About the reducibility of the variety
of complex Leibniz algebras

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Abstract
In this paper, using the notions of perturbation and contraction of Lie and Leibniz algebras, we
show that the algebraic varieties of Leibniz and nilpotent Leibniz algebras of dimension greater
than 2 are reducible.

Keywords: Leibniz algebra, perturbation, rigidity, contraction.

AMS Classification Numbers: 17A32

1 Definition and preliminary properties

The aim of this work is to prove the reducibility of the Leibniz and nilpotent Leibniz algebraic
varieties, that we will denote Leibn and LeibN respectively. First we will classify the 3-dimensional
nilpotent Leibniz algebras over the complex field. Then, using the internal set theory of Nelson [8],
we will introduce what a perturbation of a Leibniz algebra is. Such notion allows us to determine
the open components of the variety LeibN3, it will turn out to have two of them. The other algebras
are obtained as the limit by contraction of the rigid algebra or the family of rigid algebras defining
the open components. Moreover, we characterize the rigid Lie algebras over the variety L of Lie
algebras that are rigid over Leibn.

Definition 1 A Leibniz algebra law µ over C is a bilinear map µ : Cn × Cn → Cn satisfying
\[ \mu(x, \mu(y, z)) = \mu(\mu(x, y), z) - \mu(\mu(x, z), y). \] (1)

We call Leibniz algebra to any pair (Cn, µ) where µ is a Leibniz algebra law.

The previous equation is known as the Leibniz identity. From now on, the law will be identified
with its algebra and the non written products will be supposed to be zero.

Notice that if µ is anticommutative µ(x, y) = −µ(y, x), the Leibniz identity is equivalent to the
Jacobi one
\[ \mu(x, \mu(y, z)) + \mu(y, \mu(z, x)) + \mu(z, \mu(x, y)) = 0, \] (2)
as (1) is obtained by placing the element x on the first place at every element of the Jacobi identity.

Let l = (Cn, µ) be a Leibniz algebra, we define the right-decreasing central sequence as
\[ C^1(l) = l \quad C^2(l) = \mu(l, l) \quad \ldots \quad C^{k+1}(l) = \mu(C^k(l), l) \quad \ldots \]

Definition 2 A Leibniz algebra l is nilpotent if there exists some k ∈ N such that Ck(l) = {0}.
For a given nilpotent Leibniz algebra \( I = (\mathbb{C}^n, \mu) \), we define for every \( x \in \mathbb{C}^n \) the endomorphism \( R_x : \mathbb{C}^n \rightarrow \mathbb{C}^n \) as

\[
R_x(y) = \mu(y, x), \quad \forall y \in \mathbb{C}^n.
\]

It is easy to check that \( R_x \) is a nilpotent endomorphism, then for any \( x \in I \setminus \mathbb{C}^2(I) \), we write \( s_\mu(x) = (s_1(x), \ldots, s_k(x)) \) the decreasing sequence \( s_1 \geq s_2 \geq \ldots \geq s_k \) of dimensions of the Jordan blocks of the nilpotent operator \( R_x \). We may now order lexicographically \( s_\mu(x) \) for all \( x \in I \setminus \mathbb{C}^2(I) \) and denote \( s(\mu) \) its maximum which is, up to isomorphism, an invariant of the isomorphism class of the algebra \( I \). We call it characteristic sequence of \( I \). If \( x \in I \setminus \mathbb{C}^2(I) \) satisfies \( s_\mu(x) = s(\mu) \), we say that \( x \) is a characteristic vector of \( I \).

We will denote Leib\( ^n \) the set of all Leibniz algebras over \( \mathbb{C}^n \) and Leib\( ^n \) the set of nilpotent Leibniz algebras over \( \mathbb{C}^n \).

Notice that we can identify any Leibniz algebra \( \mu \) with its structure constants over a fixed base. Given \( \{e_1, \ldots, e_n\} \) a base of \( \mathbb{C}^n \), from the identity \( (1) \) we have that the coordinates defined by \( \mu(e_i, e_j) = a_{ij}^k e_k \) are the solution of

\[
da_{jk}a_{il}^m - a_{lj}^m e_k + a_{ik}^l a_{lj}^m = 0, \quad 1 \leq i, j, k, m \leq n \quad (3)
\]

As the nilpotent conditions are also polynomials, Leib\( ^n \) and Leib\( ^n \) can be endowed with an algebraic structure over \( \mathbb{C}^{n^3} \).

