Pricing and Advance Ordering Strategy for Prefabricated Building Assembler

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Abstract: This paper studies an advance order strategy for prefabricated building assembler. We study how advance strategy affect assembler’s profit, and assembler how to choose strategy to meet consumer demand. In contrast to much of the literature, we introduce time sensitivity of consumer into the demand function, and divide prefabricated components into standard and nonstandard components to highlight the characteristics of prefabricated building. Furthermore, we present two different order strategies. In one strategy, assembler orders all components once customer’s order arrives in one time, consumer have longer waiting time. In another strategy, assembler orders standard components in advance and orders nonstandard components until consumers’ order arrives. Our results show that assembler can make decision in different situations to achieve optimal profit, and satisfy consumer demand.

1. Introduction

Construction energy conservation is one of the most important part of today’s environmental protection [1]. Compared with traditional cast-in-place buildings, prefabricated buildings have the advantage of improving the quality of constructions and low building-energy consumption [2]. Thus, the demand of prefabricated building is on the increase. In the traditional ordering model, assembler orders all components from PM until consumer’s order arrives. Assembler is at risk of losing customers due to long lead times from the manufacturer in this situation. In order to address these issues, we investigate the pricing and ordering strategy of assembler in prefabricated building supply chain management.

Two research streams are closely related to this paper and will be reviewed to highlight our contributions. The first stream related to our work is the literature on the advance order strategies. Cai et al. presented two main different advance order strategies, and found that one time order strategy was easier to control but had more strict constraint on coefficients of variation [3]. Viswanath et al. found that supplier could benefit from providing adequate pricing incentives to entice buyers to commit to purchase quantities before demand information is revealed [4]. The literature abovementioned didn’t consider the decisions of assembler. Our works consider the assemblers’ advance order decisions in prefabricated supply chain. The second stream related to our work is the literature on pricing and ordering strategy of assembler. Zhai et al. presented two Stackelberg game models and an equal power model to obtain the optimal PLTH amount and crashing money [5]. Jiang et al. investigates the profit distribution model of a supply chain with cap-and-trade policy. And found out that the profits are increasing in pure competition, competition, and pure cooperation scenarios [6]. The literature above mentioned laid the foundation for our study but did not consider the standard components’ pre-order and inventory. In order to fill the gap presented by the literature review, this paper investigates the pricing and ordering strategy of assembler in prefabricated building supply chain management.
2. Model Descriptions and Assumptions

We study the pricing and advance ordering strategy for prefabricated buildings assembler, and assume that assembler has two ordering strategies, i.e. ordering all components after costumers’ order arrives or ordering standard components in advance and ordering nonstandard components after costumers’ order arrives. Ordering standard components in advance can reduce the delivery time to consumers, and PM will charge less wholesaling price to encourage assembler ordering components in advance ($\omega_1 > \omega_2$), but it will generate stock-holding cost while waiting for nonstandard components arriving. The notations of the parameters and variables in this paper are presented in Table 1.

| Notations | Descriptions |
|-----------|--------------|
| $\pi_a$   | Assembler’s profit |
| $p$       | Unit price of the prefabricated building |
| $D$       | Costumers’ demand |
| $\omega_i$| PM’s wholesaling price about all components of one prefabricated building, $i=1, 2$. $\omega_1 > \omega_2$ |
| $t_i$     | Initial PM’s components delivery time, $i=1, 2$. $t_1 > t_2$ |
| $t$       | Assembler’s required time of components delivery |
| $\alpha_s$| The proportion of standard components in all components. $\alpha_s + \alpha_{ns} = 1$ |
| $\alpha_{ns}$| The proportion of nonstandard components in all components. $\alpha_s + \alpha_{ns} = 1$ |
| $c_{ss}$  | Unit price of standard components stock-holding costs per unit time |
| $a$       | Initial demand of consumer |
| $b$       | Self-price sensitivity of consumer |
| $\lambda$| Self-time sensitivity of consumer |
| $\delta$ | Cost coefficient of delivery time |

Referring to previous research by Khanra [7], we assume that the demand function is price and time dependent. That is $D(p,t) = a - bp + \lambda(t_1 - t)$ ($a, b, \lambda > 0$), where $bp$ is the reduced demand due to an increase in price and $\lambda(t_1 - t)$ is the increased demand due to a reduce in time. We assume that assembler can require the PM to shorten the delivery time, but the PM will charge a certain crashing cost from assembler, which is only dependent on the time of delivery. That is $M(t) = \frac{1}{2}\delta(t_1 - t)^2$ ($t_1 > 0$), the larger $\delta$ means the compression cost higher per unit time.

