Design considerations for an Archimedean screw hydro turbine

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Abstract. Archimedean screws have been used for centuries, but only for pumping water. In the last decades, a growing interest appeared to also use them as hydro turbines for electricity generation. While Archimedean screws are already well optimized for working as pumps, studies are still carried out to identify those parameters that assure the best performances when they operate as turbines. This paper proposes a simple yet efficient procedure for the design of an Archimedean screw operating as a turbine. The most important formulas proposed in the paper are those for calculating the outer diameter and the rotational speed of the screw. The condition imposed for finding the diameter is the maximization of the torque produced. The calculation of the diameter requires to estimate the volume of the water buckets that form between the blades of the screw. It is practically impossible to find an analytical formula for this volume, but a rough estimate can be found easily. This rough estimate is corrected afterwards based on regression of data available for turbines that already operate with good efficiencies. The rotational speed is then determined based on the condition that the rated discharge can run through the turbine operating at the rated speed.

1. Introduction
The Archimedean screw is a design that has its origins in the ancient times, being traditionally attributed to Archimedes of Syracuse, in the 3rd century BC. Until recently, its main modern application was for pumping water mostly in wastewater treatment plants. However, the idea of also using the Archimedean screw for power generation is not so new. In 1916, Moerscher filed a patent application, which was granted in 1922, for a “water power plant or apparatus” that was making use of a screw [1]. But only at the beginning of 1990s the idea of using the Archimedean screw as a turbine for electricity generation was taken again into consideration and started to gain serious traction. As a results, more and more screw turbines were commissioned in the last decades.

The runner of an Archimedean screw turbine (AST) is usually provided with three helical blades (or flights) that are fixed on a long cylindrical hub (Fig. 1). The runner can be shrouded, being mounted inside a fixed cylindrical tube, or it can placed in a trough made either of concrete or of steel. In both cases, the runner is supported by two bearings. During its operation, the runner builds an angle with the horizontal. The water flows inside the runner at its upper inlet, forming so-called buckets in the empty space between the blades. The tangential components of the hydrostatic forces caused by the water buckets on the blades generate a moment that turns the runner. As the runner turns, the water in the buckets flows down and eventually leaves the runner at its lower outlet.
ASTs normally operate at heads between 1 m and 10 m and at discharges between 0.1 m$^3$/s and 15 m$^3$/s. The delivered power is usually below 1 MW. The operating speed is low for large runner diameters, but it increases as the diameter decreases [2]. Due to its low operating speed, an AST needs to coupled to the electrical generator through a speed increasing gear box. The maximum efficiency of ASTs is lower than that of classical turbines, such as Francis or Kaplan, normally reaching up to 80% [3]. However, the efficiency curve remains very flat down to discharges as low as only 5% of the rated discharge, which represents an important advantage on rivers that have a large annual variation of the flow rate. Besides the flat efficiency curve, there are additional advantages that recommend ASTs for low head applications:

- fish friendliness, due to the large diameter and pitch of the screw and due to the low operating speed;
- high tolerance to debris-laden water, since large solid particles – such as wood or even small stones – are able to pass through the turbine without affecting the operation or the efficiency;
- simple design, assuring low costs of manufacturing, commissioning, operation, and maintenance;
- robustness, due to the simple design, allowing an estimated operation of more than 30 years with minimum maintenance and without a visible decrease in efficiency;
- very small dimensions of the hydro power plant, resulting in low construction costs.

