Mathematical contests as seen by participants

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Abstract

Motivation to take part in mathematical competitions and sources of information concerning the competitions is analysed on a basis of a questionnaire presented to a group of students.

Key words: Identification, motivation and support of mathematical talents, mathematical competitions, mathematical education

Introduction

One of possible ways (and certainly not the only one) of identifying talented students are mathematical competitions. What do young people think of mathematical contests? Why do they enter mathematical challenges? Who primarily encourages, motivates and supports young people? Are female participants motivated by different factors than their male colleagues? We try to answer the questions. We conclude that old fashioned methods of informing on contests as printed materials, posters or newspaper announcements and even the omnipresent Internet are less effective as a source of information than teachers at school who encourage, motivate and support young talented people.

Materials and Methods

A group of about thirty students of mathematics at the State Higher Vocational School in Tarnów was asked in January 2016 if they participated in mathematical competitions in their middle school (junior high school at age 13 to 16 years) or high school (at age from 16 to 19 years) and what motivated them to start in contests. Their answers served to construct a questionnaire presented in autumn 2016 to a group of 65 students: 21 students (15 female, 6 male) at the State Higher Vocational School in Tarnów, all studying mathematics, and 44 students at the Jagiellonian University in Kraków, 28 studying mathematics (15 female, 13 male) and 16 studying biophysics (8 female, 8 male). We may consider the group of the respondents uniformly distributed between 2012 and 2016 as the graduation year is considered.

Table 1. Graduation year of respondents

| graduation year | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | total |
|-----------------|------|------|------|------|------|------|------|-------|
| number of       | 2    | 2    | 12   | 14   | 11   | 13   | 11   | 65    |
| respondents     |      |      |      |      |      |      |      |       |
Results and Discussion

The large advantage of the selected group of respondents was the completion of their high school education only a few years or even just months prior answering the questionnaire. The fresh perspective seems adequate for answering questions about participation in mathematical competitions (Table 2).

It is evident and not surprising that the most popular are the international competition Mathematical Kangaroo, which has been taking place in Poland since 1992, and other mathematical competitions organized by teachers in middle and high schools. Much smaller number of participants are attracted by the Polish Junior Mathematical Olympiad (for middle school students aged 13-16

Table 2. The number of respondents (in the group of 65 students, including 38 women and 27 men) who have participated in various mathematical competitions in middle and high school

| Type of mathematical competitions                                      | middle school (13-16 years) | high school (16-19 years) |
|------------------------------------------------------------------------|-----------------------------|----------------------------|
| International Mathematical Kangaroo                                    | 42                          | 11                         |
| Polish Junior Mathematical Olympiad (13-16 years) or Polish Mathematical Olympiad (16-19 years) | 8                           | 9                          |
| A competition organised by a local Board of Education                   | 24                          | 6                          |
| Competitions organised by respondent’s school                          | 33                          | 14                         |
| Other competitions                                                     | 8                           | 6                          |
| ‘I haven’t participated in mathematical competitions’                  | 11                          | 17                         |

Table 3. Number of mathematical competitions entered by respondents

| Number of entered competitions   | middle school (13-16 years) | high school (16-19 years) |
|---------------------------------|-----------------------------|----------------------------|
|                                 | all | female | male | all | female | male |
| none                            | 11  | 6      | 5    | 26  | 17      | 9    |
| one                             | 17  | 12     | 5    | 21  | 11      | 10   |
| two                             | 19  | 13     | 6    | 8   | 6       | 2    |
| three                           | 14  | 4      | 10   | 6   | 3       | 3    |
| four                            | 2   | 1      | 1    | 3   | 1       | 2    |
| five or more                    | 2   | 2      | 0    | 1   | 0       | 1    |
| total number of the respondents | 65  | 38     | 27   | 65  | 38      | 27   |
| mean                            | 1.77| 1.68   | 1.89 | 1.11| 0.95    | 1.33 |
| median                          | 2   | 2      | 2    | 1   | 1       | 1    |
| standard deviation              | 1.00| 0.96   | 1.03 | 0.96| 0.85    | 1.14 |
years) and the Polish Mathematical Olympiad (for high school students). Entering the Mathematical Olympiad requires not only the mathematical abilities of the participants but also a special training under teacher’s direction for a long time preceding the competition.

