Quantum sensing with superconducting circuits
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Sensing and metrology play an important role in fundamental science and applications, by fulfilling the ever-present need for more precise data sets, and by allowing to make more reliable conclusions on the validity of theoretical models. Sensors are ubiquitous, they are used in applications across a diverse range of fields including gravity imaging, geology, navigation, security, timekeeping, spectroscopy, chemistry, magnetometry, healthcare, and medicine. Current progress in quantum technologies inevitably triggers the exploration of quantum systems to be used as sensors with new and improved capabilities. This perspective initially provides a brief review of existing and tested quantum sensing systems, before discussing future possible directions of superconducting quantum circuits use for sensing and metrology: superconducting sensors including many entangled qubits and schemes employing Quantum Error Correction. The perspective also lists future research directions that could be of great value beyond quantum sensing, e.g. for applications in quantum computation and simulation.

I. INTRODUCTION

Quantum sensing is the procedure of measuring an unknown quantity of an observable using a quantum object as a probe. Quantum objects — those in which quantum-mechanical effects can manifest and be observed — are known to be highly sensitive to even tiny changes in their environment which is inevitably coupled to them. These changes can be so small that it is extremely challenging, or even impossible, to detect them employing classical measurements. Consequently, the fact that the probe/sensor is quantum endows it with extreme sensitivity. A back-action imposes a random change of a system state during the measurement. A probability of an outcome depends not only on the initial state of the system but also on the strength of the measurement. In the case of a quantum sensor, the back-action is quantum-limited, and measurement schemes where it can be evaded have been demonstrated, e.g. in Ref [2].

Quantum sensors are highly engineered systems for measurements ranging from gravitational pull, to magnetic and electric fields and propagating photons. Different quantum systems have been employed for sensing to date, we are giving a short overview in the following.

Thermal vapors of alkali atoms closed in a cell, pumped, and interrogated by near-resonant light are used to measure magnetic field [4]. This method is also known as nonlinear magneto-optical rotation magnetometry. Magnetometers of this type do not have intrinsic $1/f$-noise due to the absence of nearly degenerate energy states and do not require cryogenic cooling for operation; they offer millimetre spatial resolution and sensitivity exceeding $fT/\sqrt{\text{Hz}}$ [4]. Their accuracy is shot-noise limited and scales as $8B \sim 1/\sqrt{N\tau_f}$, where $N$ is the number of atoms, $T_2$ is the transverse relaxation (dephasing) time, and $\tau_f$ is the time of the signal acquisition. Spin-exchange relaxation free (SERF) operation can be achieved by increasing the gas density, and improves the sensitivity of atomic magnetometry [5]. Another type of magnetic field sensor utilises ensembles of nuclear spins. Although they are not as sensitive as atomic vapor sensors, they find applications in a variety of areas from archaeology to MRI systems [6] due to simplicity and robustness.

Nitrogen vacancy centres (NV-centres) in diamond — electron spin defects — have recently attracted a lot of attention as quantum sensors, with predicted sensitivity for ensembles of spins $\sim 0.25 \cdot fT/\sqrt{\text{Hz} \cdot \text{cm}^3}$ and experimentally achieved sensitivities of $\sim 1pT/\sqrt{\text{Hz}}$ [7]. With the advent of single spin in diamond readout [11]–[13], it became possible to use such single spins for magnetometry, sensing of electric field [14], or to measure pressure [15]. Demonstrations of frequency standards based on the NV defect centres in diamond [17] and nanoscale thermometry with down to $5\text{mK}/\sqrt{\text{Hz}}$ sensitivity have been made [18]–[20]. The main advantages of this type of sensors are their stability in nanostructures and superior $10–100\text{ nm}$ spacial resolution.

