Comments on D-branes in flux backgrounds

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Abstract

We construct supergravity solutions for D-branes in nontrivial flux backgrounds. We revisit the issue of charge quantization in this framework, and show that in these backgrounds, charge need not be quantized. We also show in a particular example that the semiclassical description of branes produces integral charges.

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1 Introduction

If monopoles are present in a theory, electric charge is quantized. This observation of Dirac is fundamental to any quantum-mechanical theory. Yet there appears to be a violation of this basic principle in a class of string theories.

The apparent violation occurs for D-branes in the presence of a background NSNS three-form flux. A variety of arguments show that in these theories, D-branes have a charge of the form $\sin(\pi j/k)$, where $j, k$ are integral. In this paper, we shall reexamine the arguments for this strange behaviour, and show how they are in fact compatible with Dirac’s argument. Other proposals have been made to resolve this problem: we discuss the connection of our solution to these other suggestions.

We first describe the boundary state construction of D-branes on $S^3$, where this behaviour was first noted. A semiclassical description of these branes was given in [2] and it was argued that the semiclassical description also produced nonquantized charges. Taylor [3] later argued that in fact, terms not considered by [2] made the charge integral. We review all these arguments in the following sections.

We then construct a supergravity description of such branes, which provides a complementary approach. We show that all the features of the semiclassical description are reproduced by the supergravity solution. The supergravity approach in general also allows us to generalize to the case of RR backgrounds, which cannot be treated in the boundary state approach.

In this paper, we focus on the case of M-branes in a $S^4$ background, which as we show displays all the features of D-branes in $S^3$. The supergravity description tells us the field configuration outside the M-branes. The charge of the M-brane is associated with a massless gauge field $A$ whose long distance falloff is governed by the harmonic equation

$$\frac{1}{\sqrt{G}}\partial_{A}(\sqrt{G}G^{AB}\partial_{B}A) = 0 \quad (1)$$

Here $G$ is the asymptotic metric.

The falloff therefore crucially depends on the form of the asymptotic geometry. It turns out that the falloff is much faster in the case of asymptotically $AdS$ geometries as compared to asymptotically flat geometries. Thus there is no long range interaction between a fundamental charge and a monopole in asymptotically $AdS$ spaces, and the Dirac argument does not lead to charge quantization. This is very similar to the proposal of [2] for resolving the charge quantization issue. However, we also argue that in fact, the semiclassical description produces quantized charges, in agreement with Taylor [3].

We close with a discussion of related issues. We show that deriving the flat space S-matrix from the AdS/CFT correspondence must deal with issues raised by the existence of the fractionally charged states. Also we comment on the issue of charge conservation in these theories.
2 The boundary state construction

D-branes in flat space are described as endpoints of open strings. In more general theories, it is necessary to use the more abstract language of boundary states to describe them. In this language, D-branes are described in terms of their overlap amplitudes with closed string states. The connection to open strings is made through Cardy’s condition described below.

Boundary states are typically hard to construct, but the special case of D-branes in \( AdS_3 \times S^3 \) with background NS-NS flux can be analyzed exactly, since the \( S^3 \) factor can be represented by the exactly solvable \( SU(2) \) WZW model. Boundary states in this theory were constructed in [1]. We review this construction here.

One first tries to solve the equation

\[
(J^a + J^a) |I\rangle = 0 \tag{2}
\]

The solutions to this equation are the Ishibashi states \(|I_n\rangle\), which are constructed by taking any primary \(|\phi_n\rangle\) and summing the normalized states in the Verma module. More precisely, start with a primary \(|\phi^j\rangle\) (by this we mean a state which is annihilated by all lowering operators \( J^a_n \), \( n > 0 \)). Define the Ishibashi state

\[
|I^j\rangle = M_{IJ}^{-1} J_{J^-} J_{I^-} |\phi^j\rangle \tag{3}
\]

Here \( I, J \) are ordered strings of indices \( (n_1, a_1) \ldots (n_r, a_r) \) and \( J_I = J_{n_1}^{a_1} \ldots J_{n_r}^{a_r} \). The normalization is defined by

\[
M_{IJ} = \langle \phi^j | J_I J_{-I} | \phi^j \rangle \tag{4}
\]

\( M_{IJ} \) is invertible for any module. It is straightforward to show that the states (3) satisfy the equation (2).