Several respondents in the group entered more than one mathematical competitions (Table 3).

The next table (Table 4) shows the above data divided into three groups of respondents labelled as ‘SHVS maths’ (21 students of mathematics at the State Higher Vocational School, Tarnów), ‘JU maths’ (28 students of mathematics at the Jagiellonian University, Kraków), ‘JU bio’ (16 students of biophysics at the Jagiellonian University, Kraków) and ‘all resp.’ (all 65 respondents):

It is easily seen that the median of the number of different mathematical competitions entered by a respondent is higher in middle school (mean=1.77, median=2) than a few years later in high school (mean=1.11, median=1), which is understandable. The higher the level of the competition is, the more work is requested from the participants. Since high school students didn’t have time to compete in several contests simultaneously they had to reduce the number of entered competitions.

One may use a standard Pearson’s chi-squared test (see eg. [2, 3]) and check the hypothesis that the number of entered competition has Poisson distribution with the parameter $\lambda$ equal to the sample mean versus the hypothesis that the distribution is different (Table 5).

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**Table 4.**

| number of respondents who entered | middle school (13-16 years) | high school (16-19 years) |
|----------------------------------|-----------------------------|---------------------------|
|                                  | SHVS maths | JU maths | JU bio | all resp. | SHVS maths | JU maths | JU bio | all resp. |
| none competition                 | 2           | 4        | 5      | 11        | 7           | 8        | 11      | 26        |
| one competition                  | 8           | 5        | 4      | 17        | 10          | 10       | 1       | 21        |
| two competitions                 | 8           | 8        | 3      | 19        | 3           | 3        | 2       | 8         |
| three competitions               | 3           | 7        | 4      | 14        | 0           | 4        | 2       | 6         |
| four competitions                | 0           | 2        | 0      | 2         | 1           | 2        | 0       | 3         |
| five or more                     | 0           | 2        | 0      | 2         | 0           | 1        | 0       | 1         |
| total number of respondents      | 21          | 28       | 16     | 65        | 21          | 28       | 16      | 65        |
| mean                             | 1.57        | 2.14     | 1.38   | 1.77      | 0.95        | 1.46     | 0.69    | 1.11      |
| median                           | 2           | 2        | 1      | 2         | 1           | 1        | 0       | 1         |
| standard deviation               | 0.73        | 1.10     | 1.05   | 1.00      | 0.63        | 1.17     | 0.95    | 0.96      |

**Table 5.**

|                                  | middle school (13-16 years) | high school (16-19 years) |
|----------------------------------|-----------------------------|---------------------------|
|                                  | all | female | male | all | female | Male |
| $\lambda$                        | 1.77 | 1.68   | 1.89 | 1.11 | 0.95   | 1.33 |
| p-value                          | 75.5% | 46.7%  | 27.5% | 7.8% | 43.6%  | 11.1% |
Although the sample median in the group of all respondents is the same as the median in the group of male or female respondents, the distributions of the number of competitions entered by one respondent in the two disjoint groups is different. Let \( F_{\text{male}} \) and \( F_{\text{female}} \) denote the cumulative distribution function of the sample of male and female respondents, respectively. The Wald-Wolfowitz runs test [2] of the hypothesis

\[
H_0: F_{\text{male}} \text{ and } F_{\text{female}} \text{ are identical vs. } H_1: F_{\text{male}} \text{ and } F_{\text{female}} \text{ are different}
\]

shows that hypothesis \( H_0 \) fails in both cases. The number of runs is only 11 and 10, which give insignificant p-values smaller than \( 10^{-6} \). If we restrict ourselves to the three smaller subgroups of respondents ‘SHVS maths’, ‘JU maths’ and ‘JU bio’ and fix the significance level of the Wald-Wolfowitz runs test equal 0.05, the hypothesis \( H_0 \) is to be rejected (or not) as it is shown below (Table 6).

The second part of the questionnaire contained the assessment of motivation and the availability of sources of information on contests, expressed by the respondents in the five-point scale: 5: ‘definitely yes’, 4: ‘rather yes’, 3: ‘hard to say’, 2: ‘rather no’, 1: ‘strongly no’. The following table contains the medians of the answers in the group of all respondents (65 students), female and male respondents (38 and 27 students, respectively), SHVS students of mathematics (21 persons), JU students of mathematics (28 persons) and JU students of biophysics (16 persons), arranged in descending order with respect to the sum of medians in each row (Table 7).

