ABSTRACT: An equipment replacement decision takes into account economic engineering models based on discounted cash flow (DCF) such as the Annual Equivalent Cost (AEC). Despite a large number of researches on industrial assets replacement, there is a lack of studies applied to farm goods. This study aimed at assessing an alternative model for economic decision analysis on farm machinery replacement, with no restrictions on the number of replacements and assessed goods during a defined timeline. The results of the hybrid model based on the combination of the vehicle routing problem and the equipment replacement problem (RVPSE) applied to three different farm tractors showed the model reliability, providing a wider range of decisions for management support.

KEYWORDS: economic engineering, annual equivalent cost, integer linear programming.

INTRODUCTION

A decision-making based on a rigorous analysis of available possibilities is crucial to ensure competitiveness and improve the use of resources. Replacement of capital goods is among the mandatory and relevant resolutions throughout the lifetime of companies, mainly for industries. As described by Abensur (2015), errors in these decisions could jeopardize a company survival within its dynamic competitive environment. Moreover, according to Abensur (2015), studies on equipment replacement are significantly limited, being restricted to (i) a single replacement during an analyzed timeline, and (ii) replacement comparison generally against a single substitute good.

During the last decades, capital investment associated with an increased use of farm machinery have been among the main responsible factors for a growing farm productivity in Brazil (Gasques et al., 2016).

Therefore, useful methods for deciding on the best time of replacement may reflect on profitability and financial sustainability (Vega & Abensur, 2014).

Broadly, replacing capital goods has been made due to (i) reduced performance from physical deterioration, (ii) new requirements of accuracy, speed or other specifications which are no longer met by the existing equipment or system, and (iii) technological obsolescence (Blank & Tarquin, 2011).

Considering cost models applied to farm machinery, Mercante et al. (2010) and Piacentini et al. (2012) developed computational tools based on farm costs divided into fixed (depreciation, interest, housing, insurance) and variable (fuel, lubricant, tires, grease, parts replacement). These models were developed to determine, respectively, the cost of farm operation (including labor) and the operating costs of farm machinery.

With regard to farm machinery replacement, specifically, Marques et al. (2005) developed a mathematical model based on dynamic programming to determine the optimal replacement time of a forestry machine, for a single substitute good in question. On the other hand, Zanatta & Varella (2007) designed a computational tool to ascertain the optimal replacement time of farm tractors through annual equivalent cost. Despite the relevance of this subject, there are still few studies directly related to farm machinery replacement.
Given the above, this study aimed to test and assess the application of a hybrid model of vehicle routing and equipment replacement problems (RVPSE) for farm tractors with no restrictions on the numbers of replacement and assessed goods, during a defined timeline. The simulations were compared with the results obtained by applying annual equivalent cost method, thus assessing the reliability of this model and its advantages.

MATERIAL AND METHODS

Initially, optimal replacement times were calculated by annual equivalent cost method, pairing tractors to compare the good in use with a substitute one. Subsequently, simulations were performed between the tractor elected as in use and those taken as substitutes by the RVPSE model. All simulations were carried out using Microsoft Excel, using the Solver optimization software for RVPSE simulations. We chose this software for its popularity and ease of use and implementation.

The values obtained from the annual equivalent cost method were used as a reference for the validation of results achieved through RVPSE model simulation.

Annual Equivalent Cost (AEC)

Discounted cash flow methods are based on a series of future inflows brought to present value and discounted at an interest rate considered to be of minimal attractiveness (Abensur, 2012; Araújo et al., 2011; Celuppi et al., 2014; Cunha et al., 2014; Lima et al., 2013; Rodrigues et al., 2015). Net Present Value, Discounted Payback, Internal Rate of Return, and the Annual Equivalent Cost (AEC) are among the most popular discounted cash flow methods. Particularly, AEC is a widely used method in engineering for minimizing future costs and enabling comparisons of projects with different service lives.

