QCD Phenomenology and Heavy Ion Physics

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Abstract. We describe application of saturation/Color Glass Condensate physics to heavy ion collisions, concentrating on several important observables. We show how simple saturation-inspired assumptions about particle production in heavy ion collisions allow one to accurately describe total charged particle multiplicity as a function of the collision’s centrality, of the center of mass energy, and of the particle rapidity. We will then describe how predictions of saturation physics can be tested by studying energy/rapidity dependence of particle spectra in p(d)A collisions. We concentrate on the nuclear modification factor $R_{pA}$ for gluon production. We show that at moderately high energy/rapidity the nuclear modification factor $R_{pA}$ exhibits Cronin enhancement. As the energy/rapidity increases, $R_{pA}$ decreases. At sufficiently high energy/rapidity $R_{pA}$ becomes less than 1 for all values of $p_T$ indicating the onset of suppression of gluon production due to quantum evolution effects. These predictions were confirmed by RHIC data.

1. Introduction

In this talk we will discuss saturation/Color Glass Condensate physics [1, 2, 3, 4] in the context of heavy ion collisions. Saturation physics is based on the fact that small-$x$ wave functions of ultrarelativistic hadrons and nuclei are characterized by a hard scale $Q_s$, known as the saturation scale [1, 2, 3, 4]. The scale $Q_s$ arises due to saturation of high partonic densities at small-$x$ and is an increasing function of energy and atomic number of the nucleus [5, 6, 7, 8, 4]. This large scale makes the strong coupling constant small $\alpha_s(Q_s) \ll 1$ leading to dominance of the classical gluonic fields in high energy processes [4, 9]. Therefore, the dominant gluon field in high energy collisions is given by the classical field of the scattering color charges [10]. Corresponding gluon production cross section was found for $pA$ collisions in [11] and the effects of quantum small-$x$ evolution [5] were included in it in [12]. The gluon production cross section for heavy ion collisions ($AA$) at the classical level has been studied both numerically [13] and analytically [14]. It is not quite clear at present how to include the effects of nonlinear quantum evolution [5] in the results of [13, 14]. Below we will show how one can understand many properties of the RHIC data on particle multiplicities in heavy ion collisions using general saturation principles without an exact knowledge of gluon production cross section in $AA$ [15, 16, 17].

We will proceed by discussing particle production in $p(d)A$ collisions, where, due to absence of final state interactions, predictions of saturation physics can be tested directly. Concentrating on the nuclear modification factor $R_{pA}$ we argue that the quasi-classical gluon production cross section calculated in [11] leads to Cronin enhancement [18, 19, 20, 21, 22]. This regime is probably relevant for mid-rapidity particle production in $\sqrt{s} = 200$ GeV $dAu$ collisions at RHIC. At higher (forward) rapidities the effects of CGC small-$x$ evolution would lead to suppression of $R_{pA}$ and disappearance of Cronin effect [23, 18, 24]. The suppression predicted in
[23, 18, 24] has been observed experimentally in [25, 26, 27, 28, 29], confirming the expectation of saturation/Color Glass physics. We conclude by presenting a quantitative saturation-inspired model [30] describing the data of [26].

2. Particle Multiplicity from Saturation Approach

2.1. Multiplicity at Mid-Rapidity Versus Centrality

As was argued in [4, 10], the gluon production in \( AA \) collisions is described by the classical fields. Classical field \( A_\mu \sim 1/g \) leads to produced gluon multiplicity

\[
\frac{dN}{d^2k d^2b dy} \sim \langle A_\mu A_\mu \rangle \sim \frac{1}{\alpha_s}. \tag{1}
\]

Thus, gluon transverse momentum spectrum described by a single scale \( Q_s \) can be written as

\[
\frac{dN}{d^2k d^2b dy} = \frac{1}{\alpha_s} f(k_\perp/Q_s) \tag{2}
\]

with some unknown function \( f(k_\perp/Q_s) \) to be determined by actual calculations. Integrating over \( k \) and \( b \) yields

\[
\frac{dN}{dy} = \text{const} \frac{1}{\alpha_s} \pi R^2 Q_s^2. \tag{3}
\]

where the value of the constant is determined from \( f(k_\perp/Q_s) \) and is not important here. Following [15, 16] we assume that the scale for the coupling constant in Eq. (3) is set by \( Q_s \). (This step, of course, goes beyond the classical limit and assumes that Eq. (3) is valid even when running coupling corrections are included.) Then, as \( R^2 \sim A^{2/3} \) and if \( Q_s^2 \sim A^{1/3} [4, 9] \), together with running coupling

\[
\alpha_s(Q_s) = \frac{1}{b \ln Q_s^2/A^2} \sim \frac{1}{\ln A}. \tag{4}
\]

we conclude from Eq. (3) that

\[
\frac{1}{A} \frac{dN}{d\eta} \sim \ln A. \tag{5}
\]
For heavy ion experiments at different collision centralities we substitute $A$ by the number of participants $N_{\text{part}}$ so that Eq. (5) becomes

$$\frac{1}{N_{\text{part}}} \frac{dN}{d\eta} \sim \ln N_{\text{part}}. \quad (6)$$

