On the (circular) polarization-independence of microwave-induced resistance oscillations and zero resistance state

Shenshen Wang and Tai-Kai Ng

Department of Physics, Hong Kong University of Science and Technology,
Clear Water Bay Road, Kowloon, Hong Kong

(Dated: February 5, 2008)

Abstract

The immunity of microwave-induced magneto-resistance oscillations and corresponding zero resistance regions to the direction of (circular) polarization of microwave is studied in this paper. We propose that a spontaneous circular motion of the whole electron fluid would stabilize the system and minimize the polarization sensitivity of the oscillatory DC resistance. Results of a self-consistent calculation capture the qualitative features of the experimental observation.
The observation of “Zero-(DC)-Resistance State” (ZRS) in high mobility two-dimensional electron systems (2DES) under magnetic field and microwave (MW) radiation presented a surprise to the physics community \[1, 2\]. When the system, under crossed uniform magnetic field and small dc bias, is irradiated by microwave of sufficient intensity, the longitudinal (dissipative) resistance $R_{xx}$ develops strong oscillatory dependence on the magnetic field. At low temperature and high radiation power the minima of the oscillations evolve into ZRS.

Intense interests in the physics community have been aroused by this unusual nonequilibrium phenomenon. Theoretical efforts aimed at identifying the microscopic mechanisms accounting for the MW-induced resistance oscillations (MIRO) and ZRS. Early theories employed the picture of photon-excited transport assisted by short-range scattering\[3, 4, 5\], and later the scenario of MW-induced oscillations in the nonequilibrium electron distribution function was argued to be the leading cause of MIRO\[6\]. Other mechanisms based on different physical pictures were also proposed\[7, 8, 9, 10\]. As for ZRS, though there exist proposals that do not invoke negative resistance (NR) instability\[11\], the pattern formation model\[12\], which is based on NR, remains the most popular. On the experimental side, the outstanding issues are activated temperature dependence with large energy gaps\[13\], immunity of MIRO and ZRS to the polarity of circular polarization of MW\[14\], suppression of MIRO and ZRS by in-plane magnetic fields\[15\], and multi-photon processes in the ZRS formation\[16\]; most of these findings are not readily accommodated by existing theories.

Among other issues, the polarization immunity of DC resistance poses a particularly challenging test. Experiment by Smet et al.\[14\] established that MIRO are insensitive to the polarization state of the MW radiation. However, transmission data also shows that active cyclotron resonance absorption occurs only when the circular polarization matches the magnetic field orientation (denoted as CRA), although ZRS is observed under both MW polarities. This observation was inconsistent with the two most prevailing theories, the impurity and/or phonon assisted inter-Landau level (LL) transitions model\[3, 4, 5\] and the non-equilibrium distribution function scenario\[6\]. These theories predict oscillations with correct period and phase, however, with substantially different amplitudes for the different polarizations; the MW photoconductivity of the cyclotron resonance inactive (CRI) state is smaller by a factor $((\omega - \omega_\nu)^2 + \Gamma^2) / ((\omega + \omega_\nu)^2 + \Gamma^2)$, where $\Gamma^{-1}$ is a phenomenological lifetime. The factor reflects the huge difference in the (AC) Drude conductivity for opposite circular polarities.
Polarization immunity of the DC resistance posts a big challenge to the understanding of the ZRS. To produce the “same” MIRO and ZRS, there must be an additional mechanism that compensates for the discrepancy in the energy absorption rate between the CRA and CRI states. In this paper, we propose that a spontaneous circular motion of the whole electron fluid previously proposed by Ng and Dai [17] could stabilize the system and minimize the polarization dependence of the oscillatory photoconductivity. The spontaneous circular motion arises in the cyclotron resonance (CR) favorable orientation whenever the intensity of incident MW radiation exceeds a threshold value.

