Can we use hadronic $\tau$-decay for $V_{us}$ determination

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Abstract

It is known that the discrepancy in pion spectral functions extracted from $e^+e^-$ annihilation and $\tau$-decay leads to different predictions for the muon anomalous magnetic moment. We will show that this discrepancy effects also the extraction of the Cabibbo angle from the hadronic $\tau$-decays. The corrections to the $\tau$ branching fractions, corresponding to the presence of new centi-weak tensor interactions, allow us to extract the Cabibbo angle from $\tau$ decays in agreement with its other precision determinations. Thus a more precise value of $|V_{us}| = 0.2246 \pm 0.0012$ is obtained and as a consequence $f_+(0) = 0.9645 \pm 0.0055$ and $F_K/F_\pi = 1.196 \pm 0.007$. 
1 Introduction

The vector transitions in weak decays play a special role in phenomenology of the weak interactions. According to the Standard Model (SM) the $W$-boson and the photon originate from the same multiplet of $SU(2)_L$ group and they have the same form of vector coupling with matter. Therefore, the vector weak transition could be related to its isospin analogous electromagnetic process by the CVC hypothesis [1].

For example, the branching ratio of $\tau^- \to \rho^- \nu_\tau$ decay can be predicted on the base of the pure electromagnetic process $e^+e^- \to \gamma^* \to \rho^0$ [2]. Indeed, earlier, at a moderate precision of the experimental data, the agreement between these two processes was satisfactory [3]. However, with increasing the precision in both sets of experimental data the discrepancy between them has reached an unacceptable level of 4.6 $\sigma$ [4].

At present, even after the appearing of additional high-precision data, the same level of discrepancy still remains [5]. The importance of this discrepancy increases due to new very accurate measurements of the muon anomalous magnetic moment [6]. The latter should be compared with the SM prediction, which include the evaluation of the hadronic contribution to $(g - 2)_\mu$ based on $e^+e^-$ or/and $\tau$ data. While $\tau$-based data predict a somewhat close to the experimentally measured $(g - 2)_\mu$ value, $e^+e^-$ data differs by 3.3 $\sigma$ from it.

To find a solution to the discrepancy between the description of $e^+e^-$ and $\tau$ processes, some authors [7] have assumed that the problem is mainly due to additional isospin breaking effects and not to experimental ones. However, it is still possible that both data sets could be plagued with unknown yet systematic errors, therefore, others directly advocate $\tau$ data as more reliable [8]. In this letter we assume that, in spite of some inconsistency in both data sets among different experiments, they properly describe real physical processes, and that the discrepancy stems from additional new centi-weak tensor interactions in the weak processes.

Of course, so radical solution of the problem should effect also other low-energy precise experimental data as on muon and neutron decays, and so on. It has been shown in previous papers [9, 10] that those effects are still within experimental uncertainties of the present measurements. Conspiracy of the hypothetical interactions is stipulated by their different chiral properties in comparison with $V - A$ interactions. In many cases this prevents or suppresses interference between the new interactions and the ordinary ones, which should give a leading contribution from the new interaction in the case of its weakness. The only known examples of manifestation of such interference are the radiative pion decay [11], $\pi \to e\nu_e\gamma$, and the $\tau$-decay [12], $\tau \to \rho\nu_\tau$.

In this paper it will be shown that the anomaly in the latter decay has direct relation to the problem of $V_{us}$ extraction from the $\tau$-decay: its systematically low value in comparison with other evaluations. The recent $\text{BABAR}$ [13] and Belle [14] measurements of branching fractions for several hadronic $\tau$-decay modes with kaons allow to emphasize this fact [15].

Assuming the presence of the new interactions we will analyse the most precise extractions of the Cabibbo angle from superallowed $0^+ \to 0^+$ nuclear beta transi-
tions, neutron and $\tau$ decays, and also from the ratio of kaon and pion decay widths, $\Gamma(K \rightarrow \mu\nu)/\Gamma(\pi \rightarrow \mu\nu)$, $K_{l3}$ and hyperon decays. The latter three methods strongly depend on flavor-$SU(3)$–breaking effects, which cannot be safely calculated at present, and do not provide a unique absolute determination of the Cabibbo angle. Nevertheless, using the unitarity relation we will show that all mentioned methods can lead to self-consistent results.

