Coherent single electron spin control in a slanting Zeeman field

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We consider a single electron in a 1D quantum dot with a static slanting Zeeman field. By combining the spin and orbital degrees of freedom of the electron, an effective quantum two-level (qubit) system is defined. This pseudo-spin can be coherently manipulated by the voltage applied to the gate electrodes, without the need for an external time-dependent magnetic field or spin-orbit coupling. Single qubit rotations and the C-NOT operation can be realized. We estimated relaxation ($T_1$) and coherence ($T_2$) times, and the (tunable) quality factor. This scheme implies important experimental advantages for single electron spin control.

PACS numbers: 03.67.Lx, 85.30.Wx, 76.30.-v

Stimulated by electron-spin-based proposals for quantum computation [1], a growing interest has emerged in realizing the coherent manipulation of a single electron spin in a solid-state environment. The application of the electron’s spin – rather than its charge – as a quantum bit (qubit) is motivated by its potentially long coherence time in solids and the fact that it comprises a natural two-level system. Single electron spin resonance (SESR) plays a key role in realizing electron-spin-qubit rotation. Importantly, SESR is also the prime tool for determining the single electron spin coherence time $T_2$ in confined solid-state systems such as quantum dots (QDs). The induced Rabi oscillations can be read out via electron transport [2] or optically [3], giving an estimate for $T_2$. SESR was detected in paramagnetic defects in silicon [4] and for nitrogen vacancies in diamond [5], but not in semiconductor QDs so far. Realizing SESR in QDs is hard, not least because of the necessary high-frequency ($\sim 10$ GHz) magnetic field in a cryogenic ($\sim$100 mK) setup. Waveguides and microwave cavities as used in conventional ESR [6], cause serious heating, limiting the operation temperature to $\sim$1 K. On-going work in our group focuses on generating ac magnetic fields by an on-chip microscopic coil [7].

In this Letter, we propose a new SESR scheme that eliminates the need for an externally applied ac magnetic field, and with the potential of very high and tunable quality factors. An ac voltage is applied to let an electron in a QD oscillate under a static slanting Zeeman field. This effectively provides the electron spin with the necessary time-dependent magnetic field. Note the analogy with the Stern-Gerlach experiment, where the spin and orbital degrees of freedom are coupled by employing an inhomogenous magnetic field. The spatial oscillation of the electron within the QD involves the hybridization of orbital states, as schematically depicted in Fig. 1a for the case of the two lowest orbital states, $n = 1, 2$. Charge qubits based on double QDs [8] offer great tunability, but suffer from short coherence times ($\sim$1 ns) [9]. Spin qubits on the contrary, enable long coherence times ($\sim$1 ms) [10], but are much harder to control, as pointed out above. Here, we present a hybrid charge-spin system that is promising both in terms of tunability and coherence. Analogously, the combination of the flux and charge degrees of freedom has proved to be fruitful in superconducting qubits [11]. We stress that in our system spin-orbit (SO) coupling is not required, as opposed to earlier work on electron spin control based on $g$-tensor modulation [12], and on electric fields [13].

A possible realization of the system is presented in Fig. 2. A 1D conductor like a carbon nanotube or semiconductor nanowire is gated by ferromagnetic electrodes that define both the tunnel barriers of the QD and the slanting magnetic field. Alternatively, the slanting Zeeman field could be provided by a static inhomogeneity in the nuclear spin polarization or in the $g$-factor. The total magnetic field is given by $B = b_{SL} z \hat{\mathbf{z}} + (B_0 + b_{SL} x) \hat{\mathbf{z}}$, where $B_0$ is the external uniform magnetic field paral-
FIG. 2: Model of the 1D QD in a slanting Zeeman field. Ferromagnetic gate electrodes (dark grey) are located at either end of the dot and are magnetically polarized in the plus/minus x-direction, creating a magnetic field gradient $b_{SL}$. A uniform magnetic field $B_0$ is applied in the z-direction. The spin in the dot is controlled by applying an oscillating voltage $V_{ac}$ between the two gates.

lel to the z-axis and $b_{SL}$ is the z-direction gradient of the field parallel to the x-axis (the middle of the QD corresponds to $z = 0$). A true 1D system is assumed with an electron strongly confined in the $x$ and $y$ directions. Therefore, the inhomogeneous term along the z axis, $b_{SL} x\hat{z}$, can be eliminated (which is there to let $\mathbf{B}$ obey Maxwell’s equations).

