Anomalous Tunneling of Spin Wave in Heisenberg Ferromagnet

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Abstract. The ferromagnetic spin wave (FSW) in classical Heisenberg chain exhibits the perfect transmission in the long-wavelength limit in the transmission-reflection problem with an inhomogeneity of exchange integral. In the presence of local magnetic field, on the other hand, FSW undergoes the perfect reflection in the long-wavelength limit. This difference in the long-wavelength limit is attributed to the symmetry property of the scatterers; it is crucial whether the potential preserves or breaks the spin rotation symmetry. Our result implies that the anomalous tunneling (i.e., perfect transmission in the low-energy limit) found both in scalar and spinor BECs is not specific to gapless modes in superfluids but is a common property shared with generic Nambu-Goldstone modes in the presence of a symmetry-preserving potential scatterer.

1. Introduction

For the last decade, the understanding of tunneling properties of collective excitations in Bose-Einstein condensates (BECs) has been greatly advanced [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. It has been found that the Bogoliubov excitation in a scalar BEC (BEC without internal degrees of freedom) tunnels through a potential barrier without reflection in the low-energy limit [1, 2, 3, 4, 5]. This perfect transmission in the BEC was referred to as “anomalous tunneling” in a literature [2]. In a spinor BEC (BEC with spin degrees of freedom), the Bogoliubov excitation and spin wave also show this tunneling phenomenon. Collective excitations that exhibit anomalous tunneling, such as Bogoliubov and spin wave excitations, found to be Nambu-Goldstone (NG) modes, while the non-NG mode (quadratic spin fluctuation mode) does not exhibit the anomalous tunneling [8].

Recently we found that the symmetry property of a potential barrier is important for the anomalous tunneling to occur [9]; when a NG mode results from the spontaneous breaking of a continuous symmetry $G$, the NG mode exhibits the anomalous tunneling through the barrier potential that preserves $G$. In the presence of the potential barrier that does not preserve $G$, on the other hand, the NG mode undergoes perfect reflection in the low-energy limit [9]. The symmetry $G$ should read as U(1) gauge symmetry for the Bogoliubov excitations while $G = SO(3)$ for spin wave. These findings suggest that the anomalous tunneling is a phenomenon related to the symmetry breaking and that it is not specific to BECs or superfluids but generic in symmetry-broken phases.
In this proceedings, we present an example illustrating that a NG mode even in non-superfluid or non-BEC exhibits perfect transmission through a symmetry-preserving potential barrier and exhibits perfect reflection against a symmetry-breaking potential barrier. We take the ferromagnetic classical Heisenberg model in one-dimension as a simplest example.

2. Transmission-reflection property in the presence of inhomogeneity of exchange interaction

We start with the Heisenberg equation of the local spin $S_j(t) = \exp(i\mathcal{H}t/\hbar)S_j\exp(-i\mathcal{H}t/\hbar)$ of $j$-th site

$$i\hbar \frac{dS_j(t)}{dt} = [S_j(t), \mathcal{H}],$$

with the Hamiltonian $\mathcal{H} = -\sum_j J_j S_j \cdot S_{j+1}$. Equation (1) can be reduced to

$$\frac{dS_j(t)}{dt} = -(J_j S_{j+1}(t) + J_{j-1} S_{j-1}(t)) \times S_j(t).$$

Throughout this proceedings, we take all exchange $J_j$ to be positive (ferromagnetic). As an approximation, we treat $S_j(t)$ as classical spin in the following part. The classical ground state is given by a fully polarized state. We take $S_j = S e_z$ for all $j$ as the ground state with the unit vector $e_z$ along the $z$ axis. Slowly-varying and small deviation from the polarized spin configuration (i.e. spin wave) can be considered by putting

$$S_j(t) = \sqrt{S^2 - (\delta S_j(t))^2} e_z + \delta S_j(t) \sim S e_z + \delta S_j(t),$$

with $\delta S_j(t) \cdot e_z = 0$. Linearizing (2) with respect to $\delta S_j(t)$, we obtain

$$\frac{d\delta S_j(t)}{dt} = -S \{ J_j \{ \delta S_{j+1}(t) - \delta S_j(t) \} + J_{j-1} \{ \delta S_{j-1}(t) - \delta S_j(t) \} \} \times e_z.$$  

Substituting the form $\delta S_j(t) = S_j(\omega) \exp(-i\omega t)$ of the normal-mode into (3), we obtain time-independent equations

$$\omega S_j^+(\omega) = -S \left[ J_j \left\{ S_{j+1}^+(\omega) - S_j^+(\omega) \right\} + J_{j-1} \left\{ S_{j-1}^+(\omega) - S_j^+(\omega) \right\} \right],$$

$$\omega S_j^-(\omega) = +S \left[ J_j \left\{ S_{j+1}^-(\omega) - S_j^-(\omega) \right\} + J_{j-1} \left\{ S_{j-1}^-(\omega) - S_j^-(\omega) \right\} \right],$$

with $S_j^\pm(\omega) = S_j^\mp(\omega) \pm iS_j^q(\omega)$. We focus on the case where $\omega$ is a positive real number without loss of generality; the case with negative $\omega$ can be obtained by replacing $S_j^+(\omega)$ by $S_j^-(\omega)$ for positive $\omega$.

