Newtonian approach towards mathematical modelling and tuning of a Continuously Variable Transmission

Prajwal B Bharadwaj¹ and Jeyaraj Pitchaimani¹
¹Department of Mechanical Engineering, National Institute of Technology Karnataka, Surathkal, Mangalore, India
Email: prajwalb9897@gmail.com

Abstract: Mathematical modelling of a Push - Belt Continuously Variable Transmission (CVT) has been done previously using energy-based methods. However, this paper presents a force based (Newtonian) approach as it eliminates the process of solving differential equations and hence, increases computational speed while eliminating the errors arising out of numerical solutions. The methodology for obtaining the engine rpm for optimum power is elucidated. An offset slider crank model has been used for conversion of rotary motion of flyweights to linear motion, in order to compress the pressure spring in the driving pulley. Mass of flyweights and stiffness values of pressure springs in driving and driven pulley are set as parameters which govern the constant engine rpm. The plot of engine rpm against vehicle speed is observed to be a saturating curve which gives the constant engine rpm obtained by CVT action.

1. Introduction

A CVT is a system of belt and pulleys which is used to achieve infinite gear ratios between the two transmission shafts. The CVT consists of two pulleys namely, the primary (driving) and the secondary (driven). Both the pulleys have one movable sheave and one stationary sheave. A belt (V shaped) goes over these pulleys. The movable sheave makes the belt to slide up or down in the pulley as it moves towards and away from the stationary sheave. The main reason of CVT usage is that, after a certain speed is achieved, the rpm of the engine remains constant irrespective of the increase in the speed of the vehicle.

In the clutch tuning handbook by Olav Aaen [1], CVT tuning is defined as setting a particular engine rpm as the working rpm, which is dependent upon the masses of the flyweights and the stiffness values of the pressure springs. Tuning is done in order to set the working rpm to be the one which gives the maximum power when looked up the power curve [2].

Sarah Crosby et.al [3], formulated a mathematical model for a complete vehicle equipped with CVT. A separate model for every component of the vehicle: the engine, clutch, CVT, car differentials, wheels and car body, is constructed. The differential equations for the angular velocities for every component in the system are numerically solved and the plots show how the system becomes steady after a short period of time with time.

Kim and Marshek [4], reported that, in a flat belt power transmission effective belt tension decreases with increasing belt velocity. Since the friction force between the belt and pulley is proportional to the normal pressure, the amount of torque load transmittable decreases with increasing belt velocity until the system is not able to transmit power. If the torque load is constant, the distributions of the normal pressure and tangential friction force change with belt velocity.
Oleivera and Nivaldo A Lemos [5] considered the simple case of a rope on a pulley with masses attached to the two ends of the rope. It says that ropes don’t just convey the forces from pulley to pulley. Each element exerts a force on the part of the pulley it is in contact with. The net force and torque on the pulley is the integration of the infinitesimal forces over the angle of wrap.

Abhijeet Sanchawat et. al [6] proposed a model of the CVT assuming a pressure spring in the primary pulley and the torque sensing ramp in the secondary. The governing equations have been derived using Lagrangian method assuming a conservative system. Gear ratio, Vehicle acceleration and velocity is plotted against time. Velocity is observed to saturate as time proceeds while acceleration graph is decaying with time.

This paper presents a mathematical model formulated using a force-based approach. It also incorporates the method of finding the effective normal force by integrating the infinitesimal normal forces over the angle of wrap of belt, as presented in [5]. In order to apply a force along the axis of the pressure spring, an offset slider crank mechanism has been used which converts the rotary motion of the flyweights to linear motion to compress the spring.

2. Mathematical model

The formulae to calculate belt drive variables are mentioned below.

Angles of wrap for primary and secondary pulley, \( \alpha_p \) and \( \alpha_s \) are given by,

\[
\alpha_p = \pi + 2\beta \\
\alpha_s = \pi - 2\beta
\]

Where angle \( \beta \) is,

\[
\beta = \sin^{-1}\left(\frac{d_p - d_s}{2C}\right)
\]

where \( d_p = \) primary pulley diameter, \( d_s = \) secondary pulley diameter, \( C = \) centre - centre distance between pulleys.

