Mathematical Description of THE Traction Characteristics of the Driving Devices at Spatial Stiffness Compensators of the Vibration Isolation Installations

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Abstract. During the researches the mathematical description of the traction characteristics of the stiffness compensators of the vibration isolation devices, relatively of the each axis, has been done. Representation of the compensators properties considers the variable load, thereby provide the wide enough spectrum of the action of the suggested vibration isolators. The derived expressions are valid for all three axes of space at the different stiffnesses, i.e. basic and two compensating. The research was supported by the scholarships of Russian Federation President for young scientists №184 from 10th of March 2015.

Introduction

The use of the power machine not only in industry, but and in any type of the vehicles (auto, watercraft, metro, trams, trolleybuses) inevitably leads to vibration. The vibration negatively impacts on reliability, lifetime of the machines, buildings, installations, where it were installed, and also on the system of automatic control. It is not rarely, that vibration is one of the reasons of the accidents. But the particularly harmful impacts on human body. The human, unwittingly falls under the negative influence of the vibration, during the driving of the all types of the vehicles, and which subsequently causes a variety of diseases. Therefore the issue of the decreasing of the transmitted vibrations stands very acute.

During the last time many types of the vibroprotection devices have been made [3, 7-11], but many of it does not decrease the level of the vibration down to normal level, satisfying to State Standards. Most of perspective method of decreasing of the vibration levels, created by power installations is using of the vibration isolation suspensions with floating area of zero stiffness [1, 2, 5]. The principle of the operation of the vibroprotection mechanisms is shown on figure 1. Under the limited values of the vibration isolation movement of the suspension $H$ and under the certain range of the traction changing from $P_{\text{max}}$ to $P_{\text{min}}$, transmitted from protected object to vibroprotection device, the traction characteristics of the vibroisolating mechanisms, provides the ideal vibration isolation of the system, are infinite number of the segments. These segments have a length $2A$ (magnitude of the oscillation), which in parallel of abscise axis and placed by its middles on $AB$ line, which is ramped to abscises axis under the certain angle. The tangent of this angle is equal of the stiffness of the suspension.
In such suspensions there are two basic components: resilient element and parallel compensator of the stiffness, which is device, had falling traction characteristic, with negative coefficients of the stiffness. The total stiffness of the vibration isolator is the sum of the stiffnesses of the resilient element and compensator (figure 2). The stiffness of the suspension can be decrease to zero, it can allow provide the ideal vibration isolation [3].

**Figure 1.** The traction characteristic of the tunable vibration isolation device

Calculation and simulations

In researches [4, 6] the vibration isolation device with electromagnet compensator of the stiffness was suggested, which best meets the requirements of ideal vibration isolation and has the some advantages above the early suggested of the mechanic compensators of the stiffness. Such compensator is characterized by absence of the friction forces and inertia, easy to automatization, and also has tuning system, redistributed the voltage on the electromagnets coils during the changing of the load. It allows the exclude the resonance and provides the zero of the stiffness. However, under the all its achievements the application of the vibration isolation with corrector, based on constant current electromagnets, is limited by big enough sizes. In [4, 6] for vibroprotection devices with correctors of

![Figure 2.](image-url)
the stiffness the use of the supermagnets was suggested. These magnets have the less size at the same
traction forces; it sufficiently expands the application range of such devices to decrease the vibration:
vehicles, space equipment, aircraft and etc. However, mentioned above devices, can exclude the
vibration oscillations only respectively of one axis, and as is known, the vibration is chaotic spatial
oscillations. To exclude the spatial vibrations the vibration isolation devices with electromagnetic and
supermagnet compensator of the stiffness were suggested. The principle of such devices has been built
on effect of the zero stiffness [4, 5], as is shown on figure 3, under the changing of the load the
traction characteristic of the stiffness compensator is moving in parallel itself with safe constant
stiffness. For automatic tuning of the vibration isolation device to changing load, the compensator of
the stiffness has tuning system, which taking to account the magnitude of the load [6]. At the
development of the tuning system of the spatial stiffness compensator we need to complete the
description of its forces characteristic with accounting the nonlinearity. Consider the description of the
corrector characteristic on example of one of the axis, as is shown on figure 3. For another axis the
similar description of the characteristic, but the value of the stiffness will have name compensation.

\[ F(x) = c_1 \cdot x + c_3 \cdot x^3 + \ldots + c_{2n-1} \cdot x^{2n-1} + c_n \cdot x^n, \]

where \( c_1, c_3, c_{2n-1}, c_n \) - are constant coefficients.

Under the changing of the load of the vibration isolation device, the tuning system should tune the
system, so that the traction characteristic of the compensator moves in parallel itself with constant
stiffness. At the description accept the coefficients \( c_1, c_3, c_{2n-1}, c_n \) are equal 1.

