Signal protection methods in channels with Nakagami fading

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Abstract. Existing methods for modeling the quality of digital communications based on adaptive radio links usually assume the stationarity of the parameters of the investigated system and the method of frequencies’ group use. The possibilities of these methods are largely limited due to the complexity of calculating the posterior distributions of a hypothetical environment that occurs when a signal is received. As a result of this, at present, in engineering practice, there are few models that make it possible to evaluate the quality of quasicoherent reception of adaptive radio lines even in simple situations of orthogonal signals’ transmitting. The goal of this research is to develop a way to increase the noise immunity of communication in channels with fading under the Nakagami distribution law. Analytical dependences of the error probability and the radio link utilization coefficient on the permissible signal-to-noise ratio were obtained using intermittent communication in channels with fading according to the Nakagami distribution law. The method was developed to increase the noise immunity of communication in comparison with known reception methods. For example, with the fading depth 1.4, for the error probability 10⁻⁴, the energy gain will be more than 14 dB. The proposed method can be used to provide electronic protection in various communication systems.

1. Introduction

The transmission of information in communication and broadcasting channels using a free space of radio waves is always accompanied by fluctuations in the signal amplitude. Usually, two types of amplitude fluctuations are distinguished – fast and slow fading. The reason for the fast fading is the multipath structure of the signal and interfering rays. The reason for slow fading is the shadowing of the first Fresnel half-band of the radio signal on the track due to the relief features [1]. Since the nature of fast and slow fading is different, their influence is usually considered separately.

In most cases, the probability density distribution of the envelope of a radio signal during fast fading is described by the Rayleigh law. At the same time, some published papers indicate that the signal level in urban conditions can be a subject to a deeper fading, with the deepest fading observed near the transmitting point [2]. Depending on the nature of the propagation path, the envelope of fast fading (signal fluctuations) can have different distributions.
2. Signal fading in wireless communication channels with multipath propagation
In the general case, the Nakagami law with the distribution parameters $m$ and $\mu$ is used to describe the probability density distribution of the envelope:

$$W(\mu) = \frac{2\mu^{2m-1}}{\Gamma(m)} \left( \frac{m}{\mu^2} \right)^m \exp \left( \frac{m\mu^2}{\mu_0^2} \right),$$  \hspace{1cm} (1)

where $\Gamma(m)$ is the gamma function, $m$ is the distribution parameter that determines the degree of fluctuations in the communication channel (the smaller is $m$, the deeper is the fading in the channel), $\mu = \sqrt{\mu_0^2}$ is the root-mean-square value of $\mu$. It directly follows from (1): for $m = 1/2$, there is the truncated normal distribution, for $m = 1$, there is the Rayleigh distribution, and for $m \to \infty$, the value $\mu$ is constant (there is no fluctuation). If $m$ takes values greater than unity, then (1) approximates the Rice distribution with the different ratio between the regular and fluctuating components of the random variable $\mu$.

In [3] was considered the system that combines ratios for different reception methods with different laws of signal fading in the Nakagami channel, and the ratios for calculation of the error probability when transmitting signals in the Nakagami channel. However, the dependences for the case of operation of radio links with frequency adaptation are currently studied quite a bit. Adaptive radio links (ARLs) operate in a complex stochastic environment, caused by a priori uncertainty of the propagation conditions of radio waves along radio interference and communication paths, and by the difference in interference levels at the locations of correspondents. Radio signals arriving at the receiving point are subject to various kinds of interference.

3. Methods of modeling the digital communication quality based on adaptive radio links
Existing methods for modeling the quality of digital communications based on ARL usually assume the stationarity of the parameters of the investigated system and the method of frequencies' group use [4, 5]. The possibilities of these methods are largely limited due to the complexity of calculating the posterior distributions of a hypothetical environment that occurs when a signal is received.

As a result of this, at present, in engineering practice, there are few models that make it possible to evaluate the quality of quasi-coherent reception of ARL even in simple situations of transmission of orthogonal signals' transmitting. The envelope of the received signal is a locally stationary process characteristic of decameter communication systems during ionospheric scattering.

We can assume that the Nakagami distribution is more general for the description of signal fading, and an estimation of the distribution parameters $m$ and $\mu$ is necessary to determine the characteristics of the communication channel. Indeed, the distribution parameter $\mu$ characterizes the average power of the fading signal, and the parameter $m$ characterizes the depth of fading. It is known that the estimation of the parameter $m$ is especially important in radio communication systems with a frequency adaptation. Estimation results [6–8] show that radio communication systems, which take into account differences in the depth of fading between frequency-spaced radio channels, have a significant energy gain compared to systems in which the differences are neglected.

