Vector and pseudoscalar charm meson radiative decays

B. Bajc $^a$, S. Fajfer $^{a,b}$ and Robert J. Oakes $^c$

$a)$ Physics Department, Institute "J. Stefan", Jamova 19, 61111 Ljubljana, Slovenia

$b)$ Physik Department, Technische Universität München, 85748 Garching, FRG

$c)$ Department of Physics and Astronomy Northwestern University, Evanston, Il 60208 U.S.A.

ABSTRACT

Combining heavy quark effective theory and the chiral Lagrangian approach we investigate radiative decays of pseudoscalar $D$ mesons. We first reanalyse $D^* \to D \gamma$ decays within the effective Lagrangian approach using heavy quark spin symmetry, chiral symmetry Lagrangian, but including also the light vector mesons. We then investigate $D \to V \gamma$ decays and calculate the $D^0 \to \bar{K}^{*0} \gamma$ and $D^{*+} \to \rho^+ \gamma$ partial widths and branching ratios.
1. Introduction

The list of $D$ meson decay rates is rather long and further study of their decays would eventually help to better understand their features. There has not yet been any experimental evidence for radiative decays of $D$ mesons, while $D^*$ radiative decays are known to be important. The $D^*$ decays [1, 2] can be described in a model independent framework which incorporates the appropriate constraints on the decay amplitudes. The combination of heavy quark effective theory and chiral Lagrangians have been extensively studied and applied to many $D$ mesons decays [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Wise [14] has proposed an effective Lagrangian to describe, at low momentum, the interactions of a meson containing a heavy quark with the light pseudoscalar mesons $\pi, K, \eta$. Two kinds of symmetries characterize the effective Lagrangian: the heavy quark $SU(2)$ spin symmetry and the non-linearly realized $SU(3) \otimes SU(3)$ chiral symmetry in the light sector, corresponding to spontaneous symmetry breaking of the chiral group to the diagonal $SU(3)_V$. Due to the rather large masses of the $D$ mesons, the inclusion of resonances with masses below the $D$ mesons seems necessary [11, 12, 13].

In this paper, following the requirements of heavy quark and chiral symmetry, we develop a framework for the description of heavy and light pseudoscalar and vector mesons. In section 2 we write down the most general Lagrangian in the limit of exact heavy quark and chiral symmetries. Section 3 is devoted to the higher order odd parity Lagrangian, which also describes the decay $D^* \rightarrow D\gamma$, and we reinvestigate this decay in order to learn more about the couplings in the chiral Lagrangian. In section 4 we analyse the
weak Lagrangian. Finally, as an example of the use of our model, we calculate the $D \to V\gamma$ radiative decays in section 5.

2. The chiral Lagrangian technique and heavy quark limit

The strong interaction meson Lagrangian for the light pseudoscalar octet and heavy pseudoscalar and vector triplets in the chiral and heavy quark limits was first written down by Wise \cite{14} (see also \cite{4}). The electromagnetic interactions between these mesons was described in \cite{1,2,3}. The octet of light vector mesons was included in the Wise Lagrangian \cite{14} later by Casalbuoni, et al. \cite{12} as the gauge particles associated with the hidden symmetry group SU(3)$_H$ \cite{13}. The next step is to provide a common description of both the light and heavy pseudoscalar mesons, which also includes both light and heavy vector mesons and the electromagnetic interactions. In this section we present the strong and electromagnetic Lagrangian for the description of both light and heavy pseudoscalar and vector mesons.

The light pseudoscalar mesons are described by the $3 \times 3$ unitary matrix

$$u = \exp \left( \frac{i\Pi}{f} \right)$$  \hspace{1cm} (1)

where $f \simeq 132$ MeV is the pion pseudoscalar pion constant and $\Pi$ is the pseudoscalar meson unitary matrix defined as

$$\Pi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\
\pi^- & \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\
K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta
\end{pmatrix}$$  \hspace{1cm} (2)
The octet of light vector mesons is described by the $3 \times 3$ unitary matrix

$$\hat{\rho}_\mu = i \frac{g_V}{\sqrt{2}} \rho_\mu$$  \hspace{1cm} (3)$$

where $g_V \approx 5.8 \sqrt{2/a}$ with $a = 2$ in the case of exact vector dominance) is the coupling constant of the vector meson self–interaction [15] and $\rho_\mu$ the vector meson unitary matrix

$$\rho_\mu = \begin{pmatrix} \frac{\rho_0^++\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K^{*+} \\ \rho_\mu^- & \frac{-\rho_0^+\omega_\mu}{\sqrt{2}} & K^{*0} \\ K^*_\mu^{-} & K^{*0}_\mu & \Phi_\mu \end{pmatrix}$$ \hspace{1cm} (4)$$

