A systematic study of the factors affecting central depletion in nuclei

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Abstract. A systematic study of the central depletion of proton density has been performed in the isotonic chains of nuclei with neutron numbers \(N = 20\) and \(28\) using different variants of the relativistic mean-field (RMF) models. These models include either the non-linear contributions from the mesons with the coupling constants being density independent or the non-linearity of the mesonic fields realized through the density dependent coupling strengths. The central depletion in deformed nuclei tends to disappear irrespective of the occupancy of \(2s_{1/2}\) state in contrast to the spherical nuclei in which the unoccupancy of \(2s_{1/2}\) state leads to the central depletion. Due to the differences in the strength of spin-orbit potentials in these models, the central depletions are found to be model dependent. The influence of the central depletion on the neutron-skin thickness is also investigated. It appears that the effects of the central depletion do not percolate far enough to display its fingerprints on the trends of the neutron-skin thickness.

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1. Introduction

The "Bubble Structure" or the depletion in the central density of nucleons has attracted a lot of research interest currently especially due to the availability of the advanced experimental facilities to study the exotic nuclei. The increasing experimental \cite{1} and theoretical efforts \cite{2, 3, 4, 5, 6, 7, 8, 9} in order to search and understand the bubble like structures have provided a significant amount of information on the bubble nuclei. This phenomenon of bubble is usually believed to be due to the unoccupancy of the \(s_{1/2}\)-state
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which leads to significant reduction of the density at the center. Sometimes, the bubble or the depletion in the central density is associated with the inversion of \( (2s_{1/2} \text{ and } 1d_{3/2}) \) states and \( (3s_{1/2} \text{ and } 1h_{11/2}) \) states \[10\] causing unoccupancy of \( s_{1/2} \)-state. In case of heavy and superheavy nuclei \([9 \, 11, 12, 13, 14]\), the appearance of the bubble structures has been attributed to the effects of electrostatic repulsion and the symmetry energy \[5\].

Experimental signature of empty \( 2s_{1/2} \) in \(^{34}\text{Si}\) using one-proton removal reaction technique, recently reported by Mutschler \textit{et al.} \[1\], has opened a testing ground for the already developed successful models and new avenues for the nuclear structure mechanisms related to the nucleonic central density depletion across the periodic chart. On the theoretical side, various models like the nuclear density functional theory \[5\], ab-initio self consistent Green’s function many-body method \[4\], relativistic mean-field and non-relativistic mean-field models along with shell-model \[15\], and the particle-number and angular-momentum projected Generator Coordinate Method based on Hartree-Fock-Bogoliubov + Lipkin-Nogami states with axial quadrupole deformation \[16\] etc., have been applied to study the bubble like structures. The nuclei \(^{34}\text{Si}\) and \(^{22}\text{O}\) are suggested to be the strong candidates for the proton and neutron bubbles, respectively. The tensor force effect and the pairing correlations in the bubble structure have been also investigated \([10 \, 17 \, 18 \, 19 \, 20]\). A recent work has reported the central depletion in the deformed sd-shell nuclei \[21\] where the extent of central depletion is generally found to be weaker than those in the spherical nuclei in similar mass region. The weakening of the central depletion in deformed nuclei may be due to the change in the occupancy of \( s_{1/2} \)-state. The actual mechanism of the weakening of central depletion in deformed nuclei has not been investigated in detail so far.

Henceforth, a study towards the deeper understanding and clarity on the various issues regarding the mechanisms behind the bubble phenomena discussed above, which is the need of the current research interest, is the objective of the present work. Here we perform a systematic study of the central depletion in the nuclei with neutron numbers \( N = 20 \) and 28 using relativistic mean-field (RMF) models. The RMF models employed in this work are: (a) models including the contributions from the nonlinear self- and mixed-interactions of the mesons with various coupling strengths taken to be constant \[22, 23, 24, 25\] and (b) the models with only linear contributions from the mesons, the coupling strengths are density-dependent \[26\]. The well deserved attention has been paid to the effects of deformation for which we study well deformed nuclei \(^{40}\text{Mg}, \, ^{42}\text{Si}\) and \(^{44}\text{S}\) that have shown central density depletion. Also, we perform a systematic study of \(^{24–48}\text{Si}\) isotopes that are found with oblate, spherical as well as prolate shapes with a range of \( (\beta = 0 \text{–} 0.4) \) deformation, hence provide an ideal testing ground to study central depletion variation with changing deformations and shapes. We also examine the influence of neutron to proton ratio and pairing energy in addition to occupancy in \( 2s_{1/2} \)-state on the bubble effect. The influence of central depletion on the variation of neutron-skin thickness with the asymmetry has been explored.

