Anisotropic pressure effects in hydrodynamic description of waves propagating parallel to the magnetic field in relativistically hot plasmas

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The structure of novel hydrodynamic model of plasmas with the relativistic temperatures consisted of four equations for the material fields is presented for the regime of anisotropic pressure and other tensors describing the thermal effects. Presented model constructed of equation for evolution of the concentration, the velocity field, the average reverse relativistic $\gamma$ functor, and the flux of the reverse relativistic $\gamma$ functor, which are considered as main hydrodynamic variables. Four pressure-like tensors (two second rank tensors and one fourth rank tensor) describe the thermal effects. Among them we have the flux of the particle current and the current of the flux of the reverse relativistic $\gamma$ functor. The high-frequency excitations are considered analytically in order to trace the contribution of the anisotropy of pressure-like tensors in their spectra.

Keywords: relativistic plasmas, hydrodynamics, microscopic model, arbitrary temperatures.

I. INTRODUCTION

The relativistic and nonrelativistic magnetized plasmas are under active theoretical and experimental study over several decades due to their important role in the variety phenomena existing in nature. Particularly, the astrophysical objects show large number of scenarios for the hot plasmas being in the strong magnetic field. The magnetic field creates the anisotropy in plasmas, which reveals in the anisotropic structure of tensors describing plasma dynamics at the hydrodynamic description of plasmas and anisotropy of the distribution functions at the kinetic description of plasmas (see for instance [1] and [2]). Variety of physical scenarios for the relativistic plasmas are recently considered in literature [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. Moreover, the plasmas in curved spacetime are also under consideration [13], [14], [15], [16], [17]. Majority of these scenarios involve the strong magnetic field which creates the anisotropy of the system.

In hydrodynamics the anisotropy reveals itself via the structure of pressure tensor $P$. The pressure tensor $\hat{P}$ is the symmetric second rank tensor. It can be presented via three independent functions if coordinate axis are chosen along the main axis of the system

$$\hat{P} = \begin{pmatrix} P_{xx} & 0 & 0 \\ 0 & P_{yy} & 0 \\ 0 & 0 & P_{zz} \end{pmatrix}.$$  \hspace{1cm} (1)

Otherwise the pressure is presented within six functions.

However, the symmetry of the system can decrease the number of independent functions describing the pressure tensor. For instance, the presence of the uniform magnetic field in infinite plasmas gives the axial symmetry for the system. Hence, the pressure tensor is presented within two functions

$$\hat{P} = \begin{pmatrix} P_{\perp} & 0 & 0 \\ 0 & P_{\perp} & 0 \\ 0 & 0 & P_{\parallel} \end{pmatrix}.$$  \hspace{1cm} (2)

If the magnetic field is relatively weak we can neglect the difference of the pressures $P_{\perp} \approx P_{\parallel} = p$. Hence the pressure is proportional to the unit matrix $P^{ab} = P \cdot \delta^{ab}$, where $\delta^{ab}$ is the Kronecker symbol.

Here we estimate the role of the anisotropy of the pressure (and the pressure-like tensors) of electrons in the properties of waves propagating parallel to the external magnetic filed in the relativistically hot plasmas.

Above we present the discussion of the pressure tensor, which is the flux of momentum. However, the model under development does not include the pressure itself. While two other hydrodynamic functions with the similar physical meaning exist in this model. Let us describe the structure of the suggested model. In our description we follow works [18], [19], [20], [21], [22]. The model includes the concentration of particles $n$ and equation for its evolution, which has form of the continuity equation. The continuity equation includes the velocity field $v$ defined via the current of particles $j = n v$. Therefore, the second equation of this system is the velocity field evolution equation. The velocity field evolves under the influence of the electromagnetic field presented by vector fields $E$ and $B$. The terms describing the interaction are considered in the mean-field approximation (the self-consistent field approximation). Consequently, it contains three following functions the average reverse gamma factor $\Gamma$, the average flux of reverse gamma factor $\tilde{\gamma}$, and the second rank tensor of the current of the flux of reverse gamma factor $\gamma^{ab}$. The divergence of the flux of the current of particles $\gamma^{ab}$ gives the kinetic mechanism of the evolution of the velocity field as well. The velocity field evolution equation shows that the model includes the following low rank tensors: the scalar function

