Analysis of the pseudoscalar partner of the $Y(4660)$ and related bound states

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Abstract

In this article, we study the pseudoscalar bound state $\eta_c'f_0(980)$ (irrespective of the hadro-charmonium and the molecular state) with the QCD sum rules. Considering the $SU(3)$ symmetry of the light flavor quarks and the heavy quark symmetry, we also study the bound states $\eta_c'\sigma(400-1200)$, $\eta_c'f_0(980)$ and $\eta_c''\sigma(400-1200)$, and make reasonable predictions for their masses.

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1 Introduction

In 2007, the Belle collaboration observed two resonant structures in the $\pi^+\pi^-$ invariant mass distribution in the cross section for the process $e^+e^-\rightarrow\pi^+\pi^-\psi'$ between threshold and $\sqrt{s}=5.5$ GeV using 673 fb$^{-1}$ of data on and off the $Y(4S)$ ($Y''$) resonance, one at $4361\pm9\pm9$ MeV with a width of $74\pm15\pm10$ MeV, and another at $4664\pm11\pm5$ MeV with a width of $48\pm15\pm3$ MeV (they are denoted as the $Y(4360)$ and $Y(4660)$ respectively) [1]. The structure $Y(4660)$ is neither observed in the initial state radiation (ISR) process $e^+e^-\rightarrow\gamma_{ISR}\pi^+\pi^-J/\psi$ [2], nor in the exclusive cross processes $e^+e^-\rightarrow DD,DD^*,DD^*\pi,J/\psi D^{(*)}D^{(*)}$ [3, 4, 5, 6, 7].

There have been several canonical charmonium interpretations for the $Y(4660)$, such as the $5^3S_1$ state [8], the $6^3S_1$ state [9], the $5^3S_1-4^3D_1$ mixing state [10], and some exotic interpretations, such as the radial excited state of the $\frac{1}{\sqrt{2}}(|\Lambda_c\Lambda_c\rangle+|\Sigma_c^0\Sigma_c^0\rangle)$ [11], the vector $cs\bar{s}\bar{s}$ tetraquark state [12], etc.

A critical information for understanding the structure of the charmonium-like states is wether or not the $\pi\pi$ comes from a resonance. There is some indication that only the $Y(4660)$ has a well defined intermediate state which is consistent with the scalar meson $f_0(980)$ in the $\pi\pi$ invariant mass spectra [13].

In Refs.[14, 15], Voloshin et al argue that the charmonium-like states $Y(4660)$, $Z(4430)$, $Y(4260), \cdots$ may be hadro-charmonia. The relatively compact charmonium states ($J/\psi$, $\psi'$ and $\chi_{cJ}$) can be bound inside light hadronic matter, in particular inside higher resonances made from light quarks and (or) gluons. The charmonium state in such binding retains its properties essentially, the bound system (hadro-charmonium, a special molecular state) decays into light mesons and the particular charmonium.

In Ref.[16], Guo et al assume that the $Y(4660)$ is a $\psi'f_0(980)$ bound state (molecular state), as the nominal threshold of the $\psi'-f_0(980)$ system is about $4666\pm10$ MeV [17], the $Y(4660)$ decays dominantly via the decay of the scalar meson $f_0(980)$, $Y(4660)\rightarrow\psi'f_0(980)\rightarrow\psi'\pi\pi,\psi'K\bar{K}$, the difficulties in the canonical charmonium interpretation is

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overcome. Considering the heavy quark spin symmetry, Guo et al predict an \( \eta_c f_0(980) \) bound state (\( Y(4616) \)) as the spin-doublet partner of the \( Y(4660) \) with a mass of \( 4616^{+9}_{-6} \) MeV and a width of \( 60 \pm 30 \) MeV through the prominent decay mode \( \eta_c \pi \pi \) [18].

In previous work [19], we studied the mass of the \( Y(4660) \) as a \( \psi' f_0(980) \) bound state (irrespective of the hadro-charmonium and the molecular state) using the QCD sum rules [20, 21]. In this article, we extend our previous work to study the \( \eta_c f_0(980) \) bound state \( Y(4616) \), furthermore, we take into account the \( SU(3) \) symmetry of the light flavor quarks and the heavy quark symmetry, study the related hidden charm and hidden bottom states. In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [20, 21].

The article is arranged as follows: we derive the QCD sum rules for the pseudoscalar charmonium-like state \( Y(4616) \) and the related bound states in section 2; in section 3, numerical results and discussions; section 4 is reserved for conclusion.

2 QCD sum rules for the \( Y(4616) \) and related bound states

In the following, we write down the two-point correlation functions \( \Pi(p) \) in the QCD sum rules,

\[
\Pi(p) = i \int d^4 xe^{ipx} \langle 0 | J/\eta(x) \eta(0)^\dagger | 0 \rangle,
\]

\[
J(x) = Q(x)i\gamma_5 Q(x)\bar{s}(x)s(x),
\]

\[
\eta(x) = \frac{i}{\sqrt{2}} \bar{Q}(x)i\gamma_5 Q(x) \left[ \bar{u}(x)u(x) + \bar{d}(x)d(x) \right],
\]

where the \( Q \) denotes the heavy quarks \( c \) and \( b \). We use the currents \( J(x) \) and \( \eta(x) \) \( (Q = c) \) to interpolate the bound states \( \eta_c f_0(980) \) (predicted in Ref. [18]) and \( \eta_c' \sigma(400 - 1200) \), respectively.

