Monopole action from vacuum configurations in compact QED

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Abstract

It is possible to derive a monopole action from vacuum configurations obtained in Monte-Carlo simulations extending the method developed by Swendsen. We apply the method to compact QED both in the Villain and in the Wilson forms. The action of the natural monopoles in the Villain case is in fairly good agreement with that derived by the exact dual transformation. Comparing the monopole actions, we find (1) the DeGrand-Toussaint monopole definition may be useful for $\beta_V$ larger than about 0.5, (2) the Villain model well approximates the Wilson one for $\beta$ smaller than $\beta_c$ and (3) in the Wilson action the monopole condensation occurs in the confinement phase and $\beta_c$ may be explained by the energy-entropy balance of monopole loops like in the Villain case.

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To understand confinement mechanism is very important but still unresolved problem in quantum chromodynamics (QCD). A promising idea is that the (dual) Meissner effect due to condensation of some magnetic quantity is the color confinement mechanism in QCD [1,2]. This picture is realized in the confinement phase of lattice compact QED [3–9].

The QED partition function in the Villain form [10,11] is

\[ Z_V = \prod_{s,\mu} \int_{-\pi}^{\pi} d\theta(s, \mu) \prod_{s,\mu>\nu} \sum_{n_{\mu\nu}(s)=-\infty}^{\infty} \exp(-\frac{\beta_V}{2} \sum_{s,\mu>\nu} [\theta_{\mu\nu}(s) - 2\pi n_{\mu\nu}(s)]^2), \]

where the plaquette variable \( \theta_{\mu\nu}(s) \in (-4\pi, 4\pi) \) is given by the link angles \( \theta(s, \mu) \in [-\pi, \pi) \) as \( \theta_{\mu\nu}(s) = \theta(s, \mu) + \theta(s + \hat{\mu}, \nu) - \theta(s + \hat{\nu}, \mu) - \theta(s, \nu) \). A dual transformation can be done exactly, leading us to a partition function describing a monopole Coulomb gas [4,6–8]:

\[ Z_m = (\prod_{s,\mu} \sum_{m_{\mu}(s)=-\infty}^{\infty} \delta_{\mu\nu}(m_{\mu}(s),0)) \exp(-2\pi^2 \beta_V \sum_{s,s',\mu} m_{\mu}(s) D(s-s') m_{\mu}(s')), \]

where \( D(s-s') \) is the lattice Coulomb propagator and \( \partial' \) is the backward difference. Here \( m_{\mu}(s) \) is a monopole current:

\[ m_{\mu}(s) = \frac{1}{2} \epsilon_{\mu\rho\sigma} \partial_{\nu} n_{\rho\sigma}(s + \hat{\mu}), \]

which is defined on a link of the dual lattice (the lattice with origin shifted by half a lattice distance all in four directions). \( \partial \) is the forward difference. Note that the coupling constant in (2) satisfies the Dirac quantization condition between electric and magnetic charges. One can prove the monopole condensation from energy-entropy balance using the action (2) and the (bulk) entropy \( \sim L \times \ln 7 \) where \( L \) is the monopole loop length [4,6,8]. The critical \( \beta_V \) is estimated to be \( \beta_V^c = \ln 7/(2\pi^2 D(0)) = .637 \) [4] which is in good agreement with the Monte-Carlo data \( \beta_V^c = .645 \) [12].

On the other hand, the Wilson partition function of lattice compact QED is

\[ Z_W = \prod_{s,\mu} \int_{-\pi}^{\pi} d\theta(s, \mu) \exp[-\beta_W \sum_{s,\mu>\nu} (1 - \cos \theta_{\mu\nu}(s))]. \]

Since the action of the Wilson form is the simplest and is used commonly also in non-abelian gauge theories, it is important to study the confinement mechanism of compact QED with
this action. However the exact dual transformation which enables us to make the above analytic study is impossible in this case, although this action (4) is easy to perform Monte-Carlo simulations. To speak rigorously, hence, the monopole condensation is not yet proved in the case of the Wilson action.

