Measurements of the angular and linear displacements of steered wheel

Pomiary kątowych oraz liniowych przemieszczeń koła kierowanego

Abstract
This paper concerns the use of a prototype measuring instrument for conducting measurements of the linear and angular displacements of a steered wheel in relation to the car body. The theoretical principles of the measurement are presented, as are the notation method and a solution to the system of equations of the geometric constraints of the instrument’s mechanism. In the research section, the manner in which the measurements were conducted is discussed and sample results are described. A preliminary analysis of the results is performed in the summary section.

Keywords: car, suspension, steered wheel, prototype measuring instrument, angular and linear displacements, perturbation method

Streszczenie
Praca dotyczy pomiarów przemieszczenia i orientacji koła kierowanego względem nadwozia pojazdu wykonanych za pomocą prototypowego przyrządu pomiarowego. Przedstawione zostały teoretyczne podstawy pomiaru, jak również sposób zapisu i rozwiązywania układu równań więzi wytrzymałościowych mechanizmu przedmiotowego przyrządu. W części badawczej omówiono sposób przeprowadzania pomiarów oraz przedstawiono przykładowe wyniki. W podsumowaniu została dokonana ich wstępna ocena i analiza.

Słowa kluczowe: samochód, zawieszenie, koło kierowane, prototypowy przyrząd pomiarowy, przemieszczenia kątowe oraz liniowe, metoda perturbacji
1. **Introduction**

Suspension and steering systems are two of the most important vehicle’s systems affecting the safety of the vehicle. In order to achieve the desired dynamic behaviour of cars in different driving situations, numerous computer simulations and actual road tests have been carried out using specialised measuring apparatus [1–10]. Modern car suspensions are complicated spatial mechanisms with flexible constraints [11–13] – this is one of the reasons why the real kinematic steering ratio changes in relation to the speed of the vehicle [14]. This change results in a significant difference between the actual and the theoretical steering angle. Measurements of linear and angular displacements of steered wheel taken during experimental car rides, such as the real steering angle, are very important. The results of these measurements are essential with regard to vehicle handling and stability improvements in the process of designing new suspension systems [15–17]. The measurement of the position and orientation of the steered wheel relative to the car body is very difficult and complicated – only a few studies on this topic can be found in the literature. Measured values are not obtained directly but as a result of complex calculations [15, 18–24].

2. **The prototype measuring instrument**

The proposed instrument for measuring the translation and rotation of a steered wheel is composed of two plates – external and internal – connected with nine links with linear displacement sensors $s_i, i = 1–9$ built in. The external plate is fixed to the vehicle body, while the inner plate is connected to the axis of rotation of the steered wheel. The connection is made using a bearing hub. The links of the instrument are attached to both plates via ball joints, there are 9 joints named $H_i, i = 1–9$ in the case of the external plate, and three joints named $D_j, j = 1–3$ in the case of the inner plate. A characteristic feature of the joints $D_j$ is that each of them realises the function of three ball joints with a common centre [21–24]. A schematic diagram of the prototype instrument is shown in Fig. 1.

![Schematic diagram of measuring instrument](image-url)

**Fig. 1.** Schematic diagram of a measuring instrument for the determination of the translation and rotation of a steered wheel: 1 – disc attached to a rim, 2 – disc immobilised against a stub axle. Points $B_k, k = 1–5$ are the centres of ball joints of a sample suspension. Detailed notation described in text [22]
Elongations of the instrument’s links $s_i$, $i = 1–9$ are recorded during the measurement process. They are then substituted to a system of nine equations (1)–(3), which is solved using a so-called perturbation method [25, 26]. Coordinates of the centres of the ball joints $D_j$, $j = 1–3$ are obtained as a result.

\[
\begin{align*}
\bar{r}_{D_{i,H_i}}^T \cdot \bar{r}_{D_{i,H_i}} &= (l_{D_{i,H_i}} + s_i)^2, \quad \text{for} \quad i = 1, \\
&\text{for} \quad i = 4, \\
&i = 5, \\
&i = 2, \\
\bar{r}_{D_{i,H_i}}^T \cdot \bar{r}_{D_{i,H_i}} &= (l_{D_{i,H_i}} + s_i)^2, \quad \text{for} \quad i = 6, \\
&\text{for} \quad i = 7, \\
\bar{r}_{D_{i,H_i}}^T \cdot \bar{r}_{D_{i,H_i}} &= (l_{D_{i,H_i}} + s_i)^2, \quad \text{for} \quad i = 8, \\
&\text{for} \quad i = 9, \\
\end{align*}
\]

Next, coordinates of two additional points lying on a wheel rotation axis $D_{nj}$, $n = 4, 5$ are calculated using formula (4):

\[
\bar{r}_{D_{n,D_n}}^T \cdot \bar{r}_{D_{n,D_n}} = (l_{D_{n,D_n}})^2, \quad \text{for} \quad j = 1–3, \quad n = 4, 5
\]

Knowing coordinates of points $D_j$, $j = 1–3$ and $D_{nj}$, $n = 4, 5$ it is possible to determine a unit vector lying on the wheel rotation axis. Steering $\delta$ and camber $\gamma$ angles, (see formulas (5) and (6)), as well as lateral displacements of steered wheel $\Delta K_y$ are then calculated.

\[
\begin{align*}
\delta_k &= - \arctg \left( \frac{e_{kx}}{e_{ky}} \right), \\
\gamma_k &= - \arcsin (e_{kz}),
\end{align*}
\]

