Thermal Photon Emission from QGP Fluid

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Abstract

We compare the numerical results of thermal photon distribution from the hot QCD matter produced by high energy nuclear collisions, based on hydrodynamical model, with the recent experimental data obtained by CERN WA80. Through the asymptotic value of the slope parameter of the transverse momentum distribution, we discuss the characteristic temperature of the QCD fluid.

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I. INTRODUCTION

One of the most important problems in the recent high energy physics is to analyze the quark-gluon plasma (QGP) produced in ultra-relativistic nuclear reactions [1].

In a previous paper [2], we formulated a semi-phenomenological quantum transport theory for quark-gluon plasma fluid based on an operator-valued Langevin equation. Giving a mode spectrum and a damping as the input into this theory, we can easily calculate thermodynamical quantities and transport coefficients. Along this line of thought, we have already discussed the space-time evolution of the (1+1)-dimensional viscous quark-gluon plasma fluid with phase transition [3], the (3+1)-dimensional perfect fluid quark-gluon plasma [4] and the baryon-rich quark-gluon plasma [5]. Furthermore we have also analyzed the existing experiments based on the theoretical results of the (3+1)-dimensional perfect fluid model with phase transition [6].

In high energy nuclear collisions, all secondary particles, such as hadrons, leptons and photons, come out from a hot and dense matter which is produced at the earlier stage of the nuclear collision. We first naively expect that the final distribution of those particles directly reflects information of the hot and dense matter, as the particle source. As for hadrons, however, we can say that the final distribution is seriously masked by strong final interactions. For this reason, we prefer to observe leptons and/or photons directly coming from the particle source rather than hadrons themselves, in order to study the particle source in the earlier state at higher temperature and at higher density. On the other hand, we expect to observe a sort of phase transition between the hadron and quark-gluon plasma states in the course of cooling down of the matter from higher to low temperature. For this purpose, the lepton distribution would be the best to be observed, even though the observation must not be so easy from the practical point of view. Instead, if we can distinguish experimentally photons which are thermally emitted from the particle source at higher temperature and higher density, from photons emerging through the process \( \pi^0 \rightarrow 2\gamma \), the observation of these thermal photons would be meaningful.
First imagine that we have a simple case in which the average wavelength of photons be much longer than the size of the matter, even though this case is very far from realistic cases. In this case we can expect to have a simple dipole radiation emitted from an oscillating dipole along the collision axis, and then to observe photons mainly distributing around the transverse direction. The distribution is to be compared with photons coming from the decay of neutral pions, which sharply distribute around the collision axis. This expectation must not be true, because the above assumption that the wavelength is much longer than the particle source is not justified. However, we could expect to observe a certain trend of expansion in the $p_T$-distribution of high energy photons produced in high energy nuclear collisions. In this paper, we examine this kind of trend by making use of numerical simulations of hydrodynamical expansion of high temperature and high density quark-gluon plasma fluid.

Since the thermal photon is considered to keep the information about the early stage of the hot matter produced by relativistic nuclear collisions, many theoretical analyses have already been done. Some groups [8–11] have analyzed the experimental data of CERN WA80 S+Au 200GeV/nucleon (preliminary) [13,14] so as to fit their theoretical model to the thermal photon emission data. Most of these papers except Ref. [11], however, dealt only with the photon spectrum leaving the hadron spectrum not analyzed. In this paper we analyze the photon and the hadron distribution produced by the hot QCD matter in a consistent way.

1) We first choose parameters in the hydrodynamical model so as to reproduce the hadronic spectrum, i.e. the (pseudo-)rapidity distribution and the transverse momentum distribution.

2) We derive the thermal production rate of photons from a unit space-time volume based on the finite temperature field theory.

