Optical Phase Measurement Using a Deterministic Source of Entangled Multi-photon States

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Precision measurements of optical phases have many applications in science and technology. Entangled multi-photon states have been suggested for performing such measurements with precision that significantly surpasses the shot-noise limit. Until recently, such states have been generated mainly using spontaneous parametric down-conversion – a process which is intrinsically probabilistic, counteracting the advantages that the entangled photon states might have. Here, we use a semiconductor quantum dot to generate entangled multi-photon states in a deterministic manner, using periodic timed excitation of a confined spin. This way we entangle photons one-by-one at a rate which exceeds 300 MHz. We use the resulting multi-photon state to demonstrate super-resolved optical phase measurement. Our results open up a scalable way for realizing genuine quantum enhanced super-sensitive measurements in the near future.

I. INTRODUCTION

When a light beam passes through a thin layer of transparent material, it gains a phase shift relative to the same beam in vacuum. The shift depends, in general, on the thickness of the layer, its refractive index and its birefringence. Measuring the optical phase has, therefore, numerous applications in science and technology, including microscopy, lithography and displacement measurements, to name a few.

The precision in which such measurements can be performed is typically limited to the shot-noise-limit (SNL) of $\Delta \theta_{\text{clas}} = 1/\sqrt{N}$, where $N$ is the total number of the detected beam photons. A possible way to overcome this limit is to use entangled multi-photon states, which can conceptually push the measurement precision towards the Heisenberg limit of $1/N$ [1, 2].

A well known example is the N00N state [3, 4]. Such a state of $N_{\text{ent}}$ photons can be expressed as $(|N_{\text{ent}},0\rangle + |0,N_{\text{ent}}\rangle)/\sqrt{2}$, representing a superposition of all $N_{\text{ent}}$ photons in one mode or all in another mode, with a well defined quantum mechanical phase between the two. If one mode experiences a phase shift of $\theta$ relative to the other, $\theta_{\text{ent}}$ can be accurately measured using interferometry, for example, yielding a measure of $\theta$ with an error of $\Delta \theta_{\text{ent}} = 1/N_{\text{ent}}$. Since only a single mode emerging from a single source experiences the phase shift, the N00N states also provide high spatial resolution when measuring local phase shifts [5]. Unfortunately, generating a N00N state is a very demanding and resource intensive task, thus only N00N states with $N_{\text{ent}} = 5$ photons have been reported so far [6].

The Greenberger-Horne-Zeilinger (GHZ) [7] state is yet another multi-photon entangled state that can be used for super-sensitive phase measurements. It is expressed as $(|0\rangle^\otimes N_{\text{ent}} + e^{i\alpha}|1\rangle^\otimes N_{\text{ent}})/\sqrt{2}$, describing a superposition of $N_{\text{ent}}$ photons, all in state $|0\rangle$ or all in state $|1\rangle$, with a well defined relative phase $\alpha$ between the two cases. Similarly to the N00N case, if one of the states experiences a phase shift of $\theta$ relative to the other, $\theta$ can, in principle, be measured in the Heisenberg accuracy limit.

In this work, we produce such a GHZ state where $|0\rangle$ and $|1\rangle$ are implemented in two orthogonal polarizations of the photons. GHZ states have already been produced with up to 12 [8] and 18 photonic qubits [9] using spontaneous parametric down converted (SPDC) light sources [10]. Nevertheless, these sources are probabilistic, and require inefficient post-selection in order to create the GHZ states. In addition, the spatial resolution that such GHZ states can provide, is relatively limited. This is because the generated GHZ states occupy multiple spatial modes. These and other requirements challenge the use of SPDC sources as suitable and scalable sources for super-sensitive phase measurement applications.

Single photon sources with spontaneously generated entanglement, were also considered recently for achieving super-sensitivity [11]. Single photon sources based on semiconductor quantum dots (QDs) are particularly bright and capable of deterministic production of single [12–14] and entangled [15–20] photons. Attempts to demonstrate phase super-sensitive measurements were recently reported using entangled two-photon ($N_{\text{ent}} = 2$) states from a single QD [21, 22]. Unfortunately, these methods are intrinsically limited to low numbers of entangled photons [23, 24].

