NONLINEAR SUPERSYMMETRIC GENERAL
RELATIVITY AND UNITY OF NATURE *

KAZUNARI SHIMA † and MOTOMU TSUDA ‡

Laboratory of Physics, Saitama Institute of Technology
Fukaya, Saitama 369-0293, Japan

Abstract

The basic idea and some physical implications of nonlinear supersymmetric general relativity (NLSUSY GR) are discussed, which give new insights into the origin of mass and the mysterious relations between the cosmology and the low energy particle physics, e.g. the spontaneous SUSY breaking scale, the cosmological constant, the (dark) energy density of the universe and the neutrino mass.

*Based on the talk given by K. Shima at Conference in Honor of C.N. Yang’s 85th Birthday, October 30 - November 2, 2007, Singapore.
†e-mail: shima@sit.ac.jp
‡e-mail: tsuda@sit.ac.jp
1 Introduction

The discovery of the rationale of being of Nature is the long standing problem for the particle physicist. The symmetry and its spontaneous breaking are important notions for the unified description of nature. Supersymmetry (SUSY) \[1, 2, 3\] related naturally to space-time symmetry is promising for the unification of graviton with all other (observed) particles in the single irreducible representation of the symmetry group. Therefore, the evidences of SUSY and its spontaneous breakdown \[4, 5, 6\] should be studied not only in (low energy) particle physics but also in cosmology, i.e. in the framework necessarily accommodating graviton \[7\].

Along these viewpoints, we have found by the group theoretical argument that among all $SO(N)$ super-Poincaré (SP) groups the $SO(10)$ SP group decomposed as $N = 10 = 5 + 5^*$ under $SO(10) \supset SU(5)$ may be a unique and minimal group which accommodates all observed particles, i.e. the standard model (SM) with just three generations of quarks and leptons including graviton are assigned in a single irreducible representation of $N = 10$ linear (L) SUSY \[8\], which leads to the superon quintet hypothesis \[9\].

The advocated difficulty for constructing non-trivial $SO(N > 8)$ SUSY (gravity) theory, the so called no-go theorem based on S-matrix argument \[10, 11\], can be circumvented by adopting the nonliner (NL) representation of SUSY \[12\], i.e. the vacuum degeneracy of the fundamental action. Also the NL representation of SUSY gives the (unique) action describing the spontaneous breakdown of SUSY. Volkov-Akulov (VA) model \[2\] gives the NL representation of $N = 1$ SUSY describing the dynamics of spin 1/2 Nambu-Goldstone (NG) fermion accompanying the spontaneous SUSY breaking in flat space-time.

Therefore the NLSUSY invariant generalization of the general theory of relativity gives the fundamental theory of everything in our scenario.

2 Nonlinear Supersymmetric General Relativity

For simplicity we discuss $N = 1$ without the loss of the generality. The extension to $N > 1$ is straightforward.

The fundamental action (called nonlinear supersymmetric general relativity (NL-SUSY GR)) has been constructed by extending the geometric arguments of Einstein general relativity (EGR) on Riemann space-time to new space-time inspired by NL-SUSY, where tangent space-time is specified not only by the Minkowski coordinate $x_a$ for $SO(1, 3)$ but also by the Grassmanian coordinate $\psi_\alpha$ for the isomorphic $SL(2C)$ for NLSUSY \[13, 9\], i.e. the coset parameters of $\frac{superGL(4R)}{GL(4R)}$ which can be interpreted
as NG fermions associated with the spontaneous breaking of super-GL(4R) down to GL(4R). The noncompact isomorphic groups SO(1,3) and SL(2C) for tangent space-time symmetry on curved space-time can be regarded as the generalization of the compact isomorphic groups SU(2) and SO(3) for the gauge symmetry of 't Hooft-Polyakov monopole on flat space-time.

The NLSUSY GR action [13, 9] is given by

\[ L_{\text{NLSUSYGR}}(w) = \frac{c^4}{16\pi G} |w| \{ \Omega(w) - \Lambda \}, \]

where \( G \) is the Newton gravitational constant, \( \Lambda \) is a (small) cosmological term and \( \kappa \) is an arbitrary constant of NLSUSY with the dimension (mass)\(^{-2}\). \( w^a_\mu(x) = e^a_\mu + t^a_\mu(\psi) \) and \( w^\mu_a(x) = e^\mu_a - t^\mu_a + t^\rho_\rho \rho^a - t^\rho_\sigma \sigma^a + t^\rho_\kappa t^\kappa_\sigma t^\sigma_\rho t^\mu_a \) which terminates at \( \mathcal{O}(t^4) \) are the invertible unified vierbeins of new space-time. \( e^a_\mu \) is the ordinary vierbein of EGR for the local \( SO(1,3) \) and \( t^a_\mu(\psi) \) is the stress-energy-momentum tensor (i.e. the mimic vierbein) of NG fermion \( \psi(x) \) for the local \( SL(2,C) \). (We call \( \psi(x) \) superon as the hypothetical fundamental spin \( 1/2 \) particle constituting (carrying) the supercharge of the supercurrent [15] of the global NLSUSY.) \( \Omega(w) \) is the unified scalar curvature of new space-time computed in terms of the unified vierbeins \( w^a_\mu(x) \). Note that \( e^a_\mu \) and \( t^a_\mu(\psi) \) contribute equally to the curvature of space-time, which may be regarded as the Mach’s principle in ultimate space-time. (The second index of mimic vierbein \( t \), e.g. \( \mu \) of \( t^a_\mu \), means the derivative \( \partial_\mu \).)

\[ s_{\mu\nu} \equiv w^a_\mu \eta_{ab} w^b_\nu \quad \text{and} \quad s^\mu_\nu(x) \equiv w^\mu_a(x) w^{\nu a}(x) \quad \text{are unified metric tensors of new spacetime.} \]

NLSUSY GR action (1) possesses promising large symmetries isomorphic to \( SO(N) \) (\( SO(10) \)) SP group [14, 16]; namely, \( L_{\text{NLSUSYGR}}(w) \) is invariant under

\[ [\text{new NLSUSY}] \otimes [\text{local GL}(4,R)] \]
\[ \otimes[\text{local Lorentz}] \otimes [\text{local spinor translation}] \]

for spacetime symmetries and

\[ [\text{global } SO(N)] \otimes [\text{local } U(1)^N] \]

for internal symmetries in case of \( N \) superons \( \psi^i, \ (i = 1, 2, \cdots, N) \).

