The Weakest Failure Detectors to Boost Obstruction-Freedom

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Abstract

This paper determines necessary and sufficient conditions to implement wait-free and non-blocking contention managers in a shared memory system. The necessary conditions hold even when universal objects (like compare-and-swap) or random oracles are available, whereas the sufficient ones assume only registers.

We show that failure detector ♦P is the weakest to convert any obstruction-free algorithm into a wait-free one, and ♦Ω∗, a new failure detector which we introduce in this paper, and which is strictly weaker than ♦P but strictly stronger than Ω, is the weakest to convert any obstruction-free algorithm into a non-blocking one.

1 Introduction

Multiprocessor systems are becoming more and more common nowadays. Multithreading thus becomes the norm and studying scalable and efficient synchronization methods is essential, for traditional locking-based techniques do not scale and may induce priority inversion, deadlock and fault-tolerance issues when a large number of threads is involved.

Wait-free synchronization algorithms [13] circumvent the issues of locking and guarantee individual progress even in presence of high contention. Wait-freedom is a liveness property which stipulates that every process completes every operation in a finite number of its own steps, regardless of the status of other processes, i.e., contending or even crashed. Ideal synchronization algorithms would ensure linearizability [16, 2], a safety property which provides the illusion of instantaneous operation executions, together with wait-freedom.

Alternatively, a liveness property called non-blockingness1 may be considered instead of wait-freedom. Non-blockingness guarantees global progress, i.e., that some process will complete an operation in a finite number of steps, regardless of the behavior of other processes. Non-blockingness is weaker than wait-freedom as it does not prevent some processes from starvation.

Wait-free and non-blocking algorithms are, however, notoriously difficult to design [18, 3], especially with the practical goal to be fast in low contention scenarios, which are usually considered the most common in practice. An appealing principle to reduce this difficulty consists in separating two concerns of a synchronization algorithm: (1) ensuring linearizability with a minimal conditional progress guarantee, and (2) boosting progress. More specifically, the idea is to focus on algorithms that ensure linearizability together with a weak liveness property called obstruction-freedom [15], and then combine these algorithms with separate generic oracles that boost progress, called contention managers [14, 20, 21, 9]. This separation lies at the heart of modern obstruction-

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1The term non-blocking is defined here in the traditional way [13]: “some process will complete its operation in a finite number of steps, regardless of the relative execution speeds of the processes.” This term is sometimes confused with the term lock-free. Note that non-blocking implementations provide a weaker liveness guarantee than wait-free implementations.
free) software transactional memory (STM) frameworks [14].

With obstruction-free (or OF, for short) algorithms, progress is ensured only for every process that executes in isolation for sufficiently long time. In presence of high contention, however, OF algorithms can livelock, preventing any process from terminating. Contention managers are used precisely to cope with such scenarios. When queried by a process executing an OF algorithm, a contention manager can delay the process for some time in order to boost the progress of other processes. The contention manager can neither share objects with the OF algorithm, nor return results on its behalf. If it did, the contention manager could peril the safety of the OF algorithm, hampering the overall separation of concerns principle.

In short, the goal of a contention manager is to provide processes with enough time without contention so that they can complete their operations. In its simplest form, a contention manager can be a randomized back-off protocol. More sophisticated contention management strategies have been experimented in practice [20, 21, 10]. Precisely because they are entirely devoted to progress, they can be combined or changed on the fly [9]. Most previous strategies were pragmatic, with no aim to provide worst case guarantees. In this paper we focus on contention managers that provide such guarantees. More specifically, we study contention managers that convert any OF algorithm into a non-blocking or wait-free one, and which we call, respectively, non-blocking or wait-free contention managers.

Two wait-free contention managers have recently been proposed [6, 8]. Both rely on timing assumptions to detect processes that fail in the middle of their operations. This suggests that some information about failures might inherently be needed by any wait-free contention manager. But this is not entirely clear because, in principle, a contention manager could also use randomization to schedule processes, or even powerful synchronization primitives like compare-and-swap, which is known to be universal, i.e., able to wait-free implement any other object [13]. In the parlance of [5], we would like to determine whether a failure detector is actually needed to implement a contention manager with worst case guarantees, and if it is, what is the weakest one [4]. Besides the theoretical interest, determining the minimal conditions under which a contention manager can ensure certain guarantees is, we believe, of practical relevance, for this might help portability and optimization.

