Observational characteristics and possible asphericity of overluminous Type Ia supernovae

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ABSTRACT

A few Type Ia supernovae (SNe Ia) have been suggested to be an explosion of a super-Chandrasekhar-mass white dwarf (WD) in order to account for their large luminosities, requiring a large amount of $^{56}\text{Ni}$. However, the candidate overluminous SNe Ia 2003fg, 2006gz and (moderately overluminous) SN 1991T have very different observational features: the characteristic time-scale and velocity are very different. We examine if and how the diversity can be explained, by one-dimensional spherical radiation transport calculations covering a wide range of model parameters (e.g. WD mass). The observations of SN 2006gz are naturally explained by the super-Chandrasekhar-mass model. SN 1991T represents a marginal case, which may either be a Chandrasekhar or a super-Chandrasekhar-mass WD explosion. In contrast, the low velocity and short time-scale seen in SN 2003fg indicate that the ejecta mass is smaller than the Chandrasekhar mass, which is in apparent contradiction to the large luminosity. We suggest that the problem is solved if the progenitor WD, and thus the SN explosion, is aspherical. This may reflect a rapid rotation of the progenitor star, likely a consequence of the super-Chandrasekhar-mass WD progenitor. The observed differences between SNe 2003fg and 2006gz may be attributed to different viewing orientations.

Key words: radiative transfer – supernovae: individual: SN 2006gz – supernovae: individual: SN 2003fg – supernovae: individual: SN 1991T – white dwarfs.

1 INTRODUCTION

Type Ia supernovae (SNe Ia) are currently the most mature cosmological distance indicators, and led to the discovery of the acceleration of the Universe (Riess et al. 1998; Perlmutter et al. 1999). Their use as distance indicators relies on the well-calibrated light curve characteristics, namely a phenomenological relation between the peak luminosity and the light curve width (‘Phillips relation’ or ‘stretching factor’; Phillips 1993; Perlmutter et al. 1997; Livio 2003; Hillebrandt & Niemeyer 2000; Nomoto et al. 2003).

By clarifying the nature of progenitor system(s) of SNe Ia (Livio 2003; Livio 2000; Hillebrandt & Niemeyer 2000; Nomoto et al. 2003), one expects to obtain in-depth knowledge on the origin of the light curve relation for the purpose of better luminosity calibration, as well as new applications of SNe Ia in a range of cosmological studies. The progenitor of a normal SN Ia (Branch, Fisher & Nugent 1993; Li et al. 2001) is believed to be a white dwarf (WD) having nearly the Chandrasekhar mass. Recently, a few SNe Ia have been suggested to have originated from a WD with mass that exceeds the Chandrasekhar mass of a non-rotating WD, raising a possibility that not all SNe Ia are from a single class of progenitor system.

The suggestion relies mainly on the exceptionally large luminosity of these SNe, which requires more than 1 $M_\odot$ of $^{56}\text{Ni}$ synthesized and ejected during the explosion. The thermonuclear explosion converts the initial WD composition partly to $^{56}\text{Ni}$, $\gamma$-rays from radioactive decay $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$, through Compton scattering and thermalization, power the optical emission of SNe Ia. Thus, the peak luminosity is closely related to the mass of $^{56}\text{Ni}$ initially synthesized (hereafter $M_{\text{SNi}}$).

Historically, the first observationally based suggestion for a super-Chandrasekhar WD explosion was made for SN Ia 1991T, as is summarized by Lira et al. (1998) and Fisher et al. (1999). However, the distance to this SN has been actively debated, and the smaller value is now favoured (e.g. Saha et al. 2001). As a result, the $M_{\text{SNi}}$ required to explain the peak bolometric luminosity has been reduced down. For the reddening $E(B-V)_{\text{host}} = 0.14$ and the distance modulus $M = 30.74$ as adopted by Stritzinger et al. (2006), the peak V-band magnitude is $M_V = -19.66$. Stritzinger et al. (2006) derived a mass of $^{56}\text{Ni}$ of $M_{\text{SNi}} \sim 0.87 M_\odot$ based on the peak bolometric luminosity $L_{\text{bol}} \sim 1.7 \times 10^{43}$ erg s$^{-1}$. This amount of $^{56}\text{Ni}$ could be explained even by a Chandrasekhar-mass model, although one may need an extreme, purely detonation-driven explosion (Khokhlov, Müller & Hoflich 1993).

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Progress had to wait until the discovery of overluminous SNe Ia, which require that $M_{\text{SN}} > 1 \, M_\odot$. The first candidate, SN 2003fg (SNLS-03D3bb), was reported by Howell et al. (2006). The SN reached $M_V = -19.94$ mag at peak and $L_{\text{bol, peak}} \sim 2.6 \times 10^{43}$ erg s$^{-1}$. From this, Howell et al. (2006) estimated that $M_{\text{SN}} \sim 1.29 \, M_\odot$, using an approximate relation between the peak luminosity and $M_{\text{SN}}$. Since there should be other elements required both theoretically (i.e. none of the well-studied Chandrasekhar-mass models produce more than $1 \, M_\odot$ of $^{56}$Ni; Kokhlov et al. 1993; Iwamoto et al. 1999; Röpke et al. 2007, and references therein) and observationally (i.e. the maximum-light spectrum of SN 2003fg was dominated by intermediate-mass elements), Howell et al. (2006) argued that SN 2003fg should come from a super-Chandrasekhar-mass WD. Two other overluminous SNe Ia, similarly thus candidates of the super-Chandrasekhar-mass WD explosions, have been reported: SNe Ia 2006gz (Hicken et al. 2007) and 2007if (Yuan et al. 2007). For SN 2006gz, Hicken et al. (2007) adopted $E(B - V) = 0.18$ and $\mu = 34.95$ (but see also Section 2), deriving $M_V = -19.74$ at peak and $L_{\text{bol, peak}} \sim 2.2 \times 10^{43}$ erg s$^{-1}$, and thus $M_{\text{Ni}}^{\text{bol}} \sim 1.2 \, M_\odot$.

