On Recent Weak Single Pion Production Data

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MiniBooNE [1] and MINERvA [2] charge current $\pi^+$ production data in the $\Delta$ region are discussed. It is argued that despite the differences in neutrino flux they measure the same dynamical mechanism of pion production and should be strongly correlated. The correlation is clearly seen in the Monte Carlo simulations done with NuWro generator but is missing in the data.

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I. INTRODUCTION

There has been a lot of effort to understand better the single-pion-production (SPP) reactions in neutrino-nucleon and neutrino-nucleus scattering. These studies are motivated by neutrino oscillation experiments and their demand to reduce systematic errors. In the few-GeV energy region characteristic of experiments such as T2K [3], NOvA [4], LBNE [5] and MicroBooNE [6] the SPP channels account for a large fraction of the cross section (at 1 GeV on an isoscalar target about 1/3 of the cross section).

In the 1 GeV energy region the dominant SPP mechanism is through $\Delta$ excitation. There are several challenges in the theoretical description of the SPP reaction in the $\Delta$ region. The first one comes from uncertainties in the $N-\Delta$ transition matrix element. The vector part was measured in photo- and electroproduction experiments, but precise information on its axial counterpart is still missing. In order to describe the SPP channels one also needs a significant nonresonant background contribution. Several models of the background in electro- and neutrino-production exist (e.g. [7], [8], [9], [10]). The way of describing the resonance propagator and decay vertex differs from model to model (compare e.g. [10] and [11]). The extracted $\Delta$ production form factors also differ with respect to how one defines $\Delta$ and background contributions.

In SPP on atomic nuclei many-body effects must be properly included. The most important nuclear effects going beyond Fermi motion and Pauli blocking are related to in-medium $\Delta$ self-energy. Its real part shifts the pole, whereas the imaginary part corresponds to medium-modified SPP and pionless $\Delta$ decay processes. The problem of charge current SPP on nuclei has been addressed in Refs. [12] by assuming the $\Delta$ dominance with many-body effects taken from Ref. [13]. The computations suggest a significant reduction of the pion production cross section.

On the top of all that, in the impulse approximation regime final state interactions (FSI) effects must be carefully evaluated, see e.g. [14]. FSI include: pion rescattering, absorption, charge exchange, and (for sufficiently high energies) production of additional pions. The nuclear physics uncertainties are so large that most often experimental groups do not try to measure the characteristics of neutrino-nucleon SPP process and publish the cross sections results with all the nuclear effects included for which the signal events are those with a single pion leaving a nucleus.

It is clear that more precise SPP measurements on both nucleon and nucleus target are necessary. The models of $\Delta$ excitation matrix elements and nonresonant background are still validated mainly on old low-statistics bubble chamber experiments performed at Argonne National Laboratory (ANL, [15]) and Brookhaven National Laboratory (BNL, [16]). The nonresonant background is more important in neutrino nucleon SPP channels where the cross sections are smaller than for neutrino-proton SPP reaction and the statistical uncertainties are larger [17].

In view of those limitations it is important to explore information from more recent neutrino-nucleus cross section measurements. In the case of CC $\pi^+$ reaction on mostly Carbon target interesting studies were done by MiniBooNE [1] and MINERvA [2] experiments. Both studies focus on the $\Delta$ region. The main difference is in neutrino energy. Typical MiniBooNE interacting neutrinos energies are smaller by a factor of $\sim 4$.

The main results of this paper is the following. According to Monte Carlo simulations a strong correlation between differential cross sections in pion kinetic energy and production angle in two experiments is expected. The MINERvA cross sections are expected to be larger by a factor of $\sim 2$. This correlation is absent in the published data. The data/Monte Carlo discrepancy is seen in a dramatic way when one compares ratios of differential cross sections from two experiments with the Monte Carlo predictions.

The paper is organized as follows:

In section II MiniBooNE and MINERvA SPP data are discussed and re-binning of MiniBooNE data according to MINERvA bin sizes is done. Monte Carlo-data comparison is presented in section III with the main result shown in paragraph IIIA. The paper is concluded in section IV.

