Signatures of complex new physics in $b \to c\tau\bar{\nu}$ transitions

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Recent measurements of $R_D$ and $R_{D^*}$ by Belle collaboration are in good agreement with the Standard Model (SM) predictions. After inclusion of these measurements, the tension between global averages and the SM predictions has reduced to 3.1$\sigma$. Assuming the new physics Wilson coefficients to be complex, we do a global fit to the present $b \to c\tau\bar{\nu}$ data. We find that there are only one/two/three allowed solutions depending upon three choices on upper limits 10%/30%/60% of $Br(B_c \to \tau\bar{\nu})$. We find that the forward-backward asymmetries in $B \to (D, D^*)\tau\bar{\nu}$ decays have the capability to distinguish between these solutions. Further we calculate the maximum values of CP violating triple product asymmetries in $B \to D^*\tau\bar{\nu}$ decay allowed the current data. We observe that one of the three asymmetries can be enhanced up to only $\sim 2 - 3\%$ due to presence of the allowed new physics solutions.

I. INTRODUCTION

The heavy meson decays, in particular the $B$ meson decays, are a very fertile ground to probe possible physics beyond the SM. In past few years, several measurements by BaBar, Belle and LHCb in the $B$ meson decays show significant deviations from their SM predictions. One class of such decays is mediated by charged current $b \to c\tau\bar{\nu}$ transition which occurs at tree level in the SM. In this sector, two such interesting observables are

$$R_D = \frac{B(B \to D \tau\bar{\nu})}{B(B \to D \{e/\mu\}\bar{\nu})}, \quad R_{D^*} = \frac{B(B \to D^* \tau\bar{\nu})}{B(B \to D^* \{e/\mu\}\bar{\nu})}. \quad (1)$$

These flavor ratios are consecutively measured by BaBar [1, 2], Belle [3-6] and LHCb [7-9] collaborations. The SM predicts $R_D$ to be 0.299 ± 0.003 whereas the present experimental world average is 0.340 ± 0.027 (stat.) ± 0.013 (syst.). For $R_{D^*}$, the SM prediction is 0.258 ± 0.005 and the experimental world average is 0.295 ± 0.011 (stat.) ± 0.008 (syst.). The SM predictions and the world averages are noted down from Heavy Flavor Averaging Group [10]. The present average values of $R_D$ and $R_{D^*}$ exceed the SM predictions by 1.4$\sigma$ and 2.5$\sigma$ respectively. Including the correlation of −0.38, the tension is at the level of 3.1$\sigma$. This discrepancy is an indication of lepton flavor universality (LFU) violation between $\tau$ and $\mu/e$ leptons.

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In addition, the LHCb collaboration measured another flavor ratio \( R_{J/\psi} = \frac{\Gamma(B_c \to J/\psi \tau \bar{\nu})}{\Gamma(B_c \to J/\psi \mu \bar{\nu})} \) and the measured value is \( 0.71 \pm 0.17 \text{ (stat.)} \pm 0.18 \text{ (syst.)} \) \[11\]. Even though the uncertainties are quite large, it is \( 1.7\sigma \) higher than its SM prediction \( 0.289 \pm 0.010 \) \[12\]. This is an additional hint of LFU violation in the \( b \to c l \bar{\nu} \) sector. These deviations could be due to presence of new physics (NP) either in \( b \to c \tau \bar{\nu} \) or in \( b \to c \{\mu, e\} \bar{\nu} \) transition. However, it has been shown in Ref. \[13\] that the latter possibility is ruled out by other measurements. Therefore, we assume the presence of NP only in \( b \to c \tau \bar{\nu} \) transition.

Apart from these, Belle collaboration has measured two angular observables in the \( B \to D^* \tau \bar{\nu} \) decay—(a) the \( \tau \) polarization \( P^D_\tau \) and (b) the \( D^* \) longitudinal polarization fraction \( F^D_L \). The measured values of these two quantities are \[5, 14\]

\[
P^D_\tau = -0.38 \pm 0.51 \text{ (stat.)}^{+0.21}_{-0.16} \text{ (syst.)},
\]

\[
F^D_L = 0.60 \pm 0.08 \text{ (stat.)} \pm 0.04 \text{ (syst.)}.
\]

The measured value of \( P^D_\tau \) is consistent with its SM prediction of \(-0.497 \pm 0.013 \) \[15\] whereas for \( F^D_L \) it is \( 1.6\sigma \) higher than its SM prediction of \( 0.46 \pm 0.04 \) \[16\].

