Baryonic Correlators
in the Random Instanton Vacuum

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Abstract

This is the third paper of a series devoted to a systematic study of QCD correlation functions in the framework of an instanton model for the QCD vacuum. In this paper we concentrate on the quark-quark (diquark) and three quark (baryon) channels. We have found that the quark-quark interaction resembles the one between quarks and antiquarks. Similar to the pion and rho channels, the interaction in the scalar isospin $I = 0$ and vector $I = 1$ diquark channels is completely different: the former has a much stronger attractive interaction. As a consequence, the $SU(3)$ octet (nucleon) and decuplet (delta) correlators are also found to be qualitatively different. Using a complete set of all available correlation functions, we determine masses and coupling constants for the nucleon and delta. Our results agree surprisingly well with the first lattice data on point to point correlators.
1. Introduction

This is the third paper of a series devoted to a detailed investigation of the QCD point-to-point correlation functions in the framework of the Random Instanton Liquid Model (below RILM). After considering the propagation of quarks and quark-antiquark (mesonic) states we now consider the baryon sector.

This model of the QCD vacuum was proposed by one of us in 1982. Using a variety of arguments, it was suggested that the density of 'tunneling events' (instantons and antiinstantons combined) is about $n_0 = 1 \text{ fm}^{-4}$, while their typical size is $\rho_0 = 1/3 \text{ fm}$. These two numbers were shown to reproduce two global properties of the QCD vacuum, the gluon and quark condensates.

The investigation of correlation functions in the RILM was started in reference for the pseudoscalar channels. Not only does the model explain why the OPE-based sum rules fail in the case of pseudoscalars, but it was also shown to reproduce deviations from asymptotic freedom at small distances in a quantitative way.

During the past decade significant efforts were devoted to derive the instanton picture of the QCD vacuum from first principles. First variational and later numerical methods were applied to this problem, with the hope to develop a self-consistent theory of interacting instantons. In spite of these efforts, a clear cut derivation from first principles is still missing. Recently, we have performed further studies along these lines, substituting the rather arbitrary trial functions used previously by the 'stream-line' set of configurations found in [11]. These studies have shown that we still lack an understanding of how exactly one should treat 'a bottom of the valley', the configurations with a very close instanton-anti-instanton pair. We are now preparing a separate publication, in which the current understanding of this problem will be discussed.

In the RILM we simply assume that except for the size, which we keep fixed, the distribution of the collective coordinates associated with the instantons and antiinstantons is completely random. We consider the RILM as the simplest possible model of its class.

\[^{1}\text{A detailed review on QCD correlation functions recently appeared in [1].}\]
and a benchmark for any more involved calculation with ‘correlated instantons’. Before studying more complicated ensembles, it is important to understand the predictions of the random model for physical quantities. Our previous studies of the propagation of a single quark [2] and quark-antiquark pairs (mesonic correlators) [3] in such a vacuum have significantly extended those reported in [12]: the box volume was increased by a factor 20, the correlation functions were traced down by a few orders of magnitude, etc. Furthermore, the results were shown to be in quantitative agreement with experimental data, which was not expected for this simple model. Experimentally known correlators (and eventually, masses and coupling constants of the mesons) are reproduced, in some cases literally inside the uncertainties of the calculation! What is equally exciting is the fact that we also observe good agreement with the results of recent lattice calculations [13], in spite of the quite different approaches and approximations used.

As this paper is the first exploratory study of baryon correlators in the instanton vacuum, the use of the random model appears to be well justified. Its primary goal is to check whether this model leads at least to some baryonic bound states, in the form of a resonances well separated from a multi-particle continuum.

Before we start, let us make a few general remarks about the baryonic channels. It is not surprising that quark-quark forces are even less understood than the quark-antiquark ones: ordinary (made of light quarks) baryons are not simple two-body systems. The situation is somewhat simpler if one of the quarks is ‘sterile’ (very heavy) [14], but unfortunately the experimental data on heavy-light baryons are still rather incomplete.

A direct manifestation of the ’t Hooft instanton-induced interaction [15] (which produces so spectacular effects for the scalar and pseudoscalar mesonic correlators) is also expected in the quark-quark interaction. In this case, it appears as a strong attraction in the scalar diquark channel [16]. As suggested in [17], this interaction can naturally explain all spin splittings in the octet and the decuplet, describing the spectra at least as good as traditional explanations using gluomagnetic forces between ‘constituent’ quarks. Thus, our second major question is whether baryons belonging to the two lowest $SU(3)$
multiplets, the octet and the decuplet, are similar (their differences being due to relatively small perturbative effects), or whether they are completely different. In this context we not only want to consider the \( N - \Delta \) mass splitting, but also make a comparison of the coupling constants and the general shape of the correlation functions.

The paper is structured as follows: in section 2 we introduce 'diquark' correlators, which are compared to the corresponding mesonic channels and then used (in section 3) to derive properties of heavy-light baryons. In sections 4 and 5 we deal with the nucleon and delta correlators, as representatives of the \( SU(3) \) octet and decuplet baryons. A comparison with QCD sum rules and lattice results, as well as a general discussion is contained in sections 6 and 7. Conclusions are summarized in section 8.

2. Diquarks

In this section we study correlation functions of two-quark currents. As suggested in \[14\], such currents can be thought of as part of a baryon current in which one of the quarks is 'sterile', or very heavy, so that it does not move at all. We will come back to this interpretation in the next section.

The generic diquark current \( j^a_\Gamma \) is defined by

\[
j^a_\Gamma = \epsilon^{abc} q^b_\alpha (CT)_{\alpha\beta} \tau q^c_\beta, \tag{2.1}
\]

where \( C \) is the charge conjugation matrix and \( \Gamma \) is one of the possible gamma matrix structures. Furthermore, \( q^a_\alpha \) denotes an isodoublet quark spinor with color index \( a \) and spinor index \( \alpha \). The isospin matrix \( \tau \) is given by \( \tau = \tau_2 \) for an isosinglet diquark and by \( \tau = \tau_2 \vec{\tau} \) for an isovector diquark. Note that the isospin wavefunction is determined by the symmetry of the Dirac matrix \( CT \) and the overall antisymmetry with respect to exchanging the two quarks. In particular, \( \Gamma = \{1, \gamma_5, \gamma_\mu \gamma_5\} \) corresponds to isoscalar diquarks whereas \( \Gamma = \{\gamma_\mu, \sigma_{\mu\nu}\} \) gives isovector diquarks.

Note that the parity of the Dirac matrix \( CT \) is opposite to that of the matrix \( \Gamma \): the internal parity of a \( qq \) pair is positive while the one of the \( \bar{q}q \) system is negative.
For example, if $\Gamma = \gamma_5$, the diquark is a scalar. This well known fact makes diquark notations somewhat ambiguous, one can either (i) specify the real quantum numbers of the diquarks, or (ii) indicate which gamma matrix was used in the current. We use the latter notation (e.g. call the $\Gamma = \gamma_5$ case 'the pseudoscalar correlator'), since in this notation the comparison between diquark and meson correlators is more straightforward. We hope that this will not confuse the reader.

