Area spectra of near extremal black holes

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Abstract: Motivated by Maggiore’s new interpretation of quasinormal modes, starting from the first law of thermodynamics of black holes, we investigate area spectra of a near extremal Schwarzschild de sitter black hole and a higher dimensional near extremal Reissner-Nordstrom de sitter black hole. We show that the area spectra of all these black holes are equally spaced and irrelevant to the parameters of black holes.

1. Introduction

It is widely believed that black holes play a significant role in understanding the theory of quantum gravity. In 1970s, Bekenstein discussed the quantum effects of black holes [1]. Supposing that the horizon area of a nonextreme black hole behaves as a classical adiabatic invariant which corresponds to a quantum entity with discrete spectrum, Bekenstein proposed the horizon area should be quantized. The quantized area spectrum has the following form

$$A_n = \gamma n \hbar, \ (n = 1, 2, 3 \ldots),$$

where $\gamma$ is a dimensionless constant and $\hbar$ is related to Planck length as $l_p^2 = \frac{\hbar G}{c^3}$. In this paper, we set $c = G = 1$. There are some works to quantize the area of black holes and the main difference is the value of $\gamma$. It is known that when a classical black hole is perturbed by an exterior field, the solution of the perturbational wave equation is defined as quasinormal modes (QNMs) and its relaxation is governed by a set of the modes with complex frequencies. This behavior has the same characteristic as that of a damped harmonic oscillator [2]. QNMs of black holes plays an important role not only in gravitational wave astrophysics [3] but also in the context of the AdS/CFT conjecture [4, 5]. There is a great deal of researching on QNMs. Hod found

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that the area spectrum can be obtained from the QNMs of the black hole \[6\]. In his work, by applying Bohr’s correspondence principle on the ringing frequencies which characterize a black hole, the missing “link” was found and the area spacing (that is \(4l_p^2 \ln j\), here \(j = 3\)) of a Schwarzschild black hole was derived when the real part of the QNMs introduced. This value of area spacing is consistent with both the area-entropy thermodynamic relation and statistical physics arguments. Later Dreyer derived the area spectrum from the quasinormal modes frequency from the perspective of loop quantum gravity \[7\]. The value of \(\gamma\) is in coincidence with that derived by Hod. Based on work of Hod and Dreyer, Kunstatter succeeded in deriving the Bekenstein-Hawking entropy spectrum for higher dimensional spherically symmetrical black holes, with semi-classical arguments\[8\]. In his work, the quantity

\[
I = \int \frac{dE}{\omega(E)}
\]

is treated as an adiabatic invariant, where \(E\) and \(\omega(E)\) denote the energy of a system and vibrational frequency, respectively. Applying Bohr-Sommerfeld quantization, one can get an equally spaced spectrum in the semi-classical (large \(n\)) limit, namely \(I = n\hbar\). Subsequently more progress are achieved.

Taking Hod’s conjecture into account, Maggiore very recently put forward that in the semiclassical limit, the area spectrum of a black hole ought to be determined by the asymptotic value of a physical frequency of the QNMs, defined as \[2\]:

\[
\omega_n = \sqrt{\omega_R^2 + \omega_I^2},
\]

This offers the area spectrum of black holes a new explanation. Applied this interpretation to the Schwarzschild spacetime, he concludes that the area spectrum of the horizon is equally spaced and is quantized in units \(\Delta A = \gamma l_p^2\) with \(\gamma = 8\pi\), different from that obtained by Hod, Dreyer and Kunstatter. Based on this new interpretation, there are much work appeared \[9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21\]. For example, in the research on the area spectrum of slowly rotating Kerr black holes, Vagenas found the area spectrum is not equally spaced \[9\]. Lopez-Ortega has studied the area spectrum of the higher dimensional de Sitter spacetime and spherically symmetrical black hole in small charge limit \[17\]. Area spectra of (2+1)-dimensional black holes and the large AdS black holes have been investigated by Fernando and Wei \[13, 14\]. All of these results are based on
the viewpoint that the horizon area of nonextremal black holes behaves as a classical adiabatic invariant. There are also some achievements on the area spectra of extreme black holes \[22\].

The purpose of this paper is to combine Maggiore’s new interpretation and the first law of thermodynamics to investigate the area spectra of near extremal black holes in de Sitter space. We discuss a near extremal Schwarzschild de Sitter black hole and a higher dimensional near extremal Reissner-Nordstrom de Sitter black hole. The results show that the area spectra are equally spaced and irrelevant to the parameters of the black holes.

