Equivalent Model of Active Distribution Network Considering Uncertainties of Wind Turbines, Photovoltaics and Loads

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Abstract. With the integration of large-scale distributed renewable energy, the model of active distribution network (ADN) becomes more complicated. This paper proposes an equivalent ADN model considering the uncertainties of wind turbines, photovoltaic (PV) systems and loads. The model output is composed of deterministic and uncertain components. First, the model of wind turbines, PV systems and loads are developed to describe the deterministic components and the Gaussian probability density model is used to describe the uncertain components. Then, model parameters are optimized using a reinforcement learning algorithm to track the time-varying equivalent wind turbines, PV systems and loads. Finally, case studies are carried out on a 63-bus distribution network. Simulation results demonstrate that errors of power flow calculation of the equivalent ADN model are reduced in the case that the uncertainties of wind turbines, PV systems and loads are considered.

1. Introduction
Distributed generation (DG) has the advantages of saving investment, high efficiency and variety of energy sources. Therefore, integrating the distributed generation system into the existing distribution system is the development trend of distributed generation in the future. However, the uncertainties of a large number of distributed power generation such as wind turbines, PV systems and load are significant features of an active distribution network (ADN). Therefore, it is of great importance to develop accurately equivalent ADN models.

Due to a large number of sections, branches, buses and distributed generations, the detailed real-time model of ADN is hardly obtained. Several studies have been reported on the equivalent models of ADN. Equivalent models for simplifying distribution networks can be divided into a dynamic equivalent model and a static equivalent model in terms of different conditions. For a static equivalent model, the distributed network is simplified to a ZIP model ((constant-impedance (Z), constant-current (I) and a constant-power (P)) or constant power load model (PQ model)[1]. However, those models can not reflect detailed behavior for different DG, therefore, more studies analyze the influence of each type of DG and consider the uncertainties of renewable energy resources. The equivalent generator can retain the reactive power support characteristics of DGs[2]. A negative load was used to replace distributed wind turbines and PV systems in the combination of loads and DG[3-4]. Equivalent voltage sources to represent all the DG installed in the ADN was proposed and the system loads was replaced by the equivalent impedances in both the series and parallel branches[5]. In[5], the synthesized model was applied for the place where distribution networks are unbalanced by obtaining real-time measurements. Zhang et al proposed a novel distributed generation planning methodology in active distribution network[6]. In[6], according to the idea of decomposition and coordination, the planning model was
converted to a three-layer programming model considering both demand side management and network reconfiguration. Dai et al proposed a two-tier static equivalent model, which is also composed of the equivalent generator, branch and load[7]. In the upper tier equivalent model, the consistency of sensitivity and power loss are considered, while in the lower tier equivalent model, static load characteristics are considered.

The uncertainties of wind turbines and PV systems are often ignored in ADN modeling. It was assumed that a parameterized model replaced the individual behavior of Inverter-Based Generators (IBGs) and loads, but the values of its parameters are uncertain, which are to be optimized[8]. Uncertainties affecting the parameters of the component models are normally addressed using Monte-Carlo simulations. References[9-10]described the forecasting uncertainties based on the past measurements for load, wind and solar generation. Their research ignored the impact of different spatial locations. Due to the continuous growth of distributed generation, the scale is becoming larger and larger. The joining and exiting of distributed generation cause the waste of resources. Therefore, the establishment of an ADN model considering the spatial uncertainties can effectively save the economic cost. In the meantime, distributed generation expresses different characteristics over time, and the model parameters must be updated in real time to track changing features.

The work of this paper can be summarized as follows.

1) We propose an equivalent model of the ADN, which is composed of wind turbines, PV systems and complex loads. We consider not only the time-varying characteristics of wind turbines and PV systems, but also the spatial uncertainties of them. As a result, the model output is composed of a deterministic component and an uncertain component;

2) In order to estimate the uncertainties of wind turbines and PV systems, we adopt a reinforcement learning algorithm to continuously adjust the model parameters of ADN.

2. Equivalent model of ADN

This section describes an equivalent model for the ADN. Fig 1 presents the proposed equivalent model of the ADN, which is composed of an equivalent wind turbine, an equivalent PV system and an equivalent ZIP load. In other words, the equivalent wind turbine module is equivalent to the superposition effect of all wind turbines in the ADN, as well as the equivalent PV system module and the equivalent ZIP load module.

