Algorithms for calculating the cost in the conditions of digitalization of industrial production

Tatyana Kozlova¹, Elena Zambrzhitskaia¹, Dmitry Simakov¹ and Yaroslav Balbarin²*

¹ Nosov Magnitogorsk State Technical University, Lenin St., 38, Magnitogorsk, 455000, Russia
² Group of companies “Ultra”, Profsoyznaya St., 11, Magnitogorsk, 455019, Russia

* E-mail: ybalbarin@gmail.com

Abstract. The current trends in the industrial production development are that the process chains are becoming more complex, and arranged in the network structures, and cost of production goods prevails more in prime cost. This leads to increasing digitalization of the business process - spread of corporate information systems covering various fields of enterprises. In this case, the key block of these systems is the product costing block. Due to the complexity of process relations and prevalence of costs that are not directly related to the production process, the costing is carried out in a simplified form with the exception of cause-effect relations. This article covers a number of actual oriented tools, the application of which is most effective in conditions of industry digitalization, namely, the use of iterative algorithms and a matrix model when forming a cost indicator, which are considered both in terms of labor intensity, and the capabilities of modeling various economic conditions of the enterprise. The first and second approaches are the use of an iterative algorithm using systems of linear equations both explicitly (directly mathematical model) and implicitly - the construction of a model within a spreadsheet. The third approach involves mathematical model building using the matrix algebra. Thus, these tools allow not only to accurately calculate and model product costs, but also to implement the potential of industrial production digitization and the use of corporate information systems in full.

1. Introduction

The cost indicator in modern conditions of industrial production development is not only an operational form of performance control, but also the foundation for making such strategy decisions to be noted by both Russian and foreign authors, such as:

- updating the product portfolio [1, 2];
- restructuring the industrial production in terms of separation and incorporation, creation and liquidation [3];
- reformatting process and value chains [4];
- capital investment efficiency [5];
- etc.

The key problem of obtaining adequate information when making these decisions - in industrial production, as a rule, there is a complex process structure specified by the presence of counter flows of
material and labor resources, which makes it difficult to assess the effect of implemented measures. The problem matter point is determined by the fact that in modern practice of collecting and processing information based on financial and management accounting there are practically no developed tools that can take into account counter flows and the full picture of process relations when forming cost indicator, and while focusing only on the central activities, which are noted by local and foreign researchers [6, 7]. Thus, the decision makers get distorted information on the actual business processes and this factor affects the quality of managerial decision making.

2. Methods of the study, its objectives
The lack of development of methods for processing the information relating to the occurring processes was previously specified by the absence of tools which would facilitate obtaining the result in acceptably short time and with the necessary accuracy. Overcoming the fact of available counter flows of material and labor resources proposes using the algorithms of linear [8, 9] and matrix algebra [10], which limited the possibility of their use as extreme time-consuming calculations. In the current conditions of exponentially increasing digitalization of production, widespread use of various types of corporate information systems and accumulation of information on almost all facts of business activities, the use of the above tools has become available for almost any business entity.

Thus, it is possible to define three forms to use the above tools in the framework of process digitalization, namely:

- using the iterative algorithm to be implemented based on the spreadsheets with no formal characterization of the mathematical model [11];
- using the classical iterative Gauss-Seidel method for solving simultaneous linear algebraic equations (SLAE) with the special mathematical software [12];
- using the matrix algebra by means of spreadsheets or software applications package for mathematical calculations [13, 14].

The subject of this article is the comparative characteristic of the above three tools and the definition of their applicability limits.

3. Results
The essence of the first tool to be proposed by the authors is to build a set of tables (tabular model) and to use the iterative calculations mechanism built into the spreadsheets. Thus, the tabular model substitutes the formalized SLAE that simplifies the quantitative assessment of business processes with no loss of accuracy and economic meaning.

In the framework of the tabular model (tables 1-3), the following designations are used:
CC1, CC2, ..., CCn are cost centers existing at the enterprise;
R1, R2, ..., Rn are the values of own costs falling within each cost center. Herewith, in the framework of this article, we will assume that own costs consist of two elements: a sum of the direct costs for the products and a sum of the indirect costs. The receivers of costs are within the rows, the suppliers of costs - within the columns of the tables.

