Design of Nonlinear Passive Suspension System Using an Evolutionary Programming

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Abstract. This paper proposes a methodology of finding out a set of optimal system parameters in the design of a nonlinear vehicle passive suspension system (PSS) using an evolutionary programming (EP). The difficulties for nonlinear PSS design arise due to the displacement limitation of the suspension and tire deflection, nonlinear characteristics of the system parameters, and the conflict demands for ride comfort and road holding ability. The optimal system parameters are considered under the fitness function composed of all the desired performance of a half-car model. The result of simulations shows that the PSS designed by using EP has better performances than currently-used PSS.

1. Introduction

Compared with passive suspension, active suspensions give better performance \cite{1}. However, active suspension system is also rather complex, since it requires components such as actuator, high-pressure tanks for the control fluid, sensors for detecting the system state, etc. which makes itself heavier and more energy consumptive than a passive suspension system. Hence there rises an increasing concern about whether the pursuit of higher performance of semi-active or active system is worth that much cost and complexity \cite{2}. The PSS is cost economical, reliable and it requires no power consumption which is why it is gaining more attention in the consumer’s automobile.

The research of ride and handling characteristics for vehicle suspension systems faces a multi-object optimization problem where a variety of performance measures such as ride comfort, body motion, road holding and suspension travel, need to be optimized. There has been a question as to whether the PSS is a trade-off among these performance measures. In addition, the nonlinear characteristics of the parameters for dampers and springs have significant influence on the vertical oscillations and longitudinal pitch of the vehicle.

So far most of the studies dealing with the analysis and design of suspension systems adopt a linear suspension model. Although these approaches are simply implemented, they do no allow a full exploitation of the system parameters when they are applied to the nonlinear PSS. In order to improve the suspension performances and balance between the ride comfort, road holding ability and constraint

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of suspension travels, this paper proposes the use of EP to determine the optimum set of nonlinear system parameters for the nonlinear PSS.

The principle of multi-object optimization is reaching a compromise among conflicting objectives, rather than finding a single, globally optimal solution. Evolutionary programming [3-4] is parallel to global search techniques that share the concepts of evolution theory and natural genetics. No a prior knowledge of the dynamics is assumed and no derivative or environment information is necessary. The only concern for EP is the quality of the solution produced by each parameter set. Hence, EP is rather suitable for solving complex real-world problems such as the design of a nonlinear PSS.

The vehicle system is described by nonlinear differential equations subject to unknown excitations from road surfaces. Simultaneously considering the desired performance including body acceleration, pitch acceleration, suspension deflections and tire deflections, an optimum set of system parameters can be determined by the EP.

2. Vehicle model

The half-car model has been used extensively in suspension analysis. A schematic of this model is shown in Fig. 1. The car body is supported by front and rear suspension composed of nonlinear spring and damper. The dynamical equations of the model are described by

\[ m \ddot{z} = F_{s} + F_{d} + F_{s} + F_{r} \]  
\[ m \ddot{z} = F_{s} + F_{d} - k_{s} (z_{s} - w_{s}) \]  
\[ m \ddot{z} = F_{r} + F_{d} - k_{r} (z_{r} - w_{r}) \]  
\[ I_{z} \ddot{\theta} = l_{f} (F_{s} + F_{r}) - l_{r} (F_{s} + F_{r}) \]

where \( m \) and \( I_{z} \) are the car body mass and moment of inertia, \( z_{s}, z_{r}, \theta_{s} \) the car body vertical displacement at the center of gravity and pitch angle, \( F_{s}, F_{r} \) the nonlinear damping forces of the front/rear suspensions, \( F_{s}, F_{r} \) the nonlinear restoring force of the front/rear suspensions, \( k_{s}, k_{r} \) the spring constants of the front/rear tires, \( m_{f}, m_{r} \) the masses of front/rear wheels, \( z_{f}, z_{r} \) the vertical displacements of the front/rear wheels, \( l_{f}, l_{r} \) the distances of the front/rear suspension locations, and \( w_{f}, w_{r} \) the irregular excitations from the road surface. Let the vehicle state variable and disturbance vectors be defined as:

\[ X = [x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{7}, x_{8}]^T, \quad W = [\dot{\phi}_{f} \dot{\theta}_{s}]^T \]

where each state denotes front wheel deflection (\( x_{1} = z_{s} + l_{f} \theta_{s} - z_{f} \)), rear wheel deflection (\( x_{2} = z_{s} - l_{r} \theta_{s} - z_{r} \)), front tire deflection (\( x_{3} = z_{f} - w_{f} \)), rear tire deflection (\( x_{4} = z_{r} - w_{r} \)), vertical velocity of body (\( x_{5} = \dot{\phi}_{s} \)), pitch rate of body (\( x_{6} = \dot{\theta}_{s} \)), front suspension velocity (\( x_{7} = \dot{\phi}_{f} \)), and rear suspension velocity (\( x_{8} = \dot{\phi}_{r} \)), respectively. The \( w_{f}(t) = w_{r}(t + \Delta t) \), where \( \Delta t = (l_{f} + l_{r})/V \) is irregular excitation from the road surface.
The nonlinear damping forces \( F_c \) and \( F_r \), and the nonlinear restoring forces \( F_{kr} \) and \( F_{fr} \), can be respectively represented as \( j \) pieces of linear functions [5],