In addition to the large luminosity, they are also unique in their spectra. A temporal sequence of the optical spectra of SN 2006gz is presented by Hicken et al. (2007). They identified $\lambda \lambda 4745, 5490, 6580$ and 7324 in the spectra taken 10–14 d before the maximum brightness, with the equivalent width reaching $\sim 25 \, \AA$ and at the velocity of $\sim 15 \, 500 \, \text{km s}^{-1}$. This is the strongest evidence for unburned carbon from any SNe Ia observed so far (SN 1990N by Leibundgut et al. 1991; SN 2006D by Thomas et al. 2007). Adding to this, a Si II 6355-Å absorption velocity was unusually low well before the maximum ($\sim 12 \, 500 \, \text{km s}^{-1}$ at $\sim 10 \, \text{d}$ before the maximum). Without the rapid decline observed in typical SNe Ia (Benetti et al. 2005), the Si II velocity at the maximum brightness settled down to $v_{\text{Si II}} \sim 11 \, 500 \, \text{km s}^{-1}$, a value typical of normal SNe Ia. Based on this unusually slow decline in the Si II velocity, Hicken et al. (2007) speculated that this might be the result of a dense unburned C+O envelope overlying a Si-rich region; the dense C+O envelope might be expected in the merger of two WDs leading to the formation of the super-Chandrasekhar-mass WD progenitor. Howell et al. (2006) also reported the probable detection of a C II feature in the spectrum of SN 2003fg around the maximum brightness; if pre-maximum spectra had been available, the C II feature might have been in the spectra early on. These spectroscopic features suggest that the strong C II and the slow evolution of Si II velocity may be a distinct feature of overluminous SNe Ia. However, the spectral sequence is only available for SN 2006gz, and it has not been clarified to what extent these features are common in overluminous SNe Ia. Furthermore, it is not yet clear how these features, e.g. the dense C+O envelope, are related to the progenitor system (see Maeda et al. 2009 for a caution regarding interpreting the C+O-rich region as an outcome of merging two WDs).

Summarizing, the main argument for the super-Chandrasekhar model is the large peak luminosity and the mass of $^{56}$Ni, with spectroscopic features only indicative. Detailed study on observed characteristics of super-Chandrasekhar-mass models is missing. Clearly, the luminosity is not the only quantity that is dependent on the underlying models. The suggestion by Howell et al. (2006) regarding the super-Chandrasekhar-model for SN 2003fg is actually based on another observed characteristic, the velocity of the expanding SN material as deduced from its spectra. SN 2003fg showed exceptionally small velocity around the maximum brightness ($\sim 8000 \, \text{km s}^{-1}$) compared with that of normal SNe Ia ($\gtrsim 10 \, 000 \, \text{km s}^{-1}$). At first glance, this seems fully consistent with the following expectation, the super-Chandrasekhar-mass WD provides a large binding energy, and thus a small kinetic energy per mass compared with an explosion of a Chandrasekhar-mass WD. Jeffery, Branch & Baron (2006) confirmed this statement on the relation between the WD mass and the velocity-scale.

Although the argument may sound satisfactory, it is not the whole story. Studying emission processes should add further constraints on the underlying models. There are two overluminous SNe Ia for which observed characteristics are available in the literature, i.e. 2003fg and 2006gz. An extensive observational data set for the moderately overluminous SN 1991T is also available. By examining these observations (Section 2), we clarify that they have quite different properties. For example, SN 2006gz showed a velocity similar to normal SNe Ia – the argument for the super-Chandrasekhar-mass model for SN 2006gz may therefore be flawed.¹ The aim of this paper is to examine whether the observed characteristics can be explained by super-Chandrasekhar-mass models, by means of radiation transfer calculations. We especially focus on SNe 2003fg and 2006gz. In Section 3, we describe SN Ia models and summarize a method for the radiation transfer calculations. Results based on an extensive set of model calculations are shown in Section 4. A discussion is given in Section 5, where we examine uncertainties involved in our calculations. We also discuss the nature of these peculiar overluminous SNe Ia. The emphasis is placed on our finding that not every observed characteristic can be interpreted within the context of the simple (spherical) super-Chandrasekhar-mass WD models, and on our suggestion that the progenitor should largely deviate from spherical symmetry to account for the observed features (at least for SN 2003fg). The implications for SN 1991T are also discussed. Concluding remarks are presented in Section 6.

2 OBSERVATIONAL CHARACTERISTICS

In this section, we summarize observed features relevant to this work, for SNe 1991T, 2003fg and 2006gz. Howell et al. (2006) presented observational data for SN Ia 2003fg. The multi-band light curves are fitted well by k-corrected template light curves with a stretching factor (Perlmutter et al. 1997, 1999) of $s = 1.13$. The corresponding $\Delta m_{15}^{B}$ (the magnitude change in the first 15 d past $B$ maximum; Phillips 1993) is $\Delta m_{15}^{B} = 0.84 \pm 0.02$, using equation (5) of Perlmutter et al. (1997).² We then convert $\Delta m_{15}^{B}$ to $t_{\frac{1}{2}}$, the time since the maximum luminosity to half the maximum luminosity as measured in a bolometric light curve (Contrado, Leibundgut & Vacca 2000). This is possible thanks to a correlation between these two quantities. Note that $t_{\frac{1}{2}}$ in this paper is a post-maximum quantity, not the rise time frequently used in the related field. For the conversion, we find an observationally derived set of $(\Delta m_{15}^{B}, t_{\frac{1}{2}})$ for nine nearby SNe Ia presented in Contrado et al. (2000), assuming a functional form of $t_{\frac{1}{2}} = a \Delta m_{15}^{B} + b$. We obtain $a = -4.52 \pm 0.786$ and $b = 17.3 \pm 1.03$. Using this relation, $t_{\frac{1}{2}} = 13.5 \pm 1.7$ d for SN 2003fg. The derivation of $t_{\frac{1}{2}}$ in SN Ia 2006gz is straightforward. The $UBVRi$ bolometric curve is presented by Hicken et al. (2007),³ and it directly gives $t_{\frac{1}{2}} = 18$ d. The light curve width is very different for the two SNe Ia.

¹ Indeed, it is shown in this paper that the low velocity seen in SN 2003fg creates an apparent problem in the super-Chandrasekhar-mass model.
² Hicken et al. (2007) derived $\Delta m_{15}^{B} \sim 0.9$ for SN 2003fg in their fig. 4. Although this is different from the value derived here, the expected $t_{\frac{1}{2}}$ is $\sim 13$ d, which is well within the errors of our estimate.
³ The data file is presented at http://www.cfa.harvard.edu/supernova/SNarchive.html.
The two SNe Ia are also very different in characteristic velocity. SN 2003fg showed a Si II absorption velocity ($v_{\text{Si II}}$) of 8000 $\pm$ 500 km s$^{-1}$ around maximum brightness (Howell et al. 2006). The same value for SN 2006gz is in the range 11 000–12 000 km s$^{-1}$ (Hicken et al. 2007).

Typically, normal SNe Ia have $t_{\text{1/2}}$ in the range $\sim$9–14 d (e.g. Contardo et al. 2000) and $v_{\text{Si II}}$ in the range $\sim$10 000–14 000 km s$^{-1}$ (e.g. Hachinger, Mazzali & Benetti 2006). If we focus on SNe Ia with $M_{\text{SN Ia}} \sim 0.6 M_\odot$, the Phillips relation indicates that $\Delta m_{15} \approx 1.1$ (Stritzinger et al. 2006). This corresponds to $t_{\text{1/2}} = 12.1 \pm 1.8$ using the relation we derived above. The examples are SNe 2003du and 1990N (e.g. Stritzinger et al. 2006), both having $v_{\text{Si II}} \sim 10 000$–11 000 km s$^{-1}$.

