A Holographic Model Of Hadronization

Nick Evans and Andrew Tedder

School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, UK

We study hadronization of the final state in a particle-antiparticle annihilation using a holographic gravity dual description of QCD. At the point of hadronization we match the events to a simple (Gaussian) energy distribution in the five dimensional theory. The final state multiplicities are then modelled by calculating the overlap between the Gaussian and a set of functions in the fifth dimension which represent each hadron. We compare our results to those measured in $e^+e^-$ collisions at LEP and PEP-PETRA. Hadron production numbers, which differ in range by four orders of magnitude, are reproduced to well within a factor of two.

Introduction—Since the discovery of the AdS/CFT Correspondence [1], holographic gravitational theories have been studied to shed light on strongly coupled gauge theories. Phenomenological five dimensional (5D) models in this spirit (AdS/QCD) [2] also provide a quantitative description of the QCD meson spectrum which seem to work at the level of 10% accuracy. In this paper we will apply these tools towards another notoriously difficult QCD calculation: hadronization.

Current Monte Carlo event generator models [3] of hadronization are complex with many parameters which are tuned to data in some energy regime and have limited predictive power. A simpler understanding of the process would be a boon. Recently, progress has been made by assuming that after the quarks freeze into hadrons they may be described as a hadron gas in thermodynamical equilibrium [4]. Such models provide a surprisingly good description of the multiplicities of hadrons in jets across several orders of magnitude.

Models of hadronization generically have two parts: predicting the initial yield of hadrons directly after annihilation and then allowing for decays of those particles in transit. Our model, like the thermal model [4], only addresses the first part. Modelling the decays would involve a theory of branching ratios which we do not propose. Instead we model this using the available branching ratio data from collider experiments.

AdS/QCD models provide a weakly coupled 5D gravitational theory that describes the hadrons made of confined quarks. Each hadron and its excited states (e.g. the “stack” of the $\rho$ mesons: $\rho(770), \rho(1450), \rho(1700)\ldots$) is described by a 5D field that shares the Lorentz properties and global symmetries of the hadron. In the gravitational theory one seeks solutions for these fields that separate the 3+1 dimensional dependence from the extra radial direction dependence, so they take the form $g_n(r)e^{i\kappa_n x}$ with $k_n^2 = M_n^2$. There are only regular solutions on the space for discrete masses corresponding to the meson masses. The functions $g_n(r)$ form an orthogonal basis.

The dual necessarily only describes the regime of QCD where the gauge coupling is strong. When asymptotic freedom sets in at short distances and QCD is best described by free quarks and gluons the gravitational theory is expected to become strongly coupled itself and computation becomes impossible. Our description therefore is dependent on the distribution of energy at the matching energy scale where hadronization occurs. We will see that a very simple model reproduces the data very well.

Hadronization Holographically—Our holographic model of hadronization will share with the thermal models the idea that at the point where the quarks form hadrons, the energy of the event is democratically available to all hadronic channels. We describe the initial condition as some deposition of energy into the 5D model’s stress-energy tensor. The radial dependence of the stress-energy dependence can be expanded in terms of the functions $g_n$ and will determine the relative multiplicities of each particle in a hadron stack. The $x$ dependence will determine the energy and momentum of the hadrons. In this paper we will simply concentrate on the multiplicities. There should be some kinematic limit on the maximum mass of a state produced in the shower and in principle this could vary with the centre of mass energy of the event. In practice we shall simply include all known states with mass below 1.7 GeV (but excluding the $a_0(980)$ triplet - it is thought to be a bound kaon state). Above this value the experimental data on the full spectrum becomes patchy. In addition, the high mass states in this range are only produced with very small multiplicities and have a minimal effect on the lighter particle results.

The result for the multiplicities depends on the choice of the function expanded in terms of the $g_n$ and this represents the matching to the underlying asymptotically free QCD dynamics. The simple guess we will employ is that we should treat all hadronic channels equally and pick a Gaussian for the initial condition. The height of the Gaussian determines the absolute value of each particle’s multiplicity and hence is a free parameter which we fit (this parameter can be re-expressed as the average energy per hadron, $\kappa$, which we detail below). The width is also a free parameter which we fit. Finally the thermal model requires a suppression factor on the production of strange quarks. We too include a strangeness suppression factor $\gamma_s$ which multiplies the Gaussian for each strange quark in the hadron (note that if mixing oc-
curs the strange quark content need not be an integer). \(\gamma_s\) is another fit parameter in our analysis although we find it lies close to one.

