$N_{nc}$ $Z$-open Sets and Continuity in $N_{nc}$ Topological Spaces

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Abstract. The aim of this paper is to introduce and study the notion of $N_{nc}$-$Z$-open sets and $N_{nc}$-$Z$-continuity. Some characterizations of these notions are presented. Also, some topological operations such as: $N_{nc}$-$Z$-boundary, $N_{nc}$-$Z$-exterior, $N_{nc}$-$Z$-limit etc, are introduced.

Keywords and phrases: $N_{nc}$-$Z$-open sets, locally $N_{nc}$-$Z$ closed, $N_{nc}$-$Z$-boundary, $N_{nc}$-$Z$-exterior, $N_{nc}$-$Z$-limit, $N_{nc}$-$Z$*-neighbourhood and $N_{nc}$-$Z$-continuity.

1. Introduction
Smarandache’s neutrosophic framework have wide scope of constant applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, dynamic, Medicine, Electrical & Electronic, and Management Science and so forth [1, 2, 3, 4, 19, 20]. Topology is an classical subject, as a generalization topological spaces numerous kinds of topological spaces presented throughout the year. Smarandache [13] characterized the Neutrosophic set on three segment Neutrosophic sets (T-Truth, F-Falshood, I-Indeterminacy). Neutrosophic topological spaces ($nts$’s) presented by Salama and Alblowi [10]. Lellies Thivagar et.al. [8] was given the geometric existence of $N$ topology, which is a non-empty set equipped with $N$ arbitrary topologies. Lellis Thivagar et al. [9] introduced the notion of $N_n$-open (closed) sets in $N$ neutrosophic crisp topological spaces. Al-Hamido et al. [5] investigate the chance of extending the idea of neutrosophic crisp topological spaces into $N$-neutrosophic crisp topological spaces and examine a portion of their essential properties. In 2008, Ekici [6] introduced the notion of $e$-open sets in topology. In 2020, Vadivel and John Sundar [16] introduced $N$-neutrosophic $\delta$-open, $N$-neutrosophic $\delta$-semiopen and $N$-neutrosophic $\delta$-preopen sets are introduced. The purpose of this paper is to introduce and study the notion of $N_{nc}$-$Z$-open sets and $N_{nc}$-$Z$-continuity. Some topological operations such as: $N_{nc}$-$Z$-limit, $N_{nc}$-$Z$-boundary and $N_{nc}$-$Z$-exterior etc are introduced. Also, some characterizations of these notions are presented.
2. Preliminaries

Salama and Smarandache [12] presented the idea of a neutrosophic crisp set in a set $P$ and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two neutrosophic crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty (resp., whole) set as more then two types. And they studied some properties related to neutrosophic crisp set operations. However, by selecting only one type, we define the inclusion, the intersection (union), and neutrosophic crisp empty (resp., whole) set again and discover a few properties.

**Definition 2.1** Let $P$ be a non-empty set. Then $H$ is called a neutrosophic crisp set (in short, $ncs$) in $P$ if $H$ has the form $H = (H_1, H_2, H_3)$, where $H_1, H_2,$ and $H_3$ are subsets of $P$,

The neutrosophic crisp empty (resp., whole) set, denoted by $\phi_n$ (resp., $P_n$) is an $ncs$ in $P$ defined by $\phi_n = (\phi, \phi, P)$ (resp. $P_n = (P, P, \phi)$). We will denote the set of all $ncs$'s in $P$ as $ncS(P)$.

In particular, Salama and Smarandache [11] classified a neutrosophic crisp set as the followings.

A neutrosophic crisp set $H = (H_1, H_2, H_3)$ in $P$ is called a neutrosophic crisp set of Type 1 (resp. 2 & 3) (in short, $ncs$-Type 1 (resp. 2 & 3)), if it satisfies $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ (resp. $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ and $H_1 \cup H_2 \cup H_3 = P$).

$ncS_1(P)$ ($ncS_2(P)$ and $ncS_3(P)$) means set of all $ncs$ Type 1 (resp. 2 and 3).

**Definition 2.2** Let $H = (H_1, H_2, H_3), M = (M_1, M_2, M_3) \in ncS(P)$. Then $H$ is said to be contained in (resp. equal to) $M$, denoted by $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_1, H_2 \subseteq M_2$ and $H_3 \subseteq M_3$ (resp. $H \subseteq M$ and $M \subseteq H$), $H^c = (H_3, H_2, H_1), H \cap M = (H_1 \cap M_1, H_2 \cap M_2, H_3 \cap M_3),H \cup M = (H_1 \cup M_1, H_2 \cup M_2, H_3 \cup M_3)$. Then $\bigcap_{j \in J} H_j$ (simply $\bigcap H_j$) = $(\bigcap H_{j_1}, \bigcap H_{j_2}, \bigcap H_{j_3}); \bigcup_{j \in J} H_j$ (simply $\bigcup H_j$) = $(\bigcup H_{j_1}, \bigcup H_{j_2}, \bigcup H_{j_3})$. The following are the quick consequence of Definition 2.2.

**Proposition 2.1** [7] Let $L, M, O \in ncS(P)$. Then

(i) $\phi_n \subseteq L \subseteq P_n$,
(ii) if $L \subseteq M$ and $M \subseteq O$, then $L \subseteq O$,
(iii) $L \cap M \subseteq L$ and $L \cap M \subseteq M$,
(iv) $L \subseteq L \cup M$ and $M \subseteq L \cup M$,
(v) $L \subseteq L$ iff $L \cap M = L$,
(vi) $L \subseteq M$ iff $L \cup M = M$.

Likewise the following are the quick consequence of Definition 2.2.

**Proposition 2.2** [7] Let $L, M, O \in ncS(P)$. Then

(i) $L \cup L = L, L \cap L = L$ (Idempotent laws),
(ii) $L \cup M = M \cup L, L \cap M = M \cap L$ (Commutative laws),
(iii) (Hssociative laws) : $L \cup (M \cup O) = (L \cup M) \cup O, L \cap (M \cap O) = (L \cap M) \cap O$,
(iv) (Distributive laws:) $L \cup (M \cap O) = (L \cup M) \cap (L \cap O), L \cap (M \cup O) = (L \cap M) \cup (L \cap O)$,
(v) (Hbsorption laws) : $L \cup (L \cap M) = L, L \cap (L \cup M) = L$,
(vi) (DeMorgan’s laws) : $(L \cup M)^c = L^c \cap M^c, (L \cap M)^c = L^c \cup M^c$,
(vii) $(L^c)^c = L$,
(viii) (a) $L \cup \phi_n = L, L \cap \phi_n = \phi_n$,
(b) $L \cup P_n = P_n, L \cap P_n = L$. 

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The complement of an $\mathcal{N}$ called neutrosophic crisp open sets (briefly, ncos) on a non-empty set $P$ is a family $\tau$ of nc subsets of $P$ satisfying the following axioms

(i) $\phi_n, P_n \in \tau$.
(ii) $H_1 \cap H_2 \in \tau \forall H_1, H_2 \in \tau$.
(iii) $\bigcup_{a} H_a \in \tau$, for any $\{H_a : a \in J\} \subseteq \tau$.

Then $(P, \tau)$ is a neutrosophic crisp topological space (briefly, ncsts) in $P$. The $\tau$ elements are called neutrosophic crisp open sets (briefly, ncos) in $P$. A nc $C$ is closed set (briefly, nccts) iff its complement $C^c$ is ncos.

Definition 2.4 [5] Let $P$ be a non-empty set. Then $N_1, N_2, \cdots, N_N$ are $N$-arbitrary crisp topologies defined on $P$ and the collection $N_{nc\tau} = \{H \subseteq P : H = (\bigcup_{j=1}^{N} H_j) \cup (\bigcap_{j=1}^{N} L_j), H_j, L_j \in N_{nc\tau}\}$ is called $N_{nc}$-topology on $P$ if the axioms are satisfied:

(i) $\phi_n, P_n \in N_{nc\tau}$.
(ii) $\bigcup_{j=1}^{\infty} H_j \in N_{nc\tau} \forall \{H_j\}_{j=1}^{\infty} \in N_{nc\tau}$.
(iii) $\bigcap_{j=1}^{n} H_j \in N_{nc\tau} \forall \{H_j\}_{j=1}^{n} \in N_{nc\tau}$.

Then $(P, N_{nc\tau})$ is called a $N_{nc}$-topological space (briefly, $N_{nc}$-nts) on $P$. The $N_{nc\tau}$ elements are called $N_{nc}$-open sets ($N_{nc}$-os) on $P$ and its complement is called $N_{nc}$-closed sets ($N_{nc}$-cts) on $P$. The elements of $P$ are known as $N_{nc}$-sets ($N_{nc}$-s) on $P$.

