Mode partition noise mitigation for VCSEL-MMF links by using wavefront shaping technique

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Abstract: In this paper, we propose a new method to mitigate the mode partition noise (MPN) in VCSEL-MMF based links by using wavefront shaping technique. The noise characteristic of the VCSEL-MMF links is theoretically studied and the impacts of coupling coefficients between VCSEL and MMF on the MPN and the system performance are investigated via simulation and experiment. The simulation results show that for 25-Gb/s OOK signal after 300-m MMF transmission, five orders of magnitude improvement of BER can be observed and the standard deviation of the received signal, which is the characterization of the MPN, is reduced for about an order when wavefront shaping is applied. With the help of wavefront shaping, we show that the DSP complexity has been profoundly reduced in order to achieve reliable 56-Gb/s and 112-Gb/s PAM-4 transmission by simulation. We perform experiment for 25-Gb/s OOK signal transmission over 300-m OM3 MMF with the launching optical power of -4 dBm to the fiber. About 1-dB power penalty improvement has been achieved at 7% FEC threshold after 300-m MMF transmission and the noise in the eye diagram is mitigated. The results of our simulation and experiment show the effectiveness of wavefront shaping to mitigate the MPN, therefore reduce the DSP complexity and improve transmission performance for the VCSEL and MMF based high speed short reach optical interconnects.

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1. Introduction

The virtues of low cost, high speed, and high reliability afforded by using the vertical-cavity surface-emitting lasers (VCSELs) with multimode fiber (MMF) make them well suited for intra-datacenter optical interconnect applications. Recently, the next generation 200 Gb/s and 400 Gb/s MMF PHYs are nearly finalized [1], pushing the optical transceivers into the 400G era. With the increasing requirements for the speed and transmission distance, the performance of VCSEL and MMF based links are fundamentally constrained by bandwidth limitation of devices, the inherent modal and chromatic dispersion as well as the noise impairments. There have been several proposals to compensate the bandwidth limitation and the propagation impairments [2–4]. In order to push the system further into higher speed transmission scenarios, it is important to distinguish the composition of the noise impairments at the decision circuit of the receivers and figure out corresponding solutions to particular impairments, other than simply applying digital signal processing (DSP) which will lead to noise enhancement when dealing with the channel impairments [5]. The mode partition noise (MPN), induced by the multimode characteristic and mode competition of the VCSEL as well as the VCSEL to MMF coupling [6–8], plays a significant role in the noise impairments of the VCSEL-MMF links and it would become more severe after interacting with the modal and chromatic dispersion after transmission in the MMF [6]. The MPN could be the main limitation for the multimode transmission since it will turn into an error floor after the transmission in the MMF.

In the previous work, the influence of modal-chromatic dispersion interaction (MCDI) as well as signal waveform on the MPN in the VCSEL-MMF links have been studied [5, 8, 9]. Wavefront shaping technique is known to have the ability to control the coupling coefficients and it has been widely investigated and applied in the field of imaging [10], multimode fiber nonlinearity...
control [11] and mode dispersion compensation for C-band multimode fiber communication systems with single mode launching [12]. However, it still remains blank for the research on the impact of changing the coupling coefficients, realized by wavefront shaping, on the MPN caused by complicated interaction of MPN, RIN and modal and chromatic dispersion in the VCSEL-MMF based links at the wavelength of 850 nm.

In this paper, we propose a new method to reduce the MPN in VCSEL-MMF links by using wavefront shaping technique. The noise characteristic of VCSEL-MMF links is theoretically studied and the impacts of coupling coefficients between VCSEL and MMF on the MPN and the system performance are investigated via simulation and experiment. The MPN is reduced for about an order using wavefront shaping for 25-Gb/s OOK transmission over 300-m MMF in simulation. Five orders of magnitude improvement can be found in BER after applying wavefront shaping. In the experiment, 1-dB power penalty improvement has been verified for 25-Gb/s OOK signals after 300-m OM3 MMF transmission with the launching optical power of -4 dBm to the fiber. The significant noise reduction has also been observed in the eye diagram. With the help of wavefront shaping, we show profound reduction of the DSP complexity for 56-Gb/s and 112-Gb/s PAM-4 signal over 300m and 100m transmission in simulation. The required tap number of feed-forward equalizer (FFE) in order to optimize the transmission performance is significantly reduced from 48 to 2 in the case of 56-Gb/s PAM-4 transmission over 300m MMF and the required memory length of the second order for volterra filter (VF) to reach FEC threshold is reduced to 0 for 112-Gb/s PAM-4 transmission.

