Properties of the negative effective magnetic pressure instability

K. Kemel1,2,⋆, A. Brandenburg1,2, N. Kleeorin3,1, and I. Rogachevskii3,1

1 Nordita⋆⋆, AlbaNova University Center, Roslagstullsbacken 23, SE 10691 Stockholm, Sweden
2 Department of Astronomy, AlbaNova University Center, Stockholm University, SE 10691 Stockholm, Sweden
3 Department of Mechanical Engineering, The Ben-Gurion University of the Negev, POB 653, Beer-Sheva 84105, Israel

Published online 2012 Feb 15

Key words magnetohydrodynamics (MHD) – instabilities – turbulence

1 Introduction

The concept of turbulent viscosity is often used in astrophysical and other applications in recognition of the fact that the microscopic viscosity is far too small to be relevant on the length scales under consideration. Turbulent viscosity is the simplest parameterization of the Reynolds stress tensor on the length scales under consideration. Turbulent viscosity is far too small to be relevant

As was demonstrated in earlier studies, turbulence can result in a negative contribution to the effective mean magnetic pressure, which, in turn, can cause a large-scale instability. In this study, hydromagnetic mean-field modelling is performed for an isothermally stratified layer in the presence of a horizontal magnetic field. The negative effective magnetic pressure instability (NEMPI) is comprehensively investigated. It is shown that, if the effect of turbulence on the mean magnetic tension force vanishes, which is consistent with results from direct numerical simulations of forced turbulence, the fastest growing eigenmodes of NEMPI are two-dimensional. The growth rate is found to depend on a parameter β, characterizing the turbulent contribution of the effective mean magnetic pressure for moderately strong mean magnetic fields. A fit formula is proposed that gives the growth rate as a function of turbulent kinematic viscosity, turbulent magnetic diffusivity, the density scale height, and the parameter β. The strength of the imposed magnetic field does not explicitly enter provided the location of the vertical boundaries are chosen such that the maximum of the eigenmode of NEMPI fits into the domain. The formation of sunspots and solar active regions is discussed as possible applications of NEMPI.

magnetic Reynolds numbers, but this would then be beyond the validity of their approximation. Using the spectral approach, Kleeorin et al. (1989, 1990) do indeed find this reversal of the sign of the total magnetic tension force. In addition, they find a reversal of the sign of the effective magnetic pressure term; see also Kleeorin & Rogachevskii (1994) and Kleeorin et al. (1993, 1996). Rogachevskii & Kleeorin (2007) argue that, in a stratified medium, this can lead to the formation of large-scale magnetic flux structures and perhaps even sunspots – or at least active regions.

Recently, direct numerical simulations (DNS) of both unstratified and stratified forced turbulence (Brandenburg et al. 2010, 2012; hereafter referred to as BKR and BKKR, respectively) have substantiated this idea and have demonstrated that the effective magnetic pressure can indeed change sign. Similar results have now also been obtained for turbulent convection (Käpylä et al. 2012). In addition, these papers give results of mean-field calculations illustrating that there is a negative effective magnetic pressure instability (hereafter referred to as NEMPI) when there is sufficient density stratification.

NEMPI is a convective type instability related to the interchange instability in plasmas (Tsierkovnikov 1960; Newcomb 1961; Priest 1982) and the magnetic buoyancy instability in the astrophysical context (Parker 1966). The free energy in interchange and magnetic buoyancy instabilities is drawn from the gravitational field, while in NEMPI it is provided by the small-scale turbulence.

