Optical conductivity of a superfluid density wave

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We present a calculation of the low frequency optical conductivity of a superconductor in the presence of quenched inhomogeneity in both the superfluid and normal fluid densities. We find that inhomogeneity in the superfluid density displaces spectral weight from the condensate to a frequency range that depends critically on the spatial correlation of normal and superfluid density fluctuations.

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Measurements of the optical conductivity, $\sigma(\omega)$, in the microwave and terahertz regimes have proved to be a valuable probe of quasiparticle dynamics in cuprate superconductors. Because even relatively poor samples are in the clean limit where the scattering rate is smaller than the gap frequency, or $1/\tau \ll \Delta$, the spectral weight of the dissipation due to quasiparticle scattering is far greater than that associated with pair creation. As a consequence the quasiparticle Drude peak can be clearly resolved, and its width can yield the scattering rate. Important clues as to the nature of the scattering can be obtained by comparing the rates measured by microwave conductivity with those measured by angle-resolved photoemission (ARPES).

Extracting $1/\tau$ from the microwave conductivity of a cuprate requires $\sigma(\omega)$ to be consistent with the two-fluid model for a clean, d-wave superconductor \[\text{(1)}\]. In this model the total conductivity contains two contributions: a $\delta$-function at $\omega = 0$ due to the condensate and a Drude-like peak due to quasiparticle scattering. The spectral weights of these components are the super and normal fluid densities, respectively. As the temperature $T$ tends to zero the normal fluid density, $\rho_n$, is expected to decrease linearly with $T$. The superfluid density, $\rho_s$, is expected to increase at the same rate, so that the total density remains constant. This simple behavior permits $\rho_{n,s}$ and $1/\tau$ to be determined in the superconducting state.

The two-fluid model provides an excellent description of the microwave properties of $\text{YBa}_2\text{Cu}_3\text{O}_7 - \delta$ (YBCO) single crystals \[\text{(2)}\]. The scattering rates inferred from the two-fluid analysis are remarkably small, approaching $\sim 10 \text{ GHz}$ as $T \rightarrow 0$. On the other hand, analysis of ARPES yields low temperature scattering rates that are larger by a factor of at least 100 \[\text{(3)}\]. One possible explanation is that the single particle self-energy measured by ARPES is very different from the momentum relaxation rate that determines the conductivity. However, an alternative explanation is that the ARPES data are collected almost exclusively from studies of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (BSCCO) rather than YBCO.

It would seem that the microwave data on BSCCO would hold the resolution to this question. However, analysis of such data has been problematic because the spectra are not consistent with the two-fluid model described previously. In fact the success of the two-fluid description is unique to YBCO single crystals - all other cuprates that have been measured show distinctly different properties \[\text{(4,5)}\]. In these materials the spectral weight of the Drude peak extrapolates to a large residual value as $T \rightarrow 0$, suggesting that a substantial fraction of the normal fluid fails to condense.

Recently, the frequency dependence of the conductivity was reported in optimally doped BSCCO \[\text{(6)}\]. The measurements were performed using coherent time-domain spectroscopy, a technique which can determine $\sigma(\omega)$ up to frequencies $\omega/2\pi \sim 1 \text{ THz}$. The width of the Drude peak at low temperature was determined to be $\sim 300 \text{ GHz}$. Integration of the real part of the conductivity over the experimental range showed that the uncondensed portion is at least 30% of the condensate spectral weight. A study of BSCCO for several carrier concentrations showed that this percentage increases with doping, reaching about 60% in an overdoped sample \[\text{(7)}\].

The two-fluid description of $\sigma(\omega,T)$ in such samples is not internally consistent. Fitting the spectra requires $\rho_n$ to vary with frequency and actually increase with decreasing $T$ for $\omega \lesssim 300\text{GHz}$. On the other hand, the quasiparticle conductivity could be described by physically reasonable parameters if Re $\sigma$ has a third component: a peak centered at $\omega=0$, whose spectral weight grows in proportion to $\rho_s$ as $T$ decreases.

