PENTAQUARKS IN A BREATHING MODE APPROACH TO CHIRAL SOLITONS*

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In this talk I report on a computation of the spectra of exotic pentaquarks and radial excitations of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in a chiral soliton model. In addition I present model results for the transition magnetic moments between the $N(1710)$ and the nucleon.

1. Introduction

Although chiral soliton model predictions for the mass of the lightest exotic pentaquark, the $\Theta^+$ with zero isospin and unit strangeness, have been around for some time, the study of pentaquarks as baryon resonances became popular only recently when experiments indicated their existence. These experiments were stimulated by a chiral soliton model estimate suggesting that such exotic baryons might have a width so small that it could have escaped earlier detection. These novel observations initiated exhaustive studies on the properties of pentaquarks. Comprehensive lists of such studies are, for example, collected in refs. 9, 10, 11.

In chiral soliton models states with baryon quantum numbers are generated from the soliton by canonically quantizing the collective coordinates associated with (would–be) zero modes such as $SU(3)$ flavor rotations. The lowest states are members of the flavor octet ($J^\pi = \frac{1}{2}^+$) and decuplet representations ($J^\pi = \frac{3}{2}^+$). Due to flavor symmetry breaking the physical states acquire admixtures from higher dimensional representations. For the $J^\pi = \frac{1}{2}^+$ baryons those admixtures originate dominantly from the antidecuplet, $10$, and the 27–plet. They also contain states with quantum

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* Estimates for pentaquark decays are obtained from axial current matrix elements. From what is known about the $\Delta \rightarrow \pi N$ transition such estimates may be questioned.
numbers that cannot be built as three–quark composites but contain additional quark–antiquark pairs. Hence the notion of exotic pentaquarks. So far, the $\Theta^+$ and $\Xi_{3/2}^-$ with masses of $1537\pm10\text{MeV}$\cite{2} and $1862\pm2\text{MeV}$\cite{2} have been observed, although the single observation of $\Xi_{3/2}^-$ is not undisputed.\cite{13}

Soliton models predict the quantum numbers $I(J^\pi) = 0(0^+) \text{ for } \Theta^+ \text{ and } 3(1^+)^+ \text{ for } \Xi_{3/2}^-$. These quantum numbers are yet to be confirmed experimentally.

Radial excitations\cite{14} of the octet nucleon and $\Sigma$ are expected to have masses similar $N$ and $\Sigma$ type baryons in the $\mathbf{10}$. Hence sizable mixing should occur between an octet of radial excitations and the antidecuplet.

Roughly, this corresponds to the picture that pentaquarks are members of the direct sum $\mathbf{8} \oplus \mathbf{10}$ which is also obtained in a quark–diquark approach.\cite{15}

Some time ago a dynamical model was developed\cite{16} to investigate such mixing effects and also to describe static properties of the low–lying $J^\pi = \frac{1}{2}^+$ and $J^\pi = \frac{3}{2}^+$ baryons. Essentially that model has only a single free parameter, the Skyrme constant $e$ which should be in the range $e \approx 5.0 \ldots 5.5$.

Later the mass of the recently discovered $\Theta^+$ pentaquark was predicted with reasonable accuracy in the same model.\cite{6}

In this talk I will present predictions for masses of the $\Xi_{3/2}^-$ and additional exotic baryons that originate the $\mathbf{27}$–plet from exactly that model without any further modifications. The latter may be considered as partners of $\Theta^+$ and $\Xi_{3/2}^-$ in the same way as the $\Delta$ is the partner of the nucleon. It is also interesting to see whether established nucleon resonances, such as the $N(1710)$, qualify as flavor partners of the $\Theta^+$ pentaquark. To this end, I will consider transition magnetic matrix elements between the nucleon and its excitations predicted by the model.

A more complete description of the material presented in this talk may be found in ref.\cite{17}.\n
2. Collective Quantization of the Soliton

I consider a chiral Lagrangian in flavor $SU(3)$. The basic variable is the chiral field $U = \exp(i\lambda_a \phi^a/2)$ that represents the pseudoscalar fields $\phi^a$ $(a = 0, \ldots, 8)$. Other fields may be included as well. For example, the specific model used later also contains a scalar meson. In general a chiral Lagrangian can be decomposed as a sum, $\mathcal{L} = \mathcal{L}_S + \mathcal{L}_{SB}$, of flavor symmetric and flavor symmetry breaking pieces. Denoting the (classical) soliton solution of this Lagrangian by $U_0(\vec{r})$ states with baryon quantum numbers are constructed by quantizing the flavor rotations

$$U(\vec{r}, t) = A(t)U_0(\vec{r})A^\dagger(t), \quad A(t) \in SU(3) \tag{1}$$
canonically. According to the above separation the Hamiltonian for the collective coordinates $A(t)$ can be written as $H = H_S + H_{SB}$. For unit baryon number the eigenstates of $H_S$ are the members of $SU(3)$ representations with the condition that the representation contains a state with identical spin and isospin quantum numbers. Radial excitations that potentially mix with states in higher dimensional $SU(3)$ representations are described by an additional collective coordinate $\xi(t)$.

