Quantitative damage imaging in plates based on SH$_0$ mode tomographic approach

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Abstract. Exploring the non-dispersive property, the fundamental shear horizontal mode SH$_0$ can be widely used in the detection and monitoring of defects in plate like structures. To combine an approach of ultrasonic guided waves and a tomographic technique by adapting a reconstruction algorithm for the probabilistic inspection of damage (RAPID) using a double-turn coil omnidirectional shear horizontal wave magnetostrictive patch transducer (OSH-MPT) with three different kinds of array arrangements is presented here for imaging. SH$_0$ can propagate in a composite material through bonding double-turn coil OSH-MPT. To get the information about a damage, RAPID can be utilized based on direct wave of SH$_0$ mode. It is advantageous to monitor a composite plate where the velocity of direct waves depends on the propagation direction. In this paper, a surface simulated defect, using a small cylinder, in a composite material is quantitatively evaluated by using shear horizontal mode tomographic approach. Using modified approaches, to eliminate the effect of uneven probability distribution of the array (UPDA) in RAPID, imaging is obtained. Three different array setups of transducers have been tested to monitor the influence on the images. It has been presented that the image generated by the proposed approach with the optimal transducer array and shape factor, $\beta$, calculation method has great influence on the real defect shape. To improve the imaging results threshold is selected based on the receiver operating characteristic (ROC) curve. The proposed approach can be used to detect, locate and image the surface defects in plate like structures.

1. Introduction

Composite plates are progressively used in different areas such as aerospace, automobile manufacturing, ship transportation and other fields. The applications of non-destructive testing (NDT) technology requires the detection of damage, non-uniformity of the specimen and quantitative analysis of defect without destroying the structures. The development in the structural health monitoring (SHM) is based on the modern technical inventions. However, as compared to most NDT methods, transducers are permanently fixed on the structures with some glue to perform on-line detection\cite{1}. Ultrasonic guided wave monitoring has proved its importance to efficiently and accurately extract feature analysis and real-time monitoring of plate like structures using a finite number of transducers. Ultrasonic guided waves is considered to be the most sensitive to different types of damage in plate-like structures as they can propagate in long range structures without attenuating its strength. Due to this feature, these waves are widely used in the area of NDT and SHM. Guided wave monitoring techniques have advantages of being quick testing and long range coverage to provide complete information of the inspecting structure\cite{2}. Based on the guided wave inspection of a structure, different techniques can be used to analyse the structure integrity. To monitor a structure integrity, imaging is one of the most effective approach, called the tomographic approach. Most of the imaging techniques using ultrasonic waves are...
using the classic reconstruction algorithms\cite{3}. Amongst the guided wave tomography techniques, time-of-arrival imaging method, time reversal technique, time difference-of-arrival imaging method, energy arrival and RAPID are the most highlighted\cite{4-10}. Each method with its advantages and disadvantages were presented in detail\cite{11}.

In this paper, using SH\(_0\) wave DC-MP-EMAT with three different types of transducer setup, the RAPID imaging algorithm has been specifically formulated on the composite plate. In this method wave diffraction is accounted by using an elliptical probability distribution. This SH\(_0\) mode tomographic technique was utilized to locate and size the defect in composite plate. An artificial cylindrical defect was bonded on the surface of plate. To reconstruct the image of defect, the signals of SH\(_0\) mode with defect and without defect in the specimen was obtained and used in the tomographic algorithm. The influence of transducer array setup on the imaging of defect was investigated to optimize a transducer array setup. The proposed optimal array setup can be used to apply on irregular defects to quantitatively image a defect.