II. MINIBOONE AND MINERVA SPP DATA

MiniBooNE measurement was done on the mineral oil target ($C_2H_2$). The neutrino flux is peaked at $\sim 700$ MeV with a tail extended to 3 GeV. The signal charged current...
events are defined as $1\pi^+$ and no other mesons in the final state.

MINERvA measurement was done on the $CH$ target with much higher energy flux peaked at $\sim 3$ GeV. The signal charged current events contain exactly one charged pion, almost always $\pi^+$. Due to the cut on invariant hadronic mass $W < 1.4$ GeV a contamination from $1\pi^-1\pi^0$ events is very small.

In both cases the signal includes a significant fraction of coherent $\pi^+$ production component. Even if in the case of the MiniBooNE typical neutrino energies are lower by a factor of $\sim 4$ compared to the MINERvA, in both experiments the dominant contribution comes from $\Delta(1232)$ excitation. In both cases the target consists mostly from Carbon and we expect a lot of similarity in two measurements. According to NuWro Monte Carlo simulations the only significant difference comes from ∆(1232) excitation. In both cases the signal includes a significant fraction of coherent $\pi^+$ production component.

Experimental cross section results from both experiments are shown in Figs. 1 and 2. In the case of MiniBooNE’s angular distribution the points were taken from M. Wilking PhD Thesis [18] where differential cross section in $\cos\theta$ are reported. The errors were given as a fractional uncertainty (in percent) for each bin.

Both experiments have different binning. MiniBooNE’s binning is finer than MINERvA, hence we will “translate” the data from the MiniBooNE binning to the MINERvA one.

We assume that each data point represents a random variable with expected value equal to the central value (cross section in i-th bin, $E(X_i) = \sigma(E_i)$) and variance equal to the squared error ($Var(X_i) = (\Delta\sigma_i)^2$). i-th MiniBooNE bin contributes to j-th MINERvA bin with a weight equal to the ratio of the bin’s intersection with the MINERvA bin $\alpha_{ij}$ to the MINERvA bin width $W_j$:

$$w_{ij} = \frac{\alpha_{ij}}{W_j}. \quad (1)$$

The expected value of the MiniBooNE cross section in the j-th MINERvA bin is:

$$E(Y_j) = E(\sum_i w_{ij} X_i) = \sum_i w_{ij} E(X_i). \quad (2)$$

In the above equation $E(X_i)$ represents the measured MiniBooNE cross section. The variance of the sum of $N$ random variables is:

$$Var(\sum_{i=1}^{N} w_{i,k} X_i) = \sum_{i=1}^{N} w_{i,k}^2 Var(X_i) + 2 \sum_{i>j} w_{i,k} w_{j,k} Cov(X_i, X_j). \quad (3)$$

Unfortunately, MiniBooNE experiment did not publish the covariance matrix. The experimental errors are almost entirely systematic. The biggest contribution comes from the neutrino flux uncertainty, which cannot be reduced by combining neighboring bins. The second biggest error comes from the background estimation. The simplest assumption $Cov(X_i, X_j) = 0$ would reduce the error during the rebinning operation, since $w_{ij} \leq 1$. A reasonable assumption for these systematic errors is that if one combines the neighboring bins the resulting error is a weighted average of contributing bin errors. It is easy to show, that if one sets $Cov(X_i, X_j) = \sqrt{Var(X_i)Var(X_j)}$ the resulting error will be exactly a weighted average:
\[ \text{Var} \left( \sum_{i=1}^{N} w_{i,k} X_i \right) = \sum_{i=1}^{N} w_{i,k}^2 \text{Var}(X_i) + 2 \sum_{i>j} w_{i,k} w_{j,k} \sqrt{\text{Var}(X_i) \text{Var}(X_j)} = \sum_{i=1}^{N} w_{i,k}^2 (\Delta \sigma_i)^2 + 2 \sum_{i>j} w_{i,k} w_{j,k} \Delta \sigma_i \Delta \sigma_j = \left( \sum_{i=1}^{N} w_{i,k} \Delta \sigma_i \right)^2. \] (4)

