Revisiting the phase transition of AdS-Maxwell–power-Yang–Mills black holes via AdS/CFT tools

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In the present work we investigate the Van der Waals-like phase transition of AdS black hole solution in the Einstein–Maxwell–power-Yang–Mills gravity (EMPYM) via different approaches. After reconsidering this phase structure in the entropy-thermal plane, we recall the nonlocal observables such as holographic entanglement entropy and two point correlation function to show that the both observables exhibit a Van der Waals-like behavior as the case of the thermal entropy. By checking the Maxwell’s equal area law and calculating the critical exponent for different values of charge α and nonlinearity parameter q we confirm that the first and the second order phases persist in the holographic framework. Also the validity of the Maxwell law is governed by the proximity to the critical point.

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1. Introduction

Nowadays it has become evident the pertinence of application of the AdS/CFT duality [1, 2] in a wide variety of physical themes, ranging from quantum gravity models and string theory [3–6] to condensed matter physics [7, 8]. The gauge/gravity duality could shed new light in order to have a better insight of our universe, but the complete understanding of this conjecture still in progress.

Recently, the gravity coupled to a source of matter has been aroused intense studies. Seeing the trivial coupling with scalar and Maxwell field, it is natural to couple Yang–Mills fields to gravity, where the matter is described by a non-abelian (Yang–Mills) gauge fields and constructing their black holes solutions. The first one has been found in [9] and its extension to higher dimensions and higher derivative gravity models in [10, 11]. The next relevant step is the considering a nonlinear coupling solutions and their thermodynamics [12, 13]. The nonlinearity power type plays an extremely important role in the satisfaction of the energy conditions, i.e. Weak, Strong or Dominant [14]. As in the example of Born–Infeld electrodynamics case [15] which plays a key role in the resolution of point like singularities. In addition to this, in general relativity the nonlinear terms affect black hole formation significantly, it is therefore tempting to take such combinations into account. It is our conviction that the nonlinear Yang–Mills field can establish a link with the effective cosmological parameters to contribute to the distinction between the phantom and quintessence data of our universe. Then, the Yang–Mills theory’s in non-linearity framework adds further complexity to the already nonlinear gravity, thus expectedly the theory and its accompanied solutions become rather complicated, richer and naturally attracts interest.

Since the discovery of the phase transition by Hawking and Page [16], the black hole thermodynamics has received considerable attention in recent years. More specifically, the rise of phase transitions similar to van der Waals liquid/gas transitions in Reissner–Nordstrom AdS black holes for a different model of gravity [17–26]. Another recent landmark of this analogy is the probe of the critical behavior of the black holes using the AdS/CFT tools, one can cite non-local observables such as entanglement entropy, Wilson loop and two point correlation function [27–37] which play an important role in the study quantum information science. These tools are used extensively to characterize phases, as an order parameter for phase transitions and the thermodynamical behavior [38–43].

The aim of this paper is to contribute to this topic by reconsidering the phase transition of AdS black holes in Einstein–Maxwell–power-Yang–Mills (EMPYM) gravity [12]. Specifically, we will investigate the first and second order phase transition by different approaches including the holographic one.

The outline of the rest of the work is as follows: In the next section, we discuss thermodynamic properties and stability of the EMPYM-AdS black holes in (Temperature, entropy)-plan. Then, a
similar result has been found using the holographic approach in Section 3, in other words we use the entanglement entropy and two point correlation function to check the Maxwell's equal area law in various cases and calculating the critical exponent of the specific heat capacity which is consistent with that of the mean field theory of the Van der Waals. The last section is devoted to a conclusion in which we discuss the obtained results and spotlight the main potentially interesting applications.

