Evolution of the symmetry energy of hot neutron-rich matter formed in heavy-ion reactions

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It is shown that the experimentally observed decrease of the nuclear symmetry energy with the increasing centrality or the excitation energy in isotopic scaling analyses of heavy-ion reactions can be well understood analytically within a degenerate Fermi gas model. The evolution of the symmetry energy is found to be mainly due to the variation in the freeze-out density rather than temperature. The isoscaling analyses are useful for probing the interaction part of the nuclear symmetry energy, provided that both the freeze-out temperature and density of the fragments can be inferred simultaneously from the experiments.

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I. INTRODUCTION

Information about the symmetry energy of hot neutron-rich matter is important for understanding the dynamical evolution of massive stars and the supernova explosion mechanisms, while the symmetry energy at zero temperature is important for determining properties of neutron stars at \( \beta \)-equilibrium. In particular, the electron capture rate on nuclei and/or free protons in presupernova explosions is especially sensitive to the symmetry energy at finite temperatures. The electron captures drive the collapsing core towards more neutron-rich matter. They affect not only the electron degenerate pressure working against the gravity but also the neutrino fluxes carrying away energy from the core\textsuperscript{\textsuperscript{[1]} \textsuperscript{2} \textsuperscript{3} \textsuperscript{4}}. The larger the symmetry energy, the more difficult it is for the electron captures to happen. Heavy-ion reactions are a unique means to produce in terrestrial laboratories the hot neutron-rich matter similar to those existing in many astrophysical situations. The possibility of extracting useful information about the symmetry energy from heavy-ion reactions has stimulated much interest in the nuclear physics community\textsuperscript{\textsuperscript{[5] \textsuperscript{6} \textsuperscript{7}}}.

Especially, recent analyses of the isospin diffusion data in heavy-ion reactions\textsuperscript{\textsuperscript{[8] \textsuperscript{9} \textsuperscript{10}}} and the size of neutron-skin in\textsuperscript{\textsuperscript{208}}\textsuperscript{\textsuperscript{Pb}}\textsuperscript{\textsuperscript{11} \textsuperscript{12} \textsuperscript{13}} have already put a stringent constraint on the symmetry energy of cold neutron-rich matter at subnormal densities. This has led to a significantly more refined constraint on the mass-radius correlation of neutron stars\textsuperscript{\textsuperscript{14}} including the fastest pulsar discovered very recently\textsuperscript{15}. On the other hand, the temperature dependence of the symmetry energy for hot neutron-rich matter has received so far little theoretical attention.

Among the phenomena/observables identified as potentially useful probes of the nuclear symmetry energy, the isoscaling coefficients of fragments from heavy-ion reactions\textsuperscript{16} have been most extensively studied, see e.g.,\textsuperscript{17} for a recent review. Very interestingly, it was found recently that the extracted symmetry energy from the isoscaling analyses decreases significantly from its standard value of about 25 MeV at normal nuclear matter density\textsuperscript{18} to much smaller values with the increasing excitation energy or centrality in heavy-ion reactions at both Fermi\textsuperscript{\textsuperscript{18} \textsuperscript{19} \textsuperscript{20}} and relativistic energies\textsuperscript{21} \textsuperscript{22}. Moreover, an increasing temperature of the fragmenting system was found to accompany the decreasing symmetry energy in these reactions. However, the fundamental origin of this apparent evolution of the symmetry energy is still not clear and it is particularly important and interesting to understand to what degree the evolution is due to the density and/or the temperature dependence of the symmetry energy.

In this work, it is shown that the experimentally observed evolution of the symmetry energy can be well understood within a degenerate Fermi gas model at finite temperatures. Furthermore, it is found that the evolution of the symmetry energy is mainly due to the variation in the freeze-out density rather than temperature when the fragments are emitted in the reactions carried out under different conditions.

