Advances in 3D scattering tomography of cloud micro-physics
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Abstract—We introduce new adjustments and advances in space-borne 3D volumetric scattering-tomography of cloud micro-physics. The micro-physical properties retrieved are the liquid water content and effective radius within a cloud. New adjustments include an advanced perspective polarization imager model, and the assumption of 3D variation of the effective radius. Under these assumptions, we advanced the retrieval to yield results that (compared to the simulated ground-truth) have smaller errors than the prior art. Elements of our advancement include initialization by a parametric horizontally-uniform micro-physical model. The parameters of this initialization are determined by a grid search of the cost function. Furthermore, we added viewpoints corresponding to single-scattering angles, where polarization yields enhanced sensitivity to the droplet micro-physics (i.e., the cloudbow region). In addition, we introduce an optional adjustment, in which optimization of the liquid water content and effective radius are separated to alternating periods. The suggested initialization model and additional advances have been evaluated by retrieval of a set of large-eddy simulation clouds.

Index Terms—Scattering Tomography, clouds, pySHDOM, Initialization, Polarization.

I. INTRODUCTION

THE current state of the art remote sensing of clouds cannot produce sufficient information regarding the 3D nature of small warm clouds. This is both due to the resolution of most sensors currently in orbit, in which small clouds may be below pixel resolution, and even more importantly to the assumptions of the reconstruction algorithms.

The plane-parallel assumption used in classic remote sensing effectively degenerates, to some extent, 3D light transfer to a 1D problem, as that there is effectively no horizontal light-transfer. The effect of this deficiency is particularly significant at the edges of the cloud. This leads to biased retrievals [1]–[3] and high uncertainties regarding the micro-physics and light-transfer of small clouds, regardless of the sensing resolution.

To allow a better understanding of small-cloud micro-physics and light-transfer, a 3D remote sensing approach is essential. The development of such is currently receiving growing attention [4]–[7].

Levis et al. [8], [9], have developed a 3D scattering tomography algorithm, based on the Spherical Harmonic Discrete Ordinate Method (SHDOM) for radiative-transfer [10]. Their method, pySHDOM [11], retrieves cloud properties by fitting of multi-view light intensity images to a physics based forward-model.

This method has a scattering-based computational-scattering tomography (CT) approach. It is a generalization of CT to recover scattering media such as clouds in a large scale. It is passive CT relying only on the Sun as an illumination source. The method was further developed using vSHDOM [12], for vectorized radiative-transfer, allowing consideration of the polarization properties of light. In this approach, cloud properties are retrieved by fitting of the full Stokes vector images. Levis et al. [13] have demonstrated by simulation the applicability of the method for multi-view cloud tomography.

The polarization properties of light have advantages for retrieval of cloud-droplet size distribution [14]–[20]. However, these advantages are specifically limited to conditions where single-scattering may be assumed. For this reason, methods which use polarization for such retrievals have till recently been restricted to the top 50 m layer of clouds.

Recently, a new technique has been suggested, to allow evaluation of vertical profiles [21]. This technique uses the Research Scanning Polarimeter (RSP), not only for high angular resolution of the cloud top, but also to scan the sides of the cloud, allowing an approximation of a vertical profile of the cloud-droplet size distribution.

In this paper we complement the work of Levis et al. [13] by adjustments which are necessary for a realistic space-borne optical-imager. The simulated imager was modelled as having a polarized sensor the likes of Sony Polarsens.

We introduce a new method for initialization, based on a parameterized horizontally-uniform model.

II. BACKGROUND

A. Parameter definition

We define the cloud tomography problem as the retrieval of cloud-droplet micro-physics. For small clouds we assume droplets of spherical geometry[1] of different radius values r. We pursue the three-dimensional liquid water content (LWC), and droplet size distribution n(r) per distance unit and volume unit to the extent of a chosen resolution.

The LWC is defined,

\[ \text{LWC} = \frac{4}{3} \pi \rho_w \int_0^\infty n(r)dr \quad \left[ \frac{g}{m^3} \right], \]  

(1)

This is a common approximation for cloud-droplets, where Mie theory is used to describe light scattering.

