The Imaginary Starobinsky Model

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Abstract

The recent detection by the BICEP2 collaboration of a high level of tensor modes seems to exclude the Starobinsky model of inflation. In this paper we show that this conclusion can be avoided: one can embed the Starobinsky model in supergravity and identify the inflaton field with the imaginary (instead of the real) part of the chiral scalaron multiplet in its formulation. Once coupled to matter, the Starobinsky model may then become the chaotic quadratic model with shift symmetry during inflation and is in good agreement with the current data.
1 Introduction

The recent Planck results [1] have indicated that the cosmological perturbations in the Cosmic Microwave Background (CMB) radiation are nearly gaussian and of the adiabatic type. If one insists in assuming that these scalar perturbations are to be ascribed to single-field model of inflation [2], the data put severe constraints restriction on the inflationary parameters. In particular, the Planck results have strengthened the upper limits on the tensor-to-scalar ratio, $r < 0.12$ at 95% C.L., disfavouring many inflationary models. In particular, the simplest quadratic chaotic model has been excluded at about 95% C.L.

Among the inflationary models discussed by the Planck collaboration is the Starobinsky $(R + R^2)$ theory, first presented in Refs. [3] (see also Ref. [4]). The Starobinsky model is a model that leads to a quasi-de Sitter phase and it is described by the Lagrangian

$$S_S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left( R + \frac{1}{6M^2} R^2 \right), \quad (1)$$

where $M_p$ is the reduced Planck mass. This theory does not describe only the GR degrees of freedom, i.e. the helicity-2 massless graviton, but in addition it propagates a scalar degree of freedom usually called ”scalaron”. The later is hidden in the action (1) and can be revealed in the so-called linear representation, where one writes the action in the equivalent form [4]

$$S_S = \int d^4x \sqrt{-g} \left( \frac{M_p^2}{2} R + \frac{M_p}{M} R\psi - 3\psi^2 \right), \quad (2)$$

After integrating out the field $\psi$, one gets back the original theory (1). However, the action (2) is written in a Jordan frame and it can be expressed in Einstein frame after the conformal transformation

$$g_{\mu\nu} \rightarrow e^{-\sqrt{2/3}\phi/M_p} g_{\mu\nu} = \left( 1 + \frac{2\psi}{M M_p} \right)^{-1} g_{\mu\nu} \quad (3)$$

is performed. Then, we get the equivalent scalar field version of the Starobinsky model

$$S_S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu\phi \partial^\mu\phi - \frac{3}{4} M_p^2 M^2 \left( 1 - e^{-\sqrt{2/3}\phi/M_p} \right)^2 \right]. \quad (4)$$

There is a plateau in the scalar potential for large values of $\phi$ where slow-roll inflation can be realised with a quasi-de Sitter phase driven by a vacuum energy

$$V_S = \frac{3}{4} M_p^2 M^2. \quad (5)$$

The normalization of the CMB anisotropies fixes $M \approx 10^{-5} M_p$. In addition, the scalar tilt $n_S$ and tensor-to-scalar ratio $r$ turns out to be

$$n_S - 1 \approx -\frac{2}{N}, \quad r \approx \frac{12}{N^2}. \quad (6)$$
Note that $r$ has an addition $1/N$ suppression with respect to $n_S$. Although this model looks quite ad hoc at the theoretical level, it is perfect agreement with the Planck data, basically due to an additional $1/N$ suppression ($N$ being the number of e-folds till the end of inflation) of $r$ with respect to the prediction for the scalar spectral index $n_S$.

For this reason, there has been a renewed interest on the Starobinsky model, with particular emphasis on its supergravity extensions [5–11], along the lines originated in Refs. [12–14].

This positive attitude versus the Starobinsky model has dramatically changed with the recent release of the measurement of the tensor modes from large angle CMB B-mode polarization by BICEP2 [15], implying a tensor-to-scalar ratio

$$r = 0.2^{+0.07}_{-0.05}. \quad (7)$$

Putting aside the tension with the Planck data, this result (if confirmed) puts inflation on a ground which is firmer than ever. On the other side, it is in contradiction with the predictions [6] of the Starobinsky model.

