A new MCDM-based approach using BWM and SAW for optimal search model

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1. Introduction

Search theory (ST) is the study of how to utilize the resources when attempting to find a target whose location is not known precisely (Stone, 2013). Search for lost or hidden objects happens for various real cases. Widespread applications are detection of secret hostile targets, detection of land mine and rescuing lost people (Kriheli et al., 2016; Sotoudeh-Anvari et al., 2018). ST has various models and among them, optimal search model (Ross, 1983) is one of the most broadly used forms (Benkoski et al., 1991). In the multiobjective optimal search model a single target is located in one of possible locations. The number of locations is known and the target cannot move. For each location, we determine the search cost, the search time and the probability that our target is in a given location. Also the probability that a target will be detected on one look is known (also known as overlook probability).
The aim of this problem is to gain a search policy that optimize expected cost and time in search process (Ross, 1983). Various approaches such as dynamic programming have been introduced for solving this problem (Black, 1965). But usually these approaches are complex and need significant amount of computations (Benkoski et al., 1991; Kadane, 2015; Stone, 1976; Levner, 1994). On the other hand, first, Sotoudeh-Anvari et al. (2018) formulated the optimal search problem as an MCDM problem. The aim of MCDM model of search problem is to rank the locations in the presence of some decision factors such as search cost, search time, the probability of detecting and overlook probability. It is important to mention that it cannot be claimed that the MCDM framework has the same characteristics as the original optimal search model (Ross, 1983), but it can provide important points for initial decision.

Sotoudeh-Anvari et al. (2018) suggested a new MCDM approach which was composed by Entropy method to weight criteria and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and Complex Proportional Assessment (COPRAS) to rank the locations in a search problem. Although Entropy technique as an objective weighting method has its advantages, it has some limitations too. First of all, many researchers believe that in the real problems, weights of decision factors are always subjective, returning the judgment of decision maker (DM) (Zavadskas & Podvezko, 2016; Wang & Chang, 2007). Moreover, some researchers such as Jing et al. (2012) pointed out that objective weighting methods can sometimes generate an opposite result to reasoning. Among the subjective methods, AHP is the most extensively used approach. However, AHP suffers from various drawbacks in this field. AHP is criticized for the high difficulty owing to pairwise comparisons and the lack of consistency (Rezaei 2015; Løken, 2007). Fortunately, Rezaei (2015) mentioned that these problems can be alleviated with a more structured comparison approach and introduced a new MCDM method called BWM. In this paper, BWM is used for determining the weights of the decision factors and then rankings of the locations are obtained by SAW. To our knowledge, this study presents the first usage of BWM for weighting of selection factors in search context. Also we extend SAW to rank the locations since SAW has a clear logic and very easy calculation procedure that provides the rationale of human option. Let us recall that Sotoudeh-Anvari et al. (2018) applied TOPSIS and COPRAS to tackle optimal search problem and obtained suitable results, but application of SAW in this context is new.

The reminder of our work is ordered as follows. In Section 2, a background of MCDM paradigm, BWM and SAW is presented. Section 3 focuses on the suggested MCDM-based model for search problem. A numerical case is elucidated in Section 4. Finally, the conclusion and future researches are reviewed in Section 5.

2. Methods

Here, a background of MCDM paradigm and the key methods, namely BWM and SAW are introduced.

