A Static Friction Model for Elastic-plastic Contacting Surfaces Using Statistically Homogenized Technique

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Abstract. An improved static friction model is developed for elastic-plastic contacting surfaces. Two random variables of asperity height and curvature for Gaussian isotropic contacting surfaces are assumed to govern asperities distribution and they satisfy a joint probability distribution function. Based on the contact parametric representation of the KE model proposed by Kogut and Etsion, the expected macroscopic values of normal stress and shear stress are obtained by using statistically homogenized technique. Then the effect of roughness, the energy of adhesion and material properties on static friction coefficients are studied and the results show similar trend with literatures. A comparison of the present model with the SV friction model proposed by Sista and Vemaganti shows that the adhesion force has a more significant effect for smoother surface and SV model is more suitable for heavily loaded contacts. The static friction coefficient is related to the ratio of hardness to elastic modulus $H/E$.

1. Introduction

Static friction force exists generally in applications of mechanical engineering and is defined as the force counteracting relative motion of two contacting surfaces[1]. According to the widely employed Coulomb friction model, the static friction force is calculated by multiplying the normal force by a constant static friction coefficient. In the past several decades, numerous experimental studies indicated that many factors would affect the static friction coefficient, including contacting materials, surface topography, the value of normal load and environmental conditions. To large extent it depends on the normal load and changes nonlinearly[2-4].

Bowden and Tabor[5] provided a scientific understanding of friction mechanics of two rough contacting surfaces. Tabor[6] summarized three important components of static friction without lubrication, namely, (1) real contact area, (2) nature and strength of junctions formed at the contact interface and (3) the way in which the material of the contact asperities is sheared and ruptured. Chang et al.[7] incorporated surface roughness and adhesion force into a comprehensive friction model for unlubricated metallic surfaces. The above three components can be easily understood from the definition of the static friction coefficient, $\mu$:

$$\mu = \frac{Q_{\text{max}}}{F} = \frac{Q_{\text{max}}}{P - F_s}$$

where $Q_{\text{max}}$ is the tangential load (the static friction force) related to the force breaking all the junctions to initiate sliding. $F$ is the normal load. In order to keep the force balance (see Figure 1), the normal load equals to the adhesion force $F_s$ subtracted from the actual contact force $P$. The actual contact
force $P$ is related to real contact area and the adhesion force $F_s$ is related to the strength of the junctions formed at the interface. Note that the adhesion effect is taken into account compared to the classical Coulomb model.

**Figure 1.** The forces acting on two rough contacting surfaces

Besides the experimental studies, many researchers have been focused on the estimation of static friction coefficient by the asperity-based model. An important asperity-based model was proposed by Greenwood and Williamson[8] (GW model). The basic assumptions in GW model are (1) the rough surface is isotropic, (2) asperities are spherical and (3) they have uniform radius, but heights vary randomly satisfying Gaussian distribution. Chang, Etsion and Bogy[7] (CEB model) used these assumptions to estimate the static friction coefficient. Based on the previous work, Kogut and Etsion [9-12] carried on much investigation. They employed the finite element analysis to simulate the elastic-plastic contact between a deformable sphere and a rigid plane and obtained a parametric expression of the actual contact load, adhesion force and shear coefficient of a single asperity. They extended their work to a static friction model named as the KE model[9]. But, both the CEB model and the KE model have limitations due to their basic assumptions. The rough surface of constant radius asperity may be unrealistic in nature.

The constant radius assumption was further relaxed by Nayak[13] and Francis[14]. They defined the rough surface by using two random variables represent asperity height and radius, which satisfying a joint probability distribution function (PDF). Tworzydlo et al.[15] used this approach to develop a new asperity-based model (TCOY model), and combined statistically homogenized technique with a nonlinear finite element analysis of surface asperities. Sista and Vemaganti[16](SV model) brought together the advantages of the TCOY model and the KE model. The SV model employed the contact parametric representation of a single asperity from the KE model and the statistically homogenized technique from the TCOY model. However, the simple adhesion model adopted in SV model, which is proposed by Derjaguin, Muller and Toporov[17-18](DMT model) is unable to describe the adhesion force in different deformation stages. The adhesion force can be neglected as the contact becomes more and more plastic. But in the elastic stage and in the early elastic-plastic stage, the adhesion force may be significant.