The goal of this paper is to show that this is not necessarily true: the contradiction with the tensor modes data disappear if one embeds the Starobinsky model in supergravity and identifies the inflaton field with the imaginary part of the chiral multiplet in the dual formulation of the model (instead of the real part of it, as done in all the literature so far). We dub this version of the Starobinsky theory the “Imaginary Starobinsky model” and show that it basically resembles the quadratic chaotic model during inflation [16] (for recent reviews, see [17]) once the coupling to matter is considered (a necessary condition to allow reheating in the model). It is nice that just embedding the Starobinsky model into supergravity can make it in agreement with the data.

Recently in [18] it was shown that the simplest proposal in the standard supersymmetric Starobinsky model to identify the axion $b$, partner of the scalaron, with inflaton does not work and a drastic modification to the theory must be made if the $b$ field is responsible for the inflation. We show that a plausible modification can be made which naturally leads to $b$ inflation.

The paper is organized as follows. In section 2 we recall the basics of the embedding of the Starobinsky model in supergravity, as done in the literature so far. In section 3 we describe our proposal to identify the inflaton with the partner of the “scalaron” rather than the scalaron itself. Section 4 contains the main points about the imaginary Starobinsky model. Section 5 contains our conclusions.
2 The Supergravity embedding of Starobinsky model

The bosonic Starobinsky model can be embedded in $\mathcal{N} = 1$ minimal supergravity. In fact, since it is a higher curvature theory, it can be described both in old-minimal [13] as well as in new-minimal [14] $\mathcal{N} = 1$ supergravity.

2.1 Inflaton potential embedded in new-minimal supergravity

The $(R + R^2)$ gravity dual, in the (new-minimal) off-shell formulation of supergravity corresponds to a massive vector multiplet $(1, 2(1/2), 0)$ with a self-coupling function $J(C)$ where $C$ is the scalar partner of the massive vector. In this scenario, the D-term potential is $g^2 (J'(C))^2$ and the metric for the $C$-field is just $-J''(C)$ ($J''(C) < 0$). In the Starobinsky model dual to the $(R + R^2)$ supergravity we have (in $M_p = 1$ units)

$$J(C) = \frac{3}{2} [C + \ln(-C)], \quad C = -e^{\sqrt{\frac{2}{3}} \phi}.$$  \hspace{1cm} (8)

It is only when $J(C)$ is given by (8) that the model reproduce pure $(R + R^2)$ supergravity. In all other cases, the $(R + R^2)$ theory is coupled to an extra massive vector multiplet [8]. This is similar to the findings of [20] in old-minimal formulation when we depart from a superpotential $W = ST$ to a new superpotential $SF(T)$, where $S, T$ are the chiral superfields of the old minimal formulation.

This theory is also equivalent to (new-minimal) standard supergravity coupled to a linear multiplet and a gauge field which makes the linear multiplet massive [21]. The mass terms for the vector is $m_B^2 = -J''(C) B^2$ so if we choose a free vector coupled to gravity, $J''(C) = \text{const.}$ and the D-term just generates the mass term for the vector

$$V_D = \frac{g^2}{2} C^2.$$  \hspace{1cm} (9)

Note in this model the F-I term is irrelevant because of the invariance of the kinetic term under $C \rightarrow C + \xi$.

The $\alpha$-models [8] are obtained by the Kähler potential (of the complex field $z$ where $\text{Im} \, z$ has been eaten by the vector) $K = -3\alpha \ln C$ so that the canonically normalized field becomes $C = -\exp \sqrt{\frac{2}{3\alpha}} \phi$. The curvature of $SU(2, R)/U(1)$ coset is now [8]

$$R_{zz} = -\frac{2}{3\alpha}.$$  \hspace{1cm} (10)

For each $\alpha$ there are actually three models one can construct but only one, corresponding to the gauging of the parabolic isometry, gives the potential

$$V = \frac{g^2}{2} \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \phi}\right)^2.$$  \hspace{1cm} (11)
Other gaugings produce potentials with \( \sinh \sqrt{\frac{2}{3 \alpha} \phi} \) or \( \cosh \sqrt{\frac{2}{3 \alpha} \phi} \). The \( \alpha \to \infty \) model reproduce the Stuckelberg model [23], while the \( \alpha \to 0 \) model reproduce the Freedman model [24] (where the \( U(1) \) symmetry is restored and the potential just becomes a cosmological constant).

