Low-energy three-body charge transfer reactions with Coulomb interaction in the final state

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Abstract

Three-body charge transfer reactions with Coulomb interaction in the final state are considered in the framework of coordinate-space integro-differential Faddeev-Hahn-type equations within two- and six-state close coupling approximations. The method is employed to study direct muon transfer in low-energy collisions of the muonic hydrogen $H_\mu$ by helium ($\text{He}^{++}$) and lithium ($\text{Li}^{+++}$) nuclei. The experimentally observed isotopic dependence is reproduced.
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I. INTRODUCTION

Experimental investigations of the low-energy muon-transfer reactions in collisions of muonic hydrogen $H_\mu$ (bound state of a hydrogen isotope and muon $\mu^-$) with nuclei of charge $Z_1 > 1$ are of importance for muon catalyzed fusion cycle [1]. The study of such collisions involving three charged particles is also very interesting from a theoretical point of view as an example of rearrangement scattering with Coulomb interaction in the final state. Such reactions with post-collision Coulomb interaction between clusters appear frequently in atomic and molecular physics [2]. In the following we develop a general formalism for dealing with such reactions and as an example apply it to study some muon-transfer processes.

Recently, there has been considerable experimental interest in the study of the muon-transfer reaction in collision of the muonic atoms with He$^{++}$ [3,4] and also with charges $Z_1 > 3$ [5–9], e.g. oxygen (O$^8$), neon (Ne$^{10}$), argon (Ar$^{18}$) etc. It was found that contrary to the smooth $Z$-dependence expected from the semiclassical Landau-Zener formula [10] the experimental muon transfer rates for reactions like

$$\text{(H}_\mu\text{)}_{1s} + X^Z \rightarrow X^Z_\mu + H \quad (1)$$

depend in a complicated manner on the charge $Z$ [9]. Here H stands for the hydrogen isotopes $^1\text{H}$ or $^2\text{H}$ and $X^Z$ stands for the target nuclei. Another phenomenon which has not yet found a satisfactory theoretical explanation is the measured isotope effect, e.g. the trend of the direct transition rates of reactions (1) for $X^Z = \text{O}^8$ [8], Ne$^{10}$ [9], Ar$^{18}$ [6], and Xe$^{54}$ [11]. In cases of O$^8$, Ar$^{18}$ and Xe$^{54}$ the direct transfer rate decreases with increasing the mass of the H isotope. Theoretical analyses [12] also support this trend. The experimental results for Ne$^{10}$ [9] and sulphur dioxide [8] differ considerably from the theoretical predictions. Moreover, several experiments performed in recent years have put into evidence the complex structure of the time distributions of the X-rays following transfer from muonic hydrogen isotopes to heavier elements [13].

The proper theoretical analysis of charge transfer reaction (1) becomes extremely complicated numerically as the charge $Z$ increases because of the presence of the strong Coulomb
interaction in the final state. Traditionally, in theoretical studies, such Coulombic systems with two heavy (nuclei) and one light (muon) particles are considered within the framework of the two-state molecular Born-Oppenheimer approximation [14,15]. In another study, a semiclassical model based on Faddeev-type scattering equations has been used [16]. It would be of interest to perform a full quantum mechanical consideration in view of the fact that the muon is not so light compared to the nucleon and compare with the approximate calculations mentioned above.

Here we develop a quantum mechanical approach based on Faddeev-Hahn-type equations for a careful reinvestigation of these three-body direct charge-transfer reactions with strong Coulomb repulsion in the final state. As a first step towards a model solution of this complicated problem, we apply this detailed few-body method to the study of direct muon-transfer reaction (1) for $X^Z = ^3\text{He}^{++}$, $^4\text{He}^{++}$, $^6\text{Li}^{+++}$ and $^7\text{Li}^{+++}$. This study with lighter nuclei is expected to lead to faster numerical convergence than the heavier targets. However, our approach is equally applicable for heavier targets with higher charges, although the convergence could be slow in these cases. These studies with heavier targets would be interesting future works.

