MILLISECOND MICROWAVE SPIKES: STATISTICAL STUDY AND APPLICATION FOR PLASMA DIAGNOSTICS

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ABSTRACT

We analyze a dense cluster of solar radio spikes registered at 4.5–6 GHz by the Purple Mountain Observatory spectrometer (Nanjing, China), operating in the 4.5–7.5 GHz range with 5 ms temporal resolution. To handle the data from the spectrometer, we developed a new technique that uses a nonlinear multi-Gaussian spectral fit based on $\chi^2$ criteria to extract individual spikes from the originally recorded spectra. Applying this method to the experimental raw data, we eventually identified about 3000 spikes for this event, which allows us to make a detailed statistical analysis. Various statistical characteristics of the spikes have been evaluated, including the intensity distributions, the spectral bandwidth distributions, and the distribution of the spike mean frequencies. The most striking finding of this analysis is the distributions of the spike bandwidth, which are remarkably asymmetric. To reveal the underlying microphysics, we explore the local-trap model with the renormalized theory of spectral profiles of the electron cyclotron maser (ECM) emission peak in a source with random magnetic irregularities. The distribution of the solar spike relative bandwidths calculated within the local-trap model represents an excellent fit to the experimental data. Accordingly, the developed technique may offer a new tool with which to study very low levels of magnetic turbulence in the spike sources, when the ECM mechanism of the spike cluster is confirmed.

Subject headings: acceleration of particles — radiation mechanisms: nonthermal — Sun: flares — Sun: radio radiation — turbulence

1. INTRODUCTION

Solar radio spikes are known to be a very narrowband kind of solar radio emission (Benz 1985; Stähli & Magun 1986), displaying a typical bandwidth on the order of 1%. A more detailed study of the bandwidth distribution and its correlations with other observed parameters was performed by Csillaghy & Benz (1993). They noted that the bandwidth changes significantly from one event to another, so no clear correlation between the bandwidth and the observing frequency is visible. Thus, the bandwidth is more characteristic of the event, rather than being a function of frequency.

At that time, no unique correlation between the bandwidth and the radio flux of the spikes was found: there were uncorrelated cases, as well as correlated and anticorrelated ones. On the basis of these results, Csillaghy & Benz (1993) concluded that the observed bandwidth is formed mainly by source inhomogeneity, rather than there being a natural bandwidth of the underlying emission process.

Messmer & Benz (2000) applied a more advanced approach in order to determine a minimum bandwidth of the solar radio spikes, assuming that the minimum bandwidth may correspond to the natural bandwidth of the emission process, while broader spikes are a signature of source inhomogeneity. The minimum bandwidth for two considered events was 0.17% and 0.41%, which implied that the natural bandwidth of the emission process was less than the measured values.

The currently available data for the radio spikes appearing at the main flare phase are pretty consistent with the idea that the source of a spike cluster is a loop filled by fast electrons and relatively tenuous background plasma (Fleishman et al. 2003). The trapped fast electrons have a nonthermal (power law) energy spectrum and a loss cone angular distribution. Each single spike is generated in a local source inside this loop when the local anisotropy is increased in comparison with the averaged one to produce electron cyclotron maser (ECM) emission. The assumed fluctuations of the pitch-angle distribution of fast electrons can be produced by magnetic turbulence, as in the turbulent model proposed by Bártá & Karlický (2001).

ECM emission is believed to be responsible for many kinds of solar, planetary, and stellar radiation (Stepanov 1978; Wu & Lee 1979; Holman et al. 1980; Melrose & Dulk 1982; Sharma & Vlahos 1984; Wu 1985; Wingee et al. 1988; Aschwanden & Benz 1988; Aschwanden 1990; Barrow et al. 1994; Stupp 2000; Vlasov et al. 2002; LaBelle & Treumann 2002; Treumann 2006). Recently (Fleishman & Melnikov 1998; Fleishman et al. 2003), new important evidence for solar radio spikes being produced by ECM emission has been obtained. However, although ECM emission is a good fit to many spike properties, no direct comparison between the spike spectral properties and the ECM spectral properties has yet been made.