### 2.2 Inflaton potential embedded in old-minimal supergravity

The Starobinsky Lagrangian is usually embedded [13] in the “old minimal” two-derivative formulation of supergravity with the gravitational supermultiplet coupled to a pair of additional chiral multiplets, the inflaton \( T \) and the goldstino \( S \) multiplets. In the minimal universal embedding, supergravity is actually coupled to a chiral multiplet containing the inflaton and a goldstino multiplet \( X \) replacing \( S \) [25]. The latter is a constraint superfield [26] obeying

\[
X^2 = 0. \tag{12}
\]

Such a constraint superfield has been used before for inflation [27]. This constraint in fact allows to solve the scalar of the chiral multiplet in terms of the goldstino bilinear \( GG \) and \( X \) is explicitly expressed as

\[
X = \frac{GG}{2F_X} + \sqrt{2} \theta G + \theta^2 F_X. \tag{13}
\]

The supergravity Lagrangian is then written in the conformal compensator formalism [28] as

\[
\mathcal{L} = -\left[(T + \bar{T} - |X|^2)S_0 \bar{S}_0 \right]_D + \left[(MX^2 + fX + W_0)S^3_0 + h.c. \right]_F. \tag{14}
\]

By using the identity

\[
\left[(T + \bar{T})S_0 \bar{S}_0 \right]_D = \left[T \mathcal{R} S^2_0 \right]_F + h.c., \tag{15}
\]

where \( \mathcal{R} \) is the chiral supergravity multiplet, (14) can be expressed as

\[
\mathcal{L} = \left[|X|^2 S_0 \bar{S}_0 \right]_D + \left[\left(T \left(- \frac{\mathcal{R}}{S_0} + M X \right) + f X + W_0 \right)S^3_0 + h.c. \right]_F. \tag{16}
\]

Let us note that \( T \) appears in (16) as a Lagrange multiplier, and its equation of motion is simply

\[
X = \frac{1}{M} \frac{\mathcal{R}}{S_0}. \tag{17}
\]

Due to the \( X^2 = 0 \) constraint, the chiral supergravity multiplet \( \mathcal{R} \) satisfies also

\[
\mathcal{R}^2 = 0. \tag{18}
\]

This constraint can be implemented by a chiral Lagrange multiplier \( \sigma \) and the dual action to (14) turns out to be

\[4\]
\[ e^{-1} \mathcal{L} = - \left[ S_0 \overline{S}_0 - \frac{\mathcal{R} \overline{\mathcal{R}}}{M^2} \right]_D + \left[ W_0 + \xi \frac{\mathcal{R}}{S_0} S_0^3 + \sigma \mathcal{R}^2 S_0 \right]_F. \] (19)

Then, by recalling that the components of the chiral superfield \( \mathcal{R} \) are \[ \mathcal{R} = \left( \bar{u} \equiv S + iP, \gamma^{mn} \mathcal{D}_m \psi_n, -\frac{1}{2} R - \frac{1}{3} A_m^2 + i \mathcal{D}^m A_m - \frac{1}{3} u \bar{u} \right), \] (20)

where \( u \) and \( A_m \) are the auxiliary fields of “old-minimal” \( N = 1 \) supergravity and \( \psi_n \) is the gravitino field, we find that the bosonic Lagrangian of (19) is \[ \mathcal{L} = \frac{1}{2} \left( R + \frac{2}{3} A_m^2 \right) + \frac{3}{4M^2} \left( R + \frac{2}{3} A_m^2 \right)^2 + \frac{3}{M^2} (\mathcal{D}_m A^m)^2, \] (21)

which clearly describes an \((R + R^2)\) supergravity coupled to a pseudoscalar mode coming from \( \mathcal{D}_m A^m \).

We now proceed with the dual action (14) and a simple inspection of it shows that the Kähler potential and superpotential are given by

\[ K = -3 \ln \left( T + \overline{T} - XX \right) \] (22)

and

\[ W = 3\sqrt{\lambda} XT + fX + W_0, \] (23)

respectively. Although the goldstino superfield \( X \) is not dynamical as it does not contains any elementary scalar field, it contributes to the scalar potential since \( X = 0 \)

\[ F_X = e^{\frac{K}{2}} (K_X X)^{-1} \overline{W}_X. \] (24)