The paper is organized as follows. We investigate the area spectrum of a Schwarzschild de Sitter black hole near extreme case by combining the new interpretation of Maggiore and thermodynamical properties in sect. 2. The area spectrum of a higher dimensional Reissner-Nordstrom black hole near extreme case is investigated in Sect. 3. Sect. 4 contains some discussion and conclusion.

2. Area spectrum of a near extremal Schwarzschild de Sitter black hole

In this section, following Maggiore’s work, we discuss the area spectrum of a Schwarzschild de Sitter black hole near extreme case from thermodynamical properties of black holes. The metric of the Schwarzschild-de Sitter black hole is given by

$$ds^2 = -f(r) \,dt^2 + f^{-1}(r) \,dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta \,d\phi^2 \right),$$

with

$$f(r) = 1 - \frac{2M}{r} - \frac{r^2}{a^2},$$

where $M$ represents the physical mass of the black hole and the parameter $a$ is related to the cosmological constant $\Lambda$ by $a^2 = \frac{3}{\Lambda}$. For $f(r) = 0$, there are three roots ($r_c, r_h, r_0$), corresponding to the cosmological horizon ($r = r_c$), the event horizon ($r = r_h$) and a negative one without physical interpretation ($r = r_0$), respectively. The Hawking temperature is $T = \frac{\kappa_h}{2\pi}$, where $\kappa_h$ represents the surface gravity at the event horizon

$$\kappa_h = \frac{(r_c - r_h) \,(r_h - r_0)}{2a^2 r_h}.$$  

Near the extreme case, the cosmological horizon is very close to the event horizon and the surface gravity of the event horizon can be approximated as \[23\]
\( \kappa_h \approx \frac{r_c - r_h}{2a^2 r_h} \). \hspace{1cm} (6)

As explained in the first section, once the QNMs are given, one can employ the first law of thermodynamics to calculate the area spectrum. The QNMs of the near extremal Schwarzschild de Sitter black hole was derived in [23],

\[
\omega = \kappa_h \left[ \sqrt{V_0 - \frac{1}{4}} - i \left( n + \frac{1}{2} \right) \right], \hspace{1cm} (7)
\]

with \( V_0 = l(l+1) \) for scalar as well as \( U(1) \) vector perturbations and \( V_0 = (l+2)(l-1) \) for gravitational perturbations. \( l \) stands for the angular quantum number and \( n = 0, 1, 2, 3 \cdots \). The gravitational perturbations with \( l = 2, 3 \) agree with the results of Moss and Norman [24]. In Hod’s work, the real part of the mode was adopted to derive area spectra of black holes. In this paper, following Maggiore’s arguments, we are concerned with the imaginary part in the large \( n \) limit. Maggiore argued that the physical frequency is related to the real part and imaginary part. As \( n \to \infty \), the imaginary part plays a dominant role in eq. (7), then the physical frequency can be expressed as

\[
\omega_n = |\omega_I| = n \kappa_h. \hspace{1cm} (8)
\]

When a black hole is regarded as a damped harmonic oscillator, the change of energy is related to the change of the physical frequency of the harmonic oscillator, namely \( \Delta E = \hbar \Delta \omega_n \). For the Schwarzschild de Sitter black hole, the energy is its ADM mass. Therefore we can get

\[
\Delta M = \Delta E = \hbar \Delta \omega_n. \hspace{1cm} (9)
\]

In literature, the area spectra of nonextreme black holes were studied by assuming that the horizon area behaves as an classical adiabatic invariant. The action is written as \( I = \int \frac{dE}{\omega(E)} \) for a system with energy \( E \) and vibrational frequency \( \omega(E) \) in Kunstatter’s work [8]. For the black hole near extreme case we are considering here, we introduce the first law of thermodynamics of the black hole to relate the variances of energy and entropy. The first law is

\[
dM = T \cdot dS, \hspace{1cm} (10)
\]
with finite form

$$\Delta S = \frac{\Delta M}{r}. \quad (11)$$

Combining with Eqs. (8), (9) and (11), we can get the change of the entropy spectrum,

$$\Delta S = \frac{h\Delta \omega_n}{r}$$

$$= 2\pi h,$$

where the relation $\Delta \omega_n = \omega_n - \omega_{n-1} = n\kappa_h - (n-1)\kappa_h = \kappa_h$ and the Hawking temperature at the event horizon $T = \frac{\kappa_h}{2\pi}$ were used in the last equality. It is easy to see that the entropy spectrum has the following form $S = 2\pi hn + C$, where $C$ is a constant, irrelevant to our problem. According the Bekenstein-Hawking area-entropy relation, one can easily obtain $A = 8\pi hn + 4C$. Thus

$$\Delta A = 8\pi h,$$  \quad (13)

which shows the area spectrum of the near extremal Schwarzschild de sitter black hole is equally spaced. This is in consistence with recent results and shows Maggiore’s idea can be extended to the near extremal de sitter black holes.