Hewitt et al. (1982) suggested a simple kinematic estimate for the ECM bandwidth,

$$\Delta \omega / \omega \sim (v/c)^2, \hspace{1cm} (1)$$

where $\omega$ is the central frequency of the emission line, $\Delta \omega$ is the spectral bandwidth, and $v$ is a characteristic velocity of the fast particles responsible for the ECM generation. This estimate evidently has a limited applicability region, since it does not depend on the fast electron pitch-angle distribution, viewing angle, etc. Moreover, it is not clear what value of $v$ should be used if the energy distribution of fast electrons is rather broad, as in the case of the power-law spectra typical for solar flares.

The problem of the ECM bandwidth attracted a lot of attention in connection with the fine structure of the terrestrial auroral kilometric radiation (AKR; Gurnett & Anderson 1981; Baumbach & Calvert 1987; Yoon & Weatherwax 1999; Pritchett et al. 1999). Indeed, the bandwidth of individual AKR peaks has been observed.
to be as small as $\Delta \omega/\omega \sim 10^{-3}$ (Gurnett & Anderson 1981), with extreme values down to $10^{-3}$ (Baumback & Calvert 1987). It was shown recently (Yoon & Weatherwax 1998) that the choice of a realistic distribution function for the superthermal electrons may easily provide an ECM bandwidth that is on the order of $10^{-3}$.

However, the results that have been obtained for the auroral region cannot be directly applied to the solar case because of important differences in the source conditions. First of all, the plasma frequency to gyrofrequency ratio,

$$Y = \omega_{pe}/\omega_{Be},$$

is much less than unity for the AKR source, whereas it is on the order of unity or larger for the solar corona. Therefore, the corrections to the wave dispersion provided by superthermal electrons may be important for the AKR source (Yoon & Weatherwax 1998), whereas they are typically negligible for the solar case. Then the fundamental extraordinary wave mode is the most important if $Y \ll 1$, while different wave modes (either fundamental or harmonic ordinary and harmonic extraordinary) become important if $Y \sim 1$. The distributions of the fast electrons are also believed to differ significantly for these two cases: keV electrons are responsible for the AKR, while broad distributions covering the range of $10^{-10^3}$ keV at least arise typically in solar flares.

Thus, the study of the ECM spectral properties for the conditions typical for solar flares is largely an independent problem that deserves particular attention and careful theoretical consideration. So far, the natural bandwidth of the ECM emission in the standard coronal conditions has been studied in detail by Fleishman (2004b), who demonstrated that the relative ECM bandwidth typically belongs in the range

$$\Delta \omega/\omega \sim 0.1\% - 0.4\%,$$

which is more than an order of magnitude less than the intuitive estimate (eq. [1]).

The effect of source inhomogeneity on the bandwidth of ECM peaks was studied by Platonov & Fleishman (2001) in the linear approximation; i.e., when the quasi-linear saturation and nonlinear wave-wave interactions are not important. The corresponding broadening related to the gradual nonuniformity of the solar corona was found to be rather small, whereas the effect of random inhomogeneities of the magnetic field is typically important. Fleishman (2004a) developed a renormalized theory of the ECM emission in sources with random inhomogeneities of the magnetic field and found that relatively weak magnetic inhomogeneities can provide strong broadening of the ECM peaks.

In this paper we analyze a dense cluster of solar radio spikes registered at high frequencies above 4.5 GHz. A special numerical technique was developed to decompose partly overlapping spikes in the cluster into individual Gaussian spikes and yield large, statistically significant series of the spikes. Applying this method to the experimental raw data, we eventually identified about 3000 spikes for the observed event, which allows us to make a detailed statistical analysis.

The most striking finding of this analysis is the distributions of the spike bandwidth, which are remarkably asymmetric. The overall bandwidth distribution has a characteristic skewed shape, with a rapid increase at low values of the relative bandwidth, followed by a maximum at 0.6% and a smooth tail approaching zero at approximately 3%. In order to account for the essential features of the bandwidth distributions, we explicitly use the renormalized theory of the spectral profile of the ECM emission peak. The theory accurately takes into account the fluctuations of the magnetic field in the spike source. The bandwidth distribution obtained by the proposed theory is found to agree excellently with the observed spike bandwidth distribution.

2. OBSERVATIONS

We analyze a dense cluster of solar radio spikes that were registered from 05:18:03 to 05:18:09 UT on 2001 April 10 at 4.5–6 GHz by the Purple Mountain Observatory (PMO) spectrometer (Nanjing, China), operating in the 4.5–7.5 GHz range with a temporal resolution of 5 ms. The cluster occurred during a class X2.3 flare on 2001 April 10 (Asai et al. 2003; Chernov et al. 2006) in NOAA active region 9415, which was located close to the center of the solar disk (S23, W06–08). The flare was associated with a halo coronal mass ejection (CME), meter-wavelength type II and IV bursts, and a strong microwave continuum burst.

