Direct determination of the electron-electron mean free path in diffusive mesoscopic samples using shot noise

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Using the ‘drift-diffusion-Langevin’ equation, we have recently shown that finite-frequency shot noise in diffusive mesoscopic conductors is very sensitive to the ratio \( \gamma \equiv L/l_{ee} \) between the sample length \( L \) and the electron-electron mean free path \( l_{ee} \). In this work we present numerical calculations of the noise at arbitrary value of \( \gamma \). If coupled with accurate noise measurements, the results presented here could serve as a new and independent way of determining \( l_{ee} \) in a given sample.

Electron-electron (e-e) scattering and the mean free path \( l_{ee} \) associated with it are among the most useful concepts in our understanding of interacting electron systems. In contrast, measurement schemes of \( l_{ee} \) may prove to be quite subtle. The main reason for this is that ordinary (as opposed to umklapp) electron-electron scattering does not change the total momentum of the scattered electrons, and thus does not affect the conductivity. The most widespread method to measure \( l_{ee} \) in diffusive mesoscopic (i.e., shorter than the electron-phonon mean free path) conductors is by fitting magneto-conductance data to weak-localization theories (see, e.g., Ref. 1). This method has two fundamental drawbacks. First, the measured effect is inherently small, i.e., it is of the order of the quantum conductance \( e^2/h \) which is much smaller than typical conductance of the sample. Second, weak localization measurements do not measure \( l_{ee} \) directly, but rather the dephasing length of the electrons. At some specific circumstances (even at low temperatures) the two lengths may not be the same.

The most direct effect e-e scattering has is on the distribution function of electrons. Direct measurements of this distribution function were recently performed in a superconducting tunneling spectroscopy experiment. Such a measurement scheme, while conceptually simple and elegant, uses intrusive lithography which may interfere with subsequent experiments involving the same sample. In this paper we suggest a very different measurement scheme which may lead to an accurate determination of \( l_{ee} \) in diffusive mesoscopic conductors. In particular, this measurement is non-destructive to the sample, so \( l_{ee} \) may be determined as part of the characterization of the sample before other experiments are performed.

The measurement scheme suggested is based on shot noise measurement in the conductor. We have recently shown that this type of noise is very sensitive to the strength of e-e scattering in the conductor. For the sake of definiteness, we report in this work results for a specific measurement geometry, namely, a geometry in which the conductor is located close to a ground plane, with the thickness of the conductor and the distance from the ground plane both much smaller than the length \( L \) of the conductor (The electronic density of states in the conductor can be either three-dimensional or two-dimensional). In this geometry, it was shown (see Fig. 1) that in the two limits of the parameter \( \gamma \equiv L/l_{ee} \), two measurable quantities acquire very different values. First, the spectral density of the noise \( S_I(\omega) \) saturates at high frequencies at the value \( S_I(\omega) = 0.5 \times 2eI \) (I is the dc current) for \( \gamma \to 0 \), while it grows as \( \omega^{1/4} \) when \( \gamma \to \infty \). Second, the temperature-dependence of \( S_I(\omega) \) at low lattice temperatures \( T \) is linear for \( \gamma \to 0 \) and quadratic for \( \gamma \to \infty \). In the present work results for the noise spectral density and its derivative with respect to temperature are presented at intermediate values of \( \gamma \).

![FIG. 1. (a) Frequency and (b) temperature dependence of the noise spectral density for the two limiting values of the ratio \( \gamma = L/l_{ee} \). Solid lines: \( \gamma \to 0 \). Dashed lines: \( \gamma \to \infty \). After Ref. 4.](image-url)

The basis of the noise calculations presented here is the ‘drift-diffusion-Langevin’ theory formulated in Refs. 3, 4. It is based on self-consistent solution of the Boltzmann-Langevin equation (integrated over electron momenta) together with the Poisson equation that accounts for screening in the system (the importance of screening in affecting the noise properties was first discussed by Landauer). This procedure is valid at frequencies lower than \( 1/\tau \) and \( eV/h \), with \( \tau \) the elastic scattering time and \( V \) the applied voltage. It enables one to calculate noise power at frequencies comparable to the inverse Thouless time \( \tau^{-1} \) of electron diffusion across the sample.

