Heavy-light hadrons are studied in a mass loaded flux tube model. The study indicates that the dynamics of mesons and baryons containing a $c$ quark is well described by the mass loaded flux tube. The hypothesis of good diquark-antiquark degeneracy is found reasonable in heavy-light quark systems. The spectrum of charmed ($D$) and charmed strange ($D_s$) mesons is systematically computed. $D$ and $D_s$ in $1D$ multiplets are predicted to have lower masses in comparison with other theoretical predictions. The predicted masses of the $1^+ (1^3D_1)$ and the $3^-(1^3D_3)$ $D_s$ agree well with those of recently observed $D_{1s}(2700)^\pm$ and $D_{sJ}(2860)$, respectively.

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**Keywords:** Flux tube model; Diquark; Charmed mesons; Charmed strange mesons

**I. INTRODUCTION**

Hadrons spectrum can reveal the properties of the quark dynamics such as the color confinement. So far, a great progress has been made in the lattice QCD theory, but the quark dynamics in hadrons is not very clear and the hadrons spectrum can not be extracted from the QCD theory directly. The prediction of the hadrons masses has to be made in all kinds of models, and an accurate prediction would be a great challenge in hadrons spectroscopy. For heavy-light mesons, the spectrum has been systematically computed in the relativized quark model 1, heavy quark symmetry theory 2, relativistic quark model 3, lattice QCD 4, chiral quark model 5 and some other models 6, 7, 8, 9, 10. In these calculations, it is often difficult to predict the masses of higher orbital excited states. In many cases, the predicted masses of the higher orbital excited states seems to be overestimated in comparison with experimental data.

In hadrons containing more than two quarks or antiquarks, two quarks or antiquarks may attract each other to make a diquark or anti-diquark cluster. The concept of diquark was put forth and was extensively studied in strong interactions 11, 12, 13, 14, 15, 16, 17, 18.

In terms of the diquark, a semi-classical mass loaded flux tube model 19 was recently exploited. In the model, a meson is considered a system with a massive quark $m_1$ and a massive anti-quark $m_2$ connected by a flux tube (or relativistic string) with universal constant tension $T$ rotating with angular momentum $L$. Similarly, a baryon is considered a system with a massive quark $m_1$ and a massive diquark $m_2$ connected by the flux tube. The flux tube is responsible for the color confinement. The mesons and the baryons are therefore described by the same dynamics in the same way. In addition, it is supposed that there is an approximate degeneracy between a good diquark in baryons and a relevant antiquark in mesons.

Light mesons and baryons have been studied and classified well in the model 19. After the energy $E$ and the angular momentum $L$ of the system are written down with the dynamical parameters in the model, the $E$ can be expressed in the $L$ through some reductions. The general form of $E$ is complicated, but the form is simple in some special cases.

For the light quark systems, an approximate mass formula is given 19:

$$E \approx \sqrt{\sigma L} + \kappa L^{-\frac{3}{4}} \mu^\frac{1}{2},$$

(1)

where $T = \frac{\sigma}{2\pi}$ is the string tension, $\kappa \equiv \frac{2\pi^2}{3\frac{T^2}{L}}$, and $\mu^\frac{1}{2} = m_1^\frac{1}{2} + m_2^\frac{1}{2}$ with $m_1$ and $m_2$ are quark and antiquark/diquark masses.

The parameters for the light mesons and baryons in Eq. (1) were extracted from systematical analyzes of existing data 13, 20. The analyzes indicated that the parameters for the light mesons match well with the parameters for the light baryons (see Table 7 in Ref. 19). The dynamics (especially for large $L$) of light quark systems is well described by the mass loaded flux tube 19. In the meantime, in order to account for an approximate degeneracy between the $\Lambda$ baryons and the relevant mesons, the hypothesis of "good diquark-antiquark degeneracy" was proposed 19.

In heavy-light quark systems’ case, an approximate mass formula was reduced also 19:

$$E = M + \sqrt{\frac{\sigma L}{2}} + \frac{2}{4} \kappa L^{-\frac{1}{4}} m^\frac{3}{2},$$

(2)

where $M$ is the heavy quark mass and $m$ is the light quark/diquark mass, other parameters are indicated in Eq. (1). The spin-orbit forces are ignored and $L \neq 0$ in Eq. (1) and Eq. (2).

