Reduction of boundary effects in spiral MRI experiment PROMISE

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Magnetorotational instability (MRI) is one of the most important and most common instabilities in astrophysics, it is widely accepted that it serves as a source of turbulent viscosity in accretion disks – the most energy efficient objects in the Universe. However it is very difficult to bring this process down on earth and model it in a laboratory experiment. Several different approaches have been proposed, one of the most recent is PROMISE (Potsdam-ROssendorf Magnetorotational InStability Experiment). It consists of a flow of a liquid metal between two rotating cylinders under applied current-free spiral magnetic field. The cylinders must be covered with plates which introduce additional end-effects which alter the flow and make it more difficult to clearly distinguish between MRI stable and unstable state. In this paper we propose simple and inexpensive improvement to the PROMISE experiment which would reduce those undesirable effects.

1 Introduction

Velikhov (1959) showed that for ideal magnetohydrodynamics, an axial magnetic field applied to a flow of a liquid metal between two concentric, differentially rotating cylinders (Taylor-Couette flow) can lower the critical rotation ratio or even destabilize the flow, whereas it is hydrodynamically stable (when Prandtl number is large enough). That type of instability is called magnetorotational instability and not long ago Balbus & Hawley (1991) demonstrated that it plays important role in astrophysics. MRI serves as an essential mechanism for transporting angular momentum in wide range of astrophysical objects, stellar interiors, jets and in particular it is crucial for process of accretion which resembles Keplerian disks, do not provide viscosity required to transport the angular momentum effectively and therefore this instability is ruled out as a source of viscous turbulence in the disks.

One of the most convenient laboratory models for MRI is still magnetohydrodynamical, cylindrical Taylor-Couette flow with imposed external magnetic field along the axis (Rüdiger & Zhang 2001; Ji, Goodman & Kageyama 2001). For magnetic Prandtl number \( P_m \approx 1 - 10 \) the axial field (when not too strong) reduces critical Reynolds number for instability and, more importantly, also introduces instability for flows which are always stable for nonmagnetic case (see e.g. Rüdiger, Schultz & Shalybkov 2003; Barenghi et al. 2004). The unstable state is characterized by classical Taylor vortices. Regrettably, laboratory liquid metals possess very small magnetic Prandtl number (due to low conductivity) so that for purely axial field the critical Reynolds number is of order \( O(10^6) \) and therefore vast rotation rates are necessary for MRI to grow.

Recently it has been shown by Hollerbach & Rüdiger (2005) and Rüdiger et al. (2005) that a current-free external azimuthal magnetic field in addition to the axial one can reduce the critical Reynolds number to \( O(10^3) \) and therefore makes it much easier to design a MRI experiment. Moreover due to symmetry-breaking there exists a drift of Taylor vortices associated with configuration of the applied magnetic field, its frequency can be measured and compared with theory being an important feature indicating this type of instability.

The idea of additional toroidal field was successfully implemented in PROMISE experiment by Stefani et al. (2006), where modes corresponding to so called spiral (or helical) MRI were observed for the first time (see also Rüdiger et al. 2006; Stefani et al. 2007). Results of this experiment also show that in the basic stable state, without any toroidal field, there exists a nonzero axial velocity field which arises due to presence of the rigid endplates enclosing the cylinders. These plates, undoubtedly present in any real experiment, are responsible for additional effects which do not take place for idealized infinitely long container.

The boundary layer which exists in vicinity of the end-plates consists of an Ekman layer, which is result of rotation of rigid surface, and a Hartmann layer which develops when conducting fluid is used and external axial magnetic field is applied (see e.g. Ekman 1905; Roberts 1967). Consequently global properties of the flow change when compared to idealized case of infinitely long cylinders – secondary flow, i.e. two large Ekman vortices appear and Hartmann current is drawn into the bulk of the fluid. All these effects depend on mechanical and magnetic properties of the lids. In the
PROMISE experiment one of the lids is made of copper and is attached to the outer cylinder, the other one is a stationary plexiglass plate.

In this work we review simple improvements which can reduce undesired effects induced by the lids and therefore provide possibility to more clearly distinguish between MRI stable and unstable state.