The time-scale, $t_{\text{1/2}}$, in SN 2003fg is at about the upper bound of normal SNe Ia, while $v_{\text{Si II}}$ is much lower. $t_{\text{1/2}}$ in SN 2006gz is well above the range seen in normal SNe Ia, while $v_{\text{Si II}}$ is similar to normal cases. In the case of bolometric magnitude at maximum light ($L_{\text{bol,peak}}$), SN 2003fg has been claimed to be overluminous: $L_{\text{bol,peak}} \sim (2.5–2.8) \times 10^{43}$ erg s$^{-1}$ (Howell et al. 2006). SN 2006gz also has been claimed to be overluminous: $L_{\text{bol,peak}} = (1.8–2.6) \times 10^{43}$ erg s$^{-1}$ assuming $E(B-V)_{\text{host}} = 0.18$ and $\mu = 34.95$ (Hicken et al. 2007). However, there is a caveat for $L_{\text{bol,peak}}$ in SN 2006gz; Maeda et al. (2009) pointed out that the peak luminosity of SN 2006gz derived by Hicken et al. (2007) involves a large uncertainty, because the host-galaxy extinction has not been well constrained. If the host extinction were totally negligible, then we would find $M_V \sim −19.2$ at peak and $L_{\text{bol,peak}} \sim 1.3 \times 10^{43}$ erg s$^{-1}$, which is relatively bright but not overluminous.\footnote{This uncertainty highlights the importance of investigating the radiation process and obtaining constraints independent from the peak luminosity, as is our aim in this paper.}

For SN 1991T, the following values are relevant: $t_{\text{1/2}} = 14.0$ d given by Contardo et al. (2000), based on the compilation of a multi-band light curve; $v_{\text{Si II}} \sim 10 000$–11 000 km s$^{-1}$ (Hachinger et al. 2006, and references therein). With regard to these properties, SN 1991T represents an intermediate case between SNe 2003fg and 2006gz, and indeed close to that of normal SNe Ia. The peak luminosity with the favoured distance modulus is $L_{\text{bol,peak}} \sim 1.7 \times 10^{43}$ erg s$^{-1}$ (Stritzinger et al. 2006) which is at the upper bound of normal SNe Ia, although the original suggestion for the super-Chandrasekhar-mass model was based on a brighter estimate ($L_{\text{bol,peak}} \sim 2.3 \times 10^{43}$ erg s$^{-1}$; Fisher et al. 1999; Contardo et al. 2000).

Summarizing, the two most probable candidate super-Chandrasekhar-mass WD explosions, i.e. SNe 2003fg and 2006gz, have very different observational characteristics, although this fact has not been emphasized in the literature.

### 3 METHOD AND MODELS

#### 3.1 SN Ia models

We construct SN Ia models starting with the density structure of the W7 model (Nomoto, Thielemann & Yokoi 1984), which reproduces the basic observational features of normal SNe Ia (Branch et al. 1985). Assuming a homologous expansion, which should be a good approximation for the explosion of a compact progenitor, the density distribution as a function of velocity is uniquely determined from the normalized reference density distribution (i.e. W7), by specifying the following model parameters:

(i) $M_{\text{WD}}$, the mass of the WD;
(ii) $\rho_c$, the WD central density at the burning ignition;
(iii) $f_{\text{ECE}}$, the mass fraction of Fe-peak elements produced by strong electron captures, i.e. Fe-peak elements excluding $^{56}$Ni (e.g. $^{56}$Ni, $^{56}$Fe, $^{54}$Fe);
(iv) $f_{\text{56Ni}}$, the mass fraction of $^{56}$Ni;
(v) $f_{\text{56Fe}}$, the mass fraction of partially burned intermediate-mass elements like Mg, Si, and S.

The mass fraction of unburned C+O materials ($f_{\text{CO}}$) is then simply given by $f_{\text{CO}} = 1 - f_{\text{ECE}} - f_{\text{56Ni}} - f_{\text{56Fe}}$.

The procedure for model construction follows that presented in Jeffery et al. (2006) (see also Howell et al. 2006; Maeda et al. 2009). For a given $M_{\text{WD}}$ and $\rho_c$, we compute the binding energy of the WD ($E_b$) using the formulae given by Yoon & Langer (2005), who examined a sequence of structure of a rotating WD. The energy produced by the nuclear burning ($E_{\text{nuc}}$) is given by $M_{\text{WD}} f_{\text{ECE}} f_{\text{56Ni}}$ and $f_{\text{56Fe}}$, through a simple relation:

$$E_{\text{nuc}}/10^{51} \text{erg} = (1.74 f_{\text{ECE}} + 1.56 f_{\text{56Ni}} + 1.24 f_{\text{56Fe}}) M_{\text{WD}}/M_\odot.$$  \hspace{1cm} (1)

The kinetic energy is then simply

$$E_K = E_{\text{nuc}} - E_b.$$ \hspace{1cm} (2)

The reference density structure is then scaled in a self-similar manner, i.e.

$$\rho(v) \propto M_{\text{WD}}^{1/2} E_K^{−3/2},$$ \hspace{1cm} (3)

and

$$v \propto M_{\text{WD}}^{1/2} E_K^{1/2},$$ \hspace{1cm} (4)

where $v$ is the velocity of a Lagrangian fluid element, and $\rho(v)$ the density there.

Now that we have specified the density structure, we are left to specify the distribution of elements. We examine three extreme cases so that our models should cover all real situations.

(i) Stratified (model sequence ‘a’). Characteristic burning layers are totally separated and stratified. The electron capture region (where the mass fraction of stable Fe-peak elements is set to be unity) is at the centre, surrounded by the $^{56}$Ni-rich region (the mass fraction of $^{56}$Ni is set to be unity), then by the partially burned layer (mass fractions of Si and S are set to be 0.7 and 0.3, respectively), and by the unburned C+O layer at the outermost region (mass fractions of C and O are set to be 0.5 and 0.5, respectively).

(ii) Mixing in the Fe-rich region (model sequence ‘b’). Basically the same as model sequence ‘a’, except that the innermost electron capture region and the $^{56}$Ni region are assumed to be fully mixed in the composition structure.

(iii) Full mixing (model sequence ‘c’). The materials are fully and homogeneously mixed throughout the ejecta.

