Capability measurement of non-standard parameters of manufacturing processes

R Turisova¹ and S Markulik²
¹Technical University of Kosice, Mechanical engineering faculty, Department of safety and quality, Letna 1/9, 042 00, Kosice, Slovakia
²Technical University of Kosice, Mechanical engineering faculty, Department of safety and quality, Letna 1/9, 042 00, Kosice, Slovakia

Abstract. Capability measurement of manufacturing processes creates basic attributes for the production of conformity products, those that meet the necessary conditions in terms of tolerance limits for all important parameters. They are also important in the supplier-customer relationships context as one of the important easily predictable but highly effective indicators of the quality production of delivered products. In many manufacturing companies, the capability measurement is performed by using capability indicators, where the method of calculation is based on the assumption of normality of the distribution of the measured product parameter, as well as the assumption of constant median value but also variability during the monitored period. However, in practice we often are able to see such parameters of manufacturing products or processes, which do not meet the expectations required in the classical method of capability indexes calculation. The problems are usually in a different than normal models of the distribution of a random variable representing a specific parameter of a product or process, as well as in the excessive variability of median values during the time in these so-called non-standard parameters. There will be presented on the particular manufacturing product, how certain non-conformities occur during the capability indexes measurement indices in the classical way for selected non-standard parameters. The paper describes how it is possible to modify the common method of calculating the mentioned capability indexes in a simple way so that they are applicable in specific production conditions.

1. Introduction
Process capability measurement is now routinely carried out in many manufacturing and service companies. In addition to the ppm (parts per million) parameter, which represents the number of failures per million units of output, capability indexes are probably the most common tools in process capability analysis. They measure, in a relatively simple way, to what extent a selected product parameter meets customer expectations. In doing so, they make use of some basic statistical models, namely distributions of random variables, which are also the subject of this paper. The reason why capability indexes are used quite often in practice is their simplicity and relative ease of interpretation. Using them, it is possible to estimate the actual performance of a process so that accurate information is provided to the customer. However, they are also a popular tool to use for process improvement. The analysis of process
capability and performance thus becomes an essential step in process quality management in the most general sense of the word [1,2].

The first ever metric that directly compares process variability to customer specifications was proposed by Jurana and Gryna [3] and called Capability Ratio. It is essentially an inversion of the currently used \( C_p \) capability index. Since then, a large body of literature has addressed the different properties of various PCIs (Process Capability Indexes).

In this paper, we will show, using a concrete example from engineering practice, that this frequently used indicator is perhaps not as robust and universally applicable as it is written about in many books and manuals oriented to process capability measurement. Several examples are described in the literature where various errors occur in the practical use of the mentioned index and various modifications and adjustments are proposed to at least partially if not completely eliminate these errors [4]. Before proceeding to the results of our investigation, we present some basic definitions).

2. Process Capability Index

In a capability analysis, a process is assessed using statistical methods to determine whether it meets a set of predefined specifications or requirements. Suppliers and manufacturers often use this method to assure customers that their products are of high quality with the least possible non-conformance. This is also why PCI, along with ppm, are the most widely used tools among all available capability analysis methods. The Process Capability Index measures the extent to which a process meets customer expectations. Customer expectations are usually described by numerical (specification) values within which the process is expected to work. The problem is that the customer often chooses the specific PCI and also determines the threshold value that must be met. It is then very unpopular for the manufacturer to convince the customer of the inappropriateness of the PCI chosen by it, or of the necessity of some modification of its calculation. The process capability index in the form used today was proposed by Kane [5]:

\[
C_p = \frac{USL - LSL}{6\sigma}
\]  

(1)

where \( USL \) and \( LSL \) are the upper and lower specification limits, and \( \sigma \) denotes the standard deviation of the process. If the random variable characterizing the parameter under analysis follows a normal distribution, the denominator in (1) shows the true process capability, while the numerator shows the consumer's quality requirements. This index can also be considered as an indicator characterizing the potential of the process to produce compliant products [6]. If there is a shift in the mean value of the process, a large fraction of the items may fall out of the tolerance range, while \( C_p \) may still remain high. This will be the case, for example, if the process margin shrinks when the \( C_p \) index is calculated at the same time as the mean value shifts. Thus, the \( C_p \) index does not consider possible process shifts from the target value [5, 6].

