THE ROLE OF OPTIMISM AND PESSIMISM IN THE DYNAMICS OF EMOTIONAL STATES

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Abstract. In this paper we make an attempt to study the influence of optimism and pessimism into our social life. We base on the model considered earlier by Rinaldi and Gragnani (1998) and Rinaldi et al. (2010) in the context of romantic relationships. Liebovitch et al. (2008) used the same model to describe competition between communicating people or groups of people. Considered system of non-linear differential equations assumes that the emotional state of an actor at any time is affected by the state of each actor alone, rate of return to that state, second actor’s emotional state and mutual sympathy. Using this model we describe the change of emotions of both actors as a result of a single meeting. We try to explain who wants to meet whom and why. Interpreting the results, we focus on the analysis of the impact of a person’s attitude to life (optimism or pessimism) on establishing emotional relations. It occurs that our conclusions are not always obvious from the psychological point of view. Moreover, using this model, we are able to explain such strange behavior as so-called Stockholm syndrom.

1. Introduction. People are social beings by nature. Almost all our life we talk, work or play with someone; contacts with other persons have a strong influence on our emotions. This impact can be positive or negative depending on various factors and it is most visible in the relationship during the meeting of two persons. People can like each other, and then a good mood of one person positively affects the mood of the other one, while a bad mood has a negative effect. Two persons may also not like each other, and then an emotional impact of one person to the other is opposite. Hence, a sadness of one person improves the mood of the other one and vice versa. The last possibility is that the relationship is mixed, and one person has a negative attitude to the second one, who is geared to their friendship. In such

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a relationship the emotional impact of one person to the other is a compound of previously described influences.

If we assume that our emotional state is influenced only by other people, then the obvious strategy is to be friendly for all the close-knit people and to avoid relationships with people who have negative attitude to us. In general, we hope to be like that, however if that were true, everyone would be friendly and happy, and it is not the case. Although one can try to explain this effect by saying that the feeling of pleasure is not the only desire of a man and various random events can destroy the balance, it would still not represent the full picture.

On the other hand, individual factors also have an impact on our emotional state. These are certainly: the attitude to life (optimism and pessimism) and how strong is the influence of the current mood of a person on the change of her/his emotional state.

In this paper, we make an attempt to analyse the influence of the level of optimism/pessimism on the profits from the meeting of people. We shall consider three main components which affect the changes of our emotions during the meeting with other person: the attitude to life, emotional inertia and the reaction to emotions of the person we communicate during the meeting. This suggests the usage of the model having similar components and well known in the description of dyadic interactions considered earlier by Rinaldi and Gragnani [10], Liebovitch et al. [6] and Rinaldi et al. [9]. Considered model is described by a system of non-linear differential equations, while similar model of marital interactions reflected by discrete system was proposed by Gottman et al. [4, 7].

To avoid misunderstanding, persons considered in this paper will be called actors since we do not wish to suggest that people communicating during the meeting must be involved in a romantic relationship, as it is interpreted by Rinaldi and his coauthors. Moreover, comparing to Liebovitch et al. or Rinaldi et al. we study considered model from a different point of view as we do not describe a very long time behavior but focus on a single meeting between two actors.

2. Model description. Many models describing interactions between people follow the idea of modeling romantic relationships proposed originally by Strogatz [15]. Such models can even be linear, as considered in [15, 16, 8, 3, 1, 2]. However, non-linear models seem to be more appropriate. In this paper we would like to use the model having the same structure as the one proposed by Rinaldi and Gragnani [10], which was deeply exploited by Rinaldi and his coauthors in the series of papers [10, 9, 11, 13, 12] in the context of the description of various types of romantic relationships. The text-book of Rinaldi et al. [14] gives an excellent review of the model history, mathematical analysis, and various types of couples known from the literature and movies described by this model, as well as some extensions of it. Basing on the same idea, Gottman et al. [4, 7] proposed discrete time model which was then used to predicting divorce of married couples taking part in the experimental study in Gottman’s clinic. Another interpretation of this model was given by Liebovitch et al. [6], where the authors tried to reflect emotional states of communicating people. In this context, the actors could be interpreted not only as two interacting persons, but it could be groups of persons as well.

We would like to stress that in general love relationships develop in a few weeks or months, however in this paper we wish to focus on single meetings. Therefore, our interpretation of the model and its parameters is different from those proposed
by Rinaldi et al. [14] and Gottman et al. [4, 7], as well as Liebovitch et al. [6], however is more close to the interpretation given in [6].

We consider continuous-time model reflecting the dynamics of emotional states of two actors at time \( t \). Let \( x(t) \) and \( y(t) \) reflect these emotional states. The system of differential equations we consider reads

\[
\begin{align*}
\dot{x}(t) &= -m_1 x(t) + b_1 + c_1 f_1(y(t)), \\
\dot{y}(t) &= -m_2 y(t) + b_2 + c_2 f_2(x(t)),
\end{align*}
\]

where constants \( m_1 \) and \( m_2 \) describe the rate of change of the mood of each actor in solitude, which can be also referred as to forgetting coefficients, \( b_1 \) and \( b_2 \) reflect some “ideal/reference” mood of each actor, functions \( f_1 \) and \( f_2 \) describe the impact of the emotional state of an actor \( y \) or \( x \), respectively, on the emotional state of the other actor, while constants \( c_1 \) and \( c_2 \) determine the strength and direction of these influences.

Because \( m_i \), \( i = 1, 2 \), reflects forgetting coefficient, it must be positive as describing some time-scale on which an actor is considered in solitude. Hence,

\[
m_1 > 0, \quad m_2 > 0.
\]

Therefore, if there is no influence, that is for \( c_1 = c_2 = 0 \), then after any deviation System (1) returns to the steady state \( \left( \frac{b_1}{m_1}, \frac{b_2}{m_2} \right) \), where in [4] the state \( \frac{b_i}{m_i} \) is called uninfluenced equilibrium for the \( i \)th actor.

