Intermittent dynamics, strong correlations, and bit-error-rate in multichannel optical fiber communication systems

Avner Peleg

Arizona Center for Mathematical Sciences,
University of Arizona, Tucson, Arizona 85721, USA

Abstract

We investigate the effects of delayed Raman response on pulse dynamics in massive multichannel optical fiber communication systems. Taking into account the stochastic nature of pulse sequences in different frequency channels and the Raman induced energy exchange in pulse collisions we show that the pulse parameters exhibit intermittent dynamic behavior, and that the pulse amplitudes exhibit relatively strong and long-range correlations. Moreover, we find that the Raman-induced cross frequency shift is the main intermittency-related mechanism leading to bit pattern deterioration and evaluate the bit-error-rate of the system.

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The dynamic evolution of coherent patterns in the presence of noise and nonlinearities is a rich and complex subject that is of major importance in many areas of physics. Fiber optics communication systems, which employ optical pulses to represent bits of information, serve as an excellent example for systems where noise and nonlinear effects can have an important role in the dynamics of coherent patterns. It is known that the dynamics of the parameters characterizing the pulses in optical fiber transmission systems can exhibit non-Gaussian statistics. Yet, it is commonly believed that the statistics of the optical pulse parameters is very different from the intermittent statistics encountered in strong nonlinear phenomena such as turbulence and chaotic flow. (For a review of intermittency in the context of turbulent flow, see Ref. [6]). In this Letter we present results that contrast this common belief and show that the parameters of optical pulses can exhibit intermittent dynamic behavior in massive multichannel transmission systems. Furthermore, we demonstrate that this intermittent dynamics can have important practical consequences by leading to relatively large values of the bit-error-rate (BER) characterizing the system performance.

We consider conventional optical solitons as an example for the pulses carrying the information and focus attention on the effects of delayed Raman response on the propagation. The main effect of delayed Raman response on single-soliton propagation is the self frequency shift, which is due to energy transfer from higher frequency components of the pulse to its lower frequency components. The main effect of a single two-soliton collision in the presence of delayed Raman response is an energy exchange between the colliding pulses (Raman induced cross talk), which leads to a change in their amplitudes. In addition, the frequencies of the two solitons also change as a result of the collision (Raman induced cross frequency shift).

The combined effect of Raman scattering and randomness of soliton sequences in multichannel transmission systems was considered in Refs. [14, 15], where it was found that the soliton amplitude has a lognormal distribution. It was also shown that the distribution of the cross frequency shift in a two-channel system is lognormal and that the first two normalized moments of the self frequency shift grow exponentially with increasing distance. Even though these studies implied intermittent dynamic behavior for the soliton amplitude, it was not clear whether the other soliton parameters exhibit similar dynamic behavior in a general multichannel system. Moreover, the effect of the amplitude intermittent behavior on the
main measure of system performance, the BER, was not addressed at all. In this Letter we study in detail the intermittent character of soliton dynamics. We show that the normalized $n$th moments of the self and cross frequency shifts are exponentially increasing with both propagation distance $z$ and $n^2$, i.e., the self and cross frequency shifts exhibit intermittent dynamic behavior. We find that the $n$th order equal-$z$ amplitude correlation functions have similar dependence on $z$ and $n^2$ with a typical correlation time which is much larger than the time slot width, thus showing that the system exhibits relatively strong and long-range correlations. Furthermore, we find that the cross-frequency shift is the main intermittency-related mechanism leading to an increase of the BER, and calculate the $z$-dependence of the BER for different channels.

Propagation of pulses of light through an optical fiber in the presence of delayed Raman response is described by the following perturbed nonlinear Schrödinger equation [1]:

$$i \partial_z \Psi + \partial^2_t \Psi + 2|\Psi|^2 \Psi = -\epsilon_R \Psi \partial_t |\Psi|^2. \tag{1}$$

In Eq. (1) $\Psi$ is the envelope of the electric field, $z$ is the position along the fiber, $t$ is time in the retarded reference frame and the term $-\epsilon_R \Psi \partial_t |\Psi|^2$ accounts for the effect of delayed Raman response [16]. When $\epsilon_R = 0$, the single-soliton solution of Eq. (1) in a given frequency channel $\beta$ is described by $\Psi_\beta(t, z) = \eta_\beta \exp(i \chi_\beta) \cosh^{-1}(x_\beta)$, where $x_\beta = \eta_\beta (t - y_\beta - 2\beta z)$ and $\chi_\beta = \alpha_\beta + \beta(t - y_\beta) + (\eta^2_\beta - \beta^2) z$, and $\alpha_\beta, \eta_\beta$ and $y_\beta$ are the soliton phase, amplitude, and position, respectively.