However, the heavy-light quark systems have not been systematically analyzed except that some $\Lambda_c$ baryons ($\Lambda_c(2285)$, $\Lambda_c(2625)$, $\Lambda_c(2880)$) were simply mentioned with the relevant parameters $M_c = 1600$ MeV, $m_{[ud]} = 180$ MeV and $\sigma = 0.974$ GeV$^2$ in Ref. 19.
Many topics in the mass loaded flux tube model have not been studied in the heavy-light quark systems. Whether the dynamics of the mesons and the baryons can be described well by the flux tube has not been examined. The spectrum of charmed and charmed strange mesons has not been obtained. The hypothesis of "good diquark-antiquark degeneracy", which holds in light quark systems, has not been tested.

In this article, the heavy-light quark systems are studied in the mass loaded flux model with the inclusion of the spin-orbit interactions. The spectrum of the mesons containing one heavy c quark/antiquark is systematically computed, and some possible interpretations of recently observed states are discussed.

II. CHARMED AND CHARMED STRANGE MESONS

In the conventional quark model, mesons may be marked by their quantum numbers $n^{2S+1}L_{J}$, where $n$ is the principle quantum number, $S$ is the total spin, $L$ is the orbital angular momentum, and $J$ is the total angular momentum. In most quark models, the interactions between the quark and the antiquark include the spin-independent confinement interaction, the spin-dependent interactions (spin-orbit interaction, color hyperfine interaction) and some other interactions [1, 21, 22, 23]. The spin-orbit interaction consists of a color-magnetic piece and a Thomas-precession piece. The spin-orbit interaction is often considered the dominant one except for the confinement interaction, and is sometimes simplified an $a \vec{L} \cdot \vec{S}$ coupling. This kind of spin-orbit interaction is employed in our study, while other spin-dependent interactions such as the spin-spin interaction will be ignored.

If the spin-orbit interaction was added to the energy $E$ of the system from the beginning, the final relation between the $E$ and the $L$ would be much more complicated than Eq. (2). As a good approximation, a term $a \vec{L} \cdot \vec{S}$ responsible for the spin-orbit interaction can be brought into Eq. (2) phenomenologically. The parameter $a$ is assumed a constant for the mesons having the same flavors ($a$ depends mainly on the heavy flavor). It can be determined from the fit of experimental data. The study in this article indicates that the mass loaded flux tube with the inclusion of the $a \vec{L} \cdot \vec{S}$ coupling is very potential to produce the whole $D$ and $D_s$ spectrum comparable to the experimental data.

As well known, the heavy quark symmetry applies in the heavy-light mesons. In the heavy quark limit, the mass and spin $s_q$ of the heavy quark decouples. All the mesons properties are determined by the light degrees of freedom. The spin-parity $j^P$ (total angular momentum $j = s_q + l$ of light degrees of freedom) are good quantum numbers and are conserved in strong interactions. In heavy quark effective theory (HQET), the spin-dependent interactions depend on $j$. A natural way to account for the spin-orbit interaction in HQET is to include the $a \vec{L} \cdot \vec{S}$ coupling instead of the $a \vec{L} \cdot \vec{S}$ coupling. However, from our analysis of the experimental data, the $D$ mesons spectrum is difficult to be reproduced with the simple inclusion of $a \vec{L} \cdot \vec{S}$ in Eq. (2). Besides, there may exist a spin-orbit inversion problem [24] in HQET. The heavy quark symmetry seems a little difficult to be accommodated in the present flux tube picture. This difficulty is left as an open question and is not studied here.

The reason of the inclusion of the $a \vec{L} \cdot \vec{S}$ coupling can be realized in another way. The inclusion of the $a \vec{L} \cdot \vec{S}$ in Eq. (2) will result in a nought of hyperfine splitting (spin-triplet and spin-singlet splitting) of P-wave or D-wave multiplet, which is consistent with theories and experiments. These hyperfine splitting relations have already been predicted in many quark potential models. The hyperfine splitting relations hold very well in P-wave or D-wave multiplets of charmonium (even in $1P$ multiplets of $D$ mesons) [24]. Therefore, it is reasonable to extend the mass formula of the heavy-light quark systems to

$$E = M + \sqrt{\frac{\sigma L}{2} + 2^4 \kappa L^2 - \frac{1}{2} m_s^2} + a \vec{L} \cdot \vec{S}$$  \hspace{1cm} (3)$$

