Detecting Overlapping Communities in Social Networks using Deep Learning

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Abstract:
In network analysis, the community is considered as a group of nodes that is densely connected with respect to the rest of the network. Detecting the community structure is important in any network analysis task, especially for revealing patterns between specified nodes. There are various approaches in literature for community, overlapping or disjoint, detection in networks. In recent years, many researchers have concentrated on feature learning and network embedding methods for nodes clustering. These methods map the network into a lower-dimensional representation space. In this paper, we propose a model for learning graph representation using deep neural networks. In this method, a nonlinear embedding of the original graph is fed to stacked auto-encoders for learning the model. Then an overlapping clustering algorithm is employed to extract overlapping communities. The effectiveness of the proposed model is investigated by conducting experiments on standard benchmarks and real-world datasets of varying sizes. Empirical results exhibit that the presented model outperforms some popular community detection methods.

1. INTRODUCTION

In most complex systems, data can be represented as networks like transportation networks, citation networks, World Wide Web, co-authorship networks, and social networks. Analyzing these networks yields insight into their structures. Growing attention has been paid to this analysis in recent years, particularly due to the omnipresence of such networks in real-world [1].

Social network analysis (SNA) is a novel research topic in the field of computer science. The goal of SNA is to explore associations between social entities, and to figure out the features and properties of the entire network, usually by graph theory. A graph is a means of describing relationships among a set of specified items. Given a graph \( G = (V, E) \), \( V \) is a set of \( n \) nodes (vertices), and \( E \) is a set of \( m \) edges (links). In a social network, nodes represent individuals, and their social interactions are presented by edges. The network structure, for unweighted networks, can be determined by an \( n \times n \) adjacency matrix \( A \). Each element \( A_{ij} \) of \( A \) is equivalent to 1 if there is an edge that connects nodes \( i \) and \( j \). Otherwise, it is 0.

Community structure is a shared property in many social networks. Despite the lack of a unique definition for the community, a community is assumed to represent a group of nodes where the number of connections between these nodes is greater than connecting with other nodes in the network. A community is considered as a group of nodes similar to each other, but different from other nodes in the network. Individuals that are in a similar community have comparable properties [2] such as social ties, interests, occupation, location, etc.

Community structure often unravels precious information of interest. Community detection (CD) in networks indicates an exciting problem with several applications, particularly for knowledge extraction from social networks. Several CD algorithms have been presented for the division of a social network into communities [3]. Within the community detection framework, much attention is given to the identification of disjoint communities. In disjoint (separated or non-overlapping) CD, a node may belong to only one community. However, in real-world scenario, a node may belong to more than one community that leads to overlapping communities. For instance, a person may...

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forge connections to many social groups such as friends, family, and co-workers [4]. This overlap is a remarkable feature of most real-world networks. Naturally, nodes may be a member of multiple communities. Therefore, by forcing each node into a single community, we miss much information and may obtain a misleading characterization of the underlying community structure.

A good comparison is shown in Figure 1 for this problem. Figure 1a displays an undirected social graph comprising of 18 individuals and Figure 1b shows the four isolated communities of the social network from Figure 1a. Here nodes 6, 13, and 14 in Figure 1c are overlapping nodes that can be shared by two or three communities.

Over the last decade, numerous methods have been presented for the detection of community. In the following section, these methods are explained in detail. Some of these methods have a number of drawbacks or limitations including inability to support the concept of overlapping, or high costs of computation for large graphs.

In this research, we introduce an effective algorithm to address the detection of overlapping communities by incorporating the community structure into graph embedding. Our formulation combines the strengths of random walk methods and deep learning architectures in a framework. The proposed algorithm maintains both the global and local structure of a network. Experimental results exhibit the efficacy of the presented method on some benchmarks and real-world networks.

This paper is organized as follows. Section 2 reviews the literature on community detection algorithms in networks, with emphasis on overlapping methods. Section 3 explains the design of the proposed algorithm in detail. It presents preliminaries such as the concept of node representation, deep learning components, and an overlapping clustering algorithm used on deep learning output. In Section 4, we outline our experiments on some computer-generated and real-world networks. Section 5 presents the results of experiments and compares our algorithm with a few overlapping community detection algorithms. Finally, conclusion is drawn in Section 6.

2. RELATED WORKS

The detection of overlapping and non-overlapping communities represents two distinct problems. That is, the algorithms used to detect non-overlapping communities could not be readily applied to other overlapping counterparts. Hence, a variety of algorithms have been introduced to exhibit the structures of overlapping communities [6, 7].

