SUPERSYMMETRIC “SOLUTIONS” TO COSMOLOGICAL PROBLEMS:
BARYOGENESIS AND DARK MATTER

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ABSTRACT

The possible role of supersymmetry in our understanding of big bang baryogenesis and cosmological dark matter is explored. The discussion will be limited to the out-of-equilibrium decay scenario in SUSY GUTs, the decay of scalar condensates, and lepto-baryogenesis as a means for generating the observed baryon asymmetry. Attention will also be focused on neutralino dark matter.

1. Introduction

There are several outstanding problems in cosmology which rely on particle physics solutions. If supersymmetry (broken as it may be) is realized in nature, then it is not unreasonable to expect that supersymmetry plays a non-trivial role in the solutions to these problems. The two specific problems that I will concentrate upon here are: the origin of the baryon asymmetry and the nature of dark matter. The former problem has historically been associated with Grand Unified Theories (GUTs) and among the original ideas to generate the asymmetry was the out-of-equilibrium decay scenario. I will begin, therefore, with a look back at the supersymmetric versions of this scenario. There are also purely supersymmetric solutions to baryogenesis, most notably is the decay of scalar condensates known as the the Affleck- Dine (AD) scenario which will also be briefly discussed. I will comment on the role of cosmological inflation on both the out-of-equilibrium decay and the AD scenarios. Finally, it is no longer sufficient to generate a baryon asymmetry, but one must preserve it in the face of baryon number violating interactions associated with the standard electroweak model. These interactions, however, open up new possibilities for generating an asymmetry such as the out-of-equilibrium decay of superheavy leptons. These possibilities (in the context of supersymmetry) will also be discussed.

There are many possible solutions to the dark matter problem, many of which do not involve supersymmetry (nor any new particle physics candidate). However, the minimal supersymmetric standard model (MSSM) with unbroken R-parity does offer (in much of the parameter space) a cosmologically interesting dark matter candidate.

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the lightest supersymmetric particle or LSP\textsuperscript{[5,6]}. The most likely choice being the
supersymmetric partner of the U(1)-hypercharge gauge boson, the bino. Though the
“entire” supersymmetric parameter space will be surveyed, I will focus on the bino
as the LSP. A curious possibility that the LSP is a light photino which is nearly
degenerate with the lighter stop quark\textsuperscript{[7,8]} will also be discussed.

2. Baryogenesis

Our best information on the cosmological baryon density comes from big bang
nucleosynthesis. In order to achieve consistency with the observational determinations
of the light element abundances of deuterium through \(^7\)Li, the baryon-to photon ratio
is restricted to lie in the range\textsuperscript{[9]}

\[ 2.8 \times 10^{-10} \leq \eta \leq 4 \times 10^{-10} \] (1)

where \( \eta = n_B/n_\gamma \). Combined with the lack of any observed antimatter (in primary
form), our understanding of this small number is the problem which big bang baryo-
genesis attempts to solve.

2.1. The out-of-equilibrium decay scenario

The production of a net baryon asymmetry requires baryon number violating
interactions, C and CP violation and a departure from thermal equilibrium\textsuperscript{[10]}. The
first two of these ingredients are contained in GUTs, the third can be realized in an
expanding universe where it is not uncommon that interactions come in and out of
equilibrium. In SU(5), the fact that quarks and leptons are in the same multiplets
allows for baryon non-conserving interactions such as \( e^- + d \leftrightarrow \bar{u} + \bar{u} \), etc., or decays
of the supermassive gauge bosons X and Y such as \( X \rightarrow e^- + d, \bar{u} + \bar{u} \). Although today
these interactions are very ineffective because of the very large masses of the X and Y
bosons, in the early Universe when \( T \sim M_X \sim 10^{15} \text{ GeV} \) these types of interactions
should have been very important. C and CP violation is very model dependent. In
the minimal SU(5) model the magnitude of C and CP violation is too small to yield
a useful value of \( \eta \) and in general the C and CP violation comes from the interference
between tree level and one loop corrections.

The departure from equilibrium is very common in the early Universe when inter-
action rates cannot keep up with the expansion rate. In fact, the simplest (and most
useful) scenario for baryon production makes use of the fact that a single decay rate
goes out of equilibrium. It is commonly referred to as the out of equilibrium decay scenario\textsuperscript{[11]}. The basic idea is that the gauge bosons X and Y (or Higgs bosons) may
have a lifetime long enough to insure that the inverse decays have already ceased so
that the baryon number is produced by their free decays.

