Nonlinear response of a coastal jet to an intense storm

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Abstract. The final stage of the reaction of the coastal currents on intensive short-term wind effects is considered in the framework of nonlinear equations of shallow water on the $f$-plane. An exact analytical solution describing the steady state that occurs after radiation to infinity of the inertial-gravity waves is obtained using specific lagrangian invariants. Analysis of the solution shows that coastal jet shifts onshore or offshore against its original position, depending on the wind direction. Asymmetry of stationary regimes depending on the direction of the coastal currents appears with considerable wind intensity. The most significant is the influence of the currents on the development of coastal upwelling. In particular, upwelling occurs with weaker winds if the current intensity increases when the coast is on the right side of the jet. Upwelling can be blocked if the beach is located to the left of the jet, and if it is initially close enough to the shore.

1. Introduction

Short-term intense wind effects on the surface of the ocean lead to excitation of inertial oscillations with the frequency $f$, defined by the projection of the vector of the Earth rotation on the local vertical [1]. Inertial oscillations typically induce circular rotation of a current velocity vector. However, observations often show significant variations of current velocity hodograph behavior. An exact solution of nonlinear shallow water equations on the so-called $f$-plane shows [2] that the diversity of hodographs is explained by the interaction of inertial oscillations and narrow current jet.

An interesting property of the exact solution presented in [2] is its relationship with the Galilean transformation. That is why it describes excitation of inertial oscillations on the unbounded plane in the presence of background currents arbitrarily changing over time and space.

However, the process substantially changes if coasts are taken into account. Inertial oscillations are reflected from the coast in the form of inertial-gravity waves [1]. The shallow water equations on $f$-plane can be used to describe this process. Significant simplification can be achieved in considering the case when variables do not vary along the shore. The shallow water equations have specific lagrangian invariants in this case, which allow a reduction of initial system of equations, Moreover, the new system of equations allows describing the final stationary stage after radiation of inertial-gravity waves if half – bounded plane is considered. The exact analytic solution can be found in this case if an infinitely narrow coastal jet is considered. Properties of linearized version of this solution are discussed in [3]. This paper investigates nonlinear effects. Both barotrophic and baroclinic jets are considered. The second section presents the main equations and their exact solution in fully
nonlinear case. The third section examines the final stage of the barotropic jet response to the wind onset. The fourth section discusses the response of coastal jets to the wind effect using equivalent barotropic model. In conclusion, the obtained results are discussed.

2. Basic equations and their solution
Let us consider the system of shallow water equations on the $f$-plane

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v = -g \frac{\partial h}{\partial x} + q_x \delta(t),$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + u = -g \frac{\partial h}{\partial y} + q_y \delta(t),$$

$$\frac{\partial h}{\partial t} + \frac{\partial (u \cdot h)}{\partial x} + \frac{\partial (v \cdot h)}{\partial y} = 0$$

in the half-plane $y > 0$. In equations (1) $u$ and $v$ are the zonal and meridional components of velocity, $h$ is the thickness of a layer of fluid, and $q_x, q_y$ are the components of tangential wind stress pulse which do not depend on the horizontal coordinates and time. Let initially $u = U(y)$, $v = 0$ and $h(y)$ is found from the geostrophic balance. Then the solution of equations (1) has translational symmetry and all functions depend only on meridional coordinate and time. In this case, the system of equations (1) has a specific integral due to the presence of lagrangian invariants \cite{1, 2, 3, 4, 5, 6, 7, 8}. Indeed, if all variables are independent of zonal coordinates, then the first of the equations (1) and the continuity equation give two lagrangian invariants

$$u - f \cdot y = U(y) - f \cdot y_0 + q_x \text{ and } Q(y,t) = \int_0^y h(y,t) dy = Q_0(y),$$

where $y_0$ is the lagrangian coordinate of liquid particle and $Q_0(y)$ is the volume function at $t < 0$. Let us use the second of the equations (2) to build inverse function $y = Y(Q)$. Then obtain

$$u = f \cdot y + U(Y(Q)) - f \cdot Y(Q) + q_x.$$  (3)

Excluding zonal velocity component, instead of (1) as a result, we obtain the system of two equations for meridional component of velocity of currents and fluid volume. In the work \cite{8} a similar system of equations was used to study the nonlinear geostrophic adjustment. It has been shown that for a wide range of initial conditions the solution tends to a steady state with equal zero $v$ component. We assume that in our case, the solution tends to a steady state after radiation the inertial-gravity wave to infinity. Then the final steady state is described by equation

$$f^2 y + f \cdot U(Y(Q)) - f^2 \cdot Y(Q) + f \cdot q_x = -g \frac{d^2 Q}{dy^2}$$  (4)

And boundary conditions

$$Q(0) = 0, \frac{dQ}{dy} \rightarrow H_x \text{ when } y \rightarrow \infty,$$  (5)

where $H_x$ is the unperturbed thickness of the layer at infinity.

