EXPERIMENTAL DATA ON THE SINGLE SPIN ASYMMETRY AND THEIR INTERPRETATIONS BY THE CHROMO-MAGNETIC STRING MODEL

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Abstract

An attempt is made to interpret the various existing experimental data on the single spin asymmetries in inclusive pion production by the polarized proton and antiproton beams. As the basis of analysis the chromo-magnetic string model is used. A whole measured kinematic region is covered. The successes and fails of such approach are outlined. The possible improvements of model are discussed.

Introduction

The growing amount of the experimental data on the single spin asymmetry (SSA) attracts attentions of the high energy spin community. This is understandable since the perturbative quantum chromo-dynamics (pQCD) predicts a zero spin effect at the asymptotic energy, while the experiments furnish up to now the opposite signatures. For example, SSA shows to be nonzero at the RHIC energy of $\sqrt{s} = 200$ GeV \cite{1}. Therefore the different approaches, not so strongly justified as pQCD, but able to explain all SSA effects are needed.

In 1988 the model was proposed \cite{2} for interpretation of the SSA in inclusive pion productions and $\Lambda$ inclusive polarization. The model was able to describe the scarce data existing at that time. Those experimental data came...
up from the unpolarized beams striking the polarized target and such experiments suffer from the poor statistical accuracies. Since later a lot of new data were published and mostly from the interaction of the polarized beams with pure hydrogen target. Recently the polarized Relativistic Heavy Ion Collider (pRHIC) becomes operational and the first data on the SSA for inclusive $\pi^0$ were published [1]. The PROZA experiment at IHEP, Protvino, produced for the first time the data on SSA in the polarized proton target fragmentation region using unpolarized proton and pion beams. The main goal of this article is to apply the chromo-magnetic string model (CMSM) of the paper [2] to the new experimental data on the inclusive pion asymmetries. The CMSM offers a simple analytical expression for SSA description. The original paper was devoted mostly to the interpretation of the SSA in inclusive $\pi^0$ production. In current paper we extend the CMSM to the description of the SSA in the inclusive charged pion production too. In section 1 we remind the main features of CMSM according to [2]. The formulae of CMSM are specified for two distinct kinematic regions: the central (CR) and beam fragmentation (BF) region. Section 2 describes the application of the model to the CR, while the section 3 is devoted to the BF region. Also in Section 3 we expand the model to the SSA in the inclusive charged pion productions. The unique data of E704 experiment on the SSA obtained with the polarized antiproton beam are analyzed too in scope of the isotopic spin and charge conservations. In summary we outline the successes and fails of the CMSM in description of the SSA in inclusive pion production at high energy. We make some predictions and indicate the possible improvements of the CMSM model.

1 The chromo-magnetic string model

The basis of this model is following. After collision of two hadrons the color tube (string) is stretched between them. In simple case this tube contains the flux of the chromo-electric field. But such a system is not stable. In order to make it stable the chromo-magnetic field should circulate around this tube. The interaction of this field with the color magnetic moment of the polarized quark leads to a kick transverse to the string axis. This kick depends on the spin orientation of the polarized quark. The estimate of this kick gives a magnitude of order $\delta p_T \simeq 0.1$ GeV/c [2]. If we introduce the
invariant differential cross section \( \rho = E \frac{d \sigma}{d^3 p} \) for polarized quark asymmetry we get the relation

\[ A_q = \frac{\rho(+) - \rho(-)}{\rho(+)} \],

(1)

where (+) and (-) arguments in \( \rho \) mean the up and down direction of the quark polarization. Since \( \delta p_T < p_T \), where \( p_T \) is a transverse momentum of the final pion to which the polarized quark fragments we get the following left-right asymmetry in the quark emission

\[ A_q(x) = \frac{d \sigma(p_T + \delta p_T) - d \sigma(p_T - \delta p_T)}{d \sigma(p_T + \delta p_T) + d \sigma(p_T - \delta p_T)} = \delta p_T \cdot \left( \frac{d \sigma}{d^3 p} \right) \cdot \frac{\delta}{\delta p_T} \cdot B. \]