We study diquark correlators defined by

$$\Pi_\Gamma(x - y) = \langle j_\Gamma^a(x) (j_\Gamma^a(y))^\dagger \rangle,$$  \hspace{1cm} (2.2)

where the hermitian conjugate current $(j_\Gamma^a)^\dagger$ is given by

$$(j_\Gamma^a)^\dagger = \epsilon^{abc} q_b^\alpha (\Gamma C)_{\alpha\beta}^{\tau} q_c^\beta.$$  \hspace{1cm} (2.3)

The correlator can be expressed in terms of the quark propagator resulting in

$$\Pi_\Gamma(x - y) = \text{Tr} \left[ S^{ba}(x - y) \Gamma C S^{T \, ab}(x - y) C \Gamma \right] - \text{Tr} \left[ S^{aa}(x - y) \Gamma C S^{T \, bb}(x - y) C \Gamma \right],$$  \hspace{1cm} (2.4)

where the transpose is with respect to the Dirac indices. At short distances we can evaluate the correlator using free quark propagators. Deviations from the free quark propagation have been studied using the operator product expansion (OPE), further simplified by the so called 'vacuum dominance' approximation \cite{18}. For baryonic currents it was implemented \cite{19} in a very simple way: the propagator is given by the sum of a free propagator and a quark condensate. As discussed in more detail in \cite{14}, this approximation corresponds to a 'distance dependent constituent quark mass' $m_Q = (\pi^2/3)\tau^2|\langle \bar{q}q \rangle|$ which is generated as the quark propagates in the chirally asymmetric QCD vacuum. The corresponding propagator reads\footnote{In what follows we will always work in euclidean space-time. Physically there is no preferred direction, so we can always choose $x - y$ to point in the euclidean time direction.}

$$iS(\tau) = -\frac{1}{2\pi^2\tau^3} \gamma_0 - \frac{1}{12} < \bar{q}q >.$$  \hspace{1cm} (2.5)

As we have shown in the case of the mesonic correlators, this approximation is quite useful in understanding the experimental trend of the various correlation functions, although it
is far from being quantitatively correct. For the diquark correlators introduced above we find

\[ S : \frac{\Pi_S}{\Pi_0} = 1 - \frac{\pi^4}{36} |\langle \bar{q}q \rangle|^2 \tau^6, \]  
\[ P : \frac{\Pi_P}{\Pi_0} = 1 + \frac{\pi^4}{36} |\langle \bar{q}q \rangle|^2 \tau^6, \]  
\[ V : \frac{\Pi_V}{\Pi_0} = 1 - \frac{\pi^4}{18} |\langle \bar{q}q \rangle|^2 \tau^6, \]  
\[ A : \frac{\Pi_A}{\Pi_0} = 1 - \frac{\pi^4}{18} |\langle \bar{q}q \rangle|^2 \tau^6, \]  
\[ T : \frac{\Pi_T}{\Pi_0} = \frac{\pi^4}{6} |\langle \bar{q}q \rangle|^2 \tau^6, \]  

where \( \Gamma = \{ S, P, V, A, T \} \) labels the gamma matrices in the diquark channel and \( \Pi_0 \) denotes the free correlation function. Note that these estimates for the short-distance behaviour of the diquark correlation functions exactly coincide with the corresponding approximation for the mesonic correlator in the same gamma matrix channel. This is a consequence of the fact that in the vacuum dominance approximation the propagator is diagonal in color space, which is not correct in general.

We now come to a physical interpretation of the diquark correlators. Since diquarks are color antitriplets, they cannot appear as asymptotic states. Their correlation functions are gauge-dependent and unphysical, unless either (i) a gauge fixing is implied or (ii) the additional color matrix \( U = P \exp(i g \int_C dx \mu A^a_{\mu} t^a / 2) \) is included. If the path \( C \) is a segment of a straight line going in the time direction, it corresponds to a static third quark: thus we naturally come to light-heavy baryons, to be discussed in the next section.

The specific feature of instanton models is that the matrix \( U \approx 1 \), because heavy quarks interact very little with instantons. We will therefore neglect the effects of \( U \) in our following analysis. More quantitatively, it was shown in [12] that the gauge string \( U \) produces a potential of the order of 50 MeV, which is well inside the accuracy of the present calculations. Thus, inside instanton-based vacuum models (which ignore confinement etc.), the diquark correlators defined above are 'practically gauge invariant'. In such a spirit,
one can define the coupling of the currents introduced above to 'diquark states'

\[
<0|j_{S,P}^a|\phi_{S,P}^b> = g_{S,P} \frac{\delta^{ab}}{\sqrt{3}} e^{-iqx},
\]

(2.7a)

\[
<0|j_{V,A}^a|\phi_{V,A}^b> = g_{V,A} \frac{\delta^{ab}}{\sqrt{3}} \epsilon_\mu e^{-iqx},
\]

(2.7b)

\[
<0|j_{V,A}^a|\phi_{S,P}^b> = f_{V,A} \frac{\delta^{ab}}{\sqrt{3}} q_\mu e^{-iqx},
\]

(2.7c)

where \(|\phi_\Gamma^a\rangle\) denotes a diquark state with momentum \(q_\mu\) and color index \(a\) that has the quantum numbers of the corresponding current and \(\epsilon_\mu\) is the polarization of a vector diquark. Proceeding this way we effectively treat diquarks just like mesons. In particular, we also allow for mixing of vector and scalar diquarks in the vector channel. This phenomenon is analogous to the well known \(\pi - a_1\) mixing which is observed in the axial vector channel.

In the following we want to describe the diquark correlation functions in terms of the physical intermediate states which can contribute. For this reason we represent the euclidean Fourier transform of the correlation function by a dispersion integral

\[
\int d^4x e^{iqx} \Pi_\Gamma(x) = \frac{1}{\pi} \int_0^\infty ds \Im \Pi_\Gamma(s) \frac{s}{q^2 + s},
\]

(2.8)

where \(\rho_\Gamma(s) \equiv \frac{1}{\pi} \Im \Pi_\Gamma(s)\) is the spectral density associated with the corresponding correlation function. The spectral density can be expressed as

\[
\rho_\Gamma(s = k^2) = (2\pi)^3 \sum_n \delta^4(k - q_n) <0|j_\Gamma(0)|n> <n|j_\Gamma(0)|0>,
\]

(2.9)

where \(|n>\) is a physical intermediate state with momentum \(q_n\).

Similar to the procedure in the meson sector we approximate the spectral function by the contribution of one or two diquark resonances and a free quark continuum starting at a threshold \(s_0\). This means that for long distances the correlator is dominated by the contribution of the lowest physical state whereas for short distances it can be described in terms of the propagation of free quarks. In the scalar and pseudoscalar channels we quarks and proceed directly to our discussion of nucleon and delta correlators, which are explicitly gauge invariant.
\[ \rho_{S,P}(s) = g_{S,P}^2 \delta(s - m_{S,P}^2) + \frac{3}{4\pi^2} s \Theta(s - s_0), \quad (2.10) \]

where the first term is the resonance contribution and the second term represents the continuum. On purely dimensional grounds we have \( \rho_{S,P}(s) \sim s \) for large invariant masses.

The corresponding coefficient can be calculated from the discontinuity of the free quark loop diagram appearing in eq.(2.4).

Inserting the model eq.(2.10) for the spectral function in the dispersion relation (2.8) and transforming back to coordinate space the diquark correlator is given by

\[ \Pi_{S,P}(\tau) = g_{S,P}^2 D(m_{S,P}, \tau) + \frac{3}{4\pi^2} \int_{s_0}^{\infty} ds \, s D(\sqrt{s}, \tau), \quad (2.11) \]

where \( D(m, \tau) = m/(4\pi^2\tau)K_1(m\tau) \) is the euclidean space propagator for a massive scalar particle. Proceeding in the same way in the vector channels, we find

\[ \Pi_{V,A}(\tau) = 3g_{V,A}^2 D(m_{V,A}, \tau) - m_{S,P}^2 f_{V,A}^2 D(m_{S,P}, \tau) + \frac{3}{2\pi^2} \int_{s_0}^{\infty} ds \, s D(\sqrt{s}, \tau), \quad (2.12) \]

In the tensor channel, finally, there is no continuum contribution. The dirac tensor \( \sigma_{\mu\nu} \) has components \( \sigma_{0i} \) which transform like a three-vector and \( \epsilon_{ijk}\sigma_{jk} \) which transform like a pseudovector. This means that vector as well as axial vector diquark states can contribute. Since the contributions from these two states are hard to distinguish in practice we describe the tensor diquark correlation function using a single diquark resonance with mass \( m_T \) and coupling \( g_T \)

\[ \Pi_T(\tau) = g_T^2 D(m_T, \tau). \quad (2.13) \]

We have extracted the diquark masses and coupling constants by fitting the parametrizations eq.(2.11-2.13) to the diquark correlation functions measured in the random instanton liquid. The correlation functions together with the fits are shown in figure 1 and the results of the fit are summarized in table 1.

