Towards Detection of Bottlenecks in Modular Systems

Mark Sh. Levin *

The paper describes some basic approaches to detection of bottlenecks in composite (modular) systems. The following basic system bottlenecks detection problems are examined: (1) traditional quality management approaches (Pareto chart based method, multicriteria analysis as selection of Pareto-efficient points, and/or multicriteria ranking), (2) selection of critical system elements (critical components/modules, critical component interconnection), (3) selection of interconnected system components as composite system faults (via clique-based fusion), (4) critical elements (e.g., nodes) in networks, and (5) predictive detection of system bottlenecks (detection of system components based on forecasting of their parameters). Here, heuristic solving schemes are used. Numerical examples illustrate the approaches.

**Keywords:** modular systems, system bottlenecks, engineering frameworks, combinatorial optimization, multicriteria decision making, networked systems, heuristics

1. Introduction

In recent decades, the significance of modular (multi-component) systems has been increased (e.g., [3,7,10,11,13,15,16,18,23,29,30]). This paper describes approaches to detection of bottlenecks in composite (modular) systems. Here, the following is assumed (Fig. 1):

1. The considered hierarchical modular system can be represented as a morphological structure (e.g., [15,16,18,22]) or as a network.
2. The following system elements are under examination as the bottlenecks: (i) system component (or a system component fault), (ii) interconnection between system components (compatibility), (iii) group of system components (or a group of system faults), (iv) group of interconnected system components (or a composite system faults). Thus, the system bottlenecks are considered as low quality system part(s)/component(s) or system fault(s) and their compositions.

In the paper, the following approaches are described (Table 1):

**I. Basic quality management approaches:** (1.1.) Pareto chart based method [12], (1.2.) multicriteria analysis as selection of Pareto-efficient points, and/or multicriteria ranking [14,16].

**II. Detection of low quality system parts:** (2.1.) detection of critical components/modules, (2.2.) detection of critical component interconnection (component compatibility), and (2.3.) analysis of the system structure and detecting the situation when the system structure has to be improved.

**III. Selection of interconnected system components as composite system faults** (e.g., via hierarchical morphological design [15,16,18,19] or via clique-based fusion [20,26]).

**IV. Detection of critical components in networks, detection of low quality node interconnection, detection of low quality network topology, e.g., definition of the internal network nodes via maximum leaf spanning tree problem (e.g., [15,19]), connected dominating sets problem (e.g., [15,19]), hierarchical network design problem (e.g., [20,27]).

**V. Predictive detection of system bottlenecks:** (5.1.) predictive detection of system components based on forecasting of their parameters, (5.2.) predictive detection of critical components in networks, low quality node interconnection, low quality network topology, (5.3.) predictive detection of group of interconnected system components based on clique based fusion of graph streams [20].

Mainly, composite engineering frameworks (i.e., heuristics solving schemes) are used. Numerical examples illustrate the approaches.

*Mark Sh. Levin: [http://www.mslevin.iitp.ru](http://www.mslevin.iitp.ru) email: mslevin@acm.org
Table 1. Basic approaches to detection of system bottlenecks

| Objects under examination | Basic detection methods/models | Predictive detection (forecasting/dynamics) |
|---------------------------|--------------------------------|----------------------------------------------|
| 1. System component (or system component fault) | (a) Pareto chart method [12] (b) Multicriteria analysis/ranking (sorting) [14,16] | (a) Pareto chart method based on forecast of system component parameters (b) Multicriteria analysis/ranking (sorting) based on forecast of system component parameters |
| 2. Interconnection of system components | (a) Pareto chart method [12] (b) Multicriteria analysis/ranking (sorting) | (a) Pareto chart method based on forecast of system component parameters (b) Multicriteria analysis/ranking (sorting) based on forecast of system component parameters |
| 3. Group of system components (or composite fault) | Multicriteria analysis (sorting) [14,21,28,31] | Multicriteria analysis (sorting) based on forecast of network parameters [14,21,28,31] |
| 4. Bottlenecks (e.g., critical nodes) in networks | (a) Maximum leaf spanning tree [15,19] (b) Connected dominating set [15,19] (c) Hierarchical network design [2,6,27] (d) Low-quality node interconnection | (a) Maximum leaf spanning tree based on forecast of network (b) Connected dominating set based on forecast of network (c) Hierarchical network design based on forecast of network (d) Low-quality node interconnection based on forecast of network |
| 5. Group of interconnected system components | (a) HMMD [15,16,18,19] (b) Clique-based fusion [20,26] | (a) HMMD based on system forecast [15,16,18,19] (b) Clique-based fusion over graph streams [20] |