Using the Fourier transformation of (4) and the property of the modified Bessel function, the Wilson action is approximated for very large $\beta$ and for small $\beta$ by the Villain form (5). How good the Villain approximation to the Wilson form is was studied in [10,11] extensively. In Ref. [11], physical quantities like internal energy and specific heats can be well reproduced for small $\beta_V$ up to $\beta_V^c$, if both the coupling constants satisfy

$$\beta_V^{-1} = -2\ln\left(\frac{I_1(\beta_W)}{I_0(\beta_W)}\right),$$

where $I_0$ and $I_1$ are usual modified Bessel functions. The approximation becomes rapidly worse above the transition point except for very large $\beta$. For moderately large $\beta_V$, the Villain partition function gives results rather similar to a mixed cosine action with a modified coupling relation. However even if some physical quantities are well reproduced by the Villain approximation, it does not always mean that the confinement mechanisms of both models are the same. Really it is pointed in [11] that there are physical quantities not well reproduced even for small $\beta$. Considering applications to QCD, we want to know directly the confinement mechanism in the Wilson action.

For that purpose, one has to find first monopole currents in the Wilson action where there is no natural definition like (3) contrary to the Villain case. The only known definition is that given by DeGrand and Toussaint [5]. The plaquette variable $\theta_{\mu\nu}(s)$ can be decomposed into

$$\theta_{\mu\nu}(s) = \tilde{\theta}_{\mu\nu}(s) + 2\pi m_{\mu\nu}(s),$$

where $\tilde{\theta}_{\mu\nu}(s) \in [-\pi, \pi)$ and $m_{\mu\nu}(s)$ can be regarded as a number of the Dirac string penetrating the plaquette. DeGrand and Toussaint [5] defined a monopole current as

$$m^{DG}_\mu(s) = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu m_{\rho\sigma}(s + \hat{\mu}).$$
Recently Schram and Teper \cite{13} studied how well the DGT monopoles \( m^{DG}_\mu(s) \) approximate the natural ones \( m_\mu(s) \) based on Monte-Carlo simulations of the Villain form. They suggested the DGT prescription can be meaningfully used in the studies of the U(1) phase transition.

Next we have to find a partition function described in terms of the monopole currents corresponding to the dual form of the Wilson action in order to study the energy entropy balance. Actually an exact dual transformation is impossible in almost all models where dual variables are expected to play an important role in the dynamics. It is very important to develop a method determining a dual theory for such models. It is the most important original point of this note to give a method fixing a monopole action from a given ensemble of configurations generated in Monte-Carlo simulations. To demonstrate the usefulness of our method, we first apply the method to the natural monopoles in the Villain case of \( U(1) \) lattice gauge theory where the exact monopole partition function is known as in (2). Next we derive the action for the DGT monopoles in the Villain case and test reliability of the DGT prescription. Finally we apply the method to the Wilson case to check the occurrence of the monopole condensation. A brief description of our method and preliminary applications were given also in \cite{1415}.

We extend the method of Swendsen \cite{16} to determine an action from a given ensemble of monopole loops in vacuum configurations. A theory of monopole loops is given in general by the following partition function

\[
Z = (\prod_{s,\mu} \sum_{k_\mu(s)=-\infty}^{\infty}) (\prod_s \delta_{\partial_{s\mu} k_\mu(s),0}) \exp(-S[k]), \tag{8}
\]

where \( k_\mu(s) \) is the conserved integer-valued monopole current defined above in (3) or (7) and \( S[k] \) is a monopole action describing the theory. Consider a set of all independent operators which are summed up over the whole lattice. We denote each operator as \( S_i[k] \). Then the action can be written as a linear combination of these operators:

\[
S[k] = \sum_i f_i S_i[k], \tag{9}
\]
where $f_i$ are coupling constants. Really the monopole partition function \(\mathcal{Z}\) takes this form.

