Forecasting a sharp change in traffic intensity by analysis of the third derivative of growth function of incoming requests

V Y Goichman¹, A K Levakov², N A Sokolov³, M V Nikolaeva⁴

¹ SOTSBI R&D Ltd, 14, Pestel street, Saint-Petersburg, 191028, Russia
² Center Macro-regional Branch of PJSC Rostelecom, 22, Kievskoe shosse, Moscow, 123298, Russia
³ LO ZNIIS, 11, Varshavskaya, Saint-Petersburg, 196128, Russia
⁴ Institute of oil and gas problems SB RAS, 1, Oktyabrskaya street, Yakutsk, 677000, Russia
E-mail: mv.nikolaeva@s-vfu.ru

Abstract. The article proposes a method for predicting sudden changes in the intensity of traffic entering a switching node. It is based on monitoring the nature of the input process third derivative, which is usually specified by the number of incoming requests per unit of time. If obtaining an analytical expression for the function of incoming requests is not possible, then the change in the finite differences of the third order should be controlled. The method used is an example of interdisciplinary approach, since an analogue of the third derivative in kinematics is the process called “jerk”. The proposed approach allows predicting a sharp increase in traffic and making decisions to support an acceptable quality of service. In particular, the control of the third derivative can be used in conjunction with the procedures of hysteresis control and algorithms for restricting traffic, which increases dramatically with the response of telecommunication network subscribers to extraordinary events.

1. Introduction

The traffic of modern multi-service networks has one significant feature - a sharp and sometimes very significant change in intensity, which distinguishes it from the well-studied traffic of telephone and telegraph networks. In networks similar processes are observed only in case of emergencies, as well as on certain holidays. This feature of modern networks traffic makes it difficult to predict the behavior of traffic, despite the fact that the possibility of implementing a reliable forecast would effectively use existing network resources, balance the load on the network, manage queues and ensure efficient network operation.

The importance of predicting traffic is confirmed by the fact that the list of works on predicting network traffic and building predictive models is quite extensive. It can be distinguished as the most popular models of autoregression and an integrated moving average. However, it should be noted that despite the objective advantages, these models have several serious drawbacks, since their implementation requires an analysis of the time series state and a constant calculation of the model parameters, which requires significant computational resources and additional memory for storing these states. It should be noted that at present more attention is being paid to using neural networks for prediction.

Considering that in traffic management processes, short-term forecasting methods are most in demand, it can be stated that for such processes the following most important are: comparative
simplicity, automation of the forecasting process and the ability to quickly respond to the changes. This article proposes a method for short-term forecasting of the behavior of telecommunications traffic, based on monitoring its intensity.

2. Initial data
It is known that in telecommunication networks there are periods when the intensity of the incoming flow of requests (the general concept for calls and IP packets), usually denoted as $\lambda(t)$, increases dramatically. The behavior of the function $\lambda(t)$ is monitored by measuring its values in short periods of time $\tau$. Then it is appropriate to write this function through its Laplace-Stieltjes transformation in the following form [1]:

$$\hat{\lambda}(s) = \sum_{i=0}^{N} h_i e^{-is}\tau.$$  

(1)

The value $h_i$ determines the change in the function $\lambda(t)$ fixed at the point $i\tau$. The summation limit $N$ indicates the total number of measured $h_i$ values. Within the limits of the interval $[i\tau, (i+1)\tau]$ the change in the function $\lambda(t)$ can be neglected. Based on this statement the numerical value of $\tau$ is chosen.

