Energy Management in Data Centers with Server Setup Delay: A Semi-MDP Approximation

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Abstract—The energy management schemes in multi-server data centers with setup time mostly consider thresholds on the number of idle servers or waiting jobs to switch servers on or off. An optimal energy management policy can be characterized as a Markov decision process (MDP) at large, given that the system parameters evolve Markovian. The resulting optimal reward can be defined as the weighted sum of mean power usage and mean delay of requested jobs. For large-scale data centers however, these models become intractable due to the colossal state-action space, thus making conventional algorithms inefficient in finding the optimal policy. In this paper, we propose an approximate semi-MDP (SMDP) approach, known as ‘multi-level SMDP’, based on state aggregation and Markovian analysis of the system behavior. Rather than averaging the transition probabilities of aggregated states as in typical methods, we introduce an approximate Markovian framework for calculating the transition probabilities of the proposed multi-level SMDP accurately. Moreover, near-optimal performance can be attained at the expense of increased state-space dimensionality by tuning the number of levels in the multi-level approach. Simulation results show that the proposed approach reduces the SMDP size while yielding better rewards as against existing fixed threshold-based policies and aggregation methods.

Index Terms—Data centers, energy management, Markov decision process, setup time, state aggregation, cloud computing.

1 INTRODUCTION

The inconceivable global surge in computing power demand from video streaming, cryptocurrencies, power-hungry artificial intelligence applications, and numerous cloud-connected devices has changed the operational landscape of data centers. Serving as indispensable powerhouses of the modern digital era, a large portion of the expenditure is dedicated to cooling data center servers and equipment [1]. Projected statistics on the enormous power consumption of data centers reveal a harsh reality in spite of committing to more efficient technologies [2].

In general, a server is said to be on when busy serving jobs. In absence of job requests, a server either remains idle or is turned off. While idle servers are ubiquitous in data centers, each amount for about 50 to 60 percent of the energy of its fully utilized state [3]. Energy-aware cloud data centers minimize the power wastage of idle servers by either switching them to a low-power standby state or the inactive off state. In practice, however, turning the server back to the active state (physical or virtual machine (VM)) incurs extra power consumption and transition delay, known as setup or spin-up time, which hinders immediate service to incoming job requests. Over-provisioning servers with jobs add to the energy costs while under-provisioning may result in delayed service delivery time thus, violating the service-level agreement (SLA). In order to shorten the service delay while saving energy, it is therefore necessary to determine the optimal number of idle and setup servers in cloud data centers under different load levels.

1.1 State of the Art and Prior Work

The attention drawn towards the theoretical assessment of energy management in multi-server systems has grown significantly in recent years. The authors in [4] presented a hysteresis queuing model to minimize power costs in cloud systems without explicitly considering server setup delay. The mean power consumption in systems with exponential setup time was studied in [5] wherein various operational policies such as the ON/IDLE policy that turns no server off, the ON/OFF policy that turns off all servers immediately after becoming idle and has no limit on the number of setup servers, the ON/OFF/STAG policy that allows at most one server to be in setup at any point of time, and finally, the ON/OFF/kSTAG policy that permits at most k servers to be in setup were investigated. In [6], the authors analyzed the k-staggered policy by permitting some servers to remain idle after setup, using a three-dimensional continuous-time Markov chain (CTMC). The power consumption and waiting time distributions for the ON/OFF policy have been studied in [7]. The work in [8] focused on the system queue length distribution and considered added policies that turn off servers with a finite delay, and also permit servers to go into sleep mode which, compared to the off mode, induces lower setup time and power usage. Moreover, the switch operation cost of energy-aware data centers is optimized in [9] using probability-based online algorithms that account for the expectation of job intervals. The work in [10] defines two priority queues for different levels of delay-sensitivity and in the case of peak loads, defers the jobs with lower priority to promote the quality of service (QoS) of peak loads.

By partitioning the homogeneous physical machines into three pools (hot, warm, and cold) with different power levels, the scalable model introduced in [11] used interacting Markov chains and fixed-point iteration to derive the mean
waiting time and power consumption. In [12] and [13], the authors introduced heterogeneity in the CPU cores requested by each VM, where the numbers of cores conform to uniform and general distributions, respectively. General spin-up time distribution was investigated in [14] and the mean performance measures and energy consumption of the switching policies were modeled using the M/G/1 queuing discipline. Though insightful, none of the above efforts explicitly prioritize delay over power consumption.

The optimal power-switching policy in a multi-server system, which minimizes the weighted sum of power consumption and delay, can generally be described as a Markov decision process (MDP) [15]. The MDP approach in [16] was adopted to minimize energy costs and rejected jobs in an Infrastructure-as-a-Service (IaaS) cloud system. A near-optimal solution was proposed in [17] for power switching of two dynamic servers to minimize power consumption, delay, and wear-and-tear costs using MDP and look-ahead approach. The MDP approach is also used in [18] to find the optimal policy for cost-performance trade-off in virtualized data centers, where VMs are modeled by process sharing queues. Moreover, the size of the proposed MDP is reduced using fixed thresholds to categorize the load of servers into three light, moderate, and high levels. In [19], an optimal job routing policy was proposed based on the Whittle index in a system of parallel servers, where each server follows an ON/OFF policy and is equipped with an infinite buffer. The authors in [20] conducted a detailed study on the performance of different multi-server power management policies. Denoting the job arrival and service rates as \( \lambda \) and \( \mu \), respectively, they showed that keeping \( \frac{\lambda}{\mu} + \sqrt{\frac{2}{\mu}} \) servers always on results in a near-optimal solution.

For hyperscale cloud data centers, the state-action space grows very large thus, making the value iteration and policy iteration algorithms for solving MDPs intractable. To overcome the curse of dimensionality, the state aggregation method has been widely studied in MDPs to reduce the state space size [21], [22]. This technique merges the states bearing similar transition probabilities and rewards into one meta-state to obtain a smaller MDP. Several works have been reported on the upper bound derivation for deviation of the total aggregated MDP reward from that of the original MDP in both discounted ( [23], [24]) and non-discounted [25] MDPs. However, to our best knowledge, there exists no reported work on state aggregation for optimal energy management in large-scale data centers that incorporates the stochastic characteristics of the underlying system.

