Kaon decay interferometry
as meson dynamics probes*

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Abstract

We discuss the time dependent interferences between $K_L$ and $K_S$ in the decays in $3\pi$ and $\pi\pi\gamma$, to be studied at interferometry machines such as the $\phi$-factory and LEAR. We emphasize the possibilities and the advantages of using interferences, in comparison with width measurements, to obtain information both on $CP$ conserving and $CP$ violating amplitudes. Comparison with present data and suggestions for future experiments are made.

I. INTRODUCTION

The origin of $CP$ violation is still an open problem in particle physics. Particularly interesting in this regard is the field of kaon decays, which is the only one where at least $CP$ violation in the mass-matrix has been established through the measurements of $K_L \rightarrow \pi\pi$, $K_L \rightarrow \pi l\nu$ and $K_L \rightarrow \pi\pi\gamma$ [1,2]. Further investigations are required (both experimental and theoretical) to prove the existence of direct $CP$ violation, as required by the Standard Model. To this aim, and to elucidate the mechanism of $CP$ violation, it is necessary to assess the potential manifestations of $CP$ violation in kaon decay channels alternative to $K^0 \rightarrow \pi\pi$. This requires a reliable theoretical approach to estimate the relevant hadronic matrix elements of the nonleptonic $\Delta S = 1$ weak Hamiltonian, and clearly also calls for accurate experimental studies of kaon decays (both $CP$ violating and $CP$ conserving), to

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test the theoretical description. Of course, an interesting aspect, besides the \( CP \) violation problem, would be the improvement of our understanding of meson dynamics and possibly a clarification of some fundamental issues, such as e.g. the origin of the \( \Delta I = 1/2 \) rule.

In this paper we would like to discuss some selected examples, emphasizing the role of experiments at machines such as \( \phi \)-factories \([2,3]\) and LEAR \([4]\). The special feature of this kind of machines is that they provide a well-defined initial \( K_L K_S \) quantum state, namely \( \phi \)-factories produce a \( p \)-wave \( K^0 \bar{K}^0 \) state, while tagged \( K^0 \) or \( \bar{K}^0 \) states are produced at LEAR. Therefore, time-dependent interferences in the vacuum between \( K_S \) and \( K_L \) decaying to a final state \( f \) can be accurately studied. For example, the characteristic interference factor to be studied at LEAR \([4,5]\) is of the form (assuming \( CPT \)):

\[
2e^{-\gamma t} [\text{Re}(\langle f|K_L\rangle^*\langle f|K_S\rangle) \cos \Delta m t - \text{Im}(\langle f|K_L\rangle^*\langle f|K_S\rangle) \sin \Delta m t],
\]

where \( t \) is the proper time, \( \gamma = (\gamma_L + \gamma_S)/2 \), and \( \Delta m = m_L - m_S \). By fitting the time-dependence of Eq.(1) to the experimental data, it is possible to determine independently both \( \text{Re}(\langle f|K_L\rangle^*\langle f|K_S\rangle) \) and \( \text{Im}(\langle f|K_L\rangle^*\langle f|K_S\rangle) \).

At the \( \phi \)-factory, where the initial state is produced via the decay \( \phi \rightarrow K^0 \bar{K}^0 \) with \( C = -1 \), one can measure time-correlations between \( K^0 \) decay to a final state \( f_1 \) (used as a tagging channel) at time \( t_1 \) and \( \bar{K}^0 \) decay to a state \( f_2 \) at time \( t_2 \). These correlations are expressed analogously to Eq.(1), and in terms of the same physical quantities \([2,4,7]\).

In what follows, we will discuss some interesting features of interferences in neutral kaon decays to \( 3\pi \) and \( \pi\pi\gamma \) final states, and point out their relevance and physical implications. Interferences in these channels should be measured with good statistics and precision at Da\( \phi \)ne (the Frascati \( \phi \)-factory) or at LEAR. We will also emphasize the complementary role of this kind of experimental analysis with respect to width measurements.

Specifically, in Sec. II we suggest that the recent LEAR data should already enable us to put limits on the \( 3\pi \) final state phases, which bring important information on the chiral structure of meson-meson strong interactions and, even more important, determine the size of direct \( CP \) violation asymmetries in \( K \rightarrow 3\pi \), so that their experimental determination represents an essential piece of information. In Sec. III we study \( K_{L,S} \rightarrow \pi\pi\gamma \), and complement the analysis of \([4,11]\) by extending it to the case of LEAR and including the electric “direct emission” \( CP \) conserving amplitude, not considered in that paper. In particular, we point out the relevance of interference at both LEAR and Da\( \phi \)ne in determining such amplitude. Finally, Sec. IV contains some concluding remarks.

**II. NOTATIONS AND INTERFERENCE IN \( K \rightarrow 3\pi \)**

We start by discussing the potential of LEAR, concerning the possibility of measuring the \( CP \) conserving \( K_S \rightarrow 3\pi \) amplitude as well as the final state (3\( \pi \)) phases. With the convention \( \bar{K}^0 = CP|K^0\rangle \), so that the eigenstates with definite \( CP = \pm 1 \) are \( |K_{1,2}\rangle = (|K^0\rangle \pm |\bar{K}^0\rangle)/\sqrt{2} \), the mass eigenstates are, assuming \( CPT \) invariance as we shall do throughout this paper:

\[
|K_{S,L}\rangle = p|K^0\rangle \pm q|\bar{K}^0\rangle \equiv \frac{|K_{1,2}\rangle + \xi|K_{2,1}\rangle}{\sqrt{1 + |\xi|^2}},
\]

(2)
where we adopt the same notations as in \[1\]. In particular:

$$\bar{\varepsilon} = \varepsilon - i \frac{Im A_0}{Re A_0},$$

with $A_{0,2}$ the amplitudes for $K \rightarrow \pi\pi$ with $I = 0, 2$. Consequently, the proper time evolution of initial $K^0$ or $\bar{K}^0$ states is:

$$|K^0(t)\rangle = \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2}(1 + \bar{\varepsilon})} \left[ |K_S\rangle \exp \left( -\frac{\Gamma_{st}}{2} - i m_{st} \right) + |K_L\rangle \exp \left( -\frac{\Gamma_{lt}}{2} - i m_{lt} \right) \right],$$

$$|\bar{K}^0(t)\rangle = \frac{\sqrt{1 + |\varepsilon|^2}}{\sqrt{2}(1 - \bar{\varepsilon})} \left[ |K_S\rangle \exp \left( -\frac{\Gamma_{st}}{2} - i m_{st} \right) - |K_L\rangle \exp \left( -\frac{\Gamma_{lt}}{2} - i m_{lt} \right) \right].$$

At first order in $\varepsilon$ the amplitude squared for decay to a final state $f$, as a function of time, is given in general by:

$$|A(K^0(\bar{K}^0) \rightarrow f)|^2 \simeq \frac{1}{2} \left( 1 \mp 2 Re \varepsilon \right) \left\{ \exp (-\Gamma_{st}|A_S|^2) + \exp (-\Gamma_{lt}|A_L|^2) \right\},$$

$$\pm 2 \exp (-\gamma t) \left[ Re \left( A_L A_S^* \right) \cos \Delta mt + Im \left( A_L A_S^* \right) \sin \Delta mt \right],$$

where $\Delta m$ and $\gamma$ have already been defined in connection to Eq.(1), and we use the notation $A_{S,L} \equiv A(K_{S,L} \rightarrow f)$.