2. RAPID algorithm

The RAPID algorithm is a damage imaging approach, where image reconstruction is done by comparing the differences between signals with and without defects for a number of transducer pairs\cite{12}. The signals are compared based on a damage index called signal difference coefficient (SDC). It calculates the difference of a reference signal, which passes through the material without experiencing any defect, and the measured signal through a defect. To calculate the value of SDC the following equation is used\cite{13}:

\[
SDC = 1 - \frac{\int_{t_0}^{t_0 + \Delta T} [x_y(t) - \mu]y_y(t) - \mu \int_{t_0}^{t_0 + \Delta T} [x_y(t) - \mu]y_y(t) - \mu dt}{\sqrt{\int_{t_0}^{t_0 + \Delta T} [x_y(t) - \mu]^2 dt \int_{t_0}^{t_0 + \Delta T} [y_y(t) - \mu]^2 dt}}
\]  

(1)

where \(x_y(t)\) presents the measured signal and \(y_y(t)\) presents the reference signal, \(\mu\) gives mean of the corresponding signal, \(t_0\) is the direct arrival time between a pair of transducer, \(\Delta T\) is a time window. For the same signals the value of SDC is zero whereas for the out of phase signals its value is 1. Here, it is considered that the value of SDC varies due to defect in the imaging area. The value of SDC calculates only the difference between both the damaged signal and reference signal at every transmitter-receiver path. The damage formed in the imaging area is the linear summation of SDC values calculated from each single ray path. After getting the SDC values from all transducer pairs, the RAPID algorithm is applied for the image reconstruction. Image can be extracted by the spatial distribution of each SDC value in an elliptical pattern where a transducer pair is located at the foci of the ellipse. The elliptical distribution of the ray path is shown in Figure 1. The point \((x_i, y_i)\) presents transmitter position and \((x_j, y_j)\) represents receiver position.

![Figure 1. Wave beam in transmitter-receiver path.](image)

The ellipse is controlled by a parameter \(\beta\), and its amplitude varies from its maximum value at the centre line connecting the foci to its minimum value of zero at its boundaries. This parameter \(\beta\) is the shape factor which controls the size of the ellipse. The value of each SDC depends on the size of ellipse\cite{14}. The spatial distribution function representing its value can be expressed as\cite{13}:

\[\text{Spatial Distribution Function} = \text{expression}\]
\[ S_{ij}(x, y) = \begin{cases} \frac{\beta - R_{ij}(x, y)}{1 - \beta} & \text{for } \beta > R_{ij}(x, y) \\ 0 & \text{Otherwise} \end{cases} \]  

(2)

For a variable distance between the transmitter and receiver and for an irregular shape, the shape factor may have different values.

\[ \beta_{ij} = R_{ij, max, damage_range} > 1 \]  

(3)

The shape factor depends on the propagation path between a pair of transducers. The modified spatial distribution function is\textsuperscript{(15)}:

\[ S_{ij}(x, y) = \begin{cases} \frac{\beta_{ij} - R_{ij}(x, y)}{\beta_{ij} - 1} & \text{for } \beta_{ij} > R_{ij}(x, y) \\ 0 & \text{Otherwise} \end{cases} \]  

(4)

A single pair of transducers are not enough to be used to identify damage with precision and clearly. Therefore, to form an image, image fusion approaches are used by using transducer pairs. One approach is a RAPID-based full summation imaging which is defined as\textsuperscript{(16, 17)}:

\[ P(x, y) = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{ij}(x, y) = \sum_{i=1}^{N} \sum_{j=1}^{N} A_{ij} \frac{\beta + 1 - R_{ij}(x, y)}{\beta} \]  

(5)

where \( P(x, y) \) is the expected damage probability at a point \((x, y)\), \( N \) is the total number of transducer pairs. Another approach is RAPID-based full multiplication imaging explained as\textsuperscript{(18)}:

\[ Q(x, y) = \prod_{i=1}^{N} \prod_{j=1}^{N} Q_{ij}(x, y) = \prod_{i=1}^{N} \prod_{j=1}^{N} A_{ij} \frac{\beta + 1 - R_{ij}(x, y)}{\beta} \]  

