Flavor Seesaw Mechanism

Sudip Jana,∗ Sophie Klett,† Manfred Lindner‡

Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany

Abstract

In the Standard Model, Yukawa couplings parametrize the fermion masses and mixing angles with the exception of neutrino masses. The hierarchies and apparent regularities among the quark and lepton masses are, however, otherwise a mystery. We propose a new class of models having vector-like fermions that can potentially address this problem and provide a new mechanism for fermion mass generation. The masses of the third and second generations of quarks and leptons arise at tree level via the seesaw mechanism from new physics at moderately higher scales, while loop corrections produce the masses for the first generation. This mechanism has a number of interesting and testable consequences. Among them are unavoidable flavor-violating signals at the upcoming experiments and the fact that neutrinos have naturally only Dirac masses.

∗E-mail: sudip.jana@mpi-hd.mpg.de
†E-mail: sophie.klett@mpi-hd.mpg.de
‡E-mail: lindner@mpi-hd.mpg.de
1 Introduction

The discovery of the Higgs boson in 2012 [1, 2] completes the tremendous success of the Standard Model (SM). The SM has, however, a list of theoretical and experimental problems such that it cannot be the final theory. The observed masses and mixings of quarks and leptons do not belong to this list, even though the masses span an impressive 13 orders of magnitude from the light active neutrino mass scale to the top quark mass. The observed values are readily accommodated by the SM, albeit at the cost of adding Yukawa couplings with strengths ranging from $10^{-6}$ (for the electron) to 1 (for the top quark). This is not a problem due to chiral symmetry which protects these drastically different values from big quantum corrections. The observed regularities of the masses and mixings remain, however, an unresolved mystery which may point to a mechanism beyond the SM which explains them. We present in this paper such a mechanism where new vector generations with TeV-ish masses lead to a seesaw like fermion mass matrix with tree level\textsuperscript{1} and loop contributions. Upon diagonalization the mechanism naturally produces the observed patterns without any extra ingredients.

The experimental data reveal that the quark mixing pattern differs significantly from the leptonic mixing pattern. The quark sector’s mixing angles are tiny, meaning that the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is quite close to the identity matrix. Two of the leptonic mixing angles, on the other hand, are large, and one is tiny, on the order of the Cabibbo angle, indicating a Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix that is substantially different from the identity matrix. This “flavor puzzle” is one of the features which the SM does not address. It provides motivation for investigating models with enhanced field content and expanded flavour symmetry groups to explain the existing SM fermion mass spectrum and mixing parameters.

A primary step towards a solution of the “flavor puzzle” is to comprehend the physical characteristics of the generations. Partners from various generations have universal gauge interactions, but widely differing Yukawa couplings to the Higgs field. This may imply that there is some underlying connection between gauge and Yukawa interactions while maintaining the cancellation of anomalies within a generation, which is a beautiful feature of the SM. The simplest gauge group with such qualities is widely known to be based on the difference of the baryon and lepton numbers, $U(1)_{B-L}$, provided a right-handed sterile neutrino per generation is added to cancel the anomaly. The $U(1)_{B-L}$ framework is widely studied in the literature in different contexts. In the classical framework [9–12], the $U(1)_{B-L}$ symmetry is considered to be broken at a very high scale ($\sim 10^{14}$ GeV) such that the tiny neutrino masses and mixings are generated via seesaw mechanism by

\textsuperscript{1}For tree level realization, see Refs. [3–8].
generating the lepton-number violating (LNV) Majorana mass for the sterile neutrino.
The breaking of $U(1)_{B-L}$ symmetry at low scale ($\sim$ TeV/sub-TeV) has attracted quite
a bit of interest recently [13,14]. This class is substantially different from ours in terms
of both philosophy and physics. Note also that one could identify the new abelian gauge
group as a generic $U(1)_X$. Here, the masses of the third and second generations of quarks
and leptons arise in the tree level via the seesaw mechanism, while gauge loop corrections
produce masses for the first generation (see Fig. 1). For a selection of models generating
fermion mass hierarchies from quantum loop corrections see [15–20]. In other theories, the
mechanism for generating neutrino masses and mixing are, in general, quite different than
the generation of quark and charged lepton masses. In contrast to that, our formalism
is universally applicable to the up-type quarks, down-type quarks, charged leptons and
neutrinos. It is aesthetically appealing to create such hierarchical mass pattern in a way
that allows for natural Yukawa coupling [$O(1)$] values as a result of loop suppression.
Moreover, the neutrinos in most extensions of the standard model are presumed to be
Majorana particles. The tiny neutrino masses and mixings are realized using the standard
seesaw mechanism [21–27], which generally generates an effective dimension-5 lepton
number violating operator $O_1 = \langle LLHH \rangle / \Lambda$, suppressed by the mass scale $\Lambda$ of
the heavy right-handed neutrino. However, if the lepton number is a conserved quantum
number, Majorana neutrino masses are prohibited, and the standard seesaw mechanism
does not work. It requires an alternative explanation for the smallness of the Dirac
neutrino mass. In models where the neutrino mass is zero at the tree level, it is possible
to generate a small Dirac mass as a radiative correction. Such models are available both in
the context of $SU(2) \times U(1)$ gauge theory with right-handed neutrinos and in the context
of $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories. These models [28–56], however, assume the
existence of new fermions or bosons and in most cases new discrete symmetries as well,
whose sole purpose is to provide an explanation for the small neutrino mass. We present
a model of Dirac neutrino masses which does not suffer from this unsatisfactory feature.

This paper is organized as follows: in the next section, we briefly discuss the basic
mechanism. Then we describe in detail the proposed model, its symmetries, particle spec-
trum, gauge sector, scalar potential and Yukawa interactions. Subsequently, we analyze
the resulting masses and mixings of quark and leptons and finally we discuss phenome-
logical implications before we conclude.
The Flavor Seesaw Mechanism

Let us first sketch the main ingredients of the flavor seesaw mechanism. Therefore consider the following mass matrix for the up quark sector, which has at the tree level the form:

\[
\begin{pmatrix}
0 & v_{EW} |h\rangle \\
v_S \langle h| & M_P
\end{pmatrix}.
\]

Here \(|h\rangle\) is a \(n\)-element column vector including the Yukawa couplings, \(M_P\) the explicit mass of a vector fermion, \(v_{EW}\) the electro-weak (EW) vacuum expectation value (VEV) and \(v_S\) the VEV of a suitable new scalar. A model realizing these details will be presented in the next section. For the down sector, we have a similar matrix as well. Note that the above matrix yields two nonzero and \(n-1\) zero mass-eigenvalues. If \(M_P >> v_{EW}, v_S\), which we call the seesaw limit, the nonzero eigenvalues are provided by

\[m_t \simeq a_0 \langle h | h \rangle, \ m_P \simeq M_P\]

where \(a_0 = -(v_{EW}v_S/M_P)\). The eigenvector \(|t\rangle\) corresponding to \(m_t\) is proportional to \(|h\rangle\). After including loop effects, we can write

\[M_T = \begin{pmatrix}
\delta M \\
v_S \langle \alpha | M_P
\end{pmatrix} |\alpha\rangle\]

where \(\delta M\) originates from quantum loop effects and \(|\alpha\rangle\) includes corrections. The rank of \(M_T M_T^\dagger\) or \(M_T^\dagger M_T\) which is also the rank of \(M_T\) determines the number of massive fermions in the up sector. Counting the zero eigenvalues of \(M_T\) yields its rank. We analyze the following eigenvalue equations in this regard.

\[
\delta M |x\rangle + v_{EW} |\alpha\rangle x_{n+1} = 0 \\
v_S \langle \alpha | x \rangle + M_P x_{n+1} = 0
\]

By eliminating \(x_{n+1}\), we obtain

\[(\delta M + a_0 |\alpha\rangle \langle \alpha|) |x\rangle \equiv M |x\rangle = 0\]
where we consider the combination $\delta M + a_0 |\alpha\rangle \langle \alpha|$ as $M$. If $M$ has rank $r$, then $M_T M_T^\dagger$ possesses $r + 1$ nonzero eigenvalues and $r$ generations become massive. To investigate the mass hierarchy, we use the approach of calculating $M_T$ loop by loop and determining the number of massive generations based on the rank of the mass-matrix for the light quarks and leptons, $M$. We discuss systematically scenarios with different numbers of vector-like fermion generations.

The above features are the basic mechanism which will be realized and phenomenologically analyzed in a specific model in the subsequent sections.

3 Abelian Symmetry and the Flavor Puzzle

3.1 Particle Spectrum

In order to successfully implement the flavor seesaw mechanism, we consider two additional generations of singlet vector-like (VL) fermions for each species of SM fermions. More precisely, this includes VL up- and down-type quarks $T_k, B_k$, VL charged leptons $E_k$ and VL neutral leptons $N_k$, where $k$ is the generation index of these new particles. The gauge group of the model is given by $G_{SM} \times U(1)_X$ and a successful anomaly cancellation can be achieved by the addition of three generations of right handed neutrinos $\nu_{jR}$ with $j = 1, 2, 3$. Finally, to break $U(1)_X$, a second scalar $\eta$ is required. The anomaly free charge assignments of all particles are given in Table I. We emphasize that the SM Higgs is charged under $U(1)_X$ in this scenario and the usual Higgs Yukawa couplings of the SM are forbidden. Note also that one could identify the new abelian gauge group $U(1)_X$ with $B - L$ charge.

3.2 Gauge Boson Sector

The Lagrangian of the gauge sector is given by

$$L_{\text{gauge}} = \frac{-1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu
u}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu},$$

where $G_{\mu\nu}^a, W_{\mu\nu}^i, B_{\mu\nu}$ and $X_{\mu\nu}$ denote the field strength tensors of the $SU(3)_C, SU(2)_L, U(1)_Y$ and $U(1)_X$ group respectively with $a = 1, \ldots, 8$ and $i = 1, \ldots, 3$ and we will indicate the couplings of these various gauge groups by $g_a, g, g'$ and $g_X$. Except for the VL down-type quark, all fermions are charged under $U(1)_X$ and will therefore interact with the neutral gauge boson $X$ that is associated with the new abelian gauge group. In general, the neutral current interaction involving the gauge boson $X$ is given by [57]

$$L_{\text{NC}} \supset \frac{g_X}{2} \sum_{i,j} \bar{\psi}_i \gamma_{\mu} [q_\psi P_L + q_{\bar{\psi}} P_R] \psi_j X^\mu,$$
\[
Q_{jL} = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L
\]

\[
u_{jL} = \begin{pmatrix} \nu_j \\ e_j \end{pmatrix}_L
\]

\[
\Psi_{jL} = \begin{pmatrix} \nu_j \\ \nu_j \end{pmatrix}_L
\]

\[
Q_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L
\]

\[
u_{jL} = \begin{pmatrix} \nu_j \\ e_j \end{pmatrix}_L
\]

\[
\Psi_{jL} = \begin{pmatrix} \nu_j \\ \nu_j \end{pmatrix}_L
\]

\[
T_{kL}, T_{kR} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}
\]

\[
B_{kL}, B_{kR} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}
\]

\[
N_{kL}, N_{kR} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[
E_{kL}, E_{kR} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\]

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}
\]

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}
\]

\[
\eta = \begin{pmatrix} \eta \end{pmatrix}
\]

Table I: Particle content and charge assignment under the gauge group \(G_{\text{SM}} \times U(1)_X\), where \(j = 1, 2, 3\) denotes the SM family and we introduce \(k\) generations of VL fermions. The electric charge is given by \(Q = T_3 + \frac{Y}{2}\), where \(T_3\) is the third component of weak isospin.

where \(\psi \in \{Q, \Psi, u, d, \nu, e, T, B, N, E\}\) and \(q_\psi\) denotes the respective charge under \(U(1)_X\) (see Table I). In the above equation we consider the most general case, where the couplings of \(X\) are flavor non-diagonal. We will discuss the effects of these flavor changing neutral currents (FCNCs) in more detail in Sec. 4. For now it is important to note that the generations within one fermion species are treated on an equal footing, i.e. their \(U(1)_X\) charges are family independent. Moreover, left -and right-handed fields carry the same charges.

