FROM DARK MATTER TO MOND

R.H. Sanders

Kapteyn Astronomical Institute, University of Groningen
Groningen, The Netherlands

MOND—modified Newtonian dynamics—may be viewed as an algorithm for calculating the distribution of force in an astronomical object from the observed distribution of baryonic matter. The fact that it works for galaxies is quite problematic for Cold Dark Matter. Moreover, MOND explains or subsumes systematic aspects of galaxy photometry and kinematics—aspects that CDM does not address or gets wrong. I will present evidence here in support of these assertions and claim that this is effectively a falsification of dark matter that is dynamically important on the scale of galaxies.

1 Introductory remarks

Modified Newtonian dynamics, or MOND, was proposed by Milgrom as an alternative to dark matter. Over the past 25 years a considerable lore has grown up around this idea, and now the very word seems to provoke strong reactions—pro or con—depending upon ones preconceptions or inclinations. Here I want to provide a minimalist definition of MOND—a definition which is as free as possible from emotive charge of the idea; therefore, I will avoid terms like modified inertia or modified gravity.

MOND is an algorithm that permits one to calculate the distribution of force in an object from the observed distribution of baryonic matter with only one additional fixed parameter having units of acceleration.

This algorithm works very well on the scale of galaxies. The fact that it works is problematic for Cold Dark Matter (CDM), because this is not something that dark matter can naturally do. Moreover, MOND explains or subsumes systematic aspects of galaxy photometry and kinematics—aspects which CDM does not address or gets wrong. Several of these systematics were not evident at the time that MOND was proposed, so this constitutes a predictive power going beyond the ability to explain observations a posteriori.

I will present the evidence in favor of these assertions, so this will be a discussion primarily of the phenomenology. I will, however, draw the conclusion which to me is also minimal and quite obvious: standard CDM is falsified by the existence of this successful algorithm.

2 The algorithm and its immediate consequences

MOND, in its original form, is embodied by Milgrom’s simple formula: if \( g \) is the true gravitational acceleration and \( g_N \) is the traditional Newtonian acceleration, then these are related by

\[
g \mu (|g|/a_0) = g_N
\]  

(1)
where \( a_0 \) is a new fixed parameter having units of acceleration and \( \mu \) is a function whose form is not specified but which must have the asymptotic behavior \( \mu(x) = 1 \) when \( x \gg 1 \) (the Newtonian limit) and \( \mu(x) = x \) when \( x \ll 1 \) (the MOND limit). This means that the discrepancy between the observed baryonic mass and the Newtonian dynamical mass should appear below a critical acceleration. In fact, for spiral galaxies this is the case; objects with the lowest measured centripetal acceleration (from the rotation curves) have the largest implied Newtonian dynamical mass-to-light ratios\(^2\). The critical acceleration has a value of \( a_0 \approx 10^{-8} \) cm\(\cdot\)s\(^{-2}\) or about an angstrom per second per second. This is within a factor of 10 of \( cH_0 \)– a coincidence originally pointed out by Milgrom\(^1\). I might just add here that the sort of CDM halos which emerge from cosmic N-body simulations\(^3\) do not possess such a natural acceleration scale not withstanding convoluted arguments to the contrary\(^4\).

This formula has two immediate and significant consequences. In the limit of low accelerations, the “true” gravitational acceleration would be given by

\[
g = \sqrt{a_0 g_N}.
\]  

(2)

At large distance from a point mass \( M \) this would mean that the effective gravitational attraction is

\[
g = \sqrt{GMa_0/r^2}.
\]  

(3)

If we equate this to the centripetal acceleration we have an asymptotic rotation velocity given by

\[
V_\infty^4 = GMa_0.
\]  

(4)

In other words, all rotation curves are asymptotically flat and there exists a mass-rotation velocity relation of the form \( M \propto V_\infty^4 \).

It is often said that MOND was “designed” to fit galaxy data, but Milgrom’s proposal actually predated most of the precise data on galaxy rotation curves– data such as that shown in Fig. 1. This rotation curve is derived from 21 cm line observations of the nearby spiral galaxy NGC 2403 and goes well beyond the bright optical disk\(^5\). The dashed and dotted curves are the Newtonian rotation curves calculated from the observed distribution of starlight and neutral hydrogen surface density respectively, and the solid curve is that determined from the Newtonian force using eq. 1. Here we see that the rotation curve is indeed asymptotically flat, but there is more to it. The required mass-to-light ratio in the disk (M/L=0.9) is entirely reasonable for the population of stars in this galaxy, and the calculated rotation curve even appears to reproduce structure in the observed rotation curve in the inner regions (more on this later).