Length of open belt, \( L_0 \) is given by

\[
L_0 = 2C + \frac{\pi(d_p + d_s)}{2} + \frac{(d_p - d_s)^2}{4C}
\]
Tensions in the tight and slack side of pulleys respectively, $T_1$ and $T_2$ are given by

$$\frac{(T_1 - m_b v^2)}{(T_2 - m_b v^2)} = e^{\mu \theta} \sin\left(\frac{\alpha}{2}\right)$$  \hspace{1cm} (3)

$$\frac{(T_1 - T_2)}{v} = P$$  \hspace{1cm} (4)

where $\mu$ is the friction co-efficient, $\theta$ is the angle of wrap, $\alpha$ is the angle of v-belt, $m_b$ is the mass per unit length of v-belt, $v$ is the velocity of belt and $P$ is the engine power.

Since equations (3) and (4) are linear in $T_1$ and $T_2$. They are solved simultaneously to get values of $T_1$ and $T_2$.

Equations (1), (2), (3) and (4) have been referred from “Belt Drives” Module of IIT Kharagpur Lecture series [8].

Effective normal force on the secondary pulley, $F_{n \text{ string}}$ is given in [5] as,

$$F_{n \text{ string}} = \int_{0}^{\pi + \beta} n R r d\theta = \int_{0}^{\pi + \beta} T(\theta)(\cos \theta \ x + \sin \theta \ y) d\theta$$  \hspace{1cm} (5)

$T(\theta) = \text{tension in the belt as a function of angle of wrap of secondary pulley}$

$r = \text{unit radial vector} = \cos \theta \ x + \sin \theta \ y$

$n = \text{normal force on an infinitesimal section of the belt}$

$R = \text{secondary radius}$

$d\theta = \text{angle of wrap for the section of the belt}$

Primary spring compression distance, $X$ is given by (refer figure 2)

$$R_{\text{fly}} \sin(\theta) + L \sin(\phi) = Y$$  \hspace{1cm} (6)

$$k_1 \left( L \cos(\phi) \sin(\phi) \sin(\theta) - L \cos^2(\phi) \cos(\theta) \right) + R_{\text{fly}} \cos(\theta) \sin(\phi) = F \cos(\phi) \sin(\theta)$$  \hspace{1cm} (7)

$$F = m w^2 R_{\text{fly}}$$  \hspace{1cm} (8)

$$X = L \cos(\phi) - R_{\text{fly}} \cos(\theta)$$  \hspace{1cm} (9)

where $k_1$ is the primary spring stiffness, $m$ is the effective mass of flyweights, $R_{\text{fly}}$ is the crank radius, $L$ is the length of connecting rod, $F$ is the centrifugal force on the flyweight, $Y$ is the vertical distance between stroke line CD and pivot point O of crank, $w$ is the angular velocity of primary pulley and $\theta$, $\Phi$ are the angles as mentioned in figure 2.

The non-linear simultaneous equations (6) and (7) are solved using newton raphson method to get the values of $\theta$ and $\Phi$ which can be used to calculate the spring compression using equation (9).

Belt rise in primary pulley, $\Delta r_1$ is given by

$$\Delta r_1 = \frac{x}{2 \tan(\frac{\alpha}{2})}$$  \hspace{1cm} (10)

$x = \text{Primary spring compression distance calculated from equation (9), } \alpha = \text{V-belt angle}$

Belt descent in secondary pulley, $\Delta r_2$ is given by

$$\Delta r_2 = \frac{F_{n \text{ string}} \cos(\frac{\alpha}{2})}{2 k_2 \tan(\frac{\alpha}{2})}$$  \hspace{1cm} (11)

$F_{n \text{ string}} = \text{Effective normal force on the secondary pulley calculated from equation (5)}$

$k_2 = \text{Secondary spring stiffness, } \alpha = \text{V-belt angle}$
3. Methodology

Figure 2 shows the offset slider crank mechanism used to compress the primary spring. The whole mechanism rotates about the primary pulley axis AB. The centrifugal force \( F \) pushes the flyweight outwards which pushes the slider towards left, compressing the spring. The effective mass of the flyweights \( m \) is assumed to be concentrated at the end of the crank. The equations for primary spring compression have been derived by drawing free body diagrams and force balance for zero acceleration on each link [9]. Primary avg. speed vs. Secondary speed is plotted and results are analysed. Primary avg. speed is calculated by averaging the two engine rpms encountered in an iteration of the loop. The engine rpm is increased as shown in equation (12) and (13).