To see how this could happen, we examine the general transport equation for the center of mass (CM) coordinate $\vec{R}(t)$ of the electron liquid where the effect of impurity is included to the second order [17]:

$$m \ddot{\vec{R}}(t) = (-e) \left( \vec{E}(t) + \frac{1}{c} \dot{\vec{R}}(t) \times \vec{B} \right) + \alpha \nabla_{\vec{R}(t)} \int_{-\infty}^{t} dt' \chi \left( \vec{R}(t) - \vec{R}(t') ; t - t' \right),$$

(1)

where $\alpha = n_i |u|^2 / \bar{n}$, $n_i$ is the density of impurity, $|u|^2$ indicates the strength of the impurity potential, $\bar{n} = N/V$ is the carrier density, and $\chi$ is the (equilibrium) retard density-density response function without the MW term. The equation is derived in the CM frame where the MW field is eliminated and the electron liquid sees moving impurities following the path $\sim -\vec{R}(t)$. After impurity averaging, we obtain Eq. (1) [17]. We note that similar equation has also been proposed by Lei et al. [4]. The electric field relevant to this phenomenon is of form $\vec{E}(t) = \vec{E}_1 \cos(\bar{\omega} t) + \vec{E}_2 \sin(\bar{\omega} t) + \vec{E}_d$, where $\vec{E}_2 = \pm \hat{z} \times \vec{E}_1$ and $|\vec{E}_1| = |\vec{E}_2| = E_0$. The first two terms represent a circularly-polarized MW radiation with frequency $\bar{\omega}$ and $\vec{E}_d$ stands for a small DC bias. The plus/minus sign in the expression of $\vec{E}_2$ indicates CRA/CRI state. For small DC bias $\vec{R}(t)$ can be written as $\vec{R}(t) = \vec{R}_{AC}(t) + \vec{R}_{DC}(t)$, where $\vec{R}_{AC}(t)$ is the (dominant) part induced by MW field and $\vec{R}_{DC}(t) \sim \vec{v} t$ is a small DC correction.

Eq. (1) suggests that the linear DC resistance is determined completely by $\vec{R}_{AC}(t)$ since there is nothing else in the equation. As sketched in Fig.1 (left solid circles), $\vec{R}_{AC}(t)$ for opposite polarization states have very different amplitudes of motion in the Drude model, and to explain the polarization immunity, this big difference in $\vec{R}_{AC}(t)$ must be minimized. A plausible way for this to occur is to have a spontaneous circular motion in the CRA direction for both CRA/CRI states. If this spontaneous circular motion dominates over the Drude motion, the difference in $\vec{R}_{AC}(t)$ between the CRA and CRI states will be minimized.
In reality the composite trajectory of the spontaneous and Drude motion can be rather complicated (Fig.1 right-hand solid traces) and a self-consistent numerical calculation has to be performed to test this idea.

![Schematic trajectory of the AC motion of the electron fluid in the absence (left) and presence (right) of the spontaneous circular motion (dashed circle) for CRA (upper) and CRI (lower) states.](image)

FIG. 1: Schematic trajectory of the AC motion of the electron fluid in the absence (left) and presence (right) of the spontaneous circular motion (dashed circle) for CRA (upper) and CRI (lower) states.

To test our proposal, we study a trial solution of Eq. (1) with

\[
\vec{R}(t) = \vec{v} \cdot t + \vec{R}_f(t) + \vec{R}_s(t) = \vec{v} \cdot t + \vec{A}_1 \cos(\omega t + \delta) + \vec{B}_1 \sin(\omega t + \delta) + \vec{\alpha}_1 \cos(\omega_I t) + \vec{\beta}_1 \sin(\omega_I t),
\]

where \( \vec{R}_f(t) \) describes the CM motion directly coupled to the radiation field and \( \vec{R}_s(t) \) depicts the spontaneously generated oscillatory mode with frequency \( \omega_I \). We shall call them “fast” and “slow” modes in the following in view of their frequency difference which will be shown numerically later. \( \delta \) is the phase delay of the “fast mode” with respect to the AC driving. Notice that we keep only the base harmonic modes in \( \vec{R}_f(t) \) and \( \vec{R}_s(t) \). This can be justified for the “fast” mode since in the weak radiation limit the size of the circular orbit \( R_c \sim \sqrt{A_1^2 + B_1^2} \) will be much less than the magnetic length \( l_c = \sqrt{c/eB} \) (We set \( \hbar = 1 \) in the following) which is quite long for the B field (\( \sim 1T \)) relevant to this phenomenon [17]. Notice that such an argument does not apply to the “slow” mode whose amplitude is not governed by the radiation strength.

