SPECTROSCOPY AND THREE-DIMENSIONAL IMAGING OF THE CRAB NEBULA

ANDREJ ČADEŽ
Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, Ljubljana SI-1000, Slovenia; andrej.cadez@uni-lj.si

ALBERTO CARRAMIÑANA
Instituto Nacional de Astrofísica, Óptica y Electrónica, Luis Enrique Erro 1, Tonantzintla, Puebla 72840, Mexico; alberto@inaoep.mx

AND

SIMON VIDRIH
Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, Ljubljana SI-1000, Slovenia; simon.vidrih@fmf.uni-lj.si

Received 2003 February 17; accepted 2004 March 22

ABSTRACT

Spectroscopy of the Crab Nebula along different slit directions reveals the three-dimensional structure of the optical nebula. On the basis of the linear radial expansion result first discovered by V. Trimble in 1968, we make a three-dimensional model of the optical emission. Results from a limited number of slit directions suggest that optical lines originate from a complicated array of wisps that are located in a rather thin shell pierced by a jet. The jet is certainly not prominent in optical emission lines, but the direction of the piercing is consistent with the direction of the X-ray and radio jet. The shell’s effective radius is \( \approx 79'' \), its thickness is about a third of the radius, and it is moving out with an average velocity of 1160 km s\(^{-1}\).

Subject headings: ISM: individual (Crab Nebula) — ISM: kinematics and dynamics — supernova remnants — techniques: spectroscopic

On-line material: mpg animation

1. INTRODUCTION

The Crab Nebula was an object of intense interest even before its pulsar was discovered in radio by Staelin & Reifenstein (1968) and in the optical by Cocke et al. (1969). Trimble (1968) first measured proper motions of nebular components and, combining them with spectroscopic data taken by Münch (1958), discovered that they all point to a very small, almost pointlike origin in the past. Velusamy et al. (1992) obtained a similar result from radio images of the nebula. Both observations are consistent with an outward-pointing velocity field with the proper-motion magnitude proportional to the distance from the origin. Trimble (1968), Wyckoff & Murray (1977), and Caraveo & Mignani (1999) pointed out that the position of this origin is displaced with respect to that of the pulsar. Furthermore, the proper motions of the nebular components are \( \sim 10\% \) faster than the value expected if the nebular debris were expanding freely. In radio images, the linear expansion is not as obvious and shows some anomalies farther away from the pulsar. More recently, attention has focused on phenomena connected to the Crab Nebula’s high-energy emission, its jet, and the very rapid expansion of its inner wisps discovered optically by Hester et al. (1995) and in radio by Bietenholz et al. (2001). It has become increasingly clear that the Crab pulsar and its nebula are connected through a multilayer structure of different predominant energy ranges. To understand the mutual interrelationship of these layers, it is useful to have a three-dimensional picture of at least part of the structure.

The first attempts to understand the three-dimensional structure of the Crab Nebula go back to Chevalier & Gull (1975), who studied the nebula in the light of all 13 important visible emission lines by using narrowband filter imaging. They found a structure “consisting of dense filaments from which thin clumpy sheets of gas fan out radially.” Clark et al. (1983) studied spectroscopically the radial velocity field across the nebula, to investigate the nebula’s three-dimensional properties, and found the line-emitting region to be a thick hollow shell composed of bright inner and faint outer components. They also saw the filaments as generally circumferential, but radial, “spokes” connecting the inner and outer shells. They conclude that the nebular synchrotron emission is confined within the inner, brighter shell. A comprehensive attempt to understand the geometry, composition, and mass of the Crab Nebula has been made by MacAlpine et al. (1989, 1996). They combine interference filter imaging and long-slit spectroscopy to obtain information on the composition and distribution of gas in the nebula. On the basis of a few selected spectra and using model calculations, MacAlpine et al. (1989) deduce that collisional contributions to observed exceptionally strong helium lines are insignificant. Using further model assumptions, they estimate the combined mass in the filaments to be in the range \( 6–9 \, M_\odot \). On the basis of interference filter and Fabry-Pérot imaging, Lawrence et al. (1995) interpret the \([\text{O} \, \text{iii}]\, \lambda 5007\) emission as having a spatial relationship to a “high-helium torus.”

