Alternative Awaiting and Broadcast for Two-Way Relay Fading Channels

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Abstract

We investigate a two-way relay (TWR) fading channel where two source nodes wish to exchange information with the help of a relay node. Given the traditional TWR protocols with immediate forwarding, the transmission rates in both directions are known to be limited by the hop with lower capacity, i.e., the min operations between the uplink and downlink in an identical direction. In this paper, we propose a new transmission protocol, named as alternative awaiting and broadcast (AAB), to cancel the min operations in the TWR fading channels. The process flow, average exchange sum-rate and average delay of signal transmission (ST) in the relay buffer of the proposed AAB protocol are analyzed. Moreover, we derive an achievable ergodic sum-rate (ESR) and the corresponding average delay of ST for the AAB protocol based on the well-known lattice codes. Compared with the average delay of system service (SS) in the source buffer, the average delay of ST in the relay buffer induced by the proposed AAB protocol is very small and negligible. Numerical results show that 1) the proposed AAB protocol is suitable for use in almost all TWR transmission cases, 2) the proposed AAB protocol with lattice codes significantly improves the achievable ESR with an average delay of ST of only some dozen time units compared to the traditional TWR protocols without the delay of ST, 3) the proposed AAB protocol with lattice codes approaches the new upper bound on the ergodic sum-capacity (ESC) at asymptotically large Signal-to-Noise ratio (SNR).

Index Terms

Two-way relaying, physical layer network coding, alternative awaiting and broadcast, partial decoding, ergodic sum-rate

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I. INTRODUCTION

Two-way relaying has recently obtained lots of research interests [11]–[18]. The classic two-way relay (TWR) channel consists of three nodes, wherein two source nodes exchange information with the help of a relay node. Upon receiving the bidirectional information flows, the relay node combines them together and then broadcasts to the two desired destinations. A number of TWR protocols have been proposed. Among them, four popular protocols are known as amplify-and-forward (AF) [1], [6], [15], decode-and-forward (DF) [1], [13], [15], [16], [19], [20], denoise-and-forward (DNF) [3], [13], [18] and compress-and-forward (CF) [21] respectively. Meanwhile, the operation at the relay node resembles network coding [22]. It is often referred to as analog network coding (ANC) [2] or physical layer network coding (PLNC) [6]. Some PLNC methods have also been proposed and analyzed, such as bit-level Exclusive OR (XOR) [13], [15], [19], symbol-level superposition [1], [15], [20] and superimposed XOR [16].

For the TWR Gaussian channels, Oechtering, et al. obtained the capacity region of the broadcast (BC) phase [23]. Kim, et al. further broadened the frontier of the achievable rate region by allowing time sharing between different transmission phases [3]. Subsequently, Wilson, et al. [5] and Nam, et al. [4] obtained the achievable sum-rates which approach the upper bound by the lattice codes in the multiple access (MA) phase for the symmetric and asymmetric cases, respectively. For the TWR fading channels, Rankov et al. compared the ergodic sum-rates of the AF and DF protocols [1]. Since full decoding in the MA phase and superposition in the BC phase were used in [1], both the achievable ergodic sum-rates [1, eq. (24)] and upper bound [1, eq. (69)] are relatively poor.

All the aforementioned TWR schemes carry out immediate forwarding (i.e., no buffer) on the received information flows at the relay, so both the transmission rates in two opposite directions are known to be limited by the hop with lower capacity, i.e., the \( \min \) operation between the uplink and downlink in an identical direction. As shown in Fig. 1 we have \( R_{02} = \min(R_{01}, R_{12}) \) and \( R_{20} = \min(R_{21}, R_{10}) \), where \( R_{ij} \) denote the transmission rate of the link from node \( i \) to node \( j \), for \( i, j \in \{0, 1, 2\} \). It may be unavoidable in the TWR Gaussian channels because the channel gain of the same link is stationary during the former and the latter round of information exchange, e.g., \( h_{12}[t_1] = h_{12}[t_2] \). However, due to quick variation of channel gains (e.g., \( h_{12}[t_1] \neq h_{12}[t_2] \)), the TWR fading channels have the potential to eliminate the \( \min \) operations by introducing
certain delay of partial information exchange. To the best of our knowledge, none of the works in the literatures has sufficiently exploited the potential benefits of asymmetric channel gains for the TWR fading channels.

For ease of understanding, we expound four important definitions before introducing our work. At first, we define two kinds of rounds of information exchange.

a) Round of source information exchange (SIE): In one round of SIE, there is just one round of information exchange, but the received information at each destination node may not include all the information transmitted from the corresponding source node in this round.

b) Round of desired information exchange (DIE): One round of DIE maybe contain several rounds of SIE. If the received information at each destination node includes all the information transmitted from the corresponding source node in the first round of SIE, one round of DIE is said to be completed.

One round of DIE is equivalent to one round of SIE if the relay node carries out immediate forwarding. There is no buffer at the relay node in this case. Moreover, we assume that each round of SIE occupies one time unit. Then we define two kinds of delay of information exchange. The unit of delay studied here is time unit.

a) Delay of signal transmission (ST): The delay of ST is denoted as an extra time interval. During the extra time interval and the primary one time unit occupied by the first round of SIE, all the source information transmitted in the first round of SIE is received at the destination node (i.e., the opponent source node). In this delay, we do not care at each source node whether the source information waits to be transmitted or not. We just consider that how many extra time units (begin with the second round of SIE) do we need to complete one round of DIE. We can say that all the aforementioned TWR schemes do not have the delay of ST due to the immediate forwarding at the relay node.

b) Delay of system service (SS): In general, data size of the source information which will be transmitted in the current round of SIE, is decided by the last rate of packet producing by each source node itself or the last rate of packet receiving from the other nodes of a large wireless network. We denote these two rates as packet arrival rate (PAR). Restricted by lots of applications, the PAR is always unstable. If the instantaneous PAR is greater than the instantaneous exchange rate of TWR, partial information will be delayed and transmitted in the future round of SIE. That is to say, the delay of SS is said to occur. We also denote the delay of SS as an extra
time interval during which the observed source packets have been waiting to be transmitted. Obviously, all the aforementioned TWR schemes maybe have the delay of SS.

Note that the delay of ST and SS occur at the relay node and two source nodes respectively.

In this paper, we propose a new transmission protocol, named as *alternative awaiting and broadcast* (AAB), to eliminate the min operations between the uplink and downlink in an identical direction in the TWR fading channels. The process flow, average exchange sum-rate and the average delay of ST in the relay buffer of the proposed AAB protocol are analyzed. Moreover, we derive an achievable ergodic sum-rate (ESR) and the corresponding average delay of ST for the AAB protocol based on the well-known lattice codes. Compared with the average delay of SS in the source buffer, the average delay of ST in the relay buffer induced by the proposed AAB protocol is very small and negligible. Numerical results show that 1) the proposed AAB protocol is suitable for use in almost all TWR transmission cases, 2) the proposed AAB protocol with lattice codes significantly improves the achievable ESR with an average delay of ST of only some dozen time units compared to the traditional TWR protocols without the delay of ST, 3) the proposed AAB protocol with lattice codes approaches the new upper bound on the ergodic sum-capacity (ESC) at asymptotically large Signal-to-Noise ratio (SNR).

The rest of the paper is organized as follows. In Section II, we present the system model for the TWR fading channels. Section III interprets the process flow of the proposed AAB protocol and analyzes the average exchange sum-rate and the average delay of ST. In Section IV, an achievable ESR and the corresponding average delay of ST of the AAB protocol are derived based on the well-known lattice codes. Section V makes some comparisons between the AAB protocol and the traditional TWR protocols. Numerical results are given in Section VI. Finally, we conclude the paper in Section VII.

II. SYSTEM MODEL

We consider a classic three-node TWR fading channel as shown in Fig. 1, where two source nodes, denoted as 0 and 2, wish to exchange information with the help of a relay node, denoted as 1. The channel on each communication link is assumed to be corrupted with small-scale fading, shadowing, path loss and additive white Gaussian noise (AWGN).