In the following we will also use the gauge field tensor $F_{\mu\nu}(\hat{\rho})$, defined as

$$F_{\mu\nu}(\hat{\rho}) = \partial_\mu \hat{\rho}_\nu - \partial_\nu \hat{\rho}_\mu + [\hat{\rho}_\mu, \hat{\rho}_\nu]$$ \hspace{1cm} (5)$$

The heavy mesons are $Q\bar{q}^a$ ground states, where $Q$ is a c (or b) quark and $q^1 = u$, $q^2 = d$ and $q^3 = s$. In the heavy quark limit they are described by $4 \times 4$ matrix $H_a$ ($a = 1, 2, 3$) [14]

$$H_a = \frac{1}{2} (1 + \gamma^\mu) (P^a_{\mu\gamma} \gamma^\mu - P_a \gamma_5)$$ \hspace{1cm} (6)$$

where $P^a_{\mu\gamma}$ and $P_a$ annihilate, respectively, a spin-one and spin-zero meson $Q\bar{q}^a$ of velocity $v_\mu$. The creation operators $P^a_{\mu\gamma}$ and $P^a_\gamma$ occur in [14]

$$\bar{H}_a = \gamma^0 H_a \gamma^0 = (P^a_{\mu\gamma} \gamma^\mu + P^a_\gamma \gamma_5) \frac{1}{2} (1 + \gamma^\mu)$$ \hspace{1cm} (7)$$
Following the analogy in refs. [15, 12] we introduce two currents:

\[ \mathcal{V}_\mu = \frac{1}{2}(u^\dagger D_\mu u + uD_\mu u^\dagger) \] (8)

and

\[ \mathcal{A}_\mu = \frac{1}{2}(u^\dagger D_\mu u - uD_\mu u^\dagger) \] (9)

where the covariant derivatives of \( u \) and \( u^\dagger \) are defined as

\[ D_\mu u = (\partial_\mu + \hat{B}_\mu)u \] (10)

and

\[ D_\mu u^\dagger = (\partial_\mu + \hat{B}_\mu)u^\dagger \] (11)

while

\[ \hat{B}_\mu = ieB_\mu Q \] (12)

\[ Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \] (13)

and \( B_\mu \) is the photon field. To insure that the vertices \( D^0\dagger D^0\gamma \) and \( D^{*0}\dagger D^{*0}\gamma \), or those with \( D \) replaced by \( B \) in the case of the \( b \) quark, are absent, we define the covariant derivative for the heavy meson field as
\[ D_\mu \bar{H}_a = (\partial_\mu + V_\mu - ieQ'B_\mu)\bar{H}_a \] (14)

with \( Q' = 2/3 \) for c quark \((-1/3 \text{ for } b \text{ quark})\). With these definitions we can finally write down the even parity strong and electromagnetic Lagrangian for heavy and light pseudoscalar and vector mesons:

\[
\mathcal{L}_{even} = \mathcal{L}_{light} - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + iT r(H_\mu v_\mu D^\mu \bar{H}_a) + igT r[H_b \gamma_\mu \gamma_5 (A^\mu)_{ab} \bar{H}_a] + i\beta T r[H_b v_\mu (V^\mu - \hat{\rho}_\mu)_{ab} \bar{H}_a] + \frac{\beta^2}{2f^2a} T r(\bar{H}_b H_a \bar{H}_a H_b) \tag{15}
\]

with

\[
\mathcal{L}_{light} = -\frac{f^2}{2}\{ tr(A_\mu A^\mu) + a tr[(V_\mu - \hat{\rho}_\mu)^2]\} + \frac{1}{2g_V} tr[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})] \tag{16}
\]

