Resolving the plasma profile via differential single inclusive suppression

A. Majumder

Department of Physics, Duke University, Durham, NC 27708

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The ability of experimental signatures to resolve the spatio-temporal profile of an expanding quark-gluon plasma is studied. In particular, the single inclusive suppression of high momentum hadrons versus the centrality of a heavy-ion collision and with respect to the reaction plane in non-central collisions is critically examined. Calculations are performed in the higher twist formalism for the modification of the fragmentation functions. Radically different nuclear geometries are used. The influence of different initial gluon distributions as well as different temporal evolution scenarios on the single inclusive suppression of high momentum pions are outlined. It is demonstrated that the modification versus the reaction plane is quite sensitive to the initial spatial density. Such sensitivity remains even in the presence of a strong elliptic flow.

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The goal of ultra-high-energy heavy-ion collisions is the creation and study of strongly interacting matter, heated past a temperature beyond which confinement can no longer be expected. The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL), has provided a wealth of data which call for detailed theoretical modeling and study of the produced matter. At present, computations of the wide variety of bulk observables have led to a picture that the Quark-Gluon Plasma (QGP) formed at RHIC assumes the dynamical properties of an ideal fluid. The study of hard probes complements this view by ascribing considerable opaqueness to the produced matter towards the passage of hard jets through it. This set of observables, when taken in its entirety, has led to the suggestion that the matter formed at RHIC is strongly interacting.

The object of this letter is to study the feasibility of correlating the properties of the plasma derived from investigations in the bulk sector with those gleaned from the hard probe sector. Full 3+1 dimensional hydrodynamic simulations have performed very well in comparison to experimental data in the soft sector. In essence, these simulations make predictions regarding the space-time profile (STP) of the expanding matter, but require, as input, an initial spatial density profile and are sensitive to the ansatz used. The possibility of testing such a space-time profile via an independent set of measurements afforded by the modification of the fragmentation of hard jets as they pass through the dense matter forms the focus of the current study. Such a study is similar, in spirit, to Ref. which explored the effect of the longitudinal flow on the modification of hard jets. This complementary effort will assume boost invariance and will be restricted to mid-rapidity.

There exists a wide variety of observables involving particles at high transverse momentum \( p_T \) which may offer insight regarding the space-time profile of the matter produced in a heavy-ion collision: single inclusive observables (both differential and integrated), double inclusive hadronic observables (both near and away side) as well as photon hadron correlations. In this letter, the ability of single inclusive observables to reveal the space-time profile of the plasma will be critically examined. It has already been pointed out that the nuclear modification factor \( R_{AA} \) as a function of \( p_T \) for the most central event is not very sensitive to the detailed nature of the initial gluon density or the time evolution of this density. The induction of such sensitivity requires the use of more differential probes. In this letter, the nuclear modification versus the reaction plane in non-central collisions will be examined. A simplified methodology will be followed: While both the production cross section of a jet (which depends on the number of binary collisions \( N_{\text{bin}} \)) as well as the density of the plasma at a certain transverse location (which is more closely related to the number of participants \( N_{\text{part}} \)) depend on the nuclear density profile, these will be disassociated from one another. The nuclear density profile relevant to the estimation of the number of initial binary collisions may be determined independently in \( pA \) collisions. For the current treatment, this will be approximated by a hard sphere distribution and left unchanged. As the focus is on the dependence of the jet modification on the profile of the produced matter in the final state, different nuclear densities and temporal dependences will be used as inputs to produce a space-time modulation of the produced matter. In this way, a direct measure of the density profile-dependent modification on an identical set of produced jets will be explored.

