Energy accumulation mechanism in pulsar magnetospheric plasma eigen-waves and formation of Giant Radio Pulses

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15 May 2018

ABSTRACT

In the present work we consider energy accumulation mechanism in relativistic electron-positron plasma in the magnetosphere of pulsars. Waves propagating almost across the magnetic field lines are generated that accumulate the energy of plasma particles, as the waves stay significantly long in the resonance region. The accumulated energy is transmitted in the parallel direction of the magnetic field lines as soon as the non-linear plasma processes start to operate. It is suggested that exit of the accumulated energy via waves propagating along the magnetic field lines matching the viewing angle of the observer produces giant pulses. It is shown that these waves come in the radio domain, explaining the Giant Radio Pulse phenomenon.

Key words: keyword1 – keyword2 – keyword3

1 INTRODUCTION

From several pulsars sporadic intense radio pulses are observed known as the Giant Radio Pulses (hereafter GRPs) that are much brighter than the regular pulses. This rare phenomenon has been detected only in 16 pulsars (see parameters in Kazantsev & Potapov (2018)), one of the first of them was the Crab pulsar PSR B0531+21 (Stal&Reifenstein 1968; Argyle&Gower 1972). The peak flux densities of GRPs can exceed hundreds and thousand of times the peak flux density of regular pulses and the pulses reveal extremely narrow width in comparison with the average emission of the pulsar (their duration is of the order of several microseconds down to few nanoseconds Istomin (2004); Hankins&Eilek (2007)). The ultrashort durations of the giant pulses imply very high equivalent brightness temperatures (Hankins et al. 2003) indicating that they originate from nonthermal coherent emission processes. Moreover, through the GRPs narrow-band radiation is emitted (the width of the spectrum is of the order of the frequency band; Popov&Stappers (2003)). Another distinctive feature of giant pulses from usual radio emission of pulsars is that their amplitude distribution is power law, while that of normal pulses follows a log-normal distribution (Argyle&Gower 1972). All of the listed features of giant pulses that differs them from the regular radio pulses indicates a different emission generation scenario.

Knowing the origin of giant radio pulses is extremely important for understanding the difference between “normal” radio pulsars and the ones emitting also the giant pulses. Although, several theoretical models explaining the GRPs have been proposed, the emission mechanism of giant pulses still remains unclear. Weatherall (2001), suggested that strong electrostatic turbulence in electron-positron ($e^−e^+$) plasma could produce intense radiation. Hankins et al. (2003) claimed that GRPs from the Crab pulsar is produced through the conversion of electrostatic turbulence in the pulsar magnetosphere by the mechanism of spatial collapse of nonlinear wave packets. In Petrova (2004) was proposed that GRPs are generated by induced scattering of low frequency radiation in the pulsar magnetosphere causing a redistribution of the radio emission in frequency. It has been found that the efficiency of the amplification of radiation drastically depends on radio luminosity of a pulsar. For the pulsars with GRPs, PSR B0531+21 and PSR B1937+21 the value of total radio luminosity $L_r \approx 4 \cdot 10^{31}$erg/s and 7.44 $\cdot 10^{30}$erg/s, consequently. These values are indeed higher than that for other “normal” radio pulsars Malov (2006). In Istomin (2004) it was suggested that the radio emission generation process is implemented close to the light cylinder region by the electric discharge taking place due to the magnetic reconnection of the field lines connecting the opposite magnetic poles. It is considered that the GRPs in pulsars PSR B0531+21 and B1937+21 are generated through the maser amplification of Alfvén waves. It is supposed that these objects are perpendicular rotators which provides specific charge distribution on magnetic poles of the star. This causes appearance of strong electric
field and magnetic reconnection close to the light cylinder that induces maser amplification of the amplitude of Alfvén waves. The mentioned process can develop only if the value of the magnetic field in the region of the light cylinder is sufficiently high (of the order of $10^6 \text{G}$, that is well fulfilled for the considered pulsars). The most recent explanation was provided by Lyutikov (2007) that is closest to our scenario suggested in the present work. He supposed that GRPs are generated on closed magnetic field lines in a limited small volume, where the last closed field lines approach the light cylinder via anomalous cyclotron resonance on the ordinary mode. This model is especially developed for Crab pulsars GRPs associated with the interpulse.

