We describe a plausible-speculative form of quantum computation which exploits particle (fermionic, bosonic) statistics, under a generalized, counterfactual interpretation thereof. In the idealized situation of an isolated system, it seems that this form of computation yields to NP-complete=P.

I. INTRODUCTION

Particle (fermionic, bosonic) statistics has never been applied until now in the usual (most fruitful) approach to quantum computation. We mean sequential computation, which can be characterized by the fact that a reversible Boolean network appears in the time-diagram of the computation process [1-6]. It has been applied instead in the approach expounded in ref. [7-12], which differs from the usual one:

- a different form of computation is used; its classical counterpart is simulated annealing or ground state computation; time is orthogonal to the layout of a reversible Boolean network whose input and output qubits coexist as simultaneous eigenvalues of a set of compatible observables. Therefore part of the network input and part of the output can be both constrained. Checking whether this network is satisfiable is a version of the NP-complete SAT problem.

- This framework seems suitable to applying particle statistics: the network gates and wires establish Boolean relations between the simultaneous eigenvalues of the respective input/output qubits; a particle statistics symmetry is also a constraint applying to simultaneous eigenvalues.

- However, in the conventional interpretation, a particle statistics symmetry is an initial condition which is conserved as a constant of motion by a unitary evolution. Therefore it does nothing along the course of this type of evolution. In the generalized interpretation propounded in ref. [8-12], a particle statistics symmetry is the result of a continuous projection on the Hilbert subspace satisfying the symmetry. We can thus apply a Hamiltonian $G_r$ to a part $r$ of the system, provided that particle indistinguishability is preserved. This and the continuous projection, seen as a form of “interaction” between $r$ and the remaining part of the system $s$, define a system level Hamiltonian $G_{rs}$. $G_{rs}$ induces a unitary evolution of the state of the whole system which does not violate the symmetry. In a way, this evolution is driven by $G_r$ and shaped by particle statistics seen as continuous projection.

In [12] we have shown how to map the logical constraint established by a network element (N.E., namely gate or wire), on a constraint induced by particle statistics. This constraint is seen as continuous projection on the Hilbert subspace satisfying the N.E. logical constraint. The network is prepared in an initial state satisfying the (partial) input constraints and all N.E. constraints, whereas the output constraint (it is sufficient to constrain only one output qubit) is temporarily removed. This amounts to solve a problem polynomial in network size. Then we operate on the (to be constrained) output qubit in order to bring it in match with its constraint. This operation, under the continuous projection, induces a unitary transformation leading the network state to satisfying all the constraints (with very high probability), if the network is satisfiable. This requires a time independent of network size and would yield to NP-complete=P, see [12].

However, in [12] we could not exclude the possibility that the operation performed on the output qubit pushed the network into an error state, with some probability; the rate of growth of this probability with network size was not known. An exponential growth would have completely vanified the former result.

In the current work, we show that this probability can in principle be zero, which brings to NP-complete=P in an idealized framework.
To show this, we will need to recover part of [12], namely the counterfactual interpretation of particle statistics and its application to an elementary network element (a NOT gate).

II. A COUNTERFACTUAL, GENERALIZED INTERPRETATION OF PARTICLE STATISTICS

The first part of this Section summarizes a corresponding part of [12]. The second part introduces a further development.

Let us consider the unitary evolution of a triplet state induced, for assumption, by particle statistics:

\[
|\Psi (t)\rangle = \cos^2 (\theta + \omega t) |0\rangle_1 |0\rangle_2 + \sin (\theta + \omega t) \cos (\theta + \omega t) (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2) \\
+ \sin^2 (\theta + \omega t) |1\rangle_1 |1\rangle_2
\]  

(1)

1 and 2 label two identical two-state particles with Hilbert spaces respectively \( \mathcal{H}_1 = \text{span}\{|0\rangle_1, |1\rangle_1\} \) and \( \mathcal{H}_2 = \text{span}\{|0\rangle_2, |1\rangle_2\} \) (which makes this an idealized situation).

Evolution (1) is obtained by applying an identical rotation \( \omega t \) of the states of the two particles to the initial state \( |\Psi (0)\rangle \), namely: \( Q_1 (\omega t) Q_2 (\omega t) |\Psi (0)\rangle = |\Psi (t)\rangle \), where \( Q_j (\omega t) = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix} \), with \( j = 1, 2 \).