The models ‘a’ and ‘b’ are the most likely cases, at least in normal SNe Ia, from the viewpoint of observed spectral evolution (Stehle et al. 2005; Mazzali et al. 2008). There are observational hints favouring model ‘a’ over ‘b’ for some SNe Ia (Höflich et al. 2004; Motohara et al. 2006). Examples of the density and composition structures are shown in Fig. 1.

We have examined an extensive set of models, as shown in Table 1. $M_{\text{WD}}$ is varied from 1.39 to 2.6 $M_\odot$ at intervals of 0.3 $M_\odot$. $\rho_c$ is set to be $3 \times 10^9$ g cm$^{-3}$ in most models, as the model sequence with varying $\rho_c$ can be self-similar (only different in $E_K$). The dependence of the observed behaviour on $\rho_c$ is examined only
Table 1. SN Ia models with the W7 reference density distribution. For a description of the parameters, see the main text. Each column represents a set of models for example, model SW7IM is examined with three different mixing prescription (mixing ‘a’, ‘b’, and ‘c’; see the main text). For each mixing, a set of (fECE, fIME) [(0, 0.57), (0.1, 0.47), (0.2, 0.37), (0.3, 0.27), (0.4, 0.17), (0.5, 0.07), and (0.57, 0)] is examined.

| Name   | M wd/M⊙ | ρ wd/g cm⁻³ | fECE | fSNN | fIME | fCO | M SNN/M⊙ | E K/10⁵¹ erg |
|--------|----------|--------------|------|------|------|-----|----------|-------------|
| SW7IM  | 1.39     | 3.0e9        | (0-0.57) | 0.43 | (0.57-0) | 0 | 0.6 | 1.40–1.80 |
| LW7IM  | 1.39     | 3.0e9        | (0-0.28) | 0.72 | (0.28-0) | 0 | 1.0 | 1.53–1.73 |
| Sup1.7IM | 1.70   | 3.0e9        | (0-0.41) | 0.59 | (0.41-0) | 0 | 1.0 | 1.55–1.90 |
| Sup2IM  | 2.00     | 3.0e9        | (0-0.50) | 0.50 | (0.50-0) | 0 | 1.0 | 1.56–2.06 |
| Sup2IM10 | 2.00    | 1.0e10       | (0-0.50) | 0.50 | (0.50-0) | 0 | 1.0 | 1.24–1.74 |
| Sup2.3IM | 2.30     | 3.0e9        | (0-0.57) | 0.43 | (0.57-0) | 0 | 1.0 | 1.56–2.21 |
| Sup2.6IM | 2.60     | 3.0e9        | (0-0.62) | 0.38 | (0.62-0) | 0 | 1.0 | 1.56–2.36 |
| SW7CO  | 1.39     | 3.0e9        | (0-0.57) | 0.43 | 0 | (0.57-0) | 0.6 | 0.42–1.80 |
| LW7CO  | 1.39     | 3.0e9        | (0-0.28) | 0.72 | 0 | (0.28-0) | 1.0 | 1.05–1.73 |
| Sup1.7CO | 1.70    | 3.0e9        | (0-0.41) | 0.59 | 0 | (0.41-0) | 1.0 | 0.69–1.90 |
| Sup2CO  | 2.00     | 3.0e9        | (0-0.50) | 0.50 | 0 | (0.50-0) | 1.0 | 0.32–2.06 |
| Sup2CO10 | 2.00    | 1.0e10       | (0-0.50) | 0.50 | 0 | (0.50-0) | 1.0 | 0.001–1.74 |
| Sup2.3CO | 2.30     | 3.0e9        | (0.10-0.57) | 0.43 | 0 | (0.47-0) | 1.0 | 0.33–2.21 |
| Sup2.6CO | 2.60     | 3.0e9        | (0.10-0.62) | 0.38 | 0 | (0.52-0) | 1.0 | 0.01–2.36 |
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Figure 2. Examples of the synthesized bolometric light curves. Shown here are the models with mixing ‘a’ (stratified) and \( f_{ECE} = 0.1 \) SW7IM (thick solid), LW7IM (thin solid), Sup1.7IM (thick dashed), Sup2.0IM (thin dashed), Sup2.3IM (thick dotted), and Sup2.6IM (thin dotted). The photospheric velocities at the maximum bolometric brightness \( (v_{peak}) \) are shown in parentheses.

This example shows that the relation between \( t_{1/2} \) and \( v_{peak} \) can provide a strong tool to check the validity of a model compared with the observations, although this has not been examined for SNe 2003fg and 2006gz in the previous studies. The constraints are as follows. (1) A model should reproduce the \( t_{1/2} \) derived in Section 2. (2) The photospheric velocity at the maximum luminosity in a model \( (v_{peak}) \) should be equal to or less than the observed \( v_{Si II} \), since the Si II absorption must be formed above the photosphere.

Fig. 3 shows \( (t_{1/2}, v_{peak}) \) for all the models, except for Sup2.0IM10 and Sup2.0CO10. The SW7IM/CO model sequence (with \( M_{56Ni} = 0.6 M_\odot \)) approximately satisfies the \( (t_{1/2}, v_{Si II}) \) constraint of the normal SNe Ia with \( M_{56Ni} \sim 0.6 M_\odot \) (e.g. SNe 2003du, 1990N, as the observed range shown in Fig. 3 illustrates).

An important discovery in Fig. 3 is that not even a single model satisfies the observed characteristics of SNe 2003fg. On the other hand, SN 2006gz can be explained by many models (note that the observed \( v_{Si II} \) can be larger than synthetic \( v_{peak} \)), and most naturally by super-Chandrasekhar models having \( M_{56Ni} \sim 1 M_\odot \), which results in \( v_{peak} \sim v_{Si II} \) for the favoured non-mixing case (‘a’).

We have examined an extensive set of models including extreme cases, so that our failure to reproduce the observed characteristics of SN 2003fg represents a principal difficulty in the super-Chandrasekhar model for this SN Ia.

We found that the lower boundary in the \( t_{1/2} - v_{peak} \) plot, covered by varying various parameters for given \( M_{\text{wd}} \), is represented by a model sequence of ‘a’ with only varying \( f_{ECE} \). In the mixing cases (‘b’ and ‘c’), \( ^{56}\text{Ni} \) is mixed down to the low velocity, and thus the diffusion time-scale becomes long compared with the stratified case ‘a’. This results in larger \( t_{1/2} \) in ‘b’ and ‘c’ than in ‘a’. We also found that the curves, obtained by varying \( f_{ECE} \), almost overlap between the cases ‘IM’ \( (f_{CO} = 0) \) and ‘CO’ \( (f_{IM} = 0) \) in the \( t_{1/2} - v_{peak} \) plane, and the ones with \( f_{IM} = 0 \) (e.g. SW7CO) cover a wider range of parameter space.

