Abstract

Due to the enormous technological improvements obtained in the last decades it is possible to use robotic vehicles for underwater exploration. This work describes the development of a dynamic positioning system for remotely operated underwater vehicles based. The adopted approach is developed using Lyapunov Stability Theory and enhanced by a neural network based algorithm for uncertainty and disturbance compensation. The performance of the proposed control scheme is evaluated by means of numerical simulations. Due to the enormous technological improvements obtained in the last decades it is possible to use robotic vehicles for underwater exploration. This work describes the development of a dynamic positioning system for remotely operated underwater vehicles based. The adopted approach is developed using Lyapunov Stability Theory and enhanced by a neural network based algorithm for uncertainty and disturbance compensation. The performance of the proposed control scheme is evaluated by means of numerical simulations.

I. INTRODUCTION

The control system is one of the most important pieces of a Remotely Operated underwater Vehicle (ROV), and its characteristics (advantages and disadvantages) play an essential role when one has to choose a vehicle for a specific mission. These vehicles have been substituting the divers in the accomplishment of tasks that offer risks to the human life. In this way, ROVs have been used thoroughly in the research of sub phenomena and in assembly, inspection, and repair of offshore structures. During the execution of a certain task with the robotic vehicle, the operator needs to monitor and control a series of parameters. If some of these parameters, as for instance the position and attitude of the vehicle, could be attended automatically by a control system, the teleoperation of the ROV can be enormously facilitated.

Unfortunately, the problem of designing accurate positioning systems for underwater robotic vehicles still challenges many engineers and researchers interested in this particular branch of engineering science. A growing number of papers dedicated to the position and orientation control of such vehicles confirms the necessity of the development of a controller, that could deal with the inherent nonlinear system dynamics, imprecise hydrodynamic coefficients, and external disturbances. It has already been shown (Yu et al. 1994; Golshen and Jeffreys 1990) that, in the case of underwater vehicles, the traditional control methodologies are not the most suitable choices and cannot guarantee the required tracking performance.

Intelligent control, on the other hand, has proven to be a very attractive approach to cope with uncertain nonlinear systems. By combining nonlinear control techniques, such as feedback linearization or sliding modes, with adaptive intelligent algorithms, for example fuzzy logic or artificial neural networks, the resulting intelligent control strategies can deal with the nonlinear characteristics as well as with modeling imprecisions and external disturbances that can arise.

In this work, a nonlinear controller scheme with a neural network based compensation scheme is employed for the dynamic positioning of underwater vehicles. Based on a Lyapunov stability theory, the convergence properties of the closed-loop system is analytically proven. Numerical results are also provided to confirm the control system efficacy.

II. NONLINEAR CONTROLLER DESIGN

Consider a class of $n$th-order nonlinear systems:

$$x^{(n)} = f(x) + b(x)u + d$$  \hspace{1cm} (1)

where $u$ is the control input, the scalar variable $x$ is the output of interest, $x^{(n)}$ is the $n$-th time derivative of $x$, $x = [x, \dot{x}, \ldots, x^{(n-1)}]$ is the system state vector, $d$ represents external disturbances and unmodeled dynamics, and $f, b : \mathbb{R}^n \rightarrow \mathbb{R}$ are both nonlinear functions.

As demonstrated by Bessa and Barrêto (2010), adaptive fuzzy algorithms can be properly combined with nonlinear controllers in order to improve the trajectory tracking of uncertain nonlinear systems. It has also been shown that such strategies are suitable for a variety of applications ranging from remotely operated underwater vehicles (Bessa et al. 2008, 2010) to chaos control (Bessa et al. 2009).

