Normal metal - superconductor tunnel junction as a Brownian refrigerator

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Thermal noise generated by a hot resistor (resistance $R$) can, under proper conditions, catalyze heat removal from a cold normal metal (N) in contact with a superconductor (S) via a tunnel barrier. Such a NIS junction acts as Maxwell’s demon, rectifying the heat flow. Upon reversal of the temperature gradient between the resistor and the junction the heat fluxes are reversed: this presents a regime which is not accessible in an ordinary voltage-biased NIS structure. We obtain analytical results for the cooling performance in an idealized high impedance environment, and perform numerical calculations for general $R$. We conclude by assessing the experimental feasibility of the proposed effect.

In 1867, Maxwell suggested a demon that attempts to violate the second law of thermodynamics \cite{1}. The demon acts between two containers A and B, initially at the same temperature; it exclusively allows hot particles to pass from container A to container B, and cold ones from B to A. This process would lead to a decrease of entropy if the system were isolated, and could then be used for useful work. Ever since, this thought experiment has intrigued physicists, see, e.g., Refs. \cite{1,2,3,4}. Yet the demon needs to exchange energy with the containers in order to function properly. Thereby the entropy of the whole system (including the demon) is always increasing, rendering thermodynamics intact.

In this Letter we present a particularly illustrative example of Maxwell’s demon, which can be realized experimentally in a straightforward way. Our system is a Brownian refrigerator \cite{5} in close analogy to Brownian motors and thermal ratchets \cite{6,7,8,9,10}. It conveys heat unidirectionally in response to random noise. Specifically, we consider a tunnel junction between a normal metal and a superconductor (NIS junction) subject to the thermal noise of a resistor at temperature $T_R$, see Fig. 1. The temperatures of the electrodes N and S are $T_N$ and $T_S$, respectively. The capacitance $C$ consists of that of the junction itself and the surrounding circuit. The resistor and the junction can be connected by superconducting lines which efficiently suppress electronic thermal conductance, and thus enable us to discuss solely the photonic heat exchange via the lines as in Refs. \cite{11,12}. The N side can be connected to the superconducting line via a metal-to-metal SN contact, which provides perfect electrical transmission but, due to Andreev reflection, prevents heat flow \cite{13,14}. We note already here that the presented NIS structure can be replaced by a more conventional symmetric SINIS device with two tunnel junctions back-to-back. This will be discussed towards the end of the present Letter. We also note that the idealized circuit of Fig. 1(a) as well as the thermal model presented schematically in Fig. 1(b) and mathematically below are not unrealistic for a practical on-chip device.

The heat balance between the resistor and the NIS structure can be described on the level of a single electron tunnelling event between N and S, accompanied by an exchange of energy with the junction’s resistive environment. Formally this can be done using the so-called $P(E)$ theory developed for a tunnel junction embedded in an electromagnetic environment \cite{15,16}. Assuming that for both electrodes $i = 1, 2$ the (normalized) density of states $n_i(E)$ is symmetric around the Fermi level $E = 0$ and that the corresponding energy distributions satisfy $f_i(E) = 1 - f_i(-E)$, we find that the heat flux out of 1 upon tunnelling from electrode 1 to 2 is given by

\[
\dot{Q}_1 = \frac{2}{e^2 R_T} \int dE' dE n_1(E') n_2(E) \times \\
E' f_1(E') (1 - f_2(E)) P(E' - E). \quad (1)
\]

Here, $P(E' - E)$ is the probability density of emitting energy $E' - E$ to the environment in a tunnelling process from 1 to 2. It can be calculated for a particular envi-

![FIG. 1: Schematic presentation of the system. In (a) we show the electrical diagram of the resistor at temperature $T_R$, and the tunnel junction. The parallel capacitance $C$ includes that of the junction and a possible shunt capacitor. In (b) we show the thermal diagram of the system. The NIS tunnel junction acts as a Brownian heat engine, or as Maxwell’s demon (MD) between N and S. $Q_N$ and $Q_S$ are the heat fluxes out of N and S, respectively, and $P_{\text{ext}}$ denotes the external power needed to create the temperature bias of the resistor. The thermal resistances indicate phonon coupling of the electrical subsystems.](image-url)
environment at a temperature $T_R$, once the dissipative part \( \text{Re}Z(\omega) \) of its impedance at frequency $\omega/2\pi$ is known \[16\]. The theory is perturbative in tunnel conductance, therefore the tunnel resistance $R_T$ should be of the order of the resistance quantum, $R_k = h/e^2$, or higher. Here, we have neglected the Joule dissipation in the normal metal itself, which is usually justified because N can have a very low resistance as compared to that of the tunnel junction.