The possibility to make a three-dimensional representation of the line-emitting regions in the Crab Nebula is offered by the proper-motion result of Trimble (1968) and others. On the basis of this result, one is led to speculate that all three components of the velocity vector field describing the motion of nebular components point in the outward radial direction, since the motion is ballistic. According to this assumption, the speed of this motion is proportional to the distance from the origin, i.e., we assume

\[
v = \frac{r}{T},
\]

(1)

\(^1\) Visiting Astronomer, Observatorio Astrofísico Guillermo Haro, Cananea, Sonora, Mexico.
where $\mathbf{r}$ is the velocity vector at position $r$ measured from the center of the nebula, and $T$ is the time elapsed. If the ballistic assumption were fully satisfied, $T$ would be the time since the explosion in 1054, but, as Trimble’s proper motion results show the possibility of some acceleration of the debris in the past, we consider $T$ as a model parameter that may differ somewhat from the time since the explosion. Observing spectroscopically a small patch of the nebula at an angular position $(\xi, \eta)$ from the pulsar, one expects to find multiple Doppler-shifted emission lines, each Doppler shift corresponding to a different radial velocity component $(v_z)$ of the cloud. According to equation (1), such a component corresponds to a line-of-sight coordinate $z = (v_z - v_{\text{rad}})T$, while the other two components of the vector $\mathbf{r}$ are $x = (\xi - x_0)D$ and $y = (\eta - y_0)D$, where $v_{\text{rad}}$ and $(x_0, y_0)$ are the velocity and angular coordinates of the center of the cloud, respectively, and $D$ is the distance to the nebula. Locating line-emitting regions in this way, one can construct a tentative three-dimensional image of (line-emitting) nebular components. The only parameter critical for the shape of such a reconstructed image is the ratio $T/D$. We choose to consider it as a free parameter and construct three-dimensional models of the nebula for different values of this ratio. We find it remarkable that it is possible to find a value for this parameter such that the three-dimensional model of the nebula in the radiation from the strongest optical emission lines ([N II], [S II], and Hα) appears as embedded inside a relatively thin, pierced spherical shell with a well-defined inner radius. We consider this particular ratio $T/D$ of some interest, yet we would like to emphasize that its implications for the distance to the nebula depend strongly on the ballistic assumption.

2. DATA

The spectra of the Crab Nebula were obtained on 2001 January 18 (slit orientation 0° and 90°, high resolution) and 2002 January 4 (0° and 90°, low resolution), January 5 (18°, 36°, 54°, and 72°, low resolution), January 6 (108°, 126°, 144°, and 162°, low resolution), January 7 (36°, 72°, 108°, and 144°, high resolution), and January 8 (18°, 54°, 126°, and 162°, high resolution) with the 2.12 m telescope of the Observatorio Astrofísico Guillermo Haro in Cananea, Mexico, equipped with the Boller & Chivens (B&C) spectrograph. All observations were done under photometric conditions. Spectra were taken in a low-resolution mode of 1.6 Å per pixel and a 250 μm (≈4 pixels ≈ 2″) wide slit and in a high-resolution mode of 0.43 Å per pixel and a 50 μm slit. The spatial FWHM of the pulsar profile was varying between 2" and 3" during observations and was thus wider than the projected width of the slit. The slit length was 560 pixels × 0′′.46 per pixel in both cases; however, all the pixels were not exposed properly (see Fig. 1). In the first case, the effective Doppler width of the slit was 300 km s⁻¹, and in the second case, it was about 16 km s⁻¹. Five spectra (20 minutes exposure each) were taken with the low-resolution mode and three (30 minutes exposure each) with the high-resolution mode for each of the 10 slit orientations (ψ = 0°, 18°, 36°, . . . , 162° with respect to the east-west orientation). The low-resolution spectra cover the spectral region between 5400 and 7000 Å and give an average signal-to-noise ratio (S/N) of more than 200 per pixel. The high-resolution mode covers the region between 6400 and 6840 Å and yields an average S/N of only ~2. Thus, in the low-resolution mode the nebular continuum is clearly detected with a S/N of up to ~40 per pixel, while in the high-resolution mode it is just below the detection threshold. As an example, reduced (bias subtraction, sky flat, median average) high- and low-resolution spectra for the slit orientation 36° are presented in Figure 1. In Figure 2 we also show a rescaled superposition of the two spectra, where the high-resolution spectrum is colored in red and the low-resolution one in green. A trace of the spectrum (at ~75° from the pulsar; see the lower half of the lower panel in Fig. 1) is shown in Figure 3. The rest-frame wavelengths of a number of known nebular spectral lines are marked in gray, and the Doppler-shifted lines of nitrogen, sulfur, hydrogen, oxygen, and helium are marked in blue (−1047 km s⁻¹) and red (+1729 km s⁻¹). The scale on the ordinate axis is in photon counts per pixel per 20 minutes. Finally, Figure 4 shows the continuum trace along the ψ = 36° slit at approximately 6000 Å, i.e., in the flat region between the He i and [O I] lines. The gray circle shows the position of the pulsar, where the continuum value interpolated along the dashed line is calibrated with respect to a previous measurement (Carramiñana et al. 2000). Some features in this and other continuum traces can be recognized as belonging to stars that can be identified in the image of the nebula (the peak just next to the right of the pulsar belongs to a star ~6° away). We use this information to secure accurate positions of the slits with respect to the nebula. The positions thus derived are shown in Figure 5. The (small) corrections, with respect to desired positions, obtained by these data have been taken into account in further data analysis of the low-resolution spectra. This procedure could not be applied to the high-resolution spectra because of the low S/N. In this case, we assume that the slits crossed the pulsar accurately and that the transparency of the sky was constant. We expect that the respective error in position and in flux is comparable to the shifts and calibration factors obtained in the low-resolution spectra (±4″ in position and about ±10% in flux). In constructing images we use the more precise position data from low-resolution spectra and use high-resolution spectra to reduce the velocity calibration errors of the low-resolution spectra.