The Hamiltonian is $H_0 = H_{00} + H_{0s}$, where $H_{00} = \frac{\hbar^2}{2m} V(z) - g\mu_B B_0 S_z$, and $H_{0s} = -g\mu_B b_{SL} S_z x$, with $\mathbf{S} = \frac{\hbar}{2} \sigma$, $\sigma$ the Pauli spin matrices, $q$ the effective g-factor, and $\mu_B$ the Bohr magneton \[14\]. $V(z)$ is the confinement potential of the QD with length $L$. The eigenvalues of $H_0$ are $\varepsilon_\sigma = \varepsilon_n + \frac{\pi}{2} \varepsilon_\sigma$, and the eigenfunctions $(z|n, \sigma) = \xi_\sigma \phi_n(z)$, where $n = 1, 2, \ldots$, $\sigma = \pm 1$ and $\xi_\sigma$ is the spinor. We define the Zeeman energy $\varepsilon_{0\sigma} = |g\mu_B B_0|$, which is assumed to be smaller than the orbital energy level separation $\Delta_{n,m} = \varepsilon_n - \varepsilon_m \ll U$, with $U$ the charging energy. The non-zero matrix elements of $H_{0s}$ are $(m, -\sigma|H_{0s}|n, \sigma) = \frac{1}{2} M_{m,n}$, with a coupling energy $M_{m,n} = E_{SL} \Upsilon_{m,n}$ where $E_{SL} = -g\mu_B b_{SL} L$ characterizing the strength of the slanting field, and the form factor $\Upsilon_{m,n} = \int dz \phi_m^*(z) \frac{\tilde{\xi}_\sigma}{\xi_\sigma} \phi_n(z)$.

By requiring the confining potential to have a mirror symmetry, i.e. $V(z) = V(-z)$, the diagonal coupling energy elements vanish, namely $M_{n,n} = 0$. We employ perturbation theory up to the second order in $E_{SL}$ and obtain the ground-state energy for a pseudo-spin $\sigma$, $G_\sigma = \varepsilon_1 + \frac{1}{2} \varepsilon_0 + \frac{1}{4} \sum_{l} M_{l,1}^2 (2l + 1 - \varepsilon_0)$, and its wavefunction $|G_\sigma\rangle = C_0^{(0)} |1, \sigma\rangle + \sum_{l>0} C_l^{(1)} |2l, -\sigma\rangle + \sum_{n>0} C_n^{(2)} |2n + 1, \sigma\rangle \[15\]$. Since we assumed $\Delta_{2,1} > \varepsilon_0$, the two lowest energy states $|G_+\rangle$ and $|G_-\rangle$ represent an energetically isolated qubit (see Fig. 1). We can disregard higher energy states, such as $|E_{\pm}\rangle$. For a rectangular confining potential, we find for the form factor $\Upsilon_{2n+1,2l} = -\frac{\pi}{\widetilde{\Delta}(1+2l)} (2l+1)^{2n} (-1)^{l+n}$, while for a harmonic potential $V(z) = \frac{m_0^2}{2} z^2$, we have $\Upsilon_{2n+1,2l} = \delta_{n+1,l} \sqrt{n + \frac{1}{2} + \delta_{n,l} \sqrt{n}}$, where we set $L = \sqrt{\frac{\hbar}{m_0 \varepsilon_0}}$. Therefore, $\Upsilon_{nm}$ is negligible for large $|n - m|$, and we only consider $M_{1,2}$ and $M_{2,3}$, which is exact for the harmonic potential. We define the effective Zeeman energy $\varepsilon_\sigma \equiv G_+ - G_- \sim \varepsilon_0 [1 - \frac{1}{2} \frac{M_1^2}{\Delta_{3,1} \Delta_{2,1}}]$. We consider the qubit rotation induced by an ac electric field. The time-dependent perturbation $H_1(t) = e V_{ac}(t) \cdot \hat{z}$ is applied to the system by introducing an oscillating signal $V_{ac}(t) = V_0 f(t)$ to the gate electrodes, as shown in Fig. 2 Since $H_1(t)$ is an odd function of $z$ and is independent of spin, only the off-diagonal matrix elements of $H_1$ remain, $\langle G_\sigma| H_1 |G_\sigma\rangle \sim \langle C_1^{(-1)} + C_1^{(1)} \rangle e V(t) \Upsilon_{21} \equiv \frac{1}{2} \varepsilon_x f(t)$, and the diagonal elements are zero for any order. Therefore, the effective Hamiltonian of our qubit is $H_z = \frac{1}{2} \varepsilon_z \sigma_z + \frac{1}{2} \varepsilon_x f(t) \sigma_x$, which is formally equivalent to the conventional ESR Hamiltonian \[16\]. For a sinusoidal perturbation $f(t) = \cos \omega t$ at resonance $(\hbar \omega = \varepsilon_z)$, the time required for the $\pi$-operation, i.e. $|G_+\rangle \rightarrow |G_-\rangle$, is given by

