Orbital resonance mode in superconducting iron pnictides

Wei-Cheng Lee and Philip W. Phillips

Department of Physics, University of Illinois - 1110 West Green Street, Urbana, IL 61801, USA

received on 26 August 2013; accepted by S. Savrasov on 2 September 2013
published online 16 September 2013

PACS 74.70.Xa – Pnictides and chalcogenides
PACS 74.25.Gz – Optical properties
PACS 74.20.Mn – Nonconventional mechanisms

Abstract – We show that the fluctuations associated with ferro-orbital order in the \(d_{x^2-y^2}\) and \(d_{yz}\) orbitals can develop a sharp resonance mode in the superconducting state with a nodeless gap on the Fermi surface. This orbital resonance mode appears below the particle-hole continuum and is analogous to the magnetic resonance mode found in various unconventional superconductors. If the pairing symmetry is \(s_\pm\), a dynamical coupling between the orbital ordering and the \(d\)-wave sub-dominant pairing channels is present by symmetry. Therefore, the nature of the resonance mode depends on the relative strengths of the fluctuations in these two channels, which could vary significantly for different families of the iron-based superconductors. The application of our theory to a recent observation of a new \(\delta\)-function-like peak in the \(B_{1g}\) Raman spectrum of \(\text{Ba}_2\text{Fe}_4\text{As}_2\) is discussed, and we predict that the same orbital resonance mode can be detected in electron energy loss spectroscopy (EELS).

Copyright © EPLA, 2013

Introduction. – For high-temperature superconductors, cuprates and iron pnictides, resolving the nature of the fluctuations in both the normal and superconducting states remains a crucial question as it holds the key to the pairing mechanism. In cuprates, due to the strong antiferromagnetism in the parent compounds, it is widely believed that the antiferromagnetic spin fluctuations are the most important ingredient in the pairing mechanism. One of the marquee indicators of this is the magnetic resonance mode [1] observed in the superconducting state of every cuprate. From the BCS theory, the spin-flip susceptibility of an electron scattered from \(k\) to \(k + \vec{q}\) in the superconducting state gains an extra coherence factor which is proportional to \((1 - \text{sgn}(\Delta(\vec{k})))\text{sgn}(\Delta(\vec{k} + \vec{q}))\). Since the gap symmetry of the cuprates is \(d\)-wave symmetric, if \(\Delta(\vec{k}) = -\text{sgn}(\Delta(\vec{k} + \vec{q}))\), the spin excitations near \(\vec{q} = \vec{Q}\) are compatible with superconductivity. It can be further shown [2–6] that a sharp \(\delta\)-function–like resonance mode in the spin-flip susceptibility requires an antiferromagnetic spin interaction to pull the resonance mode to an energy below the particle-hole continuum. In other words, despite some dependence on the detailed electronic structure, the existence of the magnetic resonance mode is predominantly determined by the gap symmetry and the nature of the spin interaction. As a result, it has been identified as an unambiguous [1] indicator that antiferromagnetic spin fluctuations remain strong in the superconducting state and consequently might be the driving mechanism for superconductivity.

In iron pnictides, the structural phase transition followed by stripe-like antiferromagnetism is a robust feature in the phase diagram. The onset of the structural phase transition breaks the \(C_4\) symmetry down to \(C_2\) which enables magnetic order at wave vector \((\pi, 0)\) and orbital order at zero wave vector characterized by unequal occupation in the \(d_{xz}\) and \(d_{yz}\) orbitals. These two unique consequences suggest that the dominant fluctuations might be either orbital based [7–13] or spin based [14–18]. Indeed, a magnetic resonance mode at wave vector \((\pi, 0)\) has been observed in the pnictides and has been attributed to spin fluctuations [1]. However, recent Raman scattering measurements [19], a zero–wave-vector probe, have observed a superconductivity-induced peak in the \(B_{1g}\) channel. Since this peak is sharp and occurs at zero wave vector, for two reasons, it is not likely that it is due to spin fluctuations. First, because spin nematicity corresponds to a breaking of the \(Z_2\) symmetry between spin fluctuations at \((\pi, 0)\) and \((0, \pi)\), the associated fluctuations, if any, reside in the four-spin susceptibility \((T_t(S_z^2(t) - S_y^2(t))(\overline{S_z^2(0)} - \overline{S_y^2(0)})\) [18]. Such a high-order spin susceptibility couples to the Raman vertex via complicated matrix elements.
which cannot produce a sharp delta-function-like peak in the Raman spectroscopy. Moreover, since the Raman peak in question is observed in superconducting samples which are doped away from the parent compounds, the fluctuations associated with the $Z_2$ spin nematicity are likely to diminish, and only the spin fluctuations around $(\pi, 0)$ and $(0, \pi)$ survive. It is indeed true that the Raman spectroscopy can detect large-momentum spin excitations through two-magnon processes, but this usually gives rise to broad peaks [20]. In contrast, since orbital fluctuations reside in the $B_{1g}$ charge channel, a resonance mode develops due to the orbital fluctuations which can certainly be detected by the Raman spectroscopy. Such a resonance mode can, in principle, arise from a sub-dominant $d$-wave pairing channel [19,21,22]. Our key point here is that $d$-wave sub-dominant pairing and ferro-orbital order both have the same space group symmetry of the Raman $B_{1g}$ mode. Consequently, the observed $B_{1g}$ mode in the Raman spectroscopy could arise from orbital fluctuations. We term this mode an orbital resonance mode. We work out the details of this scenario and show that the observed $B_{1g}$ mode could offer an unprecedented fingerprint of ferro-orbital order in the pnictides. The Raman experiments then supplement the current indirect evidence in the non-superconducting states that point-contact spectroscopy [23-25] and neutron scattering measurements [26,27] provide for orbital fluctuations.

