Thermal disequilibrium during melt-transport: Implications for the evolution of the lithosphere-asthenosphere boundary

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Abstract
This study explores melt-infiltration and melt-rock interaction as a means of shaping the continental lithosphere from beneath. Specifically, we focus upon the role of thermal disequilibrium between melt and the surrounding solid as a means for heating and modifying the continental lithospheric mantle. We use simple pore-network models to estimate the importance of heating of the ambient mantle by thermal disequilibrium with melt transported in channels at the base of the lithosphere. The key physical parameters that control the behavior are the fluid volume fraction (Φ), the relative fluid-solid velocity (v_{fluid}), channel spacing (d), and the timescale of episodic melt-infiltration (τ). Model results and scaling arguments suggest that for geologically-reasonable values of these parameters, disequilibrium heating may contribute more than 10^{-3} \text{W/m}^3 to the heat budget at the lithosphere-asthenosphere boundary. We find that there exists a velocity scale associated with the transient upward motion of a disequilibrium heating front. During episodic melt-infiltration, the models predict the existence of a thermal reworking zone (TRZ) associated with spatio-temporally varying disequilibrium heat exchange. The steady-state width of the TRZ is shown to scale as \sim [\phi v_{fluid} d^2 \tau^2]. The spatio-temporal scales associated with the establishment of the TRZ are comparable with those associated with migration of the lithosphere-asthenosphere boundary as inferred from geologic observations within continental intra-plate settings, such as the western US.

1 Introduction
Continental lithosphere may be profoundly altered by melt-infiltration and metasomatism, manifest in a range of thermal and chemical changes during melt-rock interaction. Indeed, melt-rock processes have been implicated in controlling the tectonic stability of continents in a number of settings (Wang et al., 2015; Wenker & Beaumont, 2017) and localities such as the North China craton (O’Reilly et al., 2001; Gao et al., 2002; Menzies et al., 2007), the Wyoming craton (Carlson et al., 2004), the western US (Plank & Forsyth, 2016; Roy et al., 2016), and the Lherz massif (LeRoux et al., 2007, 2008), among others. Melt-enhanced destruction of the lowermost continental lithosphere is also inferred as a key process in continental rifting, even within currently magma-poor rifts (e.g., Hopper et al., 2020). Although evidence for the key role of melt-rock interaction in potentially destabilizing continental lithosphere has been steadily growing, there are few constraints on the processes involved.

This study explores melt-infiltration and melt-rock interaction as a means of shaping the continental lithosphere from beneath. Specifically, we focus upon the role of thermal disequilibrium between melt and the surrounding solid as a means for heating and modifying the continental lithospheric mantle (CLM). As evidenced in volcanic eruptions, magma may be transported from depth to the surface in thermal and chemical disequilibrium with the surrounding material. The degree of thermal disequilibrium should increase with increasing fluid velocity relative to rock, degree of channelization and therefore, with decreasing depth within the lithosphere (e.g., Cashman et al., 2017; Schmeling et al., 2018). Although disequilibrium thermal and chemical interaction during transport is a complex phenomenon (Wallner & Schmeling, 2016; Oliveira et al., 2018; Schmeling et al., 2018; Bo et al., 2018; Keller & Suckale, 2019), here we simplify the problem to focus on assessing the temporal and spatial scales over which thermal disequilibrium can play a role in warming and therefore weakening the lowermost portion of the lithosphere, near the lithosphere-asthenosphere boundary (LAB). Melts generated deeper in the mantle, ascending to the LAB and interacting with the CLM are expected to be hotter than the surrounding material, transported in channels rather than in percolative flow, with transport Peclet numbers > 1 (e.g Schmeling et al., 2018; Havlin et al., 2013). The models presented are an attempt to connect first-order estimates of the rates and spatial scales of disequilibrium heating to geologic observations, for example, geochemical
and petrologic evidence for the upward migration of the LAB during melt-rock interaction in the western US (e.g., Plank & Forsyth, 2016).

2 Model of disequilibrium heat transport

The model is based on a simple, 1-D theory of heat exchange in a semi-infinite two-phase medium where we assume that fluid transport only occurs along one dimension (Figure 1; e.g., Schumann (1929)). The domain is assumed to be occupied by a constant volume fraction, $\phi$, of fluid moving with a constant (average) velocity $v$ relative to the surrounding stationary solid (with volume fraction $1-\phi$). The model domain may be thought of as co-moving with the reference frame of a constant-velocity solid matrix. The results of this model are therefore applicable to physical situations where heat transport in the fluid-flow direction dominates and where the solid motion is steady.

