Coherence Control of Adiabatic Decoherence in a Three-level Atom with Lambda Configuration

Xiao-Shu Liu$^{1,2}$, Wu Re-bing$^3$, Yang Liu$^{1,2}$, Jing Zhang$^3$ and Gui Lu Long$^{1,2,4}$

$^1$Department of Physics, Tsinghua University, Beijing 100084, P. R. China
$^2$Key Laboratory For Quantum Information and Measurements, Beijing 100084, P. R. China
$^3$Department of Automation, Tsinghua University, Beijing 100084, P. R. China
$^4$Center for Atomic and Molecular NanoSciences, Tsinghua University, Beijing 100084, P. R. China

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In this paper, we study the suppression of adiabatic decoherence in a three-level atom with Λ configuration using bang-bang control technique. We have given the decoupling bang-bang operation group, and programmed a sequence of periodic radio frequency twinborn pulses to realize the control process. Moreover, we have studied the process with non-ideal situation and established the condition for efficient suppression of adiabatic decoherence.

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I. INTRODUCTION

In recent years, there have been increased interests in the study of three-level quantum systems, for example in quantum cryptography [1], quantum communication [2], logic qubit encoding [3], entanglement measures [4], quantum control [5], quantum computation [6], error-correcting codes [7, 8, 9], logic qubit encoding [3], entanglement measures [4, 5], quantum cryptography [1], quantum communication [2], and decoherence by repetitively imposing a sequence of radio-frequency pulses on a single qubit. This active dynamical control in the bang-bang limit proves a nice tool for engineering the evolution of coupled quantum subsystems. In this paper, we will apply the bang-bang control technique to suppress the decoherence induced by pure dephasing in a three-level atom with Λ configuration.

II. PROBLEM FORMULATION

The system we consider is a three-level atom with Λ configuration under two resonant laser fields fields with frequencies

$$\omega_{20} = \frac{E_2 - E_0}{\hbar}, \quad \omega_{21} = \frac{E_2 - E_1}{\hbar},$$

respectively, as shown in Fig.1. Let $|0\rangle$, $|1\rangle$, and $|2\rangle$ be the eigenstates of the unperturbed part of the hamiltonian $\mathcal{H}_0$ of atom, and the corresponding eigenvalues are $E_0$, $E_1$ and $E_2$, respectively. Let $E_0 < E_1 < E_2$.

The two lower levels $|0\rangle$ and $|1\rangle$ are coupled to a single upper level $|2\rangle$ in the Λ type.

The total hamiltonian of the three-level atom can be expressed as $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{R.F.}$, where

$$\mathcal{H}_0 = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1| + E_2 |2\rangle\langle 2|$$

(1)

is the free hamiltonian and

$$\mathcal{H}_{R.F.} = -(g_{20}|2\rangle\langle 0| + g_{02}^*|0\rangle\langle 2|)\mathcal{E}_1 \cos \omega_{20} t - (g_{21}|2\rangle\langle 1| + g_{12}^*|1\rangle\langle 2|)\mathcal{E}_2 \cos \omega_{21} t$$

(2)
represents the interaction of the atom with the radiation fields.

Here we assume that the electric fields are linearly polarized along the $x$-axis; $g_{ij} = g_j^* = e^{i|x|j}$ $(i,j = 0,1,2)$ is the matrix element of the electric moment; $\mathcal{E}_i$ represents the amplitude of the electric field.