Recently, the anomalies in \( b \to c \tau \bar{\nu} \) transition have been studied in various model independent techniques \[17–27\]. In the most of these analysis, the NP Wilson coefficients (WCs) are assumed to be real. These NP WCs are determined by doing a fit to the data available in this sector along with the constraint on the branching ratio of \( B_c \to \tau \bar{\nu} \) decay. In Ref. \[20\], it has been shown that the NP Lorentz structure in form of \((V - A) \times (V - A)\) is the only one operator solution allowed by the present data.

In this paper we do a global fit to of all present data on \( b \to c \tau \bar{\nu} \) transition by starting with a most general effective Hamiltonian. Assuming the NP WCs to be complex, we find the allowed NP solutions with their corresponding WCs. We show that one/two/three NP solution(s) is (are) allowed if we consider three different upper limits 10%/30%/60% on the branching ratio of \( B_c \to \tau \bar{\nu} \). We calculate the predictions of angular observables in \( B \to (D, D^*) \tau \bar{\nu} \) decays and comment on their ability to distinguish between the allowed solutions. Further, we compute the predictions of the CP violating triple product asymmetries in \( B \to D^* \tau \bar{\nu} \) decay for the three NP solutions. We show that one of these three asymmetries can be enhanced up to \( \sim 2 - 3\% \) in presence the allowed NP scenarios.

The paper is organized as follows. In Section II, we describe our methodology for calculation and present our fit results. In this section, we calculate the predictions of the angular observables of \( B \to (D, D^*) \tau \bar{\nu} \) decays and discuss their distinguishing capabilities. In section III, we determine
the maximum possible CP violating triple product asymmetries in $B \to D^*\tau\bar{\nu}$ decay allowed by the current data. We present our conclusions in section IV.

II. FIT METHODOLOGY AND RESULTS

We start with the most general effective Hamiltonian for $b \to c\tau\bar{\nu}$ transition which contains all possible Lorentz structures. This is expressed as [28]

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[ O_{V_L} + \frac{\sqrt{2}}{4G_F V_{cb}} \Lambda^2 \left\{ \sum_i \left( C_i O_i + C'_i O'_i + C''_i O''_i \right) \right\} \right],$$

where $G_F$ is the Fermi coupling constant and $V_{cb}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. Here we assume that the neutrino is left chiral. We also assume the new physics scale $\Lambda = 1 \text{ TeV}$. The five unprimed operators

$$O_{V_L} = (\bar{c}\gamma_\mu P_L b)(\bar{\tau}\gamma^\mu P_L \nu), \quad O_{V_R} = (\bar{c}\gamma_\mu P_R b)(\bar{\tau}\gamma^\mu P_L \nu),$$

$$O_{S_L} = (\bar{c} P_L b)(\bar{\tau} P_L \nu), \quad O_{S_R} = (\bar{c} P_R b)(\bar{\tau} P_L \nu), \quad O_T = (\bar{c} \sigma_{\mu\nu} P_L b)(\bar{\tau} \sigma^{\mu\nu} P_L \nu),$$

are mostly being used in various analysis, whereas $O'_i$ and $O''_i$ operators could only arise in different Leptoquark models [28] depending on their spin and charge. A more rigorous discussion on all possible Leptoquarks can be found in Ref. [29]. The Lorentz structures of all these operators are described in Ref. [28]. In particular, $O'_i$ and $O''_i$ operators can be expressed in terms of five unprimed operators using Fierz identity. The constants $C_i$, $C'_i$ and $C''_i$ are the respective WCs of the NP operators in which NP effects are hidden. In this analysis, we assume these NP WCs to be complex.