When analyzing time-duplex communication systems operating on the same carrier frequency, down-and-up signal fading is correlated under the condition that $T \ll \tau$, where $T$ is the signal propagation time along the communication line, $\tau$ is the signal envelope correlation interval in time. In this case, by measuring the signal level at the input of the receiver of the subscriber station (SS) from the base station (BS), it is possible to perform gating of the transmitter at the moments most favorable for signal propagation, that is, when the signal-to-noise ratio (SNR) at the input of its receiver above a given threshold value $(h_i)$. Thanks to this, communication between subscribers is eliminated during moments of deep fading of signals, when the largest number of errors is recorded.
4. The study of the communication reliability of radio links during fading

In this work, we study the dependence of the radio lines’ communication reliability during fading \((m = 0.6; 1; 1.4; 8)\) for signals with frequency adaptation B DPSK at the required \(h = 15\) dB.

The frequency of signal fading during propagation in the troposphere is tens of hertz depending on the radio frequency. For example, at a carrier frequency of 4 GHz they will be 10-15 Hz, and at 10 Hz – 20-25 Hz. Accordingly, the duration of fading is tens of milliseconds. The indicated duration should not lead to large signal delays during intermittent communication.

The amplitude of the signal envelope \((R)\) during fading according to the Nakagami law at the receiving point is described by the probability density

\[
p_R = \frac{2}{\Gamma(m) \mu^2} R^{2m-1} \exp\left(-\frac{m}{\mu^2} R^2\right).
\]

Using the well-known relations

\[
h = \frac{R^2 T}{N}, \quad h_0 = \frac{\langle R^2 \rangle T}{N},
\]

where \(T\) is the duration of the received bit, \(N\) is the spectral power density of white Gaussian noise (WGN), \(h\) is the current ratio of the signal energy to the noise spectral density, \(h_0\) is the average ratio of the signal energy to the noise spectral density, and also the dependences

\[
p_R \cdot dR = f_h \cdot dh, \quad f_h = p_R \cdot \frac{dR}{dh}, \quad \frac{dR}{dh} = \frac{1}{2} \sqrt{\frac{N}{hT}},
\]

we determine the dependence of the probability density \(f_h\) on the SNR for known \(m\) and \(h_0\):

\[
f_h = \frac{h^{m-1}}{\Gamma(m)} \left(\frac{m}{h_0}\right) \exp\left(-\frac{h}{h_0}\right).
\]

The obtained values of the probability density function from the SNR for \(h_0 = 2.5\) and various values of the parameter \(m\) are shown in figure 1.

![Figure 1](image_url)
The values of the integral probability distribution function of the SNR for $h_0 = 2.5$ and various values of the parameter $m$ are shown in figure 2.

![Cumulative probability distribution function](image)

**Figure 2.** Values of the cumulative probability distribution function from the SNR for $h_0 = 2.5$ and various values of the parameter $m$.

For the intermittent communication option, we determine the error probability of the transmitted data and the spectral efficiency of the communication channel, which depends on the utilization of the radio link [9].

Since in case of intermittent communication, data transfer between a BS and a SS is possible only when the SNR value at the receiver input exceeds a predetermined threshold level ($h_t$), it is advisable to determine the radio link utilization coefficient ($n$) as the ratio of data transmission time to the total communication session time. For the given value of the fading parameter ($m$), it will be

$$
\eta(h_0) = \int_{h_t}^{\infty} \left( \frac{m}{h_t} \right)^m \exp \left( - \frac{h}{h_0} \right) dh = \frac{1}{\Gamma(m)} \left( \frac{m}{h_0} \right)^m \int_{h_t}^{\infty} h^{m-1} \exp \left( - \frac{hm}{h_0} \right) dh.
$$

After appropriate conversions, the utilization coefficient of the radio line becomes equal to

$$
\eta(h_0) = \frac{\Gamma \left( m, \frac{h_t}{h_0} \right)}{\Gamma(m)}.
$$

![Radio link utilization coefficient depending on the parameter m at $h_t = h_0$.](image)

**Figure 3.** Radio link utilization coefficient depending on the parameter $m$ at $h_t = h_0$. 

(5)
From the analysis of the graphical dependence in Figure 3, it follows that for small parameters $m$, the value of $n$ increases rapidly, since in this case the average value of the probability density of the SNR (Figure 1) tends to zero, and the depth of fading and the low values probability of the signal level increase.

For practical use in wireless communication channels, it is advisable to choose a threshold equal to $k h_0$. At the same time, by varying the value of $k$, the data transfer rate with acceptable noise immunity can be chosen. The radio link utilization coefficient and data transfer rate in this case (at the fixed value $k$) will depend only on the parameter $m$ characterizing the fading depth.