The proportionality to $\rho_s$ suggests that this anomalous part of $\sigma$ is related to current fluctuations of the superfluid. However, in a homogeneous superconductor the spectral weight of such fluctuations decreases exponentially with $T$, which is opposite to the growth of spectral weight that was observed. Recently, Barabash et al. \[\text{(8)}\] showed that a component of Re $\sigma$ can increase as $T \rightarrow 0$ if $\rho_s$ is spatially inhomogeneous. They considered a Josephson junction network with a quenched variation $\delta J$ about the mean coupling $J$. ($J$ is the superfluid density defined on a lattice rather than a continuum). They showed that the global phase stiffness $J$ equals $J - \langle \delta J^2 \rangle / J$. $J$ is less
than $\bar{J}$ because the phase varies more rapidly in regions where the stiffness is below the average value.

The result stated above is important to the optical conductivity because the spectral weight of the condensate $\delta$-function is proportional to $J$, whereas the total spectral weight of condensate current fluctuations is proportional to $\bar{J}$. Thus inhomogeneity of the superfluid density must displace spectral weight $\sim \langle \delta \bar{J}^2 \rangle/\bar{J}$ from the $\delta$-function to nonzero frequency. If $\delta J$ varies in proportion to $\bar{J}$ then the displaced spectral weight will track that of the condensate, in agreement with terahertz experiments \[3\].

The existence of strong spatial inhomogeneity in BSCCO has been demonstrated by scanning tunneling microscopy (STM) measurements \[11\]. The local density of states (LDOS) of BSCCO varies randomly in space, with spatial fluctuations that have a minimum wavelength of $\sim 50 \AA$. The variations in LDOS suggest quenched inhomogeneity in local carrier concentration, $x$, and therefore in the local $\rho_s$.

To determine whether quenched inhomogeneity in $\rho_s$ is responsible for the anomaly in $\sigma(\omega)$, we need to consider the spectrum of the spectral weight removed from the condensate. In a system with no normal component the spectral weight would be expected to appear at the natural oscillation frequency of the condensate, which is the Josephson plasma frequency, $\omega_s$. In optimal cuprates $\omega_s/2\pi \sim 200$ THz, whereas the anomalous dissipation is found below $\sim 1$ THz. Inhomogeneity can only explain the data if the presence of a normal fluid in addition to the superfluid causes the displaced spectral weight to appear at frequencies much smaller than $\omega_s$.

In this paper we calculate the change in $\sigma(\omega)$ due to inhomogeneity in a superconductor in which normal and superfluid coexist. This is the first calculation in which the spectral weight of the conductivity is consistent with the theorem of Ref. \[10\]. We find that $\sigma(\omega)$ is extremely sensitive to correlations of the normal and superfluid density variations in the medium. The spectrum depends crucially on whether the correlation is positive (such that regions of large $\rho_s$ have large $\rho_n$) or negative. If the correlation is positive, the displaced spectral weight indeed shifts to the plasma frequency. However, when the fluctuations of normal and superfluid density are antico- correlated, the spectral weight appears at low frequencies, in agreement with microwave and terahertz measurements on cuprate superconductors.

To treat the conductivity in the presence of inhomogeneity we apply the extended two-fluid phenomenology developed by Pethick and Smith \[12\], and Kadin and Goldman \[13\]. This approach successfully describes quenched inhomogeneity at the normal-superconductor interface and fluctuating inhomogeneity, as in the Carlson-Goldman oscillations \[14\]. In the extended two-fluid model the superfluid is accelerated by gradients of the chemical potential as well as electric fields, that is,

$$\dot{J}_s = \rho_s (\vec{E} - \nabla \mu_s).$$

The chemical potential has the subscript $s$ because in a superconductor $\mu$ is the energy per electron required to add a pair to the condensate. The corresponding equation for the normal fluid current requires solving the Boltzmann equation for the quasiparticle distribution function. However for frequencies less than $1/\tau$, the distribution function is the equilibrium distribution shifted by the ‘quasiparticle chemical potential’ or $\mu_n$. If the normal fluid is in local equilibrium with the condensate $\mu_n = \mu_s$, which differs from ”global” equilibrium where $\mu_n=0$. In the low frequency regime the constitutive relation for the normal fluid has the simple form:

$$\dot{J}_n = \rho_n (\vec{E} - \nabla \mu_n) - \bar{J}_n/\tau.$$  \hspace{1cm} (2)

