Three-body Force Effects on the Properties of Neutron-rich Nuclear Matter

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Abstract. We review our research work on the single-particle properties and the equation of state (EOS) of isospin asymmetric nuclear matter within the framework of the Brueckner-Hartree-Fock (BHF) approach extended by including a microscopic three-body force (TBF). The TBF is shown to affect significantly the nuclear matter EOS and the density dependence of nuclear symmetry energy at high densities above the normal nuclear matter density, and it is necessary for reproducing the empirical saturation property of symmetric nuclear matter in a nonrelativistic microscopic framework. The TBF-induced rearrangement effect and the ground state (g.s.) correlation effect on the s.p. properties in neutron-rich nuclear matter are investigated. Both effects turn out to be crucial for predicting reliably the s.p. properties within the Brueckner framework. The TBF effect on nucleon superfluidity in neutron star matter and neutron stars has also been discussed.

1. Introduction
To determine the equation of state (EOS) and single particle (s.p.) properties of asymmetric nuclear matter (i.e., neutron-rich nuclear matter) in a wide density range is one of the most challenging subjects in nuclear physics and nuclear astrophysics [1, 2, 3, 4, 5, 6, 7, 8, 9]. The properties of asymmetric nuclear matter and their isospin-asymmetry dependence at relatively low densities around and below normal nuclear matter density \( \rho_0 \) play a crucial role in predicting the properties of neutron-rich nuclei away from the nuclear stability line and heavy nuclei, such as the radius, the neutron-skin thickness and the density distribution. In Ref. [10], it has been shown that the neutron density distribution of neutron-rich nuclei depends sensitively on the isospin-asymmetry dependence of the equilibrium density of asymmetric nuclear matter. Theoretical investigations [11, 12, 13] have indicated that the neutron skin thickness is strongly correlated with the slope of symmetry energy around the nuclear matter saturation density. Besides the general interest in nuclear physics, the EOS of asymmetric nuclear matter, especially at supra-saturation densities, is expected to be extremely important for understanding many observational phenomena in nuclear astrophysics and neutron star physics [3, 8, 14, 15, 16, 17, 18]. For example, the EOS of asymmetric nuclear matter is a basic input for neutron star structure model (TOV equation) and it determines essentially the predicted mass-radius relation and maximum mass of neutron stars consisting of nucleons and leptons. The density dependence of symmetry energy at high densities determines the proton fraction in \( \beta \)-stable \((n,p,e,\mu)\) neutron star matter, and thus is decisive for understanding the cooling mechanism of neutron stars.
Heavy ion collisions (HIC) at intermediate and high energies provide powerful tools in laboratory for extracting information about the EOS and s.p. properties of asymmetric nuclear matter. During the dynamic evolution of HIC induced by radioactive beams, a transient state of dense and highly isospin-asymmetric nuclear matter can be formed and the isospin dynamics and observables have been shown to be quite sensitive to the isovector part of the EOS and symmetry potential [19, 20, 21, 22, 23, 24]. Since the EOS of nuclear matter can not be measured directly in the experiments of HIC, one has to compare the experimental observables with the theoretical simulations by using transport models to extract the information about the nuclear EOS indirectly. The s.p. potentials felt by protons and neutrons in asymmetric nuclear medium are basic ingredients of the transport models (such as BUU and QMD models) for HIC. Therefore, reliable information of the density-, isospin- and momentum-dependence of the s.p. potentials in neutron-rich nuclear medium is crucial for constraining the EOS of asymmetric nuclear matter from the experiments of HIC. Up to now, the density dependence of symmetry energy at low densities below \( \sim 1.2\rho_0 \) has been constrained to a certain extent by the HIC experiments and some structure information of finite nuclei [1, 21, 22]. However, the density dependence of symmetry energy at high densities remains poorly known. In 2009, Xiao et al. [25] calculated the \( \pi^-/\pi^+ \) ratio in central heavy ion collisions at at SIS/GSI energies by using an isospin- and momentum-dependent BUU model. By comparing their calculated results with the experimental data measured by the FOPI Collaboration at GSI [26], they found that a super soft density-dependence of symmetry energy at supra-saturation densities is required for reproducing the FOPI data. In 2010, Feng et al.[27] did the similar investigation by adopting an isospin-dependent QMD model and their results favor a supper stiff symmetry energy at supra-saturation densities in order to match the FOPI data. In 2011, Russotto et al. [28] studied the elliptic-flow ratio of neutrons with respect to protons or light complex particles in \( \text{Au} + \text{Au} \) collisions at 400 MeV/nucleon by using UrQMD model and compared their results with the existing FOPI/LAND data. They suggested that the density dependence of symmetry energy at high densities is most possibly between soft and stiff.

Theoretically, the EOS and s.p. properties of nuclear matter can be predicted by various many-body approaches including phenomenological methods and microscopic approaches. Although almost all theoretical approaches are able to reproduce the empirical value of symmetry energy at the saturation density, the discrepancy among the predicted density-dependence of symmetry energy at high densities by adopting different many-body approaches and/or by using different \( NN \) interactions has been shown to be quite large [5, 12, 29, 30, 31, 32]. For example, within the Skyrme-HF framework, different Skyrme parameters may lead to complete different and even opposite density-dependence of symmetry energy at supra-saturation density [12].

