Superflow of resonantly driven polaritons against a defect

E. Cancellieri,† F. M. Marchetti,† M. H. Szymańska,‡ and C. Tejedor

†Física Teórica de la Materia Condensada, Universidad Autónoma de Madrid, 28049, Spain.
‡Department of Physics, University of Warwick, Coventry, England.

(Dated: January 11, 2011)

In the linear response approximation, coherently driven microcavity polaritons in the pump-only configuration are expected to satisfy the Landau criterion for superfluidity at either strong enough pump powers or small flow velocities. Here, we solve non-perturbatively the time dependent Gross-Pitaevskii equation describing the resonantly-driven polariton system. We show that, even in the limit of asymptotically large densities, where in linear response approximation the system satisfies the Landau criterion, the fluid always experiences a residual drag force when flowing through the defect. We illustrate the result in terms of the polariton lifetime being finite, finding that the equilibrium limit of zero drag can only be recovered in the case of perfect microcavities. In general, both the drag force exerted by the defect on the fluid, as well as the height of Cherenkov radiation, and the percentage of particles scattered by the defect, show a smooth crossover rather than a sharp threshold-like behaviour typical of superfluids which obey the Landau criterion.

PACS numbers: 03.75.Kk, 71.36+c., 41.60.Bq

I. INTRODUCTION

In the past two decades, microcavity polaritons have attracted considerable interest because of the possibility of strongly coupling light and matter, leading to an easy manipulation and detection of the non-linear properties of matter via light — see, e.g., Ref.[2] and references therein. Recently, in light of their out-of-equilibrium nature, there has been a growing interest in studying polariton superfluid properties. In fact, polaritons have very short lifetimes (around ~ 30 ps even in the best available samples[2]), therefore any polariton fluid is the result of a steady state balance between pumping and decay. The effects of pump and decay in polariton systems are the subject of several recent theoretical and experimental works.

For condensates in local thermal equilibrium, such as superfluid ⁴He and the ultracold atom Bose-Einstein condensates, the concept of superfluidity is strongly linked to several paradigmatical properties, such as the Landau criterion, quantised circulation of velocity, and metastable persistent flow. In particular, the Landau criterion connects the frictionless motion of a defect at velocities smaller than a critical one, with the shape of the spectrum of elementary excitations.[11] For weakly interacting Bose gases and a microscopic weak defect, such that a perturbative linear response theory can be applied, the phonon-like dispersion of the Bogoliubov spectrum demands that the critical velocity for dissipationless flow coincides with the velocity of sound[9] However, for macroscopic defects, the critical velocity for the onset of drag is smaller than the speed of sound and most likely related to vortex nucleation[9].

In contrast, the relevance of the Landau criterion for out-of-equilibrium condensates is questionable, not least because the spectrum of excitations is now complex rather than real. For polariton fluids, one has to singularly assess the system properties in the three different pumping schemes available: i) non-resonant pumping; ii) parametric drive in the optical-parametric-oscillator (OPO) regime; iii) coherent drive in the pump-only configuration.

As far as the spectrum of quasi-particle excitations and the Landau criterion are concerned, the cases of non-resonantly pumped polaritons and parametrically driven polaritons in the OPO regime are similar: in both cases, there is a U(1) phase symmetry which is spontaneously broken above a pump power threshold. This leads to the appearance of a gapless (Goldstone) mode in the spectrum of excitations — gapless means that both real and imaginary part of the excitation energy go to zero at zero momentum. However, in both cases, the effects of pump and decay are such that the real part of the Goldstone mode energy is zero also in a finite interval at small momenta, i.e. the spectrum becomes diffusive.[9,12] This means that a strict application of the Landau criterion would lead to a zero critical velocity, where quasi-particles can be excited at any value of the fluid speed. Therefore, if one would define superfluid properties through a strict application of the Landau criterion, neither non-resonantly pumped polaritons nor parametrically driven polaritons in the OPO regime behave as superfluids. Nevertheless, in both cases there have been evidences for superfluid behaviour. For non-resonantly pumped polaritons, it has been recently shown[13] that, even though strictly speaking there cannot be superfluid behaviour, there are regimes close to equilibrium, where the drag force exerted on a small moving defect shows a sharp threshold at velocities close to the speed of sound. Otherwise, for shorter polariton lifetimes, the threshold-like behaviour of the drag force is replaced by a smooth crossover. Moreover, metastability of supercurrents in non-resonantly pumped microcavities has been theoretically demonstrated[12]. For polaritons in the OPO regime, superfluidity has instead been tested through frictionless flow of polariton bullets[14] through metastability of quan-
tum vortices and persistence of current.\cite{16,17}