We assume that an unchanging fluid-solid interface separates the phases within the domain. The detailed geometry of this interface is not specified in this 1-D calculation but, following Schumann (1929) and subsequent pore-network models (Spiga & Spiga, 1981; Kuznetsov, 1994, 1995b, 1995a, 1996), is parametrized by a heat transfer coefficient, $k$ (Figure 1). This coefficient is a proxy for the geometry of the inter-phase interface, namely the contact area per unit volume, controlled by the spatial scale of channelization, $d$. As illustrated in (Figure 1), a large value of $k$ may represent efficient fluid-solid heat exchange as in the case of many channels separated by a small distance. Conversely, a low value of $k$ would represent inefficient exchange, as in the case of a larger characteristic length scale between the channels.

Following Schumann (1929), we will assume that when the fluid and solid are in thermal disequilibrium, the thermal evolution of the system is governed mainly by heat exchange between the two phases across their interfacial surface. This heat exchange is assumed to dominate over thermal dispersion and axial conductive heat fluxes within the solid (e.g., within the walls of channels) and within the fluid (within channels). Additionally, fluid-solid heat exchange is assumed to be linearly proportional to their local temperature difference. These arguments lead to coupled equations for the temperature of the solid, $T_s$, and of the fluid, $T_f$ (Schumann, 1929):

$$\frac{\partial T_f}{\partial t} + v \frac{\partial T_f}{\partial x} = \frac{k}{\phi c_f}(T_f - T_s) = -k_f(T_f - T_s)$$

(Figure 1. Cartoon of 1D model with parameters such as: specific heat capacities ($c_p$) and densities ($\rho$), fluid velocity $v$, fluid volume fraction, $\phi$, and the fluid-solid heat transfer coefficient $k$. The heat transfer coefficient $k$ is a function of the geometry of the fluid-solid interface and scales as $d^{-2}$ (Appendix A); large $k$ corresponds to large fluid-solid contact area per unit volume (e.g., small channel spacing, $d$) and vice versa.)
\[
\frac{\partial T_s}{\partial t} = \frac{k}{(1 - \phi)c_s}(T_f - T_s) = k_s(T_f - T_s)
\]  

(2)

where \(c_f\) and \(c_s\) are the heat capacity per unit volume at constant pressure for solid and fluid, so \(c_f = c_{p,\text{fluid}} \rho_f\) and \(c_s = c_{p,\text{solid}} \rho_s\). Two independent factors on the right hand sides specify the timescales of heat exchange in the fluid, \(1/k_f = \phi c_f/k\), and in the solid, \(1/k_s = (1 - \phi)c_s/k\). Similarly, a characteristic length scale emerges out of the fluid motion, \(v/k_f\). These characteristic length- and timescales are combinations of five user-specified quantities that completely specify the model: the heat transfer coefficient, \(k\), fluid volume fraction \(\phi\), the heat capacities per volume (heat capacitances), \(c_f\), and \(c_s\), and the fluid velocity, \(v\) (Table 1).

| Name                      | Symbol           | Value or range            | Source                                      |
|---------------------------|------------------|---------------------------|---------------------------------------------|
| Fluid, solid density      | \(\rho_f, \rho_s\) | 2800, 3300 kg/m\(^3\)   | (Lesher & Spera, 2015)                      |
| Fluid specific heat capacity | \(c_{p,\text{fluid}}\) | 1400 J/(kg K)           | (Lesher & Spera, 2015)                      |
| Solid specific heat capacity | \(c_{p,\text{solid}}\) | 1250 J/(kg K)           | (Lesher & Spera, 2015)                      |
| Fluid heat capacity per volume | \(c_f\)         | 3.920 \times 10^6 J/(m\(^3\)K) | \(c_{p,\text{fluid}} \times \rho_f\)       |
| Solid heat capacity per volume | \(c_s\)         | 4.125 \times 10^6 J/(m\(^3\)K) | \(c_{p,\text{solid}} \times \rho_f\)       |
| Heat transfer coefficient | \(k\)            | \(10^{-5}\) to \(10^1\) W/m\(^3\)K | this work (section 2.1)                     |
| Fluid volume fraction     | \(\phi\)         | 0.1 to 0.2                | ref here                                    |
| Fluid average (linear) velocity | \(v\)           | 1 to 100 mm/yr           | ref here                                    |
| Weighted heat capacity ratio | \(z\)            | 0.0096 to 0.2376          | calculated                                  |
| Fluid-solid Nusselt number | \(Nu\)           | 0.1 to 12.4               | for slow flows (Handley & Heggs, 1968)      |
| Constant in Eqn A2        | \(\beta\)        | 6 to 10                   | (Dixon & Cresswell, 1979)                   |
| Constant in Eqn A2        | \(A\)            | 2 to 6                    | (Dullien, 1979)                             |
| Separation of fluid-rich channels | \(d\)          | \(10^{-1}\) to \(10^2\) m | (LeRoux et al., 2008)                      |