The scalar potential is then given by

\[ V = \frac{|MT + f|^2}{3(T + \overline{T})^2}, \] (25)

where \( M = 3\sqrt{\lambda} \) and the bosonic Lagrangian turns out to be

\[ e^{-1} \mathcal{L} = \frac{R}{2} - \frac{3}{(T + \overline{T})^2} |\partial T|^2 - \frac{|MT + f|^2}{3(T + \overline{T})^2}. \] (26)

Note that the positivity of \( V \) in (25) is due to the no-scale structure of the \( T \)-inflaton Kähler potential \[ \mathcal{K}. \]

It is standard to identify the inflaton with the real part of the complex scalar \( T \). Indeed, parametrizing the scalaron \( \text{Re}(T) \) as

\[ \text{Re} T = e^{\sqrt{2} \phi}, \] (27)

and integrating out the \( \text{Im} T \), we find that the effective bosonic theory, after appropriate shift of \( \phi \), turns out to be
This is the standard Starobinsky model in the dual theory. However, as we have already mentioned, it cannot account for the level of the gravitational waves indicated by BICEP2. As a result, one is tempting to rule out also the supersymmetric \((R + R^2)\) theory. However, unlike the non-supersymmetric case, the supersymmetric \((R + R^2)\) theory has a solution encoded in itself. This is the subject of the next section.

3 The Imaginary Supersymmetric Starobinsky Model

As we have seen above, identifying the inflaton with the real part of the \(T\)-field does not provide a large enough amount of tensor modes. However, the field \(T\) has also an imaginary part, which we have fixed before to its vacuum value \(\text{Im} \, T = 0\). In the general case of a complex \(T\),

\[
V = 3\lambda \left| T + c \right|^2 \left( T + \bar{T} \right)^2,
\]

where \(c = f/3\sqrt{\lambda}\), such that the theory is now described by the Lagrangian

\[
e^{-1} \mathcal{L} = \frac{1}{2} R - 3 \frac{|\partial T|^2}{(T + \bar{T})^2} - 3\lambda \frac{|T + c|^2}{(T + \bar{T})^2}.
\]

It is dual to the \((R + R^2)\) theory described in (21). After parametrizing the complex scalar \(T\) by two real scalar \(\phi\) and \(b\), as

\[
T = e^{\sqrt{\frac{2}{3}} \phi} + i\sqrt{\frac{2}{3}} b,
\]

we find that the effective bosonic theory turns out to be

\[
e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} e^{-2\sqrt{\frac{2}{3}} \phi} \partial_\mu b \partial^\mu b - \frac{1}{2} \lambda e^{-2\sqrt{\frac{2}{3}} \phi} b^2 - \frac{3}{4} \lambda \left( 1 - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \phi} \right)^2 .
\]

Since the field \(\phi\) is present both in the kinetic and in the mass term of the field \(b\), we can consider instead of \(b\), the new field \(\chi = e^{-\sqrt{\frac{2}{3}} \phi} b\). We consider now the initial configuration where the \(\phi\) field is close to the minimum of its potential. Its energy density is (still in Planck units) \(\Lambda \sim 1\) and it is much smaller than the one associated to the \(\chi\) field, which is \(\sim \chi^2 \gg 1\). Therefore the energy density of the \(\phi\) field is completely negligible at the beginning. However, as it has been shown in Ref. [18,19], even for an initial
large value of the $\chi$ the latter is immediately driven to oscillate around $\chi = 0$ and inflation ends almost instantaneously. The reason for this behavior is the kinetic mixing of the fields $\chi$ and $\phi$. Their kinetic terms are of the form

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \dot{\phi}^2 \left( 1 + \frac{2}{3} \chi^2 \right) + \frac{1}{2} \dot{\chi}^2 + \sqrt{\frac{2}{3}} \chi \dot{\phi}$$

and the equations of motion of the fields are

$$\frac{d}{dt} \left( \dot{\chi} + \sqrt{\frac{2}{3}} \chi \dot{\phi} \right) + 3H \left( \dot{\chi} + \sqrt{\frac{2}{3}} \chi \dot{\phi} \right) + \left( \lambda - \frac{2}{3} \dot{\phi}^2 \right) \chi - \sqrt{\frac{2}{3}} \chi \dot{\phi} = 0,$$

$$\frac{d}{dt} \left[ \dot{\phi} \left( 1 + \frac{2}{3} \chi^2 \right) + \sqrt{\frac{2}{3}} \chi \dot{\phi} \right] + 3H \left[ \dot{\phi} \left( 1 + \frac{2}{3} \chi^2 \right) + \sqrt{\frac{2}{3}} \chi \dot{\phi} \right] + \frac{3}{8} \sqrt{\frac{2}{3}} \lambda e^{-\sqrt{\frac{2}{3}} \phi} \left( 1 - \frac{1}{2} e^{-\sqrt{\frac{2}{3}} \phi} \right) = 0.$$  

