Improving SUSY Spectrum Determinations at the LHC with Wedgebox and Hidden Threshold Techniques

M. Bisset and R. Lu

Center for High Energy Physics and Department of Physics,
Tsinghua University, P.R. China

N. Kersting

Physics Department, Sichuan University, P.R. China

Abstract

The LHC has the potential not only to discover supersymmetry (SUSY), but also to permit fairly precise measurements of at least a portion of the sparticle spectrum. Proposed mass reconstruction methods rely upon either inverting invariant mass endpoint expressions or upon solving systems of mass-shell equations. These methodologies suffer from the weakness that one certain specific sparticle decay chain is assumed to account for all the events in the sample. Taking two examples of techniques utilizing mass-shell equations, it is found that also applying wedgebox and hidden threshold (HT) techniques allows for the isolation of a purer event sample, thus avoiding errors, possibly catastrophic, due to mistaken assumptions about the decay chains involved and simultaneously improving accuracy. What is innovative is using endpoint measurements (with wedgebox and HT techniques) to obtain a more homogeneous, well-understood sample set rather than just using said endpoints to constrain the values of the masses (here found by the mass-shell technique). The fusion of different established techniques in this manner represents a highly profitable option for LHC experimentalists who will soon have data to analyze.
1 Introduction

LHC experimentalists will soon determine whether or not Supersymmetry (SUSY) is a TeV-scale phenomenon: if so, colored sparticles will probably be the first to be discerned, possibly soon followed by neutralinos, charginos, and sleptons if favorable decay channels are open, though measuring the masses of these latter colorless sparticles with percent precision will be challenging \cite{1,2}. The reason for this is that every R-parity-conserving SUSY event produces at least two invisible particles (the lightest SUSY particles, or LSPs) which carry away significant missing energy and make it impossible to reconstruct mass peaks. Therefore, many SUSY mass extraction techniques depend on precise measurement of invariant mass distribution endpoints. For a sparticle decaying into an LSP and a Standard Model (SM) fermion pair, either via a three-body decay or sequential two-body decays, it is straightforward to see how the endpoint of the di-fermion invariant mass distribution yields the mass difference between the decaying sparticle and the LSP (perhaps modified by the on-mass-shell intermediate for two-body decays) \cite{4,5}. Studies attempting to fully reconstruct the actual sparticle masses from invariant mass endpoint information rely on specific longer decay chains, typically $\tilde{\chi}_1^0 b \rightarrow \tilde{\chi}_2^0 q \rightarrow \tilde{\ell}^\pm \ell^- q \rightarrow \ell^\pm \ell^- q$\cite{6,7,8,9,10,11} — each event would have two $\tilde{\chi}^0$s (for instance) produced. It is then theoretically doable to construct enough invariant mass distributions to determine the sparticle masses; however, in practice endpoint measurement may be complicated by low event rates, fitting criteria, unaccounted-for (in the simulation) higher-order and radiative \cite{12} effects, and (in particular) backgrounds.

Ideas on how to measure SUSY masses without relying on distribution endpoints have also been put forward. The work of Nojiri et al. \cite{13,14}, for example, uses mass-shell relations in a sufficiently long SUSY decay chain, e.g. $\tilde{g} \rightarrow b \bar{b} \rightarrow \tilde{\chi}_2^0 b \bar{b} \rightarrow \tilde{\ell}^\pm \ell^- b b \chi_1^0$; if some of the masses are already known in this chain the others can in principle be found by solving mass-shell relations for a small sample of events\cite{4}, which may in fact lie far from the endpoint. Another method, due to Cheng et al. \cite{15}, starts from very similar looking mass-shell relations, but instead of assuming some masses and solving for the others, scans the whole mass-space for points where these relations are most likely to be satisfied. Both methods, hereafter designated as ‘Mass Shell Techniques’ (MSTs)\cite{5}, seem quite effective in obtaining percent-level determination of the sought-after SUSY masses, at least at the parameter points considered in those works.

The accuracy of both these MSTs hinges on one critical assumption: the decay topology of choice has been isolated. In the actual LHC data, the decay topology would have to be inferred, if this is at all possible, before proceeding; MSTs would thus appear to be excellent roads to SUSY mass reconstruction which, however, begin only at a point half-way to the destination.

The present work focuses on the first half of this road; i.e., isolating a desired

---

3If R-parity is not conserved, then it may be possible to fully reconstruct events. See \cite{3} for further details.

4In practice, many events are still required.

5Refs. \cite{13,14,16} use the name ‘mass relation method’ to refer to their technique.
decay topology, and on how this affects a subsequent MST analysis. As a first foray into this potentially quite thorny task, let’s restrict ourselves to just the specific topologies considered in [13] and [15] involving a pair of neutralinos $\tilde{\chi}_i^0\tilde{\chi}_j^0$ ($i, j = 2, 3, 4$) that subsequently decay to leptons (electrons and muons) via on-shell sleptons. This situation is amenable to a wedgebox analysis [17, 18] which is based upon a scatter plot of the di-electron mass $M_{e^+e^-}$ versus the dimuon mass $M_{\mu^+\mu^-}$. A key benefit of this technique is that it allows (at least partial) separation of individual events according to the specific $(i, j)$-pair whose production gave rise to them. Given sufficient statistics, events from each such decay-type fall in distinct, easily-recognized zones of the wedgebox plot. The overall topology of the resulting wedgebox plot then tests for the significant presence of the various possible $(i, j)$ decay channels — which may for instance signal the meaningful presence of a decay channel erroneously assumed to have been insignificant as the basis for a MST analysis. Events can be selected from a specific zone of the wedgebox plot, preferably a zone populated by only one decay channel. This acts to maximize sample homogeneity and assure the basic MST assumption is satisfied.

Although the wedgebox technique relies on locating the endpoints of invariant mass distributions — just like the studies [6, 7, 8, 9, 10, 11] mentioned earlier, the information sought is radically different: the wedgebox analysis is tailor-made for event sample sets comprised of assorted produced sparticle pairs and multiple sparticle decay chains. The observed endpoints serve to delineate the zones and allow for selection of purer subset(s) from an overall sample set. (Using this endpoint information to determine a set of cuts is a far more rational course than that of arbitrarily choosing some numerical cut-off values to purify the data sample.) Virtually all previous studies presume such purification has already been accomplished either by an unspecified set of cuts or a fortuitous choice of SUSY input parameters. The wedgebox technique illustrates a concrete method of how to deal with the more general case, and the consequences which result and should not be ignored in a coupled analysis aiming to extract the sparticle masses.