2 Numerical model

We consider two concentric cylinders with radii $R_{in} = 4$ cm, $R_{out} = 8$ cm and height $H = 40$ cm which rotate with angular velocities $\Omega_{in}, \Omega_{out}$. The rotation ratio $\bar{\mu} = \Omega_{out}/\Omega_{in}$ is chosen in such a way that the flow is hydrodynamically stable, i.e. Rayleigh stability criterion $\partial_r(\bar{\mu}^2) > 0$, $r$ being distance from axis of rotation, is fulfilled. Through this paper we use $\bar{\mu} = 0.27$. For infinite cylinders the basic rotational profile for the flow is the Couette solution

$$\Omega_0(r) = a + \frac{b}{r^2},$$  \hspace{1cm} (1)

where $a, b$ are constants dependent on radii and rotation speeds

$$a = \Omega_{in} \frac{\bar{\mu} - \eta^2}{1 - \eta^2}, \quad b = \frac{1 - \bar{\mu}}{1 - \eta^2} R_{in}^2 \Omega_{in}. \hspace{1cm} (2)$$

The external magnetic field (steady, current free) has form of

$$B_0 = B_0 \left( \frac{\beta R_{in}}{r} \hat{e}_\phi + \hat{e}_z \right).$$ \hspace{1cm} (3)

its strength is measured by Hartmann number

$$Ha = \frac{B_0 \sqrt{R_{in}(R_{out} - R_{in})}}{\mu_0 \rho \eta \nu}.$$ \hspace{1cm} (4)

Magnetic properties of the conducting fluid are described by magnetic Prandtl number which is ratio of the kinematic viscosity $\nu$ to the magnetic diffusivity $\eta$, $Pm = \nu/\eta$. $\mu_0$ denotes magnetic permeability, $\rho$ denotes density. Reynolds number $Re$ is defined as $Re = \nu^{-1} \Omega_{in} \sqrt{R_{in}(R_{out} - R_{in})}$. The liquid used in the PROMISE experiment is the eutectic alloy GaInSn giving $Pm = 1.4 \times 10^{-6}$ therefore it is reasonable to solve MHD equations (dimensionless) in theirs small Prandtl number limit (Yould & Barenghi 2006; Roberts 1967; Zikanov & Thess 1998)

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = - \nabla p + \nabla^2 \mathbf{u} + Ha^2 \mathbf{b} \times \frac{B_0}{B_0},$$ \hspace{1cm} (5)

$$\nabla^2 \mathbf{b} = - \text{rot}(\mathbf{u} \times B_0/B_0),$$ \hspace{1cm} (6)

and $\nabla \mathbf{u} = 0$, $\text{div} \mathbf{b} = 0$, where $\mathbf{u}$ is velocity field and $\mathbf{b}$ is perturbated magnetic field.

We simulate the above nonlinear equations for 2D axisymmetric flow in cylindrical coordinates $(r, \phi, z)$, for details on numerical method and boundary conditions (see Yould & Barenghi 2006; Szklarski & Rüdiger 2006). The cylinders are assumed to be perfectly conducting and the endplates either conducting or insulating, for the latter pseudo-vacuum approximation is used (however even for copper the assumption for perfect conductors may be not very realistic).

3 Discussion of results

From the point of view of MRI experiment we are interested in obtaining a stable, uniform rotation profile which for the subcritical characteristic values is as close to the idealized basic state $\Omega_0$ as possible, and a clear pattern of traveling vortices for supercritical conditions. For the infinite cylinders with the external axial magnetic field and liquids with $Pm$ of our interest the base Couette profile $\Omega_0$ is not altered until a critical Reynolds number of order $O(10^5)$ is reached (this corresponds to rotation frequency $f \approx 100$ Hz). For instability due to the additional toroidal field with $\beta = 4$ we expect $Re$ to be of order $O(10^3)$ (which implies $f \approx 0.1$ Hz). $Ha$ of order $O(10)$ and therefore we search for conditions for which the flow is closest to the $\Omega_0$ profile for these parameters (for details on the critical values see Hollerbach & Rüdiger 2005; Rüdiger et al. 2005).

![Fig. 1 Profiles of $u_z(z,t)$ at $r = R_{in} + 0.6D$ for periodic cylinders just above the critical characteristic values: $Re = 1000$, $Ha = 9.5$, $\beta = 4$. The critical Reynolds number in this case is $Re_c = 842$.](image)

Figure 1 displays values of velocity component $u_z$ measured along $z$ axis at $r = R_{in} + 0.6D$ for supercritical values of rotation and magnetic fields. $R_{in} = 4$ cm, $R_{out} = 8$ cm, physical properties of gallium for the viscosity and the magnetic diffusivity are used in order to obtain values in physical scales comparable with those of the PROMISE experiment. Results in this figure are for cylinders with periodic boundary conditions so that the profiles are not constrained by the end effects and are directly comparable with results from linear theory for the infinite cylinders. We notice clear traces of the drifting Taylor vortices.

