Galactic rotation curves of spiral galaxies and dark matter in $f(R, T)$ gravity theory

Gayatri Mohan $^*$ and Umananda Dev Goswami $^\dagger$

Department of Physics, Dibrugarh University, Dibrugarh 786004, Assam, India

Galactic rotation curve is a powerful indicator of the state of the gravitational field within a galaxy. The flatness of these curves indicates the presence of dark matter in galaxies and their clusters. In this paper, we focus on the possibility of explaining the rotation curves of spiral galaxies without postulating the existence of dark matter in the framework of $f(R, T)$ gravity, where the gravitational Lagrangian is written by an arbitrary function of $R$, the Ricci scalar and of $T$, the trace of energy-momentum tensor $T_{\mu\nu}$. We derive the gravitational field equations in this gravity theory for the static spherically symmetric spacetime and solve the equations for metric coefficients using a specific model that has minimal coupling between matter and geometry. The orbital motion of a massive test particle moving in a stable circular orbit is considered and the behavior of its tangential velocity with the help of the considered model is studied. We compare the theoretical result predicted by the model with observations of a sample of nineteen galaxies by generating and fitting rotation curves for the test particle to check the viability of the model. It is observed that the model could almost successfully explain the galactic dynamics of these galaxies without the need of dark matter at large distances from the galactic center.

Keywords: Modified gravity; Galactic rotation curves; Dark matter; Minimal coupling; Tangential velocity.

I. INTRODUCTION

The significant mismatch between the observed and dynamically estimated masses of different astrophysical systems such as galaxies and clusters of galaxies raises the issue of dark matter (DM), which is one of the leading theoretical difficulties in modern astrophysics. The pioneering works and observational data show that the total mass density of the universe is not dominated by the baryonic matter, instead by some unseen, unusual form of matter called DM, which is needed for the explanation of both the galactic dynamics as well as the large scale structures in the universe [1]. The presence of such mysterious matter was first pointed out by J. H. Oort in the early 1930s [2]. He studied the doppler shifts of some stars of our galaxy moving near the galactic plane and thus by computing their velocities he argued for the presence of more mass inside the galaxy than the observed mass to hold the stars in their respective orbits. In 1933, a Swiss astronomer F. Zwicky [3] suggested similar hidden matter from his study on the Coma cluster of galaxies. By employing virial theorem he estimated the virial mass (total mass) of the cluster from the velocity dispersions of galaxies using doppler shift and then comparing the virial mass of the cluster to the luminous mass calculated from $M/L$ ratio, he found an inequality of these two masses which implies a large mass discrepancy in the cluster [3–6]. In 1939, H. Babcock [7] studied the rotation curve of M31 about 20 kpc away from the center of the galaxy and by estimating mass distribution of it, he proposed for a large amount of mass in the exterior part of the galaxy. Outstanding investigations on the mass distributions in galaxies were published during late 1960s to early 1980s through the observations of rotational velocities of stars in some galaxies by M. Roberts [8], K. Freeman [9], J. C. Jackson [10], Roberts and Rots [11], Rubin and Ford [12, 13] etc. They noted that the mass of a galaxy must be larger than its detectable mass and thus they confirmed about an undetectable, significant, additional mass density at large distances from the center. From the analyses of rotation curves based on radio observatory data, D. H. Rogstad and G. Shostak [14] also suggested the necessity of low luminosity material in the outer regions of M33, M101, NGC 6946, NGC 2403 and IC 342 galaxies.

As already clear from above, two distinguished observational findings, the mass discrepancies in clusters of galaxies and the flat shapes of rotation curves of spiral galaxies are two striking evidences for the existence of DM within these structures [15–18]. Moreover, another observational discovery within General Relativity (GR) is the gravitational lensing, in which distorted image of a distant light source is formed due to the deflection of light by a massive gravitating system such as a cluster of galaxies, is also a strong evidence of DM within the lensing cluster. It is also a powerful tool to determine the amount of mass within the lensing cluster [4, 19]. Further, rotation curves, the plots of tangential velocities of stars and gas clouds rotating around the center of galaxies as a function of the distance from the center, are directly related to the overall mass distributions of galaxies and are potent means to analyze the gravitational field within galaxies. These curves show that the tangential velocity of a star increases linearly with the distance from the center of the galaxy and attains approximately a constant value $v \approx 200 – 300$ km/s [15, 20] up to 5 – 6 times [1] of the luminous radius of a galaxy. It infers that a visible galaxy is embedded in an extended spherical matter distribution called the galactic halo (DM halo), where due to a constant $v$, the mass $M(r)$ within the radius

---

$^*$Email: gayatrimohan704@gmail.com
$^\dagger$Email: umananda2@gmail.com
increases linearly with $r$ according to the relation $M(r) = v^2r/G$ and it causes density variation as $1/r^2$ for a spherically symmetric model [21–23]. This variation is exceptional from the Newtonian gravity, which yields Keplerian decrease in velocity at a large distance from the center.