The outcome of the ‘drift-diffusion-Langevin’ approach may be summarized by the following simple recipe: for a

\[ 1000 \times 0.05 \times 1000 \]
conductor with uniform cross-section of area $A$, the spectral density of current fluctuations at any point $x$ (along the conductor’s length) is
\[
S_I(x; \omega, T) = \frac{2A}{L^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} |K(x, x'; \omega)|^2 S(x'; T) \, dx'.
\] (1)

The local noise correlator $S(x; T)$ is given by
\[
S(x; T) = 2\sigma(x) \int_0^\infty dE f_s(E, x; T) \left[ 1 - f_s(E, x; T) \right] (2)
\]
with $\sigma(x)$ the local conductivity and $f_s(E, x; T)$ the momentum-symmetric part of the local (steady state) distribution function at a total energy $E$. The response function $K(x, x'; \omega)$ equals 1 at zero frequency, but at finite frequencies it is dependent on the specific geometry of the conductor and its electrodynamic environment, always obeying the following sum rule:
\[
\frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} K(x, x'; \omega) \, dx' = 1.
\] (3)

In the geometry studied the response function for noise current in the external electrodes assumes a very simple form:
\[
K^R(x'; \omega) = \frac{L}{2} \frac{\cosh(\kappa x')}{\sinh(\kappa L/2)}.
\] (4)

Here $\kappa = \sqrt{-\omega/D'}$, $D' = D + \sigma A/C_0$, $C_0$ is the (dimensionless) linear capacitance between the conductor and the ground plane, and $D$ is the diffusion coefficient. The non-equilibrium distribution function $f_s(E, x)$ should be found by solving the stationary Boltzmann equation. At this stage it is convenient to use dimensionless quantities $\xi = x/L$, $\varepsilon = E/eV$, and $\tau = T/eV$. In the diffusion limit ($l \ll L$, with $l$ the elastic mean free path), the Boltzmann equation is
\[
-D/L^2 \frac{d^2 f_s(\varepsilon, \xi)}{d\xi^2} = I(\varepsilon, \xi)
\] (5)

with $I(\varepsilon, \xi)$ the collision integral.

The form of the collision integral for electron-electron scattering in diffusive conductors is not agreed upon. In the most simple approach, the electron wave functions are treated as plane waves. Then the collision integral assumes the familiar form
\[
I(\varepsilon, \xi) = \frac{1}{\sqrt{\Gamma}} \int d\varepsilon' \int d\omega_0 \left\{ (1 - f_s) f_0 f_0^*(1 - f_s^{\pm}) + f_0 f_0 f_0^*(1 - f_s^{\pm}) \right\}
\]
\[
-2 f_0 f_0^*(1 - f_s^{\pm})(1 - f_s^{\pm})
\] (6)

with $f_s = f_s(\varepsilon)$, $f_0 = f_0(\varepsilon)$, $f_0^* = f_0(\varepsilon + \omega_0)$, and $f_0^{\pm} = f_0(\varepsilon \pm \omega_0)$. $\Gamma^{\varepsilon} \propto V^{-2}$ is the electron-electron energy relaxation time of an electron with excess energy $eV$. The above form of the collision integral is strictly valid only at high enough voltage $eV \gg \hbar/\tau$. At lower voltages weak localization effects may become important. Then, the theory predicts $\Gamma^{\varepsilon} \propto V^{-d/2}$ ($d$ is the dimension), and the integrand in Eq. (6) depends explicitly on the transferred energy $\omega_0$. However, not even a general form of the collision integral in this regime is known. In all works we are aware of (see, e.g., Ref. [1]), only the electron relaxation rate is calculated. This rate may not be directly related to the collision integral for the strongly non-equilibrium case, since it involves only the ‘scattering out’ term, and this only at small deviations from equilibrium. Indeed, the only relevant non-equilibrium experiment reported to dated found results which are strongly inconsistent with the relaxation rate of Ref. [1]. For these reasons we restrict the present work to the form given by Eq. (6). However, it will be shown below that the main qualitative results of the work are not dependent upon the exact form of the collision integral $I$.

