A little more Gauge Mediation and the light Higgs mass

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Abstract

We consider minimal models of gauge mediated supersymmetry breaking with an extra \( U(1) \) factor in addition to the Standard Model gauge group. A \( U(1) \) charged, Standard Model singlet is assumed to be present which allows for an additional NMSSM like coupling, \( \lambda H_u H_d S \). The \( U(1) \) is assumed to be flavour universal. Anomaly cancellation in the MSSM sector requires additional coloured degrees of freedom. The \( S \) field can get a large vacuum expectation value along with consistent electroweak symmetry breaking. It is shown that the lightest CP even Higgs boson can attain mass of the order of 125 GeV.

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I. INTRODUCTION

Gauge mediated supersymmetry breaking [1–4] (for earlier works, please see, [5–11]) attractive due to several interesting features (i) flavour blind supersymmetry breaking soft terms (ii) very few parameters determine the entire spectrum (iii) different collider phenomenology compared to gravity mediated models as typically gravitino is the lightest supersymmetric particle (LSP) etc. However, phenomenologically\(^1\) the minimal versions of gauge mediation are severely constrained due to the discovery of a Higgs particle with a mass around 125 GeV. In MSSM, for the lightest CP even Higgs to be around 125 GeV would require, stop mixing parameter \(X_t\) to be large, 
\[
X_t \sim \sqrt{6} M_S, \quad \text{where} \quad M_S = \sqrt{m_{\tilde t_1} m_{\tilde t_2}}.
\]
While this holds true as long as stops are light \(\sim 1\) TeV, for very heavy stops \(\gtrsim 4\) TeV, the mixing parameter can be smaller. This would however push stops out of the reach of the LHC. In spite of theoretically appealing features, unfortunately, in minimal gauge mediation, the only way to fit a light Higgs mass \(\sim 125\) GeV is by making stops very heavy. The typical trilinear couplings in these models are very small at the mediation scale \(\sim 0\). Renormalisation group (RG) effects do generate them at the weak scale, however their magnitude is not large enough unless one makes gluinos ultra heavy \(\sim\) several TeV [15]. It should be noted that the constraints from 125 GeV Higgs boson are stronger even if one moves away from minimal mediation models to general gauge mediation models as long as \(A_t\) remains zero at the messenger scale [16].

Several possible solutions have been explored in the literature [17–34]. One of the directions which is popular with many authors is to introduce direct Yukawa couplings between messenger fields and the MSSM fields in addition to gauge interactions [35, 36]. In some cases, these interactions could also violate flavour [37]. In most of the models it is possible to generate large enough \(A_t\) at the weak scale to fit the 125 GeV light CP even Higgs boson mass. In a recent survey [19, 38] it has been pointed out that a particular class of messenger-matter interactions, messenger-stop mixings, has the least fine tuning of all the possible models which fit the light Higgs mass. Another direction which has been considered is to add additional vector like quarks close to the weak scale which couple to the Higgs superfields. These lead to additional corrections to the light Higgs boson thus lifting its mass without the need of increasing the stop masses (see for example, [30, 32]).

In the following we would like to take an alternate route. We would like to keep the minimal mediation structure in tact, thus would not like to introduce direct couplings between matter and

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\(^1\) For an early phenomenology of these models, please see, [12–14].
messenger fields. Adding an additional singlet field, like in NMSSM could help to raise the light Higgs mass. There are however, problems with electroweak symmetry breaking while incorporating NMSSM in minimal gauge mediation. These are well documented in literature \[39, 40\]. There are ways out, either by adding additional matter fields or dynamics through which NMSSM can be made viable with minimal gauge mediation \[41–50\]. Post 125 GeV Higgs boson, a model within this class has been explored in \[26\].

In the present work, we will consider an additional U(1) gauge group under which the ‘singlet’ of the NMSSM is charged. This U(1) factor also participates in gauge mediation. Anomaly cancellation requires additional vector like matter to be present. Such vector like matter is typically introduced to generate correct electroweak symmetry breaking while incorporating NMSSM in minimal mediation models \[40\]. In the present case, it is motivated from anomaly cancellation requirements. It should be noted that this kind of model has been considered earlier by the authors of Ref. \[41\]. Ours is a more explicit realisation of it in the sense that we have taken care of U(1) charges and anomaly cancellation conditions. Furthermore, we have performed a more detailed analysis of the Higgs masses in the light of 125 GeV Higgs discovery.

We found that it is possible to find an appropriate set of rational U(1) charges which satisfy the anomaly cancellation conditions as well as allow the correct set of terms in the superpotential. Electroweak symmetry breaking is possible as the U(1) charged singlet can achieve a reasonable vacuum expectation value (vev). Two factors contribute to the raise in the lightest CP even Higgs mass: the effective \(\mu\) term is sufficiently large \(\sim 0.5 – 1\) TeV and secondly the RG generated \(A_t\) term is large compared to minimal gauge mediation. The later is because at the 1-loop level, the \(SU(3)\) beta function, \(b_3\) is zero in this model and the 2-loop \(b_3\) is not sufficiently large. Together they result in sufficient \(X_t\) to ensure large mixing in the stop mass matrix. It is possible to find reasonable parameter space which gives correct lightest CP even Higgs mass and satisfy direct constraints from LHC as well as constraints from \(Z – Z’\) mixing.

The rest of the paper is organised as follows: In the next section particle spectrum and the model are presented. The details of supersymmetric spectrum and various constraints on the parameter space are discussed in section 3. Numerical results are presented in section 4. We close with an outlook in section 5.
II. MODEL AND THE PARTICLE SPECTRUM

The basic premise of the model is that the singlet of the NMSSM should no longer be a singlet, but instead, it is charged under an extension of the Standard Model gauge group such that it receives non-zero supersymmetry breaking contributions at the mediation scale. As it will be detailed in the next section this would help in attaining a large enough vacuum expectation value for the field ‘S’. In this present work, we try to do this by considering the simplest extension in terms of a $U(1)$. The relevant field $S$ is singlet under the Standard Model gauge group, but charged under the extra $U(1)$; as a consequence of which all the Standard Model fields are charged under the $U(1)$. The total gauge group is

$$G_{SM+A} = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_A$$ (1)

where the first three represent the usual Standard Model gauge group and the additional gauge group is represented by a subscript A. $U(1)_A$ is a chiral gauge group and hence introduces an extra set of anomalies which need to be canceled for a consistent quantum field theory. This imposes a set of conditions on the $U(1)_A$ charges; they are listed in Appendix A. We insist that the anomalies cancel independently for the NMSSM sector and the Messenger sector. It is easily verified that the MSSM particle spectrum along with the new field $S$ is not sufficient to cancel all the anomalies. In particular, from $(U(1)_A - [SU(3)_c]^2)$ anomaly condition we get

$$A_1(\text{exotics}) = -3s$$ (2)

where $A_1(\text{exotics})$ is the contribution of the new exotic fields which need to be added and $s$ is the $U(1)_A$ charge of the field $S$. The $U(1)_A$ charge $s$ cannot be zero as per our requirements. Furthermore, to generate the effective $\mu$ term ($\lambda S H_u H_d$) in the super-potential, the charge $s$ should be equal to

$$s = -(h_1 + h_2) \neq 0$$ (3)

where $h_1$ and $h_2$ are the $U(1)_A$ charges of $H_1$ and $H_2$ respectively. We thus need coloured exotics to satisfy $U(1)_A - [SU(3)_c]^2$ anomaly. The number of the exotics is fixed by other anomaly conditions as well as by the $U(1)_A$ gauge invariance of the super-potential. It turns out that one possible minimal set of exotic fields would be three families of $SU(2)_L$ singlet coloured exotics. We introduce a pair of colour fundamental and anti-fundamentals $D_i$ and $\bar{D}_i$, which are $SU(2)$ singlets, for each of the three families. In addition to the QCD interactions they are allowed to couple with the field
S in the super-potential. The total particle spectrum and their corresponding representations and
the $U(1)_A$ charges , in the order of Eq. (1) are given in the table below.

$$
\begin{align*}
Q_i : & \ (3, 2, \frac{1}{6}, q_i) & U_i^c : & \ (3, 1, -\frac{2}{3}, u_i), & D_i^c : & \ (3, 1, \frac{1}{3}, d_i), \\
L_i : & \ (1, 2, -\frac{1}{2}, l_i), & E_i^c : & \ (1, 1, 1, e_i), \\
H_1 : & \ (1, 2, -\frac{1}{2}, h_1) & H_2 : & \ (1, 2, \frac{1}{2}, h_2), & S : & \ (1, 1, 0, s), \\
D_i : & \ (3, 1, y_i, z_i) & \bar{D}_i : & \ (\bar{3}, 1, -y_i, \bar{z}_i), \\
\end{align*}
$$

where $i$ represents the generation index running from 1 to 3. In the rest of the paper, we will
consider all the $U(1)_A$ charges to be universal over all the generations and thus suppress the
generation index. The only exception to this rule is the $U(1)_A$ charges of exotics $z_i$. We will
consider them to be different for each of the generation, subject to the constraint that in each
generation, $z_i + \bar{z}_i = -s$. The super-potential is given by

$$
W = Y_E L E^c H_1 + Y_D Q D^c H_1 + Y_U Q U^c H_2 + \lambda S H_1 H_2 + \kappa_i S D_i \bar{D}_i 
$$

where $Y_E, Y_D, Y_U, \lambda, \kappa_i$ are Yukawa couplings and we have suppressed generation and colour
indices. Note that the field $S$ does not have cubic self interactions.

