Superluminal pions in a hadronic fluid

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We study the propagation of pions at finite temperature and finite chemical potential in the framework of the linear sigma model with 2 quark flavors and \( N_c \) colors. The velocity of massless pions in general differs from that of light. One-loop calculations show that in the chiral symmetry broken phase pions, under certain conditions, propagate faster than light.

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I. INTRODUCTION

The linear sigma model, originally proposed as a model for strong nuclear interactions, today serves as an effective model for the low-energy (low-temperature) phase of quantum chromodynamics. The model exhibits spontaneous breaking of chiral symmetry and restoration at finite temperature. In the chiral symmetry broken phase at zero temperature pions, being massless, propagate with the velocity of light. At finite temperature one would expect chiral pions to propagate slower than light, owing to medium effects. Indeed, this expectation has been confirmed by one-loop calculations in the linear sigma model with no fermions.

In this paper we discuss the propagation of pions at nonzero temperature and nonzero finite baryon density in a model with two quark flavors. It turns out that pions in the presence of fermions become superluminal in a certain range of temperature and baryon chemical potential.

A superluminal propagation has recently been studied in the context of the Casimir effect in quantum electrodynamics. Scharnhorst has demonstrated that, when vacuum fluctuations of the electromagnetic field obey periodic boundary conditions in one spatial dimension (corresponding to parallel Casimir plates), then the two-loop corrections to the photon polarization tensor lead to a superluminal propagation of photons. A similarity between the effects of Casimir plates and that of finite temperature on the propagation of photons has been discussed, and more general conditions that lead to a superluminal propagation of photons have also been identified.

A similar effect has been found for photons interacting with fermions. It has been shown that the transverse photons coupled to fermions obeying periodic boundary conditions are faster than light when propagating perpendicularly and are slower than light when propagating parallelly to the compactified dimension. Since our fermions obey the usual antiperiodic boundary conditions in the compact temporal dimension, one would expect no superluminal effects. However, we demonstrate that in a certain region of temperature and chemical potential below the symmetry restoration point, massless pions propagate faster than light. Moreover, if one of the spatial dimensions is compactified, pions will propagate superluminally or subluminally, depending on the size of the compact dimension and on the boundary conditions.

We organize the paper as follows. In Sec. II we describe the model. In Sec. III we calculate the dependence of the pion velocity on temperature and chemical potential. We present the results and discussion in Sec. IV. In the concluding section, Sec. V, we summarize our results.

II. EFFECTIVE LAGRANGIAN

Consider the linear sigma model at finite temperature and finite baryon density. The thermal bath provides a medium which may be assumed to have a homogeneous velocity field. The dynamics of mesons in such a medium is described by an effective chirally symmetric Lagrangian of the form

\[
\mathcal{L} = \frac{1}{2} (a \eta^{\mu\nu} + b u^\mu u^\nu) (\partial_\mu \sigma \partial_\nu \sigma + \partial_\mu \pi \partial_\nu \pi) \\
- m_\pi^2 (\sigma^2 + \pi^2) - \frac{\lambda}{4} (\sigma^2 + \pi^2)^2 \\
+ \bar{\psi} (c i \gamma^\mu \partial_\mu + \mu u^\mu \gamma_\mu + g (\sigma + i \tau \pi \gamma_5)) \psi,
\]

where \( u^\mu \) is the velocity of the fluid, and \( \eta_{\mu\nu} \) is the flat Lorentzian metric tensor. In the original sigma model the fermion field was a nucleon. We consider fermions to be constituent quarks, with additional \( N_c \) degrees of freedom, “colors”, from the SU(\( N_c \)) local gauge group of an underlying gauge theory (QCD). The parameters \( a, b, \) and \( c \) depend on the temperature \( T \), the chemical potential \( \mu \), and the parameters of the model \( m_0, \lambda, \) and \( g \), and may be calculated in perturbation theory. At \( T = \mu = 0 \) the medium is absent and \( a = c = 1 \) and \( b = 0 \).

