Finite-time Attitude Control with Chattering Suppression for Quadrotors Based on High-order Extended State Observer

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ABSTRACT This article investigates a finite-time attitude control with chattering suppression for quadrotors based on a high-order extended state observer (HESO). To improve the anti-disturbance capacity of quadrotors, a HESO capable of estimating fast time-varying disturbances is presented to enhance system robustness by regarding higher-order disturbance derivatives as extended arguments. Then, with the estimates of HESO, a finite-time attitude control policy including a double hyperbolical reaching law (DHRL) is introduced, alleviating control chattering and guaranteeing rapid response. The most significant feature is a finite time regulation for quadrotors without incurring severe oscillations can be obtained in the event of fast time-varying uncertainties. Moreover, the stability analysis is proved by a Lyapunov theorem. Finally, the effectiveness of proposed control scheme is demonstrated by numerical and experimental results.

INDEX TERMS High-order extended state observer, chattering suppression, disturbance estimation, double hyperbolical reaching law, quadrotors, attitude regulation.

I. INTRODUCTION

During the past decades, quadrotors have received extensive attention [1]-[5] on account of the abundant applications in military and civilian fields such as monitoring [6], patrolling [7], ground mapping, searching and rescuing [8]. They have characteristics of prominent agility, low cost and small size. However, the quadrotors are typical under-actuated systems with modelling nonlinearities accompanied by highly-coupled states, parametric uncertainties and external disturbances. Therefore, the motion control for quadrotors is always a challenging research topic. Especially, it is necessary to point out that the attitude control is an essential precondition for quadrotor applications. Researches on attitude control schemes for quadrotors have conducted abundant of advanced approaches in recent years. For instance, backstepping control [9]-[10], active disturbance rejection control [11]-[12], sliding mode control (SMC) [13]-[14] and the like.

Ascribe to the excellent anti-interference ability and strong robustness, sliding mode control (SMC) has become one of the most popular control strategies for the quadrotor attitude control [13]-[16]. Nevertheless, it is undeniable that the impact in the course of engineering practices caused by the serious oscillations of SMC is considerable. However, it is undeniable that the severe oscillations in control torques of conventional SMC is considerable in practice. So as to solve abovementioned chattering matter, researchers dedicate themselves to proposing creative SMC policies, including but not be limited to terminal SMC [17]-[18], adaptive SMC [19], disturbance estimation-based SMC [20]. A free-model-based terminal SMC method is developed for the attitude and position control for quadrotors with model parameter variations [17]. An adaptive SMC scheme is proposed for mismatched uncertainties quadrotors [19] to achieve robust tracking. By introducing an adaptive finite-time extended state observer (FTESO) to estimate the lumped disturbances, a continuous fast nonsingular terminal SMC is elaborated in [18]. In [20], focusing on actuator faults and model uncertainties, a novel SMC in combination with a neural network is investigated. As mentioned above, the disturbance estimation-based SMC has been regarded as a preferable method to retard chattering problem. The principle behind estimation-based SMC lies in the setting of switching gain value, by adjusting it larger than the error of disturbance.
estimation rather than the max value of disturbance, such that control action is assured to be continuous and smooth [21]. Absolutely, the critical point of disturbance estimation-based SMC is the choice of disturbance estimator.

A great deal of disturbance estimators are employed and combined within SMC construction to alleviate the chattering of control inputs, such as sliding mode observer (SMO) [22], extended state observer (ESO) [23]-[25] and neural network (NN) [26]. However, one should notice that the SMO design not only requires the upper bound of disturbance to be known, but also there exists obvious chattering along with its outputs. Meanwhile, for conventional ESO design, which can only handle the fixed or slow time-varying disturbances, has poor ability in dealing with fast time-varying perturbations. In addition, with respect to NN, large calculational cost is inevitable because it is a very burdensome process to adjust the appropriate parameters to prompt the weight update to converge in limited time. However, even though stronger robustness and smoother control signals are available, it worth being suggested that rapid transient convergence performance is unattainable arising from the selection of tiny switching gains [22]-[26].

In order to address the slow transient convergence problem, adaptive SMC and SMC with improved reaching law are widely applied [27]-[30]. In [27], an adaptive SMC has been proposed, which could realize rapid convergence and faster adaption for quadrotor system. In [28], a novel exponential reaching law (ERL) is invented to overcome the convergence performance dilemma in SMC. However, the appearance of additional adaptive rule leads to a huge computational burden, which occurred in the process of online updating. Moreover, although modified reaching law avoids the deficiency of the former, it still suffers from chattering problem [29]. Recently, a neoteric double hyperbolical reaching law (DHRL) [30] is proposed by means of combining hyperbolic tangent and hyperbolic cosine functions. It prompts the system convergence and eliminates the discontinuity in control performance owing to it allows the sliding mode variable enclosing to zero infinitely rather than traversing it. Hence, it is a meaningful attempt to introduce DHRL into the quadrotor attitude control design to improve the tracking performance.