$$t_\pi = \frac{2 \pi \hbar}{\varepsilon_x} \approx \frac{\pi \Delta_{2,1}}{4 |V_{ac}|} \left( 1 - \frac{\varepsilon_x^2}{\Delta_{2,1}^2} \right).$$

(1)

Since we hybridize the spin and orbital degrees of freedom, orbital relaxation processes harm the spin coherence. We find that acoustic phonon scattering is the dominant relaxation mechanism for the energies relevant in our system. The electron-phonon scattering Hamiltonian is

$$H_{e-ph} = \sum_q \lambda_q (e^{iqz} b^+_q + H.C.)$$

$$= \sum_q (\Lambda_q \sigma_x + \Lambda_q \sigma_z) b^+_q + H.C.$$

(2)

$$= \sum_q \frac{1}{2} g\mu_B (B_x(t) \sigma_x + B_z(t) \sigma_z),$$

(3)

where $q$ is the phonon wave number, $\lambda_q$ is the coupling constant and $b^+_q$ is the phonon creation operator. In Eq. 2 we project $H_{e-ph}$ to the qubit base, where the effective coupling constants are $\Lambda_0 = \lambda_q \langle |e^{iqz}|^2 \rangle (C_t^{(1)} + C_t^{(-1)})$ and $\Lambda_q = \lambda_q \langle |e^{iqz}|^2 \rangle (C_t^{(2)} - C_t^{(-2)})$. $B_r(t) (r = x, y, z)$ represents the fluctuating magnetic field caused by phonons. A standard Born-Markov approximation \[16\] gives relaxation ($T_1$) and coherence ($T_2$) times as follows,

$$T_1^{-1} = k_{xx}(\varepsilon) + k_{yy}(\varepsilon),$$

(4)

$$T_2^{-1}_{photon} = (2T_1)^{-1} + k_{zz}(0).$$

(5)

where $k_{rr}(\hbar \omega) \equiv \frac{1}{2} (g\mu_B)^2 \int dt \cos(\omega t) \langle B_r(t) B_r(t + \tau) \rangle_0$ with $(\cdots)_0$ representing the thermal average. The dephasing term $k_{zz}(0)$ related to $B_z(t) \sigma_z$ is negligible since $\frac{\Lambda_q^2}{\Delta_{1,1}^2} \approx \frac{\varepsilon_z^2 M_1^2}{\Delta_{3,1} \Delta_{2,1}} \ll 1$ and is shown to be absent in the zero frequency limit. This situation is similar to that of
a spin qubit with SO interaction \[17, 18\]. The relaxation rate \( T_1^{-1} \) is then

\[
T_1^{-1} = \frac{2\pi}{\hbar} \sum_q |\Delta_q|^2 \delta(\varepsilon_z - \hbar\omega_q) \coth \frac{\beta\varepsilon_z}{2}
\]

\[
\sim \left( \frac{E_{SL} T_{1,2} \Delta_{2,1}}{\Delta_{2,1}^2 - \frac{e^2}{\mu_0} \omega_p(e_z)} \right) \frac{1}{\tau_p(e_z)},
\]

