Sgoldstino inflation

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ABSTRACT: We discuss the possibility that inflation is driven by the sgoldstino, the superpartner of the goldstino. Unlike in generic supergravity scenarios, the sgoldstino decouples from all other fields in the theory, which allows for a simple and robust inflationary model. We argue that the two-field model given by this single complex scalar correctly captures the full multifield inflationary phenomenology. On the other hand, the assumption of stability, along the entire inflationary trajectory, of the supersymmetry–preserving sector that is integrated out leads to supplementary constraints on the parent supergravity. We investigate small field, large field and hybrid sgoldstino inflation scenarios and provide some working examples. They are subject to the usual fine-tuning issues that are common to all supergravity models of inflation. We comment on some other recently proposed sgoldstino inflation models.
1. Introduction

Scalar fields are abundant in supersymmetric theories. They all couple to each other with at least gravitational strength interactions. Planck suppressed couplings are generically unimportant when describing processes at low energies, but such a decoupling does not work for inflation. This can be most easily inferred from the slow roll parameters, which get contributions from dimension five and six operators that are unsuppressed. Describing inflation in a generic supergravity model is thus a challenging task, as generically the scalar field dynamics pose a complicated multifield problem.

There are good reasons why a single–field description is desirable. In line with Ockham’s razor, it is the simplest model that fits the data. Multifield slow–roll inflation with several (real) light fields has been studied for over a decade [1, 2, 3, 4] (see [5, 6] and references therein), and is very constrained by the observations, in particular through the tight limits on isocurvature modes and non-gaussianity [7]. On the other hand, however, it is technically challenging to obtain single-field behavior in a full multi-field set-up.
If the inflaton is the only light field in a multifield parent theory, integrating out the heavy fields should yield an effective single-field description that is accurate up to terms \( O(\partial^2 / M^2) \), with \( M \) the mass of the heavy field. Naively, if there is slow roll and a large mass hierarchy, one would assume such terms can be ignored, but this expectation is premature. In particular, if there are turns in the inflationary trajectory, derivative interactions between the inflaton and the heavy fields can become transiently strongly coupled. These lead to features and non-gaussianity in the spectrum of primordial perturbations that would not be inferred from the naive effective field theory (EFT). If the heavy fields remain sufficiently massive, the turns result in a reduced speed of sound for the adiabatic perturbations but are otherwise completely consistent with slow-roll \([8, 9, 10, 11, 12]\). Careful integration of the heavy fields recovers the general low energy effective field theory of inflation including a variable speed of sound for the adiabatic perturbations \([13, 14, 15, 16, 17]\) (see \([18]\) for a detailed discussion).

These interactions are unavoidable whenever the potential “valley” provided by the multifield potential deviates from a geodesic of the multifield sigma model metric. A corollary from the point of view of inflationary model building is that, when it comes to precision cosmology, the field space geometry of the “spectator” heavy fields (that are supposed to sit at their adiabatic minima during slow-roll inflation) is as important as their masses and couplings inferred from the potential alone.

Among the many scalars in a supersymmetric theory, the sgoldstino field stands out. The sgoldstino is the scalar partner of the goldstino, and belongs to the chiral superfield whose non-zero F-term breaks supersymmetry. It has the special property \([19, 20, 8]\) that it decouples from all other fields in the theory. This makes the sgoldstino an ideal inflaton candidate, for it allows for a description of inflation in terms of a single complex field. From the point of view of inflationary modeling this is still multifield inflation (with two real fields), but this two-field model is not a toy model, it really is the correct effective description for the full multifield system.

If inflation is effectively driven by a single real scalar field, the inflaton, this can be identified with a suitable linear combination of the real and imaginary parts of the sgoldstino field. Meanwhile, the orthogonal combination is to remain stabilized at a local minimum of the potential during inflation. The effect of turns in the trajectory on the spectrum of primordial perturbations have to be taken into account when comparing to observations, but

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1The caveats are due to other mass scales introduced by the changing background, whether in flat space or during slow-roll inflation \([8]\). This makes the details of decoupling during inflation different from particle phenomenology, where the effective field theory expansion is around a particular vacuum.

2We will not consider D-term breaking in this work.

3More precisely, setting all other superfields at the minimum of their potential is a consistent truncation from the N=1 sugra multi-field parent theory to an effective N=1 sugra with a single chiral superfield, the sgoldstino. In particular, the (real, two-dimensional) sgoldstino plane is a geodesically generated surface of the Kähler metric in the parent theory, so there are no derivative interactions with the truncated heavy fields: all turns in the inflationary trajectory are entirely confined to the sgoldstino plane. The effects of the fluctuations of the heavy fields are suppressed by their mass squared exactly as one would expect from an EFT expansion.
at least they can be calculated from the two-field model (see [21, 9, 22, 23, 24] for recent discussions and references).

Needless to say, this does not mean that all other scalars in the theory (from the susy-preserving superfields) can be completely neglected, as they have to be stabilized in a minimum of the potential during inflation. Even though the sgoldstino decouples from these fields, vice versa this is not true: the masses and couplings of all other fields generically depend on the field value of the sgoldstino field. As during inflation the sgoldstino evolves along its inflationary trajectory, the masses of the scalars change. If the inflaton is the sgoldstino, they will remain at the critical points, but they may become light or even tachyonic, triggering a waterfall-type exit from inflation that is not seen in the two-field model. Although it is still a complicated task to determine the minimum of the multifield potential along the inflationary trajectory, it is much simpler than the full multifield dynamics needed for a generic, non-sgoldstino, inflation model.

The potential energy density driving inflation breaks susy spontaneously [25, 26]. This source of susy breaking in the inflaton sector is always present during inflation, and is in principle independent of the source of vacuum susy breaking. For sgoldstino inflation there are two possibilities. First, the same superfield that drives inflation is also responsible for low energy susy breaking\(^4\). This would be the ideal situation. Not only does inflation decouple from all other fields in the theory, it also links the scale of inflation to the scale of susy breaking. The second possibility is that the two sources of susy breaking are due to different fields. Both sources may be operative during inflation, or alternatively, it may be that only after inflation has ended, a phase transition takes place generating our present-day susy breaking. In both cases the present day sgoldstino field is not the sgoldstino during inflation.

If several sources of susy breaking are present during inflation, the inflaton can only be approximately identified with the sgoldstino, and only if the vacuum susy breaking scale is much below the inflationary scale. Care should be taken in this case because, as argued in [30], only if the lightest mass in the hidden sector responsible for vacuum susy breaking is much larger than the inflaton mass and if the inflaton mass is much larger than the scale of hidden sector susy breaking, is the effect of the hidden sector on the slow-roll dynamics of the inflaton negligible. This is far from trivial; for example, it has been proven extremely hard to combine a susy breaking moduli stabilization and inflation in a consistent way [31], even in a fine-tuned setting [32].