Another capability index was introduced by Kane [5] as

\[
C_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\}
\]  

(2)

where \( \mu \) represents the process average. For a given upper tolerance \( C_{pu} \) the following applies:

\[
C_{pu} = \frac{USL - \mu}{3\sigma}
\]  

(3)

and for a given lower tolerance of \( C_{pl} \) the following applies:

\[
C_{pl} = \frac{\mu - LSL}{3\sigma}
\]  

(4)

\( C_{pk} \) shows the reduction in process capability caused by the lack of centering [7]. This means that \( C_{pk} \leq C_p \), while equality occurs assuming that \( \mu \) is exactly in the middle of the \( LSL \) and \( USL \) interval, i.e., that the process is centered at the midpoint of the interval determined by the specifications.
The characters $\mu$ and $\sigma$ were used in the calculation of the previous PCI capability index. These are two parameters of a normal distribution, and this means that the assumption for the capability indexes described in this way is the normal distribution of the investigated random variable describing some critical process quality parameter. We also note that a necessary condition for the calculation of the capability index is that the process must be statistically under control and the calculation is based on data measured over a sufficiently long period of time. In determining whether a process is under control, control charts are used, typically using averages of logistic subgroups. Based on the central limit theorem, it is quite natural to expect robustness of analyses performed in this way [8]. As for the data collection period, according to [8], for so-called long-term capability this is a period of up to about 20 working days, and the selection of the products to be measured is made in such a way that the variability of the process is fully reflected [9].

In the case of so-called medium- or short-term capability, i.e., if for various reasons we do not have the required amount of process outputs available, we can use considerably less data to determine the stability of the process. In this case, e.g., the medium-term process capability indexes $P_p, P_{pl}, P_{pu}, P_{pl}$ are used. As mentioned in the previous section, the method of calculating the capability indexes $C_p, C_{pk}, C_{pu}, C_{pl}$ as described above requires the normality of the distribution of the realized random variable.

If the measured data do not have a normal distribution, we can calculate the index of medium-term capability by using the quantiles of the expected distribution in the calculation. Their values depend, of course, on the type of distribution function used [10]:

$$P_p = \frac{\text{USL} - \text{LSL}}{(X_{0.99868} - X_{0.001355})}$$  \hspace{1cm} (5)

where USL is the upper limit of the specification, LSL is the lower limit of the specification, and $X_{0.99868}$ is the 0.99868 quantile of the given (identified) distribution of the random variable and $X_{0.001355}$ is the 0.001355 quantile of the given distribution of the random variable, and it applies that:

$$P_{pl} = \frac{X_{0.5} - \text{LSL}}{(X_{0.5} - X_{0.001355})}$$  \hspace{1cm} (6)

where $X_{0.5}$ is the 0.50 quantile (median) of the distribution of the variable $X$ in question. Further it applies:

$$P_{pu} = \frac{\text{USL} - X_{0.5}}{(X_{0.99868} - X_{0.5})}$$  \hspace{1cm} (7)

$P_{pk}$ is the minimum of $P_{pu}$ and $P_{pl}$. We note that this is a generalization of the previous formulas, which are hereby made applicable to both normal and non-normal distributions. Moreover, for a random variable with a normal distribution, the indices calculated in this way are identical to those that would have been calculated in the previous way. Namely, if $X$ follows a normal distribution, then the median of $X_{0.5}$ is equal to the $\mu$ parameter and it applies:

$$X_{0.99868} - X_{0.001355} = 6\sigma$$  \hspace{1cm} (8)

$$X_{0.5} - X_{0.001355} = X_{0.99868} - X_{0.5} = 3\sigma$$  \hspace{1cm} (9)

3. Critical Parameters on the Piston

In the following, we will look at a specific practical example related to the manufacture of compressors. Each component of a manufactured compressor has specific parameters that are crucial for the operation of the compressor and influence the quality of the final product. As an example, we will present the production process of machining a piston of the T/J series, which is one of the components of the production process in the machining section. The whole process consists of six operations:

1. First grinding of the piston casing.
2. Turning the circumferential degassing and lubrication groove and drilling the lubrication and degassing holes.
3. Reaming of the piston pin hole and drilling of the locking pin hole.
4. Inter-operative washing of pistons.
5. Second grinding of the piston casing.
6. Final grinding of the piston casing.
7. Application of the manganese phosphor coating to the entire piston.

The analysed process is carried out on the STEMUT RT 402 device. It is a single-purpose machine the task of which is the machining of the hole for the piston pin. It consists of a rotary table and five drilling heads. The individual operations for each drilling head are divided as follows: head L - horizontal drilling of the locking pin hole, head M - second drilling of the piston pin hole, head N - chamfering of the inner and outer edges of the piston pin hole and finally reaming of the piston pin hole.

