Composite Supersymmetric Axion-Dilaton-Dilatino System
and The Breaking of Supersymmetry

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PACS numbers : 11.30Pb, 12.50Lr

ABSTRACT

The spontaneous breakdown of the scale, the chiral and the superconformal symmetries for a hidden $SU(N)$ gauge group is studied in an effective lagrangean approach. The relevant low-energy degrees of freedom are taken to be the composite Goldstone particles associated with these three broken symmetries. Supersymmetry is spontaneously broken in the large $N$ limit and soft breaking terms in the observable sector are generated, together with nonrenormalisable Nambu-Jona-Lasinio type interactions.
1 - Introduction

The dilaton and the axion were introduced in field theory for a priori two very different reasons.

The axion was first postulated to exist by Weinberg and Wilczek [1] in order to solve the famous $U(1)$ problem in QCD and is usually seen as the Goldstone boson of the $U(1)$ Peccei-Quinn type symmetries [1] which allow us to dynamically set $\theta_{QCD} = 0$ by a chiral rotation. Its mass and couplings have been extensively studied [2] and the experimental searches impose a rather strong constraint on the scale $\Lambda$ associated with the spontaneous breakdown of the $U(1)$ symmetry $10^8 GeV < \Lambda < 10^{12} GeV$.

To obtain such values in a natural way (avoiding a new hierarchy problem) the composite models of axions [3] seem to be the most simple and elegant. A new gauge group is introduced with the corresponding coupling constant becoming strong at the scale $\Lambda$ producing fermion condensation and the breaking of the Peccei-Quinn symmetry. These models need new exotic coloured fermions whose only manifestation at low energies is through the composite axion.

The dilaton was originally introduced with the hope of understanding the cosmological constant problem, i.e. why the cosmological constant postulated by Einstein in the general relativity field equations is so small [4]. Even if it fails in this respect it has remarkable properties worth to be studied. At the classical level it restores the dilatation invariance and is the Goldstone boson associated with the spontaneous breakdown of this symmetry.

In a supersymmetric theory the two symmetries seem to be very close in the sense that the corresponding improved currents, together with the supersymmetry current transform into themselves under supersymmetry and form a realsupermultiplet of currents [5].

The corresponding charges together with Poincaré, supersymmetry and conformal generators give rise to the superconformal algebra [6]. Classically the superconformal group is a symmetry in the limit of zero masses and is plagued with quantum anomalies [7].

Introducing a common scale $\Lambda$ for the dynamical breaking of the three symmetries and remembering that the fermionic current was the supersymmetry cur-
rent, $\Lambda$ will correspond to the spontaneous breaking of supersymmetry with the corresponding Goldstone fermion called dilatino. The easiest way to break supersymmetry in such a way that the soft breaking terms are in the 1TeV region is to suppose that supersymmetry is broken in a hidden sector communicating very weakly with the observable one [8]. This scenario appears naturally in the heterotic superstrings models [9], in which the hidden sector couples through gravitational interactions.

The dilaton-axion system arises naturally in any superstring inspired model in the $d = 10$ supergravity multiplet and in the compactification process as internal manifold degrees of freedom [10].

The purpose of this paper is to construct a low energy-effective lagrangian of the observable matter coupled to the axion-dilaton-dilatino system in a composite model where they are dynamically generated composite particles.

Consider a $N = 1$ supergravity theory containing a hidden and an observable sector communicating only through gravitational interactions. The running of the hidden gauge group coupling constant will induce gaugino condensates $<\lambda\lambda>$ by nonperturbative effects at a scale $\Lambda$ [8]. The three symmetries which mix the observable and the hidden sector, namely $U(1)_R$, dilatation and supersymmetry transformations will be dynamically broken.

The corresponding Goldstone particles, the axion, the dilaton and the dilatino will be the only relevant degrees of freedom containing hidden gauge group gauginos at low energies. They will be composite bound states generated at the scale $\Lambda$ and their interactions will contain $\Lambda$ as the fundamental scale.

The other Goldstone particles associated with the breaking of the global symmetries concerning only the hidden gauge group have negligible interactions with the observable sector involving negative powers of the Planck mass.

The essential point is that the only remnant memory of the hidden sector is contained in the Goldstone particles related to the spontaneous breaking of those global continuous symmetries which are common to the observable and the hidden sector. It must be emphasized that this results in a different parametrization of the gaugino condensation comparing with ref.[11] The gaugino condensates give a v.e.v. to the auxiliary component of a composite superfield and will be a sign for
the susy breaking.

Because the scale \( \Lambda \) will be of the order \( 10^{11} \text{GeV} \) the supergravity multiplet will play no role in the dynamics and we will consider a flat gravitational background.