Consider a single collision between a soliton in the reference channel ($\beta = 0$) and a soliton in the $\beta$ channel. We assume that $|\beta| \gg 1$, $\epsilon_R \ll 1$ and $\epsilon_R \ll 1/|\beta|$, which is the typical situation in current multichannel transmission systems. Focusing attention on changes in the parameters of the reference channel soliton, for example, one finds that the most important effect of the collision is an $O(\epsilon_R)$ change in the soliton amplitude [10, 12]:

$$\Delta \eta_0 = 2\eta_0 \eta_\beta \text{sgn}(\beta) \epsilon_R. \tag{2}$$

The main effect of the collision in order $\epsilon_R/\beta$ is a frequency shift given by [12]: $\Delta \beta_0 = -(8\eta^2_0 \eta_\beta \epsilon_R)/(3|\beta|)$. Since $\epsilon_R \ll 1/|\beta| \ll 1$, we neglect effects of order $\epsilon^2_R$ and higher.

We now describe propagation of a soliton in the reference channel undergoing many collisions with solitons from all other frequency channels in a system with $2N + 1$ channels. We assume that the amplitudes of the latter solitons are all equal to 1. The stochastic
character of soliton sequences in different channels is taken into account by defining discrete random variables $\zeta_{ij}$, which describe the occupation state of the $j$th time slot in the $i$th channel: $\zeta_{ij} = 1$ if the slot is occupied and 0 otherwise. Therefore, the $n$th moment of $\zeta_{ij}$ satisfies $\langle \zeta^n_{ij} \rangle = s$, where $s$ is the average fraction of occupied time slots, assumed to be the same in all channels. We also assume that the occupation states of different time slots are uncorrelated: $\langle \zeta_{ij}\zeta_{i'j'} \rangle = s^2$ if $i \neq i'$ and $j \neq j'$. We denote by $\Delta \beta$ the frequency difference between neighboring channels and by $T$ the time slot width. We assume that the change in $\eta_0$ over the interval $\Delta z_c^{(1)} = T/(2\Delta \beta)$, traveled by the reference channel soliton while passing two successive time slots in the nearby channels, is small. Using Eq. (2) and summing over all collisions occurring at $\Delta z_c^{(1)}$ we arrive at

$$\frac{\Delta \eta_0}{\Delta z_c^{(1)}} \bigg|_{z=z_k} = \frac{2\epsilon_R\eta_0(z_{k-1})}{\Delta z_c^{(1)}} \sum_{i \neq 0} sgn(\beta_i) \sum_{j=(k-1)i+1} \zeta_{ij},$$

(3)

where $k - 1$ and $k$ are the indexes of the two successive time slots in the $i = -1$ channel, $z_k - z_{k-1} = \Delta z_c^{(1)}$, and the outside sum is from $-N$ to $N$. We decompose the disorder $\zeta_{ij}$ into an average part and a fluctuating part: $\zeta_{ij} = s + \tilde{\zeta}_{ij}$, where $\langle \tilde{\zeta}_{ij} \rangle = 0$, $\langle \zeta_{ij}\zeta_{i'j'} \rangle = s(1-s)\delta_{ii'}\delta_{jj'}$, and $\delta_{ii'}$ is the Kronecker delta function. Substituting $\zeta_{ij} = s + \tilde{\zeta}_{ij}$ into Eq. (3) and going to the continuum limit we obtain

$$\frac{1}{\eta_0} \frac{d\eta_0}{dz} = \frac{4s\epsilon_R\Delta \beta}{T} \sum_{i \neq 0} sgn(\beta_i)|i| + 2\epsilon_R\zeta^{(0)}(z; N),$$

(4)

where the continuous disorder field $\xi^{(0)}(z; N)$ is

$$\xi^{(0)}(z; N) = \frac{1}{\Delta z_c^{(1)}} \sum_{i \neq 0} sgn(\beta_i) \sum_{j=(k-1)i+1} \tilde{\zeta}_{ij}.$$  

(5)

