Numerical Solutions of MHD Stagnation-Point Flow over an Exponentially Stretching/Shrinking Sheet in a Nanofluid

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Abstract. This present paper analyses the steady 2-D stagnation-point flow over an exponentially stretching/shrinking sheet in nanofluids with the effect of magnetohydrodynamic (MHD). The effect of nanoparticles volume fraction parameter \( \phi \), and the stretching/shrinking sheet parameter \( \varepsilon \) was analysed over metallic nanoparticles namely as copper (Cu) in the based fluid of water with Prandtl number \( \text{Pr} = 6.2 \). The problem was solved numerically by using a similarity transformation to transform the governing partial differential equations into nonlinear ordinary differential equations and then solved using the boundary value problems solver bvp4c in Matlab software. The results reveal that the shrinking sheet is non-unique while the stretching sheet is unique. Thus, with the increment of the magnetohydrodynamic, the range of solutions is expended. As the magnetic field parameter increase, the skin friction and heat transfer also increase.

1. Introduction

In boundary layer problem, stagnation-point flow effect has been pick the enthusiasm of many researchers because of its usage in manufacturing industry for cooling purposes of devices. Stagnation-point flow is the smooth movement close to the stagnation district of a solid surface exists in two cases of a fixed or moving body in a fluid. Hiemenz [1] was the first researcher who studied the steady 2-D stagnation-point flow towards a fixed semi-infinite wall and an exact solution of the governing Navier-Stokes equations were obtained. Then, the investigation on stretching surface of boundary layer flow and heat transfer had pick interest of many researchers. The first problem of the boundary layer flow over a linearly stretching sheet had been studied by Crane [2]. The works from Hiemenz [1] and Crane [2] have been extended by Chiam [3] which is study about the stagnation-point flow over a stretching sheet. While the similarity solution of the flow equation over a shrinking sheet was explored by Miklavčič and Wang [4] and found that it depends on the external mass suction. Mahapatra and Gupta [5, 6] have been investigated the heat transfer in the stagnation-point flow over a stretching surface through a viscoelastic fluid, respectively. The investigations of the stagnation-point flow over a stretching sheet have been further study by Nazar et al. [7], Ishak et al. [8, 9] and Layek et al. [10] with different physical situations. Then, Wang [11] had been analysed the stagnation-point flow over a shrinking sheet for both cases of two-dimensional and axisymmetric. The dual and unique
solutions had been found for certain values of the ratio of shrinking and straining rates. The problems had been extended by Ishak et al. [12], Bhattacharyya and Layek [13] and Bhattacharyya et al. [14].

However, Magyari and Keller [15] and Bhattacharyya and Vajravelu [16] are the first to consider the steady boundary layers and heat transfer over an exponentially stretching continuous surface. Elbashbeshy [17] and Khan and Sanjayanad [18] have been studied the flow and heat transfer on the surface of an exponentially stretching under different reaction such as wall mass suction and viscous dissipation.

All studies mentioned above focused on a viscous fluid. Bachok et al. [19] have been investigated over an exponentially stretching/shrinking sheet on the steady 2-D stagnation-point flow in nanofluids. Nanofluids are useful in many applications due to their novel properties in heat transfer, microelectronics, fuel cells, pharmaceutical processes and hybrid-powered engines. It is also a smart fluid that can decrease and increase heat transfer freely. Alsaedi et al. [20] have been analysed the stagnation-point flow towards a linearly stretching surface in nanofluids with the heat generation/absorption and thermal boundary conditions.

It is accepted that boundary layer flow gets influence by magnetohydrodynamic (MHD). It is describing as a branch of fluid dynamics which deals the movement of an electrically conducting fluid with magnetic field effect. Ishak [21], Pop [22] and Bhattacharyya and Pop [23] have been study on an exponentially shrinking sheet in viscous fluid with the MHD effect. More importantly, due to countless applications in industrial production process, many researchers study the magnetohydrodynamic (MHD) stagnation-point flow in a nanofluid over a stretching/shrinking surface. Kandasamy et al. [24], Makinde et al. [25] and Akbar et al. [26] have been study the effect of MHD over stretching in a nanofluid. While, the effect of MHD over an exponentially stretching sheet in nanofluids have been investigated by Bhattacharyya and Layek [27]. From all the studies in nanofluids that have been mentioned above are focused on using Buongiorno [28] model. There are two models that the researchers have been always used to study the behavior of nanofluids which are Buongiorno [28] model and Tiwari and Das [29] model. While the Buongiorno [28] model highlights the heat transfer features of the Brownian motion and thermophoresis, Tiwari and Das [29] model focuses on the nanoparticles volume fraction of nanofluid.

