Large $N_c$ behavior of light resonances in meson-meson scattering

J.R. Peláez

Departamento de Física Teórica II, Universidad Complutense, 28040 Madrid, Spain

Abstract. By scaling the parameters of meson-meson unitarized Chiral Perturbation Theory amplitudes according to the QCD large $N_c$ rules, one can study the spectroscopic nature of light meson resonances. The scalars $\sigma, \kappa f_0(980)$ and, possibly, the $a_0(980)$ do not seem to behave as $\bar{q}q$ states, in contrast to the vectors $\rho(770)$ and $K^*(892)$. The behavior shown by the scalars is naturally explained in terms of diagrams with intermediate $\bar{q}qqq$-like states. Here we review our recent study and show how the results do not depend on the different fits to data.

INTRODUCTION

Although QCD is firmly established as the theory of strong interactions it becomes non-perturbative at low energies, and gives only little help to address the existence and nature of the lightest scalar mesons. Alternatively Chiral Perturbation Theory (ChPT) [1] has been devised as the QCD low energy Effective Lagrangian built as the most general derivative expansion respecting its symmetries, containing only $\pi, K$ and $\eta$ mesons. These particles are the QCD low energy degrees of freedom since they are Goldstone bosons of the QCD spontaneous chiral symmetry breaking. For meson-meson scattering ChPT is an expansion in even powers of momenta, $O(p^2), O(p^4),...$, over a scale $\Lambda \chi \sim 4\pi f_0 \approx 1$ GeV. Since the $u, d$ and $s$ quark masses are so small compared with $\Lambda \chi$ they are introduced as perturbations, giving rise to the $\pi, K$ and $\eta$ masses, counted as $O(p^2)$. At each order, ChPT is the sum of all terms compatible with the symmetries, multiplied by “chiral” parameters, that absorb loop divergences order by order, yielding finite results. The leading order is universal since there is only one parameter, $f_0$, that sets the scale of spontaneous chiral symmetry breaking. Different underlying dynamics manifest themselves with different higher order parameters. In Table I are listed the parameters that determine meson-meson scattering up to $O(p^4)$, called $L_i$. As usual after renormalization, they depend on an arbitrary regularization scale, as $L_i(\mu^2) = L_i(\mu_1) + \Gamma_i \log(\mu_1/\mu_2)/16\pi^2$, where $\Gamma_i$ are constants given in [1]. In physical observables the $\mu$ dependence is canceled with that of the loop integrals.

The large $N_c$ expansion [4] is the only analytic approximation to QCD in the whole energy region, also providing a clear definition of $\bar{q}q$ states, that become bound states when $N_c \to \infty$. The $N_c$ scaling of the $L_i$ parameters has been given in [1, 5], and is listed in Table I. In addition, the $\pi, K, \eta$ masses scale as $O(1)$ and $f_0$ as $O(\sqrt{N_c})$. However, it
is not known at what scale $\mu$ to apply the large $N_c$ scaling, and it has been pointed out that the logarithmic terms can be rather large for $N_c = 3$ \cite{6}. The scale dependence is certainly suppressed by $1/N_c$ for $L_i = L_2, L_3, L_5, L_8$, but not for $2L_1 - L_2, L_4, L_6$ and $L_7$. Customarily, the uncertainty in the $\mu$ where the $N_c$ scaling applies is estimated varying $\mu$ between 0.5 and 1 GeV \cite{11}. We will check that this estimate is correct with the vector mesons, firmly established as $\bar{q}q$ states.