Even if it is not directly implied in the dynamics we must suppose an underlying \( N = 1 \) supergravity, otherwise the observable and the hidden sector do not communicate and we will have two separate \( U(1)_R \) symmetries. As a consequence in this case the breaking of the hidden sector \( U(1)_R \) and of the corresponding supersymmetry will not be transferred to the observable part. Section 2 introduces the supercurrent and the superconformal anomalies. A very simple proof is presented in the appendix which shows that the Wess-Zumino chiral field contribution to the anomalies is chiral to all orders in the perturbation theory. Section 3 gives a supersymmetric definition of the three Goldstone particles, studies the transformation properties under the various symmetries and constructs an effective lagrangean at low energies which reproduces the symmetry behaviour of the high-energy theory in the spirit of ref. [11].

Section 4 deals with the analysis of supersymmetry breaking in this model in the large \( N \) limit, the generation of the soft-breaking terms and the radiative stabilisation of the dilaton potential. The problem of the cosmological constant is discussed.

In the conclusion some speculations about possible dynamical effects at low energies of the nonrenormalisable terms induced at the scale \( \Lambda \) are given.
2- The supercurrent and the superconformal anomalies

The currents corresponding to the three above symmetries can be put in a real superfield $V_m$ [5] with the following components:

$$V_n = C_n^5 + i\theta\chi_n - i\bar{\theta}\bar{\chi}_n + \frac{i}{2}\theta^2[M_n + iN_n]$$

$$- \frac{i}{2}\bar{\theta}^2[M_n - iN_n] - \theta\sigma^m\bar{\theta}V_{mn} + i\theta^2\bar{\theta}[\bar{\lambda}_n + \frac{i}{2}\bar{\sigma}^m\partial_m\chi_n]$$

$$- i\bar{\theta}^2[\lambda_n + \frac{i}{2}\sigma^m\partial_m\bar{\chi}_n] + \frac{1}{2}\theta^2\bar{\theta}^2[D_n + \frac{1}{2}\Box C_n^5]$$

(1)

We define $V_n = \bar{\sigma}^\alpha\alpha V_{\alpha\dot{\alpha}}$ where $\alpha$ and $\dot{\alpha}$ are two-component indices. The divergence of the supercurrent $V_n$ has anomalies [7] which can be summarized in the following formula

$$D^\alpha V_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}}A^+$$

(2)

where $A$ is a chiral superfield. A simple proof of eq.(2) will be presented in the appendix in a very convenient form. Eq.(2) tells us that $V_n$ must obey the constraint

$$D_{\beta}D^2 V_{\alpha\dot{\alpha}} = 4i(\sigma^m)_{\alpha\dot{\alpha}}\partial_m D^\beta V_{\beta\dot{\beta}}$$

(3)

which allows to eliminate $\lambda_n, D_n$ and the antisymmetric part of $V_{mn}$.

The explicit form of $V_n$ for a supersymmetric theory containing massless chiral $\phi$ and gauge fields $V$ is given by

$$V_{\alpha\dot{\alpha}} = i\phi\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \phi^* + \frac{1}{2}D_\alpha \phi D_{\dot{\alpha}} \phi^* + \frac{3}{2} Tr(e^V W_\alpha e^{-V} \bar{W}_{\dot{\alpha}})$$

(4)

Comparing the formulae (4) and (1) and using (3) we can make the identification

$$J_n^{(5)} = \frac{1}{2}C_n$$

$$J_m = (\chi_n + 2\sigma_n \bar{\sigma}^m \chi_n)$$

$$\theta_{mn} = \frac{1}{6}(V_{mn} + V_{nm} - 2\eta_{mn} V_k^k)$$

(5)

where $J_m^{(5)}, J_m$ and $\theta_{mn}$ are respectively the $U(1)_R$ current, the supersymmetry current and the energy-momentum tensor.
The transformation properties of the various fields under $U(1)_R$ are given by

$$\begin{align*}
\theta' &= e^{-\frac{3}{2}i\alpha} \theta \\
\phi'(\theta') &= e^{-i\alpha} \phi(\theta) \\
W'(\theta') &= e^{-\frac{3}{2}i\alpha} W(\theta)
\end{align*}$$

where $\phi$ are the set of all chiral superfields and $W$ all the gauge chiral superfields, hidden or observable. Under the dilatations the transformations are given by

$$\begin{align*}
x' &= e^{-\beta} x \\
\theta' &= e^{-\beta/2} \theta \\
\phi'(x', \theta') &= e^{\beta} \phi(x, \theta) \\
W'(x', \theta') &= e^{\frac{3}{2}\beta} W(x, \theta)
\end{align*}$$

Classically, in a massless theory the corresponding currents are conserved. Including the generators for Poincaré, the supersymmetry, conformal transformations and the superconformal spinorial generator they form the superconformal algebra [6].

In the quantum theory we will have anomalies conveniently described by a chiral superfield containing $\theta_m^m, \partial^m J_m^{(5)}$ and $\partial^m I_m$, where $I_m$ is the superconformal current. The corresponding charge is ”the square root” of the conformal transformations, just as the supersymmetry generator is ”the square root” of the translation generators.

Note the similarity between the following relations

$$\begin{align*}
\partial_m D_m &= \theta_m^n \\
\partial_m I_m &= \sigma_m J_m
\end{align*}$$

where $D_m$ is the dilation current defined as the first moment of the energy-momentum tensor. Similarly $I^m$ will be the first moment of the supersymmetry current $J_m$

$$\begin{align*}
D^m &= x_n \theta^{nm} \\
I^m &= (\sigma^n x_n) J^m
\end{align*}$$

Classically the right hand side in (8) is zero, but the quantum anomalies will spoil the symmetries.