In the usual interpretation, the propagator \( Q_1 (\omega t) Q_2 (\omega t) \), symmetrical for the particle permutation \( P_{12} \), commutes with \( \mathcal{S}_{12} = \frac{1}{2} (1 + P_{12}) \); therefore the triplet state symmetry is an initial condition conserved as a constant of motion.

The counterfactual interpretation given in [12] can be summarized as follows. We consider the evolution (1) is obtained by applying an identical rotation \( \omega t \) of the states of the two particles to the initial state \( |\Psi (0)\rangle \), namely: \( Q_1 (\omega t) Q_2 (\omega t) |\Psi (0)\rangle = |\Psi (t)\rangle \), where \( Q_j (\omega t) = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix} \), with \( j = 1, 2 \).

The counterfactual interpretation given in [12] can be summarized as follows. We consider the possibility that \( |\Psi (\tau)\rangle \) at any time \( \tau = t - dt \) goes out of symmetry. This is interpreted by saying that \( |\Psi (t - dt)\rangle \) is not constrained by the symmetry: it is therefore a free vector of the Hilbert space \( \mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2 \), namely:

\[
|\Psi (t - dt)\rangle = \alpha^{(t - dt)}_{00} |0\rangle_1 |0\rangle_2 + \frac{1}{2} \left( \alpha^{(t - dt)}_{01} + \alpha^{(t - dt)}_{10} \right) (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2) + \frac{1}{2} \left( \alpha^{(t - dt)}_{01} - \alpha^{(t - dt)}_{10} \right) (|0\rangle_1 |1\rangle_2 - |1\rangle_1 |0\rangle_2) + \alpha^{(t - dt)}_{11} |1\rangle_1 |1\rangle_2
\]  

(2)

the \( \alpha^{(t - dt)}_{ij} \) are free complex variables independent of each other up to \( \sum_{i,j \in \{0,1\}} |\alpha^{(t - dt)}_{ij}|^2 = 1 \); for any time \( t \). In other words, if \( t_1 \neq t_2 \), \( |\Psi (t_1)\rangle \) and \( |\Psi (t_2)\rangle \) are two independent free normalized vectors of \( \mathcal{H}_{12} \).

Of course we must assume the possibility that \( \alpha^{(t - dt)}_{01} - \alpha^{(t - dt)}_{10} \neq 0 \). If this is the case, namely if \( |\Psi (t - dt)\rangle \) is out of symmetry, this state should be “immediately” projected on the symmetric subspace

\[
\mathcal{H}^{(s)}_{12} = \text{span} \left\{ |0\rangle_1 |0\rangle_2, \frac{1}{\sqrt{2}} (|0\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2), |1\rangle_1 |1\rangle_2 \right\}.
\]

The result of this projection can be obtained by submitting \( |\Psi (t)\rangle \), another free normalized vector of \( \mathcal{H}_{12} \) independent of \( |\Psi (t - dt)\rangle \), to the following equations:

for all \( t \):

i) \( S_{12} |\Psi (t)\rangle = |\Psi (t)\rangle \),

ii) \( Max \ |\langle \Psi (t) | \Psi (t - dt)\rangle| \), which naturally means that the distance between the vector before projection \( |\Psi (t - dt)\rangle \) and the vector after projection \( |\Psi (t)\rangle \) is minimum,

where \( |\Psi (t)\rangle \) is a free normalized vector of \( \mathcal{H}_{12} \).

These equations also take into account the fact that projection goes on repeatedly in a continuous fashion.

In this way, the triplet state symmetry is seen as the result of a watchdog effect that continuously projects the system state on the symmetrical subspace \( \mathcal{H}^{(s)}_{12} \). Of course the result is that \( |\Psi (t)\rangle \) can never go out of symmetry. However, this counterfactual reasoning will have consequences (the possibility that counterfactuals yields to effective consequences in the quantum framework has been highlighted by R. Penrose[13]).

A first consequence is that we do not need to assume that symmetry is an initial condition: no state satisfying (i) for all \( t \) can ever be out of symmetry.

