Abstract

Bulk viscosity suppresses elliptic flow $v_2$, as does shear viscosity. It can thus not be neglected when extracting the shear viscosity from elliptic flow data. We here explore uncertainties in the bulk viscous contribution to viscous $v_2$ suppression that arise from presently uncontrolled uncertainties in the initial value of the bulk viscous pressure and its microscopic relaxation time.

1. Introduction

Recently, causal viscous hydrodynamics for relativistic heavy ion collisions has been experiencing rapid development. Several groups independently developed 2+1-dimensional viscous hydrodynamic codes that implement longitudinal boost invariance but allow for arbitrary dynamics in the two dimensional plane transverse to the beam [1, 2, 3, 4, 5, 6]. So far, most of these works have focussed on studying the effects caused by shear viscosity and on developing strategies for constraining the QGP shear viscosity to entropy density ratio $\eta/s$ from experimental data [1, 2, 3, 4, 5, 6, 7, 8, 9]. It was found that elliptic flow $v_2$ is very sensitive to $\eta/s$ and that, due to the rapid expansion of the fireballs created in heavy-ion collisions, even the minimal KSS bound $\eta/s = 1/4\pi$ [10] leads to a strong suppression of $v_2$ compared to the ideal fluid case [1, 2, 3, 4, 5, 6]. A first attempt by Luzum and Romatschke [5] to extract the QGP viscosity from experimental elliptic flow data, using viscous hydrodynamics simulations, indicate that $\frac{\eta}{s}_{\text{QGP}} < 5 \times \frac{1}{4\pi}$. To obtain a more precise value requires additional theoretical effort in at least the following four directions (see [11] for references): (1) connecting viscous hydrodynamics to a hadron cascade to properly account for effects from the highly viscous hadronic stage; (2) including the effects from bulk viscosity; (3) employing a more realistic equation of state (EOS) that uses the latest lattice QCD data above $T_c$ matched to a hadron resonance gas in partial chemical equilibrium below $T_c$, to properly account for chemical freeze-out at $T_{\text{chem}} \approx 165 - 170$ MeV; and (4) a better treatment of the initial conditions that not only aims to eliminate presently large uncertainties in the initial fireball eccentricity but also properly accounts for pre-equilibrium transverse flow and fluctuations in the initial fireball deformation and orientation.

In this contribution we focus on point (2) and study effects from bulk viscosity. A longer account of this work can be found in Ref. [12]. In [11], where we reported first results and to which we refer the reader for additional details, we constructed and used a function for the specific bulk viscosity $\zeta/s(T)$ that interpolates between a "minimal" value well above $T_c$, based on strong-coupling calculations using the AdS/CFT correspondence [13], to a zero value in the hadron resonance gas well below $T_c$, using a Gaussian function between these limits that peaks at $T_c$. (For a discussion of uncertainties in $\zeta/s$ below $T_c$ see [14].) We call this function "minimal bulk viscosity". To study larger bulk viscosities, we made comparison runs where this entire
function was multiplied with a coefficient $C > 1$. For twice the minimal bulk viscosity we found that the viscous $v_2$ suppression increases from 20% (for a fluid that has only minimal shear viscosity $\frac{\eta}{s} = \frac{1}{2}$) to 30% (for a fluid that additionally features bulk viscosity at twice the ”minimal” level, $C = 2$). For a given measured value of $v_2$, accounting for such a 50% relative increase in the viscous $v_2$ suppression translates into a reduction of the extracted shear viscosity $\eta/s$ by roughly a factor $\frac{3}{4}$. Consequently, bulk viscous effects cannot be ignored when extracting $\eta/s$ from experimental data.

