Introduction: A Stochastic Approach to Discounted Cash Flow

A new scientific truth does not best gain acceptance by convincing and instructing its opponents, but much more so in that its opponents gradually die out and the upcoming generation is entrusted with the truth from the beginning.

Max Planck, Autobiography

The valuation of firms is an exciting topic. It is interesting for economists engaged in either practice or theory, particularly for those in finance. Amongst practitioners, it is investment bankers and public accountants, who are regularly confronted with the question of how a firm is to be valued. The discussion about shareholder value (starting with Rappaport) indicated that one cannot tell from the numbers of traditional accounting alone whether the managers of a firm were primarily successful or did poorly. Instead, the change in the value of the firm is used in order to try and determine exactly this. That suffices for the practitioner’s interest. The reasons why academics involve themselves with questions of valuation of firms are different.

If you look more closely at how finance theoreticians use to determine the value of a firm, you quickly realize that firms are not seen by them primarily as institutions that acquire production factors and manufacture either products or services. The actual side of economic activities is not looked at in any more detail. Instead, the income which the financiers, the owners in particular, can attain is the question of interest. The ways in which firms contribute to fulfilling consumer needs are of secondary importance. What is decisive is the amount of payments and its distribution among the owners and creditors. The income a company can earn determines its value. In the end, a firm is nothing more than a risky asset, or a portfolio of assets. The valuation of firms deals with nothing else than the question as to what economic value future earnings have today. It can be principally summed up as: the more the better, the earlier the more desirable, and the more certain the more valuable.
The literature on valuation of firms recommends logical, quantitative methods, which deal with establishing today’s value of future free cash flows. In this respect, the valuation of a firm is identical with the calculation of the discounted cash flow, which is often only given by its abbreviation, DCF. There are, however, different coexistent versions that seem to compete against each other. Entity approach and equity approach are thus differentiated. Acronyms are often used, such as APV (adjusted present value) or WACC (weighted average cost of capital), whereby these two concepts are classified under the entity approach.

We see it as very important to systematically clarify the way in which these different variations of the DCF concept are related. Why are there several procedures and not just one? Do they all lead to the same result? If not, what are the economic differences? If so, for what purpose are different methods needed? And further: do the known procedures suffice? Or are there situations where none of the concepts developed up to now delivers the correct value of the firm? If so, how is the appropriate valuation formula to be found? These questions are not just interesting for theoreticians; even the practitioner who is confronted with the task of marketing his or her results has to deal with them.

When the valuation of risky assets is discussed by theoreticians, there are certain standards that get in the way of directly carrying over the results onto the problems of practical valuation of firms. Theory-based economists usually concentrate on specific details of their object of examination and leave out everything else which they consider to be less important at the moment. There have been models in which—for purposes of simplification—it is supposed that the firm to be valued will survive for exactly 1 year. Or you find models in which it is required for convenience’s sake that the firm goes on forever. But in return for that, it yields cash flows which remain the same and it does not have to pay taxes. In yet other models, it is assumed that although a very simple tax is brought to bear on the business level, the shareholders are, however, spared any taxation. It is supposed in further models that the price of an asset follows a stochastic process, which the evaluator can describe very accurately and over which he or she is methodically (mathematically) in total control. Such simplifications and specializations are part and parcel of theoretical work. They are not only usual but extremely advantageous. They are, however, not always appropriate for means of practical valuation of firms. And this is the reason why considerable efforts must be made to move in the direction away from the fundamental theoretical understanding, which derives from assumptions demanding a great deal of simplification. Instead, a move should be made towards valuation equations that are either not based on such a great deal of simplification or else based in part on far-reaching simplifications.

The article by Modigliani and Miller represents, for instance, an important basis for traditional DCF theory. Two things are characteristic for this contribution: first, a very simple corporate income tax is depicted; second, the leverage ratio of the firm is measured in market values. If you now have to value a German firm, for example, the results from Modigliani and Miller cannot simply be applied. Instead, you have to carry out appropriate adjustments. First, the German system of business taxation is somewhat more complicated. And second, it could be that the managers...
of the firm have decided in favor of a financing policy in which the leverage ratio is measured on balance sheet basis. The question must then be asked, “how are the formulas to be changed?” The theoretical literature so far offers no clear path to follow.

If the theory does not provide an answer, practitioners are left with no other choice than to ad hoc adjust the valuation equations according to their judgement so that they do justice to the present situation as far as they are convinced. For the practitioner, the theory can only be a guideline to go by anyway. They are used to taking matters into their own hands in order to make the theory at all useable.