Definition 2.5 [5] Let $(P, N_{nc\tau})$ be $N_{nc}$-nts on $P$ and $H$ be an $N_{nc}$-os on $P$, then the $N_{nc}$ interior of $H$ (briefly, $N_{nc}$int$(H)$) and $N_{nc}$ closure of $H$ (briefly, $N_{nc}$cl$(H)$) are defined as

(i) $N_{nc}$int$(H) = \bigcup\{H : H \subseteq H \& H \text{ is a } N_{nc}$os in $P\} \& N_{nc}$cl$(H) = \bigcap\{C : H \subseteq C \& C \text{ is a } N_{nc}$cts in $P\}$.
(ii) $N_{nc}$-regular open [14] set (briefly, $N_{nc}$ros) if $H = N_{nc}$int$(N_{nc}$cl$(H))$.
(iii) $N_{nc}$-pre open set (briefly, $N_{nc}$pos) if $H \subseteq N_{nc}$int$(N_{nc}$cl$(H))$.
(iv) $N_{nc}$-semi open set (briefly, $N_{nc}$sos) if $H \subseteq N_{nc}$cl$(N_{nc}$int$(H))$.
(v) $N_{nc}$-$\alpha$-open set (briefly, $N_{nc}$aos) if $H \subseteq N_{nc}$int$(N_{nc}$cl$(H))$.
(vi) $N_{nc}$-$\gamma$-open set[14] (briefly, $N_{nc}$gos) if $H \subseteq N_{nc}$cl$(N_{nc}$int$(H)) \cup N_{nc}$int$(N_{nc}$cl$(H))$.
(vii) $N_{nc}$-$\beta$-open set[15] (briefly, $N_{nc}$bos) if $H \subseteq N_{nc}$cl$(N_{nc}$int$(H))$.

The complement of an $N_{nc}$ros (resp. $N_{nc}$sos, $N_{nc}$pos, $N_{nc}$aos, $N_{nc}$bos & $N_{nc}$gos) is called an $N_{nc}$-regular (resp. $N_{nc}$-semi, $N_{nc}$-pre, $N_{nc}$- & $N_{nc}$-$\gamma$) closed set (briefly, $N_{nc}$cts (resp. $N_{nc}$scs, $N_{nc}$pcs, $N_{nc}$acs, $N_{nc}$bos, $N_{nc}$gos & $N_{nc}$cts) in $P$.

The family of all $N_{nc}$ros (resp. $N_{nc}$cts, $N_{nc}$pos, $N_{nc}$pcs, $N_{nc}$acs, $N_{nc}$bos, $N_{nc}$gos & $N_{nc}$cts) of $P$ is denoted by $N_{nc}$ROS$(P)$ (resp. $N_{nc}$RCS$(P)$, $N_{nc}$POS$(P)$, $N_{nc}$PCS$(P)$, $N_{nc}$OS$(P)$, $N_{nc}$CS$(P)$, $N_{nc}$OS$(P)$, $N_{nc}$CS$(P)$, $N_{nc}$OS$(P)$ & $N_{nc}$CS$(P)$).
Definition 2.6 [16] A set $H$ is said to be a
(i) $N_{nc}\delta$ interior of $H$ (briefly, $N_{nc}int(H)$) is defined by $N_{nc}int(H) = \cup \{H : H \subseteq H \& H$ is a $N_{nc}os\}$. 
(ii) $N_{nc}\delta$ closure of $H$ (briefly, $N_{nc}\delta cl(H)$) is defined by $N_{nc}\delta cl(H) = \cup \{x \in P : N_{nc}int(N_{nc}cl(L)) \cap H \neq \phi, x \in L \& L$ is a $N_{nc}os\}$. 

Definition 2.7 [16] A set $H$ is said to be a
(i) $N_{nc}\delta$- open set (briefly, $N_{nc}\delta os$) if $H = N_{nc}int(H)$.
(ii) $N_{nc}\delta$-pre open set (briefly, $N_{nc}\delta P os$) if $H \subseteq N_{nc}int(N_{nc}\delta cl(H))$.
(iii) $N_{nc}\delta$-semi open set (briefly, $N_{nc}\delta S os$) if $H \subseteq N_{nc}cl(N_{nc}int(H))$.
(iv) $N_{nc}\delta a$ open set (briefly, $N_{nc}\delta a os$) if $H \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(H)))$.

The complement of an $N_{nc}\delta os$ (resp. $N_{nc}\delta P os$, $N_{nc}\delta S os$ & $N_{nc}\delta a os$) is called an $N_{nc}\delta$ (resp. $N_{nc}\delta$-pre, $N_{nc}\delta$-semi & $N_{nc}\delta a$) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}\delta P cs, N_{nc}\delta S cs \& N_{nc}\delta a cs$) in $Y$.

The family of all $N_{nc}\delta os$ (resp. $N_{nc}\delta cs, N_{nc}\delta P os, N_{nc}\delta S os, N_{nc}\delta S cs, N_{nc}\delta a os \& N_{nc}\delta a cs$) of $P$ is denoted by $N_{nc}\delta OS(P)$ (resp. $N_{nc}\delta CS(P), N_{nc}\delta P OS(P), N_{nc}\delta P CS(P), N_{nc}\delta S OS(P), N_{nc}\delta S CS(P), N_{nc}\delta a OS(P) \& N_{nc}\delta a CS(P)$).

Definition 2.8 [17] Let $H$ be an $N_{nc}s$ on a $N_{nc}ts P$. Then $H$ is said to be a
(i) $N_{nc}\epsilon$-open (briefly, $N_{nc}\epsilon o$) set if $H \subseteq N_{nc}cl(N_{nc}int(H)) \cup N_{nc}int(N_{nc}\delta cl(H))$.
(ii) $N_{nc}\epsilon$-closed (briefly, $N_{nc}\epsilon c$) set if $N_{nc}cl(N_{nc}int(H)) \cap N_{nc}int(N_{nc}\delta cl(H)) \subseteq H$.

The complement of an $N_{nc}\epsilon o$ set is called an $N_{nc}\epsilon$ closed (briefly, $N_{nc}\epsilon c$) set in $P$. The family of all $N_{nc}\epsilon o$ (resp. $N_{nc}\epsilon c$) set of $P$ is denoted by $N_{nc}\epsilon OS(P)$ (resp. $N_{nc}\epsilon CS(P)$). The $N_{nc}\epsilon$-interior of $H$ (briefly, $N_{nc}\epsilon int(H)$) and $N_{nc}\epsilon$-closure of $H$ (briefly, $N_{nc}\epsilon cl(H)$) are defined as $N_{nc}int(H) = \cup \{G : G \subseteq H \& G$ is a $N_{nc}\epsilon o$ set in $P\} \& N_{nc}cl(H) = \cap \{F : H \subseteq F \& F$ is a $N_{nc}\epsilon c$ set in $P\}$.

Definition 2.9 [18] A function $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ is called $N_{nc}$ precontinuous (resp. $N_{nc}$-semicontinuous, $N_{nc}\gamma$-continuous, $N_{nc}\epsilon$-continuous) (briefly, $N_{nc}PC ts$ (resp. $N_{nc}S C ts, N_{nc}\gamma C ts \& N_{nc}\epsilon C ts$)) if $h^{-1}(V)$ is $N_{nc}\epsilon P o$ (resp. $N_{nc}\epsilon S o, N_{nc}\gamma o \& N_{nc}\epsilon o$) for each $V \subseteq \sigma$.

3. $N_{nc}Z$-open sets

Definition 3.1 Let $(P, N_{nc}\tau)$ be a $N_{nc}ts$. Let $H$ be an $N_{nc}s$ in $(P, N_{nc}\tau)$. Then $H$ is said to be a
(i) $N_{nc}Z$-open (briefly, $N_{nc}Z o$) set if $H \subseteq N_{nc}cl(N_{nc}int(H)) \cup N_{nc}int(N_{nc}\delta cl(H))$,
(ii) $N_{nc}Z$-closed (briefly, $N_{nc}Z c$) set if $N_{nc}int(N_{nc}\delta cl(H)) \cap N_{nc}cl(N_{nc}int(H)) \subseteq H$.

The family of all $N_{nc}Z o$ (resp. $N_{nc}Z c$) sets of a space $(P, N_{nc}\tau)$ will be as always denoted by $N_{nc}Z OS(P)$ (resp. $N_{nc}Z CS(P)$).