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where \( \rho_{w} = 1 \frac{k_{g}}{m^{3}} \) is the water density. Air molecular density is assumed to be known. Thus we do not try to retrieve air density.

To enable estimation of droplet size distribution, the distribution is compactly parameterized. Currently, we use a gamma distribution model to represent cloud-droplet size distribution \([22]\), defined as follows:

\[
n(r) = NC^e r^{v - 3} \exp[-r/(r_{e}v_{e})] \left[ \frac{1}{\mu m} \cdot \frac{1}{m^3} \right],
\]

where \( C = (r_{e}v_{e})^{2-v_{e}}/\Gamma(v_{e}^{-1}-2) \) is a normalization constant, \( \Gamma \) is the Gamma function and \( N \) is the droplet number concentration per volumetric unit

\[
N = \int_{0}^{\infty} n(r) dr \left[ \frac{1}{m^3} \right].
\]

The effective radius, \( r_{e} \) [\( \mu m \)], and unitless effective variance, \( v_{e} \) are defined by \([22]\):

\[
r_{e} = \frac{\int (\pi r^2) n(r) dr}{\int (\pi r^2) n(r) dr}, \quad v_{e} = \frac{\int (r-r_{e})^2 (\pi r^2) n(r) dr}{r_{e}^2 \int (\pi r^2) n(r) dr}.
\]

We currently focus on the retrieval of the LWC and \( r_{e} \) as in \([13]\).

### B. Retrieval quality measure

The quality of each retrieval is quantitatively estimated by local mean error \( \epsilon \) \([13]\) defined as:

\[
\epsilon_{LWC} = \frac{\|LWC - \hat{LWC}\|_1}{\|LWC\|_1}, \quad \epsilon_{r_{e}} = \frac{\|\hat{r}_{e} - r_{e}\|_1}{\|r_{e}\|_1},
\]

where LWC and \( \hat{r}_{e} \) are the estimated-value 3D fields, and LWC and \( r_{e} \) are the ground-truth values.

### III. SIMULATION OVERVIEW

A single simulation is constructed by the following stages:

1) **Measurements rendering.** Rendering of images by a forward model which runs a radiative-transfer simulation and then an image formation model. These are the simulation measurements (from here on, to avoid confusion, we define these images as the measurements). This stage consists of the imager setup definition, and their optics (including noise). It also consists of definition of camera positions and views. With these settings, a set of images is rendered using the ground-truth micro-physical 3D fields.

2) **Initialization.** Definition of an initial state of the medium for optimization. The initial state of the medium may be set in various ways. For instance, a medium grid at the beginning of optimization may be entirely empty. However, the more similar the initial medium is to the ground-truth data, the better and faster the optimization will be.

3) **Gradient descent based optimization.** In this stage, a set of images is again rendered by the forward model (stage 1). However, the model uses the current state (at a specific iteration) medium. These images are termed, the simulated images, which are used to fit the measurements. A cost function is defined according to the gap between the simulated and measured images.

### IV. FORWARD MODEL

This section describes the forward model which is used in the first and last stages of the simulation (see \([III]\)). It starts with the radiative-transfer simulation by pySHDOM as in \([9], [13], [23]\). Then, the image formation model of a polarized camera is used to generate the multi-view images. Each pixel gathers the simulated Stokes radiance field from a set of origins and angles and finally transfer the Stokes radiance through polarizers of the camera. In addition, we implement a Cloudbow scan approach.

#### A. Imager model

As in \([13]\), we use an imager model for both the forward and inverse models. We simulate a pinhole-camera model which is commonly used in computer-vision when the camera obeys **perspective projection**. The perspective projection is implemented in pySHDOM. Each pixel of the model’s imager gathers radiance from the scene.

