3D numerical investigations of the swirling flow in a straight diffuser for the variable speed values of the rotor obtained with a magneto-rheological brake

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Abstract. The self-induced instabilities developed in decelerated swirling flows lead to pressure fluctuations. The swirling flows generated under variable speeds of the rotor of a swirl generator using a magneto-rheological brake are numerically and experimentally investigated. Three-dimensional turbulent flow computation is performed for several rotor speeds. Two mean velocity components numerically computed are validated against LDV data on a survey axis located downstream to the rotor for two speed values. The torque on the rotor blade is examined for several speeds. The numerical results are checked against the design values to identify the benefits and limitations of the concepts applied to the design stage. The flux of momentum is modified when the rotor speed is slow down. The circulation, the total pressure and the swirl free velocity are examined on a cross section located downstream to the rotor for all investigated speeds. As a result, the changes of the hydrodynamic flow field are quantified. The unsteady pressure measured on the cone wall at four levels is acquired. The Fourier spectra associated with different self-induced instabilities developed in the cone are examined revealing the distribution of both plunging and rotating components. A low frequency plunging component in a straight diffuser is determined on a speed range of the rotor.

1. Introduction
Nowadays, the hydraulic turbines have to operate on a wide range \cite{1, 2} as well as under transient and special operating conditions (e.g. runaway speed, emergency shutdown, etc) \cite{3}. Therefore, the unsteady phenomena are developed requiring a detailed analysis to identify their causes \cite{4} and new control techniques to hinder them \cite{5}. Also, innovative design concepts have to support wide range operation under limited unsteady phenomena \cite{2, 6, 7}. Several swirling flow apparatus are developed to explore new design concepts and to tailor special swirling flow configurations with associated unsteady phenomena \cite{8-11} to understand deeper the physical phenomena.

The paper is structured as follows: a semi-active control method using a magneto-rheological brake to slow down the rotor speed is presented and the experimental setup is detailed in section 2. The numerical setup and the validation of the numerical results against experimental data for two rotor speed values are given in section 3. Several hydrodynamic quantities are defined and determined based on numerical simulation to quantify changes of the flow field at different rotor speed values in section 4. Both amplitude and frequency of the unsteady pressure signals acquired on the wall of the test rig are examined for different speed values, too. The conclusions are underlined in the last section.

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2. Experimental test rig and setup

2.1. Experimental test rig

There are two ways to investigate the phenomena that occur in Francis hydraulic turbine when operating at part load condition. The first way is to study the flow downstream to a turbine model. The second way suppose by using a swirl generator as the one designed and installed at University Politehnica Timisoara [8]. The swirl generator was specially designed together with a convergent-divergent test section [9]. This swirl apparatus is installed in a closed loop hydraulic circuit, Figure 1 left. Two bladed regions (guide vane and rotor) are included in the swirl generator (Figure 1 right) to balance the total pressure downstream to the rotor. The guide vane is designed with 13 blades while the rotor with 10 blades, respectively. The rotor was designed as a turbine closed to the hub and as a pump near to the shroud. The rotor spins freely with the runaway speed of 1020rpm at a volumetric flow rate of 30 l/s.

![Test rig available at University Politehnica Timisoara to investigate decelerated swirling flows (left) and a detailed view of swirl apparatus (right).](image)

Several active and passive control techniques were proposed to mitigate the self-induced instabilities and their unwanted effects [5]. A semi-active control method based on a magneto-rheological brake installed in the rotor hub is used in this study [12]. Constructively, the method is simple; it provides a fast response and requires low power consumption. In a simple and efficient way this control method provides several hydrodynamic regimes which are investigated in this paper.