2.1. MCDM paradigm

MCDM is one of the most popular branches of decision theory. Although a large number of MCDM techniques have been introduced, each of the them has its own characteristics and none of these methods can be selected as the best method (Løken, 2007; Mulliner et al., 2016). Nevertheless, some MCDM methods are more suitable for a given problem. On the other hand, because of the availability of many MCDM methods, selection of a suitable MCDM method can be considered as an MCDM problem (Triantaphyllou, 2000; Bernroider and Stix, 2007). On the basis of various selection factors such as simplicity, validity, computation time, problem structure (number of alternatives or criteria), final result, and rank reversal, one can deduce that COPRAS, TOPSIS and SAW can be considered as the most efficient and the most user friendly MCDM method in many fields (Dey et al., 2016; Mousavi-Nasab & Sotoudeh-Anvari, 2017; Mulliner et al., 2016; Mousavi-Nasab & Sotoudeh-Anvari, 2018; Chatterjee et al., 2011). It is very important to mention that this finding must be extended
cautiously and on the basis of the need of the given problem. For example, COPRAS, TOPSIS and SAW are compensatory methods and in these methods tradeoff among decision criteria is permissible. But in non-compensatory methods, a bad score in a decision factor cannot be compensated by a good score for the other decision factor (Mulliner et al., 2016). Clearly, in such cases, the appropriate MCDM technique should be selected according to problem structure. More formally, compatibility with the aim of problem is one of the most essential factors in choosing an MCDM approach (Roy, 1990; Mulliner et al., 2016). Also some researchers such as Løken (2007), Mulliner et al. (2016), Wang et al. (2016), Mousavi-Nasab and Sotoudeh-Anvari (2017) and Garg and Jain (2017) suggested that more than one MCDM technique should be used to a problem to obtain a more comprehensive and safe decision. For more about MCDM methods, reader is referred to Hobbs and Horn (1997), Ginevičius and Zubrecovas (2009), Mousavi-Nasab and Sotoudeh-Anvari (2017), Serrai et al. (2017), and Løken (2007).

Irrespective of preferred MCDM method for problem tackling, the key step is to gain the decision factor weights. There are different approaches for determining the criteria weights in MCDM techniques, including 1) subjective method such as AHP, 2) objective method such as Entropy method, and 3) the combination method. In subjective weighting approach, the weights are obtained according to a DM’s opinions. On the other hand, objective weights are obtained according to the information of decision matrix on the basis of mathematical models. The combination methods compute the weights by DM’s judgment and decision matrix data (Wang & Luo, 2010). Obviously, each weighting method has its own advantages and drawbacks. However, subjective weights return the opinions of DMs with the practical experience in that context and therefore, in nearly all real problems subjective weights are useful (Wang & Chang, 2007). Among the subjective methods, AHP has an extensive popularity for weighting via pairwise comparison technique. But Løken (2007) pointed out that there are various criticisms against AHP. For example, inconsistency is the key challenge in pairwise comparison approaches, which often takes place in practice (Rezaei, 2015). Also AHP is too time-consuming when the amount of decision factors is large (Løken, 2007). Rezaei (2015) revealed that the root of this problem is in the unstructured comparisons and proposed a new MCDM technique, namely BWM that gains the criteria weights in a different way. In the next sub-section, more explanation about BWM is provided.

2.2. Best-Worst Method

Although there are several MCDM techniques, the steps of these methods often are similar, including problem definition, alternatives selection, criteria selection, decision matrix construction, criteria weight determination, and ranking (Terrados et al., 2009). Because criteria weights play a very important role in the ranking of options, a vital task is to determine the weights of decision factors. There are several MCDM techniques that can be employed to weight criteria, the most conventional of which are AHP. This technique uses pairwise comparisons of the selection criteria on the basis of DM’s opinion. But as mentioned, inconsistency is a complicated problem to the pairwise comparison approaches (Rezaei, 2015). To solve this problem, Rezaei (2015) suggested BWM that is always consistent. In BWM, DM should identify the most desirable criterion and the least desirable criterion and make pairwise comparisons between these criteria and the other criteria. Finally, a maximin model is resulted to compute the weights of selection criteria (Mou et al., 2016). Clearly, in BWM fewer comparisons compared to AHP are needed and mathematically, the comparisons in BWM are reduced from $n(n-1)/2$ to $2n-3$ with $n$ decision factors (Rezaei, 2015). Furthermore, BWM provides a structured pairwise comparison which generates consistent outcomes (Rezaei, 2016). For achieving consistency and simplicity, compared to the conventional AHP, BWM has been employed broadly in diverse applications such as sustainable oil supply chain management (Ahmad et al., 2017), healthcare management (Mou et al., 2016), investment opportunities (Askarifar et al., 2018), web service selection (Serrai et al., 2017) medical tourism development (Abadi et al., 2018), technological innovation development (Ghaffari et al., 2017), research and development (Salimi and Rezaei, 2018) green supplier selection (Gupta and Barua, 2017) and urban sewage sludge (Ren et al., 2017).
2.3. Simple Additive Weight

