Dust Motions Driven by MHD Turbulence

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ABSTRACT

We discuss the relative grain motions due to MHD turbulence in interstellar medium. It has been known for decades that turbulent drag is an efficient way to induce grain relative motions. However, earlier treatments disregarded magnetic field and used Kolmogorov turbulence. Unlike hydro turbulence, MHD turbulence is anisotropic on small scales. Moreover, compressible modes are important for MHD and magnetic perturbations can directly interact with grains. We provide calculations of grain relative motion for realistic interstellar turbulence driving that is consistent with the velocity dispersions observed in diffuse gas and for realistic grain charging. We account for the turbulence cutoff arising from abmipolar drag. Our results on grain shattering are consistent with the customary accepted cutoff size. We obtain grain velocities for turbulence with parameters consistent with those in HI and dark clouds. These velocities are smaller than those in earlier papers, where MHD effects were disregarded. Finally, we consider grain velocities arising from photoelectric emission, radiation pressure and H$_2$ thrust. These are lower than relative velocities induced by turbulence. We conclude that turbulence should prevent these mechanisms from segregating grains by size.

Subject headings: ISM: dust, extinction—kinematics, dynamics—magnetic fields

1. Introduction

Dust is an important constituent of the interstellar medium (ISM). It interferes with observations in the optical range, but provides an insight to star-formation activity through far-infrared radiation. It also enables molecular hydrogen formation and traces the magnetic field via emission and extinction polarization. The basic properties of dust (optical, alignment etc.) strongly depend on its size distribution. The latter evolves as the result of grain collisions, whose frequency and consequences depend on grain relative velocities.
Grain-grain collisions can have various outcomes, e.g., coagulation, cratering, shattering, and vaporization. For collisions with $\delta v \leq 10^{-3}\text{km/s}$, grains are likely to stick or coagulate, as the potential energy due to surface forces exceeds the initial center of mass kinetic energy. Coagulation is considered the mechanism to produce large grains in dark clouds and accretion disks. Collisions with $\delta v \geq 20\text{km/s}$ have sufficient energy to vaporize at least the smaller of the colliding grains (Draine 1985). Shattering threshold is much smaller than vaporization one so that shattering dominates over vaporization in moderate energy grain-grain collisions.

It is likely that some features of the grain distribution (e.g. Kim, Martin & Hendry 1994), e.g., the cutoff at large size, are the result of fragmentation (Biermann & Harwit 1980). Even low-velocity grain collisions may have dramatic consequences by triggering grain mantle explosion (Draine 1985).

Various processes can affect the velocities of dust grains. Radiation, ambipolar diffusion, and gravitational sedimentation all can bring about a dispersion in grain velocities. It is widely believed that, except in special circumstances (e.g., near a luminous young star, or in a shock wave), none of these processes can provide substantial random velocities so as to affect the interstellar grain population via collisions (Draine 1985). Nevertheless, it was speculated in de Oliveira-Costa et al. (2000) that starlight radiation can produce the segregation of different sized grains that is necessary to explain a poor correlation of the microwave and 100$\mu\text{m}$ signals of the foreground emission (Mukherjee et al. 2001). If true it has big implications for the CMB foreground studies. However, the efficiency of this segregation depends on grain random velocities, which we study in this paper.

Interstellar gas is turbulent (see Arons & Max 1975, Scalo 1987, Lazarian, Pogosyan & Esquivel 2002). Turbulence was invoked by a number of authors (see Kusaka et al. 1970, Volk et al. 1980, Draine 1985, Ossenkopf 1993, Weidenschilling & Ruzmaikina 1994) to provide substantial relative motions of dust particles. However, they discussed hydrodynamic turbulence. It is clear that this picture cannot be applicable to the magnetized ISM as the magnetic fields substantially affect fluid dynamics. Moreover dust grains are charged, and their interactions with magnetized turbulence is very different from the hydrodynamic case. This unsatisfactory situation motivates us to revisit the problem and calculate the grain relative motions in magnetized ISM. In what follows, we use the model of MHD turbulence by Goldreich and Sridhar (1995, henceforth GS95), which is supported by recent numerical simulations (Cho & Vishniac 2000, Maron & Goldreich 2001, Cho, Lazarian & Vishniac 2002a, henceforth CLV02). We apply our results to the cold neutral medium (CNM) and a dark cloud to estimate the efficiency of coagulation, shattering and segregation of grains.
2. MHD Turbulence and Grain Motion

