Theoretical analysis for the apparent discrepancy between \( \bar{p}p \) and \( pp \) data in charged particle forward-backward multiplicity correlations

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The strength of charged particle forward-backward multiplicity correlation in \( \bar{p} + p \) and \( p + p \) collisions at \( \sqrt{s} = 200 \) GeV is studied by PYTHIA 6.4 and compared to the UA5 and STAR data correspondingly. It is turned out that a factor of 3-4 apparent discrepancy between UA5 and STAR data can be attributed to the differences in detector acceptances and observing bin interval in both experiments. A mixed event method is introduced and used to calculate the statistical correlation strength and the dynamical correlation strengths stemming from the charge conservation, four-momentum conservation, and decay, respectively. It seems that the statistical correlation is much larger than dynamical one and the charge conservation, four-momentum conservation and decay may account for most part of the dynamical correlation. In addition, we have also calculated the correlation strength by fitting the charged particle multiplicity distribution from PYTHIA to the Negative Binomial Distribution and found that the result agrees well with the correlation strength calculated by mixed events.

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Fluctuations and correlations are important observables investigating the properties of thermodynamic system and are critical tools revealing the mechanism of particle production and the formation of quark-gluon-plasma in relativistic heavy ion collisions \(^{1,2}\). Several thermodynamic quantities and the produced particle distributions show varying fluctuation patterns when system undergoes phase transition. Such as the large energy density fluctuation is expected in the first order phase transition and a second order phase transition may relate to a divergence in specific heat. The event-by-event fluctuation pattern in average transverse momentum may significantly change around a critical point, etc.

The experimental study of correlation and fluctuation becomes a hot topic in relativistic heavy ion collisions with the availability of high multiplicity event-by-event measurements at the CERN-SPS and BNL-RHIC experiments. There have accumulated an abundant experiment data \(^3\) where arisen new physics are urgent to be studied.

Recently STAR collaboration has measured the strength of charged particle forward-backward multiplicity correlation, \( b \) (defined later), in \( p + p \) collision at \( \sqrt{s} = 200 \) GeV \(^{12,13}\). It is 3-4 times smaller than the one measured by UA5 in \( \bar{p} + p \) collision at the same energy apparently \(^{12,13}\). In this paper, the PYTHIA 6.4 \(^{16}\) is employed to analyze both STAR \( p + p \) and the UA5 \( \bar{p} + p \) data. It is turned out that the above apparent discrepancy is because of the differences in detector acceptances and the interval of pseudo-rapidity bin in both experiments. In addition, a mixed event method is proposed and used to calculate the statistical correlation and dynamical correlations stemming from the charge and four-momentum conservations and the decay of unstable particles individually. We also fit the particle multiplicity distribution from PYTHIA to the Negative Binomial Distribution (NBD) and calculate the strength of charged particle forward-backward multiplicity correlation which agrees well with the one calculated by mixed events.

Following Refs \(^{13,13}\) the strength of charged particle forward-backward multiplicity correlation, \( b \), is defined

\[
b = \frac{\langle n_f n_b \rangle - \langle n_f \rangle \langle n_b \rangle}{\langle n_f^2 \rangle - \langle n_f \rangle^2} = \frac{\text{cov}(n_f, n_b)}{\text{var}(n_f)}
\]

\[
= \frac{\langle (n_f - \langle n_f \rangle)(n_b - \langle n_b \rangle) \rangle}{\langle n_f^2 \rangle - \langle n_f \rangle^2}.
\]

where \( n_f \) and \( n_b \) are, respectively, the number of charged particles in forward and backward pseudo-rapidity bins \( (\Delta \eta) \) defined relatively and symmetrically to a given pseudo-rapidity \( \eta \). The \( \langle n_f \rangle \), for instance, refers to the mean value of \( n_f \) and the \( \text{cov}(n_f, n_b) \) and \( \text{var}(n_f) \) are, respectively, the forward-backward multiplicity covariance and forward multiplicity variance. If there is no correlation between forward and backward multiplicity, then \( \langle n_f n_b \rangle = \langle n_f \rangle \langle n_b \rangle \) and \( b = 0 \). Thus \( b \) is a measure of the strength of forward-backward multiplicity correlation. As the denominator in Eq. 1 is positive, if both \( n_f \) and \( n_b \) are, respectively, larger or smaller than \( \langle n_f \rangle \) and \( \langle n_b \rangle \) simultaneously the correlation is positive, negative otherwise.

