Analysis of Relationship between Wavelength Selectivity and Angular Selectivity of Rugate coating

Zheng Guangwei\(^{1}\), Wang yang\(^{2}\)

\(^{1}\) Information and Navigation College, Air Force Engineering University, 710077, Xi’an, Shaanxi, China
\(^{2}\) China Satellite Maritime Tracking and Control Department, Jiangyin, Jiangsu, China

Email: zgw198196@126.com

Abstract. Based on Bragg law, Airy’s formulae, and second-order Taylor series expansion, the relationships between wavelength selectivity and angular selectivity of the ordinary and phase-shifted Rugate coatings are investigated, respectively. And their expressions of the wavelength selectivity bandwidth and the angular selectivity bandwidth of these two types of Rugate gratings are put forward. The results show that when the incidence angle is far away from 0 rad, the bandwidth of the wavelength selectivity is proportional to that of the angular selectivity. And when the incidence angle approaches or even equals 0 rad (frequently-used cases), the bandwidth of the wavelength selectivity is squarely proportional to that of the angular selectivity. The results are instructive for the design and application of Rugate coatings.

1. Introduction

Rugate coating is a kind of thin film with the refractive index changing continuously along its thickness\(^{[1]}\). Compared with the traditional graded coatings, Rugate coating has a lot of merits, such as no or few harmonious reflective bands, low stress between different layers, high laser damage threshold, and so on\(^{[2-4]}\). With the development of the modern fabrication methods, the characteristics of Rugate coatings are analyzed extensively\(^{[5-8]}\). Nowadays, the most widely-used spatial filter is pinhole filter. Due to its focusing characteristics, pinhole filter cannot be adjusted easily to the application for the high power laser beam. So the non-focusing method has attracted attention. Due to the fine wave vector selectivity of the grating, the non-spatial filtering for laser beam based on gratings has been put forward since 2000s\(^{[9-12]}\). The fine wave vector selectivity denotes the fine wavelength selectivity at some angular domain and the fine angular selectivity at some wavelength domain. So Rugate coatings may be a potential candidate for the pinhole filter, especially in the high power laser field, with its fine angular selectivity\(^{[13]}\). However, there is little attention paid to the wavelength selectivity for the
performance degradation of Rugate coatings’ application upon the spatial filter of laser beam, especially upon the laser pulse. In order to make the Rugate coating suitable for the spatial filter of the short pulse laser, its relationship between the angular selectivity and wavelength selectivity is analyzed in order to search for structure of Rugate coatings with the fine angular selectivity and weak wavelength selectivity. From the refractive index’s distribution, Rugate coating can be categorized into four types, such as ordinary, chirped, apodized, and phase-shifted types. In this paper, we just analyze the ordinary Rugate coating and phase-shifted Rugate coating. The other two types can be investigated with the similar method.

2. The definition of angular and wavelength selectivity bandwidths
For each Rugate grating, there is a wave vector at which the transmittance or reflectance efficiency is the highest. So we define the wave vector as the central wave vector $k_0$. And there must be the incidence angle $\theta_0$ and wavelength $\lambda_0$ in corresponding to the central wave vector. The angular selectivity bandwidth $\Delta \theta$ is defined as the two times incidence angle when the transmittance or reflectance efficiency $\eta$ descends to one half of its maximum $\eta_{\text{max}}$. And $\Delta \theta$ can be expressed as

$$\begin{align*}
\eta(\lambda_0, \theta_0) &= \eta_{\text{max}} \\
\eta(\lambda_0, \theta_0 + \Delta \theta / 2) &= \eta_{\text{max}} / 2
\end{align*}$$

(1)

And with the similar definition, the wavelength selectivity bandwidth $\Delta \lambda$ can be expressed as

$$\begin{align*}
\eta(\lambda_0, \theta_0) &= \eta_{\text{max}} \\
\eta(\lambda_0 + \Delta \lambda / 2, \theta_0) &= \eta_{\text{max}} / 2
\end{align*}$$

(2)

where the parameters has the same definition as those in equation (1).

3. The relationship between angular and wavelength selectivity of ordinary Rugate coating
The reflection of laser beam by ordinary Rugate coating is shown in figure 1.

