Delta baryons in magnetars: exploring the effects of the anomalous magnetic moment in magnetized neutron-star matter

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ABSTRACT
Strong magnetic fields can modify the microscopic composition of matter with consequences on stellar macroscopic properties. Within this context, we study, for the first time, the possibility of the appearance of spin-3/2 $\Delta$ baryons in magnetars. We make use of two different relativistic models for the equation of state of dense matter under the influence of strong magnetic fields considering the effects of Landau levels and the anomalous magnetic moment (AMM) proportional to the spin of all baryons and leptons. In particular, we analyze the effects of the AMM of $\Delta$ baryons in dense matter for the first time. We also obtain global properties corresponding to the EoS models numerically and study the corresponding role of the $\Delta$ baryons. We find that they are favored over hyperons, which causes an increase in isospin asymmetry and a decrease in spin polarization. We also find that, contrary to what generally occurs when new degrees of freedom are introduced, the $\Delta$s do not make the EoS significantly softer and magnetars less massive. Finally, the magnetic field distribution inside a given star is not affected by the presence of $\Delta$s.

Key words: dense matter – equation of state – stars: magnetars – stars: neutron

1 INTRODUCTION
Magnetars are a class of compact objects that possess the largest stable magnetic fields observed in nature, with surface magnitudes inferred for the poloidal component in the range of $10^{11} - 10^{15} \text{ G}$ at the surface (Haensel et al. 2006) and values more than one order of magnitude larger in the interior (Makishima et al. 2014; Dall’Osso et al. 2018). Although the strength of the magnetic field in the central region of these stars remains unknown, they could reach $\sim 10^{18} \text{ G}$ according to the scalar virial theorem (Lai & Shapiro 1991; Bonazzola et al. 1993), and simultaneous solutions of Einstein and Maxwell equations for poloidal (Bocquet et al. 1995; Cardall et al. 2001) and also toroidal configurations (Pili et al. 2017; Tsokaras et al. 2022). Such extreme conditions certainly play a considerable role when determining the internal composition and structure of magnetars.

The starting point for determining the macroscopic structure of compact stars is the assumption of a specific microscopic model, which leads to the calculation of an equation of state (EoS) for dense matter. The EoS encodes the particle population of baryons and leptons and how they interact through the strong interactions, constrained by equilibrium conditions, such as $\beta$-stability and charge neutrality. The extremely high energies estimated in the core of neutron stars are more than sufficient to create heavier particle species, beyond the traditional proton-neutron-electron admixture. It has become common in the literature to consider the entire spin-1/2 baryon octet (e.g. Glendenning 1982; Prakash et al. 1995; Baldo et al. 2000; Colucci & Sedrakian 2014; O. Gomes et al. 2015; Bhowmick et al. 2014; Oertel et al. 2015; Vidaña 2016; Chatterjee & Vidaña 2016; Marquez & Menezes 2017; Isaka et al. 2017; Roark et al. 2019; Stone et al. 2021; Sedrakian et al. 2020; Motta & Thomas 2022; Rather et al. 2021; Huang et al. 2022) but, recently, the role of the spin-3/2 decuplet has been slowly gaining attention (e.g. Li et al. 2018; Yang & Kim 2018; Motta et al. 2020; Ribes et al. 2019; Malfatti et al. 2020; Sahoo et al. 2020; Li et al. 2020; Raduta et al. 2020; Backes et al. 2021; Thapa & Sinha 2022; Dexheimer et al. 2021a; Marczenko et al. 2022). The lightest spin-3/2 baryons (the $\Delta$s) are only $\sim 30\%$ heavier than the nucleons (protons and neutrons) and are even lighter than the heaviest spin-1/2 baryons of the octet (the $\Xi$s). Thus, unless the $\Delta$s are subject to a very repulsive coupling, they are expected to appear at the same density range as the hyperons (around 2 or 3 times the nuclear saturation density). Not much is known about how $\Delta$ baryons couple in dense matter, but their potential for isospin-symmetric matter at saturation density is expected to be attractive and in a range of 2/3 to 1 times the potential of the nucleons, which is of order $\sim 80 \text{ MeV}$ (Drago et al. 2014; Kolomeitsev et al. 2017; Raduta 2021).

Additionally, it is of special interest to investigate how spin-3/2 $\Delta$ baryons are affected by the presence of strong magnetic fields due to the possibility of them having large electric charge and additional spin and isospin projections. The effects of Landau levels in dense stellar matter containing $\Delta$ baryons was first discussed in the context of neutron-star matter by Thapa et al. (2020) and later by Dexheimer et al. (2021a). In this work we study for the first time the effects of strong magnetic fields in $\Delta$-admixed hypernuclear stellar matter, accounting for effects due to their anomalous magnetic moments (AMMs).

For magnetic fields larger than $\sim 10^{16} \text{ G}$, the deformation of
the stellar geometry away from spherical symmetry is above 2\% (Gomes et al. 2019). Therefore, the usual relativistic hydrostatic equations usually employed when describing non-magnetised stars, i.e., the Tolman-Oppenheimer-Volkoff equations (Tolman 1939; Oppenheimer & Volkoff 1939), which assume spherical symmetry as part of their derivation from the general relativity equations, cease to be adequate. For this reason, we make use of anisotropic solutions from the Einstein and Maxwell equations to explore for the first time the macroscopic structure of magnetars with strong internal magnetic fields and containing $\Delta$-admixed hypernuclear matter. Beyond accounting for the non-spherical configurations of stars and anisotropies introduced by magnetic fields, this approach allows us to obtain an \textit{ab initio} magnetic field profile in the interior of a given star (Dexheimer et al. 2017; Chatterjee et al. 2019).

