Energy Splitting Theorems for Materials with Memory

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Abstract We extend to materials with fading memory and materials with internal variables a result previously established for materials with instantaneous memory: the additive decomposability of the total energy into an internal and a kinetic part, and a representation of the latter and the inertial forces in terms of one and the same mass tensor.

Keywords Internal energy · Kinetic energy · Simple materials · Fading memory · Internal variables

Mathematics Subject Classification (2000) 74A20 · 74D99

1 Introduction

The purpose of this paper is to extend to two classes of materials with memory a result established in [6] for materials that, as exemplified by standard thermoelastic materials, can only respond to the current values of their state variables.

The result we aim to extend is called in [6] the Energy Splitting Theorem: it is shown that the total energy and the inertia force have consistent representations, under the assumptions that (i) the power expenditure of the inertia force be linear in the velocity; and that (ii) the inertial power plus the rate of change of the energy be translationally invariant. More precisely, it is shown that the energy can be split in two parts, internal and kinetic, with the internal energy independent of velocity and the kinetic energy a quadratic form in the
velocity, based on a time-independent mass tensor, the same that determines also the work-effective part of the inertial force.

The two material classes we here consider are: the class of simple materials in the sense of Truesdell and Noll [8], whose mechanical response is determined by the history of the deformation gradient; and the class of materials with internal state variables, as considered, e.g., by Coleman and Gurtin [1] and Lubliner [5], whose evolution is governed by a generally nonlinear differential equation (that the Energy Splitting Theorem had to be extendable to this material class was suggested by M.E. Gurtin in 1994, on reading a preprint of [6]). Since these two material classes have a nonempty intersection but do not overlap, we are obliged to prove the entry part of our generalized Energy Splitting Theorem twice; we give the reasons for this at the end of next section. Luckily, as we shall see, the rest of the proof is not as sensitive to the chosen class.

Our paper is organized as follows. In Sect. 1, we introduce the quantities that are subject to a constitutive prescription, we stipulate their invariance properties, and we summarize the Energy Splitting Theorem which we aim to generalize. In Sect. 2, we provide a constructive proof of the Energy Splitting Theorem under the assumption that the constitutive functionals be smooth relative to a norm having the fading-memory property. In Sect. 3, we sketch a proof of the Energy Splitting Theorem for materials with internal variables. Apart for some technicalities that we try to explain as they arise, the structure of the proofs we give is the same as the variant of the proof in [6] given in [7].

2 Setting the Stage

We work in a referential setting. At any given body point (in this paper, we leave all space dependencies tacit), we introduce scalar volume densities of the (total) energy, denoted by τ, and of the inertial power:

\[ \pi^{\text{in}} = d^{\text{in}} \cdot v, \]

where \( v \) is the velocity vector and \( d^{\text{in}} \) is the inertia force vector. Next, we define the internal power density to be

\[ \alpha = \dot{\tau} + \pi^{\text{in}} \]  

(a superposed dot denotes time differentiation). Both \( \tau \) and \( d^{\text{in}} \) are constitutively prescribed at a later stage. For now, it suffices for us to stipulate that, in principle, they both depend on one and the same list of state variables, that we split as follows: \((\Lambda, v)\), where the list \( \Lambda \) includes only translationally invariant variables. Precisely, a translational change in observer is a mapping leaving the time line unchanged:

\[ (t, x) \mapsto (t, x^+) = (t, x^+), \]

such that, at some fixed time \( \tilde{t} \), the current shape of the body under study is pointwise preserved, while the velocity field varies by a uniform amount \( w \):

\[ x \mapsto x^+ = x + (t - \tilde{t})w, \]

\[ v \mapsto v^+ = v + w. \]

Thus, as to state-variable pairs,

\[ (\Lambda, v) \mapsto (\Lambda, v^+) = (\Lambda, v + w). \]