There is therefore a 1-1 correspondence between the Ishibashi states and the primaries. In the \( SU(2) \) WZW theory at level \( k \), the primaries are labelled by an index \( j \) that runs from 0 to \( k \). Hence there are \( k + 1 \) Ishibashi states.

One then constructs Cardy states, which are linear combinations of Ishibashi states satisfying Cardy’s condition. Cardy’s condition is that the modular transform of the overlap of any two boundary states should be interpretable as an open string partition function. This ensures that the D-branes can be reinterpreted as endpoints of open strings.

The Cardy states in \( SU(2) \) WZW theory are found in terms of the S-matrix \( S_0 \) of the theory to be

\[
|B_{ij} \rangle = \sum_n \frac{S_n^0}{\sqrt{S_0}} |I^n\rangle \tag{5}
\]
The charge of the state $|B_j\rangle$ is given by its overlap with the lowest primary, and is hence proportional to

$$Q_j = \frac{S^0_j}{\sqrt{S^0_0}} \propto \sin \frac{\pi(2j + 1)}{k + 2}$$

(6)

The charge of the brane labelled by the integer $j$ is therefore not an integer multiple of the brane labelled by $j = 1$. Hence the charges are not quantized.

3 The semiclassical approach

We now want to turn to a semiclassical description of these branes, following [2].

The boundary states that we have described satisfy $(J^a + \bar{J}^a)|B\rangle = 0$. It was shown in [10] that the semiclassical description of such branes is that of D2-branes wrapping conjugacy classes in $S^3$. This motivates the study of the dynamics of branes wrapping such conjugacy classes. As we now see, the phenomenon of fractional charges can be reproduced by this calculation, as shown by Bachas, Douglas, and Schweigert [2].

The background metric of the $S^3$ is taken to be

$$ds^2 = k\alpha'(d\theta^2 + \sin^2\theta d\chi^2 + \sin^2\theta \sin^2\chi d\phi^2)$$

(7)

and the three form flux is

$$H_{\theta\chi\phi} = 2k\alpha' \sin^2\theta \sin\chi$$

(8)

The conjugacy classes in $S^3$ are 2-spheres. Examples of such conjugacy classes are the hypersurfaces $\theta = \text{constant}$. We should therefore consider the dynamics of a D2-brane wrapped on such a hypersurface.

The brane interacts with the background curvature and 3-form flux through the Born-Infeld action

$$S = T \int d^3\sigma \sqrt{\det(G_{ab} + B_{ab} + F_{ab})}$$

(9)

Here $G_{ab} = G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$ and $B_{ab} = B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$ are respectively the pullbacks of the spacetime metric and two form field to the D-brane worldvolume. $F_{ab}$ is the worldvolume field strength.

In general, we can take the D2-brane to have a nontrivial gauge field on its worldvolume. Flux quantization implies that $\int d\sigma^\mu d\sigma^\nu F_{\mu\nu} = n$ with $n$ integral.

For a brane wrapped on the conjugacy class $\theta = \text{constant}$, we can choose a static gauge of the form

$$\sigma^0 = t \quad \sigma^1 = \phi \quad \sigma^2 = \chi$$

(10)
We then find an energy functional for the field $\theta(\sigma^0, \sigma^1, \sigma^2)$, which is

$$E_n(\theta) = 4\pi k' T \left( \sin^4 \theta + \left( \theta - \frac{\sin 2\theta}{2} - \frac{\pi n}{k} \right)^2 \right)^{1/2}$$  (11)

The equation of motion has a static solution for $\theta = \frac{\pi n}{k}$. Therefore, there are stable solutions where the 2-brane wraps a hypersurface of constant $\theta = \frac{\pi n}{k}$ for any $n$.

It can further be shown that these branes are BPS, and have a mass and induced D0-brane charge which is proportional to $\sin\left(\frac{2\pi n}{k}\right)$. We refer to [2] for details. Hence we again find the phenomenon of nonquantized charges.

### 3.1 An objection

It is however rather difficult to conclude on the basis of this semiclassical computation that the brane charge is fractional. This was pointed out by Taylor [3], who suggested that bulk interactions could compensate for the nonintegral charge, by providing another potential source for the brane charge.