1.2 Main Contributions

In this paper, a multi-server system is considered where the servers experience setup delays and the jobs arrive according to a Poisson process. In the proposed system model, a power manager is responsible for turning the servers on or off to manage the power usage of the entire system or the number of waiting jobs. We aim to find a sub-optimal power-switching policy in such a system that not only minimizes the weighted sum of power consumption of the servers and average delay of the jobs, but also characterizes the trade-off between the dimensionality of the system state space and closeness to the optimal performance. To this end, we first describe the basic semi-MDP (SMDP) for deriving the optimal power-switching policy in our system model, and then introduce the proposed approximate SMDP model. The main contributions are summarized as follows:

- By employing the state aggregation method given in [21], we propose an efficient approximate SMDP, referred to as multi-level SMDP, to reduce the system state space. Unlike the classical state aggregation approach, we use data from the stochastic system behavior analysis and the basic SMDP to define the meta-states and transition probabilities more efficiently. Increasing the number of levels in the multi-level SMDP yields higher dimensionality; the performance of the resulting policy however, becomes closer to the optimal policy derived from the basic SMDP model.

- We simulate the proposed multi-level SMDP under different settings with precedence of delay over power. Benchmarking against the uniform state aggregation method and fixed-threshold policies in [20], we show the better performance of our model in terms of the achieved average expected rewards.

The rest of this paper is structured as follows. Section 2 presents the system model and assumptions. Section 3 details the basic SMDP approach as the optimal solution, followed by the proposed state-aggregated multi-level SMDP in Section 4. Numerical results are discussed in Section 5. Finally, Section 6 concludes the paper.

2 System Model

Consider a data center comprising of \( C \) servers that serve jobs arriving at the system and a power manager that switches the servers on and off independently. When on, a server is in one of the three states: BUSY, IDLE, or SETUP. Likewise, a server is in the OFF state if powered off by the manager. The Poisson process with rate \( \lambda \) has been shown to be an acceptable approximation for job arrivals in data centers [26]. Also, the service time of each job is exponentially distributed with rate \( \mu \). A server can process a job immediately only if it is in IDLE state. Jobs that fail to receive service instantly upon arrival wait in a finite queue of capacity \( Q \) until served in a first-in-first-out (FIFO) manner. The state transition diagram of a server is shown in Fig. 1.

When a server is powered on, it enters the SETUP state and stays there for some amount of time called ‘spin-up’ time, which follows an exponential distribution with rate \( \gamma \). Upon completion of the setup process or service process of a SETUP or BUSY server, respectively, the server transitions to the BUSY state if there is a head-of-line (HoL) job in the queue or enters the IDLE state, if otherwise. Finally, an IDLE server either becomes BUSY if a job is assigned to it or is
switched off by the power manager. Note that the power manager can only turn off servers that are either in IDLE or SETUP states. Once entering the queue, a newly arrived job is served immediately only if no other job exists in the queue and an IDLE server is available. That is to say, the manager assigns the HoL job to an IDLE server as soon as it is made available. This happens whenever a SETUP server finishes its setup process or a BUSY server finishes its service time.

Though having more number of IDLE and SETUP servers leads to added power consumption on one hand, it results in fewer waiting jobs and thus, more immediate services on the other. In this regard, we refer to the average number of IDLE and SETUP servers as the power penalty and to the average number of waiting jobs as the performance penalty of the system. We thus, aim to optimize the power switching policy, governed by the power manager, such that the weighted sum of power and performance penalties is minimized.

### 3 Basic SMDP Formulation

To derive an optimal policy for minimizing the weighted sum of power and performance penalties, we formulate our optimization problem as a semi-Markov Decision process (SMDP). The system state evolves as a Markov process due to the memoryless property of the arrival, setup, and service processes in our model. We define the system state as $s = (b, i) \in \mathcal{S}$, where $\mathcal{S}$ symbolizes the state space of the SMDP. The first entry, $b \ (0 \leq b \leq C)$, denotes the number of BUSY servers, whereas the absolute value of the second entry, $|i|$, indicates the number of IDLE servers when $i \geq 0$ and the number of waiting jobs when $i < 0$. Thus, we have $-Q \leq i \leq C$. Hereafter, we refer to $i$ as I/W, where I and W stand for ‘IDLE’ and ‘waiting’, respectively. Table 1 summarizes the definitions of the notations used henceforth.

In SMDPs, decisions are made when a transition takes place in the system state, leading to a random time between decision epochs. Consequently, the manager decides on the number of servers needed to be in the SETUP and IDLE states at the instants of job arrivals, job completions, and setup completions. With $a$ denoting the action taken by the power manager, $|a|$ number of IDLE servers are turned off as well as all SETUP servers when $a < 0$. When $a \geq 0$, the manager sets exactly $a$ servers in SETUP state. To this end, if the number of current SETUP servers at the decision time is greater than $a$, then the extra servers are powered off. Otherwise, some servers are turned on to have a SETUP servers. The resultant action space at state $s$ with $i \geq 0$ is given as $A_a = \{a - i \leq a \leq C - b - i\}$. Here, the maximum number of IDLE servers that can be turned off is $i$ (thus, $a \geq -i$) and the maximum number of servers that can be in SETUP process is $C - b - i$. Also, at state $s$ with $i < 0$, we have $A_a = \{a 0 \leq a \leq C - b\}$, where no idle server exists and the manager can only manage the number of SETUP servers.