The hidden charm current \( \bar{c}(x)i\gamma_5 c(x) \) can interpolate the charmonia \( \eta_c, \eta_c', \eta_c'', \ldots \); and the hidden bottom current \( \bar{b}(x)i\gamma_5 b(x) \) can interpolate the bottomonia \( \eta_b, \eta_b', \eta_b'', \ldots \). We assume that the scalar mesons \( f_0(980) \) and \( \sigma(400 - 1200) \) are the conventional \(qq\) states, to be more precise, they have large \( q\bar{q} \) components, while in Refs. [16, 18] the scalar meson \( f_0(980) \) is taken as the \( KK \) molecular state. There are hot controversies about their nature, for example, the conventional \( q\bar{q} \) states (strongly affected by the nearby thresholds) [22], the tetraquark states, the molecular states [23, 24]. The currents \( J(x) \) and \( \eta(x) \) \( (Q = c) \) have non-vanishing couplings with the bound states \( \eta_c f_0(980), \eta_c' f_0(980), \eta_c'' f_0(980), \ldots \) and \( \eta_c \sigma(400 - 1200), \eta_c' \sigma(400 - 1200), \eta_c'' \sigma(400 - 1200), \ldots \), respectively. Considering the heavy quark symmetry, there maybe exist some hidden bottom bound states, for example, \( \eta_b f_0(980), \eta_b' f_0(980), \eta_b'' f_0(980), \eta_b'' f_0(980), \eta_b'' \sigma(400 - 1200), \eta_b'' \sigma(400 - 1200), \eta_b'' \sigma(400 - 1200), \ldots \), we study those possibilities with the currents \( J(x) \) and \( \eta(x) \) \( (Q = b) \), and make predictions for their masses.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators \( J(x) \) and \( \eta(x) \) into the correlation functions \( \Pi(p) \) to obtain the hadronic representation [20, 21]. After isolating the ground state contributions
from the pole terms of the $Y$, we get the following result,

$$\Pi(p) = \frac{\lambda_Y^2}{M_Y^2 - p^2} + \cdots,$$

(3)

where the pole residues (or couplings) $\lambda_Y$ are defined by

$$\lambda_Y = \langle 0| J/\eta(0)| Y(p) \rangle.$$

(4)

The contributions from the two-particle and many-particle reducible states are small enough to be neglected [25], for example,

$$\Pi_2(p) = i\lambda_{Y, f_0}^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{\left[q^2 - m^2_{\eta_c}\right] \left[(p - q)^2 - m^2_{f_0}\right]} + \cdots,$$

(5)

where the pole residue (or coupling) $\lambda_{Y, f_0}$ is defined by

$$\langle 0| J(0)| \eta_c f_0(p) \rangle = \lambda_{Y, f_0}.$$

(6)

The coupling $\lambda_{\eta_c f_0}$ can be written in terms of the $\eta_c$ meson decay constant $f_{\eta_c}$ and the coupling $\lambda_{f_0}$ of the scalar meson $f_0(980)$ with a tetraquark current. The coupling $\lambda_{f_0}$ should be very small as the $f_0(980)$ is a light flavor meson, and the two-particle reducible contributions can be neglected [25, 26]. Furthermore, the pseudoscalar charmonia $\eta_c$, $\eta_c'$, $\eta_c''$, $\cdots$ and the pseudoscalar bottomonia $\eta_b$, $\eta_b'$, $\eta_b''$, $\cdots$ also have Fock states with additional $q\bar{q}$ components beside the $QQ$ components. The currents $J(x)$ and $\eta(x)$ may have non-vanishing couplings with the pseudoscalar charmonia and pseudoscalar bottomonia, those couplings are supposed to be small, as the main Fock states of the pseudoscalar charmonia and pseudoscalar bottomonia are the $QQ$ components, and the pseudoscalar charmonia and pseudoscalar bottomonia have much smaller masses than the corresponding molecular states $Y$.

After performing the standard procedure of the QCD sum rules, we obtain two sum rules in the $c\bar{s}s$ and $b\bar{s}s$ channels respectively:

$$\lambda_Y^2 e^{-\frac{M_Y^2}{m^2}} = \int_{s_0}^{\infty} ds \rho(s) e^{-\frac{s}{m^2}},$$

(7)

$$\rho(s) = \rho_0(s) + \rho_{(\bar{s}s)}(s) + \rho_{(GG)}(s) \left(\frac{\alpha_s GG}{\pi}\right) + \rho_{(\bar{s}s)^2}(s) + \rho_{(GGG)}(s) \left(\frac{g^3 f_{abc} G^a G^b G^c}{s}\right).$$

(8)

The explicit expressions of the spectral densities $\rho_0(s)$, $\rho_{(\bar{s}s)}(s)$, $\rho_{(GG)}(s)$, $\rho_{(\bar{s}s)^2}(s)$, and $\rho_{(GGG)}(s)$ are presented in the appendix. The $s_0$ is the continuum threshold parameter and the $M^2$ is the Borel parameter. We can obtain two sum rules in the $c\bar{c} qq$ and $b\bar{b} qq$ channels with a simple replacement $m_q \rightarrow m_s$, $\langle \bar{s}s \rangle \rightarrow \langle s\bar{s} \rangle$ and $\langle \bar{q}g_s\sigma Gs \rangle \rightarrow \langle qg_s\sigma Gq \rangle$.