Let us denote the gauge field coupling to the D0-brane charge as $A_\mu$ and the gauge field coupling to the D2-brane charge as $C_{\mu\nu\rho}$. Furthermore, denote the field strength corresponding to $C_{\mu\nu\rho}$ by $F_{\mu\nu\rho\sigma}$. There is a term in the type II string action of the form $F_{\mu\nu\rho\sigma} A^\mu H^{\nu\rho\sigma}$, where $H^{\nu\rho\sigma}$ is the NS-NS flux. The equation of motion for $A_\mu$ therefore contains an extra contribution proportional to $F_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$. Now the presence of the D2-brane implies that $F_{\mu\nu\rho\sigma}$ must be nonzero, and of course we have a nontrivial NSNS flux in the background. So this contribution is nonzero, and presumably should be added to the charge obtained by the semiclassical computation in the previous subsection.

Taylor further argued in [3] that the total charge obtained by this procedure was indeed integral (see also [11]). This might suggest that there is no issue here; the charges of all the branes are in fact integral. However, this cannot be the complete solution. The boundary state construction is exact in string theory; in other words, it sums up all the corrections including the supergravity interactions mentioned above. Therefore since the boundary state has a fractional charge, the total charge including all bulk interactions is fractional, whether or not one trusts the semiclassical computation of [3]. We shall return to this issue in a later section.

### 4 The supergravity description

#### 4.1 Notation

We now turn to the supergravity description of these branes.
There are several reasons why such a description is useful. The supergravity approach is complementary to the boundary state and the semiclassical approaches, since it can be applied to cases where the boundary state cannot at present be constructed, and to cases where NS5-branes or M-branes are involved. We will also be able to address the issue of charge quantization in this approach.

Now, the brane in $AdS_3 \times S^3$ is a special case of a brane in the background of another brane or set of branes. The supergravity solution for this brane should be a special case of the more general case of supergravity solutions for intersecting branes. The method for constructing such solutions for brane intersections was given in [4, 5]; we will review this construction below. We shall then show that we can indeed find expanded brane solutions, similar to those of the previous sections, as a special case of these solutions.

For concreteness we will examine a special case where an M2-brane is ending on a set of M5-branes. We shall orient the M5-branes along the $x_0, x_1, x_2, x_3, x_4, x_5$ directions, and the M2-branes will be oriented along the $x_0, x_1, x_r$ directions and end on the M5-brane.

For notational purposes, we shall label the coordinates $x_0, x_1$ by $x_i$. The coordinates $x_2, x_3, x_4, x_5$ will collectively be labelled $x_a$, and the coordinates $x_7, x_8, x_9, x_{10}$ will collectively be labelled $x_\alpha$.

In the limit when the number $N$ of M5-branes is large, the supergravity solution for the M5-branes goes over to the metric

$$
\text{ds}^2 \equiv G_{\mu\nu}dX^\mu dX^\nu = \\
= \frac{r}{N^{1/3}} \left( -dx_0^2 + dx_1^2 + dx_a^2 \right) + \frac{N^{2/3}}{r^2} \left( dx_0^2 + dx_a^2 \right) \\
= \frac{r}{N^{1/3}} \left( -dx_0^2 + dx_1^2 + dx_a^2 \right) + \frac{N^{2/3}}{r^2} \left( dr^2 + N^{2/3}d\Omega_4^2 \right)
$$

which is the metric of $AdS_7 \times S^4$. Here $d\Omega_4^2$ is the metric of $S^4$.

The M2-branes (which are oriented along $x_0, x_1, x_r$) in this new metric are oriented along $x_0, x_1, x_r$ and occupy a point in the $S^4$.

The considerations of [2] suggest that the semiclassical description of the M2-branes should actually be that of M5-branes oriented along $x_0, x_1, x_r$ and wrapping a $S^3$ hypersurface in $S^4$. To stabilize this extended brane configuration, there should be a nontrivial worldvolume flux on the M5-brane. (Just as a nontrivial 2-form field strength can stabilize a brane wrapped on $S^2$, a nontrivial 3-form field strength can stabilize the M5-brane wrapped on $S^3$. ) We could therefore perform a semiclassical calculation analogous to to verify this. Instead, we will explicitly derive this from the supergravity description of the branes.

