I. INTRODUCTION

Developing suitable environments for quantum technological applications is becoming a major aim in condensed matter physics. The discovery of the striking properties of twisted bilayer graphene and, more recently, bilayer graphene represents a notable success in this direction. Similarly, the realization of normal and superconducting topological states of matter and materials with strong spin-orbit coupling has been recently achieved, and may lead to substantial advances in spintronics, superconducting spintronics, and topological quantum computation. Most of these accomplishments strongly rely on the ability to precisely control the atomic structure of matter or on the ability to perform nanostructuring, such as pressure and magnetic fields, as also demonstrated in other systems. A complementary way of engineering quantum states on demand is to perturb solid state quantum systems in a time-dependent fashion, such as manipulating system parameters in the context of quantum quenches. In particular, intense short electromagnetic pulses and periodic (Floquet) drivings have been proven to be extremely powerful tools. Short electromagnetic pulses, which in the pump-probe setups allow for the study of ultra-fast dynamics, can, for example, induce phase transitions and drive higher harmonic generation. Periodic drivings allow to induce topological band structures and boundary states, and to create new phases of matter, such as time-crystals.

An important sub-field of time-dependent quantum engineering in solids deals with the control of the superconducting order parameter. The importance of non-constant perturbations on the superconducting order parameter has been known for a long time. Recently, however, the technological progress in the generation of intense sub-picosecond laser pulses, determined a renewed interest in the field. Striking signatures of transient out of equilibrium superconductivity have been observed in cuprates and doped fullerenes: the ultimate aim is to engineer room temperature superconductivity, although serious limitations like heating still need to be overcome. All these observations have been associated to the action of the laser on lattice degrees of freedom, in particular optical phonons, e.g. to light-enhanced electron-phonon coupling. In the context of doped fullerenes, a possible electronic mechanism for intralevel pairing has been conjectured. A novel scheme for generating interband superconducting pairing in periodically driven semiconductors has been proposed in Refs. In this case, however, the laser couples to the electronic degrees of freedom, and the crucial interplay of driving and dissipation leads to a steady state characterized by superconducting correlations.

The original proposals for such a state relied on particular fermionic dissipative baths able to exchange particles with the two bands of the semiconductor involved in the interband pairing or on effective simplified master equations. Moreover, a finite value of the order parameter in the steady state required the concurrent tuning of band dispersion and electronic interactions. In our work, we develop a more realistic model for steady state interband superconductivity. We consider a two-band semiconductor resonantly driven by a laser, and we include two physically relevant intraband relaxation mechanisms, namely acoustic phonons and radiative recombination. The steady state reached by the system, for which we provide a phase diagram, can develop interband superconducting correlations. The required conditions are strong laser amplitudes and strong repulsive electron-electron interactions. Moreover, we elucidate the role of the various relaxation processes by inspecting the transient dynamics. We finally devote a section to the discussion of a Bogoliubov-de Gennes effective model that allows us to qualitatively address the physics of the...
The structure of the article is as follows: In Sec.II, we introduce the model; in Sec.III, we describe our results and discuss a simplified model. Finally, in Sec.IV, we summarize our results and draw conclusions.

II. MODEL

We consider a d-dimensional semiconductor with only two non-degenerate bands close to the Fermi energy, namely, the valence (\(\alpha = 1\)) and conduction (\(\alpha = 2\)) bands. These are coupled by means of a laser, whose frequency is tuned at resonance around a single point in the Brillouin zone (BZ), where the band distance is \(E_g\). At the same time, a superconducting pairing, whose amplitude is computed self-consistently by taking electron-electron interactions into account, is allowed. The system is described by the Hamiltonian \(H_{\text{sys}} = H_0 + H_{\text{int}}\), where \((h = 1,\) lattice constant \(a = 1)\)

\[
H_0 = \sum_{k,\alpha} E_{\alpha}(k) c_{k}^{\alpha\dagger} c_{k}^{\alpha} + \Omega(t) \sum_{k,\alpha,\beta} c_{k}^{\alpha\dagger} \sigma_{\alpha\beta}^{x} c_{k}^{\beta},
\]

\[
H_{\text{int}} = \frac{i}{2} \Delta \sum_{k,\alpha,\beta} c_{k}^{\alpha\dagger} \sigma_{\alpha\beta}^{y} c_{-k}^{\beta\dagger} - \frac{i}{2} \Delta' \sum_{k,\alpha,\beta} c_{k}^{\alpha\dagger} \sigma_{\alpha\beta}^{y} c_{-k}^{\beta\dagger}.
\]

Here \(c_{k}^{\alpha}\) is the fermionic annihilation operator in the \(\alpha\)-band with momentum \(k\), \(E_{\alpha}(k)\) is the \(\alpha\)-band dispersion relation, \(\Omega(t) = \Omega \cos(\nu t)\) is set by the laser frequency \(\nu\) and the Rabi frequency \(\Omega\). Moreover, the complex order parameter \(\Delta\) quantifies the Cooper pairing strength. We assume that phonons can induce transitions only within the same band, since interband phonons are typically suppressed due to symmetry reasons. Note that, in principle, we can consider additional optical branches without significantly affecting our results, as long as the coupling between the optical phonons and the laser is negligible.

We also take into account the possibility of interband radiative recombination processes, where a conduction band electron relaxes to the valence band and emits a photon. In this case, as opposed to the phononic bath, the emission is associated to a pseudospin-flip to obey the angular momentum selection rules and can then only take place between different bands. The corresponding contribution to the Hamiltonian is \(H_{\text{sys}}^{\text{phot-bath}} = \sum_k \omega(k) (c_{k}^{\dagger} b_k^0 + h.c.)\), where \(b_k^0\) creates a photon with energy \(\omega(k) = E_2(k) - E_1(k)\) and \(\omega\) is the coupling intensity.

To study the complete dynamics of the relevant observables (see Section III) after the laser is switched on, we focus on the time evolution of the reduced density matrix \(\rho_{\text{sys}} = \text{Tr}_{\text{bath}}(\rho_{\text{tot}})\). By means of a rotating wave approximation (RWA), which can be employed because the laser is on resonance with \(E_g\), we eliminate the explicit velocity. The electron-phonon coupling is modelled by the Fröhlich Hamiltonian

\[
H_{\text{sys}}^{\text{ph-bath}} = \sum_{k,\alpha,\omega} t_{k,q}(c_{k+q}^{\alpha\dagger} c_{-q}^{\alpha\dagger} + h.c.),
\]

where \(t_{k,q}\) represents the momentum dependent coupling strength. We assume that phonons can induce transitions only within the same band, since interband phonons are typically suppressed due to symmetry reasons. Note that, in principle, we can consider additional optical branches without significantly affecting our results, as long as the coupling between the optical phonons and the laser is negligible.

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\]
time dependence of \( H_{\text{sys}} \). Moreover we treat \( H_{\text{sys-bath}}^{\text{ph}} \) and \( H_{\text{sys-bath}}^{\text{rr}} \) perturbatively \cite{75}. Hence, we obtain a master equation of the form \[ \frac{d}{dt} \rho_{\text{sys}} = -i[H_{\text{sys}}, \rho_{\text{sys}}] + L_{\text{sys}} \rho_{\text{sys}}, \] \[ (4) \]

where the dissipative part of the time evolution is encoded in the Lindblad operator \( L_{\text{sys}} \), defined as \( L_{\text{sys}}[X] \rho_{\text{sys}} = (X \rho_{\text{sys}} X^\dagger - X^\dagger X \rho_{\text{sys}} + \text{h.c.})/2 \) \cite{75}. We further neglect Lamb shift corrections because they only renormalize the band structure \cite{74, 76}. Hereafter, we set \( E_g = 1 \) as a common energy scale. The validity range \( \delta_k \) of the RWA is inferred by solving the full dynamics in the absence of superconducting pairing. Within the full model \( \delta_k \) is the natural cut-off. However, varying \( \delta_k \) does not qualitatively change our results.