While the wedgebox plot structure does strongly indicate neutralino pair decay, the topological structure seen (such as a box, a wedge, a double-wedge, or a box within a wedge for instance) does not differentiate between neutralino decay chains with either on- or off-shell intermediate particles, though the numerical values of the endpoints may show notable sensitivity to this [4]. While the di-lepton distribution shapes for on- and off-shell decays may be theoretically distinguishable [6, 17], even if this is true the distinguishability may be lost after factoring in the effects of cuts, backgrounds, etc. If the neutralino decay is via an off-shell intermediate, then use of an MST fails simply because some of the mass-shell conditions fail. The recently discovered Hidden Threshold (HT) technique, however, can test the on- or off-shell nature of a sample of decays [19]. Here the idea is to look for correlations among

---

6The only other possibility in the MSSM involves a chargino decaying through a sneutrino, $\tilde{\chi}_2^\pm \rightarrow t^\pm \tilde{\nu} \rightarrow t^\pm l^- \tilde{\chi}_1^0$, since the kinematics are the same. The wedgebox plot must be considered effective in separating these decay topologies as well.

7This same point was made in [9]. See Discussion Point # 2 and Section 5.2 therein. This will be examined more closely in Secn. 4.
various invariant mass combinations in the four-lepton endstate – a genuine off-shell decay fulfills the correlations while an on-shell decay does not.

This paper will show that MSTs, by construction unrelated to invariant mass endpoints, can nevertheless be improved by information contained in these endpoints — specifically via the wedgebox and HT techniques. The paper is organized as follows: Sections 2 and 3 concentrate on the ‘N-MST’ method of Nojiri et al. and the ‘C-MST’ method of Cheng et al., respectively; Section 4 then offers conclusions and some additional discussion on SUSY mass spectral analyses at the LHC.

2 The N-MST Method of Nojiri et al.

For the N-MST method, the focus will be on the decay of a heavy MSSM pseudoscalar Higgs boson as considered in [13]. The specific decay chain is

\[ pp \rightarrow A^0 \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0 \rightarrow \tilde{l}_1^\pm \tilde{l}_2^\pm l_2^\mp \rightarrow l_1^\mp l_1^\mp l_2^\pm l_2^\mp \tilde{\chi}_1^0 \tilde{\chi}_1^0 \]

(1)

where the Higgs boson decays to neutralinos \((i, j = 2, 3, 4)\) via on-shell sleptons of the electron- or muon- variety (see Fig. [1]). Assuming the final leptons’ four-momenta are known while the LSP’s escape detection, (1) implies six mass-shell constraints on

---

Figure 1: Feynman diagram for the decay (1). Here \(i, j = 2, 3, 4\) while the label 1 or 2 on leptons stands for either e or \(\mu\).

---

\(^8\)Though our strategy is quite different from the ‘hybrid’ method of [16] which couples an MST with values of endpoints.
the eight unknown components of LSP four-momenta \((p^\mu\text{ and } p'^\mu)\),

\[
m_1^2 = p^2 \tag{2}
\]

\[
m_2^2 = p'^2 \tag{3}
\]

\[
m_{l_1}^2 = (p_{l_1} + p)^2 \tag{4}
\]

\[
m_{l_2}^2 = (p_{l_2} + p')^2 \tag{5}
\]

\[
m_{\bar{\chi}_1^0}^2 = (p_{l_1} + p'_{l_1} + p)^2 \tag{6}
\]

\[
m_{\bar{\chi}_2^0}^2 = (p_{l_2} + p'_{l_2} + p')^2 \tag{7}
\]

Nojiri et al. also posit two overall momentum conservation constraints

\[
(p_{l_1} + p_{l_2} + p'_{l_1} + p'_{l_2} + p + p') \cdot \vec{x} = 0 \tag{8}
\]

\[
(p_{l_1} + p_{l_2} + p'_{l_1} + p'_{l_2} + p + p') \cdot \vec{y} = 0 \tag{9}
\]

along directions \((x, y)\) transverse to the beam \((z)\), though this would appear to be contingent on the Higgs boson having no transverse momentum. If all the masses \(m_{\bar{\chi}_1^0}, m_{l_1}, m_{l_2}, m_{\bar{\chi}_2^0}\), and \(m_{\bar{\chi}_j^0}\) are known in advance, one can solve the eight equations \((2)-(9)\) for the eight unknowns and reconstruct the Higgs boson mass via

\[
m_A^2 = (p_{l_1} + p_{l_2} + p'_{l_1} + p'_{l_2} + p + p')^2 \tag{10}
\]

from just one Higgs boson event of the type \((\text{I})\). However, even in this idealized scenario which does not include detector resolution, particle widths, backgrounds, etc., there are two major caveats which prevent this procedure from being so straightforward:

- There is a 4-fold ambiguity in assigning labels \(l_{1,2}, l'_{1,2}\) to the leptons; this forms a combinatoric background.

- Combining \((2)-(9)\) leads to a quartic equation with 0, 2 or 4 solutions for the unknown momenta.

So what one must do in practice is collect a number of events and deduce the correct value of \(m_A\) from the maximum of the resulting distribution. In \([13]\) a \(10^3\) event sample (with no backgrounds) thus yielded a percent-level determination of the Higgs boson mass.

### 2.1 Addition of the Wedgebox Technique

#### 2.1.1 Box Topology

As shown in \([13]\) the programme sketched in the previous section works fairly well at Snowmass Benchmark SPS1a \([20]\) where the dominant Higgs boson decays are via \((\text{I})\) with \(i = j = 2\). Therein, 1000 events of the type \(A^0 \to \chi^0_2 \bar{\chi}^0_2 \to llll\chi^0_1\bar{\chi}^0_1\) were generated using the HERWIG6.4 \([21]\) event generator and passed through the
The only cuts required were that all four isolated leptons have \( \eta < 2.5 \), with two of the leptons also having \( p_T > 20 \text{ GeV} \) while the other two have \( p_T > 10 \text{ GeV} \). Same-flavor events such as \( e^+e^-e^+e^- \) or \( \mu^+\mu^-\mu^+\mu^- \) were also included if one of the two possible pairings of OS leptons in such configurations gave a di-lepton invariant mass beyond the \( \sim 78 \text{ GeV} \) kinematic endpoint (implying the other possible pairing is the correct one). This analysis yielded the correct Higgs boson mass with a resolution of only 6 GeV.

The following set of cuts are herein adopted in an effort to reproduce these results using ISAJET \[23\] in place of HERWIG:

1. Leptons must be hard \( (p_T > 10, 8 \text{ GeV} \text{ for } e^\pm, \mu^\pm, \text{ respectively; } |\eta^\ell| < 2.4) \), and isolated (no tracks of other charged particles in a \( r = 0.3 \text{ rad} \) cone around the lepton, and less than 3 GeV of energy deposited into the electromagnetic calorimeter for \( 0.05 \text{ rad} < r < 0.3 \text{ rad} \) around the lepton).

2. There must be missing energy in the range: \( 20 \text{ GeV} < E_T < 130 \text{ GeV} \).

3. No jets\[9\] are present with a reconstructed energy \( E_{\text{jet}} \) greater than 50 GeV.

A sufficient sample of \( pp \to A^0 \) events is collected to represent an integrated LHC luminosity of \( 300 \text{ fb}^{-1} \). With this stricter set of cuts it should be somewhat harder to extract the Higgs boson mass.