2 New interactions and direct $V_{ud}$ determination

The effective tensor interactions

\[ \mathcal{L}_T^{\text{eff}} = -\sqrt{2} f_T G_F \bar{u}_L \sigma_{ml} d_R \frac{4q^l q_m}{q^2} \bar{e}_R \sigma^{mn} \nu_L \]

\[ -\sqrt{2} f_T G_F \bar{u}_R \sigma_{ml} d_L \frac{4q^l q_m}{q^2} \bar{e}_R \sigma^{mn} \nu_L + \text{h.c. (1)} \]

naturally appear in an extension of the SM with new type of spin-1 chiral bosons described by the antisymmetric second-rank tensor fields [16]. Here $q_m$ is the momentum transfer between quark and lepton currents. The dimensionless coupling constant $f_T$ determines the strength of the new interactions relative to the ordinary weak interactions. With magnitude $f_T \simeq 10^{-2}$ these interactions allow to explain the anomaly in the radiative pion decay [17] and at the same time to escape constraints from other precise measurements, due to their peculiar form. In order to distinguish these interactions from the ordinary ones, taking into account their strength, we will refer to them as the centi-weak interactions.

Assuming flavor universality we can rewrite the interactions (1) for the third lepton family as

\[ \mathcal{L}_T^{\text{eff}} = -\sqrt{2} f_T G_F \bar{u}_R \sigma_{ml} d_L \frac{4q^l q_m}{q^2} \bar{\tau}_R \sigma^{mn} \nu_L + \text{h.c. (2)} \]

Namely, these interactions are responsible for an additional contribution to the vector transition $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ in comparison with the SM prediction, based on the CVC hypothesis. In the ref. [12] simple formula for the corresponding branching fractions has been derived

\[ R_{2\pi} = \frac{B_{2\pi}^{\text{exp}} - B_{2\pi}^{\text{CVC}}}{B_{2\pi}^{\text{CVC}}} = F_T \frac{6}{2 + r^2} + F_T^2 \frac{1 + 2r^2}{(2 + r^2)r^2}, \]

where $r = m_\tau/m_\rho$ is the mass ratio of the $\tau$ lepton and the $\rho$ meson, and the positive factor

\[ F_T = 4r f_T \frac{f_\rho^\perp}{f_\rho^\parallel} > 0, \]

is expressed through the effective coupling constant $f_T$ and the ratio of the transverse and longitudinal coupling constants of the $\rho$ meson, $f_\rho^\perp/f_\rho^\parallel = 0.703^{+0.004}_{-0.007}$ [18].

The knowledge of the experimental excess, $R_{2\pi} = (3.8 \pm 0.9)\%$ [5], allows to fix

\[ f_T = (0.69 \pm 0.16) \times 10^{-2} \]
more precisely than before, which is now up to 4σ above zero.

The interactions (1) do not effect the superallowed Fermi transitions, which are described by a vector quark current in the effective weak lagrangian

\[ \mathcal{L}_W^{\text{eff}} = -\sqrt{2} G_F V_{ud} \bar{u} \gamma_m (1 - \gamma^5) d \bar{e}_L \gamma^m \nu_L + \text{h.c.} \quad (6) \]

Therefore, the extraction of the matrix element \( V_{ud} \) from the supperallowed \( 0^+ \to 0^+ \) nuclear β-decays should give, in principle, its more reliable value. However, in reality the product \( G_F V_{ud} \) is measured, where \( G_F \) is the well known Fermi coupling constant. The latter is deduced from the muon decay, which may be effected by new interactions.

Indeed, assuming again universality of the new interactions (1) one can introduce purely leptonic tensor interactions

\[ \mathcal{L}_{\text{lepton}}^{\text{lepton}} = -\sqrt{2} f_T G_F \bar{\nu}_L \sigma_{m\mu} \mu_R \frac{4 q'_R q_m}{q^2} \bar{e}_R \sigma^{mn} \nu_L + \text{h.c.}, \quad (7) \]

which include only left-handed neutrinos and right-handed charged leptons. In accordance with the see-saw mechanism, we assume here that the right-handed neutrinos are very massive and they do not contribute to the low-energy processes. Therefore, in contrast to the semileptonic tensor interactions, the effective leptonic interactions do not include a term analogous to the second term in (1). Let us note that the interactions (7) are not present among the Michel local interactions and their effect on electron spectrum cannot be described by the Michel parameters only.

