Low voltage transport through a tunneling barrier in Tomonaga-Luttinger liquid constriction.

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Abstract

As voltage decreases d.c. conductivity of a Tomonaga-Luttinger liquid wire collapses to a small value determined by the length of the wire and its contacts with the leads. In condition that voltage drop \( V \) mostly occurs across a tunnel barrier inside the wire the tunneling density of states and, hence, the differential conductivity are shown to exhibit an interference structure resulted from the transition of the Luttinger liquid quasiparticles into free electrons at the exits from the wire. The finite length correction to the scale-invariant \( V^{2/g-2} \) dependence of the conductivity oscillates as a function of voltage with periodicities related to both right and left traversal times.

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Recently quantum transport in Tomonaga-Luttinger (TL) liquids has attracted a great deal of interest [1] as it was suggested to model both transport through a 1D constriction [2,3] and the edge state transport in the fractional quantum Hall regime [4]. The problem most often discussed in this context was a suppression of the TL transport by a point scatterer [3,4]. Finally, its exact solution has been constructed [3] which, however, could not be addressed to the 1D transport to full extent because of the importance of the finite length effect in this case [7,8]. To handle the latter effect on the transport between two leads through a 1D wire the model of the inhomogeneous TL liquid (ITL) has been invented [9–11]. It has allowed in particular to predict an interference structure in the density of states of the clean wire at low energy [12].

The aim of my present work is to examine the finite length effect on the low voltage conductivity suppression in the ITL model of the one channel wire with a point scatterer inside. Temperature dependence of the conductance in this model was considered in [13]. In spite of simplicity the model could be relevant to experiment due to its local stability against the influence of other impurities. Indeed, suppression of the current by an impurity is strengthened in \((t_L E_F)^{g-1}\) times in the TL model due to interaction. (Here \(t_L\) is the time of travelling from the impurity to the closest end of the wire, \(E_F\) is the Fermi energy in the wire and \(g < 1\) is the constant of forward scattering.) It means that the effect of the impurities located closer to the middle of the wire is more important. On the other hand, among a few closely located impurities only one could bring about the strong suppression since in that case it cuts the wire and the renormalization of the scattering strength of the other impurities would be removed by the short time of travelling to this new end of the wire. Assuming below that one impurity creates a weak link between two parts of the wire and neglecting effects of other impurities one can apply the tunneling Hamiltonian approach to describe the transport. In this approach current flowing through the weak link located, say, at \(x = 0\) inside the wire is given by the operator \(J(t) = -i [T \psi_R^+(0,t)\psi_L(0,t) - h.c.],\) \((\epsilon, \hbar = 1)\), where \(T\) is the tunneling amplitude and \(\psi_{R,L}(x,t)\) are the electron annihilation operators in the right \(0 \leq x < L_R\) and in the left \(-L_L < x \leq 0\) part of the wire, respectively.
Then the average current under voltage $V$ applied to the left lead is equal in the lowest order in $T$ to: 

$$< J > = 2\pi |T|^2 \int d\epsilon [f(\epsilon - V) - f(\epsilon)] \rho_R(\epsilon) \rho_L(\epsilon - V),$$

where $f$ is Fermi distribution. The tunneling density of the right (left) end of the junction $\rho_{R(L)}(\epsilon)$ being the sum of the particle and hole densities can be extracted from the particle correlator as

$$(1 - f(\epsilon)) \rho(\epsilon) = 1/(2\pi) \int d\epsilon e^{i\epsilon t} < \psi(0, t) \psi^+(0, 0) >$$

or from the hole one as

$$f(\epsilon) \rho(\epsilon) = 1/(2\pi) \int d\epsilon e^{i\epsilon t} < \psi^+(0, 0) \psi(0, t) >.$$