For example, \( L_{\text{NLSUSYGR}}(w) \) (1) is invariant under the following NLSUSY transformations:

\[ \delta^N L \psi = \frac{1}{\kappa} \zeta - i \kappa \gamma^a \psi \partial_\mu \psi, \quad \delta^N L e^a_\mu = i \kappa \zeta \gamma^a \psi \partial_\mu e^a_\rho, \]

where \( \zeta \) is a fermionic variational parameter.
where \( \zeta \) is a constant spinor and \( \partial_{[\mu}e_{\rho]} = \partial_{\mu}e_{\rho} - \partial_{\rho}e_{\mu} \), which induce the following GL(4R) transformations on the unified vierbein \( w^a_{\mu} \):

\[
\delta_{\zeta}w^a_{\mu} = \xi^\nu\partial_{\nu}w^a_{\mu} + \partial_{\mu}\xi^\nu w^a_{\nu}, \quad \delta_{s_{\mu\nu}} = \xi^\kappa\partial_{\kappa}s_{\mu\nu} + \partial_{\mu}\xi^\kappa s_{\kappa\nu} + \partial_{\nu}\xi^\kappa s_{\mu\kappa},
\]

(7)

where \( \xi^\mu = -i\kappa\bar{\zeta}\gamma^\mu\psi \), and under the following local Lorentz transformation:

\[
\delta_{L}w^a_{\mu} = e^b_{a}w^b_{\mu}
\]

(8)

or equivalently on \( \psi \) and \( e^a_{\mu} \)

\[
\delta_{L}\psi = \frac{i}{2}\epsilon_{ab}\gamma^b\psi, \quad \delta_{L}e^a_{\mu} = e^a_{b}e^b_{\mu} - \frac{\kappa^2}{4}\epsilon_{abcd}\bar{\psi}\gamma^5\gamma^d\psi(\partial_\mu\epsilon_{bc}),
\]

(9)

with the local parameter \( \epsilon_{ab} = (1/2)\epsilon_{[ab]}(x) \). The local Lorentz transformation forms a closed algebra, for example, on \( e^a_{\mu} \)

\[
[\delta_{L1}, \delta_{L2}]e^a_{\mu} = \beta^a_{b}e^b_{\mu} - \frac{\kappa^2}{4}\epsilon_{abcd}\bar{\psi}\gamma^5\gamma^d\psi(\partial_\mu\beta_{bc}),
\]

(10)

where \( \beta_{ab} = -\beta_{ba} \) is defined by \( \beta_{ab} = \epsilon_{2ac}\epsilon_{1b} - \epsilon_{2bc}\epsilon_{1a} \).

The commutators of two new NLSUSY transformations (6) on \( \psi \) and \( e^a_{\mu} \) are \( GL(4R) \), i.e. new NLSUSY (6) is the square-root of \( GL(4R) \);

\[
[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi = \Xi^\mu\partial_\mu\psi, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}]e^a_{\mu} = \Xi^b\partial_\rho e^a_{\mu} + e^a_{\rho}\partial_\mu\Xi^\rho,
\]

(11)

where \( \Xi^\mu = 2i\bar{\zeta_1}\gamma^\mu\zeta_2 - \zeta_1^a\zeta_2^\mu\partial_\rho e_{\rho}e_{\sigma} \). The algebra closes. The ordinary local \( GL(4R) \) invariance is trivial by the construction.

Note that the no-go theorem is overcome (circumvented) in a sense that the nontrivial \( N \)-extended SUSY gravity theory with \( N > 8 \) has been constructed in the NLSUSY invariant way.

New empty space-time for everything described by NLSUSY GR \( L_{\text{NLSUSYGR}}(w) \) of the vacuum EH type is unstable due to NLSUSY structure of tangent space-time and decays (called Big Decay [16]) spontaneously to ordinary Riemann space-time and superon (matter) described by the ordinary EH action with the cosmological constant \( \Lambda \), NLSUSY action for \( N \) superon (NG fermion) and their gravitational interactions.

The resulting action (called tentatively superon-graviton model (SGM) from the subsequent compositeness viewpoints) for \( N = 1 \), which ignites Big Bang of the present observed universe, is the following SGM action [17];

\[
L_{\text{SGM}}(e, \psi) = -\frac{e^4\Lambda}{16\pi G}\frac{c}{|e|}\frac{c}{|w|} + \frac{e^4}{16\pi G}\frac{c}{|e|}\frac{c}{|w|}R(e)
\]
\[-\frac{c^4}{16\pi G} e[w] \left[ 2t^{(\mu\nu)} R_{\mu\nu}(e) \right. \\
+ \frac{1}{2} (g^{\mu\nu} \partial^\rho \partial_\rho t^{(\mu\nu)} - t_{(\mu\nu)} \partial^\rho \partial_\rho g^{\mu\nu}) \\
+ g^{\mu\nu} \partial^\rho t_{(\mu\sigma)} \partial^\sigma g_{\rho\nu} - 2g^{\mu\nu} \partial^\rho t_{(\mu\sigma)} \partial^\sigma g_{\rho\sigma} - g^{\mu\nu} g^{\rho\sigma} \partial^\rho t_{(\mu\sigma)} \partial_\kappa g_{\kappa\mu
u}) \\
+ (t^\mu t^\nu + t^\nu t^\mu + t^{[\mu\nu]} t^{\nu\mu}) R_{\mu\nu}(e) \\
- 2t^{(\mu\rho)} t^{(\nu)} R_{\mu\nu} - t^{(\nu\rho)} t^{(\mu)} R_{\mu\nu\rho\sigma}(e) \\
- \frac{1}{2} t^{(\mu\nu)} (g^{\rho\sigma} \partial^\mu \partial^\nu t_{(\rho\sigma)} - g^{\rho\sigma} \partial^\mu \partial^\nu t_{(\sigma\nu)} + \cdots) \\
+ O(t^3) + \cdots + O(t^6) \right],
\]

