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We use the field correlator method in QCD to calculate the masses of \( \Sigma_c \), \( \Xi_c \) and recently observed \( \Sigma_b \), \( \Xi_b \) baryons and their orbital excitations.

I. INTRODUCTION

A comprehensive knowledge about the mass spectrum and spin splittings of heavy baryons is important for our understanding of quantum chromodynamics. The spectroscopy of \( c \) and \( b \) baryons has undergone a great renaissance in recent years. New results have been appearing in abundance as a result of improved experimental techniques including information on states made of both light \( (u, d, s) \) and heavy \( (c, b) \) quarks [1]. Before 2007, the only baryon with a \( b \) quark, the isospin-zero \( \Lambda_0^b \), was known. Now, we have the isospin one \( \Sigma_b \), \( \Sigma_b^* \) baryons and \( \Xi_b \). The CDF Collaboration has seen the states \( \Sigma_b^\pm \) and \( \Sigma_b^{*\pm} \) [2], while DØ [3] and CDF [4] have observed the \( \Xi_b^- \). The masses of these states are summarized in Table I.

On theoretical side there are many results on heavy baryon masses from different approaches including a number of quark model variations [5, 6, 7], HQET [8], sum rules [9] and lattice calculations [10]. Recently there have been several theoretical papers on the masses of \( \Sigma_b \), \( \Sigma_b^* \) and \( \Xi_b \) based on the modelling the color hyperfine interaction [11, 12, 13]. In the present paper we use the field correlator method (FCM) [14] to calculate the masses of the \( S \) wave baryons containing \( c \) and \( b \) quarks and orbitally excited states that will be experimentally accessible in the future. The same exercise has been applied previously to the ordinary and cascade hyperons [15]. Using FCM, substantial work has been done in the heavy-light meson sector [16]. However, so far only very few results have been reported for baryons. A further study of charmed and bottom baryons using FCM therefore seems
worthwhile.

The dynamics of the $ud$ pair plays a relevant role, being mainly responsible for the spin splitting in the strange sector. A similar contribution is expected for charmed and bottom baryons. Estimates of the one-pion exchange contribution to the baryon mass give -180 MeV both for $\Lambda$ and $\Lambda_b$. Because our approach misses the chiral physics effects we calculate in this work the masses of the $\Sigma$ and $\Xi$ states which are affected by the chiral dynamics only slightly. Note that the $b$-baryons are structurally identical to the $c$-baryons: only a charmed quark is replaced by a beauty quark. Consequently, the analysis of $b$ states and the results are only a variation of what is found for the charmed systems.

II. THE FORMALISM

FCM provides a promising formulation of the nonperturbative QCD that gives additional support of the quark model assumptions. The key ingredient of the FCM is the use of the auxiliary fields (AF) initially introduced in order to get rid of the square roots appearing in the relativistic Hamiltonian. Using the AF formalism allows to write a simple local form of the Effective Hamiltonian (EH) for the three quark system

$$\mathbf{H} = \sum_{i=1}^{3} \left( \frac{m_i^2}{2 \mu_i} + \frac{\mu_i}{2} \right) + H_0 + V, \quad (1)$$

where $H_0$ is the kinetic energy operator, $V$ is the sum of the string potential and a one gluon exchange potential, $m_i$ are the bare quark masses, and $\mu_i$ are the constant AF which are eventually treated as variational parameters. Such an approach allows one a very transparent interpretation of AF: starting from bare quark masses $m_i$ one arrives at the dynamical masses $\mu_i$ which appear due to the interaction and can be treated as constituent masses of quarks. The string potential considered in this work is

$$V_Y(r_1, r_2, r_3) = \sigma r_{\text{min}}, \quad (2)$$

where $\sigma$ is the string tension and $r_{\text{min}}$ is the minimal length corresponding to the Y–shaped string configuration.