III. NUWRO MONTE CARLO EVENT GENERATOR

NuWro is a versatile MC simulation tool describing lepton-nucleon and lepton-nucleus interactions in the energy range from \(\sim 100\ \text{MeV}\) to 1 TeV. Its main functionalities and implemented physical models are presented in [19]. Neutrino-nucleon interaction modes are: quasi-elastic (or elastic for NC) (QEL), resonant (RES) covering \(W < 1.6\ \text{GeV}\), DIS defined by \(W > 1.6\ \text{GeV}\). For a purpose of this study in \(\Delta\) decay events anisotropy in pion angular distribution has been implemented using the measurement done in ANL [15] and BNL [16] experiments.

For neutrino-nucleus scattering impulse approximation is assumed. New reaction modes, absent in neutrino-nucleon scattering, are: coherent pion production (COH) and two-body current interactions on correlated nucleon-nucleon pairs (MEC). In our simulation we used Valencia MEC model [20] with a kinematic cut on momentum transfer, \(|\mathbf{q}| < 1.2\ \text{GeV}\), as suggested in [21]. In MEC events final state nucleons are described using a model proposed in [22].

Primary interaction is followed by hadron rescatterings (FSI) simulated by custom made internuclear cascade model [19].

In the simulations discussed in this paper Carbon nucleus is treated within the relativistic Fermi Gas model. \(\Delta\) in-medium self-energy effects are included in an approximate way using the results from [23].

In MiniBooNE and MINERvA experiments pion production signal events origin from various interaction modes:

1. RES interaction, typically through \(\Delta\) excitation and decay, but also with some contribution from the nonresonant background. According to NuWro this process contributes to 87.1% and 84.7% of the signal for the MiniBooNE and MINERvA experiment respectively. There is a very important impact of FSI effects on the final state pions production rate because many pions are absorbed or suffer from charge exchange reaction inside Carbon nucleus.

2. COH process, populating 6.7% (MiniBooNE) and 10.7% (MINERvA) of the signal. NuWro uses Rein-Sehgal coherent pion production model [24].
3. DIS interactions contributes only to the MiniBooNE signal at the level of 3.6%. A typical scenario is that one out of two pions produced in the primary interaction is absorbed.

4. QEL and MEC interactions with pions produced due to nucleon rescattering reactions account for 2.7% MiniBooNE and 4.6% MINERvA signal events.

Results of NuWro simulations together with experimental points are shown in Figs. 5, 6. The results tend to overestimate the MINERvA data and underestimate MiniBooNE data at the same time. In the case of the MiniBooNE data similar problems were reported in the past by many theoretical models \cite{26, 27}. Another observation is that MC simulation predicts a large difference between MiniBooNE and MINERvA cross sections in the whole pion kinetic energy range. On the other hand, in Fig. 6 one can see, that for higher pion kinetic energies both experiments measured similar cross sections. Also, in the MC simulations the differential cross sections tend to peak at the same point in pion kinetic energy, near the threshold for $\Delta$ production, which in the pion FSI simulations leads to significant pion absorption. However, both experimentally measured cross sections seem to have maximal cross sections at different points. This is not pronounced very strongly in the kinetic energy plot, because the MINERvA errors are very large. We checked that introduction of anisotropy for pion angular distribution does not change much NuWro results, giving an effect of at most 10% in some kinetic energy or production angle bins. The shape of differential cross sections changes but there is almost no structure to it save for the kinetic energy distribution. We observe there a shift of of the cross section towards higher kinetic energies.

NuWro results are consistent with GENIE \cite{28} predictions for $\frac{d\sigma}{d\pi_-^+}$, shown in Fig. 4 of Ref. \cite{2}. NuWro and GENIE use different physical models to describe SPP (GENIE relies on resonant Rein-Sehgal model) but both predict a strong correlation between results from two experiments.

