The Fate of Hadron Masses in Dense Matter: Hidden Local Symmetry and Color-Flavor Locking

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The notion that hadron masses scale according to the scaling of the quark condensate in hadronic matter, partly supported by a number of observations in finite nuclei, can be interpreted in terms of Harada-Yamawaki’s “vector manifestation” (VM) of chiral symmetry. In this scenario, near chiral restoration, the vector meson masses drop to zero in the chiral limit with vanishing widths. This scenario appears to differ from the standard linear sigma model scenario. We exploit a link between the VM and color-flavor locking inferred by us from lattice data on quark number susceptibility (QNS) measured as a function of temperature to suggest that local flavor symmetry gets mapped to color gauge symmetry at the chiral phase transition.

I. EVIDENCE FROM FINITE NUCLEI

There are growing indications that as density increases in nuclear medium, “hadron masses” drop as proposed in \cite{1}. For instance, we have argued \cite{2} that the best indication comes from nuclear response functions such as the longitudinal response function in (\textit{e,e}p) process in nuclei \cite{3}, axial-charge transitions in heavy nuclei \cite{3}, nuclear gyromagnetic ratio \cite{4,5} etc. However these evidences involve virtual particles and here the “mass” that appears is a parameter of an effective field theory and need not be a physical quantity – as the pole mass is – except when radiative corrections are ignorable. Whether or not radiative corrections are important depend on the processes considered. The scaling mass for the $\rho$ meson as discussed in \cite{1} has been invoked \cite{6} to explain the CERES lepton pair data at invariant masses $\sim 300 – 500$ MeV. Although the explanation seems viable, it is not unique: there are alternative mechanisms such as increased widths in medium \cite{6}, quark-gluon plasma \cite{6} etc. that can equally well explain the global shift in the invariant mass. It is possible that a combination of observables in heavy ion processes such as lepton pair, direct photon… will weed out wrong mechanisms among the multitude that are present in the literature, see e.g. \cite{6}.

What is clear in the midst of the confusion is that the density does modify the properties of hadrons. The recent KEK experiment on the invariant mass spectra of the $e^+e^-$ pairs in the target rapidity region of 12 GeV $p + A$ reactions does show clearly that the vector mesons have modified properties in medium \cite{6}.

In this paper, we exploit the recent developments to describe how the properties of hadrons may change as the critical density for chiral phase transition is approached from below. Our arguments rely on two recent suggestions that come from seemingly unrelated sectors. One is the suggestion by Harada and Yamawaki \cite{7} that the phase transition from the Nambu-Goldstone phase to the Wigner-Weyl phase involves “vector manifestation (VM)” of chiral symmetry. The other is the proposal by Berges and Wetterich \cite{8,9} that color and flavor get completely locked even in the Nambu-Goldstone phase. The former involves low-energy effective field theory and the latter nonperturbative aspects of QCD proper.

II. HINT FROM LATTICE

It was suggested in \cite{10} that the quark number susceptibility (QNS) $\chi_{\pm} = (\partial/\partial \mu_u \pm \partial/\partial \mu_d)(\rho_u \pm \rho_d)$ where $\rho_u,d$ and $\mu_u,d$ are, respectively, $u,d$-quark number density and chemical potential measured on lattice as a function of temperature \cite{10} exhibited a smooth and rapid change-over in both isoscalar and isovector channels from a hidden flavor gauge symmetry to QCD color gauge symmetry at the chiral transition temperature $T_c$, implying an intricate connection between the induced symmetry and the fundamental symmetry. When interpreted in terms of the hidden gauge vector decoupling proposed by Kunihiro \cite{11}, the lattice results on QNS point to a link between Harada-Yamawaki’s “vector manifestation” in hidden local symmetry (HLS) (defined precisely below) and color-flavor locking (CFL) in QCD approaching chiral restoration from the Goldstone phase and offer a novel interpretation of BR scaling \cite{12}.