Criteria for the orbital resonance mode. – First we give a general discussion for the existence of the orbital resonance mode as was done for the magnetic resonance mode. Given that orbital fluctuations are in the charge channel, the extra coherence factor from BCS theory now becomes $\sim (1+\text{sgn}(\Delta(\mathbf{k})))\text{sgn}(\Delta(\mathbf{k}+\mathbf{q}))$. We immediately see that the sharp difference from the magnetic counterpart is that the prerequisite condition for the orbital resonance mode is $\text{sgn}(\Delta(\mathbf{k})) = \text{sgn}(\Delta(\mathbf{k}+\mathbf{q}))$. Since we focus on the case with wave vectors near zero ($\mathbf{q} \sim 0$), such a condition generally holds in any gap symmetry. In the following, we show that the orbital resonance mode generally exists provided an effective attractive interaction is present in the orbital ordering channel.

Before moving to the microscopic calculations, we prove that the orbital ordering and $d$-wave sub-dominant pairing channels are coupled in an $s$-wave superconducting state. This state of affairs obtains for two crucial reasons. First, because the singlet Cooper pair is a mixture of electrons and holes with different spins, the particle-hole excitations are intrinsically coupled to the particle-particle excitations as long as the space group symmetry allows. Second, both orbital ordering and the $d$-wave pairing channels have a sign change under a rotation of $\pi/2$ but the $s$-wave ground state does not. Therefore, a Berry phase coupling between them is allowed by symmetry as discussed in ref. [21]. This is the key physical principle underlying our work here, and we will show that such a Berry phase coupling enriches the physics of the $B_{1g}$ resonance mode.

Formalism. – We start from a general two-orbital model of the superconducting state for iron pnictides. The model Hamiltonian is given by

$$H = H_0 + H_{SO} + H_{I},$$

$$H_0 = \sum_{\mathbf{k},\sigma} \psi_{\mathbf{k},\sigma}^\dagger \left[ \epsilon_+((\mathbf{k})\hat{I} + \epsilon_-(\mathbf{k})\hat{I}_3 + \epsilon_{xy}(\mathbf{k})\hat{f}_x \right] \psi_{\mathbf{k},\sigma},$$

$$\equiv \sum_{\mathbf{k},\sigma} \phi_{\mathbf{k},\sigma}^\dagger \tilde{D}_{\mathbf{k},\sigma} \phi_{\mathbf{k},\sigma},$$

$$H_{SC} = \sum_{\mathbf{k}} \Delta(\mathbf{k}) \left[ \alpha_{+}^\dagger \alpha_{-} + \beta_{+}^\dagger \beta_{-} + h.c. \right],$$

where $\psi_{\mathbf{k},\sigma}^\dagger \equiv (d_{\mathbf{k},xz,\sigma}^\dagger, d_{\mathbf{k},yz,\sigma}^\dagger)$ are the electron creation operators for orbital $xz$ or $yz$ with spin $\sigma$, $\tilde{D}_{\mathbf{k},\sigma} = \text{diag}(E_{\sigma}^x, E_{\sigma}^y)$ the eigenenergy, and $\phi_{\mathbf{k},\sigma}^\dagger \equiv (\alpha_{\mathbf{k},\sigma}^+, \beta_{\mathbf{k},\sigma}^+)$ are the corresponding eigenvectors. We adopt the model proposed by Raghu et al. [28] for $H_0$ and use the same tight-binding parameters. $H_{SC}$ describes the model field pairing potential in the diagonalized basis and we assume the pairing symmetry to be $s_{\pm}$, which in the two-orbital model is $\Delta(\mathbf{k}) = \Delta_0 \cos(kx) \cdot \cos(ky)$. Without loss of generality, we consider two types of interactions $H_I = H_{OO} + H_{DC}$.