## 2 Classification of the nilpotent Leibniz algebra of dimension 3

Let \( I = (\mathbb{C}^3, \mu) \) be a nilpotent Leibniz algebra. According to the previous section, the possible characteristic sequences of \( I \) are \( \mathcal{C}(I) \in \{ (3), (2, 1), (1, 1, 1) \} \).

1. If \( s(I) = (3) \), there exists a characteristic vector \( e_1 \) and a base \( \{e_1, e_2, e_3\} \) such that

\[
\begin{align*}
\mu(e_1, e_1) &= e_2, \\
\mu(e_2, e_1) &= e_3.
\end{align*}
\]

As \( \mu(x, e_2) = \mu(x, \mu(e_1, e_1)) = \mu(\mu(x, e_1), e_1) - \mu(\mu(x, e_1), e_1) = 0 \), we have that \( R_{e_2} = 0 \). The Leibniz identity for \( (e_1, e_2, e_1), (e_2, e_2, e_1) \) and \( (e_3, e_2, e_1) \) shows that \( \mu(e_1, e_3) = \mu(e_2, e_3) = \mu(e_3, e_3) = 0 \). In this case thus, there only exists (up to isomorphism) one nilpotent Leibniz algebra \( \mu_1 \) of maximal characteristic sequence, which is given by

\[
\begin{align*}
\mu_1(e_1, e_1) &= e_2, \\
\mu_1(e_2, e_1) &= e_3.
\end{align*}
\]

2. If \( s(I) = (2, 1) \), we have two possibilities

(a) There exists a characteristic vector \( e_1 \) such that \( \mu(e_1, e_1) \neq 0 \).

(b) For every characteristic vector \( x \), we have \( \mu(x, x) = 0 \).

For the (a) case, we can find a base \( \{e_1, e_2, e_3\} \) such that

\[
\begin{align*}
\mu(e_1, e_1) &= e_2, \\
\mu(e_2, e_1) &= 0, \\
\mu(e_3, e_1) &= 0.
\end{align*}
\]
The Leibniz identity for \((x, e_1, e_1)\) leads again to \(\mu(x, e_2) = 0\), whereas using it for \((e_2, e_2, e_3)\) and the nilpotency of \(R_{e_3}\) lead to \(\mu(e_2, x) = 0\). Finally the Leibniz identity for \((e_1, e_1, e_3)\) and \((e_3, e_3, e_3)\) implies that

\[
\mu(e_1, e_3) = ae_2, \\
\mu(e_3, e_3) = be_2.
\]

If we consider a change of base \(\{x_1, x_2, x_3\}\) such that \(\mu(x_2, x_1) = 0, \mu(x_3, x_1) = 0\) and \(\mu(x_2, x_3) = 0\), we remain in this family of nilpotent Leibniz algebras, and the nullity or non nullity of \(a\) and \(b\) are preserved under this change of basis. Then, if \(a \neq 0\) considering \(x_1 = e_1, x_2 = e_2\) and \(x_3 = \frac{1}{a}e_3\), leads to the family of non isomorphic Leibniz algebras \(\mu_{2,b}\) given by

\[
\mu_{2,b}(e_1, e_1) = e_2, \\
\mu_{2,b}(e_3, e_3) = be_2, \\
\mu_{2,b}(e_1, e_3) = e_2.
\]

If \(a = 0\) but \(b \neq 0\), we can analogously take \(b = 1\) leading to the sole algebra \(\mu_3\) given by

\[
\mu_3(e_1, e_1) = e_2, \\
\mu_3(e_3, e_3) = e_2.
\]

Finally if \(a = b = 0\) we obtain the algebra \(\mu_4\) given by \(\mu_4(e_1, e_1) = e_2\)

For the \((b)\) case, there exists a basis \(\{e_1, e_2, e_3\}\) such that \(\mu(e_2, e_1) = e_3\). The nilpotency of \(R_{e_3}, R_{e_2}\) and the fact that there is no characteristic vector \(x\) such that \(\mu(x, x) \neq 0\), imply that the Leibniz algebra \(\mu\) is in fact a Lie algebra isomorphic to the Heisenberg algebra of dimension 3 i.e. \(\mu\) is isomorphic to \(\mu_5\) given by \(\mu_5(e_1, e_2) = -\mu_5(e_2, e_1) = -e_3\).