II. NUCLEAR SYMMETRY ENERGY AT FINITE TEMPERATURE

The Equation of State (EOS) of hot neutron-rich matter at a temperature \( T \) and an isospin asymmetry \( \delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p) \) can be written as\textsuperscript{23} \textsuperscript{24}
\[
E(\rho, T, \delta) = E(\rho, T, \delta = 0) + E_{\text{sym}}(\rho, T)\delta^2 + O(\delta^4). \tag{1}
\]

The temperature and density dependent symmetry energy \( E_{\text{sym}}(\rho, T) \) for hot neutron-rich matter can thus be extracted from \( E_{\text{sym}}(\rho, T) \equiv E(\rho, T, \delta = 1) - E(\rho, T, \delta = 0) \). The symmetry energy \( E_{\text{sym}}(\rho, T) \) is the energy cost to convert all protons in symmetry matter to neutrons at the fixed temperature \( T \) and density \( \rho \). For finite nuclei...
at temperatures below about 3 MeV, the shell structure and pairing as well as vibrations of nuclear surfaces are important and the symmetry energy was predicted to increase slightly with the increasing temperature [27, 28]. Interestingly enough, an increase by only about 8% in the symmetry energy in the range of $T$ from 0 to 1 MeV was found to affect appreciably the physics of stellar collapse, especially the neutralization processes [24]. At higher temperatures, one expects the symmetry energy to decrease as the Pauli blocking becomes less important when the nucleon Fermi surfaces become more diffused at increasingly higher temperatures [23, 24]. In this work, we use the thermal model of Mekjian, Lee and Zamick (MLZ) [27]. While all of our results have also been concurrently verified numerically by using the finite temperature Hartree-Fock (HF) approach using both Skyrme and Gogny forces [28], here we utilize the MLZ approach because of its analytically transparent properties. The results obtained within the MLZ approach are sufficient for the purposes of this work. Our studies based on the finite temperature HF calculations will be reported elsewhere [29]. The degenerate Fermi gas limit of the MLZ thermal model is appropriate for us to understand quantitatively the experimentally observed evolution of the nuclear symmetry energy. The symmetry energy $E_{\text{sym}}(\rho, T)$ has a kinetic contribution and an interaction part. In the MLZ model with Skyrme interactions, the interaction part is temperature independent, i.e.,

$$E_{\text{sym}}(\rho, T) = E_{\text{sym}}^{\text{kin}}(\rho, T) + E_{\text{sym}}^{\text{int}}(\rho). \quad (2)$$

At low temperatures, it is known that all mean field quantities are essentially temperature independent [22]. With even momentum-dependent Gogny forces, at temperatures relevant for fragment formation in heavy-ion reactions, our HF calculations indicate that the interaction part $E_{\text{sym}}^{\text{int}}$ is only slightly $T$-dependent. For temperatures much less than the Fermi energy, $T \ll E_{\text{Fermi}} \approx 36(\rho/\rho_0)^{2/3}$, the kinetic energy per nucleon of a near-degenerate two-component Fermi gas is [27]

$$E_{\text{sym}}^{\text{kin}}(\rho, T) = \frac{35}{3} u^{2/3} + \frac{32}{9} \delta^2 - \frac{8}{3} \frac{T^2}{1260 u^{2/3}} - \frac{1}{9} \delta^2. \quad (3)$$

where $u = \rho/\rho_0$ is the reduced density. The kinetic part of the symmetry energy is thus

$$E_{\text{sym}}^{\text{kin}}(\rho, T) = \frac{35}{3} u^{2/3} - \frac{8}{3} \frac{T^2}{1260 u^{2/3}}. \quad (4)$$

It is interesting to note that the $E_{\text{sym}}^{\text{kin}}(\rho, T)$ decreases with $-T^2$ with a rate depending on the density. On the other hand, for $T \gg E_{\text{Fermi}}$, the system becomes a non-degenerate classical gas with a kinetic symmetry energy of [27]

$$E_{\text{sym}}^{\text{kin}}(\rho, T) = \frac{3}{2} \lambda^2 \rho - 0.0033 \lambda^2 \rho^2, \quad (5)$$

