Broken SU(4) symmetry in a Kondo-correlated carbon nanotube

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Understanding the interplay between many-body correlations and non-equilibrium in systems with entangled spin and orbital degrees of freedom is central for many applications in nano-electronics. We demonstrate that hitherto unobserved many-body selection rules govern the Kondo effect in carbon nanotubes where spin and orbital degeneracy is broken by curvature induced spin-orbit coupling and valley mixing. They are dictated by properties of the underlying carbon nanotube spectrum at zero and finite magnetic field. Our measurements on a clean carbon nanotube are complemented by calculations based on a new approach to the non-equilibrium Kondo problem which reproduce the rich experimental observations in Kondo transport in high detail.

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I. INTRODUCTION

The Kondo effect [1] is an archetypical manifestation of strong electronic correlations in mesoscopic systems. While first observed in bulk metals with ferromagnetic impurities, it was shown to lead to a distinct zero-bias anomaly in the differential conductance of semiconductor quantum dots with odd electronic occupations [2–4]. A degeneracy of quantum states required for its occurrence is usually provided by the electronic spin degree of freedom, resulting in the so-called SU(2) Kondo behavior. Remarkably, in the Kondo regime the differential conductance obeys universal scaling as a function of temperature [5], bias voltage [6], and magnetic field [7].

Clean carbon nanotubes (CNTs) [7] provide a unique test-bed for the investigation and manipulation of the quantum dot level structure and its consequences for the Kondo effect. In CNTs an additional degeneracy in the intrinsic low energy spectrum stems from two (K, K') graphene Dirac points and enables one to study unconventional correlation phenomena such as the orbital SU(2) as well as spin plus orbital SU(4) Kondo effects both experimentally [8] and theoretically [9–15]. Besides revealing the curvature induced spin-orbit interaction [16–19], experiments indicate that also the K-K' degeneracy is frequently lifted with a finite energy $\Delta_{KK'}$ [20, 21]. While usually attributed to the presence of disorder in damaged or contaminated CNTs, it is observed also in clean carbon nanotubes as a contribution from the CNT’s longitudinal boundaries. In the following, we demonstrate how this type of SU(4) symmetry breaking leads to unconventional Kondo transport phenomena. Our results clearly show that the Kondo behavior at zero magnetic field is controlled by time reversal symmetry, which allows to identify two distinct, two-fold degenerate Kramers doublets. The analysis of the breaking of this symmetry in a magnetic field parallel ($B_{||}$) or perpendicular ($B_{\perp}$) to the CNT axis leads to a detailed understanding of the many-body processes contributing to transport in the Kondo regime. It allows to disentangle the role of the conjugation relations and goes significantly beyond earlier studies on Kondo ions [22–24], where Kondo satellites could be observed but the tunability of the spectrum by a magnetic field was not given. In particular, we elucidate the reasons for the absence of some of the many-body transitions expected from previous theoretical works [12, 14].

II. TRANSPORT MEASUREMENTS

Electronic transport measurements have been performed on a clean, freely suspended single-wall carbon nanotube (CNT) contacted with rhenium and capacitively coupled to a global back gate at milli-Kelvin temperatures. The carbon nanotube was grown by chemical vapor deposition across pre-defined trenches and electrode structures to minimize damage and contamination mechanisms [7, 25]. As seen in the low-bias conductance (Fig. 1(a)), a small band gap separates a Fabry-Pérot pattern in the highly transparent hole regime from sharp Coulomb blockade oscillations in the few electron regime ($N_{el} < 10$). With increasing gate voltage enhanced conductance is observed, leading in particular to a Kondo zero-bias anomaly in the odd electron number valleys. The electronic setup used for the measurements is sketched in Figure 1(b). A dc- and an ac-voltage are superimposed and applied as bias voltage $V_{sd} = V_{dc} + V_{ac}$ to the source contact. The current from the drain contact is converted to a voltage and measured with a lock-in amplifier. The highly positive doped silicon substrate acts as global back gate.

