An Effective field theory for non-relativistic Majorana neutrinos

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1 Introduction and Motivation

2 Effective field theories and Majorana fermions

3 Finite temperature corrections

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**A pair of open problems in cosmology**

### Dark Matter...
- 84% of the matter in the Universe is believed to be Dark Matter.
- We need a **suitable** Dark Matter candidate in agreement with **cosmological constraints**:
  \[ Q_X = 0 \quad \text{non baryonic, stable} \quad M_X \neq 0 \]
- **Standard Model neutrinos ruled out**... *no galaxies clustering*

### Baryon Asymmetry...
- The Standard Model and Standard Cosmology are not able to explain the Baryon Asymmetry in the Universe:
  \[ Y_B^{th} \ll Y_B^{exp} \]
- Look for a dynamical process to generate such an asymmetry: **Baryogenesis**

⇒ **physics beyond** Standard Model is required
Why Dark Matter?

- amount of **ordinary matter** is not able to explain the **observed gravitational effects**: rotational curves, galaxies formation ... \( \Rightarrow \) Dark Matter (?)

Some examples...

- neutralino (SUSY)
  - being the LSP \( \rightarrow \) stable

- gravitino (SUSY)
  - being the LSP \( \rightarrow \) stable

- heavy neutrinos (See-Saw Type I)
  - weakly coupled \( \rightarrow \) stable

**Common feature: Majorana fermions**

\[
\psi_M = (\psi_M)^C \quad \text{where} \quad \psi_M^C = C\gamma^0\psi^* = i\gamma_2\psi^*
\]
**Introduction and Motivation**

**Baryon Asymmetry in the Universe**

**Experimental evidences**

- Cosmic rays: $N_P/N_{\bar{P}}$ is consistent with secondary process $p + p \rightarrow 3p + \bar{p}$
- Matter-Antimatter in cluster of galaxies: detectable background of $\gamma$-radiation, **NOT DETECTED**

The BAU is accurately determined by CMB and Anisotropy measurement

$$Y_B^{CMB} = \frac{n_b - n_{\bar{b}}}{s} = (8.75 \pm 0.23) \times 10^{-11}$$

- Starting with $Y = 0$ Standard Cosmology gives: $Y_B \sim 10^{-18} \ll Y_B^{CMB}$

**Sakharov conditions, A. Sakharov (1967)**

1. baryon number ($B$) violation
2. C and CP violation
3. processes out of thermal equilibrium
Introduction and Motivation

Baryogenesis via Leptogenesis

**CP violation in quark sector is not enough, very high** $T_{rh}$

SPHALERONS: Baryons $\rightleftharpoons$ Leptons

- $B$ and $L$ well **conserved** at low temperature regime
- $T > T_{EW}$ transition between vacua of non Abelian Gauge Theory ($SU(2)$)

$$\Delta B = \Delta L = n_f \Delta N_v$$

- $100 \text{ GeV} \leq T \leq 10^{12} \text{ GeV}$: sphaleron transitions activated
- due to sphalerons properties a Baryon Asymmetry can be generated

$$B = C \cdot (B - L), \quad L = (C - 1) \cdot (B - L)$$

**Leptogenesis mechanism:** $L \neq 0 \Rightarrow (B - L) \neq 0 \Rightarrow B \neq 0$

Look for lepton number violating processes...
**Sterile Neutrinos Lagrangian**

- $\nu$ oscillation experiment $\Rightarrow$ small **mass** to $\nu$, but how?

**Standard Model Extension**

- $N$ singlet fermions $N_I$ ($I = 1, \ldots, N$) $\quad M_{N_1} \leq M_{N_2} \ldots \leq M_N$

  $$Q = 0; \quad l_W = 0; \quad Y = 0 \rightarrow \text{sterile particles}$$

- renormalizable Lagrangian with Dirac-Majorana mass term

  $$\mathcal{L} = \mathcal{L}_{SM} + i \bar{N}_I \partial_\mu \gamma^\mu N_I - \left( F_{\alpha I} \bar{L}_\alpha N_I \Phi - \frac{M_I}{2} \bar{N}_I^c N_I + h.c. \right)$$

  - R. N. Mohapatra and G. Senjanovic (1981);
  - M. Gell-Mann, P. Ramond and R. Slansky (1979)

- Majorana mass $\rightarrow$ Lepton number violation
- $F_{\alpha I}$ Yukawa couplings: complex phases $\rightarrow$ CP violation source
- *out of equilibrium* due to weak couplings
**Introduction and Motivation**