| $O(p^4)$ | $N_c$ scaling | ChPT $\mu = 770$MeV | IAM I $\mu = 770$MeV | IAM II $\mu = 770$MeV | IAM III $\mu = 770$MeV |
|---------|--------------|----------------|---------------------|---------------------|---------------------|
| $L_1$   | $O(N_c)$     | 0.4 ± 0.3     | 0.56 ± 0.10         | 0.59 ± 0.08         | 0.60 ± 0.09         |
| $L_2$   | $O(N_c)$     | 1.35 ± 0.3    | 1.21 ± 0.10         | 1.18 ± 0.10         | 1.22 ± 0.08         |
| $L_3$   | $O(N_\sigma)$| −3.5 ± 1.1    | −2.79 ± 0.14        | −2.93 ± 0.10        | −3.02 ± 0.06        |
| $L_4$   | $O(1)$       | −0.3 ± 0.5    | −0.36 ± 0.17        | 0.2 ± 0.004         | 0 (fixed)           |
| $L_5$   | $O(N_c)$     | 1.4 ± 0.5     | 1.4 ± 0.5           | 1.8 ± 0.08          | 1.9 ± 0.03          |
| $L_6$   | $O(1)$       | −0.2 ± 0.3    | 0.07 ± 0.08         | 0 ± 0.5             | −0.07 ± 0.20        |
| $L_7$   | $O(1)$       | −0.4 ± 0.2    | −0.44 ± 0.15        | −0.12 ± 0.16        | −0.25 ± 0.18        |
| $L_8$   | $O(N_c)$     | 0.9 ± 0.3     | 0.78 ± 0.18         | 0.78 ± 0.7          | 0.84 ± 0.23         |

| $2L_1 - L_2$ | $O(1)$ | −0.55 ± 0.7 | 0.09 ± 0.10 | 0.0 ± 0.1 | −0.02 ± 0.10 |

ChPT is a low energy expansion, but in recent years it has been extended to higher energies by means of unitarization \cite{3,7,8,9,10}. The main idea is that when projected into partial waves of definite angular momentum $J$ and isospin $I$, physical amplitudes $t$ should satisfy an elastic unitarity condition:

$$\text{Im} t = \sigma |t|^2 \Rightarrow \text{Im} \frac{1}{t} = -\sigma \Rightarrow t = \frac{1}{\text{Re} \ t^{-1} - i\sigma},$$

(1)

where $\sigma$ is the phase space of the two mesons, a well known function. However, from the right hand side we note that to have a unitary amplitude we only need $\text{Re} t^{-1}$, and for that we can use the ChPT expansion; this is the Inverse Amplitude Method (IAM) \cite{7}. The IAM generates the $\rho$, $K^*$, $\sigma$ and $\kappa$ resonances not initially present in ChPT, ensures unitarity in the elastic region and respects the ChPT expansion. When inelastic two-meson processes are present the IAM generalizes to $T \simeq (\text{Re} T^{-1} - i\Sigma)^{-1}$ where $T$ is a matrix containing all partial waves between all physically accessible states whereas $\Sigma$ is a diagonal matrix with their phase spaces, again well known \cite{3,8,9,10}. With this generalization it was recently shown \cite{3} that, using the one-loop ChPT calculations, the IAM generates the $\rho$, $K^*$, $\sigma$, $\kappa$, $a_0(980)$, $f_0(980)$ and the octet $\phi$, describing two body $\pi$, $K$ or $\eta$ scattering up to 1.2 GeV. Furthermore, it has the correct low energy expansion, with chiral parameters compatible with standard ChPT, shown in Table I. Different IAM fits \cite{3} are due to different ChPT truncation schemes equivalent up to $O(p^4)$ and the estimates of the data systematic error.

Those IAM results have been recently used \cite{11} to study the large $N_c$ behavior of the scattering amplitudes and the poles associated to resonances. The large $N_c$ results are similar for all IAM sets, and to illustrate it, we show here the results of set III, whereas in \cite{11} we used set II, reaching the same conclusions. Note that these ChPT amplitudes are
fully renormalized, and therefore scale independent. Hence all the QCD $N_c$ dependence appears correctly through the $L_i$ and cannot hide in any spurious parameter.

RESULTS

Let us then scale $f_0 \rightarrow f_0 \sqrt{N_c}/3$ and $L_i(\mu) \rightarrow L_i(\mu)(N_c/3)$ for $i = 2, 3, 5, 8$, keeping the masses and $2L_1 - L_2, L_4, L_6$ and $L_7$ constant. In Fig.1 we show, for increasing $N_c$, the modulus of the $(I,J) = (1,1)$ and $(1/2,1)$ amplitudes with the Breit-Wigner shape of the $\rho$ and $K^*(892)$ vector resonances, respectively. There is always a peak at an almost constant position, becoming narrower as $N_c$ increases. We also show the evolution of the $\rho$ and $K^*$ pole positions, related to their mass and width as $\sqrt{\text{pole}} \simeq M - i\Gamma/2$. We have normalized both $M$ and $\Gamma$ to their value at $N_c = 3$ in order to compare with the $\bar{q}q$ expected behavior: $M_{N_c}/M_3$ constant and $\Gamma_{N_c}/\Gamma_3 \sim 1/N_c$. The agreement is remarkable within the gray band that covers the uncertainty $\mu = 0.5 - 1\text{ GeV}$ where to apply the large $N_c$ scaling. We have checked that outside that band, the behavior starts deviating from that of $\bar{q}q$ states, which confirms that the expected scale range where the large $N_c$ scaling applies is correct.

