Anticipating Human Intention for Full-Body Motion Prediction in Object Grasping and Placing Tasks

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Fig. 1: Prediction for placing a jug on the table. A placement affordance is predicted as a probability density function on the table (a), depicted in green a full-body motion is optimized (b), which is compared to ground truth motion (c).

Abstract—Motion prediction in unstructured environments is a difficult problem and is essential for safe and efficient human-robot space sharing and collaboration. In this work, we focus on manipulation movements in environments such as homes, workplaces or restaurants, where the overall task and environment can be leveraged to produce accurate motion prediction. For these cases we propose an algorithmic framework that accounts explicitly for the environment geometry based on a model of affordances and a model of short-term human dynamics both trained on motion capture data. We propose dedicated function networks for graspability and placeability affordances and we make use of a dedicated RNN [1] for short-term motion prediction. The prediction of grasp and placement probability densities are used by a constraint-based trajectory optimizer to produce a full-body motion prediction over the entire horizon. We show by comparing to ground truth data that we achieve similar performance for full-body motion predictions as using oracle grasp and place locations.

I. INTRODUCTION

When interacting with their environment, humans model the action possibilities directly in the product space of their own capabilities and the environment. This idea of the existence of an intuitive and perceptual representation of the possibilities in an environment is known as affordances [2].

In this paper, we propose an algorithmic framework to learn and encode such affordances from data. By modeling affordances as probability density functions conditioned on the environment and the kinematic state of the human, we are able to anticipate the human intention by maximum likelihood. This intention can then be combined with a full-body motion prediction system to produce accurate predictions as seen in Fig. 1.

In our experiments grasp and place densities are defined over submanifolds of the hands pose spaces. Placeability is defined over support planes and grasps are defined over sphere surfaces around objects. Our models for each affordance derive from a common structure based on recurrent neural networks (RNNs) for modeling the human latent state and dedicated networks, i.e., Convolution Neural Networks (CNNs), for modeling the environment. The models then combine environment and human latent spaces using fully connected layers to produce densities over the sub-manifolds. Note that we use mixtures to encode multi-modal densities which is important for placeability.

Given a prediction for placements and grasps we optimize a full-body movement with a nonlinear program [3], which accounts for obstacle and goalset constraints, and models short-term movements with a data-driven dynamical system. To the best of our knowledge this paper is the first to accurately leverage the 3D geometry of the environment for combined intention and motion prediction of full-body movements.

We gathered a dataset with 5 participants using a motion capture system. Affordances and short-term motion models were trained on this dataset. Our results demonstrate superiority of our affordance densities for predicting placements and grasping locations. Finally, we show that combining goalset predictions and motion predictions compares similarly to using oracle goal locations.

This paper is structured as follows: First we discuss relevant related work in section II. Section III introduces our framework and explains the implementation. Experiments on real motion data are performed in Section IV. Finally, conclusions are drawn in Section V.
II. RELATED WORK

A. Intention and motion prediction

In prior work graphical models, such as Hidden Markov Models (HMM) and Conditional Random Fields (CRF), have been used in order to predict human motion or intention. For instance, Bennewitz et al. modeled human intention using HMMs in order to improve navigation behavior of a mobile robot [4]. Kuli´c et al. used HMMs to model full-body motion primitives and applied it to motion imitation [5]. Elfring et al. used growing HMMs in order to learn human’s goal position from data and use a social forces-based motion model to predict human motion [6]. Koppula and Saxena focused on movement prediction using conditional random fields [7]. While these approaches are sound they generally do not scale to large databases of motion capture or are limited to predict 2d motion of humans and do not deal with the full-body case.

B. Affordances

The concept of affordances stems its roots from psychology [2], [8] and relates to the action possibilities offered by a given environment to an animal or human. Jamone et al. present a survey on affordances in the field of psychology, neuroscience and robotics [9]. The field of visual affordances deals with learning affordances as a computer vision problem [10]. Roy et al. use a Convolutional Neural Network based architecture to extract affordance segmentations in RGB images [11]. Nguyen et al. model affordances using an autoencoder structure [12].