3.1 Reducing end-plates effects

All undesirable effects induced by the endplates arise as a consequence of vertical shears near the boundaries, therefore we attempt to reduce the shears by using appropriate boundary conditions. Some experiments (e.g. Ji et al.
2004; Noguchi et al. 2002) must deal with vast rotation rates since the toroidal field is not applied (i.e. $\beta = 0$) and the rigidly rotating boundaries dominate the whole flow, see Hollerbach & Fournier (2004). In this case it is necessary to split the end-plates into many independently rotating rings (Kageyama et al. 2004; Burin et al. 2006). When the rotation rates are relatively slow, as of order of $Re = O(10^3)$, the desired result can be achieved either by allowing the end-plates to rotate rigidly and independently (see e.g. Abshagen et al. 2004) or split them into two rings which are attached to the both cylinders. From technical point of view the latter configuration is easier to implement and it can be considered as a possible extension to the next spiral MRI experiment.

Firstly we consider a criterion according to which we say that the boundary conditions are more suitable. In the basic state for subcritical values for the periodic cylinders rotational profile of the fluid is $\Omega_0(r)$ and is independent on $z$, magnetic perturbations $b$ are zero everywhere. Applying the endplates leads to developments of $z$ and $r$ gradients in velocity, especially close to the boundaries where $\Omega(r)$ from bulk of the fluid must match imposed conditions for $z = 0, H$. Consequently two Ekman vortices, new currents and magnetic fields are generated (we assume the lids to be insulating unless explicitly stated otherwise). Any deviation from $\Omega_0$ will result in generating azimuthal component of magnetic field $b_\phi$ which enters the momentum equation (and in our 2D axisymmetric formulation is the only term which gives rise to the Lorentz force). Vertical profiles of $b_\phi$ in the middle, i.e. for $r = D/2$, $D = R_{out} - R_{in}$ for two different boundary conditions are shown in Fig. 2. When endplates rotate rigidly with the outer cylinder $\Omega_{end} = \Omega_{out}$ at the bottom lid Ekman circulation has clockwise direction, when they rotate with the inner cylinder $\Omega_{end} = \Omega_{in}$ counter-clockwise – all the gradients have opposite sign. We conclude that, not surprisingly, there exists $\Omega_{out} < \Omega_{end\_min} < \Omega_{in}$ for which shears are minimized and generated magnetic field as well.

We are interested in obtaining rotational profile for which energy in $b_\phi$ is minimized

$$E_b = \int \int b_\phi(r, z)^2 dr dz$$

where the integration is done over all the volume. As a measure of deviation from $\Omega_0$ one could also consider, for example, the kinetic energy of the flow but our aim is to obtain good profiles also for $Ha$ of order 10 (and $\beta = 0$) and this is not necessary good approach since the axial field can inhibit velocity whereas the rotational profile still will be significantly different from $\Omega_0$.

Figure 3 shows how $E_b$ depends on rotation of the rigid endplates $\Omega_{in} < \Omega_{end} < \Omega_{out}$. We notice that minimum occurs for $\Omega_{end\_min} \approx 0.3(\Omega_{in} - \Omega_{out}) + \Omega_{out}$ and is even three orders of magnitude smaller than for $\Omega_{end} = \Omega_{in}$.

When considering endplates divided into rings, we assume that a ring attached to the inner cylinder has width $w_1$, the other one is attached to the outer cylinder $w_2 = D - w_1$. It is not obvious what value for $w_1$ should be chosen therefore we search for optimal $w_1$, i.e. for which $E_b$ has minimum by performing simulations for several different values (Wendt 1933, for example, used straightforward $w_1/D = 0.5$). From Fig. 4 we see that the energy in induced $b_\phi$ has minimum for $w_1/D \approx 0.4$ and is roughly independent of the applied axial magnetic field. It has also been checked that the minimum holds for higher Reynolds numbers (for the Fig. 3 as well). Again we notice the improvement of $E_b$ of two to three orders of magnitude when compared to one end-ring attached either to inner ($w_1 = D$) or outer ($w_1 = 0$) cylinder and also that the minimum value is very similar to that for independently rotating endplates.

Qualitative picture of the resulting rotational profiles
Fig. 4  The magnetic energy $E_b$ as a function of radius of the inner ring for $\beta = 0$, $Re = 100$. —— Ha = 1, —— Ha = 10.