Here, we demonstrate for the first time a new approach for achieving super-sensitive optical phase measurement. This approach utilizes semiconductor QDs to deterministically generate multi-photon, polarization-entangled, GHZ states. We do it by periodic pulsed excitation of the QD, entangling photons at a rate of 330 MHz. The number of photons that can be entangled ($N_{\text{ent}}$), this way is in principle unlimited. In addition, the produced GHZ states occupy one spatial mode, providing the high-
Figure 1. Schematic and simplified description of the experimental system. A sequence of pulses is applied to the quantum dot (QD) at 76 MHz. The pulses deterministically generate a string of photons, which are polarization entangled with the spin of the dark exciton (DE) in the QD. The emitted photons pass through a liquid crystal variable retarder (LCVR), which adds adjustable relative phase difference between the two components of the light circular polarizations R and L. The polarization of the photons is then projected on two rectilinear polarizations H and V using a polarizing beam splitter (PBS). Correlation events in which two or three clicks occur during the same period are recorded by the time-tagging electronics.

est spatial resolution possible. These advantages pave the way for building a scalable method for performing super-sensitive measurements. We describe below the experiment that demonstrates the concept and discuss the conditions for achieving genuine super-sensitivity.

II. THEORETICAL BACKGROUND

A. Optical phase measurement with classical light

Consider the experimental setup described in Fig. 1. We set a liquid crystal variable retarder (LCVR) to add a relative phase of \( \theta \) between left- and right-circularly polarized light transmitted through the LCVRs. For example, rectilinear horizontally polarized light \( |H\rangle = (|R\rangle + |L\rangle) / \sqrt{2} \) accumulates a phase of \( \theta \) upon transmission through the LCVR to become \( (|R\rangle + e^{i\theta}|L\rangle) / \sqrt{2} \). A way to measure the accumulated phase \( \theta \) is to project the light on a polarizing beam splitter (PBS). One measures then the degree of rectilinear relative polarization at the output, which is given by \( D_{RP}(\theta) = I_H(\theta) - I_V(\theta) / I_H(\theta) + I_V(\theta) \), where \( I_H(\theta) \) (\( I_V(\theta) \)) is the intensity of the light transmitted (reflected) by the PBS. It is straightforward to show that

\[
I_H(V)(\theta) \propto \frac{1}{2} [1 \pm D_{RP}(\theta)]
\]

\[
D_{RP}(\theta) = D_{RP}^S \cos(\theta)
\]

where \( D_{RP}^S \) is the degree of rectilinear polarization of the light source before the LCVR (ideally \( D_{RP}^S = 1 \)).

The best uncertainty in determining \( \theta \), \( \Delta \theta \) is therefore given by:

\[
\Delta \theta = \frac{\Delta D_{RP}^S}{\partial D_{RP}^S(\theta)/\partial \theta} = \frac{\Delta D_{RP}}{D_{RP}^S \cos(\theta)|\cos(\theta) = 0} = \frac{\Delta D_{RP}}{D_{RP}^S}
\]

where one chooses the angle \( \theta \) such that \( D_{RP}(\theta) \) almost vanishes, and its slope maximizes. Here \( \Delta D_{RP} \) is the experimental uncertainty in measuring the degree of rectilinear polarization after the LCVR, for \( \theta \) close to such a point (\( \theta \approx \pi/2 \)). For classical light this uncertainty is given precisely by \( 1/\sqrt{N} \), where \( N \) is the total number of photons used for measuring \( D_{RP} \). It follows that

\[
\Delta \theta_{\text{class}} = \frac{1}{D_{RP}^S \sqrt{N}}
\]

B. Optical phase measurement with entangled light

For non-classical light composed of \( N/N_{\text{ent}} \) bunches of \( N_{\text{ent}} \) entangled photons in each bunch, forming a GHZ state, \( (|R\rangle \otimes |L\rangle + e^{i\theta}|L\rangle \otimes |R\rangle) / \sqrt{2} \), the considerations are slightly different. This time, transmission through the LCVR results in accumulated phase of \( N_{\text{ent}} \theta \) between the left and right polarization components \((|R\rangle \otimes |L\rangle + e^{iN_{\text{ent}}\theta}|L\rangle \otimes |R\rangle) / \sqrt{2} \). Measuring the degree of rectilinear polarization in this case allows the determination of \( \theta \) with higher accuracy. To see this, one obtains, as before, (see Eq. (2))

\[
I_{H(V)}(\theta) \propto \frac{1}{2} [1 \pm D_{RP}^{N_{\text{ent}}}(\theta)]
\]

