The generalized second law in universes with quantum corrected entropy relations

Ninfa Radicella and Diego Pavón
Departamento de Física, Universidad Autónoma de Barcelona, 08193 Bellaterra (Barcelona), Spain
E-mail: ninfa.radicella@uab.cat, diego.pavon@uab.es

Abstract. We apply the generalized second law of thermodynamics to discriminate among quantum corrections (whether logarithmic or power-law) to the entropy of the apparent horizon in spatially Friedmann-Robertson-Walker universes. We use the corresponding modified Friedmann equations along with either Clausius relation or the principle of equipartition of the energy to set limits on the value of a characteristic parameter entering the said corrections.

1. Introduction
As is well known, event horizons mimic black bodies and possess a nonvanishing temperature and entropy [1, 2]. Recently, it was demonstrated that cosmological apparent horizons are also endowed with thermodynamical properties, formally identical to those of event horizons [3]. The connection between gravity and thermodynamics was reinforced by Jacobson, who associated Einstein equations with Clausius’ relation [4], and later on by Padmanabhan who linked the macroscopic description of spacetime, given by Einstein equations, to microscopic degrees of freedom, $N$, through the principle of equipartition of energy [5]. On the other hand, quantum corrections to the semi-classical entropy-law have been introduced in recent years, namely, logarithmic and power-law corrections. Logarithmic corrections, arise from loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations [6],

$$ S \propto \left[ 4 \pi^{2} \frac{A}{4l_{pl}^{2}} + \alpha \ln \frac{A}{4l_{pl}^{2}} \right]. $$

(1)

On its part, power-law corrections appear in dealing with the entanglement of quantum fields in and out the horizon [7],

$$ S \propto 4 \pi^{2} \frac{A}{l_{pl}^{2}} \left[ 1 - K_{\alpha} A^{1-\alpha/2} \right]. $$

(2)

In the last two expressions, $\alpha$ denotes a dimensionless parameter whose value (in both cases) is currently under debate.

For the connection between horizons and thermodynamics to hold, these quantum entropy corrections must translate into modifications of the field equations of gravity, see e.g. [8, 3]. In any sensible physical context these modifications must fulfill the generalized second law (GSL)
of thermodynamics. The latter asserts that the entropy of the horizon plus the entropy of its surroundings must not decrease in time. As demonstrated by Bekenstein, this law is satisfied by black holes in contact with their radiation [9].

The aim of this work is to see whether the modified Friedmann equations coming from logarithmic corrections and from power-law corrections, in conjunction with Clausius relation or the equipartition principle, are compatible with the generalized second law. This sets constraints on the parameter $\alpha$ introduced above, whose value is theory dependent and rather uncertain. We hope this may be of help in discriminating among quantum corrections, via a purely classical analysis.

2. Modified Friedmann equations

As said above, when quantum modifications to the entropy-area relation are incorporated it translates into modifications of Friedmann equations when computing either Clausius relation or the equipartition principle on the apparent horizon of a flat Robertson-Walker Universe. The radius of the horizon is given by $\tilde{r}_{AH} = \frac{H}{\dot{H}}$, where $H = \dot{a}/a$ denotes the Hubble function. The detailed derivation can be found in [10], and the modified Friedmann equations read

$$H^2 [1 + g(\alpha, H)] = \frac{8\pi G}{3} \rho, \quad (3)$$
$$\dot{H} [1 + f(\alpha, H)] = -4\pi G \left( \rho + \frac{P}{c^2} \right), \quad (4)$$

the explicit expressions of $f(\alpha, H)$ and $g(\alpha, H)$ depend on both the entropy corrections and the thermodynamical relation employed, as shown in Table 1.

As can be noted, the $\alpha$ parameter directly comes from quantum corrections to the entropy and consequently affects cosmological scenarios. Its value depends on the details of the quantum calculations, and for the time being there is not agreement on it. The following analysis determines in which intervals this parameter results compatible with the GSL.