The final ingredient we require is a specific AdS/QCD model of the QCD hadrons. We will adapt a string theory derived model of chiral symmetry breaking that includes the vector mesons and pions \[5\]. That both the vector mesons and the pions are included is an important feature: the mass spectra of the pseudo-Goldstone bosons are significantly different to all other hadrons in QCD. We will then assume that the \(g_n\) functions associated with each hadron stack are not that different from the \(\rho\)-stack functions - we will simply reproduce them but with the mass of the lightest stack member tuned to the experimental value. Similarly the pion stack can be used to reproduce the towers of states associated with each pseudo-Goldstone of the chiral symmetry breaking (i.e. the pions, kaons and eta meson). The relative weighting is parameterised by \(R\), a measure of \(g^2N\), which we fit.

Putting it all together, we compare our results to that detected at LEP (\(\sqrt{s} = 91.2\text{GeV}\)) and PEP-PETRA (\(\sqrt{s} = 29\text{GeV}\)), and find that we can reproduce yields that vary over four orders of magnitude to within a factor of two. It is expected that the biggest source of error comes from the choice of holographic dual.

**Holographic Hadron Basis Functions**— Our holographic model of the hadron spectrum is based on a string theory construction of a QCD-like gauge theory consisting of a deformed D3 brane geometry with quarks included through probe D7 branes - the precise details do not need to be understood to follow this letter, but computational details are in the appendix and a more detailed analysis can be found in \[5\]. The model realizes dynamical chiral symmetry breaking. The mass of the lowest lying \(\rho\) meson can be dialed by choosing the conformal symmetry breaking scale in the model, and the mass of the lowest lying pion can be dialed by choosing the asymptotic quark mass. The excited states in both stacks are then predicted: \(m_{\pi^+} = 1737\text{MeV}, m_{\pi^-} = 1701\text{MeV}\) (c.f. experimental values of 1459 and 1300 MeV respectively). So whilst they don’t precisely reproduce the experimental values the pattern is at least roughly right.

As stated above, we assume that the \(g_n\) functions associated with each hadron stack are not that different from the \(\rho\)-stack functions, and simply rescale the \(r\) coordinate such that the mass of the lowest member of each stack is correct.

The functions representing the pions, henceforth denoted as \(f_n\), are also rescaled. This time we dial the asymptotic quark mass such that the mass of the lowest member of each stack is correct. We plot the first three functions for \(f_n, g_n\) in figure 1.

The derivation of these functions is briefly reviewed in the appendix. We stress that here we seek to describe hadronization not to provide a complete holographic model of QCD. Hopefully the functions we use, based on the string model, are a reasonable phenomenological basis.

**Overlap Computation of Multiplicities**— With our holographic functions \(f_n, g_n\) in place for each hadron stack we can now proceed with computing the expected initial yield in a hadronisation event.

We assume that all the five dimensional fields \(\Psi(r)\) (e.g. each component of a gauge field \(A_\mu\) describing the \(\rho\) mesons) have a common initial condition of a Gaussian centred at \(r = 0\) and of width \(\Lambda\). To find the multiplicities of each stack member we compute

\[
c_n = \int_0^\infty \Psi(r) w(r)g_n(r)dr
\]

where \(w(r)\) is the weighting function associated with the basis functions \(g_n\). The Gaussian receives most support from the lowest mass states, whilst very highly excited states have no overlap with the Gaussian because of their highly oscillatory nature in the IR.

The multiplicity is simply given by \(c^2_n\) multiplied by \((2J+1)\) where \(J\) is the spin of the hadron.

There are a number of special cases, which we address in turn.

**Pseudo-Goldstone Bosons**— The pseudo-Goldstones are described by a separate holographic field, \(\theta\), in our model and are represented by functions \(f_n(r)\). The relative contributions the fields make to the stress-energy tensor are

\[
T_{rr} \sim \Delta_1(r)(\partial_r^2 \theta)^2 + \Delta_2(r)\eta^\mu\nu(\partial_r A_\mu)(\partial_r A_\nu) + \ldots
\]

\(\Delta_1, \Delta_2\) are functions of \(r\), calculable from the holographic model. So that these fields see the same contribution to
the stress energy tensor we therefore rescale the Gaussian. If our standard Gaussian is $\Psi(r)$ then the pion field sees $\int dr \sqrt{\sum_i \partial_i \Psi}$. 