$$H_{OO} = \frac{1}{N} \sum_{\mathbf{q}} V_{OO}(\mathbf{q}) \hat{O}(\mathbf{q}) \hat{O}(\mathbf{q}),$$

$$H_{DC} = \frac{1}{N} \sum_{\mathbf{q}} V_d(\mathbf{q}) \left[ \Delta^d(\mathbf{q}) \right]^\dagger \Delta^d(\mathbf{q}),$$

where

$$\hat{O}(\mathbf{q}) = \sum_{\mathbf{k},\sigma} \psi_{\mathbf{k},\sigma}^\dagger \tilde{D}_{\mathbf{k},\sigma} \psi_{\mathbf{k},\sigma},$$

$$\Delta^d(\mathbf{q}) = \sum_{\mathbf{k}} d_{\mathbf{k}} \left[ \alpha_{\mathbf{k},\sigma}^+ \alpha_{-\mathbf{k},\sigma} + \beta_{\mathbf{k},\sigma}^+ \beta_{-\mathbf{k},\sigma} \right],$$

$$d_{\mathbf{k}} \equiv \cos k_x - \cos k_y.$$
Orbital resonance mode in superconducting iron pnictides

...terms can be obtained from a microscopic multi-orbital Hubbard Hamiltonian.

Since our main purpose is to study the behavior of the fluctuations due to $H_{OO}$ and $H_{SC}$ in a stable $s_{\pm}$ superconductor, we choose $V_{OO}(\vec{q} = 0)$ and $V_{d}(\vec{q} = 0)$ to be close but not exceeding critical values. The procedure for analyzing our model Hamiltonian is well established. First, the Bogoliubov transformation is performed to diagonalize $(H_{0} + H_{SC})$. Second, the random-phase approximation is applied to the two-particle correlation functions to express them in terms of the Bogoliubov quasiparticles. The relevant susceptibility is given by

$$\chi(\vec{q}, \omega)^{-1} = (\chi^{0}(\vec{q}, \omega))^{-1} - \hat{U}_{\vec{q}}, \tag{4}$$

where $\chi^{0}(\vec{q}, \omega), \hat{\chi}(\vec{q}, \omega)$, and $\hat{U}_{\vec{q}}$ are all $3 \times 3$ matrices. The bare correlation functions, $\chi^{0}(\vec{q}, \omega)$, are defined by

$$[\chi^{0}(\vec{q}, \omega)]^{ij}_{\Omega} = \frac{i}{\hbar \Omega} \int_{0}^{\infty} dt e^{i(\omega + i\delta)t} [A_{i}(t), A_{j}^{\dagger}(0)\mid 0], \tag{5}$$

where

$$A_{1}(t) = e^{i(H_{0} + H_{SC})t/\hbar} \hat{\chi}(\vec{q}) e^{-i(H_{0} + H_{SC})t/\hbar},$$

$$A_{2}(t) = e^{i(H_{0} + H_{SC})t/\hbar} \Delta^{d}(\vec{q}) - \Delta^{d\dagger}(\vec{q}),$$

$$A_{3}(t) = e^{i(H_{0} + H_{SC})t/\hbar} \Delta^{d}(\vec{q}) + \Delta^{d\dagger}(\vec{q}), \tag{6}$$

and $\hat{U}_{\vec{q}}$ is the effective interaction kernel,

$$\hat{U}_{\vec{q}} = \begin{pmatrix}
    V_{OO}(\vec{q}) & 0 & 0 \\
    0 & \frac{V_{OO}(\vec{q})}{2} & 0 \\
    0 & 0 & -\frac{V_{d}(\vec{q})}{2}
\end{pmatrix}. \tag{7}$$

To assist with the analysis, we divide the $d$-wave competing pairing channel into phase ($A_{2}$) and amplitude ($A_{3}$) modes in order to enforce the time-reversal symmetry in each of the $A_{1}$ channels. It will be shown later that because all of these three modes couple to each other due to the nature of the superconductivity and the space group symmetry, any theory ignoring these couplings is incomplete and might overlook important physics. For the details, please see the appendix.