Looking at the various diquark correlation functions one immediately realizes that there are very pronounced qualitative differences between the different channels. In particular, the scalar case (\( \Gamma = 1 \)) has very repulsive interaction and shows no indications for
a 'diquark resonance' whereas the pseudoscalar one ($\Gamma = \gamma_5$) shows a huge enhancement\(^4\) or strong attractive interaction. Determining the parameters of this resonance we find

$$m_P = 420 \pm 30 \text{ MeV}, \quad g_P = 0.23 \pm 0.01 \text{ GeV}^2.$$  \hspace{1cm} (2.14)

The dashed curve in figure 1 shows the result of the 'vacuum dominance' approximation, see [3]. As for the mesonic channels, it reproduces the qualitative differences between the scalar and pseudoscalar channels but fails to give a quantitative description at distances larger than 0.3 fm.

In the ($\Gamma = \gamma_\mu$) vector diquark channel the correlation function remains close to the perturbative one out to fairly large distances. Again, the situation is similar to the superduality phenomenon observed in the rho meson channel. The resonance parameters in the diquark channel are

$$m_V = 940 \pm 20 \text{ MeV}, \quad g_V = 0.24 \pm 0.01 \text{ GeV}^2.$$ \hspace{1cm} (2.15)

In the ($\Gamma = \gamma_\mu \gamma_5$) axialvector diquark channel, we can have mixing with the light pseudoscalar diquark. Since the contribution from this state alone can already explain the measured correlation function, we have no evidence for an axialvector 'diquark state' in the RILM. In the tensor ($\Gamma = \sigma_{\mu\nu}$) diquark channel we have fitted the correlator with a single resonance at $m_T = 570 \pm 20$ MeV, which is intermediate between the light pseudoscalar and heavy vector diquark resonances.

Of great interest for hadronic models which invoke diquark clustering in order to explain the observed spin splitting between the octet and the decuplet is the mass difference $m_V - m_P \simeq 520$ MeV. In a simple quark-diquark model [17, 20] we have $m_\Delta - m_N = \frac{1}{2}(m_V - m_P)$. Comparing with the experimental splitting $m_\Delta - m_N = 293$ MeV we do in fact get a very good agreement. We will further comment on the quark-diquark picture of the baryons in our discussion of the nucleon and delta correlation functions.

\(^4\)Which is still not as strong as for the pseudoscalar mesons, of course, which have to be massless due to Goldstone theorem. However, it is quite comparable with the effect observed for scalar isoscalar 'sigma' meson, which also has a comparable mass.
3. Heavy-light baryons

Adding a heavy quark $Q$ to the diquark currents considered in the last section we can construct currents carrying the quantum numbers of heavy baryons. Correlation functions of these currents were first considered in [14] in the framework of the QCD sum rules. Recently, further analysis along these lines was performed in [21, 22], where somewhat different (non-relativistic) currents have been used.

The heavy baryon currents have the general structure $J_{BQ \alpha} = j \Gamma (\Gamma' Q)$ where $j$ is a diquark current, $Q$ is a heavy quark spinor with dirac index $\alpha$ and $\Gamma'$ is a dirac matrix. Out of the five possible currents, three couple to the $\Lambda Q$ while the two remaining ones have the quantum numbers of the $\Sigma Q$:

\begin{align}
J_{1 \Lambda Q}^1 &= \epsilon^{abc} (u^a C d^b) \gamma_5 Q^c \\
J_{1 \Lambda Q}^2 &= \epsilon^{abc} (u^a C \gamma_5 d^b) Q^c \\
J_{1 \Sigma Q}^2 &= \epsilon^{abc} (u^a C \gamma_\mu \gamma_5 d^b) \gamma^\mu Q^c,
\end{align}

(3.1a)

\begin{align}
J_{2 \Lambda Q}^3 &= \epsilon^{abc} (u^a C \gamma_\mu d^b) \gamma^\mu Q^c \\
J_{2 \Sigma Q}^3 &= \epsilon^{abc} (u^a C \sigma_{\mu \nu} d^b) \gamma_5 \sigma^{\mu \nu} Q^c.
\end{align}

(3.1b)

In the following we will consider correlation functions constructed from these currents

$$\Pi_{\alpha \beta}(x - y) = i < J_{BQ \alpha}(x) \bar{J}_{BQ \beta}(y) >.$$ 

(3.2)

It is instructive to consider the limit in which the mass of the heavy quark is taken to infinity. In this case the correlation function simplifies to

$$\Pi_{\alpha \beta}(x - y) = (\Gamma' i S_Q(x - y) \bar{\Gamma}')_{\alpha \beta} \Pi_{\Gamma}(x - y),$$

(3.3)

where $\bar{\Gamma}' = \gamma_0 \Gamma'^* \gamma_0$ and

$$i S_Q(x - y) = \frac{1 + \gamma_0}{2} \Theta(\tau) \delta^3(\vec{x} - \vec{y}) \left( \frac{M}{2 \pi \tau} \right)^{3/2} e^{-M \tau}$$

(3.4)

denotes the heavy quark propagator in the nonrelativistic (static) limit. Using this approximation the spin structure of the correlation function becomes trivial and all the dynamical information is contained in the diquark correlator $\Pi_{\Gamma}(x - y)$. 

9
As in the last section we want to describe the Euclidean correlation function \( (3.2) \) in terms of physical intermediate states. This is achieved by means of the spectral decomposition of the correlator

\[
\Pi_{\alpha\beta}(\tau) = \int_0^\infty ds \rho_{\alpha\beta}(s) D(\sqrt{s}, \tau) \quad (3.5)
\]

Again we approximate the spectral function by the sum of a resonance and a continuum contribution, which in this case results in

\[
\rho_{\alpha\beta}(q) = \delta(q^2 - M_{BQ}^2)2M_Q\lambda^2 \sum_s Q_s^{(s)}\overline{Q}_a^{(s)} + \frac{1}{10\pi^4}(s - s_0)^{5/2}\theta(s - s_0)((\gamma \cdot q)_{\alpha\beta} + M_Q\delta_{\alpha\beta}). \quad (3.6)
\]

The coupling constant \( f_B \) is defined by

\[
<0|J_{BQ\alpha}|B_Q> = f_B\sqrt{M_Q}B_a^{(s)}, \quad (3.7)
\]

where \( B_a^{(s)} \) is a Dirac spinor with spin \( s \). The numerical factor multiplying the continuum contribution follows from asymptotic freedom.

It is convenient to measure the energies with respect to the heavy quark mass \( M_Q \). When we write \( s = (M_Q + \omega)^2 \), the non-relativistic heavy quark propagator factorizes from the spectral function, and the Euclidean diquark correlator is given by

\[
\Pi_\Gamma(\tau) = \frac{f_{BQ}^2}{2}\exp(-\epsilon_{BQ}\tau) + \frac{1}{20\pi^4}\int_{\omega_0}^\infty d\omega \omega^5\exp(-\omega\tau), \quad (3.8)
\]

where \( \epsilon_{BQ} \) is the heavy baryon energy (minus the rest mass of the heavy quark). As above we have introduced a continuum integral starting at a threshold energy \( \omega_0 \).

The heavy baryon parameters extracted from this non-relativistic interpretation of the diquark correlation functions are given in table 2, together with the predictions from QCD sum rules.

The \( \Lambda_Q \) results were obtained from the pseudoscalar diquark channel, since this current has a simple non-relativistic limit. We find

\[
\epsilon_{\Lambda_Q} = 760 \pm 30 \text{ MeV} \quad f_{\Lambda_Q} = 0.052 \pm 0.005 \text{ GeV}^3. \quad (3.9)
\]
The parameters of the Σ\textsubscript{Q} baryon can be determined from the Γ = \( \gamma_\mu \) or Γ = \( \sigma_{\mu\nu} \) diquark channels. Using the Γ = \( \gamma_\mu \) diquark channel we get

\[
\epsilon_{\Sigma Q}^V = 892 \pm 30 \text{ MeV}, \quad f_{\Sigma Q}^V = 0.011 \pm 0.002 \text{ GeV}^3. \tag{3.10}
\]

while the fit to the tensor channel gives a somewhat larger mass

\[
\epsilon_{\Sigma Q}^T = 1006 \pm 30 \text{ MeV}, \quad f_{\Sigma Q}^T = 0.044 \pm 0.008 \text{ GeV}^3. \tag{3.11}
\]

Of particular interest is the Σ\textsubscript{Q} – Λ\textsubscript{Q} mass splitting which we find to be \( m_{\Sigma Q} - m_{\Lambda Q} = 135 \text{ MeV} \) or 246 MeV, depending on the current used in the fit. This number should be compared with \( m_{\Sigma c} - m_{\Lambda c} = 170 \text{ MeV} \) for charmed baryons. Unfortunately, the corresponding value for bottom baryons is not known experimentally.

