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NEUTRINO DEGENERACY EFFECT ON NEUTRINO OSCILLATIONS
AND PRIMORDIAL HELIUM YIELD

(To my grandmother Perunika)
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Abstract

In this work we present the results of our numerical analysis of the effect of lepton asymmetries $L$ on neutrino oscillations and consequently on helium production in a model of cosmological nucleosynthesis with active-sterile neutrino oscillations. We performed detailed calculations of the neutron-proton ratio till its freezing and the subsequent synthesis of helium-4 for a wide range of values of the initial lepton asymmetry $10^{-4} - 10^{-10}$ and for the complete set of mixing parameters of the oscillation model described in [1]: $\delta m^2 \leq 10^{-7} \text{ eV}^2$ and any $\theta$.

We have found that there are significant modifications of neutrino number densities and energy distributions over a large range of values of the initial asymmetry. Hence, the previously derived (for the case of lepton asymmetry of the order of the baryon one) nucleosynthesis limits on the oscillation parameters are considerably altered. We have calculated the dependence of primordially produced helium-4 on the parameters of the model $Y_p(L, \delta m^2, \theta)$. From the isohelium contours in the $\delta m^2 - \theta$ plane, we have obtained limits on the neutrino mixing parameters corresponding to nucleosynthesis with degenerate neutrinos. An intriguing new result of our analysis is that the lepton asymmetry is able to tighten the nucleosynthesis bounds on neutrino mixing parameters, besides its well known ability to relax them.
I. INTRODUCTION

Lepton asymmetry is not directly observable. Besides, in general, large lepton numbers $L$ bigger than the baryon number $B$ ($L > B$) and even $L \gg B$ are not excluded by any profound theoretical principle.

The origin of eventual big lepton numbers is not fully understood. There exist numerous different mechanisms for their generation. Large lepton numbers can be consistent with the observed small baryon number for example in the context of Grand unification, as pointed out in [2]; there exist different models producing $L \gg B$: in a scenario of baryogenesis with baryonic charge condensate [3], where large and varying lepton asymmetry can be produced, the out-of-equilibrium decays of heavy Majorana neutrinos [4,5] can produce it as well, resonant neutrino oscillations can create considerable neutrino asymmetry at low temperature [6,1]. See also the review papers [7,8].

Besides, large lepton asymmetry has interesting implications in the early Universe: in baryogenesis models [9,4,10], for solving the monopole problem [11,12], as well as the domain wall problem [12], in primordial nucleosynthesis [13–15] for structure formation in the Universe [16], relic asymmetry can suppress neutrino transitions in the early Universe and have interesting implications for the solar neutrino problem, as well as in cosmological nucleosynthesis (CN) [17].

Therefore, in this work we consider it worthwhile to explore precisely the effect of lepton asymmetries and to study the problem of oscillations and their influence on primordial production of helium-4 in the presence of such asymmetries. Especially, in the present work we would like to show that much smaller asymmetries ($|L| < 0.1$) still can considerably affect cosmological nucleosynthesis, although indirectly through its effect on neutrino oscillations. Thus, the presence of an initial asymmetry bigger than the baryon one may considerably change the bounds on oscillation parameters obtained from primordial nucleosynthesis considerations.

The theme of lepton asymmetry in nucleosynthesis models with oscillations was previously considered in several publications [18–20,17,1,21,6]. The possibility of changing the nucleosynthesis constraints on lepton asymmetry due to oscillations was discussed within models with great $L \sim 0.1$ [20]. The dynamical evolution of $L$ due to oscillations was studied for some particular values of $L$ and neutrino mixing parameters [18,19]. The possibility for loosening the nucleosynthesis bounds on mixing parameters due to the suppression of oscillations by great enough $L$ was revealed [17]. Also the possibility of generation of asymmetry due to resonant oscillations was studied [1,6], and some scenarios where oscillations generate asymmetry which further suppresses oscillations and as a result alleviates nucleosynthesis bounds on neutrino mixing were discussed. However, usually the simplifying assumption that the evolution of the neutrino ensemble follows the average neutrino momentum was made. The nonresonant case was considered just for a certain set of parameters and for not very large initial $L$ it was shown that neutrino asymmetry adjusts dynamically to zero [19].

In this work we consider the case of nonresonant oscillations $\delta m^2 > 0$ with small mass differences for a wide range of the initial values of $L$, namely $10^{-10} - 1$, and provide a precise numerical study for the most unexplored part of it: $10^{-7} \lesssim |L| \lesssim 10^{-4}$. We insist here for the correct account for the spectral spread of neutrino, which we find to play an important role in the oscillation-asymmetry interplay, and to be decisive for the realization of low temperature resonance in $\delta m^2 > 0$ case.**

We find that even small asymmetries $|L| > 10^{-7}$ may considerably effect nucleosynthesis (contrary to the conclusions of the previous works) and that asymmetry is also able to enhance oscillations and consequently to strengthen the nucleosynthesis bounds on neutrino mixing parameters for concrete regions.