| Notation | Definition |
|----------|------------|
| $b$ | Number of BUSY servers. |
| $i$ | Number of IDLE servers (I) when $i \geq 0$ and number of waiting jobs (W) when $i < 0$. |
| $s = (b, i) / S = (B, I)$ | State of the basic/multi-level SMDP. |
| $a$ | Selected action, which is the number of SETUP servers when $a \geq 0$ or the number of IDLE servers to be powered off when $a < 0$. |
| $\mathcal{S}$ | State space/action space of the basic SMDP. |
| $\Psi(x)$ | Sum of the output rates from (or inverse of expected time spent on) state $s$ choosing action $a$ in the basic SMDP. |
| $\mathcal{A}$ | Importance factor of performance penalty (i.e. average job waiting time) over power overhead. |
| $\rho$ | Reward function given that action $a$ is taken and the state transitions to $s$. |
| $f(x; \rho)$ | Poisson distribution with rate $\rho$ and probability parameter $x$. |
| $F(x; \rho)$ | Cumulative function of Poisson distribution with rate $\rho$ and probability parameter $x$. |

| $K_1 / K_2$ | Level size of BUSY servers/both I/W component and action. |
| $B$ | BUSY level. |
| $I$ | I/W level, which is the IDLE level when $I \geq 0$ and the level of waiting jobs when $I < 0$. |
| $A$ | Selected action in multi-level SMDP, which denotes the chosen setup level when $A \geq 0$ and the SETUP level when $A < 0$. |
| $\tilde{\mathcal{S}} / \tilde{\mathcal{A}}$ | State space/action space of the multi-level SMDP. |
| $\Psi(S, A)$ | Sum of output rates from each state (or inverse of expected time spent on each state) of the multi-level SMDP. |
| $\pi_1(x) / \pi_1(x)$ | Steady-state probability of $b = x/B = x$ in the multi-level SMDP. |
| $\pi_2(S, A)$ | Probability of I/W = $i$ given that the state and action levels are $S$ and $A$, respectively. |
| $\tilde{\pi}_2(S, A)$ | Probability of being at I/W level $I$ given that the state and action levels are $S$ and $A$, respectively. |
| $d_1(B) / u_1(B)$ | Lower/upper boundary of BUSY level $B$. |
| $p^B_1(B) / p^B_1(B)$ | Probability of being at the lower/upper boundary of BUSY level $B$. |
| $p^B_2(S, A) / p^B_2(S, A)$ | Probability of being at the lower/upper bound of I/W level given the state-action level pair $(S, A)$. |
| $B_+(S, A) / B_-(S, A)$ | BUSY level increment/decrement rate given the state-action level pair $(S, A)$. |
| $\tilde{B}_+(S, A) / \tilde{B}_-(S, A)$ | I/W level increment/decrement rate given the state-action level pair $(S, A)$. |
| $J_{(+)i}(S, A)$ | Rate of simultaneous BUSY level increment and I/W level decrement given the state-action level pair $(S, A)$. |
| $J_{(-)i}(S, A)$ | Rate of simultaneous BUSY level decrement and I/W level increment given the state-action level pair $(S, A)$. |
i.e. \( a \geq 0 \). In general, the action space at state \( s \) can be written as \( A_s = \{ |a - i^+| \leq a \leq C - b - i^+ \} \), where \( i^+ = \max (i, 0) \). Based on [20 Theorem 3], the optimal policy always turns on servers following a bulk setup policy; when it decides to turn on some servers, i.e., to put them in SETUP mode, it turns on all OFF servers. Hence, the action space can be reduced to the form \( A_s = \{ a | a \in [-i^+, 0] \cup \{ C - b - i^+ \} \} \).

In order to obtain the transition probabilities of the proposed SMDP, we derive \( q(s', s, a) \), which determines the transition rate to state \( s' \) given that the current state is \( s \) and action \( a \) is taken. For the case of \( a < 0 \) and \( i > 0 \), \( q(s', s, a) \) is determined as follows:

\[
q(s', s, a) = \begin{cases} 
\lambda; & b' = b + 1, i' = i + a - 1, i + a > 0, \\
\lambda; & b' = b, i' = -1, i + a = 0, \\
b\mu; & b' = b - 1, i' = i + a + 1, b > 0.
\end{cases} \tag{1}
\]

Note that \( a \) can take negative values only when \( i > 0 \), i.e., when there exists some IDLE servers. In this case, \( |a| \) denotes the number of IDLE servers to be turned off and subsequently, we have \( i + a \geq 0 \). The first case of (1) indicates that upon arrival of a new job, the number of remaining IDLE servers decreases by one after taking action \( a \) (i.e., \( i + a > 0 \)) and hence, the number of BUSY servers increases by one. If \( i + a = 0 \), as in the second case of (1), then \( i' = -1 \) which implies that the arriving job waits in the queue. In the third case of (1), completion of a job with rate \( b\mu \) increases and decreases the number of IDLE and BUSY servers by one, respectively. Moreover, for the case when \( a \geq 0 \), when \( a \) servers are turned on, \( q(s', s, a) \) is given as:

\[
q(s', s, a) = \begin{cases} 
\lambda; & b' = b + 1, i' = i - 1, i > 0, \\
\lambda; & b' = b, i' = i - 1, -Q < i < 0, \\
\lambda; & b' = b, i' = i, i = -Q, \\
b\mu; & b' = b - 1, i' = i + 1, i > 0, \\
b\mu; & b' = b, i' = i + 1, i < 0, \\
\alpha; & b' = b, i' = i + 1, i \geq 0, \\
\alpha; & b' = b + 1, i' = i + 1, i < 0.
\end{cases} \tag{2}
\]

The first case of (2) exemplifies that if the number of IDLE servers (\( i \)) is greater than zero, then upon arrival of a new job, the number of IDLE and BUSY servers decreases and increases by one, respectively. However, when \( i \leq 0 \), as in the second case of (2), the number of BUSY servers does not change but the number of waiting jobs increases by one, i.e., we have \( i' = i - 1 \), given that \( -Q < i \) (i.e., the queue is not full). Otherwise, if \( i = -Q \), then the number of waiting jobs does not change and the newly arriving job is dropped. In the fourth and fifth cases of (2), a BUSY server becomes IDLE with rate \( b\mu \), while the last two cases signify that a SETUP server becomes IDLE with rate \( \alpha \). In either cases, the new IDLE server remains IDLE, if \( i \geq 0 \), and becomes BUSY again to serve a waiting job, if otherwise. Note that the packet arrival at state \( b = c \) is included in the second and third cases of (2). As such, when \( b = c \), we have \( i \leq 0 \) and the arrived packet awaits in the queue if it is not full.