The mechanism of NEMPI works even under isothermal conditions when entropy evolution is ignored and an
isothermal equation of state is used. This has been shown using corresponding mean-field calculations (BKKR). With this reduction to the most elementary aspects of the instability, it has recently been possible to verify the existence of NEMPI also in DNS (Brandenburg et al. 2011, hereafter referred to as BKKMR). This has been a major step forward, because now there is no doubt that one is pursuing a real effect and not just one that works only in the world of mean-field models. Essential to the paper of BKKMR has been a finding from an earlier version of the present one that only two-dimensional mean-field structures are excited. This property allowed meaningful averaging along the direction of the imposed field, making the identification of flux concentrations thus much clearer. The absence of three-dimensional mean-field structures was surprising because three-dimensional mean-field calculations have shown that the mean magnetic field develops structures along the direction of the imposed field (BKR). However, while the mean-field calculations have illustrated the nature of the instability, no systematic survey of solutions has yet been attempted. The purpose of this paper is therefore to clarify some still puzzling aspects concerning NEMPI. Note also that the large-scale flux concentrations observed in DNS of BKKMR have an amplitude of only 15% of the local equipartition field. This implies that the flux concentrations we observe in DNS are often not strong enough to be noticeable without averaging.

In addition to the structures found in BKKMR, other types of structures have recently been reported in Large-Eddy Simulations (LES), which might also be an indication of NEMPI. We have here in mind the radiation magnetoconvection simulations of Kitiashvili et al. (2010), in which one sees the formation of whirlpool-like magnetic structures. Relevant to NEMPI is also the work of Tao et al. (1998), who considered magnetoconvection in the optically thick approximation and find a horizontal segregation into magnetized and non-magnetized regions. The size of the individual regions is such that they encompass several turbulent eddies. This phenomenon might therefore well be associated with an effect that could also be modelled in terms of mean-field theory. However, before we can make such an association, we need to find out more about the properties of NEMPI. In particular, we need to know what is the optimal magnetic field strength, what are the requirements or restrictions on the turbulent velocity, and, finally, how much density stratification is needed to make NEMPI work.

To connect the aforementioned requirements to DNS, we need to have a meaningful parameterization of the turbulence effects. The work done so far has been focussing on measuring a reduction of the turbulent pressure and effective mean magnetic pressure as a function of the local mean magnetic field strength. The shape of the resulting dependence of the effective mean magnetic pressure on the mean magnetic field has been matched to a specific fit formula that can be characterized by two fit parameters that, in turn, can be linked to the minimum effective mean magnetic pressure and critical field strength above which the effect is suppressed. However, there have been indications that this parameterization is not unique and that different combinations of the two fit parameters can result in similar values of minimum effective pressure and the critical field strength. The question therefore arises whether this apparent degeneracy affects the properties of NEMPI.

We mentioned already the fact that NEMPI is capable of exciting three-dimensional structures that show variation along the direction of the mean magnetic field. This would give rise to the worry that the two-dimensional results presented so far may not reflect the properties of the fastest growing mode and may therefore not be relevant to describing NEMPI. However, as will be discussed in this paper, this is not the case, because the degree to which three-dimensional modes are excited depends on the sign and magnitude of one of the turbulence parameters, namely the term characterizing turbulence effects on the magnetic tension force, and that simulations indicate that this sign is not favorable for exciting three-dimensional modes (BKKR, Käpylä et al. 2012). Before we begin addressing the various points, we discuss first the mean-field model and basic setup.

### 2 Mean-field model

In view of further verifications of NEMPI with DNS, it is necessary to be able to reduce the essential physics to a minimum. We will therefore not make any attempt to consider other aspects that would make the model more realistic with respect to the Sun. Given that NEMPI works even under isothermal conditions (BKKR), we adopt an isothermal equation of state where the mean pressure \( \overline{p} \) is linear in the mean density \( \overline{\rho} \), with \( \overline{p} = \overline{\rho} \overline{c}_s^2 \) and \( \overline{c}_s \) being the constant isothermal sound speed. We solve the evolution equations for mean velocity \( \overline{U} \), mean density \( \overline{\rho} \), and mean vector potential \( \overline{A} \), in the form

\[
\partial_t \overline{U} = -\nabla \overline{p} - \overline{\rho} \nabla \ln \overline{\rho} + g + \mathcal{F}_M + \mathcal{F}_K, \tag{1}
\]