The following critical parameters are defined for the overall piston pin bore machining process: perpendicularity of the piston pin bore and the outer diameter of the piston, independently on both sides of the bore (side A and side B), symmetry of the piston pin bore, and position of the piston pin bore to the reference surface. These critical parameters are given increased attention in the quality control of machining. Their values are selectively measured, statistically evaluated, and compared. For our capability analysis, we have selected a critical parameter: the perpendicularity of the piston pin bore to the piston outer diameter. The measurement was performed using the Zeiss 3-D device program.

The process capability index $P_{pk}$ is used to evaluate the quality of machining. For all characteristics, it is a generally accepted rule that the value of the index must be greater than 1.33 for critical parameters. The MiniTAB statistical software is used for the calculation of this index at the plant. Measuring jigs are used to measure these parameters to monitor the quality of machining on the production lines. For a more detailed analysis, the aforementioned Zeiss 3-D measuring device is used. Each measuring system has its own measurement error. Special mechanical measuring jigs are regularly calibrated and checked by metrology technicians to eliminate this error to the minimum possible deviation. The error of the measuring systems is prescribed in the manufacturing drawings. Zeiss measuring equipment has its deviation prescribed by the manufacturer and is also subject to annual calibration and checking. In the following, we will deal with only one critical parameter of concern from the operator's point of view, namely the aforementioned perpendicularity of the piston bore for the piston pin and the outer diameter of the piston on both sides of the bore.

The identification of side A or side B depends on the way of gripping the workpiece - the cylinder head. When selecting cylinders for measurement, the operator clearly marks the A side in such a way that it cannot be confused with the unmarked B side (see Figure 1).

![Figure 1. T/J piston pin](image)

3.1. Perpendicularity of the piston pin bore and piston outer diameter

The perpendicularity parameters were measured with a measuring jig specially designed for this purpose. Forty cylinders were selected on which both measurements were made, with the operator marking side A with a coloured dot at the moment the cylinder was removed from the STEMUT RT 402 device.

The results of the measurements performed on side A and processed in histogram form along with the calculation of the fitness index assuming a normal distribution using the MiniTab software product are shown in Figure 2. The $P_{pu}$ index has a value of 1.07 and moreover, such a model assumes 649.69 ppm of non-conforming products per million units which means that the critical parameter in question does not meet the expected requirement, i.e., a value greater than or equal to 1.33. It is clear that such a
classical calculation with the assumption of normality is wrong. Thus, it is clear from the histogram that the parameter in question is inherently greater than 0 and all measured values are less than 0.09 while the USL is 0.12. However, this is the method of calculation expected by the customer, who considers it sufficiently robust and thus universally applicable to all critical parameters. From Figure 3, it is clear that the measured values do not follow a normal distribution and, moreover, when we arranged the individual consecutively manufactured products into logical subgroups of \(n=4\) elements, the Xbar Chart indicates that the process is not under control. One possible solution that the software product offers us in this situation is to use some suitable transformation.

\[
0.924 + 0.525 \times \ln \left( \frac{X - 0.03}{0.097 - X} \right).
\] (10)

After performing such a transformation, \(P_{pu} = 1.31\) was recorded which is just below the expected value of 1.33.

Another option to solve the problem is to find a more appropriate distribution model for the measured data. In Figure 5, a fit analysis of the process under the assumption that the data follow a Lognormal distribution is presented. For the stability analysis, we used a lognormal group of \(n = 3\). Based on the XBar - R Chart control chart, it came out that the process is under control with this chosen logistic subgroup.
The compliance test reached a $p$-value of 0.012 indicating that the given set does not follow even a Lognormal distribution. The $P_{pk}$ value in this case was 0.23.

In Figure 6, we performed a PCI analysis assuming that the set follows a Weibull distribution, with the data arranged into logistic subgroups of $n = 5$. Similar to the $n = 3$ case, the control charts confirmed to us that the process is under control. The compliance test with the Weibull distribution showed a $p$-value of less than 0.01 indicating that even the Weibull distribution does not correspond to the measured values. With this method of calculation, $P_{pk} = 0.61$. The PCI analysis for side B turned out to be very similar to that for side A (see Figure 7).