Fig. 5  Deviations of the averaged $\bar{\Omega}(r)$ from the basic state $\Omega_0(r)$ for different vertical boundary conditions; rigidly rotating end-plates (both with $\Omega_{\text{end}}$): —— $\Omega_{\text{end}} = \Omega_{\text{out}}$, ⋯ $\Omega_{\text{end}} = \Omega_{\text{in}}$, ⋯ $\Omega_{\text{end}} = \Omega_{\text{endmin}}$; divided into two rings: · · · $w_1 = 0.5$, ⋯ · · · $w_1 = 0.4$. (a) $Re = 1000$, $Ha = 0$, (b) $Re = 1000$, $Ha = 10$. More regular. Naturally the Ekman pumping mechanism is still present in this case and traces of two Ekman vortices can be seen, however the flow is laminar. When one considers two endplates with different rotational properties additional velocity and current gradients in vertical direction arise and further disturb the flow. It is clearly seen in Fig. 5: where the upper end-plate rotates with $\Omega_{\text{top}} = \Omega_{\text{out}}$ and the bottom one is fixed $\Omega_{\text{bot}} = 0$, that such disturbance exists. The background flow for $\beta = 0$ is highly irregular and time-dependent, especially in the middle part of the cylinder – the circulation close to the end-plates is roughly steady (and after averaging would give about zero unlike in the middle). Nonetheless the external $B_\phi$ produces, again, clear periodic motion with frequency corresponding to that of the helical MRI.

3.2 Influence of the toroidal field

Figure 6 shows values of velocity component $u_z$ similarly like Fig. 1 but for finite cylinders. The velocity field $u_z$ in the basic state, i.e. $\beta = 0$ for which there is no instability, is $u_z = 0$ everywhere when considering infinite or periodic cylinders for $Re \approx 10^3$, $Ha \approx 10$. For the enclosed cylinder this is not the case, in Fig. 6b (left) we present results for symmetrically, rigidly rotating, insulating end-plates with $\Omega_{\text{out}}$, we notice that $u_z$ is quite large and, more importantly, time dependent (this is even more evident for $u_z$ closer to the inner cylinder). The right panel in this figure displays the same flow with the toroidal field applied, $u_z(z)$ is averaged in time and subtracted in order to filter out the background. We clearly see the instability and structure of traveling vortices, frequency of this motion agrees with the predictions based on linear analysis, see e.g. Rüdiger et al. (2005).

As we have shown above, one can obtain much better basic state for the finite cylinders by dividing endplates into two rings, results for such conditions are presented in Fig. 6c. We notice that the background state quickly becomes entirely steady and the results for $\beta = 6$ are clearly gives Fig. 5 it displays deviations of $\bar{\Omega}(r)$ – angular velocity averaged in $z$ domain – from $\Omega_0(r)$ for different rotational properties of the endplates for varied $Re$ and magnetic fields. The case with independently rotating endplates refers to boundary conditions where the both lids rotate with angular velocity $\Omega_{\text{end}}$ corresponding to the minimum value of $E_b$. For comparison we also present case for two rings attached to the cylinders with equal width $w_1 = w_2 = D/2$. We see that applying independently rotating or split- ted endplates produces significantly more suitable profiles – flatter and closer to 1. We also notice that using $w_1 = 0.5D$, especially for $r > R + D/2$ where the former is almost flat.
The axial velocity $u_z(z, t)$ at $r = R_{in} + 0.6D$ as a function of time $t$ and $z$. Left: basic state $\beta = 0$, right: $\beta = 6$, the averaged $u_z(t)$ is subtracted in order to eliminate the background from the velocity field, except in (d). (a) both endplates rotate rigidly with $\Omega_{end} = \Omega_{out}$, (b) both endplates are divided into rings attached to the cylinders; the inner ring has width $0.4D$. (c), (d) the bottom endplate is stationary $\Omega_{bot} = 0$, the upper rotates with $\Omega_{top} = \Omega_{out}$. (a), (b), (c) insulating endplates, $Re = 1775$ ($\Omega_{in} = 0.377$ Hz), $Ha = 9.5$. (d) perfectly conducting endplates, $Re = 1000$, $Ha = 10$. The traveling wave frequency for (a), (b), (c) is respectively $f/\Omega_{in} = 0.0294, 0.0253, 0.0292$ whereas the linear stability analysis yields $f/\Omega_{in} = 0.0258$. 