3.3 Scalar Potential

The additional scalar \(\eta\) is a SM singlet charged under \(U(1)_X\). It couples to the SM Higgs via the scalar portal term. Thus, the Lagrangian in the scalar sector is given by

\[
L_{\text{scalar}} = (D_\mu \phi)^\dagger (D^\mu \phi) + (D_\mu \eta)^\dagger (D^\mu \eta) - V(\phi, \eta) ,
\]

\[
V(\phi, \eta) = -\mu_\phi \phi^\dagger \phi + \frac{1}{2} \lambda_\phi (\phi^\dagger \phi)^2 - \mu_\eta \eta^\dagger \eta + \frac{1}{2} \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_{\phi \eta} (\phi^\dagger \phi)(\eta^\dagger \eta) .
\]
In this notation $V(\phi, \eta)$ describes the scalar potential and the covariant derivatives take the form
\[
D_\mu \phi = \left( \partial_\mu + ig\frac{\tau^i}{2} W^i_\mu + ig' B_\mu + iq_\phi g X_\mu \right) \phi,
\]
(5)
and
\[
D_\mu \eta = \left( \partial_\mu + iq_\eta \frac{g X}{2} \right) \eta,
\]
(6)
with the $U(1)_X$ charge of the two scalars $q_\phi$ and $q_\eta$. If the scalars acquire VEVs
\[
\langle \phi \rangle = \frac{v_{EW}}{\sqrt{2}}, \quad \langle \eta \rangle = \frac{v_S}{\sqrt{2}},
\]
(7)
symmetry breaking occurs in two steps:

\[
SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\langle \phi \rangle} SU(3)_C \times SU(2)_L \times U(1)_Y
\]
\[
\xrightarrow{\langle \eta \rangle} SU(3)_C \times U(1)_{EM},
\]
(8)
where we assume $v_S \gg v_{EW}$. As a result, the gauge bosons of the broken symmetries become massive.

Since $\eta$ is a weak singlet, the electrically charged gauge bosons acquire mass solely due to the non-zero expectation value of $\phi$. Using the convention $W^{\pm}_\mu = (W^2_\mu \mp iW^1_\mu) / \sqrt{2}$, their tree level mass is given by
\[
M^2_{W^\pm} = \frac{g'^2 v_{EW}^2}{4}.
\]
(9)
We note that this result is similar to the SM case. In the neutral gauge boson sector, mixing between the SM gauge bosons and the newly added $X$ is induced by $\phi$ carrying non-zero charge under $U(1)_X$. The mass matrix spanning the neutral gauge boson space $(B, W^3, X)$ has the form
\[
\mathcal{M}^2 = \frac{1}{4} \begin{pmatrix}
g'^2 v_{EW}^2 & -gg' v_{EW}^2 & g' g_X q_\phi v_{EW}^2 \\
-gg' v_{EW}^2 & g'^2 v_{EW}^2 & -g g_X q_\phi v_{EW}^2 \\
g' g_X q_\phi v_{EW}^2 & -g g_X q_\phi v_{EW}^2 & g_X^2 \left(q_\phi^2 v_{EW}^2 + q_\eta^2 v_S^2 \right)
\end{pmatrix}.
\]
(10)
By defining the rotation angle $s_w = \sin \theta_w = g' / \sqrt{g'^2 + g'^2}$ we can transform into a more convenient basis
\[
\begin{pmatrix}
A \\
Y \\
X
\end{pmatrix} = \begin{pmatrix}
c_w & s_w & 0 \\
-s_w & c_w & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
B \\
W^3 \\
X
\end{pmatrix},
\]
(11)
where the squared mass matrix has one zero eigenvalue. This corresponds to the massless photon $A$. The submatrix mixing the two massive states $(Y, X)$ is then described by the


\[ M_{YY}^2 = \frac{v_{EW}^2}{4} \left[ g^2 c_w^2 (1 + 2 s_w^2) + g'^2 s_w^2 (1 + 2 c_w^2) \right], \]

\[ M_{XX}^2 = \frac{g_X^2 (q_s^2 v_{EW}^2 + q_t^2 v_S^2)}{4}, \]

\[ M_{YX}^2 = -\frac{g_X^2 q_t^2 (g c_w + g' s_w)}{4}. \]

Diagonalization by a further transformation

\[
\begin{pmatrix}
Z \\
Z' \\
\end{pmatrix} = \begin{pmatrix}
c_{\xi} & s_{\xi} \\
-s_{\xi} & c_{\xi} \\
\end{pmatrix} \begin{pmatrix}
Y \\
X \\
\end{pmatrix},
\]

yields the two massive eigenstates \( Z \) and \( Z' \) with masses

\[ M_{Z,Z'}^2 = \frac{1}{2} \left( M_{YY}^2 + M_{XX}^2 \mp (M_{YY}^2 - M_{XX}^2) \sqrt{1 + \tan^2 2 \xi} \right), \]

and the mixing angle defined by

\[ \tan 2 \xi = \frac{2 M_{YX}^2}{M_{YY}^2 - M_{XX}^2}. \]

We note that although \( Z - Z' \) mixing is induced in our model, the mixing angle will be suppressed by \( O(v_{EW}^2/v_S^2) \). Hence, in the limit \( v_S \gg v_{EW} \), these effects are rather small and we will neglect them in the subsequent considerations. In general, kinetic mixing may occur as well. However, we consider negligible kinetic mixing in our study.

### 3.4 Yukawa Lagrangian

Having specified the particle content and gauge interactions, it is particularly interesting to consider the explicit realization of the flavor seesaw in our model. As already noted, the charge assignments of the \( U(1)_X \) gauge group forbid Yukawa couplings between left- and right handed SM fermions via the usual Higgs mechanism. Instead, the following Yukawa couplings with the VL fermions are allowed

\[
\mathcal{L}_{Yuk} = -y_d^q \bar{Q}_{jL} \tilde{\phi} T_{kR} - y_b^q \bar{T}_{kL} \eta u_{jR} - y_c^q \bar{Q}_{jL} \phi B_{kR} - y_d^q \bar{B}_{kL} \eta^\dagger d_{jR} - y_d^\ell \bar{\Psi}_{jL} \tilde{\phi} N_{kR} - y_b^\ell \bar{N}_{kL} \eta u_{jR} - y_c^\ell \bar{Q}_{jL} \phi E_{kR} - y_d^\ell \bar{E}_{kL} \eta^\dagger e_{jR} + h.c.,
\]

where \( y_d, y_b, y_c \) and \( y_d \) denote the new Yukawa coupling matrices with superscript \( q (\ell) \) indicating the quark (lepton) sector. Besides, the VL fermions can have explicit mass terms that are given by

\[
\mathcal{L}_{\text{explicit}} = -M_T T_{kL} T_{kR} - M_B B_{kL} B_{kR} - M_N N_{kL} N_{kR} - M_E E_{kL} E_{kR} + h.c.
\]
We like to emphasize that the assigned $U(1)_X$ charges prohibit direct Majorana mass terms for neutrinos. Moreover, within our minimal particle content there is no suitable scalar field which could generate a Majorana mass term from Yukawa interactions. In fact, the LNV Weinberg operator $LLHH/\Lambda$ is protected at any loop level, due to the $U(1)_X$ gauge symmetry.

### 3.5 Generation of Quark Masses and Mixings

![Feynman diagram](image)

Figure 2: (a) Feynman diagram which contributes at 1-loop to the mass of the first generation quarks via $Z'$ boson exchange, where $i, j, m, n = 1, 2, 3$ denote the three generations of light quarks and $k$ labels the index of the VLF. (b) 1-loop contribution to the first generation masses via scalar exchange.

We start by considering the quark sector first. After spontaneous symmetry breaking, the tree level mass matrix for the fermions receives contributions from both, the VEV of $\eta$ and $\phi$. In the up-type quark sector this can be written as

$$
\mathbf{u}_L \mathbf{M}^{(0)}_u \mathbf{u}_R \equiv \begin{pmatrix} \bar{u}_{1L} & \bar{u}_{2L} & \bar{u}_{3L} & \bar{T}_{1L} & \cdots & \bar{T}_{kL} \end{pmatrix} \begin{pmatrix}
\begin{pmatrix} y^{\phi}_{a} \langle \phi \rangle \\
y^{\eta}_{a} \langle \eta \rangle \end{pmatrix} & \mathcal{M}_T \\
\end{pmatrix} \begin{pmatrix} u_{1R} \\
u_{2R} \\
u_{3R} \\
T_{1R} \\
\cdots \\
T_{kR} \\
\end{pmatrix}, \quad (18)
$$

with $y^{\phi}_{a}$ a $3 \times k$ matrix and likewise for all other Yukawa couplings. Similarly, for the
down-type quarks the mass matrix is given by

\[
\mathbf{M}_d^{(0)} = \begin{pmatrix}
d_{1L} & d_{2L} & d_{3L} & B_{1L} & \ldots & B_{kL}
\end{pmatrix}
\begin{pmatrix}
\frac{y_q}{3 \times 3} [y_d^8 \langle \phi \rangle] & \mathbf{M}_B
\end{pmatrix}
\begin{pmatrix}
d_{1R} \\
d_{2R} \\
d_{3R} \\
B_{1R} \\
\ldots \\
B_{kR}
\end{pmatrix}.
\] (19)