This Lagrangian is invariant under the following gauge transformation:

\[
\begin{align*}
H & \to e^{ieQ'(x)}Hg_0^\dagger(x) \\
\bar{H} & \to g_0(x)\bar{H}e^{-ieQ'(x)} \\
u & \to g_0(x)ug_0^\dagger(x) \\
u^\dagger & \to g_0(x)u^\dagger g_0^\dagger(x) \\
V_\mu & \to g_0(x)V_\mu g_0^\dagger(x) + g_0(x)\partial_\mu g_0^\dagger(x) \tag{17}
\end{align*}
\]
\[ A_\mu \rightarrow g_0(x) A_\mu g_0^\dagger(x) \]
\[ \dot{\rho}_\mu \rightarrow g_0(x) \dot{\rho}_\mu g_0^\dagger(x) + g_0(x) \partial_\mu g_0^\dagger(x) \]
\[ \dot{B}_\mu \rightarrow g_0(x) \dot{B}_\mu g_0^\dagger(x) + g_0(x) \partial_\mu g_0^\dagger(x) \]

where \( g_0(x) = \exp(ieQ\lambda(x)) \). The last transformation (17) together with (12)-(13) imply, of course, the usual gauge transformation for the photon field:

\[ B_\mu \rightarrow B_\mu - \partial_\mu \lambda(x) \]  

In equation (15) \( g \) and \( \beta \) are constants which should be determined from experimental data [1, 2, 11, 12, 13]. The constant \( a \) in (15)-(16) is in principle a free parameter, but we shall fix it by assuming exact vector dominance [14], for which \( a = 2 \). With exact vector dominance there are no direct vertices between the photon and two pseudoscalar mesons, so that the pseudoscalars interact with the photon only through vector mesons.

The electromagnetic field can couple to the mesons also through the anomalous interaction; i.e., through the odd parity Lagrangian. Even with the \( PP\gamma \) direct vertices absent in \( \mathcal{L}_{\text{light}} \) due to the choice \( a = 2 \), direct \( PV\gamma \) vertices are present in the odd parity Lagrangian. We write down the two contributions which are significant for our calculation:

\[
\mathcal{L}_{\text{odd}}^{(1)} = -4 \frac{C_{VV\Pi}}{f} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(\partial_\mu \rho_\nu \partial_\alpha \rho_\beta \Pi) \quad (19)
\]
\[
\mathcal{L}_{\text{odd}}^{(2)} = -4 \epsilon\sqrt{2} \frac{C_{V\pi\gamma}}{f} \epsilon^{\mu\nu\alpha\beta} \text{Tr}(\{\partial_\mu \rho_\nu, \Pi\} Q \partial_\alpha B_\beta) \quad (20)
\]
Equation (19), together with vector dominance couplings

\[ \mathcal{L}_{V-\gamma} = -m_V^2 \frac{e}{g_V} B_\mu (\rho^{0\mu} + \frac{1}{3} \omega^\mu - \frac{\sqrt{2}}{3} \Phi^\mu) \] (21)

which come from the second term in (16), describe the electromagnetic interaction assuming vector-meson dominance, while the direct photon-light vector meson-pseudoscalar interactions are contained in (20). The contributions to the odd Lagrangian (19) and (20) arise from Lagrangians of the Wess-Zumino-Witten kind \[16, 17\].

In the \( m_q \to 0 \) and \( m_Q \to \infty \) limit (\( m_q \) and \( m_Q \) are the masses of the light and heavy quarks, respectively) the strong and electromagnetic interactions of heavy and light pseudoscalar and vector mesons are thus described by the even Lagrangian (15)-(16) and by the odd Lagrangian (19)-(20). However the \( D^\ast D\gamma \) vertices are not included in the above Lagrangian since it is of the anomalous type. The terms responsible for it are of higher order \( 1/m_Q \) and they will be introduced in the next section.

3. Higher order odd Lagrangian for heavy mesons

In our approach vector meson dominance describes the couplings of light quarks and photons through the higher dimensional invariant operator

\[ \mathcal{L}_1 = i\lambda Tr[\mathcal{H}_a \sigma_{\mu\nu} F^{\mu\nu}(\hat{\rho})_{ab} \mathcal{H}_b] \] (22)

In this term the interactions of light vector mesons, heavy pseudoscalars or heavy vector \( D \) mesons are also present. The light vector meson can then
couple to the photon by the standard vector-meson dominance prescription \(^{(21)}\). These terms effectively describe the light quark-photon interaction inside the charmed (or beauty) mesons.

