The quantum Ettingshausen effect in parabolic quantum wells with in-plane magnetic field in the presence of laser radiation under the influence of confined optical phonon

Quynh Nguyen Thi Lam¹, Huong Nguyen Thu², Duc Nguyen Ba³ and Bau Nguyen Quang¹

¹Hanoi University of Science, 334 Nguyen Trai Street, Thanh Xuan District, Hanoi, Vietnam
E-mail: quynhnguyenlam@ntthnue.edu.vn (Quynh Nguyen Thi Lam)
E-mail: nguyenquangbau54@gmail.com (Bau Nguyen Quang)
²Air Defence - Air Force Academy, Kim Son commune, Son Tay town, Hanoi, Vietnam
E-mail: huong146314@yahoo.com (Huong Nguyen Thu)
³Tan Trao University, Trung Mon commune, Tan Son District, Tuyen Quang Province, Vietnam
E-mail: hieutruongdhtt@gmail.com (Duc Nguyen Ba)

Abstract. The quantum Ettingshausen effect in a parabolic quantum well subjected to a constant electric field, magnetic field in the presence of laser radiation is studied by using the quantum kinetic equation method. The analytic expressions for conductivity tensors and dynamic tensors as well as the Ettingshausen coefficient (EC) are obtained in the case of the confined electron-confined optical phonon (COP) scattering. The EC depends on specific quantities for external field (such as magnetic field, amplitude and frequency of laser radiation), temperature of the system and quantum well length, especially quantum number m characterizing the confinement of optical phonon (OP). When m is set to zero, we get the results corresponding to the case of unconfined OP. The analytic results are numerically evaluated and graphed for GaAs/AlGaAs parabolic quantum well. When examining the effect of temperature, the EC has greater values due to the COP. Meanwhile, when quantum well length and amplitude of the laser radiation (LR) increase, the EC has greater values without the confinement of OP. Because of the approach of system to bulk semiconductor structure when the quantum well length approaches micrometer-size, the EC reaches constant. The quantum number m leads to the change of resonance peaks position within the change of the magnetic field and the frequency of LR. In addition, the EC exerts non-linear dependence on the amplitude the LR. All results indicated that the COP affects not only qualitatively but also the transformation rules of the EC. It provides new insights and contributes to the orientation of research into quantum effects in low-dimensional semiconductor systems (LDS).

1. Introduction
In LDS, most of the electronic properties change significantly, especially the appearance of size effects. In quantum-sized structures, the laws of quantum mechanics take effect, the carriers are limited to the size of De Broglie wavelength. Which change the wave function
and energy spectrum of the electron. The confinement of the electron as well as its influence on kinetic effects in LDS have been studied. A great number of researches related to these issues were published. The values of absorption coefficient are dominated by the correction of electron–phonon interaction effect and the absorption peaks are made blue shift and become wider by the correction of the electron – phonon effect and the absorption peaks are made blue shift and become wider by the correction of the electron-phonon effect on the energies of the electron in cylindrical quantum wires [1]. The third-harmonic generation susceptibilities in cylindrical quantum wires is greatly influenced by the parabolic confinement potential and the applied electric field [2]. Under the drift of two-dimensional electrons in quantum well, the coupling between electrons and confined modes is a non-monotonous function of the wave vector for each of the phonon branches [3].

The magneto-thermoelectric effect called Ettingshausen effect has been attracting more and more attention. This effect has been theoretically investigated in many materials such as silicon [4], high-temperature superconductors [5], bulk semiconductor [6] and quantum well [7, 8]. In bulk semiconductor system, the generation of electron-hole pairs and their recombination at each side of the sample is the main causes of Ettingshausen effect [6]. Due to the quantization of wave function as well as energy spectrum of electrons, the results obtained for magneto-thermoelectric effect in quantum well are different from bulk semiconductor. For example: the EC in PQW has been greater than the EC in the bulk semiconductor at the same temperature region [7]; the temperature dependence of both the longitudinal and transverse Nernst-Ettingshausen effects is non-monotonic [8]. However, Ref. [7, 8] took interest in only the electron confinement but not the phonon confinement. Therefore, the theory of the Ettingshausen effect in quantum well involving both electron and phonon confinement is still an open problem.

