DILATONIC BLACK HOLES IN A S-DUALITY MODEL

S. Monni and M. Cadoni

Dipartimento di Scienze Fisiche,
Università di Cagliari,
Via Ospedale 72, I-09100 Cagliari, Italy.
and
INFN, Sezione di Cagliari.

ABSTRACT

We find exact charged black hole solutions of a string effective action that is invariant under S-duality transformations. These black hole solutions have the same causal structure as the Reissner-Nordstrom (RN) solutions. They reduce to the RN solutions for self-dual configurations of the dilaton and to the Garfinkle-Horowitz-Strominger (GHS) solution in the weak (or strong) coupling regime. Using the purely magnetic solutions of the S-duality model we also generate dyonic black hole solutions of the GHS model, which have the causal structure of the RN solutions.

PACS: 04.70.Bw, 11.25.-W
E-Mail: CADONI@CA.INFN.IT
1. Introduction.

One feature of four-dimensional string effective theory is the existence of charged black hole solutions that are drastically different from the Reissner-Nordstrom (RN) solutions of general relativity [1]. String theory predicts the existence of scalar fields, such as the dilaton and the moduli, whose couplings to the Maxwell field $F_{\mu\nu}$ enable one to circumvent the uniqueness and no hair-theorem stating that the only static black hole solution to the Einstein-Maxwell equations is the RN solution [2]. The coupling of the dilaton and the moduli to the Maxwell field is parameterized by a coupling function $f$, which therefore determines the strength of the gauge couplings. At the tree level in the string perturbation theory (spherical worldsheet topology) the function $f$ depends, exponentially, only on the dilaton [3]. The charged black hole solutions of the tree-level string action have been found by Gibbons and Maeda [4] and later rediscovered by Garfinkle, Horowitz and Strominger (GHS) [1]. These black hole solutions have features that make them drastically different from the RN solutions. They have an event horizon but no inner (Cauchy) horizon and in the extremal limit the area of the event horizon goes to zero giving zero entropy. In this limit, for a magnetically charged black hole, the event horizon moves off to infinity in all directions differently from the RN case.

This new framework for the description of charged black holes poses new questions and problems on the subject, which have been extensively debated in the literature [5]. The low-energy string effective action used by the GHS model is just a first approximation that holds at the tree level in the string perturbation theory and at the leading order in the inverse string tension $\alpha'$. Perturbative corrections to the coupling function $f$ and/or nonperturbative effects may, in principle, change the description of the charged black holes in string theory. The basic question involved here is as follows: Is the above description of charged black holes a general feature of string theory or is it just an artifact of the particular approximation? Progress in this direction has been made in ref. [6,7], but a general answer to this question is still lacking. A second and related problem is the relationship between the string and the RN description of charged black holes. Because string effective theory reduces to general relativity in the region of weak string couplings, where the dilaton $\Phi$ is approximately constant, one would expect there the RN solutions to be a good approximation to string charged black holes. However, the purely magnetic (or purely electric) GHS black hole does not reduce in any approximation to the RN solution. This fact led the authors of ref. [1] to the conclusion that the RN solution is not even an approximate solution of string theory. This behavior can be traced back to the fact that in the GHS model the dilaton has an exponential coupling to $F^2$, so every solution with $F^2 \neq 0$ must have a nonconstant dilaton. Again, this is true only for $f = \exp(-2\Phi)$ and for purely electric (or purely magnetic) configurations, with a different choice for the coupling function $f$ or for dyonic configurations the situation could change drastically. Dyonic solutions of the GHS model, which have the causal structure of the RN black hole, have been already found by Gibbons and Maeda [4].

In this paper we will tackle the previous problems by studying a model with a coupling function $f = \cosh(2g\Sigma)$. If one interprets the field $\Sigma$ as the dilaton, the model can be viewed as a way to implement the S-duality symmetry, which has been conjectured to hold in string theory [8], in the context of a low-energy string effective action.
the other hand, if one considers $\Sigma$ as a modulus, the same model can be thought as an approximation to the effective action resulting from toroidal or orbifolds compactifications. In this case the effective action is known to be invariant under the T-duality symmetry $\Sigma \rightarrow -\Sigma$ [9]. We will find exact charged black hole solutions for the models with $g^2 = 1, 3$. These black hole solutions have the same Carter-Penrose diagram as the RN solutions. In particular, the solutions with $g^2 = 1$ differ from RN case just in the areas of the spheres with $r = \text{const}, t = \text{const}$. Moreover, the following feature of these black holes will emerge: they reduce to the RN solutions for self-dual configurations of the field $\Sigma$ and to the GHS solution in the weak (or strong) coupling regime. Using the purely magnetic solutions of the S-duality model we will also be able to generate dyonic black hole solutions of the GHS model, which again have the causal structure of the RN solutions.