To resolve this issue and provide a more accurate static friction model, we integrate the SV model with an accurate general solution for elastic-plastic adhesion proposed by Kogut and Etsion[11] for parametric representation of adhesion force. Then we use the proposed model to study the static friction coefficient for metallic surfaces by varying roughness, the energy of adhesion and materials properties.

**2. Modeling and analysis of rough surfaces**

Rough surfaces are assumed as isotropic, Gaussian random distribution with asperity height and curvature as the random variables. The joint PDF of the two random variables is given by[14-16]
\begin{equation}
 f (\xi, \eta) = \frac{\sqrt{3}}{\pi \sqrt{1 - \beta^2}} \{ \eta^2 - 1 + \exp(-\eta^2) \} \\
 \quad \times \exp \left[ -\frac{1}{2(1 - \beta^2)} \{ \xi^2 - 2\beta\xi\eta + \eta^2 \} \right]
\end{equation}

in where \(\xi\) — Non-dimensional form of asperity height, 
\(\eta\) — Non-dimensional form of asperity curvature, 
\(\beta\) — Wavelength spectrum parameter. 

The above non-dimensional parameters are defined as
\begin{equation}
\xi = \frac{Z}{\sigma}, \quad \eta = \frac{\sqrt{1.5k}}{\hat{\sigma}}, \quad \beta = \sqrt{\frac{1.5\sigma^2}{\sigma\hat{\sigma}}}
\end{equation}

where \(Z\) is asperity height and \(k\) is asperity curvature, \(\sigma\), \(\hat{\sigma}\) and \(\sigma\) are standard deviations of asperity height, slopes and curvature, respectively. They can be represented by the profile spectral moments
\begin{equation}
m_0 = \sigma^2 \quad m_2 = \hat{\sigma}^2 \quad m_4 = \sigma^2
\end{equation}

Thus the non-dimensional form of asperity height, asperity curvature and wavelength spectrum parameter are given as
\begin{equation}
\xi = \frac{Z}{\sqrt{m_0}}, \quad \eta = \sqrt{\frac{1.5}{m_4}}k, \quad \beta = \sqrt{\frac{1.5m_2^2}{m_4m_4}}
\end{equation}

The asperity density can be calculated from the profile spectral moments
\begin{equation}
D_p = \frac{m_4}{6\sqrt{3\pi}m_2}
\end{equation}

In this work, the contact between two rough surfaces is modeled by a perfectly smooth rigid plane in contact with a rough surface which is called sum surface (see Figure 2). Let \(R\) and \(d\) denote the asperity radius and separation distance from rigid plane to sum surface. This equivalent method of two contacting rough surfaces has generally been used in literatures.

The interference is defined as
\begin{equation}
\omega = Z - d
\end{equation}

The interference is positive when asperity is in contact.

\textbf{Figure 2.} The equivalent model of two rough contacting surfaces

Note that the contact load \(\bar{P}\), adhesion force \(\bar{F}_S\) and the maximum static friction force \(\bar{Q}_{\text{max}}\) of a single asperity depend on its own interference. Let the separation distance \(d\) be normalized by \(\sigma\), namely \(h=d/\sigma\). Then the contact load \(P\), adhesion force \(F_S\) and the static friction force \(Q_{\text{max}}\) are functions of random variables \(\xi\) and \(\eta\). The normal load of a single asperity is given by
\begin{equation}
\bar{P} = \bar{F} + \bar{F}_S
\end{equation}

For a given separation distance, we can obtain the expected macroscopic normal stress using statistically homogenized technique[14-15]
\[ \langle \tau_N \rangle = D_F \int_{\xi=-h}^{\xi=h} \int_{\eta=-h}^{\eta=h} \tilde{F}(\xi,\eta) f(\xi,\eta) d\xi d\eta \]  
(9)

