Quantum dense key distribution

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This paper proposes a new protocol for quantum dense key distribution. This protocol embeds the benefits of a quantum dense coding and a quantum key distribution and is able to generate shared secret keys four times more efficiently than BB84 one. We hereinafter prove the security of this scheme against individual eavesdropping attacks, and we present preliminary experimental results, showing its feasibility.

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I. INTRODUCTION

Quantum dense coding (QDC) and quantum key distribution (QKD) are two direct applications of fundamental quantum mechanics, both involving two parties, Alice and Bob, exchanging some classical information.

By QKD Alice and Bob exchange secret random keys to implement a secure encryption-decryption algorithm (one-time pad) without meeting, and the security of the distributed keys is based on the laws of quantum physics.

Theoretically proposed in [2], QDC basically doubles the capacity of transmission of a classical channel by local operations on one particle of the EPR pair shared by the two parties. QDC has received only partial experimental verification [3] using polarization entangled states of photons because of the inefficiently implemented Bell’s state analysis [4] and a lack of security in the transmitted information.

Recently, Bostroem and Felbinger [5] gave birth to the original idea of a protocol encoding secure information by local operations on an EPR pair, though rather proposing a deterministic and secure transmission than implementing a truly dense coding scheme.

In this paper we propose a protocol for the quantum dense key distribution (QDKD) including the advantages of QKD and QDC in generating shared secret keys and enhancing transmission capacity. In Section II we present the protocol, in Section III we prove the security of QDKD against the individual eavesdropping attack, in Section IV the experiment proves the QDKD feasibility, and in Section V we discuss the practical efficiency of the present protocol versus the most efficient QKD ones.

FIG. 1: QDKD scheme: particle B of an EPR pair |ψAB⟩ is sent from Alice to Bob and backward. Alice’s and Bob’s encoders selecting the local operation on particle (I_B or Z_B) together with the incomplete Bell’s measurement by Alice establish the key. Some pairs are randomly selected for non-local measurement (the anticorrelation check performed by spatially separated Alice and Bob measurements), as test of the transmission security. The more general eavesdropping attack is represented by the coupling of the EPR pair with a larger Hilbert space in the initial state |ψ_E⟩ by means of unitary operators \( \hat{J}_{BE} \) and \( \hat{K}_{BE} \).

II. QDKD: THE PROTOCOL

The basic scheme of QDKD is presented in Fig. 1. Alice produces pairs of particles in the singlet state |ψ^−_{AB}\rangle = \frac{1}{\sqrt{2}}(|0_A1_B\rangle - |1_A0_B\rangle), and stores particle A in her lab, whereas she acts randomly with gate \( I_B \) or \( Z_B \) on particle B and then sends it to Bob. As \( Z_B|0_B\rangle = |0_B\rangle \), and \( Z_B|1_B\rangle = -|1_B\rangle \), Alice’s random selection of gate \( I_B \) or \( Z_B \) encodes the bits of her secret key on the EPR pair.

\begin{align}
I_B|\psi^−_{AB}\rangle &\rightarrow |\psi^−_{AB}\rangle &\rightarrow \text{bit 0} \\
Z_B|\psi^−_{AB}\rangle &\rightarrow -|\psi^−_{AB}\rangle &\rightarrow \text{bit 1}, \quad (1)
\end{align}
with $|\psi_{AB}^+\rangle = \frac{1}{\sqrt{2}}(|0_A1_B\rangle + |1_A0_B\rangle)$.

Bob randomly switches particle B towards either his measurement or his encoding apparatus. In one case Bob projects particle B on the base $\{|0_B\rangle, |1_B\rangle\}$ while in the other case, Bob, analogously to Alice, randomly acts with $1_B$ or $\hat{Z}_B$ on particle B and then sends it back to Alice. When she receives it, her measurement apparatus performs an incomplete Bell’s state analysis, i.e. a projection on $|\psi_{AB}^+\rangle$ or $|\psi_{AB}^-\rangle$ of the two-particle state composed by the previously stored particle A and particle B. When Bob’s apparatus projects particle B, Alice’s measurement apparatus has to project particle A on the base $\{|0_A\rangle, |1_A\rangle\}$ instead of performing a Bell’s state analysis. As Alice prepares only states $|\psi_{AB}^-\rangle$ and $|\psi_{AB}^+\rangle$, Alice and Bob results should be always anticorrelated. The anti-correlation check in Fig. 1, consists in comparing Alice and Bob results, and guarantees the security of the distributed keys against individual eavesdropping attack [3].

