The chiral phase transition from the exact RG

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A brief introduction is given to the concept of the effective average action. Its dependence on the averaging or coarse graining scale is governed by an exact RG equation for which nonperturbative approximation schemes are described. This formalism is applied to the computation of the equation of state for two flavor QCD within an effective linear quark meson model. Our results allow to derive the temperature and quark mass dependence of quantities like the chiral condensate or the pion mass. A precision estimate of the universal critical equation of state for the three–dimensional O(4) Heisenberg model is given. The exact RG formalism applied to the linear quark meson model is demonstrated to provide an explicit link between the O(4) universal behavior near the critical temperature and zero quark mass on one hand and the physics at low temperatures and realistic current quark masses, i.e., the domain of validity of chiral perturbation theory on the other hand.

I. THE CHIRAL TRANSITION AND THE LINEAR QUARK MESON MODEL

The chiral phase transition is very difficult to tackle analytically. The main obstacles which arise are twofold. QCD as the microscopic theory of strong interactions is formulated in terms of quarks and gluons. The IR behavior of strong interactions is, however, dominated by collective degrees of freedom like mesons. For instance, the relevant modes for the chirally broken phase are Goldstone bosons which, in QCD, are quark–antiquark bound states. Furthermore, for the relevant length scales the running gauge coupling is large and a perturbative treatment is questionable.

A popular way to circumvent the first difficulty is the use of effective field theories for the most important degrees of freedom. The most prominent example is chiral perturbation theory based on the nonlinear sigma model which describes the IR behavior of QCD in terms of the Goldstone bosons of spontaneous chiral symmetry breaking. This yields a very successful effective formulation of strong interactions dynamics for momentum scales up to several hundred MeV or temperatures of several 10 MeV. For somewhat higher scales additional degrees of freedom like the sigma meson or the light quark flavors will become important and should be included explicitly. We will therefore rather work with an effective linear quark meson model [4–7]. Here the lightest mesonic degrees of freedom are encoded in a complex scalar field matrix \( \Phi \) which can be thought of as a (color–neutral) quark–antiquark composite, \( \Phi^{ab} \sim \overline{\Psi}^a \gamma^5 \Psi^b_R \), where \( a, b \) label the \( N_f \) light quark flavors. Spontaneous chiral symmetry breaking corresponds to a scalar vacuum expectation value \( \langle \Phi^{ab} \rangle = \pi_0 \delta^{ab} \) with \( \pi_0 \neq 0 \). For the scale–dependent effective action which describes the interactions of the light scalar and pseudoscalar mesons with quarks we make the Ansatz

\[
\Gamma_k[\Phi, \Phi^\dagger, \Psi, \bar{\Psi}] = \int d^4x \left\{ Z_\Phi \text{tr} \left[ \partial_a \Phi^\dagger \partial^a \Phi \right] + Z_\Psi \text{tr} \left[ i \gamma^5 \bar{\Psi}^{\dagger a} \partial \Psi^a + U_k(\Phi, \Phi^\dagger) \right] - \frac{1}{2} \text{tr} \left( \Phi^\dagger j + j^\dagger \Phi \right) \right\}
\]

where \( k \) denotes the relevant momentum scale in a way defined more precisely below. The meson and quark wave function renormalizations, \( Z_\Phi \) and \( Z_\Psi \), respectively, are now functions of \( k \) and \( \bar{\hbar} \) is a \( k \)-dependent Yukawa coupling. The scale–dependent effective potential is denoted as \( U_k \).

We will assume that [\( \bar{\hbar} \)] describes the most important low energy degrees of freedom of strong interactions for scales \( k \) below a “compositeness scale” \( k_0 \) at which the original gauge interaction of QCD becomes strong enough to trigger the formation of the light mesonic bound states. The effect of explicit chiral symmetry breaking due to non–vanishing current quark masses is represented by an external source \( j \) for the scalar meson field \( \Phi \). It is related to the average current quark mass \( \bar{m} = (m_u + m_d) / 2 \) (we neglect here isospin violating effects) by

\[
\bar{\hbar} = 2m^2 \kappa_\Phi \bar{m}
\]

where \( \kappa_\Phi \) denotes the bare scalar mass parameter contained in \( U_k \) at the compositeness scale \( k_0 \). The main approximation made in \( \bar{\hbar} \) is the neglect of higher derivative terms as well as higher dimensional interactions among quarks and mesons. We note that this model is a generalization of the Nambu–Jona–Lasinio (NJL) model where the four–fermion interaction has been eliminated in favor of an auxiliary field \( \Phi \). Beyond the NJL–model we allow here for a non–vanishing kinetic term for the scalar mesons, i.e., \( Z_\Phi \neq 0 \), as well as an arbitrary

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of the effective average potential $U_k$ for all values of $k$.