Let \( t \) denote the \( t^{th} \) round of SIE, for all \( t \). The instantaneous SNR from node \( i \) to node \( j \) in the \( t^{th} \) round is denoted as \( \gamma_{ij}[t] = \frac{P_i|h_{ij}[t]|^2}{\sigma_j^2} \), for \( i, j \in \{0, 1, 2\} \). It counts the \( t^{th} \) channel gain.
from node $i$ to node $j$, average transmit power $P_i$ at the node $i$ and AWGN power $\sigma_j^2$ at the node $j$. Note that $| \cdot |$ stands for the magnitude of a complex scalar. The ergodic capacity $\bar{C}_{ij}$ in bit/s/Hz is determined as

$$ \bar{C}_{ij} = \mathcal{E}\left\{ C_{ij}[t] \right\} = \mathcal{E}\left\{ \log_2(1 + \gamma_{ij}[t]) \right\} $$

where $\mathcal{E}\{ \cdot \}$ represents the expectation operator. For simplicity, we assume the channel gains are reciprocal and unchanged during one round of SIE, which is defined as one time unit. Then, we have $h_{01}[t] = h_{10}[t]$, $h_{21}[t] = h_{12}[t]$. We also assume that $h_{01}[t]$ and $h_{21}[t]$ are mutually independent and subject to an identical distribution, for all $t$. That is to say, $h_{01}$ and $h_{21}$ are independent and identically distributed (i.i.d.).

In this paper, we focus on two-phase TWR system with equal time slot, which can be divided into a MA phase and a BC phase, as depicted in Fig. 1. In the MA phase, two source nodes transmit simultaneously and the relay node listens, two MA channels are also denoted as two uplinks. In the BC phase, the relay node transmits while two source nodes listen, two BC channels are also denoted as two downlinks. We assume that all the nodes operate in the half-duplex mode. Since both source nodes transmit simultaneously, they cannot directly communicate with each other even if a direct link of sufficient quality may be available. In this paper, we let $P_0 = P_1 = P_2 = P, \sigma_0^2 = \sigma_1^2 = \sigma_2^2 = \sigma^2$. We use bold upper letters to denote vectors and lower letters to denote elements.

Since we are interested in the asymmetric rate case, i.e., both source nodes wish to exchange different amount of information, we should distinguish between the rates of two source nodes separately. For simplicity, we introduce an ergodic sum-rate (ESR) to describe the performance of the TWR fading channels. An instantaneous sum-rate of $R_s[t]$ is said to be achievable if, there exist at least an encoding/decoding scheme of rate $R_0[t], R_2[t], R_0[t] + R_2[t] \leq R_s[t]$ for two source nodes respectively, with as small probability of instantaneous error in the $t^{th}$ round of information exchange as desired. An ESR of $\bar{R}_s$ is considered as the average sum-rate over all channel distributions, i.e., $\bar{R}_s = \mathcal{E}\{ R_s \}$. The instantaneous sum-capacity $C_s$ and ergodic sum-capacity (ESC) $\bar{C}_s$ are then the supremums of $R_s$ and $\bar{R}_s$, respectively.

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1 For Gaussian channel or instantaneous capacity, the computation $\mathcal{E}$ is not needed and the notation $\bar{C}$ is replaced by $C$.

2 Note that the rate region approached by partial decoding is the upper bound of the capacity region of two-phase two-way relaying up to now [3], [4]. Due to that only the equal time slot can be applied for the partial decoding if the channel gains are reciprocal, we also use this time allocation throughout this paper.
III. PROPOSED ALTERNATIVE AWAITING AND BROADCAST (AAB)

For the traditional TWR reciprocal fading channels, the transmission rate of a good uplink suffers from a poor downlink in an identical direction because the relay node can not transmit all the message received in the current time unit. In the inverse transmission direction, the transmission rate of a good downlink is also limited by a poor uplink because no enough message can be broadcasted in the current time unit. To overcome the aforementioned weakness, namely the min operations between the uplink and downlink in an identical direction, we propose a new TWR protocol, denoted as alternative awaiting and broadcast (AAB).

Due to reciprocation and quick variation of the fading channel gains, the proposed AAB protocol has the potential to significantly improve the exchange rate by trading a small average delay of ST. Next we will interpret the process flow and analyze the average delay of ST.

A. Descriptions of Process Flow

We first summarize some important notations as follows:

Let $t$ denote the $t^{th}$ round of SIE, for all $t$. We also assume that each round of SIE occupies one time unit. Then, each round of SIE is divided into two time slots. One is for a MA phase and the other is for a BC phase.

$D_i^T[t]$: the information packet to be transmitted from the node $i$ during the $t^{th}$ round of SIE, for $i \in \{0, 1, 2\}$.

$D_i^{TB}[t]$: the sub-packet split from the transmitted packet $D_i^T[t]$ and to be broadcasted immediately, for $i \in \{0, 2\}$.

$D_i^{TS}[t]$: the sub-packet split from the transmitted packet $D_i^T[t]$ and to be stored for delayed transmission, for $i \in \{0, 2\}$.

$D_i^R[t]$: the information packet to be received at the node $i$, during the $t^{th}$ round of SIE, for $i \in \{0, 1, 2\}$.

$|D_i[t]|$: the length in bits of the packet $D_i[t]$ during the $t^{th}$ round of SIE.

$B_{ij}$: the relay buffer which is set only for the transmission from node $i$ to node $j$, for $\{i, j\} \in \{0, 2\}$.

Without loss of generality, we assume that $|h_{01}[t-3]|^2 \geq |h_{21}[t-3]|^2$, $|h_{01}[t-2]|^2 \leq |h_{21}[t-2]|^2$, $|h_{01}[t-1]|^2 \leq |h_{21}[t-1]|^2$, $|h_{01}[t]|^2 \geq |h_{21}[t]|^2$, and $|h_{01}[t+1]|^2 \leq |h_{21}[t+1]|^2$. 


We also preferentially guarantee the transmission rate of the side with inferior uplink is not decreased comparing with the scheme of identical transmission rate for two source nodes.

1) $t^{th}$ round of SIE — a MA phase and a BC phase: As depicted in Fig. 2-(a), we firstly consider a MA phase in the $t^{th}$ round of SIE. Wherein, two source nodes transmit their information packet, $D^T_0[t]$ and $D^T_2[t]$, to the relay node simultaneously. Since $|h_{01}[t]|^2 \geq |h_{21}[t]|^2$, we have $|D^T_0[t]| \geq |D^T_2[t]|$. Due to the application of PLNC, the information packet $D^R_1[t]$ received at the relay node is a function of two transmitted packets, i.e., $D^R_1[t] = F\left(D^T_0[t], D^T_2[t]\right)$.

According to the capacity region in the BC phase [23] and $h_{10}[t] = h_{01}[t], h_{12}[t] = h_{21}[t]$, the desired information packet $D^R_0[t]$ can not be decoded successfully at the source node 2 if we broadcast $D^R_1[t]$ directly. At the same time interval, the source node 0 can receive more information than the transmitted packet $D^T_2[t]$.

Thus, we propose a new packet processing method which is denoted as extracting and embedding (EDE). This new method operates on the received packet at the relay node and is elaborated as follows:

a) Since that we have $|h_{10}[t]|^2 \geq |h_{12}[t]|^2$, the relay node extracts a sub-packet $D^{TS}_0[t]$ from the received packet $D^R_1[t]$ under the rule of $|D^T_0[t]| - |D^{TS}_0[t]| \leq C_{12}[t]$. Now the received packet $D^R_1[t]$ is changed into a distinct packet $D^{R'}_1[t]$ with $D^{R'}_1[t] = F\left(D^{TB}_0[t], D^T_2[t]\right)$. Therein, the sub-packet $D^{TB}_0[t]$ is remainder information of the transmitted packet $D^T_0[t]$ after extracting a sub-packet $D^{TS}_0[t]$. We have $|D^{TB}_0[t]| = |D^T_0[t]| - |D^{TS}_0[t]|$, Then we store $D^{TS}_0[t]$ in the relay buffer $B_{02}$.

b) Similar to the analysis in method a), due to $|h_{01}[t - 2]|^2 \leq |h_{21}[t - 2]|^2, |h_{01}[t - 1]|^2 \leq |h_{21}[t - 1]|^2, |h_{10}[t - 2]|^2 \leq |h_{12}[t - 2]|^2$ and $|h_{10}[t - 1]|^2 \leq |h_{12}[t - 1]|^2$, two sub-packets $D^{TS}_2[t - 1]$ and $D^{TS}_2[t - 2]$ had been stored in the relay buffer $B_{20}$ during the $(t - 1)^{th}$ and $(t - 2)^{th}$ rounds of SIE respectively. If we have $|D^{TS}_2[t - 2]| + |D^{TS}_2[t - 1]| + |D^T_2[t]| \leq C_{10}[t]$, the aforementioned two sub-packets will be picked up and embedded into the packet $D^{R'}_1[t]$. Through the secondly application of the PLNC, we generate the broadcasted packet $D^T_1[t]$ which is a function of three sub-packets (i.e., $D^{TB}_0[t], D^{TS}_2[t - 1]$ and $D^{TS}_2[t - 2]$) and an original packet $D^T_2[t]$.

