Ferromagnetic fractional quantum Hall states in a valley degenerate two-dimensional electron system

Medini Padmanabhan, T. Gokmen, and M. Shayegan

Department of Electrical Engineering, Princeton University, Princeton, NJ 08544

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We study a two-dimensional electron system where the electrons occupy two conduction band valleys with anisotropic Fermi contours and strain-tunable occupation. We observe persistent quantum Hall states at filling factors $\nu = 1/3$ and $5/3$ even at zero strain when the two valleys are degenerate. This is reminiscent of the quantum Hall ferromagnet formed at $\nu = 1$ in the same system at zero strain. In the absence of a theory for a system with anisotropic valleys, we compare the energy gaps measured at $\nu = 1/3$ and $5/3$ to the available theory developed for single-valley, two-spin systems, and find that the gaps and their rates of rise with strain are much smaller than predicted.

In a single-electron picture, there should be no quantum Hall effect (QHE) at odd-integer Landau level (LL) filling factors (\(\nu\)) in a two-dimensional electron system (2DES) with a vanishing Lande $g$-factor. In an interacting 2DES, however, a QH state exists at $\nu = 1$ even in the limit $g = 0$ [1,2]. The ground state is a ferromagnet, and it is the large Coulomb (exchange) energy cost of a spin flip that leads to a sizable energy gap separating this ground state from its excitations. These charged excitations, called skyrmions, have a nontrivial, long-range spin texture with a slow rotation of the spin as a function of distance to minimize the Coulomb energy cost [1-3].

A question naturally arises as to whether there are similar ferromagnetic ground states in the fractional quantum Hall effect (FQHE) regime. The presence of such a state with finite energy gap at $g = 0$ along with skyrmionic excitations was theoretically predicted for $\nu = 1/3$ [1,8]. Experimentally, it is challenging to tune the $g$-factor to zero while keeping the quality of the 2DES sufficiently high to exhibit FQHE. There has been only one study [3] to date probing the state at $\nu = 1/3$ under the condition of $g \rightarrow 0$, which was achieved via applying hydrostatic pressure. The data revealed no FQH resistance minimum at $\nu = 1/3$ when $g \approx 0$ [8], but from measurements at non-zero values of $g$, a finite but small energy gap was deduced for the $\nu = 1/3$ FQH state.

Here we report magnetotransport measurements for 2DESs confined to AlAs quantum wells where the electrons occupy two conduction band minima (valleys). In this system, the energy splitting between the two valleys can be controlled in situ via applying in-plane strain. Considering the valley degree of freedom as an isospin, the condition of zero valley splitting is identical to the limit of $g = 0$ in a two-spin system. Under this condition, a ferromagnetic ground state and skyrmionic excitations have already been reported at $\nu = 1$ [2,11]. Here we report the observation of a well-developed QH resistance minimum at $\nu = 1/3$ as we tune the valley splitting through zero. We also observe a pronounced resistance minimum at $\nu = 5/3$ which is the particle-hole conjugate state of $\nu = 1/3$ in our two-component (two-valley) system. These observations strongly suggest the presence of ferromagnetic FQH states at these fillings. From the temperature ($T$) dependence of the resistance minima, we deduce energy gaps for these states as a function of applied strain. The gaps increase initially and then saturate, as expected qualitatively. Quantitatively, however, both the gaps and their dependencies on strain are much weaker than the predictions of theories which were developed for the single-valley, two-spin systems.

Our samples are AlAs quantum wells grown by molecular beam epitaxy [10]. Contacts were made with GeAuNi alloy and Hall bars were defined. Metallic front and back gates were deposited on the samples, thus allowing us to control the 2D electron density, $n$. We report results for three samples from two wafers. The quantum well in sample A is 12 nm wide while samples B and C have well...
widths of 15 nm each. Measurements were done in either a 20 mK base temperature dilution refrigerator or a 300 mK $^3$He system using standard lock-in methods.