For the accounting of the traction characteristic in developing of the tuning system we should make
series Fourier expanding in range \((-b/2; b/2)\), which is equal interpolar distance of the stiffness
compensator \( b \):

\[ F(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cdot \cos \frac{n\pi x}{b/2} + b_n \cdot \sin \frac{n\pi x}{b/2} \right) \]
For representation of the function (1) in Furrier series (2) derive the coefficient:

\[
a_0 = \frac{1}{b} \cdot \frac{b^{\frac{1}{2}}}{\pi} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left( x + x^3 + \ldots + x^{n-1} + x^n \right) \cdot dx = \frac{2}{b} \cdot \left[ \frac{x^2}{2} + \frac{x^4}{4} + \ldots + \frac{x^n}{n} + \frac{x^{n+1}}{n+1} \right]_{0}^{b/2} = \frac{2}{b} \left[ \frac{b^2}{8} + \frac{b^4}{64} + \ldots + \frac{b^n}{2^n \cdot n} + \frac{b^{n+1}}{2^{n+1} \cdot (n+1)} \right] (3)
\]

\[
b_n = \frac{2}{\pi k} \int_{0}^{\pi} \int_{l}^{l} (x + x^3 + \ldots + x^{n-1} + x^n) \cdot \sin \frac{k \pi x}{l} \cdot dx = \frac{2}{\pi k} \left[ \frac{b^3}{k \pi} + \frac{(l \cdot k)^3}{k \pi} b^3 + \frac{(l \cdot k)^5}{k \pi} b^5 + \frac{(l \cdot k)^n}{k \pi} b^n \right] \sin \frac{kd}{l} = \frac{2}{\pi k} \left[ \frac{b^3}{k \pi} + \frac{(l \cdot k)^3}{k \pi} b^3 + \frac{(l \cdot k)^5}{k \pi} b^5 + \frac{(l \cdot k)^n}{k \pi} b^n \right] \sin \frac{kd}{l} = \frac{2}{\pi k} \left[ \frac{b^3}{k \pi} + \frac{(l \cdot k)^3}{k \pi} b^3 + \frac{(l \cdot k)^5}{k \pi} b^5 + \frac{(l \cdot k)^n}{k \pi} b^n \right] \sin \frac{kd}{l} (4)
\]

Finally, the coefficients for series expansion of Furrier have a view:

\[
a_0 = \frac{2}{b} \left[ \frac{b^2}{8} + \frac{b^4}{64} + \ldots + \frac{b^n}{2^n \cdot n} + \frac{b^{n+1}}{2^{n+1} \cdot (n+1)} \right] (5)
\]

\[
a_n = \sum_{n=0}^{n-1} \frac{2}{l^n} \cdot (k \pi)^{n-1} \cdot x^n \sin \frac{k \pi x}{l} \bigg|_{0}^{l} + \frac{2}{l^n} \cdot (k \pi)^{n-1} \cdot x^n \sin \frac{k \pi x}{l} \bigg|_{0}^{l} \int_{0}^{l} x^{n-1} \sin \frac{k \pi x}{l} \cdot dx; (6)
\]

\[
b_n = \sum_{n=0}^{n-1} \frac{2}{l^n} \cdot (k \pi)^{n-1} \cdot x^n \cos \frac{k \pi x}{l} \bigg|_{0}^{l} + \frac{2}{l^n} \cdot (k \pi)^{n-1} \cdot x^n \cos \frac{k \pi x}{l} \bigg|_{0}^{l} \int_{0}^{l} x^{n-1} \cos \frac{k \pi x}{l} \cdot dx; (7)
\]

On the basis of the obtained coefficients (4)-(6) the function (1)-(2) after the series expansion of furries, has a view:

\[
F(x) = \frac{1}{b} \left[ \frac{b^2}{8} + \frac{b^4}{64} + \ldots + \frac{b^n}{2^n \cdot n} + \frac{b^{n+1}}{2^{n+1} \cdot (n+1)} \right] + \sum_{n=1}^{\infty} \left[ a_n \cdot \cos \frac{mn \pi x}{l} + b_n \cdot \sin \frac{mn \pi x}{l} \right] (8)
\]

The represented characteristic, described by function (8), can be used during the description of the characteristic of the stiffness compensator under the changing of the load on the vibration isolator with taking to account of the movement.
The equation (8) graphically shows the traction characteristics of the compensator in the plane during the variation of the load, which will be taking to account for development of the 3D stiffness compensator tuning system. The derived equations further become the basis for designing of the regulators and the tuning system of the 3D electromagnetic and supermagnets stiffness compensators of the vibration devices.

**Conclusion**

In this paper the equation, described the moving in parallel to each other traction characteristics of the stiffness compensators, connected in parallel to resilient element were displayed. The stiffness in such characteristics is constant on account of control devices of the tuning correctors as under the constant loads, so and under the variable loads. The final stiffness of the vibration isolators, including the resilient element and the stiffness compensator is zero, thereby theoretically ideal vibration isolation of the protected device is provided. This derived law will be used in the development of the control device of the stiffness compensator of the vibration isolator.

The spatial vibration isolator can be used in any area of the machinery and equipment, and also can be very efficient solution for the human protection from vibration, generated by the power installation of the vehicles.

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