Many published researches show that the phonon confinement influences significantly on quantum effects in LDS [9–11]. The acoustic phonon confinement enhances the radio-electric field in a quantum well [9]. Confinement of LO-phonons has a remarkable impact on the magneto-phonon resonance condition and gives the additional resonance peaks of the conductivity tensor when investigating the Hall effect in doped semiconductor superlatices [10]. Electrons excites into confined states with a large kinetic energy strongly influence the overall capture times due to emission of confined phonons in semiconductor superlatices [11].

In this work, we have taken in-plane magnetic field and the influence of LR into account for Ettingshausen effect in PQW and considered the confined electrons–COP scattering in detail. This paper is organized as follows: we briefly present the basic expressions obtained by using the quantum kinetic equation method in Sec.2; numerical results and discussion for GaAs/AlGaAs PQW are given in Sec.3; Sec.4 shows remarks and conclusions.

2. Calculation of the EC in PQW by the quantum kinetic equation method.

We apply the quantum kinetic equation method [12, 13] to study the Ettingshausen effect in a PQW (with confinement potential $V_z = m_e\omega_z^2 z^2/2$) subjected to a crossed dc electric field $\mathbf{E}_1 = (0, 0, E_1)$ and magnetic field $\mathbf{B} = (0, B, 0)$ with a vector potential $\mathbf{A} = (zB, 0, 0)$ in the presence of LR with electric field vector $\mathbf{E} = (E_0 \sin \omega t, 0, 0)$ [7]. Due to the change of the direction of the magnetic field, the wave function and energy spectrum of electrons are different from the case of the perpendicular magnetic field (the magnetic field is perpendicular to the free-moving plane of electrons) [14]. In this case, the movement of the electrons is limited. Energy of an electron receives intermittent value:

$$\varepsilon_N \left( k_x \right) = \hbar \omega_p \left( N + \frac{1}{2} \right) + \frac{1}{2m_e} \left[ \hbar^2 k_x^2 - \left( \frac{\hbar k_x \omega_c + eE_1}{\omega_p} \right)^2 \right]$$

Here: $m_e$ and $e$ are the effective mass and the charge of a conduction electron, respectively; $\omega_p = \sqrt{\omega_z^2 + \omega_c^2}$ ($\omega_c = eB/m_e$ is the cyclotron frequency of electron, $\omega_z$ is confined frequency);
The Hamiltonian of the confined electrons–COP in the above-mentioned PQW can be written as:

\[
H = \sum_{N,n,k_x} \varepsilon_N \left( \vec{k}_x - \frac{e}{\hbar c} \vec{A} (t) \right) a^+_N a_{N,n,k_x} + \sum_{m,q_x} \hbar \omega_{m,q_x} b^+_{m,q_x} b_{m,q_x} + \sum_{q} \varphi (\vec{q}) a^+_N a_{N,n,k_x} + \sum_{m,n',q_x} D^m_{N,N',n,n'} (\vec{q}_x) a^+_N a_{N,n,k_x} (b^+_{m,-\vec{q}_x} + b_{m,\vec{q}_x})
\]  

(2.2)

Where \( \hbar \omega_{m,q_x} \) is the energy of an optical phonon with the wave vector \( \vec{q}_x = (\vec{q}_x, q_z) \) with \( q_z = \frac{m \pi}{L} \) (m is quantum number which specify the confinement of OP); \( a^+_N a_{N,n,k_x} \) and \( b^+_m b_{m,q_x} \) are the creation and annihilation operators of electron (phonons), respectively;

\[
\left| D^m_{N,N',n,n'} (\vec{q}) \right|^2 = |C_m (\vec{q})|^2 \left| I_{m,n'} (\pm \vec{q}) \right|^2 \left| J_{N,N'} (u) \right|^2 \text{ with } |C_m (\vec{q})|^2 = \frac{2\pi e^2 \hbar \omega_c}{\varepsilon_0 \varepsilon_0} \left( \frac{1}{\chi_s - \chi_0} \right) \frac{1}{q_x^2 + q_z^2}
\]

