A master-slave game model of power trading in cross-regional spot market

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Abstract. In order to solve the multi-agent equilibrium of the cross-regional power transaction, a master-slave game model considering multi-agent equilibrium is constructed with the ultimate goal of maximizing the sum of the benefits of all parties in the cross-regional power transaction.

1. Introduction
In recent years, the rapid growth of cross-regional power transactions in China, accelerating the realization of spot market construction is the top priority of reform, which plays a very effective role in optimizing resource allocation, promoting energy conservation and emission reduction, and promoting the construction of power market [1-2]. An existing article has proposed a stochastic optimization model that uses virtual bidding to generate the optimal price maker trading strategy for wind power generators. In the proposed model, virtual bidding is used to increase wind power by trading on multiple buses. Manufacturer's market influence in the day-ahead (DA) market [3].

The mature spot power market in foreign countries has basically taken shape, which is mainly constructed by day-ahead, day-ahead and real-time market [4-7]. Based on the idea of multi-time scale optimization.

To solve the problem of the master-slave game in the market [8-9], on the basis of regional power spot market trading mechanism is given priority to with the ISO, from the party to sell electricity company as well as the comprehensive energy company, Solve the model with a constrained variable scale algorithm [10-11].

In the electric power market transaction, considering carbon trading mechanism on the influence of the electric power market transaction, with carbon emissions and processing cost as constraint conditions, on the basis of safe operation, through the optimization of active efforts to meet the energy conservation and emissions reduction in the process of trading.

2. Basic model and main body of transregional electricity trade
A. The user: the profit brought to users by the implementation of cross-regional electricity trading refers to consumer surplus. The paper assumes that the end-user demand function is linear:

\[ p_g^C(q^C) = a^c - \sum_{m \in G_c} b_{g,m}^C q_m^C \]

(1)

\(p_g^C\) is the price provided by power producer G to consumer C; \(G_c\) is a collection of power generators that supply electricity; \(a^C\) is the reference electricity price; \(b_{g,m}^C\) is price elasticity; \(q_m^C\) is the electric quantity supplied by generator M to consumer C.

To get the utility function for each user, the (inverse) demand function is the first derivative of the utility function. The utility function of consumer C is:
\[ U_{Cc}(q^c) = \sum_{g \in G} a^c q_g^c - \frac{1}{2} \sum_{g,m \in G} b_{g,m}^c q_m^c q_m^c \]  
(2)

The actual price paid by the user is:
\[ P_{Cc}(q^c) = \sum_{g \in G} p_g^c q_g^c = \sum_{g \in G} a^c q_g^c - \sum_{g,m \in G} q_g^c b_{g,m}^c q_m^c \]  
(3)

Therefore, consumer surplus is:
\[ S_{Cc}(q^c) = U_{Cc}(q^c) - P_{Cc}(q^c) = \frac{1}{2} \sum_{g,m \in G} q_g^c b_{g,m}^c q_m^c \]  
(4)

B. Electricity generators: the cost of power generation can be divided into two parts: generation cost and transmission cost.

Power generation cost:
\[ G_{G_g}(q_g) = f_g + \sum_{c \in G} v_g^c q_g^c + \frac{1}{2} \sum_{c,l \in G} q_g^c e_c^l q_g^c \]  
(5)

Transmission cost:
\[ T_{G_g}(q_g) = \sum_{c \in G} \left( a_{g,p} r_{k \in R} + a_{c,r'} k \in R' + x_g^c + y_g^c \right) q_g^c \]  
(6)

\( f_g \) is the fixed generation cost of the power supplier; \( x_g^c \) is the cross-district electricity transaction network fee; \( y_g^c \) is the blocking charge of cross-district electricity transaction. The revenue of power generators is:
\[ I_{G_g}(q_g) = \sum_{c \in G} p_g^c q_g^c = \sum_{c \in G} \left( a^c q_g^c - \sum_{m \in G} q_g^c b_{g,m}^c q_m^c \right) \]  
(7)

The profit of power generator is:
\[ P_{G_g}(q) = I_{G_g}(q_g) - G_{G_g}(q_g) - T_{G_g}(q_g) \]  
(8)

C. Transmission operator: cross-regional power transaction can be divided into intra-regional power transaction and cross-regional power transaction. The transmission cost function of region \( r \) can be expressed as:
\[ T_{T_r}(q) = f_r + \sum_{(g,c) \in T_R} f_{e_{g,c}} q_g^c \]  
(9)

\( f_r \) is the fixed cost of operators in region \( r \); \( f_{e_{g,c}} \) is the power demand charge for transaction \((g, c)\) through region \( r \).