Here we present an alternative explanation of the phenomena of GRPs, which suggests the mechanism of radio emission energy growth due to nonlinear plasma processes taking place in the pulsar magnetosphere. Particularly, the GRPs generation scenario implies that the energy is accumulated in waves propagating practically across the local magnetic field lines that subsequently change the propagating direction due to induced scattering of waves on plasma particles and finally can be observed as pulsar radio emission. This happens when the wave vector is practically parallel to the direction of magnetic field lines. In general we believe that the pulsed radio emission from pulsars is generated near the light cylinder region by plasma instabilities developing in the outflowing plasma on the open field lines of the pulsar magnetosphere. Plasma can be considered as an active medium that amplifies its normal modes through the resonant wave-particle interaction. The generated $e^-e^+$ plasma eigen waves propagate along the magnetic field lines and are in this case vacuum-like electromagnetic waves so they can leave the magnetosphere directly and reach an observer as pulsar radio emission. This plasma emission model has been well developed and explains all main observational features of radio pulsars (see Lominadze et al. (1979); Machabeli & Usov (1979a); Kazbegi et al. (1991a); Lyutikov et al. (1999a); Lyutikov et al. (1999b)).

According to our scenario the formation of GRPs in pulsar magnetosphere is based on the above mentioned radio emission generation model. Therefore, it is foremost essential to introduce the latter in more details. Consequently, in Section 2, we describe the radio emission generation mechanism. This implies considering the maser-type plasma instabilities, anomalous cyclotron-Cherenkov and Cherenkov-drift resonances generating waves propagating along (Section 2) and across (Section 3) the pulsar’s magnetic field lines. The waves propagating almost across the magnetic field accumulate energy in waves until the nonlinear processes start to operate. In Section 4 the development of nonlinear processes is considered that plays the main role in producing the GRPs.

## 2 GENERATION OF WAVES PROPAGATING ALONG THE MAGNETIC FIELD - RADIO EMISSION MODEL

According to the works of Sturrock (1971) and Tademaru (1973), due to the cascade processes of pair creation, a pulsar’s magnetosphere is filled by $e^-e^+$ plasma with an anisotropic one-dimensional distribution function (see Figure 1) and consists of three components. First component is the most energetic primary beam, which consists of primary electrons (the maximum Lorentz factor of the particles $\gamma_p \approx 10^7$) extracted from the star surface by electric field induced due to rotation of the magnetized neutron star. According to Goldreich-Julian the density of the primary beam $n_b = n_{GJ} = 7 \cdot 10^{-5} B_0 P^{-1}$, where $B_0$ is the magnetic field at pulsar surface and $P$ is the spin period. The second component of pulsar magnetospheric plasma is the bulk of plasma that consists of secondary electrons and positrons produced through the cascade process of pair creation taking place near the polar cap in the vacuum gap region (Michel 1982; Ruderman & Sutherland 1975). The typical Lorentz factor of the plasma particles assuming that the pulsar has a dipole magnetic field is $\gamma_p \approx 10 – 10^2$ and for the quadrupole model for the pulsar magnetic field $\gamma_p \approx 2 – 5$ (Machabeli & Usov 1979a). Considering the quadrupole model for the Crab pulsar, its parameters $B_0 = 7.6 \cdot 10^{12} \text{G}$, $P = 33$ms and the equipartition of energy among the plasma components $2n_p \gamma_p \approx n_b \gamma_b$, one can obtain $n_p \approx 10^{20} \text{cm}^{-3}$. The third component is a tail on the distribution function with Lorentz factor $\gamma_t \sim 10^{5-5}$. Plasma with anisotropic distribution function is unstable and can cause excitation of plasma eigenwaves. The properties of the $e^-e^+$ plasma have been investigated quite thoroughly (e.g., Volokitin et al. (1985); Arons & Barnard (1986); Lominadze, et al. (1986)). There are three fundamental modes: the transverse extraordinary waves. The properties of the $e^-e^+$ plasma have been investigated quite thoroughly (e.g., Volokitin et al. (1985); Arons & Barnard (1986); Lominadze, et al. (1986)). There are three fundamental modes: the transverse extraordinary waves.
(X-mode) wave with the electric field perpendicular to the \( \mathbf{k} \) and \( \mathbf{B} \) (where \( \mathbf{k} \) is the wavevector and \( \mathbf{B} \) is the vector of the magnetic field) and the longitudinal-transverse modes with the electric field lying in the plane formed by \( \mathbf{k} \) and \( \mathbf{B} \): the ordinary (O-mode) and Alfvén (A-mode) modes. The O mode on the diagram \( \omega(k) \) begins with the Langmuir frequency (Figure 2) and when \( k_1 = 0 \), it reduces to the pure longitudinal Langmuir wave. In the laboratory frame the dispersion relations of O, A and X-modes are, respectively:

\[
\omega = kc(1 - \delta), \quad \text{X-mode (1)}
\]

\[
\omega_{lt} = k_\parallel c \left(1 - \frac{k_\perp^2 c^2}{4\gamma_p \omega_p^2}\right), \quad \text{A-mode (2)}
\]

\[
\omega_{\text{Li}}^2 = \omega_p^2 \gamma_p^3 + k_\perp^2 c^2, \quad \text{O-mode (3)}
\]

where \( \delta = \omega_p^2/4\gamma_p \omega_p^2 \omega_\parallel^2 \) and \( \omega_\parallel^2 = 4\pi n_p e^2/m \). The indexes p and b correspond to the bulk of plasma and the primary beam of electrons.

The names of the waves used above (X, A, O modes) are often used by analogy with names for the waves in non-relativistic electron-ion plasma. However, the waves in relativistic pair plasma are quite different. The low-frequency part of the X-mode is in the superluminal region, where \( \nu_{ph} > c \) and its generation is only possible in the region where it intersects the line \( \omega = kc \) on the \( \omega(k) \) diagram (See Fig. x). In the high-frequency region \( \omega \gg \omega_p/\gamma_p \), the O-mode merges with the X-mode and converts into transverse wave and for the region where \( \omega < \omega_p/\gamma_p \), the O-mode describes electrostatic Langmuir waves, the frequency of which at the intersection point with the line \( \omega = kc \) equals to \( \omega_\parallel^2 = 2\omega_p^2 \gamma_p \) (Lominadze & Mikhailovskii 1979). For more clarity we prefer here to use different designations for the X, A, and O-modes, correspondingly naming them as \( t \), \( l \t \) and \( L \) waves.

The magnetospheric plasma is anisotropic that is quite natural in the presence of strong pulsar magnetic field. The relativistic particles efficiently emit through the synchrotron mechanism, quickly losing transverse momenta near the star surface continuing to move along the magnetic field lines with the relativistic momenta \( p_L = m c \gamma \gg p_L \). Such plasma is unstable causing excitation of electromagnetic waves and as a result both types of eigen-waves \( t \), as well as \( L \) can be generated (Lominadze et al. 1979). When \( t \) and \( L \) waves are generated with the small inclination angle with respect to the magnetic field, they propagate in the same direction along the field lines and are orthogonally polarized. These are vacuum-like electromagnetic waves and can leave the magnetospheric region for an observer as pulsar emission.

When considering generation of the radio waves, for which the wavelength \( \lambda \) is much bigger than the average distance between the plasma particles \( \lambda \gg n^{-1/3} \) (here \( n \) is the density of plasma particles) the effects of wave interference should be taken into account. As already mentioned the distribution function shown on Fig. x tends to be unstable relative to some plasma instabilities. The strongest instabilities that can develop in the pulsar magnetosphere are the Cherenkov-drift and anomalous cyclotron-Cherenkov instabilities. The frequency of the waves generated via the mentioned resonances for typical pulsars falls within the radio band. The Cherenkov-drift instability develops at the resonance:\n
\[
\omega - k_\parallel u_\parallel - k_x u_x = 0,
\]

where \( u_\parallel = \nu_\parallel^2 \gamma_{res}/\omega_B R_2 \) is the drift velocity of the particles across the magnetic field caused by the magnetic field inhomogeneity and directed along the direction of the x-axis, \( \gamma_{res} \) is the Lorentz factor of the resonant particles, \( \omega_B = eB/mc \) is the cyclotron frequency and \( R_2 \) is the curvature radius of the magnetic field lines. Here the cylindrical coordinates \( x, r, \varphi \) have been chosen, x-axis directed transversely to the plane of the curved field line, with \( r \) directed along the radius of curvature of the field line and \( \varphi \) the azimuthal coordinate; \( k_\parallel \) is the component of the wavevector along the magnetic field and \( k_x = (k_x^2 + k_\parallel^2)^{1/2} \).

