Creating very slow optical gap solitons with inter-fiber coupling

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We show that gap-acoustic solitons, i.e., optical gap solitons with electrostrictive coupling to sound waves, can be produced with velocities down to less than 2.5% of the speed of light using a fiber Bragg grating that is linearly coupled to a non-Bragg fiber over a finite domain. Forward- and backward-moving light pulses in the non-Bragg fiber that reach the coupling region simultaneously couple into the Bragg fiber and form a moving soliton, which then propagates beyond the coupling region.

There is great interest in slow light [1]. Spectacular slow light results have been achieved in Bose-Einstein condensates [2], but realizations in room temperature solid state materials are desirable for many applications. Optical fiber Bragg gratings (FBGs) can support one such slow light structure, the gap-acoustic soliton (GAS), i.e., an optical gap soliton with electrostrictive coupling to sound waves, which can have velocities from zero up to the group velocity in the medium [3–7]. Gap solitons have been produced in the lab with velocities as slow as 16% of the speed of light [6], but to date not slower. Suitable experimental media for GAS propagation are available, but it can be difficult to create the correct initial and/or boundary conditions to obtain a GAS in the first place. Methods proposed for producing slow GASs include: (1) fibers with gradually varying modulation depth (apodized) [8], as was employed in Ref. [6], (2) colliding faster moving gap solitons with each other [9] or with fiber defects [10, 11], and (3) growing a soliton in-place with either distributed [12] or localized amplification [13]. We propose a method to produce GASs using a FBG and a non-Bragg fiber that are coupled over a finite distance, as illustrated in Fig. 1. We show that if light pulses, specially designed by reverse engineering, are sent into the non-Bragg fiber in the forward- and backward-moving directions such that they simultaneously reach the inter-fiber coupling region (see Fig. 2), a slow GAS can be created in the FBG. The parameters of the resulting soliton depend on the Bragg fiber parameters, the coupling to the non-Bragg fiber, and the widths, intensities, and phases of the input pulses. This inter-fiber coupling may be contrasted with soliton switching in uniformly coupled FBGs [14, 15]. One of the essential differences in this work is that here the inter-fiber coupling region is finite, and the light switches exactly once, from the non-Bragg to the Bragg fiber and then remains in the FBG.

A FBG may be coupled to the non-Bragg fiber by removing some of the cladding and bringing the fibers close together so that the evanescent waves of one fiber extend over the core of the other [16]. This results in linear coupling between the light in the two fibers, which is stronger when the fibers are closer [17, 18]. The coupling region is finite, and varies smoothly from the uncoupled to the maximally coupled region.

The dynamics of the system are described by the equations

\begin{align}
0 &= i k_0^I u_{1,t} + i u_{1,z,z} + \kappa v_1 + \lambda(z) u_2 + C(|u_1|^2 + 2|v_1|^2) u_1 + \chi_{es} w u_1, \quad (1a) \\
0 &= i k_0^II u_{2,t} - i u_{2,z,z} + \kappa v_2 + C(|u_2|^2 + 2|v_2|^2) u_2 + \chi_{es} w v_1, \quad (1b) \\
0 &= w_{t} - \beta_s^2 u_{zz} - \Gamma w_{zz} + \lambda_{es}(|u_1|^2 + |v_1|^2) z, \quad (1c) \\
0 &= i k_0^I u_{2,t} + i u_{2,z} + \lambda(z) u_1, \quad (1d) \\
0 &= i k_0^I v_{2,t} - i v_{2,z} + \lambda(z) v_1, \quad (1e)
\end{align}

