Black-hole microstate spectroscopy: ringdown, quasinormal modes, and echoes

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PRD104(2021)6, 066021
Outline

1. Introduction
2. Black hole ringdown
3. Exotic compact object ringdown
4. Numerical method
5. Results
6. Summary & Discussion
Outline

1. Introduction
   • Gravitational wave astronomy
   • Black hole vs Exotic compact object
   • Fuzzball proposal

2. Black hole ringdown

3. Exotic compact object ringdown

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Gravitational wave astronomy

- GWs give us hints of ....
  - Origin of black holes/ binaries, primordial black hole, early universe, equation of state of neutron star, BH as a particle detector, test of gravitational theory, et al
GW from binary black hole

- Three different stages
  - Inspiral
  - Merger
  - Ring-down

- How to describe ??.
  - Post-Newtonian
  - BH perturbation theory
  - Numerical relativity
  - … et al

Abbot et al (2016)
GW from binary black hole

- BH ring down is described by BH quasi-normal mode.

\[ \Psi = \sum_{n,l,m} A_{n,l,m} e^{i\omega_{nlm}t} \]

- BH no hair theorem in GR (Carter 1971, Robinson 1975)

\[ ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt_{KS}^2 - \frac{4Mra \sin^2 \theta}{\Sigma} dt_{KS} d\phi_{KS} + \frac{4Mr}{\Sigma} dt_{KS} dr \\
+ \left(1 + \frac{2Mr}{\Sigma}\right) dr^2 + \Sigma d\theta^2 - 2a \sin^2 \theta \left(1 + \frac{2Mr}{\Sigma}\right) dr d\phi_{KS} + \frac{\mathcal{F}}{\Sigma} \sin^2 \theta d\phi_{KS}^2 \]

\[ \omega^\text{QNM}_{nlm} = \omega^\text{QNM}_{nlm}(M, a) \]

\[ \begin{aligned} f^\text{QNM}_{220} &= 251^{+8}_{-8} \text{ Hz} \\
\tau^\text{QNM}_{220} &= 4.0^{+0.3}_{-0.3} \text{ ms} \end{aligned} \]

LVC, PRL 116 (2016) 221101
Q. Can we distinguish BH from exotic compact object using ringdown phase or echo?

- test of gravitational theory
- test of exotic matter

- Exotic compact object (ECO)
  - Boson stars, gravastars, wormholes, firewalls, Fuzzballs et al.
Fuzzball proposal

• BHs in GR have many theoretical problem.
  - How to describe the BH singularity.
  - Information paradox?
  - What is the origin of the BH entropy? et al
• Fuzzball proposal (Lunin and Mathur (2002))

\[ S_{\text{BH}} = \frac{A}{4} \]

\[ |\text{BH} \rangle \quad \sum |\text{microstate} \rangle \quad \text{BH entropy} \]

• BH horizon
• curvature singularity
• horizonless
• smooth (no singularities in 5-dim)
• same charge as corresponding BH
Fuzzball proposal

• 4-charge BPS BH

\[ ds^2 = -f(r)dt^2 + f(r)^{-1}\delta_{ij}dx^i dx^j \]

\[ f(r) = \left( H_1 H_2 H_3 H_4 \right)^{-1/2} \quad H_A = 1 + \frac{Q_A}{r} \]

• Microstate geometry see M.Bianch et.al (2017)

\[ ds^2 = -e^{2U}(dt^2 + \omega)^2 + e^{-2U}\delta_{ij}dx^i dx^j \]

\[ \begin{cases} 
  e^{-4U} = Z_1 Z_2 Z_3 V - \mu^2 V^2 \\
  *_3 d\omega = \frac{1}{2} \left( VdW - WdV + K^l dL_l - L_l dK^l \right) 
\end{cases} \]

\[ V = 1 + \sum_{a=1}^{N} \frac{v_a}{|x - x_a|}, \ldots \]

• multi-center

• \( Q_A = (Q_0, Q_I) \)

