Nucleon sigma term and in-medium quark condensate in the modified quark-meson coupling model

Xuemín Jin¹ and Manuel Malheiro²†
¹Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology Cambridge, Massachusetts 02139, USA
²Department of Physics, University of Maryland, College Park, MD 20742, USA

Abstract

We evaluate the nucleon sigma term and in-medium quark condensate in the modified quark-meson coupling model which features a density-dependent bag constant. We obtain a nucleon sigma term consistent with its empirical value, which requires a significant reduction of the bag constant in the nuclear medium similar to those found in the previous works. The resulting in-medium quark condensate at low densities agrees well with the model independent linear order result. At higher densities, the magnitude of the in-medium quark condensate tends to increase, indicating no tendency toward chiral symmetry restoration.

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†Permanent address: Instituto de Física, Universidade Federal Fluminense, 24210-340, Niterói, R. J., Brasil.
The physics of nuclear matter and finite nuclei is governed by the underlying theory of strong interactions of quarks and gluons, quantum chromodynamics (QCD). In reality, however, QCD is intractable at the nuclear physics energy scales due to the nonperturbative features of the theory and a realistic account of nuclear phenomena based entirely on it is not yet possible. At this stage, the best one could do is to build models that incorporate the symmetries of QCD and/or quark-gluon degrees of freedom, and hence to motivate connections between this theory and the observed nuclear phenomena and established phenomenology. Such models are necessarily quite crude.

The quark-meson coupling model (QMC) [1] treats nucleons in nuclear medium as non-overlapping MIT bags interacting through the self-consistent exchange of mesons in the mean-field approximation. It provides a simple and attractive framework to incorporate the quark structure of the nucleon in the study of nuclear phenomena [1–6]. Recently, the QMC has been modified by introducing a density-dependent bag constant [7,8]. It was demonstrated that a significant reduction of the bag constant in the nuclear matter relative to its free space value can lead to large and canceling isoscalar Lorentz scalar and vector potentials and hence strong spin-orbit force for the nucleon in nuclear matter which are comparable to those suggested by relativistic nuclear phenomenology [9,10] and finite-density QCD sum rules [11]. Such a large reduction of the bag constant can also account for the EMC effect within the framework of dynamical scaling [12]. (For further development and other applications, see Refs. [13,14]).

In this paper, we evaluate the nucleon $\sigma$ term and in-medium quark condensate in the modified quark-meson coupling model (MQMC) developed in Refs. [7,8]. The in-medium quark condensate plays an important role in studying hadron properties in nuclear medium [11] and has connections to many nuclear phenomena [15]. Its value at low densities is largely determined by the nucleon $\sigma$ term which also has attendant consequences in nuclear astrophysics [16]. In Refs. [3,4] both the nucleon $\sigma$ term and the in-medium quark condensate have been investigated in the QMC. There, it was found that the result for the nucleon $\sigma$ term is much smaller than its empirical value extracted from dispersion analysis of isospin even pion-nucleon scattering [17], and the prediction for the in-medium quark condensate based on this small $\sigma$ term is quite different from the model independent linear order result [18–20]. It is thus of interest to examine how the large reduction of the bag constant in the nuclear matter suggested in the MQMC will affect the nucleon $\sigma$ term and in-medium quark condensate.

We find that when the bag constant is significantly reduced in the nuclear matter, e.g. $B/B_0 \sim 35 - 40\%$ at the nuclear matter saturation density, the empirical value for the nucleon $\sigma$ term can be recovered and the resulting in-medium quark condensate at low densities is in good agreement with the model independent linear order result. At high densities, the magnitude of the in-medium quark condensate tends to increase, indicating no tendency of chiral symmetry restoration; this behavior has also been seen in other models [21–24]. Such a large reduction of the bag constant, as shown in previous works [7,8,12], is consistent with that required to recover large and canceling Lorentz scalar and vector potentials for the nucleon in the nuclear matter and to account for the EMC effect within the dynamical rescaling framework.
The details of the MQMC have been given in [7,8]. The two models for the in-medium bag constant are: scaling model and direct coupling model. The scaling model relates the in-medium bag constant to the in-medium nucleon mass:

\[
\frac{B}{B_0} = \left( \frac{M_N^*}{M_N} \right)^\kappa ,
\]

where \( \kappa \) is a real positive parameter and \( \kappa = 0 \) corresponds to the usual QMC model. The effect of this modification is summarized into a factor \( C(\sigma) \) that appears in the self-consistency condition for the \( \sigma \) field [7,8],

