Quantum corrected gravitational potential beyond monopole-monopole interactions

G.P. de Brito,1 M.G. Campos,1 L.P.R. Ospedal,1 and K.P.B. Veiga2

1Centro Brasileiro de Pesquisas Físicas (CBPF), Rua Dr. Xavier Sigaud 150, Urca, Rio de Janeiro, Brazil, CEP 22290-180
2Instituto Federal da Bahia (IFBA) - Campus Simões Filho, Via Universitaria s/n, Pitanguinhas, Simões Filho, BA, Brazil, CEP 43700-000

Abstract

We investigate spin- and velocity-dependent contributions to the gravitational inter-particle potential. The methodology adopted here is based on the expansion of the effective action in terms of form factors encoding quantum corrections. Restricting ourselves to corrections up to the level of the graviton propagator, we compute, in terms of general form factors, the non-relativistic gravitational potential associated with the scattering of spin-0 and -1/2 particles. We discuss comparative aspects concerning different types of scattered particles and we also establish some comparisons with the case of electromagnetic potentials. Moreover, we apply our results to explicit examples of form factors based on non-perturbative approaches for quantum gravity. Finally, the cancellation of Newtonian singularity is analysed in the presence of terms beyond the monopole-monopole sector.

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I. INTRODUCTION

The current paradigm in the description of the gravitational interaction has foundation in Einstein’s general relativity (GR), that describes gravity as a classical field theory for the space-time dynamics. The other known fundamental interactions are very well described in terms of quantum field theory (QFT), culminating in the standard model of particle physics. Combining gravity with the other fundamental interactions remains as one of the most challenging tasks in theoretical physics. In particular, a completely (self-)consistent theory of quantum gravity is still missing.

Since the space-time metric plays the role of a dynamical variable in GR, a direct approach entails a QFT treatment to the quantization of metric fluctuation around a fixed background \([1, 2]\). This approach, sometimes referred as covariant quantum gravity, was readily identified as a problematic QFT due to appearance of ultraviolet (UV) divergences that could not be absorbed by standard (perturbative) renormalization techniques. This problem, however, should not be taken as a dead end for the covariant quantum gravity approach.

- The most immediate way out to this problem relies on the interpretation of this approach as an effective field theory (EFT) \([3]\), which provides a consistent framework for quantum gravity calculations valid below some cutoff scale \(\Lambda_{\text{QG}}\).

- The problem of perturbatively non-renormalizable interactions can be circumvented by the inclusion of curvature squared terms in the action describing the gravitational dynamics \([4]\). This approach, however, seems to imply unitarity violation (and instabilities, at the classical level) due to the appearance of higher-derivative terms. In the last few years, the interest in theories with higher curvature terms was renewed with some interesting ideas that might conciliate unitarity and (perturbative) renormalizability within this framework (see, for example, Refs. \([5–9]\)).

- Beyond the perturbative paradigm, the asymptotic safety program for quantum gravity \([10, 11]\) has been investigated as a candidate for a consistent UV complete scenario for covariant quantum gravity. In this context, UV completion is achieved as a consequence of quantum scale-symmetry emerging as result of a possible fixed point in the renormalization group flow. By now, there is vast collection of results indicating the
viability of this scenario \[2, 12\] including possible phenomenological consequences (see the reviews \[13, 14\] and references therein).

An interesting consistency check in quantum gravity models based on standard QFT techniques is the investigation of quantum corrections to the Newtonian potential. This question was originally addressed in the seminal paper by Donoghue within the EFT approach for quantum gravity \[3\]. Since then EFT and other methods have been used by several authors to carry out quantum gravitational corrections to the inter-particle potentials (see, for example, Refs. \[15–21\]). Although the usual research of non-relativistic potentials concentrates in the monopole-monopole sector, a series of works in the literature also consider the contributions of spin and velocity. In this case, spin-orbit and spin-spin interactions may appear. For instance, in Ref. \[22\] the authors calculated the potentials related to one-graviton exchanged between particles with different spins. Long-range gravitational potentials and its spin-dependent interactions were obtained in Refs. \[23–26\] by taking into account gravitational scattering at one-loop approximation within the EFT formalism. In a similar way, the spin contributions of one-loop diagrams with mixed gravitational-electromagnetic scattering were investigated in Refs. \[27, 28\]. For reviews of theoretical and experimental researches on the role of spin in gravity, we point out Refs. \[29, 30\].

In this work we investigate spin- and velocity-dependent contribution to the gravitational inter-particle potential within a framework motivated by quantum gravity models. Our main goal is to present a detailed discussion on the structure of possible quantum corrections to each sector beyond the monopole-monopole interaction. For this purpose, we combine the effective action formalism with an expansion in terms of form factors to introduce quantum corrections at the level of the graviton propagator. This strategy allows us to explore structural aspects of spin- and velocity-dependent contributions without relying in any specific perturbative calculation.

This paper is organized as follows: in Section \[III\] we present our methodology and carry out the inter-particle gravitational potentials for interactions involving spin-0 or spin-1/2 external particles in terms of general form factors. After that, we analyse each sector beyond monopole-monopole interaction and discuss the comparative aspects between spin-0 and spin-1/2 cases. In addition, we also establish comparisons with the inter-particle potentials mediated by electromagnetic interaction. In Section \[III\] we apply our results to particular examples motivated by non-perturbative approaches to quantum gravity. Next, in Section
In Section IV, we discuss some aspects related to the cancellation of Newtonian singularities in higher-derivative gravity models. Finally, in Section V, we present our concluding remarks and perspectives. In the Appendix, we display some useful integrals and definitions. Throughout this work, we adopted natural units where $\hbar = c = 1$, the Minkowski metric with signature $(+,−,−,−)$. The Riemann and Ricci curvature tensors were defined as

$$R^\mu_{\nu\alpha\beta} = \partial_\alpha \Gamma^\mu_{\nu\beta} + \Gamma^\mu_{\alpha\lambda} \Gamma^\lambda_{\nu\beta} - (\alpha \leftrightarrow \beta)$$

and

$$R_{\mu\nu} = R^\alpha_{\mu\alpha\nu},$$

respectively.

II. NON-RELATIVISTIC POTENTIALS

Let us initially introduce the methodology adopted for computing inter-particle potentials and present the approximations we are dealing with. In order to obtain spin- and velocity-dependent contributions to non-relativistic (NR) potentials mediated by gravity, we employ the first Born-approximation, namely

$$V(r) = -\int \frac{d^3 \vec{q}}{(2\pi)^3} M_{NR}(\vec{q}) e^{i\vec{q} \cdot \vec{r}},$$

(1)

where $M_{NR}(\vec{q})$ indicates the NR limit of the Feynman amplitude, $M$, associated with the process $1 + 2 \rightarrow 1' + 2'$ represented in Fig. 1. Following Ref. [31], we note that the NR limit involves an appropriate normalization factor such that

$$M_{NR}(\vec{q}) = \lim_{NR} \prod_{i=1,2} (2E_i)^{-1/2} \prod_{j=1,2} (2E'_j)^{-1/2} M(\vec{q}).$$

(2)

The most direct way to include quantum corrections to the NR gravitational potential relies on the perturbative approach. In this case, the amplitude associated with the process in Fig. 1 involves all the connected Feynman diagrams up to a fixed order in perturbation theory. This approach has been successfully applied to the computation of quantum corrections to the gravitational inter-particle potential in the context of EFT [3, 16–21, 23–26].

Alternatively, one can think in terms of the effective action formalism. In this case, the amplitude associated with the process represented in Fig. 1 can be constructed as a sum over connected “tree-level” diagrams with propagator and vertices extracted from the effective action $\Gamma$ (see Fig. 2). The typical evaluation of the effective action $\Gamma$ relies on perturbative methods and, therefore, produce equivalent results with respect to the approach described
Figure 1. Representation of a process with particles labeled by 1 and 2 scattering into final states labeled by 1' and 2'. The arrows indicate the momenta assignments adopted in this paper.

Figure 2. Diagrammatic representation of the full-contribution to the scattering process represented in Fig. 1 in terms of the effective action formalism. The “blobs” indicate full vertex and propagators derived from the effective action $\Gamma$.

The effective action formalism might be useful in order to access information beyond the perturbative approach. For example, in Ref. [32], Knorr and Saueressig proposed the reconstruction of an effective action for quantum gravity starting from non-perturbative data obtained via causal dynamical triangulation. Furthermore, the effective action can be expanded in terms of form factors carrying (non-)perturbative quantum corrections. For a recent discussion on form factors for quantum gravity in connection with functional renormalization group methods, see Refs. [33, 34].

