First passage times of mesoscopic charge transport and entropy change

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The first-passage-time statistics of a stochastic process $N(t)$ for electrons transferred through a metallic double dot are considered. It is shown that in calculating the average of the first-passage time, it is necessary to take into account the changes in entropy occurring during this process. Using the example of a $DC$ bias voltage, the influence of external influences on the average first-passage time is considered. All real physical processes, including the first-passage time, occur with changes in entropy. This circumstance is not taken into account in studies of the first-passage time, but is illustrated in this paper by the example of electrons transfer through a metallic double dot.

Keywords: first passage time, changes of entropy.

1. Introduction

All real physical processes, which are modeled by stochastic processes, occur with a change in entropy. This also applies to the first-passage time processes ($FPT$). This circumstance is not taken into account in the studies of the $FPT$.

The $FPT$ are widely used in various areas [1, 2]. Changes in entropy take place in the system during the $FPT$. In general, the relationship between the $FPT$ and changes in entropy is considered in [3, 4]. In this work, this relationship is shown using the example of the process of electron transfer in mesoscopic charge transport. It is shown that taking into account such a relationship significantly affects the average $FPT$.

In [5, 6] $FPT$ was investigated until the moment when the electric charge transfer through the conductor reaches a given value. The $FPT$ distribution for the number of electrons transferred between aluminum conductor and a superconductor is considered.

This paper shows an important property of real $FPT$ s, namely, the need to take into account the change of the entropy in the system during the process of $FPT$. The usual procedure for recording the average $FPT$ does not reflect the actual situation. An analogy can be drawn with ideal and real gases.

In [5], a simple analytical approximation was obtained for the $FPT$ distribution. Advances in nanotechnology make it possible to carry out very precise experiments with the calculation of the transferred electrons [7, 8]. Therefore, it is possible theoretically and experimentally to study the distributions of the first-passage times and waiting times [9, 10].

In [5, 6], the probability distribution $P_{N}(t)$ of the first passage is studied for the process of achieving a given value of the number of electrons $N$ that have tunneling between two metal islands at time $t$. For this, the complete chronology of the events of electron transfer between two metal islands was obtained in [5, 6] (see details in [5, 6]).

The distribution of $FPT$ was obtained in [5, 6] widely used in various tasks (e.g. in queue theory [11]). This distribution has the form

$$P_{N}(t) = \frac{1}{t} N^{*} e^{\frac{C_{i}C_{j}}{C_{i}C_{j}}} \left( \frac{C_{2} + \sqrt{C_{i}C_{j}}}{C_{2} - \sqrt{C_{i}C_{j}}} \right)^{N^{*}/2} I_{n} \left( \frac{C_{1} \sqrt{C_{i}C_{j} - C_{i}C_{j}}}{C_{3} - t} \right),$$

where $N^{*} = [N^{*}(C_{i}C_{j})]$,

$C_{i} i = 1, 2, 3$ are the cumulants of the distribution $P_{n}(N)$ of the process for the number of transferred electrons at a fixed time instant $t$, $I_{n}(x)$ is the modified Bessel function
of the first kind, and \( N \) is the threshold value for the number of transferred particles. The charge transferred by them, \( e' N' \), \( e' = e\sqrt{C_1/C_i} \) gets as close as possible to the net charge of real electrons \( eN \), \( e \) is the electron charge. Here, the square brackets \([…]\) indicate the rounding function.

The average electric current \( \langle I \rangle \) and current noise are expressed in terms of the cumulants \( C_1 \) and \( C_2 \).

\[
\langle I \rangle = eC_1, \quad S_1 = 2\int dt \left\{ (I(t)I(0)) - \langle I \rangle^2 \right\} = 2e^2C_1. \tag{1}
\]

The \( C_3 \) and \( C_4 \) values represent the third and fourth cumulants of the distribution \( P_r(N) \). In [5], it was also assumed that \( C_1, C_3 > 0 \), and \( C_2^2 > C_1C_3 \). Distribution (1) was obtained in [5] from a general approach with some approximations (for sufficiently large times, an approximate form of the cumulant generating function). In [5], was noted that expression (1) is fulfilled under the conditions

\[
\frac{C_1|C_1C_4 - C_2C_3|}{12C_2^3}(\frac{N}{C_1 r} - 1)^2 \leq 1, \quad t \gg \tau_r, \quad |N| \gg 1. \tag{2}
\]

In accordance with the first condition (2), relation (1) is fulfilled better near the maximum of the distribution \( P_r(N) \), at \( t = N/C_1 \), than in the tails of the distribution. The time required for the system to return to a stationary state after an external disturbance is defined as the relaxation time \( \tau_r \).