\[
C(\sigma) = \frac{E_{\text{bag}}}{M_N^*} \left[ \left( 1 - \frac{\Omega_q}{E_{\text{bag}} R} \right) S(\sigma) + \frac{m_q^*}{E_{\text{bag}}} \right] \left[ 1 - \frac{\kappa E_{\text{bag}}}{M_N^*} \frac{4}{3} \pi R^3 B \right]^{-1} ,
\]

where \( m_q^* = m_q - g^B_{\sigma} \sigma \), and the explicit expressions for \( M_N^* \), \( E_{\text{bag}} \), \( \Omega_q \), and \( S(\sigma) \) can be found in Refs. [7,8]. The expression for \( C(\sigma) \) in the usual QMC model is obtained from Eq. (2) with \( \kappa = 0 \). The direct coupling model features a direct coupling between the bag constant and the scalar mean field

\[
\frac{B}{B_0} = \left( 1 - \frac{g^B_{\sigma}}{\delta M_N} \sigma \right)^\delta ,
\]

where \( g^B_{\sigma} \) and \( \delta \) are real positive parameters and the introduction of \( M_N \) is based on the consideration of dimension. (The case \( \delta = 1 \) was also considered by Blunden and Miller [3].) Note that \( g^B_{\sigma} \) differs from the quark-meson coupling \( g^q_{\sigma} \) (or \( g_{\sigma} \equiv 3g^q_{\sigma} \)). When \( g^B_{\sigma} = 0 \), the usual QMC model is recovered. The factor \( C(\sigma) \), in this case, is given by

\[
C(\sigma) = \frac{E_{\text{bag}}}{M_N^*} \left[ \left( 1 - \frac{\Omega_q}{E_{\text{bag}} R} \right) S(\sigma) + \frac{m_q^*}{E_{\text{bag}}} \right] + \left( \frac{g^B_{\sigma}}{g_{\sigma}} \right) \frac{E_{\text{bag}}}{M_N^*} \frac{16}{3} \pi R^3 \frac{B}{M_N} \left[ 1 - \frac{4 g^B_{\sigma}}{\delta M_N} \right]^{-1} .
\]

The nucleon \( \sigma \) term can be expressed as

\[
\sigma_N = m_q \frac{dM_N}{dm_q} = 2m_q \langle N|\overline{q}q|N \rangle .
\]

Here we neglect isospin breaking and use \( m_u = m_d \equiv m_q = \frac{1}{2}(m_u + m_d) \). Therefore, there are two ways to evaluate \( \sigma_N \) in QCD. One is to take the derivative of \( M_N \) with respect to \( m_q \). The other is to calculate the nucleon’s scalar charge \( \langle N|\overline{q}q|N \rangle \), which can be carried out by adding a term \( S(\overline{u}u + \overline{d}d) \) (with \( S \) a constant) to the QCD Hamiltonian and then extracting the response of the nucleon mass to the external field \( S \), \( dM_N(S)/dS \big|_{S \to 0} = \langle N|\overline{u}u + \overline{d}d|N \rangle \). The QMC and MQMC provide descriptions of how the nucleon mass responds to a constant scalar field. Treating \( g^B_{\sigma} \sigma \) as a constant external field, one can show

\[
\sigma_N = m_q \frac{dM_N^*(S = -g^B_{\sigma} \sigma)}{dS} \big|_{S \to 0} = 3m_q C(\sigma = 0) .
\]

where \( C(\sigma = 0) \) is related to the response of the nucleon mass to the scalar field at \( \sigma = 0 \). The explicit expressions for \( C(\sigma) \) are in the Eqs. (3) and (4).
We follow Refs. [4] and take a value of 10 MeV for \( m_q \). We expect this value to be reasonable at the scale where the MIT bag model is useful. Note that \( C(0) \) also depends on \( m_q \). Here we use \( m_q = 0 \) in evaluating \( C(0) \). It has been found in previous studies that inclusion of a small finite \( m_q \) in evaluating \( C(0) \) only leads to a negligible refinement to the \( C(0) \) value. Figure 1 shows the resulting \( \sigma_N \) from the scaling model as a function of \( \kappa \) for \( R_0 = 0.6 \) fm. When \( \kappa = 0 \), corresponding to the usual QMC, \( \sigma_N \simeq 12 \) MeV which is almost a factor of four smaller than the empirical value of 45 MeV [17]. As \( \kappa \) increases, \( \sigma_N \) increases slowly at small \( \kappa \) values and grows rapidly at large \( \kappa \) values. For \( \kappa \simeq 2.88 \), we find \( \sigma_N \simeq 45 \) MeV. We also find that the result is largely independent of \( R_0 \) in the range \( 0.6 \) fm \( \leq R_0 \leq 1.0 \) fm. The result from the direct coupling model is illustrated in Fig. 2, where \( \sigma_N \) is plotted as a function of \( g_\sigma^B/g_\sigma \) with \( R_0 = 0.6 \) fm. We see that the dependence of \( \sigma_N \) on \( g_\sigma^B/g_\sigma \) is linear. The case \( g_\sigma^B/g_\sigma = 0 \) corresponds to the usual QMC. When \( g_\sigma^B/g_\sigma \simeq 1.1 \), the empirical value of \( \sigma_N \) can be reproduced. Our results for \( \kappa \) in the scaling model and \( g_\sigma^B/g_\sigma \) in the direct coupling model in order to obtain the empirical value of \( \sigma_N \) term depend slightly on the choice we make for \( m_q \) in the vicinity of \( m_q \sim 10 \) MeV.

