Gravitational wave detection using high precision pulsar observations

G Hobbs
Australia Telescope National Facility, CSIRO, PO Box 76, Epping, NSW 1710, Australia
E-mail: george.hobbs@csiro.au

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Abstract
Pulsar timing experiments are reaching sufficient sensitivity to detect a postulated stochastic gravitational wave background generated by merging supermassive black hole systems in the cores of galaxies. We describe the techniques behind the pulsar timing detection method, provide current upper bounds on the amplitude of any gravitational wave background, describe theoretical models predicting the existence of such a background and highlight new techniques for providing a statistically rigorous detection of the background.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

It is now possible to make timing observations of millisecond pulsars to a precision of \(\sim 100\) ns. Such exquisite timing precision allows the pulsar astrometric, spin and orbital parameters to be determined with great accuracy. This has recently allowed us to limit the rate of change of the gravitational constant, \(\dot{G}/G < 2.1 \times 10^{-12}\) yr\(^{-1}\), and to obtain the most accurate astrophysical distance estimate outside of the solar system (Verbiest et al 2007). Multi-frequency pulsar observations have allowed us to study the solar corona (You et al 2007, at press) and density fluctuations in the interstellar plasma (You et al 2007b). One of the most exciting applications of such datasets is to search for the signatures of gravitational waves (GWs) passing over the Earth. This is the main goal of the Parkes pulsar timing array (PPTA) project\(^1\), which aims to observe 20 millisecond pulsars with a timing precision close to 100 ns over more than 5 years.

The existence of a GW signal can be identified by analysis of pulsar timing residuals. In brief, the pulsar timing model (see, e.g. Edwards et al 2006) allows the observed arrival times

\(^1\) http://www.atnf.csiro.au/research/pulsar/ppta.
of pulses from a radio pulsar to be compared with a model of the pulsar’s astrometric, orbital and rotational parameters. The differences between the actual arrival times and the predicted arrival times are the ‘pulsar timing residuals’. Any features observed in the timing residuals indicate the presence of unmodelled effects which may include calibration errors, (additional) orbital companions, spin-down irregularities, or GWs. The induced pulsar timing residuals due to a GW signal were first calculated by Sazhin (1978) and Detweiler (1979). They showed that a GW signal causes a fluctuation in the observed pulse frequency \( \delta \nu / \nu \) which affects pulsar timing residuals at time \( t \) from the initial observation as

\[
R(t) = - \int_0^t \frac{\delta \nu(t)}{\nu} \, dt.
\]

For a given pulsar and GW source, this effect is only dependent upon the characteristic strain at the pulsar and at the Earth. The GW strains evaluated at the positions of multiple pulsars will be uncorrelated, whereas the component at the Earth will lead to a correlated signal in the timing residuals of all pulsars. As pulsars are typically observed every few weeks for many years and standard pulsar timing techniques absorb any low-frequency GWs by fitting for the pulsar’s spin-down\(^2\), pulsar timing experiments are sensitive to GW signals in the ultra low-frequency (\( f \sim 10^{-9} \) Hz) band. This technique is therefore complementary to other GW detection methods such as the Laser Interferometer Space Antenna (LISA) and ground based interferometer systems (such as LIGO and VIRGO), which are sensitive to higher frequency GWs (\( f \sim 10^{-2.5} \) and \( f \sim 10^{2.5} \) Hz, respectively). In figure 1 we plot the sensitivity of the pulsar timing experiments, LISA and LIGO and the expected GW sources. For the pulsar timing experiments, maximum sensitivity is reached at \( f = 1/T \), where \( T \) is the data span.

\(^2\) As the intrinsic pulsar spin period and rate of spin-down is unknown, a quadratic curve is always fitted and subsequently removed from the pulsar timing residuals.
As described above, the pulsar experiments are not sensitive to GWs with \( f < 1/T \) and, for white pulsar timing residuals, the upper bound on the characteristic strain increases as \( f^{3/2} \) for \( f > 1/T \).

In this paper, we review the potential sources of GWs (section 2) before describing how upper bounds on the amplitude of any existing background can be calculated and highlighting the astrophysical implications of these bounds. In section 3, we describe how a stochastic GW background may be detected. Finally, we highlight some practical issues that need to be addressed in achieving our sensitivity goal (section 4) and describe possible future experiments (section 5).

