Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measure

Xindong Peng\textsuperscript{a, b}, Harish Garg\textsuperscript{b}

\textsuperscript{a} School of Information Science and Engineering, Shaoguan University, Shaoguan 512005, China
\textsuperscript{b} School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala 147004, India

\textbf{ABSTRACT}

This paper aims to give deeper insights into decision making problem based on interval-valued fuzzy soft set (IVFSS). Firstly, a new score function for interval-valued fuzzy number is proposed for tackling the comparison problem. Subsequently, the formulae of information measures (distance measure, similarity measure and entropy) are introduced and their transformation relations are pioneered. Then, the objective weights of various parameters are determined via new entropy method, meanwhile, we develop the combined weights, which can show both the subjective information and the objective information. Moreover, we propose three algorithms to solve interval-valued fuzzy soft decision making problem by Weighted Distance Based Approximation (WDBA), COmbinative Distance-based ASsessment (CODAS) and similarity measure. Finally, the effectiveness and feasibility of approaches are demonstrated by a mine emergency decision making problem. The salient features of the proposed methods, compared to the existing interval-valued fuzzy soft decision making methods, are (1) it can obtain the optimal alternative without counterintuitive phenomena; (2) it has a great power in distinguishing the optimal alternative; and (3) it can avoid the parameter selection problems.

\section{1. Introduction}

During the process of economic and social globalization in the 21st century, the world is undergoing an unheard-of period of crisis so far. A substantial emergency events arose, such as September 11 attacks in 2001, severe acute respiratory syndrome (SARS) in 2003, Wenchuan earthquake in 2008, Tianjin explosion accident in 2015, Jiuzhai Valley earthquake in 2017, and so on, which demolished the country or the society involving massive humans, material resources, economic, or local environment. The influences of the emergency incidents ordinar- by the ability of the affected country or society to dispose of its own resources. As a consequence, the topic about emergency response has touched off the heated discussions in recent times (Ren, Xu, & Hao, 2017). Emergency decision making, one of the significant domains in the emergency response, is urgent to be explored by building the decision making techniques to polish up the efficiency and productiveness of the emergency response, which can enormously reduce the casualties, environment disruption and the economic losses. Every barber knows that one of the momentous characters of the emergency response is timeliness, which implies that the rescue forces are required for taking impactful actions in the short time. Meanwhile, for the time-starved in the emergency response, there exists much uncertain and unknown information. Under such circumstance, the experts make decisions with a great power in distinguishing the optimal alternative rather than in low differentiated alternatives. Furthermore, the indeterminacies of the emergency circumstance determine that the experts can’t offer the precise decision making information when they confront diverse choices. While a wide diverse of the existing theories such as theory of fuzzy set (Zadeh, 1965), theory of probability, and theory of rough set (Pawlak, 1982) have been initiated to simulate vagueness. Each of these theories has its inherent insufficiencies as pointed out in (Molodtsov, 1999) nevertheless. The theory of soft set (SS), pionneered by Molodtsov (Molodtsov, 1999), has come fresh from the deficiency of the parameterization tools of those groundbreaking theories (Pawlak, 1982; Zadeh, 1965). It has potential applications in diverse fields such as rule mining (Feng, Cho, Pedrycz, Fujita, & Herawan, 2016), game theory (Deli & Çağman, 2016), feature selection (Yang, Xiao, Xu, Wang, & Pan, 2016), decision making (Aktaş & Çağman, 2016), parameter reduction (Xie, 2016).

By combining soft sets with other mathematical models, diverse extensions of soft set with their applications have been presented such as fuzzy soft set (FSS) (Alcantud, 2016a, 2016b; Alcantud & Mathew, 2017; Maji, Biswas, & Roy, 2001), Pythagorean fuzzy soft set (PFSS) (Peng, Yang, Song, & Jiang, 2015), intuitionistic fuzzy soft set (IFSS)
(Maji, Biswas, & Roy, 2001), neutrosophic soft set (NSS) (Peng & Liu, 2017), rough soft set (Zhan, Ali, & Mehmood, 2017; Zhan, Liu, & Herawan, 2017; Zhan, Zhou, & Xiang, 2017), hesitant fuzzy soft set (Peng & Dai, 2017; Wang, Li, & Chen, 2014, 2015).

The above extension models are all proposed to deal with uncertainties by taking advantages of soft set. In those extensions of SSs, the value of membership is either a fuzzy value or rough value. But in fact, the membership degree may be an interval value in a SS, so combining the interval-valued fuzzy set with soft set, Son (2007) proposed interval-valued fuzzy soft set (IVFSS). Yang, Lin, and Yang (2009) proposed choice values method for interval-valued fuzzy soft set method. Moreover, there have been some applications of interval-valued fuzzy soft sets such as decision making (Chen & Zou, 2017; Feng, Liu, & Leoreano-Fotead, 2010; Mukherjee & Sarkar, 2014; Peng, Dai, & Yuan, 2017; Peng & Yang, 2017; Xiao, Chen, & Li, 2013; Yang & Peng, 2017; Yuan & Hu, 2012), information measure (Jiang, Tang, Liu, & Chen, 2013; Mukherjee & Sarkar, 2014; Peng et al., 2017; Peng & Yang, 2015), matrix theory (Rajarajeswari & Dhanalakshmi, 2014), algebraic structure (Liu, Peng, Yager, Davvaz, & Khan, 2014; Liu, Feng & Zhang, 2014), parameter reduction (Ma, Qin, Sulaiman, Herawan, & Abawajy, 2014), medical diagnosis (Chetia & Das, 2010).

Due to the drawbacks (counterintuitive phenomena (Peng et al., 2017), discrimination problem (Feng et al., 2010; Peng et al., 2017; Yang et al., 2009; Yuan & Hu, 2012), parameter selection problems (Feng et al., 2010)) of the existing decision making methods for IVFSS, it may be difficult for decision makers to select optimal or convincible alternatives. Therefore, the aim of this paper is to tackle the three challenges mentioned above by developing three MCDM approaches to managing evaluation information for IVFSSs, which not only have a great power in distinguishing the optimal alternative, but also can obtain an optimal alternative out of counterintuitive phenomena and parameter selection problems. The decision making methods are shown in the following.

(1) Weighted Distance Based Approximation (WDBA), pioneered by Rao and Singh (2012), measures the distance from the optimal point (best value of alternatives) and non-optimal point (worst value of alternatives). Finally, the alternatives are ranked by their suitability index (SI). The alternative having the least value of SI is ranked at first position and with the maximum value at last. WDBA approach has already been applied in many fields such as E-learning websites selection (Garg & Jain, 2017; Jain, Garg, Bansal, & Saini, 2016), COTS component selection (Garg, Sharma, & Sharma, 2016), selection of software effort estimation (Bansal, Kumar, & Garg, 2017).

(2) Combinative Distance-based Assessment (CODAS), presented by Ghorabaei, Zavadskas, Turskis, and Antucheviciene (2016), measures the overall performance of an alternative by Euclidean and Hamming distances from the negative-ideal point. The CODAS utilizes the Euclidean distance as the primary measure of assessment. If the Euclidean distance of two alternatives is very close to each other, the Hamming distance is utilized to compare them. The degree of closeness of Euclidean distance is set by a threshold parameter. The Euclidean distance and Hamming distance are measured for $l^2$-norm and $l^1$-norm indifference spaces, respectively (Yoon, 1987). Therefore, in the CODAS method, we first assess the alternatives in an $l^2$-norm indifference space. If the alternatives are not comparable in this space, we go to an $l^1$-norm indifference space. To perform this process, we should compare each pair of alternatives. CODAS approach has already been applied in market segmentation (Ghorabaei, Amiri, Zavadskas, Hooshmand, & Antucheviciene, 2017).

(3) In order to compute the information measure (distance measure, similarity measure and entropy) of two interval-valued fuzzy soft sets, we propose a new method for constructing of distance measure and similarity measure. Meanwhile, some special properties are discussed in detail. Later, we apply the proposed similarity measure for decision making.

Considering that different parameters’ weights will have an effect in the ranking results of alternatives, inspired by Peng et al. (2017) and Peng and Yang (2017), we also develop a novel method to determine the parameter weights by combining the subjective elements with the objective ones. This model is not the same from the existing interval-valued fuzzy soft weight determining methods, which can be divided into two aspects: one is the subjective weighting evaluation methods and the other is the objective weighting determine methods, which can be computed by the new proposed entropy determining method. The subjective weighting methods concentrate on the preference information of the experts (Feng et al., 2010), while they overlook the objective information. The objective weighting determine methods do not take the preference of the decision maker into consideration, in other words, these methods fail to take the risk attitude of the decision maker into account (Xiao et al., 2013). The feature of our weighting model can show both the subjective information and the objective information. Consequently, combining objective weights with subjective weights, a combined model to achieve parameters’ weights is presented. Moreover, our weight determining method has keep in accordance with the references (Peng et al., 2017; Peng & Yang, 2017) when using the corresponding decision making method for achieving the final decision results.

To achieve these goals, the main contributions are as follows:

(i) A new weight determining model is proposed for avoiding the influence of subjective factor and objective factor, respectively.

