The local potential approximation for the Brueckner G-matrix

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Abstract

The Brueckner $G$-matrix for a slab of nuclear matter is analyzed in the singlet $^1S$ and triplet $^3S + ^3D$ channels. The complete Hilbert space is split into two domains, the model subspace $S_0$, in which the two-particle propagator is calculated explicitly, and the complementary one, $S'$, in which the local potential approximation is used. This kind of local approximation was previously found to be quite accurate for the $^1S$ pairing problem. A set of model spaces $S_0(E_0)$ with different values of the cut-off energy $E_0$ is considered, $E_0$ being the upper limit for the single-particle energies of the states belonging to $S_0$. The independence of the $G$-matrix of $E_0$ is assumed as a criterion of validity of the local potential approximation. Such independence is obtained within few percent for $E_0=10 \div 20$ MeV for both the channels under consideration.
1. Introduction.

Most reliable predictions of nuclear properties come from phenomenological approaches including the macroscopic-microscopic method [1, 2, 3], and, at a more fundamental level, the Finite Fermi Systems (FFS) theory [4, 5, 6], the HF method with effective forces [7, 8] and the new versions of the energy functional method [9]. These methods, being comparatively simple, permit to carry out systematic calculations for a lot of atomic nuclei. On the other hand, being phenomenological, these approaches have the necessity of introducing a set of adjustable parameters. This point appears especially delicate when one deals with new types of nuclear systems, e.g. nuclei in the drip-line vicinity. Indeed, any extrapolation of phenomenological parameters found for stable nuclei is hardly reliable in this case. Therefore the old problem of the \textit{ab initio} calculation of these parameters is of importance not only from an heuristic point of view but also from the practical one.

Application of methods of microscopic theory of nuclear matter to finite nuclei (see, e.g., the monographs [10] and [11]) are mainly limited by use of so-called local density approximation (LDA). The LDA works reasonably well inside a nucleus but fails at the surface where there is a domain of density values for which nuclear matter is unstable. However, it is impossible to develop a self-consistent nuclear theory without consistent description of the nuclear surface. Indeed, just in the surface region the nuclear mean field sharply changes from zero outside the nucleus to a constant value inside it. In terms of the FFS theory, it is associated with a sharp variation of the scalar-isoscalar Landau-Migdal amplitude $f$ from a strong attraction outside a nucleus to an almost zero inner value. A consistent description of this variation is of primary importance for the nuclear theory. We intend to develop an approach to this problem based on the Brueckner theory for nonuniform systems beyond the LDA.

Recently [12], dealing with $^1S$-pairing problem for semi-infinite nuclear matter, we developed a method of solving the Bethe-Goldstone (BG) type equation for the separable form of the Paris $NN$-potential without use of any form of local approximation. The same method was applied to the case of the slab geometry in Refs. [13, 14]. Though the separable form of the $NN$-interaction simplifies calculations significantly, they remain to be rather cumbersome and much CPU time consuming. To circumvent this difficulty, we devised a new version of the local approximation resulting from the investigation of the effective pairing interaction $V_{\text{eff}}^{\text{p}}$ [12]. To distinguish it from the standard LDA, we named it as the local potential approximation (LPA). The effective interaction is associated with splitting of the complete Hilbert space $S$ into two domains. The first one is the model subspace $S_0$, in which the gap equation is written down in terms of the effective interaction $V_{\text{eff}}^{\text{p}}$. The second one is the complementary subspace $S'$, in which the equation for $V_{\text{eff}}^{\text{p}}$ is obtained in terms of the free $NN$-interaction $V$. In this space, the pairing effects are not significant, therefore the equation for $V_{\text{eff}}^{\text{p}}$ has the form of the BG equation. Dealing with the pairing problem, the model space was taken in a form convenient for nuclear application, which includes all the single-particle states with negative energies $\varepsilon_\lambda$. The LPA is related to calculation of the two-particle propagator in the complementary space which enters the equation for $V_{\text{eff}}^{\text{p}}$. Roughly speaking, the LPA procedure consists in replacing the exact BG propagator by a suitable form taken from infinite nuclear matter. For a fixed value of the average centre-of-mass (CM) $x$-coordinate $X = (X_{12} + X_{34})$ of incoming and outgoing nucleons (the $x$-axis is perpendicular to the layer), the propagator is supposed to be equal to
the one of nuclear matter in the potential well $V(X)$. Such an approximation turned out to be accurate at a level of a few percent even at the surface [12]. It was shown by a comparison of the direct solution for $V_{\text{eff}}$ with the LPA one.

