Analysis of Mathematical Literacy on Students’ Metacognition in Conic Section Material

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Abstract. Mathematical literacy is one of the components that use mathematical concepts and applies them to an everyday situation. Mathematical literacy helps people to identify and understand the role that mathematics plays in the world, and to make the well-founded judgments and decisions required in life by constructive, engaged, and reflective citizens. By contrast, not everybody acquires sufficiently. This study aims to describe the ability of mathematical literacy especially geometry literacy in terms of student metacognition on conic section material. The problems in this study were: 1) How is the ability of mathematical literacy in terms of student metacognition on conic section material 2) What types of mistakes do students make in solving the problem of cone section and 3) What factors cause students to make mistakes in solving conic section question. This research is a qualitative descriptive study. The subjects in this study were 36 mathematics education students who took courses in Analytical Geometry. The data collected using observation, written test, and interview. Before the researcher conducted the analysis, the researcher examined the validity of the data using validity and reliability tests in order to obtain valid data. Furthermore, the valid data were analysed descriptively through identification, grouping, clarification, explanation and conclusion.

1. Introduction
Mathematical literacy is important for students to know and understand the role of mathematics in real life [1,2,3]. Mathematical literacy is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts [4]. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make well-founded judgements and decisions needed by constructive, engaged and reflective citizens [5]. By contrast, not everybody acquires sufficiently. According to Programme for International Student Assessment (PISA) organized by the Organization for Economic Cooperation and Development (OECD), Indonesia student’s average scores for mathematical literacy in 2000, 2003, 2006, 2009, 2012, and 2015 were 367, 360, 391, 371, 375, 386 points respectively [6,7]. These scores were below the average scores of OECD of 500 points for all the years. From 72 members of world countries, Indonesia is on the 65th place with the percentage of 42.3%, level 1-2 of mathematical literacy, meanwhile, only 0.8% who were on the level 5 – 6 in 2015. Besides that, the survey result of TIMSS’ study, which is done in every four years started in 2011, Indonesia placed on the 36th of 40 countries in 2015. It showed that the students in Indonesia still had not shown any satisfying prestige [8].
According to the aforementioned importance and problems, it was necessary to intensively analyse students’ mathematical literacy. Students’ mistake in solving the problem can be a hint to find out how far they acknowledge the mathematical literacy. Students’ mistake can also be useful to find out the students’ way of thinking about the concepts which they had learned. The mistake done can be reviewed from students’ metacognition. Metacognition is learner’s ability to think consciously of their cognition and has control over their cognitive processes [9]. Metacognition puts the designing, monitoring and controlling consciousness about the known information needed to solve a problem and focus on the learning activity. When students realise that they have misunderstood or made a mistake, they can correct it. Students’ mistake can be review with metacognition because metacognition investigates and measure the degree of relationship amongst mathematical literacy and student ability.

Based on the results of the observation and interview with lecturers and students in the Mathematics Department, Universitas Negeri Jakarta, one of the materials that students make many mistakes is conic section. Students find difficulty in working with the equations, the complexity of the many new terms and realising the graphs of the equation and the reflection properties of the standard conic section. Students usually think that they fail to integrate the concepts and apply geometry concepts in a real-life situation. They do not understand the question and make mistakes in the test. Some research has established too that conic section is one of the topics that students have many mistake and misinterpretations. [10,11]. The mentioned mistakes and the factors that caused them need to be determined, especially in students’ metacognition, to find a way to help to prevent any similar problems. In this paper, we will present an analysis of mathematical literation base on students’ metacognition in conic section material.

2. Materials and Method

2.1. Materials

The data collected from 36 students studying in Mathematics Education, Faculty of Mathematics and Sciences, Universitas Negeri Jakarta. All students were enrolling Analytical Geometry.

2.2. Method

This research-based on descriptive research with a qualitative approach. Descriptive research is a research used to describe, explain, and answers the questions of the phenomena as what they are and also the correlation of various variables in a phenomenon [12]. A qualitative is research which aims to understand the phenomena happened to a subject, such as the behaviours, perceptions, motivations, actions, and so on holistically, and by describing through words and languages, on a specific context which is scientific and taking advantages of various scientific methods [13].