2. Critical behavior of Einstein–Maxwell–power-Yang–Mills–AdS black holes in thermal picture

In this section, we give an overview on thermodynamics of N-dimensional for Einstein–Maxwell–power-Yang–Mills gravity with a cosmological constant $\Lambda$ described by the following action

$$I = \frac{1}{2} \int d^N x \sqrt{-g} \left( R + \frac{(N - 1)(N - 2)}{3} \Lambda - F_{\mu \nu} F^{\mu \nu} \right) - (Tr(F_{\mu \nu} F^{\mu \nu}))^Q, \quad (1)$$

where the trace element is $Tr(\cdot) = \sum_{(N - 1)(N - 2)/2} \cdot$, $R$ is the Ricci scalar, $q$ is a real positive parameter. The Yang–Mills and Maxwell fields are defined respectively as

$$F_{\mu \nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + \frac{1}{2} \sigma^{(a)} (\partial_\mu A_\nu + \partial_\nu A_\mu), \quad (2)$$

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (3)$$

With $c^{(a)}_{\mu \nu}$ is the structure constants of $(N - 1)(N - 2)/2$ parameter Lie group $G$ and $\sigma$ is a coupling constant, $A^{(a)}_\mu$ are the $SO(N - 1)$ gauge group Yang–Mills potentials, and $A_\mu$ is the usual Maxwell potential.

The corresponding metric solution for $N$ dimensional spherically symmetric line element reads as [12]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega^2, \quad (4)$$

where $d\Omega^2$ represents the volume of the unit $n$-sphere. According to [11,13], various solutions are admitted by Eq. (1). Here as [12], we will focus our study on the Einstein–Maxwell–power-Yang–Mills theory (EMPYM) with $N(= n + 2) \geq 4$ and $q \neq (n + 1)/4$, and then discuss the phase transition in thermal picture and holographic one using the entanglement entropy and the two point function correlation.

The solution of $N$-dimensional EMPYM black hole with negative cosmological constant under the condition of $q \neq (n + 1)/4$ is given by

$$f(r) = 1 - \frac{2m}{r^{n-1}} \frac{\Lambda}{3} - \frac{2(n - 1)C^2}{nr^{2n - 2}} + \frac{Q_1}{r^{4q - 2}}, \quad Q_1 = \frac{(n - 1)nQ^2}{n(4q - n - 1)}. \quad (5)$$

It is worth recalling that the parameter $m$ denotes the mass of such black hole, while $C$ and $Q$ represent the charges of Maxwell field and Yang–Mills field respectively.

Recalling the Van der Waals gas-liquid analogy, where the cosmological constant is treated as a thermodynamic pressure with

$$P = -\frac{\Lambda}{8\pi}. \quad (6)$$

In this sense, we can express the Hawking temperature, mass and entropy of black hole in term of the horizon radius $r_+$ in the extended phase space as

$$T = \frac{f'(r_+)}{4\pi} = \frac{n - 1}{4\pi r_+} + \frac{2(n + 1)P}{3} r_+ - \frac{(4q - n - 1)Q_1}{4\pi r_+^{4q - 1}}, \quad (7)$$

$$M = \frac{\rho_{\text{crit}}}{4\pi} \frac{(8\pi P r_+^{n+1} + 3r_+^{n-4q+1} Q_1 + 3r_+^{n-1} + 6(n - 1)C^2)}{nr_+^{2n-1}}. \quad (8)$$

$$S = \frac{\omega_{\text{eff}}}{4}. \quad (9)$$

The Yang–Mills potential $\Phi_Q$ and the electromagnetic one $\Phi_C$ can be written as

$$\Phi_Q = \frac{\rho_{\text{crit}} [(n - 1)Q^2 3r_+^{n-4q+1}]}{8\pi (4q - n - 1)Q_1}, \quad (10)$$

$$\Phi_C = \frac{\rho_{\text{crit}} (n - 1)C}{4\pi r_+^{n-1}}, \quad (11)$$

where $\rho_{\text{crit}} = \frac{2m(n+1)/2}{\mathcal{V}_{\text{crit}}}$ is the volume of the unit $n$-sphere. In fact, many other thermodynamical quantities can be also computed using similar techniques. Indeed, the free energy $F$ of black hole is given by

$$F = M - T \cdot S \quad (12)$$

and the heat capacity is

$$C_Q = \frac{\partial S}{\partial T} \quad (13)$$

In full generality, the quantities Eqs. (7), (8) and Eq. (9), obey to the first law of black hole thermodynamics in the extended phase space

$$M = TS + \Phi_Q Q + \Phi_C C + VP, \quad (14)$$

where $V$ is the Legendre transform of the pressure, which denotes the thermodynamic volume with $V = \left(\frac{\partial F}{\partial T}\right)_Q \Phi_Q$. The corresponding Smarr formula can be found by a scaling argument as

$$M = \frac{n}{n - 1} TS + \Phi_Q Q + \frac{2q - 1}{(n - 1)Q} \Phi_Q - \frac{2}{n - 1} VP. \quad (15)$$

