Neutrino Moments and the Magnetic Primakoff Effect

G. Domokos
and
S. Kovesi–Domokos

Department of Physics and Astronomy
The Johns Hopkins University
Baltimore, MD 21218

If different species of neutrinos possess transition magnetic moments, a conversion between species can occur in the Coulomb field of a nucleus. In the case of Dirac neutrinos this corresponds to an active to sterile conversion, whereas in the case of Majorana neutrinos, the conversion takes place between active species. The conversion cross sections grow with the energy of the incident neutrino. The formalism is also applied to a new type of experiment designed to test the existence of the “KARMEN anomaly”.

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The question whether neutrinos possess magnetic moments (both flavor diagonal and/or transition moments) is of importance in exploring physics beyond the Standard Model. There exists a large number of theoretical models in which neutrinos possess magnetic moments; also, a variety of upper limits has been placed on the magnitude of moments neutrinos of various flavors can possess. Typically, the upper bounds obtained from the various experiments and observations give $\mu \sim 10^{-11}\mu_B$, where $\mu_B$ stands for the Bohr magneton, with a varying degree of model dependence. For a recent review see e.g. [1].

Here we propose a novel method by means of which transition moments can be measured. In essence, we propose to take advantage of the magnetic analogue of the

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1E-mail: SKD@HAAR.PHA.JHU.EDU
Primakoff effect: if a neutrino possesses a transition moment, the Coulomb field of a nucleus will induce a helicity flipping transition dominantly between the incoming neutrino and one for which the transition moment is largest. (In the case of three flavors, the transition moment is a $3 \times 3$ matrix in flavor space. However, for the purposes of this work, we tentatively assume that just one moment dominates the transition.) The observed effect depends on whether neutrinos are Majorana or Dirac particles. In the case of Majorana neutrinos, the incoming neutrino flips to an active one of a different flavor. Due to the fact that neutrino beams consist mostly of muon neutrinos, the Coulomb field will induce an excess of electron neutrinos and/or tau neutrinos. Those, in turn, will create electrons or taus in a neutrino detector. By contrast, if neutrinos are Dirac particles, the magnetic transition in the Coulomb field converts them into right handed, sterile neutrinos. Hence, there will be a depletion of the active muon neutrinos from the beam.

As another application of this effect, we examine the production of the particle conjectured by the KARMEN collaboration. An anomaly in the time distribution of neutrinos measured at ISIS was interpreted as an “$x$–particle” of mass $M \approx 34\text{MeV}$. The dominant decay channel in ref. was conjectured to be

$$x \rightarrow \nu + \gamma$$

An independent experiment, performed at the Paul Scherrer Institute failed to confirm the original KARMEN result. Moreover, the original interpretation is not without problems of its own see e.g. the paper of Barger et. al. In essence, the authors of ref. argue that if the $x$-particle exists, it can only be a sterile “neutrino” mixing with an active one. For this reason, it is of importance to test the existence of the “$x$–particle” in a different type of experiment. If indeed $x$ is produced by muon neutrinos in the Coulomb field of a nucleus, the photons resulting from its decay may be observed downstream.

We give formulae for the differential and total cross sections. The calculation is carried out both for a spin 1/2 and spin 3/2 particle in the final state. There are various reasons why the possibility of a spin 1/2 $\rightarrow$ spin 3/2 transition may be of interest. We mention a few.

In a variety of theories there may exist neutral particles of spin 3/2 which mix with particles of spin 1/2; arguments why a spin 3/2 particle should be much heavier than a neutrino or the particle allegedly observed by the KARMEN collaboration are mostly based on the fact that, apparently, the fact that transitions to spin 3/2 leptons have not been observed. In addition, the idea that quarks and leptons possess a substructure keeps recurring in the literature. If a substructure indeed existed, one should consider the possibility of excited quarks and leptons. If the known spectrum of hadrons can serve as a guide, one expects the lowest lying fermionic excited state to have spin 3/2.