In the following, we focus on the intermediate coupling regime and measure the differential conductance as a function of gate voltage $V_g$ and bias voltage $V_{sd}$ (Fig-
FIG. 1. Measurement of the Kondo effect in transport through an ultra-clean carbon nanotube. (a) Low-bias conductance ($V_{sd} = 0.2 \text{ mV}$) through a clean, suspended small band gap carbon nanotube as function of applied gate voltage $V_g$. (b) Device geometry and electronic measurement setup for differential conductance measurement. (c) Differential conductance $G(V_g, V_{sd}) = dI(V_g, V_{sd})/dV_{sd}$ inside and around the gate voltage window with $N_{sd} = 21$ ($T = 30 \text{ mK}$). A sharp Kondo ridge at zero bias voltage $V_{sd} = 0$ and broader satellite ridges at finite bias $V_{sd} \approx \pm 0.5 \text{ mV}$ are clearly visible. (d) Line traces $G(V_{sd})$ at constant $V_g$ with equidistant spacing $\Delta V_g = 7 \text{ mV}$ corresponding to colored dotted lines in (c), with $V_{sd}$ rescaled by the corresponding Kondo temperature $T_K$ (see text). The central conductance peak at $V_{sd} = 0$ displays the universal Kondo behavior, while the satellite peaks move. A similar plot across different Coulomb oscillations can be found in the Supplement, Fig. S6.

ure\(1^{(c)}\)). Besides the pronounced conductance ridge at zero bias voltage, additional broad satellite peaks appear symmetrically at finite bias voltage $V_{sd} \approx \pm 0.5 \text{ mV}$, depending only weakly on the gate voltage. In analogy to the case of a broken spin degeneracy in a magnetic field, these satellite peaks at zero magnetic field signal a lifted degeneracy of the ground state, allowing inelastic transport processes to take place. Finite bias conductance peaks have already been observed in CNT quantum dots with partly-filled shells $^{20, 27, 30}$ and have been attributed to – possibly Kondo-enhanced – inelastic co-tunneling transitions.

Non-equilibrium co-tunneling is a threshold effect which gives rise to a step-like cusp in the differential conductance. Kondo correlations treated within lowest order perturbation theory yield a logarithmic enhancement of this cusp which emphasizes further the threshold effect $^{28}$. The focus of our experiment lies on the low temperature regime $T \ll T_K$, where a perturbative treatment of Kondo correlations is no longer appropriate. At such low temperatures, true Kondo peaks at finite bias, rather than co-tunneling cusps, are expected to develop in CNTs with broken $SU(4)$ symmetry $^{12, 15}$. First indications of a split Kondo peak, possibly resulting from spin-orbit interaction, have been recently reported $^{31, 32}$. The experiment of Cleuziou et al. $^{32}$ had focused on the analysis of the spectra based on the single particle theory at high magnetic fields parallel to the CNT axis. However, no systematic analysis of the conductance traces across the Kondo resonance has been performed so far. We find that the conductance traces in Figure 1(d) are dominated by nonequilibrium Kondo correlations rather than simple co-tunneling processes. As we will show, the Kondo correlations lead to novel many-body selection rules governed by the intrinsic symmetry properties of the CNT-Hamiltonian.

III. UNIVERSALITY

To unambiguously claim that a zero-bias anomaly observed in experiments has Kondo origin, the characteristic universal scaling behavior with the energy scale determined by the Kondo temperature $T_K$ has to be tested. We record conductance traces $G(V_{sd})$ at different discrete gate voltage values $V_g$ within the 21st Coulomb diamond. For each such trace, we determine the Kondo temperature $T_K(V_g)$ in non-equilibrium from the central peak in the bias voltage trace using the condition $G(k_B T_K/e) \approx 2G_0/3$ $^{33}$ and $G_0 = G(V_{sd} = 0)$.
FIG. 2. Temperature dependence of the main Kondo resonance and its satellites. (a) Measured differential conductance traces \( G(V_{sd}) \) at different temperatures, normalized by \( G_0 = G(V_{sd} = 0) \) for the lowest temperature at \( V_g = 2.39 \) V; the bias voltage is scaled with the Kondo temperature \( T_K \) = 0.86 K. The central conductance peak follows the universal behavior expected for the Kondo effect. (b) Differential conductance obtained from our field-theoretical calculation. The distance between the satellite peaks at \( T = 0 \) is \( 2\Delta = 13.8 \) \( k_B T_K \).