**Mind that, as was shown in the pioneer work [19], in models working with the mean momentum only, the resonance does not have place as far as actually the asymmetry calculated in this approximation reaches zero at resonance.
in the parameter space \((L, \delta m^2, \theta)\), besides suppressing oscillations and thus loosening the bounds on mixing parameters from nucleosynthesis. The analysis of the asymmetry effect is provided using the precise kinetic approach to the problem of neutrino oscillations and asymmetry, described in [1,21]. Actually, the present work is an extension of our previous study [1,21] for the case of a lepton asymmetry bigger than the baryon one. There we have calculated the bounds on the neutrino mixing parameters for a \(\nu_e \leftrightarrow \nu_s\) mixing scheme with small mass differences \(\delta m^2 \leq 10^{-7}\) eV\(^2\), where \(\nu_e\) is the active electron neutrino, while \(\nu_s\) denotes the sterile antineutrino. This case corresponds to nonequilibrium oscillations between electron neutrinos \(\nu_e\) and sterile neutrinos \(\nu_s\) when the latter do not thermalize till \(\nu_e\) decoupling at 2 MeV and oscillations become effective after \(\nu_e\) decoupling. We discussed the special case of nonequilibrium oscillations between weak interacting electron neutrinos and sterile neutrinos for small mass differences \(\delta m^2\), as far as the case of large \(\delta m^2\) was already studied (both concerning small asymmetries \(L \sim B\) [22-27] and bigger ones \(L > B\) [19,17,6]). Due to the correct kinetic approach accounting for neutrino depletion, neutrino spectrum distortion and neutrino asymmetry for each neutrino momentum, we have obtained more precise bounds (an order of magnitude stronger than the existing previously) to the neutrino oscillation parameters. In the present work we will show how these bounds are changed due to bigger lepton asymmetries.

We would like to emphasize that the qualitatively new result, namely that small asymmetries may enhance oscillations and consequently to strengthen the nucleosynthesis bounds, was revealed only due to the correct approach accounting for the spectral spread of neutrino momenta and energy spectrum distortion due to oscillations, advocated in our previous papers [1,21,28] and in the pioneer work of Dolgov [29]. Other studies missed this possibility as far as there it was assumed that neutrino ensemble follows the behavior of average momentum even in the presence of oscillations and asymmetry.

In this work we explore the different ways in which lepton asymmetry may affect oscillations (and vice versa - oscillations affect the asymmetry) and primordial nucleosynthesis. We have defined numerically the concrete \(L\) ranges where these effects hold, namely: (a) big enough \(L\) - total suppression of oscillations corresponding to degenerate nucleosynthesis, without oscillations (b) resonance region - \(L\) enhancing oscillations and leading to an overproduction of He-4, and (c) small \(L\) - negligible effect.

The paper is organized as follows. In the following Section II we discuss neutrino degeneracy. In Section III we present the model of nonequilibrium neutrino oscillations and analyze the neutrino evolution, using kinetic equations for the neutrino density matrix for each momentum mode. In Section IV we investigate \(\nu_e \leftrightarrow \nu_s\) the primordial production of helium in the presence of oscillations and neutrino degeneracy. The results and conclusions are presented in Section V.

II. LEPTON ASYMMETRY - PRELIMINARIES

At the epoch of interest - just prior to cosmological nucleosynthesis - at temperatures around few MeV, the lepton asymmetry is expressed through the asymmetries in the neutrino sector and the electron sector:

\[
L = \sum_{\ell} (N_{\ell} - N_{\bar{\ell}})/N_{\gamma},
\]

where \(\ell = e, \nu_e, \nu_\mu, \nu_\tau\), and \(\bar{\ell}\) corresponds to their antiparticles. Big lepton asymmetry \(L > B\) may be contained only in the neutrino sector, as far as the electron-positron asymmetry must be equal to the proton-antiproton asymmetry from the charge conservation requirements. Hence, it will be more correct to talk about neutrino asymmetry or neutrino degeneracy, as it is most often referred to. It is convenient instead of \((N_\nu - N_\bar{\nu})/N_{\gamma}\) to use the so called neutrino degeneracy parameter, \(\xi = \mu/T\), where \(\mu\) is the corresponding neutrino chemical potential. In the thermal equilibrium of the early Universe at \(T = 2\)
MeV, the neutrino degeneracy for the case $\mu/T \ll 1$ is given by

$$L = \sum_i (N_{\nu_i} - N_{\bar{\nu}_i})/N_\gamma \sim \sum_i (\pi^2/12\zeta(3))(T_{\nu_i}/T_\gamma)^3 \xi_i$$

where $i = e, \mu, \tau$.

There are no experimental or direct theoretical limitations for the magnitude or the sign of neutrino asymmetries. However, there exist indirect limits on the neutrino degeneracies in the different neutrino flavors, obtained on the basis of astrophysical and cosmological considerations. Such are the constraints obtained from the present age and density of the Universe [30]: $\xi \leq 86$ for the degeneracy in only one neutrino species; from the requirement for long enough matter dominated period, necessary for successful structure formation: $\xi \leq 6.9$, and the strongest bound on the magnitude of $L$ is obtained from primordial nucleosynthesis considerations [13,14,31]: $-0.06 \leq \xi_\nu \leq 1.1$ and $|\xi_{\bar{\nu}}| \leq 6.9$ for baryon to photon ratio $\eta \leq 1.9 \times 10^{-10}$.