\[
D_{RP}^{N_{\text{ent}}}(\theta) = D_{RP}^{S,N_{\text{ent}}} \cos(N_{\text{ent}} \theta)
\]

where \( D_{RP}^{S,N_{\text{ent}}} \) is the degree of rectilinear polarization of the entangled light source. Substituting this in the expression for \( \Delta \theta \), recalling that in this case the uncertainty in the measured polarization degree is given by the number of bunches: \( \Delta D_{RP}^{N_{\text{ent}}} = (N/N_{\text{ent}})^{-1/2} \) yields

\[
\Delta \theta_{N_{\text{ent}}} = \frac{\Delta D_{RP}^{N_{\text{ent}}}}{D_{RP}^{S,N_{\text{ent}}} \sqrt{N_{\text{ent}}}} = \frac{1}{D_{RP}^{S,N_{\text{ent}}} \sqrt{N_{\text{ent}}}}
\]

which means that if the initial degree of rectilinear polarization \( D_{RP}^{S,N_{\text{ent}}} \) of the entangled light is the same as that of the classical light \( D_{RP}^{S} = D_{RP}^{S,1} \) and if all \( N \) photons are detected, the sensitivity of the optical phase measurement with entangled light is \( \sqrt{N_{\text{ent}}} \) times better than that of the classical light.

Eq. (4) holds for the ideal case in which each bunch of \( N_{\text{ent}} \) photons is maximally entangled and the efficiency of
the photon detection, \( \eta \), is 1. In reality, however, the situation is different [25, 26]. The system detection efficiency is limited, and therefore for a finite \( \eta \), the efficiency of detecting \( N_{\text{ent}} \)-photon events is given by \( \eta^{N_{\text{ent}}} \). This means that in order to reach genuine super-sensitivity even with entangled light of only \( N_{\text{ent}} = 2 \), \( \eta \) should exceed 0.71. For super-sensitivity which is order of magnitude better than the classical limit, \( N_{\text{ent}} \) should be more than 100, and \( \eta \) should be better than 98%.

Another obstacle in reaching genuine super-sensitivity is the deviation of the multi-photon entangled state from a pure state. Typically, due to various decoherence processes in the state generation, adding photons to the multi-photon state results in greater coherence loss. This loss can often be described by a characteristic exponential decay in the degree of rectilinear polarization \( D_{\text{RP}}^{S,N_{\text{ent}}} \) of the entangled light source, as the number of entangled photons \( N_{\text{ent}} \) increases:

\[
D_{\text{RP}}^{S,N_{\text{ent}}} = D_{\text{RP}}^{S,1} e^{-(N_{\text{ent}}-1)/N_{\text{D}}} \tag{5}
\]

Here, \( D_{\text{RP}}^{S,1} \) is the degree of rectilinear polarization of the classical light beam composed of single non-entangled photons and \( N_{\text{D}} \) is a characteristic polarization decay length of the entangled photon string.

With this dependence, although the increase in the string length improves the sensitivity as \( \sqrt{N_{\text{ent}}} \), at the same time the exponential decay of the \( D_{\text{RP}} \) reduces it. For the ideal case in which \( D_{\text{RP}}^{S,1} = 1 \) and \( \eta = 1 \) it is straightforward to show that for a given \( N_{\text{D}} \) maximum sensitivity is obtained when \( N_{\text{ent}} = N_{\text{D}}/2 \). With this in hand, it follows that super-sensitivity which is about 10% better than the SNL can be achieved with \( N_{\text{ent}} = 2 \) entangled photons from an entangled light source with \( N_{\text{D}} \geq 4 \). In order to get super-sensitivity which is an order of magnitude better than the SNL, bunches longer than \( N_{\text{ent}} = 270 \) entangled photons are required from a light source with \( N_{\text{D}} \geq 540 \). The limit in which \( N_{\text{D}} \rightarrow \infty; D_{\text{RP}}^{S,1} = 1; \) and \( \eta = 1 \) is called the Heisenberg limit.

### III. EXPERIMENT

#### A. The dark exciton as a photon entangler

We use a QD to implement a scheme for deterministic generation of a string of entangled photons [27]. A QD-confined dark exciton (DE), forms a physical two-level system, effectively acting as a matter spin qubit (Fig. 2) [28].

