Quantum Rolling Friction

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An atom moving in vacuum at constant velocity parallel to a surface feels a frictional force induced by the dissipative interaction with the quantum electromagnetic fluctuations. We show that the combination of nonequilibrium dynamics, anomalous Doppler effect and spin-momentum locking of light mediates an intriguing interplay between the atom’s translational and rotational degrees of freedom. This behavior deeply affects the drag force in a way that is reminiscent of classical rolling friction. Our fully non-Markovian and nonequilibrium description highlights counterintuitive features characterizing the atom’s velocity-dependent rotational dynamics and determining the strength of the frictional force. These results prompt interesting directions for tuning the interaction and for investigating non-equilibrium dynamics as well as the properties of confined light.

Light-matter interactions at the quantum level continue to fascinate with intriguing and non-intuitive phenomena. In particular, the last years have witnessed a surge of interest in nonequilibrium systems and in the light confinement offered by photonic and plasmonic structures. Systems out of equilibrium are very common in nature and intense investigations have unraveled their relevance for both fundamental and applied research [12, 13]. Light confinement, on the other hand, is known for producing several interesting phenomena. One of them, the so-called spin-momentum locking of light [34], is currently attracting attention for its capacity of generating chiral asymmetry in light-matter interaction [5]. This effect was recently used to control light propagation in optical fibers [6] and to induce a force on a microscopic body near a structure [7–11]. In these applications, the asymmetry was induced by actively rotating the object or by preparing an atom in state with nonzero angular momentum.

Here, we show that when an atom is forced to move parallel to a surface an additional form of asymmetry solely appears due to the nonequilibrium interplay of the atomic translational and the rotational degrees of freedom. The underlying physics is somewhat reminiscent of that of a body rolling on a surface but, due to its characteristics, the interaction features many interesting counterintuitive features. To illustrate these aspects we consider a phenomenon which is intrinsically related to one of the peculiarities of quantum theory, i.e. that vacuum is not empty but rather full of roiling virtual particles. Indeed, while Lorentz’ invariance guarantees that inertial motion through the quantum vacuum is preserved, this is in general no longer true when the motion occurs with respect to another body, even if both objects are electrically neutral and non-magnetic. Due to vacuum fluctuations, light-matter interactions can lead to the occurrence of non-conservative (frictional) forces [12–13]. The physics behind the so-called quantum friction is related to the quantum Cherenkov effect through the anomalous-Doppler effect [14–17]. In this process, real photons are extracted from vacuum at the cost of the object’s kinetic energy; they are absorbed and re-emitted producing a fluctuating momentum recoil [18]. Due to the atom’s motion, the resulting stochastic process is non-isotropic and a net force appears. During the last decade this phenomenon was investigated in many scenarios [11–12, 19–23] and its connection to nonequilibrium physics was highlighted [21].

For our purpose, we consider a system at zero temperature consisting of an atom propelled at constant height $z_a > 0$ parallel to a surface in $z = 0$. We focus on the nonequilibrium steady state (NESS) characterized by a constant velocity $v$ and reached by the system when friction balances the external force. In the NESS the frictional force can be written as $F = F^t + F^r$ [25] with

$$F^t = -2\int_0^{\infty} d\omega \int \frac{d^2k}{(2\pi)^2} k \text{Tr} \left[ S^T_R (-\omega_k^-,v) \cdot G^S_R(k,z_a,\omega) \right],$$

$$F^r = -2\int_0^{\infty} d\omega \int \frac{d^2k}{(2\pi)^2} k \text{Tr} \left[ S^T_I (-\omega_k^-,v) \cdot G^S_R(k,z_a,\omega) \right],$$

where $\omega_k^\pm = \omega \pm k \cdot v$ is the Doppler-shifted frequency of the vacuum field in the atom’s comoving frame, $k$ is the component of the wave vector parallel to the surface and $G_R(k,z_a,\omega)$ is the Fourier transform of the electromagnetic Green tensor [25–26]. $S(\omega, v)$ is the velocity-dependent atomic power spectrum, i.e., the Fourier transform of the stationary two-time correlation tensor $C_{\omega,v}(\tau,v) = \langle d_\tau(\tau) d^\dagger_\tau(0) \rangle$. Here, $d(t)$ describes the full nonequilibrium dynamics of the particle’s quantum electric dipole operator. The superscripts “s” and “as”, and the subscripts “R” and “I” indicate the symmetric and the antisymmetric part of tensors and the real and the imaginary part of the corresponding quantity, respectively. “T” stands for the transposed tensor.