Microscopically the EOS and s.p. properties of asymmetric nuclear matter have been investigated extensively by adopting the Brueckner-Hartree-Fock (BHF) and the extended BHF approaches [33, 34, 35, 36, 37], the relativistic Dirac-BHF (DBHF) theory [31, 38, 39, 40], the in-medium \( T \)-matrix and Green function methods [41, 42, 43, 44], and the variational approach [45, 46]. In the present paper, we shall review systematically our research work on the properties of neutron-rich nuclear matter within the framework of the Brueckner-Bethe-Goldstone (BBG) approach extended to include a microscopic three-body force(TBF). The paper is organized as follows. In section 2, we give an introduction of the adopted theoretical approaches. The numerical results are reported and discussed in Section 3. Finally, a summary is given Section 4.

2. Theoretical Approaches
Our investigation is based on the BBG theory for asymmetric nuclear matter [33, 34]. The extension of the BBG scheme to include microscopic three-body forces can be found in Ref. [35, 47, 48]. Here we simply give a brief review for completeness. The starting point
of the BHF approach is the reaction $G$-matrix, which satisfies the following isospin dependent Bethe-Goldstone (BG) equation,

$$G(\rho, \beta, \omega) = v_{NN} + v_{NN} \sum_{k_1 k_2} \frac{|k_1 k_2| Q(k_1, k_2) \langle k_1 k_2 | G(\rho, \beta, \omega) .$$  \tag{1}

where $k_i \equiv (\vec{k}_i, \sigma_i, \tau_i)$, denotes the s.p. momentum, the $z$-component of spin and isospin, respectively. $v_{NN}$ is the realistic nucleon-nucleon ($NN$) interaction, $\omega$ is the starting energy. The asymmetry parameter is defined as $\beta = (\rho_n - \rho_p)/\rho$, where $\rho, \rho_n$, and $\rho_p$ denote the total, neutron and proton number densities, respectively. The BHF s.p. energy spectrum is given by $\epsilon(k) = \hbar^2 k^2/(2m) + U_{BHF}(k)$. In solving the BG equation for the $G$-matrix, the continuous choice [49] is adopted for the s.p. potential $U_{BHF}$ since it provides a much faster convergence of the hole-line expansion than the gap choice [50]. Under the continuous choice, the s.p. potential describes physically at the lowest BHF level the nuclear mean field felt by a nucleon in nuclear medium. The BHF s.p. potential is calculated from the real part of the on-shell $G$-matrix,

$$U_{BHF}(k) = \sum_{k'} n(k') \Re(kk'G(\epsilon(k) + \epsilon(k'))|kk')_A . \tag{2}$$

For the realistic $NN$ interaction $v_{NN}$, we adopt the Argonne $V_{18}$ (AV18) two-body interaction [51] plus a microscopic TBF [48] constructed by using the meson-exchange current approach [47]. The parameters of the TBF model have been self-consistently determined to reproduce the AV18 two-body force using the one-boson-exchange potential model and their values can be found in Ref. [48]. In our calculation, the TBF contribution has been included by reducing the BHF to an equivalently effective two-body interaction according to the standard scheme as described in Ref. [47]. In $r$-space, the equivalent two-body force $V_{3}^{\text{eff}}$ reads:

$$\langle \vec{r}_1 \vec{r}_2 | V_{3}^{\text{eff}} | \vec{r}_1 \vec{r}_2 \rangle = \frac{1}{4} \text{Tr} \sum_{n} \int d\vec{r}_3 d\vec{r}_4 \phi_n^*(\vec{r}_3') (1 - \eta(\vec{r}_3')) (1 - \eta(\vec{r}_2'))
\times W_3(\vec{r}_1' \vec{r}_2' | \vec{r}_1 \vec{r}_2 \vec{r}_3) \phi_n(\vec{r}_3) (1 - \eta(\vec{r}_1)) (1 - \eta(\vec{r}_3)). \tag{3}$$

It is worth stressing that the effective force $V_{3}^{\text{eff}}$ depends strongly on density. It is the density dependence of the $V_{3}^{\text{eff}}$ that induces the TBF rearrangement contribution to the s.p. properties in nuclear medium within the BHF framework.

At the lowest mean field level, there are two problems for the BHF approach in predicting the nuclear s.p. properties. First, the lowest-order BHF approximation destroys the Hugenholtz-Van Hove (HVH) theorem and, at densities around the saturation density, the predicted optical potential depth turns out to be too deep as compared to the empirical value [49]. This problem can be solved by taking into account the effect of ground state (g.s.) correlations [49, 52]. Second, in Ref. [53] it has been pointed out that, at high densities and high momenta, the BHF potential is too attractive and its momentum dependence turns out to be too weak for describing the experimental elliptic flow data. In order to predict reliably and realistically the s.p. properties within the Brueckner framework, we have improved the calculation of the s.p. potential in asymmetric nuclear matter by going beyond the lowest BHF approximation in two aspects. First, we have extended the calculation of the effect of g.s. correlations to the case of asymmetric nuclear matter [34]. Second, we have included the TBF-induced rearrangement contribution in calculating the s.p. properties [36, 37]. The s.p. potential in nuclear medium can be derived from the functional variation of the potential energy density with respect to the occupation probability of s.p. states [54], i.e.,

