Strong quantum interactions prevent quasiparticle decay

Ruben Verresen,1,2,3 Frank Pollmann,1 and Roderich Moessner2

1Department of Physics, T4/2, Technische Universität München, 85748 Garching, Germany
2Max-Planck-Institute for the Physics of Complex Systems, 01187 Dresden, Germany
3Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
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Quantum states of matter—such as solids, magnets and topological phases—typically exhibit collective excitations—phonons, magnons, anyons1. These involve the motion of many particles in the system, yet, remarkably, act like a single emergent entity—a quasiparticle. Known to be long-lived at the lowest energies, common wisdom says that quasiparticles become unstable when they encounter the inevitable continuum of many-particle excited states at high energies. Whilst correct for weak interactions, we show that this is far from the whole story: strong interactions generically stabilise quasiparticles by pushing them out of the continuum. This general mechanism is straightforwardly illustrated in an exactly solvable model. Using state-of-the-art numerics, we find it at work also in the spin-1/2 triangular lattice Heisenberg antiferromagnet (TLHAF) near the isotropic point—this is surprising given the common expectation of magnon decay in this paradigmatic frustrated magnet. Turning to existing experimental data, we identify the detailed phenomenology of avoided decay in the TLHAF material Ba3Co2Sb2O9, and even in liquid helium—one of the earliest instances of quasiparticle decay2. Our work unifies various phenomena above the universal low-energy regime in a comprehensive description. This broadens our window of understanding of many-body excitations, and provides a new perspective for controlling and stabilising quantum matter in the strongly-interacting regime.

It is a fundamental insight of quantum mechanics that energy levels repel. This is commonly illustrated by letting two levels with unperturbed (‘bare’) energies ±Eb interact with one another through a coupling γ, i.e.

\[
\hat{H} = \begin{pmatrix} E_b & \gamma \\ \gamma & -E_b \end{pmatrix}.
\]  

(1)

The resulting energies of \(\hat{H}\) are \(\pm \sqrt{E_b^2 + \gamma^2}\). Hence, repulsion leads to a minimal separation of the levels of \(2|\gamma|\), no matter how small the initial separation \(2|E_b|\).

A natural question is whether this extends to the case of a discrete level coupled to a continuum of states. The question might seem moot, since the common expectation is that a bare level inside a continuum will be dissolved by interactions. At best, it will become a finite-lifetime resonance. At worst, no hint of it remains.

If the bare level represents a quasiparticle, its broadening and disappearance in the many-particle continuum is known as quasiparticle decay. In the case of non-topological1 quantum magnets—where quasiparticles go under the name of magnons, or spin waves—the expectation of magnon decay has, surprisingly only recently, been borne out in inelastic neutron scattering experiments4–8, see below.

Here, we show that for strong interactions this expectation of quasiparticle decay is wrong.

Rather, with increasing interaction strength, an infinitely long-lived state re-emerges out of the continuum of states. This happens via a simple generalisation of the familiar level repulsion, Eq. (1), for a bare state |ψ⟩ with bare energy \(E_b\) coupled to a continuum of states |ψα⟩ with bare energies \(E_a\) above a threshold energy \(E_{1h}\). Physically, this model represents states with a fixed value of total momentum—the continuous index \(\alpha\) corresponds to the relative momentum of two-particle states.

Concretely, for large enough coupling \(|\gamma|\), there is a single discrete state |ψ∗⟩ with an energy below the continuum, \(E^* < E_{1h}\) (see Methods). Moreover, the contribution of the unperturbed state |ψ⟩ to this final discrete state, denoted by the weight \(Z = |\langle \psi | \psi^* \rangle|^2\), can be large—for a continuum occupying a finite range of energy, the weight approaches \(Z \sim 1/2\) for large \(|\gamma|\).

This is experimentally important: a vanishing \(Z\) implies that the state |ψ∗⟩ bears little relationship to the original quasiparticle. However, a large \(Z\) ensures that any experimental set-up—e.g., neutron scattering—for detecting the original quasiparticle |ψ⟩ also detects |ψ∗⟩. Hence, while existence of |ψ∗⟩ and finiteness of \(Z\) for this simple model have been pointed out before9, its phenomenology and in particular its relevance to quasiparticles in strongly-interacting quantum systems seem to have been underappreciated.

Fig. 1 illustrates what inelastic neutron scattering would measure for a system described by this solvable model (see Methods). It shows the weight of the bare state |ψ⟩ on the true eigenstates. The initially flat bare level (dashed line) is coupled to a continuum (shaded region). For weak interactions, the physics depends on whether the bare energy level encounters a large or small number of states upon entering the continuum. This is encoded in the density of states (DOS), \(\nu(E)\). In this example we treat the case of the two-particle continuum of particles with a parabolic dispersion (although any dispersion can be accommodated); its onset satisfies \(\nu(E_{1h} + \delta E) \sim (\delta E)^{D/2-1}\) in \(D\) spatial dimensions10.

In low dimensions \((D = 1, 2)\), where this DOS has a discontinuous onset, the infinitely-long lived state |ψ∗⟩
Avoided quasiparticle decay in a solvable model. The bare level $|\psi_k\rangle$ (short-dashed line) is coupled to a continuum (shaded). Left column is representative of gapped spectra in dimensions $D = 1, 2$; the level cannot enter the continuum, but weight is transferred into a decaying mode (long-dashed line). Right column is for $D \geq 3$. For strong interactions, the outcome is independent of dimension: a renormalised quasiparticle $|\tilde{\psi}_k\rangle$ is pushed out, whose weight $Z_k = |\langle \psi_k | \tilde{\psi}_k \rangle|^2$ approaches 1/2.