In section II, we briefly describe the different RMF models considered. The main
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results are presented in section III. In section IV, we present our conclusions.

2. Theoretical Formalisms

The effective lagrangian for the RMF models can be broadly classified in two categories: (i) in which linear terms for the mesons coupled to the nucleonic degrees of freedom and non-linear terms for mesons describe their self and mixed interactions, the coupling constants independent of the density [22, 23, 24], and (ii) contains only linear terms for the mesons and the non-linear contributions of mesons are accounted through the density dependence of the coupling constants. We use the RMF models belonging to both the categories. We consider the NL3 and NL3* parameterizations [27, 28] of the RMF model which include linear terms for the $\sigma$, $\omega$ and $\rho$ mesons and non-linear term only for the self interaction of the $\sigma$ meson. The FSU-Gold and FUS-Garnet parameterizations include in addition the non-linear self interaction of $\omega$ meson and the mixed interaction terms for $\omega$ and $\rho$ mesons [26, 29]. The RMF model belonging to the category of density dependence of coupling constants for the meson exchange are the DD-ME2 and DD-PC1 [30, 31]. The effective lagrangian for DD-PC1 model is analogous to DD-ME2 model but, it does not include the derivative term for mesonic fields and hence they are directly expressed in terms of nucleonic field.

The interaction part of the effective Lagrangian of RMF model belonging to the category (i) can be written as,

$$\mathcal{L}_{\text{int}} = \bar{\psi} \left[ g_\sigma \sigma - \gamma^\mu \left( g_\omega \omega_\mu + \frac{1}{2} g_\rho \tau \cdot \rho_\mu + \frac{e}{2} (1 + \tau_3) A_\mu \right) \right] \psi - \frac{\kappa_4}{6 M} g_\sigma m_\sigma^2 \sigma^3 - \frac{\kappa_4}{24 M^2} g_\sigma^2 m_\sigma^2 \sigma^4 - \frac{1}{24} \zeta g_\omega^2 (\omega_\mu \omega^\mu)^2 + \frac{\eta_2}{4 M^2} g_\omega^2 m_\omega^2 \omega_\mu \omega^\mu \rho_\nu \rho^\nu$$

(1)

Where the symbols have usual meaning and the details can be found in Refs. [23, 24, 25, 26].

The density-dependent meson-exchange model (DD-ME) [30] interaction part of the Lagrangian does not contain any non-linear term, but, the meson-nucleon strengths $g_\sigma$, $g_\omega$ and $g_\rho$ have an explicit density dependence in the following form:

$$g_i(\rho) = g_i(\rho_{\text{sat}}) f_i(x), \quad \text{for } i = \sigma, \omega$$

(2)

where the density dependence is given by

$$f_i(x) = \frac{1 + b_i (x + d_i)^2}{1 + c_i (x + e_i)^2}$$

(3)

in which $x$ is given by $x = \rho/\rho_{\text{sat}}$, and $\rho_{\text{sat}}$ denotes the baryon density at saturation in symmetric nuclear matter. For the $\rho$ meson, density dependence is of exponential form and given by

$$f_\rho(x) = \exp(-a_\rho(x - 1))$$

(4)
Table 1. Nuclear matter properties as saturation density $\rho_{\text{sat}}$, binding energy per nucleon $\epsilon$, effective nucleon mass $m^*/m$, incompressibility coefficient $K$, symmetry energy $J$ and slope of symmetry energy $L$ at the saturation density are given for various RMF models.

| Nuclear Matter Properties | NL3  | NL3* | FSU-Gold | FSU-Garnet | DD-ME2 | DD-PC1 |
|---------------------------|------|------|----------|------------|--------|--------|
| $\rho_{\text{sat}}$ (fm$^{-3}$) | 0.149 | 0.150 | 0.148 | 0.153 | 0.152 | 0.152 |
| $\epsilon$ (MeV)         | -16.30 | -16.31 | -16.30 | -16.23 | -16.14 | -16.06 |
| $m^*/m$                   | 0.60  | 0.59  | 0.61   | 0.58    | 0.57   | 0.66   |
| $K$ (MeV)                | 271.76 | 258.78 | 230.00 | 229.50 | 250.89 | 230.00 |
| $J$ (MeV)                | 37.4  | 38.6  | 32.6   | 30.9    | 32.3   | 33.0   |
| $L$ (MeV)                | 118.6 | 122.6 | 60.5   | 51.0    | 57.2   | 57.2   |

Interaction part of Lagrangian for the density-dependent point coupling model (DD-PC) $^{31}$ is given by

$$L_{\text{int}} = -\frac{1}{2}\alpha_S(\rho)(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\rho)(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma^\mu\psi) - \frac{1}{2}\alpha_{TV}(\rho)(\bar{\psi}\gamma^\mu\gamma^\nu\psi)(\bar{\psi}\gamma^\mu\gamma^\nu\psi) - \frac{1}{2}\delta_S(\rho)(\bar{\psi}\psi)\Box(\bar{\psi}\psi)$$

