Noninteracting Fields in the Higher Dimensional
Superconducting Cosmic String Field Source Model

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Abstract

In this paper we investigate the freely propagating fields behind the results in [6]. In [6] we calculated the magnetic dipole moment of the muon and the electric dipole moments of the muon, electron and the neutron (in a simple quark model) to first order in loop corrections in both $S_1$ and $S_1 \setminus \mathbb{Z}_2$. In these calculations in [6] we investigated the effect of fields possibly generated by higher dimensional superconducting cosmic strings [4] that interact with the charged fields on the manifold. In comparing the results in [6] with standard model precision tests for the electric and magnetic dipole moments of the various fermions in the model, we were able to obtain upper limits on the compactification size as well as an upper limit for the new $b$ parameter. Recall that in [5] we presented the theory for [6] in $M_4 \otimes S_1 \setminus \mathbb{Z}_2$ where, in [6], we allowed for external magnetic fields, that could be produced by light charged particles traveling near superconducting cosmic strings, to permeate the extra dimensional space. These fields affected the charged particles in the model resulting in a novel mechanism for parity violation in QED processes along with other phenomenological and theoretical implications [6].

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I. INTRODUCTION

For very large times, a fully interactive EW model in $M_4 \otimes S_1 \backslash Z_2$ will look like a series of field modes propagating on their own without any interactions. In our full model, what are the exact expressions for these freely propagating fields and more importantly, what are their masses as a result of the external magnetic fields that will have a flux associated with the geometry in [5]? These questions are important phenomenologically. To answer these questions we will first have to write out the Lagrangian density for our system up to quadratic order in the fields. Then we will integrate out the extra coordinate by forming the action for the model. With our effective 4-D Lagrangian density in terms of the field modes, we will then have to, rather carefully, redefine the gauge fields such that all interaction terms are eliminated up to quadratic order. The mass terms can then simply be determined from the quadratic terms of these freely propagating fields from our effective 4-D model. There will also be unwanted 5th components of the gauge fields, however these do not couple to any of the other fields in the model due to the orbifold geometry in [5].

In Sec. II the general theory of the model in [5] will be summarized. In Sec. III we will write the 5-D Lagrangian density out to quadratic order and then factor the fields in a convenient way. In Sec. IV the extra coordinate will be integrated out and further factoring of the result to make the field redefinitions more transparent. Finally in Sec. V we will present the expressions for the freely propagating fields and their masses for the model followed by a brief discussion of the results. The unphysical 5th components of the gauge fields for the model were dealt with in [5] using an orbifold geometry and thus do not couple to any of the other fields in the model.

II. SUMMARY OF THE GENERAL THEORY

In the full theory we originally had

$$\mathcal{L} = (D_A\phi)^\dagger(D^A\phi) - \frac{1}{2} Tr(F_{AB}F^{AB}) - \frac{1}{4} f_{AB} f^{AB} + \mu^2 \phi^\dagger \phi - \frac{\lambda}{2} (\phi^\dagger \phi)^2$$ (1)
where $A = 0, 1, 2, 3, 5$ with each field being a function of $x^\mu, y$ where $y$ is the extra coordinate in our system. Here the metric assumed is

$$g^{AB} = \begin{cases} 
0 & \text{if } A \neq B, \\
-1 & \text{if } A = B = 1, 2, 3, 5, \\
1 & \text{if } A = B = 0. 
\end{cases} \quad (2)$$

Then with the external flux,

$$\phi \xrightarrow{\text{flux}} B\phi \quad (3)$$

where $B = \begin{pmatrix} e^{iby/R} & 0 \\ 0 & 1 \end{pmatrix}$. We also had

$$W_A \xrightarrow{\text{flux}} BW_AB^\dagger. \quad (4)$$

It then followed that

$$\text{Tr}(F_{\mu\nu}F^{\mu\nu}) \xrightarrow{\text{flux}} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) \quad (5)$$

and

$$\text{Tr}(F_{\mu5}F^{\mu5}) \xrightarrow{\text{flux}} \frac{1}{2} F_{\mu5}^3 F^{3\mu5} - (\partial_\mu W_5^- - \partial_y W_{5^-}^- + \frac{ib}{R} W_{5^-}^-)(\partial^\mu W_5^+ - \partial_y W_{5^+}^+ - \frac{ib}{R} W_{5^+}^+)$$

$$+ \text{(cubic and quartic terms)}. \quad (6)$$

Finally with the field redefinitions required to solve the degrees of freedom problem in the general case (the $W$’s pick up a mass from the external flux before the Higg’s mechanism),