The spike cluster occurred during a local impulsive peak of a long, strong microwave burst with an absolute peak value in excess of 6000 SFU around 9.4 GHz. Highly polarized coherent emissions (left-circular polarization [LCP] at 2 GHz and right-circular polarization [RCP] at 3.75 GHz) were recorded by the Nobeyama polarimeters around the time of the spike cluster. Context data from the photospheric magnetic fields and microwave emission at 17 GHz at the time of the spike cluster are shown in Figure 1.

2.1. Instrumentation

The data that we use here were collected by the radio spectrometer at PMO, China (Xu et al. 2003). This instrument has 300 frequency channels per 3 GHz band at 4.5–7.5 GHz, with a spectral resolution of 10 MHz and a time resolution of 5 ms, and it observes daily between 1:00 and 9:00 UT.
The dynamic spectrum measured for the event studied is given in Figure 2a. Two of the most intensive regions of the spike cluster can be seen around the second and third seconds of the given dynamic spectrum. As can be seen from the figure, the spikes are not well separated in the frequency domain, but rather substantially overlap, producing a continuously fluctuating spectrum.

2.2. Spike Resolution and Identification

If the resolution of the instrument is high enough in both the temporal and spectral domains, each spike represents a two-dimensional (2D) object in the dynamic spectrum, which can, in particular, be characterized by duration, bandwidth, and spectral drift (Dąbrowski et al. 2005). In many cases, however, the resolution in either the spectral or temporal domain is insufficient to fully resolve each spike. Let us consider first the available temporal resolution against the spike duration.

Lower frequency observations, performed mainly at the decimetric spectral range, show that the spike duration is largely a function of the emission frequency, with rather weak scatter around the regression curve (Guèdel & Benz 1990; Mészárosová et al. 2002, 2003; see also Fig. 3, which gathers all currently available measurements of spike duration). The regression law found from this figure for the entire available spectral range, 237–2695 MHz,

\[ \tau \propto f^{-1.29 \pm 0.08}, \]

represents a corrected Guèdel-Benz law as established by Güdel & Benz (1990) for a limited spectral range. This law predicts that the duration of spikes at \( f > 4.5 \) GHz should be less than 2 ms, which is well below the spectrometer temporal resolution, 5 ms.

To check this prediction, we studied the cross-correlation of the recorded signal versus the time lag and found that adjacent time frames are indeed entirely uncorrelated (see Fig. 4). This means that each spike does appear only once in the dynamic spectrum in most of the cases. Therefore, the spikes are not resolved in time.

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**Fig. 2.**—Dynamic spectra for the event under study: (a) calibrated data from the PMO spectrometer, (b) the part of the spectrum selected as input for spikes analysis, (c) the fitted spectrum obtained by the summation of the extracted Gaussian spikes superimposed on the signal background level, and (d) the absolute difference between the input spectrum from panel b and the fitted spectrum from panel d. The arrows in panel b point out the time frames that are expanded in Fig. 6. Time is shown in units of seconds after 05:18:03.5 UT, 2001 April 10. The color scale ranges from white to blue to red, indicating the intensity level from zero to the maximum value.

**Fig. 3.**—Observed duration of spikes at half-maximum vs. frequency of observation, based on published data. The data points at 237, 327, 408 and 610 MHz are from Zlobec & Karlický (1998) and P. Zlobec (2002, private communication), that at 540 MHz is from Lipatov et al. (2002), those at 362, 468, 730, 770, 870, and 1010 MHz are from Güdel & Benz (1990), and those at 1420 and 2695 MHz are from Mészárosová et al. (2002, 2003). The solid line shows the best power-law fit, with an index of \(-1.29 \pm 0.08\).
by the PMO spectrometer. Thus, each time frame has to be processed independently from the adjacent time frames. This reduces a 2D fitting problem to a sequence of one-dimensional (1D) fitting problems, which is a much easier task.

On the other hand, the spike signal is well resolved in the spectral domain. Figure 5 shows the autocorrelation of the instantaneous spike signal in the frequency domain. The autocorrelation functions were calculated for each time frame that had a firm spike signal. The obtained average autocorrelation function is shown in Figure 5, labeled as “observed.” The squares connected by the solid line show the correlation coefficient calculated at lag values of $n \times 10$ MHz, where the integer $n$ denotes the number of frequency channels of the spectrometer (which has a spectral resolution of 10 MHz). Significant correlation between a few adjacent channels clearly indicates that each spike is seen through several spectral channels.

