Private Authentication with Physical Identifiers Through Broadcast Channel Measurements

Onur Günlü¹, Rafael F. Schaefer¹, and Gerhard Kramer²

¹Information Theory and Applications Chair, Technische Universität Berlin, 
{guenlue, rafael.schaefer}@tu-berlin.de
²Chair of Communications Engineering, Technical University of Munich, 
gerhard.kramer@tum.de

Abstract—A basic model for key agreement with biometric or physical identifiers is extended to include measurements of a hidden source through a general broadcast channel (BC). An inner bound for strong secrecy, maximum key rate, and minimum privacy-leakage and database-storage rates is proposed. The inner bound is shown to be tight for physically-degraded and less-noisy BCs.

I. INTRODUCTION

Secret key generation from biometric or physical identifiers such as fingerprints, uncontrollable fine variations of ring oscillator (RO) outputs, or random start-up values of static random access memories (SRAMs) is a promising alternative to key storage in a non-volatile memory (NVM) [1]. Physical identifiers for digital devices, such as Internet-of-Things (IoT) devices, can be implemented using physical unclonable functions (PUFs) [1]. One can also use PUFs at the transmitter as a source of local randomness [2, Chapter 1]. For instance, consider the wiretap channel [3] where a transmitter sends a message through a broadcast channel (BC) [4] so that a legitimate receiver can reliably decode the message, while the message is hidden from an eavesdropper by using a randomized encoder with random bits supplied by a PUF.

We use the source model for key agreement from [5], [9] to characterize rate regions for PUFs. Based on a source observation, an encoder generates a key and sends helper data to a decoder, so the key agreement at the decoder is successful. The helper data can be made public as long as the information leaked about the secret key, i.e., secrecy leakage, is negligible and the information leaked about the identifier output, i.e., privacy leakage, is small [7], [8]. The amount of storage should also be kept small to limit the hardware cost [9].

Suppose the encoder generates a key from a noisy identifier output and a decoder has access to another noisy measurement of the same identifier and the helper data to reconstruct the key. We call this model the generated-secret (GS) model. Similarly, for the chosen-secret (CS) model, an embedded key and noisy identifier measurements are combined to generate the public helper data. We consider both models to address different applications.

The source, noisy identifier, and measurement symbol strings are related by a BC with one input and two outputs. In [10], [11], the BC consists of two separate measurement channels. We extend this model to capture the effects of correlated noise in the measurements. As an example for this case, RO oscillation frequencies depend on the surrounding hardware logic, so the encoder and decoder measurements of the same RO PUFs tend to be correlated [12].

We derive achievable key-leakage-storage rate tuples for a general BC with strong secrecy. The separate-measurement case in [10], [11] corresponds to a physically-degraded (PD) BC, and the visible source model in [7], [8] corresponds to a semi-deterministic BC. The rate region for another PD BC scenario is given in this work, which does not follow from previous results due to the asymmetry in the constraints. We further establish the rate regions for less-noisy (LN) BCs and derive results with strong privacy and strong secrecy. We remark that in [7], [8], [13], a “private” key that is available to the encoder and decoder and that is hidden from the eavesdropper is considered to obtain strong privacy. This assumption is not realistic since a private key requires hardware protection against physical attacks, and if such a protection is feasible then it is not necessary to use a PUF for key generation.

This paper is organized as follows. In Section II we describe our models and the problem. We give achievable key-leakage-storage regions for the GS and CS models in Section III. The proposed inner bounds are shown in Section IV to be tight for classes of PD and LN BCs. We give an example in Section V to illustrate the effects of delay and hardware cost on the achieved rate tuples for a practical PUF design. Achievability proofs for the inner bounds and converses for a LN case are given in Sections VII and VIII respectively.

II. PROBLEM DEFINITIONS

Consider hidden identifier outputs X^n that are independent and identically distributed (i.i.d.) with respect to a probability distribution P_X. The encoder and decoder observe noisy source measurements that are outputs of a BC P_{\tilde{X}|X}. Suppose all alphabets are finite sets.

For the GS model shown in Fig. 1(a), the encoder generates the helper data W and a secret key S from \tilde{X}^n. The keys are stored in a secure database and the helper data are stored in a public database so that an eavesdropper has access only to the
helper data. This is the case for PUFs when the helper data are stored without hardware protection. The eavesdropper does not observe a random sequence correlated with the identifier output. This is a valid assumption because for many physical and biometric identifiers, invasive attacks would permanently change the identifier outputs [14]. Furthermore, there are algorithms such as in [15] to obtain almost i.i.d. outputs measured through memoryless channels.