Admittedly, neither one of these motivations is currently a very strong one; nevertheless, one should keep an open mind about the possibilities. Moreover, the measured partial width of the Z into invisible channels places very severe limitations onto the couplings (mixing angles, respectively) of such particles.
If the transition is between states of spin $1/2$ and spin $3/2$, existing limits on transition moments are somewhat weakened and, hence, some of the arguments in ref. [4] are weakened too, since a barrier penetration factor suppresses the transition rates. For most of the processes in question the suppression is not sufficiently dramatic in order to warrant attention. The only exception is the decay, $\pi \to \mu + x$. In that case, due to the very small Q value of the decay, the barrier penetration factor is about $10^{-3}$ and correspondingly, the branching ratio is suppressed by that factor.

We recall that in the case of the original Primakoff effect (i.e. photoproduction of neutral pions in the Coulomb field of a nucleus), the effective interaction is related to the anomaly of the neutral isovector axial current, viz

$$\mathcal{L}_{\text{eff}} = \frac{\alpha}{4\pi f_{\pi}} \pi^0 f_{\mu \nu} F^{\mu \nu},$$

(1)

where $f_{\mu \nu}$ and $F_{\mu \nu}$ stand for the field tensor describing the incident photon and the field tensor of the (screened) Coulomb field, respectively.

In a similar fashion, the effective Lagrangian describing the transition between two neutrino species, say, $\nu_1$ and $\nu_2$ is proportional to $F_{\mu \nu}$. We have the following expressions. Transition between spin $1/2$ neutrinos:

$$\mathcal{L}_{\text{eff}}^{s=1/2} = \frac{1}{2} \mu_B \nu_2 (\kappa_n + \kappa_a \gamma_5) \sigma_{\rho \sigma} \nu_1 F^{\rho \sigma} + \text{h.c.}$$

(2)

Transition between spin $1/2$ and spin $3/2$ neutrinos:

$$\mathcal{L}_{\text{eff}}^{s=3/2} = \frac{1}{2M} \mu_B \nu_2 (\kappa_n' + \kappa_a' \gamma_5) \sigma_{\rho \sigma} \nu_1 \partial_\lambda F^{\rho \sigma} + \text{h.c.}$$

(3)

In the equations above, the quantities $\kappa$ stand for the effective coupling strengths measured in units of a Bohr magneton, $\mu_B = e/2m_e$ and $M$ stands for the mass of the neutrino of spin $3/2$. The subscripts $n, a$ refer to normal and anomalous transitions, respectively. Finally, $\nu^\lambda$ stands for a Rarita–Schwinger spinor describing a neutrino of spin $3/2$.

It is to be noted, however that in the experiments discussed here, one cannot distinguish between the cases of normal and anomalous transitions and, apart from the interpretation of the experiments outlined above, between Majorana and Dirac neutrinos either. As an illustration consider, e.g. the inverse lifetime of a heavy, spin $1/2$ neutrino decaying into a light $\nu$ and a $\gamma$. Apart from a common phase space factor, it is given by the expression:

$$\Gamma(\nu_2 \to \nu_1 + \gamma) \propto \left( |C_n|^2 + |C_a|^2 \right),$$

where

$$|C_n|^2 = |\kappa_n|^2, \quad |C_a|^2 = |\kappa_a|^2,$$

for Dirac neutrinos, whereas

$$|C_n|^2 = 4 (\text{Im} \kappa_n)^2, \quad |C_a|^2 = 4 (\text{Re} \kappa_a)^2.$$
for Majorana neutrinos. The situation is similar in the case of *unpolarized* cross sections and for transitions involving excited neutrinos of spin 3/2.

The electromagnetic interaction conserves parity, hence, either $C_n$ or $C_a$ vanishes, depending on the relative parities of the neutrinos in the initial and final states. However, the amplitudes of normal and anomalous transitions differ in terms which are of the order of magnitude of the neutrino masses involved. (For *small* masses a neutrino is *almost* an eigenspinor of $\gamma_5$.) Henceforth, we arbitrarily set $\kappa_a = \kappa'_a = 0$ for the rest of this paper; the effective couplings will be denoted by $\kappa$ and $(\kappa')$, respectively and we express our results in terms of a single effective low energy coupling constant. Moreover, we can disregard differences between Dirac and Majorana neutrinos.