Fig. 6
Using conducting boundaries instead of insulating leads to increasing the Ekman circulation and the Hartmann current the latter being drawn from the plates. This current is significantly stronger than current generated in the Ekman-Hartmann layer and therefore we expect that experiment with conducting plates would undergo additional problems due to magnetic forces acting on the fluid. Let us consider both perfectly conducting end-plates with asymmetric rotation (again at the top \( \Omega_{\text{top}} = \Omega_{\text{out}} \) and at the bottom \( \Omega_{\text{bot}} = 0 \), then there exists an important gradient in radial current which acting in concert with the axial magnetic field is strong enough to „drag” vortices in direction in which it decreases. This situation is shown in Fig. 6, we see that there exists a periodic vertical motion even when \( \beta = 0 \). Moreover when we introduce the toroidal field with appropriate sign (i.e. positive in this case) it will act against the force due to current gradient and can reduce the periodic vertical motions in the flow (Fig. 6, right panel). When the \( B_\theta \) would have different sign both effects would interact resulting in highly irregular time-dependent behavior.

We shall notice that in the real PROMISE experiment the bottom endplate rotating with \( \Omega_{\text{out}} \) (which, after taking into account directions of rotation and the applied magnetic field, corresponds to the top endplate in our simulations) was made of copper, and the stationary top endplate (bottom in the simulations) was made of plexiglass. Therefore an additional asymmetry in the magnetic boundary conditions was present. Although copper is a good conductor it should not be directly compared with perfectly conducting boundaries used in the simulations since the latter represent stronger assumption and induce stronger currents. However it is clear that using insulating material on the both ends would prevent additional current from disturbing the flow.

### 3.3 Critical values

Noting that the background state for sufficiently fast rotation and rigidly rotating endplates \( \Omega_{\text{end}} = \Omega_{\text{out}} \) is not steady, it is interesting to investigate what happens when the spiral magnetic field with strength below the critical value is applied. One could expect that a viscous process (like the Ekman pumping) excites fluctuations which could be then amplified and, due to geometry of the applied magnetic field, drifting.

Figure 7 shows that for endplates causing strong disturbances the traveling wave can indeed be observed even for subcritical characteristic values. This is also somewhat in agreement with the experiment – traces of moving vortices were observed for states which are stable in the infinite cylinders limit. We see that amplitudes in vertical component of velocity \( u_z \) are almost unchanged when compared to the background state, Fig. 6c (left). Although the pattern of developed vortices is not very regular, there exists a clear peak in frequency of vertical traveling wave and its value, as well as direction (which is reversed upon change of sign, for example, \( \beta \)) corresponds to traveling wave frequency from the linear analysis for infinite cylinders. This leads to conclusion that although excitation do not grow due to helical MRI still the same mechanism is responsible for the drift. When the two rings are used and the basic state is steady the situation changes since the additional excitations due to endplates are minimized. In this case, on the other hand, for supercritical parameters the traveling wave, although excited for a moment, decays and the flow becomes steady, see Fig. 8. It is still possible to get sustained instability by increasing, for example, \( \beta \) (see Fig. 6b).

The reason for this damping might be height of the cylinders which is unfit for the vertical wavenumber \( k_z \). For \( \text{Re} = 1775, \beta = 5, \text{Ha} = 9.5 \) the corresponding wavelength is \( \lambda = 2\pi D/2.1728 \) and does not suit the assumed aspect ratio \( \Gamma = H/D = 10 \). When height was changed to \( \Gamma = 4\lambda = 11.57 \) the observed decay was significantly slower, so slow that after sudden turning on the external azimuthal magnetic field it could be observed with the PROMISE facility still several hours later. Bearing in mind that wavelengths for given Reynolds number are longer with decreasing beta (for \( \beta = 3, k = 1.45D^{-1}; \beta = 1, k = 0.6D^{-1} \)) the constant height of cylinders \( \Gamma = 10 \) in the experiment can be an issue when looking for critical numbers corresponding to those values of \( k \). It should be noted that due to boundary layer effective region where the traveling wave can exists for configuration with two rings is smaller than \( \Gamma \) by approximately 0.5\( D \) (distance up to about 0.25\( D \) from endplates is dominated by theirs influence).