To understand the meaning of the shape and bandwidth of the cross-correlation function obtained, we performed a very simple modeling as follows. We calculated the autocorrelation function of a pure Gaussian signal with a certain bandwidth. The corresponding Gaussian signal $g(f)$ is shown in the inset in Figure 5. Remarkably, the “observed” curve can be excellently fitted with the autocorrelation of a purely Gaussian signal of a certain width $\Gamma$ (see the curve labeled “modeled” in Fig. 5). Even though the similarity of the observed curve to the modeled curve could be a result of the central limit theorem, rather than a similarity of individual spike profiles to a pure Gaussian, it nevertheless implies the presence of a characteristic bandwidth in the spike signal. The best agreement between the modeled and observed plots is achieved for $\Gamma = 30$ MHz, which suggests that the bandwidth of the majority of the spikes is around 30 MHz.

This value is not at all unexpected. Csillaghy & Benz (1993) studied 196 individual spikes recorded in different events over a sufficiently broad spectral range to yield an empirical regression law of $\Gamma \ [\text{MHz}] = 0.66f^{0.42} \ [\text{MHz}]$, which implies a value of $\Gamma \approx 25$ MHz at the frequencies 4.5–6 GHz. Given the very large scatter of individual measurements around this regression curve in Csillaghy & Benz (1993), we conclude that our finding of the characteristic bandwidth around 30 MHz is in full agreement with the results of Csillaghy & Benz (1993). Dividing this value by the typical frequency of $\approx 5$ GHz, we get the relative bandwidth of a spike, $\Gamma_{rel} \approx 0.6\%$. Thus, we can identify this value of $\Gamma_{rel}$ with a characteristic relative bandwidth of spikes contained in the measured data.

2.3. Spike Decomposition Technique

To handle the data from the spectrometer, we developed a new technique that uses a nonlinear multi-Gaussian spectral fit based on $\chi^2$ criteria to extract individual spikes from the originally recorded spectra. At the first stage we identify the time frames in which the intensity of the signal cannot be well distinguished from the noise in the whole frequency range. Such frames are excluded from further analysis. The retained part of the dynamic spectra with distinguishable spiky signal is shown in Figure 2b. The technique described below is sequentially applied to the set of retained time frames. Recall that each instantaneous spectrum is treated as a set of unique independent spikes. The input for the elementary fitting procedure is, therefore, an instantaneous spectrum $S(f)$. The signal $S(f)$ is fitted with a sum of model spikes superposed on a zero level $z$, which does not depend on frequency but may change from frame to frame. Each spike is assumed to have a Gaussian shape described by three parameters, namely, the amplitude ($A_i$), the mean ($f_{0i}$), and the standard deviation ($\gamma_i$):

$$S^*(f) = z + \sum_{i=1}^{M} s_i(f),$$

$$s_i(f) = s_i(f, A_i, f_{0i}, \gamma_i) = A_i e^{-(f-f_{0i})^2/(2\gamma_i^2)}.$$  

No frequency-dependent background component other than the statistical noise is assumed. We will often use the bandwidth $\Gamma_i$ (full width at half-maximum) to describe a spike rather than the standard deviation; both values are related through the expression $\Gamma_i = 2(2 \ln 2)^{1/2} \gamma_i \approx 2.35 \gamma_i$. It must be noted that the bandwidths derived from the fitting (both $\Gamma$ and $\gamma$) are arbitrary real numbers, rather than integer multiples of the instrument spectral resolution of 10 MHz.

The fitting procedure starts from our “guess” set of $M$ spikes $\{s_{0i}\}$, whose number and amplitudes are those of the local maxima found in the signal above a certain noise threshold level. We
changed this noise threshold level in various runs of the fitting to check the consistency and stability of the fitting results. The constrained nonlinear minimization is performed with regard to the \(\chi^2\) statistics

\[
\chi^2 = \sum_j \frac{[S(f_j) - S^*(f_j)]^2}{\sigma_j^2}
\]

(7)

using the IDL routine CONSTRAINED_MIN. The residual of this fitting consists generally of the statistical noise (which is presumably known on the basis of the instrument characteristics) and a contribution from numerous weak unresolved spikes, whose contribution is unknown. Thus, the full uncertainties associated with a set of measurements are unknown in advance.

