Fuzzy time series Markov Chain and Fuzzy time series Chen & Hsu for forecasting

Zaenurrohman¹, S Hariyanto², T Udjiani²

¹Magister Program of Mathematics, Departement of Mathematics, Faculty of Science and Mathematics, Diponegoro University
²Departement of Mathematics, Faculty of Science and Mathematics, Diponegoro University

Corresponding author: zaenurrohman@students.undip.ac.id

Abstract. Fuzzy time series theory is a concept of artificial intelligence that can use to conduct forecasting technique. This paper discusses the fuzzy logic concept to develop the base of the fuzzy time series with time invariant and time variant methods. There are several methods of fuzzy time series, including Markov Chain method and Chen and Hsu method. The Markov Chain method combines between the fuzzy time series and the Markov Chain. This merger aims to finest opportunity of the use matrix probability transitions. Chen and Hsu method is based on the historical data difference in conducting forecasting. By using Markov Chain and Chen & Hsu methods, it may achieve forecasting outcomes with a low mistakes rate. To clarify each technique and for comparison further, it is given an example of the relevant issue to be resolved by both methods. The consequences acquired can be compared, so it can be concluded which method is better.

1. Introduction

Forecasting is an activity estimating what happens in the future for a fantastically lengthy time. Forecasting is an essential trouble spanning many fields inclusive of enterprise and industry, government, economics, environmental science, medical, social, politics, and finance[1]. In economics, forecasting is an crucial a part of the business enterprise whilst making control decisions. In forecasting is needed to a narrowest feasible error.

One of the techniques currently developed to perform forecasting is the Fuzied Time Series Technique which is classified into artificially intelligent concepts or artificial intelligence concepts that can help to perform forecasting techniques. This technique turned into first proposed by Song and Chissom who used the fuzzy logic concept to develop the basis of the fuzzy time series using the time invariant and time variant methods used to carry out forecasting[2]. Some of the fuzzy time series methods had been developed including the Markov method [3], Chen[4], Chen and Hsu[5], the weighted method [6], the multiple-attribute fuzzy time series method[7], the percentage change method [8], and the markov chain method[9]. To date there has been no really precise method of forecasting the future value of the share price.

Since there may be no truly particular approach yet, this paper will examine two methods namely Markov Chain and Chen & Hsu to be compared which method is better. The Markov Chain and Chen & Hsu strategies had been selected due to the fact they have a got an excessive diploma of accuracy.
2. Discussion

2.1. Fuzzy time series

Suppose U is a set of the universe, \( U = \{ u_1, u_2, \ldots, u_n \} \), then the fuzzy set \( A \) of U is defined as follows:

\[
A = \frac{f_A(u_1)}{u_1} + \frac{f_A(u_2)}{u_2} + \frac{f_A(u_3)}{u_3} + \cdots + \frac{f_A(u_n)}{u_n}
\]  

(1)

where \( f_A \) is a function member of the fuzzy set \( A \), \( f_A : U \to [0,1] \), \( f_A(u_i) \) denotes the grade of club of \( u_i \) in the fuzzy set \( A \), and \( 1 \leq i \leq n \) [10].

Definition 2.1.1. [5]

Suppose \((t)(t = \cdots , 0, 1, 2, \ldots )\), a subset of \( \mathbb{R} \), becomes a universe on a set of fuzzy \( f_i(t) \) \((i = 1, 2, \ldots )\). Then, \( F(t) \) is referred to as fuzzy time series on \((t)(t = \cdots , 0, 1, 2, \ldots )\).

From the above definition, it may be seen that \( F(t) \) can be considered a linguistic variable and \( f_i(t)(i = 1, 2, \ldots ) \) is seen as the linguistic value of \( F(t) \), in which \( f_i(t)(i = 1, 2, \ldots ) \) is a representation of a fuzzy set. We can see that \( F(t) \) is a function at \( t \) time.

Definition 2.1.2. [4]

If \( F(t) \) is brought on by \( F(t - 1) \), i.e., \( F(t - 1) \to F(t) \), it can be stated as follows:

\[
F(t) = F(t - 1) \times R(t, t - 1)
\]  

(2)

where \( R(t, t - 1) \) is the fuzzy relationship between \( F(t - 1) \) and \( F(t) \), and \( F(t) = F(t - 1) \cdot R(t, t - 1) \) is called the first forecasting model on \( F(t) \). The relation \( F(t - 1) \to F(t) \) is called a fuzzy logical relationship, whilst \( F(t - 1) \) is the current state and \( F(t) \) is the subsequent state.

