Can $X(5568)$ be a tetraquark state?

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Very recently, the D0 collaboration has reported the observation of a narrow structure, $X(5568)$, in the decay process $X(5568) \rightarrow B_0^0 \pi^\pm$ using the 10.4 fb$^{-1}$ data of $p\bar{p}$ collision at $\sqrt{s} = 1.96$ TeV. This structure is of great interest since it is the first hadronic state with four different valence quark flavors, $b,s,u,d$. In this work, we investigate tetraquarks with four different quark flavors. Based on the diquark-antidiquark scheme, we study the spectroscopy of the tetraquarks with one heavy bottom/charm quark and three light quarks. We find that the lowest-lying S-wave state, a tetraquark with the flavor $[su][\bar{bd}]$ and the spin-parity $J^P = 0^+$, is about 150 MeV higher than the $X(5568)$. Further detailed experimental and theoretical studies of the spectrum, production and decays of tetraquark states with four different flavors in the future are severely needed towards a better understanding its nature and the classification of hadron exotic states.

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Since the proposal of the concept of quarks by Gell-Mann [1], there have been great endeavors to test the quark model and search for exotic structures beyond this scheme. To date hundreds of hadrons were discovered, and most of them can be accommodated in the naive quark model, in which mesons and baryons are composed of a quark–antiquark pair and three quarks, respectively. No firm evidence for the existence of exotic states beyond the quark model and search for exotic structures beyond this scheme. To date hundreds of hadrons were discovered, and most of them can be accommodated in the naive quark model, in which mesons and baryons are composed of a quark–antiquark pair and three quarks, respectively. No firm evidence for the existence of exotic states beyond the quark model has been established on experimental side until the discovery of the $X(3872)$ in 2003 [2,3]. The peculiar properties of the $X(3872)$, the vicinity of its mass close to $D\bar{D}^*$ threshold, the tiny width and the large isospin violation in its production and decay, has invoked a renaissance of hadron spectroscopy studies. Since then one key topic in hadron physics is the identification of the exotic hadrons. Many new interesting structures were discovered in the mass region of heavy quarkonium, named as $XYZ$ states (for a review of these particles, see Refs. [6–9]). In particular, the charged structures with a hidden pair of heavy quark and antiquark such as the $Z_c^\pm(4430)$ [10,11], $Z_s^\pm(10610,10650)$ [12], $Z_\chi(3900)$ [13,14] and $Z_{1^+}^\pm(4020)$ [15] would be undoubtedly exotic resonances. Moreover candidates for exotic hadrons were extended to the pentaquark sector by the LHCb observations of two structures in the $J/\psi p$ invariant mass distribution with masses (widths) $(4380 \pm 8 \pm 29)$ MeV ($(205 \pm 18 \pm 86)$ MeV) and $(4499.8 \pm 1.7 \pm 2.5)$ MeV $(39 \pm 5 \pm 19)$ MeV), respectively [16]. These discoveries have opened up a new era of multi-quark spectroscopy and strengthened our belief that the hadron spectrum would be much richer than the quark model.

Most of the observed exotic $X,Y,Z$ structures so far share a common feature, i.e. they consist of a hidden heavy quark-antiquark pair, $bb$ or $cc$. Various theoretical models were motivated to explain these exotics, many of which have made use of the heavy quark symmetry and chiral symmetry. Inspired by these symmetries, various combinations of heavy and light mesons have been examined and can be searched for, of great interest is the one that is composed of a heavy meson and a light meson. Very recently, the D0 collaboration has reported the first observation of such structure in the final state $B_0^0 \pi^\pm$ [17]. Since the $B_0^0 \pi^\pm$ final state is made of four different flavors, $b,s,u,d$, the new structure is definitely exotic. Its mass and width has been determined as [17]

$$M_X = (5567.8 \pm 2.9)\text{MeV}, \quad \Gamma_X = (21.9 \pm 6.4)\text{MeV}. \quad (1)$$

The above results are obtained through a fit based on a Breit-Wigner parametrization of the S-wave decay of $X(5568) \rightarrow B_0^0 \pi^\pm$. The statistical significance including the look-elsewhere effect and systematic errors is about 5.1$\sigma$ [17].

After the first discovery, more experimental efforts to determine the properties of the $X(5568)$ are needed. Meanwhile, this also requests theoretical interpretations of its nature. The $X(5568)$ is too far from the the $B_0^0 K^\pm$ threshold (5774 MeV) to be interpreted as a hadronic molecule of $B_0^0 K^\pm$. In addition, the interaction of $B_0^0 K^\pm$ is very weak.