After repeating this protocol enough times to produce the keys, Alice discloses the results of her measurements (\(|\psi_{AB}^+\rangle, |\psi_{AB}^-\rangle, |0_A\rangle, |1_A\rangle\) publicly exposing solely to the possibility of being monitored but not modified. Alice’s measurements of $|\psi_{AB}^+\rangle$ and $|\psi_{AB}^-\rangle$ indicates the generation of the keys. In fact because Bob is aware of Alice’s measurement results (as well as of his own operation $1_B$ or $\hat{Z}_B$), he can extract the bit encoded by Alice and vice versa. Specifically, measuring $|\psi_{AB}^+\rangle$ means that Alice and Bob encoded 0 and 1 or 1 and 0, respectively (Alice performed the operation $1_B$ and Bob $\hat{Z}_B$ or vice versa). The measurement of $|\psi_{AB}^-\rangle$, instead, means that Alice and Bob encoded the same bit, 0 or 1 (Alice and Bob performed the same operations on particle $1_B$ or $\hat{Z}_B$).

Thus, two keys are generated in this process, one produced by Alice and one by Bob. The key generation is dense because any travelling particle B carries two exchanged bits, one belonging to key A and one to key B.

Since Alice’s results, disclosed publicly, to the sum (mod 2) of the bits of the two keys, i.e. $0 \oplus 0 = 1 \oplus 1 = 0$ and $0 \oplus 1 = 1 \oplus 0 = 1$, a possible eavesdropper (Eve) on the public channel can intercept some information by correlating the two keys, though she does not get information on the single key. In other words, key B is used to encrypt key A and vice versa, and the results disclosed by Alice are the ”encrypted key A” sent to Bob. In fact both keys are used twice, first to encrypt the other key, and then to encrypt Alice and Bob messages.

This obviously induces a lack of security if the one-time-pad protocol is implemented with both keys straightforwardly. For this reason only key A is used directly in the implementation of the one-time-pad protocol. The discussion on how Alice and Bob can lower Eve’s information on key B in order to generate a “new” secret key B is a typical task in the field of computer science and it is outside the scope of this paper.

Here we simply observe that Eve does not have any direct information on the single key but only on the correlation between the random bits in specific position of the two keys. If Alice and Bob have common secret algorithms for the permutation of the position of the bits of one of the two key (e.g. key B), they can strongly reduce Eve’s information on the two key. To guarantee the complete security of the encrypted message we suggest to use only key A for the one time pad, while the scrambled key B can be used as a source for a cipher to generate a new key.

### III. QDKD: The Security Proof

Let’s consider now the individual Eve’s attack (Fig. 1) consisting in coupling particle B with an ancilla system in the initial state $|\psi\rangle$ by means of a general unitary operator $\hat{J}_{BE}$. The Hilbert space $A-B$ is widened by coupling with the N-dimensional Hilbert space E of the ancilla system. After Bob’s operations, Eve couples again her ancilla to system $A-B$ by applying another unitary operator $\hat{K}_{BE}$. Eventually she performs any POVM measurement allowed by the laws of quantum mechanics on the ancilla system E. Note that this approach is general because any physical non-unitary interaction is equivalent to a unitary one with a higher dimensional ancilla space [4]. We represent the coupled system $A-B-E$ after Bob’s and Eve’s operations as

$$
\hat{K}_{BE}1_B\hat{J}_{BE}|\psi_{AB}^+\rangle \otimes |e_E\rangle = |\mu_{ABE}^+\rangle,
\hat{K}_{BE}1_B\hat{J}_{BE}|\psi_{AB}^-\rangle \otimes |e_E\rangle = |\mu_{ABE}^-\rangle,
\hat{K}_{BE}\hat{Z}_B\hat{J}_{BE}|\psi_{AB}^+\rangle \otimes |e_E\rangle = |\nu_{ABE}^+\rangle,
\hat{K}_{BE}\hat{Z}_B\hat{J}_{BE}|\psi_{AB}^-\rangle \otimes |e_E\rangle = |\nu_{ABE}^-\rangle.
$$

As Alice prepares only states $|\psi_{AB}^\pm\rangle$, with probability $\frac{1}{2}$, the results of the anti-correlation check performed by Alice and Bob should always be anticorrelated. Eve’s action induces a bit flip on particle B giving correlated results. This is basically the signature of Eve’s presence.

