Resummation and power corrections for factorized cross sections

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Threshold resummation for factorizable cross sections in hadron-hadron collisions has a number of applications and extensions. We discuss factorization scale dependence, resummation at nonleading power in the moment variable, and the implications of resummation for power corrections in the hard momentum scale.

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Abstract
Threshold resummation for factorizable cross sections in hadron-hadron collisions has a number of applications and extensions. We discuss factorization scale dependence, resummation at nonleading power in the moment variable, and the implications of resummation for power corrections in the hard momentum scale.

1. Threshold corrections
The factorization theorem for hard cross sections in QCD may be represented as

\[ \sigma_{AB\rightarrow F}(Q) = \phi_{a/A}(x_a, \mu) \otimes \phi_{b/B}(x_b, \mu) \otimes \hat{\sigma}_{ab\rightarrow F}(z \equiv Q^2/x_a x_b S, Q, \mu, \alpha_s(\mu^2)) , \tag{1} \]

with the \( \phi \)'s parton distributions and \( \hat{\sigma}_{ab\rightarrow F} \) partonic cross sections, calculable in perturbation theory. The point \( z = 1 \) corresponds to “partonic threshold”, at which the partons \( a \) and \( b \) have just enough energy to produce the observed final state, \( F \), and at which \( \hat{\sigma} \) is generally singular. These singularities may be illustrated by moments of the inclusive Drell-Yan cross section,

\[ \int_0^1 d\tau T^{N-1} \frac{1}{\sigma_0} \frac{d\sigma_{AB\rightarrow V}}{dQ^2}(\tau \equiv Q^2/S, Q^2) = \sum_{a,b=qq, g} \tilde{\phi}_{a/A}(N, \mu) \tilde{\phi}_{b/B}(N, \mu) \times \tilde{\omega}_{ab}^{DY}(N, Q, \mu, \alpha_s(\mu^2)) , \tag{2} \]

In this case, the singular corrections in \( z \) and their finite, but logarithmic, moments are of the form

\[ \ln^{2k-1}(1-z) \rightarrow \ln^{2k} N , \tag{3} \]

at \( k \)th order in \( \alpha_s \). The term threshold resummation refers to the summation of all such singular corrections to all orders in \( \alpha_s \). Such a resummed form has been known for some time for the Drell-Yan cross section \( \tilde{\omega}^{DY} \), where the hard-scattering function \( \omega(N) \) is given by an exponential, in \( \overline{\text{MS}} \) scheme as

\[ \tilde{\omega}_{ab}^{DY}(N) \sim \delta_{ab} \exp \left[ -\int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \right. \]

\[ \left. \times \int_{(1-z)^2/Q^2} \frac{d\xi^2}{\xi^2} A^{(aa)}(\alpha_s(\xi^2)) \right] , \tag{4} \]

where \( A \) is a finite function of the running coupling, known explicitly to two loops,

\[ A^{(ab)} = (C_a + C_b) \left( \frac{\alpha_s}{\pi} + \frac{1}{2} K \left( \frac{\alpha_s}{\pi} \right)^2 \right) \]

\[ K = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} \eta_f , \tag{5} \]

with \( C_q \equiv C_F, C_g \equiv C_A \). In this form, all logarithms of \( N \), and hence singular distributions in \( 1-z \), arise from the explicit integrals in Eq. (4), with the coupling running as shown.

Inverting expressions such as (4) gives estimates of the corresponding cross section that include corrections from all orders. Normally, the results do not drastically modify next-to-leading order predictions. At the same time, threshold-resummed cross sections have a much reduced dependence on the factorization scale \( \mu \) in Eq. (4). Yet another application is to study the strong coupling limit \( \xi^2 \rightarrow 0 \) in Eq. (4), as a guide to power corrections.

