Where new gravitational physics comes from: M-theory?

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Abstract

It is suggested [1] that current cosmic acceleration arises due to modification of General Relativity by the terms with negative powers of curvature. We argue that time-dependent (hyperbolic) compactifications of 4 + n-dimensional gravity (string/M-theory) lead to the effective 4d gravity which naturally contains such terms. The same may be achieved in the braneworld by the proper choice of the boundary action. Hence, such a model which seems to eliminate the need for dark energy may have the origin in M-theory.

PACS numbers: 98.80.-k,04.50.+h,11.10.Kk,11.10.Wx

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1. Introduction. Recent astrophysical data from Ia supernovae[2] and cosmic microwave radiation [3] clearly indicate that current Universe is undergoing a phase of the accelerated expansion. Despite many theoretical efforts it remains unclear what is the origin of such accelerated behaviour. Phenomenologically, the easiest possibility is to supply the standard FRW cosmology with cosmic fluid with large, negative pressure (dark energy). This dark energy is expected to carry most of the universe energy density. Various candidates for dark energy were proposed: the cosmological constant (for a review, see [4]), scalar fields, extra dimensions, etc. It is clear that such a list may be significantly extended. Unfortunately, neither of existing dark energy models is completely satisfactory.

In such a situation one can assume that cosmic speed-up is due to new gravitational physics which becomes important at very low curvatures. In other words, the Einstein gravity should be modified providing the effective dark energy contribution at late times. The model of this sort has been recently proposed in ref.[1] (for discussion of related models, see also [5]). It is a combination of the term with the scalar curvature $R$ and the inverse of $R$. Note that other terms with negative powers of curvature may be introduced as well. Such models may have problems with late time production of small/zero curvature black holes. The starting action is [1]$^3$:

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left( R - \frac{\mu^4}{R} \right).$$

By introducing an auxiliary field $A$, one may rewrite the above action (1):

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left( R - 2\mu^2 A + A^2 R \right).$$

Using the equation of motion for $A$, we find the two actions (1) and (2) are equivalent. With the redefinition of the auxiliary field $A$ by $e^{2\phi} = 1 + A^2$, the action (2) becomes:

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left( e^{2\phi} R + 2\mu^2 \sqrt{1 - e^{2\phi}} \right).$$

$^3$More generally, the theory which contains any negative power of $R$ is called the modified gravity. There are astrophysical indications [5] that power of $1/R$ term should be fractional and close to 1.
Rescaling the metric tensor $g_{\mu\nu}$ by $g_{\mu\nu} \rightarrow e^{-2\phi}g_{\mu\nu}$, the action (3) is transformed as

$$S = \frac{1}{\kappa^2} \int d^4x \sqrt{-g} \left( R - 6g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 2\mu^2 e^{-4\phi} \sqrt{1 - e^{2\phi}} \right).$$

(4)

Then the action (1) is equivalent to the Einstein action coupled with the scalar field $\phi$ with potential $2\mu^2 e^{-4\phi} \sqrt{1 - e^{2\phi}}$. As it has been demonstrated in ref.[1] (see also [5]) the late-time cosmic acceleration can really arise due to such small modification of the General Relativity (GR) by $1/R$ or by $1/R^m$ terms.

Of course, giving rise the effective dark energy, such a modification of General Relativity looks also completely ad hoc. The deep problem remains: where these curvature corrections come from? In order to find some way towards to solution of above problem it may be right time to compare the successful cosmic acceleration models where new gravitational physics appears. In particulary, recently the accelerating cosmologies from higher dimensional gravity with hyperbolic, time-dependent compactification were found [6, 7] (for related discussion and extensions, see [9, 10, 11]). Note that such solutions were also obtained earlier in a different context, as $S$-branes in refs.[8]. The influence of hyperbolic extra dimensions to 4d cosmology has been studied earlier in refs.[12]. It is remarkably, that actually string/M-theory compactification gives rise to such accelerating cosmologies [6] because the standard no-go theorems are not applied for such compactifications (for example, where these theorems are not applied to time-dependent torus compactification, see [13]). In the present Letter, we argue that time-dependent, hyperbolic compactifications of string/M-theory [6, 7, 10, 11] or braneworld scenario actually may reproduce the modifications of GR of the sort proposed in ref.[1]. This indicates (but, unfortunately, does not prove) that string/M-theory may be the source of new gravitational physics!