We then rescale the bias voltage with the respective Kondo temperature, and normalize the conductance to its maximum value \( G_0 \). The collapse of all curves \( G(\epsilon V_{sd}/k_B T_K)/G_0 \) around \( V_{sd} = 0 \) into universal behavior, as illustrated in Fig. S3 of the Supplement where we show that the universal line shape of the central Kondo resonance remains essentially unchanged also in the transition between \( SU(2) \) and \( SU(4) \).

As can also be seen in Fig. 1(d), after this rescaling the position of the satellite peaks varies, i.e., here the universality is apparently lost. This is to be expected since the Kondo temperature \( T_K \) varies within the Coulomb valley region but the splitting \( \Delta \) between central peak and satellites is gate independent. Complete universality of the differential conductance, i.e., universality in the whole range of voltages requires the ratio \( \Delta/k_B T_K \) to be invariant. In general one obtains \( T_K(\Delta) = T_K(0) f[\Delta/k_B T_K(0)] \), where \( T_K(0) = T_K(4) \) is the Kondo temperature for the \( SU(4) \) Kondo effect and \( f(x) \) depends on the strength of the \( SU(4) \) symmetry breaking. In our experiment we find \( \Delta/k_B T_K \simeq 7 \), meaning \( \Delta/k_B T_K^{SU(4)} < 7 \). As shown in Ref. [30], this implies that in our experiment the \( SU(4) \) symmetry is weakly broken. This is in contrast to the experiments in Ref. [30] where the much larger \( \Delta/k_B T_K \) ratio leads to an \( SU(2) \) Kondo effect in the odd Coulomb valleys.

Figure 2(a) displays the temperature dependence of \( G(V_{sd}, T) \) in the center of the same Coulomb diamond \( (N_{el} = 21) \), where \( V_g = 2.39 \) V. The central peak behaves in a way characteristic for the Kondo effect: it is suppressed and broadened for increasing temperatures. The satellite peaks are increasingly washed out at elevated temperatures. The Kondo temperature is determined from the differential conductance \( G(V_{sd}) \) at 30 mK to be \( T_K(V_g = 2.39 \) V \) = 0.86 K.

IV. MODELING AND NONLINEAR TRANSPORT THEORY

A quantitative analysis of our experimental results has to combine the properties of the underlying set of single-particle states with the Kondo-correlations. We have developed a nonequilibrium field theory based on the slave boson Keldysh effective action formalism \[37\]. An \( SU(2) \) formulation of this theory has been presented in Refs. [35, 39] and an \( SU(4) \) formulation (including a broken \( SU(4) \)) is presented in this work in Section II of the Supplementary Material. The theory is based on the minimal model Hamiltonian \( H_{CNT} \) for a single longitudinal mode of a CNT quantum dot in magnetic field as given in Eq. (S1) of the Supplementary Material. Accounting for the four eigenstates \( \{ |i\rangle, i = 1, 2, 3, 4 \} \) of \( H_{CNT} \), the theory provides an approximate analytical expression for the tunneling density of states \( \nu(\epsilon) \) of the quantum dot. This expression is then used to evaluate the differential conductance as a function of the temperature, bias voltage and magnetic field over the whole energy range relevant for Kondo physics according to the Meir-Wingreen formula \[40\], cf. Eq. (S58) of the Supplement. Figure 2(b) displays the differential conductance obtained from our calculation. We obtain an excellent and nearly quantitative agreement between experiment and theory; the evolution of peak amplitudes with temperature is reproduced accurately. Note that our calculation is tailored to Kondo-correlated transport and omits some of the inelastic co-tunneling processes in lowest order. This explains why the high-bias conductance tail present in the data is only partly reproduced by our calculation.