The equation (4) with boundary conditions (5) with arbitrary flow profile can only be solved numerically. However, there is a simple solution to the problem (3), (4), if the thickness of the layer initially has the shape of a step. The background current velocity in this case is presented by a delta-function imitating a narrow jet. So, let the non-perturbed thickness of the layer has the form

$$h_0(y) = \begin{cases} H_1, & 0 \leq y < 1 \\ H_2, & 1 \leq y \end{cases}.$$  (6)
Function $Y(Q)$ can be easily determined in this case and we find that function $Q = Q^-$ satisfies the equation

$$f^2 y - f^2 \cdot \frac{Q}{H_1} + f \cdot q_x = -g \frac{d^2 Q^-}{dy^2}.$$  \hfill (7)

when $0 \leq Q < H_1 \cdot l$. If $H_1 \cdot l < Q$, then function $Q = Q^+$ satisfies the equation

$$f^2 y - f^2 \cdot \frac{Q^+}{H_2} + f \cdot q_x = -g \frac{d^2 Q^+}{dy^2}.$$  \hfill (8)

where $\Delta H = H_2 - H_1$. At the position of the stream jet which is situated at an unknown point $y = y_c$, we have conservation of the volume

$$Q^- = Q^+ = H_1 \cdot l.$$  \hfill (9)

Let us mention that the thickness of the layer has a jump at $y = y_c$. Integrating equation (3) across the break point, we get the expression which allows finding its position

$$\left[ \left( \frac{dQ}{dy} \right)^2 \right] = \Delta H (H_1 + H_2).$$  \hfill (10)

where the square brackets denote the difference of functions on different sides of the break point. You should also consider that

$$Q^- (0) = 0 \text{ and } \frac{dQ^+}{dy} \to H_2 \text{ when } y \to \infty.$$  \hfill (11)

The solution of equations (7), (8) with boundary conditions (9) and (11) can be found easily. Then, substituting the found solution into the condition at the break point (9), we obtain the equation for its coordinate

$$\mu (a - z + z_c)^2 - \left[ cth z_c (a - z + z_c) - \frac{a}{cth z_c} - 1 \right]^2 = \mu^2 - 1,$$  \hfill (12)

which is written using dimensionless variables $\mu = \frac{H_2}{H_1}$, $a = \frac{q_x}{R_1}$, $z = \frac{y}{R_1}$ and $z_c = \frac{y_c}{R_1}$. Here $R_1 = \frac{\sqrt{gH_1}}{f_1}$ is Rossby deformation radius. Additionally we need also requires fulfillment of conditions that the volume of fluid increases at a distance from the coast. This condition in dimensionless variables takes the form

$$1 - \frac{a - z + z_c - ach z_c}{sh z_c} > 0$$  \hfill (13)

Features of nonlinear dynamics may be inferred from the analysis of the equations (12) and (13).

3. The response of the barotropic jet

The system of equations (1) describes the dynamics of the barotropic fluid, if $g$ refers to the acceleration of gravity. The initial jump of layer thickness through the break point should be small enough to be consistent with typical marine basins current speed. Equation (11) when $\mu$ is slightly different from the unity takes the form

$$z_c = z - a(1 - e^{-z_c})$$  \hfill (12')
and the condition of monotonic increase of the fluid volume with distance from the coast will be satisfied if

\[ 1 + a > 0 \quad (13') \]

Both of these expressions correspond to the solution of equation (6), (7) in the absence of a coastal jet. Thus, coastal current weakly affects lagrangian displacements of liquid particles. Let us mention that the barotropic Rossby radius is large even when the depth of the basin is about ten meters. Therefore parameter \( a \) is also small in barotropic case. Thus the restriction (21') is valid at any reasonable wind pulse intensity. Thus, in the barotropic case, the influence of pulse wind on the coastal jet lay in its offset of the starting position. The jet axis shifts to the shore in case of surge and off shore in opposite case.

4. Response of baroclinic jet

Much more interesting is the response of baroclinic jet to wind pulse, which is described by equations (1). However, in this case under \( g \) you should consider the reduced gravity acceleration, which decreases approximately three orders of magnitude. The thickness of the fluid layer can change significantly enough to induce currents relevant for observations. Rossby deformation radius in the coastal zone has a magnitude of about several kilometers at low magnitude of \( g \). Parameter \( a \) is no longer small at strong winds. Thus, a significantly nonlinear dynamics is observed for a baroclinic jet under the influence of strong winds. There appears asymmetry of fluid dynamics for opposite wind directions. The most significant is the impact of the jet on the development of coastal upwelling.

4.1. Onshore wind – driven transport

Consider initially briefly surge winds. In this case, solution of equation (11) matches the restriction (12) for all parameter settings, which have physical meaning. The solution shows that the wind simply shifts the jet towards the coast. If the jet is located far enough away from the shore, its shift into the dimensionless variables is equal to \( a \). The jet displacement is reduced and tends to zero when its initial position approaches the shore. The overall dependence of jet displacement from its initial position is qualitatively no different from that obtained in the work [3] in the linear approximation.