2. Refactorization and scale dependence
The reduction of scale dependence in resummed cross sections may be understood by “refactorizing” the partonic cross section in terms of parton distributions \( \psi_{a/h}(x) \) defined for a hadron (or...
parton) \( h \), of momentum \( p \), at measured partonic (a) energy \( x p_0 \), rather than fractional momentum \( x p^\mu \),

\[
\int_0^1 d\tau \tau^{N-1} \frac{1}{\sigma_0} \frac{d\sigma_{ab \to V}}{dQ^2}(\tau, Q^2) = \tilde{\phi}_{a/A}(N, \mu) \tilde{\omega}_{ab}^{DY}(N, Q, \mu) \nonumber \]

\[
= \tilde{\psi}_{a/A}(N, Q) U_{ab}(N) \left| H_{ab}(Q) \right|^2, \quad (6)
\]

where we have exhibited the distribution for incoming parton \( a \), with the corresponding factors for parton \( b \) denoted by "•". The flavor-diagonal energy distributions \( \tilde{\psi}_{a/A}(x) \) match the phase space of the annihilation process to \( O(1/N) \), and automatically absorb all leading logs in \( N \) near threshold. Most importantly, they are ultraviolet finite, and are thus independent of the factorization scale \( \mu \). The remaining logs are coherent in the incoming partons \( a \) and \( b \), and are absorbed into an "eikonal" function \( U_{ab}(N) \).

Equating the factorized and refactorized partonic cross sections in Eq. (6), we readily derive

\[
\tilde{\omega}_{ab}^{DY}(N, Q, \mu) = \left[ \frac{\tilde{\psi}_{a/A}(N, Q)}{\tilde{\phi}_{a/A}(N, \mu)} \right]^2 U_{ab} \left| H_{ab} \right|^2, \quad (7)
\]

in which the dependence on \( \mu \) appears only in the denominators \( \tilde{\phi}(N, \mu) \). The ratios of \( \tilde{\psi} \) to \( \tilde{\phi} \) may be computed, to exhibit the exponentiation of \( N \)-dependence in Eq. (6) above.

Inserting (7) into the hadronic cross section, we have, in the same notation,

\[
\sigma_{AB}^{DY}(N) = \tilde{\phi}_{a/A}(N, \mu) \tilde{\omega}_{ab}^{DY}(N, Q, \mu) \nonumber \]

\[
= \tilde{\phi}_{a/A}(N, \mu) \left[ \frac{\tilde{\psi}_{a/A}(N, Q)}{\tilde{\phi}_{a/A}(N, \mu)} \right]^2 U_{ab} \left| H_{ab} \right|^2. \quad (8)
\]

Noting, however, that the scale dependence of the \( \tilde{\phi} \)'s is universal, we have

\[
\mu \frac{d}{d\mu} \left[ \frac{\tilde{\phi}_{a/A}(N, \mu)}{\tilde{\phi}_{a/A}(N, \mu)} \right] = 0, \quad (9)
\]

so that \( \mu \)-dependence disappears (to \( O(1/N) \)), as observed in explicit applications of the formalism.

3. Resummation beyond logs of \( N \)

The resummation described above organizes all logs of the moment variable \( N \). At this, leading, power in \( N \) we may neglect mixing between parton species. We must, however, recall the long-known result that at high evolution scales, the large-\( N \) behavior of the gluon distribution is eventually dominated by the quarks,

\[
G(N, \mu) \sim \frac{\Sigma(N, \mu)}{N \ln N}, \quad (10)
\]

where \( \Sigma \) is a singlet combination of quark distributions. There is thus a need to extend the resummation formalism to \( O(1/N) \).

Contributions of the form \( \ln^m N/N \), in particular, are potentially resummable, since they correspond to integrable, but logarithmically divergent behavior in the \( x \to 1 \) limit. The phenomenological importance of these contributions has been stressed for Higgs production cross sections. So far, resummation at order \( 1/N \) has been carried out only in the case of moments of \( F_L(x, Q) \) in DIS, which begin at that power.

The extension to \( F_2 \) and hadron collisions requires an analysis that generalizes Eq. (6), by taking parton mixing into account,

\[
\tilde{\omega}_{ab}^{DY}(N, Q, \mu) = \tilde{\phi}_{c/A}(N, \mu)^{-1} \tilde{\psi}_{c/e}(N, Q) \rho_{ca}(N, Q) \nonumber \]

\[
= \tilde{\psi}_{c/e}(N, Q) U_{ab} \left| H_{ab} \right|^2 \tilde{\phi}_{a/A}(N, \mu)^{-1}, \quad (11)
\]

where the \( \tilde{\psi} \) and \( \tilde{\phi} \) are now matrices in flavor, with off-diagonal elements beginning at \( O(1/N) \). Corrections to this result are suppressed by powers of \( Q \) rather than \( N \), again because of the universality of mass factorization. The new "hard-scattering functions" \( \rho_{ca}(N, Q) \) will in general include corrections of all powers in \( 1/N \), although, as before, the factorization-scale dependence is contained in the ratios of \( \tilde{\psi} \)'s to \( \tilde{\phi} \)'s. These ratios have a form reminiscent of NLO parton distribution evolution in the singlet sector,

