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Steady Natural Convection of Non-Newtonian Power-Law Fluid in a Trapezoidal Enclosure

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ABSTRACT
Numerically investigation of free convection heat transfer in a differentially heated trapezoidal cavity filled with non-Newtonian Power-law fluid has been performed in this study. The left inclined surface is uniformly heated whereas the right inclined surface is maintained as uniformly cooled. The top and bottom surfaces are kept adiabatic with initially quiescent fluid inside the enclosure. Finite volume based commercial software FLUENT 14.5 is used to solve the governing equations. Dependency of various flow parameters of fluid flow and heat transfer is analyzed including Rayleigh number, $Ra$ ranging from $10^5$ to $10^7$, Prandtl number, $Pr$ of 100 to 10,000 and power index, $n$ of 0.6 to 1.4. Outcomes have been reported in terms of isotherms, streamline, and local Nusselt number for various $Ra$, $Pr$, $n$ and inclined angles. Grid sensitivity analysis is performed and numerically obtained results have been compared with those results available in the literature and found good agreement.

KEYWORDS: Natural convection, Non-Newtonian fluid, Trapezoidal enclosure.

INTRODUCTION
Rectangular enclosures with differentially heated vertical sidewalls are of the great importance of many field of studies in heat transfer phenomena such as natural convection. It is one of the most widely investigated configurations because of its prime importance as a benchmark geometry to
study convection effects and compare numerical techniques. Additionally, the geometry has many applications in different industrial techniques and equipments such as solar collectors, food preservation, compact heat exchangers, and electronic cooling systems among other practical applications. As a consequence of these applications, a thorough literature exists in this field of study especially in the case of Newtonian fluids [1-4].

Natural convection laminar flow of non-Newtonian Power-law fluids performs an important role in various engineering applications those are related with pseudo-plastic fluids. It should be noted that the pseudo-plastic fluid is characterized by apparent viscosity or consistency decreases instantaneously with an increase in shear rate. The study of fluid flow and heat transfer related to Power-law non-Newtonian fluids has attracted many researchers in the past half-century. An excellent research on pseudo-plastic fluid was conducted by Boger [5]. At first boundary-layer flows for such non-Newtonian fluids was investigated by Acrivos [6]. Since then, a large number of literature is created due to their wide relevance in pseudo-plastic fluids like chemicals, foods, polymers, molten plastics and petroleum production and various natural phenomena.

It is important to be noted that most of fluids employed in chemical and petrochemical processes or many other industries seems to show Non-Newtonian behavior. The natural convection of a Non-Newtonian fluid over enclosures such as a cylindrical enclosure or a heated plate has received more attention [7–14]. Several methodologies including, analytical [7], numerical [8] and experimental [9] approaches have been employed most of these studies, and the results indicated that the free convection features are considerably affected by the rheological properties of the fluid. However, the crucial issue of the buoyant convective process in various other geometries/enclosures of a Non-Newtonian fluid has remained largely unexplored.

Kim et al. [15] studied unsteady buoyant convection of a non-Newtonian Power-law fluid within a square enclosure. The authors used the finite volume technique realizing that the rheological properties have a considerable effect on the transient process. Additionally, the numerical solutions had an extensive qualitative agreement with the descriptions obtained from the scale analysis. Following their study, steady state analysis is performed in this study for a trapezoidal configuration of Non-Newtonian fluid. We have performed parametric studies by varying angle of the inclined surfaces, Rayleigh number, Prandtl number and Power-law index.
Mathematical Formulation

Consider a two dimensional trapezoidal enclosure of length (base) and height $H$, which is filled with an incompressible Power-law non-Newtonian fluid. Fig. 1 displays the enclosure with top and bottom insulated walls. The left inclined wall is heated and the right inclined wall is cooled with constant temperature. With invocation of Boussinesq approximation, governing equations take the form as below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$  \hspace{1cm} (1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{1}{\rho_0} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right),$$  \hspace{1cm} (2)

