Thermal Radiation from a Fluctuating Event Horizon

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We consider a pointlike two-level system undergoing uniformly accelerated motion. We evaluate the transition probability for a finite time interval of this system coupled to a massless scalar field near a fluctuating event horizon. Horizon fluctuations are modeled using a random noise which generates light-cone fluctuations. We study the case of centered, stationary and Gaussian random processes. The transition probability of the system is obtained from the positive-frequency Wightman function calculated to one loop order in the noise averaging process. Our results show that the fluctuating horizon modifies the thermal radiation but leaves unchanged the temperature associated with the acceleration.

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\section{I. INTRODUCTION}

Quantum field theory in curved space-time \cite{1,2} describes quantum fields propagating in a classical gravitational field background. Important processes described by the theory are vacuum polarization and particle creation in cosmological models and black-hole evaporation \cite{3}. A black hole emits thermal radiation due to quantum effects with an effective temperature inversely proportional to its mass, $\beta^{-1} = 1/8\pi M$. The basic question that naturally arises is the following: is there a way to measure such a radiation in a suitable setup? A partial answer to this question was provided by Unruh, who introduced the idea of studying analogous condensed matter systems which reproduce kinematical features of black-hole physics. Unruh showed \cite{4} that the propagation of sound waves in an irrotational and inviscid supersonic fluid is equivalent to the propagation of scalar waves in a black-hole space-time. Since this seminal paper, the possibility of simulating aspects of general relativity and quantum fields in curved space-time through analog models has been widely discussed in the literature \cite{5–13}. In particular, the possibility of constructing an acoustic analog of a black hole and measuring sound waves with thermal spectrum can provide an experimental verification of the existence of Hawking radiation in a condensed matter setting. In this respect, there exist interesting recent proposals to generate an acoustic metric with sonic horizon in atomic Bose-Einstein condensates and other superfluids \cite{14–21}.

There is, however, a serious difficulty in the semiclassical picture underlying the derivation of the thermal spectrum of back-hole radiation. Tracing the Hawking radiation back in time, one has to undo an exponentially strong gravitational red-shift in the vicinity of the horizon. This so-called trans-Planckian problem can spoil the derivation of the Hawking effect. The gravitational back reaction also raises questions on the applicability of the semiclassical theory of gravity. Within this perspective, models were formulated \cite{22,23} with the aim of studying the effects of fluctuations of the black-hole horizon on the Hawking radiation spectrum. In the absence of knowledge of the precise nature of the metric fluctuations near the horizon, the assumptions made here are that they can be treated classically and their effects on the propagation of quantum fields can be described via random differential equations. We emphasize that this is quite different from the stochastic gravity program \cite{24}, where the Einstein-Langevin equation enables one to find the dynamics of metric fluctuations generated by the fluctuations on the stress tensor of quantum fields.

Once established the analogy between black holes and fluids, it is a plausible approach to treat random media as models for fluctuations of the effective geometry of a sonic black hole. In such a scenario, recently an analog model for quantum gravity effects was proposed \cite{25}. The model builds on the work of Ford and collaborators \cite{26–30} and Hu and Shiokawa \cite{31}. Two general features of waves propagating in random fluids underlie the model. First, acoustic perturbations in a fluid define discontinuity surfaces that provide a causal structure with sound cones. Second, propagation of acoustic excitations in random media are generally described by wave equations with random speed of sound \cite{32–35}. In Ref. \cite{25}, the quantum field theory of a scalar field associated with acoustic waves was analyzed in a situation where the speed of propagation of the acoustic wave, and hence the sound cone, fluctuates. A stochastic ensemble of fluctuating geometries was assumed in that work.

In the present paper we take a pragmatic point of view towards seeking experimental consequences of quantum gravity effects in a specific scenario. Specifically, we analyze the question whether a quantum device can detect such effects by considering how fluctuations of a black-hole event horizon affect the transition rate of a two-level
system which interacts with a massless scalar field. Since we are not assuming the rotating wave approximation, the two-level system measures the vacuum noise in its world line.

