Nonequilibrium noise and current fluctuations at the superconducting phase transition

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We study non-Gaussian out-of-equilibrium current fluctuations in a mesoscopic NSN circuit at the point of a superconducting phase transition. The setup consists of a voltage-biased thin film nanobridge superconductor (S) connected to two normal-metal (N) leads by tunnel junctions. We find that above a critical temperature fluctuations of the superconducting order parameter associated with the preformed Cooper pairs mediate inelastic electron scattering that promotes strong current fluctuations. Though the conductance is suppressed due to the depletion of the quasiparticle density of states, higher cumulants of current fluctuations are parametrically enhanced. We identify experimentally relevant transport regime where excess current noise may reach or even exceed the level of the thermal noise.

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Introduction.-- Fluctuations of the order parameter associated with preformed Cooper pairs strongly influence transport properties of superconductors above the critical temperature $T_c$. Thanks to the extensive research spanned over several decades we have learned a lot about thermodynamic and kinetic properties in the fluctuation regime [1]. In the context of transport, fluctuation-induced corrections to electric, thermal, thermoelastic and thermomagnetic kinetic coefficients have been rigorously established within the linear response formalism. However, despite the old history of the subject, little is known about essentially nonlinear [2–4] or nonequilibrium domain [5–7]. In particular, the answer to the question on how superconducting fluctuations affect the noise of higher order correlation functions of various observables remains open. We address this outstanding problem by studying excess current noise in a system where superconductor is tailored to be in the fluctuation regime above $T_c$ and driven out of equilibrium by externally applied voltage. Interestingly, this problem has a very natural connection to another rich field, namely the full counting statistics (FCS) of electron transfer [8] in mesoscopic systems. It concentrates on finding a probability distribution function for the number of electrons transferred through the conductor during a given period of time. FCS yields all moments of the charge transfer, and in general it encapsulates complete information about electron transport, including effects of correlations, entanglement, and also information about large rare fluctuations. To access the FCS experimentally is a challenging task, however a great progress has been achieved during the last decade in the field of the quantum noise [9–21], where the new detection schemes enabled to extend the traditional shot noise measurements to the higher-order current correlators.

This work serves dual purpose. First, we elucidate the effect of superconducting fluctuations on the nonequilibrium transport and derive a cumulant generating function for FCS of current fluctuations in a mesoscopic proximity circuit that contains as its element a fluctuating superconductor. We find that due to a depletion of the quasiparticle density of states the conductance of the device under consideration is suppressed, however noise and higher moments of current fluctuations are enhanced due to inelastic electron scattering in a Cooper channel. It should be stressed that finding the FCS for interacting electrons is a very challenging task with only few analytical results known to date [22–28] (see also review articles [29, 30]).

The second important aspect of this paper is a derivation of the nonequilibrium variant of the time-dependent Ginzburg-Landau action (TDGL). The conventional paradigm behind TDGL phenomenology [31] and its subsequent generalizations [32–37] is to assume that electronic (quasiparticle) degrees of freedom are at equilibrium and concentrate on the dynamics of the order parameter field. While leading to the correct static averages, fluctuation-dissipation relations and gauge invariance, this wisdom of handling the problem fails to provide any prescription for calculating higher moments of observables even at equilibrium. Furthermore, existing theories miss completely the stochastic nature of electron scattering on the order parameter fluctuations. Technically, inclusion of such effects should result in stochastic noise terms (Langevin forces) which have a feedback on superconducting fluctuations. Below we elaborate on the methodology that includes all these effects.

