Residual stress determination using full-field optical methods

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Abstract
Residual stresses are created in engineering components during fabrication and processing. Such stresses can strongly influence structural behavior. They are generally found by experimental means. A widely used way of finding residual stresses is removal of a small volume of material containing stresses and measurement of the strains that develop in surrounding material as a result of stresses being released. The strains can then be used to compute residual stresses. Drilling a small shallow hole is the most common way of implementing this approach, with strains measured by nearby strain gages adhered to the surface. This paper provides an overview of how full-field optical methods can be used instead of strain gages with hole drilling, overcoming limitations associated with gages and expanding capabilities of the hole drilling approach. The methods considered are holographic and electronic speckle pattern interferometry, Moire interferometry and digital image correlation. Advantages of using optical methods to find residual stresses are shown. A variety of applications is presented, ranging from determination of stresses in underground piping to stresses in microscale specimens. In addition, optical approaches employing different ways of material removal for stress release are reviewed, as well as several non-destructive optical methods for determining residual stresses.

1. Introduction
Materials often experience stresses from externally applied loadings, but stresses can also exist in the absence of such loadings, known as residual stresses. They are generated during the forming, processing, and joining of engineering materials, for example by machining, surface treatments (e.g. shot peening), welding, and phase transformations. They are even found in biological structures such as arteries [1]. Residual stresses can be large, sometimes approaching yield strength in metallic materials. They exist over a broad range of sizes, from civil engineering structures to nano structures.

Knowledge of the magnitude and distribution of residual stresses is important to assess their effect on resistance to fatigue cracking, dimensional stability, and other important aspects of structural behavior. The prediction of residual stresses by computational modeling can be difficult owing to the complexity of processes that create those stresses, and experimental determinations of residual stresses are generally needed.

Information on how residual stresses vary with depth below a surface is frequently of interest. X-ray diffraction [2] combined with layer removal and hole drilling [3, 4] are two widely used methods for obtaining that information. In the latter method, a small (often ≈1.6 mm dia.) square bottomed hole is drilled into the surface to a shallow depth (often about half of the hole dia.). Strains develop near the hole as it releases residual stresses in the material removed by the hole. Residual stresses are then obtained from measured strains by a well-established computational methodology [3]. The hole drilling method is typically carried out by adhering a special type of strain gage rosette to the surface, depicted in figure 1, along with a guide to center the drilling of a hole in the rosette. To find how residual stresses vary with depth, a hole can be drilled in increments of depth and strains recorded as function of depth. Residual stresses vs. depth can then be computed from the strains [3, 4].

Use of strain gage rosettes restricts the hole drilling method to regions large enough to accommodate the setup shown in figure 1, provides strain data from just a few gage locations per rosette, and requires...
considerable time for installation. The primary purpose of this paper is to show how various full-field optical methods [5, 6] can overcome those drawbacks and expand potential applications of hole drilling as a valuable method for determining residual stresses. Another purpose is to provide an overview of optical methods that use different forms of stress release by material removal, as well as a number of non-destructive optical methods for finding residual stresses.

2. Holographic interferometry and hole drilling

2.1. Holographic interferometry

Holographic interferometry [7–11] enables the measurement of surface displacements on the order of tens of nanometers to tens of microns. Referring to figure 2(a), light from a laser illuminates the surface of a test object and is scattered towards a location where a hologram will be generated. Reference light also illuminates the hologram location. The reference and scattered object light interfere at the hologram location. The resulting intensity distribution can be recorded by analog means using a photographic plate, or more rapidly and conveniently using thermoplastic [10] or photorefractive materials [11].

After a hologram is recorded, suppose that it is illuminated with reference light. A reconstructed image of the test object can be seen through the hologram, even if the object is removed. Suppose that the object stays in place and is also re-illuminated. If the surface of the object deforms microscopically, the reconstructed light and light scattered from the object combine to produce optical interference fringes appearing on the surface of the object as viewed through the hologram.

Examples of fringes are shown in figures 2(b) and (c). Each fringe represents a phase difference $\Delta \varphi = 2\pi$ relative to an adjacent fringe. Phase difference data can be used to find surface displacements. As an illustration, figure 2(b) shows fringes resulting from in-plane displacements (i.e. in the plane of the figure). In that case, $\Delta \varphi = (2\pi/\lambda) d_{in}$, where $\lambda$ is the wavelength of laser light, $d_{in}$ is in-plane, fringe-to-fringe displacement, and angle $\alpha = 0^\circ$ in figure 2(a). Fringes from out-of-plane displacements (i.e. normal to the plane of the figure) with $\alpha = 90^\circ$ are shown in figure 2(c) and satisfy $\Delta \varphi = (4\pi/\lambda) d_{out}$ with $\Delta \varphi = 2\pi$. The out-of-plane, fringe-to-fringe displacement $d_{out}$ is half of that for the in-plane case. For most test objects with displacements varying over their surface, fringes with different curved contours and unequal spacing will be observed, rather than parallel lines and concentric circles in figures 2(b) and (c).

Digital holographic interferometry [7, 12–14] provides an alternative to use of analog holograms. It can utilize a setup similar to that in figure 2(a). A holographic interference pattern prior to loading can be recorded by a charge coupled device (CCD) (or complementary-metal-oxide semiconductor (CMOS)) camera and stored digitally. After surface displacements occur, another digital hologram is recorded and
Figure 2. (a) Schematic of holographic interferometry set-up, (b) interference fringes from in-plane displacements of a thin plate, stretched uniformly and (c) fringes from out-of-plane displacements of a disc supported around its circumference and loaded by a force normal to its surface and slightly off-center.

stored. Numerical reconstruction (e.g. by a Fresnel transform) of ‘before’ and ‘after’ holograms enables retrieval of phase information, which can then be used to find surface displacements.