Similarly, for a given separation distance, the expected macroscopic shear stress at the inception of sliding is given by

\[ \langle \tau_s \rangle = D_F \int_{\eta=0}^{\infty} \int_{\xi=0}^{\infty} \tilde{Q}_{\text{max}}(\xi,\eta) f(\xi,\eta) d\xi d\eta \]  
(10)

Note that the adhesion force exists in non-contacting asperities. Eq. (9) can be expressed in another form

\[ \langle \tau_N \rangle = D_F \int_{\eta=0}^{\infty} \int_{\xi=0}^{\infty} \tilde{P}(\xi,\eta) f(\xi,\eta) d\xi d\eta 
- \int_{\eta=-\infty}^{\eta=0} \int_{\xi=-\infty}^{\xi=0} \tilde{F}_s(\xi,\eta) f(\xi,\eta) d\xi d\eta \]  
(11)

The representation of \( \tilde{P}(\xi,\eta) \), \( \tilde{F}_s(\xi,\eta) \) and \( Q_{\text{max}}(\xi,\eta) \) are given in next section. Eqs. (10) and (11) can be calculated by numerical integrals. Then the static friction coefficient is obtained by

\[ \mu = \frac{\langle \tau_N \rangle}{\langle \tau_s \rangle} \]  
(12)

### 3. KE Elastic-plastic Contact parameters of a Single Asperity

The evolution of the elastic-plastic contact can be divided into three distinct stages. The first one is fully elastic contact stage for \( \omega/\omega_c < 1 \). The second one is elastic-plastic contact stage which is further divided into two regime for \( 1 \leq \omega/\omega_c \leq 6 \) and \( 6 \leq \omega/\omega_c \leq 110 \). And the third one is fully plastic contact for \( 110 < \omega/\omega_c \). \( \omega \) is the interference and \( \omega_c \) is the critical interference. The parametric representation for the normal load of a single asperity is taken from the finite element analysis[12]

\[
\bar{P} = \begin{cases} 
(\frac{\omega}{\omega_c})^{1.5} & \omega/\omega_c < 1 \\
1.03(\frac{\omega}{\omega_c})^{1.425} & 1 \leq \omega/\omega_c \leq 6 \\
1.40(\frac{\omega}{\omega_c})^{1.263} & 6 \leq \omega/\omega_c \leq 110 \\
\frac{3}{K}(\frac{\omega}{\omega_c})^{1.0} & 110 < \omega/\omega_c
\end{cases}
\]  
(13)

where \( \bar{P} = (2/3)KH\pi\omega_cR \) is the critical contact load at yielding inception(\( \omega = \omega_c \)). The critical interference \( \omega_c \) is given by

\[ \omega_c = \left( \frac{\pi KH}{2E} \right)^{2} R \]  
(14)

in where

\[ K = 0.454 + 0.42v_1 \]
\[ \frac{1}{E} = \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \]  
(15)

In Eq.(15), \( E \) and \( v \) denote elastic modulus and Poisson’s ratio. Subscript 1 and 2 represent the materials of the two contacting surfaces and material 1 is the softer one. The parametric representation for the adhesion force is given by[11,17]
And Z < \omega < -0.25 0.25 are given as information stages. And the normal stress and \( S_{nc} \) are functions of the asperity radius of curvature. This indicates that asperities have different critical interference \( \omega_c \) if they have different radius of curvature. At the same interference \( \omega_c \), the sharper and the broader asperities are in different deformation stages. And the normal stress and shear stress depend on which stage of deformation the asperity is in. Therefore, it is necessary and crucial to consider the asperity curvature as a random variable.

In the implementation, Eqs. (13), (16), (17) and (18) are first transformed to the functions of the asperity height and curvature. By substituting the transformed equations into Eqs. (10) and (11), we can calculate the expected macroscopic normal stress and shear stress. The forms of the expected macroscopic normal stress and shear stress are expressed by summation of integrals. The integrals and their corresponding limits of integration represent the contribution of the asperities in elastic, elastic-plastic and fully plastic contact, respectively. Then the static friction coefficient can be easily obtained. The non-dimensional separation distance \( d \), is 0 \leq d \leq 3 based on the basic assumptions of no interaction between neighboring asperities[9]. For a given non-dimensional separation distance, the relations of the static friction coefficient and the expected macroscopic normal stress can be plotted.