$$U(k) = \frac{\delta E_V}{\delta n_k} = \sum_{k_1} n_{k_1} \langle kk_1 | G | kk_1 \rangle_A + \frac{1}{2} \sum_{k_1 k_2} n_{k_1} n_{k_2} \langle k_1 k_2 | G | k_1 k_2 \rangle_A , \tag{4}$$
where the first term in the right hand side corresponds to the standard BHF s.p. potential. In the case without including the TBF, the above equation becomes identical with the hole-line expansion of the mass operator [49]. By the aid of the BG equation, the second term can be worked out explicitly:

$$\frac{\delta G}{\delta n_k} = \frac{\delta V_{3}^{\text{eff}}}{\delta n_k} + G \frac{Q}{\epsilon_{12}} \frac{\delta V_{3}^{\text{eff}}}{\delta n_k} G + G \frac{Q}{\epsilon_{12}} \frac{\delta V_{3}^{\text{eff}}}{\delta n_k} G + G \frac{\delta(Q/e_{12})}{\delta n_k} G. \quad (5)$$

In the right hand side, the last term comes from the density dependence of the effective interaction $G$-matrix and it leads to the g.s. correlation effect on the s.p. potential [49]. The lowest contribution of the last term corresponds to the core polarization (also called Pauli rearrangement) which affects mainly the s.p. potential around and below the Fermi surface. The first four terms arise from the effective force $V_{3}^{\text{eff}}$ which is an equivalent effective two-body interaction of the TBF and depends strongly on density. In the four terms of the TBF rearrangement contribution, the first one is predominated over the other three ones which contain the interaction matrix elements between two particle states (unoccupied) and two hole states (occupied) and are negligible as compared to the first term. Accordingly, the TBF-induced rearrangement contribution to the s.p. potential can be calculated as follows [37],

$$U_{\text{TBF}}(k) \approx \frac{1}{2} \sum_{k_1k_2} n_{k_1} n_{k_2} \left\langle k_1k_2 \left| \frac{\delta V_{3}^{\text{eff}}}{\delta n_k} \left| k_1k_2 \right\rangle \right\rangle_A. \quad (6)$$

3. Results and Discussions

3.1. EOS of symmetric nuclear matter and TBF effect

As well known, the nonrelativistic model of rigid nucleons interacting via realistic two-body forces fitting in-vacuum nucleon-nucleon scattering data misses empirical saturation properties of nuclear matter [56, 30]. In order to describe reasonably the nuclear saturation properties within the nonrelativistic BHF framework, one has to take into account the TBF effect.

In the left panel of Fig. 1 is shown the energy per nucleon of symmetric nuclear matter vs. density [48], where the dashed curve is obtained by using the AV18 two-body interaction alone;
the solid curve is the result by adopting the AV18 plus the corresponding TBF. By comparing the dashed curve and the solid curve, it is seen that the TBF contribution to the EOS is repulsive and leads to a stiffening of the EOS, especially at supra-saturation densities. At low densities, the TBF contribution turns out to be fairly small. The repulsive effect of the TBF increases monotonically and rapidly as a function of density, especially at high densities. The repulsive contribution from the TBF turns out improves remarkably the predicted saturation density of symmetric nuclear matter from $\sim 0.26 \text{fm}^{-3}$ to $\sim 0.19 \text{fm}^{-3}$, indicating that TBF is necessary for reproducing the empirical saturation property of nuclear matter in a non-relativistic microscopic framework. In Ref. [55], we have constructed a new microscopic TBF based on BonnB two-body interaction. The calculated results by using the BonnB interaction and the corresponding TBF are presented in the right panel of Fig.1. The effect of the TBF based on the BonnB potential is seen to be similar to that of the TBF based on AV18 potential. In the case of not including the TBF, the predicted saturation density and the saturation energy are seen to be $\sim 0.33 \text{fm}^{-3}$ and $\sim -22 \text{ MeV}$, far away from their empirical values. By adopting the BonnB interaction plus the corresponding TBF, the saturation density and the saturation energy are $\sim 0.167 \text{fm}^{-3}$ and $\sim -15.9 \text{ MeV}$ respectively, in satisfactory agreement with the empirical values. For both TBFs based on the AV18 and BonnB interactions, the connection between the relativistic effect in the DBHF approach and the TBF effect has been investigated [48, 55]. It has been found that the main relativistic correction to the EOS of nuclear matter in the DBHF approach can be reproduced quantitatively by the $2\sigma-N\bar{N}$ component of the microscopic TBF.

3.2. EOS of asymmetric nuclear matter and the density dependence of symmetry energy

In Fig.2 is shown the EOS of asymmetric nuclear matter at several different isospin-asymmetries. The dashed curves denote the results without including the TBF; the solid ones are obtained by adopting the AV18 two-body interaction plus the TBF. From the bottom to the top in the figure, the corresponding asymmetry parameters are $\beta = 0, 0.2, 0.4, 0.6, 0.8$ and 1, respectively. By comparing the dashed curves with the corresponding solid ones, it may be noticed that the TBF gives a repulsive contribution to the EOS in the whole asymmetry range ($0 \geq \beta \geq 1$) and inclusion of the TBF makes the EOS of asymmetric nuclear matter at high asymmetries and/or large densities much stiffer as compared to the case of not including the TBF. At any given asymmetry, the repulsive TBF contribution increases monotonically as a function of density. At a fixed density, the TBF effect turns out to be more pronounced at a higher asymmetry.

