What can we learn from $\phi_1$ and $B^0_d \to \pi^+\pi^-$?

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Abstract

We discuss what we can understand from $\phi_1$ and $B^0_d \to \pi^+\pi^-$ decay mode. Using a convention without weak phases $\phi_2$ and $\phi_3$, we can solve the parameters from the time-depended CP asymmetry. If we can put a condition the contribution from penguin except for the CKM factor including in the diagram is small, then we can lead the allowed region of $R_t$ or $\phi_2$ by using the convention.

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Measurements of CP phase $\phi_1$ by the Belle[1] and BaBar[2] collaborations established CP violation in the $B$ meson system. Measuring the other CP phase $\phi_2$ and $\phi_3$ is also very important to test the Kobayashi-Maskawa(KM) model[3]. The conventional method of measuring $\phi_2$ uses the time dependent CP asymmetry in $B^0 \to \pi^+\pi^-$ [5, 6, 7]. However this method has a difficulty of penguin contamination. If the contribution from penguin diagram is negligible, the CP asymmetry is very clean measurement to extract $\sin 2\phi_2$. But recent measurement by the Belle[9] showed the penguin contribution is likely to be sizable so that we must take account of them. In this letter, we discuss what we can learn about the CP phase $\phi_2$ from the present measurements. The data we can use in here are the branching ratio $B_{\pi\pi}$, the coefficients of $\cos \Delta m t$ and $\sin \Delta m t$ in the time dependent CP asymmetry, $S_{\pi\pi}$ and $A_{\pi\pi}$, and the weak phase $\phi_1$ measured by $B \to J/\psi K_S$.

There are two contributions in $B \to \pi^+\pi^-$ decay, which comes from tree and penguin diagrams. The amplitude is

$$A \equiv A(B^0 \to \pi^+\pi^-) = -(\{T + P_u\} V_{ub}^* V_{ud} + P_c V_{cb}^* V_{cd} + P_t V_{tb}^* V_{td}),$$

where $T$ is a tree amplitude and $P_i (i = u, c, t)$ are penguin amplitudes. Using unitarity relation of Cabbibo-Kobayashi-Maskawa(KCM) matrix, we can rewrite it to the following three type (This is called CKM ambiguity in [8]).

**Convention A:**

$$A^A = -(\{T + P_u - P_t\} V_{ub}^* V_{ud} - \{P_t - P_c\} V_{cb}^* V_{cd}) \equiv -(T_{ut} V_{ub}^* V_{ud} - P_{tc} V_{cb}^* V_{cd})$$  \hspace{1cm} (2)

**Convention B:**

$$A^B = -(\{T + P_u - P_c\} V_{ub}^* V_{ud} + \{P_t - P_c\} V_{tb}^* V_{td}) \equiv -(T_{uc} V_{ub}^* V_{ud} + P_{tc} V_{tb}^* V_{td})$$  \hspace{1cm} (3)

**Convention C:**

$$A^C = -(\{P_t - T - P_u\} V_{tb}^* V_{td} + \{P_c - T - P_u\} V_{cb}^* V_{cd}) \equiv -(T_{ut} V_{tb}^* V_{td} - T_{uc} V_{cb}^* V_{cd})$$  \hspace{1cm} (4)

Using the convention A and B[5, 6] one can extract a phase $\phi_2$ from time dependent CP asymmetry if the penguin contribution is negligible. However present situation is not so. We can not extract the value because the unknown parameters are too many[8]. So we consider how to use the remaining case C. Convention C is including only $\phi_1$ which is found from $B \to J/\psi K_S$ mode et.al. Hence we can find all parameters by using $\phi_1$ and the measurements of CP asymmetry.

We rewrite the decay amplitude for each Convention as follows:

$$A^A \equiv -|T_{ut}| |V_{ub}^* V_{ud}| e^{i\delta_{T_{ut}}} (e^{i\phi_3} + r_A e^{i\delta_A}),$$  \hspace{1cm} (5)

$$A^B \equiv -|T_{uc}| |V_{ub}^* V_{ud}| e^{i\delta_{T_{uc}}} (e^{i\phi_3} + r_B e^{i\delta_B} e^{-i\phi_1}),$$  \hspace{1cm} (6)

$$A^C \equiv -|T_{uc}| |V_{cb}^* V_{cd}| e^{i\delta_{T_{uc}}} (1 - r_C e^{i\delta_C} e^{-i\phi_1}),$$  \hspace{1cm} (7)
where \( \delta \) is the strong phase and

\[
 r_A = \frac{|P_{tc}| |V_{ub}^* V_{cd}|}{|T_{ud}| |V_{ub}^* V_{ud}|}, \quad r_B = \frac{|P_{tc}| |V_{ub}^* V_{td}|}{|T_{ud}| |V_{ub}^* V_{ud}|}, \quad r_C = \frac{|T_{uc}| |V_{ub}^* V_{td}|}{|T_{uc}| |V_{ub}^* V_{cd}|}.
\]

(8)