Also the notion that symmetry is a constant of motion can be given up. To show this, it is useful to see the continuous projection (i) and (ii) as a special, continuous form of state vector reduction. As a matter of fact, the cancelation of the amplitude \( \alpha^{(t - dt)}_{01} - \alpha^{(t - dt)}_{10} \) and the renormalization of the other amplitudes in state (2) is a partial
state vector reduction on the subspace $H_{12}^{(s)}$ (it is partial since this subspace has dimension higher than one). Of course there is no dynamics in state vector reduction, only interference – in the above form of cancelation of one amplitude and renormalization of the others.

We should note the peculiarity that such a reduction always occurs on the symmetrical subspace, never on the orthogonal subspace. This is paradoxical in counterfactual reasoning, it would mean that the result of reduction is not random but is affected by a condition placed in the immediate future, namely that the result does not violate the symmetry. As a matter of fact, the current counterfactual interpretation can be justified in a two-way (advanced and retarded) propagation model.\[78\]

From another standpoint, there is no paradox at all since, actually, $|Ψ(t)⟩$ never goes out of the subspace $H_{12}^{(s)}$. However, the consequences of this counterfactual reasoning will diverge from the conventional way of applying quantum mechanics.

We shall now reconstruct evolution (1) by resorting to conditions (i) and (ii). A first observation is that these conditions affect the overall state of the two particles, therefore the states of the individual particles may no more be defined. We must use the particle density matrices

$$\rho_i(t) = Tr_{3-i} |Ψ(t)⟩⟨Ψ(t)|,$$

where $Tr_{3-i}$ means partial trace over $3-i$, and $i = 1, 2$ is the particle label (if we use the method of random phases, $|Ψ(t)⟩$ does not need to be a pure state – anyhow it will turn out to be that).

A second observation is that the coherence elements of each density matrix – as entanglement – can also be affected by the watchdog effect (in fact they will be determined by it). We only know that the diagonal of each density matrix must show an orthogonal subspace. This is paradoxical in counterfactual reasoning, it would mean that the result of reduction is not random but is affected by a condition placed in the immediate future, namely that the result does not violate the symmetry. As a matter of fact, the current counterfactual interpretation can be justified in a two-way (advanced and retarded) propagation model.\[78\]

We should now note a fact which is essential to the current work. As readily seen, by removing either one of the two conditions (iii), evolution (1) is still obtained – the two conditions are redundant with respect to one another.

Therefore it is perfectly legitimate to say that the rotation of the state of only one particle, either one in an indistinguishable way, carries an identical rotation of the state of the other particle. In this idealized picture, particle statistics can be seen as an interaction free constraint, namely as a non-dynamic constraint operating by way of destructive interference and renormalization.

We will show how to apply (speculatively) the driving condition (iii). This is a further development with respect to [12]. Only one particle $j$, either one in an indistinguishable way, should be submitted to the Hamiltonian

$$G_j = ωσ_{yj} 1_{3-j},$$

where $σ_{yj}$ is the Pauli matrix in $H_j$

$$σ_{yj} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}_j,$$

and $1_{3-j}$ is the identity in $H_{3-j}$. The overall Hamiltonian $G_{12}$, operating in $H_{12}$, is obtained through symmetrization of $G_j$: $G_{12} = \frac{1}{2}ω(σ_{y1}1_{2} + 1_{1}σ_{y2})$. This yields

$$G_{12} = \frac{1}{2}ω \begin{pmatrix} 0 & i & i & 0 \\ -i & 0 & i & 0 \\ -i & 0 & 0 & i \\ 0 & -i & -i & 0 \end{pmatrix}.$$  \[5\]

1 An example of a unitary evolution shaped by a continuous form of state vector reduction, is the evolution of the polarization of a photon going through an infinite series of polarizing filters, each rotated by an infinitesimal constant amount with respect to the former one. In a way, we go back to the root of quantum computation (computation reversibility)\[15,\] and take an alternative branch, by exploring a strictly quantum form of reversible computation.
This is in fact the generator of \( Q_1 (\omega t) Q_2 (\omega t) \), as readily checked.