In [6] experimentally studied fluctuations of stochastic entropy production of the electric current in non-equilibrium steady-state conditions in an electronic double dot. In this paper, we consider the thermodynamic aspects of FPT moments, in particular, the relationship between the first moment and the entropy change accompanying the first achievement process.

The FPT moments classified in the theory of stochastic processes as stopping times and Markov moments [12, 13]. The set of events observed during the random time \( FPT \) \( T_{x\gamma} \) (3) [13] corresponds to a set describing Markov moments. This takes into account the dependence on the history of the system. The change in the entropy of the system depends on the events occurring at this time. \( FPT \) (3) is a multiplicative functional of a random process \( X(t) \) [12]. Dependence on the system’s past is important in distribution (4), (5).

In this work, arbitrary changes in entropy taken into account, which affect the moments of the process of \( FPT \). Using distribution (1) makes it possible to accurately calculate the impact on the system. Changes in entropy are expressed through entropy flows into the system from the outside and entropy production in the system. The production of entropy is equal to the product of thermodynamic forces and the thermodynamic flows created by these forces. The actions on the system are also described by thermodynamic forces and entropy changes.

The distribution of the first-passage time for the number of electrons reaching a certain threshold was obtained in [5]. In [3, 4], the argument of the Laplace transform of the \( FPT \) distribution is related to the change in the entropy of the system using thermodynamic relations. The average \( FPT \) value, in which the value of the argument of the Laplace transform of the \( FPT \) distribution is assumed to be zero, does not correspond to real events in which the changes in entropy and the value of this argument are not equal to zero. The task is to determine the value of the argument of the Laplace transform corresponding to the change in entropy in the real \( FPT \) process of reaching a given level.

The change in entropy is expressed in terms of thermodynamic flows and conjugate thermodynamic forces. The inverse relationship is also true: \( FPT \) affected by entropy changes caused by the introduction or change of thermodynamic forces. It is possible to formulate and study the \( FPT \) control problem. System entropy and \( FPT \) change with changes in thermodynamic forces. The processes in the system slow down or speed up.

An external \( DC \) voltage \( V_b \) is considered as external thermodynamic forces. Non-equilibrium fluctuations of the charge-state in a hybrid double point normal metal-superconductor in the regime of strong Coulomb blockade, subject to the influence of a time-independent bias voltage, were measured in [5, 6]. An external \( DC \) bias voltage, \( V_b \), which
causes the system to go to a non-equilibrium steady state and controls the net current through the double-dot, is applied between the two leads.

In the second section, the approach of works [3, 4] is briefly described. In the third section, this approach is applied to distribution (1) describing the first-passage times for the number of electrons transferred between two metallic islands.

2. Relationship between FPT and entropy change.

In [3, 4, 14-16] the first-passage time (FPT) considered. FPT is defined as the time during which the random process \( X(t) \) first reaches a certain threshold \( a \) (3). FPT is by definition equal to

\[
T_{xy} = \inf \{ t: X(t) = a \}, \quad X(0) = x > 0.
\]

(3)

A distribution that contains FPT ("lifetime" in [17]) as an additional thermodynamic parameter is introduced in [3, 4, 14-17]. The microscopic probability density in the extended phase space with cells \((u, T_{\gamma})\) has the form

\[
\rho(z; u, T_{\gamma}) = \exp[-\beta u(z) - \gamma T_{\gamma}(z)]/Z(\beta, \gamma),
\]

(4)

where \( \beta = 1/T \) is the inverse temperature of the reservoir (\( k_B = 1, k_B \) is Boltzmann constant), the partition function is equal

\[
Z(\beta, \gamma) = \int e^{-\beta u - \gamma T_{\gamma}} dz = \int dz \int dT_\gamma \omega(u, T_{\gamma}) e^{-\beta u - \gamma T_{\gamma}}.
\]