The proposed control problem is to ensure that the state vector $x$ will follow a desired trajectory $x_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the state space. Regarding the development of the control law, the following assumptions should also be made:

**Assumption 1** The state vector $x$ is available.

**Assumption 2** The desired trajectory $x_d$ is once differentiable in time. Furthermore, every element of vector $x_d$ as well as $x_d^{(n)}$, is available and with known bounds.
Let $\tilde{x} = x - x_d$ be defined as the tracking error in the variable $x$, and
\[ \tilde{x} = x - x_d = [\tilde{x}, \tilde{\dot{x}}, \ldots, \tilde{x}^{(n-1)}] \]
as the tracking error vector. Now, consider a combined tracking error measure:
\[ \varepsilon = c^T \tilde{x} \]
where $c = [c_{n-1} \lambda^{n-1}, \ldots, c_1 \lambda, c_0]$, $\lambda$ is positive constant and $c_i$ states for binomial coefficients, i.e.,
\[ c_i = \binom{n-1}{i} = \frac{(n-1)!}{(n-i-1)! i!}, \quad i = 0, 1, \ldots, n-1 \]
which makes $c_{n-1} \lambda^{n-1} + \cdots + c_1 \lambda + c_0$ a Hurwitz polynomial.

From Eq. (2), it can be easily verified that $c_0 = 1$, for $\forall n \geq 1$. Thus, for notational convenience, the time derivative of $\varepsilon$ will be written in the following form:
\[ \dot{\varepsilon} = c^T \dot{\tilde{x}} = \tilde{x}^{(n)} + \bar{c}^T \tilde{x} \]
where $\bar{c} = [0, c_{n-1} \lambda^{n-1}, \ldots, c_1 \lambda]$.

Based on Assumptions 1 and 2, the following control law can be proposed:
\[ u = \frac{1}{b} (-f - d + x_d^{(n)} - \bar{c}^T \tilde{x} - \kappa \varepsilon) \]
where $\kappa$ is a strictly positive constant.

The boundedness and convergence properties of the closed-loop system are established in the following theorem.

**Theorem 1** Consider the nonlinear system (1) and Assumptions 1–2. Then, the controller defined by (5) ensures the exponential convergence of the tracking error, i.e., $\tilde{x} \to 0$ as $t \to \infty$.

**Proof:** Let a positive definite Lyapunov function candidate $V$ be defined as
\[ V(t) = \frac{1}{2} \varepsilon^2 \]
Thus, the time derivative of $V$ is
\[ \dot{V}(t) = \varepsilon \dot{\varepsilon} = (\tilde{x}^{(n)} + \bar{c}^T \tilde{x}) \varepsilon = (x^{(n)} - x_d^{(n)} + \bar{c}^T \tilde{x}) \varepsilon = [f + b u + d - x_d^{(n)} + \bar{c}^T \tilde{x}] \varepsilon \]
By applying the proposed control law (5), one has
\[ \dot{V}(t) = -\kappa \varepsilon^2 \]
which implies $\varepsilon \to 0$ as $t \to \infty$.

From the definition of limit, it means that for every $\xi > 0$ there is a corresponding number $\tau$ such that $|\varepsilon| < \xi$ whenever $t > \tau$. According to Eq. (2) and considering that $|\varepsilon| < \xi$ may be rewritten as $-\xi < \varepsilon < \xi$, one has
\[ -\xi < c_0 \varepsilon^{(n)} + c_1 \lambda \varepsilon^{(n-2)} + \cdots + c_{n-2} \lambda^{n-2} \varepsilon + c_{n-1} \lambda^{n-1} \varepsilon < \xi \]
Multiplying (6) by $e^{\lambda t}$ and noting that
\[ \frac{d^{n-1}}{dt^{n-1}} (\varepsilon e^{\lambda t}) = (c_0 \varepsilon^{(n)} + c_1 \lambda \varepsilon^{(n-2)} + \cdots + c_{n-2} \lambda^{n-2} \varepsilon + c_{n-1} \lambda^{n-1} \varepsilon) e^{\lambda t} \]
one has
\[ -\xi e^{\lambda t} < \frac{d^{n-1}}{dt^{n-1}} (\varepsilon e^{\lambda t}) < \xi e^{\lambda t} \]
Thus, integrating (6) $n - 1$ times between 0 and $t$ gives
\[ -\xi \frac{\varepsilon}{\lambda^{n-1}} e^{\lambda t} + \left( \frac{d^{n-2}}{dt^{n-2}} (\varepsilon e^{\lambda t}) \right)_{t=0}^{t} + \frac{\varepsilon}{\lambda} + \cdots + \left( \frac{\varepsilon}{\lambda} \right)^{t} \leq \varepsilon e^{\lambda t} \leq \xi \frac{\varepsilon}{\lambda^{n-1}} e^{\lambda t} + \left( \frac{d^{n-2}}{dt^{n-2}} (\varepsilon e^{\lambda t}) \right)_{t=0}^{t} + \frac{\varepsilon}{\lambda} + \cdots + \left( \frac{\varepsilon}{\lambda} \right)^{t} \]
Furthermore, dividing (10) by $e^{\lambda t}$, it can be easily verified that the values of $\tilde{x}$ can be made arbitrarily close to 0 (within a distance $\xi$) by taking $t$ sufficiently large (larger than $\tau$), i.e., $\tilde{x} \to 0$ as $t \to \infty$. Now, considering the $(n-2)^{th}$ integral of (10), dividing again by $e^{\lambda t}$ and considering that $\dot{x}$ converges to zero, it follows that $\tilde{x} \to 0$ as $t \to \infty$. The same procedure can be successively repeated until the convergence of each component of the tracking error vector is achieved: $\tilde{x} \to 0$ as $t \to \infty$. $\square$
III. VEHICLE DYNAMICS MODEL