However, if we assume that all measurements have the same standard deviation, \(\sigma\), and that the model does provide a good fit to the data such that \(\chi^2 \approx N - 3M\), where \(N\) is the number of the data points used for the fitting, then we can estimate \(\sigma\) by first assigning an arbitrary constant value of \(\sigma_0\) to all points, next fitting for the model parameters by minimizing \(\chi^2\), and finally adopting

\[
\sigma^2 = \frac{\sum_j [S(f_j) - S^*(f_j)]^2}{N - 3M}.
\]

(8)

After the minimization procedure is completed, each extracted spike is tested for statistical significance as follows. The spike is excluded from the fitting function so that

\[
S^{**}(f) = z + \sum_{i=1}^{M-1} s_i(f).
\]

(9)

A new minimization is performed, and the new \(\chi^2\) statistics is calculated, keeping the obtained value of \(\sigma\). The spike is excluded if this procedure does not decrease the “goodness of fit” expressed by the quantity \(\chi^2^2\):

\[
\frac{\chi^2^2}{N - 3(M - 1)} \leq \frac{\chi^2}{N - 3M}.
\]

(10)

Figure 6 shows an example of the fit for three instantaneous spectra taken at the time frames indicated by the arrows in Figure 2b. The figures show very good agreement between the initial raw signal and the fit spectrum, which is the sum of the spikes extracted. Figure 6b also shows that sometimes the fitting procedure may drop a spike with a moderate intensity that is still well pronounced. While such cases are statistically rare, nevertheless we have performed a number of spike extraction procedures, varying the parameters of the constrained minimization, as well as the value of the threshold used to get Figure 2b from Figure 2a. The corresponding variation of the output statistical parameters of the extracted spike ensembles allowed us to estimate the statistical errors, as well as the mean quantities given in the next subsection.

2.4. Spike Extraction Results

The developed technique allowed us to extract about 3000 spikes. Each spike \(i\) is characterized by its amplitude \(A_i\), mean frequency \(f_{0i}\), and the bandwidth \(\Gamma_i\). The distribution of the spike mean frequencies is shown in Figure 7. In obvious agreement with the raw data spectra (Figs. 2a and 2b), the number of spikes decreases with the frequency increase. A small deviation from this tendency can be seen near \(\approx 5\) GHz. This corresponds to the sharp end of the first subcluster at about 4.9 GHz and also to the malfunction of a few frequency channels of the instrument, which can be clearly seen as a blue horizontal stripe at \(\approx 5.1\) GHz in Figure 2a. The amplitude distribution of the spikes is shown in

![Figure 6](image-url)

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**Figure 6.**—Fit examples for the three time frames marked by the arrows in Fig. 2b. Dashed lines represent the input spectra, and solid lines show the sums of the extracted spikes. The spikes are indicated by the dotted lines.
Figure 8. Note that only the decreasing part of the distribution makes sense, whereas the initial increasing part is possibly an artifact caused by skipping the spikes that are beneath the average noise level. The decreasing part of the amplitude distribution follows an exponential law (see the inset in Fig. 8), which allows us to estimate of the number of weak missing spikes to be about 30% of the extracted spikes. The spike amplitude correlates with frequency as is shown in Figure 9, which means that both more spikes and stronger spikes are produced at lower frequencies; the calculated rank correlation coefficient is \( r = -0.44 \). This tendency is also apparent from Figure 10, which presents the moving average for the spike amplitude versus the spike central frequency.

A striking finding of the presented spike analysis is the distribution of the spike relative bandwidths. This commonly used dimensionless parameter is defined as the spike width at half-maximum, divided by the central frequency of the spike. We checked and found that the bandwidth distribution has no imprint of the discreteness of the spectral channels: integer multiples of 10 MHz do not display any local peak compared with neighboring noninteger values. All values for the relative bandwidth that are present in the histogram are in excess of the minimum spike bandwidths reported in the literature for other events (Messmer & Benz 2000; Wang et al. 2003).

The distribution of the spike relative bandwidths appears to be remarkably asymmetric. It has a skewed shape, with a rapid increase at low values of the relative bandwidth, followed by a maximum at 0.6% and a smooth tail approaching zero at approximately 3% (Fig. 11). Note that the peak of the distribution (0.6%) corresponds to the characteristic spike bandwidth found in § 2.2 from the correlation analysis.