As far as primordial nucleosynthesis is particularly sensitive both to the neutrino degeneracies and to neutrino oscillations, it is interesting to consider the effect of neutrino degeneracies in a modified model of primordial nucleosynthesis with neutrino oscillations.

In this work we assume no degeneracy of muon and tau neutrinos for simplicity. However, this assumption is not essential. The results can be easily rescaled for the general case of asymmetry in all sectors. We consider the effect of neutrino asymmetries in the wide diapason of its initial values $10^{-10} - 1$. Both positive and negative values of $L$ are considered. Asymmetries smaller than $10^{-10}$ have negligible effect, therefore we have not studied them. Now we have found that small asymmetries with magnitude $|L| < 10^{-7}$ have negligible effect\(^\dagger\) also for the concrete mixing parameters of our model. We have proved that asymmetries greater than $10^{-4}$ completely suppress the oscillations and affect the nucleosynthesis due to the pure direct effects of neutrino degeneracy on nucleosynthesis - through affecting the expansion rate and the weak reaction rates. The latter effects have been studied already in the numerous publications on degeneracy and nucleosynthesis [13,14,31], therefore, we have not provided detailed numerical calculations for such big asymmetries. While asymmetries in the intermediate region, namely $10^{-7} < |L| < 10^{-4}$ have not been studied systematically.

In this work we prove that such asymmetries can both enhance and suppress oscillations depending on the concrete set of mixing parameters. For that range we have provided a detailed numerical analysis of the effect of the neutrino degeneracy on neutrino evolution and consequently on helium-4 production. Finally (as described in the following sections) we have obtained the isohelium contours for different $L$ values in the $(\delta m^2, \theta)$ plane and derived new limits on the neutrino mixing parameters for the cases with neutrino degeneracy.

III. EVOLUTION OF THE NEUTRINO ENSEMBLES IN THE PRESENCE OF INITIAL NEUTRINO DEGENERACY

A. Neutrino oscillations - the model

We suppose the existence of a sterile neutrino ($SU(2)$-singlet) $\nu_s$, and would like to explore the cosmological effect of nonresonant neutrino oscillations $\nu_e \leftrightarrow \nu_s$ on the primordial nucleosynthesis for a wide range of values of the initial lepton asymmetry.

Within the model of interest oscillations proceed effectively after the active neutrino decoupling and

\[^\dagger\]This confirmed our preliminary results stated in [21].
till then the sterile neutrinos have not yet thermalized\textsuperscript{11} so that their number density is negligible in comparison with the electron neutrino one. The model is described in detail in\cite{1,21}.

Oscillations between $\nu_s$ ($\nu_s \equiv \bar{\nu}_l$) and the active neutrinos proceed according to the Majorana\texttt{-}Dirac (M\&D) mixing scheme [32]. For simplicity mixing present just in the electron sector is assumed $\nu_l = U_{li} \nu_i$, $l = e, \mu$

\begin{align*}
\nu_1 &= c\nu_e + s\nu_s, \\
\nu_2 &= -s\nu_e + c\nu_s,
\end{align*}

where $\nu_s$ denotes the sterile electron antineutrino, $c = \cos(\theta)$, $s = \sin(\theta)$ and $\theta$ is the mixing angle in the electron sector, the mass eigenstates $\nu_1$ and $\nu_2$ are Majorana particles with masses correspondingly $m_1$ and $m_2$. We consider the nonresonant case $\delta m^2 = m_2^2 - m_1^2 > 0$, which corresponds in the small mixing angle limit to a sterile neutrino heavier than the active one. We would like to remind, that ‘nonresonant’ is used in the sense that it does not allow resonant transitions due to the finite temperature correction term (nonlocal term). However, as it is obvious, for the low temperature resonance this terminology is misleading. As far as for any sign of $\delta m^2$ then there is a possibility for a resonance due to the local term either in the neutrino sector or in the antineutrino sector.

B. The kinetics of nonequilibrium neutrino oscillations

The kinetic equations for the density matrix of the nonequilibrium oscillating neutrinos in the primeval plasma of the Universe in the epoch previous to nucleosynthesis, i.e. consisting of photons, neutrinos, electrons, nucleons, and the corresponding antiparticles, have the form:

\[ \frac{\partial \rho(t)}{\partial t} = H_p \frac{\partial \rho(t)}{\partial p} + i [H_\rho, \rho(t)] + i [H_{\text{int}}, \rho(t)], \]

where $p$ is the momentum of electron neutrino and $\rho$ is the density matrix of the massive Majorana neutrinos in momentum space.