(6)

where \( Q(x, y) \) presents the estimated damage probability value at a point \((x, y)\). Each transducer pair \( ij \) has one sensing path, one \( P_{ij}(x, y) \) and one \( Q_{ij}(x, y) \). \( R_{ij}(x, y) \) is defined as:

\[ R_{ij}(x, y) = \begin{cases} RD_{ij}(x, y), & RD_{ij}(x, y) \leq \beta + 1 \\ \beta + 1, & RD_{ij}(x, y) > \beta + 1 \end{cases} \]  

(7)

and \( RD_{ij}(x, y) \) is defined as:

\[ RD_{ij}(x, y) = \sqrt{(x_i - x)^2 + (y_i - y)^2} + \sqrt{(x_j - x)^2 + (y_j - y)^2} \]  

(8)

3. Experimental investigation

3.1. Experimental setup and specimen

Figure 2 shows experimental setup for RAPID imaging, which is composed of a high power ultrasonic measurement system RITEC-RAM-4000, an impedance matching box for transmitter and receiver, a digital oscilloscope DPO4054, a preamplifier, and an array of double-turn coil OSH-MPT\textsuperscript{(17)} located on the sample plate through epoxy glue. The sample plate is a 16-layer quasi-anisotropic carbon fiber-reinforced composite plate T300/QY8911 of dimension 1000 mm in length, 1000 mm in width and 2.24 mm in thickness. The transducer array is of 500 mm in diameter where the black circle presents transducer as illustrated in Figure 2. The blue dotted rectangle is the imaging area of dimension 600 mm × 600 mm in length and width accordingly. The center of the array is at the point (300, 300). The magnet generates static magnetic field in the radial direction which is normal to the circumferentially generated dynamic magnetic field by the excited toroidal coil. Due to magnetostriction, a shear
deformation will generate in the nickel patch due to the combine effect of static magnetic field and dynamic magnetic field.

3.2. The UPDA in RAPID

As shown in Figure 3, the probability of a defect between a pair of transducer decreases from the point of connecting both the transducers to the periphery of the ellipse formed by the transducer pair in RAPID.

If several images formed by the transducer pairs are fused, there would be a phenomenon of uneven probability distribution of the array UPDA in RAPID as shown in Figure 4. The inconsistent colour distribution in the Figure 4 shows the variation in the probability. The UPDA in RAPID is an inherent phenomenon, which cannot be eliminated because of different $\beta$ values or $A_{ij}$.

![Figure 2. Diagram of experimental system for RAPID-based imaging.](image)

4. Results and discussion

4.1. Damage imaging results

Experimental system and different array setups for the arrangement of double-turn OSH-MPT array are shown in Figure 2. A simulated cylindrical damage of size 50 mm in height and 30 mm in diameter is bonded to the composite material by epoxy resin. The center coordinates of the damage is (339, 315). Eight transducers have been used in an array, forming total 56 sensing paths. Each transducer pair is used to obtain a reference signal, without damage and a damage signal, these signals are then both compared to calculate the corresponding $A_{ij}$ value.

![Figure 3. Probability value distribution for transducer pair imaging.](image)
4.2. Imaging results at different $\beta$ values

Imaging results obtained at four different $\beta$ values are shown in Figure 5. For the values of $\beta=0.4$ and $\beta=0.2$, the dark area with greater probability value is large to make imaging contrast ratio low. At $\beta=0.02$ there exists some non-detection zone which leads to inaccuracy of the damage location. To improve the location positioning, $\beta=0.08$ can be used for the disadvantages of the above $\beta$ values. The imaging results shown proves the reason of $\beta=0.08$ value.

4.3. Influence of transducers array on the images

The effect of different transducer array setups was investigated to extract the best imaging. Figure 6 describes the different transmitter-receiver ray paths. The blue circles locate the positions of the transducers. The values of SDC can be measured based on the received time-domain signals according to Eq. 1. For the different transducer arrays there is different shape factor values because of different ray paths between the transducer pair, as shown in Figure 7. The calculated shape factors on each path defines the defect size.

