Solving Natural Distribution Ventilation Network with Homotopy Method

Wu Bing, Zhao Zihao, Ren Yuhui

Abstract

The solution of natural distribution ventilation network is the basis of ventilation network optimization. The essence of the solution is to calculate nonlinear equations with Newton method at present. The calculating precision or calculating time is influenced heavily by the initial value selection when Newton method is used. In order to solve the limitations of Newton method, a common algorithm to solve non-linear equation system–homotopy method–was discussed and the homotopy equation for solving natural distribution ventilation network was built. A ventilation network case was solved with free software–Octave–in this paper. Based on the discussion above, the relationships among initial value choice, numerical integral step and calculating precision were discussed when this method was used. The results show that homotopy method is an effective way for solving natural distribution ventilation network. In addition, solving ventilation network with Octave can effectively avoid the copyright problem which is often caused when program language such as Matlab and VB are used to solve the ventilation network.

Introduction

Currently, there are many software for underground mine ventilation network solution, but a majority of them are business software with protected intellectual property rights developed by scientific research institutes or professional software development company, such as Mvent, Avent, and so on. A fee would

* Corresponding author. Tel.: +0-086-0472-5951557; fax: +0-086-0472-5951557.
E-mail address: phyzzzh@imust.cn.
be charged for using these software and their internal solution modules are fixed so that users cannot adjust or delete according to their requirements. Therefore, it is necessary to find a free ventilation network solution program to achieve rapid development and rapid solution.

1. Introduction of Octave

Octave is a open source software used for numerical computing and mapping. In addition to general math, the software is particularly specialized in matrix operations, solution of simultaneous equations system, and calculation of matrix eigenvalues and eigenvectors. But for many practical engineering problems, data can be demonstrated with matrixes or vectors to become the solution of solution of matrixes of this kind. Octave is able to visualize data by several means. Meanwhile, Octave itself is also a programming language, so it has a strong expandability. When developed, Octave was initially an auxiliary program used for the course of chemistry for undergraduate study. The current Octave development has been led by Doctor J.W.Eation and published pursuant to GNU General Public License. Since Octave is basically compatible with Matlab commonly used in scientific research projects, its ease of use has been widely recognized.

2. Solution Principles of Homotopy Method

As we know, the solution of ventilation network is actually the solution of the energy balance equation of ventilation network loop (formula 1) and node air volume balance equation (formula 2), both in the form of matrix.

\[
F(Q_y) = CR_{\text{diag}} \begin{bmatrix} C^T Q_y \end{bmatrix}_{\text{diag}} C^T Q_y - CH_f - H_N = 0
\]

\[
Q = C^T Q_y
\]

Where:

- \( C \)——Independent loop matrix of ventilation network
- \( C^T \)——Transpose of \( C \)
- \( R_{\text{diag}} \)——Diagonal matrix of drag vector of each branch
- \( Q \)——air volume column vector of each branch
- \( Q_y \)——air volume column vector of each branch, \( y \) is a unknown variable.
- \( H_f \)——column vector of fan pressure
- \( H_N \)——column vector of natural wind pressure

Formula 1 is actually a multivariate quadratic equations system. Conventionally, normally the Newton iteration method and its variants are used for solution. However, with a very small radius of convergence, the method requires excellent initial value \( x_0 \). The homotopy method was developed in the seventies of the 20th century has a large-range convergence and can significantly expand the tolerance range of the initial value.

The idea of homotopy algorithm is to manually import a parameter \( t \) to construct a function family \( H(x,t) \) to make

\[
H(x,0) = F_0(x), H(x,1) = F(x)
\]

Suppose the solution of \( F_0(x) = 0 \) is known, then start with \( t=0 \) to solve
\[ H(x,t) \equiv 0 \quad \forall t \in [0,1] \quad \text{(3)} \]

For \( t \in [0,1] \), suppose the solution formula (3) is \( x(t) \). If \( x(t) \) can form a smooth curve, the start \( x(0) \) is the solution of \( F_0(x) = 0 \). Suppose it is a known figure, the end \( x(1) \) of curve \( x(t) \) is the solution of \( F(x) = 0 \) that we’ve tried to obtain. \( H(x,t) \) is called a homotopy, and its solution \( x(t) \) is called homotopy curve.