The algorithms for detecting communities in graphs can be divided into several categories based on diverse parameters, such as the actual operational methods [3], or the fundamental concept of the community [8]. In this section, we present an abridged overview of a few algorithms for community detection.

Community detection can be performed by detecting the denser areas of the network. A dense community is a group of vertices where the number of edges is substantially greater than the number of edges expected in a random graph (without any community structure). With this approach, community detection can be considered as optimizing a specific density function until no further augmentation is possible. Modularity [9] is a famous density function that is used to detect disjoint communities. Chen et al. [10] presented an extended version of modularity for overlapping community detection.

Communities could be assumed as parts of the graph obtained by removing all edges (i.e., bridges) that connect these parts to each other. Girvan-Newman (GN) algorithm is a famous method in this category [11]. It conducts hierarchical clustering (dividing a network into preferred numbers of clusters). GN relies on a centrality measure called betweenness. At each step, it involves counting the shortest paths between all pairs of vertices that exist in the network, sorting edges by their betweenness values, and removing edges with the most value (i.e., a bridge). Cluster Overlap Newman Girvan Algorithm (CONGA) extends GN for discovering overlapping communities in networks [12]. A central point of CONGA is the concept of split betweenness, which allows a node to split into multiple copies.

A community could be defined as a set of nodes categorized by the dissemination of a similar property, action, or data within the network. The label propagation algorithm (LPA) deals with this definition, in which...
nodes that belong to an identical label forge a community [13]. A node in the network decides to join a community where the majority of its neighbors are members of that community. With the propagation of labels, groups of nodes with tight connections obtain an exclusive label. This method is fast but does not support the overlapping concept. In the community overlap propagation algorithm (COPRA), nodes have a label and a belonging coefficient [14]. The belonging coefficient of each node is updated by averaging coefficients in all of its neighbors. A node that has an equivalent maximum number of neighbors, in at least two communities, is considered as an overlapping node. The LPA can be extended to an overlapping community detection by Speaker-listener Label Propagation Algorithm (SLPA) [15]. It gives each node a memory for the storage of the received information and the spread of labels between nodes in compliance with rules of pairwise interaction. The membership strength is defined as the possibility of complying with a label in node memory. The output of this method is the labels’ probability distribution for each node’s memory. Nodes with identical labels are then assigned to a community. A node with several labels belongs to several communities.

Another common approach to community detection defines it as an accurate structure of edges. Therefore, an algorithm defines some sort of structure and then attempts to find it efficiently within the graph. For instance, a clique is defined as a complete subgraph of a given graph. The clique of the largest possible size is called a maximal clique. The clique percolation method (CPM) [16] finds all k-cliques (clique with k nodes) in a graph and merges cliques with common k-1 nodes. A community embodies the union of all k-cliques that can be reached from each other. GCE (greedy clique expansion) [17] identifies maximum cliques as seed communities, which are then expanded through greedy optimization of a local fitness function.

Another idea that has been explored is to partition links (edges) rather than nodes for exploring community structure. According to this approach, it is the relation, not the node, which is a member of the community. Hence, since they try to cluster the network edges, the nodes belong to a set of communities forged by their edges. Link community (LC) is a well-known method in this category [18]. In LC, an original graph’s node is said to be overlapping when its connecting links are placed in at least two or more clusters.

In recent years, the methods that rely on the representation of networks in vector space (mapping graph matrices to another space) have become popular. Graph embedding seeks to offer a representation of a graph as low dimensional vectors but preserves the graph structure [19]. Finding a vector representation for nodes of a graph is challenging and poses several difficulties. A suitable vector representation of nodes needs to retain the network structure and the connection between single nodes. The network structure is represented by proximity measures such as first-order proximity (edge weights), second-order proximity (similarity of the vertices’ neighborhood structures), and higher-order proximities between nodes.

The embedding methods can be split into three general categories: methods based on (1) Factorization (2) Random Walk, and (3) Deep Learning (DL).

In the first category, connections of nodes are represented as a matrix, which is then factorized to attain the embedding. The matrices employed for the representation of the connections comprise the Laplacian matrix, Katz similarity matrix, etc. Graph Factorization GraRep [20] is an example of this category.

Several graph properties such as node similarity and centrality are approximated using random walks. They are particularly useful when only partial observation of a graph is possible. Deepwalk [21] is a method that transforms graphs into collections of linear sequences by a uniform sampling method known as the truncated random walk.