(5)

The parameters \( \beta \) and \( \gamma \) are the Lagrange multipliers. They satisfying the following expressions for the averages:

\[
< u > = -\partial \ln Z/\partial \beta, \quad < T_{\gamma} > = -\partial \ln Z/\partial \gamma.
\]

(6)

The values of energy \( u \) and FPT \( T_{\gamma} \) in expressions (4) - (6) are chosen as thermodynamic parameters. The production and flows of entropy characterize non-equilibrium processes in an open statistical system. Associated with them is the conjugate FPT parameter \( \gamma \). The non-equilibrium distribution (4) converges to equilibrium Gibbs distribution at \( \gamma = 0 \) and \( \beta = \beta_0 = T_{eq}^{-1} \), where \( T_{eq} \) is the equilibrium temperature.

The local specific entropy \( s_{\gamma} \) corresponding to distribution (4) (\( u \) is the specific internal energy) is introduced by the relation [3]

\[
s_{\gamma} = -(\ln \rho(z; u, T_{\gamma}) = \beta<u> + \gamma<T_{\gamma}> + \ln Z(\beta, \gamma); \quad ds_{\gamma} = \beta du + \gamma dT_{\gamma}.
\]

(7)

We assume that \( \omega(u, T_{\gamma}) = \omega(u) \omega_{\gamma}(u, T_{\gamma}) \), \( \omega_{\gamma}(u, T_{\gamma}) \sim f(T_{\gamma}, u) \). Here \( f(T_{\gamma}, u) \) is the FPT distribution density. Suppose that this function does not depend on the random energy \( u \). Then the variables of integration are separated. The statistical sum (5) is written as the product of equilibrium and non-equilibrium factors, \( Z(\beta, \gamma) = Z_\beta Z_{\gamma} \). The non-equilibrium part of the partition function \( Z_{\gamma} \) is the Laplace transform of the FPT distribution density. For internal energy and statistical sum the following relations are fulfilled:

\[
\bar{u} = -\partial \ln Z/\partial \beta, \quad u_\beta = -\partial \ln Z_\beta/\partial \beta, \quad u_{\gamma} = -\partial \ln Z_{\gamma}/\partial \gamma = -\int_0^\infty e^{-\gamma T} \frac{\partial f(T_{\gamma})}{\partial T_{\gamma}} dT_{\gamma} \frac{1}{Z_\beta},
\]

\[
Z(\beta, \gamma) = Z_\beta Z_{\gamma}, \quad Z_{\gamma} = \int_0^\infty e^{-\gamma T} f(T_{\gamma}) dT_{\gamma}.
\]

(8)

From expressions (7) - (8), we obtain an equation for determining the non-equilibrium parameter \( \gamma \) conjugated to the FPT:

\[
-\Delta = \beta u_\beta + \gamma T_{\gamma} + \ln Z_{\gamma}, \quad \Delta = s_{eq} - s_{\gamma},
\]

(9)

where \( s_{eq} = s_{\gamma}=0 = \beta u_\beta + \ln Z_\beta \), \( Z_\beta = \int e^{\beta u} \omega(u) du \), \( u_\beta = -\partial \ln Z_\beta/\partial \beta \), \( Z_\beta \) is "equilibrium" partition function; \( u_\beta \) is "equilibrium" energy; \( s_{eq} \) is "equilibrium" entropy.
3. Time elapsed until the electric charge transferred through a conductor reaches a given threshold value. Comparison of theory with experimental data

In [18, 19], an exact result was obtained for a one-dimensional biased random walk:

$$P_N(t) = \frac{1}{t} \left| \mathcal{N} e^{-(\Gamma_+ + \Gamma_-)} \right| \Gamma_+^{N/2} I_N(2\sqrt{\Gamma_+} \Gamma_- t).$$

(10)

On the basis of expression (10) in [5, 6] at $$\Gamma_+ = \frac{C_1}{C_3} (C_2 \pm \sqrt{C_3})$$, expression (1) is written. Here $$\Gamma_\pm$$ are the rates of jumping forward and backward. In [20], this model describes the transport of charged particles through a voltage biased tunnel junction. In [5], from (1) - (2), approximations were obtained for short times, for small values of $$N$$, for large times, as in the exact theory, [22], for a weakly non-Gaussian random process, as well as for the Gaussian limit.

