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Shear Resistance Mechanism Evaluation of RC Beams Based on Arch and Beam Actions

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Abstract

In order to enhancement accuracy of shear design of reinforced concrete (RC) beams, detail understanding of the shear resistance mechanism is required. This study evaluated the shear resistance mechanism of RC beams based on arch and beam actions by using three dimensional Rigid-Body-Spring-Method (3-D RBSM). Firstly, RC deep and slender beams with and without shear reinforcement failed in shear were tested to measure local behavior. Then, the validity of local behaviors obtained from 3-D RBSM was confirmed by comparing with the test results and the applicability of decoupling of shear resistance mechanism using simulated stress distribution was presented. Moreover, the contributions of arch and beam actions in RC beams until failure stage were investigated numerically by changing the shear reinforcement ratio and shear span to depth ratio and was compared with the current shear design recommendations in JSCE Standard Specification. As a significant finding, the numerical results upon the quantitatively evaluation of shear resistance mechanisms that the shear strength of RC beam could be evaluated without classification of deep beams and slender beams was presented.

1. Introduction

It is well-known that the shear resistance mechanism of RC beam can be decoupled into beam ($V_b$) and arch actions ($V_a$) (Park et al. 1975), where the beam action is considered as a combination of the traditional truss action ($V_t$) and concrete contribution for beam action ($V_{bc}$).

The contribution of $V_{bc}$ consists of doweling effect and aggregate interlocking and it is equivalent to shear cracking load when diagonal shear failure occurs (Kim et al. 2011). The differences in shear resistances of RC beams are attributed to the variation of beam and arch actions contribution, which are highly affected by shear span to depth ratio and shear reinforcement ratio.

In current shear design of RC beam, as the common principles, ACI 318M-14 (ACI 2015), fib Model Code 2010 (fib 2013) and JSCE Standard Specification (JSCE 2012) recommend to design the shear of slender beam and deep beam separately. In terms of the shear design for slender beam, all the above three recommendations suggest to give the shear resistance taking into account of the total concrete contribution ($V_{con}$, including arch action and concrete contribution for beam action) and the contribution by shear reinforcement ($V_s$). The design formula for concrete contributions are based on the statistical result of extensive experimental database (Kennedy 1967; Zsutty et al. 1968; Okamura and Higai 1980; Niwa et al. 1986) and the shear reinforcement contributions are assessed by truss theory, originally developed by Ritter (1899) and modified by several researches (Ramirez and Breen 1991; Collins and Mitchell 1997).

As the individual characteristics, fib code takes into account of the effect of the level of maximum aggregate size in the design of concrete contribution ($V_{con}$) and represents a variable inclination of the compression diagonal in the design of shear reinforcement contribution, while the ACI and JSCE code assumes the truss angle as 45°. With regard to the shear design for deep beams, first it should be noted that the above three codes have their individual limits of shear span to depth ratio (a/d) to define the range of deep beams, that is, the ACI and fib codes limit the a/d of deep beam smaller than 2.0 whereas the JSCE code limits it smaller than 2.5. Moreover, in the design of shear strength, ACI and fib codes apply strut-and-tie model to give required strengths of compression strut, nodal zone (strengths of loading plate zone and support zone) and tie chord, respectively, while the JSCE code is empirical formula and consider the sufficient strength of arch rib (originating from arch action) considering the width of loading plate along the longitudinal axis. As the enhancement effect of shear reinforcement for deep beams, the JSCE code (JSCE 2002) originally suggests adopting the effect of truss action directly, and the later modification (JSCE 2007)
explains that shear reinforcement improves shear strength because of the enhancement of arch action. Therefore, a strengthening factor for arch action is applied currently. In summary, it was understood that, for above three design codes, the shear designs for slender beams and deep beams are based on the completely different concepts and models. On the other side, the formula for shear design of RC beams that considers the relationship between shear resistance mechanism and critical structural variables such as shear reinforcement ratio and shear span to depth ratio, had been proposed by Architectural Institute of Japan (AIJ 1990). A combining shear design was likely to be established for slender and deep beams if the shear resistance mechanisms could be quantitatively evaluated taking into account of the effect of the main structural variables such as shear span to depth ratio. However, a full understanding of the shear resistance mechanism still remains as an unrevealed matter.

In order to evaluate the real-like behavior of different shear failure modes and further establish a rational formula for shear strength of RC beams, a quantitative evaluation of shear resistance mechanism is necessary, which signifies that the full understanding of the local stress behavior in RC beam is needed. For instance, with an intention to grasp the shear resistance mechanism of reinforced concrete, the estimation of the contributions due to concrete compressive strut, aggregate interlocking and doweling effect have been attempted (Campana et al. 2013; Cavagnis et al. 2016). Moreover, the local stress states at each deformation are the crucial factors for clarifying each contribution of shear resistance mechanisms such as beam action, arch action and truss action (Iwamoto et al. 2017; Fu et al. 2017). However, the measurement and evaluation of local stress state in RC member currently is not easy to be achieved in experimental conditions and conducting the direct evaluation of shear resistance mechanisms by this approach is a difficult issue. As an effective approach, numerical modeling and analysis appears to evaluate the shear resistance mechanism by investigating local stress state conditions. Recently, sophisticated numerical methods such as the three dimensional Rigid-Body-Spring-Method (3-D RBSM) have been developed. 3-D RBSM is one of the discrete approaches and proven to be an effective tool in simulating crack propagation realistically and visualizing stress distribution in concrete. The method has been applied several problems and success to simulate accurately not only global behavior such as load-displacement relationship but also local behavior such as crack and stress distribution (Yamamoto et al. 2010; Gedik et al. 2011, 2012; Yamamoto et al. 2014; Iwamoto et al. 2017; Fu et al. 2017). Especially, Iwamoto et al. (2017) evaluated the shear resistance mechanism of RC beams with shear span depth ratio ($a/d$) of 3.14 based on arch and beam actions by using RBSM. But the evaluation was limited only the beams of $a/d = 3.14$ and a wide range investigation for RC beams was desired.