Its two total spin (2) projections on the QD symmetry axis \( \hat{z} \) form a basis, \( |\pm Z\rangle = |\pm 2\rangle \), for the DE qubit space. The DE energy eigenstates are \( |\pm X\rangle = (|+Z\rangle \pm |-Z\rangle)/\sqrt{2} \), with an energy splitting \( \Delta \varepsilon_2 = 1.5 \mu eV \). In the Bloch sphere representation, this splitting corresponds to a coherent state-precession around the \( \hat{x} \) axis, with a period of \( T_{\text{DE}} = h/\Delta \varepsilon_2 \geq 3 \) nsec [28]. In addition to the DE, we use two states of a biexciton (BIE)—a bound state of two excitons—whose total spin projections on the spatial \( \hat{z} \) axis are either +3 or −3. The BIE eigenstates \( |\pm X_{\text{BIE}}\rangle = (|+Z_{\text{BIE}}\rangle \pm |-Z_{\text{BIE}}\rangle)/\sqrt{2} \) are also non-degenerate, having precession period of \( T_{\text{BIE}} = h/\Delta \varepsilon_2 \geq 5 \) nsec [29]. We denote these states by \( |\pm 3\rangle \). The experimental protocol relies on the optical transition rules \( |+2\rangle \leftrightarrow |+3\rangle \) and \( |-2\rangle \leftrightarrow |-3\rangle \) through right hand \( |R\rangle \) and left hand \( |L\rangle \) circularly polarized photons, respectively (see Fig. 2).

The pulse sequence for generating the \( |\text{GHZ}\rangle \) state is schematically described in the lower panel of Fig. 3. It is executed at a rate of 76 MHz, corresponding to a time window of \( \sim 13 \) nsec. Within each time window, a \( |\text{GHZ}\rangle \) state is generated and used for the optical phase measurement.

First, we deterministically initialize the DE in its spin eigenstate \( |\psi_{\text{DE}}^{0}\rangle = |-X\rangle = (|+2\rangle - |-2\rangle)/\sqrt{2} \) using a short \( \pi \)-area picosecond pulse [28]. After the initialization, we repeatedly apply a cycle containing three elements: (i) a converting laser \( \pi \)-pulse, resonantly tuned to the DE-BIE optical transition; (ii) subsequent radiative recombination of the BIE, resulting in an emission of a photon entangled with the spin of the DE which
The sequence of steps (i)–(ii) forms one full cycle. Repeating the cycle again results in a second photon, whose polarization state is entangled with that of the first photon and the spin of the remaining DE, yielding the tripartite GHZ state:

$$\sqrt{2} |\psi^{\text{DE-1ph-2ph}}\rangle = (|Z\rangle|R_1\rangle|R_2\rangle - |-Z\rangle|L_1\rangle|L_2\rangle)$$ (7)

In terms of the pulse sequence (Fig. 3), this state is generated with the emission of the second BIE photon following the second converting pulse.

This cycle can be applied $N_{\text{ent}}$ times to generate an entangled $N_{\text{ent}} + 1$ GHZ state, containing $N_{\text{ent}}$ photons and a DE:

$$\sqrt{2} |\psi^{\text{DE-1ph-2ph-...-N_{\text{ent}}}}\rangle = |Z\rangle|R_1\rangle|R_2\rangle...|R_{N_{\text{ent}}}\rangle - |-Z\rangle|L_1\rangle|L_2\rangle...|L_{N_{\text{ent}}}\rangle$$ (8)

When the emitted photons pass through the retarder of Fig. 1, the state evolves to:

$$\sqrt{2} |\psi^{\text{DE-1ph-2ph-...-N_{\text{ent}}}}\rangle = |Z\rangle|R_1\rangle|R_2\rangle...|R_{N_{\text{ent}}}\rangle - e^{iN_{\text{ent}}\theta} |-Z\rangle|L_1\rangle|L_2\rangle...|L_{N_{\text{ent}}}\rangle$$ (9)

For the conclusion of the experiment a last circularly polarized photon, quarter of a precession time after the previous pulse projects the DE spin on the $|Y\rangle$ basis, when a photon is detected. The cycle then ends in a -7 nsec long optical pulse, which depletes the QD and prepares it for the next cycle [30].