The success of the NJL-model to describe the strong chiral dynamics at low energies (see, e.g., 10 and references therein) serves as a further support for the use of \[ U_k \]. Our motivation to consider the linear quark meson model as an effective field theory for the IR behavior of QCD is, however, not only limited to the relation of \[ U_k \] to NJL-models. It can be demonstrated that, similarly to chiral perturbation theory, also within the scalar part of the linear quark meson model a systematic expansion of observables like meson masses or decay constants in powers of the light current quark masses is possible \[ U_k \]. This allows for a successful “prediction” of the light quark flavors and neglect isospin violation. Furthermore, we decouple the $\eta'$ and the $a_0$ isosinglet. This is justified by their relatively large masses and can be achieved for $N_f = 2$ in a chirally invariant manner. This leaves us with the $O(4)$-symmetric Gell–Mann–Levy model for the three pions and the sigma meson coupled here, however, to the up and the down quarks (instead of the nucleons). It should be noted, though, that our justification for the decoupling of the $\eta'$ meson is less clear at finite temperature. It has been speculated \[ U_k \] that close to the chiral transition temperature an effective restoration of the axial $U_A(1)$ symmetry might take place. In this case the $\eta'$ would become degenerate with the pions even in the spontaneously broken phase. In principle this question could be address within the linear quark meson model by allowing for a finite explicitly $U_A(1)$ breaking effective average potential $U_k$. This would lead to a finite $\eta'$ mass even in the chiral limit and the temperature dependence of the coupling $\beta(T)$ would yield the required information about an approximate restoration of $U_A(1)$ within the linear quark meson model. We leave this generalization of our model to future work.

We restrict ourselves here to the two lightest quark flavors and neglect isospin violation. Furthermore, we decouple the $\eta'$ and the $a_0$ isosinglet. This is justified by their relatively large masses and can be achieved for $N_f = 2$ in a chirally invariant manner. This leaves us with the $O(4)$-symmetric Gell–Mann–Levy model for the three pions and the sigma meson coupled here, however, to the up and the down quarks (instead of the nucleons). It should be noted, though, that our justification for the decoupling of the $\eta'$ meson is less clear at finite temperature. It has been speculated \[ U_k \] that close to the chiral transition temperature an effective restoration of the axial $U_A(1)$ symmetry might take place. In this case the $\eta'$ would become degenerate with the pions even in the spontaneously broken phase. In principle this question could be address within the linear quark meson model by allowing for a finite explicitly $U_A(1)$ breaking effective average potential $U_k$. This would lead to a finite $\eta'$ mass even in the chiral limit and the temperature dependence of the coupling $\beta(T)$ would yield the required information about an approximate restoration of $U_A(1)$ within the linear quark meson model. We leave this generalization of our model to future work.

We furthermore note that even within this relatively simple effective model we have to deal with strong couplings. For instance, the renormalized Yukawa coupling acquires a value $h \simeq 6$ to reproduce a realistic constituent quark mass of $M_q = hf_\pi/2 \simeq 300$ MeV. Thus a nonperturbative method is required which we discuss next.

II. EFFECTIVE AVERAGE ACTION AND EXACT RG

The concept of the effective average action $\Gamma_k$ is most easily introduced by considering an $O(N)$-symmetric scalar model with real fields $\chi^a$ in $d$ Euclidean dimensions and classical action $S[\chi]$. We define the scale dependent generating functional for connected Green functions as

$$ W_k[J] = \ln \int \mathcal{D}\chi \exp \left\{ -S_k[\chi] + \int d^dx J_a(x) \chi^a(x) \right\} $$

(3)

with IR cutoff scale $k$. Here $S_k[\chi] = S[\chi] + \Delta S_k[\chi]$ with $S[\chi]$ the classical action and

$$ \Delta S_k[\chi] = \frac{1}{2} \int \frac{d^dq}{(2\pi)^d} R_k(q^2) \chi^a(-q) \chi^a(q) . $$