It is instructive to compare these results with those that have been obtained from QCD sum rules. In the original paper \[14\] the predictions are \( \epsilon_{\Lambda Q} = 700 \text{ MeV} \) and \( f_{\Lambda Q} = 0.02 \text{ GeV}^3 \) together with a mass splitting \( m_{\Sigma Q} - m_{\Lambda Q} = 400 \pm 250 \text{ MeV} \). On the other hand, the values obtained in \[21\] are \( \epsilon_{\Lambda Q} = 780 \text{ MeV} \) and \( f_{\Lambda Q} = 0.022 \pm 0.005 \text{ GeV}^3 \) which is very similar, but there is no Σ\textsubscript{Q} – Λ\textsubscript{Q} splitting! Let us make a few remarks concerning these results:

(i) The two OPE-based papers give similar predictions for the mass and the coupling constant of the Λ\textsubscript{Q} when the pseudoscalar current is used. While the predicted mass agrees with our fit, the coupling constant does not. This is not surprising: the large instanton-induced attractive interaction which leads to the strong enhancement in the \( \gamma_5 \) channel is not reproduced by the OPE.

(ii) Extracting the Σ\textsubscript{Q} parameters and the Λ\textsubscript{Q} – Σ\textsubscript{Q} splitting from the OPE is very subtle: in fact the ‘vacuum dominance’ approximation (inclusion of the quark condensate term in the propagator) does not lead to an unambiguous result.

Indeed, compare for example the \( \gamma_\mu \) and \( \gamma_\mu \gamma_5 \) correlators in order to determine the Σ\textsubscript{Q} – Λ\textsubscript{Q} splitting. In this case the corrections up to order \( \langle \bar{q}q \rangle^2 \) read

\[
1 \pm \langle \bar{q}q \rangle^2 \tau^6 \pi^4/18,
\]

5 Naturally, the two coupling constants \( f_{\Sigma Q}^V \), \( f_{\Sigma Q}^T \) have different definitions and should not be equal.

6 There was a misprint in the paper: The value of \( f_{\Lambda Q} \) is one order of magnitude larger than indicated. The published version corresponds to the r.h.s. of the sum rules 100 times smaller than the l.h.s.
respectively, so that one may think there is a big splitting. However, if one considers the non-relativistic limit of the currents (see ref. [21]), using the $\vec{\gamma}$ diquark for the nonrelativistic $\Sigma_Q$ current and $\gamma_0\gamma_5$ for the $\Lambda_Q$, one finds identical OPE expressions $1 + \langle \bar{q}q \rangle^2 \tau^6 \pi^4 / 36$ in both cases, leading to zero mass splitting!

(iii) Our fits for the $\Sigma_Q$ mass in the $V$ and $T$ channels are different, but not very much so. A $\Sigma_Q - \Lambda_Q$ splitting on the order of 200 MeV is definitely predicted. What is maybe even more important is that we see a qualitative difference between the $\Sigma_Q$ and the $\Lambda_Q$: both $V$ and $T$ correlators are comparable to the perturbative ones at distances about 1 fm, while the correlator in the $\gamma_5$ channel is several times larger.

4. Nucleon correlation functions

After studying the quark-quark interaction we now proceed to correlation functions of baryonic (three quark) currents. The Euclidean correlator of two spin 1/2 nucleon currents is defined as

$$\Pi_{\alpha\beta}(x - y) = i < J_N^\alpha(x)\bar{J}_\beta^N(y) >,$$

(4.1)

where $\alpha, \beta$ are the spinor indices of the nucleon currents. Using Lorentz and parity invariance one can show that the correlator can be decomposed in terms of only two independent Dirac structures

$$\Pi_{\alpha\beta}(x - y) = \Pi_1((x - y)^2)(\gamma \cdot (x - y))_{\alpha\beta} + \Pi_2((x - y)^2)\delta_{\alpha\beta}. \tag{4.2}$$

Again we will consider the correlation function in the euclidean time direction. Only two independent correlators remain for a given current, namely $\text{Tr}(\Pi(\tau\gamma_0))$ and $\text{Tr}(\Pi(\tau))$.

Due to isospin symmetry it is sufficient to consider only one charge state of the nucleon. A proton current is constructed by coupling a $d$-quark to a $uu$-diquark. The diquark has the structure $\epsilon_{abc}u_bC\Gamma u_c$ which requires that the matrix $C\Gamma$ is symmetric. This condition

\footnote{The contribution of the gluon condensate also turns out to be identical.}
is satisfied for the $V$ and $T$ gamma matrix structures. Only two possible currents that have positive parity and spin $1/2$ can be constructed [19]. These so called Ioffe currents are given by

\begin{align}
\eta_1 &= \epsilon_{abc}(u^a C\gamma_\mu u^b)\gamma_5 \gamma_\mu d^c, \quad (4.3a) \\
\eta_2 &= \epsilon_{abc}(u^a C\sigma_\mu\nu u^b)\gamma_5 \sigma_\mu\nu d^c. \quad (4.3b)
\end{align}

In total, there exist six different nucleon correlators: the diagonal $\eta_1 \bar{\eta}_1, \eta_2 \bar{\eta}_2$ and off-diagonal $\eta_1 \bar{\eta}_2$ correlators, each contracted with either the identity or $\gamma_0$. They are defined as

\begin{align}
\Pi_1^N &= \frac{1}{4} < \text{Tr}(\eta_1 \bar{\eta}_1) >, \quad (4.4a) \\
\Pi_2^N &= \frac{1}{4} < \text{Tr}(\gamma_0 \eta_1 \bar{\eta}_1) >, \quad (4.4b) \\
\Pi_3^N &= \frac{1}{4} < \text{Tr}(\eta_2 \bar{\eta}_2) >, \quad (4.4c) \\
\Pi_4^N &= \frac{1}{4} < \text{Tr}(\gamma_0 \eta_2 \bar{\eta}_2) >, \quad (4.4d) \\
\Pi_5^N &= \frac{1}{4} < \text{Tr}(\eta_1 \bar{\eta}_2) >, \quad (4.4e) \\
\Pi_6^N &= \frac{1}{4} < \text{Tr}(\gamma_0 \eta_1 \bar{\eta}_2) >. \quad (4.4f)
\end{align}

Again we can discuss the qualitative features of these correlation functions using the vacuum dominance model (2.5). The result is

\begin{align}
\frac{\Pi_1^N}{\Pi_2^{N0}} &= \frac{\pi^2}{12} |\bar{q}q| r^3 + \frac{\pi^6}{216} |\bar{q}q|^3 r^9, \quad (4.5a) \\
\frac{\Pi_2^N}{\Pi_2^{N0}} &= 1 + \frac{\pi^4}{72} |\bar{q}q|^2 r^6, \quad (4.5b) \\
\frac{\Pi_3^N}{\Pi_4^{N0}} &= \frac{\pi^6}{216} |\bar{q}q|^3 r^9, \quad (4.5c) \\
\frac{\Pi_4^N}{\Pi_4^{N0}} &= 1, \quad (4.5d) \\
\frac{\Pi_5^N}{\Pi_4^{N0}} &= \frac{\pi^2}{2} |\bar{q}q| r^3, \quad (4.5e) \\
\frac{\Pi_6^N}{\Pi_2^{N0}} &= \frac{\pi^4}{12} |\bar{q}q|^2 r^6. \quad (4.5f)
\end{align}
In order to compare these functions with experimental information we have to analyze the correlation functions in terms of physical intermediate states. The nucleon coupling to the two Ioffe currents is given by

\[ <0|\eta_{1,2}(0)|N(q, s)> = \lambda_{1,2}^N \sqrt{\frac{2m_N}{(2\pi)^3}} u(q, s) e^{-iqx}, \]  

