Wavelet analysis of event by event fluctuations

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(Dated: August 6, 2018)

The temporal fluctuations of produced hadron density in heavy ion collisions, modelled by 2D Ising model at temperatures $T_c$ and below, are studied through a recently developed wavelet based fluctuation analysis method. At $T_c$, long-range correlated multifractal behavior, matching with the recently observed Hurst exponent $H \simeq 1$, is found. Below $T_c$ uncorrelated monofractal behavior is seen. The correlation behavior compares well with the results obtained from continuous wavelet based average wavelet co-efficient method, as well as with Fourier power spectral analysis.

PACS numbers: 05.40.-a, 05.45.Tp, 24.60.Ky

1. INTRODUCTION

The study of correlations and scaling behavior is an area of active research [1]. Various methods of analysis like rescaled range analysis [2], structure function [3], wavelet transform modulus maxima [4], detrended fluctuation analysis and its variants are used to study the correlation behavior and fractal characteristics [5, 6, 7, 8, 9, 10, 11, 12]. Recently, we have developed a new method based on discrete wavelet transform to study the scaling properties of non-stationary time series [13]. This procedure economically extracts the multifractal behavior in a time series. It has been applied to characterize current and voltage fluctuations in tokamak plasma and financial time series [14].

The goal of the present work is to apply this discrete wavelet based approach for the study of event by event fluctuations of hadron density produced during phase transition. The fact that, our method is a local one, makes it ideal for the analysis of fluctuations in a non-stationary data. The primary motivation for this work is the recent study of the event by event fluctuations of hadron multiplicities, carried out by Qin and Ta-chung, [15, 16] using rescaled range analysis, which has shown scaling behavior. The Hurst exponent $H$, has been found to be 1. The geometry of the hadronization scenario in the context of heavy ion collisions, with two flavors, has led to the description of the same through a model like two dimensional (2D) Ising model. Hwa and co-workers have analyzed the scaling behavior of hadronic fluctuations through various methods like moment analysis etc., in the context of the above model [17, 18, 19, 20].

A large number of studies have been done to analyze the characteristic behavior of hadron formation during quark hadron phase transition. Earlier studies regarding the fluctuations in particle production, like correlation analysis of hadronic fluctuations have been carried out using cluster expansion technique of Ursell and Mayer [21]. Bialas and Peschanski have studied multi-particle production during quark hadron phase transition through factorial moment analysis [22]. Using statistical methods, Ludlam and Slansky studied the mechanism of particle production during quark hadron phase transition [23].

Following Hwa et. al. [17], we have carried out the Monte-Carlo simulation of 2D Ising model to mimic the multi-particle production during quark hadron phase transition. The time series of event by event fluctuations of average hadron densities below and at phase transition have been measured for characterization. The spatio-temporal fluctuations of produced hadrons show correlation behavior which have been characterized using wavelet transform techniques. We have made use of discrete and continuous wavelet transforms to analyze the hadronic fluctuations at various temperatures below and at critical temperature. We also study the scaling behavior through power spectral analysis.

Wavelet transform is a mathematical tool which finds applications in various fields, ranging from finance [14], biology [23], to chemistry [25], and physics [4, 28, 29] etc. Wavelet transform decomposes any given function or data in a manner, which achieves optimal time-frequency localization. Wavelets are classified into

![Fig. 1: The integrated time series of average hadron densities after subtraction of the mean, [a] at $T = 1.0$, below $T_c$ and [b] $T_c = 2.27$. The difference in behavior of the fluctuations at different temperatures is clearly visible.](image-url)
2. HADRON DENSITY AND THE ISING MODEL

In the context of heavy ion collisions, a two dimensional geometry can be invoked, where 2D Ising model captures the dynamics of the phase transition in a two flavor scenario. Earlier studies by Hwa et al. describe in detail the relationship between hadron density and the 2D Ising model. In this paper, we adopt the same procedure and deal with the correlation analysis and fractal characteristics through wavelet transforms. We have studied the hadronic fluctuations using both discrete and continuous wavelet transform as a time series analysis.

The two dimensional Ising model is defined by the Hamiltonian,

\[ H = -J \sum_{\langle i,j \rangle} s_i s_j \]  

where \( s \) takes the values ±1 and the sum is over the nearest neighbors. In our Monte-Carlo simulation of 2D Ising model, we use Wolff single cluster algorithm for spin flipping in a configuration of a lattice size \( L = 256 \). We define the cell size to be \( c^2 \) with \( c = 4 \) and the whole lattice is subdivided into \( l = (L/c)^2 \) cells. The hadron density at a particular cell is defined by

\[ \rho_i = \lambda \, c_i^2 \Theta (c_i) \]  

where \( c_i \) is the net spin at cell \( i \) defined to be positive along the overall magnetization, i.e.,

\[ c_i = \left( \text{sign} \left( \sum_{j \in L^2} s_j \right) \right) \sum_{j \in A_i} s_j. \]

\( A_i \) is the cell block of size \( c^2 \) located at \( i \), \( \Theta \) is the Heaviside function and \( \lambda \) is a constant. During the hadronization process \( c_i \) fluctuates from cell to cell, which reflects in the average hadron density in event to event. The average density is calculated as:

\[ \langle \rho \rangle = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{l} \rho_i, \]

Here, \( i \) denotes the cells and \( e \) is the configurations captured, where \( N \) is the number of events simulated. We have calculated the equilibrium time series of the average hadron density over \( N = 5 \times 10^5 \) iterations.