Strangeness Suppression Factor—Since the underlying asymptotically free dynamics may distinguish the strange quark from the up and down quarks, we also multiply the Gaussian by a factor of $(\gamma_s)^\sigma$ where $\sigma$ is the strangeness content of the stack. $\gamma_s$ is then the second fit parameter in our procedure. Note that this procedure is rather crude because different members of a stack may mix to varying degrees with other states. For example the $\eta(548)$ has 32% strangeness content while the $\eta^{**}$ has 100%. In these cases we set $\gamma_s$ by the strangeness content of the lightest member of the stack.

Height of Gaussian—The normalisation of the Gaussian tells us the relative multiplicities of the various hadrons in an event. An overall multiplicative factor $\kappa$ sets the absolute number of each species and we fit this value. $\kappa$ determines the total number of final state particles (before allowing for decays in transit to the detector), and hence we express it as the average hadron energy in the collision.

A Fourth Parameter—Our choice of holographic dual also contains a free parameter, $R$, which sets the ’t Hooft coupling in the gravity dual. We fit it to the data. However, $R$ is not in the same class as $\Lambda, \gamma_s, \kappa$. $R$ has a sound theoretical background, and would be predicted if the holographic dual to QCD was known.

Decay in transit—Once we have calculated the initial yield of hadrons, we then have to allow for decays of the particles in transit from the interaction point to the detector. Branching ratios are taken from [4], and particles that can be detected at LEP (whose results we will compare to) are set as stable. All the other particles are allowed to decay through the decay channels until they reach one of the stable particles. In this way we get a list of numbers which is what our model predicts would be seen at LEP.

Predictions—We compare our results both to $e^+e^-$ collisions performed at LEP ($\sqrt{s} = 91.2$ GeV), and at PEP-PETRA ($\sqrt{s} = 29$ GeV). 

| Hadron | Model | Expt | Model | Expt |
|--------|-------|------|-------|------|
| $\pi^+$ | 5.95 | 8.5 | 4.07 | 5.35 |
| $\pi^0$ | 6.43 | 9.2 | 4.41 | 5.3 |
| $K^+$ | 1.09 | 1.2 | 0.68 | 0.7 |
| $K^0$ | 1.09 | 1.0 | 0.68 | 0.69 |
| $\eta$ | 1.06 | 0.93 | 0.66 | 0.584 |
| $\rho^0$ | 1.33 | 1.2 | 0.88 | 0.9 |
| $K^{**}$ | 0.387 | 0.36 | 0.28 | 0.31 |
| $K^{*0}$ | 0.385 | 0.37 | 0.28 | 0.28 |
| $\eta'$ | 0.042 | 0.13 | 0.03 | 0.26 |
| $p$ | 0.41 | 0.406 | 0.30 | 0.3 |
| $\phi$ | 0.03 | 0.01 | 0.02 | 0.084 |
| $\Lambda$ | 0.172 | 0.19 | 0.13 | 0.0083 |
| $\Sigma^{*+} + \Sigma^{-}$ | 0.0120 | 0.0094 | 0.0089 | 0.0083 |
| $\Xi^{-}$ | 0.012 | 0.012 | 0.0088 | 0.0083 |
| $\Xi^{*0}$ | 0.0040 | 0.0033 | n/a | n/a |
| $\Omega$ | 0.0011 | 0.0014 | 0.0008 | 0.0007 |

$\Lambda$ (MeV), $\kappa$ (GeV)  
$\gamma_s, R$  
| 150, 4.96 | 152, 2.35 |
| 0.97, 2.6 | 0.97, 2.4 |

TABLE I: Results of the model for hadron yields at $\sqrt{s} = 91.2$ GeV (centre column) and $\sqrt{s} = 29$ GeV (right column). The relevant 4 free parameter values are shown in the final row. 

Technically it is a pseudo-Goldstone boson, but instanton effects cancel out this effect. Our model just treats it as non-Goldstone boson, which is probably over simplistic. In addition, it mixes heavily with $\eta(548)$: our model contains no good parameterization of mixing.

The extra inaccuracy for the PEP-PETRA matching comes, in large part, from the $\Omega$ yield. This has a very large experimental error (see [4]), and so we may not be matching to the correct value: on comparing the PEP-PETRA and LEP experimental yields, a decrease by a factor of 10 adds doubt to the accuracy of the PEP-PETRA measurement.

Conclusions—We assumed that every hadron in QCD can in principal be represented by a function in the $r$ coordinate of the 5D holographic theory of QCD. We then proposed that hadronization can be modelled by hypothesising that the initial yield (that is before the particle created starts decaying) for any hadron is given by the square of the overlap between the function which represents the hadron, and a Gaussian, centred at the origin, with a width of $\Lambda$. In addition we have two other parameters in the theory; a strangeness suppression factor to account for the heaviness of the strange quark, and $\kappa$ which determines with what energy the particles leave the interaction point.