(34)

For initial values $\chi \gg 1$, the field $\phi$ is pushed towards the plateau of its potential and its equation is solved by $\dot{\phi} \simeq -\sqrt{3/2} \dot{\chi}/\chi$, thus canceling the friction term of the field $\chi$ and making the latter rapidly rolling to the minimum of its potential. From this discussion we can infer that one needs to strongly stabilize the field $\phi$ with a large curvature around the minimum of its potential. This can be achieved by considering the couplings of the Starobinsky model to matter. These couplings are a necessary ingredient for the Starobinsky model in order to let the universe reheat after the end of inflation. Therefore, one should include couplings of the multiplet $T$ to matter multiplets $\Phi_i$. As explained in Ref. [34], we will consider that these couplings are induced by modifying the Kähler potential as

$$K = -3 \ln \left( T + \bar{T} - X \bar{X} + (T + \bar{T})^n F(\Phi_i) + \text{h.c.} \right) + K_m(\Phi_i, \bar{\Phi}_i).$$

(35)

Assuming that all matter scalars are stabilized at $\langle \Phi_i \rangle$ with $\langle D_i W \rangle = 0$ and $F(\langle \Phi_i \rangle) = m$, the dynamics of the $T$ and $X$ multiplets are then effectively described by the Kähler potential

$$K = -3 \ln \left( T + \bar{T} - X \bar{X} + m(T + \bar{T})^n \right).$$

(36)

Different modification of the Kähler potential have been considered in [18] and shown to stabilize the $\phi$ field during inflation triggered by the $b$ field. We also assume that the superpotential $W(T, X, \Phi_i)$ is such that it takes the standard form

$$W(T, X) = W(T, X, \langle \Phi_i \rangle) = 3\sqrt{3} XT + fX.$$  

(37)

The resulting potential then turns out to be

$$V = 3\lambda \frac{|T - f|^2}{\left[ T + \bar{T} + m(T + \bar{T})^n \right]^2}.$$  

(38)

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and the Kähler metric is

\[ K_{T\bar{T}} = 3 \frac{1 + mn(T + \bar{T})^{n-2}[(3 - n)(T + \bar{T}) + m(T + \bar{T})^n]}{[T + \bar{T} + m(T + \bar{T})^n]^2}. \] (39)

In terms of the real and imaginary parts of \( T \), the potential reads

\[ V(\phi, b) = \frac{3}{4} \lambda \left( 1 - fe^{-\gamma \phi} \right)^2 + \frac{3}{4} \gamma^2 \lambda \frac{e^{-2\gamma \phi}}{\left( 1 + 2^{n-1}m e^{(n-1)\gamma \phi} \right)^2} b^2 \] (40)

where \( \gamma = \sqrt{2/3} \), and the bosonic Lagrangian turns out to be

\[ \mathcal{L} = \frac{1}{2} R - \frac{1 + mn(2e^{\gamma \phi})^{n-2}[2(3 - n)e^{\gamma \phi} + m(2e^{\gamma \phi})^n]}{2(1 + m(2e^{\gamma \phi})^{n-1})^2} \left( \partial_\mu \phi \partial^\mu \phi + e^{-2\gamma \phi} \partial_\mu b \partial^\mu b \right) - V(\phi, b). \] (41)

Note that for \( m = 0 \), the scalars parametrize the Kähler space \( SU(1,1)/U(1) \) and (41) reduces to (32). For a generic value of \( m \), the scalar manifold is deformed such that only a \( U(1) \) isometry is preserved. In this case we find that as \( m \) tends towards \( m = -(2f)^{1-n} \), the minimum of the potential in the field \( \phi \) gets steeper and steeper when \( n \) goes to unity. This behaviour is similar with that of the model considered in [18].

Figure 1: The potential (40) in terms of the canonically normalized fields and \( n = 2 \).

