Temperature Equilibration in Rapidly Heated Plasmas at Solid State Densities

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Abstract. We investigate the electron-ion energy transfer in dense two-temperature plasmas and the influence of ion acoustic modes on this process. The ion modes play an important role in temperature relaxation since, besides close binary collisions, the energy transfer through coupled collective modes is the most relevant mechanism to dissipate energy within a plasma with different temperatures. We show under which circumstances ion acoustic modes occur and how they can be incorporated.

1. Introduction
Temperature relaxation in hot plasmas at solid state densities involves a variety of physical effects. In fully ionized plasmas, a correct description incorporates at least strong collisions, spatial correlations and collective excitation. The latter yield collision processes that involve more than just two particles since in-medium effects modify the in and out channels of the collision event. As a result, collective modes can be excited and energy is transferred between the subsystems by means of them. The theoretical description must be therefore based on dynamic screening [1, 2, 3] rather than static Debye screening typically used [4, 5, 6]. The transferred energy itself can be redistributed into kinetic and potential energy, in this way modifying not only temperature but charge states and correlations as well [7, 8, 9, 10].

We focus here on the influence of coupled collective modes on the energy transfer and analyse under which conditions it can be expected to give a significant contribution. It turns out that the most important modes are the ion acoustic ones since the electronic plasmons have too high energy to be excited by ions.

2. Energy Transfer Through Coupled Collective Modes
The description of collective excitations in strongly coupled plasmas is a formidable problem not yet solved. In weakly coupled plasmas, such a theory does exist and the dynamic response of the particles is described within the random phase approximation (RPA) [11, 12]. The corresponding kinetic equation, that describes dynamic processes such as temperature relaxation, is the Lenard-Balescu equation [13, 14]. Since electrons and ions are here treated on the same footing, the influence of the electrons on the ion response and vice versa (coupled collective modes) is naturally included.

To allow for quantum effects, the basis for the description of the energy transfer rates will be the quantum version of the Lenard-Balescu equation [12] that incorporates dynamic screening...
and Pauli blocking in the collision integral. The screening is described by the retarded dielectric function in RPA

\[ \varepsilon^R(p, \omega, t) = 1 + \sum_a V_{aa}(p) \chi^0_{aa}(p, \omega, t) \]  

(1)

with the bare Coulomb potential \( V_{aa}(k) = 4\pi Z_a Z_b e^2/k^2 \) and the density response function of free particles \[1\]

\[ \chi^0_{aa}(p, \omega, t) = \int \frac{dp'}{(2\pi \hbar)^3} \frac{f_a(p' + p, t) - f_a(p', t)}{\hbar \omega + E_a(p') - E_a(p' + p) + i\epsilon} . \]  

(2)

The latter is determined by the time-dependent distribution function. Since we consider a plasma with two well-defined temperatures, the electron and ion distributions are given by Fermi and Boltzmann functions, respectively. For the energy transfer rate between the subsystems, one obtains from the Lenard-Balescu equation \[3\]

\[ \frac{\partial}{\partial t} E_{\alpha \rightarrow \beta}^{\text{trans}} = 4\hbar \sum_i \int \frac{dk}{(2\pi \hbar)^3} \int_0^\infty \frac{d\omega}{2\pi} \omega \left[ n_i^\alpha(\omega) - n_i^\beta(\omega) \right] \frac{\text{Im} \varepsilon^R_{ee}(k, \omega, t) \text{Im} \varepsilon^{\beta R}(k, \omega, t)}{|\varepsilon^R(k, \omega, t)|^2} . \]  

(3)

Here, \( n_i^\alpha = [\exp(\omega/k_BT_a) - 1] \) are Bose type functions that constitute the excitation distribution in the subsystem ‘\( \alpha \)’. Expression (3) gives the energy transfer including the effect of coupled collective modes in the electron-ion system. It is applicable for weakly coupled plasmas without restriction with respect to electron degeneracy. For most systems in the warm dense matter regime, especially hydrogen, the approximation for the dielectric function (1) is sufficient since electron-electron and electron-ion coupling is weak due to either strongly heated or highly degenerate electrons. Strong ion-ion coupling can be accounted for by inclusion of local field corrections for the ionic subsystem \[1\].

It should be pointed out that the expression (3) is equivalent to Eq. (50) derived by Dharma-wardana & Perrot \[1\] if weakly coupled plasmas are considered. In contrast to their formulation only free particle density response functions occur here in the nominator. The description of the collective modes are here fully contained in the dielectric function in the denominator. One may interpret this as a renormalisation of the bare Coulomb interaction to account for the surrounding environment (i.e., screening) and the occurrence of collective modes.

The main contribution to the integral can be expected to come from peaks created by zeros of the real part of the dielectric functions which indicate strong excitations in the spectrum of collective modes. It can however be shown that the height of these peaks is limited since the small imaginary part of the dielectric function at the excitation energy is compensated by the same function in the nominator. Thus, our equivalent, but redefined formulation proved advantageous since a numerical integration can now be set up much more easily. Nevertheless, the conditions for the occurrence of these collective excitations are worth knowing for the numerical procedure as well as a deeper understanding of the mechanisms of the energy transfer.