![Figure 2. EOS of asymmetric nuclear matter at several different isospin-asymmetries, obtained in the two cases with (solid curves) and without (dashed curves) including the TBF. Going from the bottom to the top, the asymmetry changes from 0 to 1 in a step of 0.2.](image)

In order to show more clearly the isospin dependence of the EOS of asymmetric nuclear matter, we report in Fig. 3 the isovector part of the EOS [35] (i.e., the difference between the
energy per nucleon of asymmetric nuclear matter at a given $\beta$ and the energy per nucleon of symmetric nuclear matter) as a function of $\beta^2$ for several typical densities $\rho = 0.085, 0.17, 0.34$ and $0.45 \text{fm}^{-3}$. The right panel corresponds the results without including the TBF; the left panel displays the results obtained by including the TBF. One may notice from the figure that the energy per nucleon $E_A(\rho, \beta)$ of asymmetric nuclear matter fulfills satisfactorily a linear dependence on $\beta^2$ in the whole asymmetry range of $0 \leq \beta \leq 1$. Inclusion of the TBF contribution does not destroy the linear dependence of $E_A(\rho, \beta)$ on $\beta^2$ (see the left panel). Such a linear dependence of $E_A(\rho, \beta)$ on $\beta^2$ is called $\beta^2$-law, which indicates that the EOS of asymmetric nuclear matter can be expressed as $E_A(\rho, \beta) = E_A(\rho, 0) + E_{\text{sym}}(\rho)\beta^2$ and the higher order terms are negligible. In the above expression, $E_{\text{sym}}(\rho)$ is the symmetry energy and defined generally as $E_{\text{sym}}(\rho) = \frac{1}{2} (\partial^2 E_A/\partial \beta^2)_{\beta=0}$. The $\beta^2$-law has also been obtained by Gad et al. [42] within the Green function approach. Our above result provides an microscopic support for the empirical $\beta^2$-law extracted from the nuclear mass table and extended its validity up to the highest asymmetry. The $\beta^2$-law leads to two important consequences. First, it indicates the symmetry energy can be obtained by the difference between the EOS of pure neutron matter and that of symmetric nuclear matter, i.e., $E_{\text{sym}} = E_A(\rho, \beta = 1) - E_A(\rho, \beta = 0)$. Second, the above $\beta^2$-law implies that the difference of the neutron and proton chemical potentials in $\beta$-stable neutron star matter is determined by the symmetry energy in an explicit way: $\mu_n - \mu_p = 4\beta E_{\text{sym}}$ and thus the symmetry energy plays a crucial role in predicting the composition of neutron stars. In Ref. [57] we have shown that the above-mentioned $\beta^2$-law is also valid at finite temperatures.

Nuclear symmetry energy describes the isovector part of the EOS of asymmetric nuclear matter. To see the TBF effect on the density dependence of symmetry energy, in Fig. 4 we compare the predicted symmetry energy vs. density for the two cases of including the TBF (solid curve) and without including the TBF (dashed curve) [35]. In the case of not including the TBF, the density dependence of symmetry energy is quite soft and follows approximately $E_{\text{sym}} \propto \rho^{0.6}$ in the whole density region of $\rho \leq 0.5 \text{fm}^{-3}$. The TBF effect is reasonably small at low densities around and below the saturation density. At supra-saturation density, the TBF effect on symmetry energy is repulsive and results in a stiffening of the density dependence of

![Figure 3](image-url)  
**Figure 3.** The predicted isovector part of the EOS of asymmetric nuclear matter. Take from Ref. [35].

![Figure 4](image-url)  
**Figure 4.** Symmetry energy as a function of density. Taken from Ref. [35].
symmetry energy. Inclusion of the TBF makes the stiffness of the symmetry energy at high densities become remarkably different from that at low densities, i.e., the density dependence of symmetry energy changes from a soft one (as compared with a linear dependence) to a stiff one. The thermal effect on the symmetry energy has been studied in Ref. [57]. It is shown that the thermal effect is small for temperature up to 20MeV and as the temperature increases the symmetry energy decreases slightly.

3.3. neutron and proton s.p. potentials in asymmetric nuclear matter

As pointed in Sect.2, within the Brueckner framework extended to include the microscopic TBF, the full s.p. potential includes three parts:

$$U(k) = U_{\text{BHF}}(k) + U_2(k) + U_{\text{TBF}}(k).$$

(7)

where the first part $U_{\text{BHF}}$ corresponds to the lowest-order BHF s.p. potential. The second term $U_2$ describes the effect of the g.s. correlations on s.p. potential and is called the Pauli rearrangement contribution of $G$-matrix [49], which has been investigated extensively in literature [34, 41, 42, 43, 44, 49, 52]. The third term $U_{\text{TBF}}$ denotes the rearrangement contribution induced by the TBF, i.e., Eq.(6).

