Monte Carlo simulation of the laser pulse propagation in water layers with reflection on boundaries

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Abstract. By Monte Carlo method we study peculiarities of scattering of short laser pulses in the water media with reflection from the underlying and water-air boundaries. The results of numerical modeling show that the ring-shaped structures of light may appear in the water layers.

1. Introduction

The paper deals with an optical phenomenon, which is inaccessible to the human eye because it lasts a few microseconds. After a short laser pulse passes through a sufficiently thin layer of a scattering medium, an expanding luminous ring can appear in the layer for a short time instant. Such rings of the scattered light in the atmospheric clouds were recorded by a special equipment (the wide-angle CCD-lidar detector) [1-3] and simulated by Monte Carlo methods [4-6]. We have shown [6] that similar ring-shaped structures of light can appear in layers of water media. Below, by the stochastic simulation we study specific features of the multiple scattering of a short laser pulse in a water layer taking into account the multiple reflection of light on the underlying surface and the water-air interface. In numerical experiments, we assume that the underlying surface is a Lambertian plane, while the water-air interface is a flat surface without undulations.

2. A mathematical model and Monte Carlo algorithms

The propagation of the laser pulse in a scattering medium can be described by the equation of radiative transfer in the integral form [7]

\[ \varphi(r, \omega, t) = \int_{-\infty}^{t} \int_{r}^{t} \int_{R^3} q g(\omega, \omega') e^{-\tau(r', r)} \delta\left(\omega - \frac{r - r'}{|r - r'|}\right) \times \]

\[ \times \varphi(r', \omega', t') \delta(S(t - t', r)) dr' d\omega' dt' + \psi(r, \omega, t). \]

Here \( \varphi(r, \omega, t) = I(r, \omega, t) \sigma(r) \) is the collision density, \( I(r, \omega, t) \) is the radiance at the point \( r \) in the direction \( \omega \) at the time instant \( t \), \( \sigma(r) \) is the extinction coefficient, \( q \) is the single scattering albedo, \( g(\omega', \omega) \) is the phase function, \( \delta \) is the Dirac delta function, \( S(t - t', r) \) is the set of points from which photons can reach the point \( r \) in the time \( t - t' \), \( \psi(r, \omega) \) is the density of initial collisions, and \( \tau(r, r') \) is the optical length of the interval \([r, r']\).

Monte Carlo method consists in the numerical modeling of a large sample of photon trajectories and using this sample for estimating the necessary characteristics of the radiation field. A trajectory of a
photon can be simulated as a Markov chain \( W = \{ (\mathbf{p}_n, \omega_n, r_n, t_n) : n = 0, ..., N \} \), where \( r_n \) is a point and \( t_n \) is the time instant of the n-th collision, \( \omega_n \) is the direction and \( p_n \) is the “statistical weight” of the photon after its scattering at the point \( r_n \). The statistical weight is multiplied by single scattering albedo after each scattering: \( p_n = q p_{n-1} \). The transition probabilities of the Markov chain are related to the kernel of the radiative transfer equation. The details of the corresponding Monte Carlo simulation algorithms can be found, for example, in [7, 8].

Concerning the interaction of light with the underlying surface (the bottom of the water layer), we assumed that a photon can be absorbed or reflected from the surface according to Lambert’s cosine law. In case of reflection, the cosine \( \mu \) of the angle between the external normal to the surface and the reflected photon direction has the probability density \( 2 \mu, \mu \in (0, 1) \), and it can be simulated by the method of the inverse distribution function (\( \mu = \sqrt{a} \)) or by a version of the rejection sampling (if \( \alpha_2 < \alpha_1 \), then \( \mu = \alpha_1 \), otherwise). Here by \( \alpha, \alpha_1, \alpha_2 \) we denote independent random variables uniformly distributed in (0, 1). After reflecting from the Lambert plane, the statistical photon weight is multiplied by the albedo of the underlying surface.

Some of the photons of the laser pulse scattered in the water layer may reach the water-air interface. In this case, the photon is specularly reflected by the boundary back into the water and the statistical weight of the reflected photon is multiplied by the reflection coefficient \( R \), which is calculated by the formula [7]:

\[
R = (A - B)^2 (A^2 B^2 + C^2) (A + B)^{-2} (AB + C)^{-2}.
\]

Here

\[
A = \cos \theta = -\langle \omega, s \rangle > 0,
\]

\[
B = (n_{21}^2 - 1 + A^2)^{1/2},
\]

\[
C = 1 - A^2,
\]

\( \theta \) is the incident angle, i.e. the angle between the direction \( -\omega \) and the external normal vector \( s \) to the specular surface, \( n_{21} \approx 1/1.33 \approx 0.75 \) is the relative refractive index of the air with respect to water. In the above formulas we assume that \( n_{21}^2 - 1 + A^2 > 0 \). Otherwise (for \( \theta > 49^\circ \)), we have the total internal reflection and \( R = 1 \).

