Modelling and control of electric scooter driven by induction motor

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Abstract. This paper presents the modelling and control of electric scooter driven by single-phase induction motor drive. Single-phase induction motor is employed for this design due to lower cost and little maintenance, although a complex inverter is needed for the vehicle drive system. This paper proposes an integral sliding motor controller and pole placement controller to improve the overall performance of the electric scooter system. The stability of the system using the proposed control strategies are validated with and without disturbances. The proposed controller is validated through MATLAB simulations.

Keywords: Electric Scooter, Single-phase Induction Motor, Speed control, Integral sliding mode controller, and pole placement control

1. Introduction
Depletion of fossil fuels, advancements in power converter technologies, and improved battery charging and management technologies has brought electric vehicles back. Electric vehicles are proven to be one of the promising solutions for the global environmental problem caused by air pollution [1]. The usage of a two-wheeled vehicle is predominant among city commuters due to the increase in personal transportation. Besides, these vehicles have a small footprint [2]. The traditional internal combustion engines in these vehicles produce local emissions and also poor in efficiency.

Electric scooters are becoming popular due to their higher efficiency with zero local emissions. The wide diffusion of electric vehicles demands battery charging infrastructure with fast charging capability [3] and optimal assignment of chargers [4]. Also, the range of electric vehicles depends on the capacity of the batteries. The average usage of urban vehicles is 25 km and it may go up to 40 km and hence extra battery packs can be added to increase the range which increases the cost. Instead, energy management is a more reasonable solution [5]. In [6], a contactless energy transfer system is proposed for electric scooters whereas in [7] dynamic battery swapping technique is proposed as a battery management technique. Due to the advancements in power converters, electric motors such as brushless DC motor, switched reluctance motor and induction motor are used in electric vehicles. Selection of proper electrical propulsion system with optimal energy management and optimal control strategies are important for an electric vehicle. In [8] design of an in-wheel permanent magnet, brushless DC motor is proposed for electric scooter whereas in [9] axial-flux switched reluctance motor for e-scooter is presented. In [10] speed limiter is designed for an electrically assisted kick scooter.

Induction motors have been increasingly applied in electric vehicles due to their remarkable advantages such as simple structure, high reliability, low cost, and high ruggedness. The dynamic model of induction motor is nonlinear and nonlinear control techniques such as direct torque control [11, 12, 13], backstepping, sliding mode control [14] are designed to achieve the high-performance control in electric vehicles. In [15] Lagrangian model of induction motor is used for the torque control in electric vehicles whereas in [16] speed controller is designed using model reference adaptive control. In [17] field-oriented control of multiphase induction motor is proposed. In [18], fault-tolerant cruise control of the induction motor is presented. This paper is organized as follows. Section 2 briefs about the equivalent circuit of induction motor. Section 3 details the design criteria considered for electric scooter.
Section 4 briefs about mathematical modelling of electric vehicle system while section 5 elaborates the design of control techniques. Section 6 depicts the simulation results and finally, section 7 gives some concluding remarks.

2. Motor design
The per-phase equivalent circuit of an induction motor concerning the stator side is given in ‘figure 1’. The speed of the stator revolving flux is given by $N_s = \frac{120f}{P}$; where f is the frequency in Hz and P is the number of poles.

Where,

- $R_1 =$ Resistance of stator
- $X_1 =$ Stator leakage reactance
- $I_1 =$ Stator Current
- $R_c =$ Shunt branch resistance
- $X_m =$ Magnetizing reactance
- $I_e =$ Per-phase no-load current
- $I_2 =$ Rotor Current
- $I_2' =$ Rotor current per phase referred to stator
- $X_2'$ = Standstill rotor reactance referred to stator
- $R_2' =$ Rotor resistance referred to stator
- $V_1 =$ Stator voltage
- $E_i =$ Stator induced emf
- $S =$ Slip

This paper details the development of controllers to regulate the speed of induction motor used in an electric scooter. A single phase 4 pole induction motor which is commonly used in low power applications is considered for the design of the electric scooter. The system is modelled using a state-space approach. The design parameters of the electric scooter are as follows; Total mass is 80kg, Maximum speed withstand is 20Km/hr, acceleration time is 0.5s. the effective wheel diameter is 0.2m.
The motor is fed by an inverter of 230 V ac supply. The motor parameters and the electric scooter parameter required for the EV is given in Table 1.