The decoupling of the sgoldstino from the other fields fits in with recent work on how to incorporate different fields, or sets of fields, in a sugra set-up minimizing the coupling between them [19, 33, 34, 35, 36, 37, 38, 39, 40, 41, 30, 42]. Quite commonly different sectors — e.g. the fields and couplings responsible for susy breaking, for inflation, for moduli stabilization, or making up the standard model — are combined by simply adding their respective Kähler- and superpotentials. However, following this procedure one cannot completely decouple these sectors. Even if the Kähler and superpotential do not contain direct interaction terms between

\(^4\)This possibility has been recently discussed in [27, 28, 29] but as we will show it is difficult to implement in practice.
fields in different sectors, the resulting scalar potential does. There are always at least Planck suppressed interactions between the fields, and generically the mass matrix is not block diagonal in the different sectors. This complicates the analysis of the full model enormously. Sectors are affected by the presence of others, and although they work in isolation, they may no longer do so in the full set-up. Moreover, heavy fields generically cannot be integrated out in a consistent supersymmetric way.\(^5\)

The cross-couplings between sectors can be minimized if instead of adding Kähler and superpotentials, one adds the Kähler invariant functions

\[
G = K + \ln |W|^2
\]

for the two sectors \(^4\). This approach allows to integrate out fields in a susy preserving way \(^1\). In Ref. \(^3\) the addition of sugra functions was used to couple a susy breaking moduli sector (fields \(X^i\)) to a susy preserving sector, for example the standard model (fields \(z_i\)):

\[
G_{\text{tot}}(X^i, \bar{X}^i, z_i, \bar{z}_i) = g(X^i, \bar{X}^i) + G_{\text{other}}(z_i, \bar{z}_i).
\]

In this article we want to use the same idea to couple a susy breaking inflationary sector (fields \(X^i\)) to a susy preserving sector (\(z_i\)).\(^6\). For simplicity we restrict to effectively single field inflation, and models with a single inflaton field \(X\). As susy is broken during inflation, the inflaton is then the sgoldstino. As it turns out, the ansatz (1.1) is actually too strict. We can allow for explicit couplings between the inflaton and the other fields, of the form

\[
G(X, \bar{X}, z_k, \bar{z}_k) = g(X, \bar{X}) + \frac{1}{2} \sum_{i \geq j} \left[ (z_i - (z_i)_0)(z_j - (z_j)_0)f^{(ij)}(X, \bar{X}, z_k, \bar{z}_k) + \frac{1}{2} \sum_{i \geq j} (z_i - (z_i)_0)(\bar{z}_j - (\bar{z}_j)_0)h^{(ij)}(X, \bar{X}, z_k, \bar{z}_k) + \text{h.c.} \right]
\]

with \(f, h\) arbitrary functions of its arguments. As we will show, this is the most general ansatz consistent with \(X\) being the sgoldstino. The explicit \(X\)-dependence in the second term does not spoil the decoupling of the inflaton field, the mass matrix remains block diagonal in the two sectors, as long as the fields \(z_i\) sit at the susy critical point \((z_i)_0\) during inflation. As we will show, during sgoldstino inflation the Kähler function \(G\) is well defined, maybe except from isolated points in field space.

Single field inflation can be divided into three main classes: large field, small field and hybrid inflation. We discuss whether and how sgoldstino inflation might work in these three regimes. Any sugra model of inflation has to address the \(\eta\)-problem; this puts bounds on the Kähler geometry \(^4\).\(^5\).\(^6\).

Large field sgoldstino inflation does not work, at least not for potentials that grow at most polynomial.

Hybrid inflation provides the most natural embedding for sgoldstino inflation. Indeed, usual F-term hybrid inflation is an example of having a sgoldstino inflaton. In this set-up

\(^5\)Here, once again, approximations that are justified for phenomenology applications where the background is static \(^4\) fail during inflation \(^3\), \(^3\), \(^3\), \(^8\).

\(^6\)In \(^3\) the separable form (1.1) was used to combine hybrid inflation with a susy breaking moduli sector in a successful way. It this set-up the inflaton is not the goldstino.
susy is restored in the vacuum, and there is no relation with low energy susy breaking. More complicated waterfall regimes may be devised, such that susy is broken in the minimum after inflation. However, such an analysis is multifield, and complicated multifield dynamics enters via the back door again.

Small field inflation offers the best possibility to link inflation to susy breaking. Naively all that is needed is finding and tuning a saddle or maximum in a single field potential with a susy breaking Minkowski minimum. We only find inflection points suitable for inflation rather than a maximum or saddle point. Inflection point inflation yields \[ n_s \leq 0.92 - 0.93 \] (for \( N = 50 - 60 \) efolds), which is on the verge of being ruled out by the CMB data [7]. Interestingly enough, models in which susy is broken after inflation are much easier to embed in a multi-field set-up than models with a susy preserving vacuum. Finally, we comment on recent claims in the literature for small field sgoldstino inflation \[ \text{[28, 29, 50]} \] with no or very little fine-tuning. We will explain why these models cannot work.

2. Decoupling of the sgoldstino

In this section we will show the decoupling of the sgoldstino field explicitly. In the first subsection we derive the mass matrix, which is block diagonal along the sgoldstino inflation trajectory. We will argue in subsection 2.2 that the Kähler function for a dynamical sgoldstino field can always be put in the form (1.2). In subsection 2.3 we show that this sgoldstino trajectory is independent of the field values of all the other fields. Vice versa that is not the case: the dynamics of the non-sgoldstino fields does depend on the sgoldstino field. Care must be taken so that these fields remain stabilized along the full inflationary trajectory. Finally, in subsection 2.4 we discuss the special limit of separable Kähler functions (1.1), in which the results of [35] are retrieved.

2.1 Mass matrix

We start with the general formula for the mass matrix, then specialize to sgoldstino inflation. The scalar potential can be expressed solely in terms of the Kähler function\(^7\) \( G = K + \ln |W|^2 \):

\[
V_F = e^G [G_I G^I J G_J - 3],
\]

with \( I, J \) running over all fields \( \Phi_I \). We will be working in Planck units \( M = 1 \) throughout this work. The fields span the Kähler manifold with complex metric \( G_{IJ} \). The inverse metric \( G^{IJ} \) is such that \( G_{IJ} G^{KJ} = \delta^K_I \) and \( G_{IJ} G^{IK} = \delta^I_K \). The only non-zero connection is \( \Gamma^K_{IJ} = G_{IJP} G^{PK} \) and its complex conjugate. The non-zero components of the Riemann tensor are \( R_{IJKL} = G_{SL} \partial_J G^{PK} \) and permutations thereof.

\(^7\)This procedure is ill defined for \( W = 0 \). To cure this, one can use the variable \( \phi \equiv e^G \) instead, which remains well defined [51]. However, in the next section we show that \( W = 0 \) at most in isolated points in field space.
The mass matrix is

$$\mathcal{M} = \begin{pmatrix} M^I_J & M^I_I \\ M^I_J & M^I_I \end{pmatrix}, \quad M^I_J = G^{IK} \nabla_K \nabla_J V, \quad M^I_I = G^{IK} \nabla_K \nabla_V, \quad (2.2)$$

with $\nabla_K v_L = \partial_K v_L - \Gamma^M_{KL} v_M$ the covariant derivative of some vector $v_L$. The eigenvalues and eigenvectors of the mass matrix correspond to the $m^2$-values and mass eigenstates respectively. The first derivative of the potential is

$$V_K = G_K V + e^G [G^I \nabla_K G_I + G_K] \quad (2.3)$$

where we used metric compatibility $\nabla_K G_{IJ} = 0$, $\nabla_K G^I = \delta^I_K$ and introduced the notation $V_K = \partial_K V$, $G^I = G^{IJ} G_J$. Stationarity is not assumed, as the inflaton field is displaced from its minimum during inflation. The second derivatives of the potential are

$$\nabla_L \nabla_K V = (G_{KL} - G_K G_L) V + 2G_{(KL)} + e^G [G^{JK} (\nabla_K G_I) - R_{KI} L J G^J + G_{KL}],$$

$$\nabla_L \nabla_K V = (\nabla_L G_K) - G_{(KL)} V + 2G_{(KL)} + e^G [2\nabla_K G_L + G^J (\nabla_L G_J) G_I], \quad (2.4)$$

where round brackets denote symmetrization. We used that $[\nabla_L, \nabla_K] G_I = \nabla_L \nabla_K G_I = -R_{KLI J} G^J$. Apart from the terms proportional to $V_K$, which are absent for stationary situations, these equations are the same as (2.6, 2.7) of Ref. [52].