The asymmetry of the distribution can be characterized by the skewness, which is the third moment:

\[
S = \frac{\langle (f - f_0)^3 \rangle}{\sigma^3}.
\]

For the spike ensemble under study, the overall skewness is about 1.6. The deviation from the normal distribution is estimated by the fourth moment, kurtosis, which is defined as

\[
K = \frac{\langle (f - f_0)^4 \rangle}{\sigma^4},
\]

which is \( K = 6 \) for our case, in contrast to that of the normal distribution, for which \( K_{\text{norm}} = 3 \). A rather weak correlation is found between the amplitude and the spike relative bandwidth (Fig. 9); the appropriate rank correlation coefficient is only \( r \approx -0.2 \). No correlation is observed between the mean frequency and the relative bandwidth of the spikes (the appropriate rank correlation coefficient is \( r \approx 0.04 \)).

The large number of individual spikes identified allows us to perform an analysis of the spike properties in restricted spectral regions. Accordingly, we looked for a dependence of the bandwidth distribution shape on the spike central frequency. However, no unambiguous trends have been found. Figure 12 displays the frequency dependence of the moments of the distribution, such as the mean, median, standard deviation, skewness, and kurtosis, normalized to the corresponding global values gathered in Table 1. The figure shows no significant deviation of any of these parameters \( P \) from its overall average value \( P_0 \).

3. LOCAL-TRAP MODEL

As has already been noted, most of the spike properties are consistent with a local-trap model (Fleishman & Melnikov 1998; Fleishman et al. 2003). This model adopts the assumption that a spike cluster is produced at a significant portion of a magnetic trap (Fig. 13), where a loss cone distribution of the trapped fast particles is formed due to emptying the loss cone as a result of the electron precipitation into the footpoints. The overall pitch-angle anisotropy is moderate on average, provided that the mean fast electron distribution is at about the marginal stability state with respect to ECM generation. An important ingredient of the local-trap model is magnetic turbulence, which gives rise to a local variation of the fast electron distribution anisotropy. Under favorable conditions, this turbulence will increase the anisotropy to the extent sufficient for the ECM instability to develop at some local places inside the large-scale magnetic trap. Such favorable places represent those local spike sources, which are quasi-randomly distributed over the trap.

As a by-product of the key role of the magnetic turbulence in forming the local spike sources, the model suggests that the small-scale end of the magnetic turbulence spectrum persists in each local spike source. Therefore, the spikes are formed in a source,
where random magnetic inhomogeneities are superimposed on the mean magnetic field of the source. Fleishman (2004a) developed a theory of the ECM spectral bandwidth that takes into account the stochastic irregularities of the magnetic field at the source. He found that the bandwidth of a single ECM peak generated in such a source is specified by the relation

$$\Gamma \propto \Gamma_0 \frac{a}{\tau^{1/2}}.$$  \hspace{1cm} (13)

where $\Gamma_0$ is the “natural” bandwidth of the ECM peak in the uniform source with optical depth $\tau$ in the peak maximum and

$$a = \frac{s_0^2 \langle \delta B^2 \rangle}{2 B^2}$$  \hspace{1cm} (14)

is a “turbulence parameter” defined by the ECM harmonic number $s_0$ and the magnetic turbulence energy density $\langle \delta B^2 \rangle/8\pi$ and normalized by the magnetic energy density $B^2/8\pi$. We note that the magnetic inhomogeneities give rise to quite a strong ECM broadening when $a \approx \Gamma_0^2$. Since $\Gamma_0^2 \ll 1$, rather weak random inhomogeneities of the magnetic field produce large ECM broadening.

To account for the observed spike bandwidth distribution, we developed a simple model based on the ECM emission within the local-trap model. Specifically, we make use of the fact that the natural spike bandwidth is about 0.1%–0.3%, in agreement with calculations of the ECM natural spectral bandwidth (Fleishman 2004b). We postulate a symmetric Gaussian distribution of the natural bandwidth over the local spike sources. Then we adopt the assumption that the turbulent parameter $a$ has another Gaussian distribution, not correlated with the natural bandwidth distribution.

By varying the pairs of $\Gamma_0$ and $a$ within those two parent distributions, we are able to produce artificial sets of spikes. The properties of these artificial spike distributions are specified by the parameters of the adopted Gaussian distributions; i.e., the mean values and dispersions of $\Gamma_0$ and $a$. In our modeling, we kept the mean $\Gamma_0$-value constant at a 0.2% level, while we varied the $a$-value in order to study the dependence of the distribution moments on the value of $a$. The dispersion of both values was taken.