Definition 2.1.3[9]

Suppose among \( F(t) = A_i \), is because of \( F(t - 1) = A_j \), then the fuzzy logical relationship is described as \( A_i \to A_j \).

If there are fuzzy logical relationships acquired from state \( A_2 \), then a change is made to any state \( A_j \), \( j = 1, 2, \ldots, n \), as \( A_2 \to A_1, A_2 \to A_2, \ldots, A_2 \to A_1; \) hence, the fuzzy logical relationships are grouped right into a fuzzy logical relationships group as \( A_2 \to A_1, A_2, A_3 \).

In general the steps of the fuzzy time series model include: (1) determining the universe of talks, where the fuzzy set will be defined (2) dividing the universe set into multiple intervals of the same length (3) defining the fuzzy set \( A \) (4) fuzzification of historical data (5) specifying the fuzzy logical relationship (6) grouping the fuzzy logical relationship (in step 5) (7) calculating its forecasting value.

2.2. Fuzzy Time Series Markov Chain

Markov Chain’s Fuzzy Time Series forecasting procedure is as follows [9]:

Step 1. Collecting historical data (Yt).

Step 2. Defines the U universe set of data, with \( D_1 \) and \( D_2 \) being the corresponding positive numbers. \( U = [D_{\min} - D_1, D_{\max} + D_2] \)

Step 3. Specify the number of fuzzy intervals.
Step 4. Defining the fuzzy set in the universe of discourse U, the Fuzzy A_i set declares the linguistic variable of the share price by $1 \leq i \leq n$.

Step 5. Fuzzification of historical data. If a time series data is included in the $u_{ij}$ interval, then that data is fuzzification into $A_i$.

Step 6. Specifies fuzzy logical relationships and Fuzzy Logical Relationships Group (FLRG).

Step 7. Calculate forecasting results

For time series data, using FLRG, a probability can be obtained from a state heading to the next state. So used markov probability transition matrix in calculating forecasting value, transition matrix size is $n \times n$. If state $A_i$ transition to a state $A_j$ and pass the state $A_k$, $i, j = 1, 2, ..., n$, then we will reap FLRG. The transition probability formula is as follows:

$$P_{ij} = \frac{M_{ij}}{M_i}, i, j = 1,2, ..., n, \quad (3)$$

with:

- $P_{ij}$= probability of transition from state $A_i$ to state $A_j$ one step
- $M_{ij}$= number of transitions from state $A_i$ to state $A_j$ one step
- $M_i$= the quantity of data included in the $A_i$

The probability matrix $R$ of all states can be written as follows:

$$R = \left[\begin{array}{ccc}
P_{11} & \cdots & P_{1n} \\
\vdots & \ddots & \vdots \\
P_{n1} & \cdots & P_{nn}
\end{array}\right] \quad (4)$$

Matrix $R$ displays the transition of the entire system. If $F(t-1) = A_i$, then the procedure might be described within the $A_i$ on the time of $(t-1)$, then the forecasting effects $F(t)$ will be calculated using the $[P_{i1}, P_{i2}, ..., P_{in}]$ on the matrix $R$. Forecasting results $F(t)$ is the weighted common value of the $m_1, m_2, ..., m_n$ (midpoint of $u_1, u_2, ..., u_n$). The forecasting output result value on $F(t)$ can be determined using the following rules:

a) Rule 1: if FLRG $A_i$ is one to one (assume $A_j \rightarrow A_k$ wherein $P_{ik} = 1$ and $P_{ij} = 0, j \neq k$) then the forecasting value of $F(t)$ is $m_i$ the middle value of the $u_{ik}$.

$$F(t) = m_i P_{ik} = m_i \quad (5)$$

b) Rule 2: if fuzzy logical relationship group (FLRG) $A_i$ is one to many (assume $A_j \rightarrow A_1, A_2, ..., A_n, j = (1, 2, ..., n)$, while $Y(t-1)$ at the time of $(t-1)$ blanketed within the $A_j$ then forecasting $F(t)$, is:

$$F(t) = m_1 P_{j1} + m_2 P_{j2} + \cdots + m_{j-1} P_{j(j-1)} + Y(t-1)P_{j0} + m_{j+1} P_{j(j+1)} + \cdots + m_n P_{jn} \quad (6)$$

where: $m_1, m_2, ..., m_n$ is the middle value $u_1, u_2, ..., u_n$, $Y(t-1)$ is the state value $A_i$ at the time of $t-1$.