The Archimedean screw turbines operate at roughly atmospheric pressure, hence they can be considered equal pressure turbines. The flow velocities inside the runner are small, so that the change in kinetic energy between runner inlet and runner outlet can be neglected. Consequently, these turbines convert into work mostly the potential energy of water. Independent of how the screw operates – as a pump or as a turbine – the hydrostatic forces acting on the blades and the moments generated by these forces are theoretically identical and, consequently, the power transmitted from or to the blades (i.e. only inside the runner) is the same. For this reason, the methods for designing Archimedean screw turbines were initially borrowed from screw pumps. Such methods are detailed in a design manual by Nagel [4]. Rorres [5] carried out a study for finding those geometrical and operational parameters that lead to the optimal performance of the Archimedean screw pump (ASP). However, pumps and turbines are fundamentally different from the point of view of the energy conversion that takes place inside the runner: turbines convert hydraulic energy into work, while the pumps do the opposite. Therefore, the design of screw hydro turbines based merely on the methods used for screw pumps is not expected to lead to turbines that operate the most efficiently. Different design methods have been sought and used, resulting in various designs after the first screw turbine was commissioned in 1994 [6]. It is clear that, since this turbine type is a new one, studies are still required in order to understand how the power extracted by the turbine can be maximized.
This paper proposes a design method for Archimedean screw hydro turbines, that is based partly on considerations regarding the maximization of the hydrostatic forces that act on the runner blades and partly on the similarity with screw turbines which have already been commissioned and operate with good efficiencies at different locations. Section 2 presents the parameters of an AST. In Section 3, design considerations are presented in detail. Based on these considerations, a design procedure is proposed in Section 4.

2. Parameters of an Archimedean screw hydro turbine

The geometry of an Archimedean screw turbine depends on two types of parameters:

- Internal parameters. They can be appropriately chosen and/or modified at design time.
- External parameters. They depend on the installation location, hence they cannot be changed by the designing engineer.

The designing of the turbine aims at assessing optimal values of the internal parameters based on the external ones.

Figure 1 shows the schematic of an Archimedean screw operating as a turbine, together with its main internal parameters that characterize its geometry. These parameters are the following:

- outer (exterior) diameter and radius, \( D_e \) and \( R_e \), respectively (m);
- inner (hub) diameter and radius, \( D_i \) and \( R_i \), respectively (m);
- screw length, \( L \) (m);
- installation angle, i.e. the angle build by the runner with the horizontal, \( \theta \) (degrees);
- screw slope, \( K = \tan \theta \);
- blade pitch, \( \Lambda \) (m);
- number of blades, \( N \).

Two dimensionless parameters are particularly important for the design of an Archimedean screw:

- diameter ratio, \( \delta = \frac{D_i}{D_e} = \frac{R_i}{R_e} \); (1)
- dimensionless pitch, as proposed by Rorres [5]:
  \[ \lambda = \frac{K \Lambda}{2\pi R_e} = \frac{K \Lambda}{\pi D_e}. \] (2)

From the point of view of the turbine operation, the important parameters are the following:

- turbine head, \( H \) (m);
- turbine discharge, \( Q \) (m);
- turbine speed, \( n \) (rpm);
- hydraulic power (power available in the water stream that flows through the turbine), \( P_h \) (kW);
- shaft (or mechanical) power, \( P_s \) (kW);
- turbine efficiency, \( \eta \).

Of the aforementioned parameters, the head \( H \), the discharge \( Q \), and the hydraulic power \( P_h \) are external parameters, while the turbine speed \( n \), the shaft power \( P_s \), and the turbine efficiency \( \eta \) can be considered internal parameters, since they largely depend on the design of the turbine.

3. Design considerations

3.1. Design parameters

The design parameters of an AST are usually the head \( H \) and the discharge \( Q \) that are available at the installation site. Regarding the head, a few considerations are necessary. Due to the geometry of the turbine, it can be considered that the areas of the inlet and outlet sections are roughly equal, hence the mean velocities of the water through these sections are also roughly equal. The flow through the entire
turbine has a free surface, which allows us to consider that the static pressures at turbine inlet and outlet equal the atmospheric pressure. Then, the turbine head can be estimated as follows:

\[ H = z_1 - z_2, \]  

where \( z_1 \) and \( z_2 \) are the heights of the free surface at turbine inlet and outlet, respectively, measured from a conveniently chosen reference plane.