Using Eq. (5) one can show that $\langle \xi^{(0)}(z; N) \rangle = 0$ and $\langle \xi^{(0)}(z; N)\xi^{(0)}(z'; N) \rangle = D_N^{(0)} \delta(z - z')$, where $D_N^{(0)} = N(N + 1)D_2$, $D_2 = 2\Delta \beta s(1-s)T^{-1}$, and $\delta(z)$ is the Dirac delta function. Notice that the first term on the right hand side of Eq. (3) is zero due to symmetry. Integrating both sides of Eq. (3) over $z$ we obtain

$$\eta_0(z) = \exp \left[ 2\epsilon_R x^{(0)}(z; N) \right],$$

(6)

where $x^{(0)}(z; N) = \int_0^z dz' \xi^{(0)}(z'; N)$ and we assumed $\eta_0(0) = 1$. According to the central limit theorem $x^{(0)}(z; N)$ is a Gaussian random variable with $\langle x^{(0)}(z; N) \rangle = 0$ and
\[ \langle x^{(0)2}(z; N) \rangle = D^{(0)}_N z. \] As a result, the distribution of the soliton amplitude is lognormal

\[ F(\eta_0) = (\pi D^{(0)}_N)^{-1/2} \eta_0^{-1} \exp \left[ -\ln^2 (\eta_0) / D^{(0)}_N \right], \] (7)

where \( D^{(0)}_N = 8D^{(0)}_N \epsilon_R^2 z. \) The normalized \( n \text{th} \) moment of \( F(\eta_0) \) satisfies \( \langle \eta_0^n(z) \rangle / \langle \eta_0(z) \rangle^n = \exp \left[ 2n(n-1)D^{(0)}_N \epsilon_R^2 z \right], \) from which it follows that the amplitude dynamics is intermittent.

For a soliton in the \( i \text{th} \) channel, the amplitude dynamics is given by

\[ \frac{1}{\eta_i} \frac{d\eta_i}{dz} = \frac{4s\epsilon_R \Delta \beta}{T} \sum_{i' \neq i} \text{sgn}(\beta_{i'} - \beta_i)|i' - i| + 2s\epsilon_R \xi^{(i)}(z; N), \] (8)

where

\[ \xi^{(i)}(z_k; N) = \frac{1}{\Delta z^{(1)}} \sum_{i' \neq i} \text{sgn}(\beta_{i'} - \beta_i) \sum_{j=(k-1)(i'-i)}^{k(i'-i)} \xi^{(i')}. \] (9)

From Eq. (8) it follows that \( \langle \xi^{(i)}(z; N) \rangle = 0 \) and \( \langle \xi^{(i)}(z; N) \xi^{(i)}(z'; N) \rangle = D^{(i)}_N \delta(z - z'), \) where \( D^{(i)}_N = [N(N+1) + i^2]D_2. \) Therefore, the disorder strength is different for different channels.

To solve Eq. (8) we substitute \( \eta_i(z) = \eta^{(d)}_i(z) \eta^{(f)}_i(z), \) where \( \eta^{(d)}_i \) and \( \eta^{(f)}_i \) represent the drift and fluctuating contributions due to the first and second terms on the right hand side of the equation, respectively. Assuming that the first term is compensated by appropriately adjusting the amplifiers gain, \( \eta^{(d)}_i(z) = 1 \) and \( \eta^{(f)}_i(z) = \eta^{(f)}_i(z). \) As a result, the statistics of \( \eta_i \) is described by the lognormal distribution (7), with \( D^{(i)}_N = 8D^{(i)}_N \epsilon_R^2 z \) replacing \( D^{(0)}_N. \)

The dynamics of the Raman-induced self frequency shift for the reference channel soliton is given by

\[ \beta^{(s)}_0(z) = -\frac{8\epsilon_R}{15} \int_0^z dz' \eta_0^4(z'). \] (10)

To show that Eq. (10) leads to intermittent dynamics for \( \beta^{(s)}_0 \) we first calculate the \( n \text{th} \) moment:

\[ \langle \beta_0^{(s)n}(z) \rangle = \left( -\frac{8\epsilon_R}{15} \right)^n n! \times \int_0^z dz_1 \cdots \int_0^{z_{n-1}} dz_n \langle \eta_0^4(z_1) \cdots \eta_0^4(z_n) \rangle, \] (11)