3) Accumulating the thermal production rate over the whole space-time region covered with the particle source, which is estimated by the hydrodynamical model, we evaluate the thermal photon distribution which is to be compared with the experimental data.
Assuming local equilibrium for the hot QCD matter which will be produced in high energy nuclear collisions, we will apply the hydrodynamical models [4] [6]. As for the equation of state, we discuss two different types of the model, i.e., the QGP fluid model with phase transition between QGP and hadrons and the hot hadron gas model without phase transition. Based on the assumption that the QGP fluid of the dominant mode obeys the operator-valued Langevin equation [2], we can easily deal with the thermal photon and obtain the formula for the production rate. By using the formula, we derive the thermal photon distribution in high-energy nuclear reactions, and compare the theoretical results given by the QGP fluid model with phase transition with the hot hadron gas model without phase transition in detail.

In Sec. II, we shortly review the relativistic hydrodynamical model with phase transition. In Sec. III, using the quantum Langevin equation, we derive the production rate of thermal photons from QCD matter at the high temperature region produced by high energy nuclear reactions. The transverse photon distribution and the asymptotic slope parameter will be obtained in Sec. IV. Section V is devoted to concluding remarks.

We use the natural unit ($\hbar = c = 1$ together with $k_B = 1$) throughout this paper.

**II. HYDRODYNAMICAL MODEL WITH PHASE TRANSITION AND PARTICLE PRODUCTION**

The hydrodynamical equation for perfect fluid is given by

$$\partial_\mu T^{\mu\nu} = 0,$$

and

$$T^{\mu\nu} = (E + P)U^\mu U^\nu - Pg^{\mu\nu},$$

where $E$, $P$, and $U^\mu$ are, respectively, energy density, pressure, and local four velocity. Energy density, pressure, and entropy density are given by

$$E(T) = \frac{1}{2\pi^2} \int_0^\infty p^2 dp \varepsilon(p)n(\varepsilon(p), T),$$
\[ P(T) = \frac{1}{6\pi^2} \int_0^\infty p^2 dp \frac{p^2}{\varepsilon(p)} n(\varepsilon(p), T), \quad \text{(4)} \]
\[ S(T) = \frac{E + P}{T}. \quad \text{(5)} \]

Following Ref. [4], we assume a simple model for the mode spectrum of the fluid

\[ \varepsilon(p) = A\sqrt{p^2 + M^2} \frac{1 - \tanh \frac{T - T_c}{d}}{2} + |p| \frac{1 + \tanh \frac{T - T_c}{d}}{2}. \quad \text{(6)} \]

Here we suppose that the fluid in the QGP phase is dominantly composed by u-, d-, s-quarks and gluons and that the fluid in the hadron phase is dominantly composed by pions and kaons. In this case we put \( A = 1.89, \) \( M = 200 \text{ MeV}, \) \( T_c = 160 \text{ MeV}, \) and \( d = 2 \text{ MeV} \) based on a previous analysis. With these parameters, we obtain the phase transition-like behavior of energy density (see Fig. 1), which seems to reproduce the Lattice QCD result [3].

In a previous paper [6], by making use of the following two models: 1) the QGP fluid model with phase transition, 2) the hot hadron gas model without phase transition, we have analyzed the pseudo-rapidity distribution of charged hadrons in S+Au 200 GeV/nucleon collision obtained by CERN WA80 [12]. In this paper in order to develop the hydrodynamical model furthermore, we are going to analyze the \( p_T \)-distribution of neutral pions also given by CERN WA80 [13] as well as the pseudo-rapidity distribution of charged hadrons. According to the previous analysis [4], we use the first model (the QGP fluid model with phase transition) specified by the initial temperature \( T_i = 195 \text{ MeV} \), the critical temperature \( T_c = 160 \text{ MeV} \), and the freeze-out temperature \( T_f = 140 \text{ MeV} \), and the second model (the hot hadron gas model without phase transition) specified by \( T_i = 400 \text{ MeV} \) and \( T_f = 140 \text{ MeV} \). We choose other parameters being the same as given by Ref. [6]. For these models, we obtain theoretical results of the hadronic spectrum. See Fig. 2 and Fig. 3. From Fig. 2 and Fig. 3, we observe that the both models can consistently reproduce the experimental data. The hot hadron gas model should have the initial energy of the fluid exceeding the total collision energy [6], so that we conclude that the hadron gas model is not accepted.

