Meterwavelength Single-pulse Polarimetric Emission Survey. IV. The Period Dependence of Component Widths of Pulsars

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Received 2017 June 6; revised 2018 January 4; accepted 2018 January 7; published 2018 February 21

Abstract

The core component width in normal pulsars, with periods \(P > 0.1\) s, measured at the half-power point at 1 GHz, has a lower boundary line (LBL) that closely follows the \(P^{-0.5}\) scaling relation. This result is of fundamental importance for understanding the emission process and requires extended studies over a wider frequency range. In this paper we have carried out a detailed study of the profile component widths of 123 normal pulsars observed in the Meterwavelength Single-pulse Polarimetric Emission Survey at 333 and 618 MHz. The components in the pulse profile were separated into core and conal classes. We found that at both frequencies, the core, as well as the conal component widths versus period, had a LBL that followed the \(P^{-0.5}\) relation with a similar lower boundary. The radio emission in normal pulsars has been observationally shown to arise from a narrow range of heights around a few hundred kilometers above the stellar surface. In the past the \(P^{-0.5}\) relation has been considered as evidence for emission arising from last open dipolar magnetic field lines. We show that the \(P^{-0.5}\) dependence only holds if the trailing and leading half-power points of the component are associated with the last open field line. In such a scenario we do not find any physical motivation that can explain the \(P^{-0.5}\) dependence for both core and conal components as evidence for dipolar geometry in normal pulsars. We believe the period dependence is a result of a currently unexplained physical phenomenon.

Key words: pulsars: general

1. Introduction

The coherent radio emission from pulsars is believed to arise due to the growth of instabilities in relativistically strongly magnetized plasma outflowing along open dipolar magnetic field lines well inside the light cylinder; however, the underlying physical mechanism that excites the coherent radio emission is still unidentified (see e.g., Michel 1982; Beskin et al. 1993; Melikidze et al. 2000; Melrose 2017; Mitra 2017). The radio pulsar population can be roughly divided into two groups based on their periods: millisecond pulsars with periods less than 100 ms, and normal pulsars with periods longer than this value. The presence of dipolar magnetic fields in the emission region is motivated by the widths of the profile and specific components that exhibit a dependence on the period \(P\). In this work we take a critical look into this argument, utilizing the measured profiles of 123 normal pulsars with periods greater than 0.1 s observed in the Meterwavelength Single-pulse Polarimetric Emission Survey (MSPES, Basu et al. 2016; Mitra et al. 2016a). Only for pulsars with periods larger than 0.1 s will an emission region that is located roughly 500 km above the neutron star (Blaskiewicz et al. 1991; von Hoensbroech & Xilouris 1997; Mitra & Li 2004; Weltevrede & Johnston 2008; Mitra & Rankin 2011) be largely unaffected by strong field line distortions (Dyks 2008).

The periodic radio emission in normal pulsars has typical duty cycles of \(\sim 10\%\) and the single pulses often consist of smaller structures called subpulses. In some cases the subpulses appear at the same phase within the pulse window, while in others they jitter or systematically move in phase. When the single pulses are averaged for a few thousand periods a stable integrated profile is formed that is composed of one or more distinct Gaussian-shaped components. These components are formed due to the averaging of subpulses but their properties, such as width and location, might differ from the individual subpulses depending on the subpulse dynamics. The physical origins of the subpulses in single pulses are still unknown. The most notable radio emission model that predict the presence of subpulses is that of Ruderman & Sutherland (1975), where the subpulses are associated with radiation generated from isolated plasma columns. The plasma columns are generated due to sparking discharges in an inner accelerating region characterized by high electric fields just above the polar cap. The formation of subpulses has also been speculated to arise due to non-radial oscillations in neutron stars (Clemens & Rosen 2004) or arise due to the development of instability in the outflowing plasma (Fung et al. 2006). This motivates the study of average subpulse properties in a large number of pulsars to constrain the formation of these emission structures.

The number of components in the profiles varies in the pulsar populations. The full widths of the profiles usually exhibit a frequency evolution where the widths are seen to decrease with increasing frequency and the number of components often change with frequency in a systematic manner. The profiles are highly polarized and the linear polarization position angle (PPA) executes a S-shaped traverse across them. The rotating vector model (RVM; Radhakrishnan...
Went through the calculations and observed the behavior of the PPA traverse in normal pulsars as described by Everett & Weisberg (2001), Mitra & Li (2004), Weltevrede & Johnston (2008), Mitra & Li (2004; Weltevrede & Johnston 2008; Mitra & Li 2004; Weltevrede & Johnston 2008; Mitra & Li 2004). After the observer’s line of sight cuts through the pulsar emission beam, the PPA traverse is affected by the observer’s line of sight cut through the pulsar emission beam. If the diverging magnetic field lines are ascribed to a star-centered global dipolar magnet, the RVM is expressed as a function of the dipolar geometrical angles, viz. the angle between the rotation axis and the dipole magnetic axis, where the propagation effect for the PPA traverse in normal pulsars very well, indicating that the emission originates from dipolar magnetic fields (Everett & Weisberg 2001; Mitra & Li 2004). On the other hand, these fits are very often unsuitable for estimates of the angles $\alpha$ and $\beta$ when data exhibit large error bars on PPA. The fit residuals become strongly correlated and good constraints for a unique solution are absent. However, the steepest gradient (SG) point of the PPA that lies in the plane containing the rotation and dipole magnetic axis can often be used to provide the necessary additional constraint

$$R_{\text{ppa}} = \frac{\sin(\alpha)}{\sin(\beta)},$$

where $R_{\text{ppa}}$ corresponds to the slope of the PPA at SG, for a more accurate determination of $\alpha$ and $\beta$.

A number of studies in the literature detail the interplay between the magnetospheric plasma with the field configuration, including the polar cap structure (Bai & Spitkovsky 2010), the structure of the large-scale magnetic field and plasma (Philippov & Spitkovsky 2014; Philippov et al. 2015), the location of discharge regions and plasma sources (Chen & Beloborodov 2014), the dynamics of discharge regions (Timokhin & Arons 2013), and propagation effects (Petrova & Lyubarskii 2000; Hakobyan & Beskin 2014). Additionally, the outflowing plasma in the rotating magnetosphere can also lead to effects like aberration and retardation ($A/R$ hereafter), magnetic field sweepback, etc. These can potentially distort the shape of the PPA traverse, yet the RVM corresponding to a relatively simplified model of empty pulsar magnetospheres are excellent fits to the PPA in normal pulsars (Everett & Weisberg 2001; Mitra & Li 2004). Several studies have shown that if the radio emission detaches from the magnetosphere at heights of $h_{\text{em}} < 0.1R_{\text{LC}}$, where $R_{\text{LC}} = pc/2\pi$ is the light cylinder radius, the $A/R$ effects are the only discernible observable distortion affecting the PPA traverse (Blaskiewicz et al. 1991; Hibschman & Arons 2001; Dyks 2008; Craig & Romani 2012; Kumar & Gangadhara 2012a, 2012b, 2013). In such a scenario if the radio emission detaches at a constant height across the pulsar profile, the $A/R$ effect can be approximated as a positive shift in longitude, $\Delta \phi$, between the center of the total intensity profile and the SG point of the PPA traverse, which is linearly dependent on $h_{\text{em}}$. This can in turn be used to estimate the emission heights as $h_{\text{em}} = (c/4)(\Delta \phi/360^\circ)/P$ km. Using the above relation a number of observations have found the radio emission in normal pulsars to detach from the magnetosphere around $h_{\text{em}} \sim 100-1000$ km, which is well below 0.1 $R_{\text{LC}}$ (Blaskiewicz et al. 1991; von Hoensbroech & Xilouris 1997; Mitra & Li 2004; Weltevrede & Johnston 2008; Mitra & Rankin 2011). Note that the excellent fits of the PPA traverse with the RVM indicate that the emission region across the pulse profile originates from similar heights, since significant changes in emission height can distort the PPA traverse (see for e.g., Mitra & Seiradakis 2004). The radio emission is excited in the plasma at a certain height $h_p$, where $h_p < h_{\text{em}}$, and then it propagates in the plasma and eventually detaches at $h_{\text{em}}$. The plasma properties in the propagation region can influence the properties of the radio emission and several conflicting models exist in the literature that either suggest that the radiation is unaffected by propagation (Mitra et al. 2009; Melikidze et al. 2014) or suggest that the radiation is modified due to the propagation effect (Petrova & Lyubarskii 2000; Hakobyan & Beskin 2014). However, the observational constraints from pulsar radiation can still be used to study the magnetospheric structure at the detachment point $h_{\text{em}}$. At these heights the estimates of the locus of the open dipolar field lines suggest the emission region to be close to a circular patch (Arndt & Eilek 2002; Dyks & Harding 2004). This motivates the idea of the emission components in the pulsar profile of normal pulsars as originating from an emission beam radiating approximately at a constant height and the angular dimensions of the emission component and beams can be constrained by spherical geometry.