Before carrying out the calculation, we define some notation

\begin{align}
\sigma_z^{(2,1)} &\equiv |2\rangle \langle 2| - |1\rangle \langle 1|, \\
\sigma_x^{(2,1)} &\equiv |2\rangle \langle 1| + |1\rangle \langle 2|, \\
\sigma_y^{(2,1)} &\equiv i(|2\rangle \langle 1| - |1\rangle \langle 2|), \\
\sigma_z^{(1,1)} &\equiv |2\rangle \langle 2|, \\
\sigma_x^{(1,1)} &\equiv |2\rangle \langle 1|.
\end{align}

The operators $\sigma_x^{(2,0)}$, $\sigma_z^{(1,0)}$, $\sigma_y^{(1,0)}$, $\sigma_y^{(2,0)}$, $\sigma_z^{(2,0)}$ and $\sigma_z^{(1,0)}$ can be defined similarly. With these notations, the hamiltonian can be rewritten as

\[ H_0 = \frac{\hbar \omega_{10}}{3} \sigma_z^{(1,0)} + \frac{\hbar \omega_{20}}{3} \sigma_z^{(2,0)} + \frac{\hbar \omega_{21}}{3} \sigma_z^{(2,1)} + E_0 + E_1 + E_2, \]

where

\[ \omega_{10} = \frac{E_1 - E_0}{\hbar}, \omega_{20} = \frac{E_2 - E_0}{\hbar}, \omega_{21} = \frac{E_2 - E_1}{\hbar}. \]

Usually, the constant energy $(E_0 + E_1 + E_2)/3$ is ignored.

For simplicity, we assume $g_{12} (= g_{21}^*)$ and $g_{10} (= g_{01}^*)$ are real numbers. Then

\[ \mathcal{H}_{R,F.} = -g_{21} \cos \omega_{21} t \sigma_x^{(2,1)} - g_{20} \cos \omega_{20} t \sigma_x^{(2,0)} \]
\[ \equiv u_1(t) \sigma_x^{(2,0)} + u_2(t) \sigma_x^{(2,1)}. \]

Including the decoherence of the system due to the coupling to a thermal reservoir, the total hamiltonian is

\[ \mathcal{H} = \mathcal{H}_0 \otimes \mathcal{I}_E + \mathcal{I}_S \otimes \mathcal{H}_E + \mathcal{H}_{SE} + \mathcal{H}_{R,F.}, \]

where $\mathcal{H}_0$ and $\mathcal{H}_{R,F.}$ are the hamiltonians described in [8] and [10]. $\mathcal{H}_E$ and $\mathcal{H}_I$ describe the internal hamiltonian of the environment and its coupling hamiltonian to the three-level system.

The heat bath is modelled as a large number of uncoupled bosonic modes, namely a reservoir of simple harmonic oscillators with ground state energy shifted to zero [23, 24, 25, 26, 27],

\[ \mathcal{H}_E = \sum_k \hbar \omega_k a_k^\dagger a_k. \]

The interaction hamiltonian is expressed as

\[ \mathcal{H}_{SE} = \hbar \sum_{k_1} \sigma_z^{(2,0)} (g_{k1} a_{k1}^\dagger + g_{k1}^* a_{k1}) + \hbar \sum_{k_2} \sigma_z^{(2,1)} (g_{k2} a_{k2}^\dagger + g_{k2}^* a_{k2}), \]

where $g_{k1}$ and $g_{k2}$ are the coupling constants corresponding to the virtual exchanges of excitations with the bath $|2\rangle \leftrightarrow |0\rangle$ and $|2\rangle \leftrightarrow |1\rangle$ transitions respectively.

Usually, we assume that the initial state of the total system is disentangled, i.e.

\[ \rho_{\text{total}}(0) = \rho_S(0) \otimes \rho_E(0), \]

and the thermal reservoir $\rho_E(0)$ is in thermal equilibrium state that can be factorized into the tensor product of the density operators of each mode

\[ \rho_E = \prod_k \theta_k \]

where

\[ Z_k = [1 - \exp(-\frac{\hbar \omega_k}{k_B T})] \]
\[ \theta_k = Z_k^{-1} \exp(-\frac{\hbar \omega_k a_k^\dagger a_k}{k_B T}) \]

where $k_B$ is the Boltzmann constant and $T$ is the temperature of the bath.
Dynamical Suppression of Decoherence in a Three-Level Atom in the Ideal Limits