\section{III. RESULTS}

\subsection{A. Dynamics and Phase diagram}

We solve the dynamics for the relevant observables, i.e. the populations of valence \( n_k^{11} = \langle c_k^1 c_k^1 \rangle \) and conduction \( n_k^{22} = \langle c_k^2 c_k^2 \rangle \) bands, and the ordinary \( n_k^{21} = \langle c_k^2 c_k^1 \rangle \) as well as the anomalous \( s_k^{21} = \langle c_k^{21} c_k^{11} \rangle \) interband correlations. Their complete time evolution, encoded in a set of non-linear coupled differential equations, is shown in the SM \cite{75} and has been evaluated numerically performing a 4\textsuperscript{th} order Runge-Kutta method. For clarity, we focus only on the zero temperature regime, since finite temperature corrections (for \( k_B T \ll E_g \)) do not affect qualitatively our results and interpretation. Moreover, we initialize the dynamics with the respective equilibrium state of the system and the baths, where only the valence band is populated. We also have to assume the initial interband anomalous correlations to be non-zero even though very small, in order to avoid the unstable fixed point solution of the dissipative mean field equations \( s_k^{21} = 0 \) \cite{65}. Our main result is that, in a rather generic parameter range, the anomalous interband correlator \( s_k^{21} \) reaches a non-zero steady state for any non-zero, but arbitrarily small, initial value. Consequently, a finite interband pairing \( \Delta \) can develop in the system. A phase diagram is shown in Fig. 2 where the steady state value of the order parameter is plotted as a function of laser intensity and interaction strength for fixed values of the dissipation rates. Throughout this section, we assume the momentum dependence of \( t_{k,q} \), introduced in Eq. (3), such that the corresponding scattering rates \( \Gamma_{\text{ph}} \), derived in the SM (see Eq. (S10)) \cite{75}, are approximately constant over the momentum region around the resonance \cite{8}. Thresholds in both electron-electron interactions and laser intensity are present. Moreover, in the parameter range we could access, both stronger interactions and laser intensity generally imply a larger induced superconducting pairing.

It is worth to notice the different roles played by the phononic and the photonic baths. While the phononic bath is crucial in establishing the superconducting steady-state, the radiative recombination does not qualitatively influence the phase diagram, as long as \( \Gamma_{\text{rr}} = \pi w^2 \ll \Omega \). Increasing the phononic rate \( \Gamma_{\text{ph}} \), however, qualitatively modifies the value of the superconducting order parameter \( \Delta \) achieved in the stationary state. It moves in fact, the threshold on the interaction strength to larger values. If \( \Gamma_{\text{ph}} \) is raised up even further, superconducting correlations will eventually be washed out. The dependence of the phase diagram on the other parameters involved is weaker.

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{phase_diagram.png}
\caption{Phase diagram of the system, representing the absolute value of the complex order parameter \(|\Delta|\) as a function of the laser intensity \( \Omega \) and the repulsive interaction strength \( V \). Here, \( \Gamma_{\text{ph}} = 0.02E_g \) is the phonon rate, \( \Gamma_{\text{rr}} = \Gamma_{\text{ph}}/10 \) is the radiative recombination rate and \( A_1 = 7E_g \), \( A_2 = 24E_g \) and we consider a momentum cutoff such that \(|k| \leq \delta_k = 0.2\).}
\end{figure}

In order to better understand the physics it is worth to notice that the time evolution of the anomalous interband correlator \( s_k^{21} \), and hence the superconducting pairing, is strongly dependent on the quantity \( \tilde{n}_k = n_k^{22} + n_k^{11} - 1 \) \cite{65, 66, 74}. Whenever \( \tilde{n}_k \) is close to zero, \( s_k^{21} \) behaves accordingly \cite{75}. Heuristically, the condition \( \tilde{n}_k = 0 \) implies zero probability of forming a Cooper pair of electrons (or holes) between momenta \( k \) and \(-k\). However \( \tilde{n}_k \neq 0 \) is only a necessary condition for obtaining \( \Delta \neq 0 \). The condition \( \tilde{n}_k \neq 0 \) cannot be realized by means of photonic dissipation alone, since there is no significant momentum transfer in the electron sector as a result of such processes. On the other hand, phonon scattering tends to place the electrons at the bottom of the bands, due to the fact that the bands have the same concavity at the resonance point. Hence, it is phonon scattering that generates the condition \( \tilde{n}_k \neq 0 \) necessary for the development of the superconducting correlations. The behavior of the populations is shown in Fig. 3. We find
in fact \( \tilde{n}_k \simeq 1 \) for the momenta around the minima of the quadratic bands while \( \tilde{n}_k \simeq -1 \) on the edge of the region where the rotating wave applies. Note that the transition between these two regions is not sharp, but smoothed out: In the next section we comment how this feature becomes crucial in the explanation of the Cooper pairs creation. Qualitatively speaking, the smoothing signals the (superconducting) pairing between the two bands.

In order to understand the role played by the radiative recombination, we show in Fig. 4 the real time evolution of the complex order parameter \( |\Delta| \). The role played by this relaxation mechanism is evident here: the larger the photonic relaxation rate, the shorter the time needed to reach the steady state. This behavior can be understood by noticing that photonic relaxation adds a qualitatively new (interband) dissipation channel. So, while the steady state populations are not substantially affected by the photonic relaxation, the time needed to establish them changes (see Fig. 4).

**B. Effective pairing mechanism**

As shown in Fig. 3, in the superconducting steady state the bands are qualitatively empty at the edges of the momentum region considered and filled in the center, around the minima. We can mimic this situation by employing an equilibrium system, with two bands with positive concavity and coincident minima. Indeed, the action of the laser is to effectively shift the lower band up to the upper one, balancing the gap energy difference between different bands electrons. An attractive interaction of the form \(-U(n_{k1}^{11} - 1/2)(n_{-22}^{22} - 1/2)\), with \( U > 0 \) favors the situation where, if the state \( k \) in band 1 is occupied (empty), the corresponding state \(-k\) in band 2 is occupied (empty). The chemical potential then determines whether the ground state is given by the couple of empty or filled states. If we apply the mean field approximation to the above-mentioned attractive interaction term, we obtain exactly the model studied in Ref. [67], which reads:

\[
H = \sum_{k,\alpha} \epsilon_{\alpha}(k) c_{-k}^{\alpha} c_{k}^{\alpha} - \tilde{\Delta} \sum_{k,\alpha \neq \beta} (c_{k}^{\alpha} c_{-k}^{\beta} + h.c.)
\]

where \( \alpha, \beta = 1,2 \) labels the two bands, \( \epsilon_{\alpha}(k) = A_{\alpha} k^2 - C \) and \( \tilde{\Delta} = U \Delta \). We fix \( C \simeq 0.2E_g \) to obtain a Fermi energy compatible with the steady state reached by our model as long as \( U = 0 \) (parameters in Fig. 3). Using a Bogoliubov-de Gennes representation, we are able to diagonalize the Hamiltonian and evaluate the expectation value of the population. In this picture, we observe a very similar qualitative behavior of the populations in the two bands with respect to the nonequilibrium steady state populations (see black solid line in Fig. 3). Therefore, the steady state can be effectively seen as an equilibrium two band system where the laser shifts the valence band up to the conduction band and an attractive interaction creates Cooper pairs. This simple effective model is intended to clarify the reason why the condition \( \tilde{n}_k \neq 0 \) is essential but not sufficient for the superconducting pairing. In fact, a simpler model with two bands and two different chemical potentials would lead to \( \tilde{n}_k \neq 0 \), but would not imply coupling between the bands and consequently the smearing of the populations as well as superconductivity.

A full qualitative interpretation of the rather involved results can hence be given: The laser excites electrons from the valence to the conduction band, at any \( k \) in the resonance region. The phonons let the electrons in each band get as close as possible to the bottom of the bands. Interactions between the two bands drive the system into a correlated state. A possible mean field effective
model describing the steady state reached is provided by the simple Bogoliubov-de Gennes Hamiltonian for interband superconductors, given in Eq. [9].

**IV. CONCLUSIONS**

We have shown that a non-equilibrium steady state characterized by a finite superconducting order parameter related to an anomalous interband pairing can be achieved in a semiconductor by means of a laser. Specifically, we have considered quadratic bands with the same sign of the effective mass. We have shown that an acoustic phononic bath, which is responsible of intraband transitions only, can induce an electronic distribution in the valence and conduction bands which favours the development of such an unusual pairing. Remarkably, this picture is not destroyed even if an interband relaxation with the same order of magnitude as the intraband one is switched on. Furthermore, the stationary state with $|\Delta| \neq 0$ can be achieved only for sufficiently large repulsive density-density electronic interaction ($V \gtrsim 2E_g$) and for an initial configuration where $|\Delta|$ can be vanishingly small. The Rabi frequency $\Omega$, on the other hand, has to be strong enough ($\Omega > \Gamma_{ph}^{th}$), since the laser has to be able to drive a sufficient number of electrons in the conduction band. Only in this case, the two populations can be forced to satisfy the favorable condition $\tilde{n}_k \neq 0$. The result is stable against changing the velocity $v$ of the acoustic phonons, the amplitude of the bands, the number $N_k$ of $k$ points considered and the phonon coupling $\Gamma_{ph}$, as well as the radiative recombination strength $\Gamma_e$. In this respect, we argue that our results corroborate the original proposal of Goldstein et al. [5], by putting it in a more realistic framework. This fact, together with the interpretation of the non-equilibrium pairing based on equilibrium multiband BCS theory, should facilitate the experimental detection of this novel route to light-induced insulator/superconductor transition.

