Comparative study on performance of controlling geometric nonlinearity of laminated smart composite plate using piezoelectric fiber reinforced composite & active fiber composite material

Vishwanath Khadakbhavi 1, Ashok M H 2, Shivakumar J 3, Sanjay Pujari 4, Santosh Nandurkar 5

1 Faculty, Mechanical Engineering department, Angadi Institute of Technology & Management, Belagavi, Karnataka, India
2 HOD, Mechanical Engineering department, Angadi Institute of Technology & Management, Belagavi, Karnataka, India
3 Principal, Chhattisgarh Engineering College, Durg, Chhattisgarh, India
4 Principal, Angadi Institute of Technology & Management, Belagavi, Karnataka, India
5 Faculty, Mechanical Engineering department, KLE Dr. M.S.S.CET, Belagavi, Karnataka, India

E-mail: vishi_mk@yahoo.co.in

Abstract. The present work highlights on comparative investigation for the performance of active constrained layer damping treatment using PFRC and AFC patch material for controlling the nonlinear transient vibrations of laminated composite plate. In this case for making the constraining layer of the ACLD treatment for PFRC and AFC patch material is used. For modelling of ACLD in the time domain Golla-Hughes-McTavish (GHM) method is used. The Von Karman type non-linear strain displacement relations along with a simple first-order shear deformation theory are used for deriving this electromechanical coupled problem. A finite model in 3-dimensions has been developed for smart composite plate integrated with the ACLD treated patches of PFRC and AFC. The results of both the cases have been compared and it has been found that the improved performance of AFC patch over the PFRC in subsiding the transient nonlinear vibrations of symmetric cross ply laminated composite plate.

1 Introduction

In the past three decades, more attention has been drawn by research in lightweight flexible structures using composite material in order to develop high-performing structures which have self-controlling and self-sensing capabilities[2–5]. This demands for the excess applications of structures made up of laminated composite. But light weight flexible structures are comparatively weak to withstand the vibrations having extensive decay time since they possess small internal damping. Hence to overcome these difficulties, an alternative active control system is designed with the help of sensors and actuators and integrated with the host structure to make it a smart structure. The distributed actuators and sensors made up of different piezoelectric materials are used in making of smart structures and many researchers have worked on this in recent years.
The investigation on smart materials by Shivakumar J is to understand the Performance of composite plate which is assimilated with diffused AFC having nonlinear vibrations [1]. Further an investigation is done by Majed A. Majeed on vibration analysis of reasonably thick uneven piezoelectric materials using the approach of DTF[6]. The studies are also conducted on piezoelectric laminated plate for different analytical solutions[7-10]. Various studies are also done on limited components for nonlinear assay of smart composite plates[11-17]. Though the work related to PFRC and AFC patch materials is reported in the papers but comparative investigations were not made. This study is to compare the execution of ACLD treatment made from PFRC/AFC to control the non linear vibrations of overlaid composite plates.

2 FE Model

Figure 1 shows a rectangular overlaid composite plate made up of N layers of orthotropic material. The ACLD treatment is done for rectangular patches (02 Nos.) on the top surface of the substrate plate. The thickness of PFRC/AFC layer is considered as \( h_p \) and ACLD accompanied with a viscoelastic constrained layer is represented by \( h_v \). The length of the plate is taken as \( a/2 \) and width is taken as \( b/2 \). The patches are placed centrally on the plate as shown below.

![Figure 1 Laminated composite plate assimilated with the patches by ACLD treatment.](image)

The central plane of the plate is assumed as the datum plane. The coordinates for the datum plane are considered as \((x, y, z)\) and the limits corresponding to the line of the plate is given by \( x=(0,a) \) and \( y=(0,b) \). Further, \( h_k \) and \( h_{k+1} \) are taken as thickness for the \( z \) co-ordinate, where \( k \) represents the number of layer. \( \theta \) is considered as the cover arrange framework at the edge of the substrate plate in any layer. The filaments in the PFRC/AFC are arranged so as to be parallel to plane \( xy \).