The second strongest instability in the pulsar magnetosphere the cyclotron-Cherenkov instability, which generates low frequency waves via the anomalous Doppler effect develops at the resonance:

\[
\frac{1}{2}Re = \frac{1}{2} \left( \frac{k_x}{k_\parallel} \right)^2 + \frac{k_\perp^2}{2k_\parallel^2} - \delta = - \frac{\omega_B}{\gamma_{res} \gamma_p c}.
\]

For the typical pulsar parameters the conditions (6) can be fulfilled for both the particles in the tail of the distribution function and the primary beam electrons. Although, the Cherenkov-drift resonance condition (4) is exclusively fulfilled for the beam particles. For the bulk plasma \( \gamma_{res} = \gamma_p \) the resonance can not be implemented as in this case the term \( 1/\gamma_p^2 \) is the largest.

For the wave generation via the cyclotron resonance as it follows from expression (6) it is clear that the following inequality should be fulfilled:

\[
\delta > 1 + \frac{1}{2} \left( \frac{k_x}{k_\parallel} - \frac{1}{2} \right)^2 + \frac{k_\perp^2}{2k_\parallel^2}.
\]

The parameter \( \delta \) is quite small, which makes it challenging to fulfill the condition (7). Although, the quantity \( \omega_p^2/\omega_B^2 \sim B \) and consequently grows with the growth of \( \delta \) - distance to the pulsar and for typical pulsar parameters this condition can be fulfilled for the distances of the order of light cylinder radius.

Let us estimate the frequency of the \( t \) waves generated via the cyclotron resonance. If we denote the angle between the wave vector \( \mathbf{k} \) and the magnetic field \( \mathbf{B} \) as \( \theta \) and consider the small propagation angles \( \theta \ll 1 \) and take into account that \( u_x/c \ll 1 \), after neglecting the drift term one obtains:

\[
\frac{1}{2}Re = \frac{\theta^2}{2} - \delta = - \frac{\omega_B}{\omega_{res}}.
\]

The Eq. (8) requires that \( 1/2\gamma_{res}^2 < \delta \) and \( \theta^2/2 < \delta \). The first condition implies that the resonant particles should be moving with a velocity faster than the phase velocity of the wave. The second condition limits the generated emission to small angles with respect to the magnetic field. Assuming
that $1/2y_{res}^2 \ll \delta$ and $\theta^2/2 \ll \delta$ from Eq. (8) one can estimate the frequency of the generated waves:

$$\omega \approx \omega_B \frac{\delta y_{res}}{\delta y}$$  \hspace{1cm} (9)$$

which comes in the radio frequencies for the typical pulsar parameters.

Both cyclotron-Cherenkov and Cherenkov-drift instabilities are capable to explain the main observational characteristics of pulsar radio emission. The cyclotron-Cherenkov instability is responsible for the generation of the core-type radio emission and the Cherenkov-drift instability is responsible for the generation of the cone-type emission. The excited waves propagate along the magnetic field lines and the frequency of the generated waves for typical pulsars falls within the radio band. As shown in Machabeli & Usov (1979b) the $t$ waves can be only excited by anomalous Doppler resonance, whereas Cherenkov-drift resonance can generate both $r$ and $lt$ waves. These instabilities occur in the outer parts of the magnetosphere in the region near the light cylinder. The location of the emission region is determined by the corresponding resonant condition for the instabilities. Instabilities develop in a limited region on the open field lines. The size of the emission region is determined by the curvature of the magnetic field lines, which limits the length of the resonant wave-particle interaction. The location of the cyclotron instability is restricted to those field lines with large radii of curvature, while the Cherenkov-drift instability occurs on field lines with curvature bounded both from above and from below. Thus, both instabilities produce narrow pulses, although they operate at radii where the opening angle of the open field lines is large.