Thus, the model sequence with the mixing ‘a’ and \( f_{IM} = 0 \) form the lower boundary, as we vary the value for \( f_{ECE} \) in the \( t_{1/2} - v_{peak} \) plane for given \( M_{\text{wd}} \) and \( M_{56Ni} \). If the observationally derived set of \( (t_{1/2}, v_{Si II}) \) is above the curve, the model sequence is acceptable (i.e. there are observationally acceptable combinations of parameters for given \( M_{\text{wd}} \) and \( M_{56Ni} \)). If it is below the curve, the combination of \( M_{\text{wd}} \) and \( M_{56Ni} \) should be rejected. Fig. 4 shows the lower boundary of \( v_{peak} \) as a function of \( t_{1/2} \) for different \( M_{\text{wd}} \) (note that \( M_{56Ni} \) is fixed for given \( M_{\text{wd}} \) in this paper).

Fig. 4 shows that the curve is similar for models having the same \( M_{56Ni} \). The behaviour can be explained as follows to the first approximation. The time-scale of the light curve evolution around the peak is scaled as follows (Arnett 1982):

\[
t_{1/2} \propto \kappa^{1/2} M_{\text{wd}}^{3/2} E_K^{1/4},
\]

where \( \kappa \) is the opacity averaged over the ejecta. The estimate of the photospheric velocity at the maximum brightness is complicated. For the purpose of the present demonstration, it is enough to assume that this is scaled with the average velocity, i.e.

\[
v_{peak} \propto M_{\text{wd}}^{1/2} E_K^{1/2}.
\]

Then, combining these two expressions, we obtain the following relation:

\[
v_{peak} \propto \kappa^{1/2} M_{\text{wd}}^{-1/4} t_{1/2}.
\]

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\(^{5}\) We have found that the \( (t_{1/2}, v_{peak}) \) of Sup2.0IM10/Sup2.0CO10 can be obtained by simply shifting the results for Sup2.0IM/CO along the curve defined by the mixing ‘a’ towards the bottom right. This is because larger \( \rho_e \) is equivalent to lower \( E_K \). Thus changing \( \rho_e \) can be mimicked by changing \( E_K \) (i.e. by changing \( f_{ECE} \)).
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...M is slightly smaller than SN 2003fg. It has been estimated that M_{56Ni} \sim 0.2 M_{\odot} in these faint SNe Ia (Phillips et al. 2007; Sahu et al. 2008), and a model with such a small amount of 56Ni should be considered. From equation (10), we see that t_{1/2} \propto M_{56Ni} v_{\text{peak}}. Applying our result to this relation, we expect that t_{1/2} \sim 16–20 d for M_{56Ni} \sim 0.2–0.3 M_{\odot} and for v_{\text{peak}} \sim 6000 km s^{-1}, as is consistent with the observational features of SN 2005hk.

Equations (9) and (10) are only illustrative, but already provide a solid argument, which was confirmed by a set of our model calculations. (1) For given M_{\text{sd}}, if E_k is larger, then t_{1/2} is smaller and v_{\text{peak}} is larger. (2) To reproduce the observationally derived value of t_{1/2}, one can thus vary E_k (in terms of f_{\text{ECE}}, for example). The model at the same time predicts smaller v_{\text{peak}} for smaller M_{56Ni}.

The combination (t_{1/2}, v_{\text{peak}}) of SN 2006gz is totally consistent with the expectation from M_{56Ni} \sim 1 M_{\odot}. On the other hand, the above relation between t_{1/2} and v_{\text{peak}} raises a difficulty in interpreting the observational data of SN 2003fg, since (t_{1/2}, v_{\text{peak}}) falls into the range below even the lower boundary of the Chandrasekhar model with M_{56Ni} = 0.6 M_{\odot}.

4.2 Peak luminosity and Phillips relation

The t_{1/2}–v_{\text{peak}} relation is our main focus in the present paper. We also examine the bolometric peak luminosity (L_{bol,\text{peak}}) for the self-consistency test in this section.

Fig. 5 shows L_{bol,\text{peak}} as a function of t_{1/2} for all the models (except for Sup2.0M10/CO10). Normal SNe Ia (shown as stars in Fig. 5) can basically be explained by the SW7 model sequence, if we allow dispersion in M_{56Ni} (i.e. L_{bol} \propto M_{56Ni} if other properties are the same). Our model value (M_{56Ni} \sim 1 M_{\odot}) is slightly smaller than required by the peak luminosity of SNe 2003fg. The best value is M_{56Ni} \sim 1.2 M_{\odot}, consistent with the estimate by Howell et al. (2006). For SN 2006gz, the best value is M_{56Ni} \sim 1.1 M_{\odot}, if we adopt E(B-V) = V_{\text{host}} = 0.18 and R_V = 3.1 (Hicken et al. 2007). The stringent lower limit for SN 2006gz is given by setting the host extinction negligible (Section 2); M_{56Ni} \sim 0.7 M_{\odot}.

With these values, let us go back to the discussion on the t_{1/2}–v_{\text{peak}} relation. If we take M_{56Ni} = 1.1 M_{\odot} for SN 2006gz, the lower boundary in Fig. 4 moves slightly to the right, but is still marginally consistent with SN 2006gz. For SN 2003fg, further increasing M_{56Ni} makes the situation worse: the lower limit of v_{\text{peak}} now increases, making the deviation larger. Thus, the conclusion in the previous section does not change.

So far we have seen that the properties of SN 2006gz are consistent with super-Chandrasekhar models. Can we therefore identify a set of model parameters (M_{\text{sd}} and M_{56Ni}) relevant to this SN? The t_{1/2}–v_{\text{peak}} constraint can be satisfied as long as M_{56Ni} \lesssim 1.1 M_{\odot}. This is only the upper limit corresponding to the condition v_{\text{peak}} \lesssim v_{\text{SiII}}. However, L_{\text{bol}} requires M_{56Ni} \gtrsim 1 M_{\odot} for A_V = 0.56 and M_{56Ni} \gtrsim 0.7 M_{\odot} for A_V = 0. Thus, we conclude that an explosion producing M_{56Ni} \sim 1–1.1 M_{\odot} is the most likely explanation for SN 2006gz, with a possible range of M_{56Ni} \sim 0.7–1.1 M_{\odot}. Directly deriving M_{\text{sd}}, rather than M_{56Ni}, turns out to be difficult: M_{\text{sd}} is degenerated in the t_{1/2}–v_{\text{peak}} relation. In terms of L_{\text{bol}}/M_{\text{sd}} is again basically degenerated according to the approximate relation...
The effective temperature at bolometric maximum brightness, as a function of \( t_{1/2} \), is shown by a thick solid line (equivalent to the Phillips relation) is shown by a thick solid line (excluding bright SN Ia 1991T and faint SN Ia 1991bg). An approximate linear fit to the results of the Sup2.0 sequence (with mixing ‘a’) is indicated by the thin solid line. Two lines (‘x1.1’ shown by the thin dashed line, ‘x0.8’ shown by the thin dotted line) roughly indicate what is expected for models with \( M_{\text{SN}} = 1.1 M_\odot \) and \( 0.8 M_\odot \), respectively.