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Although both STAR and UA5 experiments\cite{12, 13, 14, 15} measure the charged particle multiplicity in Non-Single-Diffractive (NSD) $p + p$ and $\bar{p} + p$ collisions at $\sqrt{s}=200$ GeV, the detector acceptances are different from each other. In UA5 experiment they are $p_T > 0$ GeV/c and $0 < |p| < 4$ GeV/c, but $p_T > 0.15$ GeV/c and $0 < |\eta| < 1.0$ in STAR experiment\cite{12, 13}. Meanwhile, the observed interval of forward-backward pseudo-rapidity bin is $\Delta \eta = 1.0$ in UA5 experiment rather than 0.2 in STAR. We shall show that those differences are the origin of apparent discrepancy between STAR and UA5 data in the strength of charged particle forward-backward multiplicity correlation.

The UA5 data of energy dependence of the correlation strength have been studied by Dual Parton Model\cite{17} and the statistical model\cite{18} and the STAR data of forward-backward charged particle multiplicity correlation have been studied recently in\cite{19}. However, the apparent discrepancy in correlation strength between UA5 and STAR data is not investigated yet. In this paper PYTHIA 6.4\cite{16} is employed to study that. Since we are not aim to reproduce the experimental data but to study the physics, we do not adjust the model parameters and default values are used in all the calculations.

The comparison of experimental strength of the charged particle forward-backward multiplicity correlation to the corresponding theoretical results is given in Figure\[1\] where the upper panel is for UA5 $\bar{p} + p$ collision and the lower panel for STAR $p + p$ collision at $\sqrt{s}=200$ GeV. One sees in this figure that the theoretical results are not so far apart from the experimental data for both the $\bar{p} + p$ and $p + p$ collisions. The theoretical correlation strength in $\bar{p} + p$ collision is also a factor of 3-4 larger than the one in $p + p$ collision, especially.

In the upper panel of Figure\[2\] the full squares are the PYTHIA results for $\bar{p} + p$ collision with same detector acceptances and $\eta$ bin interval as in UA5 experiment, whereas the open triangles are the PYTHIA results with $p_T > 0.15$ instead of $p_T > 0$ GeV/c. The open triangles are monotonously below the full squares. Middle panel of Figure\[2\] shows the PYTHIA results calculated at the same detector acceptances as UA5 but with varied pseudo-rapidity bin intervals: $\Delta \eta = 1.0$ (full squares), 0.5 (open circles), and 0.2 (open triangles-down), respectively. Here one knows that the correlation strength, $b$, declines dramatically with the decreasing of pseudo-rapidity bin interval. The PYTHIA results calculated for both the $p + p$ (full squares) and $\bar{p} + p$ (open triangles-up) collisions at STAR detector acceptances and pseudo-rapidity bin interval are given in lower panel of Figure\[2\]. We see in this panel that a factor of 3-4 apparent discrepancy nearly disappears if both STAR and UA5 experiments are performed at the same detector acceptances and pseudo-rapidity bin interval.

It is very hard to separate the statistical and dynamical correlations (fluctuations) from the measured correlations (fluctuations)\cite{2}. We introduce a mixed event method based on the real (PYTHIA) events (150 thousand events, for instance). The mixed events are generated one by one according to the real events. We assume first that the particle multiplicity $N$ in a mixed event is the same as the corresponding one in real events. However, the $N$ particles in a mixed event are sampled randomly from the particle reservoir formed by all particles in the real events. As the particles in a mixed event are separately and randomly taken from different real events, there is not any dynamical relevance among them. Thus the correlation strength, $b$, calculated by the mixed events is reasonably to be identified as the statistical correlation. Of course, we can also generate the mixed event with individual constraint, such as charge conservation, four-momentum conservation, and decay, etc. The corresponding correlation will be indicated by “statistical plus charge dynamical correlations”, the “statistical plus four-momentum dynamical correlations”, and the “statistical plus decay correlations”, etc., respectively.

