A Study of the Lyapunov Stability of an Self-Excited Induction Generator

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Abstract. The self-excited induction generators (SEIGs) can provide attractive renewable energy sources. Since it can be used in an open-loop fashion which includes no control closed-loop, its inherent stability can be counted on to allow operation over a wide range of operating conditions. Unlike classical arguments based on transient and steady-state analysis, it is proposed rigorous holistic analytical stability arguments based on the full nonlinear transient state-space model of a SEIG under two-phase stationary reference frame using dynamical systems theory. The conditions for critical transient stability of the unloaded SEIG are obtained in the sense of Lyapunov. Analytic formulas from the limit cycle conditions give the values of the range of capacitance, the range of rotor speed and the stator frequency for self-excitation. Analytic formulas from steady-state conditions give all the steady-state parameter and state values. Thus, the analytical results yield a set of simple and universal analytic formulas that can predict and evaluate the transient and steady-state stability characteristics of the SEIG. Good agreement between the experimental results and computed results validates the analytical results.

1. Introduction
The increasing concern for the environment and energy resources has motivated the world towards rationalizing the use and exploring renewable and sustainable energy sources to satisfy the ever-increasing energy demand [1]. Although synchronous generators have been used for power generation traditionally, induction generators have gained interest and are increasingly being used in recent years because of the relative advantageous features over synchronous generators, including brushless, ruggedness, low-cost, maintenance and operational simplicity, self-protection against short circuit faults, good dynamic response, and capability to generate power at varying speeds in renewable energy applications [2]. These features facilitate induction generators operation in stand-alone or isolated mode. Such induction generators generating power off-grid are referred to as self-excited induction generators (SEIGs). The research and development on SEIGs are mainly the analytical and technical considerations as well as various issues addressed in [3] towards such applications of renewable energy resources as micro/mini-hydro and wind energy.
Detailed performance studies of the SEIG under steady-state and various transient conditions are important for the optimal utilization of its advantageous features. The analysis of steady-state performance is important for ensuring good quality power and assessing the configuration for a particular application, while analysis of transient performance helps in determining the suitability of winding and effects of values of capacitor and rotor speed on the transient process. The unified analysis under steady-state and transient conditions can gain further insight into the inherent properties and operation principle of the SEIG.

A large number of articles have appeared on the steady-state analysis of the SEIG [4,5]. Based on the steady-state equivalent circuit of the induction generator, where the total loop-impedance [4] or the total nodal admittance at the magnetizing branch [5] is equated to zero, a mathematical model is developed to obtain the performance. Two nonlinear equations about real and imaginary parts of the impedance are thus obtained. There are various methods for solving unknown parameters \( f \) and \( X_m \), including numerical techniques, iterative techniques, and optimization techniques. However, there exists great limitation in the above techniques used for computing unknown parameters of steady-state performance. No solution may be found due to divergence with the times.

The transient analysis of the SEIG has appeared in many articles, most of which are related to voltage buildup due to self-excitation and load perturbation [6-9]. Various transient models have been reported to study the dynamic behavior of the SEIG using either currents or fluxes as state variables, but two phase \( dq \) model has proved to be reliable and accurate. The frequency variation and the effect of magnetic saturation with time can be explicitly observed. A standard approach uses the generalized model of the induction machine to search for parameters such that the system of differential equations describing the SEIG becomes unstable, where the SEIG voltages diverge exponentially from arbitrarily small initial conditions and converge to sinusoidal voltages, with magnitude and frequency depending on the system parameters. Adopting the complex description of the SEIG, the analytic conditions significantly reduce complexity of the coefficients [8], but increases the difficulty in proof and computation with complex, with lack in a concise and explicit physical significance of the model and a thorough recognition of its inherent property and operation principle of the SEIG.

To provide a unified analysis of the transient and steady-state stability, the paper starts by developing a transient equivalent circuit and deriving a state-space model of the unloaded squirrel-cage SEIG with nonlinearity in two phase stationary reference frame. Different from [8,9], the mathematic model in the paper carefully integrates all nonlinearities in the state-space model. Using the concept of stability in the sense of Lyapunov, it is proposed holistic analysis approaches to enable the unified stability analysis of all the operating points. By introducing nonlinear dynamic system theory, the analytic results are more concise and explicit than that of [9] and the computations are simpler. Main contributions of the paper include theoretical analysis approaches of the unified transient and steady-state stability, an analysis of transient stability and analytic conditions for critical transient stability, analysis of steady-state stability and analytic steady-state conditions, explicit formulas of parameters for the self-excitation, a concise equivalently transformed method, explicit formulas of the steady-state operating points, and the error analysis of the computed results by the steady-state conditions analytic formulas and corresponding results.

