Spontaneous photon emission stimulated
by two Bose condensates

C.M. Savage, Janne Ruostekoski and Dan F. Walls

Department of Physics, University of Auckland, Private Bag 92019,
Auckland, New Zealand

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Abstract

We show that the phase difference of two overlapping ground state Bose-Einstein condensates can effect the optical spontaneous emission rate of excited atoms. Depending on the phase difference the atom stimulated spontaneous emission rate can vary between zero and the rate corresponding to all the ground state atoms in a single condensate. Besides giving control over spontaneous emission this provides an optical method for detecting the condensate phase difference. It differs from previous methods in that no light fields are applied. Instead the light is spontaneously emitted when excited atoms make a transition into either condensate.

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Spontaneous symmetry breaking is a fundamental physical phenomenon. In particular, breaking of the global phase symmetry explains much of the interesting physics of Bose-Einstein condensates [1]. We show that this spontaneously broken symmetry can effect the rate of spontaneous emission from excited atoms. Hence we are able to link two fundamental physical phenomena: spontaneous symmetry breaking and spontaneous emission.

Other physics which effects the spontaneous emission rate are; atom stimulation by a single condensate [2], non-free space electromagnetic boundary conditions [3], due to a mirror for example, and superradiant type dipole correlations [3]. The latter occurs when the atoms are much closer than the wavelength of the emitted light. Our effect is different to these and happens when the excited atoms can decay into either of two final states which are Bose condensed. Its physical origin is the indistinguishability of the final states, which produces phase dependent interference terms in the transition probability.

Two groups have reported producing two dilute gas Bose-Einstein condensates in the same trap. Spatially overlapping condensates of the $|F = 1, m = -1\rangle$ and $|F = 2, m = 2\rangle$ hyperfine spin states of rubidium-87 have been produced by sympathetic cooling [4]. Separated condensates of the same state of sodium-23 have been produced using a far blue detuned laser sheet to divide a magnetic trap [5]. Our system requires two overlapping condensates into which an excited state can make a spontaneous transition.

Bose stimulation of transitions by *atoms* has been studied for both electronic [2] and nuclear [6,7] transitions. It was shown to be experimentally feasible for electronic transitions [2]. We apply the same idea to a system with two ground state condensates instead of one, and find that the atom stimulated emission rate depends on the condensate phase difference.

This phase difference has been observed directly by destructive optical imaging of the interference fringes in the density of overlapping condensates [5]. Previously proposed methods for optically detecting this phase difference require driving Raman transitions between the ground states. The excited state was created only through coherent excitation of the ground states and hence was not independent of the ground states. Javanainen has shown that prompt amplification of one Raman beam by light-stimulated scattering is a signature
of a condensate phase difference [8]. This is because the condensate phase difference behaves like a Raman optical coherence. Imamoğlu and Kennedy have studied light scattering from two spatially separated condensates. They showed that the optical coherence allows spontaneous Raman scattering to generate coherent light, as well as allowing the complete elimination of Raman scattering [9]. These two schemes were treated qualitatively, in contrast to the fully quantitative treatment we shall give. Ruostekoski and Walls found that the relative peak heights of the characteristic two-peaked incoherent light scattering spectrum depend on the relative condensate phase [10]. Our method is related to these three insofar as it results from the macroscopic quantum coherence between the two condensates. However no external light is applied and no Raman transitions occur. The light is emitted by the decay of independently produced excited atoms. A schematic experimental setup is shown in Fig. 1.

We consider transitions from an excited state labeled by $e$ to ground states labeled by $g_1$ and $g_2$. The excited state is separately prepared and then launched towards the ground state condensates. The ground states may differ in their internal quantum numbers and/or in their external wavefunctions. Both situations have been achieved experimentally [4,5]. We neglect radiation reaction and multiple scattering effects [11,12]. This requires that the system be optically thin at the frequency of the spontaneously emitted light [13]. We show that the recoil shift of the emitted photon makes this possible, provided the natural linewidth is sufficiently small. Hence we have in mind the decay of a metastable excited state.

