MAGNETIZED ACCRETION INSIDE THE MARGINALLY STABLE ORBIT AROUND A BLACK HOLE

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Received 1999 January 13; accepted 1999 February 19; published 1999 March 9

ABSTRACT

Qualitative arguments are presented in order to demonstrate that the energy density of magnetic fields in matter accreting onto a black hole inside the marginally stable orbit is automatically comparable to the rest-mass energy density of the accretion flow. Several consequencs follow: magnetic effects must be dynamically significant but cannot be so strong as to dominate; the outward energy transport in Alfvén waves may alter the effective efficiency of energy liberation; and vertical magnetic stresses in this region may contribute to “coronal” activity.

Subject headings: accretion, accretion disks –– black hole physics –– MHD

1. INTRODUCTION

Accretion within the last stable orbit around a black hole is a very complicated problem. Consequently, most work on the subject has been carried out in one or another simplifying limit. Some (as summarized in the text by Kato, Fukue, & Mineshige 1998) have studied what happens when the matter behaves purely hydrodynamically (i.e., without any magnetic effects). Others, in order to focus attention on those properties peculiar to magnetohydrodynamics, have taken the limit of “force-free” magnetic fields, a limit that is valid when \( B^2 / \rho c^2 \gg \) (see, e.g., Blandford & Znajek 1977; Okamoto 1992; Ghosh & Abramowicz 1997). Still another approach has been to assume an arbitrary magnetic field configuration and to compute what happens when matter is injected into the black hole magnetosphere from a specified location at a specified rate (see, e.g., Hirotani et al. 1992). In almost every study, attention has been restricted to time-steady, azimuthally symmetric situations (including, e.g., Phinney 1983 and Punsly 1991, in which the force-free restriction is relaxed). One of the few efforts to go beyond these limits is the simulation by Koide, Shibata, & Kudoh (1998), which permitted nonstationarity, but imposed azimuthal symmetry, and used arbitrary initial conditions for both the magnetic field and the gas pressure distribution.

In the portion of this literature concentrating on MHD effects, the key organizing question has been whether these processes can tap the energy stored in black hole rotation (Blandford & Znajek 1977; Phinney 1983; Takahashi et al. 1990; Punsly 1998; Livio, Ogilvie, & Pringle 1999; Meier 1998).

Unfortunately, these simplifying assumptions, attractive as they are, may be artificially limiting. Although one can formally guarantee the validity of the force-free approximation by setting the accretion rate to zero (as is sometimes done), real accretion flows with nonzero mass accretion rates may introduce enough inertia to invalidate that assumption. Similarly, if continuing accretion continually brings new plasma toward the black hole, the field configuration cannot be time steady. Furthermore, simulations of MHD turbulence in the nonrelativistic portions of accretion disks (Stone et al. 1996; Brandenburg et al. 1996) strongly indicate that the field is far from azimuthally symmetric.

It is the burden of this Letter to argue that the limitations imposed by these simplifying assumptions have prevented us from seeing a number of important elements in the dynamics of accretion onto black holes. Moreover, there has been almost no attention paid to the simple question of when these different approximations may apply. In order to get past these limita-

2. THE MAGNETIC FIELD INSIDE THE MARGINALLY STABLE ORBIT

We will estimate the state of the magnetic field inside \( r_{\text{ms}} \), the radius of the marginally stable orbit, by supposing that it is “frozen” into the accreting plasma. Its value in the plunging region can then be scaled from its value in the disk proper by studying the flow lines. This view implicitly assumes that the accretion flow carries negligible net flux, i.e., that there are hardly any field lines extending to infinity. Otherwise, the field in this region would be mostly the result of the net magnetic flux accumulated over the black hole’s accretion lifetime (as argued, e.g., in Thorne, Price, & Macdonald 1986).