![Fig. 7 Profiles of \( u_z(z, t) \) for \( \text{Re} = 1775, \text{Ha} = 9.5, \beta = 2 \) and rigidly rotating ends with \( \Omega_{\text{end}} = \Omega_{\text{out}} \). The critical \( \beta_c \) for the corresponding \( \text{Re}, \text{Ha} \) in the infinite cylinders limit is \( \beta_c = 2.56 \) so one would expect that the traveling wave decays. This is not the case for the boundary conditions shown here, clear periodic motion is visible. Its frequency \( f/\Omega_{\text{in}} = 0.0124 \) agrees with the linear analysis for marginal stability for infinite cylinders which gives \( f/\Omega_{\text{in}} = 0.0120 \), however the latter approach yields negative growth rate (exact numbers for frequencies and wavenumbers for linear results presented in this paper were provided by R.Hollerbach who used them to generate figures in Rüdiger et al. (2006).](image-url)
Although endplates clearly can serve as the source of viscous excitations and the axially traveling wave develops also for subcritical parameters, we shall notice that there are no periodic motions in the background state. In this sense the „imperfect” background state serves as a catalyst for the helical MRI instability. When the endplates are divided into rings the resulting hydrodynamic flow is laminar and only after the magnetic field is applied the periodic fluctuations occur and moreover their frequency corresponds exactly to that from the linear analysis for infinite cylinders.

Liu et al. (2006) suggested that the observed fluctuations can have theirs origin in the underlying hydrodynamical unsteady flow as reported, for example, by Kageyama et al. (2004). In the latter work the purely hydrodynamic flow for Re ≈ 1000 with rigidly rotating ends Ω_{end} = Ω_{out} and short aspect ratio Γ = 1 was already unsteady, we confirm these results with method that is used here. However when longer cylinders are used, like Γ = 10, the flow becomes steady for Re = 1000, only after imposing strong enough magnetic fields (say Ha = 12, β = 6) the traveling wave develops and its frequency matches calculations from the linear analysis.

### 4.4 Differentially rotating ends

We also have performed simulations for differentially rotating plates with ideal Couette profile for periodic cylinders so that Ω_{end}(r) = Ω_0(r). In another recent work (Liu, Goodman & Ji 2007) it has been shown that for parameters corresponding to Re = 1775, Ha ≈ 10, β ≈ 4 the traveling wave decays for such boundary conditions. We confirm this result with method that is used here, although our approach to the magnetic boundaries is simplified.

The explanation for this fact might be again the inappropriate height of the cylinders which is far from integer multiplication of the expected vertical wavelength. For these parameters λ = 3.476D according to the linear theory so that less than three wavelengths can fit in the container. On the other hand when β = 6 is used, λ = 2.49D and then Γ = 10 almost exactly corresponds to 4λ. From Fig. 9 we see that indeed persistent fluctuations exists in this case with frequency of the helical MRI instability. We have also made calculations for β = 4 with longer cylinders so that H = 4AD = 13.90D and Γ = 5AD = 17.38D, in each case sustained traveling wave has been observed. It should be mentioned that the vortices do not develop very close to the upper boundary so that it is convenient to take longer cylinders.

### 4 Summary

It is easier to perform experiment showing spiral MRI in the sense that much slower rotation of the cylinders is required for the instability to set in compared with MRI with the axial magnetic field only. Moreover, there exists additional quantity, i.e. drift frequency, which is easy to measure and serves as an important indicator for the associated phenomena. It is claimed that in the PROMISE experiment frequencies and amplitudes corresponding to the spiral MRI were observed and the results agreed with theoretical calculations both linear and nonlinear 2D simulations, see Stefani (2007) for review. However it is still possible to improve the experiment so that the basic state is a completely steady flow.

In this paper we have presented relatively simple and inexpensive modification which is suitable for such improvement. Firstly the endplates should be both made of insulating material and both should rotate in the same way so the system is symmetric in z direction, secondly it is convenient to divide the lids into two rings which can be attached to the cylinders so that no separate driving is needed. The optimal,
in the sense of minimizing the induced azimuthal magnetic field, width of the inner ring is 1.6 cm for the current experimental setup.

Our calculations also show that spiral MRI modes can be driven by endplates effects even for subcritical characteristic values, see Fig. 7. On the other side when providing a steady background flow by applying rings one has to pay more attention to height of the cylinders and take into account parameters of magnetic field giving traveling instability with short vertical wavelengths. For the current aspect ratio $\Gamma = 10$ and $Re = 1775$ it is reasonable to consider $Ha = 9.5$, $\beta = 6$ which almost exactly corresponds to $\Gamma = 4\lambda$.

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