The highest value of $P_{pk} = 1.41$ was obtained by calculating using Johnson transformation with SB Distribution Type (See Figure 8):

$$0.513 + 0.387 \times \ln((X - 0.004)/(0.091 - X)).$$

This value meets the $P_{pk}$ capability requirement of 1.33. We can consider the process in question to be capable.
3.2. Rayleigh distribution

In the search for a suitable distribution function that would characterize the measured data accurately enough for both equality parameters, we hereafter relied on the paper [11]. In it, the authors described the expected tolerance properties and their distribution models for various operations used in engineering manufacturing (see Table 1).

| Table 1. Tolerance properties and their distribution models |
|------------------------------------------------------------|
| **Characteristic** | **Symbol** | **Model layout** |
| Shape tolerances   |            |                |
| Straightness       | _          | B1<sup>a</sup> |
| Flatness           | □          | B1             |
| Roundness          | ○          | B1             |
| Cylindrical shape  | /           | B1             |
| Line shape         | □          | B1             |
| Surface shape      | △          | B1             |
| Concentricity      | △          | B1             |
| Positional tolerances |       |                |
| Parallelism        | /           | B1             |
| Perpendicularity   | ⊥           | B1             |
| Tilt               | θ           | B1             |
| Position           | ≡           | B1             |
| Coaxiality         | ⊙           | B2<sup>b</sup> |
| Symmetry           | ≡           | B1             |
| Concentricity      | △           | B2             |
| Other              |            |                |
| Roughness          | B1          |                |
| Imbalance          | B2          |                |
| Torque             | N<sup>c</sup> |               |
| Length measure     | N           |                |

<sup>a</sup>Distribution of amounts 1. Type (Rayleigh Distribution)
<sup>b</sup>Distribution of amounts 2. type (Rayleigh Distribution)
<sup>c</sup>Normal distribution

From Table 1, it can be seen that a significant proportion of the parameters that are measured in the context of engineering production do not follow a normal distribution. Therefore, based on the aforementioned paper, we have assumed that the data measured by us follow the Rayleigh distribution. In the following, we briefly describe the properties of the Rayleigh distribution.

A random variable \( X \) with Rayleigh distribution has one parameter. It is a shape parameter (\( \lambda > 0 \)). We write the above fact in the form of \( X \sim \text{Rayleigh}(\lambda) \). If \( U \sim \text{u}(0,1) \) (Uniform distribution), then Rayleigh and Uniform RVs related by the transformation \( X = \lambda \sqrt{-2 \ln(U)} \). In addition, \( x^2 \) have the Exponential distribution with parameter \( \frac{1}{\lambda^2} \).

Let \( X \sim \text{Rayleigh}(\lambda) \). Then the probability density function (PDF) and the cumulative distribution function (CDF) of \( X \) are as follows:

\[
\begin{align*}
f(x; \lambda) &= \frac{x}{\lambda^2} \exp\left(-\frac{x^2}{2\lambda^2}\right), \quad x > 0, \lambda > 0. \\
F(x; \lambda) &= 1 - \exp\left(-\frac{x^2}{2\lambda^2}\right), \quad x > 0, \lambda > 0.
\end{align*}
\]

\( q \times 100 \text{th quantile or quantile function of } x \) is:
\[ x(q; \lambda) = F^{-1}(q) = \lambda \sqrt{-2 \ln(1 - q)}, \quad 0 < q < 1 \]  
\[ \lambda \]  
where \( F^{-1}(\cdot) \) is the inverse function \( F(\cdot) \). Therefore, from the equation (14), median \( x(0.5) = \lambda \sqrt{2 \ln 2} \). The mean and variance of \( X \) are: \( E[X] = \lambda \sqrt{\pi/2} \) resp. \( Var \) \( [X] = \lambda^2 \left( 2 - \frac{\pi}{2} \right) \).

In their paper, the authors [12] derived formulas for calculating capability indexes for random variables following the Rayleigh distribution. For the purposes of this paper, we describe only the formula for calculating \( C_{pu}^R \).