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and unable to form a bounded structure [18]. In this work, we will study tetraquark states using colored components, diquark and antidiquark, and bound by the long-range color forces. Tetraquark states in the large $N_c$ limit of QCD has been explored in Refs. [19–21], which indicates that a compact tetraquark meson may have narrow decay widths scale as $1/N_c$. Thereby there are reasonable candidates for the additional spectroscopic hadron series apart from the quark model. We want to see whether the $X(5568)$ is a tetraquark state. In the past decades, tetraquark states in particular with hidden bottom and charm have been explored in Refs. [22–34] and many references therein.

The QCD confining potential for the multiquark system can be generally written as [35]

$$V(\vec{r}) = L(\vec{r}_1, \vec{r}_2, \ldots) + \sum_{i>j} I \alpha_i S_{ij},$$

where the $L(\vec{r}_1, \vec{r}_2, \ldots)$ stands for the universal binding interaction of quarks. The $S_{ij}$ is two-body Coulomb and chromomagnetic interactions, with the $I = -4/3$ and $-2/3$ as the single gluon interaction strength in quark-antiquark and quark-quark cases, respectively.

In the following, we will consider the tetraquark states with quark content $[qq']q\bar{Q}$, where $q$ and $q'$ denote the light quarks, and $Q$ denotes a heavy quark, bottom or charm. The effective Hamiltonian is composed of three kinds of interactions: spin-spin interactions of quarks in the diquark and antidiquark, and between them; the spin-orbital interaction; purely orbital interactions. An explicit model that incorporates these interactions has been established in Ref. [24]:

$$H = m_\delta + m_\delta' + H_{SS}^\delta + H_{SS}^{\delta'} + H_{LL} + H_{SL},$$

with the functions

$$H_{SS}^\delta = 2(\kappa_{qq})_{\delta}(S_q \cdot S_{q'}),$$

$$H_{SS}^{\delta'} = 2(\kappa_{QQ})_{\delta}(S_Q \cdot S_{\bar{Q}}),$$

$$H_{SS}^{\delta\delta'} = 2\kappa_{q\bar{Q}}(S_q \cdot S_{\bar{Q}}) + 2\kappa_{q'q}(S_{q'} \cdot S_q)$$

$$+ 2\kappa_{q'\bar{Q}}(S_{q'} \cdot S_Q) + 2\kappa_{q\bar{Q}}(S_q \cdot S_Q),$$

$$H_{SL} = 2A_{\delta}(S_\delta \cdot L) + 2A_{\delta'}(S_{\delta'} \cdot L),$$

$$H_{LL} = B_{\delta\delta'} \frac{L(L+1)}{2}.$$  

In the above, the $m_\delta$ and $m_\delta'$ is the constituent mass of the diquark $[qq']$ and the antidiquark $[q\bar{Q}]$, respectively. The spin-spin interaction inside the diquark and antidiquark is denoted as $H_{SS}^\delta$ and $H_{SS}^{\delta'}$, respectively. The $H_{SS}^{\delta\delta'}$ reflects the spin-spin interaction of quarks between diquark and antidiquark. The $H_{SL}$ and $H_{LL}$ is the spin-orbital and purely orbital terms. The $S_\delta$ and $S_{\delta'}$ corresponds to the spin operator of diquark and antidiquark, respectively. The spin operator of light quarks and heavy antiquark is given by $S_q(\hat{o})$ and $S_Q$, respectively. The symbol $L$ denotes the orbital angular momentum operator. The coefficients $\kappa_{q\bar{Q}}, (\kappa_{q'q})_\delta$ are the spin-spin couplings for a quark-antiquark pair and diquark in color antitriplet, respectively; $A_{\delta(\delta')}$ and $B_{\delta\delta'}$ denotes respectively spin-orbit and orbit-orbit couplings.

For the lowest-lying tetraquark states with the quark content $[qq']q\bar{Q}$, their orbital angular momenta are vanishing, i.e. $L = 0$. Among them, there are two possible tetraquark configurations with the spin-parity $J^P = 0^+$, i.e.,

$$|0_J\rangle_1 = \frac{1}{\sqrt{2}} [(|\uparrow\rangle_q(\downarrow)_{q'} - (\downarrow)_q(\uparrow)_{q'}) [(|\uparrow\rangle_Q(\downarrow)_{q} - (\downarrow)_Q(\uparrow)_{\bar{Q}})],$$

$$|0_J\rangle_2 = \frac{1}{\sqrt{2}} [(|\uparrow\rangle_q(\downarrow)_{q'} + (\downarrow)_q(\uparrow)_{q'}) [(|\uparrow\rangle_Q(\downarrow)_{q} + (\downarrow)_Q(\uparrow)_{\bar{Q}})] - \frac{1}{2} [(|\uparrow\rangle_q(\downarrow)_{q'} + (\downarrow)_q(\uparrow)_{q'}) (\uparrow)_{\bar{Q}}(\downarrow)_{\bar{Q}}],$$

$$- \frac{1}{2} [(|\uparrow\rangle_q(\downarrow)_{q'} + (\downarrow)_q(\uparrow)_{q'}) (\downarrow)_{\bar{Q}}(\uparrow)_{\bar{Q}}].$$