This work is structured as follows. In Section II, the formalism employed in the microscopic description of magnetized neutron-star matter is presented, as well as the procedure of going from the EoS employed in the microscopic description of magnetized neutron-star matter (Dexheimer et al. 2017; Chatterjee et al. 2019) to obtain an $\omega\rho$ parametrization (Lopes 2021), while

$$\omega\rho = \frac{1}{3} \left[ \rho \gamma^\mu \gamma^\nu \right]$$

where $\rho$ is the energy density, $\gamma^\mu$ the four-velocity, $\gamma^\nu$ the four-acceleration, and $\gamma^\mu$ the four-momentum of the charged baryon. Leptons are described additionally by the (free and bound) leptonic $\Delta$ interactions that allow the correct prediction of slope of the symmetry

$$L_{\text{probe}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{b,l} \tilde{q}_{b,l} \left( -q_{b,l} A^\mu - \frac{1}{4} \sigma_{\mu\nu} F^{\mu\nu} \right) \tilde{\psi}_{b,l} ,$$

which devoted to the electron charge and $\mu_N$ is the nuclear magneton

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\mu_N = e/2M_p ,
\]

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energy, neutron-star radii and tidal deformabilities. The \( \phi \) meson (with hidden strangeness) couples only to the hyperons, allowing a higher maximum mass to be reproduced for neutron stars, thus circumventing the well-known hyperon puzzle (Chamel et al. 2013), with an effect similar to the higher-order \( \omega^4 \) self interaction, also included. In addition to satisfying standard astrophysical constraints from LIGO/VIRGO and NICER, the model satisfies nuclear ground-state properties of finite nuclei and bulk properties of infinite nuclear matter. The model Lagrangian density is written as \( \mathcal{L} = \mathcal{L}_b + \mathcal{L}_m \), where the first term is the (interacting) Dirac Lagrangian density for nucleons, hyperons, and \( \Delta \), and the second term accounts for the self interaction among scalar and vector mesons\(^1\)

\[
\mathcal{L}_b = \sum_b \bar{\psi}_b \left[ i \gamma_{\mu} \partial^{\mu} - \gamma_0 \left( g_{\omega b} \omega + g_{\rho b} \rho + g_{\phi b} \phi \right) - M_b^* \right] \psi_b ,
\]

and

\[
\mathcal{L}_m = -\frac{1}{2} m^2_{\sigma} \sigma^2 - \frac{\lambda}{3} \omega^3 - \frac{K}{4} \omega^4 + \frac{\xi}{4!} \omega^b \omega^4 + \frac{1}{2} m^2_{\rho} \rho^2 + g_{\omega \rho} \omega^2 \rho^2 + \frac{1}{2} m^2_{\phi} \phi^2 ,
\]

(5)

where \( I_{3b} \) is the baryon isospin 3rd component, given in Table 1. The mass of the baryons is modified by the medium, giving rise to an effective mass \( M_b^* = M_b - g_{\rho b} \sigma \). The fittings of the self couplings \( \lambda \) and \( \kappa \), and the couplings between the mesons \( i = \{\sigma, \omega, \rho, \phi\} \) and baryons \( b \), defined in terms of the ratios \( x_{ib} = g_{ib}/8\pi \), are discussed in detail in Lopes (2021).

Relevant for this work, the scalar meson couplings are fitted to reproduce the hyperon potential depth \( U_{\chi} = -28 \) MeV (for isospin-symmetric matter at saturation) and the remaining relative strength of the coupling constants are determined by SU(3) symmetry group arguments, as proposed by Lopes & Menezes (2014), determining the complete hyperon-meson coupling scheme from a single free parameter, \( \alpha_{\chi} \). Despite the value of \( \alpha_{\chi} \), hyperons are always present in the neutron-star matter and the sequence of hyperon thresholds are always the same, with an inversely proportional relationship between \( \alpha_{\chi} \) and the stiffness of the EoS. In this work, we choose \( \alpha_{\chi} = 0.5 \), which results in values for the additional potentials \( U_{\Sigma} = +21.8 \) MeV and \( U_{\Xi} = +35.3 \) MeV, and a stiffer EoS with respect to the values \( \alpha_{\chi} = 0.75 \) or 1.0 that are considered in Lopes & Menezes (2014).