A short proof will be presented in the appendix for computing these anomalies which has the advantage of giving the chiral matter contribution in a form directly related to the superpotential and which is equivalent on-shell to the usual form.
Using eq.(2) we can readily check the following useful formulae concerning the anomalies

\[
[D^\alpha, \bar{D}^{\dot{\alpha}}]V_{\alpha\dot{\alpha}} = D^2 A + \bar{D}^2 A^+ \\
\{D^\alpha, \bar{D}^{\dot{\alpha}}\}V_{\alpha\dot{\alpha}} = -2i\partial^m V_m = D^2 A - \bar{D}^2 A^+ 
\] (10)

3- The classical definition of the axion-dilaton-dilatino system and the symmetry transformation properties

The definition that we will adopt is such that classically the three Goldstone particles will couple to the divergences of the corresponding symmetry currents. Explicitly we define their symmetry transformation properties by the following classical equations

\[
\Box(s + s^*) = -\frac{2\gamma}{M_p} \theta^m_m \\
\Box(s - s^*) = \frac{2\gamma}{M_p} \partial^m J^5_m \\
i\bar{\sigma}^m \partial_m \Psi = \frac{2\gamma}{M_p} \bar{\sigma}^m J^m = \frac{2\gamma}{M_p} \partial_m \bar{J}^m 
\] (11)

The three equations describe the dilaton \( s + s^* \), the axion \( s - s^* \) and the dilatino \( \psi \), respectively. When we write the effective action below the gaugino condensation scale \( \Lambda \) we will determine \( \gamma \) and find quantum corrections to eqn.(11) suppressed by exponentials of \( \frac{1}{\Lambda} \).

We can rewrite them in a superfield language, introducing a chiral supermultiplet \( S \). Then the equations take the form

\[
S + S^+ = \frac{\gamma}{M_p} \frac{[D^\alpha, \bar{D}^{\dot{\alpha}}]}{\Box} V_{\alpha\dot{\alpha}} 
\] (12)

The underlying theory being supergravity, the only available mass is the Planck scale \( M_p \) which should be put on the right-hand side to assure the correct dimensions. In fact, in the effective theory \( M_p \) will be replaced by \( \Lambda \) as the only relevant dynamical scale. The last of eqns.(11) reads

\[
\Box \Psi_\beta = \frac{2\gamma}{M_p} \partial_m J^m_\beta 
\] (13)

Moreover (12) gives the following constraint

\[
F_s = -\frac{\gamma}{M_p} \frac{1}{\Box} (\partial^m M_m + i\partial^m N_m) 
\] (14)
The constraint (3) tell us that

\[ M_m = \partial_m A \]
\[ N_m = \partial_m B \] (15)

with \( A \) and \( B \) scalar and pseudoscalar fields respectively.

Taking for the hidden sector only gauge and gaugino fields we find for this part, using

\[ \mathcal{V}_{\alpha\dot{\alpha}} = \frac{3}{2} Tr(e^V W_\alpha e^{-V W_\alpha}) \]

\[ A + iB = \frac{3}{2}(\bar{\lambda}\lambda) \] (16)

so

\[ F_s = \frac{3\gamma}{2M_p}(\bar{\lambda}\lambda) \] (17)

The gaugino condensation will produce a v.e.v. \( F_s \neq 0 \) which have no contribution from the observable part. Now we clearly see that the superfield \( S \) is very convenient to study the gaugino condensation, because supposing it as relevant low-energy degree of freedom will automatically connect the gaugino condensation to susy breaking.

A useful way to visualise this definition and the leading \( N \) generation of the effective action at the condensation scale is to introduce \( S \) as a static field coupled to the Ferrara-Zumino supercurrent. The lagrangean will be modified by

\[ \mathcal{L} \rightarrow \mathcal{L} - \frac{\gamma}{M_p} \int d^4\theta (S + S^+) \frac{[D^\alpha, \bar{D}^{\dot{\alpha}}]}{\Box} \mathcal{V}_{\alpha\dot{\alpha}} \] (18)

At the scale \( \Lambda \) due to the chiral symmetry breaking a supersymmetric dynamics will be induced for \( S \) of the type \( \frac{a^2A^2}{M_p^2} \int d^4\theta S^+S \), the only leading term in the \( \frac{1}{N} \) expansion if \( \gamma \sim \frac{1}{N} \). We make a one-loop computation for the dynamics with the chiral symmetry breaking effect taken into account by massive gauginos. One obtains \( a^2 = \frac{N^2m_\lambda^2}{2\pi^2\Lambda^2} \ln \frac{\Lambda^2}{m_\lambda^2} \), where \( m_\lambda \) is a typical hidden sector gaugino mass. Taking into account all the planar diagrams will add a multiplicative constant to the above expression which is not essential in our analyses. Choosing \( \gamma \) as to normalize the induced kinetic term we remark that \( \Lambda \) is the only relevant scale in the problem and \( M_p \) disappears completely from the lagrangean. Writing the field equations for \( S \) we find

\[ D^2S = \frac{4}{a\Lambda} \frac{D^2[D^\alpha, \bar{D}^{\dot{\alpha}}]}{\Box} \mathcal{V}_{\alpha\dot{\alpha}} = \frac{64}{a\Lambda} A^+ \]
which is equivalent to (12) in components if we replace $\frac{M_\gamma}{\gamma}$ by $\frac{aA}{4}$.

