Evolution of the Length and Radius of Catalyst-Free III–V Nanowires Grown by Selective Area Epitaxy
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ABSTRACT: We present a new model for the length and radius evolution of catalyst-free III–V nanowires grown by selective area epitaxy. We consider simultaneous axial and radial growth of nanowires, which is more typical for this technique compared to the vapor–liquid–solid growth of nanowires. Analytic expressions for the time evolution of the nanowire length and radius are derived, showing the following properties. As long as the nanowire length is shorter than the collection length of group III atoms on the sidewalls, the length evolves superlinearly and the radius evolves linearly with time. For longer nanowires, both the length and radius increase sublinearly with time. The scaling growth laws are controlled by a single parameter that depends on group V flux. The model fits well the data on the selective area growth of InAs and GaAs nanowires by different techniques. Overall, these results can be used for controlling the catalyst-free growth of III–V nanowires and their morphology, including ternary III–V material systems.

INTRODUCTION
Semiconductor nanowires (NWs) offer almost unlimited possibilities for the bottom-up design of their morphology, composition, and crystal phase, and enable monolithic integration of a wide range of optoelectronic III–V NW heterostructures with silicon electronic platforms.1–3 Selective area epitaxy (SAE) of III–V NWs is a promising gold-free alternative to the vapor–liquid–solid (VLS) growth method.3 The SAE growth of III–V NWs works equally well in molecular beam epitaxy (MBE) or in metal organic chemical vapor deposition (MOCVD) techniques. In this work, we try to understand the kinetics of the axial and radial growth of III–V NWs in the true SAE growth mode of catalyst-free NWs4 on patterned substrates with regular arrays of lithographically defined pores.5–9 Most importantly, we study theoretically the time evolution of the mean length and radius of SAE NWs versus the growth parameters and epitaxy technique. According to the experimental data,5–7–9 both the length L and radius R of the SAE-grown NWs usually increase with the growth time, while the radius of Au-catalyzed VLS III–V NWs is fixed by the initial size of the growth seeds and stays constant in most cases.10 Radial growth modifies very substantially the earlier models of VLS NWs at a time-independent radius R = const (see ref 10 for a review). We will show that, after a short incubation stage, the NW length and radius exhibit scaling power-law dependences on the growth time. The power exponents of these dependences are related to the growth conditions, the NW length, and the array pitch.

Model. The SAE growth and main model parameters are illustrated in Figure 1. We consider the group III and V atomic fluxes \( I_k \) in \( \text{nm}^2 \text{s}^{-1} \) (\( k = 3 \) for group III and 5 for group V atoms), arriving onto the top NW facet and its sidewalls with efficiencies \( \chi_k \) and \( \chi_d \), respectively. These \( \chi \) are defined by the beam geometries in MBE or precursor pyrolysis efficiencies in MOCVD. Highly diffusive group III adatoms are able to migrate from the NW sidewalls to the top, with the corresponding diffusion current \( j_3 \) giving the total number of the adatoms arriving to the top facet per unit time (in \( \text{s}^{-1} \)). We neglect the desorption of group III adatoms from the top part.
of a NW on the time scale of interest (assuming sufficiently low substrate temperatures).\textsuperscript{10} For highly volatile group V atoms such as arsenic and phosphorous, we neglect the surface diffusion but account for the desorption in the form of dimers (As$_2$ or P$_2$). The crystallization (or incorporation) rates of III–V pairs on the NW top and sidewalls are described by the quadratic terms, which are proportional to the product of the corresponding group III and V surface concentrations. This regular growth model assumes that the meeting of any two dissimilar adatoms on the surface immediately produces a stable III–V pair in the solid state.\textsuperscript{11} These considerations yield the steady-state material balance equations for the surface concentrations of group III and V atoms on the NW top ($n_3$, $n_5$) and sidewalls ($n'_3$, $n'_5$) (in nm$^{-2}$) of the form

\[
\dot{n}_3 + \frac{j_3}{\pi R^2} = k_{33}n_3n_5, \quad \dot{n}_5 = 2k_{55}n_3^2 + k_{53}n_3n_5 \tag{1}
\]

\[
\dot{n}'_3 - \frac{j'_3}{2\pi RL} = k_{33}'n'_3n'_5, \quad \dot{n}'_5 = 2k_{55}'(n'_3)^2 + k_{53}'n'_3n'_5 \tag{2}
\]