Now, let \( P(s'|s, a) \) denote the transition probability from state \( s = (b, i) \) to state \( s' = (b', i') \) under action \( a \) in the proposed SMDP. To derive the probabilities, we first define \( \Psi(s, a) \) as the sum of output transition rates at state \( s \) under action \( a \), which is expressed as follows:

\[
\Psi(s, a) = a^+\gamma + b\mu + \lambda. \tag{3}
\]

The transition probability \( P(s'|s, a) \) can now be written as:

\[
P(s'|s, a) = \frac{q(s'|s, a)}{\Psi(s, a)}. \tag{4}
\]

By letting \( x^- = \min (x, 0) \), the reward obtained when action \( a \) is taken is given as:

\[
r(s, a) = -\left( (a|i^-| + i^+ + \alpha'a^+) \right) \Psi(s, a). \tag{5}
\]

where \( 1/\Psi(s, a) \) is the average time spent at state \( s \) choosing action \( a \). Also, the performance penalty is given by \( \alpha|i^-| \) (i.e., \( \alpha \) times the number of waiting jobs) and \( i^+ + \alpha'a^+ \) is the weighted sum of the number of IDLE and SETUP servers that accounts for the power overhead penalty. Note that \( \alpha > 0 \) marks the importance of performance penalty over the power penalty, while \( \alpha' \) models the difference between the power consumption of SETUP and IDLE servers. It is also worth to mention that firstly, we do not model the cost of job dropouts in the reward function since, due to the precedence of performance penalty over power penalty, the optimal policy never results in considerable job dropout probabilities, especially in the presence of a finite but sufficiently large queue size. Secondly, the analytical approach presented in this paper is suited for moderate and light traffics, where the jobs do not occupy all servers almost surely and thus, the implementation of power management policies is justified.

We now find the policy that maximizes the long-term expected average reward, i.e.,

\[
\nu^\pi(s) = \lim_{n \to \infty} \frac{1}{n} \mathbb{E}_s \left[ \sum_{n=0}^{\infty} r(s_n, a_n) | s_0 = s \right], \tag{6}
\]

where \( \pi \) is the applied policy, \( t_n \) is the time of \( n \)-th event, \( s_0 \) is the initial state, and \( \mathbb{E}[\cdot] \) is the expectation operator. The optimal reward is the solution of the Bellman equation:

\[
\nu(s) = \arg \max_a \left\{ r(s, a) + \sum_{s'} P(s'|s, a) \nu(s') \right\}. \tag{7}
\]

Undertaking the approach in Chapter 11 of [27], we transform our proposed SMDP to a discrete-time MDP and solve it. Next, we partition the state-action space into multiple levels to reduce the dimensionality of the proposed SMDP.

4 Multi-level Approximate SMDP

The SMDP model in the preceding section considers the full state space in order to derive the optimal policy and thus, suffers from the curse of dimensionality. In fact, the state-action space dimension of a general MDP is \( |S| \sum_{a \in A} |A_s| \), which upon algebraic manipulation, can be shown to be \( C^2(Q + C^2)(Q + C) \) in the SMDP introduced in Section 3. Such a dimension is too high in a large-scale data center (e.g., in December 2014, Amazon Web Services operated an estimated 1.4 million servers across 28 availability zones). Therefore, instead of including the exact number of BUSY servers or I/W component, we present the multi-level SMDP wherein the system state exploits the intervals, referred to as ‘levels’, that the number of BUSY servers or I/W component belong to, respectively. Doing so significantly reduces the state space dimension to \( C' ^2(Q' + C') (Q' + C') \), where \( C' = C/K_1 \) and \( Q' = Q/K_2 \). Moreover, \( K_1 \) and \( K_2 \) are the
level sizes of the first and second components of the states, i.e. \( b \) and \( i \), respectively. For example, when \( K_1 = K_2 = 5 \) and \( K_1 = K_2 = 10 \), the approximate SMDP is 3125 and 100,000 times smaller than the original SMDP. By decreasing the level sizes in the multi-level SMDP, i.e. \( K_1 \) and \( K_2 \), we get sufficiently close to the optimal solution at the expense of increase in state space dimensionality. At \( K_1 = K_2 = 1 \), we attain the absolute optimal performance. Before delving into our model, we first need to introduce the queuing approximation for our system.

### 4.1 Queuing Model

We describe the waiting and service processes of the jobs in our system model by two sequential sub-processes, namely ‘provision’ and ‘service’ sub-processes, respectively. The provision sub-process initiates from the moment a job arrives at the system and ends when the job is assigned to a server for service. The service sub-process begins as soon as a job is assigned to a server until it is completely served and departs the system. Using the following assumptions, we approximate the provision sub-process by an \( M/G/\infty \) queue, which results in approximating the service sub-process with a \( M/M/\infty \) queue:

1. Since the power-switching policy is used for systems with low to moderate traffic, we assume that the number of servers \( (C) \) is large enough to serve all arriving jobs. Thus, the probability that all servers are BUSY is infinitesimal, i.e., each job finds at least one OFF, IDLE or SETUP server upon its arrival.

2. For states with waiting jobs \( (i < 0) \), we assume that the optimal action always turns on at least \( |i| \) servers, i.e. \( a \geq |i| \). This assumption is similar to the \( k \)-staggered policy discussed in [27]. Thus, the action set of states with \( i < 0 \), i.e. \( A_s = \{ a | 0 < a < C - b \} \), turns into \( A_s = \{ a | -i < a < C - b \} \). Hence, the action set in general is expressed as \( A_s = \{ a | -i \leq a \leq C - b - i^+ \} \).

With regard to these assumptions, if a job fails to find an IDLE server upon its arrival, a SETUP server is turned on to provide a server upon its arrival.

Thus, the provision subprocess can be approximated as an \( M/G/\infty \) queue, regarding that the job arrival to the system and thus, to the provision sub-process is Poisson. In such a queue, the departure process is also Poisson with rate \( \lambda \) [28], which is the arrival rate of jobs to the system.