Remark 3.1 One may notice that
(i) Every $N_{nc}\delta S o$ (resp. $N_{nc}\epsilon P o$) set is $N_{nc}Z o$,
(ii) Every $N_{nc}Z o$ set is $N_{nc}b o$ (resp. $N_{nc}\epsilon o$).

But not conversely.

Example 3.1 Let $P = \{a, b, c, d, e\}$, $N_{nc}\tau_1 = \{\phi, P_N, H, B, C\}$, $N_{nc}\tau_2 = \{\phi, P_N\}$. $A = \langle\{c\}, \phi\rangle, B = \langle\{a, b\}, \phi, \{c, d, e\}\rangle, C = \langle\{a, b, c\}, \phi\rangle, \{d, e\}$, then we have $N_{nc}Z = \{\phi, P_N, H, B, C\}$. Then,
(i) \( \langle \{a\}, \{\phi\}, \{b, c, d, e\} \rangle \) is a \( 2_{nc}Zos \) but not \( 2_{nc}\delta Sos \).
(ii) \( \langle \{c, d\}, \{\phi\}, \{a, b, e\} \rangle \) is a \( 2_{nc}Zos \) but not \( 2_{nc}\delta Pos \).

Example 3.2 Let \( P = \{a, b, c, d\} \), \( \nc\tau_1 = \{\phi_N, P_N, H, B, C, D\} \), \( \nc\tau_2 = \{\phi_N, P_N, E, F\} \). Let
\[ A = \langle \{a\}, \{\phi\}, \{b, c, d\} \rangle, \quad B = \langle \{c\}, \{\phi\}, \{a, b, d\} \rangle, \quad C = \langle \{a, c\}, \{\phi\}, \{b, d\} \rangle, \quad D = \langle \{a, c, d\}, \{\phi\}, \{b\} \rangle, \quad E = \langle \{a, b\}, \{\phi\}, \{c, d\} \rangle, \quad F = \langle \{a, b, c\}, \{\phi\}, \{d\} \rangle, \]
then we have \( 2_{nc}\tau = \{\phi_N, P_N, H, B, C, D, E, F\} \). The set
(i) \( \langle \{a, d\}, \{\phi\}, \{b, c\} \rangle \) is a \( 2_{nc}\gamma os \) but not \( 2_{nc}Zos \).
(ii) \( \langle \{b, c\}, \{\phi\}, \{a, d\} \rangle \) is a \( 2_{nc}\epsilon os \) but not \( 2_{nc}Zos \).

Remark 3.2 According to Definition 3.1 and Remark 3.1, the following diagram holds for a subset \( H \) of a space \( P \):

![Diagram](image-url)

Lemma 3.1 Let \( H, M \) be two \( N_{nc} \) sets of \( (P, N_{nc}\tau) \). Then:
(i) \( P \backslash (N_{nc}\int_{\delta}(H)) = N_{nc}\cl_{\delta}(P \backslash H) \) and \( N_{nc}\int_{\delta}(P \backslash H) = P \backslash (N_{nc}\cl_{\delta}(H)) \),
(ii) \( N_{nc}\cl(H) \subseteq N_{nc}\cl_{\delta}(H) \) (resp. \( N_{nc}\int_{\delta}(H) \subseteq N_{nc}\int(H) \)), for any \( N_{nc} \) set \( H \) of \( P \),
(iii) \( N_{nc}\cl(H \cup M) = N_{nc}\cl_{\delta}(H) \cup N_{nc}\cl_{\delta}(M) \), \( N_{nc}\int_{\delta}(H \cap M) = N_{nc}\int(H) \cap N_{nc}\int_{\delta}(M) \).

Proposition 3.1 Let \( H \) be a \( N_{nc} \) set of a space \( (P, N_{nc}\tau) \). Then:
(i) \( N_{nc}\cl(H) = H \cup N_{nc}\int(N_{nc}\cl(H)) \), \( N_{nc}\int(H) = H \cap N_{nc}\cl(N_{nc}\int(H)) \),
(ii) \( N_{nc}\cl_{\delta}(P \backslash H) = P \backslash N_{nc}\int_{\delta}(H) \), \( N_{nc}\int_{\delta}(H) = H \cap N_{nc}\cl(N_{nc}\int_{\delta}(H)) \),
(iii) \( N_{nc}\cl_{\delta}(P \backslash H) = P \backslash N_{nc}\int_{\delta}(H) \), \( N_{nc}\cl_{\delta}(H \cup M) \subseteq N_{nc}\cl(H) \cup N_{nc}\cl_{\delta}(M) \),
(iv) \( N_{nc}\int_{\delta}(H) = H \cup N_{nc}\cl_{\delta}(H) \), \( N_{nc}\cl_{\delta}(H \cup M) \subseteq N_{nc}\cl(H) \cup N_{nc}\cl_{\delta}(M) \).

Lemma 3.2 [17] The following hold for a \( N_{nc} \) set \( H \) of a space \( (P, N_{nc}\tau) \).
(i) \( N_{nc}\cl_{\delta}(H) = H \cup N_{nc}\cl(N_{nc}\int_{\delta}(H)) \) and \( N_{nc}\int_{\delta}(H) = H \cap N_{nc}\cl(N_{nc}\cl_{\delta}(H)) \),
(ii) \( N_{nc}\cl_{\delta}(H) = H \cup N_{nc}\cl(N_{nc}\int_{\delta}(H)) \) and \( N_{nc}\int_{\delta}(H) = H \cap N_{nc}\cl(N_{nc}\cl_{\delta}(H)) \).

Lemma 3.3 The following hold for a \( N_{nc} \) set \( H \) of a space \( (P, N_{nc}\tau) \).
\( N_{nc}\cl(N_{nc}\int_{\delta}(H)) = N_{nc}\cl_{\delta}(N_{nc}\int_{\delta}(H)) \) and \( N_{nc}\int(N_{nc}\cl_{\delta}(H)) = N_{nc}\int_{\delta}(N_{nc}\cl_{\delta}(H)) \).

Theorem 3.1 Let \( (P, N_{nc}\tau) \) be a \( N_{nc}ts \). Then:
(i) If \( H \in N_{nc}\delta OS(P) \) and \( M \in N_{nc}ZOS(P) \), then \( H \cap M \) is \( N_{nc}Zo \),
(ii) If \( H \in N_{nc}\tau \) and \( M \in N_{nc}ZOS(P) \), then \( H \cap M \) is \( N_{nc}bo \),
(iii) If \( H \in N_{nc}aO(P, N_{nc}\tau) \) and \( M \in N_{nc}ZOS(P, N_{nc}\tau) \), then \( H \cap M \in N_{nc}ZOS(P, N_{nc}\tau) \).
Proof. (i) Suppose that $H \in N_{nc} \delta O S(P)$. Then $H = N_{nc} int(H)$. Since $M \in N_{nc} ZOS(P)$, then $M \subseteq N_{nc} cl(N_{nc} int(H)) \cup N_{nc} int(N_{nc} cl(M))$ and hence

$$H \cap M \subseteq N_{nc} int(H) \cap (N_{nc} cl(N_{nc} int(H)) \cup N_{nc} int(N_{nc} cl(M)))$$

$$= (N_{nc} int(H) \cap N_{nc} cl(N_{nc} int(H))) \cup (N_{nc} int(H) \cap N_{nc} int(N_{nc} cl(M)))$$

$$\subseteq N_{nc} cl(N_{nc} int(H) \cap (N_{nc} int(H)) \cup N_{nc} int(N_{nc} cl(H) \cap N_{nc} cl(M)))$$

$$\subseteq N_{nc} cl(N_{nc} int(H) \cap M) \cup N_{nc} int(N_{nc} cl(H \cap M)).$$

Thus $H \cap M \subseteq N_{nc} cl(N_{nc} int(H \cap M)) \cup N_{nc} int(N_{nc} cl(H \cap M))$. Therefore, $H \cap M$ is $N_{nc} ZO$.

(ii) It is similar to that of (i).