The AEC method is based on the concept of economic service life, which means the number of years in which the lowest cost value occurs. The economic service life is designed to determine the years of working for an equipment and the lowest annual value, which set how long such good in use (defender) must remain before replacement. The study can also be performed considering a period previously specified as the service life of the goods (Blank & Tarquin, 2011).

Mathematically, an economic service life represents the minimum point of AEC function obtained by attempts on the values distributed throughout a cash flow. Equation (1) below expresses AEC as a function of time \((n)\) and minimum attractive rate \((i)\) as follows:

\[
AEC_{n,i} = \sum_{j=1}^{n} \left[ \frac{C_j + DI_j}{(1+i)^j} \left( \frac{1}{(1+i)^{n}} - \frac{1}{i} \right) \right]
\]

(1)

where,

- \(i\) = minimum attractive rate;
- \(C_j\) = Costs on the date \(j\);
- \(DI_j\) = outlay or investment on the date \(j\), and
- \(n\) = number of analyzed periods

RVPSE model

A vehicle routing problem may be generically perceived as a vehicle route supposed to do some stopovers (nodes), covering a sequence of predefined routes (arcs). In each node, there is (not) an activity to be performed (e.g., vehicle unloading and/ or loading), respecting certain restrictions such as travel time and/ or load capacity of each vehicle, aiming at the lowest cost.

Traditionally, a vehicle routing problem can be represented using a network, which enables several analogies. For equipment replacement, the nodes in each period (e.g. year) are taken as two possible decisions regarding the equipment; there will typically be nodes for replacement or...
permanence of the good in use. The sequence of arcs defines the trajectory of the good in use to be replaced or not, at some point in the planning horizon, by some of its substitutes. Once the problem type and its mathematical modeling are defined, a search algorithm for an optimal solution must be applied or developed.

RVPSE adapted the mathematical model proposed by Fisher & Jaikumar (1981) developed for a typical vehicle routing problem. The main changes were regarding (i) specific replacement and maintenance nodes for each good in each period, and (ii) limited replacements avoiding successive exchanges for distinct goods in each period. The sequence of arcs at the lowest cost was chosen by exhaustive enumeration using the branch-and-bound algorithm of integer linear programming, which is also available in the Microsoft Excel Solver optimization software.

The mathematical representation of the model is given by [eq. (4)], considering the costs determined in 2 and 3, as follows:

Cost to keep good in use (BU)

\[
\theta_R = IRCOBU_t^R + IRDepBU_t^R - rVRBU_t^R - CCBU_t^R - COBU_t^R \tag{2}
\]

a) Cost to replace BU

\[
\theta_S = IRCOBSi_t^S + IRDepBSi_t^S + (VRBSi_t^S - VRBU_t^S) - CCBSi_t^S - COBSi_t^S \pm LUCBU_t^S \tag{3}
\]

where,

- \( BU_t^n \) = good in use over \( t \) years in the stage \( n \);
- \( BS_i^t_n \) = substitute good \( i \) over \( t \) years in the stage \( n \);
- \( IRCO \) = income tax on operating costs;
- \( IRDep \) = income tax on depreciation;
- \( rVR \) = Minimum attractive rate on residual value of the analyzed good;
- \( VRBS_i^n_t \) = residual value of the substitute good \( i \) over \( t \) years in the stage \( n \);
- \( VRBU_n^t \) = residual value of the good in use over \( t \) years in the stage \( n \);
- \( CC \) = capital cost or loss of value of the analyzed good;
- \( CO \) = operating costs of the analyzed good, and
- \( LUC \) = accounting profit or income from sale of the good.

