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An example of numerical simulation in causal set dynamics

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Abstract. The model of a discrete pregeometry on a microscopic scale is an x-graph. This is a directed acyclic graph. An outdegree and an indegree of each vertex are not more than 2. The sets of vertices and edges of x-graph are particular cases of causal sets. The sequential growth of a graph is an addition of new vertices one by one. A simple stochastic algorithm of sequential growth of x-graph are considered. It is based on a random walk at the x-graph. The particles in this model must be self-organized repetitive structures. We introduce the method of search of such repetitive structures. It is based on a discrete Fourier transformation. An example of numerical simulation is introduced.

1. Introduction
A causal set is a directed model of spacetime at a microscopic level. A causal set approach to quantum gravity was introduced by G. ’t Hooft [1] and J. Myrheim [2] in 1978 (see e.g. [3]). Usually we try to find an approximation of a causal set as a continuous spacetime. But if we consider a causal set as a model of reality, a causal set must describe matter. Particles must be some topological fiches of a causal set.

Our particular model is a directed dyadic acyclic graph. The dyadic graph means that each vertex has two incident incoming edges and two incident outgoing edges. This model was introduced by Devid Finkelstein [4] in 1988. The acyclic graph means that there is not a directed loop. Hereinafter only such graph is considered, and it is called an x-graph. The set of vertices and the set of edges of x-graph are causal sets.

The model of the universe is an infinite x-graph. But any observer can only actually know a finite number of facts. Then we consider only finite x-graphs. In a graph theory, by definition, an edge is a relation of two vertices. Consequently some vertexes of finite x-graph have less than four incident edges. These free valences are called external edges as external lines in Feynman diagrams. They are figured as edges that are incident only to one vertex. External edges are not real edges. They are a property of vertices. But often it is useful to consider external edges as edges. There are incoming and outgoing external edges. We can prove that in the x-graphs...
the number of incoming external edges is equal to the number of outgoing external edges [5, Lemma 5].

Each such graph is a model of a part of some process. In this model, the particles must be repetitive processes. We can make a lot of repetitive graphs by hand. But such repetitive structures must be a consequence of a dynamics. These structures must be self-organized structures.

What is a dynamics of this model? The task of any dynamics is to predict the future stages of the process or to reconstruct the past stages. We can reconstruct the x-graph step by step. The minimal part is a vertex. We can add new vertices only to external edges. This procedure is called an elementary extension. There are four types of elementary extensions [6]. There are two types of elementary extensions to outgoing external edges (Fig. 1 (a) and 1 (b)). This is a reconstruction of the future of the process. In this and following figures the x-graph \( G \) is represented by a rectangle because it can have an arbitrary structure. The edges that take part in the elementary extension are figured by bold arrows. First type is an elementary extension to two outgoing external edges (Fig. 1 (a)). Second type is an elementary extension to one outgoing external edge (Fig. 1 (b)). Similarly, there are two types of elementary extensions to incoming external edges (Fig. 1 (c) and 1 (d)). These elementary extensions reconstruct the past evolution of the process. Third type is an elementary extension to two incoming external edges (Fig. 1 (c)). Fourth type is an elementary extension to one incoming external edge (Fig. 1 (d)). We can prove that we can get every connected x-graph by a sequence of elementary extensions of these four types [5, Teorem 2].

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{example.png}
\caption{The types of elementary extensions: (a) the first type, (b) the second type, (c) the third type, and (d) the fourth type.}
\end{figure}
Figure 2. The random walk at the x-graph $G$ from the outgoing external edge number $i$ to the outgoing external edge number $j$ with the U-turn in the vertex number $V$, and the addition of a new vertex number $N + 1$.