The transformation properties of the superfields $S$ under the $U(1)_R$ and the dilatation are defined by the right-hand side of (12) with the above-mentioned replacement. Being a dynamical scale, $\Lambda$ transforms as a dimension one field under dilatations. Then we obtain the following results: under $U(1)_R$

$$S + S^+ = \text{inv.},$$

(19)

and under dilatations

$$S + S^+ \to e^\beta (S + S^+).$$

(20)

The first transformation allows us to define

$$S \to S + ia\Lambda\alpha$$

under $U(1)_R$ with an arbitrary real coefficient $a$, and the second one

$$S \to e^\beta S$$

(22)

Then denoting by $d$ and $a$ the real dilaton and the axion, and using the variables $y$ and $\theta$, where $y^m = x^m + i\theta^m\bar{\theta}$, we can write the superfield $S$ as

$$S(y, \theta) = ia + e^d + \theta\psi + \theta^2 F$$

(23)

In the following using the symmetries of and the anomaly structure of the theory above the condensation scale $\Lambda$ we will find the effective lagrangian for the observable matter plus the Goldstone system.

In order to have the correct coupling to matter we must have an interaction of type

$$\frac{1}{\Lambda} \int d^4\theta (S + S^+) \frac{[D^\alpha, D^{\dot{\alpha}}]}{\Box} \mathcal{V}_{\alpha\dot{\alpha}} = \frac{2i}{\Lambda} \int d^4\theta (S - S^+) \frac{1}{\Box} \partial^m \mathcal{V}_m$$

(27)

In (27) only the observable part is retained in the current $\mathcal{V}_m$, fact that which will be tacitly assumed in the following. Then because all the induced lagrangian is generated from this primary interaction it will be a function of $S$ and $\partial^m \mathcal{V}_m$. The first terms in a $\frac{1}{\Lambda}$ expansion are of the form
\[ \mathcal{L}_{\text{eff}, \text{inv}} = \int d^4\theta \left\{ S^+ S + \frac{i}{a\Lambda}(S - S^+) \frac{1}{\Box} \partial^m \mathcal{V}_m + \frac{d}{\Lambda^2} (\partial^m \mathcal{V}_m)^2 \right\} + \left( e\Lambda^3 \int d^2\theta e^{-\frac{2\pi}{\sqrt{\kappa}}} + \text{h.c.} \right) \] (28)

The canonical kinetic term for \( S \) is fixed by the \( U(1)_R \) symmetry and is different from that of the superstring-inspired dilaton. It will be the same as those corresponding to the Standard Model dilaton [4] and axion [1]. \( \mathcal{L}_{\text{eff}} \) must be globally supersymmetric in order to obtain a spontaneous breaking of SUSY which gives only soft breaking terms [15] and no quadratic divergences. It is natural to have the condensation scale \( \Lambda \) as the fundamental mass parameter in \( \mathcal{L}_{\text{eff}} \) because the effective interaction between Goldstone particles and observable matter is generated at that scale. The third term is the lowest nonrenormalisable generated for the observable part and we will see that it can give dynamical effects at low energies. The last term is the most general superpotential for the \( S \) field invariant under \( U(1)_R \) and dilatations and \( a \) is the same coefficient appearing in the transformation of the axion (21), which is arbitrary for the moment and will be fixed by asking the compensation of the \( U(1)_R \) anomaly.

To reproduce the hidden sector anomalies at low energy we must introduce an anomalous term of the form [11]

\[ \mathcal{L}_{\text{an}} = \frac{1}{3} \int d^2\theta \mathcal{A}(\ell \frac{\mathcal{A}}{\mu^3} - 1) + \text{h.c.} \] (29)

In the above formula (29), \( \mu \) does not transform under dilatations in order to correctly reproduce the anomalies.