Firstly, we sketch the main ideas of the dynamical decoupling theory. A bang-bang operation is a unitary operation that can be performed instantaneously, namely corresponding Hamiltonian can be turned on for negligible amounts of time with arbitrarily large strength. Let $G_{B,B}$ be the group consists of the implementable bang-bang operations. The decoupling group $G$ is defined as a finite group of bang-bang decoupling operations, $G = \{g_k\} \subseteq G_{B,B}$, where $k$ belongs to some finite index set $K$. Then a decoupling-controller on $H$ is defined as the interactions of the system, including a sequence of bang-bang operations and free evolution.

Assume the cyclic time is $T_c$. Similar to the average Hamiltonian theory, we consider a given evolution between the interval $t_{\text{in}}$ and $t_{\text{out}}$. The decoupling group $G$ is defined as the interactions of the system, including a sequence of bang-bang operations and free evolution.

From these expressions, it is immediate to design a physical realization for these operations using two R.F. pulses.
as given in Eq. (10) with appropriate frequencies that interact with state transitions |2⟩ ↔ |0⟩ and |2⟩ ↔ |1⟩ respectively. For example, h1 can be realized by two twin-born pulses, i.e., applying a π-pulse with frequency ω20 at first and followed by another π-pulse with frequency ω21. h2 is realized similarly.

With the above results, we can design a procedure to effectively suppress the adiabatic decoherence with a sequence of periodic twinborn pulses. In an elementary cycle, the twinborn-pulse sequence is \{h1, h1', h2, h2'\}, as shown in Fig. 2.

![Figure 2](image)

**FIG. 2:** A sequence of twinborn pulse operating in a cycle on the three-level atom. The solid line represents the pulses with frequency of ω2, and the hollow line represents the pulses with frequency of ω20; π pulses are on the upside and −π pulses are on the downside.

In the first half of a cycle, system evolves under \( H = H_0 + H_E + H_I \) during \( t_0 \leq t \leq t_P^{(1)} \equiv t_0 + \Delta t \); at time \( t_P^{(1)} \), twinborn-pulse \( h_1 \) is applied; after 2\( \tau_P \) units of time, the pulse is switched off; then the system is governed by \( H \) during \( t_P^{(1)} + 2\tau_P \leq t \leq t_P^{(2)} \equiv t_0 + 2\Delta t \), where \( \tau_P \) is the pulse width of each sub-pulse of the twinborn-pulse. In the second half of the cycle, the twinborn-pulses \( h_1' \) and \( h_2 \) is applied and at time \( t_P^{(2)} \) and \( t_P^{(2)} + 2\tau_P \) respectively; after another 2\( \tau_P \) units of time, the pulse is switched off and the system evolves freely under \( H \) during \( t_P^{(2)} + 4\tau_P \leq t \leq t_P^{(3)} \equiv t_0 + 3\Delta t \). At time \( t_P^{(3)} \) the twinborn-pulses of \( h_2' \) begin. These complete a cycle. By repeating such sequence of elementary cycles, one can suppress the adiabatic decoherence completely in the ideal limits of \( T_c \rightarrow 0 \) and \( N \rightarrow \infty \).

**IV. DYNAMICAL SUPPRESSION OF DECOHERENCE IN A THREE-LEVEL ATOM WITH NONIDEAL CONDITIONS**

In last section, it is shown that the decoherence can be completely removed in the ideal limits of \( T_c \rightarrow 0 \) and \( N \rightarrow \infty \). However, the ideal limits that \( T_c \rightarrow 0 \) and \( N \rightarrow \infty \) cannot be exactly fulfilled in reality. In this section, we will give a quantitative analysis of effect of finite width and finite-amplitude pulses on the decoherence suppression.