In the simplest model one expects a four-fold degenerate level at the energy \( \epsilon_d \) \[41\]. This degeneracy is split by KK’ valley mixing (with the characteristic energy scale \( \Delta_{KK’} \)) and spin-orbit interaction (with the energy \( \Delta_{SO} \)). We denote the four resulting energies associated to the eigenstates \( |1\rangle, |2\rangle, |3\rangle, |4\rangle \) by \( \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \), respectively.
At zero magnetic field, the effective CNT-Hamiltonian \( \hat{H}_{\text{CNT}}^{(0)} \) (see Eq. (S2) in the Supplement) displays time-reversal (TR) symmetry \([42]\) governed by the operator \( \hat{T} \). Moreover, two additional operators, \( \hat{P} \) and \( \hat{C} \), can be introduced, which anticommute with \( \hat{H}_{\text{CNT}}^{(0)} \) and which allow to connect the states within one quadruplet in the way depicted in Fig. 3(a) (see Section I of the Supplementary Material for further details). We call the operations related to \( \hat{P} \) and \( \hat{C} \) particle-hole (PH) \([43]\) and chiral (C) conjugation, respectively.

It follows from the symmetry analysis that the diagonalization of \( \hat{H}_{\text{CNT}}^{(0)} \) results in two Kramers doublets with the spacing \( \Delta = \sqrt{\Delta_{\text{KK}}^2 + \Delta_{\text{SO}}^2} \) symmetrically located around the reference energy \( \varepsilon_d \), see Fig. 3(b) and Section I A of the Supplement. The \( \hat{T} \)-conjugated pairs of states are the Kramers doublets (1, 2) and (3, 4), with \( \varepsilon_1 = \varepsilon_2 \), and \( \varepsilon_3 = \varepsilon_4 = \varepsilon_1 - \Delta \). As seen in Fig. 3(a), the operator \( \hat{P} \) conjugates states from different Kramers doublets; the pairs are (1, 4) and (2, 3) with \( \varepsilon_1 - \varepsilon_4 = -\varepsilon_1 \) and \( \varepsilon_3 - \varepsilon_2 = -\varepsilon_1 \). If TR and PH conjugation hold, then so does in addition chiral conjugation, which is represented by the operator \( \hat{C} = \hat{P} \hat{T}^{-1} \). The chirally conjugated pairs are (1, 3) and (2, 4).

The zero-bias Kondo peak is necessarily induced by transitions between the degenerate states of the time-reversed Kramers pairs (1, 2) and (3, 4), which we call ’inter-Kramers’ transitions, see Fig. 3(c). A similar reasoning applies for the finite bias Kondo peaks at voltages equal to \( \pm \Delta/\epsilon \): the inelastic peaks are necessarily induced by transitions between distinct Kramers pairs, called in the following ’inter-Kramers’ transitions. As revealed by the evolution of the satellite peaks in magnetic field discussed below, they only involve the chiral pairs (1, 3) and (2, 4) [see Fig. 3(d)]. It is striking that the behavior in the Kondo regime is distinct from simple co-tunneling spectroscopy at higher temperatures, where also transition lines corresponding to \( \hat{P} \)-conjugated states could be observed in a magnetic field \([20]\).

**V. CONJUGATION RELATIONS IN MAGNETIC FIELD**

The central objective of our work is to show how the conjugation relations of the single particle spectrum are reflected in the many-body transitions contributing to the Kondo resonances. Figure 4(a) and 4(b) display the dispersion of the four single particle states in a magnetic field perpendicular and parallel to the tube axis. In a finite magnetic field, \( \mathbf{B} \), TR symmetry is broken. However, it is clearly visible that conjugation relations persist that lead to close connections between the single particle energies, e.g. \( \hat{T}|1, \mathbf{B}| = |2, -\mathbf{B}| \) leads in parallel field to \( \varepsilon_1(B) = \varepsilon_2(-B) \), and \( \hat{P}|1, \mathbf{B}| = |4, \mathbf{B}| \) leads to \( \varepsilon_1(B) - \varepsilon_2(B) = \Delta_1(B)/2 \), with \( \varepsilon(B) \) and \( \Delta_1(B) \) being a magnetic field dependent reference energy and level splitting, respectively (see Fig. S1 of the Supplement). A complete survey of the conjugation relations is found in Sec. I of the Supplement.