The case of coherently driven polaritons in the pump-only configuration strongly differs from the two schemes previously described: Here, there is no phase freedom any longer, because the polariton phase locks to the one of the driving pump. As a consequence, the quasiparticle excitation spectrum is always gapped — i.e., even when the real part of the spectrum energy goes to zero at the pump momentum, the corresponding imaginary part is non zero. For coherently driven polaritons in the pump-only configuration, by making use of a linearised Bogoliubov-like theory, it has been predicted that the Landau criterion can instead be satisfied at either strong enough pump powers or small flow velocities.\cite{18,19} Consequently, experiments in this configuration have been analysed in terms of the same theoretical description which is valid for equilibrium superfluids.

In this work, we show that, despite the fact coherently driven polaritons in the pump-only configuration do satisfy the Landau criterion at large enough densities or small flow velocities, because of the polariton lifetime being finite, the fluid always experiences a residual drag force even in the limit of asymptotically large densities. We show that, only in the limit of perfect microcavities (i.e. infinitely long polariton lifetimes), the residual drag force at large enough densities goes to zero, recovering therefore the equilibrium limit. Otherwise, for finite polariton lifetimes close to the current experimental values, we find that, similarly to the case of incoherently driven polaritons, both the drag force exerted on the polariton fluid by a defect, as well as the height of Cherenkov radiation, and the percentage of particles scattered by the defect show a smooth crossover rather than a sharp threshold-like behaviour which is typical of superfluids which obey the Landau criterion.

The paper is organised as follows: in Sec.\textbf{ II} we introduce the model describing polaritons coherently driven in the pump-only configuration in presence of a defect potential. In Sec.\textbf{ III} we describe the methods we use for our analysis, in particular the numerical algorithm used to evaluate quantities such as the drag force, the height of Cherenkov radiation, and the percentage of particles scattered by the defect which characterise the crossover from a superfluid-like to a supersonic-like behaviour. In addition, in Sec.\textbf{ III A} we shortly introduce the linearised Bogoliubov-like theory of Refs.\cite{18,19} which will be used later in Sec.\textbf{ IV} in order to compare the results obtained with the non-perturbative method with the results obtained in the linear response approximation. Results are discussed in Sec.\textbf{ IV} while conclusions (together with a discussion of the experimental relevance of our findings) are drawn in Sec.\textbf{ V}.

\section*{II. MODEL}

We describe the dynamics of the resonantly-driven polariton system\cite{21} via a Gross-Pitaevskii equation for coupled cavity and exciton fields $\psi_{C,X}(r,t)$, generalised to include the effects of the resonant pumping and decay ($\hbar = 1$):

\begin{equation}
\begin{aligned}
\mathbf{i} \partial_t \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} = & \begin{pmatrix} 0 \\ -F_p \end{pmatrix} + \begin{pmatrix} -iR_X + g_X|\psi_X|^2 & 0 \\ 0 & -iR_C + V_d(r) \end{pmatrix} \begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix}.
\end{aligned}
\end{equation}