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of the average reverse gamma factor \( \Gamma \), and the vector function of the average flux of reverse gamma factor \( t^a \). Therefore, the evolution equations for these functions are included in the model. The interaction in this equations is also considered in the mean-field approximation. The evolution of scalar function \( \Gamma \) is expressed via the divergence of vector \( t^a \). Moreover, the interaction contribution is expressed via the concentration, the velocity field, and the flux of current of particles \( p^{ab} \). Hence, no additional functions appear there. The evolution of vector \( t^a \) is expressed via the divergence of tensor \( t^{ab} \). The interaction contribution in this equation is presented via the concentration \( n \), current of particles \( j = n v \), the flux of current of particles \( p^{ab} \), and the fourth rank tensor \( M^{abcd} \), which is the flux of flux of tensor \( p^{ab} \). Overall, we see the contribution of three high-rank tensors \( p^{ab} \), \( t^{ab} \), and \( M^{abcd} \), which requires some equations of state. Model presented in Refs. [18, 13, 20, 21, 22] applies the equations of state appearing from relations of tensors \( p^{ab} \), \( t^{ab} \), \( M^{abcd} \) with other hydrodynamic functions obtained for the equilibrium state. These relations are found via the application of the relativistic Maxwell distribution function. In earlier papers we focus on approximately isotropic form of the second rank tensors. To this end, in those papers the equations of state are calculated using isotropic form of the relativistic Maxwellian distribution function.

There are examples of generalization of the relativistic Maxwellian distribution function [1], where the prefactor in front of the exponent contain combination of parameters giving an anisotropy. Particularly, the parameter proportional to difference of parallel and perpendicular pressures is introduced. Some methods introduce the two-temperature description of each species, where one temperature related to the motion parallel to the magnetic field and the second temperature is related to the motion perpendicular to the magnetic field. However, the temperature is the scalar function. Therefore, the effects of anisotropy cannot lead to several temperatures for the species. Each species has single temperature. While, it is well-known that the temperatures of different species (the electrons and ions, for example) can be different.

Below we present a hydrodynamic model which is derived from the microscopic model [18, 22]. The microscopic motion of classic relativistic particles is traced to obtain corresponding definitions and equations for the macroscopic functions. Most simple definition allowing to show the structure of all hydrodynamic functions is the concentration of particles \( n(r, t) \), which is defined in the arbitrary inertial frame [22, 24, 25]

\[
n(r,t) = \frac{1}{\Delta} \int \int \sum_{i=1}^{N} \delta(r + \xi - r_i(t)).
\]  

The integral operator counts the number of particles in the vicinity of the point of space. Hence, we have the number of particles in the volume \( \Delta \) around point of space \( r \) in an arbitrary moment in time \( t \). The subindex \( i \) refers to the number of particle, while \( N \) is the total number of particles in the system. Vector \( r_i(t) \) presents the radius-vector of \( i \)-th particle. Vector \( \xi \) presented under integral in (3) scans the \( \Delta \)-vicinity.

The number and form of other hydrodynamic functions are obtained during the derivation. Each step of derivation shows which functions can be applied for the extension of the model. Their definitions appear via operator

\[
\langle ... \rangle \equiv \frac{1}{\Delta} \int \int \sum_{i=1}^{N} \delta(r + \xi - r_i(t)),
\]

which is illustrated for the concentration above [18]. Therefore, we have the following set of functions: current of particles \( j = \langle v_i(t) \rangle \), which allows to introduce the velocity field \( v = \langle \frac{j}{n} \rangle \), the reverse gamma factor \( \Gamma = \langle \frac{1}{\gamma_i} \rangle \), the flux of reverse gamma factor \( t^a = \langle \frac{1}{\gamma_i} \rangle \gamma_i^a - \Gamma v^a \), the flux of current of particles \( p^{ab} = \langle v_i^a v_i^b \rangle - n v^a v^b \), the current of the flux reverse gamma factor \( t^{ab} = \langle \frac{1}{\gamma_i} \gamma_i^a v_i^b \rangle = \Gamma v^a v^b - v^a v^b \), here \( \gamma_i = 1/\sqrt{1 - v_i^2} \) is the relativistic gamma factor if \( i \)-th particle. The energy-momentum density is not presented here since it is included in the hydrodynamic model. Quantum modernization of this method is developed in literature [20, 21, 22, 23].

This paper is organized as follows. In Sec. II the relativistic hydrodynamic equations are presented and discussed for the isotropic plasmas. In Sec. III the contribution of the anisotropy in the spectra of collective excitations is considered analytically. In Sec. IV a brief summary of obtained results is presented.