$$U(\vec{r}, t) = A(t)U_0(\xi(t))A^\dagger(t).$$

(2)

Changing to $x(t) = [\xi(t)]^{-3/2}$ the flavor symmetric piece of the collective Hamiltonian for a given $SU(3)$ representation of dimension $\mu$ reads

$$H_S = \frac{-1}{2\sqrt{m \alpha \beta}} \frac{\partial}{\partial x} \left[ \frac{\alpha^3 \beta^4}{m} \frac{\partial}{\partial x} \right] + V + \left( \frac{1}{2\alpha} - \frac{1}{2\beta} \right) J(J + 1) + \frac{1}{2\beta} C_2(\mu) + s,$$

(3)

where $J$ and $C_2(\mu)$ are the spin and (quadratic) Casimir eigenvalues associated with the representation $\mu$. Note that $m = m(x), \alpha = \alpha(x), \ldots, s = s(x)$ are functions of the scaling variable to be computed in the specified soliton model. For a prescribed $\mu$ there are discrete eigenvalues $(\varepsilon_{\mu, n})$ and eigenstates $(|\mu, n\rangle)$ of $H_S$. The radial quantum number $n_\mu$ counts the number of nodes in the respective wavefunctions. The eigenstates $|\mu, n\rangle$ serve to compute matrix elements of the full Hamiltonian

$$H_{\mu, n; \mu', n'} = \varepsilon_{\mu, n\mu} \delta_{\mu, \mu'} \delta_{n, n'} - \langle \mu, n | \frac{1}{2} \text{tr}(\lambda_8 A \lambda_8 A^\dagger) s(x) | n', \mu' \rangle .$$

(4)

This “matrix” is diagonlized exactly yielding the baryonic states $|B, m\rangle = \sum_{\mu, n_\mu} C_{\mu, n_\mu}(B, m) |\mu, n_\mu\rangle$. Here $B$ refers to the specific baryon and $m$ labels its excitations. I would like to stress that quantizing the radial degree of freedom is also demanded by observing that the proper description of baryon magnetic moments requires a substantial feedback of flavor symmetry breaking on the soliton size.

3. Results

I divide the model results for the spectrum into three categories. First there are the low–lying $J = \frac{1}{2}$ and $J = \frac{3}{2}$ baryons together with their monopole excitations. Without flavor symmetry breaking these would be pure octet and decuplet states. Second are the $J = \frac{1}{2}$ states that are dominantly members of the antidecuplet. Those that are non–exotic mix with octet baryons and their monopole excitations. Third are the $J = \frac{3}{2}$ baryons that would dwell in the $27$–plet if flavor symmetry held. The $J = \frac{1}{2}$ baryons
Table 1. Mass differences of the eigenstates of the Hamiltonian with respect to the nucleon in MeV. Experimental data refer to four and three star resonances, unless otherwise noted. For the Roper resonance \(N(1440)\) I list the Breit–Wigner (BW) mass and the pole position (PP) estimate. The states \(?\) are potential isospin \(\frac{1}{2}^-\) \(\Xi\) candidates with yet undetermined spin–parity.

| B   | \(m = 0\) | \(m = 1\) | \(m = 2\) |
|-----|----------|----------|----------|
|     | \(e=5.0\) | \(e=5.5\) | \(e=5.0\) | \(e=5.5\) | \(e=5.0\) | \(e=5.5\) |
| \(N\) Input | 413 | 445 | 501 BW | 836 | 869 | 771 |
| \(\Lambda\) | 175 | 173 | 177 | 657 | 688 | 661 | 1081 | 1129 | 871 |
| \(\Sigma\) | 284 | 284 | 254 | 694 | 722 | 721 | 1068 | 1096 | 831 (*) |
| \(\Xi\) | 382 | 380 | 379 | 941 | 971 | 751 PP | 751 (?*) | 941 PP | 1515 | 1324 | — |
| \(\Delta\) | 258 | 276 | 293 | 640 | 680 | 661 | 974 | 1010 | 981 |
| \(\Sigma^*\) | 445 | 460 | 446 | 841 | 878 | 901 | 1112 | 1148 | 1141 |
| \(\Xi^*\) | 604 | 617 | 591 | 1036 | 1068 | — | 1232 | 1269 | — |
| \(\Omega\) | 730 | 745 | 733 | 1343 | 1386 | — | 1663 | 1719 | — |

from the \(27\)-plet are heavier than those with \(J = \frac{3}{2}^+\) and will thus not be studied here.