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[1] A. Acín, I. Bloch, H. Buhrman, T. Calarco, C. Eichler, J. Eisert, D. Esteve, N. Gisin, S. J. Glaser, F. Jelezko, S. Kuhr, M. Lewenstein, M. F. Riedel, P. O. Schmidt, R. Thew, A. Wallraff, I. Walmsley, and F. K. Wilhelm, New Journal of Physics **20**, 080201 (2018)

[2] D. N. Basov, R. D. Averitt, and D. Hsieh, Nature Materials **16**, 1077 (2017)

[3] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, Nature **438**, 197 (2005)

[4] A. K. Geim and K. S. Novoselov, Nature Materials **6**, 183 (2007)

[5] M. I. Katsnelson, Materials Today **10**, 20 (2007)

[6] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Nature **556**, 43 (2018)

[7] M. Yankowitz, S. Chen, H. Polshyn, Y. Zhang, K. Watanabe, T. Taniguchi, D. Graf, A. F. Young, and C. R. Dean, Science , 10.1126/science.aav1910 (2019)

[8] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007)

[9] A. Roth, C. Brüne, H. Buhmann, L. W. Molenkamp, J. Maciejko, X.-L. Qi, and S.-C. Zhang, Science **325**, 294 (2009)

[10] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature Physics **5**, 398 (2009)

[11] S.-Y. Xu, I. Belopolski, N. Alidoust, M. Neupane, G. Bian, C. Zhang, R. Sankar, G. Chang, Z. Yuan, C.-C. Lee, S.-M. Huang, H. Zheng, J. Ma, D. S. Sanchez, B. Wang, A. Bansil, F. Chou, P. P. Shibayev, H. Lin, S. Jia, and M. Z. Hasan, Science **349**, 613 (2015)

[12] B. Sacépé, J. B. Oostinga, J. Li, A. Ubaldini, N. J. G. Couto, E. Giannini, and A. F. Morpurgo, Nature Communications **2**, 575 (2011)

[13] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science **336**, 1003 (2012)

[14] S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. A. Bernevig, and A. Yazdani, Science **346**, 602 (2014)

[15] S. Hart, H. Ren, T. Wagner, P. Leubner, M. Mühlbauer, C. Brüne, H. Buhmann, L. W. Molenkamp, and A. Yacoby, Nature Physics **10**, 638 (2014)

[16] Q. L. He, L. Pan, A. L. Stern, E. C. Burks, X. Che, G. Yin, J. Wang, B. Lian, Q. Zhou, E. S. Choi, K. Murata, X. Kou, Z. Chen, T. Nie, Q. Shao, Y. Fan, S.-C. Zhang, K. Liu, J. Xia, and K. L. Wang, Science **357**, 294 (2017)

[17] C. H. L. Quay, T. L. Hughes, J. A. Sulpizio, L. N. Pfeiffer, K. W. Baldwin, K. W. West, D. Goldhaber-Gordon, and R. de Picciotto, Nature Physics **6**, 336 (2010)

[18] S. Heedt, N. Traverso Ziani, F. Crépin, W. Prost, S. Trelenkamp, J. Schubert, D. Gritzmacher, B. Trauzettel, and T. Schäpers, Nature Physics **13**, 563 (2017)

[19] D. Pesin and A. H. MacDonald, Nature Materials **11**, 409 (2012)

[20] P. Michetti and B. Trauzettel, Applied Physics Letters **102**, 063503 (2013)

[21] J. Linder and J. W. A. Robinson, Nature Physics **11**, 307 (2015)

[22] D. Breunig, P. Burset, and B. Trauzettel, Phys. Rev. Lett. **120**, 057701 (2018)

[23] C. Fleckenstein, N. T. Ziani, and B. Trauzettel, Phys. Rev. B **97**, 134523 (2018)
[24] A. Kitaev, Annals of Physics 303, 2 (2003)
[25] R. S. K. Mong, D. J. Clarke, J. Alicea, N. H. Lindner, P. Fendley, C. Nayak, Y. Oreg, A. Stern, E. Berg, K. Shtengel, and M. P. A. Fisher, Phys. Rev. X 4, 011036 (2014)
[26] J. Klinovaja and D. Loss, Phys. Rev. Lett. 112, 246403 (2014)
[27] C. Fleckenstein, N. Traverso Ziani, and B. Trauzettel, arXiv:1810.00764 (2018)
[28] F. Crétin, P. Buset, and B. Trauzettel, Phys. Rev. B 92, 100507(R) (2015)
[29] C. Fleckenstein, N. Traverso Ziani, and B. Trauzettel, Phys. Rev. B 94, 214106(R) (2016)
[30] J. Cayao, P. San-Jose, A. M. Black-Schaffer, R. Aguado, C. Fleckenstein, N. Traverso Ziani, and B. Trauzettel, Phys. Rev. B 96, 205425 (2017)
[31] J. Cayao, A. M. Black-Schaffer, E. Prada, and R. Aguado, Beilstein Journal of Nanotechnology 9, 1339 (2018)
[32] A. P. Drozdov, M. I. Eremets, I. A. Troyan, V. Ksenofontov, and S. I. Shyuin, Nature 525, 73 (2015)
[33] M. K. Chan, N. Harrison, R. D. McDonald, B. J. Ramshaw, K. A. Modic, N. Barisic, and M. Greven, Nature Communications 7, 12244 (2016)
[34] S. Gerber, H. Jiang, H. Nojiri, S. Matsuzawa, H. Yasumura, D. A. Bonn, R. Liang, W. N. Hardy, Z. Islam, A. Mehta, S. Song, M. Sikorski, D. Stefanescu, Y. Feng, S. A. Kivelson, T. P. Devereaux, Z.-X. Shen, C.-C. Kao, W.-S. Lee, D. Zhu, and J.-S. Lee, Science 350, 949 (2015)
[35] P. Calabrese and J. Cardy, Phys. Rev. Lett. 96, 136801 (2006)
[36] J. Eisert, M. Friesdorf, and C. Gogolin, Nature Physics 11, 124 (2015)
[37] L. D’Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Advances in Physics 65, 239 (2016)
[38] A. Mitra, Annual Review of Condensed Matter Physics 9, 245 (2018)
[39] S. Porta, F. M. Gambetta, F. Cavaliere, N. Traverso Ziani, and M. Sassetti, Phys. Rev. B 94, 085122 (2016)
[40] S. Porta, F. M. Gambetta, N. Traverso Ziani, D. M. Kennes, M. Sassetti, and F. Cavaliere, Phys. Rev. B 97, 035433 (2018)
[41] T. Kampfrath, K. Tanaka, and K. A. Nelson, Nature Photonics 7, 680 (2013)
[42] M. Liu, H. Y. Hwang, H. Tao, A. C. Strikwerda, K. Fan, G. R. Keiser, A. J. Sternbach, K. G. West, S. Kittiwatanakul, J. Lu, S. A. Wolf, F. G. Omenetto, X. Zhang, K. A. Nelson, and R. D. Averitt, Nature 487, 345 (2012)
[43] J. H. Mentink, K. Balzer, and M. Eckstein, Nature Communications 6, 6708 (2015)
[44] F. Schmitt, P. S. Kirchmann, U. Bovensiepen, R. G. Moore, L. Retigg, M. Krenz, J.-H. Chu, N. Ru, L. Perfetti, D. H. Lu, M. Wolf, I. R. Fisher, and Z.-X. Shen, Science 321, 1649 (2008)
[45] D. Brida, A. Tomadin, C. Manzoni, Y. J. Kim, A. Lombardo, S. Milana, R. Nair, K. S. Novoselov, A. C. Ferrari, G. Cerullo, and M. Polini, Nature Communications 4, 1987 (2013)
[46] N. H. Damrauer, G. Cerullo, A. Yeh, T. R. Boussett, C. V. Shank, and J. K. McCusker, Science 275, 54 (1997)
[47] M. Liu, H. Y. Hwang, H. Tao, A. C. Strikwerda, K. Fan, G. R. Keiser, A. J. Sternbach, K. G. West, S. Kittiwatanakul, J. Lu, S. A. Wolf, F. G. Omenetto, X. Zhang, K. A. Nelson, and R. D. Averitt, Nature 487, 345 (2012)
[48] S. Ghimire and D. A. Reis, Nature Physics 15, 10 (2019)
[49] N. H. Lindner, G. Refael, and V. Galitski, Nature Physics 7, 490 (2011)
[50] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B 84, 235108 (2011)
[51] L. Jiang, T. Kitagawa, J. Alicea, A. R. Akhmerov, D. Pekker, G. Refael, J. I. Cirac, E. Demler, M. D. Lukin, and P. Zoller, Phys. Rev. Lett. 106, 220402 (2011)
[52] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature 515, 237 (2014)
[53] N. Goldman, J. C. Budich, and P. Zoller, Nature Physics 12, 639 (2016)
[54] L. Privitera and G. E. Santoro, Phys. Rev. B 93, 241406(R) (2016)
[55] J. McIver, B. Schulte, F.-U. Stein, T. Matsuyama, G. Jotzu, G. Meier, and A. Cavalleri, arXiv preprint arXiv:1811.03522 (2018).
[56] K. Sacha and J. Zakrzewski, Reports on Progress in Physics 81, 016401 (2017)
[57] H. Watanabe and M. Oshikawa, Phys. Rev. Lett. 114, 257003 (2015)
[58] D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. 117, 090402 (2016)
[59] F. M. Gambetta, F. Carollo, M. Marcuzzi, J. P. Gaarran, and I. Lesanovsky, Phys. Rev. Lett. 122, 015701 (2019)
[60] A. F. G. Wyatt, V. M. Dmitriev, W. S. Moore, and F. W. Shepard, Phys. Rev. Lett. 116, 1166 (1966)
[61] A. H. Dayem and J. J. Wiegand, Phys. Rev. B 91, 054517 (2015)
[62] O. Hart, G. Goldstein, C. Chamon, and C. Castelnovo, arXiv preprint arXiv:1810.12309 (2018).
[63] A. Moreo, M. Daghofer, A. Nicholson, and E. Dagotto, Phys. Rev. B 80, 104509 (2009).
[64] We imagine the two bands to be separated outside the region in momentum space that we consider. They can, for instance, bend down.
[65] B. K. Ridley, Quantum processes in semiconductors (Oxford University Press, 2013).
[66] I. Esin, M. S. Rudner, G. Refael, and N. H. Lindner, Phys. Rev. B 97, 245401 (2018)
[67] N. Tandon, J. D. Albrecht, and L. R. Ram-Mohan, Journal of Applied Physics 118, 045713 (2015)
[68] M. I. Dyakonov and A. Khastsikii, Spin physics in semiconductors, Vol. 157 (Springer, 2017).
[69] R. R. Puri, Mathematical methods of quantum optics, Vol. 79 (Springer Science & Business Media, 2001).
[70] H. Haug and S. W. Koch, Quantum Theory of the Optical and Electronic Properties of Semiconductors: Fifth Edition (World Scientific Publishing Company, 2009).
[71] See Supplementary Material for details on the derivation of the dissipative equations, their full time evolution and references therein [72-74].
[72] M. A. Schlosshauer, Decoherence: and the quantum-to-classical transition (Springer Science & Business Media, 2007).
[73] N. Tandon, J. D. Albrecht, and L. R. Ram-Mohan, Journal of Applied Physics 118, 045713 (2015)
Supplemental Material for Feasible model for photo-induced interband pairing