### Table 1. Parameters for EV system

| Parameters                  | Value                  |
|-----------------------------|------------------------|
| $V_{RMS}$-inverter Voltage  | 230V                   |
| Hz-Supply frequency         | 50Hz                   |
| P-Number of poles           | 4                      |
| J-Moment of inertia         | 0.01 N·ms²/rad         |
| Motor rating                | 0.26kW                 |
| M-Mass                      | 80kg                   |
| $r_{eff}$-Effective wheel radius | 0.1m            |
| a-Acceleration constant     | 9.8m/s²                |
| $R_a$-Resistance            | 3.9Ω                   |
| $L_a$-Inductance            | 0.285H                 |
| $C_d$-Drag coefficient      | 0.2                    |
| $\rho$-Air density          | 0.129                  |
| $A_r$-Cross-sectional area of the vehicle | 0.3m²             |

### 3. Design criteria’s for Dynamic vehicle model

During the normal operating condition, the electric scooter has to overcome the disturbances such as gradient, friction, and wind (aerodynamic drag) resistance[4]. The system modelling is done by considering all the disturbances and the exploded view of an electric scooter as shown in ‘figure 2’

#### 3.1. Friction force

The friction force is an opposing force existing between the wheel and road that restricts the vehicle movement. This force can be calculated as,

$$F_f = Mgf_r$$

$$F_r = 0.01 \left( 1 + \frac{v}{447.09} \right) \text{rad/s}$$

Where M is the mass and g is the acceleration gravity.

#### 3.2. Gradient force

When the vehicle climbs up/down a hill only the gradient force will exist. The direction of movement is projected on the component of vehicle weight.

$$F_g = Mgsin(\alpha)$$

Where M is the mass, g is the acceleration gravity and $\alpha$ is the road gradient.
3.3. Aerodynamic drag force
Aerodynamic drag force is one of the speed limiting factors of a vehicle and it depends on the shape of the vehicle body or its speed.

\[ F_a = 0.5 C_D A_r \rho v^2 \]

Where \( \rho \) is the mass density of air, \( A_r \) is the cross-sectional area of the vehicle, \( C_D \) is the Drag coefficient.

![Figure 2. E-scooter](image)

4. Mathematical modelling of an electric vehicle system
For designing the EV system under dynamic environmental condition, the following equations are considered:
The mathematical modelling of IM is obtained by the physical equations governing the system [5]

The electrical equation of the machine is,

\[ V_a = I_a R_a + JX_a I_a + E_b \]  \hspace{1cm} (1)

The induced back emf \( E_b \),

\[ E_b \propto \frac{d\theta}{dt} \]  \hspace{1cm} (2)

Applying Laplace transforms for (1) and (2),

\[ V_a(s) = I_a(s)(R_a + JX_a) + E_b(s) \]  \hspace{1cm} (3)

\[ E_b = k_t s \theta_s \]  \hspace{1cm} (4)

\( I_a(s) \) is derived by substituting (4) in (3),

\[ V_a(s) = I_a(s)Z_a + k_t s \theta_s \]  \hspace{1cm} (5)

\( I_a(s) = \frac{V_a(s) - k_t s \theta_s}{Z_a} \)

Electrical torque of IM is given by,

\[ T_e = \frac{kE^2R}{R^2 + x^2} \]  \hspace{1cm} (6)

Taking Laplace of equation (6),

\[ T_e = E^2 \sin \theta \]  \hspace{1cm} (7)
Mechanical torque is given by,

\[ T_m = J \ddot{\theta} + B \theta \]  \hspace{1cm} (8)

Where \( B = r_{eff}(F_f + F_g + F_a) \) whereas, \( \theta = s; \ \ddot{\theta} = s^2 \)

Taking the Laplace transform of equation (8),

\[ T_m = Js^2 \theta(s) + r_{eff}(F_f + F_g + F_a)s\theta(s) \]

\[ T_m = (Js^2 + Bs)\theta(s) \] \hspace{1cm} (9)