4.2. Offshore wind – driven transport

It is well known that under the influence of high-intensity winds inducing offshore transport the layer thickness at the shore may be reduced to zero. The entire layer then flows from the coast and is replaced by rising cold water. This phenomenon has the name of upwelling. Model (1) cannot describe the development of upwelling in full. However, it allows identifying the critical wind intensity above which upwelling develops. by virtue of The restriction (12') gives critical parameter \( a = 1 \) in the absence of the jet. In its presence the situation changes depending on the direction and intensity of the jet.

If the coast is to the right of the direction of the jet (\( \mu > 1 \)), it is initially located close to the shore and has sufficient intensity, then the development of upwelling accelerates. For example, upwelling develops when \( a = 0.8 \) and \( \mu = 1.5 \) if the jet axis is initially located at a distance from the shore less than \( 0.14R_i \). Development of coastal upwelling begins roughly with \( a = 0.6 \) when the flow is initially very close to the shore.

The upwelling develops in any location of the jet relative to the coast if the parameter \( a > 1 \). In the absence of upwelling the jet axis is displaced offshore. The displacement may exceed initial distance if the jet from the shores in several times. For example, if the discussed above parameters \( a = 0.8 \) and \( \mu = 1.5 \), in the final steady state jet axis turns away from the shores equal to \( 0.66R_i \), The shift of the jet is equal to \( aR_i \) if it is located far enough from the shore.
The influence of the jet, for which the coast is located on the left \( (\mu < 1) \), turns out to be quite different. Upwelling is not growing, if \( a < 1 \). Upwelling is not growing even under supercritical value of parameter \( a \), if the jet has enough intensity and its axis was initially located close enough to the coast. For example, if \( a = 1.1 \) and \( \mu = 0.5 \), the upwelling does not develop if the axis of the jet was initially located at a distance from the coast of less than \( 0.64R_j \). This jet axis in final steady state turns on the distance \( 1.29R_j \). The width of the strip in which the flow blocks development of upwelling, tends to a certain value with increasing intensity of the jet. The blocking of the upwelling development for some initial locations of the jet is observed for all physically reasonable values of the wind pulse intensity.

5. Conclusion

Considered in this paper relatively simple model the impact of intense storm on the coastal current showed interesting results. First, after the passage of the storm it should be expected the jet displacement onshore or offshore depending on the wind direction. This effect is most pronounced when the impact of the storm on the coastal baroclinic current. Displacement of the baroclinic jet can reach several kilometers with the typical characteristics of the offshore area. It would be interesting to analyze the results of numerical experiments or observations to observe this effect in real physical-geographical conditions.

Another interesting effect is associated with the influence of the baroclinic coastal jet on the development of wind upwelling. Perhaps, it is difficult to identify the accelerating development of upwelling in one case and its blocking in another one from the analysis of observations. However, numerical experiments may assess the significance of identified effects. It is well known that even the synoptic upwelling usually brings biogenic elements to the sea surface and stimulates the development of productivity in the sea. Therefore, the identification of the influence of coastal currents on the upwelling may have multidisciplinary value.

Finally, let us mention that in this study we based on the assumption that equations (1) allow for the final steady state. However, results presented in [9, 10] show that in general, for large times, in addition to stationary mode, there may be a mode in which the inertial-gravity waves are captured by currents. It is not clear whether such a mode is achieved in terms of translational symmetry. It would therefore be interesting to further study the evolution of jet flows under the influence of the wind pulse with a view to identifying the possibilities of forming a non-stationary final state.

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References

[1] A E Gill Atmosphere – ocean dynamics Academic Press 1982 V.1 492 p.
[2] G K Korotaev Inertial Oscillations and Galilean Transformation Izvestiya, Atmospheric and Oceanic Physics, 2018, Vol. 54, No. 2, pp. 201–205. © Pleiades Publishing, Ltd., 2018.
[3] G K Korotaev Effect of wind impulse on the coastal current Izvestiya, Atmospheric and Oceanic Physics (in press)
[4] C G Rossby 1938 J. Mar. Res. 2 239-263.
[5] M V Kalashnik, P N Svirkin 1996 Dokl. Akad. Nauk 348(6) 811–813
[6] G K Korotaev 1997 Radiating vortices in geophysical fluid dynamics Survey in Geophysics 18 567 – 619.
[7] G K Korotaev Significance and Physics of Frontal Eddies. In: Oceanic fronts and related phenomena II Konstantin Fedorov Memorial symposium. Pushkin, St. Petersburg. 1998. P.
256 – 267.

[8] G M Reznik, V Zeitlin, M Beh Jelloul 2001 J. Fluid Mech. 445 93 – 120
[9] G M Reznik 2014 J. Fluid Mech. 743 585-605 doi:10.1017/jfm.2014.59
[10] G M Reznik 2014 J. Fluid Mech. 747 605-634. doi:10.1017/jfm.2014.166