In [5], the predictions of the exact theory [21, 22] were compared with experimental results. A perfect agreement was found at sufficiently long times determined by expression (2). In [5], a simple and universal approximation was also written for the FPT distribution (1) taking into account the non-Gaussian statistics of one-electron tunneling using the third cumulant $$C_3$$ of the distribution of the number of transmitted electrons.

The Laplace transform of the distribution (1) has the form

$$Z(s) = \left(2\Gamma_+ \right)^N \left( s + \Gamma_+ \sqrt{(s + \Gamma_+)^2 - 4\Gamma_- \Gamma_+} \right)^N / \left( s + \Gamma_+ + \sqrt{(s + \Gamma_+)^2 - 4\Gamma_- \Gamma_+} \right)$$

(11)

$$\Gamma_+ \Gamma_- = \frac{C_1^2}{4C_3} (C_2^2 - C_3 C_3)$$

The mean value of FPT determined from expressions (6), (8) is equal to $$(s=\gamma)$$

$$\bar{T}_\gamma = \frac{N^*}{\sqrt{(s + \Gamma_+)^2 - 4\Gamma_- \Gamma_+}} = \frac{N}{\sqrt{1 + \frac{s(s + 2\Gamma_+)}{(\Gamma_+ - \Gamma_-)^2}}}$$

$$T_0 = \bar{T}_{\gamma=0} = \frac{N^*}{\Gamma_+ - \Gamma_-} = \frac{N}{C_1}.$$  

(12)

By $$\gamma \geq 0$$, $$\bar{T}_\gamma \leq T_0$$. We seek the dependence $$\gamma(\Delta)$$ from equation (9), where for $$u_\gamma$$ we obtain

$$u_\gamma = u_N \frac{\partial N^*}{\partial \beta} + u_{N_1} \frac{\partial \Gamma_+}{\partial \beta} + u_{N_2} \frac{\partial \Gamma_-}{\partial \beta},$$

$$u_{N_1} = \ln(\gamma + \Gamma_+ + \sqrt{(\gamma + \Gamma_+)^2 - 4\Gamma_- \Gamma_+}) - \ln(2\Gamma_+),$$

$$u_{N_2} = N^* \frac{\gamma + \Gamma_+ + \sqrt{(\gamma + \Gamma_+)^2 - 4\Gamma_- \Gamma_+} - 2\Gamma_-}{(\gamma + \Gamma_+ + \sqrt{(\gamma + \Gamma_+)^2 - 4\Gamma_- \Gamma_+}) \sqrt{(\gamma + \Gamma_+)^2 - 4\Gamma_- \Gamma_+}}.$$  

(13)

Derivatives $$\beta \frac{\partial u}{\partial \beta}$$ and $$\beta^2 \frac{\partial u}{\partial \beta^2}$$ are included in (13). Expressions for $$\Gamma_+$$ and $$\Gamma_-$$ are written in [5, 6]. So, the expression for $$\Gamma_\gamma$$ has the form

$$\Gamma_\gamma = \Gamma_{L+\gamma}(\Delta E) = \frac{1}{e R} \left| \mathcal{N} e^{-(\Gamma_+ + \Gamma_-)} \right| \Gamma_+^{N/2} I_N(2\sqrt{\Gamma_+} \Gamma_- t),$$

(14)

where $$\Delta E = -e V_\gamma$$. Here $$R=R_{om}$$ is the resistance of the transition in which the electron jump occurs, $$n_i(E)$$ and $$f_i(E)$$ are the density of states and the distribution function in the initial electrode, $$n_t(E)$$ and $$f_t(E)$$ are the density of states and the distribution function in target electrode (is Fermi function). The density of states in superconductors has the usual form.
\[ n_s(E) = \theta([E] - \Delta)[E] / \sqrt{E^2 - \Delta^2}, \] where \( \Delta \) is the superconducting gap, and in normal metals it is equal to \( I \). The corresponding rates are obtained in [23-25].