From the computational point of view, the problem of finding the Landau-Migdal interaction amplitude in terms of the Brueckner $G$-matrix is much more complicated than the pairing problem. First, the additional triplet $^3S$-channel (coupled with the $^3D$ one) should be considered for which the calculations turned out to be more complicated than those for the singlet $^1S$-channel. Second, instead of fixing the value of the total perpendicular momentum $P_\perp = 0$ as in the pairing problem, the integral over $P_\perp$ appears in this case. Therefore, though the direct numerical solution of the problem, in principle, is possible [15], it looks very difficult. Therefore it is natural to attempt using the LPA for solving the BG equation for the $G$-matrix that simplifies calculations significantly. In the case of the singlet $^1S$-channel, the accuracy of the LPA for the BG equation is just the same as in the pairing problem because the corresponding two-particle propagators in the complementary space coincide. As to the triplet channel, applicability of the LPA is not obvious at all. The main goal of this paper is to clarify the latter point.

In the case of the BG equation, there is no evident gain from the introduction of the effective interaction and it seems more reasonable, after splitting the Hilbert space as $S = S_0 + S'$, to formulate the LPA procedure in a direct way. According to that splitting, the two-particle propagator $A$ in the BG equation can be written as a sum of $A = A_0 + A'$. The model space term, $A_0$, should be calculated exactly, whereas the second one, $A'$, within the LPA. It is clear that the applicability of the LPA depends on the choice of the model space $S_0$. Indeed, all the quantum and finite range effects originate mainly from the states nearby the Fermi surface whose contribution to the BG equation is strengthened by the small values of the energy denominator in the two-particle propagator. These contributions produce the long-range components of $A$ and should be taken into account exactly. At the same time, the individual contribution of a far-lying state is negligible and only a sum of a big number of such states is important. They produce the short-range term of $A$ and can be considered within the local approximation. Hence, the accuracy of the LPA should depend on the choice of the model space: wider $S_0$ is higher it is. We use this simple physical idea to impose a criterion of the applicability of the LPA. We define the model space $S_0(E_0)$ including all the single particle states with the energies $\epsilon_\lambda < E_0$ which is more general than that for the pairing problem [1]. It is obvious that a larger $E_0$ corresponds to a higher accuracy of the LPA. We consider the LPA to be valid at some value of $E_0$ if the $G$-matrix does not practically change with additional increase of $E_0$.

The paper is organized as follows. Sect. 2 contains the BG equation for the slab of nuclear matter with separable $NN$-forces. In Sect. 3 the splitting of the Hilbert space on the model subspace defined by the cut-off energy $E_0$ and its complementary subspace is discussed. In addition, the LPA for the BG equation is introduced. In Sec. 4 the validity of the LPA for determining the $G$-matrix in the singlet $^1S$-channel is analyzed for different values of $E_0$. In Sec. 5 the analysis is extended to the triplet $^3S + ^3D$-channel. Sec. 6 contains a summary of the results.

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1 The latter corresponds to $E_0=0$
2. Bethe-Goldstone equation for the slab system.

Let us consider the BG equation for the \( G \)-matrix of two nucleons at the Fermi surface, i.e. with the single-particle energies \( \varepsilon_\lambda = \mu \), where \( \mu \) is the chemical potential of the system under consideration. In a short notation it reads:

\[
G(E) = V + VA(E)G(E),
\]

where \( V \) is the free \( NN \)-potential, \( E = 2\mu \), and \( A \) is the two-particle propagator which is determined by the integration over the relative energy of the product \((G^pG^p)\) of two particle single-particle Green’s functions. The contribution \((G^hG^h)\) of two holes is neglected.

To speed up the convergence it is convenient to renormalize eq. (1) in terms of the free off-shell \( T \)-matrix taken at negative energy \( E = 2\mu \). The latter obeys the Lippman-Schwinger equation:

\[
T(E) = V + VA^f(E)T(E),
\]

where \( A^f(E) \) is the propagator of two free nucleons with the total energy \( E \).

The renormalized BG equation has the form:

\[
G = T + T(A - A^f)G.
\]

We consider a nuclear-matter slab of thickness \( 2L \) placed into the one-dimensional Saxon-Woods potential \( V(x) \) symmetrical with respect to the point \( x = 0 \) with depth of \( V_0 = 50 \text{ MeV} \), the diffuseness parameter \( d = 0.65 \text{ fm} \), and \( L = 8 \text{ fm} \).

We use the separable version of the Paris \( NN \)-potential which for the \(^1S_0\)-channel has the \((3 \times 3)\) form:

\[
V(k, k') = \sum_{ij} \lambda_{ij} g_i(k^2) g_j(k'^2).
\]

For the triplet \(^3S_1 + ^3D_1\)-channel, a similar \((4 \times 4)\) expansion is valid with a formal substitution \( g_i(k^2) \rightarrow \hat{g}_i(k^2) \) where the column \( \hat{g}_i \) contains two components:

\[
\hat{g}_i(k^2) = \left( \begin{array}{c} g_i^{L=0}(k^2) \\ g_i^{L=2}(k^2) \end{array} \right),
\]

\( L \) being the relative orbital moment in the CM system.

