On the optimal strategy for the hedge fund manager: An experimental investigation

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Abstract — This paper examines the empirical validity of Nicolosi’s model (2018) which investigates the optimal strategy for a hedge fund manager under a specific payment contract. The contract specifies that the manager’s payment consists of a fixed payment and a variable payment, which is based on the over-performance with respect to a pre-specified benchmark. The model assumes that the manager is an Expected Utility agent who maximises his or her expected utility by buying and selling the asset at appropriate moments. Nicolosi derives the optimal strategy for the manager. To find this, Nicolosi assumes a Black-Scholes setting where the manager can invest either in an asset or in a money account. The asset price follows geometric Brownian motion and the money account has a constant interest rate. I experimentally test Nicolosi’s model. To meet the aim of this paper, I compare the empirical support of Nicolosi’s story with other possible strategies. The results show that Nicolosi’s model receives strong empirical support for explaining the subjects’ behaviour, though not all of the subjects follow Nicolosi’s model. Having said this, it seems that the subjects somehow follow the intuitive prediction of Nicolosi’s model in which the decision-maker responds to the difference between the managed portfolio and the benchmark to determine the portfolio allocation.

Keywords: fund manager, portfolio strategy, laboratory experiment

JEL Classification: G11, C91

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1 Introduction

The hedge fund industry has grown enormously in the last few decades. It may be best defined as the private investment vehicle deploying a wide range of investment strategies in order to achieve a high rate of return, though there are alternative definitions for it (Hildebrand 2005). It has a wide variety of investments such as stock, bonds, real estate, and other commodities. The hedge fund manager is then responsible to manage the investor’s funds under a specific contract. The contract initially specifies the investor’s target (usually referred to as the benchmark), the investment period and the payment scheme. The payment typically is based on the manager’s performance with respect to a pre-specified benchmark; where the benchmark is set by the investor. The better is the manager’s performance with respect to the benchmark, the higher is the manager’s payment.

Clearly, the payment scheme determines the manager’s behaviour, given his or her risk attitude, in managing the investor’s funds (Palomino and Prat 2003). The investor employs this payment scheme to meet his or her benchmark, and the manager maximises his or her expected utility by buying and selling the asset at appropriate moments given the payment scheme. So once the contract is agreed, the manager chooses his or her portfolio strategy, given the risk attitude, to ensure beating the benchmark at maturity, in order to maximise the manager’s utility.

Much literature has explored the optimal portfolio strategy for the hedge fund manager in order to maximise his or her expected utility under a specific contract. Notable amongst these recently are Browne (1999), Carpenter (2000), Gabih et al. (2006), Hodder and Jackwerth (2007), Panageas and Westerfield (2009), Guasoni and Obloj (2016) which investigate the optimal portfolio choice for the manager in continuous-time with respective to a selected benchmark by the investor; this literature being motivated by the work of Merton (1969, 1971). One clear conclusion from this literature is that the benchmark level determines the manager’s behaviour, given his or her risk attitude (measured by the level of risk aversion). In particular, this literature investigates how the manager’s risk attitude affects his or her allocation decision: that is, how much to allocate in the risky asset. Generally, the literature shows that the manager is highly likely to hold more of the asset (that is, take on more risk) when the portfolio value is below the benchmark, in order to increase his or her chance of ending up with a higher payment. Contrariwise, the manager should reduce his or her portfolio volatility (by holding less of the asset) when the performance is relatively above the benchmark.

This paper examines the empirical validity, with a laboratory experiment, of a recent theory — that of Nicolosi (2018). This theory investigates the dynamic optimal strategy

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1 This benchmark can be either fixed or variable. The fixed benchmark usually is the expected return from the investment funds whereas the variable benchmark usually is the portfolio value at maturity following the investor’s portfolio allocation choice.

2 Electronic copy available at: https://ssrn.com/abstract=3394439
for the hedge fund manager under a performance-based payment. In his model, Nicolosi specifies two types of payment: a *fixed payment* and a *variable payment*, where the variable payment is based on the over-performance at maturity with respect to the benchmark. So the manager surely earns the fixed payment and will earn the variable payment depending on what he or she achieves. The benchmark is a linear combination of the investment invested in the risky and riskless assets, and the over-performance is achieved if the manager makes a higher portfolio value than that of the benchmark at maturity. Nicolosi imposes two important rules of the game: i) the manager allocates the fund between an asset (risky) and a money account (riskless), and ii) the manager’s performance is assessed by the value of the portfolio at maturity. He also assumes that the asset price follows geometric Brownian motion while the money account provides a constant interest rate.

Nicolosi derives the optimal portfolio strategy for the manager to maximise his or her expected utility subject to the given investment funds. The optimal strategy is dynamic portfolio choice decisions that maximizes the manager’s expected utility at maturity. The intuition behind this solution is similar to the existing literature in which the optimal strategy manages the manager’s risk-taking behaviour, given his or her level of risk aversion, in order maximise his or her expected utility at maturity. One crucial implication of Nicolosi’s story is that, during the trading period, the manager should not hold a high allocation in the asset when his or her portfolio is above the benchmark. *Mutatis mutandis*, the manager should allocate his or her portfolio to the asset when his or her portfolio value is lower than that of the benchmark. Following the optimal strategy, thus, helps the manager to end up earning both fixed and variable payments as his or her portfolio value is higher than that of the benchmark at maturity — hence receiving the maximum utility.

The aim of this paper is to investigate how close is actual behaviour to the optimal strategy of Nicolosi’s model given the estimated risk aversion. Actual behaviour is then compared with other strategies to check the empirical validity of Nicolosi’s model. I estimate the individual risk aversion — elicited from the actual choice — which best explains behaviour and use it to compute the optimal strategy and the portfolio value at maturity. In the next section, I describe Nicolosi’s model. Section 3 describes the experimental design, Section 4 describes the econometric specification, Section 5 presents the results and analysis, and Section 6 discusses and concludes.

## 2 Nicolosi’s model of the fund manager

Nicolosi explores the optimal strategy for the hedge fund manager who wants to maximise his or her expected utility subject to the investment funds. The hedge fund manager receives investment funds \( W_0 \) from the investor and takes responsibility to invest in the financial market. There are two types of the financial market where the manager can
invest, the risky asset market and the money market. The risky asset market trades an asset whose price \( S \) fluctuates over time \( t \). The money market is riskless and gives a constant return \( r \). What the manager does then is to set portfolio allocation to be invested in the asset \( \theta \) and in the money market \((1 - \theta)\).

The investor asks if the manager can achieve, at least, the benchmark \( Y \) from the investment funds over an investment period \( T \). This benchmark is the basis of the manager’s performance measure and it is used to determine his or her payment; this payment will be explained later. The investor arbitrarily sets his or her benchmark as the value at maturity of a portfolio with a constant proportion \( \beta \) invested in the asset and a constant proportion \((1 - \beta)\) in the money market. The investor then sees what would happen to his or her benchmark value at maturity \( Y_T \) following this scheme.

The manager agrees on a contract, determined by the investor, which sets the investment period and the payment scheme for the manager. The investor pays the manager depending on what the manager achieves at maturity \( W_T \). The payment \( \Pi \) consists of two terms, a fixed and a variable payment. The fixed payment is a percentage \( K \) of the initial investment funds and the variable payment is a share \( \alpha \) on the over-performance \( W_T - Y_T \) relative to the benchmark. It is assumed that there is no penalty for the manager if he under-performs relative to the benchmark. It follows that the manager will always earn non-negative payment irrespective of his or her performance. However the better is the performance compared to the benchmark, the higher is the payment for the manager.

Nicolosi assumes a Black-Scholes setting with the asset price following standard geometric Brownian motion. We can write this as:

\[
\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t
\]

where \( S_t \) is the asset price at time \( t \in [0, T] \), \( \mu \) and \( \sigma \) are trend and volatility of the asset price respectively, and \( Z \) is a standard Brownian motion which follows \( N(0,1) \). The asset price follows a geometric Brownian motion, hence it is defined as:

\[
S_t = S_0 \exp(\mu - 0.5\sigma^2)t + \sigma V_t
\]

where \( V_t = Z_t d_t^{0.5} \) is the increment of a Wiener process and \( S_0 \) is the initial asset price.