where \((\psi)^5 \equiv 0 \ (\text{for } N = 1), e = \det e^a_{\mu}, t^{(\mu\nu)} = t^{\mu\nu} + t^{\nu\mu}, t_{(\mu\nu)} = t_{\mu\nu} + t_{\nu\mu}, \) and \(|w| = \det w^a_{\mu} = \det(\delta^a_{\mu} + t^a_{\mu})\) is the flat space NLSUSY action of VA [2] containing up to \(O(t^4)\) and \(R(e), R_{\mu\nu}(e)\) and \(R_{\mu\nu\rho\sigma}(e)\) are the ordinary Riemann curvature tensors of GR. Remarkably we can observe that the first term reduces to NLSUSY action [2], i.e. the arbitrary constant \(\kappa\) of NLSUSY is now fixed to

\[\kappa^{-2} = \frac{c^4}{8\pi G} \Lambda,\]

in the Riemann-flat \(e^a_{\mu}(x) \rightarrow \delta^a_{\mu}\) space-time and the second term contains the familiar EH action of GR. These expansions describe the complementary relation of graviton and superons (matter), i.e. Mach’s principle. Note that NLSUSY GR (1) and SGM (12) possess different asymptotic flat space-time, i.e. SGM-flat \(w^a_{\mu}(x) \rightarrow \delta^a_{\mu}\) space-time and Riemann-flat \(e^a_{\mu}(x) \rightarrow \delta^a_{\mu}\) space-time, respectively.

\[L_{\text{SGM}}(e, \psi) \quad (12)\]
can be rewritten as the following familiar form

\[L_{\text{SGM}}(e, \psi) = \frac{c^4}{16\pi G} \left[ e \left\{ R(e) - \Lambda + \tilde{T}(e, \psi) \right\} \right],\]

where \(R(e)\) is the scalar curvature of ordinary EH action and \(\tilde{T}(e, \psi)\) represents the kinetic term and the gravitational interaction of superons.

Note that the NG fermion \(\psi\) can be transformed away neither by the local spinor translation, for \(\delta w = 0\) under the local spinor transformation, nor by \(GL(4R)\) on \(e^a_{\mu}\), for once the NG fermion degrees of freedom \(\psi\) were gauged away by \(GL(4R)\) on \(e^a_{\mu}\), then the subsequent ordinary \(\delta_{GL(4R)} e^a_{\mu}\) in the resulting EH action would become pathological (restricted). Also the solution of the Euler equation for \(w^a_{\mu}\) and those for \(e^a_{\mu}\) and \(\psi\) are treated independently because of the difference of (asymptotic) space-time on which they are defined.

We think that the geometric arguments of EGR principle has been generalized naturally, which accommodates geometrically spin 1/2 matter as NG fermion accompanying spontaneous SUSY breaking encoded on tangent space-time as NLSUSY.
Therefore the black hole as a singularity of space-time in EGR is an interesting object to be studied in the picture of NLSUSY GR.

We have shown qualitatively that NLSUSY GR may potentially describe a new paradigm (SGM) for the SUSY unification of space-time and matter, where particular SUSY compositeness composed of superons for all (observed) particles except the graviton emerges as an ultimate feature of nature behind the familiar LSUSY models (MSSM, SUSY GUTs) [9, 18] and SM as well. That is, all (observed) low energy particles may be eigenstates of $SO(N)$ SP expressed uniquely as the SUSY composites (eigen states) of $N$ superons. We examine these possibilities in the next section.

3 Linearizing NLSUSY and Physical Implications

Due to the high nonlinearity of the SGM action we have not yet succeeded in extracting directly (low energy) physical meanings of SGM on curved Riemann space-time.

However, considering that the SGM action reduces essentially to the $N$-extended NLSUSY action with $\kappa^2 = \left(\frac{\alpha N}{8\pi G}\right)^{-1}$ in the low energy of asymptotic Riemann-flat ($\epsilon_{\alpha \mu} \rightarrow \delta_{\alpha \mu}$) space-time, it is interesting from the viewpoint of the low energy physics on the local coordinate system to linearize the $N$-extended NLSUSY model in order to find the $N$-extended LSUSY theory equivalent (related) to the $N$-extended NLSUSY model.

The relation between $N = 1$ LSUSY representations and $N = 1$ NLSUSY representations in flat (Minkowski) space-time is well understood by using the superfield method [12, 19]. The equivalence of $N = 1$ LSUSY free theory for LSUSY supermultiplet to $N = 1$ NLSUSY VA model for NG fermion is demonstrated by many authors [20, 21, 22] and $N = 2$ case as well [23].

We have shown explicitly by the heuristic arguments for simplicity in two space-time dimensions ($d = 2$) [24, 25] that $N = 2$ LSUSY interacting QED is equivalent (related) to $N = 2$ NLSUSY model in a sense that analogous SUSY invariant relations hold, i.e. each field of LSUSY supermultiplet are expressed uniquely in terms of NLSUSY NG fermions by the arguments on SUSY transformation. (Note that the minimal realistic SUSY QED in SGM composite scenario is given by $N = 2$ SUSY [23].)

In establishing the equivalence (relations) each field of LSUSY supermultiplet is expressed uniquely as the composite of NG fermions of NLSUSY, which are called SUSY invariant relations. Consequently we are tempted to imagine some composite structure (far) behind the familiar LSUSY unified models, e.g. MSSM and SUSY GUT. In this paper we study explicitly the vacuum structure of $N = 2$ LSUSY QED
in the SGM scenario in $d = 2$ [25].