Given the matter four-velocity \( u^\alpha \), the field evolution is conveniently described (Lichnerowicz 1967) by two four-vectors: \( E^\alpha \equiv u^\beta F^{\alpha \beta} \) and \( B^\alpha \equiv u^\beta (\ast F)^{\alpha \beta} \), where \( F^{\alpha \beta} \) is the Maxwell field tensor and \( \ast F \) is its dual. Note that \( E^\alpha = (0, E) \) and \( B^\alpha = (0, B) \) in the fluid rest frame. Two of Maxwell’s equations \( [\nabla \times E = 0 \) and \( \nabla \times B = -(1/c) \partial B / \partial t] \) may be written as \( (\ast F)^{\alpha \beta} = 0 \), where we follow the usual convention and denote a covariant derivative by a semicolon. The definitions of \( E^\alpha \) and \( B^\alpha \) may be inverted in order to write \( \ast F \) in terms of the field four-vectors, so that this pair of Maxwell’s equations becomes

\[
(u^\alpha B^\beta - u^\beta B^\alpha + \epsilon^{\alpha \beta \gamma \delta} u_\gamma E_{\delta \beta}) = 0,
\]

where \( \epsilon \) is the completely antisymmetric Levi-Civita symbol. Even in these circumstances, the plasma conductivity should easily be high enough to make \( E^\alpha = 0 \) everywhere, i.e., to
ensure flux-freezing. Equation (1) then becomes

$$u^a \partial^a B_\beta = u^a \partial^a B_\beta - u^K B^K - u^K u^K B^K.$$ (2)

Here the term proportional to $B_\beta$ has been reduced by making use of the definition of $B^\beta$ and the expression for $u^a \partial^a$ implicitly given in equation (1). From equation (2), we see that the derivative of $B^K$ with respect to proper time comprises three parts: the response to shear (the term proportional to $u^K$); the response to fluid density changes (the term proportional to $u^K$); and the response to departures from free fall (the term proportional to $u^K u^K$).

If only gravitational forces acted on matter with the angular momentum and energy appropriate to a circular orbit at $r_m$, it would accelerate inward, but initially rather slowly, because both the first and the second derivatives of the “effective potential” (in the sense of Shapiro & Teukolsky 1983) are zero there. However, in a real disk, other forces can also act. Because the density of the disk drops sharply inward, there is an inward pressure gradient force (Chen & Taam 1993). In addition, the fluctuating magnetic fields that remove angular momentum from matter in the bulk of the disk (Balbus & Hawley 1998) should work in essentially the same way here. It is likely, therefore, that plasma moves from the nearly flat portion of the disk. Shapiro & Teukolsky (1983) because the magnetic forces in the disk fluctuate, we expect that the injection of matter inside $r_m$ will be both time variable and irregular as a function of azimuthal angle.

However, once in the plunging region proper, one might expect gravitational forces to be dominant again. This assumption may be tested for self-consistency by integrating equations (2) with $u^K$ as given by free fall with fixed angular momentum $L$ and energy $E$ (as evaluated at infinity; see, e.g., Shapiro & Teukolsky 1983). Because the metric is time steady and azimuthally symmetric, we set $\partial^a t = \partial^a \phi = 0$. For simplicity, we consider only motion in the equatorial plane, i.e., $u^K = 0$, an idealization appropriate to thin disks. With regard to the magnetic field, this calculation may be viewed as describing a particular small field loop accreted in a way unaffected by any adjacent streams. Equations (2), written in Boyer-Lindquist coordinates, then reduce to the single equation

$$u' \partial B^K = B^K u' - B^K u^K.$$ (3)

The field in the fluid frame is given by $B^K = K^K c^K B^K$, where the tensor $c^K$ gives the orthonormal tetrad for a locally inertial frame (Chandrasekhar 1983) and $K^K$ is the Lorentz transformation from that frame to the fluid frame.

On the other hand, the rest-mass density of matter $\rho$, as measured in the fluid frame, changes according to

$$\partial \ln \rho = - \partial (g^{1/2} u') / \partial r,$$ (4)

where $-g$ is the determinant of the metric, so that $g^{1/2}$ gives the scale factor for the differential volume element. The matter density can then be expected to fall dramatically as matter crosses $r_m$ because the radial speed outside $r_m$ is $=c(h/r)^{1/2} v_{\text{orb}}$ in a disk with thickness $h \ll r$, whereas it becomes relativistic inside $r_m$. Here $c$ is the usual Shakura-Sunyaev (1973) dimensionless stress, and $v_{\text{orb}}$ is the velocity of a circular orbit.