The simplest assumption one can make for modeling the event horizon fluctuations is to assume a wave equation with random coefficients. The differential equation governing the random wave propagation cannot be solved in closed form, but it can be treated in perturbation theory [36], using as a small expansion parameter the intensity of the noise correlation function. Using such a perturbative expansion, the wave multi-scattering processes can be interpreted in terms of Feynman diagrams. The positive-frequency Wightman function is calculated at the one loop level of the noise averaging process. With this result in hand, it is possible to calculate the distortion in transition probabilities of a two-level system induced by the fluctuating horizon.

Although the horizon fluctuations do not invalidate the semiclassical derivation of the Hawking effect, we show that predictions provided by the radiative processes in the semiclassical theory scenario may differ from those of this quantum gravity effect scenario, where the quantum fluctuations of the metric are treated using a stochastic ensemble of geometries. In this context, we would like to call the attention of the reader to the fact that Hu and Roura [37] discussed a few years ago the possibility of studying the positive-frequency two-point Wightman function in the presence of metric fluctuations and the response function of a detector coupled to the field. Also, a deviation from the thermal spectrum was found by Takahashi and Soda [38] using a different model for a fluctuating black-hole horizon.

The organization of the paper is as follows. In Section II we discuss quantum field theory in the presence of a Schwarzschild event horizon. We use the fact that, close to the horizon the Schwarzschild metric takes the form of the Rindler line element. Note that the Rindler’s line element is static, and consequently there is a straightforward way to define positive and negative frequency modes in order to impose the canonical quantization in Rindler’s spacetime. With these considerations, we describe an apparatus device which is sensitive to fluctuations of the event horizon. In Section III we discuss quantum fields in fluctuating disordered medium. In Section IV we present the distortion caused by the fluctuating horizon in the transition probabilities. Also, in this section, using topological arguments we show that fluctuations in the horizon do not change the temperature associated with the acceleration, but only the spectrum of the thermal radiation. Finally, section V contains our conclusions. To simplify presentation we assume units such that $G = h = c = k_B = 1$.

II. THE UNRUH-DEWITT DETECTOR

Our aim is to discuss a particular model for fluctuations of the black-hole event horizon. We are interested to know how such fluctuations can affect the thermal radiation due to the presence of the event horizon. Therefore, let us consider the line element of a four-dimensional Schwarzschild space-time which describes a non-rotating uncharged black hole of mass $M$:

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2 - r^2d\Omega^2, \quad (1)$$

where $d\Omega^2$ is the metric of a unit 2-sphere. Close to the horizon, $r \approx 2M$. Therefore Eq. (1) can be written as

$$ds^2 = \left(\frac{\rho}{4M}\right)^2 dt^2 - \rho^2 - 4M^2 d\Omega^2. \quad (2)$$

where $\rho(r) = \sqrt{8M(r - 2M)}$. In these coordinates the horizon is at $\rho = 0$. The quantity $4M^2 d\Omega^2$ describes the line element of a 2-sphere of radius 2M. The other two terms can be identified with the line element of the two-dimensional Rindler edge by setting $t = 4M\sigma t$ and $\rho = e^{\alpha t}/a$, for $0 < \rho < \infty$ and $-\infty < t < \infty$. Then:

$$ds^2 = e^{2\alpha t}(d\tau^2 - d\xi^2) - 4Md\Omega^2. \quad (3)$$

The null asymptotes $\xi \rightarrow -\infty$, $\tau \rightarrow \pm \infty$ act as event horizons. Note also that lines of constant $\xi$ are hyperbolic, hence they represent the world lines of uniformly accelerated observers. One sees that, close to event horizon the Schwarzschild metric takes the form of the Rindler line element. Therefore, in order to capture the essential physical features of such a situation, we consider an uniformly accelerated pointlike two-level system in a Minkowski space-time with light-cone random fluctuations.