Note that the storage and extraction in each relay buffer obey the rule of First-In First-Out (FIFO). Then, the relay node broadcasts the generated packet $D^T_1[t]$ during the coming BC phase in the $t^{th}$ round of SIE. With the help of its self-information, each destination node can successfully decode the exchanged information transmitted by the corresponding source node.
Here the self-information means the information which is generated by the destination node itself. Based on the known information packets, such as $D^T_0[t - 2], D^T_0[t - 1]$ and $D^T_0[t]$, the destination node 0 can easily decode two sub-packets (i.e., $D^{TS}_2[t - 2]$ and $D^{TS}_2[t - 1]$) and an original packet $D^T_2[t]$, from the received packet $D^R_0[t]$. At the same time, the node 2 can also successfully decode the only one sub-packet, e.g., $D^{TB}_0[t]$, from the received packet $D^R_2[t]$ with the help of a known packet $D^T_2[t]$. Wherein, we have $D^R_0[t] = D^R_2[t] = D^T_1[t]$.

2) $(t + 1)^{th}$ round of SIE — a MA phase and a BC phase: Different from the $t^{th}$ round of SIE, the instantaneous channel condition of two transmission directions in the $(t + 1)^{th}$ round of SIE has changed to $|h_{01}[t + 1]|^2 \leq |h_{21}[t + 1]|^2$.

As depicted in Fig. 2-(b), two source nodes 0 and 2 simultaneously transmit two information packets, denoted as $D^T_0[t + 1]$ and $D^T_2[t + 1]$. Similar to the analysis in Subsection III-A1, at the relay node we also need to apply the proposed EDE method as stated below:

a) The relay node extracts a sub-packet $D^{TS}_2[t + 1]$ from the received packet $D^{R}_1[t + 1]$ under the rule of $|D^T_2[t + 1]| - |D^{TS}_2[t + 1]| \leq C_{10}[t + 1]$. Now the received packet $D^{R}_1[t + 1]$ is changed into a distinct packet $D^{R}_1[t + 1]$ with $D^{R}_1[t + 1] = F(D^{TB}_2[t + 1], D^{R}_0[t + 1])$. Then we store $D^{TS}_2[t + 1]$ in the relay buffer $B_{20}$.

b) Due to $|h_{01}[t - 3]|^2 \geq |h_{21}[t - 3]|^2$, $|h_{01}[t]|^2 \geq |h_{21}[t]|^2$, two sub-packets $D^{TS}_0[t - 3]$ and $D^{TS}_0[t]$ had been stored in the relay buffer $B_{02}$ during the $(t - 3)^{th}$ and $t^{th}$ rounds of SIE respectively. If we have $|D^{TS}_0[t - 3]| + |D^{TS}_0[t]| + |D^T_0[t + 1]| \leq C_{12}[t + 1]$, the aforementioned two sub-packets will be picked up and embedded into the packet $D^{R}_1[t + 1]$. Then we generate the broadcasted packet $D^T_1[t + 1]$ which is a function of three sub-packets (i.e., $D^{TB}_2[t + 1]$, $D^{TS}_0[t - 3]$ and $D^{TS}_0[t]$) and an original packet $D^R_0[t + 1]$.

During the coming BC phase, the generated packet $D^T_1[t + 1]$ is broadcasted by the relay node. With the help of its self-information, the destination node 0 can decode successfully the only one sub-packet, e.g., $D^{TB}_2[t + 1]$. At the same time, the destination node 2 can easily decode two sub-packets (i.e., $D^{TS}_0[t - 3]$ and $D^{TS}_0[t]$) and an original packet $D^R_0[t + 1]$.

3) $t^{th}$ round of DIE: According to the definition of the round of DIE in Section II, the $t^{th}$ round of DIE contains the $t^{th}$ and $(t + 1)^{th}$ rounds of SIE. It is because that the two transmitted packets, $D^T_0[t]$ and $D^T_2[t]$, are successfully exchanged between two source nodes during the former two rounds of SIE. Note that with $D^{TB}_0[t]$ and $D^{TS}_0[t]$ node 2 can recover $D^T_0[t]$ in the
end of the \((t+1)^{th}\) rounds of SIE. In this case, we can say that the delay of ST of the \(t^{th}\) round of DIE is one time unit.

### B. Average Exchange Sum-Rate of One Round of SIE

Considering the successive rounds of SIE, it is obvious that the relay node can broadcast all received information packets by the proposed AAB protocol although it needs extra certain time units for completing each round of DIE. Next, we analyze the average exchange sum-rate of one round of SIE.

Let \(R_{ij}[t], i,j \in \{0,2\}\), denote the upper bound on instantaneous capacity of one side with transmission direction \(i \rightarrow j\) during the \(t^{th}\) round of SIE. According to the upper bound on capacity in the MA phase [3]–[5] and the exact capacity in the BC phase [23], we obtain

\[
R_{02}^u[t] = \frac{1}{2} \min \left\{ R_{01}^u[t], R_{12}[t] \right\} = \frac{1}{2} \min \left\{ C_{01}[t], C_{12}[t] \right\},
\]

\[
R_{20}^u[t] = \frac{1}{2} \min \left\{ R_{21}^u[t], R_{10}[t] \right\} = \frac{1}{2} \min \left\{ C_{21}[t], C_{10}[t] \right\}.  
\]

It is easy to note that the dominating bottlenecks of the exchange rate are two so-called min operations. Similar to the analysis given in the former subsection, we introduce the AAB protocol. The message received at the relay node with a higher rate, e.g., \(R_{01}^u[t]\), may be broadcasted in the \(t^{th}\) and some successive \(t^{th}\) rounds of SIE, where \(t' > t\). At the same time, the relay node can also broadcast both the message received with a lower rate, e.g., \(R_{21}^u[t]\), and some accumulated message during the former \(t^{th}\) rounds of SIE, where \(t'' < t\).

Then, \(R_{ij}[t]\) are modified as \(R_{ijd}[t]\) and given by

\[
R_{02d}^u[t] = \frac{1}{2} \max \left\{ R_{01}^u[t], R_{12}[t] \right\} = \frac{1}{2} \max \left\{ C_{01}[t], C_{12}[t] \right\},
\]

\[
R_{20d}^u[t] = \frac{1}{2} \max \left\{ R_{21}^u[t], R_{10}[t] \right\} = \frac{1}{2} \max \left\{ C_{21}[t], C_{10}[t] \right\}.
\]

Obviously, the Eqs. (3)-(4) cancel the bottlenecks — two min operations.

In terms of average exchange sum-rate, we attain a new upper bound on the ergodic sum-capacity of one round of SIE as follow.

\[
\bar{C}_u = \mathcal{E} \left\{ \frac{1}{2} C_{01}[t] + \frac{1}{2} C_{21}[t] \right\} = \frac{1}{2} \left( \bar{C}_{01} + \bar{C}_{21} \right).  
\]

It is obvious that the new upper bound \(\bar{C}_u\) can be achieved along with an average delay of ST in view of an achievable upper bound on instantaneous sum-capacity \(C_m^u\) in the MA phase.
C. Average Delay of ST of One Round of DIE in Relay Buffer

Since the value of the average delay of ST determines the practicability of the proposed AAB protocol, now we try to analyze the convergence behavior of the average delay of ST and give an exact expression for computing the average delay of ST of one round of DIE.

1) Convergence behavior: For simplicity, we consider an individual round of DIE — the $t^{th}$ round of DIE. Here we do not consider the influence of the delay of ST of the former one round of DIE. Without loss of generality, we assume that $|h_{01}[t]|^2 \geq |h_{21}[t]|^2$, namely $R_{01}[t] \geq R_{21}[t]$ and $R_{10}[t] \geq R_{12}[t]$. Note that one round of DIE maybe contains several rounds of SIE and each round of SIE occupies one time unit.

a) If $R_{01}[t] - R_{21}[t]$ can be successfully broadcasted during $l_{01}[t]$ time units, from the $(t + 1)^{th}$ to the $(t + l_{01}[t])^{th}$ time unit, we have

$$R_{01}[t] - R_{21}[t] \leq \sum_{l=1}^{l_{01}[t]} \phi(l) \left(C_{12}[t + l] - C_{10}[t + l]\right),$$

Wherein, $\phi(l)$ satisfies

$$\phi(l) = \begin{cases} 
1, & \text{if } |h_{01}[t+l]|^2 \leq |h_{21}[t+l]|^2, l \in [1, l_{01}[t]], \\
0, & \text{else}.
\end{cases}$$