Besides the distortion of the ordinary \( V - A \) electron spectrum, the tensor interactions give additional positive contribution to the muon decay width [19]

\[ \Gamma_\mu = \Gamma_{\text{SM}}^{\mu} \left\{ 1 + 12 \frac{m_e}{m_\mu} f_T + 3 f_T^2 \right\}. \quad (8) \]

Since \( \Gamma_\mu \propto G_F^2 \), this leads effectively to new value for the real magnitude of the Fermi coupling constant

\[ G_F = \frac{G_F^{\text{exp}}}{\sqrt{1 + 12 \frac{m_e}{m_\mu} f_T + 3 f_T^2}} = \frac{G_F^{\text{exp}}}{(1.00027 \pm 0.00008)} = (1.16605 \pm 0.00009) \times 10^{-5} \text{ GeV}^{-2} \quad (9) \]

instead of \( G_F^{\text{exp}} = (1.16637 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}. \) Therefore, the master formula for the \( V_{ud} \) determination from superallowed β decays [20] should be corrected as

\[ |V_{ud}|_\beta = (1.00027 \pm 0.00008) \sqrt{\frac{(2984.48 \pm 0.05) \text{ s}}{ft(1 + RC)}}. \quad (10) \]

The present most precise PDG value of \( |V_{ud}|_{\text{PDG}} = 0.97377 \pm 0.00027 \) [21] is based on nine best measured superallowed decays [22, 23] without the mentioned correction. However, there exist a problem with the recent precise Penning-trap measurements [23, 24] of the \( Q_{EC} \) value for the superallowed decay of \( ^{46}\text{V} \), which
has significantly affected its $f_t$ and $V_{ud}$ values. Recently, Towner and Hardy [25] have recalculated the isospin-symmetry-breaking correction including core orbitals, which remove the discrepancy with $^{46}$V, and get somewhat higher value for $|V_{ud}|_{\beta}^{\exp} = 0.97418 \pm 0.00026$.\footnote{The new calculations [25] demonstrate a good agreement among the corrected $f_t$ values for the thirteen well measured transitions, although there is a possible small discrepancy for the cases of $^{50}$Mn and $^{54}$Co, which probably are connected again to the older measurements of $Q_{EC}$ values.} If we assume the latter value as a more reliable and apply new physics correction from eq. (10), this gives a new value of

$$|V_{ud}|_{\beta} = 0.97444 \pm 0.00026 \pm 0.00008 f_T = 0.97444 \pm 0.00027.$$ \hspace{1cm} (11)

Another independent information about the matrix element $|V_{ud}|$ can be obtained from the neutron decay, which, however, besides the vector transition, includes also the axial-vector amplitude characterized by the additional phenomenological parameter $\lambda$. Therefore, besides the precise knowledge of the neutron lifetime $\tau_n$, additional measurements of $\lambda$ are necessary. The master formula for $|V_{ud}|$ determination from the neutron decay reads [20]

$$|V_{ud}|_n = \sqrt{\frac{(4908.7 \pm 1.9) \text{s}}{\tau_n (1 + 3\lambda^2)}}.$$ \hspace{1cm} (12)

At present there are severe disagreements in the experimental results both for the neutron lifetime, $\tau_n$, and $\lambda$ value and there is no sense to evaluate the averages. According to us this situation is connected to huge corrections, which have been applied in older experiments to the raw data in order to extract the final result. By our opinion the recent measurements of the neutron lifetime $\tau_n = 878.5(8)$ s [26] and parameter $\lambda_{\exp} = -1.2762(13)$ [27] are most reliable, where for the latter the applied corrections were just 0.09%, which is even below the experimental uncertainty. Using these numbers and eq. (12) one gets

$$|V_{ud}|_{n}^{\text{SM}} = 0.97430 \pm 0.00103.$$ \hspace{1cm} (13)

This value is in a very good agreement with (11), while the PDG value of the neutron lifetime $\tau_n^{\text{PDG}} = 885.7(8)$ s leads to the following $|V_{ud}|_{n} = 0.97033 \pm 0.00103$, which is about $4\sigma$ below than the reference value (11).\footnote{The new value of the neutron lifetime is favored also by cosmological reasons [28].}

