Efficiency of high-purity germanium detector at characteristic gamma energies of $^{198}$Au and $^{58}$Co and covariance analysis

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Abstract

Naturally occurring $^{197}$Au and $^{58}$Ni foils were subjected to the neutron irradiation by placing them in a dry tube-I of Kalpakkam Mini reactor to produce gamma emitting $^{198}$Au and $^{58}$Co nuclear reaction products. The efficiency study of high-purity germanium detector corresponding to characteristic gamma energies 0.4118 and 0.8107 MeV of $^{198}$Au and $^{58}$Co was carried out by the methods of calibration of $^{152}$Eu and covariance.

Keywords: Covariance, efficiency, high-purity germanium, Kalpakkam mini reactor

INTRODUCTION

The calibration source $^{152}$Eu consists of many characteristic gamma lines. It can be used for the efficiency calibration of high-purity germanium (HPGe) detector as its activity is known. It is not possible to measure directly the efficiency of HPGe detector for known energies of characteristic gamma lines of nuclear reaction products as their activities are unknown. However, it can be known using the methods of calibration of $^{152}$Eu and covariance. This finds an application in knowing the nuclear reaction cross section of $^{198}$Au and $^{58}$Co.

Geraldo and Smith$^{[1]}$ carried out work on least square methods and covariance matrix applied to the relative efficiency calibration of a Ge (Li) detector. Geraldo and Smith$^{[2]}$, carried out work on covariance analysis and fitting of germanium gamma-ray detector efficiency calibration data. Vidmar$^{[3]}$ carried out work on EFFTRAN-A monte carlo efficiency transfer code for gamma-ray spectrometry. Jose et al.$^{[4]}$ carried out work on the estimation of radioactive noble gas activity in Fast Breeder Test Reactor (FBTR)-A simple method of calibration of HPGe detector. Shivashankar$^{[5]}$, et al. carried out work on measurement and covariance analysis or reaction cross section for $^{58}$Ni(n, p)$^{58}$Co relative to cross section for the formation of $^{97}$Zr fission product in neutron-induced fission of $^{232}$Th and $^{238}$U at effective neutron energies $E_n = 5.89$, 10.11 and 15.87 MeV. Sheela$^{[6]}$, et al., carried out work on the efficiency of HPGe detector at characteristic gamma energies of $^{58}$Co and $^{115m}$ln in the reactions $^{59}$Co(n, 2n)$^{58}$Co and $^{115m}$In(n, n')$^{115}$In respectively. Further, they carried out the covariance analysis.

In the present study, the efficiency study of HPGe detector corresponding to characteristic gamma energies 0.4118
and 0.8107 MeV of $^{198}\text{Au}$ and $^{58}\text{Co}$ was carried out by the methods of calibration of $^{152}\text{Eu}$ and covariance.

**Experimental details**

Naturally occurring foils $^{197}\text{Au}$ and $^{58}\text{Ni}$ with 99.85% purity procured from Alfa Aesar, USA, have been irradiated with neutrons consisting of a spectrum of energies each with 3 h time by placing in dry Tube-I location of Kalpakkam Mini reactor to obtain nuclear reactions $^{197}\text{Au}(n, \gamma)^{198}\text{Au}$ and $^{58}\text{Ni}(n, p)^{58}\text{Co}$. The obtained products $^{198}\text{Au}$ and $^{58}\text{Co}$ are capable to emit gamma with characteristic energies 0.4118 and 0.8107 MeV respectively. Their gamma spectra have been measured using p-type co-axial vertical HPGe detector of Dounreay Stakeholder Group make available at FBTR laboratory, IGCAR. The resolution of the detector is 1.8 keV at 1332.5 keV. The measured spectra are shown in Figures 1 and 2. The measured spectrum of calibration source $^{152}\text{Eu}$ is shown in Figure 3. The point source $^{152}\text{Eu}$ capsulated in a thin disc of araldite. The analysis of the spectral data has been done both with softwares of APTEC Engineering Ltd., Canada and GENIE-2000 Canberra Industries Inc., Meriden, USA.

