Kaonic nuclei excited by the \((K^-, N)\) reaction

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Abstract

We show that kaonic nuclei can be produced by the \((K^-, p)\) and \((K^-, n)\) reactions. The reactions are shown to have cross sections experimentally measurable. The observation of the kaonic nuclei gives a kaon-nucleus potential which answers the question on the existence of kaon condensation in dense nuclear matter especially neutron stars.
The kaon-nucleon interaction at low energy region is particularly important nowadays because of the current interest in the dense nuclear matter in neutron stars where so-called kaon condensed state may be achieved by a strong attractive interaction \[1,2\]. The existence of the kaon condensed state softens the equation of state (EOS) of nuclear matter in the neutron stars and reduces their calculated maximum mass above which the neutron stars become black holes. The observed mass distribution of the neutron stars agrees with the calculation with this softening \[3\]. The introduction of strange hyperons in the EOS gives a similar softening. Strangeness is essential in both cases although quantitative understanding of the EOS requires a knowledge of both the kaon-nucleon- and hyperon-nucleon interactions at high density \[4\]. The kaon-nucleon interaction, in particular, is known quite poorly experimentally.

Recently, effective kaon mass in dense nuclear matter has been derived by the Chiral SU(3) effective Lagrangian including $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$ systems \[5\]. Such a theoretical model reproduces well the $\Lambda(1405)$ as a $\bar{K}N$ bound state due to the strong $\bar{K}N$ attractive interaction \[5,6\]. The $\bar{K}N$ interaction makes the $K^-$ feel a strong attractive potential in nuclei which consequently leads to the existence of deeply bound kaonic nuclei. The $\Lambda(1405)$, however, can also be interpreted as a three-quark state with $\ell = 1$ excitation. In this case no attractive $\bar{K}N$ interaction is relevant and the deeply bound kaonic nuclei don’t necessary exist.

Experimental data of $K^-$ optical potential mostly come from kaonic atoms. The shifts and widths of atomic levels affected by the strong interaction were reproduced by introducing an appropriate optical potential in addition to the Coulomb interaction. Recent extensive analysis of kaonic X-ray data concludes that the potential is strongly attractive \[7\]. The derived depth is around -200 MeV which opens a possibility of kaon condensation at around three times normal nuclear density. Derivation of the optical potential from the kaonic atom data is, however, subtle since the atomic state is sensitive only to the phase shift of $K^-$ wave function at the nuclear surface. The phase shift alone cannot determine the depth of the potential since the $K^-$ wave function has an ambiguity in number of nodes in the
nucleus especially when the potential depth is quite deep. The strong imaginary part of the potential further obscures the nodes. Earlier studies with a different treatment of the nuclear surface gave much shallower potentials of -80 ~ 90 MeV [7] which tend to exclude the kaon condensation in the neutron stars.

Heavy ion reactions have been studied to derive the $K^-$ optical potential [8]. Enhanced $K^-$ production in the reactions suggests a strong attractive interaction although quantitative argument requires understanding of details of the reaction mechanism [10].

The $\bar{K}N$ interaction has been derived from kaon scattering experiments. However, the available low-energy data are insufficient for unique multichannel analysis and the existence of $\Lambda(1405)$ makes the extrapolation of the amplitude below the threshold complicated [9]. Recent theoretical calculations on the kaon interaction in nuclei predict an attractive interaction although they are still controversial quantitatively and existence of kaon condensation in neutron stars is as yet inconclusive [11][12].

If $\bar{K}$-nuclear potential is as attractive as derived from the kaonic atom studies suggest [7], then deeply-bound kaonic nuclei should exist. The observation of kaonic nuclei gives directly the $K^-$ optical potential and gives decisive information on the existence of kaon condensation in neutron stars. We show the general properties of the kaonic nuclei and that the $(K^-, N)$ reaction can excite them with cross section experimentally measurable.

Energies and widths of kaonic nuclei are calculated with the potential given by the kaonic atom data. For the analysis of mesonic atoms the Klein-Gordon equation is usually used [7]. Here we use the Schrödinger equation with harmonic oscillator potential. It is a crude approximation although it is good enough for the present purpose. We are interested in gross structure of levels and an order-of-magnitude estimate of the cross section for the deeply bound state. For the moment we take the potential depth -200 MeV given by kaonic atom. It is roughly four times deeper than that for nucleon and the kaon mass is about half of that of a nucleon. Thus the major shell spacing ($\hbar\omega_K$) is $\sqrt{8}$ times the $40A^{-1/3}$ frequently used for nucleon. Since the kaon has no spin, no spin dependent splitting has to be considered.

