UNIQUE MASS TEXTURE FOR QUARKS AND LEPTONS

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Abstract

Texture specific quark mass matrices which are hermitian and hierarchical are examined in detail. In the case of texture 6 zeros matrices, out of sixteen possibilities examined by us, none is able to fit the low energy data (LED), for example, $|V_{us}| = 0.2196 \pm 0.0023$, $|V_{cb}| = 0.0395 \pm 0.0017$, $|V_{ub}|V_{cb}$ lies in the range 0.004 - 0.013 (PDG). Similarly none of the 32 texture 5 zeros mass matrices considered is able to reproduce LED. In particular, the latest data from LEP regarding $|V_{ub}|V_{cb}$ (= 0.093 ± 0.016) rules out all of them. In the texture 4 zeros case, we find that there is a unique texture structure for $U$ and $D$ mass matrices which is able to fit the data.

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The raison d’être for the existence of three well separated families of charged fermions remains ununderstood in the context of present day high energy physics. The mystery regarding the fermion masses has further deepened with the observation of “neutrino oscillations” by the Superkamiokande Collaboration [1] implying nonzero masses for the neutrinos and thus, giving for the first time a strong signal for the physics beyond the standard model. In the absence of any deeper understanding of fermion masses, attempts have been made on the one hand to understand arbitrary standard model Yukawa couplings of the fermions from more fundamental theories such as GUTs [4] composite models [5], left right symmetric models [6], etc. On the other hand, attempts have been made to discover phenomenological quark mass matrices, which are in tune with the low energy data(LED). In this regard, specific ansatz for the quark mass matrices have been tried with a fair degree of success [7] - [13] to explain quark mixing matrix. Similarly phenomenological neutrino mass matrices have been considered [13, 14] which attempt to accomodate simultaneously the Solar Neutrino Problem (SNP), Atmospheric Neutrino Problem (ANP) and Neutrino Oscillations observed by LSND.
The purpose of the present letter is to find possible textures for mass matrices which are able to accommodate LED regarding quark mixing as well as neutrino oscillations required to explain SNP, ANP and LSND oscillations. To this end, we carry out a detailed and exhaustive analysis of a large number of texture specific quark mass matrices (almost 52) and try to find out a set of quark mass matrices which can accommodate LED. After having restricted the number of quark mass matrices, we assume a similar texture structure for neutrino mass matrices and study its implications for three neutrino anomalies.

Before one can take an extensive analysis of mass matrices, it should be borne in mind that the number of free parameters available with the general mass matrices is larger than the physical observables. For example, if no restrictions are applied, there are 36 free parameters to describe 10 physical observables i.e. 6 quark masses, 3 mixing angles and one CP violating phase. Therefore to develop viable phenomenological quark mass matrices one has to limit the number of free parameters in the mass matrices. It is, therefore, desirable to invoke certain broad guiding principles based on general considerations borne out of experimental data and insight gained from the past such analyses. These guidelines constrain the present analysis to manageable number of mass matrices by restricting the number of free parameters of general quark mass matrices. In fact, it is also the purpose of present letter to consolidate and reiterate these guiding principles.

In this context, we first make use of the polar decomposition theorem of matrix algebra, by which, one can always express a general mass matrix as a product of a hermitian and a unitary matrix. Therefore, without loss of generality, we can consider quark mass matrices to be hermitian as the unitary matrix can be absorbed in the right handed quark fields. This immediately brings down the number of free parameters from 36 to 18.