We numerically integrated equations (3) and (4) along with the equation $dx^a/dr = u^a$ (where $r$ is the proper time). To mimic the effect of MHD fluctuations, we took an initial condition in which $L$ is reduced below the amount required for a circular orbit at $r_m$ just enough to give the matter an infall velocity much smaller than the orbital velocity. The energy was left fixed at the energy associated with a circular orbit at $r_m$. The result is that the magnetic field in the fluid frame does increase somewhat inside $r_m$, but not by very large factors.

When $a = 0$, $B^K$ is virtually constant until reaching radii very close to the event horizon, and increases by only a factor of 2 even there. Even if $B^K$ is zero initially, the slow growth of the dynamical instability means that the initial orbit departs only slightly from a circular orbit. Consequently, shear is relatively strong compared with the change in density, and $B^K$ can grow to be comparable to $B^K$ in a fraction of an orbit (if the initial infall velocity is $< v_{\text{orb}}$). In fact, the diminution of $B^K$ due to falling density dominates its growth due to shear only if the fluid starts out with an infall speed close to $v_{\text{orb}}$. Otherwise, in the absence of reconnection, $B^K$ becomes greater and greater as the matter takes more turns around the black hole, but it diminishes somewhat from its peak in the final approach to the event horizon. When $a = 0$, the peak $B^K$ can be as much as $\approx 30$ times the initial $B^K$. Increasing $a$ from 0 to 0.0998 from here on, dimensional expressions are written in units in which $G = c = 1$ does not change these results qualitatively, except that the peak magnitude of $B^K$ diminishes to $\approx 1.5$ times the initial value.

At the same time, the dramatic increase in radial speed quickly leads to a decrease in $\rho$ by many orders of magnitude. For example, so long as the initial infall speed is $= v_{\text{orb}}(r_m)$, when $a = 0$ (so that $r_m = 6$), $u' = -0.1$ at $r \approx 4.1$; when $a = 0.95$ at $r_m = 1.81$, the same speed is reached at $r \approx 1.47$.

These results may now be combined to estimate, in the rest frame of plunging matter, the ratio between the magnetic field energy density $U_B$ and the rest-mass density $\rho$. As we have just shown, $U_B$ is likely to be similar to, or slightly greater than, the field energy density in the disk. In terms of the magnetic contribution to the stress $\alpha M$, the magnetic energy density in the disk is $B^2 \delta(\mathbf{B}) \xi \rho$, where the quantities with subscripts $d$ are measured in the disk proper. On the other hand, the rest-mass energy density is

$$\rho = \left[ \frac{g(r_{\text{ms}})}{g(r)} \right]^{1/2} \left[ \frac{h(r_{\text{ms}})}{r_{\text{ms}}} \right]^{1/2} \left[ \frac{v_{\text{orb}}(r_{\text{ms}})}{u'(r')} \right] \left[ \frac{c}{c} \right]^{2} \alpha \rho_d,$$ (5)

where $c_s$ is the effective sound speed in the disk. Because $h/r = c_s/v_{\text{orb}}$, the ratio between magnetic energy density and rest-mass density in the region of plunging orbits is

$$U_B = \alpha M \left( \frac{B^2}{\mathbf{B} \mathbf{B}} \right) \left[ \frac{B_\delta(r)}{B^2} \right] \left[ \frac{g(r_{\text{ms}})}{g(r)} \right]^{1/2} \times \left( \frac{r_{\text{ms}}}{r_{\text{ms}}^2 + a/M} \right) u'(r').$$ (6)

Numerical MHD simulations of nonrelativistic accretion disks (see, e.g., Stone et al. 1996 and Brandenburg et al. 1996) indicate that $\alpha_M \approx \alpha$ and that $B_\delta$ is rather greater than $\langle B, B_\delta \rangle$. We have just demonstrated that in the free-fall limit,
$B_\parallel^2$ is at least as large as $B_\perp^2$. The metric determinant ratio is generally slightly less than unity; if it is reinterpreted as describing the volume element, the ratio $[g(r)/g(r_{\text{ms}})]^{1/2} = (r/r_{\text{ms}})[h(r)/h(r_{\text{ms}})]$ when the infall is contained within a thin disk centered on the black hole’s equatorial plane. We conclude, then, that, so long as magnetic torques play an important role in driving accretion in the disk proper, the magnetic field must become dynamically important in the plunging region. As a result, it also follows that the assumption of ballistic orbits is not self-consistent.