We will consider a minimal set of messengers communicating the effect of spontaneous sup-
ersymmetry breaking in the hidden sector. The spurion $X$ couples to the messengers with the
super-potential

$$
W = \eta X \Phi \Phi
$$

where $\Phi$ are messengers in fundamental representation of an $SU(N) \supset G_{SM+A}$ gauge group and
$\eta$ is some Yukawa coupling. The resultant soft terms can easily be generalised with the extra
$U(1)_A$ and can be verified with the wave-function methods of Refs. [51, 52]. The mass terms for
the gauginos and soft mass squared terms for the scalars at the mediation scale, $X$ are given as
follows$^2$:

$$
\begin{align*}
M_i(X) & \approx \frac{\Lambda}{16\pi^2} \sum_i \left( g_i^2(X) \right) \\
m_{ij}(X) & \approx \frac{2\Lambda^2}{(16\pi^2)^2} \sum_i \left( g_i^4(X) C_i(f) \right)
\end{align*}
$$

$^2$ In writing the formulae Eq.(7) we have suppressed the 1-loop and 2-loop functions. They are however taken in to
account in the numerical analysis
where through an abuse of notation, we have expanded the spurion as
\[ <X> = X + \theta^2 F_X \]
defined \( \Lambda = F_X / X \). \( C_i(f) \) are quadratic Casimirs for the fields \( f \) under the four gauge groups. The index \( i \) here runs over all the four gauge groups of Eq.(1). We denote the gauge coupling corresponding to \( U(1)_A \) as \( g_4 \) and we can see, the soft mass of \( S \) has the following non-zero value at the \( X \) scale:

\[ m_S^2(X) \approx 2s^2 \tilde{\alpha}_4^2(X) \Lambda^2, \tag{8} \]

where we used the standard notation of \( \tilde{\alpha}_i = \alpha_i / (4\pi) \) and \( \alpha_i = g_i^2 / (4\pi) \). Similarly, we christen \( M_4 \) to be the neutral gaugino corresponding to \( U(1)_A \) group. It’s mass is given by

\[ M_4 \approx \tilde{\alpha}_4(X) \Lambda \tag{9} \]

The presence of additional \( U(1)_A \) also introduces additional splittings between the mass squared terms at the mediation scale \( X \). For example, the slepton doublets and the Higgs which are degenerate at the high scale in Minimal case, get split as:

\[
\begin{align*}
&m_L^2(X) - m_{H_1,2}^2(X) = 2(l^2 - h_{1,2}^2) \tilde{\alpha}_4^2(X) \Lambda^2 \\
&m_{H_1}^2(X) - m_{H_2}^2(X) = 2(h_1^2 - h_2^2) \tilde{\alpha}_4^2(X) \Lambda^2
\end{align*} \tag{10}
\]

However, as we will see later the freedom of these splits is limited as the choice of \( U(1)_A \) is quite restricted due to phenomenological constraints and anomaly cancellation conditions. Finally, just as in the minimal messenger model, the trilinear \( A \)-terms and bilinear \( B \) terms remain zero at the mediation scale \( X \).

### III. WEAK SCALE SPECTRUM

The soft terms at the weak scale can be evaluated by using the relevant Renormalisation Group (RG) equations with the above boundary conditions, Eq.(7). One interesting aspect about the one loop beta functions for the gauge couplings is that the beta function of \( SU(3), b_3^{(1)} = 0 \). This is due to the presence of the additional colour triplets \( D, \bar{D} \) in three generations\(^3 \). As the \( \alpha_s \) does not run at the 1-loop level, most coloured particles receive larger corrections in RGE running, compared to the Minimal messenger model. This has consequences for the running of \( y_t \) and subsequently to all the parameters which depend on \( y_t \) or \( A_t \). We have used 1-loop RGE for the soft terms and

\(^3\) We have not explored in the present work about the possibility of making this model finite in the UV (see for example \([53]\)).
added 2-loop RGE’s for the gauge couplings and Yukawa couplings in this analysis. The relevant RGE for this model are given in Appendix [C].

Before proceeding further, a comment about kinetic mixing is in order. The U(1) gauge fields can mix through the kinetic terms of the type $\chi \int d\theta \, W^A W_Y$. The current bounds on $\chi$ limit it to $10^{-3}$ [54]. We expect that the implications on the phenomenology to be discussed in our paper will be minimally affected due to the presence of the kinetic mixing. For this reason, we will neglect all its effects in the subsequent discussion.

At the weak scale, $M_{SUSY} \sim 1\,\text{TeV}$, we impose electroweak symmetry breaking conditions along with the $U(1)_A$ breaking. The neutral Higgs scalar potential is given by

$$V_0 = V_F + V_D + V_{\text{soft}}$$

where

$$V_F = |\lambda H_2 \cdot H_1|^2 + |\lambda S|^2 \left( |H_1|^2 + |H_2|^2 \right)$$

$$V_D = \frac{(g_1^2 + g_2^2)}{8} \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{g_2^2}{2} \left( |H_1|^2 |H_2|^2 - |H_2 \cdot H_1|^2 \right) + \frac{g_2^2}{2} \left( h_1 |H_1|^2 + h_2 |H_2|^2 + s |S|^2 \right)^2$$

$$V_{\text{soft}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |S|^2 + (A_{\lambda} SH_2 \cdot H_1 + \text{h.c.})$$

The neutral components of the Higgs fields $H_1$ and $H_2$ get vacuum expectation values (VEV) at the weak scale, $\frac{v_1}{\sqrt{2}}$ and $\frac{v_2}{\sqrt{2}}$. The field $S$ also gets a VEV, $\frac{v_s}{\sqrt{2}}$ at the weak scale, breaking the $U(1)_A$ symmetry spontaneously. At the minima of the potential, the vevs and the soft terms along with the other parameters of the model get related. These minimisation conditions are given as

$$m_1^2 = -\frac{1}{2} \left[ \frac{G^2}{4} + h_1 g_4 \right] v_1^2 + \frac{1}{2} \left[ \frac{G^2}{4} - \lambda^2 - h_1 h_2 g_4 \right] v_2^2 - \frac{1}{2} \left[ \lambda^2 + h_1 s g_4 \right] v_s^2$$

$$+ \frac{A_{\lambda} v_2 v_s}{\sqrt{2}}$$

$$m_2^2 = \frac{1}{2} \left[ \frac{G^2}{4} - \lambda^2 - h_1 h_2 g_4 \right] v_1^2 - \frac{1}{2} \left[ \frac{G^2}{4} + h_2 g_4 \right] v_2^2 - \frac{1}{2} \left[ \lambda^2 + h_2 s g_4 \right] v_s^2$$

$$+ \frac{A_{\lambda} v_1 v_s}{\sqrt{2}}$$

$$m_3^2 = -\frac{1}{2} \left[ \lambda^2 + h_1 s g_4 \right] v_1^2 - \frac{1}{2} \left[ \lambda^2 + h_2 s g_4 \right] v_2^2 - \frac{1}{2} s g_4^2 v_s^2 + \frac{A_{\lambda} v_1 v_2}{\sqrt{2}} v_s$$

where $G^2 = g_1^2 + g_2^2$. The minimisation conditions are modified compared to the standard NMSSM case due to the presence of terms proportional to $g_4$. Subsequently, we can see from Eq. [18], that
in the limit $v_s \gg v_1, v_2$ ($v_s$ is required to be large which is discussed later in this section), we have

$$v_s^2 \approx -\frac{2}{s^2 g_4^2},$$

which is the typical vev one expects in extra U(1) models \cite{41,55}. At the high scale, $X, m_S^2$ which is positive and proportional to $\alpha^2_1 A^2$ can be driven negative at the electroweak scale by the Yukawa couplings of the exotics $k_1, k_2, k_3$.

This should be contrasted with the vev in minimal gauge mediation, without the $U(1)$ factor. See for example, Refs.\cite{39,56}. From the minimization conditions of NMSSM, we get

$$v_s^2 \approx -\frac{1}{2\kappa^2} \left( \lambda^2 (v_1^2 + v_2^2) + 2m_s^2 - 2\lambda\kappa v_1 v_2 \right)$$

(19)

which is too small to get $\mu_{eff} (\frac{\lambda v_s}{\sqrt{2}})$ of the order of electroweak symmetry breaking. To achieve a significant value either $\lambda$ has to be very large ($> 1$) or $\kappa$ has to be too small. In both the cases, achieving electroweak symmetry breaking is highly constrained \cite{57}. We now turn our attention to the Higgs sector. The CP-even tree-level Higgs mass squared matrix, $\Psi^\dagger M_+^2 \Psi$, where $\Psi^T = \{ H_1^0, H_2^0, S \}$, and the elements of the matrix are given as:

$$
(M^0_+)_{11} = \left[ \frac{G^2}{4} + h_1^2 g_4^2 \right] v_1^2 + \frac{A_\lambda v_2 v_s}{\sqrt{2}} v_1 \\
(M^0_+)_{12} = -\left[ \frac{G^2}{4} - \lambda^2 - h_1 h_2 g_4^2 \right] v_1 v_2 - \frac{A_\lambda}{\sqrt{2}} v_2 \\
(M^0_+)_{13} = \left[ \lambda^2 + h_1 s g_4^2 \right] v_1 v_s - \frac{A_\lambda}{\sqrt{2}} v_2 \\
(M^0_+)_{22} = \left[ \frac{G^2}{4} + h_2^2 g_4^2 \right] v_2^2 + \frac{A_\lambda v_1 v_s}{\sqrt{2}} v_2 \\
(M^0_+)_{23} = \left[ \lambda^2 + h_2 s g_4^2 \right] v_2 v_s - \frac{A_\lambda}{\sqrt{2}} v_1 \\
(M^0_+)_{33} = s^2 g_4^2 v_s^2 + \frac{A_\lambda v_1 v_2}{\sqrt{2}} v_s
$$