This study evaluated the shear resistance mechanism of RC beams with wide range of shear span to depth ratio and shear reinforcement ratio based on arch and beam actions by using three dimensional Rigid-Body-Spring-Method (3-D RBSM). Firstly, RC deep beams of $a/d = 1.57$ and slender beams of $a/d = 3.14$ with and without shear reinforcement failed in shear were tested to measure local behavior. Then, the validity of local behaviors obtained from 3-D RBSM was confirmed by comparing with the test results and the applicability of decoupling of shear resistance mechanism using simulated stress distribution was presented. It is noted that the results of slender beams obtained by Iwamoto et al. (2017) are referred and are described for convenience to understand applicability both deep and slender beams consistently. Moreover, the contributions of arch and beam actions in RC beams until failure stage were investigated numerically by changing the shear reinforcement ratio and shear span to depth ratio. Findings on the shear resistance mechanism were also compared with the shear design recommendations given in the current JSCE Standard Specification (2012) and discussed to denote the shear design method of RC beams.

2. Numerical method

2.1 3-D RBSM

The authors have developed the 3-D RBSM (Yamamoto et al. 2008) in order to quantitatively evaluate the mechanical responses including softening and localization fractures, and have shown that the model can well simulate the cracking and failure behaviors of RC members (Yamamoto et al. 2010; Gedik et al. 2011, 2012; Yamamoto et al. 2014; Iwamoto et al. 2017; Fu et al. 2017). In RBSM, concrete is modeled as an assemblage of rigid particles interconnected by springs at their boundary surfaces (Fig. 1a). The crack development is af-
fected by mesh design as cracks initiate and propagate through interface boundaries of particles. Therefore, a random geometry of rigid particles is generated by Voronoi tessellation (Fig. 1b), which can reduce mesh bias on the initiation and propagation of potential cracks.

The response of the spring model, which consists of one normal and two tangential springs, provides an insight into the interaction among the particles that is different from the models based on continuum mechanics. In the model, each rigid particle has three translational and three rotational degrees of freedom defined at the nuclei that generate particles according to the Voronoi tessellation.

The constitutive models for tension, compression and shear springs used in 3-D RBSM were constructed by uniaxial relationships. The details of the models and the related model parameters for monotonic loading analysis were described in the several papers (Yamamoto et al. 2008, 2010).

### 2.2 Reinforcing bar and bond models

Reinforcing bar is modeled as a series of regular beam elements (Fig. 2) that can simulate the bending effects. The rebar can be freely positioned within the member, regardless of the mesh design of concrete (Bolandar et al. 2002). At each beam node, two translational and one rotational degrees-of-freedom (DOF) are defined by means of springs. The rebar is attached to the concrete particles by zero-size link elements that provide a load-transfer mechanism between concrete particles and beam nodes. A bi-linear model is assumed for the stress-strain relationship of rebar. After yielding of rebar, the hardening coefficient is assumed as 1/100. Crack development is strongly affected by the bond interaction between concrete and linked element of rebar. For the bond-stress relation, up to the shear strength, the relation proposed by Suga et al. (2001) is applied and the relation proposed by CEB-FIB Code Model (1990) is used for the post peak behavior.

### 3. Evaluation of local and macro behavior in shear failure

#### 3.1 Experimental Setup

Two deep beams and two slender beams were designed, considering the different failure patterns due to different shear span to depth ratio, in order to investigate the local behaviors in shear failures. The details of specimens are shown in Table 1 and Fig. 3. Specimens No. 1 and No. 2 were designed for deep beams, where the shear span to depth ratios were 1.57, while Specimens No. 3 and No. 4 were designed for slender beams where the shear span to depth ratios were 3.14. As the common points of the four beams, the effective depths \((d)\) were 255 mm and the longitudinal tension reinforcement ratios \((p_t)\) were 3.36% (2 × D29 bars). In the beams, no or few shear reinforcements was arranged in shear span 2 where a shear failure was expected to generate, whereas

### Table 1 Specimen details.

| Specimen | \(a/d\) | \(\psi_t\) (%) | \(f_y\) (MPa) | \(E_t\) (GPa) | \(\psi_w\) (%) | \(f_{wy}\) (MPa) | \(E_w\) (GPa) | Compressive strength of concrete (MPa) | Design shear strength (kN) |
|----------|---------|--------------|--------------|-------------|--------------|--------------|-------------|-----------------------------------|------------------------|
| No. 1    | 1.57    | 0.00         | 363          | 179         | 31.9         | 309.1        | 309.1       |                                     | 430.9                  |
| No. 2    | 3.14    | 0.28         | 182          | 363          | 40.8         | 430.9        | 430.9       | 132.8                             | 132.8                  |
| No. 3    | 0.00    | 0.00         | 358          | 182          | 40.8         | 179          | 179         | 132.8                             | 132.8                  |
| No. 4    | 3.36    | 0.28         | 358          | 182          | 40.8         | 200.8        | 200.8       | 132.8                             | 200.8                  |