The single-particle polariton Hamiltonian $\hat{H}_0$ can be diagonalised in momentum space,

\begin{equation}
\hat{H}_0(k) = \begin{pmatrix} \omega_X(k) & \Omega_R/2 \\ \Omega_R/2 & \omega_C(k) \end{pmatrix},
\end{equation}

by rotating into the lower (LP) and upper polariton (UP) basis,

\begin{equation}
\begin{pmatrix} \psi_X \\ \psi_C \end{pmatrix} = \begin{pmatrix} \cos \theta_k & -\sin \theta_k \\ \sin \theta_k & \cos \theta_k \end{pmatrix} \begin{pmatrix} \psi_{LP} \\ \psi_{UP} \end{pmatrix},
\end{equation}

where $\tan 2\theta_k = \Omega_R/|\omega_X(k) - \omega_C(k)|$, giving the bare lower and upper polariton dispersions:

\begin{equation}
\omega_{LP,UP}(k) = \frac{\omega_X + \omega_C}{2} \pm \frac{\sqrt{2} \omega_C - \omega_X^2 + \Omega_R^2}{2}.
\end{equation}

At zero energy detuning between excitons and photons, $\omega_C(k) = \omega_X(k)$, lower and upper polaritons are an equally balanced mixture of exciton and photon, i.e. $\cos^2 \theta_k = \cos^2 \theta_k = 1/2$. In contrast, at large values of the energy detuning between photons and excitons, $\omega_C(k) - \omega_X(k) \gg \Omega_R$, i.e. $\theta_k \rightarrow 0$, the lower and upper polariton respectively coincide with the exciton and the photon.

In the following, we will neglect the exciton dispersion, $\omega_X(k) = \omega_X(0)$, and assume a quadratic dispersion for photons, $\omega_C(k) = \omega_C(0) + \sqrt{k^2 + \Omega_R^2}$, where the photon mass is $m_c = 2 \times 10^{-5} m_0$ and $m_0$ is the bare electron mass. Through the paper, we will consider the case of zero detuning at normal incidence, $\omega_X(0) = \omega_C(0)$, and fix the Rabi frequency to $\Omega_R = 4.4$ meV. The parameters $\kappa_X$ and $\kappa_C$ are respectively the excitonic and photonic de-
cence and decay rates. We will fix these parameters in order to give a polariton lifetime, \( \tau_{LP} = h / \kappa_{LP} \),
\[
\kappa_{LP}(k) = \kappa_X \cos^2 \theta_k + \kappa_C \sin^2 \theta_k ,
\]
close to the experimental values. In addition, we will consider the limit of perfect cavities \( \kappa_{LP} \rightarrow 0 \) in order to recover the equilibrium limit.

In Eq. (1), the exciton-exciton interaction strength \( g_X \) can be set to one by rescaling both fields \( \psi_{X,C} \) and pump strength \( F_p \) by \( \sqrt{\Omega R/(2 g_X)} \). The cavity field is driven by a continuous-wave pump,
\[
F_p(r, t) = F_{f, \sigma}(r) e^{i(k_p \cdot r - \omega_p t)} ,
\]
with a smoothen top-hat spatial profile with intensity \( f \) and full width at half maximum (FWHM) \( \sigma = 130 \mu m \). The pumping laser frequency \( \omega_p \) has been chosen 0.44 meV blue-detuned above the bare lower polariton dispersion at the pump momentum \( \omega_{LP}(k_p) \). The polariton flow current is determined by the pump momentum \( k_p \), which can be experimentally tuned by changing the pumping laser angle of incidence with respect to the growth direction \( \varphi_{k_p} \):
\[
k_p \omega_{LP}(k_p) \sin(\varphi_{k_p}) .
\]

Finally, in Eq. (1) the potential \( V_d(r) \) describes the defect acting on the photonic field, over which the polariton fluid scatters. Specifically, we consider:
\[
V_d(r) = V_d \theta(r - r_d) ,
\]
with \( r_d = 7 \mu m \) and \( V_d = 110 \text{ meV} \). Defects can be present naturally in the sample’s mirrors. Alternatively, defects can be artificially engineered by either growing mesas in one of the mirrors or by an additional laser.