(iii) Since

$$H \cap M \subseteq N_{nc} int(N_{nc} cl(N_{nc} int(H))) \cap (N_{nc} cl(N_{nc} int(H)) \cup N_{nc} int(N_{nc} cl(M)))$$

$$= (N_{nc} int(N_{nc} cl(N_{nc} int(H))) \cap N_{nc} cl(N_{nc} int(H)) \cup (N_{nc} int(N_{nc} cl(N_{nc} int(H))))$$

$$\cap N_{nc} int(N_{nc} cl(M))$$

$$\subseteq N_{nc} cl(N_{nc} int(H) \cap N_{nc} int(H)) \cup N_{nc} int(N_{nc} cl(N_{nc} int(H))) \cap N_{nc} int(N_{nc} cl(H \cap M)))$$

and hence

$$H \cap M \subseteq (H \cap N_{nc} cl(N_{nc} int(H) \cap N_{nc} int(H))) \cup (H \cap N_{nc} int(N_{nc} cl(H))) \cap N_{nc} int(N_{nc} cl(M)))$$

$$\subseteq N_{nc} cl_{H}(N_{nc} int(H) \cap N_{nc} int(H)) \cup N_{nc} int_{H}(N_{nc} cl(H) \cap N_{nc} int(H)) \cap N_{nc} cl_{H}(N_{nc} int(H) \cap M))$$

$$\subseteq N_{nc} cl_{H}(N_{nc} int(H) \cap N_{nc} int(H)) \cup N_{nc} int_{H}(N_{nc} cl(H) \cap N_{nc} int(H)) \cup N_{nc} int_{H}(N_{nc} cl(H \cap M))$$.

Since

$$N_{nc} int_{H}(N_{nc} int(H) \cap N_{nc} int(H)) \subseteq N_{nc} int_{H}(N_{nc} int(H)) \subseteq H$$

which is $N_{nc} \delta O$ in $H$, then

$$H \cap M \subseteq N_{nc} cl_{H}(N_{nc} int(H) \cap N_{nc} int(H)) \cup N_{nc} int_{H}(H \cap N_{nc} cl(H)) \cap N_{nc} cl_{H}(N_{nc} int(H) \cap M))$$

$$\subseteq N_{nc} cl_{H}(N_{nc} int(H) \cap M) \cup N_{nc} int_{H}(N_{nc} cl_{H}(N_{nc} int(H) \cap M))$$

Therefore $H \cap M \subseteq N_{nc} ZOS(P, N_{nc} \tau H)$.

Proposition 3.2 Let $(P, N_{nc} \tau)$ be a $N_{nc} ts$. Then the closure of a $N_{nc} ZO$ set of $P$ is $N_{nc} So$.

Proof. Let $H \in N_{nc} ZOS(P)$. Then

$$N_{nc} cl(H) \subseteq N_{nc} cl(N_{nc} cl(N_{nc} int(H)) \cup N_{nc} int(N_{nc} cl(H)))$$

$$\subseteq N_{nc} cl(N_{nc} int(H)) \cup N_{nc} cl(N_{nc} int(H)))$$

$$= N_{nc} cl(N_{nc} int(H))).$$

Therefore, $N_{nc} cl(H)$ is $N_{nc} So$.

Proposition 3.3 Let $H$ be a $N_{nc} ZO$ set of a $N_{nc} ts (P, N_{nc} \tau)$ and $N_{nc} int_{H}(H) = \phi$. Then $H$ is $N_{nc} P_{o}$.

Proof. obvious.
Lemma 3.4 Let \((P, N_{nc})\) be a \(N_{nc}\)ts. Then the following statements are hold.

(i) The union of arbitrary \(N_{nc}Zo\) sets is \(N_{nc}Zo\).

(ii) The intersection of arbitrary \(N_{nc}Zc\) sets is \(N_{nc}Zc\).

Proof. (i) Let \(\{H_i, i \in I\}\) be a family of \(N_{nc}Zo\) sets. Then \(H_i \subseteq N_{nc}\text{cl}(N_{nc}\text{int}(H_i)) \cup N_{nc}\text{int}(N_{nc}\text{cl}(H_i))\) and hence \(\bigcup_i H_i \subseteq \bigcup_i (N_{nc}\text{cl}(N_{nc}\text{int}(H_i)) \cup N_{nc}\text{int}(N_{nc}\text{cl}(H_i))) \subseteq N_{nc}\text{cl}(N_{nc}\text{int}(\bigcup_i H_i)) \cup N_{nc}\text{int}(N_{nc}\text{cl}(\bigcup_i H_i)),\) for all \(i \in I\). Thus \(\bigcup_i H_i\) is \(N_{nc}Zo\).

(ii) It follows from (i).

Remark 3.3 By the following we show that the intersection of any two \(N_{nc}Zo\) sets is not \(N_{nc}Zo\).

Example 3.3 In Example 3.1, the sets \(\langle\{a, b, e\}, \{\phi\}, \{c, d\}\rangle\) and \(\langle\{b, c, e\}, \{\phi\}, \{a, d\}\rangle\) are \(2_{nc}Zos\) but the intersection \(\langle\{b, e\}, \{\phi\}, \{a, c, d\}\rangle\) is not \(2_{nc}Zos\).

Definition 3.2 Let \((P, N_{nc})\) be a \(N_{nc}\)ts. Then:

(i) The union of all \(N_{nc}Zo\) sets of \(P\) contained in \(H\) is called the \(N_{nc}Z\) interior of \(H\) and is denoted by \(N_{nc}Zint(H)\).

(ii) The intersection of all \(N_{nc}Zc\) sets of \(P\) containing \(H\) is called the \(N_{nc}Z\)-closure of \(H\) and is denoted by \(N_{nc}Zcl(H)\).

Theorem 3.2 Let \(H, M\) be two \(N_{nc}\)ts of a \(N_{nc}\)ts \((P, N_{nc})\). Then the following are hold:

(i) \(N_{nc}Zcl(P \setminus H) = P \setminus N_{nc}Zint(H)\),

(ii) \(N_{nc}Zint(P \setminus H) = P \setminus N_{nc}Zcl(H)\),

(iii) If \(H \subseteq M\), then \(N_{nc}Zcl(H) \subseteq N_{nc}Zcl(M)\) and \(N_{nc}Zint(H) \subseteq N_{nc}Zint(M)\),

(iv) \(x \in N_{nc}Zcl(H)\) iff for each a \(N_{nc}Zo\) set \(O\) contains \(x\), \(O \cap H \neq \phi\),

(v) \(x \in N_{nc}Zint(H)\) iff there exist a \(N_{nc}Zo\) set \(W\) such that \(x \in W \subseteq H\),

(vi) \(N_{nc}Zcl(N_{nc}Zcl(H)) = N_{nc}Zcl(H)\) and \(N_{nc}Zint(N_{nc}Zint(H)) = N_{nc}Zint(H)\),

(vii) \(N_{nc}Zcl(H) \cup N_{nc}Zcl(M) \subseteq N_{nc}Zcl(H \cup M)\) and \(N_{nc}Zint(H) \cup N_{nc}Zint(M) \subseteq N_{nc}Zint(H \cup M)\),

(viii) \(N_{nc}Zint(H \cap M) \subseteq N_{nc}Zint(H) \cap N_{nc}Zint(M)\) and \(N_{nc}Zcl(H \cap M) \subseteq N_{nc}Zcl(H) \cap N_{nc}Zcl(M)\).

Proof. (i) It follows from Definition 3.2.

Remark 3.4 By the following example we show that the inclusion relation in parts (vii) and (viii) of the above theorem cannot be replaced by equality.

Example 3.4 In Example 3.1, the sets

(i) \(A = \langle\{a, b\}, \{\phi\}, \{c, d, e\}\rangle\) and \(B = \langle\{c, d\}, \{\phi\}, \{a, b, e\}\rangle\), then \(A \cup B = \langle\{a, b, c, d\}, \{\phi\}, \{e\}\rangle\). \(2_{nc}Zcl(A) = \langle\{a, b\}, \{\phi\}, \{c, d, e\}\rangle\) and \(2_{nc}Zcl(A \cup B) = \langle\{a, b, c, d\}, \{\phi\}, \{e\}\rangle\) and \(2_{nc}Zcl(A) \cap 2_{nc}Zcl(B) \subseteq 2_{nc}Zcl(A)\).

(ii) \(C = \langle\{a, c\}, \{b, d, e\}\rangle\) and \(D = \langle\{c, d\}, \{\phi\}, \{a, b, e\}\rangle\), then \(C \cap D = \langle\{c\}, \{\phi\}, \{a, b, d, e\}\rangle\). \(2_{nc}Zcl(C) = \langle\{a, c, d, e\}, \{\phi\}, \{b\}\rangle\) and \(2_{nc}Zcl(D) = \langle\{c, d\}, \{\phi\}, \{a, b, e\}\rangle\) and \(2_{nc}Zcl(C \cap D) = \langle\{c\}, \{\phi\}, \{a, b, d, e\}\rangle\). Thus \(2_{nc}Zcl(C) \cap 2_{nc}Zcl(D) \subseteq 2_{nc}Zcl(C \cap D)\).

