Supplemental document accompanying submission to *Optics Express*

**Title:** Tunable Fiber Fabry-Perot Cavities with High Passive Stability

**Authors:** Carlos Saavedra, DEEPAK PANDEY, Hannes Pfeifer, Wolfgang Alt, Dieter Meschede

**Submitted:** 10/10/2020 11:31:41
Tunable Fiber Fabry-Perot Cavities with High Passive Stability

CARLOS SAAVEDRA,1,2,* DEEPAK PANDEY,1,† WOLFGANG ALT,1 HANNES PFEIFER1 AND DIETER MESCHEDE1

1Institut für Angewandte Physik, Universität Bonn, Wegelerstr. 8, 53115 Bonn, Germany
2División de Ciencias e Ingenierías, Universidad de Guanajuato, México
*carlos.salazar@iap.uni-bonn.de, †d.pandey@iap.uni-bonn.de
http://quantum-technologies.iap.uni-bonn.de/

Abstract: In the following, we present details of the fabrication process for building the three FFPCs presented in the main manuscript. All components and materials are described such that the interested readers can easily reproduce these systems. Details about the measurement of the frequency noise spectral density are given for all three FFPCs. The methods for combining finite element simulations with the modeling of thermal noise of the FFPCs are described. At the end, the thermal tunability of the realized FFPCs is presented.

1. FFPC fabrication

Besides the fiber mirrors (see Sec. 3 main text), the important components for the demonstrated FFPCs include the glass ferrules, piezo-electric elements, glues and the electrical contact wires, which are specified below.

Fig. 1. Photograph of a full slot FFPC, a compact and miniaturized high finesse resonator with wide frequency tuning and high stability, and a 1 euro cent coin (for size comparison). The FFPC slotted region allows easy access to the cavity mode volume.
1.1. Piezo-ferrule assembly preparation

**Piezo-electric element:** The piezo-element is a $10 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm}$ PIC151 ceramic (Model-no. 51638 from PI Ceramic GmbH), with relevant piezo modulus $d_{33} \approx -2 \times 10^{-10} \text{ C N}^{-1}$. The load-free expansion is specified to be $\Delta L_p = d_{33} \cdot L_p U / t_p$, where $U$ is the applied voltage, $L_p$ the length and $t_p$ the thickness of the piezo-element. The applicable voltage range for the used piezo-element is $-400 \text{ V}$ to $2 \text{ kV}$.

**Ferrules:** We used commercially available fused silica glass ferrules from VitroCom CFQ BLOCK FERRULE with $131 \mu\text{m}$ nominal inner bore diameter and a size of $8 \text{ mm} \times 1.25 \text{ mm} \times 1.25 \text{ mm}$.

**Assembly preparation:** As a first step towards preparing the assembly, the piezo electrodes are connected to vacuum compatible and kapton insulated copper wires (diameter 0.3 mm, 311-KAP-025, Allectra GmbH) by using a small amount of vacuum compatible conductive glue (two-components silver filled Epoxy, EPO-TEK® H20-E). For curing the glue, baking at $110 ^\circ \text{C}$ for $1 \text{ hour}$ is required according to the data-sheet. In the next steps, the glass ferrules used for the FFPCs are slotted using a diamond-plated thin wire as shown in the Fig. 2 (a). For the half slot FFPC, the ferrule is sawed at the center until the partial section of the bore is reached. Following, the slotted ferrule is glued to the piezo-electric element using conductive glue and then thermally cured. Full cuts for the other two (full slot, triple slot) FFPCs are made by first slotting the ferrule at the desired full cut position up to the half of the ferrule height, Fig. 2 (a). The ferrules are then glued to the piezo with slots facing as shown in the left part of Fig. 2 (b). Finally the full cuts are finished by slotting the ferrule from the top again such that the remaining glass at the already slotted positions is completely removed, Fig. 2 (c). To avoid damaging the fragile ferrule as well as to maintain the alignment of the bore, a careful sawing with small force is required.

The slotted ferrules are cleaned to remove residual particles from the bore using an iterative process of cleaning with high pressure air, followed by distilled water and acetone, and further using a bare fiber to remove trapped dust or particles from the bore of the ferrules.