An actor characterized by a positive parameter \( b_i \) has a positive steady state and is called an optimist, while that with negative parameter – a pessimist. Rinaldi et al. (cf. [9, 14]) gave completely different interpretation of this parameter. In this interpretation \( b_1 \) reflects appeal of the actor \( y \) for \( x \), so when the actor is in solitude, this parameter is just equal to 0, as well as uninfluenced steady state, because there is no love/hate whenever there is no object of these emotions. It should be marked that although Rinaldi et al. got interesting results using this interpretation (e.g. they were able to explain the case of Beauty and the Beast [11]), we shall not follow this idea, but use the interpretation of Gottman et al. [4] and then Liebovitch et al. [6].

When both \( c_1 \) and \( c_2 \) are positive, both actors have a positive attitude to each other, while for \( c_1 = c_2 = 0 \) they have a negative attitude to each other. Clearly, for \( c_1 > 0 \) and \( c_2 < 0 \) the first actor has a positive attitude towards the second one, who has a negative attitude to the first. As we describe interactions between two actors, we mainly assume \( c_1 \cdot c_2 \neq 0 \).

Various particular influence functions \( f_i \) were considered in the literature, for details see e.g. [4, 6, 7, 11]. However, in this paper we propose \( f_i \) in general form based on the prospect theory of decision making problems [5].

We should also mark that under our interpretation the model described by System (1) reflects emotional states of actors during a single meeting, but not, for example, a series of meetings. This is because between two meetings considered actors meet other people or spend time in solitude, which affect their mood at the beginning of the meeting, and therefore also the final result.

2.1. Influence functions. The prospect theory proposed by Kahneman and Tversky [5] relates to the wider issues of risk assessment and an attitude of men to the risk. Here, we briefly introduce the assumptions, which are useful from our point of view. There are three main principles of profit and loss assessment by people. First, generally we experience losses much stronger than profits of the same value. Second,
everyone defines their own criteria with respect to which results of the decision are evaluated as a gain or loss. Third, every unit of gain is enjoyed with diminishing efficiency, and each consecutive loss is less saddening.

Although the prospect theory describes the relationship between profits or losses and satisfaction, we believe that it can be used to describe the mutual influence of actors’ emotional states. When we are alone our emotional state depends only on our character. When we meet a friend who is happy, we gain his emotions, but not in the literal sense. We react to ones smile, lively tone of voice, etc. When we get a more positive stimulus we enlarge our profit more. However, according to the prospect theory, each additional unit gives less and less profit. Therefore, influence functions $f_i$ are certainly non-linear. The first derivative of it should be positive, decreasing for positive variables and increasing for negative ones. Moreover, it should tend to 0 in $\pm\infty$, because otherwise $f_i$ become almost linear asymptotically.

Another issue is that generally we feel losses much stronger than profits of the same value. Indeed, people recognize negative emotions faster, more accurately and more strongly than positive ones. It is an adaptive process, because when we talk about surviving, the ability to recognize and quickly respond to the feeling of fear or anger is more important than joy. Hence, the negative experience makes us more sad than the same weight positive experience makes us happy.

Last feature of that theory stating that everyone assesses gains and losses from ones own point of view, actually, is not so important from our model point of view. Benchmark is always a state in solitude, that means a situation in which the environmental impact is equal to zero. However, if we do not like someone, then his negative emotions are a profit for us, and the positive emotions are our loss, while if we like someone, it is vice versa.

Concluding, to address the issues described above we assume:

$$f_i \in \mathbb{C}^2, \quad f_i(0) = 0, \quad f_i'(\xi) > 0,$$

$$f_i(\xi) \leq -f_i(-\xi) \quad \text{for} \quad \xi > 0,$$

and

$$\lim_{|\xi| \to \infty} f_i'(|\xi|) = 0, \quad \xi f_i''(\xi) < 0 \quad \text{for} \quad \xi \neq 0, \quad \text{for} \quad i = 1, 2.$$  

Moreover, to more explicitly show that the force of impact of actors on themselves depends on the value of $|c_i|$ we also assume that

$$f_i'(0) = 1, \quad \text{for} \quad i = 1, 2.$$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{influence_function.png}
\caption{Example of an influence function. We see that for $|\xi_2| = \xi_1$ we have $f(\xi_1) \leq -f(\xi_2)$.}
\end{figure}
Exemplary graph of an influence function is shown in Fig. 1. Notice that $\xi = y$ for the function $f_1$, while $\xi = x$ for $f_2$.

It is worth to notice that functions $f_i$ may have different shape because people differ in ability to distinguish emotions, and consequently – reactions to them. This means that $f'_1(y)$ and $f'_2(x)$ can change with different rates. Clearly, proposed interaction functions are defined in the general form, and hence we actually consider the whole family of functions. On the other hand, the interaction functions considered previously in the literature belong to this family. However, to the best of our knowledge, for the first time a specific psychological theory has been used to explain the form of these functions.

3. Model analysis. Properties of solutions of System (1) are known, and well described in [14]. However, for the purpose of this paper, we need to know the dynamics in the phase space for various parameter values. Therefore, we present the analysis of the phase space below. Moreover, to the best of our knowledge, the dynamics in the phase space for various parameter values. Therefore, we present the results of global stability obtained by us on the basis of this method.

Clearly, without particular forms of the influence functions we are not able to determine steady states explicitly. Hence, assume $(x_s, y_s)$ to be a solution of the system

$$
\begin{align*}
    x &= \frac{b_1 + c_1 f_1(y)}{m_1}, \\
    y &= \frac{b_2 + c_2 f_2(x)}{m_2},
\end{align*}
$$

i.e. $(x_s, y_s)$ is a steady state. The first equation of (2) describes null-cline $I_1$ for the first variable $x$, while the second one – $I_2$ for $y$. Positions of $I_1$ and $I_2$ in the phase space $(x, y)$ depend on the model parameters and specific forms of $f_i$. In the analysis presented below we treat both null-clines as functions $g_i(x)$. Therefore, $g_2$ is defined for all $x \in \mathbb{R}$ with $g'_2(x) \to 0$ as $|x| \to \infty$, while $g_1$ takes all values from $\mathbb{R}$ and $g'_1(x) \to \infty$ as $x$ tends to the end of its domain (either $\mathbb{R}$ or some bounded interval).

