Comment on the vortices of the early phase of the ELMs

F. Spineanu and M. Vlad
National Institute of Laser, Plasma and Radiation Physics
Magurele, 077125 Bucharest, Romania

Abstract

We discuss a possible perspective on the process of transition from the layer of sheared poloidal rotation in the H-mode to a set of discrete filaments that concentrate vorticity and current density. They may be precursors of the Edge Localized Modes.

The transport barrier that exists in the H mode regime in tokamak has frequently been identified in experimental observations with a layer of sheared poloidal rotation. There are theoretical basis for this. It has been shown that, in the presence of sheared poloidal rotation, the linear eigenmodes are shifted relative to the resonant surface such that their ability to extract the free energy from background gradients (i.e. $\gamma$) is diminished. Moreover, the sheared poloidal flow tears apart the radially elongated eddies of the drift wave instability and reduces the radial correlation length of the drift wave turbulence. Since the diffusion coefficient depends on the square of the radial extension of fluctuations it results a substantial reduction of the transport. A transport barrier is generated, in a narrow layer limited by the Last Closed Flux Surface.

The origin of the sheared poloidal rotation is still unclear. The loss of fast ions from the NBI, the Reynolds stress transferred by turbulent fields, the particular dependence of the parallel viscosity on rotation are possible sources; or of a radial (polarization) electric field [1]. The growth of the poloidal rotation has two difficulties to overcome: one is the inertia factor, approximatively $(1 + 2q^2)$ and the other is the damping due to the magnetic pumping. Then, if the transport barrier consists of a sheared poloidal rotation, there should be a drive with sufficient magnitude to create and then to maintain it.

In experiments it is systematically observed toroidal rotation and there is a natural tendency to look for a connection with the enhanced regimes. However the sheared poloidal rotation is much more efficient in suppressing the radial transport than the sheared toroidal rotation. On the contrary, the sheared toroidal rotation can be a source of turbulence. We will consider here that the transport barrier is due to sheared poloidal variation.
The shear of the poloidal velocity is the dependence of $v_\theta$ on the radial coordinate, $\partial v_\theta / \partial r$. At the edge this is the dominant term in

$$\nabla \times \mathbf{v} \big|_{z=1} = \frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) = \frac{v_\theta}{r} + \frac{\partial v_\theta}{\partial r}$$

where $z$ is the toroidal direction. This means that the velocity shear is the $z$ vorticity $\omega_z \approx \frac{\partial v_\theta}{\partial r}$. The formulation in terms of vorticity provides a wide range of connections. In general the vorticity is a field that has a tendency to self-organization [2], [3]. In a 2D fluid there is attraction between like-sign vortical elements, there are vortex coalescences into larger and larger vortices. There is also separation and clusterization of vorticities of opposite signs in different regions of the plane. These have been observed in experiments on neutral (Euler) fluids and have been explained theoretically. It has also been observed in non-neutral plasma and in numerical simulations of 2D plasma in strong magnetic field. Without intending to transport all this body of knowledge to tokamak plasma, we can be guided in the exploration of H mode and of Edge Localized Modes (ELM).

Adopting the vorticity as a basic analytical instrument of description, we can refer to some known results from MHD. In numerical simulations of 2D MHD it has been found that the concentration of vorticity and the concentration of the current density occur together and that asymptotically the extremum of these two fields coincide. A monopolar vortex that concentrates the initial vorticity field coincides with the maximum of the current density. This already should be seen as a suggestion that, in the H mode, the narrow layer of sheared poloidal rotation (i.e. a layer of vorticity) must also be a layer of concentration of the current density. This must be seen as independent of the connection which is usually made between the strong pressure gradient (due to the transport barrier) and the bootstrap current. The current in the layer can be different of the bootstrap value since the accumulation of current density has also a dependence on local vorticity. Numerical simulations of 2D MHD have clearly shown that, if initialized in separate regions on the plane, the current and the vorticity evolve to become overlapped and further they do not separate anymore.

We now have to examine the situation of a layer of vorticity and current, under strong transversal gradients of density and temperature. The barrier of transport exists due to this sheared rotation but this has only suppressed the convective transport that is usually realized by turbulent eddies on finite radial distances. The conductive transport remain possible although it is much smaller. We are here in a configuration analogous to the Rayleigh Benard experiment. As in that case, if the gradient of temperature/pressure increases the convective plumes that always exist from the internal surface toward the external limit of the layer begin to organize themselves into large scale convective pattern, the cells. In our case this means formation of filaments almost parallel to the magnetic field line which break up the rotation layer (and implicitly the barrier of transport). The filaments consists of vorticity and current density. This phase
should be seen as the basis for the further evolution according to the *peeling balloonning* instability.