Here, $k_3$ and $k'_3$ stand for the desorption rate constants of group V atoms from the NW top and sidewalls, whereas $k_{33}$ and $k_{33}'$ describe the crystallization rate constants of III–V pairs on the NW top and sidewalls, respectively. The net diffusion flux $j_3$ should be divided by the surface area of the top facet $\pi R^2$ to give the diffusion-induced contribution into the total flux of group III atoms arriving to the top facet. The same current should be divided by the collection area $2\pi RL$ to describe the corresponding decrease in the surface concentration of the group III adatoms per unit area of the NW sidewalls. The collection length for group III adatoms can equal either the entire NW length $L$ or only the effective diffusion length $\lambda_3$ at the NW top.\textsuperscript{12–15} In directional deposition techniques such as MBE, this $\lambda_3$ is usually limited by the shadowing effect,\textsuperscript{16} with $\lambda_3 \cong P \cot \alpha_3$, related to the interwire spacing (pitch) $P$ and the angle $\alpha_3$ of the group III beam with respect to the vertical.\textsuperscript{7} Therefore, we will use the simplified expression

\[
L_s = \min\{L, \lambda_3\} \tag{3}
\]

in what follows.

Finally, the axial and radial NW growth rates are given by

\[
\frac{dL}{dt} = \Omega_{ss}k_{33}n_3n_5, \quad \frac{dR}{dt} = \frac{L_s}{L} \Omega_{ss}k_{33}'n'_3n'_5 \tag{4}
\]

with $\Omega_{ss}$ as the elementary volume of III–V pairs in solid. In the equation for the radial growth rate, the $L_s/L$ factor accounts for the fact that crystallization of III–V pairs occurs only at the NW areas exposed to vapor fluxes but contributes into the radial growth of the entire NW. This is valid under the assumption of the cylindrical NW having a uniform radius from base to top at any moment of time, consistent with the position-independent adatom concentrations and incorporation rates in eqs 1, 2, and 4. Tapering of NWs requires a substantial generalization of the model and will be presented elsewhere.

To obtain the analytical solutions to these kinetic equations, we use two additional simplifications. First, we assume that the SAE growth proceeds under group V rich conditions; otherwise, it would quickly transition to a self-catalyzed mode with a group III droplet on top.\textsuperscript{7} Then, much more group V atoms should desorb than crystalize in solid, corresponding to negligible second terms in eqs 1 and 2 for group V atoms. This yields the simple expressions

\[
n_3 \approx \left( \frac{X_3I_3}{2k_3} \right)^{1/2}, \quad n_5 \approx \left( \frac{X_5I_5}{2k_5} \right)^{1/2} \tag{5}
\]

which directly relate the group V concentrations to the input fluxes and temperature-dependent desorption rate constants. Introducing the effective lifetimes of the group III atoms on the NW top and sidewall facets by definitions

\[
k_{33}n_3 = \frac{1}{\tau_3}, \quad k_{33}'n'_3 = \frac{1}{\tau'_3} \tag{6}
\]

Equations 1 and 4 are reduced to

\[
\dot{n}_3 + \frac{j_3}{\pi R^2} = \frac{n_3}{\tau_3}, \quad \dot{n}_5 = \frac{2n_5}{\pi R L_{wt}} \tag{7}
\]

\[
\frac{dL}{dt} = \Omega_{ss}n_3, \quad \frac{dR}{dt} = \frac{L_s}{L} \Omega_{ss}n'_3 \tag{8}
\]

Second, we should present the diffusion flux $j_3$ as a function of $n_3$. In the absence of desorption of a group III element, the simplest approximation reads

\[
j_3 = 2\pi RL\dot{n}_3(1 - \epsilon) \tag{9}
\]

with

\[
\epsilon = \frac{\dot{n}_3\tau_3}{\dot{n}'_3\tau'_3} = \frac{X_3k_{33}'(X_5k_5)^{1/2}}{X_3k_{33}X_5k_5} \tag{10}
\]

Here, the $\epsilon$ parameter is given by the ratio of the group III adatom activities on the NW top and sidewalls, similar to that in refs 10, 13, and 18 for VLS NWs. The quantity $2\pi RL\dot{n}_3$ is the total number of group III atoms collected by the top part of the NW of length $L_\tau$ per unit time, which equals the direct diffusion current from the NW sidewalls to the top. The $\epsilon$ times the same quantity equals the reverse diffusion current from the NW top to the sidewalls. The resulting diffusion current given by eq 9 equals zero under equilibrium conditions (at $\epsilon = 1$).