(4.6)

where \(|N(q, s)>\) denotes a nucleon state with momentum \(q\) and spin \(s\), \(u(q, s)\) is the corresponding free fermion spinor (normalized as in [24]) and \(\lambda_{1,2}^N\) is a coupling constant. The first two correlation functions introduced above are related to the first Ioffe current. Using the simple nucleon resonance plus continuum model introduced in section 2 we have

\[ \Pi_1^N(\tau) = \left(\lambda_1^N\right)^2 m_N D(m_N, \tau) + \frac{1}{4\pi^2} |\langle\bar{q}q\rangle| \int_{s_0}^{\infty} ds \, s D(\sqrt{s}, \tau), \]  

(4.7a)

\[ \Pi_2^N(\tau) = - \left(\lambda_1^N\right)^2 D'(m_N, \tau) - \frac{1}{2\pi^4} \int_{s_0}^{\infty} ds \, s^2 D'(\sqrt{s}, \tau), \]  

(4.7b)

where \(D(m, \tau)\) is the euclidean space propagator of a scalar particle of mass \(m\) and \(D'(m, \tau) = d/(d\tau) D(m, \tau)\). For the second correlation function \(\Pi_2^N\) the continuum corresponds to the propagation of three free quarks and can be calculated from the discontinuity of the quark loop diagram contributing to the correlation function of the two Ioffe currents.

In the case of \(\Pi_1^N\) there is no such contribution. In order to improve the description at short distances we have added the discontinuity of the diagram in which one of the quarks interacts with the quark condensate. In the language of the operator product expansion this means that we have calculated the spectral function from the imaginary parts of the diagrams which determine the coefficients of the unit and \(\langle\bar{q}q\rangle\)-operators. In a more physical picture one can check that, at least in an average sense, the continuum part of \(\Pi_1^N\) represents the contribution of the \(\pi\Delta\)-continuum and higher nucleon resonances.

The correlation functions \(\Pi_{3,4}^N\) are related to the second Ioffe current. Proceeding as above they can be written as

\[ \Pi_3^N(\tau) = \left(\lambda_2^N\right)^2 m_N D(m_N, \tau), \]  

(4.8a)
\[ \Pi_4^N(\tau) = -\left(\lambda_2^N\right)^2 D'(m_N, \tau) - \frac{3}{2\pi^4} \int_{s_0}^{\infty} ds \, s^2 D'(\sqrt{s}, \tau). \]  

(4.8b)

The last two nucleon functions are related to off-diagonal correlators between the two Ioffe currents. They are given by

\[ \Pi_5^N(\tau) = \lambda_1^N \lambda_2^N m_N D(m_N, \tau) + \frac{3}{2\pi^2} |\langle \bar{q} q \rangle| \int_{s_0}^{\infty} ds \, s D(\sqrt{s}, \tau), \]  

(4.9a)

\[ \Pi_6^N(\tau) = -\lambda_1^N \lambda_2^N D'(m_N, \tau). \]  

(4.9b)

The measured nucleon correlation functions together with the physical parametrizations introduced above are shown in figure 3. The dashed lines show the result of the vacuum dominance approximation. Nucleon parameters determined from fitting the correlation functions in the RILM are given in table 3.

The most important feature of the nucleon correlation functions is the strong enhancement over the free correlator which can be seen in \( \Pi_2^N \) and \( \Pi_4^N \). This behavior is reminiscent of the pseudoscalar diquark channel which shows a similar strong attractive interaction. Somewhat surprisingly, for the nucleon case this feature is semi-quantitatively reproduced by the vacuum dominance approximation.

This enhancement in the calculated correlators is nicely described by a nucleon contribution to the correlation function. Fitting the nucleon parameters we get

\[ m_N = 960 \pm 30 \text{ MeV}, \quad \lambda_1^N = 0.031 \pm 0.001 \text{ GeV}^3, \quad \lambda_2^N = 0.080 \pm 0.004 \text{ GeV}^3, \]  

(4.10)

with a surprisingly accurate mass value. What is even more important, the simple 'nucleon pole plus continuum' model gives a very good simultaneous description for the complete set of correlation functions.

This agreement is particularly good for correlators involving the first Ioffe current, while it is somewhat worse for the other ones. Note that the correlation functions \( \Pi_3^N \) and \( \Pi_6^N \) (which show deviations from the simple 'nucleon pole plus continuum' model) involve at least two quarks which have to flip their chirality. It might be that the RILM has problems reproducing those amplitudes. An alternative explanation (suggested by Ioffe in his original QCD sum rule analysis) might be that the second Ioffe current couples more strongly to some excited states of the nucleon.
It is important to note that the simple random instanton model, even without confinement, *does create a nucleon bound state*. The presence of a bound state is indicated by the fact that we can describe the correlation functions in terms of a single pole, *clearly separated* from the free quark continuum starting at $E_0 \simeq 2$ GeV.

In order to strengthen that point we have also considered other models for the nucleon correlation functions. The simplest one is a 'constituent quark model'. One may try to describe the correlator in terms of the propagation of three uncorrelated 'constituent' quarks. For their masses we take the coordinate dependent effective mass $m_Q = \left( \frac{\pi^2}{3} \right) \tau^2 |\langle \bar{q}q \rangle|^2$ at short distances and $m_Q = 300$ MeV at large distances. The result is shown in figure 5 for the case of $\Pi^N_2$. The conclusion is obvious: the constituent model clearly can not explain the shape and magnitude of the correlation function.

A somewhat less trivial model for the correlator is the quark-diquark description. For this purpose we have Fierz rearranged $\eta_1$ in such a way as to make the scalar and vector diquark content explicit [19]. We can now use the measured diquark correlators which show a strong enhancement in the scalar diquark channel, and describe the propagation of the third quark using the constituent quark propagator introduced above. Again, the result is presented in figure 5. The corresponding curve is much closer to the data than the pure constituent model, but it still fails to reproduce our numerical results.

We conclude form these comparisons that the RILM not only produces substantial attraction in the scalar diquark channel, but also describes the nucleon as a true three quark bound state. While the first feature is essentially a consequence of the lowest order t’Hooft interaction, the presence of a nucleon pole is a very non trivial consequence of the RILM.

5. Delta correlation functions

In the case of the $\Delta-$current, isospin symmetry allows us to restrict ourselves to the $\Delta^{++}$. In this case all quarks have the same flavors and with the help of a Fierz
transformation one can show [19] that there exists only one independent current which is given by

\[ J_{\mu}^{\Delta} = \epsilon_{abc} (u^a C \gamma_{\mu} u^b) u^c. \]  

(5.1)

However, the spin structure of the correlator

\[ \Pi_{\mu\nu;\alpha\beta}^{\Delta}(x - y) = i < J_{\mu\alpha}^{\Delta}(x) \bar{J}_{\nu\beta}^{\Delta}(y) > \]  

(5.2)

is much richer. As before the correlator is related to the spectral function by

\[ \Pi_{\mu\nu;\alpha\beta}^{\Delta}(x) = \int ds \rho_{\mu\nu;\alpha\beta} D(\sqrt{s}, x). \]  

(5.3)

For fixed values of \( \mu \) and \( \nu \) we again have a two-spinor spectral function which, as seen in section 4, reduces to two invariant structures which can be projected out by either contracting with the identity or \( \gamma_0 \). The \( \mu\nu \)-indices transform as a Lorentz vector. Five different vector structures are possible

\[ \delta_{\mu\nu}, \gamma_{\mu} \gamma_{\nu}, q_{\mu} q_{\nu}, \gamma_{\mu} q_{\nu}, \gamma_{\nu} q_{\mu}, \]  

