CP Violation in K-Meson Decays

R. N. Rogalyov

State Research Center of Russia
"Institute for High-Energy Physics", Protvino, Russia

Abstract

Here we present a pedagogical review of CP and T violation in the decays of K mesons. Diagonalization of the quark mass matrix and the emergence of the complex phase in both the standard and the left–right symmetric models is considered in great detail. A special emphasis is focused on a correct definition of CP-violating quantities: $\epsilon, \epsilon'$ etc. (with due regard for the Wu–Yang phase convention) and to formulation of the time-reversal invariance criterion in the elementary particle physics. A particular attention has been concentrated on theoretical evaluation of the parameters $\epsilon$ and $\epsilon'$ and the CP- and T-violating asymmetries in the decays $K \to \mu \nu \gamma$ and $K \to 3\pi$.

1 CKM Matrix

1.1 Diagonalization of the Mass Matrix.

The initial Lagrangian of the Standard Model (SM) involves 12 massless chiral quarks

\[
\begin{array}{cccc}
D_1^R & U_1^R & d^R & u^R \\
D_1^L & U_1^L & d^L & u^L \\
D_2^R & U_2^R & s^R & c^R \\
D_2^L & U_2^L & s^L & c^L \\
D_3^R & U_3^R & b^R & t^R \\
D_3^L & U_3^L & b^L & t^L \\
\end{array}
\]

which acquire mass due to spontaneous breaking of the $SU(2)_L$ symmetry. The most general quark mass matrix induced by the Higgs fields has the form

\[
\sum_{i,j=1}^{3} \left( D_i^L M_{ij}^R D_j^R + U_i^L N_{ij}^R U_j^R \right) + \text{H.c.},
\]

where $M_{ij}^R$ and $N_{ij}^R$ are arbitrary $3 \times 3$ matrices. This mass term can be considered as a perturbation of the initial SM Hamiltonian, the above quark states form the basis associated with the

\footnote{E-mail: rogalyov@mx.ihep.su}
12-fold degenerate zero-mass level of the unperturbed Hamiltonian. According to the quantum mechanics, one should find the basis in which the perturbation operator takes the diagonal form

\[ M' = V_d M Y_d^\dagger, \quad N' = V_u M Y_u^\dagger, \tag{2} \]

where \( M = \text{diag}(m_d, m_s, m_b) \) and \( N = \text{diag}(m_u, m_c, m_t) \); \( Y_d, Y_u, V_d, \) and \( V_u \) are unitary \( 3 \times 3 \) matrices; and

\[ d_i^R, \quad u_i^R, \quad d_i^L, \quad u_i^L, \quad (i = 1, 2, 3) \tag{3} \]

are the so-called mass eigenstates, which form the sought-for basis. The interaction eigenstates (the eigenstates of the interaction Hamiltonian) are expressed in terms of the mass eigenstates as follows:

\[ D_R^L = Y_d d_R, \quad U_R^L = Y_u u_R, \quad D_L^L = V_d d_L, \quad U_L^L = V_u u_L. \]

This being so, the interaction between left charged currents and gauge bosons

\[ \sum_{i=1}^{3} \left( \bar{D}_i^L \hat{W} U_i^L + \bar{U}_i^L \hat{W} D_i^L \right) \tag{4} \]

takes the form

\[ \sum_{i,j=1}^{3} \bar{d}_i^L (V_d^\dagger V_u)_{ik} \hat{W} U_j^L + \bar{U}_j^L (V_u^\dagger V_d)_{ik} \hat{W} d_i^L = \sum_{i=1}^{3} \bar{u}_i \gamma^\mu \left( 1 - \gamma^5 \right) W_i^\mu d_i^L + \text{H.c.}, \tag{5} \]

where

\[ d_i^L = V_{ij} d_j, \quad \text{and} \quad V = V_d^\dagger V_d \tag{6} \]

is the Cabibbo–Kobayashi–Maskawa (CKM) matrix \([1]\). **Note that the matrices \( Y_d \) and \( Y_u \) play no role when considering interaction.** Now the left doublets (that appear in the interaction Lagrangian) have the form

\[
\begin{pmatrix}
  u \\
  V_{ud} d + V_{us} s + V_{ub} b \\
\end{pmatrix}, \quad
\begin{pmatrix}
  c \\
  V_{cd} d + V_{cs} s + V_{cb} b \\
\end{pmatrix}, \quad
\begin{pmatrix}
  t \\
  V_{td} d + V_{ts} s + V_{tb} b \\
\end{pmatrix}.
\]

It should be noticed that mass eigenstates are invariant under the transformations \( V_u \rightarrow V_u \Phi_u, \quad V_d \rightarrow V_d \Phi_d \), where

\[
\Phi_u = \begin{pmatrix}
  e^{i\phi_u} & 0 & 0 \\
  0 & e^{i\phi_c} & 0 \\
  0 & 0 & e^{i\phi_t}
\end{pmatrix}, \quad
\Phi_d = \begin{pmatrix}
  e^{i\phi_d} & 0 & 0 \\
  0 & e^{i\phi_s} & 0 \\
  0 & 0 & e^{i\phi_b}
\end{pmatrix};
\]

thus any matrix of the family \( \Phi_u^\dagger V \Phi_d \) may be chosen as the quark-mixing matrix. To exclude this arbitrariness, one should fix the phases of the quark states:

\[
\begin{array}{ll}
  u & V_{ud} \\
  c & V_{cd} \\
  t & V_{td} \quad \text{real.} \\
  s & V_{us} \\
  b & V_{ub}
\end{array}
\]

The phase of the \( d \) quark is fixed by the requirement that the above matrix elements are real.
Thus 5 independent conditions can be imposed on the elements of the mixing matrix and so it has 4 independent parameters, 1 of which has to be complex (all real-valued $U(3)$ matrices belong to the $SO(3)$ group). This means that there is one $CP$- and $T$- violating parameter in the SM interaction Lagrangian, whose value cannot be determined from the general principles. In the case of $N$ flavors, a similar reasoning implies the existence of $(N - 1)(N - 2)/2$ complex phases; in the case of two generations, all the elements of the mixing matrix can be made real and the theory is $CP$ invariant.

It is instructive to show in detail how the imaginary part of the fermion–fermion–vector-boson coupling constant implies $CP$ and $T$ violation. We define the action of the $C$, $P$, and $T$ transformations on the fermion fields as follows:

$$\mathcal{P}\psi(x)\mathcal{P}^\dagger = \gamma^0\psi(\tilde{x}), \quad \mathcal{C}\psi(x)\mathcal{C}^\dagger = -i\gamma^2\psi^*(\tilde{x}), \quad \mathcal{T}\psi(x)\mathcal{T}^\dagger = i\gamma^1\gamma^3\psi(-\tilde{x}),$$

(7)

where $\tilde{x}^\mu = x^\mu$. Then the action of these operators on the fermion densities

$$S(x) = :\bar{q}(x)q(x):$$
$$V^\mu(x) = :\bar{q}(x)\gamma^\mu q(x):$$
$$T^{\mu\nu}(x) = :\bar{q}(x)\sigma^{\mu\nu}q(x):$$
$$A^\mu(x) = :\bar{q}(x)\gamma^\mu\gamma^5 q(x):$$
$$P(x) = :i\bar{q}(x)\gamma^5 q(x):$$

(8)

takes the form [2] (note that $T$ conjugates all complex constants):

**Table 1.** $\mathcal{P}$, $\mathcal{C}$, and $\mathcal{T}$ transformations of the fermion densities.

| Transformation | $S(x)$ | $V^\mu(x)$ and vector field | $T^{\mu\nu}(x)$ | $A^\mu(x)$ | $P(x)$ |
|---------------|--------|-----------------------------|-----------------|------------|--------|
| $\mathcal{P}$ | $S(\tilde{x})$ | $V_\mu(\tilde{x})$ | $T_{\mu\nu}(\tilde{x})$ | $A_\mu(\tilde{x})$ | $-P(\tilde{x})$ |
| $\mathcal{C}$ | $S(x)$ | $-V^\mu(x)$ | $-T^{\mu\nu}(x)$ | $-A^\mu(x)$ | $P(x)$ |
| $\mathcal{T}$ | $S(-\tilde{x})$ | $V_\mu(-\tilde{x})$ | $-T_{\mu\nu}(-\tilde{x})$ | $A_\mu(-\tilde{x})$ | $-P(-\tilde{x})$ |
| $\Theta = \mathcal{CPT}$ | $S(-x)$ | $-V^\mu(-x)$ | $T^{-\mu\nu}(x)$ | $-A^\mu(-x)$ | $P(x)$ |

These definitions agree with the transformations of various physical quantities under $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{T}$ (see Table 4). It is seen now that the term, say,

$$\int dx \left( V_{td} \bar{t}\gamma^\mu(1 - \gamma^5)d W^+_\mu + V^*_{td} \bar{d}\gamma^\mu(1 - \gamma^5)t W^-_\mu \right)$$

(9)

Operator $\bar{t}...d$ creates $t$ and anti-$d$ quarks and annihilates $d$ and anti-$t$ quarks.
is both $\mathcal{CP}$ and $\mathcal{T}$ violating due to imaginary part of $V_{td}$: $\mathcal{CP}$ transformation changes the fermion densities: $\bar{t}\gamma^\mu(1 - \gamma^5)dW^\mu_+ \leftrightarrow \bar{d}\gamma^\mu(1 - \gamma^5)tW^-_\mu$, whereas $\mathcal{T}$ has no effect on the operators, however, conjugates the coefficients: $V_{td} \leftrightarrow V^*_{td}$. Note that the $\mathcal{CPT}$ transformation leaves the term [1] invariant.

