On Black Ring with a Positive Cosmological Constant

Chong-Sun Chu, Shou-Huang Dai

Centre for Particle Theory and Department of Mathematics, University of Durham, Durham, DH1 3LE, UK.

chong-sun.chu@durham.ac.uk, shou-huang.dai@durham.ac.uk

Abstract

We consider black ring with a cosmological constant in the five dimensional $\mathcal{N} = 4$ de Sitter supergravity theory. Our solution preserves half of the de Sitter supersymmetries and has one rotation symmetry. Unlike the flat case, there is no angular momentum and the stability against gravitational self-attraction is balanced by the cosmological repulsion due to the cosmological constant. Our solution describes a singular black ring since although it has horizons of topology $S^1 \times S^2$, the horizons are singular. Despite the singularity, our solution displays some interesting regular physical properties: it carries a dipole charge and this charge contributes to the first law of thermodynamics; it has an entropy and mass which conform to the entropic N-bound proposal and the maximal mass conjecture. We conjecture that the Gregory-Laflamme instability leads to a resolution of the singularity and results in a regular black ring.
1 Introduction

A remarkable black hole solution, the black ring, admitting a horizon of non-spherical topology was discovered by Emparan and Reall [1] in 2001. This solution satisfies the vacuum Einstein equations in five dimensions and has a horizon of $S^1 \times S^2$ topology. The solution is neutral and rotation is needed to prevent the ring from gravitational collapse. This solution has been further generalized to charged black rings [2], and supersymmetric ones [3] using the results of [4] which provides a classification of all supersymmetric bosonic solutions of minimal supergravity in five-dimensions. Black ring exhibits infinite non-uniqueness due to its unusual dipole charge [5]. Unlike the usual black hole hair, the dipole charge is not obtained from surface integrals at infinity. Nevertheless they enter into the first law of thermodynamics for the black ring [5, 6, 7, 8]. Microscopic analysis of entropy of black ring has been considered in [9]. Relation with integrable system was noted in [10]. See [11] for further developments and a comprehensive review.

The black rings constructed so far are asymptotically flat and without a cosmological constant; black ring with a cosmological constant has not been constructed [1]. It is a challenging problem. For applications in dS/CFT or AdS/CFT correspondence, it will be very interesting to have such black rings so that one may investigate how the properties related to the non-trivial topology of horizons are encoded in terms of the dual field theory. Moreover since our universe is known to have a small but nonzero positive cosmological constant. A 5-dimensional black ring with a positive cosmological constant may lead to interesting observable effects in the 4-dimensional low energy world. The purpose of this paper is to construct a black ring with a positive cosmological constant and examine its properties.

The original black ring solution was constructed using the Kaluza-Klein reduction. A particular solution, the dilatonic C-metric [13] played a central role in the construction. The C-metric was first obtained as a special solution of the reduced 4-dimensional dilaton-Maxwell-Einstein gravity, and substituted into the Kaluza-Klein reduction formula to obtain a 5-dimensional solution. The black ring is then obtained from a double Wick rotation and by choosing appropriate range of its coordinates.

The C-metrics describes a pair of uniformly accelerated black holes in the opposite directions [14, 15]. It was initially constructed for the flat spacetime. Generalization to C-metric with a cosmological constant is not difficult [16] (see also [17, 18]) and they are obtained as solutions in the four-dimensional Maxwell-Einstein gravity with a cosmological constant. To construct a black ring in five-dimensional spacetime with a cosmological constant, it is natural to expect that these C-metrics may play a role. To make use of these C-metrics in a way similar to the idea of [1], one need to start with a five-dimensional theory which under a consistent reduction reduces to the four-dimensional Maxwell-Einstein gravity with a cosmological constant.

\footnote{An interesting black ring solution in $AdS_3 \times S^3$ has been constructed [12]. In this paper we are interested in constructing black ring solution in higher dimensional non-asymptotically flat spacetime.}
logical constant (or perhaps with a dilaton also). One may try the conventional Kaluza-Klein reduction but it is immediately clear that it does not work. Another possibility is to perform a warp compactification. Using the braneworld reduction ansatz \[19, 20, 21\], we will show that when the cosmological constant is positive, one can construct a supersymmetric solution in the 5-dimensional de Sitter supergravity \[2\] that has horizons of topology \(S^1 \times S^2\). This is our candidate of a black ring solution.

This solution is singular, however, since the \(S^2\) factor of the horizon shrinks to zero size as the warp factor vanishes \[3\]. Thus our solution describes a singular black ring. Despite the singularity, our solution display very interesting properties: it has an entropy which conform to the the N-bound proposal of Bousso \[22\]. Moreover it has a negative mass, suggesting that a maximal mass conjecture similar to that of \[23\] may hold in general for spacetime with a positive cosmological constant. These suggests that there is an underlying framework where the singularity of our solution may get resolved and the solution makes good physical sense. Indeed singular horizon is a quite generic feature of brane world black hole. In \[24\], a Schwarzschild black hole is considered on the brane, it was found that the solution is singular at the AdS horizon and it was suggested that as a result of the Gregory-Laflamme instability \[25\], the horizon will pinch off and a regular “black cigar” solution is formed. It is a challenging and still open question to construct this brane world black hole. We remark that the outcome of the Gregory-Laflamme instability is still a matter to be settled. In addition to the original proposal of Gregory and Laflamme where the horizon pinches off, Horowitz and Maeda \[26\] have argued that pinch off cannot happen; and instead, a new horizon is formed around the ”neck region” leading to a new regular horizon. In accordance with the scenario of \[26\], the Gregory-Laflamme instability for our solution could lead to a resolution of the singularity and results in a regular black ring.

The paper is organized as follows. In the next section, we give a brief review of the braneworld reduction of \[21\]. In section 3, we construct a half supersymmetric solution in the de Sitter supergravity theory. The solution has an black ring horizon as well as a cosmological horizon, both of topology \(S^1 \times S^2\). Despite the singularity of the horizon, the surface gravities and the areas (entropies) for these horizons are finite and well defined. The solution has no global charge, but it has a local dipole charge like the original black ring, and this appears in the first law of black hole thermodynamics. We discuss the fate of the singular horizon and suggest that the Gregory-Laflamme instability to lead to a regular black ring. Further discussions are given at the end of section 3 and in the section 4.