\[
\min Z = \begin{cases} \sum_{n=1}^{N} \sum_{t=0}^{H} [IRCOBU_n^t + IRDepBU_n^t - rVRBU_n^t - CCBU_n^t] (1 + r)^{-n} & \text{Keep} \\ \sum_{n=1}^{H} \sum_{i=1}^{S} \left[ IRCOBS_i^t_n + IRDepBS_i^t_n - (VRBS_i^t_n - VRBU_n^t) \right] (1 + r)^{-n} & \text{Replace} \end{cases}
\tag{4}
\]

where,

- \( ICOBU_0^0 = IRCOBS_i_0^0 = IRDepBU_0^0 = IRDepBS_i_0^0 = \cdots = COBU_0^0 = COBS_i_0^0 = 0 \)
Subject to:

\[ \sum_{i=1}^{W} x_{ik} - \sum_{i=1}^{W} x_{ki} = y_i \]  
(Continuity restriction) (5)

\[ \sum_{i=1}^{H} \sum_{w=1}^{W} y_{iw} = 1 \]  
(Number of nodes per stage) (6)

\[ \sum_{i=1}^{H} \sum_{w=1}^{W} y_{iw} \leq T_i \]  
(Maximum number of replacement nodes per stage) (7)

\[ x_{ik} \in \{0,1\} \]
\[ y_i \in \{0,1\} \]

where,

\( x_{ik} \) = arc from \( i \) to \( k \), chosen to be part of the solution;
\( y_i \) = indicates node \( i \) as the solution;
\( W \) = number of nodes in the network;
\( M \) = set of replacement nodes;
\( S \) = number of substitute goods under analysis;
\( H \) = number of stages;
\( T \) = number of replacements, and
\( N \) = longer service life estimated among the goods analyzed.

**Calculation approach**

Three different farm-tractor models from the same manufacturer and with different powers were selected as defenders (good in use) and challengers (substitute goods), as shown in table 1 below.

**TABLE 1. Tractors selected for the study.**

| Tractor | A | B | C |
|---------|---|---|---|
| Power (hp) | 15 | 18 | 30 |

The choice was made to highlight the difference between them regarding equipment replacement models since the lower power tractor should present reduced annual costs and, therefore, show favorable results. In other words, the results should converge towards the lower power tractor. This procedure has facilitated the interpretation and analysis of results, even though tractors were in practice used for different purposes. Simulations were performed using estimated values, as described below.

**Purchase value and tractor data:** provided by the manufacturer of the selected goods for characterization of the defender (good in use) and challengers (alternative goods).

**Resale value of tractors:** estimated by linear reduction, according to eqs (8) and (9).

\[ Q = \frac{1 - \frac{V_s}{V_C}}{5} \]  
(8)
\[ V_n = C(1 - n \cdot Q) \] 

where,

\( Q \) = Annual reduction index in the purchase value, estimated for compatibility of the resale value with the market price, after 5 years;

\( C \) = Purchase value;

\( V_n \) = Sale value on date \( N \), and

\( n \) = Year selected for calculation.

**Diesel, grease, and lubricant:** market prices surveyed in May 2017.

**Depreciation:** Linear (10% a year) with decision-making taken during the defined planning horizon of 5 years.

**Minimum attractive rate:** 11.25% a year.

**Income Tax (IT):** The owner was considered as a legal entity, a cooperative for example, and an IT rate of 35% was adopted.

**Work hours:** 1,000 annual work hours per tractor.

The calculation of the annual (fixed and variable) costs of each tractor was carried out using an estimate of the operating cost of farm tractors prepared by the Coordination of Integral Technical Assistance (CATI, 2012); a body linked to Department of Agriculture and Food Supply of São Paulo.

The estimated annual costs and sales values, which were used for the replacement simulations using AEC and RVPSE methods, are shown in Table 2.