### 1.2 The strong $\mathcal{CP}$ problem.

Strictly speaking, one could fix the phases of left-handed and right-handed states separately, excluding arbitrariness in the chiral phases $\alpha$:

$$q^R \rightarrow e^{i \alpha} q^R, \quad q^L \rightarrow e^{-i \alpha} q^L. \quad (10)$$

This would give rise to the terms

$$ih_1^u \bar{u}\gamma^5 u + ih_1^d \bar{d}\gamma^5 d + ... \quad (11)$$

in the interaction Lagrangian. A strong limit on the $\mathcal{CP}$-odd parameters $h_i$ comes from the mechanism of the spontaneous breaking of the chiral $SU_L(N_f) \times SU_3(N_f)$ symmetry in strong interactions. Let us assume that the vacuum is $\mathcal{CP}$-even. In this case, the v.e.v. of the mass term

$$H'_m = \sum_{i=1}^{N_F} \bar{q}_i (m_i - ih_i \gamma^5) q_i \quad (12)$$

can approach its minimum only provided that $h_u = h_d = h_s = ... = \lambda$ (a consequence of the Dashen theorem [3]). If we fix the chiral phases of quarks so that the mass term is $\gamma^5$-free, we have to consider the spontaneous breaking of the $\mathcal{CP}$ symmetry:

$$\langle 0 | \bar{U}_i^L D_j^R | 0 \rangle = \langle 0 | \bar{D}_i^L D_j^R | 0 \rangle = C e^{i\phi_i} \delta_{ij}, \quad (13)$$

where the angles $\phi_i$ are $\mathcal{CP}$-violating parameters. Thus we got rid of the $SU(N_f)$ chiral phases. The $U_A(1)$ chiral phase can be absorbed in the so called $\theta$ term giving rise to the strong $\mathcal{CP}$ violation: the measurable quantities are independent of $\theta - 2N_F \alpha$, where $\alpha$ is the chiral phase and $\theta$ is the coefficient of the $\mathcal{CP}$-odd term [4]

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^{\mu\nu} G_{\mu\nu}, \quad (14)$$

which is equivalent to (that is, can be replaced with) the "pseudoscalar mass term"

$$\sum_{i=1}^{N_F} \bar{q}_i \gamma^5_q i, \quad (15)$$

### 1.3 The CKM Matrix

Performing the above procedure, we arrive at the expression for the quark mixing matrix in the form proposed by Kobayashi and Maskawa:

$$\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
c_1 & -s_1 c_3 & -s_1 s_3 \\
s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\
s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta}
\end{pmatrix},$$

4
where \( c_i = \cos \theta_i, s_i = \sin \theta_i \). This matrix differs from the mixing matrix advocated by the Particle Data Group \[5\],
\[
\begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & \quad c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & \quad s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & \quad -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & \quad c_{23}c_{13}
\end{pmatrix},
\]
in the phases of the \( c, b, t \) quarks. A popular approximation that emphasizes the hierarchy in the size of the angles
\[
s_{12} \gg s_{23} \gg s_{13},
\]
where \( s_{12} \equiv \lambda \) is the sine of the Cabibbo angle, is that one expands the other elements in terms of the parameter \( \lambda \). Up to and including terms of order \( \lambda^3 \), the mixing matrix is given by
\[
\begin{pmatrix}
    1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
    A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix},
\]
where \( A, \rho, \eta \) are assumed to be of order unity. The unitarity of the CKM matrix implies
\[
V_{ub}V_{ud}^* + V_{cd}V_{td}^* + V_{tb}V_{td}^* = 0.
\]
(17)
In the approximation \( V_{ud} \simeq V_{tb} \simeq 1 \) one obtains
\[
\frac{V_{ub}^*}{A\lambda^3} + \frac{V_{td}}{A\lambda^3} - 1 = 0.
\]
(18)
This relation identifies a triangle in the \( \rho - \eta \) plane (see Fig. 1); the angles of this triangle are measures of \( CP \) violation. The origin of the complex CKM phase is still not clearly understood. It may be that it stems from the short-distance dynamics \[6\] or extra dimensions (or something else) \[7\].

Figure 1: The unitarity triangle.
1.4 Left-Right Symmetric Model

The fermion sector of the $SU(3)_L \times SU(3)_R \times U(1)$-symmetric model \[8\] is the same as that of the SM; the boson sector involves the left $W_L^\mu$ and right $W_R^\mu$ gauge bosons, whose interaction with quarks $\bar{q}Dq$ is determined by the covariant derivative

$$D_\mu q = \partial_\mu q + ig_L W_L^\mu \frac{1 - \gamma^5}{2} q + ig_R W_R^\mu \frac{1 + \gamma^5}{2} q,$$

and a suitable $SU(3)_L \times SU(3)_R$ multiplet of the Higgs fields

$$\begin{pmatrix}
\phi_{11} \\
\phi_{12} \\
\phi_{21} \\
\phi_{22}
\end{pmatrix}$$

interacts with quarks as follows:

$$\bar{q}_i^L (h_{ij}\phi + f_{ij}\tilde{\phi}) q_j^R + H.c.,$$

where $\tilde{\phi} = -\sigma_2 \phi^* \sigma_2$ and the indices $i, j$ denote generation. Vacuum expectation values (v.e.v.) of the Higgs fields can be taken to be

$$\langle \phi \rangle = \begin{pmatrix}
\kappa \\
0 \\
0 \\
\kappa'
\end{pmatrix}$$

giving rise to the mass term of the form

$$\sum_{i,j=1}^{3} \left( \bar{U}_i^R M'_{ij} D_j^R + \bar{U}_i^L N'_{ij} U_j^R \right),$$

where $M'_{ij} = f_{ij}\kappa + h_{ij}\kappa'$ and $N'_{ij} = f_{ij}\kappa' + h_{ij}\kappa$. Diagonalization of the mass matrices $M'$ and $N'$ can be made by the same token as in the case of the SM:

$$D_R = Y_d d_R, \quad U_R = Y_u u_R,$$

$$D_L = V_d d_L, \quad U_L = V_u u_L,$$

where

$$d_{i}^{R}, d_{i}^{L}, u_{i}^{R}, u_{i}^{L}, (i = 1, 2, 3)$$

are the mass eigenstates. Mixing in the left sector and the elimination of arbitrariness in the choice of the matrices $V_d$ and $V_u$ can be considered in the same way as in the case of the SM. The additional interaction Lagrangian of the right currents is expressed in terms of the mass eigenstates as follows:

$$\sum_{i=1}^{3} \left( \bar{U}_i^R \tilde{W} + D_i^R + H.c. \right) = \sum_{i=1}^{3} \bar{u}_i \gamma^\mu \tilde{W}_R \frac{1 + \gamma^5}{2} d''_i + H.c.,$$

where

$$d''_i = Y_{ij} d_j, \quad Y_{ij} = (Y_u^1)_{ik}(Y_d)_{kj}.$$

In the case of two flavors,

$$Y = \begin{pmatrix}
\cos\theta_R \\
-e^{i\delta_R} \sin\theta_R
\end{pmatrix},$$

In the case of $N$ flavors the number of independent $CP$-violating phases is equal to $\frac{N(N-1)}{2}$.
2 Low-Energy Effective Lagrangian

It is well to recollect that any effective Lagrangian derives from an expansion of the exact amplitudes at small external momenta. Keeping only a few terms of such expansion (denote them by \( T \)), we find the Lagrangian \( \mathcal{L}_{\text{eff}} \) such that \( T = \langle \text{out} | \mathcal{L}_{\text{eff}} | \text{in} \rangle \).