We carry out the operator product expansion (OPE) to the vacuum condensates adding up to dimension-10. In calculation, we take assumption of vacuum saturation for high dimension vacuum condensates, they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, factorization works well in the large $N_c$
In this article, we take into account the contributions from the quark condensates \( \langle s\bar{s} \rangle \), \( \langle s\bar{s} \rangle^2 \), mixed condensates \( \langle s\bar{g}_s\sigma Gs \rangle \), \( \langle s\bar{s}\rangle \langle s\bar{g}_s\sigma Gs \rangle \), \( \langle s\bar{g}_s\sigma Gs \rangle^2 \) and the gluon condensates \( \langle \frac{\alpha_s}{\pi}\rangle \), \( g_\text{\scriptsize{sea}}^3 g_\text{\scriptsize{sea}}^4 G^a G^b G^c \) (one can see the appendix for the explicit expressions). The contributions from the quark-gluon condensates \( \langle \frac{\alpha_s}{\pi}\rangle \langle s\bar{s} \rangle \), \( \langle \frac{\alpha_s}{\pi}\rangle \langle s\bar{g}_s\sigma Gs \rangle \), \( \langle g_\text{\scriptsize{sea}}^3 g_\text{\scriptsize{sea}}^4 G^a G^b G^c \rangle \) are suppressed by large denominators and would not play any significant roles. Comparing with the gluon condensate \( \langle \frac{\alpha_s}{\pi}\rangle \), the vacuum condensates \( g_\text{\scriptsize{sea}}^3 g_\text{\scriptsize{sea}}^4 G^a G^b G^c \rangle \) are of higher order in \( \alpha_s \), their contributions are greatly suppressed. In calculation, we observe that the contributions from the term \( \langle g_\text{\scriptsize{sea}}^3 g_\text{\scriptsize{sea}}^4 G^a G^b G^c \rangle \) are less than 0.2%, and can be neglected safely.

Differentiate the Eq.(7) with respect to \( \frac{1}{\omega^2} \), then eliminate the pole residue \( \lambda_Y \), we can obtain the sum rule for the mass of the bound state \( Y \),

\[
M^2_Y = \frac{\int_{s_0}^{s_f} ds \frac{d}{ds} \left( \frac{d}{d(M^2)} \rho(s) \right)}{\int_{s_0}^{s_f} ds \rho(s)} e^{-\frac{s}{\sqrt{\omega^2}}},
\]

(9)

3 Numerical results and discussions

The input parameters are taken to be the standard values \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3 \), \( \langle s\bar{s} \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle \), \( \langle sg_s\sigma Gs \rangle = m_0^2 \langle \bar{q}q \rangle \), \( \langle s\bar{g}_s\sigma Gs \rangle = m_0^3 \langle s\bar{s} \rangle \), \( m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2 \), \( \langle \frac{\alpha_s}{\pi}\rangle = (0.33 \text{ GeV})^4 \), \( g_\text{\scriptsize{sea}}^3 g_\text{\scriptsize{sea}}^4 G^a G^b G^c \rangle = 0.045 \text{ GeV}^6 \), \( m_u = m_d \approx 0 \), \( m_s = (0.14 \pm 0.01) \text{ GeV} \), \( m_c = (1.35 \pm 0.10) \text{ GeV} \) and \( m_b = (4.8 \pm 0.1) \text{ GeV} \) at the energy scale \( \mu = 1 \text{ GeV} \) [20, 21, 27].

In the conventional QCD sum rules [20, 21], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter \( M^2 \) and threshold parameter \( s_0 \). We impose the two criteria on the pseudoscalar charmonium-like states \( Y \) to choose the Borel parameter \( M^2 \) and threshold parameter \( s_0 \).

We take the threshold parameter of the pseudoscalar bound state \( \eta_c^0 f_0(980) \) as \( s_0^{0_{ss}} = (4.62 + 0.5)^2 \text{ GeV}^2 \approx 26 \text{ GeV}^2 \) tentatively to take into account possible contribution from the ground state, where the energy gap between the ground state and the first radial excited state is chosen to be 0.5 GeV. Taking into account the \( SU(3) \) symmetry of the light flavor quarks, we expect the threshold parameter \( s_0^{0_{\bar{q}q}} \) (for the bound state \( \eta_c^0 f_0(400 - 1200) \)) is slightly smaller than the \( s_0^{0_{ss}} \). Furthermore, we take into account the mass difference between the \( c \) and \( b \) quarks, the threshold parameters in the hidden bottom channels are tentatively taken as \( s_0^{0_{\bar{q}q}} = 142 \text{ GeV}^2 \) and \( s_0^{0_{ss}} = 144 \text{ GeV}^2 \). In this article, we use those values as a guide to determine the threshold parameters \( s_0 \) with the QCD sum rules.

The contributions from the high dimension vacuum condensates in the operator product expansion are shown in Figs.1-2, where (and thereafter) we use the \( \langle \bar{q}q \rangle \) to denote the quark condensates \( \langle \bar{q}q \rangle \), \( \langle s\bar{s} \rangle \) and the \( \langle \bar{g}_s\sigma Gq \rangle \) to denote the mixed condensates \( \langle \bar{q}g_s\sigma Gq \rangle \), \( \langle s\bar{g}_s\sigma Gs \rangle \). From the figures, we can see that the contributions from the high dimension condensates change quickly with variation of the Borel parameter at the values \( M^2 \leq 2.7 \text{ GeV}^2 \) and \( M^2 \leq 7.6 \text{ GeV}^2 \) in the hidden charm and hidden bottom channels respectively, such an unstable behavior cannot lead to stable sum rules, our numerical results confirm this conjecture, see Fig.4.