Using the effective Hamiltonian given in Eq. (4), we calculate the expressions of measured observables $R_D$, $R_{D^*}$, $R_{J/\psi}$, $P_{\tau}^{D^*}$ and $F_{L}^{D^*}$ as functions of the NP WCs. To obtain the values of NP WCs, we do a fit of these expressions to the measured values of the observables. In doing the fit, we take only one NP operator at a time. We define the $\chi^2$ function as follows

$$\chi^2(C_i) = \sum_{R_D,R_{D^*},R_{J/\psi},P_{\tau}^{D^*},F_{L}^{D^*}} \left( O^{\text{th}}(C_i) - O^{\text{exp}} \right) C^{-1} \left( O^{\text{th}}(C_i) - O^{\text{exp}} \right),$$

where $O^{\text{th}}(C_i)$ are NP predictions of each observable and $O^{\text{exp}}$ are the corresponding experimental central values. The $C$ denotes the covariance matrix which includes both theory and experimental correlations.
The $B \to (D, D^*) \ell \bar{\nu}$ decay distributions depend upon hadronic form-factors. The determination of these form-factors relies heavily on HQET techniques. In this work we use the HQET form factors in the form parametrized by Caprini et al. [30]. The parameters for $B \to D$ decay are determined from the lattice QCD [31] calculations and we use them in our analyses. For $B \to D^*$ decay, the HQET parameters are extracted using data from Belle and BaBar experiments along with the inputs from lattice. In this work, the numerical values of these parameters are taken from refs. [32] and [10]. The form factors for $B_c \to J/\psi$ transition and their uncertainties from ref. [33] are used in the calculation of $R_{J/\psi}$. These form factors are calculated in perturbative QCD framework.

To obtain the values of NP WCs, we minimize the $\chi^2$ function by taking non-zero value of one NP WC at a time. While doing so, we set other coefficients to be zero. This minimizations is performed by the CERN MINUIT library [34, 35]. We allow only those NP WCs which satisfy $\chi^2_{\min} \leq 4.5$. The central values of these allowed WCs of NP solutions are listed in Table I and the $1\sigma$ allowed regions for these NP solutions are shown in Fig. 1.

| NP type | Best fit value(s) | $\chi^2_{\min}$ | pull |
|---------|-------------------|-------------------|------|
| $C_{V_L}$ | $0.10 \pm 0.12 i$ | 4.55 | 4.1 |
| $C'_{S_L}$ | $0.25 \pm 0.86 i$ | 4.50 | 4.2 |
| $C''_{T}$ | $0.06 + 0.09 i$ | 3.45 | 4.3 |
| $C_{S_L}$ | $-0.82 \pm 0.45 i$ | 2.50 | 4.4 |

**TABLE I:** Best fit values of NP WCs at $\Lambda = 1$ TeV for the measurements of $R_D$, $R_{D^*}$, $R_{J/\psi}$, $P_T^{D^*}$ and $F_L^{D^*}$. We list the central values of the NP solutions with $\chi^2_{\min} \leq 4.5$. For the SM, we have $\chi^2_{\min} = 21.80$. The pull values are calculated using $\text{pull} = \sqrt{\chi^2_{\text{SM}} - \chi^2_{\min}}$.

The purely leptonic decay $B_c \to \tau \bar{\nu}$ plays a crucial role to constrain the NP solutions in this sector. This decay is subjected to helicity suppression in the SM whereas this suppression is removed for the pseudo-scalar operators. Therefore, these NP operators are highly constrained by this observable. Within the NP framework, the branching fraction of $B_c \to \tau \bar{\nu}$ can be expressed as

$$Br(B_c \to \tau \bar{\nu}) = \left| V_{cb} \right|^2 \frac{G_F^2 f_{B_c}^2 m_{B_c}^2 m_{\tau}^2 \exp}{8\pi} \left(1 - \frac{m_{\tau}^2}{m_{B_c}^2} \right)^2 \times \left| 1 + C_{V_L} - C_{V_R} + \frac{m_{B_c}^2}{m_{\tau}(m_b + m_c)} (C_{S_R} - C_{S_L}) \right|^2$$

where the decay constant $f_{B_c} = 434 \pm 15$ MeV [36] and the measured lifetime $\tau_{B_c}^{\exp} = 0.510 \pm 0.009$ ps [37]. Here $m_b$ and $m_c$ are the running quark masses evaluated at the $\mu_b = m_b$ scale. The SM predicts this branching fraction to be $\sim 2.15 \times 10^{-2}$. 
FIG. 1: The allowed 1σ regions for the complex NP WCs listed in Table I. For each plot, the blue colored region corresponds to the 1σ parameter space whereas the red dots represents the best fit values of NP WCs.