We show that the eventually perfect failure detector $\Diamond P$ [5] is the weakest to implement a wait-free contention manager.\(^2\) We also introduce a failure detector $\Omega^*$, which we show is the weakest to implement a non-blocking contention manager. Failure detector $\Omega^*$ is strictly weaker than $\Diamond P$, and strictly stronger than failure detector $\Omega$ [4], known to be the weakest to wait-free implement the (universal) consensus object [13].\(^3\)

It might be surprising that $\Omega$ is not sufficient to implement a wait-free or even a non-blocking contention manager. For example, the seminal Paxos algorithm [19] uses $\Omega$ to transform an OF implementation of consensus into a wait-free one. Each process that is eventually elected a leader by $\Omega$ is given enough time to run alone, reach a decision and communicate it to the others. This approach does not help, however, if we want to make sure that processes make progress regardless of the actual (possibly long-lived) object and its OF implementation. Intuitively, the leader elected by $\Omega$ may have no operation to perform while other processes may livelock forever. Because a contention manager cannot make processes help each other, the output of $\Omega$ is not sufficient: this is so even if randomized oracles or universal objects are available. Intuitively, wait-free contention managers need a failure detector that would take care of every non-crashed process with a pending operation so that the process can run alone for sufficiently long time. As for non-blocking contention managers, at least one process that never crashes, among the ones with pending operations, should be given enough time to run alone.

The paper is organized as follows. Section 2 presents our system model and formally defines wait-free and non-blocking contention managers. These definitions are, we believe, contributions in their own rights, for they capture precisely the interaction between a contention manager and an obstruction-free algorithm. In Sect. 3 and 4, we prove our weakest failure detector results. In each case, we first present (necessary part) a reduction algorithm [4] that extracts the output of failure detector $\Diamond P$ (respectively $\Diamond P$) using a non-blocking (respectively wait-free) contention manager im-

\(^2\)\(\Diamond P\) ensures that eventually: (1) every failure is detected by every correct (i.e., non-faulty) process and (2) there is no false detection.

\(^3\)\(\Omega\) ensures that eventually all correct (i.e., non-faulty) processes elect the same correct process as their leader.
implementation. When devising our reduction algorithms, we do not restrict what objects (or random oracles) can be used by the contention manager or the OF algorithm. Then (sufficient part), we present algorithms that implement the contention managers using the failure detectors and registers. These algorithms are devised with the sole purpose of proving our sufficiency claims. We do not seek to minimize the overhead of the interaction between the OF algorithm and the contention manager, nor do we discuss how the failure detector can itself be implemented with little synchrony assumptions and minimal overhead, unlike the transformations presented in \cite{3}. However, as we show in \cite{2}, our algorithms can be easily extended to meet these challenges.

## 2 Preliminaries

### Processes and Failure Detectors.

We consider a set of $n$ processes $\Pi = \{p_1, \ldots, p_n\}$ in a shared memory system \cite{13, 17}. A process executes the (possibly randomized) algorithm assigned to it, until the process crashes (fails) and stops executing any action. We assume the existence of a global discrete clock that is, however, inaccessible to the processes. We say that a process is correct if it never crashes. We say that process $p_i$ is alive at time $t$ if $p_i$ has not crashed by time $t$.

A failure detector \cite{5, 4} is a distributed oracle that provides every process with some information about failures. The output of a failure detector depends only on which and when processes fail, and not on computations being performed by the processes. A process $p_i$ queries a failure detector $D$ by accessing local variable $D$-output,—the output of the module of $D$ at process $p_i$. Failure detectors can be partially ordered according to the amount of information about failures they provide. A failure detector $D$ is weaker than a failure detector $D'$, and we write $D \preceq D'$, if there exists an algorithm (called a reduction algorithm) that transforms $D'$ into $D$. If $D \preceq D'$ but $D' \preceq D$, we say that $D$ is strictly weaker than $D'$, and we write $D \prec D'$.

### Base and High-Level Objects.

Processes communicate by invoking primitive operations (which we will call instructions) on base shared objects and seek to implement the operations of a high-level shared object $O$. Object $O$ is in turn used by an application, as a high-level inter-process communication mechanism. We call invocation and response events of a high-level operation $op$ on the implemented object $O$ application events and denote them by, respectively, $inv(op)$ and $ret(op)$ (or $inv_i(op)$ and $ret_i(op)$ at a process $p_i$).

An implementation of $O$ is a distributed algorithm that specifies, for every process $p_i$ and every operation $op$ of $O$, the sequences of steps that $p_i$ should take in order to complete $op$. Process $p_i$ completes operation $op$ when $p_i$ returns from $op$. Every process $p_i$ may complete any number of operations but, at any point in time, at most one operation $op$ can be pending (started and not yet completed) at $p_i$.

We consider implementations of $O$ that combine a sub-protocol that ensures a minimal liveness property, called obstruction-freedom, with a sub-protocol that boosts this liveness guarantee. The former is called an obstruction-free (OF) algorithm $A$ and the latter a contention manager $CM$. We focus on linearizable \cite{16, 2} implementations of $O$: every operation appears to the application as if it took effect instantaneously between its invocation and its return. An implementation of $O$ involves two categories of steps executed by any process $p_i$: those (executed on behalf) of $CM$ and those (executed on behalf) of $A$. In each step, a process $p_i$ either executes an instruction on a base shared object or (in case $p_i$ executes a step on behalf of $CM$) queries a failure detector.