Having shown the relevant thermodynamical quantities, we turn now to the review of the corresponding phase transition. For this purpose, we study the variation of the Hawking temperature as a function of the entropy

$$T = \frac{1}{12\pi n} \left[ -6C(1 - n)^2 \left(\frac{4S}{\omega}\right)^{\frac{1-2n}{2}} + \pi^2 + 3nQ_1(n - 4q + 1) \left(\frac{4S}{\omega}\right)^{-n} \right. \right.$$  

$$\left. + 3nQ(n - 4q + 1) \left(\frac{4S}{\omega}\right)^{-1/n} \right] \quad (16)$$

This variation plotted in Fig. 1 shows that the Hawking temperature is a monotonic function if $Q > Q_c$, but when $Q \leq Q_c$, it represents a critical point to be determined by solving the system

$$\left(\frac{\partial T}{\partial S}\right)_Q = \left(\frac{\partial^2 T}{\partial S^2}\right)_Q = 0. \quad (17)$$
Fig. 1. The temperature as a function of the entropy for different C and q, with \( n = 2 \).

Table 1

| (C, q)       | \( T_c \) | \( S_c \) | \( T_r \) |
|--------------|-----------|-----------|-----------|
| (0, 2)       | \( \frac{1}{4} \) | \( \frac{1}{4} \pi \) | \( \frac{2\sqrt{2\pi^2}}{3\sqrt{3\pi + 4\pi^2}} \) |
| \( (\frac{1}{4}, \frac{1}{3}) \) | \( \frac{\sqrt{7} + 1}{4\sqrt{2\pi^2}} \) | \( \frac{1}{4}(\sqrt{7} - 1)^2 \pi \) | \( \frac{2\sqrt{2\pi^2}}{3\sqrt{3\pi + 4\pi^2}} \) |

The solution of this equation is easily derived, the critical charge, entropy and temperature are given in Table 1 with for all the rest of the paper we keep \( n = 2 \).

The Fig. 2 displays the free energy \( F \) with respect to the Hawking temperature \( T \) for fixed value of charge \( Q \) under the critical one.

In this figure, the characteristic swallowtail behavior of the free energy shows the first order phase transition happen between large and small black holes. By the Fig. 2, we can derive the co-existence temperature \( T_c \) numerically. Substituting this value into Eq. (16), we get the values of \( S_1 \) and \( S_3 \) which are used to calculate the area \( A_1 \) and \( A_2 \) in Maxwell’s law (Eq. (18)).

\[
A_1 \equiv \int_{S_1}^{S_3} T(S)dS = T_c(S_3 - S_1) \equiv A_2
\]

The results are listed in Table 2.

It is clear that \( A_1 \) equals \( A_2 \) for different \( C \) and \( q \), so the equal area law still verified. For the second phase transition, we know that near the critical point, there is always a linear relation [12]...
log |T − T_c| = 3 log |S − S_c| + constant  

(19)

and the heat capacity behaves like

\[ C_Q \sim (T − T_c)^{−2/3} \]

(20)

where the critical exponent of the second order phase is −2/3, which is consistent with the mean field theory.

Having obtained the essential of the phase picture of thermal entropy of the AdS-Maxwell-power-Yang–Mills black hole, checked the Maxwell’s equal area law in the \((T, S)\)-plane and found the critical exponent relative to specific heat capacity, we will investigate the phase structure of the entanglement entropy and two point correlation function to see whether they have similar phase structure and critical behavior.

3. Phase transitions of Einstein–Maxwell–power-Yang–Mills-AdS black holes in holographic picture

Motivated by recent activities in holography [32–34], Here, we would like to detect whether the AdS/CFT tools like the entanglement entropy and the two point correlation function respect the critical Van der Waals-like behavior.