We consider spontaneous emission from excited atoms. “Spontaneous” specifies that the emission is not stimulated by light, although it may be stimulated by atoms. The spontaneous emission is from the excited state atom field $\hat{\psi}_e$ to ground state atom fields $\hat{\psi}_{g_i}$. The positive frequency part of the electric field operator at position $r$ is given in terms of the atom fields by Javanainen and Ruostekoski [13] as

$$\hat{E}^+(r) = \hat{E}_F^+(r) + \sum_g \int d^3r' \ K(d_g; r, r') \hat{\psi}_{g_i}^+(r') \hat{\psi}_e(r'),$$  

(1)
where \( \hat{\mathbf{E}}_F^+(r) \) is the free field, which we assume to be the vacuum so that \( \langle \hat{\mathbf{E}}_F^-(r) \cdot \hat{\mathbf{E}}_F^+(r) \rangle = 0 \). Rotation at the transition frequency \( \Omega \) has been removed from the excited atom and electric fields, so that they are slowly varying \([13]\). We ignore excited state depletion and ground state growth. The integration kernel \( K(d; r, r') \) generates the field at \( r \) due to a dipole moment \( d \) at \( r' \) \([14]\). We shall only be concerned with electromagnetic waves in the far field so that the dominant part of the kernel is that proportional to \( |r - r'|^{-1} \),

\[
K(d; r, r') \approx \frac{k^2}{4\pi \varepsilon_0} \frac{\exp(ik|r|)}{|r|} \exp(-ikr' \cdot n) D_g,
\]

where \( d_g \) is the dipole moment of the transition \( g \leftrightarrow e \), and \( k = |k| \) where \( k \) is the wavevector of the emitted light. The far field assumption, in the form \( |r| \gg |r'| \), has been used to replace \( |r - r'| \) in the denominator by \( |r| \) and to expand the exponential \( \exp(ik|r - r'|) \approx \exp(ik|r| - ikr' \cdot n) \), where \( n = r/|r| \). We have also approximated the propagation direction \( (r - r')/|r - r'| \) by \( n \), and hence introduced the components of the dipole moments perpendicular to the propagation direction \( n \),

\[
D_g = (n \times d_g) \times n = d_g - n(n \cdot d_g).
\]

With these approximations the expectation value of the field intensity, with respect to the atom field states and a vacuum free field, is

\[
I(r) = 2c\varepsilon_0 \langle \hat{\mathbf{E}}_F^-(r) \cdot \hat{\mathbf{E}}_F^+(r) \rangle \\
= \frac{\kappa}{|r|^2} \sum_{g,g'} \int d^3r' \int d^3r'' \exp[-ik(r'' - r') \cdot n] \times \mathcal{D}_g^* \cdot \mathcal{D}_{g'} \langle \hat{\psi}_e^+(r') \hat{\psi}_g(r') \hat{\psi}_{g'}^+(r'') \hat{\psi}_e(r'') \rangle,
\]

where \( \kappa = ck^4/(8\pi^2\varepsilon_0) \). As usual we consider the atom field operators to be expanded in complete sets of orthonormal mode functions \( \{ \phi_i(r) \} \) which are adapted to the system under consideration \( \hat{\psi}(r) = \sum_i \hat{a}_i \phi_i(r) \), where the \( \hat{a}_i \) are the usual mode annihilation operators. The ground state modes into which condensation occurs are typically solutions to the Gross-Pitaevskii equation \([13]\).
We assume a sufficiently low temperature that stimulation by the non-condensed atoms can be ignored. In a JILA experiment up to 80% of the trapped atoms have been measured to be condensed, at a temperature of about 140 nK [16]. Since the ratio of non-condensed to condensed atoms is then 1/4 this implies that spontaneous emission stimulated by non-condensed atoms can be reduced, at least, to 1/4 of that stimulated by the condensates. This would be a constant background emission. It would add to the unstimulated free space spontaneous emission rate $\gamma$, which is always present. Furthermore, in their two condensate experiment the JILA group reports cooling proceeding until “we can no longer see any noncondensed atoms” [4].

