Letter

Relationship between frequency power spectra and intermittent, large-amplitude bursts in the Alcator C-Mod scrape-off layer

A. Theodorsen\(^1\)\(\oplus\), O.E. Garcia\(^1\)\(\oplus\), R. Kube\(^1\), B. LaBombard\(^2\)\(\oplus\) and J.L. Terry\(^2\)\(\oplus\)

\(^1\) Department of Physics and Technology, UiT The Arctic University of Norway, N-9037 Tromsø, Norway
\(^2\) MIT Plasma Science and Fusion Center, Cambridge, MA 02139, United States of America

E-mail: odd.erik.garcia@uit.no

Received 1 February 2017, revised 2 June 2017
Accepted for publication 7 July 2017
Published 21 August 2017

Abstract

Fluctuations in the boundary region of the Alcator C-Mod tokamak have been analyzed using gas puff imaging data from a set of Ohmically heated plasma density scan experiments. It is found that the relative fluctuation amplitudes are modest and close to normally distributed at the separatrix but become increasingly larger and skewed towards the main chamber wall. The frequency power spectra are nevertheless similar for all radial positions and line-averaged densities. Predictions of a stochastic model, describing the plasma fluctuations as a super-position of uncorrelated pulses, are shown to be in excellent agreement with the measurements. This implies that the pulse duration is the same, while the degree of pulse overlap decreases radially outwards in the scrape-off layer. The universal frequency power spectral density is thus determined by the shape and duration of the large-amplitude bursts associated with blob-like structures. The model also describes the rate of threshold level crossings, for which the exponential tails underline the intermittency of the fluctuations in the far scrape-off layer.

Keywords: stochastic model, Alcator C-Mod, fluctuations, turbulence, intermittency, gas puff imaging, scrape-off layer

(Some figures may appear in colour only in the online journal)

1. Introduction

Turbulent motions and fluctuation-induced transport of particles and heat in the boundary region of magnetically confined plasmas pose numerous problems for the successful development of thermonuclear fusion power reactors [1–3]. This includes challenges for auxiliary heating and current drive by the use of radio frequency waves [4–7] and enhanced material erosion of the main chamber walls [8–10], and is likely related to the empirical discharge density limit [11–17]. At the outboard mid-plane scrape-off layer the radial transport is predominantly due to radially outwards motion of blob-like plasma filaments, which leads to order unity relative fluctuation levels, broad plasma profiles and enhanced plasma–wall interactions [15–25].

The statistical properties of plasma fluctuations in the scrape-off layer have recently been elucidated by means of exceptionally long data time series under stationary plasma conditions [20–25]. Based on this, a stochastic model of the plasma fluctuations has been developed [26–30] and its underlying assumptions and predictions are found to compare favorably with experimental measurements [20–25]. Recently, analytical expressions for the rate of level crossings were obtained from this model, that is, how frequently a realization
of the process on average crosses a given threshold level. The associated excess time statistics may be useful in order to describe threshold phenomena underlying plasma–wall interactions due to transient transport events [26, 27, 29–31].

In this contribution, fluctuations in the boundary region of the Alcator C-Mod tokamak are investigated by analysis of gas puff imaging measurements at the outboard mid-plane region [20, 21]. It is found that the relative fluctuation amplitudes are modest and nearly normal distributed at the last closed magnetic flux surface but become increasingly larger and intermittent towards the main chamber wall. Despite this, the frequency power spectral density is similar for all radial positions and line-averaged densities investigated. These results are consistent with the model predictions, implying that the temporal scale of the fluctuations is the same, while the degree of pulse overlap decreases radially outwards. The universal frequency power spectral density is determined by the shape and duration of the large-amplitude bursts associated with blob-like structures. This is in contrast to most previous work, which have interpreted the spectra in terms of scale-free inertial range cascade dynamics or self-similar processes [32–35].

