Radiative Corrections to Neutralino and Chargino Masses in the Minimal Supersymmetric Model

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Abstract

We determine the neutralino and chargino masses in the MSSM at one-loop. We perform a Feynman diagram calculation in the on-shell renormalization scheme, including quark/squark and lepton/slepton loops. We find generically the corrections are of order 6%. For a 20 GeV neutralino the corrections can be larger than 20%. The corrections change the region of $\mu$, $M_2$, $\tan\beta$ parameter space which is ruled out by LEP data. We demonstrate that, e.g., for a given $\mu$ and $\tan\beta$ the lower limit on the parameter $M_2$ can shift by 20 GeV.
1 Introduction

The Minimal Supersymmetric Model (MSSM) provides a concise theoretical framework in which supersymmetry is realized in a simple and consistent manner. The parameter space of the MSSM is somewhat constrained by present experimental data. As is well known, the region of parameter space ruled out by LEP experiments due to Higgs boson searches is dramatically altered when radiative corrections are taken into account. The large shift in the Higgs boson mass can be approximated by considering the diagram in Fig.(1a). The correction to the squared mass is approximately

$$\Delta m_h^2 \simeq 3 \cdot 4 \frac{\lambda_t^2}{16 \pi^2} m_t^2 \ln \left( \frac{\tilde{m}_t}{m_t^2} \right) \implies \frac{\Delta m_h^2}{m_h^2} \simeq 60\% \ (1)$$

(We consider $m_h = M_Z$ at tree level. Here and henceforth we set the top quark mass to 150 GeV, and the squark mass to 1 TeV.) In Eq.(1) there is a factor of 3 from color, a factor of 4 from a Dirac trace, two factors of the top quark Yukawa coupling $\lambda_t$ from the vertices, a loop factor $\frac{1}{16 \pi^2}$, two factors of $m_t$ from mass insertions, and a leading logarithm from the logarithmically divergent integral. This correction is especially important phenomenologically at LEP II, where if one did not take into account radiative corrections the MSSM would be ruled out by a negative Higgs boson search [1]. In fact, after inclusion of these corrections, the $m_A$, $\tan \beta$ parameter space is only mildly constrained [2].

Motivated by large corrections in the Higgs boson sector, one can ask if we can also expect large corrections for the Higgs super-partners. The correction to the Higgsino mass can be estimated by inspecting the diagram in Fig.(1b). In this case we find

$$\frac{\Delta M_{\tilde{h}}}{M_{\tilde{h}}} \simeq 3 \frac{\lambda_t^2}{16 \pi^2} \ln \left( \frac{\tilde{m}_t}{m_t^2} \right) \simeq 5\%.$$ 

(We consider $M_{\tilde{h}} = 90$ GeV.) The factor of 12 enhancement in the Higgs scalar case compared to the Higgsino case is due to the Dirac trace and the factor of three enhancement from
\(m_t^2/M_Z^2\). Hence we can expect Higgsino mass corrections to be mild compared to the Higgs boson mass corrections. A similar analysis for the gaugino mass correction also yields an estimate for the correction of 5%.

In the MSSM the supersymmetric partners of the \(W\), \(Z\), and photon (the gauginos) mix with the partners of the Higgs bosons (the Higgsinos) to give the mass eigenstates. The spectrum then consists of two charged states (the charginos) and four neutral states (the neutralinos). The charginos are denoted \(\chi_1^+, \chi_2^+\), while the neutralinos are \(\chi_1^0\), \(\chi_2^0\), \(\chi_3^0\), and \(\chi_4^0\). They are arranged in order of increasing mass, so that \(\chi_1^+\) (\(\chi_1^0\)) is the lightest chargino (neutralino). The explicit formulas for the neutralino masses and eigenvectors at tree level can be found in Refs. [3].

The parameter space which determines the chargino and neutralino masses at tree level includes the supersymmetric Higgs mass parameter \(\mu\), the soft–supersymmetry breaking \(U(1)_Y\) and \(SU(2)_L\) gaugino masses \(M_1\) and \(M_2\), and the ratio of Higgs boson vacuum expectation values \(\tan \beta = v_2/v_1\). In this paper we assume the GUT relation among gaugino masses, so we set \(M_1 = \frac{5}{3} \tan^2 \theta_W M_2\). Thus we will typically examine the corrections in the \(\mu, M_Z^2\) plane for fixed values of \(\tan \beta\).

