Magnetic-induced conversion between electric quadrupole radiation and quasi dipole radiation at THz band

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Abstract

Based on particle-in-cell simulation and theoretical analysis, we demonstrate the possibility of the realization of the electric quadrupole radiation (EQR) and the quasi dipole radiation (QDR) modes at the THz band from the electrostatic oscillation of the electrons driven by the two-color lasers inside the plasma. Interestingly, an implantation of the external magnetic field will induce the flexible conversion between the EQR mode and the QDR mode. The emission angle, radiation frequency, and radiation field strength of the two radiation modes can be highly modulated by adjusting the intensity of the magnetic field, the frequency of the laser pulses, and the density of the plasma. This provides a new and practical way toward the generation and conversion of the multiple radiation modes, greatly extending the potential for optical manipulation in magnetized plasma.

1. Introduction

Terahertz (THz) science has stirred enormous interest due to its many applications in physics [1], biology [2], and medicine [3]. The enhancement of the radiation field strength, the tuning of the radiation frequency and the modulation of the polarization direction of the THz source have been widely investigated [4–8]. Recently, novel challenges in the THz applications raised the need for a THz system with the high directive radiation, which not only realizes a directional THz beam to switch between multiple objects rapidly but also can reduce the wastage of THz power in unnecessary directions tremendously [9–12]. In the THz imaging, the advanced optical scanners use an elliptical main-reflector and a controllable rotating sub-reflector to modulate the propagation direction of the THz beam and realize beam scanning function generally [9, 10]. The beams from these optical devices can guarantee a quick enough imaging speed and have a wide scanning range without moving or rotating the whole system. In the THz communications, the controllable multi-beam antennas need to access many distinct far-field directions simultaneously, which can sustain the multiple point-to-point links effectively. This form of THz beam control may also be practical for the short-range radar that monitors several directions simultaneously [11, 12]. Clearly, these methods for modulating the propagation direction of the THz beam mainly rely on the transmission devices, which have significant limitations for the THz application in the aspect of costing, portability, and robustness. Therefore, how to modulate the radiation direction of the THz radiation directly based on the THz radiation source is still a challenging subject. However, the existing THz radiation sources do not have the modulation capacity for the radiation direction of the THz beam due to the inherent radiation mechanisms. For the synchrotron radiation, the THz beams are along the tangent direction of the electron’s orbital due to the incoherent electromagnetic waves are generated by relativistic electrons during their transverse acceleration in magnetic fields [4]. The Cherenkov radiation is only emitted in the forward direction, i.e., the oscillation direction of the electrons, since the group velocities of waves matching the Cherenkov condition at other angles are very small [13–15]. The spatial distribution of the THz radiation generated by laser-induced tunneling processes in gases is only a cone structure around the laser beam [6, 7].
The appearance of the new radiation mechanism with electric dipole and electric multipole radiation in single crystals, graphene hyperbolic medium, or cluster provides the possibilities to realize the controllable directive radiation, which immensely expands the THz development way and provides an unprecedented opportunity for developing miniature and tunable radiation source in the THz band [5, 8, 16]. The directional THz radiation generated by an electric dipole in graphene hyperbolic media is studied carefully [5]. The feasibility of dipole radiation at the THz band is demonstrated when a femtosecond laser pulse focuses on the cluster [8]. The terahertz radiation generated by a Gaussian distribution of parallel dipoles in a crystal with a three-layer structure is presented [16]. Moreover, the continuous-wave (CW) THz beam generated by the electric dipole and electric multipole mode can maintain a high signal-to-noise ratio and fast acquisition time [17]. The CW THz beam also have fine spectral resolution and accurate phase measurement capability, which have obvious advantages on the THz communication and THz imaging [17–19]. But these mediums used to generate THz radiation have a definite damage threshold, which results in limitations for the field strength and tuning frequency of the THz radiation.