Now we turn to the consideration of the consequences of the CP-violating phase for the hadronic physics. This can be made in the two stages:

- we construct the low-energy effective Lagrangian in terms of the quark fields, where the CKM phase gives rise to the imaginary parts of the effective coupling constants, represented by the Wilson coefficients;
- using some model assumptions, we derive the expression for the Lagrangian in terms of the meson fields.

As a result of the first stage of this evolution, one obtains the effective \( \Delta S = 1 \) Lagrangian \([1]\)

\[
\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \sum_{i=1}^{10} C_i(\mu) \, Q_i(\mu) ,
\]

which is a sum of local four–fermion operators \( Q_i \), constructed with the light degrees of freedom,

\[
Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{\pi}_\alpha d_\beta)_{V-A} ,
Q_2 = (\bar{s}_u)_{V-A} (\bar{\pi} d)_{V-A} ,
\]

where \((\bar{q}_i q_j)_{V^\pm A} \equiv \bar{q}_i \gamma^\mu (1 \pm \gamma^5) q_j \), \( \alpha \) and \( \beta \) are color indices. The operators \( Q_1 \) and \( Q_2 \) and the respective coefficients \( C_1 \) and \( C_2 \) are determined by the low-energy expansion of the diagrams in Fig. 3.
The remaining operators

\[
Q_{3,5} = \sum_q (\bar{q}q)_{V-A} ,
\]

\[
Q_{4,6} = \sum_q (\overline{q}_\beta q_\alpha)_{V-A} ,
\]

\[
Q_{7,9} = \sum_q e_q (\bar{q}q)_{V+A} ,
\]

\[
Q_{8,10} = \sum_q e_q (\overline{q}_\beta q_\alpha)_{V+A} ,
\]

come from the celebrated ‘penguin’ diagram:

Figure 3: The diagrams for the short-distance processes giving rise to the operators \(Q_1\) and \(Q_2\) in the effective Lagrangian \((24)\).

Figure 4: The ‘penguin’ diagram.
The Wilson coefficients $C_i$ can be represented in the form

$$C_i(\mu) = z_i(\mu) + \tau y_i(\mu), \quad (27)$$

where

$$\tau = - \frac{V_{td} V_{ts}^*}{V_{us} V_{us}^*}. \quad (28)$$

The CP-violating amplitudes are proportional to $y_i$.

At the hadronic level, the effective Lagrangian is expressed in terms of the meson fields:

$$\Phi = \left( \begin{array}{c} \pi^0 \frac{\sqrt{2}}{\sqrt{6}} + \eta_8 \frac{\sqrt{6}}{\sqrt{3}} + \eta_0 \frac{\sqrt{3}}{} \\ \pi^- \frac{\sqrt{2}}{\sqrt{6}} + \eta_8 \frac{\sqrt{6}}{\sqrt{3}} + \eta_0 \frac{\sqrt{3}}{} \\ K^- \frac{2\eta_8}{\sqrt{6}} + \eta_0 \frac{\sqrt{3}}{} \end{array} \right).$$

The most general effective bosonic Lagrangian of the second order in derivatives, with the same $SU(3)_L \otimes SU(3)_R$ transformation properties and quantum numbers as the short-distance Lagrangian, contains three terms \[12\]:

$$\Delta S = 1 = \frac{-G_F}{\sqrt{2}} V_{ud} V_{us}^* f^4 \left\{ g_8 \left( \langle \lambda L_{\mu} L^\mu \rangle + e^2 f^2 g_w \langle \lambda U^\dagger Q U \rangle \right) + g_{27} \left( L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) \right\}, \quad (29)$$

where the matrix

$$L_{\mu} = -iU^\dagger D_{\mu} U \quad (U = \exp(i\sqrt{2} \Phi f))$$

represents the octet of $V - A$ currents at lowest order in derivatives, $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ is the quark charge matrix, $\lambda \equiv (\lambda^0 - i\lambda^7)/2$ projects onto the $s \to d$ transition $[\lambda_{ij} = \delta_{i3}\delta_{j2}]$ and $\langle A \rangle$ denotes the flavor trace of $A$.

The chiral couplings $g_8$ and $g_{27}$ measure the strength of the two parts of the effective Lagrangian transforming as $(8_L, 1_R)$ and $(27_L, 1_R)$, respectively, under chiral rotations.

In the presence of electroweak interactions, the explicit breaking of chiral symmetry generated by the quark charge matrix $Q$ induces the $O(p^0)$ operator $\langle \lambda U^\dagger Q U \rangle$, transforming as $(8_L, 8_R)$ under the chiral group.

$$|g_8| \simeq 5.1, \quad |g_{27}| \simeq 0.29. \quad (30)$$

The huge difference between these two couplings shows the well-known enhancement of the octet $|\Delta I| = 1/2$ transitions. In the $N_c \to \infty$ limit, the real parts of these constants are expressed in terms of the Wilson coefficients as follows \[13\]:

$$g_8^\infty = - \frac{2}{5} C_1(\mu) + \frac{3}{5} C_2(\mu) + C_4(\mu) - 16 L_5 \left( \frac{\langle \bar{q}q \rangle (2)(\mu)}{f^3} \right)^2 C_6(\mu),$$

$$g_{27}^\infty = \frac{3}{5} \left[ C_1(\mu) + C_2(\mu) \right],$$

$$(g_8 e^2 g_w)^\infty = - 3 \left( \frac{\langle \bar{q}q \rangle (2)(\mu)}{f^3} \right)^2 C_8(\mu). \quad (31)$$
The imaginary parts of them are responsible for $C\bar{P}$-violating effects and will be considered further. With the effective weak Lagrangian at hand, it is very helpful to consider the evolution of a meson system to the second order in the weak interaction.

## 3 $C\bar{P}$ Violation in System of Neutral Kaons

It has become a tradition\(^3\) to begin a description of the $K^0 - \bar{K}^0$ system with writing down the most general expression for $C\bar{P}\bar{T}$-invariant Hamiltonian

$$i \frac{d}{dt} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{11} \end{pmatrix} \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix},$$

where $K^0$ and $\bar{K}^0$ are the eigenstates of the strong-interaction Hamiltonian and the matrix $H_{ij}$ is non-Hermitean.

However, it is well to recollect a derivation of this formula and formulate the assumptions made in its derivation.

- We consider weak interactions as a perturbation to the strong interactions.
- We consider evolution of the eigenstates of the strong-interaction Hamiltonian to the second order in the weak interaction and then search for the effective Hamiltonian that would give the same evolution in the leading order of perturbation theory\(^1\).

The perturbation expansion of the $S$ matrix in the $K^0 - \bar{K}^0$ system has the form

$$S_{ab} = \langle b | T \exp \left( -i \int H_{W}^{\text{int}}(t) dt \right) | a \rangle = \delta_{ab} - 2\pi i T_{ab}. \quad (32)$$

where $a, b = K^0, \bar{K}^0$, $H_{W}^{\text{int}}(t) = e^{iHt}H_W e^{-iHt}$, and $H_W$ is the weak-interaction Hamiltonian. To the second order in $H_W$, we obtain

$$T_{ab} = \langle b | H_W | a \rangle - \frac{i}{2} \int dt \langle b | T \left( H_{W}^{\text{int}}(t)H_{W}^{\text{int}}(0) \right) | a \rangle = \langle b | H_W | a \rangle + \frac{1}{2} \sum_{\lambda} \left[ \frac{\langle b | H_W | \lambda \rangle \langle \lambda | H_W | a \rangle}{E_b - E_\lambda + i\epsilon} + \frac{\langle b | H_W | \lambda \rangle \langle \lambda | H_W | a \rangle}{E_a - E_\lambda + i\epsilon} \right]. \quad (33)$$

Making use of the Sokhotsky relations one can represent the transition amplitudes $T_{ab} + \langle b | H_{\text{strong}} | a \rangle$ as the matrix elements of the effective Hamiltonian

$$H_{ab} = M_{ab} - \frac{i}{2} \Gamma_{ab}, \quad (34)$$

where

$$M_{ab} = m_K \delta_{ab} + \langle b | H_W | a \rangle + \mp \int d\lambda \frac{\langle b | H_W | \lambda \rangle \langle \lambda | H_W | a \rangle}{m_K - E_\lambda}, \quad (35)$$

$$\Gamma_{ab} = 2\pi \sum_{\lambda} \langle b | H_W | \lambda \rangle \langle \lambda | H_W | a \rangle \delta(m_K - E_\lambda).$$