Let us denote a spectral band by \( \lambda \). There, the wavelength \( \lambda \) is between \([\lambda_1, \lambda_2]\). Let us denote a spectral radiance at wavelength \( \lambda \) by \( I_{\lambda} \). It is calculated by pySHDOM and has units of \([ W/m^2/sr/nm]\). Assume a camera with lens of diameter \( D \) at distance \( f \) from the focal plane. The optics is perfect (no lens distortion), and the camera is focused on the object (i.e. clouds). Using pySHDOM, we simulate \( I_{\lambda} \) that reaches the camera lens. Assume \( I_{\lambda} \) is uniform within the area of a pixel footprint on the cloud. Geometrically, this is equivalent to saying that the irradiance on a detector pixel (area \( \pi r^2 \)) is uniform. For the rest of the text, we consider the pixels to be at optical axis of the camera.

To simulate a readout of the imager, we convert \( I_{\lambda} \) within \( \lambda \) to photo-generated electrons in the sensor\(^\text{1}\). A pixel on a sensor responds to the photons in a spectral band \( \lambda \). The pixel response depends on the sensor’s quantum efficiency, \( QE_{\lambda} [\text{electrons/photon}] \), which is a measure of the probability for a photo-electron to be created per incident photon with wavelength \( \lambda \). Let \( \tau_{\lambda} \) be the camera system efficiency due to optics losses and sensor reflection (not a part of \( QE \)) at wavelength \( \lambda \). Light energy is converted to the expected number of photons at wavelengths \( \lambda \) by the factor \( \frac{\lambda}{h c} [\text{photons/joule}] \).

\(^\text{1}\)Note that if the pixel is not at the optical axis, there is a vignetting effect, which refers to radial fall-off of pixel intensity from the centre towards the edges of the image \([24]\). For narrow field-of-view imagers, as we deal with, the vignetting is considered manageable.

\(^\text{2}\)For a sensor having a linear radiometric response, the conversion between electrons and the sensor readout value in gray-scales is by a fixed ratio. We do not deal with gray-scale values in this paper.
Let $\Delta t$ be the exposure time of the imager. The number of photo-electrons that are created in a pixel with a spectral band $\Lambda$ during exposure time $\Delta t$, is

$$N_\Lambda = p^2 \Delta t \int_\Lambda \tau_\Lambda \text{QE}_\Lambda \frac{\lambda}{\hbar c} \pi (\frac{D}{2})^2 I_\Lambda d\lambda$$

$$= \Delta t \int_\Lambda \Gamma_\Lambda I_\Lambda d\lambda \quad \text{[electrons].} \quad (6)$$

Here we define

$$\Gamma_\Lambda = \pi \tau_\Lambda (\frac{D}{2})^2 \text{QE}_\Lambda \frac{\lambda}{\hbar c} p^2 \quad \text{[electrons \cdot m^2 \cdot sr \cdot Joule]}, \quad (7)$$

which encapsulates dependencies on the optics, pixel size, QE and the pixel’s receptive solid-angle.

Ideally, to calculate the integral in Eq. (6), radiative-transfer would be run multiple times to calculate $I_\Lambda$ in the spectral band in which $\Gamma_\Lambda$ is significant (with certain wavelength resolution). There is common approximation that simplify these calculations [9], [23]. It is valid if wavelength dependencies within a spectral band are weak. This condition is met when narrow bands are considered. In this approximation, the spectral radiance is considered fixed within $\Lambda$ and Eq. (6) becomes

$$N_\Lambda = \Delta t I_\Lambda \int_\Lambda \Gamma_\Lambda L_\Lambda^{TOA} d\lambda \quad \text{[electrons].} \quad (8)$$

In Eq. (8), $I_\Lambda^{TOA}$ is calculated with input solar irradiance of $1 \frac{W}{m^2}$ and then scaled by the true solar irradiance $L_\Lambda^{TOA}$ [unitless] at the top of the atmosphere (TOA).

Now, we introduce a noise to $N_\Lambda$ to simulate a raw measurement of a camera. First we incorporate photon shot noise, which is Poisson-distributed around the $N_\Lambda$ expected value,

$$N_\Lambda^{measured} \sim \text{Poisson}\{N_\Lambda\} \quad \text{[electrons].} \quad (9)$$

The maximum number of electrons which can be contained in a pixel is referred to as the full well capacity. In our simulations, the exposure time $\Delta t$ is set to a level such that the sensor reaches 90% of its full well.