I. DERIVATION OF THE LINDBLAD MASTER EQUATION

We start by considering a d-dimensional semiconductor (or, more generally, an insulator), where a laser is used to couple the valence (α = 1) and conduction (α = 2) bands, and a density-density interaction among the electrons in the two bands is taken into account. Using a mean field approximation to handle the electronic interactions, the Hamiltonian describing the system reads (S1) \((\hbar = 1)\)

\[
H_{\text{sys}} = \sum_{k,\alpha} E_\alpha(k)c_{k\alpha}^{\dagger}c_{k\alpha} + \Omega(t) \sum_{k,\alpha,\beta} c_{k\alpha}^{\dagger} \sigma_{\alpha\beta}^{\dagger} c_{k\beta} + \frac{i}{2} \Delta \sum_{k,\alpha,\beta} \sigma_{\alpha\beta}^{\dagger} c_{-k}^{\dagger} c_{k\beta} - \frac{i}{2} \Delta^* \sum_{k,\alpha,\beta} c_{k\alpha} \sigma_{\alpha\beta} c_{-k}^{\dagger}
\]

where \(c_{k\alpha}^{\dagger}\) is the electronic annihilation operator in the \(\alpha\) band with momentum \(k\), \(E_\alpha(k)\) is the \(\alpha\) band dispersion relation, and \(\Omega(t) = \Omega \cos(\nu t)\). The complex order parameter \(\Delta\) quantifies the pairing between the bands and is in general defined as \(\Delta_{\alpha\beta} = \frac{1}{N} \sum_k V(k, k') \langle c_{k\alpha}^{\dagger} c_{k'\beta} \rangle\), where \(V(k, k') = V\) represents the strength of the interactions and its sign distinguishes between its attractive or repulsive nature, and \(N\) is the total number of electrons in the system. Furthermore, we study the effect of two different bosonic baths. We consider acoustic phonons with Hamiltonian

\[
H_{\text{ph}}^{\text{bath}} = \sum_{q,\alpha} \omega_n(q) a_{q,\alpha}^{\dagger} a_{q,\alpha} \simeq \sum_q v |q a_{q}^{\dagger} q, (S2)
\]

with constant and isotropic velocity \(v\). The coupling with the electronic system is described by means of the Fröhlich Hamiltonian

\[
H_{\text{sys} - \text{bath}}^{\text{ph}} = \sum_{k, q, \alpha} t_{k, q} (c_{k+q\alpha}^{\dagger} c_{-k-q\alpha} + h.c.) = \sum_{k, q, \alpha} (S_{k, q}^{\alpha} F_{k, q} + h.c.), (S3)
\]

where \(S_{k, q}^{\alpha} = c_{k+q\alpha}^{\dagger} c_{k\alpha}\), \(F_{k, q} = t_{k, q} a_{-q}^{\dagger}\). We assume that the phonons can induce transitions only within the same band, since interband transitions are typically suppressed [S2].

Further, we take into account the possibility of interband radiative recombination processes, where an electron in the conduction band relaxes in the valence band and emits a photon. The Hamiltonian which describes such bath and its coupling with the system [S3] can be written similarly to Eqs. (S2) and (S3):

\[
H_{\text{rr}}^{\text{bath}} = \sum_{q} |q b_{q}^{\dagger} b_{q}\rangle, (S4)
\]

\[
H_{\text{sys} - \text{bath}}^{\text{rr}} = \sum_{k, q} w_{k, q} (c_{k+q\alpha}^{\dagger} c_{-k-q\alpha}^{\dagger} + h.c.) \simeq \sum_{k} w (c_{k+q}^{\dagger} c_{k}^{\dagger} b_{q} + h.c.) , (S5)
\]

where in Eq. (S5) we focus on vertical transitions since the photon momentum is in general negligible with respect to the typical momentum discretization.

To study the dynamics of the system, we focus on the time evolution of the reduced density matrix \(\rho_{\text{sys}} = \text{Tr}_{\text{bath}} \{ \rho_{\text{tot}} \}\), where \(\rho_{\text{tot}}\) is the combined density matrix of system and reservoirs. By means of a rotating wave approximation (RWA), that is justified by the fact that the laser is tuned resonantly with the energy gap, we obtain a Lindblad type master equation of the form [S1]

\[
\frac{d}{dt} \rho_{\text{sys}} = -i[H_{\text{sys}}, \rho_{\text{sys}}] + \mathcal{L}_{\text{sys}} \rho_{\text{sys}}, (S6)
\]

where

\[
\mathcal{L}_{\text{sys}} \rho_{\text{sys}} = \sum_{k, q, k', q'} \sum_{\alpha, \beta} \left[ (S_{k, q}^{\alpha} \rho_{\text{sys}} S_{k, q}^{\beta}) W_{q, q'}^{(1)} + (S_{k, q}^{\alpha} \rho_{\text{sys}} S_{q, q'}^{\beta}) W_{q, q'}^{(2)} + h.c. \right]
\]

(S7)
In the following, we make use of the identity:
\[ W^{\alpha\beta(1)}_{q,k',q'} = \int_0^\infty e^{-i[E_{\alpha}(k')-E_{\beta}(k'+q')]} \mathcal{T}_{\text{bath}} \left\{ \mathcal{F}_{k',q'}(t-\tau)\mathcal{F}_{k',q'}^\dagger(t)\rho_{\text{bath}(0)} \right\} d\tau \simeq \Gamma_{k',q'}^{\alpha\beta}[1+n_B]\delta(q-q'), \] (S8)
\[ W^{\alpha\beta(2)}_{q,k',q'} = \int_0^\infty e^{-i[E_{\alpha}(k')-E_{\beta}(k'+q')]} \mathcal{T}_{\text{bath}} \left\{ \mathcal{F}_{k',q'}^\dagger(t-\tau)\mathcal{F}_{k',q'}(t)\rho_{\text{bath}(0)} \right\} d\tau \simeq \Gamma_{k',q'}^{\alpha\beta}n_B\delta(q-q'), \] (S9)
with
\[ \Gamma_{k',q'}^{\alpha\beta} = \pi|q_{k',q'}|^2 \delta [E_{\beta}(k') - E_{\beta}(k'+q) - v|q|] \delta_{\alpha\beta} = \Gamma^\alpha_\beta \delta_{\alpha\beta} \] (S10)
for the phononic bath and
\[ \Gamma_{k',q'}^{\alpha\beta} = \pi|u|^2 \delta_{\alpha\beta} = \Gamma^{\alpha\beta} \delta_{\alpha\beta} \] (S11)
for the photons, where \( \beta = 1 \) (2) if \( \alpha = 2 \) (1). The Bose distribution \( n_B \) is a function of the dispersion relation of phonons (S2) or photons (S4), respectively, with the chemical potential set to zero since the number of bosons (both phonons and photons) is not conserved. Note that these rates are the same we can obtain from the Fermi's golden rule, since we assume that the baths are weakly interacting with the system. Here we have neglected the principal value of the integral in Eq. (S8) and Eq. (S9), since it only slightly renormalizes the band structure (S5, S6).

