An Innovative Rough Set Model based on Fuzzy Covering for Ordered Decision Making

Ren HE¹, Wei-Wei Pan², and Liang Zheng²*

¹Haicheng College, Liaoning Radio and Television University, Anshan, Heilongjiang Province, 114200, China
²School of Sciences, Harbin Institute of Technology (Shenzhen), Shenzhen, Guangdong Province, 518055, China
*Corresponding author’s e-mail: icon_lzheng@hit.edu.cn

Abstract. Fuzzy and rough sets are considered to be complementary to each other, addressing different aspects of uncertainty and vagueness. The rough set theory has been extended and modified such that it can be applied to the preference analysis in ordered decision making. However, neither the ordered structure nor dominance degrees among the investigated objects can be revealed during the ordered decision making process. In this paper, an innovative rough set model based on fuzzy covering is developed for ordered decision making. First, the crisp preference relation rough set model and the fuzzy preference relation model for ordered decision making were introduced. Next we integrated the fuzzy preference relation with the crisp preference relation rough set model to propose an innovative rough set model based on fuzzy covering for ordered decision making. Using the innovative model, both the ordered structure and dominance degrees among the investigated objects can be revealed during the ordered decision making process. The proposed model was then compared with the crisp preference rough set model. Finally, the proposed model is utilized to analyze fuzzy preference data obtained from real experiments of ordered decision making. Results validated the effectiveness of the innovative model.

1. Introduction

The rough set theory, first described by Polish computer scientist Z. I. Pawlak in 1982 [1], is established based on a partition or an equivalence relation of the universe. In order to be applied to the preference analysis in ordered decision making, extensions and modifications have been made in recent years [2-5]. A hotspot is the fuzzy generalization of the classic rough set theory. Dubois and Prade [6], Chakrabarty et al. [7] introduced lower and upper approximations in the fuzzy set theory to set up the rough fuzzy set model and the fuzzy rough set model. He et al. [8] defined an inconsistent fuzzy decision system and their reductions, and developed discernibility matrix-based algorithms to deal with decision systems with numerical conditional attribute values and fuzzy decision attributes rather than crisp ones. Derrac et al. [9] presented a hybrid evolutionary algorithm for data reduction, using both instance and feature selection. It obtained high reduction rates on training sets which greatly enhance the behavior of the nearest neighbor classifier. Wang [10] combined the rough set theory with the fuzzy cognitive pairwise rating to construct a novel framework which is effective to construct a product platform for developing innovative tablets and efficient to derive insightful decision rules for targeting the ad-hoc user groups.
However, using the above fuzzy rough set models, neither the ordered structure nor dominance degrees among the investigated objects can be revealed in the preference analysis during the ordered decision making process. In this paper, we proposed an innovative rough set model based on the fuzzy covering to be used in the preference selection during the ordered decision making process. With our proposed model, not only the ordered structure but the dominance degrees among the investigated objects can be revealed during the process of ordered decision making.

2. An innovative rough set model based on fuzzy covering

Let $\phi$ be a fuzzy covering on the universe $U$ and $(F(U), \phi)$ is called the fuzzy covering rough space. For an arbitrary fuzzy relation $R$, $(F(U), R)$ is called the fuzzy-rough space.

**Definition 1**: Let $T$ is the fuzzy operator and $\theta$ is the implication operator of $T$. For any arbitrary fuzzy set $A \in F(U)$ and the object $x \in U$, the lower and upper approximation operators based on the fuzzy covering are defined as:

Lower approximation operator: $\underline{R}(A)(x) = \sup_{u \in U} T(R(x, u), \inf_{z \in U} \theta(R(z, u), A(z)))$; (1)

Upper approximation operator: $\overline{R}(A)(x) = \inf_{u \in U} T(R(u, x), \sup_{z \in U} \theta(R(u, z), A(z)))$. (2)

With the defining of lower and upper approximation operators, we can integrate the fuzzy preference relation with the rough set model to propose a preference rough set model based on fuzzy covering for the ordered decision making process.