The boundary conditions to be used with Eq. (5) are derived from the fact that the voltage drops entirely on the sample, and therefore the distribution function at the conductor-electrode interfaces must be equilibrium,
\[
f_s(\varepsilon, +1/2) = f_0(\varepsilon + 1/2) = \frac{1}{1 + \exp \left( \frac{\varepsilon - 1/2}{\sqrt{\Gamma}} \right)}.
\] (7)

As in the case of electron-phonon scattering, one can see from equations (6) that the dependence of $f_s(E, x)$ on the physical variables of the problem $eV$, $T$, and $L$ is only through the parameters $t = T/eV$ and $\gamma = L/L_{ee}$. From equations (6) it is then seen that for a uniform conductor $|\sigma(x)| = |\sigma|$ the only additional parameter which affects the normalized noise value
\[
\alpha = \frac{S_I}{2eI} = |\sigma|L = \sqrt{\omega_0 \tau_T}, \text{ with } \tau_T = L^2/D' \text{ the effective Thouless time.}
\]

Equation (6) was previously solved analytically in the two limiting values of the parameter $\gamma$. The distribution functions in these cases are
\[
f_s(\varepsilon, \xi) = (1/2 + \xi) f_0(\varepsilon + 1/2) + (1/2 - \xi) f_0(\varepsilon - 1/2) \quad \gamma \to 0, \quad (8a)
\]
\[
f_s(\varepsilon, \xi) = \left\{ 1 + \exp \left( \frac{\varepsilon + \xi}{t_h(\xi)} \right) \right\}^{-1} \quad \gamma \to \infty, \quad (8b)
\]

with the hot-electron temperature $t_h(\xi) = \sqrt{t^2 + 3(1 - 4\xi^2)/4\pi^2}$. Analytical expressions for the frequency and temperature dependences of the noise power in these limiting cases were given elsewhere. The main relevant results of that work are summarized in Fig. 1. The very different functional behavior of the noise in the two limits of $\gamma$ (which is what motivated the present work) is well understood and was explained in Ref. [1].

It is imprtant to note that the limiting distributions Eqs. (6) do not depend on the form of the collision integral $I$. This can be seen immediately by combining equations (6) and (7). Then, it is evident that at $\gamma \to 0$ $f_s$ is determined by evaluating the LHS of Eq. (6) (which does
not depend on $I$) to zero, while at $\gamma \to \infty$ the collision integral should vanish, which means that, for any form of $I$, $f_s$ must be given by a Fermi-Dirac function Eq. (8) ($t_h$ in this case is determined solely from conservation of total electron energy). Thus we conclude that even in the case of $eV$ comparable to or smaller than $\hbar/\tau$, for which $I$ is not known and therefore cannot be studied quantitatively, the results depicted in Fig. 1 still apply, and the noise is still very sensitive to the ratio $\gamma$.

In order to study the crossover region (i.e., at finite values of $\gamma$) we solve Eq. (5) numerically, utilizing an iterative Newton-Raphson method for the discrete Eq. (5) on a non-uniform mesh. It was found that results are accurate enough with a mesh size of up to $200 \times 200$. Results for the distribution functions for several values of $\gamma$ are presented in Figure 2 (the mesh plotted is considerably sparser than the one used for the calculations). Plots (a) and (f) correspond to the limiting cases described by equations (8a) and (8b), respectively. As seen from the figure, the step-like singularity characteristic of the distribution at $\gamma = 0$ is retained at distances smaller than $L/\gamma^{1/2}$ from the edges of the sample. Due to the form of the response function (8) the high frequency noise is sensitive to the distribution of electrons only at distances $\xi \sim 1/\kappa L = (\omega \tau T)^{-1/2}$ from the edges. Therefore, since the asymptotic behavior of the noise at $\gamma \to \infty$ (e.g., its $\omega^{1/4}$ dependence) is due to electron thermalization to temperature $t_h$, the noise should behave with this asymptotic form only at $\gamma \gg \omega \tau T$.

Figures 3 and 4 show our main results: the normalized noise spectral density and its temperature dependence as a function of $\gamma$ for various frequencies. Fig. 3 shows the results at zero frequency, while Fig. 4 includes also high frequencies. It is concluded from these figures that accurate measurements of noise should be sensitive to the ratio $\gamma$ already at low frequencies ($\omega \tau T \ll 1$), provided $\gamma$ is of the order of 1–10. On the other hand, the high-frequency noise is much more sensitive to the value of $\gamma$, and can be used to probe $\gamma$ up to a range of a few hundreds.

