Field of a moving locked charge as applied to beam-beam interactions in storage rings

Alexander J. Silenko

Research Institute for Nuclear Problems, Belarusian State University, Minsk 220030, Belarus, Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141980, Russia

(Dated: November 24, 2015)

Abstract

It is shown that the Lorentz transformation cannot in general be formally applied to potentials and fields of particles locked in a certain region. In particular, this property relates to nucleons in nuclei and to particles and nuclei in storage rings. Even if they move with high velocities, their electric fields are defined by the Coulomb law. The result obtained is rather important for the planned deuteron electric-dipole-moment experiment in storage rings.

Keywords: Lorentz transformations; deuteron; electric dipole moment
I. INTRODUCTION

A simultaneous use of two beams in electric-dipole-moment (EDM) experiments may be helpful. Injecting the beam clockwise and counter-clockwise allows one to cancel some systematical errors. However, the beam dynamics and the spin motion are affected by the beam-beam interaction. In particular, spins of particles moving clockwise rotate about the radial axis in the electric and magnetic fields of the counter-clockwise beam. To describe the beam dynamics and the spin motion in such an experiment, beam-beam interactions should be correctly described.

At first sight, this task seems to be very simple because a distribution of charges and currents is well-defined. The scalar and vector potentials of moving particles can be determined with the use of Lorentz transformations. However, the problem is rather nontrivial. We show that interactions with particles locked in a certain region have some important peculiarities.

In storage rings, particles and nuclei are locked. This circumstance influences interactions between two beams. We consider how this effect manifests in a particle spin motion and ascertain its relation to spin dynamics in electric-dipole-moment (EDM) experiments.

We use the system of units $\hbar = 1$, $c = 1$. We include $\hbar$ and $c$ into some equations when this inclusion clarifies the problem.

II. ELECTRIC FIELD OF PROTONS MOVING IN A NUCLEUS

It is instructive to consider the electric field of protons moving in a nucleus. It is well known that the scalar potential and the strength of the field of a nucleus at rest are equal to

$$\Phi = \frac{Ze}{r}, \quad E = \frac{Ze r}{r^3}.$$  

(1)

Numerous experiments have shown that Eq. (1) is valid and the measured number $Z$ is integer. Therefore, motion of charged nucleons (protons) does not affect the potential and the electric field of the nucleus.

However, this result is nontrivial. Protons and neutrons move in the nucleus with high velocities. The simplest way for a description of the electric field of the nucleus is an assumption that electric fields of all protons are independent and may be summarized. Let
the nucleus be at rest in the lab frame. In this frame, the scalar potential of the moving charge \( e \) can be obtained with the Lorentz transformation from its rest frame:

\[
\Phi = \frac{\Phi_0}{\sqrt{1 - \beta^2}} = \gamma \Phi_0 = \frac{\gamma e}{r_0}, \quad \beta = \frac{v}{c},
\]

where \( \gamma \) is the Lorentz factor. Evidently, \( r_0 = r \) when the vectors \( \mathbf{v} \) and \( \mathbf{r} \) are orthogonal and \( r_0 = \gamma r \) when these vectors are collinear. Thus, \( \Phi = e/r \) only in the latter case and \( \Phi \) is greater than \( e/r \) in other cases. In particular, \( \Phi = \Phi_0(1 + \beta^2/2) \) in the nonrelativistic approximation. The difference between \( \Phi \) and \( e/r \) can be easily checked. While the nucleons in the nucleus move in random directions and an average velocity of each nucleon is equal to zero, the resulting potential of the nucleus differs from the potential (1).

The electric field corresponding to Eq. (2) is equal to

\[
E = \left[ r - \frac{\gamma}{\gamma + 1} \beta (\beta \cdot r) \right] \frac{\gamma e}{r^3}.
\]

(3)

Averaging with respect to a proton motion in the nucleus results in

\[
\overline{E} = \frac{(2\gamma + 1)e}{3r^3}.
\]

(4)

This result also disagrees with the conventional electric field of nuclei given by Eq. (1) and with experimental data confirming this equation. The discrepancy can be reinforced with taking into account quark structure of nucleons because velocities of quarks are much closer to the light velocity as those of nucleons. Numerous experiments performed in nuclear and atomic physics explicitly demonstrate that we need not consider the internal structure of the nuclei.

The average potential of the nucleon is given by

\[
\overline{\Phi} = \int E \cdot d\mathbf{r} = \frac{(2\gamma + 1)e}{3r}
\]

and the average potential of the nucleus is equal to

\[
\overline{\Phi}_N = \frac{e}{3} \sum_{i=1}^{Z} \frac{2\gamma_i + 1}{r_i}.
\]

(5)

Evidently, it significantly differs from the conventional potential (1).

