ABSTRACT  Modern power utilization has been forced to operate close to their stability limits, which may lead to voltage instability or collapse owing to the stressed operating conditions of networks. This raises the need to develop a new technique to predict the level of voltage security under different operating conditions to detect critical lines or buses that operate close to its stability limit. This study proposes a new line voltage stability index (BVSI) to identify weak lines and buses for various loading conditions and network configurations. Comparative studies between the existing voltage stability indices (VSIs) and the proposed index have been performed to comprehensively highlight the indices’ foundation, performance, and overall behavior for several different IEEE benchmark test systems: 14-bus, 30-bus, 118-bus test systems and 33-bus radial distribution system. The sensitivity towards different loading conditions due to variables such as active power, reactive power, angular difference between sending and receiving bus voltage, line resistance, and shunt admittance, which have been neglected in the formulation of VSIs, are briefly discussed. The findings of this work are aimed to provide a general guideline for other researchers in selecting the appropriate VSIs for various applications, particularly in solving optimization problems such as distributed generation (DG) and reactive power compensation (RPC) placement, optimal power flow, optimal reactive power dispatch, optimal network reconfiguration for various loading occasions, and different networks. Lastly, the application of the proposed index has been adopted as an analytical approach for solving optimal location and sizing of DG(s) problems. The results prove the effectiveness of the proposed index in accessing the voltage stability state and stress condition of lines. Besides, the proposed approach also capable of determining the potential candidate bus for single DG location and sizing effectively.

INDEX TERMS  Voltage stability index, critical line, weak bus, voltage collapse.

I. INTRODUCTION  The modernization of power systems towards smarter grids has forced existing networks to operate close to their stability limits. Even though the advanced power systems are equipped with various control and protection schemes to prevent unpredictable events, the power grids are still experiencing emergency and maloperation conditions that are prone to major blackouts. Voltage instability has always been one of the root causes for various major blackout events worldwide.
Abnormal events such as sudden load increase, transmission line outage, or generator tripping may cause severe loading in the power system, which then contributes to voltage instability and without a proper prediction and mitigation plan, the entire system may experience cascading events that might result in voltage collapse. Various major blackout events around the world were reported in [1], and the initiating factors for all disturbances occurring in power systems are due to bad weather, faulty equipment, animals, accidents, and excessive demand. These contribute to an imbalance of real and reactive power between demand and generation, which results in the overloading of transmission lines and generators that cause the system voltage at the respective bus and system frequency to drop [1], [2].

Ability of the power system to maintain steady voltage levels at every bus after being subjected to a power disturbance from a given initial operating condition, is defined as the voltage stability [3]. Voltage instability is known as a dynamic phenomenon that occurs in power systems due to inadequate reactive power supply in power systems, which could be either a failure of reactive power sources in supplying sufficient reactive power or by a failure of transmission and distribution networks in transmitting the required reactive power, thus leading to voltage collapse, especially when networks are heavily loaded [4]. Hence, mitigation strategies are essential for avoiding voltage instability. This raised the importance of developing a method to predict the level of voltage security under different operating conditions to detect any critical lines or buses that operate close to its stability limit [5]. Based on this indication, the system operator can take necessary preventive steps if needed, and this indicator is known as the voltage stability indices (VSIs).

VSIs are not only competent in identifying the critical lines and buses under both online and offline modes through static analysis or phasor measurement units, but also provide information on voltage instability or the proximity of a collapse under various loading conditions and contingencies, such as loss of generators or lines, and indicate real-time information for voltage instability through phasor measurement unit wide area measurement system (PMU-WAMS) [6], [7], [8], [9]. The indices are also widely used as the objective function to solve various applications, such as determining the optimal location and sizing reactive power compensation (RPC) [10], [11], [12], [13], optimal distributed generation allocation (DG) [14], [15], optimal power flow [16], [17], [18], [19], optimal reactive power dispatch in [20], optimal network reconfiguration [21], and optimal locations of PMUs [22]. The reviewed works on various applications of VSIs can be found in [23], [24], [25], [26], and [27].

Numerous VSIs have been proposed by many authors in the literature for determining the proximity of a power system towards voltage instability. These indices were proposed to evaluate the performance of a power system by determining the closeness of the system to the saddle point bifurcation, also known as the voltage collapse point. The two aspects involved in analyzing voltage stability in power systems are: (i) proximity, which identifies how close the system is towards voltage instability, and (ii) the mechanism that leads to voltage instability, such as the main contributing aspects of voltage instability, voltage weak points, and areas affected [28], [29]. Proximity provides a voltage security indication, in which a useful information is generated to prevent the occurrence of voltage instability in system adjustments or operating plans.

The use of a static model is sufficient for a few studies to identify the system critical bus, line, or load margin. Therefore, a considerable number of studies in the literature have focused on the steady-state static model aspects because of the simplicity of the analysis, which offers an accurate result with less computational effort, adding some practical advantages over the dynamic study [32]. A comprehensive review of the formulation of VSIs in various aspects, such as concepts, assumptions, critical values, and equations, can be found in [30]. Similar work was done by the author in [31], where more than 40 VSIs were comprehensively reviewed based on the formulation, application, performance, and assessment measures. A comparative analysis throughout the simulation was performed to provide a general roadmap for decision-making in selecting the appropriate VSIs for weak line and bus identifications. Nevertheless, from the application perspective, in term of investigating the performance of VSIs towards operating conditions (high real power loading, high reactive power loading, line contingency, and etc.) and network topologies (radial network, mesh network, interconnection, and etc.) have not been precisely investigated. Moreover, the effect of neglecting a few variables in the formulation of some VSIs, such as line resistance, phase angle difference between sending and receiving voltages, real power changes, reactive power, and shunt admittance, has not been discovered in the literature.

In this study, a new line voltage stability index is proposed that is capable of identifying both the weak line and bus for various loading conditions and different network configurations. Comparative studies between the proposed index and other line VSIs have been comprehensively discussed to highlight the indices’ foundation, performance, and overall behavior throughout several different IEEE benchmark test systems; 14-bus, 30-bus, 118-bus, and 33-bus radial distribution systems, as well as the sensitivity towards different loading and contingency events because of variables such as active power, reactive power, angular difference between sending and receiving bus voltage, line resistance, and shunt admittance, which have been neglected in the formulation of VSIs. Then, the proposed index is employed as an analytical approach in solving the optimal location and sizing of DG(s) that aimed to enhance the voltage stability of system under normal and load growth events. The results obtained from this analytical approach are compared with other hybrid-based approaches to validate the practicability of the proposed approach in solving optimal and sizing of DG(s) problems. The findings of this work are aimed at providing a general guideline for other researchers in selecting the appropriate
VSIs for various applications, particularly in solving optimization problems such as DG and RPC placements, optimal power flow, optimal reactive power dispatch, optimal network reconfiguration for various loading and contingency occasions and different networks.