Considering the upper bound of the rate region in the MA phase may not be achieved, we set $R_{01}[t] - R_{21}[t] = \theta(C_{01}[t] - C_{21}[t]), \theta \in [0, 1]$. Here we provisionally only denote $\theta$ as the ratio of the difference of two transmission rates obtained from the capacity region to that from the upper bound. Then we obtain an instantaneous delay of ST $l_{01}[t]$ given as

$$l_{01}[t] = \min_{l_{01}[t] \in \mathbb{Z}^+} \left( \prod_{t=1}^{l_{01}[t]} \left(\frac{\sigma^2 + P|h_{12}[t+l]|^2}{\sigma^2 + P|h_{10}[t+l]|^2}\right)^{\phi(l)} \right) \frac{\left(\frac{\sigma^2 + P|h_{01}[t]|^2}{\sigma^2 + P|h_{21}[t]|^2}\right)^\theta}{\theta}. \quad (8)$$

b) If all stored message during $l_{10}[t]$ time units, from the $(t - 1)^{th}$ to the $(t - l_{10}[t])^{th}$ time unit, can be successfully broadcasted in the $t^{th}$ time unit, we have

$$C_{10}[t] - C_{12}[t] \geq \sum_{l=1}^{l_{10}[t]} \psi(l) \left(R_{21}[t - l] - R_{01}[t - l]\right),$$

Wherein, $\psi(l)$ satisfies

$$\psi(l) = \begin{cases} 
1, & \text{if } |h_{01}[t-l]|^2 \leq |h_{21}[t-l]|^2, l \in [1, l_{20}[t]], \\
0, & \text{else}.
\end{cases}$$

(10)
Here, we set $R_{21}[t-l] - R_{01}[t-l] = \theta(C_{21}[t-l] - C_{01}[t-l])$. Then we also obtain an instantaneous delay of ST $l_{10}[t]$ given as

$$l_{10}[t] = \max_{l_{10}[t] \in \mathbb{Z}^+} \left( \prod_{l=1}^{l_{10}[t]} \left( \frac{\sigma^2 + P|h_{21}[t-l]|^2}{\sigma^2 + P|h_{01}[t-l]|^2} \right)^{\theta \phi(t)} \right) \leq \frac{\sigma^2 + P|h_{10}[t]|^2}{\sigma^2 + P|h_{12}[t]|^2}, \tag{11}$$

Similarly, we can also obtain the corresponding instantaneous delay of ST $l_{12}[t]$ and $l_{21}[t]$ if $|h_{01}[t]|^2 \leq |h_{21}[t]|^2$.

The aforementioned two assumptions of a) and b) denote that, an individual round of DIE completes broadcasting the total message transmitted in the first round of SIE during the succeeding $l_{01}[t]$ time units, and the first round of SIE also broadcasts the total message accumulated during the former $l_{10}[t]$ time units. Obviously, the average delay of ST of one round of DIE is converged and bounded by a limited value if we have $\mathcal{E}\{l_{12}[t]\} \geq \mathcal{E}\{l_{01}[t]\}$ and $\mathcal{E}\{l_{10}[t]\} \geq \mathcal{E}\{l_{21}[t]\}$. Otherwise, the average delay of ST is unbounded. Fortunately, we obtain the previous conditions with an average delay of ST less than about 100 time units when $\theta < 0.97$, as depicted in the numerical results in Subsection VI-A.

Since that $R_{01}[t] - R_{21}[t] = \theta(C_{01}[t] - C_{21}[t]) = \theta(C_{10}[t] - C_{12}[t])$, we can also regard $\theta$ as the ratio of the difference of two transmission rates obtained in the MA phase to that obtained in the BC phase. This implies that the parameter $\theta$ can be used for designing a practical AAB protocol with a sufferable average delay of ST, even if we can not achieve the capacity of the TWR channels.

2) Average delay of ST of one round of DIE: Considering continuous rounds of DIE, we should take into account the influence of the delay of ST of the former one round of DIE.

Without loss of generality, we assume that $|h_{01}[t]|^2 \geq |h_{21}[t]|^2$ and $R_{01}[t] - R_{21}[t]$ can be successfully broadcasted during $l_{01}[t]$ time units — from the $(t+1)^{th}$ to the $(t+l_{01}[t])^{th}$ time unit. Let $\Gamma[t] = \left(\frac{\sigma^2 + P|h_{01}[t]|^2}{\sigma^2 + P|h_{21}[t]|^2}\right)^{\theta}$ and $\Delta[t + l(t)] = \left(\frac{\sigma^2 + P|h_{12}[t+l(t)]|^2}{\sigma^2 + P|h_{10}[t+l(t)]|^2}\right)^{\theta}$, we have

$$\log_2 \left(\Gamma[t]\right) \leq \sum_{l(t)=l_{01}[t]+1}^{l_{01}[t]} \phi(l(t)) \log_2 \left(\Delta[t + l(t)]\right) + \sum_{l(t_0)=l_{01}[t_0]+1}^{l_{01}[t_0]} \phi(l(t_0)) \log_2 \left(\Delta[t_0 + l(t_0)]\right) - \log_2 \left(\Gamma[t_0]\right), \tag{12}$$
where \( t_0 \) denotes the time unit in which adjacent former information \( R_{01}[t_0] - R_{21}[t_0] \), namely \( \log_2 \left( \Gamma[t_0] \right) \), has been generated. Moreover, \( l'_{01}[t] = l_{01}[t_0] - (t - t_0) \) denotes the extra time units should be used from the current time unit \( t \) in order to successfully broadcast the adjacent former information \( \log_2 \left( \Gamma[t_0] \right) \). Note that \( \phi(l(t)) \) satisfies \( \phi(l(t)) = 1 \) only if \( |h_{01}(t + l(t))|^2 \leq |h_{21}(t + l(t))|^2 \), for \( l(t) \in (l'_{01}[t], l_{01}[t]) \), else \( \phi(l(t)) = 0 \).

Then we obtain an instantaneous delay of \( \text{ST} \) \( l_{01}[t] \) given as

\[
\begin{align*}
l_{01}[t] & = \min_{l_{01}[t] \in \mathbb{Z}^+} \left\{ \prod_{l(t) = l'_{01}[t] + 1} \left( \Delta \left[ t + l(t) \right] \right)^{\phi(l(t))} \ 	imes \prod_{l(t_0) = l'_{01}(t_0) + 1} \left( \Delta \left[ t_0 + l(t_0) \right] \right)^{\phi(l(t_0))} \geq \Gamma[t] \times \Gamma[t_0] \right\}. \quad (13)
\end{align*}
\]

Similarly, the instantaneous delay of \( \text{ST} \) \( l_{21}[t] \) is obtained easily when \( |h_{01}[t]|^2 \leq |h_{21}[t]|^2 \). Now, the exact expression of the average delay of \( \text{ST} \) of one round of DIE can be written as \( \mathcal{L} = \{ \mathcal{E}\{l_{01}[t]\}, \mathcal{E}\{l_{21}[t]\} \} \).

If the channel conditions change slowly or remain stable, i.e., the channel conditions are unchanged during \( \iota \) rounds of SIE with \( 1 \leq \iota < +\infty \), the average delay of \( \text{ST} \) for the proposed AAB protocol is also unchanged according to Eq. (13). It is because that the unit of the average delay of \( \text{ST} \) is time unit. We can obtain the former conclusion by setting the time interval occupied by \( \iota \) rounds of SIE as one time unit. However, now the time unit is enlarged to its \( \iota \) times. In other words, the absolute value of the average delay of \( \text{ST} \) increases linearly through multiplying by \( \iota \), although Eq. (13) is still effective.

IV. Achievable Ergodic Sum-Rate and Corresponding Average Delay of ST

In this section, we derive an achievable ESR and the corresponding average delay of \( \text{ST} \) of the AAB protocol based on the well-known lattice codes.

A. Encoding and Decoding Solutions

Firstly, we consider a MA phase in the \( t^{th} \) round of SIE. Without loss of generality, we assume that \( |h_{01}[t]|^2 \geq |h_{21}[t]|^2 \), namely \( R_{01}[t] \geq R_{21}[t] \). As depicted in Fig. 3 (Left-hand), the source node 0 splits the message \( S_0(t) \) into two parts: \( S_0^1[t] \) and \( S_0^2[t] \). Therein, the length of one part,
e.g., $S_0^1[t]$, is equal to that of the message $S_2[t]$ from the source node 2. $S_0^1[t]$ and $S_0^2[t]$ are then encoded to $C_0^1[t]$ and $C_0^2[t]$ by a lattice code $L$ and a Gaussian code $G$ respectively. After operating $X_0^1[t] = (C_0^1[t] + D_0[t]) \mod \wedge^n$ and modulating $C_0^2[t]$ to $X_0^2[t]$, the source node 0 forms the transmitted signal $X_0[t] = \sqrt{\zeta[t]}X_0^1[t] + \sqrt{1 - \zeta[t]}X_0^2[t]$ by superposition and a power allocation coefficient $\zeta[t]$. At the same time, the source node 2 generates the transmitted signal $X_2[t]$ through mapping the message $S_2[t]$ to $C_2[t]$ by an identical lattice code $L$ and operating $X_2[t] = (C_2[t] + D_2[t]) \mod \wedge^n$. Note that the random dither vectors $D_0[t]$ and $D_2[t]$ are mutually independent of each other and are also known at both the relay node and two source nodes.