1.2. Optical cavity fabrication

After the successful testing of the electrical contacts, the assembly is temporally fixed on an insulating base. Two high magnification USB-cameras (Dino-Lite digital microscope) are used to observe the ferrule bore and the central slot while inserting the fibers using translation stages. The distance between the two fiber mirrors is scanned using a piezo driven translation stage to observe the cavity resonance and thereby estimating the finesse and the coupling depths, where the coupling depth is defined as the ratio of the on-resonance versus off-resonance reflected light of a probe laser from the cavity. The dependence of these optical properties in relation to the length of the cavity is shown in Fig. 3. Although, the mirrors used in all FFPCs are expected to have the same transmission properties as mentioned in the main text, there are many factors which lead to slight differences in the optical properties (see main text Tab. 1) between the three FFPCs. These factors are, e.g., the decentration of the mirror with respect to the fiber, slightly different behaviour of the mirror transmission after annealing and possible contamination of the fiber mirrors during the assembling process.

The optical properties of the resonators also depend on the relative alignment of the two fiber mirrors. However, we observe that the cavity length adjustment using the guide provided by the ferrule maintains the resonator alignment and therefore the limitation of the achievable finesse is only due to the losses in the dielectric coating and due to the clipping of the beams on the fiber mirrors. The coupling depth, on the other hand, additionally depends on the mode matching between the fiber and the cavity mode. For the assembled FFPCs, we observe that the finesse before gluing the fibers is in most cases close to the one obtained after the mirrors are glued, Fig. 3.

Here we have used a cavity length of around $93 \mu\text{m}$ for the FFPCs. The fibers are glued using a
Fig. 2. Schematics for the piezo-ferrule assembly preparation. a) First, the ferrules are slotted at specific positions (depending upon the design choice) using a diamond-platted wire. b) The slotted ferrules are glued to the piezo-element. For the half slot FFPC, the assembly is ready at this point. c) Additional slotting is required for full slot and triple slot FFPCs.
low viscosity and vacuum compatible UV-curable glue (UV16, MasterBond®) as also described in the main text. The cavity length is adjusted close to the target resonance by observing the resonance peak while scanning the piezo and successively reducing the scan range. Once the UV-light is switched on, the scan is reduced to zero. We have observed that this procedure is good enough to keep the FFPC close to the target resonance while the glue hardens. For the full slot cavity (FSC) we have observed a degradation of the Finesse after gluing the mirrors (Fig. 3) which we believe is due to a contamination of the mirrors.

Fig. 3. Comparison of the optical properties of the three cavity designs before (line) and after (dot) gluing the fiber mirrors inside the ferrule bore. a) Finesse vs length, b) Coupling depth vs length for half slot cavity (HSC), full slot cavity (FSC) and triple slot cavity (TSC).

2. Laser frequency noise

The linewidth and frequency noise of the laser is measured by using a linewidth analyser from HighFinesse®. For the unlocked laser, the measured linewidth (Δν) is 20 kHz ≤ Δν ≤ 200 kHz with a typical value around 50 kHz. The measured laser frequency noise is shown in Fig. 4. These measurements confirm that the linewidth and the frequency noise of the laser are much smaller compared to the FFPC linewidth and the frequency noise of the locked cavity (see next section).

Fig. 4. Frequency noise spectral density of the laser system as measured with a HighFinesse® Linewidth Analyzer.
3. Frequency noise spectral density

To demonstrate the high passive stability of our monolithic FFPCs, we perform the analysis of the frequency noise spectral density (FNSD), $S_\nu$, of the error signal under different lock conditions (see main text Sec. 4.3). For this purpose, we measure the noise spectral density of the locked error signal using a spectrum analyser. The noise amplitude is then converted to the frequency noise of the cavity resonance by the slope of the PDH-error signal to obtain $S_\nu$. Fig. 5 shows the FNSD for three FFPCs for a sub-Hertz (blue) and a few kHz (red curve) locking bandwidths. The gray trace is the off-resonance (detection) noise limit. To gain insight about the noise distribution, we also show the integrated rms frequency noise, $\Delta \nu_{\text{rms}}$, in the inset for the respective FFPCs. The results of these measurements are summarized in Tab. 1. $\Delta \nu_{\text{rms}}$ is only slightly increased while locking even at sub-Hertz locking bandwidth as compared to higher locking bandwidths, testifying the high passive stability of the FFPCs.