2.2. Hole drilling and holographic interferometry

With analog holography used in conjunction with hole drilling, the displacements resulting from release of residual stresses generate interference fringe patterns such as those in figure 3. The displacements are illustrated schematically by vectors $d$ in figure 4, with in-plane components parallel to the surface and out-of-plane components normal to the surface. The unsymmetrical patterns (from right to left) in figure 3 stem from the illumination arriving from one side of a hole. Phase shifts associated with the displacement vectors are sensitive to the projection of the vectors along the direction bisecting the illumination and viewing directions in figure 4. The projections differ from one side of the hole to the other. In addition to varying radially as in figure 4, displacements are also a function of circumferential position around a hole.

When a hole is drilled to a given depth over which residual stresses are uniform (or nearly so), the resulting displacements are given by [15]:

$$
\begin{bmatrix}
    u_r \\
    u_\theta \\
    u_z
\end{bmatrix} =
\begin{bmatrix}
    A + B\cos 2\theta & A - B\cos 2\theta & 2B\sin 2\theta \\
    C\sin 2\theta & -C\sin 2\theta & -2C\cos 2\theta \\
    F + G\cos 2\theta & F - G\cos 2\theta & 2G\sin 2\theta
\end{bmatrix}
\begin{bmatrix}
    \sigma_x \\
    \sigma_\theta \\
    \tau_{xy}
\end{bmatrix}
$$

(1)

where radial ($r$) and circumferential directions ($\theta$) are shown in figure 3(a), $u_r$, $u_\theta$, $u_z$ are in-plane displacements in those directions, $u_z$ is out-of-plane displacement, $\sigma_x$, $\sigma_\theta$ and $\tau_{xy}$ are residual stress components, $A = r_o(1 + \nu)a/2E$, $B = r_o b/2E$, $C = r_o c/2E$, $F = r_o f/2E$, $G = 4\nu r_o g/2E$, $r_o$ is hole radius, $E$ is modulus of elasticity, $\nu$ is Poisson’s ratio, and $a$, $b$, $c$, $f$ and $g$ are nondimensional coefficients [15] expressed in terms of $(r/r_o)$ and hole depth normalized by diameter $(h/D)$. Full-field images can provide displacement data for many different combinations of $r$ and $\theta$ for use in equation (1).

Phase differences represented by a fringe pattern from hole drilling can be related to the displacements by various approaches [e.g.,16–18]. If a fringe pattern observed in real time is perturbed by temporarily reducing the pathlength of reference light by a fraction of the wavelength of light being used, the resulting phase change causes fringes to ‘flow’ in directions that aid determination of the stresses by a simple fringe counting method [16]. The relation between displacements and stresses in equation (1) also applies if digital holographic interferometry or other full-field optical methods are used. To determine the profile of residual stresses vs. depth below a surface, a hole can be drilled incrementally, and fringe patterns recorded in real time after each increment of depth, using the same hologram. Stresses vs. depth can be found from the fringe patterns with a computational methodology [19] that relates phase shifts to displacements to stresses.
2.3. Examples of hole drilling and holographic interferometry

Holographic-hole drilling has been applied to determine residual stresses created by the widely used manufacturing process of shot peening [20]. A compact holographic system using thermoplastic recording has been used to find residual stresses in welded structures [21, 22]. Residual stresses in welded aluminum plates and tubular specimens have also been investigated [23–26]. More recently, a portable digital holographic camera has been developed and applied to determine residual stresses in a pressure vessel, aluminum cable and welds [27].
The author and a colleague (A. Makino) have used holographic-hole drilling to explore biaxial residual stresses vs. depth below the surface of a rolled, undercut fillet of a production crankshaft seen in figure 5(a). Rolling deforms surface layers elastic-plastically to produce beneficial compressive residual stresses upon unloading. A 0.8 mm diameter hole was drilled incrementally into fillets as in figure 5(b), producing a sequence of fringe patterns like those in figure 5(c). The effect of fillet curvature on fringe patterns was taken into account, and a finite element model in figure 5(d) provided a relation between fringes, surface displacements and residual stresses specific to the geometry of a fillet.

As another example, residual stresses adjacent to a weld bead were estimated by the author using the upper half of the fringe pattern in figure 6(a) and in the weld bead using the full pattern seen in figure 6(b). Use of a full-field optical method made estimates possible even with the surface undulations. (The effect of plastic deformation created by drilling into high residual stresses was taken into account [28, 29]).

As a further example, digital holographic interferometry has been applied in recent years to find residual stresses in a ceramic coating [30] by releasing stresses with holes, shallow slots and other geometries formed by pulsed laser ablation. Introducing holes by laser machining offers the prospect of access to regions that would be difficult to drill by conventional means. (Ablation would be performed in a manner to avoid generating significant residual stresses.)

3. Electronic speckle pattern interferometry (ESPI) and hole drilling

3.1. ESPI

ESPI [31–34] provides a useful alternative to holographic interferometry. The term speckle refers to a grainy image with small light and dark spots observed when laser light is scattered from a rough surface. Also referred to as digital speckle pattern interferometry, ESPI can be set up with optical components like those in figure 2(a), but with a CCD camera at the hologram location. Light scattered from a test object and reference light combine to generate a speckle interferogram that is acquired digitally. After surface displacements occur, a second interferogram is recorded. Fringes can be obtained by digital subtraction of the two interferograms.