2.2. Experimental setup

A magneto-rheological brake was designed and installed in the rotor hub to slow down its speed providing several swirling flow configurations, Figure 2 left. The magneto-rheological brake changes the apparent viscosity of the magneto-rheological fluid under variable magnetic fields. The magneto-rheological fluid is installed in a gap between a fixed part of the hub and the rotating slice linked with the rotor, respectively. A coil is installed to provide variable magnetic fields applying low current values. A cylindrical magnet is installed on each blade of the rotor [14]. A magnetic sensor was mounted on the test section to measure the rotor speed, Figure 2 right. The runner’s rotation generates a train of pulses (ten of them corresponding to one complete rotation). The pulse train frequency was acquired using the above setup to determine the rotor’s speed [14]. Thus, a constant discharge value of 30 l/s is selected for all investigated operating regimes. Then, the dimensionless flux of momentum for the swirling flow delivered by the swirl generator apparatus is plotted versus the dimensionless discharge in Figure 3. One can observe that the swirl apparatus provides a similar distribution with the FLINDT Francis turbine model tested at EPFL [15].
Figure 2. The magnetorheological brake embedded in swirling flow apparatus [16] (left) and the picture of the convergent divergent test section with magnetic sensor and pressure transducer instrumentation (right).

Figure 3. Dimensionless flux of moment of momentum: Swirl Apparatus (black) and Flindt Francis turbine (red).

Figure 4. LDV probe on W0 window (left) and the convergent-divergent section with three survey axis W0, W1 and W2 and four levels to acquire unsteady pressure signals MG0, MG1, MG2 and MG3 (right).

The flow field (both mean and unsteady parts) developed in the convergent-divergent section for several swirling flow configurations delivered by the rotor are measured. Two velocity components (meridian and circumferential) are measured along three survey axis (W0, W1 and W2) using 2D LDV system, Figure 4 left. The mean velocity components measured on W0 survey axis are used to validate the numerical results delivered by the rotor. The unsteady part of the flow field is measured on all three survey axes as well as in eight taps located on the cone wall with semi-angle of 8.5°. Eight fast response pressure sensors are placed on four levels along the cone section, Figure 4 right. The first
level denoted MG0 is placed in the throat section with the diameter of 0.1 m. Other three levels (MG1, MG2 and MG3) are placed downstream to MG0 level at 0.05 m, 0.1 m and 0.15 m, respectively. The pressure taps on the same level are positioned at 180° one to another. This experimental setup captures the unsteady pressure field generated by a single vortex rope on the wall.

3. Numerical setup. Validation of the numerical results against experimental data

3.1. Numerical setup

The three-dimensional computational domains correspond to the swirl generator apparatus from the experimental test rig, Fig. 1. The swirl apparatus (swirl generator and test section) includes four subdomains: the leaned strut, the guide vane, the rotor and the convergent-divergent test section. The first computational domain includes one single leaned strut. The circumferential extension of the leaned strut computational domain is 90°=π/2. Both guide vane and rotor computational domains correspond to one channel with circumferential extension of 27.69°=2π/13 for the guide vane domain and 36°=2π/10 for the rotor domain, respectively. The three-dimensional computational domain associated with the convergent-divergent section is fully taken into account due to the self-induced instabilities developed in the decelerated swirling flow. Structured grids are generated on all four computational domains. The following number of cells is considered on each computational domain: 63k on the leaned strut, 204k on the guide vane, 175k on the rotor and ~1.7M on the convergent-divergent part, respectively.

The following boundary conditions are imposed to compute the flow in the test section. The axial velocity component corresponding to the discharge value of 30 l/s is imposed on the inlet section while other two (circumferential and radial) velocity components are negligible. Two turbulent quantities (turbulence intensity and hydraulic diameter) are imposed on the inlet section of the leaned strut domain, too. The hydraulic diameter of 0.15 m corresponding to the inlet section is selected. The turbulent intensity of 3.38% is imposed on the inlet section. This value is determined using the equation $I [%] = 0.16(Re_Dh)^{-1/8}$ [17] where the Reynolds number of 2.47*10^5 is obtained for the discharge value and the properties associated with the water fluid. The radial equilibrium condition is imposed on the outlet surface together with the turbulent quantities (the hydraulic diameter of 0.16 m and the turbulent intensity of 3.41%) associated with the recirculation flow. A periodic condition is imposed on all side surfaces of the computational domains with spatial periodicity (e.g. leaned strut, guide vane and rotor). The wall boundary conditions are imposed on the leaned strut, on the guide vane blade, on the rotor blade and on the all walls of the swirl apparatus (e.g. hub, shroud, casing, and so on).