## II. RELATIVISTIC HYDRODYNAMIC MODEL

Here we follow Refs. [18, 22] and generalize isotropic pressure regimes applied in Refs. [19, 20, 21] where a set of relativistic hydrodynamic equations is obtained and applied for plasmas with the relativistic temperatures. The model is composed of four equations. There are other hydrodynamic models of high-temperature relativistic plasmas, where the interaction of particles is considered in terms of the momentum evolution equation [1], [2], [5], [30], [31], [32].

First equation in the presented model is the continuity equation [13]

\[
\partial_t n + \nabla \cdot (n v) = 0.
\]  

Next, the velocity field evolution equation is [18, 22]

\[
n \partial_t v^a + n (v \cdot \nabla) v^a + \frac{1}{m} \partial_b \left[ p_e e^a_B e^b_B + p_{\perp} (n^{ab} - e^a_B e^b_B) \right] = \frac{e}{m} \Gamma E^a + \frac{e}{mc} \varepsilon^{abc} (\Gamma v_b + t_B) B_c - \frac{e}{mc^2} (\Gamma v^a e^b_B + \Gamma v^a t_B e^b_B) E^a - \frac{e}{mc^2} (\bar{t}_{\parallel} e^a_B e^b_B + \bar{t}_{\perp} (n^{ab} - e^a_B e^b_B)) E^a,
\]  

where
where tensor $p^{ab} = p[B_{\text{ext}}e^a_B + p \cdot \delta^{ab} - e^a_B e^b_B]$ is the flux of the thermal velocities, and tensor $t^{ab} = t[B_{\text{ext}}e^a_B + t \cdot \delta^{ab} - e^a_B e^b_B]$ is the flux of the average reverse gamma-factor. Here we use unit vector in the direction of the external magnetic field $e_B = B_{\text{ext}}/||B_{\text{ext}}|| = e$, which coincides with the unit vector of z-axis. Therefore, if $a = x$ we find $\partial_x p^{ab} = \partial_x p \cdot e_B$ (same for $a = y$). Similarly, if $a = z$ we find $\partial_z p^{ab} = \partial_z p ||$. Parameters $m$ and $c$ are the mass and charge of particle, $c$ is the speed of light, $\delta^{ab}$ is the three-dimensional Kronecker symbol, $\epsilon^{abc}$ is the three-dimensional Levi-Civita symbol. In equation (6) and below we assume the summation on the repeating index $\sum_{b=x,y,z} = e_B$. Moreover, the metric tensor has diagonal form corresponding to the Minkovskii space, it has the following sings $\rho^{a\beta} = \{-1, +1, +1, +1\}$. Hence, we can change covariant and contravariant indexes for the three-vector indexes: $v^b_i = v_{ab}$. The Latin indexes like $a$, $b$, $c$ etc describe the three-vectors, while the Greek indexes are deposited for the four-vector notations. The Latin indexes can refer to the species $s = e$ for electrons or $s = i$ for ions. The Latin indexes can refer to the number of particle $j$ at the microscopic description.

However, the indexes related to coordinates are chosen from the beginning of the alphabet, while other indexes are chosen in accordance with their physical meaning.

The equation of evolution of the averaged reverse relativistic gamma factor includes the action of the electric field

$$\partial_t \Gamma + \partial_b (\Gamma v^b + \epsilon^b) = -\frac{e}{m c^2} n v \cdot E \left(1 - \frac{1}{c^2} \left( v^2 + \frac{p || + 2 p \perp}{n} \right) \right) + \frac{2e}{mc^2} n v_b \left( p || e^a_B e^b_B + p \perp (\delta^{ab} - e^a_B e^b_B) \right)$$

Function $\Gamma$ is also called the hydrodynamic Gamma function [15].

