Maximally symmetric D-branes in gauged WZW models

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Abstract

Gluing conditions are proposed to characterize the D-branes in gauged WZW models. From them the boundary conditions for the group-valued and the subgroup-valued fields are determined. We construct a gauged WZW action for open strings that coincides classically with those written previously, when the gluing conditions are imposed.
1 Introduction

D-branes on group manifolds are characterized by the gluing condition \( J = -R\bar{J} \) at the boundary \([1]-[3]\). It has been shown that the maximally symmetric gluing condition \( J = -\bar{J} \) (in the closed string picture) constrains the end points of open strings to conjugacy classes of the group manifold \([1]\). To make the Wess-Zumino-Witten (WZW) action for open strings well-defined, the position of the conjugacy classes has to be quantized – in perfect agreement with the Cardy boundary states. Related aspects of D-branes on group manifolds have been discussed in \([4]-[13]\) using various approaches.

D-branes on coset models have been studied more recently \([14]-[18]\). It was shown how the D-branes can be given a geometrical interpretation from the point of view of the gauged WZW model. In \([15, 16]\), more general boundary conditions were proposed for the group-valued field \( g \) in the gauged WZW model, and an algebraic classification of Cardy boundary states on a \( G/H \) coset conformal field theory (CFT) has been realized geometrically on the gauged WZW model.

In the \( G/H \) gauged WZW model, the boundary value of the group-valued field \( g \in G \) was suggested to be constrained to a product of two conjugacy classes, one for \( G \) and the other for \( H \) \([13, 16]\). This result was reached essentially by demanding that the vector (diagonal) gauge symmetries of \( g \) in the bulk are conserved when the boundary conditions are imposed. The boundary condition for the gauge field \( A \) was mentioned in \([16]\), but there \( g \) is assumed to be in a single conjugacy class of \( G \) instead of in the product of two conjugacy classes. It is interesting then to see how one can give a consistent description of the boundary conditions for the group-valued field \( g \) and the gauge field \( A \) (or equivalently, for \( g \) and fields \( U, \tilde{U} \)).

As already mentioned, in the WZW model the maximally symmetric D-branes are characterized by the gluing condition \( J = -\bar{J} \). Then the end points of open strings are restricted to the conjugacy classes of the group \( G \) \([1]\). Based on this experience with the boundary WZW model, it is natural to ask if there are similar gluing conditions for the gauged WZW model that encode the boundary conditions for \( g, U, \tilde{U} \). That is, can the corresponding D-branes be characterized by gluing conditions?

Here we propose such gluing conditions for the gauged WZW model that resemble
those for the ordinary WZW model. They enable us to characterize the D-branes explicitly, and to determine the boundary conditions for all fields in the coset model. Our results pertain to the maximally symmetric D-branes in the gauged WZW model. We recover the boundary conditions on \( g \) \[13, 16\], and also extend the result to cover gluing conditions twisted by inner automorphisms of \( G \) reducing to \( H \). We find that in the product of the twisted conjugacy classes, the twistings cancel, and we are left with the same product of (untwisted) conjugacy classes appropriate to the untwisted case.

Furthermore, from our gluing conditions, we construct the gauged WZW action for open strings. The new feature is that the two-forms \( \omega \) depend on the boundary value of one of the \( H \)-valued fields \( U, \tilde{U} \) (\( U \), say). When rewriting our action in terms of the gauge field \( A \), we find that the resulting action coincides classically with that in \[13, 16\]. This is possible because the \( U \)-dependent terms from different sources cancel in a nontrivial way; this indicates that our gluing conditions indeed describe D-branes in the gauged WZW model.

The layout of this note is as follows. In Section 2, we introduce the gluing conditions \( J = -\bar{J} \) and \( K = -\bar{K} \) in the gauged WZW model, and obtain the boundary conditions for fields \( g, U, \tilde{U} \). We then consider twisted gluing conditions, and find that inner automorphisms of \( H \) do not change the boundary restrictions of \( g \) — only \( U, \tilde{U} \) are affected. In Section 3, we construct the gauged WZW action for open strings from our gluing conditions, and compare it with that in \[13, 16\]. A short summary is presented in Section 4 as our conclusion.