---

\[2\]Our method only works for the 5-dimensional dS case. The reason will be clear as we explain our construction in section 3.

\[3\]We thank Roberto Emparan and Harvey Reall for pointing out this point to us which we missed in our original consideration.
2 Braneworld Kaluza-Klein reduction

Unlike the usual Kaluza-Klein reduction which is based on a factorizable geometry, the braneworld Kaluza-Klein reduction is based on a warp metric. Consider an embedding of the form

\[ ds_{D+1}^2 = dz^2 + f^2 ds_D^2, \]

where \( f = f(z) \) and the \( D \)-dimensional metric \( ds_D^2 \) is Lorentzian. In this paper, we will use hatted variables to indicate 5-dimensional quantities. An important observation is that the higher dimensional Ricci tensor is simply related to the lower dimensional one as

\[ \hat{R}_{zz} = \frac{D f''}{f}, \quad \hat{R}_{z\mu} = 0, \]

\[ \hat{R}_{\mu\nu} = \left[ R_{\mu\nu} + (D-1)(f''f - f'^2)g_{\mu\nu} \right] - \frac{D f''}{f} \hat{g}_{\mu\nu}, \]

for \( \mu, \nu = 1, \cdots, D \). It follows immediately that for special choices of \( f = e^{-kz}, \cosh(kz), \cos(kz), \sinh(kz) \), one can embed a lower dimensional constant curvature spacetime within a higher dimensional constant curvature spacetime, namely [21]

\[
\begin{align*}
(i) M_D \subset AdS_{D+1} : & \quad f = e^{-kz}, \quad \hat{R}_{zz} = -Dk^2, \\
& \quad \hat{R}_{\mu\nu} = g_{\mu\nu} - Dk^2 \hat{g}_{\mu\nu}, \\
(ii) AdS_D \subset AdS_{D+1} : & \quad f = \cosh(kz), \quad \hat{R}_{zz} = -Dk^2, \\
& \quad \hat{R}_{\mu\nu} = \left[ R_{\mu\nu} + (D-1)k^2g_{\mu\nu} \right] - Dk^2 \hat{g}_{\mu\nu}, \\
(iii) dS_D \subset AdS_{D+1} : & \quad f = \sinh(kz), \quad \hat{R}_{zz} = -Dk^2, \\
& \quad \hat{R}_{\mu\nu} = \left[ R_{\mu\nu} - (D-1)k^2g_{\mu\nu} \right] - Dk^2 \hat{g}_{\mu\nu}, \\
(iv) dS_D \subset dS_{D+1} : & \quad f = \cos(kz), \quad \hat{R}_{zz} = Dk^2, \\
& \quad \hat{R}_{\mu\nu} = \left[ R_{\mu\nu} - (D-1)k^2g_{\mu\nu} \right] + Dk^2 \hat{g}_{\mu\nu}.
\end{align*}
\]

An interesting feature of the reduction ansatz [1] is the change in the cosmological constant.

Moreover it has been demonstrated [21] how these reduction ansatz for the metric can be extended to the other fields of \( \mathcal{N} = 4 \) gauge supergravity in five-dimensions to obtain a consistent reduction of supergravity. The first case is basically the one considered by Randall and Sundrum. In cases (ii), (iii), one obtains gauged \( \mathcal{N} = 2, D = 4 \) AdS supergravity and, respectively, gauged \( \mathcal{N} = 2, D = 4 \) dS supergravity from the gauged \( \mathcal{N} = 4, D = 5 \) AdS supergravity. In case (iv), one obtains gauged \( \mathcal{N} = 2, D = 4 \) dS supergravity from the gauged \( \mathcal{N} = 4, D = 5 \) dS supergravity upon reduction.
2.1 The case of $dS_4 \subset dS_5$

As we will explain in the next section, the case (iv) is the only case which allows one to construct a black ring solution in a spacetime with a cosmological constant, therefore we will review the braneworld reduction for this case in more details. The five dimensional theory to start with is the five-dimensional gauged $N = 4$ de Sitter supergravity, which can be obtained by performing Kaluza-Klein reduction of type IIB theory on $H^5$ [27], where the IIB* supergravity arises by performing a T-duality transformation on type IIA on a timelike circle. As a result the Ramond-Ramond fields have “wrong-sign” kinetic terms. The bosonic fields of the theory are the metric, the dilaton field $\phi$, three $SU(2)$ Yang-Mills fields $A$ (i = 1, 2, 3), the $U(1)$ gauge potential $B$ and two 2-form potentials $A^\alpha$ (\(\alpha = 1, 2\)) which transform as a charged doublet under the $U(1)$. The bosonic Lagrangian is

$$
\mathcal{L}_{5(dS)} = \hat{R} \hat{g} - \frac{1}{2} \hat{g} \, \hat{\phi} - \frac{1}{2} \hat{X}^4 \hat{G}_2 \wedge \hat{G}_2 - \frac{1}{2} X^2 e^{ijk} \hat{A}_i \wedge \hat{A}_j \wedge \hat{A}_k \wedge \hat{B}_1 - \frac{1}{4} g^2 (X^2 + 2X^{-1}) \hat{g} \cdot \hat{1},
$$

(4)

where $X = e^{-\frac{1}{2} \sqrt{g} \phi}$, $\hat{F}_i = d\hat{A}_i - \frac{1}{\sqrt{2}} g e^{ijk} \hat{A}_j \wedge \hat{A}_k$, and $\hat{G}_2 = d\hat{B}_1$, and $\theta(\alpha) = -1$ when $\alpha = 1$ and $\theta(\alpha) = 1$ when $\alpha = 2$. The Einstein summation convention is applied over the indices $i$ and $\alpha$. The Lagrangian (4) can be obtained by applying the following analytic continuation

$$
g \rightarrow ig, \quad \hat{A}_i \rightarrow i\hat{A}_i, \quad \hat{A}_i \rightarrow i\hat{A}_i\n$$

(5)

to the gauged $AdS_5$ supergravity Lagrangian [20]. Compared to the bosonic Lagrangian of $AdS_5$ supergravity, there are opposite signs in the kinetic terms of $\hat{A}_i$ fields, the interaction terms of $\hat{A}_i$ to itself, and the term with the coupling constant $g^2$. The term $\frac{1}{\sqrt{2}} g e^{ijk} \hat{A}_j \wedge \hat{A}_k$ in $\hat{F}_i$ also has an opposite sign compared to the AdS case. For convenience, we have set the Newton constant $G = 1$.

The Lagrangian (4) gives rise to the following equations of motion

$$
d(X^{-1} \hat{g} dX) = \frac{1}{3} X^4 \hat{G}_2 \wedge \hat{G}_2 - \frac{1}{6} X^2 e^{ijk} \hat{A}_j \wedge \hat{A}_k \wedge \hat{B}_1 + \frac{1}{2} \hat{g} \hat{1},
$$

$$
d(X^4 \hat{G}_2) = -\frac{1}{2} \hat{g} \hat{A}_i \wedge \hat{A}_j \wedge \hat{A}_k \wedge \hat{B}_1 + \frac{1}{2} \hat{F}_i \wedge \hat{F}_j,
$$

$$
d(X^{-2} \hat{F}_i) = \sqrt{2} g X^2 e^{ijk} \hat{A}_j \wedge \hat{A}_k \wedge \hat{F}_i \wedge \hat{G}_2,
$$

$$
X^2 \hat{F}_i = \hat{g} \hat{A}_i,
$$

$$
\hat{R}_{MN} = 3X^{-2} \partial_M X \partial_N X + \frac{4}{3} g^2 (X^2 + 2X^{-1}) \hat{g}_{MN} + \frac{1}{2} X^4 \left[ \hat{G}_M^P \hat{G}_NP - \frac{1}{2} \hat{g}_{MN} (\hat{G}_2)^2 \right]
$$

$$
+ \frac{1}{2} X^2 \left[ -\hat{F}_M^P \hat{F}_NP + \frac{1}{6} \hat{g}_{MN} (\hat{F}_2)^2 \right] + \frac{1}{2} X^{-2} \hat{g} \left[ \hat{A}_M^P \hat{A}_NP - \frac{1}{6} \hat{g}_{MN} (\hat{A}_2)^2 \right], \quad (6)
$$