**TABLE 2. Annual costs and sales values of tractors for each year of use (R$).**

| Year | Costs/Sale values | Tractor A (15 hp) | Tractor B (18 hp) | Tractor C (30 hp) |
|------|-------------------|-------------------|-------------------|-------------------|
| 0    | Acquisition cost | 47,929.00         | 51,693.00         | 62,491.00         |
|      | Sale value        | 44,943.20         | 48,472.72         | 58,598.04         |
| 1    | Annual cost       | 18,988.33         | 21,132.46         | 28,719.26         |
|      | Sale value        | 41,957.40         | 45,252.43         | 54,705.08         |
| 2    | Annual cost       | 19,138.66         | 21,294.60         | 28,915.27         |
|      | Sale value        | 38,971.60         | 42,032.15         | 50,812.12         |
| 3    | Annual cost       | 19,289.00         | 21,456.74         | 29,111.28         |
|      | Sale value        | 35,985.80         | 38,811.87         | 46,919.16         |
| 4    | Annual cost       | 19,439.33         | 21,618.88         | 29,307.29         |
|      | Sale value        | 33,000.00         | 35,591.58         | 43,026.21         |
| 5    | Annual cost       | 19,589.67         | 21,781.02         | 29,503.30         |

RVPSE reliability was assessed through comparisons using AEC in three problems proposed for analysis. These problems are conventional and show the possibilities of replacing a tractor in use. When the options no replacement and single replacement are predominant, the results from the
models must converge since cost factors are similar. In each problem, a different tractor was selected as a defender and other two as challengers. Table 3 shows a summary of the analyzed problems.

TABLE 3. Analyzed problems.

| Problem | Owner | Defender’s age (years) | Estimated service life (years) | Defender | Challengers |
|---------|-------|------------------------|-------------------------------|----------|-------------|
| P1      | PJ    | 0                      | 5                             | Tractor A | B / C       |
| P2      | PJ    | 0                      | 5                             | Tractor B | A / C       |
| P3      | PJ    | 0                      | 5                             | Tractor C | A / B       |

RESULTS AND DISCUSSION

The analysis was carried out by determining the best moment for replacing a farm tractor in use against multiple alternatives of substitute goods, without limiting the number of replacements during the planning horizon.

The results obtained by RVPSE and AEC in the simulation of the three problems are shown in table 4.

TABLE 4. Results of the simulations using RVPSE and AEC methods.

|               | P1 murky | P2 murky | P3 murky |
|---------------|----------|----------|----------|
| Stages        | 1  2  3  4  5  1  2  3  4  5  1  2  3  4  5 | | |
| RVPSE policy  | K  K  K  K  K  R1  K  K  K  K  K  K  K  K | | |
| AEC policy    | K  K  K  K  K  R1  K  K  K  K  K  K  K  K | | |

K = keep the tractor in use; R1 = replace for 1st challenger; R2 = replace for the 2nd challenger

The results presented in Table 4 show the similarity between the proposed policies, which highlights the reliability of the RVPSE model. When there is an indication of replacement, it occurs at an early opportunity for both models.

The annual costs of each tractor were estimated based on the power and the age of the good, being directly proportional to these two variables. In other words, if three machines with the same age are compared, there is a clear advantage of tractor A (the defender of problem 1) over the other two, considering its lower operating cost, lower acquisition cost, and, consequently, lower depreciation values and a decrease of market price. Evidence can be clearly verified in the simulation results. In these tests, the tractor in use should be kept until the end when the tractor A is the defender (problem 1); however, when being the challenger (problems 2 and 3), the tractor in use should be exchanged at the earliest opportunity.

However, in a change of scenery, with the addition of other factors to the calculation, such as taxes and profit on the sale (considering the difference between the resale value of the asset used and the book value of the asset after depreciation), the tractor with the lowest operating cost may no longer be the most convenient option. The approach of the end of the service life, determined at the beginning of the problem simulation, can also change the results. In this case, although tractor A has the lowest annual cost, the scenario in which it is older imposes the need for replacement in the near future, which would eliminate tractor retention.

CONCLUSIONS

The study showed the congruence and reliability of the results of RVPSE model when compared to the AEC in analyzing the best policy of farm machinery replacement.
The logic of the model based on a network structure of decisions proved to be flexible and comprehensive, offering managers more options for their decision-making at any stage and age of the good in use, in addition to testing more than one replacement during the planning horizon.

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