is the confined electron–COP interaction constant, \( \varepsilon_0 \) is the electric constant, \( \varepsilon_0 \) is the normalization volume of specimen; \( \chi_s \) and \( \chi_0 \) are the static and the high frequency dielectric constants; \( I_{m,n'} (\vec{q}) = \langle n | e^{i\vec{q} \cdot \vec{z}} | n' \rangle \) is the form factor of electrons; \( |J_{N,N'} (u)|^2 = \frac{\pi}{N} e^{-u N^N N^N} \left( L_{N^N}^N (u) \right)^2 \) is the associated Laguerre polynomial \( L_{N^N}^N (u) \) is the associated Laguerre polynomial, \( u = l_B^2 q^2 \) with \( l_B = \sqrt{\hbar / m_e \omega_c} \) and \( \varphi (\vec{q}) = (2\pi i)^3 \left\{ e^{i\vec{E}_0} + \omega_c \left[ \vec{q}, \vec{h} \right] \frac{\partial f}{\partial \vec{q}} \right\} \) is scalar potential.

The quantum kinetic equation of average number of electrons is:

\[
i\hbar \frac{\partial f_{N,n,k_x} (t)}{\partial t} = \left\{ \left[ a^+_N a_{N,n,k_x}, H \right] \right\}_t
\]

(2.3)

in which \( f_{N,n,k_x} (t) = a^+_N a_{N,n,k_x} \).

By using Hamiltonian (2.2), and the order of calculation as in the works [7, 9, 10], the quantum kinetic equation of average number of electrons is obtained in the single (constant) scattering time approximation:

\[
\begin{align*}
&\frac{\partial f_{N,n,k_x}}{\partial t} + (\epsilon E_0 + \hbar \omega_c \left[ \vec{k}_x, \vec{h} \right]) \frac{\partial f_{N,n,k_x}}{\partial \vec{k}_x} + \frac{2\pi}{\hbar} \sum_{N',N,m,q_x} \left| D_{N,N',n,n'} (\vec{q}_x) \right|^2 \sum_{l} J_l^2 \left( \frac{\epsilon E_0}{\hbar c \omega_c^2} \right) \times \\
&\times \left\{ \left[ f_{N',n',k_x+q_x} \left(N_m q_x + 1 \right) + f_{N,n,k_x} \right] \left[ N_{m,q_x} \left(N_{m,q_x} + 1 \right) \right] \right\} \times \\
&\times \delta \left( \varepsilon_{N'} \left( \vec{p}_x + \vec{q}_x \right) - \varepsilon_N \left( \vec{k}_x \right) - \hbar \omega_{m,q_x} + \hbar \omega \right) + \left[ f_{N',n',k_x-q_x} \left(N_{m,q_x} \right) - f_{N,n,k_x} \left(N_{m,q_x} + 1 \right) \right] \\
&\times \delta \left( \varepsilon_{N'} \left( \vec{k}_x - \vec{q}_x \right) - \varepsilon_N \left( \vec{k}_x \right) - \hbar \omega_{m,q_x} + \hbar \omega \right)
\end{align*}
\]

(2.4)

Here \( N_{m,q_x} = b^+_{m,q_x} b_{m,q_x} \) is the equilibrium distribution function of the phonons; \( \frac{\hbar}{B} = \frac{\vec{k}_x}{B} \) is the unit vector in the direction of magnetic field.

In the similar way as in Ref. [7], we get the analytical expressions for the total current density and the thermal flux density. After some manipulation, the analytical expression of conductivity
tensors and dynamic tensors are achieved:

\[ \sigma_{xx}(m) = \frac{e}{m_e} b_0 \{ a + [B_1(m) + B_3(m)] b^-_1(m) \kappa^-_1 + B_2(m) + B_4(m) b^+_1(m) \kappa^+_1 + B_5(m) + B_6(m) b^-_2(m) + B_7(m) b^+_3(m) + B_8(m) b^-_3(m) \kappa^-_3 \} \]

(2.5)

\[ \sigma_{xy}(m) = -\frac{e}{m_e} b_0 \omega_c \{ a \tau(\varepsilon_F) + (B_1 + B_3) b^-_1(m) [\tau(\varepsilon_F) + \tau(\varepsilon^-_1)] + (B_2 + B_4) b^+_1(m) [\tau(\varepsilon_F) + \tau(\varepsilon^+_1)] + B_5 b^-_2(m) [\tau(\varepsilon_F) + \tau(\varepsilon^-_2)] + B_6 b^+_3(m) [\tau(\varepsilon_F) + \tau(\varepsilon^+_3)] + B_7 b^+_3(m) [\tau(\varepsilon_F) + \tau(\varepsilon^+_3)] \} \]