Reasons of this situation can be explained. We can show that standard Lorentz transformations are inapplicable to locked particles. Let us consider an interaction of a slowly
moving electron with an electric field of a nucleus. The kinetic energy of the electron slowly increases accordingly to a decrease of its potential energy. Protons in the nucleus move much more quickly than the electron. Therefore, a change of the electron position during a proton oscillation can be neglected. This means that we may consider the proton moving in the electrostatic field of the electron. The potential and kinetic energies of the proton change due to the interaction with the electron field. The field of the slow electron is characterized by the potential $-|e|/r$ and by the strength $-|e|r/r^3$. Therefore, the potential energy of an electron-proton interaction is always equal to $-e^2/r$. The interaction energy of any proton is defined by this expression while the quantity $r$ can differ for different protons. As a result, the total energy of the electron-nucleus interaction is equal to $-e^2 \sum_{i=1}^{Z} 1/r_i$. Due to a symmetric arrangement of protons in the nucleus, its potential takes the form (1).

The electric field of a moving nucleus can be obtained with an appropriate Lorentz transformation and is given by Eq. (3) with the substitution of $Ze$ for $e$. In this case, $\beta$ characterizes the velocity of the center of mass of the nucleus.

The results of this discussion relate not only to nuclei with $Z > 1$ but also to the deuteron with $Z = 1$. Their importance for the deuteron is caused by a motion of the proton about the center of mass of the two nucleus. The difference between the fields of free and locked particles relates also to proton beams in closed spaces like storage rings. This problem is considered in the next section.

III. BEAM-BEAM INTERACTIONS IN STORAGE RINGS

We can now consider the beam-beam interactions in storage rings. While the spherical symmetry is characteristic for nuclei, the cylindrical symmetry and the cylindrical coordinates are characteristic for storage rings. We analyze the case when two rings are simultaneously used. Charged particles forming the two beams are locked in the storage rings like protons in the nucleus. Thus, the scalar potential and the electric field strength of any particle in the lab frame are equal to $e/r$ and $-er/r^3$, respectively, where $r$ is also defined in the lab frame. An integration of the total electric field strength of the beam allows one to calculate the total potential of this beam. This means that the energy of the beam-beam interactions in storage rings does not depend on the momenta of the beams and is defined by the Coulomb potential.
The electric field of a particle beam is defined by the electric field of a ring of charge \([1]\). Near the particle beam (when the distance to the beam is much less than the ring radius) this field is almost orthogonal to the tangent to the ring and has a shape of a squeezed torus. The field strength can be approximated by that of an infinite charged filament formed by the particles and is given by

\[ E = \frac{E \rho}{\rho}, \quad E = \frac{2\tau}{\rho}, \quad (6) \]

where \(\rho\) is the distance to the filament and \(\tau\) is the charge of the unit of its length.

Otherwise, a formal use of Lorentz transformed potentials brings a very different result. In this case, the electric field of the infinite charged filament formed by moving particles is equal to

\[ E = \frac{2\tau\gamma}{\rho}. \quad (7) \]

A difference between Eqs. (6) and (7) is rather significant.

We should add that the motion of particles creates also an electric current which magnetic field is given by

\[ B = \frac{2\tau\beta \times \rho}{\rho^2}. \quad (8) \]

The problem directly relates to EDM experiments in storage rings. In these experiments, one can use two beams consisting from particles moving in the clockwise and counterclockwise directions with the same momentum \([2, 3]\). When the two beams are separated in space \([2]\), particles of one beam create a vertical electric field and a radial magnetic one acting on particles of another beam. The most important effect of beam-beam interactions is an action of these fields on the spin motion. The angular velocity of spin motion is defined by the Thomas-Bargmann-Michel-Telegdi equation \([4]\) extended on the EDM \([5]\):

\[ \Omega = -\frac{e}{mc} \left[ \left( G + \frac{1}{\gamma} \right) B - \frac{\gamma G}{\gamma + 1} (\beta \cdot B) \beta - \left( G + \frac{1}{\gamma + 1} \right) \beta \times E \right. \\
\left. + \frac{\eta}{2} \left( E - \frac{\gamma}{\gamma + 1} (\beta \cdot E) \beta + \beta \times B \right) \right]. \quad (9) \]

Here \(G = (g - 2)/2\), \(g = 2mc\mu/(ehs)\), \(\eta = 2mcd/(ehs)\), \(s\) is the spin quantum number, and \(\mu\) and \(d\) are the magnetic and electric dipole moments. The effects of the vertical magnetic field and the radial electric one on the EDM consist in a spin rotation about the radial axis.
Equation (9) shows that such a rotation can also be conditioned by the vertical electric field and the radial magnetic one acting on the magnetic dipole moment. Therefore, the two latter fields may cause the spin rotation imitating the presence of the EDM.