Unlike hydrodynamic turbulence, MHD turbulence is anisotropic, with eddies elongated along the magnetic field. This happens because it is easier to mix the magnetic field lines perpendicular to their direction rather than to bend them. The energy of eddies drops with the decrease of eddy size (e.g. \( v \sim l^{1/3} \) for the Kolmogorov turbulence) and it becomes more difficult for smaller eddies to bend the magnetic field lines. Therefore the eddies get more and more anisotropic as their sizes decrease. As eddies mix the magnetic field lines at the rate \( k_{\perp} v_k \), where \( k_{\perp} \) is a wavenumber measured in the direction perpendicular to the local magnetic field and \( v_k \) is the mixing velocity at this scale, the magnetic perturbations (waves) propagate along the magnetic field lines at the rate \( k_{\parallel} V_A \), where \( k_{\parallel} \) is the parallel wavenumber and \( V_A \) is the Alfvén velocity. The cornerstone of the GS95 model is a critical balance between those rates, i.e., \( k_{\perp} v_k \sim k_{\parallel} V_A \), which may be also viewed as coupling of eddies and wave-like motions. Mixing motions perpendicular to the magnetic field lines are essentially hydrodynamic (see CLV02) since magnetic field does not influence motions that do not bend it. Therefore it is not surprising that the GS95 predicted the Kolmogorov one-dimensional energy spectrum in terms of \( k_{\perp} \), i.e., \( E(k_{\perp}) \sim k_{\perp}^{-5/3} \) (see review by Cho, Lazarian & Yan 2002, henceforth CLY02).

The GS95 model describes incompressible MHD turbulence. Maron & Goldreich (2001), Cho, Lazarian & Vishniac (2002, henceforth CLV02) considered pseudo Alfvén mode, which is the limit of slow mode in incompressible medium, and find a similar scaling relation. Recent research suggests that the scaling is approximately true for the Alfvénic and slow mode in a compressible medium with Mach numbers \( M \equiv V/C_s \) of the order of unity (Lithwick & Goldreich 2001, henceforth LG01, Cho & Lazarian 2002), which is also consistent with the analysis of observational data (Lazarian & Pogosyan 2000, Stanimirovic & Lazarian 2001, CLY02). In what follows we apply the GS95 scaling\(^2\) to handle the problem of grain motions.

In MHD case, grain motions are different from those in hydrodynamic turbulence and depend on the grain charge to mass ratio. If their periods of Larmor motion \( \tau_L \) are longer than the gas drag time \( t_{\text{drag}} \), the grains do not feel magnetic field directly. Since the motions perpendicular to the local magnetic field are similar to hydrodynamic motions, the estimates obtained in the Kolmogorov hydrodynamic model should be essentially correct within a factor of order unity. Otherwise, if the Larmor time \( \tau_L \) is shorter than gas drag time \( t_{\text{drag}} \), grain

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\(^1\)Two dimensional spectrum \( E(k_{\perp}, k_{\parallel}) \) is presented in Cho, Lazarian & Vishniac (2002a).

\(^2\)Calculations in Cho & Lazarian (2002) show that fast mode have spectrum \( E(k) \sim k_{\perp}^{-3/2} \), which is not very different from the GS95 scaling. In the Appendix, we show that the motions that the fast waves produced in low \( \beta = P_{\text{gas}}/P_{\text{mag}} \) plasma are nearly perpendicular to the magnetic field.
perpendicular motions are constrained by magnetic field.

Because of turbulence anisotropy, it is convenient to consider separately grain motions parallel and perpendicular to the magnetic field. The motions perpendicular to the magnetic field are influenced by Alfvén mode, while those parallel to the magnetic field are subjected to the magnetosonic modes. The scaling relation for perpendicular motion is 
\[ v_k \propto k_{\perp}^{-1/3} \] (GS95). As the eddy turnover time is 
\[ \tau_k \propto (k_{\perp} v_k)^{-1}, \] the velocity may be expressed as 
\[ v_k \approx v_{\text{max}} (\tau_k/\tau_{\text{max}})^{1/2}, \] where 
\[ \tau_{\text{max}} = l_{\text{max}}/v_{\text{max}} \] is the time-scale for the largest eddies, for which we adopt the fiducial values 
\[ l_{\text{max}} = 10\text{pc}, \quad v_{\text{max}} = 5\text{km/s}. \]