In this paper we combine the effective action formalism with an expansion in terms of form factors in order to include quantum corrections on the NR inter-particle gravitational potential beyond monopole-monopole interactions. As a first approximation we include only
quantum corrections to the graviton propagator. In this case, the relevant contribution to
the process depicted in Fig. 1 corresponds to the diagram represented in Fig. 3. Within
this approximation, quantum corrections to the vertices and propagators associated with the
particles being scattered are not considered and the relativistic amplitude takes the form

\[ i\mathcal{M} = i T^{\mu\nu}(p_1, p'_1) \left< h_{\mu\nu}(-q) h_{\alpha\beta}(q) \right> i T^{\alpha\beta}(p_2, p'_2) \]  

(3)

where \( T^{\mu\nu} \) stands for the tree-level energy momentum tensor associated with the particles
being scattered and \( \left< h_{\mu\nu}(-q) h_{\alpha\beta}(q) \right> \) denotes the graviton full-propagator.

Figure 3. Diagrammatic representation of the approximation done in this paper. The arrow
indicate the momentum assignments adopted in the calculation of the scattering process.

Our purpose is not to compute the effective action for quantum gravity. Instead, we
assume a “template” for the effective action expanded in terms of form factors and motivated
by symmetry arguments. In general gauge theories, the effective action typically takes the
form \( \Gamma = \bar{\Gamma} + \hat{\Gamma} \), where \( \delta_{\text{gauge}} \bar{\Gamma} = 0 \) and \( \delta_{\text{gauge}} \hat{\Gamma} \neq 0 \). Nevertheless, the “symmetry breaking”
contribution \( \hat{\Gamma} \) is controlled by Slavnov-Taylor identities for \( \Gamma \). The covariant approach for
quantum gravity, thought as a QFT for the fluctuation field \( h_{\mu\nu} \) around a fixed background
with metric \( \bar{g}_{\mu\nu} \), can be faced as a gauge theory for diffeomorphism transformations. In this
case, a template for the effective action in quantum gravity should take the form

\[ \Gamma[h; \bar{g}] = \bar{\Gamma}[g] + \hat{\Gamma}[h; \bar{g}] , \]  

(4)

where \( \delta_{\text{diff}} \bar{\Gamma} = 0 \) and \( \delta_{\text{diff}} \hat{\Gamma} \neq 0 \). We note that the symmetric part, \( \bar{\Gamma}[g] \), depends only
on the full metric \( g_{\mu\nu} \), while the “symmetry breaking” sector presents separated dependence on \( \bar{g}_{\mu\nu} \) and \( h_{\mu\nu} \). In the present paper the fluctuation field \( h_{\mu\nu} \) was defined in terms
of the linear split \( g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \) (with \( \kappa = \sqrt{32\pi G} \)). In this case, there is an addi-
tional local symmetry, namely split symmetry, corresponding to the combined transforma-
tion \( \delta_{\text{split}} h_{\mu\nu}(x) = \kappa^{-1} \epsilon(x) \) and \( \delta_{\text{split}} \bar{g}_{\mu\nu}(x) = -\epsilon(x) \) that leaves the full metric invariant
\( \delta_{\text{split}} g_{\mu\nu} = 0 \) and, therefore, \( \delta_{\text{split}} \Gamma[g] = 0 \). However, the separated dependence of \( \bar{g}_{\mu\nu} \) and \( h_{\mu\nu} \) in \( \hat{\Gamma}[h; \bar{g}] \) implies \( \delta_{\text{split}} \Gamma[h; \bar{g}] \neq 0 \), leading to non-trivial Nielsen identities (or split Ward identities).

For the symmetric part, we consider a template for the effective action organized in terms of a curvature expansion, given by

\[
\hat{\Gamma}[g_{\mu\nu}] = \frac{2}{\kappa^2} \int d^4x \sqrt{-g} \left( -2\Lambda - R - \frac{1}{3}RF(\Box)R + C_{\mu\nu\alpha\beta}W(\Box)C^{\mu\nu\alpha\beta} \right) + O(R^3),
\]

where \( \Lambda \) and \( C_{\mu\nu\alpha\beta} \) denote the cosmological constant and Weyl tensor, respectively, while \( F(\Box) \) and \( W(\Box) \) correspond to form factors encoding quantum corrections contributing to the curvature squared sector. Furthermore, \( O(R^3) \) indicate all other contributions composed by curvature invariants with power larger than two. For the explicit computations performed in this paper, we have considered flat background metric, i.e., \( g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \). In this case, the relevant contributions for the full graviton propagator comes exclusively from terms up to \( O(R^2) \).

For the symmetry breaking sector, we use a template with the same functional form as a typical gauge fixing term added to classical action, namely

\[
\hat{\Gamma}[h_{\mu\nu}; \bar{g}] = \frac{1}{2\alpha} \int d^4x \sqrt{-g} \bar{g}^{\mu\nu}F_{\mu}[h; \bar{g}]F_{\nu}[h; \bar{g}],
\]

where \( F_{\mu}[h; \bar{g}] = \nabla^{\nu}h_{\mu\nu} - \frac{1}{2}\nabla^2_{\mu}h \). In a complete analysis the symmetry breaking contribution should be confronted with the appropriate Slavnov-Taylor and Nielsen identities derived from diffeomorphism invariance and split symmetry, respectively. In this sense, one can consider our approach for \( \hat{\Gamma}[h; \bar{g}] \) as a further approximation in our investigation. Nonetheless, one can argue that different choices in this sector would not affect our results since we are computing amplitudes with on-shell external legs.

Bearing in mind our template for the effective action, the graviton “full”-propagator (around flat background) can be readily computed as the inverse of the 2-point function \( \delta^2 \Gamma/\delta h^2 |_{h=0} \), resulting in the following expression

\[
\langle h_{\mu\nu}(-q)h_{\alpha\beta}(q) \rangle = \frac{i}{q^2} \left[ \frac{1}{Q_2(q^2)} P^{(2)}_{\mu\nu\alpha\beta} - \frac{1}{2Q_0(q^2)} P^{(0)}_{\mu\nu\alpha\beta} \right] + i\Delta_{\mu\nu\alpha\beta}(q),
\]

where we have defined

\[
Q_2(q^2) = 1 + \frac{2\Lambda}{q^2} + 2q^2 W(-q^2),
\]

\( q^2 = q^\mu q^\nu g_{\mu\nu} \).
\begin{equation}
Q_0(q^2) = 1 + \frac{2\Lambda}{q^2} + 2q^2 F(-q^2). \tag{8b}
\end{equation}

In addition, the tensor structures \( P^{(2)}_{\mu\nu\alpha\beta} \) and \( P^{(0)}_{\mu\nu\alpha\beta} \) were defined as

\begin{equation}
P^{(2)}_{\mu\nu\alpha\beta} = \frac{1}{2} (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha}) - \frac{1}{3} \eta_{\mu\nu} \eta_{\alpha\beta}, \tag{9a}
\end{equation}

\begin{equation}
P^{(0)}_{\mu\nu\alpha\beta} = \frac{1}{3} \eta_{\mu\nu} \eta_{\alpha\beta}. \tag{9b}
\end{equation}

The remaining terms in the graviton propagator, represented by \( i\Delta_{\mu\nu\alpha\beta}(q) \), vanish when contracted with the energy-momentum tensor of the scattered particles.

It is worthwhile mentioning that using the effective action (5), where form factors \( F(\Box) \) and \( W(\Box) \) were introduced with scalar curvature and Weyl tensor, we obtain the propagator (7) in which the contributions of these form factors are disconnected. In other words, from Eqs. (8a) and (8b), we observe that \( F(\Box) \) and \( W(\Box) \) contribute only to scalar and graviton modes, respectively.

In what follows we present our results for the NR gravitational potential, taking into account the scattering of both massive spin-0 and spin-1/2 particles, with quantum corrections being included in terms of general form factors \( F(\Box) \) and \( W(\Box) \). As usually done in the literature of spin- and velocity-dependent potentials, we adopt the center-of-mass (CM) reference frame, described in terms of the 3−momentum transfer \( \vec{q} \) and average momentum \( \vec{p} \). The CM variables are related to the momentum assignments depicted in Fig. 3 in terms of the following expressions

\begin{equation}
\vec{p}_1 = -\vec{p}_2 = \vec{p} - \frac{\vec{q}}{2}, \quad \vec{p}'_1 = -\vec{p}'_2 = \vec{p} + \frac{\vec{q}}{2}. \tag{10}
\end{equation}

We also consider an elastic scattering, \( q' = (0, \vec{q}) \). Therefore, we have \( \vec{p}' \cdot \vec{q} = 0 \), \( E_1 = E'_1 \) and \( E_2 = E'_2 \).

A. Spin-0 external particles

Within the working setup above described, we first investigate the case of gravitationally interacting spin-0 particles. The investigation performed in this paper takes into account an approximation where the full vertices, derived from the effective action \( \Gamma \), are identified with the tree-level vertices appearing in the classical action of minimally interacting gravity-matter systems. In this approximation, the relevant object to compute scattering amplitude
is the energy-momentum tensor associated with the scattered particles (see Eq. (3)). For scalar particles minimally interacting with gravity we find

$$T_{\mu\nu}(p, p') = -\frac{\kappa}{2} \left( p_\mu p'_\nu + p_\nu p'_\mu - \eta_{\mu\nu} (p \cdot p' - m^2) \right). \quad (11)$$

We have adopted conventions where the momenta \( p \) and \( p' \) were respectively assigned as incoming and outgoing with respect to the vertex.