To calculate the tunneling density to the right from the weak link $\rho_R$ let me first consider spinless fermions and apply bosonization to the $\psi$ field under condition of an elastic reflection from the boundary located at $x = 0$. (Carrying out this calculations I will omit index ”R” below.) Above boundary condition known as ”fixed” \[14\] in conformal field theory was sometimes addressed as ”open” \[15\] in other considerations. Bosonic representation of the $\psi$ field reads

$$\psi(x, t) = \sum_{a=r,l} \psi_a(x, t) = (2\pi\alpha)^{-1} \sum_{\pm} e^{i(\theta(x, t) \pm \phi(x, t))/2},$$

where $\psi_{r(l)}$ is the right (left) going chiral component of $\psi$ and the $\theta$ and $\phi$ fields are bosonic and mutually conjugated $[\theta(x, t), \phi(y, t)] = 2\pi i \text{sgn}(x - y)$. The elastic reflection means that $\psi_l(0, t) = e^{i\delta} \psi_r(0, t)$ with an appropriate phase shift $\delta$. This results in both:

$$\phi(0, t) = \delta, \quad \frac{1}{2\pi} \partial_x \theta(x, t)|_{x=0} = \psi_r^+(0, t) \psi_r(0, t) - \psi_l^+(0, t) \psi_l(0, t) = 0. \quad (1)$$

Then the density of particle states could be found from

$$\rho_p(\epsilon) = 1/(2\pi) \int d\epsilon e^{i\epsilon t} < \psi(0, t) \psi^+(0, 0) > = \rho_O E_F \int_{-\infty}^{+\infty} d\epsilon e^{i\epsilon t + \frac{1}{4}[\theta(0, t)\theta(0, 0) - \theta^2(0, 0)]}$$

where the value of the free electron tuneling density was introduced as: $\rho_O = (1 + \cos(2\delta))/(\pi v)$.

The problem reduces to finding the $\theta$ field correlator. It can be done for the finite length piece of the wire adiabatically connected to the lead making use of the ITTL model \[9\] \[11\]. In this model the Tomonaga-Luttinger interaction $(\sum_{r,l} \rho_a)^2$ is switched on in the wire $x < L_R$ and switched off outside. Then the Hamiltonian takes a bosonized form

$$\mathcal{H} = \int_0^\infty dx \frac{v}{2} \left\{ u^2(x) \left( \frac{\partial_x \phi(x)}{\sqrt{4\pi}} \right)^2 + \left( \frac{\partial_x \theta(x)}{\sqrt{4\pi}} \right)^2 \right\}$$

(3)
where function \( u(x) \) ensuing from the interaction can be approximated in the low energy limit by a step-function: \( u(x) = 1 \) if \( x > L_R \) and \( u(x) = u = 1/g < 1 \), otherwise. The correlator of the \( \theta \) field ordered in imaginary time \( T(x, y, \tau) \equiv \langle T_x \theta(x, \tau) \theta(y, 0) \rangle \) can be shown to satisfy the following equation

\[
\left\{ \frac{1}{v^2u^2(x)} \partial_x^2 + \partial_x^2 \right\} T(x, y, \tau) = -\frac{4\pi}{v} \delta(x - y) \delta(\tau)
\]

under the boundary conditions \( \partial_x T(x, y, \tau)|_{x=0} = 0 \) following from (1). Fourier transform of this correlator \( T(x, y, \omega) \) is symmetrical under \( \omega \rightarrow -\omega \). It can be compiled from the solutions of the homogeneous equation corresponding to Eq.(4)

\[
\left\{ \frac{\omega^2}{v^2u^2(x)} - \partial_x^2 \right\} f_\omega(x) = \left\{ \frac{\omega^2}{v^2u^2(x)} - \partial_x^2 \right\} h_\omega(x) = \frac{4\pi}{v} \delta(x - y)
\]

if these solutions meet boundary conditions: \( h'_\omega(0) = 0, f_\omega(x) = \exp(-\omega x/v) \) at \( x \rightarrow \infty \) and positive \( \omega \). The Wronskian \( W(\omega) \) is equal to \( -f'_\omega(0)h_\omega(0) \) and, hence, \( T(0, 0, \omega) = -\frac{4\pi}{v} 1/(lnf_\omega(0))' \). The only solution I need can be written as right going plus reflected left going waves at \( x < L_R \). The reflection amplitude \( r = -e^{-2\eta}, \tanh(\eta) = 1/u \) equals minus reflection amplitude of the \( \phi \) field \([11,16]\) due to duality symmetry and is negative for the repulsive interaction. Substituting this solution one can find \( T(0, 0, \omega) = \frac{4\pi u}{\omega} \tanh(\omega t_{LR} + \eta) \) with \( t_{LR} \) equal to the time of travelling from the junction to the right lead. Analytical continuation of this function \( [-T(0, 0, -i\omega + 0)] \) brings us the value of the retarded Green function for the \( \theta \) field. Imaginary part of the latter multiplied by the Bose distribution function for holes \( 1 + f_B(\omega) \) and by a factor \( (-2) \) coincides with the Fourier transform of the correlator at \( \omega \) what allows me to rewrite expression for the particle density of states \((4)\) in dimensionless units as