Now consider F-term breaking of susy, signaled by a non-zero $G_X \neq 0$. Here $X$ is the scalar component of the chiral superfield which also contains the goldstino. Note that one can always make a field redefinition such that only one linear combination of fields breaks susy. All other fields in the theory, denoted by $z_i$ (indexed by lower case latin letters), do not break susy. Hence, we split the fields in $\Phi_I = \{X, z_i\}$, with

$$G_X |_{z_0} \neq 0, \quad G_i |_{z_0} = 0 \quad (2.5)$$

at some point in field space $z_0 = \{(z_1)_0, (z_2)_0, \ldots\}$, the so-called susy critical point.

We are interested in a cosmological situation, in which $X(t)$ is the inflaton rolling along some trajectory with $V_X \neq 0$. While the inflaton rolls in the $X$-direction, we want all orthogonal fields $z_i$ to remain extremized at $z_0$. To that end we demand that

$$(\partial_X)^m (\partial_X)^n G_i |_{z_0} = 0, \quad \forall m, n \in \mathbb{N}. \quad (2.6)$$

Indeed, from (2.3), we then have that

$$V_i |_{z_0} = G_i V + e^G [G^P \nabla_i G_P + G_i] = e^G G^X \nabla_i G_X = 0. \quad (2.7)$$

For notational convenience we dropped the $|_{z_0}$ on the right hand side, but the reader should keep in mind that all expressions should be evaluated at $z = z_0$. Note that $i$ labels the $z_i$ fields, and capital letters label $\Phi_I$ (i.e. also running over $X$). In the first step we used (2.5), in the second $\nabla_i G_X |_{z_0} = 0$, which is a consequence of (2.6).
in contradiction with \((\ref{eq:trajectory})\). The potential \(W\) theory with two chiral fields — the extension to many fields is straightforward — with a field space. Therefore the Kähler function \(G\), which for sgoldstino inflation we nowhere have \(W\), is that it becomes undefined when \(0\). However, it is easy to show that for sgoldstino inflation we nowhere have \(W = 0\), except maybe for isolated points in field space. Therefore the Kähler function \(G\) is well defined. To illustrate this, consider a theory with two chiral fields — the extension to many fields is straightforward — with a superpotential \(W = W(X, Z)\). For sgoldstino inflation, with \(X\) the goldstino superfield, we have

\[
|D_X W|_{\{X(t), Z_0\}} \neq 0, \quad |D_Z W|_{\{X(t), Z_0\}} = 0, \tag{2.10}
\]

with \(D_X W = K X + W_X\) the Kähler covariant derivative. Setting \(W = 0\) along the whole trajectory implies

\[
|W|_{\{X(t), Z_0\}} = 0 \Rightarrow |W_X|_{\{X(t), Z_0\}} = 0 \Rightarrow |D_X W|_{\{X(t), Z_0\}} = 0 \tag{2.11}
\]

in contradiction with \((2.10)\). Therefore the superpotential can only vanish for sgoldstino inflation at accidental zeroes at isolated points in field space (possibly on the trajectory, but this does not change our conclusions).

As a side remark, note that when the inflaton is identified with the \(Z\) field rather than the goldstino field \(X\), as for example in the models of Ref. [53], it is possible to have \(W = 0\), \(D_X W|_{\{X_0, Z(t)\}} \neq 0\) and \(D_Z W|_{\{X_0, Z(t)\}} = 0\) along the whole trajectory \(\{X_0, Z(t)\}\). In this case the Kähler invariant function is not well defined, and a description in terms of \(K\) and \(W\) is needed.
Expanding the Kähler function around the susy critical point $z^i = z_0^i$, the most general form for sgoldstino inflation — satisfying (2.5) and (2.6) — can be written in the form

$$G(X, \bar{X}, z_k, \bar{z}_k) = g(X, \bar{X}) + \frac{1}{2} \sum_{i \geq j} \left[ (z_i - (z_i)0)(z_j - (z_j)0)f^{(ij)}(X, \bar{X}, z_k, \bar{z}_k) + (z_i - (z_i)0)(\bar{z}_j - (\bar{z}_j)0)h^{(ij)}(X, \bar{X}, z_k, \bar{z}_k) + h.c. \right]$$

(2.12)

with $f, h, g$ arbitrary functions of its arguments.

### 2.3 Inflationary trajectory

We have seen in subsection 2.1 that along the inflationary trajectory all non-sgoldstino fields are extremized at $z^i = z_0^i$. Since the mass matrix is block diagonal, we can determine the stability of the $z_i$ extremum from the sub-block of ${\mathcal M}$ with $z_i$ indices. It can easily be shown that the inflaton trajectory itself is independent of the field values of the other fields. Indeed, the potential along the inflationary trajectory only depends on the function $g(X, \bar{X})$ in (2.12), and is thus independent of the field values of all other fields. The height $V_0 \equiv V|_{z_0}$, slope and second derivatives of the inflaton potential are given by (2.1, 2.3, 2.4) with $I, J$ only running over $X$ and $G \rightarrow g$. For example we have

$$V_0 = e^g \left[ g_X g^X \bar{g}_X - 3 \right],$$

(2.13)

$$V_X|_{z_0} = g_X V_0 + e^g \left[ g^X \nabla_X g_X + g_X \right].$$

(2.14)

In contrast, the mass matrix along the orthogonal directions does depend on the inflaton field value. We find

$$M^i_j|_{z_0} = G^{ij} \nabla_k \nabla_j V = G^{ij} \left[ G_{jk} V_0 + e^G [G^m \nabla_k (\bar{G}_m)] (\nabla_j G_l) - R_{X \bar{X} jk} G^X G^{\bar{X}} + G_{jk} \right] = e^g \left[ \delta^i_j (b + 1) + x^n_m x^n_m + w^i_j \right],$$

(2.15)

and

$$M^i_j|_{z_0} = G^{ik} \nabla_k \nabla_j V = G^{ik} \left[ \nabla_k (\bar{G}_j) V_0 + e^G [2 \nabla_j (\bar{G}_k) + G^X \nabla_k (\bar{G}_j) G_X] \right] = e^g \left[ x^i_j (b + 2) + y^i_j \right].$$

(2.16)

Here we introduced the notation

$$b = V_0 e^{-g} = g_X g^X - 3$$

(2.17)

$$x^i_j = G^{ik} \nabla_k G_m = G^{ik} \nabla_m G_k$$

(2.18)

$$w^i_j = -G^{ik} G^X G^{\bar{X}} R_{X \bar{X} jk}$$

(2.19)

$$y^i_j = G^{ik} G^X \nabla_k (\bar{G}_j) G_X.$$
Note that $b = V_0/m_{3/2}^2$ gives the height of the potential in units of the gravitino mass. During slow-roll this is approximately $b \sim 3H^2/m_{3/2}^2$.

The functions $b, x, y, w$ can be expressed in terms of the functions $f, g, h$ appearing in the Kähler function (2.12). In general, the constraint that the squared masses should be positive is complicated, but there are two situations in which it simplifies considerably. The first one, discussed in the next section, is if the Kähler invariant function is separable [35, 36]. In this case the matrices $y$ and $w$ vanish and the constraint involves the eigenvalues of the $x$ matrix.