The relativistic scattering amplitude can be readily computed in terms of Eq. (3) along with Eqs. (7) and (11). In fact, using the conservation of the energy-momentum tensor \( q_\mu T^{\mu\nu}(p_i, p'_i) = 0 \), for on-shell in- and out-states we arrive at the intermediary result

$$i\mathcal{M}^{(s=0)} = \frac{i}{q^2} \left[ \left( \frac{1}{3Q_2} + \frac{1}{6Q_0} \right) T_1^\mu T_2^\beta - \frac{1}{Q_2} T_1^{\mu\nu} T_2^{\mu\nu} \right], \quad (12)$$

where we have adopted the shorthand notations \( T_i^{\mu\nu} \equiv T^{\mu\nu}(p_i, p'_i) \) and \( Q_i \equiv Q_i(q^2) \). After some simple algebraic manipulations using the explicit expression for the energy-momentum tensor we find the following result for the scattering amplitude

$$\mathcal{M}^{(s=0)} = \frac{\kappa^2}{6q^2 Q_2} \left( 2m_1^2 m_2^2 - 3(p_1 \cdot p_2)(p'_1 \cdot p'_2) - 3(p_1 \cdot p'_2)(p'_1 \cdot p_2) + 2(p_1 \cdot p'_1)(p_2 \cdot p'_2) - m_1^2 p_2 \cdot p'_2 - m_2^2 p_1 \cdot p'_1 \right) + \frac{\kappa^2}{6q^2 Q_0} \left( (p_1 \cdot p'_1)(p_2 \cdot p'_2) - 2m_1^2 p_2 \cdot p'_2 - 2m_2^2 p_1 \cdot p'_1 + 4m_1^2 m_2^2 \right). \quad (13)$$

In order to obtain the NR description, we use the prescription (2). In the CM reference frame with momentum attributions (10), we have

$$\mathcal{M}^{(s=0)}_{NR} = \frac{\kappa^2 m_1 m_2}{6 Q_2 q^2} \left\{ 1 + \tilde{p}^2 \left( \frac{3}{m_1 m_2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) + \frac{q^2}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \right\} - \frac{\kappa^2 m_1 m_2}{24 Q_0 q^2} \left\{ 1 - \frac{\tilde{p}^2}{2} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) - \frac{5q^2}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \right\}. \quad (14)$$

Finally, by taking the Fourier integral, Eq. (11), we promptly obtain the inter-particle gravitational potential with contributions beyond the monopole-monopole sector

$$V^{(s=0)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left\{ I_1^{(2)}(r) + \tilde{p}^2 \left( \frac{3}{m_1 m_2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) I_1^{(2)}(r) \right\} + \frac{1}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) I_1^{(2)}(r) + \frac{\kappa^2 m_1 m_2}{24} \left\{ I_1^{(0)}(r) - \frac{\tilde{p}^2}{2} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) I_1^{(0)}(r) - \frac{5}{8} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) I_1^{(0)}(r) \right\}, \quad (15)$$
where the integrals $I_n^{(a)}(r)$ are defined in Appendix, Eq. [A1] with $n = 0, 1$ and $a = 0, 2$.

### B. Spin-1/2 external particles

In present subsection, we describe the gravitational interaction between two spin-1/2 particles. The calculation follows the same line as in the case of spin-0 and the relevant energy-momentum tensor appearing in the scattering amplitude is given by

$$T_{\mu\nu}(p, p') = \kappa \left( 2\eta_{\mu\nu}((p + p')_\alpha J^\alpha(p, p') - 2m\rho(p, p')) - (p + p')_\mu J_\nu(p, p') - (p + p')_\nu J_\mu(p, p') \right),$$  \hspace{1cm} (16)

where we have defined the bi-linear structures $J^\mu(p, p') = \bar{u}(p')\gamma^\mu u(p)$ and $\rho(p, p') = \bar{u}(p')u(p)$. Combining Eqs. (3) and (7) we arrive in a similar expression as in the case of spin-0 particles, namely

$$iM^{(s=1/2)} = i\frac{\kappa}{q^2} \left[ \left( \frac{1}{3Q_2} + \frac{1}{6Q_0} \right) T_{1\mu}T_{2\beta} - \frac{1}{Q_2} T_{1\mu\nu}T_{2\mu\nu} \right],$$  \hspace{1cm} (17)

Expanding the energy-momentum tensor in terms of the bi-linears $J^\mu$ and $\rho$, we find the relativistic scattering amplitude

$$M^{(s=1/2)} = \frac{\kappa^2}{q^2Q_2} \left\{ \frac{1}{16} (p_1 + p'_1)_\mu (p_2 + p'_2)_\nu J^\mu J^\nu - \frac{m_1}{8} \rho_1(p_2 + p'_2)J^\mu - \frac{m_2}{8} \rho_2(p_1 + p'_1)J^\mu 
- \frac{1}{32} (p_1 + p'_1)_\mu (p_2 + p'_2)_\nu J^\mu J^\nu - \frac{1}{32} (p_1 + p'_1)_\mu (p_2 + p'_2)_\nu J^\mu J^\nu + \frac{m_1m_2}{3}\rho_1\rho_2 \right\}$$

$$+ \frac{\kappa^2}{q^2Q_0} \left\{ \frac{3}{32} (p_1 + p'_1)_\mu (p_2 + p'_2)_\nu J^\mu J^\nu + \frac{2m_1m_2}{3}\rho_1\rho_2 
- \frac{m_1}{4} \rho_1(p_2 + p'_2)_\mu J^\mu - \frac{m_2}{4} \rho_2(p_1 + p'_1)_\mu J^\mu \right\},$$  \hspace{1cm} (18)

where we defined the shorthand notation $\rho_j = \rho(p_j, p'_j)$ and $J_j^\mu = J^\mu(p_j, p'_j)$.

In order to extract the NR scattering amplitude, we first remember that the (on-shell) spinor coefficient $u(p)$ can be expressed in the standard Dirac representation as follows

$$u(p) = \sqrt{E + m} \begin{pmatrix} \xi \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + m} \xi \end{pmatrix}.$$

\hspace{1cm} (19)
with $\xi$ and $\bar{\sigma}$ being the basic spinor and Pauli matrices, respectively. In the NR limit, the relevant bi-linear structures $\rho$ and $\mathcal{J}^\mu$ can be written as (in the CM frame)

$$
\rho_j|_{\text{NR}} = 2 m_j \left[ 1 + \frac{1}{4m_j^2} \left( \frac{1}{2} \vec{q}^2 - i(\vec{q} \times \vec{p}) \cdot \langle \bar{\sigma}(j) \rangle \right) \right],
$$

$$(20a)$$

$$
\mathcal{J}^0_j|_{\text{NR}} = 2 m_j \left[ 1 + \frac{1}{4m_j^2} \left( 2\vec{p}^2 + i(\vec{q} \times \vec{p}) \cdot \langle \bar{\sigma}(j) \rangle \right) \right],
$$

$$(20b)$$

$$
\bar{\mathcal{J}}_j|_{\text{NR}} = 2 m_j \chi_j \left[ \frac{\vec{p}}{m_j} - \frac{i}{2m_j}(\vec{q} \times \langle \bar{\sigma}(j) \rangle) \right],
$$

$$(20c)$$

where $j$ indicates the particle label and we have defined $\chi_1 = 1$, $\chi_2 = -1$ and $\langle \bar{\sigma}(j) \rangle = \xi_j^\dagger \bar{\sigma} \xi_j$. In addition, factors of $\xi_j^\dagger \xi_j$ have been omitted.

After some algebraic manipulations, we find that

$$
\mathcal{M}_{\text{NR}}^{(s=1/2)} = \frac{\kappa^2 m_1 m_2}{6Q_2 q^2} \left\{ 1 + \vec{p}^2 \left( \frac{3}{m_1 m_2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \right. 
+ i \left[ \left( \frac{1}{m_1^2} + \frac{3}{4 m_1 m_2} \right) \vec{S}_1 + \left( \frac{1}{m_2^2} + \frac{3}{2 m_1 m_2} \right) \vec{S}_2 \right] \cdot (\vec{q} \times \vec{p}) 
- \frac{3}{4} \frac{\vec{q}^2}{m_1 m_2} \vec{S}_1 \cdot \vec{S}_2 + \frac{3}{4} \frac{1}{m_1 m_2} \left( \vec{q} \cdot \vec{S}_1 \right) \left( \vec{q} \cdot \vec{S}_2 \right) \right\} 
- \frac{\kappa^2 m_1 m_2}{24 Q_0 q^2} \left\{ 1 - \frac{\vec{p}^2}{2} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) - \frac{i}{2} \left[ \frac{1}{m_1^2} \vec{S}_1 + \frac{1}{m_2^2} \vec{S}_2 \right] \cdot (\vec{q} \times \vec{p}) \right\},
$$

$$(21)$$

where $\vec{S}_j \equiv \frac{1}{2}(\vec{\sigma}(j))$ denotes the spin of a particle.