2. Principle

The statics of modes in a VCSEL can be described by the covariance matrix [6, 7]

\[
COV \{a_i\} = \begin{bmatrix}
var_1 & cov_{12} & \cdots & cov_{1M} \\
cov_{21} & var_2 & \cdots & cov_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
cov_{M1} & cov_{M2} & \cdots & var_M
\end{bmatrix}
\]  

(1)

where \(var_i = \frac{a_i(t)^2}{\langle a_i(t) \rangle^2} - \frac{\langle a_i(t) \rangle^2}{\langle a_i(t) \rangle^2}\) is the variance of the i-th VCSEL mode and \(cov_{ij}(\tau) = \frac{a_i(t) a_j(t+\tau) - \langle a_i(t) \rangle \cdot \langle a_j(t+\tau) \rangle}{\langle a_i(t) \rangle \cdot \langle a_j(t+\tau) \rangle}\) is the covariance of the i-th and j-th VCSEL mode. \(a_i(t)\) is the normalized optical power of the i-th VCSEL mode and \(\tau\) is the relative delay of i-th and j-th VCSEL mode. The received signal in VCSEL-MMF links can be expressed as [8]:

\[
y(t) = P_{OMA} \sum_{i=1}^{M} F_i(t) a_i(t) + n(t)
\]

(2)

where \(P_{OMA}\) is the optical modulation amplitude. \(M\) is the number of VCSEL modes. \(n(t)\) consists of the additive white Gaussian noise (AWGN) of the photodetector and the random noise. \(F_i\) is the signal waveform carried by the i-th VCSEL mode after propagating in the MMF and it is given as [8]

\[
F_i(t) = \sum_{g=1}^{N_g} C_{ig} f_i(t - L \Delta t_{ig})
\]

(3)

where \(N_g\) is the number of fiber modes. \(C_{ig}\) is the coupling coefficient between the i-th VCSEL mode and the g-th fiber mode. \(f_i\) is the modulation signal of i-th VCSEL mode before coupling to MMF. \(L\) denotes the length of the MMF and \(\Delta t_{ig}\) given by \(\Delta t_{ig} = t_g - D(\lambda_i - \lambda_0)\) is the
temporal delay of the signal in unit length, where $D$ is the chromatic dispersion and $t_g$ denotes the differential mode delay of the $g$-th MMF mode. $\lambda_i$ is the wavelength of $i$-th VCSEL mode. The noise at the receiver side due to the VCSEL noise in the VCSEL-MMF links is defined as [7,8]

$$\sigma^2(t_0) = \left( \sum_{i=1}^{M} F_i(t_0) a_i - \sum_{i=1}^{M} F_i(t_0) \bar{a_i} \right)^2 = \left( \sum_{i=1}^{M} F_i(t_0) a_i \right)^2 - \left( \sum_{i=1}^{M} F_i(t_0) \bar{a_i} \right)^2$$

where $a_i$ is equal to $a_i(t)$ in Eq. (2). $t_0$ is the sampling time. $\delta \tau_{ij}$ is the relative delay of $i$-th VCSEL mode and $j$-th VCSEL mode after transmitting in MMF. The average is performed over the time window of signal capture. Due to mode spectral bias (MSB) and MCDI in VCSEL-MMF links [13], it is difficult to know the exact value of the relative delay of two VCSEL modes after propagating in MMF in our simulation to calculate the MPN. As indicated in [6], it is adequate to estimate the link performance by taking an effective $k$ value, so the MPN can be simplified as [6,8,14]

$$\sigma^2(t) = k_{eff}^2 \left( \sum_{i=1}^{M} F_i^2(t) \bar{a_i} - \left( \sum_{i=1}^{M} F_i(t) \bar{a_i} \right)^2 \right)$$

Fig. 1. Fiber transmission simulation setup. Att: attenuator. PD: photodiode. TIA: trans-impedance amplifier.