The above symmetrization is interpreted as follows. If we applied the Hamiltonian \( G_j \) to one particle assumed to be independent of the other, we would have obtained a rotation of the state of that particle by \( \omega t \). By considering the continuous projection on \( H_{12}^{(r)} \) as a form of interdependence between the two particles, the former Hamiltonian becomes \( G_{12} \) at overall system level (in a way, continuous projection is considered as a sort of interaction Hamiltonian between the two particles). \( G_{12} \) generates the operator \( Q_1 (\omega t) Q_2 (\omega t) \) which rotates the states of both particles of the same amount. Therefore we can say that the rotation of the state of only one particle (either one in an indistinguishable way), drags an identical rotation of the state of the other.

This, in the current context, is a tautological interpretation of particle statistics. However, applied to a different context (this will be two indistinguishable particles hosted by two distinguishable lattice sites), such an interpretation will yield two results diverging from the conventional way of applying quantum mechanics.

III. AN ELEMENTARY GATE AS A PROJECTOR

We consider a couple of (coexisting) qubits \( r \) and \( s \) which make up the input and the output of a NOT gate. Let

\[
\mathcal{H}_{rs} = \text{span} \{ |0\rangle_r |0\rangle_s, |0\rangle_r |1\rangle_s, |1\rangle_r |0\rangle_s, |1\rangle_r |1\rangle_s \},
\]

be the Hilbert space of the two qubits,

\[
\mathcal{H}_{rs}^{(c)} = \text{span} \{ |0\rangle_r |1\rangle_s, |1\rangle_r |0\rangle_s \}
\]

be the constrained subspace, spanned by those \( \mathcal{H}_{rs} \) basis vectors which satisfy the NOT gate, and \( A_{rs} \) be the projector from \( \mathcal{H}_{rs} \) on \( \mathcal{H}_{rs}^{(c)} \).

We shall apply the mathematical model of Section II to represent an evolution of the state of the NOT gate \(| \Psi (t) \rangle\) (this Section is purely mathematical, a plausible physical model will be given in Section IV). \(| \Psi (t) \rangle\) should be a free normalized vector of \( \mathcal{H}_{rs} \),

\[
| \Psi (t) \rangle = \sum_{i,j} \alpha_{ij}^{(t)} |i\rangle_r |j\rangle_s, \quad \text{with} \quad \sum_{i,j} |\alpha_{ij}^{(t)}|^2 = 1,
\]

subject to continuous projection on \( \mathcal{H}_{rs}^{(c)} \):

for all \( t \):

i) \( A_{rs} | \Psi (t) \rangle = | \Psi (t) \rangle \),

ii) \( \text{Max} \ | \langle \Psi (t) | \Psi (t - dt) \rangle | \).

Let \(| \Psi (0) \rangle = \cos \vartheta |0\rangle_r |1\rangle_s + \sin \vartheta |1\rangle_r |0\rangle_s \) be the gate initial state. We assume of acting on qubit \( r \) with the Hamiltonian

\[
G_r = \omega \sigma_{yr}, \quad \text{where} \quad \sigma_{yr} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}_r.
\]

We have two ways of deriving the evolution induced by \( G_r \).

a) If acting on the independent qubit \( r \), \( G_r \) would rotate its state by \( \omega t \). As we have seen in Section II, the diagonal of qubit \( r \) density matrix should not be affected by the continuous projection on \( \mathcal{H}_{rs}^{(c)} \). Thus:

iii) \( \text{diag} \rho_r (t) = \text{diag} \{ T r_s | \Psi (t) \rangle \langle \Psi (t) | \} = \cos^2 (\vartheta + \omega t) |0\rangle_r |0\rangle_s + \sin^2 (\vartheta + \omega t) |1\rangle_r |1\rangle_s \),

Conditions (i), (ii) and (iii) define the unitary evolution

\[
| \Psi (t) \rangle = \cos (\vartheta + \omega t) |0\rangle_r |1\rangle_s + \sin (\vartheta + \omega t) |1\rangle_r |0\rangle_s , \quad (6)
\]
as readily checked. Condition (iii) drives and conditions (i) and (ii) shape this evolution. We can see that the rotation of qubit \( r \) [the driving condition \( G_r \), or (iii)] induces an identical rotation of qubit \( s \):
\[ \rho_s (t) = T_r \left[ |\Psi (t)\rangle \langle \Psi (t)| \right] = \sin^2 (\theta + \omega t) |0\>_s \langle 0|_s + \cos^2 (\theta + \omega t) |1\>_s \langle 1|_s , \]

of course 0 and 1 are interchanged.