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2One may also refer this to as the “investment planning horizon”.

3Despite this assumption, there may be various implementation to be taken for the case of under-performance considering that the investor pays a relatively high amount payment for the manager. For example, a percentage deduction to the fixed payment depending on the magnitude of the under-performance.

4Black-Scholes setting has following assumptions: a) there are two types of market, the asset market (risky) and the money market (risk-free), b) asset pays no dividend and there is no transaction cost in the market, c) asset price reflects all information in the asset market, d) asset price is exogenous to all agents, e) it is possible to borrow and lend cash at riskless rate as well as doing short-selling, f) the asset price change is random with known parameters, and g) it is possible to buy and to sell asset at any time. This assumption is important in the model in order to draw stochasticity of the asset price.

5Wiener process \( (V_t) \) has the following properties: a) it is continuous, b) its change process is independent of the previous values, c) its increment process follows \( N(0, d_t) \), d) \( V_0 = 0 \).
Both the manager and the investor are aware of this process and its parameters.

Given the contract, the investor will pay the manager with a linear combination of the *fixed* and the *variable* payment which can be written as: \( \Pi = K + \alpha (W_T - Y_T)^+ \). The first term is the fixed payment \( (K) \) and the second term is the variable payment where \( \alpha \) is a proportion of the positive underlying managed portfolio minus the benchmark at maturity \( (W_T - Y_T)^+ \). So the higher is the \( (W_T - Y_T)^+ \), the higher is the manager’s payment.

The manager is assumed to be an Expected Utility (EU) agent who maximises his or her expected utility from the payment in managing the investment fund. The model assumes that the utility function of the manager is that of constant relative risk aversion (CRRA) with a parameter risk aversion \( \gamma \). In addition, it assumes that the manager is strictly risk-averse, so that \( \gamma > 0 \). Therefore the manager’s problem is written as follows:

\[
\max_{\hat{W}_T} E \left[ u \left( \alpha (W_T - Y_T)^+ + K \right) \right] \text{ s.t. } W_T = 0
\]

where \( \xi \) is the state price density \( \xi_T = \exp^{-\left( r + 0.5 \sigma^2 \right) T - X z_T} \) and \( \xi_0 = 1 \) – where \( X = \left\{ \frac{u}{\alpha} \right\} \) and \( X > 0 \). Although Equation 2 is a static problem, it is maximised through optimising the dynamic problem throughout the investment period by setting optimal allocation \( \theta^* \) subject to \( W_T \) \( \nexists \)

Crucial to this approach is to define the optimal portfolio at maturity \( (W_T^*) \). Carpenter (2000) proposes the solution of this problem in which \( W_T^* \) depends on \( Y_T \), since the manager would never want \( W_T \in (0, Y_T] \), and the realisation of \( \xi_T \). It is given by:

\[
W_T^* = \left\{ \left\lfloor I \left( \frac{\lambda^* \xi_T}{\alpha} \right) - K \right\rfloor \frac{1}{\alpha} + Y_T \right\} I_{\{\xi_T \leq \xi^*\}}
\]

where \( I (X) = \left\lfloor u^{-1} (x) \right\rfloor \) is the inverse function of the marginal utility and \( I (\_\_) \) is the indicator function over \{ \_ \} and \( \xi_T \) is the threshold state price density. As a part of the solution, there exists a unique Lagrange multiplier \( \lambda^* > 0 \) to ensure that \( E \left[ \frac{\xi_T}{\xi_0} W_T \right] = W_0 \) is satisfied for any \( W_T^* > Y_T \).

Proposition 1 of Nicolosi’s model proposes the optimal portfolio strategy throughout the investment period \( (W_T^*) \) that leads to the optimal portfolio at maturity \( (W_T^*) \). Given the manager’s risk aversion \( \gamma \), the optimal portfolio strategy \( W_T^* \) for any \( \beta \leq \beta_m \) — where \( \beta_m = \frac{X}{\sigma} \) — is:

\( \text{See Appendix 1 for the specification of CRRA utility function.} \)

\( \text{State price density contains important information on the behaviour and expectations of the market (Hardle and Hlavka 2009). It follows a log-normal distribution in the Black-Scholes setting.} \)

\( \text{Equation 2 is the implication of the martingale approach used in the model which decomposes a dynamic optimisation problem } \max_{\hat{W}_T} E \left[ u \left( \alpha (W_T - Y_T)^+ + K \right) \right] \text{ s.t. } \hat{W}_T \text{ into a static optimisation problem as in Equation 2. This determines the optimal condition at maturity. Next step is to find the portfolio strategy that leads to the optimal condition at maturity. This approach was notably developed by Pliska (1986), Karatzas et al. (1987), Cox and Huang (1989) among others.} \)
\[ W_t^* = \frac{1}{\xi_t} E_t [\xi_t W_t^*] \]  

where \( E_t [\cdot] \) is the expectation of the optimality conditional to the information at time \( t \) which is \( \xi_t \). Since \( \xi \) follows Markovian process, for which the future probability is determined by its most recent value, we can rewrite \( \xi_T \) as:

\[ \xi_T = \xi_t \exp^{-(r+0.5X^2)(T-t)-X(Z_T-Z_t)} \] (5)

The corresponding optimal strategy to achieve \( W_t^* \) as in Equation 4 given the manager’s risk aversion \( \gamma \) is:

\[
\theta_t^* = \theta_M + \frac{\beta_m}{W_t^*} \left( -\frac{1}{\gamma} C_2(t) N(d_2(t, \xi_t)) + \left( \frac{\beta}{\beta_m} - \frac{1}{\gamma} \right) C_3(t) \xi_t^{-\frac{\beta_m}{\beta}} N(d_3(t, \xi_t)) \right)
+ \frac{C_1(t) \xi_t^{-\frac{1}{2}} \exp^{-0.5d_1(t, \xi_t)^2}}{X \sqrt{2\pi (T-t)}} + \frac{C_2(t) \exp^{-0.5d_2(t, \xi_t)^2}}{X \sqrt{2\pi (T-t)}} + \frac{C_3(t) \xi_t^{-\frac{\beta_m}{\beta}} \exp^{-0.5d_3(t, \xi_t)^2}}{X \sqrt{2\pi (T-t)}} \] (6)

where \( \theta_M = \frac{\beta_m}{\gamma} \) is the Merton’s strategy (1971) in the dynamic optimisation problem without compensation scheme and \( N(.) \) is the cumulative distribution function (cdf) of the normal distribution. What \( C_1, C_2, C_3, d_1, d_2, \) and \( d_3 \) mean are defined in Appendix 2. All parameters in Nicolosi’s model are pre-determined except the risk aversion \( \gamma \); both \( \lambda^* \) and \( \hat{\xi} \) are solved from the solution to the final optimal portfolio \( (W_t^*) \) as in Equation 3. Therefore this paper reports on an experiment to see how close are the subjects’ choices, of the \( \theta_t \), to those optimal choices as in the theory and elicit the risk aversion \( \gamma \) from the subjects’ choice.

### 3 Experimental design

The actual experiment design differs in two aspects from the theoretical design: a non-consequential and a consequential difference. The non-consequential difference is that the experimental design implements a discrete approximation to the continuous time problem, due to computer system limitation. Each discrete time step has a length \( dt = 0.1 \) second — hence the asset price changes every 0.1 second. The consequential difference is that the subjects were allowed to allocate their portfolio in the asset market (\( \theta \)) only between 0\% and 100\%. By this, the subjects were not allowed to short-sell so avoiding a large negative payment for the subjects. However, the theory allows \( -\infty < \theta < \infty \).