Before being used for this research, the validity and reliability test was applied. The validity test in this research were content validity, construct validity, and empiric validity. Content validity measure represents all facets of a given construct, both of each question and overall questions. Construct validity is about how far the test items can measure what needs to be measured by the specific concept which was set [14]. The content validity and construct were tested by the lecturers. The empirical validity test used the Pearson Product-Moment Correlation shown in equation (1).

\[
 r_{xy} = \frac{N \sum_{i=1}^{N} x_i y_i - (\sum_{i=1}^{N} x_i)(\sum_{i=1}^{N} y_i)}{\sqrt{(N \sum_{i=1}^{N} x_i^2 - (\sum_{i=1}^{N} x_i)^2)(N \sum_{i=1}^{N} y_i^2 - (\sum_{i=1}^{N} y_i)^2)}}
\]  

(1)

Where \( r_{xy} \) is Pearson’s correlation coefficient, \( N \) is the number of paired scores, \( \sum_{i=1}^{N} x_i \) is the score of the first variable, \( \sum_{i=1}^{N} y_i \) is the score of the second variable, \( \sum_{i=1}^{N} x_i y_i \) is the product of the two paired scores. Correlation coefficient value which obtained for each item of question compared with the correlation coefficient value which was on the \( r \) table with \( \alpha = 0.05 \). If \( r_{xy} > r_{table} \), then the correlation coefficient value is significant and considered as empirical valid [15]. The reliability test measured with Alpha Cronbach formula shown in equation (2). Where \( r_{11} \) is alpha Cronbach coefficient, \( n \) is number of items, \( s_t^2 \) is the variance of the total score formed by summing all items, \( s_i^2 \) is total variance. If the
value of alpha is $> 0.9 = \text{Excellent}, > 0.8 = \text{Good}, > 0.7 = \text{Acceptable}, > 0.6 = \text{Questionable}, > 0.5 = \text{Poor},$ and $< 0.5 = \text{Unacceptable}$ [16].

$$r_{11} = \left[ \frac{n}{n-1} \right] \left[ 1 - \frac{\sum_{i=1}^{n} s_i^2}{s_t^2} \right]$$

(2)

After the instrument had checked as valid and reliable, then the collecting of data was done. The methods used to collect data in this research are observation, written test, and interview—the observation conducted by the researchers during the learning process in class. The test given to students consisted of questions of conic section, which were validated by experts. The test aimed to analyse the ability of students’ mathematical literacy in geometry base on students’ metacognition. The ability of mathematical literacy process in this research analysed based on the PISA indicator. They are communication, mathematically, representation, reasoning and argument, designing the strategy to solve the problem, the usage of symbols, formal language and technical, the usage of operation, and the usage of mathematics tools [17]—the analysing data of subjects’ metacognition process based on awareness, evaluation, and regulation. Metacognitive awareness is an expression of how to think mathematically. Metacognitive regulation is a student’s position in solving the problem. Metacognitive evaluation is an assessment which was created by the ways of thinking mathematically [18]. The written test takes 120 minutes for six essay questions. After the written test, the interview done by the researchers to lecturers and students.

3. Results and discussion

3.1. Instrument Analysis

The test instrument used in the research consists of six essay questions. The research instrument was tested to 30 students outside of the research samples. The result obtained from empirical validity can be seen in Table 1. From Table 1, it can be seen that the validity test of six items of essay questions fulfilled the validity criteria so that all of them could go on to the next test: the reliability test. Based on the reliability measurement instrument, about 0.879 was considered as Good, so that the instrument can be the measurement equipment because it’s valid and reliable.

Table 1. Empirical validity result

| Item Question | 1   | 2   | 3   | 4   | 5   | 6   |
|---------------|-----|-----|-----|-----|-----|-----|
| $r_{xy}$      | 0.649 | 0.715 | 0.822 | 0.723 | 0.890 | 0.636 |
| $r_{table}$  | 0.361 | 0.361 | 0.361 | 0.361 | 0.361 | 0.361 |
| Criteria      | valid | valid | valid | valid | valid | valid |

3.2. Observation

The study implemented during the second semester of the Academic Year 2019/2020 in a regular class environment. Before starting the course of analytical geometry, students gave a set of paper and pen to create their concept map in the geometry topics. Based on the created concept map, students know the basic formula of conic section and its characteristics. They can also mention the shapes of the conic section by the given pictures. They can give examples of a conic section in daily lives. They can write the general form and characteristics of circle and parabola correctly. However, they have difficulties in writing the formula of ellipse, hyperbola, and sphere. The interview was done to know the cause of their difficulties. Some of them confessed that they had not learned the conic section material at all except circle and parabola.

The beginning of observation was continued by asking students to create the writing of "UNJ" by using desmos application (www.desmos.com) in groups. With desmos, the beginning of students' ability in mathematical literacy of mathematics tool can be seen. The researchers informed the students that they are not obliged to use all the concepts in their concept mapping, and they could also integrate any other concept that they believed was relevant to the problem. We leave the student free in his choice of
formula and language. In the making of the "UNJ" writing on desmos, the students had difficulties in representing the algebra equation into the geometric form. Some groups tried to find a certain function to create the U. Some groups tried to create the U by uniting semicircle, and the equation \( x = c \), with \( c \) is a constant number. At the beginning of this research, it can be seen that mathematical literacy of planning the strategy to solve problems. Some students created the vertical line of N, and then they determined the equation line, which was suitable to connect both vertical lines. Students started to think of what gradient they would use, whether the value was positive or negative. Then, they measured the position of the line. After that, they were able to create the border of the line by the interval of N. From the making of those words, the communication between them happened (which was also considered as one of the mathematical literacy). Students created the letter J by using semicircle and vertical line. The process of making the letter J was not as hard as before, and the students did not have any difficulties.