3 GENERATION OF WAVES PROPAGATING ACROSS THE MAGNETIC FIELD - THE DRIFT WAVES

Until now we have discussed generation and propagation of waves with frequencies falling within the radio band and the rather narrow angles of propagation $\theta \approx k_x/k_\varphi \ll 1$. Taking into account the weak inhomogeneity of the magnetic field leads to drift of the plasma particles across the field lines violating the cylindrical symmetry and the permittivity tensor occurs additional terms that are proportional to the expression $1/(\omega - k_\varphi v_\varphi - k_x u_x)$ assuring generation of waves perpendicular to the magnetic field. It should be mentioned that the inhomogeneity of the medium (curvature of the magnetic field lines of pulsar) is revealed in linear approximation only in drift of the charged particles. Thus, when $kR_B \gg 1$ the waves propagating in $e^-e^+$ plasma in the pulsar magnetosphere do not feel the curvature of the field lines. Though, for the ultrarelativistic particles (e.g. primary beam electrons) one can not neglect the drift motion across the magnetic field lines when considering the wave generation processes.

Let us now investigate generation of low-frequency $lt$ waves via the Cherenkov-drift resonance propagating nearly transversely to the magnetic field $\theta = \pi/2$. In Kazbegi et al. (1991b) it is shown that for waves propagating almost across the magnetic field lines and taking into account that the parameters $\gamma_\omega/\omega_B \ll 1$, $(\alpha^2_\omega/c^2) \ll 1$ and $k_\varphi/k_x \ll 1$ are small, the dispersion relation can be written as

$$\frac{k_\varphi^2 c^2}{\omega^2 - k_\varphi^2 c^2} = e_{\varphi\varphi} = 1 + \sum_\nu \left( \frac{\omega_{p\nu}^2}{\omega} \int \frac{u_\varphi/c}{(\omega - k_x u_x - k_\varphi u_\varphi)} \frac{\partial f}{\partial \gamma} dp_\varphi \right).$$  \hspace{1cm} (10)$$

The sum is taken over all particle species, electrons and positrons of the bulk plasma, tail particles and electrons of the beam ($a = p, t, b$). Assuming that $\omega = k_x u_x + k_\varphi u_\varphi + a$, where $a$ is the small parameter and $u_x = v_{\gamma b}/\omega B R_B$ (the resonant particles are the fastest beam electrons). After integration the dispersion (10) can be rewritten as:

$$1 - \frac{3\omega_p^2}{2\gamma_B c^2} - \frac{\omega^2}{\omega_B^2} \frac{k_\varphi^2}{\omega^2} = \frac{k_\varphi^2 c^2}{\omega^2}.$$  \hspace{1cm} (12)$$

The indices $p$ and $b$ denote the bulk plasma and the beam respectively. The real part of the frequency $\text{Re}(\omega) \equiv \omega_0 = k_x u_x + k_\varphi u_\varphi = k_x u_x$ and the parameter $a$ is a complex number and the growth rate of the instability $\Gamma = \text{Im}(\omega)$ is the greatest when

$$k_\varphi^2 \leq \frac{3\omega_p^2}{2\gamma_B c^2}.$$  \hspace{1cm} (13)$$

In this case one can write

$$\Gamma \approx \left( \frac{\gamma_B}{\gamma_p} \right)^{1/2} \frac{\gamma_B^{3/2}}{\gamma_p^{1/2}} k_x u_x.$$  \hspace{1cm} (14)$$

The condition (13) implies the initial value for the wave vector $k_x \sim 1/r_D \sim 10^{-1}$ (for the Crab pulsar parameters), $r_D$ the Debye radius defines the scale of perturbations below which the condition of quasi-neutrality is violated. Consequently, this defines the maximal value for the perpendicular component of the wave vector. On the other hand the minimal value for the wave vector is confined by the perpendicular dimension of the magnetosphere in the region of the wave generation $k_{\text{min}} \sim 1/R_B^{1/3}$. The perturbations with the frequency $\omega_{dr} = k_x u_x$, which are propagating almost across the magnetic field lines we will be calling the drift waves. The drift waves are excited at the left slope of the distribution function corresponding to the beam. The wave draws energy from the longitudinal motion of the beam particles as in the case of an ordinary Cherenkov wave-particle interaction. However, the wave is excited only if $k_x u_x \neq 0$, i.e., in the presence of drift motion of the beam particles.

