Phonon-drag thermopower and hot-electron energy-loss rate in a Rashba spin–orbit coupled two-dimensional electron system

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Abstract

We theoretically study the phonon-drag contribution to the thermoelectric power and the hot-electron energy-loss rate in a Rashba spin–orbit coupled two-dimensional electron system in the Bloch–Gruneisen (BG) regime. We assume that electrons interact with longitudinal acoustic phonons through a deformation potential and with both longitudinal and transverse acoustic phonons through a piezoelectric potential. The effect of the Rashba spin–orbit interaction on the magnitude and temperature dependence of the phonon-drag thermoelectric power and hot-electron energy-loss rate is discussed. We numerically extract the exponent of temperature dependence of the phonon-drag thermopower and the energy-loss rate. We find that the exponents are suppressed due to the presence of the Rashba spin–orbit coupling.

1. Introduction

There has been a rapidly growing interest in spin–orbit coupled low-dimensional electron systems like two-dimensional electron systems (2DESs) formed at semiconductor heterostructure interfaces, quantum wires, quantum dots [1–3], etc. The usefulness of the spin–orbit coupling in condensed matter systems was realized after the proposal of a spin field effect transistor by Datta and Das [4], and thereafter various interesting theoretical and experimental studies have been reported. One main goal is to control and manipulate the spin degree of freedom of charge carriers in nanostructures so that spin-based electronic devices [5–7] and quantum information processing technology can be developed. The coupling between an electron’s spin and orbital angular momentum naturally arises when one makes a non-relativistic approximation to the relativistic Dirac equation. Various types of spin–orbit interaction (SOI) are present in semiconductor heterostructures. The most commonly used SOI is the Rashba SOI (RSOI) [8] which is due to the structural inversion asymmetry in semiconductor heterostructures such as GaAs/AlGaAs. One can also increase the strength of the spin–orbit coupling with the help of an external gate voltage [9, 10]. The RSOI modifies various properties [11–15] of a 2DES including the electron polaron effective mass, polaron binding energy, mobility, etc.

The electron–phonon interaction (EPI) plays an important role in determining the transport properties of a 2DES. Two kinds of mechanisms are mainly responsible for the EPI in semiconductor heterostructures, namely the deformation potential (DP) and the piezoelectric (PE) scattering potential. The deformation potential is the change in potential energy of an electron due to lattice deformation. In an inversion asymmetry host crystal, an electric polarization is induced due to lattice vibration and the potential corresponding to the electric polarization is known as the piezoelectric potential. The electrons are scattered by the deformation and piezoelectric potentials and provide a non-zero contribution to the momentum relaxation time, in addition to other contributions coming from disorders, impurities, etc.
Numerous studies have been devoted to probing the EPI by measuring the low temperature mobility [16–22] of a 2DES in the Bloch–Gruneisen (BG) regime. The characteristic BG temperature [23] is defined as $T_{BG} = 2\hbar v_F k_B/k_B$ (where $v_F$ is the sound velocity and $k_F$ is the Fermi wavevector). When the temperature decreases below $T_{BG}$, phonon modes with higher energy are no longer able to be thermally excited and electrons are scattered by a small fraction of acoustic phonons with wavevector $q \leq 2k_F$ due to the phase space restriction. This leads to a sharp decrease in the resistivity, in which the two-dimensional electron wavevector $k$ couples with the three-dimensional phonon wavevector $Q = (q_x, q_z)$. We consider both DP and PE scattering mechanisms responsible for the EPI. In the BG regime we find analytically that $S_T$ is proportional to $T^2$ and $T^3$ for DP and PE scattering, respectively. On the other hand, approximate analytical calculations show that the energy-loss rate is proportional to $T^5$ and $T^3$ for DP and PE scattering, respectively. However, our numerical results reveal that the exponents are strongly dependent on the electron density and the Rashba spin–orbit coupling constant.

This paper is organized as follows. In section 2 we present all the analytical results on the phonon-drag thermopower and energy-loss rate. Numerical results and discussion are given in section 3. We summarize our work in section 4.

2. Theoretical details

We consider a quasi-2DES formed at the interface of semiconductor heterostructures which has a finite thickness in the confining direction (say, $z$). Typically, the confining potential in the $z$-direction is a triangular potential. We assume that only the lowest sub-band due to transverse confinement is occupied by the electrons. Therefore, electrons are restricted to move in the $xy$ plane with wave vector $k = (k_x, k_y)$. One can write the electron’s wavefunction as $\psi(r) = \psi(x, y)e^{i\phi_0(z)}$. The Fang–Howard wavefunction [52] in the $z$-direction is given by $\psi_0(z) = \sqrt{b^3/2}\text{e}^{-b|z|/2}$ with $b = (4\pi\sigma^2a^2/\epsilon_0\hbar^2)^{1/3}(\pi_d + 11\pi_d/32)^{1/3}$ as the variational parameter. Here, $m^*$ is the effective mass of an electron, $\kappa$ is the dielectric constant. Also, $\epsilon_0$ is the permittivity of free space, $\pi_d$ is the depletion charge density and $n_e$ is the density of electrons.