We estimate now the probability $P_{corr}$ of correlated results in the anti-correlation check

$$
P_{corr} = \text{tr}(|\hat{J}_{BE}\hat{\rho}_{AB} \otimes |e_E\rangle\langle e_E|\hat{J}_{BE}^\dagger (\hat{P}_{00} + \hat{P}_{11})|),
$$

where $\hat{P}_{00} = |0_A0_B\rangle\langle 0_A0_B|$, $\hat{P}_{11} = |1_A1_B\rangle\langle 1_A1_B|$, are projection operators and $\hat{\rho}_{AB} = 1/2|\psi_{AB}^+\rangle\langle \psi_{AB}^+| + 1/2|\psi_{AB}^-\rangle\langle \psi_{AB}^-|$. From Eqs. 2 and the properties of unitary operators $\hat{K}_{BE}$ and $\hat{J}_{BE}$, it can be demonstrated that

$$
P_{corr} = \frac{1}{2}(1 + \langle \mu_{ABE}^+|\nu_{ABE}^-\rangle).
$$

The mutual information $I_{s,r}$ between the sender (s), preparing and sending states $\hat{\rho}_n$ with probability $p_n$, and the receiver (r) satisfies the Holevo bound [7]

$$
I_{s,r} \leq I_{s,r} = S(\hat{\rho}) - \sum_n p_nS(\hat{\rho}_n),
$$
Given, though in some cases too restrictively, by the keys (key A and key B) on the face of Eve’s presence, is Bob’s capability of recovering two secure cryptographic correction and privacy amplification. Alice’s and Bob distill from the noisy and possibly insecure public discussion. System A-B she only knows the results disclosed during any POVM on the final state of her system E, but on system A-B-E. This is obviously not the case, as she can perform any POVM on the final state of the whole system.

Following the same argumentation leading to Eq. (4), the Holevo bounds $I_{A:E}$ and $I_{A:B}$ are expressed in terms of $P_{corr}$ and $q = \langle \mu_{A:B}^+ | \mu_{A:B}^- \rangle$. Fig. 2 plots $I_{B:E}$ and $I_{A:E}$ versus $P_{corr}$ and $q$, showing that the anti-correlation check provides an estimate of the maximum information obtainable by Eve. From this estimate Alice and Bob establish the security level of the bits exchanged. This also implies that Eve cannot avoid to disclose herself when attacking individually the qubit on the quantum channel.

Moreover Eve’s possibilities have been overestimated in deriving $I_{B:E}$ and $I_{A:E}$. In fact Eve is assumed to perform any POVM on the final state of the whole system A-B-E. This is obviously not the case, as she can perform any POVM on the final state of her system E, but on system A-B she only knows the results disclosed during the public discussion.

In any practical implementation of QKD protocols, Alice and Bob distill from the noisy and possibly insecure key a nearly noise-free and secure key by means of error correction and privacy amplification. Alice’s and Bob’s capability of recovering two secure cryptographic keys (key A and key B) on the face of Eve’s presence, is given, though in some cases too restrictively, by the condition $I_{A:B} > I_{A:E}$ and $I_{A:B} > I_{B:E}$, where $I_{A:B}$ is the mutual information between Alice and Bob. $I_{A:B}$ can be simply calculated considering the capacity of a noisy channel of quantum bit error rate $Q$, as $I_{A:B} = 1 - H(Q)$, where $H$ is the Shannon entropy of a binary channel.

To ensure the security of the two generated keys, we replace $I_{A:E}$ and $I_{B:E}$ with the maximums of $I_{A:E}$ and $I_{B:E}$ found in $q = 0$, for any $P_{corr}$. Regardless differences in functions $I_{A:E}$ and $I_{B:E}$, it has been demonstrated that $I_{A:E}|q=0 = I_{B:E}|q=0 = H(P_{corr})$ for any values of $P_{corr}$. This means that Alice and Bob can distill common secret keys when

$$H(Q) + H(P_{corr}) < 1. \quad (6)$$

IV. QKD: THE EXPERIMENT

We implemented experimentally a proof of principle of QKD protocol by the scheme of Fig. 3.

The source of entangled photons in a singlet state is a 0.5 mm long BBO nonlinear crystal (NLC2) pumped by the pulsed laser system (GL, Ti:Sa and NLC1). The bits of the keys are encoded in the phase of the entangled state by means of half-wave plates (HWP), interference filters (IFs), fiber couplers, fibers integrated polarizing beam splitters (PBS), single-photon detectors. M stands for mirror, DM dichroic mirror, L lens, CC birefringence compensation crystals.