Although out of the scope of the present publication to answer the important question “what is a detector and what is the phenomenon of detecting particles”, it nevertheless requires some discussion. It is a known fact that there is a conceptual problem in quantum field theory in the construction of the Hilbert space of particles when particles are observer-dependent. The situation can be clarified to some extent by considering the response of an accelerated detector. In the context of quantum optics, Glauber [39], Loudon [40], and Nussenzveig [41] proposed ideal photocounter detectors. Afterwards, Unruh [42] and DeWitt [43] proposed scalar-particle detector models. Crudely speaking, the so-called “Unruh-DeWitt detector” is a two-level system with nonzero matrix elements of a monopole operator. The detector has the feature that its response to an interaction with a scalar field in the Minkowski vacuum depends on its state of motion. When in inertial motion, the detector has a vanishing asymptotic probability to wind up in an excited state, while if it moves with a constant proper acceleration, it has a finite asymptotic probability to undergo
transition to an excited state. Moreover, the accelerated detector, with proper acceleration \( \alpha \), interacting with the scalar field in the Minkowski vacuum is equivalent to the situation of the detector in inertial motion but in contact with a bath of thermal radiation at the temperature \( \beta^{-1} = \alpha / 2\pi \). Following these results, many papers appeared in the literature studying such a detector in many different situations. For the reader interested in more details, we recommend Ref. [44] (and references therein), in which the authors studied the Unruh-DeWitt detector in a situation where the probability of its excitation is evaluated over a finite time interval.

After this brief digression, let us describe our idealized model. In this paper we consider an Unruh-DeWitt detector; a two-level system coupled via a monopole interaction with a massless scalar field. We will be working in four-dimensional Minkowski space-time, whose line element is given by:

\[
d s^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2.
\]

In order to find the distortion caused by the fluctuating event horizon in the decay and excitation rates of a quantum system, let us discuss the response function of a two-level system. Let \( |g\rangle \) and \( |e\rangle \) be energy eigenstates of the system, with eigenenergies \( \omega_g \) and \( \omega_e \), and \( E = \omega_e - \omega_g \) the gap between the two states. Next, we suppose that the system is weakly coupled to a real scalar field with interaction Lagrangian [44, 45]:

\[
L_{\text{int}} = cm(\tau) \varphi(x(\tau)),
\]

where \( x^\mu(\tau) \) is the world line of the two-level system parametrized by the proper time \( \tau \), and \( m(\tau) \) is the monopole-moment operator of the two-level system. The quantity \( c \) is a small coupling constant between the detector and the scalar field. It is clear that this is an oversimplified model, with an atom represented by a two-level system that interacts with a real massless scalar field. However, this model contains all the properties needed to understand the basic features of radiative processes of atoms near a fluctuating event horizon. We can define an initial state at \( \tau = 0 \) given by \( |g\rangle \otimes |0\rangle \) and a final state \( |e\rangle \otimes |\psi_f\rangle \), at time \( \tau \). Here, where \( |0\rangle \) and \( |\psi_f\rangle \) are the vacuum and final states of the field. In first order approximation perturbation theory in the monopole coupling constant \( c \), Eq. (5), the transition probability is given by:

\[
P(\tau, 0) = c^2 |\langle e | m(0) | g\rangle|^2 F(E, \tau),
\]

where \( c^2 |\langle e | m(0) | g\rangle|^2 \) is the selectivity of the two-level system, and \( F(E, \tau) \) is the response function:

\[
F(E, \tau) = \int_0^\tau d\tau' \int_0^\tau d\tau'' e^{-iE(\tau' - \tau'')} \times |\langle 0|\varphi(x(\tau'))\varphi(x(\tau''))|0\rangle|.
\]

Using the definition of the positive-frequency Wightman function:

\[
G^+(x, x') = \langle 0|\varphi(x)\varphi(x')|0\rangle,
\]

and introducing the variables \( \zeta = \tau' - \tau'' \) and \( \eta = \tau' + \tau'' \), the response function can be rewritten as:

\[
F(E, \tau) = \frac{1}{2} \int_{-\tau}^{\tau} d\zeta e^{-iE\zeta} \int_{|\zeta|}^{2\tau - |\zeta|} d\eta G^+(\zeta, \eta).
\]