Model and results.-- We consider a superconducting diffusive wire (nano-bridge) of length $L$ connected to two normal reservoirs by tunnel junctions with dimensionless conductances $g_1$ and $g_2$, thus forming NISN-structure (Fig. 1). We assume for the conductance of the wire $g_W > g_{1,2}$ and moreover $g_{1,2} \gg 1$ so that charging effects can be neglected. The
our main result,
\[
F(\chi) = -\kappa T e \left[ 1 - \sqrt{1 - 2 \left( \chi^2 - \frac{ieV}{T_c} \right) \frac{\phi + \eta \epsilon}{\epsilon^2}} \right],
\]
which accounts for inelastic scattering of electrons on superconducting fluctuations while traversing across the wire. The proximity to a superconducting transition is controlled by the function
\[
\epsilon(\Delta T, V) = a \frac{ \Delta T }{ T_c } + b \frac{ (eV)^2 }{ T_c^2}, \quad a = \frac{8}{\pi}, \quad b = \frac{a_1 a_2}{2} \frac{14 \zeta(3)}{\pi^3},
\]
where \(a_k = g_k / (g_1 + g_2)\) and \(\zeta\) is the Riemann zeta function. At finite magnetic field the critical temperature is downshifted according to the law \(\ln(T_c/T_d) = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{1}{4\pi T_c}\right)\), where \(T_{d0} = T_{d}(H = 0)\), \(\Gamma = E_{qh} + \tau_{qh}^{-1}\) and \(\psi\) is the digamma function. Thouless energy \(E_{th} = (g_1 + g_2) \delta / 4 \pi\) is defined through the mean level spacing in the wire \(\delta\), while dephasing time \(\tau_{qh} \propto (D/2\pi)(\Phi/\Phi_0)^2 \propto H^2\) is due to orbital effects of perpendicular magnetic field, where \(\Phi\) is a total magnetic flux through the wire and \(\Phi_0\) is the flux quantum. The two dimensionless functions in Eq. (1) are defined as follows
\[
\phi = -\frac{a_1 a_2}{\pi^2} \frac{ E_{qh} }{ T_c } \left( \frac{1}{2} + \frac{ \Gamma }{ 4 \pi T_c } \right),
\]
\[
\eta = \frac{2 a_1 a_2}{\pi^3} \left[ \frac{ E_{qh} }{ \pi \Gamma } \psi'' \left( \frac{1}{2} + \frac{ \Gamma }{ 4 \pi T_c } \right) - \psi'' \left( \frac{1}{2} + \frac{ \Gamma }{ 4 \pi T_c } \right) \right].
\]
The effect of fluctuations is the most singular provided that \(E_{th} \gg \Delta T\) where \(\Delta T \gg \eta \epsilon\). In this case Eq. (1) yields a conductance correction
\[
\frac{\Delta G}{G_Q} = \frac{2 a_1 a_2}{\pi^2} \frac{ E_{qh} }{ \Delta T } \psi'' \left( \frac{1}{2} + \frac{ \Gamma }{ 4 \pi T_c } \right) \frac{ a - b v^2 }{ (a + b v^2)^2 },
\]
where we introduced a notation \(v = eV/\sqrt{T_c \Delta T}\). This result is plotted in Fig. 2 for a certain choice of parameters versus bias voltage and has BCS-like density of states profile (note that \(\Delta G\) is actually negative since \(\psi'' < 0\)). The latter should not be surprising since superconducting fluctuations deplete energy states near the Fermi level which leads to a zero-bias anomaly. In the same limit we find an excess current noise power
\[
\frac{\Delta S_I}{G_Q T_c} = \frac{4 a_1^2 a_2^2}{\pi^3} \left[ \psi'' \left( \frac{1}{2} + \frac{ \Gamma }{ 4 \pi T_c } \right) \right]^2 \frac{ E_{qh}^2 }{ \Delta T^2 } a - b v^2 \left( a + b v^2 \right)^2, \]
which is plotted in Fig. 3. The low frequency dispersion of the noise is set by \(\omega = \max(\Delta T, eV^2/T_c)\). From Eq. (1) one can extract \(n^{th}\)-moment of the current fluctuations which progressively display more singular behavior,
\[
C_n = \langle I(\omega_1) \ldots I(\omega_n) \rangle_{\omega \rightarrow 0} \propto e^{n-2} G_Q T_c \left( \frac{ E_{qh}^n }{ \Delta T } \right) \left( \frac{ T_c }{ \Delta T } \right)^{n/2 - 1}. \]
We interpret this result as bunching of electrons due to time-dependent fluctuations of the order parameter, which result in a long avalanches of charges and thus parametrically enhanced current fluctuations.