Using $W$ and the BC measurements $Y^n$, the decoder generates the key estimate $\hat{S}$ that is used for device authentication. Similar steps are applied for the CS model in Fig. 1(b), except that $S$ is embedded into the encoder.

**Definition 1.** A $(|\mathcal{V}(n)|, |\mathcal{S}(n)|, n)$-code $C_n$ for key agreement with a hidden source with BC encoder and decoder measurements consists of

- a GS model encoder $f^{(n)}_1: \hat{X}^n \to \mathcal{W}(n) \times \mathcal{S}(n)$ or a CS model encoder $f^{(n)}_2: \hat{X}^n \times \mathcal{S}(n) \to \mathcal{W}(n)$,
- a decoder $g^{(n)}: \mathcal{V}(n) \times Y^n \to \mathcal{S}(n)$.

**Definition 2.** A (secret-key, privacy-leakage, storage), or (key-leakage-storage), rate tuple $(R_s, R_t, R_w)$ is achievable for the GS and CS models with encoder and decoder measurements through the BC $P_{XY|\hat{X}}$ if, given any $\delta > 0$, there is some $n \geq 1$, and an encoder and a decoder for which $R_s = \frac{\log |\mathcal{S}|}{n}$ and

$$P[r \neq \hat{s}] \leq \delta \quad \text{(reliability)} \quad (1)$$
$$I(S; W) \leq \delta \quad \text{(strong secrecy)} \quad (2)$$
$$\frac{1}{n} I(X^n; W) \leq R_t + \delta \quad \text{(privacy)} \quad (3)$$
$$\frac{1}{n} H(S) \geq R_s - \delta \quad \text{(uniformity)} \quad (4)$$
$$\frac{1}{n} \log |W| \leq R_w + \delta \quad \text{(storage)} . \quad (5)$$

The key-leakage-storage regions $C_{gs}$ for the GS model and $C_{cs}$ for the CS model are the closures of the set of all achievable rate tuples.

### III. An Inner Bound

We are interested in characterizing the optimal trade-off among the secret-key, privacy-leakage, and storage rates with strong secrecy for BC measurements at the encoder and decoder. We give achievable rate regions for the GS and CS models in Theorem 1. The proofs are given Section VI.

**Theorem 1** (Inner Bounds for the GS and CS Models). An achievable rate region for the GS model is

$$\mathcal{R}_{gs} = \bigcup_{P_{U|\hat{x}}} \left\{ (R_s, R_t, R_w) : \begin{array}{l}
0 \leq R_s \leq I(U; Y), \\
R_t \geq \max \{0, I(U; X) - I(U; Y)\}, \\
R_w \geq I(U; \hat{X}) - I(U; Y) \end{array} \right\}$$

and an achievable rate region for the CS model is

$$\mathcal{R}_{cs} = \bigcup_{P_{U|\hat{x}}} \left\{ (R_s, R_t, R_w) : \begin{array}{l}
0 \leq R_s \leq I(U; Y), \\
R_t \geq \max \{0, I(U; X) - I(U; Y)\}, \\
R_w \geq I(U; \hat{X}) \end{array} \right\}$$

where $U - \hat{X} - XY$ forms a Markov chain for $\mathcal{R}_{gs}$ and $\mathcal{R}_{cs}$.

Let $\mathcal{R}_1$ and as $\mathcal{R}_2$ be the rate regions characterized by (6)-(8) when $\max \{0, I(U; X) - I(U; Y)\}$ is positive and zero, respectively. One can limit the cardinality of $U$ to $|U| \leq |\hat{X}| + 2$ for $\mathcal{R}_1$ and to $|U| \leq |\hat{X}| + 1$ for $\mathcal{R}_2$. Similarly, let $\mathcal{R}_3$ and $\mathcal{R}_4$ be the rate regions characterized by (9)-(11) when $\max \{0, I(U; X) - I(U; Y)\}$ is positive and zero, respectively. One can limit the cardinality of $U$ to $|U| \leq |\hat{X}| + 2$ for $\mathcal{R}_3$ and to $|U| \leq |\hat{X}| + 1$ for $\mathcal{R}_4$.

All regions are convex, and permitting randomization at the encoder and decoder does not improve the regions (see (13)(a) and (13)(b)).

### IV. Rate Regions

Suppose we view $\hat{X}^n$ as the input to the BC $P_{XY|\hat{X}}$. This perspective lets us use known results for the classic BC problem because the channel input $\hat{X}^n$ can be viewed as the encoder “output”. We characterize the key-leakage-storage regions of special classes of BCs $P_{XY|\hat{X}}$ for which the rate regions given in Theorem 1 are tight.