To simplify the discussion, let us take the particular value

\[ m = -n^{-1}(2f)^{1-n}, \] (42)

with \( n \neq 1 \), for which there is a minimum \( \phi = \phi_0 = \ln f^{1/\gamma} \) independently of the value of \( b \). The potential is drawn in Fig. 1. The value \( n = 1 \) is excluded as it the Kähler potential (36) does not depend on \( T \). At the minimum
\[ \frac{d^2 V}{d \phi^2} \bigg|_{\phi_0} = \frac{n^2 \lambda (f^2 + \gamma^2 n b^2)}{f^2 (n - 1)^2} \]  

(43)

and the field \( \phi \) is anchored there by a large curvature, so that the potential for the imaginary part of the \( T \)-field turns out to be

\[ V_{\text{eff}}(b) = V(\phi_0, b) = \frac{3 \gamma^2 n^2 \lambda}{4 f^2 (n - 1)^2} b^2. \]  

(44)

Since

\[ K_{T \bar{T}} \bigg|_{\phi_0} = \frac{3 n}{4 f^2}, \]  

(45)

the theory at \( \phi = \phi_0 \) is described by (with \( n > 0 \) and \( n \neq 1 \))

\[ \mathcal{L} = \frac{1}{2} R - \frac{3 n \gamma^2}{4 f^2} \partial_\mu b \partial^\mu b - \frac{3 \gamma^2 n^2 \lambda}{4 f^2 (n - 1)^2} b^2. \]  

(46)

Upon redefining \( \chi = b \sqrt{n/f} \) we write the Lagrangian (46) with a canonically normalized kinetic term as

\[ \mathcal{L} = \frac{1}{2} R - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m^2 \chi^2, \]  

(47)

where (since \( \lambda = M^2/9 \))

\[ m^2 = \frac{n M^2}{9(n - 1)^2}. \]  

(48)

This is just the minimal chaotic inflation with quadratic potential. It predicts

\[ n_S - 1 \approx -\frac{2}{N} = -0.04 \left( \frac{50}{N} \right), \quad r \approx \frac{8}{N} = 0.16 \left( \frac{50}{N} \right), \quad M \approx \frac{n - 1}{\sqrt{n}} 5.1 \times 10^{13} \text{ GeV}, \]  

(49)

which is in good agreement with the BICEP2 data. It is intriguing that there is no cosmological constant once the fields are settled down to their vacuum although supersymmetry is broken. This is due to the no-scale structure of the Kähler potential in Eq. (22) which prevents the appearance of a non-zero vacuum energy, even if supersymmetry is broken [31]. Furthermore, the inflaton \( \chi \sim (T - \bar{T}) \) does not appear in the Kähler potential which exhibits the global symmetry \( T \to T + ia \) where \( a \) is a real constant. This symmetry is not shared by the superpotential [23]. However, the theory is natural in the ’t Hooft sense as in the \( \lambda = 0 \) limit, the shift symmetry is recovered. The parameter \( \lambda \) can be small, originating from a more fundamental theory where there is a small breaking of the shift symmetry. As a result, we do not expect higher dimensional operators of the form \( O_n \sim \chi^n/M_{\text{pl}}^{4+n} \) to invalidate the inflationary predictions [32,33]. However, we should keep in mind that quantum gravity is not expected to respect global symmetries so that quantum gravity effects might generate such operators.
It is interesting to see if there is a dual theory written in terms of the curvature scalar \( R \) and the vector \( A_\mu \) as in the \( m = 0 \) Starobinsky case \[25\]. In terms of \( T = \tau + i\sigma \), the theory is described by

\[
L = \frac{1}{2} R - K_{TT} \left[ (\partial_\mu \tau)^2 + (\partial_\mu \sigma)^2 \right] - \frac{3\lambda}{4} \frac{(\tau - f)^2 + \sigma^2}{\tau^2 \left[ 1 + m(2\tau)^{n-1} \right]^2},
\]

(50)

where

\[
K_{TT} = \frac{3}{4} \frac{1 + m^2 n(2\tau)^{2n-2} - mn(n - 3)(2\tau)^{n-1}}{\tau^2 \left[ 1 + m(2\tau)^{n-1} \right]^2}.
\]

(51)

By performing the conformal transformation

\[
g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu},
\]

(52)

where

\[
\Omega^2 = f^{-1} \left[ y + f + 2^{n-1} m (y + f)^n \right], \quad y = \tau - f,
\]