FIG. 3: QKD set-up: photon pairs are generated by SPDC in type II nonlinear crystal (NLC2) pumped by the pulsed laser system (GL, Ti:Sa and NLC1). The bits of the keys are encoded in the phase of the entangled state by means of half-wave plates (HWP). The incomplete Bell’s state measurement is performed with an interferometer analogous to the one used in [3] consisting of a 50:50 beam splitter (BS), interference filters (IFs), fiber couplers, fibers integrated polarizing beam splitters (PBS), single-photon detectors. M stands for mirror, DM dichroic mirror, L lens, CC birefringence compensation crystals.
axis parallel to the horizontal polarization state. According to Eq. (1) the HWP on means Alice’s encoding bit 1, while off corresponds to bit 0. The same holds for Bob’s encoding apparatus. The Bell’s state analyzer is a Hong-Ou-Mandel interferometer together with two polarization-measurement systems. To compensate for the optical delays in the interferometer the 50:50 beam splitter (BS) is mounted on a micrometric translation stage. The anti-correlation check consists in polarization measurements performed separately by Alice and Bob on photons A and B respectively, when photon B is reflected by Bob’s BS towards his detection apparatus. As a matter of fact Bob decides the ratio of pairs devoted to this measurement since the reflectivity of his BS sends randomly some photons B towards his polarization measurement system. Note that Alice’s Bell-state-analyzer is also suitable for polarization measurement, namely for the anti-correlation check. Alice’s and Bob’s detection apparatuses are composed by open air-fiber couplers to collect the down-converted light in single-mode optical fibers. The detection of photons according to their polarization is guaranteed by a fiber-integrated polarizing beam splitter (PBS). Photons at the two output ports of PBS are sent to fiber coupled photon counters (Perkin-Elmer SPCM-AQR-14). These are indicated as 1h, 1v, 2h, and 2v in the Alice’s detection apparatus, and as Bh and Bv in the Bob’s anticorrelation check. Interference filters (IF, at 830 nm, 11 nm FWHM) are placed in front of the fiber couplers to reduce stray-light.

Fig. 4 shows the interference profiles when both Alice and Bob encode bit 0 (a), Alice encodes bit 1 and Bob bit 0 (b), both Alice and Bob encode bit 1 (c). Cases (a) and (c), corresponding to a final state $|\psi_{AB}\rangle$, present peaks in the coincidence counts from detectors 1h 2v or 1v 2h and dips in the coincidence from detectors 1h 1v or 2h 2v, since $|\psi_{AB}^\perp\rangle$ is spatially antisymmetric. By contrary case (b), corresponding to the spatially symmetric final state $|\psi_{AB}^\parallel\rangle$, presents the complementary behavior. When the paths of photons A and B differ more than their coherence length, no interference occurs, and one obtains classical statistics for the coincidence count rates at the detectors. For optimal positioning of BS, i.e. indistinguishable photon paths, interferences enables Alice and Bob to read the encoded information. We measured $P_{\text{corr}}$ as the ratio between the sum of coincidences from detectors Bh 1h and Bv 1v corresponding to correlated results, and the sum of coincidences between Bh 1h, Bv 1v, Bh 1v, and Bv 1h, corresponding to both correlated and anticorrelated results. We obtained $P_{\text{corr}} < 0.05$. The presence of correlated counts together with the reduction of the visibility of the interferometric profiles are basically due to losses in optics, detection, electronics, and noise. From the visibility in Fig. 4 we estimated $Q = 3.3\%$, and according with Eq. (4) $H(0.033) + H(0.05) = 0.49 < 1$. 

V. FURTHER DISCUSSION

According to Ref. [13], the theoretical efficiency of a QKD protocol is defined as $\mathcal{E} = b_s/(q_t + b_n)$, where, for every step, $b_s$ is the expected number of secret bits exchanged by the two parties, $q_t$ is the number of qubits exchanged on the quantum channel, and $b_n$ is the number of bits exchanged on the public channel. Any QKD protocol satisfies $\mathcal{E} \leq 1$, while DQKD reaches the limit value $\mathcal{E} = 1$ as $b_s = 2$, $q_t = 1$ and $b_n = 1$, and the bits used for eavesdrop checking are neglected [15].