By assumption, the dynamics of this model is a stochastic dynamics. We can only calculate probabilities of different variants of elementary extensions. In general case, these probabilities depend on the structure of the existed x-graph. In this model, causality is defined as the order of vertices and edges. But the causality has a real physical meaning only if the dynamics agrees with causality. The probability to add a new vertex to the future can only depend on the x-subgraph that precedes this vertex. Similarly, the probability to add a new vertex to the past can only depend on the x-subgraph that follows this vertex.

5 simple algorithms are introduced in [7]. The first algorithm is considered in [8, 9]. The second algorithm is considered in [10]. In this paper the third example is considered.

2. The algorithm

2.1. The random walk at the x-graph

We can describe the algorithm to calculate probabilities for this model as 3 steps. The first step is the choice of the elementary extension to the future or to the past. We assume the probability of this choice is $1/2$. The second step is the choice of the first external edge that takes part in the elementary extension. We assume the equiprobable choice. The probability of this choice is $1/n$, where $n$ is the number of outgoing (and incoming) external edges in the x-graph. The third step is the choice of second external edge that takes part in the elementary extension.

By assumption, the third step is based on a random walk at the x-graph (Fig. 2). Consider the elementary extension to the future. We have chosen the first outgoing external edge at the second step. Start the walk from this edge in the opposite direction of the edges. In each vertex we choose the U-turn or the continuation of the walk. We assume the probability of this choice is $1/2$. We make the first choice in the vertex that is incident to the first external edge. If we choose the continuation of the walk, we must choose one next edge. We assume the probability of this choice is $1/2$. If the next edge is an internal edge, we go to the next vertex and repeat this procedure. If the next edge is an incoming external edge, we must do the U-turn. After the U-turn we continue the walk in the direction of the edges without the second U-turn. In
each vertex, we choose the next edge with the probability $1/2$. The walk is finished in some outgoing external edge. By definition, this is the second outgoing external edge that take part in the elementary extension. If the first and the second outgoing external edges coincide, this is the elementary extension of the second type. The case of the elementary extension to the past is the same if we turn the direction of all edges in the x-graph.

In this algorithm, the area of the x-graph that is near to the first external edge has the main influence. The influence of the distant area exponentially decreases.

2.2. The probabilities of elementary extensions
Consider an analytic formulation of probabilities. Number outgoing external edges by lowercase Latin indices. Number incoming external edges by lowercase Greek indices. These Latin and Greek indices range from 1 to $n$, where $n$ is the number of outgoing (and incoming) external edges in the x-graph. Number vertices by capital Latin indices. These indices range from 1 to $N$, where $N$ is the number of vertices in the x-graph.

Introduce the probabilities $a_{i\alpha}$ and $a_{iV}$.

$$a_{i\alpha} = \frac{M_{i\alpha}}{\sum_{m=1}^{2^{k(m)}}}, \quad a_{iV} = \frac{M_{iV}}{\sum_{m=1}^{2^{r(m)}}}. \quad (1)$$

$a_{i\alpha}$ is a probability to reach the edge number $i$ by the random walk in the direction of the edges if we start from the edge number $\alpha$. $M_{i\alpha}$ is the number of directed paths from the incoming external edge number $\alpha$ to the outgoing external edge number $i$, and $k(m)$ is the number of vertices in the path number $m$. $a_{iV}$ is a probability to reach the edge number $i$ by the random walk in the direction of the edges if we start from the vertex number $V$. $M_{iV}$ is the number of directed paths from the vertex number $V$ to the outgoing external edge number $i$, and $r(m)$ is the number of vertices in the path number $m$ including the vertex number $V$.

Similarly, introduce the probabilities $b_{i\alpha}$ and $b_{iV}$.

$$b_{i\alpha} = \frac{M_{i\alpha}}{\sum_{m=1}^{2^{2k(m)}}}, \quad b_{iV} = \frac{M_{iV}}{\sum_{m=1}^{2^{2r(m)+1}}}. \quad (2)$$

$b_{i\alpha}$ is a probability to reach the edge number $\alpha$ by the random walk in the opposite direction of the edges if we start from the edge number $i$, and we can interrupt the walk in each vertex with probability $1/2$. $b_{iV}$ is a probability to reach the vertex number $V$ by the random walk in the opposite direction of the edges if we start from the edge number $i$, and we can interrupt the walk in each vertex with probability $1/2$.