The mass $M_B$ of a baryon is given by

$$M = M_0 + \Delta E_{HF}, \quad (3)$$
where $\Delta E_{HF}$ is the spin correction,

$$M_0 = \sum_{i=1}^{3} \left( \frac{m_i^2}{2\mu_i} + \frac{\mu_i}{2} \right) + E_0(\mu_i) + C, \quad (4)$$

$E_0(\mu_i)$ being the energy eigenvalue of the Shrödinger operator $H_0 + V$, and $\mu_i$ are defined from the minimum condition

$$\frac{\partial M_0(m_i, \mu_i)}{\partial \mu_i} = 0. \quad (5)$$

For the light quarks ($m_i \ll \sqrt{\sigma}$) $\mu_i \sim \sqrt{\sigma}(1 + \mathcal{O}(\alpha_s))$ while for the heavy quarks ($m_i \gg \sqrt{\sigma}$) $\mu_i \approx m_i$. In Eq. (4) $C$ is the quark self-energy correction which is created by the color magnetic moment of a quark propagating through the vacuum background field [17]. This correction adds an overall negative constant to the hadron masses:

$$C = -\frac{2\sigma}{\pi} \sum_i \eta(t_i) \mu_i, \quad t_i = m_i/\lambda_g, \quad (6)$$

where $\eta(t)$ is the known function [17] and $1/\lambda_g$ is the gluonic correlation length. We use $\lambda_g = 1$ GeV.

Taking the approach implemented in [15], the spin-independent masses can be obtained from (4). We solve the non-relativistic Schrödinger equation with the confining and Coulomb interactions by the hyperspherical method to determine the constituent quark masses $\mu_i$ and the zero-order baryon masses $M_0$. Then we estimate HF splittings from the perturbative color-magnetic interaction with account of the wave function corrections.

$$\Delta E_{HF} = \sum_{i<j} \frac{\sigma_i \sigma_j}{\mu_i \mu_j} \left( \frac{4\pi\alpha_s}{9} \langle \delta(r_{ij}) \rangle + \frac{\sigma^2}{4\pi} \frac{\lambda_g^2}{\langle r_{ij} \cdot K_1(\lambda_g r_{ij}) \rangle} \right) \quad (7)$$

The first term in (7) is the standard color-magnetic interaction in QCD [18], while the second term, proportional to the string tension $\sigma$, was first derived in Ref. [19].

We use the basis in which a heavy quark is singled out as quark 3 but in which the light quarks are still antisymmetrized. The calculation of the spin matrix elements in (7) is straightforward for $J = 3/2$, as the expectation value of each $\sigma_i \sigma_j$ is 1. For $J = 1/2$ $\sigma_1 \sigma_2 = 1, \sigma_3 \sigma_1 = \sigma_2 \sigma_3 = -2$ for $\Sigma_q$ and $\Xi'_q$ while for $\Xi_q$ $\sigma_1 \sigma_2 = -3, \sigma_3 \sigma_1 = \sigma_2 \sigma_3 = 0$.

The contact interaction in (7) requires the calculation of the $\delta$ function expectation values. These contact probabilities were calculated using 3-body wave functions obtained
by a hyperspherical method. E.g. for \( L = 0 \) the square of the baryon wave function at zero relative two quarks separation is
\[
\langle \delta(\mathbf{r}_{ij}) \rangle_{L=0} = \mu_{ij}^{3/2} \frac{4}{\pi^2} \gamma_0,
\]
where
\[
\gamma_0 = \int_0^\infty \frac{u_0^2(x)}{x^3} dx
\]
is universal for all quark pairs, \( u_0(x) \) is the hyperradial function normalized as
\[
\int_0^\infty u_0^2(x) dx = 1,
\]
with
\[
x^2 = \sum_i \mu_i (\mathbf{r}_i - \mathbf{R}_{cm})^2 = \frac{\mu_1 \mu_2}{M} r_{12}^2 + \frac{\mu_2 \mu_3}{M} r_{23}^2 + \frac{\mu_3 \mu_1}{M} r_{31}^2,
\]
and
\[
\mu_{ij} = \frac{\mu_i \mu_j}{\mu_i + \mu_j}, \quad M = \mu_1 + \mu_2 + \mu_3.
\]
Note that wave function corrections which influence the hyperfine splitting between the different baryons tend to affect \( \gamma_0 \) by only a few per cent: \( \gamma_0 = 0.1207 \) for \( nnc \), \( 0.1197 \) for \( nsc \), \( 0.1161 \) for \( nnb \), and \( 0.1153 \) for \( nsb \) (in units GeV\(^{3/2}\)).