The displacement occurring in the plane in the axial direction of the plate including the ACLD is built on FSD theory. The interpretations are as shown in the figure 2 & 3. Where \( u_0 \) and \( v_0 \) are the general displacements with respect to the reference point \((x, y)\). \( \theta_x \), \( \phi_x \) and \( \gamma_x \) are noted as the generalized rotations of the substrate generated between normal plane and the mid plane about \( y \) axis. Similarly \( \theta_y \), \( \phi_y \) and \( \gamma_y \) are noted as the generalized rotations about \( x \) - axis. The displacements \( u \) and \( v \) obtained at any point in the layer along both the directions \( x \) and \( y \) is as shown below.
\[ u(x, y, z, t) = u_0(x, y, t) + \left( z - \frac{h}{2} \right) \Theta_z(x, y, t) + \left( z - \frac{h}{2} \right) \Phi_z(x, y, t) + \langle z - h_{N+2} \rangle \gamma_z(x, y, t) \]  \(2.1\)

\[ v(x, y, z, t) = v_0(x, y, t) + \left( z - \frac{h}{2} \right) \Theta_y(x, y, t) + \left( z - \frac{h}{2} \right) \Phi_y(x, y, t) + \langle z - h_{N+2} \rangle \gamma_y(x, y, t) \]  \(2.2\)

\[ w(x, y, z, t) = w_0(x, y, t) + \left( z - \frac{h}{2} \right) \Theta_z(x, y, t) + \left( z - \frac{h}{2} \right) \Phi_z(x, y, t) + \langle z - h_{N+2} \rangle \gamma_z(x, y, t) \]  \(2.3\)

In which, the content within the bracket \( \langle \cdot \rangle \) at intervals the bracket shows the continuity between the 2 continua. To simplify the process, displacement variables are divided in to two variables known as translational \( \{ d_t \} \) and rotational \( \{ d_r \} \):

\[ \{ d \} = \{ d_t \} + \{ z \} \{ d_r \} \]  \(2.4\)

Where, \( \{ d \} = [u \ v \ w]^T \), \( \{ d_t \} = [u_0 \ v_0 \ w_0]^T \) \& \( \{ d_r \} = [\Theta_z \ \Theta_y \ \Phi_z \ \Phi_y \ \gamma_z \ \gamma_y]^T \)

Now to calculate the stiffness in the elemental matrix by considering transverse shear deformation theory, we need to apply the selective integration rule so as to nullify the plates locking in shear. At any point, strain experienced by the plate is split into two vectors namely longitudinal strains \( \{ \varepsilon_b \} \) and transverse strain \( \{ \varepsilon_{sh} \} \) which is given in equation number 2.5. On the same basis the stress in the plate is given in equation number 2.6.

![Figure 2](image-url)

**Figure 2** Deformation in the plate cross-section parallel to \( YZ \) plane integrated with ACLD treatment.
Figure 3 Deformation in the plate cross-section parallel to \( XZ \) plane integrated with ACLD treatment.

\[
\begin{align*}
\{ \varepsilon_b \} &= [\varepsilon_x \varepsilon_y \varepsilon_{xy} \varepsilon_z]^T & \{ \sigma_b \} &= [\sigma_x \sigma_y \sigma_{xy} \sigma_z]^T \\
\{ \varepsilon_{sh} \} &= [\varepsilon_{xz} \varepsilon_{yz}]^T & \{ \sigma_{sh} \} &= [\sigma_{xz} \sigma_{yz}]^T
\end{align*}
\] (2.5)

In which \( \sigma_x, \sigma_y, \sigma_z \) represent normal stresses acting along the directions \( x, y, z \) respectively; \( \sigma_{xy} \) represents the shear stresses along the plane whereas \( \sigma_{xz}, \sigma_{yz} \) represent shear stresses in the transverse direction. By using von Karman type the geometric nonlinear strain developed in the plates, the potential and kinetic energy for the plate in coupled form can be expressed as

\[
T_p = \frac{1}{2} \left[ \sum_{k=1}^{N+2} \int_{\Omega_k} \{ \varepsilon_b^k \}^T \{ \sigma_b^k \} + \{ \varepsilon_{sh}^k \}^T \{ \sigma_{sh}^k \} \right] d\Omega - \int_A \rho \omega^2 dA - \int_{\Omega_{N+2}} E_i D_i dv
\] (2.7)

\[
T_k = \frac{1}{2} \left[ \sum_{k=1}^{N+2} \int_{\Omega_k} \rho^k \left( \dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) d\Omega \right]
\] (2.8)

Where, \( \rho \) is load acting in the transverse direction on the surface area \( A \), \( \Omega_k \) is the volume & \( \rho^k \) is the mass density of any \( k \)th layer.