Step 8. Calculates the adjustment value ($D_t$) on the forecasting value. Here are the principles in calculating the value of adjustments:

a) If state $A_i$ is related $A_k$, beginning from state $A_i$ on the time $t-1$ declared as $F(t-1) = A_i$, and skilled a growing transition to the state $A_j$ at the time $t$ where $(i < j)$ then the adjustment values are:

$$D_{t_1} = \left(\begin{array}{c}
in \\
2\end{array}\right) \quad (7)$$
where \( l \) is the interval base.

b) If state \( A_i \) is related \( A_s \), starting from state \( A_i \) at the time \( t-1 \) declared as \( F(t-1) = A_s \), and decreasing transition to state \( A_j \) at the time \( t \) where \( (i > j) \) then the adjustment value is:

\[
D_{t1} = -\left(\frac{l}{2}\right)
\]

(8)

c) If the transition starts from the \( A_i \) at the time \( t-1 \) declared as \( F(t-1) = A_i \), and experienced a jump forward transition to the state \( A_{i+s} \) at the time \( t \) where \( (1 \leq s \leq n-1) \) then the adjustment value is:

\[
D_{t2} = s, 1 < v < i
\]

(9)

where \( s \) is the number of jump forward.

d) If the transition starts from the \( A_i \) at the time \( t-1 \) declared as \( F(t-1) = A_i \), and experienced a jump backward transition to the state \( A_{i-v} \) at the time \( t \) where \( (1 \leq v < i) \) then the adjustment value is:

\[
D_{t2} = -\left(\frac{l}{2}\right)v, 1 \leq v \leq i
\]

(10)

where \( v \) is the number of jump-backward.

Step 9. Calculate customized forecasting values

a) If FLRG \( A_i \) is one to many and state \( A_{i+1} \) accessible from the state \( A_i \) where \( A_i \) related \( A_i \) then the forecasting consequences grow to be

\[
F'(t) = F(t) + D_{t1} + D_{t2} = F(t) + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\]

(11)

b) If FLRG \( A_i \) is one to many and state \( A_{i+1} \) accessible from the \( A_i \) where state \( A_i \) unrelated to \( A_i \) then the forecasting results become

\[
F'(t) = F(t) + D_{t2} = F(t) + \left(\frac{1}{2}\right)
\]

(12)

c) If FLRG \( A_i \) is one to many and state \( A_{i-1} \) on hand from the state \( A_i \) where \( A_i \) do now no longer speak with \( A_i \) then the forecasting results become

\[
F'(t) = F(t) - D_{t2} = F(t) - \left(\frac{1}{2}\right) x 2 = F(t) - l
\]

(13)

d) When \( v \) is jump step, the general form of forecasting results are:

\[
F'(t) = F(t) \pm D_{t1} \pm D_{t2} = F(t) \pm \left(\frac{1}{2}\right) \pm \left(\frac{1}{2}\right) v
\]

(14)

Markov Chain used Tsaur to foresee the exchange rate between the Taiwan and US dollars, which concluded that the markov chain had a totally small mean absolute percentage error value. Today many studies have used markov chain, including Lintang Afidanti Nurkhasanah, et al. In his research entitled comparison of fuzzy Chen method and fuzzy markov chain to are expacting inflation statistics in Indonesia which shows that based on the value of MSE obtained, fuzzy-Markov chain method was chosen as the best method because it produces the smallest mean square error [11] and Nurul Fitri, et al. in his research mentioned that batik sales forecasting from CV. Bintang Abadi uses 48 monthly sales data from January 2014 to December 2017 using fuzzy time series markov chain providing 1072 results and mape value = 22.4803% [12].
2.3. **Fuzzy time series Chen & Hsu**

Chen and Hsu strategies are carried out withinside the following steps [5]:

**Step 1:**
- Determine the lag between \( n+1 \) and \( n \) data.
- Sum all the differences earned and then divided by the amount of data.
- Determine the length of the interval
- Determine the number of classes.
- Each class is symbolized by a universe set \( U = U_1, U_2, U_3, \ldots, U_n \) according to the number of classes.
- Defines the set of universes of each class according to the interval length of each class.

**Step 2:**
- Distribute all research data into each set of universes.
- Determine the amount of data included in each interval class.
- Redivided Interval.

**Step 3:**
- Define the fuzzy set.
- Split fuzzy sets into sections.

**Step 4:**
- Distribute fuzzy sets that have been formed into actual data tables.
- Forming Fuzzy Logical Relationship (FLR)

**Step 5:**
- Determine the Difference among \( n-1 \) and \( n-2 \) data (Diff 1-2) and Difference between \( n-2 \) and \( n-3 \) data (Diff 2-3).
- Determine the difference between (Diff 1-2) – (Diff 2-3) which is then symbolized by DIFF.
- Specifies DIFF \( x \) 2 + Data \( n-1 \) and DIFF/2 \( + \) \( n-1 \) data.