We should therefore look at the supergravity solution for M2-branes ending on M5-branes. The general approach to constructing such solutions was given in [4, 5]. One is attempting to find solutions to the Killing spinor equations,
which for instance in the case of 11-D supergravity, is
\[
\partial_\mu \epsilon - \frac{1}{4} \omega^{ab} \gamma_{ab} \epsilon + \frac{i}{288} (\gamma^a_{\mu} - 8 \omega^a_{\mu} \gamma^b) G_{abcd} \epsilon = 0 \quad (13)
\]
Here \(G_{abcd}\) is the four-form field strength of 11-dimensional supergravity.

4.2 Ansatz for the Killing equation

We will solve the Killing equation by assuming an ansatz for the Killing spinor. We take the ansatz for the Killing spinor to be
\[
\epsilon = \left( g_{00} \right)^{1/4} \epsilon_0 \quad (14)
\] where \(\epsilon_0\) is a constant spinor.

Furthermore, there are constraints on the constant spinor \(\epsilon_0\) since the supersymmetry is partly broken. Each brane imposes a constraint on the spinor \(\epsilon_0\). The way this happens is that each brane is associated with a projector \(P_i\) satisfying \(P_i * P_i = P_i\). For example the presence of a M5-brane oriented along \(x_0, x_1, x_2, x_3, x_4, x_5\) imposes the equation \(P_1 \epsilon_0 = 0\) where \(P_1 = (1 + i \gamma_{67910})/2\). Similarly the M2-brane oriented along the \(x_0, x_1, x_6\) imposes the equation \(P_2 \epsilon_0 = 0\) where \(P_2 = (1 + i \gamma_{016})/2\).

For a system containing several branes we should impose all the separate projection equations on \(\epsilon_0\). In the case we are considering, we therefore take the constraints on the spinor \(\epsilon_0\) to be \(P_1 \epsilon_0 = P_2 \epsilon_0 = 0\).

After imposing all these constraints, the Killing spinor equations reduce to a set of algebraic equations, which can be used to solve for the field strengths and the metric.

4.3 Solution

For the case of M2-branes ending on M5-branes, the solution was found in [4]. The metric was found to be of the form
\[
 ds^2 = \lambda^{-2/3} H^{-1/3} (-dx_0^2 + dx_1^2) + \lambda^{1/3} H^{-1/3} dx_a^2 \\
 + \lambda^{-2/3} H^{2/3} (dx_6 + \phi_a dx^a)^2 + \lambda^{1/3} H^{2/3} dx_a^2 \quad (15)
\]
where we must impose the constraint
\[
 \partial_6 (H \phi_a) = \partial_a H \quad (16)
\]
The field strengths are given by (all the indices are world indices)
\[
 G_{0167} = \partial_7 \left( \frac{1}{\lambda} \right) \\quad G_{0147} = \partial_7 \left( \frac{\phi_4}{\lambda} \right) \quad (17)
\]
\[
 G_{0164} = \partial_4 \left( \frac{1}{\lambda} \right) - \partial_6 \left( \frac{\phi_4}{\lambda} \right) 
\]
\[ G_{2345} = H^{-1} \partial_6 \lambda + \phi_a \partial_6 \phi_a \quad G_{2357} = -\partial_7 \phi_4 \] \hfill (18)

\[ G_{2356} = -\partial_6 \phi_4 \quad G_{2357} = -\partial_6 \phi_4 \] \hfill (19)

\[ G_{78910} = \partial_6 (H \lambda) \quad G_{68910} = -\partial_7 \partial_6 \phi_4 \] \hfill (20)

Hence there are two independent functions \( \lambda, H \) describing the solution. This is as it should be since we have two types of branes.

5 Interpretation of the solution

5.1 Source terms

What is the physical meaning of this solution? To answer this question, we must look at the various sources, which can be found by looking at the Bianchi identities and the equations of motion.

