Simulation of gamma ray deep penetration through iron based on grid weight window iteration

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Abstract. To improve the efficiency of gamma ray deep penetration shielding calculation, this paper analyzed the influence of gamma ray energies, shield thickness, shielding materials and number of simulated particles on the transport calculation of gamma ray deep penetration, and proposed a Monte Carlo simulation method for gamma ray deep penetration shielding by iterative calculation based on the combination of grid weight window and material density gradually approaching true density. The simulation results show that, by using the iterative calculation method based on grid weight window in MCNP program, it can be realized that gradual transition from non deep penetration shielding simulation to deep penetration shielding simulation. The method can effectively solve the problem of gamma ray deep penetration calculation, and it can make the statistical error of the transmission coefficient of gamma ray penetrating the iron shield with a thickness of 60 times the mean free path of gamma ray less than 5%, which can meet the demand of gamma ray deep penetration simulation.

1. Introduction
Monte Carlo simulation method has been widely used to the calculation of radiation shielding transport in physical design and analysis of nuclear technology[1]. The convergence rate of Monte Carlo simulation of radiation shielding transport is always slow, especially in the calculation of rays shielding by a very thick shield, which is called deep penetration shielding[2-3]. The researchers at home and abroad have done many researches on the deep penetration shielding of radiation, and developed a lot of variance reduction techniques[4-5]. A series of variance reduction techniques are integrated in the large general Monte Carlo particle transport simulation program MCNP[6]. There are main four types of variance reduction techniques in MCNP: truncation methods, such as geometric, energy and time truncation; the general distribution control methods, such as geometric, energy, and time related splitting and Russian roulette, and weight window, etc; modified sampling methods, such as source biasing, forced collision, exponential transform, etc; partial deterministic methods, such as point detector and DXTRAN sphere, etc[7-9]. Based on the abundant variance reduction technologies in MCNP, the researchers have carried out many investigations of the variance reduction simulations of deep penetration[10-11]. These results show that it is necessary to use a variety of variance reduction techniques to deal with the particle deep penetration transport calculation[12-13]. Han Yi proposed a simulation method to solve the gamma ray deep penetration problem based on dividing the geometric cell and simultaneously changing the thickness and density of the shield cell[14]. In the works of literature[2], the thickness and density of the shield were both reduced to unrealistic values to obtain a suitable estimate for weight window when applying the variance reduction method of weight window. Due to the change of the thickness of
the cell, the geometric cell will be re-divided each time, resulting in the preproduction of the weight window cannot be directly provided to the next calculation, which is inconvenient. Based on the analysis of the laws of gamma ray deep penetration shielding, and using the feature that the weight window can be automatically generated and optimized by MCNP, we propose a variance reduction method which can effectively solve the gamma ray deep penetration transport calculation. In the proposed method, a virtual grid weight window is used, and it does not change the thickness of shield cell, and sets density of the shield gradually from low to the realistic density in the iteration process.

2. The laws of gamma ray deep penetration shielding

There are two main factors that can affect the Monte Carlo simulation process of radiation shielding, one is physical parameters, such as particle energy, thickness of shield and shielding material, the other one is simulation parameters, such as number of simulated particles.

2.1. Effect of gamma ray energies on deep penetration transport

The mean free path (mfp) can be used to describe the ability of gamma rays penetrating a shield. The intensity of the "narrow beam" gamma ray in material decreases exponentially with the increase of the material thickness, that is

\[ I = I_0 e^{-\mu T} \]  \hspace{1cm} (1)

Where, \( T \) is the thickness of shield, in unit of cm; \( \mu \) is the linear attenuation coefficient of photon, in unit of \( \text{cm}^{-1} \), and the relationship between attenuation coefficient \( \mu \) and mfp \( l = \mu \); \( I_0 \) and \( I \) is the intensity of rays before and after it penetrates through the shield. The mfps of gamma ray with energy 1MeV in aluminum, iron, and lead are 6.03 cm, 2.12 cm and 1.25 cm, respectively.