(ii) The new information measure (similarity measure, distance measure, entropy) for interval-valued fuzzy soft set is presented and the new entropy is used for determining the weight.

(iii) A new score function is examined for tackling the comparison problem.

(iv) Three proposed algorithms with some existing algorithms (Feng et al., 2010; Peng et al., 2017; Yang et al., 2009; Yuan & Hu, 2012) are compared by some examples. Their objective evaluation is carried out, and the methods which maintain consistency of their results are chosen.

The remainder of this paper is organized as follows: In Section 2, we review some fundamental conceptions of soft sets and interval-valued fuzzy sets. In Section 3, new information measure (similarity measure, distance measure, entropy) is conceived and their inner transformation relation is studied. In Section 4, three interval-valued fuzzy soft decision making approaches based on WDBA, CODAS and similarity measure are shown and a numerical example is given to illustrate the proposed methods. In Section 5, we compare the novel proposed approaches with the existing interval-valued fuzzy soft set decision making approaches, and show the effectiveness of the proposed approaches. The paper is concluded in Section 6.

2. Preliminaries

In this section, we will briefly recall the basic concepts of interval-valued fuzzy sets, interval-valued fuzzy soft sets and score function.

Definition 2.1 (Zadeh, 1975). An interval-valued fuzzy set (IVFS) $I$ in $U$ is given by

$$I = \{x, (I^- (x), I^+(x)) \mid x \in U\},$$

where $0 \leq I^- (x) \leq I^+(x) \leq 1$. For simplicity, we call $i = \{I^- (x), I^+(x)\}$ an interval-valued fuzzy number (IVFN) denoted by $i = [i^-, i^+]$. 

Definition 2.2 (Xu and Da, 2002). Let $x = [x^- , x^+]$ and $y = [y^- , y^+]$ be two IVFNs, $\lambda \in [0,1]$, then their operational laws are defined as follows:
(1) \( x + y = [x^- x^+ ] + [y^- y^+] = [x^- + y^- x^+ + y^+] \);
(2) \( \lambda x = [\lambda x^- x^+] \);
(3) \( x^2 = ([x^-]_x^+ , [y^-]_y^+) \);
(4) \( x-y = [x^- - x^+] - [y^- - y^+] = [x^- - y^- x^+ - y^+] \);
(5) \( x = y \) if \( x^-= y^- \) and \( x^+= y^+ \);
(6) \( x^{-1} = [1-x^- -1 x^+] \);
(7) \( -x = [-x^- x^+] \).

**Remark 2.3.** From (1), Peng and Yang (2017) see that it may be not satisfied the definition of IVFN when \( x = [0.4,0.5] \) and \( y = [0.6,0.7] \), but it has functionality of computation. Moreover, for (7), although it can be easily known that it will be out of positive number range, it also has functionality of computation. That is to say, it can see IVFN as a special interval number which can satisfy the above cases.

**Definition 2.4** (Son, 2007). Let \( U \) be an initial universe and \( A \) be a set of parameters, a pair \((F,A)\) is called an interval-valued fuzzy soft set over \( \bar{P}(U) \), where \( \bar{P} \) is a mapping given by \( F: A \rightarrow \bar{P}(U) \). \( \bar{P}(U) \) denotes the set of all interval-valued fuzzy subsets of \( U \), \( \forall \in A \in \bar{F}(\xi) \) is an interval-valued fuzzy subset of \( U \), and it is called an interval-valued fuzzy value set of parameter \( \xi \). Let \( F(\xi) \) denote the membership value that object \( x \) holds parameter \( \xi \), then \( F(\xi) \) can be written as an interval-valued fuzzy set that \( F(\xi) = [x/F(\xi)(x) \in U] = [x/F(\xi)(x) \in F(\xi) (x) \in U] \).

**Definition 2.5** (Son, 2007). The complement of an interval-valued fuzzy soft set \((F,A)\), is denoted by \((F^A,A)^c\) and is defined by \((F,A)^c = (F^A,A)\). \( \forall \in A \in \bar{U} F(\xi) = [1-F(\xi)(1), 1-F(\xi)(x)] \).

In the following, Peng and Yang (2017) give a comparative law based on interval-valued fuzzy soft sets by score function and deviation function.

**Definition 2.6** (Peng and Yang, 2017). Let \((F,A)\) be an interval-valued fuzzy soft set, then the score function of \((F,A)\) is defined as follows:

\[
s_p((F,A)) = \frac{m}{2mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ F^-(\xi)(x) + F^+(\xi)(x) - 1 \right\}.
\]

**Definition 2.7** (Peng and Yang, 2017). Let \((F,A)\) be an interval-valued fuzzy soft set, then the deviation function of \((F,A)\) is defined as follows:

\[
d((F,A)) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ F^-(\xi)(x) + F^+(\xi)(x) - 1 \right\}.
\]

Based on above score function and deviation function, Peng and Yang (2017) derive the following comparative law.

**Definition 2.8** (Peng and Yang, 2017). For two interval-value fuzzy soft sets \((F,A)\) and \((G,A)\), then

(1) \( s_p((F,A)) > s_p((G,A)) \), then \((F,A)\) is superior to \((G,A)\), denoted by \((F,A)>(G,A)\);
(2) \( s_p((F,A)) < s_p((G,A)) \), then \((G,A)\) is superior to \((F,A)\), denoted by \((G,A)>(F,A)\);
(3) \( s_p((F,A)) = s_p((G,A)) \), then
   (a) if \( d_p((F,A)) > d_p((G,A)) \), then \((G,A)\) is equivalent to \((F,A)\), denoted by \((F,A)=(G,A)\);
   (b) if \( d_p((F,A)) = d_p((G,A)) \), then \((F,A)\) is equivalent to \((G,A)\), denoted by \((F,A)=(G,A)\).

**Remark 2.9** (Son, 2007). Let \( F(\xi) \) be an IVFN, then the score function of \( F(\xi) \) is defined as follows:

\[
s_p(F(\xi)) = \frac{F^-(\xi)+F^+(\xi)-1}{2}.
\]

**Definition 2.10.** Suppose that \( F(\xi) = [0.5,0.6] \) and \( G(\xi) = [0.45,0.65] \). If we use the above score function to select the desirable alternative, we can get \( s_p(F(\xi)) = s_p(G(\xi)) = 0.55 \). Hence, we can’t make a distinction between \( F(\xi) \) and \( G(\xi) \), which means that this score function can’t achieve the order in this case.

Notice the deficiencies of the score function proposed by Peng and Yang (2017), in the following, we give a superior score function which can identify the difference of two IVFNs.

**Definition 2.11.** For an IVFN \( F(\xi) = [F^-(\xi),F^+(\xi)] \), its score function is defined as follows:

\[
s_{spd}(F(\xi)) = \frac{e^{p^+(\xi)-p^-(\xi)}}{1+F^+(\xi)-F^-(\xi)}. \tag{5}
\]

**Theorem 2.12.** For an IVFN \( F(\xi) = [F^-(\xi),F^+(\xi)] \)\( s_{spd}(F(\xi)) \) monotonically increases respect to \( F^-(\xi) \) and \( F^+(\xi) \), respectively.

**Proof.** According to Eq. (5), we get the first partial derivative of \( s_{spd}(F(\xi)) \) with \( F^+(\xi) \),

\[
\frac{\delta s_{spd}(F(\xi))}{\delta F^+(\xi)} = \frac{e^{p^+(\xi)-p^-(\xi)}-1-F^-(\xi)-F^-(\xi)}{(1+F^+(\xi)-F^-(\xi))^2} \geq 0
\]

Similarly, we can obtain the first partial derivative of \( s_{spd}(F(\xi)) \) with \( F^-(\xi) \),

\[
\frac{\delta s_{spd}(F(\xi))}{\delta F^-(\xi)} = \frac{e^{p^+(\xi)-p^-(\xi)}-1+F^+(\xi)-F^+(\xi)}{(1+F^+(\xi)-F^-(\xi))^2} \geq 0.
\]

Hence, we can have \( s_{spd}(F(\xi)) \) monotonically increases respect to \( F^-(\xi) \) and \( F^+(\xi) \), respectively.

**Theorem 2.13.** For an IVFN \( F(\xi) \), a new score function \( s_{spd}(F(\xi)) \) satisfies:

(1) \( e^- \leq s_{spd}(F(\xi)) \leq e^+ \);
(2) \( s_{spd}(F(\xi)) = e^+ \) if \( F(\xi) = [1,1] \);
(3) \( s_{spd}(F(\xi)) = e^- \) if \( F(\xi) = [0,0] \).

**Proof.** According to Theorem 2.12, we can know that if we only take \( F^-(\xi) \) or \( F^+(\xi) \) into consideration, \( s_{spd}(F(\xi)) \) obtain the min-value or max-value when \( F(\xi) = [0,0] \) or \( F(\xi) = [1,1] \). That is to say, \( s_{spd}(F(\xi))_{\min} = e^- \) and \( s_{spd}(F(\xi))_{\max} = e^+ \). Therefore, \( e^- \leq s_{spd}(F(\xi)) \leq e^+ \).