Most importantly, rotation curves hint the failure of GR at large distances, as to fit the flat parts of the curves GR needs DM in the outer parts of the galaxies. But till now no experimental evidence has been found or no direct evidence of DM has been detected. Consequently, attempts have been made for searching the alternatives of Newtonian Gravity and GR. As a result, based on the modification of Newtonian gravity or of GR different theoretical models have been proposed to understand observations both at cosmological and astrophysical scales [17, 24–26]. M. Milgrom in 1983, proposed the Modified Newtonian Dynamics (MOND) [24] as a possible means to describe the hidden mass in galaxies and their clusters. Basis of this modification is that both at cosmological and astrophysical scales [17, 24–26]. M. Milgrom in the outer parts of the galaxies. But till now no experimental evidence has been found or no direct evidence of DM has been detected. Consequently, attempts have been made for searching the alternatives of Newtonian Gravity and GR. As a result, based on the modification of Newtonian gravity or of GR different theoretical models have been proposed to understand observations both at cosmological and astrophysical scales [17, 24–26].

In our present work, as we intend to study the galactic rotation curves in the framework of $f(\mathcal{R}, T)$ gravity theory and hence to explain the mystery of DM as a large-scale modification of gravity, we consider a test particle moving around a galaxy in a stable circular orbit. Here we employ the geodesic equations to obtain the rotation curves of galaxies in spherically symmetric spacetime and derive rotational velocity with a $f(\mathcal{R}, T)$ model that has minimal coupling between matter and geometry. We organize our rest of the paper as follows. In Section II, we derive the field equations in the $f(\mathcal{R}, T)$ gravity and then transform these equations in the form of modified Einstein equations. In Section III, the solutions of the modified field equations for...
II. MODIFIED EINSTEIN FIELD EQUATIONS IN $f(R, T)$ GRAVITY

The action in the $f(R, T)$ gravity theory takes the form:

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g} f(R, T) \, d^4x + S_m,$$

(1)

where $S_m = \int \sqrt{-g} \, L_m \, d^4x$ is the matter part of the action with $L_m$ as the matter Lagrangian density of matter field, $g$ is determinant of the metric $g_{\mu\nu}$ and $\kappa^2 = 8\pi G = 1/M_p^2$. $G$ and $M_p$ are the gravitational constant and Planck mass (reduced) respectively. The energy-momentum tensor of matter field can be defined as

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}} = - 2 \frac{\partial L_m}{\partial g^{\mu\nu}} + g_{\mu\nu} L_m.$$

(2)

From this equation we obtain the trace of $T_{\mu\nu}$ as

$$T = -2 g^{\mu\nu} \frac{\partial L_m}{\partial g^{\mu\nu}} + 4 L_m.$$

(3)

In this case we assumed that the matter Lagrangian density $L_m$ does not depend on the derivative of the metric tensor $g_{\mu\nu}$, instead it depends on $g_{\mu\nu}$ only. Now, the variation of action (1) with respect to $g_{\mu\nu}$ results in the field equations of $f(R, T)$ gravity as

$$F_R \frac{\delta R}{\delta g^{\mu\nu}} - \frac{1}{2} f(R, T) g^{\mu\nu} = \kappa^2 T_{\mu\nu} - F_T \frac{\delta T}{\delta g^{\mu\nu}},$$

(4)

where

$$F_R = \frac{\partial f(R, T)}{\partial R}, \quad F_T = \frac{\partial f(R, T)}{\partial T},$$

and

$$\frac{\delta R}{\delta g^{\mu\nu}} = R_{\mu\nu} - \nabla_\mu \nabla_\nu + g_{\mu\nu} \Box \text{ and } \frac{\delta T}{\delta g^{\mu\nu}} = \frac{\delta (g^{\alpha\beta} T_{\alpha\beta})}{\delta g^{\mu\nu}} = T_{\mu\nu} + \theta_{\mu\nu}. $$

(5)

Here, $R_{\mu\nu}$ is the Ricci tensor, $\nabla_\mu$ is the covariant derivative related to Levi-Civita connection $\Gamma$, $\Box = \nabla_\mu \nabla^\mu$ represents the d’Alembertian operator and

$$\theta_{\mu\nu} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} = -2 T_{\mu\nu} + g_{\mu\nu} L_m - 2 g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}} $$

(6)

is the energy tensor [43, 60]. The use of equation (5) reduces the equation (4) to the following form:

$$F_R R_{\mu\nu} - \frac{1}{2} f(R, T) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) F_R = \kappa^2 T_{\mu\nu} - F_T (T_{\mu\nu} + \theta_{\mu\nu}).$$

(7)

As already mentioned, the basis of $f(R, T)$ theory is the coupling between matter and geometry, which shows by equations (7) that the field equations of the theory depend on energy tensor $\theta_{\mu\nu}$. Thus, corresponding to different choice of $L_m$, a different set of field equations would be generated [61, 62]. $L_m$ can be written in terms of energy density $\rho$ or in terms of thermodynamic pressure $p$ [63]. The choice of the matter Lagrangian $L_m$ equal to $p$ results $\theta_{\mu\nu} = -2 T_{\mu\nu} + p g_{\mu\nu}$. Hence, from equation (7) by assuming the energy-momentum tensor $T_{\mu\nu}$ for a dustlike matter field, we finally obtain the field equations of $f(R, T)$ theory as follows [30, 64]:

$$F_R R_{\mu\nu} - \frac{1}{2} f(R, T) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) F_R = \left(\kappa^2 + F_T\right) T_{\mu\nu}. $$

(8)

With a simple algebraic manipulation, these field equations (7) can be written as the modified Einstein equations with an effective energy-momentum tensor $T_{\mu\nu}^{\text{eff}}$ as