The single electron Hamiltonian is given by

$$H = \frac{p^2}{2m^*} + \alpha \mathbf{\hat{r}} \sigma_y - \sigma_y \mathbf{\hat{r}}$$

where $p = \hbar \mathbf{k}$ is the momentum operator for the electron, $\sigma_0$ is the $2 \times 2$ identity matrix, $\alpha$ is the RSOI coupling constant and $\sigma_{xy}$ are the usual Pauli spin matrices. The energy eigenvalues and the normalized eigenstates corresponding to the above Hamiltonian are, respectively, given by

$$\epsilon_k^\pm = \frac{\hbar^2 k^2}{2m^*} + \lambda \alpha k$$

and

$$\psi_k(x, y) = \frac{1}{\sqrt{2}} \left( \frac{1}{\lambda \text{e}^{-i\phi_k}} \right) \text{e}^{ik \mathbf{r}}$$

with $\lambda = \pm$ indicating the two spin-split energy branches and $\tan \phi_k = k_y/k_x$. At a given Fermi energy $\epsilon_F$, the Fermi wavevectors for the two energy branches can be written as $k_F^\pm = \sqrt{(k_a^0)^2 - k_a^2} - \lambda k_a$ with $k_a^0 = \sqrt{2\pi n_e}$ and $k_d = m^*/\hbar^2$.

The velocity of an electron in a particular branch $\lambda$ is given by

$$\mathbf{v}_k^\pm = \frac{\hbar}{m^*} \mathbf{\hat{r}} + \lambda \alpha \mathbf{\hat{r}}$$

2.1. Phonon-drag thermopower

We consider the interaction between electrons with two-dimensional wave vector $k$ and an acoustic phonon with three-
dimensional wavevector $\mathbf{Q}$. To calculate the phonon-drag thermopower we follow the explicit formula given in [25, 26] for a 2DES. With appropriate modifications the expression for the phonon-drag thermopower in a Rashba spin–orbit coupled 2DES is given by

\begin{equation}
S_E^p = \frac{\epsilon T_p}{2\pi AK_B T_F} \sum_{k} \sum_{\mathbf{Q}} \hbar \omega \mathbf{g}(\mathbf{Q}) \left[ 1 - f(\epsilon_k^+ + \hbar \omega) \right] \times W_{\mathbf{Q}}^{*} \left( \mathbf{k}, \mathbf{k}' \right) \left[ \tau(\epsilon_k^+) \mathbf{v}_k^+ - \tau(\epsilon_{k'}^+) \mathbf{v}_{k'}^+ \right] \cdot \mathbf{v}_p.
\end{equation}

where $e$ is the electronic charge, $T_p$ is the phonon mean free time, $A$ is the area of the sample, $\sigma$ is the Drude conductivity, $k_B$ is the Boltzmann constant, $\omega_0 = \nu_0 \mathbf{Q}$ is the energy-dependent momentum relaxation time of an electron, $f(\epsilon) = [e^{\beta(\epsilon - \mu)} + 1]^{-1}$ is the Fermi–Dirac distribution function and $\beta = 1/(k_B T_F)$, $\mathbf{v}_k^+$ is the velocity of an electron in a particular branch $\lambda$, $\mathbf{v}_p$ is the phonon velocity defined as $\mathbf{v}_p = \nu_0 \mathbf{Q}/\hbar$ and $W_{\mathbf{Q}}^{*} \left( \mathbf{k}, \mathbf{k}' \right)$ is the transition probability which is responsible for making the transition of an electron from an initial state $|\mathbf{k}, \lambda\rangle$ to a final state $|\mathbf{k}', \lambda'\rangle$ with the absorption of a phonon. The explicit form of the transition probability is given by Fermi’s golden rule

\begin{equation}
W_{\mathbf{Q}}^{*} \left( \mathbf{k}, \mathbf{k}' \right) = \frac{2\pi}{\hbar} |C_{\mathbf{Q}, \lambda}^{\lambda'}^{2}|^2 \hbar \omega_0 \delta(\epsilon_{\lambda'}^+ - \epsilon_{\lambda}^- - \hbar \omega_0) \delta_{\lambda, \lambda'} \delta_{\mathbf{k}, \mathbf{k} +\mathbf{q}}.
\end{equation}