If the Larmor time 
\[ \tau_L = 2\pi m_{gr} c/|q|B \] is shorter than the gas drag time 
\[ t_{\text{drag}}, \] grain perpendicular motions are constrained by the magnetic field. In this case, grains have a velocity dispersion determined by the turbulence eddy whose turnover period is \( \sim \tau_L \), while grains move with the magnetic field on longer time scales. Since the turbulence velocity grows with the eddy size, the largest velocity difference occurs on the largest scale where grains are still decoupled. Thus, following the approach in Draine (1985), we can estimate the characteristic grain velocity relative to the fluid as the velocity of the eddy with a turnover time equal to \( \tau_L \),
\[ v_\perp(a) = \frac{v_{\text{max}}^{3/2} \rho_{gr}^{1/2} (8\pi^2 c^3/3qB)^{1/2} a^{3/2}}{l_{\text{max}}^{1/2}}, \] (1)

and the relative velocity of grains to each other should be approximately equal to the larger one of the grains’ velocities, i.e., the larger grain’s velocity,
\[ \delta v_\perp(a_1, a_2) = \frac{v_{\text{max}}^{3/2} \rho_{gr}^{1/2} (8\pi^2 c^3/3qB)^{1/2} [\text{max}(a_1, a_2)]^{3/2}}{l_{\text{max}}^{1/2}} \]
\[ = 1.4 \times 10^5 \text{cm/s} (v_5 a_5^{3/2}/(q_e l_{10} B_{\mu})^{1/2}), \] (2)

in which 
\[ v_5 = v_{\text{max}}/10^5\text{cm/s}, \quad a_5 = a/10^{-5}\text{cm}, \quad q_e = q/1\text{electron}, \quad l_{10} = l_{\text{max}}/10\text{pc}, \quad B_{\mu} = B/1\mu\text{G}, \] and the grain density is assumed to be \( \rho_{gr} = 2.6\text{g/cm}^3 \).

Grain motions parallel to the magnetic field are induced by the compressive component of slow mode with \( v_\parallel \propto k_{\parallel}^{-1/2} \) (CLV02, LG01, CLY02). The eddy turnover time is 
\[ \tau_k \propto (v_\parallel k_{\parallel})^{-1}, \] so the parallel velocity can be described as 
\[ v_\parallel \approx v_{\text{max}} \tau_k/\tau_{\text{max}}. \] For grain motions parallel to the magnetic field the Larmor precession is unimportant and the gas-grain coupling takes place on the translational drag time \( t_{\text{drag}} \). The drag time due to collisions with atoms is essentially the time for collision with the mass of gas equal to the mass of grain,
\[ t_{\text{drag}}^0 = (a \rho_{gr}/n)(\pi/8\mu kT)^{1/2}, \] where \( \mu \) is the mass of gas species. The ion-grain cross-section

\[ ^3 \text{We assume that turbulence is driven isotropically at the scale } l_{\text{max}}. \]
due to long-range Coulomb force is larger than the atom-grain cross-section. Therefore, in the presence of collisions with ions, the effective drag time decreases by the factor

\[ \alpha = \left[ \frac{1}{2} \sum_i \left( \frac{m_i}{m_H} \right)^{1/2} \sum_Z f_Z \left( \frac{Z e^2}{akT} \right)^2 \ln \left[ \frac{3(kT)^{3/2}}{2e^3 |Z| (\pi x n_H)^{1/2}} \right] \right]^{-1} \]

where \(x_i\) is the ionization of ion \(i\) with mass \(m_i\), \(f_Z\) is the probability of the grain in the state with charge \(Z\) (Draine & Salpeter 1979). The characteristic velocity of grain motions along the magnetic field is approximately equal to the parallel turbulent velocity of eddies with turnover time equal to \(t_{drag} = t_{drag0}/\alpha\)

\[ v_\parallel(a) = \alpha^{-1} \frac{v_{max}^2}{l_{max}} \left( \frac{\rho_{gr}}{4n} \right) \left( \frac{2\pi}{\mu kT} \right)^{1/2} a, \quad (3) \]

and the relative velocity of grains for \(T_{100} = T/100K\) is

\[ \delta v(a_1, a_2) = \alpha^{-1/2} \frac{v_{max}^3}{l_{max}^{1/2}} \left( \frac{\rho_{gr}}{4n} \right) \left( \frac{2\pi}{\mu kT} \right)^{1/2} \left[ max(a_1, a_2) \right]^{1/2} \]

\[ = (1.0 \times 10^6 \text{cm/s}) \alpha v^2_a \frac{n_1^2 a}{(n l_10^{-1})}, \quad (4) \]

When \(\tau_L > t_{drag}\), grains are no longer tied to the magnetic field. Since at a given scale, the largest velocity dispersion is perpendicular to the magnetic field direction, the velocity gradient over the grain mean free path is maximal in the direction perpendicular to the magnetic field direction. The corresponding scaling is analogous to the hydrodynamic case, which was discussed in Draine (1985):

\[ \delta v(a_1, a_2) = \frac{v_{max}^3}{l_{max}^{1/2}} \left( \frac{\rho_{gr}}{4n} \right) \left( \frac{2\pi}{\mu kT} \right)^{1/2} \left[ max(a_1, a_2) \right]^{1/2}. \quad (5) \]

