Synchronization of nonsolitonic Kerr combs

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Synchronization is a ubiquitous phenomenon in nature that manifests as the spectral or temporal locking of coupled nonlinear oscillators. In the field of photonics, synchronization has been implemented in various laser and oscillator systems, enabling applications including coherent beam combining and high-precision pump-probe measurements. Recent experiments have also shown time-domain synchronization of Kerr frequency combs via coupling of two separate oscillators operating in the dissipative soliton [i.e., anomalous group velocity dispersion (GVD)] regime. Here, we demonstrate all-optical synchronization of Kerr combs in the nonsolitonic, normal GVD regime in which phase-locked combs with high pump-to-comb conversion efficiencies and relatively flat spectral profiles are generated. Our results reveal the universality of Kerr comb synchronization and extend its scope beyond the soliton regime, opening a promising path toward coherently combined normal GVD Kerr combs with spectrally flat profiles and high comb-line powers in an efficient microresonator platform.

INTRODUCTION

Synchronization is a fundamental phenomenon that can occur with coupled nonlinear oscillators and allows for the operating frequencies of different oscillators to become identical (1). It has been observed throughout nature and across various disciplines of science, where many of its applications have become indispensable. For example, the irregular respiratory rhythm of a patient could be restored through entrainment by mechanical ventilation (2), and superconducting Josephson junction arrays can be frequency locked to be used as tunable local oscillators at millimeter wavelengths (3). Within the context of systems exhibiting solitonic behavior, synchronized dissipative solitons in traveling-wave field-effect transistors can be used as an efficient platform for generating phase-controlled (e.g., multiphase) pulse trains (4). In the field of photonics, synchronization has been a key enabler for numerous applications. For example, phase-locked laser arrays allow for coherent beam combining and the enhancement of optical power (5). Synchronized timing between two pulsed lasers (6) enables pump-probe measurements to operate at the shot-noise limit (7), and independent femtosecond lasers can coherently synthesize optical pulses with durations shorter than those achievable from a single laser (8). In addition, passive, all-optical synchronization schemes of soliton mode-locked lasers reduce complexity and cost (9–11) and potentially allow for long-term stable operation without the need for temperature stabilization or vibration isolation (12).

Recent demonstrations have further extended the realm of synchronization to that of soliton Kerr frequency combs, where the locking of repetition rates or equivalently the comb-line spacing is desirable for many applications. In these experiments, the Kerr combs were operating in the anomalous group velocity dispersion (GVD) regime in which dissipative cavity solitons are excited as a result of a dual balance between dispersion and nonlinearity and between loss and parametric gain (13, 14). In this regime, a clear analogy between the Kuramoto model of synchronization (15, 16) and the Lugiato-Lefever equation describing soliton comb dynamics (17) can be identified by approximating the soliton with an analytic expression (13, 14). Although such Kerr combs operating in the single-soliton regime can be accessed through simple excitation via pump detuning mechanisms, they exhibit low pump-to-comb conversion efficiencies (~1%) and follow squared hyperbolic secant spectral profiles (18) that may be less favorable for some real-world applications due to the exponential roll-off.

Alternatively, another class of Kerr combs that operate in the normal GVD regime have been shown to readily offer high pump-to-comb conversion efficiencies (>30%) and to exhibit slower spectral power falloffs over the region of interest (19, 20). All-optical synchronization of such Kerr combs could allow for coherent beam combining (21) and open a pathway to substantially increase comb-line powers in an efficient microresonator platform. Applications of microresonator-based frequency combs such as data communications (22, 23) and spectroscopy (24, 25) could benefit immensely from these high comb-line powers that are otherwise inaccessible without the use of additional amplifiers. For instance, wavelength division multiplexing (WDM) techniques have used Kerr combs as a source to replace a laser array and its limited number of channels (22, 23). Nevertheless, these techniques required the use of additional bulk amplifiers that would not be needed through the synchronization of normal GVD Kerr combs. Moreover, notable advancement in integrating microresonator-based frequency combs has been made by using low-power, electrically pumped sources (26–29). Using such electrically pumped sources and coherent combining of multiple normal GVD Kerr combs, higher comb-line powers could be synthesized directly on chip, leading to a highly efficient method to produce a large number of high-power channels.