4. Results and Discussion

The contact for steel on steel is chosen as an example. The material properties of steel are given as elastic modulus \( E = 197 \) GPa, hardness \( H = 7100 \) MPa, and Poisson’s ratio of \( \nu = 0.2920 \). Two different pairs corresponding to smooth and rough surfaces are selected to study the effect of surface roughness on the static friction coefficient.

Note that the critical interference \( \omega_c \), the normal load \( \bar{P} \), the adhesion force \( \bar{F}_s \) and the static friction force \( Q_{max} \) are functions of the asperity radius of curvature. This indicates that asperities have different critical interference \( \omega_c \) if they have different radius of curvature. At the same interference \( \omega_c \), the sharper and the broader asperities are in different deformation stages. And the normal stress and shear stress depend on which stage of deformation the asperity is in. Therefore, it is necessary and crucial to consider the asperity curvature as a random variable.

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Let \( Z_1 \) and \( Z_2 \) be asperity height of two rough surfaces. And \( Z_S \) defines the sum surface. If \( Z_1 \) and \( Z_2 \) are Gaussian and isotropic, \( Z_S \) will also be Gaussian and isotropic. Furthermore, when \( Z_1 \) and \( Z_2 \) are uncorrelated, the spectral moments of \( Z_S \) will be the sum of \( Z_1 \) and \( Z_2 \)[13,20], namely
\[
\begin{align*}
(m_0)_s &= (m_0)_i + (m_0)_2 \\
(m_2)_s &= (m_2)_i + (m_2)_2 \\
(m_4)_s &= (m_4)_i + (m_4)_2
\end{align*}
\]  

Then the statistical parameters \( m_0, m_2, \) and \( m_4 \) for sum surfaces can be derived. The statistical parameters for individual surface are listed in Table 1.

Table 1. Statistical parameters for rough surfaces

| Case      | \( (m_0)/(m_0)_2 \) (\( \mu \text{m} \)^2) | \( (m_2)/(m_2)_2 \) (\( \mu \text{m} \)^2) | \( (m_4)/(m_4)_2 \) (\( \mu \text{m} \)^2) |
|-----------|------------------------------------------|------------------------------------------|------------------------------------------|
| 1 smooth  | 6.25/6.25                                | 5.0E-5/5.0E-5                            | 1.2E-9/1.2E-9                            |
|           | 6.25/6.25                                | 5.0E-5/5.0E-5                            | 1.2E-9/1.2E-9                            |
| 2 rough   | 100.0/100.0                              | 2.0E-4/2.0E-4                            | 1.2E-9/1.2E-9                            |
|           | 100.0/100.0                              | 2.0E-4/2.0E-4                            | 1.2E-9/1.2E-9                            |

Figure 3. Static friction coefficient vs. normal stress

The energy of adhesion \( \Delta \gamma \) are 0.0 \( \text{J/m}^2 \), 1.0 \( \text{J/m}^2 \), 2.5 \( \text{J/m}^2 \). Figure 3 depicts the trend of static friction coefficient with respect to the normal stress for three different energy of adhesion for the roughness of case 1. And Figure 4 is for the roughness of case 2.

As illustrated in Figure 3 and Figure 4, with the increase of normal load, the separation distance decreases, and more asperities come into the stage of plastic deformation. According to the finding in reference 9, when the interference \( \omega \geq 6 \omega_k \), asperities cannot support any tangential load. The contact force increases faster than the static friction force and the adhesion force becomes negligible compared to the normal load. Hence, the static friction coefficient reduces.

It also can be found that the static friction coefficient increases with the increase of the energy of adhesion over the lower normal stress range and the effect diminishes as the normal stress increases. High energy of adhesion decreases the separation distance, and more asperities come into contacting for a given normal stress, thus can support larger tangential load, therefore, the static friction coefficient increase. On the other hand, for the high normal stress, the effect of adhesion can be neglected and the curves of different \( \Delta \gamma \) are superimposed.