Eq. (6) allowed the authors of [15] to correctly predict the behavior of particle multiplicity at mid-rapidity at RHIC as a function of centrality at $\sqrt{s} = 130 \text{ GeV}$. A fit of particle multiplicity at various values of rapidity at $\sqrt{s} = 130 \text{ GeV}$ taken from [16] is shown along with PHOBOS data in Fig. 1.

2.2. Multiplicity as a Function of Energy

Eq. (3) also allows one to test whether the scaling of total particle multiplicity with energy is consistent with saturation/Color Glass predictions. Using the fact that $Q_{s}^{2} \sim 1/x_{Bj}$ [5, 7] in Eq. (3) and, for the moment, dropping the slower $Q_{s}$-dependence in $\alpha_{s}$ leads to

$$\frac{dN/d\eta(\sqrt{s_{1}})}{dN/d\eta(\sqrt{s_{2}})} = \left(\frac{\sqrt{s_{1}}}{\sqrt{s_{2}}}\right)^{\lambda}. \quad (7)$$

Using PHOBOS data for total charge multiplicity at $\sqrt{s} = 130 \text{ GeV}$ for most central collisions

$$\frac{dN}{d\eta}(\sqrt{s} = 130 \text{ GeV}) = 555 \pm 12(\text{stat}) \pm 35(\text{syst}) \quad (8)$$

together with $\lambda = 0.25 \pm 0.3$ obtained in [31] by analyzing HERA data, Kharzeev and Levin [16] predicted the total charged particle multiplicity at $\sqrt{s} = 200 \text{ GeV}$ to be

$$\frac{dN}{d\eta}(\sqrt{s} = 200 \text{ GeV}) = 616 \div 634 \quad (9)$$

in agreement with the later measured PHOBOS result

$$\frac{dN}{d\eta}(\sqrt{s} = 200 \text{ GeV}) = 650 \pm 35(\text{stat}). \quad (10)$$

Figure 2. (A) Saturation model fit of the PHOBOS data on total charged particle multiplicity in $AuAu$ collisions as a function of rapidity and centrality at $\sqrt{s} = 130 \text{ GeV}$ taken from [16]. (B) Prediction of the same model compared to BRAHMS $dAu$ data.
2.3. $\frac{dN}{d\eta}$ Versus $\eta$ and $N_{\text{part}}$

Describing rapidity distribution of the produced particles requires a little modeling of the spectra. Assuming $k_T$-factorization for particle production cross section

$$\frac{d\sigma^{AA}}{d^2k \, dy} = \frac{2}{C_F} \frac{\alpha_s}{\bar{\Lambda}^2} \int d^2q \, \phi_A(q) \phi_A(k-q) \tag{11}$$

along with saturation-inspired unintegrated gluon distribution functions ($\phi_A(k) \sim \alpha_s/k^2$ if $k_\perp > Q_s$ and $\phi_A(k) \sim S/\alpha_s$ if $k_\perp < Q_s$) the authors of [16] produced an impressive fit of the PHOBOS data on charged particle multiplicities as functions of rapidity and centrality at $\sqrt{s} = 130$ GeV shown in Fig. 2A. The predictions made in [16] for similar multiplicity data at $\sqrt{s} = 200$ GeV were also in good agreement with the later published BRAHMS data. Recently, the same model was successfully used to predict rapidity distribution of charged particle multiplicity in $dA$ collisions, as shown in Fig. 2B.

Phenomenological success of the saturation models presented above does not contradict the possibility of strong final state interactions leading to formation of quark-gluon plasma. As was argued in [32], thermalization in the saturation framework would not introduce any fundamentally new scales leaving Eqs. (3) and (6) practically unchanged. (In fact, thermalization of [32] appears only to introduce a slowly-varying extra factor of $\alpha_s^{-2/5}(Q_s)$ in Eq. (3).) Late stage interactions are also not very likely to significantly modify the rapidity distribution of Fig. 2 due to causality constraints.