(5.4)

which adds up to ten different possibilities for the correlator. However, the delta current satisfies the Rarita Schwinger constraint \( \gamma_{\mu} J_{\mu}^{\Delta} = 0 \) which allows us to express the spectral function \( \rho_{\mu\nu;\alpha\beta} \) using only four independent form factors

\[ \rho_{\mu\nu;\alpha\beta} = a_1(q^2) (g_{\mu\nu} \hat{q} - 6 \frac{q_{\mu} q_{\nu}}{q^2} \hat{q} + \gamma_{\mu} q_{\nu} + \gamma_{\nu} q_{\mu}) \]

\[ + a_2(q^2) (\gamma_{\mu} \gamma_{\nu} \hat{q} - 16 \frac{q_{\mu} q_{\nu}}{q^2} \hat{q} + 2 \gamma_{\mu} q_{\nu} + 4 \gamma_{\nu} q_{\mu}) \]

\[ + b_1(q^2) (-4 g_{\mu\nu} + \gamma_{\mu} \gamma_{\nu}) \]

\[ + b_2(q^2) (g_{\mu\nu} + 2 \frac{q_{\mu} q_{\nu}}{q^2} \hat{q} - \frac{\gamma_{\mu} q_{\nu}}{q^2} \hat{q} + \frac{\gamma_{\nu} q_{\mu}}{q^2} \hat{q}). \]  

(5.5)

The above \( \Delta \)-current does not satisfy the second Rarita-Schwinger constraint \( q^{\mu} J_{\mu}^{\Delta} = 0 \). This condition would lead to two further relations between the form factors

\[ a_1(q^2) = -3 a_2(q^2), \]  

(5.6a)

\[ b_1(q^2) = b_2(q^2). \]  

(5.6b)
However, as will be shown below, the numerical values for these coefficients do not differ much from this relation. Consequently, the admixture of spin 1/2 states to the spectral function will be small.

We take the correlator in the euclidean time direction and consider the following invariant functions

\[
\Pi^\Delta_1 = \frac{1}{4} \text{Tr}(\Pi^\Delta_{\mu\nu} \delta_{\mu\nu}),
\]

(5.7a)

\[
\Pi^\Delta_2 = \frac{1}{4} \text{Tr}(\Pi^\Delta_{\mu\nu} \delta_{\mu\nu} \gamma_0),
\]

(5.7b)

\[
\Pi^\Delta_3 = \frac{1}{4} \text{Tr}(\Pi^\Delta_{\mu\nu} \delta_{\mu0} \delta_{\nu0}),
\]

(5.7c)

\[
\Pi^\Delta_4 = \frac{1}{4} \text{Tr}(\Pi^\Delta_{\mu\nu} \delta_{\mu0} \delta_{\nu0} \gamma_0).
\]

(5.7d)

As is section 3, we consider the correlators using the vacuum dominance approximation. For the four different structures we find

\[
\frac{\Pi^\Delta_1}{\Pi^\Delta_2^{1/2}} = \frac{\pi^2}{3} |\langle \bar{q}q \rangle| \tau^3 + \frac{\pi^6}{108} |\langle \bar{q}q \rangle|^3 \tau^9,
\]

(5.8a)

\[
\frac{\Pi^\Delta_2}{\Pi^\Delta_2^{1/2}} = 1 + \frac{\pi^4}{12} |\langle \bar{q}q \rangle|^2 \tau^6,
\]

(5.8b)

\[
\frac{\Pi^\Delta_3}{\Pi^\Delta_4^{1/2}} = \frac{\pi^2}{6} |\langle \bar{q}q \rangle| \tau^3 - \frac{\pi^6}{216} |\langle \bar{q}q \rangle|^3 \tau^9,
\]

(5.8c)

\[
\frac{\Pi^\Delta_4}{\Pi^\Delta_4^{1/2}} = 1 - \frac{\pi^4}{36} |\langle \bar{q}q \rangle|^2 \tau^6.
\]

(5.8d)

As usual we represent the spectral function for the delta correlator by a resonance contribution and a continuum of states starting at an invariant mass \(s_0\). The contribution of the \(\Delta\)-resonance is determined by the matrix element

\[
\langle 0 | J_{\mu\alpha}^\Delta(q, s) | \Delta(q, s) \rangle = \lambda_\Delta \sqrt{\frac{2m_\Delta}{(2\pi)^3}} u_{\mu\alpha}(q, s) e^{-i q x},
\]

(5.9)

where \(u_{\mu\alpha}(q, s)\) denotes a free Rarita-Schwinger vector spinor normalized according to

\[
\sum_s u_{\mu\alpha}(q, s) \bar{u}_{\nu\beta}(q, s) = \left[ \frac{\hat{q} + m_\Delta}{2m_\Delta} \left( g_{\mu\nu} - \frac{1}{3} \gamma_{\mu} \gamma_{\nu} - \frac{2}{3} \frac{g_{\mu\nu}}{m^2_\Delta} + \frac{1}{3} \frac{g_{\mu \gamma} \gamma_{\nu} - g_{\nu \gamma} \gamma_{\mu}}{m_\Delta} \right) \right]_{\alpha \beta}.
\]

(5.10)

The short distance behavior of the coefficients of the spectral function follows from matching the leading order singularities in the operator product expansion of the correlator with
the r.h.s. of eq. (5.5), giving
\[ a_1(q^2) = \frac{q^4}{160\pi^4}, \quad a_2(q^2) = -\frac{5}{16}a_1(q^2). \] (5.11)

Note that this result is indeed close to the pure spin 3/2 case \( a_2 = -\frac{1}{3}a_1 \). Using (5.10) and (5.11) the invariant correlation functions introduced above are given by
\[ \Pi_{\Delta}^{\Delta}(\tau) = 2m_\Delta\lambda_\Delta^2D(m_\Delta, \tau) + \frac{3|\langle \bar{q}q \rangle|}{4\pi^2} \int_{s_0}^{\infty} ds \ sD(\sqrt{s}, \tau), \] (5.12a)
\[ \Pi_{\Delta}^{\Delta}(\tau) = -2\lambda_\Delta^2D'(m_\Delta, \tau) - \frac{3}{256\pi^4} \int_{s_0}^{\infty} ds \ s^2D'(\sqrt{s}, \tau), \] (5.12b)
\[ \Pi_{\Delta}^{\Delta}(\tau) = -2\lambda_\Delta^2D'(m_\Delta, \tau) - \frac{|\langle \bar{q}q \rangle|}{2\pi^2} \int_{s_0}^{\infty} ds \ \left( \frac{1}{\tau}D'(\sqrt{s}, \tau) + \frac{s}{8}D(\sqrt{s}, \tau) \right), \] (5.12c)
\[ \Pi_{\Delta}^{\Delta}(\tau) = 2\lambda_\Delta^2 \left( -\frac{4}{m_\Delta^2\tau^2}D'(m_\Delta, \tau) + \frac{1}{\tau}D(m_\Delta, \tau) \right) \]
\[ -\frac{3}{160\pi^4} \int_{s_0}^{\infty} ds \ \left( \frac{4s}{\tau^2}D'(\sqrt{s}, \tau) - \frac{s^2}{\tau}D(\sqrt{s}, \tau) + \frac{s^2}{16}D'(\sqrt{s}, \tau) \right). \] (5.12d)

The results for the correlation functions are given in figure 4 and table 3. The most important difference as compared to the nucleon correlation function is the fact that the strong enhancement over the free correlator which we observed in \( \Pi_N^N \) and \( \Pi_N^4 \) is not present in \( \Pi_\Delta^\Delta \) and \( \Pi_\Delta^\Delta \). This means that the \( \Delta \) correlation function is qualitatively different from the nucleon one. Note that this feature is not reproduced by the vacuum dominance approximation. In particular, vacuum dominance predicts a stronger enhancement in \( \Pi_\Delta^\Delta \) than in \( \Pi_\Delta^\Delta \).

Fitting the parameters in (5.11) to the numerically calculated correlators we find
\[ m_\Delta = 1440 \pm 70 \text{ MeV}, \quad \lambda_\Delta = 0.033 \pm 0.005 \text{ GeV}^3. \] (5.13)

With these parameters, we obtain a very good description of all four correlation functions. The corresponding delta mass is somewhat bigger than the experimental one. This situation is similar to the one encountered in the meson sector [3]: the delta contains only vector diquarks, and, like the rho meson, it also turns out to be too heavy in the RILM.