**Theorem 2.14.** Let \( F(\xi) = [F^-(\xi),F^+(\xi)] \) and \( G(\xi) = [G^-(\xi),G^+(\xi)] \) be two IVFNs, if \( F^-(\xi) > G^-(\xi) \) and \( F^+(\xi) > G^+(\xi) \), then \( s_{spd}(F(\xi)) > s_{spd}(G(\xi)) \).

**Proof.** According to Theorem 2.12, we can know that \( s_{spd}(F(\xi)) \) monotonically increases respect to \( F^-(\xi) \) or \( F^+(\xi) \).

Hence, if \( F^-(\xi) > G^-(\xi) \) and \( F^+(\xi) > G^+(\xi) \), then \( s_{spd}(F(\xi)) > s_{spd}(G(\xi)) \).

To test the feasibility of the proposed score function \( s_{spd} \) for ranking IVFNs, Table 1 shows the comparison between the results achieved by the proposed score function \( s_{spd} \) and the cases of the score function \( s_p \) obtained by Peng and Yang (2017).

From Table 1, we can conclude that the proposed score function \( s_{spd} \) can effectively solve the deficiencies of \( s_p \) proposed by Peng and Yang (2017). That is to say, the proposed score function can distinguish the difference of alternatives when the existing score function \( s_p \) cannot deal with.

3. New information for interval-valued fuzzy sets

**Definition 3.1** (Liu, 1992). Let \( I_L \) and \( I_R \) be three interval-valued fuzzy sets (IVFSs) on \( \Omega \). A distance measure \( D(I_L, I_R) \) is a mapping \( D: IVFS(\Omega) \times IVFS(\Omega) \rightarrow [0,1] \), possessing the following properties:

\[
(D1) 0 \leq D(I_L, I_R) \leq 1;
\]
Theorem 3.2. Let $I_A$ and $I_B$ be two IVFSs, then $D(I_A, I_B)$ is a distance measure.

$$D(I_A, I_B) = \frac{1}{2n(t + 1)^p} \sum_{i=1}^{n} \left| |I_A^i - I_B^i| - |I_A^i + I_B^i| \right|^p,$$

where $t = 2, 3, 4, \ldots$, $p$ is parameter-varying.

Proof.

(D1) Let $I_A$ and $I_B$ be two IVFSs.

We can have the following equations for any $i$:

$$\begin{align*}
|I_A^i - I_B^i| + I_A^i - I_B^i &= ((I_A^i + I_B^i) + (I_A^i + I_B^i))^{-1}((I_A^i + I_B^i) + (I_A^i + I_B^i)), \\
|I_A^i - I_B^i| + I_A^i - I_B^i &= (I_A^i + I_B^i) + (I_A^i + I_B^i).
\end{align*}$$

It can be easily known that $0 \leq I_A^i \leq 1$, $0 \leq I_B^i \leq 1$, $0 \leq I_A^i \leq 1$.

Hence, we can have the following inequalities:

$$0 \leq (I_A^i + I_B^i) \leq t + 1, \quad -t-1 \leq (I_A^i + I_B^i) \leq 0,$$

then we have

$$-(t+1) \leq (I_A^i + I_B^i) \leq (I_A^i + I_B^i) \leq t+1.$$

That is,

$$0 \leq |(I_A^i + I_B^i) - (I_A^i + I_B^i)| \leq t + 1.$$

Furthermore,

$$0 \leq |(I_A^i + I_B^i) - (I_A^i + I_B^i)| \leq (t + 1)^p.$$

Similarly, we have

$0 \leq |(I_A^i + I_B^i) - (I_A^i + I_B^i)| \leq (t + 1)^p.$

Hence, $I_A$ is a crisp set.

(D3) Let $I_A$ and $I_B$ be two IVFSs if $I_A^i = I_B^i$, then $I_A^i = I_B^i$, $I_A^i = I_B^i$, $I_A^i = I_B^i = 0$, $I_A^i = I_B^i = 0$.

Therefore, the distance $D(I_A, I_B) = 0$.

(D4) Let $I_A$ and $I_B$ be two IVFSs.

We can have the following equations for any $i$:

$$\begin{align*}
|I_A^i - I_B^i| + I_A^i - I_B^i &= ((I_A^i + I_B^i) + (I_A^i + I_B^i))^{-1}((I_A^i + I_B^i) + (I_A^i + I_B^i)), \\
|I_A^i - I_B^i| + I_A^i - I_B^i &= (I_A^i + I_B^i) + (I_A^i + I_B^i).
\end{align*}$$

According to the definition of absolute value, we have

$$|I_A^i - I_B^i| + I_A^i - I_B^i = |(I_A^i - I_B^i) + (I_A^i - I_B^i)|,$$

and

$$|I_A^i - I_B^i| + I_A^i - I_B^i = |(I_A^i - I_B^i) + (I_A^i - I_B^i)|.$$

Therefore, we can have $D(I_A, I_B) = D(I_A, I_B)$.

(D5) Let $I_A$, $I_B$ and $I_C$ be three IVFSs. The distance measures between $I_A$ and $I_B$, and $I_A$ and $I_C$ are given as follows:

$$D(I_A, I_B) = \frac{1}{2n(t + 1)^p} \sum_{i=1}^{n} ((I_A^i - I_B^i) + (I_A^i + I_B^i)^p).$$
\[
D(I_n) = \sqrt[2n]{\frac{1}{2n(1 + p)^n} \sum_{i=1}^{n} ||(I_n - I_{n-1}) + I_{n-1} - I_{n-2}||^p + ||(I_n - I_{n-1}) + I_{n-2} - I_{n-3}||^p}.
\]

We can give the following equations:
\[
\begin{align*}
&l(I_n - I_{n-1} + I_{n-1} - I_{n-2}) = (I_n + I_{n-1}) - (I_{n-1} + I_{n-2}), \\
&l(I_n - I_{n-1} + I_{n-1} - I_{n-2}) = (I_n + I_{n-1}) - (I_{n-1} + I_{n-2}), \\
&l(I_n - I_{n-1} + I_{n-1} - I_{n-2}) = (I_n + I_{n-1}) - (I_{n-1} + I_{n-2}).
\end{align*}
\]

So it is easily known that:
\[
\begin{align*}
&l(I_n - I_{n-1} + I_{n-1} - I_{n-2}) = (I_n + I_{n-1}) - (I_{n-1} + I_{n-2}).
\end{align*}
\]

Furthermore, we can finally have the inequalities:
\[
D(I_n, I_m) \subseteq D(I_n, I_{m+1}) \subseteq D(I_n, I_{m+2}).
\]

We can conclude that
\[
D(I_n, I_m) \neq 0 \iff \mbox{ I_n \neq I_m for any } m \neq n.
\]

So we have the following inequalities:
\[
(\Omega C_1 + \Omega_i C_2) \geq (\Omega C_1 + \Omega_i C_2), \quad (\Omega C_1 + \Omega_i C_2) \geq (\Omega C_1 + \Omega_i C_2), \quad (\Omega C_1 + \Omega_i C_2) \geq (\Omega C_1 + \Omega_i C_2).
\]

Therefore, we have the following equations:
\[
\begin{align*}
&l(I_n - I_{n-1} + I_{n-1} - I_{n-2}) = (I_n + I_{n-1}) - (I_{n-1} + I_{n-2}),
\end{align*}
\]

where \( i \) identifies the level of uncertainty, \( p \) is parameter-varying, and \( w_i \) is the weights of the attributes with \( \sum_{i=1}^{n} w_i = 1 (w_i \in [0,1]) \).

**Proof.** The proof is similar to Theorem 3.2. \( \square \)

**Definition 3.7** (Zeng and Li, 2006). Let \( I_n \) be an interval-valued fuzzy set on \( \Omega \). An entropy measure \( E(I_n) \) is a mapping \( E : IVFS(\Omega) \rightarrow [0,1] \), possessing the following properties:

1. \( E(I_n) = 0 \) iff \( I_n \) is crisp set;
2. \( E(I_n) = 1 \) iff \( I_n = 1 \) for any \( i \);
3. \( E(I_n) \leq E(I_{n+1}) \) for \( I_n \in IVFS(\Omega) \);
4. \( E(I_n) \leq E(I_{n+1}) \) iff \( I_n \geq I_{n+1} \) and \( I_n \geq I_{n+1} \) for \( I_n \geq I_{n+1} \) or \( I_n \leq I_{n+1} \) and \( I_n \leq I_{n+1} \).

**Theorem 3.8.** Let \( I_n \) be an IVFS, then \( E(I_n) \) is an entropy measure.

\[
E(I_n) = 1 - \frac{1}{\sqrt[2n]{\sum_{i=1}^{n} ||(I_{n+1} - I_{n-1})||^p}}
\]

where \( p \) is parameter-varying.

**Proof.** (1) \( E(I_n) = 1 - \frac{1}{\sqrt[2n]{\sum_{i=1}^{n} ||(I_{n+1} - I_{n-1})||^p}} \geq 0 \iff ||(I_{n+1} - I_{n-1})||^p \geq 1 \)

for any \( i \) \( \iff I_n \leq I_{n+1} \) or \( I_n = I_{n+1} \) for \( I_n \leq I_{n+1} \).

Hence, \( I_n \) is a crisp set.

(2) and (3) is straightforward.