(2.6)

\[ \gamma_{xx}(m) = \frac{1}{m_e} b_0 \{ h \omega_m [B_1(m) + B_3(m)] b^-_1(m) \kappa^-_1 - h \omega_m [B_2(m) + B_4(m)] b^+_1(m) \kappa^+_1 + \varepsilon_0 B_5(m) b^-_2(m) \kappa^-_2 + \varepsilon_0 B_6(m) b^-_3(m) \kappa^-_3 \} \]

(2.7)

\[ \mu_{xx}(m) = \frac{1}{m_e} b_0 \{ -h \omega_m [B_1(m) + B_3(m)] b^-_1(m) \kappa^-_1 + h \omega_m [B_2(m) + B_4(m)] b^+_1(m) \kappa^+_1 + \varepsilon_0 B_5(m) b^-_2(m) \kappa^-_2 + \varepsilon_0 B_6(m) b^-_3(m) \kappa^-_3 \} \]

(2.8)

\[ \mu_{xy}(m) = b_0 \omega_c \{ h \omega_m [B_1(m) + B_3(m)] b^-_1(m) [\tau(\varepsilon_F) + \tau(\varepsilon^-_1)] - h \omega_m [B_2(m) + B_4(m)] b^+_1(m) [\tau(\varepsilon_F) + \tau(\varepsilon^+_1)] + \varepsilon_0 B_5(m) b^-_2(m) [\tau(\varepsilon_F) + \tau(\varepsilon^-_2)] + \varepsilon_0 B_6(m) b^-_3(m) [\tau(\varepsilon_F) + \tau(\varepsilon^-_3)] \}

(2.9)

\[ \varphi_{xx}(m) = -\frac{1}{m_e} b_0 \{ (h \omega_m)^2 [B_1(m) + B_3(m)] b^-_1(m) \kappa^-_1 + (h \omega_m)^2 [B_2(m) + B_4(m)] b^+_1(m) \kappa^+_1 + (\varepsilon_0)^2 B_5(m) b^-_2(m) \kappa^-_2 + (\varepsilon_0)^2 B_6(m) b^-_3(m) \kappa^-_3 \}

+ (\varepsilon_0)^2 B_7(m) b^+_3(m) \kappa^+_3 + (\varepsilon_0)^2 B_8(m) b^-_3(m) \kappa^-_3 \}

(2.10)

Where: \( \tau(\varepsilon) = (\tau(\varepsilon_F) / \varepsilon_F)^{1/2} \) is the momentum relaxation time; \( \varepsilon_F \) is the Fermi energy of electron; \( \omega_m = \sqrt{\omega_i^2 - v_s^2 (m \pi / L)^2} \) is the frequency of COP (\( v_s \) is velocity parameter). In addition, when we set \( m \) to zero, we obtained: \( \omega = \sqrt{\omega_i^2 - v_s^2 (0. \pi / L)^2} = \omega_0 \) and it is the frequency of unconfined optical phonon.

\[ \varepsilon_1^\pm = \varepsilon_F \pm h \omega_m; \varepsilon_i^\pm = \varepsilon_F \mp h \omega_m \pm h \omega; \varepsilon_i^\pm = \varepsilon_F \mp h \omega_m \pm h \omega (i = 2, 3) \]

\[ \varepsilon^+_0 = h \omega_m \pm h \omega; \kappa^+_i = \left[ 1 - \omega_i^2 \tau(\varepsilon_F) \tau(\varepsilon_i^\pm) \right] (j = 1 \div 3); a = -\frac{eL_e}{\pi \sum_{N,m} \sqrt{\Delta_0}} \]
\[ b_1^+ (m) = \frac{\tau (\varepsilon^+_1)}{1 + \omega_c^2 \tau^2 (\varepsilon^+_1)^2}; b_k^+ (m) = \frac{\tau (\varepsilon^+_k)}{1 + \omega_c^2 \tau^2 (\varepsilon^+_k)^2} (k = 2, 3) \]

\[ B_\eta (m) = A_1 \sum_{n,n',m} \frac{I^m_{n,n',m} L_x}{8\pi^2 \sqrt{\Delta_\eta}} \left( \frac{eB}{\hbar} \right)^3 \left\{ a_\eta^+ e^{-\frac{1}{k_B T} \frac{\hbar^2 \omega_p^2}{2m_e \omega_p^2} g (a_\eta^-) \right\} \]