Fig. 3 Test specimens. (unit: mm)
the shear reinforcement ratio ($p_w$) in shear span 1 (2 × D6 bars used with spacing of 60 mm) was 0.70% (Fig. 3). The only difference between Specimens No. 1 and No. 2 or between Specimens No. 3 and No. 4 was that shear reinforcements were also arranged in the shear span 2 of Specimen No. 2 and Specimen No. 4 with $p_w$ of 0.28% (2 × D6, with the spacing of 150 mm), as shown in Figs. 3b and 3d. Additionally, all of the other parameters and dimensions were kept same. It was a 3-point loading test with a concentrated imposed load on the middle with a loading plate having 80 mm width. The design shear strengths for the four beams were also calculated and are shown in Table 1. The calculations for No.1 and No. 2 were based on the Eq. (7) and Eq. (9) while the calculation for No. 3 and No. 4 were according to the Eq. (5) discussed in the following chapters.

During the test, the vertical displacements at loading point and supports were measured. To investigate local behavior, internal local strains of concrete and strains of rebars were also measured at several locations that are presented in Figs. 3 and 4. The strain gauges G1, G2, and G3 were arranged at the mid-height of shear reinforcements to measure strains of shear reinforcements in Specimens No.2 and No.4. To measure the internal concrete strain distributions in the beams, the acrylic bar method introduced by Nakamura et al. (2001) was adopted. The deformed acrylic bars (rectangular cross section), the side length of which was 10 mm, were embedded horizontally into the beams at four levels as shown in Fig. 4. The vertical spacing between adjacent acrylic bars was 60 mm. The strain gauges were attached to the acrylic bars along axial direction on six cross-sections numbered from [1] to [6], and at four height levels. Therefore, the local strains of concrete were measured at 24 different locations within each objective shear span.

3.2 Numerical model
Numerical models prepared for Specimens No. 1 - 4 are shown in Fig. 5. The beams were modeled by using 3-D RBSM and the rigid particles of concrete were randomly generated with an average element size of 15 mm. Concrete properties used such as Young’s modulus, compressive strength, and tensile strength were calibrated based on uniaxial compression and tension test results of standard cylinders (100 mm in diameter and 200 mm in height). The tensile strength of concrete was determined based on the formula proposed by JSCE Standard Specifications (JSCE 2012). The steel bars were modeled by using beam elements. Steel rebar properties were adopted to the models based on the test results (see Table 1). The acrylic bars used for measurement of concrete strain were also modeled by using beam elements with weaker material properties, i.e. Young’s modulus $E_s = 3000$ MPa, which were close to concrete and would not activate the change of structural behaviors of RC beams. It should be mentioned that more acrylic bars than test were modeled in numerical analysis, in order to obtain more measurements of concrete strain and stress. Thereby, 14 acrylic bars, spaced by 20 mm, were arranged in analysis.
3.3 Comparison of macro behaviours between test and analysis

(1) Specimens No. 1 and No. 2 (a/d = 1.57)

To investigate the macro behaviors in shear failure of deep beams, experimental and numerical results are compared and discussed. Load-displacement curves obtained from numerical and test results are compared for the Specimens No. 1 and No. 2 as shown in Fig. 6. In numerical results, loading stages are marked by A to D for the Specimen No. 2 and a to d for the Specimen No. 1. To be specific, the points a, b, c, d (or A, B, C, D) represent for the four loading stages: stage before inclined shear cracking, inclined shear cracking stage, stage after inclined shear cracking and peak load stage.

For Specimen No. 1, the first inclined shear crack initiated at the displacement (δ) of 0.39 mm (load P = 132.5 kN) in the test. Then, when δ reached 1.97 mm (P = 331.6 kN), the peak load was reached and the load significantly decreased hereafter in the post-peak region. In the numerical analysis, at δ of 0.30 mm (P = 128.9 kN (b)), the first inclined shear crack initiated, and when δ was increased to 1.51 mm (P = 342.4 kN (d)), the peak load was reached. The load-displacement curves of test and numerical analysis agreed significantly well. Comparison of crack propagations between the test and analysis at the peak load (d) is given in Fig. 7 where it was confirmed that the test crack pattern could be well simulated by analyses.

In the test of Specimen No. 2, an inclined shear crack initiated almost at the same displacement in Specimen No. 1. When δ was increased to 1.99 mm (P = 376 kN), the load reached the peak and then decreased in the post-peak region. The shear strength was increased by 44.4 kN, compared with that of the Specimen No. 1, which was attributed to the effect of shear reinforcement. As for the numerical analysis, the inclined shear crack initiated when δ reached 0.31 mm (P = 129.5 kN (B)). The peak load (P = 389.2 kN (D)) was reached at δ of 1.89 mm. A significantly good agreement between the test and analysis results of load-displacement curves could be noted. In Fig. 8, the comparison of crack propagations between the test and analysis at the peak load (D) is illustrated. It is shown that the crack patterns for the test and analysis came close.