Let \( x_1, \ldots, x_n \) be a random sample of size \( n \) from \( X \sim Rayleigh(\lambda) \) with observation of \( x_1, x_2, \ldots, x_n \). Then the log function - likelihood function for \( \lambda \) is:

\[ \ell(\lambda) = \sum_{i=1}^{n} \ln x_i - 2n \ln \lambda - \frac{\sum_{i=1}^{n} x_i^2}{2n} \]  
\[ \ell \]  
(15)  
If we set the derivation of equation (15) with respect to the parameter \( \lambda \) equal to 0 we obtain estimates of maximum likelihood (ML) of the parameter \( \lambda \) denoted by us as \( \hat{\lambda} \). Hence:

\[ \hat{\lambda} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{2n}} \]  
\[ \hat{\lambda} = \]  
(16)  
The estimate of the capability index based on the Rayleigh distribution is then obtained based on the relation:

\[ C_{pu}^R = \frac{2[USL-x(0.5)]}{x(q_2)-x(q_1)} = \frac{2[USL-x(0.5)]}{\sqrt{-2 \ln(1-q_2)-\sqrt{-2 \ln(1-q_1)}}} \]  
\[ C_{pu}^R = \]  
(17)  
by substituting parameter \( \lambda \) for the estimate \( \hat{\lambda} \). Based on relation (14), we have estimated the parameter \( \lambda \) for values measured by us. For side A, the given estimate has the value \( \hat{\lambda}_A = 0.03048 \). For side B, the given estimate has the \( \hat{\lambda}_B = 0.030499 \). As we described above \( USL=0.12 \) and we used the quantiles \( q_1 = 0.00135 \) and \( q_2 = 0.99865 \) as cut-off values. Substituting into relation (15), we obtained estimates of the capability indexes, for side \( C_{pu}^R(A) = 1.540288 \) and for side \( C_{pu}^R(B) = 1.53887 \). In both cases, the aforementioned capability indexes significantly exceed the expected value of 1.33 implying that the critical parameter perpendicularity on both A and B side exceeds the required value of 1.33 and hence, in terms of the aforementioned two critical parameters, the process can be safely considered as capable.

4. Conclusion
As we mentioned in the introduction, capability indexes play an important role in buyer-supplier relationships. In practice, it is encountered that even though the product achieves all the prescribed values of the capability of all critical parameters, the customer is still dissatisfied with the quality of the products in question and solves this not infrequently by increasing the requirements for the limit values of the capability indexes of critical parameters. Instead, it should perhaps carefully check the accuracy and reasonableness of the calculation in question. In this paper, we have shown how the choice of an appropriate partitioning model can substantially change the resulting capability index score and, moreover, the choice of the size of the logical subgroups can raise doubts or confirm that the process in question is under control. There are a number of approaches to the calculation of competency indices in the literature, which are intended primarily for various specific examples. Some of them are very theoretical, with the result that they are used in practice only exceptionally. Because one of the basic advantages of using indices is their simplicity and interpretability, future research is desirable on how to calculate competency indices, which have been applicable due to their simplicity and versatility, as the largest possible group of specific applications used in industrial practice.

References
[1] Pačaiova H, Sinay J, Turisova R, Hajduova Z & Markulik S 2017 Measuring the qualitative
factors on copper wire surface Measurement, 109 pp 359-365
[2] Pacaiova H, Oravec M, Smelko M, Lipovsky P & Forraj F 2018 Extra low frequency magnetic fields of welding machines and personal safety Journal of electrical engineering 69 pp 493-496
[3] Juran J M & Gryna F M 1980 Quality Planning and Analysis (New York: McGraw Hill) pp 283-295
[4] Mittag H J 1997 Measurement error effects on the performance of process capability indices Frontiers in statistical quality control (Heidelberg: Physica) pp 195-206
[5] Kane V E 1986 Process capability indices Journal of quality technology 18 1
[6] Erfanian M & Sadeghpour G B 2021 A new capability index for non-normal distributions based on linex loss function Quality Engineering 33 1
[7] Anis M Z 2008 Basic process capability indices: An expository review International Statistical Review 76 3
[8] Montgomery D C 2019 Introduction to Statistical Quality Control 8th (New York: John Wiley) p 768
[9] VDA 2 2020 Quality Assurance for Suppliers Production process and product approval PPA (Berlin: European standards) p 88
[10] Lahcene B 2018 On Recent Modifications of Extended Rayleigh Distribution and its Applications JP Journal of Fundamental and Applied Statistics 12 1 pp 1-13
[11] Streinz W, Hausberger H & Anghel C 1992 Unsymmetriegroessen erster und zweiter Art richtig auswerten - Teil 1: Unsymmetriegroessen erster Art QUALITAT UND ZUVERLASSIGKEIT 37
[12] Dariae D & Sadeghpour G B 2017 Capability Indices for Rayleigh Process Journal of System Management 3 2 pp 61-76

Acknowledgments
This contribution is the result of the projects implementation: Ministry of Education, Science, Research and Sport of the Slovak Republic APVV No. 19-0367 Framework of the Integrated Process Safety Management Approach for the Intelligent Enterprise.