Bell’s inequality for conditional probabilities as a test for quantum-like behaviour of mind

Andrei Khrennikov
International Center for Mathematical Modeling in Physics and Cognitive Sciences, MSI, University of Växjö, S-35195, Sweden
Email: Andrei.Khrennikov@msi.vxu.se

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Abstract

We define quantum-like probabilistic behaviour as behaviour which is impossible to describe by using the classical probability model. We discuss the conjecture that cognitive behaviour is quantum-like. There is presented the scheme for an experimental test for quantum-like cognitive behaviour based on a generalization of the famous Bell’s inequality. This generalization is an analogue of Bell’s inequality, but for conditional probabilities. The use of conditional probabilities (instead of simultaneous probability distributions for pairs of observables in the original Bell’s inequality) gives the possibility to separate two problems which are mixed in the original Bell’s framework: nonlocality and nonclassical (quantum-like) probabilistic behaviour. Our inequality for conditional probabilities can be used for experiments with a single system (so we need not to prepare pairs of correlated systems) to find quantum-like behaviour. This possibility is extremely important in cognitive sciences where it is practically impossible to prepare pairs of precisely correlated cognitive systems. Thus the test of the original Bell’s inequality in cognitive and social sciences, or psychology is the problem of huge complexity. On the other hand, our test based on an analogue of Bell’s inequality for conditional probabilities can be easily performed in, e.g., psychological experiments or cognitive tasks.
1 Introduction

It seems that the philosophic system of Whitehead [1]-[3] was the first attempt to establish a quantum–mental (or more precisely mental → quantum) connection. In Whitehead’s philosophy of the organism “quantum” was some feature of basic protomental elements of reality [1], especially p. 401-403. See also A. Shimony [4] for modern reconsideration of quantum counterpart of Whitehead’s philosophy of organism. Unfortunately, the main stream of “quantum-mental” investigations was directed to quantum reductionism of mental processes, see [5]-[19]. I do not want to criticize attempts to perform quantum-reduction of mental processes – reduction of mind to quantum physical processes. It is clear that this is the great program of investigations and it is too early to say anything. I just want to pay attention that, e.g., Whitehead did not have in mind quantum physical reduction of mental processes when he wrote about quantum protomental elements of reality. It is also important to mention Bohr’s ideas on complementarity in physics, psychology and other domains of science [20] (see [21] for detailed analysis). We recall the well known fact that, in fact, N. Bohr “borrowed” the principle of complementarity from psychology. In particular, reading of “Principles of Psychology” of W. James was very important for N. Bohr. But N. Bohr neither had in mind quantum-reductionism. He just wanted to emphasize that some fundamental principles of quantum mechanics can be valid outside of physics. The crucial for N. Bohr was the special information structure of measurement processes in some domains of science which can differ essentially from measurement processes in classical physics. We also pay attention to a correspondence between W. Pauli and C. G. Jung. They neither come to quantum-reductionist conclusions. We recall a practically forgotten paper of V. Orlov [22] who proposed to use quantum logic to describe functioning of brain. Orlov was not looking for reduction of mental processes to quantum physical processes. He considered brain as an information system which can in principle use rules of quantum logic. And, finally, we mention investigations of Bohm-Hiley-Pylkkänen [23]–[25] who proposed to apply the formalism of quantum pilot wave (Bohmian mechanics) to mental processes. They did not try to reduce mental processes to pilot waves coupled to quantum particles composing the brain. BHP also considered brain as a special information system which can be described by Bohmian mechanics (see [26], [27] for a rather risky application of BHP-ideas).

I also think that the most profitable applications of quantum methods to cognitive and social sciences and psychology can be obtained not via quan-

\footnote{1which was written in a concentration camp in 60th and published a few years later.}
tum physical reduction, but by considering cognitive systems as a complex information systems such that some laws of quantum theory can be applied to these systems (at least with some modifications and generalizations). At this point it is extremely important to understand: What is quantum mechanics about?, see [28]. What kind of laws of nature are encoded in quantum mathematical formalism? Despite one hundred years of development and great applications of quantum theory, there are still no commonly accepted answers to these questions. Quantum theory is characterized by a huge diversity of interpretations, see [29]. My experience as an organizer of a series of large conferences on foundations of quantum mechanics says that practically everybody elaborated its own specific interpretation of quantum mechanics. Even if people use the same name, e.g., Copenhagen interpretation, in reality they can use extremely different versions. I think that this diversity of interpretations is a sign of a deep crises in foundations of quantum mechanics.