In analogy with meson-exchange model (DD-ME) described above, this model contains isoscalar-scalar ($S$), isoscalar-vector ($V$) and isovector-vector ($TV$) interactions. The coupling constants $\alpha_i(\rho)$ are density dependent and have the form $^{31}$:

$$\alpha_i(\rho) = a_i + (b_i + C_i x) e^{-d_i x}, \text{for } i = S, V, TV$$

Some characteristic parameters associated with nuclear matter at the saturation density are listed in Table 1 for the RMF models considered. Using these RMF models we aim to examine the influence of various factors on the central depletion.

The degree of depletion in the proton or neutron densities at the center of the nuclei is usually expressed in terms of the so-called bubble parameters. These parameters are not uniquely defined. We have employed a simple definition of the bubble parameter $^{3}$,

$$b_\tau = 1 - \frac{\rho_{\tau,c}}{\rho_{\tau,max}}$$

where, $\tau = p, n$ and $\rho_{\tau,c}, \rho_{\tau,max}$ represent the central density, maximum density respectively. We also adopt the bubble parameter $b'_\tau$ defined as $^{5}$,

$$b'_\tau = 1 - \frac{\rho_{\tau,c}}{\rho_{\tau,av}}$$

where, $\rho_{\tau,av}$ is the average density defined as

$$\rho_{\tau,av} = \frac{3N_\tau}{4\pi R_{\tau,d}^3}$$

with $R_{\tau,d}$ being the diffraction radius $^{5, 32}$ and $N_\tau$ is nucleon number. It is evident from Eqs. in (6 and 7) that the parameters $b_\tau$ and $b'_\tau$ estimate the extent of depletion in the central density with respect to the maximum and the average densities, respectively. Thus, their values are not expected to be identical for a given nucleus and the model. The positive values of $b_\tau$ and $b'_\tau$ indicate the existence of the central depletion.
Table 2. A comparison of the values of bubble parameters \( b_p \) and \( b'_p \) obtained using different RMF models.

| Isotones with \( N \) | Bubble Parameters | \( b_p \) | \( b'_p \) |
|-----------------------|-------------------|---------|--------|
| \( ^{20}\text{Ne} \) | NL3 | 0.21 | 0.19 |
| \( ^{20}\text{Ne} \) | NL3* | 0.27 | 0.23 |
| \( ^{20}\text{Ne} \) | FSU-Gold | 0.23 | 0.20 |
| \( ^{20}\text{Ne} \) | FSU-Garnet | 0.17 | 0.15 |
| \( ^{20}\text{Ne} \) | DD-ME2 | 0.07 | 0.08 |
| \( ^{20}\text{Ne} \) | DD-PC1 | 0.18 | 0.17 |
| \( ^{28}\text{Ar} \) | NL3 | 0.34 | 0.30 |
| \( ^{28}\text{Ar} \) | NL3* | 0.45 | 0.35 |
| \( ^{28}\text{Ar} \) | FSU-Gold | 0.39 | 0.33 |
| \( ^{28}\text{Ar} \) | FSU-Garnet | 0.33 | 0.29 |
| \( ^{28}\text{Ar} \) | DD-ME2 | 0.29 | 0.26 |
| \( ^{28}\text{Ar} \) | DD-PC1 | 0.33 | 0.29 |
| \( ^{40}\text{S} \) | NL3 | 0.30 | 0.29 |
| \( ^{40}\text{S} \) | NL3* | 0.45 | 0.35 |
| \( ^{40}\text{S} \) | FSU-Gold | 0.33 | 0.29 |
| \( ^{40}\text{S} \) | FSU-Garnet | 0.60 | 0.54 |
| \( ^{40}\text{S} \) | DD-ME2 | 0.33 | 0.29 |
| \( ^{40}\text{S} \) | DD-PC1 | 0.00 | 0.00 |

3. Results and discussions

We now present our results for the systematics of central depletion in the isotonic chains of the nuclei with neutron numbers \( N = 20 \) and \( 28 \). The results are obtained using different variants of the RMF model as outlined briefly in the previous section. The models employed are namely NL3, NL3*, FSU-Gold, FSU-Garnet, DD-ME2 and DD-PC1 (see Table 1). We will examine here influence of various factors viz. model dependency, occupation of \( 2s_{1/2} \), neutron to proton ratio, deformation and pairing contribution, on the central depletion. A systematic study of Si isotopes (\( N = 10 \sim 34 \)) exhibiting a range of deformations and shapes is presented. We also look for the imprints of the central depletion on the systematics of the neutron-skin thickness. In particular, the effects of central depletion on the variation of neutron-skin thickness with asymmetry parameter \( (N - Z)/A \) is investigated.

