supergravity models of quintessence and supersymmetry breaking

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the issue of supersymmetry breaking in the context of supergravity models of quintessence is discussed.

1. introduction

quintessence cosmology \[1\] is probably one of the best candidates to fit the most recent data, those that point in the direction of an accelerating universe during the present epoch \[3\]. essentially it consists of a scalar field rolling down a runaway potential and providing the vacuum energy required to accelerate the universe today. most quintessence models make use of the idea of 'tracker' fields, i.e. scalar fields rolling down a potential which admits attractor solutions \[3–5\] thus relieving the problem of fine-tuning in the initial conditions in this cosmology. a successful example is given by inverse power law potentials: \[v \sim q^{-\alpha}\], where \(q\) is the quintessence scalar and \(\alpha\) is a positive number \[4\].

luckily enough, scalar potentials of that kind are found in supersymmetric gauge theories, which exhibit a superpotential of the form \[w = \lambda^{3+\alpha}q^{-\alpha}\] and a flat kähler potential, \[k = qq^*\]. in general, however, quintessence models predict that the vev of the scalar \(q\) at the present epoch is of order of the planck mass. if this is the case, we should take into account supergravity (sugra) corrections to these models.

in this talk i will discuss the issues connected with the construction of supergravity models of quintessence and also address the problem of supersymmetry breaking in this context. in particular, i will report work done in collaboration with e. j. copeland and n. j. Nunes, which is published in ref. \[7\].

2. supergravity models

the f-term of the scalar potential in a general sugra theory with \(n q_i\) fields is given by the following expression:

\[v(q_i) = |f|^2 - e^{k^2k^3}3k^2|w|^2\]

\[= e^{k^2k^3}[(w_i + k^2wk_i)k^j(i(w_j + k^2wk_j)^* - 3k^2|w|^2)]\]

\[(1)\]

where \(f_i = \partial w/\partial q_i\), the subscript \(i\) indicates the derivative with respect to the \(i\)-th field, and \(k^2 = 8\pi g = 8\pi m_{pl}^2\).

brax and Martin \[8\] discuss the case of a theory with superpotential \(w = \lambda^{3+\alpha}q^{-\alpha}\) and a flat kähler potential, \(k = qq^*\). it is straightforward to compute the resulting scalar potential:

\[v(q) = e^{k^2q^2} 3^{\alpha+\beta} \]

\[\frac{q^4}{q^\beta} \times \left( \left( \frac{\beta - 2}{4} \right)^2 - \left( \beta + 1 \right) \frac{k^2}{2} q^2 + \frac{k^4}{4} q^4 \right)\]

\[(2)\]

where \(\beta = 2\alpha + 2\). The main effect of the supergravity corrections is that the scalar potential can now become negative due to the presence of the second term. this is a serious drawback for the model, which becomes ill defined for \(q \sim m_{pl}\). they go on to propose a possible solution by imposing the condition that the expectation value of the superpotential vanishes, \(\langle w \rangle = 0\). we then see from equation \[8\] that the negative contribution to the scalar potential disappears, and it takes the form

\[v(q) = \lambda^{4+\alpha} q^{\alpha} e^{k^2q^2} \]

\[(3)\]
The condition $\langle W \rangle = 0$ is possible to realize, for example, in a model in which we allow matter fields to be present in addition to the quintessence scalar field $\tilde{Q}$. Then, if at least one of the gradients of the superpotential with respect to the matter fields is non–zero, the scalar potential will always be positive.

This is not the only possibility, though. There are two obvious problems with the potential (2): one, as already stated, is the negative term in the general expression (1) and the second is the choice of the Kähler metric which makes this term grow with the field’s vev, relative to the other terms in the potential. As mentioned above, setting $\langle W \rangle = 0$ is a tight restriction, so we will address the issue by relaxing this constraint but allow for more general forms of the Kähler metric.

Such an approach was recently adopted in [13] as a method of obtaining a minimum for the dilaton field in string theory. It had the advantage of relying on only one gaugino condensate and provided an alternative approach to the phenomenology associated with ‘racetrack’ models [14]. In this scenario, the Kähler potential acquires string inspired non-perturbative corrections. A further nice feature of these models is that it is possible to have a minimum with zero or small positive cosmological constant [10], and moreover it is possible to establish that the dilaton can be stabilized in such a minimum in a cosmological setting [13].

In general, for different choices of the Kähler metric, the negative term in (1) does not always lead to the disaster of a negative minimum in the scalar potential. For a general Kähler, we do not know a priori the shape of the potential or the location of the minimum. In fact, in what follows we will show through explicit examples that the scalar potential might always remain positive through a suitable choice of the Kähler metric. Moreover, with this approach there is no need to introduce additional fields in the model.