The sharp increase in the intensity of applications incoming flow in the telecommunications network is similar to a “jerk” in kinematics [2]. If we use this analogy, then in order to estimate the possible sharp changes in the function $\lambda(t)$, we should analyze the nature of its third derivative - $\lambda^{(3)}(t)$. The third derivative of the function $\lambda(t)$ can be obtained by various methods. One of the simplest methods is to approximate the observed process by a polynomial of degree $m$ in the range of changes in the function equal to $K\tau$. Obviously, $K \leq N$. The value of $K$ is chosen so that in the range of $K\tau$ the growth of the function $\lambda(t)$ is observed:

$$\lambda(t) \approx \sum_{j=0}^{m} a_j t^j.$$  

(2)

The values of the coefficients $a_j$ and the summation limit $m$ are calculated by the method of least squares [3]. Figure 1 illustrates the proposed approach. The nature of the change in the function $\lambda(t)$ is conditionally chosen. It allows to select three ranges of the form $K\tau – (t_0, t_1)$, $(t_2, t_3)$ and $(t_4, t_5)$. For solving practical problems, the most interesting is the period of time, which is characterized by the sharpest growth of the function $\lambda(t)$. In this example it becomes the range $(t_4, t_5)$.
Practically significant task is to predict the nature of the function \( \lambda(t) \) behavior, which requires the use of algorithm for limiting the number of applications arriving at the input of the switching node (SN) or taking other measures. It is appropriate to start developing a method for solving the set task by considering a hypothetical situation for the range \( (t_4, t_5) \).

3. The studied method

Let us suppose that the function \( \lambda(t) \) on a time interval \( (t_4, t_5) \) can be represented by one of the four continuous curves \( f_i(t) \):

\[
\begin{align*}
    f_1(t) &= a_0 + a_1 t, \\
    f_2(t) &= a_0 + a_1 t + a_2 t^2, \\
    f_3(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3, \\
    f_4(t) &= a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4. 
\end{align*}
\]

(3)

The third derivative of each function characterizes a "jerk", which determines the possible change in traffic:

\[
\frac{\partial^3 f_i(t)}{\partial t^3} = 0, \quad \frac{\partial^3 f_2(t)}{\partial t^3} = 0, \quad \frac{\partial^3 f_3(t)}{\partial t^3} = 6a_3, \quad \frac{\partial^3 f_4(t)}{\partial t^3} = 6a_3 + 24a_4 t. 
\]

(4)

The growth of the third derivative is inherent only in the function \( f_4(t) \). It means that if the increase in traffic is characterized by a polynomial with a fourth-degree term or higher, then an overload of the SN is possible. Here we should focus on the following circumstance: the growth of the third derivative, fixed at the moment of time \( i\tau \), can stop at the point \( (i+1)\tau \), and then resume. An important problem is the decision-making algorithm that should be considered as a subject of independent research.

In practice it is not always possible to successfully approximate the function \( \lambda(t) \) by a polynomial. It is simpler to analyze finite differences [4], that is data obtained from the values of \( h_i \), which are measured with a period \( \tau \). This approach eliminates errors due to approximation. The degree of influence of such errors on the final result is very difficult to assess.

For example, six samples of the function \( f(t) \), taken with an interval \( \tau \), form the following series: 0, 2, 7, 20, 99, 1000. From this range it is easy to calculate the finite differences of three orders. The results of the calculation are presented in the form of a diagonal table of differences (Table 1).
Table 1. Diagonal table of differences

|   |   |   |   |   |
|---|---|---|---|---|
| **t** | **f(t)** | **Δf(t)** | **Δ^3f(t)** | **Δ^4f(t)** |
| 0  | 0  | 2 | 5 | 5 |
| τ  | 2  | 5 | 8 | 58 |
| 2τ | 7  | 13| 66| 846|
| 3τ | 20 | 79| 912| |
| 4τ | 99 | 991| | |
| 5τ | 1000| | | |

The Δ^3f(t) values, which are finite third-order differences, form an increasing sequence. Such character of the obtained sequence, in a certain sense, is equivalent to the growth of the third derivative. It allows to conclude that there is a steady increase in the quantity being studied. Thus, the condition when a sharp increase in traffic is likely can be formulated as follows:

\[ b_1 \leq b_2 \leq \ldots \leq b_i, \quad b_i < b_{i+1}. \]  \( \text{(5)} \)

In this inequality \( b_i \), where \( i = 1, 2, \ldots, l \), determines the numerical value of the finite difference of the third order, and \( l \) is the number of members of the series obtained as a result of processing the initial data. The inequality \( b_i < b_{i+1} \) emphasizes the following requirement: some neighboring terms of expression (5) may be identical, but in general, the sequence \( b_1, b_2, \ldots, b_l \) should increase.