We can find the relation among the conventions from \( A^A = A^B = A^C \) or \( T_{uc} - T_{ud} = P_{tc} \) as following,

\[
 R_t - r_C \cos \delta^C = r_B R_b \cos \delta^B, \\
 -r_C \sin \delta^C = r_B R_b \sin \delta^B,
\]

(9)

(10)

where

\[
 R_b = \frac{|V_{ub}^* V_{td}|}{|V_{ub}^* V_{cd}|} = \frac{\sin \phi_1}{\sin \phi_2}, \\
 R_t = \frac{|V_{ub}^* V_{td}|}{|V_{ub}^* V_{cd}|} = \frac{\sin \phi_3}{\sin \phi_2} = R_b \cos \phi_2 + \cos \phi_1,
\]

(11)

(12)

and the relations among the parameters for each conventions are

\[
 r_B = r_A r_C, \\
 \delta^A = \delta^B - \delta^C.
\]

(13)

(14)

The measurements for \( B \to \pi^+ \pi^- \) are

\[
 \Gamma(B^0 \to \pi^+ \pi^-) + \Gamma(\bar{B}^0 \to \pi^+ \pi^-) \propto (|A|^2 + |\bar{A}|^2) \\
 \Gamma(B^0 \to \pi^+ \pi^-) - \Gamma(\bar{B}^0 \to \pi^+ \pi^-) \propto (|A|^2 - |\bar{A}|^2) \cos \Delta m t \\
 -2Im(e^{-2i\phi_1} A^* \bar{A}) \sin \Delta m t.
\]

(15)

(16)

The correspondence to the measurements in convention C are

\[
 B_{\pi \pi} \propto (|A|^2 + |\bar{A}|^2) = 2|T_{uc}|^2 |V_{ub}^* V_{cd}|^2 \{1 + r_C^2 - 2r_C \cos \delta^C \cos \phi_1\} \\
 A_{\pi \pi} \equiv \frac{-|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{2r_C \sin \delta^C \sin \phi_1}{1 + r_C^2 - 2r_C \cos \delta^C \cos \phi_1} \\
 S_{\pi \pi} \equiv \frac{2Im(e^{-2i\phi_1} A^* \bar{A})}{|A|^2 + |\bar{A}|^2} = \frac{-\sin 2\phi_1 + 2r_C \cos \delta^C \sin \phi_1}{1 + r_C^2 - 2r_C \cos \delta^C \cos \phi_1}
\]

(17)

(18)

(19)

From the three measurements we can find the three parameters \( |T_{uc}|, r_C \) and \( \delta^C \) by inputting the value of \( \phi_1 \) as the world average which is \( \phi_1 = 23.6^\circ \pm 2.4^\circ \)[11].

Recently, Belle collaboration reported the results[9] but they are not still consistent with the values by BaBar[10]. Hence, we discuss by using the average between Belle and BaBar.

The data are in Table 1.

The first, we find the allowed region for \( r_C \) and \( \delta^C \) from eqs. (18) and (19) by inputting \( \phi_1 = 23.6^\circ \pm 2.4^\circ \), \( S_{\pi \pi} = -0.47 \pm 0.26 \) and \( A_{\pi \pi} = 0.51 \pm 0.19 \) and it is shown in Fig. 1. The solution for the central value are

\[
 (r_C, \delta^C) = (0.816, 8.6^\circ) \quad \text{and} \quad (0.770, 71.6^\circ).
\]

(20)
Table 1: The experimental data and the average.

|                | Belle[9]       | BaBar[10]      | Average       |
|----------------|----------------|----------------|---------------|
| $Br(B \to \pi \pi) \times 10^5$ | 0.54 ± 0.12 ± 0.05 | 0.47 ± 0.06 ± 0.02 | 0.48 ± 0.05   |
| $S_{\pi\pi}$  | -1.23 ± 0.41 $^{+0.08}_{-0.07}$ | 0.02 ± 0.34 ± 0.05 | -0.47 ± 0.26  |
| $A_{\pi\pi}$  | 0.77 ± 0.27 ± 0.08 | 0.30 ± 0.25 ± 0.04 | 0.51 ± 0.19   |

We can find that the solutions have a discrete ambiguity and there are two regions in Fig.1. One of them is smaller region near $\cos \delta^C \sim 1$ and around $r_C \sim 0.8$ and this show the case of small penguin contribution. $\delta^C$ is the angle between $T - P_u - P_t$ and $T - P_u - P_c$ and it comes from the difference between $P_t$ and $P_c$. If top-penguin is very close to charm-penguin or the tree contribution $T$ dominant,

$$\frac{T - P_u - P_t}{T - P_u - P_c} \sim 1 - \frac{P_t - P_c}{T} \sim 1,$$

and then $\delta^C$ becomes to small angle and $r_C$ becomes to close $R_t$. We can guess this region is reasonable.

In the Convention C, we can solve and find the all parameters. However, because the the main target in this mode is to extract $\phi_2$, we have to consider also the other convention or to convert the solution in convention C to the others.