and

\begin{equation}
\left| C_{\lambda, \lambda'}^{\lambda} \right|^2_{DP} = \frac{D^2 h_0}{2\rho_m \nu_0(0)} \frac{1 + \lambda \lambda' \cos \gamma_{\mathbf{k}}}{2} \left( |I(\mathbf{q}_c)|^2 \right)^2
\end{equation}

where $D$ is the DP coupling constant, $h_0$ is the relevant PE tensor component, $\rho_m$ is the mass density, $\nu_0(0)$ is the longitudinal (transverse) component of the sound velocity, $\gamma_{\mathbf{k}}$ is the angle between $\mathbf{k}$ and $\mathbf{k}'$, $A_1(\mathbf{q}, \mathbf{q}_c) = 9q_c^2 q^4/[2(q_c^2 + q^2)]^3$ and $A_1(\mathbf{q}, \mathbf{q}_c) = (8q_c^2 q^4 + q^6)/[4(q_c^2 + q^2)]^3$. The Kronecker delta symbol $\delta_{\lambda, \lambda'}$ in the matrix elements implies that the EPI is spin-independent. Finally, the form factor $|I(\mathbf{q}_c)|^2$ is responsible for the finite thickness of the quasi-2DES and it has the form $|I(q_c)|^2 = \int d\zeta \xi_0^2(\zeta)e^{i\phi_0}\xi_1^2 = b^2(q_c^2 + b^2)^3$ for a triangular potential.

With the help of the Kronecker delta symbol $\delta_{\mathbf{k}, \mathbf{k}+\mathbf{q}}$ in equation (6) the summation over $\mathbf{k'}$ in equation (5) can be easily evaluated by replacing $\mathbf{k'}$ with $\mathbf{k} + \mathbf{q}$. The argument of the delta function in equation (6) confirms the conservation of energy $\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{k} + \mathbf{q}} + \hbar \omega_0$. At this point we assume that $\tau(\epsilon_{\mathbf{k}})$ is approximately constant over an energy scale of the order of $\hbar \omega_0$, so that we can write $\tau(\epsilon_{\mathbf{k}} + \hbar \omega_0) \simeq \tau(\epsilon_{\mathbf{k}})$. The summation over $\mathbf{k}$ in equation (5) can be converted into an integral over $\epsilon_{\mathbf{k}}$ by the following transformation:

\begin{equation}
\sum_{\mathbf{k}} \rightarrow \frac{A}{(2\pi)^2 h^2} \int d\epsilon_k \left( 1 - \frac{\epsilon_{\mathbf{k}}}{\epsilon_{\mathbf{k}} + \epsilon_a} \right) \int d\theta,
\end{equation}

where $\theta$ is the angle between $\mathbf{k}$ and $q$. At very low energies we can make an additional approximation as $f(\epsilon_{\mathbf{k}})[1 - f(\epsilon_{\mathbf{k}} + \hbar \omega_0)] \simeq \hbar \omega_0(N_Q + 1)\delta_{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}} - \epsilon_a}$. Now we convert the summation over $\mathbf{Q}$ into an integration over $q$ and $q_c$ as $\sum_{\mathbf{Q}} \rightarrow \int (1/4\pi^2)^2 \int dq dq_c$. In the BG regime, the phonon energy is very small compared to the Fermi energy and consequently we can make a further approximation as $q_\perp \ll 2k_F$. With all the assumptions described above taken into account one can obtain a final expression for the phonon-drag thermopower as (for intermediate steps see appendix A)

\begin{equation}
S_E^p = -\frac{eT_p m^* \tau(\epsilon_{\mathbf{k}})}{8\pi^3 h^4 \sigma k_B T_F^2 k_F^2} \left( 1 - \frac{\epsilon_{\mathbf{k}}}{\epsilon_{\mathbf{k}} + \epsilon_a} \right) \times \int \frac{d \theta}{\hbar v_{\perp}^2} \int dq dq_c \frac{C_{\lambda}^{\lambda'}^{2}}{Q} \langle h q \rangle^2 N_Q (N_Q + 1).
\end{equation}

Here, we have assumed $\tau^+ (\epsilon_{\mathbf{k}}) = \tau^- (\epsilon_{\mathbf{k}}) = \tau (\epsilon_{\mathbf{k}})$ because the difference between $\tau^+$ and $\tau^-$ is very small.