2. Experimental setup

Gas puff imaging data from the boundary region of the Alcator C-Mod tokamak in Ohmically heated, lower single null discharges have been analyzed. This comprises a four-point scan in the line-averaged density $\bar{n}_e$, with the Greenwald fraction $n_G/n_G$ ranging from 0.15 to 0.30. Here the Greenwald density is given by $n_G = (I_p/\pi a^2)10^{20} \text{ m}^{-3}$, where $I_p$ is the plasma current in units of mega-Amperes and $a$ is the plasma minor radius in units of meters. For this density scan $I_p = 0.8 \text{ MA}$ and $a = 0.21 \text{ m}$, giving $n_G = 5.26 \times 10^{20} \text{ m}^{-3}$. An on-axis toroidal magnetic field of 4.0 T was used. The condition at the outer divertor target goes from sheath limited at the lowest density to high recycling at the highest density in this density scan, and the particle density profile at the outboard mid-plane develops the familiar two-layer structure and significant broadening with increasing line-averaged density. These experiments were part of heat flux footprint studies on Alcator C-Mod, and have been extensively diagnosed and documented [18, 20, 21].

The gas puff imaging diagnostic consists of $9 \times 10$ (radial $\times$ vertical) toroidal views, coupled to an in-vessel optical fiber array. The plasma emission collected in the views is filtered for He I (587 nm) line emission that is due almost entirely to plasma excitation near the object plane of an extended He gas puff from a nearby nozzle. The fibers are coupled to high sensitivity avalanche photo diodes and the signals are digitized at a rate of $2 \times 10^6$ frames per second. The in-focus spot size is 3.8 mm for each of the individual fibre channels. The radial position of the last closed magnetic flux surface at the vertical centre of the image, $Z = -2.61 \text{ cm}$, is in the range from 89.4 to 89.7 cm for all the discharges presented here. The limiter radius mapped to this vertical position is at $R = 91.0 \text{ cm}$. For each discharge the gas puff imaging diagnostic yields 0.25 s usable data time series during the flat-top of the plasma current. By combining data from two discharges at the same $\bar{n}_e$ and two vertically adjacent views at the same radial position with identical statistical properties, time series of effectively one second duration are obtained. This allows calculation of statistical averages with high accuracy. Further information about the gas puff imaging diagnostic on Alcator C-Mod can be found in [19–21].

Excitation of neutral He gas, and thus the intensity of the gas puff imaging signals, is determined by the neutral He particle density and a combination of the local electron density and temperature. This diagnostic does therefore not give information about the absolute value of any plasma parameters but is rather a proxy for the relative plasma fluctuation level. For this reason, the measurement data for each diode view position is here rescaled such as to have vanishing mean and unit standard deviation. Thus, a measured signal $\Phi(t)$ is normalized as $\Phi(t) = (\Phi - \Phi_{\text{rms}})/\Phi_{\text{rms}}$, where $\Phi$ and $\Phi_{\text{rms}}$ are the moving average and standard deviation taken over a window of 8 ms duration in order to remove low-frequency oscillations and trends in the signals. Previous analysis of Langmuir probe data have demonstrated that the statistical properties of the ion saturation current fluctuations in the far scrape-off layer are the same as that inferred from gas puff imaging data [20–25]. The major advantage of the gas puff imaging diagnostic for these studies is the access to low fluctuation data time series for several radial positions in the scrape-off layer [19–21].

As a representative example of the plasma fluctuations in the boundary region, gas puff imaging signals recorded at three different radial positions for $Z = -2.61 \text{ cm}$ and $\bar{n}_e/n_G = 0.25$ is presented in figure 1. The innermost diode view position shown here at $R = 89.54 \text{ cm}$ is close to the separatrix and the outermost position at $R = 91.08 \text{ cm}$ is in the limiter shadow where the magnetic field lines intersect limiter structures and have a connection length of the order of one meter. It is clearly seen that the time series from the scrape-off layer are dominated by large-amplitude bursts, attributed to the radially outwards motion of blob-like plasma filaments. Further inward, pulses appear more frequently and at the separatrix

![Figure 1. Data time series recorded by the gas puff imaging diagnostic at the separatrix (top), in the main scrape-off layer (middle) and in the limiter shadow (bottom) for $n_e/n_G = 0.25$. All time series are rescaled such as to have vanishing mean and unit standard deviation.](image)
the signal resembles random noise. However, the following analysis suggests that the underlying pulse structure and temporal scale are the same for all these time series, the only difference being the degree of pulse overlap.