More specifically, let us call X, either the gauge boson or Higgs boson, which
produces the baryon asymmetry through decays. Let \( \alpha \) be its coupling to fermions.
For X a gauge boson, \( \alpha \) will be the GUT fine structure constant, while for X a Higgs
boson, \((4\pi\alpha)^{1/2}\) will be the Yukawa coupling to fermions. If the decay rate for X,
\( \Gamma_D \approx \alpha M_X \) is less than the expansion rate of the Universe, \( H \approx \sqrt{NT^2}/M_P \) (where
$N$ is the number of relativistic particles at temperature $T$ and $M_P$ is the Planck mass) at a temperature $T \sim M_X$ the decays will occur the out-of-equilibrium. Thus the condition on the superheavy mass is determined from, $\Gamma_D < H$ at $T = M_X$, or

$$M_X \gtrsim \alpha M_P (N(M_X))^{-1/2} \sim 10^{18} \alpha \text{GeV}$$

(2)

In this case, we would expect a maximal net baryon asymmetry to be produced and is given by

$$\frac{n_B}{s} \sim \frac{n_B}{N n_\gamma} \sim 10^{-2} \epsilon$$

(3)

where $s$ is the entropy density (a better quantity to compare to in an adiabatically expanding universe) and $\epsilon$ is the baryon asymmetry produced by an $X, \bar{X}$ decay and represents the degree of CP violation in the decay.

At least two Higgs five-plets are required to generate sufficient C and CP violation. (It is possible within minimal SU(5) to generate a non-vanishing $\epsilon$ at 3 loops, however its magnitude would be too small for the purpose of generating a baryon asymmetry.) With two five-plets, $H$ and $H'$, the interference of diagrams of the type in figure 1, will yield a non-vanishing $\epsilon$,

$$\epsilon \propto \text{Im}(a'^\dagger ab'^\dagger) \neq 0$$

(4)

if the couplings $a \neq a'$ and $b \neq b'$.

![Figure 1: One loop contribution to the C and CP violation with two Higgs five-plets.](image)

The out-of-equilibrium decay scenario discussed above did not include the effects of an inflationary epoch. In the context of inflation, one must in addition ensure baryogenesis after inflation as any asymmetry produced before inflation would be inflated away along with magnetic monopoles and any other unwanted relic. Reheating after inflation, may require a Higgs sector with a relatively light $O(10^{10} - 10^{11}) GeV$ Higgs boson. To see this, consider a simple model in which the inflaton potential depends on only a single dimensionful parameter $\mu$. In this case the energy density perturbations produced by inflation can be roughly estimated to be

$$\frac{\delta \rho}{\rho} \sim O(100) \frac{\mu^2}{M_P^2}$$

(5)

which when matched to the observed quadrupole moment observed in the microwave background anisotropy

$$\frac{\delta \rho}{\rho} = (5.4 \pm 1.6) \times 10^{-6}$$

(6)
fixes the coefficient $\mu$ of the inflaton potential:

$$\frac{\mu^2}{M_P^2} = \text{few} \times 10^{-8} \quad (7)$$

Fixing $(\mu^2/M_P^2)$ has immediate general consequences for inflation. For example, the Hubble parameter during inflation, $H^2 \simeq (8\pi/3)(\mu^4/M_P^2)$ so that $H \sim 10^{-7}M_P$. The duration of inflation is $\tau \simeq M_P^3/\mu^4$, and the number of e-foldings of expansion is $H\tau \sim 8\pi(M_P^2/\mu^2) \sim 10^9$. If the inflaton decay rate goes as $\Gamma \sim m_\eta^3/M_P^2 \sim \mu^6/M_P^5$, the universe recovers at a temperature $T_R \sim (\Gamma M_P)^{1/2} \sim \mu^3/M_P^2 \sim 10^{-11}M_P \sim 10^8\text{GeV}$. Thus, the light Higgs is necessary since the inflaton, $\eta$, is typically light ($m_\eta \sim \mu^2/M_P \sim O(10^{11})\text{ GeV}$), and the baryon number violating Higgs would have to be produced during inflaton decay. Note that a “light” Higgs is acceptable from the point of view of proton decay due to its reduced couplings to fermions. The out-of-equilibrium decay scenario would now be realized by Higgs boson decay rather than gauge boson decay and a different sequence of events. First the inflaton would be required to decay to Higgs bosons (triplets?) and subsequently the triplets would decay rapidly by the processes shown in figure 1. These decays would be well out of equilibrium as at reheating $T \ll m_H$ and $n_H \sim n_\eta$. In this case, the baryon asymmetry is given simply by