There were ten problems in the real experiment, all of the same type; the number of problems was chosen arbitrarily considering the experiment duration. It was preceded by
two practice problems. The subjects were given paper and on-screen instructions, and a simulation practice to generate the actual asset price with adjustable parameters ($\mu$ and $\sigma$) before going on to practice session. They were informed (in non-technical terms) that the asset price followed geometric Brownian motion, and were presented with as many simulations as they liked of such motion. Each simulation lasted for one minute; subjects could see how as many simulated asset price path as they wanted. After they were clear of what being asked to do and of how the asset price is generated, they started the practice session; after that, they started the real experiment. At the beginning of every problem, subjects were told all parameters for that problem ($S_0, K, \alpha, T, t, \beta, \mu, \sigma, W_0, r$); the initial price $S_0$, initial wealth $W_0$, and interest rate $r$ are always 25 ECU, 100 ECU, and 0 ECU respectively in every problem. They were also given six examples of the asset price chart for given parameters in every problem. Given all these information, subjects were asked to set their $\theta_0$ before the trading period.

Subjects were shown all update information during trading (the managed portfolio value in the asset, in the money account and in total, the benchmark value, the asset price, trading time and portfolio allocation in both asset and money market). They adjusted their portfolio allocation in the stock market using a slider. In addition, short instructions and the parameters used were displayed on the trading screen. They could start trading anytime they wished by clicking the “START” button. Each problem lasted for one minute in the practice session and three minutes in the real experiment. In addition, subjects were shown their performance (the managed portfolio value, the benchmark value and the payoff) by the end of every problem.

Monetary incentives were provided in accordance with the theory. One problem from the main experiment was randomly drawn for the subjects’ payment. Subjects were asked to draw a disk themselves from a closed bag containing the numbered disks from 1 to 10 — this identifies the problem number. The conversion rate is £1:3 ECU rounded up to the nearest 5 pence. The payment then will be added to a show-up fee of £3.

The experiment was conducted in the EXEC Lab, University of York. Invitation messages were sent through hroot (Hamburg registration and organization online tool) to all registered subjects in the system. 73 university members participated in this experiment: 46 males and 27 females. Composition of their educational degree was: 49 subjects were bachelor, 15 subjects were master, 7 subjects were PhD, 1 subject was diploma and 1 subject did not report his or her educational degree. I targeted the subjects who were or had been enrolled in the specific study that teaches finance and/or Brownian motion (e.g. Economics, Finance, Physics, Mathematics, and Statistics). Most of them (48 subjects) had participated in at least one economic experiment prior to this experiment. The average payment to the subjects was £8.1 and the average duration of the experiment (including reading the instructions) was around one and quarter hours. Communication was prohibited during the experiment. The experimental software was written (mainly

The paper instructions can be seen in Appendix 10.
4 Econometric specification

I use maximum likelihood to estimate the parameter of the model — risk aversion ($\gamma$), estimating subject by subject. Maximum likelihood requires a specification of the stochastic nature of the data to capture the noise or error in the subjects’ choice ($\theta_t$). I assume this error is independent in every period during trading ($t$). Since the optimal choice ($\theta^*_t$) takes any values, I consider a normal distribution to specify the stochastic story to account for this case. I assume that the choice of $\theta_t$ is normally distributed with mean $\theta^*_t$ (so that there is no bias) and standard deviation $\zeta$; I will report $s = \frac{1}{\zeta}$ which indicates the precision. My estimation takes into account the difference between the model and the actual the latter is bounded between 0 and 1 — as I have explained above.

Before I turn to the specification of the log-likelihood function, I introduce further notations which will be used in the estimation to create an interval around $\theta_t$ since the log-likelihood function is continuous, while the actual choices were discrete, with steps of 0.1 second:

$$\theta_t^+ = \theta_t + 0.005$$
$$\theta_t^- = \theta_t - 0.005$$

(7)

Given these notations, the log-likelihood function finds the probability that $\theta_t$ lies within $\theta_t^+$ and $\theta_t^-$ for any given $\gamma$ (risk aversion). Under this specification, the contribution to the likelihood of an observation $\theta_t$ is:

$$\theta_t = 0 \Leftrightarrow \Phi \left( \theta_t^+, \theta_t^*, \frac{1}{s_1} \right)$$
$$0 < \theta_t < 1 \Leftrightarrow \Phi \left( \theta_t^+, \theta_t^*, \frac{1}{s_1} \right) - \Phi \left( \theta_t^-, \theta_t^*, \frac{1}{s_1} \right)$$
$$\theta_t = 1 \Leftrightarrow 1 - \Phi \left( \theta_t^-, \theta_t^*, \frac{1}{s_1} \right)$$

(8)

where $\Phi$ is the cdf of a normal distribution with parameters $\theta^*_t$ (mean) and $\frac{1}{s_1}$ (standard deviation) given an observation $\theta_t$. For this specification, I estimate $\gamma_1$ (risk aversion) and $s_1$ (precision).

I also estimate using the average dataset. This is addressed to minimise the noise since the discrete time step ($t$) is quite fast (0.1 second). By this, I take an average of the dataset on every second, excluding the initial decision which remains as a single data — that is every 10 discrete time step ($t$). I denote subjects’ choice as $\bar{\theta}_i$ in this specification where $i$ is the average discrete time step. Given this specification, the contribution to the likelihood of an observation $\bar{\theta}_i$ is:
\[ \tilde{\theta}_i = 0 \iff \Phi \left( \tilde{\theta}_i^+, \tilde{\theta}_i^*, \frac{1}{s_2} \right) \]

\[ 0 < \tilde{\theta}_i < 1 \iff \Phi \left( \tilde{\theta}_i^+, \tilde{\theta}_i^*, \frac{1}{s_2} \right) - \Phi \left( \tilde{\theta}_i^-, \tilde{\theta}_i^*, \frac{1}{s_2} \right) \]

\[ \tilde{\theta}_i = 1 \iff 1 - \Phi \left( \tilde{\theta}_i^-, \tilde{\theta}_i^*, \frac{1}{s_2} \right) \]

(9)

where \( \tilde{\theta}_i^+ = \tilde{\theta}_i + 0.005 \) and \( \tilde{\theta}_i^- = \tilde{\theta}_i - 0.005 \). Let me call these two specifications as Nicolosi 1, following the specification in Equation 8, and Nicolosi 2, following the specification as in Equation 9. As with previous specification, I estimate \( \gamma_2 \) (risk aversion) and \( s_2 \) (precision) in this specification.

One may think of other stochastic assumptions to underlie estimation. For example, I could use a beta distribution to specify the stochastic of the subjects’ choices since they are bounded between 0 and 1. However, I start simple in this paper with a normal distribution specification.

To give a proper assessment to Nicolosi’s story, I fit the data additionally assuming both random and risk-neutral choices. The former (random choice) assumes that the choice of \( \theta_t \) is random following a normal distribution. The latter (risk-neutral choice) assumes that the choice of \( \theta_t \) follows risk-neutral behaviour. Theoretically, the risk neutrality returns either \(-Inf\) or \(Inf\), depending on the asset price change. Here I assume that \( \theta_t \) is 1 if the asset price goes up, otherwise 0 if the asset price goes down. Note crucially, neither of these alternatives involves parameter risk aversion \( \gamma \).

Again, I consider a normal distribution to specify the stochastic story for both random and risk-neutral choices. I also estimate using both all observations and the average dataset. The contribution to the log-likelihood for these specifications adopts Equation 8 and 9 as appropriate. For these specifications, let me call random choice specification as Random 1 (for the all-observation estimation) and Random 2 (for the average-dataset estimation), and Risk Neutral 1 (for the all-observation estimation) and Risk Neutral 2 (for the average-dataset estimation) for risk-neutral choice.

5 Results and analyses

One main purpose of this paper is how well Nicolosi’s model explains the subjects’ choice compared to other strategies (random and risk-neutral choices). The analyses for this purpose use all observations and average dataset from the real experiment. The former sees 1,801 decisions while the latter sees 181 decisions in each problem for each subject. However, the risk-neutral choice will see 1,800 and 180 decisions respectively, excluding the initial decision, since it is drawn following the realisation of the price change.

Additionally, I develop a simple strategy from a regression model using variables in Nicolosi’s model. This is a simplification of the theory as in Equation 6; hereinafter referred

\[^{10}\text{I use the patternsearch routine in Matlab to maximise the log-likelihood function in all specifications.}\]
to as Simple 1 (for the all-observation estimation) and Simple 2 (for the average-dataset estimation). As previously, I compare Nicolosi’s model and this simple strategy given the estimated risk aversion $\gamma$ from both the all-observation and average-dataset estimations.