III. HIDDEN LOCAL SYMMETRY IN HADRON SECTOR

Consider first the hidden local symmetry (HLS) theory of Bando et al \cite{13}. We shall consider – in the chiral limit unless otherwise noted – the symmetry group $[U(2)_L \times U(2)_R]_{\text{global}} \times U(2)_V|_{\text{local}}$ consisting of a triplet
of pions, a triplet of \( \rho \)-mesons and an \( \omega \)-meson\(^1\). Motivated by the observation that in the vacuum \( T = n = 0 \) where \( T \) and \( n \) are, respectively, temperature and density, the \( \rho \) and \( \omega \) mesons are nearly degenerate and the quartet symmetry is fairly good phenomenologically and that near the chiral restoration critical temperature \( T_c \), the \( \chi_\pm \) behave identically within the error bars, we put them into a \( U(2) \) multiplet. In this theory, baryons (proton and neutron) do not appear explicitly. They can be considered as having been integrated out. If needed, they can be re-introduced in a way consistent with HLS.

The relevant degrees of freedom in the HLS theory are the left and right chiral fields denoted by \( \xi_{L,R} \) and the hidden local gauge fields denoted by \( V_\mu \equiv V_\mu^\alpha T^\alpha = \frac{\pi^\rho}{\sqrt{2}} \rho_\mu^\rho + \frac{\omega_\mu}{2} \) with \( \text{Tr}(T^\alpha T^\beta) = \frac{1}{2} \delta^{\alpha\beta} \). If we denote the \([U(2)_L \times U(2)_R]_{\text{global}} \times [U(2)_V^\alpha]_{\text{local}}\) unitary transformations by \((g_L, g_R, h)\), then the fields transform \( \xi_{L,R} \mapsto h(x) \xi_{L,R} g_R^\dagger \) and \( V_\mu \mapsto h(x)(V_\mu - i\partial_\mu)h^\dagger(x) \). This theory has the correct symmetry structure as well as dynamical contents for low-energy excitations of hadrons. Symmetry considerations alone however do not give a unique phase structure of the theory. In fact with any given parameters of the theory, it has a multitude of flow structure as one descimates down in the Wilsonian sense\(^1\). It is however by matching with QCD at the chiral scale \( \Lambda \) that the parameters of the theory and the phase structure become unique. It was shown in\(^1\) that among the multitude of the possibilities, it is the vector manifestation with a consequent strong violation of vector dominance that is uniquely picked for the chiral phase transition.

We follow\(^1\) and consider the HLS Lagrangian as an effective Lagrangian that results when high-energy degrees of freedom above the chiral scale \( \Lambda \) are integrated out. Now the scale \( \Lambda \) will in general depend on the number of flavors \( N_f \), density \( n \) or temperature \( T \) depending upon what system is being considered. This is a bare Lagrangian in the Wilsonian sense with the parameters \( g_V(\Lambda) \) which is the hidden gauge coupling constant, \( a(\Lambda) \) which signals that chiral \( U(2) \times U(2) \) symmetry is spontaneously broken by taking the value \( a \neq 1 \) and \( f_\pi(\Lambda) \) which is the pion decay constant playing the role of the order parameter for chiral symmetry with \( f_\pi = 0 \) signaling the onset of the Wigner-Yeavel phase. These parameters can be determined\(^1\) in terms of QCD condensates by matching – à la Wilson – the vector and axial-vector correlators with the ones of QCD at the chiral scale \( \Lambda \). By following renormalization group (RG) flows to low-energy scales, one can obtain low-energy parameters that can be related to those that figure in chiral perturbation theory. An important observation here is that the (assumed) equality of the vector and axial-vector correlators \( (\Pi_V = \Pi_A) \) at chiral restoration where \( \langle \bar{q}q \rangle = 0 \) implies that the HLS theory approaches the Georgi vector limit\(^2\), namely, \( g_V = 0 \) and \( a = 1 \), plus the vanishing of \( f_\pi \), which is referred to as “vector manifestation.”

The above argument is quite general and should be applicable equally to temperature, density and \( N_f \). In terms of baryon density \( n \), this implies that at the critical density \( n = n_c \), we must have

\[ g_V(\Lambda(n_c); n_c) = 0, \quad f_\pi(\Lambda(n_c); n_c) = 0, \quad a(\Lambda(n_c); n_c) = 1 \]  

(1)

where we have indicated the density dependence of the cutoff \( \Lambda \).