In testing the accuracy of forecasting with the Fuzzy Time Series Chen and Hsu method, there are several rules that must be followed in determining the value of the forecast, namely:

**Rule 1:**
- If the data analyzed does not have \( n-2 \) and \( n-3 \) data, then the middle value of fuzzy set \( A_j \).
- If the data analyzed does not have \( n-3 \) data, then:
  a) if the difference between \( n-1 \) and \( n-2 > \) half the \( A_j \) interval, then the forecast value is expressed as upward 0.75 point interval \( A_j \).
  b) if the difference between \( n-1 \) and \( n-2 = \) half the \( A_j \) interval then the forecast value is declared as middle value interval \( A_j \).
  c) if the difference is \( n-1 \) and \( n-2 < \) half the interval of \( A_j \) then the forecast value is expressed as Downward interval \( A_j \).

**Rule 2:**
- If DIFF is positive value then:
  a) if the value (DIFF \( x \) 2 + Data \( n-1 \)) is in the \( A_j \) interval then the forecast value is expressed as upward 0.75 point interval \( A_j \).
  b) if the value (DIFF/2 + Data \( n-1 \)) is in the \( A_j \) interval then the forecast value is expressed as downward 0.25 point interval \( A_j \).
c) if point (a) and point (b) are not met then the forecast value is expressed with the Middle Value interval $A_j$.

Rule 3 :
- If DIFF is negative, then:
  a) if the value ($\text{DIFF} / 2 + \text{Data n-1}$) is in the $A_j$ interval then the forecast value is expressed as downward 0.25 point interval $A_j$.
  b) if the value ($\text{DIFF} \times 2 + \text{Data n-1}$) is in the $A_j$ interval then the forecast value is expressed as upward 0.75 point interval $A_j$.
  c) If Point (a) and point (b) are not met, then the forecast value is expressed with the Middle Value of the $A_j$ interval.

Chen & Hsu makes use of its method to forecast the weather, and the results have a high accuracy value. In its development, Chen & Hsu method has also been widely used in forecasting, among others to predict the value of Sharia Stock Exchange Index in Jakarta Islamic Index (JII) conducted by Rizka Zulfikar and Prihatin Ade Mayvita shows that Chen and Hsu techniques can be used to are expecting the trend of stock exchange indices that arise Jakarta Islamic Index with a reasonably correct of accuracy [13]. As well as being used in wulan angraeini and Indra Suyahya's research entitled rupiah exchange rate prediction against the us dollar using the fuzzy time series chen &hsu method, which resulted in a low forecasting error rate [14].

3. Conclusion
The application of the Markov Chain and Chen & Hsu fuzzy time series is very simple and, based on several studies that have been described, proven that the method has a low error rate and has a high degree of accuracy. Because this study has not compared the calculations of Markov Chain and Chen &hsu in forecasting with the same data, it is necessary to forecast using the same data as the Markov Chain and Chen & Hsu methods so as to determine or choose the best method between the two.

References
[1] Montgomery C Douglas, Jennings L Cherly and M Kulahci 2015 Introduction to Time Series Analysis and Forecasting Second Edition (New York: Wiley-Interscience)
[2] Song Q and Chisom B S 1993 Fuzzy sets Syst. 54 1 1
[3] Sullivan J and Woodall W 1994 Fuzzy sets Syst. 64 279
[4] Chen S 1996 Fuzzy sets Syst. 81 3 311
[5] Chen S and Hsu C 2004 Inter. J. Appl. Scie. Eng. 2 3 234
[6] Yu H 2005 J. Phys. A 349 609
[7] Cheng C H, Cheng G W and Wang J W 2008 Exp. Syst. Appl. 34 2 1235
[8] Stevenson M, Porter J E 2009 World Acad. Sci. Eng. Technol. 55 154
[9] Tsaur R 2012 Int. J. Innov. Comput. Inf. Cont. 8 4931
[10] Jasim H T and Salim A G J 2012 J. Appl. Appl. Math. 7 1 385
[11] Nurkhasanah, Suparti and Sudarno 2015 J. Gauss. 4 4 917
[12] Fitriyah N, Faisol T Y 2018 Proceed. Nat. Conf. Math. Sci. Educ. (NACOMSE) 163
[13] Zulfikar R, Prihatini A M 2017 Jurnal Penelitian Ilmu Ekonomi WIGA 7 108
[14] Suyahya I, Anggraeini W 2016 Jurnal Sosio-E-Kons 8 1 24