Deep learning has been particularly successful in handling images, texts, time series, and audio data in recent years [22]. Thanks to the advances in deep neural networks, these methods can produce representations for various kinds of data [23]. Deep learning can be used as a useful tool for graph analysis to produce representations that contribute to community detection.

3. PROPOSED METHOD

We propose an algorithm to investigate the problem of overlapping community detection. The overall scheme of the proposed method is shown in Figure 2.

The algorithm exploits factorized matrices, generated random walks, and deep learning architecture. It consists of three major parts: First, we introduce a model based on random walks to represent (embed) the nodes. This model captures the input graph’s structural information. It generates probabilistic co-occurrence (PCO) matrix and positive pointwise mutual information (PPMI) matrix. After that, a deep learning architecture is used to learn representations of low-dimensional nodes. The output of this part is given to an overlapping clustering method to produce overlapping communities. In this section, the major components of the proposed algorithm are discussed in detail.

3. 1. Graph Embedding  As mentioned earlier, graph (node) embedding aims to offer a representation of a graph (node) as low dimensional vectors while retaining the structure of the network. As the output of this process, any node is described with a vector.

Pioneer methods for graph embedding generalize advancements in language-modeling (sequences of words) to graphs (sequences of nodes) for representation.
The essential inputs of any language-modeling algorithm are vocabulary and a corpus. The language-modeling aims to estimate the probability of a sequence of words manifesting in a corpus [24]. Formally put, for a sequence of words in Equation (1) which \( w_i \) is an element of vocabulary:

\[
W^n_w = \{w_0, w_1, \ldots, w_n\}
\]

We would like to maximize the probability in Equation (2) over the entire corpus.

\[
\Pr(w_n \mid w_0, w_1, \ldots, w_{n-1})
\]

It is similar to calculating the probability of observing vertex \( v_i \) based on all previously visited vertices in a random walk:

\[
\Pr(v_i \mid v_1, v_2, \ldots, v_{i-1})
\]

Based on this analogy, Deepwalk [21] presented a generalization of language-modeling to examine the graph via a set of random walks. These walks could be considered as short phrases and sentences in a certain language. Deepwalk draws on local information attained from curtailed random walks. The procedure involves a slow uniform sampling process, and hyper-parameters of the method such as “walk length” and “walks per node” are not easy to determine.

Our algorithm directly constructs the PCO matrix from a graph, which avoids the expensive sampling process. A co-occurrence matrix is a giant matrix as wide as the vocabulary (graph) size. When words (nodes) occur together, they are marked with a positive entry. Otherwise, they will take a 0.

Inspired by Google’s PageRank, we consider a random walk model with a restart. First, the vertices are randomly ordered in a graph. It is assumed that the current vertex is the \( i^{th} \) vertex, and there exists a transition matrix \( T \) that takes into account the transition likelihoods between diverse vertices. Matrix \( T \) is initialized based on the adjacency matrix.

Each row vector of the PCO matrix is of the form \( p_i \), where the \( j^{th} \) entry corresponds to the likelihood of reaching the \( j^{th} \) vertex following \( k \) steps of transition. According to probability \( \alpha \), a random walk will continue (choosing the next node), and based on probability \( 1 - \alpha \), it will revert to the original vertex and the procedure will be restarted. The probabilities for arriving at different vertices after exactly \( k \) steps of transitions is specified by recurrent Equation (4):

\[
p_k = \alpha p_{k-1} T + (1 - \alpha) p_0
\]

\( p_0 \) is the initial one-hot vector of a node \( v_i \) with the \( i^{th} \) entry having a value of 1, and other entries having a value of 0. If there is no random restart in the procedure \((\alpha = 1)\), PCO is achieved using Equation (5):

\[
p_k^* = p_{k-1} T = p_0 T^k
\]

After constructing the PCO matrix, the representation \( r \) for the \( i^{th} \) node is shown in Equation (6):

\[
r = \sum_{k=0}^{K} f(k), p_k^*
\]

The \( f(\cdot) \) is a decreasing weighting function. The weight assigned to a context node is a function of the distance of that node from the current node in the graph. In our method, we use \( f(x) = 1/x \) as a common weighting function [25].