3. Stochastic model

Consider a stochastic process given by the super-position of a random sequence of \( K \) uncorrelated pulses in a time interval of duration \( T \) [26–31],

\[
\Phi_K(t) = \sum_{k=1}^{K(T)} A_k \phi(t - t_k),
\]

where for each event \( k \) the pulse amplitude is \( A_k \) and the pulse arrival time is \( t_k \). The pulse shape is taken to be the same for all events and described by a two-sided exponential function [28, 30]

\[
\phi(t) = \begin{cases} 
\exp(t/\tau_d), & t < 0, \\
\exp(-t/(1 - \lambda)\tau_d), & t \geq 0,
\end{cases}
\]

(2)

where \( \tau_d \) is the pulse duration and \( \lambda \) is the pulse asymmetry parameter restricted to the range \( 0 < \lambda < 1 \). For \( \lambda < 1/2 \) the pulse function has a shorter rise than fall time.

In agreement with previous measurements on several tokamak experiments, it is in the following assumed that the pulse amplitudes are exponentially distributed with mean value \( A \) and the waiting times between the pulses are exponentially distributed with mean value \( \tau_w \) [20–25]. It follows from exponentially distributed waiting times that \( K(T) \) is Poisson distributed with mean \( T/\tau_w \) and the pulse arrival times \( t_k \) are uniformly distributed on \([0, T]\) [36]. Introducing the scaled variable \( \tilde{\Phi} = (\Phi - \langle \Phi \rangle)/\Phi_{rms} \) with zero mean and unit standard deviation, similar to the experimental signals discussed above, the stationary probability density function for the random variable \( \tilde{\Phi} \) is a Gamma distribution, giving \( \tilde{\Phi} \) the distribution [26, 27, 29]

\[
P_{\tilde{\Phi}}(\tilde{\Phi}) = \frac{\gamma^{\gamma/2}}{\Gamma(\gamma)} \left( \gamma + \gamma^{1/2}\tilde{\Phi} \right)^{\gamma-1} \exp\left( -\gamma - \gamma^{1/2}\tilde{\Phi} \right),
\]

(3)

where \( \Gamma \) denotes the Gamma function and angular brackets denote an average over all random variables. The shape parameter \( \gamma = \tau_d/\tau_w \) determines the degree of pulse overlap. It should be noted that since \( \Phi > 0 \) it follows that \( \tilde{\Phi} > -\gamma^{1/2} \). The mean value of the variable given by equation (1) is \( \gamma \langle A \rangle \), the standard deviation is \( \Phi_{rms} = \gamma^{1/2} \langle A \rangle \), and the skewness and flatness moments are \( S_\Phi = 2/\gamma^{1/2} \) and \( F_\Phi = 3 + 6/\gamma \), respectively. Like the relative fluctuation level given by \( \Phi_{rms}^2/\langle \Phi \rangle^2 = 1/\gamma \), also the skewness and flatness moments increase with decreasing \( \gamma \). The ratio \( \gamma \) of pulse duration and waiting times is thus referred to as the intermittency parameter of the model. When \( \gamma \) is small, pulses generally appear isolated in realizations of the process, resulting in strong intermittency. When \( \gamma \) is large, there is significant overlap of pulses, realizations of the process resemble random noise and the probability density approaches as normal distribution. It should be noted that the pulse asymmetry parameter \( \lambda \) does not enter the probability density function in equation (3) or any of the moments.

In the case of the two-sided exponential pulse shape given by equation (2), the frequency power spectral density can readily be calculated [28],

\[
\Omega_\Phi(\omega) = \frac{2\tau_d}{[1 + (1 - \lambda)^2(\tau_d\omega)^2][1 + \lambda^2(\tau_d\omega)^2]^2},
\]

(4)

where \( \omega \) is the angular frequency. Thus, the spectrum is flat for low frequencies and has a steep power law tail for high frequencies. It should be noted that the intermittency parameter \( \gamma \) does not influence the shape of the power spectral density, it is determined only by the pulse shape given in equation (2) [26–29]. In the following it will be demonstrated that these predictions provide a novel interpretation of the fluctuations in the scrape-off layer of Alcator C-Mod.