The intensity variation \(I(x,y)\) over a fringe pattern produced by ESPI (or holographic interferometry) can be expressed in terms of phase difference \(\Delta \varphi(x,y)\) by:

\[
I(x,y) = c_1 + c_2 \cos \Delta \varphi(x,y)
\]  
(2)

where \(c_1\) and \(c_2\) are constants. A technique known as phase shifting [32] can retrieve \(\Delta \varphi(x,y)\). A number of fringe patterns \((n)\) is acquired, each with a different phase shift \(\beta_i\). Equation (2) then becomes

\[
I_i(x,y) = c_1 + c_2 \cos [\Delta \varphi(x,y) + \beta_i] \ (i = 1 \to n).
\]  
(3)

One way of introducing a known phase shift is to temporarily alter the path length of reference light by a carefully controlled, small amount. With different known values of \(\beta_i\), equation (3) can be solved to find \(\Delta \varphi(x,y)\), resulting in an arctangent function, which has the drawback of providing a phase map with \(2\pi\) discontinuities. Methods exist [35] to produce a continuous map, known as phase unwrapping. To achieve useful fringe patterns, an ESPI setup must be isolated sufficiently from vibration or other disturbances such
that the path lengths of reference and object light remain stable to within a small fraction of the wavelength of laser light [36]. The same applies to holographic interferometry. This constraint may not be as severe as it might appear, as illustrated in the following section.

3.2. ESPI and hole drilling
ESPI combined with hole drilling was developed in the years following introduction of holographic-hole drilling. Examples of fringe patterns generated by using ESPI-hole drilling are shown in figure 7, using a dual
Figure 7. Fringe patterns found by ESPI-hole drilling with (a) illumination and uniaxial stress in the same direction (horizontal), using and an optical set up sensitive to in-plane displacements only and (b) illumination at an angle to the direction of uniaxial stress. (Reprinted/Adapted) with permission from [43] © The Optical Society.

beam optical setup sensitive to in-phase displacements only. (Dual beams are not required to apply ESPI with hole drilling).

As with holographic interferometry, phase differences \( \Delta \varphi(x,y) \) associated with displacements from hole drilling enable residual stresses to be computed by various methods [37–40], including an incremental drilling approach [41] to find residual stresses vs. depth. Since small rigid body motions may occur in hole drilling systems (e.g. from drilling itself), computational methods [40, 42] have been developed to account for such unwelcome motions in case they occur and are of concern.

In the early 1990s, an ESPI-hole drilling system using single beam illumination became commercially available [44–46]. Various ESPI-hole drilling systems are shown in figure 8, as evidence of interest in the method. Systems have been developed for use outside of lab environments [47–49], including an instrument [50–52] in figure 8(d) with novel use of a diffractive optical element designed to be sensitive to radial in-plane displacements plus a laser diode as a light source, reducing size and cost.

3.3. Examples of use
ESPI-hole drilling been used to find residual stresses from manufacturing and fabrication processes, including shot peening [53], heat treatment [57], welding [47, 48], cold forming of pipes [58] and machining [59]. The instrument shown in figure 8(d) has measured stresses in a gas pipeline seen in figure 9, illustrating that hole drilling combined with a full-field interferometric method can be implemented in a challenging environment, without vibration isolation provided by optical tables in laboratories.

4. Moiré interferometry and hole drilling

4.1. Moiré interferometry
Moiré interferometry provides another means to determine surface displacements with high sensitivity [61–63]. Consider the Moiré interferometry setup in figure 10. A grating with finely spaced lines in both x and y directions has been applied to a flat surface region. Suppose that light from a low power laser illuminates the grating from the B1 and B2 directions (with B3 and B4 temporarily inactive). Light from each beam will be diffracted by the grating in the z-direction, towards an image plane of a recording device. Suppose that the surface region and grating displace. The relative path lengths of diffracted light will change, causing a phase difference and interference fringes to be observed at the image plane. For illumination by
beams B1 and B2, the fringes are related to in-plane \( x \)-displacements \( U_x \). If light from the B3 and B4 directions illuminates the grating instead of B1 and B2, fringes related to the \( y \)-displacements \( U_y \) will form. If a grating has, for example, a frequency of 1200 lines mm\(^{-1}\), the fringe-to-fringe displacement will be approximately 0.4 \( \mu \)m [64].

4.2. Moiré interferometry and hole drilling
An example of Moiré fringes from release of residual stress by hole drilling is shown in figure 11. As with holographic and ESPI-hole drilling, displacement data acquired from such patterns can be used to determine residual stresses [65–68]. A representative approach [67] will be summarized.
Figure 10. Moiré interferometry configuration with four beams. Reproduced with permission from [64].

Figure 11. Moiré-hole drilling fringes for \( U_x \) displacements from uniaxial stress in the horizontal direction. The pattern is nearly symmetrical since the fringes represent in-plane displacements, instead of the combination of in-plane and out-of-plane displacements in figure 3. Reproduced with permission from [67].

Figure 12 shows a schematic of \( U_y \) fringes from Moire-hole drilling. Away from the hole, the fringe order is zero. Fringe orders increase towards the hole. \((N_x \) fringe orders would be obtained from the \( U_x \) displacement field for the same hole).

For stress components \( \sigma_x \), \( \sigma_y \) and \( \tau_{xy} \) uniform with hole depth, in-plane radial displacements from equation (1) are:

\[
U_r (r, \theta) = A (\sigma_x + \sigma_y) + B \left[ (\sigma_x - \sigma_y) \cos 2\theta + 2 \tau_{xy} \sin 2\theta \right]
\]

(4)

where \( A \) and \( B \) can be computed from non-dimensional coefficients [15] as a function of hole depth-to-diameter and radial position \( (r/r_c) \). \( U_x \) and \( U_y \) displacements are related to fringe orders \( N_x \) and \( N_y \) [67] by

\[
U_x = (1/2f_s) N_x \text{ and } U_y = (1/2f_s) N_y \]

(5)

where \( f_s \) is the grating frequency.