These equations account simultaneously for the participation of neutrinos into expansion, oscillations and interactions with the medium. The first term in the equation describes the effect of expansion, the second is responsible for oscillations, the third accounts for forward neutrino scattering off the medium. The collisions (second order interactions in $G_F$ of neutrinos with the medium) are neglected, as far as we consider the case of oscillations effective after neutrino decoupling.

$H_\rho$ is the free neutrino Hamiltonian in the eigenstate basis:

\[ H_\rho = \begin{pmatrix} \sqrt{p^2 + m_1^2} & 0 \\ 0 & \sqrt{p^2 + m_2^2} \end{pmatrix}, \]

while $H_{\text{int}} = \alpha V$ is the interaction Hamiltonian, where $\alpha_{ij} = U_{ei}^* U_{ej}$, $V = \sqrt{2}G_F (+L - Q/M_W^2) N_\nu$.

The first ‘local’ term in $V$ is proportional to the fermion asymmetry of the plasma and for $L > B$ is essentially expressed through the neutrino asymmetry:

\[ \mathcal{L} \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}. \]

\textsuperscript{11}This assumption of nonthermalization of steriles till electron neutrino decoupling and the late effectiveness of oscillations limits the allowed range of oscillation parameters for the discussed model: $\sin^2(2\theta)\delta m^2 \leq 10^{-1}$ eV$^2$ [1]. However, it allowed us to examine the interesting nonequilibrium case of neutrino oscillations, in general with non Fermi-Dirac distribution, which is realized at such small mass differences. And besides, it allowed us to neglect the second order in $G_F$ terms in the equations governing the neutrino evolution, and helps simplifying the calculation process. (For the nonequilibrium case studied, this is important, as far as accounting for the neutrino spectrum distortion for that case is obligatory, and it needs much longer calculation time.)
The second ‘nonlocal’ term is momentum dependent $Q \sim E_\nu T$ [33,22]. When lepton asymmetries of the order of the baryon one are considered, the nonlocal term dominates at high temperature, while with cooling of the Universe in the process of expansion the local one becomes more important [22,19,18]. For greater asymmetries $L > B$ the role of the nonlocal term decreases in comparison with the local one, and usually for temperatures less than a few MeV it is neglected. We have precisely accounted for it in this work, having in mind the following. Due to the momentum dependence of the nonlocal term even when for the mean neutrino momentum modes at a given temperature this term is smaller than the local one, for the high energetic modes it may still play some role. We have checked numerically that the nonlocal term can change from a few up to about 14\% the results on helium-4 production. The maximum effect being, as can be easily guessed, in the case of maximum mixing. This is noticeable.

Analogues equations hold for the antineutrino density matrix, the only difference being in the sign of the fermion asymmetry: $L$ is replaced by $-L$.

Medium terms depend on neutrino energy density and neutrino degeneracy, thus introducing a nonlinear feedback mechanism. As was already observed in previous works [17,1,19], oscillations change neutrino-antineutrino asymmetry and it in turn affects oscillations. For large $L$ the evolution of neutrino and antineutrino ensembles is strongly coupled and hence, it must be considered simultaneously.

Equation (1) results into a set of coupled nonlinear integro-differential equations with time dependent coefficients for the components of the density matrix of neutrino. Due to the greater $L$ the coupling is stronger than for the case $L = 10^{-10}$. As far as for these strongly coupled nonlinear equations an analytic solution is hardly possible, we have provided an exact kinetic analysis of the neutrino evolution by a numerical integration of the kinetic equations for the neutrino density matrix for each momentum mode. The evolution of the neutrino ensembles is followed numerically from the $\nu_e$ freezing at 2 MeV till the formation of helium-4.

Our numerical analysis shows that the coupling between the neutrino and antineutrino ensembles leads to their similar behavior: Whenever the resonance condition is fulfilled for neutrino (or antineutrino), and the ensemble suffers a resonant oscillations, due to the strong coupling between the systems, the antineutrino ensemble shows the same behavior (after some negligible delay time) too.\footnote{Opposite to the traditional naive conclusions, based on observations just for certain parameter values and provided for the mean neutrino momentum, that in the low temperature resonance case (when the nonlocal term is neglected in comparison with the local one) the resonance may take place either in the neutrino sector or in the antineutrino one, but not both [19].} The results have almost negligible dependence on the sign of the initial asymmetry, as could be expected from this behavior of the ensembles. The concrete ‘spectral’ mechanism of that strong coupling between the ensembles in our case is described in detail further on.

The initial condition for the neutrino ensembles in the interaction basis is assumed of the form:

$$\rho = n_\nu^{eq} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

where $n_\nu^{eq} = \exp(\xi - E_\nu / T)/(1 + \exp(\xi - E_\nu / T))$.

It corresponds to an equilibrium distribution of active electron neutrinos, and an absence of the sterile one. We have analyzed the evolution of nonequilibrium oscillating neutrinos by numerically integrating the kinetic equations for the period after the electron neutrino decoupling till the freeze out of the neutron-proton ratio ($n/p$-ratio), i.e. for the temperature interval from 2 MeV till 0.3 MeV. We have explored the problem using the Simpson method for integration and the fourth order Runge-Kutta algorithm for the solution of the differential equations. In order to keep the appropriate precision of calculation for the wide range of initial parameter values, we used a step for the evolution with temperature decrease of the order $10^{-7} - 10^{-6}$ MeV and for the calculation of the neutrino spectrum a step for $E_\nu / T$ of the order $10^{-2} - 10^{-3}$. 