The $\hbar\omega_K$ is roughly 40 MeV, for instance, for the kaonic $^{28}_K\text{Si}$ nucleus. The 1s state
appears at around -140 ($\frac{3}{2}h\omega_K - 200$) MeV bound, which is the deepest bound state ever observed in nuclear physics. If the potential shape is closer to the square-well it appears deeper. In order to observe the state its width has to be reasonably narrow. The width is given by the imaginary part of the potential, which decreases for the deeply bound state and is around 10 MeV [3,4]. The narrow width is understandable since dominant conversion channels like $KN \rightarrow \pi \Sigma$ or $KN \rightarrow \pi \Lambda$ are energetically almost closed for such a deeply-bound state. Kaon absorption by two nucleons ($KNN \rightarrow YN$) gives little width since two nucleons have to participate to the reaction. Even though the width is twice wider the 1s state should be seen well separated since the next excited state (1p) is expected to appear 40 MeV higher.

The ($K^-, N$) reaction where a nucleon ($N$) is either a proton or a neutron is shown schematically in figure 1. The nucleon is knocked out in the forward direction leaving a kaon scattered backward in the vertex where the $K + N \rightarrow K + N$ takes place. This reaction can thus provide a virtual $K^-$ or $\bar{K}^0$ beam which excites kaonic nuclei. This feature is quite different from other strangeness transfer reactions like ($K^-, \pi$), ($\pi^\pm, K^+$) and ($\gamma, K^+$) extensively used so far. They primarily produce hyperons and thus are sensitive to states mostly composed of a hyperon and a nucleus.

The momentum transfer, which characterizes the reaction, is shown in figure 2. It depends on the binding energy of a kaon. We are interested in states well bound in a nucleus ($BE = 100 \sim 150$ MeV). The momentum transfer for the states is fairly large ($q = 0.3 \sim 0.4$ GeV/$c$) and depends little on the incident kaon momentum for $P_K = 0.5 \sim 1.5$ GeV/$c$, where intense kaon beams are available. Therefore one can choose the incident momentum for the convenience of an experiment. It is a little beyond the Fermi momentum and the reaction has characteristics similar to the ($\pi^+, K^+$) reaction for hypernuclear production where so-called stretched states are preferentially excited [13].

Recently deeply bound $\pi^-$ atoms were observed by the ($d, ^3$He) reaction [14]. A small momentum transfer ($\sim 60 MeV/c$) was vital to excite the atomic states which were typically characterized by the size of the atomic orbits. If one wishes to excite kaonic atoms, a
momentum transfer less than 100 MeV/c is desirable. It is achieved by kaon beams less than 0.4 GeV/c where available beam intensity is very small. The repulsive nature of the π-nucleus interaction allows no nuclear state although the strong attractive \( \bar{K} \)-nucleus potential makes kaonic nuclei exist. The \((K^-, N)\) reaction can excite the deeply bound kaonic nuclei with large cross section in spite of the large momentum transfer of the reaction. For the excitation of nuclear states the momentum transfer is typically characterized by the Fermi momentum.

The \((K^-, N)\) reaction on deuteron is the simplest reaction by which one can study the \( \bar{K}N \) component of excited hyperons. The \(d(K^-, p)\) reaction excites \( K^-n \) states which can only have \( I = 1 \). On the other hand \(d(K^-, n)\) reaction excites a \( K^-p \) state which can have either \( I = 1 \) or \( I = 0 \). Cross sections to the excited hyperons depend on their \( \bar{K}N \) component. For instance, the well known \( \Lambda(1405 \text{ MeV}) \) should be abundantly excited by the \((K^-, n)\) reaction if it is a \( \bar{K}N \) bound state with \( I = 0 \) as usually believed. The \(d(K^-, p)\) reaction, in particular, gives information on the \( K^-n \) interaction below the threshold, which plays decisive role on the kaon condensation in the neutron stars.

We adopt here the distorted wave impulse approximation (DWIA) to evaluate the cross section. The DWIA calculation requires: (a) distorted waves for entrance and exit channels, (b) two body transition amplitudes for the elementary \((K^-, N)\) process, and (c) a form factor given by initial nuclear and kaonic-nuclear wave functions. Relevant formulas for the calculation can be found elsewhere [13].

The differential cross section in the laboratory system for the formation of kaonic nucleus is given by

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{L,0^+}^{K^-N\rightarrow NK^-} N_{\text{eff}}.
\]

It is given by the two body laboratory cross section multiplied by the so-called effective nucleon number \((N_{\text{eff}})\).