The expectation value of an operator of monopole currents are estimated as

\[
\langle O[k] \rangle = \frac{\langle \prod_{\mu} \sum_{k_{\mu}(s)=-\infty}^{\infty} \delta \partial_{\mu} k_{\mu}(s,0) \rangle \exp(-\sum_i f_i S_i[k])}{\langle \prod_{\mu} \sum_{k_{\mu}(s)=-\infty}^{\infty} \delta \partial_{\mu} k_{\mu}(s,0) \rangle \exp(-\sum_i f_i S_i[k])}.
\] (10)

Let us now determine the monopole action using the monopole current ensemble which are calculated by vacuum configurations generated by Monte-Carlo simulations. Since the dynamical variables here are $k_{\mu}(s)$ satisfying the conservation rule, it is necessary to extend the original Swendsen method. Consider a plaquette \((s', \hat{\mu}', \tilde{\nu}')\) instead of a link on the dual lattice. Define $\tilde{S}_i[k]$ as a part of an operator $S_i[k]$ containing the monopole currents on the links in the plaquette chosen, i.e., $k_{\mu'}(s'), k_{\nu'}(s' + \hat{\mu}')$, $k_{\mu'}(s' + \tilde{\nu}')$ and $k_{\nu'}(s')$. Then we get

\[
(\prod_{\mu} \sum_{k_{\mu}(s)=-\infty}^{\infty} \delta \partial_{\mu} k_{\mu}(s,0)) S_a[k] \exp(-\sum_i f_i S_i[k])
\]

\[
= (\prod_{\mu} \sum_{k_{\mu}(s)=-\infty}^{\infty} \delta \partial_{\mu} k_{\mu}(s,0)) \prod_{\mu} \sum_{k_{\mu}(s')=-\infty}^{\infty} \sum_{k_{\mu'}(s'+\hat{\mu}')}=-\infty \sum_{k_{\nu'}(s'+\tilde{\nu}')}=-\infty \sum_{k_{\nu'}(s')}=-\infty \delta \partial_{\mu} k_{\mu}(s',0) \delta \partial_{\mu'} k_{\mu}(s'+\tilde{\nu}'),0 \delta \partial_{\mu'} k_{\mu}(s'+\hat{\mu}'),0 S_a[k] \exp(-\sum_i f_i \tilde{S}_i[k])
\]

\[
\exp(-\sum_i f_i (S_i[k] - \tilde{S}_i[k])),
\] (11)

where the primed product means removing the sites and the links in the plaquette chosen. Note here that the sum of the current conservations on the four sites $\partial_{\mu} k_{\mu}(s') + \partial_{\mu'} k_{\mu}(s' + \hat{\mu}') + \partial_{\mu'} k_{\mu}(s' + \tilde{\nu}') + \partial_{\mu'} k_{\mu}(s' + \tilde{\nu}'+\tilde{\nu}')$ does not contain any current on the four links of the plaquette adopted. Hence

\[
(11) = (\prod_{\mu} \sum_{k_{\mu}(s)=-\infty}^{\infty} \delta \partial_{\mu} k_{\mu}(s,0)) \prod_{\mu} \sum_{k_{\mu}(s')=-\infty}^{\infty} \sum_{k_{\mu'}(s'+\hat{\mu}')}=-\infty \sum_{k_{\nu'}(s'+\tilde{\nu}')}=-\infty \sum_{k_{\nu'}(s')}=-\infty \delta \partial_{\mu} k_{\mu}(s',0) \delta \partial_{\mu'} k_{\mu}(s'+\tilde{\nu}',0) \delta \partial_{\mu'} k_{\mu}(s'+\hat{\mu}',0) \delta \partial_{\mu'} k_{\mu}(s'+\tilde{\nu}'+\tilde{\nu}',0) \exp(-\sum_i f_i (S_i[k] - \tilde{S}_i[k]))
\]

\[
= (\prod_{\mu} \sum_{k_{\mu}(s)=-\infty}^{\infty} \delta \partial_{\mu} k_{\mu}(s,0)) S_a[k] \exp(-\sum_i f_i S_i[k]),
\] (12)

