Short-term optimal operation of power generators taking uncertainties into account

Wei Wu¹, Dacheng Qian, Chaoyang Pan, Hui Yu, Guangjing Hu, Bao Zhu and Huijuan Zhao

Anqing power supply company in Anhui province of State Grid, Anqing, China.

¹ Email: wuweimumu@126.com

Abstract. Under the electric power market environment, the hydroelectric power plant should synthesize the economy and the technical characteristic of hydroelectric power, consider each kind of uncertainty factor in the bidding process, carry on the decomposition and the assignment in many time intervals and in many markets to reduce the overall risk, and enable the electricity generation side to achieve the high income low risk electricity generation goal. In view of the single time interval risk measure's insufficiency, this paper proposes one new kind of multi-time interval water electricity system optimization scheduling model. This model takes the total risk of multi-time to be low as a goal under the electric power market condition, simultaneously considers the overall price fluctuation and the uncertainties of water to generation side economic efficiency influence, and takes into account the storage capacity of hydropower stations, discharge, the unit output and other constraints, in order to follow the optimal scheduling scheme. Simulation results show that the model can reflect the essential characteristics of market risk under the spatial and temporal distribution of water electricity, and ensure the system safe and reliable and the smallest risk, so as to provide a new way of thinking of optimizing the operation decision-making and risk assessment of hydropower generation company.

1. Introduction

In the power market environment, short-term optimal dispatch of hydropower stations is an important means to improve the power generation revenue of power stations. Short-term optimization scheduling can provide optimized bidding power for power stations and consider important factors for participating in market bidding, taking into account factors such as hydropower conditions, power market environment and power station characteristics. The traditional hydropower optimization scheduling schemes are often aimed at maximizing profit or minimizing costs, and do not consider the impact of risks. With the promotion of the market-oriented reform of the power industry, which is characterized by the separation of power plants and power grids, the optimal scheduling of hydropower systems faces more uncertain factors, such as water flow conditions, fluctuations in electricity prices, load growth, and uncertainty of inflows. The short-term optimal dispatching scheme of hydropower system developed under uncertainty environment inevitably carries certain risks. How to measure risk and coordinate the contradiction between profit and risk to ensure the security and economy of the system is a problem worthy of further study [1-4].
In recent years, domestic and foreign scholars have proposed a variety of methods and ideas for short-term optimal scheduling of power producers. In [5], aiming at the maximum benefit of short-term daily optimization of cascade hydropower plants, the short-term pre-generation plan of cascade hydropower plants in the market environment is studied, which provides a basis for hydropower companies to report the electricity price and electricity consumption of the next day in the previous trading market. Reference [6] considers the operation plan and constraints of actual hydropower plants, and aims to maximize the sales revenue of power producers in the market in the past. The binary and continuous variables are used to describe the performance curves of each power plant, and a mixed integer programming model is established. Literature [7] aims to achieve maximum power generation revenue under the power market conditions, and comprehensively considers the impact of peak and valley time-of-use electricity prices and environmental protection costs on the economic benefits of the power generation side, enabling the power generation side to achieve energy-saving, environmentally-friendly and high-yield power generation targets. The above research mainly focuses on how to maximize the benefits, but less research on the risks faced by power producers in the market environment.

As the theory of risk management in the financial sector has been widely introduced into power market research, many documents have discussed the risk of generators participating in the market by using Conditional Value at Risk. In [8], for the comprehensive risk management of hydropower, the linear equation is used to model discrete random variables, and the lower limit is set to control the risk. Literature [9] considers the risk measurement indicators to establish a mean-bid combination optimization model to determine the amount of electricity that the generators allocate in each market. In the above study, the time interval for power generation bidding period is a single-phase risk from 0 to T, and the measurement of multi-period risk is not considered. In [10], the single-period only considers the static risk of fixed time and the fixed rate of return, and establishes the bidding strategy optimization model of the generator-weighted multi-period portfolio market, but does not consider the uncertain factors such as market price fluctuation and load growth.