Obstruction-freedom \cite{15, 14} stipulates that if a process that invokes an operation $op$ on object $O$ and from some point in time executes steps of $A$ alone\footnote{I.e., without encountering step contention \cite{1}.}, then it eventually completes $op$. Non-blockingness stipulates that if some correct process never completes an invoked operation, then some other process completes infinitely many operations. Wait-freedom \cite{13} ensures that every correct process that invokes an operation eventually returns from the operation.

### Interaction Between Modules.

OF algorithm $A$, executed by any process $p_i$, communicates with contention manager $CM$ via calls $try_i$ and $resign_i$ implemented by $CM$ (see Fig. 1). Process $p_i$ invokes $try_i$ just after $p_i$ starts an operation, and also later (even several times before $p_i$ completes the operation) to signal possible contention. Process $p_i$ invokes $resign_i$ just before returning from an operation, and always eventually returns from this call (or crashes). Both calls, $try_i$ and $resign_i$, return $ok$. 
An example OF algorithm that uses this model of interaction with a contention manager is presented in Algorithm 1. The algorithm implements a timestamping mechanism and is based on the implementation of a splitter. It is not meant to be practical or efficient—it just shows how calls \texttt{try} and \texttt{resign} should be used.

A discussion about overhead of wait-free/non-blocking contention managers that explains when calls to \texttt{try}/\texttt{resign} can be omitted for efficiency reasons can be found in \cite{11}.

\textbf{Algorithm 1}: An example OF algorithm implementing a timestamping mechanism

\begin{verbatim}
uses: \texttt{A}[1, . . .]—unbounded array of registers, 
    \texttt{B}[1, . . .]—unbounded array of single-bit registers, 
    \texttt{L}—a register
initially: \texttt{A}[1, . . .] ← ⊥, \texttt{B}[1, . . .] ← false, \texttt{L} ← 1

upon \texttt{of-getTimestamp} do
    \texttt{CM} \texttt{.try} {
        \texttt{j} ← \texttt{L}
        while true do
            if \texttt{B}[\texttt{j}] = false then
                \texttt{B}[\texttt{j}] ← true
            if \texttt{A}[\texttt{j}] = \texttt{i} then
                \texttt{CM} \texttt{.resign} {
                    \texttt{L} ← \texttt{j}
                    \texttt{CM} \texttt{.resign}$_i$
                    \texttt{return} \texttt{j}
                }
            \texttt{CM} \texttt{.try}$_i$
            \texttt{j} ← \texttt{j} + 1
        }
    }
\end{verbatim}

We denote by \texttt{B}(\texttt{A}) and \texttt{B}(\texttt{CM}) the sets of base shared objects, always disjoint, that can be possibly accessed by steps of, respectively, \texttt{A} and \texttt{CM}, in every execution, by every process. Calls \texttt{try} and \texttt{resign} are thus the only means by which \texttt{A} and \texttt{CM} interact. The events corresponding to invocations of, and responses from, \texttt{try} and \texttt{resign} are called \texttt{cm}-events. We denote by \texttt{try}$_i$\texttt{inv} and \texttt{resign}$_i$\texttt{inv} an invocation of call \texttt{try}$_i$ and \texttt{resign}$_i$, respectively (at process \texttt{p}$_i$), and by \texttt{try}$_i$\texttt{ret} and \texttt{resign}$_i$\texttt{ret}—the corresponding responses.

\textbf{Executions and Histories}. An execution of an OF algorithm \texttt{A} combined with a contention manager \texttt{CM} is a sequence of \texttt{events} that include steps of \texttt{A}, steps of \texttt{CM}, \texttt{cm}-events and application events. Every event in an execution is associated with a unique time at which the event took place. Every execution \texttt{e} induces a \texttt{history} \texttt{H}(\texttt{e}) that includes only application events (invocations and responses of high-level operations). The corresponding CM-\texttt{history} \texttt{H}$_{\texttt{CM}}$(\texttt{e}) is the subsequence of \texttt{e} containing only application events and \texttt{cm}-events of the execution, and the corresponding OF-\texttt{history} \texttt{H}$_{\texttt{OF}}$(\texttt{e}) is the subsequence of \texttt{e} containing only application events, \texttt{cm}-events, and steps of \texttt{A}. For a sequence \texttt{s} of events, \texttt{s}$_i$\texttt{|} denotes the subsequence of \texttt{s} containing only events at process \texttt{p}$_i$.