(2)

Here \( B \) is a slope parameter of \( \rho \) as it is defined in standard way.

\[ B = \frac{\delta}{\delta p_T} \left( \frac{d \sigma}{d^3 p} \right) = \frac{\delta}{\delta p_T} (\ln \rho). \]

(3)

The general formula for description of the SSA can be expressed in the following form

\[ A_N(x) = P_q(x) \cdot A_q(x) \cdot w(x). \]

(4)

Here \( P_q(x) \) is a polarization of an initial quark, carrying a portion \( x \) of the initial polarized proton momentum, \( A_q(x) \) is so called a quark "analyzing power" defined above (2); \( w(x) \) presents a fractional contribution of a channel of interest in parton-parton interactions.

The transverse quark polarization (transversity), \( P_q(x) \), is related to the spin dependent structure function \( g_2(x) \) measured in deep inelastic scattering (DIS). Unfortunately this function has no parton interpretation. For the 100% polarized on mass-shell quark \( g_2(x) = 0 \). On the other hand it looks reasonable to expect that the \( x \)-behavior of \( P_q(x) \) is similar to that of \( g_2(x) \). Note that the present data [3] indicate the threshold like form of the function \( g_2(x) \) which becomes very small for \( x \leq 0.2 \). Similar effect is seen in experimental data for SSA in beam fragmentation region (see next chapters). Therefore such effect should be included in a more precise parametrization of \( P_q(x) \) in future. Since at moment the transversity can not be directly extracted from experimental data one needs the theoretical inputs. According
to the non-relativistic quark model (NRQM) the transversity may be taken in the following way

\[ P_q(x) = \frac{2}{3} \cdot x \text{ for } \pi^+, \quad P_q(x) = \frac{1}{3} \cdot x \text{ for } \pi^0 \text{ and } \]

\[ P_q(x) = -\frac{1}{3} \cdot x \text{ for } \pi^- . \]  

So the NRQM predicts distinct effects for the pion SSA. First, the sign of asymmetry is positive for \( \pi^+ \) and for \( \pi^0 \), while it is negative for \( \pi^- \). Second, the magnitude of asymmetry is expected to be higher for the \( \pi^+ \), than for \( \pi^0 \) and \( \pi^- \). These predictions are in general compatible with experimental data, though some deviations may happen.

Another theoretical inputs can be taken from paper [4] which aimed to explain the "spin crisis" by accounting the pion cloud around the proton. Starting with NRQM, after account for pion loops correction one gets \( \Delta u = 0.79 \) (the sum for two u-quarks), and \( \Delta d = -0.31 \) corresponding to \( P_d = -0.8P_u \).

In such a case we can take \( P_u(x) = 0.7x \) and \( P_d(x) = -0.55x \) leading to

\[ P_q(x) = 0.7 \cdot x \text{ for } \pi^+, \quad P_q(x) = 0.28 \cdot x \text{ for } \pi^0 \text{ and } \]

\[ P_q(x) = -0.55 \cdot x \text{ for } \pi^- . \]  

Comparing (6) to (5) one can see that the difference is essential only for \( \pi^- \) and this fact will be taken into account at comparisons of the calculated SSA with experimental data.

\( A_N(x) \) may depend on \( x_F \) which is a Feinman parameter, on \( p_T \) which is a transverse momentum of pion, and on \( s \) which is a square of the total energy in the center of mass system of colliding particles.

The invariant differential cross section is assumed to be measured before or simultaneously with SSA.