An analysis of all the spectra confirms the impression apparent from Figure 3, that the intense [N II] and [S II] doublets, the Hα line, and also the weak [O I] doublet always occur at a common redshift, relative intensities varying across the nebula. The forbidden [N II] λ5755 line is almost always clearly absent, but in some instances its presence may be masked by the double atmospheric feature just to the left and to the right of the red [N II] λ5755 mark (compare also Fig. 1). Only a close scrutiny revealed the presence of this weak line in the 36° spectrum at the redshift of the strongest [N II] line. It is quite difficult to automatically obtain good estimates for the ratio (λ5755 + λ6583)/λ5755 from these spectra, since the continuum is not well defined in this region because of the aforementioned presence of weak atmospheric lines. Checking spectral traces by hand and judging the intensity of the 5755 line by the value at the pixel corresponding to the velocity of the 6584 and 6583 lines, we estimate some values for this ratio, ranging from 185 ± 35 (λ5755 + λ6583) = 259 photon counts per pixel, 210 ± 35 (λ5755 + λ6583 = 487 photon counts per pixel), and 310 ± 100 (λ5755 + λ6583 = 193 photon counts per pixel), for traces where the 5755 line is visible. This value is consistent, even somewhat lower than that found by Davidson et al. (1982).

Among the nebular lines observed in our spectra, we take only the five most intense as characterizing optical emitting regions: [N II] (i = 1, 2), Hα (i = 3), and [S II] (i = 4, 5). To determine their common Doppler shift and strength, we proceed
as follows: for each slit orientation we scan the spectra along all slit-length positions \( m \) (10 pixels \(< m < 537 \) pixels for high-resolution spectra and 43 pixels \(< m < 545 \) pixels for low-resolution spectra; the slight misalignment of the slit with respect to the CCD axis is taken into account) and obtain raw spectral traces \( S_r(\lambda, m|\psi_1) \). These are flux-calibrated by making use of the fact that all slit positions cross the pulsar. Thus, we set the 6000 Å interpolated nebular continuum at the position of the pulsar to the same reference value for all spectra and analyze all spectra in units of this reference value (see Fig. 4). Pure line spectra \( S_l(\lambda, m|\psi_1) \) are then obtained by subtracting the continuum. It is automatically defined for each trace \( (m) \) as the best \( n \)th \((n = 10)\) degree polynomial fitting the points \([\lambda_c^{(e)}, S_c(\lambda_c^{(e)}, m|\psi_1)]\), where \( \lambda_c^{(e)} \) are determined by inspection of the whole spectrum at the wavelengths free of spectral lines. Lines and their common Doppler shifts are finally identified, defining a Doppler velocity–dependent weight for each of the five lines in the spectrum as

\[
w_l(v, m) = \int S_l(\lambda, m) \exp \left[ -\frac{\lambda - \lambda_c(1 - v/c)}{2\sigma^2} \right] d\lambda.
\]

3 We estimate that the relative error of this recalibration is of the order of ±3%, about the size of the gray circle in Fig. 4.
Choosing the appropriate value for the parameter \( \sigma \), we form the velocity weight function

\[
\Omega(v) = w_1(v)w_2(v) + w_cw_3(v) + w_4(v)w_5(v)
\]

and find on the interval \(-2000 \text{ km s}^{-1} < v < 2000 \text{ km s}^{-1}\) the velocity \( v_{\text{max}} \) at which \( \Omega(v) \) is maximal. Such a velocity weight function is large only if both lines of doublets are present, and it has been found experimentally on our data sample (some 55,000 spectral traces) that its maximum gives a velocity that can be considered the common velocity of the three components, H, N, and S. The characteristic strength \( w_c \), introduced with the H\( \alpha \) line, is chosen so that the contribution of this line to the velocity weight function is in the average comparable to the contribution of the two doublets. The Gaussian lines, Doppler-shifted to \( v_{\text{max}} \), are then subtracted from \( S(\lambda, m) \), and the procedure is repeated on the remaining spectrum until \( \Omega(v) \) no longer shows significant maxima. The important parameter \( \sigma \), chosen as constant in the reduction of a given spectrum, is selected before the automatic procedure starts. This is done through trial and error until most randomly chosen line multiplets subtract best in a single subtraction. In the low-resolution case, the best \( \sigma \) value is found to be essentially the effective slit width. This is expected, since no lines show evidence of being resolved. In this case, the subtractions of all of the five lines belonging to a common velocity multiplet usually leave only little more than the expected photon-counting statistical noise, except in