Provided that the column vectors of the respective Yukawa coupling matrices are linearly independent, the rank of these tree level matrices is \(2k\). Regarding the number of VL fermion generations, we categorize three different scenarios. First of all, consider the situation where the \(Z'\) couplings in Eq. (2) are flavor diagonal and thus the one-loop \(Z'\) exchange contribution that is shown in Fig. (2a) is proportional to the tensor product of Yukawa couplings, i.e., for instance, \(\sim y_d^8 (y_d^8)^T\). By introducing three generations of VL fermions, all SM generations acquire masses at tree level, which is usually referred to as the universal seesaw mechanism [3–8]. However, we find that with a proper choice of parameters the one-loop \(Z'\) exchange diagram can be the dominant source of first-generation fermion masses. Secondly, one could also think of a scenario with four generations of VL fermions, of which one is massless and does not talk to the SM generations but only to the other VL fermions due to some proper symmetry. This implies that two generations of SM fermions become massive at tree level, while the first generation masses are generated from the one-loop \(Z'\) exchange. In this work, we limit ourselves to a detailed discussion of a third case, where we consider flavor non-diagonal couplings of the \(Z'\). Two massive generations of VL fermions can then generate tree-level masses for the third and second SM generation, while the one-loop contribution of the \(Z'\) enhances the rank of the mass matrix by one, thus leading to first-generation masses. This can be understood in more detail considering the one-loop contribution of Fig. (2a) for non-diagonal \(Z'\) couplings. In the following, we denote this contribution by \(\delta M_{ij}^a = \Sigma_{ij}(p = 0)\) which is in Landau gauge given by

\[
\delta M_{ij}^a = \sum_{k=1}^{2} \sum_{m,n=1}^{3} \frac{3g^2_\chi q Q q_u [y_d^8]_{mk} [y_d^8]_{kn} v_{EW} v_S}{8} \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma_\mu (k + M_{Tk}) \gamma_\nu g^{\mu\nu}}{k^2 - (k^2 - M_{Z'}^2)((p - k)^2 - M_{Z'}^2)}
\]

\[
= \sum_{k=1}^{2} \sum_{m,n=1}^{3} \frac{3g^2_\chi q Q q_u [y_d^8]_{mk} [y_d^8]_{kn} v_{EW} v_S}{32\pi^2} \frac{M_{Tk}}{M_{Z'}^2 - M_{Tk}^2} \log \frac{M_{Z'}^2}{M_{Tk}^2},
\] (20)

where we assume \(k = 2\) from now on. We note that the one-loop contribution of this diagram is independent of the index \(i,j = 1,2,3\) as a consequence of non-diagonal \(Z'\)
couplings. Hence, the loop contribution is the same for all three generations of ordinary quarks and we can write \( \delta M_{ij}^u \equiv \delta M^u \) in the subsequent.

Using this result, the mass matrix for the up-type sector is given to one-loop order by

\[
\mathbf{u}_L \mathcal{M}_u^{(1)} \mathbf{u}_R \equiv \left( \begin{array}{cccc} u_{1L} & u_{2L} & u_{3L} & T_{1L} & T_{2L} \end{array} \right) \left( \begin{array}{c} \delta M^u \\
\frac{g_u^3 \langle \phi \rangle}{M_T} \end{array} \right) \left( \begin{array}{c} u_{1R} \\
u_{2R} \\
u_{3R} \\
T_{1R} \\
T_{2R} \end{array} \right),
\]

with \( \delta M^u \) a 3 \times 3 matrix where each entry equals \( \delta M^u \). Starting from this general matrix form, it is evident that the column vectors of the Yukawa matrix \( y_u \) need to be linearly independent from the column vectors of the loop contribution that are \( \sim (1, 1, 1)^T \). Equal statements apply to all other Yukawa couplings in order to guarantee that \( \mathcal{M}^{(1)} \) has rank five\(^2\). It should be mentioned that there is another one-loop contribution to the last column (row) of \( \mathcal{M}_u^{(1)} \), since the VL fermions themselves couple to the \( Z' \) (except for the bottom-type which carries no \( U(1)_X \) charge). However, we regard this contribution to be small compared to the tree level entries and therefore neglect it in the further calculation.

We want to emphasize that the addition of two generations of VL fermions is the most minimal setup in order to generate masses for all three generations of SM fermions. A further reduction of the number of vector generations would also reduce the rank of \( \mathcal{M}^{(1)} \) and hence lead to one generation of SM fermions being massless.

In the down-type sector, the one-loop mass matrix is similarly obtained and we quote here the result

\[
\mathbf{d}_L \mathcal{M}_d^{(1)} \mathbf{d}_R \equiv \left( \begin{array}{cccc} d_{1L} & d_{2L} & d_{3L} & B_{1L} & B_{2L} \end{array} \right) \left( \begin{array}{c} \delta M^d \\
\frac{g_d^3 \langle \phi \rangle}{M_T} \end{array} \right) \left( \begin{array}{c} d_{1R} \\
d_{2R} \\
d_{3R} \\
B_{1R} \\
B_{2R} \end{array} \right),
\]

with the obvious replacements of the loop correction in Eq. (20). For the following, we diagonalize the obtained matrices by a bi-unitary transformation, such that

\[
V_u^{(1)} \mathcal{M}_u^{(1)} (V_u^{(1)})^\dagger = \mathcal{M}_u^{\text{diag}} = \text{diag}(m_u, m_c, m_t, m_{T1}, m_{T2}),
\]

\[
V_d^{(1)} \mathcal{M}_d^{(1)} (V_d^{(1)})^\dagger = \mathcal{M}_d^{\text{diag}} = \text{diag}(m_d, m_s, m_b, m_{B1}, m_{B2}).
\]

\(^2\)There exists a second diagram (see Fig. 2b) which contributes at one-loop to the mass matrix. However we consider this diagram to be suppressed due to small scalar mixing. Furthermore its contribution is proportional to the tensor product \( \sim y_d^2 (y_u^3)^T \) and hence does not change the rank of \( \mathcal{M}_u^{(1)} \).
Hence, the fermion mass eigenstates are given by

\[ \hat{u}_{L/R} = V^u_{L/R} u_{L/R}, \]
\[ \hat{d}_{L/R} = V^d_{L/R} d_{L/R}. \]  

(24)

Evidently, there is mixing between SM quarks and their VL partners, which will effect the gauge and Yukawa interactions when transforming to the fermion mass eigenbasis.

3.6 Gauge and Yukawa Interactions

In the following section we illustrate our notation exemplary for the up-type sector. In the flavor basis, the couplings of the up-type quarks to the \( Z \) can be written as

\[ L \supset Z^\mu \left[ \hat{u}_L \gamma^\mu g^u_L(Z) u_L + \hat{u}_R \gamma^\mu g^u_R(Z) u_R \right], \]

(25)

where the coupling strength is given by

\[ g^u_L(Z) = \frac{g}{c_w} \left[ 1 - g^{u,SM}_L(Z) - \text{diag}(0, 0, 0, 1/2, 1/2) \right], \]
\[ g^u_R(Z) = \frac{g}{c_w} \left[ 1 - g^{u,SM}_R(Z) \right], \]  

(26)

with \( g^{u,SM}_L(Z) = T_3 - Q_s 2 s_w = 1/2 - 2/3 s_w^2 \) and \( g^{u,SM}_R(Z) = -Q_s 2 s_w = -2/3 s_w^2 \). If we transform to the fermion mass basis, it is evident that there will be FCNC in the left-handed sector since \( g^u_L(Z) \) is not proportional to the identity matrix and the Lagrangian becomes

\[ L \supset Z^\mu \left[ \hat{u}_L \gamma^\mu \hat{g}^u_L(Z) \hat{u}_L + \hat{u}_R \gamma^\mu \hat{g}^u_R(Z) \hat{u}_R \right], \]

(27)

with \( \hat{g}^u_L(Z) \equiv V^u_L g^u_L(Z)(V^u_L)^\dagger \) and \( \hat{g}^u_R(Z) \equiv g^u_R(Z) \).

On an equal footing, we can write down the interaction with the \( Z' \) boson in the fermion mass eigenbasis

\[ L \supset Z'^\mu \left[ \hat{u}_L \gamma^\mu \hat{g}^u_L(Z') \hat{u}_L + \hat{u}_R \gamma^\mu \hat{g}^u_R(Z') \hat{u}_R \right], \]

(28)

where

\[ \hat{g}^u_L(Z') = V_{L}^u g^u_L(Z')(V_L^u)^\dagger, \]
\[ \hat{g}^u_R(Z') = V_{R}^u g^u_R(Z')(V_R^u)^\dagger, \]  

(29)

and \( g^u_L(Z') = g^u_R(Z') \) denote the coupling strengths in the interaction basis with non-diagonal entries proportional to the \( U(1)_X \) charge of the fermion (compare Eq. (2)). Consequently, there are tree level FCNCs mediated by both, the \( Z \) and the \( Z' \) boson. However, in the case of the \( Z \) these effects are rather small since they are only induced by a small mixing between SM quarks and VL fermions.
In the mass basis, the coupling of the $W$ boson to quarks is given by

$$
\mathcal{L} \supset \frac{g}{\sqrt{2}} W^+_\mu \left[ \bar{u}_L \gamma^\mu \hat{g}_L^q (W) \dot{d}_L \right] + \text{h.c.}, \tag{30}
$$

where

$$
\hat{g}_L^q (W) = V^*_{uL} g_L^q (W) (V^*_{uL})^\dagger, \tag{31}
$$

and

$$
g_L^q (W) = \text{diag}(1, 1, 1, 0, 0). \tag{32}
$$

From these definitions we find that the mixing matrix between the generations is described by a $5 \times 5$ dimensional matrix, whose three dimensional submatrix yields the well-known CKM matrix

$$
V_{\text{CKM}} \equiv \hat{g}_L^q (W)|_{3 \times 3}. \tag{33}
$$

Finally, we consider the Yukawa couplings in the fermion mass eigenbasis. In our model fermion masses receive not only contributions from the SM Higgs, but also from the second scalar $\eta$. Therefore, the couplings of fermions to the physical Higgs boson are no longer diagonal in the mass basis and give rise to tree level FCNC. Adopting the same notation as before, the Higgs couplings in flavor basis from Eq. (16) can be rewritten as

$$
\mathcal{L} \supset -\phi^0 \bar{u}_L \hat{Y}_u u_R + \text{h.c.}, \tag{34}
$$

where the Yukawa coupling matrix is given by

$$
\hat{Y}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0_{3 \times 3} & g_u^q \\ 0_{2 \times 3} & 0_{2 \times 2} \end{pmatrix}. \tag{35}
$$

Transforming to the mass basis results in the non-diagonal coupling

$$
\mathcal{L} \supset -\phi^0 \bar{u}_L \hat{Y}_u \hat{u}_R + \text{h.c.}, \tag{36}
$$

where we defined

$$
\hat{Y}_u = V^h_L Y_u (V^h_R)^\dagger. \tag{37}
$$

We note that the scalar $\eta$ also induces FCNC. However, since its mass can be at a high scale, we do not consider these effects here.
3.7 Generation of Lepton Masses and Mixings