As shown in the insets of Fig. 5, $\Delta \nu_{\text{rms}}$ is increased close to the 50 Hz, due to the coupling to the electrical line frequency, and at around 1 kHz by ambient acoustic noise. The latter is confirmed in a separate measurement using a microphone sensor.

Table 1. Frequency noise for the three FFPCs.

| Property                  | Half slot FFPC | Full slot FFPC | Triple slot FFPC |
|---------------------------|----------------|----------------|------------------|
| $\Delta \nu_{\text{rms}}$ (800 mHz LBW) | 0.48 MHz       | 0.73 MHz       | 1.19 MHz         |
| $\Delta \nu_{\text{rms}}$ (minimum)    | 0.37 MHz$^*$   | 0.40 MHz$^#$   | 0.64 MHz$^\#$    |
| $\Delta \nu_{\text{rms}}$ noise floor | 0.31 MHz       | 0.25 MHz       | 0.24 MHz         |

$^*$ 3.7 kHz LBW; $^#$ 3.7 kHz LBW; $^\#$ 1.7 kHz LBW; $\Delta \nu_{\text{rms}}$ - integrated for 10 Hz – 1 MHz
Fig. 5. Frequency noise spectral density (FNSD) of an FFPC locked to a narrow linewidth laser. Inset shows the cumulative rms frequency noise vs $\nu_{co}$, where $\nu_{co}$ is the rms integration cut-off frequency. a) half slot, b) full slot and c) triple slot FFPC cases for different locking bandwidths.
4. Finite-element modeling and noise spectral density calculation

To estimate the influence of the mechanical modes of the devices on the frequency stability, we performed finite element simulations (COMSOL [1]). The obtained mode frequencies and displacement fields are used to calculate the optomechanical coupling of the mechanical modes to the optical cavity mode. The resulting frequency noise spectral density of the optical mode is then retrieved via the fluctuation-dissipation theorem.

4.1. COMSOL simulations

For an overview of the basic modes and their coupling, a symmetrized geometry is simulated first, see Fig. 6 (a). Here, short silica fiber pieces (diameter=125 μm) that terminate with the end of the piezo-element (1 mm × 1 mm × 10 mm, triple slot piezo width: 2 mm) are used. The silica ferrule (1.25 mm × 1.25 mm × 10 mm) holds the fibers in a fitting bore. The fiber ends are separated in the center by the approximate cavity length. The material of the piezo-element is lead zirconate titanate. Its mechanical properties are simplified by only using the Y11 component of the Young’s modulus (63 GPa). The model does not include damping, hence the simulated eigenmode frequencies are real valued. The displacement fields and eigenfrequencies of two of the lowest order mechanical modes that couple to the optical resonator for all three designs are shown in Fig. 6 (b). These modes occur in all simulated geometries and correspond to a bending and stretching mode of the devices.

From the displacement fields $u(x, y, z)$ as retrieved in the simulations the effective mass $m_{\text{eff}}$ of each mechanical mode [2] for all geometries is calculated as

$$m_{\text{eff}} = \frac{\int_V dV \rho(x, y, z) \cdot |u(x, y, z)|^2}{\max_v (|u(x, y, z)|^2)},$$

(1)

with $V$ the full simulation volume and $\rho(x, y, z)$ the local density at position $(x, y, z)$. The zero point motion $x_{\text{rpm}}$ of each of the modes is then calculated using $x_{\text{rpm}} = \sqrt{\hbar/2m_{\text{eff}}\Omega_m}$, with $\Omega_m$ the simulated angular eigenfrequency of the respective mechanical mode. The vacuum optomechanical coupling rates $g_0$ are then extracted by

$$g_0 = G \cdot x_{\text{rpm}} \left( \frac{u_{z, \text{mirror},1}}{max_v (|u|)} - \frac{u_{z, \text{mirror},2}}{max_v (|u|)} \right),$$

(2)

where $G = -2\pi v_0/L_{\text{cavity}}$ is the optomechanical coupling to the optical mode at frequency $v_0$ and $L_{\text{cavity}}$ the cavity length (here fixed at 95 μm). The factor $u_{z, \text{mirror},i}/\max_v (|u|)$ scales the displacement along the cavity axis of each of the mirrors $i$ to the full maximum displacement of the mode, since the calculated $x_{\text{rpm}}$ refers to the maximum displacement position [2].

