Sea quark QED effects and twisted mass fermions

R. Frezzotti, G.C. Rossi, N. Tantalo

Physics Dept. - University of Roma Tor Vergata
INFN - Sezione di Roma Tor Vergata – Roma, Italy

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Outline of the talk

1. Motivation & Introduction
   - About twisted mass sea fermions coupled to photons

2. EM & strong $U_A(1)$ anomalies
   - QCD+QED with no $\theta$ term at maximal twist

3. Maximal twist by symmetry recovery
   - How to fix the critical mass beyond electro-quenching

4. Strategy for leading isospin breaking (LIB) effects
   - Mixed action and RM123 insertion method
Motivation & Introduction

Q(C+E)D with maximal twisted mass (MTM) fermions: $\psi^t = (u, d)$

$$S_{F}^{Q(C+E)D}(\psi, \bar{\psi}, U, E) = a^4 \sum_{x} \bar{\psi}(x) \left[ \gamma \cdot \tilde{\nabla} - i \gamma_5 \tau_3 W_{cr} + \mu + \tau_3 \epsilon \right] \psi(x)$$

- $\gamma \cdot \tilde{\nabla} \equiv \frac{1}{2} \sum_{\mu} \gamma_{\mu}(\nabla^{*}_{\mu} + \nabla_{\mu})$, $W_{cr} \equiv -\frac{a}{2} \sum_{\mu} \nabla^{*}_{\mu} \nabla_{\mu} + M_{cr}$

- $\nabla_{\mu} \psi(x) \equiv \frac{1}{a} \left[ U_{\mu}(x) \psi(x + a \hat{\mu}) - \psi(x) \right]$

- $\nabla^{*}_{\mu} \psi(x) \equiv \frac{1}{a} \left[ \psi(x) - U_{\mu}^{\dagger}(x - a \hat{\mu}) \psi(x - a \hat{\mu}) \right]$

$$U_{\mu}(x) = E_{\mu}(x) U_{\mu}(x), \quad E_{\mu}(x) = \frac{\frac{\gamma 1 + \gamma \tau_3}{2}}{2} e^{-ieq_{up} A_{\mu}(x)} + \frac{\frac{\gamma 1 - \gamma \tau_3}{2}}{2} e^{-ieq_{dn} A_{\mu}(x)}$$

$q_{up} \neq q_{dn} \Rightarrow$ flavour structure $(\gamma 1 \pm \gamma \tau_3)/2$ in both $\bar{\psi} \gamma \cdot \tilde{\nabla} \psi$ and $\bar{\psi} W_{cr} \psi$

**Note 1** - Mass splitting with $\tau_3$ and not $\tau_1$ forced by gauge invariance

**Note 2** - $Q$ of an up-down doublet has the form $e(\gamma 1/6 + \gamma \tau_3/2)$
We want to analyze how EM and strong $U_A(1)$ anomalies affect

- the form of the (continuum fermionic) effective action, $\Gamma_{\text{F}}^{Q(\text{C+E})D}$
- its dependence on the vacuum $\theta$-angle

We start from the general action (above we had $\theta_u = -\theta_d = \frac{\pi}{2}$)

$$S_{\text{Q}(\text{C+E})D}(\psi, \bar{\psi}, U, E) = a^4 \sum_x \left[ FF|_E(x) + \text{tr}(GG)|_U(x) \right] +$$

$$+ a^4 \sum_x \bar{\psi}_u(x) \left[ \gamma \cdot \nabla u + e^{-i\theta_u \gamma_5} W_{\text{cr}}^u + M_u \right] \psi_u(x) +$$

$$+ a^4 \sum_x \bar{\psi}_d(x) \left[ \gamma \cdot \nabla d + e^{-i\theta_d \gamma_5} W_{\text{cr}}^d + M_d \right] \psi_d(x)$$