3. Particle Production in p(d)A Collisions

Here we will discuss particle production and transverse momentum spectra in $dA$ collisions. The first step in understanding particle (gluon) production in $pA$ collisions in the saturation/Color Glass Condensate framework is finding the classical gluon field of the colliding proton and nucleus in McLerran-Venugopalan model. The corresponding gluon production cross section in $pA$ was constructed in [11] yielding

$$\frac{d\sigma^{pA}}{d^2k \, dy} = \int d^2b \, a^2 \, a^2 \, \frac{1}{(2\pi)^2} \frac{\alpha_s C_F}{\pi^2} \frac{x \cdot y}{x^2 y^2} e^{-ik \cdot (x-y)} \times \left[ 1 - e^{-z^2 Q_s^2 \ln(1/(x+\Lambda))/4} - e^{-z^2 Q_s^2 \ln(1/(y+\Lambda))/4} + e^{-z^2 Q_s^2 \ln(1/(x-y+\Lambda))/4} \right], \tag{12}$$

where $k$ and $y$ are the produced gluon’s transverse momentum and rapidity, $b$ is the proton’s impact factor, $Q_s$ is the saturation scale in McLerran-Venugopalan model and $x, y$ are two-dimensional gluon transverse position vectors which are integrated over.

To find the nuclear modification factor

$$R^{pA}(k, y) = \frac{\frac{d\sigma^{pA}}{d^2k \, dy}}{A \frac{d\sigma^{pp}}{d^2k \, dy}}. \tag{13}$$

resulting from Eq. (12) one can integrate it explicitly over $x$ and $y$ neglecting the logarithms in the exponents obtaining

$$R^{pA}(k_T) = \frac{k^4}{Q_s^2} \left\{ -\frac{1}{k_T^2} + \frac{2}{k_T^2} e^{-k^2/Q_s^2} + \frac{1}{Q_s^2} e^{-k^2/Q_s^2} \left[ \ln \frac{Q_s^2}{4 \Lambda^2 k_T^2} + \text{Ei} \left( \frac{k^2}{Q_s^2} \right) \right] \right\}. \tag{14}$$

The ratio $R^{pA}(k_T)$ is plotted in Fig. 3 for $\Lambda = 0.2 \, Q_s$. It clearly exhibits an enhancement at high-$k_T$ characteristic of Cronin effect. Similar conclusions have been reached by other authors [19, 20, 21, 22]. To prove that Eq. (12) leads to Cronin effect one can expand it at high $k_T$:

$$R^{pA}(k_T) = 1 + \frac{3}{2} \frac{Q_s^2}{k_T^2} \ln \frac{k_T^2}{\Lambda^2} + \ldots, \quad k_T \to \infty. \tag{15}$$
The plus sign in front of the first correction to 1 on the right hand side of Eq. (15) implies that $R_{pA}$ approaches 1 from above at high $k_T$ indicating Cronin enhancement. Eq. (15), together with Fig. 3, can also be used to illustrate the dependence of the Cronin maximum on centrality.

As one can easily see from Eqs. (15) and (14) the position of the Cronin maximum is given by $k_T \sim Q_{s0}$ and the height of the maximum scales as $R_{pA}(k_T \sim Q_{s0}) \sim \ln Q_{s0}$. Since $Q_{s0}^2 \sim A^{1/3}$ we conclude that the position and height of the Cronin peak are increasing functions of the centrality of $pA$ collisions.

Before including quantum evolution into Eq. (12), let us first note that Eq. (12), even though it includes multiple rescatterings, can still be written in $k_T$-factorized form (!) [12, 18]

$$d\sigma^{pA}\over d^2k \, dy = 2 \alpha_s \frac{1}{C_F} \int d^2q \, \phi_p(q,0) \phi_A(k-q,0),$$

(16)

with the unintegrated “gluon distributions” given by

$$\phi_A(x, k^2) = \frac{C_F}{\alpha_s (2\pi)^2} \int d^2r \, e^{-ik \cdot r} \, \nabla^2 r \, n_G(r,b,y = \ln 1/x),$$

(17)

and

$$\phi_p(x, k^2) = \frac{C_F}{\alpha_s (2\pi)^2} \int d^2r \, e^{-ik \cdot r} \, \nabla^2 r \, n_G(r,b,y = \ln 1/x).$$

(18)

Here $N_G(r,b,y = \ln 1/x)$ and $n_G(r,b,y = \ln 1/x)$ are forward scattering amplitudes of a gluon dipole of size $r$ located at impact parameter $b$ on a nucleus and a proton correspondingly. Eq. (12) is reproduced by using $N_G(r,b,0) = 1 - e^{-r^2 Q_{s0}^2 \ln(1/r\Lambda)/4}$ and $n_G(r,b,0) = r^2 \Lambda^2 \ln(1/r\Lambda)$ [18].

As was shown in [12], Eq. (16) makes inclusion of quantum small-$x$ evolution straightforward. A tedious analysis shows that the inclusion of evolution preserves $k_T$-factorization of Eq. (16) yielding the full answer for inclusive cross section [12]

$$d\sigma^{pA}\over d^2k \, dy = 2 \alpha_s \frac{1}{C_F} \int d^2q \, \phi_p(q, Y-y) \phi_A(k-q,y),$$

(19)

where $Y$ is the full rapidity interval between the proton and the nucleus. The gluon distribution functions in Eq. (19) are still given by Eqs. (17) and (18), but now with $n_G$ given by the solution.
of the BFKL equation and with \( N_G \) given by the solution of the non-linear evolution equation [5].