The hierarchical pattern of quark masses as well as those of mixing angles immediately suggests that one should start with mass matrices whose elements follow hierarchy. This is borne out of several past and present analyses. One can think of mass matrices whose elements do not exhibit hierarchy, nevertheless are still able to reproduce the quark masses, however such mass matrices do not satisfy the criterion of “naturalness” proposed recently by Peccei and Wang. Therefore, we assume that the elements of hermitian quark mass matrices follow the pattern:

\[
M = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix},
\]

\[a_{11} < |a_{12}| \sim |a_{13}| \ll a_{22} \sim |a_{23}| \ll a_{33} \]

The famous Fritzsch ansatz and subsequent generalizations have all considered hierarchical matrices.
By using polar decomposition theorem, though the number of parameters is brought down to 18, it is still larger than the number of observables, therefore it needs to be brought down further. In this context, following Weinberg [18] and Fritzsch [5], the strategy has been to assume that mass matrices for fermions have certain “textures” imposed on them by some underlying symmetries or these could be purely phenomenological ansätze. These textures allow one to derive some interesting “predictions” which can then be compared with the experiments. Therefore, in order to keep free parameters under control, one therefore starts with texture specific ansätze.

Before proceeding further, it is perhaps natural to ask whether the assumptions of hermiticity, hierarchy and textures are preserved when one scales down from GUT scale to low energy as the mass matrices are usually derived at the GUT scale. This question has been examined in detail [8, 9, 10] and it has been shown that the hierarchical structure of mass matrices is not affected, whereas the texture structure and hermiticity is broken to a minor extent leaving phenomenological consequences unaltered. The texture structure, however, is maintained by the RG equations if it is ensured by additional symmetry. We, therefore, consider at low energy phenomenological texture specific mass matrices which are hermitian and hierarchical.

A well determined quark mixing matrix [16], in particular the knowledge of elements $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}|$, leads to the vital clues for the possible structures of mass matrices. In this context, a survey of some of the past analyses [5]-[11] as well as our own investigations [12] suggest that $|V_{us}|$ is given by $\sqrt{m_d/m_s}$ and a very small correction term $\sqrt{m_u/m_c}$ while $|V_{cb}|$ is given by $m_s/m_b$ and a correction term $m_c/m_t$. Thus $|V_{us}|$ is controlled largely by D type of quark mass matrices, whereas in the case of $|V_{cb}|$ both U and D sectors contribute significantly. These facts can give us vital clues about the positioning of the texture zeros in the phenomenological mass matrices. For the sake of simplicity, it is desirable that the U and D type of the mass matrices be taken to have identical texture structures [7, 8].

Keeping in mind above broad guidelines, one could start with the specific texture structures in the U type and D type of mass matrices [10]. Before going into the detailed structures of $3 \times 3$ hierarchical mass matrices, one would like to note that in a phenomenological ansatz, the $(1,1)$ element can always be taken to be equal to zero, because a nonzero $(1,1)$ element leads only to the rescaling of the lightest quark masses in both U and D type of mass matrices [7]. It can be very easily seen that the maximum number of texture zeros which can be considered for U or D type of mass matrices have to be three. More than three would lead to at least one of the quark masses to be zero. Therefore we start with texture 3 zeros type of matrices. In order to have non trivial mixings of three generations as well as keeping in mind guidelines mentioned above only following non trivial texture 3 zeros structures are possible:
$M_3$ (I) : with texture zeros at (1, 1), (1, 3) and (2, 3)
$M_3$ (II) : with texture zeros at (1, 1), (1, 2) and (2, 3)
$M_3$ (III) : with texture zeros at (1, 1), (2, 2) and (1, 3)
$M_3$ (IV) : with texture zeros at (1, 1), (2, 2) and (2, 3)