For the three-charged-particle system, say ($^7\text{Li}^2\text{H}^\mu$), only two asymptotic two-cluster configurations are possible, i.e. ($^2\text{H}^\mu$) - $^7\text{Li}$ and ($^7\text{Li}^\mu$) - $^2\text{H}$. For the theoretical treatment of such a three-body rearrangement process, Faddeev-type equations [17], especially the modified version proposed by Hahn [18], appear to be very suitable. The two possible asymptotic configurations of the above rearrangement problem are conveniently tackled by a set of two coupled Faddeev-Hahn-type equations for components $\Psi_1$ and $\Psi_2$ of the wave function $\Psi = \Psi_1 + \Psi_2$, where each component carries the asymptotic boundary condition for a specific configuration [19,20]. These equations are very useful to incorporate distortion potentials for specific initial and final asymptotic states [21]. It is possible to include the final-state Coulomb interaction explicitly in these equations, so that a low-order approximation to these equations produces the correct asymptotic behavior [21].

We solve the integro-differential form of the Faddeev-Hahn equation by the close-coupling approximation scheme involving up to six states. This procedure consists in expanding the
wave function components Ψ₁ and Ψ₂ in terms of eigenfunctions of subsystem Hamiltonians in initial and final channels, respectively. Although, these subsystem eigenfunctions are not orthogonal to each other, the components Ψ₁ and Ψ₂ satisfy a coupled set of equations incorporating the correct asymptotic behavior of the wave function. Consequently, there is no problem of overcompleteness as encountered in similar expansion approaches for rearrangement reactions based on the Schrödinger equation. The resultant coupled Faddeev-Hahn-type equations are then projected on the expansion functions. After a partial-wave projection this leads to a set of one-dimensional coupled integro-differential equations for the expansion coefficients, which is solved numerically.

In Sec. II we develop the formalism. We have calculated transfer rates for reaction (1) for H = ¹H or ²H and XZ = ³He++, ⁴He++, ⁶Li++ or ⁷Li+++ using a two-state close-coupling approximation, and for H = ²H and XZ =³ He++, ⁶Li+++ or ⁷Li+++ using six-state close-coupling approximations. Our results obtained for muon-transfer rates from hydrogen to helium and lithium are given in Sec. III and compared with those of other investigations. We also present a summary and outlook in the concluding part of this section.

II. THEORETICAL FORMULATION

Let us take the system of units to be \( e = \hbar = m_\mu = 1 \), where \( m_\mu \) (e) is the muonic mass (charge), and denote, the heavy nuclei (³He, ⁴He, ⁶Li, etc.) by 1, the hydrogen isotopes (¹H, ²H or ³H) by 2 and muon by 3. Below the three-body breakup threshold, following two-cluster asymptotic configurations are possible in the system 123: (23) − 1 and (13) − 2. These two configurations correspond to two distinct physical channels, also denoted by 1 and 2. These configurations are determined by the Jacobi coordinates \((\vec{r}_{j3}, \vec{r}_k)\)

\[
\vec{r}_{j3} = \vec{r}_3 - \vec{r}_j, \quad \vec{r}_k = (\vec{r}_3 + m_j \vec{r}_j)/(1 + m_j) - \vec{r}_k, \quad j \neq k = 1, 2, \tag{2}\]

\( \vec{r}_i, m_i \) are coordinates and masses of the particles \( i = 1, 2, 3 \), respectively.

Let us introduce the total three-body wave function as a sum of two components

\[
\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \Psi_1(\vec{r}_{23}, \vec{r}_1) + \Psi_2(\vec{r}_{13}, \vec{r}_2), \tag{3}\]
where \( \Psi_1(\vec{r}_{23}, \vec{\rho}_1) \) is quadratically integrable over the variable \( \vec{r}_{23} \), and \( \Psi_2(\vec{r}_{13}, \vec{\rho}_2) \) over the variable \( \vec{r}_{13} \). The components \( \Psi_1 \) and \( \Psi_2 \) carry the asymptotic boundary condition for channels 1 and 2, respectively. The second component is responsible for pure Coulomb interaction in the final state. These components satisfy the following set of two coupled equations

\[
(E - H_0 - V_{23})\Psi_1(\vec{r}_{23}, \vec{\rho}_1) = (V_{23} + V_{12} - U_C)\Psi_2(\vec{r}_{13}, \vec{\rho}_2)
\]