To analyze the nuclear modification factor given by Eq. (19) one has to separately study three regions of transverse momentum: (i) \( k_T > k_{\text{geom}} \), (ii) \( Q_s(y) < k_T \leq k_{\text{geom}} \) and (iii) \( k_T \sim Q_s(y) \).

For \( k_T \ll Q_s(y) \) one always has suppression due to parton saturation [18], as one can see already in Fig. 3. \( Q_s(y) \) is the saturation scale (such that \( Q_{s0} = Q_s(y = 0) \)) and \( k_{\text{geom}} \) is the scale describing the border of the geometric scaling region [7] \( k_{\text{geom}} \approx Q^2_s(y)/Q_{s0} \). As was shown in [18], in region (i) the double logarithmic approximation applies giving

\[
R^{pA}(k_T, y) \bigg|_{k_T > k_{\text{geom}}} \sim \exp \left[ 2 \sqrt{2} \alpha_s y \left( \sqrt{\ln \frac{k_T}{Q_{s0}}} - \sqrt{\ln \frac{k_T}{\Lambda}} \right) \right] < 1
\]  

(20)
since \( Q_{s0} \gg \Lambda \). Inside the geometric scaling region (ii) nuclear modification factor becomes [18]

\[
R^{pA}(k_T, y) \bigg|_{Q_s(y) < k_T \leq k_{\text{geom}}} \sim A^{-1/6} \exp \left[ \frac{\ln^2 \frac{k_T}{A} - \ln^2 \frac{k_T}{Q_{s0}}}{14 \zeta(3) \alpha_s y} \right] \ll 1,
\]  

(21)

so that at very high \( y \) one gets \( R^{pA} \sim A^{-1/6} \ll 1 \) for \( A \gg 1 \), reproducing the original prediction of the initial state suppression made in [23]. Finally, to understand what happens to Cronin maximum one has to study \( R^{pA} \) at \( k_T = Q_s(y) \). The analysis done in [18] gives the nuclear modification factor in the region (iii)

\[
R^{pA}(k_T = Q_s(y), y) \sim \exp \left\{ 4 \alpha_s y \left( 1 - \sqrt{1 + \frac{\ln A^{1/6}}{2 \alpha_s y}} \right) \right\} \ll 1.
\]  

(22)

Eq. (22) at very high \( y \) also gives \( R^{pA} \sim A^{-1/6} \), similar to Eq. (21), indicating that as energy increases Cronin maximum should gradually decrease and eventually disappear completely. Eqs. (20), (21) and (22) demonstrate that at very high energy/rapidity the nuclear modification factor \( R^{pA} \) should become less than 1 for all \( k_T \). Since \( R^{pA} \sim A^{-1/6} \) at very large \( y \), one should also expect that \( R^{pA} \) will become a decreasing function of centrality at high energies/rapidities.
Our conclusions regarding the variation of $R^{pA}$ with energy/centrality are summarized in Fig. 4. The top solid curve in Fig. 4 is the same as in the quasi-classical approximation shown in Fig. 3. It corresponds to moderately high energy/rapidity. As energy/rapidity increases $R^{pA}$ decreases (dash-dotted and dashed lines) and the Cronin peak flattens, eventually approaching a flat curve (lower solid line) which has suppression at all $p_T$. Similar conclusions have been reached in [24].

Using a simple model for dipole amplitudes $N_G$ and $n_G$ in Eqs. (17) and (18) and adding a valence quark contribution to Eq. (19) one can fit the $R_{dAu}$ data at various rapidities reported by the BRAHMS collaboration [25, 26], as shown in Fig. 5 [30]. At mid-rapidity ($\eta = 0$) the nuclear modification factor $R_{dAu}$ in Fig. 5 exhibits Cronin enhancement. This indicates that particle production at mid-rapidity at RHIC is best described by the multiple rescatterings in McLerran-Venugopaln model. At higher rapidities BRAHMS data exhibits strong suppression with $R_{dAu}$ becoming less than 1 for all $p_T$ at pseudorapidity $\eta = 3.2$ in qualitative agreement with our predictions shown in Fig. 4 [18] and in quantitative agreement with our fit from [30] shown in Fig. 5. This indicates the onset of quantum evolution characteristic of saturation/Color Glass Condensate at forward rapidities.

Figure 5. Nuclear modification factor $R_{dAu}$ of charged particles for different rapidities. The fit uses the model described in [30]. Data is from [26].