A reasonable model to describe the underwater vehicle’s dynamical behavior must include the rigid-body dynamics of the vehicle’s body and a representation of the surrounding fluid dynamics. Such a model must be composed of a system of ordinary differential equations, to represent rigid-body dynamics, and partial differential equations to represent both tether and fluid dynamics.

In order to overcome the computational problem of solving a system with this degree of complexity, in the majority of publications (Bessa et al., 2010, 2008; Antonelli, 2007; Hoang and Kreuzer, 2007; Smallwood and Whitcomb, 2004; Hsu et al., 2000; Kiriazov et al., 1997; Yoerger and Slotine, 1985) a lumped-parameters approach is employed to approximate vehicle’s dynamical behavior.

The equations of motion for underwater vehicles can be presented with respect to an inertial reference frame or with respect to a body-fixed reference frame. On this basis, the equations of motion for underwater vehicles can be expressed, with respect to the body-fixed reference frame, in the following vectorial form:

$$M \ddot{\nu} + k(\nu) + h(\nu) + g(x) + d = \tau \quad (11)$$

where $\nu = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]$ is the vector of linear and angular velocities in the body-fixed reference frame, $x = [x, y, z, \alpha, \beta, \gamma]$ represents the position and orientation with respect to the inertial reference frame, $M$ is the inertia matrix, which accounts not only for the rigid-body inertia but also for the so-called hydrodynamic added inertia, $k(\nu)$ is the vector of generalized Coriolis and centrifugal forces, $h(\nu)$ represents the hydrodynamic quadratic damping, $g(x)$ is the vector of generalized restoring forces (gravity and buoyancy), $d$ stands for occasional disturbances, and $\tau$ is the vector of control forces and moments.

Another important issue in the case of ROVs is the disturbance force caused by the umbilical (or tether cable). The umbilical can be treated as a continuum, discretized with the finite element method or modeled as multibody system (Bevilacqua et al., 1991; Pinto, 1996). However, the adoption of any of these approaches requires a computational effort that would be prohibitive for on-line estimation of the control action. A common way to surmount this limitation is to consider the forces and moments exerted by the tether as random, and incorporate them into the vector $d$.

On this basis, considering that the restoring forces can be passively compensated (Kiriazov et al., 1997), the most relevant hydrodynamic forces and moments acting on ROVs are discussed in the following subsections.

A. Hydrodynamic forces

Remotely operated underwater vehicles typically operate with velocities never exceeding 2 m/s. Consequently, the hydrodynamic forces ($F_h$) can be approximated using the Morison equation (Newman, 1986).
where $v$ and $\dot{v}$ are, respectively, the relative velocity and the relative acceleration between rigid-body and fluid, $\dot{v}_w$ is the acceleration of underwater currents, $A$ is a reference area, $\rho$ is the fluid density, $\nabla$ is the fluid’s displaced volume, $C_D$ and $C_M$ are coefficients that must be obtained experimentally.