FIG. 1. Avoided quasiparticle decay in a solvable model. The bare level $|\psi_k\rangle$ (short-dashed line) is coupled to a continuum (shaded). Left column is representative of gapped spectra in dimensions $D = 1, 2$; the level cannot enter the continuum, but weight is transferred into a decaying mode (long-dashed line). Right column is for $D \geq 3$. For strong interactions, the outcome is independent of dimension: a renormalised quasiparticle $|\tilde{\psi}_k\rangle$ is pushed out, whose weight $Z_k = |\langle \psi_k | \tilde{\psi}_k \rangle|^2$ approaches 1/2.

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FIG. 2. Avoided decay in an Ising ladder. (a) A paramagnet (PM) with magnon-like excitation (red arrow); an Ising ferromagnet (FM) where the quasiparticles are domain walls. By coupling the two chains, a magnon can decay into two domain walls (red dots). (b) The dynamic structure factor; the dashed line is the bare magnon dispersion and the shaded region denotes the continuum of two domain walls. At low coupling strength, the magnon decays. For strong interactions, the magnon is pushed below the continuum.
state is magnetically ordered, with neighbouring spins forming a 120° angle. However, mystery ensours its magnon excitations due to the uncontrolled nature of the available analytic and numerical methods. The most venerable of these is perhaps spin wave theory (SWT), an expansion in inverse spin, $1/S$.

We consider the spin-$\frac{1}{2}$ TLHAF

$$\hat{H} = J \sum_{(n,m)} \left( (1-\delta) \hat{S}_n \cdot \hat{S}_m - \delta \hat{S}_{n}^{\text{loc},z} \hat{S}_{m}^{\text{loc},z} \right)$$

(2)

where a small easy-axis anisotropy ($\delta = 0.05$) slightly gaps out the massless Goldstone modes, making the model more numerically tractable. Here, $\hat{S}_{n}^{\text{loc}}$ is the spin in the basis of the rotating (local) frame.

For this value of $\delta$, SWT predicts magnon decay over a large region of momentum space (shaded region in the inset of Fig. 3(a)). A magnon with momentum $k$ is then predicted to decay into two magnons with momenta $q$ and $k-q$, where $q \approx K$, the corner of the Brillouin zone (BZ). However, small spin and noncollinear order—breaking all symmetries, allowing for many interaction terms—generate strong quantum interactions. Our model thus suggests an alternative to the commonly expected scenario of magnon decay.

A recent advance in numerically simulating the dynamics of two-dimensional quantum systems allows to directly test the prediction of magnon decay in Eq. (2). Fig. 3(a) shows the out-of-plane dynamical spin structure factor along the A–B line (blue line in inset) obtained from dynamical DMRG (see Methods). Since SWT predicts decay into a K-magnon, the dotted line shows the two-magnon energy $\epsilon_q + \epsilon_K$, with $q$ along the orange line in the inset. The dashed curve is the SWT prediction of the magnon in the non-interacting limit $1/S \rightarrow 0$ (LSWT), traveling deep into the two-magnon continuum. However, the numerically-obtained $S = \frac{1}{2}$ dispersion is pushed out completely—a crisp instance of avoided magnon decay.

The dispersion is known to have a local minimum at the midpoint M of the BZ edge. This appears at higher order in SWT and in series expansion methods, as confirmed in Fig. 3(c). Our novel prediction is that the avoided decay must in turn induce a local minimum at the midpoint $Y_1$ of the magnetic BZ (MBZ) edge. This is apparent in Fig. 3(c). One should compare with $S = \frac{1}{2}$ dispersion in Fig. 3(a).

Intriguingly, this phenomenonology has already been observed in experiment. The magnetic material Ba$_3$CoSb$_2$O$_9$ is well-described by the TLHAF with a small easy-plane anisotropy, for which Fig 4(a) shows recent inelastic neutron scattering data. Since this is sensitive to the full dynamical spin structure factor, it picks up copies of the magnon dispersion translated by K. Fig. 4(a) thus shows two bands: the bottom one ($\epsilon_1$) centered at M, the top one ($\epsilon_2$) centered at $Y_1$. Neither decay and both exhibit a local minimum, in agreement with the phenomenology of Fig. 3. We thus directly reinterpret apparently unrelated experimental features as having a joint origin in avoided quasiparticle decay.

In contrast, magnon decay has experimentally been observed in a spin-2 TLHAF. This is consistent with $1/S$ being a measure of interaction strength—and avoided decay requiring strong interactions.

Level-continuum repulsion was also recently observed in the gapped spin-orbit-coupled frustrated magnet BiCu$_2$PO$_6$. This nicely fits our theoretical framework: its one-dimensional nature suggests a sharp discontinuous onset of the bare two-magnon DOS ($\gamma^2 v (E_{th} + \delta E) \sim 1/\sqrt{\delta E}$), preventing a smooth quasiparticle entry into the continuum. This is in contrast to the quasiparticle decay observed in the two-dimensional PHCC. Since the latter is spin-rotation symmetric, our earlier argument implies the effective dimensional shift $D = 2 \to D = 4$. Hence, $\gamma^2 v (E_{th} + \delta E) \sim \delta E$, consistent with the smooth entry in Fig. 4(b).

