New insights into the oscillation of the nucleons electromagnetic form factors in the timelike region

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The electromagnetic form factors of the proton and the neutron in the timelike region are investigated. The electron-positron annihilation into antinucleon-nucleon (\( \bar{NN} \)) pairs is treated in distorted wave Born approximation, taking into account the final-state interaction in the \( \bar{NN} \) system. The latter is based on amplitudes that are solutions of the Lippmann-Schwinger equation for an \( \bar{NN} \) potential derived within SU(3) chiral effective field theory. By fitting to the phase shifts and (differential) cross sections, a high quality description is obtained. With these amplitudes the oscillations of the electromagnetic form factors of the proton and the neutron are studied. It is found that each of them can be described by two fractional oscillators. One is characterized by ‘overdamping’ and dominates near the threshold, while the other is ‘underdamped’ and plays an important role in the high-energy region.

Introduction.– The electromagnetic form factors (EMFFs) of the nucleons parameterize their response to a virtual photon and play an essential role in exploring the nucleon structure. The EMFFs in the timelike region are accessible in the process of electron-positron annihilating into antinucleon-nucleon (\( \bar{p}p \) or \( \bar{n}n \)) pairs. For a review, see e.g. Ref. [1]. Over the last decade quite a few measurements have been reported [2][12]. The uncertainties of these experiment data have been greatly reduced as compared with the older measurements [13][20]. In particular, the BESIII Collaboration [12] measured the neutron EMFF with high statistics and reported in detail about the ‘oscillatory’ behavior of the EMFFs of nucleons in the energy region of [2.0–3.2] GeV, first seen for protons in Ref. [2]. Further, it is found that there should be a phase difference between the oscillations of the EMFFs of the proton and the neutron. However, at present it is too difficult to measure the differential cross section around the thresholds and the analysis of the EMFFs by Ref. [12] is restricted to the energy region above 2 GeV.

In the present work we want to extend the study of the oscillations to the low-energy region. Indeed, there has been a strong interest in the near-threshold region since long, both by theorists and experimentalists. Not only for e\(^+\)e\(^-\) \(\rightarrow\) \(\bar{NN}\), but also in electron-positron annihilation into other antibaryon-baryon pairs, there are clear enhancements around the thresholds, see e.g. Refs. [21][23], which can be attributed to the effects of the final-state interaction (FSI), for reviews see e.g. [24][25]. Accordingly, we take into account the \(\bar{NN}\) FSI to obtain reliable predictions for the EMFFs around the \(p\bar{p}\) and \(\bar{n}n\) thresholds.

As is well known, chiral effective field theory (ChEFT) provides a systematic way to deal with the dynamics of the nucleon-nucleon interaction in the low-energy region [20][24]. This approach has also been successfully extended to studies of the \(\bar{NN}\) interaction [28][29]. To cover the energy region up to 2.2 GeV, we consider \(\bar{NN}\) potentials up to next-to-leading order (NLO) within SU(3) ChEFT. Following Refs. [28][29], the \(\bar{NN}\) scattering amplitudes are obtained by solving a Lippmann-Schwinger equation. The low-energy constants (LECs) of ChEFT are fixed by fitting to the phase shifts and inelasticities of a partial wave analysis (PWA) [30] of data on \(\bar{p}p\) elastic scattering and the charge-exchange reaction \(\bar{p}p \rightarrow \bar{n}n\). Then we use these amplitudes to construct the e\(^+\)e\(^-\) \(\rightarrow\) \(\bar{NN}\) amplitude based on the distorted wave Born approximation (DWBA) [21][31][33], where the FSI in the \(\bar{NN}\) system is taken into account. By fitting to the experimental data such as cross sections and differential cross sections, the e\(^+\)e\(^-\) \(\rightarrow\) \(\bar{NN}\) amplitudes are established. With these amplitudes we can evaluate the EMFFs of nucleons from threshold up to 2.2 GeV. This framework has been also applied to study EMFFs of other baryons, specifically to e\(^+\)e\(^-\) \(\rightarrow\) \(\Lambda\Lambda\) [31], e\(^+\)e\(^-\) \(\rightarrow\) \(\Sigma\bar{\Sigma}\) [33], e\(^+\)e\(^-\) \(\rightarrow\) \(\Lambda\bar{\Lambda}\) [21]. Finally, we use fractional oscillation functions to fit to the EMFFs obtained from experiments as well as to our predictions from ChEFT, from the \(\bar{NN}\) threshold to 3.2 GeV. We find that there should be two oscillators for each nucleon, with some interesting relations between them. Note that the oscillatory behavior of the EMFFs of nucleons in the
Formalism. The partial wave amplitudes of the reaction $e^+ e^- \to \bar{N}N$ can be written as