B. Calculating the multi-qubit quantum state using the repeated cycle’s process map

The multi-qubit states that our method produces deviate from the pure wavefunctions $\psi (\theta)_{\text{DE-1ph}}$ and $\psi (\theta)_{\text{DE-1ph-2ph-...-N_{\text{ent}}}}$, described above. The proper way to describe our actual output state is within the formalism of density matrices. We can calculate the density matrix of the multi-qubit state that we produce by applying repeatedly a linear transformation $\Phi$ to the initial state of the DE. The transformation $\Phi$ is called a “process map” and it describes the evolution of the system from $N_{\text{ent}}$ qubits to $N_{\text{ent}} + 1$ [27]. For example, one can describe the evolution of any initial DE state (spanned by a $2 \times 2$ density matrix) when it is subjected to the excitation, photon emission and full periodic precession of the DE, resulting in an entangled DE-photon state (spanned by a $4 \times 4$ density matrix), by:

$$\rho^{(\text{DE+1ph})}_{\alpha\beta} = \sum_{\mu} \Phi^{(\text{DE})}_{\alpha\beta,\mu} \rho^{(\text{DE})}_{\mu}$$ (10)

Here, the density matrix elements are given in the Pauli basis, such that $\rho^{(\text{DE})}_{\alpha\beta} = \sum_{\mu} \rho^{(\text{DE})}_{\mu} \sigma_\alpha \otimes \sigma_\beta$, where $\mu, \alpha, \beta \in 0, 1, 2, 3$. The
process map has, therefore, 64 real parameters. To measure the process map, we first perform full tomography of the initialized DE in six different initialization states $|\pm X\rangle$, $|\pm Y\rangle$, $|\pm Z\rangle$ [31]. Next, we apply to these states one cycle of our protocol and perform full two-qubit tomography on the resulting entangled DE-photon states. Finally, by solving a set of linear equations, the process map $\Phi$ is fully obtained [27]. The fidelity of our measured map to the ideal one, which describes an ideal two-qubit gate and no decoherence at all, is 0.82.

Having the process map at hand, we apply it $N_{\text{ent}}$ times to the measured initialization of the DE state, simulating the resulting $(N_{\text{ent}} + 1)$ GHZ state. Then, we add a phase of $\cos(N_{\text{ent}} \theta)$ to the $|L\rangle\langle L|$ component of the density matrix relative to the $|R\rangle\langle R|$ one, imitating the action of the LCVRs on the transmitted photons. Finally, we project the simulated $N_{\text{ent}} + 1$ qubits density matrix on the orthogonal basis elements (photons on $|\pm X\rangle$ and spin on $|\pm Y\rangle$), as done in the experiment.

C. Experimental system

A simplified version of the experimental system appears in Fig. 1. A sequence of laser pulses is launched on the QD, resulting in emission of a string of single photons separated from each other by ~3 nsec. The photons are polarization entangled as explained above. By passing through the LCVR an optical phase of $\theta$ is added to $|L\rangle$ polarized photons relative to the $|R\rangle$ polarized ones. Here we used the setup to produce $N_{\text{ent}} = 1$ and $N_{\text{ent}} = 2$ entangled spin-photon and entangled spin-photon-photon ($|\text{GHZ}\rangle$) states, respectively. The photons are then projected using a standard polarizing beam splitter (PBS) and detected using superconducting single-photon detectors. In principle, one pair of detectors is enough to perform the demonstration, provided that their recovery time is shorter than the temporal separation of two sequential photons (3 nsec). In practice, since the recovery time of our detectors is longer than that (~20 nsec),
we used two more detectors, allowing us to measure up to four-photon correlations. We used a HydraHarp time-tagging device to record two- and three-photon coincidence events for projection-measurements of the $N_{\text{ent}} = 1$ and $N_{\text{ent}} = 2$ cases, respectively. We recorded the coincidence rates while scanning $\theta$ between 0 and $2\pi$. A coincidence event is registered whenever two (or three) photons are detected within the same repetition cycle of 13 nsec. The overall collection efficiency of our system, is estimated as 1%, thereby resulting in three-photon coincidence rate of $\sim$150 Hz.

### IV. RESULTS AND DISCUSSION

When a coincidence event is recorded, the data analysis proceeds as follows: The detection of the last photon which results from the last $|R \rangle$ ($|L \rangle$) circularly polarized excitation pulse, is used to project the DE spin on the $|+Y_0 \rangle$ ($|-Y_0 \rangle$) base. The preceding pulse(s), which result from $|H \rangle$ polarized excitation pulse(s) are detected in either $|H \rangle$ or $|V \rangle$ polarization, thereby projecting the detected photons on either the $|+X \rangle$ or $|-X \rangle$ basis states.

