16 with $p \approx 0.03$, while the comparison of the corrected samples gives a value of 0.31 with $p \approx 0$, indicating a slightly greater difference after correction. However, though both the X-ray and optical samples have an average $T_{90}^*$ of $\sim 10^4$ s, the radio sample has a later average $T_{90}^*$ of $\sim 10^6$ s. A KS test of the radio sample without correction versus the X-ray and optical samples without correction produces values of $KS = 0.96$ and $KS = 0.91$ with a $p$-value of $\approx 0$, respectively, while a comparison of the corrected samples produces values of $KS = 0.99$ and $KS = 0.89$, respectively, with a $p$-value of $\approx 0$ in both cases, indicating that in both the uncorrected and corrected samples the radio sample is significantly different. We discuss the possible physical mechanisms for this difference in Section 4.

4. Discussion and Conclusions

Using the largest compilation of published GRB radio afterglows to date, we achieve two main findings: (1) 18 GRBs show a feature resembling a plateau in their LCs; (2) using this sample, the Dainotti correlation still holds for this radio sample, comparable within 2.1$\sigma$. After correction for evolutionary effects and removal of outliers, we find that the Dainotti radio correlation is compatible with the corresponding X-ray and optical correlations within 1.5$\sigma$, with a slope of $a_{\text{rad}} = -0.45 \pm 0.47$. As the slope of the corrected radio correlation agrees with $-1$ within 1.17$\sigma$, similar to the slope found in X-rays, this could indicate that the energy reservoir is conserved. One likely candidate for the production of the plateau is a black hole central engine, where the energy injection is driven by fall-back accretion onto the black hole. Kumar et al. (2008) suggests that the prompt emission is driven...
by accretion of the outer stellar core, while the plateau phase
seen in X-ray LCs is caused by the accretion of the stellar
envelope of the progenitor, or is possibly driven by the
magnetar.

Another important
finding is that the radio break times occur
significantly later than those at the other wavelengths. On
average, the X-ray and optical plateaus last \( \sim 10^4 \) s, while radio
plateaus last \( \sim 10^5 \) s. However, there are cases in which longer
X-ray plateaus have been seen to last more than \( \sim 10^5 \) s, such as
GRB 060218 and GRB 980425, detailed in Dainotti et al.
(2017b). The late break times in the radio could be a result of
the peak of the spectrum appearing in the radio band at later
times as the jet decelerates, consistent with the standard
synchrotron shock model for the afterglow.

On the other hand, the late break could indicate that \( T_\alpha^* \) is
not the end of a plateau, but a break observed at radio
wavelengths. The dynamics of the afterglow emission for an
outflow propagating into a surrounding constant-density
medium has been widely explored (Sari & Piran 1995; Sari
et al. 1996, 1999; Sari 1997). The outflow transfers a large
amount of its energy to this surrounding medium during the
deceleration phase. Since the outflow launched into a cone of
opening angle \( \theta_j \) sweeps the surrounding medium, it decelerates,
thus increasing the angular size of the emitting region
\( \propto 1/\Gamma_j \). Once \( 1/\Gamma_j \approx \theta_j \), there will be no additional radiating
elements from the jet if we assume a “top hat” jet, or another
structure with a steep drop-off in energy per unit angle outside
the core, producing a steepening in the LC. This “break” is
expected to last from several hours to days (e.g., see Kumar &
Zhang 2015). After this phase, the energy flux in radio bands is
expected to evolve as \( F_\nu \propto \Gamma^{-p} \) for \( \nu_m < \nu < \nu_c \) and \( \propto \Gamma^{-1/3} \) for
\( \nu < \nu_m < \nu_c \) (Sari et al. 1999). In our sample, the \( \alpha_2 \) values,
referring to an evolving slope of the decay phase, range from
\(-0.27 \) to \(-0.9 \) with an average of \(-0.5 \), which could
potentially support a decay within the \( \nu < \nu_m < \nu_c \) regime.

Regarding whether the presence of the break is related to the
prompt emission, we investigate the distribution between \( E_{iso} \)
and \( T_{90}^* \) with and without correction for redshift evolution. We
do not observe any trend or clustering in the sample of GRBs
that present a break and those without, suggesting that \( E_{iso} \) and
\( T_{90}^* \) are not indicators of a jet break within a radio LC. More
analysis is needed to determine whether there is another
physical reason for this distinction.

In conclusion, we find:

1. After correction, the slope of the Dainotti correlation in
the radio agrees with the X-ray and optical within \( 1.5\sigma \),
instead of within \( 2.1\sigma \). When evolutionary effects are not

Figure 5. Top left: uncorrected \( E_{iso} \) vs. \( T_\alpha^* \) for the sample of 16 GRBs with a plateau considered in Section 3.1. Top right: \( E_{iso} \) vs. \( T_\alpha^* \) after correction for evolutionary
effects. Bottom panels: uncorrected (left) and corrected (right) \( E_{iso} \) vs. \( L_o \) correlation for the sample of 14 GRBs considered in the corrected Dainotti correlation.
considered. This emphasizes the importance of correcting for selection bias and that the two correlations can be interpreted within the same mechanism if we consider the slope of the correlation as a discriminant among models.

2. The time of break in the radio sample occurs later than in the X-ray and optical, and the radio sample is found to be statistically different from the X-ray and optical samples. The late break time can be a result of the passage of the synchrotron characteristic break \( (\nu_{\text{m}} \propto t^{-3/2}) \) through radio wavelengths during the lateral expansion phase. After the break, the flux in radio bands is expected to evolve first as \( F_{\nu} \propto t^{-1/3} \) for \( \nu < \nu_{\text{m}} < \nu_{c} \) and later as \( \propto t^{-9/8} \) for \( \nu_{m} < \nu < \nu_{c} \) (Sari et al. 1999). Another plausible explanation could be that the flux that dominates the radio bands is emitted in a wider decelerated shell (e.g., see Sironi & Giannios 2013; Metzg & Bower 2014; Fraija et al. 2021a).

3. Analysis of \( E_{\text{iso}} \) and \( T_{90}^{*} \) of 80 GRBs demonstrates that GRBs with and without a break in the LC appear to be drawn from the same parent population.

Figure 6. Left panels: distribution of break time \( T_{a}^{*} \) for the X-ray, optical, and radio samples. Right panels: \( T_{90}^{*} \) distribution for the sample of 80 GRBs, differentiated between GRBs with and without a break. Lower panels show the same distributions corrected for selection bias.

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