The key agreement model used in [17], [18], which is called the visible source model, corresponds to a semi-deterministic BC such that $P_{XY|X} = \mathbb{I}(X = \hat{X})P_{Y|X}$, where $\mathbb{I} \{ \cdot \}$ is the indicator function. The source model considered in [10]. [11] corresponds to a physically-degraded (PD) BC such that $P_{XY|\hat{X}} = P_{X|\hat{X}}P_{Y|X}$.

We show that the regions given in Theorem 1 are tight, i.e., they are the key-leakage-storage regions, for the following cases:

1. a PD BC such that $P_{XY|\hat{X}} = P_{Y|X}P_{X|Y}$, which corresponds to the case where the identifier measurements...
at the decoder are better than at the encoder. There are three different Markov chain conditions that can lead to different rate regions due to the asymmetry in the privacy constraint \( [3] \). Note that we consider here a PD BC that has not been studied before.

2) LN BCs \( P_{XY|\bar{X}} \) such that either \( I(U;X) \geq I(U;Y) \) or \( I(U;Y) \geq I(U;X) \) for all \( P_{U|\bar{X}} \) where \( U - \bar{X} - XY \) forms a Markov chain.

**Remark 1.** The rate regions depend on the joint conditional probability distribution \( P_{XY|\bar{X}} \) rather than only the marginal distributions \( P_{X|\bar{X}} \) and \( P_{Y|\bar{X}} \). Thus, the key-leakage-storage regions for the stochastically-degraded BCs are not necessarily equal to the regions for the corresponding PD BCs, unlike in the classic BC problem.

**Remark 2.** Since \( P_{\bar{X}XY} \) is fixed, the distinctions between the LN BCs and essentially-less noisy (ELN) BCs is not needed. Observe that the class of ELN BCs is a proper superset of the class of LN BCs [16 Claim 2].

The key-leakage-storage regions are as follows.

**Theorem 2 (PD BCs).** Suppose \( \bar{X} - Y - X \) forms a Markov chain. We have

\[
C_{gs} = R_2, \quad C_{cs} = R_4. \tag{12}
\]

**Theorem 3 (LN BCs).** Suppose \( P_{XY|\bar{X}} \) is a LN BC with \( I(U;X) \geq I(U;Y) \) for all \( P_{U|\bar{X}} \). We have

\[
C_{gs} = R_1, \quad C_{cs} = R_3. \tag{13}
\]

Suppose \( P_{XY|\bar{X}} \) is a LN BC with \( I(U;Y) \geq I(U;X) \) for all \( P_{U|\bar{X}} \). We have

\[
C_{gs} = R_2, \quad C_{cs} = R_4. \tag{14}
\]

The achievability proofs of Theorems 2 and 3 below follow from Theorem 1. In Section VII, we give the proofs of converses for the BC rate regions in (13). The proofs of the converses for Theorem 2 and the rate regions in (14) follow similarly and are omitted.

**Remark 3.** The regions \( R_2 \) and \( R_4 \) provide strong privacy, i.e., \( I(X^n;W) \) vanishes when \( n \to \infty \).

**V. Example**

We now compare two different decoder-measurement scenarios to address the practicality of PUF designs for multiple measurements. Consider the GS model with noisy measurements of uniformly-distributed and hidden ring oscillator (RO) PUF outputs \( X^n \sim \text{Bern}(\frac{1}{2}) \) through binary symmetric channels (BSCs) \( P_{\bar{X}|X} \) and \( P_{Y|\bar{X}} \) after output quantization.

Since an encoder measurement is made by a manufacturer, the temperature and voltage effects can be eliminated at the encoder, so we assume an encoder measurement through a BSC(0.05), i.e., the crossover probability is \( p_e = 0.05 \). For the decoder measurement, we assume that cheaper hardware is used to decrease the cost. We model this first case as multiple independent decoder measurements each through a BSC(0.15), where the crossover probability is \( p_d = 0.15 \). The measurement delay increases linearly with each additional measurement and with the number of ROs used.

As a second case, we can alternatively provide additional protection against environmental effects, which increases the hardware cost approximately linearly with the number of ROs used [17]. We model this second case as a single decoder measurement through a BSC(0.05), i.e., \( p_d = 0.05 \).

We consider the multiple-measurement case with two, three, and four decoder measurements through a BSC(0.15), and we compare this with a single-measurement through a BSC(0.05).

