Mean-field calculations of quasi-elastic responses in $^4\text{He}$.

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Abstract

We present calculations of the quasi–elastic responses functions in $^4\text{He}$ based upon a mean–field model used to perform analogous calculations in heavier nuclei. The meson exchange current contribution is small if compared with the results of calculations where short–range correlations are explicitly considered. It is argued that the presence of these correlations in the description of the nuclear wave functions is crucial to make meson exchange current effects appreciable.

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The evaluation of meson exchange current (MEC) effects in nuclei is a topic which has been investigated for more than twenty years. Various methods have been used to calculate these effects, and a great variety of nuclei and observables have been investigated.

A clear fact arising from the large amount of results produced in these years is that the effects of MEC are large for few body systems [1], whereas they appear to be rather small in medium and heavy nuclei [2]–[7].

In a previous work [7] we have argued that this can be ascribed to the presence of short–range correlation functions in the models describing the few body systems.
In the case of the deuteron short–range correlations in both non relativistic \cite{8} and relativistic \cite{9} calculations are explicitly included. The systems with 3 or 4 nucleons have been studied using different techniques (Faddeev equations \cite{10}, hyperspherical functions \cite{11}, Green function and variational Monte Carlo \cite{8}, etc.) but all of them consider these correlations. In these few body systems, the MEC produce large effects at any energy scale considered, either in the ground state observables \cite{1} or in the quasi-elastic response \cite{12} and even at higher energies.

For medium–heavy nuclei, nuclear models which take into account short–range correlations have been recently proposed \cite{13}. The present status of the art in this field is however quite far from the possibility of calculating MEC contributions. The effects of the MEC in these nuclei, either in the ground and low-lying states \cite{2}–\cite{3} or in the case of nuclear excitations in the continuum \cite{4}–\cite{7}, have been evaluated within the mean–field approach. Contrary to what has been found in the few body systems, in medium–heavy nuclei these effects are rather small, i.e. they are of the same order of magnitude of both theoretical and experimental uncertainties.

In this situation it should be desirable to see if mean–field models produce in light nuclei results similar to those obtained for the medium–heavy ones. This would exclude explanations of the contradictory results such as the possibility that the smallness of the global MEC effect in medium–heavy nuclei is due to the cancellations between the contributions of a large amount of particle–hole excitations.

In order to investigate this point, we have applied to the \(^4\)He nucleus the model we have used to study the quasi–elastic responses in \(^{12}\)C and \(^{40}\)Ca \cite{7}.

In this model the ground state is described as a Slater determinant of single particle wave functions produced by a mean–field potential of Woods–Saxon type. The excited states are built up as one particle–one hole (1p–1h) and 2p–2h excitations, where the particle wave functions are obtained solving the Schrödinger equation in the continuum with the same Woods-Saxon potential.

Within this model we have evaluated the quasi–elastic response functions as described in Ref. \cite{7}: the longitudinal response is produced by the one–body charge operator, while the transverse response is obtained adding to the one–body convection and magnetisation currents, the two-body MEC. These have been calculated considering the so–called seagull or contact, pionic or pion in flight, and \(\Delta\)–isobar terms.

In Table 1 we give the parameters of the Woods–Saxon potential used in our calculations and defined as in Ref. \cite{7}. The ground state properties of the \(^4\)He do not constrain the spin–orbit part of the potential, which, on the other hand, affect the continuum single particle wave functions used to calculate the responses. We have studied the sensitivity of our results on the spin-orbit potential using values taken from parametrizations considered in heavier nuclei. We found that the effect on the responses is less than 1%. All the results presented in this report have been obtained using mean field potentials without spin–orbit term.

In Fig.1 we compare some results with the experimental data of Ref. \cite{14}. The dashed lines have been obtained with a Woods–Saxon potential, the WS1 of table 1,
whose parameters have been fixed in order to reproduce the energies of the $^4$He single particle levels. This is the usual procedure followed in medium–heavy nuclei in order to choose the mean–field parameters. With this potential the charge distribution of $^4$He is not very well reproduced, as it is shown in Fig. 2 by the dashed line.

The parameters of the potential WS2 have been fixed to obtain the best fit of the charge density, compatible with the limitations of using a Woods-Saxon potential (the dashed–dotted line of Fig.2). The results obtained with this potential are presented in Fig.1 by the dashed–dotted lines.

The full lines of all the figures have been obtained with the potential WS3 whose parameters have been fixed to obtain a good agreement with the data of the longitudinal responses. The values of single particle energies and of the charge distributions obtained with this potential are rather different from the experimental ones.

The longitudinal response functions are reasonably well described by all the three calculations, while the transverse response functions are always underestimated, in spite of the fact that the MEC are included in the electromagnetic operator.

These results show a different trend with respect to the medium–heavy nuclei where the ground state properties can be described reasonably well with mean–field potentials. In the present case, we could not reproduce simultaneously the various ground state observables. The $^4$He nucleus is too small to be reasonably described by a mean–field model.

On the other hand, the aim of this work is not to produce a realistic description of this nucleus, but rather to study the possibility that MEC effects could be enhanced in few–body systems.

Our main result is presented in Fig. 3, where the relative differences between the transverse responses calculated with and without MEC are shown for the three momentum transfer considered. The left panels give the results corresponding to the three different parametrizations of the Woods–Saxon potential for $^4$He. The right panels show, for the same values of the momentum transfer, the results obtained in $^{12}$C (full lines) and in $^{40}$Ca (dashed lines) with the potentials WS1 of Refs. [6] and [7], respectively.

