Superfluid behaviour of a two-dimensional Bose gas

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Owing to thermal fluctuations, two-dimensional (2D) systems cannot undergo a conventional phase transition associated with the breaking of a continuous symmetry. Nevertheless they may exhibit a phase transition to a state with quasi-long-range order via the Berezinskii–Kosterlitz–Thouless (BKT) mechanism. A paradigm example is the 2D Bose fluid, such as a liquid helium film, which cannot condense at non-zero temperature although it becomes superfluid above a critical phase space density. The quasi-long-range coherence and the microscopic nature of the BKT transition were recently explored with ultracold atomic gases. However, a direct observation of superfluidity in terms of frictionless flow is still missing for these systems. Here we probe the superfluidity of a 2D trapped Bose gas using a moving obstacle formed by a micrometre-sized laser beam. We find a dramatic variation of the response of the fluid, depending on its degree of degeneracy at the obstacle location.

‘Flow without friction’ is a hallmark of superfluidity. It corresponds to a metastable state in which the fluid has a non-zero relative velocity with respect to an external body such as the wall of the container or an impurity. This metastable state is separated from the equilibrium state of the system (v = 0) by a large energy barrier, so that the flow can persist for a macroscopic time. The height of the barrier decreases as increases, and eventually passes below a threshold (proportional to the thermal energy) for a critical velocity . The microscopic mechanism limiting the barrier height depends on the nature of the defect and is associated with the creation of phonons and/or vortices. Whereas the quantitative comparison between experiments and theory is complicated for liquid , cold atomic gases in the weakly interacting regime are well suited for precise tests of many-body physics. In particular, superfluidity was observed in 3D atomic gases by stirring a laser beam or an optical lattice through bosonic or fermionic fluids and by observing the resulting heating or excitations. Here we transpose this search for dissipation-less motion to a disc-shaped, non-homogeneous 2D Bose gas. We use a small obstacle to locally perturb the system. The obstacle moves at constant velocity on a circle centred on the cloud, allowing us to probe the gas at a fixed density. We repeat the experiment for various atom numbers, temperatures and stirring radii and identify a critical point for superfluid behaviour.

Our experiments are performed with 2D Bose gases of confined in the vertical direction by the harmonic potential and in the horizontal plane by the radially symmetric harmonic potential (see ref. 14). The trap frequencies are Hz in the horizontal plane and Hz in the vertical direction. We use gases with temperature and central chemical potential in the range 65–120 nK and km, respectively, where is the Boltzmann constant. The interaction energy per particle is given by , where is the 2D spatial density (typically 100 atoms μm−2 in the centre), is the atomic mass, is the dimensionless interaction strength and is Planck’s constant divided by . Here .

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is the 3D scattering length and $l_c = \sqrt{n/m\omega}$. The energy $\hbar \omega_0 (k_B \times 70 \text{nK})$ is comparable to $k_B T$ and $U_{\text{int}} (\sim k_B \times 40 \text{nK at the trap centre})$. Thanks to Bose statistics, which limits to typically 10% the fractional atomic density in the axially excited states at the obstacle position, our gas is well described by the quasi-2D fluid model (see Supplementary material of ref. 14).