In the next section we describe the formalism necessary to describe the radiative corrections. In section 3 we discuss the results, and in the last section we give our conclusions.

### 2 Formalism for radiative corrections

In this section we outline our renormalization scheme. The chargino and neutralino masses are determined at tree level by the bare parameters \(x_{ib} = \{M_{Wb}^2, M_{Zb}^2, M_{tb}, M_{tb}, \mu_b, \beta_b\}\). At one-loop we must choose a renormalization prescription for each of these parameters which determines the renormalized parameter \(x_{ib}\) and the shift \(\delta x_{ib}\), where

\[x_{ib} = x_{ib} + \delta x_{ib}.\]

We choose the renormalization prescription for the parameters \(M_{Wb}^2\) and \(M_{Zb}^2\) so that at one-loop the parameters \(M_W\) and \(M_Z\) are the poles of the \(W\) and \(Z\) propagators. Hence

\[\delta M_W^2 = \text{Re } \Pi_{WW}^T(M_W^2), \quad \delta M_Z^2 = \text{Re } \Pi_{ZZ}^T(M_Z^2)\]

where \(\Pi^T\) denotes the transverse part of the boson propagator. Formulas for these gauge boson self-energies can be found in Ref. [4]. A convenient renormalization prescription for the remaining parameters \(M_1\), \(M_2\), \(\mu\), and \(\beta\) is the \(\overline{DR}\) prescription wherein the shifts \(\delta x_i\) are purely “infinite”, i.e. proportional to \((1/\epsilon + \ln 4\pi - \gamma_E)\). For these parameters we choose the \(\overline{DR}\) renormalization scale to be \(Q^2 = M_Z^2\). We have previously determined the shift \(\delta \beta\) while
studying radiative corrections in the Higgs boson sector [3]. To determine the remaining shifts $\delta M_1$, $\delta M_2$, and $\delta \mu$ we first discuss the physical masses.

The physical on-shell chargino and neutralino masses, defined as the poles of the propagators, are given by [3]

$$
M_{\chi^+_i}^{\text{phys}} = M_{\chi^+_i} + \delta M_{\chi^+_i} - \text{Re} \left( \Sigma_{1ii}^{\mu} (M_{\chi^+_i}^2) + M_{\chi^+_i} \Sigma_{1ii}^{\nu} (M_{\chi^+_i}^2) \right)
$$

(2a)

$$
M_{\chi^0_i}^{\text{phys}} = M_{\chi^0_i} + \delta M_{\chi^0_i} - \text{Re} \left( \Sigma_{1ii}^{0} (M_{\chi^0_i}^2) + M_{\chi^0_i} \Sigma_{1ii}^{\nu} (M_{\chi^0_i}^2) \right)
$$

(2b)

where we have substituted $M_{\chi^+_i} + \delta M_{\chi^+_i}$ for the bare mass $M_{\chi^+_i}$, and the $\Sigma_{ij}$’s are form factors of the one-loop fermion inverse propagator $K_{ij}$

$$
i K_{ij} = (\not{p} - M_{\chi^0_i}) \delta_{ij} + \Sigma_{1ij} + \Sigma_{5ij} \gamma_5 + \Sigma_{\gamma i} \not{p} + \Sigma_{5\gamma i} \not{p} \gamma_5
$$

(3)

The bare chargino masses $M_{\chi^+_i}^{\text{b}}$ are related to bare parameters $M_{2b}$, $\mu_b$, $\beta_b$ and $M_{W_b}$ by the equations

$$
M_{\chi^+_1}^{2b} + M_{\chi^+_2}^{2b} = M_{2b}^2 + \mu_b^2 + 2M_{W_b}^2
$$

(4a)

$$
M_{\chi^+_1}^{2b} - M_{\chi^+_2}^{2b} = \left( M_{2b} \mu_b - M_{W_b}^2 \sin(2\beta_b) \right)^2
$$

(4b)

whereas the bare neutralino masses are the absolute values of the eigenvalues of the bare mass matrix

$$
Y = \begin{pmatrix}
M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\
0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\
-M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu_b \\
-M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu_b & 0
\end{pmatrix}
$$