We remark from [10] Theorem 3 that it suffices to consider only the conditional probability distributions \( P_{\bar{X}|U} \) that are BSCs to characterize the rate regions for these channels. We plot the projections of \( R_3 \) for these scenarios onto the storage vs. privacy-leakage rate plane \( (R_w, R_l) \) in Fig. 2 and onto the storage vs. secret-key rate plane \( (R_w, R_s) \) in Fig. 3 to compare the effects of linear delay and cost increases. Every marker on each curve corresponds to evaluating the rate-region boundaries for a fixed crossover probability \( P_{\bar{X}|U} \), so Figs. 2 and 3 should be considered jointly.

Fig. 2 shows that two BSC(0.15) decoder measurements yield up to 19% less secret-key rate and up to 80% greater
privacy-leakage rate than a single BSC(0.05) decoder measurement. However, four BSC(0.15) decoder measurements result in better rate tuples than a single BSC(0.05) decoder measurement, whereas three BSC(0.15) measurements achieve similar rate points as a single BSC(0.05). Since additional costs and delays increase linearly with the number of ROs, the fair comparison is between the cases with two BSC(0.15) measurements and one BSC(0.05) decoder measurement. Thus, if hardware cost is not a bottleneck, one should improve the system design as in, e.g., [17] to obtain a better decoder-measurement channel $P_{Y|X}$ rather than measuring multiple times at the decoder.

VI. PROOF OF THEOREM 1

We provide a proof that follows from the output statistics of random binning (OSRB) [18] method by applying the steps in [19] Section 1.6.

A. Proof for the GS Model

Proof Sketch: Fix a $P_{U|X}$ and let $(U^n, \tilde{X}^n, X^n, Y^n)$ be i.i.d. according to $P_{U|X}P_XP_{XY}\tilde{X}$. Assign three random bin indices $(S, W, C)$ to each $u^n$, where $S$ is the secret key, $W$ is the helper data, and $C$ is a public index. Assume $S \in [1 : 2^{nR_S}], W \in [1 : 2^{nR_W}]$, and $C \in [1 : 2^{nR_C}]$.

The reliability constraint (1) is satisfied by using a Slepian-Wolf (SW) [20] decoder to estimate $U^n$ from $(C, W, X^n)$ if we have [18] Lemma 1

$$R_c + R_w > H(U|Y).$$

(15)

The strong secrecy and key uniformity constraints in (2) and (4), respectively, are satisfied if [18] Theorem 1

$$R_s + R_w + R_c < H(U)$$

(16)

since (16) ensures that the three random indices $(S, W, C)$ are almost mutually independent and uniformly distributed.

Similarly, the public randomness $C$ is almost independent of $\tilde{X}^n$, so it is almost independent of $(\tilde{X}^n, X^n, Y^n)$, if we have [18] Theorem 1

$$R_c < H(U|\tilde{X}).$$

(17)

To satisfy the constraints (15)-(17), we fix the rates to

$$R_s = I(U; Y) - 2\epsilon$$

(18)

$$R_w = I(U; \tilde{X}) - I(U; Y) + 2\epsilon$$

(19)

$$R_c = H(U|X) - \epsilon$$

(20)

for some $\epsilon > 0$ such that $\epsilon \to 0$ when $n \to \infty$.

Consider the privacy leakage constraint in (3). We have

$$I(X^n; W|C) \leq H(W) - H(W, C|X^n) + H(C).$$

(21)

We need to consider two different cases.

Case 1: Suppose we have

$$R_c + R_w > H(U|X)$$

(22)

i.e., $H(U|Y) \geq H(U|X)$. Then we can recover $U^n$ from $(C, W, X^n)$ by using a SW decoder [18] Lemma 1. Using (21), we have for Case 1 that

$$I(X^n; W|C)$$

$$\leq n(I(U; \tilde{X}) - I(U; Y) + 2\epsilon) + n(H(U|X) - \epsilon)$$

$$\leq n(H(U|Y) + \epsilon) - H(W, C|X^n) + n\epsilon_n$$

$$\leq n(H(U|Y) - H(U|X) + \epsilon + \epsilon_n)$$

$$= n(I(U; X) - I(U; Y) + \epsilon + \epsilon_n)$$

(23)

where (a) follows by (19) and (20), (b) follows from Fano’s inequality applied to estimate $U^n$ from $(C, W, X^n)$, given the constraint in (22), for some $\epsilon_n > 0$ such that $\epsilon_n \to 0$ when $n \to \infty$, and (c) follows because $(U^n, X^n)$ is i.i.d.

Using the selection lemma [21] Lemma 2.2, there exists a binning that achieves all rate tuples $(R_s, R_c, R_w)$ in the key-leakage-storage region $R_1$ with strong secrecy when $n \to \infty$.