\[ L_{\text{bol,peak}} \propto M_{\text{SN}}. \]

Although there is a diversity resulting from different \( M_{\text{ad}} \), the model luminosity differs up to only \( \sim 10 \) per cent for the same mixing prescription, between \( M_{\text{ad}} = 1.4 M_\odot \) and \( 2.6 M_\odot \). The variation is smaller than the observed error in \( L_{\text{bol,peak}} \) (\( \sim 20 \) per cent), and thus it is difficult to derive \( M_{\text{ad}} \) from the analysis presented in this paper.

### 4.3 Temperature and peak luminosity

An additional test on the models can be provided by the ejecta temperature. The discussion in this section is only qualitative: deriving the photospheric temperature requires detailed spectrum modelling (e.g. Hachinger et al. 2008), which is beyond the scope of the present study.

The ratio of equivalent widths of Si II \( \lambda 5972 \) to Si II \( \lambda 6355 \) provides a temperature indicator. In normal SNe Ia, the ratio is smaller for SNe with larger peak luminosity, and thus higher temperature (Nugent et al. 1995; Hachinger et al. 2008). The ratio in SN 2003fg is similar to that in relatively faint SN Ia 1994D (fig. 3 of Howell et al. 2006), and that in SN 2006gz is similar to normal SN Ia 2003du (fig. 1 of Hicken et al. 2007). These infer that the temperature of SN 2003fg is lower than those of normal SNe Ia with \( M_{\text{SN}} \sim 0.6 M_\odot \), and that of SN 2006gz is comparable to the normal case.

Fig. 6 shows the effective temperature at bolometric maximum brightness in our models. The effective temperature is generally a decreasing function of \( t_{1/2} \); although the photospheric velocity is smaller for larger \( t_{1/2} \) (Fig. 4), the peak luminosity is smaller and the photospheric radius (i.e. the photospheric velocity multiplied by the peak date) is larger for larger \( t_{1/2} \). The effect of the latter functions is more important, resulting in the dependence in Fig. 6. For models resulting in same \( t_{1/2} \), the temperature is larger for larger \( M_{\text{ad}} \) (see Fig. 4; the photospheric velocity is slightly smaller for larger \( M_{\text{ad}} \)).

The temperature of SN 2003fg looks to be lower than normal (i.e. SW7 models), and \( t_{1/2} \) is comparable to the case of normal SNe Ia (i.e. \( t_{1/2} \sim 12–14 \) d). This requires that the ejecta mass is smaller than in SW7. This is again consistent with our conclusion independently derived using the photospheric velocity. The temperature of SN 2006gz is comparable to the case of normal SNe Ia, but \( t_{1/2} \) (\( \sim 18 \) d) is larger than in normal cases. This indicates that the ejecta mass is larger than in normal SNe Ia (Fig. 6).

### 5 DISCUSSION

#### 5.1 Evaluation of uncertainties

One may question whether the uncertainty involved in our calculations might change our conclusion. In this section, we examine the dominant sources of the uncertainty in our calculations. Here we examine the effects of (i) different opacity prescription and (ii) density distribution. In short, these do not alter our conclusion.

#### 5.1.1 Opacity

We have performed the same calculations for all the models, with different opacity prescription. Here, the opacity takes the following form: \( \kappa_{\text{line}} = 0.05 \text{ cm}^2 \text{ g}^{-1} \), independent of composition. The result is shown in Fig. 7. The combination \( (t_{1/2}, v_{\text{peak}}) \) of the super-Chandrasekhar WD model is again above that of the Chandrasekhar model and above \( (t_{1/2}, v_{\text{peak}}) \) of SN 2003fg. The observed values of SN 2006gz are slightly above the lower boundary of the super-Chandrasekhar model, and thus this SN is quite consistent with the super-Chandrasekhar explosion. In this case, \( v_{\text{peak}} \propto M_{\text{ad}} \) as the...
opacity is independent of the composition. The same conclusion as in the previous section applies, but only if we use $M_{\text{wd}}$ as the major function rather than $M_{\text{SN}}$. To explain the observational characteristics of SN 2003fg, we require $M_{\text{wd}}$ smaller than in the normal SNe Ia.

### 5.1.2 Density distribution

The models with exponential or constant density distributions are examined for selected model parameters to check the uncertainty. Most of the existing explosion models (including the W7 model) predict a more or less exponential density distribution (e.g. Woosley et al. 2007). We constructed the exponential density distribution from $M_{\text{wd}}$ and $E_{\text{k}}$ according to the descriptions in Jeffery et al. (2006) and Woosley et al. (2007). The construction of the constant density distribution is trivial. These two types of density distribution are examined only for the following cases:

(i) $M_{\text{wd}} = 1.39 M_{\odot}$ and 2 $M_{\odot}$;
(ii) $\rho_c = 3 \times 10^{9}$ g cm$^{-3}$;
(iii) $f_{\text{SN}} = 0.43$ ($M_{\text{wd}} = 1.39 M_{\odot}$) or 0.50 ($M_{\text{wd}} = 2 M_{\odot}$);
(iv) $f_{\text{BME}} = 0$;
(v) mixing = ‘a’.

For each model, $\frac{v_{\text{peak}}}{f_{\text{BME}}}$ is varied to form the lower boundary in the $v_{\text{peak}} - t_{1/2}$ plot. The result is shown in Fig. 8. Exponential models are similar to the W7 density models in the $t_{1/2} - v_{\text{peak}}$ plane. The constant density distribution predicts smaller $v_{\text{peak}}$ than in the other models, but does not drastically change the result. The constant density distribution is an extreme assumption, probably resulting in the lowest value of $v_{\text{peak}}$ for given $t_{1/2}$. Thus, the uncertainty in the density distribution, as long as spherical symmetry is assumed, does not change our conclusion.