In language-modeling, we use the current word \( w \) (whose representation that we want to generate) to predict its \( c \) context in a probabilistic form. The term ‘context’ refers to the words that may appear around the current word in a sentence. With \( K \) as window size, sentences are generated using \( K \) words before and after the current word. Following this, the PCO matrix is used to compute the PMI matrix. The PMI matrix can be seen as the product of the representation matrix and the context matrix, which could be expressed by Equation (7):

\[
\text{PMI}_{w,c} = \log \left( \frac{\text{#}(w,c), |L|}{\text{#}(w), \text{#}(c)} \right)
\]

where \(|L|\) is computed using Equation (8):
\[ |L| = \sum_i \sum_c \#(w, c) \]  
\[ (8) \]

In the same vein, for graph embedding, \( v_i \) in Equation (9) refers to the current node and \( W_{v_i} \) determines the nodes that appear in a random walk rooted at \( v_i \).

\[ PMI_{v_i, W_{v_i}} = \log \left( \frac{\#(W_{v_i}) | L |}{\#(v_i), \#(W_{v_i})} \right) \]  
\[ (9) \]

To improve performance, we use a PPMI matrix instead of a PMI matrix [26]. As shown in Equation (10), each negative value is assigned to zero.

\[ PPMI_{v_i, W_{v_i}} = \max (PMI_{v_i, W_{v_i}}, 0) \]  
\[ (10) \]

Each node’s global neighborhood is expressed by a row of PPMI. For any two nodes, the PPMI values answer the question of whether the two nodes tend to occur more together or more independently.

Instead of factorized matrices used in our proposed method, Deepwalk combines a random walk and skip-gram model utilized in Word2vec [27] to learn network representations. Skip-gram is an approach that seeks to obtain similar vectors for similar words (based on their context). We compare the results in Section 5.

In some methods, Singular Value Decomposition (SVD) is used for dimensionality reduction after computing the PPMI matrix for a text corpus [26] or graph embedding [20]. SVD decomposes any specified matrix into three matrices (two orthonormal matrices and one diagonal matrix). Although SVD is an effective tool for dimensionality reduction, it also captures linear relationships between the two vectors. In our model, we replaced SVD with deep neural networks to capture a non-linear mapping between the nodes.

Inputting the PPMI matrix guarantees that the deep learning architecture can account for the proximity of the input graph’s nodes.

3.2. DL Components and Configuration

Based on unsupervised learning, deep learning employs multiple layers of processing to learn data representations with multiple abstraction levels.

An autoencoder (AE) is a form of neural network that learns features to reconstruct its input at the output layer. In [28], autoencoders are used for graph clustering tasks with the Laplacian matrix as their input. Here, the PPMI matrix was used as an input fed to the autoencoder.

An autoencoder comprises two parts: the encoder, and the decoder. The former is responsible for embedding the original input into a low-dimensional representation. The decoding layers will learn how to decode the representation and brings it back into its original form in the most accurate manner [29]. Stacked autoencoders (SAE) consist of multiple layers of such autoencoders for learning multiple layers of representations (Figure 3).

The embedding for a node in the PPMI matrix will be a \( l \times n \) vector. This is fed to the autoencoder to reduce the dimensions to \( l \times d (d < n) \) during the encoding step. We then apply function \( f(\cdot) \) to vector \( x \) in the input space so that it can be sent to a new feature space.

\[ f_{\theta_1}(x) = \sigma(W_1 x + b_1) \]  
\[ (11) \]

This result is called a code or latent variable. The term \( \theta_1 = (W_1, b_1) \) defines the parameters involved in the encoder part, where \( W_1 \) is a weight matrix and \( b_1 \) is a bias vector. An activation function \( \sigma(\cdot) \) is usually recruited to offer for modeling of the non-linearities between two vector spaces. For this purpose, we use a half-wave rectifier \( f(z) = \max (z, 0) \) known as rectified linear unit (ReLU). It typically learns very fast in networks with many layers.

The decoding step involves using a reconstruction function \( g(\cdot) \) to reconstruct the original input vectors retrieved from the latent representation space.

\[ g_{\theta_2}(y) = \sigma(W_2 y + b_2) \]  
\[ (12) \]

In Equation (12), \( y \) is the output of the encoding step, and \( \theta_2 \) defines the parameters involved in the decoder (\( W_2 \) as weight matrix and \( b_2 \) as a bias vector).

Equation (13) expresses the reconstruction error as a measure of discrepancy between input \( x \) and its reconstruction in output.

\[ \sum_i L \left( x^{(i)} , g_{\theta_2}(f_{\theta_1}(x^{(i)})) \right) \]  
\[ (13) \]

\( L \) is a loss function such as cross-entropy or mean square error (MSE). Thus, our goal is to reduce the reconstruction error by finding values of \( \theta_1 \) and \( \theta_2 \).