The time evolution of the expectation value of a generic observable \( O_p \) can be derived from the density matrix equation (S6) by multiplying it by \( O_p \) and performing the trace operation. Using the cyclic property one obtains:
\[ \frac{d}{dt}\langle O_p \rangle = -i\langle [O_p, H_{\text{sys}}] \rangle + \sum_{k,k',q} \sum_{\alpha,\beta} \Gamma_{k',q}^{\alpha\beta} \left\{ \right. \right. (1+n_B)\mathcal{S}_{k,q}^{\alpha\beta}\mathcal{S}_{k,q}^{\alpha\beta} + [\mathcal{S}_{k,q}^{\alpha\beta},O_p]\mathcal{S}_{k,q}^{\alpha\beta} + \left. \right. + n_B\mathcal{S}_{k,q}^{\alpha\beta}O_p\mathcal{S}_{k,q}^{\alpha\beta} + [\mathcal{S}_{k,q}^{\alpha\beta},O_p]\mathcal{S}_{k,q}^{\alpha\beta} \] (S12)
In the following, we make use of the identity:
\[ [AB,CD] = A\{B,C\}D - AC\{B,D\} + \{A,C\}DB - C\{A,D\}B \]
and focus on the population of the valence and conduction bands \( n_p^{\alpha\beta} = c_p^{\alpha\dagger}c_p^\alpha \), on the interband correlations \( n_p^{\alpha\beta} = c_p^{\alpha\dagger}c_p^\beta \), and on the anomalous interband correlations \( s_p^{\alpha\beta} = c_p^{\alpha\dagger}c_p^\beta \). Furthermore, note that the Wick’s theorem holds since the system we are studying is non-interacting, i.e. we use a mean field approximation (S7).

II. DISSIPATIVE TIME EVOLUTION EQUATIONS FOR THE OBSERVABLES

In this section, we obtain the time evolution equations for the relevant observables of the system. Since the first term of the right hand side of Eq. (S12) is the same as the one studied in [S1], we give only the explicit evaluation of the dissipative term of the Lindblad equation.

A. Population of the valence band

Here we focus on the derivation of the dynamics of the occupation number of the valence band, \( n_p^{11} = c_p^{1\dagger}c_p^1 \). We start by exploiting the commutators in eq. (S12) corresponding to this specific case,
\[ \mathcal{S}_{k,q}^{\alpha\beta} = \left[ c_k^{\alpha\dagger}c_{k+q}^{\beta\dagger}c_{k+p}^{\alpha\dagger}c_{k+q+p}^{\beta\dagger} \right] = \delta_{\alpha,1}\delta(k+q-p)c_{p-q}^{\dagger}c_{p+q}^{\dagger} - \delta(k-p)c_{p+q}^{\dagger}c_{p-q}^{\dagger}, \] (S13)
\[ \mathcal{S}_{k,q}^{\alpha\beta} = \left[ c_k^{\beta\dagger}c_{k+q}^{\alpha\dagger}c_{k+p}^{\alpha\dagger}c_{k+q+p}^{\beta\dagger} \right] = \delta_{\alpha,1}\delta(k+q-p)c_{p-q}^{\dagger}c_{p+q}^{\dagger} - \delta(k-p)c_{p+q}^{\dagger}c_{p-q}^{\dagger}. \] (S14)
Using Wick’s theorem, we can simplify the terms obtained after the evaluation of the commutators. Here we show the ones multiplied in Eq. (S11) by \( (1+n_B) \), i.e. the first line:
\[ \langle c_{k'}^{\beta\dagger}c_{k'+q+p}^{\alpha\dagger}c_{p-q}\rangle = \delta_{\beta,2}\delta(k'+p)\langle s_{k+p-q}^{\dagger} \rangle + \delta(k'+p+q)\left[ \delta_{\beta,1}\langle n_{p-q}^{11} \rangle (1-n_{p-q}^{11}) - \delta_{\beta,2}\langle n_{p-q}^{21} \rangle n_{p-q}^{12} \right] \] (S15)
\[
\langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p + q \rangle \rangle_{12}^\beta (k' + p + q) \langle s_{p+q}^{21} \rangle \langle s_{p+q}^{21} \rangle + \delta(k' - p) \langle \delta_{\beta,1} \langle n_{p}^{11} \rangle (1 - n_{p+q}^{11}) - \delta_{\beta,2} \langle n_{p}^{21} \rangle \rangle_{p+q}^{12} \rangle \rangle \]
\]
\[
\langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta = \langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta \tag{S17} \]
\[
\langle c^\dagger_{p} c^{\dagger}_{p+q} c^\dagger_{k' + q} c^\dagger_{k' + q} \rangle_{12}^\beta = \langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta \tag{S18} \]
while now we show the ones multiplied in Eq. (S12) by \( n_B \), i.e. the second line:
\[
\langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta = \delta_{\beta,2} \delta(k' + p + q) \langle s_{p+q}^{21} \rangle \langle s_{p+q}^{21} \rangle + \delta(k' - p + q) \delta_{\beta,1} \langle n_{p}^{11} \rangle (1 - n_{p+q}^{11}) - \delta_{\beta,2} \langle n_{p}^{21} \rangle \rangle_{p+q}^{12} \rangle \rangle \]
\[
\langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta = \delta_{\beta,2} \delta(k' + p + q) \langle s_{p+q}^{21} \rangle \langle s_{p+q}^{21} \rangle + \delta(k' - p + q) \delta_{\beta,1} \langle n_{p}^{11} \rangle (1 - n_{p+q}^{11}) - \delta_{\beta,2} \langle n_{p}^{21} \rangle \rangle_{p+q}^{12} \rangle \rangle \]
\[
\langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta = \langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta \tag{S19} \]
\[
\langle c^\dagger_{p} c^{\dagger}_{p+q} c^\dagger_{k' + q} c^\dagger_{k' + q} \rangle_{12}^\beta = \langle c^\dagger_{k' + q} c_{k' + q' + p} \langle p+q \rangle \rangle_{12}^\beta \tag{S20} \]
Hence, the dissipative part of the time evolution of \( \langle n_{p}^{11} \rangle \) due to the acoustic phonons is given by
\[
\frac{d}{dt} \langle n_{p}^{11} \rangle_{ph} = 2 \sum_{q} \left[ \left[ 1 + n_B \right] \left[ \Gamma_{p-q,q} \langle n_{p-q}^{11} \rangle \langle 1 - n_{p}^{11} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p-q}^{21} \rangle \langle n_{p}^{11} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p-q}^{12} \rangle \langle n_{p}^{21} \rangle \right\} \right] + \right.
\]
\[
- \left. \left[ 1 + n_B \right] \left[ \Gamma_{p-q,q} \langle n_{p}^{11} \rangle \langle 1 - n_{p+q}^{11} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{21} \rangle \langle n_{p}^{11} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{12} \rangle \langle n_{p}^{21} \rangle \right\} \right] \right] + \right.
\]
\[
\left. n_B \left[ \Gamma_{p-q,q} \langle n_{p+q}^{11} \rangle \langle 1 - n_{p}^{11} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p}^{21} \rangle \langle n_{p+q}^{11} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p}^{12} \rangle \langle n_{p+q}^{21} \rangle \right\} \right] \right] + \right.
\]
\[
\left. - n_B \left[ \Gamma_{p-q,q} \langle n_{p}^{11} \rangle \langle 1 - n_{p+q}^{11} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{21} \rangle \langle n_{p}^{11} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{12} \rangle \langle n_{p}^{21} \rangle \right\} \right] \right] \right]. \tag{S21} \]
Analogously, the dissipative part due to the radiative recombination reads \( \Gamma_{rr} = \pi |w|^2 \)
\[
\frac{d}{dt} \langle n_{p}^{11} \rangle_{rr} = \Gamma_{rr} \langle n_{p}^{22} \rangle \langle 1 - n_{p}^{11} \rangle. \tag{S24} \]