The NR gravitational potential associated with the scattering of spin-1/2 particles can be readily obtained by performing the Fourier integral (II), resulting in the following expression

$$
V^{(s=1/2)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left\{ I_1^{(2)}(r) + \vec{p}^2 \left( \frac{3}{m_1 m_2} + \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) I_1^{(2)}(r) \right. 
+ \left[ \left( \frac{1}{m_1^2} + \frac{3}{2 m_1 m_2} \right) \vec{S}_1 + \left( \frac{1}{m_2^2} + \frac{3}{2 m_1 m_2} \right) \vec{S}_2 \right] \cdot \frac{\vec{L}}{r} \frac{d}{dr} I_1^{(2)}(r) 
- \frac{3}{4} \frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2} I_0^{(2)}(r) + \frac{3}{4} \sum_{i,j=1}^3 \frac{(\vec{S}_1)_i (\vec{S}_2)_j}{m_1 m_2} I_{ij}^{(2)}(r) \right\} 
+ \frac{\kappa^2 m_1 m_2}{24} \left\{ I_1^{(0)}(r) - \frac{\vec{p}^2}{2} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) I_1^{(0)}(r) - \frac{1}{2} \left[ \frac{\vec{S}_1}{m_1^2} + \frac{\vec{S}_2}{m_2^2} \right] \cdot \frac{\vec{L}}{r} \frac{d}{dr} I_1^{(0)}(r) \right\},
$$

$$(22)$$

where $\vec{L} = \vec{r} \times \vec{p}$ stands for the orbital angular momentum and the anisotropic integral $I_{ij}^{(2)}(r)$ is defined in the Appendix (see Eq. (A2)).
C. Comparative aspects of spin- and velocity-dependent potentials

At this stage, it is relevant to compare structural aspects of the potentials for spin-0 and spin-1/2 cases. First of all, we note that the potential for spin-0 particles is characterized by two different sectors, monopole-monopole and velocity-velocity contributions, namely

\[
V_{\text{mon-mon}}^{(s=0)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left( I_1^{(2)}(r) - \frac{1}{4} I_1^{(0)}(r) \right) - \frac{\kappa^2 m_1 m_2}{48} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left[ I_0^{(2)}(r) + \frac{5}{4} I_0^{(0)}(r) \right],
\]

(23a)

\[
V_{\text{vel-vel}}^{(s=0)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \bar{p}^2 \left\{ \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left[ I_1^{(2)}(r) + \frac{1}{8} I_1^{(0)}(r) \right] + \frac{3}{m_1 m_2} I_1^{(2)}(r) \right\}.
\]

(23b)

The potential associated with spin-1/2 particles, on the other hand, receives contributions from four different sectors: monopole-monopole, velocity-velocity, spin-orbit and spin-spin interactions. These contributions are given by

\[
V_{\text{mon-mon}}^{(s=1/2)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left( I_1^{(2)}(r) - \frac{1}{4} I_1^{(0)}(r) \right),
\]

(24a)

\[
V_{\text{vel-vel}}^{(s=1/2)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \bar{p}^2 \left\{ \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left[ I_1^{(2)}(r) + \frac{1}{8} I_1^{(0)}(r) \right] + \frac{3}{m_1 m_2} I_1^{(2)}(r) \right\},
\]

(24b)

\[
V_{\text{spin-orbit}}^{(s=1/2)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left[ \left( \frac{1}{m_1^2} + \frac{3}{2 m_1 m_2} \right) \bar{S}_1 + \left( \frac{1}{m_2^2} + \frac{3}{2 m_1 m_2} \right) \bar{S}_2 \right] \cdot \frac{\vec{L}}{r} \frac{d}{dr} I_1^{(2)}(r)
- \frac{\kappa^2 m_1 m_2}{48} \left( \frac{1}{m_1^2} \bar{S}_1 + \frac{1}{m_2^2} \bar{S}_2 \right) \cdot \frac{\vec{L}}{r} \frac{d}{dr} I_1^{(0)}(r),
\]

(24c)

\[
V_{\text{spin-spin}}^{(s=1/2)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left[ -\frac{3}{4} \frac{\bar{S}_1 \cdot \bar{S}_2}{m_1 m_2} I_0^{(2)}(r) + \frac{3}{4} \sum_{i,j=1}^{3} \frac{(\bar{S}_1)_i (\bar{S}_2)_j}{m_1 m_2} I_0^{(2)}(r) \right].
\]

(24d)

We first note that the static limit is obtained by taking the combined limit

\[
\frac{1}{m_1 m_2} V_{\text{stat}}^{(s)}(r) = \lim_{\bar{p} \to 0, m_i \to \infty} \frac{1}{m_1 m_2} V^{(s)}(r).
\]

(25)
In this case, the only remaining contribution comes from the monopole-monopole sector, which results in

\[ V_{\text{stat.}}^{(s)}(r) = -\frac{\kappa^2 m_1 m_2}{6} \left( I_1^{(2)}(r) - \frac{1}{4} I_1^{(0)}(r) \right), \]

both for spin-0 and spin-1/2 particles. In the particular case of vanishing form factors, i.e. without deviations from the classical Einstein-Hilbert action, we recover the usual Newtonian potential

\[ V_{\text{stat.}}^{(s)}(r) = -\frac{\kappa^2 m_1 m_2}{32\pi r} \equiv -\frac{Gm_1 m_2}{r}. \]

Moving away from the static regime we note the similarities and differences between spin-0 and spin-1/2 cases. The monopole-monopole sectors, Eqs. (23a) and (24a), contain universal contributions appearing both in the spin-0 and spin-1/2. However, as we can observe from Eq. (23a), \( V_{\text{mon-mon}}^{(s=0)}(r) \) has an additional term which is not present in \( V_{\text{mon-mon}}^{(s=1/2)}(r) \). This additional term has a sub-leading behavior as we are going to see in the next section from explicit examples.

Beyond the monopole-monopole terms, we observe that the velocity-dependent sector \( V_{\text{vel-vel}}^{(s)}(r) \) has the same form both for spin-0 and spin-1/2 cases. On the other hand, spin-orbit and spin-spin interactions are present only in the potential associated with spin-1/2 particles. While spin-orbit terms (\( \sim \vec{L} \cdot \vec{S}_i \)) interact via spin-2 and spin-0 graviton modes, spin-spin contributions (\( \sim (\vec{S}_1)_i (\vec{S}_2)_j I_{ij}^{(2)} \) and \( \sim \vec{S}_1 \cdot \vec{S}_2 \)) exhibit only interactions via spin-2 graviton modes.

It is worthy to highlight that our methodology can be applied to modified (classical) theories of gravity with higher-order derivatives and other non-local functions. Once we have developed the potentials with arbitrary form factors, we just need to reinterpret the effective action as a classical one and redefine the \( Q_0 \) and \( Q_2 \) factors. We shall return to this point in Section IV. Furthermore, we comment that, for the gravitational interaction of spin-0 particles, it is possible to generalize our results to arbitrary dimensions, as already discussed in the literature for modified theories of gravity in monopole-monopole sector (see [35, 36] and references therein). However, for spin-\( \frac{1}{2} \) case and its spin-dependent contributions, this extension shall be a non-trivial task, since the definition of spin is particular to the dimension we are dealing with. For instance, when considering space-times with odd dimensions and parity symmetry (typically to electromagnetic and gravitational interactions), a reducible representation can be adopted in order to conciliate the parity symmetry with massive fermions. In these cases, new spin-dependent effects have been discussed [37, 38]. In other
words, the inclusion of the spin-dependent interactions should be carefully done for each particular dimension, especially when discrete symmetries are desired.