2. Traditional Quality Control Methods

Here, two methods for the analysis of system components are considered: (i) Pareto chart based method, (ii) multicriteria analysis (ranking). The first method is the main method to the detection of system bottlenecks in Japanese approach of quality control and consists in the analysis of system components/Parts by their reliability (or the frequency of component fault/failure/trouble/anomaly) (e.g., [12]):

*Step 1.* Definition of the initial set of system components/Part for the analysis.

*Step 2.* Assessment of reliability (i.e., frequency of the component fault.)
**Step 3.** Selection of the non-reliable system components (as system bottlenecks) by a threshold (at the Pareto chart).

Further, the method based on multicriteria description and multicriteria analysis of the system components has been suggested in [14,16]:

**Step 1.** Definition of the initial set of system components/part for the analysis.

**Step 2.** Assessment of the system components by many criteria.

**Step 3.** Multicriteria ranking (e.g., selection of Pareto-efficient elements, outranking techniques, utility function analysis) of the system components to select the most important ones from the viewpoint of the total system safety (as system bottlenecks).

Now, let us consider an illustrative example: supercharger for gas-pump aggregate [14]. The tree-like structure of the considered aggregate is the following (Fig. 2):

1. Body frame components: 1.1. external body, 1.2. body cover, 1.3. internal body with embedded elements, and 1.4. body seal.
2. Supporting block bearers.
3. Oil seals.
4. Rotor.
5. Connection units: 5.1. half-clutch, 5.2. gear hoop, and 5.3. torsion shaft.
6. System of lubrication: 6.1. oil boiler, 6.2. oil filters, 6.3. main oil pump, 6.4. start oil pump, 6.5. armature, 6.6. valve elements, 6.7. temperature regulator, 6.8. oil coolers, and 6.9. for oil coolers.
7. System of oil seals: 7.1. oil boiler, 7.2. oil filter, 7.3. main pump, 7.4. start pump, 7.5 pressure regulator, 7.6. hydro-accumulator, 7.7. stripping vessel, 7.8. oil deriving, 7.9. pipelines, 7.10. valve elements, and 7.11. gum elastic seal rings.
8. Thrust blocks: 8.1. pad, 8.2. wrapper rings, 8.3. stop rings, and 8.4. distance rings.

![Fig. 2. Structure of the examined system](image)

The following six criteria (local, systemic) are examined:
1. \( C_1 \), frequency of faults (percent);
2. \( C_2 \), time of “out of work” in the case of the component fault;
3. \( C_3 \), cost of work to repair the apparatus;
4. \( C_4 \), level of influence of component fault to other system components, scale \([0,1,2]\) (no influence: 0, influence exists: 1, strong influence: 2);
5. \( C_5 \), wideness of usage, scale \([0,1,2]\) (the component is used in the only this apparatus: 0, the component is used in other apparatus: 1, the component is used in many various systems: 2);
6. \( C_6 \), level of influence of component fault to total system safety, scale \([0,1]\) (no influence: 0, the influence exists: 1).

Table 2 contains multicriteria description (i.e., estimates upon the considered six criteria) of the considered pump system (statistical data, processing, and expert judgment) [14].
The selection of system bottlenecks by Pareto chart is illustrated in Fig. 3 (estimates upon criterion \( C_1 \)): (a) threshold 1 (6.8), system bottlenecks components are the following: 4, 7.11; (b) threshold 2 (1.5), system bottlenecks components are the following: 2, 4, 6.3, 6.8, 7.5, 7.7, 7.11.