Case 2: Suppose we have

$$R_c + R_w < H(U|X)$$

(24)

i.e., $H(U|Y) < H(U|X)$. Then $C$ and $W$ are almost independent given $X^n$ by [18] Theorem 1. We remark that [18] Theorem 1 provides an upper bound on the average variational distance between the joint distribution of $(C, W)$ given $X^n$ and the ideal joint distribution where $C$ and $W$ are independent given $X^n$. The upper bound vanishes when $n \to \infty$.

Using (21), we have for Case 2 that

$$I(X^n; W|C)$$

$$\leq H(W) - (H(W|X^n) + H(C|X^n) - \epsilon_n) + H(C)$$

$$\leq H(W) - H(W|X^n) - (H(C) - \epsilon_n) + H(C) + \epsilon'_n$$

$$\leq H(W) - (H(W) - \epsilon'''_n) + \epsilon'_n + \epsilon'''_n$$

$$= \epsilon'_n + \epsilon'''_n + \epsilon'''_n$$

(25)

where (a) follows by [22] Theorem 6 that provides an upper bound on the entropy difference between two joint probability distributions $P_{CW|X^n}$ and $P_{CW|X^n} = P_{C|X^n}P_{W|X^n}$ with a vanishing variational distance for some $\epsilon_n' > 0$ such that $\epsilon_n' \to 0$ when $n \to \infty$, (b) follows because $C$ is almost independent of $X^n$ due to (17) and by applying [22] Theorem 6 to this almost independence result for some $\epsilon_n'' > 0$ such that $\epsilon_n'' \to 0$ when $n \to \infty$, (c) follows because $R_w < H(U|X)$ for Case 2 so that $W$ is almost independent of $X^n$, so we can apply [22] Theorem 6 also to this case for some $\epsilon_n''' > 0$ such that $\epsilon_n''' \to 0$ when $n \to \infty$.

Using the selection lemma, there exists a binning that achieves all rate tuples $(R_s, R_c, R_w)$ in the key-leakage-storage region $R_2$ with strong secrecy and strong privacy when $n \to \infty$. 


B. Proof for the CS Model

We use the achievability proof for the GS model. Suppose the key $S'$, generated as in the GS model together with the helper data $W'$ and public index $C'$, have the same cardinality as the corresponding embedded secret key $S$, i.e., $|S'| = |S|$. The chosen key $S$ is independent of $(X^n, \tilde{X}^n, Y^n)$. Consider the encoder $f_2^{(n)}$ with inputs $(\tilde{X}^n, S)$ and output $W = (S' + S, W')$. Suppose the decoder $g^{(n)}$ with inputs $(Y^n, W)$ and output $\hat{S} = S' + S - S'$, where all addition and subtraction operations are modulo-$|S|$. The decoder of the GS model is used to obtain $\hat{S}'$.

We have the error probability
\[
\Pr[S \neq \hat{S}] = \Pr[S' \neq \hat{S}']
\] (26)
which is small due to the proof of achievability for the GS model.

Using (13) and (19), and from the one-time padding operation applied above, we can achieve a storage rate of
\[
R_w \geq I(U; \tilde{X})
\] (27)
for the CS model.

We have the secrecy leakage of
\[
I(S; W|C') = I(S; W'|C') + I(S; S' + S|W', C')
\]
\[
\overset{(a)}{=} H(S' + S|W', C') - H(S'|W', C')
\]
\[
\overset{(b)}{=} nR_s - H(S'|C') + I(S'; W'|C')
\]
\[
\overset{(c)}{=} nR_s - (nR_s - \epsilon_n^{(4)}) + I(S'; W'|C')
\]
\[
\overset{(d)}{=} \epsilon_n^{(4)} + \epsilon_n^{(5)}
\] (28)
where (a) follows because $S$ is independent of $(W', C', S')$. (b) follows because $|S'| = |S|$, (c) follows because $S'$ and $C'$ are almost independent and uniformly distributed due to (16) and [22] Theorem 6) provides an upper bound on the entropy differences for some $\epsilon_n^{(4)} > 0$ such that $\epsilon_n^{(4)} \to 0$ when $n \to \infty$, and (d) follows because the GS model satisfies the strong secrecy constraint (2) due to (16) for some $\epsilon_n^{(5)} > 0$ such that $\epsilon_n^{(5)} \to 0$ when $n \to \infty$.

