Estimation of strength of source in a 2D participating media using the differential evolution algorithm

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Abstract. A numerical model is developed to estimate the time varying strength of a heat source without prior information about the unknown function. The medium is considered to be gray, absorbing and emitting and the heat transfer is assumed to take place by both radiation and conduction. All the boundaries of the 2-D enclosure are considered to be black with known temperature. The finite-volume method is used for discretization of the energy as well as the radiative transfer equation. This inverse radiation-conduction problem is solved with the minimization of a performance function, which is expressed by the sum of square residuals between calculated and observed temperature, utilizing the differential evolution algorithm. The prediction of the strength of the source by the present algorithm is found to be quite reasonable.

Keywords: Inverse radiation, finite volume method, differential evolution algorithm, participating medium.

1. Introduction
A direct heat transfer problem is about solution of the heat equation depicting a model of heat transfer - Conduction, Convection, Radiation or a combination of more than one of them. These mathematical models are governed by various input factors e.g. surface temperature, heat source strength, heat flux, dimension of the conducting body / convecting media, etc. and their goal is to predict the temperature profile of the body either independent of (steady) or depending on the time (transient). These types of problems are mathematically “well posed” and, therefore, can be solved with straight forward and well-established mathematical procedures.

But in the real life situations, where it is not possible to measure the casual factors (boundary temperature or flux) directly, however the temperature can be measured inside the enclosure by a sensor/thermocouple, a natural problem becomes to predict the unknown casual factor. This leads to the inverse problem.

The determination of the unknown source term in conduction problems have been investigated by several authors due to numerous applications in engineering and science. Huang and Ozisik [1] and Neto and Ozisik [2–4], determined the unknown time wise variation of the strength of a heat source, located inside a flat plate, using the conjugate gradient method [CGM]. Yang [5] solved 2D inverse heat source problem by linear least square error model. Jin and Marin [6] employed the method of fundamental solutions (MFS) to recover the heat source in steady-state heat conduction problems. Yan et al. [7] also applied the MFS to solve an inverse heat source problem with arbitrary geometry.
Girault et al. [8] used experimentally built low order model for the estimation of time-varying heat source involving thermal diffusion with convective and radiative boundary conditions.

Recently, inverse heat transfer analysis in radiation has received much attention due to its various applications [9]. Among others, especially, the inverse conduction–radiation analysis was applied to estimate the thermal properties [10-13, 18-19] or source term in a medium [14-16] or heat flux [17]. Matthews et al. [10] estimated the extinction coefficient, back scattering fraction, and thermal conductivity when the temperature and transmittance measurements are given in one-dimensional planer layer. They have used a nonlinear parameter estimation technique. Li [11] estimated the single scattering albedo, the optical thickness, the conduction-to-radiation parameter, and the scattering phase function with the exit radiation intensities in a one-dimensional plane-parallel medium using CGM in parameter estimation approach. Park and Lee [12] estimated the spatially varying heat transfer coefficient and the absorption coefficient in a radiant cooler by improved adjoint variable method. Park and Yoon [13] estimated radiative parameters using the CGM in a 3D participating media, where radiation and conduction occur simultaneously. Salinas [14] applied CGM to estimate the strength of a heat source in a two-dimensional gray media from radiative intensities exiting in some points of boundary surfaces. Park and Lee [15] and Park and Yoo [16] also estimated the strength of a heat source using modified CGM in a furnace. Kim and Baek [17] estimated total heat flux distribution utilizing Levenberg–Marquardt method on heater surface in a two-dimensional concentric cylindrical absorbing, emitting and scattering medium.

The deterministic method like CGM is most popular method for inverse problems because being an iterative regularization method the regularization procedure is performed during the iterative processes and thus the determination of optimal regularization conditions is not needed. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, sensitivity and the adjoint problem. Deterministic methods are in general computationally faster than stochastic methods or search-based methods like Genetic algorithm (GA) and Differential Evolution (DE), although they can converge to a local minima or maxima, instead of the global one. On the other hand, stochastic algorithms can ideally converge to a global maxima or minima, although they are computationally slower than the deterministic ones.