To analyze the influence of gamma ray energies on the deep penetration shielding calculation by using mfp of gamma ray, a physical model of gamma ray shielding by circular plate shield is established. The mono-energetic gamma rays with different energies inject perpendicularly to the circular iron plate. The radius of the iron plate is 4 m, the density of iron shield is 7.87g/cm\(^3\), the model is placed in the air. The simulation is done by MCNP 4C program. The variance reduction technique is not used in the simulation. The number of simulated particles is set to be 1×10\(^7\). The results of gamma ray transmission coefficient which is normalized by the number of injected particles and its statistical error are shown in figure 1.

![Figure 1](image_url)

**Figure 1.** Transmission coefficients and its statistical errors of gamma rays with different energies shielding by iron as a function of shield thickness.

From figure 1, we can see that when the number of simulated particles is 1×10\(^7\) and the thickness of iron shield plate is more than 15 times mfp of gamma ray, the statistical error of transmission coefficient is very large, and it shows a deep penetration phenomenon. The gamma ray transmission coefficient decreases exponentially with the increase of the ratio of the thickness of the shield to mfp. As the thickness of the iron shield increasing about 10 times mfp, the number of gamma rays that can
penetrate shield reduces to about $10^4$ of the original. The simulation results shown in figure 1 are the statistics of all gamma rays penetrating through the transmission surface of iron shield for a mono-energetic single beam gamma ray source, and there are scattering accumulation factors in the simulation. Therefore, the curves in figure 1 fail to coincide with each other completely, and show the characteristics of gamma ray energy correlation.

2.2. Influence of shielding material on deep penetration transport

By replacing iron with aluminum or lead, the obtained relationships between gamma ray transmission coefficient and its statistical error with the thickness of shielding materials for the energy of 2 MeV are shown in figure 2(a) and 2(b), respectively. From figure 2, it can be seen that, whether it is a lighter density aluminum shield, a medium density iron shield, or a heavy density lead shield, the variations of gamma ray transmission coefficient with the ratio of shield thickness to mfp of gamma ray are similar. When the shield thickness is more than about 15 times mfp of gamma ray, the statistical error of transmission coefficient increases rapidly with the increase of shield thickness, shows a deep penetration phenomenon.

![Figure 2](image-url)

**Figure 2.** Gamma ray transmission coefficients and its statistical errors for energy of 2 MeV and different materials as a function of shield thickness.

The statistical error of the Monte Carlo simulation method is mainly determined by the number of particle collisions. The number of collisions is the same when the gamma rays move through a same distance expressed by times of mfp. Therefore, as long as the thicknesses of different shielding materials are expressed by mfp of gamma ray, the variations of transmission coefficients and statistical errors obtained by Monte Carlo simulation with shield thickness are similar.

2.3. Influence of the number of simulated particles on the statistical error

The statistical error of the Monte Carlo simulation method can be expressed as $C = X_\alpha \sigma / \sqrt{N}$. Where, $C$ is the statistical error; $X_\alpha$ is the value in the normal distribution table when the confidence level is given; $N$ is the number of simulated particles; and $\sigma$ is the variance of the simulation results.

It can be seen that, increasing the number of simulated particles $N$ can reduce the statistical error, but it will obviously increase the time spent in the simulation. Figure 3 shows the relationships between the statistical errors of the transmission coefficients of 2 MeV gamma ray with the shield thickness when $N$ is $1 \times 10^3$, $5 \times 10^3$, $1 \times 10^4$, and $2 \times 10^8$, respectively. From figure 3, we can see that when the shield thickness is more than 15 times the mfp of gamma ray, the statistical error of the gamma ray transmission coefficient increases with the increase of shield thickness. The effect of increasing the number of simulated particles is not significant for reducing the statistical error. In fact, the number of particles that can be simulated is limited by the computing capability in Monte Carlo simulation. It is impossible to solve the deep penetration problem by only increasing the number of simulated particles to reduce the statistical error. According to the expression of statistical error, reducing the variance of the simulation results can also reduce the statistical error.
3. Variance reduction technique based on grid weight window iteration

3.1. Simulation methods and steps
The variance reduction technique is used to shorten the computational time required for obtaining the accurate computation result. The main factors that affect the computation efficiency of the MCNP program are count type and sampling method of random walk. For the calculation of radiation shielding, the expression of the results of the transmission coefficient is

\[
<F> = \frac{1}{N} \int \int dF \int dF \cdot G(r, \nu, \tau) F(r, \nu, \tau)
\]

Figure 3. Statistical errors of transmission coefficients of 2 MeV gamma rays as a function of shield thickness at the case of different numbers of simulated particles.