To define functions \( \beta \partial \Gamma / \partial \beta \) and \( \beta \partial \Gamma / \partial \beta \) use the approximation that was used in [26] for the one-electron case, then these can be conveniently expressed, in the form:

\[ \Gamma_+ = \Gamma_0 e^{-\beta \Delta_0/2}, \quad \Gamma_- = \Gamma_0 e^{\beta \Delta_0/2}. \] (15)

In accordance with the general approach to diffusion in an external field [27], we write the value \( \Gamma_0 \) in the form \( \Gamma_0 = \Gamma_0 e^{\beta \Delta_0} \), where \( E_s \) is the maximum potential energy between the "electron islands". In [6] and [28] the expression for electrostatic energy in an electronic double dot is written. In case \( V_b = 0 \)

\[ E_{\text{tot}}(n) = \frac{E_C}{2} (n_L - n_{s,L})^2 + \frac{E_C}{2} (n_R - n_{s,R})^2 + E_{C_m} (n_R - n_{s,R}) (n_L - n_{s,L}), \] (16)

where \( E_C, E_C \) are the charging energies of the islands, \( E_{C_m} \) is the electrostatic coupling energy, \( n_L, n_R, n_{s,L}, n_{s,R} \) are the charge and gates charge states of the left and right dots.

For simplicity, let's take the values \( V_{gl} = V_{gr} = 0 \), \( n_{gl} = n_{gr} = 0 \), \( n_c = n_a = 1/2 \). From [6] we take the values \( E_{cl} = E_C = 60 \mu eV, \ E_{cr} = E_C = 40 \mu eV, \ E_{C_m} = 10 \mu eV \). Then from (16) we obtain \( E_{\text{tot}} = 15 \mu eV, T_{eff} = 1.175 K \), \( \beta^{-1} = k_B T_{eff} = 1.6215 \times 10^{-3} J, \ \beta E_a = 0.148 \). Value \( T_{eff} = 1.175 K \) taken from table 1 [6] as an average value of the\( T_{eff} \) for voltages \( V_b = 25 \mu V \) and \( V_b = -25 \mu V \).

From (15) we obtain that \( \beta \partial \Gamma_+ / \partial \beta = -\beta (E_a - eV_b / 2) \Gamma_+ \), \( \beta \partial \Gamma_- / \partial \beta = -\beta (E_a + eV_b / 2) \Gamma_- \).

To determine the value \( \beta \partial N' / \partial \beta \) we use the approximation for a biased tunnel junction, for which \( C_2 = C_1 \coth[\beta eV_b / 2kBT] \) [5].

In the course of reaching the boundary \( N \) by the random process \( N(t) \), the entropy of the system [6] changes by the value \( \Delta = \Delta_e \). These changes must be taken into account. Therefore, one cannot assume in (12) \( \gamma = 0 \), considering the value of \( T_0 \) as the average value of the time of the first achievement in the absence of impacts on the system. An internal change in entropy \( \Delta_e \) occurs and in the absence of any external influences in the system. The value of \( \Delta_e \) can be determined, for example, from the relations of extended irreversible thermodynamics [29] \( \Delta_e = \tau_{ij} / 2 \rho \sigma_i T \), where \( \tau_i \) is electric flux (current), \( \tau_i \) is relaxation time of currents, \( \sigma_i \) is electrical conductivity, or from results [6].

If there are other processes in the system that cause the corresponding changes in entropy, then \( \Delta = \Delta_e + \Delta_{oo} \), where \( \Delta_{oo} \) are the changes in entropy associated with other physical processes, for example, heat conduction.