![Figure 5. All three mixing planes (MP1, MP2 and MP3) created between the swirl apparatus parts.](image)

The three-dimensional absolute/relative flow is separately computed on all four domains of the test section using FLUENT V6.3 commercial code. An absolute steady flow is performed on both leaned strut and guide vane computational domains and a relative steady flow on the rotor computational domain, respectively. The Reynolds Average Navier Stokes (RANS) equations together with k-ω turbulence model have been solved on first three computational domains. The Reynolds Stress Model (RSM) turbulence model has been selected to compute the unsteady simulation in the convergent-
divergent test section in order to capture the flow features [18]. The flow field quantities (e.g. velocity, pressure and turbulence) are passed from one computational domain to another one using a mixing plane technique. Thus, three successive mixing planes (MP) are defined between successive domains to transfer the flow quantities, Figure 5. The convergence criterion of $10^{-5}$ is considered for steady computations while a time step of 0.1 ms and 20 inner iterations on each time step are selected for the unsteady computation based on our previous validations against experimental data [19].

3.2. Numerical results verification

The flow quantities transferred between the swirl apparatus domains is checked for the numerical investigated operating regimes with the rotor speed of 1020, 990, 960, 920, 870 and 820 rpm, respectively. On each interface the global quantities of the volumetric flow rate and the flux of moment of momentum are checked then the relative error is calculated. The relative error of the volumetric flow rate versus the rotor speed is plotted in Figure 6 left. A maximum relative error of the volumetric flow rate 0.043% is obtained for the mixing plane no. 1 (MP1) placed between the leaned strout and the guide vane domains. Other two mixing planes conserve the volumetric flow rate better than the first one having a relative deviation less than 0.03% on the MP3 and 0.025% on the MP2, respectively.

![Figure 6](image)

**Figure 6.** The quantities (volumetric flow rate and flux of moment of momentum) checked on the mixing planes to conserve the operation point.

The flux of moment of momentum is another global quantity that has to be conserve along the mixing plane. This quantity is closely related to the circumferential velocity component. The circumferential velocity component on the outlet section of the leaned strout domain is negligible. Therefore, the flux of moment of momentum is not quantified on MP1. A maximum relative error of 3.5% of the flux of moment of momentum is obtained on the MP3 between the rotor domain and the convergent-divergent domain. The parameters of the operating points are conserved on all three mixing planes. The volumetric flow rate is conserved in a limit of a relative error of 0.043% while the flux of moment of momentum with about 3.5%. However, in this paper only two regimes (920 and 870 rpm) are presented. Both plunging and rotating components associated with the pressure pulsation were found at these operating regimes [13].

3.3. Numerical results validated against experimental data

The numerical results are compared againsts experimental data for two runner speeds (920 and 870 rpm) in Figure 7. Two mean velocity components (meridian ($V_m$) and circumferential ($V_u$)) were plotted along the W0 survey axis located downstream to the runner, Figure 4. One can be observed that the mean meridian velocity component is captured quite well in both cases by the numerical results. A slight deviation can be detected in the middle section of the flow. This middle section corresponds to the transition on the rotor blade geometry from the turbine part near to the hub (large values on the survey axis) to the pump part closed to the shourd (small values on the survey axis).
4. Numerical results

4.1. On the rotor blades

The rotor is slowed down from the runaway speed value of 1020 rpm to 790 rpm using magneto-rheological brake [13]. The rotor was designed as a turbine near to the hub and a pump closed to the shroud. The rotor was designed to spin at a runaway speed of 870 rpm for a discharge of 30 l/s. The torque of the rotor is negligible at this design conditions. The torque on the rotor blades is computed based on the numerical simulations using the following equation,

\[ T = N \left[ \int_S \left( \vec{r} \times (\vec{\tau} \cdot \vec{n}) \right) dS \right] \cdot \vec{i}_z \]

where \( N = 10 \) is the number of the rotor blades, \( S \) is the surface of the blade, \( \vec{\tau} \) is the stress tensor (including the pressure and the viscous stresses), \( \vec{n} \) is the unit vector normal to the surface, \( \vec{r} \) is the position vector and \( \vec{i}_z \) is the unit vector along the rotation axis of the swirl apparatus.