For writing the constitutive relation in time domain for ACLD treatment can be used which is expressed as,

\[
\{ \sigma_{sh} \} = \int_0^t G(t-\tau) \frac{\partial \{ \varepsilon_{sh} \}}{\partial \tau} d\tau
\] (2.9)

where, \( G(t) \) is relaxation function of viscoelastic material.

Since the plate thickness is less, the generated rotary inertia is nullified. The equations of motion can be generated by using the principle of virtual work:
\[
[M^*](\ddot{d}_r^e) + [k_{tr}^e]G(t - \tau)\frac{\partial}{\partial \tau}\{d_r^e\} \dot{\tau} + [k_{trv}]G(t - \tau)\frac{\partial}{\partial \tau}\{d_r^e\} \dot{\tau} = [F^e] + \left[\{F_{tpu/a}^e\} + \{F_{rbp/a}^e\}\right]V + \{F_{tr}\}
\]

(2.10)

and,

\[
[k_{tr}^e]G(t - \tau)\frac{\partial}{\partial \tau}\{d_r^e\} \dot{\tau} = [F_{tpu/a}^e]V + \{F_{tr}\}
\]

(2.11)

For the substrate without a patch of ACLD, the values of \(\{F_{tpu}^e\}, \{F_{rbp/a}^e\}, \) and \(\{F_{rbp/a}^e\}\) will be zero.

For composite plate with a patch of ACLD the elastic behaviour in the open loop is illustrated as

\[
[M^*](\ddot{X}) + [C^*](\dot{X}) + [K^*](X) = [F^*] + \{F_p\}V + \{F_i\}
\]

(2.12)

Where

\[
[M^*] = \begin{bmatrix}
[M] & 0 & 0 \\
0 & I_z & 0 \\
0 & 0 & I_{tr}
\end{bmatrix},
[K^*] = \begin{bmatrix}
[K_x] & [K_z] & [K_{tr}] \\
-\omega^2[I_z] & \omega^2[I_z] & 0 \\
\omega^2[K_x] & -\omega^2[K_x] & \omega^2[K_x]
\end{bmatrix},
[C^*] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 2\xi\omega & 0 \\
0 & 0 & 2\xi\omega
\end{bmatrix}
\]

\[
[X] = \begin{bmatrix}
{X_1} \\
{Z_1} \\
{Z_r}
\end{bmatrix},
\quad [F^e] = \begin{bmatrix}
{F_p} \\
0 \\
0
\end{bmatrix},
\quad [F_{tr}] = \begin{bmatrix}
{F_{tr}} \\
0 \\
0
\end{bmatrix},
\quad \{F_i\} = \{F_{tr}\} - [K_{tr}]^{-1}[K_{tr}]^{-1}\{F_{tr}\}^T
\]

(2.13)

In which \(I_z\) and \(I_{tr}\) are the similarity matrices with respect to \(\{Z_1\}\) and \(\{Z_r\}\).

3 Closed loop model

According to the control voltage law the voltage developed in each patch can be shown in the form of derivatives of the global degrees of freedom which can be given as

\[
V_j = -K_d^j\dot{W} = -K_d^j[U^j]\{\dot{X}\}
\]

(3.1)

Where, \(K_d^j\) represents control gain of the patches, \(U^j\) represents the sensing location from the patch in the form of unit vector.