The second case where the constraint simplifies is for a single $z$ field, i.e. $i = \{1\}$, such that there is only one $f$ and $h$ function. Then the matrices $x,y$ and $w$ become scalars

\begin{align}
    b &= g_x g^x - 3 \\
    x &= h^{-1}(f - f_x g^x), \\
    w &= -g_x g^x h^{-1}(h_{XX} - h_x h^{-1} h_{X}), \\
    y &= h^{-1}g_x^x [f_x - 2 h_x h^{-1} f - f_x g^x + (f_x g^x_{XX} + h^{-1} h_x f_x - f_{XX}) g^x].
\end{align}

For a canonically normalized $z$ field, $h = 1, h_x = h_{XX} = 0$ which implies $w = 0$.

For single field inflation, or if the matrices $x, w, y$ can be diagonalized simultaneously, the eigenvalues of the mass matrix are given by

\begin{equation}
    m_{\pm}^2 = e^\theta [(1 + b) + |x|^2 + w \pm |(2 + b)x + y|].
\end{equation}

The $z$ eigenstates remain stabilized as long as the smallest mass is positive definite $m_{\pm}^2 > 0$.

### 2.4 Separable Kähler function

The results in the previous section are a generalization of the work [35, 36, 37], who considered a set-up with separable Kähler functions:

\begin{equation}
    G(X, \bar{X}, z_i, \bar{z}_i) = g(X, \bar{X}) + \bar{g}(z_i, \bar{z}_i),
\end{equation}

which is a special limit of the more general function (2.12). For the separable Kähler function above (2.23) all mixed derivatives of $G$, such as $G_{zzX}$, cancel. With this simplification

\begin{equation}
    b = g_x g^x - 3, \quad x^i_m = \bar{g}^\epsilon_k \bar{g}_{km}, \quad y^i_j = w^i_j = 0.
\end{equation}

We now consider the case with only one $z$ field, which turns $x^i_3$ into a scalar. As one can always diagonalize $x^i_3$, this simplification precisely gives the result along one of the eigenvectors, and thus can be straightforwardly be generalized to several $z$ fields. We recover the system studied in [35]8:

\begin{equation}
    M_z^2 |_{z_0} = e^\theta [(b + 1) + |x|^2], \quad M_{\bar{z}}^2 |_{\bar{z}_0} = e^\theta (b + 2)x,
\end{equation}

which has eigenvalues

\begin{equation}
    m^2_{\pm} |_{z_0} = e^\theta [1 + b + |x|^2 \pm |(2 + b)x|] = e^\theta \left[ \left( |x| \pm \frac{1}{2} |(2 + b)x| \right)^2 - \frac{b^2}{4} \right].
\end{equation}

8Our definition of $b$ is different from [35], which has $b \leftrightarrow b - 3$.  

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This result can also be obtained from the general expression for the mass squared eigenvalues \((2.22)\), taking the appropriate limit \(y_i^2 = w_i^2 = 0\). The function \(b\) is bigger, equal or smaller than zero for a dS, Minkowksi or AdS universe, respectively. Take \(b \geq 0\); in the opposite limit the masses \(m_\pm^2\) and \(m_\mp^2\) are exchanged. The smallest mass eigenstate is positive \(m^-_\mp > 0\), i.e., the \(z\)-field is stabilized along the inflationary trajectory, for \(|x| < 1\) or \(|x| > (1 + b)\). We will put this analysis in practice for sgoldstino inflation in subsection 3.2 (hybrid inflation) and 3.3 (small field inflation).

Close to the instability bounds \(|x| \lesssim 1\) or \(|x| \gtrsim (1 + b)\) the spectator field \(z\) is lighter than the gravitino mass and/or the Hubble scale, and cannot be integrated out. In a Minkowski vacuum after inflation either \(b = 0\) or \(b \to \infty\); the latter case may occur in a supersymmetric vacuum with \(W \to 0\). For \(b = 0\), the masses reduce to \(m_\pm^2 = m_{3/2}^2 (1 \pm |x|)^2\), with \(m_{3/2}\) the gravitino mass. For \(|x| > 1\), the lightest scalars from the supersymmetric sector are heavier than the gravitino. However, for \(|x| < 1\) the lightest of the two eigenstates is lighter than the gravitino and cannot be neglected from a low–energy description. This will play an important role later. In the supersymmetric vacuum with \(b \to \infty\) we find \(m_\pm^2 \approx V_0 (1 \pm |x|) \to 0\), and the spectators are massless. To avoid a plethora of massless fields in the theory, one has to either break the supersymmetry, or else go beyond the simple separable form of the Kähler function \((2.23)\).

### 3. Single field sgoldstino inflation

In this paper we focus on effectively single field inflation models, for simplicity. The inflaton, a real scalar, is identified with a suitable linear combination of the real and imaginary parts of the sgoldstino field; the orthogonal combination is to remain stabilized at a local minimum of the potential during inflation. Single field inflation can be divided into three classes: small field, large field and hybrid inflation. In the first two cases, if the model only contains a single chiral superfield, the inflaton is automatically the sgoldstino. If several fields are present, as is the case for hybrid inflation, one has to be more careful, as the sgoldstino does not have to coincide with the inflaton direction.

As is well known any sugra model of inflation has to address the \(\eta\)-problem \([25, 54, 55]\): the inflaton field needs to be protected from its natural tendency to become heavy, and obtain a mass of the order of the Hubble scale. This is just another manifestation of the hierarchy problem that plagues all scalars, including the standard model Higgs field. The problem is easily spotted in the sugra context. Expand the Kähler potential around \(X_0\), the inflaton field value during inflation, in \(\delta X = X - X_0\); this gives \(K = K_0 + K_{\delta X} |\delta X|^2 + ... = K_0 + |\Phi|^2 + ...\), with \(|\Phi|\) the canonically normalized complex field. The scalar potential then gives

\[
V_F = e^{i|\Phi|^2} [V_0 + ...].
\]

The \(\eta\)-parameter measures the curvature of the potential in units of the Hubble parameter along the inflationary trajectory: \(\eta = V_{\phi\phi}/V\), with \(\phi\) the canonically normalized real inflaton.
field. With the inflaton some linear combination of the real and imaginary parts of $\Phi$, it is clear that the exponent in (3.1) contributes order unity: $\eta \approx 1 + \ldots$, which spoils inflation.

The $\eta$-problem may be solved introducing symmetries which forbid an inflaton mass, and thus keep the inflaton potential flat. Such a symmetry needs to be softly broken to provide a small slope for the inflaton potential. It is far from trivial to assure that such a breaking does not introduce the $\eta$-problem again. Another solution to the $\eta$-problem is to fine-tune parameters. The order one contribution coming from the exponent in (3.1) may be tuned against all other contributions (from the ellipses) to obtain a total $\eta$-parameter that is small.

In the remainder of this section we will discuss large field, small field and hybrid sgoldstino inflation, and how the $\eta$-problem may or may not be addressed in each case.

3.1 Large field inflation

In models of large field inflation [56], the inflaton field traverses super-planckian distances in field space during inflation. For a potential dominated by a single monomial during inflation, $V \sim \lambda \phi^n$, the slow roll parameters

$$
\epsilon = \frac{1}{2} \left( \frac{V_{\phi}}{V} \right)^2, \quad \eta = \frac{V_{\phi\phi}}{V},
$$

both scale as $\eta, \epsilon \sim 1/\phi^2$, and are automatically suppressed for super-planckian field values. At first sight, no tuning of the potential is needed. However, the problem is that for such large field values all non-renormalizable operators are unsuppressed. Therefore, an explicit UV completion of the model is needed to determine whether inflation is possible.