Since we wish here to specialize Eq.(1) to $K \rightarrow 3\pi$, we adopt the familiar expansion of the amplitude for this process \[1\]:

$$A(K_L \rightarrow \pi^+\pi^-\pi^0) = (\alpha_1 + \alpha_3) \exp (i\delta_{1S}) - (\beta_1 + \beta_3) \exp (i\delta_{1M}) Y$$

$$A(K_S \rightarrow \pi^+\pi^-\pi^0) = \frac{2}{\sqrt{3}} \gamma_3 X \exp (i\delta_2).$$

In Eqs.(3) and (4), the subscripts 1 and 3 indicate $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions, respectively. Furthermore, $X = (s_2 - s_1)/m_\pi^2$ and $Y = (s_3 - s_0)/m_\pi^2$ are the Dalitz plot variables, with $s_i = (p_K - p_i)^2$, $s_0 = (s_1 + s_2 + s_3)/3$ and $p_i$ the pions momenta labelled in such a way that $i = 3$ corresponds to the “odd” charge pion (the $\pi^0$ in our case). The phases $\delta_{1S}$, $\delta_{1M}$ and $\delta_2$ originate from final-state strong interactions. The experimental values of $\alpha_i$, $\beta_i$ and $\gamma_3$ have been obtained from a fit to the experimental data on $K \rightarrow 3\pi$ differential widths \[1\,1\,2\]. The typical results, which can be used e.g. to assess expected numbers of events, are $\alpha_1 + \alpha_3 \simeq 8.5 \times 10^{-7}$, $\beta_1 + \beta_3 \simeq -2.8 \times 10^{-7}$, $\gamma_3 \simeq 2.3 \times 10^{-8}$.

The strong phases are usually neglected in the fit and therefore are not experimentally known yet. We recall that direct $CP$ violating asymmetries in $K \rightarrow 3\pi$ are dominated by the interferences between the $\Delta I = 1/2$ amplitudes $\alpha_1$ and $\beta_1$ and between $\alpha_1$ and the $\Delta I = 3/2$ amplitude $\gamma_3$. Consequently, these asymmetries are proportional to $\delta_{1S} - \delta_{1M}$ and $\delta_{1S} - \delta_2$. Therefore, the experimental determination of these phases is crucial to make estimates for direct $CP$ violation in this decay channel \[1\,1\,3\]. Another interesting aspect of this determination of the low energy (3$\pi$) phases is that it would usefully complement the measurement of $\pi$-$\pi$ phase shifts near threshold in e.g. $K_{l4}$ decays or $\pi$-$\pi$ scattering, and
thus would allow a stringent test of the current theoretical approach to meson dynamics, based on effective chiral Lagrangians. As emphasized in Sec. I and in Ref. [9], in this regard the unique advantage of interference is that it depends linearly on the \((3\pi)\) strong phases, which are expected to be small. For example, at the centre of the Dalitz plot, the theoretical expectations from both chiral loops [13] and a nonrelativistic approach [14] are:

\[
\delta_{1S} - \delta_{1M} \simeq \frac{\delta_{1S} - \delta_{2}}{2} \simeq 0.07
\]  

Conversely, width measurements only give \(\cos \delta \simeq 1 - \delta^{2}/2\) and thus are less sensitive to small \(\delta\).

An analogous linear dependence vs a quadratic one occurs also for \(A(K_{S} \to \pi^{+}\pi^{-}\pi^{0})\): one can see from (7) that the width of this process is suppressed both by the \(\Delta I = 1/2\) rule and by the angular momentum barrier. One can remark that linear dependence on the \((3\pi)\) phases \(\delta\) and on the amplitude \(\gamma_{3}\) could also obtain in a regenerator experiment [15], but compared to the possibility of observing oscillations in vacuum at LEAR or Da\(\phi\)ne this would be affected by a significant uncertainty due to the regeneration parameters.

The experimental analysis of the interference should proceed by substituting Eqs.(6) and (7) in the time evolution Eq.(5), making suitable kinematical cuts over the Dalitz plot, and fitting to the experimental time dependence [6, 9]. In the specific case, to determine the \(CP\) conserving amplitude of \(K_{S} \to 3\pi\), which according to (7) is antisymmetric in \(X\), the following weighted integral over the Dalitz plot can be considered [6, 9]:

\[
A^{+ -0}(t) = \frac{\int d\Phi \; \text{sgn}(X) \left[ |A(K^{0} \to \pi^{+}\pi^{-}\pi^{0})|^{2} - |A(K^{0} \to \pi^{+}\pi^{-}\pi^{0})|^{2} \right]}{\int d\Phi \left[ |A(K^{0} \to \pi^{+}\pi^{-}\pi^{0})|^{2} + |A(K^{0} \to \pi^{+}\pi^{-}\pi^{0})|^{2} \right]}
= \frac{a\gamma_{3}(\alpha_{1} + \alpha_{3}) \exp(-\gamma t) \left[ \cos(\delta_{1S} - \delta_{2}) \cos \Delta mt + \sin(\delta_{1S} - \delta_{2}) \sin \Delta mt \right]}{g\gamma_{3}^{2} \exp(-\Gamma_{St}) + [(\alpha_{1} + \alpha_{3})^{2} + b(\beta_{1} + \beta_{3})^{2}] \exp(-\Gamma_{Lt})},
\]  

where \(d\Phi\) denotes the phase space element. In Eq.(9) \(a, g\) and \(b\) come from phase space integrations, and their values are:

\[
a = \frac{32m_{K}Q}{9\pi m_{\pi}^{2}}; \quad g = \frac{4m_{K}Q^{2}}{9m_{\pi}^{4}}; \quad b = \frac{m_{K}Q^{2}}{9m_{\pi}^{4}},
\]  

with \(Q\) the \(Q\)-value of the reaction, \(Q^{+ -0} = 83.6\) MeV. The denominator in (9) has been chosen just to conveniently normalize the asymmetry, although different choices are quite possible. Also, one can notice that in the denominator of (9) only the second term is numerically relevant.