The sensor also introduces noise due to various causes, according to its specifications. The readout noise has a standard deviation of $\rho_{\text{read}}$ [electrons]. The dark current shot noise at $D$ $[\text{electrons/second}]$. The quantization noise is of standard deviation $\rho_{\text{digit}}$ [electrons]. Overall, a pixel readout has a signal to noise ratio (SNR) of approximately

$$\text{SNR} = \frac{N_\Lambda^{\text{measured}}}{\sqrt{N_\Lambda^{\text{measured}} + D_T \cdot \Delta t + \rho_{\text{read}}^2 + \rho_{\text{digit}}^2}}. \quad (10)$$

Noise specification in our simulation is based on Sony’s IMX250MYR sensor [25]. The pixel size is $3.45 \times 3.45$ $\mu$m$^2$, $\rho_{\text{read}}=2.31$ [electrons], $D_T = 3.51 \frac{\text{electrons}}{\text{second}}$ at 25 $^\circ$C. A pixel reaches full-well with 10,500 electrons. We use 10 bit quantization.

### B. Simulation of Stokes vector measurements

Our forward-model does not end with calculation of $N_\Lambda^{\text{measured}}$. There is a need to adjust the forward-model output, to fit the inverse-model output, so that the cost function is based on the difference between the same quantities. These quantities are for instance, the three components of the Stokes vector, $S_\Lambda = [I_\Lambda, Q_\Lambda, U_\Lambda]^T$ that correspond to linear polarization (more differences defined in Sec. V-B). In both the forward and inverse models, each component should have units of radiance.

The Sony IMX250MYR sensor [25] has four grid-wire polarizing filters which are formed on the chip in a block of four pixels. Each filter in a block has a different polarizing angle, either $90^\circ$, $45^\circ$, $135^\circ$, or $0^\circ$.

In pySHDOM, the Stokes vector is given in the meridian reference-frame. To simulate measurements of a polarized camera with four different polarization filters, we first need to convert the Stokes vector to a coordinate frame which is aligned with the camera reference-frame. Then to simulate the transmission of the converted Stokes vector through the polarizers.

We define the camera reference-frame so that it is aligned with the polarizer $0^\circ$ angle. The meridian reference-frame contains the view and zenith directions. Let $\hat{z}$ denote the zenith direction vector at every point on Earth. In the Meridian reference-frame, the electric field components are defined by direction vectors [13]

$$\mathbf{b} = \frac{\hat{z} \times \omega}{\| \hat{z} \times \omega \|}, \quad \mathbf{l} = \omega \times \mathbf{b}. \quad (11)$$

where $\omega = b \times l$ is a ray direction of a pixel and the axes $l, b$ are parallel and perpendicular to the the meridian plane (plane defined by $\omega$ [26] and $\hat{z}$), respectively. Let $S'_\Lambda = [I'_\Lambda, Q'_\Lambda, U'_\Lambda]^T$ be the Stokes vector in the camera reference-frame. The transformation between the meridian reference-frame to camera reference-frame is a multiplication of $S'_\Lambda$ by a Mueller rotation matrix

$$\mathbf{M}(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\alpha) & -\sin(2\alpha) & 0 \\ 0 & \sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (12)$$

where $\alpha$ is the rotation angle between $l$ and the direction of linear polarizer with $0^\circ$ (calculated for each pixel individually), in the anti-clockwise direction when looking in the direction of propagation [26]. Now

$$S'_\Lambda = \mathbf{M}(\alpha) S_\Lambda, \quad (13)$$

and to go back to the meridian reference-frame,

$$S_\Lambda = \mathbf{M}^{-1}(\alpha) S'_\Lambda. \quad (14)$$

We simulate the transmission of $S'_\Lambda$ through four pure linear polarizers with $90^\circ$, $45^\circ$, $135^\circ$ and $0^\circ$ angles. The transmitted radiances are

$$\begin{bmatrix} I_{\Lambda, 90} \\ I_{\Lambda, 45} \\ I_{\Lambda, 135} \\ I_{\Lambda, 0} \end{bmatrix} = \mathbf{G} \begin{bmatrix} I'_\Lambda \\ Q'_\Lambda \\ U'_\Lambda \end{bmatrix}, \quad (15)$$

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4 The radiative-transfer simulation is run on spectrally-averaged optical quantities [9], [23].