Table 2. Estimates of system components

| System part/component | Estimates upon criteria |
|-----------------------|------------------------|
|                       | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) |
| 1.1.                  | 0.25    | 12.8   | 18.0   | 0       | 0       | 1       |
| 1.2.                  | 0.00    | 12.8   | 18.0   | 0       | 0       | 1       |
| 1.3.                  | 0.87    | 12.8   | 18.0   | 1       | 0       | 0       |
| 1.4.                  | 0.25    | 12.8   | 18.0   | 0       | 0       | 1       |
| 2.                    | 1.53    | 6.4    | 5.5    | 2       | 0       | 0       |
| 3.                    | 0.30    | 9.6    | 6.4    | 1       | 0       | 0       |
| 4.                    | 6.80    | 12.8   | 18.0   | 1       | 0       | 0       |
| 5.1                   | 0.00    | 4.8    | 5.9    | 1       | 0       | 0       |
| 5.2                   | 0.16    | 4.8    | 5.9    | 1       | 0       | 0       |
| 5.3                   | 0.94    | 3.2    | 3.7    | 1       | 0       | 0       |
| 6.1                   | 0.10    | 1.6    | 1.2    | 0       | 0       | 0       |
| 6.2                   | 0.50    | 1.6    | 1.2    | 1       | 0       | 0       |
| 6.3                   | 5.00    | 3.2    | 4.0    | 1       | 0       | 0       |
| 6.4                   | 0.81    | 3.2    | 3.1    | 1       | 0       | 0       |
| 6.5                   | 0.35    | 0.8    | 1.4    | 1       | 1       | 0       |
| 6.6                   | 0.35    | 0.8    | 1.4    | 1       | 1       | 0       |
| 6.7                   | 0.20    | 0.8    | 1.4    | 1       | 1       | 0       |
| 6.8                   | 1.50    | 28.8   | 48.7   | 1       | 1       | 0       |
| 6.9                   | 0.70    | 1.6    | 2.5    | 1       | 1       | 0       |
| 7.1                   | 0.00    | 1.6    | 1.2    | 0       | 0       | 0       |
| 7.2                   | 0.35    | 1.6    | 1.2    | 1       | 0       | 0       |
| 7.3                   | 0.00    | 0.8    | 1.4    | 1       | 0       | 0       |
| 7.4                   | 0.20    | 3.2    | 3.1    | 1       | 0       | 0       |
| 7.5                   | 1.50    | 2.4    | 2.0    | 1       | 2       | 0       |
| 7.6                   | 0.00    | 0.8    | 1.9    | 1       | 0       | 0       |
| 7.7                   | 1.50    | 1.6    | 2.9    | 0       | 0       | 0       |
| 7.8                   | 1.40    | 2.4    | 2.0    | 1       | 0       | 0       |
| 7.9                   | 0.70    | 0.8    | 1.4    | 0       | 0       | 0       |
| 7.10                  | 0.20    | 0.8    | 1.4    | 1       | 0       | 0       |
| 7.11                  | 70.00   | 0.8    | 1.4    | 2       | 0       | 0       |
| 8.                    | 0.70    | 3.2    | 1.4    | 2       | 0       | 0       |
| 8.1                   | 0.20    | 3.2    | 1.4    | 2       | 0       | 0       |
| 8.2                   | 0.00    | 3.2    | 1.4    | 2       | 0       | 0       |
| 8.3                   | 0.00    | 3.2    | 1.4    | 2       | 0       | 0       |
| 8.4                   | 0.00    | 3.2    | 1.4    | 2       | 0       | 0       |

Multicriteria ranking (sorting) problem is targeted to select the most important system component(s) upon criteria as the bottleneck(s) (e.g., \([14,21,25,28,31]\)). Here, ELECTRE-like technique is used (e.g., \([21,25,28]\)) based on the following criteria weights: 1.0 (\( C_1 \)), 0.3 (\( C_2 \)), 0.4 (\( C_3 \)), 0.5 (\( C_4 \)), 0.2 (\( C_5 \)), and 3.0 (\( C_6 \)). Fig. 4 depicts the results of multicriteria ranking:

**Layer 1** (system bottlenecks): 2, 4, 6.3, 6.8, 7.11.

**Layer 2**: 1.3, 5.3, 6.4, 7.5, 7.8.

**Layer 3**: 1.1, 1.4, 3, 5.1, 5.2, 6.5, 6.6, 6.9, 7.4, 7.7, 8.1, 8.

**Layer 4**: other components.
3. Detection of Bottlenecks in Hierarchical Morphological Design

Hierarchical multicriteria morphological design (HMMD) approach for composite (multi-component, modular) systems is described in [15,16,18,19]. In HMMD approach, the resultant solution is composed from design alternatives (DAs) for system parts/components while taking into account quality if their interconnection (IC). In the basic version of HMMD, the following ordinal scales are used: (1) ordinal
scale for quality of system components (or priority) \( \eta = 1, \ldots, \nu \); 1 corresponds to the best one); (2) scale for system quality while taking into account system components ordinal estimates and ordinal compatibility estimates between the system components \( w = 0, \nu \); \( \nu \) corresponds to the best level).

For the system consisting of \( m \) parts/components, a discrete space (poset, lattice) of the system quality (excellence) on the basis of the following vector is used: \( N(S) = (w(S); n(S)) \), where \( w(S) \) is the minimum of pairwise compatibility between DAs which correspond to different system components, \( n(S) = (\eta_1, \ldots, \eta_r, \ldots, \eta_k) \), where \( \eta_r \) is the number of DAs of the \( r \)th quality in \( S \) \( \sum_{r=1}^{k} \eta_r = m \). The optimization problem is:

\[
\max N(S), \quad \max w(S), \quad w(S) \geq 0.
\]

Let us consider a numerical example (Fig. 5) for the detection of system bottlenecks in this design approach. Here, the composite four-component system is: \( S = X \star Y \star Z \star H \). For each system component, design alternatives (DAs) are depicted in in Fig. 5 (ordinal estimates of DAs quality as priorities are presented in parentheses, scale [1, 3] \( 1 \) corresponds to the best level of quality). Table 3 contains ordinal estimates of compatibility (IC) between DAs (scale [0, 3]). Poset-like scales are presented in Fig. 6.

| Table 3. Compatibility |
|-------------------------|
| Y1 | Y2 | Z1 | Z2 | H1 | H2 |
|----|----|----|----|----|----|
| X1 | 3  | 2  | 2  | 2  | 1  | 3  |
| X2 | 0  | 3  | 2  | 3  | 1  | 2  |
| Y1 | 2  | 2  | 1  | 2  |    |    |
| Y2 | 2  | 3  | 2  | 3  |    |    |
| Z1 | 1  | 2  |    |    |    |    |
| Z2 | 3  | 3  |    |    |    |    |

Fig. 5. Four-component system

\( < 4, 0, 0 > \) The ideal point
\( < 3, 1, 0 > \)
\( < 3, 0, 1 > \)
\( < 2, 2, 0 > \)
\( n(S_1) \)
\( < 2, 1, 1 > \)
\( < 1, 3, 0 > \)
\( < 2, 0, 2 > \)
\( < 1, 2, 1 > \)
\( < 0, 4, 0 > \)
\( < 1, 1, 2 > \)
\( < 0, 3, 1 > \)
\( < 1, 0, 3 > \)
\( < 0, 2, 2 > \)
\( < 0, 1, 3 > \)
\( n(S_2) \)
\( < 0, 0, 4 > \) The worst point

(a) Poset-like scale by elements \( n(S) \)

![Fig. 6. Poset-like scale for quality of system \( S \)](image-url)
Fig. 6a depicts the poset of system quality by components and Fig. 6b depicts an integrated poset with compatibility (each triangle corresponds to the poset from Fig. 6a). Two resultant composite system Pareto-efficient solutions are under examination: (i) \( S_1 = X_1 \times Y_2 \times Z_2 \times H_1 \), \( N(S_1) = (1;2,1,1) \); (ii) \( S_2 = X_2 \times Y_2 \times Z_2 \times H_2 \), \( N(S_2) = (2;0,1,3) \).

