Resistive-wall impedance effects for the new KEK Light Source

N Nakamura$^{1,2}$

$^1$High Energy Accelerator Research Organization (KEK), Tsukuba, Japan
$^2$The Graduate University for Advanced Studies (SOKENDAI), Tsukuba, Japan

E-mail : norio.nakamura@kek.jp

Abstract. Effects of resistive-wall (RW) impedance on a 3-GeV storage ring of the KEK Light Source (KEK-LS) are presented. Cu sheets used for in-vacuum undulators (IVUs) are regarded as the main source of the RW impedance. Although the calculated heating power per unit length due to the longitudinal impedance is more than 20 W, it is not serious for the water-cooled IVUs. The maximum growth rate of the coupled-bunch instability caused by the transverse impedance is calculated and as a result, a transverse feedback system with the damping rate of more than $10^4\,s^{-1}$ is required for the instability suppression. NEG coating of about 1 $\mu$m can be used for the KEK-LS vacuum pipe because increase of the heating power due to the NEG coating impedance is small and the effect on the transverse coupled-bunch instability is negligible.

1. Introduction

The KEK-LS is a 3-GeV storage ring of 20-cell HMBA (Hybrid Multi-Bend Achromat) lattice [1], which is planned to be constructed as a successor of the two existing Photon Factory storage rings (PF ring and PF-AR) in the KEK Tsukuba Campus. In this ring, a lot of IVUs with a small magnetic gap (4 mm at minimum) will be installed and Cu vacuum pipes of a small aperture (12.5 mm in inner radius) will be normally used. In addition, NEG coating, having a low electric conductivity, will be utilized for the Cu vacuum pipe to ensure a sufficient beam lifetime early in the machine commissioning. In this paper, the heating power due to the longitudinal RW impedance and the maximum growth rate of coupled-bunch instability caused by the transverse RW impedance are calculated and the effects of the RW impedance on KEK-LS are discussed.

2. Heating due to longitudinal impedance

The longitudinal RW impedance of a flat or circular metal pipe with the pipe wall thicker than the skin depth $\delta$ is approximately given by

$$Z_{\ell}(\omega) = \frac{\omega Z_0 \delta(\omega) L}{4\pi bc} \left\{ \text{sgn}(\omega) - i \right\} \quad (1)$$

where

$$\delta(\omega) = \frac{2}{\sigma_0 \mu_0 |\omega|}. \quad (2)$$

Here, $Z_0$ is the free-space impedance, $\sigma_0$ is the conductivity of the material, $\mu_0$ is the magnetic permeability, $\omega$ is the angular frequency, and $L$ is the length of the pipe.

The heating power per unit length due to the longitudinal RW impedance is

$$P_{\ell} = \frac{\pi c}{2} Re \{Z_{\ell}(\omega) \} \left| E_{\text{rms}} \right|^2 L.$$
Here $\omega$, $c$, $b$, $L$, $\sigma_c$, $\mu_0$, and $Z_0$ are angular frequency, the velocity of light, the inner radius for a circular pipe or half gap for a flat pipe, length, electric conductivity and permeability of the pipe and the characteristic impedance of vacuum. The permittivity and permeability of the metal are assumed to be equal to those of vacuum. A flat pipe corresponds to the IVU Cu sheet and a circular pipe to the Cu vacuum pipe. The heating power of the resistive-wall pipe is given by

$$P_{RW} = k_{loss} Q_b f_b = k_{loss} I_{total}^2 f_b,$$

where $k_{loss}$, $I_{total}$, $Q_b$, and $f_b$ are the loss factor, total stored current and bunch charge and frequency. For a Gaussian bunch with the bunch length $\sigma_t$, the loss factor is expressed by

$$k_{loss} = \frac{1}{\pi} \int_0^\infty \text{Re}\{Z_{eff}(\omega)\} d\omega$$

and given by the following expression:

$$Z_{eff}(\omega) = Z(\omega) \cdot \exp\left[-\left(\frac{\sigma_t}{\sigma_c}\right)^2\right].$$

The heating power per unit length is calculated for the IVU Cu sheet with a minimum gap of $g_u=2b=4$ mm and the circular Cu pipe with an inner radius of $b=12.5$ mm.

$$P_{RW}/L = 25.3[W/m] \text{ (IVU Cu sheet)} = 4.05[W/m] \text{ (Cu pipe)}$$

In equation (7), the electric conductivity of copper is $5.9 \times 10^7 \, \Omega^{-1} \, m^{-1}$ and the bunch length is assumed to be 10 ps for the KEK-LS [1]. The total stored current and the bunch frequency are 500 mA and 500 MHz in the multi-bunch operation where all the bunches are identical and the bunch number $k_b$ is equal to the harmonic number of 952. The relatively high heating of the IVU Cu sheet requires a water cooling with the same power as in the PF ring.