We discuss the thermal photon distribution based on these two models in Section IV, in which we will see that the hot hadron gas model will fail again in the analyses of the photon
III. THERMAL PRODUCTION RATE OF PHOTONS

Transition amplitude for a process involving emission of a photon with momentum $k$ and polarization $\varepsilon^{(\lambda)}$ from the QCD matter is given by

$$T_{\beta\alpha} = \langle \beta; k, \varepsilon^{(\lambda)}; \text{out} | \alpha; 0; \text{in} \rangle,$$

(7)

where $| \alpha; 0; \text{in} \rangle$ and $| \beta; k, \varepsilon^{(\lambda)}; \text{out} \rangle$ stand for, respectively, the initial state and the final state. Usual technique of the reduction formula enables us to give the following transition amplitude

$$T_{\beta\alpha} = \langle \beta; \text{out} | a^{(\lambda)}_{\text{out}}(k) | \alpha; \text{in} \rangle = ig^{\lambda\nu} \varepsilon^{(\lambda)}_{\mu} \int \frac{d^4x}{(2\pi)^3 2k_0} e^{ikx} \langle \beta; \text{out} | j^\mu_H(x) | \alpha; \text{in} \rangle = ig^{\lambda\nu} \varepsilon^{(\lambda)}_{\mu} \int \frac{d^4x}{(2\pi)^3 2k_0} e^{ikx} \langle \beta; \text{in} | T(S j^\mu_I(x)) | \alpha; \text{in} \rangle,$$

(8)

from which we obtain transition probability per unit space-time volume

$$R(k, \varepsilon^{(\lambda)}) = \frac{1}{VT} | T_{\beta\alpha} |^2 = \frac{\varepsilon^{(\lambda)}_{\mu} \varepsilon^{(\lambda)}_{\nu}}{VT} \int \frac{d^4x d^4y}{(2\pi)^3 2k_0} e^{-ik(x-y)} \times \langle \alpha; \text{in} | T^\dagger(S j^\mu_I(x)) | \beta; \text{in} \rangle \langle \beta; \text{in} | T(S j^\nu_I(y)) | \alpha; \text{in} \rangle.$$

(9)

Since we are interested in one photon state, we should sum up with respect to the final states of QCD matter denoted by $\beta$. To the lowest order in QED, we obtain

$$R(k, \varepsilon^{(\lambda)}) = \frac{\varepsilon^{(\lambda)}_{\mu} \varepsilon^{(\lambda)}_{\nu}}{VT} \int \frac{d^4x d^4y}{(2\pi)^3 2k_0} e^{-ik(x-y)} \langle \alpha; \text{in} | j^\mu(x) j^\nu(y) | \alpha; \text{in} \rangle.$$

(10)

Assuming that a certain mode is dominantly excited in a local equilibrium system of the hot QCD matter and the canonical operator of that mode obeys the quantum Langevin equation [2] (see Appendix), then we can replace the above matrix element with the ensemble average in the sense of quantum Langevin equation, i.e.,
\[ \langle \alpha; in \mid j^\mu(x)j^\nu(y) \mid \alpha; in \rangle \implies \langle j^\mu_{Q.L.}(x)j^\nu_{Q.L.}(y) \rangle_{Q.L.}, \tag{11} \]

where subscript Q.L. represents the ensemble average. Through this paper, we will omit the subscript Q.L. for the simplicity.