Radial displacements can be related to \( U_x \) and \( U_y \) by

\[
u_r (r, \theta_i) = U_x (x_i, y_i) \cos \theta_i + U_y (x_i, y_i) \sin \theta_i
\]

(6)
Figure 12. Moiré-hole drilling fringes showing radial position \((r/r_o) = 1.2\) (dashed circle) and fringe orders \(N_y\) on one side of a different hole than in figure 11. Reproduced with permission from [67].

where \((x_i, y_i)\) is a location in figure 12 with corresponding known \(N_x\) and \(N_y\) values and angle \(\theta_i = \tan^{-1}(y_i/x_i)\). Equations (4)–(6) provide a relation (equation (7)) between fringe orders and residual stress components that can be solved with three pairs of \((N_x(x_i, y_i), N_y(x_i, y_i))\) data:

\[
\begin{bmatrix}
N_x(x_i, y_i) \\
N_y(x_i, y_i)
\end{bmatrix}
\begin{bmatrix}
\cos \theta_i \\
\sin \theta_i
\end{bmatrix}
= 2f_s \begin{bmatrix}
A + B \cos 2\theta_i & A - B \cos 2\theta_i & 2B \sin 2\theta_i \\
\sigma_x & \sigma_y & \tau_{xy}
\end{bmatrix}.
\]

The sign of stress can be deduced by perturbing a fringe pattern as described in [67]. Moiré-hole drilling can also be used to determine how residual stresses vs. depth using incremental drilling and a computational methodology [67] akin to those used in the ESPI or holographic-hole drilling methods.

4.3. Examples of use

Moiré-hole drilling has been used to find residual stresses from welding [68, 69], shot peening [67, 70, 71] and thermal spraying of a coating [72]. It has also explored residual stresses vs. depth in fiber reinforced polymeric (FRP) composite laminates [73, 74]. Computational methods have been developed to find residual stresses for through holes drilled in orthotropic materials [75, 76] or drilled incrementally, layer-by-layer in FRP laminates [77]. More recently, Moiré-hole drilling has been applied to characterize residual stresses in a woven composite [78]. As a final note, Moire and ESPI-hole drilling methods were compared in a study of residual stresses in an interference fit specimen [79]. Discrepancies between stresses found by those methods and by analysis were typically on the order of 5\%, with worst-case discrepancies of approximately 15\%.

5. Digital image correlation (DIC)

5.1. DIC

DIC [80–83] offers an alternative to interferometric methods for determining surface displacements. For 2D measurements, a digital camera views a surface region illuminated by ordinary light. An initial image of the region is acquired and digitized, known as a reference image. That image is divided into smaller regions called sub-sets, each of which should have features suitable for tracking its movement with loading. (If not, features can be added, such as by applying a random pattern of tiny black dots with a contrasting background.) A second image is captured after loading, and each deformed sub-set is matched to the corresponding one in the reference image using a correlation algorithm. For 3D measurements, two cameras are spaced sufficiently apart to enable stereovision. Images acquired by the cameras can be used to determine the 3D coordinates of surface locations by triangulation. Computational algorithms that track and match sub-sets and then find displacements and strains are included in a number of commercially available 2D and...
Figure 13. Example of displacements (color coded) in the (a) $x$-direction $U_x$ and (b) $y$-direction $U_y$, from DIC-hole drilling of a polymeric specimen with stress in the $y$-direction. Reproduced from [90]. CC BY 4.0.

3D DIC systems. Calibration of a DIC system [80] is needed prior to use, which involves accounting for camera characteristics such as possible distortion and, in the case of 3D DIC, the orientation of cameras with respect to each other. The interferometric methods considered in the previous section require stringent mechanical stability of test setups to enable highly sensitive measurement of displacements. DIC relaxes that requirement, a substantial advantage, but its displacement sensitivity may be less than that of interferometric approaches [81], depending on how it is implemented. Best practices for carrying out DIC are discussed in a series of articles beginning with [84].

5.2. DIC and hole drilling

Working independently, civil [85] and mechanical [86] engineering researchers appear to have been the first to apply DIC with the hole drilling method. Images ‘before’ and ‘after’ hole drilling are obtained, and displacements found by DIC software. An example of displacements is shown in figure 13. For a hole drilled to a depth over which residual stresses are uniform (or approximately so), one approach [86] for finding residual stresses is to take radial displacements found by DIC for different radial positions ($r/r_o$) and angles $\theta_i$ around a hole and solve equation (4) (section 4.2) for $\sigma_x, \sigma_y$ and $\tau_{xy}$. A minimum of three radial displacements are needed, but it may be desirable to use more values in a least-squares solution. Unlike the interferometric approaches, there is no need for an intermediate step of converting phase changes to displacements. Prior knowledge of the analytical form of displacement fields from hole drilling can be integrated into the DIC determination of residual stresses [87, 88], improving results. To determine how residual stresses vary with depth, incremental DIC-hole drilling has been performed [89].
5.3. Examples of use
In metallic specimens, DIC-hole drilling has characterized residual stresses created by an interference fit \[86\] and by processes such as shot peening \[89\], friction stir welding \[91\], and unloading from elastic-plastic bending \[92\]. It has also found the residual stresses in an FRP composite panel \[93\] and a pultruded composite \[94\].

6. Microscale residual stress determination by hole drilling, slotting and ring coring

6.1. Hole drilling
The versions of the hole drilling method for determining residual stresses described in the previous sections were developed for use with macroscale components (centimeters or larger). At the microscale, focused ion beam (FIB) milling can be used to remove material by hole drilling or related methods. As an example, figure 14 shows a 4 µm diameter hole used to study residual stresses in a peened specimen of a metallic glass. Images of surface regions before and after material removal are obtained with a scanning electron microscope (SEM). The release of stresses generates displacements that can be found from the images using DIC algorithms.

For the frequently encountered case of residual stresses that vary with depth, FIB milling can be used to make a blind micro-hole that increases in depth incrementally. The profile of stress vs. depth can then be found with a computational methodology \[95\] akin to that for larger holes. For a thin micro-membrane with residual stress equal in all directions and constant through its thickness, in-plane displacements from drilling a through hole enable the stress to be computed from available analytical relations \[96\].

