New interactions in the radiative pion decay

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Abstract

The preliminary results of PIBETA experiment strongly suggest the presence of non $V-A$ anomalous interactions in the radiative pion decay. Without a guiding idea about the nature of these new interactions it is very difficult to interpret and fit experimental data. We assume that they arise as a result of the exchange of new intermediate chiral bosons which interact anomalously with matter. Their mixing with $W$-bosons leads effectively to a small anomalous weak moment for $W$. Based on these assumptions, we show that the most general form of the radiative pion decay rate can be parametrized by three new coupling constants of the anomalous interactions.
1 Introduction

The Standard Model (SM) is now a well established theory of the electroweak and the strong interactions. Its experimental success is impressively great and, therefore, all anomalies in the present and the past experiments are usually considered as their artifacts or statistical fluctuations. Nevertheless, from the general point of view we believe that SM is just the limit of a more general theory with new particles and an enlarged symmetry.

Many people trust that this is the supersymmetry (SUSY). The present phenomenological SUSY models are constructed by simply doubling of the known particles without introducing really new kinds of fields. Therefore, for their description the contemporary framework of the quantum field theory can be used. In such models all processes can be computed and predictions for their experimental manifestation can be made.

In the present paper we discuss a new kind of vector particles, whose full theory is not constructed yet. However, a phenomenological approach is possible, and, moreover, it is motivated by the available experimental data. The introduction of such type of particles is natural and does not contradict the basic axioms of the quantum field theory. In the paper [1] we have shown that such type of particles exists as composite quark-antiquark hadron vector resonances. Based on this idea, the dynamic properties of the vector mesons and the new mass relations among the hadron vector resonances have been derived.

The preliminary results of the PIBETA experiment [2] on the radiative pion decay (RPD) may represent one of the main indications of the existence of such particles also on a fundamental level. Using a simple phenomenological model we will show how these results can be interpreted.

2 The tensor interactions

The radiative pion decay $\pi \rightarrow \nu \gamma$ is a unique system for searching physics beyond SM. Due to chirality suppression the usual electroweak interactions are very weak while new interactions, which do not preserve chirality of fermions, can be enhanced. However, the enhancement occurs only in a certain part of the kinematically allowed region and, therefore, the previous experiments [3] have not detected the new interactions.

The first experiment, which has alarmed about deviations from SM in RPD, was ISTRA experiment [4]. The RPD in flight was investigated in a wide kinematical region with dramatic conclusions for SM. It has been found that the measured number of events is about 30% smaller than the expected ones. This cannot be explained neither by radiative corrections [5] nor by SUSY extension of SM [6]. The interpretation of these results in the framework of SM leads to a huge violation of CVC hypothesis.

In order to describe this strange result a new interaction with a tensor lepton current

$$M_T = -\frac{\epsilon G}{\sqrt{2}} F_T \epsilon_\alpha(q) q_\beta \bar{\epsilon}\sigma_{\alpha\beta}(1 - \gamma^5)\nu$$

(1)
with $F_T = -(5.6 \pm 1.7) \times 10^{-3}$ was introduced [7]. Here $G = G_F V_{ud}$ and $\varepsilon_\alpha(q)$ is the photon polarization vector. The additional interaction helps to explain the lack of events with respect to SM prediction, caused by the destructive interference of the new term with inner-bremsstrahlung (IB) amplitude. Moreover, it leads to a correct events distribution over the Dalitz plot.

Such type of a phenomenologically introduced amplitude can be induced by a four-fermion tensor interaction of quark and lepton currents

$$\mathcal{L}_T = -\frac{G}{\sqrt{2}} f_T \bar{u} \sigma_{\alpha\beta}(1-\gamma^5)d \bar{e} \sigma_{\alpha\beta}(1-\gamma^5)\nu.$$  \hspace{1cm} (2)

As far as tensor intermediate bosons are absent in SM and its popular extensions, the conclusion was made that such type of interaction can be generated only by a leptoquark exchange [8]. However, according to the Grand Unified Models (GUT) the leptoquarks should be very massive $\sim 10^{15}$ GeV and, hence, their influence at the electroweak scale should be negligibly small.