Fortunately, the excitation of a localized electrostatic oscillation of electrons in the plasma, an electric dipole or a nonzero oscillation current, has almost ideal characteristics as a radiation source to generate CW THz radiation due to the tremendous potential in terms of no-damage threshold, concise preparation process and reutilization [20–23]. A series of radiation schemes are studied and discussed widely, which generate the THz radiation from the electric dipole oscillation and nonzero oscillation current induced by the time-asymmetric ionization or the space-asymmetry of lasers in the plasma [20, 24–28]. The intense THz radiation generated by mixing the fundamental and its second harmonic laser fields in the air is initially interpreted as a four-wave difference frequency (FWDF) mixing parametric process in ionized air plasma [24, 25]. Nevertheless, comparing with the explanation of the FWDF model, it is found that the two-color ionization radiation based on the time-asymmetric ionization is more reasonable for explaining the observed THz emission driven by the nonzero oscillation current in the air plasma [26, 27]. The transition-Cherenkov radiation is generated by a femtosecond laser filament in the air and mainly depends on the longitudinal electron current driven by the longitudinal ponderomotive force of the laser [28–30].

The theoretical model of longitudinal linear-dipole-array in plasma confirmed by the systematic experiments is also proposed to control the carrier-envelope phase, elliptical and azimuthal angle of the CW THz radiation [20, 21]. As an excellent magnetized material, the magnetized plasma is used to control the field strength and the polarization direction of the CW THz radiation by modulating the oscillating modes of electrons [6, 31–33]. These radiation mechanisms based on the electric dipole or a nonzero oscillation current in the plasma have achieved unprecedented progress in improving THz quality and expanding THz application. But there is still a gap for the further exploration of the directive THz radiation based on the space-asymmetry of lasers in the plasma. Generally, the THz radiation based on the longitudinal oscillation of electrons in the unmagnetized plasma is emitted in the transverse direction since the direction of the electromagnetic radiation from the electrostatic component of the electron oscillation is perpendicular to the oscillation direction of the electrons. The THz radiation generated by the electromagnetic component of the oscillation of the electrons in the magnetized plasma propagates along the longitudinal direction. That’s, based on the electrostatic and electromagnetic components of the electron oscillation in the plasma, the flexible modulation of the radiation angle of the CW THz radiation is mainly limited by two aspects. On the one hand, the oscillating direction of the electrons is determined by the longitudinal space-asymmetry of lasers and fixed in the longitudinal direction generally. The radiation direction of the THz radiation must be parallel or perpendicular to the oscillating direction of the electrons in different radiation environments; on the other hand, it is impossible to realize the conversion of the different radiation modes generated by the electrostatic component and the electromagnetic component of the longitudinal oscillation of the electrons. Although the transverse motion of the electrons induced by the space-asymmetry of the laser is important for the interaction of the laser and plasma [34–37], there is a missing for the generation and conversion of the multiple radiation modes by modulating the electrostatic components of the transverse oscillation of electrons in the micro plasma, which will have an important effect on the control of the emission angle of the CW THz radiation.

In this paper, based on the interaction of the laser and magnetized micro plasma, we provide two novel radiation modes, i.e., the electric quadrupole radiation (EQR) and the quasi dipole radiation (QDR), which are generated by the electrostatic component of the longitudinal and transversal oscillation of the electrons in the plasma. Moreover, we also realize the conversion of the two radiation modes completely, which results in the flexible modulation of the emission angle of the CW THz radiation. Through the two-dimensional particle-in-cell (PIC) simulations, we firstly demonstrate the EQR and the QDR at the THz band excited by the two-color ps lasers inside the magnetized micro plasma. Theoretical evidence is carried out to show the physical mechanism of the generation and propagation of those CW THz radiation modes, which perfects the effects of the electrostatic components of the transverse oscillation of the
electrons on the THz radiation. The EQR generated by the electrostatic components of the transverse oscillation of the electrons will double the capacity of the THz antenna due to distinct messages can be encoded into both beams with the different phase [11]. Particularly, the flexible conversion between the EQR mode and the QDR mode is creatively realized in the micro plasma via the implantation of an external magnetic field, which means the realization of the flexible modulation of the emission angle of the CW THz radiation. Moreover, the radiation frequency and radiation field strength of the two radiation modes can also be controlled effectively.