(4) If \( E(I_n) \leq E(I_{n+1}) \), then
\[
1 - \frac{1}{\sqrt[2n]{\sum_{i=1}^{n} ||(I_{n+1} - I_{n-1})||^p}} \leq 1 - \frac{1}{\sqrt[2n]{\sum_{i=1}^{n} ||(I_{n+1} - I_{n-1})||^p}} \iff ||(I_{n+1} - I_{n-1})||^p \leq 1 \)

for \( I_n \leq I_{n+1} \) and \( I_n \geq I_{n+1} \) for \( I_n \geq I_{n+1} \) or \( I_n \leq I_{n+1} \) and \( I_n \leq I_{n+1} \).

**Theorem 3.9.** Let \( I_n \) be an IVFS, then \( E(I_n) = 1 - D(I_n, I_n') = S(I_n, I_n') \).

**Proof.** It is straightforward by three equations of similarity measure, distance measure and entropy measure. \( \square \)

4. Three algorithms for interval-valued fuzzy soft set based decision making

4.1. Problem description

Let \( U = \{x_1, x_2, \ldots, x_m\} \) be a limited set of \( m \) alternatives, \( E = \{e_1, e_2, \ldots, e_n\} \) be a finite set of \( n \) parameters, and the weight of parameter \( e_i \) is \( w_i \), \( w_i \in [0,1], \sum_{i=1}^{n} w_i = 1 \). \((F,E)\) is interval-valued fuzzy soft set which can be shown in Table 2. \( F(e_i(x_j)) = [F^-(e_i(x_j)), F^+(e_i(x_j))] \) denotes the possible IVFN of the \( i \)th alternative \( x_j \) satisfying the \( j \)th parameter \( e_i \) given by the decision maker.

4.2. The method of determining the combined weights

4.2.1. Determining the objective weights: the entropy method

Shannon entropy (Shannon, 1948) evaluates the expected information content of a certain message. The degree of uncertainty in

\[
\begin{array}{cccc}
\gamma_1 & \gamma_2 & \cdots & \gamma_n \\
F(\gamma_1) & F(\gamma_2) & \cdots & F(\gamma_n) \\
F(\gamma_2) & F(\gamma_1) & \cdots & F(\gamma_n) \\
\vdots & \vdots & \ddots & \vdots \\
F(\gamma_n) & F(\gamma_1) & \cdots & F(\gamma_{n-1}) \\
\end{array}
\]

The tabular representation of interval-valued fuzzy soft set \((F,E)\)
information can be measured using the entropy concept. Information entropy idea can regulate decision making process because it is able to measure existing contrasts between sets of data and thus clarify the intrinsic information for decision maker. Inspired by Shannon, we propose a new entropy method (Theorem 3.8) to compute objective weights.

The interval-valued fuzzy entropy $E_j(i=1,2,...,n)$ of $j$th parameter is calculated as follows:

$$E_j = 1 - \frac{1}{n} \sum_{i=1}^{n} |F^{-}(\varepsilon_j)(x_i) + F^{+}(\varepsilon_j)(x_i) - 1|^p.$$  (11)

The weight $\omega_j$ of $j$th parameter can be computed as follows:

$$\omega_j = \frac{1 - E_j}{n - \sum_{i=1}^{n} E_i}.$$  (12)

When the decision information of parameter $\varepsilon_j$ with each alternative $x_i$ is interval-valued fuzzy number, from the perspective of the original decision information, the parameter information increasingly blurred, more uncertain, the property for the amount of information available for the less, the greater entropy should be given less weight, and vice verse. Hence, the use of interval-valued fuzzy entropy to determine the weights of parameters, both to reduce the loss of information evaluation, and can reflect the willingness of decision makers.

4.2.2. Determining the combined weights: the linear weighted comprehensive method

Suppose that the vector of the subjective weight, given by the decision makers directly, is $w = [w_1, w_2, ..., w_n]$, where $\sum_{i=1}^{n} w_i = 1, 0 \leq w_i \leq 1$. The vector of the objective weight, computed by Eq. (12) directly, is $\omega = [\omega_1, \omega_2, ..., \omega_n]$, where $\sum_{i=1}^{n} \omega_i = 1, 0 \leq \omega_i \leq 1$.

Therefore, the vector of the combined weight $\sigma = [\sigma_1, \sigma_2, ..., \sigma_n]$ can be defined as follows:

$$\sigma_j = \frac{w_j \times \omega_j}{\sum_{j=1}^{n} w_j \times \omega_j}.$$  (13)

where $\sum_{j=1}^{n} \sigma_j = 1, 0 \leq \sigma_j \leq 1$.

The subjective weight and objective weight are assembled by non-linear weighted comprehensive method. Based on the multiplier effect, the greater the value of the objective weight and subjective weight are, the greater the combined weight is, or vice versa. Furthermore, we can know that the Eq. (13) breaks through the limitation of only considering either subjective or objective influence. The superiority of Eq. (13) is that the rankings of alternatives and the parameter weights can show both the objective information and the subjective information.

In the following, we will apply the WDBA, CODAS and similarity measure methods to interval-valued fuzzy soft sets.

4.2.3. The interval-valued fuzzy soft decision making approach based WDBA

To solve the MCDM problem with interval-valued fuzzy soft information, we try to present an interval-valued fuzzy soft WDBA approach, which is measured the distance from the optimal point (best value of alternatives) and non-optimal point (worst value of alternatives) and ranked the alternatives according to their suitability index.

At the beginning, we consider to normalize evaluation information because there may be some benefit parameters and cost parameters in decision matrix. The above two kinds of parameters react oppositely, in other words, the larger value reflects the better performance of a benefit parameter but means the worse performance of a cost parameter. Therefore, in order to ensure all parameters are compatible, we make further efforts to convert the cost parameters into benefit parameters by the following formula:

$$\bar{F}(\varepsilon_j)(x_i) = \begin{cases} \{1 - F^{-}(\varepsilon_j)(x_i)\}, & \varepsilon_j \text{ is benefit parameter,} \\ \{1 - F^{+}(\varepsilon_j)(x_i)\}, & \varepsilon_j \text{ is cost parameter.} \end{cases}$$  (14)

According to Eq. (15), we can achieve the normalized interval-valued fuzzy soft matrix $R = (\bar{F}(\varepsilon_j)(x_i))_{n \times m}$.

Next, we calculate the score function $t_i(i=1,2,...,m; j=1,2,...,n)$ of $\bar{F}(\varepsilon_j)(x_i)$ by Eq. (16),

$$t_i = \frac{\sum_{\varepsilon_j} \bar{F}(\varepsilon_j)(x_i)}{1 + \sum_{\varepsilon_j} \bar{F}(\varepsilon_j)(x_i)}.$$  (15)

In order to show the standardized matrix $S$ visually, we express it in an average value matrix $\varphi_j(i=1,2,...,n)$ and revised deviation matrix $SD_j(i=1,2,...,n)$. The concrete process is shown as follows:

$$S_{mj} = \frac{t_i - \varphi_j}{SD_j},$$  (16)

$$\varphi_j = \frac{1}{m} \sum_{i=1}^{m} t_i,$$  (17)

$$SD_j = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (t_i - \varphi_j)^2}.$$  (18)

The developed WDBA method is based on the concept that the selected alternative (optimum) should have the shortest distance from the ideal solution (optimal possible alternative) and be farthest from the anti-ideal solution (worst possible alternative). The ideal points are the set of parameter values ideally (most) desired. The anti-ideal points are the set of parameter values ideally not desired at all or least desirable. The ideal points, denoted by $SM^+$ and anti-ideal points, denoted by $SM^-$ are established from standardized decision matrix, which can be defined as follows:

$$SM^+ = [\max\{S_{mj}\}]_{j=1,2,...,n},$$  (19)

$$SM^- = [\min\{S_{mj}\}]_{j=1,2,...,n},$$  (20)

where $i=1,2,...,m$.

This mechanism guarantees that the top ranked alternative is closest to the ideal solution and farthest from the anti-ideal solution. Euclidean distance is the shortest distance among two points. The overall performance index score of an alternative is ascertained by its Euclidean distance between ideal solution and anti-ideal solutions. This distance is interrelated with the parameters’ weights and should be united in the distance measurement. This is reason that all alternatives are compared with ideal and anti-ideal solutions, rather than directly among themselves. Therefore, weighted Euclidean distances are considered in the developed method.