In Ref. [38], the upper limit on this branching ratio is set to be 10% from the LEP data which are admixture of \( B_c \to \tau \bar{\nu} \) and \( B_u \to \tau \bar{\nu} \) decays at \( Z \) peak. To extract the \( Br(B_c \to \tau \bar{\nu}) \), one needs to know the ratio of fragmentation functions of \( B_c \) and \( B_u \) mesons defined as \( f_c/f_u \). The value of this ratio is obtained from the data of Tevatron [39, 40] and LHCb [41]. On the other hand, the authors of Ref. [42] obtained this upper limit to be 30% by making use of the lifetime of \( B_c \) meson. This is estimated by considering that the \( B_c \to \tau \bar{\nu} \) decay rate does not exceed the fraction of the total width which is allowed by the calculation of the lifetime in the SM. In Ref. [24], the authors have argued that these two different upper limits are too conservative and these could be over-estimated. However, taking all uncertainties into account the decay width of \( B_c \) meson can be relaxed up to 60% which is not that much conservative. Therefore, we consider these three different upper limits on branching ratio of \( B_c \to \tau \bar{\nu} \) to constrain the NP parameter space. In this analysis, the NP WCs are defined at a scale \( \Lambda = 1 \) TeV. However, all these physical processes happen at \( m_b \) scale. Therefore, we include the renormalization group (RG) effects in the evolution.
of the WCs from the scale of 1 TeV to the $m_b$ scale \[43\]. In particular, these effects are important for the scalar and tensor operators.

FIG. 2: The 1\(\sigma\) regions allowed by \(b \to c\tau\bar{\nu}\) data (blue) and parameter spaces for three different upper limits 10% (green), 30% (yellow), 60% (violet) of \(Br(B_c \to \tau\bar{\nu})\) for each complex NP WC listed in Table I. In each plot, the red dots represent the best fit points.

In Fig. 2, we have shown the parameter space which span 1\(\sigma\) region allowed by present \(b \to c\tau\bar{\nu}\) data, three different ranges of branching ratio of \(B_c \to \tau\bar{\nu}\) and the best fit point for each solution listed in Table I. Only the \(O_{VL}\) solution falls within the allowed space constrained by \(Br(B_c \to \tau\bar{\nu}) < 10\%\). The allowed 1\(\sigma\) regions for \(O_{SL}'\) and \(O_{T}'\) solutions fall into the regions allowed by the constraints \(Br(B_c \to \tau\bar{\nu}) < 30\%\) and \(< 60\%\) respectively. The best fit NP WCs of \(O_{SL}\) solution do not fall into the region allowed by the constraint \(Br(B_c \to \tau\bar{\nu}) < 60\%\) whereas a small fraction of the 1\(\sigma\) region overlaps with the region allowed by \(30\% < Br(B_c \to \tau\bar{\nu}) < 60\%\). Hence we can reject the mildly allowed \(O_{SL}\) solution. We list the final three allowed NP solutions in Table II.

Using the best fit values of the allowed solutions, we provide the predicted central values of the quantities used in the fit, i.e., \(R_D, R_{D^*}, R_{J/\psi}, P_T^{D^*}\) and \(F_L^{D^*}\), for each solution. This will allow us
to see how close are the predictions of NP solutions to the experimental measurements. We note down the following observations by looking at the predictions in Table II:

- The predictions of $R_D$, $R_{D^*}$ and $P_{\tau}^{D^*}$ for the three solutions are within $1\sigma$ of the respective experimental averages.