Stochastic methods also are applied for inverse problems for their outstanding characteristics, for instance, less dependence on initial value and non-requirement of gradient information. Lobato et al. [18] used DE approach for the estimation of radiative properties in two-layer participating media. The results obtained with this methodology are compared with other approaches, namely the simulated annealing algorithm (SA), the Levenberg–Marquardt method (LM) and the hybridization simulated annealing and Levenberg–Marquardt (SA-LM). Das et al. [19, 20] used GA to solve inverse transient conduction–radiation problem for estimation of scattering albedo, conduction–radiation parameter and boundary emissivity. They used the lattice Boltzmann method for conduction and the finite volume method (FVM) for radiation. Liu [21] used modified GA for solving inverse heat conduction problem to estimate unknown transient heat source.

The DE is a structural algorithm proposed by Storn and Price [22] for optimization problems. Besides its good convergence properties and suitability for parallelization, DE’s main assets are its conceptual simplicity and ease of use. This approach is an improved version of Goldberg GA for faster optimization. There has not been extensive work done using DE approach in inverse heat transfer problems. To the knowledge of the authors, DE approach has still not been used for the estimation of strength of source in a participating medium. So, its applicability and performance for such kind of problem needs to be tested. Therefore, in this work, an inverse radiation problem is studied to estimate an unknown transient heat source using DE algorithm. The remainder of this paper is organized as follows. In Section 2 the direct and inverse heat transfer problem is described and formulated. It shows that the inverse problem can be converted into an optimization problem. Section 3 provides an overview of the DE algorithm. Some numerical results of the implementation of these algorithms for solving the inverse problem are discussed in Section 4. Section 5 summarizes this paper with certain conclusions.
2. Problem statement and formulation

2.1. Direct problem

A 2D enclosure of dimension 1m x 1m as shown in figure 1 is considered. Heat transfer within the enclosure is contributed by conduction as well as radiation. All boundaries of medium are considered as black with known temperature. Table 1 shows all the data about the thermophysical and the radiative properties taken for study. The governing equations for the temperature field are described in following subsection.

![Figure 1. Problem description](image)

| Property                  | Value      | Property              | Value     |
|---------------------------|------------|-----------------------|-----------|
| absorption coefficient, \(\alpha\) | 1.0 m\(^{-1}\) | heat capacity, \(\rho c_p\) | 1.2 kJ/kgK |
| conduction-radiation parameter, \(N = k \beta/4\sigma_s T^3\) | 0.008 | scattering coefficient, \(\sigma_s\) | 0.0 m\(^{-1}\) |
| extinction coefficient, \(\beta\) | 1.0 m\(^{-1}\) | thermal conductivity, \(k\) | 0.05 W/mK |

2.1.1. Energy equation. The transient state energy conservation equation considering conduction, radiation and heat generation can be expressed as

\[
\rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T - \nabla \cdot q_R + S_{\text{f}}(t) \delta(x - x^*)\delta(y - y^*)
\]

where the \(\nabla\) operator is with respect to some physical length \(s\), \(S_{\text{f}}(t)\) denotes the strength of heat source and \(\delta\) is the dirac delta function which determines the location of heat source. In the present work, a heat source at a location of \(x^* = 0.66\) m and \(y^* = 0.33\) m has been considered. It is assumed that the thermal conductivity \(k\) of the medium is independent of temperature \(T\) of the medium. The relevant boundary conditions are:

- At all boundaries, \(T = T_w = 300\) K

where \(T_w\) is wall temperature. The divergence of the radiative heat flux required in the energy equation (equation (1)) is given by

\[
\nabla \cdot q_R = \kappa (4\pi I_b - \int_{4\pi} I(\hat{r}, \hat{s}) d\Omega)
\]

(3)

where \(I\) is the intensity, \(I_b\) is the blackbody intensity and \(\Omega\) is the solid angle.