Phases can be moved to masses through singlet axial lattice rotations

$$\psi_u = e^{i\gamma_5 \theta_u/2} \chi_u \quad \bar{\psi}_u = \bar{\chi}_u e^{i\gamma_5 \theta_u/2}$$

$$\psi_d = e^{i\gamma_5 \theta_d/2} \chi_d \quad \bar{\psi}_d = \bar{\chi}_d e^{i\gamma_5 \theta_d/2}$$

Lattice rotations are not anomalous

$S_{\text{F}}^{Q(\text{C+E})D}$ is periodic in $\theta_u$ and $\theta_d$
The action becomes (recall $M_{u,d} > 0$, Wilson terms $W_{cr}^{u,d}$ are critical)

$$S^{Q(C+E)D}(\chi, \bar{\chi}, U, E) = a^4 \sum_x \left[ FF|_E(x) + \text{tr}(GG)|_U(x) \right] +$$

$$+ a^4 \sum_x \bar{\chi}_u(x) \left[ \gamma \cdot \tilde{\nabla}^u + W_u^{cr} + e^{i\theta_u \gamma_5} M_u^\mu \right] \chi_u(x) +$$

$$+ a^4 \sum_x \bar{\chi}_d(x) \left[ \gamma \cdot \tilde{\nabla}^d + W_d^{cr} + e^{i\theta_d \gamma_5} M_d^\mu \right] \chi_d(x)$$

Note: $M_{u,d} \cos \theta_{u,d} \equiv m_{u,d} \sim \frac{m_{u,d}^{\text{ren}}}{Z_{m_{u,d}}}$, $M_{u,d} \sin \theta_{u,d} \equiv \mu_{u,d} = \frac{\mu_{u,d}^{\text{ren}}}{Z_{\mu_{u,d}}}$

* Actually $m_{u,d}^{\text{ren}} = Z_{m_{u,d}} [m_{u,d}(1 + \rho_{m,d}^{u,d}) + m_d u \rho_{m,d}^{d,u}]$ — see e.g. Horkel & Sharpe, Phys.Rev. D92 (2015) 7, 074501

Symmetries (see below) $\Rightarrow$ continuum local effective action reads

$$\left[\#\right] \quad S_{cont}^{Q(C+E)D} = \int d^4 x \left\{ FF|_A(x) + \text{tr}(GG)|_G(x) \right\} +$$

$$+ \int d^4 x \left\{ \bar{\chi}_u(x) \left[ \gamma \cdot D^u + e^{i\hat{\theta}_u \gamma_5} \hat{M}_u \right] \chi_u(x) + \bar{\chi}_d(x) \left[ \gamma \cdot D^d + e^{i\hat{\theta}_d \gamma_5} \hat{M}_d \right] \chi_d(x) \right\}$$

$$\hat{M}_{u,d} = Z_{S_{u,d}}^{cont} \sqrt{(m_{u,d}^{\text{ren}})^2 + (\mu_{u,d}^{\text{ren}})^2} \quad \hat{\theta}_{u,d} = \arctan \left( \frac{Z_{\mu_{u,d}}}{Z_{m_{u,d}} \tan \theta_{u,d}} \right)$$
Continuum $U_A(1)$-rotations and anomaly

Recall $S_Q^{(C+E)D}_{F, cont} = \int d^4 x \bar{\chi}(x) \left[ \gamma \cdot D + e^{i\alpha \gamma_5 \mu} \right] \chi(x) =$

$$= \int d^4 x \bar{\chi}(x) e^{i\gamma_5 \frac{x}{2}} \left[ \gamma \cdot D + \mu \right] e^{i\gamma_5 \frac{x}{2}} \chi(x) \implies$$

$$\implies S_Q^{(C+E)D}_{F, cont} = \int d^4 x \bar{\psi}(x) \left[ \gamma \cdot D + \mu \right] \psi(x) +$$

$$+ i \frac{\alpha}{32\pi^2} \int d^4 x \left[ e^2 \tilde{F} F|_A(x) + g^2 \text{tr} [\tilde{G} G|_G(x)] \right]$$