Clearly, if $c_1 c_2 < 0$, then one of the null-clines is increasing, while the other is decreasing, and therefore they intersect at exactly one point. If $c_1 c_2 > 0$, then System (2) has always from one to three solutions depending on the parameter values. We briefly discuss the case $c_1, c_2 > 0$, as the case $c_1, c_2 < 0$ is symmetric. Consider first $b_1 = b_2 = 0$. Then null-clines always intersect at $x = 0$ and other intersection points appear when $g'_2(0) > g'_1(0)$, that is $c_2/m_2 > m_1/c_1$, while for $g'_2(0) \leq g'_1(0)$ there is only one intersection. For $g'_2(0) \leq g'_1(0)$ shifting null-clines does not change the situation, while for $g'_2(0) > g'_1(0)$, if we shift at least one of the null-clines sufficiently far from the origin, then additional intersection points disappear.

1. System (1) has one steady state if
   - either $c_1 c_2 \leq m_1 m_2$,
   - or $c_1 c_2 > m_1 m_2$ and there is no solution of System (2) satisfying
     $$
     f'_1(y_s) f'_2(x_s) \geq \frac{m_1 m_2}{c_1 c_2}.
     $$

2. System (1) has two steady states if $c_1 c_2 > m_1 m_2$, $b_1 \neq 0$ or $b_2 \neq 0$, and solutions of System (2) fulfill the conditions

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where $B$ is a constant, which should be chosen in appropriate way. Calculating the derivative along trajectories of System (1) we get

$$V(x, y) = (x - x_s) \left( -m_1 x + b_1 + c_1 f_1(y) \right) + (y - y_s) \left( -m_2 y + b_2 + c_2 f_2(x) \right).$$

Using the relations $b_1 = m_1 x_s - c_1 f_1(y_s)$, $b_2 = m_2 y_s - c_2 f_2(x_s)$, and the mean value theorem we obtain

$$\dot{V}(x, y) = -(m_1 (x - x_s)^2 + C m_2 (y - y_s)^2$$

$$- \left( c_1 f_1'(y_p) + C c_2 f_2'(x_p) \right) (x - x_s) (y - y_s),$$

where $x_p$ and $y_p$ are intermediate points, that is the points between $x, x_s$ and $y, y_s$, respectively, and the right-hand side could be treated as a quadratic form of $x - x_s$ and $y - y_s$. Hence, we need to study positivity of the matrix

$$M = \begin{bmatrix}
    m_1 & -\frac{1}{2} \left( c_1 f_1'(y_p) + C c_2 f_2'(x_p) \right) \\
    -\frac{1}{2} \left( c_1 f_1'(y_p) + C c_2 f_2'(x_p) \right) & C m_2
\end{bmatrix}.$$
Because $0 < f_1 f_2' \leq 1$ under our assumptions, it is enough to choose $C > 0$ such that
\[ C^2 c_2^2 - 2C(2m_1 m_2 - |c_1 c_2|) + c_1^2 < 0, \]
which under the assumption $m_1 m_2 > |c_1 c_2|$ has real positive solutions, and we can choose $C = 2m_1 m_2 - |c_1 c_2| > 0$, which gives minimum of the quadratic function above. Therefore, the function $V$ satisfies all assumptions guaranteeing the global stability of $(x_s, y_s)$.

In the case of actors having opposite attitude to each other the steady state is unique, and moreover it is globally stable independently of other model parameters.

**Theorem 3.2.** If $c_1 c_2 < 0$, then System (1) has exactly one steady state $(x_s, y_s)$ satisfying System (2), which is globally stable.

**Proof.** Uniqueness of $(x_s, y_s)$ is obvious. Proving global stability we again use the method of Lyapunov functions. We start with changing variables of System (1) such that the steady state $(x_s, y_s)$ is shifted to $(0, 0)$. We define $u = x - x_s$ and $v = y - y_s$ for which we obtain
\[
\begin{align*}
\dot{u} &= -m_1 u + c_1 (f_1(v + y_s) - f_1(y_s)), \\
\dot{v} &= -m_2 v + c_2 (f_2(u + x_s) - f_2(x_s)),
\end{align*}
\]
(3)
due to System (2). Let us consider
\[
L(u, v) = |c_2|\int_0^u (f_2(\xi + x_s) - f_2(x_s))d\xi + |c_1|\int_0^v (f_1(\xi + y_s) - f_1(y_s))d\xi, \quad u, v \in \mathbb{R}.
\]
It is obvious that $L(u, v) = 0$ iff $u = v = 0$. Because $f_2$ is increasing, for $u > 0$ we have $\xi \in [0, u]$ and the integrand in the first integral is positive, while for $u < 0$
\[
\int_0^u (f_2(\xi + x_s) - f_2(x_s))d\xi = -\int_0^u (f_2(\xi + x_s) - f_2(x_s))d\xi,
\]
and the integrand is negative as $\xi \in [u, 0]$, which again implies positivity of the first integral. The same considers the second integral. Moreover, $L(u, v) \to \infty$ for $|u| \to \infty$ or $|v| \to \infty$.

Calculating the derivative of $L$ along trajectories of System (3) we get
\[
\dot{L}(u, v) = |c_2|\frac{d}{du}\left(\int_0^u (f_2(\xi + x_s) - f_2(x_s))d\xi\right)\dot{u} + |c_1|\frac{d}{dv}\left(\int_0^v (f_1(\xi + y_s) - f_1(y_s))d\xi\right)\dot{v}.
\]
Next, using the relation $|c_1| c_2 = -|c_2| c_1$ and the mean value theorem for both functions $f_1$ we obtain
\[
\dot{L}(u, v) = -m_1 |c_2| u^2 f_2'(u_p) - m_2 |c_1| v^2 f_1'(v_p),
\]
where $v_p$ and $u_p$ are intermediate points. This yields $\dot{L}(u, v) \leq 0$ for any $u, v \in \mathbb{R}$, and $\dot{L}(u, v) = 0$ iff $u = v = 0$. Therefore, $L$ is a Lyapunov function for System (3) and $(0, 0)$ is globally stable. This proves the global stability of $(x_s, y_s)$ for System (1).
In general, we have also the following property of System (1), which is independent of the model parameters (cf. [14]).