Using (10) and the defining eq. (12) we find classically

\[ S + S^+ = \frac{1}{M_p} \frac{1}{\Box} (D^2 \mathcal{A} + \bar{D}^2 \mathcal{A}^+) \] (30)

and as a consequence

\[ \mathcal{A} = \frac{M_p}{16} (\bar{D}^2 S^+) \] (31)

Introducing this last eqn. in (29) we will find in fact \( \delta \mathcal{L}_{\text{an}} = 0 \) under both \( U(1)_R \) and dilatations. To write the effective lagrangean for \( S \) we need a relation between
$S$ and $A$ which is equivalent to an off-shell definition of the Goldstone particles contained in $S$. Lacking any natural candidate, the best thing to do is to find the most general one compatible with the symmetries. Then we are lead to the following equation

$$\mathcal{A} = b\Lambda \bar{D}^2 S^+ + c\Lambda^3 e^{-\frac{2\Lambda}{aN}}$$

(32)

We will see that taking $b = 0$ gives two supersymmetric minima, so $b$ is a crucial parameter related to the dynamical nature of the scale $\Lambda$. Indeed regarding $\Lambda$ as a fixed scale forces us to consider only the second term in the right-hand side of eq. (32) and supersymmetry is not broken. The complete induced lagrandean is defined by the eqs. (28), (29) and (32). At the one-loop level, the interaction $S$ field-observable matter will produce an effective term equivalent to replacing the anomaly equation (10a) into the lagrangean (28) which gives

$$\mathcal{L}_{\text{one-loop}} = -\frac{1}{2a\Lambda} \int d^4\theta (S - S^+)(\frac{D^2}{\partial^2} A - \bar{D} A^+) = \frac{2}{a\Lambda} (\int d^2\theta S A + \int d^2\bar{\theta} S^+ A^+)$$

(34)

We can interpret this term as a generation of a superpotential taking into account the form of the anomaly given in the appendix. To obtain that we need a nontrivial v.e.v. for the dilaton. Now we see that the transformation of $S$ under $U(1)_R$ was chosen to compensate the anomaly. The characteristic feature of this composite model is that in addition to the usual interaction with the gauge fields present in the superstring inspired supergravity models it generates an interaction with the superpotential. In components we will obtain a usual coupling of the axion-dilaton system to the matter [1-4].

We will be interested in the large $N$ limit of the theory in order to make quantitative predictions about the supersymmetry breaking. To have a nontrivial $\frac{1}{N}$ development we must be able to factorize the $N$ dependence in the lagrangean by performing field rescalings. Making $W^\alpha \rightarrow N^{\frac{1}{2}} W^\alpha$ and $S \rightarrow N^{\frac{1}{2}} S$ we fulfill this requirement if the quantities $\frac{a}{N^2}, \frac{b}{N}, \frac{c}{N^2}, \frac{e}{N}$ and $dN$ are held fixed in the large $N$ limit. To derive the large $N$ behaviour of $b$ and $c$ we used the form of the anomaly $\mathcal{A}$ given in (A.16).

Interpreting $\Lambda$ as a dynamical scale, all the terms but $\mathcal{L}_{an}$ are invariant under the dilatations such that the dilaton has really the role of restoring the symmetry at the classical level. It should be emphasized that we are dealing with two different
scale symmetries, a high energy one which was described in this paragraph and a low energy one when Λ is kept fixed. In the latter case redefining the scale transformation of \( S \) we can interpret the term in eq.(34) as being the Wess-Zumino term for the three anomalous symmetries.

4 - The dynamical breaking of supersymmetry

Usually in a globally supersymmetric theory the gaugino condensation do not breaks supersymmetry* and we need a v.e.v. for \( \theta_\mu \) to do that, according to the relation

\[
\langle 0 | -\frac{1}{8} Tr \left\{ (\bar{\sigma}^\mu J_\mu)^{\hat{\alpha}}, \bar{Q}_\alpha J \right\} | 0 \rangle = \langle \theta^m_\alpha \rangle = \frac{\beta(g)}{2g} < F^a_{mn} F^{amn}>
\]  

(35)

which vanishes in globally super Yang-Mills theory. In our case however the dynamical bound states \( S \) may change the situation. Using the formula

\[
\langle \theta^m_\alpha J \rangle = \langle \int d^2 \theta A + \int d^2 \bar{\theta} A^+ >
\]  

(36)

and using the identification of the chiral anomaly \( A \) in eq.(32) we find

\[
\frac{1}{8} < 0 | Tr \left\{ \bar{\sigma}_\mu J_\mu^\alpha, \bar{Q}_\alpha \right\} | 0 > = c \Lambda^3 < \int d^2 \theta e^{-\frac{4a}{\alpha}} + \int d^2 \bar{\theta} e^{-\frac{4a^*}{\alpha}} > =
\]

\[
= -\frac{3c}{a} \Lambda^2 < e^{-\frac{4a}{\alpha}} F_s + e^{-\frac{4a^*}{\alpha}} F_s^* >
\]  

(37)

In order to break susy we need \( < e^{-\frac{4a}{\alpha}} F_s > \neq 0 \) at the minimum of the effective action, a condition sufficient for generating soft-breaking terms in the observable sector. Writing only the terms in (28-29) relevant for the s effective potential, we obtain

\[
\mathcal{L}_s = F_s^* F_s - \frac{3e}{a} \Lambda^2 J \left( e^{-\frac{4a}{\alpha}} F_s + e^{-\frac{4a^*}{\alpha}} F_s^* \right) -
\]

\[
-\frac{c\Lambda^2}{a} \left\{ e^{-\frac{4a}{\alpha}} F_s ln \left( \frac{b\Lambda}{\mu^3} F_s^* + c \frac{\Lambda^3}{\mu^3} e^{-\frac{4a}{\alpha}} \right) + e^{-\frac{4a}{\alpha}} F_s^* ln \left( \frac{b\Lambda}{\mu^3} F_s + c \frac{\Lambda^3}{\mu^3} e^{-\frac{4a^*}{\alpha}} \right) \right\}
\]  