A closed system of equations requires continuity relations. The total charge of the superconductor separates naturally into a normal component, $Q_n$, that depends on both the coherence factors and the distribution function,

$$Q_n \equiv \sum_k q_k f_k$$ \hspace{1cm} (3)

where $q_k^2 \equiv u_k^2 - v_k^2$, and a superfluid component that depends only on coherence factors,

$$Q_s \equiv \sum_k 2ev_k^2 = 2eN_F \mu_s.$$ \hspace{1cm} (4)

Under conditions for which $\mu_n$ can be defined, the normal fluid charge is given by,

$$Q_n = 2N_F \lambda (\mu_s - \mu_n),$$ \hspace{1cm} (5)

where $N_F$ is the normal state density of states at the Fermi level. The parameter $\lambda$ relates the normal fluid charge to the shift of $\mu_n$ away from local equilibrium.

In a superconducting medium the normal and superfluid charge are not separately conserved. Interconversion of normal and superconducting charge occurs through two types of processes. In the first process, $Q_n$ changes as quasiparticles recombine or scatter. In the second, the quasiparticle charge changes even if $f_k$ remains constant. In this process, $Q_n$ varies because the quasiparticle excitation spectrum, and consequently the effective charge, adjusts to the local value of $\mu_s$. Continuity equations that include both types of exchange between the two fluids are:

$$\dot{Q}_{n,s} + \nabla \cdot \vec{J}_{n,s} = (-,+)\left(\frac{Q_n}{\tau_Q} - \lambda \dot{Q}_s\right),$$ \hspace{1cm} (6)

where $\tau_Q$ is the rate of conversion of normal charge into superfluid charge due to scattering and recombination processes. The above system of equations is closed by $\nabla \cdot \vec{E} = Q/\varepsilon_0$. 

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To see how quenched inhomogeneity affects the optical conductivity, we consider the simplest possible model: a one-dimensional sinusoidal variation in normal and superfluid density. We assume that the densities of the two components vary as $\rho_{s,n}(x) = \langle \rho_{s,n} \rangle + \text{Re}\{\rho_{sq,nq}e^{i\pi x}\}$, where $\langle \rho \rangle$ is the average density and $\rho_q$ is the amplitude of the inhomogeneity at wavevector $q$. Because the medium is inhomogeneous, a uniform applied field generates a field at $q$. Solving the extended two-fluid equations to lowest order in $\rho_{nq,sq}$, we obtain,
\[
\frac{E_q}{E_0} = -\frac{\rho_{sq} + \rho_{nq} F}{\omega_s^2 - \omega^2(1 - \lambda) + (\omega_n^2 - i\omega\lambda/\tau) F},
\]
where,
\[
F \equiv \frac{\omega_i q^2}{(\omega + i/\tau)(\omega + i/\tau_Q)} - \frac{v_s^2 q^2}{\lambda},
\]
\[
\omega_{s,n}^2 \equiv \rho_{s,n}/\epsilon_0 + v_{s,n}^2 q^2, \quad v_{s,n}^2 \equiv \rho_{s,n}/2N_F e^2, \quad \text{and} \quad \tau_Q \equiv \tau_Q(1 - \lambda).
\]

The uniform current density in response to these fields is given by $J_0 = \sigma_0 E_0 + \sigma_q E_{-q}$, where $\sigma_0$ is the uniform two-fluid conductivity, and $\sigma_q$ is the conductivity that varies with wavevector $q$. In the two-fluid model these are given by,
\[
\sigma_{0,q} = \frac{i\rho_{sq}}{\omega} + \frac{i\rho_{nq}}{\omega + i/\tau}.
\]

The effective conductivity of the medium, $\sigma = \sigma_1 + i\sigma_2$, is the ratio of the uniform current density to the uniform field, so that, $\sigma = \sigma_0 + \sigma_q E_{-q}/E_0$. The second term in this equation is the change in the optical conductivity due to inhomogeneity, or $\Delta\sigma$.

The extra term in the conductivity is particularly simple if $\rho_n = 0$, in which case,
\[
\Delta\sigma_2 = \frac{\rho_{sq}}{\omega} \frac{\rho_{sq}}{\omega_s^2 - \omega^2}.
\]

The inhomogeneity in the superfluid density indeed removes spectral weight $\rho_{sq}^2/\rho_s$ from the condensate $\delta$-function, in agreement with the results of Ref. [10]. In the absence of a normal fluid component the spectral weight reappears in a $\delta$-function at the Josephson plasma frequency.