Lastly, we consider the iconic quasiparticle dispersion of superfluid helium, Fig. 4(c). While it was originally
FIG. 4. Avoided quasiparticle decay, genuine decay, and level-continuum repulsion in experimental data: the TLHAF material Ba$_3$CoSb$_2$O$_9$, PHCC, and superfluid helium. (a) Inelastic neutron scattering data and LSWT comparison for Ba$_3$CoSb$_2$O$_9$ (see Methods for details). The neutron data picks up all magnon bands related by momentum $K$; the lower branch ($\varepsilon_1$) goes through $M$, the higher branch ($\varepsilon_2$) through $Y_1$ (see Fig. 3 for BZ labeling). Similar to Fig. 3, magnon decay is avoided, with the local minimum near $M$ inducing a local minimum near $Y_1$. (b) A scenario where the quasiparticle does decay: inelastic neutron scattering data for PHCC; white shaded region denotes the two-magnon continuum; black line traces the magnon which decays into the continuum. (c) Black dots are the phonon-roton dispersion of superfluid helium extracted from Refs. 25 and 26; inset shows single-particle weight extracted from Refs. 27 and 28 (see Methods). Our solvable model implies that the level approaches the continuum exponentially in the bare level, i.e. $E_k^b \propto \exp(-b \times k \times E_k^{bare})$ (solid red line); here $E_k^b := \varepsilon_k^b - 2\Delta_{roton}$ and $E_k^{bare} := \varepsilon_k^{bare} - 2\Delta_{roton}$, with the bare level $\varepsilon_k^{bare}$ estimated by fitting the roton minimum to a parabola. Moreover, the weight is predicted to go to zero proportional to the level approaching the continuum, i.e. $Z_k \propto a \times k \times |E_k^b|$, confirmed in the inset. In both cases we find that $a_{fit}$, $k_{roton}$ and $b_{fit}$, $\Delta_{roton}$ are comparable to the (inverse) bandwidth, in testament to the strong interactions.

thought that the quasiparticle would enter the two-roton continuum, it is now known that the dispersion instead flattens off, consistent with the discontinuous onset of the two-roton DOS. Here, we add the following quantitative insights. First, the distance to the continuum is exponentially small in the bare energy (red curve). Second, the quasiparticle weight $Z$ decays to zero linearly with this distance; the high-quality data of Refs. 25–28 allows us to extract this information to confirm this prediction, see inset of Fig. 4(c). In fact, these two seemingly unrelated predictions are unified in our theory via the Hellmann-Feynman theorem, which yields $dE^*/dE_b = Z$ (see Methods).

In conclusion, this shows that away from the universal low-energy regime, the excitations of many-body systems are not as unstructured as perhaps expected. Aside from the general message that interactions can prevent or even undo quasiparticle decay, our model can be used to derive functional relationships between a priori unrelated quantities, to extract fundamentally interesting information such as the strength of interactions from experiment—as showcased for superfluid helium. Our work also implies that the existence of quasiparticle decay is not the default option, but instead places considerable constraints on underlying physical processes.

All of these insights taken together suggest the possibility of using interactions to control, in particular to stabilise, the behaviour of quantum matter by employing, rather than combating, strong interactions.

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METHODS

Exactly solvable model. We couple a bare state |ψ⟩ with bare energy E₀ to a continuum of states |φₐ⟩ with bare energies Eₐ. I.e. ˚H = ˚H₀ + γ ˚V, where

$$\hat{H}_0 = E_0 \langle \psi | \psi \rangle + \int d\alpha E_\alpha \langle \phi_\alpha | \phi_\alpha \rangle,$$

$$\hat{V} = \int d\alpha \left( |\psi \rangle \langle \phi_\alpha | + |\phi_\alpha \rangle \langle \psi | \right).$$

The continuous label α satisfies $\langle \phi_\alpha | \phi_\beta \rangle = \delta(\alpha - \beta)$ and the density of states of the continuum is denoted as ν(E). For convenience, we define our origin to be at the onset of the continuum (i.e. in the notation of the main text, $E_{\text{th}} = 0$).

It is useful to consider the single-particle Green’s function $G(E) = \langle \psi | (E - \hat{H})^{-1} | \psi \rangle$. One can nonperturbatively derive that $G(E)^{-1} = E - E_0 - \gamma^2 g(E)$ where we have defined $g(E) := \int \frac{\nu(E) dE}{\pi}$. A detailed derivation can be found in the
Supplemental Materials. Note that \( \lim_{E \to -\infty} G(E)^{-1} = -\infty \) and \( \lim_{E \to 0^-} G(E)^{-1} = -E_0 - \gamma^2 g(0^-) \). Since \( G'(E) > 0 \), the existence of a (unique) pole at \( E^* \) below the continuum (i.e. \( E^* < 0 \)) is equivalent to \( G(0^-) > 1 \), on which its turn is equivalent to \( \gamma^2 > E_0/g(0^-) \). If \( \nu(0^+) \neq 0 \) (i.e. the DOS has a discontinuous onset), then the integral defining \( g(0^-) \) diverges, hence any nonzero \( \gamma \) will give rise to a pole below the continuum. We note that an equivalent treatment can be found in Ref. 9.