3. Basic Model

In this section, we assume that assembler orders all components until consumer’ order arrives. And we ignore the assembler’s assembling time. Assembler could delivering construction to consumers once the components arriving. Based on the model assumptions, the assembler’s profit, denoted by $\pi_a(p,t)$, is:

$$\pi_a(p,t) = (p - \omega_1)(a - bp + \lambda(t_1 - t)) - \frac{1}{2}\delta(t_1 - t)^2$$

The first term is the profit from selling constructions. The second term is the crashing money about requiring PM to shorten the delivery time.

**Proposition 1** The optimal retail price, the optimal components delivery required time of assembler and the optimal profit of assembler, denoted by $p_a$, $t_a$ and $\pi_{a_{\text{max}}}$ are:

$$p_a = \frac{a \delta + \omega_1(b \delta - \lambda^2)}{2b \delta - \lambda^2 - \lambda b \omega_1}$$

(1)

$$t_a = t_1 - \frac{\lambda a - \lambda b \omega_1}{2b \delta - \lambda^2}$$

(2)

$$\pi_{a_{\text{max}}} = \frac{\delta(a - b \omega_1)^2}{4b \delta - 2 \lambda^2}$$

(3)

**Proof.** We can obtain the first order condition of $\pi_a(p,t): \frac{\partial \pi_a(p,t)}{\partial p} = a - bp + \lambda(t_1 - t) - b(p - \omega_1)$ and $\frac{\partial \pi_a(p,t)}{\partial t} = -\lambda(p - \omega_1) + \delta(t_1 - t)$. In order to make the Hessian Matrix in the basic model
be negatively definite, we assume \( \delta > \frac{\lambda^2}{2b} \).

According to (1), we have
\[
\frac{\delta^2 \pi_a(p,t)}{\partial p^2} = -2b < 0, \quad \frac{\partial^2 \pi_a(p,t)}{\partial t^2} = \delta < 0 \quad \text{and} \quad \frac{\partial^2 \pi_a(p,t)}{\partial p \partial t} = \frac{\partial^2 \pi_a(p,t)}{\partial t \partial p} = -\lambda.
\]

Thus, we obtain
\[
\frac{\partial^2 \pi_a(p,t)}{\partial p^2} = \frac{\partial^2 \pi_a(p,t)}{\partial p} = \frac{\partial^2 \pi_a(p,t)}{\partial t} = \frac{\partial^2 \pi_a(p,t)}{\partial t^2} = 2b\delta - \lambda^2 > 0.
\]

Then, we can get that \( \pi_a(p,t) \) is jointly concave in \( p \) and \( t \). And we obtain the optimal profit of assembler by plugging optimal \( p \) and \( t \) into assembler’s profit equation.

Proposition 1 shows that assembler’s optimal retail price and components delivery time uniquely exist on basic model. According to the given cost coefficient of delivery time, assembler can optimize operational decisions to improve their own economic and environmental performances.

**Proposition 2** We find out that price and the optimal profit have positive correlation with the cost coefficient of delivery time, while the PM’s required time of components delivery has negative correlation with it in basic model.

Proof. According to equation (1), (2), (3), we take the derivative of \( \pi_a \), \( t_a \) and \( \pi_{a_{\max}} \) respectively. (\( \alpha_i < p \), \( i=1, 2 \). And \( a - bp > 0 \), then \( a - bw_i > 0 \).

\[
\frac{\partial \pi_a}{\partial \delta} = -\frac{\lambda^2(a - bw_2)}{(2b\delta - \lambda^2)^2} < 0
\]

\[
\frac{\partial t_a}{\partial \delta} = -\frac{2b\lambda(a - bw_1)}{(2b\delta - \lambda^2)} > 0
\]

\[
\frac{\partial \pi_{a_{\max}}}{\partial \delta} = -\frac{2\lambda^2(a - bw_1)^2}{(4b\delta - 2\lambda^2)^2} < 0
\]

Proposition 2 shows that assembler can optimize the price of prefabricated building and their profit by negotiating with PM to reduce the cost coefficient of delivery time. And in this situation, the PM can reduce the crashing cost because they have much more time to produce prefabricated components.