Before considering the transmission-reflection problem, we discuss the normal-mode in the spatially uniform case where $J_j = J(>0)$ for all $j$. In this case, equation (5) has the solutions

$$S_j^-(\omega) = \exp(\pm \kappa j), \quad \omega = 4JS \sinh^2(\kappa/2), \quad \kappa > 0.$$  

Equation (5) is a three-term recursive relation and thus the general solution can be given by a linear combination of the fundamental solutions (6). Similarly, (4) has the fundamental solutions

$$S_j^+(\omega) = \exp(\pm ikj), \quad \omega = 4JS \sin^2(k/2),$$

where $k \in (0, \pi)$ when $\omega \in (0, 4JS]$ and $k$ becomes a complex number when $\omega > 4JS$. Thus the plane wave solution exists only in $S_j^+(\omega)$ for $\omega \in (0, 4JS]$. We focus on this case in the following part.
Figure 1. Schematics of set-up of the transmission-reflection problem. (a) Scattering off inhomogeneity of exchange energy $J_j$ discussed in sec. 2. (b) Scattering off local magnetic field discussed in sec. 3

We consider the case where

$$J_j = \begin{cases} J_0 \equiv (1 + \delta)J, & j = 0 \\ J, & j \neq 0, \end{cases}$$

which is depicted in Fig. 1 (a). For the mode equation (4) with exchange interactions (8), we will seek for a solution satisfying “asymptotic form”

$$S_j^\pm (\omega) = \begin{cases} \exp(ikj) + r \exp(-ikj), & j \leq 0 \\ t \exp(ikj), & j \geq 1, \end{cases}$$

with $k = 2 \arcsin\left(\frac{\omega}{4JS}\right)^{\frac{1}{2}}$ for $\omega \in (0, 4JS]$. $r$ and $t$ are reflection and transmission coefficients, which are to be determined as functions of $k$ and $\delta$.

We note that the asymptotic form (9) automatically satisfies the mode equation (4) with $j \geq 2$ or $j \leq -1$. Our remaining task is thus to determine $r$ and $t$ from Eq. (4) for $j = 0, 1$:

$$\omega S_0^- (\omega) = -S \left[ J_0 \left\{ S_1^+ (\omega) - S_0^- (\omega) \right\} + J \left\{ S_1^-(\omega) - S_0^+ (\omega) \right\} \right]$$

$$\omega S_1^+ (\omega) = -S \left[ J \left\{ S_2^+(\omega) - S_1^- (\omega) \right\} + J_0 \left\{ S_0^- (\omega) - S_1^+ (\omega) \right\} \right].$$

The resultant $t$ and $r$ are given by

$$t = \frac{(1 + \delta) \cos(k/2)}{\cos(k/2) + \delta \sin(k/2)}, \quad r = \frac{i \delta \sin(k/2)}{\cos(k/2) + \delta \sin(k/2)}.$$

We can see that $|t|^2 + |r|^2 = 1$ for arbitrary $k$ and that $\lim_{k \to 0} t = 1$ and $\lim_{k \to 0} r = 0$. We conclude that the classical spin wave can transmit perfectly through the inhomogeneity of exchange interactions.

3. Transmission-reflection property in the presence of local magnetic field

We consider the case where $J_j = J > 0$ for all $j$ while a local magnetic field is exerted exclusively on $j = 0$ site (see Fig. 1 (b)). The Hamiltonian is given by

$$\mathcal{H} = -J \left[ \sum_j S_j \cdot S_{j+1} \right] + B \cdot S_0.$$
Equation of motion is given by
\[
\frac{dS_j(t)}{dt} = -J (S_{j+1}(t) + S_{j-1}(t)) \times S_j(t) + \delta_{j,0} B \times S_0(t). \tag{13}
\]
In the following, spin variables are treated as classical variables as in the previous section. We can take \( B \) to be \(-|B|\mathbf{e}_z\) without loss of generality. Then the classical ground state is the fully polarized state \( S_j = S_0 = \mathbf{e}_z \) as in the previous calculation. Substituting \( S_j(t) = S_0 = S_j(\omega)e^{-i\omega t} \) into the equation of motion (13) and discarding \( O(S_j(\omega)^2) \), we can obtain
\[
\omega S_j^+(\omega) = -JS \left\{ \{S_{j+1}^+(\omega) - S_j^+(\omega)\} + \{S_{j-1}^+(\omega) - S_j^+(\omega)\} \right\} + \delta_{j,0}|B|S_0^+(\omega). \tag{14}
\]
We note that
\[
S_j^+(\omega) = \left\{ \begin{array}{ll}
\exp(ik_j) + r \exp(-ik_j), & j \leq 0 \\
\exp(ik_j), & j \geq 0
\end{array} \right.
\]
satisfies Eq. (14) for \( j \neq 0 \). The coefficients \( r \) and \( t \) in Eq. (15) are determined by Eq. (14) for \( j = 0 \) as
\[
t = \frac{\sin k}{\sin k + ib/2}, \quad r = \frac{-ib/2}{\sin k + ib/2}, \quad b \equiv |B|/(JS). \tag{16}
\]
With keeping \( b > 0 \) fixed, we see that \( \lim_{k \to 0} t = 0 \) and \( \lim_{k \to 0} r = -1 \), i.e., magnon undergoes perfect reflection at small \( k \) limit.

4. Summaries and Discussions
We have shown that, in the long-wavelength limit, classical spin wave in one-dimensional Heisenberg Ferromagnet transmits perfectly without reflection through spatially inhomogeneity, while this spin wave reflects without transmission against the local magnetic field. This property is similar to that observed in Bogoliubov excitations and spin wave in BECs [9]. The present result is easily generalized to spin wave in three-dimensional ferromagnetic and antiferromagnetic Heisenberg models; we have confirmed that the anomalous tunneling occurs in those spin models and through these studies we infer that the anomalous tunneling is a generic property of NG modes in symmetry-breaking states. Transmission and reflection properties of spin wave are crucial factors in controllability of spin-transport in magnets, and thus the finding of the anomalous tunneling in spin wave will shed light on a new aspect of spintronics in insulating magnets.

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