Generation of mesoscopic entangled states in a cavity coupled to an atomic ensemble

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We propose a novel scheme for the efficient production of "NOON states" based on the resonant interaction of a pair of quantized cavity modes with an ensemble of atoms. We show that in the strong-coupling regime the adiabatic evolution of the system tends to a limiting state that describes mesoscopic entanglement between photons and atoms which can easily be converted to a purely photonic or atomic NOON state. We also demonstrate the remarkable property that the efficiency of this scheme increases exponentially with the cavity cooperativity factor, which gives efficient access to high number NOON states. The experimental feasibility of the scheme is discussed and its efficiency is demonstrated numerically.

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There is considerable interest in the development of mesoscopic entangled states as resources for quantum technology applications. Of particular interest are entangled states for N photons of the form \(|N_a\rangle|0_b\rangle + \sqrt{N} \langle 0_a|N_b\rangle\) (NOON states), which contain N indistinguishable particles in an equal superposition of all being in one of two possible modes a and b. A number of applications for these states have been suggested, including entanglement enhanced metrology and sub-wavelength lithography [1–3]. Yet building sources for such states is challenging because their decoherence rate increases linearly with N.

Recently NOON states with up to \(N = 3\) microwave photons have been generated deterministically [2]. In the optical domain however only few experimental realizations were reported to have generated \(N > 2\) NOON states, which were mostly based on linear optics and state projective measurements of photons which implies an exponential scaling of resources with \(N \geq 3\). For some other schemes perfect optical elements and good initial sources of quantum fields are required [10–14].

In this letter we propose a scheme which leads to a robust unitary evolution into highly nonclassical entangled states of atoms and photons. These states can then be converted into a photonic NOON state by application of a single projective measurement on the atoms which can be implemented more efficiently than projective measurements on photons. Moreover the transit time of our system between NOON states with \(N\) and \(N+1\) excitations decreases linearly in \(N\). This can be used to compensate the increasing decay and dephasing rate of NOON states and thus to generate high photon number NOON states efficiently and on demand.

We start by introducing the physical setup and the basic idea of the mechanism that we intend to exploit. Then we present analytical explanations and estimates for the feasibility range of our scheme, which are backed-up by a careful numerical study that confirms our findings. We finish off with conclusions and an outlook.

**Hardware:** The physical setup – We consider a cloud of atoms with the level structure depicted in Fig.1 that interact with a classical pump field and a pair of quantized cavity modes (Fig.1a). The pump field that runs at some angle to the cavity axis resonantly couples to the transition \(|1\rangle \leftrightarrow |2\rangle\). The transitions \(|2\rangle \leftrightarrow |3\rangle\) and \(|5\rangle \leftrightarrow |6\rangle\) are driven by cavity mode a (b). This setup can for example be realized with a cavity tuned to the D1 line of \(^{87}\text{Rb}\) atoms. Then upper states \(|2\rangle, |4\rangle, |6\rangle\) are, respectively the Zeeman \(m = 0, -2, +2\) sub-levels of hyperfine states with total

\[
\rho_N = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

where \(N = 3\) is the cavity cooperativity factor.

**FIG. 1:** (Color online) (a): Arrangement of atoms in the cavity and atomic level structure. (b): Schematic diagram of coupling of relevant \(|n\rangle\) states (see the definition in the text). Here, \(\Omega = \frac{\Omega}{\sqrt{n+1}}\) and \(g_n = g \sqrt{n(n+1)}\). The initial state \(|\psi_n\rangle\) is identical to \(|0 \rangle\). The dark state \(|\phi\rangle\) is a coherent superposition of \(|n\rangle\) states. Due to the destructive interference the population of excited states is negligible.
angular momentum $F' = 2$, lower states $|3\rangle$, $|5\rangle$ correspond to $m = -1, +1$ of $F = 2$ sublevels and $|0\rangle$ can for example be $|0\rangle$ of $F = 1$. A suitable choice for the two cavity modes are thus the two orthogonal polarizations $\sigma^+$ and $\sigma^-$ of the same frequency that are supported by the cavity. Other Zeeman sub-levels can be neglected since they are not resonantly coupled to the considered states. Such systems are routinely set up with current technology and are thus available in several laboratories

The basic physical mechanism that is proposed here also assumes that our system is initially prepared in the state $|\psi_{in}\rangle = (\alpha \hat{\sigma}_{31} + \beta \hat{\sigma}_{51}) |\text{vac}\rangle \left( |\alpha|^2 + |\beta|^2 = 1 \right)$. This initial state can be, for instance, prepared by application of a weak pump field via a cavity mediated adiabatic passage process [20]. For $\Omega = 0$, the dark state eq. (2) is identical to the initial state $|\psi_{in}\rangle$ whereas population is transferred to states $|n:\rangle$ with increasing intensity of the pumping field. Thus if the pump field is switched on adiabatically the system remains in the dark-state $|D\rangle$. Initially, when the cavity is empty the adiabaticity condition reads,

$$\frac{g^2}{\Gamma} \gg \left| \frac{\Omega(t)}{\Omega(t)} \right| \quad (4)$$

where $\Gamma$ is the relaxation rate of upper states, which for the sake of simplicity is assumed to be equal for all atomic transitions. However, when the system contains more than 1 photon the effective coupling between the atoms and the photons is increased by a factor of $\sqrt{n_{ph}}$ and one expects the adiabaticity condition to generalize to

$$\frac{g^2 n_{ph}}{\Gamma} \gg \left| \frac{\Omega(t)}{\Omega(t)} \right| \quad (5)$$