## 2. Potential sources of gravitational waves

Theoretical models predict that pulsar timing experiments will be sensitive to burst GW sources, individual sources and to a stochastic background.

### 2.1. Single sources

Recently Sudou et al (2003) reported the possible discovery of a binary supermassive black hole system in the radio galaxy 3C66B. As shown by Jenet et al (2004), the GW radiation from this postulated system would have produced clearly detectable fluctuations in the timing residuals of existing pulsar datasets. The non-detection of the expected GW signal (see figure 2) ruled out the postulated system with 95% confidence. However, detecting (or limiting the existence of) individual supermassive binary black hole systems is necessary for testing theories of hierarchical galaxy formation. Order-of-magnitude calculations demonstrate that a system with a chirp mass of \( 10^9 \) \( M_\odot \) and an orbital period of 10 year, at a distance of 20 Mpc will induce a sinusoidal signal in the timing residuals with amplitude \( \sim 100 \) ns at a frequency of twice the orbital frequency.

### 2.2. Burst sources

Sources of burst GW emission that may be detectable with current and proposed pulsar timing experiments include (1) the formation of supermassive black holes (Thorne and Braginskii...
which lead to a day-long burst of radiation, (2) highly eccentric supermassive black hole binaries (Enoki and Nagashima 2007), (3) close encounters of massive objects (Kocsis et al 2006) and (4) cosmic string cusps (Damour and Vilenkin 1). The sensitivity of a pulsar timing array experiment to burst GW emission depends upon the number of pulsars in the array, the timing precision, number of observations, the pulsar positions on the sky and the coordinates of the GW burst source. With our final datasets we expect to achieve an angular resolution of approximately 30° (Lommen, private communication). By sharing data with timing array projects being carried out in the northern hemisphere we expect to improve both the angular resolution of our detector and our sensitivity to burst sources in the northern hemisphere.

2.3. Stochastic background

A stochastic background of GWs due to binary supermassive black holes (Jaffe and Backer 2003, Wyithe and Loeb 2003, Enoki et al 2004, Sesana et al 2004), cosmic strings or relic GWs from the big bang (Maggiore 2000) are all potentially detectable. In most models, the GW strain spectrum, $h_c(f)$, can be represented by a power-law in the GW frequency, $f$,

$$h_c(f) = A \left( \frac{f}{yr^{-1}} \right)^{\alpha},$$

(2)

where the spectral exponent $\alpha = -2/3, -1$ and $-7/6$ for likely GW backgrounds due to coalescing black hole binaries, cosmic strings and relic GWs, respectively (figure 3). The energy density of the background per unit logarithmic frequency interval can be written as

$$\Omega_{GW}(f) = \frac{2}{3} \frac{\pi^2}{H_0^2} f^2 h_c(f)^2,$$

(3)

where $H_0$ is the Hubble constant.

Jenet et al (2006) introduced a method to place an upper bound on $A$ for a given $\alpha$ using actual pulsar timing residuals. In contrast with earlier techniques (e.g. Kaspi et al 1994) this method used the TEMPO2 software package (Hobbs et al 2006a) to account for all the fitting procedures undertaken during the pulsar timing process. Using observations from the PPTA project along with archival data from the Arecibo telescope, Jenet et al (2006) obtained an upper limit on the energy density per unit logarithmic frequency interval of
The expected correlation in the timing residuals of pairs of pulsars as a function of angular separation for an isotropic GW background.

\[ \Omega_{GW}[1/(8\text{yr})] h^2 \leq 2 \times 10^{-8} \text{ for } \alpha = -2/3, \]
which corresponds to \( A \leq 10^{-14} \) (see figure 3 and the ‘current limit’ in figure 1). This is the most stringent limit to date on the existence of a GW background in the nano-Hertz frequency band and has astrophysical and cosmological implications. For instance, this limit was used to (1) constrain the merger rate of massive black hole systems at high redshift, (2) rule out some relationships between the black hole mass and galactic halo mass, (3) constrain the rate of expansion in the inflationary era and (4) provide an upper bound on the dimensionless tension of cosmic strings.