The number of up-crossings \( X_\Phi \) of a given threshold level \( \Phi \) can be calculated in closed form for this stochastic process [26, 27, 29, 30]

\[
\frac{\tau_d}{T} X_\Phi(\tilde{\Phi}) = \frac{\gamma^{\gamma-1}}{\Gamma(\gamma)} \frac{\lambda(1 - \lambda)^{\gamma-1}}{\Gamma(\gamma)\Gamma(\gamma(1 - \lambda))} \left( \gamma + \gamma^{1/2}\tilde{\Phi} \right) \times \exp\left( -\gamma - \gamma^{1/2}\tilde{\Phi} \right).
\]

(5)

Here it should be noted that the pulse asymmetry parameter \( \lambda \) only appears in the pre-factor and therefore does not influence the functional form of the level crossing rate. The number of level crossings is evidently proportional to the duration \( T \) of the process and is inversely proportional to the pulse duration time \( \tau_d \). The rate of level crossings is highest for thresholds close to the mean value of the process. In the normal regime, \( \gamma \gg 1 \), there are few crossings for threshold levels much smaller or much larger than the mean value due to the low probability of large-amplitude fluctuations. The rate of level crossings is therefore a narrow Gaussian function in this limit. In the strong intermittency regime, \( \gamma \ll 1 \), the signal spends most of the time close to zero value, and virtually any pulse arrival will give rise to a level crossing for finite threshold values. The rate of level crossings therefore approaches a step function in this limit [26, 27, 29, 30].

4. Fluctuation statistics

The radial variation of the relative fluctuation level, estimated by the ratio of the sample mean and standard deviation, for all four line-averaged densities is presented in figure 2. The relative fluctuation level is small close to the separatrix, indicating significant overlap of pulse structures as is clearly seen from the raw data time series in figure 1. For all line-averaged densities, the relative fluctuation level increases radially outwards and is of order unity in the far scrape-off layer and the limiter shadow regions. This implies a small value of the intermittency parameter, consistent with the distinct large-amplitude bursts observed in the time series in figure 1.

The probability density function of the gas puff imaging data for \( n_L/n_G = 0.25 \) is presented in figure 3 for three different radial positions. The points show the estimated distribution
from the detrended measurement data time series, while the solid lines show modified Gamma distributions given by equation (3). The intermittency parameter is obtained from a least squares fit to the tail of the experimental distribution for amplitudes $\tilde{\Phi} > 2$. In the vicinity of the last closed magnetic flux surface, the fluctuations are close to normally distributed and the estimated intermittency parameter is close to 300. Radially outwards, the distribution becomes increasingly skewed and flattened, with $\gamma = 11$ estimated at $R = 90.31$ cm. In the limiter shadow region, there is a clear exponential tail for large fluctuation amplitudes and the estimated intermittency parameter is slightly less than unity. The amplitude distribution for the gas puff imaging data are well described by the predictions of the stochastic model for $\tilde{\Phi} > 2$.

for all line-averaged densities. This has an asymmetric structure with a fast rise and a slower decay which is the same for all line-averaged densities. Also presented in figure 4 is the conditionally averaged wave form for a realization of the stochastic process given by equation (1) with model parameters $\tau_d = 20 \mu s$, $\lambda = 10^{-1}$ and $\gamma = 1$. This is clearly consistent with the experimental measurement data, showing an asymmetric exponential wave form. Previous analysis have shown that the peak amplitudes and the waiting times between large-amplitude events are both exponentially distributed, as assumed for the stochastic model [20, 21].

The power spectral densities for the gas puff imaging data at the last closed magnetic flux surface, the main scrape-off layer and the limiter shadow regions are presented in figure 5 for $n_e/n_G = 0.25$. The remarkable similarity of these frequency spectra suggests that the fluctuations have the same temporal scale for all radial positions and differ only by the degree of pulse overlap seen in figure 1. The broken line in figure 5 is the prediction of the stochastic model given by equation (4).
temporal scale is clearly comparable for all radial positions in the scrape-off layer and all line-averaged densities in these experiments. This is consistent with the model prediction that the frequency power spectrum for the rescaled process $\tilde{\Phi}$ is independent of the intermittency parameter $\gamma$ and thereby the degree of pulse overlap.

The normalized rate of up-crossings of a given threshold level is presented in figure 7 for the outermost diode view position and all four line-averaged densities. The full lines are the predictions of the stochastic model given by equation (5).