The fourth and final equation in this set of hydrodynamic equations is the equation of evolution for the thermal part of current of the reverse relativistic gamma factor (the hydrodynamic Theta function):

$$\left( \partial_t + v \cdot \nabla \right) t^a + \partial_b \left[ t || e^b_B e^b_B + t \perp (\delta^{ab} - e^a_B e^b_B) \right] + \left( t \cdot \nabla \right) v^a + \Gamma (\partial_t + v \cdot \nabla) v^a =$$

$$= \frac{e}{m} n E \left[ 1 - \frac{v^2}{c^2} \right] + \frac{e}{m c^2} n v_b B_c \left[ 1 - \frac{v^2}{c^2} \right] + \frac{2e}{mc^2} \epsilon^{abc} v^d E_b \left[ p || e^a_B e^b_B e^d_B + p \perp (\delta^{ad} - e^a_B e^b_B) \right]$$

$$- \frac{2e}{mc^2} \epsilon^{abc} v^d E_c \left[ p || e^a_B e^b_B e^d_B + p \perp (\delta^{ad} - e^a_B e^b_B) \right] + \frac{2e}{mc^2} E_b v^a v_c \left[ p || e^a_B e^b_B + p \perp (\delta^{ab} - e^a_B e^b_B) \right]$$

$$+ \frac{4e}{mc} \epsilon^{abc} v^d E_c \left[ p || e^a_B e^b_B e^d_B + p \perp (\delta^{ad} - e^a_B e^b_B) \right] + \frac{2e}{mc} \epsilon^{abc} v^d E_c \left[ p || e^a_B e^b_B + p \perp (\delta^{ab} - e^a_B e^b_B) \right]$$

$$+ \frac{2e}{mc} (M || + 2M ||, e^b_B \cdot E) + \frac{2e}{mc} (M ||, || + 4M \perp \perp) (\delta^{ab} - e^a_B e^b_B) E_b.$$  

All hydrodynamic equations are obtained in the mean-field approximation (the self-consistent field approximation).

The fourth rank tensor $M^{abcd}$ entering the equation for evolution of the flux of reverse gamma factor via its partial trace $M^{abcc} = M^{abc}$. If we neglect the anisotropy we construct tensor $M^{abcd}$ of the Kronecker symbols $M^{abcd} = (M_0/3)(\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc})$. It gives $M^{xxxx} = M^{yyyy} = M^{zzzz} = M_0$. If we have two pairs of different projections we obtain $M^{xyxy} = M^{yxxy} = M^{zzzz} = M_0$. Otherwise the element of tensor $M^{abcd}$ is equal to zero. So, for the partial trace $M^{ab}$ we find $M^{ab} = (5M_0/3)\delta^{ab}$. Let us go back to the anisotropic case with the anisotropy axis directed parallel to the z-direction. So, we have $M^{xxxx} = M^{yyyy} \neq M^{zzzz}$. Moreover, we have modification of elements with two pairs of different projections: $M^{xyxy} \neq M^{yxxy} \neq M^{zzzz}$. Hence, we have four different values of elements of tensor $M^{abcd}$ instead of two values existing in the isotropic case. For the purpose of small amplitude waves in the linear approximation on the wave amplitudes we need equations for $t^x$ and $t^y$. These equations contain $M^{abc}$ and $M^{e_ab}$ which have the following form. Including the symmetric properties of the fourth rank tensor $M^{abcd}$ we obtain its following representation $M^{abcd} = M_0 e^a_B e^b_B e^c_B e^d_B + M || || (e^a_B e^b_B (\delta^{cd} - e^c_B e^d_B) + e^b_B e^c_B (\delta^{ab} - e^a_B e^b_B) + e^a_B e^d_B (\delta^{bc} - e^b_B e^c_B) +$
The equations of electromagnetic field have the traditional form presented in the three-dimensional notations

\[
\nabla \cdot \mathbf{B} = 0, \\
\n\nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}, \\
\n\nabla \cdot \mathbf{E} = 4\pi (n_i - n_e), \\
\n\nabla \times \mathbf{B} = \frac{1}{c} \partial_t \mathbf{E} + \frac{4\pi q_e}{c} n_e \mathbf{v}_e,
\]

where the ions exist as the motionless background.

III. WAVES IN THE RELATIVISTIC MAGNETIZED PLASMAS

A. Equilibrium state and the linearized hydrodynamic equations

We consider small amplitude collective excitations relatively the macroscopically motionless equilibrium state of the relativistically hot plasmas with anisotropic second rank tensors $p^{ab}$ and $t^{ab}$ (which are pressure like tensors while the pressure itself is not included in the model). This equilibrium state is described by the relativistic Maxwellian distribution. The equilibrium state is described within equilibrium concentration $n_0$ and temperature $T$. The velocity field $\mathbf{v}_0$ in the equilibrium state is equal to zero. The equilibrium electric field $\mathbf{E}_0$ is equal to zero. The plasma is located in the constant and uniform external magnetic field $\mathbf{B}_0 = B_0 \mathbf{e}_z$ which create anisotropy in the system revealing in the anisotropy of tensors $p^{ab}$, $t^{ab}$, and $M^{abcd}$. Two second rank tensors and one fourth rank tensor are involved in the description of the thermal effects. Their structure is demonstrated after equation (8) for tensors $p^{ab}$ and $t^{ab}$, and equation (8) for tensor $M^{abcd}$.