The total power can be obtained as delivered from the transmission system to ADN. In the meantime, the total generated power of wind turbines and PV system and the total consumed power and loss of load are known. The equivalent power equation of the ADN can be formulated as follows:

\[
P_{in} + jQ_{in} = (P_w + jQ_w) + (P_v + jQ_v) + (P_l + P_{loss}) + (Q_l + Q_{loss})
\]

where \(P_{in} + jQ_{in}\) is the total generated power, \((P_w + jQ_w)\) is the total generated power of wind turbines, \((P_v + jQ_v)\) is the total generated power of PV systems, and \((P_l + P_{loss}) + j(Q_l + Q_{loss})\) is the total consumed power and loss of loads.
Due to the uncertainties of wind speed, solar irradiation and temperature, wind turbine generation and PV system generation have uncertainties. They have not only the uncertainties of the input-output relationship, but also the un-certainties of their spatial position. In the ADN, renewable energy resources equipment’s locations are determined by the demand of users. Take the wind turbine as an example, there are many distributed wind turbines in a distribution network, so it is difficult to install wind speed measuring devices at each distributed wind turbines, which causes that the transmission network dispatcher is unable to obtain the wind speed data of each distributed wind turbine. At this time, it is necessary to select a distributed wind turbine as the observed wind turbine. The real-time wind speed data representing the equivalent wind speed of all distributed wind turbines in the ADN can be obtained by installing wind speed measuring device. Due to the difference between the actual wind speed and the equivalent wind speed of each wind turbine, it leads to the spatial uncertainties of the wind turbine. Therefore, for each renewable energy generation module in the ADN, the injection power of the port is composed of the deterministic power component and the uncertain power component. Next, the two components are introduced in detail.

2.1. Deterministic power component

2.1.1. Equivalent wind turbine. The power delivered by a wind turbine is usually determined by its power curve, where a relation between the wind speed and the power is established. According to the wind turbine power curve, the specific expression of the active power output of the wind turbine is defined as follows[11].

\[
P(v) = \begin{cases} 
0 & \text{if } v \leq v_{ci} \text{ or } v > v_{co} \\
q(v) & \text{if } v_{ci} < v \leq v_r \\
P_r & \text{if } v_r < v \leq v_{co}
\end{cases}
\]

(2)

where \(v_{ci}\) is the cut-in wind speed, \(v_r\) is the rated wind speed, \(v_{co}\) is the cut-out wind speed.

Since the output of all distributed wind turbines is replaced by the output of observed wind turbines, and the cut-in wind speed, rated wind speed and cut-out wind speed of each distributed wind turbine maybe different, this paper redefines the wind speed segment node. The anti-sine function is taken to fit the second wind speed curve. After the wind speed of the observed wind turbine is given, the deterministic component of the active power output of the equivalent wind turbine \(P_w\) is expressed as follows.

\[
P_w = a_4 \arcsin(a_2 v + a_3) + a_4
\]

(3)

where \(v\) is the equivalent wind speed, and \(a_i (i=1,2,3,4)\) is the parameter to be optimized. In this paper, it’s assumed that the distributed wind turbines installed in an ADN operate at unity power factor, therefore, the reactive power output of these wind turbines is zero.

2.1.2. Equivalent PV system. For simulation of PV systems, there are different approaches. Some artificial neural networks have been used for modelling the behavior of PV systems. However, the most common approaches are the ones that model PV systems as circuits. An appropriate circuit models the one that accurately reflects the electrical behavior of PV systems but also not too complex. Therefore, an ideal PV system model, whose deterministic power component expression is taken as follows[12].

\[
V_{mp}(G,T) = V_{mp,STC} + K_v(T - T_{STC}) + V_r \ln(G/G_{STC}) + \alpha \log(G/G_{STC})
\]

(4)

\[
I_{mp}(G,T) = (I_{mp,STC} + K_i(T - T_{STC}))G/G_{STC}
\]

(5)

\[
P_{mp}(G,T) = V_{mp}(G,T)I_{mp}(G,T)
\]