Before going on the main analyses, I estimate the individual risk aversion and precision in Nicolosi 1 and Nicolosi 2, which can be found in Appendix 3 and 4. The results between both estimates show that estimate in Nicolosi 2 returns less risk-averse and higher precision on average than that of estimates in Nicolosi 1. This can be a further point of interest, but I take this merely as a consequence of using different approach since the main purpose of this study is to test the empirical validity of Nicolosi’s model.

The estimated risk aversion then is used to compute the optimal portfolio value individually as in Equation 6. It should be the case that following the optimal strategy will return a better portfolio than that of the benchmark given the estimated risk aversion. There are 730 portfolios at maturity from 10 problems across 73 subjects. Appendix 5 shows comparisons of the optimal portfolio and the benchmark values at maturity across all problems in both Nicolosi 1 and 2. Results from Nicolosi 1 show that all of the optimal portfolios (730 portfolios) are better than that of the benchmark, meanwhile results from Nicolosi 2 show that 711 optimal portfolios (97.4% of the total) are better than that of the benchmark given the individual estimated risk aversion. This shows that following the optimal strategy of Nicolosi is highly likely to end up with both payments (fixed and variable payments). In addition, the optimal portfolios return the higher utility than that of the actual portfolios — as shown in Appendix 6. This is hardly surprising.

5.1 Nicolosi’s model vs random and risk-neutral strategies

Now we move on to the first comparison between Nicolosi’s model and the random and risk-neutral strategies. The concern is to find the best fitting strategy as the explanation of the individual behaviour in selecting the portfolio allocation between the asset and the money account with Nicolosi’s model as the subject to test. I measure the goodness-of-fit by maximising the log-likelihood function, as specified in the Equation 8 and 9, but we need to correct the maximised log-likelihood for the number of parameters in each specification — Nicolosi’s model has two estimated parameters while each of random and risk-neutral choices has 1 estimated parameter. In particular, I simulate 100 times each to generate the dataset for both random choices (Random 1 and 2), then take its average log-likelihood.

I use the Akaike Information Criterion (AIC) as the measure of the goodness-of-fit to

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11 The average estimated risk aversion in Nicolosi 1 is 2.3353 compared to 0.5778 from the result in Nicolosi 2. Meanwhile, the average estimated precision Nicolosi 1 is 1.3556 compared to 1.4763 from the result in Nicolosi 2; with $\theta_t$ is bounded between 0 and 1 whereas $\theta_t^*$ is unbounded.

12 This sees 535 optimal portfolios (73.29% of the total) returns the better utility than that from the actual portfolios from results in Nicolosi 1; and 665 optimal portfolios (91.1% of the total) returns the better utility than that from the actual portfolios from results in Nicolosi 2.
find the best explanation for each subject. The details of the judgment can be seen in Appendix 7 and 8. With the all observations estimation — between Nicolosi 1, Random 1 and Risk Neutral 1 — of all 73 subjects, 48 subjects are better explained with Risk Neutral 1 while the other 24 subjects are better explained with Nicolosi 1; Random 1 is always the worst. Nevertheless, with the average dataset estimation, 40 subjects are better explained with Nicolosi 2 while the other 33 subjects are better explained with Risk Neutral 1; Random 2 remains the worst. This finding obviously shows that subjects did not randomise their choice in allocating their portfolio — that they followed some specific strategies for this. In particular, averaging the dataset improves the goodness-of-fit of Nicolosi’s model. This may be the evidence that subjects somehow follow the optimal strategy as in Nicolosi’s model but having difficulties to be as precise as the theory.

So far it is obvious that the subjects did not randomise their choices, and that following the optimal strategy is highly likely to end up with a better portfolio than that of the benchmark. As it also has shown, Nicolosi’s model receives the most empirical support on the average level. Nevertheless, the subjects might find it difficult to follow the optimal strategy of Nicolosi, which involves sophisticated dynamic programming, given his or her risk aversion — calculating and implementing as precise as the optimal strategy. Results from estimated precision show that the subjects’ choices are noisy compared to the optimal strategy in both Nicolosi 1 and 2; with average estimated precisions are 1.3556 (or standard deviation 0.7377) and 1.4763 (or standard deviation 0.6774) respectively. However, they must respond to some variables shown on the screen to determine their choice.

### 5.2 Nicolosi’s model vs the simple strategy

Building on the previous results, I try to explore the determinants of the subjects’ choice in a simple way using a regression model. Following Nicolosi, the portfolio allocation in the asset ($\theta_t$) should not be constant, as the Merton’s strategy ($\theta^M = \frac{\bar{S} - \bar{Y}}{\bar{S}}$), when the managed portfolio value is lower than that of the benchmark during trading in order to increase the chance to beat the benchmark at maturity. In particular, $\theta_t$ tends to be low, during trading, when the portfolio value ($W_t$) is higher than that of the benchmark ($Y_t$), vice versa. So I involve the difference between the managed portfolio and the benchmark ($W_t - Y_t$) in the regression model; I denote this as $\Delta_t$. In addition, I also involve the asset price ($S_t$) and the benchmark value ($Y_t$) since they were shown to the subjects in the experimental interface — I denote $\bar{\theta}, \bar{S}, \bar{Y}$ and $\bar{\Delta}$ for variables used in Simple Rule 2.

The regression results from Simple Rule 1 and Simple Rule 2 are as follows:\[^{13}]\[^{13}\]

[^13]: I use a simple linear procedure in both regression models. Standard errors are in parentheses and *, **, *** denote the significance at 1%, 5%, and 10% respectively. All coefficients are jointly not equal to zero in both regression models. Adjusted $R^2$ in both models are 0.0094 and 0.0097 respectively, and the number of observations is 1,314,730 and 132,130 respectively.
\[ \theta_t = 43.649 - 0.0145 S_t + 0.115 Y_t - 0.008 \Delta t \quad (10) \]

\[ \tilde{\theta}_t = 43.672 - 0.042 \tilde{S}_t + 0.115 \tilde{Y}_t - 0.008 \tilde{\Delta}_t \quad (11) \]

I use percentage values for \( \theta \), \( t \) is time step and \( i \) is average time step. Overall results from both regression model above show that all independent variables are significant in determining the subjects’ choice — the signs of the independent variables are identical. Both the asset price and the difference between the managed portfolio and the benchmark have a negative effect to the subjects’ choice, meanwhile, the value of the benchmark has a positive effect to the subjects’ choice. These results are sensible and intuitive. Overall, the subjects tended to buy the asset when its price is low and to sell the asset when its price is high; to some extent, it is commonly known as “buy low, sell high” strategy. This strategy is possibly the most famous adage in making profits from an asset market. Moreover, the subjects tended to hold the asset as they saw the benchmark value was high. Lastly, the subjects were consistent with the theoretical prediction in which they were unlikely to hold the asset when their portfolio value was relatively far above the benchmark.

Building on the regression results, I then run the regression model individually using the same structure as in Equation 10 and 11. This is to give a comparison of which model to have a better explanation for each subject between Nicolosi’s model and the simple strategy using their measure of the goodness-of-fit; I compare between Nicolosi 1 and Simple 1, and between Nicolosi 2 and Simple 2. Since the models have different specifications, hence different degree of freedom, I calculate the AIC to correct for differing degrees of freedom. I compare them and have a conclusion accordingly for each subject.

Results from two estimation procedures (using all observations and the average dataset) produce a slightly different AIC conclusion. With the all-observation estimation, 37 subjects are better explained with Nicolosi’s model; 36 subjects are better explained with the simple strategy. Meanwhile, with the average-dataset estimation, 36 subjects are better explained with Nicolosi’s model; 37 subjects are better with the simple strategy. The details of the judgment can be seen in Appendix 9.

6 Discussion and conclusion

This paper examines Nicolosi’s story by investigating the subjects’ behaviour in order to follow the optimal strategy of Nicolosi in a controlled lab-experiment. The subjects act as if they are the hedge fund manager who takes a responsibility to manage the investor’s

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6 The AIC is given by \( 2k - 2LL \), where \( k \) is the number of estimated parameters and \( LL \) is the log-likelihood.
funds. The manager agrees on a contract, determined by the investor, who pays the
manager with two-term payment: the fixed payment and the variable payment, where the
variable payment is a share-based on the over-performance with respect to the specific
benchmark.