The general expression for weighting factor depends on the parton distribution function \( V^a(x) \) and parton fragmentation function \( D^a(x) \) (a-parton flavor). Assuming that the polarization information may be carried mostly through the polarized quark \( q \) and much less by gluons we get the weighting (or asymmetry diluting) factor through the following expression

\[ w(q) = \frac{\sigma(q)}{\sigma(q) + \sigma(g)} . \]

Assuming that the main contribution comes from \( qg' \) (for \( \sigma(q) \)) and from \( gg' \) (for \( \sigma(g) \)) scattering via the t-channel gluon exchange between the gluon \( g' \)
from unpolarized nucleon and quark \( q \) or gluon \( g \) from the polarized proton the contributions from quark and gluon may be written as

\[
\sigma(q) \propto C_F \int_x^1 V^q \left( \frac{x}{z} \right) D^q(z) \frac{dz}{z}, \\
\sigma(g) \propto C_A \int_x^1 V^g \left( \frac{x}{z} \right) D^g(z) \frac{dz}{z},
\]

(8)

where the color factors \( C_F = \frac{4}{3} \) and \( C_A = 3 \), \( z \) is a fraction of the polarized quark momentum carried by the final hadron.

The parton and gluon distribution functions were taken in the forms

\[
V^q(x) = x \cdot v(x) = 2.8 \cdot \sqrt{x} \cdot (1 - x)^2, \\
V^g(x) = x \cdot g(x) = 3.0 \cdot (1 - x)^5.
\]

(9)

The fragmentation functions for three types of inclusively produced pions were taken in the forms:

\[
D_{\pi^+}^u(z) = \frac{4}{3}(1 - z), \quad D_{\pi^0}^u(z) = \frac{2}{3}(1 - z), \quad D_{\pi^-}^u(z) \approx 0; \\
D_{\pi^+}^d(z) = D_{\pi^-}^d(z) = D_{\pi^0}^d(z) = (1 - z)^2.
\]

(10)

The conservation of the isotopic spin and charge in parton interaction leads to the relations

\[
D_{\pi^+}^u(z) = D_{\pi^-}^u(z) = D_{\pi^+}^d(z) = D_{\pi^-}^d(z), \\
D_{\pi^0}^u(z) = D_{\pi^0}^d(z) = \bar{D}_{\pi^+}^u(z) = \bar{D}_{\pi^-}^d(z).
\]

(11)

The practical applications of these relations will be given below in appropriate places.

The weighting factor (7) was calculated using the relations (8), (9), and (10). The final version of function \( w(x) \) for different pions may be approximated by

\[
w(x) = \frac{\sqrt{x}}{\sqrt{x} + c(1 - x)^{4.5}},
\]

(12)

where \( c \) is: \( c_+ = 0.48 \) for \( \pi^+ \), \( c_0 = 0.64 \) for \( \pi^0 \), and \( c_- = 0.96 \) for \( \pi^- \).

As it is seen from relations (4) and (12) the analyzing power is calculated from \( \rho \) and \( w(x) \) without any free parameter.

At lower energy, small \( p_T \) and \( x_F \) one may expect the significant contributions to the pion yields from resonance productions [5]. Therefore we arbitrary selected experimental data with \( \sqrt{s} \geq 10 \) GeV to avoid in some extent this problem.
2 Central region

At present we know 3 pieces of the published data on the SSA for inclusive $\pi^0$ production in the central region (no data for charged pions). They are:

1. Inclusive SSA for reaction

   $$p(\uparrow) + p \rightarrow \pi^0 + X.$$  \hfill (13)

   Experiment E704 at Fermilab measured this reaction at 200 GeV/c. Kinematic region: $-0.15 \leq x_F \leq 0.15$, $1.48 \leq p_T(GeV/c) \leq 4.31$ \[6\].

2. Inclusive SSA for reaction

   $$\bar{p}(\uparrow) + p \rightarrow \pi^0 + X.$$  \hfill (14)

   Experiment E704 at Fermilab measured this reaction at 200 GeV/c. Kinematic region: $-0.15 \leq x_F \leq 0.15$, $1.48 \leq p_T(GeV/c) \leq 3.35$ \[6\].

3. Inclusive SSA for reaction

   $$p + p(\uparrow) \rightarrow \pi^0 + X.$$  \hfill (15)

   Experiment PROZA-M at IHEP, Protvino, measured this reaction at 70 GeV/c. Kinematic region: $-0.15 \leq x_F \leq 0.15$, $1.05 \leq p_T(GeV/c) \leq 2.74$ \[7\].

In following we discuss these data in listed above order.