(iii) \(E = \langle\{a, d\}, \{\phi\}, \{b, c, e\}\rangle\) and \(F = \langle\{b, d\}, \{\phi\}, \{a, c, e\}\rangle\). \(2_{nc}Zint(E) = \phi\) and \(2_{nc}Zint(F) = \langle\{b\}, \{\phi\}, \{a, c, d, e\}\rangle\) and \(2_{nc}Zint(E \cup F) = \langle\{a, b, d\}, \{\phi\}, \{c, e\}\rangle\). Thus \(2_{nc}Zint(E \cup F) \not\subseteq 2_{nc}Zint(E) \cup 2_{nc}Zint(F)\).
Theorem 3.3 Let \((P, N_{nc})\) be a \(N_{nc}ts\) and \(H \subseteq P\). Then \(H\) is a \(N_{nc}Zo\) set iff \(H = N_{nc}int_{\delta}(H) \cup N_{nc}pint(H)\).

Proof. Let \(H\) be a \(N_{nc}Zo\) set. Then \(H \subseteq N_{nc}cl(N_{nc}int_{\delta}(H)) \cup N_{nc}int(N_{nc}cl(H))\) and hence by Proposition 3.1 and Lemma 3.2,

\[
N_{nc}int_{\delta}(H) \cup N_{nc}pint(H) = (H \cap N_{nc}cl(N_{nc}int_{\delta}(H))) \cup (H \cap N_{nc}int(N_{nc}cl(H))) = H \cap (N_{nc}cl(N_{nc}int_{\delta}(H)) \cup N_{nc}int(N_{nc}cl(H))) = H.
\]

The converse it follows from Proposition 3.1 and Lemma 3.2.

Proposition 3.4 Let \((P, N_{nc})\) be a \(N_{nc}ts\) and \(H \subseteq P\). Then \(H\) is a \(N_{nc}Zc\) set iff \(H = N_{nc}pcl(H) \cap N_{nc}pint(H)\).

Proof. It follows from Theorem 3.3.

Theorem 3.5 Let \(H\) be a \(N_{nc}\) set of a space \((P, N_{nc})\). Then:

(i) \(N_{nc}Zcl(H) = N_{nc}scl_{\delta}(H) \cap N_{nc}pcl(H)\),

(ii) \(N_{nc}Zint(H) = N_{nc}int_{\delta}(H) \cup N_{nc}pint(H)\).

Proof. (i) It is easy to see that \(N_{nc}Zcl(H) \subseteq N_{nc}scl_{\delta}(H) \cap N_{nc}pcl(H)\). Also, \(N_{nc}scl_{\delta}(H) \cap N_{nc}pcl(H)) = (H \cup N_{nc}int(N_{nc}cl_{\delta}(H))) \cap (H \cup N_{nc}cl(N_{nc}int(H))) = H \cup (N_{nc}int(N_{nc}cl_{\delta}(H)) \cap N_{nc}cl(N_{nc}int(H))).\) Since \(N_{nc}Zcl(H)\) is \(N_{nc}Zc\), then \(N_{nc}Zcl(H) \supseteq N_{nc}int(N_{nc}cl_{\delta}(N_{nc}Zcl(H))) \cap N_{nc}cl(N_{nc}int(N_{nc}Zcl(H))) \supseteq N_{nc}int(N_{nc}cl_{\delta}(H)) \cap N_{nc}cl(N_{nc}int(H)).\)

Thus \(H \cup (N_{nc}int(N_{nc}cl_{\delta}(H)) \cap N_{nc}cl(N_{nc}int(H))) \subseteq H \cup N_{nc}Zcl(H) = N_{nc}Zcl(H)\) and hence, \(N_{nc}scl_{\delta}(H) \cap N_{nc}pcl(H) \subseteq N_{nc}Zcl(H)\). So, \(N_{nc}Zcl(H) = N_{nc}scl_{\delta}(H) \cap N_{nc}pcl(H)\).

(ii) It follows from (i).

Theorem 3.6 Let \(H\) be a \(N_{nc}\) set of a space \((P, N_{nc})\). Then:

(i) \(H\) is a \(N_{nc}Zo\) set,

(ii) \(H\) is a \(N_{nc}Zc\) set if \(H = N_{nc}int_{\delta}(H) \cup N_{nc}pint(H)\).

Proof. (i) It follows from Theorems 3.3 & 3.4.

Lemma 3.5 Let \(H\) be a \(N_{nc}\) set of a \(N_{nc}ts\) \((P, N_{nc})\). Then the following statement are hold:

(i) \(N_{nc}pint_{\delta}(N_{nc}pcl(H)) = N_{nc}pcl(H) \cap N_{nc}int(N_{nc}cl_{\delta}(H))\),

(ii) \(N_{nc}pcl_{\delta}(N_{nc}pint(H)) = N_{nc}pint(H) \cup N_{nc}cl(N_{nc}int_{\delta}(H))\).

Proof. (i) By Lemma 3.3, \(N_{nc}pint_{\delta}(N_{nc}pcl(H)) = N_{nc}pcl(H) \cap N_{nc}int(N_{nc}cl_{\delta}(N_{nc}pcl(H))) = N_{nc}pcl(H) \cap N_{nc}int(N_{nc}cl_{\delta}(H)) = N_{nc}pcl(H) \cap N_{nc}cl(N_{nc}int_{\delta}(H))) = N_{nc}pint(H) \cup N_{nc}cl(N_{nc}int_{\delta}(H)).\)

(ii) It follows from (i).

Proposition 3.5 Let \(H\) be a \(N_{nc}\) set of a \(N_{nc}ts\) \((P, N_{nc})\). Then:

(i) \(N_{nc}Zcl(H) = H \cup N_{nc}pint_{\delta}(N_{nc}pcl(H))\),

(ii) \(N_{nc}Zint(H) = H \cap N_{nc}pcl_{\delta}(N_{nc}pint(H))\).

Proof. (i) By Lemma 3.5, \(H \cup N_{nc}pint_{\delta}(N_{nc}pcl(H)) = H \cup (N_{nc}pcl(H) \cap N_{nc}int(N_{nc}cl_{\delta}(H))) = (H \cup N_{nc}pcl(H)) \cap (H \cap N_{nc}int(N_{nc}cl_{\delta}(H))) = N_{nc}pcl(H) \cap N_{nc}pcl(N_{nc}cl_{\delta}(H)) = N_{nc}Zcl(H)\).

(ii) It follows from (i).

Theorem 3.6 Let \(H\) be a \(N_{nc}\) set of a \(N_{nc}ts\) \((P, N_{nc})\). Then the following are equivalent:

(i) \(H\) is a \(N_{nc}Zo\) set,
(ii) $H \subseteq N_{nc} pcl_{\delta}(N_{nc} pint(H))$,
(iii) there exists $O \in N_{nc} POS(P)$ such that $O \subseteq H \subseteq N_{nc} pcl_{\delta}(O)$,
(iv) $N_{nc} pcl_{\delta}(H) = N_{nc} pcl(N_{nc} pint(H))$.

**Proof.** (i) $\Rightarrow$ (ii). Let $H$ be a $N_{nc} Zo$ set. Then by Theorem 3.5, $H = N_{nc} Zint(H)$ and by Proposition 3.5, $H = H \cap N_{nc} pcl_{\delta}(N_{nc} pint(H))$ and hence, $H \subseteq N_{nc} pcl_{\delta}(N_{nc} pint(H))$.

(ii) $\Rightarrow$ (i). Let $H \subseteq N_{nc} pcl_{\delta}(N_{nc} pint(H))$. Then by Proposition 3.5, $H \subseteq H \cap N_{nc} pcl_{\delta}(N_{nc} pint(H)) = N_{nc} Zint(H)$, and hence $H = N_{nc} Zint(H)$. Thus $H$ is $N_{nc} Zo$.

(iii) $\Rightarrow$ (ii). Let there exists $O \in N_{nc} POS(P)$ such that $O \subseteq H \subseteq N_{nc} pcl_{\delta}(O)$. Since $O \subseteq H$, then $N_{nc} pcl_{\delta}(O) \subseteq N_{nc} pcl_{\delta}(N_{nc} pint(H))$, therefore $H \subseteq N_{nc} pcl_{\delta}(O) \subseteq N_{nc} pcl_{\delta}(N_{nc} pint(H))$.

(ii) $\Rightarrow$ (iv). It is clear.