3. CONSEQUENCES

The simple estimate of equation (6) leads to a number of qualitatively important consequences. First, the assumption of purely hydrodynamic flow is always inappropriate; continuity in $B$, combined with the dramatic fall in matter density, ensures that the magnetic field is dynamically significant. Simple free-fall trajectories, perhaps modified slightly by pressure forces, are not a good description of the streamlines; in other words, the accreting matter gives a significant fraction of its kinetic energy to the magnetic field. On the other hand, the force-free approximation is hardly better, except in the extreme limit of zero accretion and an independently magnetized black hole, for $B^2/8\pi$ can be no more than comparable to $\rho$. This conclusion should be very robust because it depends only on flux-freezing, mass conservation, and the assumption that $\alpha_{\text{pr}} = \alpha$. The only way it can be avoided is if there is tremendous flux loss from the plasma, which would, in its own way, also be very interesting.

Viewed another way, the estimate of equation (6) shows that the plunging matter inside $r_{\text{ms}}$ must do substantial work on the magnetic field, transferring much of its kinetic energy to it. We are then faced with the question of what happens to this energy. If it is simply carried into the black hole along with the plasma, the observational consequences would be relatively slight. Two other possibilities are both plausible and more interesting.

To the extent that magnetic forces alter the flow, momentum and energy are carried along field lines. If field lines connect matter just outside $r_{\text{ms}}$ with matter some ways inside, the inertia of the plasma outside $r_{\text{ms}}$ may cause the field lines to rotate closer to its angular frequency than the considerably larger angular frequency of matter with the same angular momentum that has fallen to a smaller radius. A torque would then be exerted in which angular momentum is carried back to the disk proper. The associated work done on the disk would ultimately be dissipated, effectively increasing the radiative efficiency of the disk. That the field likely takes the form of stretched loops does not diminish this effect: the sense of orbital shear automatically gives $BB^\ast$ the same sign, no matter what the sign of $B^\parallel$ is. Note that insisting on azimuthal symmetry would eliminate this sort of field structure.

How much energy and angular momentum is carried outward depends critically on, among other things, whether the MHD waves travel rapidly enough that waves directed outward in the fluid frame actually move outward in the coordinate frame. It is at this point in the argument that the problem of where causal disconnection sets in becomes central. To quantify this issue, let us suppose that the dispersion relation for linear waves with wavevector $k = +i\bar{r}$ in the fluid frame is $\omega^2 = v^2k^2$, so that the phase and group speeds coincide. For the waves to move outward in the coordinate frame, $\omega$ must stay positive when the wave four-momentum is transformed to that frame. By this definition, if the matter follows ballistic trajectories, the minimum wave speed in the fluid frame $v_{\text{min}} \approx |u'|$ when $a = 0$; when $a > 0$, the ratio $v_{\text{min}}/|u'|$ increases with both increasing $a$ and decreasing $r$ and can be as large as $\sim 10$ when $a = 0.998$. Since $B_{\text{in}} \sim 8\pi\rho$, we expect the Alfvén speed in the fluid frame to be $\sim c$, so it may be that $v > v_{\text{min}}$ in much of the inflow region, particularly when $a$ is relatively small. However, a proper evaluation of this mechanism’s efficiency clearly requires a much more complete calculation, and that is far beyond the scope of this Letter.

The speculation that Alfvén waves may carry away significant energy seems to be at variance with the long-held belief that the maximum energy available for radiation is the binding energy of matter at the marginally stable orbit. The traditional argument has been that so little time is required for matter to fall from $r_{\text{ms}}$ to the vicinity of the event horizon that the relatively slow processes of kinetic dissipation have no opportunity to transform any more of its energy into heat, and thence into photons. However, there is a loophole in this argument—Alfvén wave radiation is a coherent process that may transfer a substantial amount of energy on the dynamical time. The assumption of time steadiness becomes especially limiting in this regard. If one requires the field structure to be time steady, there can be no traveling waves. However, when the magnetic field inside $r_{\text{ms}}$ remains connected to the disk (as it must, at least initially, if it is carried in by accreting matter), it cannot be time steady except in some longtime average sense.