In HLS theory, the vector masses are given by the Higgs mechanism. In free space, it is of the form

\[ m_V \equiv m_\rho = m_\omega = \sqrt{a(m_V)g_V(m_V)f_\pi(m_V)} \]  

(2)

where the cutoff dependence is understood. Here the parameter \( a(m_V) \) etc means that it is the value at the scale \( m_V \) determined by an RG flow from the bare quantity \( a(\Lambda) \). Note that \( g(2) \) is similar, but not identical, to the KSRF relation \( m_\rho = \sqrt{2g_{\rho\pi\pi}f_\pi(0)} \). Now in medium with \( n \neq 0 \), if we assume that the \( U(2) \) symmetry continues to hold also in density as in temperature, we expect this mass formula to remain the same except that it will depend upon density,

\[ m_V^* \equiv m_\rho^* = m_\omega^* = \sqrt{a(m_V^*)g_V(m_V^*)f_\pi(m_V^*)} \]  

(3)

The density dependence is indicated by the star. As in the case of \( N_f \) discussed in\(^1\), the cutoff \( \Lambda \) will depend upon density, say, \( \Lambda^* \) understood in\(^1\).

The Harada-Yamawaki argument (or “theorem”) would imply that at \( n = n_c \) where \( \langle \bar{q}q \rangle = 0 \) and hence \( \Pi_V^*(n_c) = \Pi_A^*(n_c) \), the Georgi vector limit \( g_V = 0 \), \( a = 1 \) is reached together with \( f_\pi = 0 \). This means that

\[ m_V^*(n_c) = 0 \]  

(4)

At this point, the quartet scalars will be “de-Higgsed” from the vector mesons and form a degenerate multiplet with the triplet of massless pions with the massless vectors decoupled. This assures that the vector correlator is equal to the axial-vector correlator in the HLS sector matching with the QCD sector, i.e., the “vector manifestation” of chiral symmetry. In this scenario, dictated by the renormalization group equations, the vector meson masses drop as density increases. This is in agreement with Adami and Brown who arrived at the same conclusion in temperature using QCD sum rules\(^2\). Note that

\(^1\)The Harada-Yamawaki argument for the VM scenario is strictly valid for three massless flavors but not necessarily for two flavors. Just as the phase transition is known to be different for the two- and three-flavor QCD in both temperature and density, the two-flavor situation may well be quite different from the three-flavor scenario. The attitude we take here is that we are focusing on the non-strange sector of the three-flavor consideration.

\(^2\)In medium with \( n \neq 0 \), if we assume that the \( U(2) \) symmetry continues to hold also in density as in temperature, we expect this mass formula to remain the same except that it will depend upon density,
that the temperature-driven chiral transition also could as observed in lattice gauge calculations [23], we infer $N_{\text{QCD}}$. This means $T$ in the field [23].

Gest that one or both of them “melt” at chiral restoration

As in [13,15], we interpret the “measured” singlet and non-singlet QNS’s [14] to indicate that both the $\rho$ and $\omega$ couplings vanish or nearly vanish at the transition temperature $T_c$. This means that the $\omega NN$ coupling which is $\sim 3$ times the $\rho NN$ coupling at zero temperature become equal to the latter at near the critical temperature. This also means that the both vector mesons become massless and decoupled. Viewed from the CFL point of view, it follows from (5) and (6) that the condensates $\chi$ and $\Delta$ “melt” at that point. This is consistent with the observation by Wetterich [11] that for three-flavor QCD, the phase transition – which is both chiral and deconfining – occurs at $T_c$ with the melting of the color-octet condensate $\chi$. The transition is first-order for $N_f = 3$ in agreement with lattice calculations, so the vector meson mass does not go to zero smoothly but makes a jump from a finite value to zero. We expect however that in the case of $N_f = 2$ the transition will be second-order with the vector mass dropping to zero continuously.

Combined with the HLS and CFL results ([6], [8] and [10] – all of which are Higgsed), the lattice results invite us to set

\[ a g V f_\pi \approx \kappa g_c \chi \approx \kappa' g_c \Delta. \] (9)

Since in the case of $SU(3)_f$, the diquark condensate $\Delta$ must be zero, while the equality of the $\rho$ and $\omega$ masses is to still hold, this means that in the above expression, $\Delta$
must be replaced by $\chi$ in going from $SU(2)_f$ to $SU(3)_f$. This change may be interpreted as a phase transition \[1\]. The main observation of our note is that the vanishing of the hidden gauge coupling $g_V$ matches onto the vanishing of the condensates $\chi$ and $\Delta$ with the color gauge coupling $g_c \neq 0$.