To non-dimensionalize the equations, we define the normalized relative temperature of the fluid and solid, \(T'_f = (T_f - T_0)/\Delta T\) and \(T'_s = (T_s - T_0)/\Delta T\), where \(T_0\) is reference temperature and \(\Delta T\) is a temperature perturbation (described below). We also introduce the dimensionless position, \(x' = x k_f / v\), a dimensionless time, \(t' = k t\), and the heat capacitance ratio \(z = k_s/k_f = \phi c_f/(1 - \phi)c_s\). The non-dimensional versions of equations (1) and (2) are now (3) and (4):

\[
z \frac{\partial T'_f}{\partial t'} + \frac{\partial T'_f}{\partial x'} = -(T'_f - T'_s)
\]  

(3)

\[
\frac{\partial T'_s}{\partial t'} = (T'_f - T'_s)
\]  

(4)

(see also Spiga and Spiga (1981)). It is clear that for a given temperature difference, \((T'_f - T'_s)\), the behavior of this set of non-dimensional equations is governed by \(z\) (1/z is the dimensionless fluid velocity). Analytic solutions for this set of equations have been derived for a number of limiting cases, particularly for large \(k\) (Spiga & Spiga, 1981; Kuznetsov, 1994, 1995b, 1995a, 1996), and were used to benchmark the numerical calculations below.
2.1 Heat transfer coefficient

Before we can interpret the results below, we consider the meaning of the assumed heat transfer coefficient, $k$, which is related to the individual solid and fluid heat transfer coefficients, $k_s = k/(c_s (1 - \phi))$ and $k_f = k/(c_f \phi)$. Since $k_f$ and $k_s$ have dimensions of inverse time, $k$ represents the amount of heat transferred per unit time from fluid to solid, per unit volume, per unit difference in temperature (Schumann, 1929). Although the non-dimensional system above (Eqns 3 and 4) is independent of $k$, the actual physical behavior depends on the characteristic length ($v/k_f$) and time ($1/k_s$) scales, which depend on $k$.

The factors that determine $k$ are explored in Appendix A, but here we summarize: for a given fluid volume fraction, $\phi$, $k$ is strongly controlled by the length scale of channelization, parameterized by the channel spacing $d$. To get an idea of the scale of channelization at the LAB, we turn to structural, petrologic, and geochemical data from the Lherz Massif suggesting that melt-rock interaction has driven refertilization of a harzburgite body into lherzolite (LeRoux et al., 2007, 2008). In the field, the lherzolite bodies are separated from each other by distances of several tens of meters and this is also the spatial scale of isotopic disequilibrium between metasomatizing fluids and the harzburgite parent material (LeRoux et al., 2008). We take this as a proxy for the spatial separation of fluid-rich channels and choose a broad range for the relevant spatial scale of channelization, $d = 10^{-1}$ to $10^2$ m (10 cm to 100 m channel spacings). The corresponding range of the heat transfer coefficient (Table 1) is therefore $k = 10^{-5}$ to $10^3$ W m$^{-2}$K$^{-1}$, used in the models below. In the following, we shall consider modeling scenarios where material properties are fixed and the flow regime is considered to be in the channelized regime.

2.2 Response to a step-function thermal perturbation

For the first set of calculations, we assume the domain is initially at steady-state with the fluid and solid in equilibrium, $T_s = T_f = T_0$. At $t = 0$, the temperature of the fluid entering at the inflow, $x = 0$, is perturbed so that, $T_f(x = 0, t \geq 0) = T_0 + \Delta T$, introducing $\Delta T$ as a temperature scale into the problem. This disturbs the initial steady state and starting at $t = 0^+$, the fluid at the inflow is no longer in thermal equilibrium with material in the domain. Equations (3) and (4) may be solved for the thermal evolution of the fluid and solid subject to the initial and boundary condition: $T_s = T_f = 0$ initially and $T_f(x = 0, t \geq 0) = 1$. The behavior of the nondimensional system is controlled by $z$, the dimensionless fluid velocity, which is a function of both material properties $c_s$ and $c_f$ and the fluid volume fraction, $\phi$. Since the material properties $c_s$ and $c_f$ are held constant, there is a unique mapping between $z$ and $\phi$, the fluid volume fraction (we use $\phi = 0.01$ to 0.20, corresponding to values of $z = 0.0096$ to 0.2376; Table 1).

The response to a step-function is discussed in detail in Appendix B, and the primary finding is that temperature profiles within the domain exhibit a transition or disequilibrium zone, lagging behind the fluid front, but migrating inward into the domain. A key result is that the rate at which this zone migrates is independent of the heat transfer coefficient, $k$. The location of maximum disequilibrium (maximum $T_{fluid} - T_{solid}$, and therefore the greatest heat exchange) lags behind the fluid front and progresses inward into the domain at a rate determined by the fluid velocity, fluid volume fraction, and material heat capacitances (Eqn 5; Appendix B). Models using a range of $\phi$ and $v_{fluid}$ values (Table 1) show that the disequilibrium front migrates at a rate given by

$$V_{diseqm} \approx v_{fluid} \left( \frac{c_f}{c_s} \right) \left( \frac{c_f \phi}{c_f \phi + (1 - \phi)c_s} \right)$$

These results are an extension of those in Kuznetsov (1994), where an analytic expression for the migration rate is presented, in the limit that the degree of disequilibrium is
small. Although the rate of migration of the disequilibrium front is not a function of \( k \), the heat transfer coefficient, the characteristic width of this zone and the degree of disequilibrium within it are strong functions of \( k \) (Appendix B).