Embedding large field inflation in sugra provides a better control over the UV behavior of the theory. Because of the $\eta$-problem such an embedding is far from straightforward, as the potential (3.1) grows exponentially rather than polynomial. Fine-tuning $\eta$ is not an option, as $\eta$ has to be small along the whole inflationary trajectory, which spans super-planckian distances in field space $\Delta \phi > 1$. This is in contrast with small field inflation, discussed in subsection 3.3, where the $\eta$-problem can be solved by tuning $\eta$ at a single point in field space.

Instead of fine-tuning, we can try to solve the $\eta$-problem by invoking a shift symmetry [57]. Consider a Kähler function $G = \mathcal{K}(X - \bar{X})$, which is symmetric under a shift $X \to X + c$ with $c$ a real constant. Since $G$ does not depend explicitly on $\phi \propto \text{Re}(X)$, the exponent in (3.1) is independent of $\phi$ and there is no $\eta$-problem. In fact, the potential has an exactly flat direction. Since we want the system to end up after inflation in a Minkowski minimum, there is no other option than to set $V = 0$ along the flat direction, which is incompatible with having inflation.

In order to get a slope for the potential and obtain inflation, the shift symmetry needs to be weakly broken. To assure the breaking does not reintroduce exponential terms that ruin inflation, we add a logarithmic term $G = \mathcal{K}(X - \bar{X}) + \ln |W(X)|^2$ with $W$ not growing faster than power law. As we want to construct a potential that is polynomial, we forbid the linear terms in the Kähler potential $\mathcal{K} = \mathcal{K}((X - \bar{X})^2)$. Then the potential along the inflationary
trajectory is

$$V_F|_{X=\bar{X}} = W_X G^{XX} \bar{W}_X - 3|W|^2|_{X=\bar{X}}. \quad (3.3)$$

The inverse metric $G^{XX}|_{X=\bar{X}} = -1/K''(0)$ is a constant along the inflationary trajectory, as it is independent of $\phi$; it just renormalizes the field and can be absorbed by going to canonically normalized fields: $\phi^2 = -2K''(0)|X|^2$. If the superpotential during inflation is dominated by a monomial term $W \sim \lambda X^n$, we find

$$V_F|_{X=\bar{X}} \propto n^2\phi^{2n-2} - 3\phi^{2n} \quad (3.4)$$

which goes negative for large $\phi > n/\sqrt{3}$. For field values $\phi = \mathcal{O}(10)$ as needed for large field inflation, the field will run off to infinity and negative potential, rather than the Minkowski minimum at the origin. This does not give a viable inflation model. The instability occurs for every superpotential that does not grow faster than power law, such that the shift symmetry is only broken softly. Faster growing superpotentials reintroduce the $\eta$ problem.

Although we did the analysis for a single field, this straightforwardly generalizes to the multi-field case. If the inflaton is the sgoldstino, it decouples from the other fields, and its potential can be analysed independently and will always be of the form (3.4). We conclude that large field sgoldstino inflation in a sugra model does not work as it is plagued by an instability in the scalar potential.

We note that it is certainly not impossible to have large field inflation in sugra, only that it does not work with a single chiral superfield. Two-field models have been constructed that avoid the instability [53, 57], employing a shift symmetry to address the $\eta$-problem. However, in these models the inflaton is not the sgoldstino (rather the sgoldstino is the orthogonal field).

### 3.2 Hybrid inflation

Hybrid inflation is a multi-field model of inflation which in addition to the inflaton contains one or more so-called waterfall fields, which serve to end inflation [58]. During inflation the waterfall fields are stabilized in a local minimum, and inflation is effectively single field. If the inflaton field drops below a critical value one of the waterfall fields becomes tachyonic, and inflation ends with a phase transition.

Standard F-term hybrid inflation [59, 60] is an example of sgoldstino inflation. The Kähler function is of the separable form (2.23) discussed in section 2.4.

$$G = g(X, \bar{X}) + \tilde{g}(\chi_1, \bar{\chi}_1, \chi_2, \bar{\chi}_2), \quad (3.5)$$

with

$$g = X \bar{X} + k_s(X \bar{X})^2 + \ln|\kappa X|^2 + ..., \quad \tilde{g} = \chi_1 \bar{\chi}_1 + \chi_2 \bar{\chi}_2 + \ln|\chi_1 \chi_2 - \mu^2|^2 + ...$$

\footnote{To see that this setup is indeed of the general form (2.12), one can move a factor of $\ln|\mu^2|^2$ from $\tilde{g}$ to $g$ and Taylor expand the remaining $\ln|\frac{\chi_1 \chi_2}{\mu^2} - 1|^2$.}
Figure 1: (Figure adapted from [35, 37].) Stability diagram for the separable case $G = g(X, \bar{X}) + \tilde{g}(z, \bar{z})$. The variables on the axes $b, x$ are defined in (2.24), with $x$ one of the degenerate eigenvalues of the $x^i_j$ matrix. The masses of the spectator fields are positive in the shaded region, while the unstable region signals a tachyonic mode. The black arrow represents the inflationary trajectory for the proposed hybrid set-up, which ends when one of the spectator fields (the waterfall fields) becomes tachyonic. Also shown are possible inflationary trajectories for small field inflation (red arrows).

The model has an R-symmetry, which uniquely fixes the superpotential at the normalized level, and in particular it allows for a linear term in $X$ but forbids the quadratic and cubic terms in $W$. This kills large contributions to the slow roll parameters, and allows for a flat direction in the inflaton potential, which at tree level is only lifted by higher order terms in the Kähler potential.

The inflaton $\phi$ is identified with the real direction via the decomposition $X = (\phi + i\theta)/\sqrt{2}$. Inflation takes place for large $\phi > \phi_c = \sqrt{2}\mu$, and all other fields stabilized at zero field value. The potential along the inflationary trajectory is

$$V = \kappa^2 \mu^4 \left(1 - 2k_s \phi^2 + \ldots \right) + V_{1\text{-}\text{loop}}.$$  

The flatness of the potential is only lifted by higher order terms in $K$, and by the one-loop Coleman-Weinberg potential $V_{1\text{-}\text{loop}}$ [61]. The $\eta$-problem is solved via a moderate fine-tuning of $k_s \lesssim 10^{-2}$. Moreover, for the 1-loop contribution to be sufficiently small $\sqrt{\kappa^2 \mu}$ should be of the grand unified scale or smaller. During inflation $G_X = \frac{\phi^2}{\phi} + \frac{\phi^2 + k_s \phi^3}{\sqrt{2}}$ and $G_{\chi_1} = G_{\chi_2} = 0$. Hence $\phi$ is indeed the (real part of the) sgoldstino field.

The Minkowski minimum after inflation is at $X = 0$, and $|\chi_1| = |\chi_2| = \mu$. In the minimum $G_X = G_{\chi_\pm} = 0$ and susy is restored. There is no relation between inflation and low energy susy breaking. The sgoldstino during inflation is unrelated to the sgoldstino today.
The masses of waterfall fields along the inflationary trajectory can be found using the results of section 2.4. The mass eigenstates are the linear combinations $\chi_{\pm} = (\chi_1 \pm \chi_2)/\sqrt{2}$. Using these as a basis the matrix $x_{im}^i$ becomes diagonal during inflation. This shows that we can restrict our attention to only one of the complex fields $\chi_{\pm}$, the other field will give the same masses for its two real degrees of freedom. Now we can directly compute the masses from (2.26). The stability region as a function of $b$ and $|x|$ is plotted in Fig 1. The inflationary trajectory corresponds to a vertical trajectory in the plot, going upwards as the field rolls down. When it irrevocably hits the instability region (i.e. when the lower mass eigenvalue becomes negative), inflation ends.

