A gate-tunable, field-compatible fluxonium

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Circuit quantum electrodynamics, where photons are coherently coupled to artificial atoms built with superconducting circuits, has enabled the investigation and control of macroscopic quantum-mechanical phenomena in superconductors [1, 2]. Recently, hybrid circuits incorporating semiconducting nanowires [3–5] and other electrostatically-gateable elements [6–8] have provided new insights into mesoscopic superconductivity [3, 5]. Extending the capabilities of hybrid flux-based circuits to work in magnetic fields would be especially useful both as a probe of spin-polarized Andreev bound states [9–12] and as a possible platform for topological qubits [13–16].

The fluxonium [17] is particularly suitable as a readout circuit for topological qubits [16, 18] due to its unique persistent-current based eigenstates. In this Letter, we present a magnetic-field compatible hybrid fluxonium with an electrostatically-tuned semiconducting nanowire as its non-linear element. We operate the fluxonium in magnetic fields up to 1T and use it to observe the $\Phi_0$-Josephson effect. This combination of gate-tunability and field-compatibility opens avenues for the exploration and control of spin-polarized phenomena using superconducting circuits and enables the use of the fluxonium as a readout device for topological qubits.

The fluxonium consists of a Josephson junction with Josephson energy $E_J$ in parallel with a linear superinductor [19] with inductive energy $E_L$ and a capacitor characterized by the energy $E_C$, as shown in Fig. 1a. The fluxonium regime ($E_L < E_C < E_J$) is achieved by shunting the junction with a large inductance. This parameter regime results in the eigenstates of the fluxonium being composed of superpositions of persistent currents in multiple directions (Fig. 1b). Since a switch in the parity of a Majorana zero mode (MZM) corresponds to a switch in the direction of the persistent current flowing in the fluxonium circuit, it is uniquely suited to detecting the parity lifetime of a MZM. Building a fluxonium compatible with detecting topological phenomena in a magnetic field, however, presents multiple challenges. The first challenge is reaching the fluxonium regime using magnetic-field compatible materials. Recent work on magnetic-field compatible materials with a large kinetic inductance such as granular aluminium [20–22] and NbTiN [23–25] has presented a path to meeting the stringent requirements of the fluxonium superinductance.

Here we have realized a fluxonium circuit compatible with MZM-detection as shown in Figs 1c–f. All circuit elements except for the junction are fabricated using NbTiN, which has been demonstrated to have critical fields exceeding 9T and inductances exceeding 75 pH/□ [24]. The use of NbTiN, however, introduces the possibility of lossy vortices when a magnetic field is applied to the device. We mitigate the effects of these vortices by introducing vortex-pinning holes (inset, Fig. 1c) and using inductances based on thin meanders (Fig. 1e). The small widths of the meanders suppresses the emergence of vortices due to out-of-plane fields, $B_x$, up to tens of mT [26].

In addition to being composed of magnetic-field compatible materials, for use as a detector, the fluxonium must also maintain its narrow linewidth during the application of a magnetic field. The application of a magnetic field precludes the possibility of using the magnetic shielding necessary for limiting flux noise in flux-based superconducting circuits. We address this challenge by building a gradiometric superinductance as shown in Fig. 1d. Equal fluxes through each of the two loops generate equal currents that are canceled at the junction, rendering the fluxonium insensitive to flux noise due to sources larger than the fluxonium device.

The final challenge is faced when incorporating a semiconducting nanowire into a superconducting circuit. The small junction of the fluxonium is based on a semiconducting lnAs nanowire proximitized by an epitaxially-grown aluminum layer (Fig. 1f) [27]. This small junction, however, does not provide a large enough capacitance to achieve the fluxonium regime. We thus add a parallel plate capacitor (blue in Fig. 1d) to achieve the required $E_C$ for the fluxonium. We note that this fluxonium design is flexible enough to incorporate any semiconducting material as its small junction.