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{1}{\rho_0} \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \right) + g\beta(T - T_0),$$  \hspace{1cm} (3)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right),$$  \hspace{1cm} (4)

where $(u,v)$ represent velocity components in the horizontal $x$ and vertical $y$ directions; $T$ the temperature; $p$ the pressure; $g$ the gravitational acceleration; and $\rho$, $\beta$ and $\alpha$ the density, thermal expansion coefficient and thermal diffusivity of the fluid at reference temperature $T_0$. The related boundary conditions are

$$u = v = 0 \quad \text{at} \quad x\cos(\phi) + y\sin(\phi) = 0 \quad \text{and} \quad x\cos(\phi) - y\sin(\phi) = H\cos(\phi), \quad 0 \leq y \leq H, \quad 0 \leq x \leq H \quad (5a)$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, \ H \quad (5b)$$

$$T = T_H \quad \text{at} \quad x\cos(\phi) + y\sin(\phi) = 0, \ 0 \leq y \leq H, \quad (5c)$$

$$T = T_C \quad \text{at} \quad x\cos(\phi) - y\sin(\phi) = H\cos(\phi), \ 0 \leq y \leq H, \quad (5d)$$

Dimensionless forms of equation (1)-(4) can be obtained in the following fashion:

$$\left( X, Y \right) = \frac{(x,y)}{H}, \quad \left( U, V \right) = \frac{(u,v)}{\alpha}, \quad P = \frac{p}{\rho_0 \alpha^2}, \quad \theta = \frac{T - T_0}{\Delta T} \quad (6)$$

The crucial part of the formulation is to assign a suitable fundamental equation, which relates definite components of stress tensor to the relevant kinematics variables. For this purpose, a
purely viscose Power-law non-Newtonian fluid is assumed, which follows the Ostwald-De waele Power-law [7-9]:

\[ \tau_{ij} = 2\mu_D D_{ij} = 2K(2D_{kl}D_{kl})^{(n-1)/2} D_{ij}, \]  

(7)

In the above, two material parameters are involved, i.e. \( K \), the consistency factor; and \( n \) the Power-law index and \( D_{ij} \) represents rate of deformation tensor. Apparently, \( n=1 \) corresponds to those fluids of Newtonian behavior with the coefficient of viscosity \( K \), whereas \( n >1 \) indicates the dilatant (or shear thickening) behavior and \( n <1 \) shows pseudo-plastic (or shear thinning) behavior of a non-Newtonian fluid. The pseudo-plastic fluids have generally a high viscosity, and thermal variation of viscosity has also a direct effect on the thermal and flow fields. In the present set-up, the dependency of \( K \) on temperature is not assumed; a small temperature difference, \( \Delta T \) is assumed.

\( D_{ij} \) simplifies to the following equation for the two dimensional Cartesian coordinates:

\[ D_{ij} = \frac{1}{2} \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right) \]  

(8)

From equations (7) and (8), we find equation (9) for apparent viscosity [11]:

\[ \mu_a = K \left\{ 2 \left[ \frac{\partial u}{\partial x} \right]^2 + \left[ \frac{\partial v}{\partial y} \right]^2 \right\}^{n-1} \]  

(9)

Obviously for \( n=1 \), \( K \) represents the conventional viscosity. However for non unit \( n \), non-Newtonian behavior, complex dependence of viscosity to fluid's property and velocity components gradients is diagnosed. Based on the physical rationalizations and trial and error efforts, a grouping, which consists of the consistency coefficient \( K \), the Power-law index \( n \), the fluid density \( \rho_0 \) and the cavity height \( H \), emerges to be appropriate [9]:

\[ \nu' = \left( \frac{K}{\rho_0} \right)^{1/(2-n)} H^{2(1-n)/(2-n)} \]  

(10)

It is important to be noted that application of \( \dot{\psi} \), which is in dimension of m\(^2\)s\(^{-1}\), is analogous to that of kinematics viscosity of Newtonian fluids. Using equation (10), Prandtl number and Rayleigh number are defined respectively as below [12]:
\[
\text{Pr} = \left( \frac{K}{\rho_0} \right)^{1/(2-n)} \frac{H^{2(1-n)/(2-n)}}{\alpha} 
\]

\[
Ra = \frac{g\beta \Delta T H^3}{\alpha \left( \frac{K}{\rho_0} \right)^{1/(2-n)} H^{2(1-n)/(2-n)}}
\]

It is of great interest for many researchers to investigate local \( Nu \) of hot wall in many thermal systems. Similarly, local \( Nu \) is studied for left hot inclined wall which defined as follow:

\[
Nu = -\frac{\partial \theta}{\partial n}
\]

where \( n \) denotes the normal direction on left side plane.

**Numerical procedure**

Finite volume based code is used to descritize and solve the coupled set of equations (1)-(4) employing commercial software Ansys FLUENT 14.5. In this frame work, QUICK scheme was used for convective terms and SIMPLE algorithm was employed for the coupling of the pressure and velocity. Convergence criteria were set \( 10^{-5} \) for all relative residuals.