We first use the plane wave approximation to evaluate \( N_{\text{eff}}^{\text{pw}} \). At 0 degrees, where only non-spin flip amplitude is relevant, \( N_{\text{eff}}^{\text{pw}} \) is given by
\[ N_{\text{eff}}^{\mu\nu} = (2J + 1)(2j_N + 1)(2\ell_K + 1) \left( \frac{\ell_K}{0} \quad \frac{j_N}{-\frac{1}{2}} \quad \frac{J}{\frac{1}{2}} \right) F(q). \]  

(0.2)

In this equation we assumed that a nucleon in a \( j_N \) orbit is knocked out and a kaon sits \( \ell_K \) state making transition from \( 0^+ \) closed shell target to a spin \( J \) state. Here the form factor \( F(q) \) is given by the initial nucleon and final kaon wave functions as

\[ F(q) = \left( \int r^2 dr R_K(r) R_N(r) j_L(qr) \right)^2, \]

(0.3)

where \( L = J \pm \frac{1}{2} \) is the transferred angular momentum.

For an oscillator potential of radius parameter \( b \), the radial wave function is

\[ R_\ell(r) = c_\ell(r/b)\ell^\ell e^{-r^2/2b^2} \]

(0.4)

for nodeless states, where \( c_\ell = [2^{l+2}/b^3 \sqrt{\pi}(2l + 1)!!]^{1/2} \). In the present case it is enough to consider natural parity stretched states with \( L = \ell_N + \ell_K \) since the transferred momentum \( q \) is larger than the Fermi momentum. The form factor (Eq. 0.4) is well known for the harmonic oscillator wave function [13] as

\[ F(q) = \frac{(2Z)^L e^{-Z}}{[(2L + 1)!!]^{2}} \frac{[\Gamma(L + 3/2)]^2}{[\Gamma(\ell_K + 3/2)] \Gamma(\ell_N + 3/2)} \]

(0.5)

with \( Z = (bq)^2/2 \), where the radius parameter \( b = \frac{m\omega}{\hbar} \) has to be replaced by

\[ \frac{2}{b^2} = \frac{1}{b_N^2} + \frac{1}{b_K^2} \]

(0.6)

to account for the different radius parameters for the nucleon \( (b_N) \) and the kaon \( (b_K) \) where \( 1/b_K^2 = \sqrt{8}/b_N^2 \). \( N_{\text{eff}}^{\mu\nu} \) is further reduced by the distortion of incoming and outgoing waves as

\[ N_{\text{eff}} = N_{\text{eff}}^{\mu\nu} D_{\text{eik}}. \]

(0.7)

The distortion \( D_{\text{eik}} \) is estimated by the eikonal absorption where the imaginary parts of the \( K^- \) and proton optical potentials are given by their total cross sections with nucleons. At \( P_K = 1 \) GeV/c, total cross sections of \( K^- \)-nucleon and \( p \)-nucleon are almost the same and we
take both to be 40 mb. The small radius parameter $b$ indicates larger cross sections through the high momentum component; we thus evaluated $N_{eff}$ for $b_K = b_N$ also as the smallest value.

The cross section of the elementary reaction was given by the phase shift analysis of available data [13]. Here we need to consider only the non-spin flip amplitude ($f$) as explained above. Since the kaon and nucleon are isospin $\frac{1}{2}$ particles there are $I=0$ ($f^0$) and $I=1$ ($f^1$) amplitudes. The amplitudes for elastic and charge exchange scattering are represented by appropriate linear combinations of the isospin amplitudes as

$$f_{K^-n \rightarrow K^-n} = f^1,$$

$$f_{K^-p \rightarrow K^-p} = \frac{1}{2}(f^1 + f^0),$$

$$f_{K^-p \rightarrow \bar{K}^0n} = \frac{1}{2}(f^1 - f^0).$$

The CM differential cross section of the three reactions at 180° are shown in figure 3 as a function of incident kaon momentum. The cross sections depend on the incident momentum. For instance, the $K^-p \rightarrow K^-p$ reaction has a distinct peak at around 1 GeV/c. We thus take 1 GeV/c for the incident kaon momentum. Since the target nucleon is moving in a nucleus, Fermi averaging has to be made for the two body cross section which smears the fine momentum dependence. The CM cross section is reduced by 20 to 30% depending on models for this averaging. We take $\sim$10 mb/sr as the CM cross section at 1 GeV/c.