Using (1) - (2) we get the probability $p_{ij}$ to choose the outgoing external edge number $j$ at the third step if we have chosen the outgoing external edge number $i$ at the second step.

$$p_{ij} = \sum_{V=1}^{N} b_{iV}a_{jV} + \sum_{\alpha=1}^{n} b_{i\alpha}a_{j\alpha}. \quad (3)$$

By definition, if the outgoing external edges number $i$ and $j$ coincides, this is the probability $p_{ii}$ to add a new vertex to one outgoing external edge number $i$. Multiplying the probabilities of all three steps we get the probability $P_{ii}$ of the elementary extension of the second type.

$$P_{ii} = \frac{1}{2n} \left( \sum_{V=1}^{N} b_{iV}a_{iV} + \sum_{\alpha=1}^{n} b_{i\alpha}a_{i\alpha} \right). \quad (4)$$

We get the same elementary extension of the first type if we choose the outgoing external edge number $i$ at the second step and the outgoing external edge number $j$ at the third step or if we
choose the outgoing external edge number $j$ at the second step and the outgoing external edge number $i$ at the third step. Summing these two possibilities and multiplying the probabilities of all three steps we get the probability $P_{ij}$ of the elementary extension of the first type.

$$P_{ij} = \frac{1}{2^n} \left( \sum_{V=1}^{N} b_V a_j V + \sum_{a=1}^{n} b_{ia} a_j a + \sum_{V=1}^{N} b_j a_i V + \sum_{a=1}^{n} b_{ja} a_i a \right).$$ (5)

Similarly, we get the probabilities of the third and fourth types.

$$P_{a\beta} = \frac{1}{2^n} \left( \sum_{V=1}^{N} b_{Va} a_{V\beta} + \sum_{i=1}^{n} b_{ia} a_{i\beta} + \sum_{V=1}^{N} b_{V\beta} a_{Va} + \sum_{i=1}^{n} b_{i\beta} a_{ia} \right),$$ (6)

$$P_{aa} = \frac{1}{2^n} \left( \sum_{V=1}^{N} b_{Va} a_{Va} + \sum_{i=1}^{n} b_{ia} a_{ia} \right).$$ (7)

The iterative procedure to calculate probabilities is considered in Appendix A.

3. The numerical simulation

3.1. The generation of x-graphs

We consider the numerical simulation of the sequential growth from 1 vertex during 2000 steps. In Fig. 3, the result of only the first 200 steps is figured because the whole x-graph is very tangled.

In the book [11], deterministic algorithms are considered for different discrete models. The deterministic algorithms for the causal sets is considered in [12]. Some algorithms generates repetitive structures. On the contrary we consider stochastic algorithms. Such algorithms cannot generate exact repetitive structures. The x-graph in Fig. 3 looks like random structure. But it can has some approximately repetitive properties. We need the methods to search such properties.

3.2. The analysis of structures

Suppose the considered dynamics generates repetitive structures. We can analyze such structures by a Fourier transform [13]. Consider some directed path $P$ from some incoming external edge number $a$ to some outgoing external edge number $i$. We consider $P$ as a set of edges. Number the edges of $P$ by the index $t$. If $P$ consists of $T+1$ edges, $t=0$ for the incoming external edge number $a$, and $t=T$ for the outgoing external edge number $i$. Consider the probability $a_{ta}$ to reach the edge number $t$ by the random walk in the direction of the edges if we start from the edge number $a$. In this case, $i = t$ in left-hand definition (1). Number the vertexes of $P$ by the index $t$ too. By definition, the vertex has the number $t$ if it has the incident incoming edge number $t$. Consider $a_{ta}$ as a function $a(t)$ of the number $t$ of vertex in the path.