We consider propagation of perturbations in the direction parallel to the external magnetic field $k = \{0, 0, k_z\}$. Let us present the linearized equations for the plane wave excitations propagating parallel to the external magnetic field. The concentration appears as $\delta n = N_0 e^{-i \omega t + ik_z z}$, where $\omega$ is the frequency, $N_0$ is the constant amplitude. Perturbations of other functions have same structure. We start with the linearized continuity equation (9):

\[
-\omega \delta n + n_0 k_z \delta v_z = 0.
\]

Next, we show the linearized equations for the evolution of the three projections of velocity field obtained from equation (9):

\[
-\omega n_0 \delta v_x = \frac{q_e}{m} \left( \Gamma_0 - \frac{\Gamma_0}{c^2} \right) \delta E_x + \Omega_c (\Gamma_0 \delta v_y + \delta t_y),
\]

\[
-\omega n_0 \delta v_y = \frac{q_e}{m} \left( \Gamma_0 - \frac{\Gamma_0}{c^2} \right) \delta E_y - \Omega_c (\Gamma_0 \delta v_x + \delta t_x),
\]

and

\[
-\omega n_0 \delta v_z + ik_z \frac{\delta p_0}{m} = \frac{q_e}{m} \Gamma_0 \delta E_z - \frac{q_e}{mc^2} \Gamma_0 \delta E_z,
\]

where $\Omega_c = q_e B_0 / mc$ is the cyclotron frequency.

Equations (13), (14), (15), and (16) require contribution of the flux of the average reverse gamma factor. So, we use the linearized form of equation (9):

\[
-\omega \delta t_x - \omega \Gamma_0 \delta v_x
\]
waves. They lead to the following spectrum of the relativistic Langmuir wave (22) is proportional to derivative of $\Gamma_0/n_0$ to the magnetic field. They are described within equations (14), (15), (17), (18), (19), and (20). The Maxwell equations (14), (19), and (20) contain the velocity perturbations. Therefore, we need to extract the velocity perturbations from equations (14), (15), (17), and (18). They appear in the following form

$$\delta v_x = \frac{\omega}{\omega^2 - f_0\Omega^2} \left\{ \frac{q}{m} \delta E_x \left( \frac{\Gamma_0}{n_0} - \frac{t_{0,\perp}}{n_0 c^2} \right) - \frac{2q}{m} f_0 \frac{\Omega}{\omega} \delta E_y \right\},$$

and

$$\delta v_y = \frac{\omega}{\omega^2 - f_0\Omega^2} \left\{ \frac{q}{m} \delta E_y \left( \frac{\Gamma_0}{n_0} - \frac{t_{0,\perp}}{n_0 c^2} \right) - \frac{2q}{m} f_0 \frac{\Omega}{\omega} \delta E_x \right\},$$

where we use the following notations $f_0 = 1 - (4p_{0,\perp} + p_{0,\parallel})/(n_0 c^2)$, and $f_{0M} = (4M_{0,\perp}/3 + M_{0,\parallel,\perp})/(mc^2)$.

Let us write down the dispersion equation as the determinant of the corresponding part of the dielectric permeability tensor:

$$\omega^2 - k_z^2 c^2 = \frac{\omega^2 - \omega^2_{0Lc} - \frac{\omega^2_{0Lc}}{\omega^2 - f_{0M}\Omega^2} \left( \frac{\Gamma_0}{n_0} - \frac{t_{0,\perp}}{n_0 c^2} \right) \omega^2_{0Lc} f_{0M} + 2m f_{0M}}{\omega^2 - f_{0M}\Omega^2} \left( \frac{\Gamma_0}{n_0} - \frac{t_{0,\perp}}{n_0 c^2} \right)$$

\[= 0. \] (25)