### Table 1. Fiber transmission simulation parameters

| Item               | Parameter                  | Value                   |
|--------------------|----------------------------|-------------------------|
| Multimode Fiber    | Length                     | 300m                    |
|                    | Loss                       | 2.2 dB/km@850 nm        |
|                    | Group Refractive Index     | 1.48                    |
|                    | Index Contrast             | 1%                      |
|                    | Core Diameter              | 50 $\mu$m               |
|                    | Index Profile              | Graded Index (Power-law) |
| VCSEL              | 3-dB Bandwidth             | 15 GHz (4th order Bessel filter) |
| PD+TIA             | 3-dB Bandwidth             | 22 GHz (4th order Bessel filter) |
|                    | Responsivity               | 0.7A/W                  |

### 3. Simulation process

#### 3.1. Simulation setup

The fiber transmission simulation is performed in VPItransmissionMaker (VPI). The simulation setup is shown in Fig. 1. The parameters used in our simulation are listed in Table 1. The mode
field of VCSEL is calculated by using VISTAS [15, 16] in MATLAB. The radius of oxide aperture is set to 4 \( \mu m \). The equivalent core refractive index is 3.6 and the numerical aperture (NA) of the equivalent waveguide structure is 0.203. The bias current is set to 6 mA and the lateral current injection in the active area is assumed to have a disc profile. According the calculation results of the VCSEL model, the output field of the VCSEL composes of \( LP_{01}, LP_{11}, LP_{21}, LP_{02}, LP_{31} \) and \( LP_{12} \) mode. The normalized power of each mode is \( \{ \bar{a}_i \} = \{ 0.077, 0.264, 0.474, 0.185, 4.45 \times 10^{-4}, 2.90 \times 10^{-5} \} \). The phase pattern calculation process for wavefront shaping is identical to that in [17]. The 512 \times 512 pixels of the phase pattern are divided into 8 \times 8 macro-pixels with 4 phase levels which is the compromising choice considering computation complexity and performance [18]. After changing the phase of each macro-pixel, the data of mode field after spatial phase modulation is imported into VPI. The eye opening of received signal, which indicates the noise level of the signal, is measured to determine the phase value of the corresponding macro-pixel. BER is calculated after finishing the phase calculation process.

The MPN calculation is performed according to Eq. (5). The coupling coefficients between the individual VCSEL mode and the fiber modes are obtained using VPI before and after wavefront shaping. The differential mode delay (DMD) of each fiber mode can also be exported in VPI. The wavelength spacing between two nearby VCSEL mode groups is assumed to be 0.4 nm and the chromatic dispersion of the MMF is assumed to be -107 ps/(nm \( \cdot \) km) at 850 nm. The effective k value is set to 0.03 according to the results in [6].

![Fig. 2. (a) VCSEL output field. (b) Calculated phase pattern.](image)

### 3.2. Simulation results and discussion

Figure 2(a) shows the VCSEL output field. The calculated phase pattern is shown in Fig. 2(b). Figure 3 demonstrates the coupling coefficients between each VCSEL mode and fiber modes. The fiber modes are grouped in mode groups indexed \( G = m + 2n + 1 \) where \( m \) is the azimuthal index and \( n \) is the radial index of fiber \( LP_{mn} \) mode. The percentages are the power ratio of each VCSEL mode that couples to fiber modes. After applying the wavefront shaping technique, the lowest and some high-order fiber mode is prohibited from excitation. As a result, the MPN after MMF transmission can be mitigated. Figure 4 manifests the eye diagrams of 25-Gb/s OOK signal after PD in our simulation. The standard deviations (normalized to eye amplitude) of level 1 and level 0 are marked for each eye diagram. One can observe from Figs. 4(a) and 4(b) that the eye diagram is clearly opened after 300-m MMF transmission with wavefront shaping, while the wavefront shaping does not change the noise in the eye diagram when transmission distance is as short as 2 m. This indicates that the fiber dispersion induced noises can be mitigated using wavefront shaping. We measure the BER performance of the link after 300-m MMF transmission, as indicated in Fig. 5(a). It can be found that around five orders of magnitude improvement can be achieved after wavefront shaping. To show the ability of wavefront shaping to reduce the
Fig. 3. Coupling coefficients between each VCSEL mode and fiber modes.

Fig. 4. Eye diagram at received optical power of 0 dBm (a) without wavefront shaping after 300-m MMF, (b) with wavefront shaping after 300-m MMF, (c) without wavefront shaping after 2-m MMF and (d) with wavefront shaping after 2-m MMF.

MPN of VCSEL-MMF links, we calculate the standard deviation of the received signal. Figure 5(b) shows the standard deviation of the received signal. Figure 5(c) is the zoomed-in of the marked region in Fig. 5(b). The square marked points are the sampling points of the received signal, as indicated in [5,8]. There is about one order of magnitude reduction of the standard deviation of the received signal. This means that the noise will be mitigated, which is propitious to the decision circuit to retrieve the signal.