Inspired by aforementioned discussions, herein we present a HESO-based SMC with a double hyperbolical reaching law to stabilize quadrotor attitude tracking, remedy the sluggish convergence and further suppress oscillations. The contributions of this article can be listed as follows:

1) A HESO is devised by extending traditional ESO with higher extended order for quadrotor attitude control to estimate the rapid time-varying disturbances. Compare with previous SMO [22] or ESO [23]-[25], higher precision is obtained on the premise of overcoming the limitation of slow time-varying disturbances. Furthermore, unlike NN [26] suffers from complexity parameter selection and heavy computational burden, the HESO only needs to adjust the bandwidth, such that a reliable disturbance estimation could be attained.

2) Different from ecumenical disturbance estimation-based SMC for quadrotor attitude control subject to slower convergence. This article adopts a new DHRL which can guarantee rapid convergence and contain no extra oscillations contrast with orthodox RLS [28], [31].

The outline of this paper is organized as follows. Section II establishes the attitude model of quadrotors. Section III elaborates the design of the HESO-based controller with DHRL and stability analysis. Section IV discusses the simulation and experimental tests with corresponding results. At last, Section V concludes this paper.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. NOTATION

In this article, the standard definitions are presented below. $\mathbb{R}^{m \times n}$ is a set of real numbers with m rows and n columns. $[\cdot]^T$ denotes the transposed matrix. diag(\cdot) stands for the diagonal matrix. $|\cdot|$ and $\|\cdot\|$ refer to the absolute value and the Euclidian norm, respectively.

B. MODELING OF A QUADTOR

As depicted in Fig.1, the construction of quadrotor attitude system is consisted of a rigid cross frame and four rotors. Two reference frames are established to express the attitude kinematics of quadrotors, i.e., inertia frame $X_i$, $Y_i$, $Z_i$ and body frame $X_b$, $Y_b$, $Z_b$. The vector $\Theta = [\theta_1, \theta_2, \theta_3]^T \in \mathbb{R}^{3 \times 1}$ in inertia frame and the vector $\mathbf{\omega} = [\omega_x, \omega_y, \omega_z]^T \in \mathbb{R}^{3 \times 1}$ in body frame respectively represent the Euler angles and the angular velocity vector with $\theta_1$, $\theta_2$, and $\theta_3$ denoting the roll, pitch and yaw angle, severally. Quadrotors can perform multifarious attitude motions under the thrust force engendered by the rotation of four propellers. Here, the attitude dynamics of quadrotors is formulated as

$$\begin{align*}
\dot{\Theta} &= R \omega \\
J \dot{\omega} &= -\omega \times J \omega + U + d
\end{align*}
$$

$$
R = \begin{pmatrix}
\sin \theta_1 \tan \theta_2 & \cos \theta_1 \tan \theta_2 \\
0 & -\sin \theta_1 \\
\sin \theta_1 \sec \theta_2 & \cos \theta_1 \sec \theta_2
\end{pmatrix}
$$

FIGURE 1. Illustration of quadrotors with different frames.
where \( R \) is deemed to the transformation matrix between \( X_1 = Y_1 - Z_1 \) and \( X_2 = Y_2 - Z_2 \). \( J = J' + J_d \in \mathbb{R}^{3\times3} \) standing for a positive definite diagonal inertial matrix with \( J' = \text{diag}(J_{1}', J_{2}', J_{3}') \in \mathbb{R}^{3\times3} \) represent the nominal rotational matrix with \( J_d \in \mathbb{R}^{3\times3} \) being the uncertainty of rotational inertias. And \( U = [U_1, U_2, U_3]^T \in \mathbb{R}^{3\times3} \) is the control input vector. External disturbances by wind gust are represented by the vector \( d = [d_1, d_2, d_3]^T \in \mathbb{R}^{3\times3} \).

According to (1), in order to simplify succeeding analysis, the attitude dynamics of quadrotors is rewritten as:

\[
\dot{\Theta} = R\Omega + R(J')^{-1}(-\omega \times J\dot{\Omega} + d - J_d \theta) + R(J')^{-1}U
\]  

(2)

where \( M = [M_1, M_2, M_3] \in \mathbb{R}^{3\times3} \) is the unknown lumped disturbances vector and \( u = [u_1, u_2, u_3]^T \in \mathbb{R}^{3\times3} \) is relevant to the vector of control signals.