The coupling \(\lambda\) can be independently determined either from \(D^*\) decays into \(D\pi\) and \(D\gamma\) \(^{(1)}\,^{(2)}\,^{(3)}\,^{(4)}\), some ratios of which have been measured \(^{(18)}\,^{(19)}\) or from semileptonic decays of \(D\) mesons \(^{(11)}\,^{(12)}\,^{(13)}\).

The heavy quark-photon interaction is generated by the term

\[
\mathcal{L}_2 = -\lambda' e T r [H_a \sigma_{\mu\nu} F^{\mu\nu}(B) \bar{H}_a] \tag{23}
\]

According to quark models the parameter \(|\lambda'|\) can be approximately related to the charm quark magnetic moment via \(1/(6m_c)\) \(^{(1)}\,^{(2)}\,^{(3)}\,^{(4)}\). In order to reduce the error in determining the couplings we shall reanalyze the decays \(D^* \to D\pi\) and \(D^* \to D\gamma\). Experimentally one measures \(^{(19)}\) the branching fractions \(R_0^\gamma = \Gamma(D^0 \to D^0\gamma)/\Gamma(D^{*0} \to D^{0}\pi^0) = 0.572 \pm 0.057 \pm 0.081\) and \(R_+^\gamma = \Gamma(D^{*+} \to D^{+}\gamma)/\Gamma(D^{*+} \to D^{+}\pi^0) = 0.035 \pm 0.047 \pm 0.052\). Using our Lagrangian these branching ratios are

\[
R_0^\gamma = 64\pi f^2 \alpha_{EM} (\frac{\lambda'}{g} + \frac{2}{3} \frac{\lambda}{g})^2 (\frac{p_0^0}{p_0^0})^3 \tag{24}
\]

\[
R_+^\gamma = 64\pi f^2 \alpha_{EM} (\frac{\lambda'}{g} - \frac{1}{3} \frac{\lambda}{g})^2 (\frac{p_+}{p_0})^3 \tag{25}
\]

To determine \(\lambda/g\) and \(\lambda'/g\), the square-roots of the left-hand-sides of eq. \(^{(24)}\)–\(^{(25)}\) have to be taken, which introduces an ambiguity in the resulting
coupling constants. The experimental errors in the branching fractions are somewhat large but the masses of the particles involved are relatively well known. We have used the standard formulae \[18\]

\[\hat{f} = f(\hat{x}) + \frac{1}{2}\sigma^2 f''(\hat{x}) \quad \text{(26)}\]

\[\sigma_f = \sigma|f'(\hat{x})| \quad \text{(27)}\]

for \(f(x) = \sqrt{x} \) (\(x = R_0^0\) or \(R_0^+\)), where \(\hat{x}, \hat{f}\) and \(\sigma, \sigma_f\) are the mean values and standard deviations of the respective distributions. Using expressions (26)–(27) instead of their linearized versions is important due to the large experimental error in \(R_0^+\). From that data we then find

\[|\frac{\lambda'}{g} + \frac{2}{3}\frac{\lambda}{g}| = (0.863 \pm 0.075) GeV^{-1} \quad \text{(28)}\]

and

\[|\frac{\lambda'}{g} - \frac{1}{3}\frac{\lambda}{g}| = (0.089 \pm 0.178) GeV^{-1} \quad \text{(29)}\]

The two errors in eq.(28)–(29) can in principle be correlated. The main sources of correlations are the experimental efficiencies in the detection of the \(\pi^0\) and \(\gamma\). Since the contributions of the efficiencies to the net errors are small \[19\], possible correlations were neglected and the errors in the determination of single \(\lambda/g\) and \(\lambda'/g\) were combined in quadrature. There are
four solutions for the ratios $\lambda/g$ and $\lambda'/g$: 

