Phase Transition Critical Flavor Number of QCD.

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Abstract

We present an entirely perturbative QCD determination of the critical phase transition flavor number $N^c_{\tau}$ of QCD. The results obtained are compared with various determinations of $N^c_{\tau}$ by non-perturbative methods, including lattice QCD. The wider physics implication of the existence of the Banks-Zaks regime of QCD with only weakly interacting quarks, is discussed briefly.

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1 INTRODUCTION

Gies and Jaeckel in a recent paper [1] raised the question of reliable determination of the critical flavor number $N_{f}^{cr}$ separating perturbative and non-perturbative regimes of QCD from the high flavor end. They analyzed the chiral phase structure of QCD using a functional renormalization group method, and obtained a value of $N_{f}^{cr} = 10.0 \pm 0.4$. A number of earlier determinations of the same quantity by other authors exist [2-6], based on similar considerations of the chiral phase transition of QCD. Non-perturbative lattice QCD methods have also been used [7] to find $N_{f}^{cr}$. The importance of $N_{f}^{cr}$ as a critical phase transition parameter of QCD has generally been emphasized in these determinations. By noting that the take off point common to all the above formulations and determinations of $N_{f}^{cr}$ is the Banks-Zaks [8] observation of a weakly coupled two loop infrared fixed point at high flavor number in QCD, and that this feature of QCD can be exploited not only non-perturbatively but also perturbatively, we arrive at the possibility that $N_{f}^{cr}$ can also be determined by entirely perturbative methods. Such purely perturbative analysis of the Banks-Zaks (BZ) infrared phase of QCD were already considered in another context of searching for the existence of infrared fixed points in higher order QCD beta function [9-11]. Our focus in this paper is on the original two-loop Banks-Zaks infrared fixed point, presented as a tool for perturbative determination of the phase transition critical flavor number $N_{f}^{cr}$ of QCD. The results so obtained can be compared with various determinations of $N_{f}^{cr}$ by non-perturbative phase transition methods, including lattice QCD. With largely consistent firm values emerging from these different sources, we discuss the wider physics implication of the existence of the BZ regime of QCD and its only weakly interacting quarks.

2 The Banks-Zaks Domain of QCD

A basic observation made by Banks and Zaks in their paper [8] is that along side the ultraviolet asymptotically free fixed point of QCD that holds for $N_f \leq 33/2$, there exists close to this limit point in flavor space, an infrared fixed point that is induced on QCD by its two loop perturbative beta function. The QCD beta function is given by:

$$\beta(a) = -ba^2(1 + ca + c_2a^2 + c_3a^3 + c_4a^4 + \cdots \to \infty)$$  \hspace{1cm} (1)

where $a$ is the QCD coupling constant in the form $a = \alpha_s/\pi$. The first few expansion coefficients $b, c, c_2, c_3$ are known, and have the specific values
\[ b = \frac{33 - 2N_f}{6} \]  
(2)

\[ c = \frac{153 - 19N_f}{2(33 - 2N_f)} \]  
(3)

\[ c_2(\overline{MS}) = \frac{3}{16(33 - 2N_f)} \left[ \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2 \right] \]  
(4)

It is seen that the two loop beta function \( \beta(a) = -ba^2(1 + ca) = 0 \) has an infrared fixed point solution at \( a = a^* = -1/c \). The fixed point value depends on flavor, varying from \( a^* = 0.012 \) at \( N_f = 16 \) to \( a^* = 0.21 \) at \( N_f = 8.05 \). For \( N_f \leq 8 \), the infrared fixed point vanishes. Because of the smallness of the QCD coupling constant in this fixed point domain, Banks and Zaks suggested that the domain can be exploited by making perturbative expansions in the flavor parameter \( (33/2 - N_f) \).

The BZ domain has however other features which others have noted. It is chirally symmetric, and contains massless fermions and gluons [1-4]. Because of the weak coupling state of the domain, quarks are not confined there. The domain has no mass gap or condensate such as would arise from chiral symmetry breaking, and it has no physical particle spectrum such as form from \( q\bar{q} \) bound states, or Nambu-Goldstone bosons. Finally the domain being massless is scale invariant and conformally symmetric [6]. All these properties provide a variety of avenues by which one can explore the nature and physics of the BZ domain of QCD, and how this domain relates to the rest QCD.