Figure 5. Neutron and proton s.p. potentials $U_{\text{n,BHF}}$ and $U_{\text{p,BHF}}$, predicted at the lowest-order BHF approximation, in asymmetric nuclear matter at various asymmetries of $\beta = 0, 0.2, 0.4, 0.6$ and 0.8 for three typical densities \(\rho = 0.085, 0.17 \text{ and } 0.34 \text{ fm}^{-3}\). The solid curves correspond to the neutron s.p. potentials; the dashed ones denote the proton potentials. Taken from Ref. [36].

In Fig. 5 we display the lowest-order BHF s.p. potentials $U_{\text{BHF}}$ for neutrons and protons in asymmetric nuclear matter at several typical densities and asymmetries [34, 36]. It is seen that the BHF s.p. potentials are strongly attractive at low momenta. At a fixed asymmetry, the attraction of the BHF s.p. potentials turns out to increase with increasing density. In neutron-rich nuclear matter, the neutron and proton potentials are shown to become different and split from their common value in symmetric nuclear matter. For a given density, the neutron s.p. potential $U_{\text{n,BHF}}$ becomes less attractive while the proton potential becomes more attractive at
Figure 6. The contributions of g.s. correlations to the neutron and proton s.p. potentials, $U_n^2$ and $U_p^2$, in neutron-rich nuclear matter at density $\rho = 0.17{\text{fm}}^{-3}$. Taken from Ref. [34].

A higher asymmetry. The above predicted different behavior of neutron and proton potentials vs. asymmetry $\beta$ is mainly caused by the isospin-singlet $T = 0$ SD tensor component of the $NN$ interaction and is readily understood as follows. According to the experimental data on the phase shifts of $NN$ scattering, the SD channel is strongly attractive at low energies. As the neutron excess increases, the attraction of the SD interaction between two unlike nucleons becomes stronger for protons and weaker for neutrons at relatively low momenta. The isospin $T = 1$ channel contribution is associated with the variations of the neutron and proton Fermi surfaces in neutron-rich matter. Its effect on the splitting of the neutron and proton potentials is opposite and much smaller as compared with the SD channel contribution at low momenta as discussed in Ref. [36]. The attraction of the SD channel decreases with energy, so that for high enough momenta, the splitting of the neutron and proton potentials in neutron-rich matter may vanish and even become opposite due to the competition between the $T = 0$ and $T = 1$ channel effects. In Refs. [34, 36], it has been found that the lowest-order BHF neutron and proton potentials in neutron-rich matter with respect to their common value in symmetric matter fulfills almost a linear dependence on asymmetry $\beta$, which indicates that the symmetry potential, defined as $U_{sym} = (U_n - U_p)/2\beta$, is almost independent of $\beta$ at the lowest-order BHF approximation and provides a microscopic for the validity of the so-called Lane potential [58].

In Fig. 6 are plotted the contributions of g.s. correlations to the neutron and proton s.p. potentials (i.e., $U_n^2$ and $U_p^2$) at $\rho = 0.17{\text{fm}}^{-3}$ for several asymmetries $\beta = 0, 0.2, 0.4, 0.6$ and 0.8 respectively. In general, it is seen that the contribution of g.s. correlations is repulsive for both neutron and proton potentials in symmetric nuclear matter and neutron-rich nuclear matter. The g.s. correlations modify the neutron and proton potentials mainly at low momenta and below the Fermi surfaces and their contribution vanishes rapidly above the corresponding Fermi momenta. The g.s. correlations are shown to result in a strongly weakening of the momentum dependence of the neutron and proton s.p. potentials around the respective Fermi surfaces due to the rapid decreasing of their effect as a function of momentum around the Fermi momenta. In neutron-rich matter, the $U_p^2(k)$ is found to decrease rapidly as the symmetry $\beta$ increases since the proton Fermi momentum becomes smaller at a higher asymmetry. For the neutron potential $U_n^2(k)$, the isospin dependence is quite complicated due to the the coupling between the nucleon hole states and particle-hole excitations (see the discussion in Ref. [34]). The effect of g.s. correlations may destroy the linear $\beta$-dependence fulfilled by the neutron and proton potentials at the lowest-order BHF approximation [34]. The contribution of the g.s. correlations
not only play an important role in satisfactorily reproducing the depth of the empirical nuclear optical potential [49], but is also crucial for restoring the HVH theorem [34, 52] and necessary for generating a nucleon self-energy to describes realistically the s.p. strength distribution in nuclear matter and finite nuclei below the Fermi energy [59]. However, it can not provide any appreciate improvement of the high-momentum BHF s.p. potential which has been shown to be too attractive and whose momentum-dependence turns out to be too weak at high densities for describing the experimental elliptic flow data of HIC experiments at high energies [53].