where \( 0 \leq z_n \leq \cdots \leq z_1 \leq z. \) Using Eq. (5) and the fact that the integrals \( \int_{z_1}^{z_n} dz' \xi^{(0)}(z'; N) \)}
are Gaussian random variables that are independent for different $i$-values we obtain
\[
\langle \beta_0^{(s)n}(z) \rangle = \left( -\frac{8\epsilon_R}{15} \right)^n n! \times \prod_{m=1}^{n} \int_0^{z_{m-1}} dz_m \exp \left[ 32D_N^{(0)} \epsilon_R^2 (2m - 1)z_m \right],
\]
where $z_0 = z$. Thus, $\langle \beta_0^{(s)n}(z) \rangle$ is given by a sum over exponential terms of the form
\[K_m \exp \left[ 32m^2 D_N^{(0)} \epsilon_R^2 z \right] \] where $0 \leq m \leq n$ and $K_m$ are constants. To show intermittency it is sufficient to compare the leading term in the sum with the leading term in the expression for $\langle \beta_0^{(s)}(z) \rangle^n$. This calculation yields:
\[
\frac{\langle \beta_0^{(s)n}(z) \rangle}{\langle \beta_0^{(s)}(z) \rangle^n} \approx \frac{n! \exp \left[ 32n(n-1)D_N^{(0)} \epsilon_R^2 z \right]}{\prod_{m=1}^{n} \left[ n^2 - (m-1)^2 \right]}. \tag{13}
\]
Therefore, the leading term in the expression for the normalized $n$th moment of $\beta_0^{(s)}$ is exponentially increasing with both $z$ and $n^2$. To illustrate this dynamic behavior we show the $z$-dependence of the $n = 2, 3, 4$ normalized moments in Fig. 1 for a multichannel transmission system with $N = 50$, $\epsilon_R = 3 \times 10^{-4}$, $s = 1/2$, $T = 5$, and $\Delta \beta = 10$. These parameters correspond to the 101-channel system operating at 10 Gbits/s per channel discussed in detail below. One can see that the fourth moment increases much faster with increasing $z$ compared with the second and third moments. These results are of special importance since other contributions to changes in the soliton parameters are also coupled to the soliton amplitude via integrals over $z$ and thus follow the same statistics as $\beta_0^{(s)}$. In particular, the $s$-dependent contribution to the cross frequency shift discussed below and the soliton’s phase shift, which is given by $\alpha_0(z) = 2 \int_0^z dz' \eta_0^2(z')$, exhibit similar intermittent dynamics.

The dynamics of the Raman-induced cross-frequency shift $\beta_0^{(c)}$ in a two-channel system was obtained in Refs. [14, 15], where it was shown that $\beta_0^{(c)}$ is lognormally distributed. In a system with $2N + 1$ channels one obtains by a procedure similar to the one used in deriving Eq. (3)
\[
\frac{\Delta \beta_0^{(c)}}{\Delta z_c^{(1)}} \bigg|_{z=z_k} = -\frac{8\epsilon_R \eta_0^2(z_{k-1})}{3} \sum_{i \neq 0} \frac{1}{|\beta_i|} \sum_{j=(k-1)i+1}^{ki} \frac{(s + \tilde{\zeta}_{ij})}{\Delta z_c^{(1)}}. \tag{14}
\]
Equation (14) can be solved by decomposing $\beta_0^{(c)}$ into an $s$-dependent part $\beta_0^{(cd)}$ and an $s$-independent part $\beta_0^{(cf)}$. The $s$-dependent part is given by an integral of $\eta_0^2$ over $z$ and thus has similar statistics as $\beta_0^{(s)}$. To obtain the evolution of the $s$-independent part we
decompose the total disorder field $\xi^{(0)}(z; N)$ into contributions $\xi_i^{(0)}(z)$ coming from different channels: $\xi^{(0)}(z; N) = \sum_{i\neq 0} \text{sgn}(\beta_i)\xi_i^{(0)}(z)$, where $\xi_i^{(0)}(z) = \sum_j \tilde{\zeta}_{ij} / \Delta z_c^{(i)}$. Hence, the fields $\xi_i^{(0)}$ satisfy $\langle \xi_i^{(0)}(z) \rangle = 0$ and $\langle \xi_i^{(0)}(z)\xi_i^{(0)}(z') \rangle = |i|D_2\delta_{ii}\delta(z - z')$. Substituting these relations into Eq. (14), going to the continuum limit and integrating over $z$ we obtain

$$\beta_0^{(cf)}(z) = -\frac{2}{3\eta_0^2(z)} \sum_{i \neq 0} \frac{\text{sgn}(\beta_i)}{|\beta_i|} [1 - \mu_i^{-1}(z)],$$