As a warm-up exercise let us look at the simplest system, where the tunnel junction is of type NIN, i.e., both sides are normal metals. Then $n_i(E) = 1$ for both electrodes; we furthermore assume them to be kept at equal temperature $T_N$ and characterized by equilibrium Fermi distributions $f_i(E) = (1 + e^{E/k_B T_N})^{-1}$. This example closely resembles the original set-up of Johnson and Nyquist \[17\], where thermal noise acts between two resistors. At high temperatures, $k_B T_{R, N} \gg h(RC)^{-1}$, we then obtain, using Eq. \[1\], the total heat flux out of the junction as $\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = k_B (T_N - T_R) (R_T C)^{-1}$. This power is shared equally by both electrodes due to symmetry. The result for $\dot{Q}$ is consistent with the expectation that heat flows from hot to cold.

The symmetry of the previous example is vitally broken in the NIS junction that we focus on here. In particular, we have $n_1(E) = 1$ for N as before, but the BCS density of states in S, $n_2(E) = 0$ for $|E| < \Delta$ and $n_2(E) = |E|/\sqrt{E^2 - \Delta^2}$ for $|E| > \Delta$, makes this system behave as an energy selective entity in the spirit of Maxwell’s demon. Here, $\Delta$ is the energy gap of the superconductor. We again assume standard Fermi distributions, but possibly at different temperatures: $f_1(E) = (1 + e^{E/k_B T_N})^{-1}$ for N and $f_2(E) = (1 + e^{E/k_B T_S})^{-1}$ for S.

Let us first discuss a particular limit where illustrative analytical results can be obtained. We consider a very resistive environment, where $\frac{R_T k_B T_N}{E_C} \gg 1$, where $E_C = \frac{\pi^2 k^2}{2m} = \frac{2 \pi}{e^2}$ is the charging energy. Then $P(E)$ is a simple Gaussian form: $P(E) = (2 \pi \sigma)^{-1/2} \exp\left(-\frac{(E - E_C)^2}{2 \sigma^2}\right)$, where the width is given by $\sigma = 2 k_B T_R E_C$ \[16\]. We now assume that this width is small compared to the superconducting gap $\Delta$. The integration over energy in Eq. \[1\] then involves $E, E' \gg \Delta \gg k_B T_{N, S}$ and we may approximate the Fermi distributions by their Boltzmann-like exponential tails. In this limit, setting further $T_S = T_N$ for simplicity, we have

\[
\dot{Q}_N \simeq \sqrt{\frac{2 \pi \Delta k_B T_N}{e^2 R_T}} e^{-\Delta/k_B T_N} \times \\
[1 - (2 \frac{T_R}{T_N} - 1) \frac{E_C}{\Delta} \left(e^{\frac{T_R}{T_N} - 1} E_C/k_B T_N + \frac{E_C}{\Delta} - 1\right)]. \tag{2}
\]

One can similarly estimate the heat flux from the superconductor:

\[
\dot{Q}_S \simeq \sqrt{\frac{2 \pi \Delta k_B T_N}{e^2 R_T}} e^{-\Delta/k_B T_N} [1 - e^{\left(\frac{T_R}{T_N} - 1\right) E_C/k_B T_N}]. \tag{3}
\]

The expressions \[2\] and \[3\] vanish when $T_R = T_N$, as expected. The non-trivial result is that on heating the environment to temperatures $T_R > T_N$, N tends to cool down, i.e., $Q_N > 0$, and S is heated, $Q_S < 0$. On the other hand, for $T_R < T_N$ the heat flux is reversed: S tends to cool down and N to warm up. Also in this regime the heat flux between the junction and the resistor is unevenly distributed among N and S. Such cooling of S never occurs in a conventional voltage biased NIS refrigerator \[14\].