4.4. Imaging results before and after eliminating the influence of UPDA

The phenomena of UPDA in the RAPID algorithm has been analysed. As this phenomenon cannot be excluded, but the influence of it can be modified through improving the damage contrast on imaging. Ideally the value of $A_{ij}$ is zero when there is no damage, but actually it is not zero because of external interference. In this study, we have set the mean value of, $A_{avg}$, into the imaging formula to extract an image having UPDA. $A_{avg}$ is a constant value independent of the change of sensing paths. RAPID-based full summation imaging and RAPID-based full multiplication imaging approaches are applied to modify the influence of UPDA. If there is a damage, an original image with the actual value of $A_{ij}$ is generated.

![Figure 4. Phenomenon of UPDA in RAPID imaging.](image-url)
In case of RAPID-based full summation imaging approach, the image with UPDA is subtracted from the actual image. Similarly, in case of RAPID-based full multiplication imaging approach, the actual image is divided by the image with UPDA.

**Figure 5.** Imaging results at different $\beta$ values.

**Figure 6.** Different transducer setup and ray paths for specimen 1 (a) array 1 (b) array 2 (c) array 3.

In case of RAPID-based full summation imaging approach, the image with UPDA is subtracted from the actual image. Similarly, in case of RAPID-based full multiplication imaging approach, the actual image is divided by the image with UPDA.
Figure 8 shows the RAPID-based full summation imaging results before and after using a modified method with the effect of UPDA using a circular array of transducer. The actual image of detecting damage with the effect of UPDA is shown in Figure 8(a). Figure 9 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 8(a). The normalized probability value of every point in the image with its $x$-axis coordinate is shown in Figure 9(a) whereas Figure 9(b) shows the normalized probability value at each point of the $y$-axis coordinate.

Figure 8. RAPID-based full summation imaging results before and after using modified method for the influence of UPDA.
The highest normalized probability value at the $x$-axis and $y$-axis coordinates of detected damage is shown in Figure 9(a) and Figure 9(a), respectively. Figure 10 shows the probability value curves after using modified method under the influence of UPDA. By comparing the actual center coordinates of damage, (339, 315), with the results given in Figure 9 and Figure 10, the detected center coordinates of damage are (352, 304) before using modified method, whereas (338, 312) after using modified method. Figure 11 shows the RAPID-based full multiplication imaging results before and after using a modified method with the effect of UPDA. The actual image of detecting damage with the effect of UPDA is shown in Figure 11(a). Figure 12 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 11(a). The normalized probability value of every point in the image with its $x$-axis coordinate is shown in Figure 11(a) whereas Figure 11(b) shows the normalized probability value at each point of the $y$-axis coordinate. The highest normalized probability value at the $x$-axis and $y$-axis coordinates of detected damage is shown in Figure 11(a) and Figure 11(b), respectively.

Figure 13 shows the probability value curves after using modified method under the influence of UPDA. By comparing the actual center coordinates of damage, (339, 315), with the results given in Figure 12 and Figure 13, the detected center coordinates of damage are (354, 310) before using modified method, whereas (337, 312) after using modified method.

5. Selection of imaging threshold based on ROC curve

To improve the imaging accuracy by the appropriate selection of threshold value, considering the effect
of external factors like input, environmental changes, low signal-to-noise ratio. In RAPID algorithm, there exists a damage at a certain point for a specific value of threshold. This threshold value is obtained through ROC curve\textsuperscript{[19]}.

Table 1 explains how the damage may appear during the imaging process. The probability of occurring event \(E_1\) and damage \(Y_1\) at the same time is called sensitivity, whereas the probability of occurrence of event \(E_2\) and \(Y_2\) is called the specificity. In addition, the probability of event \(E_2\) and \(Y_1\) at the same time is called false positive. ROC curve is obtained by measuring the change between the sensitivity and the false positive rate at different threshold values\textsuperscript{[10,20]}. Figure 14 shows the ROC curve based on RAPID. Thresholds with a step size of 0.1 vary from 0 to 0.9 and vary from 0.9 to 1.0 with 0.01 increment size. The optimal threshold point is very close to the \((0, 1)\) point with maximum sensitivity and minimum false positive rate.