The \( \Delta \) couplings are treated more freely, as their behavior is not well known. The scarce information present in the literature allows us to infer that the nucleon-\( \Delta \) potential is slightly more attractive than the nucleon-\( \Lambda \) one, so that, \( U_{\chi} \lesssim 30 \) MeV \( \lesssim U_{\Delta} \lesssim U_{\chi} \), which implies \( x_{\chi \Lambda} \) is greater than 1. Also, the vector coupling is constrained by LQCD results as respecting the relation \( 0 \lesssim x_{\chi \Lambda} \lesssim x_{\omega \Lambda} \lesssim 0.2 \), and no constraint is put in the \( \chi \rho \Lambda \) value (Wehrberger et al. 1989; Ribes et al. 2019; Rudata 2021). Early investigations on the effect of these parameters were made in de Oliveira et al. (2000, 2007) and their role in the stellar particle composition and maximum-mass was studied considering \( x_{\omega \Delta} = 1.0 \) and 1.1, within two classes of relativistic mean-field models in Dexheimer et al. (2021a). Following the previous study, we analyse the scenarios with \( x_{\omega \Delta} = x_{\omega \Lambda} = 1.0 \) and \( x_{\chi \Lambda} = x_{\omega \Lambda} = 1.2 \), keeping \( x_{\chi \Delta} = 1.0 \), that generates, respectively, potentials \( U_{\Lambda} = -66.25 \) MeV (equal to the nucleon potential) and \( -79.50 \) MeV.

The second model we use in this work is the chiral mean-field (CMF) model, which is based on a nonlinear realization of the chiral sigma model. As in all chiral models, the masses of the baryons are generated (and not just modified) by the medium. In this way, at large temperatures and/or densities they decrease allowing chiral symmetry to be restored, in agreement with LQCD results (Aarts et al. 2017). In this work, we restrict ourselves to the hadronic version of the model (with leptons) developed by Dexheimer & Schramm (2008), and disregard the possibility of phase transitions to quark matter. We add an additional \( \omega \phi \) interaction to be in better agreement with data for the slope of the symmetry energy, neutron-star radii, and tidal deformabilities (Dexheimer et al. 2015, 2019). We also add a higher-order \( \omega^4 \) interaction to reproduce more massive neutron stars (Dexheimer et al. 2021b).

The mean-field model Lagrangian density has the terms

\[
\mathcal{L}_b = \sum_b \bar{\psi}_b \left[ i \gamma_{\mu} \partial^{\mu} - \gamma_0 \left( g_{\omega b} \omega + g_{\rho b} I_{3b} \rho + g_{\phi b} \phi \right) - M_b^* \right] \psi_b ,
\]

(7)

and

\[
\mathcal{L}_m = \frac{1}{2} \left( m^2_{\sigma} \omega^2 + m^2_{\rho} \rho^2 + m^2_{\phi} \phi^2 \right) + g_4 \left( \omega^4 + \phi^4 + 3 \omega^2 \phi^2 + 4 \omega^2 \phi + 2 \omega \phi^3 \right) \sqrt{2} \omega \phi^3 \]

\[ - k_0 (\sigma^2 + \phi^2) - k_1 (\sigma^2 + \phi^2)^2 \]

\[ - k_2 (\sigma^2 + \phi^2) - k_3 (\sigma^2 - \phi^2)^2 \]

\[ - k_4 \ln \left( \frac{\sigma^2 + \phi^2}{\sigma^2 - \phi^2} \right) - \frac{m^2_{\rho} f_{\pi} \sigma}{\sigma^2 - \phi^2} \]

\[ - \left( \frac{\sqrt{2} m^2_{\phi} f_{\chi}}{\sqrt{2} m^2_{\phi} f_{\pi}} \right) \zeta \] ,

(8)

where the effective mass of baryons is \( M_b^* = g_{\chi \rho} \sigma + g_{\chi \phi} I_{3b} \rho + g_{\chi \phi} \phi + \frac{1}{2} \phi \sigma \), including additional corrections given by the scalar-isovector and scalar-isoscalar (with hidden strangeness) \( \phi \) mesons, and a small bare mass correction \( M_0 \). The couplings were fitted to reproduce hadronic vacuum masses, decay constants, nuclear saturation properties, and to reach more than 2.1 \( M_\odot \) stars. See Roark & Dexheimer (2018) for a complete list of coupling constants. We follow the SU(3) and SU(6) coupling schemes for the scalar and vector couplings of mesons and baryons. In this way, there are only two free parameters left: one fitted to reproduce for symmetric matter at saturation the potential \( U_{\chi} = -27 \) MeV and another one \( (x_{\omega \Lambda} = x_{\omega \Lambda} = 1.25) \) fitted to reproduce under the same conditions \( U_{\Lambda} = -64 \) MeV \( \approx U_{\chi} \). They result additionally in \( U_{\Sigma} = 6 \) MeV and \( U_{\Xi} = -17 \) MeV. A much larger \( x_{\omega \Lambda} \) would suppress all \( \Delta \), while a much lower value would suppress all hyperons.

For both models, the equations of motion for the mesonic fields are obtained from the Lagrangian densities via the Euler-Lagrange conditions. Under chargeneutrality and\( \beta \)-equilibrium conditions, we can write the chemical potential of a baryon as a relation between the chemical potential of the neutron and the electron, \( \mu_n \) and \( \mu_e \), respectively, and its electric charge, i.e.,

\[
\mu_b = \mu_n - q_b \mu_e .
\]

(9)

At low (effectively zero) temperature, the Fermi energy spectra of

\( \text{MNRAS} \text{ 000,} 1–11 \text{ (2022)} \)
baryons is
\[ E_{F,b} = \mu_b - g_{\omega b} \omega - g_{\rho b} \rho - \frac{1}{2} \phi \phi \delta, \]
while for leptons it is simply \( E_{F,l} = \mu_l \).