Even the effect of the new tensor interactions cannot help to improve the situation. It is known from ref. [10] that the new interactions (1) do not effect the neutron lifetime in the leading order on $f_T$, while they change effectively $\lambda = \lambda_{\exp} + \delta \lambda$, extracted from the electron asymmetry parameter $A$, where $\delta \lambda \simeq 0.0012$ is within the experimental uncertainty for the reference value (5). If we accept this correction, we get slightly different value than (13)

$$|V_{ud}|_{n} = 0.97506 \pm 0.00103,$$ \hspace{1cm} (14)

which is also in agreement with (11).
Using the most precise value of $|V_{ud}|$ (11) and the unitarity relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$  \hspace{1cm} (15)

we can check selfconsistency of $V_{us}$ determination from different processes. This is possible due to the small value of $|V_{ub}|_{PDG} = (4.31 \pm 0.30) \times 10^{-3}$ [21] and present accuracy of $V_{ud}$ extraction (11). Indeed, neglecting $|V_{ub}|^2$ effects the unitarity relation (15) at an accuracy level of $2 \times 10^{-5}$, which is much less than the uncertainty $2|V_{ud}|^2|V_{ud}| \approx 53 \times 10^{-5}$ stems from $|V_{ud}|^2$ contribution. Therefore, the approximation by the Cabibbo angle $|V_{ud}| = \cos \theta_C$ and $|V_{us}| = \sin \theta_C$ is still very suitable for description of processes with light quarks.

Using this approximation and the corrected $V_{ud}$ value (11), extracted from the superallowed $\beta$ decay, one gets matrix element

$$|V_{us}|_{\beta}^{\text{uni}} = 0.22463 \pm 0.00112 \pm 0.00033_{f_T} = 0.22463 \pm 0.00117,$$  \hspace{1cm} (16)

which is in agreement with the PDG value of $|V_{us}|_{PDG} = 0.2257 \pm 0.0021$, but has almost two times better accuracy. Since the most precise value of $|V_{us}|$ is extracted from semileptonic kaon decays $K\mu\nu$, namely from the expression $|V_{us}f_+(0)| = 0.21666 \pm 0.00048$ [29], our result (16) indirectly confirms the reliability of the analytical evaluations [30, 31] and the lattice calculations [32] of SU(3)-breaking corrections to the form factor (see the left panel in the Fig. 1)

$$f_+(0) = 0.9645 \pm 0.0053 \pm 0.0014_{f_T} = 0.9645 \pm 0.0055,$$  \hspace{1cm} (17)

which is in a very good agreement with the naive average of the results, presented in the Fig. 1

$$f_{\text{ave}}^+ (0) = 0.9646 \pm 0.0013.$$  \hspace{1cm} (18)

Assuming the latter as an independent input, we can derive the most precise values of the matrix elements

$$|V_{us}|^{K} = 0.22462 \pm 0.00058$$  \hspace{1cm} (19)

and

$$|V_{ud}|^{K} = 0.97445 \pm 0.00013$$  \hspace{1cm} (20)

from the kaon decays only.

Another verification of the results in eqs. (11) and (16), can be found from the experimental ratio of the decay widths $\Gamma(K^+ \rightarrow \mu^+\nu_\mu(\gamma)) = 3.3716(99) \times 10^{-14}$ MeV and $\Gamma(\pi^+ \rightarrow \mu^+\nu_\mu(\gamma)) = 2.5281(5) \times 10^{-14}$ MeV [21] which is not effected by the tensor interactions and is proportional to the ratios

$$\tan \theta_C \equiv \frac{|V_{us}|}{|V_{ud}|} = 0.23052 \pm 0.00121 \pm 0.00036_{f_T} = 0.23052 \pm 0.00126$$  \hspace{1cm} (21)

and

$$\frac{F_K}{F_\pi} = 1.196 \pm 0.007 \pm 0.002_{f_T} = 1.196 \pm 0.007.$$  \hspace{1cm} (22)
Figure 1: The analytical [30, 31] and lattice [32] results for \( f_+(0) \) (left) and the unquenched \( N_f = 2 + 1 \) lattice calculations of \( F_K/F_\pi \) [33, 34] (right) versus the corresponding predicted values (17) and (22).

For the latter ratio there are several unquenched lattice calculations for \( N_f = 2 + 1 \) dynamical quark flavors [33], which are in a good accordance with eq. (22), except the CP-PACS/JLQCD result [34] (see the right panel in the Fig. 1). Their naive average

\[
\left. \frac{F_K}{F_\pi} \right|_{\text{ave}} = 1.192 \pm 0.005
\]

also agrees with eq. (22).

It is interesting to note, that many authors of the lattice calculations compare their results with the old ‘experimental’ value \((F_K/F_\pi)^{\text{exp}} = 1.223(12)\) from review [35], which is still present in the recent PDG editions and is used as an important input parameter for the chiral perturbation theory. This value is based on the old result by Leutwyler and Roos for \( |V_{us}|^{\text{old}} = 0.2196(26) \) [30] from \( K_{e3} \) decays, which is pushed up at present to a higher central value after the recent decided experiments by BNL E865 [36], KTeV, NA48, ISTRA+ and KLOE Collaborations.\(^3\) Therefore, the more precise and updated result from eq. (22) should be used.

