Polyakov Loops and Finite-Size Effects of Hadron Masses in Lattice Full QCD

S. Antonelli\(^1\), M. Bellacci\(^1\), L.A. Fernández\(^2\), A. Muñoz-Sudupe\(^2\), J.J. Ruiz-Lorenzo\(^{1,2}\), R. Sarno\(^1\), A. Tarancón\(^3\), and A. Bartoloni\(^1\), C. Battista\(^1\), N. Cabibbo\(^1\), S. Cabasino\(^1\), E. Panizzi\(^1\), P.S. Paolucci\(^1\), G.M. Todesco\(^1\), M. Torelli\(^1\), R. Tripiccione\(^4\), P. Vicini\(^1\).

\(^1\) Dipartimento di Fisica, Università di Roma “La Sapienza”, P. A. Moro, 00185 Roma, Italy and INFN, Sezione di Roma.
\(^2\) Departamento de Física Teórica, Universidad Complutense de Madrid, Ciudad Universitaria, 28040 Madrid, Spain.
\(^3\) Departamento de Física Teórica, Universidad de Zaragoza, Pza de San Francisco s/n, 50009 Zaragoza, Spain.
\(^4\) INFN Sezione di Pisa, I-56100 Pisa, Italy.

Abstract

The polarization of Polyakov type loops is responsible for the difference between quenched and unquenched finite size effects on the QCD mass spectrum. With a numerical simulation, using different sea quarks boundary conditions, we show that we can align the spatial Polyakov loops in a predefined direction. Starting from these results, we propose a procedure to partially remove the Polyakov type contributions in the meson propagators.
The finite extent of the lattice is an important source of systematic errors in lattice QCD calculations. Theoretical and numerical analysis [1, 2] faced recently the problem of finite size effects in full lattice QCD. The conclusions of these analysis are that the behaviour of the hadronic masses as a function of the lattice size $L$ follows a power law $m_L = m_\infty + cL^{-\nu}$, that only asymptotically goes to the exponential decay predicted by the effect of virtual pions emitted from a point-like hadron [3].

The magnitude of finite-size effects is much smaller for quenched QCD than for full QCD, especially below $La \sim 1\text{fm}$: $\nu = 1-2$ for the quenched case and $\nu = 2-3$ for full QCD.

The reason of the difference can be understood using, for example, the hopping parameter expansion on the valence quark mass that leads to the following relation for the meson propagator [2]:

$$\sum_C k^{l(C)}_{\text{val}} \langle W(C) \rangle + \sum_C k^{l(C)}_{\text{val}} \sigma_{\text{val}} \langle P(C) \rangle$$  \hspace{1cm} (1)

where the sums extend over all possible closed paths $(C)$ of length $l(C)$, the $W(C)$ are the Wilson loops completely contained into the lattice while $P(C)$ are valence quark loops wrapping around the lattice in the spatial directions (Polyakov-type) and $\langle \cdot \rangle$ denotes field averages; the $\sigma_{\text{val}}$ represents the spatial boundary conditions on the valence quarks: $\sigma_{\text{val}} = +1$ for the periodic and $\sigma_{\text{val}} = -1$ for the antiperiodic cases.

The averaged Polyakov loop $\langle P \rangle$ is different from zero in full QCD, while it is zero in the confined phase of quenched QCD. This means that the second term in eq.(1), due to the loops which go around the lattice, gives different finite size effects between the quenched and the full QCD value of mesonic correlations.

To obtain finite size effects comparable with those of the quenched case we have to remove the Polyakov loop contributions.

In this paper we want to demonstrate that we can align the Polyakov type loops in a predefined direction using sea quarks boundary conditions. Starting from these results we propose a procedure to remove the second term of eq.(1).