We have recently shown that the Jenet et al (2006) technique described above suffers from two limitations. First, it does not give the lowest possible upper bound and secondly, it relies on the observed spectrum being white. Unfortunately, even for millisecond pulsars, it is unusual for the timing residuals not to be affected by unexplained phenomena, such as pulsar ‘timing noise’ (see e.g. Hobbs et al 2006b). A new method has now been developed and will be described in a subsequent paper. Our new method is applicable to any pulsar timing dataset and will lead to significantly improved bounds on the GW background amplitude.

3. Detecting a background

Jenet et al (2005) developed a technique for making a definitive detection of an isotropic, stochastic, background of GWs by searching for correlated signals between pulsar datasets. For each pair of pulsars, the zero-lag correlation between the respective timing residuals can be calculated. The expected correlation as a function of angle between the pulsars for timing residuals dominated by a GW background was first calculated by Hellings and Downs (1983) and is shown in figure 4.3 Jenet et al (2005) showed, for expected amplitudes of a GW background, that this signal could be unambiguously detected if 20 or more pulsars were observed over a period of 5 years each with an rms timing residual of 100–500 ns.

3 This correlation curve assumes the general theory of relativity. We are calculating the equivalent curves for other theories of gravity. These curves and their implications will be discussed elsewhere.
Figure 5. The sensitivity of the completed PPTA project to a GW background of specified amplitude. The three curves represent no pre-filtering of the data (lowest), using an optimal low-pass filtering technique (middle) and optimally pre-whitening the datasets (highest).

We have recently generalized the initial work of Jenet et al. (2005) to produce routines that can be applied to the observed pulsar timing residuals (with the actual sampling, data lengths etc). This new technique also increases the significance of any possible detection, by optimally pre-whitening the datasets. An example of the improvement possible by pre-whitening the timing residuals is shown in figure 5 where the significance of detection is plotted versus the GW background amplitude for the PPTA goal of timing 20 pulsars with 100 ns timing precision over a period of 5 years. Without any pre-filtering of the data we will only achieve a maximum detection significance of $S \sim 3.5$ (corresponding to the sensitivity labelled as the 'PPTA limit' in figure 1). Using optimal whitening schemes this sensitivity can be significantly enhanced. Initial results suggest that if we can reduce our current timing precision for all pulsars by a factor of $\sim 2$ we will be sensitive, within 5 years, to the maximum amplitude predicted by models of an isotropic stochastic GW background caused by coalescing black holes in the centres of galaxies.

4. Practical issues

Pulsar timing is affected by the stability of terrestrial clocks, ephemeris errors and the pulsars themselves. The goal of the PPTA project, described above, to obtain datasets for 20 pulsars with approximately two-weekly sampling with rms timing residuals of $\sigma \sim 100$ ns over 5 years, is challenging. To date we have a few pulsars where we can obtain such precision. For instance, van Straten et al. (2001) obtained $\sigma = 130$ ns over 40 months of observing PSR J0437–4715. For the majority of our pulsars $\sigma \sim 1\mu s$. We have many possibilities for decreasing these timing residuals, including:

4 Note that this limit is similar to the current limits described in section 2.3 that were obtained using a few long datasets. We emphasise that, even though limits can be placed using a few datasets, detecting a GW background requires observations of $\sim 20$ pulsars.
• correcting the datasets for dispersion measure variations: You et al (2007) showed that for PSR J1939+2134, $\sigma$ decreased from 290 ns to 190 ns after correction.

• improving instrumentation: we have recently developed new digital filterbank systems for the Parkes telescope. The recently commissioned system provides 1 GHz of bandwidth at an observing frequency of 3 GHz. We are also developing a new wide-bandwidth coherent dedispersion system.

• new processing techniques: we are exploring new methods to calibrate our datasets which should lead to at least a factor of 2 improvement in our rms residuals. We have recently produced a new pulsar timing package, TEMPO2 (Hobbs et al 2006a), that is accurate (for known physics) at the 1 ns level.