We want to estimate parameters and hyper-parameters (number of layers, or neurons in each layer, etc.) to have a precise reconstruction. This process requires effort and expertise. Obviously, the output layer has \( n \) neurons such as the input layer. The encoder and decoder parts are symmetric to each other regarding the number of layers and the number of neurons in each layer.

Figure 3. Structure of an SAE: an output layer, an input layer, and three hidden layers (encoding and decoding parts) [29]
Instead of randomly initializing the weights, we use the Glorot-uniform method (Xavier method), which has much better performance. Stochastic gradient descent (SGD), with a learning rate set to 1% at the beginning of the training, is used for backpropagation.

As a regularization technique for reducing overfitting, Dropout (randomly drops some neurons together with connections formed in the neural network during training phase) is used with 0.1 as drop rate. For implementation, we use the Python deep learning library Keras.

3.3. Overlapping Clustering Algorithm  K-means is a clustering algorithm that is extensively used. It leads to the detection of a disjoint community where a user is only the member of a given community. We use a version of overlapping K-means (OKM) [30] that generalizes the k-means algorithm and applies it to the output of the DL part. OKM defines an objective function for multi-assignment, initialising as data image an arbitrary cluster prototype that has a random centroid. It uses input objects along with the prototype to estimate the average of the two vectors that are used as a threshold for assignment of objects to multiple clusters (communities). The exact number of clusters must be determined as a priori.

4. EXPERIMENTS

We conduct our experiments on two datasets of varying sizes and characteristics: real-world datasets and synthetic datasets (benchmarks).

First, the details of three publicly available real-world networks and five benchmarks are given. Then, four popular metrics (F-Score, Normalized Mutual Information (NMI), conductance, and Omega index) are reviewed. The performance of our proposed method can be evaluated in comparison with some advanced algorithms to detect overlapping communities.

4.1. Datasets  Three datasets from real-world networks are selected from the KONECT dataset collection [31] with different structures. The Advogato is an online trust community platform designed for free software developers. The Hamsterster is the site of friendships for users of the website hamsterster.com. The Virgili is the email communication network utilized at the University of Virgili in southern Spain. Each edge suggests that more than one email has been sent.

Table 1 summarizes the characteristics of these datasets. The size of each dataset is equal to \(n\) (number of nodes), \(m\) indicates the number of edges, \(k_{\text{avg}}\) is the average degree, and \(k_{\text{max}}\) is the maximum degree of nodes. Obviously, \(k_{\text{max}}\) is much greater than \(k_{\text{avg}}\). This is a common attribute of social networks, as an instance of scale-free networks [32], which follows the power-law distribution for its node degree. This distribution is expressed as \(\rho x^{-\tau}\), where \(\rho\) is a constant factor, \(x\) is the variable of distribution, and \(\tau\) is the exponent. If the degree distribution of a connected graph complies the power law, it leads to a long tail of intermittent vertices (many low-degree nodes vs. a few high-degree nodes).

The parameter \(n_{\text{GCC}}\) shows the greatest connected component of the graph (a maximal subset of nodes that are all reachable from each other). For a connected graph with one component, \(n_{\text{GCC}}\) is equal to \(n\).

The density is a measure of how tightly interconnected a graph, which is calculated as the ratio of edges to the total number of possible edges. The diameter (\(D\)) is a measure of the distance between the two most distant nodes in the network. It is the maximum number of connections required to traverse the graph.

| Dataset | \(n\) | \(m\) | \(k_{\text{avg}}\) | \(k_{\text{max}}\) | \(n_{\text{GCC}}\) | Density | \(D\) | \(CC\) | Layer Configuration |
|---------|------|------|----------------|----------------|----------------|--------|------|------|-------------------|
| Advogato | 6541 | 51127 | 15.63 | 943 | 5042 | 0.0011 | 9 | 0.0922 | 6541-1024-512-256-128-64 |
| Hamsterster | 1858 | 12534 | 13.49 | 272 | 1788 | 0.072 | 14 | 0.0904 | 1858-512-256-128-64 |
| Virgili  | 1133 | 5451 | 9.62 | 71 | 1133 | 0.0085 | 8 | 0.1660 | 1131-512-256-128-64 |