We obtain the privacy-leakage of
\[
I(X^n; W|C') \leq I(X^n; W'|C') + H(S + S'|W', C') - H(S + S'|X^n, W', C', S')
\]
\[
\overset{(a)}{=} I(X^n; W'|C') + \log(|S|) - H(S)
\]
\[
\overset{(b)}{=} I(X^n; W'|C')
\] (29)
where (a) follows because $S$ is independent of $(X^n, W', S', C')$ and $|S'| = |S|$, and (b) follows from the uniformity of $S$. We therefore have the following results for two different cases.

**Case 1:** Assume that $H(U|Y) \geq H(U|X)$ for the fixed $P_{U|\tilde{X}}$. By combining (23) and (29), we obtain
\[
I(X^n; W|C') \leq n(I(U; X) - I(U; Y) + \epsilon + \epsilon_n).
\] (30)

Using the selection lemma, there exists a binning that achieves all rate tuples $(R_s, R_{\ell}, R_w)$ in the key-leakage-storage region $R_3$ with strong secrecy when $n \to \infty$.

**Case 2:** Suppose $H(U|Y) < H(U|X)$ for the fixed $P_{U|\tilde{X}}$. By combining (23) and (29), we have
\[
I(X^n; W|C') \leq \epsilon_n + \epsilon_n' + \epsilon_n''.
\] (31)

Using the selection lemma, there exists a binning that achieves all rate tuples $(R_s, R_{\ell}, R_w)$ in the key-leakage-storage region $R_4$ with strong secrecy and strong privacy when $n \to \infty$.

C. Cardinality Bounds for Theorem 7

Using the support lemma [23], the auxiliary random variable $U$ should have $|\tilde{X}| - 1$ elements to preserve $P_{\tilde{X}}$, and three more to preserve $H(X|U), H(\tilde{X}|U)$ and $H(Y|U)$ for both theorems when $I(U; X) - I(U; Y) > 0$. When $I(U; X) - I(U; Y) \leq 0$, as in $R_2$ and $R_4$, we do not need to preserve the term $H(X|U)$.

VII. CONVERSES FOR THE BC RATE REGIONS IN 13

We use the following lemma to bound the secret-key, privacy-leakage, and storage rates for a class of LN BCs.

**Lemma 1** ([24]). For all LN BCs $P_{XY|\tilde{X}}$ with $I(U; X) \geq 0$ and $P_{U|\tilde{X}}$ and a fixed $P_{\tilde{X}}$, we have
\[
I(S, W, Y^{i-1}; Y_i) \leq I(S, W, X^{i-1}; Y_i)
\] (32)
for $i = 1, 2, \ldots, n$ if $(S, W) - X^n - (X^n, Y^n)$ forms a Markov chain.

Let $U_i \triangleq (S, W, X^{i-1})$, which satisfies the Markov chain $U_i - X_i - (X_i, Y_i)$ for all $i = 1, 2, \ldots, n$.

A. Proofs of Converses

Suppose for some $\delta_0 > 0$ and $n$, there is a pair of encoders and decoders such that (11-5) are satisfied for all LN BCs $P_{XY|\tilde{X}}$ with $I(U; X) \geq 0$ and $P_{U|\tilde{X}}$ and a fixed $P_{\tilde{X}}$ by some key-leakage-storage tuple $(R_s, R_{\ell}, R_w)$.

Using (11) and Fano’s inequality, we obtain
\[
H(S|W, Y^n) \leq H(S|\tilde{S}) \leq n\epsilon_n
\] (33)
where (a) permits randomized decoding. $\epsilon_n = \delta_0 R_s + H_b(\delta_0)/n$, where $H_b(\delta) = -\delta \log \delta - (1 - \delta) \log(1 - \delta)$ is the binary entropy function, and $\epsilon_n \to 0$ if $\delta_n \to 0$.

**Secret-key Rate:** We obtain for the GS and CS models
\[
n(R_s - \delta_n) \leq H(S) - H(S|W, Y^n) + n\epsilon_n
\]
\[
\leq n \sum_{i=1}^n [I(S, W, Y^{i-1}; Y_i) + \epsilon_n + \frac{\delta_n}{n}]
\]
\[
\leq n \sum_{i=1}^n [I(S, W, X^{i-1}; Y_i) + \epsilon_n + \frac{\delta_n}{n}]
\]
\[
\leq n \sum_{i=1}^n [I(U_i; Y_i) + \epsilon_n + \frac{\delta_n}{n}]
\] (34)
(a) follows by (3) and (33), (b) follows by (2), (c) follows from Lemma [1] and (d) follows from the definition of $U_i$.