(6)

\[
P_{pv}(G,T) = \eta P_{mp}(G,T)
\]

(7)

where \(K_v, K_i, V_{mp,STC}, I_{mp,STC}, \alpha, \eta\) are the parameters to be optimized. \(G_{STC}\) and \(T_{STC}\) are the solar irradiation and temperature corresponding to the standard test conditions respectively. And
STC 1000W / m², $T_{STC} = 25^\circ$C. $K_V$ and $K_I$ are temperature coefficient of voltage and current. $V_I$ is the diode thermal voltage. Refer to the thermal voltage, $V_I = kT / q$. $k$ is the Boltzmann constant, and $q$ is the electron charge. $\alpha$ is the coefficient of the panel which must vary with temperature and irradiation. $\eta$ is the conversion efficiency of the inverter. In this paper, it’s assumed that the distributed PV systems installed in the ADN operate at unity power factor, so the reactive power output of these PV systems is zero.

2.1.3. Equivalent ZIP load. In this paper, the static ZIP load model is selected to replace the complex loads in an ADN. A common ZIP load model is as follows.

$$ P_i(V) = a_{p1}V + a_{p2}V^2 + a_{p3} $$
$$ Q_i(V) = a_{q1}V^2 + a_{q2}V + a_{q3} $$

where $a_{pi}, a_{qi} (i=1,2,3,4)$ are the parameters to be optimized, while $V$ is the bus voltage.

2.2. Uncertain power component
Due to the time-varying characteristics and the spatial uncertainties of distributed generation, we define the uncertain component as follows.

$$ \hat{y}_y = P_y + \delta \hat{P}_y $$

where $y \in \{w, pv\}$, they are the value of wind turbine and the PV system, respectively. The deterministic component is expressed by $P_y$, which is calculated by mathematical equation. The uncertain component is expressed by $\delta \hat{P}_y$. Without considering the spatial uncertainties of loads, the output of the equivalent ZIP load is not composed of the uncertain component. Take distributed wind turbines as an example, the uncertain component is calculated as follows.

1) According to the mathematical relationship between the equivalent wind speed and the total power described by equation (3), the deterministic component of power output of equivalent wind turbine module can be obtained under a given wind speed;

2) The error $\delta \hat{P}_y$ between the deterministic component and the real measured value can be calculated by equation (9), then the probability density histogram (PDH) of the error is obtained;

3) Fit error’s PDH with probability density function (PDF). The Gaussian probability model shown by equation (10) is taken to describe the probability characteristics of uncertain components[13].

$$ F(\delta \hat{P}_y) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{(\delta \hat{P}_y - \mu)^2}{2\sigma^2} \right] $$

where $\mu$ and $\sigma$ are the parameters to be optimized refer to the error value of real measurements.

3. Online evaluation

3.1. Parameter identification
Since the power characteristics of the load, wind turbine and PV system in an ADN are time-varying, in order to effectively track their time-varying characteristics, we adopt a reinforcement learning algorithm to continuously adjust the model parameters of ADN[14]. Take the wind turbine model online identification as an example, the following is the specific steps of the algorithm.

1) An agent evaluates the power solution according to the wind speed of the observed wind turbine and the total power of the distributed wind turbine;
2) A scale reward or punishment signal (1 or -1) is then fed back to the agent, determining whether
the selected action is beneficial or not, respectively. Meanwhile, the best solution is updated and sent to
the equivalent wind turbine model;
3) The value function is then updated after obtaining reward;
4) The agent chooses the best action among its action based on the updated value functions.
Fig 2 shows the steps for wind turbine parameter identification.

3.2. Objective function
Combined with the meteorological forecast value in the next moment obtained by meteorological
observed points, we take the Latin hypercube sampling (LHS) method on the equivalent model of ADN
updated at the previous moment, and obtain \( n \) groups of output power samples. Then, these samples are
used as the probabilistic power flow of transmission network as shown Fig 3. The voltage amplitude \( V_b \)
and the apparent power \( S_b \) at the boundary bus, and the voltage amplitude \( V_r \) and branch apparent
power \( S_r \) in the transmission network are obtained. In order to verify the accuracy of the equivalent
model, it is compared with the results of probabilistic power flow in the actual system. We choose the
universal root mean square error to represent the objective function of optimization.

\[
F_y = \left[ \frac{1}{n} \sum \left( y_u - y_e(x_u) \right)^2 \right]^{1/2}
\]

(11)

where \( n \) is the total number of samples, \( X_y \) is the parameter to be optimized. \( y_u \) and \( y_e \) represent the
results of the actual system and the equivalent model respectively.

