On the effect of the degeneracy between $w_0$ and $w_a$

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Abstract

We analytically derived two models which reduce to the familiar Chevallier-Polarski-Linder parametrization with $w_a$ proportional to $1 + w_0$. The two models have only one free parameter $w_0$, which helps us eliminate the degeneracy among $w_a$, $w_0$ and $\Omega_{m0}$, and makes the models automatically consistent with the $\Lambda$CDM model. With these degeneracy relations, we test their effects on the constraints of cosmological parameters. We find that: (1) The degeneracy between $w_0$ and $w_a$ for the two models is consistent with that found for the original Chevallier-Polarski-Linder model; (2) The degeneracies have little effect on $\Omega_{m0}$; (3) The $1\sigma$ error of $\Omega_{k0}$ was reduced about 25% with the degeneracy relations; (4) The $1\sigma$ error of $w_0$ was reduced about one half with the degeneracy relations.
I. INTRODUCTION

To explain the cosmic acceleration found by the observations of type Ia supernovae (SNe Ia) in 1998 \[1, 2\], we usually introduce an exotic energy component with negative pressure to the right hand side of Einstein equation. This exotic energy component which consists about 72\% of the total energy density in the universe is dubbed as dark energy. Although the cosmological constant is the simplest candidate for dark energy and is consistent with current observations, other possibilities are also explored due to the many orders of magnitude discrepancy between the theoretical estimation and astronomical observations for the cosmological constant. Currently we still have no idea about the nature of dark energy. In particular, the question whether dark energy is the cosmological constant remains unanswered. For a recent review of dark energy, please see Ref. \[3\].

One way of studying the nature of dark energy is through the observational data. There are many model-independent studies on the nature of dark energy by using the observational data \[4–31\]. In particular, one usually parameterizes the energy density or the equation of state parameter \( w(z) \) of dark energy. The most used parametrization is the Chevallier-Polarski-Linder (CPL) parametrization \[28, 29\] with \( w(a) = w_0 + w_a(1 - a) \). Because of the degeneracies among the parameters \( \Omega_{m0}, w_0 \) and \( w_a \) in the model, complementary cosmological observations are needed to break the degeneracies. The measurement on the cosmic microwave background anisotropy, the baryon acoustic oscillation (BAO) measurement and the SNe Ia observations provide complementary data.

On the other hand, a minimally coupled scalar field \( \phi \) was often invoked to model the quintessence \[32–34\], and the phantom \[35\]. For a scalar field with a nearly flat potential, there exist approximate relations between the equation of state parameter \( w = p/\rho \) and the energy density parameter \( \Omega_\phi \) \[36–39\]. By using the generic relations, we can break the degeneracy between \( w(z) \) and \( \Omega_\phi(z) \). Furthermore, \( w(z) \) can be approximated by the CPL parametrization with \( w_a \) expressed as a function of \( w_0 \) and \( \Omega_{m0} \), so the two-parameter parametrization was reduced to one-parameter parametrization \[40\]. The CPL parametrization with analytical relations among the model parameters helps tighten the constraints on the model parameters.

In this paper, we derive two particular CPL models with \( w_a \) proportional to \( 1 + w_0 \), and study the effects of the degeneracy relations between \( w_a \) and \( w_0 \) by using the following
data: the three year Supernova Legacy Survey (SNLS3) sample of 472 SNe Ia data with systematic errors [41]; the BAO measurements from the 6dFGS [42], the distribution of galaxies [43] in the Sloan Digital Sky Survey (SDSS) and the WiggleZ dark energy survey [44]; the seven-year Wilkinson Microwave Anisotropy Probe (WMAP7) data [45]; and the Hubble parameter \( H(z) \) data [46, 47].

