Optical matching system for eliminating lateral position displacement of holographic images under varying laser wavelength

Shou Liu, Xiangsu Zhang and Xuechang Ren
Department of Physics, Xiamen University, Xiamen 361005, China
E-mail: xszhang@xmu.edu.cn

Abstract. The diffraction angle of a hologram is proportional to the wavelength so that a lateral displacement of the holographic image will occur when there is a change in wavelength. Such displacement is more serious when the image is generated at far distance. A matching system including an HOE and a hologram is proposed to eliminate the image lateral displacement under varying wavelength. Theoretical analysis demonstrates that the two elements must have the conditions: (a) They must have the same fringe periods; (b) They must be placed in parallel; (c) The diffracted beam from HOE must be collimated. The two elements were fabricated in the experiment and applied into a holographic sight. A test was carried out on the sight through watching the image lateral movement at 50-meter distance. The experimental result shows that under varying wavelength there is no image lateral displacement observed, verifying the effectiveness of the matching system.

1. Introduction
According to theory, the diffraction angle of a diffractive element is determined by the incident angle and wavelength of the light illuminating the element. Holograms are diffractive elements, so the angular positions of the holographic images are affected by laser wavelength. If the image distance is very long, slight change of diffraction angle caused by wavelength variation will generate large lateral displacement of the image.

A holographic sight uses a laser beam emitted from a laser diode (LD) to illuminate a hologram. The hologram generates a holographic virtual reticle image in the direction perpendicular to the hologram plane at long distance. Through the hologram the shooter can clearly see the reticle image on the target plane therefore can fire immediately, which greatly simplifies the aiming process thus increases the shooting speed. However, when there is a temperature drift on LD, the wavelength of the beam emitted by LD will change. This will cause lateral deviation of the reticle image from the designed position, thus largely reduces the aiming accuracy.

We have designed an optical matching system that can completely eliminate image lateral displacement under varying wavelength. The system consists of a transmission hologram and a transmission holographic optical element (HOE). The HOE is used to compensate the change of diffraction angle of the hologram caused by wavelength change. The matching system has been...
applied to holographic sights. Experimental result shows that for the image reconstructed at 50-meter distance, there is no lateral displacement at all under varying laser wavelength.

2. Theory

2.1. Lateral displacement of the holographic image caused by the change of wavelength

Based on grating equation, the angular position of a holographic image reconstructed from a hologram can be expressed as

$$\sin \phi = \frac{\lambda}{d} - \sin \theta,$$  \hspace{1cm} (1)

where $\phi$ is the image angle, $d$ is a constant representing the average fringe period of the hologram, $\theta$ and $\lambda$ are incident angle and wavelength of the laser beam on hologram respectively. It is clear from equation (1) that when $\lambda$ changes, if the light incident angle $\theta$ keeps the same, image angle $\phi$ is only affected by $\lambda$. If the image is far from the hologram, small change of $\phi$ may cause large lateral displacement as shown in Fig. 1. For the requirement of holographic sights, holographic images must be fixed in the direction of the hologram normal. So equation (1) becomes

$$0 = \frac{\lambda}{d} - \sin \theta.$$ \hspace{1cm} (2)

If the wavelength changes from $\lambda$ to $\lambda'$ while the incident beam angle is maintained, the angular position of the image has the expression

$$\sin \phi' = \frac{\lambda'}{d} - \sin \theta.$$ \hspace{1cm} (3)

Let (3) – (2), we have

$$\sin \phi' = \frac{\lambda' - \lambda}{d}.$$ \hspace{1cm} (4)

![Fig 1. Lateral displacement of the image caused by wavelength change](image)

The wavelength of LDs used in holographic sights is 535 nm. For making the holographic sight compact, the illuminating beam on hologram usually has large incidence. Suppose the beam incident angle $\theta$ is 60°, to make $\phi = 0$ under 535 nm illumination, according to equation (2) the period $d$ of the hologram must be 618 nm. Usually one degree temperature change on LD will cause about 0.3 nm
wavelength change. Suppose temperature has increased 10 degrees, the wavelength of LD will become 538 nm. Inserting $\lambda = 535$ nm, $\lambda' = 538$ nm and $d = 618$ nm into equation (4) we have $\sin \phi' = 4.9 \times 10^{-3}$. If the image distance is 50 m, the lateral displacement of the image will be 24 cm. Holographic sights are usually used for shooting the targets in the distance around tens to one hundred meters, so the lateral displacement will largely affect the aiming accuracy.

2.2. Matching system

From equation (2) we can see that to keep $\phi = 0$ while $\lambda$ is varying, the only way is to change $\theta$ accordingly so that the impact on $\phi$ by varying $\lambda$ can be canceled out. For the convenience of analyzing, equation (2) is rewritten as

$$d \sin \theta = \lambda.$$  \hspace{1cm} (5)

To make angle $\theta$ change accordingly with varying wavelength, an HOE is introduced as the compensation element to form a matching system with the hologram. The HOE is placed before the hologram in the light path, hence the diffracted beam from HOE serves as the illuminating beam to the hologram. The HOE must be able to generate a $\Delta \theta$ while there is a $\Delta \lambda$ so that the following equation is satisfied

$$d \sin(\theta + \Delta \theta) = \lambda + \Delta \lambda.$$  \hspace{1cm} (6)