The torque on the rotor blades determined based on the numerical simulation is plotted in Figure 8. The linear equation \( T(n) = 8.425 - 0.00952 \times n \) is fitted on the numerical results. The negligible torque on the rotor is yielded at 885 rpm based on the numerical simulations leading to a relative deviation of 1.72% with respect to the designed runaway speed value. The relative deviation determined above is accounted for by two causes: (i) the inviscid methodology applied to design the rotor [8] and (ii) the deviations quantified in the numerical simulations given in §3.
The runaway speed of 1020 rpm is measured on the test rig. This value leads to a relative deviation value of 17.2% with respect to the designed one. This relative deviation is most likely caused by the limited contribution of the pump part below the designed one. The limited contribution of the pump side is caused by the leakage gap between the tip blade and the cylindrical wall of the section which it is not taken into account at the design stage.

4.2. Downstream to the runner on the inlet section of the convergent-divergent section

The numerical results on the MP3 mixing plane located downstream to the rotor (see Fig. 5) are examined appraising the evolution of the hydrodynamic quantities in terms of the speed. The following hydrodynamic quantities are used to assess the evolution:

(i) the discharge fraction \( q \) is directly linked with the normalized streamfunction [6, 20]

\[
q = \frac{1}{\phi} \sum_{j=1}^{i} (r_j v_{z,j} + r_{j-1} v_{z,j-1})(r_j - r_{j-1})
\]

where \( \phi \) is the discharge coefficient, \( r=R/R_{sh} \) is the dimensionless radial coordinate and \( v_z=V_z/(\omega R_{sh}) \) is the coefficient of the axial velocity component. The shroud radius of the rotor \( R_{sh}=0.075 \) m is taken as the reference length and \( \omega R_{sh} \) is the reference velocity with \( \omega \) the angular speed associated with the rotor.

(ii) the dimensionless circulation \( k \) and the dimensionless total pressure \( h \) [21]

\[
k = \frac{R V_u}{\omega R_{sh}^2} = \frac{R_{sh}}{\omega R_{sh}} = r v_u, \quad h = \frac{p + \rho V^2/2}{\rho(\omega R_{sh})^2}
\]

where \( v_u=V_u/(\omega R_{sh}) \) is the coefficient of the circumferential velocity component, \( p \) is the static pressure, \( V \) is the absolute velocity and \( \rho=998.2 \) kg/m\(^3\) is the water density.

![Figure 9](image_url)

**Figure 9.** The dimensionless circulation (left) and the dimensionless total pressure (right) versus the discharge fraction.

Both dimensionless circulation \( k \) and dimensionless total pressure \( h \) versus the discharge fraction are plotted in Fig. 9. One can observe a linear distribution with the same slope of the circulation for all speed values. A slight deviation from the linear distribution can be observed on the last part of the blade near to the shroud. This part of the blade corresponds to the pump side. The intercept value of the linear distribution increases with the speed of the rotor. This means that the residual swirl increases with the speed of the rotor. The previous statement is supported by the flux of moment of momentum plotted in Fig. 3. The distribution of the dimensionless total pressure remains unchanged with the speed of the rotor. The same exception is quantified on the pump contribution of the rotor blade.

(iii) the swirl free velocity \( v_{sf} \) is directly linked with the relative flow angle at the trailing edge of the rotor [6, 7, 20 - 22]

\[
v_{sf} = \frac{r v_z}{r - v_u}
\]

The swirl free velocity distribution versus the discharge fraction for all speed values is given in Fig. 10. One can observed that the swirl free velocity distribution remains practically unmodified on more than
75% of the rotor blade starting from the hub. This indicates that the flow direction living the trailing edge of the rotor blade is unchanged. Contrary, a slight spread of the swirl free velocity is quantified on the pump side of the rotor blade located near to the tip. This spread of the numerical results suggests a small change of the flow angle due to the flow separate on the trailing edge of the rotor blade. This issue should be examined in a further analysis because of it is beyond the goal of this paper.