**Assumption 1:** All the quadrotor attitude details can be sensed and one can always maintain \( \phi \in (-\pi/2, \pi/2) \) and \( \theta \in (-\pi/2, \pi/2) \).

**Remark 1:** All the necessary data of quadrotor attitude system can be measured readily by the inertial measurement units (IMUs) embedded onboard. Particularly, in our experimental rig, it needs to be pointed out that quadrotors are restricted by a universal joint, which connects the quadrotor and the fixed iron shelf, so that the transformation matrix \( R \) is always nonsingular.  

The control objective of this paper is to construct a HESO-based SMC for quadrotors to accomplishing:

1. The lumped unknown disturbances could be recovered by HESO and the error of estimation is ensured to be exponentially convergent.
2. All the errors considered in this paper can accomplish ultimately uniformly bounded (UUB) results and a fast attitude tracking behavior can be achieved with a smooth control action.

**III. CONTROLLER DESIGN**

In this part, a HESO-based SMC will be design within a double hyperbolic reaching law for quadrotor attitude dynamics (2). The architecture of the proposed controller is clarified in Fig.2.

A. **DHRL DESIGN**

As the fundamental part of SMC, the reaching law plays a significant role in sliding mode surface design. Different RLs will cause various degrees of effects on system convergence behavior. For quadrotor attitude system (2), here a double hyperbolic reaching law (DHRL) including \( \cosh(\cdot) \) and \( \tanh(\cdot) \) is applied to realize fast convergence and chattering suppression of the preceding system. The form of the DHRL is as follows:

\[
\dot{s}_i = -\kappa_1[s_i \cosh(as_i) - \kappa_2 \tanh(bs_i)] \quad (3)
\]

Thereinto, \( s_i \) denotes the sliding mode variable, \( a \), \( b \), \( \kappa_1 \) and \( \kappa_2 \) are selective positive parameters. The hyperbolic cosine function and the hyperbolic tangent functions are expressed by \( \cosh(as_i) = (e^{as_i} + e^{-as_i}) / 2 \) and \( \tanh(bs_i) = (e^{bs_i} - e^{-bs_i}) / (e^{bs_i} + e^{-bs_i}) \), respectively.

**FIGURE 2.** Different parts of DHRL (3) with \( a=0.05, b=0.1, \kappa_1=0.5, \kappa_2=3 \)

For the sake of demonstrating, the effectiveness of the two hyperbolic functions of proposed DHRL are shown in Fig.3. Evidently, the hyperbolic cosine function term has a bigger value prompting \( s_i \) to converge rapidly in a considerable degree while distances between \( s_i \) and the convergency value is far. The hyperbolic tangent function term takes over control when \( s_i \) closes to the convergency value to guarantee that \( s_i \) could infinitely approach zero but never arrive at or go across it. Benefiting from the above characteristics, additional chattering cropped up in traditional RLs, will be totally averted.

**Theorem 1:** For DHRL (3), consider a region \([0, \sigma]\) small enough with \( 0 < \sigma < 1 \). Once \( s_i \) decays into it, \( s_i \) is supposed convergent and there exists \( t_0 = \ln[(\sinh(b)/\sinh(b\tanh(\kappa_1/\kappa_2))) / (bs_i)] + (e^{-\omega_0} - e^{-\omega_i}) / \alpha_k \), which is no more than the convergence time of \( s_i \), where \( \kappa_1 \) denotes the gradient of \( s_i \) at the point of \( \sigma \).

**Proof:** Assume that \( x_0 > 0 \) is the initial value of \( s_i \), the convergence time is

\[
t_0 = \int_0^\sigma \frac{1}{\kappa_1 [s_i \cosh(as_i) + \kappa_2 \tanh(bs_i)]} ds_i
\]

(4)

By the above analysis, (4) can be revised

\[
t_0 = \int_0^{s_i} \frac{1}{\kappa_1 s_i [\cosh(as_i) + \kappa_2 \tanh(bs_i)]} ds_i
\]

\[
\leq \int_0^{s_i} \frac{1}{\kappa_2 \tanh(bs_i)} ds_i + \int_0^{s_i} \frac{1}{\kappa_1 s_i \cosh(as_i)} ds_i
\]

(5)