If \( m_0^2 < 0 \), chiral symmetry will be spontaneously broken. At the classical level, the \( \sigma \) field develops a nonvanishing expectation value \( \langle \sigma \rangle = f_\pi \). At nonzero temperature the expectation value \( \langle \sigma \rangle \) depends on the temperature and vanishes at the chiral transition point. Redefin-
ing the fields as

$$\{\sigma, \pi\} \rightarrow \{\sigma, \pi\} + \{\sigma'(x), \pi'(x)\}, \quad (2)$$

where $\pi'$ and $\sigma'$ are quantum fluctuations around the constant values $\pi = 0$ and $\sigma = \langle \sigma \rangle$, respectively, we obtain the effective Lagrangian in which chiral symmetry is spontaneously broken:

$$\mathcal{L}' = \frac{1}{2}(a \eta^{\mu\nu} + b u^\mu u^\nu) \partial_\mu \pi' \partial_\nu \pi' + \frac{1}{2}(\tilde{a} \eta^{\mu\nu} + \tilde{b} u^\mu u^\nu) \partial_\mu \sigma' \partial_\nu \sigma' - \frac{m_\pi^2}{2} \sigma'^2 - \frac{m_\sigma^2}{2} \pi'^2 - g' \sigma' (\sigma'^2 + \pi'^2)$$

$$- \frac{\lambda}{4} \left( \sigma'^2 + \pi'^2 \right)^2 + \bar{\psi} (c i \gamma^\mu \partial_\mu + m_F + \mu u^\mu \gamma_\mu + g(\sigma' + i \pi' \gamma_5)) \psi. \quad (3)$$

At temperatures and chemical potentials below the chiral transition line the trilinear coupling and the masses are given by

$$m_\pi^2 = 2 \lambda \sigma^2, \quad m_F = g \sigma,$$

$$m_\sigma^2 = 0, \quad g' = \lambda \sigma. \quad (4)$$

in agreement with the Goldstone theorem.

The boson kinetic part in Eq. (3) is split in two terms since the pion and sigma self-energies get different finite temperature and chemical potential contributions in the chiral-symmetry broken phase. Hence, in general, $a \neq \tilde{a}$ and $b \neq \tilde{b}$, at temperatures and chemical potentials below the chiral transition line. The constant $c$ is related to the finite temperature and chemical potential contributions to the fermion self-energy and is irrelevant to the calculation of the pion velocity. As we shall shortly see, the only quantities relevant to the calculation of the pion velocity are the constants $a$ and $b$ that enter the pion kinetic term in Eq. (3).

The temperature dependence of the chiral condensate $\sigma$ is obtained by minimizing the thermodynamic potential $\Omega = -(T/V) \ln Z$ with respect to $\sigma$ at fixed inverse temperature $\beta$. At one-loop order, the extremum condition reads

$$\sigma^2 = f_\pi^2 - \frac{8}{\lambda} N_c \int \frac{d^3p}{(2\pi)^3} \frac{1}{2 \omega_F} n_F(\omega_F)$$

$$- 3 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\omega_\pi} n_B(\omega_\pi) - 3 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\omega_\pi} n_B(\omega_\pi), \quad (5)$$

where

$$\omega_\pi = (p^2 + m_\pi^2)^{1/2}, \quad \omega_\pi = |p|,$$

$$\omega_F = (p^2 + m_F^2)^{1/2}, \quad (6)$$

$$n_F(\omega) = \frac{1}{e^{\beta(\omega - \mu)} + 1} + \frac{1}{e^{\beta(\omega + \mu)} + 1}$$

$$n_B(\omega) = \frac{1}{e^{\beta \omega} - 1}. \quad (7)$$

The right-hand side of (5) depends on $\sigma$ through the masses $m_\pi$ and $m_F$ given by (4). Solutions to (5) are implicit functions of $T$ and $\mu$. These equations have been derived from the thermodynamic potential in which the loop corrections have been neglected [13]. This approximation corresponds to the leading order in the $1/N$ expansion, where $N$ is the number of scalar fields [10]. In our case, $N = 4$.

III. PION VELOCITY

The propagation of pions is governed by the equation of motion

$$\partial_\mu \left[ \left( a \eta^{\mu\nu} + b u^\mu u^\nu \right) \partial_\nu \pi + V(\sigma, \pi, \psi) \right] = 0, \quad (9)$$

where $V$ is the interaction potential the form of which is irrelevant to our consideration.

