In this paper we present a model for accelerated expansion of the universe, both during inflation and the present stage of the expansion, from four dimensional $N = 1$ supergravity. We evaluate the tensor-to-scalar ratio ($r \approx 0.00034$), the scalar spectral index ($n_s \approx 0.970$) and the running spetral index ($dn_s/dk \approx -6 \times 10^{-5}$), and we notice that these parameters are in agreement with Planck+WP+lensing data and with BICEP2/Keck and Planck joint analysis, at 95% CL. The number of e-folds is 50 or higher. The reheating period has an associated temperature $T_R \sim 10^{12}$ GeV, which agrees with the one required by thermal leptogenesis. Regarding the scalar field as dark energy, the autonomous system for it in the presence of a barotropic fluid provides a stable fixed point that leads to a late-time accelerated expansion of the universe, with an equation of state that mimics the cosmological constant ($w_\Lambda \approx -0.997$).

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I. INTRODUCTION

Although our universe is composed mainly of ordinary matter ($\sim 5\%$), dark matter ($\sim 27\%$) and dark energy ($\sim 68\%$) $^1$, the nature of the dark sector is still unknown. Indeed, this is one of the biggest challenges in the modern cosmology. Several models of dark energy and dark matter have been proposed (see $^2$-$^8$ and references therein), and concerning the observational evidence for accelerated expansion of the universe $^9$, $^{10}$, the simplest dark energy candidate is the cosmological constant. Its equation of state $w_\Lambda = \rho_\Lambda / p_\Lambda = -1$ is in agreement with the Planck results $^1$.

However, in this scenario occurs perhaps one of the highest discrepancies between the theoretical prediction and the observed data, a discrepancy of 120 orders of magnitude. This is the so-called cosmological constant problem and such a huge disparity motivates physicists to look into more sofisticated models. This can be done either looking for a deeper understanding of where the cosmological constant comes from, if one wants to derive it from first principles, or considering other possibilities for accelerated expansion. In the former case, an attempt is the famous KKLT model $^{11}$, and in the latter one, possibilities are even broader, with modifications of General Relativity, additional matter fields and so on (see $^2$ $^{12}$ $^{13}$ and references therein).

Among this wide range of possibilities, a scalar field is a viable candidate to be used. Indeed, it was also one of the first models of inflation $^{14}$ $^{15}$, which, in turn, solved the flatness and horizon problem in the early universe. Current results from Planck $^{16}$ and BICEP2/Keck and Planck $^{17}$ provide crucial information to rule out models of inflation and to constrain inflationary parameters. Regarding also the end of inflation, the reheating due to the scalar field decay may be constrained with the temperature of the thermal leptogenesis, which requires $T_R \gtrsim 10^8 - 10^{10}$ GeV $^{18}$ $^{19}$.

Since supergravity is the low-energy limit of the superstring theory, it is natural to investigate if it can furnish a model that describes the accelerated expansion of the universe. The canonical scalar field can be either the inflaton $^{20}$ $^{22}$ (and references therein) or dark energy $^{23}$ $^{24}$. Supergravity with four supercharges exists in four dimensions at most (that is, $N \times 2^{D/2} = 4$ for $N = 1$ in $D = 4$), so that in higher dimensions one needs more supersymmetries. However, four dimensional $N = 1$ supergravity can be seen as an effective theory in four dimensions, and can be applied to cosmology at least as a first approximation or a toy model$^1$.

The scalar potential in $N = 1$ supergravity depends on a real function $K = K(\phi^i, \phi^* i)$, called Kähler potential, and a holomorphic function $W = W(\phi^i)$, the superpotential. It is given by

$$ V = e^K [g^{ij*}(D_i W)(D_j W)^* - 3WW^*], \quad (1) $$

with

$$ D_i W = W_i + K_i W, \quad (2) $$

where lower index means derivative with respect to $\phi^i$, and $g^{ij*}$ is the inverse of the Kähler metric $g_{ij*} = K_{ij*}$. In principle, both $K$ and $W$ are free functions that can be chosen to lead particularly interesting forms of the potential. However, from a more fundamental point of view they are supposed to be provided by the full superstring theory. Thus, the choices we have made for these two

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$^1$ For some models of extended supergravities in cosmology, see $^{23}$ $^{24}$. 
functions have a string-inspired form, as we will see in the next section.