We say that a process \texttt{p}$_i$ is \texttt{blocked} at time \texttt{t} in an execution \texttt{e} if (1) \texttt{p}$_i$ is alive at time \texttt{t}, and (2) the latest event in \texttt{H}$_{\texttt{CM}}$(\texttt{e})|\texttt{t} that occurred before \texttt{t} is \texttt{try}$_i$\texttt{inv} or \texttt{resign}$_i$\texttt{inv}. A process \texttt{p}$_i$ is \texttt{busy} at time \texttt{t} in \texttt{e} if (1) \texttt{p}$_i$ is alive at time \texttt{t}, and (2) the latest event in \texttt{H}$_{\texttt{CM}}$(\texttt{e})|\texttt{t} that occurred before \texttt{t} is \texttt{try}$_i$\texttt{ret}. We say that a process \texttt{p}$_i$ is \texttt{active} at time \texttt{t} in \texttt{e} if \texttt{p}$_i$ is either busy or blocked at time \texttt{t} in \texttt{e}. We say that a process \texttt{p}$_i$ is \texttt{idle} at time \texttt{t} in \texttt{e} if \texttt{p}$_i$ is not active at time \texttt{t} in \texttt{e}.$^5$ A process \texttt{resigns} when it invokes \texttt{resign} on a contention manager.

We say that a process \texttt{p}$_i$ is \texttt{obstruction-free} in an interval \texttt{[t}, \texttt{t}′\texttt{]} in an execution \texttt{e}, if \texttt{p}$_i$ is the only process that takes steps of \texttt{A} in \texttt{[t}, \texttt{t}′\texttt{]} in \texttt{e} and \texttt{p}$_i$ is not blocked infinitely long in \texttt{[t}, \texttt{t}′\texttt{]} (if \texttt{t}′ = \infty). We say that process \texttt{p}$_i$ is \texttt{eventually obstruction-free} at time \texttt{t} in \texttt{e} if \texttt{p}$_i$ is active at time \texttt{t} or later and \texttt{p}$_i$ either resigns after \texttt{t} or is obstruction-free in the interval \texttt{[t}, \texttt{t}′\texttt{]} for some \texttt{t}′ > \texttt{t}. Note that, since algorithm \texttt{A} is obstruction-free, if an active process \texttt{p}$_i$ is eventually obstruction-free, then \texttt{p}$_i$ eventually resigns and completes its operation.

\textbf{Well-Formed Executions}. We impose certain restrictions on the way an OF algorithm \texttt{A} and a contention manager \texttt{CM} interact. In particular, we assume that no process takes steps of \texttt{A} while being blocked by \texttt{CM} or idle, and no process takes infinitely many steps of \texttt{A} without calling \texttt{CM} infinitely many times. Further, a process must inform \texttt{CM} that an operation is completed by calling \texttt{resign} before returning the response to the application.

Formally, we assume that every execution \texttt{e} is \texttt{well-formed}, i.e., \texttt{H}(\texttt{e}) is linearizable \cite{16,2}, and, for every process \texttt{p}$_i$, (1) \texttt{H}$_{\texttt{CM}}$(\texttt{e})|\texttt{t} is a prefix of a sequence \texttt{[op}$_1$, \texttt{op}$_2$|..., where each \texttt{op}$_i$ has the form \texttt{inv}$_i$(\texttt{op}$_i$), \texttt{try}$_i$\texttt{inv}, \texttt{try}$_i$\texttt{ret}, ..., \texttt{try}$_i$\texttt{inv}, \texttt{try}$_i$\texttt{ret}, \texttt{resign}$_i$\texttt{inv}, \texttt{resign}$_i$\texttt{ret}, \texttt{ret}$_i$(\texttt{op}$_i$); (2) in \texttt{H}$_{\texttt{OF}}$(\texttt{e})|\texttt{t}, no step of \texttt{A} is executed when \texttt{p}$_i$ is blocked or idle, (3) in \texttt{H}$_{\texttt{OF}}$(\texttt{e})|\texttt{t}, \texttt{inv}$_i$ can only be followed by \texttt{try}$_i$\texttt{inv}, and

$^5$Note that every process that has crashed is permanently idle.
ret_i can only be preceded by resign^ret_i; (4) if p_i is busy at time t in e, then at some t' > t, process p_i is idle or blocked. The last condition implies that every busy process p_i eventually invokes try_i (and becomes blocked), resigns or crashes. Clearly, in a well-formed execution, every process goes through the following cyclical order of modes: idle, active, idle, . . ., where each active period consists itself of a sequence blocked, busy, blocked, . . . .

Non-blocking Contention Manager. We say that a contention manager CM guarantees non-blockingness for an OF algorithm A if in each execution e of A combined with CM the following property is satisfied: if some correct process is active at a time t, then at some time t' > t some process resigns.

A non-blocking contention manager guarantees non-blockingness for every OF algorithm. Intuitively, this will happen if the contention manager allows at least one active process to be obstruction-free (and busy) for sufficiently long time, so that the process can complete its operation. More precisely, we say that a contention manager CM is non-blocking if, for every OF algorithm A, in every execution of A combined with CM the following property is ensured at every time t:

Global Progress. If some correct process is active at t, then some correct process is eventually obstruction-free at t.

Theorem 1 A contention manager CM guarantees non-blockingness for every OF algorithm if and only if CM is non-blocking.

Proof. (⇒) Consider a contention manager CM that guarantees non-blockingness for every OF algorithm. Let A be any OF algorithm and e be any execution of A combined with CM. Let some correct process be active at time t in e. Since CM guarantees non-blockingness, some active process resigns at some future time, and the Global Progress property is trivially ensured.

