Stress Analysis of a Cylinder Subjected to Thermo-mechanical Loads by Using FEM

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Abstract. Material science is the most emerging field in today’s research and is getting widespread day by day. Numerical investigation of various kinds of products by using different kinds of material and changing boundary condition is getting popular because it is less time consuming and is beneficial from the cost benefit perspective. The behavior of material subjected to thermo-mechanical loads is of interest to researchers in a wide variety of discipline. Cylinders have wide use in industrial engineering like as containing various types of liquid. They often operate under static or fluctuating mechanical and thermal loads. Various methods are introduced for increasing strength of cylinders such as compound cylinder, increasing wall thickness. In this paper the analytical and numerical analysis of single and compound thick cylinder is given. The results of the analytical approach are validated to a finite element model.

Keywords: Stress analysis, Thermo-mechanical loads, Compound cylinders, Analytical and finite element analysis.

1. Introduction
The design of thick walled pressure vessels for operation at very high temperatures and pressure is a tough task with several concerns encompassing definition of the working and acceptable stress intensities, conditions of failure, material performance etc. Cylinders have been commonly used due to their significance for containing various types of fluids. Engineers are trying to improve the design of pressure vessel in order to ensure the safety and reliability of cylinder under various thermo-mechanical loads. For storing and bypassing fluids cylinders are broadly used in various kinds of industries like chemical, petroleum, pharmaceuticals, military industries as well as in power plants. When hot fluid passes through the cylinder it exerts a certain amount of pressure and heat transfer occurs between the surface of the cylinder and the fluid. Discontinuity in model and material properties causes fracture in ductile materials. High pressure cylinders used in industries, process plants, pneumatic tanks, hydraulic reservoirs, storage for gases are the general application of thick walled cylinders [1]. Earlier many investigation have been done for increasing the efficiency of the cylinders. Numerous research studies have been carried to determine acceptable stress range, conditions of failure and material performance at the interface of bimetallic cylinder. An analytical expression to determine the thermo-mechanical stresses of a multilayered compound cylinder considering the effect of closed ends was derived by Zhang et al. [2]. Vedeld et al. [3] derived the expression for two-layer cylinders exposed
to pressure and thermal gradient for investigating stress and displacement induced in the body. It is noticeable that temperature has important influence in stress distribution. Plane stress condition was used by Sollund et al. [4] to develop analytical solutions for stress components of compound cylinders where the change along the length is assumed to be zero. The use of composite materials have widely been investigated. An overview of advanced materials was provided by David et al. [5] for high pressure hydrogen storage as storing high pressure hydrogen is important in many aspects. Design and reliability of bimetallic cylinder was inspected by Wilson et al. [6]. Kautor [7] investigated that for same thermo-mechanical loading using bimetallic pressure vessel is more convenient and the stress of a compound cylinder under pressure and temperature was also investigated. Minimization of material volume for the fabrication of multilayer cylinders subjected to pressure load was investigated by Patil [8]. For reducing hoop stress shrink fit cylinder was analyzed and bursting pressure analysis was determined by Hareram [9]. For the compound cylinder with increasing inner pressure an elastic and plastic model is developed by Tazana for determining stress and displacement fields [10]. In this paper, the resulting stress and displacement fields in both single and compound cylinders subjected to thermo-mechanical loads are presented. The analytical solutions are compared with finite element solutions.

2. Analytical model for determining the stress and displacement distribution of the cylinder

Displacement throughout the thickness of the cylinder can be obtained from following equation,

\[ u_i (r) = \frac{(1 + \nu_i)}{(1 - \nu_i)} \int_{r_m}^{r} \rho T_{c_i} d\rho + C_{i1}r + \frac{C_{i2}}{r} \]  

(1)

Where \( C_{i1} \) and \( C_{i2} \) are the constants of integration for cylinder \( C_{\nu=1,2} \).

Using the displacement given by equation (1) and stress strain relationship, the radial and hoop stress can be calculated as,

\[ \sigma_r = E_i \left[ -\frac{\alpha_i}{(1 - \nu_i)r^2} \int_{r_m}^{r} \rho T_{c_i} d\rho + \frac{C_{i1}}{(1 + \nu_i)(1 - 2\nu_i)} - \frac{C_{i2}}{(1 + \nu_i)r^2} \right] \]

\[ \sigma_\theta = E_i \left[ \frac{\alpha_i}{(1 - \nu_i)r^2} \int_{r_m}^{r} \rho T_{c_i} d\rho + \frac{C_{i1}}{(1 + \nu_i)(1 - 2\nu_i)} + \frac{C_{i2}}{(1 + \nu_i)r^2} - \frac{\alpha_i T_{c_i}}{(1 + \nu_i)} \right] \]  

(2)

The integration constants, \( C_{ij} \), are determined using the boundary conditions in the compound cylinders.