We compare in Table 2 the results for the \( b_p \) and \( b'_p \) (Eqs. 6 and 7) for some selected spherical nuclei with \( N = 20, 28 \) and 40 obtained for several RMF models. One can see that the values of \( b_p \) are larger by 20%-30% as compared to the \( b'_p \), because the maximum density \( \rho_{r,\text{max}} \) is always larger than average density \( \rho_{r,\text{av}} \). Bubble parameter \( b_p \) is easy to calculate for the spherical as well as for the deformed nuclei. Therefore, here after, we restrict ourselves to the calculations of bubble parameter \( b_p \) only. Fig. 1 shows our calculated bubble parameters \( b_p \) for \( N = 20 \) and 28 isotonic chains obtained for different RMF models considered in this work. All the nuclei belonging to \( N = 20 \) isotonic chain are found to be spherical for all the RMF models employed. For \( N = 28 \) isotonic chain, the nuclei \( ^{40}\text{Mg}, ^{42}\text{Si} \) and \( ^{44}\text{S} \) are found to be deformed in agreement with the available experimental predictions.

It is evident from the figure that the degree of central depletion is sensitive to the choice of model. For the case of \( N = 20 \) isotones, the central depletion of proton density is seen in nuclei \( ^{30}\text{Ne}, ^{32}\text{Mg} \) and \( ^{34}\text{Si} \). For these nuclei, the values of the bubble parameter for most of the models are in the range of \( \sim 0.2 \sim 0.4 \) whereas the values of \( b_p \) for the DD-PC1 model are relatively smaller. For \( N = 28 \) isotones, except for the DD-PC1 model, the significant central depletion is observed in the proton density of spherical nucleus \( ^{46}\text{Ar} \), central depletion seen in \( ^{34}\text{Si} \) and \( ^{46}\text{Ar} \) in the present work agrees with that indicated by experimental [1] and earlier theoretical [4, 5, 6, 7, 10, 15, 18, 20] works. For the deformed nuclei \( ^{40}\text{Mg}, ^{42}\text{Si} \) and \( ^{44}\text{S} \) the central depletion is observed, however, this depletion is noticeably smaller as compared to neighbouring spherical nuclei. The
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Figure 1. (Colour online) The bubble parameter $b_p$ for protons in isotonic chain with neutron numbers $N = 20$ and $28$ obtained by RMF (NL3, NL3*, FSU-Gold, FSU-Garnet, DD-ME2 and DD-PC1 force parameters). The asterisk symbols highlight the deformed nuclei in the case of $N = 28$ isotones.

Weakening of central depletion in deformed nuclei is also observed in $^{24}$Ne, $^{32}$Si and $^{32}$Ar as reported in Ref. [21]. The prolately deformed nuclei $^{40}$Mg and $^{44}$S exhibit somewhat larger central depletion in comparison to the oblate nucleus $^{42}$Si. The spread in the values of $b_p$ for in $^{40}$Mg, $^{42}$Si and $^{44}$S may be partly due to the variation in the deformation parameter obtained for different models.

Fig. 2 shows variation of $b_p$ with respect to quadrupole deformation parameter $\beta$. Straight lines are plotted to guide the eye. By and large, one can conclude that the bubble parameter decreases with increasing deformation. In the case of prolate deformation ($^{40}$Mg and $^{44}$S) the $b_p$ decreases rapidly with $\beta$ as compared to oblate nucleus ($^{42}$Si). It appears that the deformation tends to quench the central depletion. More on this will be discussed below.

Since the central depletion in the nuclei is believed to be associated with the unoccupancy of $s_{1/2}$ state [4, 10, 15, 18, 20], it is important to investigate whether one
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Figure 2. (Colour online) The bubble parameter $b_p$ as a function of deformation $\beta$ obtained by various models for $^{40}\text{Mg}$, $^{42}\text{Si}$ and $^{44}\text{S}$.