The scheme of solving the BG equation for a slab of nuclear system in the mixed coordinate-momentum representation, which has been devised in Ref.[15], is adopted also here. Therefore we write down in the explicit form only those equations which are necessary for understanding the procedure and refer to [13] for details. We consider first the singlet channel \( S = 0 \). As it was mentioned above, all relations remain valid for the triplet channel as well after replacing \( g_i(k^2) \rightarrow \hat{g}_i(k^2) \).

The separable form of the \( NN \)-potential in eqs. (1) and (2) induces similar expansions for the \( G \)-matrix:

\[
G(k^2, k'^2; x_1, x_2, x_3, x_4; E) = \sum_{ij} G_{ij}(X, X'; E, \mathbf{P}_\perp) g_i(k^2, x) g_j(k'^2, x'),
\]
and $T$-matrix

$$T(k_{\perp}^2, k_{\perp}'; x_1, x_2, x_3, x_4; E) = \sum_{ij} T_{ij}(X-X'; E, P_{\perp})g_i(k_{\perp}^2, x)g_j(k_{\perp}'^2, x').$$  \hspace{1cm} (7)

Here the form factors $g_i(k_{\perp}^2, x)$ in the mixed representation are determined by the inverse Fourier transformation of $g_i(k_{\perp}^2 + k_{\perp}'^2)$ with respect to variable $k_x$. Their analytical form can be found in [12] for the singlet channel and in [15], for the triplet one. The obvious notation for the CM and relative coordinates in the $x$-direction are used in (3) and (4). Of course, the $T_{ij}$ coefficients depend only on the difference $t = X-X'$ of the CM coordinates. In the perpendicular direction, the total momentum $P_{\perp}$ and the relative momentum $k_{\perp}$ are introduced.

Substitution of (3) and (7) into eq. (8) results in a set of one-dimensional integral equations:

$$G_{ij}(X, X'; E, P_{\perp}) = T_{ij}(X-X'; E, P_{\perp}) + \sum_{lm} \int dX_1 dX_2 T_{il}(X - X_1; E, P_{\perp}) \delta B_{lm}(X_1, X_2; E, P_{\perp}) G_{mj}(X_2, X'; E, P_{\perp}),$$

where

$$\delta B_{lm} = B_{lm} - B_{lm}^{fr}$$  \hspace{1cm} (9)

is the difference between the convolution $B_{lm}$ of the two-particle propagator $A$ with two form factors $g_l, g_m$ and the analogous convolution $B_{lm}^{fr}$ for the free propagator $A^{fr}$. The explicit form of $B_{lm}$ is as follows:

$$B_{lm}(X, X'; E, P_{\perp}) = \sum_{nn'} \int \frac{dk_{\perp}}{(2\pi)^2} \frac{1 - n\lambda}{E - P_{\perp}^2/4m - \varepsilon_n - \varepsilon_{n'} - k_{\perp}^2/m} \times g_{n,n'}^l(k_{\perp}^2, X) g_{n,n}^m(k_{\perp}^2, X').$$  \hspace{1cm} (10)

Here we have used a short notation of $\lambda = (n, P_{\perp}), \lambda' = (n', P_{\perp}')$, $P_{\perp} = P_{\perp}/2 + k_{\perp}$, $P_{\perp}' = P_{\perp}/2 - k_{\perp}$, and $n\lambda = \theta(\mu - \varepsilon)\lambda$, where $\varepsilon = \varepsilon + p_{\perp}^2/2m$, and $\varepsilon_n$ are the eigenenergies of the one-dimensional Schrödinger equation with the Saxon-Woods potential. The corresponding eigenfunctions $y_n(x)$ (they are chosen to be real) enter the matrix elements of of the form factors:

$$g_{n,n'}^l(k_{\perp}^2, X) = \int dx g_l(k_{\perp}^2, x) y_n(X+x/2)y_{n'}(X-x/2).$$  \hspace{1cm} (11)

It should be noted that the symbolic sum over $nn'$ in (10) actually includes the summation over discrete states and the integration over the continuum spectrum with the standard substitution $\sum_n \to \int dp/2\pi$.