We first demonstrate that our device behaves as expected for a fluxonium coupled to a readout resonator. Data from two similar fluxonium devices (device A and device B) will be presented in this Letter. We first focus
FIG. 1. Nanowire fluxonium. a, Circuit model. The fluxonium is composed of a Josephson junction shunted by an inductor and a capacitor. These elements are characterized by the energies $E_J$, $E_L$, and $E_C$, respectively. The value of $E_J$ can be tuned both by the external magnetic field, $B$, and by the gate voltage, $V_g$. A readout resonator (constituted by $L_r$ and $C_r$) is coupled to the fluxonium via a shared inductance $L_s$. b, Potential of the fluxonium (black), determined by $E_J$ and $E_L$, versus the phase difference across the junction, $\varphi$, at $\varphi_{\text{ext}} = 0.2\pi$. The lowest eigenenergies are indicated with horizontal lines. Arrows represent two of the transitions starting from the ground state. c, False-colored optical micrograph showing the transmission line and the lumped element resonator, with capacitive and inductive parts shaded in pink and brown respectively. Inset, scanning electron microscope (SEM) image of the resonator’s capacitive plates. d, SEM image of the fluxonium, corresponding to the area indicated by the box in c. The 9 nm-thick NbTiN superinductor (purple, enlarged in e), the shared inductance section (green), the parallel plate capacitor (blue) and the nanowire junction (red, enlarged in f), correspond, respectively, to the $E_L$, $L_s$, $E_C$ and $E_J$ elements in a. The out-of-plane component of $B$, $B_z$, is used to tune the external magnetic flux $\Phi_{\text{ext}}$. $B_z$ is the component parallel to the wire.

FIG. 2. Two-tone spectroscopy of device A, at $B_z = 0$. a, Magnitude of the transmitted readout signal as a function of the external flux and $f_{r,\text{drive}}$ (top) and $f_{l,\text{drive}}$ (bottom), showing the flux modulation of the different transitions. b, Extrema extracted from a (markers) and fitted transition frequencies (lines) obtained from the numerical diagonalization of the model Hamiltonian (Methods). Gray markers indicate extrema that are not associated with any fluxonium-resonator transitions. A value of $E_J/h = 6.7$ GHz is extracted from the fit.

on the behavior of device A. We monitored the transmission amplitude, $|S_{21}|$, at frequencies $f_{r,\text{drive}}$ around the resonator frequency $f_{g0\rightarrow g1}$, as a function of the external phase $\varphi_{\text{ext}} = \frac{2\pi}{7} \Phi_{\text{ext}}$, as shown in the top panel of Fig. 2a. Transitions are labelled as $m_i n_i \rightarrow m_e n_e$, where $m_i$ ($m_e$) and $n_i$ ($n_e$) are the initial (end) states of the fluxonium and resonator, respectively. The resonator spectrum is periodic in flux and also exhibits gaps in its visibility, which indicate that the resonator is coupled to the fluxonium. The bottom panel in Fig. 2a shows the flux dependence of the observed transition frequencies of the fluxonium-resonator system, measured by monitoring the transmission amplitude at $f_{r,\text{drive}} = f_{g0\rightarrow g1}$ while the system is driven with a second tone with frequency $f_{l,\text{drive}}$, also via the resonator. Threading a flux quantum through the gradiometric loop that comprises the fluxonium corresponds to $B_z = 550 \mu$T, which is much greater than the 15 $\mu$T that would be needed to thread a flux quantum through one of the two symmetric loops. The gradiometric geometry thus reduces the sensitivity...
of the fluxonium to magnetic field noise larger than the
fluxonium loop by more than an order of magnitude. The
residual asymmetry present is due to the nanowire place-
ment and the presence of the gates and capacitor inside
the loop.