The generation of neutrino masses succeeds analogously to the up-type quarks. At one-loop the mass term in the Lagrangian can be written as

$$\nu_L M^{(1)}_{\nu_R} \equiv \begin{pmatrix} \nu_{1L} & \nu_{2L} & \nu_{3L} & \bar{N}_{1L} & \bar{N}_{2L} \end{pmatrix} \begin{pmatrix} \delta M^{\nu} & y_{\nu}^\ell (\phi) \langle \eta \rangle M_N \end{pmatrix} \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \\ \bar{N}_{1R} \\ \bar{N}_{2R} \end{pmatrix} \tag{38}$$

where $\delta M^{\nu}$ is again a $3 \times 3$ matrix with each entry equal to $\delta M^{\nu}$. The loop contribution in the neutrino case can be inferred from Eq. (20) by replacing the appropriate gauge charges, Yukawa couplings and VLF masses. Analogously, for the charged leptons we find

$$\bar{e}_L M^{(1)}_{e_R} e_R \equiv \begin{pmatrix} \bar{e}_{1L} & \bar{e}_{2L} & \bar{e}_{3L} & \bar{E}_{1L} & \bar{E}_{2L} \end{pmatrix} \begin{pmatrix} \delta M^{e} & y_{e}^\ell (\phi) \langle \eta \rangle M_E \end{pmatrix} \begin{pmatrix} e_{1R} \\ e_{2R} \\ e_{3R} \\ E_{1R} \\ E_{2R} \end{pmatrix} \tag{39}$$

These matrices can be diagonalized by the transformations

$$V^{\nu}_{L/R} (V^{\nu}_{L/R})^\dagger = M^{\nu}_{\text{diag}} \equiv \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_{N_1}, m_{N_2}),$$

$$V^{e}_{L/R} (V^{e}_{L/R})^\dagger = M^{e}_{\text{diag}} \equiv \text{diag}(m_e, m_\mu, m_\tau, m_{E_1}, m_{E_2}),$$

where the fermion mass eigenstates are defined by

$$\hat{\nu}_{L/R} = V^{\nu}_{L/R} \nu_{L/R};$$

$$\hat{e}_{L/R} = V^{e}_{L/R} e_{L/R}.$$ (41)

Using these conventions, the charged current interaction in the mass basis reads

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} W^-_\mu \left[ \bar{e}_{L/R} \gamma^\mu \hat{g}^f_{\nu} (W) \hat{\nu}_{L/R} \right] + h.c.,$$

where

$$\hat{g}^f_{\nu} (W) = V^{\nu}_{L/R} g^f_{\nu} (W) (V^{\nu}_{L/R})^\dagger,$$ (42)

and

$$g^f_{\nu} (W) = \text{diag}(1, 1, 1, 0, 0).$$ (43)

Following the standard convention, the PMNS matrix is then given by the $3 \times 3$ submatrix:

$$U_{\text{PMNS}} = \{ \hat{g}^f_{\nu} (W) \}_{3 \times 3}. \tag{45}$$

We note that in this scenario, the PMNS matrix is no longer unitary as assumed in the SM. For the sake of brevity, we will not explicitly show here the neutral current interactions but refer the reader to the conventions outlined in Sec. 3.6.
3.8 Numerical Results

It is worth noting that the parameters for this type of flavor model cannot be chosen at random. The parameters of our model are mapped onto SM parameters and in order to show that our model can reproduce all the observable perfectly, we give two benchmark points (BPs) and their predictions. The chosen Yukawa couplings in the quark and lepton sector are shown in Table II, where $\epsilon_1 = 10^{-2}$, $\epsilon_2 = 10^{-1}\epsilon_1^2$. For simplicity, we assume that the mass of the heavy gauge boson $M_Z'$ is 300 TeV and the related gauge coupling $g_X$ equals 1 throughout our analysis. Considering the first benchmark point we select the VL quark masses $M_{T1} = 8.00$ TeV and $M_{B1} = 40.00$ TeV while the second generation is nearly degenerate with a mass difference of 1 GeV. In the lepton sector we use $M_{N1} = 7.00 \times 10^7$ TeV and $M_{N2} = 1.00 \times 10^8$ TeV for the VL neutral leptons and $M_{E1} = 50.00$ TeV and $M_{E2} = 80.00$ TeV for charged VL leptons. For the second benchmark point we assume a larger degeneracy in the quark sector with $M_{T1} = 8.00$ TeV, $M_{T2} = 17.91$ TeV, $M_{B1} = 40.00$ TeV and $M_{B2} = 65.69$ TeV. The VL neutral lepton masses are given by $M_{N1} = 1.15 \times 10^6$ TeV and $M_{N2} = 1.25 \times 10^6$ TeV while those for the charged leptons are $M_{E1} = 50.00$ TeV and $M_{E2} = 80.00$ TeV. Table III shows that these parameter settings are in accordance with the observable fermion masses and mixings.

| Yukawa Couplings | Benchmark Points |
|------------------|------------------|
|                  | BP1          | BP2         |
| $y_{u1}^2$       | 0.625 0.513 0.631 0.522 |
|                  | 0.186 $\times e^{0.05}$ 0.159 0.183 $\times e^{0.05}$ 0.158 |
|                  | 0.401 0.327 0.412 0.344 |
| $y_{d1}^2$       | 0.332 0.229 0.335 0.227 |
|                  | 0.340 0.224 0.352 0.222 |
|                  | 0.294 $\times e^{0.05}$ 0.290 $\times e^{0.05}$ |
|                  | 1.539 1.287 1.501 1.341 |
|                  | 0.195 0.538 $\epsilon_1$ 0.205 0.600 $\epsilon_1$ |
|                  | 1.060 0.754 1.061 0.795 |
|                  | 1.110 0.332 1.129 0.331 |
|                  | 1.396 12.405 1.407 13.236 |
| $y_{e1}$         | 1.396 1.494 1.235 $\times e^{0.1}$ 0.948 $\epsilon_1$ |
|                  | 0.175 1.257 $\epsilon_1^2$ 0.195 2.283 $\epsilon_1$ |
|                  | 0.538 0.333 0.443 0.347 |
|                  | 0.319 1.048 0.101 1.210 |
| $y_{e2}$         | 0.285 0.668 $\epsilon_2$ 2.871 0.947 $\epsilon_2$ |
|                  | 0.967 0.328 0.243 1.589 |
|                  | 0.678 1.166 0.484 1.468 |
|                  | 0.854 0.574 $\epsilon_1$ 0.999 1.281 $\epsilon_1$ |
|                  | 1.474 0.820 0.617 0.809 |
| $y_{e3}$         | 1.398 0.960 1.555 0.479 $\epsilon_1$ |
|                  | 0.740 0.780 $\epsilon_1$ 1.355 1.381 $\epsilon_1$ |
|                  | 0.747 1.445 0.858 0.982 |

Table II: Yukawa coupling benchmark points for our analysis.
Furthermore, as a measure of CP-violation, we calculated the Jarlskog invariant which is defined by $\mathcal{J} \equiv \text{Im} (V_{us}V_{cb}V_{ub}^*V_{cs}^*)$.

Note that while the first benchmark point correctly reproduces a normal hierarchy (NH) for neutrino masses, the second benchmark point demonstrates that our model is also capable to describe an inverted hierarchy (IH). Using VL masses of a few tens to a few hundreds of TeV, the hierarchical mass structure of the quark sector (both up and down type) can be nicely accommodated for natural Yukawa coupling values of $\mathcal{O}(10^{-2})$-$\mathcal{O}(1)$. For the charged lepton sector VL masses in the same range as in the quark sector also imply similarly moderate Yukawa couplings. For the neutral leptons VL masses of the order $\mathcal{O}(10^7)$ TeV would imply somewhat smaller Yukawa couplings $\mathcal{O}(10^{-4})$, which is still much less hierarchical than the usual values in the SM ranging from $10^{-13}$ to 1. Note, however, that VL masses can have any value without introducing a new hierarchy problem. One could therefore choose higher values for the scale of the neutral lepton VL fermions and obtain in this way Yukawa couplings $\mathcal{O}(1)$. Choosing $\mathcal{O}(1)$ Yukawa couplings would then require a VL mass scale of $\mathcal{O}(10^{17}$ GeV) which leads via the seesaw formula to the correct neutrino masses. It is also important to keep in mind that the discussed benchmark points serve only as a proof of existence and a more detailed scan of the high dimensional parameter space is beyond the scope of this work. This implies that there could be further solutions with lower scales having more effects on a variety of BSM observable. Finally, it is important to stress that the number of parameters in our specific model does not allow to predict fermion masses or mixing angles. Instead the parameters are mapped in a way such that the observed hierarchies and mixing patterns emerge naturally. Other model realizations of the flavour seesaw mechanism may, however, be more restricted and thus be predictive. One can also imagine scenarios where symmetries among Yukawa couplings explain the observed hierarchies and mixing patterns in combination with the mapping.

4 Phenomenological Implications

4.1 FCNC Processes in Quark Sector

Due to the tree level FCNCs that are present in our model, it is important to estimate the new physics effects on observables in the neutral meson mixing systems $D^0 - \overline{D^0}$, $K^0 - \overline{K^0}$, $B_d^0 - \overline{B_d^0}$ and $B_s^0 - \overline{B_s^0}$. The dominating contribution to these processes originates from three level $\phi$ and $Z'$ exchange as shown in Fig. 3.

In the following we apply an effective operator approach where contributions from

$\overline{3}$. In principle there are also tree level FCNCs mediated by the $Z$. However, these are rather small since the $Z$ couples diagonally in the interaction basis and small non-diagonal contributions arise only via mixing with the VL fermions.
Table III: SM observable for the two benchmark points. Here $\Delta m_{34}^2 \equiv \Delta m_{32}^2$ for NH and $\Delta m_{34}^2 \equiv -\Delta m_{32}^2$ for IH. Experimental ranges [58, 59] denote 3$\sigma$ intervals except for the charged lepton masses where we demand our model to fulfill the measured value within $\pm$5%.