In general case, $a(t)$ exponentially decreases. Consider $\log_2 a(t)$. This function describes properties of an Alexandrov set $A(0, t)$ of the vertices number 0 and $t$. By definition, $A(a, b) = \{c | a < c < b\}$ for the elements of any partially ordered set $\{a, b, \ldots\}$.

We found longest paths in our example of the x-graph with 2001 vertexes. These paths consist of 65 vertexes ($T = 65$). We chose one longest path and calculate $\log_2 a(t)$ at this path (Fig. 4). If there is only one directed path from the vertex number 0 to the vertex number $t$, $a(t) = a(t-1)/2$. In the considered path, such dependence is typical near both ends from the vertex number 0 to the vertex number 22 and from the vertex number 48 to the vertex number 65. If there are several directed paths from the vertex number 0 to the vertex number $t$, $a(t)$ is a sum over these paths. In this case, it is possible that $a(t) > a(t-1)$. We can see such dependence in the central part of the path. If the vertexes number $t-1$ and $t$ are connected by
Figure 3. The example of the x-graph that consists of 201 vertices. The initial vertex is figured as big vertex. For simplicity, the internal edges are shown as lines without arrows. The external edges are not shown. The up direction is a direction from the past to the future. Other spatial coordinates of the vertexes have not physical meaning.

a double edge, \( a(t) = a(t - 1) \). For example, we have a sequence of double edges between the vertices number 2 and 6.

Consider a discrete Fourier transform of \( \log_2 a(t) \).

\[
g(f) = \frac{1}{T} \sum_{t=0}^{T-1} \log_2 a(t) \exp(-i \frac{2\pi ft}{T}), \tag{8}
\]

If the directed path belongs to the repetitive structure the dominant frequency must exist. We get the power spectrum for the considered path (Fig. 5). The zero frequency is not figured because it has very high value: \( |g(0)|^2 = 5424.145 \). We get the high value for the zero frequency because \( \log_2 a(t) \) cannot be positive. Denote by \( \tau(f) \) the period of the mode with the frequency \( f \). By definition, \( \tau(f) = T/f \). The mode with the lowest frequency \( f = 1 \) has \( \tau(f) = 65 \) edges. This is the length of the path. The lowest possible period is equal to 2 edges. There are not
repetitive structures with short periods at the considered path. There is only one high maximum with \( \tau(2) = 32 \) edges. This is not a repetitive structure. This low frequency describes global properties of the x-graph. The second maximum with \( \tau(5) = 13 \) edges can be a consequence of some repetitive property of the x-graph.

4. Conclusion
In the book [11], a very big set of simple deterministic algorithms are investigated. Only few algorithms generate interesting self-organized structures. Probably only few stochastic algorithms generate interesting self-organized structures too. We must investigate a big set of simple stochastic algorithms to find such interesting cases. This is a task for further investigations.

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Appendix A. The iterative procedure for the probabilities
The number of direct calculations of all probabilities at each step of sequential growth is proportional to \( n \times N \). But if we use the probabilities at the previous step, the number of calculations is proportional to \( n \times n \).

Consider the probabilities \( p_{ij}, p_{\alpha\beta}, a_{i\alpha}, \) and \( b_{i\alpha} \) as elements of square matrices \( p_f, p_p, a, \) and \( b \) respectively. The size of these matrices is \( n \). Denote by \( G_{N,n} \) the x-graph that includes \( N \) vertex and \( n \) outgoing (and incoming) external edges. The x-graph \( G_{1,2} \) includes one vertex (\( N = 1, n = 2 \)). We have

\[
p_f(G_{1,2}) = p_p(G_{1,2}) = a(G_{1,2}) = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}, \quad (A.1)
\]

\[
b(G_{1,2}) = \begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}. \quad (A.2)
\]

If we know the probabilities for \( G_{N,n} \), we can calculate probabilities after any addition of a new vertex number \( N + 1 \).