### 2.3 Response to a sinusoidal thermal perturbation

Here we consider sinusoidal thermal pulses, introducing a timescale into the problem, the period, \( \tau \), or the non-dimensional period \( \tau k_s = \tau k/(1 - \phi)c_s \), where \( 1/k_s \) is the longest response timescale in the problem, associated with the thermal response of the solid. This class of models assume that the fluid entering the domain is hotter than the ambient initial temperature of the solid, but the thermal contrast varies sinusoidally. The time-varying inlet fluid temperature represents pulses of high temperature fluids or melt, and the non-dimensional period controls the length scale, \( \delta \), over which thermal oscillations penetrate into the domain. For the material parameters in Table 1, and channel spacing of \( d = 10 \) to 100 m leads to a response timescale \( 1/k_s \approx 1 \) to 100 yr, which is short compared to the timescales of geologic events. Oscillations with periods that are long compared to \( 1/k_s \) penetrate farther into the domain than short period oscillations (Figure 2). Therefore, periodic thermal perturbations that might represent melt infiltration pulses lasting \( 10^3 \) to \( 10^6 \) yrs will be characterized by a region of sinusoidally varying temperatures: a thermal re-working zone (TRZ) (blue curves in Figure 2a & b). The wavelength of these oscillations is set by the period \( \tau \), \( \lambda = v_{\text{fluid}}\tau \). The penetration distance of the thermal oscillations, \( \delta \), is the maximum width of the TRZ. At short times, when \( t \ll \delta/V_{\text{diseqm}} \), there is one zone of disequilibrium (the TRZ, bounded by the disequilibrium front). At longer times, the TRZ widens to a maximum width, \( \delta \), at time \( \delta/V_{\text{diseqm}} \). When \( t \gg \delta/V_{\text{diseqm}}t \), there are two zones of disequilibrium, one stationary at the inlet (the TRZ) and the migrating zone discussed above that moves at \( V_{\text{diseqm}} \) (Eqn 5; red curves in Figure 2a & b).

To illustrate the oscillatory nature of temperatures inside the TRZ, we consider temperature vs. time paths within the domain at varying distances from the inlet (Figure 2c). The amplitude of the oscillations decay with distance, but at each point in the TRZ, once the oscillations are established, the amplitude is constant in time (Figure 2c). As we might expect, the maximum width of the TRZ, \( \delta \), is set both by the non-dimensional oscillation period, \( \tau/t_s \) and by the heat transfer coefficient (Spiga & Spiga, 1981),

\[
\delta = \left( \frac{c_f \phi v_{\text{fluid}}}{k} \right) \left( \frac{\tau/t_s}{4\pi^2} \right)
\]

noting that \( k \sim d^{-2} \) (Appendix A), and \( t_s = 1/k_s = c_s(1 - \phi)/k \), the expression above suggests that, for fixed \( v_{\text{fluid}}, \delta \sim (\tau/d)^2 \) as confirmed by the numerical results (Figure 2d).

### 3 Discussion

The models presented here are highly idealized and therefore limited in their representation of the complexities of deformation and fluid-rock interactions within the Earth. In particular, we abstract the effective thermal properties and the geometry of the fluid-solid interface into a single number, the heat transfer coefficient, \( k \), which is strongly controlled by the channel spacing, \( d \). Sinuosity and other aspects of the geometry of channelization are abstracted and the details of processes at and below the scale of an average channel spacing, \( d \), are ignored. Instead, the focus here is on the effective behavior at mesoscopic spatial scales \( \gg d \). Even at these scales, we ignore spatial variations in transport, including variability in the fluid volume fraction, fluid velocity, and effective heat transfer coefficient. Additionally we have ignored time-dependent variability in transport, e.g., feedbacks due to possible phase changes during disequilibrium heating/cooling, which would affect the geometry of the fluid-solid interface (re. Keller and...
Figure 2. (a) Normalized temperature profiles, $T_s/\Delta T$ (dashed) and $T_f/\Delta T$ (solid) for a calculation with fluid velocity $v = 1$ m/yr, at time $t = 1$ Kyr. Results are shown for two cases with the same fluid velocity, $\phi$, and channel spacing $d$ indicated, but with different normalized periods, $\tau k_s = 50$ (red), 150 (blue), for the thermal perturbation. For the chosen parameters, $t_s = 1$ yr. The amplitude of spatial oscillations decreases over a decay scale $\delta$, as indicated. (b) The degree of disequilibrium ($T_f' - T_s'$) is also oscillatory over a thermal reworking zone of width $\delta$. (c) Temperature-time paths within $x < \delta$ plotted at different distances from the inlet for the case where $\tau/t_s = 150$ in (a) and (b), illustrating temporal oscillations at each location within the thermal reworking zone. (d) Width of the thermal reworking zone, $\delta$, as a function of oscillation period $\tau$ and channel spacing $d$ as indicated. Dashed lines (slope 2) are the expected analytic scaling in Eqn 6 and the squares indicate numerically derived values of $\delta$ obtained by fitting an exponential decay to the envelope of the $(T_f' - T_s')$ oscillations in the TRZ, e.g., in (b). Thin horizontal lines are at $\delta = 1$ and 10 km.