The phonon energy is given by $\epsilon_p = \hbar \omega_0 = h v_{\perp} \sqrt{q_\perp^2 + q_c^2}$, so we can write $q_\perp = \epsilon_p \cos \phi/(h v_{\perp})$ and $q_c = \epsilon_p \sin \phi/(h v_{\perp})$, so that $dq dq_c \rightarrow \epsilon_p dq_p \epsilon_p dp/(h v_{\perp})^2$. We consider that the quasi-2DES is very thin, i.e., $q_c \ll b$, so $|I(q_c)|^2$ can be approximated as $|I(q_c)|^2 \approx 1$. With these substitutions $S_E^p$ due to DP scattering becomes

\begin{equation}
S_E^p = -\frac{eT_p m^* \tau(\epsilon_{\mathbf{k}})}{8\pi^3 h^4 \sigma k_B T_F^2 k_F^2} \left( 1 - \frac{\epsilon_{\mathbf{k}}}{\epsilon_{\mathbf{k}} + \epsilon_a} \right) \frac{D^2 h}{2\rho_m} \times \frac{1}{(v_{\perp})^4} \int d\epsilon_p \epsilon_p^2 N_Q (N_Q + 1) \int d\phi \cos^2 \phi.
\end{equation}

Using the standard result $\int d\epsilon_p \epsilon_p^2 N_Q (N_Q + 1) = (n!)^2 \xi(n)$, where $n$ is the Riemann zeta function, we obtain

\begin{equation}
S_E^p = -\frac{m^* \Lambda D^2 k_B}{16\pi^2 \rho_m k^4 e \epsilon_0^2} \left( 1 - \frac{\epsilon_{\mathbf{k}}}{\epsilon_{\mathbf{k}} + \epsilon_a} \right) \times \frac{5! \xi(3)(k_B T_F)^4}{(h v_{\perp})^3},
\end{equation}

where $\Lambda = \nu_0 T_p$ is the phonon mean free path. The total phonon-drag thermopower due to DP scattering is given by

\begin{equation}
S_{dp}^p = -\frac{m^* \Lambda D^2 k_B}{8\pi^2 h^2 \rho_m e \epsilon_0^2} \sqrt{\frac{\epsilon_0^2}{\epsilon_{\mathbf{k}} - \epsilon_a}} \frac{5! \xi(3)(k_B T_F)^4}{(h v_{\perp})^3}.
\end{equation}

The total phonon-drag thermopower $S_{dp}^p$ for DP scattering is proportional to $T^4$.
2.2. Hot-electron energy-loss rate

The average energy-loss rate per electron via acoustic phonon emission is given by

\[ P = \left( \frac{\partial \epsilon}{\partial t} \right) = \frac{1}{N_e} \sum_{\lambda, Q} h\omega_Q \left[ \frac{\partial N_Q}{\partial t} \right]_{\lambda}, \]

where \( N_e \) is the total number of electrons. The rate of change of phonon occupation number for a given branch \( \lambda \) is given by

\[
\left[ \frac{\partial N_Q}{\partial t} \right]_{\lambda} = \sum_k W_{Q,\lambda}^{\lambda,\lambda} (\mathbf{k},\mathbf{k} + \mathbf{q}) \left( (N_Q + 1)f(\epsilon_k^\lambda + h\omega_Q) - N_Q f(\epsilon_k^\lambda) \right) \times \left( 1 - f(\epsilon_k^\lambda + h\omega_Q) \right),
\]

where the Fermi–Dirac distribution function \( f(\epsilon) \) is described by the temperature of hot-electrons \( T_e \) and the phonon distribution function is described by the lattice temperature \( T_l \). Obviously \( T_e \) is larger than \( T_l \) so that an electron can relax its energy via acoustic phonon emission and equilibrate to the lattice temperature.

Now we use the following identity:

\[
\frac{1 - f(\epsilon_k + h\omega_Q)}{1 - f(\epsilon_k)} = \frac{f(\epsilon_k + h\omega_Q)}{f(\epsilon_k)} e^{\beta_e h\omega_Q} \tag{18}
\]

with \( \beta_e = 1/(k_BT_e) \). Using equations (16)–(18) and taking all the assumptions made for calculating \( S_g \) into account we finally obtain the following expression for the energy-loss rate (detailed calculations are given in appendix B):

\[
P = \frac{m^2}{4\pi e^2 \hbar^2} \sum_{\lambda} \frac{1}{k_F^2} \left( 1 - \lambda \right) \left( \frac{\epsilon_a}{\epsilon_F + \epsilon_a} \right)^2 \int dq dq_z \left( \frac{1}{e^{\beta_e h\omega_Q} - 1} - \frac{1}{e^{\beta_e h\omega_Q} - 1} \right), \tag{19}
\]