**Table 1.** Products exchange between the business subdivisions.

| Suppliers of costs | CC1 | CC2 | … | CCn |
|--------------------|-----|-----|---|-----|
| Receivers of costs | CC1 | CC2 | … | CCn |
| Products (operations, services) exchange | | | | |
Table 2. Summary of costs.

| Own costs | Suppliers of costs |
|-----------|--------------------|
|           | CC1    | CC2 | …   | CCn    |
| Receivers of costs | CC1 | R1  |     |        |
|            | CC2   | R2  |     |        |
|            | …     | …   |     |        |
|            | CCn   | Rn  |     |        |

Table 3. Tabular representation of the products cost.

| Production cost |
|-----------------|
|                |
| Indicator       |
| CC1            | CC2 | …   | CCn  |

Let us consider the application of this tool (as well as the subsequent ones) based on the composed example. The flow diagram of the business process arrangement is shown in figure 1.

The diagram specificity is in the presence of both counter material flows (one of these flows is between transport and repair shops), and the special business situations when the subdivision actually supplies the products to itself (for example, Production Shop 2 processes its own production defect).

Tables 1 and 2 are initially prepared with the software in accordance with the original data. Then a costs summary table is formed, the columns of which use the following formulas:

$$Cp \cdot Vp,$$

where

- $Cp$ – intermediate unit cost;
- $Vp$ – intermediate unit consumption volume.

From all has been said it follows we got a closed cycle—calculation of the received costs requires the presence of the cost of a produced product unit; the calculation of the produced product unit cost requires the gross (full) costs of the subdivision; the calculation of the full costs of the subdivision requires the amount of costs received by the CC from other CCs; etc.

To solve the current situation, an iterative calculation algorithm built into the spreadsheets is used. Activation of this algorithm leads to the calculation of the required values. The final result is given in tables 4–6.
Figure 1. Structure of technological communications of manufacturing enterprise.

Table 4. Own costs of subdivisions and volumes of output (operations, services).

| Own costs | Total production |
|-----------|------------------|
| CC1 Transport shop | 185 | 510 |
| CC2 Repair shop | 160 | 1,402 |
| CC3 Shop management | 92 | 38 |
| CC4 Products output shop | 7,500 | 315 |
| CC5 Products manufacturing shop 2 | 1,080 | 60 |
| CC6 Finished products warehouse 1 | 110 | 300 |
| CC7 Finished products warehouse 2 | 210 | 60 |
| CC8 Enterprise top management | 310 | 59 |
| CC9 Sales Department 1 | 960 | 0 |
| CC10 Sales Department 2 | 160 | 0 |
| Total | 10,767 | x |
Table 5. Actual products exchange between the business subdivisions.

|       | CC1, thous. t-km | CC2, man-h | CC3, persons | CC4, t | CC5, t | CC6, t | CC7, t | CC8, persons | CC9, t | CC10, t |
|-------|------------------|------------|-------------|--------|--------|--------|--------|--------------|--------|---------|
| CC1   | Transport shop   | 20         | 328         | 5      | 0      | 0      | 0      | 0            | 0      | 0       |
| CC2   | Repair shop      | 60         | 50          | 8      | 0      | 0      | 0      | 0            | 0      | 0       |
| CC3   | Shop management  | 10         | 10          | 0      | 0      | 0      | 0      | 41           | 0      | 0       |
| CC4   | Products         | 200        | 820         | 20     | 0      | 0      | 0      | 0            | 0      | 0       |
|       | manufacturing    |             |             |        |        |        |        |              |        |         |
|       | shop 1           |             |             |        |        |        |        |              |        |         |
| CC5   | Products         | 50         | 164         | 5      | 15     | 0      | 0      | 0            | 0      | 0       |
|       | manufacturing    |             |             |        |        |        |        |              |        |         |
|       | shop 2           |             |             |        |        |        |        |              |        |         |
| CC6   | Finished         | 25         | 20          | 0      | 300    | 0      | 0      | 5            | 0      | 0       |
|       | products         |             |             |        |        |        |        |              |        |         |
|       | warehouse 1      |             |             |        |        |        |        |              |        |         |
| CC7   | Finished         | 0          | 0           | 0      | 60     | 0      | 0      | 10           | 0      | 0       |
|       | products         |             |             |        |        |        |        |              |        |         |
|       | warehouse 2      |             |             |        |        |        |        |              |        |         |
| CC8   | Enterprise top   | 20         | 5           | 0      | 0      | 0      | 0      | 0            | 0      | 0       |
|       | management       |             |             |        |        |        |        |              |        |         |
| CC9   | Sales            | 125        | 5           | 0      | 0      | 0      | 300    | 0            | 2      | 0       |
|       | Department 1     |             |             |        |        |        |        |              |        |         |
| CC10  | Sales            | 0          | 0           | 0      | 0      | 60     | 1      | 0            | 0      | 0       |
|       | Department 2     |             |             |        |        |        |        |              |        |         |
| Total production | 510         | 1,402      | 38          | 315    | 60     | 300    | 60     | 59           | 0      | 0       |