Recently, LEAR has produced a preliminary, direct determination of \(\gamma_{3}\) [16] from a fit of (9), neglecting the \(\sin \Delta mt\) part. Actually, we would suggest experimentalists to fit the data with the full Eq.(9), and derive (at least) an upper limit on \(\delta_{1S} - \delta_{2}\). Naively, by just imposing that the neglected term should be less or equal to the quoted statistical error of 30\% on the \(K_{S} \to 3\pi\) amplitude, one would expect an upper limit on this combination of phases of the order of 30\%. This is already at the level of discarding models claiming enhanced \(CP\) violating asymmetries from large \((3\pi)\) strong phases [17].
An analogous analysis can be performed to obtain a bound on the independent combination $\delta_{1S} - \delta_{1M}$, by considering a cut in $X \cdot Y$ [3]:

$$A^{+,-0}_{XY}(t) = \frac{\int d\Phi \text{ sgn}(X \cdot Y) \left[ |A(K^0 \to \pi^+\pi^-\pi^0)|^2 - |A(\bar{K}^0 \to \pi^+\pi^-\pi^0)|^2 \right]}{\int d\Phi \left[ |A(K^0 \to \pi^+\pi^-\pi^0)|^2 + |A(\bar{K}^0 \to \pi^+\pi^-\pi^0)|^2 \right]}$$

$$= -h \gamma_3 (\beta_1 + \beta_3) \exp (-\gamma t) \left[ \cos(\delta_{1S} - \delta_{1M}) \cos \Delta mt + \sin(\delta_{1S} - \delta_{1M}) \sin \Delta mt \right]$$

$$\times \frac{g \gamma_3^2 \exp (-\Gamma_{st}) + [(\alpha_1 + \alpha_3)\beta_1 + (\beta_1 + \beta_3)]^2 \exp (-\Gamma_{Lt})}{g \gamma_3^2 \exp (-\Gamma_{st}) + [(\alpha_1 + \alpha_3)\beta_1 + (\beta_1 + \beta_3)]^2 \exp (-\Gamma_{Lt})}$$

where $h$ is the another phase space integral

$$h = \frac{8m_K^2 Q^2}{9\pi m_h^4}.$$ (12)

In the future, the foreseen improvement in the experimental uncertainty (a factor 4 in statistics) should open the way to precise measurements of the $K \to 3\pi$ phases. Concerning $CP$ violation in this decay channel, the leading effect proportional to $Re \epsilon$ in Eq. (3) is obtained without making any cut on the Dalitz plot, because it is symmetric in $X$. Also, bounds on direct $CP$ violation can be obtained in principle. Considering the expected extreme smallness of this effect and the present capabilities, presumably this will need a really new stage in experimental accuracy.

Turning to the $\phi$-factory, the initial quantum state is represented in this case by the superposition of $K_L$ and $K_S$ states:

$$|i\rangle \equiv |K^0\bar{K}^0(C = \text{odd})\rangle = \frac{|K_L(\hat{z})K_S(\hat{z})| - |K_S(\hat{z})K_L(\hat{z})|}{2\sqrt{2}pq},$$ (13)

where $\hat{z}$ is the direction of kaons momenta in the c.m. system. The subsequent $K_L$ and $K_S$ decays are correlated, and their quantum interferences show up in relative time distributions and time asymmetries. Specifically, one considers the transition amplitude for the initial state to decay into the final states $f_1$ at time $t_1$ and $f_2$ at time $t_2$, respectively. Defining the “intensity” of time-correlated events $I(\Delta t)$ as:

$$I(f_1, f_2; \Delta t) \equiv \frac{1}{2} \int_{|\Delta t|}^{\infty} dt \langle (f_1(t_1, \hat{z}), f_2(t_2, -\hat{z})|T|i\rangle^2,$$ (14)

where $t = t_1 + t_2$ and $\Delta t = t_2 - t_1$, we find:

$$I(\Delta t < 0) = \left\{ \exp (-\Gamma_S|\Delta t|) |A_S(f_1)|^2 |A_L(f_2)|^2 + \exp (-\Gamma_L|\Delta t|) |A_L(f_1)|^2 |A_S(f_2)|^2 \right.$$

$$- 2 \exp (-\gamma|\Delta t|) \left[ Re \left( A_L(f_1)A_S(f_1)A_L(f_2)A_S(f_2) \right) \cos \Delta m|\Delta t| \right.$$  

$$+ Im \left( A_L(f_1)A_S(f_1)A_L(f_2)A_S(f_2) \right) \sin \Delta m|\Delta t| \right\} \frac{1}{16\gamma|p|^2|q|^2},$$ (15)

and
Here, we denote $A_{S,L}(f_i) \equiv A(K_{S,L} \to f_i)$ with $i = 1,2$. The theoretical analysis of the $CP$ conserving amplitude of $K_S \to 3\pi$, based on the time correlations (15) and (16), was presented in [9] with the choice $f_1 = \pi^0 h\nu$ as the tagging process and $f_2 = \pi^+\pi^-\pi^0$. $CP$ violation in the mass matrix can be studied similar to the case of Eq.(5), either with the same $f_1$ and $f_2$ as in [9] or with $f_1 = f_2 = 3\pi$ [14], and according to the previous discussion this can be regarded as an alternative to the measurement of $\Gamma(K_S \to 3\pi^0)$. Direct $CP$ violation can also be studied, although the effect is predicted to be so small in the Standard Model that presumably at best an upper limit could be achieved.

III. INTERFERENCE IN $K \to \pi\pi\gamma$

The amplitudes for $K \to \pi\pi\gamma$ decays are generally defined as the superposition of two amplitudes: internal bremsstrahlung ($A_{IB}$) and direct emission ($A_{DE}$) [18]. $A_{IB}$ accounts for bremsstrahlung from external charged particles and is predicted simply by QED in terms of the $K \to \pi\pi$ amplitude. $A_{DE}$ is obtained by subtraction of $A_{IB}$ from the total amplitude, and accounts for the possibility of direct $K \to \pi\pi\gamma$ couplings. By definition this amplitude is a test for mesonic interaction models, and in particular for the current description based on effective Lagrangians and Chiral Perturbation Theory (ChPT) [19].

For the processes $K_{S,L}(p_K) \to \pi^+(p_+)\pi^- (p_-)\gamma(q,\epsilon)$, we write

$$A^{S,L} = A^{S,L}_{IB} + A^{S,L}_{DE}$$

(17)

where

$$A^{S,L}_{IB} = eB A(K_{S,L} \to \pi^+\pi^-)$$

(18)

and

$$A^{S,L}_{DE} = eB h_E^{S,L}(E_\gamma^*,\cos \theta)A(K_S \to \pi^+\pi^-) + eB_M h_M^{S,L}(E_\gamma^*,\cos \theta).$$

(19)

Here, $E_\gamma^*$ is the photon energy in the $K_{S,L}$ rest frame and $\theta$ is the angle between the photon and the $\pi^+$ in the dipion frame. Furthermore:

$$B = \frac{\epsilon \cdot p_+}{q \cdot p_+} - \frac{\epsilon \cdot p_-}{q \cdot p_-},$$

$$\bar{B} = \epsilon \cdot p_+ q \cdot p_- - \epsilon \cdot p_- q \cdot p_+,$$

$$B_M = \epsilon_{\alpha\beta\gamma\delta} p_+^\alpha p_-^\beta q^\gamma \epsilon^\delta.$$