Determine the partial derivative of \( t \) on both sides of formula (3),

\[
\begin{aligned}
  x'(t) &= -\left[H_x(x(t),t)\right]^{-1} H_t(x,t), \quad t \in (0,1) \\
  x(0) &= x_0
\end{aligned}
\quad \text{(4)}
\]

Formula 4 is the finite solution of \( x(t) \). Very obviously, it is the initial value of ordinary differential equation, so its solution method will not be detailed here[4].

There are many ways to construct \( H(x,t) \). Here, we construct formula 1 as follows:

\[ H(Q_y,t) = t \left( Q_{f} + (1-t) F(Q_{r}) - F(Q_{y0}) \right) = F(Q_{y}) + (t-1)F(Q_{y0}) \quad \text{(5)} \]

Introduce formula 5 into formula 4, then

\[ Q_{y}'(t) = -\left[J \quad a \left(Q_{y}(t)\right) \right] F(Q_{y0}) \quad \text{(6)} \]

\( J \) in formula 6 is the Jacobian matrix. The Jacobian matrix for the energy balance equation of the ventilation network loop is

\[
\frac{\partial F}{\partial Q_{y}} = 2C \begin{bmatrix} a & C^T \end{bmatrix}_d \begin{bmatrix} i & C^T \end{bmatrix}_g \begin{bmatrix} Q_{i} \end{bmatrix}_d \begin{bmatrix} C & \frac{\partial H_f}{\partial Q_{y}} - \frac{\partial H_N}{\partial Q_{y}} \end{bmatrix}
\quad \text{(7)}
\]

Integrate formula 6 with the integral interval of \([0,t]\). After further conversion, \( Q_{y}(1) \), i.e. the solution of the energy balance equation of the ventilation network loop, can be determined.

3. Solving Ventilation Network in Octave with Homotopy Method

For brevity, this example only demonstrates the development of homotopy algorithm-based ventilation network solution tools with constant fan pressure and without consideration of natural air pressure.

The specific solution codes and descriptive text are as follows:

```octave
function tlout=tl_method(c,r,hf,x0,N)
% solve natural Distribution ventilation network with the homotopy method, the meanings of various input parameters in the function are as follows:
% c is independent loop matrix, r is drag of each branch, hf is air pressure of fan, x0 is the initial value of independent branches, N is the reciprocal of integral space distance. The larger the figure is, the integral space distance is smaller and the result is more accurate.
% the output of this function is the natural Distribution air volume of each branch.
```

\[ \]
\[ h = 1/N; x = x_0; \]
% \( f_q \) below is the matrix form of the loop air pressure balance equation.
% \( jq \) is the Jacobian determinant of the loop air pressure balance equation.
\[
f_{q} = @(q_y) c \cdot \text{diag}(r) \cdot \text{diag}(\text{abs}(c^\top q_y)) \cdot c^\top q_y - c \cdot h; \]
\[
q_j = @(q_y) 2 \cdot c \cdot \text{diag}(r) \cdot \text{diag}(\text{abs}(c^\top q_y)) \cdot c; \]
\[
f = f_{q}(x); \quad b = -h \cdot f; \]
% below is the integral calculation form of formula 6 derived by using the homotopy method
for \( i = 1:N \)
\[
a = q_{j}(x); \quad k_1 = \text{inv}(a) \cdot b; \]
\[
a = q_{j}(x+0.5 \cdot k_1); \quad k_2 = \text{inv}(a) \cdot b; \]
\[
a = q_{j}(x+0.5 \cdot k_2); \quad k_3 = \text{inv}(a) \cdot b; \]
\[
a = q_{j}(x+0.5 \cdot k_3); \quad k_4 = \text{inv}(a) \cdot b; \]
\[
x = x + (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)/6; \]
\end{verbatim}

Store the above codes to Work file folder under the installation directory of Octave, and name the file as \texttt{tl\_method.m}, then the call of this function can be realized in the Octave interactive environment.