The cut-off frequency of the longitudinal wave (22) is bound to $t_{0,\perp}$ which defines the coefficient in front of the Langmuir frequency. However, the cut-off frequencies for the transverse waves contains another coefficient in front of the Langmuir frequency which is proportional to $\Gamma_0/n_0 - t_{0,\perp}/n_0 c^2$. It includes another element of tensor $t$: $t_{0,\perp}$. The group velocity of the relativistic Langmuir wave (22) is proportional to derivative of $p_{0,\parallel}$. If we consider the electromagnetic wave propagation perpendicular to the magnetic field with the linear polarization of electric field changing parallel to the magnetic field we find the following spectrum

$$\omega^2 = \left( \frac{\Gamma_0}{n_0} - \frac{t_{0,\parallel}}{n_0 c^2} \right) \omega^2_{0Lc} + k_z^2 c^2. \] (26)

This result underline the various contribution of $t_{0,\parallel}$, $t_{0,\perp}$, and $p_{0,\parallel}$. It has same cut-off frequency as the longitudinal wave (22) propagating parallel to the magnetic field. But
they obviously have different group velocities.

IV. CONCLUSION

General derivation of the hydrodynamic model for the relativistically hot plasmas has been presented in earlier papers [18, 22]. This hydrodynamic model is based on the dynamics of four material fields: the concentration and the velocity field and the average reverse relativistic $\gamma$ factor and the flux of the reverse relativistic $\gamma$ factor. However, the final truncation has been made for the isotropic structure of the high-rank tensors entering the model. In this paper, we have presented the generalization of the model including the anisotropy of the high-rank tensors. It has been assumed that there is the single anisotropy direction related to the external magnetic field. It leads to two values of the flux of current of particles $\hat{\rho}$ and two values of the current of the flux of reverse gamma factor $\hat{\ell}$. The fourth rank tensor $\hat{M}$ has three independent elements. Experimental detection of the anisotropic effects can be partially made through the analysis of spectrum of waves including the measurement of the value of cut-off frequencies and measurement of the group-velocities of waves. Complete theoretical treatment of this problem requires further analysis of the anisotropic relativistic distribution function which can give background for the calculation of the approximate calculation of equations of state.

The background for the further analysis of the linear and nonlinear wave phenomena in the relativistically hot strongly magnetized anisotropic plasmas has been developed. Moreover, it includes the anisotropy of the tensors describing the thermal effects.

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VI. DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.