2. Initial Transient Stability Analysis

2.1. Transient Equivalent Circuits of an Unloaded SEIG

First of all, the equations of the terminal voltages and the flux linkages of the induction machine in the \( abc \) stationary frame needs to be transformed into the model type more simple to analyze in the \( a\beta \) stationary frame through the \( abc/a\beta \) reference frame transformation. If the slot effect is not taken into account, the air gap width distributed on the whole circle of the induction machine is even. The coefficient of the magnetizing inductance is thus transformed into the time invariant coefficient in the arbitrary coordinate frame, so that the magnetizing inductance is only a function of the magnetizing current, the characteristic of which never changes with the reference transformation [10].
SEIG in the two-phase $\alpha\beta$ stationary reference frame, where the positive direction is indicated and physical parameters denote real values transformed into the stator side. $M$ is the stator-rotor mutual inductance, also called magnetizing inductance. $L_{lr}$ and $L_{ls}$ are the rotor and stator leakage inductances, respectively, $R_r$ and $R_s$ are the rotor and stator resistances, respectively, $\omega$ is the rotor speed, and $C$ is the excited capacitor connected in parallel with stator windings, resulting in the additional voltage-current equations. $u_{\alpha s}, u_{\beta s}$ are the stator voltages, $i_{\alpha s}, i_{\beta s}$ are the stator currents, $i_{\alpha r}, i_{\beta r}$ are the rotor currents transformed into the stator frame of reference, and $\Psi_{\alpha r}, \Psi_{\beta r}$ are the rotor flux linkages transformed into the stator frame of reference, respectively. For the purpose of the study, the rotor speed is assumed constant. For that magnetizing inductance is a function of magnetizing current, which is consecutively varying with time, this transient equivalent circuits are nonlinear.

Seen from Fig.1 above, the self-inductances of the rotor and stator windings are $L_r = M + L_{lr}$ and $L_s = M + L_{ls}$, respectively. Thus the dynamic magnetizing current is computed as follows.

$$I_m = \sqrt{(i_{\alpha s} + i_{\alpha r})^2 + (i_{\beta s} + i_{\beta r})^2}$$  \hspace{1cm} (1)

2.2. Transient Mathematical Model of an Unloaded SEIG

A standard explicit form of the transient mathematical model for unloaded SEIGs is equation (2) as below [9].

$$\dot{X} = AX$$ \hspace{1cm} (2)

In this form, some standard numerical integration methods can be used. Nevertheless, the transient system remains nonlinear since the coefficient matrix $A$ is a function of $X$, showing that it is an autonomous system.

$$A = \begin{bmatrix}
    -\frac{R_s}{\sigma L_s} & \frac{\omega M^2}{\sigma L_s} & \frac{MR_r}{\sigma L_s} & \frac{\omega M}{\sigma L_s} & -1 & 0 \\
    -\frac{\omega M^2}{\sigma L_s} & -\frac{R_s}{\sigma L_s} & -\frac{\omega M}{\sigma L_s} & \frac{MR_r}{\sigma L_s} & 0 & -1 \\
    \frac{MR_r}{\sigma L_s} & -\frac{\omega M}{\sigma L_s} & -\frac{R_s}{\sigma L_s} & -\frac{\omega}{\sigma} & 0 & M \\
    \frac{\omega M}{\sigma L_s} & \frac{MR_r}{\sigma L_s} & -\frac{\omega}{\sigma} & \frac{\omega}{\sigma} & 0 & M \\
    \frac{1}{C} & 0 & 0 & 0 & 0 & 0 \\
    0 & \frac{1}{C} & 0 & 0 & 0 & 0
\end{bmatrix}$$

where $\sigma$ denotes the leakage coefficient $\sigma = 1 - M^2/(L_r L_s)$.