- The predicted values of $R_{J/\psi}$ and $F_L^{D^*}$ for the three solutions are within $\sim 1.6\sigma$ of the experimental measurements.

| NP type | Best fit value(s) | $R_D$  | $R_{D^*}$ | $R_{J/\psi}$ | $P_{\tau}^{D^*}$ | $F_L^{D^*}$ |
|---------|------------------|--------|-----------|--------------|------------------|--------------|
| SM      | $C_i = 0$        | 0.297 (8) | 0.253 (2) | 0.289 (8)   | −0.499 (4)       | 0.457 (5)    |
| $C_{VL}^{10\%}$  | $0.10 \pm 0.12 i$ | 0.364 | 0.294 | 0.334 | −0.499 | 0.443 |
| $C_{SL}^{30\%}$  | $0.25 \pm 0.86 i$ | 0.336 | 0.295 | 0.339 | −0.419 | 0.443 |
| $C_{T}^{60\%}$   | $0.06 + 0.09 i$ | 0.333 | 0.296 | 0.344 | −0.375 | 0.420 |

TABLE II: Central values of best fit NP WCs at $\Lambda = 1$ TeV by making use of data of $R_D$, $R_{D^*}$, $R_{J/\psi}$, $P_{\tau}^{D^*}$ and $F_L^{D^*}$. Here we allow only those solutions for which $\chi^2_{\text{min}} \leq 4.5$ as well as for three different upper limits $10\%$, $30\%$ and $60\%$ of $\text{Br}(B_c \to \tau \bar{\nu})$. We also provide the predictions of each observables which are taken into the fit.

| NP type | Best fit value(s) | $P_{\tau}^{D}$ | $A_{FB}^{D}$ | $A_{FB}^{D^*}$ |
|---------|------------------|---------------|-------------|--------------|
| SM      | $C_i = 0$        | 0.325 (1)     | 0.360 (2)   | −0.063 (5)   |
| $C_{VL}^{10\%}$  | $0.10 \pm 0.12 i$ | 0.325     | 0.360       | −0.063       |
| $C_{SL}^{30\%}$  | $0.25 \pm 0.86 i$ | 0.420 | 0.212 | 0.0001 |
| $C_{T}^{60\%}$   | $0.06 + 0.09 i$ | 0.414 | 0.100 | 0.009 |

TABLE III: Average values of angular observables $P_{\tau}^{D}$, $A_{FB}^{D}$ and $A_{FB}^{D^*}$ for the SM and three solutions listed in Table II.

We consider other angular observables in $B \to (D, D^*)\tau \bar{\nu}$ decay which are yet to be measured. In particular, we are interested in the following three observables:

- The polarization of $\tau$ lepton in $B \to D\tau \bar{\nu}$ decay, $P_{\tau}^{D}$
- The forward-backward asymmetry in $B \to D\tau \bar{\nu}$ decay, $A_{FB}^{D}$ and
- The forward-backward asymmetry in $B \to D^*\tau \bar{\nu}$ decay, $A_{FB}^{D^*}$. 
FIG. 3: The predictions of angular observables $P^D$, $A^D_{FB}$ and $A^D_{FB}^{*}$ as a function of $q^2$ (GeV$^2$) for the SM and three solutions listed in Table III. The color code for each case is shown in each plot.

We compute the average values of these three angular observables for the allowed NP solutions. The predicted values are listed in Table III. For completeness, we also plot these observables as a function of $q^2 = (p_B - p_{D(*)})^2$, where $p_B$ and $p_{D(*)}$ are the respective four momenta of $B$ and $D(*)$ mesons. These are shown in Fig. 3. From Table III and Fig. 3 we observe the following features

- The predictions of all three observables for the $O_{VL}$ solution are exactly same as those of the SM. This is because the Lorentz structure of $O_{VL}$ operator is same as the SM.

- The $P^D$ has very poor discriminating capability.

- The predictions of $A^D_{FB}$ and $A^D_{FB}^{*}$ for the $O_{SL}$ and $O_{T}$ solutions are markedly different. These two solutions can be distinguished by forward-backward asymmetries.