Based on the above approximation, the arrivals to the service sub-process, which are in fact departures from provision sub-process, are Poisson. Moreover, each job departing the provision sub-process already has an assigned server with an exponential service time with rate \( \mu \) and thus, the number of servers at the service sub-process can be assumed to be infinity. Consequently, the service sub-process is approximated by an \( M/M/\infty \) queue, in which the number of BUSY servers in steady state, denoted by \( b \), follows the Poisson distribution with rate \( \rho = \lambda/\mu \). Since \( \rho \) is large enough in our problem (e.g., \( \rho > 10 \)), the Poisson distribution behaves similar to a normal distribution with mean \( \rho \) and standard deviation \( \sqrt{\rho} \). We use the symmetry property of the normal distribution in the next section to introduce the efficient levels corresponding to the number of BUSY servers.

### 4.2 State and Action Space of the Multi-level SMDP

The state space is defined as the ordered pair \( S = (B, I) \), where \( B \) and \( I \) are the BUSY and I/W levels, respectively. In general, each level corresponding to a component indicates an interval to which that component belongs. In particular, we define \( L \) levels for the number of BUSY servers such that \( B \in \{ 0, 1, \ldots, L - 1 \} \). Hence, level \( B = l \) means that \( b \), i.e. the number of BUSY servers, is bounded by \( [d_l(l), u_l(l)] \). Moreover, we have \( \bigcup_{l=0}^{L-1} [d_l(l), u_l(l)] = [0, C] \) and \( \bigcap_{l=0}^{L-1} [d_l(l), u_l(l)] = \emptyset \). Note that sub-index 1 is chosen since \( B \) is the first component of the state. To efficiently define the endpoints of the level intervals, \( d_l(l) \) and \( u_l(l) \), we first obtain the size of the BUSY levels, i.e. \( u_l(l) - d_l(l) + 1 \), as follows. We consider the \( \epsilon \%-\)confidence interval [1] (e.g. \( \epsilon = 0.99 \)) of the number of BUSY servers in our model. Since the number of BUSY servers follows a Poisson (or normal) distribution as in Section 4.1, the above interval is derived as \( [F^{-1}\left(\frac{1 - \epsilon}{2}; \rho \right), F^{-1}\left(1 - \frac{\epsilon}{2}; \rho \right)] \), where \( F^{-1}(x; \rho) \) is the inverse of \( F(x; \rho) \), and \( F(x; \rho) \) denotes the cumulative function of the Poisson distribution with rate \( \rho \) and probability parameter \( x \). Then, \( K_1 \), the BUSY level size, is computed by dividing the \( \epsilon \%-\)confidence interval into \( L \) levels as:

\[
K_1 = \frac{F^{-1}\left(\frac{1 - \epsilon}{2}; \rho \right) - F^{-1}\left(1 - \frac{\epsilon}{2}; \rho \right)}{L},
\]

where \( [\cdot] \) is the ceiling function and is applied to yield an integral level size. Once \( K_1 \) is determined from [8], we then find the new valid confidence interval with respect to the values of \( L \) and \( K_1 \). We use the symmetry of the normal distribution of the BUSY servers around \( \rho \) to define the new lower and upper endpoints of the confidence interval, denoted by \( \beta_d \) and \( \beta_u \), respectively, as:

\[
(\beta_d, \beta_u) = \left( \left[ \rho - \frac{K_1 L}{2} \right]^+, \left[ \rho + \frac{K_1 L}{2} \right]^+ \right),
\]

where \( [\cdot]^+ \) denotes the floor function. Consequently, the extreme points of the intervals corresponding to different levels, i.e. \( d_1(B) \) and \( u_1(B) \), are derived to be:

\[
d_1(B) = \begin{cases} 0; & B = 0, \\ BK_1 + \beta_d; & \text{otherwise.} \end{cases}
\]

and

\[
\begin{align}
\text{and}
\quad u_1(B) = \begin{cases} C; & B = L - 1, \\ (B + 1)K_1 + \beta_d - 1; & \text{otherwise.} \end{cases}
\end{align}
\]

Note that in [10], we assume that all values of \( b \), where \( b < \beta_d \), are allocated to level 0, i.e. \( d_1(0) = 0 \). Moreover, in [11], all values of \( b > \beta_u \) are allocated to level \( L - 1 \), i.e. \( u_1(L - 1) = C \).

Without loss of generality, we assume the same number of levels \( (L) \) for the positive part of the I/W component, which represents the number of IDLE servers. Since the positive part can take any value from zero to \( C \), we compute the level size corresponding to the positive I/W component as \( K_2 = C/L \), where the subscript in \( K_2 \) refers to the second entry of the state. The same level size \( K_2 \) is also used for the negative.
part of the I/W component. Thus, the negative part of I/W component has \( \{Q/K_2\} \) number of levels and as a result, we have \( I \in [-Q/K_2, L-1] \), where \( I = l \) is equivalent to \( i \in [K_2 + 1, (i+1)K_2] \). With \( A \) defined as the action in the multi-level SMDP, we assume that the action level size is equal to \( K_2 \). Based on the reduced action space of the basic SMDP and the second assumption in Section 4.2, the action space at state \( S \) is defined as \( \tilde{A}_S = \{A; A \in [-I, 0] \cup \{C-u_1(B) - I^+ K_2\}\} \), where \( C-u_1(B) - I^+ K_2 \) is the number of available OFF servers, assuming \( u_1(B) \) servers are BUSY and \( I^+ K_2 \) servers are IDLE. When \( A \leq 0 \), \( AK_2 \) servers are turned on to be the SETUP state; otherwise, all SETUP and \( AK_2 \) IDLE servers are turned off.