If one employs \[2\] to find the optimum $T_R/T_N$ where the cooling power of N is maximal, one finds $T_R/T_N \simeq \Delta/2E_C$ for $E_C/\Delta \ll 1$. Although this is the right order of magnitude, the numerical results presented below show that the actual value of the ratio $T_R/T_N$ is approximately twice higher. The approximation used above for the Fermi functions leads to a counterbalance of the cooling effect as the Boltzmann tails grow exponentially at negative energies. A better estimate can be obtained by retaining the full Fermi function $f_1(E')$ and linearizing the exponent of the Gaussian $P(E' - E)$ around $E' = 0$ where $f_1$ steeply drops. We then arrive at

\[
\dot{Q}_N \simeq \frac{\pi^3 (k_B T_N)^2}{2e^2 R_T \sqrt{1 + E_C/\Delta}} \times \\
[|\Delta| + 1) T_R/T_N - 1] e^{-\frac{\sqrt{\Delta^2 T_R^2 + \pi^2 E_C^2}}{2N^2R_T}}. \tag{4}
\]

This expression predicts that for $k_B T_N, E_C \ll \Delta$ the optimal point of cooling indeed lies at $T_R/T_N \simeq \Delta/E_C$. In its range of validity this optimum as well as the overall behavior described by Eq. \[4\] are consistent with the numerical results, as will be seen below.

In order to picture the characteristics of the system in a dissipative environment of arbitrary resistance $R$, we have performed numerical calculations. The usual environment theory of single-electron tunneling was employed here also. In all the numerical calculations the ideal junction was coupled to a parallel $R_C$ circuit. In Fig. \[2\] we show a comparison of the exact numerical results for $R \gg R_K$ and the analytical approximations of Eqs. \[2\] and \[4\]. We see in the main frame that at low temperatures, in this case at $k_B T_N/\Delta = 0.03$, the approximation \[4\] works quite well over a broad range of temperature biases, whereas the Boltzmann approximation \[2\] fails at high values of $T_R/T_N$. Yet, around zero temperature bias the latter approximation works perfectly as demonstrated by the inset of Fig. \[2\].

A set of numerical results under representative conditions is collected in Fig. \[3\]. In (a) we see the influence of $E_C$ on the performance of the system. Here the Gaussian approximation of $P(E)$ was used. The maximum cooling occurs indeed at $T_R/T_N \simeq \Delta/E_C$, and the value at the maximum grows a little with increasing $\Delta/E_C$. In (b) the Gaussian approximation was abandoned. Under reduced $R$ the $P(E)$ function transforms from a broad Gaussian (width $2k_B T_R E_C$) centered around $E_C$ towards a delta-function around $E = 0$. According to Eq. \[1\] this evolution weakens the refrigeration effect. In Fig. \[3\] (b) we see that it is indeed essential to have a relatively large $R$ in order to sustain the effect, although some cooling
FIG. 2: Calculated cooling power $\dot{Q}_N$ at $k_B T_N/\Delta = 0.03$ as a function of the temperature bias $T_R/T_N$. Here we have assumed that $R \gg R_K$, $T_S = T_N$, and $\Delta/E_C = 5$. In the main frame the solid line is the result of the exact numerical calculation, and the dashed one is the analytic expression of Eq. (4) based on linearizing the exponent of the Gaussian distribution over the relevant energy interval. The dotted line diverging to negative values around $T_R/T_N \approx 3$ is the result of Eq. (4). This approximation works well, contrary to Eq. (4), at small temperature biases, as shown by the inset: the solid line is the exact result and the approximation (2) is shown by open dots.

can be observed also down to $R/R_K \sim 0.1$. We also see that with high environment resistances the cooling characteristics approach the result of a Gaussian environment as should be the case. In (c) and (d) we show the quantitative results for the heat flux $\dot{Q}_N$ and the optimum temperature bias, respectively, as a function of $T_N$ in a highly resistive environment.

Although the discussion above demonstrates countervailing heat fluxes in the system, it is straightforward to verify that they do not contradict the second law of thermodynamics. For instance, consider the resistor and the NIS junction to form an isolated system such that, referring to Fig. 1(b), $P_{\text{ext}} = 0$ and that couplings to the phonon bath vanish. Initially brought to temperatures $T_R$ and $T_N (= T_S$ for simplicity), the system then starts to relax towards a common temperature. For a small temperature difference $\Delta T = T_R - T_N$, we have $\dot{Q}_N + \dot{Q}_S \approx -\frac{\sqrt{2\pi k_B T_N}}{e^2 R_T} \langle e^2 \rangle / k_B T_N \Delta T$, according to Eqs. (2) and (4). This is the heat flux between the junction and the resistor, and it is always directed from hot to cold thereby ensuring positive entropy production.