The calculation of the differential cross section of the process

$$\nu_1 \rightarrow \nu_2$$

in a screened Coulomb field and using the effective interaction eq. (2) or eq. (3) respectively, is an elementary exercise. The only expression not given in elementary texts is the helicity sum over Rarita–Schwinger spinors, as given e.g. in ref. [6]. We do not need the full expression, only its contraction with $q_\alpha$, the momentum of the virtual photon.

We define:

$$P_{\alpha\beta}(p) = \sum_{\Lambda=\pm 3/2} \nu^\Lambda_\alpha(p) \nu^\Lambda_\beta(p),$$

where $\Lambda$ is the helicity of the spin 3/2 neutrino.

One finds for instance, for $p_0 > 0$:

$$b = P_{\alpha\beta} q^\alpha q^\beta = 2 \left( \frac{p \cdot q}{M^2} - q^2 \right) (\not p + M).$$

(One easily verifies that in a decay, e.g. $\nu_2 \rightarrow \nu_1 + \gamma$ the coefficient of $(\not p + M)$ in the expression of $b$ is just a conventional barrier penetration factor.)

We quote the differential and total cross sections for spin 1/2 and 3/2 neutrinos in the final state. The formulae are written down in the high energy limit: the beam energy is much larger than the mass of the neutrino either in the initial or in the final states. The differential cross sections are given in the laboratory frame. We have:

For $s = 1/2$ to $s = 1/2$ transition:

$$\frac{d\sigma^{1/2}}{d\Omega} \sim 8\pi Z^2 \alpha \kappa^2 \frac{2}{\sin^2 \theta/2 + \frac{\mu^2}{4E^2}}$$

$$\sigma^{1/2} \sim 8\pi^2 Z^2 \alpha \kappa^2 \mu_B^2 \left( -1 + 2 \ln \frac{2E}{\mu} \right)$$

(6) (7)

For $s = 1/2$ to $s = 3/2$ transition:

$$\frac{d\sigma^{3/2}}{d\Omega} \sim 16\pi Z^2 \alpha \kappa^2 \frac{2}{\mu_B^2} \left( \frac{E}{M} \right)^4 \sin^2 \theta/2$$

(8)
In these equations, $\mu$ stands for the inverse of the screening radius. Using the Thomas–Fermi model of atoms, it is given by the expression, $\mu = a_0^{-1}Z^{-1/3}$, where $a_0$ is the Bohr radius. (In energy units $\mu \approx 3.65 \times 10^{-3}Z^{-1/3}$MeV.) We notice that in the expressions involving a spin 3/2 final state, the screening radius does not appear in the asymptotic expressions, eq. (8) and eq. (9). This is due to the presence of the barrier penetration factor appearing in $b$.

For angles down to $\theta \approx \mu/E$, the angular distribution given by eq. (10) behaves essentially as $1/\sin^2\theta/2$. The angular distribution given by eq. (8) is quite flat, again, due to the presence of the barrier penetration factor.

There is a radical difference between the energy dependence of the total cross sections. Both the spin 1/2 and spin 3/2 cross sections grow with the beam energy $E$, but the spin 3/2 cross section grows much more rapidly. For this reason, it is important to estimate the critical energy at which eq. (9) violates the unitarity bound.