### 3 Tau decays

Let us turn now to the \( \tau \) decays. Being a heavy lepton, \( \tau \) possesses pure leptonic decays as well as hadronic decays, which to a less extent are effecting by the QCD complications than the hadronic meson decays. At the same time many of the hadronic \( \tau \) decays could be considered as cross-channels of the semileptonic meson decays or just related through the CVC hypothesis to the processes of the electromagnetic \( e^+e^- \)

\(^3\)See also Leutwyler’s remark [37] about the present experimental value of \( F_K/F_\pi \) and paper [38].
annihilation. All this makes the \( \tau \) decay investigations very sensitive to an eventual manifestation of new physics.

Indeed at present there is a serious well known 4.5\( \sigma \) discrepancy between the measured and the predicted two pion branching ratio of the \( \tau \) decay [5]. Neither SUSY nor other known models can explain this anomaly. However, it has a natural explanation [12] in the extended standard model with the effective tensor interactions (2). If we assume the presence of such interactions, it will be possible also to solve the problem with systematically low central value of \( V_{us} \) extracted from hadronic \( \tau \) decays [39].

The idea is very simple and is based on the proper accounting for the additional contributions from the new semileptonic interactions (2) and the analogous pure leptonic interactions (7) for the \( \tau \) lepton. These interactions could contribute both to the leptonic and hadronic modes of the \( \tau \) decays. It is known that only six decay modes \( \tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau \), \( \tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau \), \( \tau^- \rightarrow \pi^-\nu_\tau \), \( \tau^- \rightarrow \pi^-\pi^0\nu_\tau \), \( \tau^- \rightarrow \pi^-2\pi^0\nu_\tau \) and \( \tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau \) account for 90\% of the decays. Let us consider them in detail.

According to the eq. (8) the leptonic decay widths of the \( \tau \) lepton should increase in comparison with the SM due to new interactions by 0.02\% and 0.51\% for the electronic and muonic modes, correspondingly. This statement should be compared with the experimental averages of the electronic and muonic branching fractions

\[
B^{\exp}_e = (17.821 \pm 0.052)\%, \quad B^{\exp}_\mu = (17.332 \pm 0.049)\%,
\]

which are claimed to be known with a relative precision of 0.3\% [40] and their ratio \( B^{\exp}_\mu/B^{\exp}_e = 0.9726 \pm 0.0041 \) perfectly matches the SM value 0.972565(9). However, one should be cautious and mind the overconsistency of leptonic branching fraction measurements [41]. The predicted corrected ratio

\[
\frac{B_\mu}{B_e} = 0.9774 \pm 0.0011
\]

although higher than the experimental average, is still in agreement with the data (see the left panel in the Fig. 2).

All hadronic modes of \( \tau \) decays into final state with odd pions, which are related to the axial-vector hadronic currents, are not affected by the new interactions due to the absence of an appropriate tensor current \( \bar{u}_\sigma_{\mu\nu}\gamma^5d \) in (2). At the same time all nonstrange (\( \Delta S = 0 \)) decays, which undergo through hadron vector transitions, acquire additional contributions. A substantial effect of the new interactions has been found namely in \( \tau^- \rightarrow \pi^-\pi^0\nu_\tau \) decay [12] due to the constructive interference between the new interactions and the standard ones. The comparison of the experimental data with the corrected CVC value using eq. (3)

\[
B^{\exp} = (25.40 \pm 0.19 \pm 0.22f_\tau)\% = (25.40 \pm 0.29)\%
\]

is shown in the right panel of the Fig. 2.

The strange (\( |\Delta S| = 1 \)) hadronic \( \tau \) decays, which contain around 3\% of the total decay width, are very important input for \( V_{us} \) determination. They just concern this matrix element for Cabibbo-suppressed (axial-)vector transitions between left-handed
The quarks of the different generations. The new tensor interactions have different chiral structure than the ordinary $V-A$ interactions and, in general, should be described by a different mixing matrix between left-handed and right-handed quarks. At present there is no indication of a presence of new interactions in kaon decays [44]. Therefore, we assume a near diagonal structure of the new mixing matrix and an absence of tensor transitions between different generations.