Based on the above research, combined with the operating characteristics of hydropower system, this paper comprehensively considers a variety of constraints and establishes a short-term scheduling model for power producers in a market environment with the minimum multi-period risk as the objective function. Taking an actual hydropower plant as an example, calculate the impact of fluctuations in electricity prices and uncertainties on the economic benefits of power generation, coordinate the relationship between profit and risk, and determine the optimal output curve for the next day to solve the short-term optimization of power producers in the electricity market environment. The scheduling problem has done some exploratory research work.

2. Statement of problem

For hydropower operators, hydropower scheduling in the traditional sense only considers the randomness of runoff, which is generally solved based on stochastic dynamic programming; hydropower scheduling in market environment is based on predictive electricity price, and introduces electricity price factor in the traditional power generation maximum scheduling model [11]. To construct a mathematical model aiming at maximizing power generation revenue, the optimal solution of the scheduling can only be guaranteed when the electricity price prediction is very accurate. The subsequent bidding of the hydropower station will be based on its optimized scheduling results. Once the electricity price forecast of the hydropower supplier is too high in some time periods, it will lose the on-grid electricity during the period in the market bidding. Due to the coherence of hydropower dispatching, the water will be abandoned in the subsequent period. This will cause a large risk to the water and electricity business. Therefore, in the market environment, it is urgent to establish a hydropower optimization scheduling model that fully considers the randomness of runoff and electricity price.
3. Simulation of uncertain factors

3.1. Simulation of inbound runoff

Due to the time-dependent bearing relationship between the inflows and the inflows in different time periods, it is necessary to consider the mutual influence of \( q_{h,r} \) the runoff in adjacent periods. This paper uses the normal distribution considering the influence of adjacent time periods to randomly simulate the inflow of the hydropower station. The distribution density function is:

\[
f(q_{h,r} | q_{h,r}^{(-1)}) = \frac{1}{\sqrt{2\pi}\sigma_{q_{h,r}^{(-1)}}} \exp\left[ -\frac{(q_{h,r} - \mu_{q_{h,r}^{(-1)}})^2}{2\sigma_{q_{h,r}^{(-1)}}^2} \right]
\]

\[
\mu_{q_{h,r}^{(-1)}} = \mu_r + \sigma_{q_{h,r}^{(-1)}}(q_{h,r} - \mu_{q_{h,r}^{(-1)}})
\]

In the formula: \( q_{h,r} \) is the inflow of the reservoir \( h \) during the \( t \) period of the scheduling period; \( \mu_{q_{h,r}^{(-1)}} \) and \( \sigma_{q_{h,r}^{(-1)}} \) are the conditional expectation values and variances of \( q_{h,r} \) after considering the influence of the inflow runoff \( q_{h,r}^{(-1)} \) in the previous period.

\[
\sigma_{q_{h,r}^{(-1)}} = \sigma_r \sqrt{(1-\rho_{r}^2)}
\]

\[
\mu_r = \frac{1}{N} \sum_{i=1}^{N} q_{i,r}
\]

\[
\sigma_r = \frac{1}{\sqrt{N-1}} \left( \sum_{i=1}^{N} (q_{i,r} - \mu_r)^2 - \sigma_{q_{h,r}^{(-1)}}^2 \right)
\]

\[
\rho_r = \frac{\sum_{i=1}^{N} (q_{i,r} - \mu_r)(q_{i,r}^{(-1)} - \mu_{q_{h,r}^{(-1)}})}{\sigma_r \sigma_{q_{h,r}^{(-1)}}}
\]

In the formula: \( \mu_r, \mu_{q_{h,r}^{(-1)}} \) is the expected value of the inflow runoff \( q_{h,r}^i, q_{h,r}^{i-1} \) respectively; \( \sigma_r, \sigma_{q_{h,r}^{(-1)}} \) is the variance of the inflow runoff \( q_{h,r}^i, q_{h,r}^{i-1} \) respectively; \( \rho_r \) is the correlation coefficient of the inflow runoff \( q_{h,r}^i, q_{h,r}^{i-1} \); \( N \) is the number of scheduling periods.