### 3.1. Ordinary Baryons and their Monopole Excitations

Table shows the predictions for the mass differences with respect to the nucleon of the eigenstates of the full Hamiltonian for two values of the Skyrme parameter \(e\). The agreement with the experimental data is quite astonishing. Only the Roper resonance \(|N, 1\rangle\) is predicted a bit on the low side when compared to the empirical Breit–Wigner mass but agrees with the estimated pole position. This is common for the breathing mode approach in soliton models. All other first excited states are quite well reproduced. For the \(1^+\) baryons the energy eigenvalues for the second excitations overestimate the corresponding empirical data somewhat. In the nucleon channel the model predicts the \(m = 3\) state only about 40MeV higher than the \(m = 2\) state, i.e. still within the regime where the model is assumed to be applicable. This is interesting because empirically it is suggestive that there might exist more than only one resonance in that energy region. For the \(3^+\) baryons with \(m = 2\) the agreement with data is on the 3\% level. The particle data group lists two “three star” isospin–\(\frac{1}{2}^-\) \(\Xi\) resonances at 751 and 1011MeV above the nucleon whose spin–parity is not yet determined. The present model suggests that the latter is \(J^\pi = \frac{1}{2}^+\), while the former seems to belong to a different channel.

The present model gives fair agreement with available data and thus
Table 2. Masses of the eigenstates of the Hamiltonian \(^{(4)}\) for the exotic baryons \(\Theta^+\) and \(\Xi_{3/2}\). Energies are given in GeV with the absolute energy scale set by the nucleon mass. Experimental data are the average of refs. 2 for \(\Theta^+\) and the NA49 result for \(\Xi_{3/2}\). I also compare the predictions for the ground state \((m = 0)\) to the treatment of ref. 21.

| B        | \(e = 5.0\) | \(e = 5.5\) | expt. | \(e = 5.0\) | \(e = 5.5\) | expt. |
|----------|-------------|-------------|-------|-------------|-------------|-------|
| \(\Theta^+\) | 1.57       | 1.59       | 1.537 ± 0.010 | 1.54       | 2.02       | 2.07   | –     |
| \(\Xi_{3/2}\) | 1.89       | 1.91       | 1.802 ± 0.002 | 1.78       | 2.29       | 2.33   | –     |

supports the picture of coupled monopole and rotational modes. Most notably, the inclusion of higher dimensional \(SU_F(3)\) flavor representations in three flavor chiral models does not lead to the prediction of any novel states in the regime between 1 and 2GeV in the non–exotic channels.

### 3.2. Exotic Baryons from the Antidecuplet

Table 2 compares the model prediction for the exotics \(\Theta^+\) and \(\Xi_{3/2}\) to available data, and to a chiral soliton model calculation that does not include a dynamical treatment of the monopole excitation. In that calculation parameters have been tuned to reproduce the mass of the lightest exotic pentaquark, \(\Theta^+\). The inclusion of the monopole excitation increases the mass of the \(\Xi_{3/2}\) slightly and brings it closer to the empirical value. Furthermore, the first prediction for the mass of the \(\Xi_{3/2}\) was based on identifying \(N(1710)\) with the nucleon like state in the antidecuplet and thus resulted in a far too large mass of 2070MeV. Other chiral soliton model studies either take \(M_{\Xi_{3/2}}\) as input or are less predictive because the model parameters vary considerably.

Without any fine–tuning the model prediction is only about 30–50MeV higher than the data. In view of the approximative nature of the model this should be viewed as good agreement. Especially the mass difference between the two potentially observed exotics is reproduced within 10MeV.

### 3.3. Baryons from the 27–plet

The 27–plet contains states with the quantum numbers of the baryons that are also contained in the decuplet of the low–lying \(J = \frac{3}{2}\) baryons: \(\Delta, \Sigma^*\) and \(\Xi^*\). Under flavor symmetry breaking these states mix with the radial excitations of decuplet baryons and are already discussed in table 1. Table 3 shows the model predictions for the \(J = \frac{3}{2}\) baryons that emerge from the
Table 3. Predicted masses of the eigenstates of the Hamiltonian (4) for the exotic $J = \frac{3}{2}$ baryons with $m = 0$ and $m = 1$ that originate from the $27$-plet with hypercharge ($Y$) and isospin ($I$) quantum numbers listed. I also compare the $m = 0$ case to treatments of refs.\textsuperscript{21,22,23}. All numbers are in GeV.