| Dataset | \(n\) | \(m\) | \(k_{\text{avg}}\) | \(k_{\text{max}}\) | \(|\mu|\) | minc | maxc | \(O_o\) | \(O_o\) | Layer Configuration |
|---------|------|------|----------------|----------------|------|------|------|--------|--------|-------------------|
| Bench1  | 250  | 731  | 5 | 50 | 0.1 | 10 | 25 | 2 | 0.1 | 250-128-64-32 |
| Bench2  | 500  | 1413 | 5 | 50 | 0.1 | 20 | 100 | 2 | 0.1 | 500-128-64-32 |
| Bench3  | 1000 | 7685 | 10 | 50 | 0.1 | 20 | 100 | 2 | 0.1 | 1000-512-256-128-64 |
| Bench4  | 2000 | 15488 | 10 | 100 | 0.1 | 25 | 250 | 3 | 0.2 | 2000-512-256-128-64 |
| Bench5  | 5000 | 37769 | 15 | 250 | 0.2 | 50 | 500 | 4 | 0.3 | 5000-1024-512-256-128-64 |
By definition, the clustering coefficient (CC) for a node is the number of edges that exist between neighbors of that node, divided by the number of possible edges that can exist between its neighbors. The graph’s clustering coefficient indicates the mean clustering coefficient of all its nodes.

The number of neurons existing in the input layer and encoding part of DL (only for the first half) is specified in the last column of Table 1. The DL configuration can be obtained by adding the decoder part. For example, configuration for Virgili is 1131-512-256-128-64-128-256-512-1131.

We use another kind of dataset for our experiments. The LFR benchmark [33] is an algorithm that produces benchmark networks (realistic artificial networks that correspond to networks in the real world). LFR presents a set of parameters for controlling the network topology. It assumes that both the degree and size of community distributions in a network follow power laws displayed by exponents’ \( \tau_1 \) and \( \tau_2 \), respectively. We generate five datasets called Bench1 to Bench5 with \( \tau_1 = 2 \) and \( \tau_2 = 1 \).

Unlike the real world datasets, all the parameters are adjusted by the user (except \( m \)). In Table 2, \( m \) is defined as the mixing parameter. It controls the fraction of edges between communities. There is a common fraction (\( 1 - \mu \) of its edges) between each node and other nodes in its community as well as a fraction \( \mu \) and other nodes in the network. Here, two parameters of max. and min. are the maximum and minimum community size, respectively.

Two parameters \( O_m \) and \( O_o \) adjust the overlapping properties. \( O_m \) is for overlapping diversity, as the number of community memberships in each overlapping node. \( O_o \) stands for overlapping density, as the fraction of overlapping nodes in the graph. For a disjoint community detection algorithm, \( O_m = 1 \) and \( O_o = 0 \).

4.2. Metrics

To evaluate performance of the proposed method, we compare it with other community detection algorithms against four popular metrics such as F-score, NMI, modularity, and conductance.

F-score measures the matching between communities obtained by a method, with communities of ground truth (a predefined expected categorization). It can be defined as the harmonic average of precision and recall.

\[
F = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}
\]

In overlapping nodes, precision is defined as the proportion of accurately detected overlapping nodes to the entire number of detected nodes. The recall for overlapping nodes is defined as the proportion of accurately detected overlapping nodes to the entire number of overlapping nodes. F-score ranges from 0 to 1, with higher values, indicate a larger degree of correspondence. The ground truth is accessible in benchmark networks, so we can apply metrics such as F-score and NMI to them.

The NMI, introduced in [34], is another popular metric to evaluate the quality of a community detection algorithm. It measures the similarity between a reference (ground truth) model of communities and the community detection obtained by an algorithm.

It assessed the extent of resemblance between a cover (set of overlapping communities) as the output of a CD algorithm, with the ground truth cover. The mutual information \( I(X, Y) \) is defined in Equation (15).

\[
I(X, Y) = \sum_{x \in X} \sum_{y \in Y} P(x, y) \log \frac{P(x, y)}{P(x)P(y)} = H(X) - H(X | Y)
\]

where \( x \) and \( y \) denote the community labels of node \( i \) in covers \( X \) and \( Y \), respectively. Labels \( x \) and \( y \) indicate the values of two randomly selected variables \( X \) and \( Y \). In addition, \( I(X, Y) \) shows how much can be learned about \( Y \) if \( X \) is known, and vice versa. Based on \( I(X, Y) \), and \( H(X) \) as the entropy of \( X \), NMI is defined in Equation (16). The NMI value ranges from 0 to 1, with a higher value serving as a sign of better performance.

\[
I_{\text{NMI}}(X, Y') = \frac{2I(X, Y)}{H(X) + H(Y')}
\]

Modularity function is the most commonly used measure to evaluate the quality of a partition community structure (for non-overlapping communities). Modularity quantifies if a set of entities is more connected than a null model (randomly rewired nodes with no community structure) and thus can be considered as a community.