In the presence of a magnetic field, the Fermi momentum (squared) can be calculated from the difference between the Fermi energy (squared) and

(i) the square of the effective mass modified by the AMM for particles that are not electrically charged \((q_b = 0)\),

\[ k_{F,b}^2(s) = E_{F,b}^2 - (M_{F,b}^2 - s k_B B)^2; \]

(ii) the square of the effective mass modified by Landau quantization and AMM for particles that are electrically charged \((q_b \neq 0)\),

\[ k_{F,b}(v, s) = E_{F,b}^2 - \left( \frac{M_{F,b}^2}{M_{F,b}^2 + 2v|q_B|B - s k_B B} \right)^2. \]

For the momentum of leptons, \( M^* \) is simply \( M \). In the latter case, the Fermi momentum refers to the local direction of the magnetic field, hereafter defined as the \( z \)-axis. In the transverse direction to the local magnetic field, the Fermi momentum is restricted to discrete values \( 2v|q_B|B \), where the Landau levels \( v \) orbit the axial orbital momentum \( n \) via the relation

\[ n = v + \frac{1}{2} \frac{s}{2|q_B|}, \]

where \( n = 0, 1, 2... \). For particles with spin 1/2, the first Landau level \((v = 0)\) is occupied by a single spin projection: \( s = +1 \) for \( q_B > 0 \) and \( s = -1 \) for \( q_B < 0 \). The second level \((v = 1)\) and above are occupied by both spin projections \( s = \{ \pm 1 \} \). For the spin-3/2 positively charged \( \Delta s \), the first level \((v = 0)\) is occupied by the spin projections \( s = \{ +1, -1 \} \), the second level \((v = 1)\) by \( s = \{ +3, -3, \pm 1 \} \), and hereafter all spin states are occupied. For the negatively charged \( \Delta^* \) spin projection, signs are reversed for the lowest levels.

At zero temperature, there is a maximum Landau level allowed, beyond which the momentum in Eq. (12) becomes imaginary given by

\[ \nu_{\text{max},b}(s) = \left[ \frac{E_{F,b} + s k_B B}{2v|q_B|B} \right]^2 - M_{F,b}^2 \].

The number density for each baryon is also defined separately for electrically charged and uncharged particles, respectively,

\[ n_b^{(q_b \neq 0)} = \frac{|q_B|B}{2\pi^2} \sum_{v,s} k_{F,b}(v, s); \]

\[ n_b^{(q_b = 0)} = \bar{\psi}_b \gamma_0 \psi_b = \frac{1}{2\pi^2} \sum_{s} \left[ k_{F,b}(s) + \frac{3}{2} \left( M_{F,b}^2 - s k_B B \right) \right]; \]

\[ \frac{k_{F,b}(s)}{k_{F,b}(s) + \frac{3}{2} \left( M_{F,b}^2 - s k_B B \right)} = \frac{\pi}{2}, \]

as well as the scalar densities,

\[ n_s^{(q_b \neq 0)} = \frac{|q_B|B}{2\pi^2} \sum_{v,s} \frac{\sqrt{M_{F,b}^2 + 2v|q_B|B - s k_B B}}{M_{F,b}^2 + 2v|q_B|B} \]

\[ \times \ln \left[ \frac{k_{F,b}(v, s) + E_{F,b}^2}{\sqrt{M_{F,b}^2 + 2v|q_B|B - s k_B B}} \right]; \]

\[ n_s^{(q_b = 0)} = \frac{M_{F,b}^2}{4\pi^2} \sum_s \left[ E_{F,b}^2 k_{F,b}(s) - \left( M_{F,b}^2 - s k_B B \right)^2 \ln \left[ \frac{k_{F,b}(s) + E_{F,b}^2}{M_{F,b}^2 - s k_B B} \right] \right]. \]

The expressions for energy density and pressure are different for each model and can be obtained from the energy-momentum tensor for matter (discussed in the following).

### 2.3 Macroscopic structure

For spherically symmetric neutron stars, given an EoS \( P(\rho) \), the global structure can be obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations of hydrostatic equilibrium

\[ \frac{dM}{dr} = 4\pi r^2 c(r), \]

\[ \frac{dv}{dr} = \frac{M(r) + 4\pi r^3 P(r)}{r^2} \left( 1 - \frac{2M}{r} \right)^{-1}, \]

\[ \frac{dP}{dr} = - (e(r) + P(r)) \frac{dv}{dr}, \]

where \( M(r) \) is the stellar mass contained within the radius \( r \) and \( v(r) \) is a gravitational potential for the line element in spherical coordinates.

\[ ds^2 = -e^{2\nu(r)} dr^2 + \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]

The TOV equations cannot be applied to describe the structure of the magnetars we study in this work because the spherical symmetries assumed in Eq. (22) will not hold. This is due to the strong magnetic fields we infer for such objects, which produce highly deformed stellar shapes. Instead, the stellar structure must be determined by solving equations in General Relativity describing the stationary configuration for the fluid, coupled with Einstein field equations. The energy-momentum tensor, which contains the information on the matter properties of stars, enters the stellar structure equations as the source of the Einstein equations. Ignoring the coupling to the electromagnetic field, one generally assumes a perfect fluid and the energy-momentum tensor takes the form

\[ T^{\mu\nu}_{\text{f}} = (e + P) u^\mu u^\nu + P g^{\mu\nu}, \]

where \( e \) denotes the (matter) energy density, \( P \) the pressure, and \( u^\mu \) the fluid four-velocity.