Heavy particles are integrated out. Then, the Hamiltonian responsible for mixing in the neutral meson system reads

$$H_{\text{eff}} = -\frac{1}{\langle H \rangle} \left( \bar{q}_i \left[ (\bar{Y}_q)_{ij} \frac{1 + \gamma_5}{2} + (\bar{Y}_q^*)_{ij} \frac{1 - \gamma_5}{2} \right] q_j \right)^2 + \frac{1}{2\langle H \rangle} \left( \bar{q}_i \gamma_\mu \left[ \left[ g^q_L (Z') \right]_{ij} \frac{1 - \gamma_5}{2} + \left[ g^q_R (Z') \right]_{ij} \frac{1 + \gamma_5}{2} \right] q_j \right)^2,$$

where $\bar{q}_i$ and $q_j$ indicate the quark fields participating in the mixing. Regarding a general meson $P$, the transition matrix element is defined by $M_{12}^P = \langle P | H_{\text{eff}} | \bar{P} \rangle$ from which we can calculate the physical observable mass splitting

$$\Delta m_P = 2 \text{Re}(M_{12}^P)$$

and in the case of the kaon the CP violation parameter $|\epsilon_K| \simeq \text{Im}(M_{12}^K)/(\sqrt{2}\Delta m_K)$. For the relevant 4-fermion operators we use the hadronic transition matrix elements [60]

$$\langle P | \bar{q}_i \gamma_\mu (1 \pm \gamma_5) q_i \bar{q}_j \gamma_\mu (1 \pm \gamma_5) q_j | \bar{P} \rangle = f_P m_P \left( \frac{1}{24} + \frac{1}{4 (m_{q_i} + m_{q_j})^2} \right) B_4,$$

$$\langle P | \bar{q}_i \gamma_\mu (1 \pm \gamma_5) q_i \bar{q}_j (1 \pm \gamma_5) q_j | \bar{P} \rangle = -\frac{5}{24} f_P m_P \frac{m_P^2}{(m_{q_i} + m_{q_j})^2} B_2,$$

$$\langle P | \bar{q}_i (1 \pm \gamma_5) q_i \bar{q}_j (1 \pm \gamma_5) q_j | \bar{P} \rangle = \frac{1}{3} m_P f_P B_1,$$

which depend on the B parameters, the meson decay constant $f_P$ and the meson mass
With this at hand, the complete transition matrix element takes the form

\[ M_{12}^P = -\frac{f_P^2 m_P}{2m_\phi^2} \left[ -\frac{5}{24} \left( m_P^2 \frac{m^2}{(m_q + m_{\bar{q}})^2} \right) \cdot B_2 \cdot \eta_2(\mu) + (\hat{Y}_q)_{ij} (\hat{Y}^*_{\bar{q}})_{ij} \left( \frac{1}{12} + \frac{1}{2} \frac{m_P^2}{(m_q + m_{\bar{q}})^2} \right) \cdot B_4 \cdot \eta_4(\mu) \right] \]

\[(48)\]

In above equation \( \eta_1, \eta_2 \) and \( \eta_4 \) are QCD correction factors for the Wilson Coefficients (WC) that account for going from the heavy mass scale to the hadronic scale \( \mu \). We explain their calculation in the following.

Generally, the effective \( \Delta F = 2 \) Hamiltonian is described by

\[ \mathcal{H}_{\text{eff}}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i, \]

(49)

where \( C_i \) are the WCs for a basis of 4-fermion operators

\[ Q_1 = \overline{q}_i L \gamma_\mu q_j L \gamma^\mu q_j L, \quad Q_2 = \overline{q}_i R q_j L \tilde{q}_{\bar{q}} R q_j L, \quad Q_3 = \overline{q}_i R q_j L \overline{q}_{\bar{q}} R q_j L, \]

\[ Q_4 = \overline{q}_i R q_j L \overline{q}_{\bar{q}} R q_j L, \quad Q_5 = \overline{q}_i R q_j L \overline{q}_{\bar{q}} R q_j L, \quad (50)\]

and \( \tilde{Q}_i \) symbolizes the corresponding operator with interchanged \( L \leftrightarrow R \).

Since the WCs are evaluated at the scale of new physics, they need to be evolved down to the hadronic energy scale for consistency. Following [62], the relation between the WC

\[ ^4 \text{We use here a Fierz rearrangement for the mixed left -and right-handed current of the } Z' \text{ (see [61]).} \]
at the heavy scale $M_H$ and the WC at the scale $\mu$ is given by
\[
C_r(\mu) = \sum_i \sum_s (b_i^{(r,s)} + \eta c_i^{(r,s)}) \eta^{\alpha_i} C_s(M_H),
\]
where $\eta = \alpha_s(M_H)/\alpha_s(m_t)$ and the coefficients $b_i^{(r,s)}$, $c_i^{(r,s)}$ and $a_i$ are referred to as magic numbers.

For the $B_d$ and $B_s$ meson we take the magic numbers from [62] and use the B-parameters $(B_1, B_2, B_4) = (0.87, 0.82, 1.16)$. The decay constants are given by $f_{B_d} = 0.240$ GeV and $f_{B_s} = 0.295$ GeV while we use $m_{B_d} = 5.281$ GeV and $m_{B_s} = 5.370$ GeV for the neutral meson masses. At the scale $M_H = m_\phi = 125.1$ GeV there are contributions from the operators $Q_2$ and $Q_4$. Using Eq. (51) with $\mu = m_b$ we find
\[
C_2(\mu) = 1.650 \cdot C_2(M_H), \quad C_3(\mu) = -0.014 \cdot C_2(M_H), \quad C_4(\mu) = 2.259 \cdot C_4(M_H), \quad C_5(\mu) = 0.056 \cdot C_4(M_H).
\]

Although the operators $Q_3$ and $Q_5$ are induced via operator mixing, their contribution is negligibly small and for the correction factors we find $\eta_2(\mu) = 1.650$ and $\eta_4(\mu) = 2.259$.

Similarly, we proceed in the case of the $Z'$. Here, the operators $Q_1$ and $Q_4$ contribute at $M_H = M_{Z'} = 300$ TeV and we obtain
\[
C_1(\mu) = 0.713 \cdot C_1(M_H), \quad C_4(\mu) = 5.446 \cdot C_4(M_H), \quad C_5(\mu) = 0.165 \cdot C_4(M_H).
\]

This yields $\eta_1(\mu) = 0.713$, $\eta_4(\mu) = 5.446$ and a negligible contribution of the induced $Q_5$.

For the $K^0 - \bar{K}^0$ system we use the magic numbers from [60], $(B_1, B_2, B_4) = (0.60, 0.66, 1.03)$, $f_K = 0.160$ GeV and $m_K = 0.498$ GeV. At the scale $\mu = 2$ GeV the WC induced by the Higgs effective operators are
\[
C_2(\mu) = 2.210 \cdot C_2(M_H), \quad C_3(\mu) = 0.003 \cdot C_2(M_H), \quad C_4(\mu) = 3.523 \cdot C_4(M_H), \quad C_5(\mu) = 0.1289 \cdot C_4(M_H),
\]

which results in $\eta_2(\mu) = 2.210$ and $\eta_4(\mu) = 3.523$. For the effective operators generated by the $Z'$ we find
\[
C_1(\mu) = 0.674 \cdot C_1(M_H), \quad C_4(\mu) = 8.181 \cdot C_4(M_H), \quad C_5(\mu) = 0.329 \cdot C_4(M_H),
\]

and hence $\eta_1(\mu) = 0.674$ and $\eta_4(\mu) = 8.181$.

Finally, for the $D$ meson the B-parameters are given by $(B_1, B_2, B_4) = (0.865, 0.82, 1.08)$. The decay constant and meson mass are given by $f_D = 0.200$ GeV and $m_D = 1.864$ GeV, respectively. Using the magic numbers from [63] to evolve the WCs induced by the Higgs down to $\mu = 2.8$ GeV we find
\[
C_2(\mu) = 1.906 \cdot C_2(M_H), \quad C_3(\mu) = -0.006 \cdot C_2(M_H), \quad C_4(\mu) = 2.903 \cdot C_4(M_H), \quad C_5(\mu) = 0.097 \cdot C_4(M_H),
\]
Thus, \( \eta_2(\mu) = 1.906 \) and \( \eta_4(\mu) = 2.903 \). For the \( Z' \) similar evaluation as before reveals

\[
C_1(\mu) = 0.690 \cdot C_1(M_H), \quad C_4(\mu) = 6.939 \cdot C_4(M_H), \quad C_5(\mu) = 0.263 \cdot C_4(M_H), \tag{57}
\]

and therefore \( \eta_1(\mu) = 0.690 \) and \( \eta_4(\mu) = 6.939 \). Evidently, in all cases the induced operators play a subdominant role and therefore we will not consider them in our analysis.

Since the model reproduces the correct fermion masses and CKM mixing angles, contributions of the usual box diagrams to the neutral meson mixing are almost unaltered compared to the SM case\(^5\). Due to chirality, the new contribution of the Higgs exchange diagram cannot interfere with the purely left-handed SM contribution. Furthermore, the leading contribution from the \( Z' \) exchange originates from the operator with mixed left- and right-handed current. Thus, we will not consider any interference effects here and write the total mass difference as

\[
\Delta m^\text{tot}_P = \Delta m^\text{SM}_P + \Delta m^\text{NP}_P \tag{58}
\]

Within the theoretical and experimental uncertainties \( \Delta m^\text{tot}_P \) should agree with the measured values \( \Delta m^\text{exp}_P \). Therefore we demand the new physics contribution to agree within \( 3\sigma \) with the difference \( \Delta m^\text{exp}_P - \Delta m^\text{SM}_P \).

The Standard Model prediction for the \( B^0 \)-meson systems are \( \Delta m^\text{SM}_{B_d} = (3.475 \pm 0.513) \times 10^{-13} \) GeV and \( \Delta m^\text{SM}_{B_s} = (1.205 \pm 0.178) \times 10^{-11} \) GeV [64], while the measured values are \( \Delta m^\text{exp}_{B_d} = (3.334 \pm 0.013) \times 10^{-13} \) GeV and \( \Delta m^\text{exp}_{B_s} = (1.169 \pm 0.001) \times 10^{-11} \) GeV [58] which leads to

\[
\begin{align*}
\Delta m^\text{exp}_{B_d} - \Delta m^\text{SM}_{B_d} &= (-0.141 \pm 0.513) \times 10^{-13} \text{ GeV}, \\
\Delta m^\text{exp}_{B_s} - \Delta m^\text{SM}_{B_s} &= (-0.036 \pm 0.178) \times 10^{-11} \text{ GeV}.
\end{align*} \tag{59}
\]

For the \( K^0 \)-meson system the theoretical prediction of the short distance contribution is \( \Delta m^\text{SM}_K = 3.074 \times 10^{-15} \) GeV [65], where we will assume a 30\% uncertainty due to unknown large distance contributions that cannot be calculated from first principles [66]. Experiments measured \( \Delta m^\text{exp}_K = (3.484 \pm 0.006) \times 10^{-15} \) GeV [58] which gives

\[
\Delta m^\text{exp}_K - \Delta m^\text{SM}_K = (0.410 \pm 0.922) \times 10^{-15} \text{ GeV}. \tag{60}
\]

Since we assumed only real Yukawa couplings in the down quark sector we do not get any additional contribution to \( |\epsilon_K| \). Finally, the mass difference for neutral D-mesons is measured to be \( \Delta m^\text{exp}_D = (6.253^{+2.699}_{-2.896}) \times 10^{-15} \) GeV [58]. However, just as it is the case for \( K^0 \)-mesons, the theoretical prediction of the Standard Model contribution to neutral

\(^5\)Although there are further diagrams with an intermediate VL fermion in the loop, these contributions are highly suppressed. The VL fermions are weak singlets and a tiny coupling to \( W^\pm \) is only induced by fermion mass mixing making these small deviations negligible.
$D$-meson mixing is subject to large uncertainties [67]. We will therefore only demand the new physics contribution to be less than the error on $\Delta m_D^{\text{exp}}$.