where \( z \) is the spatial coordinate, \( t \) is time, \( u_1 \) is the amplitude of the envelope of the forward-moving electric field in the Bragg fiber, \( v_1 \) is for the corresponding backward-moving field, \( u_2 \) and \( v_2 \) are the fields in the non-Bragg fiber, \( w \) is the material density of the FBG and subscripts in the independent variables denote differentiation. The reciprocal of the group velocity is \( k_0^I = \frac{2\pi}{\omega_0} |n(\omega)\omega| c |\omega_0| \), \( \kappa \) is the Bragg scattering coefficient, which scatters light back into the same fiber, \( \lambda(z) \) is the position dependent inter-fiber coupling coefficient (co-directional; contra-directional inter-fiber coupling is taken to be zero), \( C = 2\pi \omega_0 |n(\omega_0)\omega| c^{-1} \) \( \chi(\omega_0; \omega_0, -\omega_0, \omega_0) \) is a self-phase modulation coefficient, \( \chi_{es} = (\omega_0/c) \partial n / \partial w \) and \( \lambda_{es} = \left( 2\pi \right)^{-1} n(\omega_0) w \partial n / \partial w \) are electrostrictive coefficients, \( \beta_s \) is the speed of sound in the fiber, and \( \Gamma \) is the phonon viscosity coefficient. The inter-fiber coupling is due to the overlap between the evanescent tails of the modes in the neighboring fibers [17, 18]. We have included the coupling of light (\( u_1, v_1 \)) to the material density (\( w \)) in the Bragg fiber but not for the non-Bragg fiber (\( u_2, v_2 \)) because the propagation velocities in the latter (but not the former) are too fast to be affected. Kerr effects are unimportant in the non-Bragg fiber and are neglected. We used typical physical parameters for bulk fused silica at wavelength 0.8 \( \mu \)m [19, 20]: The index of refraction is \( n_0 = 1.45 \), the Kerr coefficient is \( C = n_0^4 |n(\omega)\omega| / (2\pi) \), with \( n_0^4 = 2.8 \times 10^{-16} \) cm\(^2\)/W, the material density is \( w = 2.2 \) g/cm\(^3\), and dependence of the refractive index on it is \( \partial n / \partial w = 0.2 \) cm\(^2\)/g, the speed of sound is \( \beta_s = 5.9 \) km/s, the phonon viscosity is \( \Gamma = 6.9 \times 10^{-7} \) m\(^2\)/s, and the Bragg grating coefficient \( \kappa \) is proportional to
FIG. 1: (Color online) Schematic drawing of a Bragg fiber and a non-Bragg fiber that are brought close together so that their evanescent waves overlap and the light couples between the fibers. Light pulses are sent into both ends of the non-Bragg fiber, and reach the coupling region at the same time. The shape and intensity of the pulses are adjusted such that they create a gap-acoustic soliton in the Bragg fiber, along with additional non-soliton radiation.

the modulation depth of the grating; we use $\kappa = 2.7 \text{ cm}^{-1}$. The inter-fiber (codirectional) coupling, due to overlap of the evanescent fields with the cores of the adjacent fibers, depends on the distance between the Bragg and non-Bragg fibers. We take the coupling coefficient to be $\lambda(z) = A_{\text{cpl}} \exp[(z/z_{\text{cpl}})^4]$, with $A_{\text{cpl}} = 1 \text{ cm}^{-1}$, $z_{\text{cpl}} = 0.2 \text{ cm}$, i.e., a super-Gaussian, which is fairly flat in the middle and smoothly but quickly decreases to zero at the edges.

The inputs of light from the non-Bragg fiber that yield GASs in the Bragg fiber are determined by reverse engineering: A GAS is propagated numerically in the Bragg fiber, initially moving toward the region that is coupled to the non-Bragg fiber. When we find an instance in which an appreciable amount of the energy escapes into the non-Bragg fiber, we run the simulation in the opposite direction, inserting the obtained light pulses only into the non-Bragg fiber. The scheme is optimized further by fine-tuning the pulse parameters. One consideration is that the two input pulses must be asymmetrical to produce a soliton that moves away from the coupling region without returning the light back to the non-Bragg fiber. Another is that the initial light pulses must be stronger than the results of the time-reversal output pulses, since there is less than complete coupling of the light between the fibers. We achieved better results with stronger but shorter coupling regions.

We succeeded in creating GASs in the Bragg fiber with velocities from close to the group velocity in the medium down to less than $c/40$. Figure 2 shows an example of such a simulation. Light pulses enter the non-Bragg fiber from opposite ends [panels (a) and (b)], and they reach the region with non-zero inter-fiber coupling at the same time. After a finite interaction time, light propagates into the uncoupled region of the Bragg fiber, some of it as a very slow GAS (velocity $0.0246 c = 0.0357 v_g$), while other light propagates as faster GASs or as dispersive radiation [panel (c)]. The energy density and material density of the slowest GAS is shown in (d). Figure 3 shows the parameters of the GAS produced versus an input intensity factor $B$, where $B$ is the ratio of the input amplitude in the non-Bragg fiber divided by the output amplitude in the non-Bragg fiber after the time-reversed simulation. This illustrates the optimization we carried out on the family of results.