5. Future possibilities

The sensitivity of a pulsar timing array to a stochastic background is proportional to the average timing precision, the square root of the number of observations and to the number of pulsars in the array. This paper has concentrated on the Parkes pulsar timing array project. However, significant improvements will be made to the timing array sensitivity to GW sources by combining our datasets with northern hemisphere pulsar observations. The North American pulsar timing array project (NanoGrav) has been observing a sample of pulsars using the Arecibo and GreenBank observatories. These observations provide more than 20 years of timing PSRs B1855+09 and B1937+21 and were used to provide the first stringent limits on the stochastic gravitational wave background (e.g. Kaspi et al 1994) and to rule out the postulated binary black hole system in the radio galaxy 3C66B (Jenet et al 2004). The European pulsar timing array (EPTA) team have recently started to obtain data on a large sample of pulsars using the four major European telescopes at Jodrell Bank, Effelsberg, Nancay and Westerbork. We are currently developing techniques to combine observations from different observatories and will soon produce new limits on the existence of a GW background using all available data.

Figure 1 shows the sensitivity that should be achievable using the square kilometre array (SKA) telescope hoped to be built by the year 2020. This sensitivity curve assumes an SKA timing array project that could observe 100 millisecond pulsars with rms timing residuals around 50 ns for a period of 10 year. Provided intrinsic pulsar timing noise does not dominate the effect of the stochastic GW background then this limit corresponds to a detection limit at 3$nHz of $\Omega_{gw} \sim 10^{-13}$. Pre-whitening procedures have the potential to greatly decrease this limit.

6. Conclusion

It is possible that gravitational waves will be detected within the next decade by world-wide pulsar timing array projects. As a byproduct of these investigations stringent checks will also be placed on terrestrial time standards and the solar system ephemeris. Regular dual-frequency observations of multiple pulsars will also provide valuable information about the interstellar medium. Using a pulsar array as a gravitational wave detector is complimentary to other searches that are attempting to detect much higher frequency gravitational waves.

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References

Damour T and Vilenkin A 2001 Phys. Rev. D 71 063510
Detweiler S 1979 Astrophys. J. 234 1100
Edwards R T, Hobbs G B and Manchester R N 2006 Mon. Not R. Astron. Soc. 372 1549–74
Enoki M and Nagashima M 2007 Prog. Theor. Phys. 117 241–56
Enoki M, Inoue K T, Nagashima M and Sugiyama N 2004 Astrophys. J. 615 19–28
Hellings R W and Downs G S 1983 Astrophys. J. 265 L39
Hobbs G B, Edwards R T and Manchester R N 2006 Mon. Not R. Astron. Soc. 369 655–72
Hobbs G, Lyne A and Kramer M 2006b Chin. J. Astron. Astrophys. (Suppl 2) 6 169–75
Jaffe A H and Backer D C 2003 Astrophys. J. 583 616–31
Jenet F A, Hobbs G B, Lee K J and Manchester R N 2005 Astrophys. J. 625 L123–6
Jenet F A, Hobbs G B, van Straten W, Manchester R N, Bailes M, Verbiest J P W, Edwards R T, Hotan A W, Sarkissian J M and Ord S M 2006 Astrophys. J. 653 1571–6
Jenet F A, Lommen A, Larson S L and Wen L 2004 Astrophys. J. 606 799–803
Kaspi V M, Taylor J H and Ryba M 1994 Astrophys. J. 428 713–28
Kocsis B, Gáspár M E and Márka S 2006 Astrophys. J. 648 411–29
Lommen A N 2007 Private communication
Maggiore M 2000 Phys. Rep. 331 283–367
Sazhin M V 1978 Sov. Astron. 22 36
Sesana A, Haardt F, Madau P and Volonteri M 2004 Astrophys. J. 611 623–32
Straten W van, Bailes M, Britton M, Kulkarni S R, Anderson S B, Manchester R N and Sarkissian J 2001 Nature 412 158–60
Sudou H, Iguchi S, Murata Y and Taniguchi Y 2003 Science 300 1263–5
Thorne K S and Braginskii V B 1976 Astrophys. J. 204 L1–6
Verbiest J et al 2008 Astrophys. J. at press (Preprint 0801.2589)
Wyithe J S B and Loeb A 2003 Astrophys. J. 590 691–706
You X P et al 2007 Mon. Not R. Astron. Soc. 378 493
You X P et al 2007 Astrophys. J. 671 907