At the values \( M^2 \geq 2.7 \text{ GeV}^2 \) and \( s_0 \geq 26 \text{ GeV}^2 \), the contributions from the \( \langle \bar{q}q \rangle^2 + \langle \bar{q}q \rangle \langle \bar{g}_s\sigma Gq \rangle \) term are less than 19.5% in the \( c\bar{c}s\bar{s} \) channel, the corresponding contributions
Furthermore, we take uniform Borel windows and smear the dependence on the threshold in the hidden charm and hidden bottom channels respectively; they are small enough. In the hidden charm and hidden bottom channels, respectively. In those regions, the pole contributions are less than 7% in all the hidden bottom channels, we expect the operator product expansion is convergent in the hidden charm channels. At the values $M^2 \geq 7.6$ GeV$^2$ and $s_0 \geq 144$ GeV$^2$, the contributions from the $(\bar{q}q)^2 + (\bar{q}q)(\bar{q}g, Gq)$ term are less than 7% in the $bbs\bar{s}$ channel, the corresponding contributions are less than 18% in the $b\bar{b}qq$ channel; the contributions from the vacuum condensate of the highest dimension $(\bar{q}g, Gq)^2$ are less than (or equal) 6% in all the hidden bottom channels, we expect the operator product expansion is convergent in the hidden bottom channels.

The contributions from the gluon condensate $(\frac{\alpha_sGG}{\pi})$ are less than (or equal) 37% (26%) in the $c\bar{c}s\bar{s}$ ($c\bar{c}q\bar{q}$) channel at the values $M^2 \geq 2.7$ GeV$^2$ and $s_0 \geq 26$ GeV$^2$; while the contributions are less than 19.5% (16.5%) in the $b\bar{b}s\bar{s}$ ($b\bar{b}q\bar{q}$) channel at the values $M^2 \geq 7.6$ GeV$^2$ and $s_0 \geq 144$ GeV$^2$. The contributions from the high dimension condensates $(\frac{\alpha_sGG}{\pi}) \left[ (\bar{q}q) + (\bar{q}g, Gq) + (\bar{q}q)^2 \right]$ are small enough and neglected safely.

In the QCD sum rules for the tetraquark states (irrespective of the molecule type and the diquark-antidiquark type), the contributions from the gluon condensate are suppressed by large denominators and would not play any significant roles for the light tetraquark states [28, 29], the heavy tetraquark state [30] and the heavy molecular state [31]. In the present case, the contributions from the gluon condensate $(\frac{\alpha_sGG}{\pi})$ are rather large, just like in the sum rules for the $Y(4660)$ [19]. If we take a simple replacement $\bar{s}(x)s(x) \rightarrow \langle \bar{s}s \rangle$ and $[\bar{u}(x)u(x) + \bar{d}(x)d(x)] \rightarrow 2\langle \bar{q}q \rangle$ in the interpolating currents $J(x)$ and $\eta(x)$, we can obtain the standard pseudoscalar heavy quark current $Q(x)i\gamma_5Q(x)$, where the gluon condensate $(\frac{\alpha_sGG}{\pi})$ plays an important role in the QCD sum rules [20].

In this article, we take the uniform Borel parameter $M^2_{\text{min}}$, i.e. $M^2_{\text{min}} \geq 2.7$ GeV$^2$ and $M^2_{\text{min}} \geq 7.6$ GeV$^2$ in the hidden charm and hidden bottom channels, respectively.

In Fig.3, we show the contributions from the pole terms with variation of the Borel parameters $M^2$ and the threshold parameters $s_0$. If the pole dominance criterion is satisfied, the threshold parameter $s_0$ increases with the Borel parameter $M^2$ monotonously. From Fig.3-A, we can see that the pole dominance criterion cannot be satisfied at the values $s_0 \leq 25$ GeV$^2$ and $M^2 \geq 2.7$ GeV$^2$ in the $c\bar{c}s\bar{s}$ channel, the threshold parameter $s_0$ has to be pushed to larger value.

The pole contributions are larger than 48% at the values $M^2 \leq 3.1$ GeV$^2$ and $s_0 \geq 25$ GeV$^2$, 26 GeV$^2$ in the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$ channels respectively; and larger than 50% at the values $M^2 \leq 8.2$ GeV$^2$, $s_0 \geq 142$ GeV$^2$, 144 GeV$^2$ in the $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels respectively. Again we take the uniform Borel parameter $M^2_{\text{max}}$, i.e. $M^2_{\text{max}} \leq 3.1$ GeV$^2$ and $M^2_{\text{max}} \leq 8.2$ GeV$^2$ in the hidden charm and hidden bottom channels, respectively.

In this article, the threshold parameters are taken as $s_0 = (26 \pm 1)$ GeV$^2$, $(27 \pm 1)$ GeV$^2$, $(144 \pm 2)$ GeV$^2$ and $(146 \pm 2)$ GeV$^2$ in the $c\bar{c}q\bar{q}$, $c\bar{c}s\bar{s}$, $b\bar{b}q\bar{q}$ and $b\bar{b}s\bar{s}$ channels, respectively; the Borel parameters are taken as $M^2 = (2.7 \ldots 3.1)$ GeV$^2$ and $(7.6 \ldots 8.2)$ GeV$^2$ in the hidden charm and hidden bottom channels, respectively. In those regions, the pole contributions are about $(48 \ldots 72)$%, $(49 \ldots 72)$%, $(50 \ldots 66)$% and $(51 \ldots 66)$% in the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively; the two criteria of the QCD sum rules are fully satisfied [20, 21].