(20)

These are the only possible gauge and Lorentz invariant structures up to third order in momenta. While $B$ corresponds to the $IB$ amplitude, $\bar{B}$ and $B_M$ correspond to electric and
magnetic transitions, respectively. If the photon polarization is not measured there is no interference among electric and magnetic transitions, so that the differential width is

\[ d\Gamma = d\Gamma_{IB} + d\Gamma_{int} + d\Gamma_{M} + d\Gamma_{IB}^2, \]  

(21)

where \( int \) represents the interference between the \( IB \) and the \( DE \) electric components. On the r.h.s of Eq. (19) we have explicitly factored \( A(K_S \rightarrow \pi^+\pi^-) \) for later convenience, in order that all quantities of interest have a common factor \(|A(K_S \rightarrow \pi^+\pi^-)|^2\). Of course, in using our formulae involving \( h_E^{S,L} \) and comparing with direct emission amplitudes defined in the literature, this normalization has to be taken into account.

Concerning \( CP \) violation in this radiative nonleptonic process, and direct \( CP \) violation in particular, we recall that in the limit of \( CP \) conservation, and to the lowest contributing multipoles, for the \( K_S \) decay \( h_M^S = 0 \) and the direct emission amplitude is determined by \( h_E^S \) (electric dipole moment \( E1 \)), whereas \( K_L \) decay proceeds only through \( h_M^L \) (magnetic dipole moment \( M1 \)) [18]. However, reflecting the bremsstrahlung enhancement for \( K_L \) in particular, we recall that in the limit of \( CP \) conservation, and to the lowest contributing multipoles, for the \( K_S \) decay \( h_M^S = 0 \) and the direct emission amplitude is determined by \( h_E^S \) (electric dipole moment \( E1 \)), whereas \( K_L \) decay proceeds only through \( h_M^L \) (magnetic dipole moment \( M1 \)) [18]. However, reflecting the bremsstrahlung enhancement for \( E^*_\gamma \rightarrow 0 \), the \( CP \) violating \( A_{IB}^L \) is experimentally found to compete with the \( CP \) conserving \( DE \) amplitude \( h_M^L \) [20,21]. According to (18) \( A_{IB}^L \) is proportional to \( A(K_L \rightarrow \pi^+\pi^-) \), where the direct \( CP \) violation parameter \( e_{\pi\pi} \) is very small, being suppressed by the \( \Delta I = 1/2 \) rule. Therefore, this direct \( CP \) violation effect can hardly show up in the rate (21) for \( K_L \rightarrow \pi^+\pi^- \). Conversely, direct \( CP \) violation in \( h_E^L \) is not related to \( K_L \rightarrow \pi\pi \), and therefore in principle might give a larger effect through the \( int \) term in (21). However, unfortunately the \( IB \) enhancement cannot be beaten, and in fact such interference is expected to be strongly suppressed relative to the \( IB \) term.

This suppression factor can be qualitatively guessed \( e.g. \) by considering the total interferential width between \( A_{IB}^L \) and \( eB h_E^S(E^*_\gamma, \cos\theta)A(K_S \rightarrow \pi^+\pi^-) \). For simplicity of notations we denote it as follows:

\[ \langle Re h_E^S \rangle_{int} \equiv e^2 |A(K_S \rightarrow \pi^+\pi^-)|^2 \int d\Phi \sum_{Pol} BB \langle Re h_E^S \rangle, \]

(22)

where \( \Phi \) is the phase space, which must be cut for experimentally undetected photons. As shown in Ref. [22], at \( O(p^4) \) in ChPT \( h_E^S \) is the sum of a predicted loop amplitude and an unknown counterterm. Taking \( E^*_\gamma > 20 \) MeV and varying the counterterm in a reasonable range one obtains a negative interference, of the order of [22]:

\[ \frac{\Gamma(K_S \rightarrow \pi^+\pi^- \gamma)_{int}}{\Gamma(K_S \rightarrow \pi^+\pi^- \gamma)_{IB}} \simeq -10^{-4} \div -10^{-3}, \]

(23)

or, using \( Br(K_S \rightarrow \pi^+\pi^- \gamma, E^*_\gamma > 20 \) MeV)\( IB \simeq 4.8 \times 10^{-3} \):

\[ Br(K_S \rightarrow \pi^+\pi^- \gamma)_{int} \simeq -10^{-6} \div -10^{-5}. \]

(24)

This result strongly suppresses the expected sensitivity of the \( K_L \rightarrow \pi\pi\gamma \) width measurement to the \( CP \) violating amplitude \( h_E^L \), so that time-dependent interference analysis could represent a viable alternative in this case.

In general, besides the \( CP \) violation problem, a stringent test of the theoretical framework for the relevant hadronic matrix elements of \( H_W \) could conveniently be performed in such interference experiments. Indeed, one interesting point concerning \( A_{DE}^L \) is that higher
multipoles are in general not suppressed by CP violation, and might interfere significantly with $A_{IB}^{S,L}$. For example, being $h_{E2}^{S,L}$ odd under $\pi^+ \leftrightarrow \pi^-$ interchange ($\theta \rightarrow \theta + \pi$), by a suitable kinematical cut at the $\phi$-factory or at LEAR one could project out the interference between this amplitude and $A_{IB}$.

Regarding the current theoretical situation, both the lowest multipole amplitudes and the higher ones can be predicted in ChPT to order $p^4$ [23]. Specifically, the CP violating $O(p^4)$ amplitude $h_E^L$ is expressed, in that framework, in terms of meson loops ($h_{E,1}^L$), which are related to CP violation in $K^0 \rightarrow \pi\pi$, and of matrix elements of local counterterm operators ($h_{E,ct}^L$) not suppressed by the $\Delta I = 1/2$ rule. For the case of $K_S \rightarrow \pi^+\pi^-$, the CP conserving $O(p^4)$ direct emission amplitude $h_E^L$ is expressed analogously, in terms of loops ($h_{E,1}^S$) and counterterms ($h_{E,ct}^S$) [22]. The same counterterms contribute to $K_1 \rightarrow \pi^+\pi^-$ as well as to $K_2 \rightarrow \pi^+\pi^-$, with coefficients that are, respectively, real or imaginary in the chosen phase convention for $K^0$ and $K^0$. Similar to the case of $K \rightarrow 3\pi$ [24], the direct CP violating component of the loop amplitudes $h_{E,1}^S$ can be obtained by multiplying the $h_{E,1}^S$ by a factor containing the combination of CKM angles appropriate to $\varepsilon'_{\pi\pi}/\varepsilon$. Conversely, such a simple relation does not hold between the contributions of CP conserving and CP violating counterterms, because the actual values of the counterterm coefficients cannot be related, in general, to the process $K_L \rightarrow \pi\pi$.