[1] R. D. Hazeltine, S. M. Mahajan, ”Fluid description of relativistic, magnetized plasma”, The Astrophysical Journal 567, 1262 (2002).
[2] S. M. Mahajan, R. D. Hazeltine, ”Fluid description of a magnetized plasma”, Phys. Plasmas 9, 1882 (2002).
[3] S. M. Mahajan, Z. Yoshida, ”Relativistic generation of vortex and magnetic field”, Phys. Plasmas 18, 055701 (2011).
[4] Z. Osmanov, S. Mahajan, ”On the Heating of AGN Magnetospheres”, Universe 7, 83 (2021).
[5] N. L. Shatashvili, S. M. Mahajan , and V. I. Berezhiani, ”Nonlinear coupling of electromagnetic and electron acoustic waves in multi-species degenerate astrophysical plasma”, Phys. Plasmas 27, 012903 (2020).
[6] A. R. Soto-Chavez, S. M. Mahajan, and R. Hazeltine, ”Two-fluid temperature-dependent relativistic waves in magnetized streaming pair plasmas”, Phys. Rev. E 81, 026403 (2010).
[7] Z. Y. Liu, Y. Z. Zhang, and S. M. Mahajan, ”The effect of curvature induced broken potential vorticity conservation on drift wave turbulences”, Plasma Phys. Control. Fusion 63, 045009 (2021).
[8] L. Comisso, F. A. Asenjo, ”Thermal-Inertial Effects on Magnetic Reconnection in Relativistic Pair Plasmas”, Phys. Rev. Lett. 113, 045001 (2014).
[9] J. Heyvaerts, T. Lehner, and F. Mottez, ”Non-linear simple relativistic Alfvén waves in astrophysical plasmas”, A&A 542, A128 (2012).
[10] V. Munoz, T. Hada, and S. Matsukiyo, ”Kinetic effects on the parametric decays of Alfvén waves in relativistic pair plasmas”, Earth Planets Space, 58, 1213–1217, (2006).
[11] G. Brunetti, P. Blasi, R. Cassano, and S. Gabici, ”Alfvenic reacceleration of relativistic particles in galaxy clusters: MHD waves, leptons and hadrons”, Mon. Not. R. Astron. Soc. 350, 1174–1194 (2004).
[12] D. She, A. Huang, D. Hou, and J. Liao, ”Relativistic Viscous Hydrodynamics with Angular Momentum”, arXiv:2105.04060.
[13] C. Bhattacharjee, J. C. Feng, and S. M. Mahajan, ”Black hole in a superconducting plasma”, Phys. Rev. D 99, 024027 (2019).
[14] L. Comisso, and F. A. Asenjo, ”Generalized Magnetofluid Connections in Curved Spacetime”, arXiv:1912.12503.
[15] F. A. Asenjo, and L. Comisso, ”Gravitational electromotive force in magnetic reconnection around Schwarzschild black holes”, arXiv:1903.01203.
[16] S. Darbha, D. Kasen, F. Foucart, and D. J. Price, ”Electromagnetic Signatures from the Tidal Tail of a Black Hole–Neutron Star Merger”, arXiv:2103.03378.
[17] M. Chabanov, L. Rezzolla, and D. H. Rischke, ”General-relativistic hydrodynamics of non-perfect fluids: 3+1 conservative formulation and application to viscous black-hole accretion”, arXiv:2102.10419.
[18] P. A. Andreev, ”On the structure of relativistic hydrodynamics for hot plasmas”, arXiv:2105.10999.
[19] P. A. Andreev, ”Waves propagating parallel to the magnetic field in relativistically hot plasmas: A hydrodynamic models”, arXiv:2106.14327.
[20] P. A. Andreev, ”On a hydrodynamic description of waves propagating perpendicular to the magnetic field in relativistically hot plasmas”, arXiv:2107.13603.
[21] P. A. Andreev, ”A hydrodynamic model of Alfvénic waves and fast magneto-sound in the relativistically hot plasmas at propagation parallel to the magnetic field”, arXiv:2108.12721.
[22] P. A. Andreev, "Microscopic model for relativistic hydrodynamics of ideal plasmas", arXiv:2109.14050.

[23] L. S. Kuz'menkov, "Field form of dynamics and statistics of systems of particles with electromagnetic interaction", Theoretical and Mathematical Physics 86, 159 (1991).

[24] M. A. Drofa, L. S. Kuz'menkov, "Continual approach to multiparticle systems with long-range interaction. Hierarchy of macroscopic fields and physical consequences", Theoretical and Mathematical Physics 108, 849 (1996).

[25] L. S. Kuz'menkov and P. A. Andreev, "Microscopic Classical Hydrodynamic and Methods of Averaging", presented in PIERS Proceedings, p. 158, August 19-23, Moscow, Russia 2012.

[26] L. S. Kuz'menkov, S. G. Maksimov, "Quantum hydrodynamics of particle systems with coulomb interaction and quantum bohm potential," Theor. Math. Phys. 118, 227 (1999).

[27] P. A. Andreev, I. N. Mosaki, and M. I. Trukhanova, "Quantum hydrodynamics of the spinor Bose–Einstein condensate at non-zero temperatures", Phys. Fluids 33, 067108 (2021).

[28] P. A. Andreev, "Hydrodynamics of quantum corrections to the Coulomb interaction via the third rank tensor evolution equation: Application to the Langmuir waves and the spin-electron-acoustic waves", arXiv:2006.15656.

[29] P. A. Andreev, "Quantum hydrodynamic theory of quantum fluctuations in dipolar Bose—Einstein condensate" Chaos 31, 023120 (2021).

[30] N. L. Shatashvili, J. I. Javakhishvili, H. Kaya, "Nonlinear wave dynamics in two-temperature electron-positron-ion plasma", Astrophys Space Sci. 250, 109 (1997).

[31] N. L. Shatashvili and N. N. Rao, "Localized nonlinear structures of intense electromagnetic waves in two-electrontemperature electron–positron–ion plasmas", Phys. Plasmas 6, 66 (1999).

[32] S. M. Mahajan, "Temperature-Transformed "Minimal Coupling": Magnetofluid Unification", Phys. Rev. Lett. 90, 035001 (2003).

[33] V. I. Berezhiani, S.M. Mahajan, "Large Amplitude Localized Structures in a Relativistic Electron-Positron Ion Plasma" Phys. Rev. Lett. 73, 1110 (1994).