We next assume that $\rho_n \neq 0$ (even at $T = 0$), and describe how this assumption affects $\Delta\sigma(\omega)$. We focus on the behavior of $\Delta\sigma$ when the normal fluid density fluctuations are either perfectly correlated or anticorrelated with those of the superfluid density. We take for two-fluid parameters values that are suggested by the terahertz and microwave experiments: $\langle \rho_s/\epsilon_0 \rangle^{1/2} = 1000$ THz, $\langle \rho_n/\epsilon_0 \rangle^{1/2} = 800$ THz, and $\tau^{-1} = \tau_Q^{-1} = 3$ THz. The existence of normal fluid at $T = 0$ implies a large density of states at the Fermi level, $N_0$. While the origin of $N_0$ is outside the scope of this paper, one may speculate that it is closely related to the strong inhomogeneity observed by STM. If we make the reasonable approximation of neglecting BCS coherence factors for the quasiparticle states introduced by disorder, then $\lambda = N_0/N_F$.

![Figure 1](https://example.com/fig1.png)

**FIG. 1.** $\Delta\sigma_1$ as a function of frequency ($\omega/2\pi$) for anticorrelated variations in $\rho_s$ and $\rho_n$. Spectral weight decreases for increasing $v_s q \tau$: 0.25, 0.5, 1.0, and 2.0. The same curves are shown in a normalized plot in the inset.

We begin with the case where the density fluctuations are perfectly anticorrelated, so that $\rho_{sq} = -\rho_nq$ and the total fluid density is uniform throughout the medium. Fig. 1 shows $\Delta\sigma_1(\omega)$ with $(\rho_{sq}/\epsilon_0)^{1/2} = 100$ THz, for several values of $v_s q \tau$. $\Delta\sigma_1(\omega)$ is positive and centered at $\omega = 0$ rather than $\omega_s$. The spectra depend strongly on $v_s q \tau$. For $v_s q \tau \ll 1$, $\Delta\sigma_1$ has a Drude-like spectrum, whose width is $\sim 1/\tau$. As $v_s q \tau$ increases beyond unity, the spectral weight drops and a peak near the Carlson-Goldman frequency $v_s q$ appears in the spectrum.

The key issue is the fraction of the displaced condensate spectral weight that appears in the low-frequency peak, as opposed to $\omega \sim \omega_s$. In Fig. 2 we compare the reduction in condensate weight with the increase in dissipation at low frequency. The change in condensate spectral weight was evaluated from the lim$_{\omega \to 0}(\pi/2)\omega \Delta\sigma_1(\omega)$. The low frequency spectral weight was obtained by numerically computing the integral of $\Delta\sigma_1$ with respect to $\omega$ from 0 to 100 THz. Fig. 2 shows these two quantities, normalized to $\rho_{sq}^2/\rho_s$, as a function of $v_s q \tau$. They are equal in magnitude but opposite in sign, which shows that all of the spectral weight removed from the condensate appears at low frequency and none appears at $\omega_s$. Moreover, the decrease of condensate spectral weight coincides exactly with the prediction of Barabash et al. [10] as $v_s q \tau \to 0$, but vanishes for $v_s q \tau \gg 1$.

The surprising results presented above are a straightforward consequence of the anticorrelation of the density fluctuations. There is no dissipation near $\omega_s$ or $\omega_p \equiv (\langle \rho_s + \rho_n/\epsilon_0 \rangle^{1/2}$ because the inhomogeneous medium is dynamically homogeneous at high frequencies. For
the electric field in regions with greater than average dissipation arises ultimately from an amplification of variation in regions of low stiffness. Thus the increase in regions, which is precisely equivalent to more rapid phase coexistence in the cuprates. De
calculations upon crossing optimal doping $x_{\text{opt}}$ would indeed be expected, based on very general considerations. For a sample with $x < x_{\text{opt}}$, regions of lower than average $\rho_n$ will be strongly overdoped patches. These patches will be insulators, or at least regions with small $\rho_n$. On the other hand, the regions of low $\rho_s$ in a sample with $x > x_{\text{opt}}$ will be strongly overdoped, i.e., regions of large $\rho_n$. Thus the correlation varies systematically from positive to negative with increasing $x$, leading to a dramatic increase in the low frequency spectral weight.

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