Note that $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$. Thus the Hamiltonian

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{11} \end{pmatrix} = \begin{pmatrix} (M_{11}' + iM_{11}'') - i \frac{(\Gamma_{11}' + i\Gamma_{11}'')}{2}; & (M_{12}' - iM_{12}'') - i \frac{(\Gamma_{12}' - i\Gamma_{12}'')}{2} \\ (M_{12}' + iM_{12}'') - i \frac{(\Gamma_{12}' + i\Gamma_{12}'')}{2}; & (M_{11}' + iM_{11}'') - i \frac{(\Gamma_{11}' + i\Gamma_{11}'')}{2} \end{pmatrix},$$

\(^3\)In this Section, we follow [1, 14, 15].
is related to the Hamiltonian of the weak interactions. Eigenstates of this Hamiltonian are identified with the physical states $K_0^L$ and $K_0^S$, which are expressed through $K^0$ and $\bar{K}^0$ in terms of the parameter

$$\bar{\epsilon} = \sqrt{H_{12} - H_{21}};$$

$$K_0^L = \frac{1}{\sqrt{2(1 + |\bar{\epsilon}|^2)}} \left( \frac{1 + \bar{\epsilon}}{1 + \bar{\epsilon}} \right), \quad K_0^S = \frac{1}{\sqrt{2(1 + |\bar{\epsilon}|^2)}} \left( \frac{1 + \bar{\epsilon}}{1 - \bar{\epsilon}} \right);$$

the respective eigenvalues give the masses and widths of the $K_0^L$ and $K_0^S$ mesons:

$$\lambda_S = H_{11} - \sqrt{H_{12}H_{21}} = M_S - i \frac{\Gamma_S}{2}, \quad \lambda_L = H_{11} + \sqrt{H_{12}H_{21}} = M_L - i \frac{\Gamma_L}{2}. \quad (37)$$

$\mathcal{CP}$ transformation exchanges $|K^0\rangle$ and $|\bar{K}^0\rangle$ states:

$$\mathcal{CP}|K^0\rangle = e^{i\theta}|\bar{K}^0\rangle, \quad \mathcal{CP}|\bar{K}^0\rangle = e^{-i\theta}|K^0\rangle. \quad (38)$$

The phase factor can be chosen arbitrarily, because any quantum-mechanical state is defined up to a phase. However an interpretation of the matrix elements $H_{ab}$ and parameter $\bar{\epsilon}$ depends on a particular choice of the phase.

In the case $\theta = 0$, the parameters $M''$ and $\Gamma''$ are $\mathcal{CP}$-odd, whereas $M'$ and $\Gamma'$ are $\mathcal{CP}$-even. Assuming that $\mathcal{CP}$-odd parameters are small, $\Gamma''_{12} \ll \Gamma'_{12}$ and $M''_{12} \ll M'_{12}$, we obtain

$$\bar{\epsilon} = \frac{H_{12} - H_{21}}{4\sqrt{H_{12}H_{21} + (\sqrt{H_{12}} - \sqrt{H_{21}})^2}} \approx \frac{i M'_{12}}{\lambda_S - \lambda_L} \text{sign}(M'_{12}). \quad (39)$$

The formulas (37) agree with the experimental fact $M_{KL} > M_{KS}$ if the sign of the square root $\sqrt{H_{12}H_{21}}$ is chosen so that

$$\sqrt{H_{12}H_{21}} = M'_{12} - i \frac{\Gamma'}{2} \quad \text{for} \quad M'_{12} > 0, \quad \Delta M = M_{KS} - M_{KL} = -2 M'_{12};$$

$$\sqrt{H_{12}H_{21}} = -M'_{12} + i \frac{\Gamma'}{2} \quad \text{for} \quad M'_{12} < 0, \quad \Delta M = M_{KS} - M_{KL} = 2 M'_{12}. \quad (40)$$

Experimental data indicate that $\Delta M = -\frac{\Delta\Gamma}{2}$, hence $\lambda_S - \lambda_L \approx \Delta M (1 + i)$. Combining the above formulas, we arrive at

$$\bar{\epsilon} = -e^{i\frac{\Delta\Gamma}{4}} \frac{M'_{12}}{\sqrt{2} M_{12}} \quad (40)$$

for both $M'_{12} > 0$ and $M'_{12} < 0$. (The assumption that $M_{KL} < M_{KS}$ would give the phase factor $e^{i\frac{\Delta\Gamma}{4}}$ instead of $e^{-i\frac{\Delta\Gamma}{4}}$.)

### 3.1 Phase Convention

It is important and helpful to keep track of the phase arbitrariness stemming from the fact that

both $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} e^{i\phi} \\ 0 \end{pmatrix}$ describe the same physical state.
The transformation rules induced by the rotation of the phase of the $s$ quark

$$s \rightarrow e^{i\phi}s, \quad \bar{s} \rightarrow e^{-i\phi}\bar{s}, \quad (41)$$

are as follows:

$$K^0 \rightarrow e^{-i\phi}K^0, \quad \bar{K}^0 \rightarrow e^{i\phi}\bar{K}^0, \quad H_{12} \rightarrow e^{-2i\phi}H_{12}, \quad H_{21} \rightarrow e^{2i\phi}H_{21},$$

$$V_{us} \rightarrow e^{i\phi}V_{us} \quad \bar{\epsilon} \rightarrow \frac{\bar{\epsilon} - i \epsilon \tan \phi}{1 - i \epsilon \tan \phi}, \quad A_I \rightarrow e^{-i\phi}A_I,$$

$$\mathcal{CP} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & e^{-2i\phi} \\ e^{2i\phi} & 0 \end{pmatrix}.$$ 

Let the phases of the $K^0$ and $\bar{K}^0$ be chosen so that the matrix of $\mathcal{CP}$ transformation has the form

$$\mathcal{CP}_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(we call it the "default" phase convention). It should be compared with the widely used Wu–Yang phase convention, in which the phase of $A_0$ is set equal to zero. It should be noticed that in the Wu–Yang phase convention the operator of $\mathcal{CP}$ transformation has the form

$$\mathcal{CP}_{ab} = \begin{pmatrix} 0 & e^{-2i\alpha} \\ e^{2i\alpha} & 0 \end{pmatrix}.$$ 

It should also be emphasized that the value $\bar{\epsilon}$ as was defined above is phase-dependent and so does not measure $\mathcal{CP}$ violation, and the imaginary part of the effective weak Hamiltonian $H_W$ is not associated with $\mathcal{CP}$ violation (and so it may be larger than real).

We adopt the "default" phase convention. In this case

- the quantities $M''$ and $\Gamma''$ are $\mathcal{CP}$-odd;
- $M'$ and $\Gamma'$ are $\mathcal{CP}$-even;
- $V_{us}$ is real;

Upon fixing a phase convention, the parameter $\bar{\epsilon}$ makes physical sense and can be related to measurable quantities; the $\mathcal{CP}$-violating parameters in the effective weak Hamiltonian are $M''$ and $\Gamma''$.

3.2 Mixing of $K^0$ and $\bar{K}^0$ in the Standard Model

As has been demonstrated, such mixing is accounted for by the $\Delta S = 2$ effective weak Lagrangian.

Let us consider the computation of the diagrams in Fig. 5 (giving the transition amplitude $\bar{s}d \rightarrow s\bar{d}$).

