Study of chiral and deconfinement transition in lattice QCD with improved staggered actions

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Abstract. We present results on the chiral and deconfinement properties of the QCD transition at finite temperature. We performed calculations using asqtad and HISQ/tree actions using lattices with temporal extent $N_\tau = 6, 8$ and $12$ allowing to control the approach to the continuum limit. We analyze the chiral transition in terms of universal $O(N)$ scaling functions. From peaks in the scaling functions we perform a simultaneous continuum extrapolation for HISQ/tree and asqtad to derive the critical temperature, $T_c = 157 \pm 6$ MeV.

1. Introduction

Improved staggered fermion actions are widely used to study QCD at non-zero temperatures and densities, see e.g. Ref. [1]. However, discretization effects for staggered fermion actions are quite large in the low temperature region. To control the discretization effects it is important to perform calculations for large values of temporal extent $N_\tau$ or use actions with smallest possible discretization effects. Here we report the study of the chiral and deconfinement aspects of the finite temperature transition using the asqtad and HISQ/tree action on lattices with temporal extent $N_\tau = 6, 8$ and $12$ with light quark masses $m_l = m_s/20$ with $m_s$ the physical strange quark mass. This corresponds to the lightest pion mass of about $160$ MeV. The lattice spacing was set using the static potential. The details of the lattice setup and analysis were in part presented in [2] and will be discussed in a forthcoming publication [3].

2. Chiral transition

For vanishing light quark masses there is a chiral phase transition which is expected to be second order and thus governed by universal $O(4)$ scaling. However, even for non-vanishing light quark masses, provided they are small enough, universal scaling allows to define pseudo-critical temperatures for the chiral transition. Thus when studying the chiral transition one first needs to establish that the lattice results can be described in terms of $O(4)$ scaling. In the staggered fermion formulation there is one further complication. Since this formulation only preserves a part of the chiral symmetry, the relevant universality class in the chiral limit for non-vanishing lattice spacing is actually...
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O(2). Fortunately, in the numerical analysis the differences between $O(2)$ and $O(4)$ universality classes are small so when referring to scaling we will use the term $O(N)$ scaling. Previous studies with the p4 action provided evidence for $O(N)$ scaling \[6, 7\]. Therefore we would like to check if $O(N)$ scaling works for asqtad and HISQ/tree action. In Fig. 1 we show our numerical results obtained with HISQ/tree action for the order parameter

$$M_b \equiv \frac{m_s \langle \bar{\psi} \psi \rangle}{T^4}. \quad (1)$$

For $N_\tau = 6$ and $N_\tau = 8$ we also made use of preliminary results at smaller quark mass $m_l = m_s/40$. We performed a fit of $M_b$ to the scaling form

$$M_b(T, m_l, m_s) = h^{1/3} f_G(t/h^{1/3}) + f_{M,reg}(T, H). \quad (2)$$

Here we defined $H = m_l/m_s$ and the reduced temperature $t$ and the magnetic field $h$

$$t = \frac{1}{t_0} \left( \frac{T - T_{c0}}{T_{c0}} \right), \quad h = \frac{1}{h_0} H. \quad (3)$$

$T_{c0}$ is the critical temperature in the chiral limit. Furthermore, $f_{M,reg}(T, H) = (a_t \Delta T + b_1) H$ parametrizes the contribution from the regular part of the free energy density. By performing a 5 parameter fit of the numerical results on $M_b$, i.e. treating $T_{c0}, t_0, h_0, a_t$ and $b_1$ as fit parameters we can describe the temperature and quark mass dependence of $M_b$. Both $O(4)$ and $O(2)$ scaling fits work well. The pseudo-critical temperature can be defined as peak position of the chiral susceptibility

$$\chi_{m,l} = \frac{\partial}{\partial m_l} \langle \bar{\psi} \psi \rangle_l, \quad q = l, s. \quad (4)$$

Since the scaling Ansatz describes the quark mass dependence and temperature dependence of $M_b$ and $\chi_{m,l}$ it can be used to determine the peak positions for different $N_\tau$ for the physical value of the light quark mass $m_l/m_s = 27.3$. Then performing a
combined $1/N_t^2$ extrapolation of $T_c$ values obtained with asqtad and HISQ/tree action as shown in Fig. 2 we obtain

$$T_c = (157 \pm 4 \pm 3 \pm 1) \text{ MeV},$$

where the first error is statistical, the second error is the systematic error due to continuum extrapolations and uncertainties related to the differences in $O(2)$ and $O(4)$ scaling fits, and the last error is the overall error on the lattice scale determination. To present a combined error we add the first two errors in quadrature and then add the third one. This gives $T_c = 157 \pm 6$ MeV.

### 3. Deconfinement transition

The Polyakov loop is an order parameter for the deconfinement transition in pure gauge theory, which is governed by $Z(N)$ symmetry. For QCD this symmetry is explicitly broken by dynamical quarks. There is no obvious reason for the Polyakov loop to be sensitive to the singular behavior close to the chiral limit although speculations along these lines have been made [5]. The Polyakov loop is related to the screening properties of the medium and thus to deconfinement. After proper renormalization, the square of the Polyakov loop characterizes the long distance behavior of the static quark anti-quark free energy; it gives the excess in free energy needed to screen two well-separated color charges. The renormalized Polyakov loop has been studied in the past in pure gauge theory [8, 9] as well as in QCD with two [10], three [11] and two plus one flavors [12, 13]. The renormalized Polyakov loop, calculated on lattices with temporal extent $N_t$, is obtained from the bare Polyakov

$$L_{ren}(T) = z(\beta)^{N_t} L_{bare}(\beta) = z(\beta)^{N_t} \left\{ \frac{1}{3} \text{Tr} \prod_{x_0=0}^{N_t-1} U_0(x_0, \vec{x}) \right\},$$

where $z(\beta) = \exp(-c(\beta)/2)$ and $c(\beta)$ is the additive normalization of the static potential chosen such that it coincides with the string potential at distance $r = 1.5r_0$ with $r_0$ being the Sommer scale. The numerical results for the renormalized Polyakov loop for the HISQ/tree action are shown in the right panel of Fig. 2 as function of $T/T_c$. As one can see from the figure the cutoff ($N_t$) dependence of the renormalized Polyakov loop is small. We also compare our results with the continuum extrapolated stout results [15] and the corresponding results in pure gauge theory [8, 9]. We find good agreement between our results and the stout results. We also see that in the vicinity of the transition temperature the behavior of the renormalized Polyakov loop in QCD and in the pure gauge theory is quite different.

### 4. Conclusions

We have studied the chiral and deconfinement aspects of the finite temperature transition in QCD. The chiral pseudo-critical temperature defined as peak of the chiral susceptibility for the physical quark masses was found to be $157 \pm 6$ MeV. The chiral
transition temperature obtained in Refs. [14, 15] using different observables was found to be in the range $147 - 157$ MeV in good agreement with our result. Previous attempt to determine the chiral transition temperature by RBC-Bielefeld collaboration resulted in too high value of $T_c = (192 \pm 7 \pm 4)$ MeV because no reliable continuum extrapolation can be done by using only $N_\tau = 4$ and 6 lattices. The renormalized Polyakov loop does not show rapid change in the vicinity of the chiral transition temperature and is quite different from the pure gauge theory result. Our present findings are very similar to the previous results obtained using the stout action [14, 15].

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$T_c \text{ [MeV]}$ vs $m_l/m_s = 1/20$

- asqtad, cubic O(4) scaling
- HISQ/tree, cubic O(4) scaling