(20)

Given that the physical Higgs spectrum should be non-tachyonic, we can derive constraints on the parameter space of the model. Firstly the sign of the determinant of the matrix, in the limit $v_s \gg v_{1,2}$ is crucially dependent on the sign of the $A_\lambda$. This is obvious, by considering the full determinant of the $3 \times 3$ mass matrix, which is given by

$$Det[(M^0_+)^2] \approx \frac{A_\lambda v_s^3}{4\sqrt{2} v_1 v_2} \left[ G^2 g_4^2 s^2 (v_1^2 - v_2^2)^2 + 4 (g_1^2 h_1^2 s^2 v_1^4 - (l_1^4 + 2 g_1^2 l_2^2 (h_2 - s)) s + g_1^4 h_2^2 (2h_1 + h_2) s^2) v_1 v_2 v_s + g_1^4 h_2^2 s^2 v_2^2 \right]$$

For $A_\lambda > 0$, the region in which the sign of the determinant of the Higgs mass matrix changes is plotted in $\lambda, g_4$ plane by taking $h_1 = -\frac{1}{2}, h_2 = -\frac{5}{2}, s = 3$, and $\tan \beta = 10$. Electroweak symmetry
FIG. 1: The determinant of the CP even Higgs mass matrix is shown as a function of $g_4$ and $\lambda$. In the shaded region, the determinant is negative, thus electroweak symmetry breaking is not possible. The $U(1)$ charges used are presented in Table I and $\tan\beta$ is chosen to be 10.

breaking is not possible for the shaded region ($Det < 0$) in the parameter space. From the figure, it is seen that for $g_4 \lesssim 0.1$, large values of $\lambda \gtrsim 0.6$ are disfavoured as they do not allow electroweak symmetry breaking.

The question then arises, whether $A_\lambda > 0$? Typically the $A$ terms are negative due to the RG running from the high scale. However, in this case, $A_\lambda$ turns out to be $\mathcal{O}(10)$ and positive at the weak scale. This positive $A_\lambda$ ensures us a safe electroweak vacuum. This is shown in the left panel of Figure 2, where we have plotted $A_\lambda$ with respect to running scale. As we see from the figure 2, $A_\lambda$ initially turns negative and then increases turning positive at the weak scale. This happens because of the complicated coupling between $A_t$ and $A_\lambda$ RGE. The RGE of these parameters are presented in the Appendix C along with the other parameters. In the below, we reproduce them:

$$\frac{dA_t}{dt} \approx \frac{y_t}{16\pi^2} \left[ 2y_b A_b + A_\lambda \lambda + \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{9} g_1^2 M_1 + 4(q^2 + u^2 + h_2^2) g_3^2 M_4 \right]$$

$$\frac{dA_\lambda}{dt} \approx \frac{\lambda}{16\pi^2} \left[ 6y_tA_t + 6y_b A_b + 2A_\tau y_\tau + 6(A_{k_1} k_1 + A_{k_2} k_2 + A_{k_3} k_3) + 6g_2^2 M_2 + 2g_1^2 M_1 + 4(s^2 + h_2^2 + h_1^2) g_3^2 M_4 \right]$$

Compared to the minimal gauge mediated models, the running effects on the parameter $A_t$ are very large as $\alpha_3$ barely runs in this models. As mentioned above, $b_3 = 0$ at 1-loop and is very small, at the 2-loop. For this reason, after the SUSY threshold $M_S \sim 1$TeV, $\alpha_s$ barely runs all the
way to the mediation scale. Due to this $Y_t$ and $A_t$ receive comparatively large corrections due to the relatively large $\alpha_s$. Additional corrections from $g_4, k_i$ and $A_{k_i}$ also contribute in the running of the $A_\lambda$. This feeds into $A_\lambda$, making it positive at the weak scale. In the right panel of the Fig [2], we show the running of the $A_t$ for the same parameters.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{\(A_\lambda\) and \(A_t\) are plotted as a function of the energy scale, where free parameters are fixed as \(\lambda = 0.394, g_4 = 0.137, k_1 = 0.016, k_2 = 1.07, k_3 = 0.117, \tan \beta = 3.7\)}
\end{figure}

Let us focus our attention to the lightest Higgs mass eigenvalue. The matrix Eq.(20) gives an upper bound on the tree level lightest Higgs mass. In the present model, it has additional contribution from $\lambda$ and $g_4$ which is given as

\[
m_{h_0}^2 \leq M_Z^2 \left[ \cos 2\beta^2 + \frac{\lambda^2}{2g^2} \sin 2\beta^2 + \frac{g_4^2}{g^2} (h_1 + h_2 + (h_1 - h_2) \cos 2\beta)^2 \right]
\]

(21)

In the NMSSM, it is well known that the tree level contribution can be appreciably enhanced from the MSSM tree level values only for large values of $\lambda \gtrsim 0.7$. The above bound is thus saturated only for special values of the parameters. For most of the parameter space, however the actual eigenvalue is far below the above bound. As in MSSM, one loop corrections would play a major role.

The total number of parameters are $\Lambda$, $M_X$, $g_4$, $\lambda$ and the $U(1)$ charges. Before proceeding to present the numerical results, we discuss the possible constraints on the various parameters. The first constraint we discuss is from the neutral gauge boson mixing. The neutral gauge bosons $Z$ and $Z'$ mix with their mass matrix given by $\mathbf{L} \supset \chi^T \mathcal{M}_{Z'Z}^2 \chi$ where $\chi^T = \{Z', Z\}$ with

\[
\mathcal{M}_{Z'Z}^2 = \begin{pmatrix}
M_{Z'Z}^2 & M_{Z'Z}^2 \\
M_{Z'Z}^2 & M_{ZZ}^2
\end{pmatrix}
\]

(22)
where
\[ M_{Z'}^2 = g_4^2 (h_1^2 v_1^2 + h_2^2 v_2^2 + s^2 v_s^2), \]
\[ M_{ZZ'}^2 = g_4 \sqrt{g_1^2 + g_2^2} (v_1^2 h_1 - v_2^2 h_2), \]
\[ M_{ZZ}^2 = \frac{(g_1^2 + g_2^2) (v_1^2 + v_2^2)}{4}. \] (23)

The mixing of the matrix is given by
\[ \Theta_{ZZ'} = \frac{1}{2} \tan^{-1} \left( \frac{2M_{ZZ'}^2}{M_{ZZ}^2 - M_{Z'}^2} \right). \] (24)

The current limits on \( M_{Z'} \) require it to be greater than 1 TeV [58]. For \( g_4 \sim g_1 \), these limits already push \( v_s \) to be much larger than 1 TeV. \( \Theta_{ZZ'} \) is constrained by electroweak precision data, it should be less than \( O(10^{-3}) \) [54]. As \( v_s \) is already very heavy with \( M'_Z \) of a mass of TeV order, the constraint on mixing angle is avoided easily.

A second constraint comes from the mass spectrum of the scalar superpartners. The D-terms due to the new \( U(1)_A \) group play an important role in determining the sfermion mass spectrum due to the large vev of the \( S \) field. The strongest effects are felt in the stau mass matrix which is given as:
\[ M_{\tilde{\tau}}^2 = \begin{pmatrix} m_{\tilde{\tau}}^2 + D_L & \frac{1}{\sqrt{2}} (A_\tau v_1 - \mu y_{\tau} v_2) \\ \frac{1}{\sqrt{2}} (A_\tau v_1 - \mu y_{\tau} v_2) & m_{\tilde{e}_3}^2 + m_{\tau}^2 + D_e \end{pmatrix}, \] (25)

where
\[ D_L = \frac{1}{8} (v_1^2 - v_2^2) (-g_2^2 + g_1^2) + \frac{1}{2} g_4^2 l (h_1 v_1^2 + h_2 v_2^2 + s v_s^2) \] (26)
\[ D_e = \frac{1}{4} (v_1^2 - v_2^2) g_1^2 + \frac{1}{2} g_4^2 e (h_1 v_1^2 + h_2 v_2^2 + s v_s^2). \] (27)

Notice that for the \( D_L \) and \( D_e \) to have positive values, the products of the \( U(1)_A \) charges, \( ls \) and \( es \) should always be positive. This is because unlike \( m_Q^2, m_{\tilde{u}}^2 \) and \( m_{\tilde{d}}^2 \), the value of \( m_{\tilde{L}}^2 \) at electroweak scale due to running is very low, as it should be, owing to the fact that \( y_\tau \ll y_t \). So the sign of the diagonal terms in the stau mass squared matrix depends on the \( D_L \) and \( D_e \) which in turn depends on the dominant term \( l s g_4^2 v_s^2 \). If we choose \( U(1)_A \) charges \( l \) and \( s \) of different signs we expect tachyonic masses for stau’s.