To obtain the single-particle weight \( Z = \langle |\langle \psi|\psi^\ast\rangle|^2 \rangle \) (where \( |\psi^\ast\rangle \) is the wavefunction with energy \( E^* < 0 \)), consider that the weight of the delta function \( \delta(E - E_0 - \gamma^2 g(E)) \) is given by the inverse derivative of its argument, i.e. \( Z = 1/\gamma^2 g(E^*) \). Moreover, for large \(|\gamma|\), we have the relationship \( E^* = \gamma^2 g(E^*) \). In particular, from this one can derive that \( E^* \to -\infty \) as \(|\gamma| \to \infty \). We thus have that

\[
\lim_{|\gamma| \to \infty} Z = \lim_{E \to -\infty} \left( 1 - E \frac{g'(E)}{g(E)} \right)^{-1}.
\]  

(5)

To evaluate this, we need the asymptotic behaviour of \( g(E) \). If \( \nu(E) \) has finite support, then \( g(E) \approx \frac{1}{E} \int \nu(\varepsilon) \, d\varepsilon \) as \( |E| \to \infty \). Plugging this into Eq. (5), we obtain \( Z \to 1/2 \) as claimed in the main text.

If \( \nu(E) \) is not bounded but instead decays as \( \nu(E) \sim \beta/E^\alpha \) with \( \alpha > 0 \) as \( E \to +\infty \), then by the theory of Stieltjes transforms\( ^{13,14} \)

\[
g(E) \sim_{E \to -\infty} \begin{cases} \frac{-\beta E^{\min(1, \alpha)}}{\beta(E)} & \text{if } 0 < \alpha \neq 1 \\ \beta(E) & \text{if } \alpha = 1 \end{cases} (\beta > 0).
\]  

(6)

From these asymptotics, we obtain

\[
\lim_{|\gamma| \to \infty} Z = \begin{cases} 1/2 & \text{if } \alpha \geq 1 \\ 1/(1 + \alpha) & \text{if } 0 < \alpha < 1 \end{cases}.
\]  

(7)

Note that this is lower bounded by \( 1/2 \). In particular, for \( \nu(E) \propto 1/\sqrt{E} \), we obtain \( Z \to 2/3 \) as \(|\gamma| \to \infty \).

In Fig. 1, we plot the weight of the bare state \( |\psi\rangle \) on the excited states, i.e. \( A(E) := \sum_n \langle |\psi|n\rangle^2 \delta(E - E_n) \). We calculate it from the identity \( A(E) = \frac{1}{4} \operatorname{Im} G(E - i0^+) \). A straightforward calculation (included in the Supplemental Materials) gives

\[
A(E) = \begin{cases} \frac{1}{4} \left( \Gamma(E)^{1/2} \overline{\Gamma(E)}^{1/2} + \overline{\Gamma(E)}^{1/2} \right) & \text{if } \nu(0) = 0 \\ \frac{1}{4} \delta(E - E_b - \gamma^2 g(E)) & \text{if } \nu(0) \neq 0 \end{cases},
\]  

(8)

where \( \Gamma(E) := \gamma^2 \nu(E) \). Within the continuum (i.e. \( \nu(E) \neq 0 \)), Eq. (8) can qualitatively be interpreted as a Lorentzian with an energy-dependent HWHM \( \Gamma(E) \), and an energy-dependent mean \( E_b + \gamma^2 g(E) \). Note that for \( g(E) \) to be well-defined in the continuum, one has to interpret it as a Cauchy principal value.

More precisely, for the left column of Fig. 1, we consider the DOS

\[
\nu(E) = \begin{cases} 0 & \text{if } E \leq E_0 \\ \nu_0 \sqrt{E} & \text{if } E > E_0 
\end{cases}
\]  

(9)

which is what one expects for the two-particle continuum of a one-dimensional gapped model\( ^{10} \). A straightforward calculation gives

\[
g(E) = \nu_0 \int_0^\infty \frac{d\varepsilon}{\sqrt{\varepsilon(E - \varepsilon)}} = \begin{cases} \frac{\nu_0}{\sqrt{E}} & \text{if } E < 0 \\ 0 & \text{if } E > 0 \end{cases}.
\]  

(10)

We set \( \nu_0 = 1 \). In the top panel we take \( \gamma = 0.2 \), whereas in the second panel, \( \gamma = 0.7 \). We consider the hypothetical scenario where the onset of the continuum is at \( \omega_{\text{min}} = 2 - \cos(k) \), where \( k \) can physically be thought of as (total) momentum. Moreover, we take the bare level to be flat, \( \omega_0 = 2 \). In terms of our earlier variable, where the DOS has its onset at \( E = 0 \), we can thus say that \( E_b = \omega_0 - \omega_{\text{min}} = \cos(k) \).

For the right column of Fig. 1, we consider the DOS

\[
\nu(E) = \begin{cases} 0 & \text{if } E < 0 \text{ or } E_m < E, \\ \nu_0 \sqrt{E(E_m - E)} & \text{if } 0 \leq E \leq E_m, 
\end{cases}
\]  

(11)

which is what one expects for the two-particle continuum of a three-dimensional gapped model\( ^{10} \). This has a square-root onset at \( E = 0 \) and a square-root termination at \( E = E_m \). We obtain

\[
g(E) = \begin{cases} \frac{\pi}{2} \nu_0 (E - E_m/2) & \text{if } 0 < E < E_m, \\ \nu_0 (E - E_m/2 - \sqrt{1 - \frac{E}{E_m}}) & \text{otherwise} \end{cases}.
\]  

(12)

Given our earlier results, we know that there will not always be an isolated state below the continuum. Instead, there is a threshold value \( \gamma_{\text{th}} = \sqrt{E_b/|g(0^-)|} = \sqrt{2E_b/(\pi^2 \nu_0 E_m)} \). If \( E_b > 0 \), an isolated state exists below the continuum if and only if \(|\gamma| > \gamma_{\text{th}} \).