### 3.2. Blade geometry

The inner and outer edges of the runner blades are sinusoids having the same phase and period, but different amplitudes. The amplitudes of the two curves equal the inner radius and the outer radius, respectively. Given the pitch, \( \Lambda \), and the number of blades, \( N \), the distance between two successive blades equals \( \Lambda / N \). Then, the blades are described by the following parametric equations:

\[
\begin{align*}
x &= r \cos \left( \varphi - 2\pi \frac{k-1}{N} \right), \\
y &= r \sin \left( \varphi - 2\pi \frac{k-1}{N} \right), \\
z &= \frac{\varphi}{2\pi} \Lambda,
\end{align*}
\]

with \( r \in [R_i, R_e] \), \( \varphi \in [0, 2\pi L/\Lambda] \).

where \( k = 1, 2, \ldots, N \) is the number of the blade under consideration.

The tangents to the outer and inner edges of a blade build with the runner axis the angles \( \alpha \) and \( \beta \), respectively, as showed in Figure 1. Since the projections of the two edges on the plane \( Oyz \) are sinusoids having the equations

\[
\begin{align*}
y_e &= R_e \sin \left( \frac{2\pi z}{\Lambda} - 2\pi \frac{k-1}{N} \right) \\
y_i &= R_i \sin \left( \frac{2\pi z}{\Lambda} - 2\pi \frac{k-1}{N} \right),
\end{align*}
\]

it results that the angles \( \alpha \) and \( \beta \) calculated at \( z = \Lambda [1 - (k-1)/N] \), i.e. where the sinusoids intersect the \( z \)-axis, are given by the following relationships:

\[
\begin{align*}
\alpha &= \arctan \left( \frac{2\pi R_e}{\Lambda} \right) = \arctan \left( \frac{\pi D_e}{\Lambda} \right) \quad \text{and} \\
\beta &= \arctan \left( \frac{2\pi R_i}{\Lambda} \right) = \arctan \left( \frac{\pi D_i}{\Lambda} \right).
\end{align*}
\]

### 3.3. Installation angle

From the economical point of view, a value of the installation angle as high as possible is preferred, since a larger installation angle leads to a shorter screw, hence to lower manufacturing costs. However, the increase of the installation angle above a certain value requires the screw to be installed inside a tube to avoid the draining of the water buckets. As a results, the turbine can get completely filled with water and the Archimedean screw becomes more or less an axial turbine, being probably more efficient to replace it with a propeller-type turbine.

From the point of view of the turbine efficiency and power extracted, experimental studies carried out by Lyons [7] showed that there is no significant increase in peak efficiency when the installation angle increases roughly above 25°, while the maximum power can be obtained for installation angles that are roughly between 30° and 35°.

For practical reasons, which aim to limit the length of the screw and to install it as easily as possible, while ensuring the highest possible efficiency and maintaining the advantages offered by ASTs, an installation angle of 30° or even of 35° can be chosen. It is interesting to note that the latter value is very close to that of the screws built by Vitruvius [8], that have slopes equal to 3/4, corresponding to an installation angle of 36.87°.
3.4. Screw length

The length of the screw, \( L \), depends on turbine height and installation angle according to the following relationship:

\[
L = \frac{z_1 - z_2}{\sin \theta} = \frac{H}{\sin \theta}.
\]

(7)

In particular, when the installation angle is of 30°, the length of the screw equals twice the turbine head.

3.5. Number of blades (flights)

The criterion that should be considered when choosing the number of blades of an AST is to maximize the power transmitted to the shaft while keeping the runner as light as possible.

The increase in the number of blades of an Achimedean screw leads to an increase in the number of water buckets that form between the blades together with a decrease in the volume of each bucket. Also, the shock losses at runner entry and the viscous losses inside the runner are expected to increase [9]. A larger number of blades increases the screw weight and, consequently, the loads on the shaft and bearings, the friction losses inside the bearings, and the costs of manufacturing, transporting, and installing the turbine. Even the useful volume of the runner (i.e. the volume that allows the water flow) could be adversely affected to some extent by too large number of blades, since the blades must have a certain thickness that diminishes the useful volume.