3. If \(s(l) = (1, 1, 1)\), it turns out that \(\mu\) is the abelian algebra \(\mu_6 = 0\).

The previous analysis shows the following result

**Theorem 1** Every nilpotent complex Leibniz algebra is isomorphic to one of the algebras \(\mu_i\) with \(i = 1,3,4,5,6\) or to \(\mu_{2,b}\) with \(b \in \mathbb{C}\).

### 3 Contractions and perturbations of the Leibniz algebras

In this section \(\mathcal{L}^n\) will denote the variety of Lie algebras \(L^n\), or one of the varieties \(\text{Leib}^n\) or \(\text{LeibN}^n\). If \(\mu_0 \in \mathcal{L}^n\), we denote \(O(\mu_0)\) the orbit of \(\mu_0\) under the action of the general linear group \(\text{GL}(n, \mathbb{C})\) over \(\mathcal{L}^n:\)

\[
\begin{align*}
\text{GL}(n, \mathbb{C}) \times \mathcal{L}^n & \rightarrow \mathcal{L}^n \\
(f, \mu_0) & \mapsto f^{-1} \circ \mu_0 \circ (f \times f)
\end{align*}
\]

where \(f^{-1} \circ \mu_0 \circ (f \times f)(x, y) = f^{-1}(\mu_0(f(x), f(y))).\)

Let \(C\) be an irreducible component of \(\mathcal{L}^n\) containing \(\mu_0\), then \(O(\mu_0) \subseteq C\). We can endow naturally the variety \(\mathcal{L}^n\) with two non equivalent topologies: the metric topology induced by the inclusion of \(\mathcal{L}^n\) in \(\mathbb{C}^{n^3}\), and the Zariski topology. Notice that the latter is contained in the former. As \(C\) is closed in the Zariski topology, the adherence \(\overline{O(\mu_0)}^Z\) of the orbit \(\mu_0\) is also contained in \(C\).
In analogy with the Lie algebras we can formally define the notion of limit over the variety \( \mathcal{L}^n \) as follows: let \( f_t \in GL(n, \mathbb{C}) \) be a family of non-singular endomorphism depending on a continuous parameter \( t \), and consider some \( \mu \in \mathcal{L}^n \). If for every pair \( x, y \in \mathbb{C}^n \) the limit
\[
\mu'(x, y) := \lim_{t \to 0} \mu_t(x, y) := \lim_{t \to 0} f_t^{-1} \circ \mu(f_t(x), f_t(y))
\]
exists, then \( \mu' \) is an algebra law of \( \mathcal{L}^n \). We call this new law the contraction of \( \mu \) by \( \{ f_t \} \). Using the action of \( GL(n, \mathbb{C}) \) over the variety \( \mathcal{L}^n \), it is easy to see that a contraction of \( \mu \) corresponds to a point of the closure of the orbit \( \mathcal{O}(\mu) \).

It is important to notice that any non trivial contraction \( \mu \to \mu' \) satisfies
\[
\dim \mathcal{O}(\mu) > \dim \mathcal{O}(\mu') \, ,
\]
\[
\dim Z_R(\mu) \leq \dim Z_R(\mu') \quad \text{where} \quad Z_R(\mu) = \{ x \in \mathbb{C}^n : \mu(y, x) = 0, \forall y \in \mathbb{C}^n \} \, ,
\]
\[
s(\mu) \geq s(\mu') \quad \text{in the nilpotent case}.
\]

Therefore every component \( C \) containing \( \mu_0 \) contains also any of its contractions.

**Definition 3** Assuming the non standard analysis (I.S.T.) of Nelson [8], let \( \mu_0 \) be a standard law of \( \mathcal{L}^n \). A perturbation \( \mu \) of \( \mu_0 \) over \( \mathcal{L}^n \) is another law in \( \mathcal{L}^n \) satisfying the condition \( \mu(x, y) \sim \mu_0(x, y) \) for every standard \( x, y \) over \( \mathbb{C}^n \), where \( a \sim b \) means that the vector \( a - b \) is infinitesimally small.