Therefore, the motivation of this study is to examine the influence of magnetohydrodynamic (MHD) on stagnation-point flow over an exponentially stretching/shrinking sheet in a nanofluid by using Tiwari and Das [29] model. The effects of magnetic field parameter and nanoparticle volume fraction on heat transfer and temperature are considered. A solver bvp4c in Matlab software is used for solving the governing equations numerically. For some particular cases of the present study, the results are compared with Bachok et al. [19] to support their validity.

2. Flow analysis
A steady, incompressible, laminar, 2-D, magnetohydrodynamic (MHD) stagnation-point flow over an exponentially stretching/shrinking sheet in a water-based nanofluid containing a nanoparticle namely copper Cu is considered. The x axis is taken corresponding to the stretching/shrinking sheet in the direction of movement and the y axis is normal to it with the presence of a normal non-uniform transverse magnetic field strength \( B = B_0 \exp(\chi/2L) \), where \( B_0 \) is a uniform magnetic field strength. The induced magnetic field is neglected due to the motion of an electrically conducting field. Further, it is also assumed that the external electrical field is zero and the electric field is neglected due to the polarization of charges. The simplified 2-D MHD governing equations for the steady, laminar and incompressible nanofluid are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_s \frac{dU_s}{dx} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma B^2}{\rho_f} (U_s - u), \tag{2}
\]
along the following boundary conditions,

\[
\frac{\partial u}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2},
\]

(3)

Then, \(\mu_{nf}\) is the nanofluid’s viscosity, \(\rho_{nf}\) is the nanofluid’s density, \(\alpha_{nf}\) is the nanofluid’s thermal diffusivity, \(T\) is the nanofluid’s temperature, \(T_w\) is the sheet’s varying temperature, \(T_\infty\) is the free flow temperature thought to be constant and \(T_0\) is a constant which measuring the rate of temperature along the sheet, which are given by Oztop and Abu Nada [30] as

\[
\alpha_{nf} = \frac{k_{nf} \rho_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{\frac{2}{3}}},
\]

(4)

\[
(k_{nf} \rho_{nf}) = (1 - \varphi) (k_{nf} \rho_f) + \varphi (k_{nf} \rho_s), \quad \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi (k_f - k_s)}{(k_s + 2k_f) + \varphi (k_f - k_s)}.
\]

(5)

Here, \(\varphi\) is the nanoparticle volume fraction parameter of the nanofluid, \(k_{nf}\) is the thermal conductivity of the fluid fraction, \(k_s\) is the thermal conductivity of the nanoparticle volume fraction, \(\rho_f\) is the reference density of solid fraction, \(\mu_f\) is the viscosity of the fluid fraction and \((\rho C_p)_{nf}\) is the heat capacitance of the nanofluids, where \(C_p\) is the specific heat at constant pressure. Brinkman [31] approximated the viscosity of the nanofluid \(\mu_{nf}\) as the base fluid’s viscosity \(\mu_f\) with fine spherical particles’s diluted suspension. The stretching/shrinking velocity \(U_w\) and the outer flow velocity \(U_s\) are provided by:

\[
U_w(x) = b \exp\left(\frac{x}{L}\right) \text{ and } U_s(x) = a \exp\left(\frac{x}{L}\right),
\]

(6)

where \(b\) is rate of stretching/shrinking velocity as \(b > 0\) for stretching, \(b < 0\) for shrinking and \(a\) is a positive constant, \(x\) is the coordinate measure along the stretching/shrinking surface and \(L\) is the length of the sheet.

To obtain a similarity solution for continuity, momentum and energy equation (1) – (3), the similarity transformation is introduced:

\[
\eta = y\left(\frac{a}{2 \nu L}\right)^{1/2} \exp\left(\frac{x}{2L}\right), \psi = (2\nu_f La)^{1/2} f(\eta) \exp\left(\frac{x}{2L}\right), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}
\]

(7)

where \(\eta\) is the variable of similarity, \(\nu_f\) is the kinematic viscosity of the base fluid and a stream function \(\psi\) is defined as \(u = \partial \psi / \partial y\) and \(v = -\partial \psi / \partial x\) which identically satisfied with the continuity equation (1). By substituting variables (7) into equations (2) and (3), the transformed ordinary differential equations are obtained:

\[
\frac{1}{(1 - \varphi)^{\frac{2}{3}}(1 - \varphi + \varphi \rho_s/\rho_f)} f''' + ff'' - 2f'^2 + 2 + M(1 - f') = 0,
\]