The two terms in Eqs. (1) correspond to two distinct

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mechanisms characterizing the physics of the system (see Fig. 1). A first insight is provided by the correlation tensor: If symmetric, i.e. if $G(\tau, \nu) = G^T(\tau, \nu)$, the atomic power spectrum is necessarily real and symmetric, leading to $F^1 \neq 0$ and $F^0 = 0$ [24, 27]. This is equivalent to $\langle \vec{d}(\tau) \times \vec{d}(0) \rangle = 0$, which implies that, on average, the atomic dipole cannot rotate, absorb or emit any net angular momentum. $F^1$ corresponds to the quantum-frictional force commonly investigated in the literature, which only takes into account the atomic translational motion. $F^0$, instead, is an additional contribution which appears if the rotational atomic degrees of freedom are considered and is the main focus of this work. 

For further understanding, it is useful to analyze the Green tensor of our system. For motion along the $x$-direction ($\nu = ex$) the surface-related (scattering) part of $G(k, z_a, \omega)$ can be written as the sum of a diagonal matrix $\sigma(k, z_a, \omega)$ and a skew-symmetric matrix $-\phi(k, z_a, \omega) L_y$, where $L_y$ is the $y$-component of the usual Lie-algebra’s basis for $so(3)$ ($[L_y]_{jk} = -i \epsilon_{ijk}$) describing 3D-rotations [28]. The matrix $\sigma(k, z_a, \omega)$ and the function $\phi(k, z_a, \omega)$ are respectively even and odd in $k$. Physically, $G(k, z_a, \omega)$ and $S(\omega, \nu)$ are connected [see also Eqs. (1)]: The former characterizes the system’s electromagnetic response, while the latter is related to the atomic interaction with light. The link is provided by $G_3(k, z_a, \omega) = [G(k, z_a, \omega) - G(-k, z_a, \omega)]/2i = G_{\nu}^G(k, z_a, \omega)$ which is related to the probability that the atom absorbs ($\omega < 0$) or emits ($\omega > 0$) photons [29]. The tensor structure reveals that the interaction is sensitive to the three states of the photon’s spin: $\sigma_\nu(k, z_a, \omega) = G_{\nu}^G(k, z_a, \omega)$ is associated with processes involving linearly polarized photons (spin zero) which, due to the even parity in $k$, do not depend on the direction of propagation. The matrix $\phi_\nu(k, z_a, \omega) L_y = G_{\nu}^\phi(k, z_a, \omega)$, instead, describes emission and/or absorption of photons having a nonzero spin along the $y$-axis. The interpretation in terms of probability implies that the spin is positive when $-\phi_\nu(k, z_a, \omega) > 0$ and negative in the opposite case, linking the sign to the direction of propagation through the odd parity in $k$. This behavior, which is essentially associated with the light’s confinement at the vacuum-material interface [3–5], allows to associate $\phi^0(k, z_a, \omega)$ with a spin-dependent local density of states. All expressions simplify in near-field limit, where [25, 26]

$$G(k, z_a, \omega) \approx \Pi \frac{k}{2\epsilon_0} r(\omega) e^{-2kz_a} \quad (k = |k|),$$

(2a)

$$\Pi = \left( \begin{array}{ccc} k_x^2 & k_y k_z & -i k_x \\
    k_y k_z & k_z^2 & -i k_y \\
    i k_x & i k_y & k_z^2 \end{array} \right) \rightarrow \left( \begin{array}{ccc} k_x^2 & 0 & -i k_x \\
    0 & k_y^2 & 0 \\
    i k_x & 0 & k_z^2 \end{array} \right).$$

(2b)

$\epsilon_0$ is the vacuum permittivity and $r(\omega)$ the surface’s p-reflection coefficient. In $\Pi$ we deleted some entries since they do not contribute to $G$. As $\Pi^T = \Pi^\dagger G_3(k, z_a, \omega)$ is obtained by replacing $r(\omega)$ with $r_\nu(\omega)$ in Eq. (2a).