The EoS then relates pressure and energy density to the relevant thermodynamic quantities. In Chatterjee et al. (2015), the general expression for the energy-momentum tensor in the presence of an electromagnetic field was derived, starting from a microscopic Lagrangian including interactions between matter and the electromag-
netic field

\[ T^{\mu \nu} = T_f^{\mu \nu} + \frac{1}{\mu_0} \left( -B^\mu B^\nu + (B \cdot B) u^\mu u^\nu + \frac{1}{2} g^{\mu \nu} (B \cdot B) \right) + \frac{x}{\mu_0} \left( B^\mu B^\nu - (B \cdot B) (u^\mu u^\nu + g^{\mu \nu}) \right) \],

(24)

where \( \mu_0 \) is the vacuum permeability, \( g^{\mu \nu} \) the metric tensor, and \( x \) is the magnetisation. The electromagnetic field tensor has been expressed as \( F_{\mu \nu} = \epsilon_{\alpha \beta \mu \nu} u^\beta B^\alpha \), with \( \epsilon_{\alpha \beta \mu \nu} \) being the four-dimensional Levi-Civita symbol (Gourgoulhon 2012). Assuming an isotropic medium and a magnetisation parallel to the magnetic field, the magnetisation tensor \( M_{\mu \nu} \) can be written as

\[ M_{\mu \nu} = \epsilon_{\alpha \beta \mu \nu} u^\beta u^\alpha \],

(25)

with the magnetization four-vector defined as \( a_\mu = \frac{x}{\mu_0} B_\mu \). In the absence of magnetisation, i.e. for \( x = 0 \), this expression reduces to the standard magnetohydrodynamics form for the energy-momentum tensor (c.f. Gourgoulhon 2012).

Strong magnetic fields result in an anisotropy of the energy momentum tensor and break spherical symmetry, such that with increasing strength of the magnetic field, the shape of a magnetar departs more and more from a spherical shape. Interpreting the spatial elements of the fluid rest frame energy-momentum tensor as pressures, then there is a difference induced by the orientation of the magnetic field, commonly referred to as “parallel” and “perpendicular” pressures. Several earlier works tried to compute the mass-radius relations of strongly magnetised neutron stars through a first approach using isotropic TOV equations (Rabhi et al. 2009; Ferrer et al. 2010; Strickland et al. 2012; Lopes & Menezes 2012; Dexheimer et al. 2013; Casali et al. 2014). In these works, the components of the macroscopic energy-momentum tensor in the fluid rest frame are used to obtain the energy density \( \epsilon \), parallel (\( P_\parallel \)) and perpendicular (\( P_\perp \)) pressures. In Heaviside-Lorentz natural units, the pure electromagnetic contribution to the energy-momentum tensor, which is anisotropic, has values of \( B^2/2 \) and \( -B^2/2 \) in the perpendicular and parallel directions to the local magnetic field, respectively. However, this approach can dramatically overestimate the mass of neutron stars (as shown in Fig. 3 of Gomes et al. 2017).

Several works obtained the global structure models of magnetars by solving coupled Einstein-Maxwell equations, taking into account the anisotropy of the stress-energy tensor (Bocquet et al. 1995; Cardall et al. 2001; Oron 2002; Ioka & Sasaki 2004; Kiuchi & Yoshida 2008; Yasutake et al. 2010; Frieben & Rezzolla 2012; Yoshida et al. 2012; Pili et al. 2014; Thapa et al. 2020; Tsokaros et al. 2022). In these studies either a perfect fluid, a polytropic EoS, or a realistic EoS was assumed, but do not take into account the magnetic field modifications due to its quantisation.

Ideally, to explore magnetic field effects such as Landau quantisation and AMM on the global properties of the star, one must solve the coupled Einstein-Maxwell equations, along with a magnetic field dependent EoS. In Chatterjee et al. (2015) and Franzon et al. (2016), global numerical models for magnetars were obtained by consistently solving Einstein-Maxwell equations with magnetic field dependent quark EoS. It was however explicitly demonstrated by Chatterjee et al. (2015, 2021) that the maximum mass of a neutron star is minimally modified due to the magnetic field dependence of the microscopic EoS, even for the highest magnetic fields. Therefore in this work, we assume a non-magnetic (\( B = 0 \)) matter contribution to the EoS to compute global neutron-star models and the magnetic field enters structure calculations only through the dominant pure electromagnetic field contribution. Although it remains to be checked explicitly in future work, the effects of Landau quantisation and AMMs are not expected to sensibly affect the results of this study. Note however, that this is not the case for microscopic properties of matter, as discussed in the following.

### 3 RESULTS

#### 3.1 Matter properties

We start our discussion with the single-particle interaction potential for the \( \Delta \) baryons in dense nuclear matter, which is a measure of the energy cost of adding one particle \( b \) in a \( b \)-less matter with the given condition. For the \( L^3_\omega \rho \) model, it can be written as

\[ U_b = -g_{\sigma b} \sigma + g_{\omega b} \omega + g_{\rho b} I_3 b + g_{\phi b} \phi \],

(26)

and, for the CMF model,

\[ U_b = g_{\sigma b} \sigma + g_{\delta b} I_3 \delta + g_{\xi b} \xi - m_{b, vac} + g_{\omega b} \omega + g_{\rho b} I_3 b + g_{\phi b} \phi \].