For the CP violating $DE$ amplitude we will use in the sequel the following parametrization of $h_{E1}^L$:

$$h_{E1}^L \equiv \frac{A(K_L \rightarrow \pi^+\pi^-)_{DE,E1}}{eA(K_S \rightarrow \pi^+\pi^-)B} \approx \left(\varepsilon - i\frac{ImA_0}{ReA_0}\right)h_{E1}^S + \frac{A(K_2 \rightarrow \pi^+\pi^-)_{E1}}{eA(K_S \rightarrow \pi^+\pi^-)B} \approx \varepsilon h_{E1}^S + \varepsilon'_{\pi\pi} h_{E1,ct}^S,$$

(25)

where for phenomenological purposes we have neglected $\varepsilon'_{\pi\pi}$ with respect to $\varepsilon$. In (25) $\varepsilon'_{\pi\pi} h_{E1,ct}^S$ is the genuine, direct CP violation term in $K^0 \rightarrow \pi\pi\gamma$. We have normalized it in such a way that, if this direct CP violation is generated by the $O(p^4)$ local counterterms as we expect, then $\varepsilon'_{\pi\pi}$ does not depend on energy. Of course, this is a theoretical bias.

To discuss the opportunities of measurements at LEAR, we consider the following asymmetry as a function of proper time:

$$A_{\pi^+\pi^-\gamma} \equiv \frac{\int d\Phi \left[|A(K^0 \rightarrow \pi^+\pi^-)|^2 - |A(K^0 \rightarrow \pi^+\pi^-)|^2\right]}{\int d\Phi \left[|A(K^0 \rightarrow \pi^+\pi^-)|^2 + |A(K^0 \rightarrow \pi^+\pi^-)|^2\right]} \approx -2 Re \varepsilon$$

$$+ \frac{(2 \exp (-\gamma t)/\Gamma_S)}{\exp (-\Gamma_S t) Br(K_S \rightarrow \pi^+\pi^-) + \frac{1}{\Gamma_S} \exp (-\Gamma_L t) Br(K_L \rightarrow \pi^+\pi^-)} \times \left\{\Gamma_S Br(K_S \rightarrow \pi^+\pi^-)_{IB} [Re \eta_{+-} \cos \Delta mt + Im \eta_{+-} \sin \Delta mt] \right. $$

$$\left. + \langle Re (h_{E1}^S \eta_{+-} + h_{E1}^L) \cos \Delta mt + Im (h_{E1}^S \eta_{+-} + h_{E1}^L) \sin \Delta mt \rangle_{int}\right\},$$

(26)

where the notation $\langle \cdots \rangle_{int}$ has the same meaning as in (22), and we have introduced the familiar ratio $\eta_{+-} = A(K_L \rightarrow \pi^+\pi^-)/A(K_S \rightarrow \pi^+\pi^-)$. We have limited to the lowest significant $DE$ multipoles, and have neglected the small pure $DE$ emission contributions. Also, we have neglected in the denominator the interference term, suppressed by a CP violation
factor and always numerically smaller than the other ones. In (26), we have the mass-matrix 
$CP$ violation term $(-2Re\,\varepsilon)$, and three different time-oscillating terms.

The first time-dependent term is the one proportional to:

$$Br(K_S \to \pi^+\pi^-\gamma)_{IB}[Re\,\eta_+\cos\Delta mt + Im\,\eta_-\sin\Delta mt],$$

which measures the $CP$ violation in $K \to \pi\pi\gamma$ related to $K_L \to \pi\pi$. With the value of 
$Br(K_S \to \pi^+\pi^-\gamma)_{IB}$ leading to Eq.(24) and $Re\,\eta_+ \simeq Im\,\eta_- \sim 10^{-3}$, integrating the 
intensity of events over time (essentially a few $\tau_S$) we find that with $10^8$ initial $K^0$ or $\bar{K}^0$ 
one can expect about 100 events related to this term. We emphasize that, although this 
source of $CP$ violation is known as being related by QED to the one in $K_L \to \pi\pi$, such a measurement is still interesting because it allows to establish $CP$ violation in a different 
decay channel. Actually, this analysis might be competitive with the one at Fermilab [20], 
measuring $CP$ violation in

$$|\eta_{+\gamma}| = \left| \frac{A(K_L \to \pi^+\pi^-\gamma)_{IB+}E_1}{A(K_S \to \pi^+\pi^-\gamma)_{IB+}E_1} \right| = (2.15 \pm 0.26 \pm 0.20) \times 10^{-3},$$

$$\phi_{+\gamma} = \arg \left\{ \frac{A(K_L \to \pi^+\pi^-\gamma)_{IB+}E_1}{A(K_S \to \pi^+\pi^-\gamma)_{IB+}E_1} \right\} = (72 \pm 23 \pm 17)^\circ,$$

assuming constant $h^{S,L}_{E_1}$. From the experimental findings $\arg\varepsilon = (43.67 \pm 0.14)^\circ$ and 
$|\varepsilon_{\pi\pi}/\varepsilon| \leq 2.3 \times 10^{-3}$ [1], combining with the theoretical expectation $\arg\varepsilon_{\pi\pi}' = (43 \pm 6)^\circ$ [25], it seems reasonable to assume the approximation $Re\,\eta_+ \simeq Im\,\eta_-$, so that the term (27) 
has the characteristic time dependence:

$$Re\,\eta_+ (\cos\Delta mt + \sin\Delta mt) \equiv \sqrt{2} Re\,\eta_+ \sin (\Delta mt + \pi/4).$$

Clearly, for a more accurate estimate, one can easily include in (30) the small deviation of 
arg$\varepsilon$ from $45^\circ$.

More interesting is the second part of the interference term in Eq.(26), which according to [25] can be split in two parts. The first one is

$$2Re\varepsilon\langle Re\,h^{S}_{E_1}(\cos\Delta mt + \sin\Delta mt)\rangle_{int},$$

in the approximation $\eta_+ \simeq \varepsilon$ and $Im\,\varepsilon \simeq Re\,\varepsilon$. Eq.(31) has the same time dependence 
as the IB term (30), and therefore could be distinguished only by looking at the $E^*_\gamma$ and 
cos$\theta$ dependence, similar to the rate measurement of Eq.(24). Thus, by observing this time 
correlation one can measure $\langle Re\,h^{S}_{E_1}\rangle$, i.e. the interference between $A^L_{IB}$ and $A^S_{E_1}$. This is the 
same interference term obtained in the width measurement. With only $10^8$ initial $K^0(\bar{K}^0)$ the 
 corresponding rate of events might be too low, but anyway one could significantly improve 
existing bounds on the $DE$ amplitude for $K_S$ decay.