First type is an elementary extension to the future (Fig. 1 (a)). Two outgoing external edges numbers \( i \) and \( j \) become internal edges. We get two free numbers of outgoing external edges: \( i \)
and \( j \). Two new outgoing external edges appear. Number these new outgoing external edges by \( i \) and \( j \). We have

\[
p_{ij}(G_{N+1,n}) = p_{ji}(G_{N+1,n}) = p_{ii}(G_{N+1,n}) = p_{jj}(G_{N+1,n}) = \frac{1}{4} + \frac{1}{8}(p_{ij}(G_{N,n}) + p_{ji}(G_{N,n}) + p_{ii}(G_{N,n}) + p_{jj}(G_{N,n})),
\]

\[
p_{si}(G_{N+1,n}) = p_{sj}(G_{N+1,n}) = \frac{1}{2}(p_{si}(G_{N,n}) + p_{sj}(G_{N,n})),
\]

\[
p_{is}(G_{N+1,n}) = p_{js}(G_{N+1,n}) = \frac{1}{4}(p_{is}(G_{N,n}) + p_{js}(G_{N,n})), \text{ where } s \neq i \text{ and } s \neq j.
\]

Two new outgoing external edges and one new incoming external edge appear. Number these new outgoing external edges by \( p \) and new incoming external edge by \( \alpha \). We get

\[
p_{\alpha i}(G_{N+1,n}) = p_{ji}(G_{N+1,n}) = p_{\alpha j}(G_{N+1,n}) = \frac{1}{4} + \frac{1}{8}(p_{\alpha i}(G_{N,n}) + p_{ji}(G_{N,n}) + p_{\alpha j}(G_{N,n})),
\]

\[
p_{r s}(G_{N+1,n}) = p_{s r}(G_{N,n}), \text{ where } r \neq i, r \neq j, s \neq i, \text{ and } s \neq j.
\]

Second type is an elementary extension to the future too (Fig. 1 (b)). One outgoing external edge number \( i \) becomes an internal edge. We get \( i \) as free number of an outgoing external edge. Two new outgoing external edges and one new incoming external edge appear. Number these new outgoing external edges by \( i \) and \( n + 1 \), and new incoming external edge by \( n + 1 \). We have new column number \( n + 1 \) and new row number \( n + 1 \) in \( p_f, p_p, a, \) and \( b \). We have

\[
p_{ii}(G_{N+1,n+1}) = p_{i(n+1)}(G_{N+1,n+1}) = \frac{1}{4} + \frac{1}{8}p_{ii}(G_{N,n}),
\]

\[
p_{si}(G_{N+1,n+1}) = p_{s(n+1)}(G_{N+1,n+1}) = \frac{1}{4}p_{si}(G_{N,n}),
\]

\[
p_{is}(G_{N+1,n+1}) = p_{i(n+1)s}(G_{N+1,n+1}) = \frac{1}{4}p_{is}(G_{N,n}), \text{ where } s \neq i \text{ and } s \neq n + 1.
\]

\[
p_{rs}(G_{N+1,n+1}) = p_{rs}(G_{N,n}), \text{ where } r \neq i, r \neq n + 1, s \neq i, \text{ and } s \neq n + 1.
\]

\[
p_{\alpha i}(G_{N+1,n+1}) = p_{\alpha j}(G_{N+1,n+1}) = \frac{1}{4} + \frac{1}{8}(p_{\alpha i}(G_{N,n}) + p_{ji}(G_{N,n}) + p_{\alpha j}(G_{N,n})),
\]