**Storage Rate:** Observe for the GS model that
\[
n(R_w + \delta_n) \geq H(W|Y^n) + I(W; Y^n)
\]
\[
\geq H(S, W, Y^n) - H(S, W) - H(Y^n) + I(W; Y^n)
\]
\[
\geq I(S, W; \tilde{X}^n) - I(S, W; Y^n) - n\epsilon_n
\]
\[
\geq \sum_{i=1}^{n} [I(S, W, \tilde{X}^{i-1}; \tilde{X}_i) - I(S, W, \tilde{X}^{i-1}; Y_i) - n\epsilon_n]
\]
\[
\geq \sum_{i=1}^{n} [I(U_i; \tilde{X}_i) - I(U_i; Y_i) - n\epsilon_n]
\]
\[
(35)
\]
where (a) follows by (5), (b) follows from the encoding step, (c) follows by (33), (d) follows because the source and channel are memoryless, (e) follows from the data-processing inequality applied to the Markov chain $X^{i-1} - (S, W, X^{i-1}) - \tilde{X}_i$ and from Lemma [1] and (f) follows from the definition of $U_i$.

Similarly, observe for the CS model that
\[
n(R_w + \delta_n) \geq I(S, W; \tilde{X}^n) - H(S, W) - H(S, W|\tilde{X}^n)
\]
\[
\geq I(S, W; \tilde{X}^n) + I(S; W)
\]
\[
\geq \sum_{i=1}^{n} [I(S, W, \tilde{X}^{i-1}; \tilde{X}_i) - I(S, W, X^{i-1}; \tilde{X}_i) - I(S, W, X^{i-1}; Y_i) - n\epsilon_n]
\]
\[
\geq \sum_{i=1}^{n} [I(U_i; \tilde{X}_i) - I(U_i; Y_i) - n\epsilon_n]
\]
\[
(36)
\]
where (a) follows by (5), (b) follows because $\tilde{X}^n$ is independent of $S$, (c) follows because the source and channel are memoryless, (d) follows by applying the data-processing inequality to the Markov chain $X^{i-1} - (S, W, X^{i-1}) - \tilde{X}_i$, and (e) follows from the definition of $U_i$.

**Privacy-leakage Rate:** We have for the GS and CS models that
\[
n(R_l + \delta_n) \geq H(W|Y^n) - H(W|X^n)
\]
\[
\geq H(S, W, Y^n) - H(S, W) - H(Y^n) + H(S, W|X^n)
\]
\[
\geq I(S, W; \tilde{X}^n) - I(S, W; Y^n) - n\epsilon_n
\]
\[
\geq \sum_{i=1}^{n} [I(S, W, \tilde{X}^{i-1}; X_i) - I(S, W, \tilde{X}^{i-1}; Y_i) - \epsilon_n]
\]
\[
\geq \sum_{i=1}^{n} [I(U_i; X_i) - I(U_i; Y_i) - \epsilon_n]
\]
\[
(37)
\]
where (a) follows by (3), (b) follows by (33), (c) follows because the channel and source are memoryless and from Lemma [1] (d) follows from the definition of $U_i$.

Introduce a uniformly distributed time-sharing random variable $Q \sim \text{Unif}[1:n]$ independent of other random variables. Define $X = X_Q$, $\tilde{X} = \tilde{X}_Q$, $Y = Y_Q$, and $U = (U_Q, Q)$ so that $U - \tilde{X} - X - Y$ forms a Markov chain. The converse for all LN BCs $P_{XY|\tilde{X}}$ with $I(U; X) \geq I(U; Y)$ for all $P_{U|\tilde{X}}$ and a fixed $P_{\tilde{X}}$ for the GS model follows by using the introduced random variables in (34), (35), and (36), and letting $\delta_n \to 0$. Similarly, the converse for the same class of channels for the CS model follows by using the introduced random variables in (34), (36), and (37), and letting $\delta_n \to 0$.

**ACKNOWLEDGMENT**

O. Günüllü was supported by the German Research Foundation (DFG) through project KR 3517/9-1. O. Günüllü and R. F. Schaefer are now supported by the German Federal Ministry of Education and Research (BMBF) within the national initiative for "Post Shannon Communication (NewCom)" under the Grant 16KIS1004. G. Kramer was supported by an Alexander von Humboldt Professorship endowed by the BMBF. O. Günüllü thanks Kittipong Kittichokechai for useful discussions.