\[
(E - H_0 - V_{13} - U_C)\Psi_2(\vec{r}_{13}, \vec{\rho}_2) = (V_{13} + V_{12})\Psi_1(\vec{r}_{23}, \vec{\rho}_1),
\]

where \( E \) is the center-of-mass energy, \( H_0 \) is the total kinetic energy operator, and \( V_{ij}(r_{ij}) \) are pair-interaction potentials \( (i \neq j = 1, 2, 3) \), and \( U_C \) is a distortion interaction, e.g. Coulomb repulsion in the final state between clusters \((^3\text{He}, \mu)\) and \(^2\text{H}\) in the case of \(^3\text{He}^2\text{H}\mu\) system

\[
U_C = \frac{(Z_1 - 1)Z_2}{\rho_2}.
\]

Here \( Z_1 \) is the charge of \(^3\text{He}\) and \( Z_2(= 1) \) is the charge of the hydrogen isotope. By adding the two equations (4) we find that they are equivalent to the Schrödinger equation. For energies below the three-body breakup threshold they possess the same advantages as the Faddeev equations, since they are formulated for the wave function components with correct physical asymptotic behavior.

The component \( \Psi_1 \) carries the asymptotic behavior in elastic and inelastic channels:

\[
\Psi_1(\vec{r}_{23}, \vec{\rho}_1) \underset{\rho_1 \to +\infty}{\sim} e^{i k_1^{(1)} z} \varphi_1(\vec{r}_{23}) + \sum_n A_{n}^{\text{el/in}}(\Omega_{\rho_1}) e^{i k_n^{(1)} \rho_1} \varphi_n(\vec{r}_{23})/\rho_1.
\]

(6)

The component \( \Psi_2 \) carries the Coulomb asymptotic behavior in the transfer channels:

\[
\Psi_2(\vec{r}_{13}, \vec{\rho}_2) \underset{\rho_2 \to +\infty}{\sim} \sum_{nl} A_{nl}^{\text{tr}}(\Omega_{\rho_2}) e^{i (k_n^{(2)} \rho_2 - \pi l / 2 + \pi n / 2 k_n^{(2)} \ln 2 k_n^{(2)} \rho_2)} \varphi_n(\vec{r}_{13})/\rho_2.
\]

(7)

where \( e^{i k_1^{(1)} z} \varphi_1(\vec{r}_{23}) \) is the incident wave, \( \varphi_n(\vec{r}_{j3}) \) the \( n \)-th excited bound-state wave function of pair \((j3)\), \( k_n^{(i)} = \sqrt{2M_i(E - E_n^{(j)})} \), with \( M_i^{-1} = m_i^{-1} + (1 + m_j)^{-1} \). Here \( E_n^{(j)} \) is the binding energy of \((j3)\), \( i \neq j = 1, 2 \), \( A_{n}^{\text{el/in}}(\Omega_{\rho_1}) \) and \( A_{nl}^{\text{tr}}(\Omega_{\rho_2}) \) are the scattering amplitudes in the elastic/inelastic and transfer channels. The Coulomb parameters in the second transfer channel are [22]
τ_t = \arg\Gamma(l + 1 + \eta/2k^{(2)}_m) \quad \text{and} \quad \eta = 2M_2(Z_1 - 1)/k^{(2)}_n. \quad (8)

This approach simplifies the solution procedure and provides the correct asymptotic behavior of the solution below the 3-body breakup threshold.

Let us write down (4) in terms of the adopted notations

\[
\left[ E + \nabla^2_{\rho_k} + \nabla^2_{\bar{r}_j} - V_{j3} - U_C\delta_{k2} \right] \Psi_k(\bar{r}_j, \bar{\rho}_k) = (V_{j3} + V_{jk} - U_C\delta_{j2})\Psi_j(\bar{r}_k, \bar{\rho}_j), \quad (9)
\]

here \( j \neq k = 1, 2, \) \( M_k^{-1} = m_k^{-1} + (1 + m_j)^{-1} \) and \( \mu_j^{-1} = 1 + m_j^{-1}. \) We are using the Jacobi coordinates

\[
\bar{\rho}_j = \bar{r}_j - \beta_k\bar{r}_k, \quad \bar{r}_j = \frac{1}{\gamma}(\beta_k\bar{\rho}_k + \bar{\rho}_j) \quad \text{and} \quad \bar{r}_{jk} = \frac{1}{\gamma}(\sigma_j\bar{\rho}_j - \sigma_k\bar{\rho}_k), \quad (10)
\]

with

\[
\beta_k = \frac{m_k}{1 + m_k}, \quad \sigma_k = 1 - \beta_k \quad \text{and} \quad \gamma = 1 - \beta_k\beta_j. \quad (11)
\]

