Coupled-Map Modeling of One-Dimensional Traffic Flow†

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Abstract

We propose a new model of one-dimensional traffic flow using a coupled map lattice. In the model, each vehicle is assigned a map and changes its velocity according to it. A single map is designed so as to represent the motion of a vehicle properly, and the maps are coupled to each other through the headway distance. By simulating the model, we obtain a plot of the flow against the concentration similar to the observed data in real traffic flows. Realistic traffic jam regions are observed in space-time trajectories.

KEYWORDS: traffic flow, coupled map lattice

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Theoretical study of traffic flow has been made from two points of view, macroscopic and microscopic. In the former, traffic flow is regarded as a compressible fluid and fluid-dynamical ideas are used.\(^1\) In the latter, on the other hand, vehicles are treated individually.\(^2\)−\(^4\) Car-following models\(^2\)\(^,\)\(^4\) are typical microscopic models which are described by simultaneous differential equations. These models have been used chiefly for rather high vehicle concentration. For low concentration, they have some difficulties in realizing real traffic flow; since the velocity of vehicles is determined only by the relative velocity and the mutual headway distance, the models exhibit some unphysical behaviors, such as negative velocity and limitless acceleration. Moreover, collisions of two vehicles occur easily. They fail to describe the difference between free flow and congested flow. Recently a new car-following-type model has been proposed by Bando et al.,\(^5\) which seems to overcome some of these difficulties. While these models are constructed in continuous space-time, cellular automata (CA),\(^6\) which are discrete both in space and time, recently came into use for modeling traffic flow.\(^7\)−\(^13\) Although CA can reproduce some properties of traffic flows, they are as yet too simple as models of real traffic.

In this letter, we propose a new microscopic model of one-dimensional traffic flow using the coupled map lattice (CML) idea.\(^14\) The present model is constructed in continuous space, but in discrete time. To make the model concrete, we start by considering the motion of a single vehicle. In real traffic flow, each vehicle has its own preferred velocity \(v^F\) and adjusts its velocity so as to fit the preferred one. Once the velocity reaches \(v^F\), it fluctuates around \(v^F\). For simplicity, when the current velocity is far from \(v^F\), we assume constant acceleration or constant deceleration. The adjustment mechanism and the fluctuations of velocity are realized by the following map assigned to the vehicle:

\[
v^{t+1} = F(v^t) \equiv \gamma v^t + \beta \tanh\left(\frac{v^F - v^t}{\delta}\right) + \epsilon,\tag{1}
\]

where \(v^t\) and \(v^F\) are the velocity of the vehicle at time \(t\) and the preferred velocity of the vehicle, respectively, and \(\beta, \gamma, \delta,\) and \(\epsilon\) are the parameters. In this model, the velocity is defined as the distance traveled in a unit time step. When \(\gamma = 1\) and \(\beta \neq 0\), this map expresses constant acceleration and deceleration for velocity far from \(v^F\). We, however, take \(\gamma\) slightly greater or less than 1, since the map then becomes chaotic, and acceleration and deceleration are still approximately constant. As a result, fluctuations in velocity are naturally introduced to the model through deterministic chaos. It is worth noting that the map \(F(v)\) is closely related to that used in chaotic neural network models.\(^15\) The meaning of each parameter is as follows: \(\delta\) determines the fluctuation of velocity around \(v^F\); \(\beta\) represents the magnitude of acceleration and deceleration, and difference in their magnitude is given by \(\epsilon\). Throughout this paper, we use the values \(\beta = 0.6, \gamma = 1.001, \delta = 0.1,\) and \(\epsilon = 0.1\). The motion of one vehicle driven by
$F(v)$ is given as a space-time trajectory in Fig. 1. Weak fluctuations caused by chaos are clearly seen.

Next, we consider the case where the number of vehicles is more than one. In this situation, another deceleration mechanism is necessary to avoid collision. In fact, we assign a deceleration map to each vehicle. In real traffic flows, a vehicle changes its velocity according to, for example, the current velocity, the headway distance, and the relative velocity with another vehicle, but here we assume that the deceleration is dominated by the headway distance from the nearest vehicle ahead. Therefore the deceleration map determines the subsequent velocity from the headway distance. We introduce two deceleration processes, i.e., a sudden braking process and a slowing-down process. We call the model which includes the sudden braking process model A, and the one which includes the slowing-down process model B. The sudden braking process is very simple. Suppose a vehicle has the velocity $v_0$ and another vehicle exists at the headway distance $\Delta x - l$, where $\Delta x$ is the head-to-head distance and $l$ is the vehicle length; if $\Delta x - l < v_0$, the vehicle changes its velocity to $\Delta x - l$. Consequently, a collision of vehicles is avoided. In model B, the deceleration process is more complex; it is described by a map from the headway distance to the velocity defined as follows:

$$v_{i+1} = G(\Delta x_i, v_i) = \frac{F(v_i) - v_i}{(\alpha - 1)v_i}(\Delta x_i - l - v_i) + v_i,$$

$$v_i \leq \Delta x_i - l \leq \alpha v_i,$$

where $\Delta x_i = x_{i+1} - x_i$ with positions of the $i$th vehicle and the $(i+1)$th vehicle at time $t$ denoted by $x_{i+1}$ and $x_i$, respectively. The parameter $\alpha$ represents the range within which the vehicle uses the deceleration map $G(\Delta x, v)$. If the headway distance is less than $\alpha v$, the deceleration map $G(\Delta x, v)$ is used instead of $F(v)$. The map gives $\Delta x_i - l$ for $\Delta x_i - l = v_i$ and $F(v_i)$ for $\Delta x_i - l = \alpha v_i$. Therefore, it connects the free-motion map $F(v)$ with the sudden braking process. The velocity given by this map will be either greater or less than the current velocity, but it is always lower than the value given by the free-motion map $F(v_i)$.

We simulate the two models on a circular road. The positions and the velocity of vehicles are updated parallerly by the following process. First, the headway distance is measured for all vehicles. Then the vehicles move simultaneously according to the headway distance. If the headway distance is larger than the current velocity, the value of velocity is added to the current position. If the headway distance is less than the current velocity, on the other hand, the headway distance is added to the position. Next we determine the subsequent velocities for all the vehicles according to the maps. These processes constitute one time step. We define some quantities observed in the simulation: the average velocity $\langle v \rangle$ is defined as the distance traveled per time step per vehicle, and the traffic flow as $\rho \times \langle v \rangle$ using the vehicle concentration $\rho$. In the actual simulation, the distance is measured in the units of car length. The initial velocity and the preferred velocity
are distributed uniformly in $[2.0, 4.0]$. Initial positions are randomly chosen. In the simulation of model B, we adopt $\alpha = 4.0$.

In Fig. 2, we show the plot of the flow against the concentration for both models A and B. This diagram shows similar properties to those observed in real traffic: at low concentration, the flow increases almost linearly; after a clear peak, it decreases slowly for higher concentration. In Figs. 3(a)-3(c), typical space-time trajectories of model B for several concentrations are shown. Figure 3(a) corresponds to the concentration $\rho = 0.10$, which is lower than the peak concentration in Fig. 2. In this figure, clustering of the vehicles is seen. The cluster is led by the vehicle whose preferred velocity is lowest. This vehicle moves according to the free-motion map $F(v)$. It is seen that the fluctuation in its trajectory propagates to the following vehicles. As a result, weak shock waves form from time to time. Figure 3(b) corresponds to the concentration $\rho = 0.30$, which is higher than the the peak concentration. A jam region within which vehicles stop is seen. We call this type of jam a hard jam. The hard jam region moves backward. In real traffic flow, the hard jam is frequently observed. Figure 3(c) corresponds to $\rho = 0.20$, which is close to the peak concentration. A jam region where vehicles move more slowly than in other regions is clearly seen. We call this type of jam a soft jam. Its front moves slowly with positive velocity. A soft jam with negative velocity and hard jam are also found at the same concentration depending on the initial configuration. The soft jam with negative velocity is also observed in real traffic flows. The behavior of model A is similar to that of model B.

To summarize, we proposed a new model of one-dimensional traffic flow using the CML idea. The present model has several advantages over the traditional car-following model: the fluctuations of a single vehicle are taken into account; there is no collision of vehicles because of the discretization of time. In the space-time trajectory, the motion of vehicles escaping from the jam region seems very similar to that in real traffic. The motion of vehicles catching up to the jam region, on the other hand, is less satisfactory. We are currently trying to improve the modeling of the deceleration process. Detailed study on the model including the effect of chaos is now in progress.

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Figure Captions

**Fig. 1.** The motion of a vehicle moving freely. The initial velocity and the preferred velocity are taken to be 3.0. The system length is 500. We show the trajectory of 20 time steps after discarding 500 steps.

**Fig. 2.** The concentration vs flow of models A and B. The initial velocity and the preferred velocity of the vehicles are uniformly distributed in the range $[2.0, 4.0]$. The system length is 500. We show the average over 100 time steps after discarding 500 steps.

**Fig. 3 (a).** The space-time trajectory of model B. The initial velocity and the favorite velocity of the vehicles are uniformly distributed in the range $[2.0, 4.0]$. The system length is 100. One hundred time steps are shown after discarding 500 time steps. The number of vehicles is 10.

(b). The space-time trajectory of model B. The number of the vehicles is 30. The other parameters are the same as for (a).

(c). The space-time trajectory of model B. The number of vehicles is 20. The other parameters are the same as for (a).