Combining both electrical and mechanical torque equations, the speed equation of EV is derived and it is detailed as follows

\[ T_e = T_m = E^2\theta(s) = (Js^2 + Bs)\theta(s) \]

\[ T_m = ((Js^2 + Bs) - E^2)\theta(s) \] \hspace{1cm} (10)

\[ \frac{\theta(s)}{T_m} = \frac{1}{Js^2 + Bs - E^2} \] \hspace{1cm} (11)

Also, the torque equation is represented as,

\[ T \propto \varphi I_a \cos \theta \]

we know that,

\[ T = k\varphi I_a \] \hspace{1cm} (12)

Following the torque determination, the transfer function of EV is obtained by considering

Taking the Laplace transform of equation (12),

\[ T = k_a \theta(s)I_a(s) \] \hspace{1cm} (13)

Applying equation (5) & (10) in (13)

\[ T = k_a \left( \frac{V_a(s) - k_1s\theta(s)}{Z(s)} \right) = (Js^2 + Bs - E^2)\theta(s) \]

\[ k_aV_a(s) = (Js^2 + Bs - E^2)\theta(s)Z(s) + k_1k_a s \theta(s) \] \hspace{1cm} (14)

\[ \frac{V_a(s)}{\theta(s)} = \frac{J^2Bs - E^2\theta(s)Z(s) + k_1k_a s}{k_a} \] \hspace{1cm} (15)

\[ \frac{\theta(S)}{V_a(S)} = \frac{K_a}{(Js^2 + Bs - E^2)Z(S) + k_a k_1 S} \]

\[ \frac{\theta(S)}{V_a(S)} = \frac{K_a}{(Js^2 + Bs - E^2)(R_a + L_a S) + k_a k_1 S} \] (16)
\[
\frac{\theta(S)}{V_a(S)} = \frac{K_a}{(JS^2 + r_{\text{eff}}(F_f + F_a + F_g)S - E^2)(R_a + L_aS) + k_a k_t S}
\] (17)

\[
\frac{\theta(S)}{V_a(S)} = \frac{K_a}{(JL_a S^3 + L_a r_{\text{eff}}(F_f + F_a + F_g) + R_a J S^2) + (R_a r_{\text{eff}}(F_f + F_g + F_a) + k_a k_t) S}
\] (18)

\[
\frac{\theta(S)}{V_a(S)} = \frac{K_a}{(JL_a S^3 + a 1 S^2 + a 2 S)}
\] (19)

Equation (19) gives the transfer function model of an electric scooter. The differential equation governing the system is obtained from (19) as follows

\[
(JL_a S^3 + a 1 S^2 + a 2 S) \theta(s) = K_a V_a(s)
\] (20)

\[
JL_a s^3 \theta(s) + a_1 s^2 \theta(s) + a_2 s \theta(s) = k_a V_a(s)
\] (21)

Defining, \(\dddot{\theta} = \dot{Z}_2; \; \ddot{\theta} = \dot{Z}_1; \; \dot{\theta} = Z_1\)

The second-order state equations governing the system (19) is given by

\[
\begin{align*}
\dot{Z}_1 &= Z_2; \quad \dot{Z}_2 = \frac{-a_1 Z_2 - a_2 Z_1 + k_a}{JL_a}
\end{align*}
\] (22)

Where, \(\dot{Z}_2 = \text{speed} ; \; \dot{Z}_1 = \text{acceleration}\)

The second-order system equations in standard form are given as,

\[
\begin{bmatrix}
\dot{Z}_1 \\
\dot{Z}_2 
\end{bmatrix} =
\begin{bmatrix}
\frac{-a_1}{JL_a} & \frac{-a_2}{JL_a} \\
\frac{-a_1}{JL_a} & \frac{-a_2}{JL_a}
\end{bmatrix} \begin{bmatrix}
Z_1 \\
Z_2 
\end{bmatrix} + \begin{bmatrix}
0 \\
k_a 
\end{bmatrix} u
\] (23)

5. Control Techniques
This section details the control design procedure for the speed regulation of electric scooter using pole placement and integral sliding mode control.