Gaillard and Lee in the pioneer work [17] obtained

$$T(\bar{s}d \rightarrow s\bar{d}) = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi \sin^2 \theta_W} (\bar{d}_L \gamma^\mu s_L)(\bar{d}_L \gamma^\mu s_L) \sum_{i,j=u,c,t} \xi_i \xi_j A(x_i, x_j), \quad (42)$$
where

\[ A(x_i, x_j) = \frac{J(x_i) - J(x_j)}{x_i - x_j}, \quad J(x) \equiv \frac{1}{1 - x} + \frac{x^2 \ln x}{(1 - x)^2}, \quad (43) \]

\[ \xi_i = V_{is} V_{id}^\ast, \quad \text{and} \quad x_i = \frac{m_i^2}{M_W^2}. \]

Thus we obtain the effective \( \Delta S = 2 \) Lagrangian

\[ \mathcal{L}_{\text{eff}}^{\Delta S=2} = - \frac{G_F}{\sqrt{2}} \frac{\alpha}{16\pi \sin^2 \theta_W} Q_0 \lambda, \]

where

\[ Q_0 = \bar{d} \gamma^\mu (1 - \gamma^5) s \times \bar{d} \gamma^\mu (1 - \gamma^5) s, \quad (44) \]

\[ \lambda = \sum_{i,j} \xi_i \xi_j A(x_i, x_j). \]

With the use of this Lagrangian the mass difference is readily obtained:

\[ \Delta M = M_S - M_L \approx -2M_{12} = \frac{1}{2M_K} \langle K^0 | \mathcal{L}^{\Delta S=2} | K^0 \rangle \quad (45) \]

Now one should evaluate the matrix element

\[ \mathcal{M}_{KK} = \langle K^0 | \bar{d} \gamma^\mu (1 - \gamma^5) s \times \bar{d} \gamma^\mu (1 - \gamma^5) s | K^0 \rangle. \quad (46) \]

In early works, the matrix element was evaluated using the so called "Vacuum Insertion Approximation". The result is as follows:

\[ \mathcal{M}_{KK} = \frac{8}{3} \langle K^0 | \bar{d} \gamma^\mu \gamma^5 s | K^0 \rangle \langle K^0 | \bar{d} \gamma^\mu \gamma^5 s | K^0 \rangle = \frac{8}{3} M_K^2 F_K^2. \quad (47) \]

\[ ^4\text{To simplify these expression it is well to use the unitarity condition } \xi_u + \xi_c + \xi_t = 0. \]
The first computation of this matrix element was performed in the bag model \[18\]; it was found that it is smaller from the naive expectation of $M_{KK}$ by a factor of 2. For this reason, the factor $B_K$ in the expression

$$
\langle \bar{K}^0 | [\bar{d}\gamma^\mu(1-\gamma^5)s] [\bar{s}\gamma_\mu(1-\gamma^5)d] | K^0 \rangle = \frac{8}{3} M^2_K F^2_K B_K
$$

is named "the bag constant". The computation of the bag factors presents the major challenge in the calculations of $CP$-violating quantities in nonleptonic reactions.

The short-distance contribution to $\Delta M$ comprises $70 \div 80\%$ of the total SM contribution:

$$
\Delta M = -2 M''_{12} = -\frac{2G_F}{3\sqrt{2}} \eta B_K \frac{\alpha}{4\pi} \left( \frac{m_c}{37\text{GeV}} \right)^2 \text{Re}\lambda,
$$

where (in the approximation $x_u = 0, x_c \ll 1, x_t \sim 1$)

$$
\text{Re}\lambda = \text{Re} \left( \xi_c^2 + \xi_t^2 \frac{x_t}{x_c} \frac{(1-x_t^2 + 2x_t \ln x_t)}{(1-x_t)^3} + 2\xi_c \xi_t \left( \frac{x_t}{1-x_t} + \frac{\ln x_t}{(1-x_t)^2} - \ln x_c \right) \right)
$$

and the factor $\eta$ accounts for the corrections due to strong interactions, evaluated in perturbative QCD. The fact that $\text{Re}\lambda \approx 1$ indicates that the $c$ quark gives the dominant contribution to the mass difference. The remaining $20 \div 30\%$ are attributed to the long-distance contribution (that is the contribution of the $\pi\pi, \pi\pi\pi$ etc intermediate states in formula \[15\]), which is extremely difficult to compute exactly.

![Diagram](image.png)

Figure 6: The $K^0 - \bar{K}^0$ transition in terms of the effective Lagrangian.

The estimates of the bag constant obtained in the lattice QCD and in some models are

$$
B_K = 0.85 \pm 0.15 \quad \text{(Lattice [19]),}
$$

$$
B_K = 0.41 \pm 0.09 \quad \text{(Chiral limit, } 1/N_C \text{ [20]).}
$$

### 3.3 Basic Formula for $\epsilon$

In the above subsection we have considered in detail the determination of the real part of the amplitude of the $\Delta S = 2$ transition in terms of the short-distance contribution (the second term in the expression \[15\] for $M_{ab}$) and the bag constant $B_K$. The imaginary part of this

---

5In the real world, $x_u = 2.5 \times 10^{-11}$, $x_c = 2 \times 10^{-4}$, $x_t = 4.7$. 
amplitude, which appears in the expression (40) for \( \epsilon \), can be calculated by the same token. In the case of imaginary part, one can safely neglect the long-distance contribution due to low-lying intermediate states associated with the third term in the expression (35) for \( M_{ab} \). The result is

\[
\epsilon_{\text{teor}} = C_{\epsilon} B_{K} \exp \left( \frac{i\pi}{4} \right) \Im \xi_t [\Re \xi_c (\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)) - \Re \xi_t \eta_2 S_0(x_t)], \tag{53}
\]

where

\[
C_{\epsilon} = \frac{G_F^2 F_K^2 M_K M_{V}^2}{6 \sqrt{2} \pi^2 \Delta M_K} = 3.837 \times 10^4 \tag{54}
\]

(here we have used the experimental value of \( M_{12}' \), determined from \( \Delta M_K \), instead of the theoretical value (49)),

\[
S_0(x_c) \approx x_c, \tag{55}
\]

\[
S_0(x_t) \approx 2.46 \left( \frac{m_t}{170 \text{GeV}} \right)^{1.52},
\]

\[
S_0(x_c, x_t) \approx x_c \left[ \ln \left( \frac{x_t}{x_c} \right) - \frac{3x_t}{4(1 - x_t)} - \frac{3x_t^2 \ln x_t}{4(1 - x_t)^2} \right],
\]

\[
m_c \simeq 1.3 \text{GeV}, \quad m_t = 174.3 \pm 5.1 \text{GeV};
\]

the short-distance corrections due to the strong interactions are absorbed in the coefficients \[21\]

\[
\eta_1 = 1.38 \pm 0.20, \quad \eta_2 = 0.57 \pm 0.01, \quad \eta_3 = 0.45 \pm 0.04.
\]

Formula (53) allows to set a limitation on the \( \mathcal{CP} \)-violating parameter \( \eta \) of the SM from the experimental limitations on \( \epsilon \) (see Fig.(7)).

![Figure 7: Formula (53) confines \( \epsilon \) to lie in some vicinity of the indicated hyperbola (see [8]). The combined set of limitations makes the vertex of the unitarity triangle to lie within the indicated limits.](image)

15
### 3.4 \((K \to \pi\pi)\) Amplitudes

The \(2\pi\) decay amplitudes of the neutral Kaons in the channel with isospin \(I\) are defined by the matrix elements

\[
\langle \pi\pi, I | H_W | K^0 \rangle = \sqrt{\frac{3}{2}} (A_I + B_I) e^{i\delta_I},
\]

\[
\langle \pi\pi, I | H_W | \bar{K}^0 \rangle = \sqrt{\frac{3}{2}} (A_I^* - B_I^*) e^{i\delta_I},
\]

where \(\delta_I\) is the \(S\)-wave phase shift of the \(\pi\pi\) scattering;

\[
|\pi\pi, I \text{ out} \rangle = e^{2i\delta_I} |\pi\pi, I \text{ in} \rangle
\]

and

\[
A_I = A'_I + iA''_I, \quad B_I = B'_I + iB''_I.
\]

The properties of these amplitudes under the \(CP\)-transformation and time reversal \(T\) are seen from Table 2.

**Table 2.** Transformation properties of the isospin amplitudes.

| Transformation | \(A'\) | \(A''\) | \(B'\) | \(B''\) |
|----------------|--------|--------|--------|--------|
| \(CP\)        | +      | -      | -      | +      |
| \(T\)         | +      | -      | +      | -      |

It is seen that the amplitudes \(A_I\) are \(CP\)\(-T\)-even, whereas the \(B_I\) amplitudes are \(CP\)\(-T\)-odd.

Using the relations

\[
\frac{1}{\sqrt{2}} |\pi^+\pi^- + \pi^-\pi^+ \rangle = \frac{1}{\sqrt{3}} (\sqrt{2}|I = 0\rangle + |I = 2\rangle),
\]

\[
|\pi^0\pi^0 \rangle = \frac{1}{\sqrt{3}} (|I = 0\rangle - \sqrt{2}|I = 2\rangle),
\]

we obtain the expressions for the amplitudes \(A(K^0 \to \pi^+\pi^-)\) and \(A(K^0 \to \pi^0\pi^0)\):

\[
A(K^0 \to \pi^+\pi^-) = A_0 e^{i\delta_0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_2},
\]

\[
A(K^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{2}} A_0 e^{i\delta_0} - A_2 e^{i\delta_2}.
\]

\(A_0\) describes the transitions with \(\Delta I = 1/2\), whereas \(A_2\) describes the transitions with \(\Delta I = 3/2\).