Let us review the approximations. Equations (1) omit self-phase modulation and low wavenumber acoustic waves in the non-Bragg fiber, and interactions with high wavenumber acoustic waves (i.e., Brillouin scattering) in both fibers. Low wavenumber acoustic waves were shown in [22] to be critical for optical gap solitons when the soliton velocity is $\leq v_g/200$, because the soliton has a momentum minimum near that velocity. We did not attain such a slow velocity, but the results were within an order of magnitude of it, and with better optimization of the system parameters it might be possible to achieve a soliton velocity at which the field $\psi(z, t)$ is essential. Brillouin scattering in a waveguide is enhanced by a Bragg fiber, but—very importantly—only outside the band gap [21]. Within the band gap created
FIG. 2: (Color online) (a) Surface plot of the energy density in the non-Bragg fiber, as a function of distance \( z \) and time \( t \). (b) Initial conditions for the simulation showing the energy densities and phases of the light pulses in the non-Bragg fiber. (c) Surface plot of the energy density in the Bragg fiber propagated for 25 ns. The interactions occur over the first 4 ns because of the width of the input pulses. The output consists of one very slow GAS (velocity 0.0246 \( c \)), and the other pulses are dispersive radiation or faster GASs. (d) Light energy density and material density at 185 ns (solid curves), and the exact GAS (dashed curves), with the acoustic wave of the exact GAS scaled by a factor of \( 10^3 \) to be visible. The density in the simulation swamps the soliton wave density.

by a Bragg grating, gap solitons do not show notable Brillouin back-scattering, and the models work well without inclusion of Brillouin fields (see e.g., [6]). Reference [22] found that under realistic experimental conditions, there may be significant Brillouin scattering only when the gap solitons have velocities very close to the group velocity of light. In the non-Bragg fiber, low wavenumber acoustic waves (inclusion of which would amount to a generalized Zakharov system [23]) will be insignificant for realistic light pulses because pulses move at the group velocity of light, which is large. Brillouin scattering can be important in non-Bragg fibers, but fiber lengths, especially for short pulses, need to be fairly long. The non-Bragg fibers herein need just be long enough to bring the light to the inter-fiber coupled region, so Brillouin scattering can be avoided by not using excessively long lead-in fibers. Within the inter-fiber coupling region, the dynamics are transient, highly nonlinear, and complex. The interaction times (a few ns) was not long enough for acoustic waves to develop much, and the (transient) high wavenumber acoustic waves that result are not likely to induce an effective grating consistently in-phase with the light fields due to the messiness of the fields and non-uniformity of the fibers [note \( \lambda = \lambda(z) \)] in the inter-fiber coupled region. We omitted less important nonlinearities in order to get to the essence of the problem. However, that opto-acoustic interactions might have some effect could not be completely ruled out, so we ran simulations with all the nonlinearities described here. We used the model in [22], which is the same as in [24], but does not approximate the speed of sound as zero and does not include equations for temperature dynamics. We found only quantitative, not qualitative, modifications of the dynamics.

The reverse engineering simulations provide a framework for understanding the physics and they may help to
FIG. 3: (Color online) Plot of the velocity ($\beta$) and peak energy density ($U$) of the slowest GAS as functions of the initial intensity scaling factor ($B$), the ratio of the amplitude of the input pulses over the amplitude of the output pulses in the time-reversal simulation.

motivate guesses for experimental parameters. Ultimately, the reverse engineering simulations can be dispensed with, and one may take initial pulses as hyperbolic secants or Gaussians of suitable width, amplitude, and frequency, and relative position.

In summary, we propose and numerically demonstrate a method for creating optical gap solitons (or gap-acoustic solitons) in a fiber Bragg grating by side-coupling the light over a finite region from a non-Bragg fiber. Note that this scheme could be used as an and switch, since a soliton will only be produced if there is input from both directions of the non-Bragg fibers. The solitons we produced have velocities down to approximately 2.5% of the speed of light. Much below this velocity the solitons are subject to stronger supersonic instability [7], and velocity zero is inaccessible since the solitons need enough velocity to escape the inter-fiber coupling region. Combining this with other techniques, such as colliding solitons [9] or making positive use of the supersonic instability [7] may be used to create yet slower or even stopped light pulses.

This work was supported in part by grants from the U.S.-Israel Binational Science Foundation (No. 2006212), the Israel Science Foundation (No. 29/07), and the James Franck German-Israel Binational Program.

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