The chargino mass matrix remains unaltered compared to the MSSM whereas the neutralino mass matrix is now expanded to include the neutral gauging of \( U(1)_A \) as well as the fermionic partner of the \( S \) field. Note that the fermionic partner of the \( S \) is not exactly the singlino as it carries a \( U(1)_A \) charge unlike the NMSSM case. To summarise the constraints, we have:
TABLE I: $U(1)_A$ charges of the fields

| q  | u  | d  | 1  | e  | h1 | h2 | s  | z1 | z2 | z3 | ~z1 | ~z2 | ~z3 |
|----|----|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| 1/6| 1/3| 1/3| 1/2| 2  | -5/2| 1/2| 3  | -3 | -1 | -1 | 0   | -2  | -2  |

- For consistent electroweak breaking: we need, $\lambda = \sqrt{2} \frac{\mu_{eff}}{v_s}$ and $A_\lambda > 0$. So $\lambda$ cannot be arbitrarily large for a given $g_4$ which is evident from the Figure [1]

- From $Z - Z'$ mixing: we require that $v_s \sim O(\text{TeV}) \gg v_1, v_2$

- Sfermion masses: From the $D$-terms of the sfermion mass matrices, we require that $U(1)_A$ charges $l$ and $s$ should have opposite signs

- Landau pole: the new gauge coupling

$$g_4 < 2\pi \sqrt{\frac{2}{b_4 \log \frac{M_X}{M_Z}}}.$$  

$$g_4 \approx 0.28 \text{ for } b_4 = 145 \text{ and } M_X = 100 \text{ TeV}$$

IV. NUMERICAL RESULTS

To compute the sparticle spectrum at the weak scale, we use a modified version of the publicly available code SuSeFLAV [59] with 2-loop RGE for the gauge couplings and the Yukawa couplings. The RGE for the rest of the soft parameters are evaluated at the 1-loop level. For the Higgs spectrum, we compute the full 1-loop effective potential corrections presented in Appendix D. These corrections come from stop-top loop and the exotic quarks loop. Stop-top loop correction is the dominant contributor to the Higgs mass at the one loop. The correction due the exotic quarks is significant. It changes Higgs mass by few percent and we have checked that it is possible to get Higgs mass of 125 GeV by adding both the corrections, although we have not considered exotic quarks loop correction to the Higgs mass in the numerical analysis. The free parameters are $\Lambda, \tan \beta, \lambda, g_4, k_1, k_2$ and $k_3$. These are randomly fixed at the low energy scale, for each set of these parameters, using RGEs we obtain corresponding values at the GMSB scale $X \simeq \Lambda$. Now along with the boundary conditions for the soft masses and A-terms, the same parameters are run down to the electroweak scale to check whether they satisfy minimization conditions given in section (2) and other constraints presented in section (3). This process is repeated several times to obtain a parameter space which satisfy electroweak symmetry breaking conditions. Subsequent to
this, we impose phenomenological constraints from direct SUSY searches at LHC \[60, 61\] as well as the flavour constraints from \(b \to s + \gamma\) and \(b \to s + \mu^+ \mu^-\).

In the numerical analysis, we fix the \(U(1)_A\) charges to be as given in Table \[1\] It should be noted that these are not the only solutions available from anomaly cancellation conditions. A list of five solutions is presented in Appendix \[A\]. Of the remaining parameters, we have fixed \(\tan \beta = 10\) and varied the remaining parameters within a range presented in Table \[IV\].

| Parameter | Range |
|-----------|-------|
| \(\Lambda\) | \(1 \times 10^5 - 5 \times 10^7[GeV]\) |
| \(g_4\) | \(0.01 - 2.5\) |
| \(\lambda\) | \(0.1 - 0.9\) |
| \(\kappa_1\) | \(0.1 - 0.9\) |
| \(\kappa_2\) | \(0.1 - 0.9\) |
| \(\kappa_3\) | \(0.1 - 0.9\) |

Instead of presenting the results in terms of regions of allowed parameter space, we present the correlations of the parameters with the lightest CP even Higgs boson mass. In Fig. \(3\), we present the correlation of the light Higgs mass with respect to the \(A_t\) generated at the weak scale. The left panel presents the total Higgs mass whereas the right panel shows the 1-loop correction to the light Higgs mass. As expected we see that as \(|A_t|\) increases, the 1-loop correction to the Higgs mass increases so does the total mass. It is also surprising to see larger values for \(A_t \sim 900\ GeV\) possible in this case and accordingly the higher values for Higgs mass \(\sim 140\ GeV\). Of course, the heavier Higgs masses correspond to heavier stops. Note that we have considered only dominant 1-loop corrections to the light Higgs mass. Two loop contributions \[62\] can be important and they would give a more precise number for the light Higgs mass. However, it is clear that one can easily achieve a light Higgs mass of \(\mathcal{O}(125)\) GeV.

In Fig. \(4\), we present the correlation between \(m_h\) and \(\lambda\) in the left panel and \(m_h\) and \(g_4\) in the right panel. We find a surprising relation between \(\lambda\) and \(m_h\). The Higgs mass seems to be lower for higher values of \(\lambda\). This is contrary to expectations based on NMSSM. This is because for higher values of \(\lambda\) achieving electroweak symmetry breaking becomes harder. Similarly, larger values of \(\lambda\) typically mean lighter values of \(v_s\). Similarly, larger values of \(g_4\) are not preferred by the data as they can lead to Landau poles. This can be seen from the right panel of Fig.\(4\). Thus, the regular NMSSM like enhancement of the tree level Higgs mass is not possible in this model.
FIG. 3: Higgs mass, including one-loop correction, and only one loop correction are plotted against $A_t$. The U(1) charges are taken from Table I.

FIG. 4: Higgs mass, including one-loop correction is plotted against $\lambda$ and $g_4$

From the allowed parameter space, we now present a representative point, Point(A) which give the lightest Higgs mass to be around 125 GeV. In this point, the next to lightest supersymmetry particle (NLSP) is the A-ino, the supersymmetric partner for the extra $U(1)_A$ gauge boson.

Point (A):
The various parameters for this point are: $v_s = 2225.53$ GeV, $\tan(\beta) = 3.26$, $\lambda = 0.3439$, $g_4 = 0.1198$, $M_X = 194.22$ TeV, $\Lambda = 97.112$ TeV, $\kappa_1 = 0.1368$, $\kappa_2 = 0.7865$, $\kappa_3 = 0.7813$
V. OUTLOOK

The discovery of a Higgs boson at 125 GeV has led to strong constraints on the gauge mediated supersymmetry breaking models. Most of the present models have concentrated on generating the required large trilinear $A_t$ coupling through messenger matter interactions. In the present work, we tried a different approach of combining the ideas of an extra $U(1)$ factor and NMSSM like models. Anomaly cancellation requirement automatically determines the extra particle spectrum of the model. The coloured particles barely run in this model from the weak scale to the mediation scale due to the small value of the strong beta function. This ‘stagnation’ of $\alpha_s$ between $M_{SUSY}$ and $M_{mess}$ and the presence of additional $U(1)$ couplings helps for a larger value of the $A_t$ at the $M_{SUSY}$ even though one starts with zero at the mediation scale. Together with a reasonable value for the $\mu_{eft} = \lambda v_s$, this generates the required $X_t$ at the weak scale for the light stops.

While we have focussed on getting the right Higgs mass, the rest of the spectrum of the model is also quite interesting. There are heavy exotic coloured particles, new neutralinos which are combinations of the Standard Model singlino and the fermion of the $U(1)_A$ gauge boson. The lightest neutralino is still the LSP and could be the dark matter candidate. A study of collider signatures and dark matter issues could be interesting and will be pursued in a future work.

Finally, we have not concentrated on the issue of fine tuning in this model. Though we have not explicitly measured it, it is expected that it could be large as long as $M_X$ and $\Lambda$ are close as we have chosen. A reasonable separation between the scales can perhaps reduce the fine tuning (see for example, discussion in [63]).
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Appendix A: Anomaly Conditions

In the following we present the anomaly cancellation conditions and U(1) charges which are solutions to them. More elaborate analysis of anomaly cancellations pertinent to U(1) extensions of MSSM has been presented in [64]. To begin with, the $U(1)_A$ gauge invariance of the superpotential Eq.(5) leads to the below equations which should be satisfied by the $U(1)_A$ charges.

\begin{align*}
h_1 + q + d &= 0 \quad (A1) \\
h_2 + q + u &= 0 \quad (A2) \\
h_1 + l + e &= 0 \quad (A3) \\
s + h_1 + h_2 &= 0 \quad (A4)
\end{align*}

In addition, the following five anomaly cancellation conditions should also be satisfied.

\begin{align*}
A_1 & : U(1)_A - [SU(3)_C]^2 \\
A_2 & : U(1)_A - [SU(2)_L]^2 \\
A_3 & : U(1)_A - [U(1)_{Y}]^2 \\
A_4 & : U(1)_Y - [U(1)_A]^2 \\
A_5 & : U(1)_A^3
\end{align*}

In the following, we analyse each of these conditions and the corresponding solutions for $U(1)_A$ charges.