(5)

where $s_\beta$ $(c_\beta)$ denotes $\sin \beta$ $(\cos \beta)$, $c_W$ denotes $\cos \theta_W = M_W/M_Z$, and $s_W = \sin \theta_W$. We introduce the matrix $N$ which diagonalizes the neutralino mass matrix,

$$
N^* Y N^{-1} = \text{Diag}(M_{\chi^0_i}^{\text{b}}).
$$

(6)

The shifts in the input parameters $\delta x_i$ induce shifts in the bare masses $M_{\chi^0_i}$. From Eqs.(4) the shifts $\delta M_{\chi^+_i}$ are explicitly related to $\delta M_2$, $\delta \mu$, $\delta \beta$ and $\delta M_{W}^2$ by the following equations

$$
M_2 \delta M_2 + \mu \delta \mu = \left[ M_{\chi^+_1} \delta M_{\chi^+_1} + M_{\chi^+_2} \delta M_{\chi^+_2} + \delta M_{W}^2 \right]_{\infty}
$$

(7a)

$$
M_2 \delta \mu + \mu \delta M_2 = \left[ \frac{M_{\chi^+_1} + M_{\chi^+_2}}{M_2 \mu - M_{W}^2 \sin(2\beta)} \left( M_{\chi^+_1} \delta M_{\chi^+_1} + M_{\chi^+_2} \delta M_{\chi^+_2} \right) + 2M_{W}^2 \cos(2\beta) \delta \beta + \sin(2\beta) \delta M_{W}^2 \right]_{\infty}
$$

(7b)
where the subscript $\infty$ denotes the “infinite” part, while the shifts $\delta M_{\chi_i^0}$ are given by

$$\delta M_{\chi_i^0} = \left( N^* \delta Y N^{-1} \right)_{ii} \tag{8}$$

where $\delta Y$ is the matrix whose elements are the shifts of the elements of the matrix $Y$. We note that

$$\sum_{i=1}^{4} \eta_i \delta M_{\chi_i^0} = \sum_{i=1}^{4} (\delta Y)_{ii} = \delta M_1 + \delta M_2. \tag{9}$$

Here $\eta_i = \pm 1$ depending on whether $M_{\chi_i^0}$ is equal to $+$ or $-$ the corresponding eigenvalue of the matrix $Y$. (See Refs.[3, 4] for a detailed discussion of this technical point).

Clearly equation (2a) determines the “infinite” part of the chargino mass shift to be

$$\left[ \delta M_{\chi_i^+} \right]_{\infty} = \left[ \Sigma_{\upsilon_{ii}}^+ (M_{\chi_i^+}^2) + M_{\chi_i^+} \Sigma_{\upsilon_{ii}}^+ (M_{\chi_i^+}^2) \right]_{\infty}$$

and hence we can determine $\delta M_2$ and $\delta \mu$ from Eqs.(7). Equation(2b) determines analogously the “infinite” part of $\delta M_{\chi_i^0}$, whereby we determine $\delta M_1$ from the trace equation, Eq.(9),

$$\delta M_1 = -\delta M_2 + \left[ \sum_{i=1}^{4} \eta_i \delta M_{\chi_i^0} \right]_{\infty}$$

Having determined all the shifts in the underlying parameters, we then find the radiatively corrected chargino and neutralino on-shell masses, given by Eq.(2), are indeed finite. It is nontrivial to check that all individual neutralino masses are free of divergences.

Finally we give the explicit formulas for the chargino and neutralino self-energy form factors which are necessary for the above calculation. The top quark contribution to the chargino form factors is given by

$$\Sigma_{\upsilon_{ii}}^+ (p^2) = \frac{N_c}{16\pi^2} m_t \left( |a_{\upsilon_{ii}}^+|^2 - |b_{\upsilon_{ii}}^+|^2 \right) B_0 (p^2, m_t^2, \tilde{m}_t^2) \tag{10a}$$

$$\Sigma_{\upsilon_{ii}}^+ (p^2) = \frac{N_c}{16\pi^2} \left( |a_{\upsilon_{ii}}^+|^2 + |b_{\upsilon_{ii}}^+|^2 \right) B_1 (p^2, m_t^2, \tilde{m}_t^2) \tag{10b}$$

Here $N_c$ is the number of colors, and $B_0$ and $B_1$ are the standard integrals that appear in one-loop two-point function calculations. Explicit formulas may be found in Ref.[8].