Figure 1: The allowed region for the $r_C$ and $\delta^C$ for $S_{\pi\pi} = -0.47 \pm 0.26, A_{\pi\pi} = 0.51 \pm 0.19$ and $\phi_1 = 23.6^\circ \pm 2.4^\circ$. 
Unfortunately, we can not convert from the solutions in Convention C to the other convention case[8], because the relation to extract them are just only 2 equations (9) and (10) for 3 parameters, \( r_B, \delta B \) and \( \phi_2 \). So we consider a reasonable situation that \( r_A \) and \( r_B \) are not so small but, indeed, it was enhanced by \( 1/R_b \) and the ratio between tree and penguin without KM factor is small. We guess the ratio is order of 0.1. If so,

\[
 r_C = \frac{|T_{uc} - P_{tc}|}{|T_{uc}|} R_t
\]

\[
 \sim R_t - r_B R_b \cos \delta B + \frac{1}{2} r_B^2 \frac{R_b^2}{R_t} \left( 1 - \frac{1}{2} \cos^2 \delta B \right) + O\left( \{P/T\}^3 \right)
\]

and the higher order of \( P/T \) without KM factor is neglected. Using the relation (9) and (10), we find a equation for \( R_t \) or \( \phi_2 \),

\[
 R_t^2 + 2R_t r_C (\cos \delta - 2) + r_C^2 (2 - \cos^2 \delta) = 0
\]

From this equation, we find the allowed region on \( r_C - \phi_2 \) plane for the region we found from the averaged experimental values about \( B \to \pi \pi \) decay mode. The results for the central values are shown in Table 2. If the magnitude of penguin amplitude is negligible, then the \( R_t \) should be very close to \( r_c \). In \( r_C = R_t \) case, the dependence of \( r_c \) for \( \phi_2 \) is shown as the solid line for \( \phi_1 = 23.6^\circ \) and dashed lines for \( \phi_1 = 21.2^\circ \) and 26.0° in Figs.2-4. The close region to the line of \( r_C = R_t \) will satisfy the small penguin condition. To check this, we show \( r_B \) for \( R_t \) we obtained in Table 2. For \( (r_C, \delta C) = (0.82, 8^\circ) \), \( r_B \) is about 0.3 and the ratio between penguin and tree without KM factor \( |P_{tc}/T_{uc}|(\sim r_B R_b) \) is about 0.1. For \( (0.77, 72^\circ) \), \( r_B \) is larger than 1 and this case is out of the assumption.

| \( r_C \)  | \( \delta C \) | \( R_t \)  | \( \phi_2 \) | \( r_B \) |
|---------|-----------|---------|-----------|---------|
| 0.816   | 8.4°      | 0.812   | 104.6°    | 0.287   |
|         |           | 0.837   | 101.3°    | 0.300   |
| 0.770   | 71.6°     | 0.552   | 132.3°    | 1.46    |
|         |           | 2.041   | 19.6°     | 1.63    |

Table 2: \( R_t \) and \( \phi_2 \) for the solution of \( r_C \) and \( \delta C \) for the central values of experimental data.

If the magnitude of penguin contributions are very small, \( r_C = R_t \) and we can approximately extract the value of \( R_t \). Even if it is not so small, we can guess \( r_c \) should be near to \( R_t \) and the cosine of the strong phase \( \delta C \) should be close to 1. It is in the smaller region near 360° in Fig.1. If we take account of only region for \( \delta C \) less than 30° and 10°, then the region for \( \phi_2 \) are reduced and remain only region around \( \phi_2 = 100^\circ \). See Fig.3. When we consider the case the experimental error of \( S_{\pi\pi} \) and \( A_{\pi\pi} \) are reduced up to 0.1, the allowed region will become smaller. We show the regions in Fig.4. The region for \( r_c \) and \( \delta C \) completely separate to two parts. The region near 0° show that \( \phi_2 \) is around 100°.

In this letter we discussed how to use the solution in Convention C. Though the experimental values of \( A_{\pi\pi} \) and \( S_{\pi\pi} \) are not still consistent between Belle and BaBar, we used
Figure 2: The allowed region of $\phi_2$ obtaining from $r_C$ and $\delta^C$ in Fig.1 by using eq.(23). The lines show $r_C = R_t$ for $\phi_1 = 23.6^\circ$ (solid line) and for $\phi_1 = 21.2^\circ$ and $26.0^\circ$ (dashed lines).

Figure 3: $\phi_2$ when we cut the region of $\delta^C$ over $30^\circ$ (left) and $10^\circ$ (right).

Figure 4: The allowed region of $r_C, \delta^C$ and $\phi_2$ in the case the error of $A_{\pi\pi}$ and $S_{\pi\pi}$ is less than 0.1.
the averaged value and obtained the allowed region within the error. If the solution near $\delta^C = 0^\circ$ is true, namely the ratio of tree and penguin without KM factor is as small as order 0.1, then we can estimate $R_t$ or $\phi_2$ and $\phi_2$ is around 100$^\circ$.

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