I follow Nicolosi’s design in which there are two markets for the manager to invest: the
asset market and the money market. The asset price follows geometric Brownian motion
and the subjects were aware of all parameters used in the experiment. However, there
are non-consequential and consequential differences in the experimental setup from the
theoretical design. The former relates to the computer limitation to implement the con-
tinuous time in generating the asset price, hence I use the discrete approximation to the
continuous time with an increment of 0.1 second. The latter restricts the maximum port-
folio allocation between the asset and the money account. This is addressed to prevent
the subjects from a negative payoff from short-selling since it may have an unlimited loss.

To give a proper assessment of the empirical validity of Nicolosi’s model, I compare it
with the subjects’ risk aversion estimated from the data in two ways. First, I compare it
with random and risk-neutral choices. The random choice generates θ randomly following
a normal distribution while the risk-neutral choice generates θ as if the subject was a risk-
neutral agent; put everything on the asset if the asset price goes up, otherwise nothing if
the asset price goes down. Second, I compare Nicolosi’s model with the simple strategy,
developed using a regression model. One obvious conclusion from the first assessment is
that the subjects did not randomise their choice. They followed some specific strategies
to maximise their utility from the experiment. Of all subjects, 24 subjects are better
explained with Nicolosi’s model while the other 48 subjects are better explained with
risk-neutral choice using all the observations. We get a different conclusion if we use the
average dataset. Of all subjects, 40 subjects are better explained with Nicolosi’s model;
the other 33 subjects are better explained with the risk-neutral choice.

Building on the previous results, I then develop a regression model to provide the simple
strategy which the subjects may plausibly have followed. With this simple strategy, one
tends to hold the asset when its price is low and to sell the asset when its price is high.
In addition, one manages its portfolio value depending on the benchmark value and the
difference between the managed portfolio and the benchmark. I compare Nicolosi’s model
with this simple strategy individually — as with the previous analysis. The comparison
sees that 37 subjects are better explained with Nicolosi’s model, 36 others are better
explained with the simple strategy, using all the observations. If we use the average
dataset, we get slightly different results: of all subjects, 36 subjects are better explained
with Nicolosi’s model, while 37 others are better explained with the simple strategy.

Although the optimal strategy of Nicolosi ensures a high possibility to end up with both
fixed and variable payments, hence the maximum utility, the subjects found it difficult
to follow. As it has shown, the subjects’ choices are noisy compared with the optimal
strategy. One may argue that the subjects could have more precise computation if they
were well accommodated in the experiment since Nicolosi’s optimal strategy involves sophisticated dynamic programming. For example, we could ask the subject to specify their own strategy to be implemented during trading at the beginning of each problem, and they are free to adjust their strategy at any time. Will it improve the empirical validity of the theory? I may not think so because it depends on how subjects understand the random process in the asset price, hence determining the benchmark value. Nevertheless, Nicolosi’s model receives strong empirical support in explaining the subjects’ behaviour. In addition, the subjects follow the intuitive prediction of Nicolosi’s model where the difference between the managed portfolio and the benchmark determines the subjects’ choice.
Appendices

Appendix 1 — Specification of CRRA utility function

Nicolosi’s model assumes that the manager is utility maximiser specified with CRRA utility function. It can be written as:

$$CRRA : u(x) = \frac{x^{1-\gamma}}{1-\gamma}; \gamma > 0, \gamma \neq 1$$

The manager is assumed to be strictly risk-averse and the function is undefined when $\gamma = 1$. However I apply CRRA utility function so it is able to accommodate when $\gamma = 1$ as follows:

$$CRRA : u(x) = \begin{cases} x^{1-\gamma} ; \gamma \neq 1 \\ \log(x) ; \gamma = 1 \end{cases}$$

Appendix 2 — Definitions to Equation 6

Solution in Equation 6 contains some components ($C_1, C_2, C_3, d_1, d_2$ and $d_3$) where they are defined as follows:

$$C_1(t) = \frac{1}{\alpha} \left( \frac{X}{T-a} \right)^{-\frac{1}{2}} \exp \left[ \left( \frac{1}{\gamma} - 1 \right) \left( r + \frac{1}{2\gamma} X^2 \right) (T-t) \right]$$

$$C_2(t) = -K \frac{1}{\alpha} \exp \left[ -r (T-t) \right]$$

$$C_3(t) = Y_0 A_T \exp \left[ \left( \frac{\beta}{\beta_m} - 1 \right) \left( r + \frac{1}{2} \beta \alpha X \right) (T-t) \right]$$

$$d_1(t, \xi_t) = \frac{\ln \left( \frac{\xi}{\xi_t} \right) + (r - \frac{1}{2} X^2 \left( 1 - \frac{1}{2} \right) (T-t))}{X \sqrt{T-t}}$$

$$d_2(t, \xi_t) = \frac{\ln \left( \frac{\xi}{\xi_t} \right) + (r - \frac{1}{2} X^2)(T-t)}{X \sqrt{T-t}}$$

$$d_3(t, \xi_t) = \frac{\ln \left( \frac{\xi}{\xi_t} \right) + (r - \frac{1}{2} X^2 \left( 1 - \frac{2 \beta}{\beta_m} \right))(T-t)}{X \sqrt{T-t}}$$

$$A_T = \exp \left[ \left( r + \frac{1}{2} \beta \sigma X - \frac{1}{2} \beta^2 \sigma^2 - r \frac{\beta \sigma}{X} \right) T \right]$$

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Appendix 3 — Estimated individual risk aversion and precision in *Nicolosi 1*
Appendix 4 — Estimated individual risk aversion and precision in Nicolosi 2

![Graphs showing estimated risk aversion and precision](image)

Appendix 5 — Optimal portfolio vs the benchmark at maturity given the estimated risk aversion in Nicolosi 1 and Nicolosi 2

![Scatter plots showing optimal portfolio vs benchmark](image)

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Appendix 6 — Optimal utility vs actual utility given the estimated risk aversion in Nicolosi 1 and Nicolosi 2
### Appendix 7 — Individual AIC of Nicolosi’s model vs random and risk-neutral strategies from the all-observation estimation