Lyons [7] observed experimentally that the increase in the number of blades over \( N = 3 \) leads to only a marginal increase in efficiency, while the decrease in the number of blades to \( N = 2 \) causes a significant reduction in turbine performance. The fact that an increase in the number of blades over \( N = 3 \) does not lead to a significant increase in the performance of a screw turbine has been observed experimentally by Songin [9] as well. It can be concluded that the optimal number of blades of an AST is \( N = 3 \).

3.6. Diameter ratio

Based on experimental results obtained for ASPs, Nagel [4] recommends diameter ratios between 0.45 and 0.55. After an optimization study, Rorres [5] concluded that the diameter ratio of an ASP should be of about 0.54 when the number of blades ranges from 1 to 4.

The decrease in diameter ratio, \( \delta \), leads to an increase in blade surface, accompanied by an increase in the hydrostatic forces that act on the blades. On the one hand, the axial component of the hydrostatic force increases, causing a higher axial thrust that loads the bearings. On the other hand, the tangential component of the hydrostatic force increases as well, but it gets applied at a smaller radius, which means that the torque it produces is not expected to increase significantly. The last remark is supported by the experimental results presented by Lyons [7] for ASTs, according to which the peak torque increases as \( \delta \) decreases, but the increase in peak torque at diameter ratios below 0.5 is only marginal.

The increase in diameter ratio causes a decrease in the volume of the water buckets, which, for a given discharge, must be accompanied by an increase in runner speed. However, the speed cannot increase above an upper threshold in order to avoid the water being splashed out of the turbine. The results obtained by Lyons [7] show also that the peak torque starts to decrease rapidly as the diameter ratio increases above 0.55, while the peak efficiency varies within less than 1% for diameter ratios in the range roughly from 0.5 to 0.8, having a maximum of about 70% when \( \delta \) is around 0.67.

It can be seen that the results obtained for Archimedean screws operating both as pumps and as turbines are consistent and suggest that a good trade-off would be a diameter ratio in the range from 0.5 to 0.55. The ratio \( \delta = 0.54 \) found by Rorres as optimal for ASPs seems to be adequate for ASTs as well, as pointed out by Pandey and Karki [2].

3.7. Blade pitch

A first remark concerning the blade pitch is that it has an upper bound, that results from the condition that, during the normal operation of the turbine, the water does not flow over the outer edge of the blade
from one water bucket into the next one located downstream. From Figure 1 it results that this condition is fulfilled when

$$\theta \leq \alpha.$$  \hspace{1cm} (8)

Considering the expressions of $\theta$ and $\alpha$, the following condition can be derived for the blade pitch:

$$\Lambda \leq \frac{2\pi R_e}{K}.$$  \hspace{1cm} (9)

For obtaining optimal performances of ASPs, Nagel [4] showed that, depending on $\theta$, the blade pitch should be from 1.6 to 2.4 times higher than the outer radius $R_e$ and recommended a standard value $\Lambda = 2R_e$ that should be adopted for avoiding too many design variants and increased manufacturing costs. Rorres proposed the dimensionless pitch $\lambda$ given by Equation (2) and estimated the optimal values of this pitch depending on the number of blades, so that the volume of a water bucket is maximal. Table 1 summarizes the results of Nagel and Rorres in terms of the dimensionless pitch.

| $N$ | Dimensionless pitch, $\lambda$ |
|-----|--------------------------------|
| ASP, $\theta = 30^\circ$ (Nagel [4]) | ASP (Rorres [5]) | AST, Equation (12) |
| 2   | 0.1682                        | 0.1863              | 0.2122              |
| 3   | 0.1682                        | 0.2217              | 0.2728              |
| 4   | 0.1682                        | 0.2456              | 0.3183              |

As mentioned in the Introduction, ASPs and ASTs are fundamentally different. In case of ASPs, the optimal design means the geometry that maximizes the amount of water delivered in one turn of the screw [5]. In case of ASTs, the optimal design means the geometry that maximizes the power extracted. Since the turbine speed is limited to low values, the maximization of the power requires the maximization of the torque and, consequently, of the tangential component of the hydrostatic force. In the same time, the pitch of an AST should be kept within clear limits.

