A central problem in fractal geometry is the dimension drop conjecture that states that a self-similar subset of $\mathbb{R}^1$ has Hausdorff dimension $\min\{s,1\}$, where $s$ is the similarity dimension, unless exact overlaps occur. A landmark article by M. Hochman [Ann. Math. (2) 180, No. 2, 773–822 (2014; Zbl 1337.28015)] resolved the dimension drop conjecture for self-similar sets whenever the cylinders are not super-exponentially close. Until recently, it was not known whether there even exist self-similar systems that are superexponentially close, yet have no exact overlap. Independently, the author of this article [Adv. Math. 379, Article ID 107548, 14 p. (2021; Zbl 1457.28006)] as well as B. Bárány and A. Käenmäki [Adv. Math. 379, Article ID 107549, 23 p. (2021; Zbl 1461.28001)] proved their existence. The article under review is a follow up to the former publication and the author establishes a general sufficient condition (Theorem 2.1) under which a parametrized family of iterated function systems must contain an example of an iterated function system that has superexponentially close cylinders with no overlap.

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28A80 Fractals
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