Before we proceed with specific form factors motivated by quantum gravity models, it is interesting to compare our results with the case of NR potentials mediated by electromagnetic interaction. Adopting the same strategy as in the gravitational case, we consider the following template for the electromagnetic effective action

$$\Gamma_{EM}[A] = -\frac{1}{4} \int d^4x F_{\mu\nu}(1 + H(\Box))F^{\mu\nu} - \frac{1}{2\alpha} \int d^4x (\partial_\mu A^\mu)^2 + \mathcal{O}(F^3),$$

(27)

where $H(\Box)$ denotes a form factor modeling quantum corrections up to $\mathcal{O}(A^2)$. In this case, the photon “full”-propagator can be parameterized as

$$\langle A_\mu(-q)A_\nu(q)\rangle = -\frac{i}{q^2(1 + H(-q^2))}\eta_{\mu\nu} + i\Delta_{\mu\nu}(q),$$

(28)

where $\Delta_{\mu\nu}(q)$ indicates those contributions that vanishes when contracted with external vector currents. The photon propagator can be easily mapped in terms of quantities defined in Ref. [39]. Therefore, we can readily import the results from [39], leading to the following expressions

$$V_{EM, \text{mon-mon}}^{(s=0)}(r) = e_1 e_2 I_{1EM}^0(r),$$

(29a)

$$V_{EM, \text{vel-vel}}^{(s=0)}(r) = \frac{e_1 e_2}{m_1 m_2} \vec{p}^2 I_{1EM}^0(r),$$

(29b)

for spin-0 particles, and

$$V_{EM, \text{mon-mon}}^{(s=1/2)}(r) = e_1 e_2 \left[ I_{1EM}^0(r) - \frac{1}{8} \left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right) I_{0EM}^0(r) \right],$$

(30a)

$$V_{EM, \text{vel-vel}}^{(s=1/2)}(r) = \frac{e_1 e_2}{m_1 m_2} \vec{p}^2 I_{1EM}^0(r),$$

(30b)

$$V_{EM, \text{spin-orbit}}^{(s=1/2)}(r) = e_1 e_2 \left[ \left(\frac{1}{2m_1^2} + \frac{1}{m_1 m_2}\right) \vec{S}_1 + \left(\frac{1}{2m_2^2} + \frac{1}{m_1 m_2}\right) \vec{S}_2 \right] \cdot \frac{\vec{L}}{r} \frac{d}{dr} I_{1EM}^0(r),$$

(30c)

$$V_{EM, \text{spin-spin}}^{(s=1/2)}(r) = e_1 e_2 \left[ -\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2} I_{0EM}^0(r) + \sum_{i,j=1}^{3} \frac{(\vec{S}_1)_i (\vec{S}_2)_j}{m_1 m_2} I_{ij}^{EM}(r) \right],$$

(30d)
in the case of spin-1/2 particles. The integrals $I_n^{EM}(r)$ and $I_{ij}^{EM}(r)$ follow the same definition as Eqs. (A1) and (A2), but replacing $Q_a(\vec{q}^2)$ by $1 + H(\vec{q}^2)$.

As we can observe, the NR potentials mediated by electromagnetic interaction present some similarities in comparison with the gravitational case. In the monopole-monopole sector, Eqs. (29a) and (30a), we note the appearance of universal leading order contributions (terms involving $I_1^{EM}(r)$) both in the case of spin-0 and spin-1/2 scattered particles. On the other hand, in contrast with the gravitational case, the additional non-universal contribution (involving $I_0^{EM}(r)$) appears only in the spin-1/2 case.

Beyond the monopole-monopole contribution, we first note that in the velocity-velocity sector, as in the gravitation case, exhibits the same result both for spin-0 and spin-1/2 particles. For spin-orbit and spin-spin contributions, only present in the case of spin-1/2 particles, we observe the same kind of interaction structures (terms with $\vec{S}_1 \cdot \vec{L}$, $\vec{S}_1 \cdot \vec{S}_2$ and $(\vec{S}_1)_i (\vec{S}_2)_j I_{ij}^{(2)}$) both for electromagnetic and gravitational potentials.

III. FORM FACTORS MOTIVATED BY QUANTUM GRAVITY MODELS

The results presented in the previous section carry some model independent features at the level of the graviton propagator. It allows to study structural aspects of quantum contributions to the gravitational potential beyond the monopole-monopole sector. However, a more detailed analysis depends on the evaluation of basic integrals defined in Appendix A for specific form factors. In what follows, we work out some examples with form factors motivated by recent investigations in the context of non-perturbative approaches for quantum gravity.

A. Form factors motivated by CDT data

As a first example we consider form factors motivated by an approach of reconstruction of the effective action for quantum gravity based in data obtained via Causal Dynamical Triangulation (CDT). In Ref. [32], the authors put forward a reverse engineered procedure
to reconstruct the effective action starting from an Euclidean template of the form

$$\Gamma = \frac{2}{\kappa^2} \int d^4x \sqrt{g} \left( 2\Lambda - R - \frac{b^2}{6} R \Box^{-2} R - \frac{\tilde{b}^2}{6} C_{\mu\nu\alpha\beta} \Box^{-2} C^{\mu\nu\alpha\beta} \right),$$  \number{31}

and adjusting the free parameters $b$ and $\tilde{b}$ by matching the autocorrelation of the 3-volume operator with data from CDT. It is interesting to mention that the same class of effective action has been motivated by cosmological considerations. In fact, in Ref. \cite{40} the authors proposed an effective model with non-localities of the type $R \Box^{-2} R$ as an alternative model for dark energy. It can be found an extended version involving non-local term of the type $C_{\mu\nu\alpha\beta} \Box^{-2} C^{\mu\nu\alpha\beta}$ in Ref. \cite{41}. For an up-to-date overview on the various aspects of cosmological evolution driven by this class of non-localities see Ref. \cite{42}.

In this paper we consider form factors corresponding to the effective action given by Eq. \number{31}, namely

$$F(\Box) = -\frac{\rho_0}{\Box^2}, \quad \text{and} \quad W(\Box) = -\frac{\rho_2}{\Box^2},$$  \number{32}

with $\rho_0$ and $\rho_2$ being positive parameters. Before we proceed, it is interesting to clarify some points regarding this form factors. First of all, we note that the reconstruction approach proposed in \cite{32}, in the context of this paper, was simply used as a motivation for choosing the functional form of $F(\Box)$ and $W(\Box)$. In this sense, we will not impose any restriction on the parameters $\rho_0$ and $\rho_2$ coming from the matching template approach discussed in Ref. \cite{32}. It all important to point out that the effective action in Eq. \number{31} was written according to Euclidean signature and the passage to the Lorentzian signature was done by means of “naive” Wick rotation. We should emphasize, however, that a completely well defined Wick rotation in quantum gravity remains as an open problem and will not be addressed here.

Taking into account the class of form factors introduced above, as well as the definition of the Q-factors defined in Eqs. \number{8a} and \number{8b}, the relevant integrals contributing to the NR gravitational potential are given by

$$I_1^{(s)}(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{q^2 + \mu_s^2} e^{i\vec{q} \cdot \vec{r}} = \frac{e^{-\mu_s r}}{4\pi r},$$  \number{33a}

$$I_0^{(s)}(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{q^2}{q^2 + \mu_s^2} e^{i\vec{q} \cdot \vec{r}} = \delta^{3}(\vec{r}) - \mu_s^2 \frac{e^{-\mu_s r}}{4\pi r},$$  \number{33b}
\[ I^{(s)}_{ij}(r) = \int \frac{d^3q}{(2\pi)^3} \frac{q_i q_j}{q^2 + \mu_s^2} e^{iq \cdot r} \]
\[ = \frac{1}{3} \delta_{ij} \delta^3(\vec{r}) + \left\{ (1 + \mu_s r) \delta_{ij} - (3 + 3\mu_s r + \mu_2^2 r^2) \frac{x_i x_j}{r^2} \right\} e^{-\mu_s r} \frac{1}{4\pi r^3}, \quad (33c) \]

where we have defined \( \mu_s^2 = 2(\rho_s - \Lambda) \). We shall consider \( \rho_s > \Lambda \) such that the non-local form factors \( (32) \) introduce mass terms in the graviton propagator. The resulting contributions to the NR potential can be written as follows (throwing away Dirac delta terms)

\[ V^{(s=0)}_{\text{mon-mon}}(r) = -\frac{\kappa^2 m_1 m_2}{24\pi r} \left( e^{-\mu_2 r} - \frac{1}{4} e^{-\mu_0 r} \right) + \frac{\kappa^2 m_1 m_2}{192\pi r} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( \mu_2^2 e^{-\mu_2 r} + \frac{5}{4} \mu_0^2 e^{-\mu_0 r} \right), \quad (34a) \]

\[ V^{(s=0)}_{\text{vel-vel}}(r) = -\frac{\kappa^2 m_1 m_2}{24\pi r} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( e^{-\mu_2 r} + \frac{1}{8} e^{-\mu_0 r} \right) + \frac{3}{m_1 m_2} e^{-\mu_2 r}, \quad (34b) \]

for spin-0 particles, and

\[ V^{(s=1/2)}_{\text{mon-mon}}(r) = -\frac{\kappa^2 m_1 m_2}{24\pi r} \left( e^{-\mu_2 r} - \frac{1}{4} e^{-\mu_0 r} \right), \quad (35a) \]

\[ V^{(s=1/2)}_{\text{vel-vel}}(r) = -\frac{\kappa^2 m_1 m_2}{24\pi r} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( e^{-\mu_2 r} + \frac{1}{8} e^{-\mu_0 r} \right) + \frac{3}{m_1 m_2} e^{-\mu_2 r}, \quad (35b) \]

\[ V^{(s=1/2)}_{\text{spin-orbit}}(r) = \frac{\kappa^2 m_1 m_2}{24\pi r^3} \left\{ \frac{1}{m_1^2} \vec{S}_1 \cdot \vec{L} + \frac{1}{m_2^2} \vec{S}_2 \cdot \vec{L} + \frac{3(\vec{S}_1 + \vec{S}_2) \cdot \vec{L}}{2m_1 m_2} \right\} (1 + r\mu_2)e^{-\mu_2 r} + \frac{\kappa^2 m_1 m_2}{192\pi r^3} \left( \frac{1}{m_1^2} \vec{S}_1 \cdot \vec{L} + \frac{1}{m_2^2} \vec{S}_2 \cdot \vec{L} \right) (1 + r\mu_0)e^{-\mu_0 r}, \quad (35c) \]

\[ V^{(s=1/2)}_{\text{spin-spin}}(r) = -\frac{\kappa^2}{32\pi r^3} \vec{S}_1 \cdot \vec{S}_2 (1 + r\mu_2 + r^2\mu_2^2)e^{-\mu_2 r} + \frac{\kappa^2}{32\pi r^3} (\vec{r} \cdot \vec{S}_1) (\vec{r} \cdot \vec{S}_2) (3 + 3r\mu_2 + r^2\mu_2^2)e^{-\mu_2 r}, \quad (35d) \]

in the case of spin-1/2 scattered particles.