In this paper, we propose to choose the pitch of an AST so that the free surface in a bucket meets the downstream blade on the coordinate plane $Oxz$, that contains the turbine axis, and the upstream blade gets only very slightly wetted. This configuration is depicted in Figure 2, which shows the side projection of the resulting free surface of the water in a bucket. For the choice proposed, the negative moment produced by the hydrostatic forces on the blades (i.e. the moment opposed to the screw rotation) is practically negligible. It should be noted that the moment generated on the lower half of the upstream blade (i.e.

**Figure 2.** Choosing the blade pitch for an AST.
below the turbine axis) is entirely negative and so is also that part of the moment that would be generated on the upper half of the downstream blade (i.e. above the turbine axis). For the solution proposed, the following relationship is valid:

\[
\frac{\Lambda}{4} + \frac{\Lambda}{N} = \frac{R_c}{\tan \theta} = \frac{R_c}{K}.
\]

(10)

It results the blade pitch

\[
\Lambda = \frac{4NR_c}{K(4+N)} = \frac{2ND_e}{K(4+N)}.
\]

(11)

The estimation of the outer diameter that is required for the calculation of the pitch is detailed in the next subsection.

The corresponding dimensionless pitch becomes

\[
\lambda = \frac{4N}{2\pi(4+N)}.
\]

(12)

It can be seen that, for the solution proposed, the dimensionless pitch depends only on the number of blades, being independent of the screw geometry and installation angle. Values of \(\lambda\) calculated for ASTs with 2, 3, and 4 blades are presented in Table 1 as well. It can be seen that there is a quite fair correlation between the dimensionless pitches found by Rorres to be optimal for ASPs and those calculated for ASTs with Equation (12). There are, of course, differences, since the criteria used for ASPs are fundamentally different from those imposed for ASTs.

Considering the definition of the dimensionless pitch, namely Equation (2), it results that the condition on \(\Lambda\) given by Equation (9) is obeyed when the dimensionless pitch is less or at most equal to 1.

3.8. Outer diameter

The choice of the outer diameter of an Archimedean screw, \(D_e\), is strongly related to the turbine speed. Nagel [4] recommends that the value of the speed in rpm does not exceed the upper threshold

\[
n = \frac{50}{D_e^{2/3}}.
\]

(13)

For example, a screw with an outer diameter of 1 m should operate at a speed no higher than 50 rpm [2]. The speed limitation comes from the condition of avoiding the splashing of water outside the turbine during normal operation, which could cause an undesired and uncontrolled emptying of the water buckets. To reduce as much as possible the gear ratio and, consequently, the size of the speed increasing gearbox, the turbine speed should be chosen as close as possible to the maximum value given by Equation (13).

On the other hand, the turbine speed should be chosen so that the flow rate absorbed by the runner during a full rotation equals the discharge. Assuming that \(N\) buckets get filled with water during a full rotation, the following relationship can be derived:

\[
n = \frac{60Q}{NV_b},
\]

(14)

where \(V_b\) is the volume of a bucket.

A serious problem that arises when designing an AST is the calculation of the volume \(V_b\). Normally, this volume strongly depends on the turbine discharge and on how the blade pitch and the turbine speed are chosen.

When the blade pitch and the turbine speed are chosen as proposed before, the bucket volume is bounded by five surfaces: (i) the free surface of the water in the bucket, that builds the installation angle \(\theta\) with the turbine axis, (ii) the wetted surface of the downstream blade, (iii) the wetted surface of the upstream blade, which is, however, very small, (iv) the wetted inner surface of the trough, and
Figure 3. Projections of a water bucket in dimensionless coordinates, for $\theta = 30^\circ$.

(v) the wetted outer surface of the hub. The complexity of volume $V_b$ is depicted in Figure 3, that shows projections of this volume on three planes: (i) a vertical plane parallel to the turbine axis (side view), (ii) a plane parallel to the coordinate plane $Oxz$ inclined at the installation angle $\theta$ (top view), and (iii) a cross-sectional plane perpendicular to the turbine axis (view along $z$-axis). All coordinates in Figure 3 are dimensionless, with $\Lambda^* = \Lambda/D_e$ being the dimensionless pitch (not to be confused with the pitch ratio). The installation angle in Figure 3 is $\theta = 30^\circ$.