The outline of the paper is the following. In sect. 2 we describe the model we investigate. In sect. 3 we solve the field equations of the theory by reducing them to an equivalent, Toda molecule, dynamical system and we analyze in detail the corresponding charged black hole solutions. In sect. 4 we use our model to generate dyonic black hole solutions of the GHS model. Finally, in sect. 5 we present our conclusions.

2. The model.

We shall consider a model described by the following action:

$$A = \int d^4x \sqrt{-g} \left( R - 2(\nabla \Sigma)^2 - \cosh(2g\Sigma)F^2 \right),$$

where $R$ is the scalar curvature, $\Sigma$ a scalar field, $F$ the abelian gauge field strength and $g$ is a coupling constant. If one considers $\Sigma$ as the dilaton, this action differs from the usual low-energy 4D string action in the choice of coupling function $f(\Sigma)$ in the gauge field kinetic term $f(\Sigma)F^2$. At the tree level in the string perturbation theory $f = \exp(-2\Sigma)$ [3]. It has been conjectured that string theory is invariant under a discrete $SL(2, \mathbb{Z})$ symmetry associated with the field $S = \exp(-2\Sigma) + i\Theta$, $\Theta$ being the axion field [8]. In particular this invariance includes a symmetry $\Sigma \rightarrow -\Sigma$, which exchanges weak and strong string couplings, $g_s^2 \rightarrow 1/g_s^2$. In this context the model (2.1), which uses $f = \cosh(2g\Sigma)$, may be viewed as an S-duality invariant modification of the tree-level dilaton coupling function $f = \exp(-2\Sigma)$.

One can also interpret $\Sigma$ as a modulus field associated with an overall radius of compactification. In this case the action (2.1) represents a T-duality invariant model of the type considered in ref. [7]. It turns out that for toroidal compactifications and a large class of orbifolds, the coupling function, at the one-loop level in the string perturbation theory, can be split into the sum of the tree-level dilaton-dependent gauge kinetic function and a modulus-dependent term [9]:

$$f = e^{-2\Phi} + a \ln\left( |\eta(T)|^4(T + T^*) \right) + b,$$

where $T = \exp(2\Sigma/\sqrt{3}), \eta(T)$ is the Dedekind function, $\Phi$ is the dilaton and $a,b$ are some constants. In particular, the genus-one threshold correction term is invariant under the duality symmetry $(\Sigma \rightarrow -\Sigma)$. If one decides to study the strong coupling region ($\Phi \rightarrow \infty$)
and uses \( \cosh(2g\Sigma) \) as an approximation to the function \( f \) given by eq. (2.2), the function (2.2) reduces to the one in (2.1). In this case it is necessary to introduce in the action (2.1) a kinetic term for the dilaton. This term does not modify the solutions since the dilaton is uncoupled and, due to the no-hair theorem, it is constant (obviously, consistency requires this constant to be chosen very large). The solutions we are going to find will be examples of modulus solutions in a curved spacetime and they can be considered as an extension of some solutions previously studied in the flat space case [7].

3. Black hole solutions.

The field equations stemming from the action (2.1) are:

\[
R_{\mu\nu} = 2\nabla_\mu \Sigma \nabla_\nu \Sigma + 2 \cosh(2g\Sigma) \left( F_{\mu\rho} F^\rho_\nu - \frac{1}{4} F^2 g_{\mu\nu} \right),
\]

\[
\nabla^2 \Sigma = \frac{g}{2} \sinh(2g\Sigma) F^2,
\]

\[
\nabla_\mu \left( \cosh(2g\Sigma) F^{\mu\nu} \right) = 0.
\]

Spherically symmetric solutions of these equations can be found using an ansatz that reduces the system to a Toda-lattice form [4]:

\[
ds^2 = e^{2\nu} (-dt^2 + e^{4\rho} d\xi^2) + e^{2\rho} d\Omega^2,
\]

\[
F = Q \sin \theta \, d\theta \wedge d\varphi,
\]

where \( \nu \) and \( \rho \) are functions of \( \xi \) and \( Q \) is the magnetic charge. We consider here only magnetic monopole configurations for the electromagnetic (EM) field. Our results can be easily generalized to a purely electric configuration using the invariance of the field equations (3.1) under the EM duality transformation [10]:

\[
f \rightarrow f^{-1}, \quad F \rightarrow f F^*, \quad \Sigma \rightarrow \Sigma,
\]

with \( f = \cosh(2g\Sigma) \). The magnetic solutions of the theory with coupling function \( f \) are related to the electric solutions of the theory with coupling function \( f^{-1} \).