In the case of bosonic mode (pion or kaon) in the hadron phase, the source function is given by

\[ j^\mu(x) = i : \phi^\dagger(x)(\hat{\partial}^\mu - \partial^\mu)\phi(x) :. \tag{12} \]

The boson field operator in the theory of the quantum Langevin equation is represented as

\[ \phi(x) = \int \frac{d^3p}{\sqrt{(2\pi)^32\Omega(p)}} \int_0^\infty d\omega \sqrt{\frac{\gamma(T)}{2\pi}} \rho(p,\omega) \]
\[ \times \left( \frac{A(p,\omega)}{\omega - E(p,T)} e^{-ipx} + \frac{B^\dagger(p,\omega)}{\omega - E^*(p,T)} e^{ipx} \right), \tag{13} \]
\[ E(p,T) = \varepsilon(p) - \frac{i}{2} \gamma(T), \tag{14} \]

where \( \varepsilon(p) = \sqrt{p^2 + M^2} \) and \( \gamma(T) = cT \) are, respectively, the mode spectrum and the damping, which we have given to the theory as input. The free parameter \( c \) in \( \gamma(T) \) is so chosen as to fit the photon transverse momentum distribution to the experimental data: we have determined \( c = 0.01 \). In the case of fermionic mode (quark) in the QGP phase, a source function is given by

\[ j^\mu(x) = : \bar{\psi}(x)\gamma^\mu\psi(x) :, \tag{15} \]

where

\[ \psi(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3}} \int_0^\infty d\omega \sqrt{\frac{\gamma(T)}{2\pi}} \rho(p,\omega) \]
\[ \times \sum_{r=1,2} \left( u_r(p,\omega) \frac{A_r(p,\omega)}{\omega - E(p,T)} e^{-ipx} + v_r(p,\omega) \frac{B^\dagger_r(p,\omega)}{\omega - E^*(p,T)} e^{ipx} \right). \tag{16} \]

Here we have put \( \varepsilon(p) = |p| \) and \( \gamma(T) = cT \), with \( c = 0.01 \).

Summing up with respect to polarization and wave number, we can obtain the production rate
where we have supposed that the phase transition takes place around critical temperature $T_c$. Based on the Langevin technique mentioned in Appendix, we can easily calculate the following simple model

$$R(T) = R_{\text{had}} \frac{1 - \tanh \frac{T - T_c}{2}}{2} + R_{\text{QGP}} \frac{1 + \tanh \frac{T - T_c}{2}}{2},$$

(18)

where we have supposed that the phase transition takes place around critical temperature $T_c$ within width $d$. In the infinite $T_c$ limit, we can only keep the hot hadron gas model. Based on the Langevin technique mentioned in Appendix, we can easily calculate $R_{\text{had}}$ and $R_{\text{QGP}}$ as

$$R_{\text{had}} = \sum_s e_s^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int_0^\infty \frac{d\omega_1}{2\omega_1} \frac{\gamma(T)}{\omega_1 - E(p_1, T)}^2 \times \int \frac{d^3p_2}{(2\pi)^3} \int_0^\infty \frac{d\omega_2}{2\omega_2} \frac{\gamma(T)}{\omega_2 - E(p_2, T)}^2 \times (2\pi)^4 (-n_b(\omega_1)\{1 + n_b(\omega_2)\}(p_1 + p_2)^2\delta^4(p_1 - p_2 - k)$$

$$- \{1 + n_b(\omega_1)\}n_b(\omega_2)(p_1 + p_2)^2\delta^4(p_1 - p_2 + k)$$

$$- n_b(\omega_1) n_b(\omega_2) (p_1 - p_2)^2\delta^4(p_1 + p_2 - k)), \quad (19)$$

$$R_{\text{QGP}} = \sum_s 3e_s^2 \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3p_1}{(2\pi)^3} \int_0^\infty \frac{d\omega_1}{2\omega_1} \frac{\gamma(T)}{\omega_1 - E(p_1, T)}^2 \times \int \frac{d^3p_2}{(2\pi)^3} \int_0^\infty \frac{d\omega_2}{2\omega_2} \frac{\gamma(T)}{\omega_2 - E(p_2, T)}^2 \times (2\pi)^4 (n_f(\omega_1)\{1 - n_f(\omega_2)\}(8p_1p_2 - 16\sqrt{p_1^2 + p_2^2})\delta^4(p_1 - p_2 - k)$$

$$+ \{1 - n_f(\omega_1)\}n_f(\omega_2)(8p_1p_2 - 16\sqrt{p_1^2 + p_2^2})\delta^4(p_1 - p_2 + k)$$

$$+ n_f(\omega_1) n_f(\omega_2) (8p_1p_2 + 16\sqrt{p_1^2 + p_2^2})\delta^4(p_1 + p_2 - k)). \quad (20)$$