**B. Population of the conduction band**

The derivation is completely analogous to the previous case. For the phonons, we have to simply exchange the band index \( 1 \rightarrow 2 \). Therefore, we get:
\[
\frac{d}{dt} \langle n_{p}^{22} \rangle_{ph} = 2 \sum_{q} \left[ \left[ 1 + n_B \right] \left[ \Gamma_{p-q,q} \langle n_{p-q}^{22} \rangle \langle 1 - n_{p}^{22} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p-q}^{21} \rangle \langle n_{p}^{22} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p-q}^{12} \rangle \langle n_{p}^{22} \rangle \right\} \right] + \right.
\]
\[
- \left. \left[ 1 + n_B \right] \left[ \Gamma_{p-q,q} \langle n_{p}^{22} \rangle \langle 1 - n_{p+q}^{22} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{21} \rangle \langle n_{p}^{22} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{12} \rangle \langle n_{p}^{22} \rangle \right\} \right] \right] + \right.
\]
\[
\left. n_B \left[ \Gamma_{p-q,q} \langle n_{p+q}^{22} \rangle \langle 1 - n_{p}^{22} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p}^{21} \rangle \langle n_{p+q}^{22} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p}^{12} \rangle \langle n_{p+q}^{22} \rangle \right\} \right] \right] + \right.
\]
\[
\left. - n_B \left[ \Gamma_{p-q,q} \langle n_{p}^{22} \rangle \langle 1 - n_{p+q}^{22} \rangle - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{21} \rangle \langle n_{p}^{22} \rangle \right\} - \Gamma_{p-q,q} \text{Re} \left\{ \langle n_{p+q}^{12} \rangle \langle n_{p}^{22} \rangle \right\} \right] \right] \right]. \tag{S25} \]
while, for the radiative recombination, we obtain \( \Gamma_{rr} = \pi |w|^2 \)
\[
\frac{d}{dt} \langle n_{p}^{22} \rangle_{rr} = -\Gamma_{rr} \langle n_{p}^{22} \rangle \langle 1 - n_{p}^{11} \rangle. \tag{S26} \]
C. Standard interband correlations

Next, we focus on the time evolution of the standard interband correlations, namely \( n_{p}^{21} = c_{p}^{\dagger} c_{p}^{1\dagger} - p \cdot c_{p}^{\dagger} c_{p}^{1\dagger} \). The commutators, in this case, are:

\[
[S_{k,q}, n_{p}^{21}] = \left( \hat{c}_{k}^{\alpha_{1}} c_{k+q}^{\alpha_{2}} \right) = \delta_{\alpha_{2}, 2} \delta(k + q - p) c_{p}^{\dagger} c_{p}^{1\dagger} - \delta_{\alpha_{1}, 1} \delta(k - p) c_{p}^{\dagger} c_{p}^{1\dagger} = - [n_{p}^{21}, S_{k,q}^{\alpha}] \tag{S27}
\]

\[
[n_{p}^{21}, S_{k,q}^{\alpha}] = \left[ c_{p}^{\dagger} c_{p}^{1\dagger} \right] = \delta_{\alpha, 1} \delta(k + q - p) c_{p}^{\dagger} c_{p}^{1\dagger} - \delta_{\alpha, 2} \delta(k - p) c_{p}^{\dagger} c_{p}^{1\dagger} = - [S_{k,q}^{\alpha}, n_{p}^{21}] \tag{S28}
\]

Using Wick’s theorem, we can simplify the terms obtained after the evaluation of the commutators. We display the ones multiplied in Eq. (S12) by \((1 + n_{B})\), i.e., the first line:

\[
\langle \hat{c}_{k'}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k' - p + q) \left[ -\delta_{\beta_{1}, 1} \langle n_{p-q}^{11} \rangle \langle n_{p}^{21} \rangle + \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] \tag{S29}
\]

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k - p) \left[ -\delta_{\beta_{1}, 1} \langle n_{p-q}^{11} \rangle \langle n_{p}^{21} \rangle + \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] \tag{S30}
\]

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k - p) \left[ \delta_{\beta_{1}, 1} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{11} \rangle - \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle n_{p}^{21} \rangle \right] \tag{S31}
\]

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k - p) \left[ \delta_{\beta_{1}, 1} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{11} \rangle - \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle n_{p}^{21} \rangle \right] \tag{S32}
\]

and now the ones multiplied in Eq. (S12) by \(n_{B}\), i.e., the second line:

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k' - p + q) \left[ -\delta_{\beta_{1}, 1} \langle n_{p-q}^{11} \rangle \langle n_{p}^{21} \rangle + \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] \tag{S33}
\]

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k - p) \left[ -\delta_{\beta_{1}, 1} \langle n_{p-q}^{11} \rangle \langle n_{p}^{21} \rangle + \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] \tag{S34}
\]

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k - p) \left[ \delta_{\beta_{1}, 1} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{11} \rangle - \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle n_{p}^{21} \rangle \right] \tag{S35}
\]

\[
\langle \hat{c}_{k}^{\beta_{1}} c_{k+q}^{\beta_{2}} \rangle = \delta(k - p) \left[ \delta_{\beta_{1}, 1} \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{11} \rangle - \delta_{\beta_{2}, 2} \langle n_{p-q}^{21} \rangle \langle n_{p}^{21} \rangle \right] \tag{S36}
\]

Hence, the dissipative part of the time evolution of \( \langle n_{p}^{21} \rangle \) due to the acoustic phonons is given by

\[
\frac{d}{dt} \langle n_{p}^{21} \rangle_{ph} = \sum_{q} \left\{ [1 + n_{B}] \Gamma_{p,q}^{1} \left( \langle n_{p-q}^{11} \rangle \langle n_{p}^{21} \rangle - \langle n_{p-q}^{11} \rangle \langle n_{p}^{21} \rangle \right) + \Gamma_{p,q}^{2} \left( \langle n_{p-q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle - \langle n_{p-q}^{21} \rangle \langle n_{p}^{21} \rangle \right) + \right. \\
+ [1 + n_{p}] \left[ \Gamma_{p,q}^{1} \left( \langle n_{p}^{11} \rangle \langle n_{p-q}^{21} \rangle - \langle n_{p}^{21} \rangle \langle 1 - n_{p-q}^{11} \rangle \right) + \Gamma_{p,q}^{2} \left( \langle n_{p}^{21} \rangle \langle n_{p-q}^{21} \rangle - \langle n_{p}^{11} \rangle \langle n_{p-q}^{11} \rangle \right) \right] + \\
\left. + n_{B} \left[ \Gamma_{p,q}^{1} \left( \langle n_{p}^{11} \rangle \langle n_{p-q}^{21} \rangle - \langle n_{p}^{21} \rangle \langle 1 - n_{p-q}^{11} \rangle \right) + \Gamma_{p,q}^{2} \left( \langle n_{p-q}^{21} \rangle \langle n_{p}^{21} \rangle - \langle n_{p}^{21} \rangle \langle n_{p}^{21} \rangle \right) \right] \right\} .
\]

Analogously, the dissipative part due to the radiative recombination reads \( \Gamma_{rr} = \pi |w|^{2} \)

\[
\frac{d}{dt} \langle n_{p}^{21} \rangle_{rr} = - \Gamma_{rr} \langle n_{p}^{21} \rangle \langle 1 - n_{p}^{11} \rangle \tag{S37}
\]

D. Anomalous interband correlations

Finally, we address the time evolution of the anomalous interband correlations \( s_{p}^{21} = c_{p}^{21} c_{p}^{1\dagger} \). We start again evaluating the commutators of eq. (S12)

\[
[S_{k,q}, s_{p}^{21}] = \left[ \hat{c}_{k}^{\alpha_{1}} c_{k+q}^{\alpha_{2}} c_{p}^{21} + p \right] = \delta_{\alpha_{2}, 2} \delta(k + q - p) c_{p}^{21} c_{p}^{1\dagger} + \delta_{\alpha_{1}, 1} \delta(k + q + p) c_{p}^{21} c_{p}^{1\dagger} = - [s_{p}^{21}, S_{k,q}^{\alpha}] \tag{S39}
\]
Using Wick’s theorem, we can simplify the terms obtained after the evaluation of the commutators. Here we show the ones multiplied in Eq. (S12) by \((1 + n_B)\), i.e. the first line:

\[
\langle c_{k'}^\beta c_k^\beta \rangle = \delta_{\beta,1} \delta(k' + p) \langle s_{p-q}^{21} \rangle \langle 1 - n_{-p+q}^{11} \rangle + \delta_{\beta,2} \delta(k' - p + q) \langle s_{p+q}^{21} \rangle \langle 1 - n_{-p}^{11} \rangle
\]  

(S41)

\[
\langle c_{k'}^\beta c_{k+q}^\beta c_p^\dagger c_{p-q}^{11} \rangle = -\delta_{\alpha,1} \delta(k + p) c_p^{21} c_{p-q}^{11} - \delta_{\alpha,2} \delta(k - p) c_p^{21} c_{p-q}^{11} = -\left[ S_{k,q}^{\alpha} s_{p}^{21} \right].
\]  