We next expand the impurity-scattering induced damping force \( \vec{F}_I(t) \equiv \alpha \nabla_{\vec{R}(t)} \int_{-\infty}^{t} dt' \chi \left( \vec{R}(t) - \vec{R}(t'); t - t' \right) \) in a Bessel-Fourier series and keep only the base-frequency oscillating terms. By putting the approximate \( \vec{R}(t) \) and \( \vec{F}_I(t) \) into Eq. (1) and
compare, we obtain three force-balance equations

\[ 0 = -e\tilde{E}_d - \frac{e}{c} \bar{v} \times \bar{B} + \bar{F}_v, \]

\[ m\ddot{R}_f(t) = -e\tilde{E}(t) - \frac{e}{c} \dot{R}_f(t) \times \bar{B} + \bar{F}^{(f)}_f(t), \]

\[ m\ddot{R}_s(t) = -\frac{e}{c} \dot{R}_s(t) \times \bar{B} + \bar{F}^{(s)}_f(t). \]

Here

\[ \bar{F}_v = -\alpha \int \int d^d q\bar{q} \sum_{m,n=-\infty}^\infty Im\chi(\bar{q}, \bar{q} \cdot \bar{v} + m\bar{\omega} + n\omega_i) J_m^2(z(\bar{q})) J_n^2(y(\bar{q})) \]

is the time-averaged damping force, where \( z(\bar{q})^2 = (\bar{q} \cdot \bar{A}_1)^2 + (\bar{q} \cdot \bar{B}_1)^2 \) and \( y(\bar{q})^2 = (\bar{q} \cdot \bar{A}_1)^2 + (\bar{q} \cdot \bar{B}_1)^2 \). For the “fast mode”, \( \bar{F}^{(f)}_f(t) \sim \bar{H}_1 \cos(\bar{\omega}t + \delta) + \bar{G}_1 \sin(\bar{\omega}t + \delta) \) where \( \bar{H}_1 \equiv \pi^{(1)} \cdot \bar{A}_1 - \pi^{(2)} \cdot \bar{B}_1 \) and \( \bar{G}_1 \equiv \pi^{(1)} \cdot \bar{B}_1 + \pi^{(2)} \cdot \bar{A}_1 \) whereas for the “slow mode”, \( \bar{F}^{(s)}_f(t) \sim \bar{h}_1 \cos(\omega_i t) + \bar{g}_1 \sin(\omega_i t) \), where \( \bar{h}_1 \equiv \pi^{(1)'} \cdot \bar{\alpha}_1 - \pi^{(2)'} \cdot \bar{\beta}_1 \) and \( \bar{g}_1 \equiv \pi^{(1)'} \cdot \bar{\beta}_1 + \pi^{(2)'} \cdot \bar{\alpha}_1 \).

The \( \pi \)'s are given by

\[ \pi^{(1)} = \frac{\alpha}{2} \int \int d^d q\bar{q}^2 \sum_{m,n=-\infty}^\infty Re\chi(\bar{q}, \bar{q} \cdot \bar{v} + m\bar{\omega} + n\omega_i) \left[ J_m(z(\bar{q})) J'_m(z(\bar{q})) \right] J_n^2(y(\bar{q})), \]

\[ \pi^{(2)} = \frac{\alpha}{2} \int \int d^d q\bar{q}^2 \sum_{m,n=-\infty}^\infty Im\chi(\bar{q}, \bar{q} \cdot \bar{v} + m\bar{\omega} + n\omega_i) m \left[ J_m(z(\bar{q})) \right]^2 J_n^2(y(\bar{q})), \]

and \( \pi^{(1)'} \), \( \pi^{(2)'} \) can be obtained from \( \pi^{(1)} \), \( \pi^{(2)} \) by simply interchanging \( z \leftrightarrow y \), \( m \leftrightarrow n \) and \( \bar{\omega} \leftrightarrow \omega_i \). We shall consider the limit \( \bar{v} \to 0 \) when calculating \( \pi \)'s in the following since we are interested in the linear-response DC current.