It is not possible to find an analytical expression for the real volume of the water bucket not only because this volume has a complicated form, but also because the equations to be solved are transcendental, since the blades are described by sinusoids. Therefore, we propose to use a rough yet simple estimate of this volume. This estimate adjusted with a correction factor can then be used to find a practical formula for the outer diameter. The correction factor can be assessed based on regression of experimental data or of data from turbines that operate with good efficiencies.

To roughly assess the bucket volume, the schematic in Figure 4 can be used twice. First, a larger volume that is comprised between the free surface in the absence of the hub, the inner surface of the trough (having the radius $R_e$ when the small tip clearance of the blades is neglected), and a circular surface that replaces the downstream blade should be determined. Second, a smaller volume displaced by the hub (having the radius $R_i$) should be extracted from the volume found in the first step. Both volumes can be calculated similarly. According to Figure 4, the following relationships can be written:

$$y(x) = \sqrt{R^2 - x^2},$$
$$z(x) = \frac{y(x)}{\tan \theta} = \frac{\sqrt{R^2 - x^2}}{K},$$

(15)
where \( R \) will take the values \( R_e \) and \( R_i \). The elementary volume at coordinate \( x \) is

\[
dV = \frac{1}{2} y(x) z(x) \, dx = \frac{1}{2} \frac{R^2 - x^2}{K} \, dx.
\]  
(16)

The entire volume results by integration:

\[
V = \frac{1}{2K} \int_{R_i}^{R_e} (R^2 - x^2) \, dx = \frac{2}{3} \frac{R^3}{K}.
\]  
(17)

Now, the volume of the water bucket can be written as

\[
V_b = \frac{2}{3} k_V \frac{R_e^3 - R_i^3}{K} = \frac{2}{3} k_V \frac{R_e^3}{K} (1 - \delta^3) = \frac{1}{12} k_V \frac{D_e^3}{K} (1 - \delta^3),
\]  
(18)

where \( k_V \) is the correction factor that accounts for the rough approximations that were made. It should be noted that the value of \( k_V \) should be less than 1.

Substituting \( n \) from Equation (13) and \( V_b \) from Equation (18) in Equation (14) yields

\[
\frac{50}{D_{e/3}^3} = \frac{720 K Q}{k_V N D_e^2 (1 - \delta^3)}.
\]  
(19)

It results the following expression of the outer diameter:

\[
D_e = \left[ \frac{14.4 K Q}{k_V N (1 - \delta^3)} \right]^{3/7}.
\]  
(20)

In this paper, the correction factor \( k_V \) was estimated based on a regression analysis of the data that is publicly available for different ASTs produced and installed by the company Rehart Power at 39 locations [10]. It should be mentioned that not all the data were used, but only those of turbine-generator units that have total efficiencies higher than 60\%. The data used are presented in Table 2.

Unfortunately, the turbine speeds were not available, so it was not possible to use the unit discharge or the unit speed. Therefore, the product \( K Q \) was chosen as independent variable for the regression. The dependent variable was the outer diameter \( D_e \), while the unknown parameter was \( k_V \). The number of blades and the diameter ratio were kept constant at \( N = 3 \) and \( \delta = 0.54 \), respectively. The regression was carried out using the least-squares method. It resulted the correction factor \( k_V = 0.7730 \), for which the
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Table 2. Data available for turbine-generator units with Archimedean screw turbines produced by Rehart Power [10], having unit efficiencies, $\eta_u$, higher than 60%. Comparison with diameters calculated based on regression of these data.