can owe the differences in the extent of central depletion across the various models to the differences in the several inputs of the models. We consider the case of $^{40}\text{Ar}$ nucleus which shows strong model dependence [34] in the values of bubble parameter. We display in Fig. 3 the single particle (s.p.) energies of $^{40}\text{Ar}$ along with their occupancies calculated for several models used for the present study. The values of the bubble parameter for a given model are indicated in the figure. It is evidently seen that the bubble parameter $b_p$, quantifying the central depletion, decreases as the occupancy of $2s_{1/2}$ state increases. The occupancy of $2s_{1/2}$ state is found maximum for DD-PC1 force parameter comparative to other considered models/parameters. This difference is examined by taking into account the various inputs of the considered models. Among the inputs, coupling constants describing the interaction between mesons and nucleons are found to play a dominant role. These coupling constants predominantly determine the strength of the spin-orbit potential [35, 36] given by

$$ U_{ls} \propto \frac{1}{m^*/m} (C^2_\sigma + C^2_\omega + C^2_\rho) $$

where $m^*/m$ is the nucleon (effective) mass; the constants $C_i$ ($i = \sigma, \omega, \rho$) are defined as $C_i = mg_i/m_i$, $m_i$ and $g_i$ being the meson masses and coupling constant, respectively. For DD-ME2 and DD-PC1 models these coupling constant are evaluated at saturation density. In Fig. 3(c) the spin orbit potential strength ($U_{ls}$) is plotted against bubble parameter. The different values of $U_{ls}$ for different models are responsible for the differences in the relative separation of $2s$ and $1d$ states and hence for the differences in the pairing properties. Further, the reasonable correlation between $U_{ls}$ and bubble parameter provides plausible explanation for the existence of the model dependence. The differences in the spin-orbit splitting may be partially attributed to the differences in the deformation and hence the central depletion, as also seen in Fig. 2.
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Figure 3. (Colour online) (a) Proton s.p. energies of $^{46}$Ar, with respect to energy of $1s_{1/2}$ state, along with their occupancies from several models considered. Values of $b_p$ by these models are also indicated. (b) energy gap between $2s_{1/2}$ and $1d_{3/2}$ for $^{46}$Ar (c) strength of the spin-orbit potential $U_{ls}$ for various models/parameters. (The energies are in the unit of MeV).

To visualize the dependence on occupancy of $2s_{1/2}$ state, we show in Fig. 4 the variation of the bubble parameter $b_p$ plotted against the occupancy of $2s_{1/2}$ state for protons, obtained using different RMF models for the spherical nuclei corresponding to $N = 20$ and 28 isotonic chains. The overall trend suggests that the bubble parameter decreases as the occupancy of $2s_{1/2}$ increases. With the increase of occupancy of $2s_{1/2}$ state to 50% (i.e. $2s_{1/2}$ state has one nucleon), $b_p$ collapses to close to zero. However, many of the nuclei having almost unoccupied $2s_{1/2}$ state, show noticeable spread in the values of bubble parameter. For instance, $b_p$ for $^{34}$Si nucleus varies in the range of 0.27 to 0.41 although the $2s_{1/2}$ state is practically unoccupied for all the models. This means that there may be other factors as well which influence the central depletion in addition to the choice of model and occupancy of the s-orbit. Thus, the condition that $2s_{1/2}$ state must be practically unoccupied is only a necessary condition but not the sufficient
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Figure 4. (Colour online) Variations of bubble parameter $b_p$ as a function of the occupancy of proton $2s_{1/2}$ state for the spherical nuclei.

condition for the occurrence of the central depletion.

Nuclear deformation is one of the factors which is expected to play a key role in the central depression [11, 10, 15, 16, 17]. We investigate the quenching of central depletion due to deformation in detail for which we consider the cases of well deformed $^{40}$Mg, $^{42}$Si and $^{44}$S (see also Figs. 1 and 2). We plot (a) the binding energy, (b) the occupancy of the $2s_{1/2}$ state for the protons and (c) the bubble parameter $b_p$, as a function of the quadrupole deformation parameter $\beta$ varying from -0.6 to 0.6 computed using the DD-ME2 model in Fig. 5. The value of deformation at each point of the curve is obtained by constraining the quadrupole moment through the variational procedure [37]. The trend for the other RMF models (not shown here) have been found to be qualitatively similar to those shown in Fig. 5. The potential energy surface as displayed in the top panel shows energy minima at $\beta = 0.4$ for $^{40}$Mg and $^{44}$S, and $\beta = -0.4$ for $^{42}$Si, characterizing prolate and oblate shapes, respectively. The bubble parameter $b_p$ shown in Fig. 5(c) is maximum at $\beta = 0$ and decreases as $\beta$ increases on both prolate and oblate sides which shows quenching of central depletion due to deformation. From Fig. 5(b), it can be seen that the $2s_{1/2}$ state is highly occupied towards oblate side and it is almost unoccupied towards prolate side. It appears that the occupancy in s-orbit appears to be not playing a major role in variation of $b_p$. The $b_p$ for these nuclei is decreasing on the prolate side even though the occupancy in $2s_{1/2}$ is vanishingly small. The value of $b_p$ (highlighted by squares in 5(c)) corresponding to the respective energy minima for $^{40}$Mg, $^{44}$S and $^{42}$Si, are found to be $b_p = 0.15$, 0.16 and 0.08, respectively, even the absolute value of $\beta$ is almost same for these nuclei. The lower value of $b_p$ in $^{42}$Si (oblate) could be due to combined effect of deformation and occupancy of $2s_{1/2}$ state. Hence, deformation seems to play the predominant role in determining the $b_p$ as compared to the occupancy of $2s_{1/2}$ state. This could be due to the lowering of some of the deformed single particle
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Figure 5. (Colour online) (a) Binding energy (MeV) (b) occupancy of proton $2s_{1/2}$ state and (c) bubble parameter $b_p$ as a function of deformation $\beta$ for $^{40}$Mg, $^{42}$Si and $^{44}$S.