An important difference between hydrodynamic and MHD turbulence is the presence of the ambipolar damping of turbulence in MHD case (see Cho, Lazarian & Vishniac 2002b). If the mean free path for a neutral particle, \(l_n\), in a partially ionized gas with density \(n_{tot} = n_n + n_i\) is much less than the size of the eddies in consideration, i.e. \(l_n k_\perp \ll 1\), the damping time is

\[ t_{damp} \sim \nu_n^{-1} k_\perp^{-2} \sim (n_{tot}/n_n) (l_n v_n)^{-1} k_\perp^{-2}, \quad (6) \]
where $\nu_n$ is effective viscosity produced by neutrals. The mean free path of a neutral $l_n$ is influenced both by collisions with neutrals and with ions. The rate of a neutral colliding with ions is proportional to the density of ions, while the rate of a neutral colliding with other neutrals is proportional to the density of neutrals. The drag coefficient for neutral-neutral collisions is $\sim 1.7 \times 10^{-10} T(K)^{0.3} \text{ cm}^3 \text{ s}^{-1}$ (Spitzer 1978), while for neutral-ion collisions it is $\sim \langle v_r \sigma_{\text{in}} \rangle \approx 1.9 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$ (Draine, Roberge & Dalgarno 1983). Thus collisions with other neutrals dominate for $n_i/n_n$ less than $\sim 0.09 T^{0.3}$. In the present paper we consider cold gas and therefore the influence of ions on $l_n$ is disregarded.

Turbulent motions cascade down till the eddy turnover is of the order of $t_{\text{damp}}$. Thus the turbulence cutoff in neutral medium is

$$\tau_c \simeq \left( \frac{l_n}{v_n} \right) \left( \frac{v_n}{v_{\text{max}}} \right)^{\frac{1}{2}} \left( \frac{l_{\text{max}}}{l_n} \right)^{\frac{1}{2}} \left( \frac{V_A}{v_{\text{max}}} \right)^{\frac{1}{2}} \left( \frac{n_n}{n_{\text{tot}}} \right),$$

(7)

where $v_n$ and $V_A$ are, respectively, the velocity of a neutral and Alfvén velocity. It is easy to see that for $\tau_c$ longer than either $t_{\text{drag}}$ or $\tau_L$ the grain motions get modified. A grain samples only a part of the eddy before gaining the velocity of the ambient gas. In GS95 picture, the shear rate $dv/dl$ increases with the decrease of eddy size. Thus for $\tau_c > \text{max}\{t_{\text{drag}}, \tau_L\}$, these smallest available eddies are the most important for grain acceleration. Consider first the perpendicular motions. If $v_c$ is the velocity of the critically damped eddy, the distance traveled by the grain is $\Delta l \sim v_c \times \text{min}\{t_{\text{drag}}, \tau_L\}$. The shear rate $dv/dl$ perpendicular to the magnetic field is $\tau_k^{-1}$. Thus the grain experiences the velocity difference in the direction perpendicular to the magnetic field

$$v_\perp \sim \Delta l \times \frac{dv}{dl} \sim \frac{v_c}{\tau_c} \times \text{min}\{t_{\text{drag}}, \tau_L\}.$$  

(8)

For the parallel motions, $\Delta l \sim v_c \times t_{\text{drag}}$. And if noticing the critical balance in GS95 model $k_\parallel V_A \sim k_\perp v_\perp = \tau_k^{-1}$, the largest shear rate along the magnetic field should be $dv/dl = v_c k_\parallel \sim v_c/(V_A \tau_c)$. Therefore, in the parallel direction, grain experiences a velocity difference $V_A/v_c$ times smaller, i.e.,

$$v_\parallel \sim \frac{v_c^2}{V_A} \times \frac{t_{\text{drag}}}{\tau_c}.$$  

The viscosity due to ion-ion collisions is typically small as ion motions are constrained by the magnetic field.
3. Discussion

3.1. Shattering and Coagulation

Consider the cold neutral medium (CNM) with temperature $T = 100$K, density $n_H = 30 \text{cm}^{-3}$, electron density $n_e = 0.045 \text{cm}^{-3}$, magnetic field $B \sim 1.3 \times 10^{-5} \text{G}$ (Weingartner & Draine 2001a, hereafter WD01a). To account for the Coulomb drag, we use the results by WD01a and get the modified drag time $t_{\text{drag}} = \alpha^{-1} t_{0, \text{drag}}$. Using the electric potentials in Weingartner & Draine (2001b), we get grain charge and $\tau_L$.