The master formula for the matrix elements extraction from the $\tau$ decays has the following form

$$R_{\tau,V+A}^{00} - R_{\tau,S}^{00} \equiv \delta R_{\tau,\text{th}},$$

where $R_{\tau,V+A}^{00}$ is the ratio of the inclusive nonstrange hadronic decay width and the electron width, while $R_{\tau,S}^{00}$ is its strange counterpart. It is assumed that only the ordinary $V-A$ interactions exist. Here the difference $\delta R_{\tau,\text{th}}$ is the $SU(3)$-breaking quantity induced mainly by the strange quark mass, which can be theoretically estimated within the QCD framework [45].

The experimentally measured quantities $R_{\tau,V+A}$ and $R_{\tau,S}$ are not independent and their sum should be related to the following combination of the leptonic branching fractions

$$R_{\tau} \equiv R_{\tau,V+A} + R_{\tau,S} = \frac{1 - B_e - B_\mu}{B_e} = 3.635 \pm 0.010,$$

where the average electronic branching fraction $B_e^{\text{ave}} = (17.818 \pm 0.032)\%$ [40] and the predicted ratio (25) were used. The latter corrects the right hand side of eq. (28) for an effect of the tensor interactions in the lepton channels, however their impact is well below the uncertainty and slightly changes only the central value.

The left hand side of eq. (28) also should be corrected for the additional contributions from the new physics. Since the vector transitions occur through the
vector meson resonances $\rho$, which are effected by the new interactions, then not only $\tau^- \to \pi^-\pi^0\nu_\tau$ channel, but $\tau^- \to \pi^-3\pi^0\nu_\tau$, $\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$ and other channels with even pions states should be revised, as well. Using the recent evaluations [5]

$$B_\tau - B^{\text{CVC}}_\tau = \begin{cases} 
0.92 \pm 0.21 \% & \text{for } \tau^- \to \pi^-\pi^0\nu_\tau \\
-(0.08 \pm 0.11) \% & \text{for } \tau^- \to \pi^-3\pi^0\nu_\tau \\
0.91 \pm 0.25 \% & \text{for } \tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau
\end{cases}$$

(29)

and neglecting the higher number meson states we get $5\sigma$ deviation in

$$R_{\tau,V} - R^{\text{CVC}}_{\tau,V} = 0.098 \pm 0.019.$$  

(30)

If we assume that the undistorted vector contribution in the $\tau$ decay can be deduced from the experimental data on $e^+e^-$ annihilation by the CVC hypothesis, i.e. $R^0_{\tau,V} = R^{\text{CVC}}_{\tau,V}$, while its axial-vector and strange counterparts are not effected by the new interactions, i.e. $R^0_{\tau,A} = R_{\tau,A}$ and $R^0_{\tau,S} = R_{\tau,S}$ respectively, then we can apply corrections (30) and (28) to the master eq. (27) with

$$R^0_{\tau,V + A} \equiv R^0_{\tau} - R^0_{\tau,S} = 3.537(22) - R_{\tau,S}.$$  

(31)

The master equation is usually solved for the unknown parameter $|V_{us}|$, while the other entries are considered known inputs. Here we will show that using the unitarity relation, it is possible to extract the ratio

$$|V_{us}|^2 = \frac{\sqrt{(R^0_{\tau} - \delta R_{\tau,\text{th}})^2 + 4R^0_{\tau,S}R_{\tau,\text{th}} - (R^0_{\tau,V + A} - R^0_{\tau,S} - \delta R_{\tau,\text{th}})^2}}{2R^0_{\tau,V + A}}$$

$$\approx \frac{R^0_{\tau,S}}{R^0_{\tau,V + A} - \left(\frac{R^0_{\tau,V + A}}{R^0_{\tau}}\right) \delta R_{\tau,\text{th}}}$$

(32)

from eq. (27) and the matrix element $|V_{us}|$ with even better precision.