The students then presented ways of making the UNJ letters. The communication happened again by having two ways of communicating; the exchange of questions and answers between one group and the other ones.

After two weeks, a student enrolled in analytical geometry. The researchers did the observation in the class during the learning session and gave examples of questions with answers. Lecturers observed all of the students, reminded and approached the students who were not paying attention to the explanations, then helped them. The analytical geometry lesson was still having various problems, such as some students still could not concentrate during the learning process so they could not understand well about the material, students made mistakes in solving the problems, students still memorise the algorithm and given formulas. The learning process took 14 sessions with 150 minutes for each session. After following the learning session, six questions were applied to 36 students. The test took 120 minutes. With these questions, students’ determination of the mistakes was evaluated in line with their mathematical literacy and metacognition.

3.3. Findings and Interpretation

From various students’ answers, they got a different kind of mistakes. The classification of mistakes done based on Ashlock criteria [19] shown in Table 2.

**Table 2. Type of students’ mistakes in solving the questions**

| Item of Question | Conceptual Mistake | Operating Mistake | Algorithm Mistake | Symbolism Mistakes | Arbitrary Answer | Incomplete Answers | Mean |
|-----------------|--------------------|-------------------|-------------------|--------------------|-----------------|-------------------|------|
| 1               | 19.44%             | 0.00%             | 8.33%             | 2.78%              | 2.78%           | 5.56%             | 6.48%|
| 2               | 11.11%             | 8.33%             | 8.33%             | 8.33%              | 2.78%           | 16.67%            | 9.26%|
| 3               | 25.00%             | 11.11%            | 8.33%             | 8.33%              | 8.33%           | 25.00%            | 14.35%|
| 4               | 30.56%             | 2.78%             | 5.56%             | 13.89%             | 13.89%          | 19.44%            | 14.35%|
| 5               | 19.44%             | 2.78%             | 16.67%            | 5.56%              | 8.33%           | 19.44%            | 12.04%|
| 6               | 25.00%             | 11.11%            | 19.44%            | 16.67%             | 11.11%          | 13.89%            | 16.20%|
| **Mean**        | **21.76%**         | **6.02%**         | **11.11%**        | **9.26%**          | **7.87%**       | **16.67%**        |      |

Based on Table 2, the common mistake is a conceptual mistake with an average of 21.76%. To know the details of the mistakes’ cause effects done by students in solving given questions of Analytical Geometry and the relationship between the students’ metacognition, the author interviewed students and lecturers as respondents. The samples were the students who made mistakes in solving the given problems. The interview’s result was combined with the metacognition indicator and then be analysed. Here is the analysis’ result of mistakes done by the students.

**Question 1:** Prove the ellipse polar line equation of \( a^2x^2+b^2y^2=a^2b^2 \) drawn from the point \( T(x_1,y_1) \) is \( a^2x_1+x+b^2y_1y=a^2b^2 \)

Question 1 consists of reasoning components and arguments. Mathematical literacy involved the ability of reasoning and give reasons. This ability is based on thinking logically and doing analysis of the information and resulted reasoning conclusion. The mistakes are done in this question and students’
view about the reason shown in Figure 1. On Figure 1, it could be seen that the student could not understand the given questions well. During the interview, the student confessed that they did not know about the polar ellipse equations and they made conceptual mistakes. They only wrote the given information they knew from the lesson, without understanding the question. From the interview, the student realises that they made a mistake but still writes it down. It can be said that the student has done a metacognitive evaluation because they know the difficulties and can make an assessment of their abilities and understanding.

Figure 1. Student’s answer for question 1

Question 2: Determine the sphere equation through the circle of intersection of the sphere \( S_1 \equiv x^2 + y^2 + z^2 = 1 \) and \( S_2 \equiv x^2 + y^2 + z^2 - 4x - 5 = 0 \) through the point \( O(0,0,0) \).

Question 2 involved the reasoning and argument components, planning strategy to solve the problems, the usage of symbols, formal language and technical, and the usage of mathematics operation. This ability involved understanding, interpreting, and the ability to manipulate a mathematics context which was used to solve problems. The mistakes are done in this question shown in Figure 2. In Figure 2, students made a mistake by stating that the coefficient was equal to 0. They thought that the right side was equal to 0, so the others were the same and could be 0 = 0. Students did not use information from given questions which were to pass the point \( O(0,0,0) \), so there was a mistake. This shows that the metacognitive awareness of students was still not optimally used. They wrote the points of the sphere but did not use the information correctly.