The Borel windows $M^2_{\text{max}} - M^2_{\text{min}}$ change with variations of the threshold parameters $s_0$, see Fig.3. In this article, the Borel windows are taken as 0.4 GeV$^2$ and 0.6 GeV$^2$ in the hidden charm and hidden bottom channels respectively, they are small enough. Furthermore, we take uniform Borel windows and smear the dependence on the threshold
parameters \( s_0 \). If we take larger threshold parameters, the Borel windows are larger and the resulting masses are larger, see Fig.4. In this article, we intend to calculate the possibly lowest masses which are supposed to be the ground state masses by imposing the two criteria of the QCD sum rules.

In Fig.4, we plot the bound state masses \( M_Y \) with variation of the Borel parameters and the threshold parameters. The hidden charm current \( \bar{c}(x)i\gamma_5c(x) \) can interpolate the charmonia \( \eta_c, \eta'_c, \eta''_c, \ldots \); and the hidden bottom current \( \bar{b}(x)i\gamma_5b(x) \) can interpolate the bottomonia \( \eta_b, \eta'_b, \eta''_b, \ldots \) \cite{17}. The currents \( J(x) \) have non-vanishing couplings with the bound states \( \eta_c f_0(980), \eta'_c f_0(980), \eta''_c f_0(980), \ldots \) and \( \eta_b f_0(980), \eta'_b f_0(980), \eta''_b f_0(980), \eta'''_b f_0(980), \ldots \), respectively. The mass of the \( \eta_b \) listed in the Particle Data Group is \( M_{\eta_b} = 9388.9_{-2.3}^{+3.1} \pm 2.7 \) MeV, while the \( \eta'_b, \eta''_b, \eta'''_b, \ldots \) are not observed yet \cite{17}. In the constituent quark models, the mass splitting between the spin-singlet and spin-triplet are proportional to \( \frac{\sigma_c - \sigma_b}{M_c M_b} \), in the heavy quark limit, the \( \eta_b \) and \( \Upsilon \) degenerate. The constituent quark mass \( M_b \) is large enough, \( M_T = (9460.30 \pm 0.26) \) MeV, the energy gap between the \( \eta_b \) and \( \Upsilon \) is about 71.4 MeV, the energy gaps between the radial excited states are even smaller. In this article, we assume the masses of the \( \eta_b, \eta'_b, \eta''_b, \ldots \) are slightly smaller than ones of the \( \Upsilon, \Upsilon', \Upsilon'', \ldots \) respectively.

From Figs.3-A,3-C,4-A,4-C, we can see that the QCD sum rules support existence of the \( \eta'_c f_0(980) \) and \( \eta''_c f_0(980) \) bound states, the nominal thresholds of the \( \eta_c - f_0(980) \) and \( \Upsilon'' - f_0(980) \) systems are too low, and we cannot reproduce the \( \eta_c f_0(980) \) and \( \eta''_c f_0(980) \) bound states. Our predictions for the masses the \( Y(4660) \) \cite{19} and \( Y(4616) \) support the conjecture of Voloshin et al, i.e. a formation of hadro-charmonium is favored for higher charmonium resonances \( \psi' \) and \( \chi_{cJ} \) as compared to the lowest states \( J/\psi \) and \( \eta_c \) \cite{14}.

In this article, we intend to prove that the \( \eta'_c f_0(980) \) bound state can be reproduced by the QCD sum rules, the pseudoscalar charmonium-like state \( Y(4616) \) predicted in Ref.\cite{18} maybe exist.

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and pole residues of the pseudoscalar bound states \( Y \), which are shown in Figs.5-6 and Tables 1-2. In this article, we calculate the uncertainties \( \delta \) with the formula

\[
\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 |x_i = \bar{x}_i (x_i - \bar{x}_i)^2,}
\]

(10)

where the \( f \) denote the hadron mass \( M_Y \) and the pole residue \( \lambda_Y \), the \( x_i \) denote the input QCD parameters \( m_c, m_b, \langle q\bar{q} \rangle, \langle s\bar{s} \rangle, \ldots \), and the threshold parameter \( s_0 \) and Borel parameter \( M^2 \). As the partial derivatives \( \frac{\partial f}{\partial x_i} \) are difficult to carry out analytically, we take the approximation \( \left( \frac{\partial f}{\partial x_i} \right)^2 (x_i - \bar{x}_i)^2 \approx [f(\bar{x}_i + \Delta x_i) - f(\bar{x}_i)]^2 \) in the numerical calculations.

In table 1, we also present the nominal thresholds of the \( \eta'_c - f_0(980), \eta''_c - \sigma(400 - 1200), \Upsilon'''' - f_0(980) \) and \( \Upsilon'''' - \sigma(400 - 1200) \) systems. From the table, we can see that there maybe exist a bound state \( \eta''_c f_0(980) \) as the partner of the \( Y(4600) \). The predicted mass of the bound state \( \eta''_c \sigma(400 - 1200) \) is about \( 4.56 \pm 0.21 \) GeV, while the nominal threshold of the \( \eta''_c - \sigma(400 - 1200) \) system is about \( 4.037 - 4.837 \) GeV. There maybe exist such a bound state. The bound states \( \eta''_c f_0(980) \) and \( \eta''_c \sigma(400 - 1200) \) can be produced in the exclusive decays of the \( B \) meson through \( b \rightarrow cc\bar{s}, c\bar{c}q \) at the quark level.