At equilibrium and for ordinary materials, due the surface’s symmetry around the $z$-axis, the radiation features zero angular momentum on average. However, for a moving atom, the frequency of the radiation in the co-moving frame is Doppler-shifted by the value $k \cdot v$ [Eqs. (1)]. This induces an asymmetry in the light-matter interaction: The spin balance is altered and the atom effectively perceives spin-polarized radiation. To understand the implications of this phenomenon on the frictional force, it is useful to model the atom as a harmonic oscillator characterized by the (transition) frequency $\omega_0$. The velocity-dependent atomic polarizability tensor is then

$$\alpha(\omega, \nu) = \alpha_B(\omega) \left[ 1 - \alpha_B(\omega) \int \frac{d^3k}{(2\pi)^3} G(k, z_a, \omega_k^+)^{-1} \nu^+ ight],$$

(3)
where $\alpha_B(\omega) = \alpha_0 \omega^2 / (\omega_0^2 - \omega^2)$ and $\alpha_0$ are the bare and static oscillator’s polarizabilities, respectively. The nonequilibrium power spectrum can be written as

$$S(\omega, \mathbf{v}) = \frac{\hbar}{\pi} \left[ \theta(\omega) \mathcal{G}_\alpha(\omega) + \mathcal{J}(\omega, \mathbf{v}) \right],$$  \hspace{1cm} (4a)$$

where $\mathcal{G}_\alpha(\omega, \mathbf{v})$ is defined similarly to $\mathcal{G}_\alpha(\mathbf{k}, z; \omega)$ and

$$\mathcal{J}(\omega, \mathbf{v}) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \left[ \theta(\omega^+ - \omega^-) \right] \times \alpha(\omega, \mathbf{v}) \cdot \mathcal{G}_\alpha(\mathbf{k}, \omega^+ \mathbf{k}) \cdot \alpha^T(\omega; \mathbf{v}).$$  \hspace{1cm} (4b)$$

The nonequilibrium fluctuation-dissipation theorem (FDT) in Eqs. (4) includes the atomic rotational degrees of freedom and generalizes results reported in previous work [24]. The resulting force is represented in Fig. 2 (due to symmetry $\mathbf{F} = \mathbf{F} x$). Notice that $\mathbf{F}^e$ attenuates the force stemming from the translation. Roughly speaking, one can say that, as in classical mechanics, allowing for a “rolling dynamics” reduces the frictional force felt by the object.

For additional insight and a more quantitative analysis, let us focus on the low-velocity limit of Eqs. (1). As discussed in previous work [24, 27], the dominant contribution to the frictional force arises from $\omega \lesssim v / z_0$. If the material comprising the surface is Ohmic for these frequencies, to second order in $\alpha_0$, we have

$$\mathbf{F}^t \approx -\alpha_0^2 v^3 \frac{\hbar}{\pi} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{d^2 \mathbf{k}}{(2\pi)^2} \left[ \frac{k^4}{6} \text{Tr} \left[ \mathcal{G}_{1}^{T}(\mathbf{k}, z_0,0) \cdot \mathcal{G}_{1}^{T}(\tilde{\mathbf{k}}, z_0,0) \right] + \frac{k_x^2 k_y^2}{2} \text{Tr} \left[ \sigma_{1}^{T}(\mathbf{k}, z_0,0) \cdot \sigma_{1}^{T}(\tilde{\mathbf{k}}, z_0,0) \right] \right],$$  \hspace{1cm} (5a)$$

$$\mathbf{F}^r \approx -\alpha_0^2 v^3 \frac{\hbar}{\pi} \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{d^2 \mathbf{k}}{(2\pi)^2} \left[ \frac{k_x^2 k_y^2}{2} \text{Tr} \left[ \mathcal{L}_{\rho}^{T}(\mathbf{k}, z_0,0) \cdot \mathcal{L}_{\rho}^{T}(\tilde{\mathbf{k}}, z_0,0) \right] \right] \cdot \left[ \frac{k_x^2 k_y^2}{6} \phi_{1}^{T}(\mathbf{k}, z_0,0) \phi_{1}^{T}(\mathbf{k}, z_0,0) + \frac{k_x^2 k_y^2}{6} \phi_{1}^{T}(\mathbf{k}, z_0,0) \phi_{1}^{T}(\mathbf{k}, z_0,0) \right],$$  \hspace{1cm} (5b)$$