As we can observe, both monopole-monopole and velocity-velocity sectors are composed exclusively by terms scaling with usual \( r^{-1} \) behavior, but with an additional exponential damping as a result of mass-like terms in the graviton propagator. By a simple comparison of \( V^{(s)}_{\text{mon-mon}}(r) \) and \( V^{(s)}_{\text{vel-vel}}(r) \) we can quickly infer the suppression of velocity-velocity contribution due to the “overall” ratio \( \vec{p}^2/(m_i m_j) \) \( \ll 1 \) in the NR limit. Since both sectors
exhibit similar $r$-dependencies, the dominance of $V_{\text{mon-\text{mon}}}^{(s)}(r)$ over $V_{\text{vel-\text{vel}}}^{(s)}(r)$ is valid for all distance scales (at least, within our approximations).

Before we move on to spin-dependent contributions, let us have a closer look at the monopole-monopole sector associated with spin-0 particles. As we have anticipated in the previous section, $V_{\text{mon-\text{mon}}}^{(s=0)}(r)$ shows an additional contribution beyond the usual terms appearing in the static limit. In the present example, this extra contribution is given by

$$\Delta V_{\text{mon-\text{mon}}}^{(s=0)}(r) = \frac{\kappa^2 m_1 m_2}{192 \pi r} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( \mu_2^2 e^{-\mu_2 r} + \frac{5}{4} \mu_0^2 e^{-\mu_0 r} \right).$$

The suppression mechanism regarding this term can be readily understood in terms of some physical considerations involving the static limit, namely

$$V_{\text{static}}(r) = -\frac{\kappa^2 m_1 m_2}{24 \pi r} \left( e^{-\mu_2 r} - \frac{1}{4} e^{-\mu_0 r} \right).$$

In order to avoid significant deviations from the usual Newtonian potential within regions where the later has been experimentally verified, we have imposed upper bounds on the mass parameters $\mu_2$ and $\mu_0$. A rough estimate can be obtained by assuming the Newtonian potential as a faithful description up the solar system radius. Taking solar system radius as $r_S \sim 10 \text{ AU}$, we recover the appropriated Newtonian potential (for $r < r_S$) provided that $\mu_i r_S \ll 1$, leading to the rough limit $\mu_i \ll 10^{-25} \text{ MeV}$. In this case, the suppression of the extra term $\Delta V_{\text{mon-\text{mon}}}^{(s=0)}(r)$ occurs as a consequence of the ratios $\mu_i^2/m_j^2$ that are much smaller than one, even if we consider the scattering elementary particles (with masses of order $\sim \text{ MeV}$).

Concerning the spin-dependent contributions, $V_{\text{spin-orbit}}^{(s=1/2)}(r)$ and $V_{\text{spin-spin}}^{(s=1/2)}(r)$, we observe the appearance of terms with different scaling behaviors in comparison with the previously discussed sectors. In particular, we note that spin-orbit sector involves interactions proportional to $r^{-2}$ and $r^{-1}$ (recall that $\vec{L} \sim \vec{r}$), while spin-spin interactions also involve terms scaling with $r^{-3}$. In all cases we also observe the exponential damping.

The long-range potential is dominated by $r^{-1}$-terms, which receives contributions from all the sectors investigated in the paper. Nevertheless, even in the set of interactions scaling with $r^{-1}$, the leading order long-ranging contribution corresponds to the usual static term in the monopole-monopole sector. In this case, the remaining $r^{-1}$-terms are suppressed by factors involving $\vec{p}^2/(m_i m_j)$ and $\mu_i^2/m_j^2$.

The situation turns out to be more interesting in the short-distance regime, since in this case we observe different dominant sectors for spin-0 and spin-1/2 particles. In the spin-0
case, the leading order short-range contribution came from the usual static terms in the monopole-monopole sector,

\[ V_{\text{short-range}}^{(s=0)}(r) = -\frac{\kappa^2 m_1 m_2}{32\pi r^3} + \cdots \quad (38) \]

In the case of spin-1/2 particles, on the other hand, the dominant contribution appears with spin-spin interactions, namely

\[ V_{\text{short-range}}^{(s=1/2)}(r) = -\frac{\kappa^2}{32\pi r^3} \left( \vec{S}_1 \cdot \vec{S}_2 - 3(\hat{r} \cdot \vec{S}_1)(\hat{r} \cdot \vec{S}_2) \right) + \cdots . \quad (39) \]

It is interesting to observe that, in both cases, the leading order short-range contribution does not involve any parameter associated with the form factors considered in this example, see Eq. (32). This fact can be interpreted as direct consequence of the infrared nature of this form factors class.

**B. Form factors motivated by FRG approach for quantum gravity**

In the second explicit example we consider form factors motivated by a recent strategy employed in the functional renormalization group (FRG) approach for asymptotically safe quantum gravity \[33\]. The main idea is to adopt an expansion of a coarse-grained version of the effective action, \( \Gamma_k \), in terms of \( k \)-dependent form factors, where \( k \) stands for an infrared cutoff scale introduced in the realm of the FRG framework. Within this formulation, it is possible to use the FRG-equation in order to derive (integro-differential) flow equations for the form factors \[33, 34\]. This strategy was applied to the search of an asymptotically safe solution in terms of form factors. After some approximations, the authors of Ref. \[33\] found a fixed point solution that could be fitted into a simple functional dependence of the form factor \( W(\Box) \), namely

\[ W(\Box) = \frac{\rho}{\Box + \beta} + w, \quad (40) \]

with the parameters \( \rho, \beta \) and \( w \) being adjusted according with numerical solutions of the fixed point equations. It is worth to mention that due to approximations employed in Ref. \[33\], the form factor associated with the sector \( RF(\Box)R \) decouples from the flow equation and it was set to zero at the level of the flowing effective action \( \Gamma_k \).

Keeping this in mind, in this section we mainly focus on the contribution of \( W(\Box) \) to the NR potentials. For the sake of simplicity, in this example we set the cosmological
constant to zero ($\Lambda = 0$). Furthermore, we should also emphasize that our analysis involves two important assumptions: (i) while the result obtained in Ref. [33] is based on non-perturbative Euclidean approach, we consider a naive continuation to Minkowski spacetime; (ii) we assume that shape of the form factor $W(\Box)$ would remain the same once we integrate down to $k = 0$. For these reasons, we explore other regions of the parameter space $\rho$, $\beta$ and $w$ instead of restricting ourselves to the particular values obtained in Ref. [33].

Taking into account this class of form factors, the relevant integrals contributing to the spin-2 sector of the NR potential involve the following term

$$\frac{1}{Q_2} = -\frac{1}{2w(q^2 + A_+)(q^2 + A_-)}, \quad (41)$$

which can be mapped, by means of partial fraction decomposition, in the standard integrals reported in the Appendix A. Note that we have defined

$$A_\pm = \frac{(-1 + 2\rho + 2w\beta) \pm \sqrt{(-1 + 2\rho + 2w\beta)^2 + 8w\beta}}{4w}. \quad (42)$$

Before we discuss the main results of this section, it is important to observe that an appropriate mapping in terms of the standard integrals [A3]-[A5] requires some restrictions on $A_\pm$. Therefore, it is interesting to have a closer look in the dependence of $A_\pm$ with respect to the parameters $\rho$, $w$ and $\beta$. In particular, we want to probe the existence of regions in the parameter space $\rho$, $w$ and $\beta$ where one of the following conditions is verified

(i) $A_\pm \in \mathbb{R}$, with $A_\pm > 0$,

(ii) $A_\pm \in \mathbb{C}$, such that $\text{Re}(A_\pm) > 0$ and $A_\pm^* = A_\mp$.

In the first case, the resulting potential is composed by a sum of terms with $r$-dependency characterized by $1/r^\alpha$ and $e^{-\sqrt{A_\pm}r/r^\alpha}$ (with $\alpha = 1, 2, 3$). In the situation where $A_\pm$ take complex values we also observe oscillatory terms (modulated by an exponential dumping) coming from the imaginary part of $A_\pm$. In this case, the additional restriction $A_\pm^* = A_\mp$ appears as a reality condition for the resulting potential. In Fig. 4 we show the existence of regions in the parameter space defined by $\rho$, $w$ and $\beta$ where the aforementioned conditions are verified. We note that, since the non-trivial dependence of $A_\pm$ occurs with respect to $\rho$ and the quantity $\beta w$, we can better summarized the results in terms of two region-plots in the plane $(\rho, \beta |w|)$. Apart from $\rho$ and $\beta |w|$, the shape of viable regions depends on the sign.
of $w$. As we can observe, both signs of $w$ admit dense regions satisfying conditions of type-i (red) or type-ii (blue).