For a free massless scalar field, one has \( G^+(\zeta, \eta) = G^+(\zeta) \), and letting \( \tau \to \infty \), the double integration would reduce to a Fourier transform of the Wightman function, times an infinite time integral. Similarly, we shall show later on in the paper that random fluctuations of the light cone introduce an unbounded function \( f(\tau) \) that depends on the functional form of the noise correlation function. However, the transition probability per unit proper time should be finite. Such circumstances often arise in quantum field theory and may be dealt with by adiabatically switching off the coupling as \( \tau \to \pm \infty \). To circumvent such a problematic situation, we assume the field-detector interaction occurring during a finite time interval and, because of this, we choose to evaluate the response function over a finite proper time interval. One should recall that the detector defined above responds to the vacuum fluctuations because we do not assume the rotating wave approximation. The two-level system is measuring the vacuum noise in its world line. Consequently \( F(E, \tau) \) defines the spectrum of the vacuum noise. Determination of \( F(E, \tau) \) requires the positive frequency Wightman function \( G^+(x, x') \). In the next Section we determine \( G^+(x, x') \) and discuss the consequences of a disordered medium on the response function \( F(E, \tau) \).

### III. Perturbation Theory in a Disordered Medium

In order to access the modification caused by the fluctuating event horizon on transition probabilities, we implement a perturbation calculation similar to the one used in the context of problems of fluctuating disordered media and discussed in Refs. [25, 36]. Let us consider the random massless scalar Klein-Gordon equation in a four dimensional space-time, given by:

\[
(1 + \mu(r)) \frac{\partial^2}{\partial t^2} - \Delta \varphi(t, r) = 0,
\]

where \( \Delta \) is the three dimensional Laplacian. Note that \( \mu(r) \) works like a local refractive index, in that it perturbs locally the wave speed due to the replacement:

\[
\frac{\partial^2}{\partial t^2} \to (1 + \mu(r)) \frac{\partial^2}{\partial t^2}.
\]

For the random function \( \mu(r) \) we will take a zero-mean Gaussian random function:

\[
\langle \mu(r) \rangle = 0,
\]

with a white-noise correlation function given by:

\[
\langle \mu(r) \mu(r') \rangle = \sigma^2 \delta^{(3)}(r - r'),
\]
with \( \sigma^2 \) gives the intensity of random fluctuations. The symbol \( \langle \ldots \rangle_{\mu} \) denotes an average over all possible realizations of this random variable. On the other hand, in principle it is possible to extend the method to colored and/or non-Gaussian noise functions. Note that we assume a time-independent random function for inertial observers. However, in the accelerated detector world line \( \mu \) becomes a Rindler time-dependent random function. We will discuss this point later in the paper.

Following Refs. [25, 36], the random Klein-Gordon equation of the order (10) can be solved using a perturbation expansion in the noise function. In this way, the positive-frequency Wightman function can be written as:

\[
G^+(x,x') = G_0^+(x-x') + \sum_{n=1}^{\infty} d z_1 G_0^+(x-z_1) G^{(n)}(z_1,x'),
\]

where \( G_0^+(x-x') \) is the usual positive-frequency Wightman function without random fluctuations, and

\[
G^{(n)}(z_1,x') = (-1)^n \prod_{j=1}^{n} L_1(z_j) \int d z_{j+1} G_0^+(z_j, z_{j+1}),
\]

(14)

with \( L_1(x) \) being the random differential operator:

\[
L_1(x) = L_1(t, r) = -\mu(r) \frac{\partial^2}{\partial t^2}. \tag{16}
\]

In Eq. (15), it is to be understood that \( z_{n+1} = x' \) and that there is no integration in \( z_{n+1} \). Details on the derivations of the above expressions can be found in Ref. [36].

Due to the Gaussian nature of the noise averaging, higher order correlation functions of the form \( \langle \mu(r_1) \mu(r_2) \cdots \mu(r_p) \rangle_{\mu} \) can be easily expressed as the sum of products of two-point correlation functions corresponding to all possible partitions of \( r_1, r_2, \ldots, r_p \). An interesting feature of wave propagation in random media is Anderson localization [46]. In this context, we note that truncation of the series in Eq. (15) at a finite order \( n \) will miss the singular aspect of the localization problem, which is of a non-perturbative nature — see Refs. [47–54] for discussions on this and related subjects. For our purposes in the present paper it is sufficient to use only the first terms of the series.

