Symmetry Breaking  
and  
Time Variation of Gauge Couplings  

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Abstract  
Astrophysical indications that the fine structure constant has undergone a small time variation during the cosmological evolution are discussed within the framework of the standard model of the electroweak and strong interactions and of grand unification. A variation of the electromagnetic coupling constant could either be generated by a corresponding time variation of the unified coupling constant or by a time variation of the unification scale, or by both. The various possibilities, differing substantially in their implications for the variation of low energy physics parameters like the nuclear mass scale, are discussed. The case in which the variation is caused by a time variation of the unification scale is of special interest. It is supported in addition by recent hints towards a time change of the proton-electron mass ratio. Implications for the analysis of the Oklo remains and for quantum optics tests are discussed.

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The Standard Model of the electroweak and strong interactions has at least 18 parameters, which have to be adjusted in accordance with experimental observations. These include the three electroweak coupling strengths $g_1$, $g_2$, $g_3$, the scale of the electroweak symmetry breaking, given by the universal Fermi constant, the 9 Yukawa couplings of the six quarks and the three charged leptons, and the four electroweak mixing parameters. One parameter, the mass of the hypothetical scalar boson, is still undetermined. For the physics of stable matter, i.e. atomic physics, solid state physics and a large part of nuclear physics, only six constants are of importance: the mass of the electron, setting the scale of the Rydberg constant, the masses of the $u$ and $d$-quarks setting the scale of the breaking of isotopic spin, and the strong interaction coupling constant $\alpha_s$. The latter, often parametrized by the QCD scale parameter $\Lambda$, sets the scale for the nucleon mass. The mass of the strange quark can also be included since the mass term of the $s$-quarks is expected to contribute to the nucleon mass, although the exact amount of strangeness contribution to the nucleon mass is still being discussed - it can range from several tenth of MeV till more than 100 MeV. As far as macro-physical aspects are concerned, Newton's constant must be added, which sets the scale for the Planck units of energy, space and time.

Since within the Standard Model the number of free parameters cannot be reduced, and thus far theoretical speculations about theories beyond the model have not led to a well-defined framework, in view of lack of guidance by experiment, one may consider the possibility that these parameters are time and possibly also space variant on a cosmological scale. Speculations about a time-change of coupling constants have a long history, starting with early speculations about a cosmological time change of Newton's constant $G$ \cite{1–4}. Since in particular the masses of the fermions as well as the electroweak mass scale are related to the vacuum expectation values of a scalar field, time changes of these parameters are conceivable. In some theories beyond the Standard Model also the gauge coupling constants are related to expectation values of scalar fields which could be time dependent \cite{5}.

We should like to emphasize that a time or space variation of these coupling parameters would imply a violation of Poincaré invariance. The observed region of our universe seems to be isotropic and homogeneous to a high degree, suggesting that the subgroup of the Poincaré group concerning space translations remains a valid symmetry to a high degree. However the symmetry under time translations is certainly violated by the Big Bang about 14 billion years ago, and such a violation might show up in a time dependence of the fundamental constants. For this reason a time variation might be considered as more likely and larger in magnitude than a space variation, and the relevant time scale for a time variation should be the observed age of the universe of the order of 14 billion years.

We note that a time variation of a fundamental parameter like a gauge coupling constant is meant to be a variation with respect to the cosmological time, defined to be the time coordinate of a system, in which the cosmic background radiation is isotropic.
Recent observations in astrophysics concerning the atomic fine-structure of elements in distant objects suggest a time change of the fine structure constant $\alpha$. The data suggest that $\alpha$ was lower in the past, at a redshift of $z \approx 0.5 \ldots 3.5$:

$$\Delta \alpha / \alpha = (-0.72 \pm 0.18) \times 10^{-5}. \quad (1)$$

If $\alpha$ is indeed time dependent, the other two gauge coupling constants of the Standard Model are also expected to depend on time, as pointed out recently [6] (see also [7, 8]), if the Standard Model is embedded into a grand unified theory. Moreover the idea of a grand unification of the coupling constants leads to a relation between the time variation of the electromagnetic coupling constant and the QCD scale parameter $\Lambda$, implying a physical time variation of the nucleon mass, when measured in units given by an energy scale independent of QCD, like the electron mass or the Planck mass. The main assumption is that the physics responsible for a cosmic time evolution of the coupling constants takes place at energies above the unification scale. This allows to use the usual relations from grand unified theories to evolve the unified coupling constant down to low energy. Of particular interest is the relatively large time change of the proton mass in comparison to the time change of $\alpha$:

$$\frac{\dot{M}}{M} \approx \frac{\dot{\Lambda}}{\Lambda} \approx 38 \cdot \frac{\dot{\alpha}}{\alpha}. \quad (2)$$

Considering the six basic parameters mentioned above plus Newton’s constant $G$, one can in general consider seven relative time changes: $\dot{G}/G$, $\dot{\alpha}/\alpha$, $\dot{\Lambda}/\Lambda$, $\dot{m}_e/m_e$, $\dot{m}_u/m_u$, $\dot{m}_d/m_d$ and $\dot{m}_s/m_s$. Thus in principle seven different functions of time do enter the discussion. However not all of them could be measured, even not in principle. Only dimensionless ratios e.g. the ratio $\Lambda/m_e$ or the fine-structure constant could be considered as candidates for a time variation.

The time derivative of the ratio $\Lambda/m_e$ describes a possible time change of the atomic scale in comparison to the nuclear scale. In the absence of quark masses there is only one mass scale in QCD, unlike in atomic physics, where the two parameters $\alpha$ and $m_e$ enter. The parameter $\alpha$ is directly measurable by comparing the energy differences describing the atomic fine structure (of order $m_e c^2 \alpha^4$) to the Rydberg energy $\hbar c R_\infty = m_e c^2 \alpha^2 / 2 \approx 13.606$ eV.

The mass of the strange quark enters in QCD as a shift in the nuclear mass scale. Its effects and a possible time shift of $m_s$ can be absorbed in a time shift of the nucleon mass. The masses of the $u$ and $d$ quarks, however, influence the proton and neutron mass, as well as many effects in nuclear physics. The ratio $((\dot{m}_d - \dot{m}_u)/(m_d - m_u)) / (\dot{\Lambda}/\Lambda)$ is in principle measurable and can be considered to be the QCD analog of the fine-structure effects in atomic physics. Note that a determination of this ratio, which, for example could be seen by monitoring the ratio $(M_n - M_p)/M_p$ in time, would only give information of the relative change of the quark masses in comparison to $\Lambda$. It might imply that $\Lambda$ is changing, or the quark masses in comparison to $\Lambda$, or both.
The ratio \((m_d + m_u)/\Lambda\) is given by the magnitude of the \(\sigma\)-term, which leads to a small shift of the nucleon mass of about 40 MeV \([1]\). If \(\Lambda\) stays invariant, but the quark masses change, the effect could be seen by a time variation of the ratio \(M_p/m_e\). Thus a discovery of a time variation of this ratio would not necessarily imply that \(\Lambda\) would change in time.

Both astrophysics experiments as well as high precision experiments in atomic physics in the laboratory could in the future give indications about a time variation of three dimensionless quantities: \(\alpha\), \(M_p/m_e\) and \((M_n - M_p)/m_e\). The time variation of \(\alpha\) reported in \([6]\) implies, assuming a simple linear extrapolation, a relative rate of change per year of about \(1.0 \times 10^{-15}/\text{yr}\). This poses a problem with respect to the limit given by an analysis of the remains of the naturally occurring nuclear reactor at Oklo in Gabon (Africa), which was active close to 2 billion years ago. One finds a limit of \(\dot{\alpha}/\alpha = (-0.2 \pm 0.8) \times 10^{-17}/\text{yr}\). This limit was derived in \([12]\) under the assumption that other parameters, especially those related to the nuclear physics, did not change during the last 2 billion years. It was recently pointed out \([7, 10]\), that this limit must be reconsidered if a time change of nuclear physics parameters is taken into account. In particular it could be that the effects of a time change of \(\alpha\) are compensated by a time change of the nuclear scale parameter. For this reason we study in this paper several scenarios for time changes of the QCD scale, depending on different assumptions about the primary origin of the time variation.