Fig. 2a shows the wedgebox plot at SPS1a for \( e^+e^-\mu^+\mu^- \) events. A ‘simple box’ topology consistent with the expected \( \tilde{\chi}_2^0\tilde{\chi}_2^0 \) origin of lepton pairs is clear. Moreover, the number of flavor-balanced events \( (e^+e^-e^+e^-+\mu^+\mu^-\mu^+\mu^-+e^+e^-\mu^+\mu^-) \) exceeds the number of flavor-unbalanced events \( (e^+e^-\mu^+\mu^-+\mu^+\mu^-e^+\mu^-) \) at this parameter point \[28\]; this indicates that the events come primarily from a Higgs boson decay\[10\] with decay topology \[11\]. Though the final number of events passing cuts is somewhat small (only 30 or so compared to the 1000 which \[13\] assumes, mainly due to the inclusion of the leptonic branching ratios) the number of events in the distribution of solutions is quite a bit larger: recall the bulleted caveats earlier which potentially can yield a \( 4 \times 4 = 16 \)-fold multiplicity factor. Though only a factor of 4 or so is observed, nonetheless a fairly clear peak in the distribution emerges (see Fig. 2b) at the correct value\[11\] of \( m_A = 400 \pm 5 \text{ GeV} \).

### 2.1.2 Wedge Topology

In principle, this method should work for the case \( i \neq j \) as well; to test this we consider the following MSSM parameter point:

\[9\]Jets are defined by a cone algorithm with \( r = 0.4 \) and must have \( |\eta_j| < 2.4 \).

\[10\]Note that though in a simulation one can of course only choose to generate Higgs boson decay events, experimentalists lack this freedom.

\[11\]Also note here that the momentum-conservation constraints \[10\] do not appear to be generally true; i.e., the parent Higgs boson is often generated with significant transverse momentum in the range \( 0 \leq p_T \leq 100 \text{ GeV} \) according to this analysis. Surprisingly, this does not seem to affect the result.
Figure 2: (a) Wedgebox plot at SPS1a ($A^0$ bosons only) for $300 \text{ fb}^{-1}$ luminosity, cuts as in text. Dotted lines show locations of kinematic endpoints from $\tilde{\chi}_0^{2,3,4} \rightarrow \tilde{\chi}_1^0$ decays. (b) Distribution of solutions for SPS1a. Here the maximum is at the correct Higgs boson mass of $M_A \approx 395$ GeV.

Table 1: Relevant masses at the MSSM Test Point I,II (all masses in GeV).

| Particle | Mass (I) | Mass (II) |
|----------|----------|-----------|
| $\tilde{\chi}_1^0$ | 119.94 | 86.03 |
| $\tilde{\chi}_2^0$ | 180.33 | 143.09 |
| $\tilde{\chi}_3^0$ | 197.98 | 166.40 |
| $\tilde{\chi}_4^0$ | 317.72 | 277.27 |
| $e_R$ | 156.17 | 127.20 |
| $\mu_R$ | 156.17 | 127.20 |
| $A^0$ | 600.0 | 700.00 |
| $H^0$ | 609.28 | 705.58 |

MSSM Test Point I

$\mu = 190$ GeV  $M_2 = 280$ GeV  $\tan \beta = 10$  $M_A = 600$ GeV  
$M_{e,\mu_{L,R}} = 150$ GeV  $M_{\tilde{\mu}_{L,R}} = 250$ GeV  $M_{\tilde{\chi}^0_{1,2,3,4}} = 1000$ GeV.

which has the mass spectrum shown in Table 1. Now the main production modes contributing to the $e^+e^-\mu^+\mu^-$ signal are the Higgs boson channels $H^0/A^0 \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0, \tilde{\chi}_3^0\tilde{\chi}_3^0, \tilde{\chi}_4^0\tilde{\chi}_4^0$, ‘direct’ neutralino pair production channels, $\tilde{\chi}_2^0\tilde{\chi}_3^0, \tilde{\chi}_2^0\tilde{\chi}_4^0$, and ‘mixed’ channels involving charginos, mainly $\tilde{\chi}_2^+\tilde{\chi}_2^{-}, \tilde{\chi}_2^+\tilde{\chi}_3^0, \tilde{\chi}_2^+\tilde{\chi}_4^0$ production. A random sample of $e^+e^-\mu^+\mu^-$ events will therefore not be a clean collection of Higgs boson decays.

$^{12}$As explained in [18], direct $\tilde{\chi}_2^0\tilde{\chi}_2^0$ channels are suppressed by isospin symmetry, while $\tilde{\chi}_0^0\tilde{\chi}_4^0$ are phase-space suppressed.

$^{13}$Note that for MSSM Test Point I the staus have been set more massive than the other sleptons to avoid tau-containing decays and generate more decays to the desired leptons. This is simply done by hand here for convenience.
which for the luminosity considered here \((300 \text{ fb}^{-1})\) number about 150 against nearly 800 direct channel and 100 or so mixed channel events. This point then presents a more challenging case for applying the mass relation method.

![Figure 3: Wedgebox plot for the MSSM Test Point I defined in the text, for \(300 \text{ fb}^{-1}\). (b) origin of each four lepton event labeled as from ‘A’, ‘H’, direct production ‘DP’, or otherwise ‘Bad’. (c) specific pair of neutralinos decaying to each four lepton event identified as \(\tilde{\chi}_2^0 \tilde{\chi}_4^0\) (‘Z2Z4’), \(\tilde{\chi}_2^0 \tilde{\chi}_3^0\) (‘Z2Z3’), \(\tilde{\chi}_3^0 \tilde{\chi}_2^0\) (‘Z2Z2’), some other neutralino pair (‘Other’), or other events ‘Bad’.](image)

However, the shape of the wedgebox plot at this point suggests selecting events via their decay topology: Fig. 3a shows a clear ‘double wedge’ topology implying that events due to \(\tilde{\chi}_2^0 \tilde{\chi}_2^0\) decays are confined to the innermost box bounded by kinematic edges at \(\sim 60 \text{ GeV}\), while those due to \(\tilde{\chi}_3^0 \tilde{\chi}_3^0\) events are enclosed in the legs of the short wedge terminating at \(\sim 80 \text{ GeV}\). Events from \(\tilde{\chi}_2^0 \tilde{\chi}_4^0\) decays span both of these regions and beyond up to the final kinematic edge at \(\sim 200 \text{ GeV}\).

Fig. 3(b,c) is color-coded to show the separate distributions of events from different production channels (A, H, or DP ‘direct’) and by their assorted neutralino-pair types; these distributions agree with remarks in the previous paragraph. Note, however, the presence of ‘bad’ events which do not distribute themselves nicely within the kinematic bounds and which are typically due to chargino decays. Though nearly 10 percent of the total number of events are bad events, about half of these are excluded by rejecting events outside the overall wedge structure, nicely illustrating the strength of the wedgebox technique.