(38)

* See, for example, G. Veneziano and S. Yankielowicz in ref. [11].
It is readily seen that $c$ and $e$ are redundant parameters that can be absorbed in redefinitions of $s$ and $\mu$ such that $c = 1$ and $e = 0$. As it must be, $F_s$ is a constraint and is defined implicitly through the following equation

\[
\left(1 - \frac{b}{a} \frac{e^{-\frac{3s}{a\Lambda}}}{\Lambda^2 + e^{-\frac{3s}{a\Lambda}}}\right) F_s^* = \frac{1}{a} \Lambda^2 e^{-\frac{3s}{a\Lambda}} \ln \left( \frac{b \Lambda}{\mu^3} F_s^* + \Lambda^3 e^{-\frac{3s}{a\Lambda}} \right) \tag{39}
\]

Using it in (38) we obtain the $s$ potential

\[
V = |F_s|^2 \left\{ -\frac{b}{a} \left( \frac{e^{-\frac{3s}{a\Lambda}}}{\Lambda^2 + e^{-\frac{3s}{a\Lambda}}} + \frac{e^{-\frac{3s}{a\Lambda}}}{\Lambda^2 + e^{-\frac{3s}{a\Lambda}}} \right) + 1 \right\} \tag{40}
\]

To find the minimum we minimize $V$ with respect to $s$

\[
F_s e^{\frac{3s}{a\Lambda}} \left\{ \ln \left( \frac{b \Lambda F_s^*}{\mu^3} + \frac{\Lambda^3}{\mu^3} e^{-\frac{3s}{a\Lambda}} \right) + \Lambda^3 \frac{e^{-\frac{3s}{a\Lambda}}}{\mu^3} \right\} = 0 \tag{41}
\]

and combine it with (39). Searching for real solutions, we find the following results:

\[
i) F_s = 0, \quad <V> = 0
\]

\[ii - iii) \quad \frac{3s}{a\Lambda} = \ln \left( \frac{b \Lambda^3}{\mu^3} \right) + \frac{1}{(bA + 1)}
\]

\[
\frac{F_s}{\Lambda^2} = Ae^{-\frac{3s}{a\Lambda}}
\]

\[
<V> = -\Lambda^4 A^2 e^{-\frac{3s}{a\Lambda}} \left[ \frac{2b}{a(bA + 1)} - 1 \right]
\]

where $A = \frac{1}{2ab} \left[ b - a \pm ((b - a)^2 - 4ab)^{\frac{1}{2}} \right]$. The case $i)$ contains the runaway vacuum obtained sending $s$ to infinity and another supersymmetric solution defined by $\frac{3s}{a\Lambda} = \ln \frac{\Lambda^3}{\mu^3}$. These are the only extrema in the case $b = 0$. From (42) we remark that the only solution for a vanishing cosmological constant $<V> = 0$ is $b = 0$. In this case $F_s = 0$ and supersymmetry is not broken.

To obtain a definite result for the real minimum we will take the large $N$ limit of these equations. We get $A = \frac{1}{a}$ and $A = \frac{1}{b}$. The first value of $A$ is the minimum in the large $N$ limit and it breaks supersymmetry generating at the same time a large cosmological constant $<V> = -\Lambda^4 A^2 e^{-\frac{3s}{a\Lambda}}$. 13
The soft-breaking terms are generated from the eqs. (28) and (29). Because the $\theta^2$ and $\bar{\theta}^2$ components of the eqs (10) are identities, both of them will have the same form

$$\frac{1}{d\Lambda} F_{\lambda A_1} \lambda \lambda + A_2 w + h.c.$$  \hspace{1cm} (43)

with $w = W$, and $A_1$, $A_2$ some numerical coefficients directly calculable. Together with the soft scalar masses which will be generated by radiative corrections being no more protected by susy they constitute the most general susy breaking terms which do not produce quadratic divergences [15]. As in the usual gauging condensation scenarios [8] to obtain interesting values for the soft terms we need

$$\frac{F^*}{\Lambda} \sim \frac{\mu^3}{\Lambda^2} \sim 1 TeV$$  \hspace{1cm} (44)

which for $\Lambda = 10^{11} GeV$ requires $\mu \sim 10^8 GeV$. Then we can check that the susy breaking solution gives $s > 0$ so the generated superpotential has the required sign and we have a real value for the v.e.v. of the dilaton $d$ defined in the equation (23).

**Conclusions**

The composite axion-dilaton-dilatino system is used to parametrize the low-energy degrees of freedom below the gaugino condensation scale. We couple the corresponding superfield as a static field to the divergence of the supercurrent as in eq. (18). At the condensation scale $\Lambda$ it acquires a dynamics due to the breaking of the chiral symmetry and becomes a propagating degree of freedom, corresponding to the formation of bound states. Using symmetry considerations we have constructed an effective lagrangean which admits a supersymmetry breaking minimum in the large $N$ limit and generates soft-breaking terms for the observable sector. No fine-tuning is possible in order to have a vanishing cosmological constant and broken supersymmetry.