For the parameters given above, we find that $t_{\text{drag}}$ is larger than $\tau_c$ for grains larger than $10^{-6}$ cm, $\tau_L$ is smaller than $\tau_c$ even for grains as large as $10^{-5}$ cm. Here, we only consider grains larger than $10^{-6}$ cm, which carry most grain mass ($\sim 80\%$) in ISM, so we can still use Eq.(3) to calculate grain parallel velocities and Eq.(1) to get the perpendicular velocity for grain larger than $10^{-5}$ cm. Nevertheless, the perpendicular velocities of grains smaller than $10^{-5}$ cm should be estimated according to Eq.(8),

$$v'_\perp(a) = v_{\text{max}} \left( \frac{\tau_c}{\tau_{\text{max}}} \right)^{1/2} \left( \frac{\tau_L}{\tau_c} \right)^{1/2} = v_\perp(a) \left( \frac{\tau_L}{\tau_c} \right)^{1/2},$$

(9)

as where $v_\perp(a)$ is given by Eq.(1). The results are shown in Fig.1. The relative velocity, as we discussed earlier, is dependent on the velocity of the larger grain. Thus if $\max\{a_1, a_2\}$ is larger than $10^{-5}$ cm, the grain relative velocity should be given by Eq.(2). Otherwise, we should use Eq.(9) to estimate the grain relative velocity.

The critical sticking velocity were calculated in Chokshi et al. (1993)(see also Dominik & Tielens 1997). However, experimental work by Blum (2000) shows that the critical velocity is an order of magnitude larger than the theoretical calculation. Thus the collisions can result in coagulation for small silicate grains ($\le 3 \times 10^{-6}$ cm).

According to our result, grains won’t be shattered if the shattering thresholds for silicate is 2.7 km/s (Jones et al. 1996). Nevertheless, the grain velocities are strongly dependent on the maximal velocity of turbulence $v_{\text{max}}$ at the injection scale, which is highly uncertain. The value $v_{\text{max}} = 5$ km/s we use here is a conservative estimate. We expect grains would acquire larger velocities, therefore, more likely to be shattered during collisions if the $v_{\text{max}}$ is larger. For instance, we will get a cutoff $6 \times 10^{-5}$ cm due to shattering if $v_{\text{max}} = 10$ km/s.

For a dark cloud, the situation is different. As the density increases, the drag by

\[\footnote{There are obvious misprints in the numerical coefficient of Eq.(7) in Chokshi et al.(1993) and the power index of Young's modulus in Eq.(28) of Dominik & Tielens (1997).} \]
gas becomes stronger. Consider a typical dark cloud with temperature \( T = 20 \text{K} \), density \( n_H = 10^4 \text{cm}^{-3} \) (Chokshi et al. 1993) and magnetic field \( B \sim 2.3 \times 10^{-4} \text{G} \). Assuming that dark clouds are shielded from radiation, grains get charged by collisions with electrons: \(< q > = 0.3(r/10^{-5}\text{cm}) \) electrons. The ionization in the cloud is \( \chi = n_e/n_{\text{tot}} \sim 10^{-6} \) and the drag by neutral atoms is dominant. From Eq.(7) and the expression for the drag time and the Larmor time, we find \( \tau_L < t_{\text{drag}} \) for grains of sizes between \( 10^{-6}\text{cm} \) and \( 4 \times 10^{-6}\text{cm} \), and \( t_{\text{drag}} < \tau_L \) for grains larger than \( 4 \times 10^{-6}\text{cm} \). In both cases, turbulence cutoff \( \tau_c \) is smaller than \( t_{\text{drag}} \) and \( \tau_L \). Thus for the smaller grains, we use Eq.(1),(3) to estimate grain velocities. For larger grains, the fact \( t_{\text{drag}} < \tau_L \) means that grain velocities are similar to the hydrodynamic case and Eq(5) is used (Fig 1b).

Our results for dark clouds show only a slight difference from the earlier hydrodynamic estimates. Since the drag time \( t_{\text{drag}} \propto n^{-1} \), Larmor time \( \tau_L \propto B^{-1} \propto n^{-1/2} \), the grain motions get less affected by the magnetic field as the cloud becomes denser. Thus we agree with Chokshi’s et al. (1993) conclusion that densities well in excess of \( 10^4 \text{cm}^{-3} \) are required for coagulation to occur. Shattering will not happen because the velocities are small, so there are more large grains in dark clouds. This agrees with observations (see Mathis 1990).

Supersonic grain motion in respect to gas may result in grain alignment (see Lazarian 2000 for a review). Our present results testify that only grains with size larger than \( \sim 10^{-4}\text{cm} \) experience such velocities for the typical parameters of the CNM. Such grains are marginally important for starlight polarization (Kim & Martin 1995). This, however, does not preclude grain alignment via streaming in particular circumstances, e.g. in shocks and outflows (Roberge & Hanany 1990, Lazarian 1994) For low \( \beta = P_{\text{gas}}/P_{\text{mag}} \) is the ratio of gas pressure and magnetic pressure) media, the supersonic motions would correspond to fast mode and Alfvénic mode (Alfvén & Fälthmnnmar 1963). Alfvénic mode have velocity perpendicular to \( B \) and fast mode have velocity nearly perpendicular to \( B \). As the mechnical alignment\(^6\) minimizes grain cross section to the flow, the alignment would happen with the long axis perpendicular to the magnetic field.