In particular if \( \mu' = \lim_{t \to 0} \mu_t \) is a contraction of \( \mu \), for every \( t_0 \) infinitesimally small, the law \( \mu_{t_0} \) is isomorphic to \( \mu \) and is in fact a perturbation of \( \mu' \). Such remark encodes perfectly the link between the notions of perturbation and contraction.

**Consequence.** The invariants of the nilpotent laws characterizing the irreducibles components are the stable invariants under perturbation. In particular if \( \tilde{\mu} \) is a perturbation of \( \mu \), then
\[
\dim \mathcal{O}(\tilde{\mu}) > \dim \mathcal{O}(\mu) \, ,
\]
\[
\dim Z_R(\tilde{\mu}) \leq \dim Z_R(\mu),
\]
\[
s(\tilde{\mu}) \geq s(\mu) \quad \text{in the nilpotent case}.
\]

**Definition 4** A standard law \( \mu \in \mathcal{L}^n \) is **rigid** over \( \mathcal{L}^n \) if any perturbation of \( \mu \) is isomorphic to it.

This definition translates to the non-standard language the classic notion of rigidity. In fact, if any perturbation of \( \mu \) is isomorphic to \( \mu \), its halo (i.e. the class of laws \( \mu' \) such that \( \mu' \sim \mu \)) is contained on the orbit \( \mathcal{O}(\mu) \). This implies that the orbit is open and, by the transfer principle, we obtain the equivalence. In particular we obtain that the rigid algebras cannot be obtained by contraction and that the rigidity of \( \mu \in \mathcal{L}^n \) over \( \mathcal{L}^n \) implies that \( \mathcal{O}(\mu)^2 \) is an irreducible component of the variety \( \mathcal{L}^n \).

4 **The variety LeibN\(^3\)**

In this section, using the notions of the previous paragraph, we determine the irreducibles components of the variety LeibN\(^3\).

1. The law \( \mu_1 \) (sec. 2) is rigid over LeibN\(^3\). Clear as it is the only nilpotent Leibniz algebra with a maximal characteristic sequence.
2. \( \mu_{2,b} \) (\( b \neq 0 \)), \( \mu_3 \) and \( \mu_5 \) are not contractions of \( \mu_1 \). The dimension of the center cannot decrease with a contraction, however \( \dim(Z_R(\mu_1)) = 2 \), \( \dim(Z_R(\mu_{2,b})) = 1 \) (in the \( b \neq 0 \) case), \( \dim(Z_R(\mu_3)) = 1 \) and \( \dim(Z_R(\mu_5)) = 1 \).

3. The only contractions of \( \mu_1 \) are isomorphic to \( \mu_{2,0}, \mu_4 \) and \( \mu_6 \). It is enough to consider the following family of automorphisms of \( \mathbb{C}^3 \)

\[
\begin{align*}
| f_1(e_1) &= te_1 & g_1(e_1) &= te_1 & h_1(e_1) &= te_1 \\
| f_1(e_2) &= t^2e_2 & g_1(e_2) &= t^2e_2 & h_1(e_2) &= te_2 \\
| f_1(e_3) &= e_3 + te_1 & g_1(e_3) &= e_3, & h_1(e_3) &= te_3,
\end{align*}
\]

to obtain the contractions of \( \mu_1 \) into \( \mu_{2,0}, \mu_4 \) and \( \mu_6 \) respectively.

4. If \( b \neq 0 \) and \( \tilde{\mu} \) is a perturbation of \( \mu_{2,b} \), there exists some \( b' \in \mathbb{C} \) such that \( \tilde{\mu} \) is isomorphic to \( \mu_{2,b'} \). This means that the family \( \{\mu_{2,b}\}_{b \neq 0} \) is rigid.