3.1. Holographic entanglement entropy

For a given quantum mechanical system described by a density matrix \( \rho \), with \( A \) is some region of a Cauchy surface of spacetime. The entanglement entropy between \( A \) and its complement \( A^c \) is defined as the von Neumann entropy by

\[ S_A = −\text{Tr}_A(\rho_A \log \rho_A), \]

(21)

where \( \rho_A \) is the reduced density matrix of \( A \): \( \rho_A = \text{Tr}_{A^c}(\rho) \). Following the formula of Ryu and Takayanagi [44,45], the entanglement entropy is given by

\[ S_A = \frac{\text{Area}(\Gamma_A)}{4G_N}, \]

(22)

where \( \Gamma_A \) is a codimension-2 minimal surface with boundary condition \( \partial \Gamma_A = \partial A \), and \( G_N \) is the gravitational Newton’s constant.

For our black hole configuration we take the region \( A \) to be a spherical one on the boundary delimited by \( \theta \), and the minimal surface can be parametrized by the function \( r(\theta) \). The minimal area such that the entanglement entropy can be written in this black hole background as:

\[ A = \omega_{n−2} \int_0^{\theta_0} r^{n−2} \sin^{n−2} \theta \sqrt{\frac{(r')^2}{f(r)} + r^2} \, d\theta, \]

(23)

where \( r' = \frac{dr}{d\theta} \) and \( \theta_0 \) is the boundary of the entangling region. The function \( r(\theta) \) is obtained by viewing Eq. (23) as a Lagrangian and solving the equation of motion:

\[ 0 = r'(\theta)^2 [\sin \theta r'(\theta)]^2 f'(r) − 2 \cos \theta r'(\theta) \]

\[ −2r(\theta)f(r)[r(\theta)\sin r''(\theta) + \cos r'(\theta))] + 3 \sin \theta r'(\theta)^2 \]

\[ + 4 \sin(\theta)r(\theta)^3 f(r)^2. \]

(24)

with the following boundary conditions

\[ r'(\theta) = 0, \quad r(0) = r_0. \]

(25)

To regularize entanglement entropy we subtract the area of the minimal surface in pure AdS whose boundary is also \( \theta = \theta_0 \) with

\[ r_{\text{AdS}}(\theta) = L \left( \frac{\cos \theta}{\cos \theta_0} \right)^2 - 1 \]

(26)

To solve the Eq. (24) with the conditions Eq. (25), we perform a numerical calculation during its we take \( \theta_0 = 0.2, 0.3 \) and 0.5 with different ratio \( \frac{\omega_{n−2}}{\omega_{n−1}} = 0, 0.9, 0.4 \) and 0.2. The Ultra Violet cutoff is chosen to be \( \theta_0 = 0.199, 0.299 \) and 0.499 respectively. We present in Fig. 3 the relations between the holographic entanglement entropy and temperature for different values of parameters \((C, q)\) with a fixed charge \( Q \) and the chosen \( \theta_0 \). The red lines correspond to a charge less than the critical one, equal to the critical charge are plotted in blue dashed line and orange line for the upper than critical charge. We have also drawn the coexistence temperature \( T_c \) in dashed gray line taken from the Helmholz energy function presented in Fig. 2 and the critical temperature in green dashed line. The Van der Waals-like phase transition picture noted in Fig. 1 is observed also in all plots of the Fig. 3. Especially, the first order phase transition temperature \( T_c \) and second order phase transition temperature \( T_c \) are exactly the same as that in the thermal entropy structure.

Next, we proceed to check whether Maxwell’s equal area holds. We will refer to the two areas to be calculated as \( A_1 \) and \( A_2 \):

\[ A_1 = \int T(\Delta S_A, Q) \, d\Delta S_A - T_s(\Delta S_A^{(2)} - \Delta S_A^{(1)}) \]

(27)

\[ A_2 = T_s(\Delta S_A^{(2)} - \Delta S_A^{(1)}) - \int T(\Delta S_A, Q) \, d\Delta S_A \]

(28)

where \( \Delta S_A^{(1)}, \Delta S_A^{(2)} \) and \( \Delta S_A^{(3)} \) are respectively smallest, intermediate, and largest root of the equation \( T_s = T(\Delta S_A, Q) \), with

\[ A_1 = A_2 \]

(29)