Fig. 9.—Correlation between mean frequency and spike amplitude (left) and between spike amplitude and relative bandwidth (right). These plots show that lower frequency spikes are typically stronger and stronger spikes tend to be more narrowband, although the correlation coefficients are somewhat low.

Fig. 10.—Moving average of spike amplitudes vs. spike mean frequency. At least 100 spikes were averaged for each frequency range.

Fig. 11.—Bandwidth distribution of the extracted spikes. The bin size is taken to be 0.15% to avoid any coincidence with the spectral resolution of the instrument. Note the prominent asymmetry of the distribution.
to be about 15%. An example of the distribution produced by the model is given in Figure 14. Eventually, we generated 50 sets with 3000 artificial spikes in each set and calculated the first four moments of these distributions, which are plotted in Figure 15. Figure 15 also displays the values of the observed distribution moments. Remarkably, all the observed moment values are consistent with the corresponding model values for \( a / C_{25}^2 \) and \( 10 / C_{07}^7 \), which is especially important because the model includes only one free parameter from which to yield all four moments together with the correct values. In other words, the comparison of the observed and model moments offers an elegant method with which to study the small-scale magnetic turbulence in the sources of spike clusters. We note that the method is highly sensitive to the magnetic irregularities and is capable of detecting the turbulence at a remarkably low level of \( 10^{-7} \) or even less.

4. DISCUSSION

Although the phenomenon of the narrowband spikes and related fine spectral and temporal structures has not been well understood yet, the permanently accumulated data and their detailed and critical analysis in the context of the competing source models have given rise to a significant progress. In particular, there is currently strong evidence that narrowband millisecond spikes appearing at the main flare phase are generated by ECM emission. This follows, e.g., from detailed comparisons between the observed properties of solar radio spikes and predictions of various theoretical models (Fleishman & Melnikov 1998). Even stronger evidence has recently been found from the analysis of correlations between the spikes and the accompanying microwave continuum (Fleishman et al. 2003): they found that spike-producing radio bursts reveal smaller ratios of plasma frequency to gyrofrequency than do other radio bursts, and the strongest averaged flux of the spike emission is observed when the fast electrons display the hardest energy spectra and the most anisotropic angular distribution.

However, it is still unclear as to whether spikes in all events are produced by the same emission mechanism, or if there are different subclasses of the spikes. Indeed, only half of all the spike clusters analyzed by Aschwanden & Güdel (1992) correlate well with simultaneous hard X-ray (HXR) emission; the other half reveal weaker or no correlation. Then Benz et al. (2002) studied the location of narrowband spikes that occurred at frequencies of \( \gtrsim 450 \) MHz during the flare decay phase. The spike source was found to be located far away from the main flare location, which was interpreted as a postflare electron acceleration high in the corona. Such spike bursts are likely different in nature from the spike clusters originating during the impulsive and main flare phases, which are highly correlated with HXR and microwave flare emission. We believe that more progress toward understanding the nature of the spikes can be expected when broadband imaging radio instruments start to operate.

**TABLE 1**

| Moment of Distribution | Value          |
|------------------------|----------------|
| Mean \( f_0 \) (%)     | 0.91 ± 0.02    |
| Median (%)             | 0.77 ± 0.01    |
| Standard deviation \( \sigma \) (%) | 0.55 ± 0.04 |
| Skewness \( S \)       | 1.6 ± 0.2      |
| Kurtosis \( K \)       | 5.9 ± 0.6      |
Similar dynamic spectra of radio emission produced by different mechanisms can be understood if the (quasi-random) distribution of the spikes in frequency and time is related to the global source structure rather than the microscopic emission process. For example, Bártá & Karlický (2001) developed a turbulent model of the spike source that can result in dynamic spectra typical for spikes for different emission mechanisms. Therefore, detailed case studies, as well as statistical studies capable of distinguishing between competing emission mechanisms, are exceedingly important.