\[
F_{ci}^j(t) = c_i^j(\xi) + d_{ci}^j, \quad F_{ti}^j(t) = k_i^j(x_i) + d_{ti}^j, \quad i = f, r, \quad j = 1, \ldots, m \tag{6}
\]

where \( c_i^j \) and \( k_i^j \) are dependent upon suspension velocity \( \dot{x}_i \), \( i = f, r \) \( (\xi = \dot{x}_f, \dot{x}_r = \dot{x}_r) \) and suspension deflection \( x_i \), \( i = f, r \) \( (x_f = x_1, x_r = x_2) \), respectively, \( d_{ci}^j \) and \( d_{ti}^j \) are the parameter needed for the continuity of the two functions. The dynamic system can be described using a state space model of order 8 as:

\[
\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t) + \mathbf{E}(t)\mathbf{w}
\]

where

\[
\mathbf{A}(t) = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\eta_{f} & \eta_{r} & 0 & 0 & 0 & 0 & 0 & 1 \\
-\eta_{f} & -\eta_{r} & 0 & 0 & 0 & 0 & 0 & 1 \\
-\eta_{f} & -\eta_{r} & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad \mathbf{B}(t) = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
-\delta_{f} & -\delta_{r} & 0 & 0 & 0 & 0 \\
-\delta_{f} & -\delta_{r} & 0 & 0 & 0 & 0 \\
-\delta_{f} & -\delta_{r} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad \mathbf{E}(t) = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(7)

(8)

In this paper, the parameters are chosen as \( c_f = c_r \) between 500-2000 Ns/m and \( k_f = k_r \) between 10000-30000 N/m.

3. Evolutionary programming and the scheme for evolving PSS

Evolutionary programming techniques have been applied in solving continuous optimization problems when other techniques such as gradient-based optimization techniques are faulty [6-10]. The EP method has many significant properties, such as parallel search, no constraint on the representation, no derivative information or other auxiliary knowledge required. It adopts probabilistic transition rules, provides a number of potential solutions to a given problem, and can be applied immediately. Hence, for the task of designing PSS, the EP is applied in this study.

The proposed EP procedure is implemented as follows:

Step 1: Form an initial population \( P_0 = [s_1, s_2, \ldots, s_N] \) of \( N \) trial solutions, each solution is taken as a pair of real valued vectors. The PSS performance depends on the parameters \( c_i^j, d_{ci}^j \) and the \( k_i^j, d_{ti}^j \) of (6), thus each potential solution in the population is represented as a \( 1 \times 8 \times (4 \times i(=2) \times j) \) vector defined as \( s_i = \{o_{nj} \in \mathbb{R}\} L_n \leq o_{nj} \leq U_n, \ n = 1, 2, \ldots, 8, \ j \in N \}. \) The \( L_n \) and \( U_n \) are constants and \( o_{nj} \in \Gamma \), where \( \Gamma = \{c_i^j, d_{ci}^j, k_i^j, d_{ti}^j\} \), \( i = f, r \) and \( j = 1, \ldots, m \). Suppose we introduce variables \( v_{nj} = 0 \leq x_{nj} \leq 1 \), such that \( o_{nj} = L_n + v_{nj} (U_n - L_n) \), then \( s_i \) can be denoted as \( v_i = [v_{nj}, \ n = 1, 2, \ldots, 8, \ j \in N] \), which corresponds to the object variables. The problem for determining optimum system parameters of PSS can be stated as: \textit{optimize} \( v_i \).
Fig. 1. Structure of a PSS for a half-car.

Fig. 2. The grade of goodness for each performance measures.

Fig. 3. The final evolved characteristic functions. The nonlinear damping forces. (b) The nonlinear restoring forces. Front (EP:—), Rear (EP:-), Front (Common: — –), Rear (Common: ‘—’).
Fig. 4. Time responses to a rough road. (a) Vertical acceleration of the vehicle body (b) Pitch acceleration of the vehicle body. EP (—), Hard (-), Common (— —), Soft (…).

Fig. 5. Time responses to a rough road. (a) Front suspension deflection. (b) Rear suspension deflection. EP (—), Hard (-), Common (— —), Soft (…).

Fig. 6. Time responses to a rough road. (a) Front tire deflection. (b) Rear tire deflection. EP (—), Hard (-), Common (— —), Soft (…).