The phenomenology of particle production at very high energies in \( pp \) collisions is greatly simplified by the factorization theorems of QCD. These predict that at high transverse momentum \( p_T \) (and as a result large virtuality \( Q^2 \)) the single inclusive cross-section achieves a factorized form. For high enough transverse momentum \( p_T > 7 \text{GeV} \), further simplifications arise in the case of nuclear collisions, \( i.e., \) one may ignore a variety of initial and final state nuclear effects, such as recombination, intrinsic transverse momentum and the Cronin effect. At such high energies these are known to be small compared to the dominant effect of medium induced energy...
loss which leads to the modification of the fragmentation function \( \tilde{f} \). Such effects may have an interesting density dependence and will reappear in any effort to extend the effects discussed herein to lower \( p_T \). Assuming a factorization of initial and final state effects, the differential cross-section for the production of a high \( p_T \) hadron at midrapidity from the impact of two nuclei \( A \) and \( B \) at an impact parameter between \( b_{\text{min}}, b_{\text{max}} \) is given as,

\[
\frac{d\sigma^{AB}}{dy d^2p_T} = K \int_{b_{\text{min}}}^{b_{\text{max}}} d^2b \int d^2r_A(r + \vec{b}/2) t_B(r - \vec{b}/2) \times \int dx_A dx_B G_A(x_A, Q^2) G_B(x_B, Q^2) \times \frac{d\tilde{\sigma}_{ab \to cd}}{dt} D_B(z, Q^2) \left( \frac{\alpha_s}{\pi z} \right),
\]

where, \( G_A(x_A, Q^2) G_B(x_B, Q^2) \) represent the nuclear parton distribution functions. These are given in terms of the shadowing functions \( S_A(x_A, Q^2) \) and the parton distribution functions in a nucleon \( G_a(x_A, Q^2) \) as

\[
G_A(x_A, Q^2) = S_A(x_A, Q^2) G_a(x_A, Q^2).
\]

In Eq. \( \text{(1)} \), \( t_A(r), t_B(r) \) are the thickness functions of nuclei \( A \) and \( B \) at the transverse location \( r \equiv (x, y) \). As mentioned previously, throughout this letter, such thickness functions will be evaluated using the simple and analytically tractable hard sphere densities \( \rho(x, y, z) = \rho_0 \theta(R_A^2 - x^2 - y^2 - z^2) \). In this formulation, \( \hat{s}, \hat{t}, \hat{u} \) refer to the Mandelstam variables of the internal partonic process. Unless stated otherwise, the variables without the hats refer to the variables of the full process. The two participating partons in the initial state are referred to as \( a, b \) while the final state partons are referred to as \( c, d \). The factor \( K \sim 2 \) accounts for higher order contributions.

The most important element in Eq. \( \text{(1)} \) is the medium modified fragmentation function \( \tilde{D}(z, Q^2) \) expressed as the sum of the leading twist vacuum fragmentation function and a correction brought about by rescattering of the struck quark in the medium i.e., \( \tilde{D} = D + \Delta D \) \( \text{(1)} \). The vacuum fragmentation functions \( D(z, Q^2) \) are taken from Ref. \( \text{[20]} \). In the collinear limit, the modification is computed by isolating corrections, suppressed by powers of \( Q^2 \), which are enhanced by the length of the medium \( \text{[21]} \). At next-to-leading twist, the correction for the fragmentation of a quark has the expression (generalized from deep-inelastic scattering (DIS) \( \text{[14]} \) \( \text{[22]} \)),

\[
\Delta D(z, Q^2, \vec{r}) = \frac{\alpha_s}{2\pi} \int \frac{dt_{\perp}^2}{t_{\perp}^2} \int \frac{dy}{y} P_{q\to i}(y) 2\pi \alpha_s C_A \times T^M(\vec{b}, \vec{r}, x_A, x_B, y, l_{\perp}) D_i \left( \frac{z}{y}, Q^2 \right) \times \left[ \int_{-1}^{1} N_c t_A(\vec{r} - \vec{b}/2) t_B(\vec{r} - \vec{b}/2) \right] \times G_A(x_A) G_B(x_B) \frac{d\sigma}{dt} \left( \frac{1}{y} \right) + v.c. \quad \text{(3)}
\]