At the relay node, the received superimposed signal is given as

$$Y_1[t] = h_{01}[t]X_0[t] + h_{21}[t]X_2[t] + Z_1[t]$$

(14)

$$= h_{01}[t] \left( \sqrt{\zeta[t]}X_0^1[t] + \sqrt{1 - \zeta[t]}X_0^2[t] \right) + h_{21}[t]X_2[t] + Z_1[t]$$

(15)

$$= T[t] + \sqrt{1 - \zeta[t]}h_{01}[t]X_0^2[t] + Z_1[t].$$

(16)

As shown in Fig. 3 (Middle), the relay node first decodes $S_0^2[t]$ ($X_0^2[t]$) by treating a function $T[t]$ as noise. Subtracting $X_0^2[t]$ off its received signals, then $T[t]$ is decoded. Obviously, we should set $\zeta[t] = \frac{|h_{21}[t]|^2}{|h_{01}[t]|^2}$, $\zeta[t] \in [0, 1]$, in order to satisfy that two lattice coded signals have the same received SNR, i.e., $\gamma_{01}^1[t] = \gamma_{21}^1[t]$. Then, the relay operates $(T[t] + D_1[t]) \mod \wedge^n$ and forms $X_1^1[t]$ by a random dither vector $D_1[t]$. Due to the reciprocity between two relay channels, we have $|h_{12}[t]|^2 \leq |h_{10}[t]|^2$, namely $R_{12}[t] \leq R_{10}[t]$. Therefore, the relay stores $S_0^2[t]$ and waits a favorable channel gain, e.g., $|h_{12}[t]|^2 \geq |h_{10}[t]|^2$, to broadcast $S_0^2[t]$. At the same time, a fractional message, e.g., $S_2^2[t - \ell]$, for $\ell \in \mathbb{Z}^+$, which has been received and stored in the former $(t - \ell)^{th}$ round of SIE at the relay node, may be picked up and encoded to form $C_2^2[t]$ by a Gaussian code. $C_2^2[t]$ will be modulated to $X_2^2[t]$ and superimposed with $X_1^1[t]$ for generating the transmitted signal $X_1[t] = \sqrt{\eta[t]}X_1^1[t] + \sqrt{1 - \eta[t]}X_2^2[t]$. Here, $\eta[t]$ is also a power allocation coefficient with $\eta[t] \in [0, 1]$. The received Gaussian coded message at the relay node is decoded and stored in buffer $B_{02}$. It will be transmitted during certain rounds of SIE after the current $t^{th}$

\footnote{Note that Ong, et al. have discussed another two encoding/decoding schemes of the TWR Gaussian model without the delay of ST (24). One firstly decodes a function $T[t]$ by treating $X_0^2[t]$ as noise, the other uses the time allocation between the transmissions of lattice coded signal and the Gaussian coded signal. However, both these two schemes decrease the transmission rate of the side with inferior uplink comparing with the scheme of identical transmission rate for two source nodes.}
round of SIE while the lattice coded message $T_i[t]$, will be transmitted immediately in the current $t^{th}$ round of SIE. Note that the storage and extraction of each received Gaussian coded message obey the rule of First-In First-Out (FIFO). For example, $S_t^2[t_1 - l]$ should be broadcasted earlier than $S_t^2[t_2 - l]$, if we have $t_2 > t_1$ for $\{t_1, t_2\} \in \mathcal{N}$, $l \in \mathbb{Z}^+$, $i \in \{0, 2\}$.

In the immediate BC phase, the superimposed signal $X_1[t]$ is broadcasted to two source nodes by the relay node, as depicted in Fig. 3 (Right-hand). At two source nodes, the received signals are given as

\begin{align}
Y_i[t] &= h_{1i}[t]X_1[t] + Z_i[t], \\
&= h_{1i}[t]\left(\sqrt{\eta[t]}X_1^1[t] + \sqrt{1 - \eta[t]}X_1^2[t]\right) + Z_i[t],
\end{align}

where $i \in \{0, 2\}$. Here we have $|h_{12}[t]|^2 \leq |h_{10}[t]|^2$ when we set $|h_{10}[t]|^2 = |h_{01}[t]|^2$ and $|h_{12}[t]|^2 = |h_{21}[t]|^2$.

Since that $S_2^2[t - l]$ is known, the source node 2 first subtracts $X_2^1[t]$ off its received signal and then decodes the lattice coded message $S_2^1[t]$ by using a lattice code book $\{T, C_0^1 \in \Lambda^n\}$. At the same time, the source node 0 first decodes the lattice coded message, $S_2[t]$ ($X_1^1[t]$), by using a lattice code book $\{T, C_2 \in \Lambda^n\}$ and treating $X_1^2[t]$ as noise. Subtracting $X_1^1[t]$ off its received signal, the source node 0 then decodes the Gaussian coded message $S_2^2[t - l]$ from $X_2^1[t]$.

### B. Achievable Ergodic Sum-Rate of One Round of SIE

1) Achievable ergodic sum-rate (ESR): Let $R_{ij}[t], i, j \in \{0, 2\}$, denote the instantaneous transmission rate of one side with transmission direction $i \rightarrow j$ during the $t^{th}$ round of SIE. For a symmetrical Gaussian TWR channels, Wilson, et al. have achieved an identical transmission rate for two source nodes in [5] given as

\begin{align}
R_{02} &= R_{20} \leq \frac{1}{2} \min \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P}{\sigma^2} \right) \right]^+, \log_2 (1 + \frac{P}{\sigma^2}) \right\}.
\end{align}

\footnote{It is also possible to decode the Gaussian codeword firstly by treating the lattice codeword as noise. However, this scheme does not adequately utilize a performance improvement of $\frac{1}{2}$ bit between the rate achieved in the MA phase and that achieved in the BC phase for the lattice-coded TWR channels.}
Eq. (19) is equivalent to an achievable rate pair \( (R_{02}, R_{20}) \) given as
\[
R_{02} \leq \frac{1}{2} \min \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P_0}{\sigma_1^2} \right) \right]^+, \log_2 (1 + \frac{P_1}{\sigma_2^2}) \right\},
\]
(20)
\[
R_{20} \leq \frac{1}{2} \min \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P_2}{\sigma_1^2} \right) \right]^+, \log_2 (1 + \frac{P_1}{\sigma_2^2}) \right\},
\]
(21)
where \( P_0 = P_2 = P_1 = P, \sigma_0^2 = \sigma_2^2 = \sigma_1^2 = \sigma^2 \). Let \( P_0 = P|h_{01}|^2, P_2 = P|h_{21}|^2, \frac{P_1}{\sigma_2^2} = \frac{P|h_{12}|^2}{\sigma_2^2}, \frac{P_1}{\sigma_0^2} = \frac{P|h_{02}|^2}{\sigma_2^2} \) and \( \sigma_1^2 = \sigma^2 \). Considering the TWR fading channels, we treat variations of the channel gains \( |h_{ij}[t]|^2 \) as fluctuations of the power \( P_{ij}[t] \). Then, we extend Eqs. (20)-(21) from the Gaussian case to fading channels
\[
R_{02}[t] \leq \min \left\{ R_{01}[t], R_{12}[t] \right\}
\leq \frac{1}{2} \min \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P \min \{|h_{01}[t]|^2, |h_{21}[t]|^2\}}{\sigma^2} \right) \right]^+, \\
\log_2 (1 + \frac{P|h_{12}[t]|^2}{\sigma^2}) \right\},
\]
(22)
\[
R_{20}[t] \leq \min \left\{ R_{21}[t], R_{10}[t] \right\}
\leq \frac{1}{2} \min \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P \min \{|h_{01}[t]|^2, |h_{21}[t]|^2\}}{\sigma^2} \right) \right]^+, \\
\log_2 (1 + \frac{P|h_{10}[t]|^2}{\sigma^2}) \right\},
\]
(23)
Obviously, the bottle-neck terms are inside in the two \( \min \) operators of each equation.