6.2. Slotting
Slotting by FIB has been used to find residual stresses in thin coatings \[97, 98\], a thin membrane \[99\], small diameter steel wires \[100\] and specimens of zinc brass \[101\]. First a reference SEM image of a region is acquired, then a slot is made to release residual stresses. The resulting displacement field is found by DIC. Referring to figure 15, residual stresses normal to the slot can be determined by analytical expressions or finite element modeling with an input of displacements $U_x$ in the same direction. Similar to hole drilling, residual stresses vs. depth can be found by incremental slotting \[102\].

A variant of FIB slotting is shown in figure 16. Material is removed on either side of region of interest, releasing residual stresses and generating displacements in the remaining material between the two slots. Residual stress can then be computed from the displacements found by DIC \[103\].

6.3. Ring coring
Ring coring is an established method for finding residual stresses in macroscale applications \[104, 105\]. It too has been extended to the microscale. A small circular region (a few microns in diameter) on the surface is selected and a pattern of small dots created by FIB to facilitate DIC. A ring like that in figure 17 is milled by FIB to release residual stresses in an island (pillar). The resulting displacements on the surface of the island are found by DIC. Residual stresses vs. depth are then found from the displacements with computational approaches such as one based on an integral method \[106\].

FIB ring coring with DIC has been applied to thin coatings, including TiN on a Wc-Co substrate and Au on a Si/SiO$_2$ substrate \[108\], CrN on steel and Au on Si \[109, 110\], individual thin splats of thermally sprayed...
Figure 15. (a) Schematic of an idealized slot made in a thin coating on a substrate. Reproduced with permission from [97]. (b) An example of a wedge slot in a metallic glass specimen. Reprinted by permission from Springer Nature Customer Service Centre GmbH: Springer Nature, Metallurgical and Materials Transactions A [98]. 2010.

Figure 16. Schematic of H-slot to release residual stress.

Figure 17. Ring coring with FIB milled dots for DIC. Reprinted from [107] Copyright 2013, with permission from Elsevier.

\( \text{Al}_2\text{O}_3, \text{Ni-Al and } \text{Al}_2\text{O}_3-\text{TiO}_2-\text{Zr}_2\text{-CeO}_2 \) [111], zirconia on stainless steel [112], dental materials (ceramic on zirconia and porcelain fused to metal) [113] and a multilayer of Si\text{$_3$N$_4$/ZnO/Ag/ZnO/Si$_3$N$_4$} [114]. Rather than a circular island as in ring coring, residual stresses have also been quantified using rectangular or square
islands in a carbon doped SiO$_2$ coating [115] and in coatings of CrN on steel and Cu on Si [116]. In addition, ring coring has been carried out with picosecond laser machining [117] rather than FIB milling to find residual stresses in a thermal barrier coating. The use of ring coring extends beyond coatings. It has also been applied to study residual stresses in martensite lath crystals [118] and a shot peened aluminum alloy [119].

Factors that may affect residual stress determinations by FIB-DIC include potentially significant effects of elastic anisotropy [120] and possible damage introduced by material removal via FIB [121]. Additional important considerations for implementing FIB-DIC are discussed in [122, 123]. Also, a new computational approach has recently been presented [124, 125] as an alternative to use of the integral method [104, 126] for finding residual stresses vs. depth.

Although the combination of FIB and DIC has dominated microscale studies of residual stresses, other optical methods may potentially be used instead of DIC. For instance, figure 18 shows Moire grating lines (frequency of 5000 lines mm$^{-1}$) inscribed by FIB in three directions on an island formed by FIB ring coring. An SEM scanning Moiré method [127] was used to measure displacements and strains on the island as residual stresses in a laser peened nickel alloy were released.

**7. Other methods using residual stress release**

Full-field optical methods can also be useful for finding residual stresses when stresses are released by means other than hole drilling, ring coring or slotting. Residual stress in thin beams and plates can be found by removing layers of material containing the stresses. The resulting curvature is measured layer-after-layer, providing an input to analytical relations [128, 129] for determining how stresses vary through the thickness. Curvature can also develop when residual stresses are generated by deposition of material. Measurement of out-of-plane displacements of a surface (i.e. normal to the surface) can be used to find curvature. For example, out-of-plane displacements of thin circular discs have been measured by ESPI as layers were removed by chemical etching [130]. Fringes similar to those in figure 2(c) were observed. The same type of approach has been used to investigate residual stresses in cantilevered beams using ESPI [131] and Moiré interferometry [132]. In the case of material deposition, curvature has been measured by ESPI for cantilevered beams being plated [133] and by DIC for thin strips being coated by thermal spraying [134]. Curvature measurements by DIC have also been applied to explore residual stresses formed during additive manufacturing [135].

Sectioning is another method for finding residual stress in suitable situations. The method involves cutting an object containing residual stresses into smaller pieces and measuring strains that develop upon release of residual stresses [136]. The strains can be used to deduce residual stresses [137, 138]. Full-field optical methods have been applied from time-to-time to determine those strains. For example, Moiré fringes generated by deep slots used to release residual stresses in a railway rail are shown in figure 19. The fringe pattern provided displacement and thus strain information.
High temperature grating interferometry has been used to map displacements and strains from releasing residual stresses by annealing a slice from a railway rail \[139\]. A slice was also cut into small pieces to release residual stresses \[140\], and grating interferometry used to find strains for comparison with those from annealing, with results revealing a dependence on the method of stress release. Grating interferometry has also provided residual strain data after annealing of an explosively formed tube \[139\] and cutting of a laser weldment \[141\]. The residual stress distribution across the laser weldment was found with a hybrid experimental-finite element model that used an input of residual strains plus stress–strain data for different regions of the weldment (obtained via speckle interferometry and tensile loading) \[142\].