1. In E704 experiment up or down (to the horizontal plane) polarized proton beam of 200 GeV/c striked the unpolarized proton target. The inclusively produced $\pi^0$-s were detected by the electromagnetic calorimeters around the central region, that is, at the $\pi^0$ emission angle of $\approx 90^\circ$ in c.m.s. Since the experimental invariant differential cross section, $\rho$, is almost exponential in function of $p_T$ its slope parameter $B$ is practically constant. For example, at 200 GeV/c the slope parameter $B=(4.19\pm0.08)(GeV/c)^{-1}$ \[6\]. Then the quark analyzing power, $A_q = 0.1B = 0.419$.

According to the previous discussions \[5\] and \[6\] we take the transversity
\[ P_q(x) = \frac{1}{3} x, \text{ constant factor } c_0 = 0.64 \] and inserting all above relations into (14) we got the final result for \( \pi^0 \) asymmetry in the central region

\[ A_N(x) = \frac{0.14 \cdot x^{1.5}}{\sqrt{x} + 0.64(1 - x)^{4.5}}. \] \hspace{1cm} (16)

One can see several consequences of this formula (16). First, it is scaled on argument \( x = x_T = \frac{2p_T}{\sqrt{s}} \) assuming that the slope parameter is constant. Second, it becomes zero at \( x=0 \) mostly due to a quark polarization and in some extent due to the gluon contribution (second term in denominator). Third, when \( x \) approaches to one asymmetry increases as linear function of \( x \). All these behaviors remind the corresponding experimental asymmetry. The result of the model prediction according to the formula (16) is presented in Fig.1 by the solid line. This prediction is consistent with the experimental data at 200 GeV/c.

2. The second group of data is relevant to the SSA in inclusive \( \pi^0 \) production by the polarized antiproton beam of 200 GeV/c (E704 experiment) (14) [6]. This reaction differs from (13) by charge of quarks, that is, all quarks in polarized proton are replaced by the corresponding antiquarks in polarized antiproton. Therefore the charge conservation requires the equality of the SSA’s in these two reactions, namely

\[ A_N(\bar{p}(\uparrow) + p \rightarrow \pi^0 + X) = A_N(p(\uparrow) + p \rightarrow \pi^0 + X). \] \hspace{1cm} (17)

As seen (Fig. 1) this conclusion is consistent with the experimental data in frame of error bars.

3. Third piece of new data [7] is relevant to the reaction (15). The data were taken at 70 GeV/c unpolarized proton beam with polarized proton target. They are presented in Fig.1 with opposite sign of asymmetry (to make a comparison to the polarized beam case). The expression for asymmetry at 70 GeV/c deduced from the CMSM looks like

\[ A_N(x_T) = 0.2 \cdot \frac{x_T^{1.5}}{\sqrt{x_T} + 0.64(1 - x_T)^{4.5}}. \] \hspace{1cm} (18)

In comparison to (16) there are two changes. First is relevant to the energy difference and second- to the different slope parameters. At 70 GeV/c this parameter \( B = (5.89 \pm 0.08) GeV^{-1} \) according to [7]. The result of calculation is presented in Fig.1 by the dashed line. In this case one can see a good
Figure 1: Asymmetry in function of $p_T$ in the central region for reaction $p + p \to \pi^0 + X$. At 70 GeV/c the target is polarized, while at 200 GeV/c the beam is polarized. SSA is also shown for reaction $\bar{p}(\uparrow) + p \to \pi^0 + X$ at 200 GeV/c. The model predictions are plotted by the solid line for 200 GeV/c and by the dashed line for 70 GeV/c.
consistency of prediction and experimental data. In order to check the quantitative predictions of the CMSM one needs much better statistics.