After performing the averages over the noise function, the connected two-point positive-frequency Wightman function associated with the massless scalar field can be written in the form of a Dyson equation:

\[
\langle G^+ \rangle_{\mu} = G_0^+ - G_0^+ \Sigma \langle G^+ \rangle_{\mu}, \tag{17}
\]

with \( \Sigma \) being the self-energy. The one-loop contribution to \( \Sigma \) is obtained from noise averaging the second order contribution in the random function \( \mu \). Specifically, one can write:

\[
\langle G^+(x,x') \rangle_{\mu} = G_0^+(x-x') + \langle \bar{G}_1(x,x') \rangle_{\mu}, \tag{18}
\]

and going over to Fourier space:

\[
\langle \bar{G}_1(x,x') \rangle_{\mu} = \int \frac{dk}{(2\pi)^3} \int \frac{d\omega}{(2\pi)} e^{-ik(x-x')} \langle \bar{G}_1(\omega, k) \rangle_{\mu}, \tag{19}
\]

allows us to write:

\[
\langle \bar{G}_1(\omega, k) \rangle_{\mu} = -G_0^+(\omega, k) \Sigma(\omega) G_0^+(\omega, k), \tag{20}
\]

with \( G_0^+(\omega, k) \) being:

\[
G_0^+(\omega, k) = i/(\omega^2 - k^2), \tag{21}
\]

and

\[
\Sigma(\omega) = -\sigma^2 \omega^4 \alpha(\omega), \tag{22}
\]

where the quantity \( \alpha(\omega) \) is given by:

\[
\alpha(\omega) = \int \frac{d\omega}{(2\pi)^3} G_0^+(\omega, k) = -\frac{\omega}{4\pi}. \tag{23}
\]

Due to the Gaussian nature of the noise averaging, higher order correlation functions of the form \( \langle \mu(r_1) \mu(r_2) \cdots \mu(r_p) \rangle_{\mu} \) can be easily expressed as the sum of products of two-point correlation functions corresponding to all possible partitions of \( r_1, r_2, \ldots, r_p \). An interesting feature of wave propagation in random media is Anderson localization [46]. In this context, we note that truncation of the series in Eq. (15) at a finite order \( n \) will miss the singular aspect of the localization problem, which is of a non-perturbative nature — see Refs. [47–54] for discussions on this and related subjects. For our purposes in the present paper it is sufficient to use only the first terms of the series.

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\[
\alpha(\omega) = \int \frac{d\omega}{(2\pi)^3} G_0^+(\omega, k) = -\frac{\omega}{4\pi}. \tag{23}
\]
the two-level system:

\[ F(E, \tau) = \frac{1}{2} \int_{-\tau}^{\tau} d\zeta \, e^{-iE\zeta} \int_{|\zeta|}^{2\tau-|\zeta|} d\eta \left\langle G^+(\zeta, \eta) \right\rangle_\mu, \]

where \( \left\langle G^+(\zeta, \eta) \right\rangle_\mu \) can be obtained from Eq. (18) and (25). Fig. 2 illustrates the coordinate system for the integration over \( \eta \) and \( \zeta \). Next, one shall consider the Unruh-DeWitt detector moving in \((t, x)\) plane along a hyperbolic trajectory:

\[ x = (t^2 + \alpha^{-2})^{1/2}, \quad y = z = 0, \]

with \( \alpha \) being a constant. As well known, this represents a detector accelerating uniformly with acceleration \( \alpha \) in the frame of the detector. The detector’s proper time \( \nu \) is related to \((t, x)\) by the relations:

\[ t = \alpha^{-1} \sinh(\alpha \nu), \]

and

\[ x = \alpha^{-1} \cosh(\alpha \nu). \]

Performing such a coordinate transformation in Eq. (18), one can write the response function to one-loop order as a sum of three contributions:

\[ F(E, \tau) = F_\beta(E, \tau) + F_0(E, \tau) + F_1(E, \tau). \] (30)