The inner \( \min \) operation of each equation can be omitted by superposition. Using this method on an assumption of \( |h_{01}[t]|^2 \geq |h_{21}[t]|^2 \), we will obtain an instantaneous rate pair \( (R_{01}[t], R_{21}[t]) \) in the MA phase given by
\[
R_{01}[t] \leq \frac{1}{2} \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P|h_{21}[t]|^2}{\sigma^2} \right) \right]^+ \\
+ \log_2 (1 + \frac{P(|h_{01}[t]|^2 - |h_{21}[t]|^2)}{\sigma^2 + 2P|h_{21}[t]|^2}) \right\},
\]
(24)
\[
R_{21}[t] \leq \frac{1}{2} \left[ \log_2 \left( \frac{1}{2} + \frac{P|h_{21}[t]|^2}{\sigma^2} \right) \right]^+,
\]
(25)
and an instantaneous rate pair \( (R_{10}[t], R_{12}[t]) \) in the BC phase as
\[
R_{12}[t] \leq \frac{1}{2} \log_2 (1 + \frac{\eta[t]P|h_{12}[t]|^2}{\sigma^2}),
\]
(26)
\[
R_{10}[t] \leq \frac{1}{2} \left\{ \log_2 (1 + \frac{\eta[t]P|h_{10}[t]|^2}{\sigma^2 + (1 - \eta[t])P|h_{10}[t]|^2}) \\
+ \log_2 (1 + \frac{(1 - \eta[t])P|h_{10}[t]|^2}{\sigma^2}) \right\},
\]
(27)
where $\eta[t] \in [0,1]$.

As shown in $R_{01}[t]$ in (24), we remove the inner min operation which is needed in (22). Meantime, two outer min operations are eliminated by the proposed AAB scheme, as described in Subsections III-A and IV-A. Due to the delay of ST $l$, the transmission rates $R_{02u}[t]$ and $R_{20u}[t]$ in the $t^{th}$ round of SIE are not limited by the min operation any more. It is because that the message, received with a higher rate $R_{01}[t]$ at the relay node, can be broadcasted in the next $(t + l')^{th}$ time units, where $l' \geq 1$. At the same time, the relay node can also broadcast both the message, received with a lower rate $R_{21}[t]$, and some accumulated message, received during the former $(t - l'')^{th}$ time units, where $l'' \geq 1$.

Then, we achieve an instantaneous rate pair, denoted as $(R_{02u}[t], R_{20u}[t])$, given by

$$R_{02u}[t] \leq \max \left\{ R_{01}[t], R_{12}[t] \right\}, \quad (28)$$

$$R_{20u}[t] \leq \max \left\{ R_{21}[t], R_{10}[t] \right\}. \quad (29)$$

In general, an achievable ESR of one round of SIE based on the proposed AAB protocol, denoted as $\bar{R}_s$, is given by

$$R^{AAB}_s \leq \mathcal{E} \left\{ \left[ \log_2 \left( \frac{1}{2} + \frac{P \min \{|h_{01}[t]|^2, |h_{21}[t]|^2\}}{\sigma^2} \right) \right]^+ + \frac{1}{2} \log_2 \left( 1 + \frac{P |h_{01}[t]|^2 - |h_{21}[t]|^2}{\sigma^2 + 2P \min \{|h_{01}[t]|^2, |h_{21}[t]|^2\}} \right) \right\}. \quad (30)$$

2) Power allocation at relay node: In this subsection, we optimize the power allocation parameter, denoted as $\eta$, to maximize the achievable ESR of the AAB protocol.

Satisfying that we should not decrease the transmission rate of the side with inferior uplink, e.g., source node 2, we need

$$\frac{1}{2} \log_2 \left( 1 + \frac{\eta[t]P|h_{12}[t]|^2}{\sigma^2} \right) \geq \frac{1}{2} \log_2 \left( \frac{1}{2} + \frac{P|h_{21}[t]|^2}{\sigma^2} \right). \quad (31)$$

Then we have

$$\eta[t] \geq 1 - \frac{\sigma^2}{2P|h_{21}[t]|^2}. \quad (32)$$

Note that Nam, et al. have also omitted the inner min operation in $R_{01}[t]$ by a pure partial decoding method with the nested lattice codes for the asymmetric rate case [4]. However, it refers to a difficult question of how to embed (extract) the fractional information into (from) a network coded codeword if the AAB protocol is applied.
Due to the similar causations, we also need
\[
\frac{1}{2} \log_2 \left( 1 + \frac{\eta[t] P|h_{10}[t]|^2}{\sigma^2 + (1 - \eta[t]) P|h_{10}[t]|^2} \right) \geq \frac{1}{2} \log_2 \left( \frac{1}{2} + \frac{P|h_{21}[t]|^2}{\sigma^2} \right).
\] (33)

After simplifications, we obtain
\[
\eta[t] \geq \frac{(2P|h_{21}[t]|^2 - \sigma^2)(P|h_{10}[t]|^2 + \sigma^2)}{P|h_{10}[t]|^2(\sigma^2 + 2P|h_{21}[t]|^2)}.
\] (34)

Combining the former two inequations (32) and (34), we introduce three conditions given as
\[
\eta[t] \geq \begin{cases} 
1 - \frac{\sigma^2}{2P|h_{21}[t]|^2}, & \text{if } 0 < \frac{|h_{21}[t]|^2}{|h_{10}[t]|^2} \leq \frac{1}{2}, \frac{P|h_{21}[t]|^2}{\sigma^2} \geq \frac{1}{2}, \\
\frac{(2P|h_{21}[t]|^2 - \sigma^2)(P|h_{10}[t]|^2 + \sigma^2)}{P|h_{10}[t]|^2(\sigma^2 + 2P|h_{21}[t]|^2)}, & \text{if } \frac{1}{2} < \frac{|h_{21}[t]|^2}{|h_{10}[t]|^2} \leq 1, \frac{P|h_{21}[t]|^2}{\sigma^2} \geq \frac{1}{2}, \\
0, & \text{if } 0 \leq \frac{P|h_{21}[t]|^2}{\sigma^2} < \frac{1}{2}, 
\end{cases}
\] (35)

For maximizing the additional broadcast rate from the relay node 1 to the source node 0, we should select the minimum of \( \eta[t] \) in three considered conditions. Particularly, only \( \eta[t] = 0 \) comes into existence when \( \frac{P|h_{21}[t]|^2}{\sigma^2} < \frac{1}{2} \). In this case, the source node 2 does not transmit any message because \( \frac{1}{2} \log_2 \left( \frac{1}{2} + \frac{P|h_{21}[t]|^2}{\sigma^2} \right) \leq 0 \) and the TWR channel is degraded to an one-way relay (OWR) channel. It is important to note that the expressions of \( \eta \) are achieved in closed-forms as exhibited in (35).

C. Corresponding Average Delay of ST of One Round of DIE

In this subsection, the average delay of ST of the proposed AAB protocol with lattice codes is analyzed.

1) \( \theta \) versus \( P/\sigma^2 \): As referred in Subsection III-C1, we can regard \( \theta \) as the gap between the difference of two transmission rates obtained in the MA phase and that obtained in the BC phase. Here, we have
\[
\theta_1[t] = \frac{R_{1}^{D,MA}[t]}{R_{1}^{D,BC}[t]} = \frac{\log_2 \left( 1 + \frac{P(h_{01}[t]^2 - h_{21}[t]|^2)}{\sigma^2 + 2P|h_{21}[t]|^2} \right)}{\log_2 \left( 1 + \frac{(1-\eta[t])P|h_{10}[t]|^2}{\sigma^2} \right)},
\] (36)
\[
\theta_2[t] = \frac{R_{2}^{D,MA}[t]}{R_{2}^{D,BC}[t]} = \frac{\log_2 \left( 1 + \frac{P|h_{21}[t]|^2 - h_{01}[t]|^2}{\sigma^2 + 2P|h_{01}[t]|^2} \right)}{\log_2 \left( 1 + \frac{(1-\eta[t])P|h_{12}[t]|^2}{\sigma^2} \right)},
\] (37)
when \( |h_{01}[t]|^2 \geq |h_{21}[t]|^2 \) and \( |h_{01}[t]|^2 \leq |h_{21}[t]|^2 \), respectively. Note that we define \( R_{1}^{D,BC}[t] \) as the rate of Gaussian coded information instead of letting \( R_{1}^{D,BC}[t] = R_{10}[t] - R_{12}[t] \) subject to the lattice encoding/decoding solutions, where \( R_{12}[t] \) and \( R_{10}[t] \) are depicted in (26) and (27).
According to the analysis in Subsection III-C1, a converged and bounded average delay of ST is achieved if we have $E(\theta_1[t]) < 0.97$ and $E(\theta_2[t]) < 0.97$. Fortunately, we obtain these two inequations as described in Subsection VI-B. This implies again that the parameter $\theta$ can be used for designing a practical AAB protocol with a sufferable average delay of ST.