### 4.3 Level Boundary Probabilities

To derive the transition probabilities for the multi-level SMDP, the transition rates between levels must be calculated. The transition between levels occurs when the I/W component value or the number of BUSY servers are at the boundaries of their corresponding levels. Therefore, we first derive the probability of being at the boundaries of BUSY and I/W levels. To facilitate our analysis, we define \( \pi_1(l) = \Pr\{b = l\} \) and \( \pi_2(l) = \Pr\{i = l\} \) to be the steady-state probabilities of the first and second components of the state in the basic SMDP, which reflect the number of BUSY servers and I/W component, respectively. Similarly, notations \( \tilde{\pi}_1(l) = \Pr\{B = l\} \) and \( \tilde{\pi}_2(l) = \Pr\{I = l\} \) indicate the steady-state probabilities of the BUSY level (\( B \)) and I/W component level (\( I \)) in the multi-level SMDP, respectively. Moreover, the conditional probabilities are given as \( \pi_1(l|y) = \Pr\{b = l|y\} \), \( \pi_2(l|y) = \Pr\{i = l|y\} \), \( \tilde{\pi}_1(l|y) = \Pr\{B = l|y\} \), and \( \tilde{\pi}_2(l|y) = \Pr\{I = l|y\} \).

#### 4.3.1 Probability of BUSY Level Boundaries

Let \( f(n, \rho) \) be the probability density function (pdf) of the number of BUSY servers, which is in fact the pdf of a Poisson process with rate \( \rho \), according to Section 4.1. Then, the average number of BUSY servers at level \( B \), denoted by \( \mathbb{E}[b|B] \), is derived as:

\[
\mathbb{E}[b|B] = \sum_{n=d_1(B)}^{u_1(B)} n f(n, \rho) \tilde{\pi}_1(B),
\]

where \( \tilde{\pi}_1(B) \) is the probability that the number of BUSY servers belongs to \( [d_1(B), u_1(B)] \). Thus, \( \tilde{\pi}_0(B) \) is derived as:

\[
\tilde{\pi}_0(B) = F(u_1(B); \rho) - F(d_1(B) - 1; \rho).
\]

Given that the BUSY level is \( B \), the probabilities that \( b \) is equal to the lower and upper boundaries of the BUSY level, denoted by \( p^L_1(B) \) and \( p^U_1(B) \) respectively, are:

\[
p^L_1(B) = \pi_1(d_1(B)|B) = \frac{f(d_1(B), \rho)}{\tilde{\pi}_1(B)},
\]

\[
p^U_1(B) = \pi_1(u_1(B)|B) = \frac{f(u_1(B), \rho)}{\tilde{\pi}_1(B)}.
\]

#### 4.3.2 Probability of I/W Level Boundaries

To obtain the probability of I/W level boundaries, we first derive an approximate birth-death (BD) process. Thereby, in each multi-level state \((B, I)\) and for each \( IK_2 \leq i \leq IK_2 + K_2 - 1 \), we aggregate all states \((b, i)\) with \( d_1(B) \leq b \leq u_1(B) \) into one state, called meta-state \( i \). Fig. 2 and Fig. 3 show the transition probabilities of the states in the multi-level state \((B, I)\) when \( I > 0 \) and \( I \leq 0 \), respectively. In fact, we aggregate all states within the red dotted boxes in these two figures into one state to obtain \( K_2 \) meta-states. Also, the arrival (departure) rate of each meta-state is equal to the average of the arrival (departure) rates of
the corresponding aggregated states, respectively. When deriving these average rates, we ignore all transitions from other multi-level states into \((B, I)\) and vice versa. These transitions in fact, belong to the boundary states \((b, i)\) with \((b, i) \in \{u_1(B), d_1(B)\} \times \{IK_2, IK_2 + K_2 - 1\}\). With this approximation, the average transition rate from each metastate \(i\) to the next state \(i+1\) becomes \(A^+K_2\gamma + \mathbb{E}[b|B]\), while that from each meta-state \(i\) to the previous state \(i-1\) is equal to \(\lambda\). Since these birth and death rates are the same in all meta-states, the resulting approximated Markov chain forms a BD process with the birth and death rates equal to \(A^+K_2\gamma + \mathbb{E}[b|B]\) and \(\lambda\), respectively. In such a BD process, we thus have:

\[
\lambda\pi_2(i+1|S, A) = (\mathbb{E}[b|B]\mu + A^+K_2\gamma)\pi_2(i|S, A),
\]

where \(\lambda\) is the I/W component as in the basic SMODP and \(\pi_2(i|S, A)\) is the probability that I/W component is equal to \(i\), given that the system is at state \(S\) and action \(A\) is taken. By defining \(\eta(S, A) \triangleq (\mathbb{E}[b|B]\mu + A^+K_2\gamma)/\lambda\), we arrive at the following relationship:

\[
\pi_2(i+1|S, A) = \eta(S, A)\pi_2(i|S, A).
\]

Building on (17), the probability of being at I/W level \(I\) for the given state \(S\) and action \(A\), denoted by \(\pi_2(I|S, A)\), can be computed as follows:

\[
\tilde{\pi}_2(I|S, A) = \sum_{k=0}^{K_2-1} \eta(S, A)^k \pi_2(IK_2|S, A) \\
= \frac{\pi_2(IK_2|S, A)K_2;}{\pi_2(IK_2|S, A)1 - \eta(S, A)^{K_2};} \quad \text{when } \eta(S, A) < 1, \\
= \frac{\eta(S, A)^{K_2;}}{1 - \eta(S, A)^{K_2;}}; \quad \text{otherwise.}
\]

We have \(\tilde{\pi}_2(I|S, A) = 1\) since \(S = (B, I)\), whereas the value of \(\pi_2(IK_2|S, A)\) in (18), on the other hand, is in fact the probability of being at the lower boundary of I/W level \(I\), denoted by \(p_1^I(S, A)\). Using these facts and (18), we get:

\[
p_2^I(S, A) = \begin{cases} 
\eta(S, A) = 1, \\
1 - \eta(S, A)^{K_2;}. 
\end{cases}
\]

Furthermore, the value of \(p_2^I(S, A)\) which denotes the probability of being at the upper boundary of an I/W level, can be derived using (17) as below:

\[
p_2^S(S, A) = \pi_2((I + 1)K_2 - 1|S, A) \\
= \begin{cases} 
\eta(S, A)^{K_2-1;}1 - \eta(S, A)^{K_2;}, \\
1 - \eta(S, A)^{K_2;}. 
\end{cases}
\]