We stir the cloud with a laser beam which creates a repulsive potential with height $V_{\text{int}} \approx k_B \times 80 \text{nK}$. This is at least twice the local chemical potential $\mu_{\text{loc}}(r) = \mu - V(r)$. The beam has a Gaussian profile with a waist of $w_0 = 2.0(5) \mu\text{m}$, which is larger than the local healing length $\xi = 1/\sqrt{8\pi n} (\approx 0.3 \mu\text{m at the trap centre})$, but small compared to the size of the cloud (full width at half maximum $\approx 25 \mu\text{m}$) (see Fig. 1). We stir for typically $t_{\text{stir}} = 0.2 \text{s at constant velocity } v_n$ in a circle of radius $r_n$ centred on the cloud. The intensity of the stirring beam is ramped on and off in $\approx 5 \text{ ms without any significant additional heating}$. Once the stirring beam is switched off, we let the cloud relax for 0 s and the stirring beam at constant velocity $v_n$. We show in Fig. 3a the fitted critical temperature $T_f$ which describes the heating of a 2D superfluid in the presence of a moving point-like defect15. In equation (1) the three fit parameters are the temperature at zero velocity $T_{f,0}$, the heating coefficient $\kappa$, and the critical velocity $v_c$. In the normal state, the fit gives $v_c \approx 0$ and the according quadratic heating stems from the linear scaling of the drag force. In the absence of the stirring beam, there is no significant heating and we measure the temperature $T_0$. The presence of the stirring beam at zero velocity leads to a ‘background heating’ $T_{f,0} - T_0 \approx 10 \text{nK}$, which we attribute to photon scattering. In the following, we use the mean temperature $\bar{T} = \langle T_0 + T_{f,0} \rangle/2$ to characterize the cloud.

In Fig. 3, we summarize our data obtained for different configurations $(N, T_f, r)$. We show in Fig. 3a the fitted critical velocities versus the single parameter $\mu_{\text{loc}}(r)/k_B T$. The relevance of this parameter results from two points. First, because of the local character of the excitation, the response of the fluid to the moving perturbation is expected to be similar to that of a uniform gas with the same temperature and the chemical potential $\mu_{\text{loc}}$. Second, the scale invariance of the weakly-interacting 2D Bose gas implies that the thermodynamic properties do not depend separately on $\mu$ and $T_f$, but only on the ratio $\mu/k_B T$ (see refs 14,16,17). In particular, this ratio is univocally related to the phase space density, and thus characterizes the degree of degeneracy of the cloud.

Remarkably, the ensemble of our data for $v_c$ when plotted as a function of $\mu_{\text{loc}}/k_B T$ shows a threshold between values compatible with zero and clearly non-zero values. This threshold is located at $\mu_{\text{loc}}/k_B T \approx 0.24$, above the prediction $(\mu/k_B T)_c = 0.15$ for the superfluid phase transition in a uniform system16 with $g = 0.093$. If we assume that the stirrer must stand entirely in the superfluid core to yield a non-zero critical velocity, then the deviation can be

\[ T_f(v) = T_{f,0} + \kappa \cdot t_{\text{stir}} \cdot \max\{v_n^2 - v_c^2, 0\} \]
advantageous one (see figure 2d). For comparison, figure 2c shows the heating coefficient $\kappa$ as a function of $\mu_{\text{loc}}/k_{\text{B}} T$ for the normal data (red circles) and the superfluid data (blue circles). The red solid line shows a fit of $\kappa$ linear in the normal density, as expected from a single-particle model. The blue dashed line shows an empirical fit quadratic in the superfluid density. The calculation for the densities assumes $\bar{v} = 90 \text{ nK}$ and the densities are averaged over the size of the stirring beam. The $x$ error bars indicate the region of $\mu_{\text{loc}}/k_{\text{B}} T$ that is traced by the stirring beam due to its size (using the $1/\sqrt{\pi}$ width of the beam) and due to the ‘background heating’. The $y$ error bars are fitting errors.

The region of $\mu_{\text{loc}}/k_{\text{B}} T$ corresponding to the extent of this beam is indicated by the horizontal error bars in Fig. 3a. Note that the size of our trapped atomic clouds might also shift the BKT transition, but the effect is expected to be small (a few per cent) and in the opposite direction.

We limit the presented stirring radii to $r \geq 10 \mu$m such that the stirring frequencies $\omega = v/\bar{v}$ for the relevant velocities $v \sim v_c$ are well below $\omega_0$. Indeed, smaller radii correspond to a larger centripetal acceleration. This could lead to additional heating via the phonon analog of synchrotron radiation, as observed in the formally similar context of capillary waves generated by a rotating object.