We can speculate about the possible dynamical effects induced by the third term in the effective lagrangean (28) of the type $\frac{1}{N^2} \int d^4 \theta (\partial_{m} V_{m})^2$. Working out the components we find the local terms $C^5_m C^5 m$ and $(A^2 + B^2)$ using the notations from eqs. (1) and (15). Considering only the fermion terms we obtain Nambu-Jona-Lasinio interactions $(\lambda \lambda \bar{\lambda} \bar{\lambda})$ and $(\psi \sigma_m \bar{\psi})^2$, where $\psi$ is some observable.
fermion matter field. The second term can induce the electroweak symmetry breaking and could explain the hierarchy between the susy breaking scale and the electroweak scale by a top-antitop condensation mechanism [17]. The coupling in front of this term is of the order $G \sim \frac{1}{\Lambda^2}$ and is unable to produce the condensation for values $G < G_c \sim \frac{1}{\Sigma^2}$ where $\Sigma$ is a typical value for the soft-breaking terms. It will start to run to lower energies like an asymptotically free coupling [18] and will reach the critical value at a much lower energy. The explanation of the hierarchy is the same as the one between the QCD scale and the unification scale. The first operator $(\lambda \lambda)(\bar{\lambda} \bar{\lambda})$ is analogous to the gravity induced interaction in supergravity and could produce gaugino condensation in supersymmetric QCD.

Aknowledgments

I would like to thank P. Binétruy for many enlightening discussions and constant help. I enjoyed useful conversations with F. Pillon and C. Savoy.
APPENDIX

Consider a renormalisable theory containing arbitrary chiral multiplets \( \phi_i \) with a superpotential \( W(\phi_i) \) interacting with the gauge fields of an arbitrary gauge group. The Lagrangian is

\[
\mathcal{L} = \int d^4\theta \phi^+ e^V \phi + \int d^2\theta \left[ \frac{1}{4g^2} Tr(W^\alpha W_\alpha) + W(\phi, h_i, m_i) + h.c. \right] \quad (A.1)
\]

We will compute the vacuum energy density \( E_o \) which is a renormalisation invariant quantity depending on an arbitrary mass scale \( \mu \) and the other parameters of the lagrangean, coupling \( h_i \) and mass parameters \( m_i \).

By dimensional analysis we can write

\[
E_o = \mu^4 f(g, h_i, \frac{m_i}{\mu}) \quad (A.2)
\]

where \( g \) is the gauge coupling and \( f \) a dimensionless function. Being a physical observable, \( E_o \) obeys the renormalisation group equation

\[
\left[ \mu \frac{\partial}{\partial \mu} + m_i \gamma_i \frac{\partial}{\partial m_i} + \beta_g \frac{\partial}{\partial g} + \beta_i \frac{\partial}{\partial h_i} \right] E = 0 \quad (A.3)
\]

Combining (A.2) and (A.3) we find the equation

\[
E = -\frac{1}{4V_4} \left[ \beta_g \frac{\partial}{\partial g} + m_i(1 + \gamma_i) \frac{\partial}{\partial m_i} + \beta_i \frac{\partial}{\partial h_i} \right] \Gamma \quad (A.4)
\]

where in the right-hand side of (A.4) we replaced \( E \) by the expression \( \frac{1}{V_4} \Gamma \), \( \Gamma \) being the Legendre transform of the connected vacuum to vacuum generating functional and \( V_4 \) is the space-time volume. In principle the Legendre transform must be performed with respect to any scalar relevant degrees of freedom which could have vacuum expectation values, and in (A.4) \( \Gamma \) is computed at the saddle point value.

Using the definitions

\[
\int \mathcal{D}\phi e^{i[S(\phi) + JF(\phi)\]} = e^{iW(J)} \quad (A.4)
\]

\[
\Gamma(F_c) = W(J) - JF(\phi_c) \quad (A.5)
\]

where \( F_c = F(\phi_c) = \frac{\delta W(J)}{\delta J} \) and

\[
J = -\frac{\delta \Gamma(F_c)}{\delta F_c} \quad (A.6)
\]

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$\phi$ is the set of all quantum fields and $F(\phi)$ the scalar relevant degrees of freedom constructed out of the fundamental fields $\phi$.

At the saddle point $\frac{\delta \Gamma}{\delta F_c} = J = 0$ so using the definition (A.5) we find

$$\frac{1}{V_4} \frac{\partial \Gamma(F_c)}{\partial a_i} = \frac{1}{V_4} \frac{\partial W(J = 0)}{\partial a_i} = \frac{\partial}{\partial a_i} < \mathcal{L} > (A.7)$$

where $a_i$ is any parameter of the lagrangean, namely $g, h_i$ and $m_i$ and we used (A.4).