$$F_{LL'}^{NN,e^+ e^-} = -\frac{4\alpha}{9} f_L^{NN} f_{L'}^{e^+ e^-},$$

with $L, L'$ being the orbital angular momenta of $NN$ and $e^+ e^-$, respectively, $\alpha \approx 1/137$ is the fine-structure constant, and $N$ represents both $p$ and $n$. The EMFFs are related to these partial wave amplitudes by

$$f_S^{NN} = \frac{M_N}{\sqrt{s}} G_M, \quad f_{N^0}^{NN} = \frac{1}{\sqrt{s}} \left(G_M - \frac{2M_N}{\sqrt{s}} G_E\right),$$

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Results and discussion. — The LECs and values for $G_E^{p,0}$ and $G_E^{n,0}$ are fixed by a fit to the $ar{N}N$ phase shifts and scattering lengths, as well as to $e^+e^- 	o ar{N}N$ (differential) cross sections and EMFFs. With regard to $NN$ only the phase shifts in the very low-energy region are considered, where the FSI effects play a dominant role [31]. The parameters $G_E^{n,0}$ are mainly determined by fitting to the $e^+e^- 	o NN$ data. The uncertainty is estimated following Refs. [29, 33], generated by one class of Bayesian naturalness priors [40]. One can see that in the low-energy region, i.e., $T_{\text{lab}} \leq 25$ MeV for LO and $T_{\text{lab}} \leq 50$ MeV for NLO, the first two/three points of the PWA are reproduced rather well.

The fit to the cross sections of $e^+e^- \to \bar{p}p$, $\bar{n}n$ is presented in Fig. 2. Though no pertinent results are shown, the data on differential cross sections were considered as additional constraint for fixing the free parameters. The achieved $\chi^2$ is $\chi^2_{d.o.f.} = 1.70$. Given the partial incompatibility between the available data sets, this $\chi^2_{d.o.f.}$ is reasonable. It can be seen that the cross sections increase rapidly right above the thresholds in both reactions and decrease at higher energies. However, in case of $e^+e^- \to \bar{p}p$ the cross section remains constant until roughly 2 GeV, and then it drops slowly, while the one for $e^+e^- \to \bar{n}n$ starts to decrease already around 1.9 GeV. As a consequence the observed oscillation of the EMFFs of the neutron and the proton differ, as will be discussed below.

With the $e^+e^- \to \bar{N}N$ amplitudes fixed, we can predict the EMFFs. Here we consider the effective EMFFs which are defined by

$$|G_{\text{eff}}^N(s)| = \sqrt{\frac{\sigma_{e^+e^- \to \bar{N}N}(s)}{4\pi\alpha^2 s/3}} C(s)[1 + \frac{2M_E}{s}],$$

and follow from the assumption that $G_E^N(s) = G_N^N(s)$. The results for $|G_{\text{eff}}^p|$ and $|G_{\text{eff}}^n|$ are shown in Fig. 2. As can be seen, the effective EMFFs drop off rapidly right from the $\bar{N}N$ threshold and then decrease more slowly with increasing $\sqrt{s}$. The neutron effective EMFF is a bit smaller than that of the proton. This may be caused by the fact that the net charge of valence quarks of the neutron is zero.