\[
\rho_p(\varepsilon) = \frac{\rho_0}{2\pi \gamma} \int_{-\infty}^{+\infty} d\varepsilon p \left\{ i\varepsilon p + u \int_0^\infty d\omega e^{-\gamma|\omega|} \left( 1 + f_B(\omega) \right) \frac{e^{-i\omega p}}{\omega} - \frac{1}{\omega} \frac{Im \tan(\omega + i\eta)}{\tanh(\eta)} \right\}
\]

where the temperature and energy were scaled as \( \varepsilon = t_{LR} \varepsilon \). The dimensionless cut-off parameter \( \gamma \) equals \( (E_Ft_{LR})^{-1} \). The hole density of states \( \rho_h(\varepsilon) \) can be found as \( \rho_h(\varepsilon) = \),

\[
\rho_p(\varepsilon) = \frac{\rho_0}{2\pi \gamma} \int_{-\infty}^{+\infty} d\varepsilon p \left\{ i\varepsilon p + u \int_0^\infty d\omega e^{-\gamma|\omega|} \left( 1 + f_B(\omega) \right) \frac{e^{-i\omega p}}{\omega} - \frac{1}{\omega} \frac{Im \tan(\omega + i\eta)}{\tanh(\eta)} \right\}
\]
Below I will examine expression (6) at zero temperature when the expected effect of the interference on the density of states should be the most profound. The bosonic distribution factor $1 + f_B$ restricts the integral to positive $\omega$ and the density of states at positive $\epsilon$ coincides with the density of particle states. The form of the correlator used in (6) is equivalent to

$$<\psi_T(0,t)\psi_T^+(0,p)> = \frac{E_F}{2\pi v} \prod_n \left( \frac{\gamma + 2in}{\gamma + 2in + p} \right)^{ur_n}$$

where $p$ is dimensionless time $p = t/t_{LR}$. This expression can be easily comprehended as if it is a product of the contributions of the $2n$ length paths connecting $(0,p)$ and $(0,0)$ points and undergoing $n$ reflections from a $x = L_R$ non-elastic boundary with the negative reflection amplitude $r = -e^{-2\eta}$ and $n$ reflections from the $x = 0$ elastic boundary with unit reflection amplitude. Another form of the exponent in the left hand side of Eq. (3) $\exp\{u(S(ip + \gamma) + \int_0^\infty dz/(2z)[e^{-zp} - 1][\tanh(z + \eta) - \tanh(z - \eta)]\}$ can be obtained after rotation of the integration contour $\omega = -iz$. Here $S(ip + \gamma) = -i\pi \int_0^p ds e^{-(\eta + i\pi/2)(s-i\gamma)}(1 - e^{-i\pi(s-i\gamma)})^{-1}$ ensues from the pole contributions. For a small $\eta$ it gives expression for the correlator

$$<\psi_T(0,t)\psi_T^+(0,p)> = \frac{E_F}{2\pi v} \frac{(\pi\gamma/4)^u}{\tanh(\pi(\gamma + ip)/4)} e^{u/2} \int_0^\infty (dz/z)[e^{-zp} - 1][\tanh(z + \eta) - \tanh(z - \eta)]$$

where the divergent at $p = 4n$ part and the smooth longly decaying tail finally approaching $\text{const}/p$ at $p\eta \gg 1$ are separated. Such a long time decay brings about the finite $\rho(0)$ value predicted before [17,7,2].