Table 6. Intermediate products cost.

|       | CC1  | CC2  | CC3  | CC4  | CC5  | CC6  | CC7  | CC8  | CC9  | CC10 |
|-------|------|------|------|------|------|------|------|------|------|------|
| Unit cost, thous. rub. |     |      |      |      |      |      |      |      |      |      |
| Production cost         | 0.595 | 0.195 | 8.534 | 25.238 | 26.050 | 25.758 | 30.462 | 5.473 |      |      |
The second tool proposed by the authors implies the SLAE building based on the available data (table 4):

\[
\begin{align*}
0^{\text{CC1}}_{\text{unit}} + 328^{\text{CC2}}_{\text{unit}} + 5^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 0^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 185 &= 510^{\text{CC1}}_{\text{unit}} \\
60^{\text{CC1}}_{\text{unit}} + 50^{\text{CC2}}_{\text{unit}} + 8^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 0^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 160 &= 1402^{\text{CC2}}_{\text{unit}} \\
10^{\text{CC1}}_{\text{unit}} + 10^{\text{CC2}}_{\text{unit}} + 0^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 41^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 92 &= 38^{\text{CC3}}_{\text{unit}} \\
200^{\text{CC1}}_{\text{unit}} + 820^{\text{CC2}}_{\text{unit}} + 20^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 3150^{\text{CC4}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 1080 &= 60^{\text{CC5}}_{\text{unit}} \\
50^{\text{CC1}}_{\text{unit}} + 164^{\text{CC2}}_{\text{unit}} + 5^{\text{CC3}}_{\text{unit}} + 15^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 0^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 1100 &= 300^{\text{CC6}}_{\text{unit}} \\
0^{\text{CC1}}_{\text{unit}} + 0^{\text{CC2}}_{\text{unit}} + 0^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 60^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 0^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 210 &= 60^{\text{CC7}}_{\text{unit}} \\
20^{\text{CC1}}_{\text{unit}} + 5^{\text{CC2}}_{\text{unit}} + 0^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 0^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 310 &= 59^{\text{CC8}}_{\text{unit}} \\
125^{\text{CC1}}_{\text{unit}} + 5^{\text{CC2}}_{\text{unit}} + 0^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 300^{\text{CC6}}_{\text{unit}} + 0^{\text{CC7}}_{\text{unit}} + 2^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 960 &= 0^{\text{CC9}}_{\text{unit}} \\
0^{\text{CC1}}_{\text{unit}} + 0^{\text{CC2}}_{\text{unit}} + 0^{\text{CC3}}_{\text{unit}} + 0^{\text{CC4}}_{\text{unit}} + 0^{\text{CC5}}_{\text{unit}} + 0^{\text{CC6}}_{\text{unit}} + 60^{\text{CC7}}_{\text{unit}} + 1^{\text{CC8}}_{\text{unit}} + 0^{\text{CC9}}_{\text{unit}} + 0^{\text{CC10}}_{\text{unit}} + 160 &= 0^{\text{CC10}}_{\text{unit}}
\end{align*}
\]

After conversion and simplification, we obtain the following SLAE:

\[
\begin{align*}
490^{\text{CC1}}_{\text{unit}} - 328^{\text{CC2}}_{\text{unit}} - 5^{\text{CC3}}_{\text{unit}} - 0^{\text{CC4}}_{\text{unit}} - 0^{\text{CC5}}_{\text{unit}} - 0^{\text{CC6}}_{\text{unit}} - 0^{\text{CC7}}_{\text{unit}} - 0^{\text{CC8}}_{\text{unit}} &= 185 \\
-60^{\text{CC1}}_{\text{unit}} + 1352^{\text{CC2}}_{\text{unit}} - 8^{\text{CC3}}_{\text{unit}} - 0^{\text{CC4}}_{\text{unit}} - 0^{\text{CC5}}_{\text{unit}} - 0^{\text{CC6}}_{\text{unit}} - 0^{\text{CC7}}_{\text{unit}} - 0^{\text{CC8}}_{\text{unit}} &= 160 \\
-10^{\text{CC1}}_{\text{unit}} - 10^{\text{CC2}}_{\text{unit}} + 38^{\text{CC3}}_{\text{unit}} - 41^{\text{CC4}}_{\text{unit}} - 0^{\text{CC5}}_{\text{unit}} - 0^{\text{CC6}}_{\text{unit}} - 0^{\text{CC7}}_{\text{unit}} &= 92 \\
-20^{\text{CC1}}_{\text{unit}} - 5^{\text{CC2}}_{\text{unit}} - 0^{\text{CC3}}_{\text{unit}} + 59^{\text{CC4}}_{\text{unit}} - 0^{\text{CC5}}_{\text{unit}} - 0^{\text{CC6}}_{\text{unit}} - 0^{\text{CC7}}_{\text{unit}} &= 310
\end{align*}
\]