\[
\partial_t \overline{\rho} = -\nabla \overline{\rho} \cdot \nabla \overline{p} - \overline{\rho} \nabla \cdot \overline{U}, \tag{2}
\]

\[
\partial_t \overline{A} = \nabla \times \mathcal{B} - (\eta_k + \eta_t) \nabla \ln \overline{\rho}, \tag{3}
\]

where \( \mathcal{F}_M \) is given by

\[
\frac{\overline{p}}{\overline{\rho}} \mathcal{F}_M = -\frac{1}{2} \nabla \left[ (1 - q_p) \mathcal{B} + \mathcal{B} \cdot \nabla \mathcal{B} \right],
\]

and

\[
\mathcal{F}_K = \left( \nu_k + \nu_t \right) \left( \nabla \mathcal{B} \mathcal{B} + \frac{1}{2} \nabla \mathcal{B} \mathcal{B} \right) + 2 \nabla \ln \overline{\rho} \tag{5}
\]

is the total (turbulent plus microscopic) viscous force. Here, \( \mathcal{B} = \frac{1}{2} \mathcal{B} + \frac{1}{2} \mathcal{B} \) is the traceless rate of strain tensor of the mean flow. As in earlier work (BKR, BKK), we approximate \( q_p \) and \( q_\rho \) by simple profiles that are only functions of the ratio \( \beta \equiv \mathcal{B} / \mathcal{B}_\text{eq} \). However, in the earlier work this functional form was described by \( q_\rho(\beta) = q_\rho(0)[1 - (2/\pi) \arctan(\beta^2/\beta^2_0)] \),

\[
\mathcal{B} = \overline{\rho} \overline{c}_s^2 \tag{6}
\]
where $B_{eq}$ is the equipartition field strengths and $\sigma$ stands for subscripts $p$ and $s$, respectively. We refer to this as the arctan fit. In the present paper we use an algebraic fit of the form

$$q_\sigma(\beta) = q_{\sigma 0}[1 - (2/\pi) \arctan(\beta^2/\beta_{\sigma 0}^2)],$$

with a normalization coefficient $q_{\sigma 0}$. This formula is compatible with $B_{eq} = \rho^{1/2} \mu_{rms}$ in BKKMR, where the turbulent rms velocity, $\mu_{rms}$, was approximately constant.

The remaining parameters are the normalized imposed field strength, $B_0/B_{eq0}$, turbulent viscosity and magnetic diffusivity, as well as the parameters $q_{\sigma 0}$ and $\beta_{\sigma}$.

### 3 Results

#### 3.1 Two- and three-dimensional solutions

Earlier work has suggested that the eigenmodes of NEMPI can be three-dimensional (BKR). This could render two-dimensional calculations inadequate if the first excited mode were indeed three-dimensional. However, it turns out that the wavelength of the eigenmode in the direction of the field increases as $q_\sigma$ decreases. In BKR, where three-dimensional ($y$-dependent) solutions to NEMPI were first reported, $q_\sigma$ was chosen to be around 10, and the fastest growing mode was indeed three-dimensional. In Fig. 1 we show that the effective wavenumber of the variation of the field in the $y$ direction decreases with decreasing values of $q_\sigma$. This is shown quantitatively in Fig. 2, where we plot the dependence of the typical value of the field-aligned wavenumber, $k_y$, on the value of $q_{\sigma 0}$. Here, $k_y$ is evaluated in a layer near the surface.

We find that the typical value of $k_y$ grows with increasing values of $q_{\sigma 0}$. In addition, we find that the growth rate of the instability, $\lambda$, increases with $q_{\sigma 0}$ approximately linearly once $q_{\sigma 0}$ exceeds a value of around two. The fact that $k_y \to 0$ as $q_{\sigma 0} \to 0$ is significant, because BKKR and also Käpylä et al. (2012) found from simulations that $q_{\sigma 0} \approx 0$. In that case, the characteristic length scale along the direction of the field becomes infinite and the calculation essentially two-dimensional. Conversely, when studying NEMPI in two dimensions, changing the value of $q_{\sigma 0}$ has no effect on structure formation and the growth rate; see Table 1. However, it is now clear that this is an artifact of restricting the solutions to be two-dimensional.