Kerr frequency combs in the normal GVD regime form through the interlocking of switching waves that connect the two steady-state solutions of the bistable cavity response (30–32). Unlike solitons in the anomalous GVD regime, normal GVD combs require an initial perturbation to enable the excitation of switching waves and the comb generation process. The pair of switching waves locks to one another and leads to a phase-locked, low-noise comb state, exhibiting intricate temporal and spectral features with high conversion efficiencies and slow spectral power falloffs in the region of interest (19, 20). Normal GVD combs can be generated through pump modulation at the microresonator free spectral range (FSR) (33, 34) or
通过局部扰动来克服这种相互作用，从而实现光波的变相干涉，导致光波的频率变化。由于不同模式的相互作用导致频率变化，因此必须通过调节不同的模式来创建这种相互作用。当模式数较多时，可能会导致一种称为模式的解耦现象。在一些情况下，这种现象可能会导致光波的频率变化，使得光波的频率变化不可预测。在另一些情况下，这种现象可能会导致光波的频率变化，使得光波的频率变化可以预测。总而言之，这种现象可能会导致光波的频率变化，使得光波的频率变化不可预测。
grows as the fractional coupled power increases. We also observe a slight asymmetry in the region that we attribute to the asymmetry of the coupled spectrum (see Fig. 2A) (14).

**Experimental demonstration**

Our experimental setup consists of two oxide-clad silicon nitride (Si$_3$N$_4$) coupled ring devices with integrated platinum-resistive heaters (Fig. 1B). The waveguide cross section of our devices is 730 nm by 1000 nm, which allows for a broad region of normal GVD at the pump wavelength for the fundamental transverse electric polarization mode. Each device is pumped with 220 mW of power at 1559.79 nm. The main and auxiliary rings have an FSR of 200 and 206 GHz, respectively. The slight offset in FSR is specifically chosen to exploit the Vernier effect and induce periodic mode interactions spaced by 50 nm because only a single avoided mode crossing is required near the pump wavelength to initiate the comb generation process. To precisely control the mode-crossing position, the electrical powers through the heaters for the main and auxiliary rings are individually tuned. Increasing the heater power locally increases the temperature of the respective microresonator and thermally redshifts the associated resonances through the thermo-optic effect. The degeneracy point redshifts (blueshifts) when the smaller (larger) FSR mode family resonances are redshifted.

In addition to assisting in the comb generation process, thermally shifting the main ring resonances allows for fine-tuning of the repetition rate or equivalently the frequency spacing of the comb. By exploiting this mechanism, we can tune the relative repetition rate difference between the two combs and explore the region of synchronization for varying coupling strengths. To couple the two devices, we collect a portion of the output of the primary device and filter out the pump using a 4-\(f\) shaper. Because of the resolution of the 4-\(f\) shaper, the three neighboring higher wavelength lines from the pump are also filtered. The filtered output is then combined with the pump for the secondary device.

Experimental observation of synchronization of two normal GVD Kerr combs is shown in Fig. 3. Initially, we generate the two combs separately without the coupling link. We offset the primary comb repetition rate by tuning the main ring heater. By combining the outputs, we observe an RF beatnote and its harmonics corresponding to the repetition rate difference between the two combs. For a main ring heater power of 9.07 mW, we observe a beatnote at 2.36 MHz and the corresponding harmonics (Fig. 3A). We then initiate the unidirectional coupling and observe for a certain threshold coupling strength that the beatnotes vanish, which indicates that the repetition rates of the two combs are now identical and the

**Fig. 2. Synchronization simulation.** (A) Simulated intracavity optical spectrum of the primary system. (B) Corresponding time-domain trace. The peak is indicated by a red dashed line. (C) Time-domain evolution of a secondary structure with respect to the peak of the primary structure (red dashed line). Initial slope corresponds to a 3-MHz relative drift. Because of a fractional power transmitted \(\kappa=1\%\), the secondary structure slows down and locks to the position of the primary structure. (D) Synchronization region across various coupling strengths. A slight asymmetry is observed.