Without a specific theoretical framework for the physics beyond the Standard Model the relative time changes of the three dimensionless numbers mentioned above are unrelated. We shall incorporate the idea of grand unification and assume for simplicity the simplest model of this kind, consistent with present observations, the minimal extension of the supersymmetric version of the Standard Model (MSSM), based on the gauge group \(SU(5)\). In this model the three coupling constants of the Standard Model converge at high energies at the scale \(\Lambda_G\). In particular the QCD scale \(\Lambda\) and the fine structure constant \(\alpha\) are related to each other. In the model there are besides the electron mass and the quark masses three further scales entering, the scale for the breaking of the electroweak symmetry \(\Lambda_w\), the scale of the onset of supersymmetry \(\Lambda_s\) and the scale \(\Lambda_G\) where the grand unification sets in.

If a change of \(\alpha\) is observed, it would imply necessarily, that at least one mass scale is changing as well. The magnitudes of the three gauge coupling constants are in particular given by the value of the unified coupling constant at the scale \(\Lambda_G\). Variations of the coupling constants at low energies will result if this coupling constant changes in time, or if the unification scale \(\Lambda_G\) changes with respect to the other scales, e.g. the electron mass, or both.

According to the renormalization group equations, considered here in lowest order,
the behaviour of the coupling constants changes according to

\[ \alpha_i(\mu)^{-1} = \left( \frac{1}{\alpha_0^i(\Lambda_G)} + \frac{1}{2\pi} b_i^{SM} \ln \left( \frac{\Lambda_G}{\mu} \right) \right) \theta(\mu - \Lambda_S) \]

\[ + \left( \frac{1}{\alpha_0^i(\Lambda_S)} + \frac{1}{2\pi} b_i^{SM} \ln \left( \frac{\Lambda_S}{\mu} \right) \right) \theta(\Lambda_S - \mu), \]

where the parameters \( b_i \) are given by \( b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7) \) below the supersymmetric scale and by \( b_i^{S} = (b_1^{S}, b_2^{S}, b_3^{S}) = (33/5, 1, -3) \) when \( N = 1 \) supersymmetry is restored, and where

\[ \frac{1}{\alpha_0^i(\Lambda_S)} = \frac{1}{\alpha_0^i(M_Z)} + \frac{1}{2\pi} b_i^{SM} \ln \left( \frac{M_Z}{\Lambda_S} \right) \]

where \( M_Z \) is the Z-boson mass and \( \alpha_0^i(M_Z) \) is the value of the coupling constant under consideration measured at \( M_Z \). We use the following definitions for the coupling constants:

\[ \alpha_1 = 5/3g_1^2/(4\pi) = 5\alpha/(3\cos^2(\theta)_{\text{MS}}) \]
\[ \alpha_2 = g_2^2/(4\pi) = \alpha/\sin^2(\theta)_{\text{MS}} \]
\[ \alpha_s = g_3^2/(4\pi). \]

Assuming \( \alpha_u = \alpha_u(t) \) and \( \Lambda_G = \Lambda_G(t) \), one finds:

\[ \frac{1}{\alpha_i} \frac{\dot{\alpha}_i}{\alpha_i} = \left[ \frac{1}{\alpha_u} \frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i^{S} \dot{\Lambda}_G}{2\pi \Lambda_G} \right] \]

which leads to

\[ \frac{1}{\alpha_i} \frac{\dot{\alpha}_i}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \frac{\dot{\alpha}_s}{\alpha_s} - \frac{1}{2\pi} \left( b_2^{S} + \frac{5}{3} b_1^{S} - \frac{8}{3} b_3^{S} \right) \frac{\dot{\Lambda}_G}{\Lambda_G}. \]

One may consider the following scenarios:

1) \( \Lambda_G \) invariant, \( \alpha_u = \alpha_u(t) \). This is the case considered in [4] (see also [5]), and one finds

\[ \frac{1}{\alpha_i} \frac{\dot{\alpha}_i}{\alpha} = \frac{8}{3} \frac{1}{\alpha_s} \frac{\dot{\alpha}_s}{\alpha_s} \]

and

\[ \frac{\dot{\Lambda}}{\Lambda} = -\frac{3}{8} \frac{2\pi}{b_3^{SM}} \frac{1}{\alpha} \frac{\dot{\alpha}_i}{\alpha_i}. \]
2) $\alpha_u$ invariant, $\Lambda_G = \Lambda_G(t)$. One finds
\[ \frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi} \left( b_2^S + \frac{5}{3} b_1^S \right) \frac{\dot{\Lambda_G}}{\Lambda_G} \tag{10} \]
with
\[ \Lambda_G = \Lambda_S \left[ \frac{\Lambda}{\Lambda_S} \exp \left( -\frac{2\pi}{b_3^{SM}} \frac{1}{\alpha_u} \right) \right] \left( \frac{b_3^{SM}}{b_3^S} \right) \tag{11} \]
which follows from the extraction of the Landau pole using (3). One obtains
\[ \frac{\dot{\Lambda}}{\Lambda} = \frac{b_3^S}{b_3^{SM}} \left[ \frac{-2\pi}{b_2^S + \frac{5}{3} b_1^S} \right] \frac{1}{\alpha_u} \dot{\alpha} \approx -30.8 \frac{\dot{\alpha}}{\alpha} \tag{12} \]

3) $\alpha_u = \alpha_u(t)$ and $\Lambda_G = \Lambda_G(t)$. One has
\[ \frac{\dot{\Lambda}}{\Lambda} = -\frac{2\pi}{b_3^{SM}} \frac{1}{\alpha_u} \frac{\dot{\alpha}}{\alpha_u} + \frac{b_3^S}{b_3^{SM}} \frac{\dot{\Lambda_G}}{\Lambda_G} \tag{13} \]
\[ = -\frac{3}{8} \frac{2\pi}{b_3^{SM}} \frac{1}{\alpha_u} \frac{\dot{\alpha}}{\alpha_u} - \frac{3}{8} \frac{1}{b_3^{SM}} \left( b_2^S + \frac{5}{3} b_1^S + \frac{8}{3} b_3^S \right) \frac{\dot{\Lambda_G}}{\Lambda_G} = 46 \frac{\dot{\alpha}}{\alpha} + 1.07 \frac{\dot{\Lambda_G}}{\Lambda_G} \]
where theoretical uncertainties in the factor $R = (\dot{\Lambda}/\Lambda)/(\dot{\alpha}/\alpha) = 46$ have been discussed in [7]. The actual value of this factor is sensitive to the inclusion of the quark masses and the associated thresholds, just like in the determination of $\Lambda$. Furthermore higher order terms in the QCD evolution of $\alpha_s$ will play a role. In ref. [7] it was estimated: $R = 38 \pm 6$.

4) In a grand unified theory, the GUT scale and the unified coupling constant may be related to each other via the Planck scale e.g.
\[ \frac{1}{\alpha_u} = \frac{1}{\alpha_{Pl}} + \frac{b_G}{2\pi} \ln \left( \frac{\Lambda_{Pl}}{\Lambda_G} \right) \tag{14} \]
where $\Lambda_{Pl}$ is the Planck scale, $\alpha_{Pl}$ the value of the GUT group coupling constant at the Planck scale and $b_G$ depends on the GUT group under consideration. This leads to
\[ \frac{\dot{\Lambda_G}}{\Lambda_G} = \frac{2\pi}{b_G} \frac{1}{\alpha_u} \frac{\dot{\alpha_u}}{\alpha_u} \tag{15} \]
and thus to
\[ \frac{\dot{\Lambda}}{\Lambda} = \frac{2\pi}{-b_3^{SM}} \frac{1 - \frac{b_3^S}{b_G}}{8 - \frac{b_2^S + b_1^S}{b_G}} \frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} \tag{16} \]
\[
\frac{\dot{\alpha}}{\alpha} = \frac{-b_S^{SM}}{2\pi} \left( \frac{8}{3} - \frac{b_2^S + \frac{5}{3}b_1^S}{b_G} \right) \frac{b_G - b_3^G}{\Lambda} \dot{\Lambda}. \tag{17}
\]