The first term is the usual thermal contribution and the second is due to the switching on and off of the coupling between the two-level system and the scalar field. We are interested in the third term since it contains the correction due to fluctuating event horizon. The second term vanishes when large time intervals are considered. Using the techniques developed in Ref. [44], the first term in Eq. (30) reads:

\[ F_{\beta}(E, \tau) = \frac{\tau |E|}{2\pi} \left[ \Theta(-E) \left( 1 + \frac{1}{e^{\beta |E|} - 1} \right) \right. \]

\[ + \left. \Theta(E) \frac{1}{e^{\beta |E|} - 1} \right], \]

where \( \beta = 2\pi/\alpha \). The transition rate, i.e. the probability of spontaneous and induced decay and excitation per unit time, of the two-level system given by:

\[ R_{\beta}(E) = \frac{d}{d\tau} F_{\beta}(E, \tau), \]

can be readily computed. Clearly, the result of Eq. (31) is that one has the same effect that of a bath of thermal radiation at a temperature \( \beta^{-1} = \alpha/2\pi \) – see Refs. [42, 55]. Within the perspective of the thermalization theorem, the result can be put in the following form: the pure state which is the vacuum from the point of view of an inertial observer is a canonical ensemble from the point of view of a uniformly accelerated observer, with a temperature proportional to the magnitude of the observer’s acceleration.

The contribution \( F_1(E, \tau) \) due to the fluctuating event horizon is given by

\[ F_1(E, \tau) = \frac{1}{2} \int_{-\tau}^{\tau} d\zeta \, e^{-iE\zeta} \int_{|\zeta|}^{2\tau-|\zeta|} d\eta \left\langle G_1(\zeta, \eta) \right\rangle_\mu, \]

and can be evaluated in the same way as \( F_{\beta}(E, \tau) \). Performing the coordinate transformations given in Eqs. (28) and (29) in Eq. (25), one finds:

\[ \left\langle G_1(\zeta, \eta) \right\rangle_\mu = \frac{6i}{(2\pi)^3} \frac{\alpha}{2} \cdot \frac{5}{\sinh^5 \left( \alpha \zeta/2 - i\epsilon \right)} \cdot \frac{\hat{\sigma}^2(\eta)}{\alpha \eta + \alpha \zeta/2 - i\epsilon \alpha}, \]

where we have absorbed a positive function of \( \eta \) and \( \zeta \) into the infinitesimal parameter \( \epsilon \). Also, the quantity \( \hat{\sigma}^2(\eta) \) that gives the intensity of the horizon fluctuations and is given by:

\[ \hat{\sigma}^2(\eta) = \sigma^2 \cosh \left( \frac{5 \alpha \eta}{2} \right). \]

The \( \eta \) dependence comes from the fact that in the Wightman function, \( x \) is not an independent variable; rather, it is determined by the detector’s trajectory. In other words, inertial observers in this model experience static light-cone random fluctuations – see Eq. (10). However, for uniformly accelerated observers, such fluctuations will not be static anymore and will also depend on their proper times. On physical grounds, it is clear that for an uniformly accelerated detector the effects of the fluctuations will increase with its proper time, a result that is manifest in Eq. (35).
Using the result of Eqs. (34) and (35) in Eq. (33), one obtains the expression:

\[ F_1(E, \tau) = \frac{6i}{5(2\pi)^3} \left( \frac{\alpha}{2} \right)^4 \tilde{\sigma}^2(\tau) \int_{-\tau}^{\tau} d\zeta e^{-iE\zeta} \]

\[ \times \frac{\sinh[5\alpha(\tau - |\zeta|)/2]}{\sinh^{3}(\alpha\zeta/2 - i\alpha)} \]  

(36)

We may express the integral above as

\[ \int_{-\tau}^{\tau} d\zeta f(\zeta) = \int_{-\infty}^{\infty} d\zeta f(\zeta) \]

\[ - \left[ \int_{-\infty}^{-\tau} d\zeta f(\zeta) + \int_{\tau}^{\infty} d\zeta f(\zeta) \right], \]  

(37)

where

\[ f(\zeta) = \frac{1}{\alpha} e^{-iE\zeta/\alpha} \frac{\sinh[5(\alpha\tau - |\zeta|)/2]}{\sinh^{3}(\zeta/2)}. \]  

(38)

The last two-terms on the right-hand side of Eq. (37) can be expressed as a single integral. They have the same physical origin as the term \( F_0(E, \tau) \). In order to perform the integral

\[ I = \int_{-\infty}^{\infty} d\zeta f(\zeta), \]  

(39)

we may use contour integration – the contour to be used is shown in Fig. (3). The integral over the lower part of the contour yields \( I \) while that over the upper part yields \( \exp(2\pi E/\alpha)I \). The sum of these contributions is related to the fifth-order residue of \( f(\zeta) \) at \( \zeta = 0 \). Finally, collecting these results, we have that

\[ F_1(E, \tau) = W(E, \tau) + H(E, \tau), \]  