![Fig. 2.—Low-resolution (green) and high-resolution (red) spectra in the spectral region between 6415 and 6814 Å for the slit angle 36° in the vicinity of the [N ii] \( \lambda\lambda6548, 6583 \) doublet, H\( \alpha \) \( \lambda6563 \), and the [S ii] \( \lambda\lambda6716, 6730 \) doublet suitably rescaled and superimposed.](image)

![Fig. 3.—Trace \( S(\lambda, m) \) of the spectrum \( m = 440 \) pixels (along the line in the lower panel of Fig. 1). Some important lines are marked: the rest positions with respect to local lamp calibration are shown by gray vertical lines, and parallel blue and red lines indicate positions blue- and redshifted by \(-1047 \) and \(1729 \text{ km s}^{-1} \), respectively. Absolute velocity calibrations in low-resolution spectra are not better than \( \pm100 \text{ km s}^{-1} \), but comparisons such as the one in Fig. 2 show that calibration shifts are the same, to within the resolution of the high-resolution spectra, in the whole region of [N ii] and [S ii] lines of interest. The scale on the ordinate axis is in photon counts per pixel per 20 minutes.](image)
a few cases in which different Doppler-shifted components of Hα and [N ii] overlap. The weights \( w_i(I_{\text{max}}, m) \) form what we call reduced spectra, and the set of the five weights for a given \( I_{\text{max}} \) and \( m \) is called here a velocity component. Three examples of successive subtractions are shown in Figure 6. In this way, we detected 43,843 velocity components in all of the 10 low-resolution spectra, on average 8.5 components per spectral trace. A great majority of these are of no consequence, since their intensity is negligible. A total of 80% of the light is contributed by only 5300 (12%) of the different velocity components, and 20% account for 90% of the light. In other words, 8770 velocity components (1.7 per spectral trace) account for 90% of the light. The remaining 80% of detected velocity components contribute little, so they are barely seen in the reconstruction. Some of them belong to line wings that are not subtracted during the first run, some to weak independent lines, and some to residues of imperfect subtraction, which are due to either photon counting noise or small errors in wavelength calibration, and also to the fact that different lines sometimes overlap and their components become confused. It is difficult to precisely judge the relative contribution of these effects. We believe they are best illustrated in Figure 6, which shows typical examples of imperfect subtraction to the level of a few percent, and in Figure 7, where the observed spectrum and its reconstruction from detected velocity components can be compared.

The high-resolution spectra clearly show lines of different resolved widths. A drastic example of both an unresolved [S ii] doublet at -115 km s\(^{-1}\) and a 150 km s\(^{-1}\) broad [S ii] doublet at 814 km s\(^{-1}\) is found in the spectral trace along the line in the lower panel of Figure 1 and is shown in Figure 8.\(^4\) In this case it would be advantageous to take \( \sigma \) as a free parameter for each system of lines. However, since most lines are neither as narrow nor as wide as the example shown here and the S/N is quite low, we usually choose a moderate \( \sigma = 40 \text{ km s}^{-1} \), which subtracts most lines rather well. Only the broadest and strongest lines produce additional sidebands next to the main peak in reconstruction. A clear example of such a broad line is clearly seen in Figure 10 on top of the \( \psi = 162^\circ \) and the \( \psi = 0^\circ \) spectra.

Spectra reconstructed from velocity components are shown in Figures 9 and 10, where the intensity of the weights is color-coded, as indicated by color triangles down the middle of the figures. The intensity is defined as \( I = [1/I_{\text{max}}(\psi)] \) \((w_1 + w_2 + w_3 + w_4 + w_5)\), where \( I_{\text{max}}(\psi) \) is the maximum intensity for the given reduced spectrum. In units of interpolated continuum intensity at the pulsar position, \( I_{\text{max}}(\psi) \) is 42.5, 22.4, 61.7, 41.7, 38.3, 46.0, 30.1, 43.0, 57.5, and 74.0, in order from \( \psi = 0^\circ \) to 162°. The colors are assigned with

\[ I(\psi) = \frac{1}{I_{\text{max}}(\psi)} (w_1 + w_2 + w_3 + w_4 + w_5) \]

\[ I_{\text{max}}(\psi) = 42.5, 22.4, 61.7, 41.7, 38.3, 46.0, 30.1, 43.0, 57.5, 74.0 \]

\[ \psi = 0^\circ, 162^\circ \]

\[ \text{Color codes:} \]

\[ \text{Red (R): 0^\circ to 42.5} \]

\[ \text{Orange (O): 42.5 to 22.4} \]

\[ \text{Yellow (Y): 22.4 to 61.7} \]

\[ \text{Green (G): 61.7 to 41.7} \]