#### 3.2 Approximate degeneracy in the $q_\sigma$ fit formula

We mentioned in the introduction that recent attempts to determine $q_{\sigma 0}$ from simulations faced the difficulty that the fit formula (7) possesses an approximate degeneracy in that we

![Fig. 2](http://www.an-journal.org)
It is therefore better to measure the normalized minimum effective magnetic pressure, $P_{\text{min}} = \frac{1}{2} \min[(1 - q_p)/\beta^2]$, and its position, $\beta_{\text{min}}$. For the algebraic fit we then obtain the fit parameters

$$
\beta_p = \beta_{\text{min}}^2 \sqrt{-2P_{\text{min}}}, \; \beta_s = \beta_p + \sqrt{-2P_{\text{min}}},
$$

where we have introduced the parameter $\beta_s^2 = q_{p0}\beta_p^2$ in a modified representation

$$
q_p(\beta) = \frac{\beta_s^2}{\beta_p^2 + \beta^2},
$$

which is preferable over Eq. (8) in circumstances where $\beta_s^2 = q_{p0}/\beta_p^2$ is approximately constant. This appears to be the case in recent DNS (BKKR, Kemel et al. 2012), where $\beta_s \approx 0.2$ and $0.3$ in the absence and presence of small-scale dynamo action, respectively.

We have computed mean-field models for different combinations of parameters using the algebraic fit. We find that the resulting growth rate $\lambda$ depends on the functional form of $P_{\text{eff}}(\beta)$ near $\beta = \beta_{\text{min}}$, which manifests itself in a dependence on both $q_{p0}$ and $\beta_p$; see Fig. 4. The lower panel of this figure suggests that the dependence of the growth rate on both parameters can be collapsed onto a single dependence on $\beta_s$. This underlines the usefulness of Eq. (13) as a fit formula. As argued above, this dependence is best constrained by the fit parameters $\beta_{\text{min}}$ and $P_{\text{min}}$.

### 3.3 Onset of NEMPI

With a given prescription of $q_p(\beta)$, assuming here $q_s = 0$, we can now compute two-dimensional mean-field models. Our goal is to obtain a simple formula that can tell us how large the growth rate of the instability is, and what the critical condition for the onset of the instability is. Not much is known about the linear stability properties of NEMPI, so we have to rely on numerical determinations of the growth rates for different wavelengths for different parameters to obtain an approximate representation of the dispersion relation. Earlier work of Kemel et al. (2011) has suggested a relation of the form

$$
\lambda = \Phi(g/c_s^2, q_{p0}, \beta_p, ... - \nu_k k_p^2 - \eta_k k_y^2),
$$

where $k_p$ and $k_y$ are effective wavenumbers quantifying the effects of turbulent viscosity and turbulent magnetic diffusivity, $\Phi$ is a function of the inverse scale height, $H_p^{-1} =$...
Dependence of the normalized growth rate $\lambda/\lambda_0$, with $\lambda_0 \approx (\nu_t + \eta)H^2$ on $q_{p0}$ for $\beta_p = 0.05$ (upper panel), on $\beta_\gamma$ for $q_{p0} = 40$ (middle panel), and on $\beta_s = q_{p0}/2 \beta_p$ (lower panel) for $\beta_p = 0.05$ (red), $q_{p0} = 40$ (circles), and $\beta_p = 0.1$, $q_{p0} = 5$ (blue). Solid lines represent approximate fits given by $9(q_{p0} - 1)^{0.7}$, $(\beta_p/0.0015)^{1.3}$, and $(\beta_s/0.02)^{3/2}$, respectively.