One can define

\[
\begin{align*}
t_1 &= \int_0^{s_i} \frac{1}{\kappa_1 \tanh(bs_i)} ds_i \\
t_2 &= \int_0^{s_i} \frac{1}{\kappa_2 s_i \cosh(as_i)} ds_i
\end{align*}
\]

Then we calculate \( t_1 \) as

\[
...
HESO with the mission of acquiring the estimations of lumped disturbances can be designed as
\[
\begin{align*}
\dot{e}_i(t) &= x_i(t) - \hat{x}_i(t), \\
\dot{\hat{x}}_{i,j}(t) &= \hat{x}_{i,j}(t) + \frac{\alpha_j}{\varepsilon} e_i(t), \\
& \vdots \\
\dot{\hat{x}}_{n+1,j}(t) &= \hat{x}_{n+1,j}(t) + \frac{\alpha_j}{\varepsilon} e_i(t), \\
\dot{\hat{x}}_{n+2,j}(t) &= \frac{\alpha_j}{\varepsilon} e_i(t),
\end{align*}
\]

where \( n \) is the order of the expansion. Apparently, one can obtain the general form of traditional ESO when \( n = 1 \). \( \varepsilon = 1/w \) with \( w \) being the bandwidth of the proposed HESO, which is a positive constant parameter to be set. \( \alpha_j \) denotes the gain coefficient satisfying \( \alpha_j = (n+2)/l_i[(n+2-1)\cdot l_i(1,2,...,n+2)] \).

The proposed HESO comprises derivative estimations of the lumped disturbances, providing extra information associated with the unmeasurable disturbances, thus a developed performance of rebuilding fast time-varying disturbances can be acquired.

**Remark 2**: Although compared with traditional RLs using \( \text{sgn}(x) \), DHRL effectively overcomes the extra chattering by employing the hyperbolic tangent function, it is difficult to guarantee finite-time stability for the quadrotor attitude system because of the asymptotic property of the convergence boundary [32]-[33]. Therefore, a small set is introduced here to assist the rapid dynamic convergence of system.

**B. HESO DESIGN**

The fundamental mechanism of established ESO is that to think of the interior uncertainties and exterior unmodeled dynamics as lumped disturbances and augment the lumped disturbances as an extended item in system. In the process of conventional ESO construction, only the liner first derivative of lumped disturbances is involved, which leads to an assumption that the lumped disturbances are constant or slow changing. However, such conditions are unavailable in reality, because actual interference is always complex and time-varying.

To cope with this nonnegligible problem, inspired by the higher-order expansion of Taylor polynomials, herein a HESO is devised by extending the derivative of lumped disturbances to the \( n \)th order and regarding them as extended states of system. For the sake of subsequent exposition, define the following notations as

\[ x_{n+1} = \theta, \quad x_{n+2} = \theta, \quad x_{j} = M_j, \quad j = 1, 2, 3 \]

Then the attitude dynamics (1) is transformed into

\[
\begin{align*}
\dot{x}_{i,j}(t) &= x_{i,j}(t) \\
\dot{x}_{j}(t) &= x_{i,j}(t) + u_i(t)
\end{align*}
\]

Assuming \( M_j \) is continuously differentiable, obeying the principle of high-gain observer design [34], the particular...
**Theorem 2:** Considering (10) and supposing Assumption 2-3 are satisfied, one has a vibration parameter $0 < \varepsilon < 1$ and a positive constant $\tau_r = 2\lambda_2(\alpha + 2)\varepsilon \ln \frac{e}{\lambda_1}$, such that the following conclusion holds:

$$\sup_{0 < t < \tau_r} \left| x_{ij}(t) - \hat{x}_{ij}(t) \right| \leq O_j(e^{\alpha + 3 - \varepsilon^2}), \forall t \in [T_r, \infty)$$

where $\hat{x}_{ij}(i = 1, 2)$ is the state estimations and $\dot{x}_{ij}(i = 3, \ldots, n + 2)$ express the estimations of lumped disturbances and its higher-order derivatives. $O_j$ is positive constant in dependent of the initial values, $n$ denotes the order of extension.