For solving (9) we expand the wave function components in terms of bound states in initial and final channels, and project this equation on these bound states. The expansion of the wave function is given by

\[
\Psi_k(\bar{r}_j, \bar{\rho}_k) \approx \sum_{LM} \sum_n \frac{1}{\rho_k} f^{(k)LM}_{nl\lambda}(\rho_k) R^{(k)}_{nl}(r_j) \left\{ Y_\lambda(\hat{\rho}_k) \otimes Y_l(\hat{r}_j) \right\}_{LM}, \quad (12)
\]

where \((nl\lambda) \equiv \alpha\) are quantum numbers of a three-body state and \( L \) is the total angular momentum of the three-body system obtained by coupling \( l \) and \( \lambda, \) \( Y_{lm} 's \) are the spherical harmonics, \( R^{(k)}_{nl}(r_j) \) the radial part of the hydrogen-like bound-state wave function, \( f^{(k)LM}_{nl\lambda}(\rho_k) \) are the unknown expansion coefficients. This prescription is similar to that adopted in the close-coupling approximation. After a proper angular momentum projection, the set of integro-differential equations for the unknown expansion functions \( f^{(k)}_\alpha(\rho_k) \) can be written as

\[
\left[ \left( k^{(1)}_n \right)^2 + \frac{\partial^2}{\partial \rho_1^2} - \frac{\lambda(\lambda + 1)}{\rho_1^2} \right] f^{(1)}_\alpha(\rho_1) = g_1 \sum_{n'} \sqrt{(2\lambda + 1)(2\lambda' + 1)} \frac{Z_1}{\rho_1} \frac{1}{|\bar{r}_{23}|} \left[ -\frac{1}{|\bar{r}_{23}|} + U_C \right] R^{(2)}_{\alpha n'}(|\bar{r}_{13}|) \left| \int_0^\infty \rho_1 \rho_2 |D^L_{n m'}(0, \omega, 0)C^L_{\lambda 0 m}C^{L'}_{\lambda' 0 m'}Y_{lm}(0, \pi)Y_{lm'}(\nu_1, \pi)Y_{lm'}(\nu_2, \pi) \right|, \quad (13)
\]

\[
6
\]
\[
\left( k_n^{(2)} \right)^2 + \frac{\partial^2}{\partial \rho_2^2} - \lambda \left( \lambda + 1 \right) \rho_2^2 - U_c \right] f_\alpha^{(2)}(\rho_2) = g_2 \sum_{\alpha'} \sqrt{\frac{(2\lambda + 1)(2\lambda' + 1)}{2L + 1}} \int_0^\infty d\rho_1 f_{\alpha'}^{(1)}(\rho_1) \int_0^\pi d\omega \sin(\omega) \rho_2^{(2)}(\lfloor r_{13}\rfloor) \left[ \frac{Z_1}{|r_{13}|} + \frac{Z_1}{|r_{12}|} \right] R_{n'\nu}^{(1)}(\lfloor r_{23}\rfloor) \rho_2 \rho_1 D_{mm'}^{Lm}(0,\omega,0) C_{\nu'\nu}^{Lm'} C_{0\nu'\nu}^{Lm'} Y_{l_m}(\nu_2,\pi) Y_{l_m'}(\nu_1,\pi). \tag{14}
\]