2) Average delay of ST of one round of DIE: Similar to the analysis stated in Subsection III-C2, if $|h_{01}[t]|^2 \geq |h_{21}[t]|^2$, we gain an instantaneous delay of ST $l_{01}[t]$ of the achievable ESR through letting $\Gamma[t] = 1 + \frac{P(|h_{01}[t]|^2 - |h_{21}[t]|^2)}{\sigma^2 + 2P|h_{21}[t]|^2}$ and $\Delta[t + l(t)] = 1 + \frac{(1-\eta[t+l(t)])P|h_{21}[t+l(t)]|^2}{\sigma^2}$ in (13). Analogously, the instantaneous delay of ST $l_{21}[t]$ is also obtained when $|h_{01}[t]|^2 \leq |h_{21}[t]|^2$.

Now, the average delay of ST of the proposed AAB protocol with lattice codes can be written as $L = \{E\{l_{01}[t]\}, E\{l_{21}[t]\}\}$.

V. COMPARISONS OF AAB PROTOCOL AND TRADITIONAL TWR PROTOCOLS

In this section, we make some comparisons between the proposed AAB protocol and the traditional TWR protocols in terms of the upper bound on the ESC and the achievable ESR of one round of SIE.

A. Upper Bound on Ergodic Sum-Capacity (ESC)

According to (1) and (2), we obtain the upper bound on the ESC of the traditional TWR protocols [3]–[5], [23] given as

$$\bar{C}^{uT}_s = \mathcal{E}\left\{\frac{1}{2} \min \left\{C_{01}[t], C_{12}[t]\right\} + \frac{1}{2} \min \left\{C_{21}[t], C_{10}[t]\right\}\right\}$$

$$= \mathcal{E}\left\{\min \left\{C_{01}[t], C_{21}[t]\right\}\right\}.$$  \hspace{1cm} (38)

(39)

Compared with (5), we can see that the upper bound on the ESC of the traditional TWR protocols suffers from the min operations between the uplink and downlink in an identical direction while the proposed AAB protocol removes it from the upper bound on the ESC.

B. Achievable Ergodic Sum-Rate (ESR)

From (22) and (23), we can obtain the achievable ESR of the DNF protocol based on the lattice codes [5] given as

$$\bar{R}^{\text{DNF}}_s \leq \mathcal{E}\left\{\left[\log_2 \left(\frac{1}{2} + \frac{P \min\{|h_{01}[t]|^2, |h_{21}[t]|^2\}}{\sigma^2}\right)\right]^+\right\}.$$  \hspace{1cm} (40)
Compared with (30), we can see that the achievable ESR of the DNF protocol also suffers from the \( \min \) operations between the uplink and downlink in an identical direction. Fortunately, the proposed AAB protocol achieves extra ESR improvement through eliminating the \( \min \) operations.

VI. NUMERICAL RESULTS

Aim to vividly state the universality of the aforementioned analysis on the average delay of ST and the performance comparisons between the proposed AAB protocol and the traditional TWR protocols, in this section we present some numerical results based on three kinds of classical fading channel models: Rayleigh fading, Rice fading and Nakagami-m fading channel models.

As shown in Fig. 1, we assume that the distance between two source nodes 0 and 2 is normalized to 1 and the location of the relay is determined using the projections \( x \) and \( y \). The source nodes 0 and 2 are located at the coordinates (-0.5,0) and (0.5,0), respectively. We set \( \{x, y\} \sim \mathcal{U}[-0.5, 0.5] \), where \( \mathcal{U} \) denotes Uniform distribution. The distances from the relay to the source nodes can be computed as
\[
d_{01} = \sqrt{(x + 0.5)^2 + y^2}, \quad d_{12} = \sqrt{(x - 0.5)^2 + y^2}.
\]
Suppose that the channel gain \( h_{ij} \), for \( \{i, j\} \in \{0, 1, 2\} \), is modeled by a small-scale fading model with a distance path loss, given by
\[
h_{ij} = \alpha_{ij} \cdot d_{ij}^{-\beta/2},
\]
where \( \beta \) is the path loss exponent and fixed at 3, \( d_{ij} \) and \( \alpha_{ij} \) denote the distance between node \( i \) and \( j \) and the channel fading coefficient of the link from node \( i \) to \( j \) with \( \mathcal{E}\{|\alpha_{ij}|^2\} = 1 \), respectively. Wherein, \( \alpha_{01} \) and \( \alpha_{21} \) are i.i.d.. With \( d_{ij}[t] = d_{ji}[t] \) and a constant \( \beta \), we have \( h_{01}[t] = h_{10}[t] \) and \( h_{21}[t] = h_{12}[t] \) if let \( \alpha_{01}[t] = \alpha_{10}[t] \) and \( \alpha_{21}[t] = \alpha_{12}[t] \). Moreover, \( h_{01} \) and \( h_{21} \) are also i.i.d.. Now we obtain the desired TWR fading channel model as described in Section III. The channel gain on each link is reciprocal. In addition, each node uses the same transmission power \( P \) and the AWGN \( z_j \) at node \( j \) is subject to \( \mathcal{CN}(0, \sigma^2) \).

It is important to note that the convergence behavior of the average delay of ST for AAB protocol is constant no matter which one of three kinds of TWR fading channel models is considered. In other words, Figs. 4-8 are all effective when both \( \alpha_{01} \) and \( \alpha_{21} \) are subject to Rayleigh, Rice and Nakagami-m distributions, respectively. These phenomena are also confirmed in Figs. 12-14.

Since the time interval of one round of SIE is assumed to be 1 s/Hz in this section, i.e., one time unit is equivalent to 1 s/Hz, here the unit of the average delay of ST is also s/Hz. However, according to the analysis in former sections, the unit of the average delay of ST is only denoted...
as time unit. That is to say, the unit of the average delay of ST will changes into \( ms/Hz \) if the time interval of one round of SIE is set as 1 ms/Hz.

A. Convergence Behavior of Average Delay of ST

Figs. 4-6 illustrate the convergence behavior of the average delay of ST for the proposed AAB protocol. From Fig. 4, we see that both \( E(l_{12}[t]) \) and \( E(l_{10}[t]) \) are more than, equal to and less than the corresponding \( E(l_{01}[t]) \) and \( E(l_{21}[t]) \) if \( \theta \) is set as 0.98, 0.97 and 0.96, respectively. That is to say, \( \theta \approx 0.97 \) is the bound in which we can obtain a converged and bounded average delay of ST. It is because that the rate of accumulation in the relay buffer is more than that of reduction when \( E(l_{01}[t]) > E(l_{12}[t]) \) or \( E(l_{21}[t]) > E(l_{10}[t]) \). According to the definitions of \( \theta \), Fig. 4 confirms that the proposed AAB protocol can be used with a bounded average delay of ST if 1) the ratio of the difference of two transmission rates obtained from the capacity region to that from the upper bound is less than about 0.97, 2) or the ratio of the difference of two transmission rates obtained in the MAC phase and that obtained in the BC phase is less than about 0.97. These phenomena explicitly state that our proposed AAB protocol is suitable for use in almost all TWR transmission cases.

Fig. 5 shows the correlation of the average delay of ST and \( P/\sigma^2 \). It is observed that the average delay of ST is almost constant if \( P/\sigma^2 \) ranges from 5dB to 20dB. The average delay of ST versus \( \theta \) is depicted in Fig. 6. It can be clearly seen that the correlation between the average delay of ST and \( \theta \) is similar to an exponential function. It also shows that the average delay of ST increases laxly and is less than about 100s/Hz when \( \theta < 0.97 \) while grows sharply in the high \( \theta \) regime. It is expected because the accumulative packets in the relay buffer can not be broadcasted to two source nodes in a limited time if \( \theta < 0.97 \).

B. Average Delay of ST for AAB Protocol with Lattice Codes

Fig. 7 demonstrates the correlation of \( \theta \) and \( P/\sigma^2 \) of the AAB protocol with lattice codes. It is observed that both \( E(\theta_1[t]) \) and \( E(\theta_2[t]) \) are always less than 0.97 no matter what \( P/\sigma^2 \) is. Moreover, we can see that \( \theta \) increases slowly as \( P/\sigma^2 \) approaches 20dB.

Fig. 8 shows the variations of the average delay of ST for the AAB protocol with lattice codes when \( P/\sigma^2 \) is increasing. Comparing with Fig. 7, it can be clearly seen that the average delay of ST raises slowly as \( \theta \) is increasing and is always less than 100s/Hz for all considered \( P/\sigma^2 \).
Figs. 7-8 confirm that the parameter $\theta$ can be used for designing a practical AAB protocol with a sufferable delay of ST, even if we can not achieve the capacity of the TWR channels.