The introduction of the new tensor interaction (1) leads to another problem, following from the strong constraint on $F_T$ [9] from pion decay $\pi \to e \nu$. On one hand the tensor interaction does not contribute directly to semileptonic two-particle pion decay $\pi \to e \nu$, due to kinematical reasons. On the other hand, however, owing to electromagnetic radiative corrections, the pseudotensor current $\bar{u} \sigma_{\alpha\beta}\gamma^5d$ leads to a generation of a pseudoscalar quark current, to which the pion decay is very sensitive. As a result, $F_T$ should be two orders of magnitude smaller than its required value for the explanation of ISTRA results.

To avoid this problem a new tensor amplitude

$$M'_T = -\frac{eG}{\sqrt{2}} F'_T [q_\alpha \varepsilon_\lambda(q) - q_\lambda \varepsilon_\alpha(q)] \frac{Q_\lambda Q_\beta}{Q^2} \bar{e} \sigma_{\alpha\beta}(1-\gamma^5)\nu,$$ \hspace{1cm} (3)

was introduced [10] in addition to (1). Here $Q_\alpha = (p-q)_\alpha$ is the momentum transfer to the lepton pair. This amplitude is generated by the following four-fermion tensor interaction

$$\mathcal{L}'_T = -\frac{4G}{\sqrt{2}} f'_T \bar{u} \sigma_{\alpha\lambda}(1+\gamma^5)d \frac{Q_\lambda Q_\beta}{Q^2} \bar{e} \sigma_{\alpha\beta}(1-\gamma^5)\nu.$$ \hspace{1cm} (4)

Since this interaction includes quark current with an opposite chirality to the one in (2), and also the interaction (2) can be identically rewritten as

$$\bar{u} \sigma_{\alpha\beta}(1-\gamma^5)d \bar{e} \sigma_{\alpha\beta}(1-\gamma^5)\nu \equiv 4 \bar{u} \sigma_{\alpha\lambda}(1-\gamma^5)d \frac{Q_\lambda Q_\beta}{Q^2} \bar{e} \sigma_{\alpha\beta}(1-\gamma^5)\nu,$$ \hspace{1cm} (5)

then the pseudotensor terms $\bar{u} \sigma_{\alpha\beta}\gamma^5d$ cancel out in the sum of the eqs. (2) and (4), if $f_T = f'_T$. The tensor term $\bar{u} \sigma_{\alpha\beta}d$ does not contribute to pseudoscalar pion decay due to parity conservation in electromagnetic interactions.

Therefore, the fit of experimental data with only one amplitude (1) is not acceptable because of the contradiction with present experimental data on pion decay. Consequently, it is not a surprise that the values $F_T$ for ISTRA and PIBETA experiments, which run in different kinematical regions, are different.
3 The model

The sum of the interactions (2) and (4) is not the most general Lagrangian including different tensor currents. Following the idea of renormalizability one can generate the effective four-fermion tensor interactions (2) and (4) by exchange of new chiral vector particles, which interact with matter only anomalously weakly depend on the square of momentum transfer to the lepton pair 

\[ F_i \rightarrow \alpha \equiv \frac{e}{m_\pi} \langle \bar{u}\gamma_\alpha(1-\gamma^5)d - i\hat{Q}_\lambda \bar{u}\sigma_\lambda(g_V^T + g_A^T \gamma^5)d \rangle \] 

\[ \bar{u}\gamma_\alpha(1-\gamma^5)d - i\hat{Q}_\lambda \bar{u}\sigma_\lambda(g_V^T + g_A^T \gamma^5)d \hat{Q}_\beta \bar{u}\sigma_\beta(1-\gamma^5)d \] + h.c., (6) 

introducing in general three new independent parameters \( g_e^T, g_V^T \) and \( g_A^T \). It leads effectively to five new four-fermion interactions

\[ \mathcal{L}_{\text{eff}} = -\frac{G}{\sqrt{2}} \left[ \bar{u}\gamma_\alpha(1-\gamma^5)d - i\hat{Q}_\lambda \bar{u}\sigma_\lambda(g_V^T + g_A^T \gamma^5)d \right] \bar{e}\gamma_\alpha(1-\gamma^5)\nu \]

\[ -\frac{i\hat{g}_e^T G}{\sqrt{2}} \left[ \bar{u}\gamma_\alpha(1-\gamma^5)d - i\hat{Q}_\lambda \bar{u}\sigma_\lambda(g_V^T + g_A^T \gamma^5)d \right] \hat{Q}_\beta \bar{e}\sigma_\beta(1-\gamma^5)\nu \] (7)