(4)

We require that the IR cutoff function $R_k(q^2)$ vanishes rapidly for $q^2 \gg k^2$ whereas for $q^2 \ll k^2$ it behaves as $R_k(q^2) \simeq k^2$. This implies that all Fourier modes $\chi^a(q)$ with momenta smaller than $k$ acquire an effective mass $\sim k$ and decouple while the high momentum modes of $\chi^a$ are not affected by $R_k$. The classical fields $\Phi^a \equiv \langle \chi^a \rangle = \delta W_k[J]/\delta J_a$ now depend on $k$, and the effective average action is defined as

$$ \Gamma_k[\Phi] = -W_k[J] + \int d^dx J_a(x) \Phi^a(x) - \Delta S_k[\Phi] . $$

(5)

In order to define a reasonable coarse grained free energy we have subtracted in \[ U_k \] the infrared cutoff piece. This guarantees that the only difference between $\Gamma_k$ and $\Gamma$ is the effective IR cutoff in the fluctuations. Furthermore, it has the consequence that $\Gamma_k$ does not need to be convex for $k > 0$ whereas a pure Legendre transform is always convex. (The coarse grained free energy becomes convex only for $k \to 0$.) This is important for the description of phase transitions, in particular, first order ones. We note that the effective average action is a continuum implementation of the Wilson–Kadanoff block–spin action \[ U_k \], however, formulated here for the generating functional of $1PI$ Green functions.

For choices of the IR cutoff function $R_k$ like

$$ R_k(q^2) = \frac{Z_{\Phi}q^2}{c q^2 / k^2 - 1} $$

(6)

the effective average action $\Gamma_k$ interpolates between the classical action in the ultraviolet and the full effective action in the infrared:

$$ \lim_{k \to \infty} \Gamma_k[\Phi] = S[\Phi]$$

$$ \lim_{k \to 0} \Gamma_k[\Phi] = \Gamma[\Phi] $$

(7)

The high momentum modes are very effectively integrated out in \[ U_k \] because of the exponential decay of $R_k$ for $q^2 \gg k^2$. All symmetries of the model that are respected by the IR cutoff $\Delta_k S$ are automatically symmetries of $\Gamma_k$. Hence there is no problem incorporating chiral fermions since a chirally invariant cutoff can be formulated \[ U_k \].

The dependence of $\Gamma_k$ on the coarse graining scale $k$ is governed by an exact renormalization group (RG) equation \[ U_k \].
\[ \partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{Tr} \left\{ \frac{1}{\Gamma_k^{(2)}[\Phi]} + R_k \right\} . \] (8)

Here \( t = \ln(k/\Lambda) \) with some arbitrary momentum scale \( \Lambda \). For the linear quark meson model we will identify \( \Lambda \) for convenience with the compositeness scale \( k_\Phi \) where the mesons a thought to form within QCD. The trace includes a momentum integration as well as a summation over internal indices. The second functional derivative

\[
\left[ \Gamma_k^{(2)} \right]_{ab}(q,q') = \frac{\delta^2 \Gamma_k}{\delta \Phi^a(-q') \delta \Phi^b(q')} \] (9)

denotes the exact inverse average propagator. The exact RG equation (8) allows to start with a microscopic (classical or possibly already effective) action at some UV scale \( \Lambda \). For the linear quark meson model we will identify \( \Lambda \)\( \approx \frac{k^2}{2} Z_\Phi \rho \) it is given by

\[
\frac{\partial}{\partial t} u = -d u + (d - 2 + \eta_k) \tilde{\rho} u' + 2v d \left\{ 3 H_0(u'; \eta_k) \right\} . \] (12)

Here \( v_d^{-1} \equiv 2d+1 \pi^{d/2} \Gamma(d/2) \) and primes denote derivatives with respect to \( \tilde{\rho} \). We will always use in the following \( N_c = 3 \) for the number of quark colors and \( d = 4 \).

Eq. (12) is a partial differential equation which governs the flow of the effective average potential \( u(t, \tilde{\rho}) \) with \( k \) starting from a fixed UV scale to \( k = 0 \). Supplemented by similar (but ordinary) differential equations for the flow of the remaining parameters \( \eta_k \equiv -\ln Z_\Phi \), \( \eta_k \equiv -\frac{1}{2} \ln Z_\Psi \) and \( k^2 \equiv Z_\Phi^{-1} Z_\Psi^2 N_c \) it can be solved numerically on a computer.