(8)

\[
\frac{1}{Pr \left(1 - \varphi + \varphi (\rho C_p)_s/(\rho C_p)_{nf}\right)} \theta'' + f \theta' - f' \theta = 0.
\]

(9)

The corresponding initial and boundary conditions (4) are

\[
f(0) = 0, f'(0) = \varepsilon, \theta(0) = 1
\]

\[f'(\eta) \rightarrow 1, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty.
\]

(10)
where \( \text{Pr} = \nu_f/\alpha_f \) is the Prandtl number, \( M = 2\sigma B_0^2 L/\rho_f a \) is the magnetic field parameter and \( \varepsilon = h/a \) is the stretching/shrinking parameter where \( \varepsilon > 0 \) for stretching sheet and \( \varepsilon < 0 \) for shrinking sheet.

Quantities of practical interest which are the skin friction coefficient \( C_f \) and the local Nusselt number \( Nu_x \), are defined as

\[
C_f = \frac{\tau_w}{\rho_f U_x^2}, Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)},
\]

where \( \tau_w \) is the surface shear stress and \( q_w \) is the surface heat flux which can be expressed as

\[
\tau_w = \mu_n f \left( \frac{\partial u}{\partial y} \right)_{y=0}, q_w = -k_n f \left( \frac{\partial T}{\partial y} \right)_{y=0},
\]

with \( \mu_n \) is the nanofluids’s dynamic viscosity and \( k_n \) is nanofluids’s thermal conductivity. Using the variables of similarity (7), we obtained

\[
C_f Re_x^{1/2} = \frac{1}{(1 - \varphi)^{2.5}} f''(0),
\]

\[
Nu_x/Re_x^{1/2} = -\frac{k_n}{k_f} \theta'(0),
\]

where \( Re_x = U_x x / \nu_f \).

3. Results and discussion

From the above numerical process, a solution exists under the condition which for the stagnation-point flow over an exponentially stretching/shrinking sheet is obtained. Numerical solutions are solved numerically for the governing ordinary differential equations (8) and (9) subject to the boundary conditions (10) by using the function bvp4c from Matlab due to its effectiveness in solving the boundary value problems which are much harder than initial value problems. In this method, by setting vary starting estimate for the missing values for \( \mu_n \) and \( k_n \), the dual solutions were obtained, where all profiles satisfy the boundary conditions (10) asymptotically, thus keeping the behavior of the solution.

**Table 1** Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada [30])

| Physical properties | Fluid phase (water) | Cu   |
|---------------------|---------------------|------|
| \( c_p \) (J/kgK)   | 4179                | 385  |
| \( \rho \) (kg/m\(^3\)) | 997.1              | 8933 |
| \( k \) (W/mK)      | 0.613               | 400  |

The effect of nanoparticles volume fraction of nanofluid \( \varphi \), the Prandtl number \( \text{Pr} \) and the magnetic field parameter \( M \) are analysed for nanofluid known as copper (Cu) as the working fluids and water as the base working fluid. By following Oztop and Abu-Nada [31], the Prandtl number is taken to be \( \text{Pr} = 6.2 \) and nanoparticles volume fraction is considered from 0 to 0.2 (0 \( \leq \varphi \) \( \leq 0.2 \)), where \( \varphi = 0 \) corresponding to the regular (Newtonian) fluid. Table 1 shows that the thermophysical properties of the base fluid and the nanoparticles. A comparison of the numerical values of \( C_f Re_x^{1/2} \) and \( Nu_x/Re_x^{1/2} \) for Cu-water are shown in Table 2 and Table 3 between previous work in Bachok et al. [19] and the present work, which show a favorable agreement. The numerical computations were
performed for several values of the stretching/shrinking parameter $\varepsilon$, the magnetic field parameter $M$ and the nanoparticles volume fraction $\varphi$.