In the singlet channel, the BG equation for the G-matrix is very similar to the one for the effective pairing interaction [12]. Just as in the latter case, it is convenient to extract the singular $\delta$-form Born term from the complete $G$-matrix:

$$G = \mathcal{V} + \delta G.$$  \hspace{1cm} (12)

The equation for the correlation component $\delta G$ of the $G$-matrix can be readily found from (11):

$$\delta G = \mathcal{V} A \mathcal{V} + \mathcal{V} A \delta G.$$  \hspace{1cm} (13)
An analogous extraction of the Born term should be made also for the \( T \)-matrix:

\[
T = V + \delta T.
\]  

(14)

As a result, the renormalized eq. (3) yields the following form:

\[
\delta G = F + T(A - A^\pi)\delta G,
\]  

(15)

where the nonhomogeneous term is

\[
F = \delta T + T(A - A^\pi)V.
\]  

(16)

The explicit transformation of eqs. (15), (16) to the form similar to eq. (8) is quite obvious. A simplification of the numerical procedure for solving eq. (15) in the slab system under consideration can be made using the parity conservation which follows from the symmetry of the Hamiltonian under the inversion transformation \( x \rightarrow -x \). As a result, the eigenfunctions \( y_n \) can be separated on the even functions, \( y_n^+ \), and the odd ones, \( y_n^- \). Then the two-particle propagator in the above equations is the sum

\[
A = A^+ + A^-
\]  

(17)

of the even and odd components. The first one, \( A^+ \), originates from the terms of the sum \( (10) \) containing states \( (\lambda, \lambda') \) with the same parity, and the second one, \( A^- \), from those with opposite parity. So long as the \( NN \)-potential \( V \) does conserve the parity, the propagators \( A^+ \) and \( A^- \) are not mixed in the BG equation. Therefore the correlation part of the \( G \)-matrix is also a sum of the even and odd components,

\[
\delta G = \delta G^+ + \delta G^-,
\]  

(18)

which obey the separated equations:

\[
\delta G^\pi = VA^\pi V + VA^\pi\delta G^\pi,
\]  

(19)

\( \pi = +, - \).

It is obvious that the integral equation (19) can be reduced to the form containing positive \( x \) values only which simplifies the calculation procedure. This equation should be solved for both values of \( \pi \) separately, then the complete \( G \)-matrix is found from eqs. (12), (18).

All the above general equations remain valid for the triplet channel \( S = 1 \). The main change occurs in the definition of the convolution integral (14). For the triplet channel it has the form:

\[
B_{lm}^{S=1}(X, X'; E, P_\perp) = \sum_{mn} \int \frac{d{\bf k}_\perp}{(2\pi)^2} \frac{(1 - \nu\lambda)(1 - \nu\lambda')}{E - \frac{P_\perp^2}{4m} - \varepsilon_n - \varepsilon_{n'} - \frac{k_\perp^2}{m}} \times \left( g^{(0)l}_{nn'}(k_\perp^2, X) g^{(0)m}_{n'n}(k_{\perp'}^2, X') + g^{(0)l}_{nn'}(k_{\perp'}^2, X) g^{(0)m}_{n'n}(k_\perp^2, X') \right).
\]  

(20)

It should be noted also that all multipole expansions of eqs. (3) and (4) type become the \( 2 \times 2 \) matrices. For example, let us write down the components of the \( G \)-matrix in the explicit form:

\[
G^{LL'}(k_\perp^2, k_{\perp'}^2, P_\perp; x_1, x_2, x_3, x_4; E) = \sum_{ij} G_{ij}(X, X'; E, P_\perp) g_i^{(L)}(k_\perp^2, x) g_j^{(L')}(k_{\perp'}^2, x'),
\]  

(21)
where \( L, L' \) equal to 0 or 2.

3. Choice of the model space and the local potential approximation.

The main computational problems of solving the BG equation for the slab system are connected with the calculation of the propagators (10) and (20). The reason for introducing the model space \( S_0(E_0) \), with splitting the complete Hilbert space \( S = S_0 + S' \) and using the LPA in the complementary space \( S' \), is as follows. The subspace \( S_0(E_0) \) contains all the two-particle states \((\lambda, \lambda')\) with both one-particle energies \( \varepsilon_\lambda, \varepsilon_{\lambda'} \) being small, \( \varepsilon_\lambda, \varepsilon_{\lambda'} < E_0 \). In the complementary subspace, \( S'(E_0) \), one of these energies or both of them are large, \( \max(\varepsilon_\lambda, \varepsilon_{\lambda'}) > E_0 \). For the model space, the contribution of each individual state \((\lambda, \lambda')\) to the sum of eq.(10) or eq.(20) is strengthened, in comparison with the analogous one for the complementary space, due to a small value of the energy denominator. Such contributions induce the long-range terms of the BG propagator \( A \) and should be calculated in a direct way. On the contrary, in the complementary subspace no individual state \((\lambda, \lambda')\) is important and only wide intervals of the integration over \( k_\perp \) contribute to \( A \) significantly. The corresponding term of the BG propagator is sharply peaked and is mainly determined by the local properties of the system. Therefore it is natural to use for it some kind of local approximation. For the problem under consideration, it seems to be more natural to use the LPA rather than the LDA because the BG propagator in the vicinity of the point \( X \) is determined directly by the potential well \( V(X) \) but not by the density, \( \rho(X) \). At the same time, in the surface region there is no simple local relation between \( \rho(X) \) and \( V(X) \).