## 2 Gluing conditions in the gauged WZW model

The level-\( k \) WZW action for closed strings on a compact, connected, simply connected, Lie group \( G \) may be written

\[
\mathcal{S}^G(g) = \frac{k}{4\pi} \left( \int_{\Sigma} d^2 z L^{\text{kin}}(g) + \int_B \chi(g) \right)
\]

(1)

where \( L^{\text{kin}}(g) = tr(\partial_z g \partial_{\bar{z}} g^{-1}) \), \( \chi(g) = \frac{1}{3} tr(dgg^{-1})^3 \), and \( B \) is a three-dimensional manifold bounded by \( \Sigma \). The chiral currents are

\[
\mathcal{J}_z = -\partial_z g g^{-1}, \quad \bar{\mathcal{J}}_{\bar{z}} = \tilde{g}^{-1} \partial_{\bar{z}} \tilde{g}
\]

(2)
corresponding to the “global” symmetries $g(z, \bar{z}) \to L(z)g(z, \bar{z}), g(z, \bar{z}) \to g(z, \bar{z})R(\bar{z})^{-1}$, respectively.

One may gauge the vector (diagonal) subgroup of this “$G_L(z) \times G_R(\bar{z})$” symmetry. The resulting local symmetry transformation is then $g(z, \bar{z}) \to m(z, \bar{z})g(z, \bar{z})m(z, \bar{z})^{-1}$, with $m$ an element of the subgroup $H \subset G$. Introduce the “gauge fields” $U, \tilde{U} \in H$, transforming as $U \to mU, \tilde{U} \to m\tilde{U}$. A gauge invariant action is

$$S^{G/H} = S^G(U^{-1}g\tilde{U}) - S^H(U^{-1}\tilde{U})$$

Here $S^H$ is defined as in (1) except that the trace used $tr_H\{\cdot\}$ is that appropriate to the Lie algebra $\text{Lie} H$ of $H$, and the level is $k_H$. Let $t_H$ denote an element of $\text{Lie} H$, and let $\epsilon$ be the embedding of $\text{Lie} H$ in $\text{Lie} G$, so that $\epsilon(t_H)$ is the element of the subalgebra $\text{Lie} H \subset \text{Lie} G$ corresponding to $t_H$. Now $tr_G(\epsilon(t_H)) = \iota tr_H(t_H)$, where the subscript $G(t_H)$ is included for clarity, and $\iota$ is the index of the embedding. Therefore, by the construction of (3), $k_H = \iota k$.

Making use of the Polyakov-Wiegmann identities

$$L^\text{kin}(g_1g_2) = L^\text{kin}(g_1) + L^\text{kin}(g_2) - tr(g_1^{-1}\partial_zg_1\partial_\bar{z}g_2g_2^{-1} + g_1^{-1}\partial_\bar{z}g_1\partial_zg_2g_2^{-1})$$
$$\chi(g_1g_2) = \chi(g_1) + \chi(g_2) - dtr(g_1^{-1}dg_1dg_2g_2^{-1})$$

and defining

$$A_z = \partial_\bar{z}\tilde{U}U^{-1}, \quad A_{\bar{z}} = \partial_zUU^{-1}$$

results in the gauged WZW action [19, 20]

$$S^{G/H} = S^G + \frac{k}{2\pi} \int d^2z \text{tr} \left\{ A_z\partial_\bar{z}gg^{-1} - A_\bar{z}g^{-1}\partial_zg + A_\bar{z}gA_zg^{-1} - A_zA_{\bar{z}} \right\}$$

(6)

Note that the vector fields $A_z, A_{\bar{z}}$ take values in $\text{Lie} H$.