Substituting equation 3.1 into equation 2.12 to get the below equation which shows the closed loop dynamics for the plates accompanied with ACLD treatment.

\[
[M^*](\ddot{X}) + [C^*](\dot{X}) + [K^*](X) = [F^*] + \{F_i\}
\]

(3.2)

Where, \([C_d^*] = [C^*] + \sum_{j=1}^{m}K_d^j\{F_p^e\}[U^j]\) for active damping.
4 Numerical results under mechanical loading

The model is developed in Matlab and analysis is carried out to minimise the non-linear vibrations which are generated by applying the load in the transverse direction. The square substrate is made up of smart composite material. The patches made up of PFRC/AFC material are attached to the substrate by ACLD treatment. The thickness of the patches & plate are taken as 250 µm and 3 mm. The values estimated for $\alpha$, $\xi$ and $\omega$ by using GHM are 11.42, 1.0261e5 & 20. The load acting P in the transverse direction on the surface of the plate is given by 300. For simply supported beam the numerical outcomes which are obtained by the limiting conditions are given by

For Simply supported substrate (SSS):

$$v_0 = w_0 = 0, \quad \phi_x = \gamma_y = 0 \quad \text{at} \quad x = 0, a$$
$$u_0 = w_0 = 0, \quad \phi_x = \gamma_y = 0 \quad \text{at} \quad y = 0, b$$

The behaviour of the plate in the open and closed loop is understood by the deflection of the plate at the centre on the top layer of the surface. The load is uniformly distributed throughout the surface in the form of pulse. It is observed that the value of the non dimensional equation $Q = pa^4 / (E_s h^4)$ for the applied load exceeds 40 number, then the plate exhibits non linear deformation.

The validation is also done for the simply supported plate for open and closed loop which is included by two identical patches of PFRC/AFC for cross ply condition ($0^\circ / 90^\circ / 0^\circ$). The results obtained are shown in figure 4 and 5 which reveal that the present results match with the existing one. Thus the validation is done for the present FE model with AFC material by using ACLD treatment.

![Figure 4](image-url)

**Figure 4** Simply supported plate with cross ply ($0^\circ / 90^\circ / 0^\circ$) arranged symmetrically showing active damping ($a/h = 300$, $Q = 100$) when PFRC in the passive mode ($K_d = 0$)
Figure 5 Simply supported plate with cross ply \((0^\circ / 90^\circ / 0^\circ)\) arranged symmetrically showing active damping \((a/h = 300, Q = 100)\) when AFC in the passive mode \((K_d = 0)\) \([1]\)

The analysis is conducted for AFC and PFRC in controlling geometrically nonlinear transient vibrations of plate which is shown in following figures. In figure 6 the execution of both materials utilizing ACLD treatment is thought about in aloof mode and in figure 7 for dynamic mode for symmetric cross employ substrate. It can be watched that the adequacy of AFC material is huge than PFRC material in constricting the geometrically nonlinear transient vibrations

Figure 6 Simply supported plate with cross ply \((0^\circ / 90^\circ / 0^\circ)\) arranged symmetrically showing uncontrolled responses undergoing non linear vibrations using AFC & PFRC patches (for \(a/h = 300, Q=100\) (Passive, \(K_d = 0\) )
Figure 7 Simply supported plate with cross ply (0° / 90° / 0°) arranged symmetrically showing uncontrolled responses undergoing non linear vibrations using AFC & PFRC patches (for $a/h = 300, Q=100$) (Active, $K_d=250$)

5 Conclusion

A three dimensional FE model is developed to study the behaviour of the plate which is attached with the patches of PFRC/AFC material to minimise the geometrically nonlinear transient vibrations generated by applying the load in the time domain. The mechanics of deformation of the plate is assumed to be supported by the primary order shear deformation theory (FSDT) and also the von Karman strain-displacement relations square measure used for modelling the geometric non-linearity. For the time domain analysis, the elastic layer is shaped by the GHM technique.

The results obtained reveals that the geometrically non linear transient vibrations generated in composite plate are comparatively controlled when it is in active state as compared to passive state. The study also reveals that the AFC material shows improved control stability than that of PFRC.