The components can be separated into core and conal types, which exhibit distinct frequency evolutions as well as different polarization characteristics, with the core always being centrally located within the profile. The emission beam in pulsars has been conceptualized as a circularly symmetric structure consisting of a central core component surrounded by concentric rings of conal components (Rankin 1990, 1993a, 1993b, R90 and R93 hereafter). The principal justification for the above picture is provided by the different profile shapes in the pulsar population, which can be categorized into five distinct classes. The five component profiles consisting of a central core component flanked by two pairs of inner and outer conal outliers are classified as multiple (M) class. There are two classes of three-component profiles, namely core triple (T), which has a central core component along with one pair of conal outliers, and the conal triple (T,) where all three components are conal. The two-component profiles correspond to conal double (D) class. Finally, the single-component profiles are classified either as the core single (S) or the conal single (S,) classes. It is believed that the different profile classes arise due to different line-of-sight cuts through the emission beam and are therefore dependent on the pulsar geometry, with the beam radius ($r_p$) given as (Gil et al. 1984)

$$\sin^2(\rho_p/2) = \sin(\alpha)\sin(\alpha + \beta)\sin^2(W/4) + \sin^2(\beta/2),$$

where $W'$ is the estimated profile width at frequency $\nu$. Furthermore, by assuming $W'$ to be bound by the last open dipolar field lines, an estimate of the emission height ($h'$) above the neutron star polar cap can be computed as

$$h' = 10P\left(\frac{r_p}{10^2}\right)^2 \text{km},$$

where the radius of the opening angle of the polar cap at the stellar surface ($R = 10$ km) corresponds to $10^2\text{km}$. As discussed above the pulsar geometry, angles $\alpha$ and $\beta$ cannot be constrained using the RVM. Hence, $h'$ cannot be calculated with any certainty using Equations (2) and (3).

R90 established a dependence of the core component with pulsar period that enabled an alternative scheme for
estimating the pulsar geometry. The widths measured at 1 GHz \( W_{\text{core}}^{1 \text{GHz}} \) (estimated at 50% of peak intensity) showed the presence of a lower boundary line (LBL) scaling as \( 2.45 \, P^{-0.5} \).

R90 further established that core widths for several interpulsars (where \( \alpha \sim 90^\circ \)) lay along the LBL. Note that pulsars with core components have the line of sight passing centrally through the emission beam and hence their \( \beta \) is small. Incidentally, the boundary value \( 2.45 \) is very close to the diameter of the dipolar opening angle at the stellar surface. It was postulated that \( W_{\text{core}}^{1 \text{GHz}} \) encompasses the pulsar polar cap bounded by last open dipolar field lines. The pulsars with measured widths above the line correspond to non-orthogonal rotators with \( \sin \alpha < 1 \). This allowed an independent estimation of the angle \( \alpha \) from \( W_{\text{core}}^{1 \text{GHz}} \) using the relation

\[
W_{\text{core}}^{1 \text{GHz}} = 2.45P^{-0.5}/\sin \alpha. \tag{4}
\]

R93 expanded the above idea to estimate the geometry in a large number of pulsars with core components. The conal separations in profile classes T and M, with prominent core components, were also measured. It was proposed that the inner and outer cones originate at different heights and encompass the entire open field line region. The half-power points of inner and outer conal pairs (\( W_{\text{in, out}}^{\text{GHz}} \)) also enabled the estimation of the radius of the opening angle \( \rho^{\text{GHz}}_{\text{in, out}} \) for the conal rings as

\[
\rho^{\text{GHz}}_{\text{in}} = 4.3P^{-0.5}, \quad \rho^{\text{GHz}}_{\text{out}} = 5.28P^{-0.5}. \tag{5}
\]

R93 estimated the inner and outer cones to arise from heights (Equation (3)) of roughly 130 km and 220 km, respectively. The method of analyzing profile widths and components with R90 and R93 has an underlying assumption that all relevant widths arise from the entire open field line region, with an emphasis on the core, which is believed to fill up the entire polar cap and the inner and outer conal pairs, once again representing the entire open field line regions, etc. There are no provisions for interpreting the widths of the individual components in these schemes.

The above picture of the radio emission is difficult to reconcile with recent observations where the emission heights of the core and conal components are emitted from similar heights in the same pulsar. It has now been shown in many M and T class pulsars that the core and conal emission originates from similar heights (Mitra et al. 2007; Mitra & Rankin 2011; Smith et al. 2013; Mitra et al. 2016b). In this context, the \( P^{-0.5} \) scaling relation between \( W_{\text{core}}^{\text{GHz}} \) and \( \rho^{\text{GHz}} \) requires careful consideration. If we assume \( \beta \) to be small and \( \alpha \gg \beta \) (which is justified for central cuts with core emission), the radius of the emission beam in Equation (2) can be approximated as:

\[
\rho^{\text{GHz}} = \sqrt{4\sin^2(\alpha)} W^{\text{GHz}}/4 + \beta^2. \tag{6}
\]

Now, substituting \( \sin(\alpha) \) in terms of core width (Equation (4), which introduces the \( P^{-0.5} \) dependence) and \( \beta \) in terms of \( \sin(\alpha) \) and \( R_{\text{ppa}} \) (Equation (1)), the emission beam is estimated as

\[
\rho^{\text{GHz}} = 2.45P^{-0.5} F, \tag{7}
\]

where the factor \( F \) is given by

\[
F = \sqrt{\frac{4\sin^2(\frac{W^{\text{GHz}}}{4})}{(W_{\text{core}}^{\text{GHz}})^2} + \frac{1}{(R_{\text{ppa}} W_{\text{core}}^{\text{GHz}})^2}}. \tag{8}
\]

As is clear from the above exercise, the \( P^{-0.5} \) dependence in \( W_{\text{core}}^{\text{GHz}} \) is transferred from the \( P^{-0.5} \) dependence of the core width and is only preserved if the factor \( F \) does not have any period dependence.

To further investigate this in M and T class profiles with prominent core and conal components, we recall that \( R_{\text{ppa}} \) is usually large, hence the second term in Equation (8) can be ignored. Now, if we assume that the total width \( W^{\text{GHz}} \) to be small, \( F \) can be approximated as

\[
F \sim W_{\text{core}}^{\text{GHz}} / 2 W_{\text{core}}^{\text{GHz}}. \tag{9}
\]

The total width \( W_{\text{GHz}} \) can be separated into individual components:

\[
W_{i}^{\text{GHz}} = W_{\text{core}}^{\text{GHz}} + \sum_{i} W_{\text{con, i}}^{\text{GHz}} + \sum_{j} \delta W_{j}^{\text{GHz}}, \tag{10}
\]

where \( W_{\text{con, i}} \) is the width of the \( i \)th conal component, and \( \delta W_{j}^{\text{GHz}} \) is the \( j \)th separation between adjacent components, with the summation extending over all conal components as well as the respective separation between adjacent components. Now, for the factor \( F \) to be period-independent, \( W_{\text{GHz}}^{\text{GHz}} \) should also show a \( P^{-0.5} \) dependence. This further implies that along with \( W_{\text{core}}^{\text{GHz}} \), the \( W_{\text{GHz}}^{\text{GHz}} \) and \( \delta W_{j}^{\text{GHz}} \) should also have \( P^{-0.5} \) dependence.