3. **Coupled-bunch instability due to transverse impedance**

The transverse RW impedance of a flat metal pipe with the pipe wall thicker than the skin depth $\delta$ is approximately given by

$$Z_r(\omega) = \frac{\pi Z_0 \delta(\omega)L}{16b^3} \left\{\text{sgn}(\omega) - i\right\}.$$

In the KEK-LS ring, 5m-class and 1m-class IVUs are installed in the long and short straight sections respectively.

The growth rate of the coupled-bunch instability in the lowest mode (the rigid dipole mode) is given by [2]

$$g'_{\mu} = \frac{e I_{beam} \alpha_0 \beta_z}{4 \pi E} \sum_{p=-\infty}^{\infty} \text{Re}\left[Z_{eff}(\omega_p')\right]$$

$$Z_{eff}(\omega) = Z(\omega) \cdot \exp\left[-\left(\frac{\sigma_t}{\sigma_c}\right)^2\right].$$
\[ \omega_p'' = (p k_b + \mu + \nu_t) \omega_0, \quad \omega_0 = 2 \pi f_0 \tag{11} \]

Here \( E \), \( f_0 \), \( \nu \), and \( \beta \) are the beam energy, revolution frequency, betatron tune and betatron function at the impedance source. For \( k_b \) bunches, there are \( k_b \) coupled-bunch oscillation modes characterized by a number \( \mu = 0, 1, \ldots, (k_b - 1) \) specifying the phase shift between bunches. The chromaticity is assumed to be zero here.

Figure 1 shows the maximum growth rates of a 5-m IVU in the long straight section with the averaged betatron function of 3.25 m and a 1-m IVU in the short straight section with the averaged betatron function of 1.39 m as a function of vertical betatron tune. The transverse RW impedance dependence on frequency results in the maximum growth rate to be inversely proportional to the square root of \( 1 - \Delta \nu_y \), with \( \Delta \nu_y \) the fractional part of the vertical betatron tune. The sum of the two maximum growth rates is \( 1050 \, s^{-1} \) for 1 cell and \( 21000 \, s^{-1} \) for 20 cells at the present vertical betatron tune of 17.62. Therefore a transverse feedback system with a damping rate of more than \( 10000 \, s^{-1} \) is needed to suppress the transverse coupled-bunch instability. Such a fast transverse feedback system is operated in the PF ring for suppression of the beam instability due to ion trapping [3]. Increasing the chromaticity reduces the coupled bunch instability growth rate, but can increase higher modes instabilities grow rate, which cannot be damped by the feedback system.

![Figure 1. Maximum growth rate of the transverse instability due to a 5-m IVU in the long straight section and a 1-m IVU in the short straight section for the magnetic gap of 4 mm and the total current of 500 mA as a function of vertical betatron tune.](image)

4. Effects of NEG coating

Longitudinal resistive-wall impedance of a two-layer circular metal pipe should be used to evaluate the effect of NEG coating on the resistive-wall heating of the KEK-LS vacuum pipe and can be expressed analytically as equation (12) [4].

\[
Z_i(\omega) = \frac{-i L}{2 \pi e \rho c} \left( \frac{\omega}{\lambda_c} + \frac{\lambda_c}{\omega} \right) \left( \alpha_{12} - \frac{b \omega}{2c} \right). \tag{12} \]
\[ \alpha_{12} = \frac{J_1(\lambda_1 b) + \kappa N_1(\lambda_1 b)}{J_0(\lambda_1 b) + \kappa N_0(\lambda_1 b)} \]

\[ \lambda_i = \frac{i + \text{sgn}(\omega)}{\delta_{i,2}}, \quad \delta_{i,2} = \left( \frac{2}{\sigma_{i,2} \mu_0 \omega} \right)^{1/2} \]

Here \( b, L, d, \sigma_{1,2} \) and \( \delta_{1,2} \) are the inner radius and length of the pipe, the thickness of the inner layer, the electric conductivities and the skin depths of the inner and outer layers. The thickness of the outer layer is assumed to be infinite or thicker than the skin depth. \( J_0, J_1, N_0, N_1, H_0^{(1)}, \text{and} H_1^{(1)} \) are the 0th-order and 1st-order Bessel functions of the 1st and 2nd kinds and the 0th-order and 1st-order Hankel functions of the 1st kind. The inner and outer layers correspond to the NEG coating and the Cu pipe.

\[ \kappa = \frac{\left( \frac{\omega}{\lambda_1 c} + \frac{\lambda_2 c}{\omega} \right) H_0^{(1)}(\lambda_1(b + d)) N_0(\lambda_1(b + d)) - \left( \frac{\omega}{\lambda_2 c} + \frac{\lambda_2 c}{\omega} \right) H_1^{(1)}(\lambda_2(b + d)) N_1(\lambda_1(b + d))}{\left( \frac{\omega}{\lambda_2 c} + \frac{\lambda_2 c}{\omega} \right) H_1^{(1)}(\lambda_2(b + d)) N_0(\lambda_1(b + d)) - \left( \frac{\omega}{\lambda_2 c} + \frac{\lambda_2 c}{\omega} \right) H_0^{(1)}(\lambda_2(b + d)) N_1(\lambda_1(b + d))} \]

Figure 2. Longitudinal impedance per unit length of NEG-coated Cu pipes \((d=1, 3 \text{ and } 10 \, \mu m)\) with \(b=12.5 \, mm\) and effective impedance of the NEG-coated Cu pipe \((d=1 \, \mu m)\) shown by solid lines. The broken lines show longitudinal impedances for pure Cu and NEG pipes with \(b=12.5 \, mm\) and the conventional expression of equation (1) for the pure Cu pipe.