4.2. Noise spectral density modeling

The optomechanical coupling strengths and eigenfrequencies of the mechanical modes as retrieved from the simulations are used to calculate the expected frequency noise of the optical mode due to the thermal excitation of the mechanical resonances. This is achieved by applying the fluctuation-dissipation theorem to obtain the spectral density of displacement fluctuations of the mechanical modes of the devices. These are then coupled to the optical mode frequency by the respective optomechanical coupling strengths [3]. The resulting single-sided frequency noise spectral density $S_{\nu}(f)$ of the optical mode from a single mechanical mode at $\Omega_m$ is thereby given as

$$S_{\nu}(f)^2 = 2 \cdot \frac{S_{\text{rand}}(\Omega)}{4\pi^2} \approx \frac{2g_0^2}{4\pi^2} \cdot \frac{2\Omega_m}{\hbar} \cdot \frac{2\Gamma_m k_B T}{(\Omega^2 - \Omega_m^2)^2 + \Gamma_m^2 \Omega^2}. $$

(3)

Where the noise frequency is $f = \Omega/2\pi$, $k_B$ the Boltzmann constant, $T$ the temperature of the device and $\Gamma_m/2\pi$ the linewidth of the mechanical resonance. The rms frequency fluctuation

$$\Delta \nu = \sqrt{\int \frac{2g_0^2}{4\pi^2} \cdot \frac{2\Omega_m}{\hbar} \cdot \frac{2\Gamma_m k_B T}{(\Omega^2 - \Omega_m^2)^2 + \Gamma_m^2 \Omega^2} \frac{1}{T^2} d\Omega}.$$
Fig. 6. Simulation of the mechanical modes of the symmetrized FFPC geometry. (a) shows the different components and symmetry planes of the simulated devices. The columns in (b) show the geometry and the displacement fields of two typical mechanical modes occurring in all geometries. They are labelled with the respective simulated eigenfrequency. The last row in (b) shows a schematic depiction of the device displacement of the two modes. The frequency noise spectral densities for the three different geometries as expected from the thermal fluctuations of the simulated mechanical modes are plotted in (c). This can be translated into a rms frequency fluctuation by integrating up to a certain cutoff frequency as shown in (d). The linewidth of the mechanical modes was arbitrarily fixed at $\Gamma/2\pi = 1$ kHz for visualization. The noise for the triple slot FFPC in (c) and (d) is overestimated, since the motion of the fibers in the center ferrule part is for simplicity modelled as a free movement, which cannot accurately describe the sliding of the fibers.
up to a certain cutoff frequency $\nu_{co}$ is then calculated as $\sqrt{\int_0^{\nu_{co}} d\nu |S_\nu(f)|^2}$. Alternatively, the contributions of the mechanical modes can be used to obtain the rms frequency fluctuations by

$$\sqrt{\sum_j 2g_{0,j}^2 \langle n_j \rangle / 2\pi} [3],$$

with $g_{0,j}$ being the vacuum optomechanical coupling strength and $\langle n_j \rangle$ the expectation value of the mechanical mode occupancy ($k_B T / \hbar \Omega_{m,j}$ for $k_B T \gg \hbar \Omega_{m,j}$) of the $j$th mode. Figure 6 (c) and (d) show the resulting modeled frequency noise spectral densities of the optical mode and the corresponding integrated rms fluctuation for the three designs. Since the finite-element simulations did not take damping effects into account, the linewidth of the mechanical modes was arbitrarily fixed at $\Gamma_m/2\pi = 1$ kHz for all modes in the preliminary analysis. Note that the full ($\nu_{co} \to \infty$) rms frequency fluctuation does not depend on $\Gamma_m$.