To the best of our knowledge no such study exists in the literature that connects the conal components widths with the pulsar period. Only some hints of this effect have been discussed in Maciesiak et al. (2011, 2012), Maciesiak & Gil (2011), and Mitra et al. (2016a). In these works a distribution of the total half-power width of all available profiles with period found a LBL that scaled as \( P^{-0.5} \). Since no particular profile class was selected to obtain this relation, an LBL might exist for both core and conal widths following a similar \( P^{-0.5} \) relation. In this paper, we use the MSPES data to carry out a detailed study of the profile component widths and investigate their dependence on period. Furthermore, we explore the implications of the period dependence of component widths on the dipolar geometry.

### 2. Component Width Analysis

The principal analysis involved estimating the relevant widths of the components and the separation between adjacent components. Generally, the widths can be directly measured in the integrated profiles where the components are clearly distinguished. But in some pulsars, due to phenomena like subpulse drifting, the subpulses are seen to systematically drift in phase across the pulse window. It is also possible that in the same pulsar different components might be associated with different subpulse dynamics resulting in different widths. In such cases a more accurate estimate of the emission properties is possible by correcting for the subpulse motion across the pulse window and forming an average component from selected single pulses.

We used the MSPES data at 333 and 618 MHz for 123 pulsar with high-quality single pulses for this purpose (Mitra et al. 2016a). Specialized techniques were needed to enhance the individual components using precisely selected single pulses to make the relevant components more prominent. We have employed three different techniques (mentioned below) to generate the most prominent realization of each component in the pulsar profile. Once they were identified, the components were classified into core and conal types and their respective

\[
\text{Number of Pulsars: } 123
\]

\[
\text{Frequency: } 333 \text{ MHz and 618 MHz}
\]

\[
\text{Period range: } 6.5 \text{ s to } 28 \text{ s}
\]
widths at each frequency were measured at the 50% level of the peak intensity ($W_{\text{core}}$ and $W_{\text{cone}}$). The 50% level was selected as a representative width of the component, since any lower levels (like 25% or 10%) are usually contaminated by the adjacent components, making the estimation difficult in a large number of components.

2.1. Identifying Profile Components

Here, we discuss the three different techniques that were used to measure component widths using the single-pulse analysis. Before the single pulses were averaged, the baseline level from each single pulse was removed and the integrated profile peaks were normalized to unity after the components were formed.

1. Integrated Profile. In this method the average profiles were produced with the additional modification of selecting only significant single pulses with peaks above five times the rms noise level of the off-pulse baseline. This enhanced the signal-to-noise ratio of the components, especially in pulsars that showed nulling (Basu et al. 2017). This technique was most widely used in this work, with 89 pulsars at 333 MHz and 112 pulsars at 618 MHz where the components were characterized. Figure 1 illustrates the integrated profile for the pulsar J0304+1932 where we used this method.

2. Averaging Subpulses. In this method we separated out the components in 2 pulsars at 333 MHz and 3 pulsars at 618 MHz, which appear merged in the average profile. The subpulses corresponding to the components were seen to separate out in the single pulses and with jittering in phase. The peaks of each component were identified in the average profile using a peak detection technique. Template windows of width $2.45 P^{-0.5}$ and a center phase corresponding to the component peaks were set up. Any single pulse exceeding three times the off-pulse noise levels in each of these windows were considered for the respective average components. In addition, a further criterion for averaging was that the peak of the subpulse within the window was close to the central phase (within $5\sim10$ longitude bins). Finally, all relevant single pulses were averaged to generate a profile, with the relevant component prominently detected. A separate profile for each individual component was estimated using the minima of second derivatives of the profile curve.

Figure 1. This figure shows the measured width ($W_{\text{cone}}$) for the conal components and the separation ($W_{\text{sep}}$) between the peaks in the pulsar J0304+1932. The green and red points on the graph correspond to the peaks of the individual components, while the horizontal lines of the same color correspond to the measured component widths at the 50% level of the peak. The widths were estimated using the integrated profile (see Section 2.2). The black vertical lines indicate the measured separation between the peaks.

Figure 2. This figure shows the component estimation in the pulsar J1745−3040. The black curve shows the integrated profile. The yellow, green, and red curves correspond to the profiles used to measure the first, second, and third components using the second method described in Section 2.1. The second component could be clearly distinguished only after using these specialized techniques. The leading component is made up of sporadic emission and is unlikely to be a fully formed conal component. The width of the component is smaller than the typical conal width and is marked separately as a yellow dot in Figure 5.
generated with this technique. In Figure 2 the results of this exercise for the pulsar J1745–3040 are shown, where the second component becomes prominent after applying this technique. The leading component of this pulsar does not form a fully formed conal component, resembling a pre-cursor (Basu et al. 2015), and has not been used for component width analysis.

3. Averaging Peaks in Window. This technique was used to estimate the components in 20 pulsars at 333 MHz and 19 pulsars at 618 MHz. This was mainly useful for subpulses that showed prominent drift bands and systematic shift in phase within the pulse window. In order to measure the component widths, a technique was devised to average the peaks appearing at similar phases. The profile windows, 5–10 bins wide, were set up and spread contiguously across the whole profile. All significant subpulses (peak intensity greater than five times the baseline rms) with peaks within the relevant window were averaged to form a profile corresponding to that window. All such profiles with at least 10% of the total pulses were used for subsequent analysis. The profile separated out into individual components as shown in Figure 3 for the pulsar J2305+3100. By this method we generated several representative profiles that had one or more components. We measure the widths of all these components and produced an average component width for the pulsar. Similarly, we estimated the separation between components for all relevant cases and estimated an average $W_{sep}$.

2.2. Measuring Component Widths

Using the analysis scheme discussed above we were able to develop the best possible profile for the individual components. However, it was not always possible to clearly separate out the components in all cases. But in many of these cases it was still possible to find an estimate of the widths using certain fitting procedures. We employed three different techniques for measuring the 50% widths of the components that we describe below.

1. Full Width. This was the commonly used technique in our analysis where the 50% of peak intensity on either side of the component could be clearly measured (see Figure 1). The estimated width was the separation between the 50% points on either side of the peak. A total of 99 component widths at 333 MHz and 123 widths at 618 MHz were measured using this technique.

2. Half Width. This method was used to estimate the widths of 76 components at 333 MHz and 82 components at 618 MHz. This was utilized when the peaks were clearly seen but one side was not well-separated from the adjacent component. The separation of the peak from the 50% point of the well-resolved side was calculated and multiplied by two, to obtain an approximate estimate of the component width.

3. Gaussian Fits. In a small number of cases, 11 components at 333 MHz and 20 components at 618 MHz, the peak was not clearly seen. In these cases a Gaussian function was fitted to the component and the FWHM of the Gaussian was used as an estimate of the width. However, the Gaussian fit is not always an accurate model for the subpulses and the widths in such cases should be used with caution.

2.3. Separation between Adjacent Components

The separation between adjacent components ($W_{sep}$) in profiles with more than one component was calculated. We used the peak positions estimated in the previous subsection for each component to find the separations between them (see Figure 1 for a schematic). In some cases the exact location of the peak was uncertain due to a lower sensitivity of the emission and a Gaussian fit was used to approximate the peak position.

2.4. Error Estimation

The error ($\sigma$) in estimating the component width was evaluated as (Kijak & Gil 2003)

$$\sigma = \text{res} \times \sqrt{1 + \left(\frac{\text{rms}}{I}\right)^2},$$  \hspace{1cm} (11)

where res is the longitude resolution in degrees, rms corresponds to the baseline noise levels, and $I$ is the measured signal, which in our case corresponds to peak intensity. In case of separation between components, the error in estimating each
peak position was determined and the overall error was calculated by propagating the individual errors.