Suckale). Finally, the models presented here are 1D and ignore the 3D nature of relative motion between fluid and solid even on the mesoscale ($\gg d$).

Given these limitations, the 1D models are presented here as a useful way to frame first-order questions and develop arguments related to the consequences of disequilibrium thermal transport dominated by downstream effects in the direction of fluid flow. First, lets assume that the model domain is analogous to the lowermost lithosphere, where we expect melt or fluid transport to be channelized (e.g., Schmeling et al., 2018, Figure...
3). The region $x < 0$ considered here is analogous to a melt-rich sub-lithospheric re-

**Figure 3.** Cartoon illustrating implications for a thermal re-working zone (TRZ) that forms a modified layer at the lowermost CLM as a result of disequilibrium heating. For episodic melt-infiltration into channels of spacing $d = 100$ m, with period $\sim 10$ Kyr, the TRZ is characterized by upward-decreasing degree of disequilibrium (indicated by the color) and is $\delta \sim 10$ km wide after 0.1 to 1 Myr, for $v_{\text{fluid}} = 0.1$ to 1 m/yr.

Three key results emerge from the models above: (1) disequilibrium heating, estimated using the heat transfer coefficient, may be a significant portion of the heat budget at the LAB and the lowermost CLM, (2) a material-dependent velocity scale associated with transient disequilibrium heating, and (3) the existence of a thermal reworking zone (TRZ) associated with spatio-temporally varying disequilibrium heat exchange. We discuss each of these within the context of episodic melt-infiltration into the base of the CLM in an intra-plate setting. Specifically, an intra-plate tectonic setting such as the Basin and Range province of the western US where deformation and 3D melt-rock interaction may be simplified so that we consider the effects of dominantly vertical heat transport within a slowly deforming lithosphere.

**i. Disequilibrium heating and the heat budget at the LAB.** The relative importance of disequilibrium heating at the LAB may be established by considering the effective heat transfer coefficient, $k$, and the factor which most strongly controls it, namely the average spacing of channels, $d$. For the material parameters in Table 1, and channel spacing of $d = 1$ to 100 m, $k$ is in the range $k \approx 10^{-5}$ to $10^3$ W m$^{-3}$K$^{-1}$ (Appendix A). Physically, $k$ corresponds to fluid-solid heat transfer per unit time, per unit volume, per unit difference in temperature (Schumann, 1929). Therefore, for a 100 K excess temperature of the infiltrating melt, disequilibrium heating might contribute around $10^{-3}$ to $10^5$ W m$^{-3}$ to the heat budget at the LAB. This is a conservative estimate, given that
the temperature difference between magma and the surrounding material may be anywhere from 100 to 1000 K (e.g., in crust; (Lesher & Spera, 2015)). (While an LAB excess temperature of the melt of about 100 K is reasonable, but note that plume excess temperatures are estimated to be as large as 250 K (Wang et al., 2015).)

To put this in perspective, we now compare this estimated heat budget to the heat budget due to crystallization of melt in channels may be estimated using scaling arguments made in Havlin et al. (2013). Assuming that melt and rock are in equilibrium, Havlin et al. (2013) estimate that the heat released by a crystallization front would contribute around \( \rho HS_{dike} \), where \( \rho \) is the melt density, \( H \) is the latent heat of crystallization, \( S_{dike} \) is a volumetric flow rate out of a decompacting melt-rich LAB boundary layer due to diking. For a representative porosity of \( \phi = 0.1 \) within the dike, Havlin et al. (2013) estimate \( S_{dike} \approx 2 \times 10^{-8} \) m\(^3\)/s. Taking \( \rho = 3000 \) kg m\(^3\), and \( H = 3 \times 10^5 \) J/kg, the heat source due to the moving crystallization front would be around 10\(^2\) W for each dike. If we assume that this heating takes place within a volume that is roughly the dike height \( \times \) dike spacing \( \times \) dike length, we can determine the power per unit volume generated due to crystallization. For example, assuming dike heights of about 10\(^3\) m and dike spacing large enough for non-interacting dikes (as estimated by Havlin et al. (2013), a porosity of 0.1 would require a dike spacing of \( \sim 10^3 \) m), the heat source due to a crystallizing dike boundary layer would be \( < 10^{-4} \) W/m\(^3\) (per unit length along strike). These arguments corroborate the idea that disequilibrium heating during melt-rock interaction could be a significant portion of the heat budget at the LAB as compared to other expected processes, such as heating due to crystallization of melt in channels.