The angular relation of HOE can be expressed as

$$d'(\sin \alpha + \sin \beta) = \lambda,$$  \hspace{1cm} (7)

where $\alpha$ and $\beta$ are angles of incident and diffracted beams of HOE respectively, $d'$ is the average period of HOE. Let laser beam fall on HOE in normal direction, so $\alpha = 0$. Equation (7) becomes

$$d' \sin \beta = \lambda.$$  \hspace{1cm} (8)

When there is a wavelength change $\Delta \lambda$, the diffraction angle of HOE will have a change $\Delta \beta$. Equation (8) can be rewritten as

$$d' \sin(\beta + \Delta \beta) = \lambda + \Delta \lambda.$$  \hspace{1cm} (9)

Substituting (6) into (9) we have the condition for keeping the holographic image in the direction of hologram normal under varying wavelength

$$d' \sin(\beta + \Delta \beta) = d \sin(\theta + \Delta \theta).$$  \hspace{1cm} (10)

Comparing left and right sides in equation (10), we have

$$d' = d$$  \hspace{1cm} (11)

and

$$\sin(\beta + \Delta \beta) = \sin(\theta + \Delta \theta)$$

$$\downarrow$$

$$\beta + \Delta \beta = \theta + \Delta \theta.$$  \hspace{1cm} (12)

Equation (11) demonstrates that the periods of HOE and hologram must be equal. This can be fulfilled through the fabrication processes of the two elements. Equation (12) demonstrates that the diffraction angle of HOE and the incident angle of hologram must always be equal. This condition can
only be fulfilled if the two elements are positioned in parallel and the diffracted beam of HOE is collimated.

3. Experiment and result
The above analysis indicates that to achieve accurate aiming of holographic sights, the matching system including an HOE and the hologram can be used to maintain the holographic image in the direction of the hologram normal in spite of the wavelength change. In the matching system, the HOE and the hologram must have the following conditions: (a) They must have the same fringe periods; (b) They must be placed in parallel; (c) The diffracted beam from HOE must be collimated.

3.1. Fabrications of HOE and the hologram
In fabricating holographic elements, the periods of the elements are determined by the angles between the object and the reference beams. In order to obtain equal fringe period of the HOE and the hologram, in their recording geometries the angles between the two recording beams were set to be equal. Collimated beam was used as the reference beam in both recordings, and the recording material used in the experiment was silver-halide.

For generating collimated diffraction beam, HOE must be designed as a lens. For making the optical setup in the sight compact, in the experiment the HOE was fabricated as an off-axis transmission holographic lens[1]. Fig. 2 shows the geometry for recording HOE. A laser beam is splitted by a splitter BS into two beams: one is expanded by expander BE₁ then falls on the holographic recording plate H, with the central line of the beam along the plate normal, the other beam is expanded by expander BE₂ and collimated by lens L then falls on the plate H with large incident angle.

![Fig. 2. Recording geometry of HOE](image)

The optical geometry for fabricating hologram was different from that normally used for fabricating 2-dimensional (2D) transmission holograms. For a holographic sight, ideally the distance of the holographic image should be infinite so that clear reticle image can be seen on target plane at any distance. This requires that in recording process the 2D object should be placed at infinite distance. To realize this, a lens was used in the geometry to transform finite object distance into infinite. The lens is marked L₁ in Fig. 3, placed in between of the object and the recording plate. According to Gaussian lens formula[2], if an object is placed at the distance equal to the focal length of the imaging lens, the image distance will be infinite. So in the geometry, the distance between L₁ and the object is
chosen to be the focal length of \( L_1 \). The light path of the reference beam was the same as that used in recording HOE. In image reconstruction, the original reference beam must be used to illuminate the hologram to obtain a virtual image\(^3\).

![Fig. 3. Recording geometry of the hologram](image)

3.2. Optical setup with the matching system for holographic sights

Fig. 4 shows the optical setup with the matching system designed for holographic sights. The HOE and the hologram are both placed vertically but in different height, with the diffracted beam from HOE traveling upward to the hologram. An LD with wavelength 535 nm is placed at the focal point of the HOE with the central line of the beam propagating along HOE normal. The diffracted beam of HOE is collimated with diffraction angle \( \beta \) and illuminates the hologram with incident angle \( \theta \). It is obvious that \( \beta = \theta \). By placing the eye close to the hologram, a virtual reticle image is generated at far distance in hologram normal.

![Fig. 4. Optical setup of the holographic sight with matching system](image)
3.3. Experimental result
The optical setup with the matching system has been assembled into holographic sights. Fig. 5 is the cross-sectional view of the sight, illustrating the layout of optical components and the light path.

The test of the effect of the matching system was carried out by watching the lateral movement of the holographic image under varying wavelength. The sight was pointed at an object 50 m away through a window. After switching on the sight, through the hologram a virtual reticle image could be seen clearly on the object. The room temperature was increased to cause wavelength variation. By watching the image position carefully while the temperature was going up, no image lateral movement was observed.

![Fig. 5. Cross-sectional view of the holographic sight, illustrating the layout of the optical components and the light path](image)

4. Conclusion
An optical matching system has been proposed to eliminate lateral displacement of the holographic images under varying wavelength. The system includes an HOE and a hologram. The two elements must have the following conditions: (a) They must have the same fringe periods; (b) They must be placed in parallel; (c) The diffracted beam from the HOE must be collimated. The matching system is especially useful for far distance aiming. The experimental result obtained from a holographic sight verifies that the proposed matching system can effectively maintain the holographic image in the fixed direction despite the change of wavelength, so accurate aiming can be guaranteed.

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