An extended modularity function [10] for overlapping community detection is shown in Equation (17):

\[
Q_o = \frac{1}{2m} \sum_{i \in C} \sum_{j \in C} \left( A_{ij} - \frac{k_i k_j}{2m} \right) \frac{1}{O_i O_j}
\]

In this version, the number of communities that a node belongs to is used as a weight for \( Q \). \( A_{ij} \) is entering the adjacency matrix for nodes \( i \) and \( j \), \( c \) is a community in cover \( C \), \( k_i \) denotes the degree of node \( i \), and \( O_i \) indicates the number of communities to which node \( i \) belongs. A higher \( Q \) value for a partition (or cover), indicates a superior corresponding community structure.

Conductance is used as a simple measure to capture the community goodness [35]. It corresponds most closely to the intuition that a community comprises a set of densely linked nodes that are sparsely linked to the outside. The conductance for community \( S \) in cover \( C \) is given in Equation (18):

\[
\text{cond} (S) = \frac{c_i}{2m_{ss} + c_i}
\]

where \( m_{ss} \) shows the number of internal edges (both nodes of the edge belong to \( S \)) and \( c_i \) indicates the number of boundary edges (only one node of the edge belongs to \( S \)). Particularly, the sets of nodes that closely resemble a community are characterized by a lower conductance. It
has many inward edges and/or few edges pointing outside. The conductance of a cover is the average conductance of all its communities.

5. RESULTS AND DISCUSSION

Testing an algorithm means the analysis of a network that has a community-like structure and the recovery of its communities. In this section, performance of the presented method is empirically tested (with two values for α), in comparison with several algorithms, on networks of LFR benchmark that are characterized with real world datasets and overlapping communities.

For algorithms that have tunable parameters, we reported the results for the best setting. Considering the non-deterministic nature of COPRA and SLPA, each of them was repeated five times on each network instantiation. The maximum for the average degree in benchmarks is maintained at $k_{avg} = 15$, which resembles large social networks in terms of the order.

The results of applying the methods on the datasets are shown in Tables 3 to 6 for F-score, NMI, modularity, and conductance, respectively. Numbers in bold demonstrate the optimum performance in each row. We set $\alpha$ to 0.95 and 0.975 in our method. With $\alpha=1$, the contextual information of the nodes is not weighted based on their distance, and we obtain undesirable results.

A problem is that for real-world networks, the ground truth of overlapping communities is not accessible. Since the reference communities for ground truth of these networks need to be manually determined, they are subjective in nature. For this reason, only benchmark datasets are used for metrics such as F-score and NMI in Tables 3 and 4.

Empirical results on datasets of varying structures and sizes exhibit that the presented model excels with other advanced algorithms in the overlapping community detection tasks. The results indicate that the performance of each method worsens when the graph size increases or the communities are more similar to each other. Also, the performance worsens with the increase in $O_m, O_c$, or $\mu$ in the benchmarks.

In Table 3, the behavior of the algorithms is studied in terms of F-score. The worst results go with LC and GCE. The clique-like assumption of GCE yields high precision, but this method has a very low recall. On the other hand, LC has a poor performance in terms of F-score despite its extremely high recall. This is because the overlapping nodes claimed by LC is more than the exception (resulting in a very low precision).

As Table 4 shows, the values of NMI for networks that have a small-sized community $s = (5, 50)$ are usually greater than those for networks that have a large-sized community $s = (15, 250)$.

Modularity maximization forces small communities into larger ones (resolution limit). Thus, it does not detect communities that are smaller than the resolution limit.

As shown in Table 6, conductance is capable of identifying well-separated communities (low $c_s$ and/or high $m_s$), but it has poor performance in identifying cohesive and dense sets of nodes with high clustering coefficients.