**Theorem 3.7** Let $H$ be a $N_{nc}$ set of a $N_{nc} ts P$. Then the following are equivalent:

(i) $H$ is a $N_{nc} Zo$ set,
(ii) $N_{nc} pint_{\delta}(N_{nc} pcl(H)) \subseteq H$,
(iii) there exists $O \in N_{nc} PCS(P)$ such that $N_{nc} pint_{\delta}(O) \subseteq H \subseteq O$,
(iv) $N_{nc} pint_{\delta}(H) = N_{nc} pint_{\delta}(N_{nc} pcl(H))$.

**Proof.** It follows from Theorem 3.6.

**Proposition 3.6** If $H$ is a $N_{nc} Zo$ set of a $N_{nc} ts (P, N_{nc} \tau)$ such that $H \subseteq M \subseteq N_{nc} pcl_{\delta}(H)$, then $M$ is $N_{nc} Zo$.

**Proof.** It is clear.

**Definition 3.3** $H$ $N_{nc}$ set $H$ of a $N_{nc} ts (P, N_{nc} \tau)$ is said to be locally $N_{nc} Zc$ if $H = O \cap F$, where $O \in N_{nc} \tau$ and $F \in N_{nc} ZCS(P)$.

**Theorem 3.8** Let $H$ be a $N_{nc}$ set of a space $P$. Then $H$ is locally $N_{nc} Zc$ iff $H = O \cap N_{nc} Zcl(H)$.

**Proof.** Since $H$ is a locally $N_{nc} Zc$ set, then $H = O \cap F$, where $O \in N_{nc} \tau$ and $F \in N_{nc} ZCS(P)$ and hence $H \subseteq N_{nc} Zu(H) \subseteq N_{nc} Zcl(F) = F$. Thus $H \subseteq O \cap N_{nc} Zcl(H) \subseteq O \cap N_{nc} Zcl(F) = H$.

Therefore $H = O \cap N_{nc} Zcl(H)$. The Converse is clear.

**Theorem 3.9** Let $H$ be a locally $N_{nc} Zc$ set of a space $(P, N_{nc} \tau)$. Then the following statement are hold:

(i) $N_{nc} Zcl(H) \setminus H$ is a $N_{nc} Zc$ set,
(ii) $(H \cup (P \setminus N_{nc} Zcl(H)))$ is a $N_{nc} Zo$,
(iii) $H \subseteq N_{nc} Zint(H \cup (P \setminus N_{nc} Zcl(H)))$.

**Proof.** (i) If $H$ is a locally $N_{nc} Zc$ set, then there exists an $N_{nc} o$ set $O$ such that $H = O \cap N_{nc} Zcl(H)$. Hence, $N_{nc} Zcl(H) \setminus H = N_{nc} Zcl(H) \setminus (O \cap N_{nc} Zcl(H)) = N_{nc} Zcl(H) \cap (P \setminus (O \cap N_{nc} Zcl(H))) = N_{nc} Zcl(H) \cap (P \setminus N_{nc} Zcl(H)) = N_{nc} Zcl(H) \cap (P \setminus O)$ which is $N_{nc} Zc$.

(ii) Since $N_{nc} Zcl(H) \setminus H$ is $N_{nc} Zc$, then $P \setminus (N_{nc} Zcl(H) \setminus H)$ is a $N_{nc} Zo$ set. Since $P \setminus (N_{nc} Zcl(H) \setminus H) = (P \setminus N_{nc} Zcl(H)) \cup (P \setminus H)$, then $H \cup (P \setminus N_{nc} Zcl(H))$ is $N_{nc} Zo$.

(iii) It follows from (ii).

**Definition 3.4** $H$ $N_{nc}$ set $H$ of a space $(P, N_{nc} \tau)$ is said to be $D(c, z)$ iff $N_{nc} int(H) = N_{nc} Zint(H)$. 

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Remark 3.5 One may notice that the concepts of $N_{nc}Zo$ and $D(c,z)$ are independent and by we show this the following example.

Example 3.5 In Example 3.1, the sets
(i) $\{\{a,c\}, \{\phi \}, \{b,d,e\}\}$ is a $2_{nc}Zos$ but not $D(c,z)$.
(ii) $\{\{d,e\}, \{\phi \}, \{a,b,c\}\}$ is a $D(c,z)$ but not $2_{nc}Zos$.

Theorem 3.10 Let $H$ be a $N_{nc}$ set of $N_{nc}ts$ $P$. Then the following are equivalent:
(i) $H$ is an $N_{nc}o$ set,
(ii) $H$ is $N_{nc}Zo$ and $D(c,z)$.

Proof. Obvious.

4. Some topological operations

Definition 4.1 Let $(P, N_{nc} \tau)$ be a $N_{nc}ts$ and $H \subseteq P$. Then the $N_{nc}Z$-boundary of $H$ (briefly, $N_{nc}Zb(H)$) is defined by $N_{nc}Zb(H) = N_{nc}Zcl(H) \cap N_{nc}Zcl(P \setminus H)$.

Theorem 4.1 If $H$ is a $N_{nc}$ set of a $N_{nc}ts$ $(P, N_{nc} \tau)$, then the following statement are hold:
(i) $N_{nc}Zb(H) = N_{nc}Zb(P \setminus H)$,
(ii) $N_{nc}Zb(H) = N_{nc}Zcl(H) \setminus N_{nc}Zint(H)$,
(iii) $N_{nc}Zb(H) \cap N_{nc}Zint(H) = \phi$,
(iv) $N_{nc}Zb(H) \cup N_{nc}Zint(H) = N_{nc}Zcl(H)$.

Proof. (i) It is clear.

Theorem 4.2 If $H$ is a $N_{nc}$ set of a space $P$, then the following statement are hold:
(i) $H$ is a $N_{nc}Zo$ set iff $H \cap N_{nc}Zb(H) = \phi$,
(ii) $H$ is a $N_{nc}Zc$ set iff $N_{nc}Zb(H) \subseteq H$,
(iii) $H$ is a $N_{nc}Z$-clopen set iff $N_{nc}Zb(H) = \phi$.

Proof. (i) It follows from Theorem 4.1.

Definition 4.2 Let $(P, N_{nc} \tau)$ be a $N_{nc}ts$ and $H \subseteq P$. Then the set $P \setminus (N_{nc}Zcl(H))$ is called the $N_{nc}Z$-exterior of $H$ and is denoted by $N_{nc}Zext(H)$. A point $p \in P$ is called a $N_{nc}Z$-exterior point of $H$, if it is a $N_{nc}Z$ interior point of $P \setminus H$.

Theorem 4.3 If $H$ and $M$ are two $N_{nc}$ sets of a space $(P, N_{nc} \tau)$, then the following statement are hold:
(i) $N_{nc}Zext(H) = N_{nc}Zint(P \setminus H)$,
(ii) $N_{nc}Zext(H) \cap N_{nc}Zb(H) = \phi$,
(iii) $N_{nc}Zext(H) \cup N_{nc}Zb(H) = N_{nc}Zcl(P \setminus H)$,
(iv) $\{N_{nc}Zint(H), N_{nc}Zb(H)\}$ and $N_{nc}Z - ext(H)$ form a partition of $P$,
(v) If $H \subseteq M$, then $N_{nc}Zext(M) \subseteq N_{nc}Zext(H)$,
(vi) $N_{nc}Zext(H \cup M) \subseteq N_{nc}Zext(H) \cup N_{nc}Zext(M)$,
(vii) $N_{nc}Zext(H \cap M) \supseteq N_{nc}Zext(H) \cap N_{nc}Zext(M)$,
(viii) $N_{nc}Zext(\phi) = P$ and $N_{nc}Zext(P) = \phi$.

Proof. It follows from Theorems 3.5 and 4.1.
Remark 4.1 The inclusion relation in parts (vi) and (vii) of the above theorem cannot be replaced by equality as is shown by the following example.

Example 4.1 In Example 3.1, the sets \( A = \{c, d\}, \{\phi\}, \{a, b, e\} \) and \( B = \{c, e\}, \{\phi\}, \{a, b, d\} \). Then the following statements hold:

(i) \( 2_{nc}\text{Zext}(A) \cup 2_{nc}\text{Zext}(B) \not\subset 2_{nc}\text{Zext}(A \cup B) \).