where (1, 1) etc. correspond to zero at the position of first row and first column of mass matrix and so on. In general $M_u$ and $M_d$ could be any of the four matrices mentioned above, resulting in 16 combinations. However, if $M_u$ and $M_d$ are taken to have parallel texture structure, we are left with only four possibilities. All these matrices have been diagonalized exactly. The corresponding CKM matrices can be found easily [12] and checked against the experimental values of CKM matrix elements, for example, $|V_{us}| = 0.2196 \pm 0.0023$, $|V_{cb}| = 0.0395 \pm 0.0017$, $|V_{ub}| = 0.08 \pm 0.02$, $|V_{td}|$ lies in the range 0.004 - 0.013 [16]. In table 1, we have summarized the expressions for the CKM matrices corresponding to the four possibilities mentioned above. Without going into the details of the methodology for analyzing such CKM matrices, we refer the reader to our earlier work [12]. However, we would like to emphasize that the quark masses taken for the CKM matrix analysis correspond to masses at 1 GeV [19], for example, $m_u = 0.0051 \pm 0.0015$ GeV, $m_d = 0.0089 \pm 0.0026$ GeV, $m_s = 0.175 \pm 0.055$ GeV, $m_c = 1.35 \pm 0.05$ GeV, $m_b = 5.3 \pm 0.1$ GeV and $m_t = 300 \pm 50$ GeV.
Table 1: Expressions for CKM matrix elements $V_{us}$, $V_{cb}$ and $V_{ub}$ corresponding to $M_i$ ($i = u, d$) listed in column I. The symbols used in the table are: $a = \sqrt{m_u/m_c}$, $b = \sqrt{m_c/m_t}$, $c = \sqrt{m_d/m_s}$, $d = \sqrt{m_s/m_b}$, $A_i = |A_i|e^{i\alpha_i}$, $\phi = \alpha_u - \alpha_d$.

From the table 1, it is clear that possibilities I and II are completely ruled out. In the case of III possibility $|V_{cb}|$ cannot be fitted even after full variation of input parameters, as can be checked from the expression given in the table. The possibility IV is the famous Fritzsch ansatz, which again gives $|V_{cb}|$ much above the present experimental value, as has also been pointed out in a large number of earlier analyses. For the sake of completeness, we have also carried out investigations of the possibilities where $M_u$ and $M_d$ don’t have parallel structures. Twelve such possibilities are there, which are not listed here. We find that none of the structures is viable. It may be noted that in texture 6 zeros, Fritzsch ansatz is perhaps the best and that sets the tone for future modifications.

A strict adherence to parallel texture structure for $M_u$ and $M_d$ type of mass matrices rules out texture 5 zeros mass matrices, however, such matrices have been discussed in literature. Therefore, we have also included here a discussion of such matrices. Texture 5 zeros mass matrices would have either of the two, $M_u$ or $M_d$, being of texture 2 zeros type. As texture 3 zeros matrices have already been listed, therefore we list all the possible texture 2 zeros mass matrices compatible with the guidelines enunciated above.
\[ M_2(\text{I}) : \text{ with zeros at (1, 1) and (1, 2)} \]
\[ M_2(\text{II}) : \text{ with zeros at (1, 1) and (1, 3)} \]
\[ M_2(\text{III}) : \text{ with zeros at (1, 1) and (2, 3)} \]
\[ M_2(\text{IV}) : \text{ with zeros at (1, 1) and (2, 2)} \]

(3)

All these matrices are exactly diagonalizable, except for \( M_2(\text{IV}) \) where diagonalization can be achieved perturbatively [17]. To obtain texture 5 zeros mass matrices, one has to combine any of the matrices given in eqn. 2 with any of the matrices in eqn. 3. This leads to 32 possibilities for the texture 5 zeros mass matrices. These possibilities include the five such examples discussed by RRR [10]. The examples considered by RRR are ruled out by the present data [16], particularly the recent measurement of \( |\frac{V_{ub}}{V_{cb}}| \) at LEP [20] rules these out unambiguously. In the table 2, we present some of the examples of \( M_u \) and \( M_d \) constituting texture 5 zeros matrices and not considered by RRR.
Table 2: Expressions for CKM matrix elements $V_{us}$, $V_{cb}$ and $V_{ub}$ corresponding to $M_u$ and $M_d$ listed in column I and II respectively. The symbols used are as defined for table 1 and $B_i = |B_i|e^{i\alpha_i}$, $R_t = D_u/m_t$, $R_t' = 1 - R_t$, $R_t'' = b^2 + R_t$, $R_b = D_d/m_b$, $R_b' = 1 - R_b$, $R_b'' = d^2 + R_b$.