(⇐) By contradiction, assume that there exists a non-blocking contention manager CM such that, for some OF algorithm A, there is an execution e of A combined with CM, such that some correct process is active at t, and no active process resigns after t. By Global Progress, some correct active process p_i is eventually obstruction-free at t. Since A is obstruction-free and p_i takes infinitely many steps of A in isolation, p_i must complete its operation and resign—a contradiction.

Wait-Free Contention Manager. We say that a contention manager CM guarantees wait-freedom for an OF algorithm A if in every execution e of A combined with CM, the following property is satisfied: if a process p_i is active at a time t, then at some time t' > t, p_i becomes idle. In other words, every operation executed by a correct process eventually returns.

A wait-free contention manager guarantees wait-
freedom for every OF algorithm. Intuitively, this will happen if the contention manager makes sure that every correct active process is given “enough” time to complete its operation, regardless of how other processes behave. More precisely, a contention manager $CM$ is wait-free if, for every OF algorithm $A$, in every execution of $A$ combined with $CM$, the following property is ensured at every time $t$.

**Fairness.** If a correct process $p_i$ is active at $t$, then $p_i$ is eventually obstruction-free at $t$.

**Theorem 2** A contention manager $CM$ guarantees wait-freedom for every OF algorithm if and only if $CM$ is wait-free.

**Proof.** ($\Rightarrow$) Consider a contention manager $CM$ that guarantees wait-freedom for every OF algorithm. Let $A$ be any OF algorithm and $e$ be any execution of $A$ combined with $CM$. Since in $e$ every active process is eventually idle, every correct active process eventually resigns in $e$, and so the Fairness property is trivially satisfied.

($\Leftarrow$) Let $CM$ be a wait-free contention manager, and $A$ be any OF algorithm. Consider any execution $e$ of $A$ combined with $CM$.

Suppose, by contradiction, that some correct process $p_i$ is active at time $t$ and never completes its operation thereafter. But then, by Fairness, $p_i$ is eventually obstruction-free at $t$ and so $p_i$ is obstruction-free in period $[t', \infty)$ for some $t' > t$. Therefore, since $A$ is obstruction-free and $p_i$ takes infinitely many steps of $A$ in isolation, $p_i$ must eventually resign and complete its operation—a contradiction.

In the following, we seek to determine the weakest [4] failure detector $D$ to implement a non-blocking (resp. wait-free) contention manager $CM$. This means that (1) $D$ implements such a contention manager, i.e., there is an algorithm that implements $CM$ using $D$, and (2) $D$ is necessary to implement such a contention manager, i.e., if a failure detector $D'$ implements $CM$, then $D \preceq D'$. In our context, a reduction algorithm that transforms $D'$ into $D$ uses the $D'$-based implementation of the corresponding contention manager as a “black box” and read-write registers.

### 3 Non-blocking Contention Managers

Let $S \subseteq \Pi$ be a non-empty set of processes. Failure detector $\Omega_S$ outputs, at every process, an identifier of a process (called a *leader*), such that all correct processes in $S$ eventually agree on the identifier of the same correct process in $S$.

Failure detector $\Omega^*$ is the composition $\{\Omega_i\}_{i \in S} \cup S \neq \emptyset$: at every process $p_i$, $\Omega^*$-output$_i$ is a tuple consisting of the outputs of failure detectors $\Omega_S$. We position $\Omega^*$ in the hierarchy of failure detectors of [5] by proving the following theorem:

**Theorem 3** $\Omega < \Omega^* < \Diamond \mathcal{P}$.

**Proof.** It is immediate that $\Omega$ is weaker than $\Omega^*$: $\Omega_{\Pi}$ is equivalent to $\Omega$. In a system of three or more processes, $\Omega$ is strictly weaker than $\Omega^*$. Indeed, consider a system of three processes, $p_1, p_2,$ and $p_3$, and assume, by contradiction, that $\Omega^*$ is weaker than $\Omega$, i.e., that there exists a reduction algorithm $T_{\Omega_\Pi}$-$\Omega^*$ which extracts the output of $\Omega^*$ using $\Omega$. Take an execution $e$ of $T_{\Omega_\Pi}$-$\Omega^*$ in which $p_3$ is correct, $p_2$ is faulty, $\Omega$ always outputs $p_3$ at every process and consider the emulated output of $\Omega_{\{p_1, p_2\}}$. Since $p_1$ is the only correct process in $\{p_1, p_2\}$, there is a finite prefix $e'$ of $e$ in which $\Omega_{\{p_1, p_2\}}$ outputs $p_1$ at $p_1$. But this finite execution is indistinguishable from a finite execution $e''$ in which $p_2$ is correct but slow. Now consider a finite extension of $e''$ in which $p_1$ fails, and thus eventually $\Omega_{\{p_1, p_2\}}$ outputs $p_2$ at $p_2$. But this finite execution is indistinguishable from a finite execution in which $p_1$ is correct but slow. By repeating this argument, we obtain an infinite execution of $T_{\Omega_\Pi}$-$\Omega^*$ in which both $p_1$ and $p_2$ are correct, and the output $\Omega_{\{p_1, p_2\}}$ never stabilizes at a single correct process—a contradiction.