3. Results

In Table 2 we present the results of our analysis for the 123 pulsars observed at frequencies of 333 and 618 MHz. We have determined the number of profile components and its classification at each frequency and measured their widths wherever possible. In addition, we have also determined the separation between adjacent components. Additionally, the table also lists the measurement schemes employed for every component (see Sections 2.1 and 2.2 for details).

3.1. Component Widths and Separation

The widths were measured for 418 components in total, including 191 widths at 333 MHz and 227 widths at 618 MHz. There were 117 $W_{\text{core}}$ in total, with 54 $W_{333\text{ MHz}}$ and 63 $W_{618\text{ MHz}}$. Correspondingly, we measured 301 $W_{\text{cone}}$, with 137 $W_{333\text{ MHz}}$ and 164 $W_{618\text{ MHz}}$. In addition $W_{\text{sep}}$ had 217 measurements, with 100 $W_{333\text{ MHz}}$ and 117 $W_{618\text{ MHz}}$. The period dependences of $W_{\text{core}}$, $W_{\text{cone}}$, and $W_{\text{sep}}$ in logarithmic scales for each frequency are shown in Figures 4 and 5. There are four separate plots corresponding to all the measured widths, $W_{\text{all}}$ (top left panel), all $W_{\text{sep}}$ (top right panel), $W_{\text{core}}$ (bottom left panel), and $W_{\text{cone}}$ (bottom right panel). In each case it is clear that a lower boundary exists for the widths, as

![Figure 4](image-url)
well as the separations, though the boundary is less tightly constrained for $W_{\text{sep}}$. In addition, Figure 6 separately shows the conal widths estimated for the subpulse drifting pulsars using method 3 in Section 2.1. It is seen that the boundary estimates are not affected by these widths. A statistical approach using quantile regression (see the Appendix for a discussion on quantile regression) was employed to determine the LBL in each case.

### 3.2. The Lower Boundary Line

The LBL is of the form $W_B P^{-b}$, where $W_B$ is the boundary width and $b$ is the period dependence. Table 1 shows the estimates of the LBL for each category of measured widths. The first row represents the estimates of LBL for $W_{\text{core}}$, $W_{\text{cone}}$, and $W_{\text{all}}$ at each frequency using quantile regression. The $b$ value in each case is close to 0.5, suggesting that the $P^{-0.5}$ dependence is consistent for the component widths. We use the period dependence to exhibit the $P^{-0.5}$ relation in subsequent discussions, as a close approximation to the measured dependence. The boundary value $W_B$ was lower than $2^\circ.45$, which was the previously estimated boundary for the core widths at 1 GHz. The frequency evolution of component widths suggests that the measurements at 333 MHz and 618 MHz should be greater than the corresponding values at 1 GHz. However, the measured $W_B$ at 333 MHz was greater than that at
618 MHz, in accordance with our expectations. Assuming a frequency dependence of the component width \( (\times P^{\gamma}) \), the estimated \( \gamma \) is \(-0.15 \pm 0.01\) and the boundary at 1 GHz is \( W_0^{\text{GHP}} = 2.01 \pm 0.15 \), which is smaller than the previous estimates. In addition, the results also show that \( W_{B,\text{core}} \) \( (2.28 \pm 0.19\)\) and \( W_{B,\text{cone}} \) \( (2.24 \pm 0.11\)\) are similar. There was a possibility that the LBL was affected by the drifting widths measured using method 3 in Section 2.1. To eliminate this possibility, the boundary lines were estimated separately by removing the drifting widths. The estimated conal boundary lines for the entire population without the drifting widths are \( W_{B,\text{core},\text{nd}} \) \( 2.16 \pm 0.12\) and \( W_{B,\text{cone},\text{nd}} \) \( 2.23 \pm 0.14\), respectively. These values are similar to the overall boundaries (within measurement errors) and show that the widths of the drifting components do not bias the boundary line.

The statistical method was unable to constrain the period dependence in \( W_{\text{sep}} \). We nonetheless estimated a boundary for these quantities by assuming a \( P^{-0.5} \) dependence and estimating the boundary below which only 10% of points were present (the 0.1 quantile level; see the Appendix). The estimated boundaries of \( W_{\text{sep}} \) at 333 MHz, 618 MHz and both combined are shown in the second row of Table 1. Additionally, the lower boundary for \( W_{\text{prof}} \) in the average profiles (Mitra et al. 2016a) was also estimated at the 0.1 quantile level (assuming a \( P^{-0.5} \) dependence) and are shown in third row of Table 1.
4. Discussion

4.1. Period Dependence of Component Widths

It has been shown observationally that the radio emission in normal pulsars originate at heights $h_{\text{em}} \sim 500$ km (Blaskiewicz et al. 1991; von Hoensbroech & Xilouris 1997; Mitra & Li 2004; Weltevrede & Johnston 2008; Mitra & Rankin 2011). It has also been shown in several cases that both the core and conal components arise from similar heights within the pulsar magnetosphere (see Table 2 in Mitra et al. 2016b). In addition, we have now shown that both the core and conal components have similar LBLs, which shows a $P^{-0.5}$ dependence. Hence, the average emission beam in normal pulsars can be imagined to consist of a central core component surrounded by conal components along similar heights as shown schematically in Figure 7 (left panel). It was argued in earlier works (e.g., R90, Maciesiak et al. 2012) that $P^{-0.5}$ is an indication of the evolution of the components along the dipolar field lines, since the open field line regions in dipolar magnetic fields are associated with the light cylinder radius that has a period dependence ($R_{LC} = P c / 2 \pi$).

The equation for the field lines in a dipolar field is given as

$$r = R_{C} \sin^2 \theta.$$  \hspace{1cm} (12)

Here, $r$ and $\theta$ are the polar coordinates and $R_{C}$ is the constant of the field lines. The radio emission along the tangential direction to the field lines is given as

$$\tan \phi = \frac{3 \sin \theta \cos \theta}{3 \cos^2 \theta - 1},$$  \hspace{1cm} (13)

which for small angles can be expressed as $\phi = \frac{1}{2} \theta$. The components are bound by the dipolar field lines that follow the above equation and the only period dependence is associated with the last open field lines where $R_{C} = R_{LC}$. Thus, if the emission height is constant for different periods ($r = R_{C}$) and the components occupy inner field lines, i.e., $R_{C} = R_{LC}$, the width of the components should remain constant and not show any period dependence.

The analysis of profile component widths reported in this work suggests a deviation from the conventional view that the period dependence of widths is a consequence of the dipolar geometry in pulsars. We have shown that the $P^{-0.5}$ dependence is indeed seen individually in the core components as well as the conal components. However, this dependence is not a result of dipolar geometry if we assume that the components occupy a small area in the open field line region of pulsars. The $P^{-0.5}$ dependence follows from the light cylinder radius associated with the last open field line, hence any component with an edge not along the last open field line will not follow this dependence. Additional estimates of the underlying widths suggest that they are identical in both the core and conal components. This indicates that the emission heights in normal pulsars are similar across the pulsar magnetosphere at any given frequency, contrary to previous claims of the core and conal emission having different locations within the magnetosphere (Rankin 1990, 1993a).