Trapped ions have been employed to detect extremely small forces and displacements. To increase the solid angle of the field access to the trapped ion, an enhanced access ion trap geometry was shown [21]. A force sensitivity of $\sim 100\text{yN}/\sqrt{\text{Hz}}$ has been reached on the crystals of trapped atomic ions, with the ability to discriminate ion displacements of $\sim 18\text{nm}$ [22]. Their augmented force and displacement sensitivity are often traded against the reduced resolution. Rydberg atoms are another physical system for quantum sensing of electrical fields. Their high sensitivity is based on huge dipole moments of highly excited electronic states [23]. Rubidium atoms prepared in circular Rydberg states were used for non-destructive (quantum nondemolition [24]–[26]) measurement of single microwave photons [26], and sensitivities reaching $3\text{mV}/\sqrt{\text{Hz}}$ were achieved when Schrödinger-cat states [23]–[25] were involved in the protocol [22]. The reader is directed to reviews on atomic spectroscopy and interferometry based sensors [26], quantum metrology with single spins in diamond [23], comparative analysis of magnetic field sensors [27], and a more general and comprehensive review on quantum sensing [8].

Superconducting quantum circuits are among the leading approaches to real-world applications with quantum computers due to their controllability and reproducibility. Here we review their past and explore their future use as quantum sensors. The long-established Superconducting Quantum Inter-
Superconducting quantum sensors

II. SUPERCONDUCTING QUANTUM INTERFERENCE DEVICES

Shortly after the theoretical prediction of the Josephson effect and its experimental observation, the quantum interference of currents was demonstrated. This interference is at the core of any SQUID magnetometer operation. There are two types of SQUID-based magnetometers: dc-SQUID (Fig. 1(a)) with a pair of Josephson junctions connected in parallel in a superconducting loop, and rf-SQUID (Fig. 1(b)) with a single junction in a loop. The experimental methods and measurement schemes of magnetic field sensing with SQUID systems are diverse and extensively studied. SQUIDs became the most sensitive tools for magnetic field measurements, with applications in geophysics and neuroscience, for example, and record magnetic flux sensitivities of ~50 nΦ0/√Hz (~50 nT/√Hz) at 100 Hz and ~50 nm loop diameter. SQUID superiority in sensitivity has only recently been challenged with the advent of SERF atomic vapor magnetometers. Despite SQUIDs high sensitivity, the accuracy of measured results at low frequencies is shot-noise limited, and improves as ~ 1/√f.

III. SUPERCONDUCTING QUANTUM CIRCUIT BASED SENSORS

Superconducting circuits including macroscopic, human-designed, many-level anharmonic systems (qubits/qudits) are a well established experimental technology platform in the field of quantum computation and simulation. The field development gained significant momentum when limitations of the conventional classical paradigm of computation became apparent in the early 1980s. At present, it is undergoing a transition to the so-called Noisy Intermediate-Scale Quantum regime, and new applications of superconducting circuits comprising qubits/qudits in quantum sensing and metrology are emerging.

The first experimental works where such circuits are used as quantum sensors have recently appeared. The frequency and amplitude of a microwave signal were determined by spectroscopic means and with time-domain measurements via ac Stark shifts of qudit higher energy levels (Fig. 1(d)). Here, an external microwave signal, with a frequency ωD and an amplitude D, shifts the transitions from their unperturbed values. The change in transition frequencies allows for a measurement of D and ωD for the applied signal to be made. Furthermore, the absolute power flowing along a transmission line or distortions of microwave control pulses were measured by strong coupling to a flux qubit. Methods to use a transmon qubit as a VNA for in situ characterization of the transfer function of xy-control lines and with time-domain measurements of microwave control pulses were demonstrated recently. These methods are useful for the calibration of microwave lines and the deduction of power reaching the circuit at millikelvin temperatures. They allow for the correction of pulse imperfections and increase fidelities of control gates used in quantum computation and simulation. All of these methods are implemented on a superconducting struc-
Superconducting quantum sensors