$$\frac{n_B}{s} \sim \epsilon \frac{n_H}{T_R^3} \sim \epsilon \frac{n_\eta}{T_R^3} \sim \epsilon \frac{T_R}{m_\eta} \sim \epsilon \left(\frac{m_\eta}{M_P}\right)^{1/2} \sim \epsilon \frac{\mu}{M_P} \sim 10^{-4}\epsilon \quad (8)$$

where I have substituted for $n_\eta \sim \rho_\eta/m_\eta \sim \Gamma^2 M_P^2/m_\eta$.

In a supersymmetric grand unified SU(5) theory, the superpotential $F_Y$ can be expressed in terms of SU(5) multiplets

$$F_Y = h_d H_2 \ 5 \ 10 + h_u H_1 \ 10 \ 10 \quad (9)$$

where $10, \bar{5}, H_1$ and $H_2$ are chiral supermultiplets for the 10, and $\bar{5}$ plets of SU(5) matter fields and the Higgs 5 and $\bar{5}$ multiplets respectively. There are now new dimension 5 operators which violate baryon number and lead to proton decay as shown in figure 2. The first of these diagrams leads to effective dimension 5 Lagrangian terms such as

$$L^{(5)}_{\text{eff}} = \frac{h_u h_d}{M_H} (\bar{q} q l l) \quad (10)$$

and the resulting dimension 6 operator for proton decay

$$L_{\text{eff}} = \frac{h_u h_d}{M_H} \left( \frac{g_2^2}{M_W^2} \text{ or } \frac{g_1^2}{M_B} \right) (qqql) \quad (11)$$

As a result of these diagrams the proton decay rate scales as $\Gamma \sim h^4 g^4/M_H^2 M_G^2$ where $M_H$ is the triplet mass, and $M_G$ is a typical gaugino mass of order $\lesssim 1\text{ TeV}$. This rate however is much too large if $M_H \sim 10^{10}\text{ GeV}$.
It is however possible to have a lighter ($O(10^{10} - 10^{11})$ GeV) Higgs triplet needed for baryogenesis in the out-of-equilibrium decay scenario with inflation. One needs two pairs of Higgs five-plets ($H_1, H_2$ and $H'_1, H'_2$) which is anyway necessary to have sufficient C and CP violation in the decays. By coupling one pair ($H_2$ and $H'_1$) only to the third generation of fermions via

$$aH_1 10^{10} + bH'_1 10^3 10^3 + cH_2 10^5 5_3 + dH'_2 10^5$$  \tag{12}$$

proton decay can not be induced by the dimension five operators. Triplet decay will however generate a baryon asymmetry proportional to $\epsilon \sim \text{Im}dc^\dagger ba^\dagger$.

2.2. The Affleck-Dine Mechanism

Another mechanism for generating the cosmological baryon asymmetry is the decay of scalar condensates as first proposed by Affleck and Dine\cite{Affleck:1984fy}. This mechanism is truly a product of supersymmetry. It is straightforward though tedious to show that there are many directions in field space such that the scalar potential vanishes identically when SUSY is unbroken. SUSY breaking lifts this degeneracy so that

$$V \simeq \tilde{m}^2 \phi^2$$  \tag{13}$$

where $\tilde{m}$ is the SUSY breaking scale and $\phi$ is the direction in field space corresponding to the flat direction. For large initial values of $\phi$, $\phi_o \sim M_{\text{GUT}}$, a large baryon asymmetry can be generated\cite{Affleck:1984fy}. This requires the presence of baryon number violating operators such as $O = qgql$ which are naturally provided for in supersymmetric GUTs and such that $\langle O \rangle \neq 0$. The decay of these condensates through such an operator with an effective quartic coupling of order $\tilde{m}^2/(\phi_o^2 + M_X^2)$ can lead to a net baryon asymmetry.