In the above equation, \( l_{\perp} \) is the transverse momentum of the radiated gluon (quark) which leaves a momentum fraction \( y \) in the quark (gluon) denoted as parton \( i \) \((P_{q\to i}(y) \) is the splitting function for this process) which then fragments leading to the detected hadron. The \( v.c. \) refers to virtual corrections. The factor \( T^M \) originates from the higher twist matrix element, which encodes the information of rescattering off the soft gluon fields,

\[
T^M = t_A(\vec{r} + \vec{b}/2) t_B(\vec{r} - \vec{b}/2) G_A(x_A) G_B(x_B) \frac{d\sigma}{dt} \times \int_0^{\zeta_{\text{max}}} d\zeta x_g p_y(x_g, \hat{n} \zeta + \vec{r}) (2 - 2 \cos(\eta_L \zeta)). \quad \text{(4)}
\]

In the above equation, the factor \( \eta_L = l_{\perp}^2/[2p_T(y(1-y))] \). Where, \( p_T \) represents the transverse momentum of the produced parent jet. It should be pointed out that in the higher twist formalism used here, no assumption is made regarding the prevalent degrees of freedom of the produced matter.

Breaking with usual practice, the jet direction \( \hat{n} \) is chosen as the \( x \)-axis in the transverse \((x, y)\) plane (the \( z \)-direction is set by the beam line); the angle of the reaction plane vector \( \vec{b} \) is measured with respect to this direction. The distance travelled by the jet in this direction prior to scattering off a gluon is denoted as \( \zeta \). The gluon’s forward momentum fraction in denoted as \( x_g \). One assumes, following Ref. \( \text{[23]} \) that the relevant values of \( x_g \) are small enough that \( x_g p_y(x_g, \hat{n} \zeta) \) is almost independent of \( x_g \) and the combination will henceforth be referred to as the density \( p \). This represents the one unknown in the above set of equations and in the higher twist formalism. It is related to the more familiar quantity \( \hat{q} \) by the simple equation \( \text{[23]} \).

\[
\hat{q}(\vec{r} + \hat{n} \zeta) = \frac{4\pi^2 \alpha_s C_F}{N_c - 1} p(x_g, \vec{r} + \hat{n} \zeta). \quad \text{(5)}
\]

This is the \( \hat{q} \) at a location (and time) \( \vec{r} + \hat{n} \zeta \) in the plasma, and not the averaged value.

Ignoring the modification in the fragmentation function \( (\Delta D) \) and the factors of nuclear shadowing in Eq. \( \text{(1)} \), produce the result of binary scaling the \( p - p \) scattering cross section and produce the denominator of the nuclear modification factor,
The phenomenological input that is required to understand the variation of the nuclear modification factor with centrality or $p_T$ is the space time dependence of the gluon density $\rho(x,y,z,\tau)$. In the remainder, the focus will be on observables at midrapidity and thus the $z$-coordinate will be ignored; different forms of the gluon density profile will be invoked and their effect on the nuclear modification factor versus centrality and the reaction plane will be elucidated.

The initial spatial profile of the plasma density depends on the nucleon density brought in by the colliding nuclei. It, most likely, should be a smooth, monotonously dropping function of the transverse coordinates $(x,y)$, the time $\tau$ as well as the impact parameter of the collision $b$. The function must approach zero as any of these parameters are increased from their minimum values. A general functional form which satisfies the above constraints is

$$\rho(\vec{r},\tau;\vec{b}) = \frac{\tau_0}{\tau} \frac{N_{wn}(t_A^f(\vec{r} + \vec{b}/2),t_A^f(\vec{r} - \vec{b}/2))}{N_{wn}(t_A^f(0),t_A^f(0))}, \quad (7)$$