4. $p(\uparrow) + p \rightarrow \pi^0 + X$, $\sqrt{s} = 200$ GeV (PHENIX experiment at RHIC). At moment there are no data on the analyzing power of this reaction at $\sqrt{s} = 200$ GeV. But PHENIX collaboration recently published the precise data on the invariant differential cross section of this reaction for $1 \leq p_T \leq 13$ GeV/c region at pseudorapidity $|\eta| \leq 0.39$. This cross section was parameterized by authors in the following form

$$\rho = E \frac{d^3\sigma}{dp^3} = A \cdot \left(1 + \frac{p_T}{p_0}\right)^{-n}. \quad (19)$$

Here $A = 386$ mb $\cdot$ GeV$^{-2}$, $p_0 = 1.219$ GeV/c and $n = 9.99$. Now we can calculate the slope parameter for the PHENIX invariant differential cross section

$$B(p_T) = \frac{dln\rho}{dp_T} = \frac{n}{p_0 + p_T}. \quad (20)$$

It is seen from (20) that $B(p_T)$ decreases with growth of $p_T$ as $p_T^{-1}$ and leads to the decrease of asymmetry at large transverse momentum. This trend is similar to one stemming out at the lower initial momentum ($\approx 200$ GeV/c) [9]. The $\rho$ at momenta 100 and 200 GeV/c was presented in the following form

$$\rho = E \frac{d^3\sigma}{dp^3} \propto (p_T^2 + M^2)^N \cdot (1 - x_T)^F. \quad (21)$$

The fit to the experimental data leads to the values: $N = -5.4 \pm 0.2$, $M^2 = (2.3 \pm 0.3)$ GeV$^2$ and $F = 7.1 \pm 0.4$, $x_T = \frac{2p_T}{\sqrt{s}}$. Therefore one can find the slope parameter

$$b(p_T) = \frac{2Np_T}{p_T^2 + M^2} - \frac{2F}{\sqrt{s}(1 - x_T)}. \quad (22)$$

The slope parameter at PHENIX [20] decreases in the same way in function of $p_T$ as the slope parameter at around E704 energy [22] indicating on the independence of the slope parameter from the initial energy at $\sqrt{s} = 20-200$ GeV energy region.

One can make the prediction for asymmetry in the above reaction for PHENIX. It seems very small, less than 1%. We hope to see soon the experimental data from RHIC on $A_N(\pi^0)$.

Concluding the discussions of the SSA in central region we note that better precisions in $A_N$ of order $\leq 1\%$ are needed for testing the CMSM.
3 Beam fragmentation region

In this domain at energies $\sqrt{s} \geq 10$ GeV there are the following pieces of data:

1. The STAR result on the SSA in reaction $p(\uparrow) + p \rightarrow \pi^0 + X$ at $\sqrt{s}=200$ GeV/c. Kinematic domain of measurement: $0.18 \leq x_F \leq 0.59$, $1.5 \leq p_T(\text{GeV/c}) \leq 2.3$, $|\eta| \approx 3.8$ [1].

2. The E704 result on the SSA in reaction $p(\uparrow) + p \rightarrow \pi^0 + X$ at 200 GeV/c. Kinematic domain: $0.03 \leq x_F \leq 0.9$, $0.5 \leq p_T(\text{GeV/c}) \leq 2.0$ [10].

3. The E704 result on the SSA in reaction $\bar{p}(\uparrow) + p \rightarrow \pi^0 + X$ at 200 GeV/c. Kinematic domain: $0.03 \leq x_F \leq 0.67$, $0.5 \leq p_T(\text{GeV/c}) \leq 2.0$ [11].

4. The E704 results on the SSA in reaction $p(\uparrow) + p \rightarrow \pi^\pm + X$ at 200 GeV/c. Kinematic domain of measurements: $0.2 \leq x_F \leq 0.9$, $0.2 \leq p_T(\text{GeV/c}) \leq 1.5$ [12].

5. The E704 results on the SSA in reaction $\bar{p} \uparrow + p \rightarrow \pi^\pm + X$ at 200 GeV/c. Kinematic domain of measurements: $0.2 \leq x_F \leq 0.9$, $0.2 \leq p_T(\text{GeV/c}) \leq 1.5$ [13].