where

\[
\bar{S}_a[k] = \frac{(\sum \delta)_{\bar{k}} S_a[k, \{k\}'] \exp(-\sum_i f_i \tilde{S}_i[k, \{k\}'])}{(\sum \delta)_{\bar{k}} \exp(-\sum_i f_i \tilde{S}_i[k, \{k\}'])}.
\] (14)
Here \( \{ k \}' \) do not contain the four currents on the links of the plaquette considered and

\[
(\sum \delta)_k \equiv \sum_{k_{\mu}(s')}=-\infty \sum_{\hat{k}_{\nu}(s'+\hat{\mu})}=-\infty \sum_{\hat{k}_{\mu}(s'+\hat{\nu})}=-\infty \sum_{k_{\nu}(s')}=-\infty \delta \partial_{\nu}k_{\mu}(s'),\delta \partial_{\nu}k_{\mu}(s'+\hat{\nu}),\delta \partial_{\mu}k_{\mu}(s'+\hat{\mu}),0.
\]

Since there are current conservations at all sites in (13), we get

\[
\partial_{\mu} \hat{k}_{\mu}(s') = \partial_{\mu} k_{\mu}(s') + \hat{k}_{\nu}(s') + \hat{k}_{\mu}(s') - k_{\mu}(s') - k_{\nu}(s')
\]

(16)

\[
= \hat{k}_{\mu}(s') + \hat{k}_{\nu}(s') - k_{\nu}(s') - k_{\mu}(s').
\]

(17)

Similarly, we see

\[
\partial_{\mu} \hat{k}_{\mu}(s' + \hat{\mu}') = \hat{k}_{\nu}(s' + \hat{\mu}') - \hat{k}_{\nu}(s' + \hat{\mu}') - k_{\nu}(s' + \hat{\mu}') + k_{\nu}(s'),
\]

(18)

\[
\partial_{\mu} \hat{k}_{\mu}(s' + \hat{\nu}') = \hat{k}_{\mu}(s' + \hat{\nu}') + \hat{k}_{\mu}(s') - k_{\mu}(s') - k_{\nu}(s').
\]

(19)

Now use the following relation:

\[
\sum_{M=-\infty}^{\infty} \delta k_{\mu}(s'), k_{\nu}(s') + M = 1.
\]

(20)

Then

\[
(\sum \delta)_k S_{\alpha}[\hat{k}_{\mu}(s'), \hat{k}_{\nu}(s' + \hat{\mu}'), \hat{k}_{\mu}(s' + \hat{\nu}'), \hat{k}_{\nu}(s'), \{ k \}'] = \sum_{M=-\infty}^{\infty} \sum_{\hat{k}_{\nu}(s')}=-\infty \sum_{\hat{k}_{\mu}(s'+\hat{\nu})}=-\infty \sum_{\hat{k}_{\mu}(s'+\hat{\mu})}=-\infty \sum_{\hat{k}_{\nu}(s')}=-\infty \delta \hat{k}_{\nu}(s'), \delta k_{\mu}(s'+\hat{\mu}'), \delta \hat{k}_{\nu}(s'+\hat{\nu}'), \delta \hat{k}_{\mu}(s') - M
\]

\[
\times S_{\alpha}[\hat{k}_{\mu}(s') + M, \hat{k}_{\nu}(s' + \hat{\mu}'), M, \hat{k}_{\nu}(s' + \hat{\nu}'), -M, \hat{k}_{\nu}(s') - M, \{ k \}']
\]

(21)

\[
= \sum_{M=-\infty}^{\infty} S_{\alpha}[\hat{k}_{\mu}(s') + M, \hat{k}_{\nu}(s' + \hat{\mu}'), M, \hat{k}_{\mu}(s' + \hat{\nu}'), -M, \hat{k}_{\nu}(s') - M, \{ k \}'].
\]

(22)

When the DGT monopoles are used, the sum with respect to \( M \) is restricted from the minimum \( m_1 \) to the maximum \( m_2 \), where

\[
m_1 = -2 - \text{Min}\{ k_{\mu}(s'), k_{\nu}(s' + \hat{\mu}'), -k_{\mu}(s' + \hat{\nu}'), -k_{\nu}(s') \},
\]