We assume the Bose condensed modes may be represented by coherent states $|\sqrt{N}\exp(i\theta)\rangle$. The coherent state phase $\theta$ embodies the broken global phase symmetry. Since Hamiltonians always contain pairs of atom creation and annihilation field operators the phase is unmeasureable. However phase differences between condensates are measureable, although experimentally they are not yet controllable and hence they vary from shot to shot. The coherent states have a mean atom number of $N$ and an atom number uncertainty of $\sqrt{N}$. We further assume that all condensates are initially uncorrelated. The assumption that the excited state is condensed is not essential, however it does simplify our treatment. The atom field operator expectation value determining the intensity Eq.(4) is then

$$
\langle \hat{\psi}_e^\dagger(r') \hat{\psi}_g(r') \hat{\psi}_g^\dagger(r'') \hat{\psi}_e(r'') \rangle = N \sqrt{N_g N_g'} \exp(i\delta\theta_{gg'}) \phi_e^*(r') \phi_g(r') \phi_g^*(r'') \phi_e(r''),
$$

where $\delta\theta_{gg'} = \theta_g - \theta_{g'}$ is the ground state condensate phase difference. The atom stimulated intensity can then be written as

$$
I(r) = \frac{\kappa}{|r|^2} N \left\{ \sum_g N_g |C_g \mathcal{D}_g|^2 + 2 \sqrt{N_{g1} N_{g2}} \text{Re}\{\exp(i\delta\theta_{g1g2}) \mathcal{D}_{g1}^* \cdot \mathcal{D}_{g2} C_{g1}^* C_{g2}\} \right\},
$$

where Re{} means the real part, and we have defined Franck-Condon factors, or the overlaps of the excited and ground modes taking into account the photon recoil,

$$
C_g = \int d^3r' \exp(-ikr' \cdot n) \phi_g^*(r') \phi_e(r').
$$
The last term in the expression for the intensity Eq. (6) depends on the condensate phase difference $\delta \theta_{g_1, g_2}$ provided the Franck-Condon factors are non-zero and $D_{g_1}^* \cdot D_{g_2} \neq 0$. That is provided there is some field polarization that both dipole moments can generate at $r$. Since the electric field is a vector field this is necessary for interference to be possible. This dependence of the spontaneously emitted intensity on the condensate phase difference could in principle be used to measure it.

We now give a physical explanation based on the quantum mechanical transition amplitudes. First we recall that if the final states of two transitions are in principle distinguishable then the individual transition probabilities must be added to get the final state probability. However if the final states are indistinguishable then the transition amplitudes must be added, before taking the squared modulus to get the final state probability. The squaring produces interference terms in the final state probability. The intensity radiated in the propagation direction $n$ is proportional to the transition probability for producing a photon propagating in that direction. The first terms in Eq. (6) correspond to the individual probabilities for the transitions into each ground state. The last term corresponds to the product of amplitudes for each transition and hence represents quantum mechanical interference between them. Interference occurs because the condition $D_{g_1}^* \cdot D_{g_2} \neq 0$ ensures that the particular transition which produces photons with certain polarizations cannot be distinguished. A general treatment of the electric field polarization dependence on the atomic level scheme is given by Javanainen and Ruostekoski \cite{13}.

We next estimate the mode overlaps $C_g$. We use gaussians for the ground mode functions

$$\phi_g = (2\pi l^2)^{-3/4} \exp[-(r \mp De_z)^2/(4l^2)],$$

(8)

where $l^2$ is the one dimensional variance of the probability density $\phi_g^2$, $e_z$ is the unit vector in the $z$ direction, and we have allowed for possible displacement by the distance $\pm D$ in this direction. For the excited atoms we use the plane wave mode $\phi_e = V^{-1/2} \exp(ik_e x)$. This represents atoms propagating in the positive $x$ direction with momentum $hk_e$. $V$ is the mode volume. The plane wave usefully models modes which are much larger than the...
ground modes, so for purely calculational reasons we require \((2l)^3/V \ll 1\). We assume that the excited state atoms are completely independent of the ground state atoms. They would be prepared in a separate part of the experiment. Evaluating the integral Eq.(7) we find