(S40)

and here we show the ones multiplied in Eq. (S12) by \(n_B\), i.e. the second line:

\[
\langle c_{k'}^\beta c_k^\beta c_{p-q}^{11} c_{k+q}^\beta \rangle = \delta_{\beta,1} \delta(k' + p) \langle s_{p-q}^{21} \rangle \langle 1 - n_{-p+q}^{11} \rangle + \delta_{\beta,2} \delta(k' - p + q) \langle s_{p}^{21} \rangle \langle 1 - n_{-p}^{11} \rangle
\]  

(S45)

\[
\langle c_{k'}^\beta c_k^\beta c_{p-q}^{11} c_{k+q}^\beta \rangle = -\delta_{\beta,1} \delta(k' - p) \langle s_{p+q}^{21} \rangle \langle n_{-p}^{11} \rangle - \delta_{\beta,2} \delta(k' + p + q) \langle s_{p}^{21} \rangle \langle n_{-p}^{11} \rangle
\]  

(S46)

\[
\langle c_p^{21} c_{p-q}^{11} c_{k}^\beta c_{k+q}^\beta \rangle = -\delta_{\beta,1} \delta(k' + p) \langle s_{p}^{21} \rangle \langle n_{-p+q}^{11} \rangle - \delta_{\beta,2} \delta(k' - p + q) \langle s_{p}^{21} \rangle \langle n_{-p}^{11} \rangle
\]  

(S47)

\[
\langle c_p^{21} c_{p-q}^{11} c_{k}^\beta c_{k+q}^\beta \rangle = -\delta_{\beta,2} \delta(k' - p) \langle s_{p+q}^{21} \rangle \langle n_{-p}^{11} \rangle - \delta_{\beta,1} \delta(k' + p + q) \langle s_{p}^{21} \rangle \langle n_{-p}^{11} \rangle
\]  

(S48)

Hence, the dissipative part of the time evolution of \(\langle s_{p}^{21} \rangle\) due to the acoustic phonons is given by

\[
\frac{d}{dt} \langle s_{p}^{21} \rangle^{ph} = -\sum_q \left\{ [1 + n_B] \left[ \Gamma_{p,q}^{11} \langle s_{p}^{21} \rangle \langle 1 - n_{-p+q}^{11} \rangle + \langle n_{-p+q}^{11} \rangle \langle s_{p}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] + \right. \\
\left. + [1 + n_B] \left[ \Gamma_{p,q}^{22} \langle s_{p}^{21} \rangle \langle 1 - n_{-p+q}^{22} \rangle + \langle n_{-p+q}^{22} \rangle \langle s_{p}^{21} \rangle \langle 1 - n_{p}^{11} \rangle \right] + \\
\left. + n_B \left[ \Gamma_{p,q}^{11} \langle s_{p+q}^{21} \rangle \langle 1 - n_{-p+q}^{11} \rangle + \langle n_{-p+q}^{11} \rangle \langle s_{p+q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] + \\
\left. + n_B \left[ \Gamma_{p,q}^{22} \langle s_{p+q}^{21} \rangle \langle 1 - n_{-p+q}^{22} \rangle + \langle n_{-p+q}^{22} \rangle \langle s_{p+q}^{21} \rangle \langle 1 - n_{p}^{11} \rangle \right] + \right. \\
\left. + n_B \left[ \Gamma_{p,q}^{11} \langle s_{p+q}^{21} \rangle \langle 1 - n_{-p+q}^{11} \rangle + \langle n_{-p+q}^{11} \rangle \langle s_{p+q}^{21} \rangle \langle 1 - n_{p}^{22} \rangle \right] + \right. \\
\left. + n_B \left[ \Gamma_{p,q}^{22} \langle s_{p+q}^{21} \rangle \langle 1 - n_{-p+q}^{22} \rangle + \langle n_{-p+q}^{22} \rangle \langle s_{p+q}^{21} \rangle \langle 1 - n_{p}^{11} \rangle \right] \right\}.
\]

(S49)

Analogously, the dissipative part due to the radiative recombination reads

\[
\frac{d}{dt} \langle s_{p}^{21} \rangle^{rr} = 0.
\]  

(S50)

This feature ensures that radiative recombination is not detrimental for superconducting correlations.
E. Summary \(T = 0\)

We summarize the complete time evolution of the observables in the particular case of zero temperature, where \(n_B = 0\).

\[
\frac{d}{dt} \langle n^{12}_{p} \rangle = - \Omega \text{Im} \left\{ \langle n^{21}_{p} \rangle \right\} - 2 \text{Im} \left\{ \Delta \langle s^{21}_{p} \rangle \right\} + \Gamma^{rr} \langle n^{22}_{p} \rangle (1 - n^{11}_{p}) + \]
\[
+ 2 \sum_{q} \left\{ \left[ \Gamma^{1}_{p-q,q} \langle n^{11}_{p-q} \rangle (1 - n^{11}_{p}) - \Gamma^{2}_{p-q,q} \text{Re} \left\{ \langle n^{21}_{p-q} \rangle \langle n^{12}_{p} \rangle \right\} - \Gamma^{2}_{p-q,q} \text{Re} \left\{ \langle s^{21}_{p} \rangle \langle s^{21}_{p-q} \rangle \right\} \right]\right. + (S51)
\]

\[
\frac{d}{dt} \langle n^{22}_{p} \rangle = + \Omega \text{Im} \left\{ \langle n^{21}_{p} \rangle \right\} - 2 \text{Im} \left\{ \Delta \langle s^{21}_{p} \rangle \right\} - \Gamma^{rr} \langle n^{22}_{p} \rangle (1 - n^{11}_{p}) + \]
\[
+ 2 \sum_{q} \left\{ \left[ \Gamma^{1}_{p-q,q} \langle n^{22}_{p-q} \rangle (1 - n^{22}_{p}) - \Gamma^{1}_{p-q,q} \text{Re} \left\{ \langle n^{12}_{p} \rangle \langle n^{21}_{p} \rangle \right\} - \Gamma^{1}_{p-q,q} \text{Re} \left\{ \langle s^{12}_{p} \rangle \langle s^{12}_{p-q} \rangle \right\} \right]\right. + (S52)
\]

\[
\frac{d}{dt} \langle n^{21}_{p} \rangle = - \frac{\Omega}{2} \langle n^{22} - n^{11} \rangle + i \epsilon(p) \langle n^{21}_{p} \rangle - \Gamma^{rr} \langle n^{21}_{p} \rangle (1 - n^{11}_{p}) + \]
\[
+ \sum_{q} \left\{ \left[ \Gamma^{1}_{p-q,q} \langle n^{21}_{p-q} \rangle (1 - n^{11}_{p}) - \langle n^{11}_{p} \rangle \langle n^{21}_{p} \rangle \right] + \Gamma^{2}_{p-q,q} \left\{ \langle n^{21}_{p-q} \rangle (1 - n^{22}_{p}) - \langle n^{22}_{p} \rangle \langle n^{21}_{p} \rangle \right\} \right\] + (S53)
\]

\[
\frac{d}{dt} \langle s^{21}_{p} \rangle = + i \Delta^{*} \langle n^{22} + n^{11} \rangle - 1 + i E(p) \langle s^{21}_{p} \rangle + \]
\[
- \sum_{q} \left\{ \left[ \Gamma^{1}_{p,q} \langle s^{21}_{p} \rangle (1 - n^{11}_{p+q}) + \langle n^{11}_{p} \rangle \langle s^{21}_{p+q} \rangle \right] + \Gamma^{2}_{p,q} \left\{ \langle s^{21}_{p-q} \rangle (1 - n^{22}_{p}) + \langle n^{22}_{p} \rangle \langle s^{21}_{p} \rangle \right\} \right\] + (S54)
\]

where \(\epsilon(p) = E_2(p) - E_1(p) - \nu\) and \(E(p) = E_2(p) + E_1(p)\).

From these equations, as we discuss in the main text, we observe that interband pairing \(\langle s^{21} \rangle \) is enhanced and can be brought out of the trivial vanishing solution by the condition \(\langle n^{22} + n^{11} \rangle \neq 1\). If the dynamics of the system is efficient, i.e. is such that populations of the conduction and valence bands satisfy this condition at shorter timescales with respect to the time necessary to achieve the steady state, such state can be characterized by a finite superconducting order parameter. The aforementioned condition is, hence, necessary but not sufficient to achieve superconductivity.