A central point is that if QCD has the BZ domain controlled as it is by an infrared fixed point that exists only for certain high values of flavor number, there must exist in QCD not only a phase structure, but a critical flavor number \( N_{f}^{cr} \) at which transitions occur between the different phases of QCD. Banks and Zaks [8] sketched a phase structure for QCD, but a more authentic phase structure was later constructed by Appelquist et. al. [2,3]. Based on this QCD phase structure and various non-perturbative methods, determinations have been made of \( N_{f}^{cr} \) as follows:

The value of critical flavor obtained by Appelquist et. al. [2,3] from their chiral phase structure and non-perturbative gap equation for QCD, is \( N_{f}^{cr} = 11.9 \).
Gies and Jaeckel [1] combining a model of chiral phase structure of QCD and Nambu-Jona-Lasinio (NJL) type four fermion interactions, find $N_{cr}^f = 10.0 \pm 0.4$.

Sannino and Schechter [4] exploring the same chiral phase structure of QCD, use a non-perturbative effective potential and perturbative anomalous dimensions, to obtain a value of $N_{cr}^f = 11.7$.

Harada, et. al. [5] from analysis of meson masses in large flavor Bethe-Salpeter equation obtain a value for $N_{cr}^f = 12$.

Miransky and Yamawaki [6] from a general perspective of conformal phase transition for non-perturbative dynamics of gauge fields, including QCD and its BZ domain, obtain a value for the QCD critical flavor number $N_{cr}^f = 11.9$.

Iwasaki et. al. [7] in a lattice modelling of QCD chiral phase structure and confinement, obtain a value $N_{cr}^f \geq 7$.

We argue that comparable values for $N_{cr}^f$ can be obtained from an entirely perturbative analysis, based on the original perturbative expansions suggested by Banks and Zaks in powers of $(33/2 - N_f)$. First we note that the methods listed above rely on a suitable modelling or choice of QCD phase structure or phase diagram, before one can set up a corresponding non-perturbative QCD calculation or simulation for $N_{cr}^f$. In the perturbative approach discussed below, we use only the firmly established fact that a perturbative infrared fixed point exists in QCD at two-loop order, and for a range of high flavors.

3 Purely Perturbative Searches for $N_{cr}^f$

The BZ idea is to exploit the two-loop infrared fixed point near $N_f \leq 33/2$ by making perturbative expansions of physical quantities, around the point $N_f = 33/2$ in flavor space. For this, we rewrite the two-loop infrared fixed point coupling:

$$a^* = -\frac{1}{c} = \frac{-4}{153 - 19N_f}(33/2 - N_f)$$

in the form $a^* = \epsilon(33/2 - N_f)$ where $\epsilon = -4/(153 - 19N_f)$. We next evaluate $\epsilon$ at $N_f = 33/2$, obtaining $\epsilon = 8/321$. Finally we define a suitable BZ
perturbative expansion parameter as the quantity:

\[ a_o = a_{min} = \frac{8}{321}(33/2 - N_f) \]  

(6)

This quantity \( a_o \) can have a range of values from \( a_o = 0 \) at \( N_f = 33/2 \) to \( a_o = 0.21 \) at \( N_f = 8.05 \) where the two-loop infrared fixed point vanishes. In terms of \( a_o \), the two-loop infrared fixed point eqn(6) becomes reparametrized at any one value of \( a_o \), by the power series:

\[ a^* = -\frac{1}{c} = \frac{a_o}{1 - (19/4)a_o} = a_o + ua_o^2 + u^2a_o^3 + u^3a_o^4 + ..... \]

(7)

where \( u = 19/4 \). Correspondingly :

\[(a^*)^2 = a_o^2 + 2ua_o^3 + 3u^2a_o^4 + 4u^3a_o^5 + 5u^4a_o^6 + 6u^5a_o^7 + 7u^6a_o^8 + 8u^7a_o^9 + 9u^8a_o^{10} + ..... \]  

(8)

\[(a^*)^3 = a_o^3 + 3ua_o^4 + 6u^2a_o^5 + 10u^3a_o^6 + 15u^4a_o^7 + 21u^5a_o^8 + 28u^6a_o^9 + 36u^7a_o^{10} + ..... \]  

(9)

Any physical quantity expanded perturbatively in powers of \( a_o \) for any \( a_o \) value within the above infrared fixed point range in the BZ flavor space, should be a convergent series. While the point \( N_f = 8.05 \) provides a lower limit below which no such convergence of a perturbative series is expected, it can happen that the perturbative series in \( a_o \) breaks down even before the point \( N_f = 8.05 \) is reached, due to onset of QCD phase transition boundaries, or to specific non-perturbative QCD dynamics. Each point of break down of a physical perturbation series in \( a_o \) defines a critical flavor number \( N_c^{cr} \geq 8.05 \) in this NLO case. Thus by studying various perturbative QCD series reformulated as expansions in \( a_o \), and noting the points of break down of each series, one can deduce corresponding values of \( N_c^{cr} \) from equation (6).