III. CP VIOLATING TRIPLE PRODUCT ASYMMETRIES

If the hints of LFU violation in $b \to c\tau\bar{\nu}$ sector is indeed due to new physics, then it should definitely lead to some signatures of CP violation in the relevant decay modes. In this section, we
discuss about the possible CP violation in $B \rightarrow D^*\tau \bar{\nu}$ decay. The simplest possible CP violating observable, which one could think of, is the direct CP asymmetry between the decay and its CP conjugate mode. In order to have a non-zero value of direct CP asymmetry, we need strong phase difference between the amplitudes besides the weak phase. For $B \rightarrow D^*\tau \bar{\nu}$ decay, there is no strong phase difference in the SM because of unique final state of the decay and its CP conjugate mode. In Ref. [45], the authors suggested a mechanism where this strong phase difference could arise due to interference between the higher resonances of $D^*$ meson. They have shown that the CP violation could be as large as $\sim 10\%$ only for the tensor NP. However, the tensor NP is now ruled out by the Belle measurement on $F_L^{D^*}$.

In this work, we focus on CP violating triple product asymmetries (TPA) in $B \rightarrow D^*\tau \bar{\nu}$ decay. The full angular distribution of quasi-four body decay $B \rightarrow D^*(\rightarrow D\pi)\tau \bar{\nu}$ can be described by four independent parameters – (a) $q^2 = (p_B - p_{D^*})^2$ where $p_B$ and $p_{D^*}$ are respective four momenta of $B$ and $D^*$ meson, (b) $\theta_D$ the angle between $B$ and $D$ mesons where $D$ meson comes from $D^*$ decay, (c) $\theta_\tau$ the angle between $\tau$ momenta and $B$ meson, and (d) $\phi$ the angle between $D^*$ decay plane and the plane defined by the $\tau$ momenta [46]. The triple products (TP) are obtained by integrating the full decay distribution in different ranges of the polar angles $\theta_D$ and $\theta_\tau$. These are following [46–48]

$$\frac{d^2\Gamma^{(1)}}{dq^2 d\phi} = \int_{-1}^{1} \int_{-1}^{1} \frac{d^4 \Gamma}{dq^2 d\cos \theta_\tau d\cos \theta_D d\phi} d\cos \theta_\tau d\cos \theta_D$$

$$= \frac{1}{2\pi} \frac{d\Gamma}{dq^2} \left[ 1 + \left(A_C^{(1)} \cos 2\phi + A_T^{(1)} \sin 2\phi\right)\right],$$

$$\frac{d^2\Gamma^{(2)}}{dq^2 d\phi} = \int_{-1}^{1} d\cos \theta_\tau \left[ \int_{-1}^{0} - \int_{0}^{1} \right] \frac{d^4 \Gamma}{dq^2 d\cos \theta_\tau d\cos \theta_D d\phi} d\cos \theta_D$$

$$= \frac{1}{4} \frac{d\Gamma}{dq^2} \left[ A_C^{(2)} \cos \phi + A_T^{(2)} \sin \phi\right],$$

and

$$\frac{d^2\Gamma^{(3)}}{dq^2 d\phi} = \left[ \int_{0}^{1} - \int_{-1}^{0} \right] d\cos \theta_\tau \left[ \int_{0}^{1} - \int_{-1}^{0} \right] \frac{d^4 \Gamma}{dq^2 d\cos \theta_\tau d\cos \theta_D d\phi} d\cos \theta_D$$

$$= \frac{2}{3\pi} \frac{d\Gamma}{dq^2} \left[ A_C^{(3)} \cos \phi + A_T^{(3)} \sin \phi\right].$$

The coefficients $A_C^{(i)}$ of $\cos \phi$ and $\cos 2\phi$ are even under CP transformation and hence we are not interested in these. However, the angular coefficients $A_T^{(i)}$ of $\sin \phi$ and $\sin 2\phi$ are odd under the CP transformation which leads to these quantities to be CP violating observables. These three TPs are defined as follows [46]:

$$A_T^{(1)}(q^2) = \frac{4V_5^{0T}}{A_L + A_T}, \quad A_T^{(2)}(q^2) = \frac{V_3^{0T}}{A_L + A_T}, \quad A_T^{(3)}(q^2) = \frac{V_4^{0T}}{A_L + A_T},$$