To study the ‘oscillatory’ behavior of the EMFFs suggested by the data in Refs. [2, 12], we introduce subtracted form factors (SFFs) by subtracting the ‘dipole’ contribution [11, 2, 36]

$$G_{\text{OE}}^N(s) = |G_{\text{eff}}^N(s)| - G_D^N(s),$$

where $G_D$ is the dipole expression for the proton [11, 36]

$$G_D^p(s) = \frac{\mathcal{A}_p}{(1 + s/m_n^2)(1 - s/m_p^2)}.$$

FIG. 1. Phase shifts of the $^3S_1$ partial waves. The top graphs are for the real part of the phase shifts, while the bottom ones are for the imaginary part. The left and right panels refer to $I = 0$ and $I = 1$, respectively. The red filled circles are results of the PWA [30]. The purple dashed and black solid lines are our results up to LO and to NLO with cutoff $\Lambda = 850$ MeV. The blue and magenta bands indicate the uncertainties of the LO and NLO calculations, in order.

FIG. 2. Cross sections for $e^+e^- \to \bar{p}p$ (left) and for $e^+e^- \to \bar{n}n$ (right). The data points are from ADONE73 [14], Fenice [15–17], DM1 [18], DM2 [19], BABAR [2, 47], CMD-3 [3, 4], BESIII [7, 9–12], SND [5, 6]. Same description of curves as in Fig. 1.

FIG. 3. Results for the effective EMFFs. The experimental data sets are taken from the references as indicated in Fig. 2 with additionally Fenice 1998 [17] and PS170 [13]. Same description of curves as in Fig. 1.
with $A_p = 7.7$, $m_s^2 = 14.8$ (GeV/c)$^2$ and $\alpha_0^2 = 0.71$ (GeV/c)$^2$.

The dipole formula for the neutron is given by

$$G_D^N(s) = \frac{A_n}{\left[1 - s/q_0^2\right]^2},$$

where one has $A_n = 3.5 \pm 0.1$. Combining our amplitudes up to 2.2 GeV and the high statistics data sets in the energy region of $[2.0, 3.2]$ GeV, a complete description of the $G_D^N(s)$'s from threshold up to 3.2 GeV is achieved. Then what is the underlying physics behind these SFFs? It is found that the fractional oscillation functions [18] can be fitted to the SFFs, $G_D^N$ rather well. We have

$$G_D^N(\tilde{p}) = G^{0,N}_{osc,1} E_{\alpha_1}(-\omega_2^2{\tilde{p}}^N) + G^{0,N}_{osc,2}(0) E_{\alpha_2}(-\omega_2^2{\tilde{p}} + \tilde{p}_0^N),$$

where the Mittag-Leffler function $E_{\alpha}(z)$ can be found in Ref. [19]

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(ak + \beta)}.$$ (13)

The subscripts 1, 2 are for two kinds of oscillators. Here, $\tilde{p}$ is the momentum of the nucleon in the rest-frame of the antinucleon $\tilde{N}$, $\tilde{p} = \sqrt{\not{E} - \not{M}_N}$. $E = s/(2M_N) - M_N$. $\omega_1, \omega_2$ are the oscillation frequencies for the two oscillators, and we set each of them to be the same for proton and neutron, as inspired by Ref. [12]. This is somehow reasonable since the proton and neutron are isospin doublets. $\alpha_1, \alpha_2$ are the damping factors. Notice that when $\alpha = 1$, one has $E_{1,1}(z) = e^z$, and the fractional oscillation will restore to a normal ‘overdamped’ oscillation. It is also required that $\alpha_1^2 = \alpha_2^2$ to make sure that the oscillations of the proton and the neutron are the same but only the phase and the modulus are different. $\tilde{p}_0^N$ describes the ‘phase delay’ of the oscillation between the proton and neutron for the second oscillator. $G^{0,N}_{osc,1,2}$ are the initial values, and one simply has $G_{osc}^N = G_{osc,1}^N + G_{osc,2}^N$. We obtain $G_{osc}^P(0) = 0.2535$ and $G_{osc}^N(0) = 0.3347$. Hence, we only need two independent initial values. Specifically, the equations of motions of the fractional oscillators are given by the following integral equations

