Unitarity and nonperturbative effects
in the spin structure functions at small $x$

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Abstract: We consider low-$x$ behavior of the spin structure functions $g_1(x)$ and
$h_1(x)$ in the unitarized chiral quark model that combines the ideas on constituent
quark structure of hadrons with a geometrical scattering picture and unitarity. A
nondiffractive singular low-$x$ dependence of $g_1^p(x)$ and $g_1^n(x)$ is obtained and a diffractive
type smooth behavior of $h_1(x)$ is predicted at small $x$.

Experimental evaluation of the first moments of $g_1$ and $h_1$ (and the total nucleon helicity
carried by quarks and tensor charge respectively) in principle are sensitive to a particular
theoretical extrapolation of the structure functions $g_1(x)$ and $h_1(x)$ to $x = 0$. The essential
point in the study of low-$x$ dynamics is that the space-time structure of the scattering at small
values of $x$ involves large distances $l \sim 1/Mx$ on the light–cone [1] and the region $x \sim 0$
is therefore determined by the nonperturbative dynamics. A number of models attributes the
observed increase of $g_1(x)$ at small $x$ to the diffractive contribution. Such contribution being
dominant at smallest values of $x$ would lead to the “equal” structure functions $g_1^p(x)$ and $g_1^n(x)$
in this kinematical region, i. e.

$$g_1^p(x)/g_1^n(x) \to 1$$

at $x \to 0$. Such behavior has not been confirmed in the recent experiments. In particular, the
SMC data [2] demonstrate the following approximate relation in the region of $0.003 \leq x \leq 0.1$:

$$g_1^p(x) \simeq -g_1^n(x).$$

To consider low-$x$ region and obtain the explicit forms for the quark spin densities $\Delta q(x)$ and
$\delta q(x)$ at $x \to 0$ it is convenient to use the relations between these functions and discontinuities
of the helicity amplitudes of the antiquark–hadron forward scattering [3]. We use a nonpertur-
vative approach where unitarity is explicitly taken into account via unitary representations for
the helicity amplitudes, which follow from their relations to the $U$–matrix [4].

In the model a quark is considered as a structured hadronlike object since at small $x$ the
photon converts to a quark pair at long distance before it interacts with the hadron. At large
distances perturbative QCD vacuum undergoes transition into a nonperturbative one with for-
mation of the quark condensate. Appearance of the condensate means the spontaneous chiral
symmetry breaking and the current quark transforms into a massive quasiparticle state – a
constituent quark. Constituent quark is embedded into the nonperturbative vacuum (conden-
sate) and therefore we can treat it similar to a hadron. Spin of constituent quark $J_U$ in this
approach is given by the following sum

$$J_U = 1/2 = S_u + S_{\{\bar{q}q\}} + \langle L_{\{\bar{q}q\}} \rangle = 1/2 + S_{\{\bar{q}q\}} + \langle L_{\{\bar{q}q\}} \rangle.$$
It is also important to note the exact compensation between the spins of quark–antiquark pairs and their angular orbital momenta, i.e. $\langle L_{\bar{q}q} \rangle = -S_{\bar{q}q}$.

We consider effective lagrangian approach where gluon degrees of freedom are overintegrated. The value of the orbital momentum contribution into the spin of constituent quark can be estimated according to the relation between contributions of current quarks into a proton spin and corresponding contributions of current quarks into a spin of the constituent quarks and that of the constituent quarks into the proton spin. The existence of this orbital angular momentum, i.e. orbital motion of quark matter inside constituent quark, is the origin of the observed asymmetries in inclusive production at moderate and high transverse momenta. Mechanism of quark helicity flip in this picture is associated with the constituent quark interaction with the quark generated under interaction of the condensates [4]. Quark exchange process between the valence quark and an appropriate quark with relevant orientation of its spin and the same flavor will provide the necessary helicity flip transition, i.e. $Q^+ \rightarrow Q^-$. The helicity amplitudes $F_{1,2,3}(s,t)|_{t=0}$ at high values of $s$ and then the functional dependencies for the quark densities $q(x)$, $\Delta q(x)$ and $\delta q(x)$ at small $x$ were obtained [5].

The low-$x$ behavior of quark spin densities is as follows:

\[ q(x) \sim \frac{1}{x} \ln^2(1/x), \quad \Delta q(x) \sim \frac{1}{\sqrt{x}} \ln(1/x), \quad \delta q(x) \sim x^c \ln(1/x), \]

and correspondingly

\[ F_1^p(x)/F_1^n(x) \rightarrow 1, \quad h_1^p(x)/h_1^n(x) \rightarrow 1 \]

at $x \rightarrow 0$, with the explicit forms as follows

\[ F_1^p(x) \sim \frac{1}{x} \ln^2(1/x), \quad h_1^p(x) \sim x^c \ln(1/x). \]

Comparison of the spin structure function $g_1(x)$ with the SMC data provides a satisfactory agreement with experiment at small $x$ ($0 < x < 0.1$) and leads to the values $C^p = 2.07 \cdot 10^{-2}$ and $C^n = -2.10 \cdot 10^{-2}$ (cf. Fig. 1).

The functional dependence of the spin structure functions

\[ g_1^{p,n}(x) \sim \frac{1}{\sqrt{x}} \ln(1/x) \]

is in a good agreement with the new E154, E155 and HERMES data [4] as well. The model leads to the approximate relation

\[ g_1^p(x)/g_1^n(x) \simeq -1 \]

at small values of $x$.

The above extrapolation of $g_1(x)$ at small $x$ provides the following approximate values for the quark spin contributions:

\[ \Delta \Sigma \simeq 0.25, \quad \Delta u \simeq 0.81, \quad \Delta d \simeq -0.45, \quad \Delta s \simeq -0.11, \]

which demonstrate that the singular behavior of $g_1^n(x)$ does not lead to significant deviations from the results of the experimental analysis [2] where the smooth extrapolation of the data to $x = 0$ was used.
The obtained singular small-\(x\) behavior of \(g_1\) corresponds to the following high energy behavior of the difference of the \(\gamma N\) total cross-sections:

\[
\Delta \sigma = \sigma_{\gamma N}^{1/2} - \sigma_{\gamma N}^{3/2} \sim \ln \nu \sqrt{\nu}
\]

and gives a convergent integral in the Drell-Hearn-Gerasimov-Iddings (DHGI) sum rule. Note, however that unitarity bound [7]:

\[
\Delta \sigma \leq \ln \nu
\]

does not rule out the divergent DHGI integral.

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