- Continuum theory property above implies in our case

$$S_{cont}^{(C+E)D} = \int d^4 x \left\{ FF|_A(x) + \text{tr} (GG)|_G(x) \right\} +$$

$$= \int d^4 x \bar{\psi}_u(x) \left[ \gamma \cdot D^u + \hat{M}_u \right] \psi_u(x) + \int d^4 x \bar{\psi}_d(x) \left[ \gamma \cdot D^d + \hat{M}_d \right] \psi_d(x) +$$

$$+ i \frac{e^2}{32\pi^2} (\hat{\theta}_u + \hat{\theta}_d) \int d^4 x \tilde{F} F(x) + i \frac{g^2}{32\pi^2} (\hat{\theta}_u + \hat{\theta}_d) \int d^4 x \text{tr} [\tilde{G} G(x)]$$

- Note: $\theta_u = -\theta_d = \frac{\pi}{2}$ (MTM) $\iff \hat{\theta}_u = -\hat{\theta}_d = \frac{\pi}{2}$ (no P–breaking)
A few observations

1. $\hat{M}_u = \hat{M}_d = 0 \rightarrow$ only the trivial topological sector contributes
$\hat{\theta}_u + \hat{\theta}_d = 0 \rightarrow$ all sectors contribute with equal weight

2. No CP-breaking term if $\hat{\theta}_u = -\hat{\theta}_d$. Indeed, one could set to zero both mass phases with a non-anomalous axial-$\tau_3$ rotation

3. MTM case ($\theta_u = -\theta_d = \pi/2$ or $\mu_u > 0$, $\mu_d < 0$, $m_{u,d} \to 0^+$) amounts to $\hat{\theta}_u = -\hat{\theta}_d = \pi/2$ (in all renormalization schemes)
$\Rightarrow$ MTM quarks well suited for QCD+QED via RM123 approach

Agreement with findings by Horkel & Sharpe [Phys.Rev. D92 (2015) 7, 074501 and 9, 094514]

Incidentally: within lattice QCD the action

\[
S^{QCD}(\psi, \bar{\psi}, U) = a^4 \sum_x \text{tr}(GG)|_U(x) +
\]

\[
+ a^4 \sum_x \bar{\psi}(x) \left[ \gamma \cdot \nabla + e^{-i\theta\gamma_5} W_{cr} + M \right] \psi(x)
\]

was shown to lead to $S^{QCD}_{cont}$ with $\theta$-vacuum (Seiler&Stamatescu '81)
Form $[#]$ of $\Gamma^{Q(C+E)D}$: sketch of the proof

Setting $\hat{M}_f \cos \hat{\theta}_f \equiv \hat{m}_f$, $\hat{M}_f \sin \hat{\theta}_f \equiv \hat{\mu}_f$ the thesis reads

$$
S^{Q(C+E)D}_{cont} = \int d^4 x \left\{ FF| A(x) + \text{tr}(GG)| G(x) \right\} + \\
+ \int d^4 x \left\{ \sum_{f=u,d} \bar{\chi}_f(x) \left[ \gamma \cdot D^f + \hat{m}_f + i\gamma_5 \hat{\mu}_f \right] \chi_f(x) \right\}
$$

with $\hat{m}_f \sim Z_{S_f}^\text{cont} Z_{m_f} m_f$, $\hat{\mu}_f = Z_{S_f}^\text{cont} Z_{\mu_f} \mu_f$. Other $d = 4$ terms:

- $\bar{\chi}_f \gamma_5 \gamma \cdot D^f \chi_f$ is ruled out by charge conjugation
- $\bar{\chi}_f \gamma_5 \chi_f$, $\text{tr}(\tilde{G}G)$, $\tilde{F}F$ are ruled out by $\tilde{P} \times (\mu_f \rightarrow -\mu_f)$
- $\sum_{\mu\nu} \bar{\chi}_f \gamma_\nu \gamma_\mu D^f_{\mu} \chi_f$ are excluded by $\tilde{P} \times (\mu_f \rightarrow -\mu_f)$ and $H(4)$

while $a^{-1} \bar{\chi}_f \chi_f$ is allowed & canceled by $M^t_{cr} \bar{\chi}_f \chi_f$. Here we defined

$$
\tilde{P} : \begin{align*}
\chi_f(x) &\rightarrow \gamma_0 \chi_f(x_P), \quad \bar{\chi}_f(x) \rightarrow \bar{\chi}_f(x_P)\gamma_0, \quad x_P = (x_0, -\vec{x}) \\
U_0(x) &\rightarrow U_0(x_P), \quad U_k(x) \rightarrow U_k^\dagger(x - a\hat{k}) \\
E_0(x) &\rightarrow E_0(x_P), \quad E_k(x) \rightarrow E_k^\dagger(x - a\hat{k})
\end{align*}
$$
Critical mass $M_{cr}^{u,d}$ in Q(C+E)D with MTM quarks

Recall QCD+QED action for two distinct flavours:

$$\psi^t = (u, d)$$

$$S_F^{Q(C+E)D}(\psi, \bar{\psi}, U, E) = a^4 \sum_x \bar{\psi}(x) \left[ \gamma \cdot \tilde{\nabla} - i\gamma_5 \tau_3 W_{cr} + \mu + \tau_3 \epsilon \right] \psi(x)$$

$$\gamma \cdot \tilde{\nabla} \equiv \frac{1}{2} \sum_\mu \gamma_\mu (\nabla^*_\mu + \nabla_\mu), \quad W_{cr} \equiv -\frac{a}{2} \sum_\mu \nabla^*_\mu \nabla_\mu + M_{cr}$$

Complex quark determinant $\Rightarrow$ LIB effects via RM123 method

Here we discuss convenient conditions to fully fix $M_{cr}^u$ and $M_{cr}^d$

$$M_{cr} = M_{cr}^u \frac{\mathbb{1} + \tau_3}{2} + M_{cr}^d \frac{\mathbb{1} - \tau_3}{2} \equiv m_{cr} \mathbb{1} + \tilde{m}_{cr} \tau_3$$

$$m_{cr} = m_{cr}^{LQCD} + \alpha_{em} \frac{\delta_{em}(g^2)}{a} + O(\alpha_{em}^2)$$

$$\tilde{m}_{cr} = \alpha_{em} \frac{\tilde{\delta}_{em}(g^2)}{a} + O(\alpha_{em}^2)$$

where $m_{cr}^{LQCD} = \frac{w(g^2)}{a} + w_1(g^2) \Lambda_{QCD} + O(a)$
Leading Isospin Breaking (LIB) effects can be calculated directly by expanding the lattice path-integral in powers of $\alpha_{em}$ and $(m_d - m_u)$

$$O(\bar{g}) = \frac{\langle R[U, A; \bar{g}] O[U, A; \bar{g}] \rangle^{A, \bar{g}^0}}{\langle R[U, A; \bar{g}] \rangle^{A, \bar{g}^0}} = \frac{\langle (1 + \dot{R} + \ldots)(O + \dot{O} + \ldots) \rangle}{\langle 1 + \dot{R} + \ldots \rangle} = O(\bar{g}^0) + \Delta O$$

sea quark e.m. effects via (noisy) fermion disconnected diagrams
**M\text{cr}** determination – Non-singlet chiral WTIs

- QCD+QED continuum chiral WTIs

\[ \partial_\mu V_\mu^3 = 0 \]
\[ \partial_\mu V_\mu^1 + e \delta q A_\mu iV_\mu^2 + 2\epsilon S^2 = 0 \]
\[ \partial_\mu V_\mu^2 - e \delta q A_\mu iV_\mu^1 - 2\epsilon S^1 = 0 \]
\[ \partial_\mu A_\mu^3 - 2\mu P^3 - 2\epsilon P^0 = 0 \]
\[ \partial_\mu A_\mu^1 + e \delta q A_\mu iA_\mu^2 - 2\mu P^1 = 0 \]
\[ \partial_\mu A_\mu^2 - e \delta q A_\mu iA_\mu^1 - 2\mu P^2 = 0 \]

with e.g. \( P^3 = \bar{\psi}\gamma_5\tau_3\psi \) and \( \mu + \tau_3\epsilon = \text{diag}(M_u, M_d) \)