**Leptogenesis**  
*M. Fukugita and T. Yanagida (1986)*

- Full fill Sakarov conditions \((B \rightarrow L)\), example: Sterile Neutrinos

\[
(1) : N \rightarrow \ell_\alpha \phi^\dagger, \quad (\bar{1}) : N \rightarrow \bar{\ell}_\alpha \phi \quad \Rightarrow \quad \delta_\ell = \frac{\Gamma(1) - \Gamma(\bar{1})}{\Gamma_{\text{Tot}}}
\]

- **Different scales**: \(M_i, T, \Delta M, E_W\) ... *thermal environment*
- **General**: \(N_i\) decays are efficient in the regime \(T < M_X\)

⇒ Possible hierarchy scale: \(M \gg T \gg E_W\) → *Effective field theory*?
Effecting Field Theories and Majorana fermions

**Setting up the tools**

Dealing with problems involving **more than one energy scale**:

**Effective Field Theories**

1. a *hierarchy* of energy scales: separation of the scales, e.g. $T << M$
2. identify the *dynamical scale* ($T$) and *integrate out high energy modes* ($M$)
3. organize an *expansion* of the operators in terms of

$$\frac{T}{M} \rightarrow \text{power counting}$$

4. *dimensional analysis* helps in building the effective Lagrangian

$$\mathcal{L}_{FT} \rightarrow \mathcal{L}_{EFT} = \sum_i c_i \frac{O_i^n}{M^{n-4}}$$

**EFT strategy**

- identify the *symmetries* of the low energy Lagrangian
- identify the suitable *degrees of freedom*, ingredients of your system
- write down the low energy Lagrangian exploiting the *hierarchy of the scales*
Defining the Problem: Thermal Decay Rate

Our physical system and degrees of freedom

- hot plasma of SM particles at $T \gg E_W$ GeV: $m_i \ll T$ and $\vec{p}_i \sim T$

- Majorana neutrinos ($N, M$) are almost not affected by $T$, being $M \gg T$

$\Rightarrow N$ described by non-relativistic fields, Poincaré symmetry

Different Approaches:

1. Consider directly thermal field theory (ITF) without exploiting $M \gg T$
   - M. Laine and Y. Schroder (2012), A. Salvio, P. Lodone and S. Strumia (2011)
     - complete two loops computation in the high energy theory
     - many term $\sim e^{-M/T}$

2. EFT for heavy Majorana neutrinos
   - computation at $T=0$ via one loop diagrams $M \gg T$ hence $T \to 0$
   - thermal effects as correction via simple tadpole diagrams (RTF)
A Majorana fermion full fills

$$\psi_M = (\psi_M)^C \Rightarrow \text{self-conjugate spinor}$$

the relativistic propagator of a free Majorana particle are as follows

$$\langle 0 | T \{ \psi_a(x) \bar{\psi}_b(y) \} | 0 \rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{(\slashed{p} + M)^{ab}}{p^2 - M^2 + i\epsilon} e^{-ip(x-y)}$$

$$\langle 0 | T \{ \psi_a(x) \psi_b(y) \} | 0 \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \frac{(\slashed{p} + M)^{ab} C}{p^2 - M^2 + i\epsilon} e^{-ip(x-y)}$$

$$\langle 0 | T \{ \bar{\psi}_a(x) \bar{\psi}_b(y) \} | 0 \rangle = -i \int \frac{d^4 p}{(2\pi)^4} \frac{C(\slashed{p} + M)^{ab}}{p^2 - M^2 + i\epsilon} e^{-ip(x-y)}$$

In the low energy theory one needs

non-relativistic Majorana spinors + the non-relativistic propagator
Non-relativistic Majorana fermions

\[ N = \left( \frac{1 + \gamma^0}{2} \right) N + \left( \frac{1 - \gamma^0}{2} \right) N = N_< + N_> \]

- Projector properties

\[ N = \left( \frac{1 + \gamma^0}{2} \right) N_< + \left( \frac{1 - \gamma^0}{2} \right) N_> \]

- Hermitian conjugate expression and \( N = i \gamma_2 N^\dagger \)

\[ N = \left( \frac{1 - \gamma^0}{2} \right) i \gamma_2 N_<^\dagger + \left( \frac{1 + \gamma^0}{2} \right) i \gamma_2 N_>^\dagger \]

- Comparing one gets no disentanglement between particle and anti-particle

\[ N_< = i \gamma_2 N_>^\dagger, \quad N_> = i \gamma_2 N_<^\dagger \]

- \( N_< \) contains only annihilation operator like other known EFTs and

\[ \{ N_<^a(\vec{x}, t), N_<^{b\dagger}(\vec{y}, t) \} = \delta^3(\vec{x} - \vec{y}) \delta^{ab} \]

\[ \{ N_<^a(\vec{x}, t), N_<^b(\vec{y}, t) \} = \{ N_<^{a\dagger}(\vec{x}, t), N_<^{b\dagger}(\vec{y}, t) \} = 0 \]
**EFT strategy:**