(53)

Eq. (50) is written as

\[
L = \frac{1}{2} \Omega^2 \left[ R + 6\Omega^{-2} (\partial_\mu \Omega)^2 - 2K_{TT} (\partial_\mu y)^2 \right] - \frac{3}{4} \lambda y^2 - \frac{3}{4} \lambda \sigma^2 + A_\mu A^\mu + \frac{A_\mu A^\mu}{4K_{TT} \Omega^2},
\]

(54)

where we have introduced an auxiliary vector \( A_\mu \) \[25\]. The equations of motion of the latter give back the kinetic term of the \( \sigma \) field. We may also integrate out \( \sigma \) to get

\[
L = \frac{1}{2} \Omega^2 \left[ R + 6\Omega^{-2} (\partial_\mu \Omega)^2 - 2K_{TT} (\partial_\mu y)^2 \right] - \frac{3}{4} \lambda y^2 + \frac{\nabla_\mu A^\mu}{3\lambda} + \frac{A_\mu A^\mu}{4K_{TT} \Omega^2}
\]

(55)

or

\[
L = \frac{1}{2} \left[ y + f + 2^{n-1} m (y + f)^n \right] R - \frac{3}{4} \lambda y^2 + \frac{\nabla_\mu A^\mu}{3\lambda} + \frac{A_\mu A^\mu}{4K_{TT} \Omega^2} - (K_{TT} - 3\Omega^{-2} \Omega_{yy}^2)(\partial_\mu y)^2.
\]

(56)

In the zero-momentum limit of the \( y \) field (i.e. during inflation) we may ignore its derivatives and we can integrate it algebraically. However, although the integration cannot explicitly be performed, we can do it perturbative in \( m \). The result is

\[
L = \frac{1}{2} \left( R + \frac{2}{3} f^2 A_\mu A^\mu \right) + \frac{1}{12 f^2 \lambda} \left( R + \frac{2}{3} f^2 A_\mu A^\mu \right)^2 + \frac{\nabla_\mu A^\mu}{3\lambda} + \frac{2^{n-2}}{3n+1 (f \lambda)^n} m \left( R + \frac{2}{3} f^2 (n^2 - 3n + 1) A_\mu A^\mu \right) \left( R + \frac{2}{3} f^2 A_\mu A^\mu + 3 f^2 \lambda \right)^n + \mathcal{O}(m^2),
\]

(57)

where one sees that on the top of the standard \((R + R^2)\) term \[25\] the leading correction has a maximum higher power of curvature \( R^{n+1} \). Thus one can see that coupling the \((R + R^2)\) Starobinsky model leads to the extended Starobinsky model where, among others, an infinite series of the scalar curvature is present.
4 Conclusions

In this paper we have reconsidered the prediction of the supersymmetric Starobinsky model of inflation. In its dual formulation, although the real part of the chiral multiplet cannot generate enough tensor modes as an inflaton, its imaginary part does if appropriate couplings to matter are introduced. While its non-supersymmetric version seems to be ruled out by the recent BICEP2 data on the amount of tensor modes, we have shown that the field space of the supersymmetric theory contains inflationary directions which are in agreement with the current data once appropriate couplings to matter are considered. The reason is that, along this imaginary direction and once the couplings to matter are considered, the model may become the chaotic single-field model with a quadratic potential.

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References

[1] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].

[2] D. H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999) hep-ph/9807278.

[3] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980); V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. 33, 549 (1981)]; A. A. Starobinsky, Sov. Astron. Lett. 9, 302 (1983).

[4] B. Whitt, Phys. Lett. B 145, 176 (1984).

[5] R. Kallosh and A. Linde, JCAP 1306, 028 (2013)

[6] W. Buchmuller, V. Domcke and K. Kamada, Phys. Lett. B 726, 467 (2013) arXiv:1306.3471 [hep-th]
[7] J. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Rev. Lett. 111, 111301 (2013) [arXiv:1305.1247 [hep-th]]; J. Ellis, D. V. Nanopoulos and K. A. Olive, JCAP 1310, 009 (2013) [arXiv:1307.3537];

[8] S. Ferrara, R. Kallosh, A. Linde and M. Porrati, Phys. Rev. D 88 (2013) 085038, [arXiv:1307.7696 [hep-th]]; S. Ferrara, R. Kallosh, A. Linde and M. Porrati, JCAP 1311, 046 (2013) [arXiv:1309.1085 [hep-th]].