II. METHODOLOGY

Various techniques for voltage stability assessments have been proposed in the literature. The classification of VSIs, as discussed in [30], can be grouped into line VSIs, bus VSIs, and overall VSIs. In [31], the classification of VSI was categorized into four clusters; (i) line variable-based indices, (ii) bus variable-based indices, (iii) Jacobian matrix-based indices, and (iv) PMU-based indices. The accuracy of the overall VSIs is better than the other two types of indices, however, they are complex and required a longer computation time. Line VSIs are the simplest indices that are also suitable for solving all optimization problems because of the capability of these indices to determine both weak lines and buses. Hence, only line VSIs, where the formulation of the index is based on two bus representation concepts, are considered in this work. The mathematical formulations of several proposed methods to examine the system stability are presented here. The formulation of the VSIs is derived using the same characteristics of the voltage collapse point and is represented in the two-bus model. Fig. 1 illustrates a single line of an interconnected network, where bus i represents the sending end bus, while bus j represents the receiving end bus.

**FIGURE 1.** The sending and receiving two-bus model representation.

Where:

- \( R, X, Z, \theta \) = line resistance, line reactance, line impedance and line impedance angle between bus i and j
- \( S_i, P_i, Q_i \) = apparent, real, and reactive power at bus i, respectively
- \( S_j, P_j, Q_j \) = apparent, real, and reactive power at bus j, respectively
- \( V_i, V_j \) = magnitude of bus voltage at bus i and bus j
- \( \delta_i, \delta_j \) = voltage at bus i and bus j

The formation of all VSIs starts by determining the real and reactive powers at the receiving-end bus,

\[ S_R = V_R I^* \]  \hspace{1cm} (1)

Re-arranged (1),

\[ I = \left( \frac{S_R}{V_R} \right)^* = \frac{(P_R - jQ_R)}{V_R \angle - \delta_R} \]  \hspace{1cm} (2)

And the current through line connected between two buses is expressed using Kirchhoff Voltage Law (KVL),

\[ I = \frac{V_S \angle \delta_S - V_R \angle \delta_R}{R + jX} \]  \hspace{1cm} (3)

Substituting (3) into (2),

\[ \frac{V_S \angle \delta_S - V_R \angle \delta_R}{R + jX} = \frac{(P_R - jQ_R)}{V_R \angle - \delta_R} \]  \hspace{1cm} (4)

\[ V_S V_R \angle (\delta_S - \delta_R) - V_R^2 = P_R R - jQ_R R + jP_R X + Q_R X \]  \hspace{1cm} (5)

Let \( \delta_S - \delta_R = \delta \) and (5) can be simplified as,

\[ V_S V_R \angle \delta - V_R^2 = P_R R - jQ_R R + jP_R X + Q_R X \]  \hspace{1cm} (6)

By changing \( V_S V_R \angle \delta \) into rectangular form,

\[ V_S V_R \cos \delta + jV_S V_R \sin \delta - V_R^2 = P_R R - jQ_R R + jP_R X + Q_R X \]  \hspace{1cm} (7)

By separating (7) into real and imaginary part,

\[ V_S V_R \cos \delta - V_R^2 = P_R R + Q_R X \]  \hspace{1cm} (8)

\[ V_S V_R \sin \delta = -Q_R R + P_R X \]  \hspace{1cm} (9)

Re-arrange (8) and (9),

\[ P_R = \frac{-Q_R R + V_S V_R \cos \delta - V_R^2}{R} \]  \hspace{1cm} (10)

\[ Q_R = \frac{P_R X - V_S V_R \sin \delta}{R} \]  \hspace{1cm} (11)

Substituting (11) into (10) and (10) into (11),

\[ P_R = \left( \frac{-P_R X + V_S V_R \sin \delta}{R} \right) X + V_S V_R \cos \delta - V_R^2 \]  \hspace{1cm} (12)

\[ Q_R = \left( \frac{-Q_R X + V_S V_R \cos \delta - V_R^2}{R} \right) X - V_S V_R \sin \delta \]  \hspace{1cm} (13)

Re-arrange (12) and (13),

\[ V_R^2 - V_S V_R \left( \cos \delta + \frac{X}{R} \sin \delta \right) + P_R \left( R + \frac{X^2}{R} \right) = 0 \]  \hspace{1cm} (14)

\[ V_R^2 + V_S V_R \left( \frac{R}{X} \sin \delta - \cos \delta \right) + Q_R \left( \frac{R^2}{X} + X \right) = 0 \]  \hspace{1cm} (15)

In some VSIs, the line resistance, R and line reactance, X are replaced with the line impedance, Z and line impedance angle \( \theta \), respectively. Hence, the current through the line from Equation (3) is obtained as follows,

\[ I = \frac{V_S \angle \delta_S - V_R \angle \delta_R}{Z \angle \theta} \]  \hspace{1cm} (16)

By substituting (16) into (1), the real and reactive power at the receiving bus,

\[ P_R + jQ_R = V_R \left( \frac{V_S \angle \delta_S - V_R \angle \delta_R}{Z \angle \theta} \right)^* \]  \hspace{1cm} (17)

Let \( \delta_S - \delta_R = \delta \) and simplifying (17),

\[ P_R + jQ_R = \frac{V_S V_R}{Z} \angle (\theta - \delta) - \frac{V_R^2}{Z} \angle \theta \]  \hspace{1cm} (18)
By separating (18) into real and imaginary part,

\[ P_R = \frac{V_S V_R}{Z} \cos (\theta - \delta) - \frac{V_R^2}{Z} \cos \theta \]  
(19)

\[ Q_R = \frac{V_S V_R}{Z} \sin (\theta - \delta) - \frac{V_R^2}{Z} \sin \theta \]  
(20)

Re-arrange equation (19) and (20),

\[ V_R^2 \cos \theta - V_S V_R \cos (\theta - \delta) + P_R Z = 0 \]  
(21)

\[ V_R^2 \sin \theta - V_S V_R \sin (\theta - \delta) + Q_R Z = 0 \]  
(22)

In general, the theoretical base of all line VSIs highlighted in the next section are similar. All these indices are formulated by taking the discriminant of the voltage quadratic equation which is set to be larger than or equal to zero. However, the differences are based on the assumptions adopted in every index. The line VSIs which are proposed in literature are briefly discussed in this section.