We again consider \( \nu_0 = 1 \), \( \omega_{\text{min}} = 2 - \cos(k) \) and \( \omega_0 = 2 \), but now we also have to choose an upper threshold energy: \( \omega_{\text{max}} = 5 + \cos(k) \). The top panel has \( \gamma = 0.2 \), whereas the bottom panel has \( \gamma = 0.5 \). We note that the minimum interacting strength for which there is a state below the continuum for all values of \( k \) is \( \gamma = \sqrt{\frac{2}{\pi \nu_0} \times \sqrt{1/(2 + 3 \sec(k))}} \) at \( k = 0.357 \).

Finally, with regards to Fig. 1, we mention that we also plot the real part of complex poles when they exist. We see that their location nicely agrees with where the intensity of \( A(E) \) is largest. Moreover, the data in Fig. 1 has been convoluted with a gaussian with \( \sigma = 0.025 \) (in units shown). This is to give the delta-function outside the continuum a visible width.

**Ising ladder.** In Fig. 2(b), we plot the dynamical spin structure factor \( S^{zz}(k, \omega) = \frac{1}{2} \sum_n \langle \sigma^{zz}_{A,k}(0)|\sigma^{zz}_{A,k}(0)e^{i\omega t}|0\rangle^2 dt \) of the spin-1/2 ladder defined in the main text. This quantity is very useful, as similarly to \( A(E) \) considered in the solvable model, it tells us about weight on energy eigenstates. More precisely, \( S^{zz}(k, \omega) = \sum_n \langle \sigma^{zz}_{A,k}(0)|\sigma^{zz}_{A,k}(0)e^{i\omega t}|0\rangle^2 \). We calculated these dynamical spin-spin correlations by first using DMRG to obtain the ground state\( ^{12} \) and subsequently time-evolving \( \sigma^{zz}_{A,k}(0) \) using a matrix-product-operator-based method\( ^{13,14} \). We found that a timestep truncation of \( dt = 0.1 \) and a low bond dimension of \( \chi = 30 \) was enough to achieve converged results. We used linear prediction\( ^{32} \) and multiplication by a gaussian to soften the effects of Fourier-transforming a finite-time window. This introduces an effective broadening corresponding to a convolution with a gaussian with \( \sigma = 0.055 \) in the units shown in Fig. 2.

The values of the parameters for the top panel in Fig. 2(b) are \( g_B = 0.5 \), \( J_B = 1 \) and \( \gamma = 0.3 \). If we now ramp up the coupling strength \( \gamma \), however, this effectively renormalises the parameters of the Ising chain. This is because \( H_{\text{Isat}} \) is not purely an interaction term: it contains an \( S_z \) on the Ising chain, which attempts to condense the domain walls and cause a phase transition. To prevent this, whilst ramping up \( \gamma \) we also change the parameters \( J_B \) and \( g_B \) such that the location of the continuum (shaded region in Fig. 2(b)) remains roughly
unchanged. Thus, for the bottom panel, we arrive at $g_b = 0.9$, $J = 3$ and $\gamma = 3.4$. The location of the continuum has been determined by numerically extracting the dispersion of a single domain wall.

**Dynamics of the TLHAF.** In Fig. 3(a), we consider the out-of-plane dynamical spin structure factor $S^{yy}(t, \omega) = \frac{1}{2} \int \langle \hat{S}_y(0)\hat{S}_y(t)\rangle e^{i\omega t} dt$ of the Hamiltonian in Eq. (2), where we take the $120^\circ$ order to be in the $xz$-plane. This can be obtained by the methods mentioned in the case of the Ising ladder (including linear prediction), extended to the case of cylindrical geometry; for more details, see Refs. 21 and 22. For the data in this work, the cylinder has a circumference $L_{circ} = 6$. We checked that whilst the multimagnon continuum still had a dependence on $L_{circ}$, the single-magnon dispersion is better converged in $L_{circ} -$at least for the middle- and high-energy modes of interest. One way we checked this is by comparing energies at points which are equivalent in 2D but not on the cylinder geometry, and finding that they agree.

Due to the absence of continuous symmetry in the ground state, the large coordination number of the lattice, and the fact that the isotropic point has three Goldstone modes, it is numerically challenging to time-evolve this highly-entangled state. For this reason we are limited in the bond dimensions that we can reach: $\chi = 450$ for long-time dynamics necessary for resolving high-energy modes, and $\chi = 800$ for short-time dynamics for low-energy modes (see discussion below).

The numerical parameters for Fig. 3(a) correspond to a timestep truncation $dt = 0.05 J$, a bond dimension $\chi = 450$, and an effective gaussian broadening with $\sigma = 0.077 J$. The dotted line in Fig. 3(a) is the sum $\varepsilon_q + \varepsilon_K$, where $q$ is along the orange line in the inset. Here $\varepsilon_q$ was obtained by tracing the peak of the spectral function along that slice; $\varepsilon_K$ is a low-energy feature which could not be resolved with the bond dimension $\chi = 450$. Instead, we went up to $\chi = 800$, which limited the time-window we could obtain, leading to a larger effective broadening. However, since the low-energy mode is well-separated from other (relevant) modes, one can still reliably extract the energy from a broad response. From a scaling in bond dimension, we then obtained $\varepsilon_K \approx 0.3 J$ for the value $\delta = 0.05$. This extrapolation is represented visually in the Supplemental Materials. This is markedly lower than the LSWT prediction, $\varepsilon_K^{SWT} \approx 0.41 J$.