(40)

where

\[ W(E, \tau) = -\frac{\tau \tilde{\sigma}^2(\tau)}{(4\pi)^2} \left( \frac{24\alpha^4 - 35E^2\alpha^2 + E^4}{e^{2\pi E/\alpha} + 1} \right), \]  

(41)

and

\[ H(E, \tau) = \frac{12\tilde{\sigma}^2(\tau)}{5(2\pi)^3} \left( \frac{\alpha}{2} \right)^4 \int_{\tau}^{\infty} d\zeta \sin(E\zeta) \]

\[ \times \frac{\sinh[5\alpha(\tau - |\zeta|)/2]}{\sinh^{3}(\alpha\zeta/2)}. \]  

(42)

In Eq. (41) we consider \( \tau \) as a small quantity, so that \( \sinh(5\alpha\tau/2) \approx 5\alpha\tau/2 \).

Eqs (40)-(42) comprise our main result in this paper. We have found that the correction due to horizon fluctuations gives a thermal distribution with a temperature \( \beta^{-1} = \alpha/2\pi \), that is the same temperature as for the non-fluctuating case, but the distribution is of Fermi-Dirac form. This resembles the result obtained by Takagi [56], who studied the power spectrum of the vacuum noise measured by an accelerated detector in arbitrary dimensions and discussed the phenomenon of inversion of statistics in odd dimensions. Nevertheless, as emphasized by the literature [57], this is an apparent inversion of statistics. In our case we have a Fermi-Dirac correction to the thermal radiation of a bosonic field, but we still have the expected Bose-Einstein distribution as the leading component of the radiation. The meaning of our result is that horizon fluctuations imply in a radiation spectrum of both contributions, in that the usual Bose-Einstein distribution is perturbed by a Fermi-Dirac distribution of the same temperature. One could expect that higher loop corrections would give additional energy-dependent terms that will be neither Fermi-Dirac nor Bose-Einstein forms.

It is easy to see from our result how the fluctuating horizon will change the transition rate of a detector undergoing an inertial trajectory by taking the zero proper acceleration limit in Eq. (41):

\[ \lim_{\alpha \to 0} F_1(E, \tau) = -\tau \frac{\sigma^2}{16\pi^2} E^4 \Theta(-E) \]  

(43)

In the case of an inertial detector the intensity of the fluctuations will not change with time. The correction to the transition rate for the inertial detector is proportional to \( \Theta(-E) \), meaning that for an inertial detector we have spontaneous decay induced by the vacuum fluctuations.

Let us study the dependence of the temperature with random fluctuations in more detail. We will be using arguments developed by Christensen and Duff [58]. The Euclidean manifold associated to inertial observers has the topology of \( R^4 \), with Euler-Poincaré characteristic \( \chi = 1 \). For the case of the uniformly accelerated observer, we have a non-simply connected manifold, with topology \( R^3 \times S^1 \), with Euler-Poincaré characteristic \( \chi = 0 \). Now if we have these two distinct topological situations, we can define two different vacua, one associated with inertial observers, defined by \( |\chi = 1\rangle \) and another one associated with accelerated observers, defined by \( |\chi = 0\rangle \). Next, one can show that an accelerating observer regards the \( |\chi = 1\rangle \) vacuum as a thermal state at temperature \( \beta^{-1} = \alpha/2\pi \). The two-point Schwinger function associated with
the massless scalar field for both cases can be defined as:
\[ G^{(\chi)}(x, x') = \langle \chi | \varphi(x) \varphi(x') | \chi \rangle. \]  \hfill (44)