Figures 1, 2, 3 and 4 illustrate the variations of $f''(0)$ and $-\theta'(0)$ for the values of the magnetic field parameter $M$ and nanoparticles volume fraction $\varphi$ towards stretching/shrinking parameter $\varepsilon$ for Cu-water working fluid. From these figures, we can observe that the unique solutions are forms in the region $\varepsilon \geq -1$ and the dual solutions are occur in the region $\varepsilon_c < \varepsilon \leq -1$ and no solutions for $\varepsilon < \varepsilon_c < 0$, where $\varepsilon_c$ is the critical value of $\varepsilon$. From Figures 1 and 2, we can state that, the increasing of $M$ will increase the range of $\varepsilon_c$. Therefore, for $M = 0.2$, the range of $\varepsilon$ for which the similarity solution exists is larger, i.e. $-1.593662 \leq \varepsilon < \infty$, whereas for $M = 0$ and $M = 0.1$, the range are $-1.487069 \leq \varepsilon < \infty$ and $-1.540335 \leq \varepsilon < \infty$, respectively. However, by adding the magnetic field parameter $M$, Figures 3 and 4 also show that the increase of range of $\varepsilon$ which is $-1.540335 \leq \varepsilon < \infty$, whereas without the magnetic field parameter $M$, the range is $-1.487068 \leq \varepsilon < \infty$ (Bachok et al. [19]). From this observation, Bachok et al. [19] stated that, the first solutions are stable and physically realizable, while second solutions are not stable.

The variations of the skin friction coefficient $G_f Re_x^{1/2}$ and the local Nusselt number $Nu_x Re_x^{-1/2}$ with different values of magnetic field parameter $M$ ($M = 0, 0.1, 0.2$) for Cu-water with $\varepsilon = 0.5$ (stretching sheet) are shown in Figures 5 and 6. These quantities increase almost linearly with $\varphi$. From these figures, we can state that the increasing of $M$ will increase the both value of $G_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$.

| $M$ | $\varepsilon$ | $\varphi$ | Bachok et al. [19] | Present results |
|-----|---------------|-----------|---------------------|-----------------|
| 0   | -0.5          | 0         | 2.1182              | 2.1182          |
|     | 0.1           | 0         | 3.2381              | 3.2382          |
|     | 0.2           | 0         | 4.5071              | 4.5071          |
|     | 0             | 0.1       | 1.6872              | 1.6872          |
|     | 0.2           | 0.1       | 2.5794              | 2.5793          |
|     | 0.5           | 0.2       | 3.5901              | 3.5901          |
|     | 0.1           | 0.5       | 0.9604              | 0.9604          |
|     | 0.2           | 1         | 1.4682              | 1.4682          |
|     |               | 2.0436    |                     | 2.0436          |
| 0.1 | -0.5          | 0         | 2.1708              | 3.3186          |
|     | 0.1           | 0.1       | 4.6191              | 4.6191          |
|     | 0.2           | 0.2       | 1.7165              | 1.7165          |
|     | 0             | 0.1       | 2.6241              | 2.6241          |
|     | 0.2           | 0.1       | 3.6524              | 3.6524          |
|     | 0.5           | 0.1       | 0.9733              | 0.9733          |
|     | 0.2           | 0.2       | 1.4879              | 1.4879          |
|     |               | 2.0710    |                     | 2.0710          |
| 0.2 | -0.5          | 0         | 2.2222              | 3.3971          |
|     | 0.1           | 0.1       | 4.7284              | 4.7284          |
|     | 0.2           | 0.2       | 1.7453              | 1.7453          |
|     | 0             | 0.1       | 2.6681              | 2.6681          |
|     | 0.2           | 0.1       | 3.7137              | 3.7137          |
|     | 0.5           | 0         | 0.9860              | 0.9860          |
|     | 0.1           | 0.1       | 1.5073              | 1.5073          |
|     | 0.2           | 0.2       | 2.0980              | 2.0980          |

Further, Figures 7-10 physically show the velocity and the temperature profiles for different of $M$ and $\varphi$ which support the existence of the dual solution in Figures 1-4 for certain values of $\varepsilon$ and $\varphi$. 
Figure 7 indicates that, the increasing of $M$, increase the velocity profiles at any point $\eta$ except at the sheet where the boundary conditions confine it to value 1.0. For the first solution, the thickness of momentum boundary layer is always narrow than that of the second solution. Then, the profiles of velocity $f'(\eta)$ in Figure 9 show that the velocity increase with increasing of $\varepsilon$ and $\varphi$ in the first solution, and for the second solution, it is decrease except for very small values of $\eta$. Also, for the Figure 10, the temperature decreases as $\varepsilon$ and $\varphi$ increase in the first solution, but increase for large values of $\eta$ in the second solution. We can see that these profiles satisfy boundary conditions (10) asymptotically thus supporting the validity of the numerical results as well as the presence of the dual solutions.