- A way to fix \( M_{\text{cr}}^{u,d} \) could be to impose two continuum WTIs, e.g.

\[ \langle \partial_\mu V_\mu^1(x)P^2(0) \rangle + ie\delta q \langle A_\mu V_\mu^2(x)P^2(0) \rangle + 2i\epsilon \langle S^2(x)P^2(0) \rangle = 0 \]
\[ \langle \partial_\mu A_\mu^1(x)S^1(0) \rangle - ie\delta q \langle A_\mu A_\mu^2(x)S^1(0) \rangle - 2\mu \langle P^1(x)S^1(0) \rangle = 0 \]

with \( A_\mu = \text{photon} \). Need chiral-covariant renormalized operators ...

In the conditions above all relevant correlators are parity-violating

- Another way to fix \( M_{\text{cr}}^{u,d} \):
imposing one chiral WTI [RM123, 2013: electroquenched approximation] & minimizing \( m_\pi^\pm \) wrt bare \( m_{u,d} \) [Horkel & Sharpe, 2015]
Critical mass from parity/flavour restoring in tmLQCD

Inspiration from Wilson twisted mass lattice QCD: its effective action, $\Gamma_{\text{lat}}$, contains a local term
$$\sum_x \left[ \frac{w_0(g_0^2)}{a} - M_0 \right] [\bar{\psi} i \gamma_5 \tau_3 \psi](x)$$
plus further parity(P)-breaking local terms with $d = 5, 7, \ldots$

Enforcing P-restoration in correlators fixes $M_0 = m_{\text{cr}}^{LQCD}$ up to $O(a)$
E.g. impose
$$\sum_x \langle V^1_0(x) P^2(0) \rangle_{M_0}^{\text{lat}} = 0 \quad \text{(for all } x_0 \gg a: \text{optimal } m_{\text{cr}})$$

Lattice theory @ $M_0 \sim m_{\text{cr}}$ described by a continuum effective action
$$S_{\text{LEL}}^{\text{eff}} = \int d^4y \left\{ L_{4}^{\text{QCD}}(y) + [a^{-1} w(g^2) - M_0] [\bar{\psi} i \gamma_5 \tau_3 \psi](y) + a L_5(y) + \ldots \right\}$$
s.t. lattice correlators admit formal expansion in $a$ and $M_0 - m_{\text{cr}}$, e.g.
$$\langle V^1_0(x) P^2(0) \rangle_{M_0}^{\text{lat}} = \langle V^1_0(x) P^2(0) \rangle_{L^4}^{\text{QCD}} + O(a) + \left\{ (m_{\text{cr}} - M_0) \int d^4y \langle V^1_0(x) P^2(0) [\bar{\psi} i \gamma_5 \tau_3 \psi](y) \rangle_{L^4} \right\}$$

Note: $L_4$ is P-invariant. Here $M_0 \rightarrow m_{\text{cr}}$ limit to be taken before $a \rightarrow 0$ limit.
Critical mass from parity restoring in tmLQ(C+E)D

- Need to eliminate $a^{-1}\bar{\psi}i\gamma_5\tau_3\psi$ and $a^{-1}\bar{\psi}i\gamma_5\psi$ from $\Gamma^{QCD+QED}_{\text{latt}}$
- P-restoration fixes $m_0 \rightarrow m_{cr}$ & $\tilde{m}_0 \rightarrow \tilde{m}_{cr}$ in $M_0 = m_0 I + \tilde{m}_0 \tau_3$
- Impose $\sum_x \langle V_0^1(x)P^2(0) \rangle^{\text{latt}}_{M_0} = 0$ and $\sum_x \langle S^1(x)P^1(0) \rangle^{\text{latt}}_{M_0} = 0$