For the case of spin-photon entanglement $|\psi(\theta)_{\text{DE-1ph}}\rangle$ two-photon correlation measurements are used. In Fig. 4a, we present the measured coincidence rates as a function of $\theta$ for each one of the four possible projections. Two of them, $|+Y_0 \rangle|+X_1 \rangle$ and $|-Y_0 \rangle|-X_1 \rangle$, depend on $\theta$ through $A[1 + D_{\text{RP}}^S \cos(\theta)]$, where $A$ is the average two-photon coincidence rate (see Eqs. (1) and (2)). We call this dependence a “positive $\cos(\theta)$” dependence, referring to the plus sign coefficient of $\cos(\theta)$. The other two projections, $|+Y_0 \rangle|-X_1 \rangle$ and $|-Y_0 \rangle|+X_1 \rangle$, have a “negative $\cos(\theta)$” dependence through $A[1 - D_{\text{RP}}^S \cos(\theta)]$. Similarly, for the case of spin-photon-photon entanglement three-photon correlation measurements are used. We then project the three-qubit (GHZ) state, $|\psi(\theta)_{\text{DE-1ph-2ph}}\rangle$, on different possible polarization basis elements. Four of these projections, namely:

- $|+Y_0 \rangle|+X_1 \rangle|+X_2 \rangle$, $|+Y_0 \rangle|-X_1 \rangle|-X_2 \rangle$,
- $|-Y_0 \rangle|-X_1 \rangle|+X_2 \rangle$, $|-Y_0 \rangle|+X_1 \rangle|-X_2 \rangle$,

have positive $\cos(2\theta)$ dependence and four, obtained simply by flipping all the signs in the expressions above, have negative dependence (see Eq. (3)). Fig. 4b presents the measured three-photon coincidence rates as a function of $\theta$ for all these 8 projections.

The measured $D_{\text{RP}}^{N_{\text{ent}}} (\theta)$ for $N_{\text{ent}} = 1$, and 2 as deduced from Fig. 4a and Fig. 4b are given by the data points in Fig. 4c and in Fig. 4d respectively. As can be seen in these figures the measured data points are indeed well described by the form $D_{\text{RP}}^{S_{1}} \cos(\theta)$ and $D_{\text{RP}}^{S_{2}} \cos(2\theta)$ respectively, as presented in the figures by the calculated solid lines.

The observed frequency doubling in the three-photon correlation events as compared with the two-photon correlations, is termed “super-resolution”. It demonstrates the gain in the accuracy of the optical phase measurements, resulting from the use of the $N_{\text{ent}} = 2$ entangled photon state.

We note here that in principle, the function $D_{\text{RP}}^{N_{\text{ent}}} (\theta)$ can be measured directly by our system, without measuring first coincidence rates at various projections. This is because the sign dependence of the measured DRP in a given bunch of $N_{\text{ent}}$ photons can be deduced directly from the measurement results. In each individual bunch measurement $i$, the degree of rectilinear polarization is measured by $D_{\text{RP}} = \frac{N_H - N_V}{N_H + N_V}$ where $N_H$ ($N_V$) is the number of $|H \rangle$ ($|V \rangle$) polarized photons among the detected photons before the last detected one in a given bunch.

We conclude that in principle, the function $D_{\text{RP}}^{N_{\text{ent}}} (\theta)$ can be measured directly by our system, without measuring first coincidence rates at various projections. This is because the sign dependence of the measured DRP in a given bunch of $N_{\text{ent}}$ photons can be deduced directly from the measurement results. In each individual bunch measurement $i$, the degree of rectilinear polarization is measured by $D_{\text{RP}} = \frac{N_H - N_V}{N_H + N_V}$ where $N_H$ ($N_V$) is the number of $|H \rangle$ ($|V \rangle$) polarized photons among the detected photons before the last detected one in a given bunch. The sign of the $\cos(N_{\text{ent}} \theta)$ dependence is given by the sign of the spin projection base $|\pm Y_0 \rangle$ and the parity of the number of photons detected in $|V \rangle$ polarization. The measured degree of rectilinear polarization is therefore given by:

$$D_{\text{RP}} = \sum_{i=1}^{N_{\text{ent}}} \text{sign}(|\pm Y_0 \rangle)(-1)^{N_V} D_{\text{RP}}^{i}$$ (11)

where $N$ is the total number of photons in the experiment.
In the inset to Fig. 4c the measured (diamond-shape marks), and calculated by the process map $D_{R_P}^{N_{ent}}$ (circle-shape marks) are presented. Using the measured and calculated amplitudes in Eq. (5) one finds that the characteristic decay of the $D_{R_P}$ of our state is given by $N_D = 2.2 \pm 0.2$.