b) A second way of deriving evolution (6) consists in computing the Hamiltonian \( G_{rs} \) acting on the overall state of the two qubits. \( G_{rs} \) is originated by \( G_r \) and the continuous projection (seen as a form of interaction or better interdependence between the two qubits). Given that this latter introduces the following bijective correspondence between \( \mathcal{H}_r \) and \( \mathcal{H}_{rs} \):

\[ |0\>_r \leftrightarrow |0\>_r |1\>_s , \quad |1\>_r \leftrightarrow |1\>_r |0\>_s , \]

\( G_r \) is:

\[ G_{rs} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ; \]

generates the operator:

\[ Q_{rs} (\omega t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega t & \sin \omega t & 0 \\ 0 & -\sin \omega t & \cos \omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \]

with \( |0\>_r |1\>_s \equiv |1\>_r |0\>_s \). One can see that \( Q_{rs} (\omega t) |\Psi (0)\rangle = |\Psi (t)\rangle \) given by equation (6). We have thus obtained the same result as before – when the driving condition was the evolution of \( \text{diag}p_r (t) \).

In view of what will follow, it is important to note that, if all \( \mathcal{H}_{rs}^{(c)} \) basis vectors occur with amplitudes different from zero in the initial state \( (t = 0) \), namely if \( \theta \neq \frac{\pi}{2} \), condition (i) is redundant with respect to condition (ii). In this case, condition (ii) alone implies \( \alpha_0 |0\rangle + \alpha_1 |1\rangle = 0 \) which already satisfies condition (i).

On the contrary, condition (i) is not redundant if \( \theta = 0 \) or \( \frac{\pi}{2} \). For example, if \( \theta = 0 \), i.e. \( |\Psi (0)\rangle = |0\>_r |1\>_s \), condition (ii) implies \( \alpha_0 |0\rangle = 0 \) and \( |\alpha_0|^2 = \cos^2 (\theta + \omega t) \), as needed, while \( \alpha_1 |1\rangle = |0\rangle \) and \( \alpha_1 |1\rangle = |0\rangle \) are only subject to the constraint \( |\alpha_0|^2 + |\alpha_1|^2 = \sin^2 (\theta + \omega t) \). Thus, disregarding condition (i) would allow for the existence of the forbidden state \( \alpha_1 |1\>_r |0\>_s \).

In conclusion, we have ascertained a peculiar fact. Our “operation on a part” [this is just the mathematical condition (ii), or \( G_{rs} \) in equivalent terms, for the time being], blind to its effect on the whole, performed together with continuous \( A_{rs} \) projection, generates a unitary transformation which is, so to speak, wise to the whole state, to how it should be transformed without ever violating \( A_{rs} \) (i.e. the NOT gate). Of course \( A_{rs} \) ends up commuting with the resulting overall unitary propagator, but because this is itself Shaped by \( A_{rs} \).

**IV. Exploiting Particle Statistics**

\( A_{rs} \) projection can be shown to be an epiphenomenon of particle (fermionic or bosonic) statistics “turned on” in a special physical situation. In the following, we will adopt fermionic statistics.

In order to implement the NOT gate, we consider two identical fermionic particles 1 and 2. Just for the sake of visualization, we can think that each particle has spin 1/2 and can occupy either one of two distinguishable lattice sites \( r \) and \( s \). The generic basis vector of this system has the form \( |\chi_1\rangle_1 |\chi_2\rangle_2 |\lambda_1\rangle_1 |\lambda_2\rangle_2 \), where \( \chi_i = 0, 1 \) means that the spin of particle \( i \) is down, up and \( \lambda_i = r, s \) means that the site occupied by particle \( i \) is \( r, s \). For example, \( |0\rangle_1 |1\rangle_2 |r\rangle_1 |s\rangle_2 \) reads: particle 1 spin = 0, particle 2 spin = 1, particle 1 site = \( r \), particle 2 site = \( s \).

