Consistent Higher-Order Corrections to $\tilde{t}_i \rightarrow \tilde{b}_j H^+$ in the Complex MSSM

Sven Heinemeyer*

Instituto de Física de Cantabria (CSIC-UC), Santander, Spain
E-mail: Sven.Heinemeyer@cern.ch

Heidi Rzehak

Institut für Theoretische Physik, Karlsruhe Institute of Technology, D–76128 Karlsruhe, Germany
Albert-Ludwigs-Universität Freiburg, Physikalisches Institut, D–79104 Freiburg, Germany
E-mail: heidi.rzehak@physik.uni-freiburg.de

Christian Schappacher

Institut für Theoretische Physik, Karlsruhe Institute of Technology, D–76128 Karlsruhe, Germany
E-mail: cs@particle.uni-karlsruhe.de

We review an analysis of a consistent renormalization of the top and bottom quark/squark sector of the MSSM with complex parameters (cMSSM). Various renormalization schemes are defined, analyzed analytically and tested numerically in the decays $\tilde{t}_2 \rightarrow \tilde{b}_i H^+ / W^+ (i = 1, 2)$. No scheme is found that produces numerically acceptable results over all the cMSSM parameter space, where problems occur mostly already for real parameters. Some numerical examples for $\Gamma(\tilde{t}_2 \rightarrow \tilde{b}_1 H^+)$ in our preferred scheme, "$m_b, A_b \text{DR}^\ast"$ are shown.

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*Speaker.
1. Introduction

One of the main tasks of the LHC is to search for Supersymmetry (SUSY) [1]. The Minimal Supersymmetric Standard Model (MSSM) predicts two scalar partners for all Standard Model (SM) fermions as well as fermionic partners to all SM bosons. Of particular interest are the scalar partners of the heavy SM quarks, the scalar top quarks, $\tilde{t}_i$ ($i = 1, 2$) and scalar bottom quarks $\tilde{b}_j$ ($j = 1, 2$) due to their large Yukawa couplings. Depending on the SUSY mass patterns, possibly important decay modes of the scalar tops are,

$$\tilde{t}_i \rightarrow \tilde{b}_j H^+ \quad (i, j = 1, 2), \quad (1.1)$$

$$\tilde{t}_i \rightarrow \tilde{b}_j W^+ \quad (i, j = 1, 2), \quad (1.2)$$

where $H^+$ denotes the (positively) charged MSSM Higgs boson. These processes can constitute a large part of the total stop decay width, and, in case of decays to a Higgs boson, they can serve as a source of charged Higgs bosons in cascade decays at the LHC.

For a precise prediction of the partial decay widths corresponding to Eq. (1.1) and Eq. (1.2), at least the one-loop level contributions have to be taken into account. This in turn requires a renormalization of the relevant sectors, especially a simultaneous renormalization of the top and bottom quark/squark sector. Due to the $SU(2)_L$ invariance of the left-handed scalar top and bottom quarks, these two sectors cannot be treated independently. Within the framework of the MSSM with complex parameters (cMSSM) we review the analysis of various bottom quark/squark sector renormalization schemes [2], while for the top quark/squark sector a commonly used on-shell renormalization scheme is applied throughout all the investigations. An extensive list of earlier analyses and corresponding references can be found in Ref. [2]. The evaluation of the partial decay widths of the scalar top quarks are being implemented into the Fortran code FeynHiggs [3–6].

2. The bottom/sbottom sector and its renormalization

2.1 The generic structure

The bilinear part of the Lagrangian with top and bottom squark fields, $\tilde{t}$ and $\tilde{b}$,

$$\mathcal{L}_{\tilde{t}/\tilde{b}} \text{ mass} = - \left( \tilde{t}_L \cdot \tilde{t}_R \right) M_{\tilde{t}} \left( \tilde{t}_L \right) - \left( \tilde{b}_L \cdot \tilde{b}_R \right) M_{\tilde{b}} \left( \tilde{b}_L \right), \quad (2.1)$$

contains the stop and sbottom mass matrices $M_{\tilde{t}}$ and $M_{\tilde{b}}$, given by

$$M_q = \begin{pmatrix} M_{\tilde{Q}_L}^2 + m_q^2 + M_Z^2 c_2 \beta (Q_q^3 - Q_q^2 w) & m_q X_q^+ \\ m_q X_q & M_{\tilde{q}_a}^2 + m_q^2 + M_Z^2 c_2 \beta Q_q^2 w \end{pmatrix}, \quad (2.2)$$

with $X_q = A_q - \mu^* \kappa$ and $\kappa = \{ \cot \beta, \tan \beta \}$ for $q = \{ t, b \}$. $M_{\tilde{Q}_L}^2$ and $M_{\tilde{q}_a}^2$ are the soft SUSY-breaking mass parameters. $m_q$ is the mass of the corresponding quark. $Q_q$ and $T_q^3$ denote the charge and the isospin of $q$, and $A_q$ is the trilinear soft SUSY-breaking parameter. The mass matrix can be diagonalized with the help of a unitary transformation $U_q$,