**Theorem 3.3.** There are no periodic solutions of System (1).

**Proof.** In the proof we use the following lemma

**Lemma 3.4. Dulac–Bendixson Criterion** Consider a system of two autonomous ODEs

\[
\dot{x} = G_1(x, y), \quad \dot{y} = G_2(x, y),
\]

and assume that there exists a function \( B(x, y) \) of class \( C^1 \) with values in \( \mathbb{R} \) such that the expression

\[
\frac{\partial}{\partial x} (BG_1) + \frac{\partial}{\partial y} (BG_2)
\]

is not equivalent to \( 0 \) and does not change its sign in the simple connected region in \( \mathbb{R}^2 \), then the considered system of ODEs has no periodic orbits in this region.

Let us define \( B(x, y) \equiv 1 \). Then we have

\[
\frac{\partial}{\partial x} (BG_1) + \frac{\partial}{\partial y} (BG_2) = -(m_1 + m_2) < 0.
\]

Thus, System (1) has no periodic solutions in the whole \( \mathbb{R}^2 \).

Theorem 3.3 implies that there are not limit cycles of System (1). Moreover, whenever there is only one steady state and solutions remain in bounded regions in the phase space, then we can use another theorem (again stated as lemma below) to prove global stability.

**Lemma 3.5. Poincaré–Bendixson Theorem** Consider System (4) for differentiable functions \( G_i, i = 1, 2 \), defined in an open subset of \( \mathbb{R}^2 \). Then every non-empty compact \( \omega \)-limit set of an orbit, which contains only finitely many fixed points, is either a fixed point, a periodic orbit, or a connected set composed of a finite number of fixed points together with homoclinic and heteroclinic orbits connecting these points.

Clearly, if solutions remain in some bounded region and there is only one steady state, then any solution tends to this state or to a periodic orbit. However, if there is no periodic orbit, then the only possibility is convergence to the steady state (cf. also [14]). Notice, that this is exactly the case when influence functions are bounded and do not cross in the phase space more than once.

### 3.1. Phase space portraits

In this subsection we discuss all possible types of phase portraits of System (1) under the assumption that solutions remain in the bounded region. It should be noticed here, that we describe a single meeting between actors, such that they might be not able to achieve a steady state during the meeting. However, knowing their initial emotions we are able to predict the changes in these emotions looking at the specific phase portrait. Therefore, we are interested in studying the phase space portraits for System (1).

In Fig. 2 (a)–(b) we see two possible behaviours of solutions in the case when only one steady state exists. In most cases discussed in [14] solutions of System (1) are monotonic, which could be observed only for the steady state being a node; cf. Fig. 2 (b). However, oscillatory dynamics is also possible, as we see in Fig. 2 (a). In such a case the steady state is reached non-monotonically, meaning that the
emotions of actors change from ranges lower than the steady state to higher one and so on, or *vice versa*.

Phase portraits in Fig. 2 (c)–(d) reflect bistability (*cf.* [7]). Fig. 2 (d) illustrates typical situation when two stable steady states $S_1$ and $S_3$ are separated by a saddle $S_2$, and a stable manifold of the saddle forms separatrix dividing the whole space into basins of attraction of $S_1$ and $S_3$. A generic solution of the system starting from some initial state goes from it and tends to one of the two stable steady states $S_1$ or $S_3$. For two enemies one of the stable steady states has positive coordinate for one actor and negative for the other, and the second steady state – *vice versa*. For two friends, one stable steady state is positive for both of them, and the other one is negative (result not shown).

Fig. 2 (c) illustrates the bifurcation between one steady state and three states. This is a saddle-node bifurcation. Although this is a non-generic case, it could be also interpreted as bistability, because both states have their regions of attraction, similarly to the typical situation with three states. However, the saddle-node point, that is $S_1$ in Fig. 2 (c), is extremely sensitive to the changes of the model parameters.

Graphs in Fig. 2 were prepared using MATLAB with arctan chosen as influence functions. Clearly, $f(\xi) = \arctan(\xi)$ satisfies the assumptions we have posed, that
is sufficiently smooth, \( f(0) = 0 \), for \( \xi > 0 \) this function is concave while for \( \xi < 0 \) convex, and moreover such \( f \) is an odd function, so the condition \( f(\xi) \leq -f(-\xi) \) is satisfied. For all graphs except bifurcation one we fixed \( m_1 = m_2 = 1 \), while for bifurcation \( m_2 \) was changed and chosen to be equal to 2. Stable focus was obtained for \( b_1 = -2, b_2 = 3, c_1 = -5, c_2 = 4 \), unique stable node for \( b_1 = -5, b_2 = -4, c_1 = 5, c_2 = 3 \), saddle-node bifurcation for \( b_1 = -4, b_2 = -2, c_1 = -5, c_2 = -4 \), and three steady states for \( b_1 = -5, b_2 = -4.19, c_1 = -5, c_2 = -3 \).

4. Results and their psychological interpretation. Basing on the analysis of the considered model, we are able to give some conclusions about the influence of pessimism and optimism on our social interactions. Clearly, people want to meet if they have a chance to make a profit from the meeting. Indeed, if one of them is always feeling worse after the meeting than when being alone, he/she will avoid meetings. It happens that one of the friends always initiates the meeting and the other one sometimes agrees on it, but very unwillingly. This happens when only one of them has a profit from it. Even less chance of meeting have people who are getting worse humor after than before.

Below we present the more detailed results of the analysis of probability of maintaining friendship for various types of actors depending on if their nature is similar or not. As for the phase portraits, the results are illustrated by numerical simulations prepared in MATLAB with \( \arctan \) chosen as influence functions.