To determine residual stresses vs. depth in objects such as plates, a slitting method can be used. As depicted in figure 20, a slit is gradually deepened to release residual stresses normal to the slit. The resulting strains near the slot are measured by strain gages as a function of slit depth to provide an input to computation of residual strains \[143, 144\]. In lieu of strain gages, DIC can be applied with slitting \[145, 146\] to measure displacements and strains.

8. **Nondestructive optical methods for residual stress determination**

Several optical methods for determining residual stresses that avoid the need for material removal are possible in certain situations, as summarized below.

8.1. **Photoelasticity, imaging polarimetry and stress tomography**

Both applied and residual stresses can cause certain materials to develop birefringence, known as the photoelastic effect. In thin specimens, stress induced birefringence produces different indices of refraction \(n_1\) and \(n_2\) in directions aligned with the axes of principal stresses \(\sigma_1\) and \(\sigma_2\) acting in the plane of the specimen. Linearly polarized light passing through a specimen will emerge as two waves oriented with the \(\sigma_1\) and \(\sigma_2\) directions, with a phase shift \(\delta\) between them given by \(\delta = \frac{2\pi}{\lambda}(n_2 - n_1) t\), where \(\lambda\) = wavelength, \(t\) = thickness. The difference \((n_2 - n_1)\) is related to the principal stresses by \((n_2 - n_1) = b (\sigma_1 - \sigma_2)\) where
Figure 21. (a) Plane polariscope and (b) circular polariscope with crossed fast and slow axes of quarter wave plates.

\( b = \) relative stress-optic coefficient. When the waves pass through a second polarizer (analyzer) with its axis of polarization at 90° to that of the first polarizer, the intensity at a given location in an image will be [147]:

\[ I = I_0 \sin^2 \frac{\delta}{2} \sin^2 2\alpha \]  

(8)

where \( \alpha \) is shown in figure 21(a). For monochromatic light, the image will contain dark fringes where \( I = 0 \), which occurs when either \( \alpha = n\pi/2 \) \((n = 0, \pm 1, \pm 2, \ldots)\) or \( N\delta/2\pi = 0,1,2, \ldots \). Some of the fringes are associated with \((\sigma_1 - \sigma_2)\) and others with directions of principal stresses \((\alpha)\). In most cases, \((\sigma_1 - \sigma_2)\) is of primary interest. A circular polariscope, as in figure 21(b), is used to remove fringes associated with \( \alpha \), leaving isochromatic fringes representing contours of constant \((\sigma_1 - \sigma_2)\) given by \((\sigma_1 - \sigma_2) = NK/t\), where \( N \) is the fringe order. Constant \( K = \lambda/b \) is found by calibration using a known stress distribution. For white light illumination, fringes become sequences of colors, and a relation between different sequences and \((\sigma_1 - \sigma_2)\) values can also be established by calibration. If desired, various approaches are available to separate \( \sigma_1, \sigma_2 \) [147].

Photoelasticity has become a standard test method [148, 149] for finding residual stress in transparent or translucent plastic specimens as well as glass. It has been used, for example, to investigate residual stresses arising from injection molding of plastic objects [150–152]. Fringes like those in figure 22 have been used to analyze the residual stresses [153] in ion-exchange glass (used in smartphones), which contains compressive residual stresses near the surface to increase its strength.

Using infrared light, photoelasticity has been applied to investigate residual stresses in semiconductor wafers and microelectronic components [154]. Additional examples of the use of photoelasticity to determine residual stresses in thin (2D) specimens are numerous, but not described here for the sake of brevity.

Polarimetric imaging characterizes the state of polarization of light and can be applied to find the stress-induced birefringence across a specimen. For instance, Bajor [155] developed a polarimeter using a setup similar to that in figure 21(a) except that the axes of polarization were at 45° rather than 0 and 90° as in a plane polariscope. The polarizers were rotated simultaneously and intensity data recorded by a CCD array. Analysis of the resulting intensity data as a function of rotation yielded birefringence data, which could, in turn, be used to find \((\sigma_1 - \sigma_2)\) over a region. The polarimeter was applied to map birefringence from residual stresses in wafers of silicon, gallium arsenide, silicon and gallium phosphide [156].

Different types of polarimetry systems have been developed, such as one with a back-scattering configuration of optical components used to map applied stresses in a Plexiglas plate under compression [157]. Another system [158] used an infrared laser to scan through a rotating gallium arsenide wafer to find
phase shift, which could then be converted to a map of principal stress difference $\tau$ shown in figure 23. These are but two examples of such systems.

Polarimetry has become a standard test method [159] to determine birefringence and stresses in glass, and a number of commercial polarimetry systems are available for that purpose. In addition to glass, one of those commercial systems has been used to map residual stresses in yttria partially stabilized zirconia (Y-TZP) ceramic restorations used in dentistry [160].

Various objects containing residual stresses have 3D geometries that make determination of those stresses much more challenging. Seeking to address that challenge, photoelastic tomography [161] has been developed for application to glass and other materials suited to photoelastic characterization. An object is illuminated with polarized light, and photoelastic data (birefringence and principal stress directions) collected. Data are obtained for different orientations $\theta$ in figure 24(a). This procedure is somewhat akin to that used in x-ray computed tomography scans, but with an important difference. In x-ray scans, scalar data (attenuation) are obtained for use in imaging of body structures. However, stress depends on both magnitude and direction and can be expressed in terms of six components (e.g. $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{xz}$, $\tau_{yz}$ in Cartesian coordinates) acting in different directions, making 3D determination by photoelasticity much
more difficult. An analysis has been presented to enable stress component $\sigma_z$ (figure 24(b)) to be found [162] as well as shear stress component $\tau_{rz}$ if stresses are axisymmetric (characterized by stress components $\sigma_r$, $\sigma_z$, $\sigma_\theta$, $\tau_{rz}$). Then stress components $\sigma_r$ and $\sigma_\theta$ can be found using relations from the theory of elasticity [162]. Applications have included residual stresses in a high-pressure lamp, tempered glass tumbler and a bow-tie optical fiber [163]. A different, generalized approach has been proposed [164] for finding stress components by tomography, but has yet to be implemented as of this writing.