Fig. 1 shows Euclidean distance of an alternative to ideal and anti-ideal solutions in 2D space in case of two parameters $e_1$ and $e_2$. The rectangular box means the real domain. The Euclidean distance between points $P$ and $Q$ in $n$ dimensional space is the length of the line segment, $PQ$. In Cartesian coordinates, if $P = (p_1,p_2,...,p_n)$ and $Q = (q_1,q_2,...,q_n)$ are two points in Euclidean $n$-space, so the distance from $P$ to $Q$ is given by $d(P,Q) = \sqrt{\sum_{i=1}^{n} (p_i-q_i)^2}$. Weighted Euclidean distance (WED) between an alternative $A_i$ and ideal point $SM^+$ is expressed as $WED^+$ and between an alternative $A_i$ and anti-ideal point is expressed as $WED^-$. 

$$WED^+ = \sqrt{\sum_{j=1}^{n} \sigma_j \langle SM^- - SM^+ \rangle^2},$$  (21)
In general, interval-valued fuzzy soft WDBA approach involves the following steps:

Algorithm 1. WDBA

1: Input the interval-valued fuzzy soft decision matrix
   \( I = (F(\epsilon_j(x_i)))_{mxn} \), \( i = 1, 2, ..., m; j = 1, 2, ..., n \).
2: Transform the matrix \( I = (F(\epsilon_j(x_i)))_{mxn} \) into a normalized
   interval-valued fuzzy soft matrix \( I' = (\tilde{F}(\epsilon_j(x_i)))_{mxn} \) by Eq. (14).
3: Calculate the combined weight \( \varpi \) by Eq. (13).
4: Compute the score matrix \( T = (t_{ij})_{mxn} \) of \( I' = (\tilde{F}(\epsilon_j(x_i)))_{mxn} \) by
   Eq. (15).
5: Form the standardized matrix \( SM = (SM_j)_{mxn} \) by Eq. (16).
6: Compute the ideal points \( SM^+ \) and anti-ideal points \( SM^- \) by Eqs. (19) and (20).
7: Compute the \( WED_i^- \) and \( WED_i^+ \) by Eqs. (21) and (22).
8: Derive the suitability index value \( SI_i \) of each alternative \( A_i \) by Eq. (23).
9: Determine the ranking of the alternatives according to the
   suitability index values \( SI \).

Remark 4.1. It is indispensable to illustrate that in the interval-valued fuzzy soft WDBA approach, the information in decision matrix is expressed by IVFNs to represent the DMs’ opinions. Expressed by the membership degree with a possible interval values, the IVFNs are quite efficacious for capturing the uncertainty and imprecision of the DMs in the MCDM problem. In addition, the interval-valued fuzzy soft WDBA approach is an invaluable tool to manage the decision making problems with IVFNs possessing a great power in distinguishing the optimal alternative and obtaining the optimal alternative without counterintuitive phenomena. But other approaches (Feng et al., 2010; Peng et al., 2017; Yang et al., 2009; Yuan & Hu, 2012) to solving the decision making problems with IVFNs do not have this desirable characteristic.

4.2.4. The interval-valued fuzzy soft decision making approach based CODAS

To solve the MCDM problem with interval-valued fuzzy soft information, we try to present an interval-valued fuzzy soft CODAS approach. It is a new and efficient MCDM method which introduced by Ghorabaei et al. (2016) recently. The desirable of alternatives in the CODAS method is determined according to \( l^0\)-norm and \( l^0\)-norm indifference spaces for criteria. According to these spaces, in the procedure of this method, a combinative form of the Euclidean and Hamming distances is used for the calculation of the assessment score of alternatives. However, the Euclidean and Hamming distances are defined in a crisp environment and we can’t utilize them in interval-valued fuzzy soft set problems. The aim of this study is to develop an interval-valued fuzzy soft CODAS method. In order to achieve this aim, we use the fuzzy weighted Euclidean distance and fuzzy weighted Hamming distance, instead of the crisp distances.

At the beginning, we consider to normalize information by Eq. (14) and calculate the score function \( t_{ij} \) of \( \tilde{F}(\epsilon_j(x_i))_{i = 1, 2, ..., m; j = 1, 2, ..., n} \) by Eq. (15).

In order to compute the weighted normalized decision matrix \( R = (r_{ij})_{mxn} \), we express it in weighted normalized performance values, shown as follows:

\[
 r_{ij} = \varpi_j t_{ij},
\]

where \( w_j (0 < \varpi_j < 1) \) denotes the combined weight of \( j \)th parameter, and \( \sum_{j=1}^{n} \varpi_j = 1 \).

The proposed CODAS method is based on the negative-ideal solution (NIS). We define a negative-ideal solution as follows:

\[
 NIS = (nis_i)_{mx1},
\]

\[
 nis_i = \min_{i} [r_{ij}]
\]

where \( i = 1, 2, ..., m \).

Later, we compute the Euclidean distance measure \( E_i = (E_i)_{1x1} \) and Hamming distance \( T_i = (T_i)_{1x1} \) of alternatives from negative-ideal solution as follows:

\[
 E_i = \sqrt{\sum_{j=1}^{n} (r_{ij} - nis_i)^2},
\]

\[
 T_i = \sum_{j=1}^{n} |r_{ij} - nis_i|.
\]

Based on above two distances, we can construct the relative assessment matrix, shown as follows:

\[
 RA = (ra_{ik})_{mxn},
\]

\[
 ra_{ik} = E_i - E_k + \psi(E_i - E_k) \times (T_i - T_k).
\]

where \( k = 1, 2, ..., m \) and \( \psi \) denotes a threshold function to recognize the equality of the Euclidean distance of two alternatives, and is shown as follows:

\[
 \psi(x) = \begin{cases} 
 1, & \text{if } |x| \geq \Theta, \\
 0, & \text{if } |x| < \Theta.
\end{cases}
\]

In this function, \( \Theta \) is the threshold parameter that can be determined by decision-maker. It is suggested to determine this parameter at a value between 0.01 and 0.05 (a value close to 0). If the difference
between Euclidean distance of two alternatives is more than Θ, these two alternatives would be compared by the Hamming distance. It has been verified that when Θ = 0.02, the outcomes are consistent with the original data (Ghorabaee et al., 2017).

Now, we can collect the assessment score of each alternative. The alternative for which the value of assessment score RA is the highest is the optimal choice for the considered decision making problem shown as follows:

\[ RA_i = \sum_{k=1}^{m} r_{ik}. \] (32)

To describe the proposed method, Ghorabaee et al. (2017) utilized a simple situation with seven alternatives \([x_1, x_2, x_3, x_4, x_5, x_6, x_7]\) and two parameters \([\epsilon_1, \epsilon_2]\). Suppose that weighted normalized performance values \(\eta_i\) have been calculated. These values are dimensionless and between 0 and 1. Fig. 2 (Ghorabaee et al., 2017) shows the position of all alternatives according to these values.

In general, interval-valued fuzzy soft CODAS approach involves the following steps:

Algorithm 2. CODAS

1: It is similar to 1–3 in Algorithm 1.
2: Compute the score matrix \(T = (t_{ij})_{m \times n}\) of \(I' = (\tilde{f}(x_i))_{m \times n}\) by Eq. (15).
3: Calculate the weighted normalized decision matrix \(\tilde{N}\) by Eq. (24).
4: Determine the negative-ideal solution \(NIS\) by Eq. (25).
5: Compute the Euclidean distance \(E\) and Hamming distance \(T\) from the \(NIS\) by Eqs. (27) and (28).
6: Construct the relative assessment matrix \(RA\) by Eq. (29).
7: Calculate the assessment score of each alternative \(RA_i\) by Eq. (32).
8: Rank the alternatives according to the decreasing values of assessment \(RA\).

Remark 4.2. The interval-valued fuzzy soft CODAS approach is an invaluable tool to manage the decision making problems with IVFNs obtaining the optimal alternative without counterintuitive phenomena. But other approaches (Feng et al., 2010; Peng et al., 2017; Yang et al., 2009; Yuan & Hu, 2012) to solving the decision making problems with IVFNs do not have this desirable characteristic.

4.2.5. The interval-valued fuzzy soft decision making approach based similarity measure

In this subsection, we introduce a method for the MCDM problem by the proposed similarity measure between IVNSs. The concept of ideal point has been applied to help determine the best alternative in the decision process. Although the ideal alternative does not exist in actual cases, it does offer an invaluable theoretical construct against which to appraise alternatives. Hence, we define the ideal alternative \(x^*\) as the IVFN \(x_i^* = [1,1]\) for \(\forall j\).

Consequently, according to Eq. (8), the proposed similarity measure \(S\) between an alternative \(x_i\) and the ideal alternative \(x^*\) presented by the IVFSs is defined as follows:

\[
S(x_i,x^*) = 1 - \sqrt{\frac{1}{2(1+t)^p} \sum_{j=1}^{n} \eta_j \left[ |t(F^-(\epsilon_j)(x_i) - F^-(\epsilon_j)(x^*))| + F^+(\epsilon_j)(x_i) - F^+(\epsilon_j)(x^*)|^p + |t(F^+(\epsilon_j)(x_i) - F^+(\epsilon_j)(x^*))| + F^-(\epsilon_j)(x_i) - F^-(\epsilon_j)(x^*))|^p \right]}
\] (33)

4.3. A case study in mine emergency decision making

We will consider the emergency decision making problems of mine accidents employing the proposed decision making algorithms based on WDBA, CODAS and similarity measure. The mine explosion is one of the most hazardous dangers in mine accidents. The mine explosion enormously threatens the safety of work and life and imperils the safety production of mine. Since the explosion accidents often occur unexpectedly and suddenly, it is not easy to predict the accident to a crumb and have enough preparations and emergency actions ahead of time. Therefore, the emergency response plans and the simulations of the accidents are a requisite approach in disaster preparedness and appropriate responses. The high quality and feasibility of the emergency plans will directly influence the later emergency actions, and affect the evolution of disasters. Consequently, the evaluation and decision of the given emergency plans with simulations is considered essential for the disaster management of mine accidents (Hao, Xu, Zhao, & Fujita, 2017).
Example 4.3. Assume that there are five emergency plans $U = \{x_1, x_2, x_3, x_4, x_5\}$ to be considered for an explosion accident in the coal mine. The expert chooses the decision parameters set $E = \{e_1, e_2, e_3, e_4, e_5\}$ to be the noxious gas concentration $e_1$ (denoted as gas), reducing casualty of current events $e_2$ (denoted as casualty), the smoke and the dust level $e_3$ (denoted as smoke), the feasibility of rescue operations $e_4$ (denoted as feasibility) and repairing facility damages $e_5$ (denoted as facility). Based on the general evolving principle and the characteristics of the mine accidents, we can determine that all parameters are beneficial. Suppose that the expert has the following prior weight set given by his/her prior experience or preference: $w = (w_1, w_2, w_3, w_4, w_5) = (0.3, 0.2, 0.14, 0.16, 0.2)$. The assessments for emergency plans arising from questionnaire investigation to the expert and constructing an interval-valued fuzzy soft set $(F,E)$ with its tabular form given by Table 3.