The analysis [11] suggests that the main uncertainty stems from insufficient knowledge of the peak value of $\zeta/s$ near the phase transition [1]. However, this is only part of the story. Additional complications arise from the fact that the expected peak in $\zeta/s$ near $T_c$ is due to rapidly growing correlation lengths, associated with a ”critical slowing down” of microscopic relaxation processes near the phase transition. This probably leads near $T_c$ to a much larger relaxation time $\tau_H$ for the bulk viscous pressure than commonly used for the shear viscous pressure (which near $T_c$ is of the order of 0.5 fm/c, as obtained from kinetic theory [18,19,20] or AdS/CFT [21]). Large relaxation times can lead to strong memory effects, i.e. strong sensitivity to the initial conditions for the bulk viscous pressure. This is what we discuss here [12].

2. Setup

We use the code VISH2+1 [2,3] to solve (2+1)-dimensional transport equations for the energy momentum tensor $T^{mn}$ and the bulk viscous pressure $\Pi$:

$$d_m T^{mn} = 0, \quad T^{mn} = eu^mu^n - (p + \Pi)\Delta^{mn}, \quad (1)$$

$$D\Pi = -\frac{1}{\tau_H}(\Pi + \zeta \partial \cdot u) - \frac{1}{2}\Pi T \frac{\tau_H}{\zeta T} \left( \frac{T}{\zeta T} \right). \quad (2)$$

Here, $m, n$ denote components in $(\tau, x, y, \eta)$ coordinates, with covariant derivative $d_m$ (for details see [22]), $D = \partial^m d_m$ and $\nabla^m = \Delta^m d_l$ (where $\Delta^m = g^{mn} - u^mu^n$ is the projector transverse to the flow vector $u^m$) are the time derivative and spatial gradient in the local comoving frame, $\zeta$ is bulk viscosity, and $\tau_H$ is the corresponding relaxation time. For $\zeta(T)$ we use the phenomenological construction described in [11]. For $\tau_H$ we consider three choices: the constant values $\tau_H = 0.5$ and 5 fm/c, and the temperature dependent function $\tau_H(T) = \max[\bar{\tau} \cdot \bar{\zeta}(T), 0.1 \text{ fm/c}]$ with $\bar{\tau} = 120 \text{ fm/c}$. The last choice implements phenomenologically the concept of critical slowing down; it yields $\tau_H \approx 0.6 \text{ fm/c}$ at $T = 350 \text{ MeV}$ and $\tau_H \approx 5 \text{ fm/c}$ at $T_c$.

To study memory effects, we explore two different initializations for the bulk viscous pressure: (a) Navier-Stokes (N-S) initialization, $\Pi(\tau_0) = -\zeta \partial \cdot u$, and (b) zero initialization, $\Pi(\tau_0) = 0$. We use $\tau_0 = 0.6 \text{ fm/c}$. For all other inputs we make standard choices as discussed in Refs. [3,7] and listed in the figure below.

3. Bulk viscosity effects: uncertainties from relaxation time and bulk pressure initialization

The left panel of Fig. 1 shows the differential elliptic flow $v_2(p_T)$ of directly emitted pions (without resonance decays) for non-central Au+Au collisions at $b = 7 \text{ fm}$, calculated from ideal

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1Currently, theoretical uncertainties for the peak value of $\frac{\zeta}{s}$ near $T_c$ are very large. Extraction from lattice QCD simulations gives a peak value around 0.7 [15]. This is more than 10 time larger than the string theory prediction based on holographic models [16]. A critical discussion of the lattice QCD based extraction can be found in [17].
hydrodynamics and minimally bulk viscous hydrodynamics with identical initial and final conditions. The different lines from viscous hydrodynamics correspond to different relaxation times $\tau_\Pi$ and different initializations $\Pi(\tau_0)$. One sees that these different inputs can lead to large uncertainties for the bulk viscous $v_2$ suppression. For minimal bulk viscosity, the $v_2$ suppression at $p_T = 0.5$ GeV ranges from $\sim 2\%$ to $\sim 10\%$ compared to ideal hydrodynamics (blue dashed line in the left panel).