Personally I use so called ensemble interpretation of quantum mechanics by which quantum mechanics gives us laws of transformation of probability distributions for ensembles of physical systems. A. Einstein was one of the strongest supporters of this interpretation, see also Margenau [30], Ballentine [31]. If we use this interpretation then at the first sight quantum mechanics does not differ from classical statistical physics which also gives laws of transformation of probability distributions for ensembles of physical systems. The crucial point is that laws of transformation of probability distributions are different! The main exhibition of quantum probabilistic behaviour is the interference of probabilities of alternatives which can be easily obtained in experiments with quantum systems, e.g., in the well known two slit experiment. This interference gives the experimental basis of the superposition principle. The latter is one of the basic elements of a philosophic system of views to the physical reality which we can call quantum mysticism. Roughly speaking, since an electron can be in a superposition of states, but a car cannot, there is a huge gap between domains of application of quantum and classical probability. There is a rather common opinion that quantum randomness differs crucially from classical randomness. (the first one is “irreducible” and the second one is “reducible”, see, e.g., [32]); that quantum theory works in microworld and classical in macroworld.

These views to quantum theory and especially probability play the fundamental role in the justification of quantum reductionism. Since laws of quantum mechanics work only for quantum particles, if we try to apply these laws to mental processes then we should reduce these processes to quantum physical processes. Some supporters of quantum reductionism reply: “Not everything is so trivial. We do not claim that there is no macroscopic physical structures having quantum behaviour; consider, for example, Bose-Einstein
condensate.” This is an interesting argument, but I cannot take its seriously. Bose-Einstein condensate is a collective effect of quantum systems induced by special “quantum conditions.” By using this example we just disappoint ourselves and scientists working in neurophysiology, cognitive sciences and psychology. In fact, we say to them: “You should look for these special quantum conditions in brain!” (in particular, temperature, space and time scales). I think that R. Penrose is more fair when he wrote that an individual neuron could not be in superposition of two states and therefore we cannot use quantum mechanics on neuronal level [6]. And in his quantum-mental investigations R. Penrose operated on fantastically deep microlevel – level of quantum gravity. Roughly speaking consciousness is generated by collapses of mass superpositions. Of course, there is the huge gap between the levels of quantum cosmology and neurophysiology... Quantum reductionism is totally incompatible with modern neurophysiology which is based on the neuronal model.

It seems that “quantum mysticism” (and, in particular, mystery of quantum probability) is one of the main sources of quantum reductionism for mental processes.

Recently the problem of quantum probabilities was essentially clarified [33]-[35] when the interference of probabilities was obtained by using classical ensemble probabilities (and without to appeal to the formalism of Hilbert space). In this series of papers there was developed a calculus of contextual probabilities – probabilities depending on contexts. Contexts are complexes of physical, cognitive, social, psychological conditions. It was observed that laws of transformation of contextual probabilities coincide (under some conditions) with laws induced by the quantum formalism. But contextual probability is a classical ensemble probability. The crucial point is that context dependence of probabilities can be nontrivial. In this case we cannot use “absolute” context-independent probability measure $P$. We call nontrivial contextual probabilistic behaviour (of physical, cognitive, social systems) quantum-like.