An important nonperturbative ingredient appearing in all these flow equations and, in particular, also in \( \Gamma_k^{(2)} \) are so called mass threshold functions. Full definitions of all threshold functions relevant for the truncation \( \Gamma_k^{(2)} \) of the linear quark meson model can be found in [6]. A typical appearing in \( \Gamma_k^{(2)} \) is

\[
J_n^4 \left( \frac{M^2}{k^2}; \eta_k \right) = 8n \pi^2 k^{2n-4} \int \frac{d^4 q}{(2\pi)^4} \frac{\partial^4 (Z_k^{-1} R_k(q^2))}{[P(q^2) + M^2]^{n+4}} . \] (13)

with \( P(q^2) = q^2 + Z_\Psi^{-1} R_k(q^2) \). These functions decrease monotonically with \( M/k \) and decay \( \sim (k^2/M^2)^{n+1} \) for \( k \ll M \) where \( M \) can be seen from (14) to be a renormalized scalar mass of the model. This implies that the main effect of the threshold functions is to cut off quantum fluctuations of particles with masses \( M^2 \gg k^2 \). These functions therefore automatically and smoothly decouple massive modes from the evolution of the system whenever \( k \) becomes smaller than the \( (k \text{–dependent}) \) mass of a particle. The relevant masses typically receive contributions from arbitrarily high powers of the coupling constants of the theory and are therefore nonperturbative in nature. In fact, this nonperturbative dependence of the flow equations on the coupling constants of the model is already manifest in (8). The second functional derivative \( \Gamma_k^{(2)} \) generically depends on all couplings present in the effective average action which therefore appear in denominators of the beta functions for these very same couplings. This nonperturbative dependence of the beta functions on the couplings is an important difference to Polchinski’s exact RG equation (14). It corresponds to a resummation of the contributions from infinitely many Feynman diagrams already at the level of the beta functions. It is the main reason that nonperturbative problems like the computation of critical exponents (see below) can be carried out in a straightforward way.

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III. INITIAL CONDITIONS FOR THE RG FLOW

We will assume that the linear quark meson model is a reasonable effective description of the chiral IR dynamics of QCD for scales $k$ below a “compositeness scale” $k_0$ where the mesons are assumed to form due to strong gluonic interactions. For a solution of our coupled set of RG equations for $Z_\Phi$, $Z_{\bar{q} q}$, $\tilde{h}$ and $U_k$ we therefore need initial conditions at the scale $\Lambda = k_0$. We motivate these conditions by exploiting the relation of the linear quark meson model to the NJL–model. There the scalar meson field $\Phi$ is introduced as an auxiliary field for $k = k_0$. This implies that its wave function renormalization $Z_\Phi$ vanishes at the compositeness scale. Furthermore, in the NJL–limit the effective average potential $U_k$ is a purely quadratic function of $\Phi$ with a positive mass squared coefficient $m^2 \sim 1/G$ where $G$ is the four–fermi coupling of the NJL–model. We will be somewhat more general here and allow for a non–vanishing by small scalar wave function renormalization $Z_\Phi$ at a scale below $\Lambda = k_0$. Yet, they acquire a freedom in the model which experience confinement does effectively. The model is thus dominated by quark fluctuations for the beginning of its evolution before confinement is washed out. This implies an enormous predictive power of the exact RG analysis of the linear quark meson model which goes far beyond the chiral Ward identities and has been successfully tested [6,8].

IV. CHIRAL PHASE TRANSITION AT NON–VANISHING TEMPERATURE

The introduction of finite temperature in the exact RG formalism is straightforward. Since we are working in 4d Euclidean space time, at finite $T$ the 4d momentum integral implicitly contained in the trace in (8) is replaced in the standard way by a 3d integration times a Matsubara sum. Consequently, all scalar and fermion degrees of freedom are substituted by infinite towers of Matsubara modes with increasing $T$–dependent masses. Technically, one can treat $T$ as an external parameter and evolve the effective average action (8) for each value of $T$ from $k_0$ to $k = 0$ using [8]. In this context it is important that the initial conditions at $k_0$ for the RG flow discussed above are practically insensitive to $T$–effects up to temperatures of approximately 170 MeV [8]. It is crucial that the decoupling of massive modes through threshold functions also works for Matsubara masses. For instance, eq. (13) is now replaced by