3. Excitation Spectrum for Two Temperature Plasmas
In principle, the collective modes capable of influencing the energy transfer in dense plasmas are the electron plasmon and acoustic modes and the ion acoustic modes. Fig. 1 shows the dispersion of these modes in the right panel. An ionic plasmon-like mode can only be observed in a one component plasma (OCP); in a two component plasma, the screening of the ion-ion interaction results either in a vanishing mode or, for degenerate electrons or \( T_e \gg T_i \), in its transformation into a second ion acoustic branch \[12\]. Usually, only the upper branches are relevant since the lower ones are strongly damped. Furthermore, the electron plasmon mode is at such high energy that it is not relevant within the integral of Eq. (3). The reason is that the ionic response function in the nominator limits the integrand in \( \omega \)-space to values less than the
Figure 1. Real part of the dielectric function for three different ion temperatures (left panel) and the corresponding dispersion relation for the collective modes (right panel) for a fully ionized hydrogen plasma.

ion plasma frequency. Accordingly, only the ion acoustic modes is able to modify the energy transfer rates. The influence of the other modes is rather indirect via redistribution of spectral weight that occurs since the dielectric functions holds a number of sum rules [11, 12].

The quantity responsible for the occurrence of modes is the real part of the dielectric function while the imaginary part of the dielectric function determines the character of the modes: whether they are damped or have a quasi-particle like character. The occurrence of an ion acoustic mode depends on the appearance of a zero in the total real part of the two component dielectric function. It is supposed to lie in the ionic part of the spectrum, that is, at very small frequencies, where the electronic contribution is dominant for equilibrium situations. It is known [15] that only in non-equilibrium cases with temperatures $T_e \gg T_i$ or in highly degenerate plasmas, when the electron temperature is basically given by the Fermi temperature, an ion acoustic mode develops. This is exactly the scenario during temperature relaxation.

However, the questions to answer is how large the difference in the species temperatures has to be to allow for an ion acoustic mode. For a quantitative analysis of ion acoustic modes, we start with the real part of the dielectric function. Because of their high mass, the ions can be treated classically. In this case, the real part of the ion dielectric function is given by

$$\text{Re} \varepsilon_{ii}(p, \omega) = 1 - V_{ii}(p) \text{Re} \Pi_{ii}(p, \omega).$$

Now we take the polarization function $\Pi_{ii}$ in RPA (zeroth order in density response function, that is $\Pi_{ii} = \chi_{ii}^0$) and write this function in terms of a hypergeometric function [11, 12]

$$\text{Re} \varepsilon_{ii}(p, \omega) = 1 + \frac{4\pi e^2}{p^2 k_B T} \frac{n_i}{1 - \frac{\omega^2 m_i}{2p^2 k_B T} 1 \! F \! 1 \left[ 1, \frac{3}{2}, -\omega^2 m_i \! \frac{k_B T}{2} \right].}$$

The confluent hypergeometric function $1 \! F \! 1$ is defined over an integral [16], but may be approximated by the following Padé formula for simplicity

$$1 \! F \! 1 \left[ 1, \frac{3}{2}, -x \right] = \frac{1 + \frac{x}{2} + \frac{x^2}{7} + \frac{x^3}{24} + \frac{x^4}{120} + \frac{7x^5 + x^6}{720} + \frac{9x^7}{480} + \frac{x^8}{480}}{1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{7x^6 + x^7}{720} + \frac{x^8}{480}}.$$
We can now approximate this expression for \( p \ll \omega \) and search for the minimum in the real part of the ion dielectric function by a frequency derivative. This minimum can be seen in the left panel of Fig. 1 and is approximately located at \( \omega_0 = 2.36 p(k_B T/m_i)^{1/2} \). For an ion acoustic mode to appear, the absolute value of the real part of the ion dielectric function has at least to balance the electron contribution. The latter is given in good approximation by the zero frequency value

\[
\text{Re} \varepsilon_{ee}(p, \omega)|_{\omega=0} = 1 + \frac{\kappa_e^2}{p^2}.
\]

Using the minimum frequency \( \omega_0 \) in Eq. (5) with the hypergeometric function given by Eq. (6) yields

\[
\text{Re} \varepsilon_{pp}(p, \omega_0) = 1 - 0.27 \frac{\kappa_i^2}{p^2}.
\]

For small momenta, the first summand can be neglected and the condition for ion acoustic modes to emerge becomes

\[
\kappa_e^2 \leq 0.27 \kappa_i^2 \quad \text{or} \quad T_i \leq 0.27 T_e.
\]

In the second step, we used the classical limit of Debye screening, that is \( \kappa^2 = 4\pi e^2 n/k_B T \). In warm dense matter with degenerate ions the role of the electron temperature is taken over by the electron Fermi temperature resulting in the appearance of ion acoustic modes even in equilibrium systems. Eq. (9) quantifies the long known condition of \( T_i \ll T_e \) [15] for the appearance of ion acoustic waves. Particularly under such conditions, the energy transfer rates are expected to be dominated by coupled mode effects. A numerical evaluation of the electron-ion energy transfer (3) in RPA yields lower rates and longer relaxation times of about a factor of two than a binary collision approach. This trend, but not the magnitude, is in agreement with indications also found experimentally [17, 18, 19] which points towards correlation effects in the mode structure neglected here.

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