*Estimates.*—Let us now discuss experimentally relevant parameters to observe the effect and estimate its actual magnitude. The maximal value of the nonequilibrium excess current noise normalized to the thermal noise at $T_{c0}$ that follows from Eq. (5) is

$$\left(\frac{\Delta S}{GT_{c0}}\right)_{\text{max}} \approx \frac{1}{25g} \left(\frac{E_{Th}}{\Delta T}\right)^2. \quad (7)$$

When finding this estimate we took symmetric structure $a_1 = a_2 = 1/2$, used $\psi''(1/2) = -14\zeta(3)$ and assumed $\Gamma/4\pi T_c \ll 1$. This condition will be justified below. The minimal allowed $\Delta T$ in our theory is limited by the mean level spacing. Indeed, since $E_{Th}/\Delta T = g/2\pi$ at $\Delta T = \delta$ then the fluctuation induced correction to conductance $\Delta G$ in Eq. (4) already reaches its bare value and thus our approach breaks down for the lower $\Delta T$. At that bound the noise remains parametrically enhanced $(\Delta S/GT_{c0})_{\text{max}} \approx g/100\pi^2$ since $g \gg 1$, however large numerical factor in the denominator significantly diminishes the actual magnitude of the effect. We look for realistic numbers now. For the layout design in Fig. 1 we assume the wire of length $L \approx 0.5\mu$m and width $w \approx 100\mu$m be made of two-dimensional film of thickness $d \approx 10$nm. For aluminum nanowires typical diffusion coefficient is $D \approx 10^3$cm$^2$s$^{-1}$, Fermi velocity is $v_F = 2 \cdot 10^6$cm/s and resistivity is $\rho = 2\mu\Omega$.cm. These numbers provide Thouless energy $E_{Th} = D/L^2 \approx 0.3$K, mean free path $l = 3D/v_F \approx 15$nm, diffusive coherence length at zero temperature $\xi = \sqrt{\xi_0} \approx 440$nm, where $\xi_0 = v_F T_{c0} \approx 1.3\mu$m for the bulk aluminum $T_{c0} = 1.2$K, and sheet resistance $\rho_{c0} = \rho / d \approx 2\Omega$. The latter translates into the normal wire resistance $R_W = \rho_{c0} L / w \approx 10\Omega$ and the dimensionless conductance $g = 1 / G_0 R_W \approx 2.5 \cdot 10^6$ of the nanostructure. The corresponding mean level spacing is $\delta = 2 \pi E_{Th} / g \approx 0.75$mK while $\Gamma / T_{c0} \approx 25$. Finally, the realistic estimate for the maximal nonequilibrium noise above its thermal level is $(\Delta S/GT_{c0})_{\text{max}} \approx 2$ as shown in Fig. 3. Similar estimates can be carried out for zinc and lead nanowires. All these parameters are within the reach of current nanoscale fabrication technology and high precision measurements.

Formalism.— As a technical tool to derive Eq. (1) we use Keldysh technique built into the framework of nonlinear-sigma-model (NLσM) 53-55. For the above specified conditions separation of the length scales $l \ll \xi(0) \ll L \ll \xi(T)$ implies diffusive limit and the quantum action of the device under consideration Fig. 1 is given by the following expression $S = S_Q + S_\Lambda + S_T + S_H$, where

$$S_Q = \frac{i\pi}{\delta} \text{Tr}(-i\hat{\tau}_3 \partial_t \hat{Q} + i\Delta \hat{Q}), \quad S_\Lambda = -\frac{2}{\lambda \hbar} \text{Tr}(\Delta^\dagger \hat{s}_\Lambda \Delta), \quad (8a)$$