4
where $\hat{F}_3 = d\hat{A}_3 + g\hat{B}_1 \wedge \hat{A}_3$. To have a consistent reduction, we take $g = k$ and the following reduction ansatz

\begin{align}
\hat{d}s_4^2 &= dz^2 + \cos^2(kz) \, ds_4^2, \\
\hat{A}_{(2)}^1 &= -\frac{1}{\sqrt{2}} \cos(kz) \, *F_{(2)}, \quad \hat{A}_{(2)}^2 = -\frac{1}{\sqrt{2}} \sin(kz) \, F_{(2)}, \\
\hat{A}_{(1)}^1 &= \frac{1}{\sqrt{2}} A_{(1)},
\end{align}

(7)

(8)

(9)

where the four dimensional fields have Minkowskian signature. All other fields, $\hat{A}_{(1)}^2, \hat{A}_{(1)}^3, \hat{B}_1, \phi$, are set to zero. The ansatz (8) and (9) imply the photon field in 4 dimensions is derived from the two-form fields $\hat{A}_{(2)}$ in 5-dimensional de Sitter supergravity, which is different from the conventional Kaluza-Klein theory. The equations of the five dimensional de Sitter supergravity give the following four dimensional equations

\begin{align}
R_{\mu\nu} &= -\frac{1}{2}(F_{\mu}^{\lambda} F_{\nu\lambda} - \frac{1}{4} F_{(2)}^2 g_{\mu\nu}) + 3k^2 g_{\mu\nu}, \\
d(*F_{(2)}) &= 0.
\end{align}

(10)

(11)

These are nothing but the bosonic equations of motion for gauged $\mathcal{N} = 2$ de Sitter supergravity in four dimensions. Unlike the usual Einstein-Maxwell equations, the Maxwell term in (10) takes on an opposite sign which is characteristic of the de Sitter supergravity. The reduction for the fermionic fields goes through similarly and one obtains the full $\mathcal{N} = 2$ de Sitter supergravity in four dimensions. The solution of (10), (11) preserves [21] half of the five dimensional de Sitter supersymmetries.

3 Black ring solution with a positive cosmological constant

The following “charged de Sitter C-metric” is a solution to the equations (11) and (11): \(^4\)

\begin{align}
\hat{d}s_4^2 &= \frac{1}{A^2(x - y)^2} \left[ G(y) \, dt^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \tilde{G}(x) d\varphi^2 \right], \\
A_{\varphi} &= qx + c_0,
\end{align}

(12)

(13)

where the coefficient functions $G(\xi)$ and $\tilde{G}(\xi)$ are quartic,

\begin{align}
G(\xi) &= q^2 A^2 \xi^4 + a_3 \xi^3 + a_2 \xi^2 + a_1 \xi + a_0, \\
\tilde{G}(\xi) &= G(\xi) - k^2/A^2.
\end{align}

(14)

\(^4\)One can also take $A_t = qy$ or $A_y = qt$.  

5
Here the 4-dimensional cosmological constant is $\Lambda = 3k^2 > 0$ and the constants $c_0, a_{0,1,2,3}$ and $A > 0$ are arbitrary. The metric (12) is a C-metric with a positive cosmological constant. It describes a pair of uniformly accelerated black holes in a spacetime with a positive cosmological constant and $A$ is the uniform acceleration of the black holes [18].

One can construct a solution in the 5-dimensional de Sitter supergravity theory by substituting (12) into (7),

$$d\hat{s}_5^2 = dz^2 + \frac{\cos^2(kz)}{A^2(x-y)^2} \left[ G(y)\, dt^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{\tilde{G}(x)} + \tilde{G}(x)d\varphi^2 \right]. \quad (15)$$

By construction, our solution preserves half of the supersymmetries of the 5-dimensional de Sitter supergravity.

In the following we will choose the functions $G, \tilde{G}$ to be even. Moreover we consider the case that all the roots of $G$ and $\tilde{G}$ are real and distinct, and specify the roots as $\xi_1 < \xi_2 < \xi_3 < \xi_4$ and $\tilde{\xi}_1 < \tilde{\xi}_2 < \tilde{\xi}_3 < \tilde{\xi}_4$. It is easy to show that, by rescaling the coordinates appropriately, one can always choose $a_0 = 1 = -a_2$. Therefore we consider $G, \tilde{G}$ of the form

$$G(y) = 1 - y^2 + q^2 A^2 y^4 = q^2 A^2(y^2 - \xi_3^2)(y^2 - \xi_4^2),$$

$$\tilde{G}(x) = 1 - k^2/A^2 - x^2 + q^2 A^2 x^4 = q^2 A^2(x^2 - \tilde{\xi}_3^2)(x^2 - \tilde{\xi}_4^2). \quad (16)$$

The existence of the roots $\tilde{\xi}_2, \tilde{\xi}_3$ imposes that

$$k^2/A^2 < 1. \quad (17)$$

Explicitly the roots are given by

$$\xi_{3,4}^2 = \frac{1}{2q^2 A^2}(1 \mp \sqrt{1 - 4q^2 A^2}), \quad \tilde{\xi}_{3,4}^2 = \frac{1}{2q^2 A^2}(1 \mp \sqrt{1 - 4q^2 A^2(1 - k^2/A^2)}), \quad (18)$$

where the minus (or plus) sign corresponds to $\xi_3, \tilde{\xi}_3$ (or $\xi_4, \tilde{\xi}_4$). And $q$ is in the range

$$0 \leq q \leq \frac{1}{2A}. \quad (19)$$

When $q = 0$, $G(y) = 1 - y^2$, and the metric (12) describes a particular C-metric with a cosmological constant. In the terminology of [18], it is called the massless uncharged de Sitter C-metric. We note that although the metric satisfies the maximal symmetric space condition

$$K_{\mu\nu\lambda\rho} := R_{\mu\nu\lambda\rho} - k^2(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) = 0 \quad (20)$$

The authors of [18] claim that the spacetime is asymptotically de Sitter. We will demonstrate explicitly below, this is not true. See footnote[6] below.
when \( q = 0 \), it is not the same as de Sitter space as there is an accelerating horizon besides the cosmic horizon \( \xi \). Only when \( A = 0 \) also, (12) reduces to a pure de Sitter spacetime under a suitable coordinate transformation [18]. The solution is parametrized by \( k \) which characterizes the cosmological constant and the acceleration parameter \( A \). The corresponding five-dimensional metric (15) is locally de Sitter and can be thought of as the background upon which a general solution with \( q \neq 0 \) is obtained by turning on the the parameter \( q \).

In the following, we will consider the solution (15) with \( q \neq 0 \), \( G, \tilde{G} \) given by (16) and constraint \( k^2/A^2 < 1 \). We will show it describes a solution with a cosmological constant \( \Lambda = 6k^2 \) and admits horizons of ring topology \( S^1 \times S^2 \). The solution is parametrized by the parameters \( k, q, A \). We will see later that a non-zero \( q \) corresponds to a non-zero dipole charge for our solution.