1. Poincaré invariance, Gauge invariance
2. Non-relativistic spinors for Majorana neutrinos: \( N_\prec \equiv N (|\vec{p}| \ll M) \)
3. Low energy Lagrangian (Relevant operators for the Leptogenesis problem)

\[
\mathcal{L}_{EFT} = N^\dagger \partial_0 N + \frac{A}{M} N^\dagger N\phi^\dagger \phi + \frac{B}{M^3} N^\dagger N\bar{\psi}D_0 \psi + \frac{C}{M^3} N^\dagger NF^2 + \ldots
\]

where \( \phi \) is the Higgs doublet, \( \psi \) are fermions, \( F^2 \approx (\partial_0 A_i \partial_0 A_i) \) gauge bosons

Thermal correction of each term through **dimensional analysis**:

\[
\delta \Gamma(N)_\phi \propto \frac{T^2}{M}, \quad \delta \Gamma(N)_\psi \propto \frac{T^4}{M^3}, \quad \delta \Gamma(N)_F \propto \frac{T^4}{M^3}
\]

- A, B, C called **matching coefficients**
- the power counting \( + M \gg T \Rightarrow \text{expansion under control} \)
the effective Lagrangian produces the following diagrams

\[ \mathcal{L}_{\text{EFT}} = N^\dagger \partial_0 N + \frac{A}{M} N^\dagger N \phi^\dagger \phi + \frac{B}{M^3} N^\dagger N \bar{\psi} \partial_0 \psi + \frac{C}{M^3} N^\dagger N F^2 + \ldots \]
Effective field theories and Majorana fermions

The matching computation: an example

\[ \mathcal{L}_{EFT,\phi} = \frac{a}{M} N^\dagger N\phi^\dagger \phi + \frac{b}{M^3} N^\dagger N D_0 \phi D_0 \phi \]

- Lorentz gauge ⇒ determine a clear structure in powers of \( q^\mu \)
- manifest contributions to each effective vertex: \( \propto q^0 \) or \( \propto q^2 \)

High energy green function = EFT green function

\[ \text{Im}(a) = -i \frac{3}{8\pi} |F|^2 \lambda, \quad \text{Im}(b) = -i \frac{5}{32\pi} (3g^2 + g'^2) |F|^2 \]
We get the result for \( a \) as follows

\[
a = -i \frac{3}{8} \frac{|F|^2 \lambda}{\pi} \quad \Rightarrow \quad \mathcal{L}_{\text{NN} \phi \phi} = -i \frac{3}{8} \frac{|F|^2 \lambda}{\pi M} N^\dagger N \phi^\dagger \phi
\]

\[
N \quad \phi \quad N \quad \bar{N} \quad \phi \quad \bar{N}
\]

RTF + Heavy particles \( \Rightarrow \) only type 1 \((N. \text{Brambilla et al (2008)})\)

In a hot plasma particles are thermally excited \( \Rightarrow \) propagators affected by

\[
i \Delta_{11}(x - y) = \int \frac{d^4 K}{(2\pi)^4} \left[ \frac{i}{K^2 - m^2 + i\epsilon} + 2\pi n_B(|k_0|) \delta(K^2 - m^2) \right] e^{-iK(x - y)}
\]
Finite temperature corrections

Thermal production rate at $\mathcal{O}(\frac{T^2}{M^2})$

Thermal correction:
- the first term: $m_\phi \Rightarrow$ no scale $\Rightarrow 0$ in D-regularization
- the second term: finite contribution $\rightarrow$ thermal correction

\[ n_B = \frac{1}{e^{k/T} - 1} \Rightarrow \int_0^\infty dk \frac{k}{e^{k/T} - 1}, \quad k = xT \]

Hence one gets

\[ \Gamma_N(T) = \frac{M |F|^2}{8\pi} \left[ 1 - \lambda \frac{T^2}{M^2} + \mathcal{O}\left(\frac{T}{M}\right)^4 \right], \quad \delta\Gamma(N)_\phi \propto \frac{T^2}{M} \]
**Thermal corrections:**

- Add the contribution of gauge boson and fermions
  \[ A^\mu, B^\mu, \ell_\alpha, \nu_\alpha, t, b \]