The strength of charged particle forward-backward multiplicity correlation is calculated individually from the real events, mixed events, mixed events with charge conservation, and the mixed events with charge and four-momentum conservations. They are given in the upper panel of Figure\[3\] by the full squares (indicated as total

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The strength $b$ of the charged particle forward-backward multiplicity correlation in $\bar{p} + p$ (upper panel) and $p + p$ (lower panel) collisions at $\sqrt{s}=200$ GeV. The experimental data are taken from\cite{12} and \cite{12}, respectively.}
\end{figure}
One knows here that the dynamical correlations stemming from charge, four-momentum, and the decay may account for the most part of the total dynamical correlation.

Many experiments have indicated that the particle multiplicity distribution in hadron-hadron and the nucleus-nucleus collisions is well described by the Negative Binomial Distribution (NBD) [20, 21, 22, 23]. For
an integer \( n \) the NBD reads
\[
P(n; \mu, k) = \binom{n + k - 1}{k - 1} \frac{(\mu/k)^n}{(1 + \mu/k)^{n+k}},
\]
where \( \mu \equiv \langle n \rangle \) is a parameter, \( P(n; \mu, k) \) is normalized in \( 0 \leq n \leq \infty \), and \( k \) is another parameter responsible for the shape of the distribution. If \( k = 1 \) is a real the binomial coefficient in Eq. (2) is \( k(k+1) \cdots (k+n-1)/n! \). In NBD the variance \( \langle \sigma^2 \rangle \) and mean \( \langle \mu \rangle \) is related to \( k \) by
\[
\sigma^2 = \mu + \frac{\mu^2}{k}.
\]
An important property of NBD is that if particle multiplicity \( n \) is NBD in whole phase space and the particle has unified probability, \( p \), in a partial phase space (such as in a \( \eta \) bin here) then particle multiplicity distribution in this partial phase space is also NBD with same parameter \( k \) and the mean is equal to \( \mu p \) \[24\]. That is obviously based on the assumption that the particles are independent with each other, i.e. there is no dynamical correlation among them.

Since forward and backward pseudo-rapidity bins are symmetry relative to the investigated pseudo-rapidity \( \langle \eta \rangle \) and have same width so \( \langle n_f \rangle = \langle n_b \rangle \) and \( \text{var}(n_f) = \text{var}(n_b) \) in NBD. Using the statistic formula \[25\]
\[
\text{var}(X \pm Y) = \text{var}(X) + \text{var}(Y) \pm 2\text{cov}(X,Y)
\]
the \( b \) can be written as
\[
b = \frac{\text{cov}(n_f, n_b)}{\text{var}(n_f)} = \frac{\text{var}(n_f + n_b) - 2\text{var}(n_f)}{2\text{var}(n_f)}.
\]
Substitute Eq. (3) into Eq. (5) one has
\[
b = \frac{\langle n_f \rangle}{\langle n_f \rangle + k}.
\]
We know that the NBD becomes a Poisson distribution in the limit \( k \to \infty \), so the correlation strength, \( b \), is zero in Poisson distribution. If the charged particle multiplicity distribution in real events with decay assumption is fitted by NBD, the parameter \( k \) is obtained. As the real events are generated in NSD (Non-Single-Diffractive) indeed, the charged particle multiplicity distribution is not perfect NBD, therefore the above fit is not so sensitive to the \( k \) values within 6-7. If NBD with \( k\approx6.6 \) is assumed for the charged particle multiplicity distribution, the corresponding \( b \) can be calculated by Eq. (6) because \( \langle n_f \rangle \) can be approximated by \( dN_{ch}/d\eta \) in real event. Those \( b \) are shown in Fig. 4 by open triangles. In this figure the full squares are calculated by the mixed events with decay assumption (i.e. the open circles in upper panel of Fig. 3) and the open circles are the charged particle pseudo-rapidity distribution in real events with decay (in drawing \( dN_{ch}/d\eta \) the abscissa is identified as \( \eta \) and scaled by 2). The results of NBD agree well with the results calculated by the mixed events with decay assumption, it proves again that we are reasonable identifying the \( b \) calculated by mixed events as the statistical correlation strength. Comparing the full squares and open triangles to the open circles one knows that the statistical correlation strength may have shape similar to the charged particle pseudo-rapidity distribution.