We tabulate in Table 3 the values of the areas \( A_1 \) and \( A_2 \) for each choice of \( \theta_0 \), the charge \( C \), the exponent \( q \), the ratio of charge \( \frac{\omega_{n−2}}{\omega_{n−1}} \) and the relative error between \( A_1 \) and \( A_2 \) taken to be the difference between \( A_1 \) and \( A_2 \) divided by their average. From this table it’s obvious that \( A_1 \) is indeed almost equal to \( A_2 \) for each line near the critical point \( (\frac{\omega_{n−2}}{\omega_{n−1}} = 0.9) \) within our numerical accuracy.\footnote{Here, we have just considered in maximum three number after the decimal point. This choice is due to the calculation’s time and performances of our machine.}

### Table 2

| \((C, q)\) | \( T_s \) | \( S_1 \) | \( S_2 \) | \( S_3 \) | \( A_1 \) | \( A_2 \) |
|----------|----------|----------|----------|----------|----------|----------|
| \((0, 2)\) | 0.1658067 | 0.430617 | 1.72042  | 5.59234  | 0.855847793613 | 0.855847793614 |
| \(\frac{1}{2}, 2\) | 0.12494   | 0.743699 | 1.36918  | 2.34293  | 0.1997   | 0.1998   |

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1 Here, we have just considered in maximum three number after the decimal point. This choice is due to the calculation’s time and performances of our machine.
Fig. 3. Plot of isocharges on the \((T, \Delta S_A)\)-plane, for different values of \(\theta_0, C\) and \(q\). For all panels: the values of the charge ratio are \(Q/Q_c = 0.9\) (red), \(Q/Q_c = 0.4\) (cyan), \(Q/Q_c = 1\) (dashed blue) and \(Q/Q_c = 2\) (orange). The coexistence isotherm \(T_\ast\) (dashed gray line) is obtained from the free energy Fig. 2 and the critical temperature (dashed green line). For all curves, we also show the data points which are used to create the interpolation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

| \((C, q)\) | \(\frac{\theta}{\pi}\) | \(\theta_0\) | \(\Delta S_A^{(1)}\) | \(\Delta S_A^{(2)}\) | \(\Delta S_A^{(3)}\) | \(A_1\) | \(A_2\) | Relative error |
|---|---|---|---|---|---|---|---|---|
| \((0, 2)\) | 0.9 | 0.2 | 0.00424402 | 0.00565345 | 0.00792092 | 9.97 \times 10^{-2} | 1.01 \times 10^{-6} | 1.29% |
| | 0.3 | 0.0144572 | 0.0191749 | 0.0269562 | 3.31 \times 10^{-6} | 3.39 \times 10^{-6} | 2.38% |
| | 0.5 | 0.0709257 | 0.0926348 | 0.127731 | 0.0000147 | 0.000015 | 2.02% |
| | 0.9 | 0.2 | 0.00191022 | 0.00460008 | 0.0124709 | 0.0000558 | 0.0000444 | 22.75% |
| \((\frac{1}{3}, \frac{1}{4})\) | 0.9 | 0.2 | 0.00116043 | 0.00410067 | 0.0145447 | 0.0000145 | 0.0000937 | 42.98% |
| | 0.3 | 0.0246152 | 0.0246497 | 0.0246905 | 1.255 \times 10^{-8} | 1.235 \times 10^{-8} | 1.60% |
| | 0.5 | 0.048356 | 0.049751 | 0.050825 | 4.234 \times 10^{-9} | 4.331 \times 10^{-9} | 2.26% |
| | 0.9 | 0.2 | 0.0144326 | 0.145177 | 0.146719 | 5.9151 \times 10^{-7} | 6.729 \times 10^{-7} | 12.87% |
| \((0, 2)\) | 0.2 | 0.9 | 0.019245 | 0.0919245 | 0.0952804 | 1.490 \times 10^{-6} | 1.879 \times 10^{-6} | 22.09% |
the entanglement entropy picture. Consequently, we can predicate that the first order phase transition of entanglement entropy obeys to Maxwell’s equal area law just near the critical point.