From expression (14) follows that the increment is greater the farther from the pulsar surface the instability develops, while the drift velocity strongly increases with distance $u_x \sim r^3/R_B$. Generated in the light cylinder region the growth rate of the drift waves estimated from expression (14) $\Gamma \sim 10^{-1}$ appears quite small that is natural as number of the particles taking part in the resonance is not very big. However, the drift waves propagate near transversely to the magnetic field, encircling the magnetosphere, and stay in the resonance region for a substantial period of time. Although the particles give a small fraction of their energy to the waves and then leave the interaction region, they are continuously replaced by the new particles entering this region. The waves leave the resonance region considerably
more slowly than the particles. Hence, there is insufficient time for the inverse action of the waves on the particles. The accumulation of energy in the waves occurs without quasilinear saturation.

Note that these low-frequency waves are nearly transverse, with the electric vector being directed almost along the local magnetic field. From Maxwell’s equation
\[
\mathbf{rot} \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t},
\]

after Fourier transformation one can obtain
\[
B_r = \frac{k_x c}{\omega} E_\varphi \sim E_\varphi \frac{c}{u_x},
\]

Consequently for the drift waves \(B_r \gg E_\varphi\).

For the nonzero parallel component of the wave vector, the resonant condition (4) can be rewritten as:
\[
\omega = k u_x \sin \left(\frac{\pi}{2} - \theta'\right) + k v_\varphi \cos \left(\frac{\pi}{2} - \theta'\right),
\]
where \(\theta' = \pi/2 - \theta \ll 1\) taking into account that \(\theta \approx \pi/2\). This expression can be rewritten as, \(\omega \approx k (u_x + c \theta')\) from which one can obtain
\[
\frac{\partial \omega}{\partial k} \approx u_x + c \theta'.
\]

For the condition \(\theta' \ll u_x/c\) only a small fraction of the whole energy accumulated in drift waves is transferred in the direction along the magnetic field lines in the low frequency range. The energy is mainly transferred perpendicularly to the field lines along the circular orbits. The time of energy pumping from the beam electrons into the drift waves depends on the angle \(\theta' = k_\varphi/k_x\). This process continues on the linear stage of the turbulence until the wave does not leave the magnetosphere. The time during that the beam particles leave the pulsar magnetosphere is estimated as \(\tau \sim \tau_{LC}/c \sim 1/\Omega\), here \(\tau_{LC} = c P / 2\pi\) is the radius of the light cylinder and \(\Omega\) is the rotation frequency and for the Crab pulsar parameters \(\tau \sim 10^{-7}\) s. Consequently, the time during that the drift waves stay in the magnetosphere and can accumulate the energy is of the order of \(\tau (k_x/k_\varphi)\).

At the linear stage of the resonance interaction of the energetic beam electrons with the drift waves their drift velocity is quite large. The particle move across the magnetic field with the velocity of the order of \(u_x \sim 10^{-2}c\), give part of their energy to the waves and are carried out from the interaction region. New particles enter this region, in their turn give part of their energy to the waves and so on. The wave leaves the interaction region considerably slower than the particles. Hence there is sufficient time for the waves to accumulate energy. The energy accumulation process is maintained until the nonlinear processes start to operate. In the next section we consider the stage of development of nonlinear processes and discuss the scenario of formation of GRPs.