(23)

\[
m_2 = 2 - \text{Max}\{ k_{\mu}(s'), k_{\nu}(s' + \hat{\mu}'), -k_{\mu}(s' + \hat{\nu}'), -k_{\nu}(s') \}.
\]

(24)
Note that the DGT monopoles take integer values between -2 and 2 only as seen from the definition. Then using (10), we get

$$\langle S_a[k] \rangle = \langle \bar{S}_a[k] \rangle,$$

where

$$\bar{S}_a[k] = \frac{\sum_{M=-\infty}^{\infty} S_a[k] \exp(-\sum_i f_i \bar{S}_i[k])}{\sum_{M=-\infty}^{\infty} \exp(-\sum_i f_i \bar{S}_i[k])}$$

and

$$\bar{k}_\mu(s) \equiv k_\mu(s) + M(\delta_{s,s'} \delta_{\mu,\mu'} + \delta_{s,s'+\mu',\mu} \delta_{\mu,\nu'} - \delta_{s,s'+\mu'} \delta_{\mu,\nu} - \delta_{s,s} \delta_{\mu,\nu'}).$$

Introducing a new set of coupling constants $\{\tilde{f}_i\}$, we define

$$\tilde{S}_a[k] = \frac{\sum_{M=-\infty}^{\infty} S_a[k] \exp(-\sum_i \tilde{f}_i \bar{S}_i[k])}{\sum_{M=-\infty}^{\infty} \exp(-\sum_i \tilde{f}_i \bar{S}_i[k])},$$

where $\bar{k}$ is defined in (27). When all $\tilde{f}_i$ are equal to $f_i$, one can prove an equality from (25)

$$\langle \tilde{S}_a[k] \rangle = \langle S_a[k] \rangle.$$

When there are some $\tilde{f}_i$ not equal to $f_i$, one may expand the difference as follows:

$$\langle \tilde{S}_a - S_a \rangle = \sum_b \langle \tilde{S}_a \tilde{S}_b - \tilde{S}_a \tilde{S}_b \rangle (f_b - \tilde{f}_b),$$

where only the first order terms up to $O(f_b - \tilde{f}_b)$ are written down. This allows an iteration scheme for determination of the unknown constants $f_i$ from the ensemble of $\{k_\mu(s)\}$, which are generated in Monte-Carlo simulations.

Practically we have to restrict the number of interaction terms. We adopted quadratic interactions of up to 32 types listed in Table I in these studies. The first six terms are shown also graphically in Fig. I.

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1 All possible types of interactions are not independent, since $\partial_{\mu} k_\mu(s) = 0$. We can get rid of almost all interactions between different components of the currents from the quadratic action by use of the conservation rule.
Before getting the configurations, we checked the auto-correlation time and thermalization. For thermalization, we need more than 500 iterations in the Villain case, whereas 50 iterations are enough in the Wilson case near the critical \( \beta \). The auto-correlation quickly disappears in the Wilson case, whereas the situations in the Villain case are much worse near the critical \( \beta \). Hence using the Villain action, we generated 100 gauge field configurations separated by 50 sweeps after a thermalization of 1000 sweeps for \( \beta_V < 0.56 \). At \( \beta_V = 0.62 \), we separated 300 iterations. For more than the critical \( \beta'_V \sim 0.64 \), we performed 100 independent runs with different initial values and adopted one configuration after 1000 thermalization loops for each run. Each run in the Villain case was done on \( 8^4 \) lattice. In the Wilson case, we took 100 configurations separated by 50 sweeps after 1000 thermalization sweeps for every \( \beta_W \) on \( 8^4 \) and \( 12^4 \) lattices. The monopole currents are defined following (3) and (7). The statistical errors were estimated with the jackknife method.

Our results are the following.

1. The first coupling constants \( f_1 \sim f_6 \) of the action for the natural monopoles in the Villain case are plotted in Fig. 2 in comparison with the theoretical values given by (2). \( f_1 \) agree well with those of the theoretical values, whereas there are small discrepancies with respect to \( f_2 \sim f_6 \). The discrepancies may come from the truncation of the terms of the action taken. At the critical \( \beta'_V \sim 0.64 \), \( f_1 \) crosses the ln7 entropy line.