a. Anomaly $A_1(U(1)_A - [SU(3)_C]^2)$

\begin{align*}
3(2q + u + d) + A_1(\text{exotics}) &= 0 \quad (A5)
\end{align*}
Here first term is the contribution from three generations of the quarks in the MSSM without considering the exotic $D, \bar{D}$ quarks presented in section (1). We can show in the limit $A_1(exotics) = 0$, the $S$ field $U(1)_A$ would go to zero. This can be easily seen by considering the combination of the equations: Eq. (A5) - 3 Eq. (A2) - 3 Eq. (A1) + 3 Eq. (A4), gives us

$$A_1(exotics) = -3s$$

We assume that the exotics are triplets and anti triplets of $SU(3)_c$ with equal and opposite $U(1)_Y$ hypercharges $\pm y_i$. Eq. (A5) now becomes

$$3(2q + u + d) + \Sigma_i(z_i + \bar{z}_i) = 0$$

where $z_i$ are the $U(1)_A$ charges of the exotics. The coupling between the exotic vector like quarks the singlet is allowed under $U(1)_A$ symmetry which gives

$$s + z_i + \bar{z}_i = 0$$

Finally, to derive the number of families of exotic quarks one should add, consider the combination Eq. (A7) - 3 Eq. (A2) - 3 Eq. (A1) + 3 Eq. (A4) - $\Sigma_i$ Eq. (A1). We have $(3 - N_k)s = 0$, where $N_k$ is the number of exotic families which ends up being equal to three.

b. Anomaly $A_2(U(1)_A - [SU(2)_L]^2)$

The constraint here is given as

$$9q + 3l + h_1 + h_2 = 0$$

From Eqs. (A1), (A2), (A3), (A4) and (A9) we have 5 constraints. Without the $U(1)_A$ charges of the exotics, we have eight unknowns. Using the constraints, a general solution can be written in terms of $l, h_1, s$ as

$$
\begin{pmatrix}
    q \\
    u \\
    d \\
    e \\
    h_2
\end{pmatrix} = \frac{l}{3} + h_1 - 1 + \frac{s}{9} - 1
\begin{pmatrix}
    -1 \\
    1 \\
    1 \\
    -3 \\
    0
\end{pmatrix}
\begin{pmatrix}
    0 \\
    1 \\
    8 \\
    -1 \\
    0
\end{pmatrix}
\begin{pmatrix}
    1 \\
    -1 \\
    -1 \\
    0 \\
    -9
\end{pmatrix}

(A10)
c. Anomaly $A_3(U(1)_A - [U(1)_Y]^2)$

This anomaly condition puts constraints on the hypercharges of the exotic fields. The anomaly condition is given by

$$q + 8u + 2d + 3l - 6e + h_1 + h_2 - 6s\Sigma_i y_i^2 = 0$$  \hspace{0.5cm} (A11)

By taking the combination of Eqs. (A11) + (A9) - 8 (A2) + 2 (A1) - 6 (A3) + 6 (A4), we get

$$\Sigma_i y_i^2 = 1$$  \hspace{0.5cm} (A12)

which has several solutions. In the present work, we choose $y_i = \{-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\}$

d. Anomalies $A_4(U(1)_Y - [U(1)_A]^2)$ and $A_5[U(1)_A]^3$

The final two anomalies do not have simple algebraic solutions. These are given as $A_4$:

$$3q^2 - 6u^2 + 3d^2 - 3l^2 + 3e^2 - h_1^2 + h_2^2 + 3 \Sigma_i y_i(z_i^2 - \bar{z}_i^2) = 0$$  \hspace{0.5cm} (A13)

$A_5$:

$$18q^3 + 9u^3 + 9d^3 + 6l^3 + 3e^3 + 2h_1^3 + 2h_2^3 + s^3 + 3 \Sigma_i(z_i^3 + \bar{z}_i^3) = 0$$  \hspace{0.5cm} (A14)

We looked for integer solutions for the $U(1)_A$ charges. We could not find any as long as the charges are restricted to lie below 10. We then resorted to rational charges. There are several solutions which have been found. In Table III, we present five sample solutions which satisfy the anomaly conditions as well as the superpotential requirements. In addition to this set of charges, one can also find sets where all the $z_i$ and $\bar{z}_i$ are equal. It should also be noted that each of the set of the charges has a completely different phenomenology. This is because the charges decide the $U(1)_A$ one loop beta function, $b_4$, which could vary drastically. This in turn modifies the values of $\lambda$ and $\kappa_i$ allowed and their respective ranges.

**Appendix B: One loop corrections to the CP even Higgs mass matrix**

In the following we present the one loop corrections to the CP even Higgs mass matrix. There are two main contributions, one from the stop-top sector and the second one from from the vector like exotic quarks. To derive the one loop corrections, we use the well known effective potential
methods. The one loop effective potential is given by \[ V^1 = \frac{3}{32\pi^2} \left[ \sum_{j=1}^{2} m_{f_j}^4 \left( \ln \frac{m_{f_j}^2}{Q^2} - \frac{3}{2} \right) - 2\tilde{m}_f^4 \left( \ln \frac{\tilde{m}_f^2}{Q^2} - \frac{3}{2} \right) \right] . \] (B1)

where \( m_{f_1,2}^2 \) are the eigenvalues of the field dependent sfermion mass matrix. \( \tilde{m}_f \) is the corresponding fermion mass.

The corrections to the CP even mass matrices can be written as

\[
(M^1_+)_{ij} = \left. \frac{\partial^2 V^1}{\partial \phi_i \partial \phi_j} \right|_0 - \delta_{ij} \frac{1}{v} \left. \frac{\partial V^1}{\partial \phi_i} \right|_0 \] (B2)

By denoting

\[
\frac{\partial^2 m_{f_i}^2}{\partial \phi_i \partial \phi_j} = A'_{ij} \pm A_{ij} \quad \frac{\partial m_{f_i}^2}{\partial \phi_i} = B'_i \pm B_i
\]

mass matrix can be written as

\[
(M^1_+)_{ij} = 2k \left[ F_j (A'_{ij} - \frac{\delta_{ij}}{H_j} B'_j) + G_j (A_{ij} - \frac{\delta_{ij}}{\bar{\phi}_j} B_j) + \frac{\partial}{\partial \phi_i} (B'_i B'_j + B_i B_j) + \frac{\partial}{\partial \phi_i} (B'_i B_j + B_i B'_j) - 8 H_f y^2 \langle \phi \rangle^2 \right] \] (B3)

where

\[
F_j = -(m_{f_2}^2 + m_{f_1}^2) + (m_{f_2}^2 \log \frac{m_{f_2}^2}{Q^2} + m_{f_1}^2 \log \frac{m_{f_1}^2}{Q^2})
\]

\[
G_j = (m_{f_2}^2 - m_{f_1}^2) + (m_{f_2}^2 \log \frac{m_{f_2}^2}{Q^2} - m_{f_1}^2 \log \frac{m_{f_1}^2}{Q^2})
\]

\[
\frac{\partial}{\partial \phi_i} = \log \frac{m_{f_i}^2}{Q^2}
\]

\[
H_f = \log \frac{m_f^2}{Q^2}
\]
and \( k = \frac{3}{32\pi^2} \)

To include corrections to the Higgs mass matrix from the stop-top loop and all the three exotic quarks, we need to calculate \( B_{ij} \) in each case separately and add them. We have presented below corrections from the stop-top loop and one exotic quark.

1. Top-Stop correction

Dominant one loop correction to the Higgs mass matrix comes from the top and stop loop. The stop mass squared matrix is given as

\[
\mathcal{M}_t^2 = \begin{pmatrix} M_{\tilde{Q}}^2 + y_t^2|H_2|^2 & X_t \\ (X_t)^\dagger & M_{\tilde{U}}^2 + y_t^2|H_2|^2 \end{pmatrix},
\]

where \( X_t = (A_t H_2 - \mu_{\text{eff}} H_1 y_t) \) and \( m_t = y_t H_2 \)

\[
\begin{align*}
A_{11} &= \mu_{\text{eff}}^2 y_t^2 \left[ \frac{2}{m_{t_2}^2 - m_{t_1}^2} - \frac{8X_t^2}{m_{t_3}^2 - m_{t_1}^2} \right] \\
A_{12} &= -\mu_{\text{eff}} y_t A_t \left[ \frac{2}{m_{t_2}^2 - m_{t_1}^2} - \frac{8X_t^2}{m_{t_3}^2 - m_{t_1}^2} \right] \\
A_{13} &= \left[ \frac{-2A_t H_2 \lambda y_t}{m_{t_2}^2 - m_{t_1}^2} - \frac{8X_t^2 \mu_{\text{eff}} \lambda H_1 y_t^3}{m_{t_3}^2 - m_{t_1}^2} \right] \\
A_{22} &= A_t^2 \left[ \frac{2}{m_{t_2}^2 - m_{t_1}^2} - \frac{8X_t^2}{m_{t_3}^2 - m_{t_1}^2} \right] \\
A_{23} &= -\mu_{\text{eff}} y_t A_t \left[ \frac{2}{m_{t_2}^2 - m_{t_1}^2} - \frac{8X_t^2}{m_{t_3}^2 - m_{t_1}^2} \right] \\
A_{33} &= \lambda^2 y_t^2 H_1^2 \left[ \frac{2}{m_{t_2}^2 - m_{t_1}^2} - \frac{8X_t^2}{m_{t_3}^2 - m_{t_1}^2} \right] \\
A_{i2}' &= \delta_{i2} 2y_t^2 \\
B_1 &= -2X_t \mu_{\text{eff}} y_t \frac{m_{t_2}^2 - m_{t_1}^2}{m_{t_3}^2 - m_{t_1}^2} \\
B_2 &= 2X_t A_t \frac{m_{t_2}^2 - m_{t_1}^2}{m_{t_3}^2 - m_{t_1}^2} \\
B_3 &= -2X_t \lambda H_1 y_t \frac{m_{t_2}^2 - m_{t_1}^2}{m_{t_3}^2 - m_{t_1}^2} \\
B_{i}' &= \delta_{i2} = 2y_t^2 H_2
\]