Let us now go on to study SUGRA corrections to inverse power law quintessence models by choosing more general Kähler potentials. Consider, for example, a theory with superpotential $W = \Lambda^{3+\alpha} \tilde{Q}^{-\alpha}$ and a Kähler $K = -\ln(\kappa \tilde{Q} + \kappa \tilde{Q}^*)/\kappa^2$, the type of term which is present at tree level for both the dilaton and moduli fields in string theory. In this case, the resulting scalar potential, expressed in terms of the canonically normalized field $Q = (\ln \kappa \tilde{Q})/\sqrt{2} \kappa$, is

$$V(Q) = M^4 e^{-\sqrt{2} \beta \kappa Q}$$

where $M^4 = \Lambda^{5+\beta} \kappa^{1+\beta} (\beta^2 - 3)/2$ with $\beta = 2\alpha + 1 > \sqrt{3}$ to allow for positivity of the potential. This corresponds to the ‘scaling’ solution discussed in the introduction and so cannot lead to a negative equation of state for the field in a matter dominated regime.

Another example follows as a natural extension of the one just described and leads to potentials with more than one exponential. For a superpotential of the form $W = \Lambda^{3+\alpha} \tilde{Q}^{-\alpha} + \Lambda^{3+\beta} \tilde{Q}^{-\beta}$ and the same Kähler metric as above, then in terms of the same canonically normalized field $Q = (\ln \kappa \tilde{Q})/\sqrt{2} \kappa$, the scalar potential becomes

$$V(Q) = (M_1)^4 e^{-\sqrt{2} \gamma \kappa Q} + (M_2)^4 e^{-\sqrt{2} \beta \kappa Q} + (M_3)^4 e^{-\sqrt{2} (\gamma + \delta) \kappa Q},$$

where $\gamma = 2\alpha + 1$, $\delta = 2\beta + 1$ and

$$(M_1)^4 = \Lambda^{5+\gamma} \kappa^{1+\gamma} (\gamma^2 - 3)/2, \quad (M_2)^4 = \Lambda^{5+\delta} \kappa^{1+\delta} (\delta^2 - 3)/2, \quad (M_3)^4 = \Lambda^{5+\gamma+\delta} \kappa^{1+\gamma+\delta} (\gamma \delta - 3).$$

At first sight this appears to be of the form required in [14] in that it involves multiple exponential terms. However, closer analysis indicates that the slopes of the exponentials are not adequate to satisfy the bounds arising from nucleosynthesis constraints, whilst also providing a positive cosmological constant type contribution today.

As we mentioned earlier, it is possible to have more general Kähler potentials, and with that in mind we now consider the original model $W = \Lambda^{3+\alpha} \tilde{Q}^{-\alpha}$, but with a Kähler potential which depends on a parameter $\gamma$

$$K = \frac{1}{\kappa^2} \left[ \ln(\kappa \tilde{Q} + \kappa \tilde{Q}^*) \right]^\gamma, \quad \gamma > 1.$$  

In this case, the second derivative of the Kähler is

$$K_{Q \tilde{Q}^r} = \frac{\gamma (\gamma - 1)}{\kappa^2 (Q + Q^*)^2} \left[ \ln(\kappa \tilde{Q} + \kappa \tilde{Q}^*) \right]^\gamma.$$
and the canonically normalized field $Q$ can be obtained as a function of $\tilde{Q}$ by integrating the following expression

$$dQ = \sqrt{2K_{\tilde{Q}Q}} \, d\tilde{Q}.$$  

In order to avoid the singularity at $\tilde{Q} + \tilde{Q}^* = 1/\kappa^2$, when $ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*)$ passes through zero (see equation (5)), the only possible choice is $\gamma = 2$. We then obtain:

$$K_{\tilde{Q}Q^*} = \frac{2[1 - \ln(\kappa\tilde{Q} + \kappa\tilde{Q}^*)]}{\kappa^2(\tilde{Q} + \tilde{Q}^*)^2}$$  

and as a consequence

$$Q = -\frac{2}{3\kappa} [1 - \ln(2\kappa\tilde{Q})]^{3/2}.$$  

Implying that the theory is well defined for

$$-\infty < \ln(2\kappa\tilde{Q}) < 1$$

which corresponds to $0 < \tilde{Q} < e/2\kappa$.