where, $N_{wn}$ is the wounded nucleon participant density [14], and $t_A^f(\vec{r} \pm \vec{b}/2)$ is an input thickness function that may be varied to obtain different spatial profiles $\rho(\vec{r},\vec{b},\tau)$. The superscript $f$ on the thickness functions is meant to differentiate it from the thickness functions used to generate the number of binary collisions in the initial state. The time dependence of the density for the first set of computations will be assumed to be generated by a pure Bjorken expansion (which leads to a $1/\tau$ dependence in Eq. (4)). It may be immediately verified that even within the apparently constraining form of Eq. (4), radically different space-time profiles may be obtained by using different input thickness functions. The profiles are normalized by the maximum density achieved at $\vec{r} = 0$ in the most central collision at $\vec{b} = 0$ (the denominator of Eq. (7)). In such a form, the initial spatial modulation is contained entirely in the normalized modulation factor $M(x,y)$ i.e., $\rho(x,y) = \rho_0 M(x,y)$. By definition, the maximum value of $M(x,y) = 1$. The unknown factor, $\rho_0\tau_0$ in the above equation is set in all cases by assuming that the $R_{AA}$ for the most central collisions at a pion $p_T = 8$ GeV be 0.2. Once so normalized, the variation of the $R_{AA}$ with $p_T$ for all the different profiles considered for the most central collision is remarkably similar. The above model for the density may then be used to predict the $R_{AA}$ at a lesser centrality and also the variation of the $R_{AA}$ with the reaction plane.

In an effort to demonstrate the sensitivity of the nuclear modification factor to the space-time profile, two rather disparate input thickness functions $t_A^f$ are used: the hard sphere and Gaussian. The hard sphere thickness function is given as,

$$t_A^f(\vec{r},\vec{b}/2) = C_{HS} \sqrt{R_{AA}^2 - (x \pm b_x/2)^2 - (y \pm b_y/2)^2}. \quad (8)$$

The focus in this letter will be on the peripheral events with a centrality between 40% - 60%, there exists a considerable azimuthal asymmetry in the gluon distribution in such an event for a hard sphere distribution. The results of such a calculation are shown in Fig. 1. In this figure, the solid black line represents the $R_{AA}$ integrated over all emission angles. The other lines represent the differential $R_{AA}$ as a function of the angle of the jet with respect to the reaction plane ($\phi$) i.e., $R_{AA}(\Delta \phi)$. Due to the large azimuthal asymmetry in the initial gluon density, leading to jets facing different amounts of matter depending on emission angle, there exists a large asymmetry as well in the $R_{AA}(\Delta \phi)$. The plain dashed line ($\phi = 0 - 15^\circ$) represents emission in the reaction plane. These represent jets which pass through the least amount of matter and as a result exhibit the lowest amount of modification. The dashed line with triangles ($\phi = 75 - 90^\circ$) represents emission perpendicular to the plane. These pass through the maximum amount of matter and thus show the most modification. The lines in between represent emission at intermediate angles. This figure clearly demonstrates the sensitivity of the nuclear modification factor versus the reaction plane to the azimuthal asymmetry of the gluon distribution.

![FIG. 1: (Color online) $R_{AA}$ at 40-60% centrality (solid black line) and the variation of $R_{AA}$ as a function of the angle $\phi$ with respect to the reaction plane, assuming the initial gluon density generated by the overlap of two hard-sphere geometries. The evolution of the density is assumed to be purely due to Bjorken expansion.](image-url)
\[ t_A^f(r^f, \hat{b}) = C_G e^{-\frac{(|\vec{r} - \vec{b}|)^2}{2R_G^2}}, \]  

where \( R_G \) is an appropriately chosen Gaussian radius. As would be expected, such an initial gluon density leads to a very different variation of the \( R_{AA} \) with respect to the reaction plane. This is plotted in Fig. 2. The variation of the nuclear modification factor with \( \phi \) is much reduced as compared to that from a hard sphere thickness function. This difference testifies to the effectiveness of observables such as the \( R_{AA}(\Delta \phi) \) as probes of the initial gluon density profile. For the initial number of binary collisions, the use of a hard sphere density is continued. This ensures that all jets are produced in the same, finite, well defined, volume.