A central result of our many-body theory is now that for the many-body tunneling density of states \( \nu_j(\epsilon) \) analogous relations hold [e.g. \( \nu_1(\epsilon, B) = \nu_2(\epsilon, -B) \)], \( \nu_3(\epsilon, B) = \nu_4(\epsilon, -B) \) if the Keldysh effective action is constructed such that it obeys the underlying symmetry requirements [see Supplement, Sec. II E]. Importantly, the tunneling density of states \( \nu_j(\epsilon) \) contains many-body correlations and non-equilibrium effects through its self-energy \( \Sigma_j(\epsilon) \), which accounts for all virtual transitions which, with the help of a lead electron of energy \( \epsilon' \), change an initial dot state \( i \) to a final dot state \( j \). The energy balance is then provided by \( \epsilon = \epsilon' + \epsilon_i - \epsilon_j \). Note that \( i \) and \( j \) are not equal, since Kondo transitions always involve a flip of quantum numbers [see Figs. 3(c) and 3(d)]. As a result, our low energy expansion of the effective action does not contain the state \( j \) as final state.

Conjugation relations for \( \nu_j(\epsilon) \) also imply conjugation relations for its self-energies \( \Sigma_j(\epsilon) \), e.g., \( \Sigma_1(\epsilon, B) = \Sigma_2(\epsilon, -B) \) and \( \Sigma_3(\epsilon, B) = \Sigma_4(\epsilon, -B) \). In turn, these relations determine which final state \( i \) is accessible from an initial state \( j \): together with the constraint \( i \neq j \), the conjugation relations also imply that \( i \) is different from the PH partner of \( j \), \( \hat{P}j \) [see Eq. (S72) in the Supplement]. To see why \( \hat{P}j \) is not in the low energy expansion of the effective action, consider e.g. \( \Sigma_1(\epsilon) \), which explicitly depends on \( \Delta_1(B) \) through \( \varepsilon_1(B) \). Then \( \Sigma_1(\epsilon, -\Delta_1(B)) \) yields an energy balance \( \epsilon = \epsilon' + \epsilon_1 - \epsilon_1(\Delta_1(B)) = \epsilon' + \epsilon_1 - \epsilon_3(\Delta_1(B)), i \neq 1 \). A conjugation relation \( \Sigma_1(\epsilon, -\Delta_1(B)) = \Sigma_4(\epsilon, \Delta_1(B)) \) can then only hold if also \( i \neq 4 \). Non-Kondo transitions, which leave the state of the quantum dot unchanged, are absent in the low energy expansion of the effective action. Notice that this is different from the co-tunneling regime \( T \gg T_K \), where transitions which do not change the state of the dot also contribute to transport. In this latter case there is no requirement of a flip of a quantum number and also virtual transitions between \( \hat{P} \)-conjugated pairs are allowed.

**VI. EVOLUTION OF THE KONDO PEAKS IN MAGNETIC FIELD**

The behavior of the Kondo peaks in magnetic field, reported in Figs. 5 and 6, provides a sensitive tool that allows us to discriminate between the different types of Kondo-enhanced transitions. In fact, the positions of the Kondo peaks are related to the energy differences between the two dot states involved in the transition, and these depend very differently on direction and strength of the magnetic field for transitions between \( \hat{T}, \hat{C} \), or \( \hat{P} \) pairs. The central Kondo peak results from intra-Kramers transitions and its splitting reveals the breaking of time reversal symmetry. From our theory a spli-
FIG. 3. Conjugation relations, level spectrum, and selected Kondo transport processes. (a) Energy levels associated with a longitudinal mode of a CNT accounting for spin and valley degrees of freedom. The time-reversal operator $\hat{T}$ connects the states $(1, 2)$ and $(3, 4)$. The operators $\hat{C}$ and $\hat{P}$ govern chiral and particle-hole conjugation and provide further pairs of conjugated states. Chiral pairs are $(1, 3)$ and $(2, 4)$; particle-hole pairs are $(1, 4)$ and $(2, 3)$. (b) Spin-orbit coupling and valley mixing break the four-fold degeneracy but not the time-reversal symmetry. The spectrum splits in two degenerate Kramers doublets $(1, 2)$ and $(3, 4)$, respectively, separated by the energy difference $\Delta = \sqrt{\Delta_{KK}^2 + \Delta_{SO}^2}$. (c) The Kondo peak at zero bias is governed by virtual processes within the Kramers pairs $(1, 2)$ and $(3, 4)$ (intra-Kramers transitions). (d) The satellite peaks at finite bias, $V_{sd} = \pm \Delta/e$, result from inelastic transitions within the chiral pairs, $(1, 3)$ and $(2, 4)$ (chiral inter-Kramers transitions).