On a finite lattice with periodic boundary conditions on the gauge fields there is a symmetry in the pure gauge action consisting in multiplying all links in the $\mu$ direction at a constant $x_\mu$–plane by the complex number $z_k$, that belongs to the centre of the gauge group $(Z_3)_{\mathbb{C}}$.
Under it the Polyakov loops in the \( \mu \) direction are not invariant, but they transform as

\[
P \rightarrow z_k P
\]

In full QCD we consider both the gauge and the fermionic action. In the fermionic action

\[
S_{\text{Wilson}} = -k \sum_{x, \mu} \left( \bar{\psi}(x)(1-\gamma_\mu)U_\mu(x)\psi(x+\mu) + \bar{\psi}(x)(1+\gamma_\mu)U_\mu^\dagger(x-\mu)\psi(x-\mu) \right) + \sum_x \bar{\psi}(x)\psi(x)
\]

the kinetic part is not invariant under \( Z_3 \), since on the boundary it is not possible to find a gauge transformation of the fermionic fields that cancels out the centre transformation on the gauge fields. Thus the symmetry, that in the quenched confined case guarantees that \( \langle P \rangle = 0 \), is explicitly broken by the kinetic part of the fermionic action. Because the non-invariant term is proportional to \( k \) the amplitude of this violation is more important for light sea quarks.

It is possible to summarize what happens on the lattice boundary. Making a double expansion in the full QCD, firstly with a strong coupling expansion on \( \beta \) and secondly with a hopping parameter expansion on the sea quark masses, we obtain the 3–d Potts Model with magnetic field. The introduction of the fermionic action in QCD is equivalent to turn on a magnetic field \( h \) which breaks the \( Z_3 \) symmetry. To study the situation on the lattice boundary we introduce a simple model of a single spin \( \Pi \) that can take the three possible values:

\[
\Pi_0 = 1, \quad \Pi_1 = e^{i2\pi/3}, \quad \Pi_2 = e^{-i2\pi/3}
\]

with a Hamiltonian

\[
H = h\Pi + h^\dagger\Pi^\dagger
\]

which for \( h \neq 0 \) is not \( Z_3 \) invariant and it is composed by terms like those which break the \( Z_3 \) symmetry in the Potts Model with magnetic field.
The $\Pi$ argument is related to the phase of Polyakov loop and the value of $h$ with the sea fermionic boundary conditions.

We summarize the interesting $h$ choices in fig. 1a-1e. We see that with $h = +|h|$ (periodic boundary conditions on sea quarks) there are two preferred states, while with $h = -|h|$ (antiperiodic boundary conditions on sea quarks) only the $\Pi = 1$ state is selected. This circumstance suggests a way to select the other two $Z_3$ states, as we can see from fig. 1d and 1e.

Thus we expect that with periodic boundary conditions on sea quarks both the $e^{i2\pi/3}$ and the $e^{-i2\pi/3}$ phases of the Polyakov loops are preferred by the system. This means that for each gauge configuration there will be domains in which they will point towards either one of the two directions $e^{i2\pi/3}$ and $e^{-i2\pi/3}$. No such structures exist with the antiperiodic boundary conditions and moreover Polyakov loops are likely to point towards 1 in the $Z_3$ space.

Moreover we can align the Polyakov loop in the $e^{i2\pi/3}$ (or $e^{-i2\pi/3}$) in the $Z_3$ space if we choose $-e^{-i2\pi/3}$ (or $-e^{i2\pi/3}$) boundary conditions on the sea quarks.

To check the foregoing suggestion we performed on APE100 a full QCD simulation [5] with 2 flavors Wilson fermions at $\beta = 5.3$ on a $8^3 \times 32$ lattice with $k_{\text{sea}} = 0.1670$. We performed two different runs, one with periodic boundary conditions on the sea quarks and the other with antiperiodic boundary conditions. We collect in both cases 600 thermalization trajectories plus other 1200. On the latter we perform, every 5 trajectories, a measurement of the spatial Polyakov loops. We use the smearing procedure for the measurement of Polyakov loops [6] for 10 values of smearing.

The situation for the phases of the spatial Polyakov loops are reported in figures 2 and 3. With antiperiodic boundary condition, fig.2, we obtain that the phase is close to zero. Otherwise with periodic boundary conditions, fig.3, we obtain that the phases are spread in regions near $e^{i2\pi/3}$ and $e^{-i2\pi/3}$.