$$S_T = -\frac{i}{16} \sum_{k=1,2} g_k \text{Tr}[\hat{Q}^\dagger_k \hat{Q}], \quad S_H = -\frac{i\pi}{\delta \hbar \tau_H} \text{Tr}(\hat{\tau}_3 \hat{Q})^2. \quad (8b)$$

$$\Delta \Lambda = \Delta^\dagger \hat{s}_0 + \Delta^\dagger \hat{s}_3, \quad \Delta^\dagger = \left(\begin{array}{cc} 0 & -\Delta^\alpha \\ -\Delta^\alpha & 0 \end{array}\right), \quad \Lambda = \left(\begin{array}{c} \Delta \\ \Delta^\dagger \end{array}\right). \quad (9)$$

The action $S_T$ describes the coupling of $\hat{Q}$-matrix in the island to those in the leads

$$\hat{Q}_k^{[1]} = \left(\begin{array}{cc} \hat{h}_k & -\left(1 - \hat{h}_k\right)e^{i\chi_k} \\ -(1 + \hat{h}_k)e^{-i\chi_k} & -\hat{h}_k \end{array}\right) \hat{\tau}_3, \quad (10)$$

where $\hat{h}_k = h(e - eV_k \hat{\tau}_3) = \tanh \left(\frac{e - eV_k \hat{\tau}_3}{2\beta}\right)$ is the distribution function and $\chi_k$ is the counting field. The latter is essentially a quantum component of the vector potential which serves to generate observable current and its higher moments. Finally $S_H$ part of the action accounts for the dephasing term of Cooper pairs due to magnetic field. The action $S[\hat{Q}, \Lambda, \chi]$ defines nonequilibrium partition function via the functional integral over all possible realizations of $\hat{Q}$ and $\Lambda$

$$Z(\chi) = \int D[\hat{Q}, \Lambda] \exp(iS[\hat{Q}, \Lambda, \chi]). \quad (11)$$

Here $\delta$ is the mean level spacing in the island, $\lambda$ is the coupling constant in the Cooper channel. The two sets of Pauli matrices $\hat{\tau}_3$ and $\hat{s}_i$ are operating in the Gor’kov-Nambu (N) and Keldysh (K) subspaces, respectively. An additional remark concerning the notations, symbol Tr(...) implies trace over all matrix and continuous indices while curly brackets {...} stand for anti-commutator. The action $S_Q$ represents coupling between $\hat{Q}_{\mu}$-matrix field and superconducting order parameter field $\Delta(t)$. The former is essentially a local in space electronic Green’s function in the island which is a matrix in $\text{KeNoT}(\text{time})$ spaces. Superconducting part of the action $S_\Lambda$ stems from the Hubbard-Stratonovich decoupling of a bare four-fermion BCS interaction term which is done by introducing $\hat{\Delta}$-field. The action is subject to the nonlinear constraint $\hat{Q}^2 = 1$. As usual for the Keldysh theory 56 all fields come in doublets of classical and quantum components. The former obey equations of motions, the latter serve to generate these equations along with the corresponding stochastic noise terms. In particular

$$\hat{\Delta} = \hat{\Delta}^\dagger \hat{s}_0 + \hat{\Delta}^\dagger \hat{s}_3, \quad \hat{\Delta}^\dagger = \left(\begin{array}{cc} 0 & -\Delta^\alpha \\ -\Delta^\alpha & 0 \end{array}\right), \quad \hat{\Lambda} = \left(\begin{array}{c} \Delta \\ \Delta^\dagger \end{array}\right). \quad (9)$$

FIG. 3: Normalized fluctuation-induced nonequilibrium excess current noise in the units of thermal noise power $T_cG$, plotted versus bias voltage $v = eV / \sqrt{T_c\Delta T}$ for $\Delta T = \delta$, $g = 2.5 \cdot 10^6$ and $\Gamma/T_c \approx 0.25, 0.5, 0.75$. 