**Fig. 3. Experimental characterization of synchronization.** (A) Measured RF spectra of the combined optical outputs. Beatnotes are observed for the uncoupled case. Beatnotes vanish as the coupling is introduced and increased in strength. (B) Measured optical spectrum of combined optical outputs for the uncoupled case. (C) Measured optical spectrum of combined optical outputs for the coupled case. Frequency modulations due to a relative group delay are observed. Differences in comb-line powers between the two spectra range from −23 to 3 dB.
corresponding comb lines are perfectly overlapped with one another (Fig. 3A). In the optical domain, we observe no modulation in the combined spectrum for the uncoupled case (Fig. 3B) and strong modulation in the combined spectrum for the coupled case (Fig. 3C), which indicates interference of the two sets of comb lines when the repetition rates are synchronized. The differences in comb-line powers between the two spectra range from −23 to 3 dB. The modulation period is given by the relative group delay when combining the two outputs together. By tuning the path length difference, we can double the overall power per line or choose to enhance specific sections of the comb. Extending synchronization to multiple microresonator combs would not only allow us to achieve comb-line powers significantly greater than those possible with a single comb but also provide an approach in shaping the overall spectrum.

We further characterize the synchronization regime by measuring the maximal allowable repetition rate difference for different κ values. Figure 4A shows the measured primary beatnotes as we tune the main ring heater power. As a reference, the change in repetition rate with no coupling is shown in blue. With a coupling strength of κ = 0.3%, shown in red, we observe synchronization from 7.8 to 9.2 mW of heater power. Calibrating to the uncoupled data, this range corresponds to a total synchronization range of −2.9 to 3.6 MHz. We then measure this synchronization range for various coupling strengths and obtain a curve known as an Arnold tongue through a fit of the data (Fig. 4B). The measurements are in excellent agreement with our simulations and show the synchronization region increasing with coupling strength. In addition, the lack of symmetry about Δfrep = 0 is due to the asymmetric spectrum from the initial source and filtering system (14).

**DISCUSSION**

Our results reveal the universality of synchronization beyond the dissipative soliton regime by exploring the dynamics of two coupled normal GVD Kerr combs. Although normal GVD Kerr combs do not have solitonic profiles, they exhibit synchronization behavior. Moreover, despite the initial perturbation requirements of normal GVD Kerr combs, key beneficial features such as high pump-to-comb conversion efficiencies make synchronization of such integrated combs desirable. For example, it could allow for coherent beam combining and a marked increase in comb-line powers while maintaining high conversion efficiencies. As a passive locking mechanism with low-power requirements, all-optical synchronization can easily be extended to a fully integrated platform where multiple oscillators can be combined. Such integrated platforms could enhance current WDM schemes where power requirements are not sufficient from single comb generation (22, 23). Access to a large number of high-power channels could greatly increase the bandwidth of current data communication protocols. We envision that our system would condense complex oscillator networks into a single integrated device.

**MATERIALS AND METHODS**

**Modeling**

A single coupled ring model is implemented by mapping the electric field at the beginning of the (m + 1)th roundtrip to the electric field at the end of the mth roundtrip and by using a modified NLSE for propagation. The mapping is given by

$$
\begin{bmatrix}
    E_{\text{out}}^{(m+1)} \\
    E_{\text{in}}^{(m+1)}
\end{bmatrix}
= \mathbf{M}
\begin{bmatrix}
    E_{\text{out}}^{(m)} \\
    E_{\text{in}}^{(m)}
\end{bmatrix}
$$

Here, E_{\text{in}} is the CW driving field with power P_m = |E_{\text{in}}|^2, E_{\text{out}} is the intracavity field in the main ring, E_{\text{in}} is the intracavity field in the auxiliary ring, E_{\text{out}} is the output field, and \( \mathbf{M} = \mathbf{\Theta} \circ \mathbf{A} \circ \mathbf{\Phi} \) is the mapping matrix with \( \mathbf{\Theta} \) as the unitary coupling matrix, \( \mathbf{A} \) as the linear attenuation matrix, \( \mathbf{\Phi} \) as the linear phase accumulation matrix, and \( \circ \) as the Hadamard product. The unitary coupling matrix \( \mathbf{\Theta} \) is given by

$$
\mathbf{\Theta} = \begin{bmatrix}
    \sqrt{1 - \theta_1} & i\sqrt{\theta_1(1 - \theta_2)} & -\sqrt{\theta_1}\theta_2 \\
    i\sqrt{\theta_1(1 - \theta_2)} & \sqrt{(1 - \theta_1)(1 - \theta_2)} & i\sqrt{\theta_2(1 - \theta_1)} \\
    0 & i\sqrt{\theta_2} & \sqrt{1 - \theta_2}
\end{bmatrix}
$$

where \( \theta_1 \) is power coupling coefficient between the bus and the main ring and \( \theta_2 \) is the power coupling coefficient between the main ring and the auxiliary ring. The linear attenuation matrix \( \mathbf{A} \) is given by