\[
C_g = (8\pi)^{3/4} \left(\frac{l^3}{V}\right)^{1/2} \exp[-l^2(k^2 + k_e^2 - 2kk_e \cos \vartheta)] \times \\
\exp[\pm ikD \sin \vartheta \cos \varphi],
\]

where \(\vartheta\) is the angle between the light emission direction \(\mathbf{n}\) and the excited state propagation direction \(\mathbf{e}_x\), and \(\varphi\) is the polar angle about \(\mathbf{e}_x\). The first exponential results from momentum conservation while the second is a phase modulation due to displacement of the condensate from the coordinate origin.

Since the mode functions \(\{\phi_i\}\) are a complete orthonormal set the fraction of emissions into the solid angle \(d\Omega\) with final atomic mode \(\phi_g\) is \(|C_g|^2 \, d\Omega\). Choosing the photon and excited state momenta equal \(k = k_e\), which gives the optimal overlap, and using the realistic values \(l^3/V = 0.01\), and \(lk = 100\) gives

\[
|C_g|^2 \approx 1.3 \exp[-4 \times 10^4(1 - \cos \vartheta)] \\
\approx 1.3 \exp[-2 \times 10^4\vartheta^2], \quad (\vartheta \ll 1).
\]

This mode overlap is largest for emission into a cone about the excited state propagation direction \(\mathbf{e}_x\). For the atom stimulated emission rate to exceed the non-stimulated rate by a factor of one hundred, at an emission angle of \(\vartheta = 0.02\) radians, requires about \(100/|C_g|^2 \approx 2.3 \times 10^5\) atoms in the stimulating condensate. At \(\vartheta = 0.03\) radians the stimulated emission is negligible.

A condition for the validity of our analysis is that the spontaneously emitted light is sufficiently far off resonance to avoid cooperative effects and multiple scattering \[13\]. Assuming \(k = k_e\), the emitted photon is detuned from resonance by the recoil frequency \(\omega_R = \hbar k^2/(2m)\), where \(m\) is the atomic mass. This detuning must be at least of the order of the condensate collective linewidth, which has been estimated to be \(3N_g\gamma/(2(lk)^2)\), where \(\gamma\) is the transition’s free space natural linewidth \[17,18\]. The condition is then
\[ \omega_R > \frac{3 N_g \gamma}{2 (lk)^2}. \] (11)

With \( N_g = 10^6 \) and \( lk = 100 \) this becomes \( \omega_R > 150\gamma \). A typical recoil frequency of \( \omega_R = 10^5/s \) requires the natural line with \( \gamma < 600/s \). We note that the total transition rate is proportional to \( N_e \) which is not constrained.

We next consider some particular examples. Let the excited state have magnetic quantum number \( m \). It can decay by emission of (\( \sigma^- \)) or (\( \sigma^+ \)) circularly polarised photons, or by emission of a linearly polarised photon. The respective final states having magnetic quantum numbers \( m+1 \), \( m-1 \) and \( m \). We assume that transitions into the \( m \) ground state, which is not condensed, can be ignored. This is justified when transitions into the condensed \( m \pm 1 \) modes are dominant because of Bose stimulation.

The total radiated power is found by integrating Eq.(6), for the intensity radiated in direction \( \mathbf{n} \), over all directions. Let \( z \) be the quantization axis and let the excited atoms propagate in the positive \( x \) direction, see Fig. 1. We use the circular dipole moments \( \mathbf{d}_\pm = \mp d_\pm (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}, \) where \( d_\pm = |d_\pm| \) are the magnitudes of the dipole moments, and the \( \mathbf{e}_{x/y} \) are unit vectors in the \( x/y \) directions. In the \( z \) direction these dipoles radiate photons with opposite circular polarization. In the \( x/y \) plane, however, they radiate indistinguishable linearly polarized photons, which makes interference possible, see Fig. 1. Using the previously calculated mode overlaps Eq.(9) the total atom stimulated radiated power is found to be

\[
P = \int I(r) |r|^2 d^2 \mathbf{n} \\
= AN_e \left\{ B(N_+d_+^2 + N_-d_-^2) + 2C \sqrt{N_+N_-d_+d_- \cos \delta \theta_{+-}} \right\}. \tag{12}
\]