| Sub. | Nicolosi 1 | Random 1 | Risk Neutral 1 | Judgment     |
|------|------------|----------|----------------|--------------|
| 1    | 198,321.246| 221,535.182| 196,036.112    | Risk Neutral 1 |
| 2    | 94,621.670 | 108,686.455| 95,907.756     | Nicolosi 1    |
| 3    | 197,009.678| 217,617.004| 191,808.286    | Risk Neutral 1 |
| 4    | 192,582.322| 216,253.368| 189,739.869    | Risk Neutral 1 |
| 5    | 122,903.085| 138,235.154| 119,387.990    | Risk Neutral 1 |
| 6    | 189,562.792| 214,315.596| 189,052.356    | Risk Neutral 1 |
| 7    | 84,860.704 | 99,729.582 | 99,462.459     | Nicolosi 1    |
| 8    | 186,540.242| 207,883.918| 183,558.288    | Risk Neutral 1 |
| 9    | 182,607.437| 217,255.937| 195,904.024    | Nicolosi 1    |
| 10   | 106,135.443| 120,671.830| 115,009.369    | Nicolosi 1    |
| 11   | 196,790.120| 221,631.587| 193,273.823    | Risk Neutral 1 |
| 12   | 54,509.687 | 66,651.489 | 59,471.648     | Nicolosi 1    |
| 13   | 152,116.133| 169,983.115| 153,503.171    | Nicolosi 1    |
| 14   | 194,862.244| 222,545.901| 193,953.201    | Risk Neutral 1 |
| 15   | 186,890.991| 215,782.103| 191,504.509    | Nicolosi 1    |
| 16   | 189,016.232| 207,505.756| 181,871.617    | Risk Neutral 1 |
| 17   | 148,867.582| 164,916.650| 147,838.409    | Risk Neutral 1 |
| 18   | 176,018.071| 151,683.890| 138,768.587    | Risk Neutral 1 |
| 19   | 163,426.340| 180,998.317| 161,666.209    | Risk Neutral 1 |
| 20   | 189,294.637| 207,850.753| 184,832.636    | Risk Neutral 1 |
| 21   | 160,497.587| 177,756.712| 155,350.027    | Risk Neutral 1 |
| 22   | 186,078.435| 208,115.105| 183,769.752    | Risk Neutral 1 |
| 23   | 170,643.478| 188,881.973| 170,967.209    | Nicolosi 1    |
| 24   | 51,079.834 | 63,854.067 | 54,270.308     | Nicolosi 1    |
| 25   | 187,165.955| 209,552.850| 186,916.125    | Risk Neutral 1 |
| 26   | 126,078.497| 141,947.310| 116,499.274    | Risk Neutral 1 |
| 27   | 151,085.506| 168,136.912| 145,798.463    | Risk Neutral 1 |
| 28   | 175,190.721| 192,642.030| 169,266.254    | Risk Neutral 1 |
| 29   | 157,320.343| 174,516.666| 146,924.193    | Risk Neutral 1 |
| 30   | 190,712.679| 211,741.269| 186,811.810    | Risk Neutral 1 |
| 31   | 157,683.899| 173,815.791| 153,422.846    | Risk Neutral 1 |
| 32   | 188,010.084| 220,763.381| 195,939.363    | Nicolosi 1    |
| 33   | 171,356.837| 188,166.312| 163,075.761    | Risk Neutral 1 |
| 34   | 180,645.237| 207,768.161| 186,878.163    | Nicolosi 1    |
| 35   | 148,606.352| 165,520.165| 144,722.103    | Risk Neutral 1 |
|   |          |          |          |               |
|---|----------|----------|----------|---------------|
| 36| 149,326.154 | 168,566.469 | 154,822.340 | Nicolosi 1    |
| 37| 115,501.088 | 131,262.478 | 114,856.663 | Risk Neutral 1|
| 38| 164,300.775 | 179,758.909 | 163,826.794 | Risk Neutral 1|
| 39| 105,318.745 | 119,110.311 | 105,075.899 | Risk Neutral 1|
| 40| 193,010.105 | 215,980.369 | 191,613.115 | Risk Neutral 1|
| 41| 193,079.490 | 215,507.103 | 191,125.544 | Risk Neutral 1|
| 42| 102,158.038 | 117,273.322 | 102,023.025 | Risk Neutral 1|
| 43| 191,534.105 | 217,478.581 | 192,924.714 | Nicolosi 1    |
| 44| 100,711.684 | 115,442.352 | 96,066.439  | Risk Neutral 1|
| 45| 189,579.881 | 207,319.183 | 180,829.065 | Risk Neutral 1|
| 46| 194,783.214 | 222,833.959 | 194,219.174 | Risk Neutral 1|
| 47| 182,924.555 | 204,784.083 | 183,673.065 | Nicolosi 1    |
| 48| 124,055.332 | 139,689.192 | 119,136.480 | Risk Neutral 1|
| 49| 154,667.291 | 172,399.683 | 158,500.161 | Nicolosi 1    |
| 50| 150,204.121 | 168,634.060 | 157,427.731 | Risk Neutral 1|
| 51| 128,189.903 | 143,954.123 | 130,924.244 | Nicolosi 1    |
| 52| 135,654.533 | 151,052.930 | 138,158.201 | Nicolosi 1    |
| 53| 170,254.851 | 193,062.692 | 175,979.028 | Nicolosi 1    |
| 54| 182,041.244 | 198,634.060 | 176,427.731 | Risk Neutral 1|
| 55| 191,179.743 | 210,993.377 | 192,570.474 | Risk Neutral 1|
| 56| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 57| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 58| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 59| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 60| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 61| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 62| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 63| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 64| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 65| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 66| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 67| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 68| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 69| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 70| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 71| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 72| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|
| 73| 193,908.536 | 217,101.724 | 193,144.819 | Risk Neutral 1|

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## Appendix 8 — individual AIC of Nicolosi’s model vs random and risk-neutral strategies from the average-dataset estimation

| Sub. | Nicolosi 2 | Random 2 | Risk Neutral 2 | Judgment |
|------|------------|----------|----------------|----------|
| 1    | 19,645.443 | 22,396.753 | 19,707.556     | Nicolosi 2 |
| 2    | 10,308.974 | 11,782.616 | 10,451.865     | Nicolosi 2 |
| 3    | 19,654.644 | 21,920.540 | 19,323.457     | Risk Neutral 2 |
| 4    | 19,110.957 | 21,726.471 | 19,102.646     | Risk Neutral 2 |
| 5    | 12,721.576 | 14,567.587 | 12,584.114     | Risk Neutral 2 |
| 6    | 18,881.490 | 21,366.711 | 18,992.737     | Nicolosi 2 |
| 7    | 9,073.263  | 10,720.300 | 10,598.342     | Nicolosi 2 |
| 8    | 18,432.750 | 20,020.618 | 18,455.694     | Nicolosi 2 |
| 9    | 18,211.662 | 21,863.967 | 19,736.161     | Nicolosi 2 |
| 10   | 10,910.544 | 12,372.939 | 11,842.976     | Nicolosi 2 |
| 11   | 18,562.476 | 20,953.837 | 18,524.010     | Risk Neutral 2 |
| 12   | 7,072.808  | 8,472.030  | 7,463.668      | Nicolosi 2 |
| 13   | 15,570.811 | 17,597.844 | 15,900.244     | Nicolosi 2 |
| 14   | 19,276.702 | 22,519.796 | 19,518.378     | Nicolosi 2 |
| 15   | 18,478.606 | 21,778.457 | 19,321.530     | Nicolosi 2 |
| 16   | 18,698.295 | 20,953.837 | 18,266.338     | Risk Neutral 2 |
| 17   | 15,867.467 | 17,826.628 | 15,956.145     | Nicolosi 2 |
| 18   | 14,487.021 | 16,213.418 | 14,690.872     | Nicolosi 2 |
| 19   | 16,512.099 | 18,530.058 | 16,428.442     | Risk Neutral 2 |
| 20   | 19,139.755 | 21,299.027 | 18,857.811     | Risk Neutral 2 |
| 21   | 16,221.765 | 20,593.837 | 15,863.659     | Risk Neutral 2 |
| 22   | 18,577.961 | 20,952.206 | 18,524.010     | Risk Neutral 2 |
| 23   | 17,297.021 | 19,396.443 | 17,560.452     | Nicolosi 2 |
| 24   | 6,856.525  | 8,360.110  | 7,112.858      | Nicolosi 2 |
| 25   | 18,743.614 | 21,263.016 | 18,818.953     | Nicolosi 2 |
| 26   | 12,183.404 | 14,173.531 | 11,740.719     | Risk Neutral 2 |
| 27   | 15,094.474 | 16,840.326 | 14,710.935     | Risk Neutral 2 |
| 28   | 17,693.703 | 19,721.227 | 17,281.467     | Risk Neutral 2 |
| 29   | 15,456.541 | 17,438.657 | 14,783.131     | Risk Neutral 2 |
| 30   | 18,890.308 | 21,499.885 | 18,803.754     | Risk Neutral 2 |
| 31   | 15,805.487 | 17,789.191 | 15,689.552     | Risk Neutral 2 |
| 32   | 18,526.775 | 22,119.998 | 19,693.844     | Nicolosi 2 |
| 33   | 17,015.108 | 18,982.365 | 16,477.604     | Risk Neutral 2 |
|   | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|   | 18,015.064 | 20,749.880 | 18,891.320 | Nicolosi 2 |
|   | 15,146.999 | 17,155.569 | 14,817.561 | Risk Neutral 2 |
|   | 15,168.377 | 17,248.989 | 15,804.205 | Nicolosi 2 |
|   | 11,896.929 | 13,691.741 | 11,961.519 | Nicolosi 2 |
|   | 15,168.377 | 17,248.989 | 15,804.205 | Nicolosi 2 |
|   | 11,896.929 | 13,691.741 | 11,961.519 | Nicolosi 2 |
|   | 15,168.377 | 17,248.989 | 15,804.205 | Nicolosi 2 |
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Appendix 9 — Individual AIC of Nicolosi’s model vs the simple strategy from the all-observation and average-dataset estimations