In this work, we present a model for accelerated expansion of the universe from four dimensional $\mathcal{N} = 1$ supergravity, which may be seen as both inflation, in the very early universe, and dark energy, at late times. To do so, we estimate the slow-roll parameters and we find the tensor-to-scalar ratio ($r$), the scalar spectral index ($n_s$) and the running spectral index $(dn_s/dk)$ for the model, and compare them with Planck results [16]. Then, we consider such scalar field in a presence of a barotropic fluid, and we analyze the stability of the dynamical system.

The rest of the paper is organized in the following manner. In Section II we present our model, in Section III we apply it to describe inflation and in Section IV we show all the results of the model. In Section V is reserved for conclusions. We use Planck units ($h = c = 1 = M_{\text{pl}} = 1$) throughout the text, unless explicitly stated.

II. LOGARITHM KÄHLER POTENTIAL

The complex scalar field is $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$ and we choose the Kähler potential and the superpotential to be respectively

$$K(\phi, \phi^*) = -k \log(-i(\phi - \phi^*)), \quad W(\phi) = \mu e^{i\bar{\lambda} \phi}$$

where $k$ is a positive integer. The specific values 1 and 3 are related to the axion-dilaton field, and the radial modulus, respectively, in the Calabi-Yau three-fold compactification of the type-IIB superstring theory [27]. The above form of the superpotential may be generated by nonperturbative effects [11, 27], where $\mu$ has mass dimension 3 in natural units ($h = c = 1$) and $\bar{\lambda}$ is dimensionless.

Making use of (1), the scalar potential (1) depends only on the imaginary part of the scalar field

$$V(\phi_I) = \mu^2 e^{-\bar{\lambda} \phi_I} \left( \frac{\bar{\lambda}^2}{k} - \phi_I^2 + 2\bar{\lambda} \phi_I + k - 3 \right).$$

An interesting case is for $k = 1$, which will be explored in more details in the rest of the paper. The Lagrangian for the field $\phi_I$ with $k = 1$ is

$$e^{-1} \mathcal{L} = \frac{1}{\phi_I^2} \partial_\mu \phi_I \partial^\mu \phi_I - \mu^2 e^{-\bar{\lambda} \phi_I} \left( \bar{\lambda}^2 \phi_I + 2\bar{\lambda} - \frac{2}{\phi_I} \right).$$

The kinetic part becomes canonical with the field redefinition $\phi_I = e^{\Phi}/\sqrt{2}$.

The potential is shown in Figure [11] whose value of the scalar field at the unstable point is $\Phi \approx 3 M_{\text{pl}}$, which is the point where we assume inflation started. The values for $\bar{\lambda}$ and $\mu$ were chosen in such a way that the model best describes slow-roll inflation and dark energy, with the energy density of the scalar field similar to the cosmological constant. Both parameters were fixed and will be used and explored in Sections [III] and [IV].

Once we are considering $\mathcal{N} = 1$ supergravity in four dimension as an effective theory, we assume that the period before inflation is not in the regime of this theory. Therefore, the divergence of $V(\Phi)$ when $\Phi \to 0$ is not an issue.

III. A MODEL FOR INFLATION

To apply the previous model for the inflation period, we use the Friedmann-Robertson-Walker metric, with the scale factor $a(t)$ and the signature $(- + + +)$. The slow-roll parameters for inflation can be defined as [16]

$$\epsilon \equiv \frac{1}{2} \left( \frac{V'(\Phi)}{V(\Phi)} \right)^2,$$

$$\eta \equiv \frac{V''(\Phi)}{V(\Phi)},$$

$$\xi^2 \equiv \frac{V'(\Phi) V'''(\Phi)}{V(\Phi)^2}.$$  

These parameters are independent of $\mu$ and they are used to estimate the tensor-to-scalar ratio $r$, scalar spectral index $n_s$ and the running of the scalar spectral index $dn_s/dk$, according to the expressions [16]