The results from Tables 3 to 6 reveal the merits of our proposed model by using the weighting strategy in the random walk procedure and the effectiveness of deep architectures.

| Table 3. Results for community detection algorithms (F-score as performance measure) |
|-----------------|----------------|----------------|---------------|-------------|---------|----------|---------------|---------------|
| CONGA | COPRA | SLPA | GCE | LC | Deepwalk | Proposed ($\alpha=0.95$) | Proposed ($\alpha=0.975$) |
|-------|-------|------|-----|----|---------|----------------|----------------|
| Bench1 | 0.3213 | 0.3622 | 0.3757 | 0.2841 | 0.3092 | 0.4219 | 0.4710 | **0.5118** |
| Bench2 | 0.3185 | 0.3370 | 0.3510 | 0.2380 | 0.2738 | 0.3772 | 0.4043 | **0.4297** |
| Bench3 | 0.2957 | 0.3149 | 0.3278 | 0.2147 | 0.2409 | 0.3408 | 0.3721 | **0.3818** |
| Bench4 | 0.2753 | 0.2792 | 0.2792 | 0.1975 | 0.2140 | 0.3087 | **0.3514** | 0.3379 |
| Bench5 | 0.2314 | 0.2514 | 0.2541 | 0.1712 | 0.1946 | 0.2749 | **0.3109** | 0.3049 |

| Table 4. Results for community detection algorithms (NMI as performance measure) |
|-----------------|----------------|----------------|---------------|-------------|---------|---------------|---------------|
| CONGA | COPRA | SLPA | GCE | LC | Deepwalk | Proposed ($\alpha=0.95$) | Proposed ($\alpha=0.975$) |
|-------|-------|------|-----|----|---------|----------------|----------------|
| Bench1 | 0.3818 | 0.4417 | 0.4713 | 0.3619 | 0.3118 | 0.4629 | 0.5012 | **0.5369** |
| Bench2 | 0.2823 | 0.3742 | 0.3921 | 0.3654 | 0.2673 | 0.3376 | 0.4090 | **0.4229** |
| Bench3 | 0.2492 | 0.2944 | 0.2783 | 0.2504 | 0.2138 | 0.2887 | 0.3239 | **0.3498** |
| Bench4 | 0.1793 | 0.2286 | 0.2438 | 0.2141 | 0.1782 | 0.2381 | 0.2612 | **0.2779** |
| Bench5 | 0.1627 | 0.1940 | 0.2073 | 0.1807 | 0.1437 | 0.2028 | 0.2340 | **0.2395** |
6. CONCLUSION

In this study, we presented a deep graph representation model that deals with overlapping community detection. Our model combined the advantages of factorized matrices, random walks, and stacked autoencoders in extracting information and generating informative representations. We investigated on real world datasets and benchmarks in different tasks and showed that the proposed model outperforms several advanced algorithms. We conclude that exploring the underlying structure of data via deep learning can lead to improved representations for graphs.

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Detecting Overlapping Communities in Social Networks using Deep Learning

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چکیده
در تحلیل شبکه‌ها، مفهوم انجمن به تعدادی از گره‌هایی اطلاق می‌شود که چگالی یال‌ها آنها بیش از متوسط چگالی یال‌های دیگر شبکه است. یافتن ساختار انجمن در گراف، اهمیت زیادی در هر سطح تحلیل شبکه خصوصاً بایان ارتباط افراد گره‌هایی بررسی می‌شود که به عضویت در یک یا چند انجمن دارند. روش‌هایی برای مدل‌سازی شبکه با روش‌های مختلف برای انجمن‌بندی و نهایتاً ایجاد ساختاری برای شبکه و شناسایی انجمن‌ها پیشنهاد می‌شود. البته در سال‌های اخیر محقلینی مدل‌سازی شبکه با روش‌های یادگیری شبکه و ویژگی‌برداری برای انجمن‌بندی شبکه‌ها و یافتن الگوهای موجود در انجمن‌ها استفاده می‌شود. در این مقاله، برای انجمن‌بندی گره‌های شبکه، مدلی بر اساس شبکه‌های عصبی عمیق برای یادگیری روش شبکه‌سازی گراف معرفی می‌کنیم. در روش پیشنهادی، یک فضای تقریبی از توابع خطی در یک فضای بازنمایی داده می‌شود و به یک الگوریتم کلستر بندی شبکه‌های عصبی عمیق در یک فضای بازنمایی مربوط می‌شود. روش پیشنهادی، یک کلستر بندی غیرخطی از توابع یک‌دست نویس، شبکه‌های بازبرده و شبکه‌های نهایی حاصل شوند. برای تعریف فرمول مدل و جبرای اطمینان از دقت و دقت اعمال می‌کنیم. نتایج تجربی حاکی از آن است که روش پیشنهادی این مقاله، عملکرد بهتری از چند روش متداول انجمن‌بندی داشته است.

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