$N = 2$ NLSUSY action for two superons (NG fermions) $\psi^i (i = 1, 2)$ in $d = 2$ is written as follows,

$$L_{N=2\text{NLSUSY}} = -\frac{1}{2\kappa^2} |w|$$

$$= -\frac{1}{2\kappa^2} \left\{ 1 + t^a_b + \frac{1}{2!} (t^a_b t^b_a - t^a_b t^b_a) \right\}$$

$$= -\frac{1}{2\kappa^2} \left\{ 1 - i\kappa^2 \bar{\psi}^i \gamma^a \partial_a \psi^i - \frac{1}{2} \kappa^4 \left( \bar{\psi}^i \gamma^a \partial_a \psi^i \right) \right\}$$

(15)

where $\kappa$ is a constant whose dimension is (mass)$^{-1}$ and $|w| = \det(w^a_b) = \det(\delta^a_b + t^a_b)$, $t^a_b = -i\kappa^2 \bar{\psi}^i \gamma^a \partial_a \psi^i$.

While, the helicity states contained formally in $(d = 2)$ $N = 2$ LSUSY QED are the vector supermultiplet containing $U(1)$ gauge field

$$\begin{pmatrix} +1 \\ +\frac{1}{2} \\ +\frac{1}{2} \\ 0 \end{pmatrix} + \text{[CPT conjugate]},$$

and the scalar supermultiplet for matter fields

$$\begin{pmatrix} +\frac{1}{2} \\ 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix} + \text{[CPT conjugate]}.$$

The most general $N = 2$ LSUSY QED action for the massless case in $d = 2$, is written as follows [25],

$$L_{N=2\text{SUSYQED}} = -\frac{1}{4} (F_{ab})^2$$

$$+ \frac{i}{2} \bar{\chi} \gamma^a \phi \lambda^i + \frac{1}{2} (\partial_a A)^2 + \frac{1}{2} (\partial_a \phi)^2 + \frac{1}{2} D^2$$

$$- \frac{1}{\kappa} D + \frac{i}{2} \bar{\chi} \gamma^a \phi \lambda^i + \frac{1}{2} (\partial_a B^i)^2 + \frac{i}{2} \bar{\nu} \nu + \frac{1}{2} (F^i)^2$$

$$+ f(A \bar{\lambda}^i \lambda^i + \epsilon^{ij} \phi \bar{\lambda}_5 \lambda^j - A^2 D + \phi^2 D + \epsilon^{ab} A \phi F_{ab})$$

$$+ e \left\{ i v_a \bar{\chi} \gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \bar{\lambda}^i \chi B^i + \epsilon^{ij} \bar{\lambda}_5 \nu B^j \right\}$$

$$- \frac{1}{2} D(B^i)^2 + \frac{1}{2} (\bar{\chi} \chi + \bar{\nu} \nu) A - \bar{\chi} \gamma_5 \nu \phi$$

$$+ \frac{1}{2} e^2 (v_a^2 - A^2 - \phi^2) (B^i)^2, \quad (16)$$
where \((v^a, \lambda^i, A, \phi, D)\) \((F_{ab} = \partial_a v_b - \partial_b v_a)\) is the off-shell vector supermultiplet containing \(v^a\) for a \(U(1)\) vector field, \(\lambda^i\) for doublet (Majorana) fermions \(A\) for a scalar field in addition to \(\phi\) for another scalar field and \(D\) for an auxiliary scalar field, while \((\chi, B^i, \nu, F^i)\) is off-shell scalar supermultiplet containing \((\chi, \nu)\) for two (Majorana) fermions, \(B^i\) for doublet scalar fields and \(F^i\) for auxiliary scalar fields. The linear term of \(F\) is forbidden by the gauge invariance \([25]\). Also \(\xi\) is an arbitrary dimensionless parameter giving a magnitude of SUSY breaking mass, and \(f\) and \(e\) are Yukawa and gauge coupling constants with the dimension (mass)\(^1\), respectively.

\[N = 2\] LSUSY QED action \((16)\) is invariant under the following LSUSY transformations parametrized by \(\zeta^i\),

\[
\begin{align*}
\delta_{\zeta} v^a &= -ie^{ij} \bar{\zeta}^i \gamma^a \lambda^j, \\
\delta_{\zeta} \lambda^i &= (D - i \partial A) \zeta^i + \frac{1}{2} \epsilon^{ab} e^{ij} F_{ab} \gamma_5 \zeta^j - ie^{ij} \gamma_5 \partial \phi \zeta^j, \\
\delta_{\zeta} A &= \bar{\zeta}^i \lambda^i, \\
\delta_{\zeta} \phi &= -e^{ij} \bar{\zeta}^i \gamma_5 \lambda^j, \\
\delta_{\zeta} D &= -i \bar{\zeta}^i \partial \lambda^i, 
\end{align*}
\]

\[(17)\]

for the vector multiplet and

\[
\begin{align*}
\delta_{\zeta} \chi &= (F^i - i \partial B^i) \zeta^i - ee^{ij} V^i B^j, \\
\delta_{\zeta} B^i &= \bar{\zeta}^i \chi - e^{ij} \bar{\zeta}^j \nu, \\
\delta_{\zeta} \nu &= e^{ij} (F^i + i \partial B^i) \zeta^j + e V^i B^j, \\
\delta_{\zeta} F^i &= -i \bar{\zeta}^i \partial \chi - e^{ij} \bar{\zeta}^j \partial \nu \\
&\quad - e \{e^{ij} V^i \chi - V^i \nu + (\bar{\zeta}^i \lambda^j + \bar{\zeta}^j \lambda^i) B^j - \bar{\zeta}^j \lambda^j B^j\} 
\end{align*}
\]

\[(18)\]

with \(V^i = iv^a \gamma^a \zeta^i - e^{ij} A \zeta^j - \phi \gamma_5 \zeta^i\) for the scalar multiplet.

\[N = 2\] LSUSY QED action \((16)\) and \((18)\) can be rewritten as the familiar manifestly covariant form in terms of the complex quantities defined by

\[
\begin{align*}
\chi_D &= \frac{1}{\sqrt{2}} (\chi + i \nu), \\
B &= \frac{1}{\sqrt{2}} (B^1 + i B^2), \\
F &= \frac{1}{\sqrt{2}} (F^1 - i F^2).
\end{align*}
\]

\[(19)\]