It is immediate that $\Omega^*$ is strictly weaker than $\Diamond \mathcal{P}$: eventually each correct process $p_i$ has complete and accurate information about failures of all other processes, so $p_i$ can perform an eventually perfect leader election in each subset of processes $p_i$ belongs to.

To show that $\Omega^*$ is strictly weaker than $\Diamond \mathcal{P}$, consider a system of two processes, $p_1$ and $p_2$, and assume, by contradiction, that $\Diamond \mathcal{P}$ is weaker than

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6 This property is ensured by wait-free contention managers from the literature [6, 8].

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7 $\Omega_3$ can be seen as a restriction of the eventual leader election failure detector $\Omega$ [4] to processes in $S$. The definition of $\Omega_3$ resembles the notion of $\Gamma$-accurate failure detectors introduced in [12]. Clearly, $\Omega_{\Pi}$ is $\Omega$. 6
Lemma 5 Contention manager shown in Algorithm 3 guarantees non-blockingness for every OF algorithm.
Algorithm 2: Extracting $\Omega_S$ from a non-blocking contention manager (code for processes from set $S$; others are permanently idle)

uses: $L$—register

initially: $\Omega_S$-output$_i \leftarrow p_i$, $L \leftarrow$ some process in $S$

Launch two parallel tasks: $T_i$ and $F_i$

1. parallel task $F_i$
   2. $\Omega_S$-output$_i \leftarrow L$

2. parallel task $T_i$

   while true do
   3. issue try$_i$ and wait until busy (i.e., until call try$_i$ returns)
   4. $L \leftarrow p_i$ // announce yourself a leader

Algorithm 3: A non-blocking contention manager using $\Omega = \{\Omega_S\}_{S \subseteq \Pi, S \neq \emptyset}$

uses: $T[1, \ldots, n]$—array of single-bit registers

initially: $T[1, \ldots, n] \leftarrow$ false

1. upon try$_i$ do
   2. $T[i] \leftarrow$ true
   3. repeat
   4. $S \leftarrow \{p_j \in \Pi \mid T[j] =$ true $\}$
   5. until $\Omega_S$-output$_i = p_i$

3.6 upon resign$_i$ do
   3.7 $T[i] \leftarrow$ false

Proof. Assume, by contradiction that there exists an OF algorithm $A$ for which contention manager $CM$ implemented by Algorithm 3 does not guarantee non-blockingness, i.e., there exists an execution $e$ of $A$ combined with $CM$ in which there are a correct process $p_i$, and a time $t$, such that $p_i$ is active at $t$ but for all $t' > t$, no active process resigns at $t'$.

Take any time $t' > t$. Let us denote by $S(t')$ the set of all processes $p_j$ such that $T[j] =$ true at time $t'$ in $e$. Since no active process resigns after $t$, there is a time $t'' \geq t$ and a set $S$, such that for all $t' > t''$, $S(t') = S$. By the algorithm, $p_i$ eventually sets $T[j]$ to true. Thus, $p_i$ is in $S$, i.e., $S$ includes at least one correct process. At every correct process in $S$, $\Omega_S$ eventually outputs the same correct process $p_j$ in set $S$ (a leader).

Since every active process eventually invokes try, resigns or crashes (by the properties of OF algorithms), and no process resigns after $t''$, there is a time $t'' > t''$ after which every correct process except for $p_j$ gets permanently blocked in lines 3.3–3.5. That is because $p_j$ does not resign after $t$ and so $p_j$ does not reset $T[j]$ to false thereafter and remains the leader for set $S$ forever. Thus, $p_i$ is eventually obstruction-free at $t$. Since $p_j$ runs an obstruction-free algorithm $A$, it eventually resigns and completes its operation—a contradiction. $\Box$

From Theorem 1 and Lemma 5 we immediately obtain a proof of the following theorem:

Theorem 6 Algorithm 3 implements a non-blocking contention manager.

4 Wait-Free Contention Managers

We prove here that the weakest failure detector to implement a wait-free contention manager is $\Diamond P$. Failure detector $\Diamond P$ [5] outputs, at each time and every process, a set of suspected processes. There is a time after which (1) every crashed process is permanently suspected by every correct process and (2) no correct process is ever suspected by any correct process.

We first consider a wait-free contention manager $CM$ using a failure detector $D$, and we exhibit a reduction algorithm $T_{D-\Diamond P}$ (Algorithm 4) that, using $CM$ and $D$, emulates the output of $\Diamond P$.