Here we consider $I=0$ symmetric nuclei as targets. The ($K^-, p$) reaction produces only an $I=1$ state; on the other hand the ($K^-, n$) reaction can produce both $I=0$ and 1 states. The $\bar{K}N$ system is strongly attractive in the $I=0$ channel though not so much in the $I=1$ channel. The kaon-nucleus potential is an average of both channels and thus depends little on the total isospin of kaonic nuclei. Consequently we expect that the $I=0$ state produced by the ($K^-, n$) reaction appears at nearly the same excitation energy. The elementary cross section for the ($K^-, n$) reaction in eq (0.1) becomes the sum of the $K^-n \rightarrow K^-n$ and $K^-p \rightarrow \bar{K}^0n$ cross sections. The incoherent sum of the two cross sections may not be inappropriate for the evaluation since the $K^-$ and $\bar{K}^0$ mass difference is considered to be large on a nuclear
physics scale.

The cross section for the kaonic nuclear $1s$ states are shown in table 1. The $(\pi^+, K^+)$ reaction for the hypernuclear production shows distinct peaks corresponding to series of major shell orbits especially for target nuclei with $j_n = \ell_n + 1/2$ orbit closed. We thus take $^{12}\text{C}$ and $^{28}\text{Si}$ for the present study.

| nucleus | $N_{\text{eff}}^{\text{pw}}$ | $D_{\text{eik}}$ | $d\sigma/d\Omega$ mb/sr |
|---------|-------------------------------|-----------------|------------------------|
| $^{12}\text{C}$ | 0.055$\sim$0.26 | 0.25 | 0.72$\sim$3.3 |
| $^{28}\text{Si}$ | 0.029$\sim$0.15 | 0.16 | 0.24$\sim$1.3 |

Table 1

Calculated laboratory differential cross sections of the $1s$ states excited by the $(K^-, p)$ reactions at $P_K=1$ GeV/c for the $^{12}\text{C}$ and $^{28}\text{Si}$ targets. Range of values corresponds to the $b$ parameter (see text).

The calculated cross sections turn out to be quite large which can compensate for a low intensity kaon beam. The large cross section comes from the large cross section of the elastic $K+N \rightarrow K+N$ reaction and from the transformation of c.m. system to laboratory system.

Feasibility of the experiment depends on backgrounds. Dominant backgrounds are nucleons from knock-out reactions, where kaons are scattered backward by the quasifree process. Since the nucleons associated with the deeply-bound kaonic nuclei are much more energetic, the knock-out reactions will not be a problem.

Kaon absorption by two nucleons in nuclei can generate energetic nucleons. The process has to involve another nucleon in addition to the $(K^-, N)$ reaction. Thus one expects the process gives smaller cross section than that of the $(K^-, N)$ reaction. The process can be interpreted as a spreading width of the kaonic nuclei.

A $\Lambda$ produced in the forward direction by the quasifree $(K^-, \pi)$ reaction provides an
energetic nucleon. It would not be a serious background since its cross section is an order-of-magnitude smaller than \((K^-, N)\) reaction and no peak structure is expected.

From the experimental point of view energetic protons can be produced by knock-out reaction by pions which are contaminated in the kaon beam. This process however can be removed by the careful tuning of experimental condition.

It is shown that the \((K^-, p)\) and \((K^-, n)\) reactions can be used for the study of the kaonic nuclei. Study of the reaction requires intense low energy kaon beam for which AGS of BNL and probably PS of KEK are particularly suitable. The beam momentum can be chosen by considering the cross section, beam intensity and momentum resolution of spectrometer. There are beam lines which provide \(K^-\) beam \(0.5\sim2\ \text{GeV/c}\) at BNL and KEK. The relatively broad width \((\sim10\ \text{MeV})\) and simple structure of the state need spectrometers of only modest momentum resolution but wide momentum acceptance.

We demonstrated that the \((K^-, p)\) and \((K^-, n)\) reaction can be used to obtain direct information on the \(\bar{K}N\) interaction in nuclear matter. The calculation employed here is rather crude although it is based on well-known general concepts in nuclear physics.

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Figure 1
Diagram for the formation of kaonic nuclei via the \((K^-, N)\) reaction. The kaon, the nucleon, and the nucleus are denoted by the dashed, thin solid and multiple lines, respectively. The
kaonic nucleus is denoted by the multiple lines with the dashed line. The filled circle is the $KN \rightarrow KN$ amplitude while the open circles are the nuclear vertices. The bubbles represent distortion.

Figure 2

The momentum transfer of the $(K^-, N)$ reaction at 0 degrees is shown for four reactions. Here binding energy of kaonic nucleus $^{27}_K$Mg is taken to be -150 MeV.

Figure 3

The CM differential cross sections of the three reactions are shown as a function of incident kaon lab momentum.
$28\text{Si}(K^-,p)\text{Al}+K^-$

$28\text{Si}(K^-,p)^7\text{Mg}$

$p(K,p)K^-$

$d(K,n)\Lambda(1405)$