Figure 3 and Figure 4 show that the static friction coefficient is lower for rougher surface. This agrees with the experimental observations[21].
Figure 5. Comparisons of two static friction models

Figure 6. Comparisons of two static friction models

Figure 5 and Figure 6 depict a comparison of two friction models when the energy of adhesion $\Delta \gamma$ takes 1.0 J/m$^2$ and 2.5 J/m$^2$ for two roughness cases. Because the adhesion is stronger for smoother contacts, the effect of case 1 is more significant than case 2 and the difference becomes larger as the decreasing normal stress. It shows clearly that the adhesion force has a more significant effect on the static friction coefficient, especially for clean surfaces in contact. Note that the DMT adhesion model is a simplified model and thus the SV friction model is more acceptable for the heavily loaded contacts.

To study the effect of material properties on static friction coefficient, we calculate the contact for aluminum on aluminum case. The statistical parameters of Table 1 are still used. The material properties of aluminum are elastic modulus $E = 68.9$ GPa, hardness $H = 545$ MPa, and Poisson’s ratio $\nu = 0.3320$. The energy of adhesion $\Delta \gamma$ takes 1.0 J/m$^2$.

Figure 7. Comparisons of static friction coefficient for two materials

As shown in Figure 7, the static friction coefficient for the contact of aluminum on aluminum is considerably lower than the contact of steel on steel. Comparing the material properties, the $H/E$ ratio of steel is 0.036, whereas it is 0.0079 for aluminum. From Eq. (14), the critical interference $\omega_c$ is related to the $H/E$ ratio. According to the definition of plasticity index $\psi = (\omega_c/\sigma) - 0.5$ proposed by Greenwood and Willamson, the lower $H/E$ ratio is corresponding to the lower critical interference $\omega_c$ and higher plasticity index $\psi$. Soft materials have high $\psi$ values and the contact is mostly plastic[19]. Therefore, at a given normal stress, the contact for aluminum includes more fully yielded asperities compared to the contact for steel. As mentioned above, the yielded asperities cannot support any tangential load at the second stage of elastic-plastic deformation and have no contribution to the static friction force. Thus the static friction coefficient for the contact for steel on steel is larger.

In order to further study the effect of $H/E$ ratio, the hardness of steel is changed to 700MPa[16], the other parameters remain the same. In Figure 8, the static friction coefficient of steel is considerably lower than that of aluminum. Comparing the material properties, the $H/E$ ratio of steel is 0.0036,
whereas it is 0.0079 for aluminum. That is to say, at the same roughness, the surfaces with larger values of $H/E$ have the higher the static friction coefficient. This conclusion agrees with the analytical results by Sista and Vemaganti [16]. In addition, the values of static friction coefficient are closely related to the statistical parameters of surfaces topography, especially they decrease with an increase in the statistical parameter $m_2$ [16]. Because the above statistical parameters are hypothetical and the order of statistical parameter $m_2$ of roughness surfaces are lower than the values given in other literatures [15, 21], the values of static friction coefficient can change largely.

5. Conclusions
An improved static friction model is developed to estimate the static friction coefficient for elastic-plastic contacting surfaces. The asperity heights and radius of curvature are assumed as the random variables. A joint PDF is adopted to describe the rough surface. Based on the parametric representation of the response of a single asperity and statistically homogenized technique, the static friction coefficient is obtained. The present friction model uses the KE adhesion model instead of the DMT adhesion model. The proposed model shows that static friction coefficient is lower for rougher surface. For a given roughness, it decreases with the increase of normal load and the energy of adhesion $\Delta \gamma$. Through the comparison of SV friction model and the present model, it shows that the smooth surface has a significantly larger adhesion effect. In the low and medium load range, the difference between two models is significant. The KE adhesion model is a more accurate description of the adhesion force compared to the DMT adhesion model, in my view, the SV friction model is more acceptable for the heavily loaded contacts.

Plasticity index $\psi$ is inversely proportional to the critical interference $\omega_c$ which is proportional to the $H/E$ ratio. Under the same roughness and normal stress, the plastic deformation occurs at larger plasticity index for material with lower the ratio $H/E$ and hence the static friction coefficient is smaller.

Conflict of Interest
None declared.

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