The 2D electrons in our samples occupy two conduction band valleys with elliptical Fermi contours [10]. In our system, the Zeeman energy ($E_Z$) of electrons is typically larger than their cyclotron energy [10]. Since the FQH states around $\nu = 1/2$ and $3/2$ are formed in the first and second LLs respectively, ours is a single-spin, two-valley system. The valley degeneracy can be controllably lifted by uniaxial strain applied in the plane of the sample. Experimentally, this is achieved by gluing the sample to a piezoelectric (piezo) stack which expands in one direction and contracts in the perpendicular direction when an external voltage is applied. A schematic is shown in Fig. 1(a). The single-particle energy splitting between the two in-plane valleys is given by $E_e = \epsilon E_Z$; $\epsilon = \epsilon_{[100]} - \epsilon_{[010]}$ where $\epsilon_{[100]}$ and $\epsilon_{[010]}$ are the strains along the [100] and [010] crystal directions [10] and $E_Z$ is the deformation potential, which has a band value of 5.8 eV in AlAs. This dependence of the energy levels on strain is shown schematically in Fig. 1(b).

In Figs. 1(c) and 1(d) we show magnetoresistance traces taken for sample A at different $\epsilon$. Figure 1(c) focuses on the evolution of various FQH states around $\nu = 3/2$. In Fig. 1(d) we present data from a different cooldown showing states around $\nu = 1/2$. The varying strengths of the different FQH resistance minima around $\nu = 3/2$ as a function of $\epsilon$ have been explained by the concept of composite fermions with a valley degree of freedom [12][13]. Here, we focus on the states at $\nu = 1/3$ and $5/3$. The QH minima at both these fractions are well-developed even when $\epsilon = 0$ indicating the presence of a finite energy gap. This strongly suggests ferromagnetic ground states at $\epsilon = 0$ at these fractional fillings.

In Figs. 2(b) and 2(c) we show a summary of our measured energy gaps ($\Delta$) for $\nu = 1/3$ and $5/3$ as a function of strain-induced valley splitting $E_v = \epsilon E_Z$. These plots present the highlights of our study. For comparison, we also include in Fig. 2(a) a plot of the measured gaps for the $\nu = 1$ QH state for the same sample. In all three plots, the $x$ and $y$ axes are also shown normalized to the Coulomb energy, $V_C$. The single-particle prediction, $\Delta = \epsilon E_Z$ (with $E_Z = 5.8$ eV) is shown as dashed lines.

![Fig. 2. Energy gaps vs. $\epsilon$ for sample A for (a) $\nu = 1$, (b) $\nu = 1/3$, and (c) $\nu = 5/3$. Both the $x$ and $y$ axes are also indicated normalized to the Coulomb energy, $V_C$. The single-particle prediction, $\Delta = \epsilon E_Z$ (with $E_Z = 5.8$ eV) is shown as dashed lines.](image)

Before discussing Fig. 2 in detail, we present data used to deduce the gaps reported in Figs. 2(b) and 2(c). In Fig. 3 we show the $T$-dependence of magnetoresistance traces taken around $\nu = 1/2$. Figure 3(a) corresponds to the condition of $\epsilon = 0$ while Fig. 3(b) is for $\epsilon = -1.35 \times 10^{-4}$ which belongs to the saturated region in Fig. 2(b). The corresponding Arrhenius plots used to deduce the gaps at $\nu = 1/3$ are shown as insets. Figure 4 shows data for the case of $\nu = 5/3$. Figure 4(a) contains examples of Arrhenius plots at $\nu = 5/3$ for sample C for three different $\epsilon$. A summary of all our gap measurements at $\nu = 5/3$ from three samples is shown in Fig. 4(b). In our data, the Arrhenius plots deviate from a strictly activated behavior at low and high temperatures. This is not uncommon [14][16] and previous studies have concluded that the value of the energy gap can be deduced reasonably accurately from the maximum slope in such cases.

We now discuss various aspects of our gap data, starting with $\nu = 1$ in Fig. 2(a). A simple fan diagram of energy vs. $\epsilon$ for (spinless) non-interacting electrons is shown in Fig. 1(b). At a fixed $B$, the cyclotron energy ($E_C$) is constant while the energy levels for the [010] and [100] valleys split in response to $\epsilon$, with the splitting given by $E_v = \epsilon E_Z$. It is clear that at $\epsilon = 0$ the state at $\nu = 1$ is expected to be gapless. As the system is tuned away from $\epsilon = 0$, a QH state should appear and its gap should...
increase linearly with a slope of $E_Z$ until a LL crossing occurs. After this point, the gap is expected to be a constant equal to $E_c = \hbar e B / m_b$, where $m_b = 0.46 m_0$ is the band mass for AlAs.