with only three independent parameters. In order to get the amplitude of RPD it is necessary to calculate the following matrix elements for \( \pi^- \gamma \) transition

\[ \langle \gamma(q)|\bar{u}\gamma_\alpha(1-\gamma^5)d|\pi(p)\rangle = -\frac{e}{m_\pi} \varepsilon_\beta(q) \left\{ F_V^0 \varepsilon_{\alpha\beta\rho}P_\rho q_\sigma + i F_A^0 [(pq)g_{\alpha\beta} - q_\alpha q_\beta] \right\} \] (8)

\[ \langle \gamma(q)|\bar{u}\sigma_\alpha\gamma^5d|\pi(p)\rangle = -e F_T^0 [q_\alpha\varepsilon_\beta(q) - q_\beta\varepsilon_\alpha(q)] , \] (9)

which can be parametrized by three form factors \( F_V^0, F_A^0 \) and \( F_T^0 \). These form factors weakly depend on the square of momentum transfer to the lepton pair \( Q^2 = (p - q)^2 \) and can be taken as constants.

Assuming CVC hypothesis, the vector form factor \( F_V^0 \) is directly related to the \( \pi^0 \rightarrow \gamma \gamma \) amplitude and can be extracted from the experimental width of this decay

\[ F_V^0 = \frac{1}{\alpha} \sqrt{\frac{2\Gamma(\pi^0 \rightarrow \gamma \gamma)}{\pi m_{\pi^0}}} = 0.0262 \pm 0.0009 \] (10)

This value is in fair agreement with the calculations in the relativistic quark model (RQM) and with the leading order calculations of the chiral perturbation theory (CHPT)

\[ F_V^0 = \frac{1}{4\pi^2} \frac{m_\pi}{F_\pi} \approx 0.0270, \] (11)

where \( F_\pi = (130.7 \pm 0.4) \) MeV is the pion decay constant.

An axial form factor \( F_A^0 \approx 0.4 F_V^0 \) has been measured in previous experiments [3] in the kinematical region where the contribution of new tensor terms is not essential. This value is also in agreement with the calculations in CHPT [12].
The tensor form factor $F_T^0$ can be calculated by applying the QCD sum rules techniques [13] and the PCAC hypothesis

$$F_T^0 = \frac{1}{3} \chi \frac{\langle 0 | \bar{q} q | 0 \rangle}{F_\pi} \approx 0.2 \tag{12}$$

here $\chi = -(5.7 \pm 0.6)$ GeV$^{-2}$ is the magnetic susceptibility of the quark condensate and its vacuum expectation value is $\langle 0 | \bar{q} q | 0 \rangle \approx -(0.24 \text{ GeV})^3$.

The most general matrix element for radiative pion decay

$$M = M_{IB} + M_{SD} + M_T + M_T' \tag{13}$$

can be rewritten through four form factors $F_V, F_A, F_T$ and $F_T'$ [14]. Here the first term

$$M_{IB} = -ieG \sqrt{2} m_e \varepsilon_\alpha(q) \bar{e} \left[ \left( \frac{k_\alpha}{k q} - \frac{p_\alpha}{p q} \right) - i\sigma_{\alpha\beta} q_\beta \right] (1 - \gamma^5) \nu \tag{14}$$

is a QED radiative correction to the $\pi \to e \nu$ decay (IB) and it does not depend on any form factors. The structure-dependent amplitude

$$M_{SD} = \frac{eG}{\sqrt{2} m_\pi} \varepsilon_\beta(q) \left\{ F_V \varepsilon_\alpha\rho\sigma p_\rho q_\sigma + i F_A \left[ (pq) g_{\alpha\beta} - p_\alpha q_\beta \right] \bar{e} \gamma_\alpha (1 - \gamma^5) \nu \right\} \tag{15}$$

is parametrized by two form factors $F_V$ and $F_A$. However, due to mixed tensor-vector interactions (second term in (7)) the form factors $F_V$ and $F_A$ are not constants anymore and depend on $Q^2$ in a specific way

$$F_V = F_V^0 + \frac{m_\pi}{\sqrt{Q^2}} g_V^T F_T^0, \quad F_A = F_A^0 + \frac{m_\pi}{\sqrt{Q^2}} g_A^T F_T^0. \tag{16}$$