(15)

where $\mu_i(z) = \exp \left[ 4\text{sgn}(\beta_i)\epsilon_R x_i^{(0)}(z) \right]$, and $x_i^{(0)}(z) = \int_0^z dz' \xi_i^{(0)}(z')$. Therefore, $\beta_0^{(cf)}(z)$ is a product of a lognormal variable and a sum over independent lognormal variables. Since the terms in the sum on the right hand side of Eq. (15) decrease with frequency as $1/|\beta_i|$ it is sufficient to consider only contributions from a few neighboring channels. For a three-channel system, for example, Eq. (15) simplifies to

$$\beta_0^{(cf)}(z) = \frac{2}{3\Delta\beta} [\mu_{-1}(z) - \mu_1(z)].$$

(16)

Using Eq. (16) one finds that the leading term in the expression for the normalized $2n$th moment of $\beta_0^{(cf)}$ is $\exp \left[ 32n(n - 1)D_{N}^{(0)}\epsilon_R^2z^2 \right]/2^n$, which is exponentially growing with $z$. Hence, the cross-frequency shift in the three-channel system exhibits intermittent dynamic behavior, even though it is not lognormally distributed as in the two-channel case.

To gain further insight into the intermittent dynamic behavior exhibited by the solitons we calculate the $n$th order equal-distance amplitude correlation functions, which measure correlation between amplitudes of solitons from different time slots in the same channel. Considering the reference channel we calculate

$$C_{0j} = \langle (\eta_{00}\eta_{0j})^n \rangle / \langle (\eta_{00}^n) \langle \eta_{0j}^n \rangle \rangle - 1,$$

(17)

where $\eta_{00}$ and $\eta_{0j}$ stand for the amplitudes of the solitons in the 0th and $j$th time slots, respectively. We still assume that the amplitudes of solitons in other channels are 1. Therefore, considering collisions with solitons from the $i$th channel, for example, the difference between the dynamics of the two solitons is due to the fact that the $0j$ soliton experiences the disorder experienced by the 00 soliton with a delay given by $\Delta z_{ji} = jT/(2\beta_i)$. Using this fact and the decomposition of $\xi^{(0)}$ into the $\xi_i^{(0)}$, and assuming $j > N$, one can show that $\tilde{C}_{0j}(z) = \eta_{00}(z)\eta_{0j}(z)$ is lognormally distributed

$$F(\tilde{C}_{0j}) = \exp \left\{ -\ln^2 \left( \frac{\tilde{C}_{0j}}{[8\epsilon_R^2\tilde{D}_{N}^{(0)}(z)]} \right) \right\} \left[ 8\pi\epsilon_R^2\tilde{D}_{N}^{(0)}(z) \right]^{1/2} \tilde{C}_{0j},$$

(18)
where

$$\tilde{D}_N^{(0)}(z) = 4D_2 \left[ N(N + 1) - \frac{1}{2} |i|(|i| - 1) \right] z$$

$$-\frac{2D_2 T(N - i + 1)|j|}{\Delta \beta},$$

(19)

for $\Delta z_{ji} < z < \Delta z_{j(i-1)}$. Consequently, the normalized $n$th order equal-distance amplitude correlation functions are given by

$$C_{ij}^{(n)}(z) = \exp \left\{ 2n^2 \epsilon_R^2 \tilde{D}_N^{(0)}(z) - 2D_N^{(0)} z \right\} - 1.$$ 

(20)

In particular, for $z < \Delta z_{jN}$ $C_{ij}(z) = 0$ since the two solitons are uncorrelated. During the transient $\Delta z_{jN} < z < \Delta z_{j1}$ the solitons become correlated due to the effective collision-induced disorder. For $z > \Delta z_{j1}$, i.e., after the transient,

$$C_{ij}^{(n)}(z) = \exp \left[ 4n^2 D_N^{(0)} \epsilon_R^2 z - 8s(1 - s)n^2 N \epsilon_R^2 |j| \right] - 1.$$ 

(21)

Thus, after the transient the $n$th order correlation functions grow exponentially with both $n^2$ and $z$, in accordance with the intermittent behavior of the amplitude. Notice that $C_{ij}^{(n)}$ decays exponentially with $N|j|$, which is the total number of time slots in all other channels separating the two solitons. Using Eq. (21) with $n = 1$ one obtains $j_{cor} = 1/(2N\epsilon_R^2)$ for the typical correlation number. For a 101-channel system operating at 10Gbits/s per channel, $j_{cor} \sim 10^5$, which is much larger than the number of successive bits that can be corrected by current error correction methods ($\sim 10^3$), and much smaller than the number of time slots that are in transmission in a given channel at any given time ($\sim 10^8$). Thus, this type of effective collision-induced disorder presents a challenge for conventional error correction methods.