2. Time-dependent (hyperbolic) compactification. Let us consider now if the model (1) can be obtained from (time-dependent) compactification. We start from 4 + $n$-dimensional spacetime, whose metric is given by

$$ds^2 = \sum_{\mu,\nu=0,1,2,3} g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi(x^\mu)} \sum_{i,j=1}^n \tilde{g}_{ij} d\xi^i d\xi^j. \quad (5)$$
For simplicity, one may assume the metric $\tilde{g}_{ij}$ expresses the Einstein manifold, where the Ricci tensor $\tilde{R}_{ij}$ constructed from $\tilde{g}_{ij}$ is proportional to $\tilde{g}_{ij}$: $\tilde{R}_{ij} = k \tilde{g}_{ij}$. Here $k$ is a constant. When $n = 1$, $k$ always vanishes ($k = 0$). When $n \geq 3$, the above metric is given as the solution of $n$-dimensional Euclidean Einstein equation. When $n = 2$, since 2d Einstein equation is trivial, in the conformal gauge the above condition for Ricci tensor is the Liouville equation.

Under the above assumptions, the $4+n$ dimensional Einstein action can be rewritten as

$$
S_{4+n} = \frac{1}{\kappa^2} \int d^{4+n}x \sqrt{-g^{(4+n)}} \left( R^{(4+n)} - \Lambda \right) 
$$

$$
= \frac{V_n}{\kappa^2} \int d^4x \sqrt{-g} e^{n\phi} \left( R - \Lambda + n(n-1)g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + nke^{-2\phi} \right) .
$$

(6)

Here $V_n$ is the volume of the $n$-dimensional manifold whose metric tensor is given by $\tilde{g}_{ij}$. If we rescale the 4-dimensional metric $g_{\mu\nu}$ by $g_{\mu\nu} \rightarrow e^{-n\phi} g_{\mu\nu}$, the action (6) can be rewritten as

$$
S_{4+n} = \frac{V_n}{\kappa^2} \int d^4x \sqrt{-g} \left( R - \frac{n(n+2)}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi 
- e^{-n\phi} \Lambda + nke^{-(n+2)\phi} \right) .
$$

(7)

This is Einstein gravity coupled with the scalar field $\phi$, whose potential is $e^{-n\phi} \Lambda - nke^{-(n+2)\phi}$. The obtained action is almost identical with that in [9] where the starting action contains $p$-form field. Instead of the $p$-form field, the cosmological constant is included in the original action (6). The qualitative structure of the action (7) is the same as that in [9]. Since the potential is given by

$$
V(\phi) = e^{-n\phi} \Lambda - nke^{-(n+2)\phi} ,
$$

(8)

if $\Lambda > 0$ and $k < 0$ (this case is more attractive due to stability), the potential is monotonically decreasing function with respect to $\phi$ and vanishes

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For simplicity, instead of $p$-form contribution which is typical in M-theory we consider cosmological constant contribution. Then, the above action does not literally correspond to superstring/M-theory. However, it reflects the main features of such unified models in many respects.
at $\phi \to +\infty$. Then as in [9], for large $\phi$ with very large kinetic energy with negative velocity $\dot{\phi} < 0$, the universe starts decelerated expansion. At some point $\phi = \phi_0$, the velocity vanishes $\dot{\phi} = 0$, and after that an accelerated expansion begins.

We now further rescale the 4-dimensional metric $g_{\mu\nu}$ in the action (7) by $g_{\mu\nu} \to e^{2\phi} \frac{n(n+2)}{3} g_{\mu\nu}$, then the action (7) can be rewritten as

$$S_{4+n} = \frac{V_n}{\kappa^2} \int d^4x \sqrt{-g} \left( e^{\alpha \phi} R - e^{\phi(2n-1)} \Lambda + nk e^{\phi(2n-2)} \right).$$

(9)