Literature review shows that SAW is the simplest MCDM method to understand and to apply (Serrai et al., 2017). Moreover, this method is the most frequently used MCDM technique (Salehi & Izadikhah, 2014) and can be employed as a benchmark to evaluate the results of the other MCDM approaches (Wang et al., 2016).

SAW consists of five steps as follows,

**Step 1:** Normalize the decision matrix according to Model (1)

\[
 r_{ij} = \begin{cases} 
   x_{ij} / x_j^+ & j \in \Omega_{\text{max}} \\
   x_j^- / x_{ij} & j \in \Omega_{\text{min}}
\end{cases}
\]  

In Model (1), \( \Omega_{\text{max}} \) and \( \Omega_{\text{min}} \) are sets of positive and negative decision attributes, respectively.

**Step 2:** Identify the weight of decision factors.

**Step 3:** Compute the ranking value on the basis of Model (2):

\[
 S_i = \sum_{j=1}^{n} w_j r_{ij}
\]  

The bigger the \( S_i \), the better the option is.

3. Suggested model on the basis of BWM and SAW for search problem

Now, a new MCDM methodology as depicted in Fig. 1 is proposed for search problem. Each of these steps is introduced below:

In the first step, the potential locations of lost or hidden target and decision criteria which will be employed in the assessment process via literature and discussions with DMs are determined. Although different selection factors can be important while considering the optimal search problem, in this study, the selection attributes for the ranking of locations can be gained from the Fiedrich et al. (2000), Najafi et al. (2013) and Gutjahr and Nolz (2016). Therefore, four criteria are selected, including search cost, search time, probability that the target is in a given location and finally, the probability that a target to be detected on one look.

In the second stage, the weight of selection criteria is obtained using BWM. According to Rezaei (2015) and Rezaei (2016), BWM consists of five steps as follows.

**Step 1:** Define the set of decision factors.

In this step, DM introduces \( n \) decision factors, namely \( \{c_1, c_2, \ldots, c_n\} \) to make a decision.

**Step 2:** Specify the best decision factor and the worst decision factor.
The best decision factor is the most preferred attribute and the worst decision factor is the least preferred attribute.

**Step 3:** Identify the preference of the best decision factor over the other decision factors. In this step, integers can only be used. The best-to-others (BO) vector is:
\[ A_B = (a_{b_1}, a_{b_2}, ..., a_{b_n}) \]
In the BO vector, \( a_{bj} \) represents the preference of the best factor \( B \) over selection factor \( j \) and \( a_{bb} = 1 \).

**Step 4:** Identify the preference of all decision factors over the worst decision factor. In this step, integers can only be used. The others-to-worst (OW) vector is:
\[ A_W = (a_{1w}, a_{2w}, ..., a_{nw})^T \]
In OW vector, \( a_{jw} \) represents the preference of selection factor \( j \) over the worst decision factor \( W \) and \( a_{ww} = 1 \).

**Step 5:** Compute the best possible weights \( (w_1^*, w_2^*, ..., w_n^*) \) by Model (3) as follows:

\[
\min \max_j \left\{ \frac{w_j - a_{bj}}{w_j}, \frac{w_j - a_{jw}}{w_w} \right\}
\]
\[ s.t. \]
\[ \sum_j w_j = 1 \]
\[ w_j \geq 0 \quad \text{for all } j \]

Model (3) can be converted to Model (4) as follows:

\[
\min \xi
\]
\[ \frac{w_j - a_{bj}}{w_j} \leq \xi \quad \text{for all } j \]
\[ \frac{w_j - a_{jw}}{w_w} \leq \xi \quad \text{for all } j \]
\[ \sum_j w_j = 1 \]
\[ w_j \geq 0 \quad \text{for all } j \]