![Figure 4](image.png)

Figure 4. Regions in the space of parameters $(\rho, \beta|w|)$ for positive and negative values of $w$. The red regions correspond to the values in which $A_\pm \in \mathbb{R}$ and positive (type i). The blue region (with horizontal dashing) indicates values where type-ii restriction is verified ($A_\pm \in \mathbb{C}$, such that $\text{Re}(A_\pm) > 0$ and $A^*_\pm = A_\pm$).

Since the complete expressions are quite long, here we shall not report the full results for $V^{(s=0)}(r)$ and $V^{(s=1/2)}(r)$. Nonetheless, we note that it can be directly obtained in terms of Eqs. (15) and (22) by putting together the explicit integrals

$$I_1^{(2)}(r) = -\frac{1}{2w} \left( \frac{1}{m_+^2 - m_-^2} \right) \left\{ \left( \frac{m_+^2 - m_-^2}{m_+^2 m_-^2} \right) \frac{\beta}{4\pi r} \right. \right. $$

$$- \left. \left. \left( 1 - \frac{\beta}{m_+^2} \right) \frac{e^{-m_+ r}}{4\pi r} + \left( 1 - \frac{\beta}{m_-^2} \right) \frac{e^{-m_- r}}{4\pi r} \right\}, \quad (43a)$$

$$I_0^{(2)}(r) = -\frac{1}{2w} \left( \frac{1}{m_+^2 - m_-^2} \right) \left\{ (m_+^2 - \beta) e^{-m_+ r} - (m_-^2 - \beta) e^{-m_- r} \right\}, \quad (43b)$$

$$I_{ij}^{(2)}(r) = -\frac{1}{2w} \left( \frac{1}{m_+^2 - m_-^2} \right) \left\{ \beta \left( \frac{m_+^2 - m_-^2}{m_+^2 m_-^2} \right) \left( \delta_{ij} - \frac{3x_ix_j}{r^2} \right) \right.$$

$$\left. - \left( 1 - \frac{\beta}{m_+^2} \right) \left[ (1 + m_+ r) \delta_{ij} - (3 + 3m_+ r + m_+^2 r^2) \frac{x_ix_j}{r^2} \right] \frac{e^{-m_+ r}}{4\pi r^3} \right.$$

$$\left. + \left( 1 - \frac{\beta}{m_-^2} \right) \left[ (1 + m_- r) \delta_{ij} - (3 + 3m_- r + m_-^2 r^2) \frac{x_ix_j}{r^2} \right] \frac{e^{-m_- r}}{4\pi r^3} \right\}, \quad (43c)$$

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where we have defined $m_\pm = \sqrt{A_\pm}$ and we assume $m_\pm^2 - m_\mp^2 \neq 0$. It is interesting to observe that, in this case, the contact terms ($\sim \delta^3(\vec{r})$) resulting from integrals of the form (A4) and (A5) completely cancel out in the final expression. The scaling of the different sectors contributing to the NR potentials is summarized in Table I.

| Sector | $1/r$ | $1/r^2$ | $1/r^3$ | $e^{-m_\pm r}/r$ | $e^{-m_\pm r}/r^2$ | $e^{-m_\pm r}/r^3$ |
|--------|-------|---------|---------|----------------|-----------------|----------------|
| mon-mon | $\checkmark_{0, 1/2}$ | | | $\checkmark_{0, 1/2}$ | | |
| vel-vel | $\checkmark_{0, 1/2}$ | | | $\checkmark_{0, 1/2}$ | | |
| $\vec{L} \cdot \vec{S}_{1,2}$ | | $\checkmark_{1/2}$ | | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | |
| $\vec{S}_1 \cdot \vec{S}_2$ | | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | |
| $(\hat{r} \cdot \vec{S}_1)(\hat{r} \cdot \vec{S}_2)$ | | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ | $\checkmark_{1/2}$ |

Table I. Scaling behavior of the different sectors contributing to the NR potentials $V(s=0)(r)$ and $V(s=1/2)(r)$. The subscript indicates if the correspondent behavior appears for spin-0 and/or spin-1/2 cases.

In the static regime the only remaining contribution comes from the monopole-monopole sector, resulting in the following expression

$$V_{2, \text{static}}(r) = -\frac{\kappa^2 m_1 m_2}{24 \pi r} \left( 1 - \frac{1}{2w} \frac{1 - \beta/m_2^2}{m_+^2 - m_-^2} e^{-m_- r} + \frac{1}{2w} \frac{1 - \beta/m_2^2}{m_+^2 - m_-^2} e^{-m_+ r} \right),$$

where the subscript “2” indicates that we are taking into account only spin-2 contributions in the graviton propagator. Deviations from the $1/r$-behavior within experimentally tested scales is avoided when $m_\pm r_{\min} \gg 1$, with $r_{\min}$ being the smaller distance in which the Newtonian $1/r$-law has been validated (see Refs. [43–45] for short-distance probes of the Newtonian potential). In this case, the exponential factors strongly suppress the second and third term in Eq. (44) and the large distance behavior is dominated by the $1/r$-contribution. As it was noted in Ref. [33], the static potential in Eq. (44) has a particularly interesting behavior at small distances. In this regime, the $1/r$ terms cancel out among different contributions, result in a finite potential at $r = 0$.

Beyond the static limit, the NR potential receives multiple contributions scaling with different $r$-dependencies as it was summarized in Table I. In the large distance regime, even if we include contribution beyond the monopole-monopole sector, the leading order term
correspond to the usual \(1/r\) decay,

\[
V_{\text{2, long-range}}(r) = -\frac{\kappa^2 m_1 m_2}{24\pi r} + \cdots,
\]

both in the spin-0 and spin-1/2 cases. In this limit, all remaining terms are suppressed either by exponential decay (with \(m_\pm r \gg 1\)) or due to sub-leading behavior of \(1/r^3\) in comparison with \(1/r\).

In the short-distance regime we observe more intriguing features once we take into account contributions beyond of monopole-monopole sector. In this case, the leading order terms are given by

\[V_{\text{s=0, short-range}}(r) = \frac{\kappa^2 m_1 m_2}{384\pi w r} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) + \cdots, \quad (46a)\]

\[V_{\text{s=1/2, short-range}}(r) = -\frac{\kappa^2}{128\pi w r} \left( \vec{S}_1 \cdot \vec{S}_2 + (\hat{r} \cdot \vec{S}_1)(\hat{r} \cdot \vec{S}_2) \right) + \cdots. \quad (46b)\]

Similarly to the example explored in the previous section, we also note different leading order contributions to \(V_{\text{s=0, short-range}}(r)\) and \(V_{\text{s=1/2, short-range}}(r)\). However, a different aspect in the present case is that the leading order terms at small distances exhibit a dependence with respect to the form factor \(W(\Box)\) due to the parameter \(w\) in Eqs. (46a) and (46b). This fact indicates that the form factor studied along this section plays an important role in the UV aspect of the NR potential, even in the presence of terms beyond the monopole-monopole sector. Despite this fact, our results for short-range regime point out an important difference with respect to the static case, Eq. (44), namely, the cancellation of the Newtonian singularity at \(r = 0\) does not survive beyond the static limit.

IV. REMARKS ON THE CANCELLATION OF NEWTONIAN SINGULARITIES

The observation at the end of the previous section trigger an interesting question regarding the cancellation of Newtonian singularities. In particular, it would be interesting to investigate whether the regular behavior at \(r = 0\), observed in higher-derivative models of gravity \cite{4, 36, 46, 47}, persists after the inclusion of contributions beyond the static limit. This particular test can be easily addressed in terms of the results presented in Section III, however, it requires a slight modification in the way we interpret our framework. In the case
of higher-derivative models, the form factor expansion appears at the level of the classical action \[36, 46, 47\], given by

\[
S_{\text{HD}}[g_{\mu\nu}] = \frac{2}{\kappa^2} \int d^4 x \sqrt{-g} \left( -R - \frac{1}{3} R F(\Box) R + C_{\mu\nu\alpha\beta} W(\Box) C^{\mu\nu\alpha\beta} \right) + \mathcal{O}(\mathcal{R}^3),
\]

(47)

with polynomial form factors \((p, q \in \mathbb{N})\)

\[
F(\Box) = \sum_{n=0}^{p} f_n (-\Box)^n \quad \text{and} \quad W(\Box) = \sum_{n=0}^{q} w_n (-\Box)^n.
\]

(48)

In this case, all the results presented in Section II remains unchanged, however, keeping in mind that Eq. (7) should be interpreted as the tree-level graviton propagator.