Moreover, since bubble effect is related to the shell effects associated with the unoccupancy of $s$-orbital surrounded by single particle states with larger orbital angular momentum well separated in energy to ensure the weak dynamical correlations. The increase in deformation leads to stronger dynamical correlations, overlapping of sd-states and also less pronounced shell effect [38] which consequently disfavors formation of bubble. For better insight about the effect of deformation on bubble structure one possible way is to calculate s-wave projections of the nucleon wave functions and densities in the deformed cases [38, 39] which is left for our future work.

There have been clear indications for a relationship between the central density in nuclei and the symmetry energy [3]. The central depression of proton density in heavy nuclei is predominantly driven by Coulomb energy and the neutron to proton
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The pairing and dynamical correlations associated with quadrupole shapes have been indicated to hinder the bubble effect \[10, 17, 18, 19\] in light nuclei. Moreover, the reduced central density well below the nuclear matter saturation value is expected to result in the loss of binding energy. This invokes an investigation to explore the relationship between the central depletion and various components of the binding energy viz. Coulomb energy, pairing, deformation and isospin. It is important to point out here that the role of Coulomb interaction is already demonstrated in the superheavy bubble nuclei \[9\]. Nuclear deformation which changes with neutron to proton ratio and also influences the central depletion (as shown in Figs. 2 & 5) is an important parameter \[4, 10, 15, 16, 17\] to be investigated systematically. Hence, we look into the sensitivity of the bubble parameter to all these factors and study the full isotopic chain of Si, which consists of a wide range of deformation with many oblate, few prolate and spherical nuclei. Here, we plot and study the systematic variation of (a) bubble parameter \(b_p\), (b) occupancy (occ.) of \(2s_{1/2}\) (c) quadrupole deformation (\(\beta\)) and (d) proton pairing energy (P.P.E.) vs. neutron number \(N = 10 - 34\) \((Z = 14, A = 24 - 48)\) computed using DD-ME2 parameter in Fig. 6.

In Fig. 6, the occupancy in \(2s_{1/2}\) state is found zero for spherical \((34, 48)\) and prolate isotopes \((26, 36, 38)\) Si, shown by opaque symbol in Fig. 6(c)), whereas, it is fully occupied for oblate isotopes similar to what found in Fig. 5. At magic neutron number \(N = 20\), for the case of \(34\)Si and at \(N = 34\) for the case of \(48\)Si, which are found spherical \((\beta = 0)\), the occupancy of \(2s_{1/2}\) state is zero and \(b_p\) is maximum. However, the value of \(b_p\) for \(48\)Si is even higher than the \(b_p\) of the well known \(N = 20\) \((34\)Si) hinting a stronger bubble nature of \(48\)Si as reported very recently in Ref. \[40\]. A thorough inspection in various panels of Fig. 6 reveals that all the other parameters like proton pairing energy (PPE \(= 0\)) contribution, deformation \((\beta = 0)\) and occupancy of \(2s_{1/2}\) state \((= 0)\) are same in both the nuclei \(34\)Si and \(48\)Si. Therefore, the more \(b_p\) for \(48\)Si may be anticipated due to neutron excess indicating 'neutron to proton ratio' is one factor that also influences the central depletion and show higher \(b_p\). This was also seen in our recent work \[41\] in case of Ar isotopes where proton rich \(32\)Ar does not show central depletion, whereas \(46\)Ar has been found to exhibit bubble effect and also neutron rich \(68\)Ar has been indicated as bubble nucleus \[3, 10\].