Figure 2. Student’s answer for question 2

Question 3: There is a line crossing the point \( E(3,10) \) and perpendicular to \( g \equiv y = \frac{2}{5}x + 4 \) so it cut the ellipse \( \frac{x^2}{81} + \frac{y^2}{45} = 1 \) on point \( E \) and \( F \). Find the length of \( EF \).

Question 3 involved the representation, reasoning and argument, planning a strategy to solve problems, the usage of symbols, formal language and technical, and the usage of mathematical operations. One of the student's answers is shown in Figure 3. Based on the interview, the student could not understand the meaning of the question. The student also would not know what to do with the given
information in question 3. So, the student could not reach the level of metacognition because they even did not know how to use the information to solve the problem.

![Figure 4. Student’s answer for question 4](image)

Question 4: Line $g$ is the line that’s crossing the point $R$ on the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$ and perpendicular with the $x$-axis. Line $g$ is cutting the hyperbola asymptote on point A and B. Determine the distance multiplication from $R$ to A and $R$ to B.

Question 4 involved the representation, reasoning and argument, planning a strategy to solve problems, the usage of symbols, formal and technical language, and the usage of mathematical operations. One of the student’s answers shown in Figure 4. Based on the interview, the student could understand the meaning of the questions. They also could determine the first step of solving it, which was to find the intersection point of asymptote hyperbola. On this level, the metacognition regulation had been going on well. Nevertheless, on the next step, the student makes an algorithm mistake. This happens because the student was careless that means the metacognition evaluation not optimal yet: student did not recheck the steps of solving the problem.

![Figure 5. Student’s answer for question 5](image)

![Figure 6. Student’s answer for question 6](image)
Question 5: In Washington, D.C, there was a park named "The Ellipses". This park was located between the White House and Washington monument. The park was surrounded by a street which was ellipse-shaped with the length of minor and major 458 and 390 meters respectively. If the manager of the park wants to build a water fountain on each focus point, determine the distance between the water fountains and explain your answer.

Question 5 involved the representation, reasoning and argument, planning a strategy to solve problems, the usage of symbols, formal language and technical, and the usage of mathematical operations. One of the students' answers shown in Figure 5. On this question, the student already fulfills the planning phase. This could be seen in how the student can explain the information from question 5 and presented the problem in Cartesians plane. The student could also explain the first step, which was to look for the ellipse focus. This first step of ability is so important because, in a good understanding of comprehension, the student can also know his/her thought of solving the problem. So the awareness and regulation metacognition take apart well in the process of their learning. This could also happen because the student rechecked the answer, student could do better and realised the mistake and could fix it.

Question 6: An ellipse bridge was built above the public street. The length and height of it were 10 and 6 meters respectively. If a truck with the width 5 meters and height 5.5 meters would pass the bridge, could it pass without having any damages?

Question 6 involved the representation, reasoning and argument, planning a strategy to solve problems, the usage of symbols, formal language and technical, and the usage of mathematical operation. One of the students' answers shown in Figure 6. In Figure 6, the student did not write the full version of the given information on the question. The algorithm in solving problem was not in the correct order, and the student could not understand the question well. The student only writes the answer arbitrary, so the awareness and regulation metacognition did not apply well.

According to the answers, despite the students did not know the conceptions of conic section well. Based on the interviews done, the factors of students' mistakes were classified into two: internal and external factors. The internal factor is the physical and psychological factor. Physical factor: full schedules of college life and side job make the students lack time to rest, so their study activities were also lacking and not maximal. Psychological factor: the students did not understand the materials well and could not solve the given questions if they were kind of different than the examples. The students were not coordinative with the lecturers, so when they had any difficulties in learning Analytical Geometry, they would not let the lecturers know and preferred to do it alone. These were accumulated, and the students would still be unable to solve the problems. During the test, there were careless students and got in a hurry when they were answering the questions. They also lacked doing exercises of Analytical Geometry problems. The external factors are that some students did not have a support system of studying at home, and they were not comfortable when they studied at home.

4. Conclusion
The mistakes done by the students in solving the geometry questions were 21.76% of conceptual mistakes, 6.02% of measurement mistakes, 11.11% of algorithm mistakes, 9.26% of symbolism mistakes, 7.87% of arbitrary answers, 16.67% of incomplete answers. The factors of students' mistakes in Analytical Geometry are classified into two: internal and external factors. Internal factors are full schedules of college life and side job, the students did not understand the materials well and could not solve the given questions if they were kind of different than the examples. The students were not coordinative with the lecturers. The external factors are some students did not have a support system of studying at home, and the crowd in their homes made them feel uncomfortable.

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