The resulting action is manifestly invariant under the local \(U(1)\):

\[
(\chi_D, B, F) \rightarrow (\chi'_D, B', F')(x) = e^{i \Omega(x)} (\chi_D, B, F)(x),
\]

\[
v_a \rightarrow v'_a(x) = v_a(x) + \frac{1}{e} \partial_a \Omega(x).
\]

\[(20)\]

(For further details see ref.[25].)
For extracting the low energy particle physics contents of $N = 2$ SGM (NLSUSY GR) we consider in Riemann-flat asymptotic space-time, where $N = 2$ SGM reduces to essentially $N = 2$ NLSUSY action equivalent (related) to $N = 2$ SUSY QED action, i.e. for $e^a_{\mu} \rightarrow \delta^a_{\mu}$

\[ L_{N=2\text{SGM}} \rightarrow L_{N=2\text{NLSUSY}} + \text{[surface terms]} = L_{N=2\text{SUSYQED}}. \] (21)

The equivalence (relation) of the two theories are shown explicitly by substituting the following generalized SUSY invariant relations [25] into the LSUSY QED theory. The SUSY invariant relations for the vector supermultiplet $(v^a, \lambda^i, A, \phi, D)$ are

\begin{align*}
v^a &= -\frac{i}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma^a \psi^j |w|, \\
\lambda^i &= \xi \left[ \bar{\psi}^i |w| - \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \bar{\psi}^i \bar{\psi}^j (1 - i \kappa^2 \bar{\psi}^k \psi^k) \} \right], \\
A &= \frac{1}{2} \xi \kappa \bar{\psi}^i \psi^i |w|, \\
\phi &= -\frac{1}{2} \xi \kappa \epsilon^{ij} \bar{\psi}^i \gamma_5 \psi^j |w|, \\
D &= \frac{\xi}{\kappa} |w| - \frac{1}{8} \kappa^3 \partial_a \partial^a \left( \bar{\psi}^i \psi^j \bar{\psi}^j \psi^i \right), \tag{22}
\end{align*}

while for the scalar supermultiplet $(\chi, B^i, \nu, F^i)$ the relaxed SUSY invariant relations are

\begin{align*}
\chi &= \xi^i \left[ \bar{\psi}^i |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \bar{\psi}^i \bar{\psi}^j (1 - i \kappa^2 \bar{\psi}^k \psi^k) \} \right], \\
B^i &= -\kappa \left( \frac{1}{2} \xi \bar{\psi}^i \psi^j - \xi^i \bar{\psi}^j \psi^i \right) |w|, \\
\nu &= \xi^i \epsilon^{ij} \left[ \bar{\psi}^j |w| + \frac{i}{2} \kappa^2 \partial_a \{ \gamma^a \bar{\psi}^j \bar{\psi}^k (1 - i \kappa^2 \bar{\psi}^l \psi^l) \} \right], \\
F^i &= \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8} \kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k) \right\} \\
&\quad - i \kappa \epsilon^j \partial_a (\bar{\psi}^j \gamma^a \psi^j |w|) - \frac{1}{4} \kappa e^2 \xi \epsilon^i \bar{\psi}^j \psi^j \bar{\psi}^j \psi^i. \tag{23}
\end{align*}

$N = 2$ SUSYQED action (16) is invariant under the variations of the LSUSY component fields of two supermultiplets induced by the NLSUSY transformations of the superon $\psi$ contained in the (relaxed) SUSY invariant relations (22) and (23). Furthermore substituting these relations into the $N = 2$ SUSYQED action (16) we can show directly that $N = 2$ SUSYQED action (16) is equivalent (reduced) to $N = 2$ NLSUSY action provided $\xi^2 - (\xi^i)^2 = 1$. 

9
As for the LSUSY transformations (17) and (18), the SUSY invariant relations (22) reproduce the familiar (free case) LSUSY transformations (17) under the NL-SUSY transformations on the contained superons $\psi$. While the relaxed SUSY invariant relations (23) for the scalar supermultiplet mean that the familiar LSUSY transformations (18) of the scalar supermultiplet are not reproduced by the NL-SUSY transformations of the contained superons $\psi$ but modified by the (contact four-fermion) gauge interaction terms. It is interesting that the four-fermion self-interaction term (i.e. the condensation of $\psi^i$) appearing only in the auxiliary fields $F^i$ is the origin of the familiar local $U(1)$ gauge symmetry of LSUSY theory, which makes, however, the SUSY invariant relations relaxed. The introduction of the new auxiliary field supermultiplet may improve (clarifies) these unpleasant situations. Is the condensation of superons the origin of the local gauge interaction? Note that in the linearized theory the commutator algebra does not contain $U(1)$ gauge transformation even for the vector $U(1)$ gauge field [23].

Now we study the vacuum structure of $N=2$ SUSY QED action (16) [27]. The vacuum is determined by the minimum of the potential $V(A, \phi, B^i, D)$, §

$$V(A, \phi, B^i, D) = -\frac{1}{2} D^2 + \left\{ \frac{\xi}{\kappa} + f (A^2 - \phi^2) + \frac{1}{2} e (B^i)^2 \right\} D. \quad (24)$$

Substituting the solution of the equation of motion for the auxiliary field $D$ we obtain

$$V(A, \phi, B^i) = \frac{1}{2} f^2 \left\{ A^2 - \phi^2 + \frac{e}{2f} (B^i)^2 + \frac{\xi}{f\kappa} \right\}^2 \geq 0. \quad (25)$$

The configurations of the fields corresponding to the vacua in $(A, \phi, B^i)$-space, which are $SO(1,3)$ or $SO(3,1)$ invariant, are classified according to the signatures of the parameters $e, f, \xi, \kappa$ as follows:

(I) For $ef < 0$, $\frac{\xi}{f\kappa} < 0$ case,

$$A^2 - \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left( \tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \quad (26)$$