### III. METHODS

We numerically solve Eq. (1) on a 2D grid (256 × 256) in a 150 \( \mu m \times 150 \mu m \) box using a 5th-order adaptive-step Runge-Kutta algorithm, and evaluate both exciton and photon wave-functions \( \psi_{X,C}(r, t) \) in the steady-state regime.

We characterise the crossover from a superfluid-like to a supersonic-like regime by evaluating three different quantities. Firstly, we consider the normalised drag force \( \kappa_C \) exerted on the flowing polaritons by the defect:
\[
F_d = \frac{1}{\int \langle |\psi_C(r)|^2 \rangle} \int \langle |\psi_C(r)|^2 \rangle \nabla V_d(r) .
\]

When shining the laser pump in the \( x \)-direction, \( k_p = (k_p, 0) \), the density profile will be symmetric under the transformation \( y \rightarrow -y \), \( \psi_{X,C}(x, y) = \psi_{X,C}(x, -y) \), implying that for defect potentials symmetric under \( y \rightarrow -y \) only the \( x \)-component of the drag force can be non-zero.

Moreover, for step-like defects such as \( \delta \), only the values of the field \( |\psi_C(r)|^2 \) at a distance \( r = r_d \) contribute to the integral:
\[
F_{d,x} = \int \langle |\psi_C(r)|^2 \rangle \phi \cos \phi |\psi_C(r_d, \phi)|^2 \langle \nabla |\psi_C(r_d, \phi)|^2 \rangle .
\]

Therefore the drag force measures the degree of asymmetry of the photon (or alternatively the exciton) density profiles going from a certain angle \( \phi \rightarrow \pi/2 \) ahead (with respect to the fluid flow direction) of the defect, to an angle \( \pi - \phi > \pi/2 \) behind the defect. The asymmetry is caused by the scattering of the fluid passing through the defect. Plots of the drag force are drawn in Fig. 1.

Second, we characterise the superfluid-like behaviour by the suppression of density modulations around the defect known as Cherenkov waves — see, e.g., Refs. 23–25 and references therein for atomic Bose-Einstein condensates and Refs. 24–27 for coherently driven polaritons in the pump-only configuration. Cherenkov radiation is generated in the supersonic regime, when the fluid is passing a defect at a velocity higher than the phase velocity of the fluid elementary excitations. As mentioned later, a
simple analysis of Cherenkov radiation can be carried on by making use of perturbative Bogoliubov-like analysis: Here, in agreement with the Landau criterion, in the supersonic regime, the kinetic energy of the fluid can be dissipated radiating Bogoliubov modes, giving rise to perturbations which propagate radially from the defect, with a characteristic unperturbed region inside a Cherenkov cone which is related to the singularity of the Bogoliubov mode dispersion evaluated in the reference frame of the moving fluid. In our numerical non-perturbative analysis, we determine the value of the highest crest of the dispersion profile \( F_{f,\sigma} (r) = f \). While the mean-field equations

\[
[\omega_X (0) - \omega_p - i \kappa_X + g_X |\psi_X (0)|^2] \psi_X (0) + \frac{\Omega_R}{2} \psi_C (0) = 0
\]

\[
[\omega_C (k_p) - \omega_p - i \kappa_C] \psi_C (0) + \frac{\Omega_R}{2} |\psi_X (0)|^2 = - f
\]

allow to determine the intensity of exciton and photon fields in absence of the external potential, fluctuations above mean-field need to be introduced in order to evaluate the stability of such a solutions as well as the linear response to a weak perturbing external potential.