$$r = 16 \epsilon,$$

$$n_s - 1 = 2 \eta - 6 \epsilon,$$

$$\frac{dn_s}{dk} = -16 \epsilon \eta + 24 \epsilon^2 + 2 \xi^2.$$  

In terms of the scalar field, the number of e-folds is

$$N_s = \int_{\Phi_*}^{\Phi_+} \frac{d\Phi}{\sqrt{2 \epsilon}}.$$
decelerated phase. The scalar field at this stage is $\Phi$ becoming out of validity and the expansion entered in a decelerated way that if we increase it or decrease it, for instance, to $dn/\dot{a}$ according to Planck data, better values of $\lambda$ are shown in Figure 2, whose equation of state $w = (p_m/\rho_m)$ disagree with data [16] (e.g. $3 \pm 0.03$ or $3 \pm 0.05$). Thus, according to Planck data, better values of $n_s$ are those ones that have $\Phi_s$ closer to $3 M_{pl}$. Inflation parameters are also very sensitive to variations of $\lambda$, in such a way that if we increase it or decrease it, for instance, to $0.015$ or $0.05$, respectively, $n_s$ disagree with data $[10]$ ($-0.04$ or $0.99$, respectively).

We assume that inflation ended when slow-roll started becoming out of validity and the expansion entered in a decelerated phase. The scalar field at this stage is $\Phi \approx 6 M_{pl}$, then, the correspondent number of e-folds is around $N_e \approx 50$ for $\Phi \approx 6 M_{pl}$, and $N_e \approx 66$ for $\Phi \approx 8 M_{pl}$. From $\Phi_e \approx 6 M_{pl}$ on, universe entered in a decelerated expansion phase, so we assume that the epoch known as inflation has stopped after the time the scalar field reached this value. The time evolution of the scalar field and the $\Phi^2$ are shown in Figure 2 whose equation of motion

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0,$$

was numerically solved. As we see in the figure, inflation lasted $t \approx 10^{11} t_p$, where $t_p$ is the Planck time.

The scalar potential at $\Phi_e \approx 6 M_{pl}$ is $V(\Phi) \sim 10^{-29} M_{pl}^4$, but it reaches $V(\Phi) \sim 10^{-119} M_{pl}^4$ at $\Phi \approx 8.1 M_{pl}$ (Figure 3), which is the value for the cosmological constant today. However, $\Phi^2$ is in the range $10^{29} - 10^{32} M_{pl}^4$ for $\Phi \approx 6 - 8 M_{pl}$, implying that the universe decelerates from $\Phi \approx 6 - 8 M_{pl}$ on. Since a difference of one order of magnitude in $\mu$ increases or decreases two orders of magnitude of the potential, the scalar field mimics the cosmological constant a bit earlier or later, although the general feature does not change.

Roughly, the temperature associated with $\dot{\Phi}^2 \sim 10^{-29} - 10^{-32} M_{pl}^4$ is $T \sim 10^{-8} M_{pl} \sim 10^{10} \text{GeV}$, which agrees with the one required by thermal leptogenesis [18, 19]. However, a more careful analysis can be done if we numerically solve the equation of motion for the scalar field, considering the reheating period. In a standard scenario, the inflaton decays into the standard model particles at the end of inflation, increasing the temperature of the universe. In this period, called reheating, the Klein-Gordon equation (9) gets an extra term, due to the decay. The equation becomes

$$\ddot{\Phi} + 3H\dot{\Phi} + \Gamma_R \dot{\Phi} + V'(\Phi) = 0,$$

where $\Gamma_R$ is the decay rate, which in turn is related with the reheating temperature ($T_R$) by $T_R \sim \sqrt{\Gamma_R}$. Considering now this equation of motion we impose that this process decreases the value of $\dot{\Phi}^2$ from $10^{-29} - 10^{-32} M_{pl}^4$ at $\Phi \approx 6 - 8 M_{pl}$ to $10^{-120} M_{pl}^4$, thus lower than $V(\Phi)$ (shown in Figure 4). Then, after the reheating phase the universe restarted its accelerated expansion, and the scalar field remained at $\Phi \approx 8 M_{pl}$.