We now relax the frozen disorder assumption and take into account the dynamics of soliton amplitudes in all channels. In this case Eq. (14) is replaced by

$$\frac{1}{\eta_0} \frac{d\eta_0}{dz} = \frac{4s\epsilon_R \Delta \beta}{T} \sum_{i\neq 0} |i| \eta_i(z) \text{sgn}(\beta_i)$$

$$+ 2\epsilon_R \tilde{\xi}^{(0)}(z; N),$$ 

(22)

where $\eta_i(z) = \exp \left[ 2\epsilon_R \int_0^z dz' \xi^{(i)}(z'; N) \right]$ and

$$\tilde{\xi}^{(0)}(z; N) = \frac{1}{\Delta \zeta_c^{(1)}} \sum_{i\neq 0} \eta_i(z) \text{sgn}(\beta_i) \sum_{j=(k-1)i+1}^{ki} \tilde{\zeta}_{ij}.$$ 

(23)
Expressing $\eta_0(z)$ as a product of an $s$-dependent and an $s$-independent parts: $\eta_0(z) = \eta_0^{(d)}(z)\eta_0^{(f)}(z)$, substituting into Eq. (22), and integrating over $z$ we obtain

$$\eta_0^{(d)}(z) = \exp \left[ \frac{4\Delta \beta s \epsilon_R}{T} \sum_{i \neq 0} |i| \text{sgn}(\beta_i) \int_0^z d' \eta_i(z') \right], \quad (24)$$

and

$$\eta_0^{(f)}(z) = \exp \left[ 2\epsilon_R \int_0^z d' \hat{\xi}^{(0)}(z'; N) \right]. \quad (25)$$

It follows that $\eta_0^{(d)}(z)$ is no longer deterministic. Moreover, since $\eta_0^{(d)}(z)$ is proportional to the exponent of the integral over $\eta_i(z)$, where $\eta_i(z)$ is lognormal, one can expect the departure of the $\eta_0^{(d)}$ statistics from Gaussian statistics to be stronger than lognormal. Consider now the statistics of $\eta_0^{(f)}$. Using Eq. (23) one can show that $\langle \hat{\xi}^{(0)}(z; N) \rangle = 0$ and $\langle \hat{\xi}^{(0)}(z; N)\hat{\xi}^{(0)}(z'; N) \rangle = \hat{D}^{(0)}_N(z)\delta(z - z')$, where

$$\hat{D}^{(0)}_N(z) = D_2 \sum_{i \neq 0} |i| \exp \left[ 8D^{(i)}_N \epsilon_R^2 z \right], \quad (26)$$

and we assumed $\langle \eta_i(z)\eta_i'(z') \tilde{\zeta}_{ij}\tilde{\zeta}_{i'j'} \rangle = \langle \eta_i(z)\eta_i'(z') \rangle \langle \tilde{\zeta}_{ij}\tilde{\zeta}_{i'j'} \rangle$. As a result, the $\eta_0^{(f)}$-distribution is the lognormal distribution given by Eq. (7) with $D^{(0)}_N$ replaced by $\hat{D}^{(0)}_N(z)$, where

$$\hat{D}^{(0)}_N(z) = D_2 \sum_{i \neq 0} \frac{|i|}{D^{(i)}_N} \left[ \exp \left( 8D^{(i)}_N \epsilon_R^2 z \right) - 1 \right]. \quad (27)$$

A direct consequence of Eq. (27) is that the $n$th moments of the $\eta_0^{(f)}$-distribution are super-exponentially increasing with $z$, although the factors $8D^{(i)}_N \epsilon_R^2 z$ are much smaller than 1.