2.1.2. Radiative transfer equation (RTE). To calculate the divergence of radiative heat flux (equation (3)), the solution of RTE is required. The RTE for a gray, absorbing and emitting medium (neglecting scattering) in the direction \(\hat{s}\) can be written as

\[
\nabla \cdot (\hat{s}I) = - \beta(\hat{r}) I(\hat{r}, \hat{s}) + \kappa(\hat{r}) I_b(\hat{r})
\]

(4)

In Eqs. (3, 4), \(\hat{r}\) and \(\hat{s}\) are the position vector and the unit vector describing the radiative intensity direction respectively. \(\beta (= \kappa + \sigma_s)\) is the extinction coefficient and is equal to absorption coefficient \(\kappa\) since the scattering is absent (\(\sigma_s = 0\)) in this problem. The numerical model uses the same control volume for both conduction and radiation as both the energy equation (equation (1)) and RTE
(equation (4)) are solved with the finite volume method (FVM). The step scheme is used for spatial discretization of the RTE. The readers are referred to the work of Talukdar et al. [23, 24] for the details of the discretized equations. The boundaries are isothermal (equation (2)) and black and are given by:

$$I(\hat{r}, \hat{s}) = I_b = \sigma_b T_w^4 / \pi$$

where $\sigma_b$ is Stefan-Boltzmann constant. The direct problem is solved using the FVM. The numbers of control volumes and control angles considered are 30 x 30 and 12 x 12 respectively while using nineteen numbers of time steps. The control volume size are $\Delta x = \Delta y = 3.3$ cm for the geometry illustrated in figure 1. The total experiment duration $t_f$ is taken as 180 s and time step size $\Delta t = 10$ s.

The computational procedure for calculation of temperature field is as follows:

1. Assume a temperature field.
2. Calculate $I_b$ using the given temperature field.
3. Solve the RTE (equation (4)) using the FVM to obtain the radiation intensity $I$.
4. Calculate the divergence of the radiative heat flux from equation (3).
5. Solve equation (1) with the divergence of the radiative heat flux calculated from equation (3) as a source term to obtain the temperature field.
6. Return to step 2 with the updated temperature field.
7. Iterate until a convergence in the temperature field is obtained.

2.2 Inverse problem

In the inverse problem, strength of the source is unknown and is to be estimated. The additional information which requires for solution of this inverse problem is some temperature data inside the solution domain. In a practical problem, these temperature data are found by measuring the temperature of the enclosure at different locations using some sensors. In this work, instead of measured data, some temperature data are taken from the solution of the direct problem described in section 2.1.

The solution of present inverse problem is to be sought in such a way that the objective function $J(S_0)$ is minimized.

$$J(S_0(t)) = \sum_{i=1}^{D} \sum_{m=1}^{M} [T_c(x_m, y_m, t; S_0) - T_m(x_m, y_m, t)]^2$$

where $M$ is the number of the mounted sensors, $T_c(x_m, y_m, t; S_0)$ are the calculated temperatures at the measurement locations with estimated value of source and $T_m(x_m, y_m, t)$ are the measured temperatures taken at $D$ discrete time intervals.

The stopping criteria is specified as

$$J(S_0) < \varepsilon$$

where $\varepsilon$ is a small predefined constant. In this work $\varepsilon$ is taken as $10^{-5}$.

However, the observed temperature data may contain measurement errors. Therefore, we do not expect the functional (equation (6)) to be equal to zero at the final iteration step. We use the discrepancy principle as the stopping criterion. The following expression is obtained for stopping criteria $\varepsilon$:

$$\varepsilon = M\sigma^2 D$$

where $\sigma$ is standard deviation of measurements.

3. Differential Evolution

DE is a parallel direct search method which utilizes $Np$ $D$-dimensional parameter vectors [20]:

$$\hat{x}_{i,G} = [x_{i,1,G}, x_{i,2,G}, \ldots, x_{i,D,G}]$$

as a population for each generation $G$, i.e. for each iteration of the optimization. Each variable in the population represents strength of heat source to be estimated such as $x_{i,1,G}$, $x_{i,2,G}$, $\ldots$, $x_{i,D,G}$ represents estimated strength of heat source during different time steps ($j = 1, 2, \ldots, D$) in $i^{th}$ population and in generation $G$. The initial population is chosen randomly between the lower and the upper bounds and should try to cover the entire parameter space uniformly. The crucial idea behind DE is a scheme for generating trial parameter vectors. DE algorithm consists of three genetic operators: mutation,
crossover and selection. Mutation and crossover operators generate new individuals and selection operator determines suitable individual which gives minimum objective function values, and in this way population gets the better individuals.