We find that $\dot{\Phi}^2$ decreases from $10^{-29}$ to $10^{-120} M_{pl}^4$ (or less) for $\Gamma_R \sim 10^{-10} M_{pl}$ (Figure 4 top) or $\Gamma_R \sim 10^{-12} M_{pl}$ (Figure 4 bottom), which corresponds $T_R \sim 10^{13} \text{GeV}$ or $T_R \sim 10^{12} \text{GeV}$, respectively, in agreement with thermal leptogenesis [18, 19]. The difference between the two possibilities is how long $\dot{\Phi}^2$ takes to decrease until $10^{-120} M_{pl}^4$ or less.

IV. A MODEL FOR DARK ENERGY

In order to analyze if the scalar field can also explain the present stage of accelerated expansion of the universe, we consider it in the presence of a barotropic fluid with energy density $\rho_m$ and equation of state $w_m = p_m/\rho_m$. The discussion in this section follows closely the one presented in [2, 28].

A. Autonomous system for the model

The Friedmann equations for the canonical field are

$$H^2 = \frac{1}{3}(\rho_\Phi + \rho_m),$$

$$\dot{H} = -\frac{1}{2} \dot{\Phi}^2 + (1 + w_m)\rho_m,$$

and the continuity equation is Eq. (9), with $H$ given now by [11]. To deal with the dynamics of the system, it is convenient to define the new dimensionless variables [28].
FIG. 2. Time evolution of the scalar field $\Phi(t)$ (top) and $\dot{\Phi}^2(t)$ (bottom) in Planck units.

\[
x \equiv \frac{\dot{\Phi}}{\sqrt{6}H}, \quad y \equiv \sqrt{\frac{V(\Phi)}{3H}}, \\
\lambda \equiv -\frac{V'}{V}, \quad \Gamma \equiv \frac{VV''}{V'^2}. \tag{13}
\]

which are going to characterize an autonomous system, that is, a system of differential equations in the form $\dot{x} = f(x, y, \ldots, t)$, $\dot{y} = g(x, y, \ldots, t)$, ..., so that $f$, $g$, ..., do not depend explicitly on time. For this system, a point $(x_c, y_c, \ldots)$ is called fixed or critical point if $(f, g, \ldots)\big|_{x=x_c, y=y_c, \ldots} = 0$ and it is an attractor when $(x(t), y(t), \ldots) \to (x_c, y_c, \ldots)$ for $t \to \infty$.

The field density parameter is written in terms of these new variables as

\[
\Omega_\Phi \equiv \frac{\rho_{\phi}}{3H^2} = x^2 + y^2, \tag{14}
\]

so that (11) can be written as

\[
\Omega_\Phi + \Omega_m = 1, \tag{15}
\]

with the matter density parameter defined by $\Omega_m = \rho_m/(3H^2)$. The equation of state $w_{\phi}$ in terms of these new variables is

\[
w_{\phi} = \frac{x^2 - y^2}{x^2 + y^2}, \tag{16}
\]

and the total effective equation of state becomes

\[
w_{eff} = \frac{p_{\phi} + p_m}{\rho_{\phi} + \rho_m} = w_m + (1 - w_m)x^2 - (1 + w_m)y^2, \tag{17}
\]

where the last equality follows from (11), (14) and (15). An accelerated expansion occurs for $w_{eff} < -1/3$.

The equations of motion for the new variables $x$ and $y$ are obtained taking the derivatives with respect to $N \equiv \log a(t)$

\[
\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2}y^2 + \frac{3x}{2}(1 - w_m)x^2 + (1 + w_m)(1 - y^2), \tag{18a}
\]

\[
\frac{dy}{dN} = -\frac{\sqrt{6}}{2}\lambda xy + \frac{3y}{2}(1 - w_m)x^2 + (1 + w_m)(1 - y^2), \tag{18b}
\]

\[
\frac{d\lambda}{dN} = -\sqrt{6}\lambda^2(\Gamma - 1)x. \tag{18c}
\]
The fixed points of the system are obtained by setting $dx/dN = 0$, $dy/dN = 0$, and $d\lambda/dN = 0$ in (18). When $\Gamma = 1$, $\lambda$ is constant and the potential has an exponential form, whose fixed points are [28]:

- (a) $(x, y) = (0, 0)$:
  \[ \Omega_\phi = 0, \quad w_{\text{eff}} = w_m, \quad w_\Phi \text{ is undetermined.} \] (19a)

- (b) $(x, y) = (\pm 1, 0)$:
  \[ \Omega_\phi = 1, \quad w_{\text{eff}} = w_\Phi = 1. \] (19b)

- (c) $(x, y) = \left( \frac{\lambda}{\sqrt{6}}, \sqrt{1 - \frac{\lambda^2}{6}} \right)$:
  \[ \Omega_\phi = 1, \quad w_{\text{eff}} = w_\Phi = -1 + \frac{\lambda^2}{3}. \] (19c)

- (d) $(x, y) = \left( \sqrt{\frac{2}{3}}, \frac{1 + w_m}{\lambda}, \sqrt{\frac{3(1-w_m^2)}{2\lambda^2}} \right)$:
  \[ \Omega_\phi = \frac{3(1 + w_m)}{\lambda^2}, \quad w_{\text{eff}} = w_\Phi = w_m. \] (19d)

The point (a) corresponds to a fluid dominated solution – non-relativistic matter ($w_m = 0$) – which, however, does not provide support to the cosmic acceleration, since $w_{\text{eff}}$ is not less than $1/3$. The same also happens for solution (b). The radiation-dominated epoch corresponds to the fixed point (a) with $w_{\text{eff}} = w_m = 1/3$. The point (c) can describe the acceleration of the universe for $\lambda^2 < 2$, in which case the final state of the universe is the scalar field-dominated one ($\Omega_\phi = 1$). The point (d) is also not viable to explain a late-time acceleration, since for matter-dominated we have again $w_{\text{eff}} = 0$.

Another possibility is when $\lambda$ is not constant ($\Gamma \neq 1$), for which Eq. (18c) implies that a critical point is either $\lambda = 0$ or $x = 0$. For these possibilities we have the following

- (I) $(x, y) = (0, 0)$, for all $\lambda$:
  \[ \Omega_\phi = 0, \quad w_{\text{eff}} = w_m, \quad w_\Phi \text{ is undetermined.} \] (20a)

- (II) $(x, y) = (\pm 1, 0)$, $\lambda = 0$:
  \[ \Omega_\phi = 1, \quad w_{\text{eff}} = w_\Phi = 1. \] (20b)

- (III) $(x, y) = (0, \pm 1)$, $\lambda = 0$:
  \[ \Omega_\phi = 1, \quad w_{\text{eff}} = w_\Phi = -1. \] (20c)

All critical points are similar to those found in [28], with constant $\lambda$. Again, an accelerated expansion with scalar field dominated solution occurs only for (20c).
B. Stability around the critical points

In order to study stability of the fixed points \((x, y, \lambda) = (x_c, y_c, \lambda_c)\), we consider linear perturbations \(\delta x\), \(\delta y\) and \(\delta \lambda\) around them. The perturbations satisfy the following differential equations

\[
\frac{d}{dN} \begin{pmatrix} \delta x \\ \delta y \\ \delta \lambda \end{pmatrix} = \mathbf{M} \begin{pmatrix} \delta x \\ \delta y \\ \delta \lambda \end{pmatrix},
\]

where

\[
\mathbf{M} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial \lambda} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial \lambda} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial \lambda} \end{pmatrix}
\]

and where \(f = f(x, y, \lambda)\), \(g = g(x, y, \lambda)\) and \(h = h(x, y, \lambda)\) are the right-hand side of (18), (18b) and (18c), respectively. The matrix \(\mathbf{M}\) possesses three eigenvalues \(\mu_1, \mu_2, \mu_3\), and the general solution for the evolution of linear perturbations are

\[
\begin{align*}
\delta x &= Ae^{\mu_1 N} + Be^{\mu_2 N} + Ce^{\mu_3 N}, \\
\delta y &= De^{\mu_1 N} + Fe^{\mu_2 N} + Ge^{\mu_3 N}, \\
\delta \lambda &= Me^{\mu_1 N} + Ne^{\mu_2 N} + Fe^{\mu_3 N},
\end{align*}
\]

where the capital latin letters (A-I) are integration constants. The stability around the fixed points depends on the nature of the eigenvalues, in such a way that they are stable points if they have negative values \((\mu_1, \mu_2, \mu_3 < 0)\), unstable points if they have positive values \((\mu_1, \mu_2, \mu_3 > 0)\) and saddle points if at least one eigenvalue has positive (or negative) value, while the other ones have opposite sign. The eigenvalues \(\mu_1\) and \(\mu_2\) for the critical points (I)-(III) are equal to what was found in [28], but with \(\lambda = 0\), then we show below the results for a general \(\lambda\),