3.2. Electricity price forecast

\[
\begin{align*}
P_t &= \bar{P}_t + \Delta P_t \\
\Delta P_t &= \hat{\phi} \Delta P_{t-1} + \sigma \varepsilon_t
\end{align*}
\]

In the formula: \( P_t \) is the electricity price sequence; \( \bar{P}_t \) is the electricity price average of time \( t \); \( \Delta P_t \) is the residual sequence \( P_t \); \( \hat{\phi} \) is the regression coefficient; \( \sigma \) is the standard deviation of the electricity price series; \( \varepsilon_t \sim WN(0,1) \) (white noise, expected value is 0, variance is 1).

Since the electricity price series has Markovian characteristics, the \( \Delta P_t \) conditional probability density is the probability density of the normal distribution.

\[
f(\Delta P_t | \Delta P_{t-1}) \sim N(\hat{\phi} \Delta P_{t-1}, \sigma^2)
\]

\[
f(P_t | P_{t-1}) \sim N(\bar{P}_t + \hat{\phi}(P_{t-1} - \bar{P}_{t-1}), \sigma^2)
\]
4. Short-term scheduling of hydropower under multi-period risk measurement

4.1. Single time risk metric
Let \( f(x, y) \) be the loss function associated with the decision variable \( x \in X \subseteq \mathbb{R}^n \) and the market random factor \( y \in \mathbb{R}^m \). If the joint probability density function of the random vector \( y \) is \( p(y) \), the probability that \( f(x, y) \) does not exceed a certain holding level \( \alpha \) is

\[
\psi(x, \alpha) = \int_{f(x, y) \leq \alpha} p(y) dy
\]

\( \alpha_q(x) = \min\{\alpha \in \mathbb{R} | \psi(x, \alpha) \geq \beta\} \)

\[
\phi_{\beta}(x) = \frac{1}{1 - \beta} \int_{f(x, y) \geq \alpha_q(x)} f(x, y) p(y) dy
\]

At this time, \( \alpha_{\beta}(x) \) and \( \phi_{\beta}(x) \) are called VaR and CVaR under the confidence level \( \beta \in (0, 1) \), respectively. It can be seen that CVaR is defined on the basis of VaR. The analytical formula of \( p(y) \) is not easy to solve, then equation (12) can be approximated by the following formula:

\[
\tilde{\phi}_{\beta}(x) = \alpha + \frac{1}{q(1 - \beta)} \sum_{j=1}^{q} \left[ f(x, y^j) - \alpha \right]
\]

Where: \( [f(x, y') - \alpha] \) represents the larger value in 0 and \( f(x, y') - \alpha \); \( \tilde{\phi}_{\beta}(x) \) represents the approximate value of \( \phi_{\beta}(x) \); \( y_1, y_2, \ldots, y_q \) is historical data or \( q \) sample data simulated by Monte Carlo method.

4.2. Multi-period risk metric
It can be seen from the above formula derivation process that the single-period static risk metric only considers the change of assets in a single period, that is, only considers the change of assets from 0 to \( T \) time. Now, the time interval of the bidding period is \( T - 1 \) divided into equal parts, and the discretized time period is recorded as \( t = 1, 2, \ldots, T \), considering the case of the \( T \) period bid combination.

It is assumed that the power generation of the power producer is allocated in the \( n \) markets. For a certain period of time \( t \), \( f_t(s_t, x_t, y_t) \) is the loss function caused by \( y_t \) under the decision variable \( x_t \) and the state variable \( s_t \). Let \( x_{n,t} \geq 0 \) be the proportion of electricity generated by the power producer in the \( i \) market during the \( t \) period, satisfy \( \sum_{i=1}^{T} \sum_{t=1}^{n} x_{n,t} = 1 \) \( (t = 1, 2, \ldots, T; i = 1, 2, \ldots, n) \), \( y_{n,t} \) denotes the market yield, and its joint probability density function is \( p(y) \). Let \( x_t(s_t) \) be a set of possible decision variables under the state variable \( s_t \). The state transition equation of \( s_t \) is

\[
s_{t+1} = g(s_t, x_t), \ t = 1, 2, \ldots, T - 1
\]