• \( P_A = (P^0, P^I) \)
What we want to do

Q. How different is 4-charge BPS BH ringdown from ringdown of corresponding microstate geometries.

□ $\Phi = 0$

BH ringdown

Fuzzball ringdown?
Echoe?
Outline

1. Introduction

2. Black hole ringdown
   • Time domain
   • Frequency domain
   • Computing quasinormal mode

3. Exotic compact object ringdown

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BH Ringdown

3+1 dim evolution of test massless scalar field around BH

$\Psi \sim t^{-p}$

$\Psi = \sum_{n,l,m} A_{n,l,m} e^{i \omega_{nlm} t}$

Witek et al (2013)
Computing QNM

\[
\left( \frac{\partial^2}{\partial r_*^2} + V(r) + \omega^2 \right) \Psi = 0
\]

\[\Psi \sim e^{-i\omega(t-r_*)}\]

\[\Psi \sim e^{-i\omega(t+r_*)}\]

- Direct Integration
- Leaver’s method
- WKB approximation

\[\begin{align*}
\text{Re}(\omega) & \sim r_{LR}^{-1} \\
\text{Im}(\omega) & \sim \tau^{-1}
\end{align*}\]

\(r_{LR}\) : Light ring radius

\(\tau\) : Instability time scale
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   • Exotic compact object
   • ECO ringdown and echo
   • Several comments
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Exotic compact object

- ECO is a theoretical compact object without BH horizon.

\[ r_0 = r_H(1 + \epsilon) \]

\[ C = \frac{2M}{r_0} \]

From E.Maggio et al (2021)
Black hole vs ECO with light ring

- BH
- ECO surface/interior
- Light ring
- $\tau_{\text{echo}}$
ECO with light ring

- Evolution of test massless scalar field around ECO.

\[ \square \Phi = 0 \]

\[
ds^2 = - \alpha(r)^2 dt^2 + a(r)^2 dr^2 + r^2 d^2 \Omega
\]

\[
\alpha(r)^2 = \begin{cases} 
\text{Sch. BH} & (2M(1 + \epsilon) < r) \\
\alpha_2 & (r < 2M(1 - \epsilon))
\end{cases}
\]

also see E. Maggio et al (2020)
Comments

• **Full evolution ??**
  - Boson stars merger in GR is possible.  
    Sanchis-Gual et al (2019)
  - Other ECO may be difficult.

• **Beyond spherical symmetry**
  - These pictures are based on spherical symmetry 
    (separability of equation)
  - Non-separable case is more complicated.

• **Fuzzball**
  - Field eq. is not separable on fuzzball spacetime.
  - It is not obvious if ringdown and echo appear.
  - We need to use 3+1 dim numerical simulation.
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   - Numerical code for 4 charge BH
   - 3+1 decomposition
   - Numerical code for fuzzball
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What we want to do

Q. How different is BH ringdown from ringdown of corresponding microstate geometries.

\[ \Phi = 0 \]
4 charge BH

- **metric**

\[
\begin{aligned}
  ds^2 &= -f(r)dt^2 + f(r)^{-1}(dr^2 + r^2d^2\Omega) \\
  &= -B(q)^2dt^2 + A(q)^2dq^2 + q^2d^2\Omega
\end{aligned}
\]

- **evolution equation**

\[
\Phi = \sum_{l,m} \frac{\sigma_{lm}(t,q)}{q} Y_{lm}(\theta,\phi)
\]

\[
-\frac{\partial^2}{\partial t^2}\sigma_{lm} + \frac{\partial^2}{\partial q^2}\sigma_{lm} - V_{\text{eff}}(q)\sigma_{lm} = 0
\]

- **initial data**

\[
\begin{cases}
  \sigma_{lm}(0,q_*) = e^{-\left(\frac{q_*-q_0}{\sigma}\right)^2} \\
  \dot{\sigma}_{lm}(0,q_*) = 0
\end{cases}
\]

\[
f(r) = \left(H_1H_2H_3H_4\right)^{-1/2}
\]

\[
H_A = 1 + \frac{Q_A}{r}
\]

\[
M = \frac{Q_1 + Q_2 + Q_3 + Q_4}{4}
\]

**Effective potential**

\[
V_{\text{eff}}(q) = B^2 \left( \frac{l(l+1)}{q^2} + \frac{1}{A^2q^2} \left( \frac{\partial_q B}{B} - \frac{\partial_q A}{A} \right) \right)
\]

\[
Q_1 = Q_3, Q_2 = Q_4
\]
3+1 decomposition

- 4 dim. metric decomposes to lapse, shift, and 3-metric.