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}},$$

(9)
where \( G_{eff} = G/F_R \) and \( T_{\mu\nu}^{eff} = T_{\mu\nu} + T_{\mu\nu}^I \). \( T_{\mu\nu}^I \) represents the matter-curvature interaction term defined as interaction energy-momentum tensor and may act as a new matter source induced by the \( f(R, T) \) action that behaves as an extra effective fluid of purely geometric origin [45, 64]. It is given as

\[
T_{\mu\nu}^I = \frac{1}{k^2} \left[ F_T T_{\mu\nu} + \frac{1}{2} (f(R, T) - F_R R) g_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) F_R \right].
\] (10)

Refs. [43, 62, 64] have discussed the conservation of energy-momentum tensor and its consequences on the motion of a test particle in \( f(R, T) \) gravity. Ref. [43] shows that though an additional force perpendicular to four velocity appears in the equation of motion of a test particle in \( f(R, T) \) theory of gravity resulting the motion of the particle as non-geodesic, in case of dust (pressureless fluid), similar to GR, the conservation of the energy-momentum tensor in the theory holds good. But it has been corrected in Ref. [62] that as the covariant divergence of energy-momentum tensor is not conserved even for pressureless fluid and hence it leads to non-geodesic path to the particle. In general, the conservation equation of the effective energy-momentum tensor \( T_{\mu\nu}^{eff} \) for \( f(R, T) \) theories discussed in Ref. [64] shows that by using the usual energy-momentum conservation \( \nabla^\mu T_{\mu\nu} = 0 \) and \( \nabla^\mu G_{\mu\nu} = 0 \) obtained from the Bianchi relation for Ricci scalar, the covariant divergence equation for the \( T_{\mu\nu}^{eff} \) can be achieved in the following form [59, 64, 65]:

\[
\nabla^\mu T_{\mu\nu}^{eff} = \frac{1}{k^2} \left[ T_{\mu\nu} \nabla^\mu F_T + \frac{1}{2} F_T \nabla_\nu T + G_{\mu\nu} \nabla^\mu F_R \right].
\] (11)

Under the consideration that \( T_{\mu\nu} \nabla^\mu F_T + \frac{1}{2} F_T \nabla_\nu T = 0 \), equation (11) reduces to

\[
\nabla^\mu T_{\mu\nu}^{eff} = \frac{G_{\mu\nu}}{k^2} \nabla^\mu F_R.
\] (12)

This consideration imposes some restrictions on the choice of functionality of \( f(T) \) of \( f(R, T) \). Equation (12) indicates that conservation of interaction energy-momentum holds good only for the constant value of \( F_R \) which evidently confirms the geodesic motion of the test particle for the same form of field equations.

It is to be noted that because of the extra terms appearing in the field equations, the \( f(R, T) \) theory of gravity can be regarded as a two fluid model [44]. Further, defining \( T_{\mu\nu}^I \) for an anisotropic geometric matter distribution, we can express it as [29]

\[
T_{\mu\nu}^I = \text{diag} \left( -\rho^I, P_r^I, P_t^I, P_t^I \right),
\] (13)

where \( \rho^I \) represents interaction energy density, \( P_r^I \) and \( P_t^I \) are the radial and tangential components of interaction pressure respectively. Thus we obtain,

\[
\rho^{eff} = \rho + \rho^I, \quad P_r^{eff} = P_r^I, \quad P_t^{eff} = P_t^I.
\] (14)

Here, \( \rho^{eff} \) is effective energy density. As we have considered \( T_{\mu\nu} \) for the dustlike matter, so both radial and tangential components of the effective pressure are equal to their respective interaction terms.

### III. GRAVITATIONAL FIELD EQUATIONS FOR STATIC SPHERICALLY SYMMETRIC MASS DISTRIBUTION

As mentioned in Section I, since our study mainly concentrates on the explanation of behaviour of galactic rotation curves of spiral galaxies in galactic halo regions without considering the existence of DM in it, we need to have a suitable mass distribution for such galaxies. Simply such a galaxy can be considered to have spherically symmetric distribution of matter having smooth gravitational field [66]. So we shall consider a general static spherically symmetric metric of the form:

\[
ds^2 = -e^{2\mu} dt^2 + e^{2\nu} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta \, d\varphi^2,
\] (15)

where \( \mu \) and \( \nu \) are metric potentials and are functions of \( r \) only. In our present case the metric coefficients \( e^{2\mu} \) and \( e^{2\nu} \) can be determined by solving the modified Einstein field equations (9) as follows. The nonzero components of Einstein tensor \( G_{\mu\nu} \) for the metric (15) can be obtained as

\[
G_0^0 = - \frac{2 e^{-2\nu}}{r} + e^{-2\nu} \mu^' - \frac{1}{r^2},
\] (16)

\[
G_1^1 = \frac{2 e^{-2\nu}}{r} + \frac{e^{-2\nu}}{r^2} - \frac{1}{r^2},
\] (17)

\[
G_2^2 = G_3^3 = e^{-2\nu} \left[ (\mu')^2 - \mu \nu' \right] + \frac{e^{-2\nu}}{r} (\mu' - \nu').
\] (18)
Using these equations (16), (17) and (18) together with equation (9) and (14) we obtain the effective field equations as given by

\[ \frac{2e^{-2\nu}}{r^2} \nu' - \frac{e^{-2\nu}}{r^2} + 8\pi G_{\text{eff}} (\rho + \rho^I), \tag{19} \]

\[ \frac{2e^{-2\nu}}{r^2} \mu' + \frac{e^{-2\nu}}{r^2} - \frac{1}{r^2} = 8\pi G_{\text{eff}} P^I_r, \tag{20} \]

\[ e^{-2\nu} \left[ \mu'' + (\mu')^2 - \mu' \nu' \right] + \frac{e^{-2\nu}}{r} (\mu' - \nu') = 8\pi G_{\text{eff}} P^I_t. \tag{21} \]