Without the assistance of the wedgebox plot, one might be led to assume that events with \(M_{ee,\mu\mu} < 60 \text{ GeV}\) correspond mostly to the decay topology of (11) with \(i = j = 2\). This, however, leads to a Higgs boson mass distribution as shown in Fig. 4a. There is neither a clear peak nor any kind of distinguishing feature near the correct value of \(M_A = 600 \text{ GeV}\). Evidently, what might seem the natural choice of using events from the densest region of the wedgebox plot is not optimal.

\[\text{In the MSSM there are other possibilities for these edges aside from those of the form } \chi_i^0 \to l^+l^-\bar{\chi}_j^0, \text{ including } \chi_0^0 \to l^+l^-\bar{\chi}_j^0 \text{ and } \chi_2^\pm \to l^\mp l^-\bar{\chi}_3^\mp. \text{ Separate studies (including perhaps a Hidden Threshold analysis) can probably exclude such possibilities. For parameter set choices examined herein, these other potential decay modes have a totally negligible effect.}\]
Figure 4: (a) Distribution of solutions for events taken from the inner box region of Fig. 3a; i.e., within the first set of dotted lines at \(\sim 60\) GeV. Here the correct Higgs boson mass of \(M_A \approx 600\) GeV does not appear at the peak. (b) Distribution of solutions for events taken from the outer legs of Fig. 3a; i.e., within the regions bounded by dotted lines at \(\sim 80\) GeV and \(\sim 200\) GeV. Here the correct Higgs boson mass of \(M_A \approx 600\) GeV coincides with the peak.

Instead, events should be selected from the most homogeneous zone of the wedge-box plot, which in this case consists of the outermost legs from 80 GeV to 200 GeV, corresponding mostly \(A^0 \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_4\) (even without looking at Fig. 4). [18] predicts that events in the outer wedge of a double-wedge plot come from Higgs boson decays. As seen in Fig. 4b, the N-MST method now works splendidly, yielding an easily-identified peak at the correct Higgs boson mass, even though now an additional sparticle mass plays into the equations. The goodness of fit is more surprising considering the number of ‘bad’ events distributed throughout this zone; inspection reveals that these latter, however, often fail to give solutions to equations (2)-(9), so they do not heavily interfere with the shape of the distribution.

Improvement of the N-MST method via a wedgebox plot, at least for the Higgs boson decay topology considered here, is therefore quite straightforward.

3 The C-MST Method of Cheng et al.

Next consider the C-MST method. The process treated by Cheng et al. [15] is illustrated in Fig. 5 in which the masses \(m_1, m_2, m_{13}, m_{24}, m_{135}\), and \(m_{246}\) must
Figure 5: Event topology, taken from [15]

satisfy a set of equations precisely analogous to (2)-(9):

\begin{align}
    m_1^2 &= p_1^2 \\ 
    m_2^2 &= p_2^2 \\ 
    m_{13}^2 &= (p_1 + p_3)^2 \\ 
    m_{24}^2 &= (p_2 + p_4)^2 \\ 
    m_{135}^2 &= (p_1 + p_3 + p_5)^2 \\ 
    m_{246}^2 &= (p_2 + p_4 + p_6)^2 \\ 
    p_x^{\text{sum}} &= \sum_i p_{x i}^x \\ 
    p_y^{\text{sum}} &= \sum_i p_{y i}^y
\end{align}

where the transverse momentum sums $p_x^{\text{sum}}$ and $p_y^{\text{sum}}$ are assumed to be calculable from measurements of associated jet momenta (produced, though not shown, in the gray bubble in Fig. 5) and missing momentum (from the LSPs) necessary to balance the whole:

\begin{align}
    p_x^{\text{sum}} &= \sum_l p_l^x + p_{\text{miss}}^x \\ 
    p_y^{\text{sum}} &= \sum_l p_l^y + p_{\text{miss}}^y
\end{align}

The specific example considered in [15] was production of two neutralinos via squarks in the MSSM, followed by decay via on-shell smuons to muons and two LSPs:

$$
\tilde{q}\tilde{q} \rightarrow q q \tilde{\chi}_2^0 \tilde{\chi}_2^0 (\rightarrow \mu^+ \mu^- \rightarrow \mu^+ \mu^- \tilde{\chi}_1^0)
$$
giving
\[ m_1 = m_2 = m_{\tilde{\chi}_1^0} \]
\[ m_{13} = m_{24} = m_{\tilde{\mu}} \]
\[ m_{135} = m_{246} = m_{\tilde{\chi}_2^0} \] (21)

In the three-dimensional space of masses \((m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\mu}})\), each event gives eight equations (11)-(18) for the eight unknown LSP momenta \(p_{\tilde{\chi}}^1\) and \(p_{\tilde{\chi}}^2\), assuming the outgoing muon momenta \(p_3, p_4, p_5, p_6\) can be measured. The solution to this set of equations is again, as for the system of (2)-(9), a quartic equation with 0, 2, or 4 real roots. In contrast to the discussion of the last section, however, rather than trying to find the point in \((m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\mu}})\)-space where the density of solutions \((\equiv N(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\mu}}))\) is maximized, instead the point where the gradient of \(N\) is maximized (a heuristic argument for this is given in [15]) is sought.

In [15] this method apparently works quite well at the MSSM parameter points studied, giving all relevant sparticle masses to a few percent after collection of data samples corresponding to 300 fb\(^{-1}\) of integrated LHC luminosity. However, to test the robustness of this technique and the extent to which augmentation with the wedgebox and/or HT techniques can improve results, consider a new MSSM parameter point (see Tab. 1 again for masses):

**MSSM Test Point II**

\[
\begin{align*}
\mu &= -150 \text{ GeV} \\
M_2 &= 250 \text{ GeV} \\
M_1 &= 90 \text{ GeV} \\
\tan \beta &= 5 \\
M_{\tilde{\chi}_1^0,\tilde{\chi}_2^0,\tilde{\mu}} &= 250 \text{ GeV} \\
M_{\tilde{\chi}_1^0,\tilde{\chi}_2^0} &= 120 \text{ GeV} \\
M_A &= 700 \text{ GeV} \\
M_{\tilde{q}} &= 400 \text{ GeV} \\
M_{\tilde{g}} &= 500 \text{ GeV}
\end{align*}
\]

### 3.1 On- or Off-Shell?

First, it should be clear that this method also depends on isolating a fairly pure sample of events, in this case with the decay chain (20). From the wedgebox plot of Fig. 6, again generated using ISAJET [23] and the event selection criteria mentioned earlier (save no jet cut), one might choose to limit oneself to points \(M_{ee,\mu\mu} \gtrsim 100 \text{ GeV}\) (away from the “Z-line” at \(M_{ee,\mu\mu} \approx 91 \text{ GeV}\) [17], which is potentially populated by \(Z^0Z^0\)-background events as well as undesirable \(\tilde{\chi}_1^0 \rightarrow Z^0\tilde{\chi}_1^0\) events), since the resulting subset of the generated events should be mostly of the same \(\tilde{\chi}_1^0\tilde{\chi}_1^0\)-topology (indeed \(j = 4\) here[15]).