In what follows, we utilize the algorithms proposed above to select emergency plans under interval-valued fuzzy soft information.

According to Algorithms 1–3 shown in Table 4, we can conclude that the final decision results are the same, i.e., $x_5$ is the most desirable emergency plan. Hence, the three approaches proposed above are effective and available.

5. Comparison of the newly proposed approaches with the other approaches to interval-valued fuzzy soft set based decision making

5.1. Comparison of the newly proposed three approaches with their own

(1) Comparison of computational complexity

It is easily known that Algorithms 1 and 2 will require more computational complexity than Algorithm 3. Hence, if we take the computational complexity into consideration, the Algorithm 3 is given priority to make decision.

(2) Comparison of applied situation

When the decision maker takes ideal point into consideration in the decision process, the Algorithms 2 and 3 are given priority to make decision.

(3) Comparison of discrimination

Comparing the results in Algorithm 3 with Algorithms 1 and 2, we can see that the results of Algorithm 3 are quite close and vary from 0.6872 to 0.7349. These result of decision values can’t clearly distinguish, that is to say, the results obtained from Algorithm 3 are inconvincing (or at least not applicable). On the contrary, the Algorithms 1 and 2 have a clearly distinguish. Hence, if we take the discrimination into consideration, the Algorithms 1 and 2 are given priority to make decision. For better understanding, we give the figure form shown in Fig. 3.

5.2. Comparison of the newly proposed three methods with other approaches

5.2.1. The MABAC method (Peng et al., 2017) and its limitation

Algorithm 4. MABAC (Peng et al., 2017)

1: It is similar to 1–3 in Algorithm 1.
2: Compute the weighted matrix $T = (t_{ij})_{m \times n}$ by Eq. (35),

$$t_{ij} = \left[ t_{ij}^{-}, t_{ij}^{+} \right] = \left[ \prod_{t=1}^{n} d_{t}(t_{ij}^{+}, g), \prod_{t=1}^{n} d_{t}(t_{ij}^{-}, g) \right].$$

(34)

3: Compute the border approximation area (BAA) matrix $G = (g_{ij})_{m \times n}$. The BAA for each parameter is obtained by Eq. (36),

$$g_{ij} = \prod_{t=1}^{m} \left( t_{ij}^{-} \right)^{1/m} \prod_{t=1}^{m} \left( t_{ij}^{+} \right)^{1/m}.$$  \hspace{1cm} (35)

4: Compute the distance matrix $D = (d_{ij})_{m \times n}$ by Eq. (37),

$$d_{ij} = \begin{cases} 
\delta_{ij}(t_{ij}^{+}, g_{ij}), & \text{if } s(t_{ij}) > s(g_{ij}); \\
0, & \text{if } s(t_{ij}) = s(g_{ij}); \\
\delta_{ij}(t_{ij}^{-}, g_{ij}), & \text{if } s(t_{ij}) < s(g_{ij}). 
\end{cases}$$  \hspace{1cm} (36)

where distance measure $\delta_{ij}$ is defined in Eq. (6) ($t = 0, p = 1$ as (Peng and Yang, 2015)), and $s(t_{ij})$ and $s(g_{ij})$ are score function (Peng et al., 2017) of $t_{ij}$ and $g_{ij}$, respectively.

5: Rank the alternatives by $Q_i(i = 1,2,...,m)$. The most desired alternative is the one with the biggest value of $Q_i$.

$$Q_i = \sum_{j=1}^{n} d_{ij}, \quad i = 1,2,...,m; j = 1,2,...,n.$$  \hspace{1cm} (37)

Remark 5.1. However, we can find that the above algorithm will meet the counterintuitive phenomena when $t_{ij}^{-}$ or $t_{ij}^{+}$ is 0. That is to say, it only determines by one of them to make decision and decision information of others can be omitted.
5.2.2. The similarity method (Peng et al., 2017) and its limitation

Algorithm 5. Similarity measure (Peng et al., 2017)

1: It is similar to 1–3 in Algorithm 1.
2: Calculate the similarity measure $S(x_1,x^*) (i = 1, 2,...,m)$ by Eq. (39).

\[ S^i(x_1,x^*) = \left[ \sum_{j=1}^{n} \tilde{F}^- (e_j (x_1)) \sum_{j=1}^{n} \tilde{F}^+ (e_j (x_1)) \right] \]

3: Compute the each alternative of score function $s_i(S(x_1,x^*))$ by Eq. (4).
4: Rank the alternatives according to the decreasing values of similarity measure $s(S^i(x_1,x^*))$. The most desired alternative is the one with the biggest value of $x_i$.

Table 5
The tabular form of interval-valued fuzzy soft set $(F,E)$ in Example 5.3.

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|-------|-------|-------|-------|-------|
| $[0.8,0.1]$ | $[0.8,0.1]$ | $[0.8,0.1]$ | $[0.8,0.1]$ | $[0.8,0.1]$ |
| $[0.5,0.8]$ | $[0.5,0.8]$ | $[0.5,0.8]$ | $[0.5,0.8]$ | $[0.5,0.8]$ |
| $[0.6,0.1]$ | $[0.5,0.6]$ | $[0.3,0.4]$ | $[0.4,0.6]$ | $[0.7,0.8]$ |
| $[0.1,0.15]$ | $[0.6,0.68]$ | $[0.2,0.3]$ | $[0.4,0.5]$ | $[0.6,0.7]$ |
| $[0.13,0.15]$ | $[0.43,0.49]$ | $[0.3,0.4]$ | $[0.4,0.6]$ | $[0.4,0.6]$ |

Remark 5.2. According to above algorithm, one can establish an approach to make decision. However, the limiting condition that the probability of equality of final decision results is great. That is to say, we can hardly make decision.

Example 5.3. Suppose that we invite another expert to evaluate the coal mine in Example 5.3. The weight information given by expert has the prior weight set given by his/her prior experience or preference:

\[ w = (w_1, w_2, w_3, w_4, w_5) = (0.35, 0.15, 0.15, 0.15, 0.2). \]

The assessments for emergency plans arising from questionnaire investigation to the expert are different, which is shown in Table 5.

The final decision results and ranking are shown in Table 6. Based on the above discussion in Remarks 6.1 and 6.2, we know that the unreasonable results for MABAC method (Peng et al., 2017) owe to the “0” problem.

From Fig. 4, we observe that the proposed algorithms (WDBA, CODAS and similarity measure) are moderately correlated with similarity measure (Peng et al., 2017) and MABAC (Peng et al., 2017)(90%). The reason for such variation is implicit from the nature of formulation used by each of these methods. To further demonstrate the strength of our algorithms, we compare them to existing methods (Peng et al., 2017). Fig. 5 shows the comparison of these five methods which clarifies the power of proposed algorithms better in distinguishing the optimal alternative and obtain the optimal alternative without counterintuitive phenomena. In Fig. 6, we can find that the proposed weight determining model can effectively reflect the subjective information and objective information, but the combined weight proposed by Peng et al. (2017) can’t reveal the difference compared with the given weight.
for each alternative $\alpha \beta$ in $\xi$ may be chosen. The vector $W = (\alpha, \beta)$ is called an opinion weighting vector. The fuzzy soft set $(F, W)$ over $U$ such that $F_W(\epsilon) = [x/(\alpha F^- (\epsilon)(x) + \beta F^+ (\epsilon)(x))] x \in U$ is called the weighted reducit fuzzy soft set (WRFS) of the interval-valued fuzzy soft set $(F, A)$ with respect to the opinion weighting vector $W$.

Algorithm 7: left soft set (Feng et al., 2010)

1: It is similar to Steps 1–2 in Algorithm 1.
2: Input an opinion weighting vector $W = (\alpha, \beta)$ and calculate the WRFS $Y_W = (F_W, A)$ of the interval-valued fuzzy soft set $(F, A)$ with respect to $W$.
3: Input a threshold value $t \in [0, 1]$.
4: Compute the $t$-level soft set $L(Y_W, t)$.
5: Present the level soft set $L(Y_W, t)$ in tabular form and compute the choice value $c_i$ of $x_i$ for $\forall x_i \in U$.
6: The optimal alternative is to select $x_i$ if $c_i = \max_{x \in U} c_x$.
7: If $k$ has more than one value, then any one of $x_k$ may be chosen.