As in the case of the nucleon one can speculate whether we really observe a true bound state in the spin-isospin 3/2 channel or whether the data can also be explained in terms of
the propagation of three essentially uncorrelated quarks. This question is actually more serious in the case of the delta, since in this correlator there are no strongly attractive diquark channels present. For that we reason we have compared the measured delta correlation functions with the simple constituent model introduced in the last section. The results are shown in figure 5. Again, the contribution from three uncorrelated quarks clearly underestimates the data. We conclude that although there is less attraction in the delta channel as compared to the nucleon, the RILM predicts a bound state in the spin-isospin 3/2 channel.

6. Comparison with QCD sum rules and lattice calculations

Predictions for baryonic correlators based on QCD sum rules were obtained by several groups. We show the latest results by Belyaev and Ioffe [23], which are compared to those from Chernyak et al. (see [25] and references therein). The results of Chernyak et al. are based on somewhat different correlators and fitting procedures. In the nucleon case the two predictions agree very well (and therefore we have plotted only one of them), but they strongly disagree for the delta.

Chernyak et al. have also performed a more general calculation of 'baryonic wave functions' and have concluded that there exists a qualitative difference between the structure of octet and decuplet particles. Limited data on hard exclusive reactions involving those particles seems to confirm this important conclusion. Our results, based on the RILM, also show that the octet and the decuplet baryons have very different correlation functions. In particular the coupling constants (proportional to the probability to find all three quarks at the same point) turn out to be drastically different!

Our results are compared to QCD sum rule predictions in Fig.6. In the nucleon case the RILM results agree very well with all sum rule predictions. In the delta channel this is not the case: the predictions from Chernyak et al. are strongly favored. However, we are unable to explain why we are in agreement with certain sum rule calculations but
not with others. The whole topic is very confusing, since (as it was demonstrated above in the simplest case of diquarks) the agreement of OPE predictions with RILM results can depend on the current used, and for some quantities OPE results based on different currents do not provide consistent answers.

In general, it is hardly surprising that these two approaches do not generally agree for many particular observables. In fact, the 'vacuum dominance' hypothesis is based on the simplistic idea of a very smooth, nearly homogeneous distribution of quark and gluon fields in space time. On the contrary, the RILM is an example of a model with strongly inhomogeneous fields, with gauge field concentrated in large fluctuations of a particular structure, in this case small size instantons.

Proceeding now to a comparison with lattice calculations, let us first make some general comments. 'Traditional' lattice calculations use plane-to-plane correlation functions, and those are known to have the following two problems:

- The measured ratio \( m_N/m_\rho \) remains much closer to the 'naive quark model' value 3/2 rather than the experimental one 1.2, even for the lightest quarks studied (see e.g. [26]).

- The lattice results for the nucleon-delta splitting appear to be smaller than the experimental value \( m_\Delta - m_N = 293 \text{ MeV} \).

It was speculated in [1] that the nucleon, as measured on the lattice, seems to be 'too heavy', probably because instanton-induced effects were under-represented in those simulations, for whatever reason.

In Fig.6 we compare our results with recent lattice data [13], which for the first time looked at point-to-point correlators. Good agreement is observed, our RILM results literally coincide with the lattice data within the error bars of the calculations! This agreement is somewhat surprising, because our fit to the RILM results gives \( m_N/m_\rho = 1 \pm 0.1 \), which is smaller than the experimental value whereas our prediction for the splitting

\[ 8 \text{And sometimes even a specially chosen 'smeared' sources in order to improve the projection onto the ground states.} \]
\[ m_\Delta - m_N = 480 \text{ MeV} \] is larger than the experimental result. Thus, deviations from experimental data in the RILM results have opposite trends as compared to the deviations observed in standard lattice gauge calculations.

Provided a fit to the Negele et al. data gives resonance parameters similar to the ones obtained by us, this particular set of lattice measurements seems to show deviations from experimental data that are opposite as compared to other lattice works. One may speculate that the two sets of lattice measurements emphasize the role of confinement and instanton-induced forces differently, because plane-to-plane correlators deal with large interquark distances, while point-to-point correlators are also sensitive to smaller ones.

A controversial subject remains the question whether octet and decuplet baryons have a qualitatively different structure. Even a naive constituent quark model including a hyperfine interaction leads to significantly different mean square radii, e.g. \[ \frac{r_\Delta}{r_N} \approx 1.4. \] If so, the probability to find all three quarks at the same point should be about one order of magnitude larger in the nucleon compared to the delta. However, lattice studies have been unable to find this effect: it was concluded that the nucleon and the delta have essentially the same radius. This question certainly deserves further study.

One more comment we would like to make concerns the large finite size effects we have observed in our calculations. Going from 64 to 256 instantons, (or from a box volume of 64 fm\(^4\) to 256 fm\(^4\)) we have seen significant changes in some correlation functions, the nucleon correlators among them. Obviously, lattice measurements (which use significantly smaller boxes) should be subject to similar modifications.

Clearly, point-to-point correlation functions are much more sensitive to the dynamics of the interquark interactions at small distances. They are also more stable with respect to finite size corrections. Further efforts in this direction (using the TERAFLOP or similar next-generation projects) should be encouraged.
7. Discussion

Finally, having completed the series of three papers devoted to the RILM, we would like to make a few more general comments.

Although the RILM is a model that was originally designed only to describe the most global properties of the QCD vacuum, the gluon and quark condensates, it turns out to work very well. Not only does it reproduce small deviations from asymptotic freedom at short distances, as it was originally anticipated, but it correctly describes a large number of mesonic and baryonic correlators, some of which vary over several orders of magnitude. The results obtained imply that even in the absence of confinement the lowest states in most channels are in fact bound states of two or three quarks. If we translate the measured correlators into resonance masses and coupling constants the results agree with experimental data at the 10-15 percent level.

Does this imply that by fixing the correct magnitude of two condensates, one can reproduce all other vacuum parameters as a more or less direct consequence? Absolutely not: our experience with interacting instanton ensembles shows that even inside the instanton framework it is very easy to get drastically different sets of correlators, which are completely incompatible with experiment, even if the quark and gluon condensates are kept more or less fixed. Thus, the excellent performance of the RILM is indeed a non-trivial observation, which probably means it has a lot of truth in it.

In any case, this model definitely 'outperforms' such widely used theoretical approaches as QCD sum rules or the Nambu-Jona-Lasinio model, and therefore applying it to a broader set of questions appears to be justified. Among the most fascinating topics to be studied are nuclear forces at small distances (propagation of 6 quarks together), hadronic structure and wave functions, multi-point correlators and many others.

The central question posed at the beginning of this paper has been answered: the RILM describes the lightest baryons as some bound states.

Moreover, their spin splitting comes out naturally, without any need for additional assumptions or parameters. The splitting between scalar and vector diquarks is essentially
due to the instanton-induced attraction present in the scalar channel \cite{16, 17}. Although we have not considered hyperons in this work, the correct dependence of the instanton-induced interaction on the strange quark mass \cite{17} allows us to speculate that these splittings will also come out reasonable.

Furthermore, comparison between the calculated nucleon and delta correlators provides another important lesson, namely: the octet and the decuplet baryons have completely different wave functions, because even the coupling constants (proportional to the probability to find all three quarks at the same point) are drastically different. Note that this picture of baryons is significantly different from e.g. a 'skyrmion-based' one. According to the skyrmion picture, a nucleon and a delta are essentially the same object, just rotating with a different angular (and isotopic) momentum. According to our calculations, they are quite different already at relatively small distances due to the strong instanton-induced attraction, which is present in octet baryons but absent in the decuplet.