(2) Specimens No. 3 and No. 4 (a/d = 3.14)

Similarly, the load-displacement curves obtained from numerical and test results are compared for the slender beams, Specimens No. 3 and No. 4, as shown in Fig. 9. It is noted that the results are referred from the results of Iwamoto et al. (2017). In the numerical results, loading stages are marked by a to d for Specimen No.3 and A to D for Specimen No.4, with a range before first inclined cracking to peak load. For specimen No.3, the first inclined shear crack initiated at the displacement (δ) of 1.67 mm (P = 109 kN) in the test. Then, δ was increased to 2.14 mm (P = 121.7 kN) and the shear crack propagated toward the loading point with a slight decrease in load. Finally, when δ reached 4.86 mm (P = 180 kN), the peak load was reached and then, the load was decreased hereafter in the post-peak region. In the numerical analysis, at δ of 1.60 mm (P = 108 kN (b)), the inclined shear crack initiated, and when δ was increased to 4.60 mm (P = 194.5 kN (d)), the peak load was reached. The load-displacement curves of test and numerical analysis agreed significantly well. The comparison of crack propagations between the test and analysis at the peak load (d) is given in Fig. 10, and it was also observed that the test crack pattern after failure could be well simulated by analyses.

In the test of Specimen No. 4, an inclined shear crack initiated almost at the same displacement in Specimen No. 3. When δ was increased to 4.63 mm (P = 218.7
kN), the load reached the peak and then was decreased in the post-peak region. As for that of the numerical analysis, the inclined shear crack initiated when $\delta$ reached 1.70 mm ($P = 115$ kN (B)). The peak load ($P = 228.4$ kN (D)) was reached at $\delta$ of 5.10 mm. A significantly good agreement between the test and analysis results in terms of load-displacement curves was confirmed. In Fig. 11, from the comparison of crack propagations between the test and analysis at the peak load (D), it could be observed that crack patterns in the test and analysis came close.

Therefore, based on the above result comparisons between the test and numerical analysis, it was confirmed that 3-D RBSM could accurately simulate the shear failure behaviors of the deep beams and slender beams at macro level including load-displacement curves, crack development and effect of shear reinforcement.

3.4 Comparison of local strain behaviour results between test and analysis

To verify the ability of 3-D RBSM for simulation of local strain behavior, shear reinforcement strains obtained by gauge measurements and concrete strain growths obtained by using acrylic bars in the test were compared with the results from numerical analysis.

(1) Specimens No. 1 and No. 2 ($a/d = 1.57$)

Figure 12 illustrates the relationships between shear reinforcement strains at shear span 2 of Specimen No. 2 and the mid-span displacement, and the corresponding external loads are included in the figure as well.

In the test, the distinct developments of shear reinforcement strains of the three gauges (G1, G2 and G3) were successfully monitored. The stain by G2 began to increase after shear cracking and did not yield until the maximum load stage, the strain suddenly rising in the post-peak stage. The stains by G1 and G3 did not present dramatic growths, and apparently the stain by G3 implied a compressive deformation, as G1 and G3 were set at the mid-height of the beam, obviously not crossing the critical shear crack, and the shear reinforcements did not bear great force at the positions of G1 and G3. In analysis, the same developing trends of strains were confirmed.

Figure 13 shows the internal stress and strain distributions of concrete in cross sections [1] to [6] (stress and strain components are normal to the plane and positions are shown in Fig. 4a) for Specimen No.1 at loading stages of (a) and (d) (see Fig. 6). Dash and solid lines represent the test and numerical results, respectively. The numerical stress distributions at each cross section were also plotted for reference. Herein, the local horizontal strain was obtained from the numerical result of beam element, horizontally put in model, and it was calculated by the ratio of elongation of local beam element to its initial length, and for the local horizontal stress on each element, it was calculated by the following process: firstly, the total horizontal force resultant on the Voronoi element was calculated by adding all the horizontal force resultant on each side surface of Voronoi element in three dimensional arrangement; secondly, the local horizontal stress was calculated by the ratio of total horizontal force resultant to the area of the vertical cut surface of the element along the center point.

Prior to the initiation of inclined shear cracking (a), in test and numerical results, linear strain distribution patterns were obtained for each cross section and similar patterns for stress distributions were also observed. At loading stage (d), peak load, the strain distributions near loading plate showed linear pattern consistently. At cross-sections [1] and [2], near the loading plate, maximum strain values located at the top of cross-sections.
With the increase in the distance from loading plate (from \([1]\) to \([6]\)) however, the locations of the maximum strains moved from top levels to bottoms along the height of cross-sections and linear distribution was tailed off, which was considered caused by the occurrence of shear crack at different height of the cross-sections along beam axis. The analysis result for each cross section agreed well with the preceding test result. In addition, it was worthy note that the numerical stress distributions consistently showed similar pattern to the numerical strain distributions at pre-peak region and at the peak load. This higher accuracy in the numerical analysis was attributed to an accurate simulation of crack diagonal. Herein, only the results for Specimen No. 1 was shown and discussed, because a similar result was also observed for Specimen No. 2.

(2) Specimens No. 3 and No. 4 (\(a/d = 3.14\))

In terms of the slender beams, Fig. 14 illustrates the relationships between shear reinforcement strains at shear span 2 of Specimen No. 4 and the mid-span displacement, and the corresponding external loads were included in the figure as well. It is noted that the results are referred from the results of Iwamoto et al. (2017).

In the test, shear reinforcement strains measured by three gauges (G1, G2 and G3) began to increase significantly just after the initiation of inclined shear cracking. The strain by G1 was increased at higher rates towards the peak load and reached yielding state. In analyses, similar incremental trends were obtained for the same points with gauges G1, G2 and G3 up to the peak load. Analysis results agreed significantly well with the tests, although the increase in strain at a higher rate was obtained at G2 point.