![Figure 7. TBF rearrangement contribution $U_{\text{TBF}}$ to the s.p. potential in symmetric nuclear matter for several densities [37].](image1)

![Figure 8. TBF rearrangement contributions $U_{\text{TBF}}^p$ and $U_{\text{TBF}}^n$ in asymmetric nuclear matter for $\beta = 0.4$ (squares) and $\beta = 0.8$ (stars). Taken from Ref. [37].](image2)
compared with the corresponding isoscalar contribution. At sub-saturation densities, the isospin dependence of the $U_{\text{TBF}}^n$ and $U_{\text{TBF}}^p$ is seen negligibly small. At supra-saturation densities, the $U_{\text{TBF}}^n$ becomes more repulsive, while the $U_{\text{TBF}}^p$ becomes less repulsive at a high asymmetry.

3.4. neutron and proton effective masses

Nucleon effective mass describes the momentum dependence of of the s.p. potential felt by a nucleon in nuclear medium and is defined as:

$$\frac{m^\star(k)}{m} = \left[1 + k \frac{dU^\star(k)}{dk}\right]^{-1},$$

where $\tau$ denotes neutron or proton. The nucleon effective mass is of great interest in nuclear physics and nuclear astrophysics [62, 63, 64] since it is closely related to many nuclear phenomena and quantities such as the dynamics of HIC at intermediate and high energies[1], the damping of nuclear excitations and the giant resonances, the properties of nucleon superfluidity in nuclear matter [60], $NN$ cross sections in dense nuclear matter and the transport properties in neutron stars [65], the nuclear level density around the Fermi surface, and the physics of stellar collapse [66].

![Figure 9](image1.png)  
**Figure 9.** Nucleon effective mass as a function of density at three different level of approximations (see text). Taken from Ref. [61].

![Figure 10](image2.png)  
**Figure 10.** Neutron and proton effective masses vs. $\beta$ in neutron-rich nuclear matter. The effect of g.s. correlations has not be considered. Taken from Ref. [37].

In Fig. 9 we report the calculated effective mass $m^\star(k = k_F)$ at Fermi momentum vs. density in symmetric nuclear matter at three different level of approximations: $m_{\text{BHF}}^\star/m$ at the lowest level BHF approximation without including the effect of g.s. correlations and the TBF rearrangement contribution (dashed curve); $m_{12}^\star/m$ including the effect of g.s. correlations but without the TBF rearrangement contribution, i.e., in the Eq.(8) approximating $U$ by $U \simeq U_{\text{BHF}} + U_2$ to get $m_{12}^\star/m$ (dotted curve); the full effective mass $m^\star(k)$ by using the full s.p. potential of Eq. (7). At the lowest-order BHF level, the effective mass $m_{\text{BHF}}^\star$ decreases with increasing density and saturated at high enough densities. Inclusion of the g.s. correlation effect leads to significant enhancement of the nucleon effective mass since the g.s. correlations weakens the momentum dependence of nucleon s.p. potential around Fermi surface [34]. The TBF rearrangement effect is strongly momentum dependent and consequently it reduces significantly the nucleon effective mass, especially at large densities. One may notice from Fig. 9 that there is a strong competition between the g.s. correlation effect the TBF rearrangement effect. Without
the TBF rearrangement effect (dashed and dotted curves in the figure), the effective mass decreases at low densities below and around the saturation density 0.17fm\(^{-3}\) and its density dependence becomes quite weak at high densities. By comparing the dashed and dotted curves, it is seen that the g.s. correlations result in an overall enhancement of the effective mass in the whole density range up to \(\rho = 0.5\text{fm}^{-3}\). At low densities, the effective mass is shown to be governed mainly by the g.s. correlation effect and inclusion of the TBF rearrangement contribution reduces only slightly the effective mass. Inclusion of the TBF rearrangement contribution (solid curve) makes the effective mass become a monotonically decreasing function of density in the whole density range considered here. At relatively low densities, the full effective mass \(m^*\) is larger than the lowest order BHF one \(m^*_{\text{BHF}}\), while it becomes smaller than the \(m^*_{\text{BHF}}\) at high enough densities, which implies that the TBF-induced rearrangement effect becomes predominant over the g.s. correlation effect at high enough densities.

In neutron-rich nuclear matter, the neutron and proton effective masses are expected to split with respect to their common value in symmetric matter. In Fig. 10 is depicted the neutron and proton effective masses at their respective Fermi momenta vs. asymmetry \(\beta\) in neutron-rich nuclear matter for several densities \(\rho = 0.085, 0.17, 0.34,\) and 0.5fm\(^{-3}\), respectively. The dashed lines denote the results at the lowest-order BHF approximation; the solid ones are obtained by including the TBF rearrangement effect. The effect of g.s. correlations is not considered. It is seen from the figure that both cases with and without the TBF rearrangement contribution, as the nuclear matter becomes more neutron-rich, the neutron effective mass increases while the proton one decreases with respect to their common value in symmetric nuclear matter; i.e., the predicted neutron-proton effective mass splitting in neutron-rich nuclear matter turns out to be \(m^*_n > m^*_p\) in good agreement with the predictions by the nonrelativistic limit of the DBHF approach [39, 38]. At the lowest-order BHF approximation, the absolute magnitude of the neutron-proton effective mass splitting in neutron-rich nuclear matter depends weakly on density and remains almost the same in the whole density range considered here. Inclusion of the TBF rearrangement effect makes the absolute magnitude of the splitting become quite sensitive to the variation of the density. At sub-saturation densities, the TBF rearrangement effect on the splitting turns out to be fairly small. At supra-saturation densities, the effect of the TBF rearrangement leads to a reduction of the neutron-proton effective mass splitting. At high enough density (\(\rho = 0.5\text{fm}^{-3}\)), the TBF rearrangement effect may even suppresses almost completely the splitting.