$$
\mathbf{A} = \begin{bmatrix}
    1 & e^{-\alpha_1}e^{-\left(\frac{L_1}{4} + \frac{L_2}{4}\right)} \\
    e^{-\alpha_1}e^{-\left(\frac{L_1}{4} + \frac{L_2}{4}\right)} & 1 \\
    e^{-\alpha_2} & 1
\end{bmatrix}
$$

where \( \alpha_1 \) (\( \alpha_2 \)) and \( L_1 \) (\( L_2 \)) are the linear attenuation coefficient and cavity length of the main (auxiliary) ring. The linear phase accumulation matrix \( \mathbf{\Phi} \) is given by

$$
\mathbf{\Phi} = \begin{bmatrix}
    1 & e^{i\phi_1}e^{i\left(\frac{L_1}{2} + \phi_2\right)} \\
    e^{i\phi_1}e^{i\left(\frac{L_1}{2} + \phi_2\right)} & 1 \\
    e^{i\phi_2} & e^{i\phi_2}
\end{bmatrix}
$$

where \( \phi_1 \) (\( \phi_2 \)) is the roundtrip phase shift of the main (auxiliary) ring.

The nonlinear propagation equations are given by

$$
\frac{\partial E_1}{\partial z} = \left[ i \sum_{k \geq 2} \frac{\beta_k}{k!} \left( i \frac{\partial}{\partial \tau} \right)^k + i\gamma |E_1|^2 \right] E_1
$$

$$
\frac{\partial E_2}{\partial z} = \left[ i \sum_{k \geq 2} \frac{\beta_k}{k!} \left( i \frac{\partial}{\partial \tau} \right)^k - \delta \frac{\partial}{\partial \tau} + i\gamma |E_2|^2 \right] E_2
$$

**Fig. 4. Measurement of synchronization region.** (A) Measured beatnotes as main ring heater power is tuned for uncoupled and coupled cases. For κ = 0.3%, synchronization is observed from 7.8 to 9.2 mW of heater power, which corresponds to a synchronization range of −2.9 to 3.6 MHz. (B) Synchronization region across various coupling strengths. Similar to simulation, asymmetry is observed.
where $\beta_k$ is the $k$th order dispersion coefficient associated with the Taylor series expansion of the propagation constant $\beta$ about the pump frequency $\omega_p$ and $\gamma$ is the effective nonlinear coefficient. The FSR difference between the main and auxiliary rings is compensated by a linear phase slope across the auxiliary cavity modes $\delta = \frac{1}{L_2} (\frac{1}{\text{FSR}_2} - \frac{1}{\text{FSR}_1})$, where $\text{FSR}_1$ ($\text{FSR}_2$) is the FSR of the main (auxiliary) ring. Using roundoptimal values and the COMSOL Multiphysics mode solver package, the following parameters are used: $a_1 = 8.643 \, \text{m}^{-1}$, $\alpha_2 = 4.321 \, \text{m}^{-2}$, $\theta_1 = 0.01074$, $\theta_2 = 0.068616$, $L_1 = 699 \, \mu\text{m}$, $L_2 = 678 \, \mu\text{m}$, $P_{\text{in}} = 220 \, \text{mW}$, $\text{FSR}_1 = 200 \, \text{GHz}$, $\text{FSR}_2 = 206 \, \text{GHz}$, $\beta_2 = 40 \, \text{ps}^2/\text{km}$, $\gamma = 1.229 \, \text{W}^{-1} \, \text{m}^{-1}$, $\theta_1 = -0.05340$, and $\theta_2 = -0.1891$.

Synchronization is investigated by implementing a coupling link between two of the coupled ring modes. The coupling link filters out the pump line and the third and neighboring higher wavelengths lines of the output of the primary model before feeding a fraction of it into the input of the secondary model. The repetition rate difference $\Delta f_{\text{rep}}$ is simulated by offsetting the group velocity in the secondary coupled ring model. The group velocity offset parameter $\Delta \beta_1$ is expressed as

$$\Delta \beta_1 = \frac{t_{R_2} - t_{R_1}}{L_1} \approx \frac{1}{L_1 \text{FSR}_1} \Delta f_{\text{rep}}$$  \hspace{1cm} (7)

