Gravitational waves from phase transition in a QCD-like hidden sector

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Abstract. The gravitational wave (GW) background produced at the chiral phase transition in a conformal extension of the standard model is studied. We compute the bounce solution by the iteration method and find that the transition’s inverse duration $\beta$ normalized to the Hubble parameter $H$ is $\beta/H \gtrsim 10^3$ which implies that the sound wave period $\tau_{sw}$ as an active GW source is shorter than $1/H$. Using the factor $\tau_{sw}H$ as the reduction factor for the sound wave contribution to the total GW spectrum, we evaluate the signal-to-noise ratio for the future space-based GW interferometer experiments. In the optimistic case we find that the GW signal could be detected by Big Bang Observer.

1. Introduction

The Higgs mass term is the only dimensionful parameter in the standard model (SM). The origin of the mass is one of the key questions in particle physics and it may give us a hint of beyond the SM.

In this contribution we consider the model [1, 2, 3], in which a scale, created by the chiral symmetry breaking in a QCD-like hidden sector, transmits via a mediator to the SM sector and generates the Higgs mass term to trigger electroweak (EW) symmetry breaking. In certain region of the parameter space in the model, the chiral phase transition (PT) is of first order and the gravitational wave (GW) is expected to be produced at the PT. We therefore study the detectability of the GW background produced at the chiral PT [4, 5].

2. The model

We consider a classically scale invariant extension of the SM studied in refs. [1, 2, 3]. The model consists of a hidden $SU(3)_H$ gauge sector coupled to the SM sector via a real singlet scalar $S$. The hidden sector Lagrangian $\mathcal{L}_H$ of the total Lagrangian $\mathcal{L}_T = \mathcal{L}_H + \mathcal{L}_{SM+S}$ of the model is given as

$$\mathcal{L}_H = -\frac{1}{2} \text{Tr} F^2 + \text{Tr} \bar{\psi} (i\gamma^\mu \partial_\mu + g_H \gamma^\mu G_\mu + g' Q \gamma^\mu B_\mu - y S) \psi,$$

where $G_\mu$ is the gauge field for the hidden QCD, $B_\mu$ is the $U(1)_Y$ gauge field, $y$ is the Yukawa coupling and the hidden vector-like fermions $\psi_i$ ($i = 1, 2, 3$) belong to the fundamental
There are three contributions of the stochastic GW background at a first-order PT, \( \Omega_{3} \). Gravitational wave magnetohydrodynamic turbulence contributions, respectively, \( \Omega_{\text{dev}} \), deviation from the pure NJL model (i.e. without the singlet scalar breaking mass term. Finally, the EW symmetry breaking is triggered by the Higgs mass term \( + \frac{1}{2} \lambda_{H S} \langle S \rangle H \). 

To analyze the strongly interacting hidden sector, we replace the Lagrangian \( \mathcal{L}_{H} \) by the Nambu-Jona-Lasinio (NJL) Lagrangian [6, 7, 8]:

\[
\mathcal{L}_{\text{NJL}} = \text{Tr} \left( i \gamma^\mu \partial_\mu + g (Q \gamma^\mu B_\mu - y S) \psi + 2G \left( \Phi^\dagger \Phi + G_D \right) \right),
\]

where \( G \) and \( G_D \) are the dimensionful parameters and \( \Phi_{ij} = \psi_i(1 - \gamma_5) \psi_j \). Using the self-consistent mean-field approximation [9, 10], the mean fields \( \sigma \) and \( \phi_a(a = 0, \cdots, 8) \) are defined as \( \langle \Phi \rangle = - \frac{1}{2T} \left( \text{diag}(\sigma, \sigma, \sigma) + i \sum_{a=0}^{8} (\Lambda^a)^T \phi_a \right) \). Splitting the \( \mathcal{L}_{\text{NJL}} \) into two parts as \( \mathcal{L}_{\text{NJL}} = \mathcal{L}_{\text{MFA}} + \mathcal{L}_I \), where \( \mathcal{L}_I \) is normal ordered, the one-loop effective potential in \( \mathcal{L}_{\text{MFA}} \) can be obtained by integrating out the hidden fermions:

\[
V_{\text{NJL}}(S, \sigma) = \frac{3}{8G} \sigma^2 - \frac{G_D}{16G^2} \sigma^4 - 9I_0(M ; \Lambda_H).
\]

Here the integral \( I_0 \) is given by \( I_0(M ; \Lambda) = \frac{1}{16 \pi^2} \left( \frac{\Lambda^4 \ln \left( \frac{1 + M^2}{\Lambda^2} \right) - M^4 \ln \left( \frac{1 + \Lambda^2}{\Lambda^2} \right) + M^2 \Lambda^2}{\pi^2} \right) \).