where the prime indicates the derivative with respect to the frequency. For a motion within the near field of a surface with resistivity $\rho$ [24, 29, 31], Eqs. (5) give

$$\mathbf{F}^t \approx -\frac{63}{\pi^2} \alpha_0^2 \rho^2 \frac{v^3}{(2\pi)^2} + \mathcal{F} \approx \frac{45}{\pi^2} \alpha_0^2 \rho^2 \frac{v^3}{(2\pi)^2}.$$  \hspace{1cm} (6)$$

Notice that the contribution associated with the rotation compensates more than 70% of the force related to the translation (see inset in Fig. 2). Interestingly, one can show that, using the so-called local thermal equilibrium (LTE) approximation, at low velocity, the compensation between the translational and the rotational contributions to the frictional force is complete, leading to a vanishing force. The LTE approach is commonly used for an approximate description of nonequilibrium systems and treats each of its components as if they were locally in thermal equilibrium with their immediate surrounding. The (equilibrium) fluctuation-dissipation theorem (FDT) is then applied [32]. For our system, this is equivalent to deleting the tensor $\mathcal{J}(\omega, \mathbf{v})$ in Eq. (1a). Vanishing friction indicates that the detailed balance enforced by the FDT (instead of Eqs. (4)) incorrectly treats the processes connected with the translational and the rotational degrees of freedom on the same footing. Contrasted with Eq. (6), a zero force in the LTE approximation is not only flawed but also highlights that, as soon as the rotational degrees of freedom are included, quantum-friction is essentially a pure nonequilibrium phenomenon.

The difference in sign between $\mathbf{F}^t$ and $\mathbf{F}^r$ can be understood as a consequence of the interplay between the anomalous Doppler effect and the spin-momentum locking of light. In the NESS, for a motion along $x > 0$, the Doppler-shifted frequency $\omega_k^-$ becomes negative (“anomalous”) only for $k_x > 0$. Therefore, the atom can get excited (even at zero temperature) and emits photons due to a light–matter interaction that favors positive $k_x$. In Eq. (1a) the processes are controlled by $\sigma(\mathbf{k}, z_0, \omega)$ [see Eq. (5a)] and involve linearly polarized photons which are prevalently emitted in the direction of the motion. As a result, the net recoil force $\mathbf{F}^t$ acts against the motion [24, 25, 27]. However, allowing for the participation of the atomic rotational dynamics opens an additional channel of interaction with the surface. The
corresponding processes are associated with the function \(\phi(k, z_a, \omega)\) and the matrix \(L_y\) [Eq. 6b] and involve photons with nonzero spin. In this case, the interplay of the Doppler-shift and of the angular momentum favors an emission along the negative \(x\)-direction. This leads to the net recoil force \(F^x\) with the same sign of the velocity.

The involvement of the angular momentum has an additional implication. During the motion and the interaction with the vacuum field, the atom undergoes a stochastic process which includes rotations. The stationarity characterizing the NESS implies that all torques acting on the particle, resulting from the dissipative nonequilibrium light-matter interaction, must balance on average. The rotational stochastic motion [34] generated by the exchange of photons can be associated with a constant angular momentum \(\mathcal{L} = [\alpha_0 \omega_a^2]^{-1}(d(t) \times \ddot{d}(t))\), which can be written as [35]

\[
\mathcal{L} = \frac{1}{\alpha_0 \omega_a^2} \int_{-\infty}^{\infty} d\omega \omega \text{Tr}[\mathbb{L} \cdot \mathbb{S}(\omega, \mathbf{v})].
\] (7)

In agreement with the symmetries of our system, only the \(y\)-component is nonzero. If the stochastic process describing the rotation is to a good approximation statistically independent from the other dynamics, we can calculate the corresponding rotation frequency \(\Omega\) by multiplying \(\mathcal{L}\) by the inverse of the average atomic moment of inertia tensor, \(M_{ij} = [\alpha_0 \omega_a^2]^{-1} \langle |(d(t)|^2 \delta_{ij} - d_i(t)d_j(t)) \rangle\). With some matrix algebra we obtain

\[
\frac{\Omega(\mathbf{v})}{\frac{1}{\text{Tr}[\mathbb{L}]}} \approx \frac{\int_{-\infty}^{\infty} d\omega \omega \text{Tr}[\mathbb{S}(\omega, \mathbf{v}) \cdot \mathbb{L}_y]}{\int_{-\infty}^{\infty} d\omega \text{Tr}[\mathbb{S}(\omega, \mathbf{v}) \cdot \mathbb{L}_y^2]}.
\] (8)