4. **The Advance Order Model**

In this section, we assume that assembler orders standard components in advance and orders nonstandard components until consumers’ order arrives. Same as basic model, we also ignore the assembler’s assembling time. Based on the model assumptions, the assembler’s profit, denoted by \( \pi_b(p,t) \), is:

\[
\pi_b(p,t) = (p - \omega_2)(a - bp + \lambda(t_2 - t)) - \frac{1}{2}\delta(t_2 - t)^2 - c_{ss}t_2\alpha_s(a - bp + \lambda(t_2 - t))
\]

The first term is the profit from selling buildings. The second term is the crashing money about requiring PM to shorten the delivery time. The third term is the stock-holding cost of standard components.

**Proposition 3** The optimal retail price, the optimal components delivery time of assembler and the optimal profit of assembler, denoted by \( p_b \) and \( t_b \), are:

\[
p_b = \frac{a\delta + (b\delta - \lambda^2)(\omega_2 + c_{ss}t_2\alpha_s)}{2b\delta - \lambda^2} \tag{4}
\]

\[
t_b = t_2 - \frac{\lambda a - \lambda b(\omega_2 + c_{ss}t_2\alpha_s)}{2b\delta - \lambda^2} \tag{5}
\]

\[
\pi_{b_{\max}} = \delta(a - bp + c_{ss}t_2\alpha_s) - \frac{1}{4b\delta - 2\lambda^2} \tag{6}
\]

Proof. We can obtain the first order condition of \( \pi_b(p,t) \):
\[
\frac{\partial \pi_b(p,t)}{\partial p} = a - bp + \lambda(t_2 - t) - b(p - \omega_2) + b c_{ss}t_2\alpha_s \quad \text{and} \quad \frac{\partial \pi_b(p,t)}{\partial t} = -\lambda(p - \omega_2) + \delta(t_2 - t) + \lambda c_{ss}t_2\alpha_s.
\]

In order to make the Hessian Matrix in the basic model be negatively definite, we assume \( \delta > \frac{\lambda^2}{2b} \).

According to (3), we have
\[
\frac{\partial^2 \pi_b(p,t)}{\partial p^2} = -2b < 0, \quad \frac{\partial^2 \pi_b(p,t)}{\partial t^2} = -\delta < 0 \quad \text{and} \quad \frac{\partial^2 \pi_b(p,t)}{\partial p \partial t} = \frac{\partial^2 \pi_b(p,t)}{\partial t \partial p} = \frac{\partial^2 \pi_b(p,t)}{\partial p \partial t} = \frac{\partial^2 \pi_b(p,t)}{\partial t \partial p} = -\lambda.
\]
$$-\lambda < 0. \text{ Thus, we obtain } \frac{\partial^2 \pi_b(p,t)}{\partial p^2} \frac{\partial^2 \pi_b(p,t)}{\partial \delta^2} = 2b\delta - \lambda^2 > 0. \text{ Then, we can get that } \pi_b(p,t) \text{ is jointly concave in } p \text{ and } t. \text{ And we obtain the optimal profit of assembler by plugging optimal } p \text{ and } t \text{ into assembler’s profit equation.}

Proposition 3 shows that assembler’s optimal retail price and components delivery time uniquely exist on advance order model. According to the given cost coefficient of delivery time, assembler can optimize operational decisions to improve their own economic and environmental performances.

**Proposition 4** We find out that price and the optimal profit have positive correlation with the cost coefficient of delivery time, while the PM’s required time of components delivery has negative correlation with it in advance order model.

**Proof.** According to equation (4), (5), (6), we take the derivative of \( p_a \), \( t_a \) and \( \pi_{\text{max}}^a \) respectively. (We assume that \( p - \omega_i \geq c_{ss}t_i \), \( i=1,2 \). Which means that the holding cost of a construction won’t higher than the sale price minus whole sale price of it. And \( 0 < \alpha_s < 1 \), then \( p > \omega_i + c_{ss}t_2\alpha_s \), and \( \omega_i < p, \ a - bp > 0 \), then \( a - b(\omega_i + c_{ss}t_2\alpha_s) > 0, i=1,2. \))

\[
\frac{\partial p_b}{\partial \delta} = -\frac{\lambda^2(a - b(\omega_2 + c_{ss}t_2\alpha_s))}{(2b\delta - \lambda^2)^2} < 0
\]
\[
\frac{\partial t_b}{\partial \delta} = 2b\lambda(a - b(\omega_2 + c_{ss}t_2\alpha_s)) > 0
\]
\[
\frac{\partial \pi_{\text{max}}^b}{\partial \delta} = -\frac{2\lambda^2(a - b(\omega_2 + c_{ss}t_2\alpha_s))^2}{(2b\delta - \lambda^2)^2} < 0
\]