The transition from a layer of sheared rotation to a discrete set of vortices is a well known phenomenon in various areas.

In the physics of atmosphere, the ring-type shape of vorticity distribution in a tropical cyclone is sometimes broken and transformed into a set of vortices placed symmetrically on the circle \( [4] \). In non-neutral plasma this is observed for the electron density distribution. In fluid systems, the Kelvin Helmholtz instability creates in a layer of sheared flow discrete centers of vorticity concentration that grow and finally break up the layer.

To the vorticity concentration we must add the tendency of the layer of current density (practically the transport barrier is a current sheet) to generate concentration in a set of filaments of current.

The universal prototype for transition from a sheared layer to a set of concentrated filaments of vorticity and current is the nonlinear evolution of a Chaplygin gas with anomalous polytropic. It has been examined in detail and illustrated with many examples by Trubnikov [5]. One of the applications is the break up of a layer of density and of current in a set of filaments [6].

The geometry adopted by Trubnikov is adequate for studying the tearing of the density distribution in the layer. The width (along \( y \)) is uniform initially \( L_0 \) and it evolves to a profile \( L \) which is variable along the direction \( x \) of the layer. The coordinate \( y \) is perpendicular on the layer in the equilibrium position. We can see this as \( y \) *radial* and \( x \) *poloidal*.

The magnetic field has a shear \( B = B_z (y) = -B_0 \tanh \left( \frac{y}{\tau} \right) \) where \( B_0 \) is the main magnetic field, along \( z \). The current density is

\[
j_z = en (v_{iz} - v_{ez})
\]

Then

\[
v_{iz} - v_{ez} = \frac{cB_0}{2\pi enL (t, x)} = \frac{\text{const}}{nL}
\]

It is introduced a normalized density of plasma \( \rho (t, x) = \frac{n(t, x)}{n_0 L_0} \) and we have the usual density conservation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0
\]

Under the assumption \( v_{e,th} < v_{ez} \) we have the equation of motion

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = 1 \frac{\partial}{\partial x} (j_z B_y) = \frac{e}{m_i c} (v_{iz} - v_{ez}) \frac{\partial A}{\partial x}
\]

We consider that the system is invariant along the \( z \) direction which means that the generalized momenta of the electrons and of ions are conserved \( m_i v_{iz} + \frac{e}{\tilde{e}} A = \text{const} \) and \( m_e v_{ez} - \frac{e}{\tilde{e}} A = \text{const}' \). The equation of motion becomes

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{e}{m_i c} (v_{iz} - v_{ez}) \frac{\partial A}{\partial x} = c_0^2 \frac{1}{\rho^2} \frac{\partial \rho}{\partial x}
\]

3
Breaking of the density layer into discrete structures

Figure 1: Trubnikov solution showing the filamentation as function of time.

The constant is \( c_0^2 \) is a constant. The two equations are

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0
\]
\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = c_0^2 \frac{1}{\rho^3} \frac{\partial \rho}{\partial x}
\]

Obviously this is like a "gas with anomalous polytropic" , which means that a perturbation accumulates more and more gas, with formation of separated maxima. The equations are solved using a hodograph transformation. The solution is determined by Trubnikov and consists of a set of maxima (corresponding to filaments) that result from breaking the initially uniform distribution of density in the layer of current. The solution shows the dynamical process of replacement of the layer with the discrete filaments.

This is a useful example where the H mode barrier, which is a layer of vorticity and current density, can be broken and replaced by discrete filaments. It would be interesting to see how this dynamical process evolve to become the peeling-balloonning instability. It is suggested that the early phase of the ELM consists of the formation of filaments or vortices where the vorticity and the current density are concentrated.

**Note.** This is part of a work presented at the meeting "EFDA-TG Transport. 2nd meeting JET Culham 16 - 18 September 2009"
References

[1] Burrell et al., Phys. Plasmas 12 (2005) 056121.

[2] F. Spineanu and M. Vlad, Phys. Rev. E 67, 046309, 1-4.

[3] F. Spineanu and M. Vlad, The asymptotic quasi-stationary states of the two-dimensional magnetically confined plasma and of the planetary atmosphere, [http://arXiv.org/physics/0501020](http://arXiv.org/physics/0501020).

[4] J.P. Kossin and W.H. Schubert, J. Atmos. Sci. 58, 2196 (2001).

[5] B.A. Trubnikov and S. K. Zhdanov, Phys. Rep. 155 (1987) 137.

[6] S. V. Bulanov and P.V. Sasorov, Sov. J. Plasma Phys. 4 (1978) 418.