The NJL parameters for the hidden QCD sector are assumed to be obtained by scaling-up the values of those for the real QCD hadron world. Then, there are five independent parameters, \( \lambda_H, \lambda_S, \lambda_{H S}, y \) and \( \Lambda_H \) in the model, where effectively two of them are used to obtain \( m_h = 125 \text{ GeV} \) and \( \langle h \rangle = 246 \text{ GeV} \). We analyze the scalar potential at finite temperature and find that the chiral PT in the hidden sector which occurs in the two dimensional space \( (S, \sigma) \) is of first order for \( y \lesssim 0.06 [1, 4] \). In particular, the larger ratio \( \langle S \rangle / \langle \sigma \rangle \) is obtained, namely the larger deviation from the pure NJL model (i.e. without the singlet scalar \( S \)) is obtained, for smaller \( \lambda_S \) and larger \( y \). We study, therefore, the GW spectrum in such an optimistic parameter space.

### 3. Gravitational wave

There are three contributions of the stochastic GW background at a first-order PT, \( \Omega_{\text{GW}}(f) h^2 = [\Omega_{\varphi}(f) + \Omega_{\text{sw}}(f) + \Omega_{\text{turb}}(f)] h^2 \), where \( \Omega_{\varphi}, \Omega_{\text{sw}} \) and \( \Omega_{\text{turb}} \) are the scalar field, sound wave and magnetohydrodynamic turbulence contributions, respectively, \( h \) is the dimensionless Hubble parameter and \( f \) is the frequency of the GW at present. Two of the main quantities in discussing the stochastic GW background are \( \alpha \) and \( \beta \) with the nucleation temperature \( T_n \) [11]:

\[
\alpha = \frac{30}{\pi^2 g_s T_n^3} \left( V(T_n) - \frac{1}{4} T \frac{\partial V(T)}{\partial T} \bigg|_{T=T_n} \right), \quad \beta = H(T_n) T_n \frac{d}{dT} \left( \frac{S_3}{T} \right) \bigg|_{T=T_n}.
\]

1 The hidden chiral symmetry breaking predicts the existence of 8 Nambu-Goldstone bosons which can become DM candidates due to the remnant unbroken flavor group \( SU(3)_V \) (or its subgroup). The detailed studies on the DM are presented in [1, 2, 3].
Here $\Delta V(T) = V_{\text{EFF}}(0, 0, T) - V_{\text{EFF}}(\langle S \rangle, \langle \sigma \rangle, T)$, $g_s$ is the relativistic degrees of freedom in the symmetry phase and $S_3$ is the $O(3)$ symmetric Euclidean action. The $\beta$ corresponds to the inverse of the duration time of the first-order PT. In the left panel of Fig. 1 we show the $\beta$ normalized to the Hubble parameter $H$, $\beta/H$, for several values of $y$ with $\lambda_S$ and $\lambda_H$ fixed at 0.001 and 0, respectively. We have used the iterative method to obtain a bounce solution and found that $S_3/T$ for $x = T/T_C < 1$, where $T_C$ is the critical temperature, can be nicely fitted with a simple function [12, 13]: $S_3/T(x) = b(1-x)^{-\gamma}$. Since $\beta/H \approx 1.4 \times 10^4$ in the pure NJL model [13], the larger $y$ is, the more deviation from the pure NJL model. With increasing $y$ a new local minimum of the scalar potential, other than the true and false minima, develops near the origin. For $y \approx 0.0172$, the new local minimum becomes deeper in such a way that our iterative method breaks down.

In our model the sound wave contribution $\Omega_{\text{sw}}$ is the most dominant one. The formula for $\Omega_{\text{sw}}/h^2$ has been derived from the numerical simulations for which the duration of the sound wave period $\tau_{\text{sw}}$ is larger than $1/H$, i.e., $\tau_{\text{sw}}H > 1$ [14]. Since $\tau_{\text{sw}}H \propto (\beta/H)^{-1}$, however, $\tau_{\text{sw}}H > 1$ is unlikely satisfied in our model. In refs. [15, 16] it has been suggested, for the case that $\tau_{\text{sw}}H < 1$, to use the factor $\tau_{\text{sw}}H$ as a reduction factor for $\Omega_{\text{sw}}$. Following [15], we compute the reduction factor and obtain $\tau_{\text{sw}}H \sim 10^{-2}$ in our model.

To evaluate the detectability of the GW background of the model, we calculate the signal-to-noise ratio (SNR) [17],

$$
\text{(SNR)}^2 = 2t_{\text{obs}} \int_{f_{\text{min}}}^{f_{\text{max}}} df \left[ \frac{\Omega_{\text{GW}}(f) h^2}{\Omega_{\text{noise}}(f) h^2} \right] ^2 ,
$$

where $t_{\text{obs}}$ stands for the duration of an observation, and $(f_{\text{min}}, f_{\text{max}})$ is the frequency range of a given experiment. The $\Omega_{\text{noise}}(f) h^2$ represents the effective strain noise power spectral density for a given detector network [18, 19, 20, 21]. The SNR against $\lambda_H$ for Big Bang Observer (BBO) is shown in the middle panel of Fig. 1, where we assume that $t_{\text{obs}} = 5$ years and the speed of the bubble wall $\xi_w$ is equal to the Jouguet speed $\xi_J$ which is the wall speed for the Jouguet detonation, $\xi_J = \frac{\sqrt{\alpha(2+3\alpha)+1}}{\sqrt{2}}$. The area I and II are allowed by LHC [22]. From each allowed region we choose a benchmark point, BP1 and BP2. The SNR$^{\text{BBO}}$ of BP1 and BP2 are 11.8 and 5.7, respectively. Therefore, there is a good chance that the GW signals of our model can be detected by BBO.