The interaction energy density \(\rho^I\) and components of anisotropic interaction pressure \(P^I_r\) and \(P^I_t\) can be obtained from equation (10), which can be expressed as

\[ \rho^I = \frac{1}{8\pi G} \left[ F_{R} \rho + \frac{1}{2} (R F_{R} - f(R, T)) + \left( F''_{R} - \nu' F'_{R} + \frac{2}{r} F'_{R} \right) e^{-2\nu} \right], \tag{22} \]

\[ P^I_r = \frac{1}{8\pi G} \left[ \frac{1}{2} (f(R, T) - R F_{R}) + \left( \mu' F'_{R} - \frac{2}{r} F'_{R} \right) e^{-2\nu} \right], \tag{23} \]

\[ P^I_t = \frac{1}{8\pi G} \left[ \frac{1}{2} (f(R, T) - R F_{R}) - \left( F''_{R} + \frac{F'_{R}}{r} + (\mu' - \nu') F'_{R} \right) e^{-2\nu} \right]. \tag{24} \]

To solve equations (19), (20) and (21) for the metric coefficients, we just sum equation (19) and (20) which gives a differential equation for the functions \(\mu\) and \(\nu\) in the form:

\[ \frac{\mu' + \nu'}{r e^{2\nu}} = 4\pi G_{\text{eff}} [\rho + \rho^I + P^I_r]. \tag{25} \]

At this point it needs to be mentioned that in GR we have \(\mu' + \nu' = 0\) and hence \(e^{2\mu} e^{2\nu} = 1\). But as we are working with the \(f(R, T)\) modified theory of gravity, which is an extension of GR, so we have obtained equation (25). Thus in our case, we should have \(e^{2\mu} e^{2\nu}\) different from unity. It can be argued that if \(\mu' + \nu'\) is a well-behaved expression it should have a solution of the following form \([32]\):

\[ e^{2\mu} e^{2\nu} = X(r). \tag{26} \]

Here, function \(X(r)\) should be slightly different from unity to remain it in the vicinity of GR, so that the \(f(R, T)\) gravity behaves similarly to GR in the low curvature regimes. To differ \(X(r)\) from the unity by a small amount we assume that \([32]\)

\[ X(r) = \left( \frac{r}{h} \right)^\delta, \tag{27} \]

where \(\delta\) is a small dimensionless parameter, \(h\) is the scale length of the system. With this form of \(X(r)\) equation (26) takes the form:

\[ 2(\mu + \nu) = \ln \left( \frac{r}{h} \right)^\delta. \tag{28} \]

From which a differential equation for the metric potentials is achieved as

\[ \mu' + \nu' = \frac{\delta}{2r}. \tag{29} \]

Also

\[ e^{2\mu} e^{2\nu} = e^{2\mu + 2\nu} \cong 1 + 2(\mu + \nu). \]

Hence we may write the product \(e^{2\mu} e^{2\nu}\) approximately as

\[ e^{2\mu} e^{2\nu} = X(r) \cong 1 + \ln \left( \frac{r}{h} \right)^\delta, \tag{30} \]
Thus the product $e^{2\nu} e^{2\nu}$ is slightly greater than unity as expected. Now, using equation (29) in equation (25) we can express the metric coefficient $e^{2\nu}$ in the present case as

$$e^{2\nu} = \delta \left[ 8 \pi G_e f R^2 \left( \rho + \rho^I + P^I_r \right) \right]^{-1}.$$  

(31)

And hence the coefficient of $g_{00}$ component of the metric can be written as

$$e^{2\mu} = 8 \pi G_e f \delta^{-1} h_{-\delta} r^{\delta+2} \left( \rho + \rho^I + P^I_r \right).$$  

(32)

Later these metric coefficients will be determined in their explicit forms for our considered models of the $f(R, T)$ gravity.

It is observed that most of the baryonic matter of a normal spiral galaxy lies in the flattened disk in the form of stars, dust and interstellar gas. The mass to luminosity ratio of such galaxies increases with increasing distance from the center up to the outer regions of the galaxies [23]. We may assume a spherically symmetric model for baryonic matter distribution in which the normal mass (baryonic) density in halo has a power law density variation given as some power of the radial distance $r$ as [63, 66]

$$\rho(r) = \rho_0 \left( \frac{1}{r} \right)^{\lambda},$$  

(33)

where $\rho_0$ and $\lambda$ are positive constants, $\rho_0$ can be set as unity without loss of generality. The baryonic mass distribution within $r$ is

$$M(r) = 4\pi \int_0^r r^2 \rho(r) \, dr = 4\pi \rho_0 \frac{r^{3-\lambda}}{3-\lambda}.$$  

(34)

It is clear that the pattern of baryonic mass distribution within $r$ depends on the value of the parameter $\lambda$.