\footnote{Opposite to the traditional naive conclusions, based on observations just for certain parameter values and provided for the mean neutrino momentum, that in the low temperature resonance case (when the nonlocal term is neglected in comparison with the local one) the resonance may take place either in the neutrino sector or in the antineutrino one, but not both [19].}
The oscillation parameters range studied is $\delta m^2 \in [10^{-11}, 10^{-7}]$ eV$^2$ and $\theta \in [0, \pi/4]$, which is the full set of mixing parameters in the discussed model [21] of oscillations. The neutrino degeneracy was varied for a wide range $10^{-10} - 10^{-4}$ of $L_\nu$, as discussed in the previous section.

The distributions of electrons and positrons were taken the equilibrium ones. The neutron and proton number densities, used in the kinetic equations for neutrinos, were substituted from the numerical calculations in CN code accounting for neutrino oscillations. I.e. we have simultaneously solved the equations governing the evolution of neutrino ensembles and those describing the evolution of the nucleons (see the next section). The baryon asymmetry $\beta$, parameterized as the ratio of the baryon number density to the photon number density, was taken to be $3 \times 10^{-10}$.

IV. NUCLEOSYNTHESIS WITH NONEQUILIBRIUM OSCILLATING NEUTRINOS

We have calculated the production of helium-4 in a detailed model of primordial nucleosynthesis, accounting for the direct kinetic effects of oscillations and neutrino degeneracy on the neutron-to-proton transitions. The effect of oscillations on helium-4 has been discussed in numerous publications [34-36,18-28]. We follow the approach described in [1,21]. The numerical analysis is performed working with exact kinetic equations for the nucleon number densities and neutrino density matrix in momentum space. This enabled us to analyze the direct influence of oscillations onto the kinetics of the neutron-to-proton transfers and to account precisely for the neutrino population depletion, distortion of the neutrino spectrum and the role and dynamical evolution of neutrino-antineutrino asymmetry.

The master equation, describing the evolution of the neutron number density in momentum space $n_n$ for the case of oscillating neutrinos $\nu_e \leftrightarrow \nu_s$, reads:

$$
\left(\frac{\partial n_n}{\partial t}\right) = H p_n \left(\frac{\partial n_n}{\partial p_n}\right) + \\
+ \int d\Omega(e^-, p, \nu) [A(e^- p \rightarrow \nu n)]^2 \\
\times [n_e - n_p (1 - g_{ee}) - n_n g_{ee} (1 - n_e)] \\
- \int d\Omega(e^+ p, \nu) [A(e^+ n \rightarrow \nu \bar{n})]^2 \\
\times [n_e + n_n (1 - g_{ee}) - n_p g_{ee} (1 - n_e^+)].
$$

We have calculated the neutron-to-photon ratio evolution prior to the nucleosynthesis epoch, namely for the temperature interval $T \in [0.3, 2.0]$ MeV and for the full range of oscillation parameters of our model and $L$ in the range $10^{-10} - 10^{-4}$. The value of $g_{ee}(E_\nu/T)$ at each integration step was taken from the simultaneously performed integration of the set of equations for neutrinos, i.e. the evolution of neutrino and the nucleons was followed self consistently.

The initial values at $T = 2$ MeV for the neutron, proton and electron number densities are their equilibrium values. The parameters values of the CN model, adopted in our calculations, are the following: the mean neutron lifetime is $\tau = 887$ sec, which corresponds to the present weighted average value, the effective number of relativistic flavor types of neutrinos during the nucleosynthesis epoch $N_\nu$ is assumed equal to the standard value 3.

On the basis of this analysis the primordially produced He-4 value was obtained as a function of neutrino degeneracy parameter and neutrino mixing parameters.

V. RESULTS AND CONCLUSIONS

We have calculated the evolution of the neutron-to-nucleon number densities $X_n(t) = N_n(t)/(N_p + N_n)$ for each set of values in $(L, \delta m^2, \theta)$ parameter space. In Figs. 1 the evolution of $X_n$ is illustrated for initial asymmetry of the order of the baryon one and for $L = 10^{-5}$ and $L = 10^{-5}$. As is seen from
the figures, even relatively small asymmetry \( (L = 10^{-5}) \) can considerably effect the neutron-to nucleon ratio. In some cases, i.e. for maximal mixing (Fig. 1a) leading to underproduction of helium-4, while in others (Fig. 1b) to its overproduction in comparison with helium yields calculated in models with small asymmetry \( (L \sim 10^{-10}) \).

In Fig. 2 the dependence of the frozen neutron number density relative to nucleons \( X_n^F (L) \) on the initial neutrino asymmetry for \( \sin^2 (2\theta) = 10^{-0.05} \) and different fixed \( \delta m^2 \) is illustrated. The dependence of the frozen neutron number density relative to nucleons \( X_n^F (\theta) \) on the mixing angle for different fixed \( \delta m^2 \), is presented in Fig. 3.