In quantum information processing, precise dynamic control of the quantum states is key to increasing the circuit depth. Conventional qubit frequency tuning is achieved by applying a well-controlled magnetic flux through a split junction loop within the quantum circuit. In turn, the quantum circuit can sense these externally generated static or dynamic fields. By replacing the flux-threaded split junction with a voltage biased junction (gatesynthesized), the sensed external quantity becomes a voltage instead of a current. Note that we are using magnetic flux as an external parameter here, but the sensed quantity could be a voltage too. Superconducting circuits comprising qubits/qudits possess all the properties required to construct external field sensing quantum systems\cite{33,34,35,36,37,38,39}, they have quantized energy levels; it is possible to initialize, coherently control, and read out their quantum states; and energy levels of the circuit, $E_i(\lambda)$, can be made dependent on the external parameter, $\lambda$, to be measured (Fig. 1(e)). For frequency-tunable qubits with a split junction, the parameter $\lambda$ is an external flux, $\Phi_{\text{ext}}$. If the qubit is prepared in a superposition of basis states $\{0, 1\}$ and placed in an external field, its state will accumulate phase $\phi(\Phi_{\text{ext}}) = \Delta \omega(\Phi_{\text{ext}}) \cdot \tau$, dependent on the flux $\Phi_{\text{ext}}$. $\Delta \omega(\Phi_{\text{ext}}) = \omega_i(\Phi_{\text{ext}}) - \omega_0$ is the detuning between the qubit and the control pulse frequency used for the state preparation. By applying a second control pulse identical to the first one after some time $\tau$, and measuring the population of qubit basis states, it is possible to reveal the accumulated phase in oscillating dependencies of $P_{|0\rangle}$ and $P_{|1\rangle}$. This measurement, known as Ramsey fringes interferometry, can be employed for field sensing tasks. An equal superposition state $\langle 0 + 1 \rangle/\sqrt{2}$ provides the maximal pattern visibility here, and the best sensitivity to the field.

The Ramsey fringes pattern $P_{|1\rangle}(\Phi_{\text{ext}}, \tau)$ can be simulated or directly measured as a calibration pattern before the field sensing routine. In this scenario, the outcome $P_m$ measured during the sensing procedure will be used in conjunction with the calibration pattern to determine the unknown flux value. Fig. 2 shows the simulated dependence of the probability $P_{|1\rangle}(\Phi_{\text{ext}}, \tau)$ on the external flux at different delay times $\tau$. One can see that the longer the delay time, the higher the sensitivity of $P_{|1\rangle}$ to the external flux. This is only the case if delay time is shorter than the coherence time $T_2^*$ of the qubit; for longer delay times, the sensitivity will be reduced. However, two issues should be noted here. Firstly, for an unknown flux value, it is not possible to choose a priori the delay time $\tau^*$ with the best sensitivity. Secondly, for longer delay times, it is not possible to unambiguously determine the measured flux based on a single outcome. As shown in Fig. 2, the same result $P_m$ can correspond to many flux values $\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$, and to make the measurement unambiguous, one has to reduce the dynamic range of the sensor to the interval highlighted in the figure. This interval is substantially shorter than for the shortest delay time $\tau_1$, where the measurement is single-valued (one-to-one correspondence).

The phase estimation algorithms\cite{33,34,35,36,37,38,39,40,41} (PEA) can be employed to address both issues. They gradually tune the delay time to the value $\tau^* = T_2^*$ with the highest available sensitivity, without a reduction in the dynamic range of the sensor, and appear to be powerful tools in sensing. Kitaev\cite{42} and Fourier\cite{43} phase estimation algorithms were used with a single tunable transmon qubit to measure external flux, and experimentally demonstrated the accuracy scaling beyond the SQL\cite{44}. These algorithms involve a stepped strategy. At each step of the Kitaev algorithm, the interval of possible fluxes is reduced by a factor of two based on the measurement outcome, and a new optimal delay time is found for the next step providing improved sensitivity. The optimal delay time grows from step to step on average, and gradually tend to the coherence time. PEAs allow to approach $\sim 1/\tau$ accuracy scaling (HL). The qubit coherence $T_2^*$ serves as a quantum resource. The longer it is, the more steps of the algorithm can be made before the delay time approaches $T_2^*$, and higher accuracy can be achieved in the same sensing time. Quantum sensing algorithms employing qutrits instead of qubits have also recently been considered\cite{45}.