**Definition 2**: For a given ordered decision making system $<U, A, D>, B \subseteq A$, $R^\geq$ and $R^\leq$ represent the fuzzy preference relations generated by the attribute $B$. For any arbitrary fuzzy set $A \in F(U)$ and the object $x \in U$, the lower and approximation operators based on the fuzzy covering are defined as:

Upward fuzzy covering lower approximation operator: $\underline{R}^\geq d^\geq_i(x) = \sup_{u \in U} T(R^\geq(x, u), \inf_{z \in U} \theta(R^\geq(z, u), d^\geq_i(z)))$; (3)

Upward fuzzy covering upper approximation operator: $\overline{R}^\geq d^\geq_i(x) = \inf_{u \in U} T(R^\geq(u, x), \sup_{z \in U} \theta(R^\geq(u, z), d^\geq_i(z)))$. (4)

Downward fuzzy covering lower approximation operator: $\underline{R}^\leq d^\leq_i(x) = \sup_{u \in U} T(R^\leq(x, u), \inf_{z \in U} \theta(R^\leq(z, u), d^\leq_i(z)))$; (5)

Downward fuzzy covering upper approximation operator: $\overline{R}^\leq d^\leq_i(x) = \inf_{u \in U} T(R^\leq(u, x), \sup_{z \in U} \theta(R^\leq(u, z), d^\leq_i(z)))$; (6)

where $X(x)$ presents the extent of the object $x$ subordinate to set $X$.

**Definition 3**: For a given fuzzy preference relation ordered decision making system $<U, A, D>, B \subseteq C$, $R^\geq$ and $R^\leq$ represent the fuzzy covering preference relations generated by the attribute $B$. $S^\geq$ and $S^\leq$ represent the fuzzy covering preference relations generated by $B \cup a$ where $a \notin B$. Assuming $B$ is given, then conditional importance of $a$ approaching $D$ can be defined as:

Upward conditional importance:

$$\text{sig}^\geq(a, B, D^\geq) = \frac{\sum_i \sum_{j \in d^\geq_i} (S^\geq d^\geq_i(x) - R^\geq d^\geq_i(x))}{\sum_i |d^\geq_i|}$$ (7)

Downward conditional importance:

$$\text{sig}^\leq(a, B, D^\leq) = \frac{\sum_i \sum_{j \in d^\leq_i} (S^\leq d^\leq_i(x) - R^\leq d^\leq_i(x))}{\sum_i |d^\leq_i|}$$ (8)

Global conditional importance:
3. Experiment and Analysis

In the fuzzy preference analysis, one important task is to evaluate the influence of attributes on the decision making such that the factors with most influence can be identified and consequently the ordered decision models can be established. The proposed preference rough set model based on fuzzy covering is applied to some experiments and the effectiveness of this proposed model will be validated.

**Example:** Ten papers need to be reviewed and evaluated. Each paper can be evaluated by two conditional attributes, i.e., originality ($A_1$) and writing quality ($A_2$), and one decision attribute $D$ with the values of “accept”, “accept after revision”, and “reject”, denoting by 3, 2, and 1, respectively, as shown in Table 1. In addition, $x_i$ in the table presents the $i$th paper, etc.

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|------|------|------|------|------|------|------|------|------|------|
| $A_1$ | 0.22 | 0.25 | 0.50 | 0.38 | 0.42 | 0.60 | 0.78 | 0.68 | 0.80 | 0.85 |
| $A_2$ | 0.19 | 0.27 | 0.31 | 0.40 | 0.45 | 0.52 | 0.68 | 0.78 | 0.80 | 0.92 |
| $D$   | 1    | 1    | 2    | 2    | 2    | 2    | 2    | 3    | 3    | 3    |

The fuzzy preference relation among the investigated objects can be evaluated by:

$$
sig(a, B, D) = \frac{\sum_{i \in d_i^x} (S^x d_i^x (x) - R^x d_i^x (x)) + \sum_{i \in d_i^z} (S^z d_i^z (x) - R^z d_i^z (x))}{\sum_i (|d_i^x| + |d_i^z|)}
$$