A. FAST VOLTAGE STABILITY INDEX (FVSI)
Musirin et al. [32] developed FVSI from the concept of power transmission in a two-bus system, in which the discriminant of the voltage quadratic equation (Eq. 15) was set equal to or greater than zero. Due to the angular difference between the sending and receiving end voltages is typically small, \( \delta \approx 0 \), therefore, \( \sin \delta = 0, \cos \delta = 1 \). The shunt admittance is also neglected in the formulation of the FVSI; thus, the index is simplified to,

\[ FVSI = \frac{4Z^2 Q_j}{V_i^2 X_{ij}} \]  
(23)

B. VOLTAGE STABILITY INDEX (L_{ij})

Author in [33] proposed a similar index as FVSI by considering the assumption neglected by author in [25], in which \( \delta \) is set to zero, the index had been proposed as follows,

\[ L_{ij} = \frac{4Z^2 Q_j X}{V_i^2 (R \sin \delta - X \cos \delta)^2} \]  
(24)

The line shunt admittance is neglected.

C. LINE STABILITY INDEX (L_{mn})

L_{mn} was formulated by Moghavemmi et al. [34] using the same power transmission concept by taking the discriminant of the voltage quadratic equation from Equation (22). The effects of the real power and shunt admittance on the voltage stability were excluded from the derivation of the proposed index. The L_{mn} in a typical transmission line can be obtained as follows,

\[ L_{mn} = \frac{4XQ_j}{(V_i \sin (\theta - \delta))^2} \]  
(25)

D. LINE STABILITY FACTOR (LQP)

Mohamed et al. [35] proposed LQP for line voltage stability assessment, where the formulation of the index is similar to that in [32] and [33]. Hence, LQP is obtained as follows,

\[ LQP = 4 \left( \frac{X_{ij}}{V_i^2} \right) \left( \frac{X_{ij} P_i^2 + Q_j}{V_i^2} \right) \]  
(26)

E. LINE STABILITY INDEX (L_p)

L_p was formulated by the same author in [34], who also adopted the same power transmission concept as proposed by other authors [36]. The index is defined as follows,

\[ L_p = \frac{4RP_j}{(V_i \cos (\theta - \delta))^2} \]  
(27)

The author assumed that only real power affects the line voltage stability, hence, the index was derived by neglecting the effect of reactive power and line shunt admittance on voltage stability.

F. NOVEL LINE STABILITY INDEX (NLSI)
The NLSI was derived by Yazdanpanah-Goharrizi et al. [37] using the same power transmission concept from the quadratic equation in Equation (8). By taking the discriminate of Equation (8), which is set to be greater than or equal to zero, the NLSI is formulated as follows,

\[ NLSI = \frac{P_j R + Q_j X}{0.25V_i^2} \]  
(28)

The authors also neglected the angular voltage difference between the sending and receiving ends, \( \delta \), and line shunt admittance in the formulation of the proposed index.

G. NEW VOLTAGE STABILITY INDEX (NVSI)
The NVSI proposed by Kanimozhi et al. [38] is also based on the same power transmission concept in a single line, where the formation of index is from Equations (14) and (15). This index also neglects the line resistance (\( R = 0 \)) and shunt admittance. Using the trigonometry identity, Equations (14) and (15) are equated and simplified. By setting the discriminate to be greater than or equal to zero to obtain the real roots, the final index is formulated as follows,

\[ NVSI = \frac{2X \sqrt{P_j^2 + Q_j^2}}{2Q_j X - V_i^2} \]  
(29)

H. VOLTAGE REACTIVE POWER INDEX (VQI_{Line})

Althowibi et al. [39] derived VQI_{Line} based on the same power transmission concept where \( \delta \) and the line shunt admittance are neglected in the mathematical formulation. The index had been proposed as follows,

\[ VQI_{Line} = \frac{4Q_j}{\left| \text{Im}(1/(R + jX)) \right| V_i^2} \]  
(30)

I. VOLTAGE STABILITY LOAD INDEX (VLSI)
The VLSI proposed by Abdul Rahman et al. [40] is for line voltage stability assessment, where the formulation of the index is based on the power loss in the transmission line.
The line shunt admittance is neglected in the mathematical formulation, and the index is derived as follows,

\[ VLSI = \frac{4 \left( V_i V_j \cos (\delta) - V_j^2 \cos^2 (\delta) \right)}{V_i^2} \]  

(31)

**J. VOLTAGE STABILITY INDEX (L)**

The index proposed in [41] is a simplification of the index proposed by the author in [40], whereby the value of \( \cos \delta \) is set to 1 due to the angular difference between the sending and receiving end voltages is neglected in the formulation of the index. Therefore, \( L \) is determined by,

\[ L = \frac{4 \left( V_i V_j - V_j^2 \right)}{V_i^2} \]  

(32)

**K. VOLTAGE STABILITY INDICATOR (VSI)\(_2\)**

VSI\(_2\) proposed by Chattopadhyay et al. [42] also neglected the effect of the angular difference between the sending and receiving end voltages, where \( \cos \delta = 1 \) and \( \sin \delta = 0 \) and the line shunt admittance is neglected. Using the same power transmission concept from equation (7), the real and imaginary parts of Equation (7) are equated and simplified, and by taking the discrimination of the simplified equation that must be greater than or equal to zero, VSI\(_2\) is obtained as follows,

\[ VSI_2 = \frac{4Q_j (R + X)^2}{X \left( V_i^2 + 8RQ_i \right)} \]  

(33)

**III. THE FORMULATION OF THE NEW LINE VOLTAGE STABILITY INDEX (BVSI)**

The formulation of the proposed BVSI is based on the power transmission concept formulated in Section 2, from Equations (1) to (14). From Equation (14), the following quadratic equation is rearranged as follows,

\[ RV_R^2 - VSV_R (R \cos \delta + X \sin \delta) + P_R \left( R^2 + X^2 \right) = 0 \]  

(34)

The roots of quadratic equation are obtained as,

\[ V_R = \frac{V_S (R \cos \delta + X \sin \delta) \pm \sqrt{(V_S (R \cos \delta + X \sin \delta))^2 - 4R \left(P_R \left( R^2 + X^2 \right) \right)}}{2R} \]  

(35)

The real and nonzero values of \( V_R \) can be determined by setting a discriminant of greater than zero,

\[ (V_S (R \cos \delta + X \sin \delta))^2 - 4R \left(P_R \left( R^2 + X^2 \right) \right) \geq 0 \]  

(36)

Hence, the new index,

\[ BVSI = \frac{4RP_RZ^2}{(V_S (R \cos \delta + X \sin \delta))^2} \leq 1 \]  

(37)

The value of BVSI varies between 0 and 1, and the value must be less than unity to maintain the voltage stability of the system. Lines with BVSI go beyond unity, indicating buses where these lines are connected, and are experiencing a sudden voltage drop in which approaching towards system collapse. This is neglecting the line shunt admittance and the reactive power’s effects on voltage stability.