(27)

In isospin-symmetric nuclear matter, all families of baryons experience the same potential, since the meson field \( \rho \) (and \( \delta \)) are null in this situation. In neutron rich matter, particles with positive isospin projections (as the positively charged \( \Delta \)s) are more bound than their zero- and negative-isospin counterparts, with the largest difference occurring for pure neutron matter, that can be taken as an extrapolation of neutron-star matter in \( \beta \)-equilibrium. The first two panels of Fig. 1 show how the \( L^3_\omega \rho \) model scalar and vector interactions affect the \( \Delta \) potentials. In all cases, the particle potentials eventually become positive as the density increases, corresponding to unbound...
states, but they stay negative in the relevant interval of densities around nuclear saturation, where their depth determines how much they are bound.

For the L3ωρ model, the larger the scalar coupling value (i.e., the parameter $x_{σΔ}$), the lower the potentials are in the low density regions. Complementary to that, the larger the vector coupling (i.e., the parameter $x_{ωΔ}$), the more repulsive the potentials for $Δs$ are, which reflect in more positive curves in the high density region, where the repulsive channel dominates. Also, it can be seen that the potential depends less on the species of $Δ$, $ΔN+H$ and $ΔN+H+Δ$ with $x_{σΔ}=x_{ωΔ}=1.0$ and $x_{σΔ}=x_{ωΔ}=1.2$.

The stiffer EoSs are formally the ones with larger values of the compressibility modulus (usually referred to simply as compressibility), given by

$$K = 9 \frac{∂P}{∂n_B}. \quad (28)$$

At saturation density, compressibility values for isospin-symmetric matter can be compared with laboratory data. We find values of 256 MeV and 300 MeV for the L3ωρ and CMF models, respectively. Laboratory values range between $220 < K < 260$ MeV (Dutra et al. 2014; Lopes 2021) and $250 < K < 315$ MeV (Stone et al. 2014).

The top panel of Fig. 2 shows the effect of the inclusion of different particle species in the compressibility for the L3ωρ model, in the absence of an external magnetic field. The kinks in the curves are consequence of the onset of new particle species, which are shifted to lower densities by the inclusion of both $Δs$ and respective stronger scalar interactions. For $x_{σΔ} = 1$, the effective mass of nucleons becomes zero at $n_B \sim 0.85$ fm$^{-3}$ and, for this reason we lack solutions at higher densities. The bottom panel of Fig. 2 shows that in the CMF model the kinks are much smaller than in the L3ωρ model, with the only displacement of the curve occurring at the onset of the first non-nucleon baryon. As a consequence, the different CMF EoSs behave more similarly as the density increases.

The stiffer EoSs are formally the ones with larger values of the speed of sound $v_s$, but here we discuss stiffness with respect to $K$, related to $v_s$ through $v_s^2 = K/(9\mu)$ (Dexheimer et al. 2008). Isospin symmetric matter is softer at low densities, but becomes stiffer at large densities due to the Pauli exclusion principle because, as only nucleonic matter is considered, higher Fermi levels must be occupied (see Fig. 2). The behavior of neutron-star matter (charge neutral and in chemical equilibrium) depends on the composition, but it is always softer than the symmetric matter case after the hyperon or $Δ$ onsets, as the presence of new Fermi levels turns the EoS softer. Matter with hyperons but no $Δ$ is stiffer at intermediate densities (than matter with $Δ$), however it is softer at large densities, especially in the case of strong scalar interaction (for the L3ωρ model). This trend was noticed previously by Dexheimer et al. (2021a), where we showed that the inclusion of $Δs$ could turn the EoS stiffer (than the cases where they were absent), despite the fact that the new degrees of freedom soften the EoS. This is related to isospin asymmetry, which we discuss in the following.

We define the isospin fraction as the average 3rd isospin component of a given matter composition, weighted by the relative densities, i.e.,
Figure 4. Particle composition of neutron-star matter with Δs, with $B = 0$ (top panels) and magnetic field $B = 3 \times 10^{18}$ G (bottom panels), when considering (solid lines) or disregarding (dashed lines) the effects of the anomalous magnetic moment. The left and middle panels show results for the L3ωρ model with different interactions, while the right panel shows results for the CMF model.

Figure 5. Spin polarization fraction as a function of baryon number density for neutron-star matter with magnetic field $B = 3 \times 10^{18}$ G, when considering (solid lines) or disregarding (dashed lines) the effects of the anomalous magnetic moments and shown for different compositions and interaction strengths. The top and bottom panels show results for the L3ωρ and CMF models, respectively.

$$Y_{I^+} = \frac{\sum_b I^+_b n_b}{\sum_b n_b},$$

(29)

as shown in Fig. 3. For nucleonic matter only, $Y_{I^+} = 0$ means matter with the same amount of protons and neutrons, while $Y_{I^+} = -0.5$ means pure neutron matter. The density at which the curves with and without Δs split marks the appearance of the Δ−’s, which increase the isospin asymmetry (turn the isospin fraction more negative). The effect is much larger for the L3ωρ model (top panel) than the CMF model (bottom panel), which hints that the amount of Δs reproduced in each model is different. Both effects generated by the magnetic fields, i.e. Landau quantization and AMM, decrease the isospin asymmetry (less negative $Y_{I^+}$) at low and intermediate densities.