Three aspects shown in this figure deserve a comment.

1. In $^4$He, the contribution of the MEC at peak energies is small, of the order of a few percent, if compared with the full response. This result is rather independent from the mean–field potential used.

2. The curves for $^4$He are very similar to those found for $^{12}$C and $^{40}$Ca. In absolute value, at the peak energies, the effect of the MEC becomes bigger the heavier is the nucleus.

3. The contribution of the MEC at the peak energies, with the $\Delta$ isobar current included, is negative for transfer momenta bigger than 400 MeV/c.

These results show that the MEC contributions produced by mean–fields models in $^4$He are similar to those obtained in medium–heavy nuclei. The possibility of
an enhancement of these contributions in light nuclei due to the smallness of these systems should be excluded.

It is worthwhile to point out the similarity of our \(^4\)He results with the NT curves of Fig. 8 of Ref. [16]. Using a model quite different from ours, Leidemann and Orlandini obtained these curves with purely central short–range correlations. They also show that the addition of the tensor pieces of the correlation increases the relative contribution of MEC up to 10–15%, at the peak energy.

All these facts lead us to conclude that the small MEC effects found in medium and heavy nuclei is due to the lack of short–range correlations, and in particular their tensor components, not taken into account in the mean–field models used to describe these many–body systems.

One may claim that the contribution of MEC in heavy nuclei can be enhanced by the presence of other effects which are usually not considered, for example relativity and RPA long–range correlations.

We think relativistic effects are not playing an important role in this context, because MEC contributions in light nuclei are large even in non relativistic treatments. This idea is confirmed by a calculation done within the relativistic Fermi gas model by Blunden and Butler [5] for the quasi elastic excitation of \(^{40}\)Ca where MEC effects are evaluated to be of the order of a 10%, but they are not including the \(\Delta\)-isobar current. This is the same value we obtained with our model when we switch off this component of the two body current [6, 7].

The role of RPA correlations on the MEC in the quasi–elastic region is not clear. A recent work of the Gent group [17] shows considerable MEC effects, 20–30% of the strength of the quasi–elastic peak, within a non–relativistic Hartree–Fock–RPA model.

Full RPA calculations of MEC contributions performed at lower energies, but at the same values of the momentum transfer, show scarce sensitivity to the RPA correlations [3]. Furthermore, in continuum RPA calculations with finite range residual interactions [3, 18] the one–body quasi–elastic responses do not show sizeable differences with mean–field results.

In conclusion, we have shown that within mean–field calculation the MEC contribution in the quasi–elastic excitation of \(^4\)He is small, analogously to what happens in medium–heavy nuclei. We deduce that this results is due to the lack, in the mean–field approach, of short–range correlations. Calculations of MEC in medium–heavy nuclei with explicit treatment of the short–range correlations are desirable in order to clarify definitively the problem.

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**Table.** Parameters of the Woods–Saxon potentiala used in the various calculations described in the text and single–particle energies obtained for the proton and neutron 1s1/2 levels. The spin–orbit part is switched off and the value of the Coulomb radius is taken equal to the value of $R$. In the last row the experimental single particle energies are shown.

|     | V [MeV] | $R$ [fm] | $a$ [fm] | $\epsilon$ [MeV] |
|-----|---------|----------|----------|-------------------|
| WS1 |         |          |          |                   |
| p   | -65.83  | 1.70     | 0.60     | -19.52            |
| n   | -66.00  | 1.70     | 0.60     | -20.53            |
| WS2 |         |          |          |                   |
| p   | -52.11  | 1.80     | 0.20     | -17.24            |
| n   | -52.11  | 1.80     | 0.20     | -18.16            |
| WS3 |         |          |          |                   |
| p   | -55.00  | 1.98     | 0.85     | -17.39            |
| n   | -55.00  | 1.98     | 0.85     | -18.17            |
| exp |         |          |          |                   |
| p   |         |          |          | -19.82            |
| n   |         |          |          | -20.58            |

*a*See Ref. [7] for the definition of the potential.
Figure Captions

FIG. 1. Longitudinal and transverse response functions for different values of the momentum transfer. The dashed, dashed–dotted and full lines have been calculated with the WS1, WS2 and WS3 potential respectively. The experimental date have been taken from Refs. [14].

FIG. 2. Charge densities obtained with the WS1 (dashed line), WS2 (dashed–dotted line) and WS3 (full line) potentials compared with the experimental one (Ref. [15]).

FIG. 3. Relative differences between the transverse responses calculated with and without MEC. The value $\delta R_T = \frac{R_{T,\text{OB+MEC}} - R_{T,\text{OB}}}{R_{T,\text{OB+MEC}}}$ is plotted for different cases. In the left panels, we show the results obtained for the three mean–field potentials considered in this work for $^4\text{He}$. The curves are labelled as in Fig.1 and 2. The right panels show the results for the WS1 potentials of Refs. [6] and [7] for $^{12}\text{C}$ (full lines) and $^{40}\text{Ca}$ (dashed lines), respectively.
$\rho(r) \text{ [fm}^{-3}\text{]}$ vs. $r \text{ [fm]}$ for $^4\text{He}$.