In the above, $|0_J\rangle_1 = |0_\delta, 0_{\delta'}, 0_J\rangle, |0_J\rangle_2 = |1_\delta, 1_{\delta'}, 0_J\rangle$, and $|S_\delta, S_{\delta'}, S_J\rangle$ stands for the tetraquark; the $S_\delta$ and $S_{\delta'}$ stand for the spin of diquark $[qq']$ and antidiquark $[q\bar{Q}]$, respectively, while the $S_J$ denotes the total angular momentum of the tetraquark. In this paper, we only focus on the scalar and vector diquarks, i.e. $S_{\delta(\delta')} = 0, 1$.

Using the basis defined in Eq. (5), one can derive the mass matrix for the $J^P = 0^+$ tetraquarks
\[ M = m_\delta + m_\delta' + \left( \frac{-3}{2}((\kappa q q')3 + (\kappa Q q)3) \right) \frac{\sqrt{2}}{2}(\kappa q' Q + \kappa q q - \kappa q' q - \kappa q Q) \]

In the case \( J^P = 1^+ \), there are three possible tetraquark states, i.e.,

\[ |0_\delta, 1_\delta', 1_J \rangle = \frac{1}{\sqrt{2}} \left[ (\uparrow)_q (\downarrow)_q' - (\downarrow)_q (\uparrow)_q' \right] (\uparrow)_q (\uparrow)_Q, \]

\[ |1_\delta, 0_\delta', 1_J \rangle = \frac{1}{\sqrt{2}} \left[ (\uparrow)_q (\uparrow)_q' \right] (\uparrow)_q (\downarrow)_q' - (\downarrow)_q (\uparrow)_q' \right] \}

\[ |1_\delta, 1_\delta', 1_J \rangle = \frac{1}{2} \left\{ (\uparrow)_q (\uparrow)_q' \left[ (\uparrow)_q (\downarrow)_q + (\downarrow)_q (\uparrow)_q \right] \right. \]

\[ - \left[ (\uparrow)_q (\downarrow)_q' + (\downarrow)_q (\uparrow)_q' \right] \} (\uparrow)_q (\uparrow)_Q \} \]

Notice that unlike the heavy quarkonium-like states, the tetraquarks with the quark content \([qq'][\bar{q}Q]\) do not have any definite charge parity and thus the above three \( 1^+ \) states can mix with each other.

Using the basis defined in Eq. (7), one can obtain the mass splitting matrix \( \Delta M \) for \( J^P = 1^+ \)

\[ \Delta M = \left( \begin{array}{cc} \frac{1}{2}((\kappa Q q)3 - 3(\kappa q q')3) & \frac{1}{2}(\kappa q Q - \kappa q' q - \kappa q Q + \kappa q q) \frac{\sqrt{2}}{2}(\kappa q Q - \kappa q' q + \kappa q Q - \kappa q q) \\ \frac{2}{2}(\kappa q Q - \kappa q' q - \kappa q Q + \kappa q q) & \frac{2}{2}(\kappa q Q - \kappa q' q + \kappa q Q - \kappa q q) \frac{1}{2}((\kappa q q')3 + (\kappa Q q)3 - \kappa q Q - \kappa q' q - \kappa q Q - \kappa q q) \end{array} \right), \]

and the mass matrix is given as

\[ M = m_\delta + m_\delta' + \Delta M. \]

For the \( J^P = 2^+ \), there exits only one tetraquark configuration:

\[ |1_\delta, 1_\delta', 2_J \rangle = (\uparrow)_q (\uparrow)_q' (\uparrow)_q (\uparrow)_Q \]

with the mass

\[ M(2^+) = m_\delta + m_\delta' + \frac{1}{2}((\kappa q q')3 + (\kappa Q q)3) + \frac{1}{2}(\kappa q q + \kappa q Q + \kappa q' q + \kappa q Q). \]