The splitting of the Hilbert space \( S = S_0 + S' \) results in the representation of the BG propagator as sum:

\[
A = A_0 + A',
\]

where \( A_0 \) contains the states \((\lambda, \lambda')\) which belong to the model space, \( A' \) including the rest. In accordance with the above consideration, we calculate the model space propagator \( A_0 \) explicitly, but use the LPA for the rest term \( A' \). Obviously, the accuracy of the LPA becomes higher with the model space \( S_0 \) becoming wider. We consider the LPA to be good at some value of \( E_0 \) if the calculation results for the \( G \)-matrix do not practically change with additional increase of \( E_0 \).

The LPA procedure, in principle, is the same for both the channels under consideration and is very close to that for the pairing problem, the latter corresponding to the choice \( E_0 = 0 \). Namely, for fixed values of the CM coordinates \( X_{12}, X_{34} \), the convolution integral (10) for \( S = 0 \) (or 20, for \( S = 1 \)) is replaced by the corresponding integral for nuclear matter put in the constant potential well \( V_0 = V(X) \), where \( X = (X_{12} + X_{34})/2 \), which depends on the difference of the CM coordinates \( t = X_{12} - X_{34} \):

\[
B_{lm}^{\text{LPA}}(X_{12}, X_{34}; E, P_\perp) = B_{lm}^{\text{inf}}(V[X], E; P_\perp).
\]

In practice, for a fixed value of the chemical potential \( \mu \) and the cut-off energy \( E_0 \) and a given set of the potential depths \( V_n \), we calculate a basic set \( B_{lm}^{\text{inf}}(V_n, E = 2\mu) \) of nuclear matter propagators. In fact, we used a sequence of \( V_n = \delta V \cdot (n-1) \) with the step \( \delta V = 2 \text{ MeV} \). At

\[\text{In fact, the difference } \varepsilon_\lambda - \mu \text{ is small. Just such differences enter the denominator of eq.(10) at } E = 2\mu.\]

\[\text{The "individual" state means the fixed value of } n, n' \text{ and small interval of integration over } k_\perp.\]
a fixed coordinate set \(X_k\), the elements of the LPA propagator matrix \(B_{lm}^{\text{LPA}}(X_i, X_k)\) are found as follows. First, we find the potential depth \(V(X_0=(X_i + X_k)/2)\). Then, for a fixed value of \(t = |X_i - X_k|\), the LPA propagator is found by a linear extrapolation of two neighboring values of \(B_{lm}^{\text{inf}}([V_n], t; E), B_{lm}^{\text{inf}}([V_{n+1}], t; E)\), under the condition that the inequality \(V_n < V(X_0) < V_{n+1}\) is satisfied. The convolution integral \(B_{lm}^{\text{fr}}\) for the free propagator \(A^{\text{fr}}\), by definition, coincides with \(B_{lm}^{\text{inf}}([V_1 = 0], t; E)\). Details can be found in Refs. [12], [15].

4. Validity of the LPA for the singlet channel

Up to now, we dealt with the general form of the BG equation for the slab system containing the total perpendicular momentum \(P_{\perp}\) as a parameter. As it was discussed above, the "dangerous" terms of the propagator (or of the convolution integrals (10)) which belong to the model space and should be considered explicitly, appear due to the small value of the corresponding denominators in the sum of (10). It is obvious that they become more dangerous if the value of \(P_{\perp}\) becomes smaller. Hence the case of \(P_{\perp} = 0\) is the most crucial for validity of the LPA. Therefore we consider just this particular "bad" case for the LPA. Then, we only focus on \(\mu = -8\) MeV which is a chemical potential typical of stable nuclei. Thus, we put in all the above equations \(P_{\perp} = 0, E = -16\) MeV (omitted for brevity from now on).

As discussed above, the BG equation for the correlation part of the \(G\)-matrix (13) has a fixed parity \(\pi = (+, -)\). Therefore we deal with eq. (13) with fixed \(\pi\) which is defined only for positive \(\pi\). After finding the convolution integrals (10) and those for the free propagator \(A^{\text{fr}}\), the kernel of eq. (15) and the nonhomogeneous term (16) are derived by direct integration. Then one obtains a set of the integral equations for six independent components of \(\delta G^{\pi}_{ij}(X, X')\) (similar to eq. (8)) which can be solved numerically [12], [13]. Finally, we find the total correlation \(\delta G\)-matrix (18) with components \(\delta G_{ij}(X, X')\) or, from eq. (12), the complete \(G\)-matrix with components \(G_{ij}(X, X')\). They differ by a trivial \(\delta\)-function term:

\[
\delta G_{ij}(X, X') = G_{ij}(X, X') - \lambda_{ij}\delta(X - X').
\]  