After some algebra we obtain the equation of motion for ”fast mode”

\[ \sqrt{[m\bar{\omega}^2(1 \mp \omega_c/\bar{\omega}) + \pi^{(1)}]^2 + [\pi^{(2)}]^2} \cdot A_1 = eE_0, \]

where \(-/+\) corresponds to CRA/CRI state. The equation is of Drude form representing an oscillation driven by AC electric field under impurity-induced friction \( \pi^{(2)} \) that depends on the amplitudes of both “fast” and “slow” modes \( (z(\bar{q}) \) and \( y(\bar{q}) \). A corresponding reactive correction \( \pi^{(1)} \) also appears which can be interpreted as a mass correction[17]. The phase delay \( \delta \) is given by \( \tan \delta = \pi^{(2)} / [m\bar{\omega}^2(1 \mp \omega_c/\bar{\omega}) + \pi^{(1)}] \). Notice the explicit polarization dependence of \( A_1 \) in Eq. (8).

Since there is no external driving force, the spontaneously-generated “slow” mode is determined by the self-sustainability requirement that the corresponding frictional force
vanishes, i.e.
\[ \pi^{(2)'} = 0. \] (9)

Putting it into Eq.(5) we obtain another equation for “slow mode”,
\[ -m\omega_i^2 (1 - \omega_c/\omega_i) \alpha_1 = \pi^{(1)'} \cdot \alpha_1. \] (10)

The two equations determine self-consistently the amplitude and frequency of the “slow mode”. Notice that the spontaneous circular motion is always in the CR-favorable direction, independent of the polarization of the MW radiation. Polarization dependence enters only indirectly through \( z(\vec{q}) \) which appears in both \( \pi^{(1)'} \) and \( \pi^{(2)'} \).

Eq.s (8)–(10) form a set of self-consistent equations determining the “fast” and “slow” mode amplitudes \( A_1, \alpha_1 \) and the “slow” mode frequency \( \omega_i \). These equations are solved numerically. The longitudinal DC resistance \( R_{xx} = -\lim_{\vec{v} \to 0} [v - 2\vec{F}_v \cdot \vec{v}] \) is computed afterward.

We have employed the density-density response function of non-interacting electron gas in constant magnetic field [18] in our calculation, with Landau levels (LLs) broadened phenomenologically into Lorentzians, i.e.
\[ \delta(\varepsilon - n\omega_c) \rightarrow \pi^{-1}\Gamma/(\varepsilon - n\omega_c)^2 + \Gamma^2). \]
Since experimentally \( \bar{\omega} \sim 200 \text{GHz} \) is fixed while sweeping the B field, we use \( \bar{\omega} \) as the basic unit and set \( T \sim \bar{\omega}, \Gamma(T) \sim 0.2\bar{\omega}, E_{F} \sim 10\bar{\omega} \) and keep 20 LLs in our calculation, consistent with the low field \( (B \sim 1T) \) and intermediate temperature \( (T \sim 1K) \) setting in experiment. We shall vary magnetic field \( B \) and use the frequency ratio \( \omega_n \equiv \bar{\omega}/\omega_c \) as abscissa in presenting our results. We also define normalized radiation intensity \( I_N \equiv (eE_0/m^*\bar{\omega}\tilde{l})^2 \) where \( \tilde{l} \equiv \sqrt{\omega_c/\bar{\omega}l_c} \) \( (m^* = 0.068m_0) \) is the effective mass of conduction band electrons in GaAs as well as normalized amplitudes \( c \equiv A_1/\tilde{l}, d \equiv \alpha_1/\tilde{l} \) and renormalized frequency \( x \equiv \omega_i/\omega_c \) in our calculation.