Proposition 4 shows that assembler can also increase the price of prefabricated building and their own profit by negotiating with PM to reduce the cost coefficient of delivery time while they ordering prefabricated components in advance. And like basic model, the PM can reduce the crashing cost because they have much more time to produce prefabricated component.

5. **Impact of Advance Ordering to Consumer**

In this section, we discuss the change of prefabricated buildings’ optimal retail price and the required time of components delivery by assemblers between two situations.

**Proposition 5** Compared to basic model, using advance order strategy will cause higher retail price \( (p_b > p_a) \) and narrow the gap of optimal delivery time in two situations \( (t_a - t_b < t_1 - t_2) \) while \( \omega_2 - \omega_1 + c_{ss}t_2\alpha_s > 0. \) And it will cause lower retail price \( (p_b < p_a) \) and expand the gap of optimal delivery time in two situations \( (t_a - t_b > t_1 - t_2) \) while \( \omega_2 - \omega_1 + c_{ss}t_2\alpha_s < 0. \)

**Proof.** We compared the optimal retail price and delivery time in different situations and got: \( p_b - p_a = \frac{(b\delta - \lambda^2)(\omega_2 - \omega_1 + c_{ss}t_2\alpha_s)}{2b\delta - \lambda^2} \) and \( t_a - t_b = t_1 - t_2 - \frac{\lambda b(\omega_2 - \omega_1 + c_{ss}t_2\alpha_s)}{2b\delta - \lambda^2}. \)

Proposition 5 shows that assembler can choose best strategy to satisfy consumer demand according to the prefabricated components’ wholesale price given by PM and inventory cost.

6. **Impact of Advance Ordering to Assembler**

In this section, we discuss the impact of advance ordering, and find the optimal decision of assembler in different situation.

**Proposition 6** The advance ordering strategy should be adopted when the relationship among \( a, b \) and \( \omega_i \) satisfies the following equation:

\[
a \geq \frac{\omega_1 + \omega_2 + c_{ss}t_2\alpha_s}{2}
\]

**Proof.** We can obtain the optimal profit of assembler in both models by plugging optimal \( p \) and \( t \) into assembler’s profit equation. After adopting the advance ordering strategy, the optimal profit of assembler should be improved than without. On this account, \( \pi_{\text{max}}^b \geq \pi_{\text{max}}^a \) should be held. \( \pi_{\text{max}}^a = \)
\[
\frac{\delta(a-b\omega_1)^2}{4b\delta-2\lambda^2} < \pi_{\text{max}} = \frac{\delta(a-b(\omega_2+c_2\alpha_2\alpha_3))}{4b\delta-2\lambda^2}.
\]
And after simplification, we get:
\[
\frac{a}{b} \geq \frac{\omega_1+\omega_2+c_2\alpha_2\alpha_3}{2}.
\]

Proposition 6 shows that the assembler should order prefabricated components in advance while the advance ordering cost is low and the customer is insensitive to the price.

7. Conclusion and Future Research

The great development of the construction industry has caused a sharp rise in the consumption of environmental resources, and it has potential to exceed other industries to consume the greatest resources. With the government and general public paying more and more attention to environmental protection, the demand of prefabricated building is increasing. We propose a prefabricated building assembler’s advance ordering mode. First, we formulate the basic model without advance order. We derive the optimal price and required time of components delivery of assembler. Second, we formulate the advance order model, and derive the optimal decisions of assembler. We show that price and the optimal profit have positive correlation with the cost coefficient of delivery time, while the PM’s required time of components delivery has negative correlation with it. Finally, we investigate the impact of advance ordering to consumer and assembler.

This paper study a deterministic demand which is dependent on price and time. Actually, the consumer demand might be indeterminate. This kind demand has been formulated in many related literature. Therefore, one of the future directions is considering indeterminate demand. This will be more realistic. In addition, we ignore the manufacture time of assembler. Thus, the other direction of the paper is to consider the manufacture time of assembler, and it will influence the assembler’s ordering strategy.

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