- point (a) (point (I) for \(\lambda = 0\)):
  \[
  \mu_1 = -\frac{3}{2}(2 - \gamma), \quad \mu_2 = \frac{3}{2}\gamma,
  \]
  \[
  \therefore \text{ saddle point for } 0 < \gamma < 2,
  \]

- point (b) (point (II) for \(\lambda = 0\)):
  \[
  \mu_1 = 3 + \sqrt{\frac{6}{\gamma}}, \quad \mu_2 = 3(2 - \gamma),
  \]
  \[
  \therefore \text{ unstable point for } \lambda < \sqrt{6},
  \]

- point (c) (point (III) for \(\lambda = 0\)):
  \[
  \mu_1 = \frac{1}{2}(\lambda^2 - 6), \quad \mu_2 = \lambda^2 - 3\gamma,
  \]
  \[
  \therefore \text{ stable point for } \lambda < \sqrt{3\gamma},
  \]

where \(\gamma = 1 + w_m\). For constant \(\lambda\) there is no \(\mu_3\) and for the three cases (I)-(III) (20), \(\mu_3 = 0\), which means that perturbations over \(\lambda\) let it at the position it has been disturbed. As we see in Figure 5 \(\lambda \rightarrow 0.1\) and therefore the point (III) would not work for our model.

Therefore, the system may evolve from the saddle point (a), with \(\Omega_\phi \approx 0\), to the stable point (c) with \(\lambda = 0.1\). With this value of \(\lambda\), the system end up with \(x \approx 0.041\) and \(y \approx 0.999\), and from (11c), the equation of state for dark energy is \(w_\phi \approx -0.997\), which agrees with Planck results [1] for the cosmological constant. A phase plane for the fixed point (c) is shown in Figure 6.

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\(^2\) Cases (I)-(III) have been written here for the sake of completeness, because since \(\lambda\) is approximately constant these fixed points do not need to be considered.
FIG. 6. Phase plane for $\lambda = 0.1$. The scalar-field dominated solution (c) is a stable point at $x \approx 0.041$ and $y \approx 0.999$. The dashed line corresponds to $\Omega_\Phi \approx 0.7$ and the system evolves from $x = y = 0$ to the fixed point.

V. CONCLUSIONS

We have studied a model using four dimensional $\mathcal{N} = 1$ supergravity, with a logarithm Kähler potential and an exponential superpotential. Our model suggests an accelerated expansion of the universe, both during the very early epoch, in the period known as inflation, and during the present stage of expansion. The slow-roll and inflation parameters are in agreement with Planck+WP+lensing data [16] and BICEP2/Keck and Planck joint analysis [17], both at 95% CL. Inflation ended when the universe stopped expanding ($\Phi \approx 6 M_{\mathrm{pl}}$ on), being followed by a reheating phase during which $\Phi \approx 6 - 8 M_{\mathrm{pl}}$. The associated reheating temperature could be estimated as $T_R \sim 10^{12}$ GeV, in agreement with thermal leptogenesis [18, 19]. At the end of the reheating phase, the energy density of the scalar field is less than the energy density of the radiation. In the presence of matter/radiation the dynamical system evolves to a stable point, which is a late-time attractor, and whose equation of state for dark energy is $w_\Phi \approx -0.997$, also in agreement with Planck results [1] for the cosmological constant.

Although the model seems to describe both epochs of accelerated expansion of the universe, and the scalar field mimics the cosmological constant from the end of inflation on, the parameter $\lambda$ has been fine-tuned. The reheating process that we have assumed has this issue as well. In addition, once the scalar field mimics the cosmological constant, it seems hard from an observational point of view to distinguish the former from the latter.

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