4.1. Actors with neutral uninfluenced emotional state. We start from the case of actors with neutral uninfluenced steady state which is described by the relation \( b_1 = b_2 = 0 \).

Generally, if such an actor is alone, his solitude leads to apathy, and when two such actors meet, then the meeting does not influence their emotional states much, especially when their ability to calm emotions is greater than the strength of influence of the other actor. During the meeting of such actors, despite they are friends or enemies, their emotional states go monotonically to 0. It can happen slower or faster than in solitude, but the final result is always the same. This is associated with the existence of unique stable node, as in Fig. 2 (b). However, for actors with different attitudes to each other, there could be an oscillatory behavior (cf. Fig. 2 (a)), which is associated with the inequality

\[
2\sqrt{|c_1 c_2|} > |m_1 - m_2|.
\]  

(5)

We see that when the actors are more similar to each other and their influence each other more, then the chance of oscillatory behavior is greater; cf. Fig. 3, where an example of such oscillations is presented. Notice, that the case \( m_1 = m_2 \) is specific, as oscillations appear independently of the strength of the influence. This means that in the case of meeting of two neutral actors with the same forgetting ability their emotions oscillate around 0, taking negative and positive values alternately.

Clearly, for two neutral actors having the same attitude to each other, bistability (cf. Fig. 2 (d)) is also possible. This means that there are situations when the emotional states of such actors can change much during the meeting. It can happen for two friends or enemies, and if they are sufficiently interested in their emotions \( (c_1 c_2 > m_1 m_2) \), then depending on the initial state they could achieve some level of happiness or dissatisfaction. Two positively oriented actors reciprocate their each other emotions, while two negatively oriented actors have opposite emotions, which is visible on the phase portrait presented in Fig. 2 (d). An example of the situation
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when the emotions change is presented in Fig. 4 where the meeting of two friends with one of them having a good mood and the other having bad one is reflected. Fig. 4 shows that at the beginning the emotional state of a sad actor getting better, while the state of the other one is getting worse, but eventually their overcome the bad mood and start to be pleased together.

It can also happen that two friends having initially opposite emotions approach the neutral state after some time. From Section 3.1 we know that their emotional states should tend to one of the stable steady states, for which both coefficients have the same sign. However, if the initial state lies within the stable manifold of the saddle, then the solution goes to this saddle. When the friends have the same characteristics ($m_1 = m_2, c_1 = c_2, f_1 = f_2$), then their emotional states in solitude go to neutral state and the impact of negative and positive emotions is the same. In such a case the stable manifold is described as the straight line $y = -x$. Therefore, if the actors have initially opposite emotional states, then they react in such a way that their influence of each other is weak. This is reflected in Fig. 5.
The situation when two neutrally oriented actors meet is easier to interpret as there are only a few possibilities of emotional states of actors in this case. However, most of people are rather optimistic or pessimistic. Moreover, their emotional states almost never are the same in solitude and during the meeting with others. Typically, the actors get some profit or loss as a result of interactions. This is reflected in stabilizing their emotional states on the level higher or lower comparing to those they can achieve in solitude.

4.2. Specific types of the model dynamics. Now, we would like to point out some specific types of the model dynamics that could be achieved by specific types of actors, also for neutral actors described in the previous subsection.

Whenever two people of the opposite attitude to each other meet, there is only one steady state. For $4c_1c_2f_2'(x_s)f_1'(y_s) < -(m_1 - m_2)^2$, we have a stable focus, so when actors are more similar in terms of the rate of returning to equilibrium in solitude, the chance that their moods fluctuate at the beginning of the meeting is greater. For $4c_1c_2f_2'(x_s)f_1'(y_s) > -(m_1 - m_2)^2$, we have a stable node, and their moods at the meeting consistently approach the equilibrium typical for this particular pair. An example of such types of a behavior is presented in Fig. 6.

If the actors have the same attitude to each other, whether it is positive or negative, we get from one up to three steady states. For actors having relatively weak influence on each other (i.e. $0 < c_1c_2 < m_1m_2$) or those with strong mutual influence with both being extreme optimist or pessimist, there is only one steady state which is a stable node.

Different type of the model dynamics appears when the shift of the influence function by $b_i/m_i$ is not large enough. When this shift is reduced, null-clines intersect at three points. A steady state located between two others is a saddle (see $S_2$ in Fig. 2 (d)). Fig. 7 illustrates this situation, and we see that the actors could achieve different steady state depending on their initial emotions. Moreover, in Fig. 7 we see that actors with stronger influence and weaker forgetting, stronger experience contacts with others comparing to actors weakly influenced. What is interesting, in the presence of other people, emotions of the considered actor could be enlarged, but on the other hand too high emotions could be repressed.

Figure 5. Emotional states of two strongly dependent on each other friends with the same parameters describing both of them but with the opposite initial states.
4.3. Relationships of a pessimist. In this subsection we shall discuss relationships of a pessimist \((b_2 < 0)\) in more details. We assume that both actors are in their own uninfluenced emotional equilibrium before the meeting and we look for the change in their steady states due to the meeting. This illustrates the trends in...
Figure 8. Scheme of six possible interactions of the pessimist \((y)\) with the other actor \((x)\) who could be a pessimist or optimist. (a): \(y\) is negatively \((c_1 < 0)\) and \(x\) is positively \((c_2 > 0)\) oriented; (b): \(y\) is positively \((c_1 > 0)\) and \(x\) is negatively \((c_2 < 0)\) oriented; (c): both \(x\) and \(y\) are positively oriented \((c_1, c_2 > 0)\) and \(m_1m_2 > c_1c_2\) indicating the existence of exactly one steady state; (d): both \(x\) and \(y\) are negatively oriented \((c_1, c_2 < 0)\) and \(m_1m_2 > c_1c_2\) indicating the existence of exactly one steady state; (e): both \(x\) and \(y\) are positively oriented \((c_1, c_2 > 0)\) and \(m_1m_2 < c_1c_2\) indicating that there might exist more than one steady state; (f): both \(x\) and \(y\) are negatively oriented \((c_1, c_2 > 0)\) and \(m_1m_2 < c_1c_2\) indicating that there might exist more than one steady state.