The income of transmission operators is calculated as follows:
\[ I_{T_r}(q) = \sum_{g \in R} a_{c,k} q_g^c + \sum_{c \in R} a_{c,c'} q_r^c + \epsilon_r c_{om} \]  
(10)

\( I_{T_r}(q) \) is the revenue of operators in region \( r \); \( c_{om} \) is the total amount of compensation for inter-regional transactions; \( \epsilon_r \) is the proportion of compensation amount in region \( r \).

The profit of regional \( R \) transmission operator is:
\[ P_{T_r}(q) = I_{T_r}(q) - T_{T_r}(q) = \epsilon_r c_{om} + \sum_{g \in R} a_{c,p} q_g^c + \sum_{c \in G} c \in C_G a_{c,y} q_g^c - \sum_{g,c} f_{\hat{r},g} q_g^c - f_r \]  
(11)

![Figure 1. Framework of electricity trading market.](image-url)
3. Market trading framework and regional spot market trading mechanism

3.1. Market transaction framework
As shown in Figure 1, the market main body is made up of energy suppliers, operators and users.

3.2. Regional spot market trading mechanism
The cross-region electricity market designed in this paper consists of bilateral contract market for difference and spot market. The earnings of the electricity selling company in the contract for difference market are:

\[ B^k_f = (p_{b1} - p^k_f) q^k_f \]  \hspace{1cm} (12)

\( q^k_f \) is the electricity quantity of the financial contract signed by the k-type electricity selling company and the power generator; \( p^k_f \) is the electricity price of the financial contract signed by the k-type electricity selling company, and \( p_{b1} \) is the price cleared in the market before the day.

Regional spot market:
Prior market: In terms of clearing mechanism, in the regional (provincial) power market, public welfare ISO represents the overall interests of the society and adopts zonal Marginal price (ZPM) mechanism for clearing. After considering the electricity price of other regions and network congestion, the clearing price \( p_{b1} \) shall be assessed according to the overall maximum welfare of each entity in the wholesale market:

\[ \max F = \left[ p_{b1} \sum_{t \in T} \sum_{k \in K} q^k_{b1,t} + q^k_t p^k_t + Q_{ob}p_{ob} + Q_{ob,b}(p_{ob} - p_{b1}) \right] - \left[ \sum_{t \in T} G(l_t + r_t) + O \right] \]  \hspace{1cm} (13)

The first item represents the cost of purchasing electricity by the company selling electricity, and the second item is the cost of generating electricity. The constraint conditions consider energy balance, price constraint and blocking upper limit.

\[ L_e = \sum_{k \in K} \left( Q^k_{b1} + \sum_{t \in T} q^k_{b2,t} + q^k_t \right) + Q_{ob} \]  \hspace{1cm} (14)

\[ P_{b1,min} \leq p_{b1} \leq P_{b1,max} \]  \hspace{1cm} (15)

\[ 0 \leq Q_{ob,b} \leq Q_{ob,b,max} \]  \hspace{1cm} (16)

Real-time market: In the real-time trading part of the spot market, the trading time is very close to the actual running time. 15min before each trading hour, the distribution network reports the real-time market power purchase demand according to the economic scheduling plan, and ISO conducts security check for scheduling. ISO sends dispatching instructions to the distribution network, and the energy company or the electricity selling company that owns the distribution network completes the dispatching of the distribution network.

4. The game model of multi-agent purchasing and selling energy in cross-regional spot market
On the basis of designing the trading mechanism of regional electric power spot market, a master-slave game model considering the balance of multiple players is proposed to simulate the market behavior of...
multiple types of electricity sellers. The spot market consists of the day-ahead and real-time markets, and the market relationship of each subject is shown in Figure 2.

The game relation of one master and many followers is as follows:

a. A positive Stackelberg master-slave strategy is adopted between the decision makers of the master party and the slave party, and the master party clears the day-ahead electricity price according to the overall maximum welfare of the wholesale side. The subordinate party makes the purchase and sale energy decision which maximizes its own profit according to the upper level clearing information, and feedback the result to the upper level decision. (Note: upper decision-making (UDM) is the leading party, while lower decision-making (LDM) is the subordinate party).

b. Monopolistic competition between the parties in the retail market (for the electricity selling company to users, users to buy electricity from the electricity selling company). In the case of unknown opponent's decision, the subordinate subject makes new strategy according to the main party's decision and the opponent's early game strategy and launches price competition. The game relationship is shown in Figure 3.