IV. THE METRIC COEFFICIENTS FROM THE MODEL OF STUDY

One can perform a study on $f(R, T)$ modified theory by employing different matter-geometry coupling models. The functional $f(R, T)$ in the theory may be modelled through minimally, non-minimally or purely non-minimally coupling of the matter and geometry sectors. In the minimally coupling case the functional form of $f(R, T)$ is assumed in the form of $f(R, T) = q(R) + s(T)$, where $q(R)$ and $s(T)$ are arbitrary functions only of $R$ and $T$ respectively. Whereas in the non-minimal and purely non-minimal cases $f(R, T)$ is assumed to have forms: $f(R, T) = q(R) [1 + s(T)]$ and $f(R, T) = q(R) s(T)$ respectively [47, 58, 64]. In our work, we consider that the matter sector and geometry sector in $f(R, T)$ gravity are coupled minimally and for the fulfilment of our desired result we consider the following minimally coupled model of $f(R, T)$ gravity:

$$f(R, T) = a R - b (-T)^{\beta},$$  

(35)

where $a$ and $b$ are coupling constants that parameterize the departure of the model from GR and the constant parameter $\beta$ determines the strength of matter effect. For the existence of the model, parameters $a$, $b$ and $\beta$ should not be zero. In vacuum, beyond the galactic halo, where $T_{\mu\nu} = 0$, the model reduces to GR. Further, the model is respecting the conservation law also as discussed in Ref. [43, 64]. For this model we have,

$$F_R = a \quad \text{and} \quad F_T = b \beta \rho^\beta - 1.$$  

(36)

Thus using equation (22) the interaction energy density for this model (35) is obtained as

$$\rho^I = \frac{b \rho^\beta}{8 \pi G} \left[ \beta + \frac{1}{2} \right].$$  

(37)

It is immediately clear from equation (37) that the interaction energy density $\rho^I$ is highly model dependent and depends on the positive as well as finite values of $b$ and $\beta$. However, it does not depend on the geometry of spacetime directly for this model. We know that the galactic rotation curves show the linear increase of rotational velocity from the center of the galaxy and then remain constant giving flat curves in the halo region at large distance from the center, where a very low amount of baryonic mass is observed [55, 63, 66]. The interpretation of rotation curves at large distances is generally given by postulating the existence of DM in this region. Here, we have the relation (37), generated by our $f(R, T)$ gravity model (35) that can explain the rotation curves without the need of DM. This would be clear from the graphical analysis of the relations (37). For this we proceed by plotting interaction mass density $\rho^I$ against the baryonic mass density $\rho$ using this relation as shown in Fig. 1. The figure indicates two plausible requirements: (i) $\rho^I \geq 0$ and (ii) $\rho \ll \rho^I$ for the positive values of $b$ and $\beta$. 
In particular the requirement (ii) tells us about the domination of geometrically induced interaction mass density over the baryonic mass density at large distances from the centers of galaxies. The geometrical mass associated with this interaction mass density may be responsible for the constancy of tangential velocity in galactic halo. Furthermore, significant domination of interaction density over the baryonic mass density for larger values of $b$ and $\beta$ is seen from Fig. 1. Also, the radial interaction pressure in the model (35) takes the form:

$$P^I_r = -\frac{b}{16 \pi G} (-T)^\beta.$$  

(38)

Using expressions (37) and (38) for $\rho^I$ and $P^I_r$ respectively, we obtain,

$$\rho^I + P^I_r = \frac{b \beta}{8 \pi G} \rho^\beta.$$  

(39)

From Fig. 1 we may assume that contribution of the baryonic mass density to the total mass density exterior to the galactic disk is very less in comparison to the interaction mass density, consequently equation (31) can be written using equation (39) as

$$e^{2\nu} = \frac{\delta a}{b \beta r^2 \rho^\beta}.$$  

(40)

Accordingly $g_{00}$ component of the metric coefficient will be

$$e^{2\mu} = \frac{b \beta \rho^\beta r^{\delta + 2}}{\delta a h^\delta}.$$  

(41)

Equations (40) and (41) are the explicit forms of the metric coefficients in our specified $f(R, T)$ model (35). In the following section we will find the rotation curve according to variation of $r$ from the galactic center for the model (35).

V. GALACTIC ROTATION CURVES IN A SPHERICALLY SYMMETRIC SPACETIME

A. Orbital motion of a test particle in stable circular orbit

The tangential velocity of a star in an orbit of radius $r$ around the galactic center significantly reflects the matter distribution of the galaxy within it. Therefore, we consider a test particle (a star) moving in a static spherically symmetric spacetime described by the metric (15), which follows a time-like geodesic confined to the equatorial plane, $\theta = \pi/2$. The general geodesic equation that describes the motion of such particles has the following form [67]:

$$\frac{d^2 x^\sigma}{d\tau^2} + \Gamma^\sigma_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0,$$  

(42)
where \( \tau \) is the affine parameter along the geodesic. In such time-like geodesic it represents the proper time. The connection coefficient \( \Gamma^\sigma_{\mu \nu} \) is defined as

\[
\Gamma^\sigma_{\mu \nu} = \frac{1}{2} g^{\sigma \psi} \left[ \frac{\partial g_{\mu \psi}}{\partial x^\nu} + \frac{\partial g_{\nu \psi}}{\partial x^\mu} - \frac{\partial g_{\mu \nu}}{\partial x^\psi} \right].
\] (43)