In the simplest case of a homogeneous flow, Eq. (9) reduces to the wave equation

$$(\partial_t^2 - v^2 \Delta + \frac{v^2}{a} V) \pi = 0, \quad (10)$$

where the quantity $v$ is the pion velocity given by

$$v^2 = \left( 1 + \frac{b}{a} \right)^{-1}. \quad (11)$$

As we shall shortly demonstrate, the constants $a$ and $b$ may be derived from the finite-temperature perturbation expansion of the pion self-energy.

The pion velocity in a sigma model at finite temperature has been calculated at one-loop level by Pisarski and Tytgat in the low-temperature approximation [2] and by Son and Stephanov for temperatures close to the chiral transition point [3, 4]. It has been found that the pion velocity vanishes as one approaches the critical temperature. Here we analyze the whole range of temperatures in the chiral symmetry broken phase.

Consider the pion self-energy $\Sigma(q, T)$ in the limit when the external momentum $q$ approaches 0. The renormalized inverse pion propagator may, in the limit $q \rightarrow 0$, be
expressed in the form
\[ Z_\pi \Delta^{-1} = q^\mu q_\mu - \frac{1}{2!} q^\mu q^\nu \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q^\nu} \left( \Sigma(q, T) - \Sigma(q, 0) \right)_{\mu=0} + \ldots, \]  
(12)
where the ellipsis denotes the terms of higher order in \( q^\mu \). The \( q^\mu \) independent term of the self-energy absorbs in the renormalized pion mass, which is equal to zero in the chiral symmetry broken phase. The subtracted \( T=0 \) term has been absorbed in the wave function renormalization factor \( Z_\pi \). By comparing this equation with the inverse pion propagator derived directly from the effective Lagrangian [3]
\[ \Delta^{-1} = (a + b)q_0^2 - a q^2, \]  
(13)
we can express the parameters \( a \) and \( b \), and hence the pion velocity, in terms of the second derivatives of \( \Sigma(q, T) \) evaluated at \( q^\mu = 0 \). At one-loop level the diagrams that give a nontrivial \( q \)-dependence of \( \Sigma \) are the bubble diagrams. Subtracting the \( T=0 \) term one finds
\[ \Sigma(q) \equiv \Sigma(q, T) - \Sigma(q, 0) = \Sigma_B + \Sigma_F \]  
(14)
with the contribution of the bosonic and fermionic loops given by
\[ \Sigma_B(q) = -4g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p 2\omega_{\sigma,q}} \left\{ n_B(\omega_\pi) + n_B(\omega_{\sigma,q}) \left( \frac{1}{\omega_{\sigma,q} + \omega_\pi - q_0} + \frac{1}{\omega_{\sigma,q} + \omega_\pi + q_0} \right) \right\}, \]  
(15)
\[ \Sigma_F(q) = -8N_c g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_F 2\omega_{\sigma,q}} \left\{ \frac{-q_0 \omega_F q + q(p + q)}{\omega_F(q - \mu) + 1} \left( \frac{1}{\omega_F + \omega_{F,q} - q_0} + \frac{1}{\omega_F + \omega_{F,q} + q_0} \right) 
\right\} \]  
(16)
\[ \omega_{\sigma,q} = [ (p - q)^2 + m_\sigma^2 ]^{1/2}, \]  
\[ \omega_F,q = [ (p + q)^2 + m_F^2 ]^{1/2}. \]  
A straightforward evaluation of the second derivatives of \( \Sigma(q) \) at \( q_\mu = 0 \) yields
\[ a = 1 + a_B + a_F, \]  
(17)
\[ b = b_B + b_F, \]  
(18)
with
\[ a_B = \frac{16g^2}{m_\sigma^2} \int \frac{d^3p}{(2\pi)^3} \left[ \frac{n_B(\omega_\pi)}{4\omega_\pi} + \frac{n_B(\omega_\sigma)}{4\omega_\sigma} - \frac{1}{3} \frac{\omega_\pi^2}{m_\sigma^2} \left( \frac{n_B(\omega_\pi)}{\omega_\pi} - \frac{n_B(\omega_\sigma)}{\omega_\sigma} \right) \right], \]  
(19)
\[ b_B = \frac{16g^2}{m_\sigma^2} \int \frac{d^3p}{(2\pi)^3} \left[ \frac{\omega_{\sigma} n_B(\omega_{\sigma})}{m_\sigma^2} - \frac{\omega_\pi n_B(\omega_\pi)}{m_\sigma^2} + \frac{1}{3} \frac{\omega_\pi^2}{m_\sigma^2} \left( \frac{n_B(\omega_\pi)}{\omega_\pi} - \frac{n_B(\omega_\sigma)}{\omega_\sigma} \right) \right], \]  
(20)
\[ a_F = N_c g^2 \int \frac{d^3p}{(2\pi)^3} \frac{n_F(\omega_F)}{p^2 \omega_F}, \]  
(21)
\[ b_F = -N_c g^2 \int \frac{d^3p}{(2\pi)^3} \frac{m_F^2}{p^2} \frac{n_F(\omega_F)}{\omega_F}. \]  
(22)
The subscripts \( B \) and \( F \) denote the contributions of the
boson and fermion loops, respectively. The sign of the fermion contribution in the last equation is crucial. The pion velocity $v$ given by Eq. (14) will become larger than unity when $b < 0$, i.e., when the negative fermionic part exceeds the positive bosonic part.