(9)

$$
\begin{align*}
    r_{ij}^> &= \frac{1}{1 + e^{-k(x_i - x_j)}} \\
    r_{ij}^< &= \frac{1}{1 + e^{k(x_i - x_j)}}
\end{align*}
$$

(10)

where $k$ is a positive constant, $r_{ij}^>$ represents the degree of object $x_i$ greater than object $x_j$, and $r_{ij}^<$ represents the degree of object $x_i$ less than object $x_j$. $(1 + e^{-kx})^{-1}$ is the transfer function $logsig$ which is frequently used in BP neural network, and $k = 10$ in this example. 

$\forall x_i, x_j \in U$, the preference relation matrix of $x_i$ and $x_j$ generated by $\{A_1\} \cup \{A_2\}$ becomes:

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|------|------|------|------|------|------|------|------|------|------|
| $x_1$ | 0.50 | 0.31 | 0.06 | 0.11 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| $x_2$ | 0.57 | 0.50 | 0.08 | 0.21 | 0.14 | 0.03 | 0.00 | 0.01 | 0.00 | 0.00 |
| $x_3$ | 0.77 | 0.60 | 0.50 | 0.29 | 0.20 | 0.11 | 0.02 | 0.01 | 0.01 | 0.00 |
| $x_4$ | 0.83 | 0.79 | 0.23 | 0.50 | 0.38 | 0.10 | 0.02 | 0.02 | 0.01 | 0.01 |
| $x_5$ | 0.88 | 0.85 | 0.31 | 0.60 | 0.50 | 0.14 | 0.03 | 0.04 | 0.02 | 0.01 |
| $x_6$ | 0.96 | 0.92 | 0.73 | 0.77 | 0.67 | 0.50 | 0.14 | 0.07 | 0.06 | 0.02 |
| $x_7$ | 0.99 | 0.98 | 0.94 | 0.94 | 0.91 | 0.83 | 0.50 | 0.27 | 0.23 | 0.08 |
| $x_8$ | 0.99 | 0.99 | 0.86 | 0.95 | 0.93 | 0.69 | 0.27 | 0.50 | 0.23 | 0.15 |
| $x_9$ | 1.00 | 1.00 | 0.95 | 0.98 | 0.97 | 0.88 | 0.55 | 0.55 | 0.50 | 0.23 |
Then the upward and downward fuzzy approximation matrices can be generated according to the upward and downward fuzzy preference relation matrices, listed as below:

### Table 4. Upward fuzzy lower approximation matrix.

|     | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| $R^c d^c_1 (x_i)$ | 0.50  | 0.57  | 0.77  | 0.83  | 0.88  | 0.96  | 0.99  | 0.99  | 1.00  | 1.00     |
| $R^c d^c_2 (x_i)$ | 0.00  | 0.00  | 0.00  | 0.21  | 0.31  | 0.48  | 0.72  | 0.73  | 0.77  | 0.82     |
| $R^c d^c_3 (x_i)$ | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.23  | 0.28  | 0.53     |

### Table 5. Downward fuzzy lower approximation matrix.

|     | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| $R^c d^c_1 (x_i)$ | 0.48  | 0.31  | 0.21  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00  | 0.00     |
| $R^c d^c_2 (x_i)$ | 0.90  | 0.86  | 0.67  | 0.70  | 0.64  | 0.65  | 0.23  | 0.00  | 0.00  | 0.00     |
| $R^c d^c_3 (x_i)$ | 1.00  | 1.00  | 0.97  | 0.99  | 0.99  | 0.92  | 0.67  | 0.80  | 0.62  | 0.50     |

### Table 6. Upward fuzzy upper approximation matrix.

|     | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| $R^c d^c_1 (x_i)$ | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00     |
| $R^c d^c_2 (x_i)$ | 0.52  | 0.69  | 0.79  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00     |
| $R^c d^c_3 (x_i)$ | 0.10  | 0.14  | 0.33  | 0.30  | 0.36  | 0.44  | 0.77  | 1.00  | 1.00  | 1.00     |