Proceeding in the same manner as that of texture 6 zeros matrices, we find that all the cases considered in table 2 are ruled out. In particular the possibility V is ruled out as $|V_{us}|$ cannot be reproduced. In the cases I, III and IV, $|V_{us}|$ can be reproduced but $|V_{cb}|$ can not be fitted. In the case of possibility II, $|V_{us}|$ can be reproduced for large values of $R_b$, however that makes $|V_{cb}|$ too large to fit the data. Similarly, we have exactly derived the CKM matrix for the rest of the texture 5 zeros possibilities and found that none of these is able to reproduce full CKM matrix, even after full variation of all the parameters.

After having ruled out the texture 5 zeros mass matrices, it is natural to consider texture 4 zeros matrices. In table 3, we have listed texture 4 zeros mass matrices, where $M_u$ and $M_d$ are respectively 2 zeros type and have parallel structures.
Table 3: Expressions for CKM matrix elements $V_{us}$, $V_{cb}$ and $V_{ub}$ corresponding to $M_i$ ($i = u, d$) listed in column I. The symbols used are as defined for tables 1 and 2 and $\Delta_b = 1 - 2R_b$, $\Delta_t = 1 - 2R_t$.

Following the procedure outlined earlier, one can easily see that possibilities I, II and III are not able to reproduce $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}/V_{cb}|$ simultaneously, therefore we are left with only the possibility IV. In table 4, we have presented the numerical values of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}/V_{cb}|$ and $|V_{td}|$ as a function of independent parameters $R_t$ and $R_b$. To limit the number of possibilities we have considered only those cases where $R_b = R_t$. 

\begin{table} 
\centering 
\begin{tabular}{|c|c|c|c|c|} 
\hline
 & $M_i$ & $V_{us}$ & $V_{cb}$ & $V_{ub}$ \\
\hline 
I & $ \begin{pmatrix} 0 & 0 & A_1 \\ 0 & D_1 & |B_1| \\ A_1^* & |B_1| & C_1 \end{pmatrix}$ & $-\epsilon((1-(ab)^2)R_{Mt}^\prime R_{Mt})^\frac{1}{2}e^{i\phi}$ & $acd^2(R_{Mt}^\prime R_{Mt})^\frac{1}{2}e^{i\phi}$ & $-cd^2(R_{Mt}^\prime R_{Mt})^\frac{1}{2}e^{i\phi}$ \\
II & $ \begin{pmatrix} 0 & A_1 & |B_1| \\ A_1^* & D_1 & 0 \\ 0 & |B_1| & C_1 \end{pmatrix}$ & $+\frac{(1-(ab)^2)-R_{Mt}^\prime R_{Mt})}{8}R_{M1}^\prime R_{M1}$ & $\frac{(1-(ab)^2)-R_{Mt}^\prime R_{Mt})}{8}R_{M1}^\prime R_{M1} + R_{M1}^\prime R_{M1}$ & $\frac{(1-(ab)^2)-R_{Mt}^\prime R_{Mt})}{8}R_{M1}^\prime R_{M1} - R_{M1}^\prime R_{M1}$ \\
III & $ \begin{pmatrix} 0 & A_1 & B_1 \\ A_1^* & 0 & D_1 \\ B_1^* & D_1 & C_1 \end{pmatrix}$ & $ce^{i\phi} - a + abd$ & $2acde^{i\phi} + d$ & $2acde^{i\phi} - ad$ \\
IV & $ \begin{pmatrix} 0 & A_1 & 0 \\ A_1^* & D_1 & |B_1| \\ 0 & |B_1| & C_1 \end{pmatrix}$ & $-ce^{i\phi} + a(R_{M1}^\prime R_{M1})^\frac{1}{2}$ & $-acd^2(R_{M1}^\prime R_{M1})^\frac{1}{2}e^{i\phi}$ & $cd^2(R_{M1}^\prime R_{M1})^\frac{1}{2}e^{i\phi}$ \\
\hline
\end{tabular} 
\end{table}
Table 4: Calculated values of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|/|V_{cb}|$ and $|V_{td}|$ corresponding to $M_u$ and $M_d$ of IV row in table 3.