For the shorter relaxation time, $\tau_\Pi = 0.5$ fm/c, the bulk viscous $v_2$ suppression is insensitive to the initialization of $\Pi$, and both N-S and zero initializations show $\sim 8\%$ $v_2$ suppression relative to ideal fluids. The reason behind this becomes apparent in the right panel showing the time evolution of the average bulk pressure ($\Pi$). For short relaxation times, $\langle \Pi \rangle$ quickly loses all memory of its initial value, relaxing in both cases to the same trajectory after about $1 \sim 2$ fm/c (i.e. after a few times $\tau_\Pi$). This is similar to what we found for shear viscosity where the microscopic relaxation times are better known and short ($\tau_{\Pi}(T_c) \approx 0.2 \sim 0.5$ fm/c) and where the shear pressure tensor $\pi^{\mu\nu}$ therefore also loses memory of its initialization after about $1$ fm/c [4].

This changes if one accounts for the critical slowing down of the evolution of $\Pi$ near $T_c$. If one simply multiplies the constant relaxation time by a factor 10, setting $\tau_\Pi = 5$ fm/c, one obtains the dotted and solid magenta lines in Fig. [1] Now the bulk viscous $v_2$ suppression relative to the ideal fluid becomes very sensitive to the initialization of the bulk viscous pressure: For zero initialization $\Pi(\tau_0) = 0$, the viscous $v_2$ suppression is very small (only $\sim 2\%$ at $p_T = 0.5$ GeV/fm/c).

The right panel shows that in this case the magnitude of the (transversally averaged) bulk pressure evolves very slowly and always stays small, leading to almost ideal fluid evolution. On the other hand, if $\Pi$ is initialized with its Navier-Stokes value, which initially is large due to the strong longitudinal expansion, it decays initially more slowly than for the shorter relaxation time. Its braking effect on the flow evolution is therefore bigger, resulting in much stronger suppression of $v_2$ than for zero initialization, at $\sim 10\%$ slightly exceeding even the viscous $v_2$ suppression seen for the tenfold shorter relaxation time.

The "critical slowing down" scenario with temperature-dependent $\tau_\Pi(T)$ (black lines) interpolates between the short and long relaxation times. As for the fixed larger value $\tau_\Pi = 5$ fm/c, $v_2$ depends sensitively on the initialization of $\Pi$, but for N-S initialization the viscous $v_2$ suppression is somewhat smaller than for both short and long fixed relaxation times. The reasons for this are
subtle since now, at early times, the bulk viscous pressure $\Pi$ evolves on very different time scales in the dense core and dilute edge regions of the fireball. As a result, for N-S initialization the average value $\langle \Pi \rangle$ is smaller in magnitude than for both short and long fixed $\tau_{\Pi}$, throughout the fireball evolution (right panel, black lines).

4. Conclusions

Relaxation times and initial values for the dissipative flows (bulk and shear pressure) are required inputs in viscous hydrodynamic calculations, in addition to the transport coefficients and the EOS. Near $T_c$, the bulk viscosity $\zeta$ can exceed the shear viscosity $\eta$ of the strongly interacting matter. If the relaxation time $\tau_{\Pi}$ for the bulk viscous pressure $\Pi$ is short, it quickly loses memory of its initial value, but the relatively large peak value of $\zeta/s$ near $T_c$ can lead to a significant viscous suppression of the elliptic flow $v_2$, competing with shear viscous effects. If $\tau_{\Pi}$ grows rapidly near $T_c$, due to critical slowing down, the bulk viscous suppression effects on $v_2$ depend crucially on the initial value of $\Pi$: If $\Pi$ is zero initially, bulk viscous effects on $v_2$ are almost negligible; if $\Pi$ is initially large, however, as for the case of the N-S initialization, it remains relatively large throughout the evolution, suppressing the buildup of elliptic flow at a level that again competes with shear viscous effects. Additional research on initial conditions and relaxation times for the bulk viscous pressure is therefore necessary for a quantitative extraction of $\eta/s$ from measured data.

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