These boundary conditions are related to the application of the internal pressure, \( p_i \), the continuity of the radial displacement and the radial stress at the interface layer between the two cylinders. They are summarized in the following equation:

\[ \sigma_r^1 (a) = -p_i; \sigma_r^2 (c) = 0 \]

\[ \sigma_r^1 (b) = \sigma_r^2 (b); u_i (b) = u_z (b) \]  

(3)

Substituting the temperature distributions, stresses and displacements of the inner and outer Cylinders into the boundary conditions, the equations system, given by Eq. (4), is obtained.
\[
\begin{bmatrix}
1 & k_{12} & -1 & -k_2 \\
1 & k_{22} & k_{23} & k_{24}
\end{bmatrix}
\begin{bmatrix}
C_{11} \\
C_{21} \\
C_{12} \\
C_{22}
\end{bmatrix}
= 
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4
\end{bmatrix}
\]

(4)

With:

\[
k_{22} = \frac{(1-2\nu_1)}{b^2} = \frac{a^2}{b^2} k_{32} = \frac{c^2}{b^2} k_{44}
\]

\[
k_{12} = -k_{14} = \frac{1}{b^2}
\]

\[
k_{23} = \frac{E_2}{E_1} \left( \frac{1+\nu_1}{1+\nu_2} \right) \left( \frac{1-2\nu_1}{1-2\nu_2} \right) = -k_{24} \left( \frac{b^2}{(1-2\nu_2)} \right)
\]

\[
B_1 = \left( \frac{1+\nu_1}{1-\nu_1} \right) \frac{\alpha_2}{b^2} \int_a^b \rho T_{\gamma_1} d\rho = -B_2 \frac{1}{1-2\nu_1}
\]

\[
B_3 = \frac{(1-2\nu_1)(1+\nu_1)}{E_1} p_i
\]

\[
B_4 = \frac{\alpha_2}{c^2} \int_a^b \rho T_{\gamma_2} d\rho
\]

By obtaining the integration constant we can calculate the stress and displacements at each radial position from equation (1) and (2).

3. Finite element model

The validation of the proposed analytical model is done by using finite element model with ABAQUS software. Simulation was done by considering the cylinder subjected to thermo-mechanical loads. As this is a thermo-mechanical analysis mesh element type is used C3D8T which is an 8-node thermally coupled brick, trilinear displacement and temperature. To examine the material properties effect on the behavior of the cylinder; steel is used for single cylinder and for compound cylinder steel and aluminum is used. Loads are changed to observe the effect of loads on the stress and displacement fields. For inner cylinder steel is used and for outer cylinder aluminum is used for the analysis. Coupled temperature displacement is used for stress analysis.

3.1 Physical Aspects of the Model

In this analysis two cylindrical element have chosen to analyze. The inner cylinder having inner radius \( a = 0.01 \) m and outer radius \( b = 0.02 \) m. The outer cylinder having inner radius \( b = 0.02 \) m and outer radius \( c = 0.03 \) m. The inner cylinder is made of steel and outer cylinder is made of aluminum. Bimetallic interface occurs at \( r = b = 0.02 \) m.
Figure 1. Model showing the (a) boundary conditions (b) mesh picture of the model.

Since at the interface material property changes so bias has been used so there is more element near the interface than the inner and outer side of the cylinder. As this is a thermo-mechanical analysis mesh element type is used C3D8T which is an 8-node thermally coupled brick, trilinear displacement and temperature.

3.2 Material properties
In this analysis, material Steel and Aluminum are used. Here aluminum is used as upper material and steel as lower material. The mechanical properties of aluminum and steel are:

Table 1. Properties of the two materials used for the analysis.

| Material          | $E$ (GPa) | $\nu$ | $\alpha$ ($^\circ$C) | $K$ (W.m$^{-1}$.K$^{-1}$) |
|-------------------|-----------|-------|----------------------|---------------------------|
| Steel ASTM A564 H1150 | 210       | 0.3   | 11.6$\times$10$^{-6}$ | 19.5                      |
| Aluminum 1050A-H9 | 72        | 0.33  | 24$\times$10$^{-6}$  | 234                       |

4. Results and discussion
In this present work, ABAQUS software was used to analyze the stress and displacement variation of a thick walled cylinder under pressure and temperature. All stress components are plotted by changing loading conditions. Firstly the model was analyzed under pressure and then the model was analyzed under combined pressure and temperature loading. Symmetric boundary conditions are used as the geometry is symmetric and the boundary conditions are also symmetric. So, quarter of the model is used for analysis, resulting in fewer degrees of freedom and computational costs. The graphical presentations and the contours are shown below. All simulated results are plotted against r to investigate their stress and displacement characteristics. The graphical representations are shown below.

4.1 Mesh sensitivity analysis of the work
In case of numerical analysis mesh sensitivity analysis is the most important issue. The main objective of numerical analysis is to obtain the most accurate result in the shortest possible time. Here different mesh sizes and different number of elements were imposed on the model to verify the accuracy of work.
From the figure it is clear that results are not so accurate using 2 lakh elements. Accuracy of the result is increased as element number is increased from 2 lakh to 10 lakhs. It was observed that the results obtained using 9 lakh elements is same as the result obtained 10 lakh elements. So, there is no reason to use 10 lakh elements and increase the computational time. So the optimum number of elements is found 9 lakh. Further analysis was done using this optimum number of elements.