Finally, there is the term which can measure the direct $CP$ violation parameter $\varepsilon_{\pi\pi\gamma}$:

$$\langle Re(\varepsilon_{\pi\pi\gamma}'h^{S}_{E_1,ct}) \cos\Delta mt + Im(\varepsilon_{\pi\pi\gamma}'h^{S}_{E_1,ct}) \sin\Delta mt\rangle_{int}.$$


The important point is that this term has a different time behavior, compared to (33) and (32), and consequently should be disentangled from the others by accurate time dependence studies. Taking into account the imaginary character of the counterterm coefficients, one can notice that $\varepsilon'_{\pi\pi\gamma}h_{E2,cl}^S$ should have a real part, due to our definition (13) and final state strong interactions which make $A(K_S \to \pi^+\pi^-)$ complex. Although some theoretical models seem to indicate a suppression of $\varepsilon'_{\pi\pi\gamma}$ [20], we nevertheless believe that a value of the order of $10^{-4} \sim 10^{-5}$ is not completely unreasonable. Then, multiplying Eq. (32) by the interference factor $\sim 10^{-5}$ of Eq. (24), we see that it should be possible to put interesting bounds on this direct $CP$ violation parameter with $10^8$-$10^9$ initial $K^0(\overline{K^0})$. Probably, this can be more easily done by studying the interference in the kinematical region where the $IB$ contribution is less important, which is the region of maximum photon energy.

Interferometry experiments might provide also a unique method to detect higher multipole transition amplitudes via the interference with the $IB$ amplitude. For example, by considering the following cut in the Dalitz plot:

$$\int d\Phi^0 \equiv \int d\Phi \; \text{sgn}(\sin \theta), \quad (33)$$

one can define an asymmetry to extract $h_{E2}^L$:

$$A_{\pi^+\pi^-}^\theta \equiv \frac{\int d\Phi^0 \left[ |A(K^0 \to \pi^+\pi^-)|^2 - |A(\overline{K^0} \to \pi^+\pi^-)|^2 \right]}{\int d\Phi \left[ |A(K^0 \to \pi^+\pi^-)|^2 + |A(\overline{K^0} \to \pi^+\pi^-)|^2 \right]} \approx \frac{2\exp(-\gamma t)\exp(-\Gamma_S t)Br(K_S \to \pi^+\pi^-)}{\Gamma_S} \frac{\langle Re h_{E2}^L \cos \Delta m t + Im h_{E2}^L \sin \Delta m t \rangle_{\text{int}}}{\Gamma_{S,\text{int}}} \int d\Phi \left( \sum_{Pol} B_{\bar{B}} \right) \text{sgn}(\sin \theta) Re h_{E}^L. \quad (34)$$

Here, with an obvious extension of the notation in Eq. (24), we have introduced the interference:

$$\langle Re h_{E2}^L \rangle_{\text{int}}^\theta \equiv e^2 |A(K_S \to \pi^+\pi^-)|^2 \int d\Phi \left( \sum_{Pol} B_{\bar{B}} \right) \text{sgn}(\sin \theta) Re h_{E}^L. \quad (35)$$

To derive Eq. (34), we have used the well-known fact that $|A_{IB}|^2$ is even in $\sin \theta$ and $h_{E2}^L$ is independent of $\sin \theta$ to $O(p^4)$ in ChPT, and have neglected terms proportional to $CP$ violation.

The $O(p^4)$ one-loop $CP$ conserving amplitude $h_{E2}^L A(K_S \to \pi^+\pi^-)$ has been computed in ChPT [23]. In this theoretical framework, this product does not have absorptive part, i.e. it should be purely real. By looking at the values of this amplitude over the Dalitz plot, the authors of Ref. [23] suggest

$$\left| \frac{eh_{E2}^L A(K_S \to \pi^+\pi^-)B}{A(K_L \to \pi^+\pi^-)_{IB}} \right| \leq 10^{-2}. \quad (36)$$

As mentioned above, due to final state interactions $A(K_S \to \pi^+\pi^-)$ has both real and imaginary parts. Consequently, both terms in (34) are present and possibly might be separately measured. Assuming optimistically the upper bound (36) to be saturated all over the Dalitz plot, with $10^8$-$10^9$ initial kaons few events should be available for this kind of analysis.
At the $\phi$-factory, it is possible to study $K \to \pi\pi\gamma$ decays through interferences by choosing in Eqs.\((15)\) and \((16)\), $f_1 = \pi^+ \pi^-, \pi^0 \pi^0$ or $\pi^\pm l^\mp \bar{\nu}(\nu)$ as tagging channels, and $f_2 = \pi^+ \pi^- \gamma$. For the case $f_1 = \pi^+ \pi^-$ one obtains:

$$I(\Delta t < 0)_{\pi^+ \pi^-} = \frac{\Gamma(K_S \to \pi^+ \pi^-) \Gamma(K_S \to \pi^+ \pi^-)_{IB}}{16\gamma|p|^2|q|^2} \times \left\{ \exp(-\Gamma_S |\Delta t|) R_L + \exp(-\Gamma_L |\Delta t|)|\eta_{+-}|^2 R_S - 2 \exp(-\gamma |\Delta t|) \left[ (|\eta_{+-}|^2 + \frac{\text{Re}(\eta_{+-} \langle h_{E1}^L + \eta_{+-} h_{E1}^S \rangle_{\text{int}})}{\Gamma(K_S \to \pi^+ \pi^-)_{IB}}) \cos \Delta m |\Delta t| \right] \right\}.$$

\((37)\)

$$I(\Delta t > 0)_{\pi^+ \pi^-} = \frac{\Gamma(K_S \to \pi^+ \pi^-) \Gamma(K_S \to \pi^+ \pi^-)_{IB}}{16\gamma|p|^2|q|^2} \times \left\{ \exp(-\Gamma_L |\Delta t|) R_L + \exp(-\Gamma_S |\Delta t|)|\eta_{+-}|^2 R_S - 2 \exp(-\gamma |\Delta t|) \left[ (|\eta_{+-}|^2 + \frac{\text{Re}(\eta_{+-} \langle h_{E1}^L + \eta_{+-} h_{E1}^S \rangle_{\text{int}})}{\Gamma(K_S \to \pi^+ \pi^-)_{IB}}) \cos \Delta m |\Delta t| \right] \right\}.$$

\((38)\)

and similar formulae hold for $f_1 = \pi^0 \pi^0$. For convenience we have introduced the ratios:

$$R_L = \frac{\Gamma(K_L \to \pi^+ \pi^- \gamma)}{\Gamma(K_S \to \pi^+ \pi^-)_{IB}}; \quad R_S = \frac{\Gamma(K_S \to \pi^+ \pi^- \gamma)}{\Gamma(K_S \to \pi^+ \pi^-)_{IB}}.$$

\((39)\)