The theoreticians are not under the same time constraints as the practitioners. They are obligated to the truth, not their mandates. This is the reason it is not allowed for them to simply change valuation equations ad hoc, which were developed under the specific circumstances of a model, if the original conditions no longer prevail. They must instead abide by the rules, which make it possible to check to see if its assertions are correct. As a first step, the new conditions are to be described in an orderly way. On this basis and that of further contradiction-free assumptions, the theoretician must try to derive the valuation equation relevant for this case with the help of logical and mathematical operations. It is only in observing these conventions that they have the right to recommend a specific valuation equation as appropriate. And it is only in observing these rules that a third party has the possibility to check whether a valuation procedure in fact deserves the predicate “suitable.” We are convinced that whoever does otherwise risks being accused of having no scientific ground to stand on.

We will attempt in this book to stick to the line of thought just described. We will thus not ad hoc draw up valuation equations, but rather call to mind the theoretical groundwork upon which these were gotten. We see no other serious alternative in this matter. Readers who take the trouble to follow along with us will indeed be rewarded with a lot of discoveries, which are at once formally precise and also economically interesting.

In closing, we want to get a little more concrete regarding the “long way round” that we have propagated and impart what we regard as particularly characteristic of our methodology. More than anything else there are four points, which differentiate our depiction of the DCF concept from that of the literature up until now:

No arbitrage Certain paradigms dominate finance theory today. These include, for instance, expected utility, the concept of perfect markets, the postulate that the markets are free of arbitrage, and the equilibrium concept, just to name a few. No reality-grounded theoretician would maintain that any of these are empirically representative. We are well aware that managers and investors do not always behave rationally. This presents the basis for an interesting development, which is today referred to as behavioral finance. The investors’ assumption of homogenous expectations is characteristic of perfect markets. It is clear to us that in reality the market participants are working with asymmetric information. Principal–agent models, which take exactly that into consideration, have made a lot of headway in finance theory in the last 40 years.
But as far as the development of valuation equations is concerned, finance theoreticians have only been successful when they have strictly followed the guidelines of the neoclassical paradigm. And this paradigm is based without ifs, ands, or buts on the assumption that there is no free lunch in the market. Although we are in reality observing arbitrageurs, theoreticians have never attempted to make these arbitrage opportunities the object of independent theories. On the contrary, the principle of no free lunch represents the indispensable cornerstone of the neoclassical paradigm. That is why all valuation equations in this book are based on this condition. We are sure that this principle has not always been consequently paid attention to by other authors regarding the issue of valuation of firms and we will make that pointedly clear at the appropriate spot.

Cost of capital Cost of capital is definitely one of the key concepts in finance. Surprisingly, there is no definition of this term to be found in the literature that is precise enough that it can be used with logical operations to get valuation equations in particular in a multiperiod context. Since we regard cost of capital as a central term, we have chosen to begin our considerations with its clarification. But then, several statements that are considered in the literature to be obviously true have to be proven by us.

Data given As far as we can perceive, the information that the evaluator has on the firm to be valued plays no systematic role within the DCF literature. But it is in fact exactly this that can decide how the evaluator is to calculate. That is why we will always very precisely describe what information is supposed to be available when we develop a valuation equation.

Uncertainty of Cash Flows We have found that when evaluating a company, many practitioners attach little or no importance to the stochastic structure of future cash flows. Practitioners are much more likely to limit themselves to estimating the expectations of cash flows. If we were to fully understand the significance of this, then valuation concepts explicitly based on the stochastic structure of payment patterns are ruled out.

We are thus concentrating on models, where the expectation of the cash flows is of central interest and extraneous information about the distribution of cash flows is left out. That will play an important role in our analysis.

The first version of our book has been published some 10 years ago. Since then we have worked continuously on the subject and added new insights to the second edition. We removed mistakes and typos, added a new chapter on transversality and were able to enrich our findings on default. New literature was also included. Some of our ideas have since then found its way into textbooks (see, among others, Fan and Yo (2017, p. 346 ff.)), although we still think that a systematic and concise approach as we prefer it is still missing.

In the first edition of the book we marked all random variables with a tilde, i.e., \( \tilde{X} \). Today, such a notation clearly looks old-fashioned, typographically complex, and in the modern literature the use of the tilde is discouraged. Notwithstanding this fact, we decided to mark random variables with a tilde and this mainly for
didactic reasons. Otherwise, it would require some time for the inexperienced reader to notice that an equation like

\[ E[X \cdot Y] = X \cdot E[Y] \]

needs several assumptions for being correct.

We have provided extensive supplementary material for those who want to use this book for teaching. You find slides and additional material on our website http://www.wacc.info/.

Reference

Fan J, Yo Q (2017) The elements of financial econometrics. Cambridge University Press, Cambridge (UK)

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