\[ + a^- e^{-\frac{1}{k_B T} \frac{\hbar^2 \omega_p^2}{2m_e \omega_p^2} g (a^-) \right\} \]

\[ B_\eta (m) = A_2 \sum_{n,n',m} \frac{I^m_{n,n',m} L_x}{8\pi^2 \sqrt{\Delta_\eta}} \left( \frac{eB}{\hbar} \right)^3 \left\{ a_\eta^+ e^{-\frac{1}{k_B T} \frac{\hbar^2 \omega_p^2}{2m_e \omega_p^2} g (a_\eta^-) \right\} \]

\[ + a^- e^{-\frac{1}{k_B T} \frac{\hbar^2 \omega_p^2}{2m_e \omega_p^2} g (a^-) \right\} \]

\[ B_\eta (m) = A_3 \sum_{n,n',m} \frac{I^m_{n,n',m} L_x}{8\pi^2 \sqrt{\Delta_\eta}} \left( \frac{eB}{\hbar} \right)^3 \left\{ a_\eta^+ e^{-\frac{1}{k_B T} \frac{\hbar^2 \omega_p^2}{2m_e \omega_p^2} g (a_\eta^-) \right\} \]

\[ + a^- e^{-\frac{1}{k_B T} \frac{\hbar^2 \omega_p^2}{2m_e \omega_p^2} g (a^-) \right\} \]

In which:

\[ g (a_\eta^+) = \left[ \left( \frac{a_\eta^+}{e} \right)^2 - \frac{2eE_1 \omega_c}{\hbar \omega^2} \left( \frac{a_\eta^+}{e} \right) + \frac{2m_e (N + 1/2) \hbar \omega_p^3 - e^2 E_1^2 - 2m_e \omega_p^2 \varepsilon_F}{\hbar^2 \omega^2} \right] \]

\[ a_\eta^+ = \frac{eE_1 \omega_c}{\hbar \omega^2} \pm \sqrt{\Delta_\eta}; \Delta_0 = \left( \frac{eE_1 \omega_c}{\hbar \omega^2} \right)^2 - \frac{2m_e (N + 1/2) \hbar \omega_p^3 - e^2 E_1^2 - 2m_e \omega_p^2 \varepsilon_F}{\hbar^2 \omega^2} \]

\[ \Delta_\eta = \left( \frac{eE_1 \omega_c}{\hbar \omega^2} \right)^2 - \frac{2m_e (N + 1/2) \hbar \omega_p^3 - e^2 E_1^2 - 2m_e \omega_p^2 \varepsilon_F}{\hbar^2 \omega^2} \]

\[ \Delta_\eta = \left( \frac{eE_1 \omega_c}{\hbar \omega^2} \right)^2 - \frac{2m_e (N + 1/2) \hbar \omega_p^3 - e^2 E_1^2 - 2m_e \omega_p^2 \varepsilon_F}{\hbar^2 \omega^2} \]

\[ A_1 = - \frac{4\pi^2 e^3 \omega_p^2 \varepsilon_0}{\varepsilon_0 \hbar \omega^2 \left( \frac{\hbar \omega_p}{eB} - 1 \right)} \left( \frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right); A_2 = - \frac{1}{2} \left( \frac{eE_0}{m_e \omega^2} \right)^2 A_1 \]

\[ A_3 = \frac{1}{4} \left( \frac{eE_0}{m_e \omega^2} \right)^2 A_1; I = \left( \sqrt{N + 1/2} + \sqrt{N + 1/2} \right) \frac{l_B}{2} \]

\[ I^m_{n,n'} = \left| \frac{1}{L \sqrt{2\pi n! n''!}} \int_{-\infty}^{+\infty} e^{i \frac{m \pi \eta}{2}} e^{-\frac{z^2}{2}} H_n (\xi) H_{n} (\xi) d\xi \right|^2 \]

of n-th order.
The expression of the EC is given by:

\[ E_C = \frac{1}{4 \times 10^{-7} \pi B \sigma_{xx}(m) \mu_{xy}(m) - \sigma_{xy}(m) \mu_{xx}(m)} \frac{\sigma_{xx}(m) \mu_{xy}(m) - \sigma_{xy}(m) \mu_{xx}(m)}{\gamma_{xx}(m) \mu_{xx}(m) - \sigma_{xx}(m) [\varphi_{xx}(m) - K_L]} \]  

(2.11)

with \( K_L \) is lattice heat conductivity [7]. In quantum well, the value of \( K_L \) is very small and it should be neglected for simplicity [15].