**Fig. 5** Dependence of $\lambda$ on $\nu_t + \eta$, normalized appropriately in terms of $g$ and $c_\gamma$. The negative slope gives $k_\lambda^2$ with $k_\lambda \approx 2.5/H_\rho$, where $H_\rho = c_\gamma^2/g$.

$g/c_\gamma^2$, and other parameters describing the functional form of $q_{p0}$.

We now need to determine the various unknowns. We begin by determining $k_\nu$ and $k_\eta$ by varying either only $\nu_t$ or only $\eta$ at a time. It turns out that $k_\nu = k_\eta \equiv k_\lambda$ satisfies. In this way we obtain a linear fit for the growth rate $\lambda = \text{const} - (\nu_t + \eta)k_\lambda^2$, giving us $k_\lambda$ as the slope of this graph; see Fig.5 We find $k_\lambda \approx 2.5/H_\rho$, superseding earlier results by Kemel et al. (2011) for a different $B_{eq}(z)$ profile.

**Fig. 6** Dependence of $\Phi = \lambda + (\nu_t + \eta)k_\lambda^2$ on $(k_1H_\rho)^{-3/4}$ (upper panel), $\lambda$ on $B_0/B_{eq0}$ (middle panel), as well as $z_B/H_\rho$ and $z_\gamma/H_\rho$ on $B_0/B_{eq0}$ (lower panel). Here, $\beta_{eq0} = 0.0083$ is a fit parameter and $\lambda_d = (\nu_t + \eta)k_d^2$ is used for normalization.

Accepting now the fit parameter $k_d$ as measured, we can proceed to determining the dependence of $\Phi$ on $H_\rho$ (top panel of Fig.6). A convenient non-dimensional quantity is $k_1H_\rho$, where $k_1$ is the wavenumber of the energy-carrying eddies of the turbulence, which is related to $\eta = u_{rms}/3k_1$ with $u_{rms} = B_{eq0}/\rho_0^{1/2}$. Note that $\Phi \propto (k_1H_\rho)^{-3/4}$. Combining this with the $\beta_s^{3/2}$ scaling of Fig.4 we suggest

$$\lambda \approx \left[ (\beta_s/\beta_{eq0})^{3/2}(k_1H_\rho)^{-3/4} - 1 \right] (\nu_t + \eta)k_d^2,$$

with $\beta_{eq0} \approx 0.008$ being yet another fit parameter. Interestingly, $\lambda$ is independent of the imposed field strength, $B_0/B_{eq0}$, provided the bulk of the eigenmode ($z = z_B$) fits well within the domain (middle panel of Fig.6). This has here been achieved by adjusting the positions of the boundaries, $z_{\text{top}}$ and $z_{\text{bot}}$. Indeed, as $B_0/B_{eq0}$ is increased, $z_B$ is found to decrease approximately like $z_B/H_\rho \approx -2 \ln B_0/B_{eq0} + \text{const}$. It turns out that $z_B$ is about 2–3 scale heights below the location $z_\gamma$ where $\mathcal{P}(z)$ attains its minimum value. Contrarily, when $z_\gamma > z_{\text{bot}} > z_B$ or when $z_{\text{top}} < z_B$ NEMPI will depend on respectively $\beta_{\text{bot}}$ or $\beta_{\text{top}}$, when $z_\gamma < z_{\text{bot}}$ there is no instability.

Some comments about the horizontal dimensions are in order. In all cases with $q_{p0} = 0$, we find that in three-dimensional calculations with finite $y$ extent, the value of $L_y$ does not affect the growth rates. On the other hand, dou-
bling the $x$ extent yields two pairs of rolls. This has also been confirmed for DNS; see Kemel et al. (2012).