**ii. Progression of a disequilibrium heating zone/front at a rate \( V_{diseqmn} \).**

The importance of a reduced velocity for the migration rate of either a disequilibrium front (Figure B1a,b,c) or widening of a thermal reworking zone (Figure 2b) is that this limits the rate at which the lowermost CLM may be modified by thermal disequilibrium. It is important to note that this reduced velocity (Eqn 5) is independent of temperature contrast between the CLM and infiltrating melt and depends only on the relative fluid volume fraction, fluid velocity and material properties. Assuming a fluid volume fraction of 1 to 10 \% at the LAB, and material properties in Table 1, we would expect that \( V_{diseqmn} \) would be around 1 to 10 \% of the fluid velocity (see Appendix B and Figures B1, 3). For relative fluid velocity in the range of 0.01 to 1 m/yr, we would predict that disequilibrium heating front at the LAB would migrate upward at a rate of \( \approx 1 \) to 10\(^2\) km/Myr, which is comparable to rates of CLM thinning predicted by heating due to the upward motion of a dike boundary layer (1 to 6 km/Myr in Havlin et al. (2013)). Interestingly, an upward-moving disequilibrium heating zone with \( V_{diseqmn} \approx 1 \) to 10\(^2\) km/Myr brackets the rate of upward migration of the LAB at 10-20 km/Myr inferred from the pressure and temperature of last equilibration of Cenozoic basalts in the Big Pine volcanic Field in the western US (Plank & Forsyth, 2016). An implication of the models here, therefore, is that disequilibrium heating may produce lithosphere modification at geologically-relevant spatial and temporal scales provided that the melt velocity in channels at the LAB is on the order of \( 10^{-2} \) to \( 10^{-1} \) m/yr (Figure 3).

**iii. Thermal reworking zone (TRZ).** A key result that may be relevant to the evolution of the LAB is that episodic infiltration of melts that are hotter than the surrounding CLM would lead to a long-lived region of disequilibrium heating within a thermal reworking zone or TRZ. Although the models suggest that the TRZ would undergo a phase of transient widening (at a rate given by Eqn 5), it will reach a steady-state width \( \delta \) that should scale as \( \delta \sim [\phi v_{fluid} d^2 \tau^2] \) where \( d \) is a characteristic scale of channelization and \( \tau \) is a timescale associated with the episodicity of melt-infiltration (Figures 2d and 3). This scaling gives us a way to conceptualize the modification of the lowermost CLM as a zone that encompasses a variable thickness TRZ, depending on \( v_{fluid} \) and on the timescale of melt-infiltration (Figure 3). Regions where the timescale of episodic melt-infiltration is longer are predicted to have a thicker zone of modification at the LAB.
For example, for a channel spacing of $d = 10^2$ m, disequilibrium heating by repeated melt pulses that last around 10 Kyrs implies a maximum thickness of roughly 10 km for the zone of modification (Figure 2d). In this scenario, the TRZ grows to this maximum width over a timescale governed by $\delta/V_{diseqm}$; for $V_{diseqm} = 10$ km/Myr, which corresponds to melt velocity of roughly 0.1 m/yr (see (ii) above), the 10 km wide TRZ would be established within about 1 Myr (Figure 3), comparable to that comparable to rates of CLM modification inferred from observations in Plank and Forsyth (2016).

4 Conclusions

In summary, the spatial and temporal scales associated with the establishment of the TRZ are comparable to those for CLM modification inferred from geochemical and petrologic observations intra-plate settings, e.g., the western US (Plank & Forsyth, 2016). These scaling arguments lead to the idea that perhaps the TRZ represents a zone of thermal modification at the base of the CLM that may also correspond to (or encompass) a zone of rheologic weakening and/or in-situ melting if the infiltrating fluids are hotter than the ambient material. The dynamic evolution of the LAB during episodic pulses of melt-infiltration is beyond the scope of the simple models above (which assume a stationary, undeforming matrix). However, assuming mantle material that obeys a temperature and pressure-dependent viscosity scaling relation such as in Hirth and Kohlstedt (2003), at an LAB depth of about 75 km where we assume that the ambient mantle is cooler than the dry solidus (e.g., $1100^\circ$C + 3.5$^\circ$C/km; Plank and Forsyth (2016)), we would expect a significant viscosity reduction during heating (e.g., factor $\approx 1/62$ for a temperature increase of 100 K). This effect is weaker, but still important for a deeper LAB; e.g., at 125 km depth, the viscosity reduction would be a factor $\approx 1/18$ for a temperature increase of 100 K.