A better understanding of the effects of the inclusion of Δ baryons, magnetic fields, and AMM in neutron-star matter subject to strong magnetic fields can be obtained from Fig. 4. Comparing the top row ($B = 0$) with the lower one ($B = 3 \times 10^{18}$ G), we can see that some of the charged particles are favored when magnetic field effects without AMMs are considered, an effect that is more pronounced for protons, whose onset density is pulled to very low densities for both models. As a consequence, their population becomes more similar to the neutron one in densities below ~0.05 fm$^{-3}$, turning $Y_{I^+}$ less negative. The inclusion of AMM enhances this effect. This explains why the isospin asymmetry depends both on the magnetic field and on the AMM in the lower density region, as shown in Fig. 3. The Δ− threshold (at densities around 0.3 fm$^{-3}$) coincides with the region at intermediate densities below which the N+H+Δ EoSs are softer than the respective N+H EoSs. The Δ (and the Σ in the CMF model) hyperons appear at larger densities than the Δ−’s. The remaining Δs appear at much larger densities and in amounts that depend on the interactions in the L3ωρ model.

To discriminate AMM effects on the particle composition is not trivial, as they depend on the AMM coupling strength and sign, on the particle mass, charge, and density. Additionally, different spin...
projections are separately enhanced or suppressed, but this cannot be clearly seen in Fig. 4, as it follows the usual convention and shows the sum of all spin projections for each particle. For this reason, we make use of a quantity that reveals the degree of spin polarization, more suited to discuss spin projection asymmetry of fermions.

We define the total spin polarization of a given matter composition, weighted by the relative densities, in analogy to Eq. (29), i.e.,

$$Y_{\text{spin}} = \frac{\sum_{b,s} s n_b(s)}{\sum_{b,s} n_b(s)},$$

and shown the results in Fig. 5. For a fixed magnetic field strength, all charged particles are fully spin polarized at low densities: only spin projection $1/2$ for protons and spin projection $3/2$ for positive $\Delta$s, only spin projection $-1/2$ for leptons and negative $\Sigma$s, and spin projection $-3/2$ for negative $\Delta$s. When AMMs are considered, neutral particles obey the same logic, presenting only positive (negative) spin projections according to their positive (negative) sign of $\kappa_b$. At intermediate densities, full polarization is broken for more massive particles, but not for leptons and $\Delta$s. But, regardless, the polarization never goes to zero, meaning that partial spin projection imbalance remains at high densities. Overall, spin polarization fraction is much stronger for the CMF model (bottom panel) than for the L3$\omega\rho$ model (top panel). Full polarization can be understood from Eqs. (11), (12), and (13), which explains why particles with different isospin projections present different momenta and why particles occupying the first Landau level ($\gamma = 0$) are more abundant when only a few levels are occupied. This happens for strong magnetic fields and low particle densities, or simply less massive particles.

It is a well-established concept that the magnetic field is not constant within neutron stars, but increases towards their centers where the density is larger. But, before we discuss stellar configurations with macroscopic magnetic fields in detail, we study how one more relevant quantity changes as a function of magnetic field strength. The fraction of exotic particles can be defined as the following quantity

$$Y_i = \frac{\sum_{b,i} n_b}{\sum_b n_b},$$

for $i = H$ or $\Delta$, shown in Fig. 6. On the left panel for the L3$\omega\rho$ model, the amount of $\Delta$s is slightly reduced at a given density but then increases tremendously at the larger density when the AMM is included, a behaviour quantitatively not reproduced with larger coupling constants, as seen on the middle panel. The amount of hyperons, on the other hand, is not significantly modified by the magnetic field, only slightly decreases in the presence of AMM and is affected by the small fluctuations related to the De Haas-Van Alphen oscillations (De Haas & Van Alphen 1930). The right panel shows the same qualitative behavior for the CMF model, which has a more clear substitution of hyperons in favor of deltas for higher values of $B$, independently of the density or accounting for the AMM.

### 3.2 Macroscopic structure

In a previous study, Dexheimer et al. (2021a) obtained results on the effects of the inclusion of $\Delta$s in neutron stars for both the L3$\omega\rho$ and CMF models without magnetic fields, using standard TOV equations. As expected from the discussion regarding the hyperon puzzle (Chamel et al. 2013), pure nucleonic stars are always the most massive configuration, and the inclusion of hyperons decreases the maximum
mass obtained. It was noticed that the inclusion of Δs, however, does not modify significantly (and in some cases increases) the maximum stellar mass in relation to the composition with only nucleons and hyperons, and this effect is more obvious for the cases where Δs are more abundant. It is argued that the addition of Δs decreases the fraction of nucleons (in fact, neutrons) and hyperons (mostly Δs) to create Δs and some protons, in a way that the overall increase in isospin asymmetry turns the EoS stiffer, even when more species are present.