II. CPL PARAMETRIZATION WITH DEGENERATED \( w_0 \) AND \( w_a \)

For scalar fields satisfying the slow-roll conditions,

\[
\left( \frac{1}{V} \frac{dV}{d\phi} \right)^2 \ll 1, \quad \frac{1}{V} \frac{d^2 V}{d\phi^2} \ll 1, \quad (1)
\]

a general relationship between \( w \) and the energy density \( \Omega_\phi = 8\pi G \rho_\phi / 3H^2 \) was found [36, 37],

\[
1 + w = (1 + w_0) \left[ \frac{1}{\sqrt{\Omega_\phi}} - \left( \frac{1}{\Omega_\phi} - 1 \right) \tanh^{-1}(\sqrt{\Omega_\phi}) \right]^2 \times \left[ \frac{1}{\sqrt{\Omega_{\phi0}} - (\Omega_{\phi0}^{-1} - 1) \tanh^{-1} \sqrt{\Omega_{\phi0}}} \right]^{-2}. \quad (2)
\]

Note that the above result does not depend on the specific form of the potential \( V(\phi) \) and holds for thawing models [48] with the potentials satisfying the slow roll conditions [1].

Following Refs. [36, 37, 39], we first approximate the energy density \( \Omega_\phi \) as

\[
\Omega_\phi = \left[ 1 + (\Omega_{\phi0}^{-1} - 1)a^{-3} \right]^{-1}, \quad (3)
\]

and we get

\[
w(a) = -1 + (1 + w_0) \left[ \frac{1}{\sqrt{\Omega_{\phi0}}} - (\Omega_{\phi0}^{-1} - 1) \tanh^{-1} \sqrt{\Omega_{\phi0}} \right]^{-2} \times \left[ \sqrt{1 + (\Omega_{\phi0}^{-1} - 1)a^{-3} - (\Omega_{\phi0}^{-1} - 1)a^{-3} \tanh^{-1}[1 + (\Omega_{\phi0}^{-1} - 1)a^{-3}]^{-1/2}} \right]^2. \quad (4)
\]

In other words, we now consider a scalar field with \( w(a) \) given by equation (4). If we Taylor expand \( \Omega_\phi(a) \) and \( w(a) \) around \( a = 1 \), we get

\[
\Omega_\phi \approx \Omega_{\phi0}[1 - 3(1 - \Omega_{\phi0})(1 - a)], \quad (5)
\]

and

\[
w = w_0 + 6(1 + w_0) \frac{\Omega_{\phi0}^{-1/2} - \sqrt{\Omega_{\phi0} - (\Omega_{\phi0}^{-1} - 1) \tanh^{-1}(\sqrt{\Omega_{\phi0}})}}{\Omega_{\phi0}^{-1/2} - (\Omega_{\phi0}^{-1} - 1) \tanh^{-1}(\sqrt{\Omega_{\phi0}})}(1 - a). \quad (6)
\]
Therefore, we derive the CPL parametrization with \( w_a \) determined by \( w_0 \) and \( \Omega_{\phi 0} \) starting from equation (2). We call this model as SSLCPL model. In particular, we get

\[
w_a = 6(1 + w_0) \frac{(\Omega_{\phi 0}^{-1} - 1)[\sqrt{\Omega_{\phi 0}} - \tanh^{-1}(\sqrt{\Omega_{\phi 0}})]}{\Omega_{\phi 0}^{-1/2} - (\Omega_{\phi 0}^{-1} - 1) \tanh^{-1}(\sqrt{\Omega_{\phi 0}})}.
\] (7)

When \( \Omega_{\phi 0} = 0.7 \), we get \( w_a = -1.42(1 + w_0) \) which is consistent with the numerical result \( w \approx w_0 - 1.5(1 + w_0)(1 - a) \) obtained in [36, 37]. For the SSLCPL model, we only have two model parameters \( \Omega_{m0} \) and \( w_0 \) for the spatially flat case and three model parameters \( \Omega_{m0}, \Omega_{k0} \) and \( w_0 \) for the spatially curved case.