The integral is over the spherical surface of radius \( |r| \) centered on the origin. We have defined the coefficients

\[
A = \kappa (8\pi)^{3/2} \frac{l^3}{\sqrt{2\pi}} \exp[-2l^2(k^2 + k_e^2)],
B = \cosh \alpha + \frac{(\alpha^2 - 1) \sinh \alpha}{\alpha^3}, \tag{13}
\]

\( \kappa \) is the loop radius of the atom.
where $\alpha = 4l^2kk_e$. For the case of complete overlap $D = 0$ the remaining coefficient has the simple form

$$C_{D=0} = 3B - 4\frac{\sinh \alpha}{\alpha}. \tag{14}$$

In the limit of ground state mode functions much larger than the wavelength of light, $\alpha \gg 1$, the power reduces to

$$P_{D=0} = N_eN_gk(8\pi)^{3/2}l^3\frac{2\pi}{V4(\kappa l)^2}d^2(1 - \cos \delta \theta_{+-}), \tag{15}$$

where for simplicity we have assumed here, and hereafter, that $N_+ = N_- = N_g$, $d_+ = d_- = d$, and $k = k_e$. This expression shows that the total spontaneously emitted power may vary substantially with the condensate phase difference $\delta \theta_{+-}$. We note that the excited state decay rate is proportional to the total emitted power. This means that the depletion of the excited state, as measured by direct detection of excited state atoms, could also be used to detect the phase difference.

The power Eq.(15) can be scaled to an emission rate by dividing by the photon energy $\hbar ck$. Using the free space spontaneous emission rate $\gamma = d^2k^3/(3\pi\varepsilon_0\hbar)$ and the numerical values preceding Eq.(10) gives the total atom stimulated emission rate

$$\gamma_{stim} = \gamma N_eN_g\frac{3(8\pi)^{3/2}}{16}\frac{l^3}{(kl)^2}\left(\frac{l}{V}\right)(1 - \cos \delta \theta_{+-})$$

$$= \gamma N_eN_g(2.4 \times 10^{-5})(1 - \cos \delta \theta_{+-}). \tag{16}$$

For $\cos \delta \theta_{+-} = 0$ this is one hundred times the free space emission rate $\gamma N_e$ when $N_g \approx 4.2 \times 10^6$.

We next consider spatially separated condensates, that is with $D \neq 0$ in Eq.(8). The coefficient $C$ of the interference term in Eq.(12) for the total radiated power is then

$$C = \sqrt{\pi}\sum_{j=0}^{\infty} \frac{(-1)^j}{j!}\left(\frac{D}{\sqrt{2l}}\right)^{2j}\left\{-I_{j+\frac{1}{2}}(\alpha) + 2\frac{j + \frac{3}{2}}{\alpha^2}I_{j+\frac{3}{2}}(\alpha)\right\}, \tag{17}$$

where $I_n$ is the modified Bessel function of the first kind of order $n$. The terms in the series proportional to $I_{j+\frac{1}{2}}$ result from the overlap of the gaussian wavefunctions. These terms are
dominant for wavefunction widths much bigger than the wavelength, $\alpha \gg 1$. A graph of this coefficient as a function of the ratio of condensate width to separation is shown in Fig. 2, for $kl = 4\pi$. This shows that the effect of the interference on the total radiated power disappears once the condensates are separated.