Contextuality of probabilities has no direct coupling with microworld. Even ensembles of macroscopic systems can demonstrate nontrivial contextual probabilistic behaviour. Thus quantum-like behaviour can be in principle found not only for, e.g., electrons or photons, but also for, e.g., human beings or some ensembles of neurons.\footnote{We do not speak about a quantum-like superposition of states of an individual neuron; in our approach interference of probabilities and superposition are ensemble features.}

In [36] it was proposed to investigate probabilistic quantum-like behaviour for cognitive and social systems. As the first step we should really show
that cognitive systems may exhibit quantum-like behaviour. This can be proved only by using experiments (in cognitive science, psychology,...). Thus we need experimental tests of quantum-like behaviour. In [36] I proposed a simple test of quantum-like behaviour based on interference of probabilities for two questions \(a\) and \(b\) which can be asked to people. In [37] there were performed experiments with students giving preliminary confirmation of quantum-like behaviour of people. Up to now there have not been performed experiments with animals, fishes or insects. Thus there is no experimental data on quantum-like behaviour for these biological organisms. It is natural to suppose that only biological organism having a rather high level of mental development can exhibit quantum-like behaviour. For example, we can speculate that insects would always behave classically. We can measure numerically the level of quantum-like behaviour by the magnitude of the coefficient interference of probabilities, see [36].

In this paper I propose a new more complex test of quantum-like behaviour for people (but similar experiments can be also performed for animals) based on three questions (or cognitive tasks) \(a, b, c\). This test is based on a generalization of the famous Bell’s inequality, see, e.g., [38]. This generalization is an analogue of Bell’s inequality, but for conditional probabilities. It was derived in [39]. The use of conditional probabilities (instead of simultaneous probability distributions for pairs of observables in the original Bell’s inequality) gives the possibility to separate two problems which are mixed in the original Bell’s framework: nonlocality and nonclassical (quantum-like) probabilistic behaviour. J. Bell (who was nonlocal realist) wanted to show that quantum world is nonlocal: by performing a measurement on one of correlated particles we change the state of the second particle. We are not interested in the use of Bell’s inequality for investigation of mental nonlocality.\(^3\) We are very pragmatic. We “just” want to show that some cognitive systems are quantum-like. This will give us the possibility to apply powerful methods of quantum mechanics to cognitive and social sciences, psychology, economy....

Our inequality for conditional probabilities can be used for experiments with a single cognitive system to find quantum-like behaviour (so we need not to prepare pairs of “correlated cognitive systems”). This possibility is extremely important in cognitive sciences where it is practically impossible to prepare pairs of precisely correlated cognitive systems. Thus the test of the original Bell’s inequality in cognitive and social sciences, or psychology is the problem of huge complexity. On the other hand, our test based on an analogue of Bell’s inequality for conditional probabilities can be easily

\(^3\)We do not reject such a possibility, see, e.g., [40] on interesting results in this direction.
performed in psychological experiments or cognitive tasks for animals and people.

We hope that this paper will attract attention of scientists working in experimental cognitive science, psychology, social sciences and that there will be really found violations of an analogue of Bell’s inequality for conditional probabilities presented in this paper.

2 Description of test

The main aim of our paper is to attract attention of experimenters working in cognitive science, psychology, sociology to our test of quantum-like behaviour. Therefore we would not like to disturb these people by mathematical derivations and discussions on the analogy with quantum physics. The test by itself is very simple and we present it here, see section 3 for derivations and discussions.

Let \( a, b, c \) be three questions which can be asked to people (it can also be some cognitive tasks for people or animals). The answers “yes” and “no” are encoded by +1 and -1, respectively.

1). We collected an ensemble \( S \) of people which are “prepared” in the same mental state with respect to these questions.\(^4\)

The crucial in our test is that an ensemble \( S \) should be homogeneous with respect to questions \( a, b, c \) in the sense that for all questions the probabilities of answers are equal:

\[
P(u = +1) = P(v = -1) = \frac{1}{2}, \text{ where } u = a, b, c.
\]

In our experimental test we consider frequencies instead of probabilities, \( P(u = +1) \approx \nu(u = +1), u = a, b, c \), where:

\[
\nu(u = +1) = \frac{\text{the number of people who answered “yes” to the question } u}{\text{the total number of people in } S}
\]

2). We split \( S \) into two subensembles \( U \) and \( V \) containing equal numbers of people.

3). The question \( b \) is asked to people from the ensemble \( U \) and the question \( c \) is asked to people from the ensemble \( V \). Then we select in the ensemble \( U \) people who answered “yes” and people who answered “no”. Corresponding subensembles are denoted by symbols \( U^b_+ \) and \( U^b_- \), respectively. We do the same with people from \( V \), and get subensembles \( U^c_+ \) and \( U^c_- \).