FIG. 4. Evolution of the energy levels in a magnetic field. (a,b) Single particle level spectrum in a magnetic field (a) perpendicular or (b) parallel to the CNT axis. The energy eigenstates are labeled $1 \sim 4$. At $B = 0$ there are two energy degenerate Kramers doublets $(1, 2)$ and $(3, 4)$ separated by $\Delta$. The energy difference within each chiral pair, i.e., $(1, 3)$ and $(2, 4)$, remains essentially independent of $B_{\perp}$, provided that $B_{\perp} \lesssim \Delta_{KK}/g_s\mu_B$. The level splitting within each Kramers doublet is $\delta_{\perp} = g_s\mu_B B_{\perp}$. An avoided level crossing of the particle-hole pair $(2, 3)$ occurs when $B_{\perp} \approx \Delta_{KK}/g_s\mu_B$. For $B_{\parallel}$ the Aharonov-Bohm effect induces a level crossing of the pair $(3, 4)$, as indicated by the blue arrows. (c) Visualization of the level separations from (a) and (b) for values of $B_{\perp}$ and $B_{\parallel}$ corresponding to the dashed lines.
ting of the central Kondo resonance is expected once the energetic separation within a Kramers doublet exceeds a threshold value $\varepsilon_c = 0.6\, k_B T_K$ as observed in Figs. 5(a),(b) and 5(d),(e).

Puzzling at first glance is the independence of the positions of the satellite Kondo peaks on the perpendicular field [Figs. 5(a),(b) and 5(a)]. This is in strong contrast to the co-tunneling regime investigated earlier (see Fig. 3 in Ref. 20), where a splitting of the inelastic co-tunneling line was observed as a result of two possible sets of transitions: within the pair (4, 2) or (4, 1) for positive, and within pair (3, 1) or (3, 2) for negative field orientation [cf. Fig. 3(b)]. In our case only the transitions between the $\mathcal{C}$-conjugated states, (4, 2) and (3, 1), are observed while the transitions between the $\mathcal{P}$-conjugated states, (4, 1) and (3, 2), are absent. This observation is in nearly perfect agreement with the results of our many-body theory plotted in Fig. 5(c). No Kondo-enhancement of the virtual transitions (4, 1) and (3, 2) occurs as a consequence of the symmetry constraints imposed onto the Keldysh action. These constraints reduce the allowed number of Kondo peaks expected in a perpendicular magnetic field with respect to earlier theoretical predictions (cf. Refs. 12, 13).

On the other hand, a splitting of the satellite Kondo peaks is expected in a magnetic field parallel to the tube axis because, according to Figs. 5(b),(c), the energy levels of single particle states 2 and 4 (1 and 3) no longer evolve in parallel at low fields, but are mutually tilted by the Aharonov-Bohm effect. Inspection of Figs. 5(d) and 5(e) shows that a splitting of the Kondo satellites is indeed observed, in qualitative agreement with our theoretical result displayed in Fig. 5(f). Note that the following pa-
parameters of our model Hamiltonian are extracted from the experimental data: the ratio \( \Delta / k_B T_K \) (see Fig. 2), the ratio \( \Delta_{\text{SO}} / k_B T_K \), which is obtained from the evolution of the splitted central Kondo peaks according to Eqs. (S27) and (S40), and, for the parallel field case only, the ratio \( g_{\text{orb}} / g_s \) of orbital and spin \( g \)-factor, which is computed from Eq. (S41). These three parameters are used to generate the theoretical curves in Figs. 5(c) and 5(f).

Finally we focus on the critical behavior of the Kondo peaks in a magnetic field \( \vec{B} \). A single Kondo peak is expected as long as the level separation of the underlying single particle states does not exceed the already mentioned threshold value \( \varepsilon_c \). In order to check our prediction we compare the maxima of the \( G(V_{\text{sd}}) \) traces [orange and blue dots in Fig. 5(a),(b)] with the results of the many body theory and the energy differences of the underlying single particle levels [thick and thin lines in in Fig. 5(a),(b)], respectively. We used the same independently determined parameters as in Fig. 5. It is clearly seen that the many body calculation matches our experimental data: the ratio \( \Delta / k_B T_K \) of orbital and spin \( g \)-factor, which is computed from Eq. (S41).