The predictions of a stochastic model are here shown to reproduce many of the statistical properties of the gas puff imaging measurements on Alcator C-Mod. This includes the probability density function, the power spectral density and the rate of threshold crossings. Previous investigations have demonstrated that the assumptions underlying the model are supported by the measurement data, namely a superposition of uncorrelated exponential pulses with an exponential distribution of pulse amplitudes [20–25].

The analysis presented here clearly demonstrate that the plasma fluctuations in the boundary region of Alcator C-Mod become increasingly intermittent with radial distance into the scrape-off layer. Despite this, there is a remarkable similarity of the frequency power spectra for all radial positions and line-averaged densities investigated. Based on the predictions of the stochastic model, this implies that the temporal scale for the fluctuations set by the pulse duration time $\tau_d$ is the same in all cases. The frequency spectra are therefore entirely determined by the shape and duration of the underlying pulses [28]. Near the last closed magnetic flux surface, there is significant overlap of pulse structures leading to normally distributed fluctuation amplitudes and low intermittency. In the far scrape-off layer, the pulses appear separated and intermittency is pronounced.

The much stronger degree of pulse overlap in the vicinity of the last closed magnetic flux surface is likely due to the dominant poloidal flow in the edge and near scrape-off layer regions. An example of this is presented in figure 8, which is based on gas puff imaging by a $64 \times 64$ pixel Phantom fast-framing camera in a plasma with similar conditions as those investigated above [37]. The camera field of view is divided into smaller square regions of $10 \times 10$ pixels, and the cross-correlation between the pixels in each square is evaluated. From the time delay of the maximum of the correlation it is possible to compute the two-dimensional velocity field. This
clearly shows a dominantly poloidal velocity in the vicinity of the last closed magnetic flux surface and a dominantly radial velocity in the far scrape-off layer \cite{37,38}.

Moreover, some of the blob-like structures that propagate through the scrape-off layer will likely be subject to non-linear dispersion and break up, resulting in small-amplitude noise which enhances the rate of level crossings for small threshold values \cite{39–43}. However, most large-amplitude structures are expected to propagate through the scrape-off layer and into the limiter shadow, resulting in an exponential tail in the probability density function and frequent crossings of large threshold levels.

The stochastic model promises to be a valuable tool for predicting threshold phenomena like plasma–wall interactions due to transient transport events \cite{8–10,26–31}. The frequency and duration of large-amplitude events depend on the intermittency parameter \( \gamma \), the pulse duration time \( \tau_d \), the asymmetry parameter \( \lambda \) and the mean amplitude \( \langle A \rangle \). The results presented here suggest that \( \tau_d \), \( \lambda \) and \( \gamma \) have values which are independent of the plasma parameters in the far scrape-off layer. Thus, only an estimation of the mean value \( \langle A \rangle \) is required in order to assess the intermittency effects. This is exactly the output from two-dimensional transport modelling of the boundary region, and clearly demonstrates the potential value of the stochastic model for predicting plasma–wall interactions due to transient transport events.

Gas puff imaging data does not provide a proxy for any plasma parameter due to the strong dependence of the intensity on the neutral gas density. This motivates a systematic investigation of the fluctuation statistics using the novel mirror Langmuir probe system implemented on Alcator C-Mod \cite{44,45}. Moreover, the analysis presented here will be extended to include measurement data from high performance plasmas and improved confinement modes. Finally, a systematic multimachine comparison should be made in order to confirm the universality of the fluctuations and reveal any dependence of the model parameters on for example machine size.

**Acknowledgments**

This work was supported by financial subvention from the Research Council of Norway under grant 240510/F20 and the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, using User Facility Alcator C-Mod, under Award Number DE-FC02-99ER54512-CMOD. A.T. and O.E.G. acknowledge the generous hospitality of the MIT Plasma Science and Fusion Center where this work was conducted.

**ORCID iDs**

A. Theodorsen  
https://orcid.org/0000-0002-5142-3113  
O.E. Garcia  
https://orcid.org/0000-0002-2377-8718  
B. LaBombard  
https://orcid.org/0000-0002-7841-9261  
J.L. Terry  
https://orcid.org/0000-0003-4255-5509

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