The possibility to use the results presented in this work as an actual probe of $L/l_{ee}$ relies on the advancement of accurate noise measurements in diffusive conductors. I believe that the present situation is very close to the necessary accuracy. Steinbach et al. performed low frequency measurements in which they could unambiguously distinguish between the two limiting noise values of Fig. 3(a). Schoelkopf et al. performed high-

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![Figure 2](image2.png)

**FIG. 2.** The symmetric part of the electron distribution function $f_s(\xi, \xi)$ for different values of $\gamma = L/l_{ee}$. For all curves, $T = 0.01$ eV.

![Figure 3](image3.png)

**FIG. 3.** The dependence of the noise spectral density (a) and its derivative with respect to temperature (b) on $\gamma$ for $t = T/eV = 0.01$ at zero frequency. The units of panel (a) are normalized to the 'full' shot noise $2eI$ while those of panel (b) are normalized to the temperature derivative of the equilibrium noise $4/R$, with $R$ the resistance.

![Figure 4](image4.png)

**FIG. 4.** Same as Figure 3, but for zero and finite frequencies.
frequency noise measurements with excellent accuracy. The samples in the latter experiments were made of well-conducting gold, so the crossover frequency \(1/\tau_T\) was very high. In samples with poorer conductivity the 'high-frequency' regime may be reached at \(100/2\pi\tau_T \sim 10\) GHZ, so the curves of Fig. 4 can be experimentally verified at microwave frequencies. In addition to absolute measurements of noise, the temperature dependence shown in Figures 3(b), 4(b) should be relatively simple to detect.

The results presented here assume that the sample is strictly in the mesoscopic regime, so that no thermalization by phonons occurs. Practically, at \(\gamma > 100\) phonon relaxation may start to be appreciable even at relatively low temperatures. Theoretically, this means that the electron-phonon collision term should be added to the collision integral \(W\). However, the subsequent suppression of the noise compared to the results presented here may not be substantial because significant reduction of the noise, particularly at high frequencies, happens only at a very large ratio of \(L\) and the electron-phonon relaxation length \(l_{ee}\). To summarize, we have performed numerical calculations of the non-equilibrium noise in mesoscopic samples with strong elastic scattering and with an arbitrary strength of electron-electron scattering. We have shown that the spectral density of the high frequency noise and its functional dependence on temperature are very sensitive to the ratio \(\gamma \equiv L/l_{ee}\). The results presented here, if coupled with accurate noise measurements, can thus serve as a new and independent way to identify this ratio in a given sample.

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1. B. L. Altshuler and A. G. Aronov in *Electron-electron interactions in disordered systems*, Edited by A. L. Efros and M. Pollak (Elsevier, Amsterdam, 1985).
2. B. L. Altshuler, M. E. Gershenson, and I. L. Aleiner, cond-mat/9803125 (1998).
3. H. Pothier, S. Gueron, N. O. Birge, D. Esteve, and M. H. Devoret, Phys. Rev. Lett. 79, 3490 (1997).
4. Y. Naveh, D.V. Averin, and K.K. Likharev, cond-mat/9801188 [Phys. Rev. B. (to be published)].
5. Y. Naveh, D.V. Averin, and K.K. Likharev, Phys. Rev. Lett. 79, 3482 (1997).
6. Sh.M. Kogan and A.Ya. Shul’man, Zh. Eksp. Teor. Fiz. 56, 862 (1969) [Sov. Phys. JETP 29, 467 (1969)];
7. R. Landauer, Ann. New York Acad. Sci. 755, 417 (1995); Physica B, 227, 156 (1996).
8. V. F. Gantmakher and Y. B. Levinson, *Carrier Scattering in Metals and Semiconductors* (North-Holland, Amsterdam, 1987).
9. H. Fukuyama and E. Abrahams, Phys. Rev. B 27, 5976 (1983).
10. Y. Naveh, D.V. Averin, and K.K. Likharev, cond-mat/9803335 [Phys. Rev. B. (to be published)].
11. K.E. Nagaev, Phys. Lett. A 169, 103 (1992).
12. K. E. Nagaev, Phys. Rev. B 52, 4740 (1995).
13. V. I. Kozub and A. M. Rudin, Surf. Sci. 361/362, 722 (1996).