We note that the $D_{R_P}$ of the single photon beam that we produced $D_{R_P}^{N_{ent}} \leq 0.4$, is relatively low. The reason for this is attributed to the limited efficiency (~75%) by which we deplete the QD before the DE preparation [30]. This inefficient depletion, can be measured directly by the PL emission intensity at the end of the depletion pulse (see Fig. 3). The limited depletion reduces both the fidelity of DE state preparation and the fidelity of the pulse (see Fig. 3). The limited depletion reduces both the fidelity of DE state preparation and the fidelity of the DE spin projection, resulting in the measured $D_{R_P}^{N_{ent}}$ of 0.4 only. In fact, if one takes this inefficiency into account corrects the initial state for it and applies the process map on a fully depleted QD, the characteristic DRP decay length becomes $N_D = 3.8 \pm 0.2$, as can be seen in the inset to Fig. 4c. With this decay length genuine super-sensitivity of a few percents can be achieved already with $N_{ent} = 2$ entangled photons (and $\eta \geq 0.71$).

To see this we display in Fig. 5 the calculated enhancement of the optical phase resolution relative to the SNL, as a function of $N_{ent}$. We display it for several sources of varying quality, characterized by their DRP characteristic decay lengths, $N_D$. As can be seen in Fig. 5 for a given source quality, an optimum is obtained if the number of entangled photon used ($N_{ent}$) equals half of the characteristic decay length. Using this condition we plot in the inset to Fig. 5 the enhancement in the optical phase measurement with respect to the SNL as a function of the number of entangled photons in a bunch under this condition ($N_{ent} = N_D/2$). The few percent super-resolution that we achieved is represented by the data point in Fig. 5 and in its inset. The case in which all the photons in a given bunch are maximally entangled ($N_D \rightarrow \infty$) is represented in the inset by a dash line (Heisenberg limit).

In summary, we have demonstrated a novel way for achieving super-sensitivity in optical phase measurement using deterministically prepared entangled multi photon GHZ state. We outlined the required conditions for achieving genuine super-sensitivity and showed that there are no conceptual physical barriers which prevent achieving this long-desired technological goal.