Our results for the two benchmark points are given in Table IV and agree with the demanded limits. The additional contribution to $B^0_d - \bar{B}^0_d$ mixing is closest to experimental limits and could therefore provide an excellent channel to observe new physics effects in upcoming experiments.

| Observable (in GeV) | Model Prediction |
|---------------------|------------------|
| $\Delta m_{B_d}^{\text{NP}}$ | $-1.402 \times 10^{-13}$ | $-1.495 \times 10^{-14}$ |
| $\Delta m_{B_s}^{\text{NP}}$ | $2.663 \times 10^{-14}$ | $3.003 \times 10^{-14}$ |
| $\Delta m_{D}^{\text{NP}}$ | $2.405 \times 10^{-15}$ | $2.036 \times 10^{-15}$ |
| $\Delta m_{K}^{\text{NP}}$ | $0.504 \times 10^{-15}$ | $0.109 \times 10^{-15}$ |

Table IV: New physics contribution to observable in neutral meson mixing for the two given benchmark points.

4.2 Charged Lepton Flavor Violation

![Diagram](image)

Figure 4: One-loop contributions to the decay $\ell_i \rightarrow \ell_j \gamma$ via (a) SM Higgs exchange and (b) $Z'$ exchange.

The flavor violating couplings of the Higgs and the $Z'$ equally impact observable in the charged lepton sector. Considering the process $\ell_i \rightarrow \ell_j \gamma$, there are two diagrams contributing at one-loop to the decay and we illustrate them in Fig. 4. The contribution from the one-loop diagram mediated by the Higgs $\phi$ is given by [68]

$$
\Gamma(\ell_i \rightarrow \ell_j \gamma) = \sum_k \frac{(m_i^2 - m_k^2)^3 (|\sigma_L|^2 + |\sigma_R|^2)}{16\pi m_i^3}
$$

(61)
with
\[
\sigma_L = \frac{iQ_k}{16\pi^2 m_\phi^2} [ (\rho m_i + \lambda m_j) F_1(t) + \nu m_k F_2(t) ]
\] (62)
\[
\sigma_R = \frac{iQ_k}{16\pi^2 m_\phi^2} [ (\lambda m_i + \rho m_j) F_1(t) + \zeta m_k F_2(t) ]
\] (63)
and we sum over all internal fermions \( \ell_k \) with electric charge \( Q_k \). The functions \( F_1(t) \) and \( F_2(t) \) are defined by
\[
F_1(t) = \frac{t^2 - 5t - 2}{12(t - 1)^3} + \frac{t \ln t}{2(t - 1)^4},
\]
\[
F_2(t) = \frac{t - 3}{2(t - 1)^2} + \frac{\ln t}{(t - 1)^3},
\] (64)
with \( t = m_k^2/m_\phi^2 \) and the couplings \( \rho = (\tilde{Y}_e)_{kj}^* (\tilde{Y}_e)_{ki} \), \( \lambda = (\tilde{Y}_e)_{jk} (\tilde{Y}_e)_{ik}^* \), \( \nu = (\tilde{Y}_e)_{kj}^* (\tilde{Y}_e)_{ik} \) and \( \zeta = (\tilde{Y}_e)_{jk} (\tilde{Y}_e)_{ki}^* \). For the contribution of the \( Z' \) exchange Eq. (61) equally applies but with the replacement
\[
\sigma_L = \frac{iQ_k}{16\pi^2 M_Z^2} \left[ (\rho m_i + \lambda m_j) F_3(t) + \nu m_k F_4(t) - \zeta m_k F_5(t) \right]
\] (65)
\[
\sigma_L = \frac{iQ_k}{16\pi^2 M_Z^2} \left[ (\lambda m_i + \rho m_j) F_3(t) + \zeta m_k F_4(t) - \nu m_k F_5(t) \right]
\] (66)
where \( t = m_k^2/M_Z^2 \), and we defined the functions
\[
F_3(t) = \frac{-5t^3 + 9t^2 - 30t + 8}{12(t - 1)^3} + \frac{3t^2 \ln t}{2(t - 1)^4},
\]
\[
F_4(t) = \frac{t^2 + t + 4}{2(t - 1)^2} - \frac{3t \ln t}{(t - 1)^3},
\] (67)
\[
F_5(t) = \frac{-2t^2 + 7t - 11}{6(t - 1)^3} + \frac{\ln t}{(t - 1)^4}.
\]
The couplings in this notation are given by \( \lambda' = [\tilde{g}_L^{t'}(Z')]_{kj}^* [\tilde{g}_L^t(Z')]_{ki} \), \( \rho' = [\tilde{g}_R^t(Z')]_{kj}^* [\tilde{g}_R^t(Z')]_{ki} \), \( \zeta' = [\tilde{g}_L^{t'}(Z')]_{kj} [\tilde{g}_R^{t'}(Z')]_{ki}^* \) and \( \nu' = [\tilde{g}_R^{t'}(Z')]_{kj} [\tilde{g}_L^{t'}(Z')]_{ki}^* \).

In addition, the flavor non-diagonal couplings of the \( Z' \) and \( \phi \) give rise to tree level contributions to the three lepton decay \( \ell_i \to \bar{\ell}_j \ell_k \ell_l \). The partial decay width mediated by \( \phi \) exchange is given by
\[
\Gamma(\ell_i \to \bar{\ell}_j \ell_k \ell_l) = \frac{1}{1536\pi^3} \frac{m_\phi^5}{M_Z^2} S |(\tilde{Y}_e)_{ij}^* (\tilde{Y}_e)_{kl}|^2,
\] (68)
with a factor \( S = 1 \) for two equally charged fermions in the final state \( (k = l) \) and \( S = 2 \) for \( k \neq l \). The contribution from tree level \( Z' \) exchange (see Fig. 5) can be written like
\[
\Gamma(\ell_i \to \bar{\ell}_j \ell_k \ell_l) = \frac{1}{1536\pi^3} \frac{m_\phi^5}{M_Z^2} S \left[ \frac{2}{S} |C_{LL}|^2 + \frac{2}{S} |C_{RR}|^2 + |C_{RL}|^2 + |C_{LR}|^2 \right]_{ij}^{kl}
\] (69)
where we use the abbreviation

\[ [C_{XY}]_{kl}^{ij} = \hat{g}^X_{(Z')ij}[\hat{g}^X_{(Z')}ij]_{kl}, \]

(70)

with \( X, Y \in \{L, R\} \) and \( \hat{g}^X(Z') \) denoting the coupling matrix of the \( Z' \) to charged leptons with chirality \( X \) [69]. The additional factor two for the coefficients \( C_{LL} \) and \( C_{RR} \) arises due to the fact that diagrams which differ by the replacement of two identical leptons can interfere with each other if the two currents are of same chirality. From our analysis we find that the \( Z' \) contribution to this process is much larger than that of the Higgs. The reason behind this is that the Higgs boson has not only suppressed flavor non-diagonal couplings but also small flavor diagonal couplings to first and second generation leptons.

From the total decay widths of the muon and tau lepton, \( \Gamma_\mu^{tot} = 3.00 \times 10^{-19} \text{GeV} \) and \( \Gamma_\tau^{tot} = 2.27 \times 10^{-12} \text{GeV} \) [58], the branching ratios for the decays \( \mu^- \rightarrow e^-\gamma, \tau^- \rightarrow e^-\gamma, \tau^- \rightarrow \mu^-\gamma, \tau^- \rightarrow \mu^-e^+e^-, \tau^- \rightarrow e^-e^+e^- \) and \( \mu^- \rightarrow e^-e^+e^- \) are calculated for the two benchmark points and can be seen in Table V. In this scenario, all expected branching ratios are below the current experimental limits. However, the experimental sensitivity for the process \( \mu^- \rightarrow e^-\gamma \) is not far from our model prediction and may be tested in future experiments. Finally, we emphasize the interesting fact that these flavor violating observable are approximately equal for each generation.

### 4.3 Other Implications

The presence of a heavy neutral gauge boson and heavy vector-like fermions in our framework will have a plethora of implications at collider experiments. Throughout our analysis, we choose \( Z' \) mass scale of \( \mathcal{O}(100) \) TeV, although the vector-like fermions might exist at a much lower scale (a few TeV), accessible to the future collider experiments. Note that the mass scales of these new heavy particles can be accommodated at a much lower scale with proper adjustment of benchmark parameters and which is beyond the scope of this study. However, we will briefly mention the collider implications and testable consequences on

![Figure 5: Tree level contribution of \( Z' \) mediated tri-lepton decay.](image-url)
| Process                                  | Experimental Limit | Model Prediction |
|------------------------------------------|--------------------|------------------|
| $\text{BR}(\mu^- \to e^- \gamma)$      | $< 4.2 \times 10^{-13}$ | $1.8 \times 10^{-14}$ | $6.3 \times 10^{-15}$ |
| $\text{BR}(\tau^- \to e^- \gamma)$     | $< 3.3 \times 10^{-8}$  | $2.2 \times 10^{-14}$ | $2.2 \times 10^{-14}$ |
| $\text{BR}(\tau^- \to \mu^- \gamma)$  | $< 4.4 \times 10^{-8}$  | $6.8 \times 10^{-15}$ | $3.0 \times 10^{-15}$ |
| $\text{BR}(\mu^- \to e^-e^+e^-)$       | $< 1.0 \times 10^{-12}$ | $1.3 \times 10^{-18}$ | $3.9 \times 10^{-19}$ |
| $\text{BR}(\tau^- \to e^-e^+e^-)$      | $< 2.7 \times 10^{-8}$  | $2.2 \times 10^{-17}$ | $1.8 \times 10^{-17}$ |
| $\text{BR}(\tau^- \to \mu^-e^+e^-)$    | $< 1.8 \times 10^{-8}$  | $9.5 \times 10^{-18}$ | $4.1 \times 10^{-18}$ |

Table V: Predicted branching ratios for certain LFV processes and current experimental limits [58].

flavor anomalies for the low-scale realization. After being pair produced through the $s-$channel $Z/\gamma$ exchange at the $pp$ collider, singly charged vector-like leptons will lead to $2l+\not{E}_T$, $4l+\not{E}_T$ signatures and vector-like quarks will lead to promising $jj+4l$, $b\bar{b}+4l$, $t\bar{t}+4l$ signatures at the LHC. There has been an extensive study of vector-like lepton searches at future collider experiments (such as HL-LHC, HE-LHC, and FCC-hh), and it has been shown that a $SU(2)_L$ singlet vector-like lepton can be probed up to a mass of $\sim 3$ TeV at 100 TeV future collider looking at multi-leptons, and $\not{E}_T$ final state signature [70]. Moreover, Ref. [71] has studied the discovery prospects of vector-like quarks looking at multi-lepton final state signatures (in association with jets or $t/b$ quarks) at future collider experiments, and a mass up to $\sim 7$ TeV vector-like quark can be probed at 100 TeV collider. Test of these vector-like families will be a good test for our model at future collider experiments. It is worth noting that there has been an increase in attention since there are various flavor anomalies in the muon sectors, such as muon $g - 2$ [72], $R^{(e)}_K$ [73–76] anomalies. We find that it is very difficult to address the $R^{(e)}_K$ [73–76] anomalies at tree level in our framework since it suffers from strong flavor constraints from $B_S - \bar{B}_S$ mixing. However, it is worth noting that muon $g - 2$ anomaly can be easily addressed with correct strength and sign in our framework due to chiral enhancement from vector-like family for heavy vector-like fermions inside the loop. However, it is worth mentioning that it suffers from fine-tuning in the absence of additional symmetries since this results in substantial muon mass corrections at loop level. We should note that the muon $g - 2$ contributions from the two BPs are three orders of magnitude lower than the experimental finding and muon mass correction is under control. Although the mass of the neutral heavy gauge boson $Z'$ in our theory is on the order of $O(100)$ TeV, complementary probe from two