\[ \text{Blue (B): 41.7 to 38.3} \]

\[ \text{Cyan (C): 38.3 to 46.0} \]

\[ \text{Magenta (M): 46.0 to 30.1} \]

\[ \text{Purple (P): 30.1 to 43.0} \]

\[ \text{Brown (B): 43.0 to 57.5} \]

\[ \text{Gray (G): 57.5 to 74.0} \]

\[ \text{White (W): 74.0 to 0^\circ} \]
with respect to the “average bright spot mixture of intensities,” which is coded gray [center of the triangle, \((H:N:S) = (0.18:0.42:0.40)\)], while the corners of the triangles correspond to pure H, N, or S, as indicated on the topmost triangle in Figure 9. Comparing Figures 9 and 10, note the advantages and disadvantages of high and low resolution. The dominating features are clear and accurately correspond in both figures. However, the global features of the radiating gas distribution are better appreciated in low resolution because of the higher S/N. On the other hand, only the high resolution shows that the radiating gas is organized in exquisitely fine sheets, tubes, or blobs.

Reduced spectral intensities are considered the line intensities at the position

\[
\begin{align*}
\xi &= \hat{m} \cos \psi + \Delta \xi(\psi) \\
\eta &= \hat{m} \sin \psi + \Delta \eta(\psi) \\
v &= v_{\text{max}} + \Delta v(\psi)
\end{align*}
\]

of the three-dimensional image and are stored accordingly. Here \(\hat{m} = [m - m_0(\psi)] \times 0.46\) per pixel, and \(m_0(\psi)\) is the value of \(m\) in the spectrum \(S_i(\lambda, m)\), where the spectral trace belonging to pulsar light is strongest. Here \(\Delta \xi(\psi)\) and \(\Delta \eta(\psi)\) are coordinates of the vector connecting the slit position.
[\text{m}_0(\psi)] with the center of the pulsar in the image \cite[see the lower image of Fig. 5, for \Delta q(0) and \Delta \psi(0)]{00}. The \Delta \xi, \Delta \eta \text{ values are of the order of } 2^\circ \text{ except for the slit } \psi = 72^\circ \text{ and } 162^\circ, \text{ where the star next to the pulsar was mistaken for the pulsar. The velocity calibration error of low-resolution spectra } \Delta v(\psi) \text{ was determined by graphically correlating low-resolution and high-resolution synthetic spectra, assuming that high-resolution spectra velocity error is negligible. The values found are between } -180 \text{ km s}^{-1} \text{ and } 0 \text{ km s}^{-1}. \text{ From high-resolution spectra, } \Delta \xi(\psi), \Delta \eta(\psi), \text{ and } \Delta v(\psi) \text{ cannot be obtained, but comparison with low-resolution spectra suggests that the pointing error was not larger than the one reached in taking low-resolution data.}

3. IMAGES

On the basis of the spectral information alone, we first reconstruct (1) the continuum image of the nebula at 6000 Å and (2) the image of the nebula in the light of the five strong lines. They are shown together with the optical, radio, and X-ray images in Figure 11. The black and white image corresponding to the 6000 Å continuum \textit{(bottom left)} is formed by assigning the values for the continuum intensity at measured points \((\xi_k, \eta_k)\), and values in between are assigned by linear interpolation, i.e., \(S(\lambda, \Delta m + m_0(\psi_i))_{/>\psi} = S_1\) and \(S(\lambda, \Delta m + m_0(\psi_i))_{/>\psi} + \delta v = S_2\), then \(S(\lambda, \Delta m + m_0(\psi_i))_{/>\psi} + \delta v = S_1 + (S_2 - S_1)/(\delta v/\Delta v)\), where \(\Delta v = 18^\circ\) and \(0 \leq \delta v < \Delta v\). The coordinates \(\xi\) and \(\eta\) belonging to points \(m = \Delta m + m_0(\psi)\), \(\psi = \psi_i + \delta \psi\) are calculated according to equation (4).

The emission-line image \textit{(bottom right)} is formed in a similar fashion but in color, so that the green intensity is proportional to the intensity of nitrogen regardless of velocity, the red component comes from H\(\alpha\), and the blue from sulfur. The color contributions are normalized so that the strongest line of any color gives the maximum display value for this color (255).

In order to appreciate the level of detail that such synthetic images can give, we also display in Figure 11 a blend of the visual image and the X-ray image \textit{(top left)}, the radio image \textit{(top center)}, and an enhanced visual image \textit{(top right)} of the nebula. We find the continuum image reminiscent of the X-ray image, while the emission lines image clearly highlights the filament structure embedded in the nebula, as does the radio image.