The models above suggest that disequilibrium heating may contribute more than $10^{-3}$W/m$^3$ to the heat-budget at the LAB and, for fluid velocity of 0.1 to 1 m/yr in channels that are roughly $10^2$ m apart, a 10 km wide TRZ may be established within 1 Myr. Disequilibrium heating, therefore, is associated with a timescale that may be relevant for regional tectonics in intra-plate settings. Interestingly, geochemical evidence from Cenozoic basalts from the western US, particularly space-time variations in volcanic rock Ta/Th and Nd isotopic compositions suggest that the timescale of modification and removal of the lowermost CLM is on the order of $10^4$ Myrs (Farmer et al., 2020). These authors argue that the observed transition from low to intermediate to high Ta/Th ratios indicates a change from: arc/subduction-related magmatism, to magmatism associated with in-situ melting of a metasomatized CLM (the “ignimbrite flare-up”), to magmatism due to decompression and upwelling after removal of the lowermost CLM. If correct, these interpretations and observations are promising and provide an important avenue for exploring the role of thermal and chemical disequilibrium during melt-rock interaction and destabilization of the CLM in an intra-plate setting. Disequilibrium heating during melt-infiltration may be an important process for modifying the lowermost CLM and may play a role in the rheologic weakening that may be needed to mobilize and possibly remove the lowermost CLM.

Appendix A Heat transfer coefficient, $k$

The factors that determine $k$ can be illustrated by considering that the fluid-solid heat transfer rate must depend on the geometry of the inter-phase interface and also on the effective thermal conductivity of the porous medium. Although the geometry of the fluid-solid interface may be complex, this model considers one aspect of it: the specific fluid-solid interface surface area (contact area per unit volume), $a_{sf}$, which is a function of the length-scale of channelization in the solid. In the porous flow case for example, if the solid matrix is made of spheres with an average particle diameter $d$, then the spe-
specific area for a grain is $S_0 = 6/d$, so $a_{sf} = S_0(1 - \phi) = 6(1 - \phi)/d$ (Dullien, 1979). This sets a limit for channels, where we shall assume that the specific surface area is $a_{sf} \sim A(1 - \phi)/d$, where $A$ is a number that is between 2 (as for a single cylindrical channel with small volume fraction $\phi$) and 6 (Dixon & Cresswell, 1979). Whereas the specific contact area is a geometric factor, the effective conductivity of the medium depends on the Nusselt number, $Nu$. Theoretical arguments in Dixon and Cresswell (1979) show that the effective thermal conductivity may be written in terms of the individual fluid and solid thermal conductivities $\lambda_f$ and $\lambda_s$ (basically taking the fluid and solid in parallel):

$$\frac{1}{C_{eff}} = \left[ \frac{1}{Nu\lambda_f} - \frac{1}{\beta \lambda_s} \right]$$

(A1)

where $\beta = 10$ for spherical matrix grains, 8 for cylinders, and 6 for slabs (Dixon & Cresswell, 1979). For the rest of this work, we will take the range of $\beta$ to be 6 to 10 representing the highly-channelized vs porous flow end-member geometries. For slow flows (Reynolds number $Re \ll 100$), Handley and Heggs (1968) argue that $Nu$ ranges from 0.1 to 12.4 (Dixon & Cresswell, 1979) (Table 1). The relevant quantity that determines $k$ is an effective “conductance” $C_{eff}/d$, so that $k = C_{eff}a_{sf}/d$,

$$k = \frac{1}{d} \left[ \frac{1}{Nu\lambda_f} - \frac{1}{\beta \lambda_s} \right]^{-1} \frac{A(1 - \phi)}{d}$$

(A2)

a product of a material-dependent quantity and a geometry-dependent quantity. Turning now to physical properties relevant to the transport of melts through the lithosphere, using a reasonable conductivity for basaltic magma of $\lambda_f = 1 \text{ W/(m K)}$ (Lesher & Spera, 2015), $\lambda_s = 2.5 \text{ W/(m K)}$ for the solid, and taking $Nu = 0.1$ to 12.4, $\beta = 6, 8,$ or 10, and $A = 6$, we find that the effective conductivity $C_{eff}, a_{sf}$ and $k$ are within the ranges shown in Figure A1. (Note that choosing $A \approx 2$, for a more channelized geometry, would change $k$ by less than an order of magnitude.) Specifically, Eqn (A2) shows that $k \sim d^{-2}$ and strongly decreases with increasing spatial scale of channels characterizing the fluid-solid interaction; Figure A1c.

**Appendix B** Response to a step-function

As fluid with a perturbed temperature enters at $x = 0$, the solid heats up while the fluid cools (for positive perturbation $\Delta T$). The perturbation “front”, the farthest extent of the fluid that entered $x = 0$ at $t = 0$ with perturbed temperature, is always at $x_{front} = vt$ (or $x'_{front} = t'/z$; where the dimensionless velocity of the fluid is $1/z$). The response to a step-function is essentially a transient disequilibrium front, traveling inward at $V_{diseq}$, behind which the solid and fluid equilibrate at the new inlet temperature.