Taking into account relations (6) - (8), (11), (12), equation (9) is written in the form

\[ -\Delta = \beta \frac{\partial N'}{\partial \beta} \left[ \ln(s + \Gamma_+ + \sqrt{(s + \Gamma_+)^2 - 4\Gamma_+}) - \ln(2\Gamma_+) \right] + \]

\[ \beta \frac{\partial \Gamma_+}{\partial \beta} N' \left[ \frac{s + \Gamma_+ - 2\Gamma_+ + \sqrt{(s + \Gamma_+)^2 - 4\Gamma_+}}{(s + \Gamma_+ + \sqrt{(s + \Gamma_+)^2 - 4\Gamma_+})} \right] = \frac{1}{\Gamma_+} + \]

\[ \beta \frac{\partial \Gamma_-}{\partial \beta} N' \left[ \frac{s + \Gamma_- - 2\Gamma_- + \sqrt{(s + \Gamma_-)^2 - 4\Gamma_-}}{(s + \Gamma_- + \sqrt{(s + \Gamma_-)^2 - 4\Gamma_-})} \right] + \frac{(\Gamma_+ - \Gamma_-)sT_0}{\sqrt{(s + \Gamma_+)^2 - 4\Gamma_+}} + \]

\[ N'[\ln(2\Gamma_+) - \ln(s + \Gamma_+ + \sqrt{(s + \Gamma_+)^2 - 4\Gamma_+})] \]

or
\[ e^{-\lambda/n_1} = 2\Gamma_n \exp\left(\frac{1}{1 - 4\Gamma_n \Gamma_n x^2} \left[ \frac{T_n L}{n_1} (1 - 2ax + 4\Gamma_n \Gamma_n x^2) - (1 - 2\Gamma_n x)(a_+ / \Gamma_+ - 2a_+) \right] \right) \]

\[ a_+ = \beta \frac{\partial \Gamma_+}{\partial \beta} \frac{N^*}{n_1}, \quad a_- = \beta \frac{\partial \Gamma_-}{\partial \beta} \frac{N^*}{n_1}, \quad n_1 = N^* - \beta \frac{\partial N^*}{\partial \beta}, \quad a = \Gamma_+ + \Gamma_-, \quad L = \Gamma_+ - \Gamma_- \quad \text{(18)} \]

\[ x = \frac{1}{s + a + \sqrt{(s + a)^2 - 4\Gamma_+ \Gamma_-}} \quad s = \frac{1}{2x} (1 - 2ax + 4\Gamma_n \Gamma_n x^2) \]

The solution of the transcendental Eq. (18) makes it possible to find the parameters \( x \) and \( s \) from (18). Substitution of these parameters into expression (12) makes it possible to find the value of the average \( FPT \) \( T_\gamma \) at \( s = \gamma \), obtained from distribution (4) - (6), depending on the change in entropy \( \Delta e \) in accordance with Eq. (9).

The values of \( \Delta e \) change in entropy at different voltages are taken from Fig. 4, 12 and Table II [6]. Fig. 1 shows the dependences \( T_0(N) = N/C_1 \) (dashed line) and \( \bar{T}_\gamma(\Delta e) = \bar{T}_\gamma(\Delta e) \) for different values of \( N \) - solid line. Let us consider external influences using the example of the applied bias voltage \( V_b \). Fig. 2 shows the dependence of \( T_\gamma(\Delta e) \) at \( T_0(N=10) = 2.174 \) on the applied voltage \( V_b \).

**Fig. 1.** Dependencies \( T_1 = T_0(N) = N/C_1 \) (dashed line) and \( T_\gamma = \bar{T}_\gamma(\Delta e) \) (solid line approximating the calculated points) for the value \( N \) of the process \( N(t) \) to take at a fixed time \( t \). Regarding the random process of achieving a given value \( N \), \( N(t) \) is the net number of transmitted electrons [5, 6]. Bias voltage \( V_b = 90 \mu V \).
Fig. 2. Calculated from relations (7)-(16) and table 1 dependences of the average time $T_1=\bar{T}_{\gamma(\lambda_n)}$ to reach the level $N = 10$ for different values of $V_b$ (25, 50, 65, 90 $\mu$V).

The figure is symmetrical about the y-axis for negative $V_b$ values.