4 Conclusions

The present work has clarified a number of puzzling aspects of NEMPI. Firstly, it is now clear that we can proceed with two-dimensional mean-field simulations as long as we know that $q_{00} = 0$ (or negative). However, this may not always be the case. The fact that three-dimensional structures can emerge from NEMPI was initially thought to be an interesting aspect, because it could readily explain the formation of bipolar regions (BKR). However, given that simulations now indicate that $q_0 \approx 0$ (or perhaps even negative), this proposal would thus not be an option, unless some other as yet unexplored effect begins to play a role.

In principle, all turbulent transport processes are nonlocal (relative), this proposal would thus not be an option, unless different. This means that it is not sufficient to measure only $\min(\beta)$ and $\beta_{\text{crit}}$, but the growth rates can still be quite different. This means that it is not sufficient to measure only $\min(\beta)$ and $\beta_{\text{crit}}$. Instead, to characterize the functional form of $\beta$ more accurately, we need some other characteristics to represent the dependence of this function near $\beta = 0$. One such possibility is to use the field strength $\beta_{\text{min}}$ for which the minimum of the effective magnetic pressure is reached.

4 Conclusions

The present work has clarified a number of puzzling aspects of NEMPI. Firstly, it is now clear that we can proceed with two-dimensional mean-field simulations as long as we know that $q_{00} = 0$ (or negative). However, this may not always be the case. The fact that three-dimensional structures can emerge from NEMPI was initially thought to be an interesting aspect, because it could readily explain the formation of bipolar regions (BKR). However, given that simulations now indicate that $q_0 \approx 0$ (or perhaps even negative), this proposal would thus not be an option, unless some other as yet unexplored effect begins to play a role.

In principle, all turbulent transport processes are nonlocal (relative), this proposal would thus not be an option, unless different. This means that it is not sufficient to measure only $\min(\beta)$ and $\beta_{\text{crit}}$, but the growth rates can still be quite different. This means that it is not sufficient to measure only $\min(\beta)$ and $\beta_{\text{crit}}$. Instead, to characterize the functional form of $\beta$ more accurately, we need some other characteristics to represent the dependence of this function near $\beta = 0$. One such possibility is to use the field strength $\beta_{\text{min}}$ for which the minimum of the effective magnetic pressure is reached.

Next, we have seen that the degeneracy in the fit formula used for $q_{00}(\beta)$ and $P_{\text{eff}}(\beta)$ is significant in that different combinations of $q_{00}$ and $\beta_{\text{p}}$ result in similar values of $\min(\beta)$ and $\beta_{\text{crit}}$, but the growth rates can still be quite different. This means that it is not sufficient to measure only $\min(\beta)$ and $\beta_{\text{crit}}$. Instead, to characterize the functional form of $\beta(\beta)$ more accurately, we need some other characteristics to represent the dependence of this function near $\beta = 0$. One such possibility is to use the field strength $\beta_{\text{min}}$ for which the minimum of the effective magnetic pressure is reached.

Knowing the value of $\beta_{\text{min}}$ has particular relevance in determining the height where NEMPI occurs. For a given value of the imposed field strength $B_0$, the condition $B_0 / B_{\text{eq}}(z_p) = \beta_{\text{min}}$ determines the height $z_p$, where the effective magnetic pressure attains a minimum, and thus the height $z_B$, which tends to be 2–3 scale heights below $z_{\text{min}}$; see the lower panel of Fig. 6. Therefore, the value of $B_0$ does not directly affect the growth rate of NEMPI.

Finally, we have tried to establish an approximate dispersion relation to estimate the growth rate of NEMPI as a function of turbulent viscosity, turbulent magnetic diffusivity, mean field strength, and the strength of stratification. This formula may serve as a first orientation and can hopefully be improved further with future simulations. This formula can also be useful in connection with analytic estimates concerning the regimes when NEMPI is expected in DNS or under other more realistic circumstances.

Acknowledgements. We thank the anonymous reviewer for making useful suggestions. We acknowledge the NORDITA dynamo programs of 2009 and 2011 for providing a stimulating scientific atmosphere. Computing resources provided by the Swedish National Allocations Committee at the Center for Parallel Computers at the Royal Institute of Technology in Stockholm, the National Supercomputing Center in Linköping, and the High Performance Computing Center North in Umeå. This work was supported in part by the European Research Council under the AstroDyn project 227952 and the Swedish Research Council grant 621-2011-5076.