A grid of \( 81\times81 \) has been required for obtaining acceptable results, as shown in Table 1, a refinement to \( 101\times101 \) leads to maximum difference of \( 2.04\% \) and \( 0.35\% \) in terms of maximum stream function (\( \psi_{\text{Max}} \)) and average (\( Nu_{\text{avg}} \)) for \( Pr=100 \) of a square enclosure. As an additional check of the results accuracy, the present solution has been validated against Benchmark solutions obtained, in the case of the classical Newtonian fluids and non-Newtonian fluids in a square enclosure. Nusselt number of some certain cases is compared in Table 2.

**Results and Discussion**

In this section, the results corresponding to the influence of important parameters, namely inclination angle \( (0 \leq \varphi \leq 60) \), Power-law index \( (0.6 \leq n \leq 1.4) \), Rayleigh number \( (10^4 \leq Ra \leq 10^6) \) and Prandtl number \( (100 \leq Pr \leq 10,000) \) on heat transfer and fluid flow. The results are presented as a form of local Nusselt number and isotherm and stream function for the above parameters. Figure 2 illustrates isotherms and streamlines of various angles for trapezoidal enclosure of \( Ra = 10^5, Pr = 100 \) and \( n = 1 \). As expected due to presence of hot and cold walls, fluid rise up from
bottom horizontal edge, adjacent to the hot inclined wall and flow up along it reaching the top horizontal edge. Then the fluid flows down beside the oblique cold wall forming a roll with clockwise rotation inside the cavity. By the increase of angle, horizontal isotherms occupy much area of the enclosure. Also the formed roll is elongated toward the side walls by the increment of trapezoidal angle. Figure 3 indicates local \( Nu \) of hot wall for three angles of \( Pr = 100, Ra = 10^5 \) and \( n = 1 \). For square enclosure (\( \varphi = 0 \)) local \( Nu \) has a maximum value of nearly 14 at the top end of hot inclined side wall. By tilting the angle to \( 30^\circ \), maximum \( Nu \) is reduced to almost 8 and its position is near top end again. By further increase in angle value (\( \varphi = 60 \)), \( Nu \) value reduced more and many positions may be regarded to have maximum \( Nu \) of nearly 2. It is concluded that by the increment of trapezoidal angle, average \( Nu \) is reduced and this may be attributed to the increase of mean distance between two differentially heated inclined side walls.

Figure 4 displays isotherms and streamlines of distinct Power-law index \( n \), from 0.6 to 1.4 for \( Pr = 100, Ra = 10^5 \) and \( \varphi = 30 \). When shear thinning behavior is converted to the shear thickening behavior by the increment of Power-law index, maximum stream function is reduced from nearly 0.3kg/s to \( 9.3 \times 10^{-6} \)kg/s. This reveals that for \( n > 1 \), fluid gradually rises up to the top edge and we would expect lower \( Nu \) for \( n > 1 \) with respect to those \( n < 1 \). Isotherm lines displays that the intrusion of fluid at top and bottom edge is thickened by the increment of Power-law index. This fact shows that much part of fluid inside the enclosure is expressed for the case of higher \( n \). Local \( Nu \) is displayed in Figure 5 for different \( n \) of \( Pr = 100, Ra = 10^6 \) and \( \varphi = 30 \). Shear thinning fluid has the larger value of maximum \( Nu \) than that of shear thickening fluid. This maximum value is located near top edge of sloped hot wall for all \( n \).

Figure 6 represents isotherms and stream functions of different \( Ra \) for \( Pr = 100, n = 1 \) and \( \varphi = 30 \). Stream lines reveal that for \( Ra = 10^4 \) a clock wise roll is formed within the enclosure and with the increase of \( Ra \), this roll is extruded and elongated toward the side walls generating two small rolls. Also maximum stream function is increased from 0.0022 kg/s for \( Ra = 10^4 \) to 0.01 kg/s for \( Ra = 10^6 \). It is clear that larger value of \( Ra \), results in higher \( Nu \) due to higher rate of heat transfer from hot wall to the cold wall and Figure 7 reveals this fact for \( Pr = 100, n = 1 \) and \( \varphi = 30 \). The \( Pr \) effect on flow behavior is also investigated. Note that the values of \( Pr \) is much larger than unity for non-Newtonian fluids and it has been shown that an increase of this parameter makes the contribution of convective terms in Eq. 4 negligible [15], but to have better insight to the fluid flow, we present numerical results of distinct \( Pr \). Figure 8 displays isotherms and
streamlines of fluid for different \( Pr \) of \( Ra = 10^5 \), \( n = 1 \) and \( \varphi = 30 \). There exists negligible difference of isotherms and stream functions. Stream function for \( Pr = 100 \) is reduced from 0.0052\( \text{kg/s} \) to \( 5.2 \times 10^{-5} \text{kg/s} \) for \( Pr = 10,000 \). Note that \( Nu \) never changes as the Power-law index is unity for different \( Pr \) [15].