**REFERENCES**

[1] B. Gassend, “Physical random functions,” Master’s thesis, M.I.T., Cambridge, MA, Jan. 2003.
[2] O. Günüllü, “Key agreement with physical uncloneable functions and biometric identifiers,” Ph.D. dissertation, TU Munich, Germany, Nov. 2018, published by Dr. Hut Verlag.
[3] A. D. Wyner, “The wire-tap channel,” Bell Labs Tech. J., vol. 54, no. 8, pp. 1355–1387, Oct. 1975.
[4] T. M. Cover and J. A. Thomas, Elements of Information Theory, 2nd ed. Hoboken, NJ: John Wiley & Sons, 2012.
[5] R. Ahlswede and I. Csiszár, “Common randomness in information theory and cryptography - Part I: Secret sharing,” IEEE Trans. Inf. Theory, vol. 39, no. 4, pp. 1121–1132, July 1993.
[6] U. M. Maurer, “Secret key agreement by public discussion from common information,” IEEE Trans. Inf. Theory, vol. 39, no. 3, pp. 2733–742, May 1993.
[7] T. Ignatenko and F. M. J. Willems, “Biometric systems: Privacy and secrecy aspects,” IEEE Trans. Inf. Forensics Security, vol. 4, no. 4, pp. 956–973, Dec. 2009.
[8] L. Lai, S.-W. Ho, and H. V. Poor, “Privacy-security trade-offs in biometric security systems - Part I: Single use case,” IEEE Trans. Inf. Forensics Security, vol. 6, no. 1, pp. 122–139, Mar. 2011.
[9] I. Csiszár and P. Narayan, “Common randomness and secret key generation with a helper,” IEEE Trans. Inf. Theory, vol. 46, no. 2, pp. 344–366, Mar. 2000.
[10] O. Günüllü and G. Kramer, “Privacy, secrecy, and storage with multiple noisy measurements of identifiers,” IEEE Trans. Inf. Forensics Security, vol. 13, no. 11, pp. 2872–2883, Nov. 2018.
[11] O. Günüllü, K. Kittichokechai, R. F. Schaefer, and G. Caire, “Controllable identifier measurements for private authentication with secret keys,” IEEE Trans. Inf. Forensics Security, vol. 13, no. 8, pp. 1945–1959, Aug. 2018.
[12] D. Merli, F. Stumpf, and C. Eckert, “Improving the quality of ring oscillator PUFs on FPGAs,” in ACM Workshop Embedded Sys. Security, New York, NY, Oct. 2010, pp. 9:1–9:9.
[13] R. A. Chou, M. R. Bloch, and E. Abbe, “Polar coding for secret-key generation,” IEEE Trans. Inf. Theory, vol. 61, no. 11, pp. 6213–6237, Nov. 2015.
[14] R. Pappu, “Physical one-way functions,” Ph.D. dissertation, M.I.T., Cambridge, MA, Oct. 2001.
[15] O. Günlü, T. Kernetzky, O. İşcan, V. Sidorenko, G. Kramer, and R. F. Schaefer, “Secure and reliable key agreement with physical unclonable functions,” Entropy, vol. 20, no. 5, May 2018.

[16] C. Nair, “Capacity regions of two new classes of two-receiver broadcast channels,” IEEE Trans. Inf. Theory, vol. 56, no. 9, pp. 4207–4214, Sep. 2010.

[17] O. Günlü, O. İşcan, and G. Kramer, “Reliable secret key generation from physical unclonable functions under varying environmental conditions,” in IEEE Int. Workshop Inf. Forensics Security, Rome, Italy, Nov. 2015, pp. 1–6.

[18] M. H. Yassaei, M. R. Aref, and A. Gohari, “Achievability proof via output statistics of random binning,” IEEE Trans. Inf. Theory, vol. 60, no. 11, pp. 6760–6786, Nov. 2014.

[19] M. Bloch, Lecture Notes in Information-Theoretic Security. Atlanta, GA: Georgia Inst. Technol., July 2018.

[20] D. Slepian and J. Wolf, “Noiseless coding of correlated information sources,” IEEE Trans. Inf. Theory, vol. 19, no. 4, pp. 471–480, July 1973.

[21] M. Bloch and J. Barros, Physical-layer Security. Cambridge, U.K.: Cambridge Uni. Press, 2011.

[22] S. Ho and R. W. Yeung, “The interplay between entropy and variational distance,” IEEE Trans. Inf. Theory, vol. 56, no. 12, pp. 5906–5929, Dec. 2010.

[23] I. Csiszár and J. Körner, Information Theory: Coding Theorems for Discrete Memoryless Systems, 2nd ed. Cambridge, U.K.: Cambridge Uni. Press, 2011.

[24] Z. V. Wang and C. Nair, “The capacity region of a class of broadcast channels with a sequence of less noisy receivers,” in IEEE Int. Symp. Inf. Theory, Austin, TX, June 2010, pp. 595–598.