The baryon asymmetry produced, is computed by tracking the evolution of the sfermion condensate, which is determined by

$$\ddot{\phi} + 3H\dot{\phi} = -\tilde{m}^2\phi$$  \tag{14}$$

If it is assumed that the energy density of the Universe is dominated by $\phi$, then the oscillations will cease, when

$$\Gamma_\phi \simeq \frac{\tilde{m}^3}{\dot{\phi}^2} \simeq H \simeq \frac{\rho_\phi^{1/2}}{M_P} \simeq \frac{\tilde{m}\phi}{M_P}$$  \tag{15}$$
or when the amplitude of oscillations has dropped to $\phi_D \simeq (M_P \tilde{m}^2)^{1/3}$. Note that the decay rate is suppressed as fields coupled directly to $\phi$ gain masses $\propto \phi$. It is now straightforward to compute the baryon to entropy ratio,

$$\frac{n_B}{s} = \frac{n_B}{\rho_\phi^{3/4}} \simeq \frac{\lambda \phi_o^2 \phi_D^2}{\tilde{m}^{5/2} \phi_D^{3/2}} = \frac{\lambda \phi_o^2}{\tilde{m}^2} \left( \frac{M_P}{\tilde{m}} \right)^{1/6}$$

and after inserting the quartic coupling, $\lambda$,

$$\frac{n_B}{s} \simeq \epsilon \frac{\phi_o^2}{(M_X^2 + \phi_o^2)} \left( \frac{M_P}{\tilde{m}} \right)^{1/6}$$

which could be quite large.

In the context of inflation, a couple of significant changes to the scenario take place. First, it is more likely that the energy density is dominated by the inflaton rather than the sfermion condensate. Second, the initial value (after inflation) of the condensate $\phi$ can be determined by the inflaton mass $m_\eta$, $\phi_o^2 \simeq H^3 \tau \simeq m_\eta M_P$.

The baryon asymmetry in the Affleck-Dine scenario with inflation becomes

$$\frac{n_B}{s} \sim \epsilon \frac{\phi_o^4 \eta^3}{M_X^2 M_{P,5/2} \tilde{m}} \sim \frac{\epsilon m_\eta^{7/2}}{M_X^2 M_{P,1/2} \tilde{m}} \sim (10^{-6} - 1) \epsilon$$

for $\tilde{m} \sim (10^{-17} - 10^{-16}) M_P$, and $M_X \sim (10^{-4} - 10^{-3}) M_P$ and $m_\eta \sim (10^{-8} - 10^{-7}) M_P$.

### 2.3. Lepto-baryogenesis

#### 2.3.1. Preservation of the asymmetry

The realization of significant baryon number violation at high temperature within the standard model, has opened the door for many new possibilities for the generation of a net baryon asymmetry. Electroweak baryon number violation occurs through non-perturbative interactions mediated by “sphalerons”, which violate $B + L$ and conserve $B - L$. For this reason, any GUT produced asymmetry with $B - L = 0$ may be subsequently erased by sphaleron interactions.

With $B - L = 0$, it is relatively straightforward to see that the equilibrium conditions including sphaleron interactions gives zero net baryon number. By assigning each particle species a chemical potential, and using gauge and Higgs interactions as conditions on these potentials (with generation indices suppressed),

$$\begin{align*}
\mu_\eta + \mu_0 &= \mu_W \\
\mu_{u_R} - \mu_{u_L} &= \mu_0 \\
\mu_{d_R} - \mu_{d_L} &= -\mu_0 \\
\mu_{t_R} - \mu_{t_L} &= -\mu_0 \\
\mu_{d_L} - \mu_{u_L} &= \mu_W \\
\mu_{l}, - \mu_{\nu} &= \mu_W
\end{align*}$$

one can write down a simple set of equations for the baryon and lepton numbers and electric charge which reduce to:

$$\begin{align*}
B &= 12 \mu_{u_L} \\
L &= 3 \mu - 3 \mu_0 \\
Q &= 6 \mu_{u_L} - 2 \mu + 14 \mu_0
\end{align*}$$

(19)
where $\mu = \sum \mu_i$. In (20), the constraint on the weak isospin charge, $Q_3 \propto \mu_W = 0$ has been employed. Though the charges $B, L$, and $Q$ have been written as chemical potentials, since for small asymmetries, an asymmetry $(n_f - n_{\bar{f}})/s \propto \mu_f/T$, we can regard these quantities as net number densities.