16 combinations like make up the basis of the Hilbert space \( \mathcal{H}_{12} \). However, there are only six antisymmetrical combinations (not violating fermion statistics) which make up the basis of the antisymmetrical subspace \( \mathcal{H}_{12}^{(a)} \).

These basis vectors are represented in second quantization and, when there is exactly one particle per site, in qubit notation (\( \chi \) and \( \lambda \) stand respectively for the qubit eigenvalue and label), \( |0\rangle \) is the vacuum vector:

\[ |a\rangle = a_{0r}^{\dagger} a_{1r}^{\dagger} |0\rangle , \]

\[ |b\rangle = a_{0s}^{\dagger} a_{1s}^{\dagger} |0\rangle ; \]
\[ |c\rangle = a_{0r}^\dagger a_{1s}^\dagger |0\rangle = |0\rangle_r |0\rangle_s, \]
\[ |d\rangle = a_{1r}^\dagger a_{1s}^\dagger |0\rangle = |1\rangle_r |1\rangle_s, \]
\[ |e\rangle = \frac{1}{\sqrt{2}} \left( a_{0r}^\dagger a_{1s} + a_{1r}^\dagger a_{0s}^\dagger \right) |0\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_r |1\rangle_s + |1\rangle_r |0\rangle_s \right). \]
\[ |f\rangle = \frac{1}{\sqrt{2}} \left( a_{0r}^\dagger a_{1s}^\dagger - a_{1r}^\dagger a_{0s}^\dagger \right) |0\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_r |1\rangle_s - |1\rangle_r |0\rangle_s \right). \]
\[ a_{\chi}^\dagger \text{ creates a particle of spin } \chi \text{ in site } \lambda; \text{ creation/annihilation operators are subject to: } \{a_{\alpha}^\dagger, a_{\beta}^\dagger\} = \delta_{\alpha\beta}, \quad \{a_{\alpha}^\dagger, a_{\beta}\} = 0, \quad \{a_{\alpha}, a_{\beta}\} = \delta_{\alpha\beta}. \]
Under the condition that there is exactly one particle per site, they generate a \textit{qubit algebra}.

Now we introduce the Hamiltonian
\[ H_{rs} = -\left( E_a a_{0r}^\dagger a_{1s} + E_b a_{0s}^\dagger a_{1r} + E_c a_{0r}^\dagger a_{1s} + E_d a_{0s}^\dagger a_{1r} \right) + \text{diag } \rho, \]
with \( E_a, E_b, E_c, E_d \geq 0 \) discretely above 0. This leaves us with two degenerate ground eigenstates:
\[ |e\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_r |1\rangle_s + |1\rangle_r |0\rangle_s \right) \quad \text{and} \quad |f\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_r |1\rangle_s - |1\rangle_r |0\rangle_s \right). \]
The generic ground state is thus:
\[ |\Psi\rangle = \alpha |0\rangle_r |1\rangle_s + \beta |1\rangle_r |0\rangle_s, \quad \text{with} \quad |\alpha|^2 + |\beta|^2 = 1. \] (7)

Of course \(|\Psi\rangle\) satisfies \( A_{rs} |\Psi\rangle = |\Psi\rangle \), and belongs to \( H_{12}^{rs} \) (Section III), a subspace of \( H_{12}^a \).

Let \( A_{12} |\Psi\rangle = \frac{1}{2} \left( 1 - P_{12} \right) |\Psi\rangle \) be the usual antisymmetrization projector. Due to the above anticommutation relations:
\[ A_{12} |0\rangle_r |1\rangle_s = |0\rangle_r |1\rangle_s \quad \text{and} \quad A_{12} |1\rangle_r |0\rangle_s = |1\rangle_r |0\rangle_s, \quad \text{moreover} \]
\[ A_{12} |0\rangle_r |0\rangle_s = |0\rangle_r |0\rangle_s \quad \text{and} \quad A_{12} |1\rangle_r |1\rangle_s = |1\rangle_r |1\rangle_s, \]
without forgetting that \(|0\rangle_r |0\rangle_s = |c\rangle\) and \(|1\rangle_r |1\rangle_s = |d\rangle\) are \textit{excited states}.