### 3.2. Grain Segregation and Turbulent Mixing

Our results are also relevant to grain segregation. Grains are the major carrier of heavy elements in the ISM. The issue of grain segregation may have significant influence on the ISM metallicity. Subjected to external forcing, e.g., due to radiation pressure, grains gain size-

\(^6\)For grains rotating thermally the Gold alignment theory (Gold 1952, Roberge et al. 1993, Lazarian1997) is applicable, while cross sectional and crossover (Lazarian 1995, Lazarian & Efroimsky 1996) is applicable.
dependent velocities with respect to gas. WD01a have considered the forces on dust grains exposed to anisotropic interstellar radiation fields. They included photoelectric emission, photodesorption as well as radiation pressure, and calculated the drift velocity for grains of different sizes. The velocities they got for silicate grains in the CNM range from 0.1 cm/s to 10³ cm/s. Fig.1 shows that the turbulence produces larger velocity dispersions. Thus the grain segregation of very small and large grains speculated in de Oliveira-Costa et al. (2000) is unlikely to happen for typical CNM conditions.

A different mechanism of driving grain motions is a residual imbalance in “rocket thrust” between the opposite surfaces of a rotating grain (Purcell 1979). This mechanism can provide grain relative motions and preferentially move grains into molecular clouds. It is easy to see that due to averaging caused by grain rotation, the rocket thrust is parallel to the rotation axis. Three causes for the thrust were suggested by Purcell (1979): spatial variation of the accommodation coefficient for impinging atoms, photoelectric emission, and H₂ formation. The latter was shown to be the strongest among the three. The uncompensated force in this case arises from the difference of the number of catalytic active sites for H₂ formation on the opposite grain surfaces. The nascent H₂ molecules leave the active sites with kinetic energy E and the grain experiences a push in the opposite directions. The number of active sites varies from one grain to another, and we should deal with the expectation value of the force for a given distribution of active sites.

Adopting the approach in Lazarian & Draine (1997), we get the mean square root force of H₂ formation on a grain in the shape of a square prism with dimensions $b \times b \times a$ (with $b > a$)

$$\langle F_zH \rangle = r^{3/2}(r + 1)^{1/2}\gamma(1 - y)n_H v_H a^2 \left(\frac{2m_H E}{\nu}\right)^{1/2},$$

where $r = b/2a$, $n_H \equiv n(H) + 2n(H_2)$, $y = 2n(H_2)/n_H$ is the H₂ fraction, $\gamma$ is the fraction of impinging H atoms and $\nu$ is the number of active sites over the grain surface. Due to internal relaxation of energy (Purcell 1979, Spitzer & McGlynn 1979, Lazarian & Efroimsky 1999, Lazarian & Draine 1999a,b) the grain rotational axis tends to be perpendicular to the largest $b - b$ surface, the uncompensated force above is parallel to the rotational axis. The other components of force are being averaged out due to grain fast rotation. The

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7If reconnection is fast (see Lazarian & Vishniac 1999), the mixing of grains over large scales is provided by turbulent diffusivity $\sim v_{max} l_{max}$. On small scales the grain decoupled motions are important.

8This statement is true up to grain thermal wiggling (Lazarian 1994, Lazarian & Roberge 1997) and occasional flipping (Lazarian & Draine 1999).
expected grain velocity is \( v = \langle F_z \rangle t_{\text{drag}}/m \). In the CNM we consider, \( y = 0 \), adopting the characteristic values in Lazarian & Draine (1997), \( r = 1 \), \( \gamma = 0.2 \), \( E = 0.2eV \), and the density of active sites \( 10^{11}\text{cm}^{-2} \) so that \( \nu = 80(a/10^{-5}\text{cm})^2r(r+1) \),\(^9\) we get the “optimistic” velocity, \( v \simeq 330(10^{-5}\text{cm}/\sqrt{a})\text{cm/s} \), shown in Fig 1. The scaling is approximate due to the complexity of coefficient \( \alpha \) (see WD01a Fig.16). For maximal active site density \( 10^{15}\text{cm}^{-2} \), we get the lower boundary of grain velocity \( v \simeq 3.3(10^{-5}\text{cm}/a)^{1/2}\text{cm/s} \). Such velocities may have noticeable effects. For instance, \( \text{H}_2 \) forces and therefore \( \text{H}_2 \) driven velocities are expected to drop in molecular clouds, resulting in preferential diffusion of heavy elements into molecular clouds. Due to the size dependence of the velocities, one could also expect a segregation of grains of different sizes.