Here $\alpha \equiv \sqrt{\frac{n(n+2)}{3}}$. Since the kinetic term for $\phi$ vanishes, one may regard $\phi$ as an auxiliary field. By the variation of the action (9) with respect to $\phi$, we obtain

$$R = f(\phi) \equiv \left( 2 - \frac{n}{\alpha} \right) e^{(a-n)\phi} \Lambda - \left( 2 - \frac{n+2}{\alpha} \right) nk e^{(a-n-2)\phi}. \tag{10}$$

Solving (10) with respect to $\phi$ when $n \neq 0$, one may delete $\phi$ in (9). The resulting action looks like

$$S_{4+n} = \frac{V_n}{\kappa^2} \int d^4x \sqrt{-g} F(R). \tag{11}$$

With no matter and for the Ricci tensor $R_{\mu\nu}$ being covariantly constant, the equation of motion is:

$$0 = 2F(R) - RF'(R), \tag{12}$$

which is the algebraic equation with respect to $R$. Solving (12) with respect to $R$, one may find $\phi$ from (10).

When $n = 6$, Eq.(10) can be exactly solved as

$$e^{\phi} = \sqrt{\frac{2R}{\Lambda}}, \tag{13}$$

and we obtain

$$F(R) = -\frac{\Lambda}{2R} + 6k. \tag{14}$$
The strange property of above formula is that Einstein term does not appear there. There may be several mechanisms behind such a behaviour:

1. Our $4+n$-dimensional gravity is not effective string theory even for $n = 6$. One can expect that in full superstring theory where there are contributions from other fields the Einstein term will be restored.

2. Another proposal could be related with the compactification under the discussion. For instance, the physical compactification could be somehow different, say, like $H_n/\Gamma$.

When $|k| \ll |\Lambda|$, Eq.(10) can be solved as

$$ e^\phi \sim \left\{ \frac{\alpha}{2\alpha - n} \frac{R}{\Lambda} \right\}^{\frac{1}{\alpha - n}}. \quad (15) $$

and one gets

$$ F(R) = f_0^{(n)} \left( \frac{R}{\Lambda} \right)^{2+\frac{n-2}{\alpha-n}} + f_1^{(n)} \left( \frac{k}{\Lambda} \right) \left( \frac{R}{\Lambda} \right)^{2+\frac{n-2}{\alpha-n}} + f_2^{(n)} \left( \frac{k}{\Lambda} \right)^2 \left( \frac{R}{\Lambda} \right)^{2+\frac{n-4}{\alpha-n}} + \cdots. \quad (16) $$

Here $f_i^{(n)}$’s are coefficients depending on $n$. Especially when $n = 7$, which may correspond to M-theory, we numerically obtain

$$ F(R) = f_0^{(7)} \left( \frac{R}{\Lambda} \right)^{-0.86} + f_1^{(7)} \left( \frac{k}{\Lambda} \right) \left( \frac{R}{\Lambda} \right)^{-0.04} + f_2^{(7)} \left( \frac{k}{\Lambda} \right)^2 \left( \frac{R}{\Lambda} \right)^{0.77} + \cdots. \quad (17) $$

We again see that the terms with fractional negative powers of $R$ appear. Such the action may produce the required cosmic acceleration. However, the Einstein term is induced only approximately. (Note the astrophysical data suggest that power of $R$ in Einstein gravity may vary around 1 with 10% accuracy!) As in previous case of $n = 6$ one can expect that in full M-theory with account of p-form contribution the correct Einstein term may be induced within existing cosmological bounds. Or, once more the physical compactification which corresponds to our evolving FRW universe is somehow different from the simple case under consideration.

On the other hand, when $|k| \gg |\Lambda|$, Eq.(10) can be solved as

$$ e^\phi \sim \left\{ \frac{\alpha}{2\alpha - (n - 2)} \frac{R}{nk} \right\}^{\frac{1}{\alpha - n - 2}}. \quad (18) $$
and

\[ F(R) \propto R^{2 + \frac{n+2}{\alpha-n-2}}. \tag{19} \]

We should note \(2 + \frac{n}{\alpha-n} = \infty\) and \(2 + \frac{n+2}{\alpha-n-2} > 0\) when \(n = 1\) and \(2 + \frac{n}{\alpha-n} > 0\) for \(0 < n < 1\) and \(2 + \frac{n}{\alpha-n} < 0\) for \(n > 1\). We should also note \(2 + \frac{n+2}{\alpha-n-2} = 0\) for \(n = 6\), \(2 + \frac{n+2}{\alpha-n-2} > 0\) for \(n > 6\) and \(2 + \frac{n+2}{\alpha-n-2} < 0\) for \(n < 6\). When \(n \geq 2\), there effectively appears the negative power of the curvature in 4d action.