We now consider some standard QCD perturbation series, and find their radius of convergence in the BZ flavor space.

(i) As our first perturbation series we take the ratio quantity:

\[ R_{e^+e^-}(Q) = \frac{\sigma_{tot}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]

(10)

Its QCD perturbation expansion is written:

\[ R_{e^+e^-} = 3 \sum_{i=1}^{N_f} Q_i^2(1 + R) \]

(11)
where
\[ R = a + r_2a^2 + r_3a^3 + r_4a^4 + \ldots \] (12)

with [16-20]
\[ r_2 = 1.986 - 0.115N_f \]
\[ r_3 = -6.637 - 1.200N_f - 0.005N_f^2 - 1.2395(\Sigma Q_f)^2/(3\Sigma Q_f^2) \]

We evaluate this QCD perturbative series \( R_{e^+e^-} \) in the specific BZ region of high flavors and NLO infrared fixed point. For this we set \( a = a^* \) in equation (12); also we rewrite each of the coefficients \( r_i \) in terms of \( a_o \) using equation (6). We obtain
\[ r_1 = 0.0885 + 4.6144a_o \] (13)
\[ r_2 = -27.798 + 54.77a_o - 8.05a_o^2 \] (14)

The BZ perturbation series for \( R_{e^+e^-} \) is now obtained by substituting equations (7-9) and equations (13-14) into equation (12). We obtain:
\[
R_{e^+e^-} = R(a^*) = a_o + 4.84a_o^2 + 0.2172a_o^3 - 184.35a_o^4 - 2131.445a_o^5
- 17870.74a_o^6 - 13.01468 \times 10^4a_o^7 - 87.33858 \times 10^4a_o^8
- 55.5123 \times 10^5a_o^9 - 33.9406 \times 10^6a_o^{10} \ldots
\] (15)

One can next use our requirement that the perturbation series equation (15) defined as it is in a fixed point domain, must be a convergent series, to place a bound on \( a_o \) and correspondingly on \( N_f \). Before discussing equation (15) further, we write down the analogous BZ expansions for other QCD physical perturbation series.

(ii) Our second example is the Bjorken Sum rule for polarized electron nucleon deep inelastic scattering, whose QCD perturbation series is [21]
\[
R_{Br}^e = a + k_1a^2 + k_2a^3 + \ldots
\] (16)

where
\[ k_1 = \frac{55}{12} - \frac{1}{3}N_f = -0.917 + 13.375a_o \] (17)
and
\[ k_2 = 41.4399 - 7.6073N_f + 0.17747N_f^2 = -35.7641 + 70.2495a_o + 285.729a_o^2 \] (18)
By plugging equations (7-9) and (17-18) into equation (16), we obtain the BZ expansion for the Bjorken sum rule:

\[
R^e_{Bj} = R^e(a^*) = a_o + 3.833a_o^2 - 8.5405a_o^3 - 267.2234a_o^4 - 2533.49a_o^5 \\
- 18.9297a_o^6 - 12.6894 \times 10^4a_o^7 - 79.8579 \times 10^4a_o^8 \\
- 48.1868 \times 10^5a_o^9 - 28.2121 \times 10^6a_o^{10} \cdots
\]  

(19)

(iii) As our third QCD perturbation series, we take the Bjorken Sum rule for neutrino scattering on nucleons. Its QCD perturbation series is [22]

\[
R^{(\nu)}_{Bj} = a + f_1a^2 + f_2a^3 + ..... 
\]  

(20)

where

\[
f_1 = 5.75 - 0.44N_f = 1.58 + 17.833a_o
\]  

(21)

and

\[
f_2 = 54.2298 - 9.4968N_f + 0.2392N_f^2 = -37.34588 + 64.332a_o + 385.1117a_o^2
\]  

(22)