(11)
where $V$’s are the angular coefficients and $A_L$ and $A_T$ are the longitudinal and transverse amplitudes respectively, defined in Ref. [46]. The SM predictions of these TP are almost zero. Therefore, the complex NP WCs can predict a non-zero value for these quantities. For the CP conjugate decay, the definitions in Eq. 11 take the following forms

$$\bar{A}_T^{(1)}(q^2) = -\frac{4\bar{V}_5^T}{A_L + A_T}, \quad \bar{A}_T^{(2)}(q^2) = \frac{\bar{V}_3^0 T}{A_L + A_T}, \quad \bar{A}_T^{(3)}(q^2) = -\frac{\bar{V}_4^0 T}{A_L + A_T}. \quad (12)$$

Using Eq. 11 and 12, three asymmetries can be defined between the corresponding TPs of the decay and its CP conjugate. These TPs are defined as follows

$$\langle A_T^{(1)}(q^2) \rangle = \frac{1}{2} \left( A_T^{(1)}(q^2) + \bar{A}_T^{(1)}(q^2) \right),$$

$$\langle A_T^{(2)}(q^2) \rangle = \frac{1}{2} \left( A_T^{(2)}(q^2) - \bar{A}_T^{(2)}(q^2) \right),$$

$$\langle A_T^{(3)}(q^2) \rangle = \frac{1}{2} \left( A_T^{(3)}(q^2) + \bar{A}_T^{(3)}(q^2) \right). \quad (13)$$

First we calculate the predictions of these TPs for the SM and the three best fit NP solutions

![FIG. 4: The TPAs are plotted as a function of $q^2$ (GeV$^2$) for the SM and three best fit NP WCs listed in Table II. The color code for each plot is shown in figure.](image)

listed in Table II as a function of $q^2$. These predictions are shown in Fig. 4. From this figure, we make the following observations
• The TPAs \( \langle A_T^{(1)}(q^2) \rangle \) and \( \langle A_T^{(3)}(q^2) \rangle \) depend only on the \( \mathcal{O}_{V_L} \) operator and it has the same Lorentz structure as the SM. Therefore, the \( \mathcal{O}_{V_L} \) solution predicts these two asymmetries to be zero for whole \( q^2 \) range. For other two NP solutions, the predictions are zero because these two asymmetries do not depend on those NP WCs.

• The TPA \( \langle A_T^{(2)}(q^2) \rangle \) depends on \( \mathcal{O}_{V_L}, \mathcal{O}_{S_L} \) and \( \mathcal{O}_{S_R} \) operators. The \( \mathcal{O}_{V_L} \) operator has the same Lorentz structure as the SM. Hence, the prediction of this TPA is zero for the \( \mathcal{O}_{V_L} \) solution for whole \( q^2 \) range. The \( \mathcal{O}'_{S_L} \) and \( \mathcal{O}_T'' \) operators are linear combinations of \( \mathcal{O}_{S_L} \) and \( \mathcal{O}_T \). Therefore, we get some non-zero value of this TPA for these two solutions. For the \( \mathcal{O}'_{S_L} \) solution, \( \langle A_T^{(2)}(q^2) \rangle \) reaches a maximum value of \( \sim 0.7\% \) at \( q^2 \simeq 6 \text{ GeV}^2 \) and decreases to zero at \( q_{\text{max}}^2 \). For the \( \mathcal{O}_T'' \) solution, \( \langle A_T^{(2)}(q^2) \rangle \) reaches a maximum value of \( \sim 1.7\% \) at \( q^2 \simeq 5.4 \text{ GeV}^2 \) and decreases to zero at \( q_{\text{max}}^2 \).

![Graph 1](image1.png)

**FIG. 5:** The second TPA is plotted as a function of \( q^2 \) (GeV\(^2\)) for three benchmark NP WCs \( C_{S_L}' = 0.24 + i \) (blue curve), \( C_T'' = 0.06 + 0.098i \) (black curve) and \( C_{S_L} = -0.35 - 0.60i \) (red curve).