On the other side, for neutron number \(N = 28\) and also for \(N = 14\), where the deformation shows its maximum with oblate shapes, the \(b_p\) is lowest which once again indicates the effect of oblate deformation and full occupancy of \(2s_{1/2}\), strengthening our outcome from Fig. 5. The nuclear deformation shown in Fig. 6(c) for the full chain of Si isotopes along with experimental data \[33\] show good agreement. While comparing the bubble parameter \(b_p\) (Fig. 6(a)) and deformation \(\beta\) values, we find that an inverse trend between \(b_p\) and \(\beta\) authenticate the quenching effect of deformation on bubble nuclei. The proton pairing energy plotted in Fig. 6(d) is found to be zero for all the Si isotopes except for a few cases \((N = 12\) and 24\). Therefore, \(Z = 14\) is found to be a shell closure as per RMF approach for most of the isotopes and hence gives doubly magic character to \(34\)Si \[1\] and \(48\)Si \[6, 40\]. Since the pairing correlations are expected
to quench the bubble, therefore, for $N = 12$ and 24 the non-zero pairing energy or rather the pairing correlations may be one of the cause for the lower value of $b_p$ in nuclei $^{26,38}\text{Si}$ as reflected from Figs. 6(a) and (d). With a close watch of Fig. 6 one can find that moving from $N = 10$ to $N = 12$, shape changes from oblate to prolate and $2s_{1/2}$ state becomes fully occupied to unoccupied which favour central depletion but the value of deformation increases and also proton pairing energy (PPE) reaches non-zero which consequently reduces the bubble parameter $b_p$. In a similar manner, while moving from $N = 22$ to $N = 24$, even if shape remains prolate and occupancy remains zero but value of deformation increases along with non-zero contribution of PPE which lead to a sharp drop in the value of $b_p$ from $^{36}\text{Si}$ to $^{38}\text{Si}$. Hence the reduction of central depletion is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.png}
\caption{Variations of bubble parameter $b_p$, occupancy of $2s_{1/2}$ state, deformation and pairing energy contribution (in MeV) as a function of neutron number for full isotopic chain of Si.}
\end{figure}
Figure 7. (Colour online) $\Delta r_{np}(X) - \Delta r_{np}^{(208\text{Pb})}$ ($\Delta r_{np}(X)$ is the neutron-skin thickness for nucleus 'X' in question) plotted as a function of asymmetry $[(N-Z)/A]$ obtained for several RMF models as considered. The hollow symbols indicate nuclei with central depletion in proton density and filled symbols are for nuclei without central depletion.

attributed due to the pairing and deformation correlations in these cases. Therefore, this analysis shows that the central depletion is actually a complex phenomenon, which is affected by the combined effects of N to Z ratio, pairing correlations, deformation, and the occupancy of $s_{1/2}$ state. Among these factors, the occupancy of $s_{1/2}$ state and the deformation influence significantly on the value of $b_p$.

Another important aspect relevant to the present study is to check the possible imprint of the central depletion in the proton density on the systematics of the neutron-skin thickness, is presented here. The neutron-skin thickness $\Delta r_{np}$ is given by

$$\Delta r_{np} = \langle r_{n}^2 \rangle^{1/2} - \langle r_{p}^2 \rangle^{1/2},$$

which is the difference between the rms radii for the point neutrons and the protons density distributions. Although the neutron skin thickness is a surface effect, the central depletion in the proton density might affect the rms radii for the proton distributions. Further, the change in the proton density might also modify the neutron distributions in the nuclei due to the self-consistency of the mean-field. The neutron and proton densities at the center tend to be more or less the same in the absence of the central depletion and the excess neutrons are pushed to the surface causing the neutron-skin. However, one does not know a priori, the influence of central depletion of proton density on the neutron distributions or the neutron-skin thickness. One often considers the variations of the neutron-skin thickness with the asymmetry $(N - Z)/A$ parameter. The correlations among the neutron-skin thickness of different nuclei are also usually considered. These systematics enable one to assess the information content.
of the neutron-skin thickness of a single nucleus. It is thus important to examine how the systematics of the neutron-skin thickness might get influenced in the presence of depletion of the central density in the nuclei. In Fig. 7, we plot neutron-skin thickness as a function of the asymmetry \((N - Z)/A\) parameter for several spherical nuclei for different RMF models considered. The value of neutron-skin for a nucleus is plotted in reference to its value for \(^{208}\text{Pb}\) nucleus for a given model. It is interesting to note that the difference \(\Delta r_{np}(X) - \Delta r_{np}(^{208}\text{Pb})\) linearly depends on the asymmetry parameter irrespective of the model. The nuclei having central depletion in the proton density (e.g., \(^{34}\text{Si}, ^{46,58}\text{Ar}\) and \(^{56}\text{S}\), see Table 2) follow the trend similar to those having no central depletion. Central density for the protons in the nuclei with central depletion may be as small as half of its maximum density (i.e., \(b_p \sim 0.5\)). The results presented in Fig. 7 clearly suggests that the depletion in the central density of the nuclei does not affect the systematics of the neutron-skin thickness in a noticeable manner.