Changes of moods of both actors. For a chosen, fixed pessimist there are six possible configurations. He could like the other actor or not, as well as the other actor could like him or not. Moreover, for two friends or two enemies there could be only one steady state or the change from one steady state to bistability may occur. All possibilities are illustrated in Fig. 8, where various vertical curves reflect the fact that the other actor could be a pessimist or optimist. In each graph there is a null-cline for the pessimist (chosen as \(y\)), that is the curve described by the relation \(\frac{dy}{dx} = 0\).
or \( y = \frac{b_2}{m_2} + \frac{c_2}{m_2} f(x) \) equivalently. This curve is shifted down comparing to \( f(x) \), as \( b_2 < 0 \), and could be either increasing (for \( c_2 > 0 \)), when the pessimist is positively oriented towards the other actor, or decreasing (for \( c_2 < 0 \)) in the opposite situation. Other curves reflect null-clines \( \frac{dy}{dt} = 0 \). In each graph these null-clines differ only due to translation, which means that they reflect actors differing on the scale optimism-pessimism.

First two graphs (a) and (b) reflect the case when the actors have opposite attitude to each other, and then only one steady state exists. The point \( C \) reflects uninfluenced steady state for the pessimist \( y \) and is taken as a reference value. Graph (a) illustrates the situation, when the negatively oriented pessimist \( y \) (\( c_2 < 0 \)) meets with the positively oriented actor \( x \) (\( c_1 > 0 \)). If the actor \( x \) is sufficiently pessimistic, then the meeting is beneficial for both of them. The point \( A \) reflects this situation, as we have \( y_A > y_C \) and \( x_A > x_E \), where \( E \) is the state in solitude for \( x \). This means that two pessimists can benefit from the meeting even if one of the actors does not reciprocate the friendship of the other. However, if the actor \( x \) becomes less pessimistic, then the situation changes. The point \( B \) reflects the benefit only for the pessimist \( y \), while the other actor has the same steady state as in solitude. If we change \( b_1 \) such that the actor \( x \) changes from a slight pessimist (as for the point \( B \)) through neutral to a slight optimist, then the pessimist \( y \) still gains but the other actor loses, till the point \( C \) which reflects loss for the optimist \( x \), while the pessimist \( y \) remains at the same steady state. For more optimistic actor \( x \) both of them lose as a result of the meeting; cf. the point \( D \).

Graph (b) reflects the “symmetric” situation, when the considered pessimist \( y \) is positively oriented (\( c_2 > 0 \)), and he meets with negatively oriented actor \( x \) (\( c_1 < 0 \)). However, this symmetry could be related only to the graphs of null-clines and not to the interpretation. If \( x \) is highly optimistic, then the pessimist \( y \) gains and the optimistic actor \( x \) loses, as reflected by the point \( A \). At the point \( B \) only the pessimist \( y \) benefits, while the actor \( x \) remains at the same steady state as in solitude. On the other hand, if \( x \) changes his attitude to life from slightly optimistic to slightly pessimistic we obtain the range of parameters \( b_1 \) for which both of the actors benefit. At the point \( C \) the situation changes again and only the actor \( x \) benefits, while the steady state for \( y \) remains unchanged comparing to the uninfluenced case. If \( x \) is more and more pessimistic, he gains and the pessimist \( y \) loses, like at the point \( D \).

Next graphs reflect the situation when both actors have the same attitude to each other, that is both are positively or negatively oriented. Graphs (c) and (d) describes the case when the actors do not influence each other much, and there is only one steady state (\( m_1 m_2 > c_1 c_2 \)). Notice that now the uninfluenced equilibrium of the pessimist \( y \) is reflected by the point \( B \). In Graphs (e) and (f) the change from one steady state to bistability (\( m_1 m_2 < c_1 c_2 \)) is observed, as the actors have strong influence on each other, and therefore there is a region of parameters for which the outcome of the meeting depends crucially on the initial states of both actors.

In Graph (c) both actors are positively oriented (\( c_1, c_2 > 0 \)). If the actor \( x \) is sufficiently optimistic, then both actors gain as a result of the meeting. This proves that a friendship between pessimists and optimists is possible. Clearly, the point \( D \) reflects this situation. On the other hand, the point \( A \) describes the reverse case – the actor \( x \) is a pessimist now, and both of the actors lose. As before, the path from \( A \) to \( D \) goes through the points \( B \) and \( C \), where one of the actors has unchanged steady state. More precisely, at the point \( B \) the actor \( x \) who is slightly optimistic
still loses and the pessimist \( y \) does not change the state. If \( x \) is more optimistic, then he loses but the pessimist \( y \) starts to gain, till the point \( C \), where the actor \( x \) changes his result of the meeting from loss to benefit due to increase of his optimism level.

Graph (d) illustrates the “symmetric” case again. Now both actors are negatively oriented \((c_1, c_2 < 0)\). If the actor \( x \) is an optimist, then he benefits and the pessimists \( y \) loses; cf. the point \( A \). The situation changes at the point \( B \), where \( y \) starts to gain, and till the point \( C \) both the actors gain. This is associated with slightly pessimistic behaviour of the actor \( x \). If \( x \) is more pessimistic, then he loses and the chosen pessimist \( y \) still gains.

In Graph (e) both actors are positively oriented \((c_1, c_2 > 0)\). In this case, when the actor \( x \) is sufficiently optimistic, then both actors have a benefit (cf. the point \( D \)) and a friendship is possible, like for the case described in Graph (c). However, if \( x \) is less optimistic, then we have the region (with the boundaries at \( B_1, C_1, C_2, B_2 \)) where, depending on the initial state, they can benefit or lose. Eventually, when \( x \) is less optimistic or is just a pessimist, then both lose during the meeting, as the point \( A \) shows.