Remark 5.7. The level soft set method (Feng et al., 2010) is in fact an adjustable algorithm which seize a key feature for decision making in a vague circumstances: some of these issues are essentially humanistic and thus subjective in nature; there does not exist a uniform criterion for evaluating the alternatives. Apparently, the level soft set algorithm (Feng et al., 2010) can’t be applied to IVFSS based decision making in some special cases.

Example 5.8. Suppose that $(F, E)$ is an interval-valued fuzzy soft set. For obtaining the choice value $c_i$, we set $W = (0.5, 0.5)$ (Step 2), $t = 0.6$ (Step 3) in Feng et al. (2010). Then Table 9 gives its tabular representation with choice values.

Table 9

| $q_1$ | $q_2$ | $q_3$ | $q_4$ | Choice value $c_i$ |
|-------|-------|-------|-------|------------------|
| 0.550, 0.70 | 0.30, 0.5 | 0.50, 0.6 | 0.480, 0.51 | 1 |
| 0.550, 0.65 | 0.340, 0.47 | 0.360, 0.64 | 0.360, 0.64 | 1 |
| 0.450, 0.80 | 0.370, 0.43 | 0.350, 0.65 | 0.460, 0.54 | 1 |
| 0.530, 0.67 | 0.360, 0.46 | 0.450, 0.65 | 0.380, 0.62 | 1 |

to diverse parameters is not always rational. In fact, it just like the addition of weight and height.

The following example will illustrate that algorithm in Yang et al. (2009) may not be successfully applied to some soft decision making problems.

Example 5.5. Suppose that $(F, E)$ is an interval-valued fuzzy soft set and Table 7 gives its tabular representation with choice values and scores.

From Table 7, we can find that the scores of all alternatives are identical, in other words, all of them could be selected as the best choice by algorithm in Yang et al. (2009). Hence, it is unreasonable in current case. But in fact, we can achieve an optimal alternative by our pioneered algorithms and the existing algorithms (Peng et al., 2017). Suppose that we use Eq. (13) to determine the objective weight instead of combined weight by our algorithms and use Eq. (7) in Peng et al. (2017) to determine the objective weight instead of combined weight by Peng and Dai’s algorithms Peng et al. (2017). Meanwhile, assume that there is no discriminative power of cost parameters and benefit parameters, i.e., we think that the parameters are benefit parameters. Therefore, we can have the decision results by our algorithms and the existing algorithms (Peng et al., 2017), which is presented in Table 8.

Table 8

| Algorithms | Ranking | Optimal alternative |
|------------|---------|---------------------|
| Algorithm 6: choice values (Peng & Dai, 2017) | x_1 \sim x_2 \sim x_3 \sim x_4 | * |
| Algorithm 1: WDBA | x_2 > x_1 > x_3 > x_4 | x_2 |
| Algorithm 2: CODAS | x_2 > x_1 > x_3 > x_4 | x_2 |
| Algorithm 3: Similarity measure | x_2 > x_1 > x_4 > x_3 | x_2 |
| Algorithm 4: MABAC (Yang & Peng, 2017) | x_2 > x_1 > x_3 > x_4 | x_2 |
| Algorithm 5: Similarity measure (Yang & Peng, 2017) | x_2 > x_1 > x_3 > x_4 | x_2 |

Table 7

The tabular form of interval-valued fuzzy soft set $(F, E)$ in Example 5.5.

5.2.4. The level soft set method (Feng et al., 2010) and its limitation

Definition 5.6 (Feng et al., 2010). Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta = 1$. The vector $W = (\alpha, \beta)$ is called an opinion weighting vector. The fuzzy soft set $(F_W, A)$ over $U$ such that $F_W(\epsilon) = [x/(\alpha F^- (\epsilon)(x) + \beta F^+ (\epsilon)(x))] x \in U$ is called the weighted reducit fuzzy soft set (WRFS) of the interval-valued fuzzy soft set $(F, A)$ with respect to the opinion weighting vector $W$.

Algorithm 7: left soft set (Feng et al., 2010)

1: It is similar to Steps 1–2 in Algorithm 1.
2: Input an opinion weighting vector $W = (\alpha, \beta)$ and calculate the WRFS $Y_W = (F_W, A)$ of the interval-valued fuzzy soft set $(F, A)$ with respect to $W$.
3: Input a threshold value $t \in [0, 1]$.
4: Compute the $t$-level soft set $L(Y_W, t)$.
5: Present the level soft set $L(Y_W, t)$ in tabular form and compute the choice value $c_i$ of $x_i$ for $\forall x_i \in U$.
6: The optimal alternative is to select $x_i$ if $c_i = \max_{x \in U} c_x$.
7: If $k$ has more than one value, then any one of $x_k$ may be chosen.

Remark 5.7. The level soft set method (Feng et al., 2010) is in fact an adjustable algorithm which seize a key feature for decision making in a vague circumstances: some of these issues are essentially humanistic and thus subjective in nature; there does not exist a uniform criterion for evaluating the alternatives. Apparently, the level soft set algorithm in Feng et al. (2010) can’t be applied to IVFSS based decision making in some special cases.

Example 5.8. Suppose that $(F, E)$ is an interval-valued fuzzy soft set. For obtaining the choice value $c_i$, we set $W = (0.5, 0.5)$ (Step 2), $t = 0.6$ (Step 3) in Feng et al. (2010). Then Table 9 gives its tabular representation with choice values.
Table 10

A comparison study with some existing methods in Example 5.8.

| Algorithms                     | Ranking | Optimal alternative |
|-------------------------------|---------|---------------------|
| Algorithm 7: left soft set     |         | *                   |
| (Xiao et al., 2013)            |         |                     |
| Algorithm 1: WDBA              |         | x₁ > x₂ > x₃ > x₄   |
| Algorithm 2: CODAS             |         | x₁ > x₂ > x₃ > x₄   |
| Algorithm 4: MABAC             |         | x₁ > x₂ > x₃ > x₄   |
| Algorithm 5: Similarity measure|         |                     |
| (Yang & Peng, 2017)            |         |                     |

“*” denotes that there is no unified alternative to selected.

From Table 9, we can find that the choice values of all alternatives are identical, in other words, all of them could be selected as the best choice by algorithm in Peng et al. (2010). Hence, it is unreasonable in current case. But in fact, we can achieve an optimal alternative by our pioneered algorithms and the existing algorithms (Peng et al., 2017). Suppose that we use Eq. (13) to determine the objective weight instead of combined weight by our algorithms and use Eq. (7) in Peng et al. (2017) to determine the objective weight instead of combined weight by Peng and Dai’s algorithms (Peng et al., 2017). Meanwhile, assume that there is no discriminative power of cost parameters and benefit parameters, i.e., we think that the parameters are benefit parameters. Therefore, we can have the decision results by our algorithms and the existing algorithms (Peng et al., 2010; Peng et al., 2017), which is presented in Table 10.

5.2.5. The comparison table method (Yuan & Hu, 2012) and its limitation

Definition 5.9 (Yuan and Hu, 2012). Let U be an initial universe, E be a parameter set, A ⊆ E, (F,A) is an interval-valued fuzzy soft set on U. If F⁺(e)(x) ≤ F⁻(e)(x₂), then we say the lower membership of x₁ related to e₁ is less than the lower membership of x₂, its corresponding characteristic function is given as follows:

\[ f_{\delta}^L(x₁, x₂) = \begin{cases} 1, & F^-(e)(x₁) \leq F^-(e)(x₂) \\ 0, & F^-(e)(x₁) > F^-(e)(x₂). \end{cases} \]  (39)

Definition 5.10 (Yuan and Hu, 2012). Let U be an initial universe, E be a parameter set, A ⊆ E, (F,A) is an interval-valued fuzzy soft set on U. If F⁺(e)(x₁) ≤ F⁺(e)(x₂), then we say the upper membership of x₁ related to e₁ is less than the upper membership of x₂, its corresponding characteristic function is given as follows:

\[ f_{\delta}^U(x₁, x₂) = \begin{cases} 1, & F^+(e)(x₁) \leq F^+(e)(x₂) \\ 0, & F^+(e)(x₁) > F^+(e)(x₂). \end{cases} \]  (40)

Definition 5.11 (Yuan and Hu, 2012). The comparison table of interval-valued fuzzy soft set means that row numbers and column numbers are equal. C_{̂} is given as follows:

\[ C_{̂} = \sum_{j=1}^{n} (f_{\delta}^L(xᵢ, xⱼ) + f_{\delta}^U(xᵢ, xⱼ)). \]  (41)

j is the number of parameters. Obviously, 0 ⩽ C_{̂} ⩽ 2n, and C_{̂} = 2n.

Algorithm 8: comparison table (Yuan & Hu, 2012)

1. It is similar to Steps 1–2 in Algorithm 1.
2. Compute the comparison table by Eqs. (40) and (41).
3. Compute the P₁ = ∑_{i=1}^{m} C_{̂} and Q₁ = ∑_{i=1}^{m} C_{̂}.
4. Compute the score S₁ = P₁ – Q₁.
5. The optimal alternative is to select xᵢ if S₁ = max_{xᵢ∈X}.S₁.

