The analysis of energy conservation employed in the control of linear compressor

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Abstract. Linear compressors have received more and more attention for their compact structure and high efficiency. They are highly sensitive to the working condition, thus the control system is essential to ensure linear compressors are able to operate reliably and efficiently. Operating in a resonant state would improve linear compressor efficiency, while the natural frequency is also sensitive to the operating condition. This paper proposes the analysis of energy transfer process during linear compressor operation. Compared with force analysis, energy analysis uses the electromechanical analogy to show the energy conversion from electrical energy into mechanical energy during linear compressor operation. The principle for the control to achieve high efficiency is proposed and verified in a refrigeration system. The result shows it is practical and reliable to applied energy analysis in the control of linear compressor to achieve highly efficient driving.

1. Introduction

Linear compressor technology provides an alternative choice for domestic refrigerators, as it owns compacted structure, high efficiency and excellently variable cooling capacity. Linear compressors has been used in pulse tube cryocoolers with several years [1, 2], and also begin to fabricated for domestic refrigerators [3-5], air conditionings [6] and electronics cooling [7, 8]. Because linear compressor design with free piston structure, the dynamic characteristics of piston, including the stroke and the equilibrium position, are sensitive to the operating condition. Thus, a control system for linear compressor operating reliable and efficient is a crucial component.

In linear compressor, the piston coupled with mechanical springs is directly assembled into the linear motor on the axis of reciprocation. The model of linear compressor usually is divided into an electrical circuit and a dynamical model with pressure force linearized. Choe and Kim [9] took the describing function approach and Fourier transform to derive equivalent parameters, and studied the static and dynamic characteristics of compressor. Kim et al. [10] investigated the COP of a linear compressor in a refrigeration system, and they identified that link the operating frequency to the system natural frequency would improve the compressor efficiency. Redlich proposed a tubular structure named Sunpower type linear motor as a preferable driver for linear compressor [11]. And he also suggested a control method with the Sunpower type linear motor as a sensor [12]. A closed-loop sensorless stroke...
control system for a linear compressor had been designed in which the motor parameters were identified as a function of the piston position and the motor current [13].

As the gas load is time varying and nonlinear, it brings the dynamic characteristic of linear compressor nonlinear, such as the natural frequency, while the natural frequency is important for the efficient operating. The authors once employed phasor algorithm for stroke and natural frequency estimation for linear compressor [14], and using this online measuring method investigated the dynamic characteristics of gas kinetic parameters of linear compressor [15].

This paper proposes the analysis of energy transfer process when linear compressor operates, and proposed the principle for the control system to calculate the natural frequency. Then, we verified the energy method experimentally in refrigeration system.

2. Theoretical analysis

2.1. The structure of the compressor

Fig. 1 (a) shows the schematic diagram of the linear motor which employed in linear compressor. The moving-magnet linear oscillation motor is composed of an outer yoke, an inner yoke, a coil, springs and a moving magnet assembly.

![Schematic diagram of Sunpower type linear motor and compressor](image)

(a) Linear motor

(b) Linear compressor

Fig. 1 Schematic diagram of Sunpower type linear motor and compressor

The schematic diagram of a typical linear compressor is shown in Fig. 1(b). A piston is attached in the moving magnet assembly. The piston reciprocates in the cylinder directly driven by the linear motor. Refrigerant sucks into the compression chamber through a suction valve which is located on the piston head. After compression, the refrigerant is discharged through a pre-pressurized valve attached on the terminal of cylinder. The linear compressor is suspension supported by vibration isolation springs in a hermetic shell.

2.2. Theoretical model with energy method

Fig. 2 shows the energy process of a typical motor when it is operating. As the voltage applied on the motor, the AC power provides the total energy, and a small part of the energy is transferred into heat loss in motor, the left part is converted into mechanical energy by the electromagnetic force. As the mechanical damper will consume a small part of mechanical energy, then the left part will output, and this part energy converted into internal energy of the refrigerant in compressor.
As for the linear compressor, the gas load is nonlinear. Fig. 3 shows the pressure in compression chamber changes with the piston displacement. By taking equivalent gas load parameters with energy conservation, the energy stored in the compressed gas could be expressed as follows:

\[ W_{\text{gas}} = \phi(P_r, A, x, \dot{x}) = \frac{1}{2} c_g \dot{x}^2 + \frac{1}{2} k_g x^2 \]  

(1)

Where, \( c_g \) is gas equivalent damping coefficient, \( k_g \) is gas equivalent stiffness, \( x \) is displacement.