1. $\lambda/|g| = k(0.77 \pm 0.19) \text{GeV}^{-1}$, $\lambda'/|g| = k(0.35 \pm 0.12) \text{GeV}^{-1}$ and $b) \lambda/|g| = k(0.95 \pm 0.19) \text{GeV}^{-1}$, $\lambda'/|g| = k(0.23 \pm 0.12) \text{GeV}^{-1}$, where $k$ can be $+1$ or $-1$. For the individual determination of $\lambda$, $\lambda'$ and $g$, one has to fix one parameter. Choosing $|\lambda| = (0.60 \pm 0.11) \text{GeV}^{-1}$ as determined in [13] minimized the errors in these quantities. We present in Table 1 the four possible solutions of (28)–(29) for $\lambda$ and $|g|$, together with the combinations $\lambda' + 2\lambda/3$ and $\lambda' - \lambda/3$, which will be used in our calculations in section 5.

\begin{table} [H]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
1 & $+0.60 \pm 0.11$ & $+0.29 \pm 0.12$ & $0.82 \pm 0.20$ & $+0.71 \pm 0.18$ & $+0.07 \pm 0.15$ \\
2 & $-0.60 \pm 0.11$ & $-0.29 \pm 0.12$ & $0.82 \pm 0.20$ & $-0.71 \pm 0.18$ & $-0.07 \pm 0.15$ \\
3 & $+0.60 \pm 0.11$ & $+0.15 \pm 0.09$ & $0.66 \pm 0.13$ & $+0.57 \pm 0.12$ & $-0.06 \pm 0.12$ \\
4 & $-0.60 \pm 0.11$ & $-0.15 \pm 0.09$ & $0.66 \pm 0.13$ & $-0.57 \pm 0.12$ & $+0.06 \pm 0.12$ \\
\hline
\end{tabular}
\caption{Four possible solutions from (28)-(29) with $\lambda$ determined by [13].}
\end{table}

The experimental value $|g| = 0.57 \pm 0.13$ (see for example [13] and references therein for a discussion of these experimental values) and the approximate validity of the equation $|\lambda'| \simeq 1/(6m_c)$ with $m_c \simeq 1.5 \text{ GeV}$ slightly favour solutions 3) and 4).

Our approach is different from [1], [2] and [6], since we do not use any quark model prediction for the parameter $\lambda'$ but treat it on an equal footing with the parameter $\lambda$, so that both are considered as purely phenomenological. Nevertheless we were able to obtain reasonably good precision in the determination of model parameters.
4. Weak Lagrangian for light and heavy mesons

In addition to strong and electromagnetic interactions, we must also address
the weak decays within these scheme. We will follow the approach of [12, 14]
and use an effective current between the heavy mesons and the light mesons.
The weak current is

\[ L_{Q_a}^\mu = \bar{q}_a \gamma^\mu (1 - \gamma_5) Q \]

and it transforms as \((\bar{3}_L, 1_R)\). The lowest dimension operator with the same transformation properties but with
meson degrees of freedom is

\[ J_{Q_a}^\mu = \frac{1}{2} i \alpha T r [\gamma^\mu (1 - \gamma_5) H_b u_{ba}^\dagger] \]

\[ + \alpha_1 T r [\gamma_5 H_b (\hat{\rho}^\mu - V^\mu)_{bc} u_{ca}^\dagger] + \cdots \]

where the ellipsis denote terms vanishing in the limit \(m_q \to 0, m_Q \to \infty\) or
terms with derivatives.

The light mesons decay constants \(f_{P,V}\) are defined by the usual relations

\[ < 0 | J_{q_{\mu}}^{ab}(0) | P_i(p) > = i f_P \frac{\lambda_{i}^{ab}}{\sqrt{2}} p_\mu \]

\[ < 0 | J_{q_{\mu}}^{ab}(0) | V_i(\epsilon, p) > = f_V \frac{\lambda_{i}^{ab}}{\sqrt{2}} m_V \epsilon_\mu \]

(31)

Here \(\lambda_{i}^{ab}, i = 1, \ldots, 8, a, b = 1, 2, 3\) are the usual eight Gell-Mann \(3 \times 3\)
matrices, normalized as \(T r (\lambda_i \lambda_j) = 2 \delta_{ij}\). In the chiral limit \(m_q \to 0\) the
above decay constants are related through \(f_P = f_V / \sqrt{a} = f\) for all \(P, V\).