4. Conclusion
We have numerically analyzed how magnetic field parameter $M$ affects the flow of stagnation-point over an exponentially stretching/shrinking sheet in a nanofluid. The analysis effect of nanoparticles volume fraction parameter $\varphi$ and heat transfer characteristics for Cu-water were solved numerically with Prandtl number $Pr = 6.2$. The study reveals that the shrinking sheet is non-unique while the stretching sheet is unique. And the range of solutions is widely expanded with the increment of the magnetohydrodynamic. The skin friction and heat transfer also increase as the magnetic field parameter increase.

| $M$ | $\varepsilon$ | $\varphi$ | $\text{Bachok et al. [19]}$ | $\text{Present results}$ |
|-----|---------------|-----------|-----------------------------|-------------------------|
|     |               |           | Cu-water                    | Cu-water                |
| 0   | -0.5          | 0         | 0.6870                      | 0.6870                  |
|     |               | 0.1       | 1.1432                      | 1.1432                  |
|     |               | 0.2       | 1.5184                      | 1.5185                  |
|     |               | 0.1       | 2.1358                      | 2.1358                  |
|     |               | 0.2       | 2.5400                      | 2.5400                  |
| 0.5 |               | 0         | 2.4874                      | 2.4874                  |
|     |               | 0.1       | 2.9150                      | 2.9149                  |
|     |               | 0.2       | 3.3565                      | 3.3565                  |
| 0.1 | -0.5          | 0         | 0.7079                      |                          |
|     |               | 0.1       | 1.1649                      |                          |
|     |               | 0.2       | 1.5419                      |                          |
|     |               | 0         | 1.7220                      |                          |
|     |               | 0.1       | 2.1442                      |                          |
|     |               | 0.2       | 2.5494                      |                          |
| 0.5 |               | 0         | 2.4897                      |                          |
|     |               | 0.1       | 2.9177                      |                          |
|     |               | 0.2       | 3.3597                      |                          |
| 0.2 | -0.5          | 0         | 0.7279                      |                          |
|     |               | 0.1       | 1.1857                      |                          |
|     |               | 0.2       | 1.5642                      |                          |
|     |               | 0         | 1.7291                      |                          |
|     |               | 0.1       | 2.1524                      |                          |
|     |               | 0.2       | 2.5586                      |                          |
| 0.5 |               | 0         | 2.4919                      |                          |
|     |               | 0.1       | 2.9205                      |                          |
|     |               | 0.2       | 3.3629                      |                          |
Figure 1 $f''(0)$ with $\varepsilon$ for several values of $M$ for Cu in water-based nanofluid, $Pr = 6.2$ and $\varphi = 0.1$.

Figure 2 $-\theta'(0)$ with $\varepsilon$ for several values of $M$ for Cu in water-based nanofluid, $Pr = 6.2$ and $\varphi = 0.1$.

Figure 3 $f''''(0)$ with $\varepsilon$ for several values of $\varphi(0 \leq \varphi \leq 0.2)$ for Cu in water-based nanofluid, $Pr = 6.2$ and $M = 0.1$.

Figure 4 $-\theta(0)$ with $\varepsilon$ for several values of $\varphi(0 \leq \varphi \leq 0.2)$ for Cu in water-based nanofluid, $Pr = 6.2$ and $M = 0.1$.

Figure 5 Skin friction coefficient $C_{f}Re_{x}^{1/2}$ with $\varphi$ for different magnetic field $M$ for Cu in water-based nanofluid, $Pr = 6.2$ and $\varepsilon = 0.5$. 

Figure 6 Local Nusselt number $N_u_x R_e^{-1/2}$ with $\varphi$ for different $M$ for Cu in water-based nanofluid, $Pr = 6.2$ and $\varepsilon = 0.5$.

Figure 7 Velocity profiles for various values of $M$ for Cu in water-based nanofluid, $\varphi = 0.1, \varepsilon = -1.25$ and $Pr = 6.2$.

Figure 8 Temperature profiles for various values of $M$ for Cu in water-based nanofluid, $\varphi = 0.1, \varepsilon = -1.25$ and $Pr = 6.2$.

Figure 9 Velocity profiles for several values of $\varphi$ ($0 \leq \varphi \leq 0.2$) for Cu in water-based nanofluid, $M = 0.2$ and $Pr = 6.2$.

Figure 10 Temperature profiles for several values of $\varphi$ ($0 \leq \varphi \leq 0.2$) for Cu in water-based nanofluid, $M = 0.2$ and $Pr = 6.2$. 
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