If significant energy can be removed from plunging matter, carried back out to the disk, and dissipated just outside $r_{\text{ms}}$, the effective efficiency of accretion may be greater than that given by the binding energy at $r_{\text{ms}}$. In a Schwarzschild metric, in principle, one might imagine that the efficiency might approach unity; in a Kerr metric, it is even imaginable that the outward flux of angular momentum carried by the magnetic field is so strong that the matter between $r_{\text{ms}}$ and the event horizon is put on negative energy orbits. If that occurs, the efficiency would become nominally greater than unity because the accumulated spin energy of the black hole is being tapped. In other words, there is the potential here for a realization of the Penrose process, made feasible by the long-range action of magnetic fields (Phinney 1983; Okamoto 1992; Yokosawa 1993). Both upper bounds, however, are somewhat unlikely to be realized in practice. Efficient removal of energy would mean that, at any given radial coordinate, the kinetic energy of the flow would be smaller than in free fall. If this occurs through a reduction in $|u'|$, the relative importance of magnetic forces would be diminished, perhaps leading to a self-limiting of this process. Similarly, the “Penrose process” would likely require the orbital frequency of the matter inside $r_{\text{ms}}$ to be reduced below the orbital frequency of the matter in the disk to which it is donating its angular momentum.

Whether Alfvén waves can carry a significant amount of energy and angular momentum outward to a “normal” accretion disk is also an issue in the theory of advection-dominated accretion flows (ADAFs). As described, for example, by Narayan & Yi (1995), these are accretion flows in which the inflow speed is close to free fall and in which angular momentum is efficiently removed to larger radii by magnetic stresses. Their mechanics, therefore, are very similar to the mechanics of accretion inside $r_{\text{ms}}$ as envisaged here. The difference here is that there is a “normal” disk at radii only factors of a few outside the region of interest, where the energy carried outward can be dissipated. In contrast, most ADAFs are supposed to span a large range in radius.

“Coronal” activity is a second possible outlet for the mag-
magnetic energy. If $B^2 \sim 8\pi \rho$ in the accreting matter but is weaker for $|\mathbf{z}| > h$, the vertical component of the $\nabla B^2$ force would be $\sim rh$ times the vertical gravity of the black hole. Strong upwelling of matter must then result, leading directly to the creation of a significant vertical component in the magnetic field. Although these vertical motions are, like all the motions in this region, relativistic, flux loss into the corona cannot reduce the field strength by more than a factor of a few because the inward plasma flow amplifying the field is equally rapid.

As magnetic loops rise vertically, the shearing of their footpoints can be expected to lead to reconnection in exactly the same fashion as expected in the nonrelativistic portions of accretion disks (see, e.g., Galeev, Rosner, & Vaiana 1979 and Romanova et al. 1998). So much energy is available that the associated dissipation could be a major contributor to the coronal activity (i.e., strong hard X-ray emission) seen so often in accreting black hole systems. Because Compton scattering is such an efficient energy-loss mechanism, the dissipated heat could be radiated in less than a dynamical time:

$$t_{\text{Compt}} \sim \left( \frac{m_e}{\mu_e} \right) \left( \frac{L}{L_E} \right)^{-1},$$

where $\mu_e$ is the mean mass per electron and $L/L_E$ is the luminosity of seed photons in terms of the Eddington luminosity. Of course, in normal plasma, $m_e/\mu_e \sim 10^{-3}$.

A further consequence of the strong vertical forces that may be expected is the possible excitation of jets. This mechanism appears to provide an efficient way both to eject magnetized plasma along the rotation axis of the system and to heat it at the same time.

In sum, many of the simplifying assumptions on which our intuition about accretion inside the marginally stable orbit has been built may have misled us: supposing the flow to be time steady prevents recognition of both traveling waves and unsteady behavior like magnetic reconnection; supposing it to be azimuthally symmetric forbids consideration of magnetic field topologies with stretched loops; supposing it to be purely hydrodynamic eliminates the possibility of magnetically mediated dynamics; and supposing that the magnetic field is force free rules out consideration of how infall dynamics might alter the field configuration.

I am happy to acknowledge stimulating and instructive conversations with Eric Agol, Mitch Begelman, Roger Blandford, Jim Pringle, and Ethan Vishniac. This work was partially supported by NSF grant AST 96-16922 and NASA grant NAG5-3929.

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