**Results.** From now on, we study the case of zero wave vector ($\vec{q} = 0$) and neglect the subscript $\vec{q}$. In principle, our formalism can be applied for finite $\vec{q}$, but detailed knowledge in the $\vec{q}$-dependent interaction will be necessary in order to obtain the correct dispersion of the resonance mode. For proof of principle, an analysis of the zero-wave-vector case suffices. It is now clear, from the form of $[\chi^{0}(\vec{q}, \omega)]^{ij}_{\Omega}$, that the orbital ordering and the $d$-wave pairing channels are coupled because $[\chi^{0}(\omega)]^{ij}_{i}$ is non-zero even for $i \neq j$. In fig. 1, we plot $[\chi^{0}(\omega)]^{12}_{12}$ and $[\chi^{0}(\omega)]^{13}_{13}$ for $\omega$ smaller than the particle-hole continuum edge $\Omega_{ph} \approx 2\Delta(k_{F})$, which show the typical behavior for a Berry phase coupling: $\chi(\omega) \sim \omega$ [6]. A similar coupling occurs in the magnetic resonance mode of the cuprates in which the $\pi$-particle and spin-flip channels are coupled by symmetry [2,5,6].

Since an $s_{\pm}$ superconductor is fully gapped on the Fermi surface, any of the two-particle correlation functions should have zero spectral weight below $\Omega_{ph}$ unless there exists a resonance mode with frequency $\omega_{res}$ which is a solution to

$$\det([\chi(\omega_{res})^{-1}] = 0. \tag{8}$$

If $\Im\chi^{0}(\omega)$ has a sudden increase from zero at $\omega = \Omega_{ph}$, $\Re\chi^{0}(\omega)$ develops a weak logarithmic divergence at $\omega = \Omega_{ph}$ as can be shown from the Kramers-Kronig relations. In this case, eq. (8) is satisfied if there is an effective attractive interaction in the orbital ordering channel, i.e., $V_{OO}(0) < 0$. In other words, an orbital resonance mode can generally arise if the fluctuations from the orbital ordering transition persist as the transition to the finally apped superconducting state obtains. This orbital resonance mode is an analogue of the magnetic resonance mode in the cuprates.

In fig. 2, we plot the spectral weight of the bare correlation function in the orbital ordering channel. Clearly a jump from zero to a finite value occurs at the edge of the particle-hole continuum. This signifies that $\omega_{res}$ always exists below $\Omega_{ph}$ for $V_{OO}(0) < 0$. Let us discuss first the case without a residual $d$-wave pairing interaction $V_{d}(0) = 0$. Generally speaking, if the interaction strength $|V_{OO}(0)|$ is small, $\omega_{res}/\Omega_{ph} \approx 1$ and the spectral weight in this resonance mode is small. On the other hand, there exists a critical strength of $|V_{OO}(0)|$ for which eq. (8) yields a solution of $\omega_{res} = 0$ (in this calculation, this critical strength is $|V_{OO}(0)| \approx 1.2$). This marks the instability towards the orbital ordering transition. Since we are only interested
in the superconducting state without orbital ordering, we have $0 < \omega_{\text{res}}/\Omega_{\text{ph}} < 1$. This behavior is confirmed by our RPA calculations which are summarized in fig. 3.

Note that the orbital resonance mode discussed so far will not be present for a superconducting gap with nodes. The reason is that the orbital resonance mode emerges near zero wave vector. Consequently, the particle-hole continuum edge is roughly $2\Delta_{\text{min}}$, where $\Delta_{\text{min}}$ is the minimal gap on the Fermi surface. If $\Delta_{\text{min}} = 0$, which is the case of the gap symmetry with nodes on the Fermi surface, eq. (8) can never be satisfied. Instead, a peak could still arise, but it will be damped or even completely washed out by the particle-hole continuum, the precise details of which depend on the electronic structure of the material and thus is not symmetry protected. The aforementioned case might be realized in cuprates in which nematic fluctuations (identical to the orbital fluctuations [25]) are argued to be present [33,34] and the $d$-wave gap symmetry has nodes on the Fermi surface. This is in a sharp contrast to the magnetic resonance mode which usually appears near finite wave vector, for example, $(\pi, \pi)$ for the cuprates and $(\pi, 0)$ for iron pnictides. In this case, the particle-hole edge, $\Omega_{\text{ph}}$, could be finite for both nodal and nodeless gap symmetry, and the existence of the magnetic resonance modes is determined primarily by the sign difference between $\Delta(\vec{k})$ and $\Delta(\vec{k} + \vec{q})$.