(6)

with \( \hbar\omega_q \) the phonon energy, \( \beta = 1/(k_B T) \), and the relaxation time \( \tau_p(E) \) defined in analogy with Fermi's Golden rule for a transition from level 2 to 1 with energy transfer \( E \). The coherence time is obtained by

\[
T_{2\text{total}}^{-1} = T_{2\text{phonon}}^{-1} + T_{2\text{spin}}^{-1},
\]

(7)

where we included the generic spin coherence time \( T_{2\text{spin}} \), which is the ‘pure’ coherence time of the electron spin in the QD. The upper bound of the quality factor of the one-qubit operation is characterized by \( 2T_1 \) divided by \( t_\pi \) of Eq. \[14\],

\[
Q \sim \frac{2\Delta_{2,1} \tau_p(e_z) \varepsilon V_0}{\pi \hbar |E_{SL}| (1 - \frac{e^2}{\Delta_{2,1}^2})}.
\]

(8)

Importantly, the quality factor is tunable by controlling the amplitude of the ac voltage modulation, \( V_0 \).

For the practical implementation of our scheme, 1D systems with small electron-phonon coupling and/or weak SO coupling are favorable. Single wall carbon nanotube QDs are very suitable, because of the absence of piezoelectricity and the weak deformation potential coupling \[19\]. SO coupling does not play a role either. QDs of semiconductor (e.g. SiGe) nanowires are also good candidates, since 1D phonons couple weakly to the electron orbitals. Here we estimate \( T_1 \) and \( Q \) of GaAs 1D QDs embedded in bulk AlGaAs. Since the phonon character is 3D in this system, the results are worse than for the more suitable systems given above. Figure 3a shows the low-temperature (\( \beta\varepsilon_z \gg 1 \)) relaxation rate caused by bulk acoustic phonons in a QD with longitudinal parabolic confinement \( \hbar\omega_0 = 1 \text{ meV} \) and a transversal confinement of 10 meV. \( E_{SL} = 1 \text{ meV} \), corresponding to \( b_{SL} = 1.16 \text{ T}/\mu\text{m} \), which can be realized with a ferromagnetic material \[20\]. Of the three acoustic phonon scattering mechanisms, transversal piezoelectric scattering is dominant for low \( B_0 \), where \( T_1 \) is of the order of 10 ns. For comparison, the typical relaxation time from higher levels, i.e. \( |E_\pm\rangle \to |G_\pm\rangle \) in Fig. 1, is much shorter, \( \sim 10 \text{ ns} \), dominated by the deformation potential scattering. Note that the contribution of SO interaction (Dresselhaus coupling) is very small (\( T_{2SO} \sim 10^5 \text{ s} \)) in 1D QDs, in contrast to disk-shaped dots as examined in Ref. \[17\]. Figure 3b shows \( Q \) for various confinement potentials (dot length \( L \)) with \( b_{SL} = 1.16 \text{ T}/\mu\text{m} \), \( V_0 = 10 \mu\text{V} \) \[21, 22\], and \( t_p \sim 400 \text{ ns} \) for \( \hbar\omega_0 = 1 \text{ meV} \). A quality factor \( Q \gg 10^4 \) is often used as a threshold for viable quantum computation \[23\].

We study the time evolution of the density matrix of the four levels \( |G_\pm\rangle, |E_\pm\rangle \), including \( V_{ac}(t) \) and phonon scattering. Near the resonant condition \( \hbar\omega = \varepsilon_z, |E_\pm\rangle \) are almost empty and do not contribute to the qubit dynamics at all. \( T_{2\text{total}} \) can be evaluated using a time-resolved measurement of the Rabi oscillation (see e.g. \[24\]). After rotating the qubit over a certain angle, a projection measurement is done into \( |G_-\rangle \) or \( |G_+\rangle \) using a single-shot read out scheme based on spin-to-charge conversion \[24\]. Both energy selectivity and spin selectivity of tunneling out of the QD are applicable for our pseudo-spin qubit system. The read-out error introduced by level mixing to \( n = 2 \) by the slanting field is negligible, namely of the order of \( (C_{1e})^2 \sim 10^{-6} \). Importantly, the pure electron spin coherence time \( T_{2\text{spin}} \) can be evaluated by extrapolating the \( T_{2\text{total}} \)-dependence on \( E_{SL} \) where \( T_{2\text{phonon}}^{-1} \propto E_{SL}^2 \) (see Eq. \[10\]), as shown in the inset to Fig. 3a.