with \( \beta_l = 1/(k_BT_l) \). The integration over \( q \) and \( q_z \) in equation (19) can be easily evaluated by the same technique as described in the previous sub-section. The expressions for the total energy relaxation rates for DP, longitudinal PE and transverse PE scattering are respectively given by

\[
P_{DP} = \frac{m^2}{2\pi^2} \sum_{\lambda, P} 4\xi(5) \sqrt{\frac{\epsilon_0^F}{\epsilon_0^F - \epsilon_a}} (T_e^5 - T_l^5), \tag{20}
\]

\[
P_{PE} = \frac{9m^2}{64\pi^2 n_e \rho_m \hbar^3 v_F^2 k_F^0} \left( \frac{\epsilon_0^F}{\epsilon_0^F - \epsilon_a} \right)^2 (T_e^3 - T_l^3), \tag{21}
\]

and

\[
P_{TT} = \frac{13m^2}{128\pi^2 n_e \rho_m \hbar^3 v_F^2 k_F^0} \left( \frac{\epsilon_0^F}{\epsilon_0^F - \epsilon_a} \right)^2 (T_e^3 - T_l^3). \tag{22}
\]

In the BG regime \( P_{DP} \) is proportional to \( T_e^5 \) for DP scattering and \( P_{PE} \sim T_e^3 \) for PE scattering.

3. Numerical results

In the previous section we have presented approximated analytical results for the phonon-drag thermopower and energy-loss rate. In this section we discuss numerical results for the phonon-drag thermopower and hot-electron

![Figure 1. Plots of the phonon-drag thermopower due to DP scattering versus temperature for different values of the density. Here, the solid, dotted and dashed lines represent \( n_s = 3n_0, n_c = 5n_0 \) and \( n_e = 7n_0 \), respectively.](image-url)
energy-loss rate. To do this we solve equations (5) and (16) numerically for both DP and PE scattering mechanisms in the low temperature regime. For the numerical calculations we consider the material parameters of GaAs/AlGaAs heterostructures as $m^* = 0.067 m_e$ with free electron mass $m_e$, $\rho_m = 5.31 \times 10^3$ kg m$^{-3}$, $v_{sl} = 5.12 \times 10^3$ ms$^{-1}$, $v_{st} = 3.04 \times 10^3$ ms$^{-1}$, $D = 12$ eV, $h_{14} = 1.2 \times 10^9$ V m$^{-1}$, $\kappa = 12.91$, $n_d = 10^{14}$ m$^{-2}$, $a_0 = 10^{-11}$ eV m and $n_0 = 10^{15}$ m$^{-3}$.

We estimate the effective exponent from the log–log plot of the phonon-drag thermopower versus temperature due to DP scattering in figure 1 for both $\alpha = 0$ and $\alpha = \alpha_0$ with different densities. At very low temperature ($T \sim 1–3$ K), we obtain $\nu = 3.449, 3.942$ and $4.139$ for $n_e = 3n_0, 5n_0$ and $7n_0$, respectively, when $\alpha = 0$. On the other hand, we obtain $\nu = 2.846, 3.294$ and $3.547$ for $n_e = 3n_0, 5n_0$ and $7n_0$, respectively, when $\alpha = \alpha_0$. The value of $\nu$ with $\alpha = \alpha_0$ is lower than that with $\alpha = 0$. Figure 1 also shows that the magnitude of $S_g$ with $\alpha = \alpha_0$ is less in comparison with $\alpha = 0$.

In figure 2, the log–log plot of the phonon-drag thermopower versus temperature due to PE scattering is presented for different values of $\alpha$ and $n_e$. When $\alpha = 0$, we obtain $\nu = 1.688, 1.961$ and $2.096$ for $n_e = 3n_0, 5n_0$ and $7n_0$, respectively. When $\alpha = \alpha_0$, we obtain $\nu = 1.290, 1.520$ and $1.658$ for $n_e = 5n_0, 7n_0$ and $7n_0$, respectively. Comparing figures 1 and 2, one can conclude that the magnitude of $S_g$ due to DP scattering is larger than that due to PE scattering.

In figure 3 we plot $S_g$ due to DP and PE scattering versus $T$ for $\alpha = 0$ and $\alpha = \alpha_0$ at a fixed density $n_e = 5n_0$. Figure 3 clearly shows that the slope of the line with $\alpha = \alpha_0$ is less than that with $\alpha = 0$. 

![Figure 2](image1.png)  
Figure 2. Plots of the phonon-drag thermopower due to PE scattering versus temperature for different values of the density. Here, the solid, dotted and dashed lines represent $n_e = 3n_0, 5n_0$ and $7n_0$, respectively. 