The Berry phase coupling gives rise to rich physics in the context of the resonance mode. If only one of these two channels has strong fluctuations, the resonance mode has an increased weight in the dominant channel. Therefore, the $B_{1g}$ mode can be viewed as either an orbital resonance mode or a $d$-wave excitonic pairing mode despite the non-zero dynamical coupling between them. If both channels, however, are equally strong, then the resonance mode has comparable weight in both channels. As a result, the nature of the resonance mode becomes plasmonic [5,6], and the resonance frequency tends to be smaller as shown in fig. 3. It is interesting to point out that in iron pnictides, the relative strength of the orbital ordering and the $d$-wave pairing fluctuations could vary significantly from material to material. This means that the nature of the resonance mode discussed above could be different for different families of the iron-based superconductors. In light of this observation, we argue that the interpretation of a $d$-like peak recently found in the Raman scattering measurement on Ba$_2$K$_4$Fe$_2$As$_2$ and Rb$_2$Fe$_1$As$_2$ needs further consideration.

While Kretzschmar et al. [19] have given a detailed fit to their data based on a model consisting of only competing pairing potentials, we would like to offer an alternative viewpoint based on the present theory. We note that from fig. 3 in ref. [19], all of the peaks observed in Rb$_2$Fe$_1$As$_2$ appear both above and below the superconducting transition temperature $T_c$, while in fig. 4(c), the peaks in the $B_{1g}$ symmetry in Ba$_2$K$_4$Fe$_2$As$_2$ are present only in the superconducting state. This indicates that only Ba$_2$K$_4$Fe$_2$As$_2$ has truly superconductivity-induced collective modes of the $B_{1g}$ symmetry. Since the orbital order parameter, $O(q = 0)$, defined in eq. (3) changes sign upon a rotation by $\pi/2$, it has $B_{1g}$ symmetry and consequently the $B_{1g}$ Raman spectroscopy is the ideal probe of the orbital resonance mode discussed in this paper. The first peak at $\omega = 190$ cm$^{-1}$ is the feature of the particle-hole continuum edge, and we interpret the second peak at $\omega = 140$ cm$^{-1}$ to be the orbital resonance mode. For Rb$_2$Fe$_1$As$_2$, because a structural phase transition is not observed and theoretical calculations also suggest an absence of ferro-orbital ordering [35–37], the orbital resonance mode should not exist in this material.
In principle, the orbital resonance mode can be detected either by probes in the charge channel which are sensitive to the $B_1g$ symmetry or electron energy loss spectroscopy (EELS) because of its momentum resolution. EELS can directly measure the density-density correlation function $\chi(\vec{q}, \omega)$ at finite momentum $\vec{q}$ [38]. From a symmetry point of view, if $\vec{q}$ is parallel to either the $\hat{x}$ or $\hat{y}$ directions, $\chi(\vec{q}, \omega)$ will break the $C_4$ symmetry and thus it can couple to orbital fluctuations. On the other hand, if $\vec{q}$ is along the off-diagonal direction with respect to the $\hat{x}$ and $\hat{y}$ axes, $\chi(\vec{q}, \omega)$ will still have $C_4$ symmetry so that it will not couple to orbital fluctuations. As a result, for finite but small momentum $\vec{q}$, EELS can also reveal the orbital resonance mode discussed in this paper.

***

We thank E. Fradkin and A. J. Leggett for helpful discussions. We are also grateful to P. Abbamonte for a very useful discussion on EELS measurements. This work is supported by the Center for Emergent Superconductivity, a DOE Energy Frontier Research Center, Grant No. DE-AC0298CH1088.

**APPENDIX**

Here we provide all the details of the formalism used in the paper. First, the tight-binding Hamiltonian $H_0$ is defined as

$$H_0 = \sum_{\vec{k}, \sigma} \psi_{\vec{k}, \sigma}^{\dagger} \left[ \epsilon_+ (\vec{k}) \hat{I} + \epsilon_- (\vec{k}) \hat{\tau}_3 + \epsilon_{xy} (\vec{k}) \hat{\tau}_x \right] \psi_{\vec{k}, \sigma},$$