\[\text{Page 20}\]
2. Correction due to Exotic quarks

The one loop correction due to the exotic quarks changes Higgs mass by few percent. The exotic quark mass matrix given by

\[
M^2_{\tilde{D}_i} = \begin{pmatrix}
M^2_{\tilde{D}_i} + k_i^2 |S|^2 & X_{d_i} \\
(X_{d_i})^\dagger & M^2_{\tilde{D}_i} + k_i^2 |S|^2
\end{pmatrix},
\]

where \(X_{d_i} = (A_k S - \lambda k_i H_1 H_2)\) and \(m_{\tilde{D}_i} = k_i S\)

\[
A_{11} = (\lambda k_i H_2)^2 \left[ \frac{2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} - \frac{8X_{d_i}^2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} \right]
\]

\[
A_{22} = (\lambda k_i H_1)^2 \left[ \frac{2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} - \frac{8X_{d_i}^2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} \right]
\]

\[
A_{33} = A_{k_i}^2 \left[ \frac{2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} - \frac{8X_{d_i}^2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} \right]
\]

\[
A_{12} = \left[ \frac{\lambda^2 k_i^2 H_1 H_2 - 2\lambda k_i A_k S}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} - \frac{2X_{d_i}^2 \lambda^2 k_i^2 H_1 H_2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} \right]
\]

\[
A_{13} = \lambda k_i H_2 A_k \left[ \frac{2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} - \frac{8X_{d_i}^2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} \right]
\]

\[
A_{23} = \lambda k_i H_1 A_k \left[ \frac{2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} - \frac{8X_{d_i}^2}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}} \right]
\]

\[
A_{i3}' = \delta_{i3} 2k_i^2
\]

\[
B_1 = -2\lambda k_i H_2 \frac{X_{d_i}}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}}
\]

\[
B_2 = -2\lambda k_i H_1 \frac{X_{d_i}}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}}
\]

\[
B_3 = 2A_k \frac{X_{d_i}}{m^2_{\tilde{D}_2} - m^2_{\tilde{D}_1}}
\]

\[
B_3' = \delta_{i3} 2k_i^2 S
\]

Appendix C: RG Equations

In the last section of the appendix we present the renormalisation equations for the various superpotential and gauge parameters as well as soft terms. To derive the formulae we use the standard formulae available in the literature\[62, 66\]. The notation we use is \(t = \text{Log}(\frac{\mu}{M_{\text{Susy}}})\).
\[
\frac{dg_i}{dt} = \frac{1}{16\pi^2} \beta_i^{(1)} + \frac{1}{(16\pi^2)^2} \beta_i^{(2)}
\]
\[
\frac{dy_j}{dt} = \frac{y_i}{16\pi^2} \gamma_i^{(1)} + \frac{y_i}{(16\pi^2)^2} \gamma_i^{(2)}
\]
\[
\beta_a^{(1)} = b_a g_a^3,
\]

where \( b_a = \{17, 1, 0\} \) and \( b4 = 18q^2 + 6l^2 + 9(u^2 + d^2) + 3e^2 + s^2 + 2(h_1^2 + h_2^2) + 3(z_1^2 + z_2^2 + z_3^2 + (s + z_1)^2 + (s + z_2)^2 + (s + z_3)^2) \),

\[
\beta_1^{(2)} = 4g_1^3 \left( \frac{287}{36} g_1^2 + \frac{9}{4} g_2^2 + \frac{46}{3} g_3^2 + (q^2/2 + d^2 + 4u^2 + 3l^2/2 + (h_1^2 + h_2^2)/2 + 3e^2
\right.
\]
\[
+ \frac{1}{3} (z_1^2 + (s + z_1)^2 + 4(z_2^2 + z_3^2 + (s + z_2)^2 + (s + z_3)^2)) - \frac{1}{4} \left( \frac{26}{3} g_1^2 + \frac{14}{3} g_2^2 + 6g_7^2
\right.
\]
\[
+ 2\lambda^2 + \frac{4}{3} k_1^2 + \frac{16}{3} (k_2^2 + k_3^2) \right)
\]
\[
\beta_2^{(2)} = 4g_2^5 + g_2^3 (3g_1^2 + 4g_2^2 + 24g_3^2 + g_4^2 (18q^2 + 6l^2 + 4(h_1^2 + h_2^2)) - 6(y_1^2 + y_2^2) - 2(y_1^2 + \lambda^2))
\]
\[
\beta_3^{(2)} = -54g_3^5 + 4g_3^3 \left( \frac{47}{12} g_1^2 + \frac{9}{4} g_2^2 + \frac{21}{2} g_3^2 + g_4^2 (3q^2 + \frac{3}{2} (u^2 + d^2) + \frac{1}{2} (z_1^2 + z_2^2
\right.
\]
\[
+ z_3^2 + (s + z_1)^2 + (s + z_2)^2 + (s + z_3)^2) - \frac{1}{4} (y_1^2 + y_2^2) - \frac{4}{3} \lambda^2 - 3(k_1^2 + k_2^2 + k_3^2) \right)
\]
\[
\beta_4^{(2)} = 4g_4^3 \left( g_1^2 \left( \frac{g_2^2}{2} + 4u^2 + d^2 + \frac{3l^2}{2} + 3e^2 + \frac{1}{2} (h_1^2 + h_2^2) + \frac{3}{9} (z_1^2 + (s + z_1)^2 + 4(z_2^2 + z_3^2 + (s + z_2)^2
\right.
\]
\[
+ (s + z_3)^2 + 9}(q^2 + u^2 + h_1^2)^2 + 4z_1^2 + z_2^2 + z_3^2 + (s + z_1)^2
\right.
\]
\[
+ (s + z_2)^2 + (s + z_3)^2) + g_2^2 (24q^2 + 12(u^2 + d^2) + 4(z_1^2 + z_2^2 + z_3^2 + (s + z_1)^2
\right.
\]
\[
+ (s + z_2)^2 + (s + z_3)^2) + g_3^2 (18q^2 + 9(u^4 + d^4) + 6l^4 + 3e^4 + 2(h_1^2 + h_2^2) + s^4 + 3(z_1^4 + z_2^4 + z_3^4
\right.
\]
\[
+ (s + z_1)^4 + (s + z_2)^4 + (s + z_3)^4) - \frac{1}{4} (12y_1^2 (q^2 + u^2 + h_1^2)) + 12y_2^2 (q^2 + d^2 + h_3^2) + 4y_3^2 (l^2 + e^2 + h_1^2
\right.
\]
\[
+ 4\lambda^2 (s^2 + h_1^2 + h_2^2) + 6k_1^2 (s^2 + z_1^2 + (s + z_1)^2) + 6k_2^2 (s^2 + z_2^2 + (s + z_2)^2) + 6k_3^2 (s^2 + z_3^2 + (s + z_3)^2))\right)
\]
\[
\gamma_l^{(1)} = \left[ \lambda^2 + 6y_1^2 + y_2^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{9} g_1^2 - 2g_7^2 (q^2 + u^2 + h_2^2) \right]
\]
\[
\gamma_b^{(1)} = \left[ \lambda^2 + 6y_1^2 + y_2^2 + y_7^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{9} g_1^2 - 2g_7^2 (q^2 + d^2 + h_1^2) \right]
\]
$$
\gamma_{r}^{(1)} = \left[ \lambda^2 + 3y_6^2 + 4y_r^2 - 3g_2^2 - 3g_1^2 - 2g_4^2(l^2 + e^2 + h_1^2) \right]
$$

$$
\gamma_{\lambda}^{(1)} = \left[ 4\lambda^2 + 3(k_1^2 + k_2^2 + k_3^2) + 3(y_6^2 + y_6^2 + y_r^2 - g_1^2 - 2g_4^2(s^2 + h_2^2 + h_1^2) \right]
$$

$$
\gamma_{k_1}\,^{(1)} = \left[ 2\lambda^2 + 5k_1^2 - \frac{16}{3} g_3^2 - \frac{4}{9} g_1^2 - 2g_4^2(s^2 + z_1^2 + (s + z_1)^2) \right]
$$

$$
\gamma_{k_2}\,^{(1)} = \left[ 2\lambda^2 + 5k_2^2 - \frac{16}{3} g_3^2 - \frac{8}{9} g_1^2 - 2g_4^2(s^2 + z_2^2 + (s + z_2)^2) \right]
$$

$$
\gamma_{k_3}\,^{(1)} = \left[ 2\lambda^2 + 5k_3^2 - \frac{16}{3} g_3^2 - \frac{8}{9} g_1^2 - 2g_4^2(s^2 + z_3^2 + (s + z_3)^2) \right]
$$