The results with $\mu = 0$ in 3+1 dimensional space-time with one spatial dimension, e.g., in the $z$-direction, compactified to the size $L \equiv \beta$. In this case, the effective Lagrangian may be written in the form of Eq. (4) with $b$ replaced by $-b$ and the velocity $u_\mu$ replaced by a spacelike vector $n_\mu$ parallel to the compactified dimension and normalized as $n_\mu n_\mu = -1$. The inverse propagator is now given by

$$\Delta^{-1} = a(q_0^2 - q_x^2 - q_y^2) - (a + b)q_z^2 \quad (23)$$

and hence the velocity in the compact direction precisely equals the inverse of $v$ given by (14). Clearly, the velocity in the subspace orthogonal to the compact dimension is equal to the velocity of light. Recall that the extension of field theory to nonzero temperature is equivalent to a compactification of the time coordinate. Now, the compactified spatial coordinate and the time coordinate exchange their roles. However, since there is no time-ordering restriction in spatial coordinates, fermions may be chosen to be periodic or antiperiodic in the compact dimension. Hence, in the case of antiperiodic fermions, all the calculations of the self-energy are exactly the same.

In this case further restricts this available range.

In order to proceed with a quantitative analysis, we have to choose the input parameters from particle physics phenomenology. For the constituent quark mass we take $m_F = 340$ MeV. The coupling $g$ is then fixed by $g = m_F/f_\pi$. The Particle Data Group gives a rather wide range 400-1200 MeV of the sigma meson masses. We shall shortly see that the analysis of the $T = 0$, $\mu \neq 0$ case further restricts this available range.

At $T = 0$ the extremum condition (13) for nonnegative $\mu$ and $N_c = 3$ reads

$$\sigma^2 = f_\pi^2 - \frac{3g^2}{\lambda\pi^2} v(\mu - g\sigma) \left[ \mu\sqrt{\mu^2 - g^2\sigma^2} + g^2\sigma^2 \ln \frac{g\sigma}{\mu + \sqrt{\mu^2 - g^2\sigma^2}} \right], \quad (30)$$

which determines the extremum of the thermodynamic potential

$$\Omega(\sigma, \mu) = \frac{\lambda}{4} \sigma^4 - \frac{\lambda}{2} f_\pi^2 \sigma^2 - \frac{\partial(\mu - g\sigma)}{(2\pi)^2} \left[ \mu\sqrt{\mu^2 - g^2\sigma^2}(2\mu^2 - 5g^2\sigma^2) + 3g^4 \sigma^4 \ln \frac{\mu + \sqrt{\mu^2 - g^2\sigma^2}}{g\sigma} \right]. \quad (31)$$

The chemical potential $\mu_c$ at which chiral symmetry gets restored is given by the condition $\Omega(0, \mu_c) = \Omega(\sigma, \mu_c)$.
For $\mu_c < g f_\pi$, the solution of (30) is $\sigma = f_\pi$, so (30) leads to

$$\mu_c = \left(\frac{\lambda \pi^2}{2}\right)^{1/4} f_\pi.$$  \hfill (32)

Since $\mu_c$ is the threshold where extended nuclear matter forms, a reasonable assumption, confirmed by the strong-coupling QCD analysis \cite{18,19}, is

$$\mu_c \leq m_F.$$  \hfill (33)

It may be easily verified that with this assumption the pion velocity is constant and equal to light velocity up to the transition point $\mu_c$, where it drops to zero.