The second term in (7) is expressed in terms of the integrals
\[
\langle \mathbf{r}_{ij} \cdot K_1(\lambda r_{ij}) \rangle = \frac{16}{\pi \lambda_g} \int_0^\infty u^2(x) \left( \int_0^{\frac{\pi}{2}} \xi K_1(\xi) \sin^2 \theta \cos^2 \theta d\theta \right) dx, \quad \xi = \frac{\lambda_g x \sin \theta}{\sqrt{\mu_{ij}}}.
\]

III. THE RESULTS

We employ some typical values of the string tension \( \sigma \) and the strong coupling constant \( \alpha_s \) that have been used for the description of the ground state baryons: \( \sigma = 0.15 \) GeV\(^2\) and \( \alpha_s = 0.39 \). We neglect the mass difference between \( u \) and \( d \) quarks, writing \( n \) to stand for either \( u \) or \( d \). We use the current light quark masses \( m_n = 7 \) MeV and the (slightly updated) strange quark mass \( m_s = 185 \) MeV found previously from the fit to \( D_s \) spectra [16]. However, our predictions need an additional input for the bare quark masses \( m_c \) and \( m_b \). These were fixed from the masses of \( \Sigma_c \) and \( \Sigma_b \), respectively, \( m_c = 1359 \) MeV and \( m_b = 4712 \) MeV.
The result of the calculation of the $S$ wave states is given in Table II. In this Table we also present the dynamical quark masses $\mu_n$, $\mu_s$ and $\mu_Q$ for various baryons. The latters are computed solely in terms of the bare quark masses, $\sigma$ and $\alpha_s$ and marginally depend on a baryon. We also display the results obtained without the HF corrections. The baryon masses are for the isospin averaged states. The result show good agreement between data and theoretical predictions.

In the $\Xi_Q$ (with $Q$ standing for either $c$ or $b$) the light quarks are approximately in a state with $S = 0$, while another heavier state $\Xi'_Q$ is expected in which the light quarks mainly have $S = 1$. Both have total $J = 1/2$. The effect of $\Xi - \Xi'$ mixing due to the spin-spin interaction is negligible \[12\]. There is also a state $\Xi^*_Q$ expected with total $J = 3/2$. The hyperfine splitting between $\Xi^*_c$ and $\Xi'_c$ is found to be 69 MeV that agrees with the experimental value ($\sim 70$ MeV) \[20\], while the predicted mass difference $\Xi'_b - \Xi^*_b = 26$ MeV agrees with the finding of Ref. \[12\]. However, our perturbative calculations do not reproduce the observed $\Xi'_c - \Xi_c$ mass difference. The large hyperfine splitting between axial and scalar $ns$ diquarks is usually described by the smeared $\delta$-function that requires additional model-dependent assumptions about the structure of interquark forces.

A similar calculations were performed for the P-wave orbitally-excited states, see Table IV. Our basis states diagonalize the confinement problem with eigenfunctions that correspond to separate excitations of the light and heavy quarks ($\rho$- and $\lambda$- excitations, respectively). Excitation of the $\lambda$ variable unlike excitation in $\rho$ involves the excitation of the “odd” heavy quark. For states with one unit of orbital angular momentum between $Q$ quark and the two light quarks we obtain $M(\Sigma_c) = 2832$ MeV, $M(\Xi_c) = 2867$ MeV, $M(\Sigma_b) = 6132$ MeV, and $M(\Xi_b) = 6164$ MeV, while the states with one units of orbital momentum between the two light quarks are typically $\sim 100$ MeV heavier. Note that zero order results of Table IV do not include the spin corrections (which are smaller than those for the $S$-wave states) and the (negative) string corrections contributing into the masses of the orbitally excited baryons \[21\]. Our preliminary analysis of the latters shows that the string corrections tend to decrease the masses of the P-wave states by $\sim 30$ MeV. A more complete analysis will be given elsewhere.
IV. CONCLUSIONS

We have calculated the masses of heavy baryons systematically using the FCM and the perturbative color-magnetic interaction. There are two main points in which we differ from other approaches to the same problem based on various relativistic Hamiltonians and equations with local potentials. The first point is that we do not introduce the constituent mass by hand. On the contrary, starting from the bare quark mass we arrive to the dynamical quark mass that appears due to the interaction. The second point is that for the first time we calculate the hyperfine splitting with account of the nonperturbative spin-spin forces between quarks in a baryon. We find our numerical results to be in agreement with experimental data and calculation in other approaches.