The spin-spin couplings have been extensively explored in the previous analyses of mesons, baryons and the \( XYZ \) spectra in quark model and diquark model. We quote the results from Refs. [24, 26, 28, 29] and summarize them in Table I. The masses of diquarks \([cq]\) and \([bq]\) are determined through the analysis of the \( X(3872) \) with \( J^{PC} = 1^{++} \) and \( Y(10890) \) with \( J^{PC} = 1^{--} \) in the diquark model, respectively. We quote \( m_{[cq]} = 1.932 \text{GeV} \) and \( m_{[bq]} = 5.249 \text{GeV} \) [26, 28, 30]. Using these results for the spin-spin, spin-orbit and orbit-orbit couplings and diquark masses, we can obtain the tetraquark spectrum. The tetraquark spectra with quantum number \( J^P = 0^+, 1^+, \) and \( 2^+ \) are depicted in Fig. 1 in which the left and right panel corresponds to the tetraquark with a bottom and a charm quark respectively. The masses given in the figure are in units of GeV. The thresholds of the \( B_s \pi, B_c \pi, B_{c2} \pi \) and their charm analogues are shown in dashed lines. The masses of the \( X(5568) \) and the \( D_{s0}^*(2317) \) are also given in the figure.

| Quark-antiquark Couplings \((\kappa_{ij})_0\) | Diquark Couplings \((\kappa_{ij})_3\) |
|-----------------|-----------------|
| \(qq\) | \(ss\) \(qq\) \(cs\) \(cc\) \(bq\) \(bs\) | \(qq\) \(ss\) \(qq\) \(cs\) \(cc\) \(bq\) |
| 315 121 195 70 72 59 23 23 | 103 72 64 22 25 6.6 |

Our results for tetraquarks with a charm quark in Fig. 1 are consistent with Ref. [22], in which we find that the lowest tetraquark state is about 60 MeV higher than the discovered \( D_{s0}^*(2317) \). Switching to the bottom sector, we find this difference gets bigger: our prediction for the mass of the lower S-wave \( 0^+ \) tetraquark is about 150 MeV larger than the experimental result by D0 for the mass of the \( X(5568) \) in Eq. (7). The deviation on mass of the \( X(5568) \) still exists when reducing the diquark masses in a reasonable region. In the charm sector, the \( D_{s0}^*(2317) \) is about 70
MeV higher than the $D_s^*\pi$ threshold, while in the bottom sector, the observed $X(5568)$ by D0 is only about 10 MeV higher than the $B_s^*\pi$ threshold. In the heavy quark limit, the mass difference between the lowest tetraquark state and the $D_s^*/B_s^*\pi$ presumably arises from the excitation of the light system, and might be at the same magnitude in the bottom and charm sector. Thus data may indicate that the $X(5568)$ is too light to be the partner of the $D_{s0}^*(2317)$. In order to simultaneously describe the tetraquarks with four different flavors, a more comprehensive analysis is called for in future. In addition, the open bottom (charm) tetraquark states with strangeness number $S = -1$ constitute an isospin triplet and a singlet. Therein the tetraquark states with the quark content $[su][bd]$, $[sd][\bar{b}\bar{u}]$ and $1/\sqrt{2}([su][\bar{b}\bar{u}] - [sd][bd])$ constitute an isospin triplet, while the tetraquark with $1/\sqrt{2}([su][\bar{b}\bar{u}] + [sd][bd])$ is an isospin singlet. Due to the isospin symmetry, the masses of these tetraquark partners are identical to the values given in Fig. 1.

In the past decades, the spectroscopy study of hadron exotics has played an important role in uncovering the hadron inner structure, and examining various models for hadrons with fundamental freedom. Many of the recently observed structures defy an ordinary interpretation as a $\bar{q}q$ meson or a $qqq$ baryon. In this work, we have explored the tetraquarks with four different quark flavors. Based on the diquark-antidiquark scheme, we have calculated the spectroscopy of the tetraquarks with one heavy bottom/charm quark and three light quarks. We find that the lowest-lying S-wave state, a $J^P = 0^+$ tetraquark with the flavor $[su][bd]$, lies at around 5.7GeV and is about 150 MeV higher than the $X(5568)$. The identification of the $X(5568)$ as a tetraquark with spin-parity $J^P = 0^+$ is a challenge to the tetraquark model. Actually, the LHCb Collaboration did not see a signal for the $X(5568)$, where the invariant mass of $B_{s0}^0\pi^\pm$ is scanned between 5.5GeV and 5.7GeV\cite{pdg}. So it is worth to search tetraquark states with four different flavors in the invariant mass of $B_{s0}^0\pi^\pm$ beyond 5.7GeV. Further detailed experimental and theoretical studies of the spectrum, production and decays of tetraquark states with four different flavors in the future are severely called for towards a better understanding its nature and the classification of hadron exotics.

Note added: After this work was finished, a series of papers also investigated the structure of the $X(5568)$, where a diquark-antidiquark interpretation is employed in Refs. \cite{38,43}; threshold rescattering effect and is studied in Ref. \cite{44}, and Ref. \cite{45}, respectively; decay relations in flavor SU(3) symmetry is studied in Ref. \cite{46}.

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