The spectrum of the quasi-particle excitations can be found by introducing particle-like \( u_{X,C} \) and hole-like \( v_{X,C} \) excitations in both the exciton and photon fluctuation fields:

\[
\delta \psi_{X,C} (r, t) = \sum_k \left( e^{-i \omega t} e^{i k \cdot r} u_{X,C; k} + e^{i \omega t} e^{-i (k - 2 k_p) \cdot r} v_{X,C; k} \right),
\]

and by solving the eigenvalue equation:

\[
[(\omega + \omega_p) I - L] \begin{pmatrix} u_{X,k} & u_{C,k} \\ v_{X,k} & v_{C,k} \end{pmatrix}^T = 0,
\]

where \( L \) is a 4 \times 4 matrix given by:

\[
L = \begin{pmatrix}
\omega_X (0) + 2 g_X |\psi_X (0)|^2 - i \kappa_X & \Omega_R / 2 & \Omega_R / 2 \\
\Omega_R / 2 & \omega_C (k) - i \kappa_C & 0 \\
-g_X |\psi_X (0)|^2 & 0 & 2 \omega_p - \omega_X (0) - 2 g_X |\psi_X (0)|^2 - i \kappa_X \\
0 & 0 & -\Omega_R / 2 \\
2 g_X |\psi_X (0)|^2 & \Omega_R / 2 & 2 \omega_p - \omega_C (2 k_p - k) - i \kappa_C
\end{pmatrix}.
\]

Eq. (10) admits four complex eigenvalues for each \( k \) which we indicate as \( \omega + \omega_p = LP^\pm (k), UP^\pm (k) \). Note that the spectrum obtained this way is already evaluated in the polariton flow moving frame. This becomes evident in the limit where the pump momentum \( k_p \) is small enough so that the LP dispersion can be approximated as parabolic, and the UP dispersion can be neglected. In fact, here, one can show that the spectrum of excitation reduces to

\[
LP^\pm (k) - \omega_L \approx v_f \cdot (k - k_p) - i \kappa_{LP} \\
\pm \sqrt{(\varepsilon_k - \Delta)} \left( \varepsilon_k - \Delta + 2 g_{LP} |\psi_{LP} (0)|^2 \right),
\]

where

\[
v_f = \frac{k_p}{m_{LP} \kappa_{LP}}
\]

is the polariton fluid velocity at the pump momentum, \( \varepsilon_k = (k - k_p)^2 / (2 m_{LP}) \), \( m_{LP} = m_C / \sin^2 \theta_{kp} \) is the LP mass, \( \Delta = \omega_p - \omega_{LP} (k_p) - g_{LP} |\psi_{LP} (0)|^2 \) is the pump detuning renormalised by the interaction, and \( g_{LP} = g_X \cos \theta_{kp} \). For more details, we refer the reader to the original calculation of Ref.\(^{10}\). What we would like to stress here is how to generalise the Landau criterion to the complex spectrum of elementary excitations obtained from Eq. (10). In particular, in the linear approximation,
we expect that the perturbation introduced by the defect is able to excite Bogoliubov-like quasi-particle states with momentum $k$, when the condition
\[ \Re[\text{LP}^+(k)] - \omega_p < 0 \] (13)
is satisfied. Note that, in the limit where the approximation (11) is valid and at resonance, $\Delta = 0$, this recovers the Landau criterion in its original formulation for a conservative system, i.e. close to $k_p$, $\Re[\text{LP}^+(k)] - \omega_p \simeq (c_s \pm v_f)|k - k_p|$ and the critical fluid velocity coincides with the sound velocity given by the usual expression:
\[ c_s \equiv \sqrt{g_{\text{LP}} n_{\text{LP}}/m_{\text{LP}}}, \] (14)
where $n_{\text{LP}} = |\psi_{\text{LP}}(0)|^2$ is the mean-field polariton density. In order to be able to draw an analogy with the equilibrium limit, we will later present our results in terms of the ratio $c_s/v_f$.