Yet this is not enough; these events must also arise from the decay of on-shell sleptons (see Fig. 4 of [17] for an example of an off-shell wedgebox plot, which is visually quite similar), since if the sleptons were off-shell the equalities (13) and (14) would be false. Without the assistance of these equations, the technique of [15] does not work at all, since there are not enough constraints to solve for the unknown LSP momenta.

\[[15] \text{Judging from the wedgebox plot alone, this could be interpreted as } j = 3 \text{ or even a chargino decay as previously mentioned; this distinction is irrelevant to the present discussion.}\]
Here the HT method can very easily be applied. As explained in \cite{19}, for each four-lepton event compute the invariants $M_{4l}$, $M_{22l}$, $M_{3l}$, and $M_{ll}$ (definitions of these reproduced in Appendix A), and plot each of these versus $M_{ee,\mu\mu}$ (see Fig. 7). Drawing a line at the kinematic edge of $M_{ee,\mu\mu} \sim 182$ GeV, presumed to be equal to the mass difference $m_{\chi_j^0} - m_{\chi_1^0}$, the minimum values of each invariant may be read off and compared to the expected functions of $m_{\chi_j^0}$ and $m_{\chi_1^0}$ (given in Appendix B). If this were a genuine off-shell decay, the four sets of solutions would be mutually consistent. However, it is easy to check in this case that they are not (the $M_{4l}$, $M_{3l}$, and $M_{ll}$ minima all imply different negative values of $m_{\chi_j^0}$). Therefore, the event sample is consistent with an on-shell decay, and it is safe to apply the C-MST methodology\cite{16}.

3.2 Wedgebox selection

The wedgebox structure\cite{17} of Fig. 6 is compartmentalized into four substantially event-populated regions by the shown red-dashed lines to which will be applied the

\footnote{A similar analysis can be done if one slepton is on-, one off-shell.}
\footnote{MSSM Test Point II is clearly representative of the general case in the MSSM, where $pp$ collisions yield a ‘mixed bag’ of concomitant neutralino decays. Events on this plot pass the same cuts as in Section 2.1 minus the jet cut.}
Figure 7: Correlations (at 90 fb$^{-1}$) among two-lepton and various four-lepton invariant masses (see Appendix A for definitions). The off-shell character of decays corresponding to the two edges of Fig. 6 at $M_{ee,\mu\mu} \sim 47$ GeV and $M_{ee,\mu\mu} \sim 182$ GeV may be tested by drawing lines at these values and measuring the corresponding minimum values of $M_{4l}$, $M_{2\ell 2\ell}$, $M_{3\ell}$, and $M_{4\ell}$. Since these values do not match what is expected from off-shell formulæ the decays must instead be on-shell.
The Zone 1 box, with \([M_{ee} \text{ and } M_{\mu\mu}] < 47 \text{ GeV}\), is the most densely-populated region of the wedgebox plot and should include all \(\tilde{\chi}^0_2\tilde{\chi}^0_2\) events.

Zone 2 is composed of two rectangles (the legs of wedges) running outwards along both axes from the Zone 1 box — satisfying the condition that \([M_{ee} \text{ and } M_{\mu\mu}] < 182 \text{ GeV}\) and \([M_{ee} \text{ or } M_{\mu\mu} \text{ but not both}] < 47 \text{ GeV}\). Events due to \(\tilde{\chi}^0_2\tilde{\chi}^0_2\) and \(\tilde{\chi}^0_2\tilde{\chi}^0_4\) not residing in Zone 1 will fall in Zone 2.

Zone 3, with \(47 \text{ GeV} < [M_{ee} \text{ and } M_{\mu\mu}] < 182 \text{ GeV}\), lies outside of Zones 1 & 2 and should only be populated by \(\tilde{\chi}^0_3\tilde{\chi}^0_3\), \(\tilde{\chi}^0_3\tilde{\chi}^0_4\) and \(\tilde{\chi}^0_4\tilde{\chi}^0_4\) events.

Consider first using four-lepton events across the whole wedgebox plot — this would include several different kinds of neutralino pair events. A scan was performed over the \((m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\mu}})\) mass space, and the resulting values projected onto the \((m_{\tilde{\chi}^0_1}, m_{\tilde{\mu}})\)-plane. This is shown in Fig. 8 from which it is apparent that the gradient of \(N\) is maximized along the line \(m_{\tilde{\chi}^0_1} = m_{\tilde{\mu}}\), which is far from the correct values: the C-MST method badly fails in this attempt to find the sparticle masses. Since the wedgebox plot clearly indicates that \(\tilde{\chi}^0_2\tilde{\chi}^0_3\), \(\tilde{\chi}^0_2\tilde{\chi}^0_4\) and \(\tilde{\chi}^0_4\tilde{\chi}^0_4\) production is substantial, the failure of the C-MST is not surprising given the inhomogeneity of the data set.

Now instead we perform an analysis for events in Zone 3 — which should be more homogeneous. Yet before proceeding one more feature of the wedgebox plot should be taken into account: there is a clear \(Z\)-line at \([M_{ee} \text{ or } M_{\mu\mu}] \simeq 91 \text{ GeV}\). This is due to neutralinos decaying to an LSP and a pair of leptons via an on-shell \(Z^0\) rather than

---

\(^{18}\)With the \(\tilde{\chi}^0_2\tilde{\chi}^0_3\) \((\tilde{\chi}^0_2\tilde{\chi}^0_4)\) events terminating at \(M_{ee} \text{ or } M_{\mu\mu} \simeq 80 \text{ GeV} (182 \text{ GeV})\). The endlines at \(\simeq 80 \text{ GeV}\) are faintly discernible in Fig. 6 however, the forthcoming analysis does not rely upon this.

---

Figure 8: Mass space scan for MSSM Test Point II, using 1000 random events from the wedgebox plot. The correct position of the masses is shown by the dotted lines.
via a slepton\textsuperscript{19}. The situation is greatly simplified if the events in Zone 3 are further curtailed to encompass only those for which \(100 \text{ GeV} < [M_{e\mu} \text{ and } M_{\mu\mu}] < 182 \text{ GeV}\) (as was done in the previous section). This will exclude the on-shell \(Z^0\) events along with those due to \(\tilde{\chi}_3^0\tilde{\chi}_3^0\) or \(\tilde{\chi}_3^0\tilde{\chi}_4^0\). The remaining fairly homogeneous subset of events still yields 1000+ signal events (corresponding to \(90 \text{ fb}^{-1}\) of LHC integrated luminosity), 80\% of which are in fact due to \(\tilde{\chi}_4^0\tilde{\chi}_4^0\) pair production (the remainder mostly involve colored sparticle decays into the heavier chargino, \(\tilde{\chi}_2^\pm\)). With this subset of events chosen with the wedgebox plot, a plot analogous to Fig. 8 is generated, as depicted in Fig. 9. This time the contours do cluster more closely in the vicinity of the correct mass point, though the steepest descent of \(N\) is still spread over a line segment of the form \(m_{\mu} = m_{\chi_1^0} + \Delta\).