Finally, it should be mentioned that the scale of supersymmetry could also vary with time. One obtains:

\[
\frac{1}{\alpha_i} \frac{\dot{\alpha}_i}{\alpha_i} = \left[ \frac{1}{\alpha_u \alpha_u} - \frac{b_i^S \dot{\Lambda}_G}{2\pi \Lambda_G} \right] + \frac{1}{2\pi} (b_i^S - b_i^{SM}) \frac{\dot{\Lambda}_S}{\Lambda_S} \theta (\Lambda_S - \mu). \tag{18}
\]

However without a specific model for supersymmetry breaking relating the supersymmetry breaking scale to e.g. the GUT scale, this expression is not very useful.

One should also mention that in principle all the other parameters, i.e. vacuum expectation values of Higgs fields, Yukawa couplings, Higgs bosons masses may have a time dependence.

The case in which the time variation of \( \alpha \) is not related to a time variation of the unified coupling constant, but rather to a time variation of the unification scale, is of particular interest. Unified theories, in which the Standard Model arises as a low energy approximation, might well provide a numerical value for the unified coupling constant, but allow for a smooth time variation of the unification scale, related in specific models to vacuum expectation values of scalar fields. Since the universe expands, one might expect a decrease of the unification scale due to a dilution of the scalar field. A lowering of \( \Lambda_G \) implies according to (10)

\[
\frac{\dot{\alpha}}{\alpha} = -\frac{1}{2\pi} \alpha \left( \frac{b_2^S}{b_G} + \frac{5}{3}b_1^S \right) \frac{\dot{\Lambda}_G}{\Lambda_G} = -0.014 \frac{\dot{\Lambda}_G}{\Lambda_G}. \tag{19}
\]

If \( \dot{\Lambda}_G/\Lambda_G \) is negative, \( \dot{\alpha}/\alpha \) increases in time, consistent with the experimental observation. Taking \( \Delta \alpha/\alpha = -0.72 \times 10^{-5} \), we would conclude \( \Delta \Lambda_G/\Lambda_G = 5.1 \times 10^{-4} \), i.e. the scale of grand unification about 8 billion years ago was about \( 8.3 \times 10^{12} \) GeV higher than today. If the rate of change is extrapolated linearly, \( \Lambda_G \) is decreasing at a rate \( \frac{\dot{\Lambda}_G}{\Lambda_G} = -7 \times 10^{-14}/\text{yr} \).

According to (12) the relative changes of \( \Lambda \) and \( \alpha \) are opposite in sign. While \( \alpha \) is increasing with a rate of \( 1.0 \times 10^{-15}/\text{yr} \), \( \Lambda \) and the nucleon mass is decreasing, \( \Lambda \) and the nucleon mass are decreasing, e.g. with a rate of \( 1.9 \times 10^{-14}/\text{yr} \). The magnetic moments of the proton \( \mu_p \) as well of nuclei would increase according to

\[
\frac{\dot{\mu}_p}{\mu_p} = 30.8 \frac{\dot{\alpha}}{\alpha} \approx 3.1 \times 10^{-14}/\text{yr}. \tag{20}
\]

The effect can be seen by monitoring the ratio \( \mu = M_p/m_e \). Measuring the vibrational lines of H\(_2\), a small effect was seen \[13\] recently. The data allow two different interpretations:
a) $\Delta \mu/\mu = (5.7 \pm 3.8) \times 10^{-5}$

b) $\Delta \mu/\mu = (12.5 \pm 4.5) \times 10^{-5}.$

The interpretation b) agrees with the expectation based on (12):

$$\frac{\Delta \mu}{\mu} = 22 \times 10^{-5}. \quad (21)$$

It is interesting that the data suggest that $\mu$ is indeed decreasing, while $\alpha$ seems to increase. If confirmed, this would be a strong indication that the time variation of $\alpha$ at low energies is caused by a time variation of the unification scale.