The sphaleron process yields the additional condition,

$$9\mu_{ul} + \mu = 0 \quad (21)$$

which allows one to solve for $L$ and $B - L$ in terms of $\mu_{ul}$, ultimately giving

$$B = \frac{28}{79} (B - L) \quad (22)$$

Thus, in the absence of a primordial $B - L$ asymmetry, the baryon number is erased by equilibrium processes. Note that barring new interactions (in an extended model) the quantities $\frac{1}{3}B - L_e$, $\frac{1}{3}B - L_\mu$, and $\frac{1}{3}B - L_\tau$ remain conserved.

With the possible erasure of the baryon asymmetry when $B - L = 0$ in mind, since minimal SU(5) preserves $B - L$, electroweak effects require GUTs beyond SU(5) for the asymmetry generated by the out-of-equilibrium decay scenario to survive. GUTs such as SO(10) where a primordial $B - L$ asymmetry can be generated becomes a promising choice. The same holds true in the Affleck-Dine mechanism for generating a baryon asymmetry. In larger GUTs there are baryon number violating operators and associated flat directions. A specific example in SO(10) was worked out in detail by Morgan.

Another possibility for preserving a primordial baryon asymmetry when $B - L = 0$ arises if the asymmetry produced by scalar condensates in the Affleck-Dine mechanism is large ($n_B/s \gtrsim 10^{-2}$). After the decay of the A-D condensate, the baryon number is shared among fermion and boson superpartners. However, in equilibrium, there is a maximum chemical potential $\mu_f = \mu_B = \tilde{m}$ and for a large asymmetry, the baryon number density stored in fermions, $n_{B_f} = \frac{g_f}{6} \mu_f T^2$ is much less than the total baryon density. The bulk of the baryon asymmetry is driven into the $p = 0$ bosonic modes and a Bose-Einstein condensate is formed. The critical temperature for the formation of this condensate is given by $n_B \simeq n_{B_b} + n_{B_c} = \frac{g_b}{3} \tilde{m} T^2_c$ so that,

$$n_{B_c} = \frac{g_b}{3} \left( 1 - \left( \frac{T}{T_c} \right)^2 \right) T^2_c \quad (23)$$

At $T < T_c$, most of the baryon number remains in a condensate and for large $n_B$, the condensate persists down to temperatures of order 100 GeV. Thus sphaleron interactions are shut off and a primordial baryon asymmetry is maintained even with $B - L = 0$. One should note however that additional sources of entropy are required to bring $\eta$ down to acceptable levels.

2.3.2. Generating a baryon asymmetry from a primordial lepton asymmetry

Sphaleron interactions also allow for new mechanisms to produce a baryon asymmetry. The simplest of such mechanisms is based on the decay of a right handed
This mechanism is certainly novel in that it does not require grand unification at all. By simply adding to the Lagrangian a Dirac and Majorana mass term for a new right-handed neutrino state,

\[ \mathcal{L} \ni M_{\nu} \bar{\nu} \nu + \lambda H \bar{\nu} \nu \]  

(24)

the out-of-equilibrium decays \( \nu^c \rightarrow L + H^* \) and \( \nu^c \rightarrow L^* + H \) will generate a non-zero lepton number \( L \neq 0 \). The out-of-equilibrium condition for these decays translates to \( 10^{-3} \lambda^2 M_P < M \) and \( M \) could be as low as \( 10(10) \) TeV. (Note that once again in order to have a non-vanishing contribution to the C and CP violation in this process at 1-loop, at least 2 flavors of \( \nu^c \) are required. For the generation of masses of all three neutrino flavors, 3 flavors of \( \nu^c \) are required.) Sphaleron effects can transfer this lepton asymmetry into a baryon asymmetry since now \( B - L \neq 0 \). A supersymmetric version of this scenario has also been described. \(^{13,26} \)

The survival of the asymmetry, of course depends on whether or not electroweak sphalerons can wash away the asymmetry. The persistence of lepton number violating interactions in conjunction with electroweak sphaleron effects could wipe out both the baryon and lepton asymmetry in the mechanism described above through effective operators of the form \( \lambda^2 LLHH/M \). In terms of chemical potentials, this interaction adds the condition \( \mu_\nu + \mu_0 = 0 \). The constraint comes about by requiring that this interaction be out of equilibrium at the time when sphalerons are in equilibrium. Otherwise, the additional condition on the chemical potentials would force the solution \( B = L = 0 \). To prevent the erasure of the baryon asymmetry, the constraint on \( M/\lambda^2 \gtrsim 3 \times 10^9 \) GeV obtained by requiring the \( B+L \) violating operators to remain out of equilibrium at least until right-handed electrons come into equilibrium \(^{25} \) leads to a bound on neutrino masses, \( m_\nu \sim \lambda^2 v^2/M \lesssim 20 \) keV, where \( v = 247 \) GeV is the Higgs vev. Similar constraints can be derived on \( R \)-parity violating operators \(^{29} \).