All listed above experimental data are shown in Fig.2 and Fig.3. The expression for asymmetry [4] is still correct, as well as for the weighting factor $w(x)$. The main change consists in replacing the argument $x$ by $x_F$, which presents now the portion of momentum acquired by hadron in the fragmentation of polarized quark. SSA may be written in the following way (see (4))

$$A_N(x_F, p_T, s) = P_q(x_F) \cdot A_q(p_T, x_F, s) \cdot w(x_F, c).$$

(23)

The factor $c$ was defined earlier (see [12]).

In order to calculate $A_q$ we must know the slope parameter $B$ defined earlier [3] for each reaction at the appropriate kinematic domain. In following we discuss point by point the above experimental data on SSA’s in the beam fragmentation region.

1. STAR made a measurement of SSA and $\rho$ (invariant differential cross-section) at $\sqrt{s}=200$ GeV and the pseudorapidity $|\bar{\eta}| = 3.8$. In order to
extract the slope parameter the $\rho$ measured at the same experiment as function of $x_F$ was presented as the exponential function of $p_T$ and a fit to the data resulted in the slope parameter $B=(5.94 \pm 0.016) \text{GeV}^{-1}$ with $\chi^2 = 27$ at number of points $=5$. Sure the fit may be improved by adding a new parameter quadratic in $p_T$ but for our goal the linear approximation in $p_T$ is acceptable.

So the final expression for $\pi^0 \text{ SSA}$ at $\sqrt{s} = 200 \text{ GeV}$ looks like

$$A_N(x_F) = \frac{0.2 \cdot x_F^{1.5}}{\sqrt{x_F} + 0.64(1 - x_F)^{4.5}}.$$  \hspace{1cm} (24)

The result of calculation is shown in Fig.2 by the dotted line and it is consistent with the first STAR experimental data.

2. Since the E704 collaboration did not make the direct measurement of the invariant differential cross section simultaneously with SSA measurement at 200 GeV/c we took such data from paper [14]. In this paper the $\rho$ was measured in the interval of 50-400 GeV/c initial proton momentum and for the different laboratory angle of the photon emission $30 \leq \theta_L (\text{mrad}) \leq 275$. The measured photon spectrum was converted to $\rho(\pi^0)$ by applying the Sternheimer’s method. The transverse momenta of $\pi^0$ were in interval 0.3-4.0 GeV/c. The closest to our kinematic domain are data at laboratory $\pi^0$ production angle $\theta_L = 30 \text{ mrad}$. We took data in the region $0.6 < p_T (\text{GeV/c}) < 2$. By using the exponential function with slope parameter $B$ we found by fit that $B = (4.75 \pm 0.07) \text{ (GeV/c)}^{-1}$. For 9 experimental points $\chi^2/\text{d.o.f.} = 0.7$. Using the relation $x_F = 2p_T/\sqrt{s \cdot \tan \theta \text{ cms}}$ the $x_F$ region for experiment was defined as 0.03-0.27. In frame of these data it is impossible to go farther in $x_F$.

Taking the quark polarization as $P_q(x_F) = (1/3) \cdot x_F$ the final formula for SSA at 200 GeV/c (E704 experiment) for $\pi^0$ is

$$A_N(x_F) = \frac{0.16 x_F^{1.5}}{x_F^{0.5} + 0.64(1 - x_F)^{4.5}}.$$  \hspace{1cm} (25)

The result of calculation is presented in Fig.2 as a solid line. One can see a qualitative consistency between the model prediction and the data of E704 experiment.

3. In the same Fig.2 we present the SSA data for reaction $\bar{p}(\uparrow) + p \rightarrow \pi^0 + X$ at 200 GeV/c. The model prediction is simplified by using the relation [17]
Figure 2: Asymmetry in function of $x_F$ in the polarized beam fragmentation region for reaction $p(\uparrow) + p \rightarrow \pi^0 + X$ at $\sqrt{s} = 19.4$ and 200 GeV. The data for reaction $\bar{p}(\uparrow) + p \rightarrow \pi^0 + X$ are also shown. The solid line is the CMSM prediction for $\sqrt{s} = 19.4$ GeV data, the dotted line is for $\sqrt{s} = 200$ GeV data.
which is a consequence of transformation from quarks to antiquarks. As seen from Fig.2 there is a good consistency of model prediction (solid line) with experimental data.