The problem can be reformulated in the interaction picture. Let \( H^0 = H_0 + H_E \), then under the standard state transformation \( \exp(-i\hat{H}^0t) \), the interaction \( \hat{H}_{SE} \) reads

\[
\hat{H}_{SE} = \hbar \sum_{k1} \sigma_z^{(2,0)}(g_{k1}a_{k1}^\dagger e^{i\omega_{k1}t} + g_{k1}^* a_{k1} e^{-i\omega_{k1}t}) + \hbar \sum_{k2} \sigma_z^{(2,1)}(g_{k2}a_{k2}^\dagger e^{i\omega_{k2}t} + g_{k2}^* a_{k2} e^{-i\omega_{k2}t}),
\]

(20)

and the free unitary evolution of the composite system is

\[
\tilde{U}(t_0, t) = \exp \left[ \sum_{k1} \xi_{k1}(t - t_0) - \h.c. \right] \times \exp \left[ \sum_{k2} \xi_{k2}(t - t_0) - \h.c. \right]
\]

(21)

where

\[
\xi_{k1}(\Delta t) = \frac{2g_{k1}}{\omega_{k1}} \left( 1 - e^{i\omega_{k1}\Delta t} \right)
\]

\[
\xi_{k2}(\Delta t) = \frac{2g_{k2}}{\omega_{k2}} \left( 1 - e^{i\omega_{k2}\Delta t} \right).
\]

During an elementary cycle between time \( t_0 \) and \( t_1 = t_0 + 3\Delta t + 2\tau_P \), the state propagator can be written as

\[
\tilde{U}_P(t_0, t_1) = \tilde{U}_P 4 \tilde{U}^{(2)}(t_P^{(2)} + 2\tau_P, t_P^{(2)}) \tilde{U}_P 3 \tilde{U}^{(2)}(t_P^{(1)} + 2\tau_P, t_P^{(1)} + \tau_P) \tilde{U}_P 1 \tilde{U}^{(2)}(t_P^{(1)} + \tau_P, t_P^{(1)})
\]

(22)

where

\[
\tilde{U}_P 1 = e^{i\hat{H}_P^{(1)}(t_P^{(1)} + \tau_P)} e^{i\hat{H}_I^{[2,1]}(t_P^{(1)} + \tau_P)} e^{-i\hat{H}_P^{(1)}(t_P^{(1)})} \\
\times e^{i\hat{H}_P^{(1)}(t_P^{(1)})} e^{-i\hat{H}_P^{(1)}(t_P^{(1)})}
\]

\[
\approx e^{i\hat{H}_P^{(1)}(t_P^{(1)})} h_1 e^{-i\hat{H}_P^{(1)}(t_P^{(1)})}
\]

(23)

and likewise,

\[
\tilde{U}_P 2 \approx e^{i\hat{H}_P^{(2)}(t_P^{(2)})} h_2 e^{-i\hat{H}_P^{(2)}(t_P^{(2)})}
\]

\[
\tilde{U}_P 3 \approx e^{i\hat{H}_P^{(2)}(t_P^{(2)})} h_2 e^{-i\hat{H}_P^{(2)}(t_P^{(2)})}
\]

\[
\tilde{U}_P 4 \approx e^{i\hat{H}_P^{(2)}(t_P^{(2)})} h_2 e^{-i\hat{H}_P^{(2)}(t_P^{(2)})}
\]

(24)
Substituting Eqs. (18,19) into Eq. (24), we obtain

\[
\tilde{U}(t_0, t) = e^{\frac{(2,0)}{2} \sum_{k_i} a_{i}^\dagger e^{i\omega_{k_i} t_0} \xi_{k_i}(\Delta t) - h.c.}\]

\[
\times \left[ h_1^{(2,0)} h_1 \sum_{i_k} [a_{i_k} e^{i\omega_{k_1} t_0} e^{i\omega_{k_1} \Delta t} \xi_{k_1}(\Delta t) - h.c.] \right]
\times \left[ h_1^{(2,0)} h_2 \sum_{i_k} [a_{k_2} e^{i\omega_{k_2} t} e^{i\omega_{k_1} 2\Delta t} \xi_{k_2}(\Delta t) - h.c.] \right]
\times \left[ e^{\frac{(2,1)}{2} \sum_{i_k} [a_{k_2}^\dagger e^{i\omega_{k_2} t_0} \xi_{k_2}(t_0) - h.c.]} \right]
\times e^{iH_0(t_1-t_0)}. \tag{25}
\]