4.2. Period Dependence of Profile Widths

It is worthwhile to look at the implications of the component widths on the overall profile width and their period dependence. First, it is unlikely that the measured full width at 50% points stretches out to the last open field lines. As we discuss above, the $P^{-0.5}$ scaling of overall widths is not likely due to the effect of dipolar geometry. However, seeing the effect of the LBL on full profile widths, especially in pulsars with more than one component, is difficult, as the LBL is dominated by $S_i$ and $S_d$ classes with single components. As discussed in Section 1 a $P^{-0.5}$ dependence is reported for the opening angle $\rho'$ corresponding to measured widths for multiple component profiles (M and T classes). We argued that the dependence is an effect of the estimation process of $\rho'$ and follows from the period dependence in the core widths. In order to preserve this relation we found that the quantity $F$ defined in Equation (5) should be period-independent. Based on our analysis we are now in a position to verify this claim. As shown earlier, the full width at any frequency $\nu$ can be broken down as $W_{\nu} = W_{\text{comp}} + 2n W_{\text{cone}} + 2n \delta W_{\nu}$, with $n = 1, 2$ for the inner and outer cones, respectively. We have assumed identical widths for all the conal components (which is very likely given the presence of boundary line), as well as all the separations between components (less well-constrained due to the absence of a clear boundary). We have now shown that both $W_{\text{comp}}$ and $W_{\text{cone}}$ follow a $P^{-0.5}$ dependence individually and are more or less identical (which we call $W_{\text{comp}}$). The final part gives the distance between adjacent components and can be represented as $\delta W_{\nu} = W_{\nu} - W_{\text{comp}}$, where $W_{\text{sep}}$ is the separation between the adjacent peaks of components. Our estimates of $W_{\text{sep}}$ in Section 3 did not show an explicit period dependence as seen in the components. But boundary values $W_{\text{sep}} = 2.66$ and $W_{\text{comp}} = 2.25$ imply $\delta W = 0.41$; i.e., $\delta W \approx 0.18 W_{\text{comp}}$. Thus, the distance between the components forms a small fraction of the component widths. Hence, even if $\delta W$ do not show a $P^{-0.5}$ dependence, their contribution to the overall width will be small enough. This means that the factor $F$ in Equation (9) can be approximated to be largely period-independent.
| No | PSR       | $P$ (s) | $N_c$ | Method | $W_{90}$ (degree) | $W_{sep}$ (degree) | No | Method | $W_{90}$ (degree) | $W_{sep}$ (degree) |
|----|-----------|---------|-------|--------|------------------|-------------------|----|--------|------------------|-------------------|
| 1  | B0031−07  | 0.94    | 2     | 3-a/b  | 11.29 ± 0.13     | 17.14 ± 0.13      | 2  | 3-a/b  | 9.61 ± 0.13      | 16.21 ± 0.13      |
| 2  | J0134−2937| 0.14    | 1     | 1-a    | 16.20 ± 0.60     | 12.72 ± 0.13      | 1  | 1-a    | 18.80 ± 0.90     | ⋯                 |
| 3  | B0148−06  | 1.46    | 2     | 1-a    | 6.89 ± 0.09      | 30.51 ± 0.09      | 2  | 1-a    | 6.49 ± 0.09      | 27.01 ± 0.09      |
| 4  | B0149−16  | 0.83    | 2     | 1-a, b | 3.38 ± 0.15      | 5.72 ± 0.15       | 2  | 1-a, b | 2.75 ± 0.15      | 5.36 ± 0.15       |
| 5  | B0203−40  | 0.63    | 2     | 1-a, c | 3.83 ± 0.20      | 6.46 ± 0.20       | 1  | 1-a    | 4.13 ± 1.00      | ⋯                 |
| 6  | B0301−19  | 1.39    | 2     | 1-a    | 3.20 ± 0.09      | 12.69 ± 0.09      | 2  | 1-a    | 4.75 ± 0.09      | 10.27 ± 0.09      |
| 7  | B0450−18  | 0.55    | 4     | 1/3-a, b, b | 4.19 ± 0.23    | 7.90 ± 0.23       | 4  | 1-b, c, b | 4.03 ± 0.23    | 7.58 ± 0.23       |
| 8  | B0523+11  | 0.35    | 2     | 1-a, b | 4.00 ± 0.35      | 12.24 ± 0.35      | 2  | 1-a, a | 7.83 ± 0.35      | 11.24 ± 0.35      |
| 9  | B0525+21  | 3.75    | 2     | 1-a, a | 2.78 ± 0.03      | 14.69 ± 0.03      | ⋯  | ⋯     | ⋯                | ⋯                 |
| 10 | B0540+23  | 0.25    | 2     | 1/3-a  | 8.34 ± 0.51      | 18.72 ± 0.51      | 2  | 1/3-a  | 7.40 ± 0.51      | 10.44 ± 0.51      |
| 11 | B0611+22  | 0.33    | 1     | 1-a    | 6.95 ± 0.37      | ⋯                 | 1  | 1-a    | 5.91 ± 0.38      | ⋯                 |
| 12 | B0626+24  | 0.48    | 2     | 1-a, b | 5.86 ± 0.26      | 8.17 ± 0.26       | 2  | 1-a    | 6.36 ± 0.26      | 6.13 ± 0.26       |
| 13 | B0628−28  | 1.24    | 1     | 1-a    | 19.00 ± 0.10     | ⋯                 | 1  | 1-a    | 18.33 ± 0.10     | ⋯                 |
| 14 | B0656+14  | 0.38    | ⋯     | 1-a    | 15.60 ± 0.30     | ⋯                 | 1  | 1-a    | 14.30 ± 0.60     | ⋯                 |
| 15 | B0727−18  | 0.51    | 3     | 1-a, b | 3.17 ± 0.25      | ⋯                 | 3  | 1-a, b | 3.38 ± 0.25      | ⋯                 |
| 16 | B0736−40  | 0.37    | ⋯     | ⋯     | ⋯                 | ⋯                 | 3  | 1-b, b | 7.99 ± 0.33      | 17.72 ± 0.33      |
| 17 | B0740−28  | 0.17    | 3     | 1-b, b | 6.40 ± 0.75      | 5.74 ± 0.75       | 4  | 1-b, b | 5.39 ± 0.75      | 2.66 ± 0.75       |
| 18 | B0756−15  | 0.68    | 2     | 1-b, b | 3.95 ± 0.18      | 2.08 ± 0.18       | 2  | 1-b, b | 2.68 ± 0.18      | 2.01 ± 0.18       |
| 19 | B0818−13  | 1.24    | 2     | 3-a/b  | 3.61 ± 0.10      | 2.64 ± 0.10       | 2  | 3-a/b  | 3.60 ± 0.10      | 3.07 ± 0.10       |
| 20 | B0818−41  | 0.55    | ⋯     | ⋯     | ⋯                 | ⋯                 | 2  | 1-b, b | 4.94 ± 2.33      | 72.00 ± 2.33      |
| 21 | B0834+06  | 1.27    | 2     | 1-a, b | 2.87 ± 0.10      | 4.59 ± 0.10       | 2  | 1-a, b | 2.86 ± 0.10      | 4.93 ± 0.10       |
| 22 | B0844−35  | 1.12    | 4/5   | 1-b, a, b | 6.83 ± 0.56    | 6.75 ± 0.56       | 4/5 | 1-b, a, b | 5.64 ± 0.57    | 5.56 ± 0.56       |
| 23 | J0905−4536| 0.99    | ⋯     | ⋯     | ⋯                 | ⋯                 | 2  | 1-a, a | 12.87 ± 1.92     | 48.54 ± 1.91      |