We first define the quantities \( P, Q \) through

\[ P = -\partial_6 \phi_4 + H^{-1} \partial_6 \lambda + \phi_a \partial_6 \phi_a \] \hfill (20)

and

\[ \partial_0 Q = \partial_6^2 (H \lambda) + \partial_0^2 H \] \hfill (21)

\( Q \) also satisfies

\[ \partial_a Q = \partial_6 \partial_a (H \lambda) + \partial_0 \partial_a (H \phi_4) \] \hfill (22)

The nontrivial Bianchi identities are then found to be

\[ \partial_6 G_{2345} + \text{cyclic} = \partial_6 P \] \hfill (23)

\[ \partial_7 G_{2345} + \text{cyclic} = \partial_7 P \] \hfill (24)

\[ \partial_4 G_{78910} + \text{cyclic} = \partial_4 Q \] \hfill (25)

\[ \partial_6 G_{78910} + \text{cyclic} = \partial_6 Q \] \hfill (26)

while the nontrivial equations of motion are

\[ \partial_A (\sqrt{g} G^{A235}) = - \frac{1}{2(24)^2} \epsilon^{235abcdefg} G_{abcd} G_{efgh} = \frac{1}{\lambda} (\partial_4 Q - \phi_4 \partial_6 Q) \] \hfill (27)

\[ \partial_A (\sqrt{g} G^{A8910}) = - \frac{1}{2(24)^2} \epsilon^{8910abcdefg} G_{abcd} G_{efgh} = \frac{1}{\lambda} \partial_7 P \] \hfill (28)

\[ \partial_A (\sqrt{g} G^{A017}) = - \frac{1}{2(24)^2} \epsilon^{017abcdefg} G_{abcd} G_{efgh} = -H \partial_7 P \] \hfill (29)

\[ \partial_A (\sqrt{g} G^{A914}) = - \frac{1}{2(24)^2} \epsilon^{914abcdefg} G_{abcd} G_{efgh} = \phi_4 \partial_6 Q - \partial_4 Q \] \hfill (30)
\[
\partial_A(\sqrt{g}G^{A016}) + \frac{1}{2.(24)^2} \varepsilon^{016abcdefgh} G_{abcdefg} G_{efgh} = P \partial_6(H) - \phi_a \partial_a Q + \int (dx_6)(\partial_6^2 Q - \partial_a^2 (HP)) \tag{31}
\]

The RHS of the Bianchi identities and the equations of motion should be interpreted as sources like M5-branes and M2-branes.

For instance if there are no M2-branes, and \(N\) M5-branes oriented along \(x_1, x_2, x_3, x_4, x_5\), then only the RHS of (26) should be nonzero. In this case we have

\[
\partial_6 Q = N \delta(x_6) \delta(x_{\alpha}) \quad P = \phi_4 = 0 \quad \lambda = 1 \tag{32}
\]

### 5.2 Interpretation of sources

We now present the interpretation of these sources in the more general case when we have both M5-branes and M2-branes. In this more general case, both \(P\) and \(Q\) are nonzero, and this produces source terms for all the Bianchi identities and the equations of motion. To simplify the analysis, we will focus on the limit where the number \(n_2\) of M2-branes is much smaller than the number \(N\) of M5-branes, so that the picture of branes in the metric (12) is still valid.

Some of the sources are easy to interpret. As we have already seen, the RHS of (26) is the density of M5-branes oriented along \(x_0, x_1, x_2, x_3, x_4, x_5\). Similarly the RHS of (31) is the density of M2-branes oriented along \(x_0, x_1, x_6\).

However, it would seem strange to interpret the source term on the RHS of (30) as the density of M2-branes oriented along \(x_0, x_1, x_4\). Luckily, there is a better interpretation. There is a 2-form gauge field on the M5-brane worldvolume, which couples to the spacetime gauge fields through the term \(\int d^6 \sigma A^{(3)} h^{(3)}\). A nonzero flux \(h_{014}\) thus provides a source for the field \(A^{(3)}\) i.e. we can set the right hand of (30) to be

\[
h_{014} = \phi_4 \partial_6 Q - \partial_4 Q \tag{33}
\]

Similarly the right hand side of (27) can be identified with \(h_{235}\), which is equal to \(h_{014}\) since the 3-form is self dual.

The source terms on the RHS of (23,24) are very interesting. The source in (23) has a natural interpretation as M5-branes oriented along \(x_0, x_1, x_7, x_8, x_9, x_{10}\). Similarly, the source in (24) has a natural interpretation as M5-branes oriented along \(x_0, x_1, x_6, x_8, x_9, x_{10}\).