We note that a similar stability analysis can be done for all models of sgoldstino inflation. Whereas hybrid inflation critically makes use of the instability regions, for any non-hybrid scenario — being it small or large field inflation — the inflationary trajectory would have to stop before reaching the instability region. This is automatic for $|x| < 1$, otherwise the stability conditions place an upper bound on $b$ during inflation. We will return to this point shortly when discussing small field inflation.

3.3 Small field inflation

Inflation in small field models [62, 63] takes place for sub-Planckian values of the inflaton field. This allows for Taylor expanding the inflaton potential around its Minkowski minimum. If one term in the polynomial expansion dominates during inflation the slow roll parameters blow up: $\epsilon, \eta \sim 1/\phi^2$ in the small field limit, prohibiting inflation. As before, $\phi$ is the canonically normalized inflaton field. The only way to get around this conclusion is that several terms in the expansion conspire together to nearly cancel, thus obtaining small slow roll parameters.

This motivates to consider inflation near an extremum — a maximum, saddle point or inflection point — of the potential. This assures that the first slow roll parameter $\epsilon$ vanishes. The $\eta$-parameter can be made small by tuning the parameters in the potential. Since the inflaton field traverses only small, sub-planckian distances in field space, tuning the curvature of the potential at a single point, at the extremum, suffices. This is in contrast with large field inflation, where $\eta$ needs to be small along the full, super-planckian inflationary trajectory. The tuning of the parameters in the potential is typically of the 1-permille level, dictated by the need to get $\eta \lesssim 10^{-2}$. Note that in a sugra the $\eta$-parameter cannot be tuned for arbitrary Kähler geometry [45, 46, 47]. In our example below we will assume an (approximately) canonical Kähler potential, for which there are no obstacles. Ref. [47] considered modular inflation near a maximum; we come back to this model at the end of this section.

Symmetries generically do not help in solving the $\eta$ problem in the small field models. For example, a shift symmetry $K = K(X - \bar{X})$, so useful in large field models, does not do anything in the small field regime. By Taylor expanding the Kähler potential and performing a Kähler transformation, it becomes equivalent to a non shift symmetric $K = K(X\bar{X})$. R-symmetries may help in providing a flat potential, but the R-symmetry breaking, which is necessary to obtain a Minkowski vacuum, also tends to spoil the flatness. This is what kills the model proposed in [50], on which we will comment in a bit more detail below.
We were able to construct a fine-tuned small field inflation model in sugra containing only a single chiral field. In such a set-up the inflaton is automatically the sgoldstino, and our example is an existence proof for small field sgoldstino inflation. Consider a model with

\[ K = \sum_n \alpha_n (X \bar{X})^n, \quad W = \sum_n \lambda_n X^n. \]  

(3.7)

We decompose the complex scalar \( X = (\phi + i\theta)/\sqrt{2} \) with \( \phi \) the inflaton field. The model parameters \( \lambda_n, \alpha_n \) can be tuned in such a way that the potential allows for inflation near an inflection point which, without loss of generality, is located at the origin \((\phi, \theta) = (0, 0)\), and a Minkowski minimum at finite field value \((\phi, \theta) = (\phi_0, 0)\). In particular, we demand

- Vanishing slope and curvature of the potential at the origin 1) \( V_{\phi}\big|_{(0,0)} = 0 \) and 2) \( V_{\phi\phi}\big|_{(0,0)} = 0 \), to assure zero slow roll parameters \( \epsilon = \eta = 0 \). The condition on \( \eta \) may be relaxed to \( \eta \lesssim 10^{-2} \).

- The height 3) \( V\big|_{(0,0)} \equiv V_0 \) of the potential at the origin is fixed by the COBE normalization of the inflaton perturbations.

- After inflation the inflaton settles in a local Minkowski minimum with 4) \( V\big|_{(\phi_0,0)} = 0 \) and 5) \( V_{\phi}\big|_{(\phi_0,0)} = 0 \). Moreover, the masses are positive definite 6) \( m^2_{\phi}\big|_{(\phi_0,0)} > 0 \).

- Along the whole trajectory, from the extremum to the minimum, the orthogonal field is stabilized 7) \( V_{\theta} = V_{\phi\theta} = 0 \) and 8) \( m^2_{\theta} \gtrsim H^2 \).

We consider solutions with canonical kinetic terms, i.e. we set \( \alpha_1 = 1 \) and \( \alpha_i = 0 \) for \( i \neq 1 \). To satisfy conditions 1-5 we need at least five parameters and choose them accordingly. We take all \( \lambda_i \) real, and consider the first five in the expansion. Tuning is required to satisfy conditions (2) and (4) — the smallness of \( \eta \) parameter and of the cosmological constant — in the usual sense that large contributions should nearly cancel. Conditions 6-8 are then checked for consistency, but do not require any new input. Setting the minimum at \( \phi_0 = 1 \) we find two inflationary inflection point solutions

\[ \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \sqrt{\frac{V_0}{23}} \times \{3, -5\sqrt{2}, 3, 0, 2\}, \]  

(3.8)

and

\[ \{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \frac{\sqrt{V_0}}{19\sqrt{73}} \times \]  

\[ \left\{3\sqrt{39287} - 1464\sqrt{6}, \sqrt{2 \left(543551 - 19764\sqrt{6}\right)}, 3\sqrt{39287} - 1464\sqrt{6}, 0, -2\sqrt{4943 - 1152\sqrt{6}} \right\}, \]  

(3.9)

and all other \( \lambda_i \) are zero.

\(^{10}\)This ansatz \((3.7)\) is equivalent to \( G = \sum_{n=1}^{\alpha_n (X \bar{X})^n + \log |\sum_{n=0}^{\lambda_n X^n|^2} \). 

\(^{11}\)\( \lambda_3 = 0 \) only vanishes for \( \phi_0 = 1 \), but is non-zero for other positions of the minima.
Both examples above correspond to inflection point inflation, rather than to inflation near a maximum or saddle point. This is unfortunate, as for inflection point inflation the spectral index is bounded to be \( n_s \lesssim 0.92 \), which is on the verge of being ruled out. We review this argument in appendix A.

The spectral index can be larger if the cubic term is absent or unnaturally small, as is the case for inflation at a maximum rather than an inflection point. Then the correction to the spectral index (A.4) is set by the quartic term in the Taylor expansion around the extremum, rather than by cubic term, with an upper bound \( n_s \lesssim 0.95 \). In our set-up this would require an extra tuning condition \( V_{\phi\phi\phi}\approx 0 \); without it we always find a saddle point.

The first solution above (3.8) has a supersymmetric Minkowski minimum. In this scenario the susy breaking observed today is not related to the susy breaking during inflation. The second solution (3.9), however, does end in a susy breaking minimum, and the gravitino mass today can be related to the inflationary scale. The gravitino mass is \( m_{3/2} \sim 10^{-7} \), see appendix A.