The relevant integrals appearing in Eqs. (15) and (22) can be easily computed by means of the partial decomposition

\[
\frac{1}{\vec{q}^2 Q_a(-\vec{q}^2)} = \frac{1}{\vec{q}^2} + \sum_{i=1}^{N_a} \frac{\mathcal{R}^{(a)}_i}{\vec{q}^2 + \mu_{a,i}^2}, \quad \text{with } a = 0, 2,
\]

(49)

where \(N_a = p \delta_{a,0} + q \delta_{a,2} + 1\) and we have defined the residues

\[
\mathcal{R}^{(a)}_n = -\prod_{l \neq n} \frac{\mu_{a,l}^2}{\mu_{a,l}^2 - \mu_{a,n}^2}.
\]

(50)

The mass parameters \(\mu_{a,l}\) were defined as the zeros of the \(Q_a\)-factors, namely \(Q_a(\mu_{a,l}^2) = 0\). In order to avoid complications with degenerate poles we assume \(\mu_{a,i}^2 \neq \mu_{a,j}^2\) if \(i \neq j\). In such a case, we can decompose \(I^{(a)}_n(r)\) and \(I^{(a)}_{ij}(r)\) in terms of the standard integrals (A3)-(A5), as displayed below

\[
I^{(a)}_n(r) = \mathcal{I}_n(r, 0) + \sum_{l=1}^{N_a} \mathcal{R}^{(a)}_l \mathcal{I}_n(r, \mu_{a,l}),
\]

(51a)

\[
I^{(a)}_{ij}(r) = \mathcal{I}_{ij}(r, 0) + \sum_{l=1}^{N_a} \mathcal{R}^{(a)}_l \mathcal{I}_{ij}(r, \mu_{a,l}).
\]

(51b)

The explicit NR potentials can be directly obtained by using Eqs. (51a) and (51b). The scaling dependence of the different sectors exhibits the same behavior of the previous subsection (see Table I).
The static limit (Eq. (26)) has been computed before, e.g. see Refs. [36, 46, 47], resulting in the following expression

\[ V_{\text{static}}(r) = -\frac{\kappa^2 m_1 m_2}{32\pi r} \left( 1 + \frac{4}{3} \sum_{l=1}^{q+1} \mathcal{R}_l^{(2)} e^{-\mu_2 l r} - \frac{1}{3} \sum_{l=1}^{p+1} \mathcal{R}_l^{(0)} e^{-\mu_0 l r} \right), \]

\[ = -\frac{\kappa^2 m_1 m_2}{32\pi r} \left( 1 + \frac{4}{3} \sum_{l=1}^{q+1} \mathcal{R}_l^{(2)} - \frac{1}{3} \sum_{l=1}^{p+1} \mathcal{R}_l^{(0)} \right) + \text{finite}. \] (52)

The cancellation of the 1/r singularity follows from the property \( \sum_{l=1}^{N} \mathcal{R}_l^{(a)} = -1 \) (see Ref. [47]).

Once we take into account contributions beyond the static sector, the regularity of the NR potential at \( r = 0 \) become more subtle. As an example, we consider the particular case corresponding to Stelle’s Quadratic Gravity \( (p = q = 0) \) [4]. In such a case, the leading order short-distance contribution is given by

\[ V_{\text{Stelle}}^{(s=0)}(r) = -\frac{\kappa^2 m_1 m_2}{192\pi r} \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) \left( \mu_2^2 + \frac{5}{4} \mu_0^2 \right) + \text{finite}, \] (53a)

\[ V_{\text{Stelle}}^{(s=1/2)}(r) = \frac{\kappa^2 \mu_2^2}{64\pi r} \left( \vec{S}_1 \cdot \vec{S}_2 + (\hat{r} \cdot \vec{S}_1)(\hat{r} \cdot \vec{S}_2) \right) + \text{finite}. \] (53b)

We note quite a similar behavior in comparison with the example of the previous section. As we can observe, the 1/r singularity reappears once we include contributions beyond the static limit. This result indicate that additional UV modifications should be included in order to keep the NR potential finite at \( r = 0 \). Indeed, this is actually the case as one can easily see by taking into account higher terms in the polynomial form factor defined in Eq. (48). The simple example is the sixth-order higher-derivative gravity \( (p = q = 1) \) which result in a singularity-free potential, even after the inclusion of contributions beyond the static limit. The same behavior is also observed for any \( p, q \geq 1. \)

V. CONCLUDING COMMENTS

In this paper, we investigate quantum effects in the NR gravitational inter-particle potential, including contributions beyond the static regime. We consider both the gravitational scattering of spin-0 and spin-1/2 particles. Our results are based on the form-factor expansion of the effective action in the covariant approach for quantum gravity. Within this
formalism, the quantum corrections are encoded in the form-factors $F(\Box)$ and $W(\Box)$ associated with curvature squared terms in the effective action. Taking into account metric fluctuations around flat background, these form-factors capture all the relevant information concerning the (flat) graviton propagator. Our main results can be summarized as follows:

- In the monopole-monopole sector, the NR potentials associated with spin-0 and -1/2 particles exhibit a universal leading-order contribution but differ with respect to a sub-leading term. The velocity-velocity sector exhibits the same result both for spin-0 and spin-1/2 particles.

- The NR potential associated with the scattering of spin-1/2 particles also involves spin-orbit and spin-spin interactions. We observe that both form factors $F(\Box)$ and $W(\Box)$ may contribute to spin-orbit, while only $W(\Box)$ can generate corrections to spin-spin interactions.

- Comparing our results with previous investigations concerning the electromagnetic NR potential, we observe similar interaction structures appearing in both cases.

We apply the results obtained in Sec. II to explicit examples of form factors motivated by non-perturbative approaches for quantum gravity. In the first example, we consider form factors motivated by an approach where the effective action was obtained by matching a predefined template with CDT data [32]. In the second one, we explore a form factor obtained in the context of the FRG approach for asymptotically safe quantum gravity [33]. In both cases, the contributions to the NR potentials reduce to the form $1/r^\alpha$ or $e^{-mr}/r^\alpha$ with $\alpha = 1, 2, 3$. Furthermore, we also observe that dominant short-range contributions depend on the type of particle being scattered.

An interesting finding concerning the example studied in Sec. III B was the reappearance of the singularity at $r = 0$ once we take into account contributions beyond the static regime. Motivated by this result, in Sec. IV we revisit the cancellation of Newtonian singularities in higher-derivative models. Within this class of models, our results indicate that the cancellation of singularities at $r = 0$ requires a higher number of derivatives when compared with the static approximation.

Finally, we highlight some comments. The analysis performed here only includes quantum corrections at the level of the graviton propagator, while the vertices are taken to be
tree-level ones. This was an important approximation in our approach and deserves further investigation. In principle, we could also adopt a form factor expansion in order to capture quantum corrections at the level of gravity-matter systems (see, for example, Ref. [34]). However, this approach increases considerably the calculations of the inter-particle potentials. In addition, we only consider the scattering of spin-0 and spin-1/2 particles, but we could also include the case of spin-1 particle. As discussed in Ref. [26], the NR potentials associated with spin-1 scattered particle exhibit new interactions involving the polarization, besides the velocity- and spin-dependent contributions. These points remain to be investigated in a future work.

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Appendix A: Integrals

Along this paper we have used the following definitions

\[ I_{n}^{(a)}(r) = \int \frac{d^3 \tilde{q}}{(2\pi)^3} \frac{e^{i\tilde{q} \cdot \tilde{r}}}{(\tilde{q}^2)^n Q_a}, \quad (A1) \]

\[ I_{ij}^{(a)}(r) = \int \frac{d^3 \tilde{q}}{(2\pi)^3} \frac{e^{i\tilde{q} \cdot \tilde{r}}}{\tilde{q}_i \tilde{q}_j}, \quad (A2) \]

where \( n \in \mathbb{N} \) and \( a = 0, 2 \).

In special, it is useful to have in mind some particular cases corresponding to standard integrals appearing along the calculations performed in Sec. III, namely

\[ I_1(r, m) \equiv \int \frac{d^3 \tilde{q}}{(2\pi)^3} \frac{e^{i\tilde{q} \cdot \tilde{r}}}{\tilde{q}^2 + m^2} = \frac{e^{-mr}}{4\pi r}, \quad (A3) \]

\[ I_0(r, m) \equiv \int \frac{d^3 \tilde{q}}{(2\pi)^3} \frac{\tilde{q}^2}{\tilde{q}^2 + m^2} = \frac{m^2}{4\pi r}e^{-mr}, \quad (A4) \]
\[ I_{ij}(r, m) \equiv \int \frac{d^3 \vec{q}}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} \frac{\vec{q}_i \vec{q}_j}{q^2 + m^2} = \frac{1}{3} \delta_{ij} \delta^3(\vec{r}) + \left\{ (1 + mr)\delta_{ij} - (3 + 3mr + m^2 r^2)\frac{x_i x_j}{r^2} \right\} e^{-mr} \frac{4\pi r^3}{3} \tag{A5} \]

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