To further simplify calculation and analysis, we make another approximation of keeping only \( |n|, |m| \leq 1 \) terms (zero- and one-photon processes) in the impurity induced forces in our calculation; this is consistent with keeping only base harmonics in our trial trajectory. Correspondingly, we restrict our calculation to the frequency range \( \omega_n = \bar{\omega}/\omega_c \sim 1–2 \) since MIRO occur at the weak B field side \( \bar{\omega} > \omega_c \) and higher frequency range will be dominated by transitions between higher LLs \( (|n|, |m| > 1 \text{ processes}) \). The approximation of keeping only single photon process in the ”fast mode” can be justified in the weak radiation field limit. However the approximation can not be justified a priori for the spontaneous (slow) mode. We have estimated the effect of multi-photon processes associated with the “slow
mode” on our self-consistent equations and found that it does not affect terms associated with $\text{Im}\chi(\vec{q}, \vec{q} \cdot \vec{v} + n\omega_i + \bar{\omega})$ but would enhance terms associated with $\text{Im}\chi(\vec{q}, \vec{q} \cdot \vec{v} + n\omega_i)$ and $\text{Im}\chi(\vec{q}, \vec{q} \cdot \vec{v} + n\omega_i - \bar{\omega})$. To mimic these effects we introduce a correction factor $a > 1$ that multiplies the latter two terms in $\pi^{(2)'}$. The same factor is introduced to the counterparts in $\pi^{(1)'}$ and $\vec{F}_v(\vec{R}_{xx})$ to ensure consistency in our calculation. As long as $a$ does not deviate from $a = 1$ too much, the qualitative behavior of the solutions is not sensitive to $a$. We shall present our calculation results with $a = 1.2$ in the following.

Numerically we find that we may roughly divide the region $\omega_n \sim 1–2$ into negative resistance (NR) region that centers about $\omega_n \sim 1.2$, and positive resistance (PR) region that peaks at $\omega_n \sim 1.7$. For radiation intensity $(I_N)_{\text{th}} < I_N < (I_N)_{\text{multi} - \bar{\omega}}$, where $(I_N)_{\text{th}} \sim 0.001$ numerically (corresponding to $E_0 \sim 7\,\text{V/cm}$) is a threshold value above which “slow” mode appears and $(I_N)_{\text{multi} - \bar{\omega}} \sim 0.02$ ($E_0 \sim 32\,\text{V/cm}$) is a value where multi-photon processes of MW radiation become important, we find that the solutions for $c$, $d$ and $x$ fall within the ranges listed in Table 1.

| polarity | frequency range | $d$  | $c$  | $x$  |
|----------|----------------|------|------|------|
| <CRA>    | NR($\omega_n : 1.1–1.5$) | 0.1–0.2 | 0.2–0.4 | $\sim 0.3$ |
|          | PR($\omega_n : 1.5–1.9$) | 0.2–0.3 | 0.2–0.4 | $\sim 1.1$ |
| <CRI>    | ($\omega_n : 1.1–2.0$) | $\sim 0.1$ | 0.03–0.08 | $\sim 0.3$ |

**TABLE I:** The numerical range of the self-consistently determined $d$, $c$ and $x$ for CRA and CRI states in the regime $\omega_n \sim 1–2$.

Notice that the amplitude of “fast mode” ($c$) is much smaller in the CRI state when compared with CRA state as expected. However, the amplitudes $d$ of the “slow mode” in the two polarization states are comparable. The low-frequency “slow mode” ($\omega_i \sim 0.3\omega_c$) appears in the whole frequency range of CRI state and the NR region of CRA state. Unexpectedly, $d \neq 0$ solutions also exist at the PR region of CRA state with rather large amplitude and near-CR frequency.

The effect of the “slow mode” on the polarization-dependence of DC resistance and ZRS can be roughly understood as follows: when the incident radiation is sufficiently strong and NR instability shows up, a spontaneous circular motion is generated in the electron fluid.
in CR favorable direction. Mathematically, low-frequency fictitious photons represented by \( n \neq 0 \) processes are spontaneously generated and additional photon-assisted transport channels open up. As a result normal dissipation is enhanced and NR is cured. This mechanism dominates the whole frequency range of CRI state and the NR region of CRA state. The spontaneous circular motion also turns the original fast-rotating CRA orbit (left-hand solid circle of Fig.1) into the “lace” of a slowly-rotating (\( \omega_i \)) orbit (right-hand dashed circle). The strong mixing of the two orbits destroys the identity of well-defined Landau orbit and suppresses photon-assisted scattering in the CRA state, thus drawing the two states more close to each other. Mathematically, the suppression comes through the increase in the argument (\( y(\vec{q}) \)) of Bessel functions.