Some of the results above can be obtained approximately by straightforward classical estimates. In particular, we may consider an ordinary NIS junction with large resistance $R_T$ and evaluate its cooling power when biased by a fluctuating voltage with vanishing mean value and Gaussian variance $\langle (eV)^2 \rangle \equiv \sigma = 2k_B T_R E_C$. In the limit of low frequencies (small $E_C$), we may make a quasi-stationary averaging over the fluctuations, such that the expected cooling power of $N$ is $\langle \dot{Q}_N \rangle \approx \sigma \int:\nabla p(eV)\dot{Q}_N d(eV)$, where $p(eV) = (2\pi\sigma)^{-1/2} e^{-(eV)^2/(2\sigma)}$ is the distribution of fluctuations and $\dot{Q}_N(eV)$ is the cooling power of the NIS junction at a static bias voltage $V$. For $|eV| \gg k_B T_N$, one obtains $\dot{Q}_N(eV) \approx \frac{\sqrt{2\pi k_B T_N}}{e^2 R_T} \langle \Delta - |eV| \rangle e^{-\langle \Delta - |eV| \rangle / k_B T_N}$ [18]. Performing the averaging yields then $\langle \dot{Q}_N \rangle \approx \frac{\sqrt{2\pi k_B T_N}}{e^2 R_T} \Delta e^{-\Delta/k_B T_N} (1 - 2T_R T_N E_C) e^{E_C/k_B T_N}$. This result resembles very closely that of Eq. (2), in particular for $T_R > T_N$. Yet the classical result above neglects the feedback of heat from the junction to the resistor, which is an important contribution especially when $T_R \lesssim T_N$.

Next we discuss the possible experimental demonstration of the presented effect. The basic concept can be implemented for instance in a standard on-chip configuration by employing common N and S metals like copper and aluminium, respectively. The resistance can be formed using a strip of resistive metal such as chromium or a metallic alloy. Figure 3 demonstrates the calculated temperature reduction, $T_N/T_S$, of the N island assuming that the superconductor is well thermalized at the phonon bath temperature $T_0$. The results are shown specifically for the case of aluminium as a superconduc-
tor \((\Delta = 200 \text{ meV}, \text{transition temperature} T_C \approx 1 \text{ K})\). We have chosen the ratio \(\Delta/E_C = 5\), which, on one hand corresponds to realistic junction parameters, and on the other hand allows maximum cooling to occur at not excessively high resistor temperatures: the largest temperature reduction of almost 40\% occurs at \(T_0 \approx 0.2\) K corresponding to \(T_R \approx 0.7\) K. The specific material parameter for the electron-phonon coupling in the normal metal was chosen to be \(\Sigma = 1 \cdot 10^9 \text{ W K}^{-2} \text{ m}^{-3}\) [14]. The N island volume of \(\Omega = 1 \cdot 10^{-21} \text{ m}^3\) can be achieved by a standard process.

In the analysis above the charging energy \(E_C\) is due to the parallel connection of the junction capacitance and an optional shunt capacitance. We have seen that the optimum cooling results do not depend particularly strongly on \(E_C\), as long as \(E_C < \Delta\) holds. Some improvement in performance can, however, be obtained by decreasing \(E_C\), see Fig. 3 because the parallel capacitance filters out the harmful high frequency tail of the noise spectrum. Yet the ratio \(T_R/T_N \sim \Delta/E_C\) reflects the fact that with large shunting capacitance one needs to have a hotter noise source to induce sufficiently strong fluctuations. From the practical point of view this is not advantageous, at least in a conventional on-chip solution, because the high temperature of the environment leads to parasitic heating of N via the phonons of the substrate. Therefore, in a practical realization the choice of \(C\) is a trade-off.

We believe that the presented parallel RC environment can be realized almost exactly as long as \(C\) is kept small, say on the level of few fF arising from the junction itself. Then the series inductance of a circuit of sub-100 \(\mu\)m dimensions can be neglected, because the corresponding LC-frequency remains high as compared to the frequency band of thermal radiation at sub-K temperatures.

Finally, the effect can also be realized in a symmetric SINIS configuration, instead of using a single NIS junction with an NS Andreev mirror. There the same theoretical analysis can be carried over by replacing the environment resistance by an effective resistance \(R/4\), and the capacitance by \(2C\), where \(R\) is the true environment resistance and \(C\) is the capacitance of one junction. To take into account the possible charging effects on the island, a more involved analysis needs to be performed, however.

In summary, we have discussed a Brownian refrigerator of electrons, which offers an illustrative example of Maxwell’s demon in the form of a tunnel junction with a superconducting energy gap. Its operation is based on building blocks whose characteristics and implementation are well known. We expect that it yields a substantial temperature reduction in a straightforward on-chip realization.

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