Equation (1) shows that the magnetizing inductance $M$ is a definite function of the magnetizing current $I_m$, which depends on the magnetizing curve of the unloaded induction machine. There exist the magnetic saturation characteristics in magnetic circuits of induction machines. When the magnetizing current increases sufficiently high so that the magnetic saturation is encountered, with the magnetizing inductance $M$ reduced, the magnetizing curve becomes a nonlinear shape. However, different opinions on the general shape of a magnetizing curve lie in the shape of the curve at low currents. Most of studies often neglect the ascending part of the curve at low currents in the magnetic unsaturation region, while several other works, for accuracy of modeling self-excitation in induction generators, take it into account. So the accurate complete curve includes an ascending part rising from an initial value to the maximum, a more or less flat part at the maximum corresponding to a linear magnetic regime, and a descending part corresponding to the magnetic saturation. Therefore, equation (2) of the unloaded SEIG system only in the linear magnetic regime of the curve is linear, while that in others is nonlinear.
3. Transient and Steady-State Stability Analysis

3.1. Transient Stability Analysis

There are various kinds of stability problems that arise in the study of dynamical systems. Transient stability is concerned with stability of equilibrium points, while steady-state stability is concerned with stability of periodic orbits. Both stability of equilibrium points and periodic orbits are usually characterized in the sense of Lyapunov. An equilibrium point is stable if all solutions starting at nearby points stay nearby; otherwise, it is unstable. It is asymptotically stable if all solutions starting at nearby points not only stay nearby, but also tend to the equilibrium point as time approaches infinity. In the case of an initial transient system, the currents are zero. Since this transient system is an autonomous system, it is necessary for self-excitation that the initial state is not zero, which is called disturbance. The trace of transient operating points is obviously nonlinear, so it is needed to linearize around the corresponding transient point. Suppose that the initial state is zero, the linearization is no longer needed, the determinant of the state coefficients matrix is then directly obtained.

\[
\det(A) = (\omega^2 L_x^2 + R_x^2)/(C^2 L_y^2 \sigma^2) > 0
\]  

(3)

It is obtained from \(AX=0\) the system of the unloaded SEIG has only one equilibrium point, \(X_e=0\). Self-excitation is a process of self-oscillation. This process is surely not along with energy degradation, which is the basic characteristic of asymptotically stability in the sense of Lyapunov. If this initial transient system is asymptotically stable, an arbitrary initial state \(X_0\) around the equilibrium point of the initial transient system will converge to \(X_e=0\) ultimately. Therefore, an initial transient operating point may behavior three possible modes, trending to the equilibrium point \(X_e=0\) infinitely, a limit cycle, and diverging away from \(X_e\). The critical analytic conditions for the initial transient stability of the unloaded SEIG are considered to be a limit cycle, which is a type of equilibrium states of energy conversion. However, when the initial state of this initial transient system is zero, \(X_0=0\), coinciding with the equilibrium point, even the transient linear system is unstable, it stays the original point regardless of whether this system is unstable.

3.2. Steady-state Stability Analysis

The determination of steady-state stability is a bit different from that of transient stability, not only for the steady states is a type of transient operating points of critical transient stability, but for they represent equilibrium states of energy conversion from external drive mechanical energy added with the initial electromagnetic energy to electromagnetic and thermal energy produced through self-excitation. The determination of the steady-state stability depends on the transient stability of the operating point during self-excitation buildup before arrival at the energy equilibrium states.

Three cases are classified as follows.

Case 1: If the linearized system around the transient operating point a moment before arrival at steady states is asymptotically stable, the limit cycle and all the steady states are unstable. An arbitrary small perturbation will draw the steady-state operating points to the stable \(X_e=0\). But this case is unwanted, because the self-excitation will decay to zero.

Case 2: If the linearized system around the transient operating point a moment before arrival at steady states is stable on the boundary, the steady states are also stable. Obviously, the SEIG is operating in the linear magnetizing region, with magnetic unsaturated and the magnetizing inductance kept the maximum value. So the periodic orbit of limit cycle is stable.

Case 3: If the linearized system around the transient operating point a moment before arrival at steady states is unstable, the steady states are stable. As long as \(d>0\), the operating point \(X\) keeps diverging away from the equilibrium point of the original point, while the energy conversion makes the electromagnetic energy increase until the energy equilibrium of the SEIG system reaches, with \(d=0\). The limit cycle is also stable.

To facilitate computations, an analytic approximation of the magnetizing characteristic curve obtained experimentally is used. By experimental measurement, the magnetizing curve or relationship of the magnetizing inductance \(M\) represented by the magnetizing current \(I_m\) is given by
4. Example Analysis and Results Comparison

4.1. Example analysis

The example is used to test the validation of the analytic formulas obtained from the steady-state conditions, and proves that both the limit cycle and steady-state operating point conditions are essential for the steady states. A squirrel-cage three-phase induction machine is used as an example. Based on the stability analysis above, the paper may neglect nonlinearity in the magnetic unsaturated region of the unloaded induction machine. However, the analytic formulas obtained from the steady-state conditions are applicable to not only some special form of the magnetizing curve, but to all forms of the magnetizing curves.