Table 2
Details of the Measurements of Widths in Pulsars
| No | PSR     | $P$ (s) | $N_c$ | Method | $W_{333}$ (degree) | $W_{618}$ (degree) |
|----|---------|---------|-------|--------|-------------------|-------------------|
| 24 | J0905−5127 | 0.35   | 1     | 1/2-b, b | 9.73 ± 0.36       | 4.37 ± 0.36       |
| 25 | B0906−17 | 0.40   | 2     | 1-a    | 8.05 ± 0.31       | 2.77 ± 0.36       |
| 26 | B0919+06 | 0.43   | 3     | 1-b, b | 5.86 ± 0.29       | 4.59 ± 0.31       |
| 27 | B0942−13 | 0.57   | 2     | 1/3-b  | 2.95 ± 0.22       | 3.29 ± 0.29       |
| 28 | B0950+08 | 0.25   | 2     | 1/3-a, a | 16.06 ± 0.50   | 3.08 ± 0.20       |
| 29 | B0957−47 | 0.67   |       |        |                   | 18.93 ± 0.88      |
| 30 | J1034−3224 | 1.15   | 5     | 1-a, b, b, b | 6.52 ± 0.11 | 13.77 ± 0.11     |
| 31 | B1039−19 | 1.39   |       |        |                   | 5.70 ± 0.20       |
| 32 | B1114−41 | 0.94   | 1     | 1-a    | 5.16 ± 0.11       | 1.76 ± 0.11       |
| 33 | B1133+16 | 1.19   | 2     | 1-a, a | 1.95 ± 0.11       | 7.60 ± 0.11       |
| 34 | B1237+25 | 1.38   | 5     | 1-a, b, a, c, b | 1.51 ± 0.09 | 2.56 ± 0.09       |
| 35 | B1254−10 | 0.62   | 2     | 1-a, a | 2.95 ± 0.20       | 10.61 ± 0.20      |
| 36 | B1322+83 | 0.67   |       |        |                   | 2.37 ± 0.19       |
| 37 | B1325−43 | 0.53   | 2     | 1-b, b | 4.59 ± 0.24       | 6.81 ± 0.24       |
| 38 | B1325−49 | 1.48   | 5     | 1-b, b, a, a, b | 2.51 ± 0.09 | 2.45 ± 0.09       |
| 39 | J1415−3921 | 1.10  |       |        |                   | 4.52 ± 0.57       |
| 40 | B1504−43 | 0.29   | 1     | 1-a    | 7.40 ± 0.80       | 6.20 ± 0.80       |
| 41 | B1524−39 | 2.42   | 2     | 1-a, a | 1.89 ± 0.05       | 1.90 ± 0.19       |
| 42 | J1549−4848 | 0.29  | 1     | 1-a    | 6.40 ± 0.80       | 7.10 ± 0.80       |
| 43 | B1555−31 | 0.52   | 2     | 1-a, a | 4.84 ± 0.24       | 15.73 ± 0.24      |
| 44 | J1557−4258 | 0.33   |       |        |                   | 4.17 ± 0.41       |
Table 2
(Continued)

| No  | PSR    | $P$  | $N_r$ | Method | $W_{30}$ (degree) | $W_{exp}$ (degree) | $N_r$ | Method | $W_{30}$ (degree) | $W_{exp}$ (degree) |
|-----|--------|------|-------|--------|-------------------|-------------------|-------|--------|-------------------|-------------------|
| 45  | B1556−44 | 0.26 | 3     | 1/3-b, a | 10.76 ± 0.49 | 8.04 ± 0.49 | 13.70 ± 0.49 | 6.49 ± 0.49 | 8.12 ± 0.49 | 13.10 ± 0.49 |
| 46  | B1558−50  | 0.86 | …    | …       | …               | …                 | 2     | 1-a, b | 2.73 ± 0.14 | 3.77 ± 0.14 | 4.51 ± 0.14 |
| 47  | J1603−2531 | 0.28 | 1     | 1-a     | 10.60 ± 0.80 | …                 | 1     | 1-a    | 8.80 ± 0.80 | …                 |
| 48  | B1600−49   | 0.33 | …    | …       | …               | …                 | 3     | 1-c, a, c | 3.52 ± 0.39 | 4.88 ± 0.38 | 7.80 ± 0.38 |
| 49  | J1625−4048 | 2.36 | …    | …       | …               | …                 | 3     | 1-a, a, a | 1.73 ± 0.05 | 3.02 ± 0.05 | 6.15 ± 0.05 |
| 50  | B1634−45   | 0.12 | 1     | …       | …               | …                 | 1     | 1-a    | 16.40 ± 1.90 | …                 |
| 51  | B1642−03   | 0.39 | …    | …       | …               | …                 | 3     | 1/3-a, b | 3.63 ± 0.32 | 5.60 ± 0.32 | 8.68 ± 0.32 |
| 52  | J1648−3256 | 0.72 | 1     | 1-a     | 5.98 ± 0.52 | …                 | 1     | 1-a    | 5.71 ± 0.52 | …                 |
| 53  | J1700−3312 | 1.36 | 3     | 1-b, b  | 4.32 ± 0.09 | 4.58 ± 0.10 | 5.05 ± 0.10 | 4.61 ± 0.10 | 4.78 ± 0.10 | 4.57 ± 0.10 |
| 54  | B1700−32   | 1.21 | 3     | 1-b, c, d | 5.05 ± 0.10 | 4.80 ± 0.10 | 5.40 ± 0.10 | 5.76 ± 0.10 | 4.78 ± 0.10 | 4.57 ± 0.10 |
| 55  | J1705−3423 | 0.26 | …    | …       | …               | …                 | 1     | 1-a    | 20.09 ± 0.49 | …                 |
| 56  | B1706−16   | 0.65 | 1     | 1-a     | 5.80 ± 0.19 | …                 | 1     | 1-a    | 5.57 ± 0.19 | …                 |
| 57  | B1706−44   | 0.10 | 1     | 1-a     | 42.30 ± 2.20 | …                 | 1     | 1-a    | 31.90 ± 2.20 | …                 |
| 58  | B1717−29   | 0.62 | 3     | 3-a/b   | 4.99 ± 0.21 | 4.99 ± 0.20 | 6.53 ± 0.20 | 4.24 ± 0.20 | 7.56 ± 0.20 | 8.70 ± 0.20 |
| 59  | B1718−32   | 0.48 | …    | …       | …               | …                 | 2     | 1-b, b | 3.81 ± 0.26 | 7.81 ± 0.26 | 5.57 ± 0.26 |
| 60  | B1719−37   | 0.24 | 1     | 1-a     | 11.85 ± 1.60 | …                 | 1     | 1-a    | 7.52 ± 1.60 | …                 |
| 61  | J1727−2739 | 1.29 | 2     | 1-a     | 13.51 ± 1.50 | 10.04 ± 1.47 | 25.79 ± 1.46 | 6.51 ± 1.47 | 6.30 ± 1.77 | 22.69 ± 1.46 |
| 62  | B1727−47   | 0.83 | 3     | 1-b, b  | 2.71 ± 0.15 | 2.87 ± 0.15 | …       | 2.50 ± 0.15 | …                 |
| 63  | B1730−22   | 0.87 | 4     | 1-a, c, a | 5.13 ± 0.14 | 6.02 ± 0.14 | 10.46 ± 0.14 | 6.65 ± 0.29 | 10.05 ± 0.29 | 10.67 ± 0.29 |
| 64  | B1730−37   | 0.34 | …    | …       | …               | …                 | 2     | 1-a, a | 9.16 ± 1.88 | 14.95 ± 1.91 | 40.88 ± 1.86 |
| 65  | B1732−07   | 0.42 | 3     | 1-b, a, b | 4.31 ± 0.30 | 3.83 ± 0.30 | 9.71 ± 0.30 | 3.69 ± 0.30 | 9.28 ± 0.30 | 4.86 ± 0.30 |
| 66  | B1736−29   | 0.32 | …    | …       | …               | …                 | 1     | 1-a    | 8.16 ± 0.40 | …                 |
| 67  | B1737+13   | 0.80 | 5     | 1/3-a, c, b, b | …       | …                 | 5     | 1/3-a, c, b, b | …                 |
| No  | PSR       | $P$ (s) | $N_e$ | Method       | $W_0$ (degree) | $W_{sep}$ (degree) | No  | PSR       | $P$ (s) | $N_e$ | Method       | $W_0$ (degree) | $W_{sep}$ (degree) |
|-----|-----------|---------|------|--------------|----------------|------------------|-----|-----------|---------|------|--------------|----------------|------------------|
|     |           |         |      |              |                |                  |     |           |         |      |              |                |                  |
| 68  | B1738−08  | 2.04    | 4    | 1-b, b      | 2.72 ± 0.06    | ...              | 2   | B1737−39  | 0.51    | 2    | 1-b         | 10.88 ± 0.24    | ...              |
| 70  | B1742−30  | 0.37    | 4    | 1-a, a      | 5.86 ± 0.35    | ...              | 3   | B1745−12  | 0.39    | 3    | 1-b, b, c   | 3.62 ± 0.32      | 4.49 ± 0.32      |
| 72  | J1750−3503| 0.68    | 1    | 1-a         | 38.30 ± 0.30   | ...              | 1   | J1754−24  | 0.23    | ... | ...         | ...             | ...              |
| 73  | B1747−46  | 0.74    | 2    | 1-b, b      | 3.55 ± 0.17    | 4.17 ± 0.17      | 2   | B1758−03  | 0.92    | 1/3 | 1-a         | 4.73 ± 0.14      | ...              |
| 74  | B1749−28  | 0.56    | 3    | 1/3-b, b    | 3.23 ± 0.22    | 8.03 ± 0.22      | 3   | B1758−29  | 1.08    | 3    | 1-a, a, a  | 3.23 ± 0.35      | 9.33 ± 0.35      |
| 78  | B1804−08  | 0.16    | 3    | 1-a         | 10.87 ± 0.77   | ...              | 3   | B1804−21  | 0.87    | 5    | 1-a         | 5.82 ± 0.32      | 5.70 ± 0.32      |
| 79  | J1808−0813| 0.88    | 1    | 1-a         | 14.90 ± 0.30   | ...              | 1   | J1817−3837| 0.38    | ... | ...         | ...             | ...              |
| 80  | B1813−26  | 0.59    | 2    | 1-a, a      | 11.12 ± 1.06   | 26.20 ± 1.06     | 2   | B1813−36  | 0.39    | 3    | 1/3-a       | ...             | ...              |
| 82  | J1817−3837| 0.38    | ...  | ...         | ...             | ...              | 3   | B1818−04  | 0.60    | 2    | 1-b         | 4.75 ± 0.21      | ...              |
| 84  | B1819−22  | 1.87    | 2    | 3-b/a       | 6.13 ± 0.20    | 8.60 ± 0.20      | 2   | B1819−22  | 1.87    | 2    | 3-b/a       | 6.13 ± 0.20      | 8.60 ± 0.20      |
| 85  | J1823−0154| 0.76    | 2    | 1-a         | 60 ± 0.30      | ...              | 2   | B1821+05  | 0.75    | 3    | 1-a, a, b  | 3.10 ± 0.17      | 13.99 ± 0.17     |
| 86  | B1821+05  | 0.75    | 3    | 1-a, a, b   | 3.10 ± 0.17    | 13.99 ± 0.17     | 3   | B1820−31  | 0.28    | 2    | 1/3-a       | 6.40 ± 0.44      | 4.06 ± 0.44      |
| 87  | B1820−31  | 0.28    | 2    | 1/3-a       | 6.40 ± 0.44    | ...              | 2   | B1831−04  | 0.29    | 5    | 1-a, a, a  | 5.80 ± 0.44      | 3.74 ± 0.44      |