The three-dimensional image can be understood on a computer screen if the structure is rotated and displayed as a movie \cite{00} which can be obtained in the electronic version of this paper; see also Fig. 12 based on the low-resolution data. The color code is the inverse of that used in Figure 9, and the intensity is coded as the density of radiating fog multiplied by \(\sin \theta, \text{ where } \theta\) is the angle between the \(z\)-axis and the direction from the coordinate-system origin to the observed cloud. This is to compensate for the difference in sampling density. The \(x\)- and \(y\)-axes are oriented as in Figure 5 and their length is 90\(^\circ\), while the \(z\)-axis points away from the observer and its length is 1300 km s\(^{-1}\). When observing this animation, it becomes quite apparent that the structure is shell-like and that it is possible to adjust the \(T/D\) value so that the structure looks round when rolled on the screen. Therefore, we fit a spherical shell to the distribution of line radiators by minimizing the action:

\[
A(\xi_k, \eta_k, \psi_k, \Delta, \alpha) = \frac{1}{\sum_k u_k} \sum_k \left[ \sqrt{(\xi_k - \xi_0)^2 + (\eta_k - \eta_0)^2 + \alpha^2(\psi_k - \psi_0)^2 - \Delta} \right]^2 u_k. \tag{5}
\]

Here \(u_k\) is the weight of the \(k\)th radiating blob formed from line intensities\(^5\) \(w_k(\xi_k, \eta_k, \psi_k, \Delta, \alpha)\), and the variational parameters

\(^5\) The conversions are cyan \(\rightarrow\) red, yellow \(\rightarrow\) blue, magenta \(\rightarrow\) green.

\(^6\) For high-resolution spectra, \(u_k = \sum_{i=1}^{n} w_i(\xi_k, \eta_k, \psi_k)\). In the case of low-resolution spectra, inspection of Figures 9 and 11 makes obvious the fact that the \(\rho\)-plane of the nebula has not been scanned exhaustively; some waning filaments have clearly escaped detection, since the slit of the spectrograph is not long enough to reach to the farthest radiating regions. We note, however (see Fig. 9), that the intensity generally decreases with distance from the shell, so that weighting brighter clouds more heavily than the dimmer ones might not displace the center of the nebula, but it does overcome the geometric selection problem. Therefore, we either exclude all clouds with less than a few percent of maximum intensity or by weighting \(u_k\) proportional to \(w_k^2\). Both methods give similar results that differ only by about 1\% in \(\Delta, \text{ by } \sim 5\%\) in \(\xi, \eta, \alpha, \psi\), but almost 30\% in \(\alpha\), and agree with those from the high-resolution spectra.
Fig. 9.—Reduced low resolution spectra for all slit orientations ($\psi = 0\degree, 18\degree, \ldots, 162\degree$, numbered in red). The intensity $w_i(\psi, v)$, coded according to the color intensity code shown by the triangles down the middle of the figure, is shown as a function of velocity (abscissa: $v$ with ticks at 100 km s$^{-1}$) and position along the slit (ordinate: $\tilde{m}$, major ticks spaced by 1'). The positive axis of the velocity coordinate points away from the observer, and the positive axis of the slit coordinate points toward the numbers in Fig. 5. The large orange cross indicates the true origin of the coordinate system centered on the pulsar at $v = 0$, cf. eq. (4). The light gray ring with a dot at its center indicates the projection of the spherical shell volume ($R_{\text{in}} = 65\degree, R_{\text{out}} = 90\degree$) found by the fit. The colors are assigned with respect to the “average bright spot mixture of intensities,” which is coded gray [center of the triangle, $(H:N:S) = (0.18:0.42:0.40)$; in the average hydrogen is much stronger], while the corners of the triangles correspond to pure H, N, or S as indicated on the triangle at the top of the figure. Intensity is coded relative to each reduced spectrum as the fraction of the maximum intensity of the given reduced spectrum. For respective maximum values, see text. The minimum intensity seen is $\sim 5\%$ of the maximum intensity, which is significantly above noise; remember, however, that because of line interference the deconvolution is not unique (compare Fig. 7).
Fig. 10.—Same as Fig. 9 for the high-resolution spectra. The S/N for this spectroscopy is quite low (see text), so that some specks are due to noise fluctuations; in particular, the trace $\psi = 18'$ is the noisiest, since it has the lowest maximum intensity (see text). This spectroscopy is mainly useful in conjunction with low-resolution spectroscopy for better velocity calibration. Compare also with Fig. 2.