$$
I_n^4(\frac{M^2}{k^2}, \frac{T}{k}, \eta_\Phi) = 8n\pi^2k^{2n-4}\sum_{l\in\mathbb{Z}} \int \frac{d^4q}{(2\pi)^4} \frac{\partial_l(Z^{-1}_\Phi R_k(q^2))}{[P(q^2) + M^2]^{n+1}}
$$

where

$$
q^2 = (2\pi T)^2 + q^2 .
$$

Using $P(q^2) = q^2 + Z_{\Phi}^{-1} R_k(q^2)$ we see that the squared mass $M^2$ is effectively replaced in (14) by the Matsubara masses $M^2 + (2\pi T)^2$, $l \in \mathbb{Z}$. The threshold functions therefore automatically decouple all massive Matsubara modes as $k$ is lowered and the model is dimensionally reduced in a smooth way. This procedure yields, for instance, the full effective potential as a function of $\Phi$, $T$ and the external source $j$ and therefore the equation of state $\partial U(\langle \Phi \rangle, T)/\partial \Phi = j$ which contains the $T$–dependence of various observables. In fig. 1 we show the chiral condensate $\langle \bar{\Psi} \Psi \rangle$ as a function of $T/T_c$. Lines $(a)$, $(b)$, $(c)$, $(d)$ correspond to $m_\pi(T = 0) = 0, 45$ MeV, 135 MeV, 230 MeV, respectively. For each pair of curves the lower one represents the full $T$–dependence of $\langle \bar{\Psi} \Psi \rangle$ whereas the upper one shows for comparison the universal scaling form of the equation of state for the O(4) Heisenberg model. We see a sharp second order phase transition in the chiral limit (this claim will be substantiated further by demonstrating the appropriate critical behavior near $T_c$).
For non–vanishing quark masses this changes into a smooth crossover which becomes smoother for increasing $\hat{m}$. The values for the respective (pseudo)critical temperatures are given in table I. It is remarkable that $T_c$ for the largest pion mass, computed within this very simple effective model, is quite close to the result of current full two flavor QCD Monte Carlo simulations performed for comparable quark masses [20]. We consider this as an indication that our model yields indeed even quantitatively a reasonable description of the chiral transition. This hold for non–universal as well as for universal properties of the transition as will be demonstrated below. This claim is further substantiated by the following consideration: Because of the relatively large sigma mass [6,8], for low temperatures the linear quark meson model is well approximated by the nonlinear sigma model. In fact, our $\langle \bar{\Psi}\Psi \rangle (T)/\langle \bar{\Psi}\Psi \rangle (0)$ curve rather accurately agrees with that of chiral perturbation theory for $T < \sim 60$ MeV as shown in figure 2. Here the solid line displays the exact RG result for vanishing average current quark mass $\hat{m} = 0$ whereas the dashed line shows the corresponding result of three–loop chiral perturbation theory [21,22] On the other hand, in the vicinity of $T_c$ the universal behavior of the model sets in which is independent of the details of the effective action used in our approach and also accurately described by our method as demonstrated below.

**TABLE I.** The table shows the critical and “pseudocritical” temperatures for various values of the zero temperature pion mass.

| $m_\pi$ MeV | 0  | 45 | 135 | 230 |
|----------------|----|----|-----|-----|
| $T_c$ MeV       | 100.7 | $\approx 110$ | $\approx 130$ | $\approx 150$ |

In order to demonstrate our ability to compute the complete temperature dependent effective meson potential we plot in figure 3 $\partial U(T)/\partial \phi_R$ as a function of the renormalized field variable $\phi_R = (Z_\Phi \rho/2)^{1/2}$ for different values of $T$. The first curve on the left corresponds to $T = 175$ MeV. The successive curves to the right differ in temperature by $\Delta T = 10$ MeV down to $T = 5$ MeV. One nicely observes the convexity of the potential even deep in the spontaneously broken phase, i.e. for small enough temperatures. This property of $U$ is rather difficult to reproduce correctly in perturbation theory.

The effective potential $U(\phi_R, T)$ also contains the necessary information to compute the renormalized masses
(or rather inverse spatial correlation lengths) of the pions and the sigma meson. These are obtained from $U$ as second derivatives with respect to the appropriate scalar field components. Our results for the $T$–dependence of these two quantities are plotted in figures 4 and 5, respectively, for three different values of $M_\pi(T = 0)$.