where $\lambda = \sqrt{2\pi\hbar^2/mT}$ is the thermal wavelength of nucleons with an average mass $m$. It is seen that the symmetry energy decreases approximately according to $E_{\text{sym}}(\rho, T) \propto \rho/\sqrt{T}$ for $\lambda^2 \rho \ll 1$. The genuine feature of a decreasing symmetry energy associated with an increasing temperature at both the degenerate and non-degenerate Fermi gas limits is consistent with predictions of the microscopic and/or phenomenological many-body theories [23, 24, 25, 29, 30]. To evaluate the relative importance of the $T$-dependent part $\Delta E_T \equiv \pi^2 T^2/(1260 u^{2/3})$ with respect to the total symmetry energy for cold neutron-rich matter, we show in Fig. 1 the ratio $\Delta E_T/E_{\text{sym}}(\rho, T = 0)$. The analyses of the isospin diffusion data from NSCL/MSU [8, 9] and the study on the size of neutron-skin in $^{208}$Pb [11, 12, 13] have recently consistently constrained the symmetry energy of cold matter to be around $32(\rho/\rho_0)^{0.7} \leq E_{\text{sym}}(\rho, T = 0) \leq 32(\rho/\rho_0)^{1.1}$ at sub-normal densities. Using the above two limits for the $E_{\text{sym}}(\rho, T = 0)$ at a typical freeze-out temperature of $T = 5$ MeV for the fragment emission in heavy-ion reactions, it is seen that the ratio $\Delta E_T/E_{\text{sym}}(\rho, T = 0)$ increases quickly with decreasing density. The $T$-dependent part of the symmetry energy becomes increasingly more appreciable, e.g., up to about 35% at $\rho = 0.1\rho_0$ for $E_{\text{sym}}(\rho, T = 0) = 32(\rho/\rho_0)^{1.1}$. Moreover, since the ratio increases quadratically with $T$, the effect will be much larger at higher temperatures. This result also shows the magnitude, especially at low densities and/or high temperatures, of some artificial effects that would be introduced should one attribute the entire evolution of the symmetry energy to its density dependence while neglecting its intrinsic temperature dependence.

![FIG. 1](image)

**FIG. 1:** (Color online) The relative importance of the temperature-dependent part of the nuclear symmetry energy.

Corresponding to the above two limits for the $E_{\text{sym}}(\rho, T = 0)$, the interaction part of the symmetry energy $E_{\text{sym}}^{\text{int}}(\rho)$ is constrained between $E_{\text{sym}}^{\text{int}}(\rho) = \frac{35}{3} u^{2/3} - \frac{8}{3} \frac{T^2}{1260 u^{2/3}}$ and $E_{\text{sym}}^{\text{int}}(\rho) = \frac{3}{2} \lambda^2 \rho - 0.0033 \lambda^2 \rho^2$. The genuine feature of a decreasing symmetry energy associated with an increasing temperature at both the degenerate and non-degenerate Fermi gas limits is consistent with predictions of the microscopic and/or phenomenological many-body theories [23, 24, 25, 29, 30]. To evaluate the relative importance of the $T$-dependent part $\Delta E_T \equiv \pi^2 T^2/(1260 u^{2/3})$ with respect to the total symmetry energy for cold neutron-rich matter, we show in Fig. 1 the ratio $\Delta E_T/E_{\text{sym}}(\rho, T = 0)$. The analyses of the isospin diffusion data from NSCL/MSU [8, 9] and the study on the size of neutron-skin in $^{208}$Pb [11, 12, 13] have recently consistently constrained the symmetry energy of cold matter to be around $32(\rho/\rho_0)^{0.7} \leq E_{\text{sym}}(\rho, T = 0) \leq 32(\rho/\rho_0)^{1.1}$ at sub-normal densities. Using the above two limits for the $E_{\text{sym}}(\rho, T = 0)$ at a typical freeze-out temperature of $T = 5$ MeV for the fragment emission in heavy-ion reactions, it is seen that the ratio $\Delta E_T/E_{\text{sym}}(\rho, T = 0)$ increases quickly with decreasing density. The $T$-dependent part of the symmetry energy becomes increasingly more appreciable, e.g., up to about 35% at $\rho = 0.1\rho_0$ for $E_{\text{sym}}(\rho, T = 0) = 32(\rho/\rho_0)^{1.1}$. Moreover, since the ratio increases quadratically with $T$, the effect will be much larger at higher temperatures. This result also shows the magnitude, especially at low densities and/or high temperatures, of some artificial effects that would be introduced should one attribute the entire evolution of the symmetry energy to its density dependence while neglecting its intrinsic temperature dependence.