(II) For $ef > 0$, $\frac{\xi}{f\kappa} < 0$ case,

$$A^2 - \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left( \tilde{B}^i = \sqrt{\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} \right) \quad (27)$$

(III) For $ef < 0$, $\frac{\xi}{f\kappa} > 0$ case,

$$-A^2 + \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left( \tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = \frac{\xi}{f\kappa} \right) \quad (28)$$

§The terms, $\frac{1}{4} e^2 (A^2 + \phi^2)(B^i)^2$, should be added to Eqs.(24) and (25) but they do not change the final results.
(IV) For $ef > 0$, $\frac{\xi}{f} > 0$ case,

$$-A^2 + \phi^2 - (\tilde{B}^i)^2 = k^2.$$ \hfill (29)

We find that the vacua (I) and (IV) with $SO(1, 3)$ isometry in $(A, \phi, B^i)$-space are unphysical, for they produce the pathological wrong sign kinetic terms for the fields induced around the vacuum.

As for the cases (II) and (III) we perform the similar arguments as shown below and find that two different physical vacua appear. The physical particle spectrum is obtained by expanding the field $(A, \phi, B^i)$ around the vacuum with $SO(3, 1)$ isometry.

For case (II), the following expressions (IIa) and (IIb) are considered:

Case (IIa)

\[
\begin{align*}
A &= (k + \rho) \sin \theta \cosh \omega \\
\phi &= (k + \rho) \sinh \omega \\
\tilde{B}^1 &= (k + \rho) \cos \theta \cos \varphi \cosh \omega \\
\tilde{B}^2 &= (k + \rho) \cos \theta \sin \varphi \cosh \omega.
\end{align*}
\]

and

Case (IIb)

\[
\begin{align*}
A &= -(k + \rho) \cos \theta \cos \varphi \cosh \omega \\
\phi &= (k + \rho) \sinh \omega \\
\tilde{B}^1 &= (k + \rho) \sin \theta \cosh \omega \\
\tilde{B}^2 &= (k + \rho) \cos \theta \sin \varphi \cosh \omega.
\end{align*}
\]

Note that for the case (III) the arguments are the same by exchanging $A$ and $\phi$, which we call (IIIa) and (IIIb). Substituting these expressions into $L_{N=2SUSYQED}$ $(A, \phi, B^i)$ and expanding the action around the vacuum configuration we obtain the physical particle contents. For the cases (IIa) and (IIIa) we obtain

\[
L_{N=2SUSYQED}
\begin{align*}
&= \frac{1}{2} \left\{(\partial_a \rho)^2 - 2(ef)k^2 \rho^2\right\} \\
&+ \frac{1}{2} \left\{(\partial_a \theta)^2 + (\partial_a \omega)^2 - 2(ef)k^2(\theta^2 + \omega^2)\right\} \\
&+ \frac{1}{2} (\partial_a \varphi)^2 \\
&- \frac{1}{4} (F_{ab})^2 + (ef)k^2 \nu_a^2 \\
&+ \frac{i}{2} \bar{\lambda}^i \gamma^i \lambda^i + \frac{i}{2} \bar{\chi} \gamma \chi + \frac{i}{2} \bar{\nu} \gamma \nu
\end{align*}
\]
\[ + \sqrt{2ef(\bar{\lambda}^1 \chi - \bar{\lambda}^2 \nu)} + \cdots, \]  
(30)

and the consequent mass generation

\[
\begin{align*}
    m_\rho^2 &= m_\theta^2 = m_\omega^2 = m_{\nu_a}^2 = 2(ef)k^2 = -\frac{2\xi e}{\kappa}, \\
    m_{\chi^i} &= m_\chi = m_\nu = m_\varphi = 0. 
\end{align*}
\]  
(31)

Note that \( \varphi \) is NG boson for the spontaneous breaking of \( U(1) \) symmetry, i.e. the \( U(1) \) phase of \( B \), and totally gauged away by the Higgs-Kibble mechanism with \( \Omega(x) = \sqrt{e\kappa/2\varphi(x)} \) for \( U(1) \) gauge (20). The vacuum breaks both SUSY and the local \( U(1) \) spontaneously. All bosons have the same mass and remarkably all fermions remain massless. \( \lambda^i \) transforming inhomogeneously \( \delta \lambda^i = \hat{\xi} \zeta^i + \cdots \) in the true vacuum are NG fermions for the spontaneous \( N = 2 \) SUSY breaking. The physical implication of the off-diagonal mass terms \( \sqrt{2ef(\bar{\lambda}^1 \chi - \bar{\lambda}^2 \nu)} = \sqrt{2ef(\bar{\chi}\lambda + \bar{\lambda}\chi_D)} \) for fermions is unclear (pathological?) and deserves further investigations, which would induce the mixings of fermions and/or the lepton (baryon) flavour violations.

By the similar computations for (IIb) and (IIIb) we obtain

\[
\begin{align*}
L_{N=2\text{SUSYQED}} &= \frac{1}{2} \left\{ (\partial_a \rho)^2 - 4f^2k^2\rho^2 \right\} \\
&\quad + \frac{1}{2} \left\{ (\partial_a \theta)^2 + (\partial_a \varphi)^2 - e^2k^2(\theta^2 + \varphi^2) \right\} \\
&\quad + \frac{1}{2} (\partial_a \omega)^2 \\
&\quad - \frac{1}{4} (F_{ab})^2 \\
&\quad + \frac{1}{2} (i\bar{\lambda}^i \gamma^a \lambda^i - 2f k \bar{\lambda}^i \lambda^i) \\
&\quad + \frac{1}{2} \left\{ i(\bar{\chi} \gamma^a \chi + \bar{\nu} \gamma^a \nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu) \right\} + \cdots. 
\end{align*}
\]  
(32)

and the following mass spectrum which indicates the spontaneous breakdown of \( N = 2 \) SUSY;

\[
\begin{align*}
    m_\rho^2 &= m_\lambda^2 = 4f^2k^2 = -\frac{4\xi f}{\kappa}, \\
    m_\theta^2 &= m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2 = -\frac{\xi e^2}{\kappa f}, \\
    m_{\nu_a} &= m_\omega = 0. 
\end{align*}
\]  
(33)
which can produce mass hierarchy by the factor $\frac{f}{f}$. Interestingly all fermions acquire masses through the spontaneous SUSY breaking. The local $U(1)$ gauge symmetry is not broken. The massless scalar $\omega$ is a NG boson for the degeneracy of the vacuum in $(A, \tilde{B}_2)$-space, which is gauged away provided the local gauge symmetry between the vector and the scalar multiplet is introduced.