\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu \]
\[ = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \]

\[ K_{ij} = -\frac{1}{2\alpha}\left( \partial_t - \mathcal{L}_\beta \right)\gamma_{ij} \quad \text{: extrinsic curvature} \]

- Klein-Gordon eq.

\[ \Box \Phi = 0 \quad \rightarrow \quad \begin{cases} 
(\partial_t - \beta^i \partial_i)\Phi = -2\alpha K_{\Phi} \\
(\partial_t - \beta^i \partial_i)K_{\Phi} - \frac{\alpha}{2}D^2\Phi + \alpha KK_{\Phi} - \frac{1}{2}D^i\alpha D_i\Phi 
\end{cases} \]
Fuzzball geometry

- Fuzzball metric (3 center)

\[ ds^2 = -e^{2U}(dt + \omega)^2 + e^{-2U} \sum_{i=1}^{3} \delta_{ij}dx^i dx^j \]

\[
\begin{align*}
e^{-4U} &= Z_1Z_2Z_3V - \mu^2V^2 \\
*_{3}d\omega &= \frac{1}{2} \left( VdW - WdV + K^I dL_I - L_I dK^I \right) \\
V &= 1 + \sum_{a=1}^{N} \frac{v_a}{|x - x_a|}, \ldots \quad N = 3
\end{align*}
\]

- Geometry is regular in 5 dim. but, singular in 4 dim.

\[
\frac{1}{|x - x_a|} \rightarrow \text{erf} \left( \frac{|x - x_a|}{e} \right)
\]
Einstein ToolKit

- Numerical code (Einstein ToolKit)
  - Einstein Toolkit is software platform for numerical relativity.
  - To resolve the multicenter, we use mesh refinement provided by Carpet.
  - For long simulations, we use multipatch infrastructure provided by Llama.
  - 4th order Runge-Kutta method
  - Spatial derivative is evaluated by 4th order finite differences.
  - 2nd order interpolation in mesh refinement.
Numerical code

- We extract multipole mode of scalar field during evolution.

\[
\Phi = \sum_{l,m} \Phi_{lm}(r, t)Y_{lm}(\theta, \phi)
\]
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QNM in 4 charge BH

\[ \beta = \sqrt{Q_2/Q_1}, \quad Q_1 = Q_3, \quad Q_2 = Q_4 \]

\[ \omega_{IM} \sim \mathcal{O}(0.1) \]
**Light ring around Fuzzball**

- **WKB approximation**
  - The unstable photon sphere in axisymmetric fuzzball.
    
    \[
    \text{Re}(\omega_{\text{QNM}}) \quad \leftrightarrow \quad \text{Light ring radius}
    \]
    
    \[
    \text{Im}(\omega_{\text{QNM}}) \quad \leftrightarrow \quad \text{Instability time scale}
    \]
    
  - echo time scale
    
    \[
    \tau_{\text{echo}} = 2 \int_{r_c}^{r_t} \frac{dr}{dr/dt}
    \]
3+1 simulations

- Axisymmetric Fuzzball
Waveform from Axisymmetric FB

\[ \Phi(0,x) \propto e^{-\left(\frac{r-r_0}{\sigma}\right)^2} \]

- Ringdown, echo, and QNM appear.
- Ringdown and echo time scale for \((l,m)=(2,2)\) mode are agreement with geodesic approximation.
3+1 simulations

- Scaling Fuzzball
Waveform from Scaling solution

$L = 0.67M$

$\Phi(0,x) \propto e^{-\left(\frac{r-r_0}{\sigma}\right)^2}$

$L = 0.27M$

QNM dominates soon.
Time evolution for each case

4-charge BH

an axisymmetric fuzzball

scaling fuzzball
Summary

- 4 charge BH
  - 1+1 dim. simulation of test massless scalar field
  - QNM using direct integration and WKB approximation.

- Fuzzball
  - 3+1 dim. simulations of test massless scalar field
  - ringdown associated with light ring
  - echo
  - QNM of fuzzball

- Fuzzball is the linearly stable (no ergoregion).
Discussion

- Physical interpretation ??
- Boundary condition on center ??
  - We regularized the singularities.
  - But, the boundary condition should be determined from 5 dim. picture.

Each fuzzball have ringdown and echo.

$$\sum |\text{microstate} >$$

Ringdown and echo still exist after quantum average ?
Thank you