\[
\tilde{\phi}_{c/e}(N, Q) \tilde{\psi}_{c/e}(N, Q) = \mathcal{P} \exp \left\{ \int_0^{Q^2} \frac{d\xi^2}{\xi^2} \right\}, \quad (12)
\]

as path-order exponentials in terms of a matrix of anomalous dimensions, \( \Gamma_{ab} \), which is entirely of order \( 1/N \), and a diagonal matrix \( A_{ab} \).

4. Power corrections from resummation

As is evident in Eq. (10), resummed cross sections involve integrals for which the argument of the running coupling vanishes. Over the past few years, it has been realized that these results give a way to predict the form of power corrections to cross sections to which the operator product expansion does not directly apply.

\[\]
In $e^+e^-$ annihilation, this approach has been employed to organize power corrections to event shapes. For the thrust, the analysis of resummation formulas similar to Eq. (4) leads to a convolution form for the cross section that organizes all terms of order $O([N/Q]^0)$ in moment space: [11]

$$\frac{d\sigma(t)}{dt} = \int_0^{gt} \frac{\alpha_s}{2\pi} \frac{d\psi_{\phi}}{d^3\psi_{\phi}}(t - \epsilon/Q) + O \left( \frac{1}{IQ^2} \right),$$

(13)

with $f(\epsilon)$ a new nonperturbative function. The well-known prescription of a nonperturbative “shift” ($t \to t - \lambda t/Q$) in the resummed thrust distribution [12] follows in the limit of a very narrow $f(\epsilon) \sim \delta(\epsilon - \lambda t)$.

A related question, which has seen less attention, is the nature of power corrections in hadronic inclusive cross sections, such as Drell-Yan. [6] In particular, it was suggested in Ref. [6] on the basis of Eq. (4) that all integer powers $(N/Q)^n$ occur in Drell-Yan. Subsequently, however, it was shown that the coefficient of the linear, $N/Q$, term in this series vanishes, when the anomalous dimension $A$ is calculated to all orders in perturbation theory in the large-$n_f$ limit [10], a procedure not practical in full QCD.

We have returned to this issue recently, verifying that the $N/Q$ correction is indeed absent in full QCD. We can see this on the basis of rather general arguments, without recourse to explicit all-orders calculations, although we must generalize the form of Eq. (4) somewhat.

The basic observation is illustrated by the following expression for the ratio of $\tilde{\psi}$ to $\tilde{\phi}$ functions, relevant to resummed Drell-Yan, given here at lowest order in $A$:

$$\ln \left[ \frac{\tilde{\psi}(N,Q)}{\tilde{\phi}(N,Q)} \right] \sim \int_0^1 \frac{dz}{1-z} \int_0^{2(1-z)^2Q^2} \frac{dk_T^2}{k_T^2} \left[ e^{-N(1-z)} - 1 \right] \int_{2(1-z)^2Q^2} f^2 \frac{dk^2}{k^2} \frac{\alpha_s}{2\pi} \ln^2 \left( \frac{Q}{k} \right).$$

(14)

The first integral on the right-hand side illustrates the finite remainder after the cancellation of collinear divergences between the energy ($\tilde{\psi}$) and MS ($\tilde{\phi}$) distributions. It has no logs of $N$. This term is explicitly present in the analysis of Ref. [6], but was treated only to lowest order, and absorbed into an overall constant in the resummed cross section, Eq. (4). To study power corrections due to the full $O(N^0)$ cross section, however, it is necessary to consider the running of the coupling in this contribution. The second integral on the right-hand side contributes directly to the resummed cross section of Eq. (4), and contains all logs of $N$.

Power corrections in the ratio of $\tilde{\psi}/\tilde{\phi}$ can be identified by expanding the $z$ integrals of Eq. (14) in powers of $k_T/Q$ in the strong-coupling ($k_T \to 0$) limit, running the coupling with scale $k_T$. [4] In Eq. (14), we verify that the $1/Q$ terms found in this way cancel between the first and second integrals. Power corrections in the ratio, and in the full cross section, thus begin at $(N/Q)^2$. This analysis can be extended to all orders in perturbation theory.

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