The second consequence, the mass-rotation velocity relationship, forms the basis of the observed Tully-Fisher law– a correlation between rotation velocity and luminosity in spiral galaxies. This observed relation has been converted by McGaugh\(^6\) to a baryonic Tully-Fisher relation shown in Fig. 2. Here the stellar luminosity is re-expressed as a stellar mass, and the directly observed mass of gas, important in dwarf galaxies, is also included. The solid line is not a fit but the expectation from MOND with \( a_0 = 10^{-8} \) cm\(\cdot\)s\(^{-2}\). Thus MOND subsumes the baryonic Tully-Fisher law: it exhibits the observed slope, and the correct zero-point, and the relationship is exact (the only scatter is observational).

In the context of dark matter, the baryonic Tully-Fisher relation would be a correlation between the tiny bit of remaining baryonic mass (after blowout) in the central regions of a vast halo and the circular velocity established by that halo. How does CDM explain this near perfect correlation? In CDM cosmological simulations all halos at a given cosmic time have about the same density (with considerable scatter). If we combine this with the Newtonian virial theorem we find that \( M \propto V^3 \) (with considerable scatter\(^7\). This, in itself, is inconsistent with observations as is illustrated in Fig. 2 by the dotted line. To bring CDM expectation in line with the observations, protogalaxies must lose a fraction of their baryonic mass which is greater for
lower mass galaxies. And it must do this in a manner which reduces the scatter. There has been a great deal of work trying to accomplish this (with so-called “semi-analytic galaxy formation” programs), but it remains an exercise characterized by parameter tuning in order to achieve the desired result. It is characteristic of the CDM paradigm with respect to explaining global scaling relations: The scaling relations arise from aspects of galaxy formation. The pure N-body simulations do not explain the observed relations, so advocates fall back on “complicated baryonic physics” or “gastrophysics” to push the expectations to conform to observations. With MOND, scaling laws like Tully-Fisher do not result from the details of galaxy formation but from existent dynamics. The same is true of the general trends which I enumerate below.

3 General trends embodied by MOND

1. The MOND acceleration may be written as a surface density

\[ \Sigma_c = a_0/G \]  

Numerically this corresponds to 860 \( M_\odot/pc^2 \), or, if we assume a mass-to-light ratio of one to two in solar units in the blue band, to a surface brightness of \( \mu_B \approx 22 \) mag/arcsec\(^2\). When an astronomical system has a surface brightness comparable or larger than this value, it implies that \( \Sigma \geq \Sigma_c \) and the object is in the high-acceleration or Newtonian regime. Then there should be no significant discrepancy between the Newtonian dynamical mass and the detectable baryonic mass, at least not within the bright regions of the system. Examples of high surface brightness systems are globular star clusters and luminous elliptical galaxies. It is well known that the Newtonian dynamical mass-to-light ratio in globular clusters is entirely consistent with normal stellar populations—there is no evidence for dark matter\[^8\]. Furthermore, MOND predicts a “dearth of dark matter” in luminous elliptical galaxies, a dearth that has been recently confirmed\[^9\][10\]. To falsify MOND, one need only find high surface brightness systems which require dark matter within the optically visible image.
Figure 2: The baryonic Tully-Fisher relation derived by McGaugh [7]. This is the baryonic mass plotted against the rotation velocity for a sample of spiral galaxies. The squares are dwarf galaxies where gas makes a dominant contribution to the baryonic mass. The triangular points are the deduced circular velocity for clusters of galaxies. The solid line is the MOND relation and the dotted line is that predicted from CDM simulations.

On the other hand, for low surface brightness systems $\Sigma < \Sigma_c$ and the object is in the low acceleration or MOND regime. This means that in such systems there should be a large discrepancy between the Newtonian mass and the baryonic mass. This is certainly true. In dwarf spheroidal galaxies, the very faint low surface brightness companions of the Milky Way, there is indeed a very large discrepancy to the Newtonian mass. Since Milgrom’s original papers, many low surface brightness disk galaxies, with well measured rotation curves, have been discovered. These objects, without exception, exhibit a large discrepancy within the visible disk.

2. It has been known for many years that rotationally supported Newtonian disks tend to be unstable; in fact, this was an original motivation for pressure supported dark halos around spiral galaxies. But, in the context of MOND, disks with surface densities below $\Sigma_c$ are not Newtonian and thus stable. Therefore, MOND implies that there should be an upper limit to the surface density or surface brightness of rotationally supported disks. There is: it is called the Freeman limit and is roughly 22 mag/arcsec$^2$, corresponding to $\Sigma_c$.