IV. RESULTS

We now turn to the non-perturbative numerical analysis of the Gross-Pitaevskii equation (1) and the analysis of the behaviour of the superfluid properties of the resonantly driven polariton system in the pump-only regime when changing the laser pump strength. In particular, by fixing the pump momentum $k_p$ and energy $\omega_p$, we evaluate the dependence of the drag force (7), the height of the Cherenkov waves $h_\text{Ch}$, and the percentage of scattered particles $S\text{out}$, on the the steady-state average density of polaritons, $n_{\text{LP}} = \int d\mathbf{r} |\psi_{\text{LP}}(\mathbf{r})|^2/\Omega$, pumped into the cavity — here, $\Omega$ is the circle pumping area we are averaging the density over. Rather than as a function of $n_{\text{LP}}$, we present the results as a function of the ratio between sound and polariton fluid velocities, $c_s/v_f$, defined in Eqs. (12) and (14).

We plot in Fig. 1 the drag force exerted on the polariton fluid by the defect (6) in the $x$-direction, $F_{dx}$ (7), as a function of the ratio $c_s/v_f$ (14), for different values of the pump wavevector $k_p$. We find that the drag force decreases fast as a function of the polariton density, when $c_s/v_f < 1$, while, for finite values of the polariton lifetime, reaches a finite asymptotic value, a residual drag force, at large densities, when $c_s/v_f \gg 1$. While for conservative superfluid systems the drag force has a threshold-like behaviour and in particular, for perturbatively weak delta-like defects, is finite only for superfluid velocities above the speed of sound (6), we now observe a smooth crossover as a function of $c_s/v_f$. In addition, as

\[ \Re[\text{LP}^+(k)] - \omega_p < 0 \] (13)
is satisfied. Note that, in the limit where the approximation (11) is valid and at resonance, $\Delta = 0$, this recovers the Landau criterion in its original formulation for a conservative system, i.e. close to $k_p$, $\Re[\text{LP}^+(k)] - \omega_p \simeq (c_s \pm v_f)|k - k_p|$ and the critical fluid velocity coincides with the sound velocity given by the usual expression:
\[ c_s \equiv \sqrt{g_{\text{LP}} n_{\text{LP}}/m_{\text{LP}}}, \] (14)
where $n_{\text{LP}} = |\psi_{\text{LP}}(0)|^2$ is the mean-field polariton density. In order to be able to draw an analogy with the equilibrium limit, we will later present our results in terms of the ratio $c_s/v_f$.

IV. RESULTS

We now turn to the non-perturbative numerical analysis of the Gross-Pitaevskii equation (1) and the analysis of the behaviour of the superfluid properties of the resonantly driven polariton system in the pump-only regime when changing the laser pump strength. In particular, by fixing the pump momentum $k_p$ and energy $\omega_p$, we evaluate the dependence of the drag force (7), the height of the Cherenkov waves $h_\text{Ch}$, and the percentage of scattered particles $S\text{out}$, on the the steady-state average density of polaritons, $n_{\text{LP}} = \int d\mathbf{r} |\psi_{\text{LP}}(\mathbf{r})|^2/\Omega$, pumped into the cavity — here, $\Omega$ is the circle pumping area we are averaging the density over. Rather than as a function of $n_{\text{LP}}$, we present the results as a function of the ratio between sound and polariton fluid velocities, $c_s/v_f$, defined in Eqs. (12) and (14).
height and decay rates are finite.

ical reasons. A clear conclusion is drawn from Fig. 2: have been done with negative values of the residual drag force on k.

V.

k.

V.

k.

k.

κ.

κ.

κ.

κ.

κ.

k.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.

κ.
while eventually the Cherenkov waves disappear at large enough densities, the persistence of asymmetric perturbations around the defect, which we ascribe to finite lifetime effects, contributes to give a finite drag force. This is also apparent in Fig. 5, where we plot the height of the Cherenkov waves as a function of $c_s/v_f$ for the same system parameters as Fig. 4. Here, it is clear that the Cherenkov waves height is strongly suppressed for $c_s/v_f \simeq 1$, and go to zero for $c_s/v_f \gg 1$.