From the practical point of view it is important to understand the influence of the intermittent dynamic behavior of the soliton parameters on the BER. The contribution of the collision-induced pulse decay to the BER was discussed in detail in previous works (see Ref. [17] and references therein). Moreover, the small-$\eta$ tail of the lognormal distribution lies below the corresponding tail of the Gaussian distribution, whereas the large-$\eta$ lognormal tail lies above the corresponding Gaussian tail. As a result, strong effects due to deviations from Gaussian statistics are related to relatively large $\eta$-values. When the position dynamics or the frequency dynamics are coupled to the amplitude dynamics such large $\eta$-values can lead to significant increase in the BER due to walk-off of the soliton from its assigned time slot. Therefore, we focus our attention on contributions to the BER due to the large-$\eta$
We consider a 101-channel system operating at 10 Gbits/s per channel and emphasize that state-of-the-art experiments with dispersion-managed solitons demonstrated multichannel transmission with 109 channels at 10 Gbits/s per channel over a distance of $2 \times 10^4$ km \[18\]. We use the following parameters, which are similar to the ones used in multichannel soliton transmission experiments \[19\]. Assuming that $T = 5$, $\Delta \beta = 10$ and $s = 1/2$, the pulse width is 20 ps, $\epsilon_R = 3 \times 10^{-4}$, the channel spacing is 75 GHz, and $D_2 = 1$. Taking $\beta_2 = -1 \text{ps}^2/\text{km}$, the soliton-peak-power is $P_0 = 1.25 \text{mW}$. For these values the disorder strength is $D_0^{(0)}(z) = 1.8 \times 10^{-3}z$ for the reference channel and $D_0^{(50)}(z) = 3.6 \times 10^{-3}z$ for the two outermost channels. For $z = 25$, corresponding to transmission over $2 \times 10^4$ km, $D_0^{(0)}(25) = 0.046$ and $D_0^{(50)}(25) = 0.091$.

For this system we evaluated the contributions to BER due to the Raman-induced cross frequency shift, the Raman-induced self frequency shift and the “ideal” component of the collision-induced position shift, i.e., the position shift due to soliton collisions in the absence of perturbations. The calculations show that the dominant contribution to the BER is due to the $s$-dependent part of the cross-frequency shift $\beta_0^{(cd)}$. The position shift induced by $\beta_0^{(cd)}$ is obtained by taking the continuum limit in Eq. (14) and integrating the $s$-dependent term twice with respect to $z$:

$$y_0^{(cd)}(z) = -\frac{64N\epsilon_R s}{3T} \int_0^z dz' \int_0^{z'} dz'' \eta_0^2(z''). \quad (28)$$

The position shift with a fixed amplitude $\eta_0(z) = 1$ is $\tilde{y}_0^{(cd)}(z) = -(32N\epsilon_R sz_0^2)/(3T)$ and the relative position shift is $\Delta y_0^{(cd)}(z) = y_0^{(cd)}(z) - \tilde{y}_0^{(cd)}(z)$. We assume that $\tilde{y}_0^{(cd)}$ can be compensated by employing filters. Therefore, the total energy of the soliton at a distance $z$ is

$$I(\eta_0, \Delta y_0^{(cd)}) = \eta_0^2 \int_{-T/2}^{T/2} dt \cosh^{-2}[\eta_0(t - \Delta y_0^{(cd)})]. \quad (29)$$

Occupied time slots are considered to be in error, if $I(\eta_0, \Delta y_0^{(cd)}) \leq I(z = 0)/2 \simeq 1$. To estimate the BER we numerically integrate Eq. (28) coupled to Eq. (6) for different realizations of the disorder $\xi^{(0)}(z, N)$ and calculate the fraction of errored occupied time slots. The BER in a generic channel $i \neq 0$ is calculated in a similar manner, where $\eta_0$ and $\xi^{(0)}(z, N)$ are replaced by $\eta_i$ and $\xi^{(i)}(z, N)$, respectively. Figure 2 shows the $z$-dependence of the BER in channels $i = 0$ (the reference channel), $i = 25$, and $i = 50$ (the outermost channel) for the aforementioned system. One can see that the BER in the reference channel increases...
from values smaller than $10^{-5}$ for $z < 15$ ($x < 1.2 \times 10^4$ km) to about $8.2 \times 10^{-2}$ at $z = 25.0$ ($x = 2 \times 10^4$ km). Furthermore, for intermediate distances $15 < z < 20$, the BER value in the outermost channels can exceed that in the reference channel by several orders of magnitude, even though, the disorder strengths differ by only a factor of 2. This behavior presents another challenge to conventional error correction methods based on knowledge gained from single- or few-channel transmission systems.