**Step 2:** Evaluate the fitness scores for each member $v_i$, $i \in \{1,2,..., N\}$, of the population based on the associated objective function. The assignment of the fitness in EP serves as a guidance to lead the search toward the optimal solution. The objective of the nonlinear PSS design is to improve the vehicle dynamical performance expressed in terms of the ride comfort, handling ability, and the suspension travels. Thus, the fitness function is defined as
\[ f_i = \sum_{\tau=1}^{\Omega} \left[ g_1(x_1(\tau)) + g_2(x_2(\tau)) + g_3(x_3(\tau)) + g_4(x_4(\tau)) + g_5(\delta(\tau)) + g_6(\delta(\tau)) \right] / \Omega \]  

(9)

where \( \tau \) is sampling number and \( g_i, i = 1, \ldots, 6 \), are used to express the grade of goodness of each performance measure. The body displacements \( x_1 \) and \( x_2 \) represent a measure of design constraint on available suspension stroke, the front/rear tire deflections \( x_3 \) and \( x_4 \) are related to the handling ability, while the body acceleration \( \delta \) and the pitch acceleration \( \delta \) account for the ride comfort. The grades of goodness for each performance measures are described in Fig. 2. It is noticed that these evaluative functions are flexible to adjust for each performance measure according to designer’s requirements. The fitness function \( f_i \) is used to evaluate the potential solution to reflect how well the individual solves the task. In other words, as \( f_i \) is increased as greatly as possible, the desired performance of the PSS corresponding to the string will behave as well as possible.

**Step 3:** Generate one offspring \( \mathbf{v}_i(t) \) from each parent \( \mathbf{v}_i(t) \) as follows:

\[ \mathbf{v}_i(t) = \mathbf{v}_i^n(t) = \mathbf{v}_i(t) + N_1(\delta, \sigma^2_j, f_i, f_i^2) , \quad \forall n \in \{1, \ldots, 8j\} \]  

(10)

where \( t \) is the current generation number varying from zero to maximum generation number \( T \), \( \mathbf{v}_i(t) \) denotes the \( n \)th component in the \( i \)th vector among \( N \) vectors at the \( t \)th generation, \( N_1(\delta, \sigma^2_j, f_i, f_i^2) \) presents a Gaussian random variable with a mean \( \delta \) and variance \( \sigma^2_j \), \( f_i^2 \) is the sum of fitness in the generation \( t \), \( \alpha \) is constant and \( \beta \) is a coefficient to scale \( f_i^2 / f_i^2 \). The concept of equation (10) reveals that the worse performing individuals change more often than the better ones. At the beginning of the EP iterations, the population is diverse due to the initial randomness. The fitness-based Gaussian random variable can help the population mutate towards the higher fitness. Over iterations, the population should converge to smaller sections in the search space.

**Step 4:** Evaluate the new fitness for each offspring \( \mathbf{v}_i(t) \), \( \forall i \in \{1, \ldots, N\} \).

**Step 5:** Conduct pairwise comparison over all the solutions \( 2N \). For each solution, \( q \) randomly selected opponents are chosen from \( \mathbf{v}_i(t) \) with equal probability. In each comparison, if the conditioned solution offers at least as good performance as the randomly selected opponent, it receives a “win”.

**Step 6:** Select the \( N \) solutions out of all the \( 2N \) solutions that have the most wins to be as parents of the next generation.

**Step 7:** Go to Step 3 until the acceptable solution has been discovered or the maximum number of iterations is met.

4. Simulation and results

For the half-car suspension system discussed in Section 2, the commonly used half-car suspension parameters [5] are adopted in this study. The suspension springs and dampers operate chiefly on the driving comfort and operating safety. To find the fittest characteristic functions of the nonlinear damping forces and the nonlinear restoring forces so that the optimum performance of the PSS can be achieved, these parameters \( c_j, d_j, k_j \) and \( d_j \) are considered as the object variables in the EP searching. For practical implementation, these two nonlinear functions are represented as piece-wise linear functions, i.e. \( j = 1, 2, 3 \), respectively.
In off-line evolution, we construct a 6 second tested road profile [11] that includes various terrains for the training of EP. Once the EP evolution procedure converges, the final evolved characteristic functions of the nonlinear damping forces and the nonlinear restoring forces are depicted in Fig. 3. To the evolved nonlinear functions, the time responses of the vertical and pitch accelerations of the vehicle body, the suspension deflections, and the tire deflections for this kind of road are depicted in figures 4-6, respectively. We can tell from these figures that the EP is able to find an optimum set of system parameters to improve the system performances and obtain the best compromise in comparison with the soft, hard and commonly used suspension systems. The soft or hard suspensions in this paper mean that the parameter characteristics are reduced or increased 3 times from the commonly used one, respectively.

As a whole, the evolved system by EP has the highest fitness score, which suggests it can help the suspension system to improve system performance and achieve a best compromise between the designer’s demands. The simulations have shown that the overall performance is effectively improved by the set of parameters determined by the EP.

5. Conclusions
In this paper, we have exploited evolutionary programming techniques for the optimization problem of the half-car PSS. The benefits of EP are that this method can utilize multiple objective functions, it does not need extra mathematical analysis, and no derivative or environment knowledge is necessary. According to the desired performance, we have proposed an integrated fitness function that contains the body and pitch accelerations, suspension deflections, and tire deflections. The simulation results indicate that the proposed training scheme is capable of discovering an optimum set of system parameters for the PSS to achieve satisfactory performance in the ride comfort, road holding ability and available suspension stroke under various vehicle speeds.

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