Table 2 (Continued)
Table 2
(Continued)

| No | PSR       | $P$ (s) | $N_\ell$ | Method | $W_\ell$ (degree) | $W_{\text{exp}}$ (degree) | $N_\ell$ | Method | $W_\ell$ (degree) | $W_{\text{exp}}$ (degree) |
|----|-----------|---------|----------|--------|------------------|--------------------------|----------|--------|------------------|--------------------------|
| 89 | J1835−1020 | 0.30    | ...      | ...    | ...              | ...                      | 1        | 1-a    | 10.80 ± 0.70     | ...                      |
| 90 | J1835−1106 | 0.17    | 1        | 1-a    | 26.70 ± 1.30     | ...                      | 1        | 1-a    | 13.30 ± 1.30     | ...                      |
| 91 | B1839+09   | 0.38    | 2        | 1/3-a, b | 3.97 ± 0.33     | 4.31 ± 0.33              | 2        | 1-a    | 6.40 ± 0.33      | ...                      |
| 92 | B1839−04   | 1.84    | 2        | 1-a, a | 45.48 ± 2.08     | 55.16 ± 0.20             | 2        | 1-a, a | 18.68 ± 0.20     | 53.26 ± 0.20             |
| 93 | J1843−0000 | 0.88    | ...      | ...    | ...              | ...                      | 1        | 1-a    | 12.23 ± 0.14     | ...                      |
| 94 | B1842+14   | 0.38    | 2        | 1-a    | 8.24 ± 0.33      | ...                      | 1/2      | 1-a    | 9.30 ± 0.33      | ...                      |
| 95 | B1844−04   | 0.60    | ...      | ...    | ...              | ...                      | 2        | 1-b, b | 8.44 ± 0.21      | 5.48 ± 0.21              |
| 96 | B1845−01   | 0.66    | ...      | ...    | ...              | ...                      | ...      | ...    | ...              | ...                      |
| 97 | J1848−1414 | 0.30    | 1        | 1-a    | 19.03 ± 3.70     | ...                      | 1        | 1-a    | 13.4 ± 3.70      | ...                      |
| 98 | B1846−06   | 1.45    | ...      | ...    | 3                | 3.30 ± 0.09              | 3        | 1-a    | ...              | 17.41 ± 1.16             |
| 99 | J1852−2610 | 0.34    | 2        | 1-a, a | 5.80 ± 0.38      | 19.75 ± 0.37             | 2        | 1-b, a | 8.60 ± 1.16      | 17.41 ± 1.16             |
| 100| B1857−26   | 0.61    | 5        | 1-b, c, a, c, a | 6.28 ± 0.20     | 10.25 ± 0.20             | 5        | 1-b, c, c, b  | 9.73 ± 0.20      | ...                      |
| 101| J1901−0906 | 1.78    | 2        | 1-a, a | 2.48 ± 0.07      | 9.29 ± 0.07              | 2        | 1-a, a | 2.05 ± 0.07      | 8.49 ± 0.07              |
| 102| B1907+10   | 0.28    | 3        | 1-a    | 6.19 ± 0.44      | ...                      | 3        | 1-c, a | 5.48 ± 0.47      | 9.05 ± 0.44              |
| 103| B1907+03   | 2.33    | 3        | 1-c, a, a | 17.07 ± 0.27     | 25.45 ± 0.27             | 3        | 1-b, c, a | 16.38 ± 0.27     | 23.1 ± 0.27              |
| 104| B1911−04   | 0.83    | 2        | 1/3-a  | 2.98 ± 0.15      | 3.35 ± 0.15              | 2        | 1/3-a | 3.21 ± 0.15      | 3.44 ± 0.15              |
| 105| B1914+09   | 0.27    | 2        | 1-a, b | 6.18 ± 0.46      | 7.21 ± 0.46              | 2        | 1-a, a | 5.42 ± 0.47      | 8.2 ± 0.46               |
| 106| B1915+13   | 0.19    | 2        | 1/3-a  | 5.22 ± 0.64      | ...                      | 2        | 1/3-a | 4.75 ± 0.65      | ...                      |
| 107| B1917+00   | 1.27    | 3        | 1-b, a | 3.13 ± 0.10      | 3.34 ± 0.10              | 3        | 1-b, a, b | 2.67 ± 0.10      | 3.27 ± 0.10              |
| 108| J1919+0134 | 1.60    | ...      | ...    | ...              | ...                      | 2        | 1-a, a | 9.17 ± 0.24      | 10.10 ± 0.23             |
| 109| B1918+19   | 0.82    | 4        | 1-b, bc | 5.42 ± 0.46      | 14.24 ± 0.46             | 4        | 1-b, b | 5.15 ± 0.46      | 11.01 ± 0.46             |
### Table 2 (Continued)