**Figure 15** shows the internal strain distributions of concrete in cross-sections [1] to [6] (positions are shown in Fig. 4b) in Specimen No. 3 at loading stages of (a) and (d) (see Fig. 6). Dash and solid lines represent the test and numerical results, respectively. Numerical stress distribution values at each cross section were also plotted for reference. Prior to the initiation of inclined shear cracking (a), in both test and numerical results, a linear strain distribution pattern was obtained for each cross section. A similar pattern for stress distribution was also observed. At loading stage (d), which is the peak, the strain distributions near loading plate showed a linear pattern consistently. At cross-section [1], near the loading plate, maximum strain values occurred at the top of cross-section. With the increase in the distance from...
loading plate (from [1] to [6]), similar to those of the Specimen No. 1, the locations of the maximum strains moved from top levels to bottoms along the height of cross-sections and linear distribution was tailed off. In analysis, it was clarified that the change of strain distributions along beam axis of slender beam could be captured and the numerical stress distributions consistently showed similar pattern to the numerical strain distributions. Only the results for Specimen No. 3 were shown here, since similar results were also observed for Specimen No. 4.  

Thus, it was demonstrated that 3-D RBSM could well compute the local stress state in the RC beams with varying shear span to depth ratio, and this was the basement of the assessment of shear resistance mechanism depending on the calculation using local stress information.

4. Decoupling of shear resistance mechanisms

4.1 Decoupling method of shear resistance mechanisms

It has been mentioned that the shear resistance mechanism of RC member consists of beam and arch actions. The decoupling method of beam and arch actions based on the results of 3-D RBSM was already applied by Fu et al. (2017). On the basis of the previous work by Park et al. (1975), the mechanical equilibrium of the cross section of a RC member, as shown in Fig. 16, could be expressed by Eq. (1).

\[ M = \left( T_c + C_c \right) \frac{j_c}{2} + C_c \cdot j_{c,c} + T_c \cdot j_{c,c} \]  

where, \( M \) is the bending moment acting on the cross section, \( T_c \) and \( C_c \) are forces sustained by longitudinal reinforcing bars in tensile and compression regions, \( C_c \) is the resultant compressive force in concrete; \( T_c \) is the resultant tensile force in concrete; \( j_t \) is the arm length between the centroids of tensile and compressive longitudinal reinforcing bars; \( j_{c,c} \) is the arm length between concrete compressive resultant and the beam axis and \( j_{c,c} \) is the arm length between concrete tensile resultant and the beam axis.

If we differentiate Eq. (1) by a small segment \( \Delta x \) between two adjacent cross sections as shown in Fig. 17a, the equation for calculation of shear resistance can be derived and the shear resistance can be divided into the two contributions of beam action and arch action expressed by Eq. (2) and Eq. (3).
Fig. 18

The contribution of $V_b$ is defined as the beam action, which consists of incremental change of rebar tension $dT_r$, and rebar compression $dC_r$ and concrete compressive and tensile resultants of $dC_c$ and $dT_c$ between two adjacent cross sections (Fig. 17b). The contribution of $V_c$ is defined as the arch action, which is caused by incremental change for centroids of concrete compressive ($dC_c$) and tensile ($dT_c$) resultants, along the member axis (Fig. 17c).

Concrete stress distributions along a beam axis are shown conceptually in Fig. 18 on a longitudinal beam section. It was clear that if the intensity of stresses and centroids of compressive stress block were determined section. It was clear that if the intensity of stresses and centroids of compressive stress block were determined for two adjacent cross sections with a small space of $dx$, by using the above equations, the beam and arch actions could be calculated separately. The variables required for that calculation of beam action and arch action could be obtained from the local stress results by 3-D RBSM analysis, which was a difficult task to investigate by test method directly. In this study, a small increment of 50 mm for $dx$ was adopted for an accurate evaluation of decoupling shear resistance mechanism, i.e. decoupling of beam and arch actions in the objective shear span of a RC beam.

Based on the truss theory, which was one part of beam action, the shear resistance of truss action ($V_s$) provided by shear reinforcement could be evaluated by Eq. (4),

$$V_s = \frac{A_w \cdot \sigma_{w} \cdot j \cdot d}{s}$$

where, $A_w$ is the cross sectional area of shear reinforcement with a spacing of $s$; $\sigma_{w}$ is the stress intensity of shear reinforcement; $j \cdot d$ is the effective arm length, where $j = 1 / 1.15$ and $d$ is the effective depth of RC beam.

In current study, an individual truss action for each shear reinforcement arranged in the shear span was calculated using the maximum stress values in beam elements of shear reinforcements in analyses by assuming that the diagonal shear crack angle was 45°. Then, the truss action ($V_s$) was subtracted from the beam action ($V_b$) at each displacement and the concrete contribution ($V_{bc}$) for beam action could be determined. In addition, the combination effect of concrete contribution ($V_{bc}$) and the arch action ($V_c$) was considered as the shear resistance provided by concrete ($V_{con}$), which would be discussed in the following section.

4.2 Decoupling results of shear resistance mechanisms

According to the decoupling method given in chapter 4.1, the development of shear bearing force of the four beams were decoupled into crucial shear resistance mechanisms, and as typical result, the decoupling results of Specimen No. 3 and No. 4 of slender beam would be introduced, for understanding of the developing process of each shear component and the reinforcing effect of shear reinforcements. It is noted that the results are referred from the results of Iwamoto et al. (2017).