| $R_t$ | $R_b$ | $|V_{us}|$ | $|V_{cb}|$ | $|V_{ub}|/|V_{cb}|$ | $|V_{td}|$ |
|-------|-------|-----------|-----------|-----------------|-----------|
| 0.075 | 0.075 | 0.22      | 0.044     | 0.0829          | 0.009     |
| 0.080 | 0.080 | 0.22      | 0.043     | 0.085           | 0.009     |
| 0.085 | 0.085 | 0.22      | 0.042     | 0.087           | 0.009     |
| 0.090 | 0.090 | 0.22      | 0.041     | 0.089           | 0.009     |
| 0.950 | 0.950 | 0.22      | 0.040     | 0.091           | 0.008     |
| 0.100 | 0.100 | 0.22      | 0.039     | 0.094           | 0.008     |
| 0.105 | 0.105 | 0.22      | 0.038     | 0.096           | 0.008     |
| 0.110 | 0.110 | 0.22      | 0.037     | 0.099           | 0.008     |

A quick look at the table 4 shows that the calculated values of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|/|V_{cb}|$ and $|V_{td}|$ are well within the experimental bounds [16]. The details of the CKM phenomenology with CKM matrix derived from texture 4 zeros mass matrices will be discussed elsewhere.

After having found a viable texture structure for quark mass matrices, it is natural to ask whether a similar texture could be used for neutrino mass matrices or not. In particular, one would like to examine whether such a texture can generate the required appearance and disappearance probabilities for explaining the three neutrino anomalies. In this context, recently an analysis has been carried out by Barenboim and Scheck [21]. In particular, they find a mixing matrix, which is able to provide a simultaneous fit to the SNP, ANP
and LSND oscillations, for example,

\[
\begin{pmatrix}
0.793 & 0.566 & 0.226 \\
-0.601 & 0.662 & 0.447 \\
0.103 & -0.490 & 0.865
\end{pmatrix}.
\]

In our analysis based on texture 4 zeros mass matrices, the above matrix can be reproduced by considering the following eigenvalues of neutrino masses:

- \(m_1 = 0.53 \times 10^{-3} eV\)
- \(m_2 = 0.1 \times 10^{-1} eV\)
- \(m_3 = 0.5 eV\)

Without going into the details (to be discussed elsewhere), we would like to emphasize that hierarchical neutrino masses can generate the required mixing matrix within the texture four zero scenario.

In conclusion, we would like to mention that an extensive analysis of a large number of texture specific quark mass matrices, which are hermitian and hierarchical has been carried out. Interestingly, there are no quark mass matrices with texture 6 zeros and texture 5 zeros which can fit LED. In the case of texture 4 zeros, there is a unique texture with parallel texture structure for \(M_u\) and \(M_d\) which fits the data. When a similar texture structure is assumed for neutrinos, we are able to reproduce a mixing matrix which can accommodate Solar Neutrino Problem, Atmospheric Neutrino Problem and the oscillations observed at LSND.

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References

[1] Superkamiokande collaboration; Y. Fukuda et. al., Phys. Lett. B433, 9(1998) and B436, 33(1998);
T Kajita, Talk presented at Neutrino-98, Takayama, Japan (1998).

[2] H. Georgi and C. Jarlskog, Phys. Lett. B86, 297(1979); J. Harvey, P. Ramond and D. Reiss, Phys. Lett. B92, 309(1980); S. Dimopoulos, L.J. Hall and S. Raby, Phys. Rev. Lett. 68, 1984(1992); Phys. Rev. D45, 4195(1992); P. Ramond, in On Klauder’s path: a field trip, ed. G. Emch, G. Hegerfeldt and L. Streit(World Scientific, Singapore, 1992); H. Arason, D.J. Castano, P. Ramond and E.J. Piard, Phys. Rev. D47, 232(1993); G.F. Giudice, Mod. Phys. Lett. A7, 2429(1992).