4. Results

The idea of using third order finite differences to estimate traffic growth was formed as a result of a search for solutions to similar problems in other disciplines. In particular, a discussion of such questions was found in papers on kinematics. In other words, an interdisciplinary approach was used [5], which is rightfully considered one of the areas of promising research. From this point of view, it is appropriate to mention the class of problems where finite differences or derivatives of the fourth, fifth and sixth order are used.

The calculation of third-order finite differences is of interest for the development of forecasting methods used in the study of telecommunication systems [6]. To create different profiles of packet traffic, the proposed approach was used to implement a traffic generator [7]. It forms a stream of IP packets with arbitrary characteristics that have practical meaning. In fact, the traffic generator for a certain \( i \)-th experiment creates a flow of \( \lambda(t) \) applications in accordance with the Tukey-Huber model [8]:

\[ \lambda(t) = (1 - \varepsilon_i) \lambda_i(t) + \varepsilon_i \xi_i(t). \]  \( \text{(6)} \)

The factor \( \lambda_i(t) \) allows to describe the expected behavior of the process under study. The value of \( \varepsilon_i \) determines the probability describing a sharp increase in traffic is carried out according to the law \( \xi_i(t) \). For the function \( \xi_i(t) \) that various properties of finite differences of the third order are interesting. As a result, the traffic generator allows to experimentally determine the ability of a switching node or telecommunications network to cope with a significant increase in traffic.
5. Conclusion
The growth of the third derivative of the function that characterizes the increase in traffic intensity seems to be a logical criterion for detecting congestion in individual components of telecommunication networks. The proposed approach, based on the calculation of third-order finite differences, can be used in combination with other methods of estimating the growth of traffic intensity and telecommunication network control algorithms. In particular, its combined use with hysteresis control algorithms [9-12] seems promising.

Thus, it can be argued that the development of the approach proposed in this article will allow to solve a number of important practical problems that relate to the technical operation of modern communication networks. In particular, similar algorithms have been proposed to control the avalanche-like traffic arising in emergency situations.

References
[1] Ditkin V A, Prudnikov A P 1974 Integral transformations and operational calculus (Moscow: Science)
[2] Kirsanov M N 2015 Solving problems on theoretical mechanics (Moscow: INFRA-M)
[3] Samara A A, Gulin A V 1989 Numerical methods (Moscow: Science)
[4] Gelfond A O 1959 Calculus of finite differences (Moscow: State Publishing House of Physics and Mathematics)
[5] Moiseev N N 2003 Selected Works. Interdisciplinary research of global problems. Publicism and social issues (Moscow: Taydeks Ko)
[6] Vanston L K, Hodges R L 2004 Technology forecasting for telecommunications Telektronikk, 100(4) pp 32-42
[7] Goichman V, Esalov K, Sokolov N 2016 Using specialized computer systems to study the characteristics of telecommunication networks Proceedings of the FRUCT’18 Saint-Petersburg, Russia, 18-22 April (Saint-Petersburg: ITMO University) pp 456 - 462
[8] Huber P 1981 Robust Statistics (New York: Wiley)
[9] Krasnoselsky M A, Pokrovsky A V 1983 Systems with hysteresis (Moscow: Nauka)
[10] Dudin A N 2002 Optimal hysteresis control of an unreliable BMAP/SM/1 system with two modes of operation Automatics and telemechanics 10 58-72
[11] Levakov A K 2013 Aspects of preventive communication network preparation for work after an emergency Telecommunications 4 42-44
[12] Levakov A, Sokolov N 2012 Access to Emergency services during overload traffic period 12th International Conference "Internet of Things, Smart Spaces, and Next Generation Networking" (Saint Petersburg: Springer) pp 424-428.