To demonstrate our claim, we need to specify an appropriate range of \( x, y \). We choose \( x \) to be \( \tilde{\xi}_2 \leq x \leq \tilde{\xi}_3 \) so that \( \tilde{G}(x) \geq 0 \), and \( \tilde{\xi}_3 \leq y < \infty \). See the figure. In 5 dimensions, for the constant \((z, t, y)\) slices of (15), conical singularities appear at \( x = \tilde{\xi}_2 \) and \( x = \tilde{\xi}_3 \) as \( g_{\varphi \varphi} = 0 \) at

---

6 By examining the behaviour of the curvature invariants similar to that of (24) below, [18] claims that the metric (12) approaches the 4-dimensional de Sitter space asymptotically as \( r \to \infty \). However this statement is wrong. In fact the tensor \( K_{\mu \nu \lambda \rho} \) is nonvanishing when \( q \neq 0 \). For example, it is \( K_{x\varphi \varphi} = q^2(4Ax + 1) + a_3 r/A \). For the massless-uncharged C-metric, \( G = 1 - y^2 \) and one can check that \( K_{\mu \nu \lambda \rho} = 0 \) identically.

7 The situation is similar to the difference between the Minkowskian space and the Rindler space: locally they are the same, but the Rindler space has an acceleration horizon and a temperature parametrized by the acceleration parameter.
these points. To avoid conical singularities we identify \( \varphi \) with the period

\[
\Delta \varphi \bigg|_{x=\hat{\xi}_2} = \frac{4\pi}{|G'(\hat{\xi}_2)|}, \quad \Delta \varphi \bigg|_{x=\hat{\xi}_3} = \frac{4\pi}{|G'(\hat{\xi}_3)|}.
\]

(21)

Since we have chosen \( G \) to be even, the two periods are automatically equal and we have

\[
\Delta \varphi = \frac{2\pi}{q^2 A^2 \left( \hat{\xi}_3 (\hat{\xi}_2^2 - \hat{\xi}_3^2) \right)}.
\]

(22)

With this period, the constant \((z, t, y)\) surface has an \( S^2 \) topology spanned by \( x \) and \( \varphi \), with the north and south poles at \( x = \hat{\xi}_2 \) and \( x = \hat{\xi}_3 = -\hat{\xi}_2 \). Due to the warp factor \( \cos^2(kz) \) in the metric, the coordinate \( z \) is periodic \(-\pi/2k < z \leq \pi/2k\) with period

\[
\Delta z = \pi/k.
\]

(23)

Therefore the sections at constant \( t, y \) has a ring topology \( S^1 \times S^2 \). It should also be clear now why a ring topology of horizon is possible only for the case (iv) as the warp factor in all the other cases is not a periodic function, implying the warp coordinate is non-periodic. But our construction can give new black tube solution with horizon of \( S^2 \times R \) topology for all cases of the brane world reduction. The interested readers can fill out the details easily. See also [28] for some black string solutions with positive cosmological constant.

Next we examine the asymptotic behaviour of the metric as \( x \to y \to \hat{\xi}_3 \). It is convenient to apply the coordinate transformation [18]

\[
0 < r = \frac{1}{A(y-x)} < \infty
\]

(24)

to (15). For the more general form of \( G \) and \( \tilde{G} \) as in (14), the curvature invariants are (for general \( x, y, z \)):

\[
\hat{\mathcal{R}} = 20k^2, \quad \hat{R}_{MN}\hat{R}^{MN} = \frac{4q^4}{\cos^4(kz)r^8}, \quad \hat{R}_{MNPQ}\hat{R}^{MNPQ} = 40k^4 + \frac{192q^4 A^2 x^2 + 96q^2 a_3 x + 12a_3/A^2}{\cos^4(kz)r^6} + \frac{192q^4 A x + 48q^2 a_3/A}{\cos^4(kz)r^7} + \frac{56q^4}{\cos^4(kz)r^8}.
\]

(25)

As \( r \to \infty \), the terms inverse proportional to higher order \( r \) vanish in (25), and the curvature invariants approach those of \( dS_5 \) spaces, \( \hat{\mathcal{R}} \to 20k^2, \quad \hat{R}_{MN}\hat{R}^{MN} \to 80k^4, \quad \hat{R}_{MNPQ}\hat{R}^{MNPQ} \to 40k^4 \). It is suggestive that the metric may approaches \( dS_5 \) as \( r \to \infty \). To check this, we should check whether the tensor

\[
\hat{K}_{\mu\nu\lambda\rho} := \hat{R}_{\mu\nu\lambda\rho} - k^2 (\hat{g}_{\mu\lambda}\hat{g}_{\nu\rho} - \hat{g}_{\mu\rho}\hat{g}_{\nu\lambda})
\]

(26)
approaches zero or not. One can easily check that it is not as long as 

\[ q \neq 0 \]

For example,

\[ \hat{K}_{x\varphi} = \cos(kz)^2 (4Aq^2xr + a_3r/A + q^2) \]

Thus our metric (15) does not approach \( dS_5 \). However, it is

\[
\begin{align*}
\dot{R}_{zz} & = 4k^2, \\
\dot{R}_{tt} & = \frac{\dot{R}_{yy}}{\dot{g}_{yy}} = 4k^2 + \frac{q^2}{r^4 \cos^2(kz)}, \\
\dot{R}_{xx} & = \frac{\dot{R}_{\varphi\varphi}}{\dot{g}_{\varphi\varphi}} = 4k^2 - \frac{q^2}{r^4 \cos^2(kz)}. 
\end{align*}
\] (27)

Therefore as \( r \to \infty \), the metric satisfies the Einstein equation with a cosmological constant \( \hat{\Lambda} = 6k^2 \) and we have a solution with a positive cosmological constant.

Our metric (15) has the Killing vectors \( \partial/\partial t \) and \( \partial/\partial \varphi \). In the region \( \xi_3 \leq y \leq \xi_4 \), \( G(y) < 0 \), \( t \) is timelike and \( y \) is spacelike. At the endpoints \( y = \xi_3 \) or \( \xi_4 \), the coordinates break down. Let us introduce new coordinate \( v \) by

\[ dv = dt + \frac{dy}{G(y)}. \] (28)

In the new coordinates, the metric taking the form

\[
d\hat{s}_5^2 = dz^2 + \frac{\cos^2(kz)}{A^2(x-y)^2} \left[ G(y) dv^2 - 2dv dy + \frac{dx^2}{G(x)} + \tilde{G}(x)d\varphi^2 \right] \] (29)

is regular. It is easy to show that the surface \( y = y_0 \) (where \( y_0 = \xi_3 \) or \( \xi_4 \)) is a Killing horizon of the Killing vector field \( \eta = \frac{\partial}{\partial v} \). \( \tag{30} \)