The non-linear dispersion of the positions of the Kondo peaks reflects the protection of the Kondo state against perturbations on energy scales below \( k_B T_K \). The absence of a splitting of the satellite peaks in the perpendicular field is again very prominent in Fig. 5(a), where the position of the satellite peaks is essentially free of dispersion. A careful inspection of the second derivative of the \( I(V_{\text{sd}}) \) confirms that the lines corresponding to transitions between \( \vec{P} \)-conjugated states are indeed absent. On the other hand, the satellite peaks do split in a parallel magnetic field, where the Aharonov-Bohm effect acts differently on the two pairs of \( \vec{C} \)-conjugated states (see Fig. S1 in the Supplement). The agreement is only qualitative in this case, indicating most probably that our simple model Hamiltonian becomes insufficient at higher magnetic fields.

An additional discussion of the evolution of the conductance peaks for a larger range of bias voltages and parallel magnetic fields can be found in Section III of the Supplementary Material.

VII. CONCLUSION

In conclusion, our work provides a first systematic experimental and theoretical investigation of the Kondo effect in carbon nanotubes in the presence of both spin-orbit coupling and valley mixing. The wide tunability of the carbon nanotube spectrum by magnetic fields allows to elucidate the role of symmetry and conjugation relations. Despite the symmetry breaking by spin-orbit interaction and valley mixing the underlying operators still give rise to conjugation relations between certain states that have to be respected by the transport theory. The presence of these relations in non equilibrium dynamics leads to many-body selection rules that explain...
unexpected “missing” resonances in the Kondo transport spectrum.