If \( \alpha = n + 1 \),

\[
p_{(n+1)i}(G_{N+1,n+1}) = \frac{1}{2},
\]

\[
a_{i\alpha}(G_{N+1,n+1}) = a_{(n+1)\alpha}(G_{N+1,n+1}) = \frac{1}{2}a_{i\alpha}(G_{N,n}).
\]
\[ b_{i\alpha}(G_{N+1,n+1}) = b_{(n+1)\alpha}(G_{N+1,n+1}) = \frac{1}{4} b_{i\alpha}(G_{N,n}), \text{ where } \alpha \neq n + 1. \quad (A.19) \]

\[ a_{i(n+1)}(G_{N+1,n+1}) = a_{(n+1)(n+1)}(G_{N+1,n+1}) = \frac{1}{2}, \quad (A.20) \]

\[ b_{i(n+1)}(G_{N+1,n+1}) = b_{(n+1)(n+1)}(G_{N+1,n+1}) = \frac{1}{4}, \quad (A.21) \]

\[ a_{s(n+1)}(G_{N+1,n+1}) = 0, \ b_{s(n+1)}(G_{N+1,n+1}) = 0, \text{ where } s \neq i \text{ and } s \neq n + 1. \quad (A.22) \]

If we interchange the Latin and Greek indices in (A.1) - (A.22), we get the equations for the elementary extensions of the third and fourth types.

Third type is an elementary extension to the past (Fig. 1 (c)). Two incoming external edges numbers \( \alpha \) and \( \beta \) become internal edges. We get two free numbers of incoming external edges: \( \alpha \) and \( \beta \). Two new incoming external edges appear. Number these new incoming external edges by \( \alpha \) and \( \beta \). We have

\[ p_{\alpha\beta}(G_{N+1,n}) = p_{\beta\alpha}(G_{N+1,n}) = p_{\alpha\alpha}(G_{N+1,n}) = p_{\beta\beta}(G_{N+1,n}) = \frac{1}{4} + \frac{1}{8}(p_{\alpha\beta}(G_{N,n}) + p_{\beta\alpha}(G_{N,n}) + p_{\alpha\alpha}(G_{N,n}) + p_{\beta\beta}(G_{N,n})), \quad (A.23) \]

\[ p_{\gamma\alpha}(G_{N+1,n}) = p_{\gamma\beta}(G_{N+1,n}) = \frac{1}{2}(p_{\gamma\alpha}(G_{N,n}) + p_{\gamma\beta}(G_{N,n})), \quad (A.24) \]

\[ p_{\alpha\gamma}(G_{N+1,n}) = p_{\beta\gamma}(G_{N+1,n}) = \frac{1}{4}(p_{\alpha\gamma}(G_{N,n}) + p_{\beta\gamma}(G_{N,n})), \text{ where } \gamma \neq \alpha \text{ and } \gamma \neq \beta. \quad (A.25) \]

\[ p_{\delta\gamma}(G_{N+1,n}) = p_{\delta\gamma}(G_{N,n}), \text{ where } \delta \neq \alpha, \delta \neq \beta, \gamma \neq \alpha, \text{ and } \gamma \neq \beta. \quad (A.26) \]

\[ p_{ij}(G_{N+1,n}) = p_{ij}(G_{N,n}) - \frac{1}{2}(b_{i\alpha}(G_{N,n})a_{j\beta}(G_{N,n})) + b_{i\beta}(G_{N,n})a_{j\alpha}(G_{N,n}) - b_{i\alpha}(G_{N,n})a_{j\beta}(G_{N,n}) - b_{i\beta}(G_{N,n})a_{j\alpha}(G_{N,n})). \quad (A.27) \]

\[ a_{i\alpha}(G_{N+1,n}) = a_{i\beta}(G_{N+1,n}) = \frac{1}{2}(a_{i\alpha}(G_{N,n}) + a_{i\beta}(G_{N,n})), \quad (A.28) \]

\[ b_{i\alpha}(G_{N+1,n}) = b_{i\beta}(G_{N+1,n}) = \frac{1}{4}(b_{i\alpha}(G_{N,n}) + b_{i\beta}(G_{N,n})). \quad (A.29) \]