$$\tilde{W}_\mu^- = W_\mu^- - \Lambda \partial_\mu W_5^- , \quad (7)$$

$$\tilde{W}_\mu^+ = W_\mu^+ - \beta \partial_\mu W_5^+ , \quad (8)$$

we then had

$$\text{Tr}(F_{\mu5}F^{\mu5}) \xrightarrow{\text{flux}} \frac{1}{2} F_{\mu5}^3 F^{3\mu5} - (\partial_\mu \tilde{W}_\mu^- - \partial_y \tilde{W}_\mu^-^- + \frac{ib}{R} \tilde{W}_\mu^-^-)(\partial^\mu \tilde{W}_\mu^+ - \partial_y \tilde{W}_\mu^+^+ + \frac{ib}{R} \tilde{W}_\mu^+^+)$$

$$+ \text{(cubic and quartic terms)}. \quad (9)$$

Here $\Lambda = (\partial_y - \frac{ib}{R})^{-1}$ and $\beta = (\partial_y + \frac{ib}{R})^{-1}$. For the details of the general theory, please see [5].
III. FREELY PROPAGATING FIELDS

Now we want to look at our system for very large times, in which the fields should all propagate by themselves with no interactions taking place. In this limit, we need only look at the quadratic pieces of our model and then ask ourselves, what are the freely propagating fields and what are the masses for these fields? To answer this question, we have

\[
\mathcal{L} = (D_A \varphi)^\dagger (D^A \varphi) - \frac{1}{2} Tr(F_{AB} F^{AB}) - \frac{1}{4} f_{AB} f^{AB} + \mu^2 \varphi^\dagger \varphi - \frac{\lambda}{2} (\varphi^\dagger \varphi)^2
\]

\[
= \phi^\dagger \left[ \partial_\mu + ig \tilde{W}_\mu + ig \partial_\mu T + \frac{i}{2} g' B_\mu \right] \left[ \partial^\mu - ig \tilde{W}^\mu - ig \partial^\mu T - \frac{i}{2} g' B^\mu \right] \phi
\]

\[- \phi^\dagger \left[ \partial_\mu + (\partial_\mu B^\dagger) B + ig W_5 + \frac{i}{2} g' B_5 \right] \left[ \partial^\mu + B^\dagger (\partial_\mu B) - ig W_5 - \frac{i}{2} g' B_5 \right] \phi
\]

\[- \frac{\lambda}{2} (\varphi^\dagger \varphi)^2 + \text{(cubic and quartic terms)}.
\]

In terms of the vacuum expectation value \( v \) we then let

\[
\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ v + \phi_2 \end{pmatrix},
\]

which then gives, after a considerable amount algebra,

\[
\mathcal{L} = \frac{g^2 v^2}{4} \left[ \tilde{W}^{\mu+} + \frac{i \sqrt{2}}{g \nu} \partial^\mu \phi_1 + \beta (\partial^\mu W_5^+) \right] \left[ \tilde{W}^- - \frac{i \sqrt{2}}{g \nu} \partial_\mu \phi_1 + \Lambda (\partial_\mu W_5^-) \right]
\]

\[+ \frac{v^2}{8} (g^2 + g'^2) \left[ Z^\mu - \frac{2i}{v \sqrt{g^2 + g'^2}} (\partial^\mu \phi_2) \right] \left[ Z_\mu + \frac{2i}{v \sqrt{g^2 + g'^2}} (\partial_\mu \phi_2^*) \right]
\]

\[- \frac{1}{2} \left[ \partial_\nu \phi_1 + \frac{ib}{R} \phi_1 - ig \nu \sqrt{2} W_5^+ \right] \left[ \partial_\nu \phi_1^* - \frac{ib}{R} \phi_1^* + ig \nu \sqrt{2} W_5^- \right]
\]

\[- \frac{v^2}{8} (g^2 + g'^2) \left[ Z_5 - \frac{2i}{v \sqrt{g^2 + g'^2}} (\partial_5 \phi_2) \right] \left[ Z_5 + \frac{2i}{v \sqrt{g^2 + g'^2}} (\partial_5 \phi_2^*) \right]
\]

\[- (\partial_\mu \tilde{W}^- - \frac{ib}{R} \tilde{W}_\mu^- (\partial_\mu \tilde{W}^{\mu+} + \frac{ib}{R} \tilde{W}^{\mu+}) - \frac{1}{2} F_{\mu 5}^3 F^{3 \mu 5} - \frac{1}{2} Tr(F_{\mu \nu} F^{\mu \nu}) - \frac{1}{4} f_{AB} f^{AB}
\]

\[+ \mu^2 \varphi^\dagger \varphi - \frac{\lambda}{2} (\varphi^\dagger \varphi)^2 + \text{(cubic and quartic terms)}.
\]
IV. THE FIELD MODES