Besides making use of the above form of the correlator in (3) it is convenient to represent the density as a sum $\rho(\epsilon)/\rho_O = a^{u-1}n(\sin(\pi u)\Gamma(1-P)\epsilon^{u-1} + 2r(\epsilon))$ of a scale invariant density of the infinite long wire and the finite length correction. Then calculation of the latter reduces to Fourier transform of the integrable function at least if $u < 3$. It has been done numerically and results are depicted in Fig.1a. They show that as the interaction is strengthened the dimensionless finite length correction to the density of states changes...
its period of oscillations from \( \pi/t_{LR} \) for \( u = 1.4 \) to \( \pi/(2t_{LR}) \) for \( u = 1.86 \). At larger \( u \) the oscillations become more profound and practically cease to decay with increase of the energy. As \( u \) decreases to 1 the finite length correction \( r(\varepsilon) \) goes to zero everywhere except for \( \varepsilon = 0 \) where the value \( r(0) \) approaches \( \pi/2 \). Albeit the amplitude of the \( r \) function oscillations becomes much smaller than 1 in this limit it decreases slowly with increase of the energy.

To generalize expression (6) for the density of states to the case of the spin electrons one can notice that the \( \theta_\sigma \) arising in Eq. (2) in this case may be represented as a sum over the charge and spin fields \( \theta_\sigma = (\theta_c \pm \theta_s)/\sqrt{2} \). Dynamics of the charge field is described by the Hamiltonian (3) as before while dynamics of the spin field is not affected by the interaction. It means that the spin generalization of the density of states requires change of \( Im[\tan(\omega + i\eta)]/\tanh(\eta) \) in (3) into \( (1 + Im[\tan(\omega + i\eta)]/\tanh(\eta))/2 \). The results of calculations for the spin electrons are shown in Fig.1b. They behave similar to the results of the spinless case, however, the interference structure is weaker than in the spin case at the same value of the interaction constant.

Calculating the differential conductivity one can use the same form where the finite length correction is separated from the scale invariant infinite length contribution: \( \partial J/\partial V = R_O^{-1} \gamma^{2(u-1)}[(2u - 1)\sqrt{\pi}2^{1-2u}(\Gamma(u)\Gamma(1/2 + u))^{-1}v^{2(u-1)} + \partial j(v)] \). Here \( R_O^{-1} \) is a free electron conductance of the junction and both the cut-off \( \gamma \) and the applied voltage \( V \) are measured in the unit of the full traversal time \( t_L = t_{LR} + t_{LL} \) so that \( v = Vt_L \). The finite length correction \( \partial j \) to the conductivity depends on relation \( t_{LR}/t_{LL} \) between the travelings times to the right and left leads. Its behavior is illustrated by Fig.2a,b. They show that as the voltage increases the correction grows up if \( u \) is larger than \( u \approx 2 \) and just oscillates otherwise. It is reasonable since the leading contribution to the correction comes from convolution of the oscillating \( r(\varepsilon) \) function and the infinite length \( \varepsilon^{u-1} \) function. The amplitude of the oscillations behaves similar to the one of the density of states. It is on the order of the conductance for large \( u \) and smaller than conductance for \( u \) close to 1. However, in both cases the oscillations decay very slowly as the voltage increases what hopefully makes them observable.

In summary, I have shown that the ITLL model accounting for the finite length of the
wire and predicting a finite zero temperature conductance also brings about the oscillating interference structure in the differential conductivity. The latter survives at energies much higher than the one corresponding to the wire length $1/t_L$. The period of the oscillations of the tunneling density of states corresponds to the right (left) traversal time for a weak interaction $u \approx 1$ and becomes two times shorter at larger $u$. The differential conductivity in turn oscillates with both periods of the right and left tunneling density of states.

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FIGURES

Dependences of the finite length correction \((E_F t_L)^{1-u} 2r(E)/\pi\) to the density of states on the energy \(E\) measured in \(\pi\) over traversal time \(t_L\) unit. In the spinless case (a) solid, dot dashed and dashed lines correspond to constant of interaction \(u\) equal to \(u = 1.396, 1.862, 2.825\), appropriately. In the spin case (b) solid and dashed lines correspond to \(u = 1.862, 2.825\). Long decay of the oscillations at \(u=1.396\) is shown in the inset.

Dependences of the finite length correction \((E_F t_L)^{2(1-u)} \partial j(v)/R_O\) to the differential conductivity on voltage \(v\) measured in \(\pi\) over traversal time \(t_L\) unit. For spin electrons (a) \(u = 1.396\) the solid line relates to the symmetrical position of the weak link, the dotted line to the case when relation between the lengths of the right and left shoulders is 1/4. In the spinless case (b) the lines 1,2 correspond to \(u = 2.164, 1.862\), respectively.
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Fig. 1a
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Fig. 1b
Fig. 2a
Fig. 2b