![Figure 3](image2.png)  
Figure 3. Plots of the phonon-drag thermopower due to DP and PE scattering versus temperature for a fixed density $n_e = 5n_0$. Here, the solid and dashed lines represent $\alpha = 0$ and $\alpha = \alpha_0$, respectively.
Figure 4. Plots of the phonon-drag thermopower due to DP and PE scattering versus $\alpha$ for different values of the density at fixed temperature $T = 2$ K. Here, the solid, dotted and dashed lines represent $n_e = 3n_0$, $n_e = 5n_0$ and $n_e = 7n_0$, respectively.

Figure 5. Plots of the energy-loss rate due to DP scattering as a function of electron temperature $T_e$ for $\alpha = 0$ and $\alpha = \alpha_0$ for different densities. We set the lattice temperature $T_l = 0$. Here, the solid, dotted and dashed lines represent $n_e = 3n_0$, $n_e = 5n_0$ and $n_e = 7n_0$, respectively.

The variation of $S_g$ due to DP and PE scattering as a function of the Rashba coupling constant ($\alpha$) is shown in figure 4. The phonon-drag thermopower $S_g$ decreases very slowly with increase of $\alpha$ for both DP and PE scattering.

It is important to compare the orders of magnitude of the diffusion and phonon-drag contributions to the total thermopower. The diffusion thermopower [50] is given by

$$S_d = -\frac{\pi^2 k_B^2 T}{3|\epsilon_F|} \left( p + 1 - \frac{\epsilon_\alpha}{\epsilon_F} \right),$$

where the parameter $p$ depends on various scattering mechanisms. We calculate $S_d$ from equation (23) and $S_g$ from numerical evaluation of equation (5). With $\alpha = \alpha_0$ and $n = 5n_0$ at $T = 2$ K we obtain $S_d \sim -5.464$ $\mu$V K$^{-1}$, $S_g^{\text{DP}} \sim -18.526$ $\mu$V K$^{-1}$ and $S_g^{\text{PE}} \sim -7.135$ $\mu$V K$^{-1}$. It is clear that at $T = 2$ K the phonon-drag thermopower due to DP scattering dominates over both PE scattering induced phonon-drag thermopower and diffusion thermopower.

Now we present the numerical calculations for the energy-loss rate of hot-electrons in the BG regime. In figures 5 and 6 we show the dependence of $P$ on the electron temperature $T_e$ for DP and PE scattering, respectively.

In general, the energy-loss rate of hot-electrons is given by $P = \Gamma(T_e - T_l^0)$ with $\Gamma$ as the proportionality constant. By taking the lattice temperature $T_l = 0$ we determine the values of $\delta$ for $\alpha = 0$ and $\alpha = \alpha_0$ from figures 5 and 6. For DP scattering with $\alpha = 0$ the values of $\delta$ are $\delta = 4.707, 4.916$ and $4.998$ for $n_e = 3n_0, 5n_0$ and $7n_0$, respectively. When $\alpha = \alpha_0$
Figure 6. Plots of the energy-loss rate due to PE scattering versus electron temperature $T_e$ for $\alpha = 0$ and $\alpha = \alpha_0$ for different densities. Here, the lattice temperature is fixed to $T_l = 0$. The solid, dotted and dashed lines represent $n_e = 3n_0$, $n_e = 5n_0$ and $n_e = 7n_0$, respectively.

Figure 7. The energy-loss rate due to DP scattering is plotted as a function of $T_e - T_l$ with $\alpha = \alpha_0$ for two lattice temperatures $T_l = 3.4$ K and $T_l = 5.5$ K. Here, the solid, dotted and dashed lines represent $n_e = 3n_0$, $n_e = 5n_0$ and $n_e = 7n_0$, respectively.

we obtain $\delta = 4.268$, $4.511$ and $4.661$ for $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively. For PE scattering we find $\delta = 2.945$, $3.078$ and $3.139$ for $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively at $\alpha = 0$. With $\alpha = \alpha_0$ the values of $\delta$ are obtained as $\delta = 2.635$, $2.790$ and $2.880$ for $n_e = 3n_0$, $5n_0$ and $7n_0$, respectively.

Our numerical calculations reveal that the effective exponents of the temperature dependence of the phonon-drag thermopower and energy-loss rate strongly depend on the electron density and the Rashba spin–orbit coupling constant.

In figures 7 and 8 we plot $P$ due to DP and PE scattering as a function of $T_e - T_1$ with $\alpha = \alpha_0$ for different densities. Different values of $T_1$ have been considered here. The magnitude of $P$ is higher at higher values of $T_1$ and it decreases with increase of the electron density. Comparing figures 7 and 8, we see that the energy-loss rate due to DP scattering is much higher than that due to PE scattering.