<ref>[77]</ref>, it was demonstrated in a different framework that the muon $g - 2$ anomaly can be addressed with a $45$ TeV of vector-like lepton within the loop.
body scattering might be found in a future muon collider experiment. It has recently been demonstrated that a 100 (400) TeV $Z'$ scenario can be probed at a future muon collider experiment with a center of mass energy of $\sqrt{s} = 3$ TeV and an integrated luminosity of $\mathcal{L} = 1$ ab$^{-1}$ looking at $\mu^+\mu^- \rightarrow \mu^+\mu^-$ signature while considering the strength of the gauge coupling $g_X = 1 (\sqrt{4\pi})$ [78, 79]. Note that there will be additional contributions to the Higgs observable as well. However, these contributions will be negligible due to new physics at moderately higher scales as mentioned in the BPs. For example, our framework leads to SM Higgs mediated flavor violating signals like $h \rightarrow e\tau, \mu\tau$ which can be a clear sign of new physics. We analyze and find that the strength of these Higgs-mediated flavor violating signals is many orders of magnitude lower than the present experimental sensitivity.

5 Conclusions

We propose a flavor seesaw mechanism where large hierarchies and patterns of mixings of quarks and leptons arise naturally. The mechanism requires two additional vector generations of fermions with TeV-ish masses, which lead to a seesaw-like fermion mass matrices with tree level and loop contributions. The masses of the third and second generations of quarks and leptons arises at tree level, while gauge boson mediated loop corrections generate the masses of the first generation. We realize the flavor seesaw mechanism in a model where the SM is extended by an extra $U(1)_X$ symmetry. Our fermion mass generation mechanism is universally applicable to the up-type quarks, down-type quarks, charged leptons, and neutrinos.

Counting parameters one can see that the model does not predict masses. Instead it maps the observed hierarchies and patterns of SM fermion masses onto the extended theory with Yukawa couplings $O(1)$. An impressive 13 orders of magnitude of the SM from the light active neutrino mass scale to the top quark mass becomes thus much simpler. Note that the flavor seesaw mechanism does not require any additional ingredient such as additional discrete flavor symmetries or multiple scalars. The mechanism automatically creates the exhibited patterns after diagonalization and it is therefore quite appealing to explain the hierarchical mass pattern and mixing regularities of the SM in such a manner. For our model we explicitly show how the mapping works numerically for $M_Z' = 300$ TeV.

The model has a plethora of implications and testable effects. Among these, the most promising one is the inescapable and complementary flavor-violating signal in upcoming experiments. We have analyzed all these flavor violating signals in the quark as well as lepton sector and find that $l_i \rightarrow l_j\gamma$ process and $B^0_d - \overline{B^0_d}$ mixing is closest to current experimental limits and could therefore provide an excellent signal to observe new physics
effects in upcoming experiments. The presence of heavy vector-like fermions at a few TeV scale in our model will result in promising multi-lepton signals associated with jets or missing energy, which can be probed in future collider experiments like FCC-hh. Note that the discussed benchmark point does not imply that other solutions with lower $M'_Z$ could not exist. Such solutions would lead to further phenomenological consequences and their inter-dependence could be an interesting method to test the model further. For instance, due to chiral enhancement inside the loop, a relatively light $Z'$ can potentially address the observed discrepancy in muon $g-2$ measurement. Another interesting feature is that lepton number is conserved such that neutrinos have naturally only Dirac masses.

The seesaw flavor mechanism can be implemented in different ways. We demonstrated the mechanism for an extra $U(1)_X$ symmetry, but other solutions should exist. A generalization of the flavor seesaw mechanism to non-abelian theories seems also possible and embeddings into GUT groups appear also possible. This will be studied in a forthcoming paper.

References

[1] ATLAS Collaboration, G. Aad et al., “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B 716 (2012) 1–29, arXiv:1207.7214 [hep-ex].

[2] CMS Collaboration, S. Chatrchyan et al., “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” Phys. Lett. B 716 (2012) 30–61, arXiv:1207.7235 [hep-ex].

[3] Z. G. Berezhiani, “The Weak Mixing Angles in Gauge Models with Horizontal Symmetry: A New Approach to Quark and Lepton Masses,” Phys. Lett. B 129 (1983) 99–102.

[4] D. Chang and R. N. Mohapatra, “Small and Calculable Dirac Neutrino Mass,” Phys. Rev. Lett. 58 (1987) 1600.

[5] A. Davidson and K. C. Wali, “Universal Seesaw Mechanism?,” Phys. Rev. Lett. 59 (1987) 393.

[6] S. Rajpoot, “See-saw masses for quarks and leptons in an ambidextrous electroweak interaction model,” Mod. Phys. Lett. A 2 no. 5, (1987) 307–315. [Erratum: Mod.Phys.Lett.A 2, 541 (1987)].

[7] K. S. Babu and R. N. Mohapatra, “CP Violation in Seesaw Models of Quark Masses,” Phys. Rev. Lett. 62 (1989) 1079.
[8] K. S. Babu and R. N. Mohapatra, “A Solution to the Strong CP Problem Without an Axion,” Phys. Rev. D 41 (1990) 1286.

[9] J. C. Pati and A. Salam, “Lepton Number as the Fourth Color,” Phys. Rev. D 10 (1974) 275–289. [Erratum: Phys.Rev.D 11, 703–703 (1975)].

[10] R. E. Marshak and R. N. Mohapatra, “Quark - Lepton Symmetry and B-L as the U(1) Generator of the Electroweak Symmetry Group,” Phys. Lett. B 91 (1980) 222–224.

[11] F. Wilczek and A. Zee, “Conservation or Violation of $B^{-1}$ in Proton Decay,” Phys. Lett. B 88 (1979) 311–314.

[12] R. N. Mohapatra and R. E. Marshak, “Local B-L Symmetry of Electroweak Interactions, Majorana Neutrinos and Neutron Oscillations,” Phys. Rev. Lett. 44 (1980) 1316–1319. [Erratum: Phys.Rev.Lett. 44, 1643 (1980)].

[13] A. E. Nelson and J. Walsh, “Short Baseline Neutrino Oscillations and a New Light Gauge Boson,” Phys. Rev. D 77 (2008) 033001, arXiv:0711.1363 [hep-ph].

[14] K. S. Babu, A. Friedland, P. A. N. Machado, and I. Mocioiu, “Flavor Gauge Models Below the Fermi Scale,” JHEP 12 (2017) 096, arXiv:1705.01822 [hep-ph].

[15] B. S. Balakrishna, “Fermion Mass Hierarchy From Radiative Corrections,” Phys. Rev. Lett. 60 (1988) 1602.

[16] K. S. Babu and X.-G. He, “Fermion Mass Hierarchy and the Strong CP Problem,” Phys. Lett. B 219 (1989) 342–346.

[17] B. A. Dobrescu and P. J. Fox, “Quark and lepton masses from top loops,” JHEP 08 (2008) 100, arXiv:0805.0822 [hep-ph].

[18] S. Weinberg, “Models of Lepton and Quark Masses,” Phys. Rev. D 101 no. 3, (2020) 035020, arXiv:2001.06582 [hep-th].

[19] P. W. Graham and S. Rajendran, “A Domino Theory of Flavor,” Phys. Rev. D 81 (2010) 033002, arXiv:0906.4657 [hep-ph].

[20] E. Ma, “Syndetic Model of Fundamental Interactions,” Phys. Lett. B 741 (2015) 202–204, arXiv:1411.6679 [hep-ph].

[21] P. Minkowski, “$\mu \rightarrow e\gamma$ at a Rate of One Out of $10^9$ Muon Decays?,” Phys. Lett. B 67 (1977) 421–428.
[22] T. Yanagida, “Horizontal gauge symmetry and masses of neutrinos,” *Conf. Proc. C 7902131* (1979) 95–99.

[23] M. Gell-Mann, P. Ramond, and R. Slansky, “Complex Spinors and Unified Theories,” *Conf. Proc. C 790927* (1979) 315–321, arXiv:1306.4669 [hep-th].

[24] R. N. Mohapatra and G. Senjanovic, “Neutrino Mass and Spontaneous Parity Nonconservation,” *Phys. Rev. Lett. 44* (1980) 912.

[25] J. Schechter and J. W. F. Valle, “Neutrino Masses in SU(2) x U(1) Theories,” *Phys. Rev. D 22* (1980) 2227.

[26] J. Schechter and J. W. F. Valle, “Neutrino Decay and Spontaneous Violation of Lepton Number,” *Phys. Rev. D 25* (1982) 774.

[27] R. Foot, H. Lew, X. G. He, and G. C. Joshi, “Seesaw Neutrino Masses Induced by a Triplet of Leptons,” *Z. Phys. C 44* (1989) 441.

[28] R. N. Mohapatra, “A Model for Dirac Neutrino Masses and Mixings,” *Phys. Lett. B 198* (1987) 69–72.

[29] R. N. Mohapatra, “Left-right Symmetry and Finite One Loop Dirac Neutrino Mass,” *Phys. Lett. B 201* (1988) 517–524.

[30] B. S. Balakrishna and R. N. Mohapatra, “Radiative Fermion Masses From New Physics at Tev Scale,” *Phys. Lett. B 216* (1989) 349–352.

[31] G. C. Branco and G. Senjanovic, “The Question of Neutrino Mass,” *Phys. Rev. D 18* (1978) 1621.

[32] K. S. Babu and X. G. He, “DIRAC NEUTRINO MASSES AS TWO LOOP RADIATIVE CORRECTIONS,” *Mod. Phys. Lett. A 4* (1989) 61.

[33] P.-H. Gu and U. Sarkar, “Radiative Neutrino Mass, Dark Matter and Leptogenesis,” *Phys. Rev. D 77* (2008) 105031, arXiv:0712.2933 [hep-ph].

[34] Y. Farzan and E. Ma, “Dirac neutrino mass generation from dark matter,” *Phys. Rev. D 86* (2012) 033007, arXiv:1204.4890 [hep-ph].