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[1] H. Lee, P. Kok, and J. Dowling, Journal of Modern Optics 49, 2325 (2002).
[2] V. Giovannetti, S. Lloyd, and L. Maccone, Nature Photonics 5, 222 EP (2011).
[3] A. Boto, P. Kok, D. Abrams, S. Braunstein, C. Williams, and J. Dowling, Physical Review Letters 85, 2733 (1999).
[4] M. W. Mitchell, J. S. Lundeen, and A. M. Steinberg, Nature 429, 161 (2004).
[5] J. P. Dowling, Contemporary Physics 49, 125 (2008).
[6] I. Afek, O. Ambar, and Y. Silberberg, Science 328, 879 (2010).
[7] P. Walther, J.-W. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni, and A. Zeilinger, Nature 429, 158 (2004).
[8] H.-S. Zhong, Y. Li, W. Li, L.-C. Peng, Z.-E. Su, Y. Hu, Y.-M. He, X. Ding, W. Zhang, H. Li, L. Zhang, Z. Wang, L. You, X.-L. Wang, X. Jiang, L. Li, Y.-A. Chen, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 121, 250505 (2018).
[9] X.-L. Wang, Y.-H. Luo, H.-L. Huang, M.-C. Chen, Z.-E. Su, C. Liu, C. Chen, W. Li, Y.-Q. Fang, X. Jiang, J. Zhang, L. Li, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 120, 260502 (2018).
[10] J.-W. Pan, T. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Zukowski, Reviews of Modern Physics 84, 777 (2008).
[11] K. R. Motes, J. P. Olson, E. J. Rabeaux, J. P. Dowling, S. J. Olson, and P. P. Rohde, Phys. Rev. Lett. 114, 170802 (2015).
[12] E. Dekel, D. Gershoni, E. Ehrenfreund, J. M. Garcia, and P. M. Petroff, Phys. Rev. B 61, 11009 (2000).
[13] N. Somaschi, V. Giesz, L. De Santis, J. C. Loredo, M. P. Almeida, G. Hornecker, S. L. Portalupi, T. Grange, C. Antón, J. Demory, C. Gómez, I. Sagnes, N. D. Lanzillotti-Kimura, A. Lemaître, A. Auffeves, A. G. White, L. Lanco, and P. Senellart, Nature Photonics 10, 340 EP (2016).
[14] X. Ding, Y. He, Z.-C. Duan, N. Gregersen, M.-C. Chen, S. Unsleber, S. Maier, C. Schneider, M. Kamp, S. Höfling, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 116, 020401 (2016).
[15] N. Akopian, N. H. Lindner, E. Poem, Y. Berlatzky, J. Avron, D. Gershoni, B. D. Gerardot, and P. M. Petroff, Phys. Rev. Lett. 96, 130501 (2006).
[16] R. J. Young, R. M. Stevenson, P. Atkinson, K. Cooper, D. A. Ritchie, and A. J. Shields, New Journal of Physics 8, 29 (2006).
[17] M. Müller, S. Bounouar, K. D. Jöns, M. Glässl, and P. Michler, Nature Photonics 8, 224 (2014).
[18] R. Winik, D. Cogan, Y. Don, I. Schwartz, L. Gantz, E. R. Schmidgall, N. Livneh, R. Rappaport, E. Buks, and D. Gershoni, Phys. Rev. B 95, 235435 (2017).
[19] J. Liu, R. Su, Y. Wei, B. Yao, S. F. C. d. Silva, Y. Yu, J. Iles-Smith, K. Srinivasan, A. Rastelli, J. Li, and X. Wang, Nature Nanotechnology 14, 586 (2019).
[20] H. Wang, H. Hu, T.-H. Chung, J. Qin, X. Yang, J.-P. Li, R.-Z. Liu, H.-S. Zhong, Y.-M. He, X. Ding, Y.-H. Deng, Q. Dai, Y.-H. Huo, S. Höfling, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 122, 113602 (2019).
[21] A. J. Bennett, J. P. Lee, D. J. P. Ellis, T. Meany, E. Murray, F. F. Floether, J. P. Griffiths, I. Farrer, D. A. Ritchie, and A. J. Shields, Science Advances 2 (2016).

[22] M. Müller, H. Vural, C. Schneider, A. Rastelli, O. G. Schmidt, S. Höfling, and P. Michler, Phys. Rev. Lett. 118, 257402 (2017).

[23] J. P. Olson, K. R. Motes, P. M. Birchall, N. M. Studer, M. LaBorde, T. Moulder, P. P. Rohde, and J. P. Dowling, Phys. Rev. A 96, 013810 (2017).

[24] Z.-E. Su, Y. Li, P. P. Rohde, H.-L. Huang, X.-L. Wang, L. Li, N.-L. Liu, J. P. Dowling, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 119, 080502 (2017).

[25] K. J. Resch, K. L. Pregnall, R. Prevedel, A. Gilchrist, G. J. Pryde, J. L. O’Brien, and A. G. White, Phys. Rev. Lett. 98, 223601 (2007).

[26] T. Nagata, R. Okamoto, J. L. O’Brien, K. Sasaki, and S. Takeuchi, Science 316, 726 (2007).

[27] I. Schwartz, D. Cogan, E. R. Schmidgall, Y. Don, L. Gantz, O. Kenneth, N. H. Lindner, and D. Gershoni, Science 354, 434 (2016).

[28] I. Schwartz, E. R. Schmidgall, L. Gantz, D. Cogan, E. Bordo, Y. Don, M. Zielinski, and D. Gershoni, Phys. Rev. X 5, 011009 (2015).

[29] D. Cogan, O. Kenneth, N. H. Lindner, G. Peniakov, C. Hopffmann, D. Dalacu, P. J. Poole, P. Hawrylak, and D. Gershoni, Phys. Rev. X 8, 041050 (2018).

[30] E. R. Schmidgall, I. Schwartz, D. Cogan, L. Gantz, T. Heindel, S. Reitzenstein, and D. Gershoni, Applied Physics Letters 106, 193101 (2015).

[31] D. Cogan, G. Peniakov, Z.-E. Su, and D. Gershoni, Phys. Rev. B 101, 035424 (2020).