Defining \( \zeta = \nu + \rho \) and using the ansatz (3.2), the field equations (3.1) become (\( ' = d/d\xi \)):

\[
\zeta'' = e^{2\zeta},
\]

\[
\Sigma'' = gQ^2 e^{2\nu} \sinh(2g\Sigma),
\]

\[
\nu'' = Q^2 e^{2\nu} \cosh(2g\Sigma),
\]

with the constraint

\[
\zeta'^2 - \nu'^2 - \Sigma'^2 - e^{2\zeta} + Q^2 e^{2\nu} \cosh(2g\Sigma) = 0.
\]

Integrating the first equation in (3.3), the remaining eqs. (3.3) and the constraint (3.4) are equivalent, respectively, to the equations of motion and to the Hamiltonian constraint derived from the Lagrangian:

\[
L = \frac{\nu'^2}{2} + \frac{\Sigma'^2}{2} - V,
\]
where the potential $V$ is given by

$$V = -\frac{Q^2}{2}e^{2\nu}\cosh(2g\Sigma).$$

This lagrangian describes two particles of mass equal to one moving on a line and interacting through the potential $V$. The system (3.3) can be solved exactly for $g^2 = 0, 1, 3$. When $g^2 = 0$ the solutions are the RN solutions of general relativity. For $g^2 = 1, 3$ the equivalent dynamical systems represent the Toda molecule $SU(2) \times SU(2)$ and $SU(3)$ respectively [11]. We treat the two cases separately.

3.1 $g^2 = 1$.

After some manipulations, we find a three-parameter class of solutions describing asymptotically flat black holes with a regular event horizon:

$$e^{2\Sigma} = e^{2\Sigma_{\infty}} \left(1 + \frac{2\sigma}{r}\right),$$
$$ds^2 = -\frac{(r-r_-(r-r_+)}{r(r+2\sigma)} dt^2 + \frac{r(r+2\sigma)}{(r-r_-(r-r_+)} dr^2 + r(r+2\sigma)d\Omega^2. \tag{3.5}$$

The constants $\sigma$ and $\Sigma_{\infty}$ are respectively the scalar charge and the asymptotic value of the field $\Sigma$. They are defined through the asymptotic behavior $\Sigma \to \Sigma_{\infty} + \sigma/r$. The constants $r_+$ and $r_-$ are related to the mass, magnetic and scalar charges of the black hole through:

$$r_\pm = M - \sigma \pm \sqrt{M^2 + \sigma^2 - Q^2 \cosh(2\Sigma_{\infty})}. \tag{3.6}$$

The parameters $M$, $\sigma$, $Q$ and $\Sigma_{\infty}$ are not independent but are constrained by

$$\sigma = -\frac{Q^2}{2M} \sinh 2\Sigma_{\infty}. \tag{3.7}$$

The duality symmetry $\Sigma \to -\Sigma$ of the action (2.1) acts on the space of the solutions transforming $\sigma \to -\sigma$ and $\Sigma_{\infty} \to -\Sigma_{\infty}$. Therefore, we can restrict our discussion to the case $\sigma > 0$, $\Sigma_{\infty} < 0$. The solutions (3.5) describe black holes only for

$$M^2 + \sigma^2 - Q^2 \cosh 2\Sigma_{\infty} \geq 0. \tag{3.8}$$

We have a curvature singularity at $r = 0$ shielded by an inner (Cauchy) horizon at $r = r_-$ and by an outer (event) horizon at $r = r_+$. The equality in (3.8) holds in the extremal limit, $r_+ = r_-$. Using (3.7) the condition of extremality can be written in another form:

$$M^2 - \sigma^2 - Q^2 = 0. \tag{3.9}$$

This means that in the extremal limit the gravitational attraction is balanced by the repulsive forces of the magnetic and scalar fields. The solutions (3.5) represent a three-parameter class of solutions generalizing the well-known RN solutions, to which they reduce when we have a self-dual configuration for the field $\Sigma$, i.e for $\sigma = \Sigma_{\infty} = 0$. The presence
of the scalar charge modifies the area of the spheres \( r = \text{const}, t = \text{const} \) in the RN solution, but the main features of the latter are still preserved. In particular, one can easily verify, performing the usual Kruskal extension of the solutions (3.5), that the causal structure of the spacetime (the Penrose diagram) is exactly the same as in the RN case. The closed resemblance with the RN solution is also confirmed by the calculation of the thermodynamical parameters associated with the black hole. For the temperature and the entropy of the hole we find:

\[
T = \frac{1}{4\pi r_+(r_+ + 2\sigma)} , \quad S = \pi r_+(r_+ + 2\sigma).
\]

These formulae differ from the ones for the RN case only in the area of the spheres with \( r = \text{const}, t = \text{const} \).

It is also interesting to compare our solutions to the GHS solutions. The action (2.1) becomes the GHS action in the weak coupling regime \( \Sigma \to -\infty \) (due to the duality invariance of the action (2.1) also in the strong coupling regime \( \Sigma \to \infty \)). This regime can be studied by considering the behavior of the solutions (3.5) for \( \Sigma_\infty \to -\infty \). Using the eqs. (3.6) and (3.7), one easily finds that in this limit the inner horizon disappears. After the translation \( r \to r - 2\sigma \), one has \( r_- = 2\sigma = Q^2 \exp(-2\Sigma_\infty)/2M, r_+ = 2M \) and the solutions (3.5) become

\[
e^{-2\Sigma} = e^{-2\Sigma_\infty} \left(1 - \frac{Q^2 e^{-2\Sigma_\infty}}{2M r}\right),
\]

\[
ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r \left(r - \frac{Q^2 e^{-2\Sigma_\infty}}{2M}\right) d\Omega^2,
\]

which is the GHS solution (the redefinition \( Q^2 \to 2Q^2 \) is needed to match the conventions of ref. [1]). It is important to notice that the strong (or weak) coupling regime we consider here, is slightly different from the strong (or weak) string coupling region that one usually considers in the GHS model. Normally, one takes the asymptotic value \( \Sigma_\infty \) of the dilaton constant, the strong coupling region is then obtained by considering a spacetime region near the singularity where the dilaton diverges. Our strong coupling regime is obtained just by acting on the parameter \( \Sigma_\infty \), without any reference to a particular spacetime region. In particular, this means that in the strong coupling regime the theory is strong coupled even in the asymptotically flat, \( r \to \infty \) region. The lesson to be learned here is that the parameter \( \Sigma_\infty \) is crucial to understand fully the parameter-space of the black hole solutions in string effective theory. The relevance of this parameter is also evident if one considers it as the vacuum expectation value of the dilaton. It is well-known that at the tree level in the string perturbation theory this parameter is undetermined, though nonperturbative effect may fix it to some value, and that it is related to different possible string vacua.

\[3.2 \quad g^2 = 3.\]

Also here one can use the equivalent dynamical system given by the Toda molecule \( SU(3) \), to find the solutions of the field equations (3.3). This case has been treated by several authors (see for example [12] and references therein), here we will use a form of the solutions that is particularly suitable for our purposes. The asymptotically flat black hole solutions can be written in the form:
\[ e^{2\sqrt{3}\Sigma} = e^{2\sqrt{3}\Sigma_\infty} \left( \frac{P_2(r)}{P_1(r)} \right)^{3/2}, \quad (3.9) \]

\[ ds^2 = -\frac{(r-M)^2 - q^2}{\sqrt{P_1(r)P_2(r)}} \, dt^2 + \frac{\sqrt{P_1(r)P_2(r)}}{(r-M)^2 - q^2} \, dr^2 + \sqrt{P_1(r)P_2(r)} \, d\Omega^2, \quad (3.10) \]

where:

\[ P_1(r) = \left( r - \frac{\sigma}{\sqrt{3}} \right)^2 - \frac{Q^2 \sigma e^{2\sqrt{3}\Sigma_\infty}}{(\sigma - \sqrt{3}M)}, \]
\[ P_2(r) = \left( r + \frac{\sigma}{\sqrt{3}} \right)^2 - \frac{Q^2 \sigma e^{-2\sqrt{3}\Sigma_\infty}}{(\sigma + \sqrt{3}M)}, \]
\[ q^2 = M^2 + \sigma^2 - Q^2 \cosh(2\sqrt{3}\Sigma_\infty). \]