Similarly we can define the heavy meson decay constants by [14],

\[ < 0 | J_{Q_{\mu}}^{a}(0) | D^b(p) > = - i f_D \delta^{ab} m_D v_\mu \]

\[ < 0 | J_{Q_{\mu}}^{a}(0) | D^{ab}(\epsilon, p) > = i f_D \delta^{ab} m_D \epsilon_\mu \]

(32)
In the heavy quark limit $m_Q \to \infty$ we have $m_D = m_{D^*} \to \infty$ and $f_D = f_{D^*}$. The constant $\alpha$ in eq. (30) can be fixed in this limit by taking the matrix elements of $J_\mu^a$ between the heavy meson state and the vacuum, with the result

$$\alpha = f_P \sqrt{m_P} \quad (33)$$

Unfortunately, up to now, there does not exist either a theoretical prediction or experimental data for the other parameter, $\alpha_1$, in the current (30). The situation is different and better in the light sector, where a well known prescription exists, for deriving the weak current directly from the strong Lagrangian [20]. Since the quark weak current $L_{q_{ab}}^\mu = \bar{q}_b \gamma^\mu (1 - \gamma_5) q_a$ must transform as $(\bar{3}_L, 3_R)$ the light meson weak current with these transformation properties can be obtained from the Lagrangian (16). Of course, one has to properly define the covariant derivative for pseudoscalars, enlarging the gauge group to include the $W$-boson contributions [21]. The resulting light meson part of the weak current is

$$J_q^\mu = i f^2 u [\mathcal{A}^\mu + a(\mathcal{V}^\mu - \mathcal{\hat{J}}^\mu)] u^\dagger \quad (34)$$

The part of the weak Lagrangian for the pseudoscalar and vector, light and heavy mesons, which we will use, can be written as [22, 23, 24]

$$\mathcal{L}_{WW}^{eff}(\Delta c = \Delta s = 1) = -\frac{G}{\sqrt{2}} V_{ud} V_{cs}^* \left[ a_1(\bar{u}d)^\mu_{V-A}(\bar{s}c)_{V-A,\mu} + a_2(\bar{s}d)^\mu_{V-A}(\bar{u}c)_{V-A,\mu} \right] \quad (35)$$
where $V_{ud}$, etc. are the relevant CKM mixing parameters, while $a_1$ and $a_2$ are the QCD Wilson coefficients, which depend on a scale $\mu$. One expects the scale to be the heavy quark mass and we take $\mu \simeq 1.5 \text{ GeV}$ which gives $a_1 = 1.2$ and $a_2 = -0.5$, with an approximate 20% error. In the factorization model the quark currents are approximated by the corresponding meson currents defined in eqs. (30) and (34):

\begin{equation}
(q_a Q)_V^{\mu A} \equiv \bar{q}_a \gamma^\mu (1 - \gamma^5) Q \simeq J_{Q_a}^\mu
\end{equation}

\begin{equation}
(q_b q_a)_V^{\mu A} \equiv \bar{q}_b \gamma^\mu (1 - \gamma^5) q_a \simeq J_{qa}^\mu
\end{equation}

Many heavy meson weak nonleptonic amplitudes \cite{22, 23, 24, 25} have been calculated using the factorization approximation. It has been shown in \cite{22}, however, that for some of the $D$ meson decays there are rather important final state interactions and the factorization approximation can be improved by the inclusion of the $SU(3)$ symmetry breaking effects \cite{26}.

The authors of ref. \cite{23} have classified the weak nonleptonic decays into three classes: decays determined by $a_1$ only (class I), decays determined by $a_2$ only (class II) and decays where $a_1$ and $a_2$ amplitudes interfere (class III). Factorization can therefore be tested in several ways. There are the following two categories of decays: ”quark decays”, in which the heavy quark decays while remaining antiquark acts as a spectator, and ”annihilation processes” in which heavy and light quarks annihilate and two new quarks are created. For the annihilation processes the factorization approximation is usually only a small contribution \cite{23, 24, 22}.
We are forced to use this approximation in our calculations, since there are no better approaches developed so far for nonleptonic weak decays.