5 Sensor suppliers specify it in units of [electrons].
Additional images are obtained without changing the number of imagers, i.e. one or two satellites take more than one image of a single cloud field.

The idea is that one or two satellites capture additional images in viewing angles within the cloudbow, as illustrated in Fig. 4. The angular resolution of sampling is between 1° to 1.5°, which is a realistic assumption for satellite attitude control. The algorithm we implemented finds these one or two satellites, per solar illumination direction.

On average, we find the cloudbow scan decreases the mean errors $\epsilon_{\text{LWC}}$ and $\epsilon_{r_c}$, by 2% and 1%. We find this is a consistent improvement. However, further sensitivity study may improve the choice of scanned scattering angles and resolution.

\section{Initialization}

This section describes the stage 2 of the simulation (see 3), definition of the initial medium for optimization. The cloud initialization has been found to be highly influential in gradient based CT optimization. The prior initialization method $[13]$, assumes a simple homogeneous model, using typical values of effective radius, $r_e$, and LWC (in $[13]$, it is $r_e = 12 \mu m$, and LWC = 0.01 $LWC_{\text{min}}$). We denote this method as $H_{\text{Typical}}$.

We now describe a horizontally-homogeneous parametric cloud model. The parameter values used for each initialization are found by optimization of the initial cost function, which would result from the parameter choice.

We emphasize the model is suggested strictly for initialization purposes. Once initialized, the optimization is no longer restricted to a horizontally-homogeneous assumption.

\subsection{Monotonous model}

The suggested cloud model assumes monotonic profiles of LWC and $r_e$ with altitude $Z \geq Z_0$ within a cloud, $Z_0$ being the cloud-base height $[27]$. The LWC is assumed to have a linear profile:

$$LWC \propto (Z - Z_0),$$

and $r_e$ is assumed proportional to the cubic root of $Z$ $[27]$:

$$r_e \propto (Z - Z_0)^{\frac{1}{3}}.$$

This assumption has been compared to Large-Eddy Simulations (LES) based on the Barbados Oceanographic and Meteorology Experiment (BOMEX), and is suitable for steady-state cumulus convection. The profile equations set fixed minimum values for LWC ($LWC_{\text{min}}$) and $r_e$ ($r_{e_{\text{min}}}$). The profiles are defined as follows:

$$LWC = \alpha_1(Z - Z_0) + LWC_{\text{min}}$$

$$r_e = \alpha_e(Z - Z_0)^{\frac{1}{3}} + r_{e_{\text{min}}}$$

Here $\alpha_1$ and $\alpha_e$ are slope parameters of the LWC and $r_e$, respectively. Here $LWC_{\text{min}}$ and $r_{e_{\text{min}}}$ are set to the values 0.0001 $LWC_{\text{min}}$, and 2.5 $\mu m$, respectively.

The value is determined by the lower altitude of a 3D mask$^6$.

\footnote{Note that when using a mask determined by space-carving there is uncertainty in $Z_0$.}
B. Parameter choice

Let \( N_{\text{meas}} \) be the length of the measurement vector, i.e. \( N_{\text{meas}} = N_{\text{view}} N_{\text{pixels}} \) (total number of viewpoints multiplied by a number of pixels of each view). The slope parameters \( \alpha_l \) and \( \alpha_r \) used for a monotonous model are found by minimization of a cost function. We follow the notations of [13].