We should take into account that the average vertical force acting on the beam is zero. When magnetic focusing is used, the resulting radial magnetic field is equal to the sum of the magnetic fields created by the motion of particles of another beam [see Eq. (8)] and by focusing magnets. The resulting radial magnetic field causes the force which should be equal on the average to the force originated in the vertical electric field. The latter field is defined by Eq. (6) [or, alternatively, by Eq. (7)]. When one uses electric focusing, the average force caused by the summary electric field of the beam and of the focusing electric quadrupoles counterbalances the force conditioned by the magnetic field (8). The spin turn about the radial axis takes place in the both cases.

IV. SPIN MOTION CAUSED BY THE BEAM-BEAM INTERACTIONS IN THE DEUTERON EDM EXPERIMENT

One plans to use two beams circulating clockwise and counterclockwise in the deuteron [2] and proton [3] EDM experiments in storage rings. Therefore, it is important to calculate an effect of electric field of one beam on the particle spin motion in another beam. To describe the spin motion in storage ring EDM experiments, it is convenient to present Eq. (9) in cylindrical coordinates [6]:

$$\Omega' = -\frac{e}{mc} \left\{ GB - \frac{\gamma G}{\gamma + 1} (\beta \cdot B) \beta + \left( \frac{1}{\gamma^2 - 1} - G \right) (\beta \times E) + \frac{1}{\gamma} \left[ B_\parallel - \frac{1}{\beta^2} (\beta \times E)_\parallel \right] \right. $$

$$+ \left. \frac{\eta}{2} \left( E - \frac{\gamma}{\gamma + 1} (\beta \cdot E) \beta + \beta \times B \right) \right\}. $$

(10)

The sign $||$ means a horizontal projection for any vector.

Equation (10) shows that the special connection between the vertical magnetic field and the radial electric one,

$$E_r = -\frac{G\beta\gamma^2 B_z}{1 - G\beta^2\gamma^2},$$

cancels the spin rotation about the vertical axis conditioned by the magnetic moment. However, there is the spin rotation about the radial axis. This rotation is caused by the EDM.
and is given by
\[ \Omega'_{EDM} = \frac{en}{2mc} \cdot \frac{\beta B_z}{1 - G\beta^2 \gamma^2} e_r = \frac{d}{\hbar} \cdot \frac{\beta B_z}{1 - G\beta^2 \gamma^2} e_r. \] (11)

The vertical electric field is counterbalanced by the radial magnetic field \( B_r = -E_z/\beta \).

Cumbersome but simple calculations define the overall effect of these two fields on the spin motion:
\[ \Omega'_{b-b} = \frac{e(1 + G)}{mc\beta \gamma^2} E_z e_r. \] (12)

These calculations are based on Eq. (6). The use of Eq. (7) increases the angular velocity of the spin rotation by the factor of \( \gamma \).

In the deuteron EDM experiment, one plans to have two independent rings located on top of each other, about 40 cm apart \[2\]. Each ring will contain \( 1 \times 10^{11} \) nuclei rotating clockwise in one ring and counterclockwise in another one. Since the circumferences of the rings are 82.955 m \[2\], the vertical electric field is 8.7 V. For a ring with the beam momentum \( p = 1 \) GeV/c and the vertical magnetic field \( B_z = 0.5 \) T \[2\], a comparison of Eqs. (11) and (12) shows that the spin rotation defined by Eqs. (6) and (12) corresponds to the deuteron EDM \( d = 1.9 \times 10^{-21} \) e·cm. The effect of the electric field conditioned by the moving beam is therefore rather important and the use of the right equation (6) is necessary. The large value of the systematical correction for the beam-beam interactions (i.e., for the vertical electric field originated from another beam) as compared with the planned experimental sensitivity of \( d = 1 \times 10^{-29} \) e·cm proves a necessity of shielding this field.

Shielding the influence of the electric field of one beam on another beam is stipulated in the deuteron EDM experiment \[2\]. When the two rings are made of a conductive material, they shield the electric and magnetic fields of the stored beams. In this case, one needs to take into account a beam-wall interaction caused by a mirror current running on the walls of the rings.

