Optimizing Students Combinatorial Thinking Skill Through Design-based Research

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Abstract
The aim of this study is to construct and compose an instructional design in combinatorial learning, especially in concept of counting. The learning design that is composed is expected to optimize combinatorial thinking skills of students. This research was conducted at the first author’s workplace, in combinatorial and graph consisting of 12 students. This study consists of three phases. The first phase is preliminary research, the authors analyzes the needs and context of learning activities, study of literature, and develops a framework of thinking. The second phases is development, the authors validates the instructional design for further improvement, application in class, and improvement based on learning activities in class. The third phase is assessment, the authors provides evaluation questions to students to find out the result of the application of the design related to the optimization of combinatorial thinking. As a result, an instructional design in counting begins with review of concept of finite sets, small group discussion, class discussion, self-reflection, and evaluation

Keywords
Combinatorial thinking, Combinatorics, Design-based Research

1. Introduction

As a prospective mathematics educator, students of mathematics education study program must be able to master all the concept that will be taught later. On the other hand, a mathematics educator must be able to teach high order thinking skills [9][10].

Combinatorics can be used as a means to teach high order thinking skills. Besides being needed in other disciplines such as computers science, science engineering, statistics, etc [13], combinatorics is rich in problem material, so it can train student's high order thinking skill.

Combinatorial thinking is a result from [15]. There is four level in combinatorial thinking[9]. The first level is investigating some cases, in this level students can investigate certain and special cases. The second is "How am I sure that I have counted all cases?". in this level, students argue about their investigation. The third is "generating all cases". In this level, students can make a generalization problem systematically, from certain simple cases to more complex general problem. And Finally, level changing problem. In this level, students can solve another problem related to and relevant to the problem that has been previously worked on. The following is a brief explanation of the
steps in question:
1. Investigating some cases, at this level students are able to investigate a particular and special cases;
2. How am I sure that I have counted all cases ?, at this level students are able justify the argument that the answer (at level 1) does not exist countless cases;
3. Systematically generates all cases, at this level students are able Generating problems systematically ; and
4. Changing the problem into another combinatorial problem, at this level students are able to use the concepts that have been obtained in completion previously given questions to solve other relevant problems.

This study is aimed to construct and compose an instructional design in combinatorial learning, especiaaly in concept of counting. The learning design that is composed is expected to optimize combinatorial thinking skills of students.

2. Methodology

The method used in this study is design-based research. Consists of three main stages, namely preliminary research, developing, and assessment. This research is a design research whose stages refer to [14]. Figure 1 is the phase or stage of design research according to [14] (in [10]).

At the preliminary research stage the researcher analyzes the curriculum requirements of the study program that are combined with combinatorial thinking skills. The researcher also analyzed the learning needs related to student profiles which were known based on the initial test of combinatorial abilities and previous learning outcomes (addition and multiplication rules). The next activity of the researcher is a review of the permutation material literature from [6] and [2] regarding the concept of permutation, and from [15], and [6] regarding combinatorial thinking. The next researcher made a learning design framework that the researchers arranged according to the task-based learning concept [17] and guided learning concepts [8].

At the developing stage, the researcher provides learning design to the expert team for later validation. Based on the results of the validation, a revised learning design was carried out to then be applied in classroom learning. The results of learning activities in the research class are made as revised material (stage 2) learning design.

At the assessment stage the researcher gives test questions to be completed by each student. The question given is a test to determine the level of mastery of student combinatorial thinking.

The place for conducting this research is the even semester combinatorics and graph classes 2017-2018 academic year at the place where the researcher works. The class
consists of 12 students consisting of 5 men and 7 women. Each of these students has taken courses which are prerequisites for Combinatorics and Graph courses.

3. Result and Finding

This research in general has two results. The first result is the design of permutation material learning related to combinatorial thinking abilities. The second result is the result of a combinatorial thinking ability level evaluation.

The permutation learning design that the researcher obtained consisted of 3 main activities namely the initial activity, the core, and the closing. The initial activity of the lecture begins with the preparation of learning activities in the form of a review of the prerequisite concepts that have been studied. Furthermore, students are directed to review the concept of finite sets.

In core activities students are directed to complete tasks in groups (1 group consists of 2 students). The task given is in the form of problems related to the concept of permutation. Problems are presented in 3 consecutive questions. The first question is related to a simple problem, assuming each student can finish briefly. This first question accommodates the assessment of mastery level 1 of student combinatorial thinking skills.

The second question directs students to provide arguments for their answers to the first question. This second part of the question accommodates the assessment of mastery of level 2 combinatorial thinking skills. Then the third question directs students to summarize their findings, thus combinatorial level 3 thinking skills can be assessed or observed.

In an effort to direct students to find valid concepts about permutations, the learning process is continued at the stage of class discussion. Two selected groups presented the results of their respective group discussions. Groups that do not appear are directed to give responses.

In the closing activity each student is given a question of evaluation. It was given 15 minutes for each student to complete the evaluation question. Giving the question is intended to find out students' mastery of the concept of permutation. Furthermore, with the evaluation of researchers can measure the mastery of student combinatorial thinking skills, especially level 4.

For further analysis of combinatorial thinking skills, each student is given an assignment to be completed outside of lecture hours. In addition to analyzing combinatorial thinking skills, especially level 4, assignments are given to provide an overview of the next learning material.

The second result that the researcher obtained was a profile of mastery of student combinatorial thinking skills. These results are obtained from the analysis of learning activities and evaluation activities. The most portion of the analysis is the evaluation of the results of the evaluation questions. Figure 2 below is a mastery diagram of students' combinatorial thinking abilities which are known based on the test results.

![Mastery Diagram](image_url)
4. Conclusion

After going through the research process, obtained a conclusion that is the result of research conducted by the author. The first conclusion is about the design of permutation material learning. The design of the intended learning is as follows:

1. Initial activities (preparation, then proceed with a review of the concept of finite sets);
2. Core Activities (group discussions, class discussions, conclusions)
3. Closing learning activities (evaluation and assignment)

The second conclusion obtained is about the level of mastery of student combinatorial thinking. Of the 12 students who joined the Combinatorics and Graph classes, only 9 people who master the four levels of combinatorial thinking skills. Furthermore, out of 3 students who have not mastered optimal combinatorial skills, 2 of them have mastered level 3, the rest only master level 2.

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