We first use eq 9 in eq 7 for $n_3/\tau_3$ and insert the result into eq 8 for d$L$/dt, which yields the straightforward result for the axial growth rate. Using eq 9 in eq 7 for $n'_3/\tau'_3$ gives $n'_3/\tau'_3 = \epsilon \dot{n}'_3$, and then eq 8 for d$R$/dt yields the result for the radial growth rate. Thus, our final equations describing the simultaneous axial and radial growth of cylindrical NW have the form

\[
\frac{dL}{dt} = F_3[\dot{K}_5 + \dot{\lambda}_3 \frac{2L_\tau}{R}(1 - \epsilon)] \tag{11}
\]

with $F_3 = \Omega_{ss}I_3$ as the deposition rate of group III atoms in nm/s. This model should be relevant for sufficiently long NWs, while for shorter ones, we should additionally account for a diffusion flux from the substrate surface.\textsuperscript{14–18} While negligible for the growth on rough substrates with a parasitic layer,\textsuperscript{19} this flux can be much more important in the case of atomically flat patterned SiO$_x$/Si(111) substrates and can be essential for understanding of the growth start.\textsuperscript{15} This important question will be considered elsewhere. However, for NWs whose length is much larger than the group III diffusion length on the oxide
surface, eq 11 should give a reasonable approximation for the growth kinetics (this is guaranteed for the axial growth rate when \( L > \lambda_0 \), and hence, the diffusion flux from the substrate no longer contributes into the NW elongation\(^{12,18-21} \)).

Equation 11 for \( \text{d}L/\text{d}t \) has a similar form as the conventional expression for the axial growth rate of VLS III–V NWs\(^{10,18-21} \) or self-induced GaN NWs\(^{12,22} \) under group V rich conditions. The parameter \( \epsilon \) directs the diffusion flux of group III adatoms from the NW sidewalls to the top or in the opposite direction, depending on whether the adatom activity on the sidewall facets \( (\chi_{i}', \tau_i') \) is larger or smaller than that on the top facet \( (\chi_{3}, \tau_3) \), as in refs 13 and 18. However, there are two important differences. First, most growth models originally developed for VLS III–V NWs completely ignored the influence of a group V element, while our parameter \( \epsilon \) contains explicitly the adsorption–desorption and crystallization rates of group V atoms. Second, the NW radius \( R \) increases due to incorporation of the material at the NW sidewalls, so eq 11 for \( L(t) \) and \( R(t) \) should now be considered as the system of two connected differential equations.

Equation 10 shows that \( \epsilon \) decreases when \( k_{i}' / k_{i} \) decreases (that is, when the crystallization rate on the NW sidewalls is much lower than on the top) or \( k_{i} / k_{i}' \) increases (that is, when the desorption rate of group V atoms from the NW sidewalls is much higher than that from the top). This shows the importance of a catalyst droplet in enhancing the axial and suppressing the radial growth rate of VLS NWs, because the droplet should collect group V atoms more efficiently and at the same time, increase the crystallization rate of III–V pairs with respect to the vapor–solid growth. For the SAE NW growth, any differences in the crystallization or evaporation rates on the sidewalls and top facets should arise due to their different crystallographic structures, and hence, the \( \epsilon \) parameter may be on the order of unity. We will therefore treat this \( \epsilon \) as the free parameter of the model, whose value can be deduced from fitting the experimental data on the axial and radial NW growth rates under different conditions and in different material systems.

### RESULTS AND DISCUSSION

Equation 11 for “short” NWs \(( L < \lambda_0 \) is solved as follows. At \( L = L_0 \), integration of eq 11 for \( R \) readily yields \( R = R_0 + \chi \epsilon \), where \( R_0 \) corresponds to the initial radius of the pore or pre-existing NW stem. Using \( \text{d}t = \text{d}R/(F_{\chi \epsilon} \), eq 11 for \( L \) can be put in the form

\[
\frac{\text{d}L}{\text{d}R} = \frac{2(1 - \epsilon)}{\epsilon} \frac{L}{R} + \frac{\chi}{\chi_R \epsilon}
\]