Graph (f) is “symmetric” again, and again we could compare extreme cases to those illustrated in Graph (d). If \( x \) is optimistic, then he gains and the pessimist \( y \) loses; cf. the point \( A \). When \( x \) is less optimistic or slightly pessimistic, then the outcome depends on the initial state (cf. the points \( B_1, C_1, C_2, B_2 \)), and eventually for more pessimistic actor \( x \) he always loses and the pessimist \( y \) gains; cf. the point \( D \). Notice, that the threshold values for the change of the actors behavior could be positive or negative, depending on the parameters describing both actors. This means that sometimes the change will be observed when the actor \( x \) is a pessimist, and sometimes when he is an optimist.

On the basis of the analysis presented above we shall discuss which types of relationships of negatively and positively oriented pessimist have a chance to develop.

4.3.1. Relationships of a negatively oriented pessimist. As it turns out, a pessimist negatively oriented towards the other actor may sustain a relationship with another pessimist who is positively oriented towards him (cf. Fig. 8 (a)). He can also be
in a relationship with a pessimist similar to him whose reciprocal attitude is also negative, but in this case both of them take a risk, as small change of the parameters could change the situation from benefit for both of the actors to loss of one of them (cf. Fig. 8 (d)). Even more risky is a relationship between pessimists having a strong influence on each other (cf. Fig. 8 (f)). As a result, sometimes the chosen pessimist will achieve positive and sometimes negative steady state. The switch in emotions in such a case is shown in Fig. 9, where taking the risk the actor \( y \) loses. Favorable situation for both negatively oriented pessimists is shown in Fig. 10, where the thin dotted straight line illustrates the steady state which is achieved by each of the actors when spending time alone. Curves representing the emotional states of these two actors are above the dotted line after some time, so they have benefits from the meeting.

4.3.2. Relationships of a positively oriented pessimist. When a pessimist has a positive attitude to the other actor, he has a chance to form a relationship with a slight optimist or pessimist being his enemy (cf. Fig. 8 (b)), while if this enemy becomes more optimistic, then he will avoid contacts although our pessimist would like to meet. He may also be in a relationship with a great optimist who is also positively oriented (cf. Fig. 8 (c) and (e)), however for a moderate optimist there is a risk of loss. The more extreme optimist the other actor would be, the better result could be achieved. This is the best company for a pessimist. The only problem may be that optimists have more profit from contacts with other optimists, so meeting with a pessimist may be disadvantageous, despite the seeming gain.

4.4. Relationships of an optimist. As above for a pessimist, we can also discuss the possible relationships of an optimist. However, all the relationships accept those between two optimists are discussed in the context of a pessimist. Therefore, in this subsection we discuss in more details only the possible outcomes for two optimists. Moreover, we assume that fixed actor \( y \) is a moderate optimist and the outcome of the meeting may depend on it. As in Fig. 8, each graph corresponds to various attitudes of the actors to each other, and we change the level of optimism for the other actor \( x \).
Figure 11. Relationships between two optimists: (g) \( y \) is positively while \( x \) is negatively oriented \((c_1 < 0, c_2 > 0)\); (h) \( y \) is negatively while \( x \) is positively oriented \((c_1 > 0, c_2 < 0)\); (i) both actors are negatively oriented \((c_1, c_2 < 0)\) and \( m_1 m_2 > c_1 c_2 \) indicating the existence of exactly one steady state; (j) both actors are negatively oriented \((c_1, c_2 < 0)\) and \( m_1 m_2 < c_1 c_2 \) indicating the existence of up to three steady states; (k) both actors are positively oriented \((c_1, c_2 > 0)\) and \( m_1 m_2 > c_1 c_2 \) indicating the existence of exactly one steady state; (l) both actors are positively oriented \((c_1, c_2 > 0)\) and \( m_1 m_2 < c_1 c_2 \) indicating the existence of up to three steady states.

Graph (g) reflects relationships between positively oriented actor \( y \) \((c_2 > 0)\) and negatively oriented actor \( x \) \((c_1 < 0)\). Clearly, there is only one steady state in this case. If \( x \) is highly optimistic, like for the point \( D \), then he loses \((x_B > x_D)\) and \( y \) gains \((y_D > y_C)\), which is the result of negative attitude of the actor \( x \) to \( y \). If the actor \( x \) is less optimistic, then \( y \) gains less and less, until the point \( C \), when there is no benefit for \( y \). If \( x \) is a slight optimist, then both the actors lose as a result of the meeting.

In Graph (h) the opposite case is illustrated. Negatively oriented actor \( y \) \((c_2 < 0)\) always loses when meets positively oriented actor \( x \) \((c_1 > 0)\). Clearly, the point \( A \)
reflecting uninfluenced equilibrium for $y$ is always above the steady state established as a result of the relationship. On the other hand, the actor $x$ can lose or gain, depending on his level of optimism. If he is highly optimistic, then he loses (cf. the point $D$). The point $C$ reflects the threshold level of optimism; if $x$ is less optimistic, then he gains.

In Graph (i) both the actors are negatively oriented ($c_1, c_2 < 0$), but they do not influence each other much ($m_1m_2 > c_1c_2$) and there is only one steady state in this case. If $x$ is highly optimistic, then he gains and the actor $y$ loses. The point $C$ reflects the case when $x$ does not change his steady state, while $y$ still loses. For $x$ being less optimistic, both actors lose until the point $B$ at which $y$ does not change his steady state. Eventually, if $x$ is close to be neutral, then he loses and $y$ gains as a result of the meeting.

For Graph (j) the situation is similar till there is only one steady state. However, in the region with borders $C_1, B_1, B_2, C_2$ there are three steady states and depending on the initial moods either $x$ gains and $y$ loses or vice versa.

Graph (k) shows that whenever two positively oriented optimists meet both always gain, which is reflected by the point $D$. The last graph (l) indicates that if these optimists strongly influence each other, then when $x$ is sufficiently optimistic, then there is only one steady state and both of them gain, while when $x$ is less optimistic, then two steady states appear (border of this region are $C_1$ and $C_2$) and either both actors have a benefit (this is always the case when both have relatively large positive initial emotions) or both lose (always when both have initially relatively large negative emotions).