By plugging equations (7-9) and (21-22) into equation (20), we obtain the BZ expansion for the Bjorken neutrino sum rule:

\[
R^{(\nu)}_{Bj} = R^{(\nu)}(a^*) = a_o + 6.33a_o^2 + 18.057a_o^3 - 84.315a_o^4 - 1360.3677a_o^5 \\
- 11742.8a_o^6 - 8.429 \times 10^4a_o^7 - 55.2112 \times 10^4a_o^8 \\
- 34.2104 \times 10^5a_o^9 - 20.4055 \times 10^6a_o^{10} \cdots
\]  

(23)

(iv) Our fourth case is the QCD perturbation correction to \(\tau\) hadronic decay:

\[
R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^-\bar{\nu}_e)}
\]  

(24)

Its QCD perturbation series is [20,23]:

\[
R_\tau = a + \tau_1a^2 + \tau_2a^3 + ..... 
\]  

(25)

where

\[
\tau_1 = F_3 - \frac{19}{24}\beta_1 = 0.08325 + 15.21494a_o
\]  

(26)

with \(F_3 = 1.9852 - 0.1153N_f\) ; and \(\beta_1 = (2N_f - 33)/6\). Also we have

\[
\tau_2 = -38.44269 + 107.0978a_o + 254.2058a_o^2
\]  

(27)
By plugging equations (7-9) and (26-27) into equation (25), we obtain the BZ expansion for the \( \tau \) hadronic decay width:

\[
R_{\tau}^{(h)} = R^\tau (a^*) = a_o + 4.83325a_o^2 + 0.1223a_o^3 - 183.36a_o^4 - 1849.236a_o^5 \\
- 13.927.0a_o^6 - 9.29142 \times 10^4 a_o^7 - 5.7986 \times 10^5 a_o^8 \\
- 34.647 \times 10^5 a_o^9 - 20.087 \times 10^6 a_o^{10} \cdots \tag{28}
\]

(v) The last QCD perturbation series we consider is QCD correction to scalar Higgs boson hadronic decay width described by the QCD perturbation series [24]:

\[
R_H(a) = 3[1 + h_1 a + h_2 a^2 + h_3 a^3 + \ldots..] \tag{29}
\]

where \( h_1 = 5.66667 \)

\[
h_2 = 35.93996 - 1.35865N_f = 13.52235 + 54.5158a_o \tag{30}
\]

and

\[
h_3 = 164.139 - 25.771N_f + 0.25897N_f^2 = -190.57975 + 691.1551a_o + 416.952a_o^2 \tag{31}
\]

By plugging equations (7-9) and (30-31) into equation (29), we obtain the BZ expansion for the hadronic Higgs decay width:

\[
R_{H}^{(h)} = R_H(a^*) = 3 + 17.0a_o + 121.37a_o^2 + 360.754a_o^3 + 47.7a_o^4 - 9.4869 \times 10^3 a_o^5 \\
- 9.972 \times 10^4 a_o^6 - 7.742 \times 10^5 a_o^7 - 52.975 \times 10^5 a_o^8 \\
- 33.7675 \times 10^6 a_o^9 - 20.449 \times 10^7 a_o^{10} \cdots \tag{32}
\]

4 Analysis of BZ Perturbation Series of Physical Quantities

We now look at equations (15), (19), (23), (28) and (32) and make our deductions. First we comment on the structure of these BZ series. We notice that the first few terms of each series have signs opposite to the rest of the infinite series. Furthermore, simple ratio test of convergence with these early terms does not yield a consistent result. Since we are interested in the overall convergence of each infinite series, we can drop any finite number of terms without affecting the convergence property of each series. Accordingly we shall drop the first four terms of each series and impose our convergence requirement from the fifth term upwards. As for ratio test of convergence, we use the more sensitive Raabe ratio test [25]. This states that for a convergent infinite series of terms: \( \ldots..u_n, u_{n+1} \ldots.. \), the quantity:
\[ n \left( \frac{u_n}{u_{n+1}} - 1 \right) \geq P > 1 \quad (33) \]