Several more remarks about Eq.(10) are in order. We should note that \(2 - \frac{n}{\alpha}\) is always positive. On the other hand, \(2 - \frac{n+2}{\alpha} < 0\) when \(n < 6\), \(2 - \frac{n+2}{\alpha} > 0\) when \(n > 6\), and \(2 - \frac{n+2}{\alpha} = 0\) when \(n = 6\). \(n = \alpha < n + 2\) when \(n = 1\) and \(\alpha < n < n + 2\) when \(n \geq 2\).

We first consider \(n = 1\) case, where \(k = 0\) and \(\alpha = 1\). Then (10) gives \(R = \Lambda\). One cannot solve (10) with respect to \(\phi\). Consider \(n \geq 2\) case. If \(k > 0\) and \(n < 6\) \((k < 0\) and \(n > 6\)), \(f(\phi) \rightarrow +\infty\) \((f(\phi) \rightarrow -\infty)\) when \(\phi \rightarrow -\infty\) and \(f(\phi) \rightarrow 0\) when \(\phi \rightarrow +\infty\). If the scalar curvature is large, \(F(R)\) in the action behaves as (19) and if the scalar curvature is small, \(F(R)\) behaves as (16). When \(n = 6\), it is described by (16), that is \(F(R) \propto \frac{1}{R}\).

When \(n \geq 6\), for both, large or small curvature, the action is described by the positive power of \(R\). On the other hand, when \(n < 6\), the action corresponds to the positive power of \(R\) when \(R\) is small but to the negative power of \(R\) when \(R\) is large. This is the inverse of the behaviour in (1).

When \(\Lambda > 0\), \(k < 0\), and \(n < 6\) \((\Lambda < 0\), \(k > 0\), and \(n < 6\)), or \(\Lambda > 0\), \(k > 0\), and \(n < 6\) \((\Lambda < 0\), \(k < 0\), and \(n > 6\)) \(f(\phi)\) (10) is a monotonically decreasing \((increasing)\) function. When \(\Lambda, k > 0\) and \(n < 6\) \((\Lambda, k < 0\) and \(n < 6\)), or \(\Lambda > 0\), \(k < 0\) and \(n > 6\) \((\Lambda < 0\), \(k > 0\) and \(n > 6\)), \(f(\phi)\) has a zero at finite \(\phi\): \(f(\phi_0) = 0\). If \(R\) is small, there is also a solution \(\phi \sim \phi_0\) in (10). Then the action (9) takes the form of the Einstein action with cosmological constant.

Thus, we showed that there are similarities between effective 4d gravitational action \((after\ time-dependent\ (hyperbolic)\ compactification\ of\ M-theory)\) and the starting \(1/R\) action.

3. Product hyperbolic compactifications. One may consider the prod-
uct compactification more general than (5) as in [11].:

\[ ds^2 = \sum_{\mu,\nu=0,1,2,3} g_{\mu\nu} dx^\mu dx^\nu + e^{\beta_1 \phi(x^\nu)} \sum_{i,j=1}^n g_{i,j}^{(1)} d\xi^i d\xi^j + e^{\beta_2 \phi(x^\nu)} \sum_{I,J=1}^m g_{I,J}^{(2)} d\xi^I d\xi^J. \]  

(20)

(As a further generalization, some function \( \sigma(\phi) \) may be introduced in the exponent. Then, the negative power of the scalar curvature could appear more naturally.) Starting from 4 + \( n + m \)-dimensional Einstein action and rescaling the 4-dimensional metric \( g_{\mu\nu} \) by \( g_{\mu\nu} \rightarrow e^{-\frac{n \beta_1 + m \beta_2}{2} \phi} g_{\mu\nu} \), the classical action becomes

\[ S_{4+n+m} = \frac{V_n V_m}{\kappa^2} \int d^4 x \sqrt{-g} \left\{ R - \Lambda e^{-\frac{n \beta_1 + m \beta_2}{2} \phi} \right. 
- \left( \frac{\beta_1^2}{4} n + \frac{\beta_2^2 m}{8} + \frac{\beta_1 \beta_2 m (n+2)}{2} \right) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi
+ \left. n k^{(1)} e^{-\frac{n \beta_1 + (m+2) \beta_2}{2} \phi} + m k^{(2)} e^{-\frac{n \beta_1 + m \beta_2}{2} \phi} \right\}. \]