$$D_q = U_q M_q U_q^+ = \begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix}, \quad U_q = \begin{pmatrix} U_{q_{11}} & U_{q_{12}} \\ U_{q_{21}} & U_{q_{22}} \end{pmatrix}. \quad (2.3)$$
The scalar quark masses, $m_{\tilde{q}_1}$ and $m_{\tilde{q}_2}$, will always be mass ordered, i.e. $m_{\tilde{q}_1} \leq m_{\tilde{q}_2}$:

$$m_{\tilde{q}_{12}}^2 = \frac{1}{2} \left( M_{\tilde{q}_1}^2 + M_{\tilde{q}_2}^2 \right) + m_{\tilde{q}}^2 + \frac{1}{2} T_q M_Z^2 c_2 \beta \right) + \frac{1}{2} \left( M_{\tilde{q}_1}^2 - M_{\tilde{q}_2}^2 + M_Z^2 c_2 \beta (T_q^3 - 2 Q_q s_w^2) \right)^2 + 4 m_{\tilde{q}}^2 |X_q|^2. \quad (2.4)$$

### 2.2 Renormalization of the bottom/sbottom sector

The field renormalization constants of the bottom/sbottom (as well as of the top/stop) sector are chosen according to an on-shell prescription [2].

The parameter renormalization can be performed as follows, 

$$M_{\tilde{q}} \rightarrow M_{\tilde{q}} + \delta M_{\tilde{q}} \quad (2.5)$$

which means that the parameters in the mass matrix $M_{\tilde{q}}$ are replaced by the renormalized parameters and a counterterm. After the expansion $\delta M_{\tilde{q}}$ contains the counterterm part,

$$\delta M_{\tilde{q}_{11}} = \delta M_{\tilde{q}_1}^2 + 2 m_{\tilde{q}} \delta m_{\tilde{q}} - M_Z^2 c_2 \beta Q_q \delta \tilde{s}_w^2 + (T_q^3 - Q_q s_w^2) (c_2 \beta \delta M_Z^2 + M_Z^2 \delta c_2 \beta), \quad (2.6)$$

$$\delta M_{\tilde{q}_{12}} = (A_q^1 - \mu \kappa) \delta m_{\tilde{q}} + m_{\tilde{q}} (\delta A_q^1 - \mu \delta \kappa - \kappa \delta \mu), \quad (2.7)$$

$$\delta M_{\tilde{q}_{12}} = \delta M_{\tilde{q}_{12}}^2 + 2 m_{\tilde{q}} \delta m_{\tilde{q}} + M_Z^2 c_2 \beta Q_q \delta \tilde{s}_w^2 + Q_q s_w^2 (c_2 \beta \delta M_Z^2 + M_Z^2 \delta c_2 \beta). \quad (2.9)$$

Another possibility for the parameter renormalization is to start out with the physical parameters which corresponds to the replacement:

$$U_q M_{\tilde{q}} U_q^\dagger \rightarrow U_q M_{\tilde{q}} U_q^\dagger + U_q \delta M_{\tilde{q}} U_q^\dagger = \left( \begin{array}{cc} m_{\tilde{q}_1}^2 & Y_q \\ Y_q^* & m_{\tilde{q}_2}^2 \end{array} \right) + \left( \begin{array}{cc} \delta m_{\tilde{q}_1}^2 & \delta Y_q \\ \delta Y_q^* & \delta m_{\tilde{q}_2}^2 \end{array} \right), \quad (2.10)$$

where $\delta m_{\tilde{q}_1}^2$ and $\delta m_{\tilde{q}_2}^2$ are the counterterms of the squark masses squared. $\delta Y_q$ is the counterterm to the squark mixing parameter $Y_q$ (which vanishes at tree level, $Y_q = 0$), and corresponds to the off-diagonal entries in $D_q = U_q M_{\tilde{q}} U_q^\dagger$, see Eq. (2.3). Using Eq. (2.10) one can express $\delta M_{\tilde{q}}$ by the counterterms $\delta m_{\tilde{q}_1}^2$, $\delta m_{\tilde{q}_2}^2$, and $\delta Y_q$. Especially for $\delta M_{\tilde{q}_{12}}$ one yields

$$\delta M_{\tilde{q}_{12}} = U_q \delta m_{\tilde{q}_1}^2 U_{\tilde{q}_{12}}^2 + U_q \delta m_{\tilde{q}_2}^2 U_{\tilde{q}_{12}} + U_q \delta Y_q U_{\tilde{q}_{12}}^2 + U_q \delta Y_q^* U_{\tilde{q}_{12}}^2. \quad (2.11)$$

For the top/stop sector we use an on-shell renormalization, see e.g. Refs. [2, 7, 8]. The various options to renormalize the bottom/sbottom sector are listed in Tab. [I].