In the \( b\bar{b}c\bar{s} \) channel, the numerical result \( M_Y = 11.42 \pm 0.21 \) GeV indicates that there
The LHC will be the world’s most copious source of proton-proton collisions at bound states. We cannot draw decisive conclusion with the QCD sum rules alone. Considering the $SU(3)$ symmetry of the light flavor quarks and the heavy quark symmetry, we can obtain the conclusion tentatively that there maybe exist the bound states $\eta_c \sigma(400 - 1200)$ and $\eta_c'' \sigma(400 - 1200)$ which lie in the regions $(4.037 - 4.837)\text{GeV}$ and $(10.979 - 11.779)\text{GeV}$, respectively. As the energy gaps between the $\Upsilon$'s are rather small and the scalar meson $\sigma(400 - 1200)$ is broad enough, there maybe exist the $\eta_b \sigma(400 - 1200)$, $\eta_c'' \sigma(400 - 1200)$ and $\eta_c''' \sigma(400 - 1200)$ bound states. We cannot draw decisive conclusion with the QCD sum rules alone.

The LHCb is a dedicated $b$ and $c$-physics precision experiment at the LHC (large hadron collider). The LHC will be the world’s most copious source of the $b$ hadrons, and a complete spectrum of the $b$ hadrons will be available through gluon fusion. In proton-proton collisions at $\sqrt{s} = 14\text{TeV}$, the $b \bar{b}$ cross section is expected to be $\sim 500\mu\text{b}$ producing $10^{12}$ $b \bar{b}$ pairs in a standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ [52]. The pseudoscalar bound states $\eta_c''' f_0(980)$ and $\eta_c''' \sigma(400 - 1200)$ predicted in the present work may be observed at the LHCb, if they exist indeed. We can search for those bound states in the $\eta_b \pi \pi$, $\eta_c' \pi \pi$, $\eta_c'' \pi \pi$, $\eta_c''' \pi \pi$, $\eta_b K \bar{K}$, $\eta_c' K \bar{K}$, $\eta_c'' K \bar{K}$, $\eta_c''' K \bar{K}$, ··· invariant mass distributions.

| bound states | $M_Y (\text{GeV})$ | $M_{\eta_c'}/\Upsilon + M_{f_0}/\sigma (\text{GeV})$ |
|-------------|-------------------|-----------------------------------------------|
| $c \bar{c} s s$ | $4.68 \pm 0.29$ | $4.617$ |
| $c \bar{c} q \bar{q}$ | $4.56 \pm 0.21$ | $4.037 - 4.837$ |
| $b \bar{b} s s$ | $11.42 \pm 0.21$ | $11.559$ |
| $b \bar{b} q \bar{q}$ | $11.36 \pm 0.18$ | $10.979 - 11.779$ |

Table 1: The masses of the pseudoscalar bound states.

| bound states | $\lambda_Y (10^{-2}\text{GeV}^2)$ |
|-------------|-------------------|
| $c \bar{c} s s$ | $3.63 \pm 1.80$ |
| $c \bar{c} q \bar{q}$ | $3.41 \pm 1.37$ |
| $b \bar{b} s s$ | $20.7 \pm 8.0$ |
| $b \bar{b} q \bar{q}$ | $20.4 \pm 6.5$ |

Table 2: The pole residues of the pseudoscalar bound states.

In this article, we study the pseudoscalar bound state $\eta_c' f_0(980)$ (irrespective of the hadro-charmonium and the molecular state) with the QCD sum rules, the numerical result $M_Y = 4.68 \pm 0.29\text{GeV}$ is consistent with the value $4616^{+4}_{-6}\text{MeV}$ predicted by Guo et al. Considering the $SU(3)$ symmetry of the light flavor quarks and the heavy quark symmetry, we also study the bound states $\eta_c' \sigma(400 - 1200)$, $\eta_c''' f_0(980)$ and $\eta_c''' \sigma(400 - 1200)$ with the
QCD sum rules, and make reasonable predictions for their masses. Our predictions depend heavily on the two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules. We can search for those bound states at the LHCb, the KEK-B or the Fermi-lab Tevatron.

Appendix

The spectral densities at the level of the quark-gluon degrees of freedom:

\[
\rho_0(s) = \frac{3}{2048\pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta)^2(s - \bar{m}_Q^2)^3(3s + \bar{m}_Q^2), \quad (11)
\]

\[
\rho_{\langle ss \rangle}(s) = \frac{9m_s\langle \bar{s}s \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta s(s - \bar{m}_Q^2) - \frac{m_s\langle \bar{s}g_s\sigma G_s \rangle}{32\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha (1 - \alpha)(3s - \bar{m}_Q^2), \quad (12)
\]