2. Model and PIC simulation

The sketch of our model is shown in figure 1. The two-color lasers of the frequencies $\omega_1 = 1040$ THz and $\omega_2 = 1017$ THz emitted by two highly synchronized lasers [38] or the interaction of the laser and the highly nonlinear fiber [39] propagate along the $z$ direction in the magnetized micro plasma. The lasers have the same spot radius $b_0 = 33 \mu m$, duration $\tau = 0.26 \, \text{ps}$, and electric field $E_L = 6 \times 10^{10} \, V \cdot m^{-1}$. The micro plasma with a length $l = 35 \mu m$ and width $w = 46 \mu m$ is performed and the environment beyond the plasma is vacuum. In general, the plasma filaments are generated by the dynamic competition from the several linear and nonlinear effects when the pre-laser pulses focus on the argon gas or ambient air [28, 40]. By modulating the related optical device parameters (such as: the effective focal length of the plano-convex lens), the competition of the nonlinear effects (optical Kerr effect, self-focusing effect and defocusing effect) can control the size of the plasma filament effectively to generate the micro plasma [23, 41]. The magnetization for the micro plasma is realized by the pulsed magnetic field generated by discharging a high-voltage capacitor through a small wire-wound coil [42] or laser-driven capacitor–coil targets [43, 44]. The force of the electric field accompanied with the pulsed magnetic field is far less than the Lorentz force of the magnetic field, which can be neglected completely. When the laser pulses pass through the magnetized micro plasma with a uniform density $n_0 = 3 \times 10^{17} \, \text{cm}^{-3}$, the free electrons will be driven effectively and have a significant oriented drift distance which will result in the generation of charge separation inside the plasma. Finally, the free electrons will oscillate with the frequency $\omega = (\omega_p^2 + \omega_c^2)^{1/2}$ under the restoring force and Lorentz force due to the charge separation and the external magnetic field, where $\omega_p$ is the plasma frequency and $\omega_c$ is the cyclotron frequency of the electrons. When the size of the plasma is on the order of a skin depth wide (i.e., the micro plasma), the electrostatic components of the oscillation of the electrons will generate the EQR and the QDR at the THz band. The free-space electro-optic sampling can detect the THz radiation effectively [18]. In the PIC simulation, the resolution is 0.1 \mu m. Due to the performed plasma is fully ionized and collisionless, the large number of macro particles per cell is not needed to treat the medium and strong collision problem [6, 22, 45–48]. There are three macro particles placed in per cell. The convergence of the simulation is checked with more macro particles per cell. The PIC simulation results are shown in figures 2–4.

When the plasma is unmagnetized ($B = 0$), the EQR is generated effectively, which means the formation of the two CW THz beams with the same field strength and the different radiation direction (figure 2(a)). In this case, the spatial distribution of the two radiation beams is symmetric about the axis of the lasers $y = 0$. When the external static magnetic field of $B = 20$ T is imposed along the $+x$ direction (figure 1(a)), the QDR mode is generated with the field strength of $5.1 \, \text{MV} \cdot \text{m}^{-1}$ and asymmetric spatial distribution (figure 2(d)), which is completely different from the EQR mode (figure 2(a)). To further show the variation of the radiation characteristics in different radiation environments, the THz frequency spectrum and the temporal waveform observed at different spatial locations are illustrated in figures 2(b), (e), (f) and (g). At the location of probe 1 (figures 2(b) and (f)), the radiation frequency of the CW THz beam is focused on the 29.85 THz, which means the generation of the narrowband THz radiation. However, the external magnetic field magnifies the amplitude of the THz beam significantly, which results in enhancement of the intensity of the electric field at 29.85 THz. At the location of probe 2 (figures 2(e) and (g)), comparing with the afterglow of the lasers in $B = 0$ case, the THz radiation with strong field strength is generated by the coupled effect of the lasers and the external magnetic field. Interestingly, as shown in figure 2(c), the emission angle is also different for two radiation modes. The radiation direction of the two THz beams of EQR ($B = 0$) mainly focuses on $\theta \approx \pm 52.5^\circ$, which has a narrow distribution of the emission angle ($\Delta \theta \approx \pm 8^\circ$) with the presence of two lobes located at the range of $-90^\circ \leq \theta \leq 90^\circ$. For the THz radiation generated by the QDR mode ($B > 0$), there is only one strong lobe with a wide distribution of the emission angle ($\Delta \theta \approx \pm 30^\circ$) (see figure 2(c)). Clearly, by the implantation of an external magnetic field, there is not only a modulation for the radiation direction (i.e., the emission angle) of the CW THz beams, but also a conversion between the EQR and QDR modes.