Using the Lagrange’s motion equation is an effective way to bridge the gap between the dynamic and the energy analysis for an electromechanical system. As for the electromechanical system with the component of energy consumption, the Lagrange’s motion could be expressed as follows:

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_j} \right) - \frac{\partial L}{\partial x_j} + \frac{\partial D}{\partial \dot{x}_j} = f_j(t) \]  

(2)

Where, \( L \) is the Lagrange’s function, \( D \) is the dissipation function, \( f(t) \) is external force. For the Sunpower type linear motor, the Lagrange’s function could be expressed as follows [16].

\[ L = W_c + \frac{1}{2} m \dot{x}^2 - \frac{1}{2} (k_c + k_g) x^2 - \frac{1}{2} c \ddot{q}^2 \]  

(3)

Where \( W_c \) is magnetic coenergy, which could obtain from the magnetic circuit analysis, shown as follows. \( C \) is capacity in linear compressor electric circuit.
\[ W_c = N^2 i^2 \frac{(Bs + Al)}{4AB} + NizH_s \frac{(l + s)x}{Bs + Al} + z^2 H_s^2 \frac{(l + s)x}{2(Bs + Al)} \]  

(4)

Where \( N \) is number of windings, \( i \) is current, \( z \) is thickness of magnet, \( H_s \) is magnetic potential, \( x \) is displacement, \( l \) is half length of magnet, \( s \) is the length of air gap.

In the formula (4), \( A \) and \( B \) are expressed as follows:

\[ A = \frac{g_1 + g_2 + z}{\mu_0 \pi D_3} \]  

(5)

\[ B = \frac{g_1}{\mu_0 \pi D_1} + \frac{g_2}{\mu_0 \pi D_2} + \frac{z}{\mu_m \pi D_3} \]  

(6)

Where \( g_1 \) and \( g_2 \) is the thickness of air gap, \( D_1, D_2 \) and \( D_3 \) is diameters which shown in Fig. 1. is relative magnetic permeability.

The dissipation function of linear compressor could be expressed as:

\[ D = \frac{1}{2} R_e i^2 + \frac{1}{2} (c_e + c) x^2 \]  

(7)

Where \( R_e \) is equivalent resistance of linear motor, \( i \) is current.

The external force in linear compressor, \( f(t) \) is the voltage that applied on linear motor.

Then the model of linear compressor could obtain as follows from expression (1)-(7).

\[
\begin{align*}
\frac{d^2 x}{dt^2} + (k_e + k_s) x - NzH_s \frac{(l + s)}{Bs + pl} i - z^2 H_s^2 \frac{(l + s)}{(Bs + Al)} x + (c_e + c) \nu &= 0 \\
N^2 \frac{(Bs + Al)}{2AB} \frac{d^2 \nu}{dt^2} + NzH_s \frac{(l + s)}{Bs + Al} \frac{dx}{dt} + iR_e + \frac{1}{C} q &= u
\end{align*}
\]  

(8)

If we define the following symbol:

\[ K_e = NzH_s \frac{(l + s)}{Bs + pl} \]  

(9)

\[ L_e = N^2 \frac{(Bs + Al)}{2AB} \]  

(10)

\[ k_m = z^2 H_s^2 \frac{(l + s)}{(Bs + Al)} \]  

(11)

Then the linear compressor model could be expressed as

\[
\begin{align*}
\frac{d^2 \nu}{dt^2} + (c_e + c) \nu - K_e \frac{dx}{dt} + (k_e + k_m) \nu &= 0 \\
L_e \frac{di}{dt} + K_e \nu + iR_e + \frac{1}{C} i dt &= u
\end{align*}
\]  

(12)
We could see similarity in formula (12), then we could take the complex mechanical system analogous to a circuitry. Then from the point of energy conversion, when linear compressor operates, we could obtain the following equations.

\[
\begin{align*}
P_U &= P_K + P_{x_s} \\
Q_U &= Q_K + Q_{x_s} \\
P_{x_s} &= P_{(x_s,v_s)} \\
Q_{x_s} &= Q_m + Q_{(x_s,v_s,v_s)}
\end{align*}
\]

(13)

(14)

Where \( P \) and \( Q \) represent active power and reactive power that consumed in the different elements of linear compressor model respectively.

Thus, if we could measure the active power and the reactive power when the compressor operates, then we could measure the damping coefficient and the stiffness of linear compress system on time, even if the operating condition changes rapidly. So, the natural frequency of linear compressor could be calculated on time according the following formula.