Inserting Eq. (4a) in Eq. (8) we obtain at the leading order in the near field [36]

\[
\frac{\Omega(\mathbf{v})}{\frac{1}{\text{Tr}[\mathbb{L}]}} \approx \frac{v}{1 + \frac{2}{3} \frac{r_l |\omega_a|}{r_s |\omega_a|}} z_a.
\] (9)

Equation (9) indicates that, in the NESS, while propelled by a constant external force within the near field of the surface, translation and rotation couple and the atom rotates clockwise around the \(y\)-axis despite no external torque is applied on its center of mass (see Fig. 4). This last result contradicts our classical intuition, which, for a motion along the positive \(x\)-axis, would instead suggest a counterclockwise rotation. It also differs from evaluations on rotating non-translating metallic nanoparticles [11, 22], which in the same limit agree with the classical prescription. Once again, however, the sense of rotation can be explained as resulting from the motion-induced asymmetry in the light-matter interaction. In the atomic excitation process the anomalous Doppler effect favors the absorption of photons propagating along the positive \(x\)-axis. In the near-field they have negative spin, resulting in the absorption of negative angular momentum and a clockwise rotation of the atom. This also provides a better understanding of the sign of \(F^x\): During the dissipative process associated with the frictional force, in order to keep \(\mathcal{L}\) constant, the atom emits photons with positive spin, absorbing a negative angular momentum recoil. Due to the spin-dependent density of states, in the near field these photons can be absorbed by the environment (essentially the surface) if they are emitted along the negative \(x\)-axis, favoring therefore a positive momentum recoil and a positive \(F^x\). Still, because of the Doppler-shift, the process is less effective than the one associated with linearly polarized photons, whose absorption rate does not depend on the sign of the wave vector, explaining why \(|F^x| < |F^y|\).

It is important to highlight that our description takes into account the full nonequilibrium electromagnetic backaction on the microscopic object, setting it apart from other related studies. Nonequilibrium backaction is often not included in perturbative approaches for atoms [7–9] and it is commonly neglected for metallic nanoparticles [11, 22], due to the strong intrinsic dissipation of the metal [25]. In our case, however, this feature ultimately characterizes important quantities such as the atomic power spectrum or polarizability and affects the spin-sensitive atom-surface interaction. Disregarding the backaction, removes the intrinsic velocity-dependence in these quantities, making them coincide with their bare or equilibrium expressions. This leads to a description which, to a large extent, is equivalent to the LTE approximation for which some of the above effects disappear.

In conclusion, we have shown that a combination of the nonequilibrium dynamics, the anomalous Doppler effect and the spin-momentum locking of light induces quantum rolling friction on an atom moving parallel to a surface. During the zero-temperature dissipative process, the atom performs a driven Brownian-like motion that involves all its internal degrees of freedom and depends on the three states of the photon’s spin. The atom absorbs and emits photons, promoting an exchange of translational and angular momentum within the light-matter interaction. As in the classical rolling motion, the interplay between atomic translational and rotational degrees of freedom sensibly diminishes the drag force with respect to the case where the rotation is not considered. Nonetheless, quantum rolling friction remains always nonzero. This result invalidates the commonly used local equilibrium approximation predicting instead a vanishing force at low velocities. The reduction in strength of the drag force is connected with a steady atomic rotation. However, the sense of rotation is the opposite of what one would expect from classical intuition. This behavior is related with the invariance of the atomic angular momentum in the nonequilibrium steady-state and with spin-momentum locking characterizing the field confined near the surface.

Our analysis offers novel perspectives for experimen-
tally investigating nonequilibrium interactions and confined light. The above description also prompts several options for tuning the quantum frictional interaction via an enhancement of its asymmetry using, for instance, chiral atoms [37] or special materials [38] or even by introducing static magnetic fields [39].

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