| Location         | Manufacturer’s data | Calculated data | Regression |
|------------------|---------------------|-----------------|------------|
|                  | $Q$ (m$^3$/s) | $H$ (m) | $P_e$ (kW) | $L$ (m) | $D_e$ (m) | $\theta$ (degrees) | $\eta_u$ (%) | $D_e$ (m) | $\varepsilon_D$ (%) |
| Haddo            | 0.5                 | 5              | 15.9       | 10.47   | 1.4       | 28.5              | 64.8         | 1.35     | 3.7          |
| Indore           | 0.6                 | 5.3            | 19         | 10.23   | 1.4       | 31.2              | 60.9         | 1.53     | 8.5          |
| Schnaittach      | 0.8                 | 1.35           | 7.5        | 3.2     | 1.6       | 25                | 70.8         | 1.54     | 3.9          |
| Herrenhof        | 0.9                 | 2.1            | 13.9       | 5.71    | 1.6       | 21.6              | 75           | 1.51     | 6            |
| Gennkikungou     | 0.99                | 1.05           | 7.3        | 3.02    | 1.6       | 20.3              | 71.6         | 1.53     | 4.6          |
| Bischofsmais     | 1                   | 3.16           | 21         | 7.43    | 1.6       | 25.2              | 67.7         | 1.7      | 5.9          |
| Mühlen           | 1                   | 3              | 21         | 6       | 1.5       | 30                | 71.4         | 1.86     | 19.4         |
| Vadodara         | 1                   | 5              | 33         | 10.35   | 1.7       | 28.9              | 67.3         | 1.82     | 6.6          |
| Eitting          | 1.2                 | 3.57           | 29         | 9.6     | 1.8       | 21.8              | 69           | 1.72     | 4.7          |
| Erding           | 1.2                 | 1.75           | 13.9       | 4.52    | 1.8       | 22.8              | 67.5         | 1.75     | 2.9          |
| St. Michael      | 1.2                 | 3.2            | 26.92      | 6.67    | 1.7       | 28.7              | 71.5         | 1.96     | 13.3         |
| Viersbühl        | 1.2                 | 1              | 3           | 3.1     | 1.6       | 18.8              | 68           | 1.6      | 0            |
| Colditz          | 1.5                 | 3              | 33         | 8.7     | 2.2       | 20.2              | 74.8         | 1.82     | 20.9         |
| Niedermühle      | 1.5                 | 3.17           | 33         | 8.46    | 1.9       | 22                | 70.7         | 1.9       | 0            |
| Flatford Mill    | 1.6                 | 1.1            | 12.6       | 3.13    | 1.9       | 20.6              | 73           | 1.89     | 0.5          |
| Gescher          | 1.8                 | 3.45           | 46         | 9.4     | 2         | 21.5              | 75.5         | 2.03     | 1.5          |
| Yvoir            | 2                   | 1.8            | 26         | 4.83    | 2.1       | 21.9              | 73.6         | 2.14     | 1.9          |
| Stimpfach        | 2.3                 | 2.55           | 44         | 6.57    | 2.3       | 22.8              | 76.5         | 2.32     | 0.9          |
| Dautphetal       | 2.5                 | 2.55           | 45.8       | 7.3     | 2.6       | 20.4              | 73.2         | 2.28     | 14           |
| Untermünkheim    | 2.5                 | 1.8            | 31         | 5       | 2.4       | 21.1              | 70.2         | 2.32     | 3.4          |
| Turbury Mill     | 2.8                 | 2.1            | 43         | 6.4     | 2.5       | 19.2              | 74.5         | 2.33     | 7.3          |
| Pilsing          | 3.2                 | 3.6            | 8          | 8.1     | 2.9       | 26.4              | 70.8         | 2.87     | 1            |
| Wiener Neustadt  | 3.5                 | 4.05           | 98         | 9.76    | 2.8       | 24.5              | 70.5         | 2.88     | 2.8          |
| Bairesdorff      | 4.5                 | 1.5            | 48.1       | 5.08    | 3.2       | 17.2              | 72.6         | 2.71     | 18.1         |
| Shanes Castle    | 5.5                 | 5              | 192        | 12.85   | 3.4       | 22.9              | 71.2         | 3.38     | 0.6          |
| Hauser           | 6                   | 5.8            | 250        | 15.3    | 3.4       | 22.3              | 73.2         | 3.46     | 1.7          |
| Kirchberg        | 6                   | 2.97           | 130        | 6.83    | 3.4       | 25.8              | 74.4         | 3.71     | 8.4          |
| Wien             | 7.1                 | 1.7            | 84         | 4.54    | 3.6       | 22                | 70.9         | 3.7      | 2.7          |
| Maple Durham     | 8                   | 1.73           | 99         | 5.27    | 3.6       | 19.2              | 72.9         | 3.65     | 1.4          |