The last term of Eq. (12) is the so-called Proude-Kryloff force and will not be considered in this work due the fact, that at normal working depths, the acceleration of the underwater currents is negligible. In this way, the coefficient $C_M\rho\nabla$ of the second term will be called hydrodynamic added mass. The first term represents the nonlinear hydrodynamic quadratic damping. Experimental tests [Kleczka et al., 1992] show that Morison equation describes with sufficient accuracy the hydrodynamic effects due to the relative motion between rigid-bodies and water.

1) Quadratic Damping: The effects of the hydrodynamic damping $h(\nu)$ over the vehicle, due not only to the translational but also to rotational motions, can be described in the body-fixed reference frame by:

$$h(\nu) = \frac{1}{2} \rho [C_{D_x}v_x|v_x|, C_{D_y}v_y|v_y|, C_{D_z}v_z|v_z|, C_{D_\gamma}\omega_z|\omega_z|]$$

where the parameters $C_{D_x}, C_{D_y}, C_{D_z}$ and $C_{D_\gamma}$ depend on the geometry of the vehicle and should be obtained experimentally in a wind tunnel [Pinto, 1996], or on-line estimated with adaptive algorithms in a water tank [Smallwood and Whitcomb, 2003].

2) Added Inertia: Considering that an underwater vehicle typically operates at low speeds, the added inertia matrix, $M_A \in \mathbb{R}^{4 \times 4}$, could be assumed as diagonally dominant and described as follows:

$$M_A = \text{diag} \{C_{M_x}, \rho \nabla, C_{M_y}, \rho \nabla, C_{M_z}, \rho \nabla\}$$

As with the computation of the hydrodynamic damping, the coefficients $C_{M_x}, C_{M_y}, C_{M_z}$ and $C_{M_\gamma}$ should be determined experimentally. The matrix $M_A$ must be combined with the rigid-body inertia matrix in order to obtain the matrix $M$ of Eq. (11).

IV. DYNAMIC POSITIONING SYSTEM

The dynamic positioning of underwater robotic vehicles is essentially a multivariable control problem. Nevertheless, as demonstrated by Slotine [1983], the variable structure control methodology allows different control strategies to be separately designed for each degree of freedom (DOF). Over the past decades, decentralized control strategies have been successfully applied to the dynamic positioning of underwater vehicles [Sebastian and Sotelo, 2007; Chatchanayuenyong and Parmichkun, 2007; Smallwood and Whitcomb, 2004; Kiriazov et al., 1997; Da Cunha et al., 1995; Yoeger and Slotine, 1985].

Considering that the control law for each degree of freedom can be easily designed with respect to the inertial reference frame, Eq. (11) should be rewritten in this coordinate system.

Remembering that

$$\dot{x} = J(x)\nu$$

where $J(x)$ is the Jacobian transformation matrix, it can be directly implied that

$$\nu = J^{-1}(x)\dot{x}$$

and

$$\dot{\nu} = J^{-1}\dot{x} + J^{-1}\ddot{x}$$

Therefore, the equations of motion of an underwater vehicle, with respect to the inertial reference frame, becomes

$$\ddot{M}\ddot{x} + \ddot{\kappa} + \ddot{h} + \ddot{d} = \ddot{\tau}$$

where $M = J^{-T}MJ^{-1}$, $\kappa = J^{-T}\kappa$, $\dot{h} = J^{-T}\dot{h}$, $\ddot{d} = J^{-T}\ddot{d}$ and $\ddot{\tau} = J^{-T}\tau$.

In order to develop the control law with a decentralized approach, Eq. (18) can be rewritten as follows:

$$\ddot{x}_i = \ddot{m}_i^{-1}(\ddot{\tau}_i - \ddot{k}_i - \ddot{h}_i - \ddot{d}_i); \quad i = 1, 2, 3, 4.$$  

(19)

where $x_i$, $\ddot{\tau}_i$, $\ddot{k}_i$, $\ddot{h}_i$, and $\ddot{d}_i$ are the components of $x = [x, y, z, \gamma]$, $\ddot{\tau}$, $\ddot{k}$, $\ddot{h}$ and $\ddot{d}$, respectively. Concerning $\ddot{m}_i$, it represents the main diagonal terms of $J^{-T}MJ^{-1}$. The off-diagonal terms of $J^{-T}MJ^{-1}$ are incorporated in the vector $\ddot{d}$.