4 NONLINEAR INTERACTION OF DRIFT WAVES

The energy of the excited drift waves grows when propagating across the magnetic field and encircling the region of the open field lines until the nonlinear process of induced scattering of waves on plasma particles develops. From all nonlinear processes the probability of the three-wave interaction and the induced scattering of waves on plasma particles is the highest. However, the second-order current describing the decay interaction is proportional to \(e^2\). Hence the contribution of electrons and positrons is compensated if their distribution functions coincide. At the same time the induced scattering is proportional to the even charge power \((e^2\))

So that the induced scattering on particles appears significant, it is necessary that the number of particles taking part in the resonance is sufficiently large, i.e. the bulk plasma particles with the Lorentz factors \(\gamma_p \sim 2 - 5\) should be the resonant particles. The drift velocity for the particles with small Lorentz factors is considerably small and the particles move practically along the field lines. Consequently, the nonlinear interaction occurs along the pulsar magnetic field and the resonance condition is written as (Kazbegi et al. 1991b):
\[
\omega - \omega' - (k_\varphi - k_\varphi') v_\varphi = 0.
\]

After taking into account that \(\omega = k_x u_{xb} + k_\varphi v_{\varphi b}\) and \(v_{\varphi b} \approx c(1 - 1/2\gamma_p^2)\), one can rewrite the resonance condition (19) as
\[
\frac{k_x - k_\varphi}{k_\varphi} \frac{\gamma_p}{k_\varphi \gamma_p} = -\frac{c - 1}{u_{xb} 2\gamma_p^2}.
\]

This equation implies two cases: \((k_x - k_\varphi') > 0\) and \((k_x - k_\varphi') < 0\), or \((k_x - k_\varphi') > 0\) and \((k_x - k_\varphi') > 0\). In the first case \(k_x' < k_x\), the perpendicular component of the wave vector is reducing via the scattering process and the parallel component grows \(k_\varphi' > k_\varphi\). For the second case \(k_x' > k_x\), \(k_\varphi' > k_\varphi\). Both of these processes are equiprobable. However, at the initial stage of the development of scattering process the second case is unlikely to develop. In Sec. 3 we have shown that for the linearly generated drift waves propagating almost across the magnetic field the wave vector, i.e. \(k_x\) is maximal. The minimum possible value for the wave vector is defined from the size of the magnetosphere and the minimal value for the parallel component of the wave vector for the Crab parameters \(k_\varphi \sim 1/r_{LC} \sim 10^{-8}\) cm. Consequently, before the scattering takes place \(\theta' \approx k_\varphi/k_x \approx 10^{-7}\) and the time during that the waves stay in the pulsar magnetosphere accumulating energy will be of the order of \(\tau (k_x'/k_\varphi') \approx 10^5\) s.

The scattering event causes growth of the parallel component of the wave vector and the perpendicular component reduces at the same time. At some point one can reach the case when \(k_x' \ll k_\varphi'\), in the other words the wave will be propagating with the small inclination angle with respect to the magnetic field lines. This must cause appearance of the radio emission and supposedly in this case the GRP will be observed. The time of escape of such waves from the pulsar magnetosphere is of the order of \(r_{LC}/c \sim 10^{-7}\) s, as they are propagating almost along the magnetic field (the angle \(\theta \approx k_\varphi'/k_x' \ll 1\)). The drift waves spend much more time encircling the open field lines and accumulating energy than escaping the magnetosphere to reach an observer.

5 APPLICATION OF THE MODEL AND CONCLUSIONS

The GRPs are the brightest sources of radio emission from astrophysical objects. They do not affect the average radio emission characteristics of the given pulsar and are detected
from pulsar with parameters differing in wide range. The GRPs are distinguished from pulsar’s ordinary pulsed radio emission by several special properties. The main property, as mentioned above is that the peak intensities of GRPs greatly exceed the peak intensities of the ordinary average pulses. For the Crab pulsar’s strongest GRP, the peak flux density exceeds the mean flux density of regular pulses by a factor of $5 \times 10^5$ (Kostyuk et al. 2003). Let us estimate the energy gained by drift waves at the phase of perpendicular propagation. The energy source of the process is the kinetic energy of the beam particles with the typical Lorentz factors $\gamma_b \sim 10^6 - 10^9$ drifting in the perpendicular direction of the inhomogeneous. Consequently, for the maximum accumulated energy by the drift waves through their propagation in the pulsar magnetosphere one can write

$$\mu_c^2 \gamma_b \eta_b \frac{k_x}{k_{\perp}} \tau.$$  \hspace{1cm} (21)

here $\tau (k_x/k_{\perp})$ is the accumulation time and $\mu_c^2 \gamma_b \eta_b$ is the energy density of the beam particles. In order to estimate the excess of the GRP peak intensity over the ordinary pulse, one needs to consider the a different generation mechanism. In particular, as we suggest the ordinary radio emission is generated via the plasma instabilities. Considering that the ordinary radio pulses in the Crab pulsar are produced through the cyclotron-Cherenkov resonance, the mean pulse peak energy can be estimated as

$$mc^2 \gamma_b \eta_b \tau.$$  \hspace{1cm} (22)