The parameters \( M, \sigma \) and \( \Sigma_\infty \) appearing in the previous equations are respectively the mass, the scalar charge and the asymptotic value of the field \( \Sigma \). (3.9) and (3.10) are solutions of the field equations only if these parameters are related to the magnetic charge by

\[ Q^2 \left( e^{2\sqrt{3}\Sigma_\infty} (\sigma + \sqrt{3}M) + e^{-2\sqrt{3}\Sigma_\infty} (\sigma - \sqrt{3}M) \right) = \frac{4}{3} \sigma (\sigma^2 - 3M^2). \quad (3.11) \]

We have a three-parameter class of solutions. In the same way as for \( g^2 = 1 \) the duality symmetry of the action (2.1) relates solutions with opposite signs of \( \sigma \) and \( \Sigma_\infty \). We will therefore consider only solutions with \( \sigma > 0, \Sigma_\infty < 0 \). The solutions (3.9), (3.10) represent black holes only for

\[ M^2 + \sigma^2 - Q^2 \cosh(2\sqrt{3}\Sigma_\infty) \geq 0, \]
\[ e^{2\sqrt{3}\Sigma_\infty} \leq \left( \frac{\sqrt{3}M - \sigma}{\sqrt{3}M + \sigma} \right)^{3/2}, \quad \sigma < \sqrt{3}M. \quad (3.12) \]

For these values of the parameters, \( P_1(r) \) is always positive whereas \( P_2(r) \) has two zeroes \( r_1, r_2 \), with \( r_1 < r_2 \). The solutions are defined for \( r > r_2 \). The spacetime has two horizons at \( r_\pm = M \pm q \), the inner of which screens the timelike singularity at \( r = r_2 \) to any observer in the exterior region, just like in a RN black hole. The Carter-Penrose diagram of the spacetime is therefore the same as in the RN case. When the equalities in eqs. (3.12) hold, the black hole becomes extremal and the spacetime has the causal structure of the extremal RN black hole. The solutions (3.10) reduce to the RN solutions in the self-dual configuration of the field \( \Sigma \), i.e. when the scalar charge \( \sigma \) vanishes, which implies from (3.11) also \( \Sigma_\infty = 0 \). In the weak (or strong) coupling regime \( \Sigma_\infty \to -\infty \) the solutions reduce to that found by GHS in ref. [1] for the model with coupling function \( f = \exp(-2\sqrt{3}\Sigma) \). In this regime the solutions (3.9),(3.10) and the constraint (3.11) become respectively (we rescale \( Q^2 \to 2Q^2 \) to match the conventions of ref. [1])

\[ e^{-2\sqrt{3}\Sigma} = e^{-2\sqrt{3}\Sigma_\infty} \left( 1 - \frac{r_\pm}{r} \right)^{3/2}, \]
\[ ds^2 = -\left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{-1/2} dt^2 + \left(1 - \frac{r_+}{r}\right)^{-1} \left(1 - \frac{r_-}{r}\right)^{1/2} dr^2 + r^2 \left(1 - \frac{r_-}{r}\right)^{3/2} d\Omega^2, \]

\[ Q^2 e^{-2\sqrt{3} \Sigma} = \frac{r_+ - r_-}{4}, \]

with the mass of the solutions given by \( 2M = r_+ - r_-/2 \).

To conclude this section let us calculate the thermodynamical parameters associated with the black hole solution (3.10). We have for the temperature and the entropy:

\[ T = \frac{1}{2\pi} \frac{r_+ - r_-}{\sqrt{P_1(M + q)P_2(M + q)}}, \]

\[ S = \pi \sqrt{P_1(M + q)P_2(M + q)}. \]

4. Dyonic black holes.

In the previous section we have seen that the implementation of a S-duality symmetry at the string effective action level changes drastically the structure of the black hole solutions with respect to those of the GHS model. We found not only that the black hole solutions of the action (2.1) have the causal structure of the RN solutions but also that the GHS solutions emerge as an approximation in the weak (or strong) coupling regime. Up to now nobody has shown that S-duality is a symmetry of string theory, it remains just a conjecture. One could therefore object that our results are just a peculiarity of our model (2.1) and that the true description of charged black holes in string effective theory is that given by the GHS model. In the following we shall show that our model (2.1) can be used to generate black hole solutions of the GHS model with both magnetic and electric charges. Surprisingly enough, these dyonic solutions turn out to be similar to our solution (3.5).