The factor $R_L$ contains all contributions to Eq.\((21)\). As already mentioned, the IB contribution is suppressed by $|\eta_{+-}|^2$ and it turns out that it is comparable to the magnetic contribution \((21)\), so that the numerical value of $R_L$ will be a factor times $|\eta_{+-}|^2$. Ideally, experiments should be able to fit all the coefficients of the three time-dependent terms in \((37)\) and \((38)\), so that the corresponding interesting physics could be measured. Using \((25)\) and the approximation $\eta_{+-} \approx \varepsilon$, we can write the leading contribution to the interference as:

$$I(\Delta t \lesssim 0)_{\pi^+ \pi^-}^{\text{int}} = \frac{\Gamma(K_S \to \pi^+ \pi^-) \Gamma(K_S \to \pi^+ \pi^-)_{IB}}{16\gamma|p|^2|q|^2} \times \left\{ -2 \exp(-\gamma |\Delta t|) |\varepsilon |^2 \left. \left[ (1 + \frac{\langle 2 \text{Re} h_{E1}^L + \varepsilon \langle \varepsilon_{\pi\pi\gamma} h_{E1,e1 \pi} \rangle_{\text{int}} \rangle}{\Gamma(K_S \to \pi^+ \pi^-)_{IB}}) \cos \Delta m |\Delta t| \right) \left. \right| \right\}.$$

\((40)\)

Integrating over all times (essentially a few $\tau_S$), the intensity resulting from direct $CP$ violation is of the order of $\sim 10^{-9} \varepsilon^2$. Indeed, direct $CP$ violation could be disentangled by considering $I(\Delta t < 0)_{\pi^+ \pi^-}^{\text{int}} - I(\Delta t > 0)_{\pi^+ \pi^-}^{\text{int}}$. Although depressed by low statistics (both at LEAR and at $\phi$-factories), these measurements have the advantage over experiments at
fixed-target beams \[20\] that the different time behavior greatly helps in distinguishing the various terms. Consequently, it might be not so surprising if better measurements or bounds on \(\varepsilon_{\pi^+\pi^-}^t, h_{E1,ct}/\varepsilon\) would come from interferometry machines.

A difficulty is that all terms in (37) and (38) are suppressed by at least \(|\varepsilon|^2\), and in addition there is the problem that to distinguish them from each other requires the accurate knowledge of the factors \(R_L\) and \(R_S\). In this regard, as an alternative to \(\pi^+\pi^-\) tagging one could use the semileptonic decay \(\pi^\pm l^\mp \bar{\nu}(\nu)\). In this case one obtains:

\[
I(\Delta t < 0)_{\pi^\pm l^\mp \bar{\nu}(\nu)} = \frac{\Gamma(K^0(K^0) \to \pi^\pm l^\mp \bar{\nu}(\nu))\Gamma(K_S \to \pi^+\pi^-)_{IB}}{16\gamma|\frac{S}{q}|^2}
\times \left\{ \exp(-\Gamma_S|\Delta t|)R_L + \exp(-\Gamma_L|\Delta t|)R_S
\right.
\]
\[
\pm 2\exp(-\gamma|\Delta t|)\left[ Re\left(\eta_{+-}^* + \frac{\langle h_{E1}^L + \eta_{+-}^* h_{E1}^S \rangle_{int}}{\Gamma(K_S \to \pi^+\pi^-)_{IB}}\right)\cos \Delta m|\Delta t| + Im\left(\eta_{+-}^* + \frac{\langle h_{E1}^L + \eta_{+-}^* h_{E1}^S \rangle_{int}}{\Gamma(K_S \to \pi^+\pi^-)_{IB}}\right)\sin \Delta m|\Delta t| \right]\right\}, \tag{41}
\]

and

\[
I(\Delta t > 0)_{\pi^\pm l^\mp \bar{\nu}(\nu)} = \frac{\Gamma(K^0(K^0) \to \pi^\pm l^\mp \bar{\nu}(\nu))\Gamma(K_S \to \pi^+\pi^-)_{IB}}{16\gamma|\frac{S}{q}|^2}
\times \left\{ \exp(-\Gamma_L|\Delta t|)R_L + \exp(-\Gamma_S|\Delta t|)R_S
\right.
\]
\[
\pm 2\exp(-\gamma|\Delta t|)\left[ Re\left(\eta_{+-}^* + \frac{\langle h_{E1}^L + \eta_{+-}^* h_{E1}^S \rangle_{int}}{\Gamma(K_S \to \pi^+\pi^-)_{IB}}\right)\cos \Delta m\Delta t
\right.
\]
\[
- Im\left(\eta_{+-}^* + \frac{\langle h_{E1}^L + \eta_{+-}^* h_{E1}^S \rangle_{int}}{\Gamma(K_S \to \pi^+\pi^-)_{IB}}\right)\sin \Delta m\Delta t \right]\right\}. \tag{42}
\]

Here, the interference term is not suppressed by \(|\varepsilon|^2\). Unfortunately, the cost is the smaller \(Br(K_S \to \pi^\pm l^\mp \bar{\nu}(\nu)) \sim 10^{-4}\), instead of \(Br(K_S \to \pi^+\pi^-)\) which appears in (40). This gives a depressing factor \(Br(K_S \to \pi l\nu) \cdot Br(K_S \to \pi^+\pi^-) \sim 10^{-7}\). Comparing (42) with (26), we see that the interference terms have the same form, while (41) compared to (26) has just the opposite sign for the imaginary part. Similar to the case of LEAR, we have here three different time-dependent terms of the form (27), (31) and (32). However, the \(\phi\)-factory has the advantages that \(i\): in principle one can select the interference term by considering the asymmetry between opposite charge modes, \(A(\Delta t \lesssim 0) \equiv I(\Delta t \lesssim 0)_{\pi^+ l^-} - I(\Delta t \lesssim 0)_{\pi^- l^+}\); and \(ii\): the imaginary part of the interference can be separately studied by considering the difference \(A(\Delta t < 0) - A(\Delta t > 0)\). Concerning statistics, taking into account the suppression factor mentioned above, it appears that, with \(e.g.\) \(10^{12}\) \(\phi\)'s one might obtain about 100 events related to the term (27), and (at least) put some significant constraints on the other two terms. In any case, this analysis should enable to give limits on \(\varepsilon_{\pi^+\pi^-}^t\) in a way complementary to the direct measurement of the charge asymmetry in \(K^\pm \to \pi^\pm \pi^0\gamma\) (21).