For notational simplicity the index $i$ will be suppressed in Eq. (19) and, in this way, the equation of motion for each degree of freedom (DOF) becomes:

$$\ddot{x} = \ddot{m}^{-1}(\ddot{\tau} - \ddot{k} - \ddot{h} - \ddot{d})$$

(20)

On this basis, according to Eq. (5) and considering $\varepsilon = \dot{x} + \lambda_t\ddot{x}$, the following could be proposed for each DOF:

$$\ddot{x} = \ddot{k} + \ddot{h} + \ddot{d} + \ddot{m}(\ddot{x}_d - \lambda_d\ddot{x}) - \kappa\varepsilon$$

(21)

But since disturbances are unknown, in this work the value of $d$ will be estimated using a three-layer artificial neural network.
\[ \dot{d} = W^T \sigma(V^T \theta) \]  

(22)

where \( W \) and \( V \) are the weight matrices in, respectively, hidden and output layers. The vector \( \theta \) represents the network input. Standard sigmoid functions are used in hidden layer and the output layer has a linear activation function. The weights could be online updated using the conventional backpropagation scheme.

In this way, the resulting control law could be written as follows:

\[ \tau = \bar{k} + \bar{h} + \hat{d} + m \ddot{x}_d - \lambda \dot{x} - \kappa \varepsilon \]  

(23)

V. SIMULATION RESULTS

In order to evaluate the control system performance, three different numerical simulations were performed for the depth regulation of an underwater robotic vehicle. The obtained results were presented from Fig. 2 to Fig. 5.

In the first case, it was considered that the initial state coincides with the initial desired state, \( \tilde{z}(0) = [\tilde{z}(0), \dot{\tilde{z}}(0)]^T = 0 \). Figure 2 gives the corresponding results for the tracking of \( z_d = 0.5[1 - \cos(0.1\pi t)] \). Regarding controller and model parameters, the following values were chosen: \( \bar{m} = 50 \text{ kg}, \bar{h} = 250 \text{ Ns}^2/\text{m}^2, \kappa = 3.5 \) and \( \lambda = 3.5 \). The capability of the proposed scheme to deal with uncertainties was appraised by choosing the parameters for the controller based on the assumption that exact values are not known but with a maximal uncertainty of \( \pm 10\% \) over adopted values for the model parameters.

![Figure 2: Tracking with \( z_d = 0.5[1 - \cos(0.1\pi t)] \) and \( \tilde{z}(0) = 0 \).](attachment:image2.png)

As observed in Fig. 2, the proposed control scheme allows the underwater robotic vehicle to track the desired trajectory with a small tracking error. Through the comparative analysis showed in Fig. 2(a), the improved performance of the proposed controller over the uncompensated counterpart can be easily ascertained.

In the second simulation the initial state and initial desired state are not equal, \( \tilde{z}(0) = [0.1, 0.0] \). The controller and model parameters and the desired trajectory were defined as before. Figures 3 and 4 show the corresponding results, respectively, with and without the neural network compensation scheme.

![Figure 3: Tracking with \( \tilde{z}(0) = [0.1, 0.0] \) and ANN compensation.](attachment:image3.png)
VI. CONCLUDING REMARKS

In this paper, a nonlinear controller was proposed to deal with the dynamic positioning system for underwater robotic vehicles. To enhance the tracking performance the adopted strategy embedded an artificial neural-network based scheme within the conventional nonlinear controller for uncertainty/disturbance compensation. The stability and convergence properties of the closed-loop systems were analytically proven using Lyapunov stability theory. Through numerical simulations, the improved performance over the uncompensated control strategy was demonstrated.

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REFERENCES

Antonelli, G., 2007. “On the use of adaptive/integral actions for six-degrees-of-freedom control of autonomous underwater vehicles”. IEEE Journal of Oceanic Engineering, Vol. 32, No. 2, pp. 300–312.
Bessa, W.M., Kreuzer, E., Lange, J., Pick, M.A. and Solowjow, E., 2017. “Design and adaptive depth control of a micro diving agent”. *IEEE Robotics and Automation Letters*, Vol. 2, No. 4, pp. 1871–1877. doi:10.1109/LRA.2017.2714142.

