Search for optimal conditions for exploring double-parton scattering in four-jet production: $k_T$-factorization approach

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Abstract

In the present paper we discuss how to maximize the double-parton scattering (DPS) contribution in four-jet production by selecting kinematical cuts. Here both single-parton and double-parton scattering effects are calculated in the $k_T$-factorization approach, following our recent developments of relevant methods and tools. Several differential distributions are shown and discussed in the context of future searches for DPS effects, such as rapidity of jets, rapidity distance, and azimuthal correlations between jets. The dependences of the relative DPS amount is studied as function of those observables. The regions with an enhanced DPS contribution are identified. Future experimental explorations could extract more precise values of $\sigma_{eff}$ and its potential dependence on kinematical variables.

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I. INTRODUCTION

The relative amount of hard double-parton scattering (DPS) grows with energy. This is because the density of partons (sea quarks and antiquarks and gluons) grows with decreasing values of the longitudinal momentum fractions $x_1$ and $x_2$ of the first and second hadron momenta, respectively. The larger the energy, the smaller the values of the longitudinal momentum fractions necessary for hard scattering to take place are.

This is particularly true for processes induced by gluon-gluon fusion, like charm production for instance [1–3]. So far, most practical calculations of DPS contributions were performed within the so-called factorized ansatz often called pocket-formula. In this approach, the (differential) cross section for DPS is a product of the corresponding (differential) cross sections for single-parton scatterings (SPS). This is an approximation which is not well under control yet. A better formalism exists in principle, but predictions are not easy, as they require unknown input(s) related to the correlation of partons in configuration space, spin, etc [4]. The latter are explored to a far lesser extent than single-parton distributions. In this situation we may try to explore the problem by first collecting a sufficient amount of empirical facts to draw practical conclusions. As proposed by two of us some time ago, double $c\bar{c}$ production is a good place to explore DPS [1]. A new analysis shows that even there the situation may be not that simple [3]. Four-jet production seems a natural case to look for hard DPS effects [5–9].

A year ago two of us analyzed how to find optimal conditions for the observation and exploration of DPS effects in four-jet production [9]. In those analyses only the leading-order (LO) approach was applied both to SPS and DPS. It is expected that higher order effects are provided, already at tree level, by the $k_T$-factorization approach. At high energy the small-$x$ values region opens up which is a further motivation to apply this approach.

Very recently, we have performed for the first time a calculation of four-jet production for both single-parton and double-parton mechanism within $k_T$-factorization [10]. It was shown that the effective inclusion of higher-order effects leads to a substantial damping of the double-scattering contribution with respect to the SPS one, especially for symmetric (identical) cuts on the transverse momenta of all jets. In a leading-order approach to $2 \rightarrow 2$ processes, the transverse momenta of the final state jets must have the same size. Either of them passing the cut automatically implies that the second one is accepted too.
The situation is subtler in the next-to-leading order (NLO) collinear approach or in the tree-level $k_T$-factorization approach, which is the reason why, as will be recalled also in this paper, asymmetric cuts should do a better job in searches for DPS. For the purpose of the present analysis, we will take into account higher-order virtual effects via K-factors deduced from NLO calculation in collinear factorization.

As discussed in Ref. [9], jets with a large rapidity separation seem more promising than others in exploring DPS effects in four-jet production. In the following we shall concentrate on the study of this and other more optimal observables to pin down DPS in the $k_T$-factorization framework. Obviously, low cuts on the transverse momenta of jets favour DPS.

II. A SKETCH OF THE THEORETICAL FORMALISM

We will briefly recall the theoretical formalism we use to obtain our predictions. This has already been discussed extensively in Ref. [10], to which we refer for further details, including references, on both the Transverse Momentum Dependent parton distribution functions (TMDs) and the scattering amplitudes with off-shell initial state partons.

The high-energy-factorization (HEF) [11] formula for the calculation of the inclusive partonic four-jet cross section at the Born level reads

$$\sigma_B^{4-\text{jets}} = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d^2k_{T1}d^2k_{T2} \mathcal{F}_i(x_1, k_{T1}, \mu_F) \mathcal{F}_j(x_2, k_{T2}, \mu_F) \times \frac{1}{2^8} \prod_{l=1}^{4} \frac{d^3k_l}{(2\pi)^3 2E_l} \Theta_{4-\text{jet}} (2\pi)^4 \delta \left( x_1 P_1 + x_2 P_2 + \vec{k}_{T1} + \vec{k}_{T2} - \sum_{l=1}^{4} k_l \right) |\mathcal{M}(i^*, j^* \rightarrow 4\text{ part.})|^2.$$

(II.1)