The absolute value of the scattering amplitude is given by:

$$|f_{3/2}(\theta)| = \left(\frac{d\sigma_{3/2}}{d\Omega}\right)^{1/2}$$

From here we find the estimate for the S-wave phase shift:

$$|\sin \delta_0| \sim Z \sqrt[3]{\frac{8\pi}{3}} \left(\frac{\kappa}{\mu B} \right)^{2/3} M \left(\frac{E}{M}\right)^{3}$$

At the critical energy, $E_c$, $|\sin \delta_0| \approx 1$. Depending on the estimates on $\kappa$ and $M$ (see [11]), one gets various critical energies. Using conservative estimates taken from that reference, one gets $E_c \approx 10^6$GeV or so. This value of $E_c$ is comfortably high for the purposes of any terrestrial beam. However, neutrinos emerging from active galactic nuclei may have comparable energies, cf. e.g. [7]. For that reason, more careful estimates are needed (using $K$–matrix unitarization etc.) for the highest energies.

From the experimental point of view, if the particle observed by the KARMEN collaboration exists (and its dominant decay mode is what has been conjectured by the collaboration), its spin can be determined from the energy dependence of its production cross section via the magnetic Primakoff effect. In general, the energy and $Z$ dependence of the conversion probability provide important clues in determining whether a conversion occurs due to oscillations or due to the magnetic Primakoff effect. In fact, the oscillation length increases linearly with the neutrino energy (except perhaps in the neighborhood of a MSW resonance), whereas the comparable quantity, the mean free path of the magnetic Primakoff effect always decreases with the neutrino energy: it is, roughly, proportional to $1/\ln E$ or $1/E^4$ depending on the spins of the particles involved, cf. the expressions of the cross sections. In Fig. 1 we displayed the total cross section as a function of the energy. We remark that for moderate energies, ($E \approx 15$GeV) the spin 1/2 to spin 1/2 transition has a cross section comparable to a weak cross section within the framework of the standard model (see [8]) for $\kappa \approx 2 \times 10^{-8}/Z$. 

\[ \sigma_{3/2} \sim \frac{32\pi^2}{3}Z^2\alpha^2\mu_B^2 \left(\frac{E}{M}\right)^4 \]
Figure 1: Plot of the cross sections of the magnetic Primakof effect as a function of the energy. The cross sections are plotted for Z=26. Dashed line: spin 1/2 final state. Full line: spin 3/2 final state. The symbols labeling the axes mean the following: $\sigma = \sigma/10^{−10}\text{mb}$, $\kappa = 10^{11}\kappa$. The cross section of the spin 1/2 to spin 3/2 transition was plotted for $M=34\text{MeV}$. 
We notice that the transition to a spin 3/2 final state has a much larger cross section than the one to spin 1/2. The fact that, in all probability, no such transition has been detected so far, puts very stringent upper limits on the transition moments. (We have not yet analyzed existing data from this point of view.)

A further severe limitation on the existence (more precisely, the couplings) of a light spin 3/2 particle arises from the measured invisible decay width of the Z. The decay width of the Z into a pair of spin 1/2 particles with $M \ll M_Z$ is given by the expression:

$$\Gamma(Z \rightarrow \nu + \bar{\nu}) \approx \frac{G_\mu M_Z^3}{12\sqrt{2}} \left( g_L^2 + g_R^2 \right).$$

Here $G_\mu$ is the muon decay constant, and $g_L$ and $g_R$ are the effective left and right handed couplings of the Z to the hypothetical extra neutrino, respectively. Assuming that the error on the invisible width of the Z leaves room for an extra neutrino, one gets the upper limit,

$$\sqrt{g_L^2 + g_R^2} \lesssim 5 \times 10^{-3}.$$

However, if the extra neutrinos had spin 3/2, the expression has to be multiplied by a factor,

$$\frac{1}{36} \left( \frac{M_Z}{M} \right)^8.$$

(This factor arises from eq. (5) in the limit $M_Z \gg M$.)

For all practical purposes therefore, the coupling of the Z to a light, spin 3/2 particle is zero. Of course, in the absence of a compelling theory, there is no a priori reason to exclude the existence of such a particle. However, if it exists, it is extremely unlikely that it is coupled to any of the known particles.

After the calculation reported here was finished, we became aware of the work described in ref. [9]. The authors of that work developed ideas similar to the ones described here, but in a rather different context.

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