The chargino–top quark–bottom squark coupling $g_{t\tilde{b}_1 \chi_i^+}$ is parameterized by $a_{t\upsilon_{ii}}^+$ and $b_{t\upsilon_{ii}}^+$ as $g_{t\tilde{b}_1 \chi_i^+} = a_{t\upsilon_{ii}}^+ + b_{t\upsilon_{ii}}^+ \gamma_5$. In the couplings $a_{t\upsilon_{ii}}^+$, $b_{t\upsilon_{ii}}^+$ and in Eqs.(10) we suppress the squark index. We implicitly sum over the bottom squarks $\tilde{b}_1$ and $\tilde{b}_2$ in Eqs.(10). It is straightforward to generalize Eqs.(10) to include the contributions from the other quarks and leptons.
For the neutralinos we have similar formulas. The top quark contribution to the neutrino form factors is

\[ \Sigma_{ii}^0(p^2) = \frac{N_c}{8\pi^2} m_t \left( |a_{iti}|^2 - |b_{iti}|^2 \right) B_0(p^2, m_t^2, \tilde{m}_t^2) \]  

\[ \Sigma_{ii}^0(p^2) = \frac{N_c}{8\pi^2} \left( |a_{iti}|^2 + |b_{iti}|^2 \right) B_1(p^2, m_t^2, \tilde{m}_t^2) \]  

Here again we suppress the squark index. Implicit in Eqs.(11) is a sum over top squarks \( \tilde{t}_1 \) and \( \tilde{t}_2 \).

As a check on the calculation, we find that the \( \beta \)-constants derived from Eqs.(2b,11b) in the limit of a pure bino, wino, or Higgsino eigenstate agree with the standard RGE equations for the parameters \( M_1, M_2, \) and \( \mu \). Additionally, we checked that the correction Eq.(11a) in the appropriate limit agrees with the results of Ref.\[10\] obtained for a massless photino. (As previously pointed out in Ref.\[11\] there is a \( \tilde{t}_1, \tilde{t}_2 \) top squark mixing angle factor \( \sin 2\theta_t \) missing from Eq.(4) of Ref.\[10\], which is unity in the context of Ref.\[11\].) The chargino and neutralino couplings can be found in Ref.\[7\].

3 Results

At tree level the neutralino and chargino masses are invariant under \( \mu \rightarrow -\mu, \ M_2 \rightarrow -M_2 \) (and \( M_1 \rightarrow -M_1 \)). At one-loop level this invariance is violated by typically less than 0.1%. It is weakly broken only because the squark masses are not invariant under \( \mu \rightarrow -\mu \). Hence, we shall show results only for \( M_2 > 0 \). In the results shown here we set the soft supersymmetry breaking squark and slepton mass parameters \( M_Q = M_U = M_D = M_E = M_L = 1 \) TeV, \( A = 200 \) GeV, and the top quark mass is set to 150 GeV.

In Figs.(2a-f) we show contours of \( \chi \) masses at tree level (dashed lines) and one-loop level (solid lines) in the \( \mu, M_2 \) plane with \( \tan \beta = 2 \). Note that results shown for the lightest chargino and neutralino in Figs.(2a,c) are qualitatively similar, and the contours for the heaviest chargino and neutralino in Figs.(2b,f) are quantitatively similar, both at tree level and at one-loop.