Recently, the forward-backward multiplicity covariance in \( p+p \) collision at \( \sqrt{s}=200 \) GeV has been studied in Ref. \[19\]. They assumed the back-to-back partonic scattering is the origin of hadronic correlation, related that partonic scattering angles to a Gaussian like hadronization function, and derived the forward-backward multiplicity covariance. Without more dynamical inputs their results are well comparing with STAR data \[12\]. Therefore they conclude that the correlation length might have no fundamental significance. We plan to investigate the partonic origin of forward-backward multiplicity correlation by transport model in next study.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Charged particle forward-backward multiplicity statistical correlation strength \( b \) and the charged particle pseudo-rapidity distribution in \( p+p \) collision at \( \sqrt{s}=200 \) GeV.}
\end{figure}

In summary, we have calculated the strength of charged particle forward-backward multiplicity correlation in \( p+p \) and \( p+p \) collisions at \( \sqrt{s}=200 \) GeV by PYTHIA 6.4 \[16\] and compared with UA5 data \[15\] and STAR data \[12\], respectively. It is turned out that a factor of 3-4 apparent discrepancy between UA5 and STAR data can be attributed to the differences in detector acceptances and the interval of observed \( \eta \) bin in both experiments. A mixed event method is introduced and used to calculate the statistical correlation strength and the individual dynamical correlations stemming from charge conservation, four-momentum conservation, and the decay. It seems that the statistical correlation is much larger than the dynamical one, and the charge, four-momentum, and decay may account for the main part of the dynamical correlation. The NBD \( b \) results agree well with the ones calculated by mixed events proves again that one is reasonable to identify the correlation in mixed events as a statistical correlation.

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[1] R. C. Hwa, nucl-th/701053.
[2] T. K. Nayak, J. of Phys. G 32, S187 (2006); nucl-ex/0608021.
[3] H. Appelshäuser, et al., NA49 Collaboration, Phys. Lett. B 459, 679 (1999).
[4] S. V. Afanasiev, et al., NA49 Collaboration, Phys. Rev. Lett. 86, 1965 (2001).
[5] J. Adams et al., STAR Collaboration, Phys. Rev. C 68, 044905 (2003).
[6] J. Adams et al., STAR Collaboration, J. Phys. G 32, L37 (2006).
[7] J. Adams et al., STAR Collaboration, Phys. Rev. C 75, 034901 (2007).
[8] K. Adcox et al., PHENIX Collaboration, Phys. Rev. Lett. 89, 082301 (2002).
[9] S. S. Adler et al., PHENIX Collaboration, Phys. Rev. Lett. 93, 092301 (2004).
[10] A. Adare et al., PHENIX Collaboration, Phys. Rev. Lett. 98, 232302 (2007).
[11] Zheng-Wei Chai, et al., PHOBOS Collaboration, J. of Phys.: Conference Series 27, 128 (2005).
[12] B. K. Srivastav, STAR Collaboration, nucl-ex/0702054.
[13] T. Tarnowsky, STAR Collaboration, nucl-ex/0702056.
[14] K. Alpgård et al., UA5 Collaboration, Phys. Lett. B 123, 361 (1983).
[15] R. E. Ansorge et al., UA5 Collaboration, Z. Phys. C 37, 191 (1988).
[16] T. Söjstrand, S. Mrenna, and P. Skands, J. High Energy Phys. JHEP05, 026 (2006); hep-ph/0603175.
[17] A. Capella, and J. Tran Thanh Van, Z. Phys. C 18, 85 (1983).
[18] Liu Lian-sou and Meng Ta-chung, Phys. Rev. D 27, 2640 (1983), ibid D 33, 1287 (1986).
[19] R. C. Hwa and C. B. Yang, arXiv:0705.3073.
[20] G. J. Alner et al., UA5 Collaboration, Phys. Lett. B 160, 193 (1985).
[21] G. J. Alner et al., UA5 Collaboration, Phys. Rep. 154, 2663 (1995).
[22] T. Abbott et al., E802 Collaboration, Phys. Rev. C 52, 247 (1997).
[23] J. Mitchell, PHENIX Collaboration, nucl-ex/0701080.
[24] K. Adcox, et al., PHENIX Collaboration, Nucl. Phys. A 757, 184 (2005).
[25] R. E. Walpole and R. H. Myers, “Probability and statistics for engineers and scientists”, 2-th Ed., P71, 1978, Macmillan Publishing Co., Inc. New York.