3. WAVELET BASED FLUCTUATION ANALYSIS

Considering the time series of average hadron density \( \langle \rho \rangle \) from \( i = 1, ..., N \), we capture configurations at various temperatures like below, and at critical temperature. In most applications, the index \( i \) will correspond to the time of the measurements. The profile \( Y_i \) of the time series is obtained by subtracting the mean and taking cumulative sum of the average hadron density. The profiles have been analyzed by the wavelet based fluctuation analysis method to study the scaling behavior. We calculate the scaling exponents \( h(q) \) for various moments \( q \), in this case \( q \) varies from \(-10\) to \(10\). For \( q = 2 \), \( h(q) \) is the Hurst exponent, which is one of the measures of correlation behavior in the time series analysis: \( 0 \leq H \leq 1 \). For persistent time series \( H > 0.5 \), \( h = 0.5 \) for uncorrelated time series and \( H < 0.5 \) for anti-persistent behavior. Recently, the anti-persistent regime \( 0 < H < 0.5 \) has been mapped to \( F_q \) statistics, widely employed in random matrix theory. The main aim here is to study the fractal characteristics of the time series and its multifractal nature. For the time series possessing multifractal behavior, \( h(q) \) decreases with increasing values of \( q \). For mono-fractal time series \( h(q) \) is constant for all values of \( q \). Fig. 1 depicts the profile of the time series of event by event fluctuations of average hadron density, [a] below \( T_c \), and [b] at \( T_c \).

We use a discrete wavelet based fluctuation analysis method (which is analogous to multifractal detrended fluctuation analysis method) to study the existence of scaling behavior of the time series. Our earlier paper gives the detail procedure of this approach. The fluctuations are extracted using a wavelet (Db-8) from Daubechies family. The fluctuation function \( F_q(s) \) is obtained for various scales \( s \). The power law manifests itself as a straight line in the log-log plot of \( F_q(s) \) versus \( s \), for each value of \( q \).

\[ F_q(s) \sim s^{h(q)} \]  

From the analysis of hadron density time series, below \( T_c \), uncorrelated mono-fractal behavior and long-range correlated multifractal behavior at \( T_c \) are found. These results are shown in Fig. 2. It was found that the Hurst exponent \( H = h(q = 2) \simeq 1 \) matches with that of Qin et. al. [7]. Very interestingly, multifractal behavior is clearly seen for event by event fluctuations at \( T_c \).
4. AVERAGE WAVELET COEFFICIENT METHOD

We now make use of average wavelet coefficient method to corroborate the above findings. This is a continuous wavelet based approach, which has been used to study the temporal correlations of the fluctuations in time series. We obtain the Hurst measure by transforming the time series, \( f(t) \) into wavelet domain. The continuous wavelet transform of a function \( f(x) \) is given by

\[
W[f](a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi_{a,b}^*(x) f(x) dx, \tag{6}
\]

where the mother wavelet \( \psi_{a,b} \) is defined as,

\[
\psi_{a,b}(x) = \psi \left( \frac{x-b}{a} \right). \tag{7}
\]

Here \( a \) is the scaling parameter \( (a > 0) \) and \( b \) is the translation parameter \( (-\infty < b < \infty) \) and \( \psi^*(x) \) is the complex conjugate of \( \psi(x) \).

It has been shown that [34],

\[
W[f](a) = \langle |W[f](a,b)| \rangle \simeq a^{\frac{d}{2} + H}. \tag{8}
\]

At a given scale, the wavelet energy of all locations have been averaged yielding \( W[f](a) \). The same averaging is followed for all scales. Thus for a self-affine time series, the scaling exponent \( \left( \frac{d}{2} + H \right) \) is measured from the slope of the log-log plot of \( W[f](a) \) versus scale \( a \). The results are shown in Fig. 3. They clearly indicate that the fluctuations possess self-affine nature and the Hurst exponent is the measure of correlation. From the obtained \( H \), we found that the hadronic fluctuations are uncorrelated below \( T_c \) (\( H = 0.5 \)) and at \( T_c \) it possesses long-range correlation (\( H \simeq 1 \)). This matches with our previous discrete wavelet analysis.

We have also analyzed the scaling behavior through Fourier power spectral analysis,

\[
P(s) = \left| \int Y(t) \exp(-2\pi ist) \ dt \right|^2. \tag{9}
\]

Here \( Y(t) \) is the accumulated fluctuations after subtracting the mean \( \langle Y \rangle \). It is well known that, \( P(s) \sim s^{-\alpha} \). The obtained scaling exponent \( \alpha \) is compared with Hurst exponent by the relation \( \alpha = 2H + 1 \). For the time series far below from \( T_c \), the scaling exponent \( \alpha = 2 \) which reveals uncorrelated Brownian motion behavior and at \( T_c \), the scaling exponent \( \alpha = 3 \), which reveals long range correlated fractional Brownian motion behavior as shown in Fig. 4.
5. CONCLUSION

We have found that the event by event fluctuations of hadron density, as modelled by 2D Ising spins, possess long-range correlated multifractal and fractional Brownian motion behavior at $T_c$. This is due to the correlation arising from phase transition. Uncorrelated monofractal and Brownian motion behavior below $T_c$ is observed as per physical expectations. The analysis has been carried out through discrete wavelet wavelet based fluctuation analysis method, which is well suited for non-stationary data. The continuous wavelet based average wavelet coefficient method and Fourier analysis fully supported the above conclusion.

We thank Prof. J. C. Parikh and M. S. Santhanam for useful discussions.

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