The horizon has \( S^2 \times S^1 \) topology and has the surface gravity

\[ \kappa = \frac{|G'(y_0)|}{2} = q^2 A^2 y_0 (\xi_4^2 - \xi_3^2) \] (31)

and the horizon area

\[ A = \frac{2\pi^2}{kq^2A^4} \cdot \frac{1}{(y_0^2 - \xi_3^2)(\xi_4^2 - \xi_3^2)} \] (32)

for \( y_0 = \xi_3 \) or \( \xi_4 \). In terms of \( q \), we have

\[ \kappa_i = \frac{\sqrt{1 - 4q^2A^2}}{\sqrt{2qA}} \] (33)

\[ \text{We will justify the normalization of the normalization of the Killing vector later.} \]
and 
\[ A_i = \frac{4\pi^2 q^2}{k} \frac{1}{\sqrt{1 - 4q^2A^2(1 - k^2/A^2)} \mp \sqrt{1 - 4q^2A^2}} \sqrt{1 - 4q^2A^2(1 - k^2/A^2)} \] (34)

where \( \text{sign } i = - \) (resp. +) is for \( y_0 = \xi_3 \) (resp. \( y_0 = \xi_4 \)). Note that
\[ A_- > A_+ > 0. \] (35)

The horizon at \( y = \xi_3 \) is the cosmological horizon. It is at a larger \( r \) and has a larger area. As one decreases \( y \) (i.e. increases \( r \)) to the range \( \hat{\xi}_3 \leq y < \xi_3 \), \( G(y) \) becomes positive, \( y \) becomes timelike and \( t \) becomes spacelike. The situation is similar to the de Sitter space where the timelike Killing vector becomes spacelike as one goes outside the cosmological horizon. On the other hand, if one increases \( y \) (i.e. decreases \( r \)) until \( \xi_4 \), we reach the the black ring event horizon. The black ring horizon is at a smaller \( r \) and has a smaller area. The metric can be continued beyond the black ring horizon to \( y > \xi_4 \), until \( y = \infty \) (correspondingly \( r = 0 \)), which is a curvature singularity.

As is clear from (25), the horizons are singular at \( z = \pi/2k \). In fact the \( \text{S}^2 \) factor of the horizon shrinks to zero size there. However the solution is unstable near the tip due to the Gregory-Laflamme instability \[25\]. Two different possible final states has been suggested in the literature, different in whether the horizon will pinch off \[25\] or not \[26\]. In our case, if the horizon pinches off at the tip, then a black hole solution with a regular horizon of topology \( \text{S}^3 \) will be formed. On the other hand, if the horizon does not pinch off, then a new horizon is expected to form to surround the neck region and a black ring with a regular horizon of topology \( \text{S}^1 \times \text{S}^2 \) will be formed. We tend to believe the Horowitz-Maeda scenario is more likely for our case. Moreover, if we allow rotation in \( \varphi \), then, at least for sufficiently large angular momentum, the second scenario appears to be more favorable. We conjecture this is the case. If this is true, our solution with a singular ring horizon can be interpreted as evidence that a black ring in de Sitter space do exists.

Despite the singularity, the surface gravities and the areas are finite and one may consider the thermodynamics associated with the horizon. The horizons have the Hawking temperatures
\[ T_i = \frac{\kappa_i}{2\pi} = \frac{\sqrt{1 - 4q^2A^2} \sqrt{1 \mp \sqrt{1 - 4q^2A^2}}}{2\sqrt{2\pi qA}}. \] (36)

The same temperature can also be obtained by Wick rotating the metric (15) with \( t = i\tau \). The \( y-\tau \) part of the Euclidean metric has conical singularities at \( y_0 \) unless \( \tau \) is periodic with period
\[ \Delta \tau = \frac{4\pi}{|G'(y_0)|} = \frac{2\pi}{\kappa}. \] (37)

This gives immediately the temperature (36). This also justify the choice of the normalization of the Killing vector (30).
As usual, the horizons carry a Bekenstein-Hawking entropy

\[ S_i = \frac{A_i}{4} = \frac{\pi^2 q^2}{k} \sqrt{\frac{1}{1 - 4q^2 A^2 (1 - k^2/A^2)}} \mp \sqrt{\frac{1}{1 - 4q^2 A^2 (1 - k^2/A^2)}} \]

(38)

where the sign \((-+)\) is for the cosmological (black ring) horizon. For small \(q\), the entropy for the cosmological horizon \(S_c\) behaves as

\[ S_c = \frac{\pi^2}{2k^3} - \frac{\pi^2}{2k} q^2 - \frac{\pi^2}{2k} (4A^2 - 3k^2)q^4 + O(q^6), \]

(39)

and is a decreasing function of \(q\). In fact it is easy to verify that

\[ S_c \leq \frac{\pi^2}{2k^3} := S_{\text{de Sitter}}, \]

(40)

where \(S_{\text{de Sitter}}\) is the entropy for 5-dimensional pure de Sitter space of the same cosmological constant. This reminds us of the N-bound proposal \[22\] of Bousso which states that the total entropy of a spacetime with a cosmological constant is bounded by the entropy of the pure de Sitter space of the same cosmological constant. In our case, the total entropy \(S_T\) is the sum of the entropy of the black ring and the entropy of the cosmological horizon\(^9\). Quite remarkably, we obtain the result

\[ S_T = S_{\text{de Sitter}}. \]

(41)

Thus the N-bound is precisely saturated. This is amazing especially because there are matters violating the usual form of energy condition due to their negative kinetic terms in the dS supergravity; and as a result one may expect the N-bound to be violated. We view this as an evidence that there is a sensible quantum gravity description of the singular black ring.

We also note that the specific heat

\[ C_i := T \frac{\partial S_i}{\partial T} = \frac{\pi^2 q^2}{k} \frac{\sqrt{1 - 4q^2 A^2 (1 - k^2/A^2)}}{(1 - 4q^2 A^2 (1 - k^2/A^2))^{3/2}} \]

(42)

is positive for the cosmological horizon and negative for the black ring horizon, meaning that the cosmological horizon is thermally stable while the black ring horizon is thermally unstable. This is similar to that of a de Sitter Schwarzschild black hole.

In addition to the metric, our solution is supported by nontrivial two-form \(\hat{F}^1_{(2)}\) and three-forms \(\hat{F}^\alpha_{(3)}\) (\(\alpha = 1, 2\)). They obey the equation of motion: \(d(\star \hat{F}^1_{(2)}) = 0\) and \(\star \hat{F}^\alpha_{(3)} = k A^\alpha_{(2)}\). The

\(^9\)The entropy is defined by \(S = \beta(\partial/\partial \beta - 1)I\), where \(I\) is the action of the Euclidean solution. As in the Schwarzschild-de Sitter black hole, both the black ring (or the black hole) horizon and the cosmological horizon appear in the real Euclidean geometry and so they both contribute to the entropy of our solution. We thank Simon Ross for explanation of this point.
non-standard form of equation of motion of the three-forms does not lead to any conserved charge. As in [5], the two-form leads to the dipole charge:

\[ q_e = \frac{\sqrt{2}}{4\pi} \int_{S^2} \hat{\star} \hat{H}, \]  

(43)

where the integral is carried over any \( S^2 \) which can be deformed to an \( S^2 \) on the cosmological horizon \( ^{11} \). Here \( \hat{H} \) is the dual field strength

\[ \hat{H} = d\hat{B} = \hat{\star} \hat{F}^1_{(2)}. \]