To further reveal the modulation for the radiation direction and the flexible conversion of the different modes, the effect of the magnetic field on the main emission angle (i.e., the emission angle with
the max field strength of one beam of the two beams) and the amplitude difference of the field strength between the two radiation beams with the different emission angle $\Delta E = |E_{01} - E_{02}|$ are shown in figure 3(a). With the increase of the magnetic field, the main emission angle (the amplitude difference $\Delta E$) will be modulated from $\theta = 52.5^\circ$ to $\theta = 32.4^\circ$ ($\Delta E = 0.073$ to $\Delta E = 3.85$ MV m$^{-1}$) (see also figure 2(c)). Clearly, the radiation angle is effectively modulated, and the two radiation beams generated by the EQR mode (characterised with $\Delta E > 0$) will be modulated to form the quasi-single radiation beam corresponding the radiation characteristics of the QDR mode (characterised with $\Delta E = 0$) when the plasma is magnetized gradually, which presents the unique tuning characteristics of this radiation scheme based on the electrostatic oscillation of the electrons. The increase of the lasers’ intensity has an obvious effect on the amplitude difference of the two radiation beams $\Delta E$, which does not generate any significant effects on the main emission angle. Furthermore, the relative amplitude difference of the THz field $|\Delta E|/(E_{01} + E_{02}) = |E_{01} - E_{02}|/(E_{01} + E_{02})$ against the laser intensity (figure 3(b)). Clearly, the relative amplitude difference of the THz field cannot be affected by the laser intensity and is improved effectively by the increase of the intensity of the magnetic field. A fitting formula is obtained by $|\Delta E|/(E_{01} + E_{02}) = f_1(E_{11}, B) = \chi_1 E_{11} + \chi_2 B + \chi_3$, where $\chi_1$ is the correlation coefficient of the laser intensity, $\chi_2$ is the correlation coefficient of the intensity of the magnetic field and $\chi_3$ is a constant. The corresponding fitting parameters of $f_1$ are shown in table 1. Clearly, $\chi_1 \ll \chi_2$, the dependence of the relative amplitude difference of the THz field on the intensity of the laser is far less than that on the intensity of the magnetic field. That’s, comparing with the effect of the intensity of the magnetic field on the relative amplitude difference of the THz field, the effects of the laser intensity can be ignored completely. The fitting formula can be simplified with $|\Delta E|/(E_{01} + E_{02}) = f_1(E_{11}, B) \approx \chi_2 B + \chi_3$. The relative amplitude difference of the THz field mainly depends on the intensity of the magnetic field and not the laser intensity (figure 3(b)). Note that, in our theoretical prediction, the relative amplitude difference of the THz field is 0 (figure 2(c)) when the magnetic field is 0. But for the fitting of the PIC simulation results, the amplitude difference and the relative amplitude difference of the THz field is $\chi_3 (\approx 0.000 163)$ when the magnetic field is 0, which is caused by some nonlinear effects (the local deformation of the lasers, the afterglow of the lasers, and the heating effect of the electrons) in the PIC simulation process, which is not considered in the theoretical model. This magnetized modulation way can obtain a steady modulation region of the emission angle and relative amplitude difference of the THz field in the different intensities of the lasers. That is, the strong point of this THz technique is its robustness and flexibility for the modulation of the emission angle and the selection between the EQR and QDR active modes.

