Limits on a strongly interacting Higgs sector*

J. J. van der Bij

Institut für Physik, Albert-Ludwigs Universität Freiburg

H. Herderstr. 3, 79104 Freiburg i.B., Deutschland

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Using the classical argument about tree level unitarity breakdown in combination with the precision electroweak data, it is shown, that if part of the Higgs sector is strongly interacting, this part is small and is out of range of the LHC.

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I. INTRODUCTION

Recently some renewed interest in the possibility of a strongly interacting light Higgs sector has appeared. It was proposed to parametrize the effects of the strong sector by anomalous couplings arising in the form of higher dimensional operators at low energy. Of course these operators should come from an ultraviolet completion of the theory that gives rise to these effects. The most important operators acting only on the Higgs field itself are:

\[ \mathcal{O}_1 = \partial_\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi \Phi^\dagger). \]

\[ \mathcal{O}_2 = (\Phi \Phi^\dagger)^3. \]

These operators are automatically invariant under the custodial SU_L(2) × SU_R(2) symmetry, which is highly desirable on phenomenological grounds. The phenomenological effects are as follows. After a rescaling of the fields \( \mathcal{O}_1 \) gives rise to anomalous Higgs boson couplings, namely every coupling of standard model particles to the Higgs field is multiplied with a common factor. Furthermore \( \mathcal{O}_2 \) gives rise to a change in the Higgs selfcoupling. Measuring \( \mathcal{O}_2 \) would amount to measuring the Higgs selfcoupling, which is notoriously difficult at the LHC. In precision tests at LEP, the effects of \( \mathcal{O}_2 \) appear only at the two-loop level and happen to be actually finite, even though the full theory is non-renormalizable. The effect is however very small. In this paper we focus on \( \mathcal{O}_1 \), since the presence of this operator affects the precision electroweak variables at the one-loop level and can therefore be constrained more easily. Actually the theory with this operator is non-renormalizable and this shows up already at the one-loop level in the electroweak precision tests. The relevant corrections are logarithmically divergent. In order to realistically constrain the theory one therefore has to start with an ultraviolet completion, that gives rise to this operator at low energy.

II. THE HILL MODEL

Such a completion is given by the Hill model, which is actually the simplest possible renormalizable extension of the standard model, having only two extra parameters. The Hill model is described by the following Lagrangian:

\[ \mathcal{L} = -\frac{1}{2} (D_\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{2} (\partial_\mu H)^2 \]

\[ -\frac{\lambda_0}{8} (\Phi^\dagger \Phi - f_0^2)^2 - \frac{\lambda_1}{8} (2f_1 H - \Phi^\dagger \Phi)^2. \]

Working in the unitary gauge one writes \( \Phi^\dagger = (\sigma, 0) \), where the \( \sigma \)-field is the physical standard model Higgs field. Both the SM Higgs field \( \sigma \) and the Hill field \( H \) receive vacuum expectation values and one ends up with a two-by-two mass matrix to diagonalize, thereby ending with two masses \( m_- \) and \( m_+ \) and a mixing angle \( \alpha \). There are two equivalent ways to describe this situation. One is to say that one has two Higgs fields with reduced couplings \( g \) to standard model particles:

\[ g_- = g_{SM} \cos(\alpha), \quad g_+ = g_{SM} \sin(\alpha). \]

The standard model would correspond to \( \alpha = 0 \) with the light Higgs the standard model Higgs. The other way, which has some practical advantages is not to diagonalize the propagator, but simply keep the \( \sigma - \sigma \) propagator explicitly. One simply replaces the standard model Higgs propagator, in all calculations of experimental cross section, by:

\[ D_{\sigma \sigma}(k^2) = \cos^2(\alpha)/(k^2 + m_-^2) + \sin^2(\alpha)/(k^2 + m_+^2). \]

The generalization to an arbitrary set of fields \( H_k \) is straightforward, one simply replaces the singlet-doublet interaction term by:

\[ L_{H\Phi} = -\sum \frac{\lambda_i}{8} (2f_i H_k - \Phi^\dagger \Phi)^2. \]

For a finite number of fields \( H_k \) no essentially new aspects appear, however dividing the Higgs signal over even a small number of peaks, can make the detection of the Higgs at the LHC very challenging. Having an infinite number of Higgs fields one can also make a continuum, which would make the Higgs field effectively undetectable.