In fact, on one hand we have that \( \tilde{\mu} \notin O(\mu_1) \) and thus \( s(\tilde{\mu}) = (2,1) \). On the other hand, by the transfer property [8] we can assume that \( b, \mu_{2,b} \) and \( \{e_1, e_2, e_3\} \) are standard and therefore

\[
\begin{align*}
\tilde{\mu}(e_1, e_1) &\sim \mu_{2,b}(e_1, e_1), \\
\tilde{\mu}(e_1, e_3) &\sim \mu_{2,b}(e_1, e_3), \\
\tilde{\mu}(e_3, e_3) &\sim \mu_{2,b}(e_3, e_3),
\end{align*}
\]

and the result follows.

5. The algebras \( \mu_{2,0}, \mu_3, \mu_4, \mu_5 \) and \( \mu_6 \) can be perturbed over the laws of the family \( \{\mu_{2,b}\}_{b \neq 0} \). In order to obtain perturbed algebras isomorphic to the one of the family \( \{\mu_{2,b}\}_{b \neq 0} \), it is enough to consider the bilinear maps defined by

\[
\begin{align*}
\varphi_2(e_3, e_3) &= e_2, & \varphi_3(e_1, e_3) &= e_2, \\
\varphi_4(e_3, e_3) &= \varphi_3(e_1, e_3) = e_2, & \varphi_5(e_1, e_1) &= e_1,
\end{align*}
\]

and the laws of LeibN\( ^3 \) given by \( \mu_{2,0} + \varepsilon \varphi_2, \mu_i + \varepsilon \varphi_i \) for \( i = 3, 4, 5 \), where \( \varepsilon \sim 0 \) is non zero.

Analogously, we can show that the only contraction of \( \mu_3 \) and \( \mu_{2,0} \) are isomorphic to \( \mu_4 \) and \( \mu_6 \), and that the only contraction of \( \mu_4 \) and \( \mu_5 \) is \( \mu_6 \). We can summarize all these results in the following diagram, where the arrows represent contractions and hence the rigid elements are those for which no arrow finishes at them

![Diagram](image_url)

After this study, we can classify the components of the variety as follows
Theorem 2 The variety \( \text{LeibN}^3 \) is the union of the two irreducibles components \( \overline{O(\mu_1)^Z} \) and \( \bigcup_{b \in \mathbb{C}} \overline{O(\mu_{2,b})}^Z \).

Remark 1 In reference [1], the authors claim that the law \( \lambda_5 \) of \( \text{LeibN}^3 \) defined (over the basis \( \{x_1, x_2, x_3\} \)) by
\[
\lambda_5(x_2, x_2) = \lambda_5(x_3, x_2) = \lambda_5(x_2, x_3) = x_1,
\]
is rigid. Notice however that \( \lambda_5 \) is isomorphic to \( \mu_3 \) via the change of basis \( e_1 = x_2, e_2 = x_1 \) and \( e_3 = -ix_2 + ix_3 \). As \( \mu_3 \) can be perturbed into the family \( \{\mu_{2,b}\} \), such claim is not correct.

5 The reducibility of the varieties \( \text{LeibN}^n \) and \( \text{Leib}^n \)

Let \( \mu_0 \in \text{LeibN}^n \) be a law with characteristic sequence \( s(\mu_0) = (n) \). In that case there exists a basis \( \{e_1, \ldots, e_n\} \) of \( \mathbb{C}^n \) such that \( \mu_0(e_i, e_1) = e_{i+1} \) for \( i = 1, 2, \ldots, n-1 \). Once again, applying the Leibniz identity to \( (x, e_1, e_1) \) we obtain that \( R_{e_2} = 0 \). In fact every \( R_{e_k} = 0 \) for \( k \geq 2 \), as by induction \( \mu_0(x, e_{k+1}) = \mu_0(x, \mu(e_1, e_k)) = \mu_0(\mu_0(x, e_1), e_k) - \mu_0(\mu_0(x, e_k), e_1) = 0 \).

Proposition 1 Any nilpotent Leibniz algebra \( \mu \) of dimension \( n \) and characteristic sequence \( s(\mu) = (n) \) is isomorphic to \( \mu_0 \), where \( \mu_0(e_i, e_1) = e_{i+1} \) for \( i = 1, \ldots, n-1 \).

Remark 2 The law \( \mu \) satisfying \( \dim(\mathcal{O}^i(\mu)) - \dim(\mathcal{O}^{i+1}(\mu)) = 1 \) for every \( i = 1, \ldots, n \) is called null-filiform in reference [4].