### 2.3 Summary of the renormalization scheme analysis

A bottom quark/squark sector renormalization scheme always contains dependent counterterms which can be expressed by the independent ones. According to our six definitions, these dependent parameters can be $\delta m_{\tilde{b}}$, $\delta A_b$ or $\delta Y_b$. A problem can occur when the MSSM parameters are chosen such that the independent counterterms (nearly) drop out of the relation determining the

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\[1\] The unitary matrix $U_q$ can be expressed by a mixing angle $\theta_q$ and a corresponding phase $\varphi_q$. Then the counterterm $\delta Y_q$ can be related to the counterterms of the mixing angle and the phase (see Ref. [7]).
dependent counterterms. This can lead to (unphysically) large counterterm contributions in such a case. As it was shown in Ref. [2] it is possible already in very generic SUSY scenarios to find a set of MSSM parameters which show this behaviour for each of the chosen renormalization schemes. Consequently, it appears to be difficult by construction to define a renormalization scheme for the bottom quark/squark sector (once the top quark/squark sector has been defined) that behaves well for the full MSSM parameter space. One possible exception could be a pure DR scheme, which, however, is not well suited for processes with external top squarks and/or bottom squarks.

The analytical and numerical analysis performed in Ref. [2] identified RS2 as “preferred scheme”. This scheme showed the “relatively most stable” behavior, problems only occur for maximal sbottom mixing, |U_{\tilde{b}_1}| = |U_{\tilde{b}_2}|, where a divergence in \delta Y_b appears. On the other hand, other schemes with \delta m_b or \delta A_b as dependent counterterms generally exhibit problems in larger parts of the parameter MSSM space and may induce large effects, since m_b (or the bottom Yukawa coupling) and A_b enter prominently into the various couplings of the Higgs bosons to other particles.

### 3. Numerical Example

In this section we show some example results for \Gamma(\tilde{t}_2 \rightarrow \tilde{b}_1 H^+) [2]. This decay mode can serve potentially as a source of charged MSSM Higgs bosons in SUSY cascade decays. The parameters are chosen according to the two scenarios S1 and S2 as defined in Tab. 2.

In Fig. 1 we show the partial decay width \Gamma(\tilde{t}_2 \rightarrow \tilde{b}_1 H^+) as a function of tan \beta (upper left), as a function of A_b (upper right), as a function of \mu (lower left) and as a function of \phi_{A_b} (lower right plot). “tree” denotes the tree-level value and “full” is the decay width including all one-loop corrections (including hard QED and QCD radiation, see Ref. [2] for details). For S1 the grey region is excluded and for S2 the dark grey region is excluded. The spikes and dips visible in the lower left plot are due to various particle thresholds, while the first dip in S1 is due to |U_{\tilde{b}_1}| \approx |U_{\tilde{b}_2}|.

[2] Corrections from imaginary parts of external leg self-energy contributions [10] are not included.
Table 2: MSSM parameters for the initial numerical investigation; all parameters are in GeV. We always set $m_{\tilde{t}}^{\text{min}}(m_t) = 4.2$ GeV. In our analysis we use $M_{\tilde{Q}_L}(\tilde{t}) = M_{\tilde{R}_L} = M_{\tilde{B}_R} = M_{\text{SUSY}}$, where $M_{\text{SUSY}}$ is chosen such that the above value of $m_{\tilde{t}}^2$ is realized. The parameters entering the scalar lepton sector and/or the first two generations do not play a relevant role in our analysis. The values for $A_t$ and $A_b$ are chosen such that charge-or color-breaking minima are avoided.

| Scen | $M_{H^+}$ | $m_{\tilde{t}}$ | $\mu$ | $A_t$ | $A_b$ | $M_1$ | $M_2$ | $M_3$ |
|------|-----------|----------------|-------|-------|------|-------|-------|-------|
| S1   | 150       | 600            | 200   | 900   | 400  | 200   | 300   | 800   |
| S2   | 180       | 900            | 300   | 1800  | 1600 | 150   | 200   | 400   |

Figure 1: $\Gamma(\tilde{t}_2 \rightarrow \tilde{b}_1 H^+)$. Tree-level and full one-loop corrected partial decay widths for the renormalization scheme RS2. The parameters are chosen according to the scenarios S1 and S2. For S1 the grey region is excluded and for S2 the dark grey region is excluded. Upper left plot: $\tan\beta$ varied. Upper right plot: $\tan\beta = 20$ and $|A_b|$ varied. Lower left plot: $\tan\beta = 20$ and $|\mu|$ varied. Lower right plot: $\tan\beta = 20$ and $\varphi_{A_b}$ varied.
The two spikes in the lower right plot are also due to $|U_{h_1\tilde{b}_1}| \approx |U_{h_1\tilde{b}_2}|$, which leads to a divergence in RS2, which, however, is confined to very narrow intervals. The loop corrections, as can be observed in all four plots, are relatively modest, staying below $\sim 25\%$ for all parameters. The fact of relatively small one-loop corrections shows that no unphysically large contributions via large counterterms are introduced, a characteristic of a suitable renormalization scheme.

The real quantity of interest at the LHC is the BR($\tilde{t}_2 \rightarrow \tilde{b}_1 H^+$). This, however, requires the evaluation of all decay modes (at the same level of accuracy). The corresponding results will be presented elsewhere [11].

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