Then the system of equations is solved by the Gauss-Seidel method, at the 6th step the iterative process is finished, and the following values are obtained: \( \text{CC1} = 0.595; \text{CC2} = 0.195; \text{CC3} = 8.534; \text{CC4} = 25.238; \text{CC5} = 26.050; \text{CC6} = 25.758; \text{CC7} = 30.462; \text{CC8} = 5.473. \) The results are similar to the previously obtained ones (table 6).

The advantage of this tool lies in the fact that with a small amount of input data, the calculation can be made using the applied mathematical software. It should also be noted that it is possible to reduce any rather complex business situation to the elementary SLAE, and solve it with this algorithm.

The third approach proposed by the authors [15] is to form a specific set of matrices that will be used for further calculations (table 7).
Table 7. Extended tabular representation of the model.

| Resources                                      | Cost for production | Shipping to the third party | Total output | Resource price vectors |
|------------------------------------------------|---------------------|-----------------------------|--------------|------------------------|
| Direct costs                                   | A - a matrix for consumption of semi-finished products and services of own production per unit of output | T - a column vector of shipment of goods (services provision) of own production to the third party | V - a vector-column of enterprise gross output | Ss - a column vector of the cost of produced goods unit |
| Semi-finished products and services of own production | B - a matrix for consumption of feed stocks, materials and energy from the third party per unit of output | - | - | P - a column vector of prices for feed stock, materials, and energy from a third party |
| Feed stocks, materials and energy from the third party | C - a matrix of indirect costs (by elements) for the entire output for a certain period | - | - | - |
| Indirect costs                                 | -                   | -                           | -            | -                      |

Table 7 contains the following designations:

\( P_1, \ldots, P_n \) – the range of products (denominations of the producing subdivisions).

Based on the original data, it is formed a matrix of resource consumption per unit of output for each subdivision.

According to the condition, the production program is 300 tons of products of shop 1 and 60 tons of products from shop 2. The rated values of indirect costs and resources from a third party are represented by the values of their own costs.

First, it is required to determine the gross output in accordance with the formula below:

\[
V = E - A^{-1} * T
\]  

(2)

where \( E \) is a unit matrix (the matrix consisting of units along the main diagonal and zeros).

Then, an intrafactory turnover is calculated:

\[
A' = A * diag(V)
\]  

(3)

where \( diag(V) \) is the diagonal matrix of gross output.

After that, the total cost for production of all types of goods and semi-finished ones is calculated according to the formula:

\[
Ss = diag \ V - A'^{-1} * (R)^T
\]  

(4)

where \( R \) is a row-vector of resources for distribution.

After all calculations, we get the cost values given in the table below.
Table 8. The result of the cost calculation.

| Unit cost, thous. rub. | CC1   | CC2   | CC3   | CC4   | CC5   | CC6   | CC7   | CC8   | CC9   | CC10  |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Production cost       | 0.595 | 0.195 | 8.534 | 25.238| 26.050| 25.758| 30.462| 5.473 | 29.246| 33.2203|

We again obtained values similar to the previously obtained ones using two other tools.

4. Conclusions
Based on the obtained results, we can make the conclusions on the tools used.

The first tool with the use of spreadsheets has a significant disadvantage - the tabular representation is not visual and actually it is a “black box”. This tool can not be used for operational and tactical cost management, and is intended for periodic calculations for long periods, for example, the formation of a business plan for several years ahead, that is, covering only the strategic level of planning.

In view of the tool using the SLAE and iterative calculations is rather time-consuming in use and cannot be represented as a compact mathematical model (multiply expanding with increasing a number of variables), it does not imply operational use. Its main purpose is to make one-time calculations covering long-term intervals (implying the stability of the indicators used) or at the stage of processes design, i.e. at the tactical or strategic levels of management and planning. Due to its unambiguous definiteness from the descriptive point of view (formulas cover the entire technological process), the tool based on matrix algebra can be used both in operational calculations for any periods of time and in strategic planning, which contributes to improving the quality of intracompany planning in industrial enterprises, industries and complexes.

5. Trends of further studies
The trends of further studies are the development of industrial recommendations for using the considered tools. In this regard, a separate problem requiring the study is the process of integration into the existing corporate information systems.

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