1. Adding Entanglement

Quantum entanglement can provide improvements in attainable sensitivity\cite{46,47,48,49,50} for short interrogation times $\tau$ since entangled state of $N$ qubits, used as the probe state, allows for an $N$-fold speed-up in phase accumulation. Experimentally, this has been demonstrated for systems including three trapped $^{9}$Be$^{+}$ ions\cite{51}, four-entangled photons\cite{52}, ten nuclear spins\cite{53}, or single bosonic mode of a superconducting resonator\cite{54}. Though these experiments clearly demonstrate the improvement of sensitivity beyond the SQL with the number of entangled qubits, they do not yet provide an explicit metrologic route to allow the measurement of an unknown external field. To this end, we analyze the probability pattern $P_{|10\rangle}$ of a two-qubit state for sensing with PEAs. We use entangling conditional phase gate\cite{55,56,57,58} and simulate the evolution of the two-qubit state in QuTiP\cite{59}. Fig. 3(a) shows a time scheme used for the simulation of the pattern, where $CP_{ij}$ denotes the c-Phase gate inverting the sign of only the $|ij\rangle$ state. Relaxation $\sqrt{1/10} \hat{\sigma}_{+}$ and pure dephasing $\sqrt{1/2} \hat{\sigma}_{\phi}$ processes were taken into account in the simulation with the identical value used for both qubits decoherence rates. The flux dependence of the qubit transition frequency is assumed to be the same for
both qubits. They are equally detuned to the flux point where \(d\Delta\omega/d\Phi_{\text{ext}} \neq 0\). Starting from both qubits in the ground state, we create the \(|\Phi^+\rangle\) Bell state, apply the external flux we want to measure to both qubits, and allow the system to evolve for a variable time \(\tau\). After that, we convert the entangled state to a separable state, projecting the entangled state phase to the phase of the first qubit, shown in Eq. (1).

\[|\Phi^+\rangle = \frac{|00\rangle + e^{i\Phi_{\text{ext}}}|11\rangle}{\sqrt{2}} \]

Subsequent measurement of both qubit states, for different delay times \(\tau\) and different external fluxes \(\Phi_{\text{ext}}\), results in a pattern \(P_{10}\), shown in the Fig. 3(b), and allows for the determination of the probabilities of all four possible two-qubit states. The pattern closely resembles that of the Ramsey fringes, but has double the frequency of \(P_{10}\) oscillations, \(\phi = 2 \times \Delta\omega \times \tau\) (Fig. 3(c)).

The doubling of phase accumulation speed results in two times better accuracy of flux sensing at the same short sensing times. However, the pattern contrast also reduces more quickly (Fig. 3(c)) in comparison with a single qubit case, making the advantage less impressive for long measurement times. The quicker reduction of pattern contrast originates from the shortening of the coherence time \(T_{\text{2}}\) with the growth of the system size \(N\). It was pointed out\(^{38}\) that for entanglement only pure dephasing provides the same maximal sensitivity reached at a shorter delay time \(\tau\), in comparison with the standard Ramsey fringes scheme, because the coherence time shortens proportionally to the size of the system (∼\(N\)). However, experimental tests on up to eight trapped ion qubits, under the influence of correlated noise\(^{39}\), demonstrated a quicker coherence reduction (∼\(N^2\)). So, the pure dephasing rate can be proportional to ∼\(N^\alpha\), with \(\alpha = 1\) representing non-correlated noise and \(\alpha = 2\) representing correlated noise acting on all qubits. Experimental investigations into noise correlations between two or more superconducting qubits have only recently started to appear\(^{39,40}\). These results are important for quantum computation and quantum-enhanced sensing, and further experiments would be of great value. The dependence of the entangled state coherence time on the number of entangled qubits useful for quantum sensing has not been studied for superconducting qubits thus far.