Finding residual stresses by reflection photoelasticity combined with hole drilling was explored more than five decades ago [165–168]. In that approach, a thin coating of photoelastically sensitive material is attached to a surface using a reflective adhesive (or backing) and a hole drilled through it into underlying material containing stresses. The coating is illuminated with light that first passes through a linear polarizer and quarter wave plate (as in a circular polariscope). The light then traverses the coating and is reflected. The reflected light is observed after passing through another quarter wave plate and analyzer. Fringes that develop in the coating from release of stress by a hole can be analyzed to find stresses [169] for the case of a hole drilled through stresses uniform with depth. A capability to determine residual stresses that vary with depth does not seem to have been developed. The photoelastic version of the hole drilling method has been relatively dormant until quite recently [90].

### 8.2. Thermoelastic stress analysis

When a specimen is tested by applying cyclic stresses under adiabatic conditions, the surface will experience a small cyclic temperature change $\Delta T$ that can be related to the change in the sum of principal stresses [170, 171] by

$$\Delta T = -\frac{\alpha T_0}{\rho c_p} \Delta (\sigma_1 + \sigma_2)$$

(9)

where $\alpha = \text{coefficient of thermal expansion}$, $T_0 = \text{absolute temperature}$, $\rho = \text{density}$, and $c_p = \text{specific heat}$ (at constant pressure). This relation assumes elastic stresses and isotropic material behavior but can be modified to accommodate orthotropic materials [170]. Adiabatic conditions are typically achieved by cycling at frequencies in the range of approximately 10–25 Hz [172]. The magnitude of $\Delta T$ can be quite small, with measurement resolution on the order of 0.001 C often needed [171]. The temperature change results in a flux of photons $\Delta \Phi$ emitted from a surface in the infrared regime. The flux is monitored by a highly
sensitive infrared camera (photon detector), resulting in a voltage signal. The change in principal stress sum can be expressed in terms of a camera signal $S$ as \[ \Delta(\sigma_1 + \sigma_2) = AS \] (10)

where constant $A$ combines parameters such as detector responsivity, system amplification, surface emissivity, etc. and can be established by calibration. Thermoelastic data can be obtained over a desired surface region to map stress. Important practical considerations involved in making thermoelastic stress measurements are described in \[172\] Thermoelastic stress analysis has become a well-established method with numerous uses. A few examples include determination of fracture mechanics parameters \[173\], detection and monitoring of crack growth \[174\], and evaluation of the quality of adhesive joints \[175\]. Several methods have been developed for separating principal stresses $\sigma_1$ and $\sigma_2$ if desired \[170, 172\].

In addition to cyclic stress, a mean value of stress may be present in many practical situations. Several experimental studies reviewed in \[176\] indicate that mean stress can influence the thermoelastic behavior of materials. A theory has been proposed to explain the effect by considering the dependence of elastic constants such as modulus of elasticity on temperature \[177\]. If thermoelastic data can be analyzed to find mean stress separately from cyclic stress, then the prospect of determining residual stress (which may be considered a mean stress) exists. However, the mean stress effect in thermoelasticity is relatively small, making its use to find residual stresses problematic \[176, 178\]. Studies of the mean stress effect continue \[179, 180\] but, as of this writing, the ability of thermoelasticity to determine residual stresses remains to be demonstrated.

8.3. Piezospectroscopic methods

When a solid is illuminated with monochromatic light, some of the light is scattered. The bulk of scattering is elastic (Rayleigh) with the same wavelength as the light source, but a small portion is scattered inelastically with wavelengths that differ from that of the light source, known as Raman scattering \[181, 182\]. The inelastically scattered light can be processed with a spectrometer and plotted as a spectrum of intensity vs. wave number (reciprocal of wavelength). The wave numbers of peaks in the spectrum shift in response to applied or residual stresses \[183\]. Although the Raman effect is not active for all materials, it has been applied to characterize residual stresses in a variety of technologically important materials. Examples include silicon \[183–185\], ceramics \[184, 186\], polycrystalline graphite \[187\], amorphous carbon \[188\], epoxy \[189\] and nickel with silicon carbide particles serving as stress sensors \[190\].

In certain cases, illumination of a solid with light can cause a jump to a higher electronic state accompanied by a release of photons upon a return to a lower energy level. This phenomenon is known as photoluminescence and detailed descriptions of it can be found in references such as \[191\]. The emitted light can be processed to provide a spectrum of intensity vs. wavenumber. The wave numbers of luminescent peaks in the spectrum shift in response to applied or residual stresses \[192, 193\]. Residual stress studies have utilized small amounts of chromium ions typically present in aluminum oxide as a photoluminescent substance \[194\]. Examples of applications include exploration of residual stresses in thin aluminum oxide layers that form within thermal barrier coatings that protect components such as gas turbine blades \[195, 196\]. A portable stress measurement system \[197\] has been developed to investigate residual stresses in thermite welds in railway rails. Thermite welds also contain aluminum oxide. In another application, residual stresses have been characterized in semiconductor gallium arsenide wafers with chromium doping \[198\]. Chromium ions are not the only stress-sensitive photoluminescent substance. For instance, rare earth Europium ions introduced into a thermal barrier coating have also provided a means to determine residual stresses \[199\].