4. Case of Solution

Take the ventilation network shown in Figure 1 as an example. The drag in this network:
\[
\begin{align*}
    r_1 &= 0.5 \text{ kg/m}^7, & r_2 &= 0.005 \text{ kg/m}^7, & r_3 &= 0.004 \text{ kg/m}^7, & r_4 &= 0.005 \text{ kg/m}^7, & r_5 &= 0.001 \text{ kg/m}^7, \\
    r_6 &= 0.02 \text{ kg/m}^7, & r_7 &= 0, \end{align*}
\]
respectively, fan pressure is 500pa.

Fig.1. Ventilation Network
Enter the following commands in the Octave interactive environment:

c=[1 -1 1 0 0 0 0; 0 1 1 0 1 0 0; 0 0 1 1 0 1 1]; % independent loop matrix of the ventilation network
r=[.5 .005 .004 .005 .01 .02 0]'; % drag column vector of branches in the ventilation network
hf=[0 0 0 0 0 500]'; % column vector of fan pressure in the ventilation network
x0=[10 30 60]'; % initial value of air volume of remaining branches, is an arbitrary value

\[ t_l\text{method}(c,r,hf,x0) \]

to solve the air volume of each branch in this ventilation network. In this case, the solution result is

\[ [7.5201, 126.1737, 106.9637, 233.1374, 133.6938, 99.4437, 233.1374] \]

5. Impact of Solution Efficiency and Initial Value Selection on Precision of Results

To measure the impact of different initial values on the precision of solution, the ventilation network shown in Figure 1 is also taken as an example. Assume the output with the initial value \( x0=[10,30,60] \) and iteration count of 1000 is the standard result, recorded as \( Q \). Take the air volume of corresponding remaining branches in \( Q \) as the standard initial value, recorded as \( X \). Randomly generate initial value \( xi \) on computer (between -1000 and 1000), set \( N=20 \), introduce and call the solution module of the homotopy method, the output is \( qi \), record the deviation of initial value \( \frac{\text{sum}((X-xi)^2)}{\text{sum}(X^2)} \) as the abscissa and deviation of results \( \frac{\text{sum}((Q-qi)^2)}{\text{sum}(Q^2)} \) as the ordinate, repeat the process for several times, the initial value deviation-result deviation image is as shown in Figure 2.

![Initial Value Deviation-Result Deviation Image](image-url)
It can be seen from Figure 2 that after 20 steps of integral calculation, for any initial value, the result deviation is smaller than the initial value deviation, indicating that the calculation method can effectively approach accurate results. Meanwhile, it also indicates that the larger the initial value deviation is, the result deviation will also be larger after the same integral steps.

Set the number of integral steps between 10 and 200, respectively, repeat the above process in order, take the mean of the corresponding result deviation under each integral step. Map the step count-average result deviation, as shown in Figure 3.

It can be seen from Figure 3 that with the increase of the number of integral steps, the average result deviation gradually approaches 0. The calculation results and field practices indicate that when the number of integral steps reaches 100, the average result deviation is 0.013411, which reaches the practical precision for engineering. Therefore, when using the homotopy method for solution, 100 integral steps can fully meet the requirement for precision.

![Fig. 3. Integral Step Count-Average Result Deviation](image-url)
6. Conclusions

(1) The homotopy method is one of the best methods to solve non-linear equation systems. Nonetheless, no example of its application to ventilation network solution has been reported. Through demonstration, the paper proves that the method can be properly applied to ventilation network solution, and with a remarkable tolerance range of initial value, can overcome the shortcoming of the Newton method in the selection of initial value.

(2) Octave is a free and open source software, which means that there will be great flexibility in using the software to solve ventilation network. In other words, the source codes developed by others or even the source codes of Octave can be utilized without worrying about infringing on copyright. Octave enjoys outstanding matrix computing capacity and the solution of ventilation network is in essence the solution of matrixes, therefore using Octave can well complete the task of ventilation network solution.

References

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