(A.1)

where $\psi_{\vec{k}, \sigma} \equiv (d_{\vec{k}, \sigma}^x, d_{\vec{k}, \sigma}^y)$, $\hat{D}_{\vec{k}, \sigma} = \text{diag}(E_{\vec{k}, \sigma}^x, E_{\vec{k}, \sigma}^y)$, and $\hat{D}_{\vec{k}, \sigma} \equiv (\alpha_{\vec{k}, \sigma}^x, \beta_{\vec{k}, \sigma}^y)$. We introduce a unitary transformation $\hat{U}_{\vec{k}, \sigma}$ such that

$$\hat{U}_{\vec{k}, \sigma} \left[ \epsilon_+ (\vec{k}) \hat{I} + \epsilon_- (\vec{k}) \hat{\tau}_3 + \epsilon_{xy} (\vec{k}) \hat{\tau}_x \right] \hat{U}_{\vec{k}, \sigma} = \hat{D}_{\vec{k}, \sigma},$$

(A.2)

and it can be derived that

$$\hat{U}_{\vec{k}, \sigma} = \begin{pmatrix} \cos \theta_k & \sin \theta_k \\ -\sin \theta_k & \cos \theta_k \end{pmatrix},$$

(A.3)

where

$$\cos 2\theta_k = \frac{\epsilon_+ (\vec{k}) - \epsilon_- (\vec{k})}{H (\vec{k})}, \quad \sin 2\theta_k = \frac{\epsilon_{xy} (\vec{k})}{H (\vec{k})}.$$  

(A.4)

The eigenvalues are

$$E_{\vec{k}, \sigma}^x = \epsilon_+ (\vec{k}) + H (\vec{k}), \quad E_{\vec{k}, \sigma}^y = \epsilon_- (\vec{k}) - H (\vec{k}),$$

$$\epsilon_+ (\vec{k}) = \frac{\epsilon_{xz} (\vec{k}) + \epsilon_{yz} (\vec{k})}{2},$$

$$\epsilon_- (\vec{k}) = \frac{\epsilon_{xz} (\vec{k}) - \epsilon_{yz} (\vec{k})}{2},$$

$$H (\vec{k}) = \sqrt{\epsilon_+ (\vec{k})^2 + \epsilon_{xy}^2},$$

(A.5)

and the corresponding eigenvectors can be expressed as $\psi_{\vec{k}, \sigma} = U_{\vec{k}, \sigma} \phi_{\vec{k}, \sigma}$.

The next step is to perform a Bogoliubov transformation for $H' = H_0 + H_{SC}$. We define $\Psi_{\mu, \vec{k}}^\dagger \equiv (\alpha_{\vec{k} + \vec{\epsilon}, \sigma}^x, \alpha_{-\vec{k}, \sigma}^x)$ and $\Psi_{\mu, \vec{k}}^\dagger \equiv (\beta_{\vec{k} + \vec{\epsilon}, \sigma}^y, \beta_{-\vec{k}, \sigma}^y)$. Then we can rewrite

$$H_0 + H_{SC} = \sum_{\vec{k}, \mu, \alpha, \beta} \Psi_{\mu, \vec{k}}^\dagger \left[ E_{\mu, \vec{k}}^x \hat{\tau}_3 + \Delta_{\mu, \vec{k}} \hat{\tau}_1 \right] \Psi_{\mu, \vec{k}},$$

(A.6)

where $\phi_{\vec{k}, \vec{\epsilon}} \equiv (\alpha_{\vec{k} + \vec{\epsilon}, \sigma}^x, \alpha_{-\vec{k}, \sigma}^x)$ and $\phi_{\vec{k}, \vec{\epsilon}} \equiv (\beta_{\vec{k} + \vec{\epsilon}, \sigma}^y, \beta_{-\vec{k}, \sigma}^y)$ are the Bogoliubov quasiparticles in the $\alpha$ or $\beta$ bands. The energies and the corresponding eigenvectors of the Bogoliubov quasiparticles are

$$\Psi_{\mu, \vec{k}} = \left( \begin{array}{c} \cos \omega_{\mu, \vec{k}} \sin \omega_{\mu, \vec{k}} \\ -\sin \omega_{\mu, \vec{k}} \cos \omega_{\mu, \vec{k}} \end{array} \right),$$

(A.8)

where

$$\cos 2\omega_{\mu, \vec{k}} = \frac{E_{\mu, \vec{k}}^x}{E_{SC, \mu, \vec{k}}}, \quad \sin 2\omega_{\mu, \vec{k}} = \frac{\Delta_{\mu, \vec{k}}}{E_{SC, \mu, \vec{k}}}.$$  

(A.9)