It is easy to see that there is virtually no difference in the ordering of the motivation to take the competition between the disjoint groups of female and male respondents or between the disjoint groups of JU students of mathematics, JU students of biophysics or SHVS of mathematics, which are almost the same as in the group of all respondents.

**Table 6.**

| group of respondents | female | male | critical region | number of runs in Wald-Wolfowitz test | \( H_0 \) rejected |
|----------------------|--------|------|-----------------|--------------------------------------|-------------------|
| all resp.            | 38     | 27   | [2, 25]         | middle school: 11, high school: 10  | yes               |
|                      |        |      |                 | middle school: yes, high school: yes |                  |
| SHVS maths           | 15     | 6    | [2, 6]          | middle school: 8, high school: 6    | No                |
|                      |        |      |                 | middle school: yes, high school: yes |                  |
| JU maths             | 15     | 13   | [2, 10]         | middle school: 11, high school: 20  | No                |
|                      |        |      |                 | middle school: no, high school: no   |                  |
| JU bio               | 8      | 8    | [2, 5]          | middle school: 6, high school: 8    | No                |
|                      |        |      |                 | middle school: no, high school: no   |                  |

**Conclusions**

The passion for the subject of the competition is the most important motivation to take it. A necessity of proving of own abilities is almost as important. A striking negligible influence of parents, siblings, classmates on the decision to enter a competition gives a lot to think about. Is the school the only and primary support for gifted students? The main source of information about competitions seems to be a direct conversation with the teacher. Press information, posters and even messages posted on websites of competitions seem to be completely unnoticed by middle and high school students.
Table 7.

| Reason for participation | Middle School (13-16 years) | High School (16-19 years) |
|--------------------------|-----------------------------|---------------------------|
|                          | all resp. | female | male | SHVS | JU maths | all resp. | female | male | SHVS | JU maths | JU bio |
| I liked the subject of the competition | 5 | 5 | 5 | 5 | 5 | 4 | 5 | 5 | 5 | 5 | 5 | 5 |
| I just wanted to check my abilities | 4 | 4 | 4 | 5 | 4 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| I decided myself to participate in the competition | 4 | 4 | 4 | 5 | 4 | 5 | 5 | 5 | 5 | 5 | 5 | 4.5 | 5 |
| My teacher encouraged me to enter the competition | 4 | 4 | 4 | 5 | 4 | 4 | 3 | 3 | 4 | 4 | 4 | 4 |
| My parents encouraged me | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2.5 | 3 | 2 | 3 | 3 |
| The promise of a better assessment in my school | 3 | 3 | 3 | 3 | 3 | 4 | 2 | 2 | 2 | 2.5 | 2 | 3 |
| Additional points in the recruitment to high school or university | 3 | 3 | 1 | 2 | 3 | 3 | 2 | 2 | 2 | 2 | 2 | 4 |
| An attractive prize foreseen for the winner or finalist | 2 | 3 | 2 | 3 | 2 | 2 | 2 | 2.5 | 2 | 2 | 2 | 2 |
| The promise of exemption from a final examination | | | | | | | | 2 | 2 | 2 | 2 | 1 | 3 |
| An information founded on a poster in school | 1.5 | 2 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 2 | 1 | 2 |
| A classmate encouraged me | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 1 | 1 |
| An information on a website | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| My older brother or sister encouraged me | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| An information in newspapers | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
Additional comments left by some of the respondents shed some light on the motivation to take part in competitions and on sources of information about them. One respondent has written “In my middle school I had many classmates good at mathematics and we were racing in the number of competitions in which we started. If we had a teacher in high school who could help me and my colleagues in preparation for the Mathematical Olympiad, the preparation would be both fun and more effective”. A few more respondents also complained about the lack of sufficient support from teachers in high school.

The talented individuals who love problem solving and finding non-standard new solutions are a treasure which must not be squandered. The modern society focusing on harmonious development is trying to identify young potentially talented people and to deliver them the most adequate education. Properly trained teachers taking care about gifted students in middle and high school are currently the basic support for the talented individuals.

In the present note we do not develop the problem of the proper preparation of teachers who support talented students. It is widely discussed in several publications, see eg. [1] and references given there.

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