Averaging the error probabilities of incoherent reception of the $BDPSK$ signal for the channel with WGN according to the statistics of received fading (4) over all values of $h$ for continuous communication and $h \geq h_t$ for intermittent communication, we determine the probabilities of erroneous reception in the channel with fading of the signal according to the Nakagami law for continuous (6) and intermittent (7) communication.

$$p_{nc}(h_0) = \frac{1}{2} \int_0^{h_0} \exp\left(\alpha h \frac{h^{m-1}}{m} \left(\frac{m}{h_0}\right)^m \exp\left(-\frac{h}{h_0}\right)\right)^m dh = \frac{1}{\Gamma(m)} \left(\frac{m}{h_0}\right)^m \int_0^{h_0} \exp\left(-\frac{h}{h_0} + \alpha\right) dh,$$

$$p_{nc,f}(h_0) = \frac{1}{2} \int_{h_0}^{\infty} \exp\left(\alpha h \frac{h^{m-1}}{m} \left(\frac{m}{h_0}\right)^m \exp\left(-\frac{h}{h_0}\right)\right)^m dh = \frac{1}{\Gamma(m)} \left(\frac{m}{h_0}\right)^m \int_{h_0}^{\infty} \exp\left(-\frac{h}{h_0} + \alpha h\right) dh,$$

where $\alpha = 1$ and 0.5, respectively, for incoherent reception of phase- and frequency-manipulated signals, $p_{nc}(h_0), p_{nc,f}(h_0)$ are, respectively, the probabilities of signal reception errors during continuous and discontinuous communication in a fading channel. In formula (7), $h_t = h_0$.

The radio link utilization coefficient $(n)$ in the lower integration limit (7) shows that the transmitter power is unchanged, that is, it does not increase inversely with $n$, and, accordingly, the energy of each bit and the average SNR decrease $n$ times.

After appropriate transformations, the probability of erroneous reception of the signal in a channel with fading according to the Nakagami law with continuous (6) and intermittent (7) communications become equal, respectively:

$$p_{nc}(h_0) = \frac{1}{2} \left(\frac{m}{h_0}\right)^m \left(\frac{m}{h_0} + \alpha\right)^m,$$

$$p_{nc,f}(h_0) = \frac{1}{2} \left(\frac{m}{h_0}\right)^m \left(\frac{m}{h_0} \frac{\Gamma(m)}{\Gamma(m,m)} + \alpha\right)^m \Gamma\left(m + \alpha h_0, \frac{\Gamma(m)}{\Gamma(m,m)}\right).$$

Figure 4 shows the error probability values of incoherent reception of the $BDPSK$ signal in channels with fading according to the Nakagami law.

The analysis of the curves in Figure 4 shows that, with a high SNR, the noise immunity of intermittent communication is much better than continuous. For small values of the parameter $m$, this gain occurs at lower values of the SNR, since in this case, data transmission is limited to deeper fading, which is the main cause of errors in signal reception.
Figure 4. Error probabilities of incoherent reception of the BDPSK signal in channels with fading according to the Nakagami law with continuous (solid lines) and intermittent (dashed lines) communications.

5. Conclusions

Analytical dependences of the error probability and the radio link utilization coefficient on the permissible signal-to-interference ratio are obtained using intermittent communication in channels with fading according to the Nakagami distribution law.

As the result of the research, the method was developed to increase the noise immunity of communication in comparison with known reception methods. For example, with the fading depth 1.4, for the error probability $10^{-4}$, the energy gain will be more than 14 dB.

The proposed method can be used to provide electronic protection in various communication systems.

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References

[1] Sklar Bernard 2001 Digital Communication. Fundamentals and Application. (Prentice Hall)
[2] Lipatnikov V A 2018 Methods of radio monitoring. Theory and practice (Saint Petersburg)
[3] Chuchin E V 2014 System of quality models for transmitting digital signals over radio channels with Nakagami fading Scientific journal of Kursk State University 1 42-6
[4] Kiselev I G, Andrianov M N 2007 About increasing the noise immunity of the transmission of discrete messages in channels with fading Mobile systems 4 13-6
[5] Andrianov M N 2009 Features of intermittent communication in a channel with lognormal fading Metrology 5 35-43
[6] Rabin A V, Dobroselskij M A and Lipatnikov V A 2018 Research of characteristics of noise immunity with use of orthogonal coding Questions of radio electronics 10 6 80-5
[7] Rabin A V 2019 Orthogonal coding as a way to increase noise immunity when transmitting signals over multipath channels with fading Sensors and systems 4 (235) 7-12

[8] Lipatnikov V A, Kuzin P I 2016 Method of increasing the efficiency of changing adaptation parameters when receiving information in HF and VHF radio communication systems Automation of management processes 4(46) 18-22

[9] Bitkov A N, Zhilin A V, Kasibin S V, Komarovich V F, Kuznetsov S I and Lipatnikov V A 2005 Automatic frequency selection device Patent for invention dated 13.07.2005 no 2295761 RU (Moscow: Rospatent)