To fit the spectroscopy data (markers in Fig. 2b),
we diagonalize the Hamiltonian for the circuit shown in
Fig. 1a [28], leaving all circuit parameters free except
for \( C_r = 26 \, \text{fF} \), which we extract from electromag-
netic simulations. The parameters obtained from the fit are
shown in Tab. I and the fitted transition frequencies are
denoted with lines in Fig. 2b. Each state is identified by
the closest state in energy for the uncoupled system.
In addition to transitions originating from the ground state,
g0, we also observe transitions for which the initial state is
g1, with one photon in the resonator (dashed lines).
This is due to the continuous drive used to monitor \(|S_{21}\)
and thus does not limit the possible observable phe-
nomena. We first measure\( E_0 \) versus \( V_j \) range. We do note, how-
ever, that at higher magnetic fields, the thermal occu-
lation of the excited state of the fluxonium does appear
to be higher since we observe transitions from this state
even when the \( g0 \rightarrow e0 \) frequency is above 1 GHz. Impor-
tantly, we can still fit the spectroscopy data accurately
in this regime, indicating that the fluxonium-resonator
Hamiltonian remains valid at high magnetic fields, with
\( E_j \) being the only parameter largely affected by \( B_z \). Fit
parameters for device B are shown in Tab. I.

Next, we explore the magnetic field compatibility of
the nanowire fluxonium. The magnetic field behavior of
the device strongly depends on the microscopic details
of the nanowire junction. In order to demonstrate the
field compatibility of the fluxonium circuit elements, we
here show data from device B whose parameters were op-
timized for magnetic field compatibility. The magnetic
field behavior of device A is provided in the supplement.
Spectroscopy measurements at two different \( V_j \) and \( B_z \)
points are shown in Figs. 4a and b. We continue to be
able to perform spectroscopy on the fluxonium over the
full range in \( \varphi_{\text{ext}} \) at fields up to 1 T. We do note, how-
ever, that at higher magnetic fields, the thermal occu-
pation of the excited state of the fluxonium does appear
to be higher since we observe transitions from this state
even when the \( g0 \rightarrow e0 \) frequency is above 1 GHz. Impor-
tantly, we can still fit the spectroscopy data accurately
in this regime, indicating that the fluxonium-resonator
Hamiltonian remains valid at high magnetic fields, with
\( E_j \) being the only parameter largely affected by \( B_z \). Fit
parameters for device B are shown in Tab. I.

We finally use the nanowire fluxonium to investigate
the behaviour of spin-orbit coupled superconducting junc-
tions in magnetic field. We perform spectroscopy mea-
surements at gate voltages ranging from 3.5 to 4.8 V and
fields ranging from 0 to 1 T. From the spectroscopy we
extract \( E_j \) as a function of \( B_z \) at two different gate points,
which is shown in Fig. 4c. A non-monotonic decrease of
\( E_j \) with field is observed at both gate points. We ex-
pect an overall decrease in \( E_j \) versus \( B_z \) due to the su-
perconducting gap closing at high magnetic fields. The
non-monotonic behaviour of \( E_j \), however, suggests the
presence of interference between different modes in the
junction [29].

A shifting of the zero-flux point in the spectroscopy of
the fluxonium device at high fields can be used to deter-
mine the breaking of multiple symmetries in the semicon-
ducting junction [30]. This phase shift is known as the
\( \varphi_0 \)-Josephson effect, which occurs when chiral and time-
reversal symmetries are both broken in the junction. In
InSb- and InAs-based junctions, this symmetry-breaking
originates from the interplay between the presence of
multiple channels in the junction, spin-orbit coupling,
FIG. 3. Gate tuning of $E_J$ in device A, at $B_z = 0$. a, Gate dependence of the resonator’s resonant frequency, $f_{0\rightarrow g1}$, at $\varphi_{ext} = 1.25\pi$. b–d, Fluxonium spectra at three different gate points, indicated with vertical lines in a. The markers correspond to the peaks extracted from the measured resonator (top) and two-tone (bottom) transmission data. The fitted transition frequencies $f_{g0\rightarrow e0}$, $f_{g0\rightarrow f0}$ and $f_{g0\rightarrow h0}$ (black lines) are obtained by fitting the darker markers. The values of $E_J/h$ extracted are 0.2, 3.8 and 9.6 GHz respectively. The insets in b, c and d show sections of the measured transmission magnitude. In b we observe gaps in visibility at zero and half flux in the $g0\rightarrow f0$ transition.