5.1. Pole placement controller
The pole placement technique is based on classical control theory [19], in which poles of the closed-loop system are placed at the desired location to obtain stability and desired performance.

\[
\dot{x} = A x + B u, x(0) = x_0
\]
Using equation (23), the system matrices \( A \) and \( B \) are obtained based on the parameters in Table (1) as follows,

\[
A = \begin{bmatrix}
0 & 1 \\
-3.333 & -13.684
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 \\
0.5415
\end{bmatrix}
\]

Using pole placement controller the controller gains \( K_1, K_2 \) are obtained as \( K_1 = 86.181 \) and \( K_2 = 2.430 \). Hence,

\[
V_a = -86.181(z_1 - z_{1\text{des}}) - 2.4303(z_2)
\] (24)

5.2. Sliding mode control

The sliding mode control (SMC) is one of the robust nonlinear control techniques commonly used in uncertain systems due to its ease in design and the ability to discard uncertainties parameter. It can alter the dynamics of a nonlinear system utilizing a switching control. It is designed in two steps; 1. Design of a surface called a sliding surface; 2. Design of a discontinuous control. The design of the surface is such that once the state trajectory is considered on it, the controlled system will exhibit the desired performance\cite{20}. The discontinuous control forces the system to reach the surface in a finite time. Once the system trajectory reaches the surface, it slides along the surface to reach the origin as shown in ‘figure 3’.

![Figure 3. State trajectory of a second-order system under SMC\cite{21}](image)

5.2.1 ISMC design for E-scooter. ISMC is an advanced version of SMC which gives a fast dynamic response to the system during external disturbances \cite{22}-\cite{25}. The ISM control method can ensure less chattering and also provide compensation for both matched and unmatched disturbances.

The sliding surface is defined as

\[
s = c_1 x_1 + c_2 x_2 + c_3 \int (x_1 - x_{1\text{ref}})
\]

Where, \( x_1 = \text{Speed} \); \( x_2 = \text{acceleration} \);

\[
\dot{s} = c_1 \dot{x}_1 + c_2 \dot{x}_2 + c_3(x_1 - x_{1\text{ref}})
\]

Where, \( \dot{x}_1 = z_1 \); \( \dot{x}_2 = z_2 \)

This \( c_1, c_2, c_3 \) are the Surface constant and gain “k” are presented in the “Integral Sliding mode controller” for both with and without disturbance. The values for without disturbance \( c_1 \) is 1 and the
with disturbance $c_1$ is 3, then $c_2$ and $c_3$ values are represented as 0.5 and 0.001. The gain k value for without disturbance is 500 while with disturbance condition to achieve the expected speed to increase the gain value as 1000.

The second-order equation,

$$-k\text{sign}(s) = c_1z_1 + c_2\left(\frac{-a_2}{JL_a}z_1 - \frac{a_1}{JL_a}z_2 + \frac{k}{JL_a}V_a\right) + c_3(x_1 - x_{1\text{ref}})$$

$$V_a = \frac{-k\text{sign}(s) - c_1z_1 - c_2\left(\frac{-a_2}{JL_a}z_1 - \frac{a_1}{JL_a}z_2\right) - c_3(z_2 - z_{2\text{ref}})}{c_2\frac{k}{JL_a}}$$

Whereas, $\dot{z}_2$ = speed ; $\dot{z}_1$ = acceleration

6. Simulation results

The proposed control techniques are validated by numerical simulations in MATLAB. The results obtained are given in this section. The simulation results for pole placement controller are given in ‘figure 4’ and ‘figure 5’. The simulation results for sliding mode controller are given in ‘figure 6’ and ‘figure 7’. The reference speed is chosen as 20km/hr. ‘Figure 5’ and ‘figure 7’ shows that, speed can be maintained to 19.1Km/hr for pole placement and 19.8Km/hr for ISMC.

6.1 Pole placement Controller

![Figure 4. Without disturbance](image-url)
6.2 ISMC Controller

Figure 5. With disturbance

Figure 6. Without disturbance
Comparative analyses of the results show that ISMC (Integral sliding mode controller) gives better performance when compared to the pole placement system.

7. Conclusion
In this paper, modeling and control of electric vehicle driven by a single-phase induction motor has been presented. Simulations have been performed and results validate the effectiveness of proposed nonlinear control. The analysis shows ISMC system design provides a comparatively better response than the pole placement controller. Also, it is observed that the ISMC design offers a perfect speed range even under external disturbances. Hence, it is evident from the simulation results that the Single-phase induction motor can be utilized to drive the electric scooter with good performances.

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