Therefore, it is necessary to have \( \kappa > 0 \) and \( g_\sigma^B/g_\sigma > 0 \) or a reduction of the bag constant in the nuclear medium in order to recover the empirical value of \( \sigma_N \) from the small value predicted by the usual QMC. If one had assumed the opposite behavior, the resulting \( \sigma_N \) would be even smaller. This solidifies the MQMC model. It is the coupling of the bag constant to the scalar field featured in the MQMC that describes how the bag constant responds to the external field and provides a new source of contribution to the nucleon’s scalar charge and hence the nucleon \( \sigma \) term.

For \( \kappa \sim 2.9 \) (in the scaling model) and \( g_\sigma^B/g_\sigma \sim 1.1 \) (in the direct coupling model), \( B/B_0 \simeq 35 - 40\% \) at the saturation density [8]. It is rewarding that such a significant reduction of the bag constant in the nuclear matter relative to its free-space value coincides with that required to recover the relativistic nuclear phenomenology and account for the EMC

![Graph](image)

**FIG. 1.** Nucleon \( \sigma \) term as a function of \( \kappa \) resulting from the scaling model. The case \( \kappa = 0 \) corresponds to the usual QMC.
FIG. 2. Nucleon $\sigma$ term as a function of $g^B/\sigma$ resulting from the direct coupling model. The case $g^B/\sigma = 0$ corresponds to the usual QMC.

effect [7,8,12]. Conversely, the empirical value $\sigma_N = 45$ MeV provides an extra constraint to the MQMC. In fact, there will be no free parameter in the scaling model and only $\delta$ is left as a free parameter in the direct coupling model when $\sigma_N$ is treated as an input. Then the resulting scalar and vector potentials for the nucleon in the nuclear matter will be comparable to those suggested by the relativistic nuclear phenomenology and finite-density QCD sum rules, and the predictions for the rescaling parameter will be consistent with that required to explain the EMC effect within the dynamical rescaling approach.

We now turn to the in-medium quark condensate. Following Ref. [19], we can write the in-medium quark condensate as

$$\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{q}q \rangle_0 + \frac{1}{2} \frac{d\mathcal{E}}{dm_q} = \langle \bar{q}q \rangle_0 + \frac{1}{2} \left( \frac{\partial \mathcal{E}}{\partial M_N} \frac{dM_N^*}{dm_q} + \chi_\sigma \frac{\sigma_N}{m_q} \frac{\partial \mathcal{E}}{\partial m_\sigma} + \chi_\omega \frac{\sigma_N}{m_q} \frac{\partial \mathcal{E}}{\partial m_\omega} \right)$$

where $\mathcal{E}$ is the energy density of the nuclear medium, $\chi_\sigma \sigma_N/m_q \equiv dm_\sigma/dm_q$, $\chi_\omega \sigma_N/m_q \equiv dm_\omega/dm_q$, and $\langle \bar{q}q \rangle_{\rho_N}$ and $\langle \bar{q}q \rangle_0$ denote the quark condensates in the nuclear medium and vacuum, respectively. Here we have followed Ref. [19] and neglected the dependence of various couplings on $m_q$. Using the Gell-Mann–Oakes–Renner relation, $2m_q \langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2$, one finds [3]

$$R_\rho \equiv \frac{\langle \bar{q}q \rangle_{\rho_N}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} \left[ \frac{m_\sigma^2 \bar{\sigma}}{g_\sigma C'(0) \rho_N} + \chi_\sigma \frac{m_\sigma \bar{\sigma}^2}{\rho_N} - \chi_\omega \frac{g_\omega^2}{m_\omega^2} \rho_N \right],$$

where $m_\pi$ is the pion mass (138 MeV) and $f_\pi$ the pion decay constant (93 MeV). Here we have used $dM_N^*/dm_q = 3m_q C(\bar{\sigma})$ [3,4], which can be obtained by following the same discussion leading to Eq. (7).