This two-point function obeys
\[ \Delta G^{(\chi)}(x, x') = -\delta^4(x - x'), \]  \hfill (45)
where \( \Delta \) is now the four dimensional Laplacian. For simplicity, let us take the case where the two points \( x \) and \( x' \) belong to the accelerated world-line \( \xi = \xi' = \alpha^{-1}, \) \( y = y' \) and \( z = z' \). The generalization for two arbitrary points can be found in Ref. [59]. We have:
\[ G^{(1)}(\tau, \tau') = \frac{1}{16\pi^2} \frac{\alpha^2}{\sin^2(\alpha \Delta \tau/2)}, \]  \hfill (46)
and, since \( G^{(1)}(\tau, \tau') \) must be periodic in \( \tau \) with period \( 2\pi/\alpha \), we have:
\[ G^{(1)}(\tau, \tau') = G^{(1)}(\tau, \tau' + 2\pi/\alpha). \]  \hfill (47)

In the case where \( \chi = 0 \), paths winding around the origin have topologically distinct classes with winding number \( n \). In this case we have
\[ G^{(1)}(\tau, \tau') = \sum_{n=-\infty}^{\infty} G^{(0)}(\tau, \tau' + 2\pi n/\alpha). \]  \hfill (48)

Using the KMS condition, we have that the finite temperature Schwinger function satisfies
\[ G_\beta(\tau, \tau') = G_\beta(\tau, \tau' + \beta), \]  \hfill (49)
and using the identification \( \beta^{-1} = \alpha/2\pi \) we have that \( G^{(0)} = G_{\beta \to \infty} \) and \( G^{(1)} = G_\beta \). We conclude that the ground state of the accelerated observer is the \( |\chi = 0\rangle \) vacuum state, relative to which the \( |\chi = 1\rangle \) vacuum is a thermal state. In the presence of noise, it is easy to see that the above arguments can be used in the same way. Recalling Eq. (17), we get:
\[ \langle G^{(1)}(\tau, \tau') \rangle_\mu = \langle G^{(1)}(\tau, \tau' + 2\pi/\alpha) \rangle_\mu. \]  \hfill (50)

In the case where \( \chi = 0 \), we also have
\[ \langle G^{(1)}(\tau, \tau') \rangle_\mu = \sum_{n=-\infty}^{\infty} \langle G^{(0)}(\tau, \tau' + 2\pi n/\alpha) \rangle_\mu. \]  \hfill (51)

Using the KMS condition, we have that the finite temperature Schwinger function satisfies
\[ \langle G_\beta(\tau, \tau') \rangle_\mu = \langle G_\beta(\tau, \tau' + \beta) \rangle_\mu, \]  \hfill (52)
and using the same identification \( \beta^{-1} = \alpha/2\pi \) we have that \( \langle G^{(0)} \rangle_\mu = \langle G_{\beta \to \infty} \rangle_\mu \) and \( \langle G^{(1)} \rangle_\mu = \langle G_\beta \rangle_\mu \). Therefore, the temperature associated with the acceleration remains the same, but the fluctuating horizon does change its emitted thermal radiation.

V. CONCLUSIONS

Recently, an analog model for quantum gravity effects in a condensed matter scenario was proposed in Ref. [25]. In Ref. [36] this discussion was extended to a more general case, namely to a massive real scalar field. In the present paper such a model was used to study a massless scalar field near a four-dimensional Schwarzschild black hole with fluctuations in the event horizon. Using a perturbation theory similar to the one used in problems of fluctuating disordered media, we obtain the two-point positive-frequency Wightman function associated with a real scalar field. After performing the averages over the noise function, we discuss the thermal radiation near the fluctuating event horizon. We obtained the modification of the transition probabilities caused by the fluctuating horizon on the decay and excitation processes. We showed that horizon fluctuations imply that the usual Bose-Einstein distribution is perturbed by a Fermi-Dirac distribution of the same temperature. Our results are obtained by assuming that the mechanical disturbances caused by the radiative processes belongs to an energy scale much smaller than the mass of the accelerated two-level system. This means that its world line does not change due to light-cone random fluctuations. We conclude our discussions noting that the previous treatment can be presented using the Fermi Golden Rule and the fact that the density of states per unit volume is given by \( -1/\pi \text{Im} G_R(r, r') \), where \( G_R \) is the retarded Green function associated with the massless scalar field.

Finally, we remark that random-matrix theory can be used to find how the spectral density near the fluctuating event horizon is modified. In a similar situation, in Ref. [60] the transition rate of a two-level atom within a chaotic cavity was presented using random matrices. The use of random matrices to find how the spectral density near the fluctuating event horizon is modified is under investigation by the authors.

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