We run several instances of $CM$. These instances use disjoint sets of base shared objects and do not directly interact. Basically, in each instance, only two processes are active and all other processes are idle. One of the two processes, say $p_j$, gets active and never resigns thereafter, while the other, say $p_i$, permanently alternates between being active and idle. To $CM$ it looks like $p_j$ is always obstructed by $p_i$. Thus, to guarantee wait-freedom, the instance of $CM$ has to eventually block $p_i$ and let $p_i$ run obstruction-free until $p_i$ resigns or crashes. Therefore, when $p_i$ is blocked, $p_i$
can assume that \( p_j \) is alive and when \( p_i \) is busy, \( p_i \) can suspect \( p_j \) of having crashed, until \( p_j \) eventually observes \( p_j \)'s “heartbeat” signal, which \( p_j \) periodically broadcasts using a register. This ensures the properties of \( \lozenge P \) at process \( p_i \), provided that \( p_j \) never resigns.

As in Sect. 3, we face the following issue. If \( p_j \) is correct, \( p_i \) will be eventually blocked forever and \( p_j \) will thus be eventually obstruction-free. Hence, in the corresponding execution, obstruction-freedom is violated, i.e., the execution cannot be produced by any OF algorithm combined with CM. One might argue then that CM is not obliged to preserve Fairness with respect to \( p_j \). However, we show that, since CM does not “know” how much time a process executing an OF algorithm requires to complete its operation, CM has to provide \( p_j \) with unbounded time to run in isolation.

More precisely, the processes in Algorithm 4 run \( n(n - 1) \) parallel instances of CM, denoted each \( CM_{jk} \), where \( j, k \in \{1, \ldots, n\}, j \neq k \). We denote the events that process \( p_i \) issues in instance \( CM_{jk} \) by \( \text{try}_{ij}^k \) and \( \text{resign}_{ij}^k \). Besides, every process \( p_i \) runs \( 2n - 1 \) parallel tasks: \( T_{ij}, T_{ji} \), where \( j \in \{1, \ldots, n\}, i \neq j \), and \( F_i \). Every task \( T_{ij} \) executed by \( p_j \) is responsible for detecting failures of process \( p_j \). Every task \( T_{ji} \) executed by \( p_i \) is responsible for preventing \( p_i \) from falsely suspecting \( p_j \). In task \( F_i \), \( p_i \) periodically writes ever-increasing “heartbeat” values in a shared register \( R[i] \).

In every instance \( CM_{ji} \), there can be only two active processes: \( p_i \) and \( p_j \). Process \( p_i \) cyclically gets active (line 4.7) and resigns (line 4.8), and process \( p_j \) gets active once and keeps getting blocked (line 4.12). Each time before \( p_i \) gets active, \( p_i \) removes \( p_j \) from the list of suspected processes (line 4.6). Each time \( p_i \) stops being blocked, \( p_i \) starts suspecting \( p_j \) (line 4.9) and waits until \( p_j \) observes a “new” step of \( p_j \) (line 4.10). Once such a step of \( p_j \) is observed, \( p_i \) stops suspecting \( p_j \) and gets active again.

**Theorem 7** Every wait-free contention manager can be used to implement failure detector \( \lozenge P \).

*Proof.* Consider any execution \( e \) of \( TD_{\lozenge P} \), and let \( p_i \) be any correct process. We show that, in \( e \), \( \lozenge P\)-output \( i \) satisfies the properties of \( \lozenge P \), i.e., \( p_i \) eventually permanently suspects every non-active process and stops suspecting every correct process. (Note that if a process \( p_j \) is not correct, then \( \lozenge P\)-output \( i \) trivially satisfies the properties of \( \lozenge P \).)

Let \( p_i \) be any process distinct from \( p_j \). Assume \( p_j \) is not correct. Thus \( p_i \) is the only correct active process in instance \( CM_{ji} \). By the Fairness property of CM, \( p_i \) is eventually obstruction-free every time \( p_i \) becomes active, and so \( p_i \) cannot be blocked infinitely long in line 4.7. Since there is a time after which \( p_j \) stops taking steps, eventually \( p_i \) starts
suspecting \( p_i \) (line 4.9) and suspends in line 4.10, waiting until \( p_i \) takes a new step. Thus, \( p_i \) eventually suspects \( p_j \) forever.

Assume now that \( p_i \) is correct. We claim that \( p_i \) must eventually get permanently blocked so that \( p_i \) would run obstruction-free from some point in time forever. Suppose not. But then we obtain an execution in which \( p_i \) alternates between active and idle modes infinitely many times, and \( p_j \) stays active and runs obstruction-free only for bounded periods of time. But the CM-history of this execution could be produced by an execution \( e' \) of some OF algorithm combined with CM in which \( p_i \) never completes its operation because \( p_i \) never runs long enough in isolation. Thus, Fairness is violated in execution \( e' \) and this contradicts the assumption that CM is wait-free. Hence, eventually \( p_i \) gets permanently blocked in line 4.7. Since each time \( p_i \) is about to get blocked, \( p_i \) stops suspecting \( p_j \) in line 4.6, there is a time after which \( p_i \) never suspects \( p_j \).