In addition to the modulation of the magnetic field on the emission angle and field strength of the THz radiation, the plasma frequency and the frequency shift of the lasers also have the obvious effects on the radiation frequency and field strength of the THz radiation. When the size of the plasma is on the order of a skin depth wide (i.e., the micro plasma), the electrostatic oscillation of the electrons under the restoring force will generate the THz radiation with the plasma frequency. Because of $\omega_p = (ne^2/\varepsilon_0 m)^{1/2}$, the plasma density can modulate the radiation frequency of the THz radiation effectively. As shown in figure 4(a), the plasma frequency is improved significantly with the increase of the plasma density. Accordingly, the frequency of the CW THz radiation is consistent with the variation trends of the plasma frequency given by...
Figure 2. (a) and (d) Snapshots of the THz electric fields at the time of 1.8 ps and $z = 5 \mu m$ after the plasma-vacuum interface when $B = 0$ and $B = 20 T$, respectively. (b) and (e) The spectra of the THz electric fields at the location of the probe 1 and probe 2 and (e) and (f) the corresponding temporal waveforms. (c) The theoretical results and the PIC simulation of the emission angle of the THz electric fields.

Figure 3. (a) The emission angle $\theta$ and the amplitude difference of the two radiation peaks ($\Delta E = |E_\theta^1 - E_\theta^2|$) against the magnetic field with different intensity of laser pulses. (b) The relative amplitude difference of the THz field ($(|\Delta E|/(E_\theta^1 + E_\theta^2))$ against the magnetic field with different intensity of laser pulses. The black line is the fitting results.

Figure 4. (a) The frequency of the THz radiation against the density of the plasma with different frequency of laser pulses. The red line is the theoretical result. (b) The intensity of the THz radiation against the frequency shift of the laser pulses ($\Delta \omega = \omega_1 - \omega_2$) in the different density of the plasma. The different color lines in (b) are the fitting results with different plasma density. $B = 0$.

the theoretical analysis (red line). Although the frequency shift of the lasers has the effects on the radiation frequency of the THz radiation (figure 4(a)), the intensity of the THz radiation is modulated by the frequency shift of the lasers $\Delta \omega = \omega_1 - \omega_2$ effectively (figure 4(b)). As the increase of the frequency shift of the lasers, there is an obvious peak value of the field strength of the THz radiation in the different plasma density. However, for different plasma density, the peak value of the field strength of the THz radiation is also different, which includes the difference in the intensity and the location of the peak value. As the increase of the plasma density, the peak value of the THz radiation increases and the location of the peak value of the THz radiation moves along the high frequency direction. Moreover, a fitting formula for the THz radiation intensity $E(MV/m)$ as a function of the laser frequency shift $\Delta \omega (10 \text{THz})$ and the plasma density $n(10^{17} \text{cm}^{-3})$ is obtained completely and shown in figure 4(b), which is $E(MV/m) = f_2(n, \Delta \omega) = \gamma_1n + \gamma_2\Delta \omega + \gamma_3n^2 + \gamma_4\Delta \omega^2 + \gamma_5n\Delta \omega + \gamma_6$. The corresponding fitting parameters of $f_2$ are shown in
In the following, we will clarify these new THz radiation modes observed in PIC simulations by the 

Table 1. The fitting parameters of $f_1$ and $f_2$.

| $f_1(k_{l1}, R)$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $x_1$ | 0.0014 | 0.02745 | 0.000163 | — | — | — | — | — | — |
| $x_2$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma_5$ | $\gamma_6$ | $\gamma_7$ | $\gamma_8$ | $\gamma_9$ |
| $f_2(n, \Delta \omega)$ | —0.4122 | 3.4488 | —0.2943 | —1.0457 | 0.8354 | —2.8419 |

table 1. The fitting formula is in good agreement with the PIC simulation, which can be used as empirical 
formulæ to subtly modulate the intensity of the THz field in the experiment. Clearly, the plasma frequency 
and the frequency shift of the lasers can also modulate the radiation frequency and field strength of the CW 
THz radiation.