3. Low surface brightness disk galaxies are in the deep MOND regime. Therefore the rotation curve should slowly rise to the asymptotic value which is the maximum rotational velocity. On the other hand, high surface brightness disks are in the Newtonian regime. Therefore the rotation curves, after an abrupt rise, should slowly decline in a near Keplerian fashion to the final asymptotic velocity. These predictions have been subsequently confirmed by observations of LSB and HSB disk galaxies; examples are shown in Fig. 3.

4. Not all galaxies are rotationally supported disks; there are also galaxies held up against gravity by the random motion of the stars—pressure supported systems. In fact, not only galaxies fall into this category but there are pressure supported systems ranging from globular star clusters (with $10^5$ $M_\odot$) to the great clusters of galaxies with a baryonic content consisting primarily of hot gas having a total mass approaching $10^{14}$ $M_\odot$. A fair approximation for such systems is the isothermal sphere— an object with a constant velocity dispersion. When we solve the equation of hydrostatic equilibrium for such systems, using Milgrom’s formula, we find that unlike their strictly Newtonian counterparts, isothermal spheres have finite mass. In the inner high acceleration regime, the density falls as $1/r^2$ as in a Newtonian isothermal sphere,
Figure 3: Observed rotation curves of a low surface brightness (Broeils [15]) and a high surface brightness galaxy (Begeman [6]). Here the dotted curve is the Newtonian rotation curve of the stellar component and the dashed curve for the gas. The solid curve is the MOND rotation curve. The mean surface brightness and the implied mass-to-light ratios are indicated.

but then at distance where the acceleration approaches \( a_0 \) \( (r = \sqrt{GM/a_0}) \) the sphere rather abruptly truncates (density \( \rightarrow 1/r^4 \)). This means that the average internal acceleration in MOND isothermal spheres should be near \( a_0 \).

Fig. 4 shows the observations. This is log-log plot of velocity dispersion vs. size for systems spanning many orders of magnitude from sub-galactic to super-galactic systems (the identity of the systems is noted in the caption). The straight line is not a fit but rather the locus of \( \sigma^2/r = a_0 \). We see that the internal acceleration of these systems all lie with in a factor of a few of \( a_0 \). How does dark matter accomplish this?

There is another prediction with respect to pressure supported systems originally pointed out by Milgrom. It is an easy matter to show, directly from the hydrostatic gas equation that MOND implies that the mass and the velocity dispersion are related as

\[
\frac{M}{10^{11} M_\odot} \approx \left[ \frac{\sigma}{100 \text{ km/s}} \right]^4
\]  

This relation is not so precise as the Tully-Fisher law because the scaling is quite sensitive to deviations from an isothermal state or isotropy of the velocity distribution. It forms the basis of the Faber-Jackson relation for elliptical galaxies—an observed \( L \propto \sigma^4 \) correlation. But it goes beyond this; it applies to any pressure supported, near isothermal system, and it tells us that an object with a velocity dispersion of 5 km/s will have a mass of about \( 10^5 M_\odot \) (a globular cluster), or that an object with a velocity dispersion of 100 km/s will have a galaxy mass, or an object with a velocity dispersion of 1000 km/s will have a mass (\( \approx 10^{14} M_\odot \)) of a cluster of galaxies.
Figure 4: A log-log plot of the velocity dispersion in hot stellar systems against the characteristic size for different classes of objects. The star points are globular clusters, the solid round points are massive molecular clouds in the Galaxy, the crosses are luminous elliptical galaxies, the triangles are dwarf spheroidal galaxies, the squares are X-ray emitting clusters of galaxies. The solid line corresponds to $\sigma^2/r = a_0$ and the dashed lines show a factor of 2.2 variation on each side. The references to the relevant observational papers may be found in [2].

4 Rotation curves

Certainly the most remarkable aspect of MOND is its ability to predict the form of rotation curves from the observed distribution of baryonic matter. The procedure has been discussed many times before. We assume that the stellar mass is traced exactly by the surface brightness distribution (in the near-infrared preferably). We then include the observed distribution of gas (which can make a significant contribution in low mass galaxies), assume that the stars and gas are distributed in a thin disk (apart from some spiral galaxies which possess a significant luminous bulge or central spheroidal component), apply the traditional Poisson equation to determine the Newtonian force ($g_N$) and finally use the MOND formula to calculate the “true” gravitational force. There is generally one free parameter in all of this—the mass-to-light ratio of the visible disk which is adjusted to achieve the best fit.