Fig1. Reflection of laser beam by ordinary Rugate coating
In figure 1, $\tilde{N}$ denotes the normal to the front surface. $n_a$ and $n_s$ denote the refractive indexes of the ambient media and substrate, respectively. $d$ is the thickness. $K$ denotes the coating’s vector, and is equal to $2\pi / \Lambda$, where $\Lambda$ is the period of the coating. $k_0$ denotes the
central wave vector of the incident laser. And $\theta_0$ is the incidence angle. The refractive index of ordinary Rugate coating is sinusoidally modulated. It can be expressed as

$$n_r(z) = n_0 + n_1 \sin(Kz)$$  \hspace{1cm} (3)

where $n_0$ and $n_1$ denote the average and modulated refractive index, respectively. From equation (3), we can conclude that the structure of the ordinary Rugate coating is the same as that of reflecting volume phase grating \cite{14}.

Due to the fact that the structure of the ordinary Rugate coating is the same as that of the reflecting volume phase grating, the reflecting characteristics of ordinary Rugate coating can be analyzed by Bragg law. From Snell’s theorem and Bragg law, the central wave vector must satisfy the following equations, which can be expressed as:

$$n_a \sin \theta_0 = n_0 \sin \theta_0'$$ \hspace{1cm} (4)

$$2n_o \Lambda \cos \theta_0' = \lambda_0$$ \hspace{1cm} (5)

where $\theta_0'$ denotes the angle of the incidence angle $\theta_0$ in the Rugate coating.

We make the second order Taylor series expansion for $\lambda$ with the variable $\theta$ at the point $(\lambda_0, \theta_0')$. And then $\lambda$ can be expressed as:

$$\Delta \lambda = -2n_0 \Lambda \sin \theta_0' \Delta \theta' - n_0 \Lambda \cos \theta_0' \Delta \theta'^2 + o[\Delta \theta'^2]$$ \hspace{1cm} (6)

where $\Delta \theta'$ denotes $\theta' - \theta_0'$, while $\Delta \lambda$ denotes $\lambda - \lambda_0$. $o[\Delta \theta'^2]$ denotes the higher-order infinitesimal of $\Delta \theta'^2$.

According to the angle $\theta_0$, there are three situations classified for the relationship between wavelength and angular selectivity.

3.1. $\theta_0 = 0$

When $\theta_0$ is equal to 0, the equation (6) can be expressed as:

$$\Delta \lambda = -n_0 \Lambda \Delta \theta'^2$$ \hspace{1cm} (7)

where the higher-order infinitesimal of $\Delta \theta'^2$ is omitted.

We make the derivative for the equation (4), which is expressed as:

$$\Delta \theta' = [n_a \cos \theta_0'/ (n_0 \cos \theta_0')] \Delta \theta$$ \hspace{1cm} (8)

Due to $\theta_0 = 0$, $\Delta \theta' = (n_a / n_0) \Delta \theta$.

So the relationship between wavelength selectivity bandwidth and angular selectivity bandwidth can be expressed as:

$$\Delta \lambda = -(n_0^2 \Lambda / n_0) \Delta \theta^2$$ \hspace{1cm} (9)

The result shows that the wavelength selectivity bandwidth is squarely proportional to the angular one, when the central vector is normally incident into the ordinary Rugate coating.

3.2. $\theta_0 \in (0, 0.1 \text{rad})$

According to equations (4) and (8), the absolute quotient of the first two terms of the right side of equation (6) can be expressed as:

$$|2n_0 \Lambda \sin \theta_0' \Delta \theta' / (n_0 \Lambda \cos \theta_0' \Delta \theta'^2)| = |2 \tan \theta_0' / \Delta \theta'| = |2 \tan \theta_0 / \Delta \theta|$$ \hspace{1cm} (10)