Another interesting issue to be pursued at the \(\phi\)-factory is the intensity with the \(\theta\)-cut Dalitz plot defined in (33), giving access to \(h_{E1}^L\). For \(f_1 = \pi^+\pi^-\) tagging, this can be expressed as:
\[ I(\Delta t < 0)^\theta_{\pi^+\pi^-} = \frac{\Gamma(K_S \to \pi^+\pi^-)}{16\gamma |p|^2 |q|^2} \times \left\{ 2(\text{Re}(h_{E2}^L\epsilon^*))_{\text{int}}^\theta \exp(-\Gamma_S|\Delta t|) - 2\exp(-\gamma|\Delta t|) \right\} \times \left[ (\text{Re}(\epsilon^*h_{E2}^L) \cos \Delta m|\Delta t| - \text{Im}(\epsilon^*h_{E2}^L) \sin \Delta m|\Delta t|)_{\text{int}}^\theta \right]. \] (43)

and

\[ I(\Delta t > 0)^\theta_{\pi^+\pi^-} = \frac{\Gamma(K_S \to \pi^+\pi^-)}{16\gamma |p|^2 |q|^2} \times \left\{ 2(\text{Re}(h_{E2}^L\epsilon^*))_{\text{int}}^\theta \exp(-\Gamma_L|\Delta t|) - 2\exp(-\gamma|\Delta t|) \right\} \times \left[ (\text{Re}(\epsilon^*h_{E2}^L) \cos \Delta m|\Delta t| + \text{Im}(\epsilon^*h_{E2}^L) \sin \Delta m|\Delta t|)_{\text{int}}^\theta \right]. \] (44)

Analogously, for semileptonic tagging:

\[ I(\Delta t < 0)^\theta_{\pi^0 l^\mp\bar{\nu}(\nu)} = \frac{\Gamma(K^0(K^0) \to \pi^0 l^\mp\bar{\nu}(\nu))}{16\gamma |p|^2 |q|^2} \times \left\{ 2(\text{Re}(h_{E2}^L\epsilon^*))_{\text{int}}^\theta \exp(-\Gamma_S|\Delta t|) \right\} \times \left[ (\text{Re}(\epsilon^*h_{E2}^L) \cos \Delta m|\Delta t| - \text{Im}(\epsilon^*h_{E2}^L) \sin \Delta m|\Delta t|)_{\text{int}}^\theta \right]. \] (45)

and

\[ I(\Delta t > 0)^\theta_{\pi^0 l^\mp\bar{\nu}(\nu)} = \frac{\Gamma(K^0(K^0) \to \pi^0 l^\mp\bar{\nu}(\nu))}{16\gamma |p|^2 |q|^2} \times \left\{ 2(\text{Re}(h_{E2}^L\epsilon^*))_{\text{int}}^\theta \exp(-\Gamma_L|\Delta t|) \right\} \times \left[ (\text{Re}(\epsilon^*h_{E2}^L) \cos \Delta m|\Delta t| + \text{Im}(\epsilon^*h_{E2}^L) \sin \Delta m|\Delta t|)_{\text{int}}^\theta \right]. \] (46)

We notice that phenomenologically \((h_{E2}^L\epsilon^*)\) tends to be almost purely imaginary in the framework considered above, \(i.e.\) ChPT to order \(p^4\), since \(\arg \epsilon \sim 44^\circ\) and, from the definition \([19]\), final state interactions determine \(\arg h_{E2}^L \sim -\delta_0\) with \(\delta_0\) is the \(I = l = 0\ \pi\pi\) phase shift. It turns out that this angle is about \((39 \pm 5)^\circ\) \([25]\), which implies \(\arg h_{E2}^L\epsilon^* \sim -83^\circ\). Consequently, only the last term in \((43)\) and \((44)\) is substantially different from zero. Instead, with the semileptonic tagging, both oscillating terms in \((45)\) and \((46)\) are different from zero, while also in this case the purely exponential term tends to be almost vanishing. Anyway, we can more precisely dispose of this term and select the oscillating interference by considering the difference between intensities with opposite lepton charges in the tagging channel. Furthermore, \(\sin \Delta m|\Delta t|\) can be isolated by the time asymmetry. Concerning the needed statistics, at least \(10^{12}\) \(\phi\)'s should be required for this kind of analysis.

**IV. CONCLUSIONS**

In this paper we have analyzed the advantages of using interferometry kaon machines like LEAR or the \(\phi\)-factory in the study of the decays \(K_{L,S} \to 3\pi\) and \(K_{L,S} \to \pi\pi\gamma\). Typical interference patterns can be obtained by studying as a function of time the difference \(\Gamma(K^0 \to f) - \Gamma(K^0 \to f)\) at LEAR, and at the \(\phi\)-factory the intensity for \(\phi\)-decaying to the \(K_SK_L\) system and this in turn to two final states \(f_1\) and \(f_2\). Generally, at the \(\phi\)-factory one of the final states is chosen to be a tagging channel \([3]\), and we have considered here both two pions and semileptonic decays tagging.

Concerning the channel \(K \to 3\pi\), we have seen how LEAR could measure the \((3\pi)\) phase shifts, by just fitting the correlation \(A(K_L \to 3\pi)^*A(K_S \to 3\pi)\) to the data as a function of
time. Actually, it seems already possible to put some interesting limit on these phases using the present data. This analysis extends the previous discussion of this issue, presented for the case of the $\phi$-factory in Ref. [9].

As to the channel $K \to \pi\pi\gamma$, the correlation $A(K_L \to \pi^+\pi^-\gamma)^* A(K_S \to \pi^+\pi^-\gamma)$ has a richer structure, which we have tried to analyze for both the $\phi$-factory and LEAR. This correlation can be either symmetric or antisymmetric under exchange of pion four-momenta. Accordingly, to select the corresponding physics, we suggest to analyze the data by integrating either over the full Dalitz plot or with an antisymmetric kinematical cut. In the symmetric correlation, several interesting physical effects can be either studied or significantly constrained. These include: the $CP$ violation in $A(K_L \to \pi^+\pi^-\gamma)_{IB}$, proportional to $A(K_L \to \pi^+\pi^-)$; the $CP$ conserving structure dependent amplitude $A(K_S \to \pi^+\pi^-\gamma)_{DE}$; the direct $CP$ violation $\varepsilon'_{\pi\pi\gamma}$, in general not suppressed by the $\Delta I = 1/2$ rule. Our discussion indicates that interferometry machines should be quite useful to study these effects, and that time-dependence in correlations provides a convenient way to disentangle direct $CP$ violation from other contributions. Furthermore, in the case of the $\phi$-factory with semileptonic tagging, one can define time asymmetries which are directly proportional to $\varepsilon'_{\pi\pi\gamma}$. Also, intensities integrated asymmetrically over the phase space both at LEAR and at the $\phi$-factory should be an efficient tool to measure $CP$ conserving higher multipole amplitudes.

A related analysis was performed in Ref. [10], where quantum correlations in $K \to \pi^+\pi^-\gamma$ were studied, limiting to the $\phi$-factory with two pion tagging and (mostly) to large time intervals $\Delta t \to \infty$, where interference is not so important. Here, we complement that analysis in several directions, namely we consider also the case of LEAR and include higher multipole amplitudes in the analysis. In addition, for the $\phi$-factory, we are mostly concerned with finite time intervals $\Delta t$, for which interference plays a crucial role. Furthermore, also the semileptonic tagging has been exploited in the present paper.

In conclusion, we expect that very likely, by time-dependence measurements, interferometry machines will measure the three-pion phase shifts, will improve the existing value of the $CP$ conserving $K \to \pi\pi\gamma$ amplitude and put a stringent limit on $\varepsilon'_{\pi\pi\gamma}$. These time-dependence measurements should usefully complement higher statistics experiments at fixed-target kaon beams.
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