III. SOLUTION FOR QUADRATIC BANDS AT \(T = 0\)

We now focus on a specific form of the semiconductor bands (assuming spherical symmetry, so that the \(d\)-dimensional model can be reduced to an equivalent one-dimensional model):

\[
E_1(p) = A_1p^2 - E_g/2 \quad \quad E_2(p) = A_2p^2 + E_g/2 \quad \quad (S55)
\]

where \(E_g\) represents the gap amplitude in \(p = 0\). We are able to identify the allowed \(q\) selected by the different \(\Gamma_{p,q}\) in the time evolution equations. Note that, as we discuss in the main text, since we are considering the zero temperature case phonons can only be emitted: the only possible transitions are the ones where the electron acquires a momentum \(\pm q\) and loses an amount of energy equal to \(\nu |q|\). According to this picture, if the band concavity is positive then the phonon transitions tend to move the electronic population to the center of the band, while, in the
The time evolution equations in Section II E can be solved numerically. In contrast to standard mean-field calculations, there is no need to solve it self-consistently together with an equation for the order parameter. Indeed the "self-consistency" is taken into account by the non-linearity of our master equation: $\Delta$ is a time-dependent quantity and there is no need for a self-consistency loop. In order to numerically solve Eqs. (S51, S52, S53, S54), we have to consider the momentum $p$ as a discrete variable and we convert the energy conservation Dirac deltas into the momentum representation:

$$\delta \left[ E_\beta(p - q) - E_\beta(p) - v|q| \right] = \frac{1}{2A_{\beta p} + v} \delta \left[ q - 2p - \frac{v}{A_{\beta}} \right] \Theta \left( p + \frac{v}{2A_{\beta}} \right) + \frac{1}{2A_{\beta p} - v} \delta \left[ q - 2p + \frac{v}{A_{\beta}} \right] \Theta \left( -p + \frac{v}{2A_{\beta}} \right),$$

$$\delta \left[ E_\beta(p) - E_\beta(p + q) - v|q| \right] = \frac{1}{2A_{\beta p} + v} \delta \left[ q + 2p + \frac{v}{A_{\beta}} \right] \Theta \left( -p - \frac{v}{2A_{\beta}} \right) + \frac{1}{2A_{\beta p} - v} \delta \left[ q + 2p - \frac{v}{A_{\beta}} \right] \Theta \left( p - \frac{v}{2A_{\beta}} \right),$$

$$\delta \left[ E_\beta(-p - q) - E_\beta(-p) - v|q| \right] = \frac{1}{2A_{\beta p} - v} \delta \left[ q + 2p - \frac{v}{A_{\beta}} \right] \Theta \left( -p - \frac{v}{2A_{\beta}} \right) + \frac{1}{2A_{\beta p} + v} \delta \left[ q + 2p + \frac{v}{A_{\beta}} \right] \Theta \left( p + \frac{v}{2A_{\beta}} \right),$$

$$\delta \left[ E_\beta(-p) - E_\beta(-p + q) - v|q| \right] = \frac{1}{2A_{\beta p} - v} \delta \left[ q - 2p + \frac{v}{A_{\beta}} \right] \Theta \left( p - \frac{v}{2A_{\beta}} \right) + \frac{1}{2A_{\beta p} + v} \delta \left[ q - 2p - \frac{v}{A_{\beta}} \right] \Theta \left( -p - \frac{v}{2A_{\beta}} \right).$$

Note that every electronic state $p$ is influenced by dissipation and phonons induce a constant momentum exchange given by their linear dispersion relation.

When we convert the energy conservation Dirac delta from energy to momentum space, we obtain the full $k$ dependence of the scattering rate $\Gamma_{ph}$. Such dependence is due to both the density of states and the electron-phonon coupling strength $t_{k,q}$. In order to mimic realistic scenarios, which apply at low temperature for example to prototypical semiconductors like Silicon and Germanium, we set the coupling constants $t_{k,q}$ such that the corresponding scattering rate $\Gamma_{ph}$ is approximately constant over the momentum window close to the resonance.

A problem which arises in this picture is the implementation of the particle number conservation in the system, which has to hold in the $\Delta = 0$ regime. To fulfill this condition, we ensure that our equations satisfy the principle of detailed balance. Furthermore, since we are considering a finite region in the momentum space near the laser resonance, where the rotating wave approximation works, states near the two edges of this box can only lose(receive) particles if the concavity of the band is positive(negative).

Note that all the energies are measured with respect to the gap amplitude $E_g$ at $k = 0$, which is set to one for simplicity and is our unit of energy. We have performed all calculations with a standard 4-th order Runge-Kutta method.

### A. Numerical solution without superconductivity

In this section we show the numerical solution of the system in presence of dissipation and electric field. The latter is tuned at the resonant $k = 0$ point, where both bands have their minimum. In particular this condition amounts to fixing the laser frequency to $\nu = E_g = 1$. We use the same parameters as in the previous section, the amplitude of the laser is set to $\Omega = 0.25E_g$, and the initial conditions coincide with the ground state of the system: $\langle n_{k1}^{11}(t = 0) \rangle = 1$, $\langle n_{k2}^{22}(t = 0) \rangle = 0$, and $\langle n_{k1}^{21}(t = 0) \rangle = 0$.

The resulting stationary state achieved starting from these initial conditions is shown in Figs. S1 (a) and (b).

The stationary state is characterized by a very similarly populated conduction and valence bands and a vanishing interband correlation. Analogous results are obtained by $\Gamma^{tr} = \Gamma_{ph}/10$.

### B. Complete numerical solution

In this section we focus on the full system of coupled equation described in Section II E. The superconducting order parameter $\Delta$, which derives from the mean field approximation performed in the density-density interaction...
FIG. S1. a) Occupation number of valence \( \langle n_{1k}^{11} \rangle \) and conduction \( \langle n_{k}^{22} \rangle \) bands in the stationary state. b) Real and imaginary part of the interband correlation \( \langle n_{k}^{21} \rangle \) in the stationary state. Parameters: \( A_1 = 8E_g, A_2 = 23E_g, k_c = 0.2, N_k = 100, \Gamma_{ph} = 0.01E_g, \Gamma_{rr} = \Gamma_{ph}, \) and \( \Omega = 0.25E_g \).

Hamiltonian in Eq. (S1), is given by:

\[
\Delta = \frac{V}{N} \sum_{k} \langle s_{k}^{21} \rangle \xrightarrow{N \to \infty} \frac{V}{2\pi} \int_{-k_c}^{k_c} \langle s_{k}^{21} \rangle \, dk
\]  

(S60)

Note that we do not need any correction to this formula to achieve a non-zero \( \Delta \) in the stationary state reached by the system. Conversely, in Goldstein et al.'s derivation [S1], this is a necessary condition.

We start with the initial conditions used in the previous sections and in addition we set \( \langle s_{k}^{21}(t = 0) \rangle = 10^{-20}(1 + i) \). Note that the time evolution is strongly independent from this value: \( \Delta \), as a function of \( t \), approaches always a value numerically compatible with zero before having a jump and stabilizing to its stationary value (see for example Figure S3 (b), where the intensity of the repulsive interaction is set to \( V = 5E_g \)). We observe in Figure S2 that in

the stationary state the populations are smoothed and do not resemble a box function. Furthermore the interband correlations are not zero in the region where the populations of the valence and conduction bands become slightly different. In this region, where both the bands are partially filled, we find that \( \langle n_{k}^{21} \rangle \neq 0 \). Then, since the laser
FIG. S3. (a) Real and imaginary part of the anomalous interband correlation $\langle s_{k}^{21} \rangle$ in the stationary state. (b) Modulus of the superconducting gap $\Delta$ as a function of time. Parameters: $A_1 = 8E_g$, $A_2 = 23E_g$, $k_c = 0.2$, $N_k = 100$, $\Gamma^{\text{ph}} = 0.01E_g$, $\Omega = 0.25E_g$ and $V = 5E_g$. Here the interband dissipation is switched on and has the same intensity of the intraband one ($\Gamma^{rr} = \Gamma^{\text{ph}}$).

cancels out the energy difference between the valence and conduction bands, the electrons occupy with almost the same probability the two bands. This is also the region where the modulus of the anomalous interband correlator has its maxima (see Fig. S3 (a)). Note that $\langle s_{k}^{21} \rangle \neq 0$ in the whole momentum region considered, but tends to zero near the boundaries of the momentum space region considered.

[S1] G. Goldstein, C. Aron, and C. Chamon, Phys. Rev. B 91, 054517 (2015).
[S2] M. I. Dyakonov, Spin Physics in Semiconductors, Springer Series in Solid-State Sciences (2017).
[S3] I. Esin, M. Rudner, G. Rafael, and N. Lindner, Phys. Rev. B 97, 245401 (2018).
[S4] R. R. Puri, Mathematical methods of quantum optics, Springer (2001).
[S5] H. Haug and W. Koch, Quantum theory of the optical and electronics properties of semiconductors, World Scientific (2004).
[S6] O. Hart, G. Goldstein, C. Chamon, and C. Castelnovo, arXiv:1810.12309
[S7] R. van Leeuwen and G. Stefanucci, Phys. Rev. B 85, 115119 (2012).
[S8] N. Tandon, J. D. Albrecht and L. R. Ram-Mohan, Journal of Applied Physics 118, 045713 (2015)