In Robotics, affordances can be used to model the actions a robot is able to perform [13], [14], [15]. For example, Montesano et al. use Bayesian networks to encode affordances and demonstrate how a humanoid robot can use it to interact with objects [13]. For Human Robot Interaction affordance models are used to model human action possibilities and to be able to infer human intent [16], [7]. Koppula and Saxena define object affordance as potential functions depending upon how the object will be interacted with [7].

In this paper, we design and implement a system to understand human object affordances in a real world table setup task performed in a motion capture environment. As we aim to use the affordance model in order to predict human motion, we use a probabilistic model. Given the human state and the scene context, it predicts a density of interaction possibilities for the corresponding affordance. In particular, we concentrate on graspability and placeability affordances, and model them using a probabilistic neural network framework.

C. Neural Network Human Motion Prediction

Prior work on full-body human motion prediction for has focused on recurrent neural network (RNN) architectures. Fragkiadaki et al. proposed a Long Short Term Memory (LSTM) based model that is able to train across multiple subjects [17]. Martinez et al. introduced a gated recurrent unit (GRU) based approach [18]. A residual connection forces the network to predict velocities and thus improves the generalization capability of the network. Pavllo et al. changed the joint angle representation to quaternions which further improved the predictions [19]. Recently Wang and Feng introduced a position-velocity recurrent encoder-decoder model (VRED) [1]. Their model adds an additional velocity connection as an input to the GRU cell in the recurrent structure. Motion prediction aproaches based on recurrent neural networks show good results on forecasting of purely human motion. However, they do not handle environmental context, an issue we tackle in this paper due to encoding the environment in our affordance model.

D. Motion Optimization

Gradient-based optimization algorihtms are widely used in the field of robotics and optimal control [20], [21], [22] for optimizing trajectories.

Moreover, motion optimization techniques have been used for human motion synthesis. For example, Mordatch et al. use motion optimization approaches to synthesize realistic motions and animate human behavior [23], [24].

In our prior work we propose to use motion optimization in order to improve short-term motion prediction [25], [3]. We built on the VRED model and used the trajectory optimization technique to change the prediction in order to adapt for specific constraints [3]. In this paper we will use this proposed method in order to predict full-body motion towards a goal state that is sampled from a separate affordance model. This makes it possible to take environmental context into account.
encoding of both: the object type the human has in the hand and the surface we compute the affordance for, and a grid that covers the plane state.

The network additionally takes plane features as input which are a 24 × 24 grid consisting of a binary occupancy map, a 2d position of the planes reference frame and a signed distance field (SDF) (see Figure 4).

a) Multi-modal placements: Placeability is fundamentally multi-modal. For instance in our experiments we consider a table setting scenario such as found in a home or restaurant, four people can sit next to the table, therefore there are four possible locations where the human can place a plate.

A standard approach to model multi-modal distributions are Mixture Density Networks (MDN) [26], which we make use of for modeling placement distributions:

\[
p(x|d) = \sum_{i=1}^{m} \alpha_i \phi_i(x|d)
\]

where \(m\) indicates the count of the components in the mixture model, \(\alpha_i\) are the mixing coefficients. \(\phi_i\) are functions representing conditional densities for the \(i^{th}\) kernel.

We use multivariate Gaussian kernels with diagonal covariance. We use 7 kernels in output, which gave good empirical results on our dataset. The network is trained using a neg-log likelihood (NLL) loss with the 2d place position on the surface as ground truth.

b) Constraining affordances to free regions: We improve our placeability model with the intention of making it more robust against violating regions where objects are already placed. We consider 2 approaches to tackle the issue. In the penalty approach we modify the cost function to include a penalty term penalizing placement in invalid regions using the value of the SDF map. In the transfer learning approach we learn environment features related to plane occupancy separately. To achieve this, we build an autoencoder network with inputs being the 4 feature maps and the one-hot encoding vector. The encoder uses two convolutional layers with maxpooling to downsample, the decoder upsamples and uses three convolutional layers. It is trained to output the binary occupancy map of the plane after the object is placed on the plane. We train the autoencoder using a standard mean squared loss.