The system component (DA) or compatibility between a pair of DAs can be considered as the system bottleneck(s). The following solving schemes (frameworks) can be considered:

**Scheme 1.** Multicriteria ranking of system components (DAs).

**Scheme 2.** Multicriteria ranking of component interconnections.

**Scheme 3.** Joint multicriteria ranking of DAs and their interconnections.

**Scheme 4.** Detection of interconnected system component (as a composite fault): clique-based fusion.

Fig. 7 depicts the system solution \( S_1 = X_1 \times Y_2 \times Z_2 \times H_1 \) (including estimates of DAs and their compatibility). Table 4 contains six bottlenecks (components, their compatibility). Evidently, each bottleneck above has to be assessed by some criteria in the case of its improvement (e.g., possible profit for system quality, required cost). Further, it is reasonable to use multicriteria ranking of the bottlenecks while taking into account the above-mentioned criteria and to select the most ‘prospective’ bottleneck(s).

![Fig. 7. Concentric presentation of solution \( S_1 \)](image)

| Composite DAs | Bottlenecks | Actions w/ι |
|---------------|-------------|--------------|
| \( S_1 = X_1 \times Y_2 \times Z_2 \times H_1 \) | \( Y_1 \) | 2 \( \Rightarrow \) 1 |
| \( S_2 = X_2 \times Y_2 \times Z_2 \times H_1 \) | \( Z_2 \) | 2 \( \Rightarrow \) 1 |
| \( X_1 \times Y_2 \times Z_2 \times H_1 \) | \( (X_1, Y_2) \) | 2 \( \Rightarrow \) 3 |
| \( X_1 \times Y_2 \times Z_2 \times H_1 \) | \( (X_1, Z_2) \) | 2 \( \Rightarrow \) 3 |
| \( X_1 \times Y_2 \times Z_2 \times H_1 \) | \( (X_2, H_1) \) | 1 \( \Rightarrow \) 3 |
| \( X_1 \times Y_2 \times Z_2 \times H_1 \) | \( (Y_2, H_1) \) | 2 \( \Rightarrow \) 3 |

The detection of the system bottleneck as a group of interconnected system components can be considered as revelation of a set of low quality components which are connected at the high level compatibility. This situation corresponds to a new type of a composite system fault which was suggested in [20,26]. Here, some weak system faults are interconnected (at the high level) and this combination can lead to a significant composite system fault. In our case, the composite bottleneck (as the composite fault) corresponds to the combination of low quality components with high-level component compatibility. Thus, the following two-criteria optimization problem can be examined:

\[
\min n(B), \quad \max w(B); \quad B \text{ is a subsolution of a system solution for } S.
\]

Fig. 8 illustrates a composite solution \( S_2 = X_2 \times Y_2 \times Z_2 \times H_1 \), \( N(S_2) = (2;0,1,3) \) (from example in Fig. 5). For this four-component solution, it is possible to examine four three-component subsystems: \( B_1 = X_2 \times Y_2 \times Z_2 \), \( N(B_1) = (2;0,1,2) \); \( B_2 = X_2 \times Z_2 \times H_2 \), \( N(B_2) = (2;0,0,3) \); \( B_3 = X_2 \times Y_2 \times H_2 \), \( N(B_3) = (2;0,2,1) \); \( B_4 = Y_2 \times Z_2 \times H_2 \), \( N(B_4) = (3;0,2,1) \). Two Pareto-efficient subsystems as composite bottlenecks are (Fig. 9): \( B_2 \) and \( B_4 \).
4. Critical Elements in Multilayer Structures/Networks

Generally, it is reasonable to examine multi-layer structures/networks (Fig. 10) (e.g., [22]).

Here, the following kinds of problems for detection of system bottlenecks can be examined:

Kind I: for structure/network layer:
(i) detection of critical nodes in networks (e.g., maximum leaf spanning tree problem, connecting connected dominating sets problem),
(ii) detection of group of critical network nodes,
(iii) detection of group of critical interconnected network nodes, and
(iv) detection of low quality layer topology.
Type II: for neighbor layers: detection of critical connection between nodes of neighbor layers.