For a universal set of quantum gates a two-qubit gate is required. Here we present a realization of a two-qubit gate based on two coupled dots in series \[8\]. Although
it has been pointed out that an inhomogeneous magnetic field introduces swap errors [25, 26], we show that correct swap operation is possible in our system. The two-qubit Hamiltonian is \( H = \sum_{i=L,R} H_{\text{oi}} + H_T + H_V \), where \( H_{\text{oi}} \) is the single-dot Hamiltonian \( i = L, R \) (ac field is off, \( \varepsilon_x = 0 \)) , \( H_T \) represents the tunneling between the dots, and \( H_V \) represents the inter-dot interaction \( V \). By projecting the Hamiltonian onto the qubits, we find

\[
H_{\text{oi}} = \varepsilon_i\frac{2}{\sigma} \sum \sigma c_{i\sigma}^\dagger c_{i\sigma} + U n_{i\uparrow} n_{i\downarrow},
\]

\[
H_T = \sum_{\sigma} \left[ t_{\sigma} c_{L\sigma}^\dagger c_{R\sigma} + \sigma \right] c_{L\sigma} c_{R\sigma} + H.c.,
\]

\[
H_V = \sum_{\sigma\sigma'} n_{L\sigma} n_{R\sigma'}
\]

where \( c_{i\sigma} \) annihilates an electron of pseudo-spin \( \sigma \) in dot \( i \). A spin-dependent tunneling term \( t_{\sigma} \) and a tunneling term with spin flip \( s_{\sigma} \) emerge, which are defined by \( t_{\sigma} = C_{\sigma}^{(1)L} t_{12} + C_{\sigma}^{(1)R} t_{21} + 2 C_{\sigma}^{(2)} t_{13} \), \( s_{\sigma} = (C^{(1)L}_{\sigma} + C^{(1)R}_{\sigma}) t_{12} \), where \( t_{nm} \) represents the tunneling amplitude from level \( n \) in dot \( L \) to level \( m \) in dot \( R \). The relevant lowest four eigenenergies and their eigenfunctions are obtained by the effective exchange Hamiltonian using local spin operators:

\[
H_{\text{EX}} = J_{||}(S_{Lz} S_{Rz} + J_{\perp}(S_{Lx} S_{Rx} + S_{Ly} S_{Ry}) + \varepsilon_z(S_{Lz} + S_{Rz}),
\]

where \( J_{||} = \frac{2(t_{z}^2 + t_{z}^2)}{U - V} - \frac{2s^2(U - V)}{(U - V)^2} \), \( J_{\perp} = \frac{4t_{z}^2}{U - V} \), and \( \varepsilon_z = \varepsilon_z(1 - \frac{s^2}{(U - V)^2}) \). It is well known that the SO interaction makes the exchange Hamiltonian anisotropic [27]. In contrast to the SO case, where the anti-symmetric term dominates, the dominant anisotropic correction of \( H_{\text{EX}} \) in a slanting field is the symmetric term. Nevertheless, C-NOT operation can be accomplished by this anisotropic exchange Hamiltonian simply by replacing \( J \) of the Heisenberg Hamiltonian by \( J_{||} \), and single-qubit operation (SESR) with replacing \( \varepsilon_{0z} \) as \( \varepsilon_{0z} \), as is shown in Refs [26, 27].

In conclusion, we propose a viable qubit based on combining the orbital and spin degrees of freedom of an electron in a QD placed in a slanting Zeeman field. Both single-qubit rotation and the C-NOT operation are demonstrated. This qubit is easier to manipulate than a spin qubit and has a better quality factor than a charge qubit. The concept is general and can be applied to a range of systems such as single wall carbon nanotubes, GaAs, and SiGe QDs. This scheme also allows for the measurement of the intrinsic single electron spin coherence time.

We thank Y. Avislai, A. Khaetskii and M. Pioro-Ladriere for discussions. We acknowledge financial support from DARPA grant number DAAD19-01-1-0659 of the QuIST program and SORST-JST.