5. $D \to V\gamma$ decays

The simplest radiative decays of D mesons are into a light meson and a photon. Since the process $D \to P\gamma$ ($P$ is a light pseudoscalar) is forbidden due to the requirement of gauge invariance and chiral symmetry \cite{27}, as well as angular momentum conservation, we will concentrate on the $D \to V\gamma$ ($V$ is a light vector meson) decays. Although there are not yet any data on such processes we can predict the partial widths and branching ratios. We consider the only two processes which are possible at tree level and are not Cabibbo supressed, namely $D^0 \to \bar{K}^*\gamma$ and $D^{*+} \to \rho^\pm\gamma$. Both processes have contributions from the odd-parity interaction Lagrangian. The second one has, in addition, a direct emission term, due to the charged initial and final mesons.

Regarding the anomalous term, there are two contributions which are important. The photon can first be emitted from the $D^0$ ($D^{*+}$) meson, which becomes a $D^{*0}$ ($D^{*+}$) and then $D^{*0}$ ($D^{*+}$) decays weakly into $\bar{K}^{*0}$ ($\rho^+$). The other contribution comes from the weak decay of $D^0$ ($D^{*+}$) first into an off-shell $\bar{K}^0$ ($\pi^+$), which then decays into $\bar{K}^{*0}\gamma$ ($\rho^+\gamma$). Both contributions are proportional to $a_2$ ($a_1$) \cite{35}. For the description of this amplitude we need the $D^*D\gamma$ and $K^*K\gamma$ ($\rho\pi\gamma$) couplings and these couplings were obtained in the previous section.

The amplitude for the $D^0 \to \bar{K}^{*0}\gamma$ is
\[ A(D^0(p_{D^0}) \rightarrow \bar{K}^{*0}(p_{K^{*0}})\gamma(q)) = e \frac{G}{\sqrt{2}} V_{ud} V_{cs}^{*} a_2 \] (38)

where

\[ C_{D^0K^{*0}\gamma}^{(1)} = \left[ C_{V}^{(1)} \frac{1}{g_{V}} + C_{V}^{(1)} \right] \frac{f_{D^0} f_{K^{*0} m_{D^0}^{2}}}{(m_{D^0}^{2} - m_{K}^{*0}^{2})} \frac{8 \sqrt{2} m_{D^0}}{3 f} \]

\[ + \frac{4}{3} \frac{\lambda'}{\lambda} f_{D^0} f_{K^{*0}} m_{D^0} m_{K^{*0}} \frac{m_{D^0}^{2} - m_{K^{*0}}^{2}}{m_{D^0}^{2} - m_{K^{*0}}^{2}} \] (39)

Similarly, the amplitude for \( D^{*+} \rightarrow \rho^{+} \gamma \) is

\[ A(D^{*+}(p_{D^{*}}) \rightarrow \rho^{+}(p_{\rho})\gamma(q)) = e \frac{G}{\sqrt{2}} V_{ud} V_{cs}^{*} a_1 \] (40)

where

\[ C_{D^{*+}\rho\gamma}^{(1)} = \left[ C_{D^{*+}\rho\gamma}^{(1)} \right] \frac{f_{D^{*}} f_{\pi} m_{D^{*}}^{2}}{(m_{D^{*}}^{2} - m_{\pi}^{2})} \frac{4 \sqrt{2} m_{D^{*}}}{3 f} \]

\[ + \frac{4}{3} \frac{\lambda'}{\lambda} f_{D^{*+}} f_{\rho} m_{D^{*}} m_{\rho} \frac{m_{D^{*}}^{2} - m_{\rho}^{2}}{m_{D^{*}}^{2} - m_{\rho}^{2}} \] (41)

and

\[ C_{D^{*+}\rho\gamma}^{(2)} = f_{D^{*}} f_{\rho} \] (42)
In our numerical calculations we used the following numerical values $C_{VV\Pi} = 0.423$, $C_{V\Pi\gamma} = -3.26 \times 10^{-2}$, $g_{V} = 5.8$ [12], $f \approx f_{\pi} = 132$ MeV, and the other decay constants $f_{P,V}$ were taken from [25]. It is straightforward to calculate the decay widths. The result, of course, depends on which numerical value we take for $(\lambda' + 2\lambda/3)$ and $(\lambda' - \lambda/3)$. The numerical predictions for the decay widths and branching ratios are given in Table 2, where all the four possible choices of parameters (see Table 1) are considered.