Motion in the equatorial plane \((\theta = \pi/2 \rightarrow \dot{\theta} = \ddot{\theta} = 0)\) immediately results in three equations of motion for the coordinates \( t, r, \phi \) respectively corresponding to the non-vanishing components of \( \Gamma^\sigma_{\mu \nu} \) as follows:

\[
\frac{d^2 t}{d\tau^2} + 2 \Gamma^0_{01} \frac{dt}{d\tau} \frac{dr}{d\tau} = 0,
\] (44)

\[
\frac{d^2 r}{d\tau^2} + \Gamma^1_{00} \left( \frac{dt}{d\tau} \right)^2 + \Gamma^1_{11} \left( \frac{dr}{d\tau} \right)^2 + \Gamma^1_{33} \left( \frac{d\phi}{d\tau} \right)^2 = 0,
\] (45)

\[
\frac{d^2 \phi}{d\tau^2} + 2 \Gamma^3_{13} \frac{dr}{d\tau} \frac{d\phi}{d\tau} = 0,
\] (46)

where we take \( x^0 = t, x^1 = r, x^2 = \theta \) and \( x^3 = \phi \). For a stable circular orbit, after insertion of non-zero values of the connection coefficients, equations (44) and (46) take the forms respectively as

\[
\frac{d}{d\tau} \left( e^{2\mu} \frac{dt}{d\tau} \right) = 0 \quad \text{and} \quad \frac{d}{d\tau} \left( r^2 \frac{d\phi}{d\tau} \right) = 0.
\]

The integration of above two equations lead two constants of motion along the geodesic, viz. energy, \( E = e^{2\mu} \frac{dt}{d\tau} \) and angular momentum, \( l = r^2 \frac{d\phi}{d\tau} \). But at distances far from the spherically symmetric body, the spacetime is almost flat and the field is weaker. At this weak field limit GR agrees with the Newtonian gravity [68] and the particle moves with a considerably slower speed relative to the speed of light. In this low speed approximation, all the spatial components of particle’s four velocity \( u^k = \frac{dx^k}{d\tau} \), where \( k = 1, 2, 3 \) are dominated by its time component \( u^0 = \frac{dx^0}{d\tau} \) [69]. Thus, for such a sufficiently slow moving particle, \( u^k = \frac{dx^k}{d\tau} \ll \frac{dx^0}{d\tau} \), and the proper time \( \tau \) may be approximated to the coordinate time \( t \) and the four velocity \( \frac{dx^\sigma}{d\tau} (\frac{dx^0}{d\tau}, \frac{dx^1}{d\tau}, \frac{dx^2}{d\tau}, \frac{dx^3}{d\tau}) \) as \((1, 0, 0, 0)\) [70]. With this approximation none of the spatial component of the four velocity will appear in the geodesic equation [70] and hence equation (42) takes the form as

\[
\frac{d^2 x^\sigma}{dt^2} + \Gamma^\sigma_{00} = 0.
\] (47)

Here the connection \( \Gamma^\sigma_{00} \) can be calculated from the metric (15) using the relation (43) as

\[
\Gamma^\sigma_{00} = \frac{1}{2} g^{\sigma \psi} \left[ \frac{\partial g_{0 \psi}}{\partial x^0} + \frac{\partial g_{0 \psi}}{\partial x^0} - \frac{\partial g_{00}}{\partial x^\psi} \right] = -\frac{1}{2} g^{\sigma \psi} \frac{\partial g_{00}}{\partial x^\psi}.
\]

Now, the equation of motion for the radial component will be

\[
\frac{d^2 r}{dt^2} + \Gamma^1_{00} = 0.
\] (48)

In the Newtonian limit, the radial or centripetal acceleration \( a \) of the test particle moving with the tangential (circular) velocity \( v \) in an orbit of radius \( r \) is given as

\[
a = -\frac{v^2}{r}.
\] (49)

Equations (48) and (49) together give \( v^2 = r \Gamma^1_{00} \), where after substitution of corresponding metric elements of the metric (15) we obtain \( \Gamma^1_{00} = \mu e^{2\mu - 2v} \) and hence

\[
v^2 = r \mu e^{2\mu - 2v}.
\] (50)

This equation relates the tangential velocity of a test particle to the radial coordinate \( r \), the \( g_{00} \) and \( g_{11} \) components of the metric (15).
The metric coefficients $e^{2\nu}$ and $e^{2\mu}$ in terms of $r$ using equation (33) can be rewritten from equations (40) and (41) in the following forms:

$$e^{2\nu} = \frac{\delta a}{b \beta r^{\lambda \beta + 2}} \quad \text{and} \quad e^{2\mu} = \frac{b \beta r^{1-\lambda \beta + \delta}}{\delta a h^2}.$$

(51)

Next, we have to find $\mu'$ to express the tangential velocity of the particle in terms of the model parameters. It may be obtained from the differentiation of the second relation of (51) with respect to $r$ as

$$\mu' = \frac{1}{2 e^{2\nu}} \left[ \frac{b \beta (2 + \delta - \lambda \beta) r^{1-\lambda \beta + \delta}}{h^2 a \delta} \right].$$

(52)

Thus, insertion of relations (51) and equation (52) in equation (50) we get the tangential velocity of a test particle rotating around a galaxy in terms of radial coordinate $r$ for our assumed $f(R, T)$ gravity model (35) as

$$v^2 = \frac{b^2 \beta^2 (2 + \delta - \lambda \beta) r^{4+\delta - 2\lambda \beta}}{2 h^3 a^2 \delta^2}.$$

(53)

Relation (53) is the main equation for fulfilment of the aim of the study. In the following we will compare this result with some observed rotation velocity data to test the viability of the model considered for the study.