Here the summation $s$ is over the source particle, and $e_s$ stands for the electric charge of a source particle. These formulas contain three kinds of photon emission processes, as shown in Fig. 4. The double line stands for off-shell particles in thermal bath. Taking into account the structure of $R_{\text{had}}$ and $R_{\text{QGP}}$ having factors $\gamma(T)/|\omega - E(p, T)|^2$, we can understand
that the particle spectrum in heat bath is spread due to the presence of random force. The production rate as a function of temperature $T$ is shown in Fig. 5.

In order to obtain the theoretical formula for hydrodynamical quantities and transport-theoretical coefficient, we have assumed that the local system is in uniform thermal-equilibrium on a microscopic space-time scale. On the other hand, the QGP fluid hydrodynamically evolves in the macroscopic space-time region, specified by space-time coordinate $x$. Solving the hydrodynamical equation, we obtain the $x$-dependence temperature $T(x)$ and local four velocity $U^\mu(x)$. Following the well-known procedure, we first relate the production rate in each local system of the QGP fluid with one in the whole center-of-mass system of the QGP fluid. Since the number of photons emitted from the local system is Lorentz-invariant, we obtain the following relation between the local system at space-time point $x$ and the overall center-of-mass system

$$k_0 \frac{d^3 R_{c.m.}}{dk^3} = k'_0 \frac{d^3 R(T(x))}{dk^3} \bigg|_{k'_0 = U^\mu(x)k_\mu}.$$  

(21)

Integrating the above $R_{c.m.}$ over the whole space-time volume in which the particle source exists, we obtain momentum and transverse momentum distributions

$$\frac{d^3 N}{dk^3} = \int d^4 x k'_0 \frac{d^3 R(T(x))}{dk^3} \bigg|_{k'_0 = U^\mu(x)k_\mu},$$  

(22)

$$\frac{1}{k_T} \frac{dN}{dk_T} = \int d\phi dk_L \frac{d^3 N}{dk^3}(k_T, k_L, \phi),$$  

(23)

which are to be compared with experimental data. Here temperature $T(x)$ and local four velocity $U^\mu(x)$ at space-time point $x$ are given by the numerical solution of the hydrodynamical model.

**IV. RESULTS AND DISCUSSION**

Figure 6 shows the numerical results of Eq. (23) compared with the experimental data (S+Au 200 GeV/nucleon collision) obtained by CERN WA80 [15]. The solid curve and the
dotted curve are, respectively, the contribution of the hadron phase region and that of the QGP phase region in the QGP fluid model. The whole thermal photon distribution given by our QGP fluid (with phase transition) model is the sum of them, but we can easily see that the contribution of the QGP phase region is negligibly small. Using our hydrodynamical model for the QGP fluid with phase transition, we also see in Fig. 6 that we reproduce the experimental data of WA80 well. The dashed curve stands for the photon distribution given by our hot hadron gas model. The theoretical curve (the dashed curve) deviates from the experimental data in both absolute value and slope. Our formalism keeps a free parameter $c$ in the damping function which is directly related to the intensity of the photon emission but which is almost independent of the slope. Even if we choose another value for a free parameter $c$, the hot hadron gas model cannot reproduce the experimental data due to the deviation of the slope. Therefore, we say that we reproduce the WA80 experimental data consistently only with the QGP fluid model with phase transition.