| Sub. | Nicolosi 1  | Simple 1  | Judgment | Nicolosi 2  | Simple 2  | Judgment |
|------|-------------|-----------|----------|-------------|-----------|----------|
| 1    | 198,321.246 | 174,453.096 | Simple 1 | 19,645.443 | 17,455.645 | Simple 2 |
| 2    | 94,621.670  | 187,219.239 | Nicolosi 1 | 10,308.974 | 18,728.059 | Nicolosi 2 |
| 3    | 197,009.678 | 177,602.827 | Simple 1 | 19,654.644 | 17,758.940 | Simple 2 |
| 4    | 192,582.322 | 166,821.787 | Simple 1 | 19,110.957 | 16,733.459 | Simple 2 |
| 5    | 122,903.085 | 182,955.881 | Nicolosi 1 | 12,721.576 | 18,304.908 | Nicolosi 2 |
| 6    | 189,562.792 | 168,015.790 | Simple 1 | 18,881.490 | 16,891.658 | Simple 2 |
| 7    | 84,860.704  | 171,150.393 | Nicolosi 1 | 9,073.263  | 17,002.175 | Nicolosi 2 |
| 8    | 186,540.242 | 171,222.098 | Simple 1 | 18,432.750 | 17,215.338 | Simple 2 |
| 9    | 106,135.443 | 182,277.380 | Nicolosi 1 | 10,910.544 | 18,228.427 | Nicolosi 2 |
| 10   | 194,862.244 | 149,449.397 | Simple 1 | 19,276.702 | 15,026.829 | Simple 2 |
| 11   | 186,890.991 | 140,797.022 | Simple 1 | 18,478.606 | 14,131.838 | Simple 2 |
| 12   | 189,016.232 | 170,370.559 | Simple 1 | 18,698.295 | 17,123.570 | Simple 2 |
| 13   | 152,116.133 | 174,479.333 | Nicolosi 1 | 15,570.811 | 17,457.335 | Nicolosi 2 |
| 14   | 148,867.582 | 182,301.089 | Nicolosi 1 | 15,867.467 | 18,154.031 | Nicolosi 2 |
| 15   | 176,018.071 | 188,021.265 | Nicolosi 1 | 14,487.021 | 18,768.873 | Nicolosi 2 |
| 16   | 163,426.340 | 163,742.014 | Nicolosi 1 | 16,512.099 | 16,425.107 | Simple 2 |
| 17   | 189,294.637 | 182,459.465 | Simple 1 | 19,139.755 | 18,084.611 | Simple 2 |
| 18   | 160,497.587 | 180,933.397 | Nicolosi 1 | 16,221.765 | 16,410.910 | Nicolosi 2 |
| 19   | 186,078.435 | 172,963.107 | Simple 1 | 18,577.961 | 17,361.759 | Simple 2 |
| 20   | 170,643.478 | 183,885.162 | Nicolosi 1 | 17,297.021 | 18,371.147 | Nicolosi 2 |
| 21   | 51,079.834  | 189,411.817 | Nicolosi 1 | 6,858.525  | 18,873.701 | Nicolosi 2 |
| 22   | 187,165.955 | 176,686.211 | Simple 1 | 18,743.614 | 17,630.047 | Simple 2 |
| 23   | 126,078.497 | 173,352.631 | Nicolosi 1 | 12,183.404 | 17,410.910 | Nicolosi 2 |
| 24   | 151,085.506 | 180,397.210 | Nicolosi 1 | 15,094.474 | 18,129.940 | Nicolosi 2 |
| 25   | 175,190.721 | 176,939.549 | Nicolosi 1 | 17,693.703 | 17,721.956 | Nicolosi 2 |
| 26   | 157,320.343 | 153,376.336 | Simple 1 | 15,465.541 | 15,409.923 | Simple 2 |
| 27   | 190,712.679 | 170,171.255 | Simple 1 | 18,890.308 | 17,041.947 | Simple 2 |
| 28   | 157,683.899 | 184,829.529 | Nicolosi 1 | 15,895.487 | 18,514.293 | Nicolosi 2 |
| 29   | 188,010.084 | 161,722.966 | Simple 1 | 18,526.775 | 16,250.788 | Simple 2 |
| 30   | 171,356.837 | 166,172.225 | Simple 1 | 17,015.108 | 16,676.372 | Simple 2 |
|   | 180,645.237 | 170,523.007 | Simple 1 | 18,015.064 | 17,085.953 |
|---|-------------|-------------|----------|-------------|-------------|
| 35 | 148,606.352 | 181,124.271 | Nicolosi 1 | 15,146.999 | 18,123.924 |
| 36 | 149,326.154 | 173,965.530 | Nicolosi 1 | 15,168.377 | 17,377.665 |
| 37 | 115,501.088 | 185,358.890 | Nicolosi 1 | 11,896.929 | 18,521.134 |
| 38 | 164,300.775 | 190,106.051 | Nicolosi 1 | 16,989.139 | 18,786.662 |
| 39 | 149,326.154 | 173,965.530 | Nicolosi 1 | 15,168.377 | 17,377.665 |
| 40 | 148,606.352 | 181,124.271 | Nicolosi 1 | 15,146.999 | 18,123.924 |
| 41 | 127,173.519 | 186,266.268 | Nicolosi 1 | 13,006.015 | 18,667.986 |
| 73 | 191,689.817 | 170,526.544 | Simple 1 | 19,224.107 | 17,077.428 | Simple 2 |
|----|-------------|-------------|---------|------------|------------|---------|

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Appendix 10 — Instructions

Instructions

Preamble
Welcome to this experiment and thank you for coming. Please read these Instructions carefully. They will help you to understand what the experiment is all about and what you are being asked to do during the experiment. This experiment gives you the opportunity to earn money which will be paid to you in cash after you have completed the experiment. However there is no participation fee in this experiment; what you earn in this experiment is what you will be paid. So you must take this experiment seriously. Your payment is described below and it will be added to a show-up fee of £3 that you will be paid independently of your answers.

The Experiment
This experiment is interested in your decision as a fund manager who manages the investor’s funds. You are entrusted to manage his or her funds and then the investor will see if you can perform better than that of the investor’s benchmark. What the investor’s benchmark means will be explained later. There are two types of market that you can invest, the money market and the asset market. The money market is risk-free that gives you a constant interest rate, whereas the asset market is risky that trades assets continuously in time.

The asset price process in the asset market contains the Brownian motion. It means that the asset price changes randomly over time t. There are two parameters in the asset price process, the Drift (mean) and the Scale (sigma). Please notice that parameter Drift determines the realisation of the asset price by the maturity time, whereas parameter Scale determines the fluctuation of the asset price over all period. Those parameters remain constant in a particular problem. However the asset price is non-negative (either zero or positive) in this experiment. The asset price change therefore can be written as: $dS_t = S_t(\mu dt + \sigma dZ_t)$; where S is the asset price, $\mu$ is the drift, $\sigma$ is the scale and Z is the Brownian motion. There will be examples of the asset price process that help you to understand how it works given the parameters. Please do not ignore these examples! Moreover they are addressed to help you make your decision during the experiment.
You will be presented with a sequence of 10 problems, all of the same type, in the main experiment, preceded by 2 practice problems. Please do not waste the practice session! Each problem has its unique circumstances depending on its initial setup. In the beginning of each problem, you are endowed by an initial fund for you to manage in a certain time. You are told the initial asset price in the asset market and the interest rate in the money market. Given these information, you will be asked your initial allocation, of your portfolio, in each market. Then, you can adjust your allocation in both markets as a response to the current asset price in the asset market and the current benchmark portfolio.