Next, we simulate the flux sensing routines based on the Kitaev PEA run with a single qubit, and with two and three qubits prepared in the GHZ entangled state. We compare the accuracy of flux sensing achieved by employing entangled states to that of a single qubit, for the cases when \(\alpha = 1\) and 2 in the pure dephasing rates. To perform the simulation we compute the probability patterns \(P_{1000}(N, \Delta\omega, \tau)\) of the \(N\)-qubit states for \(N = 1, 2,\) and 3 as

\[P_{1000}(N, \Delta\omega, \tau) = \frac{1}{2} + \frac{1}{2}e^{-\left(\frac{N\tau_1}{2} + N\alpha\pi\Gamma_{\Phi}\right)\tau} \cos(N\Delta\omega\tau).\]  

These probabilities are obtained after projecting the phase accumulated by the GHZ \(N\)-qubit state during the evolution in the external magnetic field to the first qubit. The dependencies of the qubits’ spectra on the flux are assumed to be identical. The total relaxation rate and the total pure dephasing rate are \(\Gamma_{1,N} = N\Gamma_1\) and \(\Gamma_{\Phi,N} = N^\alpha\Gamma_{\Phi}\), with \(\Gamma_1 = 0.2\) MHz and \(\Gamma_{\Phi} = 0.034\) MHz. We use the equidistant flux grids in the computation of probability patterns with 2048, 3072, and 6144 values for 1-, 2-, and 3-qubit cases, respectively. If the sensor is exposed to the measured field only during the phase accumulation time, the dynamic range of fluxes measured with \(N\) entangled qubits is ∼\(\Delta\Phi_{\text{ext}} \sim \pi/N\min\), where \(\tau_{\min}\) is the minimal time required for switching the external field on and off. Thus, for a sensor with \(N\) entangled qubits, the dynamic range is reduced as ∼\(1/N\) in comparison with a single qubit sensor. The flux grids for 2- and 3-qubit sensors form the subsets of the grid for the single-qubit sensor (Fig. 3(a)). We choose \(F = 256\) flux values to be measured from the flux grid of the 3-qubit sensor so that it is also possible to measure them with the two other sensors. As the flux interval of possible values is reduced by 2 at each step of the Kitaev PEA, the
chosen flux grids allow us to make 10 steps of the algorithm. We repeat the algorithm $M = 24$ times at each of the $F = 256$ flux values. Fig. 4(b) shows the obtained delay times for every step of the algorithm averaged, first, over all $M = 24$ repetitions and, then, over all $F = 256$ flux values. One can see that the delay times grow on average from step to step, and tend toward the coherence time of the sensor $T_2^*$. With the reduction of the coherence time for $N = 2$, 3 or $\alpha$ going from 1 to 2 the delay times start to saturate at the earlier steps.

Fig. 4(c) shows the results of the simulations. We compute the phase accumulation time $\tau_{jkl}$ for every flux value ($j$), repetition ($k$), and the step ($l$), and then the averaged total phase accumulation time, $\overline{\tau}$, for every step as

$$\overline{\tau} = \frac{1}{F} \sum_{j=1}^{F} \frac{1}{M} \sum_{l=1}^{M} \tau_{jkl}, \quad \tau_{jkl} = \sum_{i=1}^{l} \tau_{i}^{(j,k)} n_{i}^{(j,k)}.$$  \hspace{1cm} (3)

Here, $\tau_{i}^{(j,k)}$ and $n_{i}^{(j,k)}$ are the delay time for the step number $i$ and the number of measurements done at this step for the $j$-th flux value in the $k$-th repetition, respectively. In our simulations, we use $\sigma_0 = \sigma_1 = 1.5$ for the widths of measurement outcome normal distributions for states $|0\rangle$ and $|1\rangle$, and $\varepsilon = 0.01\%$ for the error probability. These determine the number of measurements $n_i$ done at each step, and also the condition to terminate the step and discard less probable flux values. By the end of step $l$ of the algorithm, we have a probability distribution for the remaining most probable fluxes for every flux value $\Phi_j$, $j \in [1,F]$ chosen to be measured, and every repetition $k \in [1,M]$. We use this distribution to compute the mean flux values $\overline{\Phi}_{jkl}$ and find the averaged flux accuracy for every step as

$$\frac{(\delta \Phi)(\Phi_0)}{l} = \left[ \frac{1}{\overline{\Phi}_0 F} \sum_{j=1}^{F} \frac{1}{M-1} \sum_{k=1}^{M} (\Phi_{jkl} - \overline{\Phi}_j)^2 \right].$$  \hspace{1cm} (4)