[1] D. Loss and D.P. DiVincenzo, Phys. Rev. A 57 120 (1998); R. Vrijen et al., Phys. Rev. A 62 012306 (2000).
[2] H.-A. Engel and D. Loss, Phys. Rev. Lett. 86 4648 (2001); H.-A. Engel and D. Loss, Phys. Rev. B 65 195321 (2002).
[3] O. Gywat et al., Phys. Rev. B 69 205303 (2004); O. Gywat, H.-A. Engel and D. Loss, cond-mat/0408451 (2004).
[4] M. Xiao et al., Nature (London) 430 435 (2004).
[5] F. Jelezko et al., Phys. Rev. Lett. 92 076401 (2004).
[6] M. Dobers, K. von Klitzing, J. Schneider, G. Weimann and K. Ploog, Phys. Rev. Lett. 61 1650 (1988).
[7] T. Koder, et al., Physica E 22 518 (2004); T. Koder et al., Fabrication and characterization of quantum dot single electron spin resonance devices, World Scientific Publishing (in press).
[8] W.G. van der Wiel et al., Rev. Mod. Phys. 75 1 (2003).
[9] T. Hayashi et al., Phys. Rev. Lett. 91 226804 (2003).
[10] J.R. Petta et al., Science Express [DOI: 10.1126/science.1116955] (2005).
[11] D. Vion et al., Science 296 886 (2002).
[12] Y. Kato et al., Science 299 1201 (2003).
[13] E.I. Rashba and Al. L. Efros, Phys. Rev. Lett. 91 126405 (2003).
[14] The equivalent Hamiltonian is obtained for a 1D QD with a relatively weak Rashba SO interaction. L. S. Levitov and E. I. Rashba, Phys. Rev. B 67 115324 (2003). Namely, by taking the limit \( \xi \to 0 \) in Eq.(2) of this reference.
[15] The explicit form of the coefficients read \( C_{\sigma}^{(0)} = 1 - \frac{s}{2} \sum_{n} \frac{M_{2n}^{(2)} + \epsilon_{n\sigma}}{M_{2n}^{(2)} + \epsilon_{n\sigma}^2} \), \( C_{\sigma}^{(1)} = -\frac{1}{2} \frac{M_{2n}^{(2)}}{M_{2n}^{(2)} + \epsilon_{n\sigma}^2} \), and \( C_{\sigma}^{(2)} = \frac{s}{2} \sum \frac{M_{2n}^{(2)} + \epsilon_{n\sigma}^2}{M_{2n}^{(2)} + \epsilon_{n\sigma}^2} \).
[16] In conclusion, we propose a viable qubit based on combining the orbital and spin degrees of freedom of an electron in a QD placed in a slanting Zeeman field. The single-qubit rotation and the C-NOT operation is general and can be applied to a range of systems such as single wall carbon nanotubes, GaAs, and SiGe QDs. This scheme also allows for the measurement of the intrinsic single electron spin coherence time.
[17] A.V. Khaetskii and Yu.V. Nazarov, Phys. Rev. B 64 251361 (2001).
[18] V.N. Golovach, A. Khaetskii, and D. Loss, Phys. Rev. Lett. 93 16601 (2004).
[19] H. Suzuura and T. Ando, Phys. Rev. B 65 235412 (2002).
[20] J. Wrobel et al., Phys. Rev. Lett. 93 246601 (2004).
[21] W.G. van der Wiel, et al., Physica B 272 31 (1999).
[22] W.G. van der Wiel et al., Photon Assisted Tunneling in Quantum Dots in Strongly Correlated Fermions and Bosons in Low-Dimensional Disordered Systems, eds. I.V. Lerner et al., pp. 43-68, Kluwer (2002).
[23] A.M. Steane, quant-ph/0207119 and references therein.
[24] J.M. Elzerman, et al., Nature 430 431 (2004); R. Hanson et al., Phys. Rev. Lett. 94 196802 (2005).
[25] R. de Sousa, X. Hu, and S. Das Sarma, Phys. Rev. B 64 42307 (2001).
[26] X. Hu, R. de Sousa, and S. Das Sarma, Phys. Rev. Lett., 86 918 (2001).
[27] G. Burkard and D. Loss, Phys. Rev. Lett., 88 47903 (2002).