Here $\mathcal{F}_i(x_k, k_{Tk}, \mu_F)$ is the TMD for a given parton ($k$ numbers the parton type), $x_k$ are the longitudinal momentum fractions, $\mu_F$ is a factorization scale, $\vec{k}_{Tk}$ are the parton’s transverse momenta, perpendicular to the collision axis. In the calculations we use the DLC2016v2 TMD set [10]. $\mathcal{M}(i^*, j^* \rightarrow 4\text{ part.})$ is the gauge invariant matrix element for $2 \rightarrow 4$ particle scattering with two initial off-shell legs. They are evaluated numerically

1 Available by request from krzysztof.kutak@ifj.edu.pl. The difference to the DLC2016 set is due to promoting running coupling to NLO accuracy. This change affected slightly the observables we study as compared to [10].
with AVHLIB \cite{12}, which also provides the other necessary Monte Carlo tools for the calculation. In the calculation, the scales are set to $\mu_F = \mu_R = \frac{\hat{H}_T}{2} = \frac{1}{2} \sum_{i=1}^{4} k_i^2$, and we use the $n_F = 5$ flavour scheme.

The so-called pocket-formula for DPS cross sections (for a four-parton final state) is given by

\[
\frac{d\sigma^{B}_{-\text{jet},\text{DPS}}}{d\zeta_1 d\zeta_2} = \frac{m}{\sigma_{\text{eff}}^{i_1 j_1 \rightarrow k_1 l_1}} \sum_{i_2 j_2 \rightarrow k_2 l_2} \frac{d\sigma^{B}(i_1 j_1 \rightarrow k_1 l_1)}{d\zeta_1} \frac{d\sigma^{B}(i_2 j_2 \rightarrow k_2 l_2)}{d\zeta_2}, \tag{II.2}
\]

where the $\sigma(ab \rightarrow cd)$ cross sections are obtained by restricting (II.1) to a single channel and the symmetry factor $m$ is 1/2 if the two hard scatterings are identical, to prevent double counting them. Finally, $\zeta_1$ and $\zeta_2$ stand for generic kinematical variables for the first and second scattering, respectively. It goes without saying that such a formula is a phenomenology-motivated approximation. The effective cross section $\sigma_{\text{eff}}$ can be loosely interpreted as a measure of the transverse correlation of the two partons inside the hadrons, whereas the possible longitudinal correlations are usually neglected. As for our previous paper \cite{10}, we use the value $\sigma_{\text{eff}} = 15$ mb, although this value may be questioned \cite{3} when all SPS mechanisms of double charm production are included. For recent developments in the formal theory of DPS in the collinear factorization framework, we refer the interested reader to \cite{13}.

### III. DETAILED STUDIES

#### A. Comparison to the CMS data

We start our analysis by confronting our approach with the existing data for relatively low cuts on jet transverse momenta. In this context, the CMS data \cite{14} appear to be more suitable than any other available experimental analysis of multi-jet production, as they are the only ones featuring sufficiently soft cuts on the transverse momenta for DPS to stand out. The cuts on transverse momenta are in this case $|p_T| > 50$ GeV for the two hardest jets and $|p_T| > 20$ GeV for the third and fourth ones; the rapidity region is defined

\footnote{As customary in the literature, we use the $\hat{H}_T$ notation to refer to the energies of the final state partons, not jets, despite this is obviously the same in the LO analysis.}
by $|\eta| < 4.7$ and the constraint on the jet radius parameter is $\Delta R > 0.5$. The situation is shown in Fig. 1 where we plot rapidity distributions for jets ordered by their transverse momenta (leading, 2nd, 3rd, 4th).

The $k_T$-factorization approach includes higher-order corrections through the resummation in the PDFs, neglecting fixed order loop effects. Therefore, we allow for an effective $K$-factor. From [15], the NLO $K$-factors are known to be smaller than unity for three- and four-jet production in the collinear case with the hard cuts on the transverse momenta chosen by the ATLAS collaboration in [16]. To describe the CMS data, we also need $K$-factors smaller than unity for the SPS contributions, as expected. Concerning the DPS contribution, instead, we do not include $K$ factors and the motivation is as follows. The theoretical $K$-factor for the 2-jet inclusive cross section in the collinear case and for the same cuts as above is known to be 1.18 or 1.25, depending on whether one includes or not non perturbative hadronization effects on top of the NLO calculation. But, contrary to the three- and four-jet cases, the NLO predictions for the inclusive cross section is further away from the measured value than the LO one [15]. This is due to a phase space effect which is specific to 2-jet production at fixed perturbative order and affects primarily the lowest $p_T$ bins, as first discussed in [17] and remarked, from another point of view, in [10] (for another recent discussion of such effect in two jet production in the context of DIS, see [18]). The resulting overestimation of the cross section is the reason why the theoretical 2-jet $K$-factors would lead to an overestimation of DPS.