3. The Generalized second law of Thermodynamics

Equipped with the entropy expressions (1) and (2), we set out to study whether the GSL is satisfied when the modified Friedmann equations (3) and (4) are employed.
Since the entropy depends on the area of the apparent horizon, \( A_H \propto H^{-2} \), it varies as \( \dot{S}_H \propto F(H)H \). Using eq.(4), it can be cast in terms of the Hubble parameter and the energy density and pressure of the fluid that fills the universe:

\[
\dot{S}_H = K \frac{F(H)}{H^3} \left( \rho + \frac{P}{c^2} \right),
\]

(5)

where \( K = \frac{8\pi G k_B}{h} \) and \( F(H) \) depends on the entropy corrections and the thermodynamic relation used to derive Friedmann equations, namely

\[
F(H) = 1 + \frac{\alpha \frac{\rho H^2}{c^2}}{1 + f(\alpha, H)},
\]

(6)

for logarithmic entropy corrections, and

\[
F(H) = \frac{1 + \alpha (r_c H)^{\alpha - 2}/2}{1 + f(\alpha, H)},
\]

(7)

for power-law corrections.

For the sake of clarity in what follows we split the analysis for the two classes of entropy corrections, but we will only consider perfect fluids assuming that the dominant energy condition (DEC) holds true (i.e., \( \rho + P/c^2 > 0 \)) all along the expansion. Then, in view of eq.(5) the GSL is satisfied provided \( F(H) \) is non-negative which occurs only for some values of the parameter \( \alpha \).

3.1. Logarithmic entropy corrections

For convenience, we introduce the dimensionless variable \( x = l_p^2/A_{AH} \) so that \( x \sim 1 \) at the quantum regime and, provided \( \dot{H} < 0 \), it decreases as time goes on. We require \( F(\alpha, x) \) to be non-negative, if the GSL is to hold, as well as \( \rho \geq 0 \), not to deal with ghosts.

Left panel of fig.(1) depicts the regions in the plane \((x, \alpha)\) where \( F > 0 \) is fulfilled and those in which \( F < 0 \). We believe that the allowed values for \( \alpha \) are just those such that the GSL holds throughout the expansion of the Universe. The upper bound on \( \alpha \) is given by the local minimum of the dashed curve, that is \( \alpha = 4e^3 \), and the lower bound by the intersection of the dotted curve with the line \( x = 1 \) (i.e., when horizon area equals Planck’s area), that is \( \alpha = -1/4 \) (not shown). Positive values of \( \alpha \) seem to be largely favored, which is also consistent with some quantum calculations of the entropy corrections [11].

By using Clausius relation, instead, and adopting the above defined variable \( x \), as seen in the right panel of Fig.1, the GSL is satisfied, \( (F(\alpha, x) \geq 0) \), for \( \alpha > -1/(4x) \).

3.2. Power-law entropy correction

Before applying the GSL let us look at the Friedmann equations for power-law entropy correction. Inspection of eq.(2) shows that the values of \( \alpha = 0 \) and \( \alpha = 2 \) are special, in the sense that for \( \alpha = 0 \) there are no entropy corrections and the equations reduce to the corresponding general relativity expressions with a cosmological constant. Likewise \( \alpha = 2 \) represents just a renormalisation of Newton constant, \( G \). Bearing this in mind, we start the analysis by introducing the dimensionless variable \( x = (r_c H)^{-1} \), and identifying the crossover scale \( r_c \) with \( H_0^{-1} \). Thus \( x \) tends to zero in the far past and its today value is \( x_0 = 1 \) (provided, again, that \( \dot{H} < 0 \)).

Performing a similar analysis as before of the sign of the \( F \) function and on the energy density, as found in [10], constraints on the \( \alpha \) parameter follow. These are shown in Fig. 2. In the case of Clausius relation, the GSL remains valid throughout the expansion so long as \(-2 \leq \alpha < 2 \) while equipartition of energy allows a wider range for the parameter \( \alpha \) which can now lie in the range \( 3 \leq \alpha < 4 \), as well.
Figure 1. Plot of the sign of $\mathcal{F}$ depending on $\alpha$ along the Universe expansion. On the left panel the case of the logarithmic correction and equipartition principle is shown. The right panel corresponds to the case of using Clausius relation instead. The plots are restricted to the range $0 < x < 0.1$ but in the remaining region the curves behave monotonically.

Figure 2. Plots of the sign of $\mathcal{F}$ depending on $\alpha$ along the Universe expansion. On the left panel the case of the power-law correction and the equipartition principle is shown. The right panel depicts the case when Clausius’ relation is used instead.

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