The pre-trained encoder model is connected to the main placeability model. The encoder model weights are made non-trainable when the overall placeability network is trained. The intuition here is that, with the autoencoder, we

Fig. 4: Plane features. From left to right: A visualization of the table with several objects, the corresponding binary occupancy map, a visualization of the signed distance field.
capture the latent representation that are unique for different combinations of the occupancy map. With the pre-trained encoder network producing distinctive feature representations, the main model should learn to not predict outputs in invalid regions.

C. Graspability Affordance

We model the graspability affordance as follows: Given that the subject wants to grasp an object of a particular type from its current resting surface, predict the likelihood of the right wrist position for successful grasp action. The model can then be queried for every object in the scene to get the complete dynamic mapping of the grasp affordance from a human’s perspective.

The choice of the posterior distribution influences how the affordance is modeled. We will compare two probability distributions: The Gaussian distribution and the von Mises-Fisher (vMF) distribution. The VMF distribution describes distributions depending on whether we want to model a Gaussian or a vMF distribution.

a) Gaussian posterior: In the Gaussian network type, the wrist position is modeled as a 3D position in Euclidean space and the final layer of this network outputs the parameters of a Gaussian distribution having a diagonal covariance structure. We use a NLL cost function similar to [27]:

\[
L_{Gauss}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{3}{2} \ln(2\pi) + \frac{1}{2} \ln(\text{det}(C(d_i))) + \frac{1}{2} (y_i - \mu(d_i))^T C^{-1}(d_i)(y_i - \mu(d_i)) \right)
\]

with \((d_i, y_i)\) being data points and labels and diagonal covariance matrix \(C\). The number of output neurons are 6, owing to 3 dimensional mean and variances.

b) vMF posterior: The intention for the vMF formulation is to model grasp points as a distribution on a 2D manifold defined on the surface of a sphere:

\[
p_{vMF}(x; \mu, \kappa) = C_p(\kappa) \exp(\kappa \mu^T x)
\]

where \(\mu \in \mathbb{R}^p, ||\mu|| = 1\) is the mean direction, with \(p = 3\) and \(\kappa \geq 0\) is the concentration parameter, which defines the spread of the distribution on the surface the hypersphere in the direction of \(\mu\). \(C_p(\kappa)\) is the normalizing constant given by

\[
C_p(\kappa) = \frac{\kappa^{p/2-1}}{(2\pi)^{p/2} I_{p/2-1}(\kappa)}.
\]

where \(I_s\) denotes the modified Bessel function of the first kind at order \(s\).

We have 4 output neurons for the vMF model: 3 for the mean direction and 1 for the \(\kappa\) term that defines the spread. Additionally, the output neurons corresponding to the mean direction should satisfy the unit norm constraint.

D. Full-Body Prediction

The goal for full-body prediction is to find a trajectory of human motion \(h_{t+1:T}\) of future states, given a trajectory \(h_{0:t}\) of already observed states and our affordance model. For this purpose we use a trajectory prediction framework introduced in our prior work [3]. The framework works in 2 phases: 1) Offline, a VRED model \(f\) [1] is trained to predict purely kinematic trajectories only based on human motion. \(f(h_{0:t}, \delta) = h_{t+1:T}\). 2) Online, trajectory optimization techniques are used to adapt to environmental objectives while being close to the prediction. This is done by changing additional controls \(\delta\) that are added to the VRED architecture. In this paper we use the low-level objective and the goalset objective as described in [3]:

\[
c_{\text{low-level}}(\delta) = ||\delta||^2
\]

The low-level objective \(c_{\text{low-level}}\) ensures that the deltas are close to zero and therefore the deviation from what the network predicts is small.

\[
c_{\text{goalset}}(\delta) = ||\phi_{\text{FK}}(f(h_{0:t}, \delta)_{T}) - p^*||^2
\]

The goalset objective \(c_{\text{goalset}}\) optimizes the position of the hand of the human to end up close to position \(p^*\), with \(\phi_{\text{FK}}\) being the forward kinematics map, mapping the last human state to the hand position.