(21)

We now further rescale the 4-dimensional metric \( g_{\mu\nu} \) in the action (21) by \( g_{\mu\nu} \rightarrow e^{\phi} \sqrt{\frac{\beta_1^2 n + \beta_2^2 m}{6} + \frac{\beta_1 \beta_2 m (n+2)}{12}} g_{\mu\nu} \). The action (21) can be rewritten as

\[ S_{4+n+m} = \frac{V_n V_m}{\kappa^2} \int d^4 x \sqrt{-g} \left( e^{\gamma \phi} R 
- e^{\phi (2 \gamma - \frac{n \beta_1 + m \beta_2}{2})} \Lambda + n k^{(1)} e^{\phi (2 \gamma - \frac{n \beta_1 + m \beta_2}{2})} 
+ m k^{(2)} e^{\phi (2 \gamma - \frac{n \beta_1 + (m+2) \beta_2}{2})} \right). \]

(22)

Here

\[ \gamma \equiv \sqrt{\frac{\beta_1^2 n + \beta_2^2 m}{6} + \frac{\beta_1 \beta_2 m (n+2)}{12}}. \]

(23)

Let us consider the situation that

\[ 3 \gamma = (n+2) \beta_1 + m \beta_2, \]

\[ \tilde{\gamma} \equiv -2 \gamma + \frac{n \beta_1 + (m+2) \beta_2}{2} \gg \gamma. \]

(24)
It is chosen $\Lambda = 0$. Then the action (22) has the following form

$$S_{4+n+m} = \frac{V_n V_m}{k^2} \int d^4 x \sqrt{-g} \left( e^{\gamma \phi} R + n k^{(1)} e^{\tilde{\gamma} \phi} + n k^{(2)} e^{-\tilde{\gamma} \phi} \right).$$

By the variation over $\phi$, one finds

$$R = \tilde{f}(\phi) \equiv -\frac{n\gamma k^{(1)}}{2} e^{-\tilde{\gamma} \phi} - n\tilde{\gamma} k^{(2)} e^{(-\tilde{\gamma}-\gamma)\phi}. \quad (26)$$

Then if $k^{(1)}, k^{(2)} > 0$ ($k^{(1)}, k^{(2)} < 0$), $\tilde{f}(\phi)$ is a monotonically increasing (decreasing) function. When $\phi \to +\infty$, the first term in $\tilde{f}(\phi)$ becomes dominant and $\tilde{f}(\phi) \to 0$. When $R$ is small $e^{\tilde{\gamma} \phi} \sim \frac{1}{R}$, and

$$S_{4+n+m} \sim \int d^4 x \sqrt{-g} \left( \frac{1}{R} \right). \quad (27)$$

On the other hand, when $\phi \to -\infty$, the second term in $\tilde{f}(\phi)$ becomes dominant and goes to infinity ($|\tilde{f}(\phi)| \to \infty$). If $R$ is large, one gets $e^{\tilde{\gamma} \phi} \sim R^{1-\tilde{\gamma}+\gamma}$, and

$$S_{4+n+m} \sim \int d^4 x \sqrt{-g} R^{1-\tilde{\gamma}+\gamma}. \quad (28)$$

Then in the limit $\tilde{\gamma} \gg \gamma$, Eqs. (27) and (28) reproduce the behaviour in (1).

How realistic is a condition (24)? The first equation $3\gamma = (n+2)\beta_1 + m\beta_2$ (24) can be always solved with respect to $\frac{\beta_1}{\beta_2}$. It is difficult, however, to realize the second condition $\tilde{\gamma} \gg \gamma$ (24) since $\tilde{\gamma} - \frac{\gamma}{2} = \beta_1$. It is more easier to get the fractional negative power of $R$.