We also transmit 56-Gb/s PAM-4 signal over 300-m MMF [19] and 112-Gb/s PAM-4 signal over 100-m MMF by combining the wavefront shaping technique and advanced equalization technologies. The DSP based equalization is able to compensate bandwidth limitations of electrical and optical components and also to eliminate the modal and chromatic dispersion which wavefront shaping could not fully tackle with. On the other hand, as the wavefront shaping technology would reduce MPN profoundly, it may help to reduce the DSP complexity. The simulation parameters of 56-Gb/s PAM-4 transmission are the same as that of the simulation...
for OOK signal. When performing simulation for 112-Gb/s PAM-4 signal, the bandwidth of the VCSEL and PD is changed to 25 GHz and 30 GHz appropriately. The BER curves are shown in Fig. 6. Only using wavefront shaping can lower the BER but the performance is not so satisfactory since that the PAM-4 transmission sacrifices the signal-to-noise ratio (SNR) to double the data rate and in this case, there is not much margin to improve the performance. The BER can be lower than 7% FEC threshold by applying FFE but the minimum tap number required is 48. After applying optical equalization, the minimum tap number required for BER below 7% FEC threshold becomes 2. For 112-Gb/s PAM-4 signal, the required memory length for the VF to reach FEC threshold without wavefront shaping is (51, 9, 0) for the terms of first three order while the memory length of the second order of VF can vanish to 0 after wavefront shaping. The simulation results of PAM-4 signal transmission shows that using wavefront shaping can greatly reduce the DSP complexity when the data rate becomes higher.

4. Experimental demonstration

4.1. Experimental setup

The experimental setup is depicted in Fig. 7. Due to the insertion loss of the free space optics, we cannot get enough SNR to perform experiment for PAM-4 signal. So we use OOK signal to verify
our proposed method. The 25-Gb/s PRBS-11 OOK signal generated from a Keysight M8195A arbitrary waveform generator (AWG, 3-dB bandwidth: 25 GHz), together with the DC signal, are combined and fed to a multi-mode VCSEL chip via a 40-GHz GGB probe (40A-SG-100-DP). The L-I-V curve and the spectrum of the VCSEL we use are depicted in Fig. 8. The VCSEL chip is biased at 6 mA and the amplitude of AC signal is 900 mVpp. The 850-nm light emitted from the VCSEL chip is butt-coupled to a multi-mode fiber patch cord. The output power of the patch cord is around 3 dBm. The output beam of the fiber patch cord is collimated by a bi-convex lens which has a focal length of 4.5 mm. The collimated light spot is then expanded by two lenses (focal length is 25 mm and 100 mm, respectively) to ensure enough illumination area on the Holoeye Spatial Light Modulator (SLM, PLUTO-NIR-011). The phase of the light is manipulated by the SLM to achieve wavefront shaping. The resolution of the SLM is 1920×1080 pixels with pixel pitch of 8 µm. The phase level of the SLM is 256 (8-bit) levels. The average reflectivity of the SLM is 63% at 850 nm. A polarizer is applied to ensure that the polarization direction of the incident light agrees with the long axis of liquid crystal in the SLM screen. The light reflected by the SLM is then coupled into 300-m OM3 multimode fiber via a bi-convex lens which has a focal length of 18.4 mm. The total loss of the free space optics system is around 7 dB including the insertion loss of polarizer and the reflection loss of the SLM. The loss of the free space optics system leads to a launching optical power of -4 dBm to the fiber. After transmitting in the fiber, the signal is detected by a photoreceiver (Newfocus 1484-A-50) with 22-GHz bandwidth and the eye diagram is analyzed by a sampling oscilloscope (Keysight 86100D). The signal data is captured by a real-time oscilloscope with sampling rate of 80-GSa/s and 33-GHz bandwidth. Due to the low received optical power, the output signal of the photoreceiver cannot reach the input sensitivity of our BER tester. Adding an amplifier would amplify the noise of photoreceiver and bring excessive noise, so offline BER calculation in MATLAB is used.