Definition 4.3 Let \( H \) is a \( N_{nc} \) set of a \( N_{nc}\text{ts} (P, N_{nc}\tau) \). Then a point \( p \in P \) is called a \( N_{nc}\text{Z-limit} \) point of a set \( H \subseteq P \) if every \( N_{nc}\text{Zo} \) set \( G \subseteq P \) containing \( p \) contains a point of \( H \) other than \( p \). The set of all \( N_{nc}\text{Z-limit} \) points of \( H \) is called a \( N_{nc}\text{Z-derived} \) set of \( H \) and is denoted by \( N_{nc}\text{Zd}(H) \).

Theorem 4.4 If \( H \) and \( M \) are two \( N_{nc} \) sets of a space \( P \), then the following statements are hold:

(i) If \( H \subseteq M \), then \( N_{nc}\text{Zd}(H) \subseteq N_{nc}\text{Zd}(M) \).

(ii) \( N_{nc}\text{Zd}(H) \cup N_{nc}\text{Zd}(M) \subseteq N_{nc}\text{Zd}(H \cup M) \).

(iii) \( N_{nc}\text{Zd}(H \cap M) \subseteq N_{nc}\text{Zd}(H) \cap N_{nc}\text{Zd}(M) \).

(iv) \( H \) is a \( N_{nc}\text{Zc} \) set iff it contains each of its \( N_{nc}\text{Z-limit} \) points.

(v) \( N_{nc}\text{Zd}(H) = H \cup N_{nc}\text{Zd}(H) \).

Proof. It is clear.

Definition 4.4 A \( N_{nc} \) set \( N \) of a \( N_{nc}\text{ts} (P, N_{nc}\tau) \) is called a \( N_{nc}\text{Z-neighbourhood} \) (briefly, \( N_{nc}\text{Znbd} \)) of a point \( p \in P \) if there exists a \( N_{nc}\text{Zo} \) set \( W \) such that \( p \in W \subseteq N \). The class of all \( N_{nc}\text{Znbd}'s \) of \( p \in P \) is called the \( N_{nc}\text{Z-neighbourhood} \) system of \( p \) and denoted by \( N_{nc}\text{ZN}_{p} \).

Theorem 4.5 \( H \) is a \( N_{nc}\text{Zo} \) if and only if \( N_{nc}\text{Znbd} \) for every point \( p \in G \).

Proof. It is clear.

Theorem 4.6 In a \( N_{nc}\text{ts} (P, N_{nc}\tau) \), let \( N_{nc}\text{ZN}_{p} \), be the \( N_{nc}\text{Znbd} \). System of a point \( p \in P \), then the following statements hold:

(i) \( N_{nc}\text{ZN}_{p} \) is not empty and \( p \) belongs to each member of \( N_{nc}\text{ZN}_{p} \).

(ii) Each superset of members of \( N_{nc}\text{ZN}_{p} \) belongs to \( N_{nc}\text{ZN}_{p} \).

(iii) Each member \( N \in N_{nc}\text{ZN}_{p} \) is a superset of a member \( W \in N_{nc}\text{ZN}_{p} \), where \( W \) is \( N_{nc}\text{Znbd} \) of each point \( p \in W \).

(iv) The intersection of \( N_{nc}\text{Znbd} \) of a point \( p \) and \( N_{nc}\text{Znbd} \) of \( p \) is \( N_{nc}\text{Znbd} \) of \( p \).

Proof. (iv) It follows from Theorem 3.1.

5. \( N_{nc}\text{Z-continuous} \) mappings

Definition 5.1 A mapping \( h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma) \) is called \( N_{nc}\text{Z-continuous} \) (briefly, \( N_{nc}\text{ZCts} \)) if the inverse image of each member of \( (Q, N_{nc}\sigma) \) is \( N_{nc}\text{Zo} \) in \( (P, N_{nc}\tau) \).

Remark 5.1 Let \( F : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma) \) be mapping from a space \( (P, N_{nc}\tau) \) into a space \( (Q, N_{nc}\sigma) \), the following diagram hold:

\[ \begin{array}{ccc}
N_{nc}\text{ZCts} & \rightarrow & N_{nc}\gamma\text{Cts} \\
\downarrow & & \downarrow \\
N_{nc}\text{PCts} & \rightarrow & N_{nc}\epsilon\text{Cts}
\end{array} \]
Now, the following examples show that these implication are not reversible.

**Example 5.1** Let $P = \{a, b, c, d, e\}$, $nct_1 = \{\phi_N, P_N, H, B, C\}$, $nct_2 = \{\phi_N, P_N\}$. $A = \{\{c\}, \{\phi\}, \{a, b, d, e\}\}$, $B = \{\{a, b\}, \{\phi\}, \{b, c, d, e\}\}$, $C = \{\{a, b, c\}, \{\phi\}, \{d, e\}\}$, then we have $2_{nc}\tau = \{\phi_N, P_N, H, B, C\}$. Define $h : (P, 2_{nc}\tau) \rightarrow (P, 2_{nc}\tau)$ as identity map, then

(i) $2_{nc}ZCts$ but not $2_{nc}\delta SCts$, the set $h^{-1}(\{(a), \{\phi\}, \{b, c, d, e\}\}) = \{(a), \{\phi\}, \{b, c, d, e\}\}$ is a $2_{nc}Zos$ but not $2_{nc}\delta Sos$.

(ii) $2_{nc}ZCts$ but not $2_{nc}PCts$, the set $h^{-1}(\{(c, d), \{\phi\}, \{a, b, e\}\}) = \{(c, d), \{\phi\}, \{a, b, e\}\}$ is a $2_{nc}Zos$ but not $2_{nc}Ptos$.

**Example 5.2** Let $P = \{a, b, c, d\}$, $nct_1 = \{\phi_N, P_N, H, B, C, D\}$, $nct_2 = \{\phi_N, P_N, E, F\}$. $A = \{\{a\}, \{\phi\}, \{b, c, d\}\}$, $B = \{\{c\}, \{\phi\}, \{a, b, d\}\}$, $C = \{\{a, c\}, \{\phi\}, \{b, d\}\}$, $D = \{\{a, c\}, \{\phi\}, \{b\}\}$, $E = \{\{a, b\}, \{\phi\}, \{c, d\}\}$, $F = \{\{a, b, c\}, \{\phi\}, \{d\}\}$, then we have $2_{nc}\tau = \{\phi_N, P_N, H, B, C, D, E, F\}$. Define $h : (P, 2_{nc}\tau) \rightarrow (P, 2_{nc}\tau)$ as identity map, then

(i) $2_{nc}\gamma Cts$ but not $2_{nc}ZCts$, the set $h^{-1}(\{(a, d), \{\phi\}, \{b, c\}\}) = \{(a, d), \{\phi\}, \{b, c\}\}$ is a $2_{nc}\gamma os$ but not $2_{nc}Zos$.

(ii) $2_{nc}eCts$ but not $2_{nc}ZCts$, the set $h^{-1}(\{\{b, c\}, \{\phi\}, \{a, d\}\}) = \{\{b, c\}, \{\phi\}, \{a, d\}\}$ is a $2_{nc}eos$ but not $2_{nc}Zos$.

**Theorem 5.1** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be a mapping, then the following statements are equivalent:

(i) $h$ is $N_{nc}ZCts$,

(ii) For each $p \in P$ and $V \in N_{nc}\sigma$ containing $h(P)$, there exists $O \in N_{nc}ZOS(P)$ containing $x$ such that $h(O) \subseteq V$,

(iii) The inverse image of each $N_{nc}\sigma$ set in $Q$ is $N_{nc}Zc$ in $P$,

(iv) $N_{nc}int(N_{nc}cl(h^{-1}(M))) \cap N_{nc}cl(N_{nc}int(h^{-1}(M))) \subseteq h^{-1}(N_{nc}cl(M))$, for each $M \subseteq Q$,

(v) $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}cl(N_{nc}int(h^{-1}(M))) \cup N_{nc}int(N_{nc}cl(h^{-1}(M)))$, for each $M \subseteq Q$,

(vi) If $h$ is bijective, then $N_{nc}int(h(H)) \subseteq h(N_{nc}cl(N_{nc}int(H))) \cup h(N_{nc}int(N_{nc}cl(H)))$, for each $H \subseteq P$,

(vii) If $h$ is bijective, then $h(N_{nc}int(N_{nc}cl(H))) \cap h(N_{nc}cl(N_{nc}int(H))) \subseteq h_{nc}cl(h(H))$, for each $H \subseteq P$.

**Proof.** (i) $\iff$ (ii) and (i) $\iff$ (iii) are obvious.

(iii) $\Rightarrow$ (iv). Let $M \subseteq Q$, then by (iii) $h^{-1}(N_{nc}cl(M))$ is $N_{nc}Zc$. This means $h^{-1}(N_{nc}cl(M)) \supseteq N_{nc}int(N_{nc}cl(h^{-1}(N_{nc}cl(M)))) \cap N_{nc}cl(N_{nc}int(h^{-1}(M))) \subseteq N_{nc}int(N_{nc}cl(h^{-1}(M)))$.