The expressions for the tensor amplitudes $M_T$ and $M_T'$ coincide with (1) and (3), but the form factors $F_T$ and $F_T'$ are not constants. In our model they have strong $Q^2$ dependence and due to vector-tensor mixed interactions (third term in (7)) they depend also on the vector $F_V^0$ and the axial-vector $F_A^0$ form factors as well

$$F_T = -g_e^T \left[ g_V^T F_T^0 + \frac{\sqrt{Q^2}}{m_\pi} F_A^0 \right], \quad F_T' = -g_e^T \left[ (g_V^T + g_A^T) F_T^0 + \frac{\sqrt{Q^2}}{m_\pi} (F_V^0 + F_A^0) \right]. \tag{17}$$

4 The decay rate

In the general case the decay rate depends on the squares of the amplitudes and their various interference terms. In our case, when the electron mass is vanishingly small, the interference between the SD amplitude on the one hand, and the IB amplitude or the tensor amplitudes on the other hand, can be safely neglected.
In this approximation the differential decay width is

\[
\frac{d^2\Gamma}{dx d\lambda} = \frac{\alpha}{2\pi} \Gamma_{\pi\to e\nu} \left\{ IB(x, \lambda) + a_{SD}^2 \left[ (F_V + F_A)^2 SD^+(x, \lambda) + (F_V - F_A)^2 SD^-(x, \lambda) \right] \right. \\
+ 2a_{SD} \left[ 2(F_T - F'_T) + F'_T x \right] I(x, \lambda) + 2a_{SD}^2 \left[ 2F_T(F_T - F'_T) + F_T^2 \right] T(x, \lambda) \left. \right\} (18)
\]

where \(a_{SD} = \frac{m^2}{2F_\pi m_e} \approx 145.8\),

\[
IB(x, \lambda) = \frac{(1-x)^2 + 1 - \lambda}{x}, \quad SD^+(x, \lambda) = (1-x)x^3\lambda^2, \\
SD^-(x, \lambda) = (1-x)x^3(1-\lambda)^2, \quad I(x, \lambda) = x(1-\lambda), \quad T(x, \lambda) = x^3(1-\lambda)\lambda. \quad (19)
\]

In the pion rest frame the variables \(x\) and \(\lambda\) are defined as \(x = 2E_\gamma/m_\pi\) and \(\lambda = 2E_e/m_\pi \sin^2(\theta_{e\gamma}/2)\).

To analyse the contribution of the new terms into the decay rate (18) one can use the analytical expressions for the Dalitz plot densities (19) and the form factors dependences (16) and (17). The squared momentum transfer to the lepton pair \(Q^2 = m^2_\pi(1-x) \geq m^2_\pi\) is constrained from below and the maximal effects of the new terms on \(F_V\) and \(F_A\) reveal exactly at this low limit. In other words, the events with the maximal photon energy \(x \approx 1\) represent the interesting region of the Dalitz plot, where the new terms effect is maximal. To avoid the contradiction with the discussed constraint from the pion decay \(\pi \to e\nu\), the anomalous axial coupling constant \(g^T_A\) should be very small. Hence, for fitting one can use actually only two of the three independent parameters.

In the case when the anomalous tensor coupling constant \(g^T_V\) is negative, one can get effectively lower value for the vector form factor \(F_V\) than the CVC predicted one. Noticeably, exactly such effect has been observed in ISTRA [4] and PIBETA [15] fits.

Another manifestation of the new interactions can be observed as a destructive interference between the IB amplitude and the amplitudes with the tensor lepton currents at \(x \approx 1\). This result immediately follows from the expressions (17) for the tensor form factors, in case the natural universality hypothesis about the sign of the tensor coupling constants \(g^T_e\) and \(g^T_V\) is accepted. The effect of the destructive interference is the maximal one at \(\lambda \approx 0\).

5 Conclusions

In this paper we present a simple model for an explanation of ISTRA and PIBETA results on the radiative pion decay. A small admixture of anomalous couplings for the \(W\)-boson interactions with a specific form (6) allows to explain the essential features of the observed deviations from SM in these experiments. Hopefully, the correct fitting according to this model of experimental data of high statistics PIBETA experiment will reveal new directions for probing physics beyond the Standard Model.
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