Our next aim is to compute the maximum CP violation allowed by the present \( b \to c\tau\bar{\nu} \) data. To calculate this, we choose a benchmark point from the 1\( \sigma \) allowed parameter space of each NP solution. From Fig. 4, we have learned that for any complex value of \( C_{V_L} \) three TPAs lead to zero. Only the second TPA \( \langle A_T^{(2)}(q^2) \rangle \) is non-zero for the \( \mathcal{O}'_{S_L} \) and \( \mathcal{O}_T'' \) solutions. Therefore, we pick a benchmark points from Fig. 4 for each of these two solutions. These points are \( C_{S_L}' = 0.24 \pm i \) and \( C_T'' = 0.06 + 0.098i \), which can lead to the maximum value of the TPA \( \langle A_T^{(2)}(q^2) \rangle \) in \( B \to D^*\tau\bar{\nu} \) decay. In the left panel of Fig. 5, we plot the TPA \( \langle A_T^{(2)}(q^2) \rangle \) as a function of \( q^2 \) for these two benchmark points of \( \mathcal{O}'_{S_L} \) and \( \mathcal{O}_T'' \) solutions. From this plot, we observe that it has almost same features which are obtained from the plot of \( \langle A_T^{(2)}(q^2) \rangle \) in Fig. 4. We have not got much larger value of TPA \( \langle A_T^{(2)}(q^2) \rangle \) than what we got for the best fit NP solutions.
As per discussion in Sec II, the $\mathcal{O}_{SL}$ solution listed in Table I is marginally disfavored because the best fit values of $C_{SL}$ does not satisfy the constraint of $Br(B_c \to \tau \bar{\nu}) < 60\%$. However, a small fraction of the $1\sigma$ region of this solution falls on the region spanned by the constraint $30\% < Br(B_c \to \tau \bar{\nu}) < 60\%$. For completeness, we calculate the predictions of TPAs for this solution. We can get a allowed value of $C_{SL}$ which can give to maximum possible TPA for the $\langle A_T^{(2)}(q^2) \rangle$. We choose a benchmark point $C_{SL} = -0.35-0.60i$ from the allowed region and calculate the second TPA. In right panel of Fig. 5 we plot $\langle A_T^{(2)}(q^2) \rangle$ as a function of $q^2$ for the benchmark point of $C_{SL}$. From this plot, we observe that the second TPA reaches a maximum value of $\sim 2.6\%$ at $q^2 \approx 5 \text{ GeV}^2$ and decreases to zero at $q_{\text{max}}^2$. In fact, this is the maximum value of $\langle A_T^{(2)}(q^2) \rangle$ predicted by the scalar operator solution among all the predictions made by allowed NP solutions.

IV. CONCLUSIONS

In this work, we have done a global fit of $b \to c\tau\bar{\nu}$ data assuming NP WCs to complex. We find that the $\mathcal{O}_{VL}$ solution is the only NP solution allowed by the constraint $Br(B_c \to \tau \bar{\nu}) < 10\%$. If we relax the constraint to 30\% or 60\%, then we get one or two additional allowed NP solutions. We calculate the predictions of angular observables in $B \to (D, D^*)\tau \bar{\nu}$ decays. We find that the forward-backward asymmetries in these two decays are quite useful to distinguish the two solutions other than the $\mathcal{O}_{VL}$ solution.

We then compute the maximum values of CP violating TPAs in $B \to D^*\tau \bar{\nu}$ decay for the allowed NP solutions. We find that the predictions of first and third TPAs are zero for all NP solutions whereas the second TPA reaches a maximum value of $\sim 1.9\%$ for the $\mathcal{O}'_{SL}$ solution and $\sim 0.9\%$ for the $\mathcal{O}'_{T}$ solution. The mildly favored NP solution $\mathcal{O}_{SL}$ predicts a maximum value of $\sim 2.6\%$ for the second TPA which is the maximum predicted value among all the NP predictions.

To measure the angular observables and TPAs, the reconstruction of the $\tau$ lepton momentum is crucial. This is quite difficult because of the missing neutrinos. But LHCb collaboration has taken up this challenge for the near future [49]. Recently in Ref. [50], the author discussed an outline to measure the full angular distribution and the CP violating TPAs for $B \to D^* l \bar{\nu}$ decays at the collider experiments.
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