**Figure 11.** RAPID-based full multiplication imaging results before and after using modified method for the influence of UPDA.

**Figure 12.** Probability value curves before using modified method for the influence of UPDA.
Table 1. Four events of damage identification.

| Events   | Y1: Damaged | Y2: Undamaged |
|----------|-------------|---------------|
| E1: Detected | E1 and Y1 occur | E1 and Y2 occur |
| E2: Undetected | E2 and Y1 occur | E2 and Y2 occur |

Figure 15 presents the RAPID imaging results at threshold value. The imaging threshold has greatly reduced the positioning range. Figure 16 shows the RAPID-based full summation imaging results before and after using a modified method with the effect of UPDA using a rectangular array of transducer. The actual image of detecting damage with the effect of UPDA is shown in Figure 16(a). Figure 17 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 16(a). The normalized probability value of every point in the image with its x-axis coordinate is shown in Figure 17(a) whereas Figure 17(b) shows the normalized probability value at each point of the y-axis coordinate. The highest normalized probability value at the x-axis and y-axis coordinates of detected damage is shown in Figure 17(a) and Figure 17(b), respectively. Figure 18 shows the probability value curves after using modified method under the influence of UPDA. By comparing the actual center coordinates of damage, (339, 315), with the results given in Figure 17 and Figure 18, the detected center coordinates of damage are (361, 299) before using modified method, whereas (340, 307) after using modified method. Figure 19 shows the RAPID-based...
full multiplication imaging results before and after using a modified method with the effect of UPDA. The actual image of detecting damage with the effect of UPDA is shown in Figure 19(a).

![Figure 15. RAPID imaging results with threshold.](image)

![Figure 16. RAPID-based full summation imaging results before and after using modified method for the influence of UPDA.](image)

![Figure 17. Probability value curves before using modified method for the influence of UPDA.](image)

Figure 20 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 19(a). The normalized probability value of every point in the image with its \(x\)-axis coordinate is shown in Figure 20(a) whereas Figure 20(b) shows the normalized probability value at each point of the \(y\)-axis coordinate. The highest normalized probability value at the \(x\)-axis and \(y\)-axis coordinates of detected damage is shown in Figure 20(a) and Figure 20(b), respectively.
Figure 21 shows the probability value curves after using modified method under the influence of UPDA. By comparing the actual center coordinates of damage, (339, 315), with the results given in Figure 20 and Figure 21, the detected center coordinates of damage are (367, 299) before using modified method, whereas (347, 308) after using modified method. Figure 22 shows the RAPID imaging results at threshold value. Figure 23 shows the RAPID-based full summation imaging results before and after using a modified method with the effect of UPDA using a linear array of transducers.
The actual image of detecting damage with the effect of UPDA is shown in Figure 23(a).

![Figure 21](image1.png)

**Figure 21.** Probability value curves after using modified method for the influence of UPDA.

![Figure 22](image2.png)

**Figure 22.** RAPID imaging results with threshold.

![Figure 23](image3.png)

**Figure 23.** RAPID-based full summation imaging results before and after using modified method for the influence of UPDA.

Figure 24 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 23(a). The normalized probability value of every point in the image with its x-axis coordinate is shown in Figure 24(a) whereas Figure 24(b) shows the normalized probability value at each point of the y-axis coordinate. The highest normalized probability value at the x-axis and y-axis coordinates of detected damage is shown in Figure 24(a) and Figure 24(b), respectively. Figure 25 shows the probability value curves after using modified method under the influence of UPDA. By comparing the actual center coordinates of damage, (339, 315), with the results given in Figure 24 and Figure 25, the detected center coordinates of damage are (351, 307) before using
modified method, whereas (330, 344) after using modified method. Figure 26 shows the RAPID-based full multiplication imaging results before and after using a modified method with the effect of UPDA. The actual image of detecting damage with the effect of UPDA is shown in Figure 26(a).