![Graph showing DC resistance versus frequency ratio](image)

**FIG. 2:** DC resistance versus frequency ratio for CRA and CRI states under two values of radiation intensity: (a) \( I_N = 0.005 \) (moderate intensity) and (b) \( I_N = 0.01 \) (high intensity). \((R)\) — with slow mode included in calculation, \((r)\) — slow mode is excluded.

We show in Fig. 2 the calculated DC resistance versus frequency ratio both with \((R)\) and without \((r)\) “slow mode” included in our model. Notice the different plotting scale for the two calculated resistances in the figure. We see that when the radiation is of moderate intensity \( I_N = 0.005(\varepsilon_0 \sim 16 \text{ V/cm}) \) (Fig.2a), NR region becomes pronounced in the absence of the “slow mode” for both CRA and CRI states and the disparity in the oscillatory amplitude of the two polarization states is large.

The entering of the “slow mode” suppresses the oscillatory amplitudes, especially for CRA state, which effectively reduces the amplitude discrepancy between the two polarizations;
meanwhile the NR instability is almost completely healed by the $\omega_I$-photon-excited inter-LL transitions in our highly simplified model. As the radiation strength further increases to $I_N = 0.01(E_0 \sim 22 \text{V/cm})$ (Fig.2b), oscillation becomes stronger for CRI state but is approaching saturation for CRA state in the absence of the “slow mode”. In this case, the suppression effect plus the photon-assisted transport processes associated with the “slow mode” make the oscillatory shape of the two polarization states even closer and lift the NR region to ZR for both polarities. We note also that at the B field region near CR where significant absorption takes place, no $d \neq 0$ solution is found in our calculation and the oscillation amplitude of CRA and CRI curves are considerably different, in agreement with experiment [14].

In conclusion, we propose and demonstrate numerically that a spontaneous slowly-rotating circulating current in a 2DES under magnetic field will be generated when the incident circularly-polarized MW radiation exceeds certain threshold intensity. The spontaneous mode can cure the NR problem and provides a plausible explanation for the observed ZRS and polarization immunity of the DC resistance. Our calculation is crude because of the many approximations we made and the results are only in semi-quantitative agreement with experiment. Nevertheless, we believe our theory has provided a promising starting point to understand the physics behind the ZRS phenomenon and associated polarization immunity.

We acknowledge support from HKUGC through grant CA05/06.SC04.

[1] R. G. Mani et al., Nature 420, 646 (2002).
[2] M. A. Zudov et al., Phys. Rev. Lett. 90, 046807 (2003).
[3] A. C. Durst et al., Phys. Rev. Lett. 91, 086803 (2003).
[4] X. L. Lei and S. Y. Liu, Phys. Rev. Lett. 91, 226805 (2003).
[5] M. G. Vavilov, I. L. Aleiner, Phys. Rev. B 69, 035303 (2004).
[6] I. A. Dmitriev et al., Phys. Rev. B 71, 115316 (2005).
[7] Junren Shi and X. C. Xie, Phys. Rev. Lett. 91, 086801 (2003).
[8] P. H. Rivera and P. A. Schulz, Phys. Rev. B 70, 075314 (2004).
[9] S. A. Mikhailov, Phys. Rev. B 70, 165311 (2004).
[10] C. Joas et al., Phys. Rev. B 70, 235302 (2004).
[11] D. H. Lee and J. M. Leinaas, Phys. Rev. B 69, 115336 (2004).
[12] A. V. Andreev et al., Phys. Rev. Lett. 91, 056803 (2003).
[13] R. L. Willett, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 93, 026804 (2004).
[14] J. H. Smet et al., Phys. Rev. Lett. 95, 116804 (2005).
[15] C. L. Yang, R. R. Du, L. N. Pfeiffer and K. W. West, Physica E 34, 232 (2006).
[16] M. A. Zudov et al., Phys. Rev. B 73, 041303(R) (2006); M. A. Zudov et al., Phys. Rev. Lett. 96, 236804 (2006).
[17] T. K. Ng and L. Dai, Phys. Rev. B 72, 235333 (2005).
[18] C. S. Ting, S. C. Ying and J. J. Quinn, Phys. Rev. B 16, 5394 (1977).