From the above analytical expression, we can see clearly that the results obtained is correct for all temperatures which are greater than 0K. According to Eq.2.11, the EC in PQW depends on many specific quantities for external field such as amplitude \( (E_0) \) and frequency \( (\omega) \) of the LR, magnetic field, temperature. From analytic expressions, we can see that the EC also depends on the specific quantities for the structure of PQW (quantum well length, confined frequency). Especially, the EC depends in a complicated way on the \( m \) - quantum number specific confinement of COP. It means the change of OP for wave vector and frequency due to confinement effect leads to abundant theoretical results and being added to resonance condition in PQW. The results for the case of unconfined optical phonons (OP) are obtained when we set \( m \) to zero in particular.

3. Numerical results and discussion

To clarify the influence of the COP on the EC in the presence of the LR in detail, we have considered a PQW of GaAs/GaAsAl. The characteristic parameters of sample are:

\[ v_s = 87300ms^{-1}, \ m_e = 0.067m_0, \ e = 2.07\varepsilon_0, \ \chi_0 = 12.9, \ \chi_\infty = 10.9, \ \varepsilon_F = 50meV, \ N = 1; N' = 3, \ n \text{ and } n' \text{ rate from 1 to 3}, \ m_e = 0.067m_0. \]

![Figure 1](image1.png)

**Figure 1.** The dependence of EC on temperature

As we see in Fig.1, the EC depends on temperature in a non-linear way in both cases (with and without the confinement of OP). At high temperature domain: the influence of the LR is less remarkable because of the chaotic movement of electrons; so, the EC decreases slowly and approaches to zero. Within confinement of OP, the value of the EC obtained is greater than the EC in the case of unconfined OP. The result for the case of unconfined OP is obtained by setting the quantum number \( m \) to zero and it could be found in Ref. [7].

Fig.2 describes clearly the change of the EC when the magnetic field increases. The graph shows that both lines oscillate and reach resonant point. The peaks of the red line (with COP) are not only higher but also not in different position from the peaks of the blue line (with unconfined OP).

These results can be explained as follows: both COP’s energy and interaction constant of electron-phonon systems depend on the quantum number \( m \) because of the
confinement of OP; these resonance peaks correspond to the condition: 
\[ 2\hbar \sqrt{(eB/m_e)^2 + \omega_z^2} \pm \hbar \sqrt{\omega_o^2 - v_s^2(m\pi/L)^2} \pm \hbar \omega = 0. \]

In Fig. 2, from the left to the right, resonance peaks of the EC without the confinement of OP correspond to the condition: 

\[ B = \frac{m}{4\pi}\left[(\omega_0 - \omega)^2 - \omega_z^2\right]^{\frac{1}{2}}, \]

\[ B = \frac{m}{4\pi}\left[(\omega_0 + \omega)^2 + \omega_z^2\right]^{\frac{1}{2}}. \]

In the case of confined OP are confined (m = 3), COP’s frequency is modified to 
\[ \omega_{COP} = \sqrt{\omega_o^2 - v_s^2(3\pi/L)^2}. \]

It not only gives the additional resonance peaks of the EC but also makes the resonance peaks position shift to higher magnetic field.

**Figure 2.** The dependence of EC on magnetic field

**Figure 3.** The dependence of EC on the LR
Fig. 3 indicates that the confinement of OP has significant influence on the EC in PQW when the specific quantities for the LR change. It is well-known that the presence of LR changes the wave function and the energy spectrum of the electron in PQW. Which leads to the change of the electron-phonon interaction constant and scattering area.

According to Fig. 3a, both lines fluctuate and reach resonance point. The EC changes both qualitatively and quantitatively due to the confinement of OP. We can see clearly that the red line (with COP) not only has more resonance peaks than the blue line (with unconfined OP) but peaks of the red line are also about 10 times taller than the blue line’s. Furthermore, the peak positions of the red line are shifted to the left in comparison with the blue line. It is easy to explain this result. The position of each peaks is determined by the condition: \( \omega = \pm \omega_p \mp \omega_{OP} \). Within confinement of OP, the frequency of LR is defined: \( \omega = \pm \omega_p \mp \sqrt{\omega_0^2 - v_s^2 (m \pi / L)^2} \). It means that the value of quantum number \( m \) plays important role in locating the resonance peaks.