Finally we should like to mention the case of unification based on $SU(5)$ or $SO(10)$ without supersymmetry. Although in this case no unification is achieved, based on the experimental data, one may discuss a unification of the electromagnetic and the strong interactions at a scale of $1.0 \times 10^{13}$ GeV. Varying this scale, one finds:

$$\frac{\dot{\Lambda}}{\Lambda} = \left[ \frac{-2\pi}{b_2^2 - \frac{2}{3} b_3^2} \right] \frac{1}{\alpha} \frac{\dot{\alpha}}{\alpha} = -234.8 \frac{\dot{\alpha}}{\alpha}. \quad (22)$$

This shows that the case with and without supersymmetry differ from each other by about a factor 7.6, as far as the relative time changes are concerned.

The fact that we find opposite signs for the time changes of $\alpha$ and $\Lambda$ is interesting with respect to the limit on the time change of $\alpha$ deduced from the Oklo reactors remains. The Oklo constraint comes from the fact that the neutron capture cross section for thermal neutrons off Samarium 149 is dominated by a nuclear resonance just above threshold. According to the analysis given in [12] the position of the resonance could not have changed much during the last 2 billion years. This gives a constraint for a time change of alpha, given above. Due to the Coulomb repulsion in the nucleus an increase of $\alpha$ would increase the energy of the resonance. However a corresponding decrease of $\alpha_s$ would have the opposite effect. Thus in any case the Oklo constraint will be less restrictive, and there might even be a nearly complete cancellation of the effect. A more detailed analysis would be beyond the scope of this paper and will be discussed elsewhere. We emphasize, however, that a partial or complete cancellation would not be in sight, if both time changes have the same sign, as e.g. in eq. (8).

The time variation of the ratio $M_p/m_e$ and $\alpha$ discussed here are such that they could by discovered by precise measurements in quantum optics. The wave length of the light emitted in hyperfine transitions, e.g. the ones used in the cesium clocks being proportional to $\alpha^4 m_e/\Lambda$ will vary in time like

$$\frac{\dot{\lambda}_{hf}}{\lambda_{hf}} = 4 \frac{\dot{\alpha}}{\alpha} - \frac{\dot{\Lambda}}{\Lambda} \approx 3.5 \times 10^{-14}/\text{yr} \quad (23)$$
taking $\dot{\alpha}/\alpha \approx 1.0 \times 10^{-15}/\text{yr}$ [1]. The wavelength of the light emitted in atomic transitions varies like $\alpha^{-2}$:

$$\frac{\dot{\lambda}_{at}}{\lambda_{at}} = -2\frac{\dot{\alpha}}{\alpha}.$$  \hfill (24)

One has $\dot{\lambda}_{at}/\lambda_{at} \approx -2.0 \times 10^{-15}/\text{yr}$. A comparison gives:

$$\frac{\dot{\lambda}_{hf}/\lambda_{hf}}{\lambda_{at}/\lambda_{at}} = -\frac{4\dot{\alpha}/\alpha - \dot{\Lambda}/\Lambda}{2\dot{\alpha}/\alpha} \approx -17.4.$$  \hfill (25)

At present the time unit second is defined as the duration of 6.192.631.770 cycles of microwave light emitted or absorbed by the hyperfine transmission of cesium-133 atoms. If $\Lambda$ indeed changes, as described in (12), it would imply that the time flow measured by the cesium clocks does not fully correspond with the time flow defined by atomic transitions.

It remains to be seen whether the effects discussed in this paper can soon be observed in astrophysics or in quantum optics. A determination of the double ratio $(\dot{\Lambda}/\Lambda)/(\dot{\alpha}/\alpha) = R$ would be of crucial importance, both in sign and in magnitude. If one finds the ratio to be about $-20$, it would be considered as a strong indication of a unification of the strong and electroweak interactions based on a supersymmetric extension of the Standard Model. In any case the numerical value of $R$ would be of high interest towards a better theoretical understanding of time variation and unification.

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