The magnon dispersion in Fig. 3(c) was obtained by tracing the low-energy peak of the spectral function—having verified that the magnon branch was resolved enough for this to be sensible. At low energies, this was supplemented by the aforementioned approach where we could go up to $\chi = 800$. Due to the cylindrical geometry on which our method is based, the dispersion we obtain is continuous along one direction, and discrete along the other. We then superimposed the momentum cuts along three different orientations and subsequently interpolated this to the full two-dimensional Brillouin zone. The fact that where these cuts intersected, they agreed, is a confirmation that the circumference $L_{circ} = 6$ is large enough for the single-magnon dispersion to resemble the true two-dimensional result. As a sanity check for our interpolation method, we have verified that it gives the correct result when applied to the LSWT dispersion, as shown in the Supplemental Materials.

**Experimental data for the TLHAF.** In the inelastic neutron scattering data for $\text{Ba}_3\text{CoSb}_2\text{O}_9$ in Fig. 4(a), the momentum-cut is along $K$–$K'$. In the inset of Fig. 3(a), $K'$ is shown as a corner point of the (first) BZ. However, in the experiment, $K'$ was taken in the second BZ (which differs from the other choice by a reciprocal lattice vector). This difference has no bearing on the bands one picks up, so for our purposes this distinction is irrelevant. It does, however, affect the precise value of the intensity. This explains why Fig. 4(a) is not left-right symmetric.

**Subtleties near and at the isotropic point of the TLHAF.** The decay process $k \rightarrow K + (k - K)$ accounts for the complete decay region (as predicted by LSWT) only at the isotropic point ($i.e. \delta = 0$). For $\delta \neq 0$, this process represents the core of the decay region, which is then slightly extended by considering $k \rightarrow q + (k - q)$ with $q \approx K$. One consequence is that the minimum predicted by the principle of avoided decay is only precisely at $Y_1$ at the isotropic point. Indeed, in Fig. 3(c) one can see that the minimum (for $\delta = 0.05$) has been slightly shifted inward, albeit not very substantially so.

Interestingly, at the isotropic point $\delta = 0$, absence of decay is equivalent to the magnon dispersion $\varepsilon_K$ being periodic with respect to the magnetic BZ—which is three times smaller than the original BZ. (This can be derived from the fact that $\varepsilon_K = 0$ for $\delta = 0$.) This powerful criterion might help to figure out the extent of (avoided) decay at the isotropic point, be it using numerical or experimental methods.

**Relationship between $E_b$, $E^*$ and $Z$.** In the main text, we alluded to the general relationship $dE^*/dE_b = Z$. This is a general property of our model. To prove this, first rewrite

$$\frac{dE^*}{dE_b} = \frac{d}{dE_b} \langle \psi^* | \hat{H} | \psi \rangle = \langle \psi^* | \frac{d\hat{H}}{dE_b} | \psi \rangle,$$

where we used the Hellmann-Feynman theorem to move the derivative inside. The proof is finished by noting that Eq. (3) implies $\frac{d\hat{H}}{dE_b} = |\psi\rangle \langle \psi|$. A few comments relevant to the case of superfluid helium. As shown in the Supplemental Materials, the two-roton continuum has a jump discontinuity. Hence, let us consider the case where $\nu(E)$ has a discontinuous onset $\nu_0$. Then a straight-forward computation shows that $g(E) \sim \nu_0 \ln(-E) + \text{const}$, for $E$ small and negative. Hence, remembering the condition we derived above ($E^* = E_b + \gamma^2 g(E^*)$), we see that as $E^* \rightarrow 0^-$, we have the functional relationship $\nu_0 \ln(-E^*) = E_b + \text{const}$, i.e. $E^* \propto \exp(-E_b/\nu_0)$. Using the fact that $\nu_0 \sim 1/k$, we obtain the formula mentioned in the main text. Using the general relationship $dE^*/dE_b = Z$, we also directly obtain the other prediction. In particular, this means that the values of $a$ and $b$ (the parameters mentioned in the main text) should be equal. However, it does not make sense to expect this for the experimental data, as the weight $Z$ extracted in that setting is usually only defined up to a global (momentum-independent) multiplicative factor.

**Experimental data for helium.** With regard to the experimental data for helium, the quasiparticle dispersion relation was straightforwardly extracted from Refs. 25 and 26. The weight, however, is more subtle: Refs. 27 and 28 showed the data as a function of momentum, which we extracted and interpolated. We then evaluated this interpolated function at the same momenta for which Refs. 25 and 26 quoted values for the energy. This allowed us to plot $Z$ as a function of energy in the inset of Fig. 4(c).
Finally, by Cramer’s rule, we can express
\[ g \]
In other words,
\[ g \]
Since \( \phi \)
Supplemental Materials: “Strong quantum interactions prevent quasiparticle decay”

I. THE INTERACTING SINGLE-PARTICLE GREEN’S FUNCTION AND SPECTRAL FUNCTION

We will first calculate \( G(E) = \langle \psi | (E - \hat{H})^{-1} | \psi \rangle \). If we think of \( E - \hat{H} \) as a matrix (with indices labeled by \( | \psi \rangle \) and \( | \varphi_\alpha \rangle \)), then \( G(E) \) is the top left element of its inverse. This is easily calculated. Schematically, first write
\[
E - \hat{H} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}
\]
with \( A = E - E_b \), \( D_{\alpha\beta} = (E - E_\alpha) \delta(\alpha - \beta) \) and \( B_\alpha = C_\alpha = -\gamma \). (S1)

Since \( D \) is diagonal, one can apply the well-known result that
\[
\det(E - \hat{H}) = \det(A - BD^{-1}C) \det D = \left( E - E_b - \gamma^2 \int d\alpha \frac{1}{E - E_\alpha} \right) \det D.
\]
Finally, by Cramer’s rule, we can express
\[
G(E) = \langle \psi | (E - \hat{H})^{-1} | \psi \rangle = \frac{\det D}{\det(E - \hat{H})} = \frac{1}{E - E_b - \gamma^2 g(E)},
\]
where we have introduced the Cauchy principal value \( g(E) := \int_0^\infty \frac{\nu(\epsilon)}{E - \epsilon} d\epsilon \), as in the main text.