### 5.2 SN 2003fg

In this section (Section 5.2) and the following two sections (Sections 5.3 and 5.4), we summarize our results for individual objects, and discuss the implications. For SN 2003fg, we have found that the observational characteristics cannot be put into the supernova explosion model. Most importantly, the observed $v_{\text{peak}}$ vs $t_{1/2}$ relation (Section 4.1) requires that $M_{\text{SN}}$ or $M_{\text{wd}}$ (or both) should be smaller than even the Chandrasekhar mass, contrary to the earlier expectations (Howell et al. 2006). Additional support is provided by the ejecta temperature (Section 4.3), which also indicates that the ejecta mass (i.e. $M_{\text{wd}}$) is smaller than the Chandrasekhar mass. On the other hand, the large peak luminosity requires that $M_{\text{WD}} \sim 1.1 M_{\odot}$ (Section 4.2; Howell et al. 2006).

This is apparently a contradiction. To remedy the problem, we suggest that the ejecta structure is far from spherical. Our radiation transfer calculations in this paper assume spherical symmetry, and for example the proportional coefficient in equation (8) for $v_{\text{peak}}$ should be a function of the viewing angle in the presence of large deviation from spherical symmetry. If the viewing angle is such that the effective (isotropic) mass is small along the line of sight, this may effectively look like an explosion with a small amount of $^{56}\text{Ni}$ and/or $^{56}\text{Co}$, although the large luminosity can probably be provided by the $^{56}\text{Ni}$ in the whole ejecta, not only along the line of sight (note that the photospheric velocity is more sensitively affected than the peak luminosity in asymmetric SN models; Maeda, Mazzali & Nomoto 2006; Tanaka et al. 2007).

The important finding here is that the mass should be effectively small as viewed from an observer to satisfy the $v_{\text{peak}}$ vs $t_{1/2}$ constraint. Such an explosion can not be a strongly jetted explosion of a spherically symmetric progenitor star. The jet-type explosion should yield smaller $t_{1/2}$ for an observer closer to the jet direction (thanks to the large isotropic $E_{\text{k}}/(M_{\text{WD}})$) but at the same time result in larger $v_{\text{peak}}$ (for the same reason; see e.g. Maeda et al. 2006 but also Hillebrandt, Sim & Röpke 2007). Alternatively, we suggest that the progenitor star is highly aspherical. This may actually be
consistent with the super-Chandrasekhar model, as such a massive WD should rotate rapidly to support an excessive mass.

Although an explosion based on such a deformed configuration has not been examined except for the purely detonation model (Steinmetz, Müller & Hillebrandt 1992), we believe it is rational to assume that the structure after the explosion preserves the initial configuration to some extent. The disc-like structure has effectively small isotropic mass if viewed along the axis of the rotational symmetry. We suggest this is a situation in SN 2003fg.

Hillebrandt et al. (2007) suggested an off-centre explosion model (a kind of one-sided jet-like explosion model) within the context of a Chandrasekhar-mass WD explosion as an alternative explanation for SN 2003fg (see also Sim et al. 2007). Their argument is based on the viewing angle effect on the light curve features, i.e. the larger luminosity and smaller diffusion time-scale for an observer closer to the direction of the $^{56}$Ni-rich blob. We emphasize the differences between our work and theirs: (1) our suggestion on the ejecta asymmetry is based on the combination of the light curve and spectral features, and (2) the disc-like/oblate geometry that we favour in this work is different from their suggestion.

Although we have clarified the need for the ejecta asymmetry for SN 2003fg, the progenitor WD mass ($M_{\text{wd}}$) is not conclusively constrained by the present study. This will require detailed multi-dimensional radiation transfer calculations; in particular, we need to understand the dependence of the luminosity on the geometry and the viewing angle.

### 5.3 SN 2006gz

We have found that the observed features of SN 2006gz are consistent with expectations from the super-Chandrasekhar-mass WD explosion scenario. A difficulty in identifying the underlying model for SN 2006gz is the uncertainty regarding its host’s extinction and luminosity. Although we cannot completely reject the possibility that this is an explosion of a Chandrasekhar WD, we favour the suggestion that this is indeed a super-Chandrasekhar WD explosion; the $t_{1/2}$.peak relation (Section 4.1) and the ejecta temperature (Section 4.3) are consistent with the expectations from a super-Chandrasekhar WD explosion.

For SN 2006gz, it is not necessary to introduce ejecta asymmetry, unlike SN 2003fg. These two SNe may be intrinsically different, or SN 2006gz may be explained by a configuration similar to that of SN 2003fg, but viewed at a different angle. Here, we point out that the differences in the observational features of these two SNe are qualitatively consistent with the expected differences arising from the same configuration but viewed at different orientations. These two SNe Ia require similar amount of $^{56}$Ni as estimated from the peak luminosity, and the main difference is in the spectroscopic feature ($v_{\text{peak}}$) and the light curve width ($t_{1+2}$). The latter two quantities are expected to be more sensitive dependent on the viewing angle than the estimated value of $M(^{56}\text{Ni})$ (Maeda et al. 2006). For the disc-like/oblate configuration, we expect larger $v_{\text{peak}}$ and larger $t_{1+2}$ for the larger inclination. The two values for SNe 2003fg and 2006gz follow this tendency. This is, however, only a qualitative argument, and discriminating between these two possibilities (the difference in the ejecta shape or the viewing angle) needs more detailed, multi-dimensional study.

Maeda et al. (2009) reported that SN 2006gz is also peculiar at late phases, $\gtrsim 300 \text{ d}$ after the explosion. Applying the standard $^{56}$Ni/Co/Fe heating scenario, they estimated $M_{\text{SN}}(\text{NI})$ smaller by a factor of 5 than that estimated with the early-phase data. This contradiction could mean either of the following two possibilities: (1) SN 2006gz was not powered by decays of $^{56}$Ni/Co/Fe at the early phases (which then casts doubt on the super-Chandrasekhar-mass WD progenitor), or (2) it was powered by the decay, but for some reason (e.g. thermal catastrophe or dust formation) the bulk of the emission was shifted to the near-/mid-infrared. Based on the results in this paper, we favour the second possibility.

### 5.4 SN 1991T

For SN 1991T, $t_{1+2}$ is at the upper boundary of normal SNe Ia, and $v_{\text{SN}}$ is at the lower boundary. This is marginally explained by the SW7 model sequence with $M_{\text{SN}}(\text{NI}) = 0.6 M_{\odot}$. Thus, this constraint results in $M_{\text{SN}}(\text{NI}) \lesssim 0.6 M_{\odot}$. On the other hand, the other constraint from $t_{\text{SN}}.\text{peak}$ results in $M_{\text{SN}}(\text{NI}) \gtrsim 0.8 M_{\odot}$ (Fig. 5).

Strictly speaking, these two constraints are not mutually consistent, and the same argument as for SN 2003fg can apply to SN 1991T. However, the deviation between the observations and models is not as large as in SN 2003fg, and SN 1991T may be marginally consistent with a spherical explosion with $M_{\text{SN}}(\text{NI}) \sim 0.8 M_{\odot}$ within the uncertainties involved in our model calculations.