4.2 Variation of stress for a 2D single cylinder

When fluid passes through cylinders it exerts uniform internal pressure on the inner surface of the cylinder. Here various magnitude of pressure was applied in the inner surface of the cylinder and the variation of radial and hoop stress throughout the radius was investigated.

Figure 3 indicates the variation of radial stress and hoop stress along the radius subjected to internal pressure only. Both radial and hoop stress is maximum at the inner radius and minimum at the outer radius as pressure is applied on the inner surface of the cylinder only. It is also noticeable from the graph that increasing internal pressure causes increase in radial stress and hoop stress.
Figure 4. Variation of (a) radial and (b) hoop stress along radius subjected to external pressure only.

In this graph it is seen that as the direction of applying pressure load changes to external surface of the cylinder only. Radial stress is nearly zero at the inner radius that satisfies the boundary condition. By increasing the magnitude of pressure, stress developed is also increased.

Figure 5. Variation of (a) radial and (b) hoop stress along the radius subjected to thermo-mechanical loads.

In this graph the variation of radial and hoop stress subjected to thermo-mechanical loads is represented. The temperature gradient between the inner and outer surface of the cylinder creates radial displacements in each radial positions. When the temperature difference between inside and outside is large, the radial stress is more significant for all radial positions as illustrated.
4.3 Variation of stress for a 2D compound cylinder

Figure 6. Variation of radial stress along the radius subjected to pressure and temperature.

Figure 7. Variation of hoop stress along the radius subjected to pressure and temperature.

From these figures it is clear that maximum hoop stress occur at the interface. As hoop stress is responsible for the failure of the cylinder, extra material should be added at the interface to prevent failure. Extra material may be provided by using adhesive near the interface region. Stress continuity is a must for bimetallic joint otherwise joint will break. It is clear that using compound cylinder increases the pressure carrying capacity of the cylinder and extra material should be provided at the interface region to prevent failure.
5. Conclusion
In this work the displacement and stress distribution along radius subjected to internal and external pressure is investigated. The boundary conditions like pressure load and temperature are changed to observe the stress and displacement variation. Analytical solutions are presented. The results obtained in this analysis leads to the following conclusions:

a) In order to ensure the dependability of the cylinder, pressure and temperature gradient should be considered during design operations.

b) The tangential stress component is more responsive to temperature than the pressure.

c) The value of stresses are higher in the interface section than the other spaces.

d) Extra material should be added in the interface to prevent failure.

e) The internal pressure effect is more important when they are made from similar materials than dissimilar materials.

6. References
[1] Choudhury Susanta, Roy H., 2010 “Stress analysis of a thick walled cylinder”. B Tech thesis, National Institute of Technology India.

[2] Q. Zhang, Z. W. Wang, C. Y. Tang, D. P. Hu, P. Q. Liu and L. Z. Xia, 2012., “Analytical solution of the thermo-mechanical stresses in a multilayered composite pressure vessel considering the influence of the closed ends”, International Journal of Pressure Vessels and Piping, Vol. 98, PP.102-110.

[3] K. Vedeld, H. A. Sollund and J. Hellesland, 2015,“Closed analytical expressions for stress distributions in two-layer cylinders and their application to offshore lined and clad pipes”, Journal of Offshore Mechanics and Arctic Engineering, Vol. 137, Issue 2 PP. 021702-021702-9.

[4] H. A. Sollund, K. Vedeld and J. Hellesland, 2014, “Efficient analytical solution for heated and pressurized multi-layer cylinder Ocean Engineering”, Vol.92, PP.285-295.

[5] E. David, 2005. “An overview of advanced materials for hydrogen storage”, Journal of Material Processing Technology, Vol.162–163 PP.169-177.

[6] W. R. D. Wilson and W. J. Skelton, 1967, “Design of bi-metallic high pressure cylinders”, Proc. Inst. Mech. Eng., Vol.3, PP.1-10.

[7] Bahoum Kaoutar, Dinay, 2017, “Stress analysis of compound cylinders subjected to thermo-mechanical loads”. J. Mechanical Science and Technology, Vol.31, Issue 4, PP. 1805-1811.

[8] Patil S.A., Miraje, 2011, “Minimization of material volume of three layer compound cylinder having same materials subjected to internal pressure”. Int. J. Engineering Science and Technology Vol.3, NO.8, PP.26-40.

[9] Lohar Hariram, 2013, “Steees analysis and brust pressure determination of two layered compound cylinder”. IJEST Vol 5.

[10] Vasko Tajana, Franjo., 2014 “Stress and deformation analysis of compound cylinders in elastic plastic range”. PhD Thesis, University of Osijek.