Here \( g_2 = 4\pi M_k/\gamma^3 \), \( \gamma = 1 - m_k m_j / ((1 + m_k)(1 + m_j)) \), \( \alpha' \equiv (n'l'\lambda') \), \( D_{mm'}^{Lm}(0,\omega,0) \) the Wigner function, \( C_{\nu'\nu}^{Lm'} \) the Clebsh-Gordon coefficient, \( \omega \) is the angle between the Jacobi coordinates \( \vec{r}_i \) and \( \vec{r}_{i'} \), \( \nu_i \) is the angle between \( \vec{r}_{i3} \) and \( \vec{r}_i \), \( \nu_{i'} \) is the angle between \( \vec{r}_{i3} \) and \( \vec{r}_{i'} \). The following relations are useful for numerical treatment

\[
\sin \nu_i = \frac{\rho_{i'}}{\gamma r_{i3}} \sin \omega \quad \text{and} \quad \cos \nu_i = \frac{1}{\gamma r_{i3}} (\beta_i \rho_i + \rho_{i'} \cos \omega) \quad (i \neq i' = 1, 2). \tag{15}
\]

To find unique solution to (13)–(14), appropriate boundary conditions are to be considered. First we impose \( f_{nl}^{(i)}(0) = 0 \). For the present scattering problem with \( 1 + (23) \) as the initial state, in the asymptotic region, two solutions to (13)–(14) satisfy the following boundary conditions

\[
\begin{aligned}
& f_{1s}^{(1)}(\rho_1) \sim \sin(k_1^{(1)} \rho_1) + K_{11} \cos(k_1^{(1)} \rho_1) , \\
& f_{1s}^{(2)}(\rho_2) \sim \sqrt{v_1/v_2} K_{12} \cos(k_1^{(2)} \rho_2) - \eta/2k_1^{(2)} \ln 2k_1^{(2)} \rho_2 ,
\end{aligned} \tag{16}
\]

where \( K_{ij} \) are the appropriate coefficients. For scattering with \( 2 + (13) \) as the initial state, we have the following conditions

\[
\begin{aligned}
& f_{1s}^{(1)}(\rho_1) \sim \sqrt{v_2/v_1} K_{21} \cos(k_1^{(1)} \rho_1) , \\
& f_{1s}^{(2)}(\rho_2) \sim \sin(k_1^{(2)} \rho_2 - \eta/2k_1^{(2)} \ln 2k_1^{(2)} \rho_2) + K_{22} \cos(k_1^{(2)} \rho_2 - \eta/2k_1^{(2)} \ln 2k_1^{(2)} \rho_2) ,
\end{aligned}
\]

where \( v_i \) \((i = 1, 2)\) are velocities in channel \( i \). In the absence of Coulomb interaction \( U_c \) in the final channel, \( K_{ij} \) are the components of the on-shell \( K \)-matrix \([22]\). With the following change of variables in (13)–(14)

\[
\begin{aligned}
& f_{1s}^{(1)}(\rho_1) = f_{1s}^{(1)}(\rho_1) - \sin(k_1^{(1)} \rho_1) , \\
& f_{1s}^{(2)}(\rho_2) = f_{1s}^{(2)}(\rho_2) - \sin(k_1^{(2)} \rho_2 - \eta/2k_1^{(2)} \ln 2k_1^{(2)} \rho_2) , \tag{17}
\end{aligned}
\]
we obtain two sets of inhomogeneous equations which are solved numerically. The coefficients $K_{ij}$ are obtained from the numerical solution of the Faddeev-Hahn-type equations. The cross sections are given by

$$
\sigma_{ij} = \frac{4\pi}{k_1^{(i)2}} \frac{\delta_{ij}D^2 + K_{ij}^2}{(D - 1)^2 + (K_{11} + K_{22})^2},
$$

where $i, j = 1, 2$ refer to the two channels and $D = K_{11}K_{22} - K_{12}K_{21}$. When $k_1^{(1)} \rightarrow 0$: $\sigma_{tr} \equiv \sigma_{12} \sim 1/k_1^{(1)}$. For comparison with experimental low-energy data it is very useful to calculate the transfer rates

$$
\lambda_{tr} = \sigma_{tr} v N_0,
$$

with $v$ being the relative velocity of the incident fragments and $N_0$ the liquid-hydrogen density chosen here as $4.25 \times 10^{22}$ cm$^{-3}$, because $\lambda_{tr}(k_1^{(1)} \rightarrow 0) \sim const$.