**Proof:** Firstly, define the vector of observation error $e_i(t) = [x_{ij}(t) - \hat{x}_{ij}(t), x_{ij}(t) - \hat{x}_{ij}(t), \ldots, x_{ij}(t) - \hat{x}_{ij}(t)]$

By virtue of Assumption 2, construct $z_{ij}(t) = e_i(t)/e^{\alpha + 3 - \varepsilon^2}$, and take the derivative of $z_{ij}(t)$ and $z_{ij+1}(t)$ yields

$$\frac{dz_{ij}(t)}{dt} = \frac{d}{dt} \left( \frac{x_{ij}(t) - \hat{x}_{ij}(t)}{e^{\alpha + 3 - \varepsilon^2}} \right) = \frac{x_{ij+1}(t) - \hat{x}_{ij+1}(t)}{e^{\alpha + 3 - \varepsilon^2}} - g_i(z_{ij}(t))$$

And the observation error dynamics can be described in the following form:

$$\dot{z}_{ij}(t) = z_{ij}(t) - g_i(z_{ij}(t)), \quad z_{ij}(0) = \frac{e_{ij}(0)}{e^{\alpha + 3 - \varepsilon^2}}$$

$$\dot{z}_{ij+1}(t) = z_{ij+1}(t) - g_{ij+1}(z_{ij+1}(t)), \quad z_{ij+1}(0) = \frac{e_{ij+1}(0)}{e^{\alpha + 3 - \varepsilon^2}}$$

(11)

Then constructing a Lyapunov function $V(z(t))$, differentiating $V(z(t))$ according to time gives

$$\frac{d}{dt} V(z(t)) = \sum_{i=1}^{n} \frac{\partial V}{\partial z_{ij}} (z_{ij}(t) - g_i(z_{ij}(t))) - \frac{\partial V}{\partial z_{ij+1}} g_{ij+1}(z_{ij+1}(t)) + \frac{\partial V}{\partial z_{ij+2}} g_{ij+2}(z_{ij+2}(t))$$

$$\leq -W(z) + \beta \left\| \frac{d^2 G(t)}{dt^2} \right\|$$

Consider $\left\| \frac{d^2 G(t)}{dt^2} \right\| \leq \lambda_1 \sqrt{V(z)}$, one can obtain

$$V(z(t)) \leq \frac{\lambda_1}{2} \int_0^t e^{-\lambda_1 s} \left\| \frac{d^2 G(t)}{dt^2} \right\| ds$$

Hence, the observation error satisfies the following relationship:

$$\left\| e_{ij}(t) \right\| = e^{\alpha + 3 - \varepsilon^2} \left( 1 + \frac{\lambda_1}{2} \int_0^T e^{-\lambda_1 s} ds \right)$$

This completes the proof. When $i = 3$, $e_{ij}$ denotes the estimation error of the lumped disturbances $M_j$.

**Remark 3:** By using above proposed HESO, disturbance estimation with high precision can be obtained through a proper selection of the extension order $n$. Different from traditional ESO, whose estimation performance is limited to single parameter regulation, proposed HESO has a higher degree of freedom due to the selection of extension order $n$ can provide an assistance when bandwidth adjusting is insufficient.

### C. HESO-based SMC Design

According to above analysis, in this part, a HESO-based SMC containing the DHRL will be stated for quadrotor attitude dynamics.

Here, the attitude command is given as $\Theta = [\theta_1, \theta_2, \theta_3]^T \in \mathbb{R}^{3 \times 1}$. To investigate the controller, tracking error is defined as

$$e = \Theta - \Theta = [e_1, e_2, e_3]^T \in \mathbb{R}^{3 \times 1}$$

(12)

And define the sliding mode variable as

$$s_j = \dot{e}_j + \lambda e_j$$

(13)

where $\lambda > 0(i = 1, 2, 3)$ is selective constant parameters.

Calculating $\dot{s}_j$, one has

$$\dot{s}_j = \dot{e}_j + \lambda \dot{e}_j = \ddot{\theta}_j - \dot{\theta}_j + \lambda \dot{e}_j$$

(14)

Considering DHRL (3) and HESO design (9), the attitude controller $u_i$ is constructed as

$$u_i = \ddot{\theta}_j - \dot{\theta}_j - k_i e_i$$

(15)

where $k_1, k_2, k_3$ are positive controller gains to be set.

**Theorem 3:** For the relevant quadrotor attitude dynamics (2), along with HESO design (9), sliding mode surface (13) and proposed control law (15), supposing all assumptions are met, then the estimation error $e_{ij}$ and sliding mode variable $s_j$ are both UUB.