4.4.1. Relationships of a negatively oriented optimist. Surprisingly, negatively oriented optimist can have a benefit from the meeting with a pessimist (cf. Fig. 8 (b), (d) and (f)), however if actors have the same attitude to each other, then the pessimist would lose or both would need to take a risk and still they do not gain at the same time, thus it would be hard to establish a friendship. On the other hand, a friendship of negatively oriented optimist with a pessimist who has an opposite attitude to him would be possible if the optimist is close to be neutral. Moreover, negatively oriented optimist always loses due to the meeting with negatively oriented highly optimistic actor or positively oriented optimist. In addition, a friendship between two negatively oriented optimists such that both of them are sufficiently interested in their emotions is not possible since the meeting is beneficiary at most to one of them and each time our actor needs to take a risk of loss.

4.4.2. Relationships of a positively oriented optimist. Positively oriented optimist can have a benefit from the meeting with a pessimist (cf. Fig. 8 (c) and (e)). However, only a relationship between a great optimist and positively oriented pessimist when the actors are not interested in their emotions much can be developed. If actors are sufficiently interested in their emotions, then they can lose or gain from the meeting, however they need to take a risk again. Positively oriented optimist would prefer to meet negatively oriented towards him great optimist or positively oriented optimist not strongly interested in their emotions. However, in the first case the relationship would be not continued, since the second actor would lose from the meetings. On the other hand, if positively oriented optimist meets an actor with the same attitude to him and the same attitude to life, and moreover
they are strongly interested in their emotions both of them will lose or gain from the meeting.

![Graph showing possible changes of emotional states](image)

**Figure 12.** Possible changes of the dynamics of emotional states initiated by one of the interacting actors.

4.5. **Change of dynamics initiated by one of the actors.** As we have mentioned above, two enemies can calm their emotions or one of the actors can enjoy the misfortune of the other. Clearly, this situation is comfortable only for the first actor. The second actor have two ways out. Firstly, he can finish the meeting, secondly, he can reverse the situation, however the second possibility could be achieved only under some circumstances. This actor should pretend to be a friend,
and if Inequality (5) is satisfied, then oscillations of emotions will lead to neutral state. However, if the “pro temporary friend” comes back to his real state, then the final emotional state is the reverse of the initial one. The proper moment to change the behavior from temporary friendship to hostility depends on the model parameters. The possible changes of the emotional states in time are presented in Fig. 12. The graphs show the changes of the emotional states for two enemies. Top graph shows the solution of System (1) till $t = 6$ (with parameters $m_1 = -1, m_2 = -2, b_1 = b_2 = 0, c_1 = -4, c_2 = -3$), and at $t = 6$ the first actor $x$ changes his attitude to another actor from negative to positive of the same strength. This leads to improving of the emotional state of the actor $x$ and decreasing of satisfaction of the actor $y$ to 0. The middle graph shows the unsuccessful attempt of the change of the emotional state of the actor $x$. This attempt is unsuccessful due to the too early change of the attitude (at time less then 7) back to negative by the actor $x$. The bottom graph shows the situation in which the actor $x$ changes his attitude to negative again at $t = 7$ (after the first change of attitude from negative to positive one at time 6). This leads to increase his positive emotions together with negative emotions of the second actor. It should be noticed that even if Inequality (5) is not satisfied, the unhappy actor should change his attitude to the other (being his enemy), as both actors calm their emotions in such a case, as presented in the bottom graph in Fig. 6.

The line of reasoning presented above is able to explain so-called Stockholm syndrome, when kidnapped person starts to feel positive emotions for the kidnapper. This is just a smart defence mechanism, which is able to calm emotions of both the kidnapped and kidnapper, and decrease the dangerous of kidnapper.

5. Conclusions. From the model analysis several conclusions appear. First, only actors with neutral uninfluenced steady state are able to feel similarly in solitude and being with the partner. Second, for two enemies, it is enough that one of the actors changes his/her behaviour to obtain complete change of the emotional states of both of them. Next, pessimists, which is not surprising, may have greater difficulties with finding a friend. Negatively oriented pessimist is unable to sustain a friendship with an optimist. He can be in mutual relationship only with positively oriented pessimist. However, he can also be in some kind of relationship in which the second partner do not change his equilibrium in solitude. The positively oriented pessimists can be in a relationship which is profitable for both actors if they are highly positively oriented optimists or even with close to being neutral negatively oriented optimists or pessimists. Positively oriented optimists can take mutual advantages from meeting with another optimists. However, for optimists with negative attitude to second actor it would be hard to sustain friendship with other actors except the case of being almost neutral and meeting positively oriented pessimist.

It appears that often in order to make a profit chosen actor must have better mood than a partner before the meeting to gain from it. An example of such situation is illustrated in Fig. 7. Graphs show the relation between emotional states of two actors negatively oriented towards each other. The differences in these graphs are only due to different initial states of actors. In the first graph the actors approach the steady state favorable to the actor $x$ being an optimist. This is despite the better initial mood of the actor $y$ being a pessimist. In the second case, pessimism of the actor $y$ causes that the actor “has won” his emotional state not too highly.
An interesting fact which has stemmed from our analysis is that the pessimistic enemies may be not friendly to each other and have it enhance their moods, while pessimistic friends will mutually worsen their moods. Fig. 13 perfectly illustrates an example in which for two pessimists it is better to have a different relationship to each other than friendship. The left graph shows that a meeting of two friends always causes they approach an unprofitable state due to the strong pessimistic tendency of them both. This state is more negative than they could achieve being alone. As shown in the right graph, it would be advantageous for both of them to change attitudes to the partner who is more pessimistic. Then they both could gain from the meeting.

At the end we should mark that people do not like to feel diametrically changing emotions, and therefore almost all people prefer to interact with actors having the same attitude. It particularly considers optimistic actors.

The model presented in this paper is very simple but clear, and much more conclusions could be drawn from it. As we have noticed in Introduction and Model Description sections, it gives a possibility of different interpretations, such that we expect it could be farther exploited in the future.

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