Now, one can find the critical exponent using the slope of the relation between \( \log |T - T_c| \) and \( \log |\Delta S_A - \Delta S_{A_c}| \), where \( \Delta S_{A_c} \) is the critical entanglement entropy given numerically by an equation \( T(\Delta S_A) = T_c \). For different charge and exponent \((C, q)\), we plot the relationship between \( \log |T - T_c| \) and \( \log |\Delta S_A - \Delta S_{A_c}| \) in Fig. 4, the corresponding results can be fitted for \( \theta_0 = 0.2 \) as

\[
\log |T - T_c| = \begin{cases} 
12.9649 + 3.03402 \log |\Delta S_A - \Delta S_{A_c}| & (C, q) = (0, 2) \\
16.0061 + 3.0343 \log |\Delta S_A - \Delta S_{A_c}| & (C, q) = (1, 1) 
\end{cases}
\] (30)

From the results shown in Fig. 3 and especially the Table 3 we can see that the area \(A_1\) and \(A_2\) are almost equal near the critical point within our numerical accuracy. The behavior of the relative error, confirm the conservation of Maxwell’s equal area law observed in the first order phase transition structure for the values of the ratio \(\frac{r_0}{r_1}\) approaching the unit. The Eq. (30), give a critical exponent around 3 in good agreement with that in Eq. (19) and the mean field of Van der Waals theory. As results, we can confirm that the holographic entanglement entropy does exist first and second order phase transition and the validity of the Maxwell equal area is governed by the proximity to the critical point.

Now, we will pay attention in the next section to investigate whether the two point correlation function has the similar behavior as that of the entanglement entropy picture.

3.2. Two point function correlation

According to the Maldacena conjecture, the time two point correlation function can be written under the saddle-point approximation and in the large limit of \( \Delta \) as [46]

\[
\langle O(t_0, x_i)O(t_0, x_j) \rangle \approx e^{-\Delta L},
\] (31)

where \( \Delta \) is the conformal dimension of the scalar operator \( O \) in the dual field theory, \( L \) is the length of the bulk geodesic between the points \((t_0, x_i)\) and \((t_0, x_j)\) on the AdS boundary. By the space-time symmetry of the such black hole, we can rename \( x_i \) as \( \theta \) with the boundary \( \theta_0 \) and employ it to parameterize the trajectory. In this case the proper length reads as

\[
L = \int_0^{\theta_0} L(r(\theta), \theta) d\theta, \quad L = \sqrt{\left(\frac{r'(\theta)}{r(\theta)}\right)^2 + r(\theta)^2},
\] (32)

where \( r' = dr/d\theta \). The motion equation for \( r(\theta) \) when treating \( L \) as Lagrangian and \( \theta \) as time reads as

\[
r'(\theta)^2 r'(\theta) - 2 f(r(\theta)) r''(\theta) + 2 r(\theta) f(r(\theta))^2 = 0.
\] (33)

Using the same boundary conditions of the previous section [Eq. (25)] to solve this equation and choosing the same values of \( \theta_0 \) with the same UV cutoff in the dual field theory. We label the regularized two point correlation function as \( \Delta L_{A} = L - L_0 \), with \( L_0 \) is the geodesic length in pure AdS under the same boundary region and show the behavior of the temperature \( T \) in function of \( \Delta L_A \) in the Fig. 5, from which we can see that the Van der Waals-like phase transition is held as the thermal and the holographic entanglement entropy pictures.

Similar to the case of the entanglement entropy, for different \( \theta_0 \), and the charge ratio \(\frac{Q}{C}\), the results of \( \Delta L_{A_1, 2} \), \( A_1, A_2 \) and the relative error are listed in Table 4 which are necessary to check the equal area law for the first phase transition.

Analyzing this table, we can conclude that just when the ratio \(\frac{Q}{C}\) approaches the unit, Maxwell's law also holds under our numerical accuracy implying \( A_1 \) almost equals \( A_2 \), which is consistent with the Fig. 5. In other words, like the entanglement entropy, the two point correlation function also exhibits the first order phase transition as that of the thermal entropy, however, the validity of the Maxwell law is controlled by the charge ratio.

For the second phase transition, we recall the quantities \( \log |T - T_c| \) and \( \log |\Delta L_A - \Delta L_{A_c}| \) with \( \Delta L_{A_c} \) is obtained numerically by solving \( \beta(\Delta L_A) = T_c \). The results linked the logarithm of \( |T - T_c| \) and \( |\Delta L_A - \Delta L_{A_c}| \) are illustrated in Fig. 6.