Integrating this with the initial condition \( L(R = R_0) = L_0 \) with \( L_0 \) as the length of the initial NW stem at \( t = 0 \), we obtain

\[
L = \left( L_0 + \frac{\chi R_0}{\chi_R \epsilon(a - 1)} \right)^{\frac{1}{a}} - \frac{\chi R_0}{\chi_R \epsilon(a - 1)}
\]

Therefore, the solutions can be presented as

\[
L = L_0 + R_0 \left( 1 + \frac{t}{t_0} \right)^{\frac{1}{a}} - R_0 \left( 1 + \frac{t}{t_0} \right), R = R_0 \left( 1 + \frac{t}{t_0} \right)^{\frac{1}{a-1}}
\]

with

\[
a = \frac{2(1 - \epsilon)}{\epsilon}, R_c = \frac{\chi_0}{\chi_R} \frac{R_0}{t_0}, t_0 = \frac{R_0}{F_{\chi \epsilon} \epsilon}
\]

The \( L_0 \) can in principle be put to zero; however, we reiterate that our model may not be suited to describe the initial stage of NW growth evolving from the substrate surface. Also, some NWs can only be grown on pre-existing stems of a different material (for example, InAsSb NWs on InAs stems\(^{1} \)), in which case \( L_0 \) represents the length of the InAs stem.

For “long” NWs \(( L \geq \lambda_0 \), the solutions are obtained as follows. At \( L = \lambda_0 \), after dividing \( \text{d}L/\text{d}t \) by \( \text{d}R/\text{d}t \), eq 11 gives

\[
\frac{\text{d}L}{\text{d}R} = \left( \frac{a}{R} + b \right) L
\]

with \( b = \chi_{i}' / (\chi_3 \lambda_0 \epsilon) \). Integrating this with the initial condition \( L(R_c) = \lambda_0 \), we get

\[
L = \frac{R}{R_c} \left( \frac{a}{b} \right)^{\frac{a}{b}} \left( \frac{R}{R_c} \right)^b e^{(R - R_c)}
\]

Here, \( R_c \) denotes the NW radius at the moment of time \( t_c \), where \( L \) becomes equal to \( \lambda_0 \). This expression is further reduced to \( L = \lambda_0 (R / R_c)^a \) at \( R - R_c \ll \lambda_0 \), which is the reasonable approximation unless the growth time is very long. Substitution of this power-law dependence to eq 11 for \( R \) yields \( \text{d}R / \text{d}t = A / R^\gamma \), with \( A = F_{\chi \epsilon} \lambda_0 e R_c^2 / \lambda_0 \). Upon integration with the initial condition \( R(t_c) = R_c \) and presenting both length and radius as functions of time, the resulting expressions take the form

\[
L \approx \lambda_0 \left( 1 + \frac{t - t_c}{t_1} \right)^{a/(a+1)}, R = R_c \left( 1 + \frac{t - t_c}{t_1} \right)^{1/(a+1)}
\]

with

\[
t_1 = \frac{R_c}{F_{\chi \epsilon} \left( 2 - \epsilon \right)}
\]

Equation 14 is similar to the one obtained previously for the self-induced GaN NWs\(^{12} \) with a short diffusion length \( \lambda_0 \) on the order of 40–50 nm.