**Experimental setup**

Two independent $\text{Si}_3\text{N}_4$ chips each with a coupled ring device is pumped with 220 mW of power in the bus waveguide. The pump sources for both coupled ring devices are derived from a single CW laser at 1559.79 nm with a narrow linewidth of <1 kHz. The laser output is amplified by an erbium-doped fiber amplifier and split into two light fields with a fused fiber coupler. One field is coupled to the bus waveguide of the primary device using a lensed fiber. Using a nonpolarizing beam splitter, the other field is combined with a filtered and attenuated portion of the primary output. To ensure that the polarization state of the filtered primary output is collinear with that of the secondary pump, a quarter-wave plate and a half-wave plate is used. The combined light is coupled to the secondary device using a microscope objective. The unfiltered portion of the primary output and an attenuated secondary output are combined with a nonpolarizing beam splitter before being collected into an optical fiber using a fiber collimator package. The fiber output is then split with a 90/10 fused fiber coupler. Light from the higher-power arm is detected by a 10-GHz photodiode, and the RF signal is observed on an RF spectrum analyzer. Light from the lower-power arm is used to observe the combined spectrum on an optical spectrum analyzer.

**REFERENCES AND NOTES**

1. S. H. Strogatz, Exploring complex networks. *Nature* 410, 268–276 (2001).
2. C. Graves, L. Glass, D. L. Lecomte, R. Meloche, A. Grassino, Respiratory phase locking during mechanical ventilation in anesthetized human subjects. *Am. J. Physiol. Regul. Integr. Comp. Physiol.* 250, R902–R909 (1986).
3. K. Wiesenfeld, P. Colet, S. H. Strogatz, Frequency locking in Josephson arrays: Connection with the Kuramoto model. *Phys. Rev. E* 57, 1563–1569 (1998).
4. K. Nara, Synchronization of dissipative solitons in a system of closed traveling-wave field-effect transistors. *Nonlin. Dyn.* 94, 711–721 (2018).
5. M. Nixon, M. Fridman, E. Ronen, A. A. Friesem, N. Davidson, I. Kanter, Controlling synchronization in large laser networks. *Phys. Rev. Lett.* 108, 214101 (2012).
6. L.-S. Ma, R. K. Shilton, H. C. Kaptyn, M. M. Murnane, J. Ye, Sub-10-femtosecond active synchronization of two passively mode-locked Ti:sapphire oscillators. *Phys. Rev. A* 64, 021802 (2001).

Kim et al., *Sci. Adv.* 2021; 7 : eabi4362     20 October 2021
33. V. E. Lobanov, G. Lihachev, M. L. Gorodetsky, Generation of platicons and frequency combs in optical microresonators with normal GVD by modulated pump. Europhys. Lett. 112, 54008 (2015).

34. Y. Xu, A. Sharples, J. Fatome, S. Coen, M. Erkintalo, S. G. Murdoch, Frequency comb generation in a pulse-pumped normal dispersion Kerr mini-resonator. Opt. Lett. 46, 512–515 (2021).

35. W. Liang, A. A. Savchenkov, V. S. Ilchenko, D. Eliyahu, D. Seidel, A. B. Matsko, L. Maleki, Generation of a coherent near-infrared Kerr frequency comb in a monolithic microresonator with normal GVD. Opt. Lett. 39, 2920–2923 (2014).

36. J. K. Jang, Y. Okawachi, M. Yu, K. Luke, X. Ji, M. Lipson, A. L. Gaeta, Dynamics of mode-coupling-induced microresonator frequency combs in normal dispersion. Opt. Express 24, 28794–28803 (2016).

37. Y. Liu, Y. Xuan, X. Xue, P.-H. Wang, S. Chen, A. J. Metcalf, J. Wang, D. E. Leaird, M. Qi, A. M. Weiner, Investigation of mode coupling in normal-dispersion silicon nitride microresonators for Kerr frequency comb generation. Optica 1, 137–144 (2014).

38. T. Carmon, H. G. L. Schwefel, L. Yang, M. Oxborrow, A. D. Stone, K. J. Vahala, Static envelope patterns in composite resonances generated by level crossing in optical toroidal microcavities. Phys. Rev. Lett. 100, 103905 (2008).

39. S. Ramelow, A. Farsi, S. Clemmen, J. S. Levy, A. R. Johnson, Y. Okawachi, M. R. E. Lamont, M. Lipson, A. L. Gaeta, Strong polarization mode coupling in microresonators. Opt. Lett. 39, 5134–5137 (2014).

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