3.5. Nucleon superfluidity in neutron star matter and neutron stars

Nucleon superfluidity plays a crucial role in understanding many astrophysical phenomena in neutron stars. [67, 68, 69, 70, 71]. In the inner crust of a neutron star where the total baryon density is low, neutron superfluidity is expected to exist in the singlet \(^1S_0\) channel. In the outer core part, protons may form a superfluid in the \(^1S_0\) partial wave states due to the small proton fraction, and neutron superfluidity may be formed in the \(^3PF_2\) coupled channel since the \(NN\) scattering data indicate that the \(^3PF_2\) component of the \(NN\) interaction is attractive at relatively high energies. It is generally expected that the cooling processes via neutrino emission [67, 68, 69, 70], the magnetic properties, the properties of rotating dynamics, the post-glitch timing observations [9], and the possible vertex pinning [71] of neutron stars are very sensitive to the presence of neutron and proton superfluid phases as well as to their pairing strength.

In Fig. 11 we show the TBF effect on the proton \(^1S_0\) pairing gap \(\Delta_F = \Delta(k_F^p)\) in \(\beta\)-stable neutron star matter predicted within the BHF + BCS framework [72]. In the figure, the solid curve is predicted by using the AV18 interaction plus the TBF, and the dashed curve by using the AV18 two-body interaction alone. As expected, in the case of not including the TBF, due to the small proton fraction in \(\beta\)-stable neutron star matter, the \(^1S_0\) proton superfluid phase may extend to considerably high baryon densities up to \(\rho = 0.45\text{fm}^{-3}\) with a peak gap value of
0.95MeV at $\rho \simeq 0.2\text{fm}^{-3}$. Inclusion of the TBF leads to a strong suppression of the $^1S_0$ proton superfluidity in neutron star matter, especially at high baryon densities. On the one hand, the TBF reduces significantly the peak value of the $^1S_0$ proton pairing gap by about 50% from $\sim 0.95\text{MeV}$ to $\sim 0.55\text{MeV}$ and shifts the peak to a much lower baryon density from $\sim 0.2\text{fm}^{-3}$ to $\sim 0.09\text{fm}^{-3}$. On the other hand, inclusion of the TBF results in a remarkably shrinking of the density region for the $^1S_0$ proton superfluid phase to $\rho < 0.3\text{fm}^{-3}$. In spite of the small proton fractions in $\beta$-stable neutron star matter which correspond to small proton densities in the matter, the above predicted TBF suppression of the $^1S_0$ proton superfluidity can be readily understood as follows. Since proton pairs are embedded inside the medium of neutrons and protons, both the surrounding protons and neutrons contribute to the TBF renormalization of the proton-proton pair interaction. And consequently the relevant density to the TBF effect on proton pairing is the total baryon density, but not the proton one, which can be verified from Fig. 11 that the TBF-induced reduction of the pairing gap becomes stronger at a higher total baryon density. The strong weakening of the $^1S_0$ proton superfluidity induced by the TBF may has important implication for modeling the neutron-star cooling scenario [70].

In Fig. 12 it is shown the $^3PF_2$ neutron pairing gap in $\beta$-stable neutron star matter as a function of density. Right panel: distribution of the $^3PF_2$ neutron pairing gap inside a typical neutron star. Taken from Ref. [60].

Figure 11. Proton $^1S_0$ pairing gap in $\beta$-stable neutron star matter. The solid curve is predicted by using the AV18 interaction plus the TBF, and the dashed curve by using the pure AV18 two-body force alone. Taken from Ref. [72].

Figure 12. Left panel: $^3PF_2$ neutron pairing gap in $\beta$-stable neutron star matter as a function of density. Right panel: distribution of the $^3PF_2$ neutron pairing gap inside a typical neutron star. Taken from Ref. [60].
mass of $M = M_\odot$ (right panel) [60]. In the figure, the empty squares corresponds to the results by using the pure $AV18$ two-body interaction; the filled squares are predicted by adopting the $AV18$ interaction plus the microscopic TBF; the empty triangles denote the results obtained by Zhou et al. [73] by using the $AV18$ plus the semi-phenomenological UIX TBF. In the case of not including any TBF, it is seen from the left panel of Fig. 12 that in $\beta$-stable neutron star matter, the pairing gap increases first as a function of density, reaching a peak value of about 0.2 MeV, and then decreases with increasing density in the region of $\rho \geq 0.24$ fm$^{-3}$. By comparing the empty squares and the filled squares, one may notice that inclusion of the TBF enhances remarkably the $^3PF_2$ neutron superfluidity in $\beta$-stable neutron star matter, especially at high densities, and makes the $^3PF_2$ neutron pairing gap become a monotonically increasing function of density. From the right panel, one may see that the predicted distribution of the $^3PF_2$ neutron pairing gap in a neutron star by adopting purely the $AV18$ two-body interaction is similar with the density dependence of the pairing gap in $\beta$ neutron star matter, i.e., the pairing gap first increases going from the outer part to the inner part of the neutron star, reaches its maximum value of about 0.2MeV at $R \simeq 8.4$km, then starts to decrease and finally vanishes at $R < 4.7$km in the inner part of the neutron star where the baryon density is expected to be high enough. By comparing the empty squares and the filled squares, it is clear that inclusion of the microscopic TBF leads to a significantly overall enhancement of the $^3PF_2$ neutron superfluidity inside neutron stars and makes the superfluid phase spread throughout the whole neutron star. The microscopic TBF turns out to enlarge the maximum strength of the pairing gap from $\sim 0.2$ to $\sim 0.5$MeV. We notice that the semi-phenomenological Urbana UIX TBF leads to an extremely strong $^3PF_2$ neutron superfluidity in neutron stars which is much stronger as compared with our prediction by adopting the microscopic TBF (for a detailed discussion, see Ref. [60]).