and the Zeeman splitting due to the applied magnetic field [30–32]. We observe such a shift in the zero-flux point of the spectroscopy lines as $V_j$ is varied in a $B_z$ field (indicated by $\varphi_0$ in Fig. 4a.) The $\varphi_0$-shift as a function of $V_j$ is shown, at $B_z = 0$ and at $B_z = 0.5$ T, in Fig. 4d; the $\varphi_0$ value at $V_j = 4.80$ V is taken as the $\varphi_0 = 0$ reference. At $B_z = 0$ the zero-flux point does not change, while it changes continuously with $V_j$ when a $B_z$ field is applied. We remark that since the observed phase shift appears as a function of $V_j$ at fixed magnetic fields, we can exclude trivial effects such as misalignment of the magnetic field.

In conclusion, we have successfully realized a gate-tunable fluxonium resilient to high magnetic fields. We have combined a gate-controlled junction with magnetic field-compatible materials and a novel gradiometric design to build the hybrid fluxonium. We are able to perform spectroscopy over a large range of gate voltages and in-plane magnetic fields. We have used the fluxonium to investigate the behavior of an InAs Josephson junction in a magnetic field and observed a non-monotonic decrease of the $E_J$ of the junction as well as the $\varphi_0$-Josephson effect.

Our magnetic-field compatible hybrid fluxonium can now be used as a detector for the $4\pi$-periodic Josephson effect [18] and of Majorana parity dynamics. We emphasize that the hybrid fluxonium is not limited to the exploration of nanowires but constitutes a new platform for exploring the behavior of topological materials as well as superconductivity in the presence of a magnetic field. Proposals to probe superconductor-proximitized edge states in a quantum spin Hall insulator [33, 34] or field-dependent spin-polarized correlated insulating phases [35] are also now accessible using our hybrid circuit.

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FIG. 4. Behavior of device B in magnetic field. a, b, Fluxonium spectra at two different \( V_j \) and \( B_z \) points. The \( E_j \) value extracted from the fit is lower at higher magnetic field. In a we observe a \( \varphi_0 = -0.16\pi \) phase shift with respect to a reference \( \varphi_{\text{ext}} \) taken at the same field at \( V_j = 4.80 \) V. c, \( E_j \) versus \( B_z \) at two different gate voltage points. \( E_j \) decreases non-monotonically with field. d, \( \varphi_0 \) versus \( V_j \) at two different magnetic field points. At \( B_z = 0.0 \) T, \( \varphi_0 \) stays constant for the whole \( V_j \) range. At \( B_z = 0.5 \) T, however, there is a continuous \( \varphi_0 \) shift that ranges from 0 to \(-\pi\). The two insets show the zero-flux spectroscopy feature shifted from zero at two different gate points. In c and d, the points corresponding to the spectra in a and b are highlighted with matching colors.

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**AUTHOR CONTRIBUTIONS**

M.P.-V., G.d.L, and A.K. conceived the experiment. M.P.-V., A.B., and N.H. performed device simulations. M.P.-V. performed the measurements and analysed the data with input from W.P., L.P.K., and A.K. P.K. grew the InAs nanowires. M.P.-V. and A.K wrote the manuscript with input from all authors.