All that is left is a series of long but straightforward calculations. We express all the three channels $A_1, A_2, A_3$ in terms of the Bogoliubov quasiparticles $\{\alpha_{\vec{k}, \sigma}^x, \beta_{\vec{k}, \sigma}^y\}$, and then the bare correlation functions are

$$[\chi^{\alpha} (\vec{q}, \omega)]_{IJ} = \frac{1}{\Omega} \sum_\vec{k} \left[ -\frac{A_I A_J}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^x + E_{SC, \vec{k}}^x)} \right] B_{IJ}$$

$$\left[ \frac{C_I C_J}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^y + E_{SC, \vec{k}}^y)} \right]$$

$$+ \left[ \frac{D_I D_J}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^z + E_{SC, \vec{k}}^z)} \right]$$

$$+ \left[ \frac{1}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^\alpha + E_{SC, \vec{k}}^\alpha)} \right]$$

$$+ \left[ \frac{1}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^\beta + E_{SC, \vec{k}}^\beta)} \right]$$

$$+ \left[ \frac{1}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^\gamma + E_{SC, \vec{k}}^\gamma)} \right]$$

$$+ \left[ \frac{1}{\hbar \omega + i\delta - (E_{SC, \vec{k} - \vec{q}}^\delta + E_{SC, \vec{k}}^\delta)} \right]$$

(A.10)
where
\[ A_1 = B_1 = \cos(\theta_{k - \mathbf{q}} + \theta_{k}) \times \sin \left( \omega_{\alpha, \mathbf{k}} + \omega_{\alpha, \mathbf{k} - \mathbf{q}} \right), \]
\[ A_2 = -B_2 = -d_{\mathbf{k}} \times \cos \left( \omega_{\alpha, \mathbf{k}} + \omega_{\alpha, \mathbf{k} - \mathbf{q}} \right), \]
\[ A_3 = -B_3 = -d_{\mathbf{k}} \times \cos \left( \omega_{\alpha, \mathbf{k}} - \omega_{\alpha, \mathbf{k} - \mathbf{q}} \right), \]
\[ C_1 = D_1 = -\cos(\theta_{k - \mathbf{q}} + \theta_{k}) \times \sin \left( \omega_{\beta, \mathbf{k}} + \omega_{\beta, \mathbf{k} - \mathbf{q}} \right), \]
\[ C_2 = -D_2 = -d_{\mathbf{k}} \times \cos \left( \omega_{\beta, \mathbf{k}} + \omega_{\beta, \mathbf{k} - \mathbf{q}} \right), \]
\[ C_3 = -D_3 = -d_{\mathbf{k}} \times \cos \left( \omega_{\beta, \mathbf{k}} - \omega_{\beta, \mathbf{k} - \mathbf{q}} \right), \]
\[ E_1 = \sin(\theta_{k - \mathbf{q}} + \theta_{k}) \times \sin \left( \omega_{\alpha, \mathbf{k}} + \omega_{\alpha, \mathbf{k} - \mathbf{q}} \right), \]
\[ E_2 = E_3 = 0, \]
\[ F_1 = \sin(\theta_{k - \mathbf{q}} + \theta_{k}) \times \sin \left( \omega_{\alpha, \mathbf{k}} + \omega_{\beta, \mathbf{k} - \mathbf{q}} \right), \]
\[ F_2 = F_3 = 0. \quad (A.11) \]