The last step is to use the renormalized Schwinger principle which allow us to act with the derivatives inside the brackets using appropriate renormalized expressions for the singular product of the composite operators. For example in the $MS$ scheme using the dimensional reduction as regularisation [12]

$$\frac{1}{V_4} \frac{\partial}{\partial g} W(J = 0) = -\frac{1}{2g^3} \int d^2 \theta Tr\{N(W^\alpha W_\alpha)\} (A.8)$$

when $N$ denotes the renormalized product of operators in the sense of Zimmermann [13].

After a trivial rescaling of the gauge field the final result is obtained (we will omit the symbol $N$ in the following, understanding that all the composite operators are renormalized)

$$\mathcal{E}_o = \frac{1}{4} \int d^2 \theta < -\frac{\beta(g)}{2g} Tr(W^\alpha W_\alpha) + m_i(1 + \gamma_i) \frac{\partial W}{\partial m_i} + \beta_i \frac{\partial W}{\partial h_i} > + h.c. (A.8)$$

The result is that for the most general renormalisable lagrangean we can express $\mathcal{E}_o$ as an integral over a holomorphic function. We will introduce the chiral field of anomalies $\mathcal{A}$

$$\mathcal{A} = -\frac{\beta(g)}{2g} Tr(W^\alpha W_\alpha) + m_i(1 + \gamma_i) \frac{\partial W}{\partial m_i} + \beta_i \frac{\partial W}{\partial m_i} (A.9)$$

The anomalous transformations of the lagrangian under a $U(1)$ transformation is given by

$$\delta_{U(1)}^r \mathcal{L} = i\alpha(\int d^2 \theta A - \int d^2 \theta A^+) (A.10)$$

and under dilatations

$$\delta \mathcal{L} = \beta(\int d^2 \theta A + \int d^2 \bar{\theta} A^+) (A.11)$$
To check the equivalence with the usual definition of the chiral anomaly $A$ [7] we take an arbitrary gauge model with chiral superfields $\phi_i$ and the most general renormalisable lagrangian

$$\mathcal{L} = \int d^4\theta [\phi_i^+ (e^V)_{ij} \phi_j] + \left( \int d^2\theta \left[ \frac{1}{4g^2} Tr(W^\alpha W_\alpha) + \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{3} g_{ijk} \phi^i \phi^j \phi^k \right] + h.c. \right)$$

Define the wave function renormalisation $Z^i_j$ and the renormalisation group $\beta_{ijk}$, $X_{ij}$ and $\gamma_{ij}$ functions by

$$\phi^i = Z^i_j \phi^j$$
$$X^\ell_i = \mu \frac{\partial}{\partial \mu} Z^i_j$$
$$m^\ell_i \gamma_{\ell j} = \mu \frac{\partial}{\partial \mu} m_{ij}$$
$$\beta_{ijk} = \mu \frac{\partial}{\partial \mu} g_{ijk}$$

(A.12)

The only renormalisation of $m_{ijk}$ comes from the wave-function renormalisation due to the nonrenormalisation theorem [14]. Then we obtain in the lowest order of the perturbation theory

$$m^\ell_i \gamma_{\ell j} = X^\ell_i m_{\ell j} + X^\ell_j m_{\ell i}$$
$$\beta_{ijk} = X^\ell_i g_{\ell jk} + X^\ell_j g_{\ell ik} + X^\ell_k g_{\ell ij}$$

(A.13)

Introducing the equations (A.13) in the formulae for the chiral multiplet of anomalies

$$A = - \frac{1}{2} \frac{\beta(g)}{g} Tr(W^\alpha W_\alpha) + \frac{1}{2} m^\ell_i \gamma_{\ell j} \phi^i \phi^j + \frac{1}{3} \beta_{ijk} \phi^i \phi^j \phi^k$$

and using the symmetry of $m_{ij}$ and $g_{ijk}$ we obtain

$$A = - \frac{1}{2} \frac{\beta(g)}{g} Tr(W^\alpha W_\alpha) + X^\ell_i (m_{\ell j} \phi^i \phi^j + g_{\ell ij} \phi^i \phi^j \phi^k)$$

(A.14)

We will rewrite the chiral superfield contributions using the classical field equations

$$\bar{D}^2 [\phi^+_k (e^V)_1^k] - 4(m_{ij} \phi^j) + g_{ijk} \phi^i \phi^j \phi^k = 0$$

(A.15)

with the result

$$A = - \frac{1}{2} \frac{\beta(g)}{g} Tr(W^\alpha W_\alpha) + \frac{1}{4} \bar{D}^2 [(\phi^+_i e^V)_i X^\ell_i \phi^j]$$

(A.16)
in agreement with previous results [7].

Using the formula

\[ E_o = \frac{1}{4} < \theta_m^m > \]  \hspace{1cm} (A.17)

we see that \( \theta_m^m \) is the real part of the highest component of the chiral superfield \( \mathcal{A} \). It is easily checked that the chiral anomaly is the corresponding imaginary part and that the \( \theta \) component correspond to the anomaly in the divergence of the superconformal current \( \partial_m I^m \).

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