To compute by Monte Carlo method the density of the space-time distribution of the laser pulse energy absorbed and scattered in a water layer, we use the so-called “collision estimator” [7, 8]. Consider a sample of \( M \) trajectories

\[
W_i = \{ (\mathbf{p}_n^{(i)}, \omega_n^{(i)}, r_n^{(i)}, t_n^{(i)}) : n = 0, ..., N(i) \}, \quad i = 1, 2, ..., M.
\]

Assume that it is required to calculate the space-time density of the light energy absorbed and scattered in the neighborhood of the point \( r_n \) at the time instant \( t_n \). Then we take a “small” area \( V \) of the volume \( |V| \) containing the point \( r_n \) and the time interval \( T \) of length \( |T| \) such that \( t_n \in T \). For each trajectory \( W_i \) we sum up the weights for collisions that have occurred in the volume \( V \) during the time interval \( T \),

\[
\sum_{n > 0, r_n^{(i)} \in V, t_n^{(i)} \in T} p_{n-1}^{(i)}.
\]

Finally, we take the sum over all trajectories and normalize it:

\[
\frac{dE}{dxdydzdt}(r_n, t_n) \approx \frac{1}{|V||T|} \frac{E_0}{M} \sum_{i=1}^M \sum_{n > 0, r_n^{(i)} \in V, t_n^{(i)} \in T} p_{n-1}^{(i)}.
\]

Here \( E_0 \) is the total energy of the emitted laser pulse. The space-time density of the energy distribution has the dimension \( J/(m^3s) \). Below, we call this function

\[
\Psi(x, y, z, t) = \frac{dE}{dxdydzdt}(x, y, z, t)
\]

the space-time interaction density of the laser pulse with the scattering medium. In addition, for the whole scattering layer we estimate the temporal interaction density \( \Psi_L(t) \) by the formula

\[
\Psi_L(t) \approx \frac{1}{|T|} \frac{E_0}{M} \sum_{i=1}^M \sum_{n > 0, r_n^{(i)} \in T} p_{n-1}^{(i)}.
\]
3. Simulation results
Numerous scientific monographs and papers deal with optical properties of water media (see, for example, [9-15]). Below, we present the results for optical model M13 from [14, 15] based on the observations in the Strait of Gibraltar in May 1998. For this optical model of the water medium, the extinction coefficient is equal to 0.20723, the single scattering albedo is 0.86667, and the scattering phase function is presented in Figure 1 (the wavelength is 520 nm).

![Figure 1. The scattering phase function of the water media for model M13 from [15].](image)

In Figures 2-5, we present the results for the numerical experiment where we assume that the laser pulse is emitted from the upper boundary downwards perpendicular to a water layer of height 50 m. For simplicity we suppose that the laser pulse has the delta distribution in time. The laser emitter field of view is equal to 2 mrad. The underlying surface (the lower boundary) reflects the light by Lambert’s cosine law with probability 0.8 (that is albedo of the underlying surface). Figure 2 shows the temporal density \( \Psi_{L}(t) \) of the laser pulse interaction with the water layer. The function is strictly monotonically decreasing. Note that probability of the collision-induced absorption in a water medium is comparatively high (it is equal to 0.13333). A sharp descent near 222 ns corresponds to the moment when a pulse reaches the underlying surface (photons are absorbed here with probability 0.2). The distributions of the photons scattered in a water medium at sequential time instants are shown in Figures 3, 5. Every image displays a distribution averaged over 18.6 ns time interval, 1m intervals of vertical and horizontal axes. Figure 5 shows that after reflecting the laser pulse from the underlying surface, the domain with a high concentration of photons has the form of a hemisphere. After some time (see images in Figure 5) photons reach the water-air interface. Part of these photons is reflected back into the layer, and another part (the refracted light) goes out from the layer. The images in Figure 5 for the time moments after 477 ns demonstrate the process of generating an expanding ring-shaped luminous structure.

Figure 4 shows “average directions” of the photons in the water layer at the time instant 920 ns after the laser pulse hits the layer (as compared with image (g) in Figure 5). We calculated the average directions in the following way. For each photon collision we find the projection \( \sqrt{a^2 + b^2, c} \) of the photon direction \((a, b, c)\) before the collision onto the plane passing through the laser beam and the photon collision point. Then we take an average of the projections over the corresponding cells taking into account the statistical weights of photons.
Figure 2. The temporal density $\Psi_L(t)$ of the laser pulse interaction with the water layer. The layer thickness is 50 m and the optical thickness is 10.36. The horizontal axis represents the time $t$ in ns from the beginning of the pulse propagation in the layer.

Figure 3. Spatial distributions of the laser pulse photons scattered in the water layer of height 50 m at the time instants 388 (a), 477 (b), 565 (c) and 654 (d) nanoseconds after the laser pulse hits the water layer. The images represent the horizontal projections of 3D-distributions for a 400 m wide part of the water layer.

Figure 4. Average directions of the laser pulse photon motion in the water layer of 50 m thickness at the time instant 920 ns after the laser pulse hits the water layer. The image represents the vertical sections of a 400 m wide part of the layer.
Figure 5. Spatial distributions of the laser pulse photons scattered in the water layer of height 50 m at the time instants 388 (a), 477 (b), 565 (c), 654 (d), 743 (e), 831 (f), 920 (g), 1009 (h), 1097 (i) nanoseconds after the laser pulse hits the water layer. The images represent the vertical sections of 3D-distributions corresponding to a 400 m wide part of the layer. To increase the contrast, for images we use a map color legend with blue-brown colors for small-large values.
4. Conclusion
In this paper, we have shown that a short laser pulse going through a water layer can generate expanding the ring-shaped luminous structures. This phenomenon is relative to expanding the rings of the light generated by laser pulses in layers of the atmospheric clouds. By Monte Carlo method, we have studied specific features of the laser pulse scattering in water layers caused by the multiple reflection of light from the layer boundaries. In our opinion, the studied optical phenomenon can be useful for the development of the novel remote sensing methods and solving inverse problems of the water optics.

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