Next we approximate \( \Omega_\phi \) by the model with constant equation of state \( w = w_0 \),

\[
\Omega_\phi = \frac{\Omega_{\phi 0} a^{-3w}}{1 - \Omega_{\phi 0} + \Omega_{\phi 0} a^{-3w}},
\] (8)

the Taylor expansion around \( a = 1 \) gives

\[
\Omega_\phi = \{1 + (\Omega_{\phi 0}^{-1} - 1)[1 - 3w(1 - a)]\}^{-1}.
\] (9)

Substituting the result (9) into equation (2), we then obtain,

\[
w(a) = w_0 - 6w_0(1 + w_0) \frac{(\Omega_{\phi 0}^{-1} - 1)[\sqrt{\Omega_{\phi 0}} - \tanh^{-1}(\sqrt{\Omega_{\phi 0}})](1 - a)}{\Omega_{\phi 0}^{-1/2} - (\Omega_{\phi 0}^{-1} - 1) \tanh^{-1}(\sqrt{\Omega_{\phi 0}})},
\] (10)

so again we get CPL parametrization with

\[
w_a = -6w_0(1 + w_0) \frac{(\Omega_{\phi 0}^{-1} - 1)[\sqrt{\Omega_{\phi 0}} - \tanh^{-1}(\sqrt{\Omega_{\phi 0}})]}{\Omega_{\phi 0}^{-1/2} - (\Omega_{\phi 0}^{-1} - 1) \tanh^{-1}(\sqrt{\Omega_{\phi 0}})}.
\] (11)

We call this model as SSWCPL model. For the SSWCPL model, we only have two model parameters \( \Omega_{m0} \) and \( w_0 \) for the spatially flat case and three model parameters \( \Omega_{m0}, \Omega_{k0} \) and \( w_0 \) for the spatially curved case. For both the SSLCPL and SSWCPL models, we find that \( w_a \propto 1 + w_0 \), so the models are automatically consistent with \( \Lambda \)CDM model with \( w_0 = -1 \) and \( w_a = 0 \).

III. OBSERVATIONAL CONSTRAINTS

We apply the SNe Ia, BAO, WMAP7 and the Hubble parameter \( H(z) \) data to test the effects of the degeneracy relations (7) and (11) on the constraints of \( \Omega_{m0}, \Omega_{k0} \) and \( w_0 \). The SNLS3 SNe Ia data consists of 123 low-redshift SNe Ia data with \( z \lesssim 0.1 \) mainly from
Calan/Tololo, CfAI, CfAII, CfAIII and CSP, 242 SNe Ia over the redshift range $0.08 < z < 1.06$ observed from the SNLS [41], 93 intermediate-redshift SNe Ia data with $0.06 \lesssim z \lesssim 0.4$ observed during the first season of SDSS-II supernova survey [49], and 14 high-redshift SNe Ia data with $z \gtrsim 0.8$ from Hubble Space Telescope [50]. For the fitting to the SNLS3 data, we need to add two more nuisance parameters $\alpha$ and $\beta$.

The BAO data [44] consists of the measurement at the redshift $z = 0.106$ from the 6dFGS [42], the measurements of the distribution of galaxies at two redshifts $z = 0.2$ and $z = 0.35$ [43] in the SDSS and the measurements of the acoustic parameter at three redshifts $z = 0.44$, $z = 0.6$ and $z = 0.73$ from WiggleZ dark energy survey [44]. For the BAO data, we need to add two more nuisance parameters $\Omega_b h^2$ and $\Omega_m h^2$.

For the WMAP7 data, we use the measurements of the shift parameter and the acoustic index at the recombination redshift [45], and we need to add two more nuisance parameters $\Omega_b h^2$ and $\Omega_m h^2$.

The Hubble parameter $H(z)$ data consists of the measurements of $H(z)$ at 11 different redshifts obtained from the differential ages of passively evolving galaxies [47, 51], and three data points at redshifts $z = 0.24$, $z = 0.34$ and $z = 0.43$, determined by taking the BAO scale as a standard ruler in the radial direction [46]. The $H(z)$ data spans out to the redshift regions $z = 1.75$.