REFERENCES

[1] Scalapino D. J., Rev. Mod. Phys., 84 (2012) 1383.
[2] Demler E., Kohno H. and Zhang S.-C., Phys. Rev. B, 58 (1998) 5719.
[3] Brinkmann J. and Lee P. A., Phys. Rev. Lett., 82 (1999) 2915.
[4] Tchernyshyov O., Norman M. R. and Chubukov A. V., Phys. Rev. B, 63 (2001) 144507.
[5] Lee W.-C., Sinova J., Borkov A. A., Joglekar Y. and MacDonald A. H., Phys. Rev. B, 77 (2008) 245148.
[6] Lee W.-C. and MacDonald A. H., Phys. Rev. B, 78 (2008) 174506.
[7] Lv W., Wu J. and Phillips P., Phys. Rev. B, 80 (2009) 224506.
[8] Krüger F., Kumar S., Zaanen J. and van den Brink J., Phys. Rev. B, 79 (2009) 054504.
[9] Lee W.-C. and Wu C., Phys. Rev. Lett., 103 (2009) 176101.
[10] Lee C.-C., Yin W.-G. and Xu W., Phys. Rev. Lett., 103 (2009) 267001.
[11] Chen C.-C., Maciejko J., Sorini A. P., Moritz B., Singh R. R. P. and Devereaux T. P., Phys. Rev. B, 82 (2010) 100540.
[12] Lv W., Krüger F. and Phillips P., Phys. Rev. B, 82 (2010) 045125.
[13] Kontani H., Saito T. and Onari S., Phys. Rev. B, 84 (2011) 024528.
[14] Yildirim T., Phys. Rev. Lett., 101 (2008) 057010.
[15] Xu C., Müller M. and Sachdev S., Phys. Rev. B, 78 (2008) 020501.
[16] Fang C., Yao H., Tsai W.-F., Hu J. and Kivelson S. A., Phys. Rev. B, 77 (2008) 224509.
[17] Fernandes R. M., VanBebber L. H., Bhattacharya S., Chandra P., Keppens V., Mandrus D., McGuire M. A., Sales B. C., Sefat A. S. and Schmalian J., Phys. Rev. Lett., 105 (2010) 157003.
[18] Fernandes R. M., Chubukov A. V., Knolle J., Eremeev I. and Schmalian J., Phys. Rev. B, 85 (2012) 024534.
[19] Kretzschmar F., Muschler B., Bohm T., Baum A., Hackl R., Wen H.-H., Tsurkan V., Deisenhofer J. and LoiDL A., Phys. Rev. Lett., 110 (2013) 187002.
[20] Devereaux T. P. and Hackl R., Rev. Mod. Phys., 79 (2007) 175.
[21] Lee W.-C., Zhang S.-C. and Wu C., Phys. Rev. Lett., 102 (2009) 217002.
[22] Scalapino D. J. and Devereaux T. P., Phys. Rev. B, 80 (2009) 140512.
[23] Arham H. Z., Hunt C. R., Park W. K., Gillett J., Das S. D., Sebastian S. E., Xu Z. J., Wen J. S., Lin Z. W., Li Q., Gu G., Thaler A., Budko S. L., Canfield P. C. and Greene L. H., arXiv:1108.2749 (2011).
[24] Arham H. Z., Hunt C. R., Park W. K., Gillett J., Das S. D., Sebastian S. E., Xu Z. J., Wen J. S., Lin Z. W., Li Q., Gu G., Thaler A., Ran S., Bud’ko S. L., Canfield P. C., Chung D. Y., Kanatzidis M. G. and Greene L. H., Phys. Rev. B, 85 (2012) 214515.
[25] Lee W.-C. and Phillips P. W., Phys. Rev. B, 86 (2012) 245113.
[26] Xu Z., Wen J., Zhao Y., Matsuda M., Ku W., Liu X., Gu G., Lee D.-H., Birgeneau R. J., Tranquada J. M. and Xu G., Phys. Rev. Lett., 109 (2012) 227002.
[27] Lee W.-C., Lv W., Tranquada J. M. and Phillips P. W., Phys. Rev. B, 86 (2012) 094516.
[28] Raghu S., Qi X.-L., Liu C.-X., Scalapino D. and Zhang S.-C., Phys. Rev. B, 77 (2008) 220503(R).
[29] Kuboki K., Onari S., Arta R., USui H., Tanaka Y., Kontani H. and Aoki H., Phys. Rev. Lett., 101 (2008) 087004.
[30] Wang F., Zhai H., Ran Y., Vishwanath A. and Lee D.-H., Phys. Rev. Lett., 102 (2009) 047005.
[31] SEO K., Bernevig B. A. and Hu J., Phys. Rev. Lett., 101 (2008) 206404.
[32] Graser S., Maier T. A., Hirschfeld P. J. and Scalapino D. J., New J. Phys., 11 (2009) 025016.
[33] Oganesyan V., Kivelson S. A. and Fradkin E., Phys. Rev. B, 64 (2001) 195109.
[34] Lawler M. J., Barci D. G., Fernández V., Fradkin E. and Oxman L., Phys. Rev. B, 73 (2006) 085101.
[35] Lv W., Lee W.-C. and Phillips P., Phys. Rev. B, 84 (2011) 155107.
[36] Luo Q., Nicholson A., Riera J., Yao D.-X., Moreo A. and Dagotto E., Phys. Rev. B, 84 (2011) 140506.
[37] Yin W.-G., Lin C.-H. and Ku W., Phys. Rev. B, 86 (2012) 081106.
[38] Garcia de Abajo F. J., Rev. Mod. Phys., 82 (2010) 209.