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130u − 111.4u^{1.1}$ (labeled as $x = 0$) and $E^{\text{sym}}_{\text{int}}(\rho) = 3.7u + 14.9u^{1.6}$ (labeled as $x = −1$).\[10\] With this $E^{\text{sym}}_{\text{int}}(\rho)$ and Eq. (4), we can now examine the evolution of the total symmetry energy (Eq. (2)) as functions of density and temperature. Shown in Fig. 2 is the evolution of the normalized symmetry energy $[E_{\text{sym}}(\rho, T) − E_{\text{sym}}(\rho_0, 0)]/E_{\text{sym}}(\rho_0, 0)$ as a function of temperature at sub-saturation densities. It is seen that in the temperature range considered here, the symmetry energy does not change much with temperature at a given density. It is interesting to mention that using the same Gogny interactions corresponding to $x = 0$ and $x = −1$\[32\] in the finite temperature HF approach, all of the major results discussed above are qualitatively reproduced\[28\]. The major physical features given by the MLZ thermal model are thus rather general.

### III. ISOSCALING IN HEAVY-ION COLLISIONS

It has been observed in many types of reactions that the ratio $R_{21}(N, Z)$ of yields of a fragment with proton number $N$ and neutron number $Z$ form two reactions reaching about the same temperature $T$ satisfies an exponential relationship $R_{21}(N, Z) \propto \exp(\alpha N)$\[10\],\[10\] \[20\],\[21\],\[33\],\[34\],\[35\],\[36\]. In several statistical and dynamical models under some assumptions\[33\],\[34\],\[35\],\[36\],\[37\], it has been shown that the scaling coefficient $\alpha$ is related to the symmetry energy $C_{\text{sym}}(\rho, T)$ via

\[
\alpha = \frac{4C_{\text{sym}}(\rho, T)}{T} \triangle [(Z/A)^2], \tag{6}
\]

where $\triangle [(Z/A)^2] \equiv (Z/A_1)^2 − (Z/A_2)^2$ is the difference between the $(Z/A)^2$ values of the two fragmenting sources created in the two reactions.

Before we proceed, a few comments and discussions regarding the validity of Eq. (6) and the physical meaning of $C_{\text{sym}}$ are in order. First of all, we notice that Eq. (6) is an approximation in equilibrium models and an empirical assumption in dynamical models where isoscaling is observed in generated events. Because of the different assumptions used in the various derivations, the validity of this equation is still disputable as to whether and when the $C_{\text{sym}}$ is actually the symmetry energy or the symmetry free energy. Moreover, the physical interpretation of the $C_{\text{sym}}(\rho, T)$ is also not clear, sometimes even contradictory, in the literature. The main issue is whether the $C_{\text{sym}}$ measures the symmetry energy of the fragmentating source or that of the fragments formed at freeze-out. This ambiguity is also due to the fact that the derivation of Eq. (6) is not unique. In particular, within the grand canonical statistical model for multifragmentation\[38\],\[39\], the $C_{\text{sym}}$ refers to the symmetry energy of primary fragments. While within the sequential Weisskopf model in the grand canonical limit\[32\], it refers to the symmetry energy of the emission source.

Based on the grand canonical model for multifragmentation, some experts take the view that the $C_{\text{sym}}$ is the symmetry energy of the fragments at the freeze-out. Moreover, they use the finite size effects, such as the surface symmetry energy and its temperature dependence, to explain the observed smaller value of $C_{\text{sym}}$ compared to the symmetry energy of about 30 MeV for cold nuclear matter at the saturation density. However, from the very nature of the isoscaling phenomenon itself that isotopes/isotones having very different mass numbers (sizes) fall on the same curve described by a single scaling coefficient, it is hard to believe that the finite-size effects have any influence on the $C_{\text{sym}}$ at all. In another word, unless the finite size effects on both the $C_{\text{sym}}$ and the temperature $T$, if there is any at all, are completely cancelled out, the isoscaling phenomenon should not have been observed in multifragmentation in the first place. Indeed, in the AMD analyses of isoscaling in multifragmentation, it was found that “the extracted symmetry energy shows almost no surface effect in it, which suggests that the properties of infinite nuclear matter can be directly obtained from the information of fragmentation”\[28\].