The Fig. 3b shows that the confinement of OP is the cause of the decrease in magnitude of the EC. In addition, the EC has positive values and increases in non-linear way because of the increase of amplitude of the LR. These results obtained are the same whether OP is confined or not. Thus, the confinement of OP only changes the magnitude of the EC without changing the EC’s transformation rules under the influence of the LR.

![Figure 4.](image)

**Figure 4.** The dependence of the EC on the quantum well length

Fig. 4 shows the EC plotted as a function of the quantum well length. The dependence of the EC on quantum well length is non-linear. When the system is survey at nano-size, the EC decreases rapidly due to the increase of quantum well length. The EC reaches constant when the quantum well length increases and approaches micrometer-size with every state of OP due to the approach to bulk semiconductor structure of our model. In both case (with and without the
confinement of OP), the EC has positive values and decreases due to the increase of quantum well length. When the quantum number m is set to zero, we obtained the result for the case of unconfined OP. This result is completely consistent with the previous research about the dependence of the EC on the quantum well length in the same PQW [7].

4. Conclusions
In this work, we have studied the Ettingshausen effect in PQW subjected to a crossed dc electric and magnetic field in the presence of LR. The confined electron–COP scattering is taken into account. The analytic expression for the EC is obtained by using the quantum kinetic equation method. The influence of LR is interpreted by the dependence of the EC on the amplitude $E_0$ and the frequency $\omega$ of the LR besides the dependence on the magnetic $B$, the temperature $T$ and the quantum well length, quantum number $m$ as in the ordinary Ettingshausen effect. Due to significant contribution of the COP, theoretical results are different from the previous researches for Ettingshausen effect in LDS [7, 8]. All numerical results for GaAs/AlGaAs PQW indicate that the EC depends on the external field, the temperature and parameters of system. When we carry out the survey in the case of COP, the magnitude of EC is different from the case where OP are unconfined. On the other hand, the confinement of OP leads to the displacement of resonance peaks when examining the dependence of the EC on the magnetic field. We also get the results that fit to the case of unconfined OP when the quantum number $m$ is set to zero. Finally, the confinement of OP creates surprising changes of the Ettingshausen effect in the PQW and the results obtained contribute to the theory of quantum effect in LDS, especially in PQW.

Acknowledgements
This work was completed with financial support from Vietnam National University, Hanoi (QG.17.38).

References
[1] You-Bin Y U 2008 Commun. Theor. Phys. 49 1615–1618
[2] Wang G 2005 Phys. Rev. B 72 155329–155334
[3] Komirenko S M, Kim K W, Demidenko A A, Kochelap V A and Stroscio M A 2000 Phys. Rev 62 7459–7469
[4] Mette H, Gartner W W and Loscoce C 1960 Phys. Rev 117 1491–1493
[5] Ullah S and Dorsey A T 1990 Phys. Rev 65 2066–2069
[6] Paranjape B V and Levinger J S 1960 Phys. Rev 120 437–441
[7] Hang D T, Ha D T, Thanh D T T and Bau N Q 2016 Photonics Letters of Poland 8 79–81
[8] Hashimzade F M, Babayev M M, Mehdiyev B H and Hasanov K A 2010 Journal of Physics: Conference Series 245 12–15
[9] Long D T and Bau N Q 2015 Journal of Physics: Conference Series 627 12–19
[10] Bau N Q and Long D T 2018 Physica B:Condensed Matter 532 149–154
[11] Paula A M and Weber G 1995 J. Appl. Phys. 77 6306–6312
[12] Epshtein E M 1976 Sov. J. Thier. Phys. Lett 2 234–237
[13] Epshtein E M 1980 Sov. Phys. Tech. Semiconductors 14 1600–1601
[14] Quynh N T L, Ba C T V and Bau N Q 2019 VNU Journal of Science: Mathematics Physics 35 67–73
[15] Hashimzade F M, Hasanov K A, Mehdiyev B H and S C 2010 Physica Scripta 81 1–8