While we have not proven this condition analytically due to the complex energy level structure, it is in excellent agreement with our numerical results.

From eq. (2) one observes that the population of states with $n_{ph}$ excitations depends on the amplitude of the pumping field. Moreover when the pump field becomes larger than $g$ the main part of the population accumulates in states with $n_{ph} \approx \frac{\Omega}{g} \quad (6)$

Thus the adiabaticity condition (4) is easier to satisfy when the amplitude of the pumping field is larger. From eq. (2) one can also see that if $\Omega \gg gM$ all the population is transferred to the states with the highest possible number of excitations ($n = M - 1$). Then our dark state is reduced to

$$|D\rangle = |(M - 1) :\rangle \quad (7)$$

which is a maximally entangled state with $M$ excitations either in the left leg ($\hat{a}$-photons and atoms in level $|3\rangle$) or the right leg ($\hat{b}$-photons and atoms in level $|5\rangle$) of the system.

Although this state is an entangled state of atoms and photons it can easily be converted to purely photonic (atomic) NOON states by projective measurements and local unitary operations. For example resonant quick $\pi/2$ microwave pulses between Zeeman sub-levels $|3\rangle$ and $|5\rangle$ can be used to map $\hat{\sigma}_{31} \rightarrow \frac{1}{\sqrt{2}}(\hat{\sigma}_{31} + \hat{\sigma}_{51})$
and $\hat{\sigma}_{31} \rightarrow \frac{1}{\sqrt{2}}(\hat{\sigma}_{31} - \hat{\sigma}_{31})$. This implies $|D\rangle \rightarrow \left\{ \alpha(\hat{a})^{M-1}(\hat{\sigma}_{31} + \hat{\sigma}_{31})^{M} + \beta(\hat{b})^{M-1}(\hat{\sigma}_{31} - \hat{\sigma}_{31})^{M} \right\} |\text{vac}\rangle = \left[ \alpha(\hat{a})^{M-1} + \beta(\hat{b})^{M-1} \right] \sum_{k=0}^{M/2} \left( \frac{M}{2k} \right) \hat{\sigma}_{31}^{M-2k+2k} |\text{vac}\rangle + \left[ \alpha(\hat{a})^{M-1} - \beta(\hat{b})^{M-1} \right] \sum_{k=0}^{M/2} \left( \frac{M}{2k-1} \right) \hat{\sigma}_{31}^{M-2k-1+2k-1} |\text{vac}\rangle$

where we have skipped the normalization for simplicity. Hence the detection of $K$ quanta in level $|3\rangle$ projects the photons into the pure photon NOON state.

$|\text{NOON}\rangle = \frac{1}{\sqrt{\alpha^{2} + \beta^{2}}} \left[ \alpha(\hat{a})^{M-1} - (1)^{K} \beta(\hat{b})^{M-1} \right] |\text{vac}\rangle$

Making use of the cyclic transitions $|3\rangle \leftrightarrow |5\rangle$ the projective measurement can be done with high fidelity. If the initial number of atoms is known the detection of $K$ atoms in level $|3\rangle$ produces a NOON state with probability proportional to $p^{K}(\frac{M-K}{2}) \sum_{m=K/2}^{M} (1-p)^{2m} \left( \frac{M}{2m} \right)$ where, $p$ is the probability for the successful detection of an excitation in level $|3\rangle$. It should be noted here that the efficiency of atomic state detection ($p > 0.9$) is much higher than the typical efficiency of single-photon detectors ($p \sim 0.4$). Hence for the preparation of high NOON states, which on average require the detection of several quanta in state $|3\rangle$, our scheme performs dramatically better than schemes that rely on the detection of multiple individual photons.