Within the grand canonical model for multifragmentation, another possible reason for the extracted small value of $C_{\text{sym}}$ is that fragments themselves at freeze-out are dilute. This picture, however, seems to contradict the basic Fisher hypothesis that correlations inside a dilute medium are exhausted by clusterization. One possible explanation put forward was that primary fragments formed in heavy-ion reactions are hot and thus also expanded\[18\],\[34\]. However, this explanation is insufficient to explain the much smaller value of $C_{\text{sym}}$ observed in central collisions. Moreover, the isoscaling phenomenon is actually observed experimentally for cold fragments. The sequential decay of hot primary fragments may not affect much the isoscaling coefficient. We notice that this is still a matter of hot debate depending on the model calculations\[39\],\[40\]. Therefore, with this view the small value of $C_{\text{sym}}$ for cold fragments extracted in the isoscaling experiments would indeed indicate that the fragments have dilute internal density. This would require a deep reconsideration of the statistical models from which Eq. (6) was derived.

To this end, it is necessary to repeat here some remarks made by one of us at the WC13 meeting\[41\]. While the Eq. (4) is a good approximation within the grand canonical statistical model for multifragmentation, the isoscaling coefficient $\alpha$ is sensitive to the density dependence of the symmetry energy not because of the $C_{\text{sym}}$ which should be the symmetry energy of normal nuclear matter, but rather because of the $Z/A$ ratios of the fragmenting sources through the dynamical isospin fractionation\[12\] in the early stage of the reaction. This was also pointed out recently in ref.\[43\]. Unfortunately, since the $Z/A$ ratio of the effective fragmentating source, if exists at all, is not directly accessible experimentally, simplified assumptions are normally made in the data analyses within both statistical and dynamical models. The efforts are normally misplaced on extracting the $C_{\text{sym}}$ as if it is the one depending on the density and temperature. Conse-
quently, the extracted $C_{\text{sym}}$ may thus contain some information about the density dependence of the symmetry energy that is actually carried by the $Z/A$ ratios of the fragmenting sources.

On the other hand, within the sequential Weisskopf model in the grand canonical limit, all quantities in Eq. 6 refer to the emission source. In particular, the $C_{\text{sym}}$ itself in Eq. 6 reflects the bulk symmetry energy of the low density fragmenting source. Besides, because of the way by which the data is analyzed as we mentioned above, the experimentally extracted $C_{\text{sym}}$ also contains information about the symmetry energy through the $Z/A$ ratios of the fragmenting sources. Here we thus use broadly a working assumption that the $C_{\text{sym}}$ reflects the symmetry energy of bulk nuclear matter. We notice that this assumption can be justified only in the sequential weisskopf model. Nevertheless, it is interesting and reassuring to note that this assumption is consistent with the statement in Ref. 16 that the $C_{\text{sym}}$ is the symmetry energy of uniform nuclear matter at a reduced density. Within this picture it is natural for the $C_{\text{sym}}$ to have values smaller than the symmetry energy of normal nuclear matter. One would thus have no difficulty with the Fisher hypothesis for fragment formation. At freeze-out, fragments are formed at their normal density in a large volume. Only the average density in the freeze-out volume is small.

We would also like to stress here that the $C_{\text{sym}}(\rho, T)$ extracted from studying the isoscaling coefficient $\alpha$ is the total symmetry energy at the finite temperature $T$. It should be distinguished from the symmetry energy at zero temperature extracted from transport model analyses of dynamical observables, such as the isospin diffusion, or that from studying the neutron-skims of heavy nuclei. In transport models, the nucleon mean field or effective interaction for cold nuclear matter is used as an input. The zero temperature symmetry energy as a function of density can thus be constructed analytically from the particular mean fields or effective interactions used in the calculations. Special cares should thus be taken when comparing the density functions for nuclear symmetry energy extracted from different approaches and/or results from the same approach but for reactions at different temperatures, especially at low densities.