For the specific purpose of optimizing the initialization, two cost function were considered for a polarized imaging. The first is defined by errors of all linear polarization components of the Stokes vector. For this purpose cost functions were defined for the components of the Stokes vector, \( S = [I, Q, U]^T \), as the errors between the simulated components \( S_i \), and the measured components \( y_{S_i} \), at measurement index \( k \) (pixel) and component index \( i \in [1, 3] \):

\[
D_{\text{radiance}} = \frac{1}{2} \sum_{k=1}^{N_{\text{meas}}} (I[k] - y_I[k])^2, \tag{21}
\]

\[
D_Q = \frac{1}{2} \sum_{k=1}^{N_{\text{meas}}} (Q[k] - y_Q[k])^2, \tag{22}
\]

\[
D_U = \frac{1}{2} \sum_{k=1}^{N_{\text{meas}}} (U[k] - y_U[k])^2. \tag{23}
\]

The combined Stokes vector cost function, \( D_{\text{Stokes}} \) is defined as the sum of the separate component cost functions:

\[
D_{\text{Stokes}} = D_{\text{radiance}} + D_Q + D_U. \tag{24}
\]

We define a second initialization method, \( M_{\text{Stokes}} \), which uses the monotonous model, with parameters \( \alpha_l \) and \( \alpha_r \) set by a grid-scan search of \( D_{\text{Stokes}} \).

The second considered cost is defined by the error of the degree of linear polarization (DoLP) where:

\[
\text{DoLP} = \frac{\sqrt{Q^2 + U^2}}{I}. \tag{25}
\]

We define the simulated DoLP at measurement index \( k \):

\[
\text{DoLP}[k] = \frac{\sqrt{Q[k]^2 + U[k]^2}}{I[k]}, \tag{26}
\]

and the measured DoLP at measurement index \( k \):

\[
y_{\text{DoLP}}[k] = \frac{\sqrt{y_Q[k]^2 + y_U[k]^2}}{y_I[k]}. \tag{27}
\]

Accordingly, the cost function is:

\[
D_{\text{DoLP}} = \frac{1}{2} \sum_{k=1}^{N_{\text{meas}}} (\text{DoLP}[k] - y_{\text{DoLP}}[k])^2. \tag{28}
\]

We define a third initialization method, \( M_{\text{DoLP}} \), which uses the monotonous model, with parameters \( \alpha_l \) and \( \alpha_r \) set by a grid-scan search of \( D_{\text{DoLP}} \).

Examples of the cost function grid-planes are presented in Fig. [3] The values of \( D_{\text{radiance}} \) in this example are a result of a simulated visible light camera. The values of \( D_{\text{Stokes}} \) and \( D_{\text{DoLP}} \) are a result of a simulated camera having a polarization sensor.

This example presents a potential advantage in using the DoLP for setting \( \alpha_l \) and \( \alpha_r \). The cost \( D_{\text{DoLP}} \) shows a distinct minimum, while \( D_{\text{Stokes}} \) results in a “valley” and is less explicit in defining the best parameters for initialization CT.

For both \( M_{\text{Stokes}} \) and \( M_{\text{DoLP}} \) methods, the model parameters are set, by a grid scan. For \( \alpha_l \), sixteen equally spaced values were considered between 0.1 \( \frac{\text{mrad}}{\text{m}} \) and 1.6 \( \frac{\text{mrad}}{\text{m}} \). For \( \alpha_r \), sixteen values were considered between 3 \( \frac{\text{mrad}}{\text{m}} \) and 15 \( \frac{\text{mrad}}{\text{m}} \).

The grid scan can also be used for a homogeneous model. We define a fourth initialization method, \( M_{\text{Stokes}} \), which uses the homogeneous model, with values \( LWC \) and \( r_c \) set by grid-scan search of \( D_{\text{Stokes}} \).

In this case the scan is over homogeneous values of the LWC and \( r_c \). For comparison, we present retrieval results from this method as well. The grid search in this case is over the range of \( \alpha_l \) between 0.1 \( \frac{\text{mrad}}{\text{m}} \) and 1.6 \( \frac{\text{mrad}}{\text{m}} \) and \( \alpha_r \) values between 3 \( \frac{\text{mrad}}{\text{m}} \) and 15 \( \frac{\text{mrad}}{\text{m}} \).