It is interesting to note, that, as illustrated in Fig. 3 depending on the mixing angle, it is possible to weaken or to strengthen the limits in comparison with the model of nucleosynthesis with oscillations without a considerable lepton asymmetry. Namely, for large mixing, the presence of neutrino degeneracy of the order \( 10^{-6} \), for example, leads to less strong constraints on the mixing parameters, weakening the oscillation effect of overproducing helium. While for smaller mixing angles the same asymmetry leads to stronger limits on mixing parameters, as a result of the enhancement of oscillations, and hence increasing the helium overproduction.

These results can be understood as follows: As it was shown in [21] in case of baryon like small lepton asymmetry the kinetic effects (neutrino population depletion and distortion of neutrino spectrum) due to oscillations play an important role and lead to a considerable overproduction of helium.

In the presence of a bigger asymmetry both the neutrino depletion and spectral distortion are changed due to the interconnections between oscillations and asymmetry. Large enough asymmetries (in our case \( L > 10^{-7} \)) affect oscillations, while strong enough oscillations also influence asymmetry and lead to its dynamical evolution. We would like to note, that even for relatively small \( |L| < 10^{-5} \) the effect of asymmetry is considerable (contrary to the thought before), and also that even oscillations with small mass differences are able to influence neutrino asymmetry. A characteristic situation in our model is that great asymmetries cannot be created: neutrino asymmetries typically of an order of magnitude greater than the initial one are generated, however they are rapidly oscillating, the average value being zero. In our case of nonresonant oscillations, the presence of the second system (for antineutrino) hinders the continuous growth of asymmetry. In more detail asymmetry-oscillation relations will be analyzed in a following paper. Our main purpose here was the study of asymmetry effect on primordial production of helium-4, and derivation of the nucleosynthesis limits on neutrino mixing in the presence of neutrino degeneracy.

For the oscillation model discussed we have found three interesting regions for the range of the lepton asymmetry. To be more clear let us consider nearly maximal mixing: \( \sin^2 (2\theta) = 10^{-0.05} \) (see Fig. 2 for illustration). Then these regions are as follows.

A. In the range \( |L_\nu| \geq 5 \times 10^{-5} \) the asymmetry fully suppresses neutrino oscillations.*** The yield of helium-4 coincides with the values obtained in the models of primordial nucleosynthesis without oscillations. Therefore, the nucleosynthesis bounds on the mixing parameters in the presence of such large asymmetries are waved away.

For \( |L| \leq 10^{-2} \) the direct kinetic effect of such asymmetries on the neutron-to-proton transfer is almost negligible, i.e. for the range \( 10^{-4} - 10^{-2} \) the only role of asymmetry is to suppress oscillations. Asymmetries greater than that effect nucleosynthesis by directly changing the kinetics of nucleons and are exhaustively studied in previous works [13,14,31]. And the effect of the asymmetry is well known: Neutrino degeneracy effects element production in two ways [13]. First, the nonzero chemical potential results to an increase of neutrino energy density and hence to an increase of the expansion rate of the

***This result is in accordance with the estimations made in [17], where it was shown that a big neutrino asymmetry may lead to suppression of oscillations.
Universe thus allowing less time for the nuclear reactions to proceed. Second, it alters the reaction rates governing the neutron-proton transitions, as far as these rates are extremely sensitive to the neutrino spectrum. Namely, electron neutrino degeneracy results into preponderance of protons, hence into a drop of primordial helium-4 abundance, while electron antineutrino degeneracy leads to the increase of neutrons. Therefore for $L > 10^{-4}$ we have not provided detailed numerical calculations.

B. The range of smaller than $10^{-4}$ asymmetries was more appealing to us as far as it was totally unexplored till now. Therefore, we numerically analyzed the problem for the initial values of the neutrino asymmetry in the range $10^{-10} - 10^{-4}$, and for the full mixing parameter space of the active-sterile oscillations model, described in [1], i.e. for all mass differences $\delta m^2 \leq 10^{-7}$ eV$^2$ and mixing angles $\theta$.

The zone $10^{-7} \leq |L| \leq 5 \times 10^{-6}$ is the most interesting one. The asymmetry with magnitudes of that order is strong enough to influence neutrino oscillations. And although the neutrino asymmetry is too small to effect nucleosynthesis directly, it does effect nucleosynthesis yields in an indirect way through its influence on the neutrino oscillations pattern. The asymmetry values in that range, depending on the concrete values of mixing parameters, are able to enhance oscillations. So, depending on the mixing angle for some $L$ in this interval the ensemble of neutrinos expires a resonance. For nearly maximal mixing it is roughly around $L \sim 10^{-6}$ (Fig. 2). This enhancement of oscillations is big enough to influence considerably the electron neutrino and electron antineutrino number density and spectrum i.e. it leads to an enhanced depletion of the number densities of neutrinos and antineutrinos(!) and a decrease in their mean energy, as well as nontrivial evolution of the asymmetry itself. Hence influencing the helium-4 primordial value, which is extremely sensitive to it. The result is overproduction of helium-4 in comparison with the case of primordial nucleosynthesis in the presence of oscillations with negligibly small $L$. This in its turn leads to a strengthening of the nucleosynthesis bounds on the neutrino mixing parameters for that certain set of parameters $(L, \delta m^2, \theta)$.