In addition, in Fig. 5 we plot the percentage of the particles scattered by the defect. Here, like for the drag force, we find a residual value of the percentage of scattered particles at asymptotically high polariton densities. The difference with the drag force is here that, even for perfect cavities when the drag goes to zero, the percentage of scattered particles keeps retaining a residual value (see panel (c) of Fig. 3).

In Fig. 6 we plot the photon density profiles $|\psi_C(r)|^2$ (left panels) for increasing pump power, obtained by solving the time dependent Gross-Pitaevskii equation (1), together with the spectrum of linear excitations obtained by solving the eigenvalue problem (10) (right panels). Left panels show Cherenkov waves evolving smoothly from a ‘closed’ to an ‘open’ shape till they disappear when the subsonic superfluid regime is reached. In particular, the angle formed between the waves and the propagation direction increases by increasing the polariton density. Note also that, while qualitatively the phenomenology of the Cherenkov waves seems to be well described by the linear approximation theory, the transition from the supersonic to the subsonic superfluid regime is not. In particular, we find a value range of the pump power (like the one shown in the third row of Fig. 6), where no scattering and therefore no Cherenkov waves are allowed in the linear approximation description, while the full numerical solution of the time dependent Gross-Pitaevskii equation (1) still allows scattering of polaritons and the associated Cherenkov waves.

V. CONCLUSIONS AND DISCUSSION

In this paper we have proposed three different ways to analyse the superfluid properties of coherently driven polaritons in the pump-only configuration in presence of a defect potential. We have evaluated the drag force when the fluid passes the defect, the height of Cherenkov waves, and the percentage of particles scattered by the defect. By making a comparison with the linearised Bogoliubov-Dozier theory introduced in Refs. 18, 19, we have found that, the disappearance of the Cherenkov waves, characterising the transition from a supersonic to a subsonic superfluid behaviour, is not well described by the linear theory. In addition, we have found that non-equilibrium effects which go beyond the linear approximation cause a finite residual drag force even at asymptotically large polariton densities, where the Landau criterion predicts that no elementary excitation can be emitted by the defect. Only in the limit of infinitely long polariton lifetimes, the residual drag force at large enough densities goes to zero, recovering the equilibrium limit. The drag force exerted on the polariton fluid by a defect, as well as the height of Cherenkov radiation, and the percentage of particles scattered by the defect show a smooth crossover rather than a sharp threshold-like behaviour which is typical of superfluids obeying the Landau criterion. We have characterised this crossover as a function of the fluid velocity, the polariton density and the polariton lifetime.

The three observables which we here evaluate theoretically, can in principle be measured in current state-of-the art experiments on semiconductor microcavities. For example, the defect can be carefully engineered by either patterning a metal grating on the microcavity top mirror or growing mesas in one of the mirrors. This allows having a predetermined shape and size of the defect, suitable for a direct comparison with our theoretical analysis. Alternatively, the defect can be switched on and off externally by an additional laser. The second scheme would
allow a direct comparison between the fluid motion in presence and absence of the defect. Both schemes are within the current experimental reach, and so with this work we intend to motivate further experimental investigations, which would lead to a better understanding of the novel non-equilibrium superfluid phenomena in microcavities.

Acknowledgments

We are grateful to I. Carusotto, J. Keeling, D. Sanvitto, and L. Viña for continuous stimulating discussions. This research has been supported by the Spanish MEC (MAT2008-01555, QOIT-CSD2006-00019) and CAM (S-2009/ESP-1503). F.M.M. acknowledges financial support from the programs Ramón y Cajal and INTENBIOMAT (ESF).