Figure 9: Same as the previous plot, but now using 1000 events from Zone 3 away from the \(Z\)-line, i.e., \(M_{e\mu} > 100 \text{ GeV}\). Contours bunch together closer to the correct point, but the point of steepest descent is still ambiguous.

One reason for persisting inaccuracy is the fact that the momentum conservation relation (19) is only approximately true: the MC simulation indicates that each event actually has extra \(p_T\) (presumably from undetectable soft parton exchanges) in the range \(-30 \text{ GeV} < p_T < 30 \text{ GeV}\) (see Fig. 10 for an example distribution). This is formidable in light of the fact that a small inaccuracy in \(p_T\) (say \(2 \text{ GeV}\)) can change the number of solutions at a given mass point by 10\%. Using the correct \(p_T\) values, fits to masses are quite good (easily within 1\%). However, it is impossible in practice to know the precise \(p_T\) imbalance, and the best that can be done is to perform a scan over this extra \(p_T\) for each event, taking for instance \(-10 \text{ GeV} < p_T < 10 \text{ GeV}\). This is done in making Fig. 11 whose maximum does lie somewhat closer to the actual LSP mass; however, since the correct value of \(\Delta = m_{\mu} - m_{\chi_1^0}\) was assumed in this 1-D

\textsuperscript{19}Although the missing energy cut should eliminate most SM \(Z^0Z^0\), \(Z^0Z^0^*\) events, any such remnant background surviving would also populate this line.
Figure 10: $p_T$ distribution for one of the transverse directions. Here “real” includes all particles (except the LSPs), while “calculated” includes leptons and jets (after the selection criteria) only.

projection, the result is somewhat better than what would be obtained in practice.

Figure 11: 1-D projection of Fig. 9, assuming the correct value of $m_{\tilde{\nu}} - m_{\chi_1^0}$, and scanning over $-10 \text{ GeV} < p_T < 10 \text{ GeV}$. The maximum of the curve, $m_{\chi_1^0} = 98 \pm 5 \text{ GeV}$, roughly approximates the actual LSP mass (86 GeV), given by the red dashed line.

The fitted LSP mass in Fig. 11 has a small error but with a slightly up-biased central value (primarily due to the fact that the neutralino pairs do not have a fixed CM energy). The HT technique can in fact be used to check this result. In [19] it was discovered that the four-lepton invariant masses $M_4l$, $M_{22l}$, $M_{12l}$, and $M_{3l}$ are correlated in a unique way. In particular, a plot of $M_{3l}$ versus $M_4l \times M_{12l}$ or $M_{3l}$...
versus $M_4 \times \overline{M}_{2/2}$ for a suitably large event sample produces a visible \textit{curve} whose shape is strictly controlled by the slepton and LSP masses in the decay chain (20). Fig. 12 shows what these plots look like for \textbf{MSSM Test Point II}. Measurement of several points along these curves, (i.e., upper envelopes of the shapes formed in these scatter plots), yields correct central values for the LSP and slepton masses albeit with slightly larger uncertainties ($\pm 7$ GeV) than those found when just using C-MST.

4 Conclusions

Three ways to increase the accuracy and efficacy of the N-MST and C-MST mass reconstruction methods have been illustrated:

1. Use a wedgebox plot to select the most homogeneous sample of events. \textit{If the events analyzed do not mostly share the same decay topology, both techniques fail. If the wedgebox is a simple box, there is nothing to do}\footnote{It is true that at the Snowmass Benchmark point SPS1a \cite{20}, a simple box does describe the wedgebox topology of a few processes studied \textit{at this point}. But these SPS1a studies \cite{7, 11, 13} are certainly not representative of many other perfectly allowable parameter set choices and/or signature selections.}. However, over much of the allowable MSSM parameter space the topology of the wedgebox plot is not merely a lone box — if a wedge or composite structure is observed, then selecting events from the legs of the wedge or the outer areas proves the most effective. Note that this runs counter to the choices made in all the N-MST and C-MST publications. In \cite{10} for instance, some care is taken to describe the desirability of so-called ‘symmetric events’ — where both legs from the original

![Figure 12: HT plots for MSSM Test Point II: points on the top of each curve correspond to threshold endpoints which, when matched to formulae in \cite{19}, yield the LSP and slepton masses to a few percent.](image)
parent particle contain the same intermediate particle states. The present work, on the other hand, makes the case that the benefits from using un-symmetric decay legs, e.g. efficient isolation of events with the same decay chain structure, may well trump the convenience of symmetric events in the subsequent MST analysis, and therefore unsymmetric events should not be ignored or viewed as an unnecessary complication.

2. Use the Hidden Threshold technique to check decays are indeed on-shell. Since the pivotal system of equations (2)-(7) or (11)-(18) assume decays through on-shell sleptons, it is important to verify this assumption. The HT method can be employed to seek a ‘null’ result to the fit of various 4-lepton invariant mass minima.

3. Scan over the CM $p_T$ in a $\pm 10$ GeV-window. Assumptions that the partonic CM has no transverse momentum (as implied by equations (9) and (19)) are basically incorrect; while in the N-MST method this does not seem to matter, the C-MST method is much more sensitive to this parameter. An ‘averaging’ over $p_T$ improves the result, but perhaps a more detailed analysis should eventually be performed as the latest set of structure functions and other knowledge of QCD becomes available.

Note that the time scale required to collect a sufficient number of events to generate a wedgebox diagram or apply HT tests is roughly comparable to that needed to perform the MST analysis. This is in spite of the fact that the wedgebox and HT techniques rely upon populating scatter plots while an MST analysis in principle only requires collection of enough events to simultaneously solve the requisite equations. In practice, ambiguity in assigning the leptons and multiple solutions to the resulting quartic equation (see bulleted items in Secn. 2) as well as experimental factors (also enumerated earlier) necessitate a far larger sample of events to perform either of the MST analyses discussed. Further, and even more compellingly, without augmentation by the wedgebox and HT techniques, applying an MST analysis to a quite limited data set is tantamount to wild speculation as to what SUSY channels are actually present and the results of such an analysis must be viewed most cautiously.