[9] F. Farakos, A. Kehagias and A. Riotto, Nucl. Phys. B 876, 187 (2013) [arXiv:1307.1137];

[10] A. Kehagias, A. M. Dizgah and A. Riotto, Phys. Rev. D 89, 043527 (2014)

[11] S. V. Ketov and A. A. Starobinsky, Phys. Rev. D 83, 063512 (2011) [arXiv:1011.0240 [hep-th]]; S. V. Ketov and A. A. Starobinsky, JCAP 1208, 022 (2012) [arXiv:1203.0805 [hep-th]]; S. V. Ketov and S. Tsujikawa, Phys. Rev. D 86, 023529 (2012) [arXiv:1205.2918 [hep-th]]; S. V. Ketov and T. Terada, JHEP 1312, 040 (2013) [arXiv:1309.7494 [hep-th]].

[12] S. Ferrara, M. T. Grisaru and P. Van Nieuwenhuizen, Nucl. Phys. B 138 (1978) 430.

[13] S. Cecotti, Phys. Lett. B 190, 86 (1987).

[14] S. Cecotti, S. Ferrara, M. Porrati and S. Sabharwal, Nucl. Phys. B 306, 160 (1988).

[15] P. A. R. Ade et al. [BICEP2 Collaboration], [arXiv:1403.3985 [astro-ph.CO]].

[16] A. Linde, Phys. Lett B 129, 177(1983).

[17] R. Kallosh, [arXiv:1402.0527 [hep-th]]; R. Kallosh, [arXiv:1402.0527 [hep-th]]; A. Linde, [arXiv:1402.0526 [hep-th]].

[18] R. Kallosh, A. Linde, B. Vercnocke and W. Chemissany, [arXiv:1403.7189 [hep-th]].

[19] K. Hamaguchi, T. Moroi and T. Terada, [arXiv:1403.7521 [hep-ph]].

[20] S. Cecotti and R. Kallosh, [arXiv:1403.2932 [hep-th]].

[21] S. Cecotti, S. Ferrara and L. Girardello, Nucl. Phys. B 294 (1987) 537.

[22] S. Ferrara, P. Fre and A. S. Sorin, [arXiv:1401.1201 [hep-th]]; [arXiv:1311.5059 [hep-th]].

[23] R. Kallosh, A. Linde and D. Roest, JHEP 1311, 198 (2013) [arXiv:1311.0472 [hep-th]].

[24] D. Z. Freedman, Phys. Rev. D 15, 1173 (1977).

[25] I. Antoniadis, E. Dudas, S. Ferrara and A. Sagnotti, [arXiv:1403.3269 [hep-th]].
[26] M. Rocek, Phys. Rev. Lett. 41 (1978) 451; U. Lindstrom, M. Rocek, Phys. Rev. D 19 (1979) 2300; R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio and R. Gatto, Phys. Lett. B 220 (1989) 569; Z. Komargodski and N. Seiberg, JHEP 0909 (2009) 066 [arXiv:0907.2441 [hep-th]].

[27] L. Alvarez-Gaume, C. Gomez and R. Jimenez, Phys. Lett. B 690, 68 (2010) [arXiv:1001.0010 [hep-th]]; JCAP 1103, 027 (2011) [arXiv:1101.4948 [hep-th]].

[28] For a review see: D. Z. Freedman and A. Van Proeyen, Cambridge, UK: Cambridge Univ. Pr. (2012) 607 p.

[29] S. Ferrara, R. Kallosh and A. Van Proeyen, JHEP 1311 (2013) 134 [arXiv:1309.4052 [hep-th]].

[30] S. Ferrara, A. Kehagias and M. Porrati, Phys. Lett. B 727, 314 (2013) [arXiv:1310.0399 [hep-th]].

[31] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B 133 (1983) 61; J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B 134 (1984) 429. [arXiv:1403.3985 [astro-ph.CO]].

[32] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. 85, 3572 (2000) [hep-ph/0004243].

[33] R. Kallosh, A. Linde, K. A. Olive and T. Rube, Phys. Rev. D 84, 083519 (2011) [arXiv:1106.6025 [hep-th]].

[34] J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B 241, 406 (1984).