**RESULTS AND DISCUSSION**

Efficiency calibration of high-purity germanium for $^{152}\text{Eu}$

The full energy peak efficiency of HPGe detector was measured by considering nine gamma-ray energies ($E_i, 1 \leq i \leq 9$) of $^{152}\text{Eu}$ source. The source in our experiment was placed 8 cm from the detector. Hence, the correction factor $k_c$ due to coincidence summing effect was estimated using Monte Carlo simulation code EFFTRAN. The source was procured on March 1, 1999 with initial activity $A_0$ was 45500 Bq. The efficiency $\varepsilon(E_\gamma)$ of detector is given by

$$\varepsilon(E_\gamma) = \frac{CK\varepsilon_k}{I_\gamma A_0 e^{-0.693/T_{1/2}}}$$

(1)

Where $E_\gamma$ is $\gamma$ energy, $C$ is counts obtained from the measured $^{152}\text{Eu}$ gamma spectrum, $I_\gamma$ is $\gamma$ abundance, $T_{1/2}$ is half-life ($13.517 \pm 0.014$), $t$ is time elapsed between source and detector calibrations (17.92y). The decay data $I_\gamma$ at each of the mentioned energies and $T_{1/2}$ are retrieved from ENSDF data sets maintained by National Nuclear Data Center. The input data and the obtained detector efficiency $\varepsilon(E_\gamma)$ at each of the gamma ray energy of $^{152}\text{Eu}$ are listed in Table 1. The comparison of the detector efficiencies with and without correction factor due to coincidence summing effect is shown in Figure 4.

Covariance analysis for $^{152}\text{Eu}$

The uncertainty\(^{[4]}\) $(\Delta \varepsilon_i)$ in efficiency $\varepsilon_i$ ranging from $\varepsilon_1 (E_{\gamma1})$ to $\varepsilon_9 (E_{\gamma9})$ is obtained using the following relation.

$$\left(\Delta \varepsilon_i \right)^2 = \left(\frac{\Delta C_i}{C_i} \varepsilon_i \right)^2 + \left(\frac{\Delta I_\gamma}{I_\gamma} \varepsilon_i \right)^2 + \left(\frac{\Delta A_0}{A_0} / e_{\text{C}} \right)^2 + \left(\frac{\Delta \lambda_i}{\lambda_i} / e_{\lambda_i} \right)^2$$

(2)

where $\Delta C_i$, $\Delta I_\gamma$, $\Delta A_0$ and $\Delta \lambda_i$ are partial uncertainties $(e_{\text{C}}, e_{\lambda_i})$ in $C_i$, $I_\gamma$, $A_0$ and $\lambda_i$, respectively. $e_{\text{C}}$ is obtained with the following relation.
The obtained values of $\Delta \varepsilon_i$ and $\varepsilon_r$ are shown in Table 2.

The microcorrelation matrix $S_{ij}$ is shown in Table 3.

The covariance matrix is obtained with the following relation and the obtained values are shown in Table 4.

$$V_{\varepsilon \gamma} = \sum_{r=1}^{4} S_{ij} (r) \varepsilon_{r}, \quad i, j = 1, 2, \ldots, 9, 1 \leq r \leq 4$$

Where $\delta \varepsilon_i$ and $\Delta \varepsilon_j$ are partial uncertainties in efficiency $\varepsilon_i$ and partial uncertainty in $\gamma$, respectively.

The macro-correlation matrix $C_{\varepsilon \gamma}$ is obtained with the following relation and its values are shown in Table 5.

$$C_{\varepsilon \gamma} = \frac{V_{\varepsilon \gamma}}{\Delta \varepsilon_i \Delta \varepsilon_j}$$

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The covariance matrix $V_{\varepsilon \gamma}$ is obtained with the following relation and its values are shown in Table 6.
The linear parametric matrix \( Z \), design matrix \( A \) and fitting parameter matrix \( P \) are related by

\[
Z = P A \tag{7}
\]

Matrices \( Z \), \( A \) and \( P \) can be obtained with the following relations.

\[
Z = Z_i = \ln(E_i) = \sum_{k=1}^{m} \rho_k (\ln[E_{\gamma_k}])^{k-1} \tag{8}
\]

\[
P = \rho_k \tag{9}
\]

\[
A = A_k = (\ln[E_{\gamma_k}])^{k-1} \tag{10}
\]

Where \( 1 \leq i \leq 9 \), \( 1 \leq k \leq m \), \( m = 2, 3.8 \).

The covariance matrix \( V_\gamma \) can be obtained with the following relation.

\[
V_\gamma = [A^T \times V^{-1}_{\gamma} \times A]^{-1} \tag{11}
\]

\( A \) and \( V_\gamma \) matrices can be generated for various \( m \) values lying between 2 and 8. In the present case, \( m = 5 \) is considered. Their values are shown in Tables 7 and 8, respectively.

Fitting parameter matrix \( P \) is obtained with the following relation and its values are shown in Table 9.

\[
P = \left( V_\gamma A^T V^{-1}_{\gamma} \right) Z_j \tag{12}
\]

The least square condition states that the best estimate for parameter vector in the model is the one which minimizes the Chi-square statistics given by

\[
\chi^2 = (Z - AP)^T V^{-1}_{\gamma} [Z - AP] \tag{13}
\]

The obtained \( \chi^2 \) values for \( m = 2, 3, 4, 5, 6, 7 \) are 63.4, 18.6, 3.8, 1.45, 0.29, and 0.568, respectively. The two values for \( m = 5 \) and 6 are closer to one. \( m = 5 \) was considered for further calculation as this was less than the other.