There is a huge difference between the two solutions when combined with other spectator fields. The first solution has a susy preserving vacuum in which \( W \to 0 \). Although at this exact point our description in terms of a Kähler function \( G \) breaks down, we can nevertheless
Figure 3: Stability plot of the spectator $z$-fields for a separable Kähler function $G = g(X, \bar{X}) + \tilde{g}(z, \bar{z})$. The trajectories for small field inflation are vertical lines, going upward (red) to infinity for solution (3.8) which has a susy preserving vacuum, and downward (black) to zero for (3.9) which has a susy breaking vacuum. Dashed lines indicate unstable trajectories. The position on the horizontal axis depends on the specifics of the spectator sector. Solution (3.8) always leads to an instability for $|x| > 1$.

describe the behavior of the potential as we approach this singular limit. We find that $b \propto V_0/W_0 \to \infty$, with $b$ defined in (2.21). This implies that if we draw the stability diagram for the simplified case of separable Kähler functions (2.23), see Fig. 3, this inflationary model corresponds to vertical trajectories going upwards to infinity.

The position on the horizontal axis given by $|x|$ depends on the specifics of the spectator sector, but it is clear that for all $|x| > 1$ one of the fields becomes tachyonic as the inflaton approaches its minimum, and the potential is unstable. Hence, solution (3.8) with a susy vacuum can only be combined with different fields if this extra sector has $|x| < 1$ (for several fields the eigenvalues of the $|x|^2$ matrix should all be less than unity). This puts enormous limitations on the spectator sector. For $|x| < 1$ the masses of the spectator fields vanish in the vacuum, as discussed at the end of section (2.4). However, in a subsequent susy breaking phase transition they may pick up a soft mass term.

This disastrous conclusion may be avoided by going to the most generic Kähler function for sgoldstino inflation (2.12) rather than sticking to the separable case (2.23); it is hard to make a general prediction as in the $b \to \infty$ limit also the other quantities $x, w, y$ in the mass matrix (2.22) may blow up.

In contrast, solution (3.9) has a susy breaking vacuum, and the parameter $b = V_0/W = 0$ vanishes in the minimum. The inflaton trajectory again corresponds to a vertical trajectory in the stability diagram, but now going downwards. Except for a small region near $|x| = 1$
there are no instabilities in the potential, and at least for the separable Kähler function (2.23) sgoldstino inflation can straightforwardly be combined with a spectator sector. In the region $|x| > 1$ the spectator fields are heavy in the vacuum and can be integrated out to get a low energy EFT. In the other limit $|x| < 1$ the spectator fields are of the same order as the gravitino mass (see the discussion at the end of section 2.4), and are relatively light.

Ref. [47] constructed a single-field potential with a maximum, rather than an inflection point, suitable for inflation. As remarked above, this set-up gives a spectral index in better agreement with the WMAP data than our inflection point model. The flat maximum was obtained by only allowing odd powers in the superpotential $W = \sum \lambda_{2n+1} \phi^{2n+1}$, and fine-tuning the lowest four $\lambda_{2n+1}$ parameters. In the absence of a symmetry that can guarantee this form of the superpotential, this model is more fine-tuned than the inflection point set-up, as it also requires tuning the even parameters $\lambda_{2n} = 0$; not only the $\eta$-parameter is tuned, but also $V_{\phi\phi\phi}$ at the extremum should vanish. We further note that in this set-up $W \to 0$ at the maximum, and thus $b \to \infty$. As discussed above, this puts very strong constraints on the spectator sector, and may make it harder to embed the inflaton model in a larger parent theory.

### 3.3.1 Recent proposals for small field sgoldstino inflation

In the recent literature there have been claims for small field sgoldstino inflation, with no or very little fine-tuning of the parameters in the potential. As argued in this paper, unless some symmetry principle is invoked, this is not possible as the slow roll parameters generically blow up in the small field limit. Indeed we find that these proposals do not work, although the devil is sometimes in the details.

Refs. [28, 29] propose a model of sgoldstino inflation in a single field set-up without tuning of parameters. To address the $\eta$ problem they add a logarithmic term to the Kähler potential

$$K = X \bar{X} + aX \bar{X} (X + \bar{X}) + b(X \bar{X})^2 + \ldots - 2 \ln(1 + X + \bar{X}),$$

$$W = fX + f_\eta M. \quad (3.10)$$

However, in the small field regime the logarithm can simply be expanded and does not alter the qualitative structure of the potential. It also does not enhance the symmetry.

Taking arbitrary parameters, except for the constraint that the minimum at the origin is stable and has zero cosmological constant, both the epsilon and eta-parameter exceed unity throughout the whole field space $|X| < 1$. Slow roll inflation cannot happen. In [28] it is actually claimed that $\epsilon < 1$, but what they calculate is $\epsilon_\theta = g^{\theta\theta} (V_\theta/V)^2$, where we again decomposed the field $X = (\phi + i\theta)/\sqrt{2}$ and $g_{ij}$ is the metric in field space. However, in a situation where the potential falls much steeper in the $\phi$-direction than in the $\theta$-direction, this is not the relevant slow roll parameter. Instead, one should use the more general multi-field generalization $\epsilon = g^{ij} V_i V_j / V^2$.

Ref. [29] shows inflationary trajectories with a large number of efolds $N > 60$. However, their trajectories are calculated in the — non-applicable — slow roll approximation. For all
initial points in field space proposed in [28, 29] we have solved the full two-dimensional field equations and the slow-roll approximations to them. In all cases the slow roll solutions wildly diverge from the full solutions, which can only give inflation for less than an efold, confirming once more that this setup does not provide a slow roll regime.

The only way to get inflation in the set-up of [28, 29] is to tune parameters near an extremum, along the lines of our example (3.7).

Ref. [50] proposes a model with an approximate R-symmetry:

\begin{equation}
K = S \bar{S} + \alpha (S \bar{S})^2, \quad W = W_0 + \mu^2 S - \frac{\lambda}{2(n+1)} S^{n+1}.
\end{equation}

The R-symmetry is only broken by \( W_0 \) and the higher order term in the superpotential. In the absence of the constant \( W_0 \), this assures that the potential is nearly flat near the origin, as there is no quadratic and cubic term in the superpotential. The potential is only lifted by the higher order quartic term in the Kähler (which must be tuned \(|\alpha| < 10^{-2}\)), and the 1-loop Coleman-Weinberg correction (which vanishes at the origin).

The set-up looks ideal for inflation. However, the \( n \) degenerate minima of the potential are all anti-de Sitter. To get a Minkowskian minimum after inflation, the constant \( W_0 \) has to be turned on. And although this is a small correction to the potential near the minimum, it is the dominant correction to the inflationary plateau at the origin, and gives rise to non-zero slow roll parameters \( \epsilon \) and \( \eta \). We find that the resulting potential is too steep to generate 60 e-folds of inflation (at most a single efold is possible). Moreover, the tilt of the classical potential (not including the one-loop contribution, which may change this) is such that, unless there is some initial velocity to make it roll uphill, the inflaton will not end in the minimum which is lifted to \( V = 0 \), but rather in one of the other AdS minima.