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TABLE I: The masses of bottom baryons observed by CDF and DØ collaborations.

| Baryon | Mass (MeV)                                      | Collaboration |
|--------|------------------------------------------------|---------------|
| Σ⁺₇   | $5808_{-2.3}^{+2.0}$ (stat.) ± 1.7 (syst.)     | CDF[2]        |
| Σ⁻₇   | $5816_{-1.0}^{+1.0}$ (stat.) ± 1.7 (syst.)     |               |
| Σ⁺₇   | $5829_{-1.8}^{+1.6}$ (stat.) ± 1.7 (syst.)     |               |
| Σ⁻₇   | $5837_{-1.9}^{+2.1}$ (stat.) ± 1.7 (syst.)     |               |
| Ξ⁻₇   | $5774 ± 11$ (stat.) ± 15 (syst.)               | DØ[3]         |
| Ξ⁻₇   | $5793 ± 2.5$ (stat.) ± 1.7 (syst.)             | CDF[4]        |

TABLE II: Heavy Baryons with $L = 0$. The values of the bare quark masses used in this calculation are $m_n = 7$, $m_s = 185$, $m_c = 1359$, and $m_b = 4712$ MeV. The underlined masses have been used to fix $m_c$ and $m_b$. The dynamical quark masses $\mu_i$ are defined by Eq. [5].

| Baryon | $\mu_n$ | $\mu_s$ | $\mu_h$ | $M_0$ | $\Delta E_{HF}^{(p)}$ | $\Delta E_{HF}^{(np)}$ | $M$   |
|--------|---------|---------|---------|-------|------------------------|-----------------------|-------|
| Σ⁺₇   | 470     | 1455    | 2479    | -19   | -6                     | 2454                  |       |
| Σ⁺₇   | 470     | 1455    | 2479    | 30    | 13                     | 2522                  |       |
| Ξ⁺₇   | 476     | 522     | 1458    | -39   | -20                    | 2460                  |       |
| Ξ⁻₇   | 509     | 4749    | 5806    | 0     | 2                      | 5808                  |       |
| Ξ⁺₇   | 509     | 4749    | 5806    | 19    | 8                      | 5833                  |       |
| Ξ⁻₇   | 514     | 615     | 4751    | -36   | -17                    | 5791                  |       |
TABLE III: Masses of the heavy baryons from the present work and other approaches and the comparison with experimental data (in MeV).

|       | [5] | [6] | [8] | [10] | [7] | [9] | this work | exp       |
|-------|-----|-----|-----|------|-----|-----|-----------|-----------|
| Σ_c   | 2440| 2453| 2452| 2439 | 2411| 2454| 2454      | 2454 ± 0.18 |
| Σ_c^* | 2495| 2520| 2538| 2518 | 2534| 2522| 2518.4 = 0.6 |
| Ξ_c   | 2468| 2473| 2481| 2432| 2460| 2467.9 ± 0.4 |

|       | [5] | [6] | [8] | [10] | [7] | [9] | this work | exp       |
|-------|-----|-----|-----|------|-----|-----|-----------|-----------|
| Σ_b   | 5795| 5820| 5824.2| 5847| 5805| 5809| 5808      | 5808      |
| Σ_b^* | 5805| 5850| 5840.0| 5871| 5834| 5835| 5833      | 5829      |
| Ξ_b   | 5810| 5805.7| 5788| 5812| 5780|     | 5791      | 5774 ± 20 |
|       |     |     |     |     |     |     |           | 5793 ± 3  |
TABLE IV: Heavy Baryons. $L = 1$. The $\lambda$ excitations involve the excitation of the “odd” quark ($c$ for $\Lambda_c$, $\Xi_c$ or $b$ for $\Sigma_b$, $\Xi_b$), while the $\rho$ excitations involve the excitation of the $ud$ diquarks. The bare quark masses are the same as in Table II.

| Baryon | $L_\alpha$ | $\mu_n$ | $\mu_s$ | $\mu_b$ | $E_0$ | $M$ |
|--------|------------|---------|---------|---------|------|-----|
| $nnc$  | $1_\rho$   | 536     | 1452    | 1397    | 2920 |     |
| $nnc$  | $1_\lambda$| 495     | 1491    | 1377    | 2832 |     |
| $nsc$  | $1_\rho$   | 542     | 582     | 1455    | 1372 | 2954|
| $nsc$  | $1_\lambda$| 497     | 544     | 1494    | 1353 | 2867|
| $nnb$  | $1_\rho$   | 570     | 4746    | 1294    | 6240 |     |
| $nnb$  | $1_\lambda$| 540     | 4764    | 1234    | 6132 |     |
| $nsb$  | $1_\rho$   | 574     | 615     | 4748    | 1271 | 6272|
| $nsb$  | $1_\lambda$| 542     | 588     | 4765    | 1211 | 6164|