**Proof:** The Lyapunov function of the controller is built as

$$V = \frac{1}{2} \sum_{i=1}^{n} s_i^2$$

(16)

Then differentiating $V$,

$$\dot{V} = \sum_{i=1}^{n} \dot{s}_i^2 = \sum_{i=1}^{n} s_i \dot{s}_i = \sum_{i=1}^{n} s_i \dot{s}_j - u_i \dot{\theta}_j + \lambda \dot{e}_j$$

(17)

Subsisting the control rule (15) into (17), then $\dot{V}$ becomes

$$\dot{V} = \sum_{i=1}^{n} \left[ s_i \left( -k_i e_i \right) \right] \left[ \cosh(\alpha s_i) + k_i \tanh(b s_i) + k_i s_i \right]$$

(18)
According to Young’s inequality, one obtains $s_\varepsilon_1 \leq s_\varepsilon_1^2/2 + s_\varepsilon_2^2/2$, then $\dot{V}$ is rewritten in the following form:

$$\dot{V} \leq \sum_{i=1}^3 \frac{s_\varepsilon_1^2}{2} - (\kappa_n \varepsilon_1^2 + \varepsilon_2^2) \leq \sum_{i=1}^3 \left(\frac{-\kappa_n + 0.5}{2} \varepsilon_2^2 + \varepsilon_2^2\right) \leq \left(1 - 2\kappa_n\right) \frac{1}{2} \sum_{i=1}^3 \varepsilon_2^2$$

(19)

Once $\kappa_n > 0.5$ and $0 < \varepsilon < 1$ are satisfied, $\dot{V}$ becomes the subsequent form:

$$\dot{V} \leq -\xi V + \zeta$$

where $\xi = \max(1 - 2\kappa_n) > 0$ and $\zeta$ is a positive constant related to $O_{ij}$. Solving $\dot{V}$ then one obtains

$$V(t) \leq V(0) + \frac{\zeta}{\xi} e^{\xi t} + \frac{\zeta}{\xi} V(0) e^{\xi t} \leq V(0) e^{\xi t} + \frac{\zeta}{\xi} V(0).$$

(20)

Up to now, estimation error and sliding mode variable are both proved UUB. Besides, referring to (13), one can derive that the attitude tracking error $\varepsilon$ is also ultimately bounded. In conclusion, all errors in the researched quadrotor attitude system are proved bounded.

**Remark 4:** Some experience about parameter tuning is given here to help the selection of the controller parameters for acquiring satisfactory control performance, which are extracted as follows:

1) For controller module, the control gains $\kappa_{ij}$, $\kappa_{ij}$, and $\kappa_{ij}$ should be selected larger enough to guarantee fast convergence of control signals and achievement of tracking mission. Nevertheless, it should be noted that overlarge gains may cause unexpected chattering into control inputs. Therefore, suitable control parameters cannot be more significant for controller to realize an admirable attitude tracking performance.

2) In terms of proposed HESO, a higher extended order $n$ results to an estimation with higher precision, but the sensitivity to measurement noise of HESO will inevitably increase and unanticipated oscillations may appear. In consequence, a compromise between estimation accuracy and system robustness should be carefully considered.

### IV. NUMERICAL AND EXPERIMENTAL RESULTS

#### A. NUMERICAL RESULTS

**TABLE 1. Parameter selection of proposed control scheme**

| Indices | Values | n = 3, w = 25 |
|---------|--------|--------------|
| DHRL    | $a = 3, b = 0.05$ | $k_{i1} = k_{i2} = 5$ |
|         | $k_{i2} = 2.5, k_{i2} = 3.5, k_{i2} = 3$ | $k_{i1} = k_{i2} = 3$ |
| HESO    | $n = 3, w = 25$ | $k_{i1} = 1.5, k_{i2} = 1, k_{i3} = 1$ |
| SMC     | $\lambda_i = \lambda_i = \lambda_i = 3$ | $\lambda_i = \lambda_i = \lambda_i = 3$ |

To validate the effectiveness of the proposed control algorithm, a range of simulations are proceeded in this part. The initial attitude states of quadrotor attitude system are given as $\Theta(0) = [5, 5, 4]^T$ deg and $\omega(0) = [0, 0, 0]^T$ deg/s. The attitude tracking command is chosen as

$$\begin{align*}
\Theta_d &= 10 \sin(0.5 t) (1 + \cos(0.5 t)) \text{deg} \\
\omega_d &= 8 \sin(0.4 t) (1 + \cos(0.4 t)) \text{deg} \\
\omega_{d3} &= 9 \cos(0.5 t) (1 + \sin(0.5 t)) \text{deg} \\
\end{align*}$$

Then supposing an external time-varying disturbance is selected as $d = [\cos(1.5 t) \sin(t), 1 + 0.5 \cos(t), 1 - \cos(1.2 t)]^T \text{N} \cdot \text{m}$. The nominal inertia moments and uncertainties of rotational inertias are set as $J^* = \text{diag}(0.16, 0.16, 0.32) \text{kg} \cdot \text{m}^2$ and $J = \text{diag}(0.032, -0.032, 0.064) \text{kg} \cdot \text{m}^2$, respectively. Related control parameters are set as TABLE 1. The simulations are operated in MATLAB/Simulink with a fixed sampling time of 0.01s.