In order to account for our affordance model, we compute the expected prediction position \(p^*\) from the affordance model \(P_{0:a}(x|h, s)\). Thus, the trajectory will be optimized to end up at this position.

The gradient based optimization algorithm L-BFGS [28] is used to optimize the trajectory with the loss:

\[
L = \alpha_1 c_{\text{low-level}} + \alpha_2 c_{\text{goalset}}(\delta)
\]

with \(\alpha\) being hyperparameters. The gradients are calculated using automatic differentiation functionalities from tensorflow.

IV. RESULTS

The models were implemented using Keras [29] functional API, with TensorFlow [30] as backend. To develop the loss functions used to train our custom models, we used the Tensorflow distributions [31] package.

\[
\begin{array}{|c|c|c|c|}
\hline
& \text{Train set} & \text{Test set} \\
\hline
\text{Models} & \text{NLL} & \text{MSE} & \text{NLL} & \text{MSE} \\
\hline
\text{Baseline} & - & 0.0799 & - & 0.1051 \\
\text{MDN+no CNN features} & -3.1769 & 0.0184 & -2.6904 & 0.0239 \\
\text{MDN+CNN features} & -4.3029 & 0.0136 & -2.6378 & 0.0213 \\
\text{MDN+CNN+penalty} & -3.7081 & 0.0151 & -1.6122 & 0.0245 \\
\text{MDN+transfer learning} & -4.1168 & 0.0115 & -2.7793 & 0.0194 \\
\text{MDN+transfer+penalty} & -4.2717 & 0.0107 & -2.4688 & 0.0196 \\
\hline
\end{array}
\]

TABLE I: Results obtained for Placeability models.
Fig. 5: Valid region percentages of placeability over time before the placement happens.

A. Dataset

In our setup an Optitrack\textsuperscript{1} Motion capture system was used. The environment has a total size of 4 × 4 meters. The human subjects were asked to wear a motion capture suit with 50 markers attached to it. There are objects in the scene that are each attached with markers for tracking and can be categorized into two types: The first type of objects are the ones that the users can directly interact with, such as cups, plates, jug and bowl. The second type of objects remain stationary in a given recording session and also acts as supporting bodies over which the first type of objects can be placed, namely a table, a big shelf and a small shelf. We model affordances for the first type of objects. Participants were asked to perform tasks related to setting up the table and clearing it. In the collected data, the users were subject to two affordances, namely graspability and placeability.

A total of 5 users participated in the recording session, with each session being approximately 25 minutes long. We extracted a total of 1551 grasp-place sequences. For training the models, we split the data based on the subjects. We used data of 3 of the subjects for training and 2 for testing.

B. Placeability

We computed results on different variants of our MDN networks showing the NLL loss and the mean squared error between the mean of the MDN and the ground truth. Results can be seen in Table I. The results are computed on place sequences extracted from the training data. The sequences include place actions for several planes, namely the table and the planes of the big and the small shelf. We model affordances for the first type of objects. Participants were asked to perform tasks related to setting up the table and clearing it. In the collected data, the users were subject to two affordances, namely graspability and placeability.

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1\textsuperscript{1}https://optitrack.com/
TABLE II: Results obtained for Graspability models.

| Models   | Train Set MSE | Test Set MSE |
|----------|---------------|--------------|
| Gaussian model | 0.0025 | 0.0043 |
| vMF model    | 0.0042 | 0.0070 |
| Baseline    | 0.0188 | 0.0250 |

C. Graspability

We compare the MSE for the vMF model, the Gaussian Model and a baseline. The results can be seen in Table II. The baseline for the grasp affordance is based on maximum likelihood, wherein for all combinations of object types and surfaces, the mean distances of the wrist from the object being grasped is computed. During inference, starting from the object, the unit vector along the direction of the right wrist is calculated and the grasp position is computed using this and the corresponding mean distance.

For calculating the MSE with the Gaussian model, the output parameters of the network that correspond to the 3D mean are considered as prediction point, and this is used to compare against the ground truth grasp point. For the vMF model, based on the predicted mean direction and distance, we calculate the 3D position and consider it as the prediction point. Table II shows the best results obtained from the network. The results are computed at 1s before grasping.