Imposing the above pulse sequences repeatedly, we then get the general expression of the evolution under \( N \) bang-bang control cycles

\[
\tilde{U}^{(N)}_p(t_0, ..., t_N) = \tilde{U}_p(t_{N-1}, t_N) ... \tilde{U}_p(t_1, t_2) \tilde{U}_p(t_0, t_1),
\]

where \( t_n = t_0 + 3n\Delta t (n = 1, ..., N) \) is the ending time of the \( n \)-th bang-bang control cycle.

With Eq. (26) and more careful calculations, we arrive at

\[
\tilde{U}^{(N)}_p = e^{\frac{(2,0)}{2} \sum_{k_i} [a_{i}^\dagger e^{i\omega_{k_i} t_0} f(n, N, \omega_k, \Delta t) \xi_{k_i}(\Delta t) - h.c.]}
\times \left[ \frac{1}{2} \sum_{k_1} [a_{i}^\dagger e^{i\omega_{k_1} t_0} f(n, N, \omega_k, \Delta t) \xi_{k_1}(\Delta t) e^{i\omega_{k_1} \Delta t} - h.c.] \right]
\times \left[ \frac{1}{2} \sum_{k_2} [a_{k_2} e^{i\omega_{k_2} t} f(n, N, \omega_k, 2\Delta t) \xi_{k_2}(\Delta t) e^{i\omega_{k_2} 2\Delta t} - h.c.] \right]
\times \left[ e^{\frac{(2,1)}{2} \sum_{i_k} [a_{k_2}^\dagger e^{i\omega_{k_2} t_0} f(n, N, \omega_k, 2\Delta t) \xi_{k_2}(t_0) - h.c.]} \right]
\times e^{iN\xi_0 3\Delta t}, \tag{26}
\]

where \( f(n, N, \omega, \Delta t) = \sum_{n=1}^{N} e^{3i\omega (n-1)\Delta t}. \)

Now we can give a quantitative estimation of the decoherence rate according to the time dependence of non-diagonal matrix elements of the reduced density matrix of the three-level atom. For example, the coherence between the level 0 and 2 is represented by

\[
\tilde{\rho}_{02}^S(t) = TrE\{\langle 0|\tilde{U}^{(N)}_p |\tilde{\rho}_{02}^S(0) \otimes \tilde{\rho}_E(0) \tilde{U}^{(N)}_p \rangle |2\rangle\}
\]

\[
= \tilde{\rho}_{02}^S(0) TrE \left[ \tilde{\rho}_E(0) \exp\left\{ \sum_{k_1} a_{k_1}^\dagger e^{i\omega_{k_1} t_0} \eta_{k_1}(\Delta t) + \sum_{k_2} a_{k_2}^\dagger e^{i\omega_{k_2} t_0} \eta_{k_2}(\Delta t) - h.c. \right\} \right] \exp\{ -i3N\omega_0 2\Delta t \}
\]

\[
= \tilde{\rho}_{02}^S(0) \exp\{ -i3N\omega_0 2\Delta t - \Gamma(k_1, \Delta t) - \Gamma(k_2, \Delta t) \},
\]

where

\[
\Gamma(k_1, \Delta t) = \sum_{k_1} \left| \frac{\eta_{k_1}(\Delta t)}{2} \right|^2 \coth \left( \frac{\omega_{k_1}}{2T} \right),
\]

\[
\Gamma(k_2, \Delta t) = \sum_{k_2} \left| \frac{\eta_{k_2}(\Delta t)}{2} \right|^2 \coth \left( \frac{\omega_{k_2}}{2T} \right),
\]