To be self-consistent, we use the predictions for $\sigma_N$ from Eq. (7). (In Refs. [3,4], the empirical value of $\sigma_N$ was used in evaluating the in-medium quark condensate.) The two
FIG. 3. Ratio \( R_\rho = \langle \mathcal{T}_q \rangle / \langle \mathcal{T}_q \rangle_0 \) as a function of the medium density from the scaling model with \( R_0 = 0.6 \) fm. The \( \sigma_N \) value is predicted from Eq. (3). The solid curve represents the linear order result with \( \sigma_N = 45 \) MeV [19], and the other three curves correspond to \( \kappa = 0 \) (dashed) (usual QMC, \( \sigma_N \approx 12.6 \) MeV), 2.0 (long-dashed) (\( \sigma_N \approx 25.2 \) MeV), and 2.88 (dot-dashed) (\( \sigma_N \approx 45.0 \) MeV), respectively.

The resulting \( R_\rho \) from the direct coupling model is plotted in Fig. 4. The results are for \( \delta = 4 \) and different values of \( g^2_\sigma \). For \( g^2_\sigma \approx 1.93 \) (\( \sigma_N \approx 45 \) MeV), the prediction for \( R_\rho \) is in good agreement with the linear order result below \( 1.2 \rho_N^0 \); at \( \rho_N \sim 1.9 \rho_N^0 \), \( R_\rho \) starts to increase with density. As \( g^2_\sigma \) gets larger (smaller), the \( \sigma_N \) becomes smaller (larger) and hence \( R_\rho \) becomes larger (smaller). At higher densities, \( R_\rho \) increases with increasing density with a large (small) rate for small (large) \( g^2_\sigma \) values. We also tested the sensitivity to the \( \delta \) value. For a given \( g^2_\sigma \), the result is not sensitive to \( \delta \) value in the regime \( \rho_N < 2.5 \rho_N^0 \), and saturates at \( \delta \sim 12 \).
FIG. 4. Ratio $R_\rho = \langle \overline{q}q \rangle_{\rho N}/\langle \overline{q}q \rangle_0$ as a function of the medium density from the direct coupling model with $\delta = 4$ and $R_0 = 0.6$ fm. The $\sigma_N$ value is predicted from Eq. (6). The solid curve represents the linear order result with $\sigma_N = 45$ MeV [19], and the other three curves correspond to $g_\sigma^q = 5.309$ (dashed) (usual QMC, $\sigma_N \simeq 12.6$ MeV), 1.93 (long-dashed) ($\sigma_N \simeq 45.0$ MeV), and 1.5 (dot-dashed) ($\sigma_N \simeq 61.2$ MeV), respectively.

We observe that the ratio $R_\rho$ at low densities ($\rho_N \leq \rho_0^N$) is essentially determined by the $\sigma_N$ value and the linear order result is robust. This is consistent with that found in the usual QMC [3,4] and in hadronic models [19,21–24]. At higher densities, the nonlinear higher-order contributions become increasingly important. In particular, the last term in large parentheses in Eq. (8) becomes dominant, leading to a hindrance of chiral symmetry restoration. This behavior is also seen in hadronic models [21–24]. An exception [23,24] is the ZM model [25], which features density-dependent meson-nucleon coupling constants. However, since the parameters in the MQMC and in hadronic models are chosen to fit only the nuclear matter properties at the saturation density, the reliability of their predictions for the in-medium quark condensate at high densities is unknown. Moreover, the high-density behavior of $R_\rho$ is sensitive to $\chi_\sigma$ and $\chi_\omega$, which are not well determined [22]. The possible dependence of the couplings, $g_\sigma^q$, $g_\sigma^B$, and $g_\omega$, on $m_q$ is also neglected, which would give extra contributions to $R_\rho$.

In summary, we have evaluated the nucleon $\sigma$ term and in-medium chiral quark condensate in the modified quark-meson coupling model. The coupling of the bag constant to the scalar mean field featured in the MQMC gives rise to additional contribution to the nucleon $\sigma$ term compared to the usual QMC. This contribution can lead to the recovery of the empirical value of the nucleon $\sigma$ term when the reduction of the bag constant in the nuclear matter relative to its free space value is significant, i.e., $B/B_0 \simeq 35–40\%$ at $\rho_N = \rho_0^N$. Such a large reduction of the bag constant in the nuclear matter is consistent with that required to recover large and canceling Lorentz scalar and vector potentials for the nucleon in the nuclear matter and to account for the EMC effect within the dynamical rescaling framework.
The resulting in-medium quark condensate at low densities agrees well with the model independent linear order result; at higher densities, the magnitude of the in-medium quark condensate tends to increase, indicating no tendency toward chiral symmetry restoration.

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