\[
\rho_{\langle ss \rangle^2}(s) = -\frac{\langle \bar{s}s \rangle^2}{16\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha (1 - \alpha)(3s - \bar{m}_Q^2) + \frac{\langle \bar{s}s \rangle\langle \bar{s}g_s\sigma G_s \rangle}{32\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha (1 - \alpha) \left[ 6 + \left( \frac{s^2}{M^2} \right) \delta(s - \bar{m}_Q^2) \right] + \frac{m_s^2\langle \bar{s}s \rangle\langle \bar{s}g_s\sigma G_s \rangle}{32\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \left[ 1 + \frac{s^2}{M^2} \right] \delta(s - \bar{m}_Q^2) - \frac{3\langle \bar{s}g_s\sigma G_s \rangle^2}{128\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha (1 - \alpha) \left[ 1 + \frac{s^2}{2M^4} + \frac{s^3}{6M^6} \right] \delta(s - \bar{m}_Q^2) - \frac{3m_s^2\langle \bar{s}g_s\sigma G_s \rangle^2}{768\pi^2M^6} \int_{\alpha_i}^{\alpha_f} d\alpha s^2 \delta(s - \bar{m}_Q^2), \quad (13)
\]

\[
\rho_{\langle GG \rangle}(s) = \frac{3}{256\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta s(s - \bar{m}_Q^2) + \frac{3}{512\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta)^2 s(s - \bar{m}_Q^2) - \frac{m_s^2}{1024\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{\alpha}{\beta^2 + \alpha^2} \right] (1 - \alpha - \beta)^2 (3s - 2\bar{m}_Q^2) - \frac{m_s^4}{1024\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^3 + \beta^3} \right] (1 - \alpha - \beta)^2 + \frac{3m_s^2}{1024\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^2 + \beta^2} \right] (1 - \alpha - \beta)^2 (s - \bar{m}_Q^2), \quad (14)
\]
Figure 1: The contributions from different terms with variation of the Borel parameter $M^2$ in the operator product expansion. The $A$, $B$ and $C$ correspond to the contributions from the $\langle qg_\sigma Gq \rangle^2$ term, the $\langle q\bar{q} \rangle^2 + \langle q\bar{q} \rangle \langle qg_\sigma Gq \rangle$ term and the $\langle \frac{\alpha}{\pi}GG \rangle$ term, respectively. The (I) and (II) denote the $cc\bar{s}s$ and $cc\bar{q}q$ channels, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 23\text{ GeV}^2$, $24\text{ GeV}^2$, $25\text{ GeV}^2$, $26\text{ GeV}^2$, $27\text{ GeV}^2$ and $28\text{ GeV}^2$, respectively.
Figure 2: The contributions from different terms with variation of the Borel parameter $M^2$ in the operator product expansion. The $A$, $B$ and $C$ correspond to the contributions from the $\langle \bar{q}g_\sigma Gq \rangle^2$ term, the $\langle \bar{q}q \rangle^2 + \langle \bar{q}q \rangle \langle \bar{q}g_\sigma Gq \rangle$ term and the $\langle \alpha_{GG} \rangle$ term, respectively. The (I) and (II) denote the $b\bar{b}s\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively. The notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 138$ GeV$^2$, 140 GeV$^2$, 142 GeV$^2$, 144 GeV$^2$, 146 GeV$^2$ and 148 GeV$^2$, respectively.
Figure 3: The contributions from the pole terms with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$ and $b\bar{b}q\bar{q}$ channels, respectively. In the hidden charm channels, the notations $\alpha$, $\beta$, $\gamma$, $\lambda$, $\rho$ and $\tau$ correspond to the threshold parameters $s_0 = 23$ GeV$^2$, 24 GeV$^2$, 25 GeV$^2$, 26 GeV$^2$, 27 GeV$^2$ and 28 GeV$^2$, respectively; while in the hidden bottom channels they correspond to the threshold parameters $s_0 = 138$ GeV$^2$, 140 GeV$^2$, 142 GeV$^2$, 144 GeV$^2$, 146 GeV$^2$ and 148 GeV$^2$, respectively.
Figure 4: The masses of the pseudoscalar bound states with variation of the Borel parameter \( M^2 \) and threshold parameter \( s_0 \). The \( A, B, C, \) and \( D \) denote the \( c\bar{c}s\bar{s}, c\bar{c}q\bar{q}, b\bar{b}s\bar{s}, \) and \( b\bar{b}q\bar{q} \) channels, respectively. In the hidden charm channels, the notations \( \alpha, \beta, \gamma, \lambda, \rho \) and \( \tau \) correspond to the threshold parameters \( s_0 = 23 \text{ GeV}^2, 24 \text{ GeV}^2, 25 \text{ GeV}^2, 26 \text{ GeV}^2, 27 \text{ GeV}^2 \) and \( 28 \text{ GeV}^2 \), respectively; while in the hidden bottom channels they correspond to the threshold parameters \( s_0 = 138 \text{ GeV}^2, 140 \text{ GeV}^2, 142 \text{ GeV}^2, 144 \text{ GeV}^2, 146 \text{ GeV}^2 \) and \( 148 \text{ GeV}^2 \), respectively. The \( \xi \) and \( \mu \) denote the \( \eta_c - f_0(980) \) and \( \eta_c' - f_0(980) \) thresholds respectively in the \( c\bar{c}s\bar{s} \) channel, while in the \( b\bar{b}s\bar{s} \) channel they correspond to \( \Upsilon'' - f_0(980) \) and \( \Upsilon''' - f_0(980) \) thresholds respectively.
Figure 5: The masses of the pseudoscalar bound states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, and $b\bar{b}q\bar{q}$ channels, respectively.
Figure 6: The pole residues of the pseudoscalar bound states with variation of the Borel parameter $M^2$. The $A$, $B$, $C$, and $D$ denote the $c\bar{c}s\bar{s}$, $c\bar{c}q\bar{q}$, $b\bar{b}s\bar{s}$, and $b\bar{b}q\bar{q}$ channels, respectively.
\[ \rho_{GGG}(s) = \frac{m_Q^4}{8192 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( 1 - \alpha - \beta \right)^2 \left[ \frac{1}{\alpha^4} + \frac{1}{\beta^4} \right] \delta(s - \bar{m}_Q^2) \]
\[ - \frac{3m_Q^2}{8192 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( 1 - \alpha - \beta \right)^2 \left[ \frac{1}{\alpha^3 \beta} + \frac{1}{\alpha \beta^3} \right] \delta(s - \bar{m}_Q^2) \]
\[ + \frac{m_Q^2}{8192 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 2 + \bar{m}_Q^2 \delta(s - \bar{m}_Q^2) \right] \]
\[ (1 - \alpha - \beta)^2 \left[ \frac{\alpha}{\beta^3} + \frac{\beta}{\alpha^3} \right] \]
\[ - \frac{1}{16384 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 3s - 2\bar{m}_Q^2 \right] (1 - \alpha - \beta)^2 \left[ \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right] \]
\[ - \frac{m_Q^4}{8192 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( 1 - \alpha - \beta \right)^2 \left[ \frac{1}{\alpha^2 \beta} + \frac{1}{\alpha \beta^2} \right] \delta(s - \bar{m}_Q^2) \]
\[ + \frac{3m_Q^2}{4096 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left( 1 - \alpha - \beta \right)^2 \left[ \frac{1}{\alpha} + \frac{1}{\beta} \right] \]
\[ (1 - \alpha - \beta)^2 \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \]
\[ - \frac{3}{16384 \pi^6} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 3s - 2\bar{m}_Q^2 \right] (1 - \alpha - \beta)^2 \left[ \frac{1}{\alpha} + \frac{1}{\beta} \right], \quad (15) \]