The results have been impressive, particularly considering all the intrinsic uncertainties in galaxies (warps, bars, distance uncertainties, effects of companions). In about 90% of the 100 or so rotation curves considered, MOND produces a very reasonable version of the observed rotation curve. Moreover, the implied mass-to-light ratios of the stellar components are not only sensible, but in complete agreement with population synthesis models.

I have already shown three examples of observed rotation curves compared to the curve calculated with the MOND formula using the observed distribution of detectable baryons (Figs. 1 & 3). I show one more in Fig. 5 because it illustrates very well a point that I wish to emphasize. UGC 7524 is a dwarf low surface brightness galaxy [15]. In the top figure I show the logarithm of the surface density in stars and gas as a function of radius (the stellar surface density is determined from the surface brightness distribution with MOND value of M/L=1.6). In the bottom figure I show again the observed rotation curve (points), the Newtonian rotation curves of stars and gas, and MOND rotation curve. We see that, for both stars and gas, there is an enhancement in the surface density between 1.5 and 2.0 kpc, and of course, there is a corresponding feature in the Newtonian rotation curves. But we see that there is also a feature at this position in
the total rotation curve, even though there is a significant discrepancy between the Newtonian and detectable mass. The total rotation curve perfectly reflects details in the observed mass distribution even though the object is “dominated by dark matter” in the inner regions.

This is an empirical point emphasized repeatedly by Sancisi [16]. For every feature in the surface brightness distribution (or gas surface density distribution) there is a corresponding feature in the observed rotation curve (and vice versa). I would add that with dark matter this seems rather unnatural. How is it that the dark matter distribution could match so perfectly the baryonic matter distribution? But with MOND, it is expected. What you see is all there is!

5 Concluding remarks

Although eq. 1 predicts the detailed distribution of force in galaxies from the observed distribution of baryonic matter, it appears to break down in clusters of galaxies. Applying the MOND formula in the hydrostatic gas equation, we find that, for X-ray emitting clusters, MOND reduces the mass discrepancy by a factor of two, but there still remains a factor of two or three more mass than is directly observed in hot gas and stars in galaxies. Formally, this is not a falsification because we may always find more mass in clusters (it would be a falsification if MOND predicted less mass than is observed), but this is seen by some as devastating for a proposed alternative to dark matter.

I take quite the opposite point of view. The existence of an algorithm which precisely predicts the force in galaxies from the observed distribution of baryonic matter is devastating for dark matter which clusters on the scale of galaxies, CDM. In fact, it constitutes a falsification. To explain the MOND phenomenology with dark matter would require an intimate dark matter-baryon coupling which is totally at odds with the proposed nature of CDM. Baryons behave...
quite differently from CDM: they dissipate and collapse to the center of a system; they are blown out by supernovae; they are left behind in collisions between galaxies or clusters of galaxies. The intimate connection of dark matter and baryons implied by the phenomenology of rotation curves is incomprehensible in terms of CDM.

In the context of CDM, global scaling relations, such as the Tully-Fisher or Faber-Jackson relation, have their origin in aspects of galaxy formation. Yet, galaxy formation, as emphasized by Milgrom\(^{17}\), is quite a haphazard process with each galaxy having its own unique history of formation, interaction, and evolution. It is difficult to imagine that the ratio of baryonic to dark mass would be a constant in galaxies, or even vary systematically with galaxy mass. And yet this is required, in a very precise way, to explain the baryonic Tully-Fisher relation—an exact correlation between the baryonic mass and the asymptotic rotation velocity which supposedly is a property of the dark matter halo. Any initial intrinsic velocity-mass relation of proto-galaxies would surely be erased in the stochastic process of galaxy formation. To believe that vague processes such as “feedback” or “self-regulation” can restore even tighter correlations is equivalent to faith in the tooth fairy.

And finally, there is the ubiquitous appearance of \(a_0 \approx cH_0\). How do CDM halos, which embody no intrinsic acceleration scale, account for the facts that \(a_0\) determines the normalization of the Tully-Fisher relation for spiral galaxies and the Faber-Jackson relation for hot systems, that \(a_0\) is the characteristic internal acceleration of spheroidal systems ranging from sub-galactic objects to clusters of galaxies, that \(a_0\) defines a critical surface brightness below which the discrepancy is present.

This body of evidence cannot be ignored and constitutes a profound case against CDM. Moreover, this phenomenology implies that there is something essentially correct about MOND. Although I have avoided the subject here, the implications are far-reaching.

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