**METHODS**

**Materials and fabrication details**

All circuit elements, except the Josephson junction, are fabricated using a 9 nm-thick sputtered NbTiN film, with a kinetic inductance of 41 pH/□. The elements are defined by e-beam lithography and reactive ion etching. The superinductive loop of the fluxonium is made with a 50 nm-wide NbTiN strip line and has a total inductance of ~100 nH. The fluxonium capacitor consists of two square NbTiN plates sandwiching a 29 nm-thick PECVD SiN dielectric. The junction is based on a semiconducting InAs nanowire with an epitaxially grown Al layer. The
nanowire is deterministically deposited on top of the pre-patterned leads of the inductor and capacitor using a micromanipulator. The Josephson junction is defined by etching away an Al section of around 80 nm on top of the junction gate. The wire is contacted to the leads using 150 nm-thick sputtered NbTiN.

The fluxonium is inductively coupled to a readout LC-resonator by a small shared inductance $L_s$. The resonator (shown in Fig. 1b) is composed of a lumped element capacitor with vortex-pinning holes and an inductor formed from a 200 nm-wide meandering strip. The gate and transmission lines are made out of 100 nm-thick sputtered NbTiN. The resonator is capacitively coupled to a transmission line to allow readout of the fluxonium using standard dispersive readout techniques.

**Fitting procedure**

To find the relative extrema we apply a peak finding algorithm to the raw data. This algorithm first smooths the data in the frequency axis to avoid errors in peak finding due to noise. A minimum peak height is specified.

We fit the extracted data with the Hamiltonian corresponding to the circuit model in Fig. 1. The fluxonium Hamiltonian, $\hat{H}_f$, and the total Hamiltonian of the coupled readout-fluxonium system, $\hat{H}$, can be written in terms of two degrees of freedom, $\hat{\phi}_f$ and $\hat{\phi}_r$ (the phase drops across the fluxonium junction and across $C_r$, respectively), and their conjugated charges $\hat{n}_f$ and $\hat{n}_r$ [28]. In the limit $L_f \gg L_s, L_r$ (where $L_f = \frac{\Phi_0^2}{4\pi^2 E_L}$ and $\Phi_0 = h/2e$),

$$\hat{H}_f = 4E_C\hat{n}_f^2 - E_J(V_J, B)\cos(\hat{\phi}_f) + \frac{1}{2} E_L(\hat{\phi}_f - \phi_{ext})^2$$

and

$$\hat{H} = \frac{2e^2}{C_r} \hat{n}_r^2 + \frac{1}{2} \frac{(\Phi_0/2\pi)^2}{(L_r + L_s)} \hat{\phi}_r^2 - \frac{1}{2} \frac{(\Phi_0/2\pi)^2 L_s}{L_f(L_r + L_s)} \hat{\phi}_r \hat{\phi}_f + \hat{H}_f$$

The first two terms of $\hat{H}$ describe the uncoupled resonator, while the third term accounts for the coupling between resonator and fluxonium.

We diagonalize Hamiltonian 2 using the numerical method in [28]. All the spectra for the same device are fitted simultaneously. The free parameters $E_C, E_L, L_r$ and $L_s$ are common for all spectra corresponding to the same device. The free parameter $E_J$, however, has a different value for each spectrum.

All markers shown in Figures 2, 3 and 4 are included without distinction in the fits. The markers colors are assigned by association to the different transitions. Grey markers do not correspond to any transition present in the fit.
I. THEORETICAL MODEL FOR THE UNCOPLED FLUXONIUM

The Hamiltonian for the uncoupled fluxonium, $\hat{H}_f$, can be written in terms of the phase drop across the junction, $\hat{\phi}_f$, and its conjugated charge $\hat{n}_f$ [1, 2]

$$\hat{H}_f = 4E_C\hat{n}_f^2 - E_J(V_1, B)\cos(\hat{\phi}_f) + \frac{1}{2}E_L(\hat{\phi}_f - \varphi_{\text{ext}})^2.$$  \hspace{1cm} (1)

Each of the terms in $\hat{H}_f$ results from one of the three characteristic energies of fluxonium: $E_C$, $E_L$, and $E_J$. The two conjugated variables in this Hamiltonian, $\hat{\phi}_f$ and $\hat{n}_f$, are analogous to position and momentum, respectively. With this interpretation, the two terms involving $\hat{\phi}_f$ constitute a phase-dependent potential

$$V(\varphi) = -E_J\cos(\varphi) + \frac{1}{2}E_L(\varphi - \varphi_{\text{ext}})^2.$$  \hspace{1cm} (2)