In order to avoid the above difficulty, we now consider further generalization of the product compactification like

$$ds^2 = \sum_{\mu, \nu = 0, 1, 2, 3} g_{\mu\nu} dx^\mu dx^\nu + \sum_{l=1}^L e^{\beta_l(x^\mu)} \sum_{i,j=1}^{n_l} g^{(l)}_{ij} d\xi^i d\xi^j. \quad (29)$$

Then starting from $D = 4 + \sum_{l=1}^L n_l$-dimensional Einstein gravity, instead of (22), one introduces the parameter

$$\tilde{\gamma} \equiv \sqrt{\frac{\sum_{l=1}^L n_l \beta_l^2}{6}} + \left( \frac{\sum_{l=1}^L n_l \beta_l}{12} \right)^2. \quad (30)$$
The Ricci tensor $R_{ij}^{(l)}$ constructed from $g_{ij}^{(1)}$ is assumed to be proportional to $g_{ij}^{(1)}$: $R_{ij}^{(l)} = k^{(l)} g_{ij}^{(1)}$. With number of higher-dimensional parameters, by adjusting them, we may satisfy the condition corresponding to (24):

$$3\tilde{\gamma} = \sum_{l=1}^{L} n_l \beta_l + \beta_1 , \quad \tilde{\gamma} \equiv -2\tilde{\gamma} + \frac{\sum_{l=1}^{L} n_l \beta_l + 2\beta_2}{2} \gg \tilde{\gamma} . \quad (31)$$

Finally, putting $\Lambda = k^{(3)} = k^{(4)} = \cdots = k^{(L)} = 0$ the effective action (25) behaves as (27) for small $R$ and as (28) for large $R$.

4. Discussion. It is shown that time-dependent (hyperbolic) compactification of $4+n$-dimensional gravity leads to the effective 4d gravity action which naturally contains the (fractional) negative powers of curvature. However, there are problems to get such an effective 4d gravity which also contains the Einstein term on the same time. In our explicit example of ten- and eleven-dimensional gravity, the corresponding power of $R$ is 0 or about 0.8 (which is still below the admitted value between 0.9-1.1). There may be several explanations of such behaviour.

1. The full string/M-theory calculation with the account of all fields may presumably improve the situation.
2. The question which compactification is physical one remains so far unresolved in full string/M-theory.
3. The orthodox point of view that results of second and third sections of present work have nothing to do with string/M-theory may be also admitted. Nevertheless, the fact that negative power of $R$ effectively appears is quite convincing.

It is also interesting that term like $\frac{1}{R}$ (1) might appear in the braneworld scenario [14] which is believed to be related with string/M-theory too. Let the 3-brane is embedded into the 5d bulk space as in [15]. Let $g_{\mu\nu}$ be the metric tensor of the bulk space and $n_\mu$ be the unit vector normal to the 3-brane. Then the metric $q_{\mu\nu}$ induced on the brane is $q_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$. Our starting action is:

$$S = \int d^5 x \sqrt{-g} \left\{ \frac{1}{\kappa^2_5} R^{(5)} - 2\Lambda + \cdots \right\} + S_{\text{brane}}(q) \, . \quad (32)$$

Neglecting the terms with the higher powers of the curvature, the effective
Einstein equation is given by (for instance, [15])

\[
\frac{1}{\kappa_5^2} \left( R^{(4)}_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R^{(4)} \right) = -\frac{1}{2} \left( \Lambda + \frac{\kappa_5^2 \lambda^2}{6} \right) q_{\mu\nu} + \frac{\kappa_5^2 \lambda}{6} \tau_{\mu\nu}.
\]

(33)

Here \(\lambda\) is the tension of the brane. Then the corresponding effective 4d action follows

\[
S = \frac{1}{\kappa_4^2} \int d^4 x \sqrt{-q} \left\{ R^{(4)} - \Lambda_4 \right\} + S_{\text{brane}}(q).
\]

(34)

Here

\[
\frac{1}{\kappa_4^2} \equiv \frac{6}{\kappa_5^2 \lambda}, \quad \Lambda_4 \equiv \kappa_5^2 \left( \Lambda + \frac{\kappa_5^2 \lambda^2}{6} \right).
\]