[3] For a review see, E. Farhi and J. Susskond, Phys. Rev. 74, 277(1981) and references therein; J. C. Pati, Phys. lett. B228, 228(1989); J. C. Pati, M. Cvetic and H. Sharatchandra, Phys. Rev. Lett. B58, 851(1987); K.
S. Babu, J. C. Pati, H. Stremnitzer, Phys. Lett. B256, 206(1991); K. S. Babu, J. C. Pati, H. Stremnitzer, Phys. Rev. Lett. 67, 1688(1991).

[4] J. Pati and A. Salam, Phys. Rev. Lett. 31, 661(1973); R. Mohapatra and J. Pati, Phys. Rev. D11, 566(1975); G. Senjanovic and R. Mohapatra, Phys. Rev. 12, 1502(1975); H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61(1976); M.Beg, R. Budny, R. Mohapatra and A. Sirlin, Phys. Rev. Lett. 38, 1252(1977).

[5] H. Fritzsch, Phys. Lett. 73B, 317(1978); Nucl. Phys. B 155, 189(1979); Phys. Lett. 166B, 423(1986).

[6] B. Stech, Phys. Lett. 130B,189 (1983); M. Shin, Phys. Lett. 145B, (1984); M. Gronau, R. Johnson and J. Schechter, Phys. Rev. Lett. 54, 2176(1985); L. Lavoura, Phys. Lett. 228B, 245(1989); T. Ito, N. Okamura, M. Tanimoto, Phys. Rev. D 58, 077301(1998); M. Baillargeon, F. Boudjema, C. Hamzaoui and J. Lindig, hep-ph/9809207.

[7] C.H. Albright and M. Lindner, Z. Phys. C 44, 673(1989); C.H. Albright, Phys. Lett. B246, 451(1990); Phys. Lett. B246, 451(1990).

[8] H. Fritzsch and Z.Z. Xing, Phys. Lett. B353, 114(1995); Z.Z. Xing, J. Phys. G 23, 1563(1997).

[9] R. Peccei and K. Wang, Phys. Rev. D53, 2712(1996).

[10] P. Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. B406, 19(1993), hereafter referred to as RRR.

[11] Suraj N. Gupta and J.M. Johnson, Phys. Rev. D 44, 2110(1991); S. Rajpoot, Mod. Phys. Lett. A7, 309(1992).

[12] P.S. Gill and M. Gupta, Mod. Phys. Lett. 9, 2461(1994); J. Phys.G: Nucl. Part. Phys. 21, 1(1995); Parmania 45, 333(1995); Int. J. Mod. Phys. A11, 333(1996); Jol. Phys. G23, 1(1997); Phys. Rev. D56, 3143(1997); Mod. Phys. Lett. A13, 2445(1998).

[13] Kyungsik Kang and Sin Kyu Kang, Phys. Rev. D56, 1511(1997); K. Kang, S.K. Kang, C.S. Kim and S.M. Kim, hep-ph/9808419 and references therein.

[14] H. Fritzsch and Z.Z. Xing, Phys. Lett. 372, 265(1996); P.S. Gill and M. Gupta, Phys. Rev. D57, 3971(1998).

[15] The term texture zeros has been used in the sense of P. Ramond et al. in ref. [10]. For example, texture four zeros corresponds to the no. of diagonal zeros plus half of the symmetrically placed off diagonal zeros in U and D type mass matrices considered in Eq. 3.
[16] Particle Data Group, The Euro. Phys. Journal C3, (1998).

[17] Andrija Rasin, Phys. Rev. D58, 096012(1998).

[18] S. Weinberg, Transaction of the New York Academy of Sciences, Series II, 38, 185(1977).

[19] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77(1982); S. Narison, Phys. Lett. 197B, 405(1987).

[20] CLEO Collab., A. Bean et al. Phys. Rev. Lett. 70, 2861(1993); CLEO Collab., J.P. Alexander et al., Phys. Rev. Lett. 77, 5000(1996).

[21] G. Barenboim and F. Scheck, Phys. Lett. B440, 332(1998).