In addition to the mechanism described earlier utilizing a right-handed neutrino decay, several others are now also available. In a supersymmetric extension of the standard model including a right-handed neutrino, there are numerous possibilities. Along the lines of the right-handed neutrino decay, the scalar partner or a condensate of \( \tilde{\nu}^c \)'s will easily generate a lepton asymmetry. Furthermore if the superpotential contains terms such as \( \nu^c_3 + \nu^c H_1 H_2 \), there will be a flat direction violating lepton number à la Affleck and Dine. While none of these scenarios require GUTs, those that involve the out-of-equilibrium decay of either fermions, scalars or condensates must have the mass scale of the right-handed neutrino between \( 10^9 \) and about \( 10^{11} \) GeV, to avoid washing out the baryon asymmetry later and to be produced after inflation respectively. In contrast, the decay of the flat direction condensate (which involves other fields in addition to \( \tilde{\nu}^c \)) only works for \( 10^{11} < M < 10^{15} \) GeV.

3. Dark Matter

There are several reasons for postulating the existence of dark matter. On the theoretical side, if the cosmological density parameter is one, then the upper bound on
the fraction of Ω in baryons is restricted by nucleosynthesis to take values \( \Omega_B < 0.08 \) leaving the remainder as non-baryonic dark matter. Also on the theoretical side, is the effect of dark matter on the growth of density perturbations. The problem of making galaxies and clusters is exasperated without dark matter. There are also several observational pieces of evidence which include: galactic rotation curves, X-ray emitting hot gas from elliptical galaxies and clusters, as well as gravitational lensing by dark halos. What portion of the dark matter is truly non-baryonic is still unknown, but if in fact \( \Omega = 1 \), most of the dark matter would be in the form a new particle candidate. I will here concentrate only the supersymmetric candidates. For a more general recent review see: ref. (31).

Supersymmetric theories introduce several possible candidates. If \( R \)-parity (which distinguishes between “normal” matter and the supersymmetric partners) is unbroken there is at least one supersymmetric particle which must be stable. I will assume \( R \)-parity conservation. The stable particle (usually called the LSP) is most probably some linear combination of the only \( R = –1 \) neutral fermions, the neutralinos: the wino, \( \tilde{W}^3 \), the partner of the 3rd component of the \( SU(2)_L \) gauge boson; the bino, \( \tilde{B} \), the partner of the \( U(1)_Y \) gauge boson; and the two neutral Higgsinos \( \tilde{H}_1 \) and \( \tilde{H}_2 \). Gluinos are expected to be heavier, \( m_{\tilde{g}} = \left( \frac{\alpha_3}{\alpha} \right) \sin^2 \theta_W M_2 \) and do not mix with the other states (\( M_2 \) is the soft SUSY breaking \( SU(2) \) gaugino mass). The sneutrino is also a possibility but has been excluded as a dark matter candidate by direct searches, indirect searches, and accelerator searches. For a recent examination of very heavy sneutrino candidates see ref.(36).

The only parameters which determine the mass and composition of the LSP are; \( M_2 \), \( \mu \) and \( \tan \beta \) (assuming the GUT relations among the soft SUSY breaking gaugino masses). The latter two are the supersymmetric Higgsino mixing mass and the ratio of the Higgs scalar vevs respectively. However, for the relic abundance of LSP’s, it is necessary to specify the Higgs (scalar) masses, and the sfermions masses. The LSP can be expressed as a linear combination
\[
\chi = a\tilde{W}^3 + b\tilde{B} + c\tilde{H}_1 + d\tilde{H}_2
\]
(25)