(35)

We now choose \(S_{\text{brane}}(q)\) to be an action of a dilaton gravity:

\[
S_{\text{brane}}(q) = -\frac{1}{\kappa_4^2} \int d^4 x \sqrt{-q} e^{2\phi} \left\{ R^{(4)} + 4 q^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi \right\}.
\]

(36)

Under the conformal transformation \(g_{\mu\nu} \rightarrow e^\sigma g_{\mu\nu}\), with the choice \(\sigma = \ln \left( 1 + e^\phi \right)\), the action (34) with (36) can be transformed as

\[
S = \frac{1}{\kappa_4^2} \int d^4 x \sqrt{-q} \left\{ A^2 (2 - A) R^{(4)} + AA_4 \right\}.
\]

(37)

Here \(A \equiv 1 + e^\phi = e^\sigma\). In (37), the kinetic term of \(\phi\) vanishes. \(A\) can be regarded as an auxiliary field. By the variation of the action with respect to \(A\), we obtain

\[
(4A - 3A^2) R^{(4)} + \Lambda_4 = 0.
\]

(38)

When \(R\) is large, \(A\) behaves as \(A \rightarrow \frac{4}{3}\) or \(\frac{\Lambda_4}{2R}\). For first branch \(A \rightarrow \frac{4}{3}\) the Einstein gravity is reproduced:

\[
S \rightarrow \frac{1}{\kappa_4^2} \int d^4 x \sqrt{-q} \left\{ \frac{32}{27} R^{(4)} + \frac{4}{3} \Lambda_4 \right\}.
\]

(39)

On the other hand, for second branch \(A \rightarrow \frac{\Lambda_4}{2R}\), \(\frac{1}{R}\)-gravity is reproduced:

\[
S \rightarrow \frac{1}{\kappa_4^2} \int d^4 x \sqrt{-q} \left\{ \frac{\Lambda^2}{R} \right\}.
\]

(40)
Therefore when $R$ is large, there are two branches: where gravity is the Einstein one or where $\frac{1}{R}$-gravity occurs. We may consider the limit that $R$ is small, where $A \rightarrow \pm \sqrt{\frac{\Lambda}{3R}}$. The 4d action (37) behaves as $\frac{1}{\sqrt{R}}$-gravity:

$$S \rightarrow \frac{1}{\kappa_4^2} \left( 1 \pm \frac{1}{3} \right) \Lambda \int d^4x \sqrt{-q} \sqrt{\frac{\Lambda}{R}}.$$ (41)

In the same way, changing the brane action (boundary terms in AdS/CFT) one can get other negative powers of the curvature.

Thus, there is some ground to believe that gravitational alternative to dark energy (which may be called the effective dark energy) may be produced by the modification of GR which is dictated by string/M-theory. In such case, the mysterious cosmic fluid with negative pressure does not occur in the accord with the proposal [1]. It is remarkable that the terms with positive higher derivative curvature\(^5\) like those in trace anomaly-driven inflation [16] (for a recent discussion in relation with dark energy, see [17]) may produce the early time inflation. These terms may also have string/M-theory origin. Hence, GR is modified in a different way at early and late times. It remains to be the standard GR in intermediate epoch only. The appearing new gravitational physics helps to realize both phases: early time inflation and current cosmic acceleration. We presented the arguments that such modifications of GR may be predicted by M-theory. However, the complete proof of this statement in full string/M-theory is still missing.

**Acknowledgments.** The research is supported in part by the Ministry of Education, Science, Sports and Culture of Japan under the grant n.13135208 (S.N.), DGI/SGPI (Spain) project BFM2000-0810 (S.D.O.), RFBR grant 03-01-00105 (S.D.O.) and LRSS grant 1252.2003.2 (S.D.O.).

\(^5\)The number of instabilities are predicted for original $1/R$ theory. Moreover, this theory seems non-realistic as equivalent scalar-tensor theory may not pass the simplest solar system tests. However, the addition of higher derivative terms like $R^2$ in modified gravity may help to resolve the above instabilities and to make the scalar mass very heavy by simply tuning of the coefficient of $R^2$ term [18].
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