The phase pattern is calculated using a feedback system. The feedback calculation is achieved by using LabVIEW. The sampling oscilloscope is connected to a laptop via local area network (LAN) and the SLM is connected to the same laptop via RS232. We use 1024×1024 pixels in
Fig. 9. Final pixel information shown on the SLM

the center and these pixels are divided into 8×8 macro-pixels. The incident light is adjusted to
illuminate the central area to ensure effective phase manipulation. We choose eye height of the
signal, which is related to the noise level [20], to be our objective function. The phase value
which leads to the largest eye height is chosen to be the phase value of corresponding macro-pixel.

Fig. 10. Eye diagram of 25-Gb/s OOK signal after 300-m OM3 MMF (a) before wavefront
shaping and (b) after wavefront shaping. (c) SNR values before and after wavefront shaping.
(d) BER curves of 25-Gb/s OOK signal after 300-m MMF transmission.

4.2. Experimental results and discussion

Figure 9 shows the phase distribution of the SLM pixels. Figures 10(a) and 10(b) depict the eye
diagrams before and after wavefront shaping. It can be observed that after wavefront shaping,
the eye opening becomes larger and the noises in the center of the eye and the crossing area are
reduced. We perform several SNR measurements in 15 minutes. As indicated in Fig. 10(c),
during the measurement time, the SNR of the signal after wavefront shaping always exceeds that
without wavefront shaping. Then we measure the BER of 25-Gb/s OOK signal after transmission
over 300-m MMF. When using real-time oscilloscope to acquire data, we pick up 16M data
points to make the offline BER calculation more precise and for each received optical power,
we collect three data streams and calculate three BER values then take an average to determine
the BER of each received optical power. As shown in Fig. 10(d), there is 1-dB power penalty improvement after wavefront shaping.

It should be noted that the performance enhancement may also come from the mitigation of dispersion induced inter-symbol interference (ISI) via wavefront shaping. In order to verify that there is MPN mitigation when the wavefront shaping technique is applied, we send a fix bit sequence (periodical bit stream of 8 zeros and 8 ones) at 10 Gb/s to eliminate the impact of dispersion induced ISI and capture the eye diagram after 300-m MMF transmission then compare the SNR without and with wavefront shaping. The eye diagrams are shown in Fig. 11, revealing that the SNR of the low speed fixed bit sequence is improved after wavefont shaping is applied. This can support that wavefront shaping has the ability of mitigating MPN and the performance improvement of 25-Gb/s OOK transmission could be the result of the mitigation of both dispersion induced ISI and MPN.

We notice that there are some differences between our simulation results and experimental results. In order to achieve effective phase manipulation, the beam from the VCSEL must be expanded to illuminate enough area on the SLM screen by using free space optics. This will lead to a large insertion loss and there would be selective mode excitation when coupling to the MMF after propagating in free space which leads to an inadequate performance improvement space for the whole link when applying wavefront shaping. Furthermore, the mode distribution in our simulation is calculated by a simplified equivalent waveguide model, while the mode distribution in real VCSELs are more complex and the index change in the z-direction should be taken into account, as indicated in [21]. The different transverse mode distribution may lead to an inferior performance in the experiment compared with simulation since in the simulation an ideal mode distribution is considered. Although the gap exists, the simulation can provide an illustration of the limit performance of wavefront shaping to reduce the MPN and the experimental results can verify the validity of our proposed scheme.

5. Conclusion

In this paper, we propose a new MPN mitigation scheme for the VCSEL and MMF based links by using wavefront shaping technique. The noise characteristic of the VCSEL-MMF links is theoretically studied and the impacts of coupling coefficients between VCSEL and MMF on the MPN and the system performance are investigated via simulation and experiment. We prove the validity of wavefront shaping to reduce the MPN for 25-Gb/s OOK after 300-m multimode fiber in simulation. Five orders of magnitude improvement of BER can be observed and the MPN is reduced for about an order with wavefront shaping. The DSP complexity is also shown to be reduced for 56-Gb/s and 112-Gb/s PAM-4 signal with the help of wavefront shaping. We also perform experiment for 25-Gb/s OOK transmission over 300-m OM3 MMF with the launching
power of -4 dBm to the fiber. About 1-dB power penalty improvement has been achieved at 7% FEC threshold and noise in the eye diagram can be observed to be mitigated. To show the ability of wavefront shaping to mitigate MPN, we measure the SNR of a 10-Gb/s fixed sequence. The SNR is found to be improved with wavefront shaping. Results of our simulations and experiment bespeak the ability of wavefront shaping to mitigate the MPN and reduce the DSP complexity for the VCSEL and MMF based links in high speed transmission scenarios.

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