We also impose the boundary condition $-e^{-i2\pi/3}$ and $-e^{i2\pi/3}$ on the sea quarks in a quick simulation on a $4^3 \times 6$ lattice with $\beta = 3.0$ and $k_{\text{sea}} = 0.1670$ and we verified that Polyakov loops point towards the $e^{i2\pi/3}$ and $e^{-i2\pi/3}$ correspondingly, see fig. 4.

From eq.(1) (see ref. [2]), we do not expect large differences between the meson masses calculated either with periodic boundary conditions on sea and valence quarks or antiperiodic boundary conditions on sea and valence quarks, because the sign (not the amplitude) of the second term is the same.
in both cases. But due to our results on the polarization of the Polyakov loops we conclude that the best boundary conditions are the antiperiodic, because their Polyakov loops have lower dispersion than with the periodic ones.

Moreover the previous analysis gives us a procedure to partially eliminate the Polyakov loops that contribute to eq.(1). In fact we can choose the antiperiodic boundary conditions on the sea quarks. In this way all the gauge configurations have Polyakov loops that point towards the 1 $\text{Z}(3)$ state. Then we invert three times the fermionic valence operator: with antiperiodic, with $-e^{i2\pi/3}$ and with $-e^{-i2\pi/3}$. The average of the meson propagators obtained with the three inversions has a factor $\sigma_{\text{val}} = 0$ in the second term of eq.(1). We note that the required CPU time for valence fermionic inversion is negligible compared to the CPU time needed to obtain the full QCD configuration and that in the full QCD simulations the CPU time scales as $\text{Volume}^{5/4}$.

The numerical simulations of this work have been done with configurations obtained using 2 months of CPU time of a 128 nodes APE100 machine.

**Acknowledgement**

We thank G. Parisi for many suggestions concerning this work and for many discussions. We acknowledge interesting discussions with E. Marinari.

We would like to thank F. Marzano, J. Pech, F. Rapuano for encouragements and support.

L.A.F., A.M., J.J.R.L. and A.T. acknowledge CICyT (Spain) for partial financial support. J.J.R.L. is also supported by a grant of MEC (Spain).
FIGURE CAPTIONS

Figure 1. The energy levels of the model of eq.(6) for different values of $h$. The notation for the states of $\Pi$ is that of eq.(5).

Figure 2. Histogram of the phase of the x, y and z components of the Polyakov loop for antiperiodic boundary conditions on the sea quarks. Data are from trajectory 440 to trajectory 1800. The value of smearing is 10. The lattice is $8^3 \times 32$.

Figure 3. The same of Figure 2. for periodic boundary conditions on the sea quarks.

Figure 4. Behavior of the phase of the average of the three spatial Polyakov loops as a function of the HCMA trajectories. Data are from trajectory 205 to trajectory 600. The value of smearing is 0. The lattice is $4^3 \times 6$. In fig.4a we report the case of periodic boundary conditions on the sea quarks; in fig.4b we report the case of antiperiodic boundary conditions on the sea quarks; fig.4c is obtained from the boundary condition $-e^{-i2\pi/3}$ and fig.4d from the boundary condition $-e^{i2\pi/3}$.

References

[1] M. Fukugita, H. Mino, M. Okawa, G. Parisi, A. Ukawa Phys. Lett. B 294 (1992) 380.

[2] S. Aoki et al. Talk presented at Lattice 93 Conference Dallas 1993.

[3] M. Lusher, Commun. Math. Phys. 104, 177 (1986)

[4] G. ’t Hooft, Nucl. Phys. B 153 (1979) 141.

[5] S. Antonelli, M. Bellacci, A. Donini, R. Sarno “Full QCD on APE100 Machines” Preprint n.972/93 Dep. of Phys. Universitá di Roma “La Sapienza”; submitted to Comp. Phys. Comm.

[6] M.Albanese et al. Phys. Lett B 192(1987)163.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-lat/9405012v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-lat/9405012v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-lat/9405012v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-lat/9405012v1