Figure 24. Probability value curves before using modified method for the influence of UPDA.

Figure 25. Probability value curves before using modified method for the influence of UPDA.

Figure 26. RAPID-based full multiplication imaging results before and after using modified method for the influence of UPDA.
Figure 27 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 26(a). The normalized probability value of every point in the image with its x-axis coordinate is shown in Figure 27(a) whereas Figure 27(b) shows the normalized probability value at each point of the y-axis coordinate. The highest normalized probability value at the x-axis and y-axis coordinates of detected damage is shown in Figure 27(a) and Figure 27(b),

![Graphs showing probability value curves before using modified method](image)

**Figure 27.** Probability value curves before using modified method for the influence of UPDA.

![Graphs showing probability value curves after using modified method](image)

**Figure 28.** Probability value curves after using modified method for the influence of UPDA.

![RAPID imaging results with threshold](image)

**Figure 29.** RAPID imaging results with threshold.

Figure 27 describes the probability value curves before using modified method under the influence of UPDA, corresponding to the image in Figure 26(a). The normalized probability value of every point in the image with its x-axis coordinate is shown in Figure 27(a) whereas Figure 27(b) shows the normalized probability value at each point of the y-axis coordinate. The highest normalized probability value at the x-axis and y-axis coordinates of detected damage is shown in Figure 27(a) and Figure 27(b),
respectively. Figure 28 shows the probability value curves after using modified method under the influence of UPDA. By comparing the actual center coordinates of damage, (339, 315), with the results given in Figure 27 and Figure 28, the detected center coordinates of damage are (355, 302) before using modified method, whereas (351, 307) after using modified method.

Figure 29 shows the RAPID imaging results at threshold value. Under different transducer array setups the obtained imaging results are quite changed from each other, which specifies that using the guided wave tomographic approach with RAPID algorithm an optimal transducer array setup is necessary to quantitatively image a defect. For some of the defects of having very small width, choosing an appropriate imaging path can have a major influence on imaging results. The imaging results are completely affected by the distribution of shape factor $\beta$. In this analysis, it is clearly shown that a surface damage of better quality can be obtained by the optimization of transducer setup. In addition, it is quite useful to use the same optimized setup of array for the same types of defect, such as surface crack, even they are dissimilar in their shapes. Hence, an initial optimized transducer array setup can be used as a reference for the transducer setup for the other specimen with unknown defect. However, it is still required and important to be studied the optimized transducer setup for different types of defects, such as surface cracks and corrosion. The tomographic approach employed for surface defect imaging in composite materials can only obtain the location and shape of the defect, while the information about the depth of damage is not delivered through this method. The defect here was of dimension 30 mm in diameter and 50 mm in height, which was a simulated cylindrical damage. Some complex types of defects are needed to be studied using magnetostrictive patch transducer.

6. Conclusion
A combination of ultrasonic guided wave SH$_0$ mode and tomographic technique by employing the RAPID approach was investigated for defect imaging in plate. Special EMAT transducers were used to propagate and receive the SH$_0$ mode signals in plate, through experiments. To extract an image of the defect the signal difference of SH$_0$ wave propagation in the specimen with and without defect was used in a tomographic algorithm. It was concluded that the reconstructed damage image can be expressively affected by different transducer array setup. Optimum transducer array setups are suggested to obtain the ultrasonic guided wave signals and measure the various defect shape factors. It was concluded that the image extracted by the proposed arrangements has a good influence on the real defect. The proposed arrangements can be further investigated, using various number of transducers, to detect, locate and image the defects in plate.

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