(44)

The dual 2-form potential is

\[ \sqrt{2} \hat{B}_{tx} = qy + c_1, \]

(45)

where \( c_1 \) is a constant. \( q_e \) is well defined due to the equation of motion \( d(\hat{\star} \hat{H}) = 0 \). For our solution, \( \hat{F}^1_{x\phi} = q/\sqrt{2} \) and the dipole charge is

\[ q_e = \frac{q}{\sqrt{1 - 4q^2 A(1 - k^2/A^2)}}. \]

(46)

Next we would like to determine the mass of our solution. Generally to determine the mass associated with a given gravitational configuration, a well known procedure due to Brown and York is to start with a suitably defined quasi local stress tensor on the boundary of a given region of spacetime \[ ^{29} \]

\[ T^{\mu\nu} := \frac{2}{\sqrt{-\gamma}} \frac{\delta S_{grav}}{\delta \gamma^{\mu\nu}}. \]

(47)

Here \( \gamma^{\mu\nu} \) is the boundary metric and \( S_{grav} \) is the gravitational action thought of as a functional of the boundary metric. The resulting stress tensor typically diverges as the boundary is taken to infinity. To obtain a finite stress tensor, one may try to add an appropriate boundary term which does not affect the bulk equation of motion to cancel the divergences. The original proposal of Brown-York utilises a subtraction derived by embedding the boundary with the same intrinsic metric \( \gamma^{\mu\nu} \) in some reference spacetime. However this is not always possible. Due to interests in AdS/CFT and dS/CFT correspondence, the quasi local stress tensor for spacetime which is asymptotically Anti-de Sitter or asymptotically de Sitter has been worked out in \[ ^{30, 23} \] using a different subtraction procedure. The required counter terms are constructed in terms of the boundary curvature invariants and are fixed essentially uniquely by the finiteness requirement of the stress tensor. Unfortunately none of these methods help with our present case where the asymptotic behaviour of the metric is rather complicated. Thus instead of trying to look for a first principle determination of the mass, we will assume the validity of the first law of thermodynamics and use it to determine the mass of our solution.

\[ ^{10} \text{The extra factor of } \sqrt{2} \text{ here and in (48) below is due to the nonstandard normalization of fields in the Lagrangian (4).} \]

\[ ^{11} \text{The inner black ring horizon carries exactly the same dipole charge. However as we will be interested in the thermodynamics of the cosmological horizon only, it will not be relevant for our consideration.} \]
In particular since the mass should be defined from the asymptotic infinity, i.e. outside of the cosmological horizon, the relevant first law of thermodynamics shall be the one related to the cosmological horizon.

The first law of thermodynamics for de Sitter black hole was first obtained by [31], and the form presented contains contributions from the black hole horizon as well as the cosmological horizon. It has been noted [32] that for a wide class of de Sitter black holes, the cosmological horizon satisfies a first law of thermodynamics by itself remarkably. And it is natural to guess that given an appropriate definition of the energy, the cosmological horizon will always satisfy the first law of thermodynamics. We will assume so for our case and use it to derive the mass of the solution. For dipole ring, there is however a new ingredient. It was first noted by Emparan [5] that the dipole charge appears in the first law of thermodynamics in the same manner as a global charge. This is not expected from the general derivation of the first law by Sudarsky and Ward [33] and the explanation has been given by Copsey and Horowitz [6]. Usually the gauge potential $B$ can be globally defined and non-singular everywhere outside and on the horizon. However this is not compatible with the assumptions of a non-vanishing dipole charge and that $B$ is invariant under spacetime symmetries. Since $B$ is defined up to gauge transformation, one can choose any gauge to try to determine the consequence of this incompatibility. A particularly transparent gauge is to have $B_{t\hat{\psi}}$ as the only nonzero component and to have $B_{t\hat{\psi}}$ vanishes at the infinity. As demonstrated by [6], this implies that $B_{t\hat{\psi}}$ vanishes at the rotation axis (hence violation of the above stated conditions) and $B_{t\hat{\psi}}$ necessarily diverges at the horizon. As a result a new dipole term arises in the first law. In our case, we also have a dipole charge. For it to make contribution to the first law, we need to examine the behaviour of the dipole potential

$$\phi_e = -\frac{\sqrt{2\pi}}{2} B_{t\hat{z}} \bigg|_{\text{horizon}}, \quad (48)$$

where $\hat{z} = (\pi/\Delta z) z = k z$ is the canonical normalized angular variable and it is evaluated at the cosmological or the black ring horizon. Note that $B_{t\hat{z}} = \eta^{\mu} B_{\mu\nu}(\partial/\partial z)^{\nu}$ and so we expect that $B_{t\hat{z}}$ to be vanishing at both horizons. However this is impossible for ours (45). This means $B$ must be singular somewhere. Choosing the constant $c_1$ such that $\phi_e = 0$ at infinity and follows the same argument as [6], we obtain a contribution $\phi_e dq_e$ to the first law of thermodynamics for the cosmological horizon

$$dE = TdS + \phi_e dq_e. \quad (49)$$

Here $q_e$ is the dipole charge (46) and $\phi_e$ is the dipole potential evaluated at the cosmological horizon

$$\phi_e = -\frac{\pi q}{2k} (\xi_3 - \tilde{\xi}_3). \quad (50)$$

We note that $dE$ is an exact differential and $E$ is well defined. In fact although our solution is parametrized by the three parameters $k$, $A$ and $q$, one should think of $k$, $A$ as specifying the background and $q$ as specific to the solution. By increasing $q$ from 0 to a nonzero value, we
get to our solution. We have

$$M := E - E_0 = \int_0^q (T \frac{dS}{dq} + \phi_e \frac{dq_e}{dq})dq,$$  \hfill (51)

where $E_0$ is the energy of the background (i.e. when $q = 0$). The mass $M$ is given by the difference of energy $E - E_0$ above the background. The first integral

$$\int_0^q T \frac{dS}{dq}dq = -\frac{\pi}{2\sqrt{2kA}} \int_0^q \sqrt{1 - \sqrt{1 - 4q^2A^2}} dq$$  \hfill (52)

is manifestly negative. For the second integral

$$\int_0^q \phi_e \frac{dq_e}{dq}dq = -\frac{\pi}{2k} \int_0^q q\xi_3 \frac{dq_e}{dq}dq + \frac{\pi}{2k} \int_0^q q\tilde{\xi}_3 \frac{dq_e}{dq}dq,$$  \hfill (53)

it is easy to show that the first piece

$$-\frac{\pi}{2k} \int_0^q q\xi_3 \frac{dq_e}{dq}dq = \frac{\pi}{2\sqrt{2kA}} \int_0^q \sqrt{1 - \sqrt{1 - 4q^2A^2}} dq = -\int_0^q T \frac{dS}{dq}dq$$  \hfill (54)

cancels exactly the entropy contribution (52) and so

$$M = \frac{\pi}{2k} \int_0^q \tilde{\xi}_3 \frac{dq_e}{dq}dq$$

$$= -\frac{\pi}{4\sqrt{2kA^2}} \left( \sqrt{1 + Y} - \sqrt{2Y} \right) - \frac{1}{2} \ln \left( \frac{\sqrt{1 + Y} + \sqrt{2 - 1}}{\frac{1}{2} \sqrt{1 + Y} - 1 \sqrt{2 + 1}} \right),$$  \hfill (55)

where

$$Y := \sqrt{1 - 4q^2A^2}(1 - k^2/A^2) = |q/q_e| \leq 1.$$  \hfill (56)