Working backwards then, the gauged WZW action (6) may be rewritten as a coset action [19, 20]

$$S^{G/H} = S^G(\tilde{g}) - S^H(h)$$

(7)

with

$$\tilde{g} = U^{-1}g\tilde{U} \in G, \quad h = U^{-1}\tilde{U} \in H, \quad g \in G, \quad U, \tilde{U} \in H$$

(8)
Each WZW action in the gauged WZW action $S^{G/H} = S^G(\tilde{g}) - S^H(h)$ is well defined for the fields $\tilde{g}$ and $h$ \cite{19}, and the associated currents are

$$J = -\partial_z \tilde{g} \tilde{g}^{-1}, \quad \bar{J} = \tilde{g}^{-1} \partial_{\bar{z}} \tilde{g}$$
$$K = \partial_z hh^{-1}, \quad \bar{K} = -h^{-1} \partial_{\bar{z}} h$$

(9)

$J$ and $K$ are associated with the groups $G$ and $H$, respectively, and the sign difference on the currents stems from the minus sign in front of $S^H$ in (7). The two currents are not independent \cite{20}, however, since there is a first class constraint. Recall that the Lie algebra embedding is denoted $\epsilon : \text{Lie} H \rightarrow \text{Lie} G$. The components of $J$ and $\epsilon(\bar{K})$ in the Lie subalgebra $H \subset G$ are clearly related. At the quantum level, the constraint can be expressed as

$$\langle \psi' | \text{tr} \{\epsilon(t_H)(J + \epsilon(K))\} | \psi \rangle = 0$$

(10)

Here $\langle \psi |, \langle \psi' |$ are ghost-free states and $t_H$ is any element of the Lie algebra of $H$. In terms of $J$ and $K$, the conformal stress-tensor is $T = \frac{1}{2k+c_G} : \text{tr}_G(JJ) : -\frac{1}{2k_H+c_H} : \text{tr}_H(KK) :$, $c_G$ and $c_H$ are the eigenvalues of the quadratic Casimir operators in the adjoint representation of $G$ and $H$, respectively. Exploiting the constraint, we have $\langle \psi' | T(z) | \psi \rangle = \langle \psi' | T^G(z) - T^H(z) | \psi \rangle$ with $T^G = \frac{1}{2k+c_G} : \text{tr}_G(JJ) :$ and $T^H = \frac{1}{2k+c_H} : \text{tr}_G(\epsilon(K)\epsilon(K)) :$. This is a field-theoretic version of the GKO stress-tensor, confirming that the current $K$ is related to the current of the Kac-Moody subalgebra via the constraint \cite{21}.

In order to describe maximally symmetric D-branes in the gauged WZW model, we will henceforth consider world sheets $\Sigma$ with nonvanishing boundary, $\partial \Sigma \neq 0$. We propose the gluing conditions

$$J|_{\partial \Sigma} = -R\bar{J}|_{\partial \Sigma}, \quad K|_{\partial \Sigma} = -R\bar{K}|_{\partial \Sigma}$$

(11)

with $R$ an automorphism of the Lie algebra of $G$ reducing to $H$. When $g = 1$, $\tilde{g}$ reduces to $h$, so the gluing condition $K|_{\partial \Sigma} = -R\bar{K}|_{\partial \Sigma}$ can be induced from $J|_{\partial \Sigma} = -R\bar{J}|_{\partial \Sigma}$. Initially, we shall focus on $R = 1$:

$$J|_{\partial \Sigma} = -\bar{J}|_{\partial \Sigma}, \quad K|_{\partial \Sigma} = -\bar{K}|_{\partial \Sigma}$$

(12)

\footnote{For simple illustration, we have omitted the contribution from the ghost fields.}
In that case, we see that the gluing conditions may be recast into
\[
\begin{align*}
\tilde{g}^{-1}\partial_{\tau}\tilde{g} &= \frac{1 + \text{Ad}(\tilde{g})}{1 - \text{Ad}(\tilde{g})}\tilde{g}^{-1}\partial_{\sigma}\tilde{g} \\
h^{-1}\partial_{\tau}h &= \frac{1 + \text{Ad}(h)}{1 - \text{Ad}(h)}h^{-1}\partial_{\sigma}h
\end{align*}
\] (13)

with \(\text{Ad}(g)y = g yg^{-1}\). The coordinates are related by \(\partial_{z} = \partial_{\tau} + \partial_{\sigma}\), \(\partial_{\bar{z}} = \partial_{\tau} - \partial_{\sigma}\).