The situation is very different in the proton EDM experiment. In this experiment, the clockwise and counterclockwise beams are joined (the split between them is of the order of a picometer \[3\]). Therefore, we should consider a particle motion and a spin rotation in the resulting fields created by the two beams. The magnetic field is almost zero in the lab frame due to a clockwise and counterclockwise motion of particles. The electric field acting on a particle is the field of a charged cylinder. The cylinder radius \( \rho_0 \) is equal to the beam
radius. In the considered case, the electric field is given by

\[ E = 2\pi \eta \rho, \quad \rho \leq \rho_0, \quad (13) \]

\[ E = \frac{2\pi \eta \rho_0^2 \rho}{\rho^2}, \quad \rho > \rho_0, \quad (14) \]

where \( \eta \) is the charge density. As compared with Eq. \( (6) \), \( \tau = \pi \rho_0^2 \eta \).

Equations \( (13), (14) \) define the fields acting in both the vertical and radial directions \( (E_z = E \cos \theta, \ E_r = E \sin \theta) \). It is easy to show that the action of these vertical and radial fields consists in a change of frequencies of the vertical and radial betatron oscillations caused by focusing fields. Equations \( (13), (14) \) demonstrate that the average electric field of the two beams acting on the spin is almost zero. Its negligible nonzero value is conditioned by the small asymmetry of beam positions.

We should underline that Eq. \( (6) \) defines the electric field which acts not only on another beam but also on ring magnets, electric plates and so on. The electric field in an inertial frame moving relatively the storage ring can be obtained with the corresponding Lorentz transformation (cf. the end of Sec. \( \Box \)).

V. SUMMARY

We have ascertained that the Lorentz transformation cannot in general be formally applied to potentials and fields of particles locked in a certain region. In particular, this property relates to nucleons in nuclei and to particles and nuclei in storage rings. Even if they move with high velocities, their electric fields are defined by the Coulomb law. The result obtained is rather important for the planned deuteron EDM experiment in storage rings.

Acknowledgements

The author is grateful to Y.K. Semertzidis for useful discussions. The work was supported in part by the Belarusian Republican Foundation for Fundamental Research (Grant No. \( \Phi 14D-007 \)) and by the Heisenberg-Landau program of the German Ministry for Science and
[1] F. R. Zypman, Off-axis electric field of a ring of charge. Am. J. Phys. 74, 295 (2006); I. Mandre, Off-axis electric field of a ring of charge. http://www.mare.ee/indrek/ephi/efield_ring_of_charge.pdf

[2] D. Anastassopoulos et al (Storage Ring Electric Dipole Moment Collaboration), AGS Proposal: Search for a permanent electric dipole moment of the deuteron nucleus at the $10^{-29}$ e·cm level. http://www.bnl.gov/edm/deuteron_proposal_080423_final.pdf

[3] V. Anastassopoulos et al (Storage Ring Electric Dipole Moment Collaboration), A Storage Ring Experiment to Detect a Proton Electric Dipole Moment. arXiv:1502.04317.

[4] L.H. Thomas, The Motion of the Spinning Electron. Nature (London) 117, 514 (1926); I. The Kinematics of an Electron with an Axis. Philos. Mag. 3, 1 (1927); V. Bargmann, L. Michel, V.L. Telegdi, Precession of the Polarization of Particles Moving in a Homogeneous Electromagnetic Field. Phys. Rev. Lett. 2, 435 (1959). This equation has also been obtained by J. Frenkel, Die Elektrodynamik des rotierenden Elektrons. Zeits. f. Phys. 37, 243 (1926)

[5] D.F. Nelson, A.A. Schupp, R.W. Pidd and H.R. Crane, Search for an Electric Dipole Moment of the Electron. Phys. Rev. Lett. 2, 492 (1959); I.B. Khriplovich, Feasibility of search for nuclear electric dipole moments at ion storage rings. Phys. Lett. B 444, 98 (1998); T. Fukuyama, A.J. Silenko, Derivation of Generalized Thomas-Bargmann-Michel-Telegdi Equation for a Particle with Electric Dipole Moment. Int. J. Mod. Phys. A 28, 1350147 (2013); A.J. Silenko, Spin precession of a particle with an electric dipole moment: contributions from classical electrodynamics and from the Thomas effect. Phys. Scr. 90, 065303 (2015).

[6] A. J. Silenko, Equation of spin motion in storage rings in the cylindrical coordinate system. Phys. Rev. ST Accel. Beams 9, 034003 (2006).