| B    | $Y$ | $I$ | $m = 0$ | $m = 1$ |
|------|-----|-----|---------|---------|
|      |     |     | $\epsilon =$ 5.0 | $\epsilon =$ 5.5 |
|      |     |     | $\text{WK}^{21}$ | $\text{BFK}^{22}$ | $\text{WM}^{23}$ | $\epsilon =$ 5.0 | $\epsilon =$ 5.5 |
| $\Theta_{27}$ | 2   | 1   | 1.66  | 1.69  | 1.67  | 1.60  | 2.10  | 2.14  |
| $N_{27}$    | 1   | 1/2 | 1.82  | 1.84  | 1.76  | --   | 1.73  | 2.28  | 2.33  |
| $\Lambda_{27}$ | 0   | 0   | 1.95  | 1.98  | 1.86  | --   | 1.86  | 2.50  | 2.56  |
| $\Gamma_{27}$ | 0   | 2   | 1.70  | 1.73  | 1.70  | 1.68  | 2.12  | 2.17  |
| $\Pi_{27}$  | -1  | 3/2 | 1.90  | 1.92  | 1.84  | 1.88  | 1.87  | 2.35  | 2.40  |
| $\Omega_{27}$ | -2  | 1   | 2.08  | 2.10  | 1.99  | 2.06  | 2.07  | 2.54  | 2.59  |

$27$-plet but do not have partners in the decuplet. Again, the experimental nucleon mass is used to set the mass scale. Let me remark that the particle data group\textsuperscript{19} lists two states with the quantum numbers of $N_{27}$ and $\Lambda_{27}$ at 1.72 and 1.89 GeV, respectively, that fit reasonably well into the model calculation. In all channels the $m = 1$ states turn out to be about 500 MeV heavier than the exotic ground states.

3.4. Magnetic Moment Transition Matrix Elements

Table 4 shows the model prediction for magnetic moment transition matrix elements for states with nucleon quantum numbers. I expect the model to reliably predict these matrix element because it also gives a good account of the magnetic moments of the spin–$\frac{1}{2}$ baryons, in particular with regard to deviations from flavor symmetric relations\textsuperscript{16}. It is especially interesting to compare them with the result originating from the assumption that the $N(1710)$ be a pure antidecuplet state\textsuperscript{24}. This assumption yields a proton channel transition matrix element much smaller than in the neutron channel. While I do confirm this result for the case of omitted configuration mixing (entry $|10,0\rangle \rightarrow |8,0\rangle$) it no longer holds true when the effects of

Table 4. Transition magnetic moments of excited nucleons in the proton and neutron channels. Results are given in nucleon magnetons (n.m.) and with respect to the proton magnetic moment, $\mu_p$.

| $e =$ 5.0 | proton | neutron |
|-----------|--------|---------|
| $m$       | $\mu$ [n.m. | $\mu/\mu_p$ | $\mu$ [n.m. | $\mu/\mu_p$ |
| 1 (Roper) | -0.90  | -0.41 | 0.89 | 0.40 |
| 2 ($N(1710)$ | -0.28 | -0.13 | -0.17 | -0.08 |
| 3        | -0.24  | -0.11 | -0.19 | -0.09 |
| $|8,1\rangle \rightarrow |8,0\rangle$ | -0.55 | -0.24 | 0.40 | 0.18 |
| $|10,0\rangle \rightarrow |8,0\rangle$ | 0.00 | 0.00 | -0.62 | -0.28 |
flavor symmetry breaking are included. Then the transition matrix elements in the proton and neutron channels for the $N(1710)$ candidate state ($m = 2$) are of similar magnitude. This difference to the pure SU picture for the $N(1710)$ should be large enough that data on electromagnetic properties could test the proposed mixing scheme.

4. Conclusion

In this talk I have discussed the interplay between rotational and monopole excitations for the spectrum of pentaquarks in a chiral soliton model. In this approach the scaling degree of freedom has been elevated to a dynamical quantity which has been quantized canonically at the same footing as the (flavor) rotational modes. Then not only the ground states in individual irreducible $SU_F(3)$ representations are eigenstates of the (flavor–symmetric part of the) Hamiltonian but also all their radial excitations. I have treated flavor symmetry breaking exactly rather then only at first order. Thus, even though the chiral soliton approach initiates from a flavor–symmetric formulation, it is capable of accounting for large deviations thereof.

The spectrum of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons is reasonably well reproduced. Also, the model results for various static properties are in acceptable agreement with the empirical data. This makes the model reliable to study the spectrum of the excited states. Indeed the model states can clearly be identified with observed baryon excitations; except maybe an additional P11 nucleon state although there exist analyses with such a resonance. Otherwise, this model calculation did not indicate the existence of yet unobserved baryon states with quantum numbers of three–quark composites. Here the mass difference between mainly octet and mainly antidecuplet baryons is a prediction while it is an input quantity in most other approaches, and the computed masses for the exotic $\Theta^+$ and $\Xi_{3/2}$ baryons nicely agree with the recent observation for these pentaquarks. The present predictions for the masses of the spin–$\frac{3}{2}$ pentaquarks should be sensible as well and are roughly expect between 1.6 and 2.1GeV.

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