and

\[
\eta_{k_1}(\Delta t) = -\frac{1}{2} f(n, N, \omega_{k_1}, \Delta t) \xi_{k_1}(\Delta t) (2 - e^{i\omega_{k_1} \Delta t} - e^{i2\omega_{k_1} \Delta t}),
\]

and

\[
\eta_{k_2}(\Delta t) = -\frac{1}{2} f(n, N, \omega_{k_2}, \Delta t) \xi_{k_2}(\Delta t) (1 + e^{i\omega_{k_1} \Delta t} - 2 e^{i\omega_{k_2} \Delta t}).
\]

On the other hand, the density matrix without the bang-bang control is

\[
\tilde{\rho}_{02}^{SW}(t) = \tilde{\rho}_{02}^{SW}(0) e^{-\Gamma(k_1, \Delta t) - \Gamma(k_2, \Delta t)}, \tag{27}
\]

\[
\text{where}
\Gamma(k_1, \Delta t) = \sum_{k_1} \left| \frac{\xi_{k_1}(N, \Delta t)}{2} \right|^2 \coth \left( \frac{\omega_{k_1}}{2T} \right),
\]

\[
\xi_{k_1}(N, \Delta t) = f(n, N, \omega_{k_1}, \Delta t) \xi_{k_1}(\Delta t),
\]

and

\[
\Gamma(k_2, \Delta t) = \sum_{k_2} \left| \frac{\xi_{k_2}(N, \Delta t)}{2} \right|^2 \coth \left( \frac{\omega_{k_2}}{2T} \right),
\]

\[
\xi_{k_2}(N, \Delta t) = f(n, N, \omega_{k_2}, \Delta t) \xi_{k_2}(\Delta t),
\]

Physically, there exists a finite cutoff frequency of the environment \( \omega_c \). For a single mode of frequency \( \omega_k \), the time needed to produce appreciable dephasing is \( \tau_k = \omega_k^{-1} \), so \( \tau_c \sim \omega^{-1} \) sets the shortest time scale (or memory time) of the environment. When \( \omega_k \Delta t \in [0, \arccos(\frac{3}{4})] \), we get that \( \Gamma(k, \Delta t) \leq \Gamma'(k, \Delta t) \), which means in the quiet regime \( \Delta t \leq \tau_c \arccos(\frac{3}{4}) \) the pulses will effectively suppress the decoherence.
In addition, $\Gamma(k, \Delta t)$ depends monotonically on the cycle time $\Delta t$. In the ideal limit of $\Delta t \to 0 (N \to \infty)$, the decoherence is completely suppressed as a result of symmetrization. To show this, we numerically simulate one of the dephasing factors, $\Gamma(k, \Delta t)$. In the continuum limit of the bath mode, we can see

$$\Gamma_2 \equiv \Gamma(k, \Delta t) = \int_0^{+\infty} d\omega I(\omega) \times \frac{\left| \eta k_2(\Delta t) \right|^2}{2} \coth \frac{\omega}{2T},$$

(28)

where $I(\omega) = \frac{4}{\pi} \omega^n e^{-\omega/\omega_c}$, and $\alpha$ measures the strength of the system-bath interaction and the index $n$ classifies different environmental behaviors. For instance, the Ohmic environment corresponds to $n = 1$. From Fig. 3, we can see that the bigger the $N$ (or smaller the $\Delta t$), the more effective the bang-bang operation in suppressing the dephasing decoherence.

V. SUMMARY

In this paper, we have studied the suppression of adiabatic decoherence of the three-level atom with $\Lambda$ configuration using bang-bang control technique. The decoupling bang-bang operation group is found, and the sequence of periodic R.F. twinborn pulses is developed for the realization of the control strategy. Moreover, we give a quantitative estimation of the decoherence suppression in non-ideal limits. We also give the condition of effectively suppressing this decoherence.

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