Bessa, W.M., Brinkmann, G., Duecker, D.A., Kreuzer, E. and Solowjow, E., 2018. “A biologically inspired framework for the intelligent control of mechatronic systems and its application to a micro diving agent”. *Mathematical Problems in Engineering*, Vol. 2018, pp. 1–16. doi:10.1155/2018/9648126.

Bessa, W.M., 2005. “Controle por Modos Deslizantes de Sistemas Dinâmicos com Zona Morta Aplicado ao Posicionamento de ROVs”. Tese (D.Sc.), COPPE/UFPRJ, Rio de Janeiro, Brasil.

Bessa, W.M. and Barrêto, R.S.S., 2010. “Adaptive fuzzy sliding mode control of uncertain nonlinear systems”. *Controle & Automação*, Vol. 21, No. 2, pp. 117–126.

Bessa, W.M., De Paula, A.S. and Savi, M.A., 2009. “Chaos control using an adaptive fuzzy sliding mode controller with application to a nonlinear pendulum”. *Chaos, Solitons & Fractals*, Vol. 42, No. 2, pp. 784–791.

Bessa, W.M., De Paula, A.S. and Savi, M.A., 2012. “Sliding mode control with adaptive fuzzy dead-zone compensation for uncertain chaotic systems”. *Nonlinear Dynamics*, Vol. 70, No. 3, pp. 1989–2001. doi:10.1007/s11071-012-0591-z.

Bessa, W.M., De Paula, A.S. and Savi, M.A., 2014. “Adaptive fuzzy sliding mode control of a chaotic pendulum with noisy signals”. *ZAMM – Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 94, No. 3, pp. 256–263. doi:10.1002/zamm.201200214.

Bessa, W.M., Dutra, M.S. and Kreuzer, E., 2005. “Thruster dynamics compensation for the positioning of underwater robotic vehicles through a fuzzy sliding mode based approach”. In *COBEM 2005 – Proceedings of the 18th International Congress of Mechanical Engineering*. Ouro Preto, Brasil.

Bessa, W.M., Dutra, M.S. and Kreuzer, E., 2008. “Depth control of remotely operated underwater vehicles using an adaptive fuzzy sliding mode controller”. *Robotics and Autonomous Systems*, Vol. 56, No. 8, pp. 670–677.

Bessa, W.M., Dutra, M.S. and Kreuzer, E., 2010. “An adaptive fuzzy sliding mode controller for remotely operated underwater vehicles”. *Robotics and Autonomous Systems*, Vol. 58, No. 1, pp. 16–26.

Bessa, W.M., Otto, S., Kreuzer, E. and Seifried, R., 2019. “An adaptive fuzzy sliding mode controller for uncertain underactuated mechanical systems”. *Journal of Vibration and Control*, Vol. 25, No. 9, pp. 1521–1535. doi:10.1177/1077546319873903.

Bevilacqua, L., Kleczwa, W. and Kreuzer, E., 1991. “On the mathematical modeling of ROVs”. In I. Troch, K. Desoyer and P. Kopacek, eds., *Proceedings of the Symposium on Robot Control*. Vienna, Austria, pp. 595–598.

Chatchanayuenyong, T. and Parnichkun, M., 2007. “Neural network based-time optimal sliding mode control for an autonomous underwater robot”. *Mechatronica*, Vol. 16, pp. 471–478.

Da Cunha, J.P.V.S., Costa, R.R. and Hsu, L., 1995. “Design of a high performance variable structure control of ROVs”. *IEEE Journal of Oceanic Engineering*, Vol. 20, No. 1, pp. 42–55.

Dos Santos, J.D.B. and Bessa, W.M., 2019. “Intelligent control for accurate position tracking of electrohydraulic actuators”. *Electronics Letters*, Vol. 55, No. 2, pp. 78–80. doi:10.1049/el.2018.7218.

Goheen, K.R. and Jeffreys, E.R., 1990. “Multivariable self-tuning autopilots for autonomous and remotely operated underwater vehicles”. *IEEE Journal of Oceanic Engineering*, Vol. 15, No. 3, pp. 144–151.