Latt. theory @ $\epsilon = 0$, $(m_0, \tilde{m}_0) \sim (m_{cr}, \tilde{m}_{cr})$ described by cont. LEL

$L_4^{QCD+QED}(y) + [m_{cr} - m_0] [\bar{\psi}i\gamma_5\tau_3\psi](y) + [\tilde{m}_{cr} - \tilde{m}_0] [\bar{\psi}i\gamma_5\psi](y) + aL_5(y) + \ldots$

s.t. correlators admit a formal expansion in $a$, $m_0 - m_{cr}$, $\tilde{m}_0 - \tilde{m}_{cr}$, e.g.

$$\langle V_0^1(x)P^2(0) \rangle^{\text{latt}}_{M_0} = (m_{cr}^{LQCD} + \alpha_{em}a^{-1}\delta_{em} - m_0) \int d^4z \langle V_0^1(x)P^2(0)\bar{\psi}i\gamma_5\tau_3\psi(z) \rangle|^{L4} +$$

$$+ (\alpha_{em}a^{-1}\delta_{em} - \tilde{m}_0) \int d^4z \langle V_0^1(x)P^2(0)\bar{\psi}i\gamma_5\psi(z) \rangle|^{L4} + O(a)$$

$$\langle S^1(x)P^1(0) \rangle^{\text{latt}}_{M_0} = (\alpha_{em}a^{-1}\tilde{\delta}_{em} - \tilde{m}_0) \int d^4z \langle S^1(x)P^1(0)\bar{\psi}i\gamma_5\psi(z) \rangle|^{L4} +$$

$$+ (m_{cr}^{LQCD} + \alpha_{em}a^{-1}\delta_{em} - m_0) \int d^4z \langle S^1(x)P^1(0)\bar{\psi}i\gamma_5\tau_3\psi(z) \rangle|^{L4} + O(a)$$

P-invariance (isospin symm. as $\alpha_{em} \rightarrow 0$) of $L_4^{QCD+QED}$ was (can be) used

Frezzotti-Rossi-Tantalo (Roma - Tor Vergata) Unqueching QED July 26, 2016 - Southampton 13 / 15
RM123 approach for $N_f = 2+1+1$ MTM LQ(C+E)D

Action with two-flavours Dirac operators $D_{33}^{h,\ell} \supset -i\gamma_5\tau_3 + \mu_{h,\ell} + \epsilon_{h,\ell}\tau_3$

- is necessary to preserve e.m. gauge invariance
- in general has complex fermionic determinant

The RM123 method for LIB effects allows to extract physical info from correlation functions (with suitable operator insertions) evaluated

★ in isosymmetric lattice theory for $N_f = 2$: real fermionic determinant

★ in a mixed action lattice theory with $e = \epsilon_{\ell} = 0$ for $N_f = 2 + 1 + 1$:

$$S_{\text{mix}} = S_{33}^{h,\ell} \big|_{e = \epsilon_{\ell} = 0} + \left( \bar{\psi}_{\text{sea}}^{h} \left[ \gamma \cdot \tilde{\nabla} - i\gamma_5\tau_3 W_{\text{cr}} + \mu_{h} + \epsilon_{h}\tau_1 \right] \psi_{\text{sea}}^{h} \right) \big|_{e = 0} + S_{\text{ghost}}(\Phi_{h}; \mu_{h} + \epsilon_{h}\tau_3) \big|_{e = 0}$$

has real fermionic determinant too

Suitable operator insertions in correlators reproduce all LIB effects due to $\epsilon_{\ell} \neq 0$ and $\alpha_{\text{em}} > 0$, included those from electro-unquenching (which need fermion disconnected diagrams evaluation) with only $O(a^2)$ lattice artifacts

[Proof along the lines of Phys.Rev. D87 (2013) 11, 114505 (RM123) & JHEP 0410 (2004) 070 (Frezzotti-Rossi)]
Thank you