The data used in the calculations are summarized in Table 1.

| $V_b$ (\$\mu$V) | 90   | 65   | 50   | 25   |
|-----------------|------|------|------|------|
| $-\beta \epsilon V_b/2$ (\$\mu$eV) | 0.522 | 0.377 | 0.29 | 0.145 |
| $C_1 \beta \beta^{\gamma} / \partial \beta$ | 4.6  | 3.25 | 2.48 | 1.23 |
| $C_2$ Hz | 9.27 | 8.725 | 8.48 | 8.23 |
| $\Gamma_+ Hz$ | 2.18 | 1.54 | 1.18 | 0.583 |
| $\Gamma_-$ Hz | 13.12 | 11.54 | 10.7 | 9.82 |
| $\beta \beta \alpha^{\gamma} / \partial \beta$ Hz | 4.94 | 2.64 | 1.52 | 0.029 |
| $\beta \beta^{\gamma} / \partial \beta$ Hz | -4.3 | -3.59 | -3.1 | -2.21 |
| $(\beta \beta^{N^+} / \partial \beta) / N$ | 0.2821 | 0.179 | 0.123 | 0.0515 |
| $n_i/N$ | 1.1679 | 1.27 | 1.33 | 1.4 |
| $N^{+}/n_1$ | 1.2445 | 1.14 | 1.09 | 1.037 |
| $a_+$ | 6.148 | 3.01 | 1.656 | -0.03 |
| $a_-$ | -5.364 | -4.09 | -3.379 | -2.29 |
| $s$ Hz | 9.4 | 2.83 | 1.34 | 0.022 |
| $T_0$ c | 2.17588 | 3.09648 | 4.037 | 6.4 |
| $\bar{T}_{\gamma(\lambda_n)}$ c | 0.6495 | 1.25492 | 1.84 | 5.975 |

Table 1. Used in the calculations of Fig. 2, the data obtained from relations (13) - (15) and expressions for the cumulant $C_i=1,2,3$ [5,6].

4. Conclusion

The average $FPT$ is associated with changes in entropy that occur during this process. The effect of such changes on the average $FPT$ is shown using electrons transferred through a metallic double dot in the Coulomb-blockade as an example. The proposed approach makes it possible to take into account any other changes in entropy that correspond to other possible processes occurring in the system. The mean $FPT$ values in accordance with expression (12) decrease when the system is influenced (for this process and distribution (1)).

It is shown that the average $FPT$ with a zero value of the argument of the Laplace transform of the $FPT$ distribution density does not reflect the effect of real processes on the average $FPT$. In fig. 1 shows how taking into account the changes in entropy accompanying the process of the $FPT$ affects the average value of $FPT$. In fig. 2 shows how the applied voltage affects the system.

In this paper, the possibilities of taking into account the influence of entropy changes on the average $FPT$ are shown. This is done on the basis of experimental results. It can be shown that this effect is different for different physical systems. For example, for neutrons in a nuclear reactor, the effect of changes in entropy on the reactor period during this period is negligible for almost all values of the multiplication factor (with some exceptions). The influence of external influences is shown on the example of voltage affects. However, the effects can be very diverse (temperature effects, mechanical, etc.). The approach proposed in [3, 4] opens up opportunities for the study of arbitrary influences by including them in changes in entropy. There are also opportunities to control the $FTP$, as noted in [3].
References