From these arguments we conclude that $N = 2$ SUSY QED is equivalent (related) to $N = 2$ NLSUSY action, which is the matter sector of $N = 2$ SGM produced by Big Decay (phase transition) of $N = 2$ NLSUSY GR (new space-time). It possesses two different vacua, the type (a) and (b) in the $SO(3,1)$ isometry of (II) and (III).

The resulting models describe:
for the type (a); two charged chiral fermions ($\psi_L^{cj} \sim (\tilde{\chi}_{DL}, \tilde{\nu}_{DL})$) ($j = 1, 2$), two neutral chiral fermions ($\lambda_L^{ij} \sim \tilde{\lambda}_{DL}^{ij}$), one massive vector ($v_a$), one charged massive scalar ($\phi^c \sim \theta + i\omega$), and one massive scalar ($\phi^0 \sim \rho$), where ($\chi, \nu, \lambda$) are written by left-handed Dirac fields and

for the type (b); one charged Dirac fermion ($\psi_D^c \sim \chi + iv$), one neutral (Dirac) fermion ($\lambda_D^0 \sim \lambda^1 - i\lambda^2$), one massless vector (a photon) ($v_a$), one charged scalar ($\phi^c \sim \theta + i\varphi$) and one neutral complex scalar ($\phi^0 \sim \rho + i\omega$),

which are the composites of superons.

4 Cosmology of NLSUSY GR

Now we discuss the cosmological implications of $N = 2$ NLSUSY GR (or $N = 2$ SGM from the composite viewpoints). NLSUSY GR spacetime of (1) is unstable and spontaneously breaks down to Riemann spacetime with superon (massless NG fermion) matter of (14), which may be the birth of the present universe (space-time and matter) by the quantum effect Big Decays in advance of the inflation and/or the Big Bang. The variation of (14) with respect to $e^a_\mu$ gives the equation of motion for $e^a_\mu$ recasted as follows

$$R_{\mu\nu}(e) - \frac{1}{2} g_{\mu\nu} R(e) = \frac{8\pi G}{c^4} \left\{ \tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \right\}, \tag{34}$$

where $\tilde{T}_{\mu\nu}(e, \psi)$ abbreviates the stress-energy-momentum of superon (NG fermion) matter including the gravitational interactions. Note that $-\frac{c^4 \Lambda}{8\pi G}$ can be interpreted as the negative energy density of empty spacetime, i.e. the dark energy density $\rho_D$. (The negative sign determined uniquely and produces the correct kinetic term of superons in NLSUSY.)

While, on tangent spacetime, the low energy theorem of the particle physics gives
the following superon (massless NG fermion)-vacuum coupling
\[ < \psi^j_{\alpha}(q)|J^{k\mu}_{\beta}|0> = i \sqrt{\frac{c^4 A}{8\pi G}} (\gamma^\mu)^{\alpha\beta} \delta^{jk} e^{iqx} + \cdots, \] (35)

where \( J^{k\mu} = i \sqrt{\frac{c^4 A}{8\pi G}} \gamma^\mu \psi^k + \cdots \) is the conserved supercurrent obtained by applying the Noether theorem [15] and \( \sqrt{\frac{c^4 A}{8\pi G}} \) is the coupling constant \( g_{sv} \) of superon with the vacuum via the supercurrent. Further we have seen in the preceding section that the right hand side of (34) for \( N = 2 \) is essentially \( N = 2 \) NLSUSY V A action. And it is equivalent to the broken \( N = 2 \) LSUSYQED action (16) with the vacuum expectation value of the auxiliary field (Fayet-Iliopoulos term). For the case (b) it gives the SUSY breaking masse,

\[ M_{SUSY}^2 \sim < D > \sim \sqrt{\frac{c^4 A}{8\pi G}}, \] (36)

to the component fields of the (massless) LSUSY supermultiplet, provided \( -f\xi \sim \mathcal{O}(1) \). We find NLSUSY GR (SGM) scenario gives interesting relations among the important quantities of the cosmology and the low energy particle physics, i.e.

\[ \rho_D \sim \frac{c^4 A}{8\pi G} \sim < D >^2 \sim g_{sv}^2, \] (37)

Suppose that in the (low energy) LSUSY supermultiplet the stable and the lightest massive particle retains the mass of the order of the spontaneous SUSY breaking. And if we identify the neutrino with such a particle and with \( \lambda(x) \), i.e.

\[ m_\nu^2 \sim \sqrt{\frac{c^4 A}{8\pi G}}, \] (38)

then SGM predicts the observed value of the (dark) energy density of the universe and naturally explains the mysterious numerical relations between \( m_\nu \) and \( \rho_D^{obs} \):

\[ \rho_D^{obs} \sim (10^{-12}GeV)^4 \sim m_\nu^4 \sim g_{sv}^2. \] (39)

The tiny neutrino mass is the direct evidence of SUSY (breaking), i.e., the spontaneous phase transition of SGM spacetime. NLSUSY GR gives in general

\[ \Lambda \sim M_{SUSY}^2 \left( \frac{M_{SUSY}}{M_{Planck}} \right)^2. \] (40)

The large mass scales and the non-abelian gauge symmetry necessary for building the realistic and interacting broken LSUSY model will appear by the extension to the large \( N \) SUSY and by the linearization of \( \tilde{T}_{\mu\nu}(e, \psi) \) which contains the mass scale \( \Lambda^{-1} \) in the higher order with \( \psi \) [26].
5 Conclusions

We have proposed a new paradigm for describing the unity of nature, where the ultimate real shape of nature is new vastable space-time described by \( L_{\text{NLSUSYGR}}(w) \) in the form of the free EH action for empty space with the constant energy density which may materialize the Mach’s principle. We have shown that the vacua of SGM composite scenario created by the Big Decay of \( N \)-extended NLSUSY GR action possess rich structures promising for the unified description of nature. In fact, we have shown explicitly that \( N = 2 \) LSUSY theory for the realistic \( U(1) \) gauge theory appears as the physical field configurations on the vacuum of \( N = 2 \) NLSUSY theory on Minkowski tangent space-time, which gives new insights into the origin of mass and the cosmological problems. The cosmological implications of the composite SGM scenario deserve further studies.