![Graph showing normalized fluctuation-induced nonequilibrium excess current noise](image-url)
The knowledge of $Z(\chi)$ yields all desired cumulants for current fluctuation by simple differentiation $\langle I^n \rangle = (e/\hbar)(-i\hbar)^n \ln Z(\chi)$.

**Technicalities.**—When computing path-integral in Eq. (11) we need to identify such a configuration of the $\hat{Q}$-matrix field that realizes the saddle point of the action Eq. (8a). For this purpose one needs a parametrization of the $\hat{Q}$-field which explicitly resolves the nonlinear constraint $\hat{Q}^2 = 1$. We adopt the exponential parametrization $\hat{Q} = e^{-i\hat{W}} \hat{Q}_{0} e^{i\hat{W}}$ with $\langle \hat{W}, \hat{Q}_{0} \rangle = 0$, where the matrix multiplication in the time space is implicitly assumed. New matrix field $\hat{W}$ accounts for the rapid fluctuations of $\hat{Q}$ associated with the electronic degrees of freedom and is to be integrated out, while $\hat{Q}_{0}$ is the stationary Green’s function. Minimizing the action Eq. (8a) with respect to $\hat{W}$ one finds following saddle point equation for $\hat{Q}_{0}$

$$\frac{\delta}{8\pi} \sum_{k} g_k \{ \hat{Q}_{0}, \hat{Q}_{0}^{[vk]} \} = -\{ \hat{\tau}_3 \partial_t, \hat{Q}_{0} \} + i\{ \hat{\Delta}, \hat{Q}_{0} \} + \frac{1}{4\tau_H} \{ \hat{\tau}_3 \hat{Q}_{0} \hat{\tau}_3, \hat{Q}_{0} \}$$

(12)

which is nothing else but zero-dimensional version of the Usadel equation. In the stationary case and without superconducting correlations Eq. (12) is solved by such a $\hat{Q}_{0}$ that nullifies commutator in the left-hand-side. This immediately suggests solution for $\hat{Q}_{0}$ that has to be chosen as a linear combination of the $\hat{Q}$-matrices in the leads

$$\hat{Q}_{0} = \frac{1}{g_{1} + g_{2}} \left( g_{1}^{[1]} \hat{Q}^{[1]} + g_{2}^{[1]} \hat{Q}^{[2]} \right) / \sqrt{N_{k}},$$

(13)

$$N_{k} = \frac{1}{(g_{1} + g_{2})^{2}} \left( g_{1}^{[1]} + g_{2}^{[1]} + g_{1}g_{2} \{ \hat{Q}_{0}^{[1]}, \hat{Q}_{0}^{[2]} \} \right),$$

(14)

where factor $N_{k}$ ensures proper normalization. If one uses now Eqs. (13)-(14) back in the action Eq. (8a), then partition function of the normal double tunnel junction follows immediately in agreement with Ref. [29].

The next step is to integrate out fluctuations around the saddle point. To this end, we linearize Eq. (12) with respect to $\delta \hat{Q}_{0} = 2i \hat{W}, \hat{W}$ and solve for Cooperon matrix field $\hat{W}$ to the linear order in superconducting field $\hat{\Delta}$ by passing to Fourier space to invert the matrix equation. The result is

$$\hat{W}_{\omega, e} = \frac{i \Gamma_{\omega, e} (\varepsilon + \varepsilon') \hat{\tau}_3 \hat{Q}_{0}}{2 \Gamma_{\omega} + (\varepsilon + \varepsilon')^{2}} \{ \hat{\Delta}, \hat{Q}_{0} \},$$

(15)

where $\Gamma_{\omega} = \tau_{R} \sqrt{N_{k}}$. Integrating over $\hat{W}$ at the Gaussian level in Eq. (11), \[ D[\hat{W}] \exp(is[I(W, \Delta, \chi)] = \exp(is[I(\Delta, \chi)]) \] one arrives at the effective action written in terms of the superconducting order parameter only

$$S[\Delta, \chi] = S_{\alpha}[\Delta, \chi] + S_{\beta}[\Delta, \chi] + S_{\chi},$$