**Estimate of limitations** – Important practical limitations of the present scheme result from dissipation in the form of cavity damping and spontaneous emission. Cavity damping comes into play as soon as the cavity mode is excited and causes decoherence. It can be neglected if

$$\kappa \int_{0}^{T} n_{ph}(t) \, dt \ll 1$$

(8)

where, $\kappa$ is the cavity decay rate, $T$ is the duration of the process and $n_{ph}(t)$ is the number of photons in the cavity at time $t$. Spontaneous decay can be disregarded if the interaction is adiabatic i.e. satisfying [5]. Thus the scheme works if both conditions [6] and [8] are satisfied. Integrating both sides of inequality [5] yields

$$\frac{\Delta^{2}}{4} \int_{0}^{T} n_{ph}(t) \, dt \gg \ln \left( \frac{\Omega(T)}{g} \right)$$

where $t_{1}$ is the time when the cavity contains one photon ($\Omega(t_{1}) = g$). Then by making use of relations [6] and [8] one can show that the scheme is expected to work if

$$\frac{\Delta^{2}}{\kappa T} \gg \ln M.$$  

(9)

The term on the left hand side is the so called cooperativity parameter which represents the coupling strength between single atom and a cavity photon. Thus in the presented system the size of the achievable NOON state increases exponentially for increasing cooperativity. State-of-the-art technology enables production of cavities with cooperativity of order of 10, which according to condition [6] will allow efficient generation of NOON states with $n_{ph} \lesssim 10$.

**Numerical verification** – We will now present a numerical analysis of the system to assess its feasibility in detail. For an initial state of the form $|\psi_{in}\rangle = (\alpha\hat{\sigma}_{31} + \beta\hat{\sigma}_{31}) |\text{vac}\rangle$, all relevant states of our system can be written as

$$|\psi\rangle = \sum_{m=0}^{M} \sum_{l=0}^{M-m} \sum_{k=0}^{\min(k,l)} \sum_{q,r=0}^{\min(k,l)} [\alpha A_{m,k,l,q,r}(t) \hat{\sigma}_{31} + \beta B_{m,k,l,q,r}(t) \hat{\sigma}_{31}] |m, k, l, q, r\rangle$$

where $A_{m,k,l,q,r}(t)$ and $B_{m,k,l,q,r}(t)$ are the time dependent probability amplitudes and $|m, k, l, q, r\rangle = \hat{\sigma}_{21}^{m} \hat{\sigma}_{31}^{k-q} \hat{\sigma}_{41}^{l-r} \hat{\sigma}_{51}^{q} \hat{\sigma}_{61}^{r} (\hat{a})^{k-r} |\text{vac}\rangle$ The evolution of the system is described by the time dependent Schrödinger equation with Hamiltonian [11]. To include the decay of excited states [2], [4], and [6] we assume that the amplitudes containing these states decay exponentially with rate $(m + q + r)\Gamma$, where the term in the brackets is the number of atoms in the excited states.

For our numerical calculations we have neglected the cavity loss and assumed $g = \Gamma$. The results are presented in Fig2 and Fig3. From these numerical examples several conclusions can be drawn. First of all one can see that the main part of population from initial state $|0\rangle$ is adiabatically transferred into higher $|n\rangle$ states when the amplitude of the pumping field is increased. One can also see that the main part of population stays in the dark state even for relatively large number of atoms $M = 8$. Better transfer can in principle be obtained with field changes that satisfy the adiabaticity condition more strongly.

In Fig3 numerical results obtained with pump field $\Omega(t) \sim \frac{1}{t}$ are presented. One might expect that these type of pump fields are very inefficient due to large nonadiabatic interactions arising in the region where the derivative of $\Omega(t)$ is very large. However in our system the adiabaticity of the process increases with $\Omega$ as can be concluded from eq.6 which allows us to apply pulses with very steep shapes. In this case the time that the system spends in states with large value of $n_{ph}$ is smaller which strongly reduces the cavity loss parameter [8].

For the results presented in Fig3 the inequalities [10] and [8] are very well satisfied. The value of integral from [5] is $\Gamma \int_{0}^{T} n_{ph}(t) \, dt \approx 5.3$ (a); 8.6 (b) and 12.2 (c). Thus proof of principle experiments achieving non-trivial NOON states sizes can be realized if $\Delta^{2} / \kappa T \approx 10$, although the influence of cavity decay cannot be completely disregarded. Much better results can in principle be obtained with ultrahigh finesse cavities with $\Delta^{2} / \kappa T > 60$ [21, 22].

Finally we would like to mention that the presented scheme works very much in the same manner if initially
all atoms of the system are in level |1⟩ and the cavity contains a photon in arbitrary superposition of modes a and b (ψin = (α^a† + β^b†)|vac⟩). It should also be noted that the presented scheme can be used to generate mesoscopic entanglement in other systems such as trapped ions, provided they are described by the Hamiltonian (1).

To summarize we have discussed the evolution of two quantized modes interacting with a medium of six level atoms realizable as Zeeman sub-levels. We have shown that this system has a dark eigenstate which is a mesoscopic entangled state of photons and atoms. In the limit of strong coupling the system can adiabatically be transferred to a dark state that can be converted to a purely photonic or atomic NOON states by application of simple local operations. We have also demonstrated that the efficiency of NOON state generation increases exponentially with the single atom cooperativity factor, thus allowing the preparation of large NOON states with moderate, experimentally realized, cooperativity factors.

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