**Conclusions**

A numerical study has been performed on steady natural convection of Non Newtonian fluids within a trapezoidal cavity with differentially heated walls. The main objective of the present work was to observe the influence of parameters namely Power-index, trapezoidal angle, \( Pr \) and \( Ra \) in terms of isotherms, streamlines and local Nusselt number. Main outcomes of the study are as follow:

- By the increase of trapezoidal angle, the formed roll within the enclosure is elongated and extruded toward the side walls. Additionally maximum \( Nu \) on left hot wall is reduced by the increase of trapezoidal angle.
- Shear thinning behavior of the working fluid has higher \( Nu \) value than that of shear thickening. This may be attributed to the lower maximum stream function of higher \( Pr \).
- Increment of \( Ra \) enhances local \( Nu \) and makes the generated roll at the core of enclosure be extruded to the side walls.
- \( Pr \) variation has not significant effect on \( Nu \) as the most non-Newtonian fluids contain higher value of \( Pr \). Also maximum stream function is reduced by the increase of \( Pr \).

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Table 1. Preliminary tests on the grid size effect ($Ra = 10^5$, $Pr=100$ and $\phi=0$).

| Grid sizes | 61×61 | 81×81 | 101×101 |
|------------|-------|-------|---------|
| $n$        | $|\psi_{Max}|$ | $Nu_{avg}$ | $|\psi_{Max}|$ | $Nu_{avg}$ | $|\psi_{Max}|$ | $Nu_{avg}$ |
| 0.6        | 0.1225 | 7.085 | 0.1223 | 7.045 | 0.1223 | 7.020 |
| 1          | 0.0049 | 4.771 | 0.0049 | 4.752 | 0.0048 | 4.741 |
| 1.4        | 4.954e-6 | 3.785 | 4.947e-6 | 3.775 | 4.940e-6 | 3.770 |

Table 2. Validation of the numerical code for a square enclosure.

| $Ra$ | $Pr$ | $n$ | Present $Nu_{avg}$ | $Nu_{avg}$ [15] | Error (%) |
|------|------|-----|-------------------|------------------|----------|
| $10^4$ | 100  | 0.6 | 40.91             | 41.02            | 0.26     |
| $10^5$ | 100  | 0.6 | 17.46             | 17.51            | 0.28     |
| $10^5$ | 100  | 0.8 | 24.87             | 24.97            | 0.40     |
| $10^7$ | 100  | 1   | 17.45             | 17.52            | 0.39     |
| $10^7$ | $10^4$ | 0.6 | 21.89             | 22.05            | 0.72     |
| $10^5$ | 100  | 0.6 | 6.12              | 6.15             | 0.48     |
Fig. 1: Schematic of the cavity and the coordinate systems
Fig. 2: Isotherms (left) and streamlines (right) for different inclination when $Ra = 10^5$, $Pr = 100$ and $n = 1$. 
Fig. 3: Local $Nu$ for 3 different angles of $Pr = 100, n = 1$ and $Ra = 10^5$. 
Fig. 4: Isotherms (left) and streamlines (right) for different $n$ when $Ra = 10^6$, $Pr = 100$ and $\phi = 30^\circ$. 
Fig. 5: Local $Nu$ for different $n$ when $Pr = 100$, $\varphi = 30$ and $Ra = 10^6$. 
Fig. 6: Isotherms (left) and streamlines (right) for different $Ra$ when $n=1$, $Pr = 100$ and $\phi = 30$. 

$Ra = 1.0 \times 10^4$

$Ra = 1.0 \times 10^5$

$Ra = 1.0 \times 10^6$

$\psi_{\text{max}} = 0.00222$

$\psi_{\text{max}} = 0.00521$

$\psi_{\text{max}} = 0.01014$
Fig. 7: Local $Nu$ for different $Ra$ when $Pr=100$, $\varphi=30$ and $n=1$. 
Fig. 8: Isotherms (left) and streamlines (right) for different Pr when $n=1$, $Ra = 10^5$ and $\phi=30$. 

- Pr = 100
- Pr = 1000
- Pr = 10,000