The mutation operator used in this article is:

$$\hat{x}_{i,G} = \hat{x}_{i,G} + F_1 \cdot (\hat{x}_{\text{best},G} - \hat{x}_{i,G}) + F_2 \cdot (\hat{x}_{r_1,G} - \hat{x}_{r_2,G})$$  

where the indexes $r_1$ and $r_2$ represent the random and mutually different integers generated within range $(1, N_p)$ and also different from index $i$. $F_1$ and $F_2$ are the mutation scale factors usually less than 1. $\hat{x}_{\text{best},G}$ is the best vector in generation $G$.

After mutation, the new individuals are generated using the following scheme in crossover procedure:

$$u_{i,j,G} = \begin{cases} u_{i,j,G} & \text{if rand (0,1) } \leq CR \\ x_{i,j,G} & \text{otherwise} \end{cases}$$  

where $CR$ is a crossover parameter or factor usually less than 1 and presents the probability of creating parameters for trial vector from a mutant vector.

In selection procedure for each target individual $\hat{x}_{i,G}$, the fitness value of the trial individual $\hat{u}_{i,G}$ is compared with that of the target individual $\hat{x}_{i,G}$, and the individual with the maximum fitness value is selected for the next generation. Selection is given by equation (12),

$$\hat{x}_{i,G+1} = \begin{cases} \hat{u}_{i,G} & \text{if } f(\hat{u}_{i,G}) < f(\hat{x}_{i,G}) \\ \hat{x}_{i,G} & \text{otherwise} \end{cases}$$

The overall computational procedure is given in flow chart (Figure 2). The parameters used for DE algorithm in this work are $N_p = 10$, $F_1 = 0.8$, $F_2$ randomly varying in range $(0, 1)$ and $CR = 0.5$. The upper and lower limit of source strength for inverse problem taken is $10,000$ W/m$^2$ and $0$ W/m$^2$ respectively at all time steps.

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**Figure 2.** Computational procedure
4. Results and Discussion
The objective of this work is to estimate the strength of a transient heat source without any prior information on the functional form of the source. The unknown strength of source is assumed to be any of the three different forms of function listed below to examine the accuracy of the proposed model.

\[
\begin{align*}
S_0(t) &= 30000 \sin \left( \frac{\pi t}{180} \right), \text{ for } 0 \leq t \leq 180 \\
S_0(t) &= 30000 \left( \frac{t}{180} \right), \text{ for } 0 \leq t \leq 90 \\
S_0(t) &= 30000 \left( 1 - \frac{t}{180} \right), \text{ for } 90 < t \leq 180 \\
S_0(t) &= 1000, \text{ for } t \leq 30 \text{ and } t \geq 150 \\
S_0(t) &= 30000, \text{ for } 30 < t < 150
\end{align*}
\]

The effects of number of measurements on the accuracy of estimations are investigated. Figure 1 shows schematic of five thermocouples (\(M = 5\)) arrangement in y direction at \(x = 20 \Delta x = 0.66 \text{m}\) with spacing of \(7 \Delta y\) between each two sensors and with the first and last sensor placed at \(y = 0.5 \Delta y\) and \(29.5 \Delta y\) respectively. Similar type of arrangement at same position of \(x = 20 \Delta x\) is assumed for \(M = 8\) with corresponding spacing of \(4 \Delta y\) respectively. The accuracy of estimations is also investigated with single sensor positioned at source location. It is also noted that with \(M = 8\) third sensor from bottom placed exactly at source position while with \(M = 5\), no sensor comes at source position.