With the known deposition rate \( F_t \) and geometry (the \( \chi \) coefficients and the pore or stem radius \( R_0 \)), the obtained expressions are controlled by the single parameter \( \epsilon \). It determines the power exponents in the scaling power-law dependences of the length and radius on the growth time as well as the characteristic times \( t_0 \) and \( t_1 \) for the axial elongation and radial extension in different stages. After a short incubation stage (involving different types of incubation processes such as the NW nucleation on the substrate surface\(^{3,10} \) or in the patterned holes\(^{5} \) diffusion of group III adatoms from the substrate surface to the NW top\(^{3,10,12,14} \) or shape transformation\(^{25} \)), the NW radius evolves linearly with time as long as \( L < \lambda_0 \), while the length increases superlinearly for \( \epsilon < 2/3 \) or \( a > 1 \). The limiting case of the highest \( \epsilon_{min} = 2/3 \), the power exponent for both length and radius at \( L > \lambda_0 \) equals 1/2. The standard VLS-like growth is recovered in the second limiting regime of the lowest \( \epsilon_{min} = 0 \), corresponding to \( a \to \infty \). In this case, the NW radius stays constant, while the length increases first exponentially and then linearly with time\(^{3,14} \).
We first consider the experimental data on SAE InAs NWs grown by MBE in patterned arrays on SiO$_2$/Si(111) at 480 °C, with a fixed pore radius of 40 nm and variable pitches of the array. The evolution of NW length and diameter shown in Figure 2 can be seen how a fast axial growth at a constant radius at $\varepsilon = 0$ is transitioned to a much slower one as the $\varepsilon$ parameter increases, with the length and radius becoming comparable at $\varepsilon = 0.64$. The NW length increases superlinearly with time until it reaches the diffusion length of 1000 nm, after which the time evolution of length becomes sublinear. The further increase of $\varepsilon$ up to its maximum value of 2/3 may lead to situations where the NW length becomes sublinear. This corresponds to the effective deposition rate on the NW sidewalls $2F_0/\varepsilon < 1.45$ nm/min. As discussed in detail in ref 8, the axial NW growth rate is faster at the beginning of growth, which can be attributed to the additional collection of indium from the oxide surface. Leaving aside this initial stage, we use eq 12 for the NW length starting from $L_0 = 170$ nm. The best fit to the length data for a 3000 nm pitch shown in Figure 3a is then obtained with $R_0 = 17.5$ nm and $a = 1.14$, corresponding to $\varepsilon = 0.637$. This large $\varepsilon$ value yields only a slight superlinearity of the length versus time. The data for smaller pitches clearly show the sublinear behavior of the NW length and diameter, both decreasing with decreasing pitch. All sets of data for 250 to 1000 nm pitches are well fitted by eq 14 at $L > \lambda_0$ if we assume that the indium collection length decreases from ~1.85 $\mu$m for a 1000 nm pitch to ~0.5 $\mu$m for a 250 nm pitch, which seem plausible. Very importantly, all curves in Figure 3 correspond to the same $\varepsilon = 0.637$, related to a fixed V/III flux ratio, with the characteristic times $t_f$ in the range from 24 to 76 min. Overall, our model reproduces the data very well, explaining why the length and diameter are almost linear in time for the largest pitch and become more and more sublinear for smaller pitches after the NW length exceeds the collection length of indium. According to these results, the diffusion length of indium adatoms on the sidewalls of InAs NWs at 480 °C is more than 3.5 $\mu$m, and the indium collection on the sidewalls is limited only by the shadowing effect. This is consistent with the earlier observations for MBE-grown InP$_{1-x}$As$_x$ NWs. 

$$L = \left\{ \begin{array}{ll} \frac{\chi'_3}{\chi'_0} R_0 \left[ \exp \left( \frac{2F_0\chi'_3}{R_0} \right) - 1 \right], & L < \lambda_3 \\ \lambda_3 \left[ 1 + \frac{2F_0\chi'_3 (t - t_0)}{R_0} \right], & L \geq \lambda_3 \end{array} \right. \quad (16)$$

These expressions are written for $L_0 = 0$. Typical time dependences of the NW lengths and radii for the fixed $R_0 = 50$ nm, $F_0 = 2$ nm/min, $\lambda_3 = 1000$ nm, $\chi'_3 = \chi'_0$, $L_0 = 0$, and different $\varepsilon$ are shown in Figure 2. It can be seen how the data very well, explaining why the length and diameter are fitted by the model (lines). For the largest pitch of 3000 nm, the length evolution is slightly superlinear, and the diameter is linear in time in the absence of shadowing. Decreasing the pitch leads to the decrease of the indium collection length due to the shadowing effect, from more than 3500 nm for 3000 nm pitch down to 500 nm for 250 nm pitch. This leads to the sublinear behavior of both length and diameter after the NW length exceeds the collection length of indium.
Next, we consider the data of ref 9 for the length and diameter evolution of GaAs NWs grown by SAE MOCVD at 750 °C on patterned SiO$_2$/GaAs substrates, with a 600 nm pitch and variable pore diameter from 125 to 225 nm. These curves, shown in Figure 4, feature an interesting nonlinear evolution of the NW lengths. Similar to Figure 2, the NW length first increases superlinearly with time, converging to a sublinear behavior for longer times. We speculate that this effect is due to a limited diffusion length of gallium on the NW sidewalls. At a high growth temperature of 750 °C, a fraction of gallium adatoms desorbs before reaching the NW top, corresponding to the transition from the growth regime at $L < \lambda_3$ to the one at $L > \lambda_3$. The pronounced superlinear increase of the NW length for shorter times requires sufficiently large $\alpha$ according to eq 12. The NW diameter in this stage is linear in time regardless of $\chi$. All theoretical curves shown in Figure 4 correspond to a fixed $\alpha = 1.83$ or $\varepsilon = 0.522$ at $L_0 = 0$. Other parameters are summarized in Table 1 and provide the excellent fits to the data. From the fits, we can deduce the values of $F_{\chi}(=5.44)$ nm/min and $\chi_3/\chi=1.78$, showing that the adsorption of gallium is better on the top NW facet than on the sidewalls. The observed decrease of the NW length for larger pore size is explained simply by the fact that the NW elongation is controlled by the surface diffusion of gallium, which contributes more into the axial growth rate of thinner NWs. Most importantly, the gallium diffusion length on the NW sidewalls at 750 °C appears very close to 2 μm in all cases, as shown in Figure 4.