The expressions (13) are sensible because, as in the WZW model [1], when \(\tilde{g}\) and \(h\) are restricted to conjugacy classes of \(G\) and \(H\), respectively, the operators \((1 - \text{Ad}(\tilde{g}))\) and \((1 - \text{Ad}(h))\) are invertible when acting on \(\tilde{g}^{-1}\partial_{\sigma}\tilde{g}\) and \(h^{-1}\partial_{\sigma}h\). On the boundary, we may therefore choose the parametrizations
\[
\begin{align*}
\tilde{g}(\tau) &= (U^{-1}n)f(U^{-1}n)^{-1}(\tau), \quad n, f \in G \\
h(\tau) &= (U^{-1}p)l^{-1}(U^{-1}p)^{-1}(\tau), \quad p, l \in H
\end{align*}
\] (14)

with \(U \in H\). We choose this parametrization so that the boundary value of \(g\) in (15) below agrees with that in [16], up to the change in notation replacing their \(k\) with \(n\).

Exploiting (8), we then arrive at
\[
g(\tau) = nf^{-1}p^{-1}(\tau)
\] (15)

and
\[
\tilde{U}(\tau) = pl^{-1}p^{-1}U(\tau)
\] (16)

Thus, the boundary condition (15) for \(g\) derived from our gluing conditions (12) agrees exactly with that of [15, 16]: \(g\) is restricted to a product of two conjugacy classes, one for \(G\) and one for \(H\). Furthermore, and as expected, we see that the fields \(U\) and \(\tilde{U}\) are related on the boundary (16).

Now, since all elements of \(G\) and \(H\) are conjugate to such elements, we can put
\[
f = e^{2\pi i \lambda_{G}/k} \text{ and } l = e^{2\pi i \lambda_{H}/k_{H}} \text{ in (15)}, \] where \(\lambda_{G}\) and \(\lambda_{H}\) are elements of the Cartan

\footnote{If we choose to write expressions for the fields in terms of \(\tilde{U}\), instead of \(U\), we find that \(g\) is a product of elements of conjugacy classes of \(H\) and \(G\), instead of the reverse order: \(G\) and \(H\). The two conjugacy classes are identical in the two cases, however. This is important for the Cardy correspondence between boundary conditions and bulk primary fields, as we'll see below.}
subalgebras of the Lie algebras of $G$ and $H$, and the factors $2\pi/k,2\pi/k_H$ are included for later convenience. Extended Weyl invariance implies that we can restrict $\lambda_G$ to

$$\bar{P}^k(G) = \{\lambda_G | 0 \leq \alpha(\lambda) \leq k, \forall \alpha \in R_>(G)\}$$  \hspace{1cm} (17)$$

where $R_>(G)$ denotes the set of positive coroots of Lie $G$. Similarly, we can restrict to $\lambda_H \in \bar{P}^k_H(H)$.

Let us now consider the twisted gluing conditions

$$J = -\text{Ad}(r)J, \quad K = -\text{Ad}(r)K$$  \hspace{1cm} (18)$$

where $r$ is a fixed but arbitrary element of $H$. $R$ is thus chosen to be an inner automorphism of the Lie algebra of $G$ reducing to $H$ \cite{11}. In this case, the elements $\tilde{g}$ and $h$ are constrained to twisted conjugacy classes of $G$ and $H$ at the boundary, respectively, and may be parametrized as

