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Global stable splittings of Stiefel manifolds. (English) Zbl 07538269
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Summary: We prove global equivariant refinements of Miller’s stable splittings of the infinite orthogonal, unitary and symplectic groups, and more generally of the spaces $O/O(m), U/U(m)$ and $Sp/Sp(m)$. As such, our results encode compatible equivariant stable splittings, for all compact Lie groups, of specific equivariant refinements of these spaces. In the unitary and symplectic case, we also take the actions of the Galois groups into account. To properly formulate these Galois-global statements, we introduce a generalization of global stable homotopy theory in the presence of an extrinsic action of an additional topological group.

MSC:

55N91 Equivariant homology and cohomology in algebraic topology
55P91 Equivariant homotopy theory in algebraic topology

Keywords:

global homotopy theory; stable splitting; Stiefel manifold

Full Text: DOI

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