We note that $M$ is negative, showing that our solution has a smaller energy with respect to the background’s. This is similar to the case of black holes in asymptotically de Sitter space where the pure de Sitter spaces is always more massive than the Schwarzschild-de Sitter black holes in the corresponding dimensions. This has leaded [23] to the conjecture that any asymptotically dS space whose mass exceeds that of pure dS space must contain a cosmological singularity. The fact that our solution carries a negative mass in a spacetime with a positive cosmological constant leans support to the following generalized maximal mass conjecture: In any spacetime with a positive cosmological constant, if the addition of matter leads to an increase in energy, the resulting solution must contain a cosmological singularity.

We remark, however, that this cannot be the complete statement and presumably more specific conditions needed to be specified[12]. Our solution provides an explicit example which satisfies both the N-bound proposal and the maximal mass conjecture, and may help one to identify the appropriate conditions.

[12] Explicit examples are known which violate the N-bound proposal [34, 35] as well as the maximal mass conjecture [35]. Presumably this is due to the fact that certain asymptotic energy condition is not satisfied. To our knowledge, the precise condition for the N-bound proposal or the maximal mass conjecture to hold has not been formulated explicitly. We thank Simon Ross for discussions on this matter.
4 Discussions

In this paper we have constructed a singular black ring solution in the 5-dimensional de Sitter supergravity theory. The solution has singular horizons whose singularity occurs when the warp factor vanishes. Despite the singularity in the horizon, thermodynamic quantities such as temperature and entropy are well defined and finite. It is quite amazing to find that the entropy and mass of our solution are consistent with what one would expect for solution in a spacetime with a positive cosmological constant. Moreover the N-bound is precisely satuated for our solution. It will be interesting to understand its physical significance.

Our construction relies on the brane-world reduction ansatz. In the case of a negative cosmological constant, this ansatz does not give any solution with ring topology, not even singular one. This can be easily understood as our metric ansatz does not include rotation, while we expect a black ring in a spacetime with negative cosmological constant to be rotating since there is no repulsive force from the background. It maybe possible to generalize our ansatz to include rotation and use this to construct a black ring with a negative cosmological constant.

Our solution has a metric which is not asymptotically de Sitter and so one cannot apply the usual subtraction procedure to the Brown-York stress tensor. We remark that in case the deviation from asymptotically (anti-)de Sitter behaviour is caused by a nontrivial dilaton potential, a well defined boundary stress tensor can be constructed \cite{36}. Our solution is deformed from the asymptotically de Sitter behaviour by the presence of nontrivial form fields in the action. It may be possible to devise a similar procedure to construct a well defined boundary stress tensor and to obtain the mass. With this one can verify the first law of thermodynamics \cite{49} independently. It is also interesting to derive the first law of thermodynamics by generalizing and extending the procedure of \cite{33} to include a positive cosmological constant.

De Sitter supergravity contains matter fields with wrong sign kinetic terms, one may therefore worries about the stability of our solution. As argued in \cite{27}, it is possible that the ghost modes are artifacts purely because of a truncation to the supergravity limit and are not present in the full string theory. Therefore one can expect that this unpleasant feature of the de Sitter supergravity will be go away once all the string modes and string corrections are included; and the black ring will be a solution to the full string equations of motion which are manifestly free of any ghost modes. In particular since our solution is supersymmetric, we do expect our solution to be stable, at least classically. It has been argued \cite{37} that ghosts fields for some solutions in de Sitter supergravity does not lead to instability. A similar analysis may be performed which support our guess.

Incidently a very recent preprint \cite{38} appears which deals with the existence of black ring in Anti-de Sitter space. These authors find that the most general supersymmetric, asymptotically AdS black hole that admits two rotation symmetries may have a horizon of topology $S^1 \times S^2$.
However there is always a conical singularity presents in the $S^2$ factor, thus ruling out the possibility of such black ring. The authors also suggest that nonsupersymmetric black ring may exists by increasing the angular momentum of the singular solution they found. In our case, our solution is 1/2 BPS and has one rotation symmetry. Since a positive cosmological constant is repulsive, it should be possible for a de Sitter black ring to exist without any rotation and we conjecture this is the case. It is possible one has to break more supersymmetries. It remains a challenging problem to construct explicitly a regular black ring with either positive or negative cosmological constant.

Acknowledgements

We would like to thank Mohsen Alishahiha, Roberto Emparan, Ruth Gregory, Veronika Hubeny, Takeo Inami, Clifford Johnson, James Lucieti, Mukund Rangamani, Harvey Reall and especially Simon Ross for helpful discussions. We are particularly grateful to Roberto Emparan and Simon Ross for reading the revised version of the manuscript and for giving us numerous valuable suggestions and comments. CSC thanks the theory group of CERN for hospitality where part of the work was carried out. CSC acknowledges the support of a PPARC rolling grant and an EPSRC advanced fellowship. SHD acknowledges support from the Ministry of Education of Taiwan for a research student fellowship.

References

[1] R. Emparan and H. S. Reall, “A rotating black ring in five dimensions,” Phys. Rev. Lett. 88, 101101 (2002) [arXiv:hep-th/0110260].

[2] H. Elvang, “A charged rotating black ring,” Phys. Rev. D 68, 124016 (2003) [arXiv:hep-th/0305247].

H. Elvang and R. Emparan, “Black rings, supertubes, and a stringy resolution of black hole non-uniqueness,” JHEP 0311, 035 (2003) [arXiv:hep-th/0310008].

[3] H. Elvang, R. Emparan, D. Mateos and H. S. Reall, “A supersymmetric black ring,” Phys. Rev. Lett. 93 (2004) 211302 [arXiv:hep-th/0407065].

I. Bena and N. P. Warner, “One ring to rule them all ... and in the darkness bind them?,” arXiv:hep-th/0408106.

H. Elvang, R. Emparan, D. Mateos and H. S. Reall, “Supersymmetric black rings and three-charge supertubes,” Phys. Rev. D 71 (2005) 024033 [arXiv:hep-th/0408120].

16
[4] J. P. Gauntlett, J. B. Gutowski, C. M. Hull, S. Pakis and H. S. Reall, “All supersymmetric solutions of minimal supergravity in five dimensions,” Class. Quant. Grav. 20 (2003) 4587 [arXiv:hep-th/0209114].

[5] R. Emparan, “Rotating circular strings, and infinite non-uniqueness of black rings,” JHEP 0403 (2004) 064 [arXiv:hep-th/0402149].

[6] K. Copsey and G. T. Horowitz, “The role of dipole charges in black hole thermodynamics,” Phys. Rev. D 73 (2006) 024015 [arXiv:hep-th/0505278].