### Table I: Sample of observed Galaxies.

| Galaxy Name | Type | Distance ($D$) (Mpc) | Luminosity $L_B$ | Scale length ($h$) (kpc) | Data Sources |
|-------------|------|----------------------|-----------------|------------------------|-------------|
| F 583-1     | LSB  | 35.4                 | 0.064           | 1.6                    | [71] [71]   |
| DDO 154     | LBS  | 4.04                 | 0.008           | 0.54                   | [71] [71]   |
| UGC 128     | LSB  | 64.5                 | 0.597           | 6.9                    | [71] [71]   |
| UGC 1230    | LSB  | 53.7                 | 0.366           | 4.7                    | [71] [71]   |
| UGC 1281    | LSB  | 5.1                  | 0.017           | 1.6                    | [71] [37]   |
| UGC 6446    | LSB  | 12.0                 | 0.263           | 1.9                    | [71] [71]   |
| NGC 0247    | LSB  | 3.7                  | 0.512           | 4.2                    | [71] [71]   |
| NGC 0300    | LSB  | 2.08                 | 0.271           | 2.1                    | [71] [71]   |
| NGC 1003    | LSB  | 11.4                 | 1.480           | 1.9                    | [71] [71]   |
| NGC 2976    | LSB  | 3.58                 | 0.201           | 1.2                    | [71] [71]   |
| NGC 2403    | HSB  | 3.16                 | 1.647           | 2.7                    | [71] [71]   |
| NGC 2998    | HSB  | 68.1                 | 5.186           | 4.8                    | [71] [71]   |
| NGC 3198    | HSB  | 13.8                 | 3.241           | 4.0                    | [71] [71]   |
| NGC 3521    | HSB  | 7.7                  | 2.048           | 3.3                    | [71] [71]   |
| UGC 3580    | HSB  | 20.7                 | 10.12           | 2.4                    | [71] [71]   |
| NGC 4088    | HSB  | 18.0                 | 2.957           | 2.8                    | [71] [71]   |
| NGC 4183    | HSB  | 18.0                 | 1.042           | 2.9                    | [71] [71]   |
| NGC 5585    | HSB  | 7.1                  | 0.333           | 2.0                    | [71] [71]   |
| NGC 7793    | HSB  | 3.61                 | 0.910           | 1.7                    | [71] [71]   |

LSB → Low surface brightness; HSB → High surface brightness.

### B. Fitting of Rotation Curve

We have generated the rotation curves (see Figs. 2 and 3) for different values of the model parameters according to equation (53) for the test particle moving around the galactic center and test the model by fitting the predicted results to observational data...
of a sample of 19 galaxies [71], 10 LSB and 9 HSB, listed in Table I with the data extracted from the references indicated in the table. As different types of galaxies exhibit separate morphologies, we chose these few galaxies as typical galaxies to represent the morphologies of similar kinds of other galaxies. In Table I, data of the adopted distance $D$ of sample galaxies measured in the unit of Mpc, the B-band luminosity $L_B$ and scale length $h$ in kpc are also listed. It is seen that for all samples the predicted rotation curves are almost in good agreement with observed data showing well fitted results between the model and observations. The reduced $\chi^2 (\chi^2_{red})$ value of fitting for each selected galaxy is calculated and indicated in the table Table II. Eventually $\chi^2_{red}$ values for galaxies DDO 154, NGC 2403 and NGC 5585 are found to be large (≈ 11.1, 8.5 and 6.6 respectively) and for galaxies UGC 128, NGC 0247, NGC 1003, UGC 3580, NGC 2998, UGC 1290 are high around 3.6, 3.4, 3.03, 2.5, 2.2, 2.1 respectively. But the shape of rotation curves are not affected by these large or high $\chi^2_{red}$ values, which are depicted in Figs. 2 and 3. Table II also shows the best fitted values of model parameters for each sample galaxy and also the value of $\delta$, which must take a very small value as mentioned after the equation (27). For all well fitted curves the values of $\delta$ are obtained in the order of $10^{-6} – 10^{-5}$. Values of parameters $b$ and $\beta$ are not too large, $a$ lies in 700 – 800 and the product $\lambda \beta$ is in between 1.28 and 2. Both the HSB galaxies which have steeply rising rotation curves and the LSB galaxies which are supposed to be DM dominated at small radii are seen to be well fitted in the model considered for the study.