The distribution function \( \psi_t(s_t, x_t) \) whose loss function \( f_t(s_t, x_t, y_t) \) does not exceed the critical value \( \alpha \) is:

\[
\psi_t(s_t, x_t) = P\{f_t(s_t, x_t, y_t) \leq \alpha\} = \int_{f_t(s_t, x_t, y_t) \leq \alpha} p(y) dy
\]

\( \alpha_{\beta}(s_t, x_t) = \min\{\alpha \in \mathbb{R} | \psi_t(s_t, x_t, \alpha) \geq \beta\} \)

\[
\phi_{\beta}(s_t, x_t) = \frac{1}{1 - \beta} \int_{f_t(s_t, x_t, y_t) \geq \alpha_{\beta}(s_t, x_t)} f_t(s_t, x_t, y_t) p(y) dy
\]

For any period of time \( t = 1, 2, \ldots, T \), \( \alpha_{\beta}(s_t, x_t) \) and \( \phi_{\beta}(s_t, x_t) \) are \( \alpha_{\beta}(s_t, x_t) \) and \( \phi_{\beta}(s_t, x_t) \) under the confidence level \( \beta \in (0, 1) \), respectively. Since the VaR function \( \alpha_{\beta}(s_t, x_t) \) term in
equation (17), its analytical expression is difficult to find, and a relatively simple function $F_\beta(s, x, \alpha)$ is introduced instead of $\phi_\beta(s, x)$ to calculate CVaR:

$$F_\beta(s, x, \alpha) = q + \frac{1}{1-\beta} \int_{\alpha}^{s} [f_t(s, x, y_t) - \alpha] p(y_t)dy$$

(18)

The loss function of the power generation combination bid in the $t$ period state is:

$$f_t(s, x_t, y_t) = \sum_{i=1}^{q} f(s_i, x_{n_i}, y_{n_i}) = -\sum_{i=1}^{q} (x_n + \lambda s_i T_t y_t)$$

(19)

Where: $\lambda \in (0, 1)$ is the transfer rate and $\lambda s_i T_t y_t$ is the transfer loss. Take the sample value $y_t^1, y_t^2, \ldots, y_t^q$ of the market income $y_t$, and estimate the CVaR loss function for a certain period of time as:

$$\tilde{F}_t(s, x, \alpha) = q + \frac{1}{1-\beta} \sum_{i=1}^{q} [-(x_i + \lambda s_i T_t y_t^i)^T y_t^i - \alpha_i]$$

(20)

$$\min \sum_{i=1}^{T} \tilde{F}_t(s, x, \alpha) = \sum_{i=1}^{T} \left\{ \alpha_i + \frac{1}{q(1-\beta)} \sum_{j=1}^{q} [-(x_i + \lambda s_i T_t y_t^i)^T y_t^i - \alpha_i] \right\}$$

(21)

$$z_i \geq 0 \quad z_i \geq -(x_i + \lambda s_i T_t y_t^i)^T y_t^i - \alpha_i$$

(22)

$$s_i \geq 0 \quad \sum_{i=1}^{T} s_i = 1$$

(23)

$$s_{t+1} = g_t(s_t, x_t) \quad t = 1, 2, \ldots, k$$

(24)

$$E[R(s_t, x_t, y_t)] \geq e$$

(25)

Equation (22) is the constraint formula for ensuring risk CVaR to reduce the loss below; formula (23) is the total power generation constraint; formula (24) is the state transfer formula; formula (25) is the expected power generation yield constraint, The $e$ is the lower bound of the income, and $O \leq e \leq 1$; $x_t = AQ_t R_t \Delta T / G$, $A$ is the comprehensive output coefficient, $Q_t$ and $R_t$ are respectively expressed as the power generation flow and the average power generation net head in the $t$ period, $\Delta T = 0.25h$, $G$ is the total daily power generation.