Such sources represent a M5-brane occupying a hypersurface in the \(x_0, x_1, x_6, x_7, x_8, x_9, x_{10}\) directions. If we go to the coordinates in the metric (12), we see that the brane occupies the \(x_0, x_1\) directions and traces a hypersurface in the \(x_r \times S^4\) directions. If we wish to ensure that the brane occupies a hypersurface in \(S^4\) alone, we must take

\[
r \partial_r P = x_6 \partial_6 + x_\alpha \partial_\alpha P = 0 \tag{34}
\]
The brane now occupies the $x_0, x_1, x_r$ directions and a $\theta = \theta_0$ hypersurface. This provides a connection between our solution and the semiclassical description of section (3).

Remarkably, the correspondence can be made even more explicit. A flux $h_{8910}$ on the worldvolume of this brane produces the source in equation (28), and its self-dual partner sources equation (29). In the metric (12), we find that this is exactly a constant worldvolume field strength on the M5-brane. This makes the correspondence with section (3) precise.

The remaining source terms can be understood as deformations of the M5-branes.

We thus see that the branes discussed in [2] represent a special subclass of the general supergravity solution given above.

5.3 Long distance behaviour

Now we look at the long distance behaviour of the field configuration. This requires the usual assumption that the sources are localized in a finite region. Hence in the long distance region, we can set $P = Q = 0$.

We will expand around the background geometry of the M5-brane

$$H = H_0 \quad \lambda = 1 \quad \phi_a = 0$$

To leading order the perturbations

$$\delta H = H - H_0 \quad \delta \lambda = \lambda - 1 \quad \delta \phi_a = \phi_a$$

satisfy the system of equations (16, 20, 21) with $P = Q = 0$. The solution to this system of equations is

$$\delta H = \partial_6^2 \tau \quad H_0 \phi_a = \partial_6 \partial_a \tau \quad \delta \lambda = \partial_6^2 \tau$$

where $\tau$ satisfies

$$(\partial_6^2 + \partial_a^2 + H_0 \partial_a^2) \tau = 0$$

This is more recognizable as the harmonic equation

$$\frac{1}{\sqrt{G}} \partial_A (\sqrt{G} G^{AB} \partial_B \tau) = 0$$

where $G$ is the unperturbed metric for the M5-branes defined in (12).

Hence $\tau$ satisfies the equation for a massless field in $AdS_7 \times S^4$. (Note that this is different from [2], where it was suggested that $\tau$ was massive.)

This is the solution far away from the branes. More generally, the sources $P, Q$ will appear on the RHS of equation (38). The general solution for $\tau$ is then

$$\tau = \int dx' H(x, x') \rho(x')$$
where $H(x, x')$ is the Green’s function satisfying

$$\frac{1}{\sqrt{G}} \partial_A (\sqrt{G} G^{AB} \partial_B H(x, x')) = \delta(x - x') \quad (41)$$

and $\rho(x')$ is a source density.

6 Discussion

6.1 Charge quantization

We now examine the behaviour of $\tau$. On shorter distance scales, much less than the radius of the $AdS_7$, the space is locally flat (if $N > 1$) and so $\tau$ has the usual flat space behaviour.

For distances larger than the radius of the $AdS_7$, the curvature becomes important. The long-distance behaviour depends on the form of $H_0$. If $H_0 \to 1$ at long distances, then asymptotically $\tau$ again has the flat space behaviour.

But we can also consider the near-horizon limit of the M5-brane, where $H_0 = \frac{r^3}{\sqrt{r r_0^2 - (x_a - x_0^a)^2}}$. Then the curvature does not vanish asymptotically, and the equation for $\tau$ does not go over to the flat space equation.

We would like to understand the long distance behaviour of $\tau$ in this case. Unfortunately, in this case, we cannot solve for the Green’s function explicitly. We instead note that the equation (39) is now the harmonic equation for a massless field in $AdS_7 \times S^4$. We can therefore expand the fields in harmonics on the $S^4$.

The higher harmonics are massive and hence fall off exponentially in $AdS_7$. The long distance behaviour is therefore dominated by the lowest harmonic, and hence we only need to consider the lowest harmonic on the $S^4$. Note that this is similar to the discussion in section 2, where the charge is defined as the overlap of the boundary state with the lowest harmonic.