In figure 9, we present the energy-loss rate due to DP and PE scattering versus $\alpha$ for different lattice temperatures. It shows that the energy-loss rates increase monotonically with $\alpha$ in both cases. This behavior is quite different from the behavior of the phonon-drag thermopower versus $\alpha$ shown in figure 4.

4. Summary

In this section we summarize the main results of this work. In the BG regime the phonon-drag thermopower and hot-electron energy-loss rate have been calculated for a quasi-2DES formed at the interface of a GaAs/AlGaAs heterojunction. Both the DP and the PE scattering mechanisms have been taken into account separately. It is shown that the effective exponent of the temperature dependence
Figure 8. The energy-loss rate due to PE scattering is plotted as a function of $T_e - T_l$ with $\alpha = \alpha_0$ for two lattice temperatures $T_l = 3.4$ K and $T_l = 5.5$ K. Here, the solid, dotted and dashed lines represent $n_e = 3n_0$, $n_e = 5n_0$ and $n_e = 7n_0$, respectively.

Figure 9. Plots of the energy-loss rate due to DP and PE scattering versus $\alpha$ for different values of the lattice temperature $T_l$. We fix the electron temperature at $T_e = 6$ K. Here, the solid, dotted and dashed lines represent $T_l = 3.4$ K, $T_l = 4.2$ K and $T_l = 5.5$ K, respectively.

Table 1. The effective exponent of the temperature dependence of $S_g$ in the BG regime for various values of $n_e$ and $\alpha$.

| Density ($n_e$) | DP $\alpha = 0$ | $\alpha = \alpha_0$ | PE $\alpha = 0$ | $\alpha = \alpha_0$ |
|---------------|----------------|------------------|----------------|------------------|
| $3n_0$        | 3.449          | 2.846            | 1.688          | 1.290            |
| $5n_0$        | 3.942          | 3.294            | 1.961          | 1.520            |
| $7n_0$        | 4.139          | 3.547            | 2.096          | 1.658            |

Table 2. The effective exponent of the temperature dependence of $P$ in the BG regime for various values of $n_e$ and $\alpha$.

| Density ($n_e$) | DP $\alpha = 0$ | $\alpha = \alpha_0$ | PE $\alpha = 0$ | $\alpha = \alpha_0$ |
|---------------|----------------|------------------|----------------|------------------|
| $3n_0$        | 4.707          | 4.268            | 2.945          | 2.635            |
| $5n_0$        | 4.916          | 4.511            | 3.078          | 2.790            |
| $7n_0$        | 4.998          | 4.661            | 3.139          | 2.880            |

of the phonon-drag thermopower and the energy-loss rate strongly depend on the electron density and the Rashba spin–orbit coupling constant. For DP and PE scattering in the BG regime, the values of the effective exponents of $T$ of the thermoelectric power and the hot-electron energy-loss rate for different values of $n_e$ and $\alpha$ are summarized in Tables 1 and 2.

It is shown that the orders of magnitude of the phonon-drag thermopower due to DP and PE scattering are almost the same and the phonon-drag thermopower dominates over the diffusion thermopower in the BG regime. The
phonon-drag thermopower due to DP and PE scattering decreases very slowly with increase of $\alpha$.

The order of magnitude of the energy-loss rate due to DP scattering is much higher than that of PE scattering. The energy-loss rate due to DP and PE scattering increases monotonically with $\alpha$.

Appendix A

In this appendix, we briefly sketch the derivation of the phonon-drag thermopower. Using equations (5) and (9) we can write

$$S^J_g = \frac{e^2 \nu_m^*}{8\pi^2 \hbar^2 \alpha T^2} \sum_Q \int d\epsilon_k \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_k + \epsilon_a}} \right)$$

\[ \times \int d\theta \ h \omega_Q \omega^{\lambda/2}_Q(k, \mathbf{k} + \mathbf{q}) f(\epsilon_k) \]

\[ \times \left( 1 - f(\epsilon_k + h \omega_Q) \right) \]

\[ \times \tau(\epsilon_k^2) (\mathbf{v}_k^2 - \mathbf{v}_{k+q}^2) \cdot \mathbf{v}_p. \]  \hspace{1cm} (A.1)