Thus, there is a time after which, if \( p_i \) is correct, then \( p_i \) stops being suspected by every correct process, and if \( p_i \) is non-correct, then every correct process permanently suspects \( p_j \).

We describe an implementation of a wait-free contention manager using \( \Diamond \mathcal{P} \) and registers in Algorithm 5. The algorithm relies on a (wait-free) primitive \( \text{GetTimestamp}() \) that generates unique, locally increasing timestamps and makes sure that if a process gets a timestamp \( ts \), then no process can get timestamps lower than \( ts \) infinitely many times (this primitive can be implemented in an asynchronous system using read-write registers). The idea of the algorithm is the following. Every process \( p_i \) that gets active receives a timestamp in line 5.2 and announces the timestamp in register \( T[i] \). Every active process that invokes \( \text{try} \) repeatedly runs a leader election mechanism (lines 5.3–5.6): the non-suspected (by \( \Diamond \mathcal{P} \)) process that announced the lowest (non-\( \bot \)) timestamp is elected a leader. If a process \( p_i \) is elected, \( p_i \) returns from \( \text{try} \) and becomes busy. \( \Diamond \mathcal{P} \) guarantees that eventually the same correct active process is elected by all active processes. All other active processes stay blocked until the process resigns and resets its timestamp in line 5.8. The leader executes steps obstruction-free then. Since the leader runs an OF algorithm, the leader eventually resigns and resets its timestamp in line 5.8 so that another active process, which now has the lowest timestamp in \( T \), can become a leader.

Lemma 8 Contention manager implemented by Algorithm 5 guarantees wait-freedom for all OF algorithms.

Proof. Consider an execution \( e \) of any OF algorithm \( A \) combined with contention manager CM implemented by Algorithm 5. By contradiction, assume that in \( e \), some correct process is active at some time \( t \), and never resigns after \( t \). Let \( V \) denote the non-empty set of correct processes that are active at \( t \) but never resign (in line 5.8) and complete their operations thereafter, i.e., that remain active after \( t \) forever. Recall that every process in \( V \) either invokes \( \text{try} \) infinitely many times or invokes \( \text{try} \) and stays blocked forever (by the properties of OF algorithms). Let \( t^* > t \) be time at which every process in \( V \) invoked \( \text{try} \) and reached line 5.3 at least once. For every \( p_j \in V \), let \( ts_j^* \) denote the value of \( T[j] \) at time \( t^* \). Note that since every \( ts_j^* \neq \bot \) and no process in \( V \) resigns after time \( t^* \), \( T[j] = ts_j^* \) at all times \( t' \geq t^* \).

Let \( p_i \) be the process in \( V \) having the lowest timestamp in \( \{ ts_k^* \mid p_k \in V \} \) (there is exactly one such process since timestamps are unique). We establish a contradiction by showing that \( p_i \) has to eventually resign.

Let us consider time \( t' > t^* \) after which:
- at every correct process, failure detector \( \Diamond \mathcal{P} \) outputs the list of all non-correct processes (by the properties of \( \Diamond \mathcal{P} \), this eventually happens),
- all non-correct processes have crashed,
- for every correct process \( p_j \neq p_i \), if \( T[j] \neq \bot \), then \( T[j] > ts_j^* \).

The last condition eventually holds, because timestamps are unique, no process can receive a timestamp lower that \( ts_j^* \) infinitely many times and \( p_i \) has the lowest timestamp among processes in \( V \) (that retain their timestamps infinitely long).

Thus, after \( t' \), \( p_i \) is always elected a leader, and every correct process \( p_j \) other than \( p_i \) that gets blocked after time \( t' \) will remain blocked in lines 3.3–3.5, as long as \( p_i \) does not resign.

Hence, eventually \( p_i \) will be the only active process that is not blocked and thus \( p_i \) will be given unbounded time to perform steps of \( A \) in isolation. Since \( A \) is obstruction-free, \( p_i \) eventually resigns and completes its operation—a contradiction.

From Theorem 2 and Lemma 8 we immediately obtain a proof of the following theorem:

Theorem 9 Algorithm 5 implements a wait-free contention manager.
Algorithm 5: A wait-free contention manager using $\Diamond P$

uses: $T[1, \ldots, N]$—array of registers (other variables are local)

initially: $T[1, \ldots, N] \leftarrow \perp$

5.1 upon try $i$ do
5.2   if $T[i] = \perp$ then $T[i] \leftarrow \text{GetTimestamp()}$
5.3   repeat
5.4     $sact_i \leftarrow \{ j \mid T[j] \neq \perp \land p_j \not\in \Diamond P\text{-output}_i \}$
5.5     $leader_i \leftarrow \argmin_{j \in sact_i} T[j]$
5.6     until $leader_i = i$
5.7 upon resign $i$ do
5.8     $T[i] \leftarrow \perp$

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