(16a)

$$S_{\alpha} = \frac{\pi}{2\delta} \text{Tr} \left\{ \hat{C}_{\omega, e} (\hat{Q}_{0} (\varepsilon) \hat{\Delta}_{\omega, e} - \hat{\Delta}_{\omega, e} \hat{Q}_{0} (\varepsilon')) + \hat{\Delta}_{\omega, e} - \hat{\Delta}_{\omega, e} \hat{Q}_{0} (\varepsilon') \hat{\Delta}_{\omega, e} \right\},$$

(16b)

$$S_{\beta} = \frac{\pi}{2\delta} \text{Tr} \left\{ \hat{C}_{\omega, e} (\hat{\Delta}_{\omega, e} - \hat{\Delta}_{\omega, e} \hat{Q}_{0} (\varepsilon') \hat{\Delta}_{\omega, e} \hat{Q}_{0} (\varepsilon')) \right\},$$

(16c)

where $\hat{C}_{\omega, e} = i (\varepsilon + \varepsilon') \hat{\tau}_3 / [(\varepsilon + \varepsilon')^2 + \Gamma_{\omega}^2]$ and $\hat{C}_{\omega, e} = \Gamma_{\omega} \hat{\tau}_0 / [(\varepsilon + \varepsilon')^2 + \Gamma_{\omega}^2]$ are Cooperon propagators. For the technical reasons of convenience with the intermediate steps of calculations we choose to work in the gauge $\chi_{1} = \alpha_{2} \chi$ and $\chi_{2} = -\alpha_{1} \chi$, and similarly for the voltages $V_{1} = \alpha_{2} V$ and $V_{2} = -\alpha_{1} V$. Carrying out matrix products, traces and integrations with the help of Eqs. (9), (10), and (13), one finds eventually

$$S[\Delta, \chi] = \frac{\pi}{4\delta} \text{Tr} \left( \hat{\Delta}_{0}^{\dagger} \hat{\Pi}_{w}(V, \Delta T, \chi) \hat{\Delta}_{0} \right),$$

(17a)

$$\hat{\Pi}_{w} = \left( \frac{\varepsilon - \mu + i\omega/T_{c} - \chi_{e}^{2} \eta}{\varepsilon - \mu + i\omega/T_{c} - \chi_{e}^{2} \eta} \right).$$

(17b)

Here we have used the notation $\chi_{e}^{2} = \chi^{2} - ieV / T_{c}$. Eq. (17a) represents time-dependent Ginzburg-Landau action for nonequilibrium superconducting fluctuations. Off-diagonal elements (retarded and advanced blocks) of the propagator matrix $\hat{\Pi}_{w}$ carry information about the excitation spectrum of fluctuations. Keldysh block (quantum-quantity element of the matrix $\propto \Delta^2 \Delta^\alpha$) ensures fluctuation-dissipation relations. Anomalous classical-classical block accounts for the feedback of stochastic Langevin forces of fluctuations due to nonequilibrium quasiparticle background.

Performing remaining path-integration over $\Delta$ in Eq. (11) with the action from Eq. (17a) one realizes that corresponding cumulant generation function for current fluctuations is governed by the determinant of the Ginzburg-Landau propagator [Eq. (17b)], namely $\ln(\Delta Z) \propto \det \hat{\Pi}_{w}$. We regularize $\det \hat{\Pi}_{w}$ by normalizing it to itself taken at zero counting field, $\det \hat{\Pi}_{w} \rightarrow \det \hat{\Pi}_{w}(\chi)/\det \hat{\Pi}_{w}(0)$, and thereby find

$$\ln(\Delta Z(\chi)) = t_{0} \int \frac{d\omega}{2\pi} \ln \left| 1 - \frac{2\gamma^{2}(\phi + \varepsilon \eta)}{\varepsilon^{2} + \omega^{2}/T_{c}^{2}} \right|$$

(18)

which upon final integration reduces to Eq. (1).
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