The extraction of the latter makes the quantity under consideration more convenient for analysis and graphical representation. Therefore, as a rule, we deal with the only correlation part of the \(G\)-matrix, and not with the complete one. One more remark should be also made before discussing the results. Following [12], we change the original normalization [16, 17] of the expansion (4) in such a way that the identity \(g_i(0) = 1\) holds true. In this case, the absolute values of the \(\lambda_{ij}\)-coefficients give direct information on the strength of the corresponding terms of the force. Their values (in MeV·fm\(^3\)) are as follows: \(\lambda_{11} = -3.659 \cdot 10^3\), \(\lambda_{12} = 2.169 \cdot 10^3\), \(\lambda_{22} = -1.485 \cdot 10^3\) and \(\lambda_{13} = -2.36 \cdot 10^1\), \(\lambda_{23} = 5.76 \cdot 10^1\), \(\lambda_{33} = 1.72 \cdot 10^4\). The strengths of all the components containing only the indices \(i = 1, 2\) are much stronger (by two orders) than those with the index \(i = 3\). Therefore the latter are important only for large momenta which come virtually to the BG equation or the Lippman-Schwinger one. If one analyzes the matrix elements of the \(G\)-matrix over the nuclear wave functions, the typical momenta \(k \approx k_F\) appear for which the contribution of the small components is negligible. Therefore we, as a rule, concentrate on the "big" components in a qualitative analysis. Of course, in the calculations all the terms \(\lambda_{ik}\) are taken into account.

We made a series of calculations of the \(G\)-matrix for several values of the cut-off energy \(E_0 = 0, 10, 20\) MeV to analyze a dependence of the results on this parameter. To present the
results we draw the profile functions $\delta G_{ij}(X, X'=X_0)$ of the correlation term of the $G$-matrix at several values of $X_0$ and the zero moment of the $G$-matrix:

$$\bar{G}_{ij}(X) = \int_{-\infty}^{\infty} dt \, G_{ij}(X, X+t).\quad (25)$$

A typical example of the profile function of $\delta G_{11}(X, X'=0)$ is shown in Fig. 1 for the case of the model space with the cut-off energy $E_0=0$. It has a sharp peak at the point $X=X'$. In such a scale, similar curves for $E_0=10$ and 20 MeV are distinguishable from that for $E_0=0$ only after magnification. Such magnified profile functions for large components with $ij=11, 12, 22$ are shown in Fig. 2, for $X'=0$, and in Fig. 3, for $X'=8$ fm. It is easily seen that already the difference between the curves for $E_0=0$ and $E_0=10$ MeV is rather small. As to that for $E_0=10$ MeV and $E_0=20$ MeV, it looks negligible.

To analyze the dependence of the $G$-matrix on $E_0$ in a more quantitative way, it is worth to compare each other the zero moments (25) at different values of $E_0$. They are shown in Fig. 4 for the same large components and, as an example, for one small component, $ij=13$. One sees that a difference, at a level of a few percent, exists between the curves for $E_0=0$ and $E_0=10$ MeV and again the additional increase of $E_0$ from 10 MeV to 20 MeV does not practically influences the results. This is true not only for big components, but also for small ones.

Finally, we calculated the ”Fermi averaged” $G$-matrix in the $1S$-channel:

$$<G_F>_{S=0}(X) = \sum_{ij} \bar{G}_{ij}(X) g_i(k_F^2(X)) g_j(k_F^2(X)),\quad (26)$$

where we have introduced the local Fermi momentum as $k_F(X)=\sqrt{2m(\mu-V(X))}$ at $\mu-V(X)>0$ and which otherwise takes zero value. Such an average appears if one calculates the Landau-Migdal amplitude in terms of the $G$-matrix [15]. To this respect, one remark should be made. Though the profile functions $G_{ij}(X, X')$ are strongly peaked at the point $X=X'$, the long range ”tails”, which are hardly seen ”by eyes” in Fig. 1, also contribute to the zero moment (25). These terms of the $G$-matrix appear due to states entering the model space and their contribution to the integral (25) was analyzed in [13] for the case of $E_0 = 0$. When one deals with the problem of evaluation of the Landau-Migdal amplitude which is supposed to be a short-range coordinate function it is natural to cut these tales. In Ref. [13] a recipe was suggested to use the Fermi averaged $G$-matrix by eq.(26) with the zero moments ”with cut-off” which are defined by the integral of the (25) type, but with limits of $|t|<t_c$, $t_c=3$ fm. Of course, for the problem of the validity of the LPA it is not important what kind of the zero moment is used in (25). However, we use here the same recipe for the Fermi averaged $G$-matrix as in [13] because it is more physical. This quantity is shown in Fig. 5 for the same three values of $E_0$ together with the analogous average value of the free off-shell $T$-matrix:

$$<T_F>_{S=0}(X) = \sum_{ij} \bar{T}_{ij}(E=2\mu) g_i(k_F^2(X)) g_j(k_F^2(X)),\quad (27)$$

\[4\] This contribution depends on $ij$ and is, as a rule, not greater than $10 \div 20\%.$
where the zero moments $T_{ij}$ of the $T$-matrix are defined similar to (25). In this case, the introduction of the cut-off with $t_c=3$ fm does not practically change the integral. Of course, it is $X$-independent.

Again, the difference between the Fermi-averaged $G$-matrix for $E_0=10$ MeV and that for $E_0=20$ MeV is negligible. Their deviation from the one corresponding to $E_0=0$ is also very small everywhere except in the surface region. It should be noted that the difference between the average $G$-matrix and $T$-matrix is rather small. A similar property was found previously [13], [14] for the effective pairing interaction in the $1S$-channel.

Analysis of Figs. 2 - 5 leads us to the conclusion that for the singlet channel $S = 0$ the LPA works perfectly well for $E_0=10 \div 20$ MeV. Moreover, within the accuracy of a few percent, it is also valid for $E_0=0$. The latter agrees with the analysis of [12] where the LPA was introduced for the pairing problem in the $1S$-channel.

5. Validity of the LPA for the triplet channel

In general, the calculation scheme for the triplet $3S + 3D$-channel is very similar to that for the singlet one, though the calculations become more cumbersome in this case. Indeed, first, we get ten independent components $G_{ij}(X, X')$ and ten integral equations (8) for them instead of the six of the singlet case. Second, the calculation of the convolution integral (20) in the triplet channel is also more difficult than that of (11). Therefore the problem of simplifying these calculations is even more important than in the singlet channel.

Contrarily to the singlet case, now it is difficult to separate the multipole terms into the "large" and "small" ones. Again we changed the original normalization [16] of the expansion (4), (5) to guarantee the identity $g_i^{L=0}(0) = 1$ (it should be noted that $g_i^{L=2}(0) = 0$). Then the strengths of the corresponding terms of the force (in MeV·fm$^3$) are as follows: $\lambda_{11} = -1.618 \times 10^3$, $\lambda_{12} = -1.296 \times 10^3$, $\lambda_{13} = 8.921 \times 10^2$, $\lambda_{14} = 4.271 \times 10^1$, $\lambda_{22} = 7.848 \times 10^2$, $\lambda_{23} = 1.394 \times 10^3$, $\lambda_{24} = -7.860 \times 10^2$, $\lambda_{33} = -7.450 \times 10^2$, $\lambda_{34} = -5.723 \times 10^2$, $\lambda_{44} = 1.865 \times 10^3$. These values show that, though the strengths of different components vary significantly, only one of them, $\lambda_{14}$, is less by two orders of magnitude as compared to the largest ones. Therefore almost all the terms are significant. We take several typical components to illustrate the calculation results.

The profile functions and zero moments are shown in Figs. 6 - 8. One can see that now the results with increase of the cut-off energy from $E_0=0$ to $E_0=10$ MeV change more sizably than in the singlet channel, especially in the surface region. At the same time, the subsequent increase of $E_0$ up to 20 MeV does not practically influence the $G$-matrix, the maximum variation being of a few percent. Hence, once more one may conclude that the LPA is sufficiently accurate if the cut-off energy is $E_0=10 \div 20$ MeV. But, contrarily to the singlet case, the accuracy of the LPA is rather poor when the model space is limited to $E_0=0$, since at the surface the $G$-matrix must tend to the off-shell free $T$-matrix. But the latter has a virtual pole at small energy. It is then clear that an accurate account of the contribution of the single-particle states with small positive energies is important for correct description of this pole behavior. Therefore these states should be included into the model space $S_0$. This does occur if one chooses the cut-off energy $E_0 \geq 10$ MeV, but it does not occur if one takes $E_0=0$.

Let us now consider the Fermi averaged $G$-matrix in the triplet channel, which now is a
2 × 2 matrix in the orbital angular momentum space:

\[ < G_F^{LL'}_{S=1} (X) > = \sum_{ij} \bar{G}^{S=1}_{ij} (X) g_i^{(L)} (k_F^2 (X)) g_j^{(L')} (k_F^2 (X)), \]  

(28)

where \( L, L' = 0, 2 \). Just as in the singlet case, the quantity \( \bar{G}_{ij} \) in (28) has the meaning of the zero moment "with cut-off". The components of this matrix are shown in Fig. 9 for all three values of the cut-off energy \( E_0 \). The component \( < G_F^{00}_{S=1} > \) is significantly larger than those containing \( L=2 \), especially at the surface region where the form factors \( g^{(2)}_i \) vanish. Again all the components of the Fermi-averaged \( G \)-matrix calculated for \( E_0=10 \) MeV coincide practically with those for \( E_0=20 \) MeV, though deviations from the \( E_0=0 \) case can be noticeable.