### 4.4 Transition Rates Between Levels

Now that the probabilities of being at the boundaries of BUSY and I/W levels have been derived, we calculate the transition rates when the BUSY level, I/W level, or both change. In what follows, we derive the level transition probabilities resulting from the positive part of the action, \(A^+\). These transitions, corresponding to the cases \(I > 0\) and \(I \leq 0\), are shown on the outer dotted boxes in Fig.2 and Fig.3, respectively. The value of \(A^-\) does not affect the BUSY level \(B\) and only changes the I/W level \(I\) to \(I + A^-\) instantly with probability one. Therefore, its effect is considered in the constraints of the final transition probabilities given in (20).
The resulting multi-level SMDP transition probabilities are embedded DTMC of the given CTMC is derived. The sum of Similar to the approach used in solving the basic SMDP, the

4.5 Solving Multi-level Approximate SMDP

The transition rate from \( S = (B, I) \) to \((B + 1, I - 1)\) under \( A^+ \) is denoted by \( J_{(+,-)}(S, A^+) \), as seen in Fig. 4 This transition occurs with rate \( \lambda \) only when \( I \geq 0 \) and the values of BUSY and IDLE servers are equal to the upper and lower boundaries of the corresponding levels, respectively. Thus,

\[
J_{(+,-)}(S, A^+) = \begin{cases} 
\lambda p^2_1(B)p^2_2(S, A); & I \geq 0, \\
0; & I < 0.
\end{cases}
\]  

(25)

4.4.3 Joint BUSY and I/W Level Transition Rates

The transition rate from \( S = (B, I) \) to \((B + 1, I + 1)\) is given by \( J_{(-,+)}(S, A^+) \) and takes place only when \( I \geq 0 \) and the number of BUSY and IDLE servers are equal to the upper and lower boundaries of the corresponding levels, respectively. Also, the rate of transition is \( d_1(B)\mu \) since the number of BUSY servers equals \( d_1(B) \). Thus, \( J_{(-,+)}(S, A^+) \) is obtained to be:

\[
J_{(-,+)}(S, A^+) = \begin{cases} 
\mu d_1(B)p^2_1(B)p^2_2(S, A); & I \geq 0, \\
0; & I < 0.
\end{cases}
\]  

(27)

Finally, the transition rate from state \( S = (B, I) \) to state \((B+1, I+1)\) is denoted by \( J_{(+,+)}(S, A^+) \) and is given as:

\[
J_{(+,+)}(S, A^+) = \begin{cases} 
A^+K_2\gamma p^2_1(B)p^2_2(S, A); & I < 0, \\
0; & I \geq 0.
\end{cases}
\]  

(28)

5 Simulation Results and Discussions

In this section, we evaluate our proposed multi-level SMDP through numerical and simulation results. The results are compared with the staggered threshold and bulk setup policies with parameters given in [20], and the uniform state-aggregation method in [21]. The SMDPs are solved using linear programming approaches for solving multi-chain SMDPs [27] and Gurobi java plugin [29]. Hereafter, we assume \( \alpha' = 2 \), implying that each SETUP server consumes twice the power of an IDLE server.

Fig. 4 compares the multi-level SMDP model with the basic SMDP when the number of levels is equal to the number of servers, i.e., \( L = C = 100 \) and thus, \( K_1 = K_2 = 1 \). Here, we set \( Q = C = 100 \). The optimal expected reward is plotted versus the arrival rate \( \lambda \), where \( \mu \) and \( \alpha \) are considered to be fixed (\( \mu = 1, \alpha = 50 \)). For varying setup and arrival rates, we observe that the performance of the proposed multi-level SMDP is exactly the same as that of the basic SMDP.

Two fixed-threshold methods, namely staggered threshold and bulk setup policies [20], have been reported in the literature for power management in multi-server systems with setup times. Both policies represent a threshold called ‘static ON’ servers, which is denoted by \( C_s \), and is the number of servers that should always be powered on. As such, when the number of ON servers (either BUSY, IDLE or SETUP) goes beyond \( C_s \) and no waiting job exists, all SETUP or IDLE servers will be turned off in both policies to reach \( C_s \).
number of ON servers. However, when there exist waiting jobs, these two policies behave differently. In the bulk setup policy, when the number of waiting jobs becomes equal or greater than the threshold parameter \( k \) (equivalently, when \( i \leq -k \) in our model), all \( \text{OFF} \) servers are turned \( \text{ON} \), and when it goes below \( k \), i.e. when \( i > k \), all \( \text{SETUP} \) servers will be turned off until the number of \( \text{ON} \) servers equals \( C_s \). Contrarily, in the staggered threshold policy, for each of the \( k \) waiting jobs, one server will be in \( \text{SETUP} \) mode. In both policies, greater values of \( k \) means higher priority of power over delay. Here, we set the threshold \( k = 1 \) to get the highest priority of delay over power. According to \( (29) \), we assume \( C_s = \rho + \sqrt{\rho} \), which is the optimal number of ‘static \( \text{ON} \)’ servers. Mathematically, these two policies are described as:

\[
\begin{align*}
  a_{\text{bulk}}(s) & = \begin{cases} 
    0, & \text{if } b + i > C_s, \text{ } (C_s - b)^+ - i \leq C_s, \\
    b + i > C_s, i > -1, & \text{if } (C_s - b)^+ - i > 1, \\
    c - b, & \text{if } b > C_s, i \leq -1.
  \end{cases} \\
  a_{\text{stag}}(s) & = \begin{cases} 
    0, & \text{if } b + i > C_s, \text{ } (C_s - b)^+ - i \leq C_s, \\
    b + i > C_s, i > -1, & \text{if } (C_s - b)^+ - i > 1, \\
    |i| & \text{if } b > C_s, i < 0.
  \end{cases}
\end{align*}
\]