Remark 5.12. As stated in Yuan and Hu (2012), the given approach is in fact a comparable table method. Some of these problems are essentially humanistic and thus subjective in nature; there does not exist a uniform criterion for evaluating the alternatives. Apparently, the comparable table proposed by Yuan and Hu (2012) cannot be applied to IVFSS based decision making in some cases.

Example 5.13 (Peng et al., 2017). Assume that (F,E) is an interval-valued fuzzy soft set. Then Table 11 gives its tabular representation and their decision results. From Table 11, we can find that the score values of all alternatives are identical, in other words, all of them could be selected as the best choice by algorithm in Yuan and Hu (2012). Hence, it is unreasonable in current case. But in fact, we can achieve an optimal alternative by our proposed algorithms and the existing algorithms (Peng et al., 2017). Suppose that we use Eq. (13) to determine the objective weight instead of combined weight by our algorithms and use Eq. (7) in Peng et al. (2017) to determine the objective weight instead of combined weight by Peng and Dai’s algorithms Peng et al. (2017). Meanwhile, assume that there is no discriminative power of cost parameters and benefit parameters, i.e., we think that the parameters are benefit parameters. Therefore, we can have the decision results by our algorithms and the existing algorithms (Peng et al., 2017; Yuan & Hu, 2012), which is presented in Table 12.

6. Conclusion

The major contributions in this paper can be summarized as follows:

1. We construct a new score function for interval-valued fuzzy number. Comparing with the existing literature (Peng et al., 2017; Peng & Yang, 2017), it can distinguish the difference of alternatives when the existing score functions (Peng et al., 2017; Peng & Yang, 2017) cannot work.
2. A new information measure (distance measure, similarity measure and entropy) are introduced and their transformation relations are pioneered.
3. We give new entropy method for obtaining the objective weight, meanwhile, the combined weights are developed, which can effectively reflect the subjective information and objective information, but the combined weight proposed by Peng et al. (2017) can’t reveal the difference compared with the given weight information.
4. We propose three algorithms to solve interval-valued fuzzy soft decision making problem by Weighted Distance Based Approximation (WDBA), Combinative Distance-based ASsessment (CODAS) and similarity measure. Compared to the existing interval-valued fuzzy soft decision making methods, (i) it can obtain the
optimal alternative without counterintuitive phenomena (Peng et al., 2017); (ii) it has a great power in distinguishing the optimal alternative (Feng et al., 2010; Peng et al., 2017; Yang et al., 2009; Yuan & Hu, 2012); (iii) it can avoid the parameter selection problems (Feng et al., 2010).

In the future, we shall apply more groundbreaking theories into interval-valued fuzzy soft set and also take different fuzzy environment (Li & Wang, 2017; Nie, Wang, & Li, 2017; Peng & Dai, 2017; Peng, Wang, Yang, & Qian, 2017; Peng & Yang, 2015; Yang & Wang, 2018; Zhou, Wang, & Zhang, 2019) into consideration by proposed algorithms (WDBA and CODAS) for solving more emergency decision-making problems.

Acknowledgement

The authors are very appreciative to the reviewers for their precious comments which enormously ameliorated the quality of this paper. Our work is sponsored by the National Natural Science Foundation of China (No. 61462019), Natural Science Foundation of Hunan Province (No. 11JJ3074), the General Project of Shaoguan University (No. SY2016KJ11).

References

Aktaş, H., & Çağman, N. (2016). Soft decision making methods based on fuzzy sets and soft sets. Journal of Intelligent & Fuzzy Systems, 30, 2797–2803.
Alcantud, J. C. R. (2016a). A novel algorithm for fuzzy soft set based decision making from multiserver input parameter data set. Information Fusion, 29, 142–148.
Alcantud, J. C. R. (2016b). Some formal relationships among soft sets, fuzzy sets, and their extensions. Applied Soft Computing, 68, 45–53.
Alcantud, J. C. R., & Mathew, T. J. (2017). Separable fuzzy soft sets and decision making with positive and negative attributes. Applied Soft Computing, 59, 586–595.
Bansal, A., Kumar, B., & Garg, R. (2017). Multi-criteria decision making approach for the selection of software effort estimation model. Management Science Letters, 7, 285–296.
Chen, W. J., & Zou, Y. (2017). Rational decision making models with incomplete weight information. Annals of Fuzzy Mathematics and Informatics, 8, 447–460.
Nie, R. X., Wang, J. Q., & Li, L. (2017). A shareholder voting method for proxy advisory firm selection based on 2-tuple linguistic picture preference relation. Applied Soft Computing, 60, 520–539.
Pawlak, Z. (1982). Rough sets. International Journal of Computer and Information Sciences, 11, 341–356.
Peng, X. D., & Dai, J. G. (2017). Hesitant fuzzy soft decision making methods based on WASPAS, MABAC and COPRAS with combined weights. Journal of Intelligent & Fuzzy Systems, 33, 1313–1325.
Peng, X. D., & Yang, Y. (2015). Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight. Applied Soft Computing, 54, 415–430.
Peng, X. D., Yang, Y., Song, J. P., & Jiang, Y. (2015). Pythagorean fuzzy soft set and its application. Computer Engineering, 41, 224–229.
Rajasekaran, P., & Dhanalakshmi, P. (2014). Interval-valued fuzzy soft matrix theory. Annals of Pure and Applied Mathematics, 7, 61–72.
Rao, R. V., & Singh, D. (2012). Weighted Euclidean distance based approach as a multiple attribute decision making method for plant or facility layout design selection. International Journal of Industrial Engineering Computations, 3, 365–382.
Ren, P. J., Xu, Z. S., & Hao, Z. N. (2017). Hesitant fuzzy thermodynamic method for emergency decision making based on prospect theory. IEEE Transactions on Fuzzy Systems, 25, 1313–1325.
Shannon, C. E. (1948). A mathematical theory of communication. Bell System Technical Journal, 27, 379–423.
Son, M. J. (2007). Interval-valued fuzzy soft sets. Journal of Korean Institute of Intelligent Systems, 17, 557–562.
Wang, F. Q., Li, H. X., & Chen, X. H. (2014). Hesitant fuzzy set soft and its applications in multicriteria decision making. Journal of Applied Mathematics, 2014, 1–10.
Wang, Q. J., Li, X. E., & Chen, X. H. (2015). Hesitant fuzzy soft sets with application in multicriteria group decision making problems. The Scientific World Journal, 2015, 1–14.
Xiao, Z., Chen, W. J., & Li, L. L. (2013). A method based on interval-valued fuzzy soft set for multi-attribute group decision-making problems under uncertain environment. Knowledge and Information Systems, 34, 653–669.
Xie, N. X. (2016). An algorithm on the parameter reduction of soft sets. Fuzzy Information and Engineering, 8, 127–145.
Xu, Z. S., & Da, Q. L. (2002). The uncertain OWA operator. International Journal of Intelligent Systems, 17, 557–562.
Yang, X. B., Lin, T. Y., & Yang, J. Y. (2009). Combination of interval-valued fuzzy set and soft set. Computers and Mathematics with Applications, 58, 521–527.
Yang, Y., & Peng, X. D. (2017). A revised TOPSIS method based on interval fuzzy soft set model with incomplete weight information. Fundamenta Informaticae, 152, 297–309.
Yang, Y., & Wang, J. Q. (2018). SMAA-based model for decision aiding using regret theory in discrete Z-number context. Applied Soft Computing, 65, 590–602.
Yang, D. L., Xiao, Z., Xu, W., Wang, X. N., & Pan, Y. C. (2016). A novel soft set approach for feature selection. International Journal of Database Theory and Application, 9, 77–90.
Yoon, K. (1987). A reconciliation among discrete compromise solutions. Journal of the Operational Research Society, 38, 277–286.
Yuan, F., & Hu, M. J. (2012). Application of interval-valued fuzzy soft sets in evaluation of teaching quality. Journal of Hunan Institute of Science and Technology, 25, 28–30.
Zadeh, L. A. (1965). Fuzzy sets. Information Control, 8, 338–353.
Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning. Information Sciences, 8, 199–249.
Zeng, W. Y., & Li, H. X. (2006). Relationship between similarity and entropy of interval-valued fuzzy sets. *Fuzzy Sets and Systems*, 157, 1477-1484.

Zhan, J. M., Ali, M. I., & Mehmood, N. (2017). On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods. *Applied Soft Computing*, 56, 446-457.

Zhan, J. M., Liu, Q., & Herawan, T. (2017). A novel soft rough set: Soft rough hemirings and corresponding multicriteria group decision making. *Applied Soft Computing*, 54, 393-402.

Zhan, J. M., Zhou, X. W., & Xiang, D. J. (2017). Rough soft n-ary semigroups based on a novel congruence relation and corresponding decision making. *Journal of Intelligent & Fuzzy Systems*, 33, 693-703.

Zhou, H., Wang, J. Q., & Zhang, H. Y. (2018). Multi-criteria decision-making approaches based on distance measures for linguistic hesitant fuzzy sets. *Journal of the Operational Research Society*, 69, 661-675.