In this work, various voltage stability assessments using line VSIs are done to identify both the weak line and bus for various loading conditions and different network configurations. The comparative studies between the proposed BVSI and other line VSIs are comprehensively discussed to highlight the indices’ foundation, performance, and overall behavior throughout several different IEEE benchmark test systems. These comparative studies are done to determine the sensitiveness of every index towards different loading and contingency events as well as to validate the impact of neglecting few variables such as active power, reactive power, angular difference between sending and receiving bus voltage, line resistance, and shunt admittance, in the formulation of VSIs.

Next, the proposed index is employed as an objective function in solving the optimal location and sizing of distributed generations, DG(s) that aimed to enhance the voltage stability of system under normal and load growth events. The details of the mathematical expression of the objective functions and constraints are given below,

**Objective function:**

\[ F = \min \sum_{j=1}^{nl} \frac{4RP_RZ^2}{(V_S (R \cos \delta + X \sin \delta))^2} = \min \sum BVSI \]  

(38)

The equality and inequality constraints:

\[ P_{Gi} - P_{Li} = \sum_{i=1}^{nl} \sum_{j=1}^{nl} V_i V_j \left(G_{ij} (\delta_i - \delta_j) + B_{ij} \sin (\delta_i - \delta_j) \right) \]  

(39)

\[ Q_{Gi} - Q_{Li} = \sum_{i=1}^{nl} \sum_{j=1}^{nl} V_i V_j \left(G_{ij} \sin (\delta_i - \delta_j) - B_{ij} \cos (\delta_i - \delta_j) \right) \]  

(40)

Voltage bus limits, \( V_i \) at each bus,

\[ V_i^{\min} \leq V_i \leq V_i^{\max}, \quad i = 1, \ldots, Nb \]  

(41)

Real and reactive power generation limits, \( P_{Gi} \) and \( Q_{Gi} \),

\[ P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \quad i = 1, \ldots, Ngen \]  

(42)

\[ Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \quad i = 1, \ldots, Ngen \]  

(43)

where, \( P_{Gi} \) and \( Q_{Gi} \) are the real and reactive power generation at bus \( i \), \( P_{Li} \) and \( Q_{Li} \) are the real and reactive power load demand at bus \( i \), \( B_{ij} \) is the susceptance of line \( ij \) and \( \delta_i \) and \( \delta_j \) are the voltage angle at bus \( i \) and bus \( j \), respectively. The proposed algorithm for solving the optimal location and sizing DG(s) problem is clearly shown in flowchart presented in Fig. 2.
IV. SIMULATION RESULTS AND DISCUSSION

In this section, the proposed index has been thoroughly examined under various operating and system events; Case I: high loading occasions, Case II: line contingency events, and lastly Case III: Application of BVSI in solving the optimal location and sizing problems for DG(s) placement. The effectiveness of the proposed algorithm had been rigorously compared with those obtained using FVSI, Lij, Lmn, LQP, Lp, NLSI, NVSI, VQILine, VLSI, L, and VSII in under various high loading events (Case I) and several N-1 and N-2 line contingencies (Case II). Finally, the proposed index BVSI is adopted in solving the optimal location and sizing of DG(s) in Case III.

A. CASE I: HIGH LOADING OCCASIONS

In this section, the effectiveness of the proposed BVSI has been tested under various loading conditions, such as heavy real power loading, heavy reactive power loading, and heavy real and reactive power loading. The results are compared with line VSIs to validate the feasibility of the proposed index in three different benchmark test systems; the IEEE 14-bus, 30-bus, 118-bus and 33-bus radial distribution systems. The findings have been comprehensively discussed to highlight the indices’ foundation, performance, and overall behavior across different network configurations (radial, mesh, inter-connection, etc.), as well as the sensitivity towards different loading conditions due to variables such as active power, reactive power, angular difference between sending and receiving bus voltage, line resistance, and shunt admittance, which have been neglected in the formulation of indices. Finally, the most appropriate VSIs for different networks and loading occasions are proposed.

1) IEEE 14 BUS TEST SYSTEM

IEEE 14 bus test system consists of 14 buses, 11 loads, five generators, 15 lines, and five transformers. Under normal loading conditions, the total required loads are 259MW and 73.5MVAR that operate at four different voltage levels; 132kV, 33kV, 11kV and 1kV. Fig 3 shows that under normal loading, the values of all indices are near zero, indicating that the system is operating at a stable point. Based on these findings, the highest indices are recorded at line 9-14, followed by 6-13, 6-11 and 13-14. The results are slightly different when the load at all load buses was increased to 2 times of the normal loading for all three cases; active power (P) loading only, reactive power (Q) loading only, and both active and reactive power (PQ) loading. It is observed that line 13-14 records the highest VSIs in all P, Q, and PQ loading cases, followed by line 6-11, 6-13, 10-11 and 9-14 are among the top lines with the highest indices, as shown in Fig. 4-6.

Location-wise is one of the contributing factors for higher VSIs in line. The line connecting buses 13 and 14 shows the highest VSIs in the line for all cases because of the location of these buses, which are far from the main supply. Another factor contributing to higher VSIs is the load connected to a particular bus. It is observed that the load connected at bus 14 is among the highest total loads connected at the 33kV bus. These two factors are the reasons why line 13-14 appears as the most critical line and bus 14 appears as the most critical bus, based on the voltage stability perspective. Moreover, bus 14 has always been highlighted as the most critical bus in this network [43], [44], [45]. Lines 6-11, 6-13 and 9-14 are three lines connected to the 33kV substation; buses 6 and 9 and the flow of real and reactive power carried by these lines are high. This factor also influenced to higher VSIs in the lines.

Next, the system loads are gradually increased from the initial value till the load flow is diverged, where at this point, the voltage collapse occur in the system. This procedure is performed to determine the voltage stability limit at different loading occasions; PQ loading, P loading, and Q loading. For simplicity, only the results for the critical line; line 13-14 are presented as shown in Fig. 7-9. In addition, power flow limits in lines are ignored since the concern of this work is to validate the performance of the indices with respect to the voltage stability limit of the system. Based on the results obtained, the loading for both PQ and P loads can be increased up to 300% of the nominal load, whereas for Q loads can be further increased up to a maximum of 920%. This is because the total loads in the Q loading event (259 MW and 749.7MVAR) are not as high as those in PQ (1036 MW and 294 MVAR) and P loading (1036 MW and 73.5 MVAR). Hence, the reactive power load variation in this system can be increased up to 9.2 times from the base load values.