Using the approximation $f(\epsilon_k)[1 - f(\epsilon_k + h \omega_Q)] \approx h \omega_Q(N_Q + 1) \delta(\epsilon_k - \epsilon_f)$, the integration over $\epsilon_k$ in equation (A.1) can be easily performed and we obtain

$$S^J_g = \frac{e^2 \nu_m^* \tau(\epsilon_f)}{8\pi^2 \hbar^2 \alpha T^2} \sum_Q \int d\theta \ h \omega_Q \omega^{\lambda/2}_Q(k, k_F + \mathbf{q})$$

\[ \times \left( \mathbf{v}_{k_F}^2 - \mathbf{v}_{k_F+q}^2 \right) \cdot \mathbf{v}_p. \] \hspace{1cm} (A.2)

Using equation (4) and taking $q \ll k_F$ we can write the difference between the two velocities as

$$\mathbf{v}_{k_F}^2 - \mathbf{v}_{k_F+q}^2 \approx -\hbar q \frac{m^*}{m} - \lambda \frac{\alpha}{h} \left( q \frac{k_F}{k_F} + \frac{k_F^3}{k_F^3} \right). \hspace{1cm} (A.3)$$

Similarly, the difference between the two energies can be approximated as

$$\epsilon_{k_F+q}^2 - \epsilon_{k_F}^2 \approx \left( \frac{\hbar^2 k_F^2}{m^*} + \lambda \alpha \right) q \cos \theta$$

\[ + \left( \frac{\hbar^2}{2m^*} + \frac{\alpha}{2k_F} \right) q^2. \] \hspace{1cm} (A.4)

where $\theta$ is the angle between $k_F$ and $q$. Converting the summation over $Q$ into an integration over $q$, $\mathbf{q}$, and performing the $\theta$ integration we can simplify equation (A.2) as

$$S^J_g = -\tau(\epsilon_f) \frac{e^2 \nu_m^* \tau(\epsilon_f)}{8\pi^2 \hbar^2 \alpha T^2} \sum_Q \int d\mathbf{q} d\mathbf{q}_z \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_f + \epsilon_a}} \right)^2$$

\[ \times \left[ \frac{|C_q|^2}{Q G(k_F^2, Q)} \left( \frac{\hbar k_F^2}{m^*} + \lambda \alpha \right) - \frac{\lambda}{h} \right] \] \hspace{1cm} \[ \times \left[ \frac{m^* \nu_v}{\hbar k_F^2} Q \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_f + \epsilon_a}} \right) - \frac{q}{2k_F^2} \right]^2. \] \hspace{1cm} (A.5)

with

$$G(k_F^2, Q) = \left( 1 \right) - q^2 \frac{2}{4k_F^2} \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_f + \epsilon_a}} \right)$$

\[ \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_f + \epsilon_a}} \right)^2 \] \hspace{1cm} \[ \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_f + \epsilon_a}} \right)^2 \] \hspace{1cm} (A.6)

With the assumptions $q \ll 2k_F$ and $m^* \nu_v \ll \hbar k_F$ we have $G(k_F^2, Q) \approx 1$. Equation (10) can be derived easily from equation (A.5) with all the above mentioned approximations taken into account.

Appendix B

In this appendix, we briefly outline the derivation of the energy-loss rate of hot-electrons. Using equation (9) and after performing the integration over $\theta$, equation (17) can be re-written as

$$\left[ \frac{\partial N_Q}{\partial t} \right]_\lambda = \frac{m^* A}{\pi \hbar^2 q} \int d\epsilon_k \left( 1 - \sqrt{\frac{\epsilon_a}{\epsilon_k + \epsilon_a}} \right)^2$$

\[ \times \frac{|C_q|^2}{k^2 G(k^2, Q)} Q G(T_z) \left[ e^{(f_\lambda - f_\lambda) h \omega_Q} - 1 \right] \] \hspace{1cm} \[ \times f(\epsilon_k)[1 - f(\epsilon_k + h \omega_Q)]. \] \hspace{1cm} (B.1)

At low temperature we use the same approximation as made in appendix A. After performing the integration over $\epsilon_k$ in equation (B.1) finally equation (16) becomes

$$P = \frac{m^*}{4\pi^2 n \hbar^2} \sum_Q \left( 1 - \lambda \sqrt{\frac{\epsilon_a}{\epsilon_f + \epsilon_a}} \right)^2 \int dq dq_z (h \omega_Q)^2$$

\[ \times \frac{|C_q|^2}{k^2 G(k^2, Q)} N_Q(T_z) \left[ e^{(f_\lambda - f_\lambda) h \omega_Q} - 1 \right]. \] \hspace{1cm} (B.2)

By taking the approximation $G(k_F^2, Q) \approx 1$ into account at low temperature and after evaluating the integrations over $q$ and $\mathbf{q}_z$ in equation (B.2) it is not difficult to obtain the conclusions (20)–(22).

References

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