Comment on ”Critical behavior of the Pauli spin susceptibility...”
by A. A. Shashkin et al., cond-mat/0409100

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The paper by A. A. Shashkin et al. reports measurements of the thermodynamic magnetization of two-dimensional electrons in silicon. Although the experimental data is very similar to that reported by us \[1\] more then two years ago, the authors arrive at an opposite conclusion regarding the spin susceptibility “critical behavior” and spin instability in the vicinity of the metal-insulator transition. We show that this interpretation is based on a flawed analysis of the experimental data.

The state of strongly correlated fermions at low temperatures, and in particular its magnetic properties, that has been a subject of active interest, is still poorly understood. Some theories predict that in the absence of disorder exchange effects can lead to spontaneous magnetization, but such instability has never been observed, see \[1\] for references. Several years ago we proposed a novel technique for measuring the thermodynamic magnetization of 2d electrons and applied it to a high mobility silicon MOSFET. Notwithstanding the enhanced spin susceptibility observed in the experiment, the magnetization vanished smoothly as the magnetic field was reduced, thus excluding the possibility of a spin instability at zero field.

Ref. \[2\] repeats our measurements and reproduces the data, extending it to 20% lower carrier concentration. Albeit the similarity in the data, the conclusion of Ref. \[2\] is opposite to ours. Based on extrapolation from strong fields authors infer magnetic instability at zero field. In what follows we show how this erroneous conclusion emerges from a wrong assumption.

The experimental method is described in Prus et al. \[1\]. It yields directly \(\partial \mu / \partial B\) which by the Maxwell relation equals \(-\partial M / \partial n\), with \(\mu, n\) the electron chemical potential and density, and \(M, B\) the magnetization and magnetic field. Fig. 2 in Ref. \[2\] presents \(\partial \mu / \partial B\) for \(B\) between 1.5 T and 7 T. The curves \[2\] for \(B = 1.5, 7\) T are reproduced as \(\partial M / \partial n\) by solid lines in Fig. 1. In Ref. \[2\] the vanishing of \(\partial \mu / \partial B\) is identified with the full spin polarization. The corresponding \(B\) values plotted versus density (Fig. 4a in Ref. \[2\]) align along a straight line that, when extrapolated to \(B = 0\), crosses the density axis at a finite \(n\) close to the critical density \(n_c(0)\) below which the system becomes strongly localized at \(B = 0\). From such an extrapolation Ref. \[2\] concludes that the crossing indicates spontaneous spin polarization at zero field.

We first note that the association of \(\partial \mu / \partial B = 0\) with full spin polarization is baseless. According to the Maxwell relation, full spin polarization would occur at \(\partial \mu / \partial B = -\partial M / \partial n = -\mu_B\), where \(\mu_B\) is the Bohr magneton. In contrast, \(\partial M / \partial n = 0\) occurs at the maximal magnetization rather then at the full one, and we denote the corresponding density \(n_m\). Fig. 2a, taken from \[1\], depicts spin magnetization vs. density obtained by integration of \(\partial M / \partial n\) data. We note that the density \(n_m(B)\) is always smaller than \(n_c(B)\), and the magnetization at this density, \(M(B, n_m(B))\), is always below the full one, \(\mu_B n_m(B)\).

The degree to which the assumption of full spin polarization at \(n_m\) is invalid is also evident directly from the data of Ref. \[2\]. The dependence \(M(n_m)\) can be obtained by integration: \(M(n_m) = \int_0^{n_m} \partial M / \partial n\). Since the measured \(\partial M / \partial n\) (Fig. 1) never approaches \(\mu_B\), the maximal magnetization is significantly lower than the full one, and therefore the susceptibility determined in \[2\] as \(\mu_B n_m(B) / B\) and plotted in fig. 4b of \[2\] is also significantly overestimated.

We also remark on the risk of drawing conclusions at \(B = 0\) based on extrapolation from high \(B\). It is quite easy to fall into the trap of arguing for instability at low \(B\) by attempting to judge on the faith of the maximal magnetization or the magnetization at \(n_c\) by the behavior at high field. However, as clearly seen in Fig. 2b, both go smoothly to zero with vanishing magnetic field.
FIG. 2: (a) Spin magnetization as a function of density at different magnetic fields and temperatures 0.2, 0.8, 2.5 and 4.2 K; higher magnetization corresponds to lower temperature. Critical densities, $n_c$, are marked by circles. Thick blue line - full magnetization, thick red line - magnetization of a degenerate ideal electron gas at $B = 6$ T. (b) Maximal spin magnetization and spin magnetization at the critical densities plotted against magnetic field. Dashed line - extrapolation from high magnetic fields (reproduced from [1]).

Finally we note that one should not ignore the diamagnetic contribution to $M$. This contribution can be roughly estimated from the experimental data by assuming that the maximal magnetization at high fields, e.g. 7 T, approaches $1 \mu_B$ per electron. To obtain such value from Fig. 2 the curve should be shifted by approximately $0.7 \mu_B$. This will also increase $\partial M/\partial n$ for 1.5 T, keeping it still well below the full spin polarization value (see dashed lines in Fig. 2). This approximation for the diamagnetic shift is supported by theoretical calculations [1].

In conclusion, the data presented in Ref. 2, properly analyzed, indicates that the 20% reduction in the accessible density compared to the earlier studied samples does not lead to spin instability. Such an instability, anticipated by theory, may still occur at a much lower density.

[1] O. Prus, Y. Yaish, M. Reznikov, U. Sivan and V. Pudalov, Phys. Rev. B 67, 205407(2003), cond-mat/0209142.
[2] A. A. Shashkin, S. Anissimova, M. R. Sakr, S. V. Kravchenko, V. T. Dolgopolov and T. M. Klapwijk, cond-mat/0409100.
[3] For parabolic confining potential $U(x) = m^* \omega_0^2 x^2 / 2$, in the presence of parallel magnetic field with the cyclotron frequency $\omega_c$, the electron energy is $E_0 = \frac{1}{2} \hbar (\omega_0^2 + \omega_c^2)^{1/2}$. Expanding $(\omega_0^2 + \omega_c^2)^{1/2} = \omega_0 + \omega_c^2/(2\omega_0)$, with $\partial \omega_c/\partial B = e/m^* c$, the ground state energy field dependence is

$$\frac{\partial E_0}{\partial B} \approx \frac{(m/m^*)}{(\omega_c/\omega_0)} \times \mu_B$$

The diamagnetic contribution to the energy of spatial quantization is linear in $B$, as expected. For the chemical potential set by $E_0$ we obtain $\partial \mu/\partial B \approx \partial E_0/\partial B$. The ratio $\omega_c/\omega_0$ can be estimated as $(d/\lambda)^2$, with $d$ the 2DEG thickness and $\lambda$ the magnetic length. For $d \approx 30 \lambda$ and $B = 10$ T the diamagnetic contribution is comparable to $\mu_B$ and thus cannot be ignored.