Average 71.2
Standard deviation 3.4

The coefficient of determination is $R^2 = 0.9696$. The regression curve is presented in Figure 5 together with the data on which it is based. The outer diameter can be expressed now as

$$D_e = \left[ \frac{18.63KQ}{N(1-\delta^3)} \right]^{3/7}.$$  \hspace{1cm} (21)

Diameters recalculated with Equation (21) for the Rehart turbines and their relative differences with respect to the real diameters,

$$\varepsilon_D = \left[ \frac{D_{e,\text{Rehart}} - D_{e,\text{calc}}}{D_{e,\text{calc}}} \right] \times 100 \, [\%],$$  \hspace{1cm} (22)

are also presented in Table 2 for comparison. In most cases, the differences are below 5%, with two cases in which the diameters are identical. Only in five situations the differences are higher than 10%.
3.9. Turbine speed

By substituting the bucket volume given by Equation (18) in Equation (14), the following expression of the turbine speed is obtained:

\[ n = \frac{931.4KQ}{ND_e^3(1 - \delta^3)}. \] \hspace{1cm} (23)

The speed calculated with the previous formula must be lower than the upper threshold

\[ n_{\text{lim}} = \frac{50}{D_e^{2/3}}. \] \hspace{1cm} (24)

To avoid not fulfilling the condition mentioned above, we recommend to always round up, not down, the diameter calculated with Equation (21).

Since a speed increasing gearbox must be used anyway, it is not necessary to round the value of the turbine speed to match the speed of the electrical generator, unless the gearbox is supplied by a third party manufacturer and has a speed increasing ratio that requires the adjustment of the turbine speed. When the speed needs to be adjusted, an adequate value that is lower than that calculated with Equation (23) should be chosen.

3.10. Turbine efficiency

The selected turbine-generator units for which data are presented in Table 2 have efficiencies ranging roughly from 60% to 76%. The average efficiency of the units is of 71.2% with a standard deviation of 3.4%. Since these efficiencies are of complete units, that include not only the AST, but also the electrical generator and the speed increasing gear box, it can be expected for a well designed AST to have a total efficiency \( \eta \) of about 70–75%.

3.11. Shaft power

After estimating the total efficiency, the shaft power of an AST, i.e. the power delivered, is evaluated with the formula used for turbines:

\[ P_s = 9.81\eta QH \text{ [kW]}. \] \hspace{1cm} (25)

4. Design procedure

The considerations presented in the previous sections allow the formulation of a simple design procedure for ASTs. This procedure is presented in form of a flowchart in Figure 6. Once the turbine head and
5. Conclusions
The paper presents useful considerations regarding the choosing of the parameters of an Archimedean screw turbine during its design. A formula for calculating the outer diameter of the screw was derived based on a simple approximation of the bucket volume and the correction factor that accounts for the approximation made was derived based on regression of data available for ASTs that operate with efficiencies higher than 60%. A comparison between diameters calculated with this formula and the real diameters of the ASTs considered shows a good agreement in most of the cases. The expression of the outer diameter was subsequently used to find a formula for calculating the turbine speed. Based on the considerations presented in the paper, a simple design procedure for ASTs was proposed.

A useful extension of this work would be a simple procedure for estimating the axial and tangential components of the hydrostatic forces that act on the blades of the Archimedean screw turbine. These components could be subsequently used to better estimate the power that can be delivered by the turbine and to more accurately assess the loads on the bearings.

Figure 6. Design procedure for Archimedean screw turbines.

If $H$ and $Q$ are known, the design procedure is straightforward and allows to calculate step by step all parameters detailed in Section 3.
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