(1) Specimen No. 3

For specimen No. 3, no shear reinforcements arranged, the shear loads provided by beam and arch actions along the beam axis of shear span without shear reinforcements at the loading stages of a, c and d (see Fig. 9) are plotted in Fig. 19. The left and right ends of the horizontal axis represent the positions of loading point and the support in the objective shear span 2, respectively. The beam and arch actions varied along the beam axis singly where their combined resistances were equivalent to the external shear loads that demonstrates the reliability of the decoupling method. Prior to the inclined shear cracking (a) (see Fig. 19a), it was observed that the total shear load was almost entirely sustained by the beam action while the arch action had so minor effect on the shear resistance. After inclined shear cracking (c) (see Fig. 19b), the arch action increased significantly and on the contrary, the beam action was decreased comparing with the case at stage (a). At the peak load (d) (see Fig. 19c), the arch action was further increased, while a few changes were observed on the beam action contribution compared to that at stage (c).

The relationship between the contributions of beam and arch actions and the displacement of loading point is shown in Fig. 19d. Herein, the average beam and arch actions of all elements $dx$ along shear span were adopted. Due to extreme stress concentration occurred at the areas within 50 mm from loading and support points, the results of these areas were excluded. The
results show that, after the occurrence of shear crack (b), the arch action was increased, whereas the beam action was decreased rapidly. Then, the beam action stopped declining at the stage (c) and remained constant until the peak load. On the other hand, the arch action was constantly increased until the peak load (d), and then the load started to decrease due to the decline of arch action.

(2) Specimen No. 4
For specimen No. 4, the shear loads provided by beam and arch actions along the beam axis of shear span with shear reinforcements at the loading stages of A, C and D (see Fig. 9) are plotted in Fig. 20. Prior to the occurrence of inclined shear crack (A) (see Fig. 20a), similar to that of Specimen No.3, it was observed that the total shear load was mostly sustained by the beam action while the arch action had so limited effect on the shear strength. After inclined shear cracking (C) (see Fig. 20b), the arch action was increased significantly, while the beam action remained constant which was a different behavior from that observed in Specimen No. 3. That was due to the truss action provided by the shear reinforcements in the shear span 2 in this case. At this stage, some part of shear reinforcements reached the yielding stage. At the peak load (D) (see Fig. 20c), the arch action rose further, while the beam action slightly declined. It was obvious that all of the shear reinforcements were yielded except for those close to support point and hence the truss action remained constant thereafter. Similarly, the relationship between the average contributions of beam and arch actions and the displacement of loading point is shown in Fig. 20d. It was clear that, after the shear cracking, the arch action was increased at a stable rate until the peak load (D), while a gradual decrease of the beam action was observed. On the other hand, the truss action started to rise rapidly, just after the shear cracking and it remained constant after the yielding of shear reinforcements. The majority of beam action was contributed by the truss action near the peak load while the concrete contribution \( V_{bc} \) was almost zero.

Therefore, by the same way, the development of shear resistance components of RC beams could be precisely assessed, and it became accessible to evaluate the effect of main structural variables on the capacities of each shear component.

5. Discussion of effect of shear span to depth ratio and shear reinforcement ratio

It was widely reported that the change of shear span to depth ratio \( \frac{a}{d} \) could lead to transformation of shear failure mode. That is, deep beams, considered as \( \frac{a}{d} < 2.5 \) in JSCE Standard Specification (JSCE 2012) and \( \frac{a}{d} < 2.0 \) in ACI 318M-14 (ACI 2014) and fib Model Code (fib 2010), ordinarily suffer from shear compression failure in web zones, whereas a shear tension failure usually occurs in slender beams due to dramatic devel-
opment of inclined shear crack. Thus, based on dimensions of the tested RC beams, the beams with a wide range of $a/d$ ratios (i.e., 1.57, 2.35, 3.14, 3.92, 4.31, 4.71), including deep beams and slender beams, were analyzed. The compressive strengths for all cases were taken as 40.8 MPa, which is same as that of No. 3 and No. 4 specimens. In addition, arrangement of shear reinforcements is fundamental of shear design and effect of shear reinforcements should be investigated more by considering combination of $a/d$. Therefore, for each $a/d$ beam series, five classes of shear reinforcements ratio $p_{w}$ ($0.00\%$, $0.19\%$, $0.28\%$, $0.42\%$, $0.56\%$), were taken into account. The results for evaluation of shear resistance mechanisms depending on $a/d$ ratios and shear reinforcement ratio $p_{w}$ would be discussed and compared with the current JSCE Standard Specification (JSCE 2012).

In the JSCE Standard Specification (JSCE 2012), a modified truss theory is applied as represented by the following Eq. (5) and Eq. (6):

$$V_c = V_{u1} + V_{u2}$$

$$V_c = 0.20 \cdot f_{c'}^{1/3} \cdot p_{l}^{1/3} \cdot d^{-1/4} \cdot (0.75 + 1.4/(a/d)) \cdot b_o \cdot d$$

where, $V_c$ is the shear strength of a RC member; $V_{u1}$ is design shear capacity of linear members without shear reinforcements; $V_{u2}$ is the shear reinforcement contribution as introduced in Eq. (4); $f_{c'}$ is the concrete compressive strength; $p_{l}$ is the longitudinal tension reinforcement ratio. $b_o$ is the width of a beam member, $a$ is shear span and $d$ is the effective depth. Herein, $V_c$ is usually considered to be equivalent to the inclined shear cracking load of a RC member without shear reinforcement.