For the heaviest neutralino \( (\chi_0^4) \) and chargino \( (\chi^+_2) \) the radiative corrections yield positive shifts in the mass \( \Delta M_\chi \) of \( \sim 2 \) GeV for \( \mu \) and \( |M_2| \sim 50 \) GeV increasing to 30 GeV for \( |M_2| \sim 500 \) GeV. We show in Fig.(3a) contours of the correction \( \Delta M_{\chi^+_2} \) in the \( \mu, M_2 \) plane with \( \tan \beta = 4 \). Figure (3a) is nearly identical to the corresponding figure for the heaviest neutralino. In the region \( |M_2| > |\mu| \) the heaviest neutralino and chargino are predominantly wino and they couple to the matter particles via the \( SU(2) \) gauge coupling. Hence the
Figure 2: Contours of chargino and neutralino masses in the $\mu$, $M_2$ plane at tree (dashed lines) and one-loop (solid) level with $\tan\beta = 2$. The axes and contours are labeled in GeV units.
correction in this region is nearly independent of the top quark mass and $\tan \beta$. In the region $|\mu| > |M_2|$ the heaviest chargino and neutralino is dominantly Higgsino and the correction is proportional to the top quark mass and decreases with increasing $\tan \beta$. For example, in Fig.(3a) in the region $|\mu| > |M_2|$ the contour $\Delta M_{\chi^+} = 14$ GeV near $|\mu| = 430$ GeV increases to $\Delta M_{\chi^+} = 26$ GeV when $\tan \beta = 1$. We show the percent change in the $\chi_4^0$ mass in the $\mu, M_2$ plane at $\tan \beta = 4$ in Fig.(3b). The percentage change of the $\chi_2^+$ and $\chi_4^0$ mass increases from 2% for $|\mu|$ and $|M_2| \simeq 50$ GeV to 6% for $|M_2| \simeq 500$ GeV.

For the lightest neutralino ($\chi_1^0$) and chargino ($\chi_1^+$) the percentage change in the mass is largest in the $\mu, M_2$ plane along the upper right contour $M_{\chi_1^+} = 45$ GeV. In this region the $\chi_1^0$ and $\chi_1^+$ masses typically increase by 10-20% (8-10%) for $\tan \beta \simeq 1$ ($\tan \beta \gtrsim 4$). We illustrate this in Figs.(4a,b). In Fig.(4a) we show the tree and one-loop level $\chi_1^0$ and $\chi_1^+$ masses vs. $M_2$ for $\mu = 100$ GeV and $\tan \beta = 1$. The LEP limit on the parameter $M_2$ shifts from 161 GeV to 141 GeV, so that a smaller region of parameter space is ruled out after radiative corrections are considered. Given this new limit on $M_2$ we can obtain a new limit for the mass of the lightest neutralino. In this case, however, we find that the $\chi_1^0$ mass limit changes by only 1 GeV as compared to its tree level value. The contour $\chi_1^+ = 45$ GeV at $\tan \beta = 1$ is shown in Fig.(4b). Note that the lower left 45 GeV contour is practically unchanged while the upper right 45 GeV contour shifts appreciably; by 10 GeV for $\mu$ or $M_2 \simeq 200$ GeV and by 20 GeV if $\mu$ or $M_2 \simeq 100$ GeV.

In the region $2|\mu| > |M_2|$, $\chi_2^0$ is dominantly gaugino and hence the typical 6-8% change
Figure 4: (a) The tree level (dashed) and one-loop level (solid) $\chi^0_1$ and $\chi^+_1$ masses vs. $M^2$. The region to the left of the vertical dotted line at $M^2=141$ (161) GeV is ruled out at one-loop (tree) level. (b) Contours of the light chargino mass at tree and one-loop level. The contours are labeled in GeV.

in the mass $M_{\chi_2^0}$ in this region is essentially independent of tan $\beta$. In the region $|M^2| > 2|\mu|$ the percentage change varies from 3-6% for tan $\beta = 1$ to 2-4% for tan $\beta \geq 4$. These same comments hold for the third neutralino $\chi^0_3$, provided $2\mu$ and $M^2$ are interchanged.

4 Conclusions

We have computed corrections to the neutralino and chargino masses in the MSSM. We find that typically the corrections are of order 6%. These corrections are of the order expected by simple examination of the relevant Feynman diagrams. The largest corrections, of order 20% for the lightest neutralino, occur on the boundary of the region in the $\mu$, $M^2$ plane excluded by LEP measurements. These corrections can increase somewhat the region of parameter space allowed by the latest data.

Here we included only quark/squark and lepton/slepton loops. We can expect the corrections from the gauge/Higgs/gaugino/Higgsino sector to be of this same order of magnitude. While the basic pattern of $\chi$ masses remains unaltered by including radiative corrections, they should be included in the extraction of the parameters $\mu$ and $M^2$ in the fortunate circumstance that the charginos and neutralinos are discovered.

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