The variation of the \( R_{AA} \) with \( \phi \) may also be sensitive to the temporal evolution of the plasma. In both of the preceding calculations, the temporal dependence was taken to be that due to Bjorken expansion. In real heavy-ion collisions, there is a considerable amount of transverse (radial and elliptic) flow of the matter caused by the large pressures built up in the dense matter. To have a completely realistic description of such a scenario requires that the density modulation factor \( M(x, y, \tau) \) be extracted from full three dimensional hydrodynamical simulations [23]. In this effort, a simple scaling form will be employed: the two transverse coordinates are replaced by \( x/|r(\tau)\epsilon(\tau)| \) and \( y\epsilon(\tau)/r(\tau) \), where \( r(\tau) \) describes the radial expansion and \( \epsilon(\tau) \) generates the azimuthally symmetric expansion. It should be noted that there already exists an asymmetry in the density distribution from Eq. (7) where the hard sphere thickness function (Eq. 8) is used as input to calculate the gluon density. The factors \( r, \epsilon \) are meant to introduce a further time dependent modulation. Thus at \( \tau = 0 \), both these factors must be set to unity. A simple form for the time dependence of \( r \) and \( \epsilon \) may be obtained in a toy model:

\[ r(\tau) = 1 + \frac{\nu}{R_A} \tau \]  

and  

\[ \epsilon(\tau)^2 = 1 + \frac{\nu}{2R_A} \tau. \]  

The result of such a model calculation is shown in Fig. 3. As is clearly demonstrated, the large asymmetric flow, leads to an \( R_{AA} \) versus reaction plane with a lesser spread as a function of \( \phi \) than in Fig. 1. The ordering of the contributions remains unchanged. Thus even with a realistic amount of radial and elliptic flow, the \( R_{AA}(\Delta \phi) \) still maintains a sensitivity to the initial asymmetry. If the asymmetric expansion is increased, this leads to a further reduction in the spread. This is shown in Fig. 4, where the \( R_{AA} \), along with the \( R_{AA} \) in reaction plane and out of plane are plotted as a function of \( f_x \), an overall multiplicative factor used to modify the asymmetric velocity \( v_x \). In the figure, \( f_x = 0 \) corresponds to the case of Fig. 1 with a radial flow, \( i.e. \), with no elliptic flow. The case of \( v_x = 1 \) corresponds to the case of Fig. 3 \( i.e. \), with elliptic flow parametrized from Ref. 4. Higher values of \( f \) represent multiples of the asymmetric velocity used in Fig. 3. As expected, raising the asymmetric velocity leads to a decrease in the spread of the \( R_{AA} \) as a function of the reaction plane. It should be pointed out that in

![FIG. 2: (Color online) Same as Fig. 1 with the hard sphere density replaced with a Gaussian profile.](image)

![FIG. 3: (Color online) Same as Fig. 1 including transverse flow.](image)
both Figs. [1] and [3], the final state gluon density profile starts out with a considerable azimuthal asymmetry as a hard sphere thickness function is used as input in Eq. [4].

The calculations presented above offer sufficient evidence that differential probes such as the $R_{AA}$ versus the reaction plane offer deeper insight into the space-time profile of the matter produced at RHIC. Sharper eccentricities lead to large variations of the $R_{AA}$ with respect to the emission angle. The effect of large elliptic flow is discernable. It is true that such observations alone will not distinguish the effect of a large elliptic flow from that of a lower eccentricity in the initial condition. To discern between such facets will require multiple differential probes to be compared in unison. The nuclear modification factor versus the reaction plane carry more information than the azimuthal asymmetry of high $p_T$ hadrons (the $v_2$ at high $p_T$). The reader will note that the $v_2$ obtained from the $R_{AA}(\Delta \phi)$ (Figs. [1] [3]) is insensitive to the overall magnitude of the $R_{AA}$.

The current exercise is meant to serve as a first attempt in the resolution of the spatio-temporal profile of the hot plasma. As such, a variety of approximations were made, $k_T$ broadening was ignored, and the number of initial binary collisions was taken from a hard sphere nuclear density distribution. This was done in the interest of focussing the study on the effect of the plasma profile on the variation of $R_{AA}$ with respect to the jet emission angle. The inclusion of such effects will result is slender modifications to the results outlined in this letter and are left for a future, more rigorous comparison with data.

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