As we might expect, the behavior of the nondimensional system (Eqns 3 and 4) is governed by $z$, the heat capacitance ratio. In response to a step-function increase in the fluid temperature at the inlet, temperature profiles within the domain exhibit a transition or disequilibrium zone, lagging behind the fluid front (Figure B1). Ahead of this disequilibrium zone, the fluid and solid are in equilibrium at the initial ambient temperature of the solid, $T'_{s} = T'_{f} = 0$. Behind this zone, the fluid and solid are in equilibrium at the incoming fluid temperature, $T'_{s} = T'_{f} = 1.x$

Following an initial lag time (when the maximum disequilibrium is at $x' = 0$), the disequilibrium zone migrates inward migration at a steady speed, a fixed fraction of the fluid velocity, $v$ (Figure B1c and B1d). The ratio of the migration rate of the disequilibrium zone to the fluid velocity is controlled by the heat capacity ratio, $z$, and therefore by the fluid volume fraction, $\phi$ (Figure B1). In the limit that the fluid and solid are nearly in equilibrium ($T'_{f} - T'_{s} \approx 0$), Kuznetsov (1994) shows that the shape of the temperature difference function (Figure B1b) approaches a Gaussian with width that depends...
Figure A1. (a) Effective thermal conductivity, $C_{\text{eff}}$, as a function of Nusselt number, (b) geometric factor, $a_{\text{sf}}$, as a function of transport length scale in the solid, $d$, and (c) fluid-solid heat transfer coefficient, $k$, as a function of transport length scale in the solid, $d$. For a fixed $d$ value, the dashed lines in (c) delineate the variation in $k$ for the range of $\beta$ values in (a) and $\phi$ values in (b), illustrating that $k$ is mainly controlled by $d$, rather than the other parameters.

on $\sqrt{T}$ and the zone of disequilibrium migrates at speed $vc_f/(\phi c_f + (1-\phi)c_s)$. Our models show that, when there is significant disequilibrium, the zone of disequilibrium migrates with a rate given by Eqn 5 (Figure B1d), which does not depend on $k$, the heat transfer coefficient.

We illustrate the dependence on $k$ for the specific case where we take the fluid velocity $v=1$ m/yr, and consider channel spacings, $d = 50$ to 150 m, which correspond to $k = 5 \times 10^{-3}$ and $5 \times 10^{-4}$ W m$^{-3}$ K$^{-1}$, respectively (Figure B2).

The migration of the disequilibrium front may be thought of as the motion of the locus of maximum heating, which moves at speed $V_{\text{dissym}} \approx v/10$, for $\phi = 0.1$ in Figure B2. The degree of disequilibrium within the migrating disequilibrium zone, $D$, decays as $1/\sqrt{T}$ (e.g., Kuznetsov, 1994) and is controlled by the heat transfer coefficient, $k$. $D$ scales as $1/\sqrt{k}$ and therefore is a linear function of $d$, the channel spacing (Figure B2b). This transient is apparent in temperature-time paths (Figure B2c and B2d), where the approach to steady-state occurs on a timescale governed by $\phi v_{\text{fluid}}/k$.

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Figure B1. (a) Normalized temperature as a within the fluid (solid lines) and solid (dashed lines) at different values of the dimensionless time, $t'$, as indicated. The colors represent two different fluid volume fractions, $\phi$, and therefore different $z$. (b) Normalized temperature difference between fluid and solid as a function of dimensionless position, shown for the cases considered in (a). (c) The same profiles as in (b), but now plotted as a function of position normalized by the fluid front location. The fluid front is always at $x/\text{front}=t'/z$; stationarity of the disequilibrium zone in this plot indicates that the disequilibrium zone migrates at a constant, $z$-dependent fraction of the fluid velocity. (d) Normalized migration rate of the zone of disequilibrium as a function of fluid volume fraction, $\phi$. Red dot is for $\phi \approx 0.1$, corresponding to models shown in Figures B2 and 2.

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Figure B2. (a) Normalized temperature profiles, $T_s/\Delta T$ (dashed) and $T_f/\Delta T$ (solid) for a calculation with fluid velocity $v = 1$ m/yr, at times $t = 1, 12.5,$ and $25$ Kyr. The temperature profiles transition between the incoming fluid temperature (=1, on the left) and the initial ambient temperature (=0, on the right). Results are shown for two cases with different channel spacings, $d$, corresponding to different heat transfer coefficients, $k$, as indicated. The transition region (e.g., highlighted in gray at $t=12.5$ Kyr, is characterized by a width, $w$, that is larger for smaller $k$ (large $d$) and increases over time. (b) The degree of disequilibrium within the transition zone is characterized by the maximum difference, $D$, between the solid and fluid temperature profiles. $D$ is greater for smaller $k$ and decreases as a function of time. (c) and (d), Normalized (c) solid and (d) fluid temperature as a function of dimensionless time since first contact with the perturbation front for the results in blah.
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