An example of initialization mean profiles of LWC and \( r_c \), as set by the different methods (summarized in Table [I]), are presented in Fig. [3].

In this example, the model initialization of \( M_{\text{Stokes}} \) and \( M_{\text{DoLP}} \) yield different optimal initializations (\( \alpha_l \), \( \alpha_r \)). This is not the case for all clouds. Anyway, conclusion regarding...
which initialization is better can be drawn only from resulting retrievals as described next.

VI. PRECONDITIONING AND OPTIMIZATION IN PARTS

This section relates to the third stage of the simulation (see III).

The optimization attempts to solve a problem which consists of unknowns of two different orders of magnitude. In small clouds (up to a volume of $1 \times 1 \times 1$ km$^3$), typical maximum values of LWC and $r_e$ are $1 \text{ g m}^{-3}$ and $15 \mu m$, respectively. A gradient-descent based optimization method is highly affected by this relative scale.

One way to overcome this effect is a preconditioning of the scales of the variables, which sets them at the same order of magnitude. Levis et al. [13] have shown the efficiency of this method, for preconditioning factors of $\Pi_{\text{LWC}} = 15$ and $\Pi_{r_e} = 0.01$. However, we find that the preconditioning is case sensitive, and the ideal factors may vary greatly, depending on the characteristics of the cloud in question. For 3D variation in $r_e$, we recommend preconditioning factors of $\Pi_{\text{LWC}} = 10$ and $\Pi_{r_e} = 0.1$.

In addition, we propose an optional different approach. The gap in order of magnitude of each variable may be overcome by defining separate episodes of retrieval of the LWC and $r_e$. The algorithm is as follows:

1) Initialization of both LWC and $r_e$.
2) Gradient descent of LWC until convergence, while $r_e$ is kept constant with values from initialization.
3) Gradient descent of $r_e$ until convergence, while LWC is kept constant with values from the previous estimation.
4) Gradient descent of LWC until convergence, while $r_e$ is kept constant with values from the previous estimation.
5) Repeat stages 3 and 4 until the difference in the cost function between stages decreases below a set value.

VII. SIMULATIONS AND DISCUSSION

The dataset used as ground-truth is a set of realistic 3D LWC and $r_e$ arrays of six BOMEX-based LES-generated small clouds. The volumetric resolution of the ground-truth is $20 \times 20 \times 20$ m$^3$. The effective variance $v_e$ is assumed constant, $v_e = 0.1$. The characteristics of the cloud-set is summed in Table III.

The configuration of satellites is constant in all demonstrations. A set of ten imagers in separate satellites is considered. The satellites are assumed to be in a string of pearls configuration, moving northward consecutively, as illustrated in Fig. 1. The altitude of the satellites is 500 km and the uniform distance between each consecutive pair is 100 km (on orbit arc). The viewing angles are between $-46^\circ$ and $39.3^\circ$ relative to the zenith.

The coordinate system is local Cartesian coordinate system East, North, Up (ENU). In our convention, the East axis is labeled $Y$, the North $X$, and the zenith $Z$. The satellites move in the positive direction of $X$ and $Y = 0$. Each imager is aimed to the center of the cloud field. No satellite pointing error is introduced.

The angular position of the sun in the simulations is at 25° from the zenith, and at azimuth of 90° (i.e., in the East).

The simulated imager has a narrow red waveband channel, between 620 nm and 670 nm. There, absorption by water

| TABLE I |
| Summary of Methods |
| Method Representation |
| Homogeneous with typical values $H_{\text{Typical}}$ |
| Homogeneous with $D_{\text{Stokes}}$ grid search $H_{\text{Stokes}}$ |
| Monotonous with $D_{\text{Stokes}}$ grid search $M_{\text{Stokes}}$ |
| Monotonous with $D_{\text{DoLP}}$ grid search $M_{\text{DoLP}}$ |