A. Ratio of MINERV A/MINIBOONE cross sections

Correlations of both $\pi^+$ production measurements should be seen in ratios of cross sections. In this paper we will look at ratios of differential cross sections in pion kinetic energy and also production angle relative to the neutrino flux. The shapes of both ratios do not depend on flux normalization factors in both experiments and are interesting observables to investigate.

In order to calculate ratios of both measurements together with appropriate errors we consequently treat the processed data points as random variables $X$ and $Y$ with known expected values and variances. One has to compute $E\left(\frac{X}{Y}\right)$ and $Var\left(\frac{X}{Y}\right)$. For independent variables:

$$E(X \cdot Z) = E(X)E(Z)$$

$$Var(X \cdot Z) = Var(X)Var(Z) + E(X)^2Var(Z) + E(Z)^2Var(X)$$

and replacement $Z = \frac{1}{Y}$ must still be done.

The assumption that two experiments are independent is rather conservative because errors coming from neutrino interaction models are in both cases correlated.

The most difficult task is to calculate $E\left(\frac{1}{Y}\right)$ and $Var\left(\frac{1}{Y}\right)$, because $E\left(\frac{1}{Y}\right) \neq \frac{1}{E(Y)}$ unless the probability distribution function of $Y$ is given by the Dirac delta function, $P(Y) = \delta(Y - Y_0)$. We must introduce some...
model-dependence, which fortunately will be shown to be negligible.

We investigated several assumptions for \( P(Y) \):

- flat distribution
- linear distribution
- quadratic distribution
- log normal distribution

In all the cases \( P(Y) \) is monotonous below and above its expected value. The other assumption is that \( P(Y \leq 0) = 0 \) and \( P(Y) \) drops to 0 faster, than \( Y^2 \) as \( Y \) approaches 0 since the cross section cannot be negative and we do not want the integral to give indefinite values for \( E(1/Y) \) and \( E(1/Y^2) \).

We tested the model dependence of ratios using above probability distribution hypotheses by calculating both the expected ratio value as well as its error. We compared them also to a “naive” approximation, in which \( E(1/Y) \approx \frac{1}{E(Y)} \) and \( Var(1/Y) \approx \frac{Var(Y)}{E(Y)^2} \).

We verified that the expectation values and variances coming from various probability distribution hypotheses do not differ in any significant manner. The only exception is the “naive” approach, leading to a few-percent effect on expected value and enlargement of variance. From the above described models we chose the log-normal distributions as it allows any value of random variable along the positive real semiaxis and has well-defined probability maximum:

\[
P(Y) = \frac{1}{\sqrt{2\pi}bY} \exp \left[ -\frac{(\ln(Y) - a)^2}{2b^2} \right] \Theta(Y)
\]

\[
E(Y) = \exp(b^2/2 + a)
\]

\[
Var(Y) = \exp(2b^2 + 2a)
\]

We get \( E(\frac{1}{Y}) = \exp(b^2/2 - a) \) and \( Var(\frac{1}{Y}) = \exp(b^2 - 2a) [\exp(b^2) - 1] \).

The procedure is to generate samples with NuWro generator for both experiments and compare the resulting ratio of Monte Carlo cross sections to the experimental one. In order to maintain statistically meaningful samples each dynamical channel contributing to MINERvA and MiniBooNE signals has been generated separately.

We tried to estimate errors of both ratios as calculated by NuWro. We distinguish systematic and statistical errors coming from implemented theoretical models. We run the code with very high event rate in order to minimize statistical fluctuations. We obtained at least 8 000 events in each bin with typical value of the order of \( 10^4 \) – \( 10^5 \) events/bin. We found the resulting impact of statistical errors on the predicted ratio to be negligible.

In order to establish the leading systematic error contribution we searched for the dominant dynamical process giving rise to the signal in both experiments. We checked that contributions from RES channel usually exceed 85-90\%, save for the lowest angle bins, see Figs 7 and 8. The effect for small pion production angles can be explained by coherent process, which tends to produce very forward-going pions and gives significant contribution for the forward-peaked bins. This provides a strong support for the claim, that MiniBooNE and MINERvA signals originate from almost the same physical processes.