You will receive payoffs depending on your performance with respect to the investor’s benchmark by the end of each problem. The benchmark is a constant portfolio consisting of the proportions of the funds invested in the money market and in the asset market. These proportions are determined by the investor in the beginning of the problem and remain fixed throughout the problem. There are two types of the payoff that you will receive, a fixed payoff and an additional payoff. The fixed payoff is paid to you independently of your performance. The additional payoff is paid proportionally to the profit earned by you if the final portfolio you manage is higher than that of the final benchmark portfolio. You will be told these information in the beginning of each problem along with other information. Given this, your payoff function is:

$$\alpha (W_T - Y_T)^+ + K$$

where $K$ is the fixed payoff and $\alpha$ is the proportion of positive margin of the final portfolio to the final benchmark $(W_T - Y_T)^+$ that you will earn—where $W_T$ and $Y_T$ are your portfolio and the benchmark portfolio respectively by the maturity time. All the currency used in this experiment is in ECU (Experimental Currency Unit). Therefore, you can earn more payoffs if your final portfolio is higher than that of final benchmark portfolio.

Here is to give you an example of how you will receive payoff in a particular problem:

**Example:** You are asked by the investor if you can optimise his or her funds in both money market and asset market. The initial fund that you can manage is 100 ECU. The investor has his or her benchmark to the portfolio allocation in both money market and asset market. He or she would allocate 0% of the initial fund to the money market, and 100% of the initial fund to the asset market. The money market return is 0%, whereas the asset market trades asset in continuously period. The investor would let his or her portfolio grows given his or her allocation to the maturity time. This will be the final benchmark portfolio and a basis to the fund manager’s performance.

You will receive two types of payoffs as a part of your effort by the end of the problem: a fixed payoff and an additional payoff. The fixed payoff for you is 10 ECU. The additional payoff is 25% of the positive margin of your final portfolio minus the final benchmark portfolio. Given this information, you will start this problem by setting your initial
portfolio allocation in each market. Then you can manage your portfolio by adjusting your portfolio allocation in each market for a given certain time—as an example, the total time in this problem is 3 minutes.

If, for example, your final portfolio is 190 ECU and the final benchmark portfolio is 150 ECU by the end of a problem. Your payoff is calculated as follow: \(0.25(190 - 150) + 10 = 20\) ECU. You receive both fixed payoff and additional payoff because you have managed to have a higher final portfolio than that of the final benchmark portfolio.

If, for example, your final portfolio is 190 ECU and the final benchmark portfolio is 200 ECU by the end of a problem. Your payoff, therefore, is 10 ECU. You receive only a fixed payoff because you have failed to have a higher final portfolio than that of the final benchmark portfolio.

The Interface

When you arrive at the laboratory, you will find the screen displaying the EXEC logo. Do not touch the computer until all the participants have read the instructions. The screen remains inactive. When all have done so, the experimenter will let you know to go to the instructions screen. There are three instruction screens that will help you to understand the experiment.

After you think you are clear of what you are asked to do, you can practice the asset price simulation, which contains the Brownian motion, to get your feeling on how the asset price is generated. Below is the practice screen for the asset price simulation.

You can adjust both parameters Drift and Scale and click “Start” button to display the asset price path (which contains Brownian motion). There is no time limit for you in this simulation. Make sure you clearly understand and get the feeling of how the asset price path is produced for any given parameters. After you have understood and got your feeling, please click the “NEXT” button to move on to the next page (as shown in the figure below).
There will be two practice sessions for you before going on to the main experiment. But notice that these practice sessions do not count for your payment. You can continue to the practice session by clicking on the “Start Practice!” button if you think you are clear so far.

You will be told any necessary information in the particular problem. It includes the initial funds to manage, the interest rate in the money market, time maturity (in minute), the initial price per unit asset in the asset market, the investor’s allocation strategy in each market, your payoffs and the asset price process parameters. Once you have understood the tasks and all information, you can continue by clicking the “NEXT” button (as shown in the figure below).

Next screen (the figure below) is examples of asset price path using the same parameters as stated in a particular problem (in the previous screen). Click “NEXT” button to move on to the next page if you are clear so far.
Then, in the next page, you will be asked your initial allocation of the funds to be invested in the asset market given all necessary information that you have acquired in the previous screen. You do so by making an adjustment in the “Initial Allocation in the Asset Market” box. Its default value is 50 percent, but you can adjust it up to two decimal places. However you can only set the initial allocation in the asset market between 0 and 100. Once you have decided your initial allocation, you can continue to have your practice by clicking the “NEXT” button as shown in the figure below.

The next screen (the figure below) is the main screen of the practice session. On the left panel there are updating information in continuously period—your portfolio value, the benchmark portfolio value and the unit asset hold. There is a short instruction that tells you the parameter in the particular problem. On the right panel there are two figures that show you the asset price chart and the portfolio value chart continuously in time. The blue line in the portfolio value chart shows your updated portfolio value and the green line shows the updated benchmark portfolio value.
The “START” button will be inactive once you click that. You can adjust your portfolio allocation in the stock market by moving the slider below “Next Adjustment on Allocation in the Stock Market”. Again, you can only adjust your allocation in the stock market between 0 and 100. Bottom right are the information of your portfolio in the stock market and in the money market, and their proportion to your total portfolio. Notice that the better the Portfolio Value than that of the Benchmark Value, the higher the payoff you will earn in this particular problem.

You will be told your gain value (Portfolio Value – Benchmark Value) after the problem is finished (as shown in the figure above). Please click the “NEXT” button to go to the next practice session and finish this session. If you have understood the experiment after finishing the practice session you can click the “Start the Main Experiment!” button (as shown in the figure below). Otherwise you should raise up your hand and the experimenter will come to you to answer any of your questions.
Notice that you cannot go back to the previous screen once you click the “Start the Main Experiment!” button. So please make sure that you clearly understand of what you are asked to do during the experiment. You will be told any necessary information and are asked to set your initial allocation in the asset market as in the practice session.

The Payment

It is important for you to take this experiment seriously because it determines your payment by the end of this experiment. Notice that you will earn cash from this experiment. Only problems in the main experiment will be basis of your payment. You will draw a disk by yourself from a closed bag containing the number disks from 1 to 10, each indicating the number of a problem. The drawn disk determine which problem to be a basis of your payment. The exchange rate of your payment is £1:3 ECU. That means if your payoff in a particular problem is 3 ECU, then you will receive £1 from this experiment. However this will be round up by 5 pence. Your payment in this experiment then will be added to a show-up fee of £3.

How Long Will the Experiment Last?

It is important for you to understand the problem carefully. You can start the experiment as you wish. However, as the timing for each problem is fixed, I estimate that the experiment will take at least 60 minutes of your time. Please notice that you cannot go back to the previous screen as you move on to the next screen, and any kind of communication is prohibited during this experiment.
Questionnaire

Subject Number: 

Please provide us the following information about you.

Q. Sex: Male/Female (Cycle the right one)

Q. Age: What is your age?

Q. Ethnicity origin: Please specify your ethnicity. (Cycle the right one)

• White • Hispanic or Latino • Black or African American • Native American or American Indian • Asian / Pacific Islander • Other

Q. Education: What is the highest degree or level of school you have completed? If currently enrolled, highest degree received. (Cycle the right one)

• No schooling completed • Nursery school to 8th grade • Some high school, no diploma • High school graduate, diploma or the equivalent (for example: GED) • Some college credit, no degree • Trade/technical/vocational training • Associate degree • Bachelor’s degree • Master’s degree • Professional degree • Doctorate degree

Please answer also the following questions

Q. Are you currently a student? If so, in which level you are currently enrolled?

Q. What are you studying?

Q. Do you have any work experience in Economics? If so, for how long did/do you work in this field and which was/is your job title/titles?

Q. Have you participated in economics experiments in the past?

Q. Did you feel impatience during the experiment? (Cycle the right one)

1: Not at all 2: Mainly disagree 3: Neither agree nor disagree 4: Mainly agree 5: Totally agree

Q. Did you feel stress during the experiment? (Cycle the right one)
1: Not at all 2: Mainly disagree 3: Neither agree nor disagree 4: Mainly agree 5: Totally agree

Q. Which is your risk aversion level? From 1 to 5 the risk aversion level is increasing.

(Cycle the right one)

1  2  3  4  5

Q. What did you like in the experiment?

Q. What you did not like in the experiment?

Q. Any suggestions for improvement?

Thank you for your participation!

Yudistira Permana

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