Within our working assumption that the extracted $C_{\text{sym}}$ from isoscaling analyses is the symmetry energy of the fragmenting source, we can then compare the experimental $C_{\text{sym}}$ with our calculations of the symmetry energy for uniform nuclear matter. In Fig. 2, the experimental data taken from the SJY group at Texas A&G University (filled circles) 19 and the INDRA-ALADIN collaboration at GSI (open squares) 20,22 are compared with the calculations. In the TAMU experiments, combinations of projectile-like fragments from peripheral to semiperipheral collisions of 25 MeV/nucleon $^{86}$Kr and $^{64}$Ni beams on several neutron-rich targets were used in the isoscaling analyses. The symmetry energy was found to decrease quickly from about 25 MeV to 19 MeV as the temperature increases from about 4.8 MeV to 5.8 MeV. The INDRA@GSI data were obtained from the fragmentation of target-like residues following collisions of $^{12}$C on $^{112,124}$Sn targets at a beam energy of 300 MeV/nucleon. The INDRA@GSI data indicate that the symmetry energy decreases from about 26 MeV to 16 MeV as the temperature increases from about 6 MeV to 9 MeV when the reaction goes from peripheral to central collisions 20.

It is very interesting to compare the calculations using both the $x = 0$ (upper window) and the $x = -1$ (lower window) interactions with the experimental data. The comparison then allows us to estimate the required density of the fragment emitting source. At the respectively low temperatures reached in the peripheral reactions at TAMU and GSI, the calculated results indicate that the fragments are emitted from sources at densities only slightly below $\rho_n$. In the peripheral reactions, either the projectile-like or target-like residue is only slightly excited with little expansion. While at the higher temperatures reached in the more central reactions, the fragments are emitted from significantly diluted sources with densities depending on the interaction used. This picture is consistent with dynamical model calculations of nuclear multifragmentations in the energy range considered. More interestingly, it is seen that the experimentally observed evolution of the symmetry energy is mainly due to the change in density rather than temperature. Around the typical freeze-out temperatures reached in both the TAMU and the GSI experiments, the evolution of the symmetry energy due to the change in temperature at a given density is rather small. It implies that the underlying origin of the observed decrease of the symmetry energy with the apparently increasing temperature...
or centrality in these experiments is actually due to the accompanying decrease in freeze-out density. Comparing the two sets of data, it is seen that they actually parallel with each other in the common density range. Since the evolution of the symmetry energy is essentially independent of the temperature for the experiments considered, the two sets of data thus indicate the same density-dependence of the symmetry energy as one expects. On the other hand, within the view that the extracted $C_{\text{sym}}$ reflects the symmetry energy of the fragments at freeze-out, the two data sets are incompatible.

From the above discussions, we can see that the evolution of the symmetry energy can be useful in exploring the density dependence of the interaction part of the nuclear symmetry energy. The latter is most uncertain but very important for many interesting questions in astrophysics. With the interaction labeled $x = 0$, the hottest point requires an average freeze-out density of about $0.62\rho_0$ and $0.49\rho_0$ for the TAMU-SJY and the INDRA@GSI data, respectively. While using the interaction labeled $x = -1$, the corresponding average freeze-out density is about $0.8\rho_0$ and $0.68\rho_0$, respectively. Therefore, the required freeze-out density depends strongly on the interaction part of the symmetry energy. The effect of using interactions of $x = 0$ and $x = -1$ is about 30% and 40% for the TAMU and the INDRA@GSI experiment, respectively. Of course, for the purpose of extracting the interaction part of the symmetry energy from the isoscaling analyses, it is necessary to know not only the freeze-out temperature but also the density when the fragments are emitted from the reactions. Fortunately, inferring both the freeze-out temperature and density in isoscaling analyses has been shown feasible very recently by the TAMU-JBN group [31], albeit largely based on model calculations. Since an independent determination of the freeze-out density will be very useful, observables known to be sensitive to the freeze-out density, such as the fragment correlation functions [11,15] or source functions from imaging techniques [11,17], may be explored together with the isoscaling analyses.

IV. SUMMARY

In summary, within the degenerate Fermi gas model of Mekjian, Lee and Zamick it is shown that the experimentally observed evolution of the symmetry energy can be well understood. Furthermore, the evolution is found to be mainly due to the variation in the freeze-out density rather than temperature in the reactions carried out under different conditions. The isoscaling analyses are thus useful for probing the interaction part of the nuclear symmetry energy, provided that both the freeze-out temperature and density can also be inferred.

Acknowledgments

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