| TABLE II |
| Summary of ground-truth characteristics of cloud-set for simulations. |
| Maximum $r_e$ [\(\mu m\)] | 8.8 - 15.4 |
| Mean $r_e$ [\(\mu m\)] | 6.2 - 10 |
| Maximum cloud optical depth | 19.2 - 71.8 |
| Minimum cloud base altitude [m] | 550 |
| Maximum cloud top altitude [m] | 1710 |
The simulated (normalized) radiance \( I \), Stokes elements \( Q \) and \( U \), and the DoLP, as viewed by the satellite setup described in Fig. 1. The leftmost images correspond to the view of satellite no. 1, the rightmost to that of satellite no. 10. The DoLP is visibly more noisy than the radiance. In addition, the high values of DoLP viewed by satellites no. 2 and no. 8, indicate the viewing angles of these satellites correspond to cloudbow scattering angles.

droplets is negligible, and Rayleigh scattering is relatively low, reducing both airlight and sky polarization. The imager resolution is 20 m on the ground at nadir.

The simulations include a cloudbow-scan. A single imager captures ten additional views within the scattering angles of \([135^\circ, 150^\circ]\). The retrieval assumes a ground-truth 3D mask, which is extracted from the 3D extinction field of the cloud-data.

A. Initialization method

For each initialization method, the \( \epsilon \) error values of the cloud-set retrievals are averaged. The mean \( \epsilon \) values of the different methods are compared.

A comparison of the mean \( \epsilon \) errors of the different methods (presented in Fig. 5), demonstrates the superiority of the \( M_{\text{Stokes}} \) and \( M_{\text{DoLP}} \) methods for initialization, which use the monotonous model.

The \( H_{\text{Stokes}} \) method is also an improvement with respect to \( H_{\text{Typical}} \). Even so, the errors of both the LWC and \( r_c \) are even lower using the monotonous model methods.

The difference in the mean of \( \epsilon \) errors of \( M_{\text{Stokes}} \) and \( M_{\text{DoLP}} \) is very small, with a slight advantage to \( M_{\text{DoLP}} \).

As mentioned in section V-B these two methods did not result in different initialization profiles for all cloud samples. For the few that yielded significantly different parameters we find an improvement in favor of the \( M_{\text{DoLP}} \) method. For this reason, in spite of the seemingly small mean advantage, we recommend the use of \( M_{\text{DoLP}} \) for the initialization method.

B. Optimization in parts

Assuming the \( M_{\text{DoLP}} \) initialization method, we find that separating optimization periods for LWC and \( r_c \) (see section VI) yields an additional improvement to the retrieval.

The average errors decrease from \( \epsilon_{\text{LWC}} = 0.42, \epsilon_{r_c} = 0.09 \) to \( \epsilon_{\text{LWC}} = 0.39, \epsilon_{r_c} = 0.08 \). However, the separation does take a toll on the time of convergence, increasing it up to double.

We find that for some cases the \( r_c \) optimization does but barely change the mean error. Even so, the LWC error may continue to decrease, in it’s following optimization period.

We find that the initial mean profiles of \( r_c \), as set by \( M_{\text{DoLP}} \), have very low initial errors. Therefore when optimizing in parts, which begins with optimization of the LWC, conditions for LWC optimization are optimal. We notice two important implications in this respect.

First, beginning the optimization from \( r_c \) would not be as successful, as we have also observed (not presented here). Secondly, the 1D \( r_c \) model proves to be a good assumption, and the parameter grid search may be sufficient for \( r_c \) estimation. We may optionally reduce the optimization by gradient descent to estimation of LWC, while assuming a 1D \( r_c \) profile, retrieved by \( M_{\text{DoLP}} \).
VIII. CONCLUSION

We have presented adjustments to the pySHDOM 3D microphysical scattering tomography code. The major adjustments include implementation of a realistic polarized imager model, and a new initialization method.

We have demonstrated the superiority of our proposed initialization method, under the conditions of the implemented imager model.

In addition, we have suggested two optional adjustments, cloudbow scanning, and optimization of the LWC and $r_m$ in separate periods, which both have been demonstrated to reduce the mean error of the retrieval.

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