(iv) $\Rightarrow$ (v). By replacing $Q\setminus M$ instead of $M$ in (iv), we have $N_{nc}int(N_{nc}cl(h^{-1}(Q\setminus M))) \cap N_{nc}cl(N_{nc}int(h^{-1}(Q\setminus M))) \subseteq h^{-1}(N_{nc}cl(Q\setminus M))$ and therefore $h^{-1}(N_{nc}int(M)) \subseteq N_{nc}cl(N_{nc}int(h^{-1}(M))) \cup N_{nc}int(N_{nc}cl(h^{-1}(M)))$.

(v) $\Rightarrow$ (vi). Follows directly by replacing $H$ instead of $h^{-1}(M)$ in (v) and applying the bijection of $h$.

(vi) $\Rightarrow$ (vii). By complementation of (vi) and applying the bijective of $h$, we have $h(N_{nc}int(\delta N_{nc}cl(P\setminus H))) \cap h(N_{nc}cl(N_{nc}int(P\setminus H))) \subseteq h(N_{nc}cl(h(P\setminus H)))$. We obtain the required by replacing $H$ instead of $P\setminus H$.

(vii) $\Rightarrow$ (i). Let $V \in N_{nc}\sigma$. Set $W = Y \setminus V$, by (vii), we have $h(N_{nc}int(N_{nc}cl(h^{-1}(W)))) \cap f(N_{nc}cl(N_{nc}int(h^{-1}(W)))) \subseteq h_{nc}cl(h^{-1}(W)) \subseteq h_{nc}cl(W) = W$. So $N_{nc}int(N_{nc}cl(h^{-1}(W))) \cap N_{nc}cl(N_{nc}int(h^{-1}(W))) \subseteq h^{-1}(W)$ implies $h^{-1}(W)$ is $N_{nc}Zc$ and therefore $h^{-1}(V) \in N_{nc}ZOS(P)$.  

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**Theorem 5.2** Let \( h : (P, N_{nc} \tau) \to (Q, N_{nc} \sigma) \) be a mapping, then the following statements are equivalent:

(i) \( h \) is \( N_{nc}ZCts \),

(ii) \( N_{nc}Zc(l^{-1}(M)) \subseteq h^{-1}(N_{nc}cl(M)) \), for each \( M \subseteq Q \),

(iii) \( h(N_{nc}Zc(l(H)) \subseteq N_{nc}cl(h(H)) \), for each \( H \subseteq P \),

(iv) If \( h \) is bijective, then \( N_{nc}int(h(H)) \subseteq h(N_{nc}Zint(H)) \), for each \( H \subseteq P \),

(v) If \( h \) is bijective, then \( h^{-1}(N_{nc}int(M)) \subseteq N_{nc}Zint(h^{-1}(M)) \), for each \( H \subseteq P \).

**Proof.** (i) \( \Rightarrow \) (ii). Let \( M \subseteq Q \), \( h^{-1}(N_{nc}cl(M)) \) is \( N_{nc}Zc \) in \( P \), then \( N_{nc}Zc(l^{-1}(M)) \subseteq h(N_{nc}Zc(l^{-1}(M))) \)

\[ = h^{-1}(N_{nc}cl(M)). \]

(ii) \( \Rightarrow \) (iii). Let \( H \subseteq P \), then \( h(H) \subseteq Q \), by (ii), \( h^{-1}(N_{nc}cl(h(H))) \supseteq h^{-1}(h(H)) \supseteq h^{-1}(h(h(H))) \supseteq h(N_{nc}Zc(H)). \)

Therefore, \( N_{nc}cl(h(H)) \supseteq h^{-1}(N_{nc}cl(h(H))) \supseteq h^{-1}(h(h(H))) \supseteq h(N_{nc}Zc(H)). \)

(iii) \( \Rightarrow \) (iv). Follows directly by complementation of (iii) and applying the bijection of \( h \).

(iv) \( \Rightarrow \) (v). By replacing \( h^{-1}(M) \) instead of \( H \) in (iv) and using the bijection, we have \( N_{nc}int(M) = N_{nc}int(h^{-1}(M)) \subseteq h(N_{nc}Zint(h^{-1}(M))), \) therefore \( h^{-1}(N_{nc}int(M)) \subseteq N_{nc}Zint(h^{-1}(M)). \)

(v) \( \Rightarrow \) (i). Let \( V \in N_{nc}\sigma \), by (v), \( h^{-1}(V) = h^{-1}(N_{nc}int(V)) \subseteq N_{nc}Zint(h^{-1}(V)), \) therefore \( h^{-1}(V) \in N_{nc}ZOS(P). \)

**Definition 5.2** Let \( P \) and \( Q \) be spaces. A mapping \( h : P \to Q \) is called \( N_{nc}Z \)-continuous (briefly, \( N_{nc}ZCts \)) at a point \( p \in P \) if the inverse image of each \( N_{nc}Znbhd \) of \( h(p) \) is \( N_{nc}Znbhd \) of \( p \).

**Theorem 5.3** Let \( P \) and \( Q \) be spaces. Then the mapping \( h : P \to Q \) is \( N_{nc}ZCts \) iff it is \( N_{nc}ZCts \) at every point \( x \in P \).

**Proof.** Let \( H \subseteq Q \) be an \( N_{nc}O \) set containing \( h(p) \). Then \( p \in h^{-1}(H) \), but \( h \) is \( N_{nc}ZCts \), hence \( h^{-1}(H) \) is an \( N_{nc}Zo \) of \( P \) containing \( p \), therefore, \( h \) is \( N_{nc}ZCts \) at every point \( p \in P \).

On the other hand suppose that \( G \subseteq Q \) is \( N_{nc}O \) for every \( p \in h^{-1}(G) \) and \( h \) is \( N_{nc}ZCts \) at every point \( p \in P \). Then there exists an \( N_{nc}Zo \) set \( H \) containing \( p \) such that \( p \in G \subseteq h^{-1}(G) \), i.e., \( h^{-1}(G) = \bigcup \{H : p \in h^{-1}(G), \ H \ \text{is} \ N_{nc}Zo\} \), then \( h^{-1}(G) \subseteq P \) is \( N_{nc}Zo \). SO, \( h \) is \( N_{nc}ZCts \).

**Remark 5.2** The composition of two \( N_{nc}ZCts \) mappings need not be \( N_{nc}ZCts \) as show by the following example.

**Example 5.3** Let \( P = \{a, b, c\} = Z, Q = \{a, b, c, d\} \), \( N_{nc}\tau_1 = \{\phi_N, P_N, A\}, N_{nc}\sigma_1 = \{\phi_N, P_N, B\} \), \( N_{nc}\mu_1 = \{\phi_N, Z_N, C, D\} \), then we have \( N_{nc}\tau = \{\phi_N, P_N, A\}, N_{nc}\sigma = \{\phi_N, P_N, B\} \), \( N_{nc}\mu = \{\phi_N, Z_N, C, D\} \).

Let \( h : (P, N_{nc}\tau) \to (Q, N_{nc}\sigma) \) be an identity function and \( g : (Q, N_{nc}\sigma) \to (Z, N_{nc}\mu) \) defined as \( g(a) = a, g(b) = g(d) = b \) and \( g(c) = c \). It is clear that \( f \) and \( g \) are \( N_{nc}ZCts \) but \( g \circ f \) is not \( N_{nc}ZCts \).

**Theorem 5.4** The restriction mapping \( h|_H : (H, N_{nc}\tau_H) \to (Q, N_{nc}\sigma) \) of a \( N_{nc}ZCts \) mapping \( h : (P, N_{nc}\tau) \to (Q, N_{nc}\sigma) \) is \( N_{nc}ZCts \) if \( H \in N_{nc}O(P, N_{nc}\tau) \).

**Proof.** Let \( O \in N_{nc}\sigma \) and \( h \) be a \( N_{nc}ZCts \) mapping. Then \( h^{-1}(O) \in N_{nc}ZOS(P, N_{nc}\tau) \). Since \( H \in N_{nc}O(P, N_{nc}\tau) \), then by Theorem 3.1, \( (h|_H)^{-1}(O) = H \cap h^{-1}(O) \in N_{nc}ZOS(P, N_{nc}\tau) \), therefore \( h|_H \) is \( N_{nc}ZCts \).
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