For a better insight of the robustness of systematics of neutron-skin thickness with respect to the central depletion, we compare the neutrons and protons density profiles for \(^{34}\text{Si}\) and \(^{46}\text{Ar}\) with those of nuclei \(^{48}\text{Ca}, ^{132}\text{Sn}\) and \(^{208}\text{Pb}\) in Fig. 8. These densities are plotted as a function of the radial co-ordinate \(r\) scaled by a factor of \(A^{1/3}\). The density distributions for the protons and neutrons for the \(^{34}\text{Si}\) and \(^{46}\text{Ar}\) show similar trends in Fig. 8 where the central depletion in proton density for \(^{34}\text{Si}\) and \(^{46}\text{Ar}\) is clearly seen which is at variance with the densities of \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\). The effects of such depletion in proton density are partially compensated by neutron density as can be seen from Fig. 8 (b). It may be noted that for the values of \(r \sim 0.75A^{1/3}\) fm, neutron densities of \(^{34}\text{Si}\) and \(^{46}\text{Ar}\) remain constant compared to neutron densities of \(^{48}\text{Ca}, ^{132}\text{Sn}\) and \(^{208}\text{Pb}\) nuclei. One might thus expect these differences in the density distributions to affect the neutron-skin thickness in the bubble nuclei.

We plot the differences between the neutron and proton densities \((\rho_n - \rho_p)\) and \(r^2(\rho_n - \rho_p)\) as a function of \(r/A^{1/3}\) in Fig. 9. The latter quantity may be more appropriate in order to assess the influence of central depletion in the proton density on the neutron-skin thickness \(\Delta r_{np}\). The differences between neutron and proton densities \(\rho_n - \rho_p\) for \(^{34}\text{Si}\) and \(^{46}\text{Ar}\) are maximum at center, whereas it tends to be small in case of \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\) nuclei at the center. The behaviour of \((\rho_n - \rho_p)\) for \(^{132}\text{Sn}\) nucleus close to the center is similar to those for \(^{34}\text{Si}\) and \(^{46}\text{Ar}\) nuclei, but at moderate value of \(r/A^{1/3}\), it follows the trend as those of \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\). The differences \(r^2(\rho_n - \rho_p)\), however, look pretty much the same for the nuclei with or without the central depletion as can be noticed from Fig. 9 (b). The values of \(r^2(\rho_n - \rho_p)\) peak around \(r \sim A^{1/3}\) fm. The peak heights are mainly governed by the number of excess neutrons, i.e., \(N - Z\). The dissimilarities in the \((\rho_n - \rho_p)\), arising due to the central depletion, may not have any imprints on the values of the neutron-skin thickness. In other words, the value of the neutron-skin thickness is mainly governed by the differences between the neutron and proton densities around the surface region and the effects of the central depletion do not percolate that far.
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4. Conclusions

The existence of depletion in the central density is explored in the isotonic chains of nuclei with the neutron numbers \( N = 20 \) and 28 using different variants of the relativistic mean-field model. The RMF models considered are the ones (a) which include contributions from the non-linear self- and mixed-interactions of the mesons with constant coupling strengths and (b) with only linear contributions from the mesons where the coupling strengths are density-dependent. The central depletion of proton...
density is observed in several spherical as well as in deformed nuclei and found to be affected not only by occupancy of $2s_{1/2}$ state as usually believed, but, also by pairing correlations, deformation and neutron to proton ratio. Bubble parameter shows the inverse dependence on deformation and the occupancy in the s-orbit. The depletion in spherical nuclei are found to disappear if the occupancy of $2s_{1/2}$ becomes more than $\sim 50\%$, while, it vanishes in deformed nuclei even though the occupancy of $2s_{1/2}$ is almost zero. On the prolate side the bubble parameter decreases even though the occupancy of $2s_{1/2}$ state is almost zero whereas on oblate side, decrease in the bubble parameter is associated with the combined effects of deformation and the increase in occupancy of $2s_{1/2}$ state.

We find that the density-dependent point coupling model yields smaller central depletion in general. For instance, the density-dependent point coupling model results in practically no central depletion in the case of $^{40}$Ar nucleus, whereas other RMF models show strong central depletion in the proton density. This model dependence is traced back to be associated with the differences in the strength of the spin-orbit potentials in these models.

The imprints of the central depletion on the systematics of the neutron-skin thickness are investigated. It is found that the nuclei with the proton central density as small as half of its maximum density do not alter the systematics of neutron-skin thickness. The variations of neutron-skin thickness with the asymmetry for the nuclei with central depletion are very much in the harmony with those of the normal nuclei (i.e., no central depletion). The profiles for neutron and proton densities for the nuclei with central depletion do seem to be at variance with those of the normal nuclei. The effects of the central depletion, however, do not percolate to the surface regions which contributes maximally to the determination of the neutron-skin thickness.

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