- Consider the **Fermi-Dirac thermal distribution** for fermions

Finally we get the decay rate with **thermal corrections**

M. Laine, Y. Schroder (2012)

\[
\Gamma = \frac{|F|^2 M}{8\pi} \left\{ 1 - \lambda \left( \frac{T}{M} \right)^2 - \frac{\pi^2}{80} \left( \frac{T}{M} \right)^4 \left( 3g^2 + g'^2 \right) - \frac{7\pi^2}{60} \left( \frac{T}{M} \right)^4 |\lambda_{tb}|^2 \right\}
\]
What is next? Lepton asymmetry

- thermal effects in the N decays may be important for leptogenesis
- get in touch with an observable

\[ Y_B \approx \frac{135\zeta(3)}{4\pi^2} \times \epsilon \times \eta \times C \]

G.F. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia (2004)

**EFT formalism → Thermal corrections to lepton asymmetry**

\[ \epsilon \rightarrow \epsilon(T) \]
Finite temperature corrections

Thermal correction to lepton asymmetry, resonant case

**Leading thermal correction**

\[ \mathcal{L} = \frac{a'}{M} N^\dagger N \phi^\dagger \phi + \sum_i \frac{c_i}{M^n} O_i , \quad n > 2 \]

- the definition of lepton asymmetry from \( N \) decays is

\[
\epsilon = \sum_{i, \alpha} \frac{\Gamma(N_i \rightarrow \ell_\alpha) - \Gamma(N_i \rightarrow \bar{\ell}_\alpha)}{\Gamma(N_i \rightarrow \ell_\alpha) + \Gamma(N_i \rightarrow \bar{\ell}_\alpha)} = 2 \frac{\text{Im}(B) \text{Im}[(F_1 F_2^*)^2]}{|F_1|^2}
\]

1) ![Diagram](image1.png)

2) ![Diagram](image2.png)

- thermal corrections in \( \text{Im}(B) \) due to the thermal Higgs (tadpole as before)

\[ \Rightarrow \epsilon(T) \simeq \epsilon \left[ 1 + \lambda \left( \frac{T}{M} \right)^2 \right] \]
Conclusions

Dark matter and Leptogenesis as open problems in Cosmology

Majorana particles involved

an EFT for non-relativistic Majorana fermions is built

focus on heavy neutrinos considered so far

the thermal decay rate for one kind of heavy neutrino is reproduced

the leading thermal correction to the lepton asymmetry via EFTs (work in progress)
**Thermal production of Dark Matter (X)**

**Thermal approach to the Dark Matter problem**

- **X** produced in a hot dense plasma at high temperature
- Cold Dark Matter $\simeq \mathcal{O}(100)$ GeV or warm Dark Matter $\simeq \mathcal{O}(10)$ KeV
- Decays of $X'$ play an important role in the regime $T < M'$

$\Rightarrow$ finite temperature treatment, $T \neq 0$
Low energy Lagrangian and Wilson coefficient

- The low energy Lagrangian in more details reads:

\[ L_{\text{int}} = \frac{a}{M^3} N^\dagger N \phi^\dagger \phi + \frac{b}{M^3} N^\dagger N D_0 \phi^\dagger D_0 \phi + c_1 (N a_R \bar{L}_\alpha) (i D_0 L_\beta a_L N^\dagger) + \frac{c_2}{M^3} (N a_R \gamma_\mu \gamma_\nu \bar{L}_\alpha) (i D_0 L_\beta \gamma^\nu \gamma^\mu a_L N^\dagger) + \frac{c_3}{M^3} N^\dagger N (\bar{t} a_L \gamma^0 i D_0 t) + \frac{c_4}{M^3} N^\dagger N (\bar{q} a_L \gamma^0 i D_0 q) + \frac{d_1}{M^3} \text{tr} \{ T^a T^b \} N^\dagger N F_{0i}^a F_{0i}^b + \frac{d_2}{M^3} N^\dagger N W_{0i} W_{0i} \]

- The Wilson coefficients are:

| Decay Rate | Imaginary Part |
|------------|----------------|
| \( a = -i \frac{3}{8\pi} |F|^2 \lambda \) | \( b = -i \frac{5}{32\pi} (3g^2 + g'^2)|F|^2 \) |
| \( c_1 = i \frac{3}{8\pi} |\lambda_{tb}|^2 |F|^2 \) | \( c_2 = i \frac{3}{16\pi} (3g^2 + g'^2)|F|^2 \) |
| \( c_3 = -i \frac{1}{24\pi} |\lambda_{tb}|^2 |F|^2 \) | \( c_4 = -i \frac{1}{48\pi} |\lambda_{tb}|^2 |F|^2 \) |
| \( d_1 = -i \frac{1}{48\pi} g^2 |F|^2 \) | \( d_2 = -i \frac{1}{96\pi} g'^2 |F|^2 \) | \( |F|^2 = F_{1,\alpha} F^\ast_{1,\alpha} \) |