**FIGURE 4. Reference signal and actual states in simulation**

**FIGURE 5. Control signals in simulation**

**FIGURE 6. Actual and estimated value of disturbances in simulation**

The simulation results are plotted in Figs. 4-6. Apparently, for pre-given composite command with unavailable external...
disturbances, the performance of attitude tracking is comparatively perfect. Furthermore, owing to the concerned DHRL, the convergence process of attitude tracking errors has been shortened and chattering in the control inputs can be suppressed effectively. The precise estimations of lumped disturbances are guaranteed by selecting a proper bandwidth of HESO. In summary, by implementing DHRL and HESO in attitude tracking control of quadrotors, satisfactory tracking property, rapid dynamic response along with precise disturbance estimation can be concurrently realized.

B. COMPARISON SIMULATIONS

To show up the advantages of proposed control scheme, a comparison is taken here between HESO-based SMC and continuous nonlinear terminal SMC (CNTSMC) [22]. In order to ensure the fairness of comparison, the convergence speed of two SMCs is demanded to be identical by tuning the related parameters advisedly. Figs. 7-8 show the comparative results of attitude tracking with same disturbances and quantized in TABLE 2. By comparing the quantitative comparative performance indices, it can be found clearly that under the premise of compensating the lumped disturbances and tracking the desired commands accurately, one can obtain a more accurate tracking performance contributed by proposed method. Additionally, the control oscillations, which are occurred in CNTSMC severely, are effectively alleviated with the aid of devised HESO and DHRL whether in transient or steady state. Hence, the control consumption is reduced naturally.

TABLE 2. Comparison of quantitative indexes between CNTSMC [22] and HESO-based SMC in simulation

| Index                        | HESO-based SMC | CNTSMC [22] |
|------------------------------|----------------|-------------|
| Convergence time\(^1\)      |                |             |
| \(e_1\)                      | 1.61s          | 3.58s       |
| \(e_2\)                      | 1.49s          | 3.62s       |
| \(e_3\)                      | 1.51s          | 2.73s       |
| Steady error (Standard deviation) |               |             |
| \(e_1\)                      | 0.0178         | 0.0549      |
| \(e_2\)                      | 0.0198         | 0.0528      |
| \(e_3\)                      | 0.0167         | 0.0429      |
| IAU\(^2\)                    |                |             |
| \(u_1\)                      | 736            | 755.1       |
| \(u_2\)                      | 1086           | 1147        |
| \(u_3\)                      | 258.5          | 337.9       |
| IADU\(^3\)                   |                |             |
| \(u_1\)                      | 673.5          | 707.8       |
| \(u_2\)                      | 1042           | 1047        |
| \(u_3\)                      | 218.9          | 312.5       |

1. Convergence time means the instant when tracking error arriving at the set (-2, 2) for the first time.
2. IAU can be obtained through integrating the absolute value of control signals.
3. IADU is computed by integrating the absolute of difference value between control signal and its average.

Moreover, to emphasize the advancements of HESO, remaining other units unchanged, a prevailing ESO [35] is introduced as a substitution of HESO to estimate the lumped disturbances. Figs. 9-10 and TABLE 3 show the comparison results. Even though both ESO and designed HESO can reconstruct the disturbances online, the single derivative of lumped disturbances leads to a severe limitation of the estimation accuracy of ESO, which is notably avoided in HESO by introducing the higher-order derivative of lumped disturbances. Besides, the quantization of comparative results revealed by TABLE 3 further demonstrating the advantage of HESO.
The initial attitude states are set as \( \Theta(0) = [5, 5, 4]^T \text{deg} \) and \( \omega(0) = [0, 0, 0]^T \text{deg/s} \) in the experiment. The rotational inertias are given as \( J_x = 0.023 \text{kg\cdotm}^2 \), \( J_y = 0.021 \text{kg\cdotm}^2 \) and \( J_z = 0.039 \text{kg\cdotm}^2 \), which are offered by the Links-UAV Testbench. The attitude command is selected the same as before. Other parameter selection of experimental validation is identical with that in simulation showed in TABLE 1.