Needless to say, the initial temperature or the phase transition temperature are very helpful to understand high energy nuclear reactions, if they are derived from theoretical analyses as mentioned above. We naively expect the thermal photon distribution to directly reflect the characteristic temperature, but we have to pay attention to the following points: 1) temperature $T$ cannot be invariant under Lorentz transformations, as was mentioned in Sec. III, 2) each local system in the QGP fluid has different temperature. For these reasons, we introduce a little mathematical manipulation in the following way. The thermal distribution from the volume element with velocity $v$ and temperature $T$ should read as

$$\exp\left(-\frac{k_0}{T}\right)_{\text{local system}} \Rightarrow \exp\left(-\frac{k_\mu U\mu}{T}\right)_{\text{c.m. system}} = \exp\left(-\frac{k_0 U_0 - k_L U_L - k_T U_R \cos \theta}{T}\right),$$

(24)

where $U\mu = (\frac{1}{\sqrt{1-v^2}}, \frac{v}{\sqrt{1-v^2}})$. In order to pick up the most dominant contribution to the transverse momentum distribution, we put $k_L = 0$ and rewrite Eq. (24) as

$$\exp\left(-\frac{k_T}{\sqrt{1-v^2}}\right) = \exp\left(-\frac{k_T}{\sqrt{1-v^2} \sqrt{1-v^2} T}\right),$$

(25)
by which we can define the effective temperature $T_{\text{eff}}$ of the fluid at the volume element with velocity $v_L, v_T$ and temperature $T$ by

$$T_{\text{eff}} = \sqrt{1 - \frac{v_L^2}{1 - v_T^2}} \sqrt{\frac{1 + v_T}{1 - v_T} T}.$$  \hspace{1cm} (26)$$

Furthermore we have assumed that the largest value of $T_{\text{eff}}$ dominates in the slope of the transverse momentum distribution for extremely large $k_T$ region. The transverse momentum distribution at rapidity $y = 0$ and its slope parameters are, respectively, shown in Fig. 7 and Fig. 8. Table 1 shows the maximum $T_{\text{eff}}$ given by our numerical results of hydrodynamical simulation and the slope parameter at $k_T = 20$ GeV for the above two models. We see in Fig. 8 that the slope parameter $T_s$ of each model asymptotically tends to maximum value of $T_{\text{eff}}$ shown in Table 1. Through comparison of $T_{\text{eff}}$ evaluated by the numerical results of hydrodynamical simulation with the asymptotic slope parameter $T_s$ of the transverse momentum distribution in Table 1, we know that the asymptotic slope parameter $T_s$ has possible origins different from each other for the above two models: The critical temperature dominates the asymptotic slope parameter in the QGP fluid model with phase transition, while the initial temperature dominates the asymptotic slope parameter in the hot hadron gas model without phase transition.

**V. CONCLUDING REMARKS**

We have derived the thermal photon distribution emitted from a hot matter produced by the high energy nuclear collisions, based on hydrodynamical model, and compared these theoretical results with S+Au 200 GeV/nucleon data obtained by CERN WA80. We have observed that only the QGP fluid (with phase transition) model can consistently reproduce the above experimental data. Furthermore we have determined the asymptotic slope parameter of the transverse photon distribution through comparison of our numerical results of hydrodynamical simulation with the thermal photon distribution.

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APPENDIX : QUANTUM LANGEVIN EQUATION

We introduce a quantum Langevin equation for the annihilation operator

\[
i \frac{d}{dt}a(p, t) = \int K(p, t')a(p, t')dt' + f(p, t), \tag{27}\]

and its hermit conjugate, where \(K(p, t)\) and \(f(p, t)\) are input parameter and random force, respectively. Canonical operators \(a(p, t)\) and \(a^\dagger(p, t)\) should satisfy following two requirements:

1) Equal time commutation relation

\[
[a(p, t), a^\dagger(p, t)] = \delta^3(p - p'), \tag{28}\]

where \(\mp\) stand for boson and fermion respectively.

2) The Kubo-Martin-Schwinger condition

\[
\langle a^\dagger(p, t)a(p', t' + i\beta)\rangle = \langle a(p', t')a^\dagger(p, t)\rangle e^{i\beta\mu}. \tag{29}\]

We can fix the detailed properties of \(a\) and \(f\) so as to satisfy above requirements.