\[
\text{engine rpm}_{\text{new}} = \text{engine rpm}_{\text{old}} + c \tag{12}
\]

\[
\text{engine rpm}_{\text{new}} = \text{engine rpm}_{\text{old}} + c \left( \frac{\text{secondary radius}_{\text{old}}}{\text{primary radius}_{\text{old}}} \right) \tag{13}
\]

where \( c \) is a constant. In equation (12), the engine rpm is increased in equal steps in every iteration. In equation (13), \( c \) is taken to be the constant increase in secondary speed and hence is multiplied by the reciprocal of gear ratio while incrementing the engine rpm. Here, the increase in engine rpm is such that there is constant increase in secondary speed in every loop iteration. Error in belt length is calculated since the engine rpm increase is not continuous. Primary speed and engine rpm, secondary speed and vehicle rpm are used interchangeably in the following lines and each of them is expressed in terms of rpm.

3.1. Parameters [10]

1. \( k_1 \) (Stiffness of primary pulley spring) = 5 kN/m, 10 kN/m, 15 kN/m
2. \( k_2 \) (Stiffness of secondary pulley spring) = 20 kN/m, 25 kN/m, 30 kN/m
3. \( m \) (Effective mass of flyweights) = 78g
4. \( \alpha \) (Angle of V- belt) = 12°
5. \( C \) (centre-centre distance between pulleys) = 310 mm

3.2. Assumptions

1. The vehicle rpm remains constant during the gear ratio change.
2. The belt is assumed to be non-elastic and rigid.
3. Slippage is neglected.
4. Friction of the belt and pulley in radial direction is neglected [6].
5. There are no losses in power transmission.

![Figure 4. Methodology of modelling the processes](image)

4. Results

Figure 5, 6, 7 and 8 show the plots where the engine rpm is increasing in accordance with equation (12). In figure 5, each of the plots are saturating upon reaching their respective constant working rpms. It is observed that the constant working rpm increases with increase in primary spring stiffness \( (k_2) \).
figure 6. too, it is observed that the constant working rpm increases with increase in secondary spring stiffness ($k_s$). The saturation observed in figure 5 and figure 6 is similar to figure 3.

Figure 7 shows that the error belt length is increasing with secondary speed. The error has arisen since the increase in primary radius and decrease in secondary radius are such that belt length calculated in the new iteration is lesser than the actual belt length i.e., the secondary radius is decreasing at a faster rate than desired. In figure 8, it is observed that the constant working rpm increases with decrease in flyweight mass.
Figure 9 and figure 10 show the plots where the engine rpm is increasing in accordance with equation (13). Upon comparison of these two figures with figure 5 and figure 6 respectively, it is observed that the curves are very similar to each other and that the method of using equation (12) leads to earlier saturation of the curve.

5. Conclusions

A force-based mathematical model for the working of a CVT has been developed in this paper. In order to utilize the centrifugal force to compress the spring in the primary pulley, an offset slider crank mechanism has been implemented for converting the rotary motion of flyweights to linear motion. Governing equations for belt rise and fall in the pulleys are formulated by using free body diagrams for zero acceleration case. Belt-pulley equations are used to calculate the tensions on the tight and slack side. Integration method is used to calculate the effective normal force of the belt on the secondary pulley. Since increase of engine rpm during the working of a real CVT happens continuously, in order to mimic the desired characteristics, two methods of increasing the engine rpm have been explored. One of the methods being constant increase of engine rpm in every iteration. In the second method, the engine rpm is increased such that the secondary speed increase will be constant in every iteration of the loop.

It is observed from the results that both the methods lead to saturation of the curve, however the first method leads to early saturation. The constant working rpm is found to be increasing with increase in either primary or secondary spring stiffness whereas it is decreasing with increase in the effective flyweight mass. This is in conjunction with the literature presented in [1]. Since the engine rpm is not increased continuously but in steps, there is bound to be a difference in the belt lengths calculated in two consecutive iterations. The increase of error in belt length with respect to secondary speed indicates that the secondary radius is decreasing at a faster rate than desired.

6. References

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