The straight blue line in Fig. 6 is fitted as follow

\[
\log |T - T_c| = \begin{cases} 
20.0115 + 3.08895 \log |\Delta L_A - \Delta L_{A_c}| & (C, q) = (0, 2) \\
14.5296 + 3.01268 \log |\Delta L_A - \Delta L_{A_c}| & (C, q) = (\frac{1}{2}, \frac{1}{2}) 
\end{cases}
\] (34)

Again, like the holographic entanglement entropy structure, we discovered a slope around 3, then the critical exponent of the specific heat capacity is consistent with that of the mean field theory of the Van der Waals. Therefore, we conclude that two point correlation function of such black hole exists a second order phase transition at the critical temperature \( T_c \).
Fig. 5. Plot of isocharges on the \((T, \Delta L_A)\)-plan, for \(C = 0\) (left), and \(C \neq 0\) (right). For all panels: the values of the charge are \(Q = 0.9Q_c\) (red), \(Q = 0.4Q_c\) (cyan), \(Q = Q_c\) (dashed blue) and \(Q = 2Q_c\) (orange). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 4
Comparison of \(A_1\) and \(A_2\) for the EMPYM-AdS black hole using two point correlation function.

| \((C, q)\) | \(Q/Q_c\) | \(\theta_q\) | \(\Delta L_A^{(1)}\) | \(\Delta L_A^{(2)}\) | \(\Delta L_A^{(3)}\) | \(A_1\) | \(A_2\) | Relative error |
|-----------|-------------|-------------|-----------------|-----------------|-----------------|--------|--------|---------------|
| \((0, 2)\) | 0.9         | 0.2         | 1.34983         | 1.34991         | 1.35006         | 0.000285 | 0.000279 | 2.12%         |
|           | 0.3         | 0.5         | 0.881824        | 0.882015        | 0.882324        | 0.000119 | 0.000122 | 2.48%         |
|           | 0.5         | 0.4923      | 0.4935          | 0.49537         | 0.0000325      | 0.000334 | 2.73%    |               |
|           | 0.4         | 0.2         | 1.34968         | 1.34985         | 1.35033         | 0.000050 | 0.000463 | 17.17%        |
|           | 0.2         | 0.2         | 1.34964         | 1.34982         | 1.35047         | 0.000610 | 0.00418  | 37.35%        |
| \((\frac{1}{4}, \frac{1}{4})\) | 0.9         | 0.2         | 1.36054         | 1.36056         | 1.36059         | 0.0000976 | 0.0000991 | 1.52%         |
|           | 0.3         | 0.3         | 0.893827        | 0.893972        | 0.893524        | 0.0000236 | 0.0000242 | 2.51%         |
|           | 0.5         | 0.506145    | 0.50629         | 0.506402        | 0.000683        | 0.000668 | 2.22%    |               |
|           | 0.4         | 0.2         | 1.35602         | 1.35608         | 1.35617         | 0.000231 | 0.000309 | 28.88%        |
|           | 0.2         | 0.2         | 1.3537          | 1.35377         | 1.3539          | 0.000283 | 0.000494 | 54.31%        |
4. Conclusion

In this work we have investigated the Van der Waals-like phase transition of AdS black hole in the Einstein–Maxwell–power-Yang–Mills gravity. With a fixed charge ensemble, we first investigated the phase structure of the thermal entropy in the \((T, S)\)-plane and found that the phase structure agrees with the study made in [12], where the authors consider the thermodynamics of such black hole in the \((P, V)\)-plane notably the critical behavior and the analogy with the Van der Waals gas. For the charge under critical one, there is always an unstable hole interpolating between the small stable black hole and large stable black hole. The transition from the small black hole to the large one is first order and the equal area law holds. As the charge of the black hole increases to the critical value, the unstable black hole merges into an inflexion point where the phase transition is second order. While the charge is larger than the critical charge, the black hole is always stable.

We also found that this phase structure of the EMPYM-AdS black hole can be probed by the two point correlation function and holographic entanglement entropy in our numerical accuracy, in addition to this we can claim the validity the Maxwell law is governed by the proximity to the critical point, which is in agreement with the recent work [47] which asserts the breakdown of the equal area law in the holographic framework, this proposal remains an open question. At the same time this approach provides a new approach to our understanding the phase picture from the point view of holography.

At the end, in the further works, we can extend our study to reconsider higher dimensions and higher derivative Yang–Mills gravity models.

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