For concreteness, we can choose to uplift the AdS minimum at positive values of \( \phi \) to a Minkowski minimum (with \( X = (\phi + i\theta)/\sqrt{2} \)). Moreover, just as [50], we take the parameters in the superpotential real, which simplifies the analysis. The resulting potential will have a positive slope at the origin, as argued above, which kills inflation at the origin. However, there will always be a maximum of the potential in between the origin and the minimum. Can we do inflation here? Although the R-symmetry has lost all of its power here (as it can only help to keep the potential flat near the origin), this is still a possibility. However, although the epsilon parameter vanishes at the maximum, the \( \eta \) parameter naturally exceeds unity. Of course, \( \eta \) can be tuned, but as follows from our analysis in section 3.3, to satisfy all constraints one needs at least five parameters. The potential of [50] has not enough freedom to do so. Moreover, adding extra, say, higher order terms, and trying to tune \( \eta \), we find that the maximum morphs into an inflection point (although we did by no means an exhaustive study). This is as expected, there is no reason, no symmetry, which assures that when expanded around the extremum as in (A.1), the cubic term should vanish.
4. Conclusions

Inflationary models in supergravity, where the inflaton sits in a complex scalar superfield, necessarily involve a multifield analysis. Any extra fields present during inflation must be integrated out to give an effective single-field slow-roll dynamics that is consistent with the CMB. However, even very heavy fields can leave a detectable imprint in the spectrum of primordial perturbations, in particular through a reduction in the speed of sound of the adiabatic perturbations. The correct effective field theory for the adiabatic mode has a variable speed of sound that depends on the background trajectory. A necessary condition to recover the standard single-field slow roll description is that the trajectory should have no turns into the heavy directions. In this case, the speed of sound is unity, equal to the speed of light, and integrating out the extra fields gives the same effective action as truncating the heavy fields at their adiabatic minima.

In supersymmetric models there is an extra complication. One has to integrate out whole supermultiplets in order to obtain an effective supergravity description for the remaining superfields. This is only possible if the superfields that are being integrated out are in configurations that do not contribute to susy breaking.

Sgoldstino inflation naturally implements these two conditions. The full inflationary dynamics is confined to the sgoldstino plane. Putting the scalar components of all other superfields at their minima is a consistent truncation of the parent theory. This makes sgoldstino inflationary models extremely attractive, because of their simplicity and robustness.

We have analysed sgoldstino inflation scenarios exploiting the fact that the Kähler invariant function \( G = K + \log |W|^2 \) has a relatively simple form (2.12) which allows some aspects to be analysed in a model–independent way. We derived a necessary and sufficient condition on the Kähler function (2.22) for the stability of the susy-preserving sector, the spectator fields that are integrated out. Figure 1 shows the constraint for a separable Kähler function, in particular for hybrid F-term inflation (which is a well studied case of sgoldstino inflation).

In the case of small field sgoldstino inflation we were able to provide some viable fine-tuned examples around inflection points. The spectral index is rather low, on the verge of being ruled out by the CMB data. A higher spectral index would be possible with additional fine–tuning. Rather surprisingly, the inflationary model can only be straightforwardly combined with a spectator sector if the minimum after inflation breaks susy. In our inflation example with a susy preserving Minkowski vacuum the spectator sector is very constrained by the condition that there should be no tachyonic modes in the system. This is illustrated in Figure 3. These constraints would also affect the hilltop inflation examples in [47].

One of the motivations for this study was the interesting suggestion, put forward in [28], that a relatively simple supergravity model with a single chiral sgoldstino superfield could account for both inflation and susy breaking in the vacuum. Contrary to claims in [28, 29], our conclusion is that this minimal scenario is very tightly constrained and requires the usual level of fine–tuning that is expected on general grounds. Another interesting model was proposed in [50], in which the flatness of the inflationary plateau follows from an R-symmetry. However
we find that the R-symmetry breaking needed to obtain a Minkowski vacuum introduces an unacceptable tilt in the potential, and prevents inflation. It is possible that variations of this model may still work with some extra fine-tuning.

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A. Small spectral index for inflection point inflation

In this appendix we derive the spectral index and power spectrum for inflection point inflation, following the work of Refs. [48, 49]. To a very good approximation the inflationary observables only depend on the \( \eta \)-parameter at the extremum and on the number of efolds.

Expanding the potential around the inflection point gives:

\[
V = V_0 (1 + 1/2\eta_0 \phi^2 + C_3 \phi^3 + C_4 \phi^4 + ...),
\]

\( \text{(A.1)} \)

with \( \eta, C_3 < 0 \) so that the field rolls towards the minimum at positive \( \phi \) values. Inflation ends when the \( C_3 \) term becomes important, and \( \epsilon \approx 1 \), which occurs for field values \( \phi_f^2 \sim \sqrt{2}/(3|C_3|) \). We can calculate the number of efolds

\[
N \approx \int_{\phi_f}^{\phi_N} \frac{V}{V'} = \frac{1}{\eta} \log \left[ \frac{\phi}{3C_3 \phi + \eta} \right]_{\phi_f}^{\phi_N},
\]

\( \text{(A.2)} \)

where we used \( V \approx V_0 \) above. The above expression can be inverted to obtain the value of the inflaton field \( N \) efolds before the end of inflation \( \phi_N \):

\[
\phi_N = \left( \frac{e^{N \eta_0} \eta_0 / C_3}{-3(e^{N \eta_0} - 1) - \eta_0 / (\phi_f C_3)} \right) \approx \left( \frac{e^{N \eta_0} \eta_0}{-3C_3 (e^{N \eta_0} - 1)} \right),
\]

\( \text{(A.3)} \)

where in the second step we used \( \eta_0 / (\phi_f |C_3|) \ll 1 \). This is a good approximation as \( \eta_0 \ll 1 \) is fine-tuned, whereas \( C_3 \), and thus \( \phi_f \), is naturally of order one\(^\text{12}\). Note that in this limit, the number of efolds is independent of the end of inflation, as \( \phi_f \) has dropped out of the

\(^{12}\text{To be precise, } C_3 = \mathcal{O}(1) \text{ for } \phi_0 \sim 1. \text{ For minima at smaller field values generically } C_3 \text{ increases, as a sharper turnover of the potential is needed. We do not find valid solutions for minima for } \phi_0 \gg 1 \text{ much larger, as then other local minima at smaller field values appear.}\)
equation. As a result the inflationary observables are insensitive to the precise coefficients of the higher order terms in (A.1). The spectral index is

\[ n_s \approx 1 + 2\eta \approx 1 + 2\eta_0 + 12C_3\phi_N \approx 1 - 2\eta_0 \left( \frac{e^{\eta_0}N + 1}{e^{\eta_0}N - 1} \right), \]

(A.4)

where we used that \( \epsilon \ll \eta \). For \( N < 50 - 60 \) one finds \( n_s < 0.92 - 0.93 \) for the whole range of \( |\eta_0| \lesssim 10^{-2} \). The power spectrum is

\[ P_\zeta = \frac{V}{150\pi^2\epsilon} = \frac{3C_3^2e^{-4\eta_0}(e^{N\eta_0} - 1)^4V_0}{25\pi^2\eta_0^4} \]

(A.5)

with \( P_\zeta = 4 \times 10^{-10} \) measured by WMAP.

For the first example (3.8) in the text \( \eta_0 = 0 \) and \( C_3 = -2.39 \). For \( \eta_0 = 0 \), the expressions simplify to

\[ n_s - 1 = \frac{4}{N}, \quad P_\zeta = \frac{3C_3^2N^4V_0}{25\pi^2}, \quad (\text{for } \eta_0 = 0). \]

(A.6)

Choosing \( N = 50 \) this gives \( n_s = 0.92 \) and \( V_0 = 9 \times 10^{-16} \). The second example (3.9) has \( C_3 = -3.69 \), and gives the same spectral index and similar \( V_0 = 4 \times 10^{-16} \). The gravitino mass today is related to the inflationary scale via \( m_{3/2} = e^{K/2}|W|_{\text{min}} \sim 10^2\sqrt{V_0} \sim 10^{-7} \), far above the electroweak scale.

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