For the final combination of measured frequency noise spectral densities and simulations, the frequencies and linewidths of the mechanical modes of the exemplarily chosen full slot FFPC were fitted in the measured spectrum. The fitted noise peaks were then attributed to mechanical modes in the simulations at close-by frequencies. To get a final estimate of the noise caused by the thermal fluctuations, $S_\nu(f)$ was calculated using the fitted mode frequencies and linewidths together with the vacuum optomechanical coupling strengths of the attributed mechanical modes from the simulations. Thereby slight shifts of the mechanical mode frequencies caused by deviations from idealized geometry are captured. Also, the corresponding linewidths are extracted without the necessity to include mechanical damping in the finite-element simulations.

4.3. Effect of structure asymmetries

In order to get a more realistic prediction of the expected frequency noise, a detailed model without the symmetry simplification and including some more properties of the final devices and some common imperfections was set up exemplarily for the full slot FFPC geometry, see Fig. 7. The introduced asymmetry causes more modes to couple to the optical resonance, e.g. sideward bending modes, whose coupling would otherwise be prevented by the symmetry. The longer fiber parts and the included silver glue pads at the bottom of the piezo cause the mechanical mode frequencies to slightly shift. More importantly, also the optomechanical coupling of the mechanical modes changes since the effective mass and the displacement fields are altered. For the model presented here, slightly smaller coupling strengths and thereby also smaller rms noise was retrieved from the more realistic model. Some pronounced noise peaks in the spectrum slightly below 100 kHz appear, which can also be found in the measured noise spectra. The final combination of measurement and simulation as presented in the main text used the more detailed model with the parameters $(\delta_{\text{longitudinal}}, \delta_{\text{lateral}}, \theta_{\text{tilt}}) = (1.9 \text{ mm}, 0.05 \text{ mm}, 0.2°)$ and a height of 0.5 mm for the glue pads on the bottom of the piezo (see Fig. 7).
Fig. 7. Effect of asymmetries on the modeled spectra and rms noise. (a) shows an exaggerated sketch of the asymmetries and imperfections that were included in the finite-element simulations. To get a more realistic estimate of the spectrum the parameters were chosen as \((\delta_{\text{longitudinal}}, \delta_{\text{lateral}}, \theta_{\text{tilt}}) = (1.9 \text{ mm}, 0.15 \text{ mm}, 0.3^\circ))\) here. In addition the length of the fiber pieces outside the ferrule was increased to 5 mm and the conductive glue points were included as silver pads at the bottom of the device (glue height: 0.7 mm, width fitting to the piezo-element). The comparison of the resulting spectra (exemplary \(\Gamma_{m,j}/2\pi = 1 \text{ kHz}\)) is plotted in (b). The asymmetry leads to a number of weakly coupled additional resonances as well as to a small reduction of the optomechanical coupling strengths, which results in a smaller estimate for the rms fluctuations in (c). Aside the integrated spectra also the separated contribution of each individual mode \(j\) to the rms noise \(\sqrt{2\hbar\omega_{0,j}^2} (\langle n_j \rangle) / 2\pi\) for both the original, simplified symmetric and for the more realistic asymmetric model are indicated.
5. Other FFPC properties

5.1. Thermal tuning

In addition to the fast piezo tuning as described in the main text, thermal tuning can also be used to change the cavity length. Fig. 8 shows the change in the cavity resonance frequency as the ambient temperature of single slot FFPC is varied. Here the FFPC is placed inside a thermally isolated box which has a Peltier element and a sensor to change and stabilize the temperature to a desired value. From the slope of the curve, the measured temperature tunability is 8 GHz/K.

![Graph showing thermal tuning](image)

Fig. 8. Thermal tuning of a temperature controlled and isolated Half slot FFPC. Tunability of 8 GHz/K is obtained from the measurement shown here.

References

1. COMSOL AB, “COMSOL Multiphysics® v. 5.2,” www.comsol.com. Stockholm, Sweden.
2. M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, *Cavity optomechanics: nano- and micromechanical resonators interacting with light* (Springer, 2014).
3. M. Gorodetsky, A. Schliesser, G. Anetsberger, S. Deleglise, and T. J. Kippenberg, “Determination of the vacuum optomechanical coupling rate using frequency noise calibration,” Opt. express 18, 23236–23246 (2010).