Type III: for multi-layers: detection of wrong or low quality assignment of nodes into structure/network layers.

Let us consider some of the problems above for the structure layer level. First, detection of critical node(s) in the structure/network layer may be based on the methods which were described in previous sections (e.g., Pareto chart based method, multicriteria analysis/ranking, detection of interconnected nodes as clique fusion). Second, three well-known combinatorial optimization problems can be considered. Fig. 11 illustrates this type of combinatorial problems: (a) maximum leaf/terminal nodes problem, (b) minimum internal nodes problem, and (c) hierarchical two-level network design problem. Here, the set of internal structure/network nodes can be considered as some crucial nodes (e.g., for improvement, for testing) or 'bottlenecks'.

![Fig. 11. Maximum leaf/ minimum internal nodes](image)

The “maximum leaf spanning tree” problem is the following (e.g., [1,5,9]):

Find a spanning tree of an input graph so that the number of the tree leafs is maximal.

Generally, the spanning tree of a graph contains the following types of nodes: (a) root, (b) internal nodes (the internal nodes may be considered as a virtual “bus” in networking), and (c) leaf nodes. Thus, the problem consists in maximizing the number of leaf nodes or minimizing the number of internal nodes. The problem is one of the basic NP-hard problems [9].

In sense of exact algorithms, this problem is equivalent to “connected dominating set” problem (NP-hard) (e.g., [4,5,9]):

Find a minimum set of vertices $D \subseteq A$ of input graph $G = (A, E)$ that the induced by $D$ subgraph $G' = (D, E')$ ($E' \subseteq E$) is connected dominated set and $D$ is a dominating set of $G$.

A recent survey on the connected dominating set problem is presented in [4].

The basic hierarchical two-level network design problem is (e.g., [2,6,27]):

Find a minimum cost two-level spanning network, consisting of two parts: (i) main (internal) path (or several paths, tree, ring) (ii) secondary trees.

Thus, the initial network is divided into two parts:

(a) main part (i.e., the higher level part): a path (or several paths, tree, ring) composed of primary arcs, which visits some of the nodes of the network (i.e., primary nodes);

(b) secondary part (i.e., secondary nodes, secondary trees): the part is composed of one or more trees whose arcs, termed secondary, are less expensive to build than the primary arcs.

Here, each arc has a cost ($d_{ij}$, $\forall i, j \in A$, $A$ is the set of nodes). The total cost of the selected arcs in the spanning structure is used as the minimized objective function. The problem is formulated as combinatorial optimization model (e.g., [6]), it is NP-hard [2].

Evidently, similar problems can be considered for detection of critical arcs in networks.

In the above-mentioned problems kind II and kind III, the solution consists in assignment of elements into positions (i.e., assignment/allocation problems). Here, new advanced combinatorial problem statements are required for the detection of low quality assignment(s) in the existing solution(s). Note, usage of HMMD approach to an extended assignment problem has been suggested in [15,17]. Thus, detection of bottlenecks in hierarchical morphological design, described in previous section, can be used for the assignment/allocation problems as well.
Detection of low quality network topology requires special additional study. The augmentation problem (e.g., [8]) can be considered as a version of this approach.

5. Predictive Detection of System Bottlenecks

A predictive detection of system bottleneck(s) can be considered as the following (Fig. 12):

Step 1. Study of existing changes and/or future changes of systems parameters and/or system structure (i.e., parameters for system components, parameters for system structure).

Step 2. Analysis of system evolution (i.e., the corresponding trajectories for system, system parameters).

Step 3. Forecasting of the system parameters to build the system forecast.

Step 4. Detection of the system bottleneck(s) on the basis of the future system parameters (i.e., system forecast, system parameters forecasts).

Evidently, the same system objects can be under examination: system component(s), group of interconnected system components, system structure.

In the case of network-like system, the pointed out predictive detection problems can be complicated.

5.1. Predictive Detection of System Components

The predictive detection of system bottlenecks as system component(s) can be based on the same methods (i.e., Pareto chart method, multicriteria ranking). In this case, system parameters forecasts are used as the initial information. In the considered example for aggregate (Fig. 2, Table 2, Fig. 3, Fig. 4), forecasts of the data from Table 2 have to be used.

