Longitudinal Structure Function at Intermediate $x$ and the Gluon Density

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Abstract

Calculations are presented of the longitudinal structure function $F_L(x, Q^2)$. We use next-to-leading order expressions in QCD ($\mathcal{O}(\alpha_s^2)$) plus parton densities determined previously from global fits to data on deep inelastic lepton scattering, prompt photon production, and bottom quark production. Anticipated data from the DESY ep collider HERA should provide discriminating information on the gluon density, particularly for values of $x < 0.05$.

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In perturbative quantum chromodynamics (QCD), the longitudinal structure function $F_L(x, Q^2)$, measured in deep inelastic lepton scattering, may be expressed to leading order as a sum of integrals over the gluon and quark densities of the nucleon[1]. As usual $Q^2$ is the square of the four-vector momentum exchange, and $x$ is the Bjorken scaling variable. At small values of $x$, the gluon contribution dominates, and, to fair approximation,

$$F_L(x, Q^2) \approx \frac{2\alpha_s(Q^2)}{3\pi} x G(x, Q^2).$$

(1)

Here $G(x, Q^2)$ is the gluon density, $\alpha_s(Q^2)$ is the QCD coupling strength, and $a$ is a parameter whose value is about 0.4 for $F_L$[2]. Correspondingly, measurement of $F_L$ has long been advocated as perhaps the most direct probe of the gluon density at small $x$. Studies indicate that precise measurements of $F_L$ should be feasible at the HERA ep collider at DESY for values of $x$ of order $10^{-2}$, or less, and values of $Q^2$ in the range 10 to 100 GeV$^2$[3].

Recently a full next-to-leading order calculation ($\mathcal{O}(\alpha_s^2)$) in QCD has been published for the deep inelastic structure functions $F_2$ and $F_L$[4]. These analytic expressions allow us to extend the exploration of the sensitivity of $F_L$ to G beyond lowest order. In addition, new sets of parton densities evolved at two-loop level[5,6] have been determined from global fits to recent data from deep inelastic muon and neutrino scattering[7], prompt photon production[8], and, for the first time[5] bottom quark production cross sections at collider energies[9]. There are significant and interesting differences among the gluon densities in these different fits. We use these various distributions in our calculations of $F_L$, and, as we will show, measurements of $F_L$ at HERA should be discriminating and instructive at both intermediate ($0.005 < x < 0.1$) and smaller $x$.

Complementary to prior phenomenological studies [10], we call attention in this paper principally to the region of intermediate $x$. As we have described previously[5], within the context of $\mathcal{O}(\alpha_s^3)$ perturbative QCD, an acceptable fit to the CDF data on bottom quark production at Fermilab hadron collider energies requires that the gluon density be enhanced in the vicinity of $x = 0.05$, relative to the value it tends to take on if these data are omitted from the fit. No other data yet constrain the gluon density directly in the neighborhood of $x = 0.05$. As we will show, the enhancement in $G(x, Q^2)$ shows up clearly in $F_L(x, Q^2)$. Distinct from an enhanced gluon density at intermediate $x$, alternative theoretical contributions, including the role of off-shell initial gluons [11], may help explain
the bottom quark data. Thus, independent information on \( G(x, Q^2) \) from data on \( F_L \) is highly desirable.

In all global fits done to extract parton densities, specific functional forms are adopted for the gluon density \( G(x, Q^2) \) and other parton densities at the reference value \( Q_0^2 \). Free parameters are varied to obtain a best fit to the data set selected. (The chosen functional forms are somewhat limiting since they may not in all cases be versatile enough to match the form of the data). One constraint imposed on the parton densities is the momentum sum rule. If attention is restricted to the gluon density, this sum rule takes the approximate form

\[
\int_0^1 xG(x, Q^2)dx \approx 0.5
\]

Correspondingly, an enhancement of \( G(x, Q^2) \) in the neighborhood of \( x = 0.05 \) necessarily requires diminution of \( G(x, Q^2) \) elsewhere in \( x \), either at large \( x \) or at small \( x \). In all of our global fits[5], we require good agreement with data on deep inelastic lepton scattering, including the most recent muon and neutrino data from the NMC and CCFR collaborations[7]. Experimental support for the gluon density at \( x > 0.2 \) is provided by data on the production of prompt photons in nucleon-nucleon interactions at fixed target energies[8]. We have determined two different sets of parton densities[5], sets A and B, that differ according to the extent to which we insist on a good fit to the prompt photon data. In Tables 1 and 2, we provide the values of chi-squared for these two fits, and in Fig. 1 we compare the gluon densities from the two fits at \( Q = 5 \) GeV. Although hadronic jet data were not included in the fits, it is of interest that the two sets of densities yield good fits[12] to the inclusive hadronic jet cross section data at \( \sqrt{s} = 1.8 \) TeV[13] and do as well as other published densities at representing the jet data at \( \sqrt{s} = 546 \) GeV[14].

The prompt photon data require a relatively large gluon density for \( x > 0.2 \). As indicated in Fig. 1, set A that provides the best simultaneous representation of the bottom quark and prompt photon data has a gluon density that is relatively suppressed at small \( x \). The relative suppression is most apparent at small values of \( Q^2 \); differences tend to be washed out after evolution to large \( Q^2 \), as is apparent in the curves of \( F_L(x, Q^2) \) discussed below.