For small grains \((< 3 \times 10^{-6}\text{cm})\), the “optimistic” velocity obtained from \( \text{H}_2 \) formation is higher than that arising from our turbulent estimates. Does this mean that such grains will be segregated? Our calculation implicitly assumes that grains do not flip. Therefore, they experience constant forcing through \( t_{\text{drag}} \). However, Lazarian & Draine (1999a) have shown that subjected to \( \text{H}_2 \) torques alone small grains \( \leq 10^{-5}\text{cm} \) should experience frequent thermal flipping, which means that the forces are averaged out by this effect. This flipping results from coupling of grain rotational and vibrational degrees of freedom through internal relaxation. The Barnett relaxation discovered by Purcell (1979) was used for the calculations. Further research by Lazarian & Draine (1999b) resulted in a discovery of nuclear relaxation that made flipping efficient for grains up to \( 10^{-4}\text{cm} \). Does this mean that the \( \text{H}_2 \) forces on interstellar grains are completely averaged out? Probably not. The flipping efficiency depends on the value of the grain angular momentum (Lazarian & Draine 1999a). If grains are already spun up to sufficient velocities, they get immune to thermal flipping. Radiative torques (Dolginov & Mytrophanov 1975, Draine 1996, Draine & Weingartner 1996) provide a possibility of spinning grains up to high velocities. For them to be active, the grain size should be comparable to the wavelength. Therefore, for a typical interstellar diffuse radiation field radiative torques are expected to spin up grains with sizes larger than \( \sim 4 \times 10^{-6}\text{cm} \). They will also align grains with rotational axes parallel to the magnetic field. Thus grains should acquire velocities along the magnetic field lines and the corresponding velocities should be compared with those arising from turbulent motions parallel to the magnetic field. It is clear from Fig.1 that for the chosen set of parameters the effect of \( \text{H}_2 \) thrust is limited. All in all, we conclude that the radiation effects and \( \text{H}_2 \) thrust are not efficient for segregating grains in typical ISM conditions.

It is possible to show that for the CNM the effect of variation of accommodation coeff-

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\(^9\)The number of \( \text{H}_2 \) formation sites is highly uncertain. It may also depend on the interplay of the processes of photodesorption and poisoning (Lazarian 1995, 1996).
ficient is less important than the $\text{H}_2$ formation. Photoelectric emission does not vanish as grains flip and the corresponding motion was considered by WD01a, but the velocity due to the effect is smaller than that arising from turbulent motions. Thus, we think segregation is marginal in such a CNM. Some effect on metallicity might be present and worth quantifying elsewhere.

The percentage of atomic hydrogen is reduced in dark clouds, and the radiation field is weak. Thus the only force we should pay attention to is the bombardment by gas atoms. According to Purcell (1979), the force should be $\langle F_{zb} \rangle = \sqrt{8a^2/s \Delta T/2\delta s^2}$, for a grain with a deviation $\delta$ in accommodation coefficient by a scale of $s$ if the temperature difference between gas and grain is $\Delta T$. This temperature difference is usually small ($<10$ K). Moreover, because of insufficient radiation grains are expected to flip frequently. Therefore, the velocities driven by the variation of the accommodation coefficient are always much smaller than those due to turbulence drag so that grains in dark clouds should be fully mixed.

### 3.3. Resonant Grain Acceleration

In the treatment above we disregarded the possibility of direct acceleration of grains through their interactions with the fluctuating electromagnetic field in the MHD turbulence. As we mentioned in §2, the turbulent motions parallel to the magnetic field are like MHD waves. The charged grains interact with the waves effectively when the wave frequency is a multiple of the gyrofrequency of the grain\textsuperscript{10}, $\omega - k_{\|}v_{\mu} = n\Omega$, this is so called gyroresonance. This resonant process is important for a highly ionized medium. In our next paper we will discuss the stochastic grain acceleration by gyroresonance with different MHD modes in warm ionized medium. In partially ionized medium, the turbulence is damped due to ion-neutral collisions (see the discussion around Eq.(6)). For the CNM we consider, we find in that the turbulence is cut off at $\tau_c = 2\pi/\omega$ beyond the Larmor time $\tau_L = 2\pi/\Omega$ of the grain less than $10^{-5}$cm (see Fig. 1a). Since the grain($<10^{-5}$cm) velocities are much smaller than Alfvén velocity $V_A = 10^5$cm/s, the resonant condition is not satisfied for these grains. Indeed, there are still some magnetic structures below the cutoff according to Cho, Lazarian & Vishniac (2002b). However, these structure are static so that they do not contribute to grain acceleration. Therefore, while the velocities of larger grains($>10^{-5}$cm) may be modified by gyroresonance, the velocities of those smaller grains($<10^{-5}$cm) should remain the same as shown in Fig.1.