Assuming that the Hamiltonian for the transitions \(K \to 2\pi\) contains only terms with quantum numbers \(I = 1/2, I_3 = 1/2\) and \(I = 3/2, I_3 = 1/2\) (that is, it does not contain the terms with
Proceeding as indicated above, we obtain

\[
\eta_{00} = \frac{i [\sin \alpha - \tilde{\omega} \sin(\alpha + \chi)] + \tilde{\epsilon} [\cos \alpha - \tilde{\omega} \cos(\alpha + \chi)]}{[\cos \alpha - \tilde{\omega} \cos(\alpha + \chi)] + i \tilde{\epsilon} [\sin \alpha - \tilde{\omega} \sin(\alpha + \chi)]}
\]  

and a similar expression for \(\eta_{+-}\).
Assuming that CP-violating parameters are small, that is \( \bar{\epsilon}, \alpha, \chi \ll 1 \), we arrive at

\[
\eta_{00} = \bar{\epsilon} + i\alpha - i \frac{\chi\bar{\omega}}{1 - \bar{\omega}},
\]

where

\[
\chi \approx \frac{A''_0}{A_2} - \frac{A''_0}{A_0}.
\]

In a similar way, one obtains

\[
\eta_{+-} = \frac{\langle \pi^+ \pi^- | K^0_L \rangle}{\langle \pi^+ \pi^- | K^0_S \rangle} = \bar{\epsilon} + i\alpha - i \frac{\chi\bar{\omega}}{1 - \bar{\omega}},
\]

Let us introduce the parameters, which are conventionally used for a description of the effects of CP violation:

\[
\epsilon = \frac{\langle \pi \pi, I = 0 | K^0_L \rangle}{\langle \pi \pi, I = 0 | K^0_S \rangle} = \bar{\epsilon} + i\alpha,
\]

\[
\epsilon' = \frac{i}{\sqrt{2}} e^{i\delta} \text{Im} \frac{A_2}{A_0} = \frac{i}{2} \bar{\omega} \left( \frac{A''_0}{A_2} - \frac{A''_0}{A_0} \right).
\]

Note that, in contrast to \( \bar{\epsilon} \), the parameter \( \epsilon \) is independent of the phase convention. These values coincide only in the Wu–Yang phase convention. In terms of the introduced parameters, we have

\[
\eta_{00} = \frac{\langle \pi^0 \pi^0 | K^0_L \rangle}{\langle \pi^0 \pi^0 | K^0_S \rangle} = \epsilon - \frac{2\epsilon'}{1 - |\omega| e^{i\delta} \sqrt{2}} \approx \epsilon - 2\epsilon'
\]

\[
\eta_{+-} = \frac{\langle \pi^+ \pi^- | K^0_L \rangle}{\langle \pi^+ \pi^- | K^0_S \rangle} = \epsilon + \frac{\epsilon'}{1 + |\omega| e^{i\delta} \sqrt{2}} \approx \epsilon + \epsilon',
\]

where

\[
|\omega| = \text{Re} \frac{A_2}{A_0} \approx 0.045, \quad \delta \approx 45^\circ.
\]

From the above it follows that

\[
\text{Re} \frac{\epsilon'}{\epsilon} \approx \frac{1 - |\omega|}{6} \left( 1 - \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \right).
\]

Note that \( \epsilon'/\epsilon \) is approximately real. Using the short-distance Lagrangian, the CP-violating ratio \( \epsilon'/\epsilon \) can be written as follows [22]:

\[
\frac{\epsilon'}{\epsilon} = \text{Im} (V_{ts}^* V_{td}) e^{i\phi} \frac{G_F}{2|\epsilon| |\text{Re}(A_0)|} \omega \left( P^{(0)} (1 - \Omega_{1B}) - \frac{1}{\omega} P^{(2)} \right),
\]

where the quantities

\[
P^{(l)} = \sum_i y_i(\mu) \langle \langle \pi \pi \rangle_I | Q_i | K \rangle
\]

where
contain the contributions from hadronic matrix elements with isospin $I$ and

$$\Omega_{IB} = \frac{1}{\omega} \frac{\text{Im}(A_2)_{|IB}}{\text{Im}(A_0)}$$  \hspace{1cm} (74)$$

parameterizes isospin breaking corrections. The factor $1/\omega$ enhances the relative weight of the $I = 2$ contributions.

The hadronic matrix elements $\langle (\pi\pi)_I|Q_i|K \rangle$ are usually parameterized in terms of the bag parameters $B_i$, which measure them in units of their vacuum insertion approximation values. In the SM, $P^{(0)}$ and $P^{(2)}$ turn out to be dominated by the contributions from the QCD penguin operator $Q_6$ and the electroweak penguin operator $Q_8$, respectively [23]. Thus, to a very good approximation, $\epsilon'/\epsilon$ can be written (up to global factors) as [24, 25]

$$\frac{\epsilon'}{\epsilon} \sim \left[ B_6^{(1/2)} (1 - \Omega_{IB}) - 0.4 B_8^{(3/2)} \right]. \hspace{1cm} (75)$$

The isospin–breaking correction coming from $\pi^0$-$\eta$ mixing was originally estimated to be $\Omega_{IB}^{\pi^0\eta} = 0.25$ [23]. Together with the usual ansatz $B_i \sim 1$, this produces a large numerical cancellation in (72) [27] leading to low values of $\epsilon'/\epsilon$ around $7 \times 10^{-4}$. A recent improved calculation of $\pi^0$-$\eta$ mixing at $O(p^4)$ in $\chi$PT has found the result [28]

$$\Omega_{IB}^{\pi^0\eta} = 0.16 \pm 0.03. \hspace{1cm} (76)$$

This smaller number, slightly increases the naive estimate of $\epsilon'/\epsilon$.

**Table 3.** Theoretical and experimental values of $\epsilon'/\epsilon$.

| Year       | Theory (models: $1/N_c$, unitarization) | Theory (lattice) | Experiment |
|------------|----------------------------------------|------------------|------------|
| ~ 1988     | 0.01 $\pm$ 0.03 [1]                    |                  | (3.2 $\pm$ 1.0) $\times 10^{-3}$ [31] |
| 1995       | (6.7 $\pm$ 2.6) $\times 10^{-4}$ [24]  | (3.1 $\pm$ 2.5) $\times 10^{-4}$ [24] | (1.5 $\pm$ 0.8) $\times 10^{-3}$ [32] |
| 1999–2000  | $(-1 \div 35) \times 10^{-4}$ [24]    | (0.44 $\div$ 2.1) $\times 10^{-3}$ [30] * | (2.1 $\pm$ 1.5) $\times 10^{-3}$ [8] |
| 2001       | (1.7 $\pm$ 0.9) $\times 10^{-3}$ [27]  |                  | (1.53 $\pm$ 0.24) $\times 10^{-3}$ [33] ** |

* This value depends crucially on the mass of the $s$ quark: $0.44 \times 10^{-3}$ for $m_s(m_c) = 150$ MeV and $2.1 \times 10^{-3}$ for $m_s(m_c) = 80$ MeV

** The most recent data: $\text{Re} \frac{\epsilon'}{\epsilon} = (1.73 \pm 0.18) \times 10^{-3}$

[http://na48.web.cern.ch/NA48/Welcome/images/talks/win02/win02.pdf](http://na48.web.cern.ch/NA48/Welcome/images/talks/win02/win02.pdf)
4 Time-Reversal Invariance

Throughout this Section it is assumed that all the processes under consideration are adequately described by some local quantum field theory, that is, \(CPT\) is an exact symmetry of the theory. In this case \(CP\)-violation is equivalent to \(T\)-violation and so we turn to the consideration of the time-reversal invariance.

![Figure 8: The reversal of time in classical mechanics.](image)

**Time-reversal invariance in the classical mechanics** (Fig. 8):

- motion from \(A\) to \(B\) \(\{x(t) : x(T_1) = A, x(T_2) = B\}\)
- motion from \(B\) to \(A\) \(\{x(t) : x(T_1) = B, x(T_2) = A\}\)

are described with the same Hamiltonian

This is the case provided that \(H(p, x) = H(-p, x)\).