9. Discussion

Although nondestructive optical methods have numerous useful applications, x-ray diffraction and the hole drilling method with strain gages are the mainstays of residual stress measurement. Use of non-contacting, full field optical methods with hole drilling instead of strain gages offer significant advantages such as (a) avoiding the need for a relatively smooth and flat (or mildly curved) surface to bond a hole drilling rosette and allowing stresses to be found for geometries and surfaces unreceptive to strain rosettes, (b) eliminating the time and costs needed to attach a rosette(s) and a hole drilling guide, (c) avoiding errors when a hole is drilled off center in a rosette, (d) use in elevated temperature environments and (e) providing more data for use in determining residual stresses than from three locations in a strain gage rosette. The sensitive interferometric methods considered here are well matched to the displacements of a few microns generated in typical hole drilling applications. However, the methods do require a coherent light source and adequate mechanical stability in optical setups.

The DIC-hole drilling approach can be carried out with ordinary light illumination, avoids interferometric-level mechanical stability, and can correct for rigid body motions. Drawbacks of the
approach are the need to calibrate cameras prior to use (which is not burdensome) and generally less sensitivity to displacements than interferometric techniques.

The hole drilling method is typically limited to finding residual stresses vs. depth to depths of roughly half of a hole diameter [4]. This results from surface displacements being less responsive to stresses released as depth increases.

The following experimental approach may allow the hole drilling method to find stresses at greater depths. It becomes feasible with non-contacting, full-field optical means. It represents an improved version of an earlier approach proposed by Makino et al [200] by enabling: (a) measurement to be made over much smaller areas, (b) use of readily available small end mills for making holes, and (c) illumination directed normal to the surface. First, a small diameter hole would be milled incrementally in depth into a region of interest to find residual stress vs. depth, as usual. The smaller hole would reach a depth of one-half of its diameter. Next, a somewhat larger diameter hole would be milled over the smaller hole, removing it, as depicted in figure 25. The bottom of the larger hole would provide a fresh surface for illumination. A smaller diameter hole would then be milled incrementally into the bottom of the larger hole. Displacement data generated by the new smaller hole would be obtained on the bottom by DIC or from fringes (phase changes) using interferometric methods. The data would provide an input to compute stresses vs. depth beneath the bottom. A model for computing stresses probed by the smaller hole would need to correct for removal of material containing stresses by the previous larger hole. It would also have to account for the effect of the walls of the larger hole on the displacements measured on its bottom surface. Next, a second larger hole would be milled to remove the second smaller hole, providing a new surface for the next smaller hole. This process would continue until a desired total depth is reached.

Feasibility experiments were performed with smaller holes of 2.38 mm (0.094 in.) diameter. Larger ones were made with an end mill of 6.35 mm (0.25 in.) diameter. Holographic interferometry was used with a steep angle of illumination to allow the diameter of larger holes to have a relatively small footprint. Steep angles also minimized shadows cast by the walls of larger holes that could otherwise darken a substantial portion of fringe patterns observed on the bottoms of larger holes. A drawback of using steep angles is that fringe patterns from hole drilling then depend more on out-of-plane displacements than in-plane displacements, and out-of-plane displacements from hole drilling are smaller than in-plane displacements [16]. The purpose of the experiments was to see if enough fringes (phase differences) would be observed on the bottoms of larger holes to make the proposed approach feasible. The larger holes were gently milled using thin layers as the final depth of each hole was approached to minimize possible effects of residual stresses created by milling. Fringe patterns that resulted from reaching a depth of 4.75 mm (0.19 in.) in a 7075-T651 aluminum alloy specimen with a uniaxial stress of 190 MPa (approx. 36% of yield stress and nearly constant...
Figure 26. Fringe patterns at two ratios of hole depth-to-diameter (h/D) for (a) the first smaller hole drilled into the specimen surface, (b) the second smaller hole, drilled into the bottom of the first larger hole, (c) the third smaller hole, drilled into the bottom of the second larger hole and (d) the fourth smaller hole, drilled into the bottom of the third larger hole. Smaller and larger hole diameters = 2.38 and 6.35 mm, respectively.

In depth) are shown in figure 26. It appears that sufficient fringes/phase changes occur to make the approach feasible, at least in this case. This 'hole-within-a-hole' approach may provide an interesting opportunity for further experimental exploration and development of a computational model to reconstruct the residual stresses. Although holographic interferometry was used here, other non-contacting full-field optical methods could be considered instead.
10. Summary

Residual stresses are often measured by the hole drilling method, in which a rosette pattern of strain gages is attached to a surface and a small, shallow hole drilled in the center of the pattern, releasing residual stresses in the material removed by the hole. The resulting strains are measured and used to compute residual stresses. The use of strain gage rosettes restricts the approach to relatively flat and smooth surfaces large enough for the rosettes. Also, typical rosettes for use with the hole drilling method have had three strain gages, limiting the input of strain data for computing residual stresses to just a few locations.

The hole drilling approach can be based on displacements found by full-field optical methods and overcome limitations associated with strain gages. It also provides much more data than available from several gages. Holographic and ESPI, as well as Moire interferometry, have been applied successfully with hole drilling to find residual stresses and how they vary with depth below a surface. Their ability to determine surface displacements with high sensitivity is well-suited to typical magnitudes of displacements encountered with the hole drilling method. However, they do require optical setups with sufficient mechanical stability. DIC may have a lower sensitivity to displacements than the interferometric methods but can still be used successfully to find residual stresses with hole drilling, while avoiding the need for stringent mechanical stability. DIC has also been applied to microscale specimens, using SEM images and FIB milling to create holes or similar ways of releasing residual stresses (e.g. ring coring). Non-contacting optical methods allow access to many different surface geometries unreceptive to strain gages and can be applied from the macro- to micro-scale.

A limitation of stress release by the current hole drilling method is that it can determine stresses vs. depth to about half of the hole diameter, but this limitation could potentially be overcome by further development of an optically based ‘hole-within-a-hole’ method outlined in this paper.

Non-destructive optical methods are also available for determining residual stresses, including photoelasticity and imaging polarimetry for transparent objects, and Raman and luminescent piezospectroscopy for suitable materials.

Data availability statement

No new data were created or analyzed in this study.

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