The performance of the indices for both P and PQ loading cases are quite similar. VSIs, such as Lp and BVSI, which include the real power in the formulation of the index, are sensitive towards P and PQ loading. Hence, these two indices record the highest value. For the Q loading event, the proposed index is among the highest at the beginning. As the system approaches toward instability point, line indices such as Lmn, Lij, VSI_2, and FVSI offer higher values than other indices. This undoubtedly reveals that the indices that consider the reactive power flows in the formulation of the index provide an accurate tool for predicting the voltage stability for heavy Q loading cases. However, these indices offer lower values for P and PQ loading cases due to few important variables such as line resistance, angle differences between sending and receiving end voltage, and real power flows are neglected in the formulation of these indices. Fig. 10 shows the voltage level at each load bus under various loading values. The voltage profile at the load buses drastically decreased during the Q loading event due to the massive increase in reactive power loads may lead to a progressive drop in the voltage at every load bus.

2) IEEE 30 BUS TEST SYSTEM

IEEE 30 bus test system consists of 30 buses, 24 loads, six generators, 41 lines, seven transformers, and two shunt capacitors. Under normal loading conditions, the total required loads are 283.4 MW and 126.2MVAR that operate at four different voltage levels; 132 kV, 33 kV, 18 kV and 1kV. For simplicity, only the results obtained for all the 33kV lines
FIGURE 2. The flowchart of solving the optimal location and sizing.

are presented in this section. It is noticeable from the results of the VSIs in Fig. 11 that under normal loading, the values of all indices are close to zero, which indicates that the system operates at a stable point. Based on the finding, it is observed that the highest indices are recorded at line 27-30, followed by line 27-29, 10-20, 12-15 and 29-30. For the following cases, only the results from the highest lines are presented, as shown in Fig. 12-14, for simplicity reason. All loading events increased to 80% of the base loading. Similar findings are obtained for all given loading occasions; P, Q, and PQ loading where line 27-30 appears the highest, followed by the same sequence of results in the normal loading case.

It is observed that line 27-30 records the highest value in all VSIs solely due to the fact that the total real power load at bus 30 is among the largest value. This leads to a few VSIs that are sensitive towards changes in the real power flow in line record higher values. In addition, the values of both line resistance and reactance at line 27-30 are high, which shows that it is a valid contributing factor for the higher values of VSIs measured in this line. All of these factors also contributed to the high value of VSIs in line 27-29 and 25-26. Due to location wise, line 27-30, 27-29 and 25-26 have always been highlighted in the literature [46], [47], [48], [49], [50], [51] as the weakest lines for this network due to its distant location. Next, lines that are connected to the substation bus (bus 10 and bus 12), line 10-20 and 12-15 also appear with higher VSIs as a result of the large amount of real and reactive power flow carried by these lines.

Subsequently, the system loadings are gradually increased to the maximum to determine the voltage stability limit at different loading occasions; PQ loading, P loading, and Q loading. The results for line 27-30 which is the critical line for this network, are presented in this section. Based on the results shown in Fig. 15-17, it is observed that the loading for PQ loads could be increased up to 80% of the nominal load, whereas the P and Q loads could be further increased up to 110% and 200%, respectively. It is clearly shown that the proposed index, BVSI, and Lp record the highest value, while the other indices predict an inaccurate state of voltage stability in all loading events.

The proposed index indeed is a good and accurate indicator for predicting the voltage stability for all the given scenarios. Even though the formulation of the proposed index neglected the changes in reactive power, the index still provided the highest value for the Q loading event. This is due to a significant drop in voltage at most of the load buses that are located far from the main source. Fig. 18 illustrates the voltage profile in every bus for all loading events, and it obviously shows that the voltage at the load buses is at the lowest during Q loading event. Although the amount of loads under Q loading events (283.4MW and 378.6MV) are considered lower than both P (595.14MW and 126.2MV) and PQ loading (510.12MW and 227.16MV), the impact of heavily Q loading may drastically causes a progressive and uncontrollable drop in voltage as shown in Fig.18 [46].

3) IEEE 118 BUS TEST SYSTEM

The applicability of the proposed index is also tested in a large-scale power system. This is crucial to validate the consistency of the results to verify the superiority of the proposed index over other existing VSIs. The IEEE 118 bus test system is a large power network comprising 118 buses, 90 loads, fifty-four generators, nine transformers, and 186 lines. Under base loading, the total loads connected are 3677MW and 138MV that operates at two different voltage levels; 345kV and 138kV. Analyzing voltage stability in a large-scale power system is very complex due to the large number of generators connected, plus the locations of these generators are scattered throughout the networks. Consequently, it is very difficult to identify the weak areas of buses, as well as the critical lines. Due to the fact that this benchmark test system consists of many generators connected, therefore lines with a generator attached either at the sending or receiving end bus,
which recorded high value of index are excluded from the list of critical lines.

Fig. 19-21 presents the selected lines with the highest VSIIs recorded under several high loading events, where the loads at every bus are increased by 1.8 times of the base loading. Under both P and PQ loading, line 43-44 appears as the most critical line, as this line measures the highest index in all VSIIs, followed by line 33-37 and line 22-23. For Q loading, line 96-97 appears as the highest value, followed by line 22-23 and line 33-37. A comparison of the indices shows that the proposed index, BVSI, and Lp are closest to unity at the point of bifurcation, while other indices are far from unity, which is incapable of identifying the critical point of voltage instability in most of the lines.

Loadability tests in diverse loading events are also performed to investigate the versatility of VSIIs towards the collapse point. The permissible load scaling factor for PQ is 1.8 times of the base loading (6222MW and 2588.4MVAR), whereas for P loading, the real power loads can be increased up to 1.9 times (6954MW and 1438MVAR). Under the Q loading event, the scaling factor can be increased to 3.8 (3660MW and 5464.4MVAR). For simplicity reason, the results from the critical line, line 43-44 are presented in this work. The consistency of the obtained results validates the adaptability of the proposed index under diverse operating conditions, as shown in Fig. 23-25.

The proposed index, BVSI, and Lp provide the highest values for all loadings, followed by VSII2 and NLSI.
remaining indices are far below the critical values for all loadings, which reveals the inaccuracy of the indices in predicting the voltage instability state for large-scale networks. It is observed that there are changes in the direction of reactive power flow that affect all indices, considering the reactive power changes in the formulation of the index during the P loading event, as shown in Fig. 23. Consequently, there is a slight drop in some VSIs, such as FVSI, Lij, Lmn, VQILine.
and VSI$_2$. This could be due to the large number of generators connected, which are scattered, and the increase in loads may cause the direction of the power flow to change accordingly. Moreover, if the direction of the real and reactive power flows in line is opposite, the value of VSI is low due to the resistive voltage drop is subtracted from the reactive voltage drop [46].

Fig. 25 shows the voltage profiles for every bus under various loading events. It is observed that the voltage at some buses drops tremendously during a high Q loading event. This clearly shows that the impact of heavy Q loading can also be observed in large-scale power systems. Even though the amount of real power required by loads are relatively large compared to reactive power loads, the impact of a sudden increase in reactive power loading may drastically cause a progressive and uncontrollable drop in voltage.
FIGURE 14. Line VSIs of all 33 kV transmission lines for IEEE 30 bus system under heavy PQ loading.