Fig. 1i shows the potential at $\varphi_{\text{ext}} = 0$ for the three $E_J$ values in Fig. 3 in the main text. The $E_L$ term results in a parabolic background, common for the three cases. The $E_J$ adds a periodic modulation on top of it, which becomes more noticeable as $E_J$ increases. $E_C$ can be seen as a mass term and, together with the potential $V$, determines the eigenstates of fluxonium. The lowest energy eigenstates are labelled $g$, $e$, $f$ and $h$. Their energies at $\varphi_{\text{ext}} = 0$ are shown as colored lines in Fig. 1i. Fig. 1ii shows how these energies disperse with $\varphi_{\text{ext}}$. For small $E_J$ values, the variation of $\varphi_{\text{ext}}$ results in weak oscillations of the eigenenergies, while for large $E_J$’s the $\varphi_{\text{ext}}$-dependence is much stronger. In the limit of small $E_J$ the eigenstates are vibrational modes of the harmonic LC oscillator determined by $E_L$ and $E_C$. Therefore, their energies are evenly spaced with a separation determined by the plasma frequency $\sqrt{E_C E_L}/\hbar$. In the limit of large $E_J$ the eigenstates become superpositions of persistent currents localized in phase.

Fig. 1iii shows the transition frequencies between different pairs of eigenstates, which are the quantities that can be addressed experimentally. For simplicity, only the transitions starting from the ground state are shown here. The two-tone spectroscopy data in Figures 2, 3 and 4 in the main text shows the measured transition frequencies for the fluxonium-resonator coupled system.
FIG. 1. Potential, energy spectrum and transition frequencies of the uncoupled fluxonium. Rows a, b and c correspond to $E_J/h = 0.2$, 3.8 and 9.6 GHz, respectively, for the model parameters of device A. Column I shows the fluxonium potential in equation 2, at $\phi_{\text{ext}} = 0$, in black lines. The lowest eigenenergies of Hamiltonian 1 at $\phi_{\text{ext}} = 0$ are superimposed as horizontal dashed lines. Column II shows how these energies disperse as $\phi_{\text{ext}}$ is varied. The solid arrows indicate transitions starting from the ground state, $g$. The energies of these transitions are plotted in column III as a function of $\phi_{\text{ext}}$. 
II. FIELD DATA FOR DEVICE A

Fig. 2 shows the spectroscopy data for device A under in-plane magnetic field, up to 0.3 T. We note the presence of phase-independent lines crossing the fluxonium spectrum at frequencies above 6 GHz at 0.3 T. Beyond 0.3 T, the transition frequencies of the fluxonium came within 1 GHz of the resonator frequency, which resulted in the spectroscopy of the fluxonium becoming unmeasurable.

**FIG. 2.** Spectroscopy data for device A under in-plane magnetic field. Spectra in a and b are taken at $B_z = 0.1$ T and 0.3 T, respectively, at the same gate voltage, $V_j = 2.20$ V, as the spectrum in Fig. 3c in the main text. c and d show the extracted peaks (markers) and the fitted transition frequencies (solid lines). The $E_j/h$ values obtained from the fit are 4.8 and 3.7 GHz, respectively.
III. ADDITIONAL FIELD AND GATE DATA FOR DEVICE B

FIG. 3. Extra data for device B in magnetic field. a $E_j$ versus $B_z$ and $V_j$. The data shown in Fig. 4c in the main text corresponds to two line-cuts at $V_j = 3.97$ V and $V_j = 4.63$ V. b $\phi_0$ versus $B_z$ and $V_j$. The data shown in Fig. 4d in the main text corresponds to two line-cuts at $B_z = 0.5$ T and $B_z = 1.0$ T.
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