4. Case study

4.1. Simulation system
The case in this paper is a joint system of a coupled transmission network and a distribution system. The
transmission side of the system is a loop network modified from the IEEE 30-bus system. The data of
IEEE 30-bus system can be referred to[15]. Assume that the load on bus 20 of the IEEE 30-bus system is replaced by an active distribution network, the rest data are consistent with that in[15]. The distribution side of coupled transmission and distribution system is an ADN based on IEEE 33-bus system. As shown in Fig 5, the ADN includes ten wind turbines and ten PV systems. The internal load of ADN is set as a ZIP load.

The data on wind speed, solar irradiation and temperature used in this paper is from historical measurements. The sampling interval of data is 5 minutes and simulation time is 24 h. Suppose that the total output power of distributed wind turbines, the total output power of distributed PV systems, the voltage amplitude at bus 31 and the injected power, as well as the total power injected into the ADN are known. Selecting bus 42 as the observed bus, we know the wind speed, solar irradiation and ambient temperature of bus 42.

4.2. Accuracy evaluation

The spatial uncertainties of renewable energy resources in the actual system is simulated according to the existing wind speed and solar irradiation data. Take the wind speed as an example, the equivalent wind speed $v$ of the observed wind turbine is known, and the wind speed of the rest wind turbine installed at different locations is calculated. The sampling value of wind speed and the active power output of these distributed wind turbines are calculated by the Gaussian joint distribution.

In order to evaluate the accuracy of the equivalent model of ADN considering the spatial uncertainties, we use the LHS sampling method to sample the Gaussian probability distribution in the equivalent model, and obtain the uncertain components of 500 groups of equivalent wind turbines and equivalent PV system. Given the equivalent wind speed, equivalent solar irradiation and temperature, the deterministic components of equivalent wind turbine and equivalent PV system can be obtained. Finally, the active power output of renewable energy generation can be obtained by adding the deterministic and uncertain components. By inputting 500 groups of samples into the actual system and the equivalent system, the voltage amplitude $V_b$ and the injected apparent power of bus 31 can be obtained. In the meantime, we can also obtain the probability distributions of $V_b$ and $S_b$ for the two systems, which are compared with the actual system.
Fig 6 and Fig 7 show the voltage amplitude and the injected apparent power at the boundary bus in the equivalent system and the actual system. Equivalent* represents an equivalent model that considers the uncertainties, while equivalent denotes an equivalent model that does not consider the uncertainties. We can see the error of equivalent* is smaller than equivalent and it is clear in the apparent power of the boundary bus. We also evaluate the accuracy of the proposed equivalent model using 1000 samples at the hour 8. The PDFs of $V_b$ and $S_b$ are given in Fig 8 and Fig 9 respectively. According to these two figures, we can see that the PDFs of $V_b$ and $S_b$ obtained from the equivalent system and the actual system are almost consistent. It demonstrates that the proposed equivalent model can accurately represent ADN.

![PDF of $V_b$](image1)

![PDF of $S_b$](image2)

Fig 10. Errors of the mean value of $V_r$.

Similarly, in order to evaluate the error of probabilistic power flow in the transmission network of equivalent model of active distribution network considering the spatial uncertainties, we compare the voltage amplitude’s error of 30 buses and the branch power flow’s error, then compare the mean values of errors $E_{mean}$ in the two systems. In Fig 10 and Fig 11, equivalent* represents a model that considers the uncertainties. In the meanwhile, the maximum error of the mean value of $V_r$ is no more than 0.005%, and the minimum value is zero. Furthermore, the errors of the equivalent model that consider the uncertainties is less than the equivalent model that does not consider the uncertainties. The maximum error of the mean value of $S_r$ is no more than 4%, and the minimum value is zero, too. It demonstrates that the proposed equivalent model’s accuracy is high.