Interestingly the physical particle states of \( N = 2 \) SUSY as a whole look the similar structure to the lepton sector of ordinary SM with the local \( U(1) \) and the implicit global \( SU(2) \) [23] disregarding the R-parity. (Note that \( \omega \) in (32) is a NG boson and disappears provided the corresponding local gauge symmetry is introduced.)

We anticipate that the physical cosequences obtained in \( d = 2 \) hold in \( d = 4 \) as well, for the both have the similar structures of on-shell helicity states of \( N = 2 \) supermultiplet though scalar fields and off-shell (auxiliary field) structures are modified (extended). However the similar investigations in \( d = 4 \) are urgent for the realistic model building based upon SUSY.

Further investigations on the spontaneous symmetry breaking for \( N \geq 2 \) SUSY remains to be studied.

The extension to large \( N \), especially to \( N = 5 \) is important for superon quintet hypothesis of SGM scenario with \( N = 10 = 5 + 5^* \) for equipping the \( SU(5) \) GUT structure [9] and to \( N = 4 \) may shed new light on the mathematical structures of the anomaly free non-trivial \( d = 4 \) field theory.

Also NLSUSY GR with extra space-time dimensions equipped with the Big Decay is an interesting problem, which can give the framework for describing all observed particles as elementary \( \text{à la} \) Kaluza-Klein.

Linearizing SGM action \( L_{\text{SGM}}(e, \psi) \) on curved space-time, which elucidates the topological structure of space-time [28], is a challenge.

The physical and mathematical meanings of the black hole as a singularity of space-time and the role of the equivalence principle are to be studied in detail in NLSUSY GR and SGM scenario .

Finally we just mention that NLSUSY GR and the subsequent SGM scenario for the spin \( \frac{3}{2} \) NG fermion [14, 29] is in the same scope.
6 Acknowledgements

It is a great pleasure for the authors to dedicate this work to the 85th birthday of Professor C. N. Yang. One of the authors (K.S.) would like to thank Professor K. K. Phua for inviting him to the Conference in Honor of C. N. Yang’s 85th Birthday and for the encouraging invitation to contributing to the proceedings.
References

[1] J. Wess and B. Zumino, *Phys. Lett. B* **49**, 52 (1974).
[2] D.V. Volkov and V.P. Akulov, *Phys. Lett. B* **46**, 109 (1973).
[3] Yu. A. Golfand and E.S. Likhtman, *JETP Lett.* **13**, 323 (1971).
[4] A. Salam and J. Strathdee, *Phys. Lett. B* **49**, 465 (1974).
[5] P. Fayet and J. Iliopoulos, *Phys. Lett. B* **51**, 461 (1974).
[6] L. O’Raifeartaigh, *Nucl. Phys. B* **96**, 331 (1975).
[7] M. Gell-Mann, *Proceedings of supergravity workshop at Stony Brook*, eds. P. van Nieuwenhuisen and D. Z. Freedman (North Holland, Amsterdam, 1977).
[8] K. Shima, *Z. Phys. C* **18**, 25 (1983).
[9] K. Shima, *European Phys. J. C* **7**, 341 (1999).
[10] S. Coleman and J. Mandula, *Phys. Rev.* **159**, 1251 (1967).
[11] R. Haag, J. Lopuszanski and M. Sohnius, *Nucl. Phys. B* **88**, 257 (1975).
[12] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, 1992).
[13] K. Shima, *Phys. Lett. B* **501**, 237 (2001).
[14] K. Shima and M. Tsuda, *Phys. Lett. B* **507**, 260 (2001).
[15] K. Shima, *Phys. Rev. D* **20**, 574 (1979).
[16] K. Shima and M. Tsuda, *PoS HEP2005*, 011 (2006); K. Shima and M. Tsuda, *Phys. Lett. B* **645**, 455 (2007).
[17] $L_{SGM}$ is called the nonlinear supergravity $L_{NLSUGRA}$ by Y. Cho.
[18] K. Shima, M. Tsuda and M. Sawaguchi, *Int. J. Mod. Phys. E* **13**, 539 (2004).
[19] E.A. Ivanov and A.A. Kapustnikov, *J. Phys. A* **11**, 2375 (1978).
[20] M. Roček, *Phys. Rev. Lett.* **41**, 451 (1978).
[21] T. Uematsu and C.K. Zachos, *Nucl. Phys. B* **201**, 250 (1982).
[22] K. Shima, Y. Tani and M. Tsuda, *Phys. Lett. B* **525**, 183 (2002).
[23] K. Shima, Y. Tanii and M. Tsuda, Phys. Lett. B 546, 162 (2002).
[24] K. Shima and M. Tsuda, Mod. Phys. Lett. A 22, 1085 (2007).
[25] K. Shima and M. Tsuda, Mod. Phys. Lett. A 22, 3027 (2007).
[26] K. Shima and M. Tsuda, Class. Quant. Grav. 19, 5101 (2002).
[27] K. Shima, M. Tsuda and W. Lang, Phys. Lett. B 659, 741 (2007).
[28] K. Shima and M. Kasuya, Phys. Rev. D 22, 290 (1980).
[29] N. S. Baaklini, Phys. Lett. B 67, 335 (1977).