| Solution | $D^{0} \rightarrow \bar{K}^{*0}\gamma$ | $D^{s+} \rightarrow \rho^{+}\gamma$ |
|----------|---------------------------------|---------------------------------|
|          | $\Gamma \ [10^{-7}\text{eV}]$ | BR $[10^{-4}]$ | $\Gamma \ [10^{-7}\text{eV}]$ | BR $[10^{-4}]$ |
| 1        | $7.8 \pm 2.5$                 | $4.9 \pm 1.6$ | $4.9 \pm 2.0$ | $3.3 \pm 1.3$ |
| 2        | $0.8 \pm 0.7$                 | $0.5 \pm 0.4$ | $7.6 \pm 3.7$ | $5.1 \pm 2.5$ |
| 3        | $5.9 \pm 1.4$                 | $3.7 \pm 0.9$ | $7.0 \pm 2.9$ | $4.7 \pm 1.9$ |
| 4        | $0.3 \pm 0.3$                 | $0.2 \pm 0.2$ | $4.7 \pm 1.7$ | $3.2 \pm 1.1$ |

Table 2: Different possible predictions for the $D^{0} \rightarrow \bar{K}^{*0}\gamma$ and $D^{s+} \rightarrow \rho^{+}\gamma$ decays. The errors are due to uncertainties in the determination of the combinations $(\lambda' + 2\lambda/3)$ and $(\lambda' - \lambda/3)$.

An interesting feature can be seen from Table 2: A not very precise measurement of the $D^{0} \rightarrow \bar{K}^{*0}\gamma$ decay rate is sufficient to differentiate between solutions 1)-3) and solutions 2)-4), which are predicted to be of one order of magnitude different. Unfortunately, due to the much larger branching ratio for the weak decay $D^{0} \rightarrow \bar{K}^{*0}\pi^{0}$ and the difficulty in differentiating the photon from the $\pi^{0}$ in this energy range, the decay $D^{0} \rightarrow \bar{K}^{*}\gamma$ has not been seen by the ARGUS collaboration [30]. The situation is similar for the detection of $D^{s+} \rightarrow \rho^{+}\gamma$ by the ARGUS collaboration: due to the very small branching ratio, low detector’s acceptance (3γ events have to be measured)
and poor mass resolution, the ARGUS data are not likely to find this decay. But, hopefully, some experimental signals for these radiative decays will come from the CLEO data. The experimental measurement of this branching ratio will determine the relative sign between the first and the second contributions, giving in such a way new information on the parameters $\lambda$, $\lambda'$ and $g$.

In conclusion, we used both chiral symmetry and heavy quark symmetry to obtain an effective strong, EM and weak Lagrangian for the description of both light and heavy pseudoscalar and vector mesons. In this framework we reanalyzed the $D^*$ strong and radiative decays, obtaining without any reference to quark models, a good determination of some of the parameters in the effective Lagrangian. Within the same framework and with these values for the parameters we calculated the $D \to V\gamma$ decay widths, providing numerical predictions. These results can be used to test the validity of the approximations that were made in the context of the heavy quark effective theory. At the least, our numerical results are reasonable estimates and provide some guidance. In the framework developed here other D meson radiative non-leptonic decays ($D \to VP\gamma$ or $PP\gamma$) can also be calculated \[31\], giving estimates for future experiments and further tests of the applicability of HQET.

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*Note added.* After completing of this work the preprint of P. Jain, A. Momen and J. Schechter ([hep-ph/9406338](hep-ph/9406338)), where some of these results have been independently obtained, came to our attention. Their approach is quite similar to ours in section 3, but they fix the parameter $\lambda'$ to be proportional to $1/m_c$, while we leave it free. Also we noticed a preprint by H.Y. Cheng, C.Y. Cheung, G.L. Lin, Y.C. Lin, T.M. Yan and H.L. Yu ([hep-ph/9407303](hep-ph/9407303)). In this preprint the $D \rightarrow K^* \gamma$ decay rate was estimated using an effective electromagnetic and weak Lagrangian developed from quark diagrams for $b\bar{d} \rightarrow c\bar{u}\gamma$. They then made the replacement $b \rightarrow c$ and $c \rightarrow s$. Their result is comparable with our second case in Table 2.

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