To finalize the matrix elements extraction we need to define two more inputs $R^0_{\tau,S}$ and $\delta R_{\tau,\text{th}}$. The latter depends on the strange quark mass $m_s$. The recent $m_s$ evaluations, disregarding the $\tau$ decays because they are most effected by the new interactions, are shown in the Fig. 3, which lead to the following average mass

$$m_s(2 \text{ GeV}) = (99.6 \pm 3.3) \text{ MeV},$$

(33)

where the uncertainty is corrected by the scale factor $S = 1.3$, and its corresponding

$$\delta R_{\tau,\text{th}} = 0.1602 \pm 0.0046 + (6.08 \pm 1.00)\frac{m_s^2}{\text{GeV}^2} = 0.221 \pm 0.012.$$  

(34)

The most critical part is the experimental value of $R_{\tau,S}$, which after LEP epoch was

$$R^{\text{LEP}}_{\tau,S} = 0.1686 \pm 0.0047,$$  

(35)
and at present after the new BABAR [13] and Belle [14] measurements is

\[ R_{\tau,S}^{\text{new}} = 0.1617 \pm 0.0040. \] (36)

Of course, these two values lead to different magnitudes for the ratio

\[ \frac{|V_{us}|}{|V_{ud}|_{\tau}} = \begin{cases} 
0.2310 \pm 0.0032 & \text{from eq. (35)} \\
0.2260 \pm 0.0028 & \text{from eq. (36)} 
\end{cases} \] (37)

and for the matrix elements

\[ |V_{us}|_{\tau} = \begin{cases} 
0.2251 \pm 0.0029 & \text{from eq. (35)} \\
0.2205 \pm 0.0026 & \text{from eq. (36)} 
\end{cases} \] (38)

and

\[ |V_{ud}|_{\tau} = \begin{cases} 
0.97434 \pm 0.00068 & \text{from eq. (35)} \\
0.97540 \pm 0.00058 & \text{from eq. (36)} 
\end{cases} \] (39)

Comparison with the eqs. (11), (16) and (21) shows, that the extracted from the hadronic \( \tau \) decays matrix elements, based on the old data, are in a perfect agreement with the present most precise measurements from the superallowed beta decays, while the new BABAR and Belle results lead to the low value for \( |V_{us}| \), but still statistically acceptable. This agreement induces us to make doubt of the BABAR and Belle measurements, moreover, they have analyzed only several Cabibbo-suppressed hadronic \( \tau \) decay modes and all their results show systematically low values. Perhaps we need to wait for more serious investigations which include thorough analysis of the main \( \tau \) decay channels as well.

Let us compare now our derivation (32) of the matrix elements from the hadronic \( \tau \) decays with the usually used one [49]. First of all let us note that the eq. (32) for

![Figure 3: The recent analytical [46] and lattice \( N_f = 2 \) [47] and \( N_f = 2 + 1 \) [48] evaluations of \( m_s(2 \text{ GeV}) \).](image-url)
\[ |V_{us}|/|V_{ud}| \] determination is very similar to the formula for \(|V_{us}|\) and even has a slightly lower uncertainty. However, we get the main gain in precision for \(|V_{us}|\) determination from the ratio \(|V_{us}|/|V_{ud}|\) using unitarity. The simple evaluations show that, if the precisions for \(|V_{us}|\) and \(\tan \theta_C \equiv |V_{us}|/|V_{ud}|\) are the same, we get around 8% better accuracy for \(|V_{us}|\) determination from the ratio \((\delta|V_{us}| = \delta \tan \theta_C/(1 + \tan^2 \theta_C)^{3/2})\). The second advantage of the eq. (32) consists in a possibility of a simultaneous extraction of \(|V_{us}|\) and \(|V_{ud}|\) as well with a good precision. Although it cannot still compete with the accuracy of the superallowed beta decays, we get \(|V_{ud}|\) (39) with even less uncertainty than from the neutron decays (14).

The surprising agreement between the matrix elements, extracted from the superallowed beta decays and from the hadronic \(\tau\) decays, results from the essential corrections of the hadronic decay width, \(R_\tau\), (28) and (30) due to the new tensor interactions. It should be noted that uncorrected \(R_\tau\) input leads approximately to one standard deviation less central value for the \(|V_{us}|\), what has been pointed out in \([50]\). To support our suggestions we will draw attention to the results of ref. \([51]\), where the weighted spectral integrals have been used. It has been shown that using spectral weights which suppress the contributions from the region of the spectrum with high hadronic masses, the central values of the extracted \(|V_{us}|\) are systematically higher than for the unweighted case. It is just in concordance with our results from \([12]\), which predict a monotonically increasing deviation of the \(\tau\) spectral function from the SM case with the increase of the hadron invariant mass. Therefore, a suppression of this part of the spectrum allows to make more reliable predictions.