| Galaxy Name | $a$  | $b$  | $\lambda$ | $\delta$ | $\beta$ | $\lambda \beta$ | $\chi^2_{red}$ |
|-------------|-----|-----|---------|--------|------|----------|---------|
| F583-1      | 710 | 0.46| 2.0     | $9 \times 10^{-6}$ | 0.730 | 1.46     | 0.37    |
| DDO 154     | 750 | 0.40| 1.95    | $8 \times 10^{-6}$ | 0.800 | 1.56     | 11.1    |
| UGC 128     | 700 | 0.82| 2.0     | $8 \times 10^{-6}$ | 0.840 | 1.68     | 3.6     |
| UGC 1230    | 800 | 1.50| 2.0     | $7 \times 10^{-6}$ | 0.920 | 1.84     | 2.1     |
| UGC 1281    | 700 | 0.20| 2.0     | $4 \times 10^{-6}$ | 0.640 | 1.28     | 0.28    |
| UGC 6446    | 730 | 0.89| 2.0     | $7 \times 10^{-6}$ | 0.890 | 1.78     | 0.58    |
| NGC 0247    | 800 | 0.40| 2.0     | $5 \times 10^{-6}$ | 0.790 | 1.58     | 3.4     |
| NGC 0300    | 790 | 0.80| 2.0     | $7 \times 10^{-6}$ | 0.850 | 1.70     | 1.4     |
| NGC 1003    | 710 | 1.15| 2.0     | $9 \times 10^{-6}$ | 0.890 | 1.78     | 3.03    |
| NGC 2403    | 798 | 2.30| 2.0     | $9.2 \times 10^{-6}$ | 0.917 | 1.83     | 8.5     |
| NGC 2998    | 720 | 3.80| 2.0     | $8 \times 10^{-6}$ | 0.940 | 1.88     | 2.2     |
| NGC 2976    | 700 | 1.58| 2.0     | $9 \times 10^{-6}$ | 0.870 | 1.74     | 0.94    |
| NGC 3198    | 700 | 0.82| 2.0     | $8 \times 10^{-6}$ | 0.760 | 1.52     | 1.13    |
| NGC 3521    | 710 | 5.60| 2.0     | $8 \times 10^{-6}$ | 0.962 | 1.92     | 0.95    |
| UGC 3580    | 700 | 1.30| 2.0     | $8 \times 10^{-6}$ | 0.895 | 1.79     | 2.5     |
| NGC 4088    | 720 | 1.50| 2.0     | $9 \times 10^{-6}$ | 0.830 | 1.66     | 0.92    |
| NGC 4183    | 710 | 1.30| 2.0     | $9 \times 10^{-6}$ | 0.891 | 1.78     | 1.8     |
| NGC 5585    | 720 | 0.68| 2.0     | $8 \times 10^{-6}$ | 0.820 | 1.64     | 6.6     |
| NGC 7793    | 720 | 0.88| 2.0     | $8 \times 10^{-6}$ | 0.760 | 1.52     | 0.56    |

**VI. CONCLUSIONS**

MTGs provide alternative gravitational approaches in contrast to the GR to understand and determine the flatness of rotation curves corresponding to stars that are moving around the galactic nucleus at far distances away from it. Thus MTGs can be used to give the geometric interpretation of the so-called DM. In this present work, we have developed such an approach to address the DM problem by considering the $f(R, T)$ modified theory of gravity especially in the galactic scale and emphasis is given to the possibility for elucidating the rotation curves of spiral galaxies without considering the need for the exotic DM. Here, we have employed a minimally coupled $f(R, T)$ gravity model of the form $af(R) + bf(T)$, which could explain the additional...
FIG. 2: Fitting of rotation curves generated from the model (35) to rotation velocities of samples of 10 LSB galaxies with their quoted errors. The data points are observational values of rotational velocities extracted from Ref. [71].

matter content needed for the observational agreement with the rotation curves of galaxies. In fact, the extra terms appearing in the field equations of the $f(R, T)$ gravity theory may be treated as an extra fluid, whose source is due to the coupling between
FIG. 3: Fitting of rotation curves generated from the model (35) to rotation velocities of samples of 9 HSB galaxies with their quoted errors. The data points are observational values of rotational velocities extracted from Ref. [71].

Assuming the first fluid (the physical matter fluid) in the two fluid structure of $f(R, T)$ gravity theory as a dust-like perfect fluid, the modified Einstein equations have been solved and subsequently obtained the metric coefficients for a static spherically symmetric spacetime in the vicinity of GR. We have fixed the metric coefficients in terms of model parameters and derived tangential velocity relation for the test particle moving around the galaxy. The rotation curves generated by the relation (53) are nicely comparable to the observational data and they constrain $\delta$ to the order of $10^{-6} - 10^{-5}$. After constraining the model parameters through fitting of theoretical and observed rotational curves of a sample of nineteen galaxies (10 LSB and 9 HSB), the best fit range of parameter $\beta$, that determines the strength of matter is found as $0.63 < \beta < 0.97$ and the product of $\lambda \beta$ is $< 2$. Similarly, the parameter $a$ which determines the curvature coupling is found to lie within the values of $700 - 800$ and the matter coupling parameter $b$ is found to remain within the range $0.2 - 5.6$. With these values of parameters our $f(R, T)$ model (35) explains the behavior of rotation curves of the selected sample of galaxies well. Further, in conformity to be with the observed data on the galactic rotation curves this result suggests that the parameter $\delta$ must be a considerably smaller one (of the order of $10^{-6} - 10^{-5}$) by confirming our assumption that $X(r)$ should slightly be different from the unity.

Indeed, our specified model results in more or less the constant tangential velocity of a test particle needed for fitting observed rotation curves far away from the galactic center. The flatness behavior of rotation curves suggests a considerable amount of
hidden matter in galactic halo and in other way, the $f(R, T)$ model we have considered leads to a desired result about the rotation curve through the modification of gravity. It therefore nullifies the presence of DM within the halo. In this study, no attempt has been made to include the Tully-Fisher relation in our work. We plan to do this in future with an attempt to extend the work choosing a non-minimally coupled $f(R, T)$ model by constraining the model parameters with the available observational data.

Acknowledgments

UDG is thankful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for the Visiting Associateship of the institute.