\textsuperscript{10}We should take into account resonance broadening if the dynamical time scale of turbulence is comparable to the time scale of grain motion.
4. Summary

We have calculated relative motions of dust grains in a magnetized turbulent fluid taking into account turbulence anisotropy, turbulence damping and grain coupling with the magnetic field. We find that these effects decrease the relative velocities of dust grains compared to the earlier hydrodynamic-based calculations. The difference is substantial in CNM, but less important for dark clouds. For CNM we find that coagulations of silicate grains happen for sizes \( \leq 3 \times 10^{-6}\) cm. The force due to \( \text{H}_2 \) formation on grain surface might drive small grains \((< 3 \times 10^{-6}\) cm\) to larger velocities if thermal flipping of grains is suppressed by grain rapid rotation. This may be possible in the presence of radiative torques. However, radiative torques are suppressed for grains smaller than wavelength and therefore grains are expected to be well mixed due to turbulence.

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A. Angle between B and v

A fundamental question arises from the fact, that in MHD turbulence wave vectors are not aligned along magnetic field lines, as this is the case for pure Alfvénic waves. We need to analyze relative position of three vectors: magnetic field vector \( \mathbf{B} \), wave vector \( \mathbf{k} \), and the displacement velocity vector \( \mathbf{v} \). In what following, we shall study how the angle \( \gamma \) between \( \mathbf{v} \) and \( \mathbf{B} \) changes with plasma \( \beta = P_{\text{gas}}/P_{\text{mag}} = 2C_s^2/V_A^2 \).

It is shown in Alfvén H. & Fälthmmar (1963) that the angle \( \Psi \) between \( \mathbf{v} \) and \( \mathbf{k} \) can be expressed as follows:

\[
\tan \Psi = \frac{\sin \alpha \cos \alpha}{\cos^2 \alpha - v_p^2/V_A^2},
\]

where \( v_p \) is the phase velocity, which is related to the Alfvénic velocity \( V_A \) and the sound velocity \( C_s \) through the dispersion relation

\[
v_p^4 - (V_A^2 + C_s^2)v_p^2 + C_s^2 V_A^2 \cos^2 \alpha = 0.
\]

Solving this equation in respect to \( \epsilon = v_f^2/v_A^2 \),

\[
\epsilon(\beta) = \frac{1}{2} \left( 1 + \beta/2 \pm \sqrt{(1 - \beta/2)^2 + 2\beta \sin^2 \alpha} \right),
\]

where \( '+' \) gives the result for fast mode and \( '-' \) represents slow mode. Thus the angle \( \gamma \) can be calculated as

\[
\gamma = \alpha - \arctan \frac{\sin \alpha \cos \alpha}{\cos^2 \alpha - \epsilon(\xi)}.
\]
and the corresponding plot is shown in Fig. 2. It is evident that for low $\beta$ plasma the velocity $v$ of the fast mode is directed nearly perpendicular to $B$ whatever is the direction of $k$, while the velocity of the slow mode is nearly parallel to the magnetic field. So the parallel motions we got from slow mode are essentially correct, while the perpendicular motions are also subjected to fast mode. Since the scaling of fast mode according to Cho & Lazarian (2002) is $v_k \propto k^{-1/4}$, which is not so different from the scaling we use, $v_k \propto k^{-1/3}$, we expect that our result for perpendicular motions wouldn’t be affected substantially.

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Fig. 1.— Grain relative velocities as a function of radii (solid line) (a) in CNM, (b) in DC. Dashdot line represents parallel velocity due to the drag by compressible modes, dotted line refers to perpendicular velocity from the contribution of the drag by Alfvén mode, also plotted is the earlier hydrodynamic result (dashed line). The change of the slope in (a) is due to the cutoff of turbulence by ambipolar diffusion. In (b), for grains larger than $\sim 4 \times 10^{-6}$ cm, the grains velocities are similar with the hyro case (see text). The grain velocity driven by H$_2$ thrust is plotted to illustrate the issue of grain segregation in the CNM (see text), the part marked by 'o' is nonphysical because thermal flipping is not taken into account.

Fig. 2.— The range of angles between $\mathbf{B}$ and $\mathbf{v}$ The solid line refers to the fast mode, the dotted line is plotted for fast mode. This range is produced when angle between $\mathbf{k}$ and $\mathbf{B}$ changes in the range 0 to $\pi$. 
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