Dependencies of the averaged flux accuracy $(\delta \Phi/\Phi_0)$, on the averaged total phase accumulation time $\overline{\tau}$ are shown in Fig. 4(c).

We compare the improvements of the flux sensing accuracy with the phase accumulation time for the sensors comprising a single qubit, or 2 and 3 entangled qubits with $\alpha = 1$ or 2, in Fig. 4(b). One can see that for the first algorithm steps, the scaling of the flux accuracy is close to the HL scaling for all considered sensors. When the averaged delay time approaches the coherence time and starts to saturate (Fig. 4(b)), the accuracy scaling deviates from the HL scaling and returns gradually back to the SQL scaling. The shorter the coherence time of the sensor, the sooner this transition happens, so that the sensors with 2 and 3 entangled qubits deviate from the HL scaling at the earlier steps of the algorithm. Nevertheless, the accuracies at the same phase accumulation time achieved by the sensors with the entangled qubits are always better than that of the sensor based on a single qubit, and the sensor with 3 entangled qubits proves to be better than with 2 entangled qubits. The advantage in the accuracy is reduced as the crossover from the HL scaling to the SQL scaling occurs, but the advantage from the earlier steps of the algorithm is not completely lost at the later steps even when $\alpha > 1$. Importantly, for all pure dephasing rates with $\alpha \in [1,2]$, there is an accuracy improvement caused by the use of the entangled sensor.

In practice, the calibration of a sensor employing PEA – the measurement of the probability pattern $P_{|0\rangle\langle0|}$ can take a long time. To mitigate this, FPGA-based electronics can be used for fast reset of the sensor qubits. If the duration of control pulses and the time to read out and reset the qubits are...
much shorter than the coherence time of the sensor, the total sensing time will almost entirely consist of the phase accumulation time. This will noticeably shorten the calibration and speed up the sensing itself.

Another experimental aspect of employing entangled states for sensing is the sensitivity of the control pulses to the external field being measured. Conditional phase gates realized via flux control pulses are very sensitive to external magnetic fields, which makes it necessary to allow the external field to act on the system only during the phase accumulation time. Otherwise, the initial entangled $N$-qubit state will not be the desired one. With regard to this, all-microwave entangling gates\cite{42} can be considered as an alternative way of the preparation of the sensing state. If they appear to be more resilient to the field being measured, it will be possible to keep the field continuously present, simplifying the operation.

It is important to emphasize that QEC schemes rely on the realization of fast and full quantum control, where qubit read-out, analysis, and reaction times are much shorter than the coherence time of the sensor. The development of experiments where PEAs are combined with QEC is of great importance, as such experiments can contribute substantially to future progress of quantum sensing and metrology.

IV. CONCLUSIONS

Two strategies involving entanglement for sensing and metrology with superconducting quantum circuits are considered, with the aim of going beyond the SQL scaling in the time domain. The first is based on the increased speed of phase accumulation for a large-scale entangled state of $N$ sensors. The advantages seen for this strategy depend on the characteristics of noise seen by the entangled state. Future experimental studies of the coherence reduction with the size of the entangled state $N$ are interesting and necessary, but have not yet been undertaken with superconducting qubits. The second strategy is to use QEC on entangled pairs (sensor-ancillary) of superconducting qubits, with the idea of enhancing the coherence time of the sensor qubits. To this end, implementation of metrological protocols combining QEC with one of the PEAs could experimentally demonstrate magnetic field sensing beyond the SQL scaling in time. Proposed experiments are of high value for quantum metrology and sensing.

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Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Superconducting quantum sensors

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