In summary, the proposed approach allows one to treat simultaneously the axial elongation and radial extension of SAE III–V NWs. The most important parameter of the model, $\varepsilon$, determines the power exponents in the scaling time dependence of the NW length and average radius and can be finely tuned by the growth conditions including the V/III flux ratio and temperature. We plan to generalize the model by considering more accurately the initial stage of NW growth from the substrate surface, as well as spatially inhomogeneous incorporation rates and tapered NW geometries. The developed model yields simple analytical solutions with a minimum number of parameters and therefore can be useful for understanding and controlling the SAE growth in different material systems, including ternary III–V NWs such as InGaAs, InAsP, InAsSb, and their heterostructures. In particular, ref 26 gives an example of application of a similar growth model to SAE InAsSb NWs, where the $\varepsilon$ parameter depends on the InSb fraction in a ternary NW. More studies of ternary III–V NWs grown in the SAE mode will be presented elsewhere.

Table 1. Parameters of GaAs NWs Used in Modeling

| pore size (2R$_0$ (nm)) | $\varepsilon$ | $R_0/R_0$ | $\lambda_3$ (nm) | $t_0$ (min) | $t_1$ (min) | $t_2$ (min) |
|-------------------------|--------------|-----------|-----------------|-------------|-------------|-------------|
| 125                     | 0.522        | 3.73      | 2100            | 22          | 44          | 44          |
| 150                     | 0.522        | 3.73      | 2000            | 27          | 46          | 46          |
| 175                     | 0.522        | 3.73      | 1950            | 37          | 47          | 57          |
| 200                     | 0.522        | 3.73      | 1800            | 43          | 47          | 60          |
| 225                     | 0.522        | 3.73      | 1900            | 53.5        | 47          | 69          |

![Figure 4. Length and diameter evolution of SAE GaAs NWs grown by MOCVD in patterned arrays on SiO$_2$/GaAs with different pore sizes from 125 to 225 nm (symbols), fitted by the model (solid lines) for the length and dashed lines for the diameter) with the parameters summarized in Table 1. The time dependence of the NW length converges from superlinear to sublinear at around 2 μm, corresponding to the transition from the growth regime at $L < \lambda_3$ to the one at $L > \lambda_3$. The pronounced superlinear increase of the NW length for shorter times requires sufficiently large $\alpha$ according to eq 12. The NW diameter in this stage is linear in time regardless of $\chi$. All theoretical curves shown in Figure 4 correspond to a fixed $\alpha = 1.83$ or $\varepsilon = 0.522$ at $L_0 = 0$. Other parameters are summarized in Table 1 and provide the excellent fits to the data. From the fits, we can deduce the values of $F_{\chi}(=5.44)$ nm/min and $\chi_3/\chi=1.78$, showing that the adsorption of gallium is better on the top NW facet than on the sidewalls. The observed decrease of the NW length for larger pore size is explained simply by the fact that the NW elongation is controlled by the surface diffusion of gallium, which contributes more into the axial growth rate of thinner NWs. Most importantly, the gallium diffusion length on the NW sidewalls at 750 °C appears very close to 2 μm in all cases, as shown in Figure 4.

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| 175                     | 0.522        | 3.73      | 1950            | 37          | 47          | 57          |
| 200                     | 0.522        | 3.73      | 1800            | 43          | 47          | 60          |
| 225                     | 0.522        | 3.73      | 1900            | 53.5        | 47          | 69          |
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