As noted earlier, the MST analyses presented here assume that the decay chain involved is a series of two-body decays via intermediate on-mass-shell sleptons. This need not be the case, and this assumption should be tested. The di-lepton distribution shapes for on- and off-shell decays are not identical \cite{6,17}, and this could be used to distinguish between the two possibilities; however, the effects of cuts, backgrounds and a finite-sized data set must be considered. Ref. \cite{24} notes that distribution shapes for on-shell (sequential two-body decays) and off-shell (three-body decays) are effectively indistinguishable for some parameter choices. Also, Ref. \cite{25} finds that the shape of the di-lepton distribution may be affected by the nature of the neutralinos (the extent to which they are gauginos or higgsinos), illustrating how dynamical issues arising from the nature of the coupling involved in a decay may not be separable from purely kinematic issues associated with the relevant masses. So
an alternative to a straightforward examination of the di-lepton distribution shape is desirable \[9\]. In this study the hidden threshold technique \[19\] is used quite effectively to fill this need. Herein has been presented (see the C-MST study in Secn. \[3\]) a concrete example of how cross-correlating different invariant mass distributions resolves the on-shell vs. off-shell issue (as well as actually improving the accuracy of the final masses obtained). Alternative ideas have been put forward in other works. In \[20\] a rudimentary sketch of a very Dalitz-esque technique to look for the presence of two-body decay chains is presented. This concept is certainly related to the HT methodology, and a realistic study applying this idea would be interesting. Refs. \[9\] \[27\] instead champion a ‘Markov chain’ approach to analyzing the event sample where “no assumption is made about the processes causing the observed endpoints.” Supposedly then the issue of whether the sleptons are on-mass-shell or off-mass-shell is rendered moot to the more modest goal of determining a region of parameter space consistent with the data in a non-MST analysis \[22\].

The present work may be thought of as an initial installment of a much grander programme to fuse all known kinematic mass reconstruction methods together. The make-up of this programme consists not only of combining fits from different methods for a static event sample set, but also of improving the composition of the event set under consideration. In the present work where wedgebox and HT techniques are used to select (to the degree possible) events due to a specific \(\tilde{\chi}^0_i\tilde{\chi}^0_j\) neutralino pair decaying through on-shell sleptons \[23\]. Once a fairly homogeneous event sample is obtained \[23\] then it becomes straightforward to apply various mass reconstruction methods and cross-check them. For example, in the case of Higgs boson decay considered here, one could try matching the N-MST results presented here to those from a study of 4-lepton invariant mass endpoints \[28\] — it would be especially instructive to compare results at the SPS1a parameter point, for example, where both techniques, in the total ignorance of sparticle masses, give poor results individually, but may give a stronger result in unison \[29\].

An MST analysis is then an attractive option if enough mass-shell conditions can be found to match the number of mass components of the invisible final-state particles, as is the case for the LSP-generating SUSY decay chains considered herein. Further though, the present work shows how endpoint information funneled through wedgebox and HT techniques can positively supplement such an MST analysis, as in the augmented C-MST study presented in Secn. \[3\]. No mass-reconstruction technique is immune from possessing potentially faulty assumptions, and so coupling several complementary analysis techniques will in general improve reliability as well.

---

\[21\] See pages 50–51 therein.

\[22\] Though suggested, this issue is not explored in any detail in either of these works.

\[23\] Additional possibilities, e.g. that only one slepton is off-shell, or \(m_{\tilde{e}} \not= m_{\tilde{\mu}}\), are similarly testable utilizing wedgebox and HT techniques.

\[24\] An alternative track is attempted in Ref. \[27\], wherein the idea is to deal with all of the complexity of a mixed data set in the mass analysis program, rather than bifurcate the analysis into a purifying stage and then an analysis stage. The inherent weakness of this approach is that results from studying just the simplest subset of the events are impeded by the need to disentangle more confusing events.
as accuracy\textsuperscript{25}.

Likewise, consideration of suitable inclusive variables, such as the $m_{\gamma 2}$ variable \textsuperscript{30} and its variants \textsuperscript{31}, to augment either an MST study \textsuperscript{10} or an endpoint analysis \textsuperscript{10, 11, 32-35} has been shown to be beneficial, at least in some cases. Then there is the entire array of dynamical (and thus model-dependent) information associated with cross-sections and the shapes/spread of events plotted against one(or more) parameters. As noted earlier, kinematics can never really be totally divorced from the present dynamics. MSSM/mSUGRA studies combining information from cross-sections \textsuperscript{27} or distribution shapes \textsuperscript{35, 36} with that from an endpoint analysis have also been performed and are no doubt the vanguard of many more such studies, at least if initial LHC results prove favorable. And, when those first major blocks of data from the LHC become available, application of numerous analysis techniques — including wedgebox and HT techniques, would be a good idea.

References

[1] ATLAS Detector and Physics Performance Technical Design Report 2, Chapter 20, CERN-LHCC-99-015,ATLAS-TDR-15, May, 1999, http://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/TDR/access.html

[2] CMS Physics TDR 8.2 Volume II: Physics Performance, CERN/LHCC 2006-021.

[3] B. C. Allanach et al., JHEP 0103, 048 (2001).

[4] F. Paige, arXiv:9609373 [hep-ph]; I. Hinchliffe et al., Phys.Rev.D55, 5520 (1997). F. Gianotti, ATLAS note ATL-PHYS-97-110; I. Hinchliffe and F. E. Paige, Phys.Rev.D60, 095002 (1999). Phys.Rev.D61, 095011 (2000).

[5] For recent, more detailed experimentalist studies of such invariant mass differences, see for example B. Mura, CMS-TS-2007/004; N. Mohr, CMS-TS-2008/005.

[6] H. Bachacou, I. Hinchliffe and F. E. Paige, Phys.Rev.D62, 015009 (2000).

[7] B. K. Gjelsten, D. J. Miller and P. Osland, JHEP 0412, 003 (2004), JHEP 0506, 015 (2005), “Resolving ambiguities in mass determinations at future colliders,” In: Proc. 2005 Internatl. Linear Collider Workshop, Stanford, California March 2005, p. 211, arXiv:0507232 [hep-ph], arXiv:0511008 [hep-ph]; B. K. Gjelsten et al., ATLAS note ATL-PHYS-2004-007; B. K. Gjelsten, D. J. Miller, P. Osland and A. R. Raklev, “Mass ambiguities in cascade decays,” In: Proc. ICHEP 2006, Moscow, p. 1171, arXiv:0611080 [hep-ph].

\textsuperscript{25} Also minimizing overlapping information content between the analysis components will increase efficiency. Whether or not this is a significant issue would depend on how cpu-intensive the techniques are and on the computing resources available.
[8] D. J. Miller, P. Osland and A. R. Raklev, JHEP 0603, 034 (2006); E. Lytken, ATLAS note ATL-PHYS-COM-2004-001; J. M. Butterworth, J. Ellis and A. R. Raklev, JHEP 0705, 033 (2007);

[9] C. G. Lester, M. A. Parker and M. J. White, JHEP 0710, 051 (2007);

[10] B. C. Allanach, C.G. Lester, M. A. Parker and B. R. Webber, JHEP 0009, 004 (2000); D. R. Tovey, Czech. J. Phys. 53, A23 (2003), ATLAS note ATL-PHYS-CONF-2003-005; I. Borjanovic, J. Krstic, and D. Popovic, Czech. J. Phys. 53, A21 (2003), ATLAS-PHYS-CONF-2006-010.

[11] D. R. Tovey, JHEP 0804, 034 (2008);

[12] R. Horsky, M. Krämer, A. Mück and P. M. Zerwas, arXiv:0803.2603 [hep-ph].