5. Conclusion

This paper has proposed an equivalent ADN model considering the uncertainties of wind turbines, PV systems and loads. The equivalent ADN model is composed of an equivalent wind turbine module, an equivalent PV system module and an equivalent ZIP load module. In order to describe the time-varying characteristics and the spatial uncertainties of them, deterministic and uncertain components are
A reinforcement learning algorithm has been developed to optimize model parameters according to real-time measurements. We carry out simulation tests on the boundary bus and the transmission network. The simulation results of the voltage amplitude and the apparent power show that the proposed equivalent ADN model is more accurate than the equivalent model without considering the spatial uncertainties. Moreover, the model proposed in this paper can be applied on a cross border network, when involving in numerous renewable energy resources.

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References
[1] A. Debs. (1975) Estimation of external network equivalents from internal system data, IEEE Trans Power App Syst, vol. 94, no. 2, pp. 273–279.
[2] B. K. Tamimi B, Canizares C. (2013) System stability impact of large-scale and distributed solar photovoltaic generation: the case of ontario canada. IEEE Trans Sustain Energy, vol. 4, no. 3, pp. 680–688.
[3] J. V. Milanovic, K. Yamashita, S. M. Villanueva, S. Z. Djokic, and L. M. Korunovic. (2013) International industry practice on power system load modeling. IEEE Transactions on Power Systems, vol. 28, no. 3, pp. 3038–3046.
[4] A. Samadi, L. Soder, E. Shayesteh, and R. Eriksson. (2015) Static equivalent of distribution grids with high penetration of pv systems. IEEE Transactions on Smart Grid, vol. 6, no. 4, pp. 1763–1774.
[5] F. Mahmood, H. Hooshyar, J. Lavenius, P. Lund, and L. Vanfretti. (2016) Real-time reduced steady state model synthesis of active distribution networks using pmu measurements. IEEE Transactions on Power Delivery, vol. PP, no. 1, pp. 1–1.
[6] S. Zhang, H. Cheng, D. Wang, L. Zhang, F. Li, and L. Yao. (2018) Distributed generation planning in active distribution network considering demand side management and network reconfiguration. Applied Energy, vol.228, pp. 1921–1936.
[7] L. X. L. W. Dai W, Yu J. (2018) Two-tier static equivalent method of active distribution networks considering sensitivity, power loss and static load characteristics. Int J Electr Power Energy Syst, vol. 100, pp. 193–200.
[8] T. V. C. Gilles Chaspiere, Patrick Panciatici. (2018) Modelling active distribution networks under uncertainty: Extracting parameter sets from randomized dynamic responses. In: 2018 Power Systems Computation Conference (PSCC). Dublin. pp. 1-7.
[9] F. Golestaneh, P. Pinson, and H. B. Gooi. (2016) Very short-term nonpara-metric probabilistic forecasting of renewable energy generation with application to solar energy. IEEE Transactions on Power Systems, vol. 31, no. 5, pp. 3850–3863.
[10] F. M. P. M. (2016) Net load forecasting in presence of renewable power curtailment. In: 13th IEEE Int. Conf. on European Energy Market (EEM). Porto. pp. 165–178.
[11] C. Carrillo, A. F. O. Montano, J. Cidras, and E. Diaz-Dorado. (2013) Review of power curve modelling for wind turbines. Renewable and Sustainable Energy Reviews, vol. 21, no. may, pp. 572–581.
[12] N. Rajasekar, N. K. Kumar, and R. Venugopalan. (2013) Bacterial foraging algorithm based solar pv parameter estimation. Solar Energy, vol. 97, no. 10, pp. 255–265.
[13] R. Doherty and M. O’Malley. (2005) A new approach to quantify reserve demand in systems with significant installed wind capacity. IEEE Transactions on Power Systems, vol. 20, no. 2, pp. 587–595.
[14] X. Shang, Z. Li, T. Ji, P. Wu, and Q. Wu. (2017) Online area load modeling in power systems using enhanced reinforcement learning. Energies, vol. 10, no. 11, pp. 1852.

[15] R. W. Ferrero, S. M. Shahidehpour, and V. C. Ramesh. (2002) Transaction analysis in deregulated power systems using game theory. IEEE Transactions on Power Systems, vol. 12, no. 3, pp. 1340–1347.