$$\tilde{g}(\tau) = (U^{-1}n)f(U^{-1}n)^{-1}(\tau)r$$

$$h(\tau) = (U^{-1}p)l^{-1}(U^{-1}p)^{-1}(\tau)r$$  \hspace{1cm} (19)$$

As before, these boundary conditions are easily translated into those on $g,U,\tilde{U}$:

$$g(\tau) = nf n^{-1}p l p^{-1}(\tau)$$  \hspace{1cm} (20)$$

and

$$\tilde{U}(\tau) = p l^{-1} p^{-1} U(\tau)r$$  \hspace{1cm} (21)$$

We see that twisting the gluing conditions by an inner automorphism of the Lie algebra of $H$ does not affect the boundary conditions on $g$, even though the boundary conditions in each of the sectors (represented by $\tilde{g},h$) are affected \cite{13}. The boundary value of $\tilde{U}$ depends on the automorphism through \cite{21}.

3 Gauged WZW action for open strings

Let us turn to the gauged WZW action for open strings. The WZW term is not well-defined for a worldsheet $\Sigma$ with a boundary. The remedy is to introduce an auxiliary disc
$D$ for each hole in $\Sigma$ with boundaries common with those of $\Sigma$. For simplicity, we consider the situation with a single hole. The map $g$ from $\Sigma$ to $G$ is then extended \cite{footnote} to a map from the extended worldsheet $\Sigma \cup D$. The disc $D$ is mapped into the conjugacy classes for $\tilde{g}$ and $h$ on the boundary. Following the strategy in \cite{footnote} and exploiting our gluing conditions, we propose

$$S^{G/H} = \frac{k}{4\pi} \left( \int_{\Sigma} L_{\text{kin}}(\tilde{g}) + \int_{B} \chi(\tilde{g}) - \int_{D} \omega(\tilde{g}) \right) - \frac{k_{H}}{4\pi} \left( \int_{\Sigma} L_{\text{kin}}(h) + \int_{B} \chi(h) - \int_{D} \omega(h) \right)$$

(22)

as the gauged WZW action for open strings. $B$ is a three-dimensional manifold bounded by $\Sigma \cup D$. $\omega(\tilde{g})$ and $\omega(h)$ are the Alekseev-Schomerus two-forms defined on the conjugacy classes of $G$ and $H$, respectively. With our parametrizations, they become

$$\omega(\tilde{g}) = \text{tr} \left\{ (U^{-1}n)^{-1}d(U^{-1}n)f(U^{-1}n)^{-1}d(U^{-1}n)f^{-1} \right\}$$

$$\omega(h) = \text{tr} \left\{ (U^{-1}p)^{-1}d(U^{-1}p)l^{-1}(U^{-1}p)^{-1}d(U^{-1}p)l \right\}$$

(23)

On the conjugacy classes, the three-forms $\chi$ satisfy

$$d\omega(\tilde{g}) = \chi(\tilde{g}), \quad d\omega(h) = \chi(h)$$

(24)

The action (22) is ambiguous, since one could choose a different auxiliary disk $D'$, while keeping the same boundary conditions on $\partial \Sigma$. Topologically, $D \cup (-D') \cong S^2$, and so the difference in (22) with the two choices is related to embeddings of $S^2$ into $G$ and $H$, subject to the boundary conditions on $\tilde{g}$ and $h$, respectively. As explained in \cite{footnote1, footnote2}, these latter are characterized by coroot lattice vectors $s_{G}$ of $G$, and $s_{H}$ of $H$, respectively. The corresponding ambiguities in the action (22) are then \cite{footnote1, footnote2}

$$\Delta_{G}s^{G/H} = \frac{k}{4\pi} \left( \int_{B} \chi(\tilde{g}) - \int_{S^2} \omega(\tilde{g}) \right) = 2\pi s_{G}(\lambda_{G})$$

(25)

and

$$\Delta_{H}s^{G/H} = \frac{k_{H}}{4\pi} \left( \int_{B} \chi(h) - \int_{S^2} \omega(h) \right) = 2\pi s_{H}(\lambda_{H})$$

(26)