The ability of $L$ within that range to enhance the oscillations looks quite amazing at a first glance. The naive picture one expects as a result of increasing the initial $L$ is a gradual suppression of oscillations proportional to $L$. As far as $L$ value, calculated for the mean neutrino momentum is by orders of magnitude bigger than the necessary one for a resonance transfer $L \gg L_r$.

However, the detailed numerical analysis, accounting for the momentum spread of neutrino ensemble, showed a more complex picture. Varying $L$ for fixed mixing parameters we have observed a resonance region, i.e. an enhancement of oscillations, as seen in Fig. 2 and Fig. 3. And besides that, when a resonance for a given $L$ value was observed for neutrino it was followed with a negligible time delay by a resonance in antineutrino! This leads to a considerable depletion of number densities and to further overproduction of helium-4. These amazing results can be explained as follows.

The system of nonlinear differential equations cannot be solved analytically without radical assumptions. However, the qualitative behavior of the ensembles and the obtained results concerning helium-4 can be guessed from the following simplified considerations.

The resonant conditions for the neutrino looks like [36]:

$$\cos(2\theta)(\delta m^2/2E_\nu) = \sqrt{2}G_F(L - Q/M_W^2)N_\gamma$$

while for antineutrino the sign before the $L$ term is the opposite one.

For the mean neutrino momenta $\bar{p} = 3.15 T$ in our model

$$|L| \gg Q/M_W^2$$

$$|L| \gg \cos(2\theta)\delta m^2/(2\sqrt{2}E_\nu G_F N_\gamma).$$

Now let us consider the possibility that at a given temperature, for neutrino of a given momentum $p < \bar{p}$ the resonance condition is fulfilled, i.e. $L$ has the resonance value $L_r(p)$. Then neutrinos with this momentum suffer a resonant transfer, leading to a decrease in the number densities of neutrinos (in favour of the sterile neutrinos). As far as for antineutrinos $L$ has the opposite sign, oscillations remain
suppressed, and the number densities of antineutrinos do not change. Hence, the net result is a decrease in \( L \) due to this resonant transition. This decrease makes possible the fulfillment of the resonant condition for more energetic neutrinos, leading to further decrease of \( L \) and so on; due to this ‘resonance wave’ passing to neutrinos with higher and higher momenta, the neutrino number densities expire a considerable depletion, consequently, having in mind the small initial values of the neutrino asymmetry \( (\delta N_\nu \sim L_\nu N_\nu) \) soon this resonant wave leads to a change in the sign of \( L \). This suppresses further resonant transfer for neutrinos, however, for the antineutrino ensemble now there appears the possibility for a resonance. So, the same process follows in the antineutrino ensemble. The only difference being that contrary to the neutrino system, for antineutrinos the resonance is first fulfilled for the high energy neutrinos and then passes to the low energetic ones - i.e. it reaches more rapidly neutrinos with mean momentum, and the process is more avalanche like. This rapid decrease of antineutrinos leads to a rapid bump of \( L \) which again becomes positive. This pendulum like process proceeds relatively fast, the oscillation of the asymmetry proceeds much faster than the temperature decrease. The total effect is that the resonant transfer both for neutrino and antineutrino system is realized even for a considerable initial asymmetry values (nonresonant ones, when calculated for the mean neutrino momentum), and due to the enhanced transfer of active to sterile neutrinos a considerable ‘resonant’ production of helium-4 is observed.

C. For small asymmetries \( |L| < 10^{-7} \) we have confirmed our preliminary results from [21], namely that asymmetry with these values has a too weak effect on nucleosynthesis to be considered.

Finally, we have calculated the \( ^4\text{He} \) dependence on the neutrino asymmetry for the whole range of mixing parameters of our model and we have obtained the primordial helium yields \( Y_p(L, \delta m^2, \vartheta) \). This enabled us to obtain isohelium contours in the \( \delta m^2 - \vartheta \) plane. Some of these constant helium contours, for \( L = 10^{-10} \) and \( L = 10^{-6} \), are presented in Fig. 4.

On the basis of these results, requiring an agreement between the theoretically predicted and the inferred from observations values of primordial helium, we have obtained cosmological constraints on the neutrino mixing parameters corresponding to different initial lepton asymmetry values. Assuming the conventional observational bound on primordial \( ^4\text{He} \) \( Y_p = 0.24 \) the cosmologically excluded region for the oscillation parameters on the plane \( \sin^2(2\vartheta) - \delta m^2 \) in Fig. 4 lies to the right of the \( Y_p = 0.245 \) curve, which gives 5% overproduction of helium in comparison with the standard value. For comparison the curves corresponding to \( L = 10^{-10} \) and \( L = 10^{-6} \) are plotted.