8. Conclusions

We have performed a numerical analysis of diquark and baryonic correlation functions in the framework of the random instanton liquid model (RILM). Our main findings can be summarized as follows:

(i) The quark-quark interaction between light quarks is very much channel-dependent, and it qualitatively resembles the one in the quark-antiquark channels (with opposite parity). In particular, similar to the situation in the pion and rho channels, the interaction in the isospin I=0 (scalar diquarks) and I=1 (vector diquarks) channels are completely different: the scalar channel has a much stronger attractive interaction than the vector one, with the correlator reaching a maximum value which is about an order of magnitude larger than the free quark correlator.

(ii) If diquarks are treated as physical particles, the scalar and vector diquarks are found to have masses of 420 and 940 MeV, respectively. The non-relativistic description,
related to masses of heavy-light baryons (minus the mass of the heavy quark), have produced $\epsilon_{\Lambda Q} = 760 \text{ MeV}$ and $\epsilon_{\Sigma Q} = 900 - 1000 \text{ MeV}$, in reasonable agreement with the limited experimental information and QCD sum rule calculations.

(iii) Calculation of all six nucleon and four delta correlation functions have been performed and analyzed. The octet (nucleon) and decuplet (delta) correlators are found to be qualitatively different, with the main difference being due to attractive interaction for scalar I=0 diquarks mentioned above.

(iv) The nucleon parameters agree well with the predictions from different versions of the QCD sum rules, but in the case of the delta the results from Chernyak et al. (see [25]) are strongly preferred.

(v) Our results were also found to be in agreement with the first lattice data on point-to-point correlation functions [13]. Moreover, this agreement is surprisingly good, the two calculations practically agree inside error bars.

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**Figure Captions**

1. Diquark correlation functions in the RILM, measured as a function of distance. All correlators are normalized to the perturbative results. The diquark channels are labeled by the Dirac matrix $\Gamma$ defining the current. The solid lines show a fit using the diquark model discussed in section 2 and the dotted curves are the prediction from the vacuum dominance model.

2. Correlation functions for $\Lambda$-type diquarks (a-b) and $\Sigma$-type diquarks (c-d). The channels are labeled as in figure 1. The solid lines now correspond to the heavy-light parametrization discussed in section 3. The dotted and dashed lines in figure (a) correspond to Grosin-Yakovlev prediction and their OPE expression, respectively.

3. Nucleon correlation functions in the RILM. Channels are labeled as in section 4 of the text. The solid line shows the 'nucleon pole plus continuum' model, while the dotted curve is the vacuum dominance approximation.

4. Delta Correlation functions in the RILM. Curves are as in figure 3.

5. Comparison of nucleon and delta correlation functions with simple quark-diquark models. Figures (a) and (b) show the correlation functions $\Pi^N_2$ (nucleon) and $\Pi^\Delta_2$ (delta), respectively. Data and solid curves are as shown in figs. 3 and 4. The dotted curves show the constituent model introduced in section 4 of the text. In figure 5a, the quark-diquark model is given by the dashed curve.

5. Comparison with QCD sum rules and lattice data for the two correlation functions $\Pi^N_2$ (nucleon) and $\Pi^\Delta_2$ (delta). The RILM results are given by the solid triangles, the lattice data from [13] by the open squares. QCD sum rules predictions from Belyaev and Ioffe [23] are shown as long dashed lines, results from Chernyak et al. [25] as short dashed lines.
Table Captions

1. Numerical results from fitting the diquark correlation functions in the RILM with a 'diquark resonance plus continuum' model. The parameters are defined in section 2 of the text.

2. Results from fitting the diquark correlators using the non relativistic parametrization related to the masses of heavy light baryons introduced in section 3. The two different values for the $\Sigma_Q$ correspond to the results obtained from the $\Gamma = \gamma_\mu$ and $\sigma_{\mu\nu}$ diquark channels. For comparison, we also quote the results from two QCD sum rule calculations.

3. Nucleon and Delta parameters obtained from performing a global fit to the six nucleon and four delta correlation functions defined in section 4 and 5. Results are also compared to the experimental values for the masses and QCD sum rule predictions for the coupling constants.
\[ m_S \quad 420 \pm 30 \text{ MeV} \quad 234 \text{ MeV} \quad \text{NJL model} \quad [27] \\
m_{AV} \quad 940 \pm 20 \text{ MeV} \quad 824 \text{ MeV} \quad \text{NJL model} \quad [27] \\
m_T \quad 570 \pm 20 \text{ MeV} \\
g_S \quad 0.225 \pm 0.011 \text{ GeV}^2 \quad 0.135 \pm 0.025 \text{ GeV}^2 \quad \text{QCD sum rules} \quad [28] \\
g_{AV} \quad 0.244 \pm 0.010 \text{ GeV}^2 \\
g_T \quad 0.134 \pm 0.004 \text{ GeV}^2 \\
\begin{array}{|c|c|c|} \hline 
\text{this work} & \text{other information} & \text{comment} \\
\hline 
m_S & 420 \pm 30 \text{ MeV} & 234 \text{ MeV} \\
m_{AV} & 940 \pm 20 \text{ MeV} & 824 \text{ MeV} \\
m_T & 570 \pm 20 \text{ MeV} & \\
g_S & 0.225 \pm 0.011 \text{ GeV}^2 & 0.135 \pm 0.025 \text{ GeV}^2 \\
g_{AV} & 0.244 \pm 0.010 \text{ GeV}^2 & \\
g_T & 0.134 \pm 0.004 \text{ GeV}^2 & \\
\hline 
\end{array}

\text{table 1}

\[ \epsilon_{\Lambda_Q} \quad 760 \pm 30 \text{ MeV} \quad 700 \pm 150 \text{ MeV} \quad 780 \text{ MeV} \quad \text{QCD sum rules} \quad [14] \\
\epsilon_{\Sigma_Q} \quad 890 \pm 30 \text{ MeV} \quad 1005 \pm 30 \text{ MeV} \quad 1100 \pm 200 \text{ MeV} \quad 780 \text{ MeV} \quad \text{QCD sum rules} \quad [14] \\
f_{\Lambda_Q} \quad 0.052 \pm 0.005 \text{ GeV}^3 \quad 0.02 \text{ GeV}^4 \quad 0.0225 \pm 0.0045 \text{ GeV}^3 \quad \text{QCD sum rules} \quad [14] \\
f_{\Sigma_Q} \quad 0.011 \pm 0.002 \text{ GeV}^3 \quad 0.06 \text{ GeV}^3 \quad 0.0225 \pm 0.0045 \text{ GeV}^3 \quad \text{QCD sum rules} \quad [14] \\
\begin{array}{|c|c|c|} \hline 
\text{this work} & \text{other information} & \text{comment} \\
\hline 
\epsilon_{\Lambda_Q} & 760 \pm 30 \text{ MeV} & 700 \pm 150 \text{ MeV} \\
\epsilon_{\Sigma_Q} & 890 \pm 30 \text{ MeV} & 1005 \pm 30 \text{ MeV} \\
f_{\Lambda_Q} & 0.052 \pm 0.005 \text{ GeV}^3 & 0.02 \text{ GeV}^4 \\
f_{\Sigma_Q} & 0.011 \pm 0.002 \text{ GeV}^3 & 0.06 \text{ GeV}^3 \\
\hline 
\end{array}

\text{table 2}
\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
 & this work & other information & comment \\
\hline
$m_N$ & $960 \pm 30$ MeV & 939 MeV & experiment \\
\hline
$\lambda_1$ & $0.032 \pm 0.001$ GeV$^3$ & $0.035 \pm 0.008$ GeV$^3$ & QCD sum rules [19] \\
\hline
$\lambda_2$ & $0.080 \pm 0.004$ GeV$^3$ & & \\
\hline
$E_0$ & $1920 \pm 50$ MeV & & \\
\hline
$m_\Delta$ & $1440 \pm 70$ MeV & 1232 MeV & experiment \\
\hline
$\lambda_\Delta$ & $0.033 \pm 0.005$ GeV$^3$ & $0.050 \pm 0.013$ GeV$^3$ & QCD sum rules [19] \\
& & $0.035$ GeV$^3$ & QCD sum rules [25] \\
\hline
$E_0$ & $1962 \pm 101$ MeV & & \\
\hline
\end{tabular}
\end{table}

table 3
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