\footnotemark[3]

\footnotetext[3]{By a common abuse of notation, we use the same symbols to denote such extensions as well as the original maps.}
Single-valuedness of path integrals involving the action \((22)\) therefore leads to
\[
\alpha_G(\lambda_G) \in \mathbb{Z} \tag{27}
\]
\[
\alpha_H(\lambda_H) \in \mathbb{Z} \tag{28}
\]
for any coroots \(\alpha_G\) and \(\alpha_H\) of the Lie algebras of \(G\) and \(H\). Therefore \(\lambda_G\) and \(\lambda_H\) are quantized to be associated with integral weights of those Lie algebras. Furthermore, the allowed set
\[
P^k(G) = \{\lambda_G | \alpha(\lambda_G) \in \{0, 1, \ldots, k\}, \forall \alpha \in R_> (G)\} \tag{29}
\]
lables the set of bulk primary fields in the WZW model, and similarly for \(\lambda_H\) in \(P^k_H(H)\).

Now, pairs of such Cartan subalgebra elements (and so their associated weights) \(\{\lambda_G, \lambda_H\}\) label the bulk coset primary fields. In the simplest cases, this verifies Cardy’s assertion that the boundary conditions are in one-to-one correspondence with bulk primary fields. When \(G\) and \(H\) share central elements, however, there are the complications of selection rules and their dual field identifications, and there is a so-called fixed point problem in some cosets. All these were discussed in [15, 16]. Remarkably, Cardy’s correspondence continues to hold. Possible exceptions are the so-called maverick cosets [22], that are problematic even in the closed string case.

In summary, the gluing condition \(J = -\bar{J}\) \((K = -\bar{K})\) implies that \(\tilde{g} = U^{-1}g\tilde{U}\) \((h = U^{-1}\tilde{U})\) is restricted to a conjugacy class of the group \(G\) (subgroup \(H\)), labelled by \(\lambda_G \in \tilde{P}^k(G)\) \((\lambda_H \in \tilde{P}^k_H(H))\). The constraint (14) then indicates that \(g\) is restricted to be a product of elements labelled by \(\lambda_G\) and \(\lambda_H\). The quantization conditions (27) and (28) then show that we can label the boundary states in the coset theory by \(\{\lambda_G, \lambda_H\}\) with \(\lambda_G \in P^k(G)\) and \(\lambda_H \in P^k_H(H)\). Our gluing conditions \(J = -\bar{J}\) and \(K = -\bar{K}\), with the quantization conditions (27) and (28), verify Cardy’s correspondence.

The local gauge transformation
\[
g \rightarrow mgm^{-1}, \quad U \rightarrow mU, \quad \tilde{U} \rightarrow m\tilde{U} \tag{30}
\]
with \(m \in H\), reduces on the boundary to
\[
n \rightarrow mn, \quad p \rightarrow mp, \quad U \rightarrow mU, \quad \tilde{U} = pl^{-1}p^{-1}U \rightarrow mpl^{-1}p^{-1}U \tag{31}
\]
Since $\omega(\tilde{g})$ and $\omega(h)$ are invariant under the (reduced) local gauge transformations (31), the gauged WZW action for open strings (22) is invariant under the transformation (30).

We may rewrite the gauged WZW action (22) in terms of the gauge fields $A$ and the related $H$-valued fields $U, \tilde{U}$. Classically, i.e., neglecting possible quantum corrections from the path-integral measure, we arrive at

\[
S^G/H = S^G(g) + \frac{k}{2\pi} \int_{\Sigma} d^2z \text{tr} \left\{ A_{\bar{z}} \partial_z gg^{-1} - A_{\bar{z}}g^{-1}\partial_z g + A_{\bar{z}}gA_{\bar{z}}g^{-1} - A_{\bar{z}}A_{\bar{z}} \right\}
\]

\[
+ \frac{k}{4\pi} \int_D d^2z C
\]

with

\[
C = -\omega(\tilde{g}) + \omega(h) - \text{tr} \left\{ g^{-1}dgd\bar{U}U^{-1} - dUU^{-1}(dgg^{-1} + gd\bar{U}\bar{U}^{-1}g^{-1}) \right\}
\]

\[
+ dUU^{-1}d\bar{U}\bar{U}^{-1}
\]  