The design \( A \), linear parametric \( Z \), efficiency covariance \( V_{\gamma} \), and correlation \( C_{\gamma} \) matrices,\(^8\)

For \(^{198}\)Au (0.411802 MeV) and \(^{58}\)Co (0.810759 MeV) were obtained with the following relations.

\[
A_i = (\ln \left[ E_{\gamma_i} \right])^{k-1}, \quad Z_i = A P, \quad \epsilon_i = e^\epsilon_i = 1, 2 \quad \text{and} \quad 1 \leq k \leq 2 \tag{14}
\]

\[
V_{\gamma i} = A_i^T V_{\gamma} A_i, \quad V_{\gamma i} = (\epsilon_i). (\epsilon_{\gamma_j}) \quad \text{for} \quad i = 1, 2, \text{and} \quad j = 1, 2, \tag{15}
\]

\[
C_{\gamma i} = \frac{V_{\gamma i}}{(\Delta \epsilon_i)(\Delta \epsilon_{\gamma j})} \tag{16}
\]

The obtained values of \( \epsilon_i \), \( V_{\gamma i} \) and \( C_{\gamma i} \) are shown in the Table 10.

**CONCLUSIONS**

The energy depended efficiency calibration of HPGe detector corresponding to characteristic gamma energies of \(^{152}\)Eu has been carried out. The fitting parameters have been estimated by Chi-square test. The efficiency corresponding to characteristic gamma energies 0.4118 and 0.8107 MeV.
Table 6: Covariance matrix

|      | V_{ij}      |
|------|-------------|
| 5.84E-05 | 7.26E-05   |
| 2.28E-05 | 2.82E-05   |
| 2.28E-05 | 2.28E-05   |
| 2.28E-05 | 2.28E-05   |
| 2.28E-05 | 7.78E-05   |
| 2.28E-05 | 2.28E-05   |
| 86.00      | 0.00540    |
| 2.28E-05 | 0.756      |
| 2.28E-05 | 2.28E-05   |
| 2.28E-05 | 2.28E-05   |
| 2.28E-05 | 2.28E-05   |
| 2.28E-05 | 7.01E-05   |
| 2.28E-05 | 4.39E-05   |
| 2.28E-05 | 8.73E-05   |
| 0.00301   | 9.0E-10    |
| 2.28E-05 | 1.00       |
| 2.28E-05 | 8.45E-05   |
| 2.28E-05 | 2.98E-09   |

Table 7: Design matrix A

\[
\begin{bmatrix}
1 & -2.10 & 4.43 & -9.33 & 19.65 \\
1 & -1.40 & 1.98 & -2.78 & 3.95 \\
1 & -1.06 & 1.13 & -1.21 & 1.29 \\
1 & -0.24 & 0.062 & -0.01 & 0.0038 \\
1 & -0.03 & 0.001 & -4.9E-5 & 1.8E-6 \\
1 & 0.082 & 0.006 & 0.0005 & 4.6E-5 \\
1 & 0.106 & 0.011 & 0.0011 & 0.00012 \\
1 & 0.261 & 0.068 & 0.068 & 0.0046 \\
1 & 0.342 & 0.177 & 0.177 & 0.013 \\
\end{bmatrix}
\]

Table 8: Covariance matrix V_p

\[
\begin{bmatrix}
3.9E-5 & -6.4E-6 & -7.7E-5 & 7.9E-5 & 1.7E-5 \\
-6.4E-6 & 3.0E-4 & -1.6E-4 & -5.7E-4 & -2.0E-3 \\
-7.7E-5 & -1.6E-4 & 1.2E-3 & 1.6E-3 & 4.7E-4 \\
7.9E-5 & -5.7E-4 & 1.6E-3 & 2.6E-3 & 8.3E-4 \\
1.7E-5 & -2.0E-3 & 4.7E-4 & 8.3E-4 & 2.6E-4 \\
\end{bmatrix}
\]

Table 9: Fitting parameters P

\[
\begin{bmatrix}
-5.973 \\
-0.809 \\
-0.017 \\
-0.120 \\
-0.059 \\
\end{bmatrix}
\]

Table 10: Estimated parameters at two gamma energies of the activated foils

| Gamma energy (MeV) | Efficiency | Covariance matrix | Correlation matrix |
|--------------------|------------|-------------------|--------------------|
| 0.4118E02          | 0.00450    | 2.98E-9           | 1.00               |
| 0.8107E59          | 0.00301    | 9.0E-10           | 0.756              |

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