Figure 1. Left: $\beta/H$ against the Yukawa coupling $y$ for $\lambda_S = 0.001$ and $\lambda_H = 0$. Middle: SNR$^{\text{BBO}}$ against $\lambda_H$ with 5 years observation. The area I and II are allowed by LHC. The SNR$^{\text{BBO}}$ (5 yrs) of the benchmark points BP1 (purple star) and BP2 (green star), are also plotted. Right: The GW spectrum for the benchmark points BP1 (purple), BP2 (green) and the power-law-integrated sensitivity of BBO (red dashed curve) as well as DECIGO (blue dashed curve). The threshold SNR is 5 with 5 years observation. The dashed purple and green lines present, respectively, the GW spectrum of BP1 and BP2, for which the reduction factor $\tau_{\text{sw}}H$ due to the short sound wave period is ignored.
In the right panel of Fig. 1 we present the GW spectra for BP1 (solid purple line) and BP2 (solid green line) with $\xi_w = \xi_J$. The power-law-integrated sensitivity of BBO (red dashed curve) and Deci-Hertz Interferometer Gravitational Wave Observatory (DECIGO) (blue dashed curve) are also shown, where we assume that the threshold SNR is 5 with 5 years observation. Since a part of the spectral curves for BP1 and BP2 runs over the sensitivity curve of BBO, we see once again that their signals could be detected at BBO, while for DECIGO it would be very difficult. For comparison, the GW spectra without the reduction factor $\tau_{sw}H$ are also presented (dashed purple and green lines), for which the SNR are given in the parentheses.

4. Summary
We have studied the stochastic GW background produced at the cosmological chiral PT in a conformal extension of the SM. Using the obtained fitting function for $S_3/T$, we have computed the ratio $\beta/H$ and found $\beta/H \simeq (4 - 9) \times 10^3$ in the optimistic parameter space. This implies a short sound wave period $\tau_{sw}$ as an active GW source; $\tau_{sw}H \sim 10^{-2}$. We have used $\tau_{sw}H$ as the reduction factor for the sound wave contribution which is nevertheless the most dominant contribution to $\Omega_{GW}$. Evaluating the SNR for BBO and DECIGO, we conclude that the GW signal in the optimistic case could be detected by BBO.

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References
[1] M. Holthausen, J. Kubo, K. S. Lim and M. Lindner, JHEP 1312 (2013) 076.
[2] J. Kubo, K. S. Lim and M. Lindner, JHEP 1409 (2014) 016.
[3] Y. Ametani, M. Aoki, H. Goto and J. Kubo, Phys. Rev. D 91 (2015) no.11, 115007.
[4] M. Aoki, H. Goto and J. Kubo, Phys. Rev. D 96 (2017) no.7, 075045.
[5] M. Aoki and J. Kubo, arXiv:1910.05025 [hep-ph].
[6] Y. Nambu, Phys. Rev. Lett. 4 (1960) 380.
[7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.
[8] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124 (1961) 246.
[9] T. Kunihiro and T. Hatsuda, Prog. Theor. Phys. 71 (1984) 1332.
[10] T. Hatsuda and T. Kunihiro, Phys. Rept. 247 (1994) 221.
[11] J. R. Espinosa, T. Konstandin, J. M. No and G. Servant, JCAP 1006 (2010) 028.
[12] C. J. Hogan, Phys. Lett. 133B (1983) 172.
[13] A. J. Helmboldt, J. Kubo and S. van der Woude, Phys. Rev. D 100 (2019) no.5, 055025.
[14] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, Phys. Rev. D 96 (2017) no.10, 103520.
[15] J. Ellis, M. Lewicki and J. M. No, JCAP 1904 (2019) 003.
[16] J. Ellis, M. Lewicki, J. M. No and V. Vaskonen, JCAP 1906 (2019) 024.
[17] E. Thrane and J. D. Romano, Phys. Rev. D 88 (2013) no.12, 124032.
[18] C. J. Moore, R. H. Cole and C. P. L. Berry, Class. Quant. Grav. 32 (2015) no.1, 015014.
[19] K. Yagi, N. Tanahashi and T. Tanaka, Phys. Rev. D 83 (2011) 084036.
[20] K. Yagi, Int. J. Mod. Phys. D 22 (2013) 1341013.
[21] S. Isoyama, H. Nakano and T. Nakamura, PTEP 2018 (2018) no.7, 073E01.
[22] T. Robens and T. Stefaniak, Eur. Phys. J. C 76 (2016) no.5, 268.