After obtaining the constraints on the model parameters, we reconstruct $w(z)$ and apply the Om diagnostic [52] to detect the deviation from the ΛCDM model. $Om(z)$ is defined as

$$\text{Om}(z) = \frac{E^2(z) - 1}{(1 + z)^3 - 1},$$

where the dimensionless Hubble parameter $E(z) = H(z)/H(z = 0)$.

We first consider the effects of the degeneracy relations (7) and (11) on $\Omega_{m0}$ and $w_0$, so we take the spatially flat case $\Omega_{k0} = 0$. Fitting the SSLCPL model to the observational data, we get the marginalized $1\sigma$ constraints $\Omega_{m0} = 0.275^{+0.015}_{-0.011}$ and $w_0 = -1.08^{+0.11}_{-0.09}$ with $\chi^2 = 432.6$. By using the degeneracy relation (7) and the correlation between $\Omega_{m0}$ and $w_0$, we derived the marginalized $1\sigma$ constraint $w_a = 0.11^{+0.12}_{-0.14}$. We show the marginalized $1\sigma$ and $2\sigma$ contours of $\Omega_{m0}$ and $w_0$, and $w_0$ and $w_a$ in Figure [1]. By using the correlation between $\Omega_{m0}$ and $w_0$, we reconstruct the evolutions of $w(z)$ and $Om(z)$ in Figure [1]. Fitting the SSWCPL model to the observational data, we get the marginalized $1\sigma$ constraints $\Omega_{m0} = 0.276^{+0.014}_{-0.013}$ and $w_0 = -1.09 \pm 0.10$ with $\chi^2 = 432.6$. By using the degeneracy relation (11) and the
correlation between $\Omega_{m0}$ and $w_0$, we derived the marginalized 1σ constraint $w_a = 0.12^{+0.16}_{-0.15}$. We show the marginalized 1σ and 2σ contours of $\Omega_{m0}$ and $w_0$, and $w_0$ and $w_a$ in Figure 2. By using the correlation between $\Omega_{m0}$ and $w_0$, we reconstruct the evolutions of $w(z)$ and $Om(z)$ in Figure 2. Fitting the CPL model to the observational data, we get the marginalized 1σ constraints $\Omega_{m0} = 0.278^{+0.018}_{-0.011}$, $w_0 = -1.0^{+0.17}_{-0.13}$ and $w_a = -0.33^{+0.53}_{-1.03}$ with $\chi^2 = 432.4$. We show the marginalized 1σ and 2σ contours of $\Omega_{m0}$ and $w_0$, and $w_0$ and $w_a$ in Figure 3. By using the correlations among the parameters, we reconstruct the evolutions of $w(z)$ and $Om(z)$ in Figure 3. These results are summarized in Table I. Comparing the results from three models, we find that all three models fit the observational data well because they give almost the same value of $\chi^2$. In terms of the Akaike information criterion (AIC) [53] or Bayesian information criterion (BIC) [54], the SSLCPL and SSWCPL model fit the observational data a little better than the CPL model does. All three models are consistent with $\Lambda$CDM model at the 1σ level, as shown explicitly by the $w(z)$ and $Om(z)$ plots in Figs. 1-3. From the contour plots for $w_0$ and $w_a$ in Figs. 1-3, we see that the degeneracy relations (7) for the SSLCPL model and (11) for the SSWCPL model are consistent with that for the CPL model and the variation of $w(z)$ is constrained much tighter. The effects of the degeneracy relations (7) and (11) on $\Omega_{m0}$ are minimal. The 1σ errors of $w_0$ for SSLCPL
FIG. 2. The marginalized 1σ and 2σ constraints on the flat SSWCPL model. The upper panels are for the 1σ and 2σ contour plots of $\Omega_m - w_0$ and $w_0 - w_a$. The lower panels show the reconstructions of $w(z)$ and $\Omega_m(z)$ by using the constraints on $\Omega_m$ and $w_0$.