Figure 10. Swirl free velocity versus the discharge fraction.

4.3. In the cone section

The unsteady pressure signals measured on all four levels (MG0, MG1, MG2 and MG3) located along the cone are acquired for all rotor speeds. Then, the decomposition procedure developed by Bosioc et al. [23] is applied to discriminate both rotating and plunging components associated with the unsteady pressure field. The plunging component propagates as standing waves along the entire hydraulic passage while the rotating one is locally acting in the cone section. The rotating component is associated with the precession motion of the flow instabilities (e.g. vortex rope).

Two dimensionless quantities (the Strouhal number ($Sh$) and the dimensionless maximum amplitude ($a$)) associated with both rotating and plunging components are defined according to the next equations:

$$Sh = \frac{fR_{sh}}{(\omega R_{sh})}, \quad a = \frac{A_M}{\rho(\omega R_{sh})^2}$$

where $f$ is the fundamental frequency of the rotating component or the frequency of the plunging one, the rotor shroud radius of $R_{sh}$=0.075 m is taken as the reference length, $(\omega R_{sh})$ is the reference velocity with $\omega$ the angular speed associated with the rotor, $A_M$ is the maximum amplitude corresponding to the each frequency value on all levels located along the cone and $\rho$=998.2 kg/m$^3$ is the water density.

The Strouhal number ($Sh$) and the dimensionless maximum amplitude on all levels along the cone ($a$) versus the rotor speed for all investigated regimes are given in Figure 11.

Figure 11. The Strouhal number (left) and the dimensionless maximum amplitude together with the level on it is determined (right) versus the rotor speed.
The plunging component with low frequency associated with the self-induced instability of the decelerated swirling flow in a straight pipe was already identified at the rotor speed of 920 rpm [14]. The mechanism identified for this type of plunging component is the deformation of the precessing vortex core filament [24]. One can observe that this low frequency plunging component is detected on the speed range from 870 rpm to 970 rpm. The ratio between the plunging component and the fundamental frequency of the rotating component varies with 40% from 0.125 to 0.172 on the speed range mentioned above. The ratio between the maximum amplitude associated with the plunging component and maximum amplitude corresponding to the fundamental frequency of the rotating component changes by three times on the same speed range. The helical vortex breakdown associated with this speed range most like correspond to the type regime II indentified by Nishi et al. [25]. The following features are associated with this helical vortex breakdown type [26]: (i) various shapes of the self-induced instabilities appear irregularly; (ii) the axial location of the stagnation point in the inlet conical diffuser varies irregularly. A detailed analysis of this helical vortex breakdown morphology will be performed in our further investigations.

5. Conclusions
The paper focuses on numerical and experimental investigation of the swirling flows induced by a rotor with variable speed. The speed of the rotor is controlled using a magneto-rheological brake. The three-dimensional turbulent flow computation in a swirl apparatus is performed for several rotor speeds. In this way, different swirling flow configurations are delivered on the inlet section of the convergent-divergent section. Two mean velocity (meridian and circumferential) components numerically computed are validated against LDV data on W0 survey axis located downstream to the rotor for two speed values. The torque on the rotor blade is examined for several speeds identifying the runaway speed of 885 rpm. The deviation of the numerical value of the runaway speed in relation to the value measured on the test rig is caused by the design constrains. Several hydrodynamic quantities (circulation, total pressure and swirl free velocity) are defined and examined on a cross section located downstream to the rotor for several speed values. The rotor was designed as turbine near to the hub while a pump close to the shroud. The flow downstream to the rotor remains unchanged on the turbine side while a slight deviation of the flow is obtained on the pump part. The unsteady pressure signals are measured on the cone wall at four levels. Both plunging (synchronous) and rotating (asyncronous) components associated with different self-induced instabilities developed in the cone are determined. A low frequency plunging component in a straight diffuser is determined on a speed range of the rotor from 870 rpm to 970 rpm. A helical vortex breakdown is associated with the self-induced instability on this speed range.

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