It is remarkable that Eq. (33) yields an upper bound on the sigma mass

$$m_\sigma \leq \frac{2m_F^2}{\pi f_\pi}.$$  \hfill (34)

If we take $f_\pi = 92.4$ MeV, $m_F = 340$ MeV, and saturate the bound, we find $m_\sigma = 796.5$ MeV. In the following numerical analysis we take this value as an input parameter. Any other choice below this value would not alter our qualitative picture.

Next we analyze the case of nonzero temperature. In Fig. 1 we plot the pion velocity $v$ as a function of temperature for various fixed $\mu$ (upper plots) and as a function of chemical potential for various fixed $T$ (lower plots). For each $T$ and $\mu$ the chiral condensate $\sigma$ is calculated by solving Eq. (5) numerically. The dashed line represents the velocity obtained with thermodynamically unstable solutions for $\sigma$. This line follows a similar backwards going line in the plot for $\sigma$ \cite{13}, which is typical of the first order phase transition. The actual phase transition takes place at the point $T_c$ (for a given $\mu = \mu_c$) or at $\mu_c$ (for a given $T = T_c$) where the two minima of the thermodynamic potential at $\sigma = 0$ and $\sigma(T_c, \mu_c)$ are leveled \cite{13}. At the critical point, the pion velocity drops abruptly to zero. We note that there is always a region of temperatures or chemical potentials below the critical point where the pion velocity becomes superluminal. The temperature $T = 50$ MeV and the chemical potential $\mu = 160$ MeV are chosen to represent a typical behavior below the critical temperature and the critical chemical potential, respectively. The value $T = 3$ MeV is chosen to show the low-temperature behavior near the point $T = 0$ which we have discussed above. If we increase

FIG. 1:  Pion velocity squared as a function of temperature and chemical potential. The dashed line corresponds to a thermodynamically unstable solution. $T$ and $\mu$ are in MeV.
the temperature, starting from the point \( (T = 0, \mu = 0) \) where \( v = 1 \), the velocity at the beginning drops slightly below 1 and at some temperature \( T_{SL} \) of the order of 50 MeV, it becomes superluminal.

It is important to check how sensitive the onset of superluminal propagation is to the variation of the parameters of the model. In Fig. 2 we plot the superluminal onset temperature \( T_{SL} \) as a function of the fermion-boson coupling \( g \) keeping the boson self-coupling \( \lambda \) fixed. Contrary to what one would naively expect, the superluminal onset shifts to larger temperatures with increasing \( g \). The reason is that in this temperature range the contribution of fermions (22), relative to that of bosons (20), decreases with increasing \( g \) owing to the fermion mass dependence of the distribution function.

The behavior of the pion velocity near the critical temperature should be analyzed with special care. It is well known that the phase transition in the SU(2) \( \times \) SU(2) linear sigma model at \( \mu = 0 \) is of second order [20] and that the one-loop approximation violates this prediction by producing a weak first-order transition [13, 21]. Calculations of the chiral condensate in the chiral perturbation theory [22] show the deviation from nonperturbative renormalization group calculations [23] at temperatures of about 0.5\( T_c \). As our results depicted in Fig. 1 (upper left plot) show that the superluminal pion propagation starts at temperatures of about 50 MeV, one would perhaps be inclined to think that the superluminal effect was an artifact of the one-loop approximation. However, as we have already pointed out, the superluminal pion velocity is a consequence of the negative fermion loop contribution. The proportion of the fermion part to that of the boson depends on the parameters of the model. For example, a decreasing of \( g \) with other free parameters fixed, increases the negative fermion contribution given by Eq. (22), which in turn yields a superluminal propagation even at temperatures much below 50 MeV (Fig. 2). Hence, we believe that the onset of superluminal propagation coincides only accidentally with the window where perturbation theory starts to deviate from nonperturbative calculations. For a firm judgment whether a superluminal pion propagation is a genuine effect or an artifact of the perturbation expansion one should go beyond the one-loop approximation or apply a nonperturbative technique such as Monte Carlo calculations on the lattice or the \( 1/N \) expansion of the O(\( N + 1 \))/O(\( N \)) nonlinear sigma model with fermions.