In conclusion we would like to stress once again that lepton asymmetries \( |L| \geq 10^{-7} \) can both enhance or suppress (completely or partially, depending on the concrete values of the model parameters) neutrino oscillations and consequently can strengthen or weaken the oscillation effect on primordial nucleosynthesis. Therefore, the constraints on the oscillation parameters obtained from nucleosynthesis with lepton asymmetry of the order of the baryon one [18-26,1] can be considerably changed - either relaxed (or even totally removed) or strengthened for (note!) not very large values of the initial lepton asymmetry. This may have interesting astrophysical and cosmological implications.

Vice versa, having definite values for the neutrino mixing parameters, it will be possible, on the basis of requirement of agreement between the theoretically calculated and the extracted from observations values of helium-4, to put cosmological constraints on \( L \) even within that range of exclusively small magnitudes.

Finally we would like to point to the essential differences between our work and the previous ones: we have considered the nonresonant case of oscillations with very small mass differences; the oscillations proceed effectively relatively late i.e. after neutrino decoupling and the change in the lepton number is dominated by oscillations, not by collisions; we have accounted precisely for the momentum spread of neutrinos.

Having in mind the latest news from SuperKamioka concerning the solar neutrino oscillation solutions [37], the very small mass differences concerned in our paper may look much more attractive, and hence this study will be interesting for the solar neutrino audience too. On the other hand we would like to stress that models exploiting the suppression of oscillations due to \( L \) or the generation of \( L \) which further suppresses oscillations must be exploited with caution and be carefully calculated for the concrete
\((L, \delta m^2, \theta)\) values of interest, as far as it was shown here that the effect of asymmetry is not straightforward. Asymmetry - oscillations interplay reveals greater complexity than the rough estimations suggest and should be studied carefully.

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Figure 1a: The evolution of the neutron number density relative to nucleons $X_n(t) = N_n(t)/(N_p + N_n)$ for the case of oscillations with maximal mixing and $\delta m^2 = 10^{-7} \text{ eV}^2$ for different values of the initial asymmetry ($L = 10^{-6}$, $L = 10^{-5}$ and $L = 10^{-10}$) is plotted.
Figure 1b: The evolution of the neutron number density relative to nucleons \( X_n(t) = N_n(t)/(N_p + N_n) \) for the case of oscillations with \( \sin^2(2\theta) = 10^{-0.05} \) and \( \delta m^2 = 10^{-7} \) eV\(^2\) for different values of the initial asymmetry (\( L = 10^{-6}, L = 10^{-5} \) and \( L = 10^{-10} \)) is plotted.
Figure 2: The figure illustrates the dependence of the frozen neutron number density relative to nucleons $X_n = N_n/(N_p + N_n)$ on the value of the initial neutrino asymmetry for $\sin^2(2\theta) = 10^{-0.05}$, and different mass differences. For comparison the standard curve is plotted also with dashed line.
Figure 3: The dependence of the neutron to nucleon freezing ratio on the mixing angle for $L = 10^{-6}$ for different mass differences $\delta m^2 = 10^{-7} \text{ eV}^2$ and $\delta m^2 = 10^{-8} \text{ eV}^2$ is shown. For comparison with dashed lines the corresponding curves with small asymmetry $L = 10^{-10}$ are presented.
Figure 4: On the $\delta m^2 - \vartheta$ plane the constant helium contours calculated in the discussed model of cosmological nucleosynthesis with neutrino oscillations for $L = 10^{-6}$ and $L = 10^{-10}$ are shown.
Fig. 4

\[
\log(\delta m^2 / \text{eV}^2) \quad \log(\sin^2 2\theta)
\]

- Solid line: \(L_\nu = 10^{-6}\)
- Dashed line: \(L_\nu = 10^{-10}\)
Fig. 3

$X_n$

- $L_\nu = 10^{-6}$
- $L_\nu = 10^{-10}$

$\delta m^2 = 10^{-7} \text{ eV}^2$

$\delta m^2 = 10^{-8} \text{ eV}^2$

$log(sin^22\theta)$
$\log (\sin^2 2\nu) = -0.05$

\begin{align*}
\delta m^2 &= 10^{-7} \text{ eV}^2 \\
\delta m^2 &= 10^{-8} \text{ eV}^2 \\
\delta m^2 &= 10^{-9} \text{ eV}^2
\end{align*}

Fig. 2 $\log(L_{\nu})$
\log(\sin^2 2\beta) = -0.05

\delta m^2 = 10^{-7} \text{ eV}^2

\begin{align*}
X_n & \\
\text{Fig. 1b} & \quad t [\text{sec}] \\
- - - & \quad L_\nu = 10^{-10} \\
\quad & \quad L_\nu = 10^{-6} \\
\quad & \quad L_\nu = 10^{-5}
\end{align*}
log(sin^22\theta) = 0
\delta m^2 = 10^{-7} \text{ eV}^2

Fig. 1a  \quad t \text{ [sec]}