3. Theoretical analysis

In the following, we will clarify these new THz radiation modes observed in PIC simulations by the 

the basic set of equations consisting of the momentum equation, the continuity equation and Poisson’s 
equation for the cold electron plasma

where

where $\omega = \omega_1 - \omega_2$, $k = k_1 - k_2$, $m$ is the mass of the electron. The dynamics of the electron are described 
by the following $y$ and $z$ components of the ponderomotive force

where $\omega_c$ is the electron’s density and $\omega_i$ is the electron’s cyclotron frequency. All the physical quantities can be expressed as affine combinations of the uniform equilibrium value and the perturbed one, i.e., $\eta = n_0 + n_1, u = u_0 + u_1$ and $E = E_0 + E_1$. Then, the perturbed equations can be expressed as

where $r = \tilde{r}(t) \exp [i(kz - \omega t)]$ is the drift distance of the free electrons, $\tilde{r}$ is the amplitude of the electrons’

The equation of the electron’s motion yields

where $r = \tilde{r}(t) \exp [i(kz - \omega t)]$ is the drift distance of the free electrons, $\tilde{r}$ is the amplitude of the electrons’
oscillation and $u_0 = \tilde{r}, \omega_p = (n_0e^2/\varepsilon_0m)^{1/2}$ is the plasma frequency. Equation (8) shows that the electron 
dynamic is driven by external parameters such as the ponderomotive force of the lasers and the external
magnetic field. This effective control for the electron dynamics can generate the THz radiation and realize
the flexible conversion between the EQR and the QDR mode.

When the plasma is not magnetized, i.e., $B_0 = 0, \omega_c = 0$, the equation of motion equation (8) can be 
seen as a typical Helmholzt equation, which can be solved analytically. The Green function of the harmonic
opposite (see figure 5(a)), i.e., the classic electric quadrupole oscillation. The longitudinal radiation current, \( J \), where, \( \rho \), electrons can be expressed by

\[
J = \frac{e n_0}{2} |\mathbf{E}| \cdot \mathbf{B} \quad \text{in the THz regime.}
\]

Clearly, the transverse radiation current \( J_\perp \) against the transverse location. (g) The inset is the net radiation current \( \Delta J = |J_{\perp} - J_{\parallel}| \) with the \( B = 0 \) (blue column) and \( B = 20 \, \mu T \) (red column). The theoretical results of the field strength of the THz radiation (MV m\(^{-1}\)) excited by the transverse momentum \( p_x \) (c) and (d) and the total momentum \( p_x p_y \) (e) and (f) of the electrons at the time of 1.8 ps. The left (right) column for \( B = 0 \) (\( B = 20 \, \mu T \)).

Figure 5. (a) and (b) The PIC simulation of the number of the electrons in the \( y-v \) plane when \( B = 0 \) and \( B = 20 \, \mu T \) respectively, the red solid line represents the transverse current \( J_{\perp} \) against the transverse location. (g) The inset is the net radiation current \( \Delta J = |J_{\perp} - J_{\parallel}| \) with the \( B = 0 \) (blue column) and \( B = 20 \, \mu T \) (red column). The theoretical results of the field strength of the THz radiation (MV m\(^{-1}\)) excited by the transverse momentum \( p_x \) (c) and (d) and the total momentum \( p_x p_y \) (e) and (f) of the electrons at the time of 1.8 ps. The left (right) column for \( B = 0 \) (\( B = 20 \, \mu T \)).