III. NUMERICAL RESULTS

We employ muonic atomic unit: distances are measured in units of $a_\mu$, where $a_\mu$ is the radius of muonic hydrogen atom. The integro-differential equations were solved by discretizing them into a linear system of equations. The integrals in Eqs. (13) and (14) are discretized using the trapezoidal rule and the partial derivatives are discretized using a three-point rule [23]. The discretized equation is subsequently solved by Gauss elimination method. As we are concerned with the low-energy limit only the total angular momentum $L = 0$ is taken into account. Even at zero incident energy, the transfer channels are open and their wave functions are rapidly oscillating Coulomb waves. In order to get a converged solution we needed a large number of discretization points (up to 900) adequately distributed between 0 to 40$a_\mu$. More points are taken near the origin where the interaction potentials are large; a smaller number of points are needed at large distances. For example, near the origin we took up to 40 equally spaced points per an unit length interval $a_\mu$; in the intermediate region ($\rho = 10 - 20a_\mu$) we took up to 25 equally spaced points per unit length interval $a_\mu$, and in the asymptotic region ($\rho = 20 - 40a_\mu$) we took up to 15 equally spaced points per
unit length interval $a_\mu$. The following mass values are used in the unit of electron mass:

$m(^1\text{H}) = 1836.152$, $m(^2\text{H}) = 3670.481$, $m(^3\text{He}) = 5495.882$, $m(^4\text{He}) = 7294.295$, $m(^6\text{Li}) = 10961.892$, $m(^7\text{Li}) = 12786.385$ and the muon mass is $m_\mu = 206.769$.

We present muon-transfer rates $\lambda_{tr}$ calculated using the formulation of last section for processes (1). First, we restrict ourselves to a two-level approximation by choosing in the relevant close-coupling expansion the hydrogen-like ground states $(\text{H}_\mu)_{1s}$ and $(\text{X}_Z)_\mu$,

where $\text{H} = ^1\text{H}$ and $^2\text{H}$, and $\text{X}^Z = ^3\text{He}^{++}$, $^4\text{He}^{++}$, $^6\text{Li}^{+++}$ and $^7\text{Li}^{+++}$. Numerically stable and converged results were obtained in these cases. The rates $\lambda_{tr} \times 10^6$ sec$^{-1}$ at low energies are presented in table 1 together with the results of [14–16]. The results in this case converged to the precision shown in this table, except in the case of $^2\text{H}_\mu + ^4\text{He}^{++}$, where it was difficult to get converged result. The present results are consistent with the experimentally observed isotope effect [3–9], e.g., the rate decreases from $^1\text{H}$ to $^2\text{H}$.

In table 2 we present our results for transition rate of reaction (1) to $(^3\text{He}^{++}_\mu)_{1s}$, $(^6\text{Li}^{+++}_\mu)_{1s}$ and $(^7\text{Li}^{+++}_\mu)_{1s}$ from $(^2\text{H}_\mu)_{1s}$ using the six-state close-coupling model. The six states are $\text{H}_\mu(1s,2s,2p)$ and $\text{X}_Z\mu(1s,2s,2p)$. The results so obtained are consistent with the measured isotope effect. The effect of including the $(2s,2p)$ states in the calculational scheme is also explicit there.

The results reported in table 1 and 2 demonstrate the efficiency of the present few-body model in describing muon transfer from H isotopes to nuclei of charge $Z_1 = 2$. Its application to nuclei involving higher charges, therefore, is also expected to be justified. The present calculation with $^6\text{Li}^{+++}$ or $^7\text{Li}^{+++}$ represents the first examples for such a full quantum-mechanical extension within the six-state close-coupling model.

The study of three-body charge transfer reactions with Coulomb repulsion in the final state has been the subject of this work. We have studied such reactions employing a detailed few-body description of the rearrangement scattering problem by solving the Faddeev-Hahn-type equations in coordinate space. To provide correct asymptotic form in the final state the pure Coulomb interaction has been incorporated directly into the equations. It is shown that within this formalism, the application of a close-coupling-type ansatz leads to satisfactory results already in low-order approximations for direct muon-transfer reactions between
hydrogen isotopes and light nuclei He$^{++}$ and Li$^{+++}$. Because of computational difficulties, in this preliminary application we have considered up to six states in the expansion scheme (1s,2s,2p on each center – (H$_\mu$) and X$_\mu^Z$), which may not always be adequate. Further calculations with larger basis sets are needed to obtain accurate converged results. However, the inclusion of three basis states on each center is expected to build in a satisfactory account of the polarization potential in the model. It has been observed [24] in studies of positron and positronium scattering using close-coupling type approach that once the 1s,2s,2p states of positronium and target states are included, a good account of scattering including transfer reaction is obtained (estimated error of 10 – 20%). However, the inclusion of only the 1s basis functions do not lead to the converged results. A similar conclusion can be obtained from tables 1 and 2. In view of the results of ref. [24] we do not believe the results of table 2 to be very different from the converged ones, although we cannot provide a quantitative measure of convergence. If the above conclusion based on the works of ref. [24] hold in this case we expect a maximum error of 20% in table 2.