FIGURE 15. The index values for different VSIs with respect to PQ loading for line 27-30 in IEEE 30 bus system.

FIGURE 16. The index values for different VSIs with respect to P loading for line 27-30 in IEEE 30 bus system.

FIGURE 17. The index values for different VSIs with respect to Q loading for line 27-30 in IEEE 30 bus system.
4) IEEE 33 BUS RADIAL DISTRIBUTION TEST SYSTEM

Distribution networks are passive networks that are typically designed in radial configurations in nature. The electrical power in distribution networks is fed at only one point via a substation, and the power flow is unidirectional. The resistance-to-reactance (R/X) ratio in a radial distribution system (RDS) is typically high [53], [54]. Therefore, some indices that neglect line resistance in the formulation of VSIs may not produce an accurate result. Hence, the effectiveness of the line voltage stability indices, including the proposed index, is also examined in the RDS to validate the impact of excluding certain variables such as line resistance and angular differences between the sending and receiving voltages.

IEEE 33 bus RDS consist of 33 buses, 32 loads, and 32 lines. Under normal operating conditions, the total required loads are 3.715MW and 2.3MV AR and all buses operate at the same voltage level; 12.66kV. For simplicity, only the six lines with the highest VSIs are presented in
this section. Fig. 26-28 illustrate the VSIs at the selected lines for P and Q loading, where the loads at every bus are increased up to 270% of the nominal load. It is observed that for the P loading case, the proposed index and Lp provided the highest value of index in most of the lines, followed by VSI₂. For the Q loading event, VSI₂ measured the highest index in every line, followed by FVSI and Lmn.

In order to validate the adaptability of VSIs under diverse operating conditions, the system loadings are gradually increased to a maximum, and only the results obtained from the critical line, line 5-6, are presented in this section.
The maximum acceptable load scaling factor for PQ loading is 2.6 times of the base loading (10.14195MW and 8.2446MVAR) while for P loading, the real power loads can be increased up to 3.2 times (15.603MW and 3.02MVAR) and lastly, for Q loading, the scaling factor can be increased up to 2.8 (3.715MW and 11.7176MVAR). It is obvious that the VSIs are at the highest values during the Q loading event, and this shows that the voltage stability limit for Q loading is less than that for both P and PQ loadings due to the enormous increased in Q loading, which leads to a progressive drop in the load bus voltage.

The results presented in Fig. 29-31 reveal that the proposed index, BVSI, and Lp recorded the highest indices for P loading, whereas for both PQ and Q loading, $VSI_2$ had the highest value. For radial networks, $VSI_2$ provides an enormous indication under various loading events, whereas BVSI and Lp show the best performance for P loading events. This is because of the amount of real and reactive power...
required by all loads, which are almost the same whereas in transmission networks, the total reactive power loads are typically less than half of the total real power loads.

The effect of neglecting the line resistance in the formulation of the index can be observed in the radial configuration. Both the LQP and NVSI are recorded as the lowest values for all loading occasions, as the formulation of both VSIs assumed that the line resistance is typically small ($R \approx 0$) and can be neglected. Hence, these VSIs are not suitable for radial networks, since the line resistances in all radial configurations.
are typically higher than the line reactance. Next, the effect of excluding the angular difference between the sending and receiving voltages ($\delta = 0$) can be found between FVSI and $L_{ij}$, and also between VLSI and $L$. Both FVSI and $L$ are simplifications of $L_{ij}$ and VLSI with an angular difference is set to zero. It is observed that the value of FVSI measured in all cases is slightly higher than $L_{ij}$, however, both $L$ and VLSI provided the same value in all given loading occasions. In the FVSI formulation, when $\delta = 0$, the following variables in $L_{ij}$ are simplified to $R \sin \delta = 0$ and $X \cos \delta = X$ and these assumptions are not valid for the radial configuration due $R/X$ ratio in the radial distribution system is characteristically high. Owing to that reason, FVSI is measured slightly higher than $L_{ij}$, and technically, this index can be considered less accurate. However, for $L$ index, only the following variable is set to be zero; $\cos \delta = 0$. Since $\delta$ is very small, thus $L$ provides the same value as recorded by the VLSI.

### B. CASE II: LINE CONTINGENCIES

Contingencies are one of the contributing factors in voltage instability that could be resulted by single or multiple outages of lines or generators. Therefore, the contingency analysis is one of the important aspects in power system’s planning. It capable to predict the impact of various contingencies on the overall performance of power system and this helps to initiate proper mitigation plans. The severity of contingencies can be determined through a process of determining the performance indices for every contingency. Various severity indices had been proposed in literature and static voltage stability-based indices such as line VSIs were also adopted in most of studies for contingency rank [55].

The contingency rank is done to determine the severity of voltage stability condition in power system. The procedure of this contingency ranking is conducted by obtaining VSIs in every line for each particular line outage. Lines with highest value VSIs in every line outage event are classified as the weak lines and the most severe contingencies are determined based on the highest values of VSIs recorded every line when that particular line is disconnected.

In this case, the effectiveness of the proposed BVSI has been tested under N-1 and N-2 outages using two benchmark test systems; IEEE 14-bus and 30-bus systems. The findings have been comprehensively compared with the other line VSIs. The results obtained from the most severe cases are presented in the following sub-section.

#### 1) IEEE 14 BUS TEST SYSTEM

The bar graphs presented in Fig. 32-35 are the most severe cases for N-1 line outage, and line 6-13 outage appears as the most severe contingency under normal loading in IEEE 14 bus system. By observing all line VSIs recorded in all given N-1 line outage cases, line 9-14 records the highest value, followed by line 6-13, 6-12 and 13-14. In N-2 contingencies line outage events, the most severe contingency is recorded during line 6-12 and 6-13 outage. Most of the lines provide the highest value of indices under this event as shown in Fig. 36-38. The highest index is recorded in line 9-14 under all N-2 line outage events, followed by line 13-14.

The results are quite similar as obtained in previous high loading cases where line 9-14, 13-14, 6-12 and 6-13 appear as the weak lines for this network. Line 6-13 and 6-12 outage events are the most severe cases due to high amount of real and reactive power flow through these lines since they are connected to 33kV substation. The absence of these lines contributes to voltage instability of a power system.
From the indices performance’s point of view, it obviously shows that the proposed index, BVSI and Lp provide higher indication compared to the rest indices. This shows that BVSI provides a good and accurate indication in predicting the current state of voltage stability under various circumstances including high loading and contingency events.
2) IEEE 30 BUS TEST SYSTEM

The previous said procedure is repeated for large network, IEEE 30 bus system to validate the performance of proposed index, BVSI. From the results obtained, the most severe case for N-1 line outage is recorded for case when line 12-15 disconnected followed by lines 15-18, 27-30, 25-27 as shown in...
Fig. 39-42. It shows that most of the lines especially the lines connected nearby to these lines experienced higher value of VSIIs. The main reason could be because of the location of these lines which are connected to substation (bus 12 and bus 27) where the large amount of real and reactive power is feed to these lines. Hence, tripping of these lines contributes to the increase of real and reactive power flow at other lines, and this significantly led to the increase of VSIIs in most of the lines especially lines connected closed to these tripped lines.