\(^4\)For example, if questions \( a, b, c \) about politics then it is assumed that people from \( S \) live in the same country, have the same social and cultural level, the same age, sex and so on.
4). We ask the question $a$ to people belonging to the ensembles $U^+_b, V^+_c$ and the question $c$ to people belonging to the ensemble $U^-_b$. By counting frequencies we find approximately conditional probabilities; for example, $P(a = +1/b = +1) \approx \nu(a = +1/b = +1)$, where

\[ \nu(a = +1/b = +1) = \frac{\text{the number of people who answered \textquoteleft yes\textquoteright\ to } a}{\text{the total number of people in } U^+_b} \]

Our text of quantum-like behaviour is based on the violation of the following inequality (an analogue of Bell’s inequality for conditional probabilities), see section 3:

\[ P(a = +1/b = +1) + P(c = +1/b = -1) \geq P(a = +1/c = +1) \] (1)

If this inequality is violated than people (or animals) behave in the quantum-like way with respect to questions $a, b, c$.

In statistical investigation we should fix small $\delta > 0$ and probability $p \approx 1$. Calculate

\[ \Delta(a, b, c) = \nu(a = +1/c = +1) - \nu(a = +1/b = +1) - \nu(c = +1/b = -1) \]

and apply the criteria $\chi^2$ for the hypothesis that $\Delta(a, b, c) \geq \delta$.

3 Inequality for conditional probabilities

3.1. Quantum-like probabilistic behaviour. Since the first days of quantum mechanics there is intensively discussed the problem of realistic description – the possibility to use in quantum mechanics the same realistic viewpoint to observables as in classical physics: values of physical observables as properties of objects. We recall that quantum mechanics is a statistical theory and it cannot say anything about behaviour of individual quantum systems (at least we use such a viewpoint to quantum mechanics in this paper). It seems that first time the rigorous mathematical definition of statistical realism was given by J. Bell [38]. By analogy with quantum theory we give the following definition:

**Definition 1.** Let $\mathcal{O} = \{a, b, \ldots c\}$ be a family of observables (physical, or cognitive, or psychological, or social). This family permits the realistic statistical description if there exists a Kolmogorov probability space $\mathcal{K} = (\Omega, F, P)$ such that all observables belonging to $\mathcal{O}$ can be represented by random variables on $\mathcal{K}$.

Here $\Omega$ is an ensemble of, e.g., physical systems, or neurons, or neural networks, or animals, or people, or collectives of people,... (in mathematical
model \( \Omega \) is an ideal infinite ensemble; \( \mathcal{F} \) is a \( \sigma \)-field of subsets of \( \Omega \) and \( P \) is a probability measure on \( \mathcal{F} \). We recall that conditional probabilities in the Kolmogorov model are defined by the Bayes’ formula:

\[
P(a = a_1/b = b_1) = \frac{P(a = a_1, b = b_1)}{P(b = b_1)}.
\]

**Definition 2.** A family of observables \( \mathcal{O} = \{a, b, \ldots c\} \) which does not permit the realistic statistical description is called quantum-like.

Let \( \mathcal{O} \) is a quantum-like family of observables and observations are performed on some class of systems (e.g., physical systems, or neurons, or neural networks, or animals, or people, or collectives of people,...). We say that such systems exhibit quantum-like behaviour. Our aim is to present an experimental test for quantum-like behaviour for three observables, \( a, b, c \).

In quantum mechanics J. Bell studied similar (but not the same!) problem. He considered the famous Einstein-Podolsky-Rosen experiment [41].\(^5\) There are prepared pairs of correlated systems \( \omega = (s, s') \). Let \( a, b, c \) be three dichotomous observables on \( s \) and \( a', b', c' \) corresponding observables on \( s' \). For example \( a, b, c \) are measurements on \( s \) of spin projections corresponding to angles \( \theta_1, \theta_2, \theta_3 \) and \( a', b', c' \) are spin projections corresponding to same angles \( \theta_1, \theta_2, \theta_3 \), but the latter observations are performed on \( s' \). It is supposed that we have precise anti-correlations:

\[
a(s) = -a'(s'), \ b(s) = -b'(s'), \ c(s) = -c'(s'). \quad (2)
\]

Then J. Bell proposed to consider pair wise correlations of variables \( u = a, b, c \) and \( v = a', b', c' \) and starting with assumption of realism and locality he proved inequality for these correlations. This inequality is violated by quantum correlations.