where \( n \) is chosen to be \( n \geq N \) and \( N \) is a fixed positive integer, chosen here to be the fifth term of each series. We can then choose in succession \( n = 5, 6, \ldots, 10 \ldots \) in equation (33), obtaining values of \( P \), and deducing the constraint values on \( a_0 \). The corresponding constraint values on \( N_f \) yielding \( N_f^{cr} \) are obtained from equation (6). These values of \( N_f^{cr} \) obtained for the various QCD perturbation series we considered above, are summarized in Table 1. We find a fairly consistent set of values for the critical flavor number \( N_f^{cr} \). These values compare very well with the results quoted earlier from different non-perturbative QCD methods. One feature of our result is that the value of \( N_f^{cr} \) obtained for each physical quantity studied, decreases gradually with increasing \( n \) along each series. The trend is that for \( n \to \infty \), the value of \( N_f^{cr} \) converges to 8.05 like \( a^* \) itself in eqn.(7). In the process we find we reproduce all the results of various non-perturbative calculations quoted earlier. Our results do not yield the lower lattice QCD limit of \( N_f^{cr} \geq 7 \), reported in [7], nor any values \( N_f^{cr} \leq 6 \) suggested in [9,10]. Other independent investigations using Pade approximants [26] place the QCD infrared phase transition boundary well above \( N_f \geq 6 \) in agreement with our result.

5 Summary and Conclusions

It is certain from several independent calculations and sources that the critical phase transition flavor number of QCD lies well above \( N_f \geq 8 \), associated with the BZ two-loop infrared fixed point in QCD.

A remarkable fact coming out of the existence of NLO BZ infrared fixed point region in QCD, is that quarks of flavor \( N_f \geq 8 \) if they exist at all, can only be weakly interacting, unable to form bound states of mesons or hadrons. Therefore they must remain unconfined and observable in their naked color and fractional charge. While various searches for conventional quarks of fractional charge and naked color appear so far to yield no positive results [27], the existence of the BZ region of QCD is sufficiently indicative that these quarks may still be found in the form of very high flavor quarks, probably in cosmic rays.

Additionally, if we remain in the infrared low energy region of high flavor QCD, the BZ analysis shows that the inter-quark coupling \( q\bar{q} \) weakens with increasing flavor variety. Thus quark pairs at \( N_f^{cr} \) are more strongly paired
than quark pairs of \( N_f > N_f^{cr} \) meaning:
\[
\frac{\partial}{\partial N_f} \langle q\bar{q} \rangle = \langle 0 \tag{34} \]

If this trend is assumed to hold continuously across the phase transition boundary at \( N_f \leq N_f^{cr} \), one may infer a regime of ordering of the inter-quark couplings: \( t\bar{t} < b\bar{b} < c\bar{c} < s\bar{s} < d\bar{d} \approx u\bar{u} \). In turn this will reflect in observable flavor dependence of quark condensates, and on the ratios of these condensates. Dosch [28] for example obtained that
\[
\frac{\langle s\bar{s} \rangle}{\langle u\bar{u} \rangle} = 0.65 < 1. \tag{35}
\]
However not enough data presently exist on the values of \( \langle c\bar{c} \rangle, \langle b\bar{b} \rangle, \langle t\bar{t} \rangle \) condensates, to check the above BZ extrapolation. Data on these higher flavor quarkonium states, \( q\bar{q} \) potentials and decay widths will also be helpful.

| BZ series | n = 5 | n = 6 | n = 7 | n = 8 | n = 9 |
|-----------|-------|-------|-------|-------|-------|
| \( R_{e+e^-} \) | \( N_f \geq 12.51 \) | \( N_f \geq 11.78 \) | \( N_f \geq 11.27 \) | \( N_f \geq 10.89 \) | \( N_f \geq 10.59 \) |
| \( R_{Bj}^{e+e^-} \) | \( N_f \geq 12.02 \) | \( N_f \geq 11.37 \) | \( N_f \geq 10.92 \) | \( N_f \geq 10.59 \) | \( N_f \geq 10.33 \) |
| \( R_{Bj}^{e+e^-} \) | \( N_f \geq 12.36 \) | \( N_f \geq 11.71 \) | \( N_f \geq 11.14 \) | \( N_f \geq 10.74 \) | \( N_f \geq 10.45 \) |
| \( R_{Bj}^{e(b)} \) | \( N_f \geq 12.06 \) | \( N_f \geq 11.34 \) | \( N_f \geq 10.87 \) | \( N_f \geq 10.53 \) | \( N_f \geq 10.27 \) |
| \( R_{H}^{e(b)} \) | \( N_f \geq 13.3 \) | \( N_f \geq 12.07 \) | \( N_f \geq 11.37 \) | \( N_f \geq 10.90 \) | \( N_f \geq 10.54 \) |

Table caption: Raabe [25] ratios of convergence equation (33), calculated for various values of \( u_n/u_{n+1} \) and for different QCD physical perturbation series.

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