On the other hand, Niwa et al. (1983) proposed the following equation to evaluate the shear strength for deep beams without shear reinforcement.

$$V_{u2} = \frac{0.24 \cdot f_{c'}^{1/3} \cdot (1 + \sqrt[3]{100p_{w}}) \cdot (1 + 3.33r/d) \cdot b_o \cdot d}{1 + (a/d)^{3/2}}$$

where, $V_{u2}$ is the shear strength of RC deep beam without shear reinforcement; and $r$ is the length of loading plates along beam axis.

In the case of deep beams with shear reinforcements, it was proved that the shear reinforcement contribution is confirmed but a direct use of the truss theory is not proper due to the contribution reduction for smaller $a/d$ (Hayashikawa et al. 1990). To consider this, an empirical coefficient of $\phi$ was adopted to reduce the shear reinforcement contribution $V_c$ by the following method (JSCE 2002):

$$V_{u2} = V_{u2} + \phi \cdot V_1$$

$$\phi = -0.17 + 0.3a/d + 0.33/(100p_{w})$$

where, $V_{u2}$ is the shear strength of a RC deep beam with shear reinforcement.

Tanimura et al. (2004) further presented a study based on FEM analyses and demonstrated that the shear reinforcement effect on deep beams was attributed to the increase of compression zone, where an arch action develops. This consideration was also adopted to the later JSCE Standard Specification (JSCE 2007) as given in Eq. (9) to design the shear strength of RC deep beams with shear reinforcements, where the calculation $V_{u2}$ in Eq. (7) was magnified by a coefficient $k$ to calculate the overall strength.

$$V_{u2} = V_{u1} + \phi \cdot V_{c}$$

$$\phi = -0.17 + 0.3a/d + 0.33/(100p_{w})$$

As described in the chapter 1, it is a fact that there are difference basic concept among the various shear design codes. In particular, the shear design equations between deep beams and slender beams are found being discontinuous as their basic resistance mechanisms are different. In the shear design for deep beam recommended by JSCE Standard Specification (JSCE 2002), the concept of truss action, as one part of beam action, is adopted to consider the improvement effect of shear reinforcement as described by Eq. (8). But in the later modification (JSCE 2007), the effect of shear reinforcement is considered being attributed to the improvement of arch action described by Eq. (9). Therefore, the basic concept of shear design completely changed. Corresponding to this problem, in this study, each contributions of arch action, beam action, concrete contribution and shear reinforcement contribution for shear strength evaluation was clarified, in the cases of different $a/d$ ratios and $p_{w}$ ratios.

Figure 21 shows the change of shear resistance components for different shear span to depth ratio for beams series $p_{w} = 0.00\%, 0.28\%$. Results of $a/d = 1.57$ and $a/d = 3.14$ correspond tested specimens dimension.

The shear strength result of the beam series without shear reinforcement ($p_{w} = 0.00\%$) from analyses are compared with the ones calculated by design equations discussed before, i.e. $V_{u2}$ of Eq. (7) and $V_c$ of Eq. (6), which are given in Fig. 21a. The shear strength values from analyses matched well with the $V_{u2}$ calculation for beams with smaller $a/d$ ratios ($1.57$ and $2.35$), whereas the beams with larger $a/d$ ratios ($a/d > 3.91$) demonstrates a similar result with $V_c$ calculation. It also was noted that the shear strength of the beam with $a/d = 3.14$ was greater than $V_{u2}$ and $V_c$ calculations. The reason was that the $V_c$ equation reflected a shear tension failure considering inclined shear cracking, but a shear compression failure occurred here; and for $V_{u2}$ calculation, this equation was proposed for the beams with $a/d$ less than $2.5$, therefore it also could not evaluate the beam with $a/d = 3.14$. Another important finding was that the arch action values were very close to the shear strength distribution for the beams with $a/d$ less than $4.31$, which demonstrated the arch action was dominant on the development of shear strengths of the RC beams without...
shear reinforcements. For the beam with \( \frac{a}{d} = 4.71 \), a limited proportion of the arch action in the shear strength was attributed to the early shear failure after diagonal cracking, namely, and the arch action was unable to develop further like in other beams.

Numerical results of the beam series with \( p_w = 0.28\% \) are shown in Fig. 21b, and compared with the ones calculated by design equations, i.e. \( V_u = \frac{a}{d} \) of Eq. (9) and \( V_c + V_s \) of Eq. (5) discussed, for beams with shear reinforcement. \( V_u \) calculation is shown on the figure as well for reference. The shear strength results of the beams with small \( \frac{a}{d} \) ratios (1.57, 2.35) matched well with \( V_u \) calculations, whereas, the results for the beams with great \( \frac{a}{d} \) ratios (> 3.91) agreed well with \( V_c + V_s \) calculations. Analysis result of the beam with \( \frac{a}{d} = 3.14 \) gave an overestimation of shear strength compared to \( V_c + V_s \) calculation, due to the same reason as mentioned for Fig. 21a. The concrete shear resistance \( V_{con} \) (= \( V_c + V_s \)), which was mainly provided by the arch action \( V_a \), was decreased with the increase in \( \frac{a}{d} \) ratio, and obviously, the downward trend of shear strength was governed by the decrease in \( V_{con} \). On the other hand, the concrete shear resistance \( V_{con} \) for small \( \frac{a}{d} \) ratios (1.57 and 2.35) approached to \( V_u \) calculation. Additionally, the beam action \( V_b \), which was almost entirely governed by truss action \( V_s \), was not varied by different \( \frac{a}{d} \) ratios. That is, truss action contributes to shear strength even in deep beam.