As described here in section 2.3, to compute the effect of the strong magnetic fields on the structure of the magnetars, one must solve the coupled Einstein–Maxwell equations with the equations of state (described in section 3.1). For the chosen poloidal field geometry, we solve the Einstein–Maxwell equations within the numerical library LORENE using a multi-domain spectral method. In Fig. 7, we show the mass radius relations for the L3ωp and the CMF models, with and without Δs, as a function of equatorial radius for sequences of constant stellar central magnetic field. Despite the fact that, for the choices discussed here for xσΔ and xoΔ parameters in the L3ωp model, the masses of N+H+Δ stars never surpass the respective N+H configurations, we still observe that the the maximum mass (shown in Fig. 7) follows the same ordering of a large (and most relevant) portion of Fig. 3 for the compressibility.

In Fig. 7, any differences between the mass-radius curves for the B = 0 case (solid lines) arise from the differences in the (non-magnetic) EoS, while the differences with magnetic field come from the pure electromagnetic field contribution. We know that the Lorentz force originating from the pure electromagnetic field affects the low density part of the EoS. This is why the maximum mass of very massive stars does not change with increasing magnetic field strength, but the mass and radius of less massive stars increase significantly. For the L3ωp model, the inclusion of Δs decreases modestly the maximum stellar mass, especially for the larger coupling. However, for the CMF model, we do not see meaningful changes on the mass-radius diagram with the inclusion of Δs. From Table 2 we see that, keeping the radius of the neutron star fixed (going up vertically in Fig. 7), the increase in the strength of the central magnetic field increases both the central baryon and energy densities, as a larger matter pressure is necessary to balance the Lorentz force. The addition of Δs decreases both quantities, as these stars are naturally (at B=0) smaller.

At this point, we note that the maximum mass value of the stellar family described by the L3ωp model with Δs and xσΔ = 1.0 (the yellow curve) has not been attained because, as explained earlier, the EoS numerical code stops converging at large densities due to reaching zero nucleon masses. Such behavior indicates that hadronic matter is no longer stable at this point and deconfinement to quark matter must be considered. We leave such analysis to a future work. But, since the trend of the yellow curve is quite obvious, we can conclude that its maximum mass is lower when compared to the other coupling and composition.

Using the full numerical solution, we also study the effect of the EoS on the magnetic field configurations inside a given star. We decompose the magnetic field norm in terms of spherical harmonics

\[ B(r, \theta) \approx \sum_{l=0}^{l_{\text{max}}} B_l r^l Y_l^0(\theta), \]

and plot the first four even multipoles (l = 0, 2, 4, 6) as function of coordinate radius for both the EoS models and coupling strengths in Fig. 8. We also plot the profile of the dominant monopolar, spherically

\[ \Delta \]

symmetric, term (l = 0) inside the star in Fig. 9. For L3ωp model, specially if we include Δs, the magnetic field norm decreases slightly inside the star but, for CMF model, we do not see any considerable changes.

4 CONCLUSIONS

In the present work, we have used two different relativistic models, namely, one of the parametrizations of the Walecka model with nonlinear terms called L3ωp and the chiral mean field (CMF) model, to investigate the effects of the presence of Δ baryons in dense matter. When the L3ωp is used, the unknown meson-hyperon coupling

\[ \text{http://www.lorene.obspm.fr} \]
constants can be extracted from both phenomenology and symmetry group considerations, but the choice of meson-Δ couplings is still flexible. The CMF model, on the other hand, fixes the coupling to baryons and Δ baryon potentials and the compressibility away from saturation are quantitatively different for both models, although they qualitatively present the same behavior. This is a consequence mainly of the different hyperon and Δ composition at intermediate and large densities. Those features are carried out when strong magnetic fields and anomalous magnetic moments (AMMs) are incorporated.

We carefully investigated particle composition and spin polarization when Δ baryons are included in neutron-star matter under the influence of strong magnetic fields with and without AMM corrections. Due to the effects of charge conservation and chemical equilibrium, there is no common behavior for all the particles (as predicted by their AMM signs and strengths). However, in general, while the population of charged particles increases with the inclusion of AMM, the population of neutral particles tends to decrease.

The macroscopic properties of magnetars for the above choice of EoS models were obtained by solving Einstein-Maxwell equations within the LORENE library. It was found that maximum masses as high as 2M⊙ can be attained even on inclusion of Δ particles. This is due to isospin readjustment at large densities, which turns the EoS stiffer. The Δs also respond more strongly to the AMM, which is expected due to the fact that they present additional electric charges and isospin projections. As a consequence, Δ-admixed hypernuclear stellar matter, possesses larger spin polarization. The latter effect is more dramatic for the L3ωρ model, which presents a larger number of exotic particles than the CMF model.

Considering strong magnetic fields, heavy stars tend to contain more Δs in their interiors. They are not necessarily more massive (than their B=0 counterparts), but are larger and, for a given radius, present larger central number density and energy density. While Δs modify the magnetic field distribution very little inside stars, they decrease their radii, improving the agreement with modern observational data of neutron-star radii and tidal deformability (Miller et al. 2019; Riley et al. 2019; Miller et al. 2021; Riley et al. 2021; Abbott et al. 2019).

Our results do not show the significant increase of stiffness of neutron-star matter with Δs at large density, as initially observed by Dexheimer et al. (2021a). In a more in depth analysis (to appear in a separate publication Marquez (2022)), we will show that this effect can indeed occur and reproduce more massive magnetars, but is very sensitive to model parameters.

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