FIG. 3. The marginalized 1σ and 2σ constraints on the flat CPL model. The upper panels are for the 1σ and 2σ contour plots of $\Omega_m - w_0$ and $w_0 - w_a$. The lower panels are the reconstructions of $w(z)$ and $\Omega_m(z)$ by using the constraints on $\Omega_m$, $w_0$ and $w_a$. 
and SSWCPL models are reduced around 30% with the degeneracy relations (7) and (11) compared with that in CPL model.

| Model          | $\Omega_m$   | $\Omega_k$ | $w_0$    | $w_a$    |
|----------------|--------------|------------|----------|----------|
| flat SSLCPL    | $0.275^{+0.015}_{-0.011}$ | $-1.08^{+0.11}_{-0.09}$ | $0.11^{+0.12}_{-0.14}$ |
| flat SSWCPL    | $0.276^{+0.014}_{-0.013}$ | $-1.09 \pm 0.10$ | $0.12^{+0.16}_{-0.15}$ |
| flat CPL       | $0.278^{+0.018}_{-0.011}$ | $-1.0^{+0.17}_{-0.13}$ | $-0.33^{+0.53}_{-1.03}$ |
| curved SSLCPL  | $0.277^{+0.014}_{-0.013}$ | $0.0002^{+0.0046}_{-0.0044}$ | $-1.05^{+0.11}_{-0.09}$ | $0.11^{+0.12}_{-0.15}$ |
| curved SSWCPL  | $0.276^{+0.015}_{-0.012}$ | $-0.00004^{+0.0049}_{-0.0040}$ | $-1.07^{+0.09}_{-0.11}$ | $0.10^{+0.18}_{-0.13}$ |
| curved CPL     | $0.278^{+0.018}_{-0.011}$ | $-0.004^{+0.006}_{-0.007}$ | $-0.97^{+0.27}_{-0.12}$ | $-0.57^{+0.65}_{-1.86}$ |

Now we consider the impact of $\Omega_{k0}$. Fitting the curved SSLCPL model to the observational data, we get the marginalized 1σ constraints $\Omega_{m0} = 0.277^{+0.014}_{-0.013}$, $\Omega_{k0} = 0.0002^{+0.0046}_{-0.0044}$ and $w_0 = -1.08^{+0.11}_{-0.09}$ with $\chi^2 = 432.6$. By using the degeneracy relation (7) and the correlations among $\Omega_{m0}$, $\Omega_{k0}$ and $w_0$, we derived the marginalized 1σ constraint $w_a = 0.11^{+0.12}_{-0.15}$. It can be seen that the curvature $\Omega_{k0}$ has little effect on $w_0$. We show the marginalized 1σ and 2σ contours of $\Omega_{m0}$ and $\Omega_{k0}$, $\Omega_{m0}$ and $w_0$, and $w_0$ and $w_a$ in Figure 4. By using the correlations among $\Omega_{m0}$, $\Omega_{k0}$ and $w_0$, we reconstruct the evolution of $w(z)$ in Figure 4.

Fitting the curved SSWCPL model to the observational data, we get the marginalized 1σ constraints $\Omega_{m0} = 0.276^{+0.015}_{-0.012}$, $\Omega_{k0} = -0.00004^{+0.0049}_{-0.0040}$ and $w_0 = -1.07^{+0.09}_{-0.11}$ with $\chi^2 = 432.7$. By using the degeneracy relation (11) and the correlations among $\Omega_{m0}$, $\Omega_{k0}$ and $w_0$, we derived the marginalized 1σ constraint $w_a = 0.10^{+0.18}_{-0.13}$. Again the curvature $\Omega_{k0}$ has little effect on $w_0$. We show the marginalized 1σ and 2σ contours of $\Omega_{m0}$ and $\Omega_{k0}$, $\Omega_{m0}$ and $w_0$, and $w_0$ and $w_a$ in Figure 5. By using the correlations among $\Omega_{m0}$, $\Omega_{k0}$ and $w_0$, we reconstruct the evolution of $w(z)$ in Figure 5.