![Figure 3. Master-slave game structure.](image)

4.1. Principal Party (public welfare ISO) clearing pricing model
The specific expression of the clearing electricity price has been given in the previous article.

4.2. The decision-making model of purchasing and selling electricity from the subordinate party (Electricity and Energy Company)

4.2.1. Purchasing and selling energy decision-making model of single electricity selling company. A single electricity selling company can make profit from the price difference of electricity purchasing and selling by making reasonable electricity purchasing and selling strategies, and its decision-making objective function of electricity purchasing and selling is as follows:

\[
\max S^l = p^l T^l_e - \left( p_{b1} \sum_{t \in T} q^l_{b1,t} + \sum_{t \in T} p_{b2,t} q^l_{b2,t} + q^l_{f,t} p_{b1} + p_{ob} Q^l_{ob} \right) - K_t Q^l_{ob} (p_{ob} - p_{b1})
\]  

(17)

The first item in the above formula is the power purchase cost of the electricity selling company, the second item is the income of the contract for difference market, the third item is the sum of the power purchase expenses within and outside the region, and the last item is the total blocking cost borne.

Constraints include price constraints and balance constraints on electricity purchase and sale:

\[
p_{s,\text{min}} \leq p_s \leq p_{s,\text{max}}
\]  

(18)

\[
l^l_e = q^l_{b1,t} + q^l_{b2,t} + q^l_{f,t} + q^l_{ob,t}
\]  

(19)

\[
L^l_e = \sum_{t \in T} l^l_{e,t}
\]  

(20)

\[
q^l_{b1,t}, q^l_{b2,t}, q^l_{f,t}, q^l_{ob,t} \geq 0
\]  

(21)

4.2.2. Bidding operation decision model of integrated energy company. The integrated energy system takes electricity, natural gas and heat as the main energy supply. The integrated energy system is shown in Figure 4.
The energy conversion matrix of the system is expressed as (the energy matrix is composed of input matrix $I$, conversion matrix $T$ and output matrix $C$):

$$
\begin{pmatrix}
    l_{e,t}^	ext{II} \\
    l_{h,t}^	ext{II}
\end{pmatrix} =
\begin{pmatrix}
    K_e \\
    K_g (\eta_g^e + \eta_g^h p_g \eta \text{ORC})
\end{pmatrix}
\begin{pmatrix}
    (1 - K_e) \eta_h^h \\
    (1 - K_g) \eta_g^{g2h} + K_g \eta_g^h (1 - R_g)
\end{pmatrix}
\begin{pmatrix}
    q_{e,t}^	ext{II} \\
    q_{g,t}^	ext{II}
\end{pmatrix}
$$

(22)

$l_{e,t}^	ext{II}, l_{h,t}^	ext{II}$ are the output electrical power and thermal power at $t$ period respectively; $q_{e,t}^	ext{II}, q_{g,t}^	ext{II}$ are respectively the input electrical power and natural gas power at $t$ period; $\eta_e^h$ is heating efficiency of electric heating boiler; $\eta_g^{g2h}$ is gas boiler thermal efficiency; $K_g$ is the proportion of direct electricity purchase; $R_g$ is ORC power generation rate.

Due to the introduction of carbon transaction cost, the operating cost of the system will change to some extent.

In order to ensure the reduction of carbon emission, the national emission reduction plan can be realized if the total carbon emission of each unit in the system is less than the specified amount during the unit time of power generation:

$$
\sum_{i=1}^{n} C_{ei} P_{gi} \leq C_j
$$

(23)

$C_{ei}$ is the carbon emission intensity of unit $I$; $P_{gi}$ represents controllable power generation output of unit $I$; $C_j$ represents carbon emission quota; $n$ indicates the number of system generators.

The carbon emission permits are specifically quantified as a special commodity, and the units with low emissions are encouraged to sell the remaining carbon emission permits for profit, so as to improve the enthusiasm of the industry in emission reduction. Set the carbon emission price as $k_c$, and the optimization model is:

$$
k_w = k_c k_1
$$

(24)

$k_w$ represents the carbon emission cost, and $k_1$ represents the total carbon emission per unit electric quantity produced by the generator set.