### Table 7. Downward fuzzy upper approximation matrix.

|     | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
| $R^c d^c_1 (x_i)$ | 1.00  | 1.00  | 1.00  | 0.79  | 0.69  | 0.52  | 0.28  | 0.27  | 0.23  | 0.18     |
| $R^c d^c_2 (x_i)$ | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 0.77  | 0.72  | 0.47  |          |
| $R^c d^c_3 (x_i)$ | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00  | 1.00     |

Based on the above results, the following conclusions can be made:
It can be seen that these conclusions agree with the classic fuzzy set theory.

In order to show the difference and advantage of our proposed fuzzy covering preference rough set model, the dependency of the decision $D$ on attributes $A_1$ and $A_2$ is calculated using crisp preference rough set model and the fuzzy covering preference rough set respectively, as listed in Table 8. As seen in Table 8, the upward, downward, and global dependencies on $A_2$ are all the same as 1 using the crisp preference rough set, whereas are different as 0.9159, 0.9362, and 0.9261 respectively, if our fuzzy covering preference rough set model are utilized.

Then we change the value of $x_3$ for attribute $A_1$ (in Table 1) from 0.5 to 0.55, and recalculate the dependency of the decision $D$ on attributes $A_1$ and $A_2$, with new results shown in Table 9. One can see from Table 9 that the upward, downward, and global dependencies on $A_1$ remain the same as before changing, if the crisp preference rough set model is utilized. However, all of the upward, downward, and global dependencies on $A_1$ were reduced after the change, via the use of our proposed model.

Table 8. Dependency of the decision $D$ on attributes $A_1$ and $A_2$ in the Example.

|                | Crisp preference rough set | Fuzzy covering preference rough set |
|----------------|----------------------------|------------------------------------|
|                | $A_1$                      | $A_2$                              | $A_1$                      | $A_2$                     |
| upward         | 0.7000                     | 1.0000                             | 0.8242                     | 0.9159                    |
| downward       | 0.8000                     | 1.0000                             | 0.8763                     | 0.9362                    |
| global         | 0.7500                     | 1.0000                             | 0.8503                     | 0.9261                    |

Table 9. Dependency of the decision $D$ after a change made in Table 1

|                | Crisp preference rough set | Fuzzy covering preference rough set |
|----------------|----------------------------|------------------------------------|
|                | $A_1$                      | $A_2$                              | $A_1$                      | $A_2$                     |
| upward         | 0.7000                     | 1.0000                             | 0.8025                     | 0.9159                    |
| downward       | 0.8000                     | 1.0000                             | 0.8689                     | 0.9362                    |
| global         | 0.7500                     | 1.0000                             | 0.8357                     | 0.9261                    |

This example and the analysis indicate that our proposed fuzzy covering preference rough set model can reveal not only the dominance among the investigated objects but the degree of dominance among the objects. Furthermore, our proposed model has a much better precision on reflecting the variation of the data than the crisp preference rough set model.

4. Conclusions

In this paper, we developed an innovative rough set model based on fuzzy covering for the ordered decision making process. The crisp preference relation rough set model and the fuzzy preference relation for ordered decision making were introduced, then the fuzzy preference relation was integrated with the rough set model to propose a preference relation rough set model based on fuzzy covering for ordered decision making. Using the innovative model, both the ordered structure and dominance degrees among the investigated objects can be revealed during the process of ordered decision making. The proposed fuzzy-rough set model was then compared with the crisp preference rough set model. Comparison indicated that our proposed model has a much better precision on reflecting the variation of the data than the crisp preference rough set model. Finally, the proposed model is utilized to analyze the fuzzy preference data obtained from real ordered decision making experiments. Results validated the effectiveness of our proposed innovative model. This proposed model can be used in the fields of feature selection, medical imaging, fault diagnosis, etc.
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