References

Brandenburg, A., von Rekowsky, B.: 2001, A&A 379, 1153
Brandenburg, A., Kleeorin, N., Rogachevskii, I.: 2010, AN 331, 5 (BKR)
Brandenburg, A., Kemel, K., Kleeorin, N., Mitra, D., Rogachevskii, I.: 2011, ApJ 740, L50 (BKKMR)
Brandenburg, A., Kemel, K., Kleeorin, N., Rogachevskii, I.: 2012, ApJ, submitted, astro-ph/1005.5700 (BKKR)
Brandenburg, A., Rädler, K.-H., Schrinner, M.: 2008, A&A 482, 739
Courvoisier, A., Hughes, D.W., Proctor, M.R.E.: 2010, Proc. Roy. Soc. Lond. 466, 583
Frisch, U., She, Z.S., Sulem, P.L.: 1987, Physica 28D, 382
Käpylä, P.J., Brandenburg, A., Kleeorin, N., Mäntele, M.J., Rogachevskii, I.: 2012, MNRAS, submitted, astro-ph/1104.4541
Kemel, K., Brandenburg, A., Kleeorin, N., Rogachevskii, I.: 2011, in: A. Bonanno, E. de Gouveia dal Pino, A. Kosovichev (eds.), Advances in Plasma Astrophysics, IAU Symp. 274, p.473
Kemel, K., Brandenburg, A., Kleeorin, N., Mitra, D., & Rogachevskii, I.: 2012, Sol. Phys., submitted, astro-ph/1112.0279
Kitaishvili, I.N., Kosovichev, A.G., Wray, A.A., Mansour, N.N.: 2010, ApJ 719, 307
Kleeorin, N., Rogachevskii, I.: 1994, Phys. Rev. E 50, 2716
Kleeorin, N., Mond, M., Rogachevskii, I.: 1993, Phys. Fluids B, 5, 4128
Kleeorin, N., Mond, M., Rogachevskii, I.: 1996, A&A 307, 293
Kleeorin, N.I., Rogachevskii, I.V., Ruzmaikin, A.A.: 1989, Sov. Astron. Lett. 15, 274
Kleeorin, N.I., Rogachevskii, I.V., Ruzmaikin, A.A.: 1990, Sov. Phys. JETP 70, 878
Newcomb, W.A.: 1961, Phys. Fluids 4, 391
Parker, E.N.: 1966, ApJ 145, 811
Priest, E.R.: 1982, Solar Magnetohydrodynamics, D. Reidel Publ. Co., Dordrecht
Rädler, K.-H.: 1974, AN 295, 265
Roberts, P.H., Soward, A.M.: 1975, AN 296, 49
Rogachevskii, I., Kleeorin, N.: 2007, Phys. Rev. E 76, 056307
Rüdiger, G.: 1974, AN 295, 275
Rüdiger, G.: 1980, Geophys. Astrophys. Fluid Dyn. 16, 239
Rüdiger, G.: 1989, Differential Rotation and Stellar Convection: Sun and Solar-Type Stars, Gordon & Breach, New York
Rüdiger, G., Hollerbach, R.: 2004, The Magnetic Universe: Geophysical and Astrophysical Dynamo Theory, Wiley-VCH, Berlin
Rüdiger, G., Tuominen, I., Krause, F., Virtanen, H.: 1986, A&A 166, 306
Rüdiger, G., Kitchatinov, L.L., Schultz, M.: 2012, AN 333, 84
Tao, L., Weiss, N.O., Brownjohn, D.P., Proctor, M.R.E.: 1998, ApJ 496, L39
Tserkovnikov, Y.A.: 1960, Sov. Phys. Dokl. 5, 87