It is necessary to know the initial configuration of the instrument mechanism at the start of the measurement process. The coordinates of ball joints centres $H_i$, $i = 1–9$ were determined using the coordinate measurement method, while the coordinates of the centres of the ball joints $D_j$, $j = 1–3$ were calculated using mathematical dependences. The theoretical analysis of the measurement of angular and linear displacements of a steered wheel using the prototype instrument and the determination of the initial configuration of the instrument mechanism have been widely described in earlier works [20–22, 24].
The initial configuration of the prototype instrument mechanism is shown below in millimetres:

\[
\begin{align*}
H_1(113.6, -11.2, 61.0); & & H_4(0.0, 1.3, 36.5); & & H_7(-113.9, -12.9, 59.3); \\
H_2(177.9, -11.8, -50.0); & & H_5(-64.9, -11.1, 146.8); & & H_8(-177.6, -12.4, -50.6); \\
H_3(50.1, 1.1, -50.2); & & H_6(63.7, -10.3, 147.3); & & H_9(-49.8, 1.1, -50.0); \\
D_1(69.3, -232.2, -40.0); & & D_2(-69.3, -232.2, -40.0); & & D_3(0.0, -232.2, 80.0); 
\end{align*}
\]

3. Measurements of the linear and angular displacements of a steered wheel

The test bench measurements of linear and angular displacements of a steered wheel were conducted on a car with independent (MacPherson) front wheel suspension. During the execution of the measurements the front wheels of the car were placed on turntables, while the rear wheels were placed on plates of the same height. An internal plate of the instrument was kinematically attached to the wheel using a bearing hub. The possibility of the plate rotating against its lateral axis \( y_w \) was taken away. The external plate, parallel to the previous plate, was attached to the body of the car. Figure 2 shows an overview of a vehicle with a prototype measuring instrument mounted on the front left wheel.

It was important to appropriately configure the instrument before starting the measurement process. The configuration consisted of:

- setting up the internal and external plates in a position parallel to each other;
- unambiguous determination of a origin of the coordinate system;
- setting up proper angular position of internal plate in relation to the external plate.

In addition to the procedure described above, measurements of the steering rack displacements were carried out. The position of the steering rack was measured using optical linear displacement sensor with an accuracy of 0.02 mm. The method of mounting the sensor in the test car is shown in Fig. 3.

![Fig. 2. An overview of a tested vehicle with the prototype measuring instrument mounted on the front left wheel](image)

![Fig. 3. Optical sensor of the linear displacements of the steering rack mounted in a test car: 1 – analogue-to-digital converter, 2 – sensor's housing, 3 – movable strip of a sensor, 4 – rack and pinion housing](image)
Data from all sensors went to an analogue-to-digital converter and then to a notebook. The scheme of the measuring configuration is shown in Fig. 4, an overview is presented in Fig. 5.

During the execution of the measurements the wheels were being turned left, then right, then left again, while the instrument links elongations and steering rack displacements were being simultaneously registered. Changes of linear dimensions of instrument links were used to calculate the steering and camber angles and the lateral displacements of the car wheel. At the same time, the steering angle was measured using a universal protractor with a vernier scale of 0.05’. Data registration was made at approximately every 2° of steering angle. The measurements were conducted for three different suspension deflections: neutral position, 43 mm compression and 57 mm rebound.
4. Results of measurements

4.1. Instrument links elongations

Example characteristics of instrument links elongations against steering rack displacement $u_p$ and suspension deflection $q$ are shown in Fig. 6.

![Fig. 6. Elongations of links $s_i$ and $s_j$ of prototype instrument against steering rack displacement $u_p$ and suspension deflection $q$](image)

4.2. Characteristics of suspension

Instrument links elongations $s_i$, $i = 1–9$ were used to calculate the characteristics of the suspension: camber $\gamma$ and steering $\delta$ angle and lateral displacements of steered wheel $\Delta K_y$ against steering rack displacement $u_p$ and suspension deflection $q$. Figures 7 to 9 show objective suspension characteristics.

Comparative values of steering angle $\delta$ obtained using the prototype measuring instrument and an universal protractor for three different suspension deflections are shown in Figs. 10 to 12.

![Fig. 7. Steering angle $\delta$ against steering rack displacement $u_p$ and suspension deflection $q$](image)

![Fig. 8. Camber angle $\gamma$ against steering rack displacement $u_p$ and suspension deflection $q$](image)
Fig. 9. Lateral displacements of steered wheel $\Delta K_y$ against steering rack displacement $u_p$ and suspension deflection $q$

Fig. 10. Steering angle $\delta$ against steering rack displacement $u_p$ for 57 mm rebound

Fig. 11. Steering angle $\delta$ against steering rack displacement $u_p$ for neutral position of suspension

Fig. 12. Steering angle $\delta$ against steering rack displacement $u_p$ for 43 mm compression
5. Summary

Results of preliminary measurements of translation and rotation of a steered wheel have been presented in this paper. The measurements were conducted using a prototype measuring instrument. As a result, camber $\gamma$ and steering $\delta$ angle, and lateral displacements of a steered wheel $\Delta K_y$ against steering rack displacement $u_p$ and suspension deflection $q$, were obtained. Steering angle $\delta$ values, obtained through the use of the prototype instrument, were compared with analogous values achieved with an universal protractor. An analysis of the comparison for three different suspension deflections shows that, qualitatively, the results are very similar to each other. There is also a quantitative similarity, although some differences are visible especially with regard to marginal measured values of the steering rack displacement in the case of 43 mm compression of suspension.

The presented results show that the proposed method of measurement of linear and angular displacements of the steered wheel is accurate, despite the presence of some differences between values achieved using the prototype instrument and those achieved with a universal protractor.

The discussed measurements were preliminary, their results will be helpful for optimisation of the proposed method of measurement.

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