6. Conclusion

The applicability of the LPA has been analyzed for the Brueckner \( G \)-matrix. Previously \[12\] this kind of the local approximation proved to be quite accurate in the problem of the microscopic evaluation of the effective pairing interaction in the \( 1S \)-channel. The BG equation for a slab of nuclear matter has been solved for the singlet \( 1S \) and triplet \( 3S + 3D \) channels using the separable representation \[16, 17\] of the Paris potential. The complete Hilbert space has been split into two domains separated by the energy \( E_0 \). The model subspace \( S_0(E_0) \), in which the two-particle BG propagator is calculated explicitly, contains all the two-particle states with both the single-particle energies \( \varepsilon_{\lambda}, \varepsilon_{\lambda'} < E_0 \). In the complementary subspace, \( S'(E_0) \), the LPA for the BG propagator has been used. A qualitative analysis shows that the accuracy of the LPA becomes higher with increase of the energy \( E_0 \). It should be also higher for larger values of the perpendicular total momentum \( P_\perp \), therefore we limit ourselves to the most "dangerous" case of \( P_\perp=0 \).

For either channel under consideration, a set of calculations of the \( G \)-matrix has been made for different values of the cut-off energy \( E_0 \). The LPA has been assumed to be valid starting from the value of \( E_0 \) for which the \( G \)-matrix does not practically change any longer. An approximate independence of results on the value of \( E_0 \), at a level of a few percent, was found for \( E_0=10 \div 20 \) MeV for both the channels. It should be mentioned that in the singlet channel the accuracy of the LPA is sufficiently high even at \( E_0=0 \), in accordance with \[12\]. On the contrary, in the triplet channel the LPA is not practically applicable at \( E_0=0 \).

A similar analysis could be made also for the channels with \( L > 0 \). Estimates show that in this case conditions for validity of the LPA are even better than those for \( L=0 \). We believe that the LPA gives a good device for microscopic describing finite nuclear systems.

In this paper, we limit ourselves to one value of the chemical potential \( \mu = -8 \) MeV which is typical for stable nuclei. In principle, for smaller values of \( \mu \) the analysis should be repeated. However, as some estimates show, even in the drip-line vicinity where \( \mu \to 0 \) the LPA should be rather good for \( E_0=10 \div 20 \) MeV in either channel. At the same time, at \( E_0=0 \) it should become inapplicable even in the singlet channel.

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Figure captions

1. The profile function $\delta G_{11}(X, X' = 0)$ in the singlet channel for $E_0 = 20$ MeV.

2. The profile functions $\delta G_{ij}(X, X' = 0)$ in the singlet channel for $E_0 = 0$ (dotted lines), $E_0 = 10$ MeV (dashed lines), and $E_0 = 20$ MeV (solid lines).

3. The profile functions $\delta G_{ij}(X, X' = 8)$ in the singlet channel for $E_0 = 0$ (dotted lines), $E_0 = 10$ MeV (dashed lines), and $E_0 = 20$ MeV (solid lines).

4. The zero moments $\bar{G}_{ij}(X)$ in the singlet channel for $E_0 = 0$ (dotted lines), $E_0 = 10$ MeV (dashed lines), and $E_0 = 20$ MeV (solid lines).

5. The Fermi-averaged $G$-matrix in the singlet channel $< G_F >_{S=0}(X)$ for $E_0 = 0$ (dotted line), $E_0 = 10$ MeV (dashed line), and $E_0 = 20$ MeV (solid line) and the Fermi-averaged $T$-matrix (thin solid line).

6. The same as in Fig. 2 for the triplet channel.

7. The same as in Fig. 3 for the triplet channel.

8. The same as in Fig. 4 for the triplet channel.

9. The same as in Fig. 5 for the triplet channel.
$G_{11}(X, X'=0), \text{ GeV fm}^3$
\( \delta G_{ij}(X, X'=0), \text{GeV fm}^3 \)

\( i j = 11 \)

\( i j = 12 \)

\( i j = 23 \)
\[ \delta G_{ij}(X,X'=8), \text{GeV fm}^3 \]

\( i j = 11 \)

\( i j = 12 \)

\( i j = 23 \)

\( X, \text{fm} \)
$\langle G_F \rangle_{S=1}(X)$, GeV fm$^3$

$LL' = 00$

$LL' = 02$

$LL' = 22$

$X$, fm