Alternatively, SN 1991T may also be an aspherical, disc-like explosion such as we suggest for SN 2003fg. However, the intrinsic properties (e.g. the explosion geometry, the mass of $^{56}$Ni) of SN 1991T are likely different from those of SN 2003fg (note that there is a possibility that SN 2006gz is intrinsically similar to SN 2003fg). The estimated value of $M(^{56}\text{Ni})$ is different, and other features ($v_{\text{peak}}, t_{1+2}$) do not seem to be consistent with the expectation from the disc-like/oblate geometry that we favour for SN 2003fg.

### 6 CONCLUDING REMARKS

In this paper, we critically examined super-Chandrasekhar-mass WD models for (candidate) overluminous SNe Ia 2003fg, 2006gz, and moderately overluminous SN Ia 1991T. Our new approach is to use two observed features, $t_{1+2}$ and $v_{\text{peak}}$. This is equivalent to making use of combined information from the light curve and spectra, and thus is more powerful than previous studies, which relied mainly on the peak luminosity. Our conclusions are summarized as follows.

(i) Somewhat negatively, the observations of SN 2003fg are not readily explained by the standard $^{56}$Ni/Co/Fe heating scenario. The combination of relatively small $t_{1+2}$ and small $v_{\text{peak}}$ requires that either $M_{\text{wd}}$ or $M_{\text{SN}}(\text{NI})$ (or both) should be smaller than in normal SNe Ia.

(ii) The observations of SN 2006gz can be naturally accounted for by the (spherical) super-Chandrasekhar-mass model with $M_{\text{SN}}(\text{NI}) \sim 1-1.1 M_{\odot}$. Although the peak luminosity of SN 2006gz is largely uncertain (Section 2), we found that the super-Chandrasekhar-mass model is consistent with various observational features of SN 2006gz, and thus we favour the interpretation that SN 2006gz was indeed overluminous at peak as suggested by Hicken et al. (2007).

(iii) The observed features of SN 1991T are marginally explained by a spherical explosion of a WD and $M(^{56}\text{Ni}) \sim 0.8 M_{\odot}$. This may be either a Chandrasekhar WD or a super-Chandrasekhar explosion.

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6 This is independent of the uncertainty in the host extinction.
The failure in fitting SN 2003fg is not a result of the $M_{\text{bol}} - E_V$ relation expected for SNe Ia. In short, the large amount of $^{56}\text{Ni}$ will inevitably yield too large a diffusion time-scale. One can then try a model with large $E_V$ to reduce the diffusion time-scale, but this inevitably leads to a large velocity, hence the contradiction. Therefore, the observed characteristics of SN 2003fg are indeed inconsistent with any parameter set within the standard $^{56}\text{Ni}/\text{Co}/\text{Fe}$ heating scenario.

We suggest a solution to remedy the problem – the ejecta asymmetry resulting from a disc-like or oblate density structure of a rapidly rotating progenitor WD. This may indeed be consistent with the super-Chandrasekhar WD scenario. Within the results obtained by the present calculations, however, we can not conclusively derive the progenitor WD mass. An interesting possibility is that the different observational features of SNe 2003fg and 2006gz may be unified into one scheme, i.e. an explosion of a super-Chandrasekhar WD viewed at different directions.

Thomas et al. (2002) pointed out that a signature of ejecta asymmetry can be identified in the absorption strength of $\text{Si}II \lambda 6355$. They concluded that the typical scale of inhomogeneity should be smaller than the size of the photosphere in normal SNe Ia, in order to account for uniformity of the absorption depth of the $\text{Si}II$. From fig. 1 of Hicken et al. (2007), we see that the absorption strength is indeed different between SNe 2003fg and 2006gz; SN 2003fg seems to show the weaker absorption. This may support the hypothetical configuration mentioned above, i.e. disc-like ejecta structure with SN 2003fg viewed closer to the polar than SN 2006gz. In the polar direction, there is a small amount of material to absorb the light emitted at the photosphere, possibly leading to the weaker absorption strength seen in SN 2003fg.

Our results can be tested by future observations of overluminous SNe Ia. Foremost, we expect a large degree of polarization in SN 2003fg-like overluminous SNe Ia, depending on the viewing direction. If SN 2006gz is an explosion similar to SN 2003fg, but viewed at different orientation, we also expect a large degree of polarization for SN 2006gz-like SNe Ia. Next, late-time nebular spectra may be used to infer the distribution of $^{56}\text{Ni}$ directly1 (Motohara et al. 2006; Maeda et al. 2008; Modjaz et al. 2008). Furthermore, if we plot a number of overluminous SNe Ia on the $t_{\text{end}} - \text{he} - \text{peak}^2$ plot, it will illustrate in a statistical way how large the deviation from spherical symmetry is on average, and how great the intrinsic diversity (e.g. $M_{\text{bol}}$) in super-Chandrasekhar-mass WD explosions is.

More detailed, multi-dimensional studies are necessary to confirm our suggestions and speculations. First, multi-dimensional hydrodynamic simulations, based on a rapidly rotating super-Chandrasekhar WD, should be useful to check whether our preferred geometry, i.e. the disc-like or oblate distribution of density and $^{56}\text{Ni}$, can indeed be realized. Secondly, multi-dimensional radiation transport calculations should be useful in determining whether the $t_{\text{end}} - \text{he} - \text{peak}^2$ constraint can indeed be satisfied. We postpone these studies to the future, since in this paper we concentrate on examining a large parameter space to clarify the applicability and difficulty of spherically symmetric models (practically impossible to do in an expensive multi-dimensional study), and to clarify the need for non-spherical ejecta (for SN 2003fg).

Identifying the geometry should be useful in evaluating possible contamination of SNe Ia, which do not follow the Phillips relation, in cosmological study. SN 2006gz is roughly consistent with the Phillips relation if we adopt the peak luminosity derived by Hicken et al. (2007) (Sections 4.2 and 4.3; Fig. 5), while SN 2003fg is not (Howell et al. 2006). Our scenario for the disc-like/oblate ejecta infers that overluminous SNe Ia that do not follow the Phillips relation (e.g. SN 2003fg) should also show a peculiar low photospheric velocity. Thus, by performing spectroscopy, the SNe Ia with large deviation from the Phillips relation can be easily identified, and can be removed from samples for cosmological study.

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1 However, according to late-time observations of SN 2006gz, it did not show strong [Fe II] to probe the distribution of $^{56}\text{Ni}$ at ~1 yr after the explosion (Maeda et al. 2009). Some unidentified emission process at the late phase might cause this unexpected behaviour (Section 3.3), and spectroscopy at relatively early epochs is therefore recommended for the study of geometry for these peculiar SNe Ia.