operator in the left-hand side is given by \( G(t, t') = \sin[\omega_p t - \omega_p t'] / \omega_p \). Then the drift distance of the free electrons can be expressed by

\[
\mathbf{r} = \frac{1}{m} \int_{-\infty}^{\infty} \mathbf{F}(t) G(t, t') \, dt = \frac{\eta W_p}{(T_w + W_p)^2} \left[ 1 - 3T_w^2 W_p^2 \right] \left( \frac{j k^2 - 4y}{b^2 y} \right),
\]

where \( W_p = \frac{e \omega_p^2}{2 \eta} \) and \( \eta = \sqrt{2e^2 E_0^2 r / 8m^2 \omega_0 \omega_w} \), \( T_w = \frac{\sqrt{2e^2 / \omega_0 \omega_v}}{\omega_w} \approx \frac{\sqrt{2e^2}}{\omega_w} \) due to the propagation time of laser in the micro plasma is far less than the half wave width of the laser, i.e., \( z/c \ll t_w \). Equation (9) clearly shows that the drift distance \( \mathbf{r} \) of the free electrons in the \( x \)-direction is not affected by the space location \( \mathbf{r} \), which is a general longitudinal dipole oscillation investigated in relevant references [16, 18, 20]. But the drift distance \( \mathbf{r}_y \) of the free electrons in the \( y \)-direction will be divided into the positive part with \( y > 0 \) and the negative part with \( y < 0 \). When the plasma is magnetized (\( B > 0 \)), we use the Fourth order Runge–Kutta method to obtain the numerical solution of the drift distance of the electrons given by equation (8). Through the continuity equation (2), the perturbed density of the electrons can be obtained by \( n_1 = -n_0 \nabla \mathbf{r} \). Accordingly, the radiation current can be expressed by

\[
J_{\perp} = -n_0 e u_{t_{\perp}} - n_1 e u_{t_{\parallel}} = -(n_0 + n_0 \nabla \mathbf{r}) e_{t_{\perp}},
\]

\[
J_{\parallel} = -n_0 e u_{t_{\parallel}} - n_1 e u_{t_{\parallel}} = -(n_0 + n_0 \nabla \mathbf{r}) e_{t_{\parallel}}.
\]

Clearly, the transverse radiation current \( J_{\perp} \) mainly depends on the transverse drift distance of the electrons \( r_{\perp} \), which means the oscillation direction of the radiation current located at either side of the laser axis is opposite (see figure 5(a)), i.e., the classic electric quadrupole oscillation. The longitudinal radiation current \( J_{\parallel} \) depends on the longitudinal drift distance of the electrons \( r_{\parallel} \), which has been investigated in relevant references.

After the lasers pass through the micro plasma, the electrons oscillate with the amplitude \( r \) and frequency \( \omega \) under the restoring force and Lorentz force of the magnetic field, which means the formation of the dipole array in the transverse and longitudinal direction. The vector potential of the THz radiation generated by a dipole is

\[
\mathbf{A}(\mathbf{r}) = \frac{i \mu_0}{4 \pi R} \int \mathbf{J}(\mathbf{r}) \, dV',
\]

where, \( R \) is the propagation distance of the THz wave from the radiation source and \( \mu_0 \) is the permeability of vacuum. Due to the integral part of equation (11) can be expressed by \( \int_j \mathbf{J}(\mathbf{r}) \, dV' = \hat{p} \), the vector...
potential of the radiation field generated by a dipole is

\[ \mathbf{A}(\mathbf{r}) = \frac{\mu_0 k}{4\pi R} \mathbf{p} = \frac{\mu_0 k^2}{4\pi R} (\hat{p}_y + \hat{p}_z), \]  

(12)

where, \( p_{yx} = n_1 e r_{yx} \) is the dipole moment of the electrons. For a dipole, the electric field of the radiation field generated by the dipole oscillation can be expressed by

\[ d\mathbf{E}(y, z, t) = \frac{ic}{k} d[\nabla \times (\nabla \times \mathbf{A})] = \frac{e^{ikR}}{4\pi \varepsilon_0 c R} [d\hat{p}_y \cos \theta + d\hat{p}_z \sin \theta] \hat{\theta}. \]  

(13)

where \( k(=\omega/c) \) is the wave vector of the THz radiation, \( \theta \) is the emission angle. For the whole plasma, the final THz field can be obtained by integrating the radiation field from each dipole oscillation inside the plasma \( E_{THz} = \int_0^{l_w/2} d\mathbf{E}(y, z, t) \).