Here we have taken into account that the speed of the beam electrons approximately equals $c$ and multiplied the energy density by the escaping time of the particles from the magnetosphere $\tau$. Now for the ration of the peak flux densities of the GRP and the mean ordinary pulse (OP) we obtain

$$\frac{S_{\text{GRP}}}{S_{\text{OP}}} \approx \left( \frac{\mu_c^2}{c} \right)^2 \left( \frac{k_x}{k_{\perp}} \right).$$  \hspace{1cm} (23)

For the Crab pulsar parameters $S_{\text{GRP}}/S_{\text{OP}} \sim 10^5$ that matches the observations.

The GRPs are very bright and short. As shown in Hankins et al. (2003) the GRPs from the Crab pulsar are as short as 2ns. The resonant particles are moving along the field lines to the observer and the angular distribution of the radiation reaching the observer is mainly concentrated within the small angle (Landau & Lifshitz 1971; Schwinger et al. 1976)

$$a_{\text{rad}} \approx \frac{1}{\tau_{\text{res}}}.$$  \hspace{1cm} (24)

around the field line. If the size of the emitting spot is negligible (compared to the size of the magnetosphere) then the duration of emission received by an observer is

$$\tau_{\text{rad}} \approx \frac{1}{\Omega_{\text{res}}}.$$  \hspace{1cm} (25)

Noting that for the crab pulsar $\Omega \approx 200 s^{-1}$ and for the particles of primary beam duration of the pulse is $\tau_{\text{rad}} \sim 10^{-9}s$ of the order of nanosecond.

As we can see two main characteristics, the excess of flux density relative to an average pulse ones and a short pulse time-scale compared to an average pulse is well explained in the framework of the present scenario. The radio emission that we receive as giant radio pulses is generated via the Cherenkov-drift resonance near the light cylinder similarly as the ordinary radio pulses. The only difference is the propagation direction of the generated waves. In case of ordinary radio pulses the generated waves are propagating almost along the magnetic field lines leaving the pulsar magnetosphere and consequently the resonant region considerably fast. On the other hand the same resonance provides generation of almost perpendicular waves that encircle the region of open magnetic field lines and propagate in the direction of the magnetic field with the very low speed of the order of $c k_{\perp}/k_x$. This ensures presence of the waves in the resonant region for a sufficiently long time. The resonant particles leave the wave-particle interaction region quite fast, though the new particles enter this region continuously and the waves appear to have sufficient time to accumulate energy. The accumulated energy is received by observer as pulsar radio emission due to change of the direction of the mentioned perpendicular waves via nonlinear scattering processes. As these type of waves have much more time to gain particles’ kinetic energy than the waves that directly leave the magnetosphere, the pulses with very large energy excess are observed. The observations reveal that for the Crab pulsar the GRPs can occur anywhere within the average pulse, which could be caused by excitement of the drift waves at different altitudes and magnetic field lines with. This could also cause little changes in observed frequency of the waves. One more fact in favor of our model is the nondetection of correlation of GRPs and emission in higher frequencies (Shearer et al. 2003). The mechanism of generation of high frequency emission in pulsars differs from that of radio emission (see Chkheidze et al. (2013)) and is connected with the appearance of pitch angles due to development of cyclotron-Cherenkov and Cherenkov-drift resonances, switching on the synchrotron emission generation mechanism. The high frequency emission fulfills $\lambda < n^{-1/3}$ condition leaving the generation region freely without taking part in any process of energy accumulation by waves.

ACKNOWLEDGMENTS

This research was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [grant number FR/516/6-300/14]. Basic Research Program of the Presidium of the Russian Academy of Sciences “Transitional and Explosive Processes in Astrophysics(P41)” and Russian Foundation for Basic Research (grant 16-02-00954).

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