The rated values of the experimental induction machine give \( P_N = 2.2 \text{ kW}, \) \( U_N = 380 \text{ V (Y-connection)}, \) \( I_N = 5 \text{ A}, \) \( f_{SN} = 50 \text{ Hz}. \) The rated speed for the induction motor tested as a generator is \( n_N = 1430 \text{ rpm}, \) the parameters are \( R_s = 3.383 \Omega, \) \( R_r = 2.973 \Omega, \) \( L_{ls} = 0.008479 \text{ H}, \) \( L_{lr} = 0.008479 \text{ H}, \) and the approximation of the magnetizing curve is developed and verified by experimental.

\[
M = f(I_m) \quad (4)
\]

\[
\begin{align*}
M &= \frac{0.2875}{I_m} \quad \text{if} \quad I_m \leq 1.163 \\
&= \frac{3.500}{(I_m + 11.01)} - 1.163 < I_m \leq 2.162 \\
&= \frac{3.099}{(I_m + 9.503)} - 2.162 < I_m \leq 3.046 \\
&= \frac{2.519}{(I_m + 7.152)} - 3.046 < I_m \leq 4.553 \\
&= \frac{1.527}{(I_m + 2.544)} \quad \text{if} \quad I_m \geq 4.553
\end{align*}
\]

4.2. Results Comparison of Boundaries of Self-excitation

Figure 1 shows the upper and lower limits of the excited capacitance as functions of the rotor speed value. The figure is also consistent with the Figure 2 of [8].

![Figure 1](image)

Figure 1 Critical excited capacitances for self-excitation buildup.

Consider the generator of [9], with \( R_s = 1.7 \Omega, \) \( R_r = 2.7 \Omega, \) \( L_{ls} = L_{lr} = 0.0114 \text{ H}, \) and \( M = 0.180 \text{ H}, n_p = 2. \) Let \( C = 300 \text{ mF}, \) the analytic formulas gives \( \omega_{\min} = 133.4 \text{ rad/s} \) and \( \omega_{\max} = 930.7 \text{ rad/s}. \) The minimum mechanical speed of 66.70 rad/s and the maximum of 465.4 rad/s are consistent with that under no-load of the paper. Other comparison results of the unloaded SEIG are all completely same. For \( C = 100 \text{ mF}, \) the range of rotor speeds for self-excitation is increased to 229.4-1615 rad/s, and the range vanishes for \( C > 28.9 \text{ mF}. \) Lower capacitance requires a higher speed although, for low sufficient capacitance, no rotor speed produces self-excitation. Using the analysis results, one may plot the combinations of rotor speed and capacitor values for the self-excitation. Figure 2 shows the upper and lower limits of rotor speed as functions of the capacitance. This figure is also consistent with the figure of [9].
Although the critical analytic formulas of the initial transient stability for the unloaded SEIG can be used as the criterion for which the process of self-excitation occurs, the approach to the initial transient stability of the unloaded SEIG can be used in other type of transient systems, where it only needs to locally linearize the original nonlinear state-space equation around a steady-state operating point. For this, the limit cycle condition becomes the critical states of the transient and steady-state operating points for the process of self-excitation buildup.

5. Conclusion

The paper presents a transient equivalent circuit model and then derives a state-space transient model of a squirrel-cage unloaded SEIG in two-phase stationary reference frame. By introducing the dynamical system theory, holistic analysis approaches based on Lyapunov stability theory are proposed to analyze the unified transient and steady-state stability of the SEIG. It is regarded that the transient stability is the stability of an equilibrium point, while the steady-state stability is the stability of a periodic orbit. Through transient stability analysis, it is found that the sufficient conditions for self-excitation are the limit cycle and a non-zero initial state, $X_0 \neq 0$. The critical analytic formulas and ranges of parameters for self-excitation of the unloaded SEIG are obtained. The steady-state conditions include the limit cycle and steady-state operating points. An equivalently transformed method from the complex frame to the real frame is proposed to compute all steady-state operating points. Good agreement is observed between the computed and experimental results. The analytic formulas are more concise and explicit with no complicated derivations. The theoretical approaches unify the transient and steady-state analysis of the SEIGs, showing the dynamical system theory can help gaining insights into the inherent principle of induction generators.

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