The NOT gate can be implemented by suitably operating on the ground state (7). We assume this to be initially:
\[ |\Psi(0)\rangle = \cos \theta |0\rangle_r |1\rangle_s + \sin \theta |1\rangle_r |0\rangle_s. \] (8)

The evolution induced by the Hamiltonian \( G_r \) is obtained by adopting the projection interpretation of particle statistics. This means that \(|\Psi(t)\rangle\) is continuously projected on the antisymmetric subspace \( H_{12}^{(c)} \) while \( \text{diag } \rho_r(t) \) evolves as if \( G_r \) were applied to the independent qubit \( r \):
\[ \text{for all } t: \]
\[ \text{i) } A_{12} |\Psi(t)\rangle = |\Psi(t)\rangle, \]
\[ \text{ii) } Max \{|\langle \Psi(t)\rangle |\delta_r (t - dt)\rangle\}, \]
\[ \text{iii) } \text{diag } \rho_r(t) = \text{diag } \{ T_r \{ |\Psi(t)\rangle \langle \Psi(t)| \} \} = \cos^2 (\theta + \omega t) |0\rangle_r |0\rangle_s + \sin^2 (\theta + \omega t) |1\rangle_r |1\rangle_s, \]
\[ \text{iv) } \langle \xi_s(t) | = \langle \Psi(t) | H_{rs} |\Psi(t)\rangle = 0, \]
where \(|\Psi(t)\rangle\) is a free normalized vector of \( H_{12} \).

The solution of the above equations is the desired evolution (6), repeated here for convenience:
\[ |\Psi(t)\rangle = \cos (\theta + \omega t) |0\rangle_r |1\rangle_s + \sin (\theta + \omega t) |1\rangle_r |0\rangle_s. \]

Conditions (i), (ii) and (iii) mean that the link state undergoes a transformation [driven by (iii) or \( G_r \)] under continuous state vector reduction on the \textit{antisymmetric} subspace \( H_{12}^{(c)} \). If \( \theta \neq 0, \pi, \) namely if the preparation (4) comprises \textit{all} the basis vectors of \( H_{12}^{(c)} \), condition (ii) alone keeps the link evolution inside \( H_{12}^{(c)} \) (Section III). The link state remains ground and consequently the link expected energy \( \langle \xi_s(t) | \rangle \) is always zero. By excluding \( \theta = 0, \pi, \) \textit{condition (iv) is a consequence of the former conditions}.

Mathematically, conditions (i) and (iv) give the constraint \( A_{rs} |\Psi(t)\rangle = |\Psi(t)\rangle \). In conclusion the above conditions (i) through (iv) (which imply interpreting fermionic antisymmetry \( A_{12} \) as \textit{continuous projection} on the antisymmetric subspace) are equivalent to condition (i) through (iii) of Section III. This gives in fact the evolution (6).

Since \(|G_r, H_{rs}\rangle = 0\), as readily checked, the application of \( G_r \) to qubit \( r \) does not disturb in principle the ground state of \( H_{rs} \), in constrast with what hypothesized in ref. [12]. This means \( \text{NP-complete=P}^{[12]} \) under the current form of computation. Of course we should keep in mind that we are in the same idealized and speculative context of the former work.
V. CONCLUSIONS

By using the generalized interpretation of particle statistics (viewed as continuous projection on a constrained subspace), we have obtained an evolution of the NOT gate by acting only on qubit \( r \) (an input or an output qubit of the gate, indifferently so given that the two coexist) and without violating the NOT gate logical constraint.

This would be impossible under the conventional interpretation of particle statistics. In this context, since all \( \mathcal{H}_{rs} \) basis vectors are already antisymmetrical, particle statistics would do nothing. Any operation performed only on qubit \( r \) in the initial entangled state (8), would necessarily originate the terms \(|0\rangle_r |0\rangle_s \) and \(|1\rangle_r |1\rangle_s \), thus violating the NOT gate logical constraint.

If the model Hamiltonians used in this work could be substituted by more concrete Hamiltonians – say implementable in a laboratory – in principle such a divergence between the two interpretations of particle statistics could be verified.

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