By observing the MSE, we can see that both neural network models beat the baseline by a significant margin. The Gaussian model achieves a bit better results than the vMF model on the MSE. However, the models digress in the manner of uncertainty estimation. With the Gaussian model, we get a spherical covariance structure indicating the confidence interval around the mean position. This interval gives the possible locations in 3D space, where the human wrist should position, in order to grasp the object. In case of the vMF model, uncertainty is defined on a 2D manifold, i.e. the surface of a sphere with its center at object centroid, and radius being the predicted distance. The output vMF parameters inform on the direction and spread of the distribution on this manifold. Since this gives a density on a surface around the object, it reflects on the possible approach angle of the wrist for successful grasping.

D. Full-Body Prediction

In order to test the full-body prediction we use the prediction framework introduced in [3]. We train the position-velocity model (VRED) on the training set. From the test data we extract 27 trajectories for placing on the table. We use the place affordance model and extract the expected place point \( p^* \) from the MDN. We predict for 1.5sec of motion and optimize the prediction to end up above \( p^* \). Table III shows the distance to the ground truth at different times in the future for our method and several baselines. In the first part of the table the sum over distances of key joints (wrists, elbows, knees, ankles and pelvis) is shown, in the second part only the distance of the wrist to the ground truth is shown. Values are averaged over the 27 trajectories.

The zero velocity baseline just keeps the current state as prediction for future timesteps. The VRED baseline just

| Models   | 250  | 500  | 750  | 1000 | 1250 | 1500 |
|----------|------|------|------|------|------|------|
| Zerovel (b) | 2.38 | 5.18 | 7.76 | 9.68 | 11.09 | 11.94 |
| VRED (b)   | 0.88 | 1.70 | 2.82 | 4.01 | 5.27 | 6.30 |
| ours (b)   | 0.86 | 1.43 | 2.00 | 2.42 | 2.70 | 2.80 |
| oracle (b) | 0.86 | 1.44 | 1.84 | 2.03 | 2.19 | 2.18 |

TABLE III: Error of state prediction per time step for whole body (b) and right wrist (w). Reported values are in meters. For the whole body the sum distance of 9 key joints is shown. The zero velocity baseline just keeps the current state as prediction for future timesteps. The VRED baseline just unrolls the recurrent neural network. Our method takes the affordance prediction into account and optimizes to end up at \( p^* \). The oracle has additional oracle information about the true endposition of the wrist.

It can be seen that the oracle prediction performs best, which is not surprising, as it uses information that is not available at prediction time. Our method using the place point prediction performs second best and outperforms the prediction without any optimization at all time steps.

Figure 7 shows an example trajectory for predicting motion to place a cup. The top row shows our method, the bottom row shows a uninformed prediction using VRED. It can be seen that our method is very close to the ground truth, while the uninformed predicts that the human only moves forward a bit and keeps position afterwards.

V. CONCLUSIONS

We presented a system to learn human object affordances for human motion prediction. We demonstrate that the method can be used to predict full-body trajectories.

A user study was conducted to collect a dataset in a motion-capture setup on a table setup task, in which the actors were subjected to two affordances, namely graspability and placeability.

We modeled the two affordances as conditional probability distributions using deep learning methods by capturing the implicit uncertainty. For the grasp affordance we use a vMF model. The uncertainty encodes the possible approach angles of the human hand for a successful grasp action. The place affordance was modeled with a MDN model. The uncertainty is encoded as possible regions on the surface where the object can be placed.

Testing within our experimental framework shows the good results of the proposed method. Furthermore, our experiments prove that the affordances can be used to improve full-body motion prediction within a state-of-the-art motion prediction framework.

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Fig. 7: Full-body prediction example. From left to right the trajectories after 33ms, 66ms, 100ms and 133ms. Prediction is depicted in green, ground-truth in gray. The top row shows the trajectory with goal optimization towards the affordance, the bottom trajectory is the trajectory without goal optimization.

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