We use $\sigma_{eff} = 15$ mb in the pocket-formula (II.2) to calculate the DPS contribution. This is a typical value known from the world systematics [19]. However, in the present study we consider larger energies and we explore a slightly different region, and such a value does not need to be universal. Larger values of $\sigma_{eff}$ were obtained recently, for example, for $D$ meson production when including $g \to D$ fragmentation [3].

In the following, we will propose a set of observables that we find particularly convenient to identify DPS effects in four-jet production, both for symmetric and asymmetric cuts.

Some comments are in order concerning Fig. 2 showing the plot of distribution in the variable which was proposed as a potential smoking gun for DPS in four-jet production [14] which is the azimuthal angle separation between the hardest and softest pair of jets. This variable is defined as a ratio between a differential and a total cross section, which
FIG. 1: Rapidity distribution of the leading, 2nd, 3rd and 4th jets. The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line.

makes it insensitive to possibly constant K-factors from higher order corrections; only a phase-space dependence of the K-factors could have an impact on this distribution. Setting this hypothesis aside for the moment, as one can see, the SPS contribution computed with our $k_T$-factorization approach describes the data pretty well within uncertainties, except for two of the highest bins. The situation in the highest bins does not seem significantly improved by the DPS contribution, which otherwise leads to overestimation of the data in the lower bins.

Considering the mentioned proviso on phase space dependence, our conclusion is that it is best to propose other variables which, on the ground of the theoretical calculation, seem potentially useful in discriminating more clearly between SPS and DPS in four-jet production.
B. Symmetric cuts

In this section we introduce our proposed optimal observables for the study of DPS. We start with a completely symmetric cuts scenario, $p_T > 20$ GeV for all the four leading jets, moving on to the asymmetric case in the following section. In both this and the following section the cuts on rapidity and jet radius parameter stay the same as for the CMS case. In Fig. 3 we show our predictions for the rapidity distributions. In contrast to the previous case (Fig. 1), which featured a harder cut on the two hardest jets, the shapes of the SPS and DPS rapidity distributions are rather similar. There is only a small relative enhancement of the DPS contribution for larger jet rapidities $|\eta|$. This is also different than the result obtained in the leading-order collinear approach [9].

Elaborating on the results of [20], it was shown in Ref. [9] in a collinear approach that two more observables are potentially useful to nail down DPS, namely the maximum rapidity distance

$$\Delta Y \equiv \max_{i,j \in \{1,2,3,4\}} \left| \eta_i - \eta_j \right|$$  \hspace{1cm} (III.1)

and the azimuthal correlations between the jets which are most remote in rapidity

$$\varphi_{ij} \equiv \left| \varphi_i - \varphi_j \right|, \quad \text{for} \quad |\eta_i - \eta_j| = \Delta Y. \hspace{1cm} (III.2)
FIG. 3: Rapidity distribution of leading and subleading jets for $\sqrt{s} = 7$ TeV (left column) and $\sqrt{s} = 13$ TeV (right column) for the symmetric cuts. The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

One can see in Fig. 4 that the relative DPS contribution gradually increases with $\Delta Y$ which, for the CMS collaboration, can be as large as 9.4. A potential failure of the SPS contribution to describe such a plot would therefore be a signal of the presence of a sizable DPS contribution.

Fig. 5 depicts azimuthal correlations between the jets most remote in rapidity. While at $\sqrt{s} = 7$ TeV the SPS contribution is always larger than the DPS contribution, at $\sqrt{s} = 13$ TeV the DPS contribution dominates over the SPS contribution for $\phi_{jj} < \pi/2$. The relative DPS contribution is shown again in the lower extra panels.

In Fig. 6 we show distribution in the $\Delta S$ variable already discussed for the CMS cuts (see Fig. 2). Here the relative contribution of the DPS is bigger than for the CMS experi-
\[ p p \rightarrow 4 \text{ jets} + X \]  
for \( \sqrt{s} = 7 \text{ TeV} \) (left) and \( \sqrt{s} = 13 \text{ TeV} \) (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

FIG. 4: Distribution in rapidity distance between the most remote jets for the symmetric cut with \( p_T > 20 \text{ GeV} \) for \( \sqrt{s} = 7 \text{ TeV} \) (left) and \( \sqrt{s} = 13 \text{ TeV} \) (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

\[ p p \rightarrow 4 \text{ jets} + X \]  
for \( \sqrt{s} = 7 \text{ TeV} \) (left) and \( \sqrt{s} = 13 \text{ TeV} \) (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

FIG. 5: Distribution in relative azimuthal angle between the most remote jets for the symmetric cut with \( p_T > 20 \text{ GeV} \) for \( \sqrt{s} = 7 \text{ TeV} \) (left) and \( \sqrt{s} = 13 \text{ TeV} \) (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

ment. For \( \sqrt{s} = 13 \text{ TeV} \) the DPS component wins with the SPS one for \( \Delta S < \frac{\pi}{2} \).