The solid lines in both cases correspond to the realistic values $\hat{m} = \hat{m}_{\text{phys}}$ whereas the dotted line represents the situation without explicit chiral symmetry breaking, $\hat{m} = 0$. The intermediate dashed lines are computed for $\hat{m} = \hat{m}_{\text{phys}}/10$. We note that, even for the case of the physical current quark mass, $m_\pi$ is to a good approximation a monotonically increasing function of $T$ for all three quark masses. This implies, in particular, that for two light quark flavors there is no long pion correlation length in thermal equilibrium. This is interesting in view of recent speculations [23,24] that such long correlation lengths might lead to spectacular experimental signatures in heavy ion collision experiments due to the formation of large regimes of a disoriented chiral condensate. A further interesting result is the dip in the $T$–dependence of the sigma mass. For vanishing temperature the sigma decays predominantly into two pions making it extremely wide through the large scalar self–interaction. From figure 5 we see that for increasing temperature the phase space for this decay decreases until at temperatures around $\sim 100 \text{ MeV}$ this decay becomes kinematically impossible. One would therefore expect to see a strong $T$–dependence of the sigma decay widths at least in thermal equilibrium.

V. UNIVERSAL CRITICAL BEHAVIOR

In addition to the non–universal results discussed above also universal properties of the second order phase transition are described accurately by the exact RG method. Using the equation of state allows to substantiate the claim that the chiral transition in the chiral limit $\hat{m} = 0$ is indeed of second order and to compute the associated critical exponents. Table II shows our results which correspond to the critical exponents of the three–dimensional $O(4)$–Heisenberg model. Our results are denoted by “ERG” whereas “MC” labels the exponents obtained by lattice simulations [25]. The agreement is within a few percent except for the anomalous dimension. The latter deviation can be understood at least qualitatively as a consequence of the rather crude approximation of the momentum dependence of the scalar propagator in (1). In fact, if the first non–trivial order of the derivative expansion in the scalar sector of the model is completed by including in the Ansatz (1) a term $\sim Y_0(\rho)\partial_\mu \rho \partial^\mu \rho$ the values of the critical exponents for general $O(N)$–models can be seen to improve [26]. Yet, the exact RG method is even capable of computing the full Widom scaling form of the equation of state. In fig. 6 a comparison of our results, denoted by “ERG”, with results of other methods for the scaling function of the three–dimensional $O(4)$–Heisenberg model is shown (for notation see [8]). The constants $B$ and $D$ specify non–universal amplitudes of the model. The curve labeled by “MC” represents a fit to lattice Monte Carlo data [27]. The second order epsilon expansion [28] and mean field results are denoted by “$\epsilon$” and “MF”, respectively. Apart from our results the curves are taken from [27].

| $\nu \gamma \delta \beta \eta$ |
|---|---|---|---|---|---|
| ERG 0.787 | 1.548 | 4.80 | 0.407 | 0.0344 |
| MC 0.7479(90) | 1.477(18) | 4.851(22) | 0.3836(46) | 0.0254(38) |
We should emphasize, perhaps, that these universal results by no means prove that two flavor QCD is in the universality class of the 3d $O(4)$–symmetric Heisenberg model. As we pointed out before there is the possibility that the axial $U_A(1)$ symmetry might be effectively restored at high temperatures in which case one would rather expect a $U_L(2) \times U_R(2)$–symmetric behavior near the chiral transition. Our results rather confirm that the exact RG method is capable of computing highly nonperturbative quantities like critical exponents or the scaling form of the equation of state with good accuracy. Assuming that the axial $U_A(1)$ symmetry remains sufficiently broken even at high $T$ we would then expect our results to be in good qualitative and quantitative agreement with the universal as well as the non–universal properties of the chiral phase transition of full two flavor QCD. This is based on the fact that our method provides a smooth link between two temperature regimes where the properties of strongly interacting matter are well understood:

- low temperatures where chiral perturbation theory is expected to yield a sound description of the dynamics of the Goldstone modes
- temperature in the vicinity of $T_c$ where the universal critical behavior of the model sets in and the system becomes independent of almost all the details of the underlying microscopic description.

The plots 1 and 2 demonstrate that the intermediate range of temperature, where in principle sizeable quantitative deviations from full two flavor QCD are possible, is not very large.

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