Solving Model (4), the best possible weights \( (w_1^*, w_2^*, ..., w_n^*) \) and \( \xi \) are achieved. Also the consistency ratio (CR) can be calculated on the basis of Model (5):

\[
\text{Consistency ratio} = \frac{\xi}{\text{Consistency Index}}
\]

The “Consistency Index” can be seen in Table 1 (Rezaei, 2015; Rezaei, 2016).
Table 1

| $a_{ij}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|---|---|---|---|---|---|---|---|
| Consistency Index | 0 | 0.44 | 1 | 1.63 | 2.3 | 3 | 3.73 | 4.47 | 5.23 |

A CR close to zero indicates more consistency and CR close to one indicates less consistency (Rezaei, 2016). As mentioned, always the result of BWM is consistent. However, Rezaei (2015) pointed out that in AHP, CR is used to verify the validity of comparisons, but CR in BWM is used to determine the degree of reliability.

Finally, location rankings are identified by SAW.

Fig. 1. Flowchart of the suggested model for search problem

4. Numerical example

The application of the suggested MCDM-based model in this numerical case will signify its validity. After an earthquake, a search team is organized to search a missing family. Probably, they have been confined in one of the following locations: a cottage (location 1), a forest (location 2), a mountain (location 3), a shopping center (location 4) and amusement park (location 5). The probability that they are in each of these locations, overlook probability, the search cost and the search time of these locations are provided as decision matrix in Table 2. The objective of the aforementioned problem is the ranking of these five locations for search operation.
In the first step, the weight of decision criteria should be determined. In this study, the weights are obtained by BWM as follows:

The first stage in BWM is specified the best selection factor and the worst selection factor. In this problem, “probability of finding (C3)” and “search cost (C1)” is selected as the best and the worst criteria, respectively. In the second stage, the preferences of the best factor over the criteria should be determined by integers (as can be found in Table 3).

| Criteria | C1 | C2 | C3 | C4 |
|----------|----|----|----|----|
| Best criterion: C3 | 4 | 3 | 2 |

In the third step, the preferences of criteria over the least important factor should be determined by integers (as can be found in Table 4).

| Criteria | C1 | C3 | C4 |
|----------|----|----|----|
| Worst criterion: C1 | 2 | 4 | 3 |

According to Table 3 and Table 4 and by solving Model (4), the weights can be gained as $w_1 = 0.1036$, $w_2 = 0.1706$, $w_3 = 0.4514$, $w_4 = 0.2742$ and $ξ = 0.354$. On the basis of Model (5), we have $CR = 0.21$ that this indicates a very good degree of reliability. According to this numerical case, the ease and consistency of BWM is clear.

Now, SAW is used to rank these locations. According to SAW, the ranking of locations is gained as 3-1-4-5-2. On the other words, the aforementioned locations are searched in order 3-1-4-5-2 until the family is detected. On the other hand, when the same search problem is worked out by TOPSIS and COPRAS, the arrangement of locations is achieved as 3-1-4-5-2 too. Clearly, the ranking of locations exactly match in SAW, COPRAS and TOPSIS. This numerical example denotes that the suggested model is very easy and produces comparable consequence with other MCDM approaches.

5. Conclusion

In this paper, we have suggested a new MCDM-based framework for solving optimal search problem. First, BWM was applied to obtain the criteria weights and then SAW is used to find the final ranking. Compared to AHP, BWM has various advantages such as very structured pairwise comparison as well as trustworthy outcomes. Moreover, the comparison times in BWM was reduced considerably. Hence,
BWM can be used well as a subjective technique for weighting of decision factors in search problem. Fortunately, the suggested approach is very straightforward and can be developed to handle other MCDM problems.

As a future study, BWM will be extended to various fuzzy environments and we will employ this fuzzy BWM in a number of real problems. Also the possible shortcomings of BWM will be discussed.

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