We also find that another variable, introduced in the high transverse momenta analysis of 4 jets production presented in Ref. [21], can be very interesting for the scrutiny of DPS effects. It is defined as follows

\[
\Delta \varphi_{3j}^{\text{min}} \equiv \min_{i,j,k \in \{1,2,3,4\}} \left( |\varphi_i - \varphi_j| + |\varphi_j - \varphi_k| \right) .
\]  \hspace{1cm} (III.3)
As three out of four azimuthal angles are always entering in (III.3), configurations featuring one jet recoiling against the other three are necessarily characterised by lower values of $\Delta \phi_{3j}$ with respect to the two-against-two topology; the minimum, in fact, will be obtained in the first case for $i, j, k$ denoting the three jets in the same half hemisphere, whereas such a situation is not possible for the second configuration. Obviously, the first case would be allowed only by SPS in a collinear tree-level framework, whereas the second should be enhanced by DPS. In the $k_T$-factorization approach, this situation is smeared out by the presence of transverse momenta of the initial state partons. For our TMDs, the corresponding distributions are shown in Fig. 7. In contrast to the naive expectations, similar shapes are obtained for DPS and SPS contributions.

C. Asymmetric cuts

So far we have addressed the problem of identifying DPS with low and completely symmetric cuts on the transverse momenta. Nevertheless, as already remarked above, it was also pointed out in [10], that the two-jet production mechanism which accounts for DPS is affected by a severe underestimation of the cross section when higher order effects are included. It is thus desirable, in order to get rid of this phase-space effect when looking for DPS, to employ asymmetric cuts on the jets transverse momenta, especially when considering $p_T$ distributions.
FIG. 7: Distribution in $\Delta \phi_{3j}^{\text{min}}$ angle for the symmetric cut with $p_T > 20$ GeV for $\sqrt{s} = 7$ TeV (left) and $\sqrt{s} = 13$ TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

In order for our analysis to be complete, we present here the same variables discussed in the previous section in such an asymmetric setup. In Figs. 8, 9, 10 and 12 we show our predictions for the following cuts: $p_T > 35$ GeV for the leading jet, and $p_T > 20$ GeV for the remaining jets. In our $k_T$-factorization framework and for these particular variables, the situation appears to be very similar to the situation of symmetric cuts.
FIG. 8: Rapidity distribution of leading and subleading jets for $\sqrt{s} = 7$ TeV (left column) and $\sqrt{s} = 13$ TeV (right column) for the asymmetric cuts. The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.
FIG. 9: Distribution in rapidity distance between the most remote jets for the asymmetric cut for $\sqrt{s} = 7$ TeV (left) and $\sqrt{s} = 13$ TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

FIG. 10: Distribution in relative azimuthal angle between the most remote jets for the asymmetric cut for $\sqrt{s} = 7$ TeV (left) and $\sqrt{s} = 13$ TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.
FIG. 11: Distribution in ΔS for the asymmetric cut for √s = 7 TeV (left) and √s = 13 TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.

FIG. 12: Distribution in Δφ_{3j}^{min} angle for the asymmetric cut for √s = 7 TeV (left) and √s = 13 TeV (right). The SPS contribution is shown by the dotted line while the DPS contribution by the dashed line. The relative contribution of DPS is shown in the extra lower panels.
IV. CONCLUSIONS

In the present study we have discussed how to explore DPS effects in four jet production. We have used results obtained in the $k_T$-factorization formalism, for both single-parton scattering and double-parton scattering, and we have discussed how to maximize their role.

Here we have shown that our approach is able to describe existing CMS data on jet rapidity distributions and we have presented our predictions for rapidity distributions, distribution in the distance between the most remote jets, azimuthal angle between the most remote jets and a new $\Delta \phi_{3j}^{min}$ variable.

We find that, for sufficiently small cuts on the transverse momenta, DPS effects are enhanced relative to the SPS contribution

- when rapidities of jets are large,
- for large rapidity distances between the most remote jets,
- for small azimuthal angles between the two jets most remote in rapidity,
- for large values of $\Delta \phi_{3j}^{min}$.

In general, the relative effects of DPS in the $k_T$-factorization approach are somewhat smaller than those found previously in the LO collinear approach.

Both the CMS and ATLAS collaborations could perform corresponding analyses. Future exploration of DPS effects could help in finding a new, more precise value for $\sigma_{eff}$ for the proton and/or finding a signal of a dependence of $\sigma_{eff}$ on kinematical variables. Such a dependence was predicted e.g. in a two-component model with perturbative-parton-splitting mechanism [22] but has not been clearly identified experimentally yet.

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