*for Chris Quigg’s 65th birthday
at the LHC. A mini-review of this type of models is given in [4]. There may even be evidence at LEP2 that this possibility is realized in nature [5]. For the purpose of this paper the precise form of the Higgs propagator is irrelevant. Important is that there is a light piece in the Higgs sector, which is weakly interacting and a heavy piece that is strongly interacting. For this purpose the simple Hill model is sufficient.

III. LIMITS FOR A STRONGLY INTERACTING HIGGS SECTOR

In order to determine whether the Higgs sector can become strongly interacting we adopt the classical analysis of [6, 7] to our case. The breakdown of tree level unitarity is used as a criterium for the presence or absence of strong interactions. Studying partial wave unitarity the adapted classical analysis from [6, 7] gives the limit:

$$\cos^2(\alpha) m_-^2 + \sin^2(\alpha) m_+^2 \leq \frac{8\pi\sqrt{2}}{3G_F}. \quad (8)$$

in order to have tree level unitarity. Since we demand that the Higgs sector becomes strongly interacting at high energies we demand that this bound is broken. For this to happen one needs a sufficiently large combination $m_+ \sin(\alpha)$. However one cannot have an arbitrarily large value here, since the correction to a typical electroweak precision observable $\delta_{EW}$ behaves like:

$$\delta_{EW} \approx \log(m^2_+/m^2_Z) + \sin^2(\alpha) \log(m^2_+/m^2_-). \quad (9)$$

This must then be smaller than the limit for the standard model

$$\delta_{EW} \leq \log(m^2_{up}/m^2_Z). \quad (10)$$

where $m_{up}$ is the upper limit for the Higgs boson mass. From the electroweak working group we take $m_{up} = 157$ GeV We define $x = m_+/m_-$ and $m_{min}$ the minimal allowed Higgs mass, which we take to be $m_{min} = 115$ GeV from the direct search. The expectation value $v$ of the Higgs field is given by $v^2 = G_F^{-1}/\sqrt{2} = (246$ GeV)$^2$. One then derives

$$\frac{x - 1}{\log(x)} \geq \frac{16\pi v^2}{3m_-^2 \log(m_{up}/m_-^2)}. \quad (11)$$

Taking $m_- = m_{min}$ one finds the weakest limits. With the above values one finds:

$$m_+ \geq 3285 \text{ GeV.} \quad (12)$$

and

$$\sin^2(\alpha) \leq 0.093. \quad (13)$$

So the lowest energy where one can find a strongly interacting part of the Higgs sector is at 3285 GeV with a production cross section of only 9.3% of the one for a standard model Higgs field with the same mass. This is clearly out of range for the capabilities of the LHC, since this heavy Higgs boson is produced too little because of its high mass and the reduced coupling to the standard model particles. Moreover it is also wide, so there is no clear signal above the background. The above analysis is of course somewhat simplified and could be improved in many ways, for instance improving the unitarity bound, applying more accurate formulas for the electroweak tests etc. However such refinements will not change the conclusion, that strong interactions can only play a very small part in the Higgs propagator in a very high energy region, that is out of the range of the LHC or any machine that is at present under consideration.

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[1] A. Pomarol; in XLIIInd Rencontres de Moriond, Electroweak interactions and unified theories, La Thuile 2007.
[2] J. J. van der Bij; Nucl. Phys. B 267, 557 (1986).
[3] A. Hill and J. J. van der Bij; Phys. Rev. D 36, 3463 (1987).
[4] J. J. van der Bij; Phys. Lett. B 636, 56 (2006).
[5] S. Dilcher and J. J. van der Bij; Phys. Lett. B 638, 234 (2006).
[6] B. W. Lee, C. Quigg and H. B. Thacker; Phys. Rev. Lett. 38, 883 (1977).
[7] B. W. Lee, C. Quigg and H. B. Thacker; Phys. Rev. D 16, 1519 (1977).