**Time-reversal invariance in the quantum mechanics:**

- evolution from \(|A\rangle\) to \(|B\rangle\) \(\{|\psi(t)\rangle : |\psi(T_1)\rangle = |A\rangle, \langle\psi(T_2)| = \langle B|\}\)
- evolution from \(|B\rangle\) to \(|A\rangle\) \(\{|\psi(t)\rangle : |\psi(T_1)\rangle = |B\rangle, \langle\psi(T_2)| = \langle A|\}\)

are described with the same Hamiltonian

This is the case provided that \(H = H^*\).
Table 4. $\mathcal{P}$ and $\mathcal{T}$ transformations for various quantities.
Action of these operators on the quantum-mechanical states is determined by the condition that the $\mathcal{T}$- (or $\mathcal{P}$-)transformed states are characterized by the $\mathcal{T}$- (or $\mathcal{P}$-)transformed (eigen)values of the respective operators.

| Value                  | Notation | $\mathcal{P}$-transformed value | $\mathcal{T}$-transformed value | Comment                                      |
|-----------------------|----------|---------------------------------|---------------------------------|----------------------------------------------|
| Coordinate            | $\vec{x}$ | $-\vec{x}$                      | $\vec{x}$                      |                                              |
| Momentum              | $\vec{p}$ | $-\vec{p}$                      | $-\vec{p}$                     | $\vec{p} = m \frac{d\vec{x}}{dt}$          |
| Angular momentum      | $\vec{l}$ | $\vec{l}$                        | $-\vec{l}$                     | $(\vec{r} \times \vec{p})$                 |
| Spin                  | $\vec{s}$ | $\vec{s}$                        | $-\vec{s}$                     | Like $\vec{l}$                              |
| Electric field        | $\vec{E}$ | $-\vec{E}$                      | $\vec{E}$                      | $\vec{E} = - \frac{dA_0}{d\vec{r}}$        |
| Magnetic field        | $\vec{B}$ | $\vec{B}$                        | $-\vec{B}$                     | $\vec{B} \sim \vec{r} \times \vec{j}$     |
| Potential             | $A_0$    | $A_0$                           | $A_0$                          |                                              |
| Vector potential      | $\vec{A}$ | $-\vec{A}$                      | $-\vec{A}$                     |                                              |
| Helicity              | $\lambda$ | $-\lambda$                      | $\lambda$                      | $\lambda = \vec{s}\vec{p}$                 |
| Transverse polarization* | $\xi$    | $\xi$                           | $-\xi$                         | $\xi = (\vec{s}, [\vec{p}_1 \times \vec{p}_2])$ |
| Triple correlation**  | $\eta$   | $-\eta$                         | $-\eta$                        | $\eta = (\vec{p}_1, [\vec{p}_2 \times \vec{p}_3])$ |

* A characteristic of a three-particle state, if at least one particle has a nonzero spin.
** A characteristic of a multiparticle state (number of particles must be greater than 3).
Time-reversal invariance in the $S$-matrix approach:

The system described by the quantum field theory is invariant under the time reversal if

- the space of $in$-states ('ket'-vectors) is isomorphic to the space of $out$ states ('bra'-vectors):
  $\forall |I\rangle \in in \ \exists \langle I| \in out$;

- There exists (anti-unitary) operator of time reversal
  $\mathcal{T} : in \rightarrow out, \ \mathcal{T} : out \rightarrow in$, such that $\langle \mathcal{T}I|\mathcal{T}F \rangle = \langle F|I \rangle = \langle I|F \rangle^*$,

which changes the signs of spins and momenta of all particles; this condition can be cast in the form $(\phi, \mathcal{T}\psi) = (\mathcal{T}\phi, \psi)^*$ for all vectors $\phi$ and $\psi \in \mathcal{H}$;

- the transition from the state $|I\rangle$ in the state $\langle F|$ and the transition from the state $|TF\rangle$ to the state $\langle TI|$ are described with the same $S$-matrix:

$$M_{I \rightarrow F} = \langle F|S|I \rangle = \langle TI|SF \rangle = M_{TF \rightarrow TI}.$$  \hfill (77)

Having in mind that

$$\langle TI|SF \rangle = (TI, S\mathcal{T}F) = (S^\dagger TI, \mathcal{T}F) = (\mathcal{T}F, S^\dagger TI)^* = \langle TF|S^\dagger TI \rangle^*,$$  \hfill (78)

we obtain the condition of $\mathcal{T}$-invariance in terms of the decay and scattering amplitudes:

$$\langle F|S|I \rangle = \langle TF|S^\dagger TI \rangle^*.$$  \hfill (79)

In perturbation theory, the $S$ matrix is expanded in the coupling constant $g$, which is assumed to be a small parameter:

$$S = 1 + igT_1 + g^2T_2 + ...$$  \hfill (80)

---

6This isomorphism is nothing but assumption; however, it allows to consider the vectors from $in$ and $out$ spaces as the vectors of the same Hilbert space $\mathcal{H}$, identified with $in$. 

---

Figure 9: The reversal of time in the $S$-matrix approach.
Here $T_1$ is nothing but the interaction Lagrangian, $T_1 = \mathcal{L}_{\text{int}}$; the unitarity of the $S$ matrix implies Hermiticity of the interaction Lagrangian $T_1$. Thus the $\mathcal{T}$-invariance condition in the leading order of perturbation theory takes the form

$$\langle TF|\mathcal{L}_{\text{int}}|TI \rangle = \langle F|\mathcal{L}_{\text{int}}|I \rangle^*.$$  

To put it differently, if the complex-conjugated amplitude of the transition between the states $I$ and $F$ differs in the leading order of perturbation theory from the amplitude of the transition between the states $TI$ and $TF$ then the dynamics of such system is not invariant under the time reversal.

Let me illustrate this statement by considering the example of the decay $K \to \mu\nu\gamma$; for definiteness, we consider the reference frame comoving with the kaon. Let the average transverse polarization of the muon $\xi \neq 0$. This is possible only if the probabilities of the decay into the states with positive and negative transverse polarizations of the muon differ from each other:

$$|\langle \mu(\vec{k}, \sigma)\nu(\vec{k'})\gamma(\vec{q}, \vec{\epsilon})|S|K \rangle|^2 \neq |\langle \mu(\vec{k}, -\sigma)\nu(\vec{k'})\gamma(\vec{q}, \vec{\epsilon})|S|K \rangle|^2.$$  

Note that the state $\langle \mu(\vec{k}, -\sigma)\nu(\vec{k'})\gamma(\vec{q}, \vec{\epsilon}) \rangle$ can be obtained from the state

$$\langle \mu(-\vec{k}, -\sigma)\nu(-\vec{k'})\gamma(-\vec{q}, -\vec{\epsilon}) \rangle = \langle T \mu(\vec{k}, \sigma)\nu(\vec{k'})\gamma(\vec{q}, \vec{\epsilon}) \rangle$$

as the result of the rotation by the angle of $180^\circ$ in the reaction plane. For this reason,

$$\langle \mu(\vec{k}, -\xi)\nu(\vec{k'})\gamma(\vec{q}, \vec{\epsilon})|S|K \rangle = \langle T \mu(\vec{k}, \xi)\nu(\vec{k'})\gamma(\vec{q}, \vec{\epsilon})|S|TK \rangle.$$  

The equations (82) and (83) imply the conclusion as follows: if the transverse polarization of the muon emerges in the first order of perturbation theory, then

$$\langle T \mu\nu\gamma|\mathcal{L}_{\text{int}}|TK \rangle \neq \langle \mu\nu\gamma|\mathcal{L}_{\text{int}}|K \rangle,$$

that is, the dynamics is not invariant under the time reversal.

However, we should take into account the following reasoning. Since the transverse polarization of the muon is determined by the imaginary part of the decay amplitude, which does not vanish in higher orders of perturbation theory due to unitarity condition, the transverse polarization of the muon emerges in higher orders even in the case of $\mathcal{T}$-even interactions.

Thus the transverse polarization of the muon in the decay $K \to \mu\nu\gamma$ can be caused by both electromagnetic and $\mathcal{T}$-odd interactions (beyond the SM)

$$\xi = \xi_{EM} + \xi_{\text{odd}},$$  

where $\xi_{EM}$ is the electromagnetic contribution to the transverse polarization of the muon and $\xi_{\text{odd}}$ is the contribution of the $\mathcal{T}$-odd (and, therefore $\mathcal{CP}$-odd) interactions.

The $\mathcal{CP}$-violating interactions can be accounted for by the imaginary parts of the coupling constants in the effective quark–lepton Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}\sin\theta_c\bar{s}\gamma^\alpha(1 - \gamma_5)u\bar{u}\gamma_\alpha(1 - \gamma_5)\mu +$$

$$+G_S\bar{s}\bar{u}(1 + \gamma_5)\mu + G_P\bar{s}\gamma_5u\bar{u}(1 + \gamma_5)\mu +$$

$$+G_V\bar{s}\gamma^\alpha\bar{u}\gamma_\alpha(1 - \gamma_5)\mu + G_A\bar{s}\gamma^\alpha\gamma_5u\bar{u}\gamma_\alpha(1 - \gamma_5)\mu +$$

$$+G_T\bar{s}\sigma^{\alpha\beta}(1 - \gamma_5)u\bar{u}\sigma_{\alpha\beta}(1 - \gamma_5)\mu + \text{H.c.}.$$  

The interactions in (86) arise from new physics. Nonvanishing imaginary parts in the effective coupling constants $G_P, G_V, G_A, G_T$ gives rise to the imaginary parts of the form factors $F_{1B}, F_A, F_V, F_T$ parameterizing the matrix element of the decay $K^+(p) \to \mu^+(k)\nu(k')\gamma(q)$.