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[1] J. Kondo, Prog. Theo. Phys. 32, 37 (1964).
[2] D. Goldhaber-Gordon et al., Nature 391, 156 (1998).
[3] D. Goldhaber-Gordon et al., Phys. Rev. Lett. 81, 5225 (1998).
[4] S. M. Cronenwett, T. H. Oosterkamp, and L. P. Kouwenhoven, Science 281, 540 (1998).
[5] M. Grobis, I. G. Rau, R. M. Potok, H. Shtrikman, and D. Goldhaber-Gordon, Phys. Rev. Lett. 100, 246601 (2008).
[6] M. Gaass, A. K. Hüttel, K. Kang, I. Weymann, J. von Delft, and C. Strunk, Phys. Rev. Lett. 107, 176808 (2011).
[7] J. Cao, Q. Wang, and H. Dai, Nature Mat. 4, 745 (2005).
[8] F. Jarillo-Herrero, J. Kong, H. S. J. van der Zant, C. Dekker, L. P. Kouwenhoven, and S. De Franceschi, Nature 434, 484 (2005).
[9] L. Borda, G. Zarand, W. Hofstetter, B. I. Halperin, and J. von Delft, Phys. Rev. Lett. 90, 026602 (2003).
[10] M.-S. Choi, R. López, and R. Aguado, Phys. Rev. Lett. 95, 067204 (2005).
[11] J. S. Lim, M.-S. Choi, M. Y. Choi, R. López, and R. Aguado, Phys. Rev. B 74, 205119 (2006).
[12] T.-F. Fang, W. Zuo, and H.-G. Luo, Phys. Rev. Lett. 101, 246805 (2008).
[13] T.-F. Fang, W. Zuo, and H.-G. Luo, Phys. Rev. Lett. 104, 169902 (2010).
[14] J. S. Lim, R. López, G. L. Giorgi, and D. Sánchez, Phys. Rev. B 83, 155325 (2011).
[15] M. R. Galpin, F. W. Jayatilaka, D. E. Logan, and F. B. Anders, Phys. Rev. B 81, 075437 (2010).
[16] F. Kuemmeth, S. Ilani, D. C. Ralph, and P. L. McEuen, Nature 452, 448 (2008).
[17] M. Dell Valle, M. Marginańska, and M. Grifoni, Phys. Rev. B 84, 155427 (2011).
[18] G. A. Steele, F. Pei, E. A. Laird, J. M. Jol, H. B. Meierwaldt, and L. P. Kouwenhoven, Nature Comm. 4, 1573 (2013).
[19] T. Ando, J. Phys. Soc. Jpn. 69, 1757 (2000).
[20] T. S. Jespersen, K. Grove-Rasmussen, J. Paaske, K. Muraki, T. Fujisawa, J. Nygård, and K. Flensberg, Nature Physics 7, 348 (2011).
[21] K. Grove-Rasmussen, S. Grap, J. Paaske, K. Flensberg, S. Andergassen, V. Meden, H. I. Jorgensen, K. Muraki, and T. Fujisawa, Phys. Rev. Lett. 108, 176802 (2012).
[22] M. Garnier, K. Breuer, D. Purdike, M. Hengsberger, Y. Baer, and B. Delley, Phys. Rev. Lett. 78, 4127 (1997).
[23] F. Reinert, D. Ehm, S. Schmidt, G. Nicolay, S. Hüfner, J. Kroha, O. Trovarelli, and C. Geibel, Phys. Rev. Lett. 87, 106401 (2001).
[24] S. Ernst, S. Kirchner, C. Krellner, C. Geibel, G. Zwicknagl, F. Steglich, and S. Wirth, [Nature 474, 362 (2011)]
[25] J. Kong, H. T. Soh, A. M. Cassell, C. F. Quate, and H. Dai, Nature 395, 878 (1998).
[26] W. Liang, M. Bockrath, D. Bozovic, J. H. Hafner, M. Tinkham, and H. Park, Nature 411, 665 (2001).
[27] P. Jarillo-Herrero, J. Kong, H. van der Zant, C. Dekker, L. Kouwenhoven, and S. De Franceschi, Phys. Rev. Lett. 94, 156802 (2005).
[28] J. Paaske, A. Rosch, P. Wölfle, N. Mason, C. M. Marcus, and J. Nygård, Nature Physics 2, 460 (2006).
[29] A. Makarovski, A. Zhukov, J. Liu, and G. Finkelstein, Phys. Rev. B 75, 241407 (2007).
[30] C. Quay, J. Cumings, S. Gamble, R. de Picciotto, H. Kautara, and D. Goldhaber-Gordon, Phys. Rev. B 76, 245311 (2007).
[31] Y.-W. Lan, K. Aravind, C.-S. Wu, C.-H. Kuan, K.-S. Chang-Liao, and C.-D. Chen, Carbon 50, 3748 (2012).
[32] J. P. Cleuziou, N. S. Wingreen, and Y. Meir, Phys. Rev. B 55, 1757 (2000).
[33] Y. Baer, and B. Delley, Phys. Rev. Lett. 87, 4127 (1997).
[34] F. Kuemmeth, S. Ilani, D. C. Ralph, and P. L. McEuen, Nature Comm. 4, 1573 (2013).
[35] A. Altland and B. Simons, Condensed Matter Field Theory, 2nd ed. (Cambridge University Press, Cambridge, 2010).
[36] S. Smirnov and M. Grifoni, Phys. Rev. B 87, 121302(R) (2013).
[37] S. Smirnov and M. Grifoni, New J. Phys. 15, 073047 (2013).
[38] N. S. Wingreen, and J. Nygård, Phys. Rev. Lett. 111, 136803 (2013).
[39] The precise expression is extracted from the many-body theory calculations, yielding $G(kT_F/e) = 0.612 G_0$.
[40] A. V. Kretinin, H. Shtrikman, and D. Mahalu, Phys. Rev. B 85, 201301(R) (2012).
[41] M. Pletyukhov and H. Schoeller, Phys. Rev. Lett. 108, 266601 (2012).
[42] K. Yamada, K. Yosida, and K. Hanazawa, Prog. Theo. Phys. 71, 1450 (1984).
[43] The term particle-hole symmetry refers here to the fact that the corresponding operator $\mathcal{P}$ exchanges states at energies located symmetrically around a reference energy $\varepsilon_0$. It must not be confused with the particle-hole symmetry around the band gap.
[44] The asymmetry of the peaks in the experiment is attributed to the difference in the coupling strength to the deutschen Volkes.
left and right contacts; for simplicity, in the theoretical curves the coupling to the left and right contacts is assumed to be of the same strength.