Considering the cost of carbon emission, the decision-making objective function of integrated energy company is:

$$
\max S = p_e L_e + p_e \sum_{i \in I} l_i + B_f - [(p_0 q_{e,i} + q_{f,i} p_{b,i} + \sum_{i \in I} p_{g,i} q_{g,i} + \sum_{i \in I} p_{b,i} q_{b,i}) - K_c Q_{eb}(p_{eb} - p_{bi})] - k_w
$$

(25)

The mathematical model of the master-slave game is:
\[
\begin{align*}
    x &= (p_{b1}, q_{b1}, q_{bk}, q, Q_{ob}) \\
    y_1 &= (p_{b1}, p_s^1, p_s^2, L^c, q^1, Q_{ob}) \\
    y_2 &= (p_{b1}, p_s^2, L^c, L^h, q^2, q_{bk}, q, Q_{ob}) \\
    y &= (y_1, y_2)
\end{align*}
\]

\[
\begin{align*}
    \max F(x) \quad \text{s.t.} \quad (x) \in \Omega_0 \\
    \max S^1(y_1) \quad \text{s.t.} \quad (y_1) \in \Omega_1 \\
    \max S^{II}(y_2) \quad \text{s.t.} \quad (y_2) \in \Omega_{II}
\end{align*}
\] (26) (27)

The objective function of UDM is nonlinear. The nonlinear source comes from the product of the clearing price and the purchased quantity in the market and the product of the blocking quantity and the clearing price. At the same time, the slave equilibrium problem contains coupling constraints. The interactive constrained variable scale algorithm is used to solve the master-slave game problem with nonlinear multi-objective equilibrium.

5. Example analysis

5.1. Market conditions

In the regional power market, the maximum capacity of inter-regional connection line is 600MW, the maximum blocking capacity is 200 MW, the minimum purchased power in the region is 30% of the total purchased power, the spare capacity is configured with 5% margin of load, other social costs are 5% of the power generation cost, and the blocking allocation coefficient is 0.6. The market price of natural gas is 2.5(yuan /m3), and the market price of heat load is 57.11 yuan /GJ. In the calculation process, the energy price is uniformly converted into electricity price unit (yuan /(MW·h)) through energy. In order to verify the effectiveness of the model and algorithm, and analyze the influence of market environment changes on the market power of the two-oligopoly electricity selling companies in the spot market, four market environments are set up:

a. Scenario 1. In the wet season, the oversupply of regional power market with abundant water resources leads to the low price of electricity in the external regional market.

b. Scenario 2. In dry season, the shortage of power market in regions rich in hydraulic resources leads to the high price of electricity in external regional markets.

c. Scenario 3. The demand for heat load decreases in summer, and the heating market price is low.

d. Scenario 4. Winter heat load demand increases, heating market price is stable.

5.2. Result analysis

5.2.1. Market comparison. According to the above scenario conditions, the market equilibrium solution under each scenario is solved, and the clearing electricity price and social welfare value before and after the establishment of trans-regional market trading mechanism are compared. As shown in Table 1 and Figure 5.

After the establishment of trans-regional transactions, the clearing price in the region has been reduced to a certain extent in most scenarios compared with before the reform. At the same time, the social welfare of the region with trans-regional trading mode is significantly better than that of the spot market without trans-regional trading.

| Scenario | No cross-regional market/(yuan/(MW-h)) | under cross-regional markets/(yuan/(MW-h)) | Out-of-region electricity price/(yuan/(MW-h)) | Group competition power average/(y(MH-h)/h) |
|----------|----------------------------------------|-------------------------------------------|---------------------------------------------|---------------------------------------------|
| 1        | 379.3                                  | 345.6                                    | 301                                         | 182.3                                       |
| 2        | 332.4                                  | 382.8                                    | 443                                         | 0                                           |
| 3        | 367.1                                  | 367.3                                    | 412                                         | 0                                           |
| 4        | 364.2                                  | 351.9                                    | 412                                         | 0                                           |

Table 1. Clearing price and average blocking value.
5.2.2. Comparison of market power of electricity selling (energy) companies. Analyse the changes of power purchase strategies of two types of electricity selling companies in different market environments. As shown in Figure 6.

For category I electricity sales companies. When the market price of electricity in external region is significantly lower than the local clearing price, Category I electricity sales companies will purchase a large amount of electric energy in external region, and the proportion of external electric energy exceeds that of local electric energy. A certain amount of local power needs to be purchased to avoid the high cost of blocking. When the price of electric energy in the outer region is high, the electricity selling company mainly purchases local electric energy and a small amount of electric energy in the outer region as a hedge against market risks. Category II sell electricity companies have more flexibility to buy can choose under different scenarios, scene 1 when local electricity price is higher, to increase the capacity of energy system and improve the external regional electricity purchasing proportion of abundant energy sources in two ways, avoid blocking link blocking causes losses, at the same time the user retail side also increase their market share. When the heat load is relatively cheap in summer, energy companies buy more power from the power market. Meanwhile, integrated energy uses the ORC system to realize thermoelectric decoupling and produce power with a higher electric-heating ratio to compensate for the compression of profit margins caused by the high real-time electricity price during peak load periods. In winter heat load peak, through the energy system to achieve cogeneration, achieve the maximum profit of the heat market.