C. Comparisons of Considered TWR Protocols

In this subsection, we present a performance study of the proposed AAB protocol with lattice codes in terms of two metrics: ergodic sum-rate (ESR) and average delay of system service (SS). For comparison, the performance of the upper bound without the delay of ST [3] and the DNF protocol with lattice codes derived in [5] are also shown in the figures.

1) Ergodic sum-rate (ESR) versus $P/\sigma^2$: Fig. 9 shows the ESRs of different TWR protocols when $P/\sigma^2$ increases from $-10$ dB to 20 dB in the TWR Rayleigh fading channels. It is observed that no matter what $P/\sigma^2$ is, both the proposed upper bound with the delay of ST and the AAB protocol with lattice codes always obtain the larger ESRs than the other protocols, i.e., the upper bound without the delay of ST and the DNF protocol with lattice codes. Moreover, we can see that the DNF with lattice approaches the upper bound without the delay of ST, especially in the high $P/\sigma^2$ regime. Although the proposed AAB with lattice yet does not approach the upper bound with the delay of ST even if $P/\sigma^2$ is 20 dB, the gaps between them are decreased slowly when $P/\sigma^2$ is increasing. So, we can also claim that the AAB with lattice has the ability to approach the upper bound with delay of ST at certain higher $P/\sigma^2$. At the same time, all the gaps between the considered protocols with the delay of ST and that without the delay of ST are enlarged because of the influence of inferior channel gains. As shown in Fig. 9, the upper bound with the delay of ST outperforms the upper bound without the delay of ST and the DNF with lattice about 2 b/s/Hz when $P/\sigma^2$ at 20 dB. It is also shown that the proposed AAB with lattice is only inferior to the upper bound with the delay of ST about 0.4 b/s/Hz and is superior to the upper bound without the delay of ST about 1.6 b/s/Hz at $P/\sigma^2 = 20$ dB. Generally, we can say that the performance improvement obtained by the proposed AAB protocol is obvious and significant.

Figs. 10-11 show the ESRs of different TWR protocols in the TWR Rice and Nakagami-m fading channels respectively. The general trends of the performance lines in these two figures are the same as that depicted in Fig. 9. The main difference among these three cases are only the performance gaps between the considered protocols with the delay of ST (proposed AAB protocol) and that without the delay of ST (traditional TWR protocols).
2) **Average delay of system service (SS) versus packet arrival rate (PAR):** Note that we do not care at each source node whether the source information waits to be transmitted or not in terms of the delay of ST. In this subsection we will study this phenomenon through considering the average delay of SS versus PAR. Suppose that both two source nodes have buffers for all considered TWR protocols while the relay node has buffer only for the proposed AAB protocol with lattice codes. We assume that the PAR, scaled by $p/s/Hz$, at two source nodes follows Poisson distribution with mean $\rho$, the length of each packet is fixed as $\Omega$ bits. Let $Q^s(t-1) = [Q^s_0(t-1), Q^s_2(t-1)]$ and $Q^r(t-1) = [Q^r_0(t-1), Q^r_2(t-1)]$ represent the remaining bits in the queues after the $(t-1)^{th}$ time unit at two source nodes and relay node respectively. Then,

$$Q^s(t) = [Q^s_0(t), Q^s_2(t)] = [Q^s_0(t-1) - R_{02}(t) + \rho_{02}(t)\Omega, Q^s_2(t-1) - R_{20}(t) + \rho_{20}(t)\Omega],$$

where $R_{ij}(t), \rho_{ij}(t)$ ($\{i, j\} \in \{0, 2\}$) denote the system service rates and packet arrival rates of node $i$ in the $t^{th}$ time unit. In addition, $Q^r(t) = [Q^r_0(t), Q^r_2(t)] = [Q^r_0(t-1) - \tilde{R}_{12}(t) + (R_{01}(t) - R_{21}(t)), Q^r_2(t-1) - \tilde{R}_{10}(t) + (R_{21}(t) - R_{01}(t))]$. Wherein, $\tilde{R}_{10}(t) = D_1[2k+2] \neq R_{10}(t) - R_{12}(t)$ and $\tilde{R}_{12}(t) = D_2[2k+2] \neq R_{12}(t) - R_{10}(t)$, as depicted in Eqs. (36) and (37).

Fig. 12 shows the average delay of SS with $\Omega = 10$ in the TWR Rayleigh fading channels. It can be clearly seen that the proposed AAB with lattice always outperforms the other two TWR protocols significantly. It is because that the AAB protocol can cancel the $\text{min}$ operations between the uplink and downlink in an identical direction while the traditional protocols can not. The characteristic can be easily seen from two kinds of comparisons between Eqs. (1-2) and Eqs. (3-4), Eqs. (22-23) and Eqs. (28-29). For all considered TWR protocols, both the average delay of SS for two transmission directions in the source buffers are exponentially increased to about $10^4 s/Hz$ as the rates of information exchange are approaching the corresponding maximum values. However, it also shows that the average delay of ST in the relay buffer stops increasing and maintains at about $45 s/Hz$ as $\rho$ is approaching 0.26. Compared with the average delay of SS in the source buffer, the delay of ST in the relay buffer induced by the proposed AAB protocol is very small and negligible. These phenomena confirm that the proposed AAB protocol with lattice codes can significantly improve the rate of information exchange with a sufferable delay of ST.

Figs. 13-14 show that the average delay of SS with $\Omega = 10$ in the TWR Rice and Nakagami-m fading channels respectively. We can see the same observations as that depicted in Fig. 12. In particular, both the average delay of ST in the relay buffer in Figs. 13-14 stop increasing and
maintain at about 45s/Hz. These phenomena confirm again that the convergence behavior of the average delay of ST for the proposed AAB protocol is constant in all considered TWR fading channels.

VII. CONCLUSION

In this research, we considered joint network coding and opportunistic transmission for TWR fading channels. Under the model of the MA and BC phases using equal time allocation and reciprocal channel gains, we presented a new TWR protocol, named as alternative awaiting and broadcast (AAB). The process flow, average exchange sum-rate and average delay of ST in the relay buffer of the proposed AAB protocol are analyzed. A new upper bound on the ESC obtained by the AAB protocol is no longer limited by the poor channel compared to the traditional upper bound without the delay of ST. That is to say, the min operations between the uplink and downlink in an identical direction are canceled. We further derive an achievable ESR and the corresponding average delay of ST for the AAB protocol based on the well-known lattice codes. Compared with the average delay of SS in the source buffer, the average delay of ST in the relay buffer induced by the proposed AAB protocol is very small and negligible. Numerical results show that 1) the proposed AAB protocol is suitable for use in almost all TWR transmission cases, 2) the proposed AAB protocol with lattice codes significantly improves the achievable ESR with an average delay of ST of only some dozen time units compared to the traditional TWR protocols without the delay of ST, 3) the proposed AAB protocol with lattice codes approaches the new upper bound on the ESC at asymptotically large SNR.

In the following research, we will propose a practical solution based on the AAB protocol through asymmetrical modulation and channel coding with pure partial decoding. The coming study is different from that presented in this paper. The well-known MPSK/QAM and LDPC codes will be combined with the AAB protocol skillfully.

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Fig. 1: System model of two-way relay fading channels.
Fig. 2: Process flow of proposed alternative awaiting and broadcast (AAB) protocol.

Fig. 3: Encoding and decoding solutions of AAB protocol with lattice codes.
Fig. 4: Convergence behavior of average delay of ST for AAB protocol.
Fig. 5: Average delay of ST versus $P/\sigma^2$ with different $\theta$ (Eq. (13)).

Fig. 6: Average delay of ST versus $\theta$ when $P/\sigma^2 = 10dB$ (Eq. (13)).
Fig. 7: $\theta$ versus $P/\sigma^2$ for AAB protocol with lattice codes (Eqs. (36) and (37)).

Fig. 8: Average delay of ST versus $P/\sigma^2$ for AAB protocol with lattice codes (Eq. (13) modified according to Subsection IV-C2.
Fig. 9: Ergodic sum-rates (ESR) versus $P/\sigma^2$ in TWR Rayleigh fading channels.

Fig. 10: Ergodic sum-rates (ESR) versus $P/\sigma^2$ in TWR Rice fading channels.
Fig. 11: Ergodic sum-rates (ESR) versus $P/\sigma^2$ in TWR Nakagami-m fading channels.

Fig. 12: Delay of SS/ST versus packet arrival rate in TWR Rayleigh fading channels.
Fig. 13: Delay of SS/ST versus packet arrival rate in TWR Rice fading channels.

Fig. 14: Delay of SS/ST versus packet arrival rate in TWR Nakagami-m fading channels.