![Results](image)

**FIGURE 1.** Left: Modulus of $\pi \pi$ and $\pi K$ elastic amplitudes versus $\sqrt{s}$ for $(I,J) = (1,1), (1/2,1)$: $N_c = 3$ (thick line), $N_c = 5$ (thin line) and $N_c = 10$ (dotted line), scaled at $\mu = 770\text{ MeV}$. Right: $\rho(770)$ and $K^*(892)$ pole positions: $\sqrt{\text{pole}} \equiv M - i\Gamma/2$ versus $N_c$. The gray areas cover the uncertainty $N_c = 0.5 - 1\text{ GeV}$. The dotted lines show the expected $\bar{q}q$ large $N_c$ scaling.

In Fig.2, in contrast, all over the $\sigma$ and $\kappa$ regions the $(0,0)$ and $(1/2,0)$ amplitudes decrease as $N_c \rightarrow \infty$. Their associated poles show a totally different behavior, since their width grows with $N_c$, in conflict with a $\bar{q}q$ interpretation. (We keep the $M$, $\Gamma$ notation, but now as definitions). This is also suggested using the ChPT leading order unitarized amplitudes with a regularization scale $[10,12]$. In order to determine their spectroscopic nature, we note that in the whole $\sigma$ and $\kappa$ regions, $\text{Im} t \sim O(1/N_c^2)$ and $\text{Re} t \sim O(1/N_c)$. Imaginary parts are generated from s-channel intermediate physical states. If it was a $\bar{q}q$ meson, with mass $M \sim O(1)$ and $\Gamma \sim 1/N_c$, we would expect $\text{Im} t \sim O(1)$ and a peak at $\sqrt{s} \simeq M$, as it is indeed the case of the $\rho$ and $K^*$. Therefore, from $\bar{q}q$ states, the $\sigma$ and $\kappa$ can only get real contributions from $\rho$ or $K^*$ t-channel exchange, respectively. The leading s-channel contribution for the $\kappa$ comes from $\bar{q}qqq$ (or two meson) states, which are predicted to unbound and become the meson-meson continuum when $N_c \rightarrow \infty$ [13]. The same interpretation holds for the sigma, but in the large $N_c$ limit $\bar{q}qqq$ and glueball exchange count the same. Given the fact that glueballs are expected to have masses above 1 GeV, and that the $\kappa$ is a natural SU(3) partner of the $\sigma$, a dominant $\bar{q}qqq$ component for the $\sigma$ seems the most natural interpretation, although it could certainly have some glueball mixing.
The large $N_c$ behavior of the $(0,0)$ amplitude in the vicinity of the $f_0(980)$ is shown in Fig.3. This resonance and the $a_0(980)$ are more complicated due to the distortions caused by the nearby KK threshold. We see that the characteristic sharp dip of the $f_0(980)$ vanishes when $N_c \to \infty$, at variance with a $\bar{q}q$ state. For $N_c > 5$ it follows again the $1/N_c^2$ scaling compatible with $\bar{q}q\bar{q}q$ states or glueballs. The $a_0(980)$ behavior, shown in Fig.4, is more complicated. When we apply the large $N_c$ scaling at $\mu = 0.55 - 1$ GeV, its peak disappears, suggesting that this is not a $\bar{q}q$ state, and $\Im t_{10}$ follows roughly the $1/N_c^2$ behavior in the whole region. However, as shown in Fig.5, the peak does not vanish at large $N_c$ if we take $\mu = 0.5$ GeV. Thus we cannot rule out a possible $\bar{q}q$ nature, or a sizable mixing, although it shows up in an extreme corner of our uncertainty band. For other recent large $N_c$ arguments in a chiral context see [14].