Moreover, the pion kinetic energy distribution produced in RES process before FSI is quite similar in both cases.

We propose a simplified MC systematic error analysis based on uncertainties in the RES process, which should cover the leading error of MC predictions. Two error sources are taken into account:

1. \( \Delta \) production rate uncertainty driven by \( C^A_0 \) and \( M_{A\Delta} \) parameters.
2. \( \Delta \) decay uncertainty coming from pion angular correlations.
We vary the axial coupling of the $\Delta$ resonance $C_A^\Delta(0) = 1.19 \pm 0.08$ and $M_{A\Delta} = 0.94 \pm 0.03$ (GeV) within the limits found in [29] and treat the maximum variation as a systematic error $\delta_{C_A^\Delta}$, $\delta_{M_{A\Delta}}$. We compared descriptions of angular anisotropy reported by ANL and BNL experiments also took the maximum variation from both parameterizations as another systematic error $\delta_{\text{decay}}$. We combined these errors in quadrature and obtained the estimate of the total error in NuWro simulations $\delta_{MC} = \sqrt{\delta_{C_A^\Delta}^2 + \delta_{M_{A\Delta}}^2 + \delta_{\text{decay}}^2}$.

In Figs 10 and 11 we show the final results for

$$\frac{\frac{d\sigma}{dT_\pi}}{d\sigma/dT_\pi}^{\text{MINERvA}} \text{ and } \frac{\frac{d\sigma}{dT_\pi}}{d\sigma/dT_\pi}^{\text{MiniBooNE}}$$

where experimental results are compared to NuWro predictions. Central results are predicted by NuWro with BNL angular correlations and default values of $C_A^\Delta(0) = 1.19$ and $M_{A\Delta} = 0.94$ (GeV).

It is very interesting to see that Monte Carlo predictions are very different from the experimental results. Differences in shape are much more important than differences in overall scale which can be due to uncertainties in overall neutrino fluxes in both experiments.

NuWro predicts almost constant small drop of the ratio with growing pion production angle. The data, in contrary, form a visible dip for the intermediate angles and the ratio grows on both ends. For the pion kinetic energy distribution MC predicts almost constant ratio $\sim 2$, but the ratio of data tends to drop from $\sim 3$ in the lowest energy bin to $\sim 1$ starting from 100 MeV pion kinetic energy. The disagreement of data and MC is huge, and it is clearly over $3\sigma$ effect at many points.

There are two plausible explanations of the discrepancy.

Perhaps the way nuclear effects are modelled in Monte Carlo simulation tools suffers from unexpected nuclear effect. It is important to remind that other Monte Carlo generators/computation tools like GENIE [28] or GiBUU [26] also have problems with reproducing both data sets. In the case of GiBUU in [26] a paradoxical conclusion is presented that the MiniBooNE data is reproduced well only if FSI effects are neglected. Note also that the underlying SPP and FSI models in all the generators are quite different. Even though, it seems unlikely that large data/Monte Carlo discrepancy is caused by deficiencies of theoretical models implemented in NuWro. Our main argument is that in the cross section ratios all the deficiencies should cancel each other because in both cases the dominant dynamical mechanism: $\Delta$ excitation and decay is the same.

It is also possible that something is missing in the understanding of experimental data coming from both SPP measurements. Not being experimentalists, we are reluctant to present any speculations along these lines.
though.

IV. CONCLUSIONS

We are still far away from a good understanding of SPP channels in neutrino scattering. Interpretation of old ANL and BNL deuteron target experiments is not straightforward because of apparent differences in measured cross sections (see, however a discussion in [29]) and problems with modeling nonresonant contribution [17].

More recent experimental papers on SPP report results that are apparently very difficult to understand using widely used models implemented in Monte Carlo event generators. It is clear that more dedicated effort aiming to measure pion production reactions together with nuclear effects is needed.

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