For a homogeneous system, the value of the critical velocity is limited by two dissipation mechanisms, the excitation of phonons or vortices. For a point-like obstacle, phonon excitation dominates and $v_c$ is equal to the speed of sound, given in the zero-temperature limit by $v_c = h / \sqrt{\pi \hbar n} / m$ ($\sim 1.6 \text{ mm s}^{-1}$ for $n = 50 \text{ atoms mm}^{-2}$) (this situation is described by the celebrated Landau criterion). When the obstacle size $w_0$ increases and becomes comparable to $\xi$, dissipation via the nucleation of vortex–antivortex pairs (vortex rings in 3D) becomes significant. The corresponding $v_c$ is then notably reduced with respect to $v_c$. In the limit of very large obstacles ($w_0 \gg \xi$), an analytical analysis of the superfluid flow stability yields $v_c \sim \hbar / mw_0 \ll v_c$ (see refs. 22,23). With an obstacle size $w_0 \gtrsim \xi$, our experimental situation is intermediate between these two asymptotic regimes. For a non-homogeneous system such as ours with the stirring obstacle close to the border of the expected superfluid regime, one can also excite surface modes$^{24,25}$, which constitute a further dissipation mechanism.

Our measured critical velocities are in the range 0.5–1.0 mm s$^{-1}$, that is, $v_c / c_s \approx 0.3–0.6$. By contrast, previous experiments in 3D clouds found lower fractions, $v_c / c_s \approx 0.1$ (see ref. 9). The difference may be due to the larger size of the obstacles that were used, and to the average along the axis of the stirring beam of the density distribution in the 3D gas. The dominant dissipation mechanism could be revealed, for example, by directly observing the created vortex pairs as in ref. 12 or interferometrically detecting the Cerenkov-like wave pattern for $v > v_c$ as in experiments with a non-equilibrium 2D superfluid of exciton–polariton quasi-particles.

Figure 3b shows the fitted heating coefficients $\kappa$ for the normal (red circles) and superfluid data (blue circles). In the normal region, we expect the heating to scale linearly with the normal density $n_{\text{no}}$ (see ref. 10). Using the prediction of ref. 16 for $n_{\text{no}}$ (averaged over the size of the stirring beam) we fit $\kappa = v_c / c_s n_{\text{no}}$ and obtain $v_c \approx 3 \times 10^{-4}$ nK s. This value is in reasonable agreement with the prediction of a model$^{10}$ of a single particle with a thermal velocity distribution of mean $\bar{v} = (\hbar / k_{\text{B}} T) / 2m$ colliding with a moving hard wall of width $L = w_0$, yielding $v_c = 16mL^2 / \pi k_{\text{B}} T \approx 6 \times 10^{-4}$ nK s (for $N = 65,000$ and $T = 90 \text{ nK}$). In particular our data nicely reproduce the maximum of $n_{\text{no}}$ around the expected superfluid transition point. In the superfluid case and $v > v_c$, we empirically fit a quadratic scaling of the heating with density $\kappa = v_c / c_s n_{\text{no}}$ and find $v_c = 8 \times 10^{-3}$ nK s $\mu$m$^{-2}$. In principle, one could develop a more refined model to describe the superfluid region, by taking into account the coexistence of the normal and superfluid states via the sum of two heating terms. However, within the accuracy of our data, we did not find any evidence for the need of such a more refined description.

We have presented a direct proof of the superfluid character of a trapped 2D Bose gas. An interesting extension of our work would be the study of superfluidity from the complementary point of view of persistent currents, by adapting to 2D the pioneering experiments performed in 3D toroidal traps.

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Author contributions
The experiments were carried out by R.D., L.C., T.Y. and C.W.; the stirring laser set-up was developed by J.L. and J.B.; C.W., R.D. and L.C. analysed the data; C.W. wrote the manuscript with the input from all coauthors; J.B. and J.D. planned and supervised the project.

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Competing financial interests
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