The random force operator can be expanded as

\[
f(p, t) = \int_{-\infty}^{+\infty} d\omega' [\frac{\gamma(p, \omega')}{2\pi}]^{1/2} \rho(p, \omega')A(p, \omega')e^{-i\omega't}, \tag{30}\]

where \(A(p, \omega)\) is a canonical random operator whose canonical commutation relation is given by

\[
[A(p, \omega), A^\dagger(p', \omega')] = \delta^3(p - p')\delta(\omega - \omega'). \tag{31}\]

The stationary solution of Eq. (27) is given as
\[ a(p, t) = \int_{-\infty}^{\infty} d\omega \sqrt{\frac{\gamma(p, \omega)}{2\pi}} \frac{\rho(p, \omega) A(p, \omega)}{\omega - E(p, \omega)} e^{-i\omega t}. \] (32)

The Kubo-Martin-Schwinger condition (29) can be satisfied by the following ansatz:

\[ \langle A(p, \omega) \rangle = \langle A^\dagger(p, \omega) \rangle = 0, \]
\[ \langle A^\dagger(p, \omega) A(p', \omega') \rangle = \delta^3(p - p') \delta(\omega - \omega') n(\omega, T), \]
\[ \langle A(p, \omega) A(p', \omega') \rangle = \delta^3(p - p') \delta(\omega - \omega') [1 + \xi n(\omega, T)], \]
\[ \langle A(p, \omega) A(p, \omega) \rangle = \langle A^\dagger(p, \omega) A^\dagger(p, \omega) \rangle = 0, \] (33)

\[ n(\omega, T) = \frac{1}{\exp\left(\frac{\omega - \mu}{T}\right) - \xi}, \] (34)

where \( \xi \) takes +1 for boson and −1 for fermion.

From (30) and (33), quantum fluctuation-dissipation theorem for the operator-valued Langevin equation (27) is obtained

\[ \langle f(k, t) \rangle = \langle f^\dagger(p, t) \rangle = 0 \]
\[ \langle f^\dagger(p, t) f(p', t') \rangle = \delta^3(p - p') \int_0^{\infty} \frac{d\omega}{2\pi} \rho^2(p, \omega) \gamma(p, \omega) n(\omega, T) e^{-i\omega(t-t')} \]
\[ \langle f(p, t) f^\dagger(p', t') \rangle = \delta^3(p - p') \int_0^{\infty} \frac{d\omega}{2\pi} \rho^2(p, \omega) \gamma(p, \omega) [1 + \xi n(\omega, T)] e^{+i\omega(t-t')} \]
\[ \langle f(p, t) f(p', t') \rangle = \langle f^\dagger(p, t) f^\dagger(p', t') \rangle = 0 \] (35)

We have formulated a quantum Langevin equation consistently. We make thermal field operators for bosons (13) and fermions (16) by means of the above operator \( a(p, t) \) and \( a^\dagger(p, t) \).
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| Model                          | $T$ (MeV) | $v_T$ | $v_L$ | $T_{\text{eff}}$ (MeV) | $T_s$ (MeV) |
|-------------------------------|-----------|-------|-------|------------------------|-------------|
| QGP fluid (QGP phase)         | 157.5     | 0.53  | 0.11  | 280.8                  | 273.2       |
| QGP fluid (hadron phase)      | 157.5     | 0.53  | 0.11  | 280.8                  | 270.5       |
| Hot hadron gas                | 400.0     | 0.0   | 0.0   | 400.0                  | 390.1       |

**TABLE CAPTION**

The maximum $T_{\text{eff}}$ for the QGP fluid model and the hot hadron gas model in our hydrodynamical simulation, and the slope parameters $T_s$ at $k_T = 20$ GeV in Fig. 8. $T_{\text{eff}}$ is evaluated by Eq. (26).

**FIGURE CAPTION**

FIG. 1 The phase transition-like behavior of energy density distribution. The solid curve stands for our QGP fluid model with phase transition, the dotted curve for the fluid of the hot hadron gas composed of massive pions and kaons, and the dashed curve for the fluid of the quark-gluon plasma composed of massless u-, d- s-quarks and gluons. The critical temperature $T_c = 160$ MeV.