2. The same coupling constants of the action for the DGT monopoles in the Villain case are plotted in Fig. 3. Both \( f_1 \) of the natural and the DGT monopole actions are compared in Fig. 4. We see there is a large deviation for small \( \beta \). For \( \beta_V > 0.5 \), we may use the DGT prescription. Schram and Teper [13] recently showed the difference of the natural and the DGT monopoles can be reproduced by a random distribution of dipoles having trivial long-distance behaviors. They concluded therefore the DGT prescription can be used for \( \beta_V > 0.3 \). \( f_1 \) is the dominant part of the action and it plays an important role in the energy-entropy balance. There is a large gap with respect to \( f_1 \) at \( \beta_V = 0.3 \). It seems dangerous to use the DGT definition for such small
\( \beta_V \). Fortunately the DGT prescription looks rather good around the critical \( \beta_V \).

3. Using the DGT definition, we get the monopole action in the Wilson case. The first six coupling constants of the action on \( 12^4 \) lattice are plotted in Fig. 5. There is a small volume dependence for \( \beta_W \) larger than the critical value (\( \sim 1.0 \)). \( f_1 \) on \( 8^4 \) and \( 12^4 \) lattices are compared in Fig. 4. The energy of a monopole loop of length \( L \) may be approximated by a self-energy part \( f_1 L \) when \( L \) is large. The entropy line \( \ln \frac{7}{1} \) and the \( f_1 \) line crosses at \( \beta_W = 1.05 \) which is very near to the critical \( \beta_W \sim 1.0 \). Namely the critical coupling constant determined from the monopole condensation due to the energy-entropy balance of monopole loops agrees to that of the deconfinement transition. The monopole condensation really occurs in the confinement phase also in the Wilson model. This is the first direct evidence for the occurrence of the monopole condensation in the Wilson action without the use of the Villain approximation.

4. Similarity of the Villain and the Wilson partition functions is studied by comparing both \( f_1 \) in Fig. 4, where \( \beta_W \) is transformed into \( \beta_V \) using (3). Both are in good agreement for small \( \beta \) less than the critical value \( \sim 0.64 \). For larger \( \beta \), both begin to deviate rapidly. This is consistent with the conclusion given in [11].

Finally we have seen that our method is very useful in the study of compact QED. The method can be applied also to more interesting QCD cases. For preliminary reports, see [14,15]. The detail of the results will be published elsewhere [18].

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TABLE I. The quadratic terms of the monopole action adopted. Only the partner of the current multiplied by $k(\mu(s)$ are listed. All terms in which the relation of the two currents is equivalent are added to make each $S_i[k]$ invariant under translation and rotation. Here $\hat{a} \equiv \hat{\nu} + \hat{\rho}, \hat{b} \equiv \hat{\nu} + \hat{\rho} + \hat{\omega}, \hat{c} \equiv \hat{\mu} + \hat{\nu} + \hat{\rho} + \hat{\omega}, \hat{d} \equiv 2\hat{\mu} + \hat{\nu}, \hat{e} \equiv 2\hat{\nu} + \hat{\mu}$ and $\hat{f} \equiv 2\hat{\nu} + \hat{\omega}$, where $\hat{\mu}, \hat{\nu}, \hat{\rho}$ and $\hat{\omega}$ denote unit vectors in four different directions.