4. Figure 3 presents the experimental data from E704 experiment on the SSA for the inclusively produced pions at 200 GeV/c in the following reactions:

\[ p(\uparrow) + p \rightarrow \pi^{\pm,0} + X, \quad \bar{p}(\uparrow) + p \rightarrow \pi^{\pm,0} + X. \]  \tag{26}

For calculation of the quark analyzing power, \( A_q \), we must estimate the slope parameters \( B^+ \) for positive and \( B^- \) for negative pion productions. This task was done by using the data from paper [15]. The invariant differential cross-section was taken at \( \sqrt{s} = 31 \text{ GeV} \), closest to E704 data. The slope parameters were estimated by taking two values of \( p_T = 0.82 \text{ GeV/c} \) and \( p_T = 1.42 \text{ GeV/c} \) and averaged over region \( 0.35 < x_F < 0.65 \). Finally we took \( B^+ = 5.55 \text{ GeV}^{-1} \) and \( B^- = 5.33 \text{ GeV}^{-1} \). So the expression for \( \pi^+ \) SSA taking \( P_q(x) = (2/3) \cdot x \) looks like

\[ A_N^{\pi^+}(x_F) = \frac{0.37x_F^{1.5}}{x_F^{0.5} + 0.48(1-x_F)^{4.5}}. \]  \tag{27}

Prediction of this formula for SSA in inclusive \( \pi^+ \) production by polarized protons is presented in Fig. 3a by the solid line. Data (closed circles) confirm this prediction fairly well excluding the region \( x_F < 0.4 \).

Fig.3b shows the SSA for inclusive \( \pi^- \) production in pp collisions at 200 GeV/c. According to the CMSM taking \( P_q(x) = -(1/3) \cdot x \) the following relation for \( \pi^- \) SSA can be written

\[ A_N^{\pi^-}(x_F) = -\frac{0.18x_F^{1.5}}{x_F^{0.5} + 0.96(1-x_F)^{4.5}}. \]  \tag{28}

Following the non-relativistic quark model we assumed the negative sign of the d-quark polarization. As seen from Fig.3b the CMSM prediction (solid line) follows qualitatively the experimental data (closed circles) but it does not give a quantitative agreement. A better description of SSA at the large \( x_F \) furnishes the dotted line corresponding to the relation \( P_q(x) = -0.55x \) stemming out from paper [4]. But in both cases the model needs some improvements in order to describe better the smallest \( x_F \) region.

5. Fig.3c presents the SSA versus \( x_F \) for inclusive \( \pi^0 \) productions by the polarized proton (closed circles) and antiproton (open circles) beams. As seen from this Fig.3c the charge conjugation rule (17) is well respected by the
experiment. For SSA in the inclusive charged pion productions the following charge conjugation rule should be respected

$$A_N(\bar{p}(\uparrow) + p \rightarrow \pi^\pm + X) \approx A_N(p(\uparrow) + p \rightarrow \pi^\mp + X).$$  \hspace{1cm} (29)

According to this relation one can predict the inclusive charged pion asymmetries produced by the polarized antiproton beam without any free parameter. The comparison of model prediction with experimental data is illustrated in Fig.3. The solid lines are model predictions. The experimental data for pp-collisions are shown by the closed circles, while data for $\bar{p}p$ collision are indicated by the open circles. Relation (29) is consistent with data in general (Fig.3) but there are the drastic discrepancies in some cases which are not yet understood. For example, Fig.3a shows, that the experimental data violate the rule (29), while model gives a fairly well description of the SSA for $\pi^+$ production. In Fig.3b the model (5) gives a better description of data than parametrization (6). The data in Fig.3c are consistent with expectation and the CMSM gives a good description of SSA.