In conclusion, we would like also to compare our results with \(|V_{us}|\) extraction from strangeness-changing semileptonic hyperon decays, which, as we assumed, are not influenced by the new interactions in contrast with Cabibbo-allowed decays. A simple phenomenological fit of the decay rates and axial-vector to vector form factors ratios, up to \(SU(3)\) breaking effects, leads to the following result \([52]\)

\[ |f_1 V_{us}|_{SU(3)} = 0.2250 \pm 0.0027, \]  

where \(f_1\) is the ratio of the vector formfactor \(f_1(0)\) to its \(SU(3)\) predicted value. This analysis has been confirmed in ref. \([53]\). However, the authors suggest two times bigger uncertainty due to the disagreement of the different theoretical estimations of \(f_1\). Comparing the result (40) with the reference value (16) one can conclude, that \(f_1\) is equal to one up to second-order \(SU(3)\) breaking effects in agreement with the Ademollo–Gatto theorem \([54]\). It means that the \(SU(3)\) breaking corrections in hyperon decays at present accuracy are still not essential and their account results in a smaller central value of \(|V_{us}| = 0.2199 \pm 0.0026 \) \([55]\).

4 Conclusions

The main purpose of this paper is to point out the necessity for the account of the new tensor interactions in the weak processes. For many processes their effect is still well below the experimental uncertainties, however, for some of them their account is urgently needed. It happens due to the weakness of the new interactions and their
different chiral structure in comparison with the SM. They can reveal themselves, for example, in chirally suppressed pion decays, or in result of the interference of the tensor currents from the centi-weak interactions with transverse hadronic currents, as in tau decays. At the same time, due to the complexity of the Lorentz structure of such interactions, they do not contribute to the two-particle pion decay as the more simple hypothetical pseudoscalar interaction would. All these properties of the new tensor interactions allow them often to escape stringent experimental constraints.

In this paper we have analyzed of the extraction of the matrix elements between light quark species. Using the available experimental data and the unitarity relation, we have shown that all results can be come selfconsistent, if

(a) the new data for the neutron decay are used,
(b) the old branching fractions of strangeness-changing $\tau$ decays are correct,
(c) the additional tensor interactions are taken into account.

The latter removes the disagreement between the spectral functions extracted from the electromagnetic $e^+e^-$ data and the $\tau$ decays. It gives us the possibility to use both data for the evaluation of the hadronic contribution into the muon anomalous magnetic moment. This could strengthen the present discrepancy between its predicted and experimentally measured values.

Taking this discrepancy seriously we can constrain other sector of our model connected with new neutral tensor currents, which should also be present in the extension of the SM by doublets of the tensor particles [16]. The new charged and neutral tensor current interactions appear as effective interactions resulting from the exchanges of the massive charged and neutral tensor particles, respectively. The doublet structure of the new particles stems from the symmetry of the SM and that the tensor interactions as the (pseudo)scalar ones mix the left and right handed fermions from different representations of $SU(2)_L \times U(1)_Y$ group. In order to compensate the contributions of the new couplings of the tensor particles into the chiral anomaly, the doubling of the doublets both for the tensor and Higgs particles is needed. Besides this, in order to prevent also the presence of flavor-changing neutral currents, up and down type fermions should couple different doublets of the Higgs [56] and tensor particles.

In ref. [57] the masses of the new tensor particles have been estimated. Two mass scales around 700 GeV and 1 TeV for the different doublets have been derived on the basis of the dynamical principle and the value of the effective coupling constant $f_T$, which shows the relative strength of the new interactions in comparison with the SM. Due to additional mixing the lightest charged tensor boson has a mass around 500 GeV and can effect nonnegligibly the charged current SM processes, while the lightest neutral tensor boson couples just to the up type fermions and can effect leptonic processes with neutrinos. Additional contributions into neutral current processes with charged leptons come from the exchanges of the heavy neutral tensor particles and are around four times weaker than in the charged tensor current sector.

Nevertheless, an eventual mixing of the heavy neutral tensor particles with photons can effect electromagnetic characteristics of charged leptons like their anomalous magnetic moments, which have been measured with unprecedent precision for the electron and muon. It has been shown [58] that such mixing can explain the sign of the discrepancy between the predicted and measured muon anomalous magnetic
moment. In contrast to the new heavy particles, which contribute through the loops and effect the anomalous magnetic moments as $m^2_{\ell}/M^2_H$, the effect of the mixing is proportional to $m_\ell/M_H$. Therefore, the effect of the new physics on the electron anomalous magnetic moment could be stronger than have been expected and can be tested in independent measurements of the fine structure constant $\alpha$. On the other hand the mass scale of the new tensor particles offers a unique possibility for their discovery at the Large Hadron Collider.

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