Let us consider the following action:

\[ A = \int d^4 x \sqrt{-g} \left( R - b(\nabla \Sigma)^2 - f(\Sigma) F^2 \right). \]  

(4.1)

where \( b \) is an arbitrary parameter. One can shown that the spherically symmetric solutions

\[ ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + e^{2\rho} d\Omega^2, \]  

(4.2)

of the action (4.1) with a purely magnetic configuration for the EM field and coupling function \( f \) given as follows:

\[ f = h + \frac{1}{h}, \]

\[ F = Q_M \sin \theta d\theta \wedge d\varphi, \]

(4.3)

coincide with spherically symmetric solution of the action (4.1) with the following dyonic configuration for the EM field and coupling function \( f \):

\[ f = h, \]

\[ F = -\frac{Q_M}{h} e^{\nu + \lambda - 2\rho} dt \wedge dr + Q_M \sin \theta d\theta \wedge d\varphi. \]  

(4.4)
In fact, the field equations for the EM field:

\[ \nabla_\nu (f F^{\nu \mu}) = 0 \]

are identically satisfied both for \( f, F \) given by (4.3) or by (4.4). The field equations for the metric and the field \( \Sigma \):

\[
R_{\mu \nu} = b \nabla_\mu \Sigma \nabla_\nu \Sigma + 2f \left( F_{\mu \rho} F^\rho_\nu - \frac{1}{4} F^2 g_{\mu \nu} \right),
\]

\[
\nabla^2 \Sigma = \frac{1}{2b} \frac{df}{d\Sigma} F^2
\]

remain invariant inserting for \( f \) and \( F \) the expressions (4.3) or (4.4). Using this equivalence, we can generate dyonic solutions for the GHS theory, i.e for \( f = h = \exp(-2\Sigma) \), from the magnetic solution of the model (2.1) with \( g^2 = 1 \). These dyonic solutions are given by (3.5), with \( Q^2 = 2Q^2_M \), and by the EM form:

\[
F = -\frac{Q_M}{r^2} e^{2\Sigma_\infty} dt \wedge dr + Q_M \sin \theta d\theta \wedge d\varphi.
\]

The electric charge \( Q_E \) of the solutions is related to the magnetic charge by

\[
Q_E = Q_M e^{2\Sigma_\infty}.
\]

It is also evident from the construction of the solutions that the S-duality symmetry of the action (2.1) is related to the EM duality of the field equations of GHS model. In fact, the field equations of the latter are invariant under the EM duality transformation:

\[ \Sigma \rightarrow -\Sigma, \quad F \rightarrow hF^*. \]

These dyonic solutions of the GHS model have the spacetime structure of the solutions with \( g = 1 \) discussed in sect. 3. In particular, as pointed out is sect. 3, they are very similar to the RN solutions and differ drastically from the purely magnetic (or electric) solutions found in ref. [1]. Using eq. (4.7) and from the discussion of sect. 3, one finds that the purely magnetic (electric) solutions of the GHS model can be found as a limit of the dyonic ones in the weak (strong) coupling regime \( \Sigma_\infty \rightarrow -\infty (\Sigma_\infty \rightarrow \infty) \). The dyonic solutions, (4.6), (3.5) have been already found by Gibbons [12] and Gibbons and Maeda [4]. In the latter paper it was also pointed out that the solutions have the same Penrose diagram as the RN solutions. Furthermore, the solutions in an explicit form and with a nonvanishing asymptotic value of the scalar field, as (3.5), have been lately found in [13]. As far as the case with \( g^2 = 3 \) is concerned, one can show, using arguments similar to those used for \( g^2 = 1 \), that the solutions (3.9), (3.10) also represent Kaluza-Klein black holes with \( Q_E = Q_M \exp(2\sqrt{3}\Sigma_\infty) \). These black hole solutions are similar to the Kaluza-Klein black hole solutions found in refs. [12,14].