(32)

Since the images of $D$ are also constrained to the conjugacy classes, we extend the boundary conditions (15) and (16) to $D$. $C$ is defined on $D$ only, so after straightforward calculation, it can be rewritten as

\[
C = - \left( \omega(n) + \omega(p) + \text{tr}(dc_2c_1^{-1}dc_1) \right)
\]  

(34)

Here $c_1 = nf^{-1}$ and $c_2 = plp^{-1}$, elements of conjugacy classes of $G$ and $H$, respectively, and

\[
\omega(n) = \text{tr}(n^{-1}dnf^{-1}dnf^{-1}), \quad \omega(p) = \text{tr}(p^{-1}dplp^{-1}dpl^{-1})
\]  

(35)

Notice that $\omega(n)$ and $\omega(p)$ just written are different from $\omega(\tilde{g})$ and $\omega(h)$ given in (23) above. Eqs. (34) and (35) show that all the $U$-dependent terms in (33) cancel in a nontrivial way. Inserting (34) in the action (32), we recover the action of [15, 16] (see (2.9) of [15] and (3.11) of [16]). Thus, the open string action (22) constructed from our gluing conditions can be reduced classically to those written previously in a different approach.

To conclude this section, let us discuss the relation of the gluing conditions to the currents

\[
j = -\partial_z gg^{-1} - gA_{\bar{z}}g^{-1} + A_{\bar{z}}
\]

\[
\tilde{j} = g^{-1}\partial_z g - g^{-1}A_{\bar{z}}g + A_{\bar{z}}
\]  

(36)
defined by varying the gauge fields $A$ \[21\] in a gauged WZW action. One might hope that our gluing conditions could be expressed in a natural way using $j, \bar{j}$. This does not turn out to be the case, however. The only simple relations between the currents $j, \bar{j}$ and $J, \bar{J}, K, \bar{K}$ are

$$ U^{-1}jU = J + K $$
$$ \tilde{U}^{-1}\bar{j}\tilde{U} = \bar{J} + \bar{K} $$

and they also involve the fields $U, \tilde{U}$. The complete set of gluing conditions \[12\] cannot be rewritten in terms of $j, \bar{j}$, only, and so D-branes in the gauged WZW model cannot be characterized in that manner. While the gluing conditions can be rewritten in terms of $j, \bar{j}$ and $U, \tilde{U}$, their form is not enlightening.

### 4 Conclusion

We have proposed the gluing conditions $J = -\bar{J}$ and $K = -\bar{K}$ to describe D-branes in the gauged WZW model. The gauged WZW action can be written as $S^{G/H} = S^G(\tilde{g}) - S^H(h)$, showing that the fields $\tilde{g}, h$ formally decouple – we can therefore introduce two currents $J$ and $K$ for the group $G$ and the subgroup $H$. The boundary conditions for $g, U, \tilde{U}$ have been derived from the gluing conditions, and the boundary condition on $g$ proposed in \[15, 16\] has been recovered.

We have also considered gluing conditions twisted by inner automorphisms of $G$ reducing to $H$. It was found that in the product of the twisted conjugacy classes, the twistings cancel, and we are left with the same product of (untwisted) conjugacy classes as in the untwisted case.

From our gluing conditions, we have constructed the gauged WZW action for open strings, with the new feature that the two-forms $\omega$ depend on the boundary value of one of the $H$-valued fields $U, \tilde{U}$. When rewriting our action in terms of the gauge field $A$, we have shown that all the $U$-dependent terms cancel each other in a nontrivial way, and the resulting action coincides classically with that in \[13, 16\]. This indicates that these gluing conditions indeed describe D-branes in the gauged WZW model.
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