It is also observed that under most of N-1 line outage events, lines 27-30, 27-29 and 10-20 measure the highest index and these lines are highlighted as the weakest line for this case. These results support the findings obtained in previous high loading events since the results obtained in all given scenarios are similar.

The study is also conducted for N-2 line outage to identify the most severe case for N-2 line outage event and the weak lines. Under line 12-15 and 27-30 outage event, it is observed that almost all lines are at the highest values of VSIIs as shown in Fig. 43-45. This reveals that this event is the most severe case for N-2 line outage. As stated before, these lines are connected to the substation (bus 12 and bus 27) and removing these lines have greatly affected the entire system’s voltage stability. Furthermore, lines 27-29, 27-30 and 10-20 are remained as the weakest line for this case as well since these lines provide the highest VSIs in all N-2 contingencies line outage events. Fig. 42-44 presents the VSIs of selected lines under three most severe cases for N-2 line outage events and it is obviously shows that lines 27-29, 27-30 and 10-20 appear as the highest line VSIs.

In terms of the performance of proposed index, BVSI and Lp are also recorded higher values compared to other line VSIs. This proven that the proposed index effectively provides a good indication not only under various high loading and contingency events but also for all given network configurations.

**C. CASE III: EFFECT OF DG/RPC INTEGRATION**

Integrating distributed generation (DG) or reactive power compensation (RPC) unit in a power system has always been one of the mitigation plans in improving the overall system performances. They are capable of enhancing the voltage profiles, decreasing the network losses, improving the power factor as well as improving the voltage stability and security of the power system. These can be only achieved with an appropriate sizing and location of DG or RPC integration especially when installing several units of DGs or RPCs. Therefore, the performance of the proposed index, BVSI is also tested on various penetration level (low, medium and high) of DG and RPC and the findings are compared with those obtained other line VSIs.

This study employs the Photovoltaic generator with a unity power factor as the source for DG unit and static synchronous compensator (STATCOM) as the shunt compensation for RPC unit. The location of DG/STATCOM is placed at the bus where the weakest lines are connected. Based on the findings obtained in high loading (Case I) and line contingency events (Case II), line 9-14 and 13-14 are weak lines for IEEE 14-bus network. Hence, DG/STATCOM is placed at Bus 14. The results obtained from the two weakest lines are presented in this section as illustrated in Fig. 46-49.

Under high real power injected by DG unit, the amount of real power flow in lines which are located close to DG unit are changed accordingly. This also proportionally increased the angular difference between the sending and receiving end voltages, \( \delta \) especially at bus where DG unit is placed. It is observed that the index considering few important variables such as effect of real power, line resistance and \( \delta \) in the...
formulation of index such as BVSI and Lp, provides higher value compared to other VSIs. Moreover, VSIs that neglect the angular difference between sending and receiving end voltage, \( \delta \) such as FVSI and L, provide inaccurate value. It can be seen that FVSI records slightly lower than Lij and L records slightly higher than VSIL in lines with higher value of \( \delta \).

In case when the system is highly compensated, most of VSIs provide higher value compared to DG. This is because of the effect of both reactive power and bus voltage are considered in the formulation of index such as FVSI, Lij, Lmn, NLSI, VQIline and VSI2. It is observed that VSI2 records the highest index which clearly shows the sensitiveness of index towards the changes in reactive power in the system. The proposed index, BVSI and Lp consistently provide high value in all cases. The voltage profile at buses which are located close to bus where STATCOM is placed are greatly improved as shown in Fig. 50. This undoubtedly reveals the role of reactive power in regulating the bus voltage.

Based on the outcome obtained in these cases, the voltage stability of the entire system is improved with the integration of DG or RPC unit. However, without a proper sizing and location, the benefits such as reducing power losses, enhancing voltage profile and improving voltage stability cannot
be fully maximized. Therefore, the application of BVSI in solving the optimal location and sizing problem is proposed in the next section.

D. CASE IV: APPLICATION OF BVSI IN SOLVING THE OPTIMAL LOCATION AND SIZING OF DG PROBLEMS

Voltage stability assessment is one of the important aspects in optimizing the performance of power system. The use of VSIs in solving the optimization problems such as DG/RPC allocation, network reconfiguration, optimal power flow, optimal reactive power dispatch and etc., has greatly drawn the attention of many researchers over the past decades. The ability of VSIs in determining the weak lines and buses effectively, has gained popularity especially in solving the DG allocation problems. The simple procedure of determining the optimal location of DG is started by examining the VSI in every line and lines with the highest value of index is identified as the weak lines. Next, the weak bus is selected based on the following two criteria: i. the receiving end bus, bus i that located at the weak lines and, ii. must be a load bus only (exclude generator bus and reference bus).

In this section, the effectiveness of proposed index in solving optimal location and sizing problems is tested in IEEE 33 radial distribution system. The procedure of selecting the optimal location and sizing of DG using the proposed index, BVSI are clearly described in flowchart in Fig. 2. Several case studies are adopted to validate the most optimal location and sizing of DG(s) with consideration of both normal and load growth events. The consideration of load growth event is essential in solving the optimal and sizing of DG problem as the global electrical power consumption is expected to increase with the average of 2~2.2% annually as the impact of rapidly evolving technologies [56]. The following case studies are proposed to analyze the voltage stability performance of power system;

Case 1: with single DG integration normal loading
Case 2: with double DGs integration normal loading
Case 3: with triple DGs integration normal loading
Case 4: with single DG integration high loading
Case 5: with double DGs integration high loading
Case 6: with triple DGs integration high loading