A similar fan diagram can be used in the case of single-valley, two-spin systems with the role of $E_n$ taken over by $E_Z$. Since this simple picture is inadequate to explain experimental results, a comprehensive theory including the role of interaction was developed for single-valley, two-spin systems [1, 17]. Here we assume that the valley degree of freedom is an isospin, and compare our experimental results to the theory developed for the two-spin case. We point out, however, that in our system the Fermi contours are anisotropic at $B = 0$. This introduces an anisotropic Coulomb interaction, the consequences of which are unknown.

The theoretically relevant parameter in the two-spin case is the ratio of the Zeeman energy to Coulomb energy, $\eta = E_Z / V_C$. In our system, we redefine $\eta = E_c / V_C$. In the limit of $\eta = 0$, the ground state at $\nu = 1$ is expected to be a QH ferromagnet with an energy gap (for an ideal 2DES) equal to $0.62 V_C$ [1, 17]. Experimentally determined gaps, including the value measured in our experiments (0.034$V_C$ in Fig. 2(a)), however, are much smaller [2, 9]. Finite layer thickness, LL mixing, and disorder are possible causes for this reduction [2, 8, 10]. We note that LL mixing is particularly severe for electrons in AlAs because of their large band mass. The proximity of the second LL can be appreciated by noting that $E_c$ in our AlAs 2DES (0.17$V_C$) is a very small fraction of $V_C$.

For small enough $\eta$, for a single-valley, two-spin system, the excitations above the ferromagnetic ground state are macroscopic spin textures (skyrmions) [1]. The rate at which the $\nu = 1$ QH gap rises as a function of $E_Z$ gives direct information about the size of the skyrmions [2, 4, 3, 9]. In an ideal 2DES, the initial slope of $\Delta$ (measured in units of $V_C$) vs. $\eta$ should be infinite, then gradually decrease with increasing $\eta$ (signaling finite-size skyrmions) and, beyond a critical $\eta$ ($\approx 0.054$), should be equal to one [17]. In Fig. 2(a) we find an initial slope of 13, implying that if the excitations are skyrmions [11], they are large but finite-sized. Finite skyrmion size has also been reported for spin skyrmions near zero $E_Z$ [2], and is presumably because of disorder. We would like to emphasize that the saturation value of the energy gap in Fig. 2(a) ($\approx 0.19 V_C$), is close to the cyclotron energy (0.17$V_C$) in our 2DES. This is not surprising since in our system we expect the gap to be limited by the cyclotron gap (see fan diagram in Fig. 1(b)).

We now discuss our gap data at $\nu = 1/3$ (Fig. 2(b)). Theoretically, for an ideal single-valley, two-spin 2DES, a ferromagnetic ground state [8] with an energy gap of about $0.024 V_C$ [1] and skyrmionic excitations has been predicted at $\eta = 0$ [1, 20, 21]. As in the case of $\nu = 1$, the gap is expected to quickly increase with $\eta$, signaling the presence of skyrmions. Our measured $\nu = 1/3$ gaps, plotted in Fig. 2(b), are qualitatively consistent with the above expectations. The gap at $\eta = 0$ ($\approx 0.006 V_C$), however, is only about one-forth of the gap predicted for an ideal single-valley, two-spin 2DES. Also, the slope of $\Delta / V_C$ vs. $\eta$ is only slightly greater than one ($\approx 1.5$) suggesting that, if the excitations are skyrmions, their size is quite small. Finally, for very large strains the $\nu = 1/3$ gap should saturate at a constant, expected to be $\approx 0.1 V_C$ for an ideal, single-valley, single-spin 2DES [22]. Our data of Fig. 2(b) are again qualitatively consistent with this expectation except that the saturation value we measure ($\approx 0.015 V_C$) is much smaller than 0.1$V_C$. It is likely that the much reduced values of the gaps and the slope originate from finite-layer thickness, LL mixing, disorder [13] and Fermi contour anisotropy. We also remark that in our system an overall strain inhomogeneity cannot be ruled out. At $\eta = 0$, this might lead to
the formation of ferromagnetic domains of opposite polarity. Charged excitations with energy gaps less than that of the 2D bulk skyrmions have been predicted at these domain walls [22].