\footnote{That is, transverse with respect to the momenta of the outgoing particles}
Current limitations on the $\mathcal{T}$-violation parameters in various extensions of the SM allow the transverse polarization of the muon in the decay $K \to \mu\nu\gamma$ to be rather large: the left-right symmetric models based on the symmetry group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with one doublet $\Phi$ and two triplets $\Delta_{L,R}$ of Higgs bosons can give $\xi_{\text{odd}} \sim 5 \times 10^{-3}$, supersymmetric models—$\xi_{\text{odd}} \sim 5 \times 10^{-3}$ [34].

The respective $\mathcal{T}$-even contribution to the transverse polarization of the muon is determined by the imaginary part of the decay amplitude (see Fig. [10]) and emerges in the second order in $\alpha_{\text{em}}$. Straightforward computations of $\xi_{\text{em}}$ were made by several authors; recent results [35] agree with each other and give the average value ($\xi_{\text{em}}$) $\sim 0.5 \times 10^{-3}$ (with the photon cutoff energy $\sim 25$ MeV). The previous computations [36] are incomplete: either diagrams in Fig. [10] or the diagrams in Fig. [10] were not taken into account.

A similar $\mathcal{T}$-even contribution to the correlation $\eta = (\vec{p}_1 \times [\vec{p}_2 \times \vec{p}_3])$ in the decay $K^+ \to \pi^0\mu^+\nu\gamma$ is given by similar diagrams and has the same order of magnitude $1.1 \times 10^{-4}$ [37].

4.1 $\mathcal{CP}$ and $\mathcal{T}$ violation in the decays $K \to 3\pi$.

The imaginary part of the effective coupling constants $g_8$, $g_{27}$ and $g_{\text{ew}}$ in the effective weak Lagrangian (29) gives rise to the $\mathcal{CP}$-violating effects in the decays $K \to 3\pi$.

The kinematical variables used to describe the decay $K(p) \to \pi_1(p_1)\pi_2(p_2)\pi_3(p_3)$ are as follows:

$$s_i = (p - p_i)^2, \quad X = \frac{(s_1 - s_2)^2}{m_{\pi}^2}, \quad Y = \frac{(s_3 - s_0)}{m_{\pi}^2},$$

where "3" is the "odd" pion in either the $\tau(\pi^+\pi^+\pi^-)$ or $\tau'(\pi^+\pi^0\pi^0)$ decay mode. The slope parameters $g$ and $j$ are defined by the formula for the differential probability of the decay:

$$|A(K \to 3\pi)|^2 \sim 1 + gY + jX + hY^2 + kX^2. \quad (87)$$

The $\mathcal{CP}$-violating quantities are as follows:

$$\delta_{\Gamma} = \frac{\Gamma(K^+ \to 3\pi) - \Gamma(K^- \to 3\pi)}{\Gamma(K^+ \to 3\pi) + \Gamma(K^- \to 3\pi)}, \quad (88)$$

and

$$\delta_g = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)} \quad (89)$$

With the assumption that $\Delta I \leq 1/2$, the relevant $K \to 3\pi$ amplitudes can be expanded as follows:

$$A(K^+ \to \pi^+\pi^+\pi^-) = 2a_c \left(1 + i\alpha^0 + \frac{i\alpha'_0}{2}Y\right) + \left[b_c(1 + i\beta_0) + b_2(1 + i\delta_0)\right] Y, \quad (90)$$

$$A(K^+ \to \pi^0\pi^0\pi^+) = a_c \left(1 + i\alpha^0 - i\alpha'_0 Y\right) - \left[b_c(1 + i\beta_0) + b_2(1 + i\delta_0)\right] Y.$$

From here on we restrict our attention to the slope asymmetry (89) in the $\tau$ decay mode. As it usually is, this asymmetry is determined by the interplay of (i) the imaginary parts of the parameters $a_c$ and $b_2$, stemming from the $\mathcal{CP}$-odd effective weak Lagrangian (29) and (ii) the imaginary part coming about the $\mathcal{CP}$-even final-state interactions (38):

$$(\delta g)_\tau = \frac{\alpha_0 - \beta_0}{a_c(b_c + b_2)}(a_c\text{Im}b_c - b_c\text{Im}a_c) + \frac{\alpha_0 - \delta_0}{a_c(b_c + b_2)}(a_c\text{Im}b_2 - b_2\text{Im}a_c). \quad (91)$$
Figure 10: Diagrams giving a contribution to the imaginary part of the amplitude of the decay $K \rightarrow \mu\nu\gamma$. 
An evaluation of the strong rescattering phases in the one-loop approximation of the χPT gives

\[
\alpha_0 = \frac{1 - (4m^2/s_0)}{32\pi F^2}(2s_0 + m^2) \approx 0.13, \quad (92)
\]

\[
\beta_0 = -\delta_0 = \frac{1 - (4m^2/s_0)}{32\pi F^2}(s_0 - m^2) \approx 0.05.
\]

The values \( a_c, b_c, b_2, \alpha_0, \beta_0, \) and \( \delta_0 \) can be expanded in powers of the χPT expansion parameter \( \lambda \simeq p/(4\pi F) \), where \( p \) defines the momentum scale and \( F = 93 \text{ MeV} \). In the case of K-meson decays, \( \lambda \simeq 0.4 \). In each order of the chiral expansion it is helpful to isolate the \( \Delta I = 1/2 \) and \( \Delta I = 3/2 \) contributions to \( \Delta g \), the latter contribution being suppressed by the factor \( \omega = 0.045 \).

The point is that, in the order \( O(p^2) \), the neglect of the \( \Delta I = 3/2 \) contribution gives rise to the relations \( a_c = -\frac{1}{3} \frac{M^2_K}{M^2_{\pi}} b_c \) and \( b_2 = 0 \), which, in their turn imply that \( \Delta g_{O(p^2)}, \Delta I = 1/2 \) = 0. However, the \( \Delta I = 1/2 \) contribution does not vanish in the order \( O(p^4) \) and so it dominates the total \( O(p^4) \) contribution. It is natural to assume that it is enhanced by the factor \( \omega^{-1} = 22.5 \) (see [61]) as compared to the \( \Delta I = 3/2 \) contribution in the order \( O(p^2) \) of the χPT. The \( O(p^4) \) \( \Delta I = 3/2 \) contribution is suppressed by the χPT expansion parameter \( \lambda^2 \) as compared to the \( O(p^2) \) \( \Delta I = 3/2 \) contribution \( A \). The above reasoning is summarized in Table 5.

**Table 5.** Various contributions to \( \delta_g = \frac{\Delta g}{2g} \).

| Order of χPT | \( \Delta I = 1/2 \) | \( \Delta I = 3/2 \) | Numerical estimate |
|--------------|-----------------|-----------------|------------------|
| \( O(p^2) \) | 0               | \( A \)         | \((1 ÷ 3) \times 10^{-6}\) |
| \( O(p^4) \) | \( \sim \frac{\lambda^2 A}{\omega} \) | \( \sim \lambda^2 A \) | \((0.4 ÷ 1) \times 10^{-5}\) |
| \( O(p^6) \) | \( \sim \frac{\lambda^4 A}{\omega} \) | \( \sim \lambda^4 A \) | \((0.8 ÷ 2) \times 10^{-6}\) |

We see that the total contribution is dominated by the \( O(p^4) \) contribution, which is \( \omega^{-1} \lambda^2 \simeq 4 \) times greater than the \( O(p^2) \) contribution—due to vanishing of the \( O(p^2) \) \( \Delta I = 1/2 \) contribution. Maiani and Paver [38] assume that the enhancement factor may run up to \( 10 ÷ 20 \). Therewith, the conclusion by Bel’kov et al. [39] that the \( O(p^6) \) corrections increase the enhancement factor by the order of magnitude appears, in view of the above reasoning, highly questionable. It should also be noticed that the multi-Higgs models allow a two-fold increase of the parameter \( \delta_g \) as compared with the SM prediction [40].

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