[7] D. Astefanesei and E. Radu, “Quasilocal formalism and black ring thermodynamics,” Phys. Rev. D 73 (2006) 044014 [arXiv:hep-th/0509144].

[8] M. Rogatko, “Black rings and the physical process version of the first law of thermodynamics,” Phys. Rev. D 72 (2005) 074008 [Erratum-ibid. D 72 (2005) 089901] [arXiv:hep-th/0509150]; “First law of black rings thermodynamics in higher dimensional dilaton gravity with p+1 strength forms,” Phys. Rev. D 73 (2006) 024022 [arXiv:hep-th/0601055]; “First Law of Black Rings Thermodynamics in Higher Dimensional Chern-Simons Gravity,” arXiv:hep-th/0611260.

[9] M. Cyrier, M. Guica, D. Mateos and A. Strominger, “Microscopic entropy of the black ring,” Phys. Rev. Lett. 94 (2005) 191601 [arXiv:hep-th/0411187].

F. Larsen, “Entropy of thermally excited black rings,” JHEP 0510 (2005) 100 [arXiv:hep-th/0505152].

S. Giusto, S. D. Mathur and Y. K. Srivastava, “A microstate for the 3-charge black ring,” arXiv:hep-th/0601193.

[10] S. S. Yazadjiev, “Completely integrable sector in 5D Einstein-Maxwell gravity and derivation of the dipole black ring solutions,” Phys. Rev. D 73 (2006) 104007 [arXiv:hep-th/0602116].

H. Iguchi and T. Mishima, “Solitonic generation of five-dimensional black ring solution,” Phys. Rev. D 73 (2006) 121501 [arXiv:hep-th/0604050].

S. Tomizawa and M. Nozawa, “Vacuum solutions of five-dimensional Einstein equations generated by inverse scattering method. II: Production of black ring solution,” Phys. Rev. D 73 (2006) 124034 [arXiv:hep-th/0604067].

[11] R. Emparan and H. S. Reall, “Black rings,” Class. Quant. Grav. 23 (2006) R169 [arXiv:hep-th/0608012].

[12] I. Bena and P. Kraus, “Microscopic description of black rings in AdS/CFT,” JHEP 0412 (2004) 070 [arXiv:hep-th/0408186].

[13] F. Dowker, J. P. Gauntlett, D. A. Kastor and J. H. Traschen, “Pair creation of dilaton black holes,” Phys. Rev. D 49 (1994) 2909 [arXiv:hep-th/9309075].
[14] W. Kinnersley and M. Walker, “Uniformly Accelerating Charged Mass In General Relativity,” Phys. Rev. D 2 (1970) 1359.

[15] See for example, H. Stephani, D. Kramer, M. MacCallum, C. Hoenselaers and E. Herlt, “Exact solutions of Einstein’s field equations,” Cambridge, UK: Univ. Pr. (2003).

[16] J. F. Plebanski and M. Demianski, “Rotating, Charged, And Uniformly Accelerating Mass In General Relativity,” Annals Phys. 98 (1976) 98.

[17] O. J. C. Dias and J. P. S. Lemos, “Pair of accelerated black holes in anti-de Sitter background: The AdS C-metric,” Phys. Rev. D 67 (2003) 064001 [arXiv:hep-th/0210065].

[18] O. J. C. Dias and J. P. S. Lemos, “Pair of accelerated black holes in a de Sitter background: The dS C-metric,” Phys. Rev. D 67 (2003) 084018 [arXiv:hep-th/0301046].

[19] L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. 83 (1999) 4690 [arXiv:hep-th/9906064].

[20] H. Lu and C. N. Pope, “Branes on the brane,” Nucl. Phys. B 598 (2001) 492 [arXiv:hep-th/0008050].

[21] I. Y. Park, C. N. Pope and A. Sadrzadeh, “AdS braneworld Kaluza-Klein reduction,” Class. Quant. Grav. 19 (2002) 6237 [arXiv:hep-th/0110238].

[22] R. Bousso, “Positive vacuum energy and the N-bound,” JHEP 0011 (2000) 038 [arXiv:hep-th/0010252].

[23] V. Balasubramanian, J. de Boer and D. Minic, “Mass, entropy and holography in asymptotically de Sitter spaces,” Phys. Rev. D 65 (2002) 123508 [arXiv:hep-th/0110108].

[24] A. Chamblin, S. W. Hawking and H. S. Reall, “Brane-world black holes,” Phys. Rev. D 61 (2000) 065007 [arXiv:hep-th/9909205].

[25] R. Gregory and R. Laflamme, “Black strings and p-branes are unstable,” Phys. Rev. Lett. 70 (1993) 2837 [arXiv:hep-th/9301052].

[26] G. T. Horowitz and K. Maeda, “Fate of the black string instability,” Phys. Rev. Lett. 87 (2001) 131301 [arXiv:hep-th/0105111].

[27] C. M. Hull, “Timelike T-duality, de Sitter space, large N gauge theories and topological field theory,” JHEP 9807 (1998) 021 [arXiv:hep-th/9806146].

[28] Y. Brihaye and T. Delsate, “Black strings and solitons in five dimensional space-time with positive [arXiv:hep-th/0611195].

[29] J. D. Brown and J. W. York, “Quasilocal energy and conserved charges derived from the gravitational action,” Phys. Rev. D 47 (1993) 1407.
[30] V. Balasubramanian and P. Kraus, “A stress tensor for anti-de Sitter gravity,” Commun. Math. Phys. 208 (1999) 413 [arXiv:hep-th/9902121].

[31] G. W. Gibbons and S. W. Hawking, “Cosmological Event Horizons, Thermodynamics, And Particle Creation,” Phys. Rev. D 15 (1977) 2738.

[32] A. M. Ghezelbash and R. B. Mann, “Entropy and mass bounds of Kerr-de Sitter spacetimes,” Phys. Rev. D 72 (2005) 064024 [arXiv:hep-th/0412300].

[33] D. Sudarsky and R. M. Wald, “Extrema of mass, stationarity, and staticity, and solutions to the Einstein Yang-Mills equations,” Phys. Rev. D 46 (1992) 1453.

[34] R. Bousso, O. DeWolfe and R. C. Myers, “Unbounded entropy in spacetimes with positive cosmological constant,” Found. Phys. 33 (2003) 297 [arXiv:hep-th/0205080].

[35] R. Clarkson, A. M. Ghezelbash and R. B. Mann, “Entropic N-bound and maximal mass conjectures violation in four dimensional Taub-Bolt(NUT)-dS spacetimes,” Nucl. Phys. B 674 (2003) 329 [arXiv:hep-th/0307059].

[36] R. G. Cai and N. Ohta, “Surface counterterms and boundary stress-energy tensors for asymptotically non-anti-de Sitter spaces,” Phys. Rev. D 62 (2000) 024006 [arXiv:hep-th/9912013].

[37] J. H. Cho and S. Nam, “Living near de Sitter bubble walls,” [arXiv:hep-th/0607098]

[38] H. K. Kunduri, J. Lucietti and H. S. Reall, “Do supersymmetric anti-de Sitter black rings exist?,” [arXiv:hep-th/0611351]