5. Conclusions.

In this paper we have found charged black hole solutions of a string effective theory invariant under S-duality transformations. The picture that has emerged from the study
of these solutions is rather intriguing: the black hole solutions are similar to the RN solutions of general relativity, in particular they share with the last-named the causal structure, reduce to the RN case for self-dual configurations of the dilaton and to the GHS black holes in the weak (strong) coupling regime $\Sigma_\infty \to -\infty$ ($\Sigma_\infty \to \infty$). We have seen that this picture of charged black holes emerges also in the context of the GHS model if one considers dyonic configurations for the EM field. In view of these results one is led to conclude that the description of charged black holes in string theory can be reconciled with the RN description if one goes beyond the tree-level approximation for the coupling function $f$ or, even in this approximation, if one considers black holes with both electric and magnetic charges. The statement, affirming that the RN solutions are not an approximate solutions of string theory is, therefore, only true at the tree level in the string perturbation theory, where the coupling function $f = \exp(-2\Sigma)$ and if one considers only purely magnetic (or purely electric) EM field configurations. As we have shown considering a S-duality model, this statement can be invalidated both with a different choice for $f$ or with dyonic configurations for the EM field.

Though reasonable our description of charged black holes is still incomplete and far from giving a definitive answer to the question about the true description of charged black holes in string theory. Our results rely heavily on the existence of a S-duality symmetry of the string effective action, which exchanges strong and weak string couplings. This is just a conjectured symmetry of string theory and one cannot be sure that it really holds. On the other hand the dyonic solutions we have found for the GHS model represent a special case of the generic dyonic solution of this model. We are therefore not allowed to draw general conclusions from this particular case, without having full control of the general solution.

Even though one could prove that S-duality is a symmetry of string theory, it is not evident a priori that our choice for the coupling function $f$ is even a good approximation to the exact S-duality invariant coupling function. Apart from the fact that the coupling function $f$ has the form of a series of powers in the string coupling function $g_s^2 = \exp(2\Sigma)$ and that the genus-$n$ string-loop contributions contain the factor $g_s^{2(n-1)}$, little is known about the exact form of $f$. However, the main features of the solutions we found (existence of RN solutions for self-dual configurations of the dilaton, existence of a strong or weak coupling regime in which the solutions have the GHS form) seem to be consequence of the symmetry of the model, namely the S-duality symmetry. One would therefore expect that these main features do not depend on the particular functional form of $f$.

Acknowledgments.

We thank S. Mignemi for useful comments.

References.

[1] D. Garfinkle, G.T. Horowitz and A. Strominger, *Phys. Rev. D* 43 (1991) 3140.

[2] W. Israel, *Commun. Math. Phys.* 8, 245 (1968); J. Chase, *Commun. Math. Phys.* 19 (1970) 276.
[3] E. S. Fradkin, A. A. Tseytlyn, *Phys. Lett. B* **158**, 316 (1985); C. G. Callan, D. Friedan, E. J. Martinec and M. S. Perry, *Nucl. Phys. B* **262** (1985) 593.

[4] G.W. Gibbons and K. Maeda, *Nucl. Phys. B* **298** (1988) 748.

[5] G. T. Horowitz, *The Brill Festschrift*, B. L. Hu and T. A. Jacobson, eds, Cambridge University Press (1993); C.F.E. Holzhey and F. Wilczek, *Nucl. Phys. B* **380** (1992) 447; A. Shapere, S. Trivedi and F. Wilczek, *Mod. Phys. Lett. A* **29** (1991) 2677; A. G. Agnese, M. La Camera *Phys. Rev. D* **49** (1994) 2126.

[6] M. Cadoni, S. Mignemi *Phys. Rev. D* **48** (1993) 5536.

[7] M. Cvetič, A. A. Tseytlin, *Nucl. Phys. B* **416** (1994) 137.

[8] A. Font, L. E. Ibáñez, D. Lüst and F. Quevedo, *Phys. Lett. B* **249** (1990) 35; J.H. Schwarz, A. Sen, *Phys. Lett. B* **312** (1993) 105; *Nucl. Phys. B* **411** (1994) 35.

[9] L. J. Dixon, V. S. Kaplunovsky, J. Louis, *Nucl. Phys. B* **355** (1991) 649.

[10] M. K. Gaillard, B. Zumino, *Nucl. Phys. B* **193** (1981) 221.

[11] M.A. Olshanetsky and A.M. Perelomov, *Phys. Rep.* **71** (1981) 313.

[12] G.W. Gibbons, *Nucl. Phys. B* **207** (1982) 337.

[13] R. Kallosh, A. Linde, T. Ortín, A. Peet, *Phys. Rev. D* **46** (1992) 5278.

[14] G.W. Gibbons and D.L. Whiltshire, *Ann. Physics* (N.Y.) **167** (1986) 201.