To better understand error generation due to $\beta_0^{(cd)}$ we analyzed the $z$-dependence of contributions to the BER coming from different regions in the $\eta_0 - \Delta y_0^{(cd)}$ plane. The results of this analysis are presented in Fig. 3. At $z = 15$ the dominant contribution to the BER comes from the domain $\eta_0 < 0.7$ and $\Delta y_0^{(cd)} > 0$, i.e., from decaying solitons with relatively large positive values of $\Delta y_0^{(cd)}$. For $z \geq 17$ the dominant contribution comes from the region $\eta_0 > 1.0$ and $\Delta y_0^{(cd)} < 0$, which corresponds to solitons with relatively large amplitudes and large negative values of $\Delta y_0^{(cd)}$. The latter contribution is associated with the large-$\eta$ lognormal tail of the amplitude distribution. Figure 4 shows the mutual distribution function $G(\eta_0, \Delta y_0^{(cd)})$ at $z = 25$ and the two domains giving the main contributions to the BER. It can be seen that this distribution is very different from the one observed for single-channel soliton propagation in the presence of amplifier noise (see Fig. 1 in Ref. 3). While the latter distribution is approximately symmetric about $\Delta y_0 = 0$ and $\eta_0 = 1$, the former is strongly asymmetric with an extended tail in the large-$\eta_0$ and large-negative-$\Delta y_0^{(cd)}$ region. The strong asymmetric form of $G(\eta_0, \Delta y_0^{(cd)})$ in our case is due to the strong coupling between the position dynamics and the amplitude dynamics and the lognormal statistics of the soliton amplitude. Thus, we find that amplitude dynamics plays a dominant role in error generation in massive multichannel optical fiber transmission systems, a situation which is very different from the one observed in single-channel transmission systems [3].

In summary, we studied soliton propagation in massive multichannel optical fiber communication systems taking into account the effects of delayed Raman response and the random character of pulse sequences. We found that the soliton parameters exhibit intermittent dynamic behavior and showed that the cross frequency shift is the main mechanism leading to bit pattern deterioration and to relatively large values of the bit-error-rate. We emphasize that similar dynamic behavior is expected in massive dispersion-managed multichannel transmission systems as well. In such systems the Raman-induced energy exchange in collisions will lead to lognormal statistics for the pulse amplitudes. In addition, the frequency
and position dynamics will be affected by a variety of amplitude-dependent perturbations due to Kerr nonlinearity. The coupling of the frequency and position dynamics to the amplitude dynamics will lead to intermittent dynamics of the pulse frequency and position and to relatively large values of the bit-error-rate.

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[16] The dimensionless $z$ in Eq. (1) is $z = (\beta_2 x)/(2\tau_0)$, where $x$ is the actual position, $\tau_0$ is the soliton width, and $\beta_2$ is the second order dispersion coefficient. The dimensionless retarded time is $t = \tau/\tau_0$, where $\tau$ is the retarded time. The spectral width is $\nu_0 = 1/(\pi^2 \tau_0)$ and the channel spacing is $\Delta \nu = (\pi \Delta \beta \nu_0)/2$. $\Psi = E/\sqrt{P_0}$, where $E$ is the electric field and $P_0$ is the peak power. The dimensionless second order dispersion coefficient is $d = -1 = \beta_2/(\gamma P_0 \tau_0^2)$, where $\gamma$ is the Kerr nonlinearity coefficient. The dimensionless Raman coefficient is $\epsilon_R = 0.006/\tau_0$, where $\tau_0$ is in picoseconds.
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FIG. 1: Normalized moments of the reference-channel soliton self frequency shift $\frac{\langle \beta_s^{(s)n} \rangle}{\langle \beta_0^{(s)} \rangle^n}$ vs propagation distance $z$ for a 101-channel system operating at 10 Gbits/s per channel. The solid, dashed, and dotted lines correspond to the $n = 2, 3, 4$ normalized moments calculated by using Eq. (12), respectively.
FIG. 2: BER vs propagation distance $z$ for a 101-channel transmission system operating at 10 Gbits/s per channel. The squares, circles, and triangles represent the BER at channels $i = 0$ (central), $i = 25$, and $i = 50$ (outermost), respectively.
FIG. 3: The $z$-dependence of different contributions to the BER for the reference channel in a 101-channel system operating at 10 Gbits/s per channel. The up triangles correspond to the total BER. The squares, circles, and down triangles correspond to contributions coming from the regions $\eta > 1.0$ and $y < 0$, $\eta < 0.7$ and $y > 0$, and $0.7 < \eta < 1.0$, respectively, in the $\eta_0 - \Delta y_0^{(cd)}$ plane.
FIG. 4: Mutual distribution function $G(\eta_0, \Delta y_0^{(cd)})$ for a reference channel soliton at $z=25$ in a 101-channel system operating at 10 Gbits/s per channel. The arrows A and B show the domains giving the main contributions to the BER.