| i  | current partner   | i  | current partner | i  | current partner           |
|----|-------------------|----|-----------------|----|---------------------------|
| 1  | $k_\mu(s)$        | 13 | $k_\mu(s + \hat{e})$ | 25 | $k_\mu(s + 3\hat{\mu} + \hat{a})$ |
| 2  | $k_\mu(s + \hat{\mu})$ | 14 | $k_\mu(s + 2\hat{\nu})$ | 26 | $k_\nu(s + \hat{d} + 2\hat{\omega})$ |
| 3  | $k_\mu(s + \hat{\nu})$ | 15 | $k_\mu(s + \hat{\mu} + \hat{\rho})$ | 27 | $k_\mu(s + \hat{\nu} + \hat{\omega})$ |
| 4  | $k_\mu(s + \hat{\mu} + \hat{\nu})$ | 16 | $k_\mu(s + 3\hat{\mu})$ | 28 | $k_\mu(s + 3\hat{\mu} + \hat{b})$ |
| 5  | $k_\mu(s + \hat{\mu})$ | 17 | $k_\mu(s + \hat{\nu} + \hat{\mu})$ | 29 | $k_\mu(s + \hat{\nu} + 2\hat{\mu})$ |
| 6  | $k_\mu(s + 2\hat{\mu})$ | 18 | $k_\mu(s + \hat{\mu})$ | 30 | $k_\mu(s + \hat{\nu} + \hat{\omega})$ |
| 7  | $k_\mu(s + \hat{\mu} + \hat{\nu})$ | 19 | $k_\mu(s + 2\hat{\nu})$ | 31 | $k_\mu(s + \hat{\nu} + 2\hat{\mu})$ |
| 8  | $k_\mu(s + \hat{\nu})$ | 20 | $k_\mu(s + \hat{\mu} + \hat{\omega})$ | 32 | $k_\nu(s + \hat{\mu} + 3\hat{\mu})$ |
| 9  | $k_\mu(s + \hat{\nu})$ | 21 | $k_\mu(s + \hat{\mu} + \hat{\nu})$ | 33 | $k_\mu(s + \hat{\nu} + 2\hat{\mu})$ |
| 10 | $k_\mu(s + \hat{\nu})$ | 22 | $k_\mu(s + \hat{\mu} + \hat{\nu})$ | 34 | $k_\mu(s + \hat{\nu} + 2\hat{\mu})$ |
| 11 | $k_\mu(s + \hat{\nu})$ | 23 | $k_\mu(s + \hat{\mu} + \hat{\nu})$ | 35 | $k_\mu(s + \hat{\nu} + 2\hat{\mu})$ |
| 12 | $k_\mu(s + 3\hat{\mu} + \hat{a})$ | 24 | $k_\mu(s + 2\hat{\mu} + \hat{b})$ | 36 | $k_\mu(s + 3\hat{\mu} + \hat{b})$ |
FIGURES

FIG. 1. The first six terms of the monopole action adopted.

FIG. 2. Coupling constants $f_i$ versus $\beta$ in the Villain model when the natural monopole currents $m_{\mu}(s)$ are used. The solid (dotted, dashed and dott-dashed ) line denotes the theoretical curve of $f_1(f_2 = f_3; f_4 = f_5$ and $f_6)$.

FIG. 3. Coupling constants $f_i$ versus $\beta$ in the Villain model when the Degrand-Toussaint monopole currents $m_{DG}\mu(s)$ are used. The lines show the theoretical curves predicted by the monopole action for the natural monopoles.

FIG. 4. Coupling constants $f_1$ versus $\beta$ in the Villain model for both the natural and the Degrand-Toussaint monopole currents.

FIG. 5. Coupling constants $f_i$ versus $\beta$ in the Wilson model on $12^4$ lattice where the Degrand-Toussaint monopole currents $m_{DG}\mu(s)$ are used.

FIG. 6. Coupling constants $f_1$ versus $\beta$ in the Wilson model both on $8^4$ and $12^4$ lattices.

FIG. 7. Coupling constants $f_1$ versus $\beta$ in the Villain and the Wilson models, where the DGT monopoles are used. The solid line denotes the theoretical value of $f_1$ of the action for the natural monopoles.
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REAL MONOPOLE
DEGRAND–TOUSSAINT
THEORY

\( \ln 7 \)

\( (\beta)_{\nu} \)
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Wilson

\( \ln 7 \)

\( 12^4 \)

\( f_1 \bullet \)

\( f_2 \circ \)

\( f_3 \blacksquare \)

\( f_4 \Box \)

\( f_5 \diamond \)

\( f_6 \bigcirc \)