3. Area spectrum of a higher dimensional near extremal Reissner-Nordstrom de Sitter black hole

In this section, we calculate the area spectrum of the charged de sitter black hole for near extreme case. The metric of the Reissner-Nordstrom de sitter black hole is given by the line element

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2d\Omega_n^2, \quad (14)$$

with

$$f(r) = 1 - \lambda r^2 - \frac{2M}{r^{n-1}} + \frac{q^2}{r^{2n-2}},$$

where $d\Omega_n^2$ is the line element of a $n$ sphere. The parameters $M$ and $q$ are the physical mass and electric charge of the black hole respectively \[25\]. $\lambda$ is related to the cosmological constant. There are three positive roots for $f(r) = 0$, representing the Cauchy horizon $r_a$, the event horizon $r_h$ and the
cosmological horizon $r_c$, respectively. Near extreme case, the cosmological horizon is very close to the event horizon. Then the surface gravity associated the event horizon is

$$\kappa_h \approx \frac{(r_c - r_h)(n - 1)}{2r_h^2} \left(1 - nq^2\right). \tag{15}$$

The QNMs obtained by Cardoso [26] is

$$\omega = \kappa_h \left[\sqrt{\frac{V_0}{\kappa_h^2} - \frac{1}{4} - i \left(n + \frac{1}{2}\right)}\right], \tag{16}$$

where $V_0 = V_{S\pm}(r_h) \cosh(\kappa_hr_e)^2$ and $n = 0, 1, 2, 3 \cdots$. Parallel to the strategy in section two, we know the area spectrum is related to the physical frequency. From the first law of thermodynamics of the black hole, we can get

$$dM - \Phi \cdot dQ - \Omega \cdot dJ = T \cdot dS. \tag{17}$$

Since the black hole does not rotate, one has $\Omega \cdot dJ = 0$. The change of energy of the black hole is given by $dE = dM - \Phi \cdot dQ$. Therefore, at the event horizon, one obtains

$$T \cdot \Delta S = \Delta M - \Phi \cdot \Delta Q = \Delta E = \hbar \Delta \omega_n, \tag{18}$$

where the physical frequency $\omega_n = \sqrt{\omega_R^2 + \omega_I^2}$ is obtained from eq.(16). When $n \gg 1$, the frequency is mainly relied on the imaginary part of the QNMs, namely $\omega_n = |\omega_I|_n$, and there is $\Delta \omega_n = |\omega_I|_n - |\omega_I|_{n-1} = \kappa_h$. From eq. (18), the change of the entropy spectrum can be obtained as

$$\Delta S = \frac{\hbar \Delta \omega_n}{T} = 2\pi \hbar. \tag{19}$$

By the same procedure as in section two, it is straightforward to find

$$\Delta A = 8\pi \hbar, \tag{20}$$

which shows the area spectrum of the higher dimensional Reissner-Nordstrom de sitter black hole near extreme case is equally spaced. This is in accordance with the results of charged and higher black holes [13, 17].

4. Discussion and Conclusion

In this paper, from thermodynamical properties of the black holes, by adopting Maggiore’s new interpretation of QNMs, we investigated entropy
spectra and area spectra of the near extremal Schwarzschild de sitter black hole and the higher dimensional near extremal Reissner-Nordstrom de sitter black hole. The results show that the area spectra are all equally spaced. In literature\[9\], there is a logarithmic term appeared in the area spectrum of the slowly rotating Kerr black hole, where Vagenas explained it as the logarithmic correction. When the logarithmic term becomes dominant in some certain condition, it introduces difficulty to the interpretation of entropy spectrum. In fact, the logarithmic term can be avoided appearing in the area spectrum, as addressed in the work of Medved and Myung respectively \[10, 18\]. When applied to the slowly rotating Kerr black hole, our argument is parallel to that suggested by Medved \[10\]. Therefore, the area spectrum of the slowly rotating Kerr black hole is also equally spaced in the limit $J/M \rightarrow 0$ where $J$ stands for the angular momentum.

In conclusion, we investigated the area spectrum of near extremal black holes and found that the area spectra of these black holes are all equally spaced and irrelevant to the parameters of these black holes.

Note added: When we are in the final stage of writing the manuscript, paper \[27\] appears in the preprint archive. The authors discuss the area spectrum of a near extreme Schwarzschild de sitter black hole by introducing the adiabatic invariant, which is different from our method.

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