2 The idea of this work and the pole movements were presented by the author in two workshops [11]. While completing the calculations and the manuscript the results without the scale uncertainties have been confirmed [15] for all resonances, using the approximated IAM [8].
FIGURE 4. Right: Modulus of a $(I,J) = (1,0)$ scattering amplitude, versus $\sqrt{s}$, for $N_c = 3$ (thick), $N_c = 5$ (thin), $N_c = 10$ (dashed) and $N_c = 25$ (dotted), scaled at $\mu = 770\text{MeV}$. Right: scaled at $\mu = 500\text{MeV}$. Center: $\text{Im} t_{00}$ versus $N_c$. The dark gray area covers the uncertainty $\mu = 0.55 - 1\text{GeV}$, the light gray area from $\mu = 0.5$ to $0.55\text{GeV}$.

CONCLUSION

We have shown that the QCD large $N_c$ scaling of the unitarized meson-meson amplitudes of Chiral Perturbation Theory is in conflict with a $\bar{q}q$ nature for the lightest scalars (not so conclusively for the $a_0(980)$), and strongly suggests a $\bar{q}qq$ or two meson main component, maybe with some mixing with glueballs, when possible.

ACKNOWLEDGMENTS

I thank A. Andrianov, D. Espriu, A. Gómez Nicola, F. Kleefeld, R. Jaffe, E. Oset, J. Soto and M. Uehara for their comments and support from the Spanish CICYT projects, BFM2000-1326, BFM2002-01003 and the E.U. EURIDICE network contract no. HPRN-CT-2002-00311.

REFERENCES

1. S. Weinberg, Physica A96 (1979) 327. J. Gasser and H. Leutwyler, Annals Phys. 158 (1984) 142; Nucl. Phys. B 250 (1985) 465.
2. J. Bijnens, G. Colangelo and J. Gasser, Nucl. Phys. B427 (1994) 427.
3. A. Gómez Nicola and J. R. Peláez, Phys. Rev. D 65 (2002) 054009 and AIP Conf. Proc. 660 (2003) 102 [hep-ph/0301049].
4. G. ’t Hooft, Nucl. Phys. B 72 (1974) 461. E. Witten, Annals Phys. 128 (1980) 363.
5. A. A. Andrianov, Phys. Lett. B 157, 425 (1985). A. A. Andrianov and L. Bonora, Nucl. Phys. B 233, 232 (1984). D. Espriu, E. de Rafael and J. Taron, Nucl. Phys. B 345 (1990) 22 S. Peris and E. de Rafael, Phys. Lett. B 348 (1995) 539
6. A. Pich, [hep-ph/0205030]
7. T. N. Truong, Phys. Rev. Lett. 61 (1988) 2526. Phys. Rev. Lett. 67, (1991) 2260; A. Dobado, M.J.Herrero and T.N. Truong, Phys. Lett. B235 (1990) 134. A. Dobado and J. R. Pelaez, Phys. Rev. D 47 (1993) 4883. Phys. Rev. D 56 (1997) 3057.
8. J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. 80 (1998) 3452; Phys. Rev. D 59 (1999) 074001 and Phys. Rev. D 62 (2000) 114017. M. Uehara, [hep-ph/0204020]
9. F. Guerrero and J. A. Oller, Nucl. Phys. B 537 (1999) 459 [Erratum-ibid. B 602 (2001) 641].
10. J. A. Oller and E. Oset, Nucl. Phys. A 620 (1997) 438; Phys. Rev. D 60 (1999) 074023.
11. J. R. Pelaez, [hep-ph/0309292] [hep-ph/0307018] and [hep-ph/0306063].
12. M. Harada, F. Sannino and J. Schechter, hep-ph/0309206.
13. R. L. Jaffe, Proceedings of the Intl. Symposium on Lepton and Photon Interactions at High Energies.
   Physikalisches Institut, University of Bonn (1981). ISBN: 3-9800625-0-3
14. V. Cirigliano et al. hep-ph/0305311  N. N. Achasov, hep-ph/0309118  T. Schaefer, hep-ph/0309158
15. M. Uehara, hep-ph/0308241