with $$\vec{L} \cdot \vec{S} = \frac{J(J+1) - L(L+1) - S(S+1)}{2}.$$ With this formula in hand, we go ahead with the study of the heavy-light mesons. Firstly, we examine whether the hypothesis of "good diquark-antiquark degeneracy" is favored. For this purpose, the parameters $M_s = 1600$ MeV and $\sigma = 0.974$ GeV$^2$ extracted from the charmed baryons [19] are used as our inputs to compute the spectrum of the $D$ mesons. Under the hypothesis of good diquark-antiquark degeneracy, $m_{u,d} = m_{\bar{u},\bar{d}} = 180$ MeV. We obtained $m(1^3P_1) = 2.406$ GeV for one $1P$ D meson. This predicted mass agrees well with that of the experimentally observed $D_1(2430)^0$ [25]. Spectrum of other charmed mesons ($L > 0$) can be subsequently computed after $a = 24.6$ MeV has been fitted from other three $1P$ D triplets. In terms of these parameters, $m_s = 320$ MeV is determined from $D_{s1}(2536)^\pm$ and $D_{s2}(2573)^\pm$. The spectrum of $D_s$ mesons can be systematically computed (the results are not given here for the reason mentioned below).

The experimental spectrum of $D$ and $D_s$ mesons can be well reproduced by the same group of parameters from the charmed baryons except that the predicted $1^3P_0$ and $1^1P_1$ $D_s$ mesons are much heavier in comparison with possible experimental candidates. The fact that the spectrum of the $D$, $D_s$ mesons and the charmed baryons is successfully obtained by the same formula and parameters indicates explicitly that the dynamics of the mesons and the baryons containing one heavy $c$ quark is well described by the flux tube. It is found that the hypothesis of "good diquark-antiquark degeneracy" is favored in the heavy-light quark systems.
In Ref. 19, the $\sigma$s are a little different for different kinds of light mesons and baryons, for which there are two reasons. One reason is that the $\sigma$s in the reference were extracted with spin-orbit interactions ignored. The other reason is that the string tension (the string is responsible for the dynamics) may be different for hadrons containing different flavors. Therefore, the $\sigma$s for mesons may be different from the $\sigma$s for baryons. In order to compare the spectrum of $D$ and $D_s$ in a more reasonable way, the parameters of $\sigma$ and $a$ have to be refitted from the confirmed $D$ mesons.

For a consistent study, the masses of the $c$ quark and the light $u,d$ quarks are regarded universal parameters for the mesons and the baryons in our fitting processes. That is to say, the parameters $m_c = 1.6$ GeV and $m_{u,d} = 180$ MeV extracted from the $\Lambda_c$ baryons are used as inputs to predict the spectrum of the charmed mesons. Other parameters $\sigma = 1.10$ GeV$^2$ and $a = 37.9$ MeV are extracted from the four $1P$ charmed mesons candidates (to extract these two parameters, the minimum of mean square error of the mass of the four $1P$ charmed mesons is applied). In terms of these parameters, it is straightforward to get the spectrum of $1D$ and $1F$ charmed mesons from Eq. 3.

In experimental side, each observed state has a mass uncertainty. The mass uncertainties of observed states may result in some uncertainties to our predictions. If a mass uncertainty $\pm 30$ MeV is assumed for each $1P$ charmed candidate, the $\sigma$ will have an uncertainty $\pm 0.09$ GeV$^2$. This assumed uncertainty may result in $\pm 30$ MeV, $\pm 44$ MeV and $\pm 54$ MeV uncertainties to the masses of the $1P$, $1D$ and $1F$ charmed and charmed strange multiplets, respectively.

Our results for the charmed mesons are obtained in Table 1. In the table, possible candidates for the $D$ mesons of each state are displayed. For some states, there is no one to one correspondence between the $j^P$ and the $n^{2S+1}L_J$ notation. To compare our results with other theoretical predictions explicitly, we listed the results of two typical computations [1, 5]. In Ref. [1], the notation $n^{2S+1}L_J$ was used, and this notation is employed in our calculation. The calculation was performed in HQET and the notation $j^P$ was used in Ref. [3]. For simplicity, quantum numbers $J^P$, $j^P$ and $n^{2S+1}L_J$ are all labeled in the table. A parenthesis is put for the $j^P$ when there is no one to one correspondence between the $j^P$ and the $n^{2S+1}L_J$ notation. A dash ”-” is put in the entry where the corresponding state has not been computed in the two models. A ”?” indicates that there is no observed candidate corresponding to the assignment at present time.

Our results for the $1P$ states are comparable with those in Refs. [1, 3] and experiments. For the $1D$ states, our results are much lower in comparison with those in Refs. [1, 3]. This obvious difference may provide a way to examine whether the mass loaded flux tube model is reasonable. It may give people a hint to find an underlying dynamics of hadrons.

In terms of the parameters $\sigma = 1.10$ GeV$^2$, $m_c = 1.6$ GeV and $a = 37.9$ MeV extracted from the charmed mesons and baryons, the strange quark mass $m_s = 288$ MeV is determined from two $1P$ charmed strange mesons: $D_{s1}(2536)^\pm$ and $D_{s2}(2573)^\mp$. The spectrum of the $D_s$ is subsequently computed and listed in Table 2.