Because of the present promising results for the muon-transfer rates of (1) for $Z_1 < 4$, it seems useful to make future applications of the present formulation for larger targets with $Z_1 \geq 4$. Such calculations involving nuclei of higher charge are in progress. The present approach should also be useful in rearrangement collision involving electron, e.g., such as in H(1s) + He$^{++}$ → H$^+$ + He$^+$(1s), considered in [25].

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Table 1. Low energy muon transfer rates $\lambda_{tr}/10^6$ sec$^{-1}$ from proton ($^1$H$\mu$)$_{1s}$ and deuteron ($^2$H$\mu$)$_{1s}$ to hydrogen-like ground state ($^3$He$^+_\mu$)$_{1s}$, ($^4$He$^+_\mu$)$_{1s}$, ($^6$Li$^{++}\mu$)$_{1s}$ and ($^7$Li$^{++}\mu$)$_{1s}$ within two-state close-coupling model.

| System          | Energy (eV) | Present Results | \[16\] | \[15\] | \[14\] |
|-----------------|-------------|-----------------|-------|-------|-------|
| $^1$H$\mu$ + $^3$He$^{++}$ | $\leq 0.04$ | 8.4             | 7.25  | 10.9  | 6.3   |
|                 | 0.1         | 8.3             |       |       |       |
|                 | 1.0         | 8.1             |       |       |       |
| $^1$H$\mu$ + $^4$He$^{++}$ | $\leq 0.04$ | 6.8             | 6.65  | 10.7  | 5.5   |
| $^2$H$\mu$ + $^3$He$^{++}$ | $\leq 0.04$ | 5.2             | 4.77  | 9.6   | 1.3   |
|                 | 0.1         | 5.1             |       |       |       |
|                 | 1.0         | 4.7             |       |       |       |
| $^2$H$\mu$ + $^4$He$^{++}$ | $\leq 0.04$ | 5.0 ± 0.3       | 4.17  | 9.6   | 1.0   |
| $^2$H$\mu$ + $^6$Li$^{+++}$ | $\leq 0.04$ | 1.2             | 1.01  |       |       |
|                 | 0.1         | 1.2             |       |       |       |
|                 | 1.0         | 1.1             |       |       |       |
| $^2$H$\mu$ + $^7$Li$^{+++}$ | $\leq 0.04$ | 1.12            | 0.96  |       |       |
|                 | 0.1         | 1.12            |       |       |       |
|                 | 1.0         | 1.06            |       |       |       |
Table 2. Low energy muon transfer rates $\lambda_{tr}/10^6$ sec$^{-1}$ from $^{2}$H$\mu$ to hydrogen-like ground state $^{3}$He$^{+}$, ($^{6}$Li$^{++}$) and ($^{7}$Li$^{+++}$) within six-state close-coupling model.

| System                  | Energy (eV) | Present Results |
|-------------------------|-------------|-----------------|
| $^{2}$H$\mu + ^3$He$^{++}$ | $\leq 0.04$ | 9.0±0.2         |
|                         | 0.1         | 8.8±0.2         |
|                         | 1.0         | 5.0±0.2         |
| $^{2}$H$\mu + ^6$Li$^{+++}$ | $\leq 0.04$ | 1.9±0.1         |
|                         | 0.1         | 1.9±0.1         |
|                         | 1.0         | 1.2±0.1         |
| $^{2}$H$\mu + ^7$Li$^{+++}$ | $\leq 0.04$ | 1.6±0.1         |
|                         | 0.1         | 1.6±0.1         |
|                         | 1.0         | 1.2±0.1         |