Fourth type is an elementary extension to the past too (Fig. 1 (b)). One incoming external edge number \( \alpha \) becomes an internal edge. We get \( \alpha \) as free number of an incoming external edge. Two new incoming external edges and one new outgoing external edge appear. Number these new incoming external edges by \( \alpha \) and \( n + 1 \), and new outgoing external edge by \( n + 1 \). We have new column number \( n + 1 \) and new row number \( n + 1 \) in \( p_p, p_f, a, \) and \( b \). We have

\[ p_{\alpha\alpha}(G_{N+1,n+1}) = p_{\alpha(n+1)}(G_{N+1,n+1}) = \left( p_{(n+1)\alpha}(G_{N+1,n+1}) = p_{(n+1)(n+1)}(G_{N+1,n+1}) = \frac{1}{4} + \frac{1}{8} p_{\alpha\alpha}(G_{N,n}), \right. \]

\[ p_{\gamma\alpha}(G_{N+1,n+1}) = p_{\gamma(n+1)}(G_{N+1,n+1}) = \frac{1}{2} p_{\gamma\alpha}(G_{N,n}), \quad (A.30) \]

\[ p_{\alpha\gamma}(G_{N+1,n+1}) = p_{\alpha(n+1)\gamma}(G_{N+1,n+1}) = \frac{1}{4} p_{\alpha\gamma}(G_{N,n}), \text{ where } \gamma \neq \alpha \text{ and } \gamma \neq n + 1. \quad (A.31) \]

\[ p_{\delta\gamma}(G_{N+1,n+1}) = p_{\delta\gamma}(G_{N,n}), \text{ where } \delta \neq \alpha, \delta \neq n + 1, \gamma \neq \alpha, \text{ and } \gamma \neq n + 1. \quad (A.32) \]
$$p_{ij}(G_{N+1,n+1}) = p_{ij}(G_{N,n}) - \frac{1}{2} b_{ja}(G_{N,n}) a_{ja}(G_{N,n}) =$$

$$= p_{ij}(G_{N,n}) - 4 b_{ja}(G_{N+1,n+1}) a_{ja}(G_{N+1,n+1}). \quad (A.34)$$

$$p_{i(n+1)}(G_{N+1,n+1}) = \frac{1}{2} b_{ia}(G_{N,n}) = 2 b_{ia}(G_{N+1,n+1}), \quad (A.35)$$

$$p_{(n+1)i}(G_{N+1,n+1}) = \frac{1}{2} a_{ia}(G_{N,n}) = a_{ia}(G_{N+1,n+1}). \quad (A.36)$$

If \( i = n + 1, \)

$$p_{(n+1)(n+1)}(G_{N+1,n+1}) = \frac{1}{2} \quad (A.37)$$

$$a_{ia}(G_{N+1,n+1}) = a_{i(n+1)}(G_{N+1,n+1}) = \frac{1}{2} a_{ia}(G_{N,n}), \quad (A.38)$$

$$b_{ia}(G_{N+1,n+1}) = b_{i(n+1)}(G_{N+1,n+1}) = \frac{1}{4} b_{ia}(G_{N,n}), \text{ where } i \neq n + 1. \quad (A.39)$$

$$a_{(n+1)a}(G_{N+1,n+1}) = a_{(n+1)(n+1)}(G_{N+1,n+1}) = \frac{1}{2}, \quad (A.40)$$

$$b_{(n+1)a}(G_{N+1,n+1}) = b_{(n+1)(n+1)}(G_{N+1,n+1}) = \frac{1}{4}, \quad (A.41)$$

$$a_{(n+1)\gamma}(G_{N+1,n+1}) = 0, b_{(n+1)\gamma}(G_{N+1,n+1}) = 0, \text{ where } \gamma \neq \alpha \text{ and } \gamma \neq n + 1. \quad (A.42)$$

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