Note that
\[
g(E - i\eta) = \int_0^\infty \nu(\epsilon) \frac{E - \epsilon + i\eta}{(E - \epsilon)^2 + \eta^2} d\epsilon, \quad \text{hence} \quad \frac{1}{\pi} \text{Im} \ g(E - i0^+) = \int_0^\infty \nu(\epsilon) \delta(E - \epsilon) = \nu(E).
\]
In other words, \( g(E - i0^+) = g(E) + i\pi \nu(E) \).

We can now calculate \( \mathcal{A}(E) = \frac{1}{\pi} \text{Im} G(E - i0^+) \) as follows,
\[
\mathcal{A}(E) = \frac{1}{\pi} \text{Im} \left( \frac{1}{E - E_b - \gamma^2 g(E - i0^+) - i0^+} \right) = \frac{1}{\pi} \text{Im} \left( \frac{1}{E - E_b - \gamma^2 g(E) - i\pi \nu(E) - i0^+} \right)
\]
\[
= \begin{cases} 
\frac{\gamma^2 \nu(E)}{(E - E_b - \gamma^2 g(E))^2 + (\pi \nu(E))^2} & \text{if } \nu(E) \neq 0, \\
\delta(E - E_b - \gamma^2 g(E)) & \text{if } \nu(E) = 0.
\end{cases}
\]

II. TWO-PARTICLE DOS

A. Quadratic dispersion

Here we calculate the two-particle DOS in various dimensions \( D \) for the single-particle dispersion
\[
\varepsilon_k = \Delta + \frac{|k|^2}{2m}.
\]

1. \( D = 1 \)

\[
\rho_2^{(1D)}(k, \varepsilon) \propto \int \delta(\varepsilon_q + \varepsilon_{k-q} - \varepsilon) dq = \int \delta \left( 2\Delta - \varepsilon + \frac{q^2 + (k - q)^2}{2m} \right) dq
\]
\[
= \int \delta \left( 2\Delta - \varepsilon + \frac{1}{2m} \left( 2 \left( q - \frac{k}{2} \right)^2 + \frac{k^2}{2} \right) \right) dq
\]
\[
\propto \frac{1}{m} \times \left. \frac{1}{q - \frac{k}{2}} \right|_{q = k/2} = \frac{1}{\sqrt{\varepsilon - 2\Delta - \frac{k^2}{4m}}}. \quad (S11)
\]
\(\rho^{(2D)}_2(k, \varepsilon) \propto \int \delta(\varepsilon_q + \varepsilon_{k-q} - \varepsilon) \, d^2q = \int \delta \left( 2\Delta - \varepsilon + \frac{q_x^2}{2m} + \frac{(k_x - q_x)^2}{2m} + \frac{(k_y - q_y)^2}{2m} \right) \, dq_x dq_y \) (S12)

\[= \int \rho_2^{(1D)} \left( k_x, \varepsilon - \frac{q_y^2}{2m} - \frac{(k_y - q_y)^2}{2m} \right) \, dq_y = \int \sqrt{\frac{m}{\varepsilon - 2\Delta - \frac{q_y^2}{2m} - \frac{(k_y - q_y)^2}{2m} - \frac{k_x^2}{2m}}} \, dq_y \] (S13)

\[= \int \frac{m}{\sqrt{\varepsilon - 2\Delta - (k_x^2 + k_y^2)/4m} - (q_y - k_y/2)^2} \, dq_y \propto \begin{cases} m & \text{if } \varepsilon > 2\Delta + \frac{k_x^2 + k_y^2}{4m} \varepsilon, \\ 0 & \text{otherwise}. \end{cases} \] (S14)

3. \(D \geq 3\)

\[\rho_2^{(3D)}(k, \varepsilon) \propto \int \delta(\varepsilon_q + \varepsilon_{k-q} - \varepsilon) \, d^Dq \propto \int \left( \frac{1}{|\partial_\theta \varepsilon_{k-q}|} \sin \theta \right)_{|\varepsilon_q + \varepsilon_{k-q} = \varepsilon} q^{D-1} dq \] (S15)