FIG. 2 The pseudo-rapidity distribution of charged hadrons in S+Au 200 GeV/nucleon collision. The experimental data was obtained by CERN WA80[12]. The solid curve and the dashed curve stand for, respectively, the QGP fluid model with phase transition and the hot hadron gas model without phase transition.

FIG. 3 The transverse momentum distribution of neutral pions in S+Au 200 GeV/nucleon collision. The experimental data was obtained by CERN WA80[13]. The solid curve and the dashed curve stand for, respectively, the phase transition model and the hot hadron gas model.

FIG. 4 Three kinds of photon emission processes represented by Eqs. (19) and (20). The double line stands for off-shell particles in the heat bath. The wavy line stands for the external line of the photon.
FIG. 5 The production rate as a function of temperature represented by Eq. (18). The solid curve stands for the phase transition model, the dotted curve for the production rate of the hadron phase and the dashed curve for the production rate of quark-gluon plasma phase. The critical temperature $T_c = 160$ MeV.

FIG. 6 The transverse momentum distribution of thermal photon in S+Au 200 GeV/nucleon collision. The upper limit of experimental data was estimated by CERN WA80[15]. The solid curve and the dotted curve are, respectively, the contribution of the hadron phase region and the QGP phase region in the QGP fluid model with phase transition. The dashed curve is the photon distribution of the hot hadron gas model without phase transition.

FIG. 7 The transverse momentum distribution at rapidity $y = 0$. The solid curve and the dotted curve are, respectively, the contribution of the hadron phase region and the QGP phase region in the phase transition model. The dashed curve is the photon distribution of the hot hadron gas model.

FIG. 8 The slope parameter of the transverse momentum distribution which is shown in Fig. 7. The solid curve and the dotted curve stand for, respectively, the hadron phase and the QGP phase in the QGP fluid model with phase transition. The dashed curve is the photon distribution of the hot hadron gas model without phase transition. The solid bold line and the dashed bold line stand for, respectively, $T_{eff}$ of the QGP fluid model and $T_{eff}$ of the hot hadron gas model. The values of the asymptotic slope parameter $T_{eff}$ are evaluated by Eq. (26) and shown in Table 1.
Fig. 1
Phase transition scenario
hadron gas scenario
experimental data (charged hadron)

S+Au 200 GeV/A data from WA80
Fig. 3

The graph shows the 

\[ \frac{1}{p_T} \frac{dN}{dP_T} \text{(GeV}^2) \]

as a function of 

\[ p_T \text{ (GeV)} \]

with data from WA80 for S+Au 200 GeV/A. The graph includes three scenarios:

- Solid line: phase transition scenario
- Dashed line: hadron gas scenario
- Diamond markers: experimental data π0
Fig. 4

off-shell source

photon
Fig. 5

A graph showing the variation of \( \frac{dR}{dk} \) with temperature \( T \) in MeV. The graph includes curves for QGP fluid with phase transition, QGP phase, and hadron phase. The x-axis represents temperature in MeV, ranging from 100 to 200, and the y-axis represents \( \frac{dR}{dk} \) in \( \text{GeV}^{-1} \), ranging from \( 10^{-8} \) to \( 10^{-2} \).
Fig. 6

$S + Au \ 200 \ \text{GeV/A}$

- hadron phase
- QGP phase
- hot hadron gas model

upper limit
estimated by CERN WA80
Fig. 7

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure7}
\caption{\textbf{Fig. 7} The figure shows the distribution of hadron production as a function of transverse momentum ($k_T$) for different scenarios: hadron gas scenario, hadron phase, and QGP phase. The y-axis represents $\frac{1}{k_T} \frac{dN}{dk_T} \big|_{y=0}$ in units of $(\text{GeV}^{-2})$, while the x-axis represents $k_T$ in units of (GeV).}
\end{figure}
Fig. 8