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{k_s + k_x + k_m}{m}}
\]

(15)

3. Test Bench

The configuration of the test bench is shown in Fig. 4. A linear compressor, driven by the controller with a pulse width modulation (PWM), operates in a residential refrigerator with refrigerant R600a. The PWM controller receives the set values (frequency and amplitude of voltage) from the LabVIEW program to drive the linear compressor with frequency adjustment and voltage controller, and the LabVIEW program collects the supply current, voltage, input power and frequency for calculation. Then the LabVIEW program calculates the gas equivalent coefficients based on the energy method, and presents the calculation results on its front panel in real time, and executes the control strategies by sending set values to the PWM controller. As linear compressor operates in resonance, it could achieve the maximum coefficient of performance (COP), which had been verified [10]. What’s more, the motor efficiency also gets the maximum value. Thus, in the experiment, we monitored the motor efficiency on time to verify the compressor working in resonance states as the voltage increased. That’s means, when the motor efficiency reaches the maximum value, the linear compressor operating in resonance. and in this condition, the calculated natural frequency should equal to the operating frequency.

In the experiment, the motor efficiency could be obtained by:

\[
\eta = \frac{P - fR}{P}
\]

(16)

Where, \( \eta \) is efficiency of the oscillating motor, \( P \) is the power of AC power source input to the linear compressor.

Thus, in the experiment, the linear compressor was installed in a refrigerator, shown as in Fig. 4. Then we start to increase the voltage applied on the linear compressor with a constant frequency, which makes the stroke of compressor increases from zero to the set point. In each applied voltage, we measure the voltage, current, and the energy of the compressor consumed. Then the natural frequency could be calculated with energy method employed equation (13), (14) and (15) in LabVIEW. At the same time, we could obtain the corresponding motor efficiency and the suction pressure and the discharge pressure of the applied voltage in the LabVIEW.

In this paper, we established the test bench to verify the estimated natural frequency of a linear compressor, based on an electromechanical analogy to describe the conversion of electrical energy into
mechanical energy. As the indicator diagram area and the mass flow rate are not used for the natural
frequency estimation, so we didn’t measure them in the experiment.

4. Results and discussion

Fig. 5 (a) shows the relationship between the equivalent stiffness of compressor and the voltage. The equivalent stiffness, influenced by the piston stroke, rises as the voltage increases firstly. And then, as the voltage increases continuously, the stiffness decreases. After that the stiffness increases as the voltage continue to rises. The reason is that the stiffness reflects the linear compressor operating regimes [15]. Fig. 5 (b) shows the relationship between the equivalent damping coefficient and the voltage. Similar to the equivalent stiffness, With the increase of the voltage, the equivalent damping coefficient of linear compressor decreases firstly. Then it increases rapidly, before the piston gets to TDC point. After that, it begins to decrease again. This variation trend of gas equivalent damping coefficient is also determined by the operating regimes of linear compressor [15]. The gas equivalent stiffness and damping coefficient are affected not only by the voltage but also by the gas pressure. With different operating frequency, the equivalent stiffness and the equivalent damping coefficient has a litter gap due to the suction and discharge pressure has changed which is shown in Fig. 6(a).

Fig. 6 (a) shows the pressure curves change with the voltage under different operating condition. As the voltage increased, the stroke increased gradually in several minutes, thus the pressure would have a slight change. The pressure ratio is about 8 which shows in Fig. 6 (a). Fig. 6 (b) shows the corresponding tested motor efficiency changes with the ration of the calculated natural frequency $f_n$ to the operating frequency $f$. As we could see that the tested motor efficiency gets a maximum efficiency when the ration of frequency gets close to 1 under different operating frequency. The motor reaches maximum motor efficiency suggests linear compressor works in a resonant state. Thus, it denotes the calculated natural
frequency using energy method for controller could achieve the compressor operating in an optimum efficiency.

5. Conclusions
The analysis of energy transfer process is proposed in the paper when linear compressor operates. It shows the energy conversion from electricity to mechanical energy when linear compressor operates. Thus, though the gas load is nonlinear, it could calculate the equivalent parameters of the gas loads on time with energy method. Then the principle for the control to achieve high efficiency is proposed and verified in refrigeration system. The result shows the tested motor efficiency gets a maximum efficiency when the ration of frequency gets close to 1 under different operating frequency. Thus, it is practical and reliable to applied energy analysis in the control of linear compressor to achieve highly efficient operating.

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