Fitting the curved CPL model to the observational data, we get the marginalized 1σ constraints $\Omega_{m0} = 0.278^{+0.018}_{-0.011}$, $\Omega_{k0} = -0.004^{+0.006}_{-0.007}$, $w_0 = -0.97^{+0.27}_{-0.12}$ and $w_a = -0.57^{+0.65}_{-1.86}$ with $\chi^2 = 432.1$. We show the marginalized 1σ and 2σ contours of $\Omega_{m0}$ and $\Omega_{k0}$, $\Omega_{m0}$ and $w_0$, and $w_0$ and $w_a$ in Figure 6. By using the correlation between $w_0$ and $w_a$, we reconstruct the evolution of $w(z)$ in Figure 6. The addition of $\Omega_{k0}$ has little impact on $\Omega_{m0}$ for all three models, but it increases the 1σ error on $w_0$ for CPL model. On the other hand, the degeneracy relations (7) and (11) help reduce the 1σ error on $\Omega_{k0}$. 
FIG. 4. The marginalized 1σ and 2σ constraints on the curved SSLCPL model. The upper and the lower left panels are for the 1σ and 2σ contour plots of $\Omega_m - \Omega_k$, $\Omega_m - w_0$ and $w_0 - w_a$. The lower right panel is the reconstruction of $w(z)$ from the curved SSLCPL model.

FIG. 5. The marginalized 1σ and 2σ constraints on the curved SSWCPL model. The upper and the lower left panels are for the 1σ and 2σ contour plots of $\Omega_m - \Omega_k$, $\Omega_m - w_0$ and $w_0 - w_a$. The lower right panel is the reconstruction of $w(z)$ from the curved SSWCPL model.
FIG. 6. The marginalized 1σ and 2σ constraints on the curved CPL model. The upper and the lower left panels are for the 1σ and 2σ contour plots of $\Omega_m - \Omega_k$, $\Omega_m - w_0$ and $w_0 - w_a$. The lower right panel is the reconstruction of $w(z)$ by using the constraints on $w_0$ and $w_a$ from the curved CPL model.

These results are summarized in Table I. At the 1σ level, ΛCDM model is consistent with the observational data as shown explicitly by the $w(z)$ plots in Figs. 4–6. From the contour plots for $w_0$ and $w_a$ in Figs. 4–6, we see that the degeneracy relations (7) for the SSLCPL model and (11) for the SSWCPL model are consistent with that found for the CPL model.

IV. CONCLUSIONS

From the relationship (2) for thawing models with a nearly flat potential, we derived the SSLCPL and SSWCPL models which break the degeneracies among $\Omega_{\phi0}$, $w_0$ and $w_a$. The two models reduce to the CPL model with only one free parameter $w_0$. The proposed degeneracy relations for $w_0$ and $w_a$ are consistent with that found for the original CPL model, so the SSLCPL and SSWCPL models are self consistent. Both models give almost the same minimum $\chi^2$ as the original CPL does when fitting to the observational data. In terms of AIC or BIC, the models fit the observational data a litter better than the original CPL does. With the help of relations (7) and (11), the 1σ error bar of $w_0$ is reduced about 30% and 50% for the flat and curved cases respectively, and the 1σ error bar of $\Omega_k$ is reduced about 25%.
Since the parameters $\Omega_{m0}$ and $w_0$ have little change for the SSLCPL and SSWCPL models when we consider the curved case, this effect is unexpected and further understanding of it needs to be studied. Both SSLCPL and SSWCPL models have only one free parameter and they help tighten the constraint on $\Omega_{k0}$, so they can be applied to probe the curvature of the universe.

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