$$G_{osc}^N(\tilde{p}) = G_{osc,1}^N(\tilde{p}) + G_{osc,2}^N(\tilde{p}),$$

$$G_{osc,1}^N(\tilde{p}) = G^{0,N}_{osc,1} - \frac{\omega_1^2}{\Gamma(\alpha_1)} \int_0^{\tilde{p}} (\tilde{p} - t) a_1^{\alpha_1 - 1} G_{osc,1}^N(t) dt,$$

$$G_{osc,2}^N(\tilde{p}) = G^{0,N}_{osc,2} - \frac{\omega_2^2}{\Gamma(\alpha_2)} \int_0^{\tilde{p}} (\tilde{p} - t) a_2^{\alpha_2 - 1} G_{osc,2}^N(t) dt,$$

where $\tilde{p} = \tilde{p} + \tilde{p}_0^N$. The first formula of Eq. (14) indicates that the two oscillations can be combined together, whereas for the second type we require a phase delay and thus must differentiate between the proton and the neutron.

The SFFs, i.e. the $G_D^N(\tilde{p})$’s, based on the very recent data sets [2] [6] [11] and also our own predictions including the $NN$ FSI effects from ChEFT are described rather well with two fractional oscillators, see Fig. 4. The data points of Refs. [10, 47] are also superimposed for the reader’s convenience. The first term in Eq. (12) dominates the oscillation behavior around the threshold, and then it decreases rapidly with increasing energy. Its oscillation is very similar to the ‘overdamped vibration’, of which the magnitude decreases quickly and no further fluctuations are seen. Indeed we have $\alpha_1^2 = 1.21$ and $\alpha_1^2 = 1.03$, close to 1. Therefore, we call this an ‘over-damped’ oscillator. The second term describes a much slower decreasing oscillatory behavior and dominates in the high-energy region, and thus we call it ‘under-damped’ oscillator. To see clearly the contributions of each kind of oscillators, we draw the individual contributions in Fig. 5 for the proton and the neutron. As can be seen, the ‘under-damped’ oscillators, cf. the red solid lines in Fig. 5, start from different positions for the proton and the neutron while they have the same period function, where there is a translation/delay on the momentum/energy. This confirms the ‘phase delay’ proposed in Ref. [12]. For the ‘overdamped’ oscillators, cf. the blue dashed lines in Fig. 5 that of the proton still shows ‘oscillation’ around

![FIG. 4. Results for the SFFs $G_D^N(\tilde{p})$, see Eq. (11), with fractional oscillation functions. The dashed line is the $G_{osc}$ in the low energy region obtained by ChEFT up to NLO.](image-url)
the threshold, whereas the one of the neutron just decreases. This is consistent with the fit parameters that the damping factor $\alpha_2$ is larger than $\alpha_0$. Further, it dominates in the low energy region, and should be related to the strong interactions. Our model can not only explain the oscillation behaviour above 2 GeV, but also around the threshold. Further, it suggests that the EMFFs also have an ‘overdamped’ behaviour around the threshold. This can be checked by future experiments.

**Summary.**— In this letter we have investigated the EMFFs of the nucleons in the timelike region. The FSI in the antinucleon-nucleon system has been taken into account in distorted wave Born approximation, based on an $NN$ interaction derived within SU(3) ChEFT up to NLO. The experimental data of (differential) cross sections of $e^+e^- \rightarrow NN$ as well as the phase shifts of $NN$ scattering are used as input to fix free parameters. A high quality description of the $e^+e^- \rightarrow NN$ data has been obtained. Specifically, the strong enhancements near the thresholds, seen in the $\bar{p}p$ as well as in the $\bar{n}n$ channel, can be reproduced.

A more detailed analysis of the electromagnetic form factors of the nucleons suggested that the oscillations seen for the proton and the neutron should be a combined effect of two fractional oscillators. One is ‘overdamped’, dominating near the threshold, and the other is ‘underdamped’, dominating in the high-energy region. The ‘phase delay’ of the ‘underdamped’ oscillators between proton and neutron is confirmed and should be relevant to the inner structure of the nucleons. The ‘overdamped’ oscillator reveals the strong interactions of the antinucleon-nucleon pairs. It would be thus highly relevant to perform measurements in the low-energy region to check the oscillation behaviour of the nucleon EMFFs, further deepening our understanding of the physics underlying the nucleon structure.

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