(2)

Where, \( N \) is the number of simulated particles; \( F \) is count function which is a \( \delta \) function, when particle reaches the count area, \( F=1 \), otherwise, \( F=0 \); \( G \) is the density function of particles.

If we simply sampled according to physical rules, many of the simulated particles could not penetrate the shield and could not be recorded. The computation efficiency will be very low. We must calculate huge number of particles to make the statistical errors of results small. In fact, during the sampling process, we can extract more events with \( F=1 \) and extract less events with \( F=0 \), but to keep the \( <F> \) of statistical results unchanged, we need to adjust the weight of simulated particles accordingly, which is the basic principle of variance reduction technology. The weight of the particle \( w \) refers to the contribution of the particle to the final statistical result, and \( w \) can be less than 1. The principle of the weight window is to set up a series of weight intervals in space, energy or time area, and then reduce the variance of the simulation results.

We propose a method of variance reduction based on grid weight window iteration. In the method, by using the function of automatic generation and automatic optimization of weight window in MCNP program, artificially setting the density of the shield in the simulation from a small value to realistic density gradually, and using the weight window obtained by the simulation of non deep penetration shielding to generate a better weight window which is suitable for the deep penetration simulation step by step, then the non deep penetration shielding simulation is transited gradually to the deep penetration shielding simulation.

3.2. Example results and discussions
To use the variance reduction proposed here, we take iron shield as an example. The iron shield cell is firstly divided into virtual grid by utilizing MESH card in MCNP program. The grid size is one or two times mfp of 1MeV gamma ray. In our simulation, the iron shield is divided into 48 virtual layers along the gamma ray incident direction. The thickness of each layer in the 12 layers nearest to the gamma ray source, in the middle 15 layers, and in the last 20 layers are 3.33 cm, 2.67 cm, and 2.5 cm, respectively. Each layer represents a virtual grid. The sketch of shielding model is shown in figure 4.

The number of simulated particles is \( 1 \times 10^7 \). The density of iron shield in the simulation is set to be 0.2, 0.5 to 1.0 times the realistic density of iron, while Monte Carlo simulation by MCNP is carried out in turn, and the grid weight window calculated at the lower density condition is used as the input
grid weight window for the following calculation, so that the iterative calculation can be carried out and the densities in the simulations are gradually approaching to the realistic density of iron.

![Diagram](image)

**Figure 4.** Sketch of shielding calculation model based on MESH grid division of cell.

The densities set in the simulation process, and the transmission coefficients and its statistical errors calculated under the corresponding conditions are shown in table 1. It can be seen that the statistical errors of the iterative calculation are all within 15%, and the statistical error of the last calculation is less than 5%, which meet the requirement of statistical error.

**Table 1.** The densities set in the calculation process and the transmission coefficients calculated under the corresponding conditions.

| Times of iron density | Defined density g/cm³ | Transmission coefficient $I_t$ | Error |
|-----------------------|-----------------------|-------------------------------|-------|
| 0.20                  | 1.57                  | $9.11 \times 10^{-05}$       | 0.0331|
| 0.35                  | 2.76                  | $2.02 \times 10^{-08}$       | 0.0576|
| 0.5                   | 3.94                  | $3.59 \times 10^{-12}$       | 0.0907|
| 0.65                  | 5.12                  | $6.02 \times 10^{-16}$       | 0.1351|
| 0.75                  | 5.91                  | $1.69 \times 10^{-18}$       | 0.0623|
| 0.88                  | 6.93                  | $6.30 \times 10^{-22}$       | 0.1572|
| 0.94                  | 7.40                  | $2.29 \times 10^{-23}$       | 0.0389|
| 1.0                   | 7.87                  | $6.09 \times 10^{-25}$       | 0.0472|