[35] H. Okada, “Two loop Induced Dirac Neutrino Model and Dark Matters with Global U(1)’ Symmetry,” arXiv:1404.0280 [hep-ph].

[36] C. Bonilla, E. Ma, E. Peinado, and J. W. F. Valle, “Two-loop Dirac neutrino mass and WIMP dark matter,” *Phys. Lett. B 762* (2016) 214–218, arXiv:1607.03931 [hep-ph].
[37] W. Wang and Z.-L. Han, “Naturally Small Dirac Neutrino Mass with Intermediate $SU(2)_L$ Multiplet Fields,” *JHEP* **04** (2017) 166, arXiv:1611.03240 [hep-ph].

[38] E. Ma and U. Sarkar, “Radiative Left-Right Dirac Neutrino Mass,” *Phys. Lett. B* **776** (2018) 54–57, arXiv:1707.07698 [hep-ph].

[39] W. Wang, R. Wang, Z.-L. Han, and J.-Z. Han, “The $B - L$ Scotogenic Models for Dirac Neutrino Masses,” *Eur. Phys. J. C* **77** no. 12, (2017) 889, arXiv:1705.00414 [hep-ph].

[40] J. C. Helo, M. Hirsch, and T. Ota, “Proton decay and light sterile neutrinos,” *JHEP* **06** (2018) 047, arXiv:1803.00035 [hep-ph].

[41] M. Reig, D. Restrepo, J. W. F. Valle, and O. Zapata, “Bound-state dark matter and Dirac neutrino masses,” *Phys. Rev. D* **97** no. 11, (2018) 115032, arXiv:1803.08528 [hep-ph].

[42] Z.-L. Han and W. Wang, “$Z'$ Portal Dark Matter in $B – L$ Scotogenic Dirac Model,” *Eur. Phys. J. C* **78** no. 10, (2018) 839, arXiv:1805.02025 [hep-ph].

[43] S. K. Kang and O. Popov, “Radiative neutrino mass via fermion kinetic mixing,” *Phys. Rev. D* **98** no. 11, (2018) 115025, arXiv:1807.07988 [hep-ph].

[44] C. Bonilla, S. Centelles-Chuliá, R. Cepedello, E. Peinado, and R. Srivastava, “Dark matter stability and Dirac neutrinos using only Standard Model symmetries,” *Phys. Rev. D* **101** no. 3, (2020) 033011, arXiv:1812.01599 [hep-ph].

[45] J. Calle, D. Restrepo, C. E. Yaguna, and O. Zapata, “Minimal radiative Dirac neutrino mass models,” *Phys. Rev. D* **99** no. 7, (2019) 075008, arXiv:1812.05523 [hep-ph].

[46] C. D. R. Carvajal and O. Zapata, “One-loop Dirac neutrino mass and mixed axion-WIMP dark matter,” *Phys. Rev. D* **99** no. 7, (2019) 075009, arXiv:1812.06364 [hep-ph].

[47] E. Ma, “Scotogenic $U(1)_X$ Dirac neutrinos,” *Phys. Lett. B* **793** (2019) 411–414, arXiv:1901.09091 [hep-ph].

[48] P. D. Bolton, F. F. Deppisch, C. Hati, S. Patra, and U. Sarkar, “Alternative formulation of left-right symmetry with $B - L$ conservation and purely Dirac neutrinos,” *Phys. Rev. D* **100** no. 3, (2019) 035013, arXiv:1902.05802 [hep-ph].

[49] S. Saad, “Simplest Radiative Dirac Neutrino Mass Models,” *Nucl. Phys. B* **943** (2019) 114636, arXiv:1902.07259 [hep-ph].
[50] R. Srivastava, C. Bonilla, and E. Peinado, “The role of residual symmetries in dark matter stability and the neutrino nature,” *LHEP* 2 no. 1, (2019) 124, arXiv:1903.01477 [hep-ph].

[51] A. Dasgupta, S. K. Kang, and O. Popov, “Radiative Dirac neutrino mass, neutrinoless quadruple beta decay, and dark matter in B-L extension of the standard model,” *Phys. Rev. D* 100 no. 7, (2019) 075030, arXiv:1903.12558 [hep-ph].

[52] S. Jana, P. K. Vishnu, and S. Saad, “Minimal dirac neutrino mass models from U(1)_{R} gauge symmetry and left–right asymmetry at colliders,” *Eur. Phys. J. C* 79 no. 11, (2019) 916, arXiv:1904.07407 [hep-ph].

[53] K. Enomoto, S. Kanemura, K. Sakurai, and H. Sugiyama, “New model for radiatively generated Dirac neutrino masses and lepton flavor violating decays of the Higgs boson,” *Phys. Rev. D* 100 no. 1, (2019) 015044, arXiv:1904.07039 [hep-ph].

[54] E. Ma, “Two-loop Z_{4} Dirac neutrino masses and mixing, with self-interacting dark matter,” *Nucl. Phys. B* 946 (2019) 114725, arXiv:1907.04665 [hep-ph].

[55] S. Jana, P. K. Vishnu, and S. Saad, “Minimal realizations of Dirac neutrino mass from generic one-loop and two-loop topologies at d = 5,” *JCAP* 04 (2020) 018, arXiv:1910.09537 [hep-ph].

[56] D. Restrepo, A. Rivera, and W. Tangarife, “Singlet-Doublet Dirac Dark Matter and Neutrino Masses,” *Phys. Rev. D* 100 no. 3, (2019) 035029, arXiv:1906.09685 [hep-ph].

[57] P. Langacker and M. Plumacher, “Flavor changing effects in theories with a heavy Z’ boson with family nonuniversal couplings,” *Phys. Rev. D* 62 (2000) 013006, arXiv:hep-ph/0001204.

[58] **Particle Data Group** Collaboration, P. A. Zyla et al., “Review of Particle Physics,” *PTEP* 2020 no. 8, (2020) 083C01.

[59] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, “The fate of hints: updated global analysis of three-flavor neutrino oscillations,” *JHEP* 09 (2020) 178, arXiv:2007.14792 [hep-ph].

[60] M. Ciuchini et al., “Delta M(K) and epsilon(K) in SUSY at the next-to-leading order,” *JHEP* 10 (1998) 008, arXiv:hep-ph/9808328.
[61] P. Arnan, A. Crivellin, M. Fedele, and F. Mescia, “Generic Loop Effects of New Scalars and Fermions in \( b \to s \ell^+ \ell^- \), \( (g - 2)_\mu \) and a Vector-like 4th Generation,” *JHEP* **06** (2019) 118, arXiv:1904.05890 [hep-ph].

[62] D. Becirevic, M. Ciuchini, E. Franco, V. Gimenez, G. Martinelli, A. Masiero, M. Papinutto, J. Reyes, and L. Silvestrini, “\( B_d - \bar{B}_d \) mixing and the \( B_d \to J/\psi K_s \) asymmetry in general SUSY models,” *Nucl. Phys. B* **634** (2002) 105–119, arXiv:hep-ph/0112303.

[63] UTfit Collaboration, M. Bona *et al.*, “Model-independent constraints on \( \Delta F = 2 \) operators and the scale of new physics,” *JHEP* **03** (2008) 049, arXiv:0707.0636 [hep-ph].

[64] M. Artuso, G. Borissov, and A. Lenz, “\( \varepsilon'/\varepsilon \) in 331 Models,” *Rev. Mod. Phys.* **88** no. 4, (2016) 045002, arXiv:1511.09466 [hep-ph]. [Addendum: Rev.Mod.Phys. 91, 049901 (2019)].

[65] A. J. Buras and F. De Fazio, “\( \varepsilon'/\varepsilon \) in 331 Models,” *JHEP* **03** (2016) 010, arXiv:1512.02869 [hep-ph].

[66] D. Nguyen Tuan, T. Inami, and H. Do Thi, “Physical constraints derived from FCNC in the 3-3-1-1 model,” *Eur. Phys. J. C* **81** no. 9, (2021) 813, arXiv:2009.09698 [hep-ph].

[67] E. Golowich, J. Hewett, S. Pakvasa, and A. A. Petrov, “Implications of \( D^0 - \bar{D}^0 \) Mixing for New Physics,” *Phys. Rev. D* **76** (2007) 095009, arXiv:0705.3650 [hep-ph].

[68] L. Lavoura, “General formulae for \( f(1) \leftrightarrow f(2) \) gamma,” *Eur. Phys. J. C* **29** (2003) 191–195, arXiv:hep-ph/0302221.

[69] J. Aebischer, A. J. Buras, M. Cerdà-Sevilla, and F. De Fazio, “Quark-lepton connections in \( Z' \) mediated FCNC processes: gauge anomaly cancellations at work,” *JHEP* **02** (2020) 183, arXiv:1912.09308 [hep-ph].

[70] P. N. Bhattiprolu and S. P. Martin, “Prospects for vectorlike leptons at future proton-proton colliders,” *Phys. Rev. D* **100** no. 1, (2019) 015033, arXiv:1905.00498 [hep-ph].

[71] M. Chala, R. Gröber, and M. Spannowsky, “Searches for vector-like quarks at future colliders and implications for composite Higgs models with dark matter,” *JHEP* **03** (2018) 040, arXiv:1801.06537 [hep-ph].
[72] Muon g-2 Collaboration, B. Abi et al., “Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm,” Phys. Rev. Lett. 126 no. 14, (2021) 141801, arXiv:2104.03281 [hep-ex].

[73] LHCb Collaboration, R. Aaij et al., “Test of lepton universality in beauty-quark decays,” arXiv:2103.11769 [hep-ex].

[74] LHCb Collaboration, R. Aaij et al., “Test of lepton universality with $B^0 \to K^{*0} \ell^+\ell^-$ decays,” JHEP 08 (2017) 055, arXiv:1705.05802 [hep-ex].

[75] LHCb Collaboration, R. Aaij et al., “Search for lepton-universality violation in $B^+ \to K^+ \ell^+\ell^-$ decays,” Phys. Rev. Lett. 122 no. 19, (2019) 191801, arXiv:1903.09252 [hep-ex].

[76] Belle Collaboration, A. Abdesselam et al., “Measurement of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ with a semileptonic tagging method,” arXiv:1904.08794 [hep-ex].

[77] R. Dermisek, K. Hermanek, N. McGinnis, and N. McGinnis, “Highly Enhanced Contributions of Heavy Higgs Bosons and New Leptons to Muon $g-2$ and Prospects at Future Colliders,” Phys. Rev. Lett. 126 no. 19, (2021) 191801, arXiv:2011.11812 [hep-ph].

[78] G.-y. Huang, S. Jana, F. S. Queiroz, and W. Rodejohann, “Probing the $R_{K^{(*)}}$ Anomaly at a Muon Collider,” arXiv:2103.01617 [hep-ph].

[79] G.-y. Huang, F. S. Queiroz, and W. Rodejohann, “Gauged $L_\mu - L_\tau$ at a muon collider,” Phys. Rev. D 103 no. 9, (2021) 095005, arXiv:2101.04956 [hep-ph].