Figure 22 presents the effect of critical factors of \( \frac{a}{d} \) and \( p_w \) on the shear resistance mechanisms. To clarify
the effect, the results for all beam series and different mechanisms are comparatively discussed. Figure 22a shows the distributions of beam action $V_\text{b}$, $V_\text{a}$ values for all $p_\text{w}$ series nearly remained uniform with varying $a/d$ ratios. The beam action $V_\text{a}$ was enhanced with increase in $p_\text{w}$ ratio, except for the beam with $a/d = 1.57$. Figure 22b shows the analysis results for the truss action $V_\text{c}$, along with the strength values calculated by design equations assuming that all shear reinforcements were yielded (Eq. (4)). Similar to the $V_\text{a}$ cases, $V_\text{c}$ values for each $p_\text{w}$ series almost was not affected with the varying $a/d$ ratios. It was observed that, the truss actions $V_\text{c}$ for the beam series with $p_\text{w} = 0.14%$ and $0.28%$ were very close to the calculated values based on the truss theory. The results for the beam series with $p_\text{w} = 0.42%$ and $0.56%$ were slightly lower than the calculated values. This was because in the beam series with large $p_\text{w}$ ratios (0.42%, 0.56%), shear reinforcements only yielded locally until failure stage and the existence of non-yielding shear reinforcements with relatively small stress intensity ultimately resulted in lower $V_\text{c}$ results. The results for concrete contribution $V_{\text{con}}$ of the beam action mechanism are plotted in Fig. 22c. It was observed that the shear resistance provided by $V_{\text{con}}$ was nearly negligible for all beam series and it was independent on $a/d$ ratio or $p_\text{w}$ ratio. The results became evident that the shear strength provided by the beam action $V_\text{a}$ could be assessed by truss action $V_\text{c}$.

Figure 22d shows the analysis results for the arch action $V_\text{a}$ along with $V_{\text{ul}}$ and $V_\text{c}$ calculations. $V_\text{a}$ values for all $p_\text{w}$ series were gradually decreased with the increase in $a/d$ ratio and agreed well with $V_{\text{ul}}$ calculations. It was observed that the arch action $V_\text{a}$ was enhanced with the increase in $p_\text{w}$ ratio. Figure 22e compares the analysis results for the concrete contribution $V_{\text{con}}$ combined with $V_{\text{ul}}$ and $V_\text{c}$ design calculations. Similar to $V_\text{c}$ case, the concrete contributions $V_{\text{con}}$ were also decreased with the increased $a/d$ ratio and enhanced with the increase in $p_\text{w}$ ratio. The $V_{\text{con}}$ distributions matched well with the $V_{\text{ul}}$ calculations for the beams with small $a/d$ ratios (1.57, 2.35), and approached to $V_\text{c}$ calculation for large $a/d$ cases (> 3.91). It was confirmed that the concrete contribution $V_{\text{con}}$ was mainly sustained by the arch action $V_\text{a}$.

From the results presented in Fig. 22, it could be concluded that regardless of the varying $a/d$ ratios and $p_\text{w}$ ratios, the concrete contribution $V_{\text{con}}$ to the shear strength was almost completely provided by the arch action $V_\text{a}$. These results suggest that concrete contribution can be evaluated without classification between deep and slender beams. On the other hand, the effect of shear reinforcement could be accurately evaluated by truss theory and the shear resistance mechanism corresponding to shear strength could be evaluated by a combination of arch action $V_\text{a}$ and truss action $V_\text{c}$.

6. Conclusions

This study presented numerical results of the shear failure of RC beams performed by 3-D RBSM and investigated the decoupling results in terms of beam and arch action mechanisms. The major outcomes of the study were presented as following.

1. Based on the comparisons between the test and numerical analysis, it was confirmed that 3-D RBSM could accurately simulate the shear failure behaviors of the deep beams and slender beams at macro level including load-displacement curves, crack development and effect of shear reinforcement.

2. By using the local stress results obtained by 3-D RBSM, beam, arch and truss actions, which was difficult to evaluate by test methods directly, could be decoupled for RC beams.

3. Prior to the occurrence of inclined shear crack, total shear load was mostly sustained by the beam action while the arch action had so limited effect on the shear strength. After inclined shear cracking, the arch action was increased significantly, while the beam action of concrete was reduced. When shear reinforcements are arranged, the change of concrete contribution of arch and beam actions was almost same with no shear reinforcement case. The effect of shear reinforcement was that the beam action is sustained by truss action provided by the shear reinforcements. These behaviors appear in both deep and slender beam.

4. The effect of the shear span to depth ratio ($a/d$) and shear reinforcement ratio ($p_\text{w}$) on the shear resistance mechanisms was assessed based on the decoupling results, and the sequent shear resistance mechanisms were compared with the current shear design recommendations given in Japanese codes. It was understood that the shear strength of a RC beam, irrelevant to $a/d$ ratio and $p_\text{w}$ ratio, was nearly completely provided by arch action $V_\text{a}$ and truss action $V_\text{c}$. It was found that the shear strength of RC member is evaluated without classification of deep beams and slender beams, through the quantitatively evaluation of shear resistance mechanisms.

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