$$
\gamma_{l}\,^{(2)} = \left[ -22y_1^4 - 5y_6^4 - y_l^2(3\lambda^2 + 5y_6^2) - y_6^2 y_r^2 - 3\lambda^4 - 4y_6^2 \lambda^2 - \lambda^2 y_r^2 - 3\lambda^2(k_1^2 + k_2^2 + k_3^2) + y_r^2(2g_2^2 + 6g_2^2 + 16g_2^2 + g_3^2(8q^2 + 4u^2)) + y_6^2\left( \frac{2}{3} g_2^2 + 2g_4^2(d^2 - h_1^2 - q^2) \right) + 2\lambda^2 g_4^2(h_1^2 + s^2 - h_2^2) + \frac{3679}{162} g_4^4 + \frac{15}{2} g_4^2 + \frac{416}{9} g_4^4 + g_1^4(2s_4(q^2 + u^2 + h_2^2)) + 4(q^4 + u^4 + h_2^2) + \frac{5}{3} g_1^2 g_2^2 + \frac{136}{27} g_1^2 g_3^2 + 8(h_2^2 + \frac{q^2}{4} + \frac{4u^2}{9} + g_1^2 g_3^2 + 8g_2 g_3^2 + 6g_2 g_4^2 (q^2 + h_2^2) + \frac{32}{3} (q^2 + u^2) g_3 g_4^2 \right]
$$

$$
\gamma_{b}\,^{(2)} = \left[ -22y_6^4 - 5y_6^4 - 4y_6^2 \lambda^2 - y_6^2(3\lambda^2 + 5y_6^2 + y_6^2 - 3\lambda^4 - 3y_6^2 \lambda^2 - \lambda^2 y_6^2 - 3\lambda^2(k_1^2 + k_2^2 + k_3^2) + y_6^2\left( \frac{4}{3} g_1^2 + \frac{9}{2} g_2^2 + 16g_3^2 + g_3^2(6q^2 + 6d^2 + 2h_2^2) \right) + 2y_6^2\left( \frac{4}{3} g_1^2 + 2g_3^2(u^2 + h_2^2 - q^2) \right) + 2\lambda^2 g_4^2(s^2 + h_1^2 - h_2^2) + y_r^2(2g_2^2 + 2g_2^2(l^2 + e^2 - h_2^2)) + \frac{1939}{162} g_4^4 + \frac{15}{2} g_4^2 + \frac{416}{9} g_4^4 + g_1^4(2s_4(q^2 + d^2 + h_1^2) + 4(q^4 + d^4 + h_1^2) + \frac{5}{3} g_1^2 g_2^2 + \frac{40}{27} g_1^2 g_3^2 + 8(h_1^2 + \frac{q^2}{4} + \frac{4u^2}{9}) g_1^2 g_3^2 + 8g_2 g_3^2 + 6g_2 g_4^2 (q^2 + h_1^2) + \frac{32}{3} (q^2 + d^2) g_3 g_4^2 \right]
$$

$$
\gamma_{r}\,^{(2)} = \left[ -9y_6^4 - 3\lambda^4 - 10y_r^4 - 3y_r^2 y_6^2 - 3\lambda^2 y_r^2 - 3y_r^2(\lambda^2 + 3y_6^2) - 3\lambda^2(k_1^2 + k_2^2 + k_3^2) + y_6^2\left( -\frac{2}{3} g_1^2 + 16g_3^2 + 6g_4^2(q^2 + d^2 - h_1^2) + 2\lambda^2 g_4^2(s^2 + h_1^2 - h_2^2) + y_r^2(2g_2^2 + 6g_2^2 + 4g_4^2(l^2 + h_1^2)) + \frac{99}{2} g_4^4 \right) + \frac{15}{2} g_2^2 + g_1^4(2s_4(e^2 + l^2 + h_2^2) + 4(l^4 + h_1^4 + e^4)) + 3g_1^2 g_2^2 + g_1^2 g_3^2(2h_1^2 + 2l^2 + 8e^2) + 6g_2 g_4^2 (h_1^2 + l^2) \right]
$$
\[
\gamma^{(2)}_\lambda = \left[ -9y_4^4 - 9y_4^2 - 10\lambda^4 - 3y_4^4 - 6y_4^2 + 2\lambda^2(9y_4^2 + 9y_4^2 + 3y_4^2 + 6(k_1^2 + k_2^2 + k_3^2)) - 6(k_1^4 + k_2^4 + k_3^4)
+ 2g_1^4(3g_1^2 + 16g_3^2 + 6g_4^4(u^2 + q^2 - h_3^2)) + \lambda^2(2g_2^4 + 6g_2^2 + 4g_4^2(h_1^2 + h_2^2)) + k_1^2(4g_1^2
+ 16g_2^2 + 6g_2^2(s^2 + (s + z_1)^2 - s^2)) + k_2^2(16g_3^4 + 16g_3^2 + 6g_3^2(s^2 + (s + z_2)^2 - s^2)) + k_3^2(16g_3^2
+ 16g_2^2((s + z_3)^2) + g_2^2(-\frac{2}{3}g_1^2 + 3g_2^2 + 16g_3^2 + 6g_2^2(q^2 + d^2 - h_1^2)) + 2y_4^4(3g_1^2 + g_1^2
+ 2g_1^2g_3^2(h_1^2 + h_2^2) + 6g_2^2g_4^2(h_1^2 + h_2^2))]
\]
\[
\gamma^{(2)}_{k_1} = \left[ -6k_1^2\lambda^2 - 6k_1^4 - 4\lambda^2(2y_1^2 + 6y_2^2 + 6y_4^2) - 6k_2^2(k_1^2 + k_2^2 + k_3^2) + k_1^2(4g_1^2 + 16g_3^2 + 2g_4^2(z_1^2
+ (s + z_1)^2 - s^2)) + \lambda^2(2g_1^2 + 6g_2^2 + 2g_2^2(h_1^2 + h_2^2) - s^2)) + \frac{542}{81}g_1^4 + \frac{416}{9}g_3^4 + g_4^4(2s_4(z_1^2
+ (s + z_1)^2) + 4(z_1^4 + (s + z_1)^4) + \frac{64}{27}g_1^2g_3^2 + \frac{8}{9}(z_1^2 + (s + z_1)^2)g_1^2g_4^2 + \frac{32}{3}(z_1^2 + (s + z_1)^2)g_1^2g_3^2
\right]
\]
\[
\gamma^{(2)}_{k_2} = \left[ -6k_2^2\lambda^2 - 6k_2^4 - 4\lambda^2(2y_2^2 + 6y_2^2 + 6y_4^2) - 6k_3^2(k_1^2 + k_2^2 + k_3^2) + k_2^2(16g_1^2 + 16g_3^2 + 2g_4^2(z_2^2
+ (s + z_2)^2 - s^2)) + \lambda^2(2g_1^2 + 6g_2^2 + 2g_2^2(h_1^2 + h_2^2) - s^2)) + \frac{2168}{81}g_1^4 + \frac{416}{9}g_3^4 + g_4^4(2s_4(z_2^2
+ (s + z_2)^2) + 4(z_2^4 + (s + z_2)^4) + \frac{256}{27}g_1^2g_3^2 + \frac{32}{9}(z_2^2 + (s + z_2)^2)g_1^2g_4^2 + \frac{32}{3}(z_2^2 + (s + z_2)^2)g_1^2g_3^2
\right]
\]
\[
\gamma^{(2)}_{k_3} = \left[ -6k_3^2\lambda^2 - 6k_3^4 - 4\lambda^2(2y_3^2 + 6y_2^2 + 6y_4^2) - 6k_3^2(k_1^2 + k_2^2 + k_3^2) + k_3^2(16g_1^2 + 16g_3^2 + 2g_4^2(z_3^2
+ (s + z_3)^2 - s^2)) + \lambda^2(2g_1^2 + 6g_2^2 + 2g_2^2(h_1^2 + h_2^2) - s^2)) + \frac{2168}{81}g_1^4 + \frac{416}{9}g_3^4 + g_4^4(2s_4(z_3^2
+ (s + z_3)^2) + 4(z_3^4 + (s + z_3)^4) + \frac{256}{27}g_1^2g_3^2 + \frac{32}{9}(z_3^2 + (s + z_3)^2)g_1^2g_4^2 + \frac{32}{3}(z_3^2 + (s + z_3)^2)g_1^2g_3^2
\right]
\]
where \(s_4 = 18q^2 + 9(u^2 + d^2) + 6l^2 + 2(h_1^2 + h_2^2) + 3(z_1^2 + z_2^2 + z_3^2 + (s + z_1)^2 + (s + z_2)^2 + (s + z_3)^2) + e^2 + s^2\),