Since \(|k - q|^2 = k^2 + q^2 - 2kq \cos \theta\), we have that \(\partial_\theta \varepsilon_{k-q} = \frac{kq}{m} \sin \theta\). Hence \(\rho_2(k, \varepsilon) \propto \frac{m}{k} \int q^{D-2} dq\). To determine the range of integration, it is useful to first define \(\delta\) through \(\varepsilon = 2\Delta + \frac{k_x^2}{4m} + \frac{\delta}{m}\). From the condition that \(\varepsilon = \varepsilon_q + \varepsilon_{k-q}\) and that \(|\cos \theta| \leq 1\), we obtain the condition on \(q\), i.e. \(|\sqrt{\delta} - k/2| \leq q \leq \sqrt{\delta} + k/2\). Note that this only makes sense if \(\delta \geq 0\), i.e. it is the correct variable to use to describe the onset of the DOS. Plugging this in and using that \(\delta\) is small, we obtain

\[\rho_2(k, \varepsilon) \propto \frac{m}{k} q^{D-1} |\sqrt{\delta} - k/2| \propto \frac{m}{k} q^{D-1} \left[ \left( 1 + \frac{\sqrt{\delta}}{k} \right) - \left( 1 - \frac{\sqrt{\delta}}{k} \right) \right] \approx \frac{m}{k} q^{D-1} \left[ \left( 1 + (D-1) \frac{\sqrt{\delta}}{k} \right) - \left( 1 - (D-1) \frac{\sqrt{\delta}}{k} \right) \right] \approx \frac{mk^{D-3}\sqrt{\delta}}{k^{D-3}} \propto m^{3/2} k^{D-3}\sqrt{\varepsilon - 2\Delta - \frac{k^2}{4m}}. \] (S17)

Hence, for \(D \geq 3\), the onset always has a square-root onset. However, the above derivation is only for a narrow window near the onset, and this window vanishes as one approaches \(k \to 0\). Indeed, one can straightforwardly calculate that

\[\rho_2(k = 0, \varepsilon) \propto \int \delta \left( 2\Delta - \varepsilon + \frac{q^2}{m} \right) q^{D-1} dq \propto m^{D-2} \varepsilon = m^{D/2} (\varepsilon - 2\Delta)^{D/2-1}. \] (S18)

This is physically the behaviour that will dominate near \(k \approx 0\).

**B. Roton minima**

We now consider the dispersion relevant to the roton minimum appearing in, for example, superfluid helium,

\[\varepsilon_k = \Delta + \frac{1}{2m} (|k| - K)^2. \] (S19)

Here we restrict ourselves to \(D \geq 3\), where we can use Eq. (S15). Since \(|k - q| = \sqrt{k^2 + q^2 - 2kq \cos \theta}\), we have that

\[\partial_\theta \varepsilon_{k-q} = \frac{1}{m} \times (|k - q| - K) \times \frac{1}{2|k - q|} \times 2kq \sin \theta. \] (S20)
We are interested in $0 < k < 2K$, where the threshold is near $\varepsilon \approx 2\Delta$, which forces the decay products to be very close to the roton minimum, i.e. $q \approx K$ and $|k-q| \approx K$. Hence, near the threshold we have

$$\rho_2(k, \varepsilon) \propto m \int \left( \frac{|k-q|}{|k-q-K|} \right)^{\frac{q^D-2}{k}} dq \approx \frac{mK^{D-1}}{k} \int \left( \frac{1}{\sqrt{2m(\varepsilon_k - q - \Delta)}} \right)^{\varepsilon_k+\varepsilon_{k-q}=\varepsilon} dq$$  \hspace{1cm} (S21)

$$= \frac{mK^{D-1}}{k} \int_{K-\sqrt{2m(\varepsilon-2\Delta)}}^{K+\sqrt{2m(\varepsilon-2\Delta)}} \frac{1}{\sqrt{2m(\varepsilon - 2\Delta) - (q - K)^2}} dq$$  \hspace{1cm} (S22)

$$= \frac{mK^{D-1}}{k} \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} dx \left( \text{where } x := \frac{q - K}{\sqrt{2m(\varepsilon - 2\Delta)}} \right)$$  \hspace{1cm} (S23)

$$\propto \left\{ \begin{array}{ll}
0 & \text{if } \delta < 0 \\
\frac{mK^{D-1}}{k} & \text{if } \delta > 0
\end{array} \right. \text{ where } \varepsilon \approx 2\Delta + \delta.$$  \hspace{1cm} (S24)

We repeat that the above derivation is for $0 < k < 2K$. We conclude that in this regime, there is a jump discontinuity at the onset.

III. DMRG ANALYSIS OF 2D TLHAF

![FIG. S1](image)

(a) By scaling in bond dimension ($\chi = 400, 500, 600, 700, 800$), we find that $\varepsilon_K \approx 0.3J$ for $L_{\text{circ}} = 6$ and $\delta = 0.05$. (b) The dispersion for $\delta = 0.05$ that we obtained numerically (in units of $J$) before interpolating to 2D. (c) The LSWT prediction for $\delta = 0.05$ (in units of $J$). (d) The result of first restricting the aforementioned LSWT prediction onto the grid shown in (b) and then using our 2D interpolation method; the fact that this closely agrees with (c) is an indication that our interpolation method is reliable.

Fig. S1(a) shows how by scaling in bond dimension, we can get an estimate $\varepsilon_K \approx 0.3J$ for $\delta = 0.05$.

Fig. S1(b) shows the data that we can numerically obtain for the magnon dispersion for a cylinder with circumference $L_{\text{circ}} = 6$, where we have rotated and superimposed the data along three different directions. This is for $\delta = 0.05$. The fact that the values roughly agree when they spatially overlap indicates that the finite-circumference effects are not too strong. We interpolated this data to the two-dimensional BZ to generate Fig. 3(c) in the main text.

To test our interpolation method, we can take the LSWT prediction (Fig. S1(c)) as a test case: restricting this to the same grid as is shown in Fig. S1(b) and then using our 2D interpolation method, we produce Fig. S1(d). We see that this closely agrees with the original dispersion in Fig. S1(c).