Figure 5. Comparison of social welfare values with and without regional transaction mode.

Figure 6. Power purchase strategies and load curves of the two types of electricity selling subjects in four scenarios.
5.2.3. *Carbon trading constrains the bidding output of each unit in the energy market*. MT has good economy and output stability. However, in the context of the introduction of the carbon trading mechanism into the electricity market and the increasing perfection of the carbon trading market, MT needs to pay a high cost of purchasing carbon trading rights. Figures 7-8 show the influence of the introduction of carbon trading on the bidding output of each unit.

![Figure 7](image1.png)
**Figure 7.** No carbon trading restrains the bidding output of each energy market unit.

![Figure 8](image2.png)
**Figure 8.** Consider carbon trading constraints on the bidding output of each energy market unit.

6. **Conclusions**

A. The regional power market mechanism not only promotes fair competition among power generation companies, but also breaks the monopoly of power generation subjects in the region and effectively reduces the regional electricity price.

B. The proposed model can effectively analyse the purchasing and selling energy market behaviour of each subject under the new situation. The change of clearing price before and after the introduction of trans-regional trading mechanism is analysed and compared, which verifies that the new trading mechanism can reduce the price fluctuation and effectively reduce the price. This model can provide reference for power market trading mechanism planning and investment and operation of integrated energy companies.

C. The introduction of carbon trading mechanism increases the operating cost of MT. In order to reduce the cost of purchasing carbon emission rights, MT bidding power should be reduced. The introduction of carbon trading mechanism ensures the economic benefits of operators to a certain extent, and also effectively reduces the emission level of pollutants and greenhouse gases in the region, which has economic and social significance.

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References

[1] Yang Haoqin, Wei Zhenbo, Huang Yuhan, Sui Dongxu, et al. 2019 Multi-agent Purchase and Sale Master-slave Game Model in Cross-regional Spot Market[J]. *Power System Technology* **43**(8) 2781-2791 (in Chinese)

[2] Zhang S, Jing Z and Xiao D 2021 Cross-Provincial/Regional Transmission Pricing Mechanism to Facilitate Regional Integration Development in China: Analyses and Suggestions[C]. 2021 *5th International Conference on Smart Grid and Smart Cities (ICSGSC)* IEEE 134-140

[3] Xiao D, AlAshery M K and Qiao W 2021 Optimal Price-Maker Trading Strategy of Wind Power Producer using Virtual Bidding[J]. *Journal of Modern Power Systems and Clean Energy*

[4] Dou Xiaobo, Xu Minhui, Dong Jianda, et al. 2016 Microgrid improved multi-time scale energy management model[J]. *Automation of Electric Power Systems* **40**(9) 48-55

[5] Wang Chengshan, Lu Chaoxian, Li Peng, et al. 2019 Multiple time scale optimal scheduling of community integrated energy system based on model predictive technology *Proceedings of the CSEE* **39**(23) 6791-6803 (in Chinese)

[6] Gu Wei, Lu Shuai, Yao Shuai, et al. 2019 Hybrid time scale operation optimization of integrated energy system[J]. *Electric Power Automation Equipment* **39**(8) 203-213

[7] Wei Wei, Chen Yue, Liu Feng, et al. 2015 Agent pricing strategy and electric vehicle charging management in smart residential areas based on master-servant game[J]. *Power Grid Technology* **39**(4) 939-945 (in Chinese)

[8] Gan Yuxiang, Jiang Chuanwen, Bai Hongkun, et al. 2018 Optimal Quotation and Operation Optimization of Park E-commerce Under Market Environment[J]. *Power Grid Technology* **42**(3) 707-714 (in Chinese)

[9] Shimizu K and Lu M 1995 A global optimization method for the Stackelberg problem with convex functions via problem transformation and concave Programming[J]. *IEEE Transactions on Systems, Man and Cybernetics* **25**(12) 1635-1640

[10] Zhang X, Bao T, Yu T, et al. 2017 Deep Transfer Q-Learning with Virtual Leader-Follower For Supply-Demand Stackelberg Game of Smart Grid[J]. *Energy* **133** 348-365

[11] Liu Xin, Wu Hongbin, Wang Jingjie, et al. 2020 Economic scheduling of pseudo-power plants considering Demand Response in Market Environment[J]. *China Electric Power* **53**(9) 172-180