Our results for $1D$ and $1F$ are two ”exotic” states. They were firstly observed by BaBar [25, 26] and CLEO [25, 27] and were once interpreted as the $0^+ \frac{1}{2}^+$ and the $1^+ \frac{1}{2}^+$ $D_s$ mesons, respectively. However, there are different interpretations to them. One difficulty of the $D_s$ mesons interpretation is that they have lower masses in comparison with theoretically predicted masses. So far, this two states have not yet been pinned down definitely. In our article, they are not used as inputs to determine the mass of the strange quark. The difficulty of the $D_s$ mesons interpretation is not yet solved in the mass loaded flux tube. $D_{s0}(2317)^\pm$ and $D_{s1}(2460)^\pm$ are really difficult to be interpreted as the pure $1^3P_0$ and $1^3P_1$ mix with $1^1P_0$ $D_s$ mesons.

The situation of the $D_s$ mesons is similar to that of the $D$ mesons. The predicted masses of the $1D$ $D_s$ are much lower than those in Refs. [1, 5].

Recently, two new $D_s$ candidates were observed. $D_{s1}(2860)$ was first reported by BaBar [28] in $D_{sJ}(2860) \rightarrow D^0 K^+, D^+ K_s^0$ with $M = 2856.6 \pm 1.5(stat) \pm 5.0(syst)$ MeV and $\Gamma = 48 \pm 7(stat) \pm 10(syst)$ MeV. For its natural spin-parity: $J^P = 0^+, 1^-, \ldots$, this state was explained as the first radial excitation of the $D_{s0}(2317)$ or the $3^-(1^D)_{D_s}$ [24, 28, 29, 31, 32].

$X(2690)$ was also reported by BaBar [28], but the significance of the signal was not stated. $D_{sJ}(2700)$ was first observed by Belle [32] in $B^+ \rightarrow \bar{D}^0 D_{sJ} \rightarrow \bar{D}^0 D^0 K^+$.

| Candidates [25] | $J^P$ | $j^P$ | $n^{2S+1}L_J$ | GI [1] | PE [5] | our paper |
|----------------|-------|-------|----------------|--------|--------|----------|
| $D_s^0$ | 0$^+$ | 0 | $1^3S_0$ | 1.88 | 1.68 | - |
| $D_s^0(2007)0$ | 1$^-$ | 1 | $1^3S_0$ | 2.04 | 2.00 | 5 |
| $D_s(2400)^0$ | 0 | 1$^+$ | $1^3P_0$ | 2.40 | 2.377 | 2.370 |
| $D_s(2420)^0$ | 1$^+$ | ($\frac{7}{2}^-$) | $1^3P_1$ | 2.49 | 2.417 | 2.408 |
| $D_s(2430)^0$ | 1$^+$ | ($\frac{5}{2}^-$) | $1^3P_1$ | 2.44 | 2.49 | 2.446 |
| $D_s(2460)^0$ | 2$^+$ | $1^3P_2$ | 2.50 | 2.46 | 2.484 |

TABLE I: Spectrum of $D$ mesons (GeV) with parameters $\sigma = 1.10$ GeV$^2$, $m_c = 1.6$ GeV, $m_{u,d} = 180$ MeV and $a = 37.9$ MeV.
The mass and the decay width change a little in their with MeV.