Next, as shown in \[2\], above the chiral transition temperature $T \gtrsim T_c$, the QNS’s can be well described by perturbative gluon exchange with a small gluon coupling constant $\frac{g^2}{4\pi} \ll 1$. This implies that the flavor gauge symmetry cedes to the fundamental QCD gauge symmetry at the phase transition. We propose that Eq. \[2\] describes the relax that takes place in terms of the hidden flavor gauge coupling $g_V$ on one side and the color gauge coupling $g_c$ on the other side. Now above $T_c$, the color and flavor must unknot, with the gluons becoming massless and releasing the scalar Goldstones. The way the two condensates melt as temperature is increased is a dynamical issue that cannot be addressed within the present scheme.

At present, unlike hot matter, there is no guidance from lattice on dense matter. We shall therefore assume that the above scenario holds in density up to $n = n_c$. As suggested by Schäfer and Wilczek \[23\], one possibility is that the three-flavor color-flavor locking operative at asymptotic density continues all the way down to the “chiral transition density” ($n_c$) in which case there will be no real phase change since there will then be a one-to-one mapping – although symmetries are “twisted” – between hadrons and quark/gluons, e.g., in the sense of hadron-quark continuity. In nature however the non-negligible strange-quark mass is likely to spoil the ideal three-flavor consideration. The alternative scenario that we adopt here is that viewed from “bottom-up,” the hadronic phase with $\chi \neq 0$, $\sigma \neq 0$ and $\Delta \neq 0$ goes over to that with $\chi = \sigma = 0$ and $\Delta \neq 0$ corresponding to the two-flavor color superconducting (2csc) phase \[2\]. In this case we will preserve the mass formula \[3\] as one approaches $n_c$.

**V. BR SCALING AND LANDAU FERMI LIQUID**

In the Harada-Yamawaki scenario, at density approaching critical, the width should become narrower, decreasing like $\sim g_V^2$, with increasing density. Then the vector mesons would behave more like a quasiparticle at higher density than at lower density. This is our new interpretation of the BR scaling originally formulated with the Skyrme Lagrangian. The situation near the normal matter density is undoubtedly more complicated. Nonetheless, several cases evidencing BR scaling are discussed in a recent review \[2\]. Some are somewhat model-dependent and hence subject to objections. The most direct case is the $(e,e'p)$ response functions in nuclei \[3\] where the effect of BR scaling is more prominently exposed.

Thus far, we have argued for a link between the color-flavor-locked condensates and hidden gauge symmetry. This will constitute a major progress in the field if confirmed. Here we propose an even more remarkable connection between QCD “vacuum” properties encoded in BR scaling and many-body nuclear interactions. This comes about because nuclear matter owes its stability to a Fermi-liquid fixed point \[4\] as a consequence of which certain interesting nuclear properties turn out be calculable in terms of the Fermi-liquid fixed point parameters. Specifically, it has been shown that the Landau parameter $F_1$ – which is a component of quasiparticle interactions – can be expressed in terms of the BR scaling factor $\Phi(n) \equiv m_{\pi}^*(n)/m_{\rho}(0)$. An observable that probes this relation is the anomalous gyromagnetic ratio $\delta g_1$ in heavy nuclei which takes the form \[1\]

$$\delta g_1 = \frac{4}{9} \left[ \Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1(\pi) \right] \tau_3 \quad (10)$$

where $\tilde{F}_1(\pi)$ is the pionic contribution to $F_1$ which is completely given for any density by chiral symmetry. We should stress that \[10\] is valid only near nuclear matter density. At nuclear matter density, it takes the value $\tilde{F}_1(\pi)|_{n=n_c} = -0.153$. Note that \[10\] depends on only one parameter, $\Phi$. This parameter can be extracted either from nuclear matter saturation ($m^*_\pi$) or from Gell-Mann-Oakes-Renner formula for in-medium pion ($f_\pi^*$) or from a QCD sum rule for the $\rho$ meson ($m^*_\rho$). All give about the same value, $\Phi(n_0) \approx 0.78$. Given $\Phi$ at nuclear matter density, Eq. \[10\] makes a simple and clear-cut prediction, $\delta g_1 = 0.23 \tau_3$. This was confirmed by a measurement for proton in the Pb region \[3\], $\delta g_1^p = 0.23 \pm 0.03$. Turning the reasoning around, we could consider this a quantitative determination of the scaling factor $\Phi$ at nuclear matter density.

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