This harmonic (which we denote $\tau_0$) satisfies the equation

$$(y^2 + 2y)\partial_y^2 \tau_0 + (7y + 7)\partial_y \tau_0 = 0 \quad (42)$$

where we have defined

$$y = \frac{\sqrt{r r_0}}{2} \left( \frac{(\sqrt{r} - \sqrt{r_0})^2}{r r_0} + \frac{(x^a - x_0^a)^2}{4N} \right) \quad (43)$$

Here $x_0^a, r_0$ are constants labelling the position of the branes.

We see that for large $y$ the solution has the behaviour $\tau_0 \sim y^{-6}$, while for small $y$, the solution has the behaviour $\tau_0 \sim y^{-5/2}$. Clearly the falloff is much steeper for large $y$.

Note that for small $y$, the falloff is appropriate for a massless field in 6+1 dimensions. (This is consistent with the idea that for short distances $AdS_7$
resembles $R^{6,1})$. For large $y$ the falloff is faster, so $\tau$ behaves as if it has an effective mass.

The long distance behaviour of the gauge fields is controlled by the behaviour of $\tau$ through the supergravity solution. Since $\tau$ has a steep falloff at long distances, we correspondingly find a steep falloff of the gauge field. In effect, the massless gauge field appears to get an effective mass due to the background curvature. Due to this behaviour, we find that there is no requirement of charge quantization for asymptotically $AdS$ spaces. The fields fall off too quickly to have a long distance interaction. This explanation is similar to that of [2].

To summarize, in the case of asymptotically flat geometries, we have the usual quantization condition. In the case of asymptotically $AdS$ spaces, there is no quantization condition, since the fields fall off too fast.

6.2 Integral charges

There are other puzzles raised by this situation, though. We are claiming that for very large but finite $N$, the semiclassical picture produces quantized charge. Yet, in the limit $N = \infty$, the semiclassical solution becomes nonquantized. A local observer in $AdS_7 \times S^4$ would then be able to determine the asymptotic geometry from local measurements. This is therefore a gross violation of locality.

The natural objection to this is that in this limit, the nonquantized spectrum, which is of the form $Q_n = \sin\left(\frac{n\pi}{N}\right)$ goes over to $Q_n = \frac{n\pi}{N}$, a quantized spectrum. But clearly, if we scale $n$ with $N$, this is not the case. Therefore the problem remains.

The solution to this puzzle is that demanding locality actually requires quantized charges. This is easily seen from the supergravity solution. The field strength on the brane is a constant. It is obtained from the RHS of equation (28). The shape of the brane in $S^4$, on the other hand, is given by (24). Clearly to linear order in perturbations, they are proportional with a fixed constant of proportionality (in fact, they are exactly proportional even at the nonlinear level, once the effects of the curved geometry are taken into account.)

Hence the field strength is constant on the brane, and the constant does not depend on the size of the brane. Since the total flux is quantized, this implies that the volume of the brane is quantized. This further implies that the mass and charge of the brane are quantized.

Note that this argument does not depend on the asymptotic geometry. It only requires that the local observer, sees branes that are interpretable as flat space branes in the large $N$ limit. Such an observer must necessarily see quantized charges. This is furthermore in agreement with Taylor’s argument in [3], where he argued that the semiclassical picture a la [2] should always result in quantized charges.

There is no inconsistency with the boundary state analysis. We have already shown that charges need not be quantized in asymptotically curved spaces. The only issue is whether there is a semiclassical description of the boundary states.
Our analysis indicates that there is not such a description. More precisely, it would appear that the boundary states constructed in section 2 correspond to states in the semiclassical description where the flux on the brane is not quantized.

6.3 On the flat space S-matrix

The AdS/CFT correspondence provides a nonperturbative definition of string theory on AdS spaces. It has been proposed \cite{footnote2} that this indirectly also provides a nonperturbative definition of string theory on flat space by considering a limit where the interaction region is small compared to the scale of the AdS space. Intuitively, the curvature of the space should be invisible in this limit, and we should reproduce scattering in flat space.

However, if the BPS spectrum in the AdS space differs from the flat space BPS spectrum, then the interactions will not be reproduced. The existence of the fractionally charged objects might then lead to a breakdown of the map from AdS space to flat space.