In experiments, there could be errors in the measurement of temperature. Since in the present work, these temperature data are taken from the solution of direct problem, they are error free. In order to be more realistic, errors are added randomly in these temperature data so as to represent them more like an experimental data. Thus, the measured temperature data \(T_m\) are represented by adding random errors to the exact temperature \(T_e\) computed from the solution of the direct problem as:

\[
T_m = T_e + \omega \sigma
\]

where \(\omega\) is a random variable of normal distribution with zero mean and unit standard deviation. The value of \(\omega\) is calculated by the Fortran subroutine ‘random_number’ and chosen over the range \(-2.576 < \omega < 2.576\). Standard deviation, \(\sigma\) can be related to relative measurement error as follows:

\[
e_{\text{rel}} = \frac{2.576 \times \sigma}{T_e}
\]

The \(\sigma\) is adjusted such that \(e_{\text{rel}}\) is zero, 1\%, 3\% and 5\% respectively.

The RMS error is calculated to compare the quality of estimation of strength of source as:

\[
RMS = \frac{1}{D} \sqrt{\sum_{i=1}^{D} \left[ S_{0\text{est}}(t) - S_{0\text{ex}}(t) \right]^2}
\]

where \(S_{0\text{ex}}\) and \(S_{0\text{est}}\) are the exact and estimated strength of source values.
As the first test case of the present work, an idealized situation is considered in which there are no measurement errors i.e. $e_{rel} = 0$. Figures 3(a), 3(b) and 3(c) show the exact and estimated profiles of the heat source function $S_0(t)$ for the three functional forms and with three different set of measurements as described above. In the same figures, the RMS error and number of iterations to satisfy stopping criterion are also indicated. The RMS error with one and eight measurements set are almost similar. With $M = 5$ sensors possesses relatively low sensitivity of heat source strength as no sensor placed at source position therefore it takes more number of iterations for stopping criteria and RMS error is also high compared with $M = 1$ and $M = 8$.

The next consideration is the effect of variation of conduction-radiation parameter $N (= k/4\pi \sigma T^4)$ on the accuracy of estimation. Figure 4 shows the exact and estimated profiles of the sinusoidal heat source function with eight errorless measurements and different values of $N$. In addition to the value of $N (= 0.008)$ as given in Table 1, two other values of $N (= 0.1$ and $1.0)$ are considered in this parametric study. When the value of $N$ increases, the conduction heat transfer gradually increases and for $N = 1$, the contribution of conduction heat transfer becomes much more dominant than radiation heat transfer. It is observed from the results that as the value of $N$ increases the RMS error increases. This is due to the fact that the sensitivity value becomes smaller when the conduction heat transfer becomes more dominant than radiative heat transfer as also pointed by Kim and Baek [17].

Figure 3: The exact and estimated profiles of heat source function (a) sinusoidal, (b) triangular and (c) step, with different number of measurements and $e_{rel} = 0$

Figure 4: The exact and estimated sinusoidal profiles of heat source function with eight errorless ($e_{rel} = 0$) measurements and different values of $N$. 

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Finally, the effect of measurement errors on the accuracy of estimation is investigated. Figure 5 shows the exact and estimated profiles of the sinusoidal heat source function $S_o(t)$ with measurement error, $e_{rel} = 1\%$, 3\% and 5\%. When $e_{rel}$ is 1\%, RMS error with $M = 5$ is 639.31 as shown in figure 5(a). It is also seen that the RMS error increases with increase in $e_{rel}$. Therefore, for $M = 5$, the RMS errors becomes very large when $e_{rel} = 3\%$ and 5\%, and hence it is not shown in Figures 5(b)-(c) to avoid congestion. Same is the case with $M = 1$ when $e_{rel} = 5\%$ and is not shown in figure 5(c). In all cases of measurement errors, results show better estimation with $M = 8$.

5. Conclusions
The Differential evolution (DE) algorithm is successfully applied for the solution of inverse radiation problems to estimate the unknown source in a 2D participating media. Several test cases involving different functions of heat source, different numbers of thermocouples, different values of conduction-radiation parameter and introduction of measurement errors are considered. Even a single accurate
sensor embedded at source location is sufficient to estimate the heat source profile by the DE algorithm. When a sensor is placed other than source position both the RMS error and number of iteration to reach stopping criterion increases as with case $M = 5$. Higher the value of conduction-radiation parameter higher is the RMS. It is also observed when the error in the measurements (direct problem temperature data) are added, eight set of measurements gives better estimation. The overall results show that the inverse solutions obtained by DE remain stable and found to be quite reasonable.

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