As \( \mu_0 \) is the nilpotent Leibniz algebra with a maximal characteristic sequence, it has to be rigid and \( \overline{\mathcal{O}(\mu_0)^Z} \) is an irreducible component of the variety \( \text{LeibN}^n \). On the other hand if \( \mu \) is a non abelian Lie algebra, \( \dim(Z(\mu)) \leq n - 2 \) and \( \dim(Z_R(\mu_0)) = n - 1 \) implying that \( \mu \) cannot be a contraction of \( \mu_0 \). From this considerations we have the following theorem

Theorem 3 The variety \( \text{LeibN}^n \) for \( n \geq 3 \) is reducible.

Remark 3 \( \text{LeibN}^2 \) is irreducible and the only irreducible component is \( \overline{\mathcal{O}(\mu)^Z} \), where \( \mu \) is the law defined over the basis \( \{e_1, e_2\} \) by \( \mu(e_1, e_1) = e_2 \).

Let \( I = (\mathbb{C}^n, \mu) \) be a Leibniz algebra. It is clear that \( Z_R(\mu) \) is an ideal of \( I \) that contains the elements of the form \( \mu(x, y) + \mu(y, x), \mu(x, x) \) and \( \mu(\mu(x, y), \mu(y, x)) \) with \( x, y \in \mathbb{C}^n \). Thus \( I/Z_R(\mu) \) is a Lie algebra, which shows the following claim

Every Leibniz algebra which is not a Lie algebra verifies \( Z_R(\mu) \neq 0 \).

Theorem 4 A \( L^n \)-rigid Lie algebra without center is also rigid over \( \text{Leib}^n \).

Proof. Let \( \mu \) be a rigid Lie algebra without center. Let \( \tilde{\mu} \) be a perturbation of \( \mu \) in \( \text{LeibN}^n \). As \( Z(\mu) = 0 \) then \( Z_R(\tilde{\mu}) = 0 \) and \( \tilde{\mu} \in L^n \). By the rigidity of \( \mu \) over \( L^n \), \( \tilde{\mu} \) is isomorphic to \( \mu \).

Theorem 5 A Lie algebra with non null center cannot be rigid over \( \text{Leib}^n \).

Proof. Let \( \mu \) be a Lie algebra with non null center. We may assume \( n \) and \( \mu_0 \) standard. Let \( x \) be a generator of the Lie algebra, \( y \) a non zero vector of the center and \( \varphi \) the bilinear algebra such that its only non vanishing product is \( \varphi(x, x) = y \). Thus the perturbation \( \tilde{\mu} \) of \( \mu \) given by \( \tilde{\mu} = \mu + \varepsilon \varphi \) where \( \varepsilon \sim 0 \) is non zero, is a Leibniz algebra that is not a Lie algebra, then \( \tilde{\mu} \) cannot be isomorphic to \( \mu \).
Corollary 1 The variety $\text{Leib}^n$ is reducible for $n \geq 2$. In fact,
- $\text{Leib}^6$ has at least 5 irreducible components,
- $\text{Leib}^7$ has at least 8 irreducible components,
- $\text{Leib}^8$ has at least 33 irreducible components.
- $\text{Leib}^9$ has at least 41 irreducible components.

For $n \geq 81$, the number of irreducible components of $\text{Leib}^n$ is lower from below by $\Gamma(\sqrt{n})$, where $\Gamma$ is Euler gamma function (see [5] and [6]).

Remark 4 The variety $\text{Leib}^2$ is the union of the two irreducibles components $O(\varphi_1)^Z$ and $O(\varphi_2)^Z$, where the laws are defined, in the basis $\{e_1, e_2\}$ of $\mathbb{C}^2$, by $\varphi_1(e_1,e_2) = -\varphi_1(e_2,e_1) = e_2$ (Lie algebra) and $\varphi(e_2,e_1) = e_2$.

Acknowledgments

The first author is supported by the research project MTM2006-09152 of the Ministerio de Eucación y Ciencia. This work is a translation from the french version, made by the second author, of the paper Sur la Rédductibilité des Variétés des Lois d’Algèbres de Leibniz Complexes published at the J. Lie Theory 17 (2007), No. 3, 617–624.

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