Figure 2 shows the real part of the resistive-wall impedances of NEG-coated Cu pipes calculated from equation (12). The DC electric conductivity of NEG is assumed to be \(5 \times 10^5 \, \Omega^{-1}m^{-1}\) \([5]\). The effective longitudinal impedance in equation (5) for the NEG-coating thickness of 1 \(\mu m\) and the bunch length of 10 ps is also shown in the same figure. The broken lines show longitudinal impedances for pure Cu and NEG pipes with the same inner radius and the conventional expression of equation (1) for the pure Cu pipe.

These results show that the longitudinal impedances of the NEG-coated Cu pipes deviate at high frequencies from that of the pure Cu pipe \((d=0 \, \mu m)\) and approach to that of the pure NEG pipe \((d \rightarrow \infty)\) around the frequency \(f_d\) where the skin depth of NEG is equal to the NEG coating thickness. However
the effective impedance of the NEG-coated Cu pipe with $d=1\ \mu m$ is not so changed from that of the pure Cu pipe because the bunch length of 10 ps is longer than $1/2\pi f_d$ and consequently the loss factor and heating power are expected to only slightly change.

Figure 3 shows the calculated heating power per unit length as a function of NEG-coating thickness. The heating power significantly increases above the NEG thickness of $\sim 2\ \mu m$ and saturates around 10 $\mu m$. On the other hand, as expected from the effective impedance in figure 2, the heating power for NEG coating thickness of 1 $\mu m$ is almost unchanged and is higher by only less than 10 percent than that of the uncoated Cu pipe ($d=0\ \mu m$).

![Figure 3](image)

**Figure 3.** Heating power per unit length of the NEG-coated Cu pipe with $b=12.5\ \text{mm}$ as a function of NEG thickness $d$.

The transverse impedance of a two-layer circular pipe can also be obtained with an analytical expression [6], not shown here for lack of space. Figure 4 shows the real part of the calculated transverse impedances of NEG-coated Cu pipes ($d=1, 3$ and 10 $\mu m$) and the effective impedance for $d=1\ \mu m$ and $\tau=10\ \text{ps}$. As for the longitudinal impedance, the transverse impedances of the NEG-coated pipes change from the one of the pure Cu pipe to the one of the pure NEG pipe at frequencies around $f_d$. The effective impedance of the NEG-coated Cu pipe with the coating thickness of 1 $\mu m$ remains almost unchanged from that of the uncoated Cu pipe because of the relatively long 10 ps bunch length. Moreover the dominant frequency for the growth rate of the transverse RW coupled-bunch instability is $(1-\Delta \nu, \nu/f_d$ and much lower than $f_d$. Therefore the effect of NEG coating of 1 $\mu m$ at least on the transverse coupled-bunch resistive-wall instability is neglected.
Figure 4. Transverse impedance per unit length of NEG-coated Cu pipes (d=1, 3 and 10 µm) with b=12.5 mm and effective impedance of the NEG-coated Cu pipe (d=1 µm) shown by solid lines. The broken lines show transverse impedances for pure Cu and NEG pipes with b=12.5 mm and the conventional expression for the pure Cu pipe.

5. Conclusions
The heating power per unit length reaches about 25 W at the IVUs of KEK-LS. However it is tolerable in the normal multi-bunch operation for the water-cooled IVU system. From the calculated maximum growth rate of the transverse coupled-bunch instability, a transverse feedback system with the maximum damping rate of higher than $10^4$ s$^{-1}$ is needed for suppressing the instability. NEG coating of about 1 µm or less can be applied to the vacuum pipe of the KEK-LS ring to achieve ultra high vacuum against the small aperture of the vacuum pipe, because the consequent increase of the heating power and of the instability growth rate can be considered negligible.

References
[1] Harada K et al. 2016 Proc. IPAC2016 (Busan Korea) pp 3251-3253
[2] Zisman M S, Chattopadhyay S and Bisognano J J 1986 LBL-21270/UC28
[3] Takai R et al. 2009 Proc. DIPAC2009 (Basel Switzerland) pp 59-61
[4] Nakamura N 2009 Proc. ERL2009 (Ithaca USA) pp 85-89
[5] Koukovini-Platia E et al. 2015 Proc. IPAC2015 (Richmond USA) pp 3097-3099
[6] Nakamura N unpublished