5. Case analysis

Assume that a hydropower plant has an installed capacity of 3000 MW, a guaranteed output of 1 000 MW, a combined output coefficient of 8.7, a weighted average head of 170 m, a maximum reference flow of 2,400 m$^3$/s, a reservoir with a normal water level of 1,200 m and a corresponding storage capacity of 5.8 billion m$^3$. The minimum operating water level for power generation is 1155 m and the check flood level is 1203.5 m.

According to historical data, the current electricity price and water supply of hydropower stations are predicted. The predicted value of the inflow runoff for 24 hours in a day is shown in Figure 1. The relevant statistical parameters of the market electricity price 24 hours ago are shown in Figure 2.

Assume that the unit cost of hydropower generation is 190 yuan/(MW・h), for a certain period of time $t$, the market electricity price is $p_t$, then the market rate of return $y_t = (p_t - c) / c$. For each time period $t$, the sample value $y_t^1, y_t^2, \ldots, y_t^q$ of the market return $y_t$ is taken, and the water level at the beginning and end of the decision day is known. According to the predicted runoff, the total power generation of the power plant on the decision day can be calculated $G = AR_\Delta T \sum q_t$. The lower bound of the expected yield $e$ is 0.20, and the confidence level $\beta$ is 0.95. The simplex method is used to solve the linear programming problem.
The simulation results show that the power generation yield is 0.242, and the overall system risk is 1.784, which means that within 24 time periods, the average allowable generation of excess losses for all time periods is greater than 1.784. Figure 4 shows the optimal power generation curve of the generator 24h. It can be seen from the comparison between Figure 2 and Figure 3. When the peak price is high, the power station increases the output. When the electricity price is low, the output is small. Most of the power of the power station is arranged at the peak time very little electricity. These results are reasonable and in line with the laws of the market economy. It can be seen that it is feasible to use this method to arrange a short-term pre-generation plan for hydropower plants.

When the electricity price distribution characteristics and the confidence level are constant, the effective frontiers of VaR and CVaR are calculated, as shown in Figure 4. Both effective frontier curves are monotonically increasing, indicating that on the effective frontier curve, the greater the expected rate of return of the generator, the greater the risk loss to be assumed, which is in line with the real situation of market behavior; The leading edge of the optimized CVaR is on the right side of the leading edge of VaR, because the CVaR metric is the average loss over VaR, which is better than the var risk metric.

When the electricity price distribution characteristics and expected returns are constant, the values of VaR and CVaR at different confidence levels are calculated, as shown in Figure 5. Under the same expected return level, as the confidence level increases, the risk corresponding to the best advantage is greater, indicating that the risk aversion of the generator is greater; on the contrary, the lower the confidence level, at the same expected return level. The lower the risk borne by the system, the lower the risk aversion of the power producer. Therefore, generators can choose different confidence levels based on their own risk preferences.
6. Conclusions
In order to reduce risk and maximize profit, this paper establishes a short-term optimal scheduling model that combines expected benefits with hydropower system constraints, and studies the risk metrics of generators in a continuous period of time in a day to determine that generators are under different conditions. The optimal power allocation for the time period and the following conclusions are drawn:

(1) Electricity price fluctuations and inflow water runoff uncertainty are the two major risk factors affecting hydropower dispatching. Accurately predicting market price information and analyzing hydropower coming water conditions are prerequisites for making decisions;

(2) For generators, if you only consider the maximization of the utility of a certain bid period, then this is only a static process and is not accurate. The model proposed in this paper considers the multi-period risk of short-term optimal dispatching of hydropower, and involves the calculation of the distribution of electricity in multiple time periods in a space, which is a dynamic risk measure and more accurate.

(3) The level of confidence, the size of the expected interest rate, and the degree of aversion of the generator to the risk will affect the power generation's allocation decision. In addition to the risk factors mentioned in this paper, power producers must also face the impact of various uncertain factors such as competitors' quotations, market rules, unit maintenance, network transmission capacity, and load demand. How to consider the multi-risk impact of these uncertain factors, the author will further explore and study in future research.

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