In figure 5, the transverse distribution of the current density \( J_\perp \) inside the plasma and the radiation field are obtained theoretically and the number of the electrons in the \( (y - v_y) \) plane is generated by the PIC simulations. When the plasma is unmagnetized, i.e., \( B = 0 \), the spatial distribution of the electrons number (the electrons velocity) on two sides of the laser axis is quasi-symmetrical (antisymmetric), correspondingly, the transverse total current density on both sides of the laser axis is centrosymmetric about point \( (J_\perp, y) = (0, 0) \) (figure 5(a)). In this case, the oscillation mode of the electrons is similar to two large charges oscillating with the opposite directions on both sides of the lasers’ axis \( y = 0 \), i.e., EQR is generated. When the plasma is magnetized, i.e., \( B > 0 \), the symmetry of the electrons’ distribution is broken by the increase of the electrons’ number in the \( y < 0 \) region with the same oscillation direction, which results in the spatial symmetry of the current density is broken with the increase (decrease) of the region of the positive (negative) current (figure 5(b)). The oscillation mode of the electrons is similar to single large charges oscillating with the same direction, i.e., QDR is generated. Meanwhile, the net current density \( \Delta J = |J_{\perp+} - J_{\perp-}| \) inside the magnetized plasma is greater than that inside the unmagnetized plasma (figure 5(g)), which evolves in time and emits the strong QDR. To further prove the importance of the transverse current density \( J_\perp \), the THz radiation pattern without the effects of the longitudinal current density \( J_\parallel \) in different radiation environments is obtained theoretically (figures 5(c) and (d)). In the case of \( B = 0 \), two narrowband THz beams with different emission directions and opposite electric field are excited effectively (figure 5(c)). But the two narrowband THz beams will be modulated to form the strong single THz beam in \( B = 20 \) T case (figure 5(d)). Clearly, the transverse behavior of electrons is the key factor for the realization of the EQR and QDR modes and the magnetic-induced transformation between the two radiation modes. Finally, by synthetically considering both the longitudinal current and the transverse current, the total electric field of the THz radiation of the EQR and QDR modes in the different plasma environment is shown in the figures 5(e) and (f). For the EQR in the unmagnetized plasma, there is an enhancement of the intensity of the CW THz radiation due to the effects of the longitudinal component of the current density. But the spatial distribution of the narrowband THz radiation is not changed significantly (figure 5(e)). However, the spatial distribution of the QDR in magnetized plasma will be replaced by an asymmetrical spatial distribution due to the variation of the radiation region and the drift of the emission angle by magnetic field (figure 5(f)), which is in good agreement with the PIC simulation (see figures 2(a) and (d)). Moreover, the theoretical results also show that the emission angle of the THz radiation can be modulated from two emission angles to one emission angle in the different radiation environments, which is also in good agreement with the PIC simulation and corresponding to the conversion between the EQR and QDR mode (figure 2(c)).

4. Conclusion

In summary, we demonstrated the flexible conversion between the EQR and QDR mode through the interaction of the two-color laser pulses and the plasma immersed in the uniform magnetic field. The electrons involved in oscillation show the completely different oscillation modes including the difference of the number, velocity and space distribution of the electrons in different magnetized environments. Thus, the corresponding oscillation modes of the radiation current are modulated by the coupled effect of the laser pulses and the external magnetic field effectively. The modulation mechanism can be well understood by the magnetized electrostatic oscillation of the electrons (i.e., the electrostatic component of the charge separation field), which is the principle of the flexible conversion between the EQR and QDR modes at the THz frequency. Clearly, this opens a key way toward light magnetized-plasma interaction beyond the state of art about the excitation and conversion of the multiple radiation modes.
Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant Nos. 11865014, 11765017, 11764039, 11475027, 11274255, and 11305132.

Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

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