5.2. Predictive Detection of Interconnected System Components

The predictive detection of bottlenecks in hierarchical morphological design can be considered analogically (i.e., analysis of the system evolution, computing a system forecast, detection of system bottleneck(s) via the methods above for the system forecast).

Let us consider a simplified example for detection of a composite bottleneck (as a subsystem) for four-component system $S = X \ast Y \ast Z \ast H$ from Fig. 5. Fig. 13 depicts an illustrative numerical example for evolution and forecasting of solution $S_2 = X_2 \ast Y_2 \ast Z_2 \ast H_2$ ($N(S_2) = 2; 0, 1, 3$). Here, the following time axe is considered: basic time point $t = \tau_0$, next time point (evolution) $t = \tau_1$, forecast time point $t = \tau_f$ (i.e., $t = \tau_f$).

Fig. 12. Predictive detection of system bottleneck(s)

Fig. 13. Evolution of solution $S_2$ and forecast
Note, for the basic time point ($\tau_0$), two subsystems (as composite bottlenecks) have obtained (Fig. 9): $B_2 = X_2 \ast Z_2 \ast H_2$, $N(B_2) = (2; 0, 0, 3)$; $B_4 = Y_2 \ast Z_2 \ast H_2$, $N(B_4) = (3; 0, 2, 1)$. For next time points, the following poset-like estimates are obtained: (i) $t = \tau_1$: $N(B_1^{\tau_1}) = (3; 0, 1, 2)$, $N(B_2^{\tau_1}) = (2; 0, 1, 2)$, $N(B_3^{\tau_1}) = (2; 0, 2, 1)$, $N(B_4^{\tau_1}) = (2; 0, 2, 1)$; (ii) $t = \tau_f(\tau_2)$: $N(B_1^{\tau_2}) = (3; 0, 0, 3)$, $N(B_2^{\tau_2}) = (3; 1, 1, 1)$, $N(B_3^{\tau_2}) = (2; 1, 0, 2)$, $N(B_4^{\tau_2}) = (2; 1, 1, 1)$. As a result, the following subsystems are obtained as composite bottlenecks:

(a) $t = \tau_1$ (Fig. 14a): $B_1^{\tau_1} = X_2 \ast Y_2 \ast Z_2$, $N(B_1^{\tau_1}) = (3; 0, 1, 2)$;
(b) $t = \tau_f(\tau_2)$ (Fig. 14b): $B_1^{\tau_2} = X_2 \ast Y_2 \ast Z_2$, $N(B_1^{\tau_2}) = (3; 0, 0, 3)$.

Thus, the forecast bottleneck is: $B_1^{\tau_2} = X_2 \ast Y_2 \ast Z_2$. Fig. 15 depicts a trajectory of the bottleneck.

![Fig. 14. Poset-like scale for subsystem B](image)

![Fig. 15. Trajectory of composite bottlenecks](image)

Generally, it is reasonable to examine for the detection of the composite system bottlenecks an approach 'clique-based fusion based on graph streams' that was presented in [20].

6. Conclusion

The paper describes basic approaches to detection of bottlenecks in composite (modular) systems. In general, detection of system bottlenecks has to be used as a first stage of the system improvement/development process. The described approaches to detection of system bottlenecks are significant preliminary stage for the analysis and new design (redesign) of various systems. On the other hand, the considered approaches to detection of system bottlenecks are very close to system testing procedures (e.g., multi-function system testing [20,26]). In the future, it may be reasonable to consider the following research directions: (1) examination of various real-world applications; (2) examination of multi-stage frameworks for detection of system bottlenecks; (3) examination of system bottlenecks as system component(s) 'trajectories'; (4) additional study of detection of bottlenecks in hierarchical (multi-layer) networks, for example: (i) detection of low quality layer topology, (ii) detection of low quality connection between nodes of neighbor layers, (iii) detection of wrong or low quality assignment of nodes into structure/network layers. (5) taking into account uncertainty; and (6) usage of the described system approaches in education (computer science, engineering, applied mathematics).

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