Usually, $\Delta \theta$ is less than 0.1 rad. So when $\theta_0 \in (0, 0.1 \text{rad})$, the quantity of (10) is less than 1. The second term of the right side of (6) cannot be omitted. And with the omission of the higher order infinitesimal of $\Delta \theta'^2$ and equation (4), (5), and (8), $\Delta \lambda$ can be expressed as:

$$\Delta \lambda = [(4n_0^2 \Lambda^2 - \lambda_0^2) / (4(n_0^2 - n_0^2) \Lambda^2 + \lambda_0^2)]^{1/2} \Delta \theta$$

$$+ [4(n_0^2 - n_0^2) \Lambda^2 + \lambda_0^2] \frac{1}{2 \lambda_0} \Delta \theta^2$$ \hspace{1cm} \left(\frac{\lambda_0}{2n_0} < \Lambda < \frac{\lambda_0}{2(n_0^2 - n_0^2)^2}\right) \hspace{1cm} (11)
3.3. $\theta_0 \in [0.1, \pi/2\text{rad}]$

When $\theta_0 \in [0.1, \pi/2\text{rad})$, the quantity of (10) is much larger than 1. So the second term of the right side of (10) and the higher order infinitesimal of $\Delta \theta'^2$ can be omitted, $\Delta \lambda$ can be expressed as:

$$\Delta \lambda = \left\{ \left( 4n_0^2\Lambda^2 - \lambda_0^2 \right) \left[ 4(n_0^2 - n_d^2)\Lambda^2 + \lambda_0^2 \right] \right\}^{1/2} \frac{1}{\lambda_0} \Delta \theta \quad \left( \frac{\lambda_0}{2n_0} < \Lambda < \frac{\lambda_0}{2(n_0^2 - n_d^2)^{1/2}} \right) \quad (12)$$

The result shows that the wavelength selectivity bandwidth is proportional to the angular one at this situation.

4. The relationship between angular and wavelength selectivity of phase-shifted Rugate coating

The transmission of a laser beam by phase-shifted Rugate coating is shown in figure 2.

Fig2. Transmission of a laser beam by phase-shifted Rugate coating

The parameters in figure 2 are defined similarly as that in figure 1. Region 1 and region 2 are identical to each other. And their average refractive index is the same as $n$.

From the point that the two highly reflective coatings are placed parallel, the structure of the phase-shifted Rugate coating is similar to the Fabry-Perot interferometer. And their optical characteristics are similar, except that the Fabry-Perot interferometer has transmission harmonics, while Rugate coating has not. We just analyze the relationship between the wavelength bandwidth and angular bandwidth of the phased-shifted Rugate coating. So by Airy’s formulae and without loss, the intensity of the transmitted laser beam can be expressed as:

$$I_T = \left( 1 + F\sin^2 \frac{\delta}{2} \right)^{-1} \quad (13)$$

where

$$F = \frac{4R}{(1 - R)^2} \quad (14)$$

$$\delta = 4\pi n h \cos \theta_1 / \lambda \quad (15)$$

$R$ denotes the reflectivity of region 1.

When $I_T$ reduces to the half of its maximum, $\delta$ changes to $\delta'$, which can be expressed as:

$$\delta' = 2\arcsin(F^{-0.5}) \quad (16)$$

From equation (15), the relationship between $\lambda$ and $\theta_1$ is expressed as:

$$2\pi n h \cos \theta_1 / \arcsin(F^{-0.5}) = \lambda \quad (17)$$
Comparing equation (17) and (5), the relationships between \( \lambda \) and \( \theta_0 \) of these two types of Rugate coatings are similar to each other. So through the similar method, we can deduce the relationship between the wavelength selectivity bandwidth and angular selectivity bandwidth of phase-shifted Rugate coating. The results are as follows.

When \( \theta_0 = 0 \), their relationship can be expressed as

\[
\Delta \lambda = -\left(2n_a^2\pi h/[n_0\arcsin(F - 0.5)]\right)\Delta \theta^2
\]

(18)

When \( \theta_0 \in (0, 0.1\, \text{rad}) \), their relationship can be expressed as

\[
\Delta \lambda = \left\{(4n_0^2\arcsin(F - 0.5)^2 - \lambda_0^2)\right\}\left\{4(n_a^2 - n_0^2)\arcsin(F - 0.5)^2 + \lambda_0^2\right\}^{1/2}\frac{1}{\lambda_0} \Delta \theta
\]

\[
+\left[4(n_a^2 - n_0^2)\arcsin(F - 0.5)^2 + \lambda_0^2\right]^{1/2}\frac{1}{2\lambda_0} \Delta \theta^2
\]

\[
\left(\frac{\lambda_0}{2n_0} < \frac{\pi h}{\arcsin(F - 0.5)} < \frac{\lambda_0}{2(n_a^2 - n_0^2)^{1/2}}\right)
\]