Now let us integrate out the extra coordinate, keeping in mind that for the gauge fields the modes are the same fields as the anti modes. This is because equation (1) is invariant under parity even for nonzero flux, then it must also be parity invariant when we integrate out the extra coordinate and this condition forces the gauge field modes to be the same fields as the anti modes. If we add the QED portions, then the model becomes parity violating for nonzero flux. Please note that $\beta \rightarrow \beta_n = -\frac{iR}{n+b}$ and $\Lambda \rightarrow \Lambda_n = \frac{iR}{n+b}$ once we integrate out the modes. Then we have, after a considerable amount of factoring,

$$\mathcal{L}_{\text{effective}} = \sum_{n = -\infty}^{\infty} \left\{ \frac{1}{2} \left[ \frac{g^2 \nu^2}{2} + \frac{(n + b)^2}{R^2} \right] \left[ \tilde{W}_{n}^+ \mathcal{W}^+_n \right. \right.$$

$$\left. + \frac{i g^2 \nu}{\sqrt{2}} \frac{\partial^\nu \phi_{1, n}^*}{\sqrt{2}} + \frac{g^2 \nu^2}{2} \lambda_n (\partial \nu W_{5, n}^-) \right] \right.$$

$$\left. + \frac{i g^2}{\sqrt{2}} \beta_n \partial \nu W_{5, n}^+ \right\}$$

$$- \frac{1}{2} \frac{(n + b)^2}{R^2} (\phi_{1, n} - \frac{i g^2}{\sqrt{2}} \beta_n W_{5, n}^-) (\partial \nu \phi_{1, n} + \frac{i g^2}{\sqrt{2}} \lambda_n \partial \nu W_{5, n}^-)$$

$$+ \frac{1}{2} (\partial \nu \phi_{2, n} - \frac{i g^2}{\sqrt{2}} \beta_n \partial \nu W_{5, n}^+) (\partial \nu \phi_{2, n}^* + \frac{i g^2}{\sqrt{2}} \lambda_n \partial \nu W_{5, n}^-) + \frac{v^2}{8} (g^2 + g'^2) \left[ Z_{5, n} + \frac{2n}{v R \sqrt{g^2 + g'^2}} \partial \nu \phi_{2, n} \right]$$

$$+ \frac{2n}{v R \sqrt{g^2 + g'^2}} \partial \nu \phi_{2, n}^* \right\}$$

Please note that

$$F_{\mu 5, n} F_{5, n}^- = - (\partial \nu W_{5, n}^3 - \frac{i n}{R} W_{5, n}^3) (\partial \nu W_{5, n}^- + \frac{i n}{R} W_{5, n}^-)$$

and

$$f_{\mu 5, n} f_{5, n}^- = - (\partial \nu B_{5, n} - \frac{i n}{R} B_{5, n}) (\partial \nu B_{5, n} + \frac{i n}{R} B_{5, n}).$$
V. FIELD REDEFINITIONS

Let us define the following fields:

\[ \tilde{Z}_{5,n} = Z_{5,n} + \frac{2n}{vR \sqrt{g^2 + g'^2}} \text{Re}\phi_{2,n}, \]  
\[ \tilde{Z}_{\mu,n} = Z_{\mu,n} + \frac{2}{v \sqrt{g^2 + g'^2}} \text{Im}(\partial_\mu \phi_{2,n}), \]  
\[ W'_{\mu} = \tilde{W}'_{\mu} + \frac{igv}{\sqrt{2}}(\partial^\mu \phi_{1,n}) + \frac{g^2}{2} \beta_n (\partial^\mu W_{5,n}^+), \]  
\[ \chi_n = \phi_{1,n} - \frac{igv}{\sqrt{2}} \beta_n W_{5,n}^+. \]