* Electronic address: emiliano.cancellieri@uam.es
† also at London Centre for Nanotechnology, UK

1 C. Weisbuch, M. Nishioka, A. Ishikawa, and Y. Arakawa, Phys. Rev. Lett. 69, 3314 (1992).
2 J. Keeling, F. M. Marchetti, M. H. Szymańska, and P. B. Littlewood, Semicond. Sci. Technol. 22, R1 (2006).
3 J. Keeling and N. G. Berloff, Nature 457, 273 (2009).
4 E. Wertz, L. Ferrier, D. Solnyshkov, R. Johne, D. Sanvitto, A. Lemaître, I. Sagnes, R. Grousson, A. V. Kavokin, P. Senellart, et al., arXiv:1004.4084.
5 L. P. Pitaevskii and S. Stringari, Bose-Einstein Condensation (Clarendon Press, Oxford, 2003).
6 G. E. Astrakharchik and L. P. Pitaevskii, Phys. Rev. A 70, 013608 (2004).
7 R. Onofrio, C. Raman, J. M. Vogels, J. R. Abo-Shaeer, A. P. Chikkatur, and W. Ketterle, Phys. Rev. Lett. 85, 2228 (2000).
8 J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeanbrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymańska, R. André, J. L. Staehli, et al., Nature 443 (2006).
9 P. G. Savvidis, J. J. Baumberg, R. M. Stevenson, M. S. Skolnick, D. M. Whittaker, and J. S. Roberts, Phys. Rev. Lett. 84, 1547 (2000).
10 M. H. Szymańska, J. Keeling, and P. B. Littlewood, Phys. Rev. Lett. 96, 230602 (2006).
11 M. Wouters and I. Carusotto, Phys. Rev. Lett. 99, 140402 (2007).
12 M. Wouters and I. Carusotto, Phys. Rev. A 76, 043807 (2007).
13 M. Wouters and I. Carusotto, arXiv:1001.0660.
14 M. Wouters and V. Savona, Phys. Rev. B 81, 054508 (2010).
15 A. Amo, D. Sanvitto, F. P. Laussy, D. Ballarini, E. del Valle, M. D. Martin, A. Lemaître, J. Bloch, D. N. Krizhanovskii, M. S. Skolnick, et al., Nature 457, 291 (2009).
16 D. Sanvitto, F. Marchetti, M. Szymańska, G. Tosi, M. Baudisch, F. Laussy, D. Krizhanovskii, M. Skolnick, L. Marrucci, A. Lemaître, et al., Nature Physics 6, 527 (2010).
17 F. M. Marchetti, M. H. Szymańska, C. Tejedor, and D. M. Whittaker, Phys. Rev. Lett. 105, 063902 (2010).
18 I. Carusotto and C. Ciuti, Phys. Rev. Lett. 93, 166401 (2004).
19 C. Ciuti and I. Carusotto, Physica Status Solidi B 242, 2224 (2005).
20 A. Amo, J. Lefrère, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. Houdré, E. Giacobino, and A. Bramati, Nat. Phys. 5, 805 (2009).
21 C. Ciuti, P. Schwendimann, and A. Quattropani, Semicond. Sci. Technol. 18, S279 (2003).
22 O. E. Däif1, A. Baas, T. Guillet, J.-P. Brantut, R. I. Kaitouni, J. L. Staehli, F. Morier-Genoud, , and B. Deveaud, Appl. Phys. Lett. 88, 061105 (2006).
23 A. Amo, S. Pigeon, C. Adrados, R. Houdré, E. Giacobino, C. Ciuti, and A. Bramati, arXiv:1003.0131.
24 I. Carusotto, S. X. Hu, L. A. Collins, and A. Smerzi, Phys. Rev. Lett. 97, 260403 (2006).
25 Y. G. Gladhush, G. A. El, A. Gammal, and A. M. Kamchatnov, Phys. Rev. A 75, 033619 (2007).
26 S. Iasenelli, C. Menotti, and A. Smerzi, J. Phys. B: At. Mol. Opt. Phys. 39, S135 (2006).