[13] M. M. Nojiri, G. Polesello and D.R. Tovey, Proc. 3rd Les Houches Workshop, Les Houches, France, May 26 - June 6, 2003, arXiv:0312317 [hep-ph].

[14] K. Kawagoe, M. M. Nojiri and G. Polesello, Phys.Rev.D71, 035008 (2005).

[15] H.-C. Cheng et al., JHEP 0712, 076 (2007).

[16] M.M. Nojiri, G. Polesello and D.R. Tovey, arXiv:0712.2718 [hep-ph].

[17] M. Bisset et al., Eur.Phys.J.C45, 477 (2006).

[18] G. Bian et al., Eur.Phys.J.C53, 429 (2008).

[19] P. Huang, N. Kersting and H.H. Yang, arXiv:0802.0022 [hep-ph].

[20] B.C. Allenbach et al., “The Snowmass points and slopes: Benchmarks for SUSY searches,” In: N. Graf (ed.), Snowmass 2001: Proc. of APS/DPF/DPB Summer Study on the Future of Particle Physics, Snowmass, Colorado, July, 2001, p. 125, Eur. Phys. J. C 25, 113 (2002).

[21] G. Corcella et al., JHEP 0101, 010 (2001); G. Corcella et al., hep-ph/0210213
S. Moretti et al., JHEP 0204, 028 (2002).

[22] E. Richter-Was, D. Froidevaux and L. Poggioli, ATLAS note ATL-PHYS-98-131.

[23] H. Baer, F.E. Paige, S.D. Protopopescu and X. Tata, hep-ph/0001086

[24] C. G. Lester, M. A. Parker and M. J. White, JHEP 0601, 080 (2006).
[28] P. Huang, N. Kersting and H.H. Yang, Phys.Rev. D77, 075011 (2008).

[29] Work in progress.

[30] C. G. Lester and D. J. Summers, Phys.Lett. B463, 99 (1999); A. Barr, C. Lester and P. Stephens, J. Phys. G29, 2343 (2003); C. Lester and A. Barr, JHEP 0712, 102 (2007).

[31] A. J. Barr et al., JHEP 0303, 045 (2003); M. Serna, arXiv:0804.3344 [hep-ph].

[32] M. M. Nojiri, Y. Shimizu, S. Okada and K. Kawagoe, arXiv:0802.2412 [hep-ph].

[33] W. S. Cho, K. Choi, Y. G. Kim and C. B. Park, JHEP 0802, 035 (2008), Phys. Rev. Lett. 100, 171801 (2008).

[34] B. Griparios, JHEP 0802, 053 (2008); A. J. Barr, B. Griparios and C. G. Lester, JHEP 0802, 014 (2008).

[35] G. G. Ross and M. Serna, arXiv:0712.0943 [hep-ph].

[36] B. K. Gjelsten, D. J. Miller, P. Osland and A. R. Raklev, “Mass determination in cascade decays using shape formulas.” In: Proc. SUSY 06, Irvine, California, June 2006, p. 257, AIP Conf. Proc. 903, 257 (2007), arXiv:0611259 [hep-ph]; C. G. Lester, Phys.Lett. B655, 34 (2007);

Appendix A

The specific definitions of the 4-lepton kinematic invariants are as follows (lepton momenta labeled as in Fig. 1):

\[
M_{ul}^2 \equiv (p_1 + p_{1\nu} + p_2 + p_{2\nu})^2
\]

\[
\overline{M}_{22l}^4 \equiv \frac{\{(p_1 + p_{1\nu} - p_2 - p_{2\nu})^4 + (p_1 + p_{2\nu} - p_2 - p_{1\nu})^4 + (p_1 + p_2 - p_{1\nu} - p_{2\nu})^4\}}{3}
\]

\[
\overline{M}_{12l}^4 \equiv \frac{\{(p_1 + p_{1\nu} - p_2)^4 + (p_1 + p_{2\nu} - p_2)^4 + (p_1 + p_2 - p_{1\nu} - p_{2\nu})^4 + (p_1 - p_{1\nu} + p_{2\nu})^4 + (p_1 - p_{2\nu} + p_{1\nu})^4 + (p_2 - p_{1\nu} + p_{2\nu})^4 + (p_2 - p_{2\nu} + p_{1\nu})^4 + (p_2 - p_{1\nu} + p_{2\nu})^4 + (p_2 - p_{2\nu} + p_{1\nu})^4\}}{12}
\]

\[
\overline{M}_{3l}^4 \equiv \frac{\{(p_1 + p_{1\nu} + p_2)^4 + (p_1 + p_{2\nu} + p_2)^4 + (p_1 + p_{2\nu} + p_{2\nu})^4 + (p_1 + p_{2\nu} + p_{1\nu})^4\}}{4}
\]

\[
\overline{M}_{ul}^4 \equiv \frac{\{(p_1 + p_{1\nu})^4 + (p_1 + p_{2\nu})^4 + (p_1 + p_2)^4 + (p_2 + p_{2\nu})^4 + (p_2 + p_{1\nu})^4 + (p_{1\nu} + p_{2\nu})^4\}}{6}
\]
Appendix B

The off-shell minima are:

\[
M_{4l}^{\text{min}} = (m_j - m_1) \sqrt{2 + \frac{m_1}{m_j}}
\]

(23)

\[
\overline{M}_{2l2l}^{\text{min}} = (m_j - m_1) \left( \frac{2 + \frac{m_1^2}{m_j}}{3} \right)^{\frac{1}{4}}
\]

(24)

\[
\overline{M}_{3l}^{\text{min}} = (m_j - m_1) \left( \frac{11 + 10 \frac{m_1}{m_j} + 3 \frac{m_1^2}{m_j}}{8} \right)^{\frac{1}{4}}
\]

(25)

\[
\overline{M}_{ll}^{\text{min}} = (m_j - m_1) \left( \frac{3 + 2 \frac{m_1}{m_j} + \frac{m_1^2}{m_j}}{48} \right)^{\frac{1}{4}}
\]

(26)

For MSSM Test Point II, with \(m_j - m_1 = 182\) GeV, Fig. 7 yields the following values:

\[
M_{4l}^{\text{min}} = 215 \pm 5, \text{GeV}, \quad M_{2l2l}^{\text{min}} = 162 \pm 5, \text{GeV}, \quad M_{3l}^{\text{min}} = 170 \pm 5, \text{GeV}, \quad M_{ll}^{\text{min}} = 115 \pm 5, \text{GeV},
\]

where generous allowance has been allotted for measurement error. Matching each value against the corresponding four formulae above yields, \(m_1 < 0, m_1 < 102, \text{GeV}, m_1 < 0,\) and \(m_1 < 0,\) respectively. This is nonsense since the LSP cannot have negative mass (moreover, these negative values are all different).

The decay corresponding to the edge at \(M_{ee,\mu\mu} \sim 47\) GeV can be similarly analyzed, with the same conclusion.