Our results find a natural interpretation in light of the composite fermion (CF) picture [20, 22, 24, 25] where the fractional QHE of electrons is interpreted as the integer QHE of CFs. Every electronic fractional filling factor \( \nu \) has an integer CF counterpart \( p \). The fan diagram in Fig. 1(b) can in fact be applied to single-spin, two-valley CFs with the modification that \( E_c \) denotes the cyclotron energy for CFs. Note that \( \nu = 1/3 \) of electrons maps to \( p = 1 \) of CFs, consistent with the qualitative similarities seen in the gap data for \( \nu = 1 \) and 1/3 (Fig. 2). In a two-spin system, at \( \eta = 0 \), interaction between the CFs is expected to lead to a ferromagnetic QH state and skyrmionic excitations at \( \nu = 1/3 \) [20, 22]. This is qualitatively consistent with our observations. Note that according to the fan diagram of Fig. 1(b), the saturation gap for \( p = 1 \) is given by the CF cyclotron energy (\( \approx 0.17 \hbar c \) for an ideal 2DES) [22, 24].

The results of our gap measurements at \( \nu = 5/3 \) (Figs. 2(c) and 4(b)) are qualitatively similar to the \( \nu = 1/3 \) data except that the gaps are even smaller. The slope of gap vs. \( \eta \) in Fig. 2(c) is also smaller than that in Fig. 2(b). Within the theoretical framework, ideally, there is no distinction between the FQH states at \( \nu = 1/3 \) and 5/3 since they are related by particle-hole symmetry. However, in real systems this symmetry is broken by factors such as LL mixing which is particularly significant in our system because of the large effective mass. We would like to add that in our AlAs 2DES, at much higher strains (\( |\epsilon| \gtrsim 2.3 \times 10^{-4} \) for Fig. 1(c)), we observe a crossing of the electron valley LLs as a result of which the sample resistance near \( \nu = 5/3 \) dramatically increases thereby destroying the QH state [26].

We now compare our results to those of a previous study of the role of spin splitting on the \( \nu = 1/3 \) QHE in GaAs [17]. Leadley et al. measured the \( \nu = 1/3 \) energy gap as they tuned the \( g \)-factor (and thus \( \eta \)) through zero via the application of hydrostatic pressure. The magnetoresistance trace taken at \( \eta = 0 \) did not show a resistance minimum at \( \nu = 1/3 \) but, extrapolating the gaps measured at finite \( \eta \), they concluded a finite gap (\( \approx 0.017 \hbar c \)) at \( \eta = 0 \). In contrast, our data shows a deep resistance minimum with a strong \( T \)-dependence at \( \eta = 0 \). Also, in Ref. [7], an initial slope of 3 was deduced from the plot of gap vs. \( \eta \), about a factor of two larger than ours.

In other relevant studies [27, 28], enhanced slopes were reported for the \( \nu = 5/3 \) case, but the gaps extrapolate to a vanishing value in the limit of \( \eta = 0 \).

Besides AlAs, another system in which the electrons have a valley degree of freedom is a Si/SiGe 2DES where the two out-of-plane valleys are almost degenerate. Here however, the valley splitting cannot be easily tuned and a small valley splitting can be present [29]. In Ref. [29] it is shown that the QH state at \( \nu = 5/3 \) is absent, while the state at \( \nu = 1/3 \) is present. We point out that in our AlAs samples, near \( \epsilon = 0 \), we see a distinct resistance minimum at \( \nu = 5/3 \) only in highest quality samples and at very low temperatures. Notably, in an 11 nm-wide AlAs quantum well [12] at 50 mK, the resistance minimum at \( \nu = 5/3 \) is almost absent when the valleys are degenerate, suggesting a very small gap. We attribute this to the lower sample quality where the 2DES suffers more from interface roughness scattering due to the smaller well-width.

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