With these new field definitions we can diagonalize the effective 4-D Lagrangian density up to quadratic order. Substituting these field redefinitions, which are the freely propagating fields for the model, we then have finally

\[
\mathcal{L}_{\text{effective}} = \sum_{n=-\infty}^{\infty} \left\{ \frac{1}{2} \left( \frac{g^2 v^2}{2} + \frac{(n+b)^2}{R^2} \right) W'_{\mu,n} W'^{-\mu}_{\mu,n} 
+ \frac{1}{2} (\partial_\mu \chi_n)(\partial^\mu \chi_n^*) (1 - \frac{g^2 v^2}{2(\frac{g^2 v^2}{2} + \frac{(n+b)^2}{R^2})}) - \frac{1}{2} (\frac{n+b}{R^2})^2 (\chi_n)(\chi_n^*) + \frac{1}{2} (\partial_\mu h_{-n})(\partial^\mu h_{-n}) 
- \frac{1}{2} (4\mu^2) h_{-n} h_{-n} + \frac{v^2}{8}(g^2 + g'^2) \tilde{Z}_{\mu,n} \tilde{Z}_n^\mu + \frac{v^2}{8}(g^2 + g'^2) Z_{5,n} \tilde{Z}_n^5 
- \frac{1}{2} \frac{n^2}{R^2} l_{-n} l_n - \frac{1}{2} F_{\mu,\nu,n}^3 F_{\nu,\mu,n}^3 - \frac{1}{2} Tr(F_{\mu,\nu,n} F_{\nu,\mu,n}) - \frac{1}{2} f_{\mu,\nu,n} f_{\nu,\mu,n}^5 
- \frac{1}{4} f_{\mu,\nu,n} f_{\nu,\mu,n}^5 \} + \text{(cubic and quartic terms)}. \]

Here \( h = \text{Re}(\phi_2) \) and \( l = \text{Im}(\phi_2) \). Notice that \( \frac{1}{2} F_{\mu,5,n}^3 F_{\nu,5,n}^3, \frac{1}{2} Tr(F_{\mu,\nu,n} F_{\nu,\mu,n}^5) \) and \( \frac{1}{4} f_{\mu,\nu,n} f_{\mu,\nu,n}^5 \) are invariant under the above transformations up to quadratic order when cast in the \( Z \) and \( \tilde{W} \) basis.

Reading off the mass terms for the modes from equation (15) we have

\[ m_{W'_n} = \sqrt{\frac{g^2 v^2}{2} + \frac{(n+b)^2}{R^2}} \]  
\[ m_{Z_n} = \sqrt{\frac{v^2 (g^2 + g'^2)}{4} + \frac{n^2}{R^2}} \]  
\[ m_{A_n} = \frac{|n|}{R} \]
\[ m_{Z_{5,n}} = \frac{v}{2} \sqrt{g^2 + g'^2} \]  
\[ m_{h_n} = \sqrt{4\mu^2 + \frac{n^2}{R^2}} \]  
\[ m_{\chi_n} = \left| \frac{n + b}{R} \right| . \]  

The \( Z_{5,n} \) does not couple to the other modes due to the orbifold geometry and is thus phenomenologically absent from the diagrams involving the other fields in the model \cite{5}. The mode dependence for \( m_{Z_n} \) and \( m_{A_n} \) comes from \(-\frac{1}{2} F_{\mu5,n}^3 F_{\rho\mu5} - \frac{1}{2} f_{\mu5,n} f_{\rho5} \). Notice that there is a mass term for \( l_n \), but this is not really a mass term because this field does not have a kinetic term associated with it and thus it carries no degrees of freedom of motion. It does however act as a constraint to the system such that it imparts an additional contribution to the mass of \( m_{h_n} \).

To see this, simply write \( h_n \) and \( l_n \) as a linear combination of two new fields such that there are no cross terms for the field derivatives. These new fields will couple to one another and the equation of motion for one of the fields will impart an additional mass contribution to the other field. This is where the mode dependence for \( m_{h_n} \) comes from (note that we have not written the final result in terms of the new fields for convenience, it does not matter what we call these fields anyway). The remaining unwanted mass term vanishes in the zero mode limit. It is also clear that in the zero mode and zero flux limit, equation (15) and the mass terms reduce to the standard model results once the fifth component for each gauge field is orbifolded away \cite{5}. It should be noted however that the field redefinition of \( \tilde{W}_{n}' \) is undefined in the zero mode limit when \( b = 0 \) just as in the general case \cite{5}.

[1] For an early study of universal extra TeV scale dimensions see I. Antoniadis, Phys. Lett. B 246 (1990) 377.
[2] For extensive recent introductions and reviews to both UED dimensions and ones where only gravity propagates into the bulk see K. Hwang and J. E. Kim, arXiv:hep-ph/0411286 and G. D. Kribs, arXiv:hep-ph/0605325.
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