This is most easily exhibited in the case of AdS\(_3 \times \Sigma^3 \times K3\), when the K3 is taken to have a size \(l_s\) (the string length), while AdS\(_3 \times \Sigma^3\) has a large radius \(kl_s\) with \(k\) large. If we consider interactions occurring in a small, string-scale sized local region of AdS\(_3 \times \Sigma^3\), then we might expect to reproduce scattering in \(R^{5,1} \times K3\).

Now in \(R^{5,1} \times K3\) there are no bound states of two D0-branes. In AdS\(_3 \times \Sigma^3 \times K3\), on the other hand there does exist such a state. This has a charge \(k \pi \sin \frac{2\pi}{k} \sim 2\) times the D0-brane charge. The semiclassical construction of \cite{footnote2} provides an explicit description of another state, which has charge exactly 2 times the D0-brane charge. We know the size of this second state; it is of order \(l_s\).

Now if either of these bound states appears in the interaction then it will produce a pole (or cut) in the AdS\(_3 \times \Sigma^3 \times K3\) correlations which would not appear in the flat space S-matrix. This would then indicate that the flat space S-matrix would not be obtained as a limit of the AdS scattering. Since the second state above has a small size, of order \(l_s\), there is no obvious reason it should not appear in the interactions. This therefore seems to lead to a breakdown of the derivation of the flat space S-matrix (at least in this particular case, but similar issues exist in the cases with greater supersymmetry).

It would be very interesting to see if there is a way that this issue is resolved, or whether it is really the case that the flat space S-matrix cannot be obtained from the AdS theory.

6.4 Charge quantization

Finally, we comment on the issue of charge quantization. Since we now have branes of charge \(k \pi \sin \frac{2\pi}{k} \sim 2\), one can ask if two branes of charge 1 can decay
to a bound state with the charge given above.

We wish to argue that this in fact cannot occur. The charge is still coupled to a massless gauge field. The consistency of the gauge transformation implies charge conservation. In other words, the lack of charge conservation leads to a gauge anomaly. This would appear to be fraught with difficulties.

It was argued in \[9\] that in fact one could find a transition between a set of separated D0-branes and a bound state which is of the form described in section \(6\). We have argued that in fact this new state is also integrally charged, and hence charge conservation is maintained. (A similar argument was given in \[11\]). Presumably there is no transition between separated D0-branes and a fractionally charged state.

7 Summary and Conclusions

We have presented a supergravity solution for branes in $AdS_7 \times S^4$ which displays the fact that branes which are naively pointlike in the $S^4$ are in fact extended objects on the $S^4$. The existence of a constant flux on the worldvolume was found as a direct consequence of the supergravity solution. The construction works similarly for branes in other $AdS$ geometries.

Using this construction, we were able to resolve several confusing issues regarding these branes. We showed that the charges of these branes were not quantized despite the fact that they were coupled to a massless gauge field. This was because the curvature of the $AdS$ space led to an effective mass for the gauge field at long distances.

Furthermore, we were able to show that the semiclassical description \[2\] of this brane always produced integral charges. This is most directly seen in the construction we have given. Other calculations require a careful accounting of bulk charges and brane charges, which is automatically taken care of here.

Finally, we have commented on several other issues, including the problems of reproducing the flat space S-matrix, and charge conservation in these theories.

References

[1] J. L. Cardy, Nucl. Phys. B 324, 581 (1989).

[2] C. Bachas, M. R. Douglas and C. Schweigert, JHEP 0005, 048 (2000) [arXiv:hep-th/0003037].

[3] W. Taylor, JHEP 0007, 039 (2000) [arXiv:hep-th/0004141].

[4] A. Rajaraman, JHEP 0109, 018 (2001) [arXiv:hep-th/0007241].

[5] A. Rajaraman, arXiv:hep-th/0011279.
[6] N. Ishibashi, Mod. Phys. Lett. A 4, 251 (1989).

[7] J. Polchinski, arXiv:hep-th/9901076.

[8] L. Susskind, arXiv:hep-th/9901079.

[9] S. Fredenhagen and V. Schomerus, JHEP 0104, 007 (2001) arXiv:hep-th/0012164.

[10] A. Y. Alekseev and V. Schomerus, Phys. Rev. D 60, 061901 (1999) arXiv:hep-th/9812193.

[11] A. Alekseev and V. Schomerus, arXiv:hep-th/0007098.