The scalar potential in terms of the canonically normalized field $Q$ reads

$$V = M^4 \left[ 2x^2 + (4\alpha - 7)x + 2(\alpha - 1)^2 \right] \frac{1}{x} \times \exp[(1 - x)^2 - 2\alpha(1 - x)]$$

where for notational convenience we have defined the quantities

$$x = \left( -\frac{3}{2} \kappa Q \right)^{2/3} = 1 - \ln(2\kappa\tilde{Q}),$$

$$M^4 = \Lambda^{6+2\alpha} \kappa^{2+2\alpha} 2^{2\alpha}.$$  

Note that the canonically normalized field $Q$ has a range $-\infty < Q < 0$.

We can see from equations (12) and (13) that the scalar potential behaves like an exponential for $|Q| \gg 1$ and like an inverse power law for $|Q| \ll 1$, and thus develops a minimum at a finite value $Q_m$. Note that the potential is always positive for any $\alpha > 1.25$. Thus, we have found that in this case the supergravity corrections induce a finite minimum in the potential but do not spoil its positivity. Note also that the field’s value in the minimum is exactly in the region where we expect the supergravity corrections to become relevant. For example, with $\alpha = 5$ we obtain $Q_m \simeq -0.02$ (in $8\pi G = 1$ units), which corresponds to $\tilde{Q} \simeq 1.2$. Imposing that the minimum of the potential equals the critical energy density today, we can also estimate the mass scale $\Lambda$, depending on $\alpha$. In the case $\alpha = 5$ we have that $V(Q = Q_m) \simeq 10^{-47}GeV^4$ which corresponds to $\Lambda \simeq 6 \times 10^{10}$ GeV.

3. SUPERSYMMETRY BREAKING

If supersymmetry is to be realized in nature, it must be broken at a mass scale $M_S$ such that $M_S^2 \sim \langle F \rangle \gtrsim (10^{10}GeV)^2$ or $M_S^2 \sim \langle F \rangle \gtrsim (10^4GeV)^2$ (for gravity and gauge mediated cases respectively), in order to lift the supersymmetric scalar particle masses above $10^6GeV$.

This then requires the superpotential in (1) to be $W \sim \langle F \rangle \kappa^{-1} \sim m_{3/2} \kappa^{-2}$ in order to cancel the F-term contribution and consequently to give a negligible total vacuum energy density ($m_{3/2}$ is the gravitino mass).

From the discussion in the last section, it is clear that the dynamical cosmological constant provided by the quintessence potential cannot be the dominant source of SUSY breaking in the Universe as $W \sim \Lambda^{3+\alpha} \kappa^{-\alpha} \sim (10^{-3}eV)^2 \kappa^{-1} \ll \langle F \rangle \kappa^{-1}$. Therefore, we need some additional source of SUSY breaking. If we consider now the superpotential,

$$W = \Lambda^{3+\alpha} Q^{-\alpha} + m_{3/2} \kappa^{-2},$$

then one gains a correction to the scalar potential in (12) of,

$$\delta V \sim \Lambda^{3+\alpha} m_{3/2} \kappa^{-\alpha} + m_{3/2}^2 \kappa^{-2},$$

for $Q \sim M_{Pl}$ today.

The first term can in principle be controlled for sufficiently large $\alpha$, however, the constant second term unavoidably leads to a disruption of the quintessence potential. This is a very serious problem of all supergravity models in quintessence. (In order to avoid the SUGRA corrections problem, Choi proposes a Goldstone-type quintessence model inspired in heterotic M-Theory.)
The situation gets worse, since, as pointed out in [16], it appears that even if we imagine that the amount of SUSY breaking that we observe in the universe today comes from a hidden sector other than the quintessence one, there will still be gravitational couplings between the two sectors that rekindle the original problem.

However, some recent proposals point in a slightly different direction for solving this problem. The basic idea is that the traditional approach to SUSY breaking might not be the best way to explain the world we live in. For example, the mass difference between the superpartners could arise in a 4D world with unbroken SUSY through some higher dimensional effects [17]. If this is the case, we would not need to break supersymmetry, and the quintessence potential would be preserved. Another possibility [18] is that the relation between the SUSY breaking scale $M_S$ and the cosmological constant $\langle F \rangle^2$ is not what is usually considered.

If $M_S = \kappa^{-1} \langle F \rangle^{2 \kappa^4 \beta}$ and $\beta = 1/8$, instead of the usual 1/4, then the observed cosmological constant would provide just the right amount of SUSY breaking. In this case we wouldn’t have any dangerous F-term of order $\sim \kappa^{-2} M_S^2$ which spoils the quintessence potential.

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