| Candidates [25] | $J^P$ | $j^P$ | $n^{L+L_J}$ | GI | PE [5] | our paper |
|----------------|------|------|-------------|----|--------|-----------|
| $D_s^+(1969)$ | 0 $\frac{1}{2}^+$ | 1$^3$S$_0$ | 1.98 | 1.965 | - |
| $D_s^+(2112)^0$ | 1 $\frac{1}{2}^-$ | 1$^3$S$_1$ | 2.13 | 2.113 | - |
| $D_{s0}^+(2317)^\pm$ | 0 $\frac{1}{2}^+$ | 1$^3$P$_0$ | 2.48 | 2.487 | 2.478 |
| $D_{s1}^+(2536)^\pm$ | 1 $\frac{1}{2}^+$ | 1$^3$P$_1$ | 2.57 | 2.535 | 2.516 |
| $D_{s1}^+(2460)^\pm$ | 1 $\frac{1}{2}^+$ | 1$^3$P$_1$ | 2.53 | 2.605 | 2.554 |
| $D_{s2}^+(2573)^\pm$ | 2 $\frac{1}{2}^+$ | 1$^3$P$_2$ | 2.59 | 2.581 | 2.592 |
| $D_s(2700)^\pm$ | 1 $\frac{1}{2}^+$ | 1$^3$D$_1$ | 2.90 | 2.913 | 2.714 |
| $?_{J=1}(2860)$ | 2 $\frac{1}{2}^+$ | 1$^3$F$_2$ | - | 2.900 | 2.789 |
| $?_{J=1}(2860)$ | 2 $\frac{1}{2}^+$ | 1$^3$F$_2$ | - | 2.900 | 2.789 |
| $D_{s1}(2860)$ | 3 $\frac{1}{2}^+$ | 1$^3$D$_3$ | 2.92 | 2.925 | 2.903 |
| $?_{J=1}(2860)$ | 2 $\frac{1}{2}^+$ | 1$^3$F$_3$ | - | 3.203 | 3.046 |
| $?_{J=1}(2860)$ | 2 $\frac{1}{2}^+$ | 1$^3$F$_3$ | - | 3.203 | 3.046 |
| $?_{J=1}(2860)$ | 3 $\frac{1}{2}^+$ | 1$^3$F$_4$ | 3.19 | 3.220 | 3.160 |

TABLE II: Spectrum of $D_s$ mesons(GeV) with parameters $\sigma = 1.10$ GeV$^2$, $m_c = 1.6$ GeV, $m_s = 288$ MeV and $a = 37.9$ MeV.

with $M = 2715 \pm 11_{14}^{+11}$ and $\Gamma = 115 \pm 20_{32}^{+36}$ MeV. The mass and the decay width change a little in their published version [33]. For its $J^P = 1^+$, this state was interpreted as a mixture of the 2$^3$S$_1$ and the 1$^3$D$_1$ [30] or the 1$^3$D$_1$ [31].

In these interpretations, one difficulty for the 1$^3$D$_1$ and the 1$^3$D$_3$ interpretations is that the masses of $D_{s1}(2700)^\pm$ and $D_{s1}(2860)$ are 100 $\pm$ 200 MeV lower than the theoretical predictions. In the mass loaded flux tube model(Table 2), there is no difficulty to these interpretations at all. The predicted mass of the 1$^3$D$_1$ $D_s$ is around 2714 $\pm$ 30 MeV and the predicted mass of the 1$^3$D$_3$ $D_s$ is around 2903 $\pm$ 44 MeV. As the masses and the decays modes considered, it is very possible that $D_{s1}(2700)^\pm$ and $D_{s1}(2860)$ are the 1$^3$D$_1$ and the 1$^3$D$_3$ charmed strange mesons, respectively.

III. CONCLUSIONS

In summary, the mass loaded flux tube with the inclusion of the spin-orbit interaction is studied. In heavy-light quark systems, the mesons and the baryons are well described by the mass loaded flux. Experimental data (spectrum) for the masses of the mesons and the baryons containing one heavy c quark are reproduced well by the same formula and the same parameters. It is found that the hypothesis of "good diquark-antiquark degeneracy" is a reasonable and consistent hypothesis in heavy-light quark systems.

Our results indicate that $D_{s0}^+(2317)^\pm$ and $D_{s1}^+(2460)^\pm$ are unlike the pure 1$^3$P$_0$ and 1$^3$P$_1$ charmed strange mesons, respectively.

Our predictions of masses of the 1D $D_s$ are much lower in comparison with other theoretical predictions. The predicted masses of the 1$^-(1^3D_1)$ and the 3$^-(1^3D_3)$ charmed strange mesons agree well with those of the recently observed $D_{s1}(2700)^\pm$ and $D_{s1}(2860)$ states, respectively. Other two 1D charmed strange mesons around 2800 MeV are expected to exist, which is left for the confirmation of future experiments.

Of course, many observed states are mixed states in the real world. Under mixing, how to interpret the observed states with the pure states is not clear, which deserves more study.

The heavy-light quark systems containing one b quark have not been analyzed. The heavy quarkonium has not been explored either. The systems with radial excitation or excitation inside the string are not yet involved. How to extend the model to compute the spectrum of all kinds of mesons and baryons would be an interesting work. Furthermore, how to develop the model to describe the production and decay dynamics deserves further exploration. It will be more important to find whether there is an underlying dynamics in the mass loaded flux tube model different from existing QCD inspired models.

The mass loaded flux tube model is a semi-classical one, it will be interesting to study the mass loaded flux tube in a "fundamental" theory such as the string theory(some features such as the Regge trajectory behavior in the mass loaded flux tube model have already been obtained in the string theory).

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