To our knowledge only two other protocols reach the limit value of $\mathcal{E} = 1$, one proposed by Cabello (high capacity Cabello protocol, HCCP) [14], and one by Long and Liu (high capacity Long Liu protocol, HCLLP) [16]. Both protocols exploit the fact that a possible eavesdropper, with no access to the whole quantum system at the same time, cannot recover the whole information without being detected, and both employ a larger alphabet, a four dimensional orthogonal basis of pure states. HCLLP appears hardly implementable as the detection system should perform a complete discrimination between the four Bell’s states [4]. As to HCCP the detection system is completely analogous to the interferometer implemented in the DQKD experiment. On the contrary, in DQKD the source of photons and the encoding apparatuses are quite simple (a type II parametric down-conversion (PDC) source and two local operations), while they are much more complicated in HCCP, where they could be implemented by using a system to switch between two PDC sources (one type I, and one type II) and three local operations. In conclusion DQKD is a new maximally efficient QKD protocol fully exploiting QDC instead of a larger alphabet, and incorporating the practical advantages of HCCP and HCLLP.

In order to evaluate the effective potential applications of DQKD we analyze the effects of losses, considering the length $L$ of the communication channel between Alice and Bob. As the photon sent from Alice to Bob backwards travels over a distance $2L$, the other photon of the pair should be stored by Alice for a time $2L/c$ (c is the speed of light in the quantum channel), for example in a storage fiber ring of length $2L$. Thus, for DQKD the two photons should be kept decoherence-free for a time $2L/c$, as for HCCP, while for HCLLP they should be kept decoherence-free for a time $3L/c$.

In this respect we define a probability $\mathcal{P}$ that a photon is transmitted over the distance $L$. The practical efficiency, $\eta$, of a QKD protocol can be evaluated by the product between the theoretical efficiency $\mathcal{E}$ and the probability that the photons are not lost in the communication channels. In the case of DQKD (as well as of HCCP) $\eta = \mathcal{P}^2\mathcal{P}$, because both photons travel for a time $2L/c$. In the case of BB84 protocol $\eta = 0.25\mathcal{P}$, as there is only one photon travelling for the path $L$, and the theoretical efficiency is only $\mathcal{E} = 0.25$. An optimized version of the BB84 exploits the possibility that Bob measures the photon sent by Alice only after Al-
ice has disclosed the measurement basis. In this case $\mathcal{E} = 0.5$, but after travelling for the distance $L$, the photon should be stored by Bob for an additional time $L/c$, waiting for Alice classical bits, resulting $\eta = 0.5P^2$.

Eventually, the practical efficiency of a QKD protocol depends on the quality of the quantum channel: if $P \geq 71\%$ the most efficient protocols is still DQKD (as well as HCCP), if $50\% \leq P < 71\%$ the more efficient is the optimized BB84, while if $P < 50\%$ the best performance are obtainable by the original BB84 protocol. For example, if we consider 1550 nm commercial telecommunication fiber, whose attenuation factor is 0.2 dB/km, DQKD protocol is more efficient than BB84 protocols for distance $L$ shorter than 7.4 km, for $L > 15$ km the most efficient is the original BB84, while for intermediate distances the more suitable choice is the optimized BB84. However, the forthcoming advent of a new generation of optical fibers, such as photonic-cristal fibers (PCF) and Bragg fibers with predicted attenuation factors between $10^{-2}$ dB/km and $10^{-4}$ dB/km or less, will lead DQKD protocol advantageous over much longer distances, namely up to $L \simeq 150$ km corresponding to attenuation of $10^{-2}$ dB/km and up to $L \simeq 15000$ km for attenuation of $10^{-4}$ dB/km.

VI. CONCLUSION

In conclusion the proposed protocol embeds the main advantages of two quantum communication applications, namely QKD and QDC, as it allows the generation of secure cryptographic keys using only one travelling-qubit for two bit of classical information.

The protocol has been proved to be secure against individual eavesdropping attack by using a non-local measurement. We proved also that QDKD reaches the maximum of the efficiency of QKD protocols.

We performed a experimental implementation establishing the conditions for a secure QDKD. The experiment in fact proved the feasibility of QDKD by showing high-visibility interferometric profiles as well as low-noise anticorrelation check. This is only the proof of principle of the system described above for QDKD, as we are aware that the actual system is not yet fast enough in switching the encoding operations. We are now focused on the realization of fast switching $\tilde{Z}_B$ gates by using Pockell’s cells and HWPs.

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