The Fig.3d, 3e, and 3f show the $p_T$ dependencies of the SSA for $\pi^{\pm,0}$ inclusively produced by protons and antiprotons. In the explicit form such dependencies were presented in the original papers only for reactions $\bar{p}(\uparrow)p \rightarrow \pi^\pm X$ \[13\] (Fig.3d and Fig.3e, open circles). Other figures were restored by us using the tables of data in the original papers \[10\], \[11\], and \[12\]. In function of $p_T$ all SSA are consistent with the charge conjugation rule (compare the closed and open circles in Fig.3d, 3e, and 3f).

The solid lines are predictions of the CMSM. These lines were calculated by using the relation $x_F \approx a + b p_T$ deduced from the tables of data in the original papers. Constants $a$ and $b$ were defined by fit for each pair of SSA as they are presented in figures. The consistencies between data and predictions are reasonable, though in Fig.3d and Fig.3e the discrepancies are seen at low $p_T$ between model prediction and the experimental data. At such low $p_T$ some non-perturbative interaction (in particular, instanton induced) may affect our predictions by changing the weighting factor $w(x)$ and/or by flipping quark helicity.

Concluding this section we note that in general the CMSM gave the qualitative description of most of the SSA effects in a whole measured kinematic domain. Nevertheless it fails in the interpretations of the one feature of SSA, namely, the evidence of $A_N \approx 0$ in the interval $0 < x_F < 0.4$. This is illustrated by the $A_N(p(\uparrow)p \rightarrow \pi^- + X)$ (see Fig.3b) and $A_N(\bar{p}(\uparrow)p \rightarrow \pi^+ + X)$ (see
Figure 3: Test of the charge conjugation rules (17) and (29) for SSA in inclusive pion productions in $p(\uparrow)p^-$ (closed circles) and $\bar{p}(\uparrow)p^-$ (open circles) collisions at 200 GeV/c. SSA are given in function of $x_F$ in Figures a), b), c) and in function of $p_T$ in Figures d), e), and f). Solid lines are the CMSM predictions for the initial quark polarizations (5) and the dashed lines are for conditions (6).
Conclusion

We propose a Chromo-Magnetic String Model producing the simple analytic expressions for the single spin asymmetries of inclusive charged and neutral pion productions. It describes all known experimental data in the central and beam fragmentation regions. The following experimental facts are naturally explained on the basis of the parton model, conservation of the charge and the invariance of the parton interaction with respect to the charge conjugation: 

\[ A_N(\bar{p}(\uparrow) + p \rightarrow \pi^\pm + X) \approx A_N(p(\uparrow) + p \rightarrow \pi^\mp + X) \]

and

\[ A_N(\bar{p}(\uparrow) + p \rightarrow \pi^0 + X) \approx A_N(p(\uparrow) + p \rightarrow \pi^0 + X). \]

The qualitative descriptions of the \( p_T, x_F \) and energy dependencies of the single spin asymmetry seem reasonable. The CMSM should be applied beyond the domain of resonance production and out of the threshold phenomena.

To improve the model one has to use the quark and gluon distributions \( V(x) \) given by the global parton analysis and the fragmentation functions \( D(z) \) measured in \( e^-e^+ \)-annihilation together with the NLO expressions for the hard parton-parton scattering cross sections. This will provide a more precise weighting factor \( w(x) \) \(^7\).

Next (and more important) is to use a more realistic formula for the quark polarization \( P_q(x) \). Unfortunately we can not use the spin dependent structure function \( g_2(x) \) measured in the deep inelastic scattering (DIS) directly. This function has no parton interpretation and for 100% polarized on mass-shell quark \( g_2(x)=0 \). Therefore, the value of \( g_2(x) \) measured in DIS is expected to be smaller than \( P_q(x) \). On the other hand it looks reasonable to choose \( P_q(x) = c \cdot g_2(x) \) with such a coefficient 'c' that (according to "spin crisis") the whole polarization carried by valence quark will be about 1/4 of the incoming proton polarization. In this way, thanks to the \( x \) behavior of \( g_2(x) \), we hope to reproduce the behavior of SSA observed in the data at \( x_F \leq 0.3 \).

We expect that the coming soon data from the polarized RHIC will test our model more precisely.

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