The physical situation is related to the Young’s two slit interference experiment with slits of width $2l$ and separation $2D$. In the one dimensional case the interference pattern is the product of a sinc function due to the single slit diffraction and a cosine due to the interference

$$I(m) = \left\{ \frac{\sin(kDm)}{kDm} \cos(klm + \theta_d) \right\}^2,$$  

where $m = \sin \vartheta$, $\vartheta$ is the angle of observation of the interference pattern, and $\theta_d$ is the phase difference between the light waves at each slit. This phase difference determines the relative positions of the interference pattern and the diffraction pattern. For the two slit experiment $D > l$. However overlapping condensates have $D < l$. Consider the effect of varying the source phase difference $\theta_d$ in this case. Since the interference maxima are further apart than the sinc function zeros, Eq. (18) shows that the interference maxima can shift in and out of under the maxima of the sinc function as $\theta_d$ varies. This is the physical origin of the modulation of the total power by the condensate phase difference. When $D > l$ this does not happen because the interference maxima are closer together. Hence when one maximum shifts out another shifts in to replace it and the total power is approximately constant. The overall effect is well approximated by considering the effect on the total power to arise from the overlapping portions of the condensates.

As a final example we consider the polarization dependence of the radiation for another geometry. The positive frequency part of the electric field with polarization $\mathbf{e}_i$, perpendicular to $\mathbf{n}$, is the scalar product $\mathbf{E}^+(\mathbf{r}) \cdot \mathbf{e}_i^*$, with $\mathbf{E}^+(\mathbf{r})$ given by Eq. (1). The intensity of the polarised field is given by similar expressions to Eq. (4) and Eq. (6) with $\mathbf{D}_g$ replaced by $\mathbf{D}_g \cdot \mathbf{e}_i^* = \mathbf{d}_g \cdot \mathbf{e}_i^*$. Let the quantization, observation, and excited atom propagation directions all be $z$. With this geometry the transitions are distinguishable by the polarization of the
emitted photon and the total intensity does not depend on the relative condensate phase. However the intensity in particular linear polarizations does depend on the relative phase, as we now show. Define the linear polarization vectors $\mathbf{e}_\beta = e_x \cos \beta + e_y \sin \beta$. Then the intensity in polarization $\mathbf{e}_\beta$ is

$$I_\beta = N_e N_g \frac{d^2}{2} \frac{\kappa}{|\mathbf{r}|^2} \left( |C_+|^2 + |C_-|^2 - \text{Re}\{C_+^* C_- \exp[i(2\beta + \delta \theta_{+-})]\} \right). \tag{19}$$

This intensity is polarization dependent due to the final term, which also depends on the ground state condensate phase difference. This polarization dependence is diagnostic of broken symmetry in the form of a relative phase between the ground state condensates. When the relative phases of the transition dipoles of different atoms are independent there is no such polarization dependence. Independence is impossible for condensed atoms, which are all in the same state. However for non-condensed atoms independence arises through collisions or other dephasing perturbations.

We summarise our results by listing measurable quantities that may depend on the ground state condensate phase difference: the intensities of particular electromagnetic field modes, the total radiated power, and the total transition rate. The physical origin of these effects is the indistinguishability of the final states.

An experiment relying on atom stimulated emission into two condensates will be more difficult than the corresponding single condensate experiment. However our numerical estimates, and the recent experimental interest in two condensates in the same trap [4,5], give cause for optimism concerning the feasibility of observing the effect of spontaneously broken phase symmetry on spontaneous emission.

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FIGURES

FIG. 1. Schematic diagram of an experiment investigating the condensate phase dependence of spontaneously emitted light. The solid ellipses represent the ground state condensates and the dashed ellipse represents the excited mode. The excited mode has momentum $\hbar k$ in the $x$ direction. The dipole moments are quantized in the $z$ direction and are shown as arrows and directed circles. Note that when viewed from the $x$ direction both dipoles appear to oscillate linearly.

FIG. 2. Graph of the interference coefficient $C$, Eq. (17), as a function of condensate separation $D$. Parameters are $kl = 4\pi$ and $\alpha = 4(4\pi)^2$, corresponding to condensates larger than a wavelength in size.
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* Permanent address: Department of Physics and Theoretical Physics, Faculty of Science, Australian National University, ACT 0200, Australia.

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Savage et al., PRA, Figure 2