$$(\xi_c, \eta_c), v_c, \Delta, and \alpha$$ are the position of the center of the shell with respect to the pulsar, the velocity of the shell center with respect to the observer, the effective shell radius, and the parameter values $$(\xi_c, \eta_c, \Delta, v_c) = (10^5, 17^h, 79^m, 198 \text{ km s}^{-1})$$ with the action value of $$13^2 \times 13^2/2$$. In the high-resolution case, the parameter values $$(\xi_c, \eta_c, \Delta, v_c) = (111^h, 13^h, 74^m, 189 \text{ km s}^{-1})$$ for the action value of $$10^1 \times 10^1$$. The best-fit parameters were used to plot projections of the shell on the $v$-$\rho$ planes in Figures 9 and 10. The dynamical center of the nebula as found by our fit $$(\xi_c, \eta_c, \Delta, v_c)$$ is plotted as a cross in Figure 11 (top right); both the high-resolution and the low-resolution spectra obtain a center within the box defined by the size of the cross ($\sim 10''$). The best value for $\alpha$ is $0.0068 \text{ km}^{-1} \text{s}$. This position of the center of the nebula is puzzling. The vector from our center of the nebula to the pulsar points almost exactly opposite to the proper-motion vector multiplied by time since explosion found by Caraveo & Mignani (1999). This could be explained if the nebula was moving with respect to us with twice the velocity of the pulsar, which seems unlikely, since the Crab Nebula is considered a Galactic-disk object. Therefore, we checked the assumption that the center of the nebula is where Caraveo & Mignani’s proper-motion result would put it; assuming that the proper motion of the nebula is negligible. We constructed a movie rotating about this center. In this case, wobbling of the three-dimensional image is beyond any doubt. Note, however, that because of slit-length limitations, the radial distribution is cut at $\approx 120''$ from the center, where the intensity is not completely negligible. This should be taken as a warning that the obtained fit values may not yet have a very simple physical meaning. If we assume in equation (1) that $T$ is the time elapsed since the supernova explosion, we obtain 3.1 kpc as the best value for the distance, somewhat larger than the accepted value. Finally, in Figure 13 we show the radial distribution of intensity (proportional to the power radiated in a line in the shell between $\rho$ and $\rho + \Delta \rho$) in the three line sets, as a function of the $``\text{radial distance}'' \rho = (\xi - \xi_c)^2 + (\eta - \eta_c)^2 + \alpha^2 (v - v_c)^2)^{1/2}$. The sharp rise at $\approx 70''$ is followed by gradual and regular falloff.

Our three-dimensional image suggests that the volume within $\approx 55''$ of the center of the nebula is almost void of line emitters. A notable exception is a very weak trail of spots on the line connecting the upper left and lower right brightest spots and going through the center of the nebula in the spectra of slit orientation 144°, which is the orientation of the X-ray and radio jet in the image (see Fig. 5). These spots are less than 5% of maximum intensity and are likely located along the jet. A saturated image of this reduced spectrum is shown in Figure 14.

4. CONCLUSION

We obtained multiple long slit spectra of the Crab Nebula in the light of the optical emission lines of H$\alpha$, [N II] $\lambda\lambda 6548, 6583$, and [S II] $\lambda\lambda 6716, 6730$. The simple assumption of ballistic expansion, expressed by equation (1) and suggested by the work of Trimble (1968), allows a three-dimensional representation of the nebula, although through an arbitrary scale in the line of sight. Such a three-dimensional representation suggests that line-emitting regions are organized in a relatively thin shell, regardless of the line-of-sight scale. Therefore, we make the assumption that the shell is likely to be quasispherical and adjust the unknown scale parameter accordingly. The resulting three-dimensional image indeed
Fig. 11.—Crab Nebula image reconstruction from the spectral data. *Top left:* Blend of optical and X-ray images (Weisskopf et al. 2000); *top center:* radio image, (Bietenholz et al. 2001); *top right:* optical image with the dynamical center of the nebula and the projection of the main line emitting lobe; *bottom left:* 6000 Å continuum; and *bottom right:* [N ii] (green), H α (red), and [S ii] (blue) spectral components.
Fig. 12.—Image (i.e., a single movie frame) of the Crab Nebula from a different perspective. The observer on the Earth is in the direction of the negative $z$-axis. The color coding in the movie is explained in § 3. [This figure is available as an mpg file in the electronic edition of the Journal.]
appears to fill a spherical shell with an inner radius of \( \approx 55'' \), as already shown by the results of Lawrence et al. (1995). The shell is pierced by the jet in the southeast direction and at \( \approx 75' \) with respect to the line of sight. The center of this shell is well aligned with the two-dimensional projection of the center of the optical and X-ray continuum emissions. It is clearly displaced from the pulsar but appears to be connected to it through the northwestern extension of the jet. The three-dimensional distribution of the optical continuum and the X-ray emission cannot be determined, but the observed two-dimensional projections of these are very similar and consistent with the idea that the optical and X-ray continuum come from the same, rather flat, oblate structure centered in the spherical shell. The optical continuum emission drops regularly with distance from the center of the nebula (Fig. 4), becoming weak when reaching the spherical shell. Based on the absence of the \([\text{N} \, \text{ii}] \lambda 5755\) line, we estimate that the effective gas temperature in the shell is below 7500 K. The filling of the 55'' shell seems patchy. We note that some bright spots are only arcseconds wide (cf. Fig. 1) and possibly less than 16 km s\(^{-1}\) “deep” (Fig. 8), so that we expect that a more detailed high-resolution three-dimensional view would show that the shell is woven with delicate threads. Note that the

Fig. 13.—Radial distribution of intensity in the three line sets as a function of the “radial distance” \( \rho \).