With the full holographic dual to QCD currently unknown, we made some reasonable assumptions to achieve a full set of functions which represent every hadron. We then compared the results to $e^+e^-$ collisions made at
LEP and PEP-PETRA. The results are surprisingly good suggesting the broad framework is correct.

The model of hadronization presented here is applicable to all particle-antiparticle annihilation events, where the fireball after the collision has no residual quantum numbers. Broadening this model to include events such as deep inelastic proton-proton scattering, and heavy ion collisions would clearly be desirable. A natural way to do this would be to include enhancement factors on the multiplicities of stacks contributing to the quantum numbers that are non zero in the final state. We leave such an analysis for the future.

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APPENDIX

String Theory Progenitor— The phenomenological model used here is based on the AdS/CFT Correspondence realization of chiral symmetry breaking in [3]. That model consists of a dilaton flow deformed AdS geometry, in the Einstein frame of

\[ ds^2 = H^{-1/2} f^{1/4} dx^2 + R^2 f \frac{H^{1/2} f^{2(1-\delta)/4} w^4 b - b^4}{w^4} \sum_{i=1}^6 dw_i^2 \]

where \( H = f^{1-\delta} \), \( f = w^4 + b^4 \), \( c^2 = c^2 f \Delta \).

There are formally two free parameters, \( R \) and \( b \), since \( \delta = 1/2b^4 \) and \( \Delta^2 = 10 - \delta^2 \). The parameter \( b \) sets the conformal symmetry breaking scale and we set it equal to one from this point onwards. We will use the \( \rho(770) \) mass to set an absolute mass scale. \( R \) sets the 't Hooft coupling which we treat as a free parameter.

Quarks are introduced by including probe D7 branes into the geometry. We minimize the D7’s world-volume mass to set an absolute mass scale.

Substituting from the geometry above we can find the equation of motion for the radial separation, \( \sigma \), of the two branes in the 8 – 9 directions as a function of the radial coordinate \( r \) in the 4 – 7 directions. We first calculate the background solution as in [5]. The large \( r \) asymptotic solutions take the form \( \sigma_0 = m + c/r^2 \) with \( m \) representing the quark mass and \( c \) the quark condensate. The regular solutions have non-zero \( c \) even when \( m = 0 \) and describe chiral symmetry breaking.

\[
\frac{\partial}{\partial r} \left( \frac{e^G}{\sqrt{1 + (\sigma_0)^2}} \frac{\sigma_0^2 f}{f} \right) + M^2 w_f f = 0 \tag{6}
\]

with \( w_f = \frac{R^2 e^G}{\sqrt{1 + \sigma_0}} f^{1/4} \frac{(r^2 + \sigma_0^2)^2 - 1}{(r^2 + \sigma_0^2)^2} \sigma_0^2 \).

Finding regular solutions to \( \delta \) is simply a Sturm-Liouville eigenvalue problem, and hence we know that the regular solutions \( f_n \) will form a basis under the weighting function \( w_f \).

Similarly, if we write \( A_\mu = g(r) \sin(kx) \epsilon_\mu \), the equation of motion for \( g \) is

\[
\frac{\partial}{\partial r} \left( \frac{e^G}{\sqrt{1 + \sigma_0^2}} \frac{w^4}{\sqrt{(w^4 + 1)(w^4 - 1)}} \right) + M^2 w_g g = 0 \tag{8}
\]

\[
w_g = \frac{R^2 e^G}{\sqrt{1 + \sigma_0^2}} f^{1/4} \frac{r^2 + \sigma_0^2}{r^2 + \sigma_0^2} \sigma_0^2 \tag{9}
\]

Where once again \( g_n \) form a basis under the weighting function \( w_g \).

Stress-energy tensor— We also need to know the stress-energy contributions,

\[
T_{rr} = -\frac{2}{g} \frac{\delta}{g^\mu g^\nu} \left( e^G \sqrt{-g} \mathcal{L} \right) \tag{10}
\]

We have to be careful, and use [1], not [5], before expanding to second order. On doing so, we find

\[
\frac{\Delta_2}{\Delta_1} = \frac{g^\mu g^\nu}{\sigma^2} \frac{\sigma^2 F^2}{F^2} \tag{11}
\]

with \( \Delta_1, \Delta_2 \) defined in equation [2].

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