We remark that in quantum theory measurements have huge disturbance effect therefore we cannot measure, e.g., \( a \) on \( s \) without to disturb \( s \). Thus we should use different \( s \) for different measurements (for, e.g., \( a \) and \( b \)). For a single system \( s \) we cannot perform the simultaneous measurement of \( a \) and \( b \); we cannot find the simultaneous probability distribution of \( a \) and \( b \) and their correlation. Therefore Einstein, Podolsky and Rosen proposed to consider pairs of correlated particles and therefore J. Bell was successful with his test for correlations (for observations on correlated particles).

We would like to study observables in cognitive science, psychology, sociology. In these frameworks it seems to be impossible to realize preparations of

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\(^5\)In fact, the Bohm-version of the original EPR-experiment. In the EPR-experiment there are considered continuous variables, position and momentum; in the EPR-Bohm-experiment dichotomous variables, e.g., spin projections.
ensembles which would satisfy to the condition of precise (anti)-correlation. Therefore we need Bell's like tests for measurements performed on a single system. Of course, such a framework has nothing to do with investigations on mental nonlocality. We study only quantum-like behaviour of mental systems.

3.2. Wigner inequality. We shall use the following simple mathematical result:

**Theorem 1.** (Wigner inequality) *Let \( a, b, c = \pm 1 \) be arbitrary dichotomous random variables on a single Kolmogorov space \( K \). Then the following inequality holds true.*

\[
P(a = +1, b = +1) + P(b = -1, c = +1) \geq P(a = +1, c = +1) \tag{3}
\]

The proof of this theorem in purely mathematical framework can be found e.g., in my book [42], p. 89-90. However, the inequality (3) is, in fact, the well Wigner's inequality; see [43]. Wigner proved (3) for three random variables, but then he applied (3) to the EPR-Bohm experiment for correlated particles. By using the condition of the precise anticorrelation (2) he wrote (3) in the form:

\[
P(a = +1, b' = +1) + P(b = -1, c' = +1) \geq P(a = +1, c' = +1) \tag{4}
\]

It is easy to see that (4) is violated for an appropriated choice of spin projectors.

By reasons presented in section 2.1 we would not like to use (4) as an experimental test. But we neither can use directly (3). It is evident that, e.g., psychological observations can also have disturbance effect and we cannot find a simultaneous probability distribution for some pairs of questions \( a \) and \( b \).

2.3. Inequality for conditional probabilities. As a simple consequence of Theorem 1 we obtain:

**Theorem 2.** (“Bell’s inequality for conditional probabilities”). *Let \( a, b, c = \pm 1 \) be dichotomous symmetrically distributed random variables on a single Kolmogorov space. Then the following inequality holds true:*

\[
P(a = +1/b = +1) + P(c = +1/b = -1) \geq P(a = +1/c = +1) \tag{5}
\]

**Proof.** We have \( P(b = +1) = P(b = -1) = P(a = +1) = P(a = -1) = P(c = +1) = P(c = -1) = 1/2 \).

Thus

\[
P(a = +1/b = +1) + P(c = +1/b = -1) = 2P(a = +1, b = +1) + 2P(c = +1, b = -1)
\]

and

\[
P(a = +1/c = +1) = 2P(a = +1, c = +1).
\]
Hence by (3) we get (5).

As was shown in [39], the inequality (5) is violated in quantum mechanics (by 1/2-spin projections).

We underline again that the main distinguishing feature of (5) is the presence of only conditional probabilities. Conditional probabilities can always be calculated: we first ask the question \( b \), then select all people who answered “yes”, and then ask them the question \( a \).

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