This paper reports a spike cluster at high frequencies, 4.5–6 GHz, which is a relatively rare type of event. No statistics of spikes has yet been made available at this spectral range. To remedy this situation, we developed a method that is capable of extracting spikes from dense clusters of overlapping spikes. Applying this method, we extracted a few thousand spikes, sufficient for a detailed analysis of spike properties. In particular, besides finding the mean and the variance of the distribution of the spike relative bandwidth, we were able to confidently determine higher moments, namely, the skewness and kurtosis. We found this distribution to be highly asymmetric and to deviate confidently from the normal distribution. Although similar asymmetric distributions at lower frequencies have already been reported on the basis of manual selections of well-isolated Gaussian spikes in less dense regions of the spike clusters (Csilaghy & Benz 1993; Messmer & Benz 2000), those previous studies involved much lower numbers of manually selected spikes, and therefore the statistics was less significant.

To complement these works, we developed a simplified theoretical model of spike cluster generation in a magnetic trap with random inhomogeneities based on the renormalized theory of the ECM spectral properties that takes these inhomogeneities into account. The simulated spike distribution is remarkably similar to the observed one for this particular event, which is a strong point of evidence in favor of the ECM mechanism of the spike generation within the local-trap model with magnetic turbulence.

Thus, the demonstrated agreement between the model and observations may suggest an efficient new tool for measuring weak magnetic inhomogeneities in the spike sources. Since the turbulence parameter $\alpha$ depends on the emission harmonics number, it is highly desirable to determine at what harmonics of the gyrofrequency the spikes are produced. A direct answer to this question could be obtained from imaging observations of the spike source, which are currently unavailable at the considered frequency range. Therefore, we have to use an indirect approach.

The ECM emission requires both nonthermal electrons and a sufficiently strong magnetic field to coexist at the spike source. Figure 1 displays the photospheric magnetic field regions corresponding to the fundamental, second, and third gyroharmonics falling to the 4.5–5.6 GHz range and contours of the optically thin microwave radiation at 17 GHz, indicating the region where the fast electrons are present. Although the spikes are produced in the coronal rather than photospheric sources, we believe that the spatial locations of strong coronal magnetic fields are correlated with regions of strong photospheric magnetic fields. Thus, the presence of a strong photospheric magnetic field is a necessary condition for the presence of a strong coronal field nearby. Inspection of this figure shows that generation of the fundamental ordinary mode ECM emission is almost certainly excluded, while the fundamental extraordinary mode ECM emission is not too probable, even though it cannot be confidently excluded.

Apparently, the most probable location of the spike source is the position of the peak of the positive Stokes $V$ distribution overlapping with a tongue of the Stokes $I$ distribution occurring on top of the strong positive magnetic field region, because high circular polarization ($\sim 50\%$) of the microwave continuum radiation is a strong indication in favor of a large magnetic field in this part of the radio source (Bastian et al. 1998). Thus, the second extraordinary harmonics is the most probable emission mode in the spike cluster studied, although the third extraordinary and the second ordinary harmonics cannot be firmly excluded. In any case, the performed study involving the forward modeling offers a sensitive new tool for detecting and measuring the magnetic turbulence in the spike sources.

5. CONCLUSIONS

We report a high-frequency dense cluster consisting of many overlapping spikes and suggest a tool with which to decompose the cluster onto individual Gaussian spikes. The large number of identified spikes allows us to make a much more detailed statistical analysis of them than has ever been published. Then we develop a theoretical model of the spikes as being produced by the ECM mechanism in numerous local sources with random magnetic inhomogeneities. We find that the model and the observed distributions of the spike relative bandwidth agree excellently with each other; therefore, the presented observations support the adopted local-trap model with magnetic turbulence.
Since a specific shape of the model distribution is highly sensitive to the magnetic turbulence, this study suggests an elegant and straightforward way of studying magnetic turbulence in the spike sources. In favorable conditions, this study can be performed in the spectral and temporal domains giving rise to the information of the spatial distribution and evolution of the magnetic turbulence. We note that the level of the magnetic turbulence in the considered event is very weak, $a \ll 10^{-6}$, and most probably is not enough for bulk particle acceleration in the spike source. We interpret this result as an important indication that the spikes are a secondary phenomenon, rather than a manifestation of the primary energy release and particle acceleration.

The developed tool, while powerful and promising, requires further analysis and development. In particular, we need detailed modeling in order to better understand its capability and the limits of its applicability to denser spike clusters. This is especially important for studying long-lasting clusters in which the spike density strongly changes in time. In that case, analysis of a larger number of the spike clusters throughout the whole spectral range in which the spikes are observed will be necessary in order to convert this tool into a kind of routine flare diagnostic.

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