| No | PSR     | $P$ (s) | $N_c$ | Method | $W_{90}$ (degree) | $W_{30}$ (degree) | $W_{exp}$ (degree) | $N_c$ | Method | $W_{90}$ (degree) | $W_{30}$ (degree) | $W_{exp}$ (degree) |
|----|---------|---------|-------|--------|-------------------|-------------------|-------------------|-------|--------|-------------------|-------------------|-------------------|
| 110 | B1919+21 | 1.34    | ...   | ...    | ...               | ...               | ...               | 3/2   | 1/2/3-b, b | 2.29 ± 0.09 | ...               | 3.41 ± 0.09 |
| 111 | B1929+10 | 0.23    | 2      | 1-a, b | 5.32 ± 0.55       | 6.18 ± 0.55       | 5.09 ± 0.55       | 2     | 1-a, b   | 5.90 ± 0.55 | 4.58 ± 0.55   | 3.52 ± 0.55 |
| 112 | B1937–26 | 0.40    | 2      | 1-a, b | 3.38 ± 0.31       | 3.50 ± 0.31       | 5.05 ± 0.31       | 2     | 1-a, b   | 2.54 ± 0.31 | 2.57 ± 0.31   | 5.05 ± 0.31 |
| 113 | B1944+17 | 0.44    | 3      | 3-b/a  | 7.89 ± 0.28       | 6.37 ± 0.28       | 11.98 ± 0.28      | 3     | 3-b/a    | 11.55 ± 0.28 | 7.88 ± 0.28   | 9.51 ± 0.28 |
| 114 | B2003–08 | 0.58    | 5      | 1-b, b, a, b, b | 8.70 ± 0.65 | 6.73 ± 0.65 | 13.28 ± 0.65 | 5     | 1-b, a, a, b | 14.72 ± 0.66 | 7.16 ± 0.66 | 8.24 ± 0.66 |
| 115 | B2043–04 | 1.55    | 3      | 3-b/a  | 2.45 ± 0.08       | 2.12 ± 0.08       | 2.89 ± 0.08       | 3     | 3-b/a    | 1.80 ± 0.08 | 1.49 ± 0.08   | 2.86 ± 0.08 |
| 116 | B2044+15 | 1.14    | ...    | ...    | ...               | ...               | ...               | 2     | 1-c, a   | 5.70 ± 0.11 | 2.69 ± 0.11   | 9.17 ± 0.11 |
| 117 | B2045–16 | 1.96    | 3      | 1-a, b, a | 2.17 ± 0.06 | 3.86 ± 0.06 | 9.06 ± 0.06 | 3     | 1-a, b, a | 2.67 ± 0.06 | 4.45 ± 0.06 | 8.52 ± 0.06 |
| 118 | J2144–3933 | 8.51   | 1      | 1-a    | 0.80 ± 0.06       | ...               | ...               | 1     | 1-a      | 0.76 ± 0.12 | ...               | ...               |
| 119 | B2303+30 | 1.58    | 2      | 3-a/b  | 3.48 ± 0.08       | 2.40 ± 0.08       | 3.93 ± 0.08       | ...   | ...      | ...               | ...               | ...               |
| 120 | B2310+42 | 0.35    | 4      | 1-b, b, b | 3.89 ± 0.36 | 5.88 ± 0.36 | 5.07 ± 0.36 | 2     | 3-a      | 2.05 ± 0.26 | 2.54 ± 0.26 | 2.39 ± 0.26 |
| 121 | B2315+21 | 1.44    | 2      | 3-a/b  | 2.18 ± 0.09       | 3.36 ± 0.09       | 2.24 ± 0.09       | 2     | 3-a      | 2.05 ± 0.26 | 2.54 ± 0.26 | 2.39 ± 0.26 |
| 122 | B2327–20 | 1.64    | 3      | 1-a, a, b | 1.44 ± 0.08 | 1.86 ± 0.08 | 2.70 ± 0.08 | 3     | 1-a, c   | 1.49 ± 0.08 | 2.06 ± 0.08 | 2.79 ± 0.08 |
| 123 | J2346–0609 | 1.18   | 2      | 1-a, a | 2.24 ± 0.11       | 2.83 ± 0.11       | 16.48 ± 0.11      | 2     | 1-a, a   | 2.27 ± 0.21 | 3.23 ± 0.21 | 14.04 ± 0.21 |

**Note.** The measurements include widths of components ($W_{90}$) and separation between adjacent components ($W_{30}$). Column 1 presents the name, and column 2 present the period ($P$). Columns 4–7 give the measurements at 333 MHz, which include the number of components $N_c$, the technique used to estimate the component (1—integrated profile, 2—averaging subpulses, 3—averaging peaks in window), along the width estimation method (a—full width, b—half width, c—fitting Gaussian). Columns 8–11 give the corresponding values for 618 MHz data.

### 5. Summary

In this work we have carried out detailed measurements to characterize the widths of the core and conal components separately for a large number of normal pulsars. We have shown that the component widths, when distributed with period, show the presence of identical LBLs for both the core and conal components, which follow a $P^{-0.5}$ dependence. These results firmly establish the cone and the conal components to be equivalent within the pulsar profile and eliminate any requirement for two different emission mechanisms for core and conal emission at different heights (Rankin 1990, 1993a; Melrose 2017). We have also highlighted that in normal pulsars where the radio emission is supposed to originate at heights of around 500 km from the stellar surface throughout the pulsar profile, the $P^{-0.5}$ dependence of the LBL is a result of the physical mechanism and not an outcome of the dipolar nature of the magnetic field in the emission region. In the absence of
Figure 7. This figure shows a schematic of the core and conal components along the dipolar open field line region in a normal pulsar. The component widths and their separation are estimated from the lower boundary lines for the distribution of widths with period. The emission is assumed to originate at a constant height above the surface for all the emission components.

Further constraints from observations, more detailed modeling of the emission processes are required to explain the period dependence of components.

We thank our second referee for the insightful comments, which helped to improve the paper. We thank Mihir Arjunwadkar for the discussion on the LBL analysis techniques. We also thank the staff of the Giant Meterwave Radio Telescope and National Center for Radio Astrophysics for providing valuable support in carrying out this project. This work was supported by grants DEC-2012/05/B/ST9/03924, DEC-2013/09/B/ST9/02177, and UMO-2014/13/B/ST9/00570 of the Polish National Science Centre.

Appendix
Quantile Regression

The quantile regression is distinct from the traditional least-squares method. The least-square method finds the conditional mean function, $E(y|x)$, where $y$ corresponds to the mean value for the data at variable $x$. The quantile regression, on the other hand, gives a more generalized response function, $Q_{q}(y|x)$, where the quantile $q$ is such that $y$ splits the data at any $x$ with a $q$ fraction below $y$ and a $1 - q$ fraction above $y$. If the prediction error of a model function is $\epsilon_i$ for the $i$th variable, for the least-squares method the minimization of the term $\sum \epsilon_i^2$ is carried out. In quantile regression, asymmetric penalties are sought with weights $(1 - q)|\epsilon_i|$ for overprediction and $q|\epsilon_i|$ for underprediction.

Let us consider $\hat{y}(x)$ to denote the prediction function and $\epsilon(x) = y - \hat{y}(x)$ to denote the prediction error. Then $E(\epsilon(x)) = E(y - \hat{y}(x))$ denotes the loss associated with the prediction error. In the least-squares formulation the loss function is $E(\epsilon) = \epsilon^2$. In quantile regression, if $\hat{y}_q(x)$ is the $q$th quantile prediction, then the loss associated with the prediction error is asymmetric with either $E_{\hat{y}_q}(y - \hat{y}(x)) = q|y_i - \hat{y}_q(x)|$ if $y_i \geq \hat{y}_q(x)$ or $E_{\hat{y}_q}(y - \hat{y}(x)) = (1 - q)|y_i - \hat{y}_q(x)|$ if $y_i < \hat{y}_q(x)$. The objective function for minimization is given as

$$Q(\hat{y}_q) = \sum_{y_i \geq \hat{y}_q(x)} q|y_i - \hat{y}_q(x)| + \sum_{y_i < \hat{y}_q(x)} (1 - q)|y_i - \hat{y}_q(x)|.$$  

The above function is non-differentiable but can be minimized using the simplex method to find the optimal solution for $\hat{y}_q(x)$.

The LBL in the component width distribution with period was estimated using quantile regression. The data were converted to a logarithmic scale in order to make the boundary line ($\hat{y}_q(x) = ax + b$) a linear function of period. The quantile regression is implemented in the statsmodel Python module’s QuantReg class, which was used for these estimates. We investigated the boundary line for multiple definitions of the quantile $q = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5$. The results for each case are shown in Figure 8, where all the component widths have been considered. As seen in the figure, the boundary line has a slope close to $-0.5$ for all $q \geq 0.1$, reproducing the period dependence. Hence, we have used $q = 0.1$ as our quantile level for estimating the LBL.

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