(19)

When \( \theta_0 \in [0.1, \pi/2\, \text{rad}) \), their relationship can be expressed as

\[
\Delta \lambda = \left\{(4n_0^2\arcsin(F - 0.5)^2 - \lambda_0^2)\right\}\left\{4(n_a^2 - n_0^2)\arcsin(F - 0.5)^2 + \lambda_0^2\right\}^{1/2}\frac{1}{\lambda_0} \Delta \theta
\]

\[
+\left[4(n_a^2 - n_0^2)\arcsin(F - 0.5)^2 + \lambda_0^2\right]^{1/2}\frac{1}{2\lambda_0} \Delta \theta^2
\]

\[
\left(\frac{\lambda_0}{2n_0} < \frac{\pi h}{\arcsin(F - 0.5)} < \frac{\lambda_0}{2(n_a^2 - n_0^2)^{1/2}}\right)
\]

(20)

5. Conclusion

According to equation (9), (11), (18) and (19), we can conclude that for ordinary or phase-shifted Rugate coatings, when \( \theta_0 \in [0, 0.1\, \text{rad}) \), the bandwidth of the wavelength selectivity is almost squarely proportional to that of the angular selectivity. It means that when the incidence angle approaches or even equals 0 rad, both ordinary and phase-shifted Rugate coatings have fine wavelength selectivity and weak angular selectivity. So they can not be used as the spatial filter for the laser pulse at this situation. When the incidence angle is far away from 0 rad, the bandwidth of the wavelength selectivity is proportional to that of the angular selectivity. In order to analyze the potential substitute for pin-hole spatial filter by Rugate coating, we should design Rugate coating with fine angular selectivity and weak wavelength selectivity. With the similar method to design Rugate coating in the optical spectrum domain, we can use needle algorithm to synthesize the well-performed coating structure in the angular spectrum domain.

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References

[1] Bertrand G. Bovard 1993 Rugate filter theory: an overview Appl. Opt 32 5427
[2] W. H. Southwell and Randolph L. Hall 1989 Rugate filter sidelobe suppression using quintic and rugated quintic matching layers Appl. Opt 28 2949
[3] W. H. Southwell 1988 Spectral response calculations of rugate filters using coupled-wave
theory Appl. Opt 28 2949

[4] W. H. Southwell 1989 Using apodization functions to reduce sidelobes in rugate filters Appl. Opt. 28 5091

[5] Andy C. van Popta et al 2004 Gradient-index narrow-bandpass filter fabricated with glancing-angle deposition Optics Letters 29 2545

[6] Stephan Fahr et al 2008 Rugate filter for light-trapping in solar cells Optics Express 16 9332

[7] P. V. Usik et al 2009 Spatial and spatial-frequency filtering using one-dimensional graded-index lattices with defects Optics Communications 282 4490

[8] Julien Lumeau et al 2008 Phase-shifted volume Bragg gratings in photo-thermo-refractive glass Proc. Of SPIE 6890 68900A-1

[9] Ivan Moreno and J. Jesus Araiza 2004 Thin-film optical filters for spatial frequencies Proc. of SPIE 5524 409

[10] Ivan Moreno et al 2005 Thin-film spatial filters Optics Letters 30 914

[11] Zhang Ying et al 2015 An improved transmitting multi-layer thin-film filter Chinese Physics B 24 054212-1

[12] Zhang Ying et al 2015 Analysis of the spatial filter of a dielectric multilayer film reflective cutoff filter-combination device Chinese Physics B 24 104216-1

[13] Luo Zhaoming et al 2010 Low-pass rugate spatial filters for beam smoothing Optics communications 283 2665

[14] H. Kogelnik 1969 Coupled wave theory for thick hologram gratings The Bell Syst. Technol. J 48 2909

[15] Max Born and Emil Wolf 1999 Principles of Optics(7th edition) Cambridge University Press 325