References:
1. Shivakumar J. and Ashok M. H., Performance analysis of smart laminated composite plate integrated with distributed AFC material undergoing geometrically nonlinear transient vibrations, IOP Conf. Series: Materials Science and Engineering, Ref. 310 012100,2018.
2. Shivakumar J. and Ray M. C., Nonlinear Analysis of Smart Cross Composite Plates Integrated with a Distributed Piezoelectric Fiber Reinforced Composite Actuator, Mechanics of Advanced Materials and Structures, ISSN 1537-6494, vol. 15, No.1, 2008, pp. 40-52.
3. R Suresh Kumar, M C Ray, Smart damping of geometrically nonlinear vibrations of functionally graded sandwich plates using 1–3 piezoelectric composites, Journal of Mechanics of Advanced Materials and Structures Vol. 23, 2016 - Issue 6, Pages 652-669.
4. Saroj Kumar Sarangiand Ray M. C., Smart Control Of Nonlinear Vibrations Of Laminated Plates Using Active Fiber Composites, Vol. 12, No. 6, 2012, DOI:10.1142/S0219455412500502
5. Majed A. Majeed, Ayech Benjeddou, Mohammed A. Al-Ajmi, Free vibration analysis of moderately thick asymmetric piezoelectric adaptive cantilever beams using the distributed transfer function approach, Journal of Sound and Vibration, Volume 333, Issue 15, 21 July 2014, pp. 3339-3355
6. Seyedeh Marzieh Hosseini, Hamed Kalhori, Alireza Shooshtari, Nima Mahmoodi S., Analytical solution for nonlinear forced response of a viscoelastic piezoelectric cantilever beam resting on a
nonlinear elastic foundation to an external harmonic excitation, *Composites Part B: Engineering*, Volume 67, 2014, pp. 464-471

7. Mehran Shahraeeni, Rezgar Shakeri, Seyyed Mohammad Hasheminejad, An analytical solution for free and forced vibration of a piezoelectric laminated plate coupled with an acoustic enclosure, *Computers & Mathematics with Applications*, Volume 69, Issue 11, 2015, pp. 1329-1341

8. Lin Li, Shunhua Yin, Xue Liu, Jun Li, Enhanced electromechanical coupling of piezoelectric system for multimodal vibration, *Mechatronics*, Volume 31, October 2015, pp. 205-214

9. Khdeir A.A., Aldraihem O.J., Exact analysis for static response of cross ply laminated smart shells, *Composite Structures*, Volume 94, Issue 1, 2011, pp. 92-101

10. Vahid Tajeddini, Anastasia Muliana, Nonlinear deformations of piezoelectric composite beams, *Composite Structures*, Volume 132, 2015, pp. 1085-1093

11. Zhangjian Wu, Chao Han, Zhongrong Niu, A 3D exact analysis of the boundary layer effect of asymmetric piezoelectric laminates with electromechanical coupling, *International Journal of Solids and Structures*, Volume 72, 2015, pp. 118-129

12. Fallah N., Ebrahimnejad M., Finite volume analysis of adaptive beams with piezoelectric sensors and actuators, *Applied Mathematical Modelling*, Volume 38, Issue 2, 2014, pp. 722-737

13. Mareishi S., Rafiee M., He X.Q., Liew K.M., Nonlinear free vibration, postbuckling and nonlinear static deflection of piezoelectric fiber-reinforced laminated composite beams, *Composite Structures*, Volume 59, 2014, pp. 123-132

14. Ahmad Eduardo Guennam, Bibiana M. Luccioni, Model for piezoelectric/ferroelectric composites polarized with interdigitated electrodes, *Composite Structures*, Volume 131, 2015, pp. 312-324

15. Milazzo A., Variable kinematics models and finite elements for nonlinear analysis of multilayered smart plates, *Composite Structures*, Volume 122, 2015, pp. 537-545

16. Zhang, Shun-Qi, Li, Ya-Xi, Schmidt, Rüdiger, Active shape and vibration control for piezoelectric bonded composite structures using various geometric nonlinearities, *Composite Structures*, Volume 122, April 2015, pp. 239-249

17. M. H. Ashok, Geometrically nonlinear transient vibrations of actively damped antisymmetric angle ply laminated composite shallow shell using active fiber composite (AFC) actuators, *IOP Conf. Series: Materials Science and Engineering*, Ref. 310 0121001, 2018