Next we discuss the case \( T = 0 \) and \( \mu = 0 \) with one spatial dimension compactified to the size \( L \equiv \beta \). The pion velocity \( v_{||} \) parallel to the compact direction is plotted in Fig. 3 for both periodic and antiperiodic fermions. In the case of antiperiodic fermions, the compactification size plays the role of inverse temperature and chiral symmetry gets restored at the critical size \( L_c = 1/T_c \). In this case, the velocity plot is just the inverse of the first plot in Fig. 1.

For periodic fermions we find the minimum of the thermodynamic potential corresponding to a nonzero sigma to be always below the minimum corresponding to \( \sigma = 0 \). In this case, the right-hand side of (29) is negative and no restoration of chiral symmetry takes place. Owing to the opposite sign of the fermion contribution the velo-

FIG. 2: Superluminal onset temperature as a function of the fermion-boson coupling \( g \).
ity $v_{||}$ is always superluminal, monotonously approaching the velocity of light as the size of the compactification $L$ approaches infinity.

To gain a physical insight into why the pions become superluminal it is worthwhile making a comparison with the similar effects found for photons propagating in a medium and in a Casimir vacuum. In QED at either finite temperature or finite boundaries the genuine change of the speed of massless photons is a two loop effect related to the coherent light-by-light scattering $^{14,15}$. However, a one loop superluminal effect has been found in QED with one space dimension compactified $^{16}$. Modifications of the vacuum that change the population of real and virtual particles introduce coherent scattering which decreases or increases the speed of massless photons. In a Casimir vacuum some of the virtual modes are eliminated and consequently their would be scattering $^{17}$. As a result the speed of massless photons is increased. Roughly speaking, the negative Casimir energy density results in massless photons being superluminal. In our case, a superluminal propagation is, formally, a consequence of the negative fermion one-loop contribution to the $q^2$ coefficient in the inverse pion propagator $^{18}$. It is conceivable that, for the parameters close to the phase transition point, the specific interaction between the chiral fields and fermions reduces the effective vacuum energy density and as a result the massless pions become superluminal.

V. SUMMARY AND CONCLUSION

We have analyzed the propagation of massless pions at nonzero temperature and nonzero finite baryon density in a sigma model with two quark flavors. By calculating the pion dispersion relation at one-loop order we have shown that pions in the presence of fermions become superluminal in a certain range of temperatures and baryon chemical potential.

Furthermore, we have studied the case when one of the spatial dimensions is compactified with fermions obeying periodic or antiperiodic boundary conditions. Restricting attention to $T = \mu = 0$, we have calculated the pion velocity $v_{||}$ along the compact direction. We have found that for antiperiodic fermions, pions will propagate superluminally or subluminally, depending on the size of the compact dimension. With periodic fermions, the velocity $v_{||}$ is always larger than the velocity of light.

A superluminal propagation of massless particles may naively seem to contradict special relativity and Lorentz invariance. The most disturbing consequence of a superluminal propagation would be an apparent violation of causality. However, it has been convincingly argued that superluminal effects realized in quantum field theories with a nontrivial vacuum do not lead to causal paradoxes $^{24}$. The basic argument goes as follows: once the conditions that determine the vacuum fluctuations are fixed, the propagation velocity in a given reference frame is unique. This implies that it is not possible to send signals both forwards and backwards in time within one reference frame. In other words, the causal loops are forbidden.

It is important to bear in mind that we have considered an idealized situation when there is no explicit chiral symmetry breaking of the original Lagrangian $^{11}$ and pions are exactly massless in the broken symmetry phase. In reality, chiral symmetry is explicitly broken owing to nonvanishing current quark masses. To make the pions massive we could have, as usual, introduced an explicit chiral symmetry breaking term in the original Lagrangian. In that case the quantity $v$, defined in $^{11}$ and calculated in the limit $q \to 0$, would no longer have the meaning of the pion velocity. Instead, given the dispersion relation $\omega = \sqrt{v^2 q^2 + m^2}$, with both $v$ and $m$ being $T$ and $\mu$ dependent, one would define a group velocity $v_g = \partial \omega / \partial q$. It may easily be verified that for $q \ll m$ the quantity $v_g$ is below one even if $v > 1$. Of course, the situation could change for $q \simeq m$. That would involve a calculation of $v$ at non-zero momenta which is beyond the scope of the present paper.

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