1. R. Metzler, G. Oshanin and S. Redner (ed), First-Passage Phenomena and Their Applications, Singapore: World Scientific, 2014, 608 p.
2. J. Masoliver, Random Processes: First-Passage and Escape, Singapore: World Scientific, 2018, 388 p.
3. V. V. Ryazanov, First-passage time and change of entropy, European Physical Journal B 94, Article number: 242 (2021).
4. V. V. Ryazanov, First passage time and statistical thermodynamics, arXiv:2109.11823.
5. S. Singh, P. Menczel, D. S. Golubev, I. M. Khaymovich, J. T. Peltonen, C. Flindt, K. Saito, É. Roldán & J. P. Pekola, Universal First-Passage-Time Distribution of Non-Gaussian Currents, Phys. Rev. Lett. 122(23): 230602, 1-6 (2019).
6. S. Singh, É. Roldán, I. Neri, I. M. Khaymovich, D. S. Golubev, V. F. Maisi, J. T. Peltonen, F. Jülicher & J. P. Pekola, Extreme reductions of entropy in an electronic double dot, Physical Review B, 99, 115422, pp. 1-15 (2019). https://doi.org/10.1103/PhysRevB. 99.115422, arXiv:1712.01693 (2017).
7. W. Lu, Z. Ji, L. Pfeiffer, K. W. West, and A. J. Rimberg, Real-time detection of electron tunnelling in a quantum dot. Nature, 423, 422-425 EP (2003).
8. C. Flindt, C. Fricke, F. Hohls, T. Novotný, K. Netočný, T. Brandes, and R. J. Haug, Universal oscillations in counting statistics, Proc. Natl. Acad. Sci., 106, 10116 (2009).
9. M. Albert, C. Flindt, and M. Büttiker, Distributions of Waiting Times of Dynamic Single-Electron Emitters, Phys. Rev. Lett. 107, 086805 (2011).
10. D. Dasenbrook, C. Flindt, and M. Buttiker, Floquet Theory of Electron Waiting Times in Quantum-Coherent Conductors, Phys. Rev. Lett. 112, 146801 (2014).
11. N. U. Prabhu, Queues and Inventories. John Wiley & Sons, Inc., New York, London 1965. 275 p.
12. I. I. Gichman, A. V. Skorochod, The theory of stochastic processes, II, New-York, Springer-Verlag, 1974.
13. A. N. Shiryaev, Statistical Sequential Analysis, Amer. Mathematical Society, 1973, 174 p.
14. V. V. Ryazanov, S. G. Shpyrko, First-passage time: a conception leading to superstatistics, Condensed Matter Physics, 9, 1(45), 71-80 (2006).
15. V. V. Ryazanov, First-passage time: a conception leading to superstatistics. I. Superstatistics with discrete distributions. Preprint: physics/0509098, 2005.
16. V. V. Ryazanov, First-passage time: a conception leading to superstatistics. II. Continuous distributions and their applications. Preprint: physics/0509099, 2005.
17. V. V. Ryazanov, Lifetime distributions in the methods of non-equilibrium statistical operator and superstatistics. European Physical Journal B, 72, 629–639 (2009).
18. S. Redner, A Guide to First-Passage Processes (Cambridge University Press, Cambridge, UK, 2001).
19. W. Feller, An Introduction to Probability Theory and Its Applications (John Wiley, New York, 1957).
20. S. Levitov, H. Lee, and G. B. Lesovik, Electron counting statistics and coherent states of electric current, J. Math. Phys. 37, 4845 (1996).
21. K. Saito and A. Dhar, Waiting for rare entropic fluctuations, Europhys. Lett., 114, 50004 (2016).
22. K. Ptaszyński, First-passage times in renewal and nonrenewal systems, Phys. Rev. E, 97, 012127 (2018).
23. D. V. Averin and A. A. Odintsov, Macroscopic quantum tunneling of the electric charge in small tunnel junctions, Phys. Lett. A, 140, 251-257 (1989).
24. D. V. Averin and Y. V. Nazarov, Single - electron charging of a superconducting island, Phys. Rev. Lett., 69, 1993-1996 (1992).
25. J. König, H. Schoeller, and G. Schön, Co tunneling at Resonance for the Single-Electron Transistor, *Phys. Rev. Lett.*, **78**, 4482 (1997).

26. O. Maillet, P. A. Erdman, V. Cavina, B. Bhandari, E. T. Mannila, J. T. Peltonen, A. Mari, F. Taddei, C. Jarzynski, V. Giovannetti, and J. P. Pekola, Optimal Probabilistic Work Extraction beyond the Free Energy Difference with a Single-Electron Device, *Phys. Rev. Letters*, **122**, 150604 (2019).

27. N. G. van Kampen, *Stochastic processes in physics and chemistry.* (New York: Elsevier North-Holland, 1981, 419 p.).

28. W. G. van der Wiel, S. De Franceschi, J. M. Elzerman, T. Fujisawa, S. Tarucha, and L. P. Kouwenhoven, Electron transport through double quantum dots, *Rev. Mod. Phys.* **75**, 1 (2002).

29. D. Jou, J. Casas-Vazquez, G. Lebon, *Extended Irreversible Thermodynamics.* Berlin: Springer. 2010. 442 p.