\[
\frac{dm^2_{q_1}}{dt} = \frac{1}{16\pi^2} \left[ 2g_1^2(m^2_{q_1} + m^2_{q_2} + m^2_{q_3}) + 2A_1^2 + 2g_2^4(m^2_{q_3} + m^2_{q_3} + m^2_{q_7}) + 2A_1^2
- \frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - 2\frac{g_1^2M_1^2}{9} - 8g_1^2g_4^2M_1^2 + \frac{1}{3}g_1^2M_1^2 + 2qg_2^2\xi + g_4^2\xi\right]
\]
\[
\frac{dm^2_{q_2}}{dt} = \frac{1}{16\pi^2} \left[ 4g_1^2(m^2_{q_1} + m^2_{q_2} + m^2_{q_3}) + 4A_1^2 - \frac{32}{3}g_3^2M_3^2 - 8\frac{4}{9}g_1^2M_1^2 - 8u^2g_4^2M_1^2 + \frac{4}{3}g_1^2M_1^2 + 2ug_2^2\xi + g_4^2\xi\right]
\]
\[
\frac{dm^2_{q_3}}{dt} = \frac{1}{16\pi^2} \left[ 4g_1^2(m^2_{q_1} + m^2_{q_2} + m^2_{q_3}) + 4A_1^2 - \frac{32}{3}g_3^2M_3^2 - 8\frac{1}{9}g_1^2M_1^2 - 8d^2g_4^2M_1^2 + \frac{2}{3}g_1^2\xi + 2dg_2^2\xi\right]
\]
\[
\frac{dm^2_{l_1}}{dt} = \frac{1}{16\pi^2} \left[ 2g_1^2(m^2_{q_1} + m^2_{q_2} + m^2_{q_3}) + 2A_1^2 - 6g_2^2M_2^2 - 8\frac{1}{4}g_1^2M_1^2 - 8l^2g_4^2M_1^2 - g_1^2\xi + 2lg_2^2\xi\right]
\]
\[
\frac{dm_2}{dt} = \frac{1}{16\pi^2} \left[ 4y_1^2(m_3^2 + m_7^2 + m_8^2) + 4A_1^2 - 8g_4^2M_1^2 - 8e^2g_4^2M_4^2 + 2g_1^2\xi + 2eg_4^2\xi' \right]
\]

\[
\frac{dm_3}{dt} = \frac{1}{16\pi^2} \left[ 6y_2^2(m_3^2 + m_4^2 + m_1^2) + 6A_2^2 + 2y_2^2(m_2^2 + m_3^2 + m_1^2) + 2A_2 \right.
\]
\[
+2\lambda^2(m_1^2 + m_2^2 + m_3^2) + 2A_1\lambda - 6g_2^2M_2^2 - 2g_1^2M_1^2 - 8h_2^2g_4^2M_4^2 - g_1^2\xi + 2h_1g_4^2\xi' \right]
\]

\[
\frac{dm_4}{dt} = \frac{1}{16\pi^2} \left[ 6y_2^2(m_3^2 + m_4^2 + m_2^2) + 6A_1^2 + 2\lambda^2(m_1^2 + m_2^2 + m_3^2) \right.
\]
\[
+2A_1\lambda - 6g_2^2M_2^2 - 2g_1^2M_1^2 - 8h_2^2g_4^2M_4^2 + g_1^2\xi + 2h_1g_4^2\xi' \right]
\]

\[
\frac{dm_1}{dt} = \frac{1}{16\pi^2} \left[ 2k_1^2(m_1^2 + m_3^2 + m_1^2D_1) + 2A_1k_1 - \frac{32}{3}g_3^2M_3^2 - \frac{8}{9}g_1^2M_1^2 - 8z_1^2g_4^2M_4^2 - \frac{2}{3}g_1^2\xi + 2z_1g_4^2\xi' \right]
\]

\[
\frac{dm_2}{dt} = \frac{1}{16\pi^2} \left[ 2k_2^2(m_2^2 + m_3^2 + m_2^2D_2) + 2A_2k_2 - \frac{32}{3}g_3^2M_3^2 - \frac{32}{9}g_2^2M_2^2 - 8z_2^2g_4^2M_4^2 + \frac{4}{3}g_1^2\xi + 2z_2g_4^2\xi' \right]
\]

\[
\frac{dm_3}{dt} = \frac{1}{16\pi^2} \left[ 2k_3^2(m_3^2 + m_3^2 + m_3^2D_3) + 2A_3k_3 - \frac{32}{3}g_3^2M_3^2 - \frac{32}{9}g_2^2M_2^2 - 8z_3^2g_4^2M_4^2 + \frac{4}{3}g_1^2\xi + 2z_3g_4^2\xi' \right]
\]

\[
\frac{dm_4}{dt} = \frac{1}{16\pi^2} \left[ 2k_4^2(m_4^2 + m_4^2 + m_4^2D_4) + 2A_4k_4 - \frac{32}{3}g_3^2M_3^2 - \frac{8}{9}g_1^2M_1^2 - 8(s + z_1)^2g_4^2M_4^2 + \frac{2}{3}g_1^2\xi \right.
\]
\[
+2(s + z_2)g_4^2\xi' \right]
\]

\[
\frac{dm_5}{dt} = \frac{1}{16\pi^2} \left[ 2k_5^2(m_5^2 + m_5^2 + m_5^2D_5) + 2A_5k_5 - \frac{32}{3}g_3^2M_3^2 - \frac{32}{9}g_2^2M_2^2 - 8(s + z_2)^2g_4^2M_4^2 - \frac{4}{3}g_1^2\xi \right.
\]
\[
+2(s + z_3)g_4^2\xi' \right]
\]

\[
\frac{dm_6}{dt} = \frac{1}{16\pi^2} \left[ 2k_6^2(m_6^2 + m_6^2 + m_6^2D_6) + 2A_6k_6 - \frac{32}{3}g_3^2M_3^2 - \frac{32}{9}g_2^2M_2^2 - 8(s + z_3)^2g_4^2M_4^2 - \frac{4}{3}g_1^2\xi \right.
\]
\[
+2(s + z_4)g_4^2\xi' \right]
\]

\[
\frac{dA_1}{dt} = \frac{A_1}{16\pi^2} \left[ 18y_1^2 + y_6^2 + \lambda^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2 - 2(q^2 + u^2 + h^2)g_1^2 \right]
\]
\[
+ \frac{y_b}{16\pi^2} \left[ 2y_bA_b + A_1\lambda + \frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{9}g_1^2M_1 + 4(q^2 + u^2 + h^2)g_4^2M_4 \right]
\]
\[
\frac{dA_b}{dt} = \frac{A_b}{16\pi^2} \left[ 18y_b^2 + y_t^2 + y_r^2 + \lambda^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{9} g_1^2 - 2(q^2 + d^2 + h_1^2)g_4^2 \right]
+ \frac{y_b}{16\pi^2} \left[ 2y_tA_t + 2A_r y_r + 2A_\lambda \lambda + \frac{32}{3} g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{9} g_1^2 M_1 + 4(q^2 + d^2 + h_1^2)g_4^2 M_4 \right]
\]

\[
\frac{dA_r}{dt} = \frac{A_r}{16\pi^2} \left[ 12y_r^2 + 3y_t^2 + \lambda^2 - 3g_2^2 - 2(q^2 + d^2 + h_1^2)g_4^2 \right]
+ \frac{y_r}{16\pi^2} \left[ 6y_b A_b + 2A_\lambda \lambda + 6g_2^2 M_2 + 6g_1^2 M_1 + 4(q^2 + d^2 + h_1^2)g_4^2 M_4 \right]
\]

\[
\frac{dA_\lambda}{dt} = \frac{A_\lambda}{16\pi^2} \left[ 3y_b^2 + 3y_t^2 + y_r^2 + 12\lambda^2 + 3(k_1^2 + k_2^2 + k_3^2) - 3g_2^2 - g_1^2 - 2(s^2 + h_2^2 + h_1^2)g_4^2 \right]
+ \frac{\lambda}{16\pi^2} \left[ 6y_tA_t + 6y_b A_b + 2A_r y_r + 6(A_k, k_1 + A_k, k_2 + A_k, k_3) + 6g_2^2 M_2 + 2g_1^2 M_1 + 4(s^2 + h_2^2 + h_1^2)g_4^2 M_4 \right]
\]

\[
\frac{dA_{k_1}}{dt} = \frac{A_{k_1}}{16\pi^2} \left[ 3k_1^2 + \lambda^2 - \frac{16}{3} g_3^2 - \frac{4}{9} g_1^2 - 2(s^2 + z_1^2 + (s + z_1)^2)g_4^2 \right]
+ \frac{k_1}{16\pi^2} \left[ 4\lambda A_\lambda + \frac{32}{3} g_3^2 M_3 + \frac{8}{9} g_1^2 M_1 + 4(s^2 + z_1^2 + (s + z_1)^2)g_4^2 M_4 \right]
\]

\[
\frac{dA_{k_2}}{dt} = \frac{A_{k_2}}{16\pi^2} \left[ 3k_2^2 + \lambda^2 - \frac{16}{3} g_3^2 - \frac{16}{9} g_1^2 - 2(s^2 + z_2^2 + (s + z_2)^2)g_4^2 \right]
+ \frac{k_2}{16\pi^2} \left[ 4\lambda A_\lambda + \frac{32}{3} g_3^2 M_3 + \frac{32}{9} g_1^2 M_1 + 4(s^2 + z_2^2 + (s + z_2)^2)g_4^2 M_4 \right]
\]

\[
\frac{dA_{k_3}}{dt} = \frac{A_{k_3}}{16\pi^2} \left[ 3k_3^2 + \lambda^2 - \frac{16}{3} g_3^2 - \frac{16}{9} g_1^2 - 2(s^2 + z_3^2 + (s + z_3)^2)g_4^2 \right]
+ \frac{k_3}{16\pi^2} \left[ 4\lambda A_\lambda + \frac{32}{3} g_3^2 M_3 + \frac{32}{9} g_1^2 M_1 + 4(s^2 + z_3^2 + (s + z_3)^2)g_4^2 M_4 \right]
\]

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