Guo, J., Chiu, F.C. and Huang, C.C., 2003. “Design of a sliding mode fuzzy controller for the guidance and control of an autonomous underwater vehicle”. *Ocean Engineering*, Vol. 30, pp. 2137–2155.

Hoang, N.Q. and Kreuzer, E., 2007. “Adaptive PD-controller for positioning of a remotely operated underwater vehicle”. *Control Engineering Practice*, Vol. 15, pp. 411–419.

Hsu, L., Costa, R.R., Lizarralde, F. and Da Cunha, J.P.V.S., 2000. “Dynamic positioning of remotely operated underwater vehicles”. *IEEE Robotics and Automation Magazine*, Vol. 7, No. 3, pp. 21–31.

Kiriazov, P., Kreuzer, E. and Pinto, F.C., 1997. “Robust feedback stabilization of underwater robotic vehicles”. *Robotics and Autonomous Systems*, Vol. 21, pp. 415–423.

Kleczcza, W., Kreuzer, E. and Pinto, F.C., 1992. “Analytic-numeric study of a submerged double pendulum”. In M.P. Paidousis and M.K. Au-Yang, eds., *International Symposium on Flow-Induced Vibration and Noise*. Anaheim, USA.

Lima, G.S., Bessa, W.M. and Trimpe, S., 2018. “Depth control of underwater robots using sliding modes and gaussian process regression”. In *LARS 2018 – Proceedings of the Latin American Robotic Symposium*. João Pessoa, Brazil. doi:10.1109/LARS/SBR/WRE.2018.00012.

Lima, G.S., Porto, D.R., de Oliveira, A.J. and Bessa, W.M., 2021. “Intelligent control of a single-link flexible manipulator using sliding modes and artificial neural networks”. *Electronics Letters*, Vol. 57, No. 3, pp. 869–872. doi:10.1049/el.2021.12300.

Lima, G.S., Trimpe, S. and Bessa, W.M., 2020. “Sliding mode control with gaussian process regression for underwater robots”. *Journal of Intelligent & Robotic Systems*, Vol. 99, No. 3, pp. 487–498. doi:10.1007/s10846-019-01128-5.

Newman, J.N., 1986. *Marine Hydrodynamics*. MIT Press, Massachusetts, 5th edition.

Pinto, F.C., 1996. *Theoretische und experimentelle Untersuchung zur Sensorik und Regelung von Unterwasserfahrzeugen*. VDI Verlag, Düsseldorf.

Sebastián, E. and Sotelo, M.A., 2007. “Adaptive fuzzy sliding mode controller for the kinematic variables of an underwater vehicle”. *Journal of Intelligent and Robotic Systems*, Vol. 49, pp. 189–215.

Slotine, J.J.E., 1983. *Tracking Control of Nonlinear Systems Using Sliding Surfaces*. Tese (Ph.D.), Massachusetts Institute of Technology, Cambridge.

Smallwood, D.A. and Whitcomb, L.L., 2003. “Adaptive identification of dynamically positioned underwater robotic vehicles”. *IEEE Transactions on Control Systems Technology*, Vol. 11, No. 4, pp. 505–515.

Smallwood, D.A. and Whitcomb, L.L., 2004. “Model-based dynamic positioning of underwater robotic vehicles: Theory and experiment”. *IEEE Journal of Oceanic Engineering*, Vol. 29, No. 1, pp. 169–186.
Tanaka, M.C., de Macedo Fernandes, J.M. and Bessa, W.M., 2013. “Feedback linearization with fuzzy compensation for uncertain nonlinear systems”. *International Journal of Computers, Communications & Control*, Vol. 8, No. 5, pp. 736–743.

Yoeger, D.R. and Slotine, J.J.E., 1985. “Robust trajectory control of underwater vehicles”. *IEEE Journal of Oceanic Engineering*, Vol. 10, No. 4, pp. 462–470.

Yuh, J., 1994. “Learning control for underwater robotic vehicles”. *IEEE Control Systems Magazine*, Vol. 14, No. 2, pp. 39–46.

Zanoli, S.M. and Conte, G., 2003. “Remotely operated vehicle depth control”. *Control Engineering Practice*, Vol. 11, pp. 453–459.