Pure state LSP possibilities are: The photino, when \( M_2 \to 0 \)
\[
\tilde{\gamma} = \tilde{W}^3 \sin \theta_W + \tilde{B} \cos \theta_W
\]
(26)

and
\[
m_{\tilde{\gamma}} \to \frac{8}{3} \left( \frac{g_1^2}{g_1^2 + g_2^2} \right) M_2
\]
(27)

the Higgsino, \( \tilde{S}^0 \), when \( \mu \to 0 \)
\[
\tilde{S}^0 = \tilde{H}_1 \cos \beta + \tilde{H}_2 \sin \beta
\]
(28)

and
\[
m_{\tilde{S}} \to \frac{2v_1 v_2}{v^2} \mu
\]
(29)
When $M_2$ is large and $M_2 \ll \mu$ then the binol, $\tilde{B}$, is the LSP and
\[ m_{\tilde{B}} \simeq M_1 = \frac{5}{3} \frac{\alpha_1}{\alpha_2} M_2 \] (30)

Finally when $\mu$ is large and $\mu \ll M_2$ either the Higgsino state $\tilde{H}$
\[ \tilde{H}_{(12)} = \frac{1}{\sqrt{2}} (\tilde{H}_1^0 + \tilde{H}_2^0) \quad \mu < 0 \] (31)
or the state
\[ \tilde{H}_{[12]} = \frac{1}{\sqrt{2}} (\tilde{H}_1^0 - \tilde{H}_2^0) \quad \mu > 0 \] (32)
is the LSP depending on the sign of $\epsilon$ and
\[ m_{\tilde{H}} \simeq |\mu| \] (33)

Figure 3: The relic neutralino density, $\Omega h^2$, in the $M_2 - \mu (= -\epsilon)$ plane.

The relic abundance of LSP’s is controlled by annihilations until freeze out. The value of $\Omega h^2$ is roughly proportional to $1/\langle \sigma v \rangle_{\text{ann}}$ and is determined by solving the Boltzmann equation for the LSP number density in an expanding Universe. The technique used is similar to that for computing the relic abundance of massive neutrinos. For binos, as was the case for photinos, it is possible to adjust the sfermion masses $m_{\tilde{f}}$ to obtain closure density. Adjusting the sfermion mixing parameter allows even greater freedom. In figure 3, the relic abundance ($\Omega h^2$) is shown in the $M_2 - \mu$ plane with $\tan \beta = 2, \mu < 0$, the Higgs pseudoscalar mass $m_0 = 50 \text{GeV}$, $m_t = 100 \text{GeV}$ and $m_{\tilde{f}} = 200 \text{GeV}$. Binos (which occupy the upper triangular quarter of figure 3 as the LSP), are cosmologically significant in the mass range $25 \sim 300 \text{ GeV}$. The lower bound coming from the requirement that for large $\mu$, $M_2 \gtrsim 45 \text{ GeV}$ to avoid a light chargino (the shaded regions at either large $\mu$ or $M_2$) and the upper bound coming from the bound on $\Omega h^2$ (heavier binos would require sfermions with masses $m_{\tilde{f}} < m_{\tilde{B}}$). As annihilations as well as scatterings proceed through sfermion exchange, detection rates for binos are expected to be somewhat low, $\lesssim 0.1/\text{kg/day}$. Clearly the minimal model offers sufficient room to solve the dark matter problem. Similar results have been found by other groups. In figure 3, in the higgsino sector $\tilde{H}_{12}$ marked off by the dashed line, co-annihilations between $\tilde{H}_{(12)}$ and $\tilde{H}_{[12]}$ were not included. These tend to lower significantly the relic abundance in much of this sector.
There is also a curious possibility which has been recently suggested in which the photino is the LSP and is light and nearly degenerate with a light stop. For example, it is still experimentally possible that the lighter stop quark has a mass in the range 20-40 GeV if the stop mixing angle $\theta_t \simeq 0.98$. At or near this value, the stop does not couple to the $Z^0$. For a photino with a mass in the range 16-33 GeV (i.e. nearly degenerate with the stop), the stop is nearly invisible. The relic density of the light photinos is acceptable even though all other sfermion masses may be very high, because the co-annihilation process $\tilde{\gamma} + c \rightarrow \tilde{t}$ and $\tilde{t}^* \rightarrow X$ is efficient if $m_\tilde{t} - m_\tilde{\gamma} \sim 3 GeV$. The relic density of photinos in this case is shown in figure 4. However, if all other SUSY mass scales are high, this photino is virtually undetectable although this sector may have consequences for the top quark branching ratio.

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