The analytic behavior of parton densities in the region of very small \( x \) has been the subject of considerable theoretical discussion. As emphasized above, our interest is focused
here principally on the region of intermediate $x$. Nevertheless, it is useful to summarize the nature of the debate at very small $x$. The "conventional" behavior $xG(x, Q_0^2) \to \text{constant}$ has been assumed in many parametrizations over the years, motivated by considerations founded in Regge pole phenomenology. More recently, the "singular" behavior $xG(x, Q_0^2) \to x^{-1/2}$ has been suggested on the basis of studies[15] of the Lipatov evolution equation, and sets of parton densities incorporating this analytic form have been derived. We have not made fits in which the singular form was imposed, and, as a result, we have little to add to earlier work [10] showing that data on $F_L(x, Q^2)$ below $x = 0.005$ would help to distinguish the conventional and singular behaviors. The relative enhancement to which we call attention should begin to be apparent at more easily accessible intermediate values of $x$, $0.005 < x < 0.1$.

The complete expressions for $F_L$ in order $\alpha_s^2$ may be found in the paper of Zijlstra and van Neerven [4]. Owing to their length we do not reproduce them here. The analysis in this paper is carried out in the MS factorization scheme. We assume five flavors of initial quarks with mass corrections for the bottom quark.

Our numerical results are presented in Fig. 2 as functions of $x$ for four values of $Q^2$: 10, 40, 80, and 160 GeV$^2$. We show predictions for $F_L(x, Q^2)$ obtained from our sets A and B along with expectations from the non-singular set $D_0'$ of MRS. The enhancement in the vicinity of $x = 0.05$ in the gluon densities of sets A and B, visible in Fig. 1, is also apparent in the curves of $F_L(x, Q^2)$ for all four values of $Q^2$. These results strengthen and extend to a larger domain in $x$ previous conclusions [3,10] that measurements of $F_L$ will provide discriminating information on $G(x, Q^2)$. (We caution that owing to the important role of the $\mathcal{O}(\alpha_s^2)$ contributions, the leading order approximation in Eq. (1) should not be applied to deduce $G(x, Q^2)$ from the data.)

As remarked above, the enhancement in the gluon density near $x = 0.05$ arises from $\mathcal{O}(\alpha_s^3)$ QCD fits to data on bottom quark production at $\sqrt{s} = 1.8$ TeV. Alternative explanations of the bottom quark data may involve contributions to the cross section in order $\alpha_s^4$ and beyond, including the effects of off-shell initial gluons[11]. An independent determination of $G(x, Q^2)$ from data on $F_L(x, Q^2)$ would therefore be particularly instructive. Knowledge of the gluon density at small $x$ is of further value for more reliable estimates of rates for hard scattering processes at supercollider energies[16].

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Table 1: Values of $\chi^2$ from the combined fit A to the prompt photon, bottom quark, deep inelastic lepton scattering and massive lepton pair production data. The fit to the deep inelastic data is restricted to points with $Q > 3.16$ GeV and $W > 4$ GeV. The top line specifies the data sets, and the second line lists the values of $\chi^2$ divided by the number of data points. The BCDMS data are renormalized by a factor 0.985. The NMC data are renormalized by a factor 1.020. The CCFR data are renormalized by a factor 0.965.

| Set A          | UA1 | CDF | WA70 | E706 | BCDMS | CDHSW | NMC | CCFR | E605 |
|----------------|-----|-----|------|------|-------|-------|-----|------|------|
| $F'_2^H$      | 0.69| 1.08| 1.28 | 1.45 | 0.65  | 1.05  | 0.47| 0.52 | 1.52 |
| $F'_2^D$      | 1.02| 0.86| 0.67 | 1.02 | 0.50  | 0.53  | 1.61| 1.41 | 1.86 |
| $F_2$         |     | 0.77| 1.14 |      |       |       |     |      |      |

Table 2: Values of $\chi^2$ from the combined fit B to the bottom quark, deep inelastic lepton scattering and massive lepton pair production data. The fit to the deep inelastic data is restricted to points with $Q > 3.16$ GeV and $W > 4$ GeV. The top line specifies the data sets, and the second line lists the values of $\chi^2$ divided by the number of data points. The BCDMS data are renormalized by a factor 0.985. The NMC data are renormalized by a factor 1.020. The CCFR data are renormalized by a factor 0.965.

| Set B          | UA1 | CDF | BCDMS | CDHSW | NMC | CCFR | E605 |
|----------------|-----|-----|-------|-------|-----|------|------|
| $F'_2^H$      | 1.02| 0.86| 0.67  | 1.02  | 0.50| 1.61 | 1.41 |
| $F'_2^D$      | 0.50| 0.53| 1.61  | 1.41  | 1.86| 0.77 | 1.14 |
| $F_2$         |     | 0.77| 1.14  |      |     |      |      |

**Figure Captions**

Fig. 1. Solid and dashed curves show the behavior of the gluon densities from our fits A and B as a function of $x$ for $Q = 5$ GeV. They are compared to the density (dotted) from the conventional set $D'_0$ of MRS, ref. [6].

Fig. 2. Solid and dashed curves show the predicted behavior of the longitudinal structure function $F_L(x, Q^2)$ derived from our fits A and B as a function of $x$ at the $Q^2$ values a) 10 GeV$^2$, b) 40 GeV$^2$, c) 80 GeV$^2$, and d) 160 GeV$^2$. Also shown are values calculated from the conventional set $D'_0$ of MRS, ref. [6].