Fig. 14.—Saturated reduced low-resolution spectrum for the slit orientation \( \psi_s = 144' \), showing the trail of the jet. The color triangle corresponds to the intensity 5%. The signal along the jet is up to 25% of maximum intensity, yet it is important to realize that the spectrum in this region is crowded, so that some line confusion is probable. Higher resolution and better S/N spectroscopy of this region is required.
small size of line-emitting regions indicates high emission coefficients in the bright regions; in particular, we estimate that the region shown in Figures 1 and 8 is only about 0.02 lt-yr thick, which leads to the $H_\alpha$ emission coefficient of the order $j_{H_\alpha} \rho \approx 10^{-21}$ ergs cm$^{-3}$ s$^{-1}$, which is an order of magnitude higher than the average emission coefficient at these wavelengths of the whole nebula including the synchrotron cloud. Hints of threads are suggested in the superimposition of the radio image by Velusamy et al. (1992) on the optical image and are clearly seen in the newer radio image by Bietenholz et al. (2001) shown in Figure 11. Note that optical filaments closely map the smoother radio loops. The jet, which is so prominent in X-rays and in radio, is also marked by optical continuum emission and also seems to be threaded by weak line emission (see Fig. 9, in particular at the 144$^\circ$ orientation, and Fig. 14). Line emission from regions where the jet hits the 55$''$ shell is particularly strong—such as the upper-left brightest spot in the reduced spectra $\psi = 144^\circ$ and also $\psi = 162^\circ$, both in Figure 9.

In concluding, we would like to call attention to two observations: the roundness and average regularity of the spherical shell, as expressed by the sharpness and simplicity of the radial distribution of line-emitting regions in Figure 13, is quite remarkable. The delicate radiating tubes or blobs filling this relatively thin shell closely follow radio loops on the large scale, even if, as emphasized by Sankrit et al. (1998) and Hester et al. (1996), on the short scale they are dominated by the magnetic Rayleigh-Taylor instabilities. The three-dimensional image, even if it still lacks in detail, suggests that the filaments are wound on the shell along long, smooth threads, possibly following the magnetic field lines.

This work was done as a collaboration of the Instituto Nacional de Astrofísica, Optica y Electrónica (INAOE) and the University of Ljubljana and was supported in part by CONACyT grant 25539E and by the Ministry of Science, Education, and Sport of the Republic of Slovenia under grant 1554-501. The essential support to this project by the technical staff of the OAGH in Cananea is appreciated. The movie production is a courtesy of ARTREBEL9.

**REFERENCES**

Bietenholz, M. F., Frail, D. A., & Hester, J. J. 2001, ApJ, 560, 254
Cadež, A., Vidrih, S., Galičič, M., & Carramiñana, A. 2001, A&A, 366, 930
Caraveo, P. A., & Mignani, R. 1999, A&A, 344, 367
Carramiñana, A., Cadež, A., & Zwitter, T. 2000, ApJ, 542, 974
Chevalier, R. A., & Gull, T. R. 1975, ApJ, 200, 399
Clark, D. H., Murdin, P., Wood, R., Gilmozzi, R., Danziger, J., & Furr, A. W. 1983, MNRAS, 204, 415
Cocke, W. J., Disney, M. J., & Taylor, D. J. 1969, Nature, 221, 525
Davidson, K., et al. 1982, ApJ, 253, 696
Hester, J. J., et al. 1995, ApJ, 448, 240
———. 1996, ApJ, 456, 225
Lawrence, S. S., MacAlpine, G. M., Uomoto, A., Woodgate, B. E., Brown, L. W., Oliversen, R. J., Lowenthal, J. D., & Liu, C. 1995, AJ, 109, 2635
MacAlpine, G. M., Lawrence, S. S., Sears, R. L., Sosin, M. S., & Henry, R. B. C. 1996, ApJ, 463, 650
MacAlpine, G. M., McGaugh, S. S., Mazzarella, J. M., & Uomoto, A. 1989, ApJ, 342, 364
Münch, G., 1958, Rev. Mod. Phys., 30, 1042
Sankrit, R., et al. 1998, ApJ, 504, 344
Staelin, D. H., & Reifenstein, E. C. 1968, Science, 162, 1481
Trimble, V. 1968, AJ, 73, 535
Velusamy, T., Roshi, D., & Venugopal, V. R. 1992, MNRAS, 255, 210
Véron-Cetty, M. P., & Woltjer, L. 1993, A&A, 270, 370
Weisskopf, M. C. et al. 2000, ApJ, 536, L81
Wyckoff, S., & Murray, C. A. 1977, MNRAS, 180, 717