where \( \alpha_f = \frac{1 + \sqrt{1 - 4m_Q^2/s}}{2}, \ \alpha_i = \frac{1 - \sqrt{1 - 4m_Q^2/s}}{2}, \ \beta_i = \frac{\alpha m_Q^2}{\alpha - \bar{m}_Q^2}, \ \bar{m}_Q^2 = \frac{(\alpha + \beta) m_Q^2}{\alpha \beta}, \ \bar{m}_Q^2 = \frac{m_Q^2}{\alpha(1 - \alpha)}, \)

and \( \Delta = 4(m_Q + m_s)^2. \)

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References

[1] X. L. Wang et al, Phys. Rev. Lett. 99, 142002 (2007).
[2] C. Z. Yuan et al, Phys. Rev. Lett. 99, 182004 (2007).
[3] G. Pakhlova et al, Phys. Rev. Lett. 98, 092001 (2007).
[4] G. Pakhlova et al, Phys. Rev. Lett. 100, 062001 (2008).
[5] G. Pakhlova et al, Phys. Rev. D 77, 011103 (2008).
[6] B. Aubert et al, arXiv:0710.1371.
[7] P. Pakhlov et al, Phys. Rev. Lett. 100, 202001 (2008).
[8] G. J. Ding, J. J. Zhu and M. L. Yan, Phys. Rev. D 77, 014033 (2008).
[9] B. Q. Li and K. T. Chao, Phys. Rev. D 79 (2009) 094004.
[10] A. M. Badalian, B. L. G. Bakker and I. V. Danilkin, Phys. Atom. Nucl. 72 (2009) 638.
[11] C. F. Qiao, J. Phys. G35, 075008 (2008).
[12] R. M. Albuquerque and M. Nielsen, Nucl. Phys. A 815, 53 (2009).
[13] R. Faccini, arXiv:0801.2679.
[14] S. Dubynskiy and M. B. Voloshin, Phys. Lett. B 666, 344 (2008).
[15] M. B. Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455.
[16] F. K. Guo, C. Hanhart and Ulf-G. Meissner, Phys. Lett. B 665, 26 (2008).
[17] C. Amsler et al, Phys. Lett. B667, 1 (2008).
[18] F. K. Guo, C. Hanhart and U. G. Meissner, Phys. Rev. Lett. 102 (2009) 242004.
[19] Z. G. Wang and X. H. Zhang, arXiv:0905.3784.
[20] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.
[21] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. 127 (1985) 1.
[22] Z. G. Wang, W. M. Yang and S. L. Wan, Eur. Phys. J. C37, 223 (2004).
[23] F. E. Close and N. A. Tornqvist, J. Phys. G28 (2002) R249.
[24] C. Amsler and N. A. Tornqvist, Phys. Rept. 389 (2004) 61.
[25] S. H. Lee, H. Kim and Y. Kwon, Phys. Lett. B609 (2005) 252.
[26] R. D. Matheus, F. S. Navarra, M. Nielsen and C. M. Zanetti, Phys. Rev. D80 (2009) 056002.
[27] B. L. Ioffe, Prog. Part. Nucl. Phys. 56 (2006) 232.
[28] Z. G. Wang, Nucl. Phys. A791 (2007) 106.
[29] Z. G. Wang, W. M. Yang and S. L. Wan, J. Phys. G31 (2005) 971.
[30] Z. G. Wang, Eur. Phys. J. C62 (2009) 375.
[31] Z. G. Wang, Eur. Phys. J. C63 (2009) 115.
[32] G. Kane and A. Pierce, “Perspectives On LHC Physics”, World Scientific Publishing Company, 2008.