We also compare our SMDP model with the uniform state-aggregation approach, which derives the reward and transition probability of a meta-state by averaging those of the corresponding aggregated states. By defining \( L_u \) as the number of levels in this method, we have \( K = K_1 = K_2 = C/L_u \). Hence, the state-action space and the transition probabilities of the uniform state-aggregation method can be expressed as follows, where \( K^4 \) is divided in \( (39) \) for averaging over the transition probabilities:

\[
\begin{align*}
  B & \in [0, L_u - 1], \quad B = l \Leftrightarrow b \in [lK, (l+1)K - 1], & \quad (36) \\
  I & \in [-L_u, L_u - 1], \quad I = l \Leftrightarrow i \in \{lK, (l+1)K - 1\}, & \quad (37) \\
  A & = x \Leftrightarrow a = xK, & \quad (38) \\
  p(S' \mid S, A) & = \sum_{w=0}^{K-1} \sum_{x=0}^{K-1} \sum_{y=0}^{K-1} \sum_{z=0}^{K-1} p((B', K', x) \mid (B + w, I'K + x), AK)/K^4. & \quad (39)
\end{align*}
\]

In what follows, terms \( \mathbb{E}[W] \) and \( \mathbb{E}[P] \) refer to the average delay and average power of the policies per time unit, respectively. However, in regard to the definition of the reward in \( (5) \), the term \( \mathbb{E}[W] \) essentially denotes the average number of waiting jobs \( (|j^-|) \), and \( \mathbb{E}[P] \) is the average weighted sum of the number of \( \text{IDLE} \) and \( \text{SETUP} \) servers \((i^+ + \alpha a^+)\).

Fig. 5 compares the policies discussed in terms of \( \mathbb{E}[W] \), \( \mathbb{E}[P] \), and expected reward \( \mathbb{E}[R] \) for different values of \( \alpha \in \{1, 2, 5, 10, 20, 50, 100\} \) and \( \lambda = 30, \mu = 1, \gamma = 2 \). Since the bulk setup policy turns on all \( \text{OFF} \) servers whenever the
The optimal approximation of the basic SMDP as compared to uniform level SMDP method. This shows that our method is a better is at most equal to the reward from the proposed multi-level SMDP, which indicates that the reward of the uniform state-aggregation method is better than both bulk and staggered threshold policy and thus, consumes more power and the jobs receive service with less delay (see Fig. 5a and Fig. 5b). Furthermore, for larger \( \alpha \) values (\( \alpha \geq 50 \)), which indicate delay being prioritized over power, the bulk setup policy results in higher reward since it prioritizes delay, while for smaller \( \alpha \) values, the staggered threshold policy outperforms the bulk setup policy. Since the bulk and staggered threshold policies are independent of \( \alpha \), the power and delay in these methods do not change for different \( \alpha \) values. Moreover, it is evident in Fig. 5 that the absolute value reward of the multi-level SMDP decreases with \( L \) since at larger values of \( L \), we have a more accurate model. Moreover, even for the smallest \( L \) value (\( L = 10 \)) in our experiment, the reward achieved from multi-level SMDP is better than both bulk and staggered threshold methods. It can also be noted in Fig. 5 that the reward of the uniform state-aggregation method is at most equal to the reward from the proposed multi-level SMDP method. This shows that our method is a better approximation of the basic SMDP as compared to uniform state aggregation method.

For varying values of \( \lambda \in \{10, 20, 30, 40, 50\} \), \( \mathbb{E}[W] \), \( \mathbb{E}[P] \), and \( \mathbb{E}[R] \) of the different policies are compared in Fig. 6. The parameters \( \alpha = 100, \mu = 1, \text{and} \gamma = 2 \) are set to be fixed. As shown in Fig. 6a and Fig. 6b, the mean delays of both bulk and staggered threshold policies are much greater (about 100 times) than optimal and multi-level approaches. The power and delay of bulk and staggered threshold policies increase with \( \lambda \) in Fig. 6b and Fig. 6c, thus leading to lower rewards as depicted in Fig. 6d. Indeed, by increasing \( \lambda \), the traffic density of the system increases, which results in longer average waiting time. On the other hand, more servers will be in SETUP state to deal with higher traffic which leads to more energy consumption too. However, such a monotonic increase in power and delay cannot be observed in SMDP-based approaches since they minimize the weighted sum of power and performance penalties and thus, improving one component may affect the other component for different \( \lambda \) values. Nevertheless, it is apparent in Fig. 6d that the reward decreases with \( \lambda \) in SMDP-based approaches.

Finally, Fig. 7 compares the mean delay, power, and reward calculated for different values of \( \gamma \in \{0.1, 0.5, 1, 2, 5\} \), where \( \alpha = 100, \mu = 1, \text{and} \lambda = 30 \). In Fig. 7a and Fig. 7b, it can be observed that for bulk setup and staggered threshold policies, increasing \( \gamma \) results in the decrease of the delay, because
the setup process finishes faster and the jobs experience less delay. The power also drops with $\gamma$ as shown in Fig. 7 since decrease in the SETUP delay brings about reduction in the number of SETUP servers. Similar to Fig. 6 for SMDP-based approaches, such a monotonic decrease can not be observed for delay and power separately, but the increase in reward with respect to $\gamma$ is clearly evident in Fig. 7a.

6 CONCLUSION

In this paper, we have proposed an approximate SMDP model built on state aggregation in order to achieve near-optimal policies for power management in cloud data centers with setup time. Our aggregation method reduces the state-space of the SMDP through aggregating multiple states of busy servers, idle servers, and waiting jobs into distinct levels. It also takes advantage of approximate Markovian behavior of the system model to derive the transition probabilities of the approximate multi-level SMDP. The considerable reduction in state-action space of the model makes it well-suited for scalable systems. Through numerical simulations, we not only validated the correctness of the presented model with the basic optimal SMDP model, but also demonstrated better performance of the model in terms of achievable near-optimal rewards with respect to existing fixed threshold methods and the uniform state aggregation method.

The work can be further extended by considering explicit power consumption for physical machines, which is determined by the virtual machines running on them, or managing the power expenditure of physical and virtual machines separately. Another improvement is to include the server power switching cost, known as ‘wear-and-tear’ cost, to this model.

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