### TABLE 3. Comparison of quantitative indexes between ESO [35]-based and HESO-based controllers

| Index          | HESO   | ESO [35] | Improvement |
|----------------|--------|----------|-------------|
| Estimation error (Standard deviation) | \( e_{3,1} \) | 3.189 | 3.734 | 14.6% |
|                | \( e_{3,2} \) | 2.884 | 4.58 | 37% |
|                | \( e_{3,3} \) | 0.156 | 0.511 | 69.5% |
| Tracking error (Standard deviation) | \( e_1 \) | 0.43 | 0.4783 | 10.1% |
|                | \( e_2 \) | 0.4416 | 0.6075 | 27% |
|                | \( e_3 \) | 0.4267 | 0.6436 | 33.7% |

### C. EXPERIMENTAL VALIDATION

Some experimental validation will be executed in this section based on the Links-UAV Testbench produced by Beijing Links Co., Ltd to manifest the practicability of suggested SMC ulteriorly. Figs.11-12 depict the experiment system and connection between each functional part. The attitude information of quadrotor can be acquired by two IMUs (max sensitivity error: \( \pm 3\% \)), which are inbuilt to monitor angle states and angular velocities. The proposed control algorithm is executed by the slave computer, which is played by Pixhawk V3x with 4ms sampling time and could generate pulse-width modulation (PWM) signals. The electronic speed controllers (ESCs) actuate the four propellers according to the received PWM signals, so that the quadrotor can track the desired attitude moments. The host computer in charge of producing the desired commands and sending them to the slave computer, meanwhile, plotting the tracking performance figure when receiving the attitude information from IMUs. The operations are all completed in Simulink Library. All the communications between host computer and the quadrotor are realized via a 2.4GHz WIFI. What is more, to get closer to real situations, a gust of wind with a constant speed being 5m/s is generated by the blast blower to simulate the external natural wind.

![Experimental platform](image1.png)

**FIGURE 11.** Experimental platform

- Host Computer
- Quadrotor Testbench
- Blast Blower
- SIMULINK Library
- Pixhawk
- PWM
- IMU

![Connection between each functional part](image2.png)

**FIGURE 12.** Connection between each functional part

![External Disturbance](image3.png)

**FIGURE 13.** Attitude tracking performance in experiment

![Control signals in experiment](image4.png)

**FIGURE 14.** Control signals in experiment

Experimental results are demonstrated in Figs.13-14. One can find an accurate and fast attitude tracking performance produced by the proposed control method due to the merit of HESO and DHRL. According to Fig.14, it is obviously to see that continuous control actions with a slight oscillation can be obtained by proposed control method, which is acceptable because of the measurement noises involved in attitude information measured by a low cost IMU. Definitely, a costly IMU will effectively improve measurement accuracy and reduce measurement noises, as a consequence, one should make a compromise between control performance and equipment cost.

### D. EXPERIMENTAL COMPARISONS

To emphasize the excellence of the proposed SMC, a comparative experiment is executed with CNTSMC devised in [22]. The relevant parameters stay the same with previous simulation. As a matter of convenience, the pitch angle is chose as a typical example for comparison and analysis.
V. CONCLUSION
In this article, a HESO-based SMC with a DHRL is presented for quadrotor attitude tracking under unknown disturbances. Focusing on improving control accuracy, a HESO with single parameter regulation is devised via a higher-order expansion with respect to the derivatives of lumped disturbances to obtain a more accurate disturbance estimation. To avoid extra chattering brought by traditional reaching law, an original DHRL consisting of two different hyperbolical functions is employed, which produces neither additional oscillation nor sluggish dynamic response. The analysis of the total system stability is specified based on the Lyapunov theorem. Simulations along with experiments have been implemented and adequate results have been acquired to indicate the prominent characteristics of proposed control scheme.

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FIGURE 15. Comparative results in experiment

TABLE 4. Quantitative comparative results in experiment

| Indices               | CNTSMC [22] | Proposed Scheme | Improvement   |
|-----------------------|-------------|-----------------|---------------|
| Steady error (std dev) | e₂         | 1.66            | 0.54          | -67.47%        |
| Convergence time      | e₂         | 1.116s          | 0.116s        | -89.60%        |
| IADU                  | r₂         | 253.96          | 207.26        | -18.34%        |

Experiment comparative performances are displayed in Fig.15 and quantized TABLE 4. By the picture one can see that both controllers could produce a continuous attitude tracking performance under the specified timevary command. Evidently, the CNTSMC has nonnegligible deficiencies in tracking accuracy and control stability. The proposed HESO-based SMC provides a more accurate tracking behavior and a smoother control action.
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