In the simulation process, there will be a weight window on each virtual grid, and the upper limit of the weight window is set to be 5 times the lower limit according to experience. The weight of the particle entering the virtual grid is $w_{in}$, when $w_{in}$ is higher than the upper limit of the weight window, the particle will be split into more particles, but the corresponding weight will be reduced; when $w_{in}$ is smaller than the lower limit of the weight window, the particle will be betted on the roulette, the number of particles will be reduced and the weight will be increased correspondingly; when $w_{in}$ is between the upper and lower bounds of the weight window, the particle will be transported continuously. In each calculation, the MCNP program automatically calculates the importance of each virtual grid based on the contributions of the particles in the virtual grid to the count results, and the lower limit of the weight window is the reciprocal of the particle importance. The weight window function is obtained by regarding the lower limit of the weight window as a discrete function of the virtual grid coordinates. The relationships between weight window function with the virtual grid coordinate in each iterative calculation are shown in figure 5. From figure 5, we can see that the weight window function decreases exponentially with the virtual grid coordinate. With the increase of the density of
the iron shield, the falling rate of the weight window function with the virtual grid coordinate is getting faster and faster.

Figure 5. The relationship between weight window function with the virtual grid coordinate in iteration.

Figure 6 is the gamma ray transmission coefficient of iron shield with a thickness of 60 times mfp of gamma ray. From figure 6(a), we can see that the gamma ray transmission coefficients with statistical errors less than 2% decreases exponentially with the thickness of iron shield. The law of exponential decay is consistent with that in previous literatures [2, 14]. Therefore, the results are believable, which shows that the method in this paper can solve the problem of gamma ray deep penetration through iron. Figure 6(b) is a fitting relation of the transmission coefficient of gamma ray with the thickness of the iron, the fitting expression is

$$\log(I_t) = 0.793 - 0.191T$$  

(3)

Where, $I_t$ is the transmission coefficient of gamma ray; $T$ is the thickness of the iron shield, in unit of cm. The fitting formula is suitable for the case that the thickness of the iron shield is not more than 130cm. It is shown that an iron shield with thickness of 5.24cm will decrease the intensity of the mono-energetic beam gamma ray with energy of 1MeV by 1/10. According formula (1), if $I/I_0 = 1/10$, then $\mu T = 2.23$, so the thickness needed to reduce the intensity of narrow beam gamma by 1/10 is 2.23 mfp. That is 4.88 cm because the mfp of 1 MeV gamma ray in iron. In fact, all the gamma rays penetrating through the transmission surface are recorded. It is not a narrow beam gamma ray in the simulation due to the scattering accumulation factors. The simulation results in figure 6(a) are basically consistent with the theoretical results. By the way, the method in this paper is also applicable to the deep penetration shielding calculation of isotropic gamma ray sources.

![Figure 6](image_url)

Figure 6. Transmission coefficient and its statistical error as a function of thickness of iron.

4. Conclusions
In this paper, the influence of the energies of gamma rays, the thickness of the shield, the type of shielding material and the number of simulated particles on gamma ray deep penetration calculation is analyzed. The results show that when the number of simulated particles is 10 million, the deep
penetration will occur when the shield thickness is more than 15 times the mfp of gamma ray. For different shielding materials, the laws of transmission coefficients of gamma rays as a function of shield thickness are similar as long as the shield thickness is represented by the multiplier of mfp of gamma rays. The problem of deep penetration shielding transport simulation can’t be solved only by increasing the number of simulated particles. A method of variance reduction which can be used to simulate the gamma ray deep penetration shielding transport based on grid weight window iteration was proposed in this paper. In the method, by using the function of automatic generation and automatic optimization of weight window in MCNP program, artificially setting the density of the shield in the simulation from a small value to realistic density gradually, and using the weight window obtained in the simulation of non deep penetration problem to generate a better weight window which is suitable for the deep penetration simulation step by step, it can be used to solve the transport calculation of gamma ray deep penetration shielding by the iron with a thickness of 60 times the mfp of gamma ray. This method is also suitable for the simulation of isotropic point gamma ray source. The obtained fitting formula of the transmission coefficient of gamma ray with the thickness of the iron can be used to guide the design of radiation shielding in practice.

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