4. Summary

We have reviewed our research work on the EOS and the s.p. properties of asymmetric nuclear matter within the framework of the Brueckner approach extended by including the microscopic TBF. The TBF is shown to provide a repulsive contribution to the EOS of asymmetric nuclear matter and its repulsion increases monotonically and rapidly as a function of density and asymmetry at supra-saturation densities. The repulsive contribution of the TBF leads to a significantly stiffening of the EOS of asymmetric nuclear matter and the density dependence of symmetry energy at high densities above the normal nuclear matter density, and it turns out to be necessary for reproducing the empirical saturation property of symmetric nuclear matter within a nonrelativistic microscopic many-body framework. The EOS of asymmetric nuclear matter is proved to fulfill satisfactorily a linear dependence on $\beta^2$ in the whole asymmetry range of $0 \leq \beta \leq 1$, which supports microscopically the empirical $\beta^2$-law extracted from the nuclear mass table and extended its validity up to the highest isospin asymmetry. Inclusion of the TBF does not destroy the $\beta^2$-law. Inclusion of the TBF makes the density dependence of symmetry energy at high densities become much stiffer than that at sub-saturation densities.

In predicting the s.p. properties in asymmetric nuclear matter, we have extended and improved the Brueckner approach in two aspects. One is to extend the calculation of the effect of g.s. correlations to asymmetric nuclear matter; the second is to include the rearrangement contribution induced by the TBF. Both the TBF rearrangement contribution and the g.s. correlation effect are shown to be crucial and necessary for predicting reliably the s.p. properties in neutron-rich nuclear matter within the microscopic Brueckner framework. The g.s. state correlations give a repulsive contribution to the neutron and proton s.p. potentials, and modify mainly the s.p. properties at low momenta around and below the Fermi surfaces. The rapid decreasing of the effect of g.s. correlations as a function of momentum around the Fermi momenta $k_F$ may weakening considerably the momentum dependence of the s.p. potentials and results in a significant enhancement of the nucleon effective masses around $k_F$. Inclusion of the
contribution of g.s. correlations destroys the linear $\beta$-dependence fulfilled by the neutron and proton potentials at the lowest-order BHF approximation. The TBF is found to induce a strongly repulsive momentum dependent contribution to the s.p. potentials at high densities and large momenta. Being different from the effect of g.s. correlations, the TBF-induced rearrangement repulsion increases rapidly as a function of density and momentum. The TBF rearrangement effect enhances strongly the repulsion of the s.p. potentials and turns out to be necessary for reducing the disagreement of the large-density and high-momentum BHF s.p. potential in symmetric matter with the parameterized potential extracted for describing the elliptic flow data in HIC and those predicted by the DBHF approach. The strong momentum-dependence of the TBF rearrangement contribution leads to a significant reduction of the nucleon effective mass and makes the effective mass decrease monotonically and rapidly as a function of density at supra-saturation densities. At sub-saturation densities, the effective mass is shown to be governed mainly by the g.s. correlation effect. Whereas, the TBF-induced rearrangement effect becomes predominant over the g.s. correlation effect at high enough densities. In neutron-rich nuclear matter, the neutron effective mass turns out to be larger than the proton one in both cases with and without including the TBF rearrangement contribution. The TBF rearrangement effect is shown to reduce remarkably the isospin splitting of the neutron and proton effective masses in high-density neutron-rich matter.

The TBF effect on nucleon superfluidity in $\beta$-stable neutron star matter and neutron stars has also been discussed. On the one hand, the TBF is shown to suppress remarkably the $^1S_0$ proton superfluidity in neutron star matter, especially at high baryon densities. It reduces significantly the peak value of the $^1S_0$ proton pairing gap by about 50% from $\sim 0.95\text{MeV}$ to $\sim 0.55\text{MeV}$ and shifts the peak to a much lower baryon density from $\sim 0.2\text{fm}^{-3}$ to $\sim 0.09\text{fm}^{-3}$. On the other hand, inclusion of the TBF turns out to enhance considerably the predicted $^3PF_2$ neutron superfluidity in neutron star matter and neutron stars. The microscopic TBF leads to a strongly overall enhancement of $^3PF_2$ neutron superfluidity and makes the corresponding superfluid phase spread throughout inside the whole inner part of neutron stars.

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