The results obtained in both normal and load growth events are tabulated in Table 1. Under normal loading, the DG(s) are optimally located at bus 6 for Case 1, bus 6 and bus 28 for Case 2 and lastly, bus 6, bus 28 and bus 24 for Case 3. It is obviously revealed that the overall system’s performances are further enhanced when number of DGs installed are increased. The percentage of voltage stability improvement is further enhanced to 28.89% in Case 1, 34.47% in Case 2 and 41.38% in Case 3. Besides, the percentage of both real and reactive power loss reduction is further increased with multiple DGs installed at different locations with the optimal installed size. The real and reactive power loss are at the lowest when DGs are installed at three different locations in Case 3 with 55.43% and 51.93% reduction. The voltage levels in every bus are also significantly increased as shown in Fig. 46, with the minimum voltage of 0.9598pu and the average voltage of 0.9841pu in Case 3.
In load growth event, both of the real and reactive power loads are increased up to 1.5 times of the base loading and the total real and reactive power loads under this event are 5572.5kW and 3450kVAR. Based on the findings, the DGs are optimally placed at the same location as obtained in normal loading. However, the optimal capacity of DGs is slightly higher in this case compared to normal loading event as recorded in Table 1. It is observed that the performance of both voltage stability and bus voltage profile of the entire networks are greatly improved with the increase of number of DGs installed in the benchmark test system. The percentage of voltage stability’s improvement in Case 6 is 43.81%, which is the highest value recorded, followed by 36.38% in Case 5 and 32.10% in Case 4. The minimum and average bus voltage are also further enhanced in Case 4 when DGs with the optimal size are installed at three different locations as shown in Fig. 47. The voltage profile in every bus is well improved with the minimum voltage of 0.9373pu and average voltage of 0.9735pu are recorded after multiple DGs integration (Case III.IV) compared to without DG integration wherein the minimum and average voltage are at 0.8634pu and 0.9192pu.

The real and reactive power loss are also enormously reduced in all cases with the highest reduction of real and reactive power loss are recorded in Case 6. It is observed that the reduction of real and reactive power loss is between 49∼58% with single and multiple DG(s) integration. Based on the outcomes, it clearly shown that the triple DGs integration in Case 3 for normal loading and Case 6 for load growth event significantly enhanced the voltage stability, improved voltage profile at every bus and reduced the real and reactive power loss compared to other cases. The proposed analytical approach through the proposed index, BVSI provides a good estimation of the location and size for DG integration with the effective results as shown in Table 2. It is observed that the proposed method significantly enhanced the voltage profile and improved the voltage stability of the benchmark test system. Although the results are not optimum as obtained in hybrid-based approaches, the structure of algorithm is simple and also effectively provides the optimal results for single DG integration but not for solving multiple DGs integrations or multi-objective optimization problems. Moreover, index-based approach or also known as analytical approach or sensitive-based approach is reliable and computing.
efficiency algorithm which capable to provide an optimal or near-optimal global solution but lack of consideration of non-linearity and complexity of system [16]. This limitation can be solved by combining the proposed index, BVSI with a metaheuristic algorithm which formed the hybrid-based approach that will be the future work of this project.

In order to validate the practicability of the proposed index BVSI in solving the optimal location and sizing problem, the results are compared with the other few existing methods using hybrid analytical - metaheuristic algorithm or also known as two-stage approach. In all previous works proposed by authors in [57], [58], and [59], the candidate buses for DGs location were pre-determined initially using analytical approach via loss sensitivity factor (LSF). This helps to reduce the search space for solving the optimization problems in the next stage. Based on these pre-determined candidate locations, the optimal location and sizing of DGs were finally determined using few metaheuristic algorithms such as Bacteria Foraging Optimization Algorithm (BFOA) [57], Dragonfly Algorithm (DA) [58] and Improved Particle Swarm Optimization (IPSO) [59].

V. CONCLUSION

In this study, a new line voltage stability index, BVSI, has been proposed for voltage stability assessment under diverse operating conditions. Comparative studies between the proposed index and other line VSIs have been comprehensively discussed to highlight the indices’ foundation, performance, and overall behavior throughout several different IEEE benchmark test systems 14-bus, 30-bus, 118-bus, and 33-bus radial distribution systems. Besides, the sensitivity of indices towards various high loading occasions, N-1 and N-2 line contingencies and the impact of low and high integration level of DG/RPC have been validated to determine the consequences of neglecting few variables such as active power, reactive power, angular difference between sending and receiving bus voltage, line resistance, and shunt admittance, in the formulation of VSIs.

Based on these findings, the proposed index consistently and accurately predicts the voltage stability of the mesh and large-scale networks (IEEE 14-bus, IEEE 30-bus, and IEEE 118-bus) under various operating conditions. For a radial network, the proposed index provides an erroneous indication for analyzing the voltage stability under high real and reactive power loadings and high real power loading. This shows the superiority of the proposed index, BVSI, in accurately predicting the critical line, weak bus, and area in all the given networks and diverse loading and contingency events as well as validating the impact of integrating DG/RPC in the existing network. The findings of this work were aimed to be general guidance for researchers in selecting appropriate VSIs for various applications, especially in solving optimization problems such as DG and RPC placement, optimal power flow, optimal reactive power dispatch, optimal network reconfiguration for various loading occasions, and different networks. The following conditions are highlighted to be general guidance for researchers in selecting appropriate VSIs for solving various applications:

- VSIs that excluded the effect of real power, angular differences between the sending and receiving end voltages, \( \delta \) and line resistance may not be feasible for analyzing any problems related to the changes of real power in system such as both P and PQ loading events, line contingencies, DG or microgrid integration, optimal power flow, and etc.

- VSIs that include reactive power and bus voltage in the index’s formulation are suitable for analyzing problems related to reactive power such as RPC integration, Q loading, reactive power dispatch and etc.

- The assumption of line resistance is zero, \( R \approx 0 \) may not be viable for radial distribution networks due to the \( R/X \) ratio at distribution level is high.

- Neglecting \( \delta \) in all high real power loading events may not feasible especially for long distance transmission line where \( \delta \neq 0 \).

The contributing factor on lines that appear low or high value of VSIs are emphasized as follows:

- Location-wise; Buses located far from the main source potentially have higher VSIs in the line.

- Large amounts of loads connected at bus; Lines connected to this particular bus are most likely to have higher VSIs in line.
• Lines connect to substations; these lines have to carry a large amount of real and reactive power flow that may also contribute to higher values of VSIs.
• The direction of real power flow and reactive power flow in a line is crucial; lines with different real and reactive power flow directions measure low values of VSIs due to the resistive voltage drop is subtracted from the reactive voltage drop.
• Effect of neglecting line resistance in a radial distribution network; VSIs that exclude line resistance may produce inaccurate results in radial networks.
• Effect of neglecting angle differences – some indices provide a higher value than the actual value, especially in the radial network, which is; less accurate

The application of proposed index, BVSI has been also adopted as an analytical approach for solving the optimal location and sizing of DG(s) and the results have been compared with other hybrid-based approaches to validate the practicability of the proposed index. The proposed index is capable of determining the potential candidate bus for single DG location and sizing effectively via analytical approach. However, the proposed analytical approach is not efficiently and effectively in handling multiple DGs allocation as well as solving complex multi-objective optimization problems. This can be done by combining the proposed index with other metaheuristic algorithm that formed a hybrid-based approach or also known as two-stage approach that will be future work of this project.

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